Synthesis of the Reflective Surface on an Elastic Shell

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Abstract. We consider the method for obtaining a surface that asymptotically approaches the necessary for converging convergent beams into parallel ones by directed action on the elastic shell, in particular, on a flat flexible disk. A condition is formulated for the coefficients of the power series expansion of the solution of the differential equation of bending, under which the resulting surface will (from the center to the edges) asymptotically approach the surface of a paraboloid of revolution, which gives an exact solution. We formulate this condition by the theorem about the limit of a complex-valued function given by the vector of coefficients of its analytic element. The physical basis of the method is the effect of pulling or pushing elements, the selection of which ensures the necessary transformation of the existing shell.

1. Introduction

The history of flexible optical elements begins with Bernhard Schmidt's idea concerning making a corrector lens for his telescope system from strained glass [1],[2]. Later it was planned to use deformable mirror in space telescope [3], but the project became too difficult for that time and is not realized yet. Nowadays adaptive optics using the same principle of controlled deformation is applied [4], [5] in the largest professional systems such as VLT telescopes (Paranal, Chile), Large Binocular Telescope (Mt. Graham, USA), Magellan Telescopes (Las Campanas, Chile), Subaru (Hawaii, USA), Keck (Hawaii, USA) [6], future Giant Magellan Telescope [7] and some others. But all of these telescopes require sophisticated real time control system including laser-made “artificial star” in the upper atmosphere. The problem is to apply flexible optics approach to smaller systems, including non-telescopic (antennas, etc), using its great cost-saving ability [8] comparing to rigid optics systems.

An exact solution to the problem of converting a plane wave front into a spherical one while reflection from a given surface is achieved using the surface of a paraboloid of revolution. The deviation of the mirror from this paraboloid, ensuring the fulfillment of the Rayleigh criterion (conversion error of not more than 1/4 of the wavelength of the incident radiation), should not exceed 1/8 of the wavelength. At the same time, while the size of such a surface increasing, the complexity of its technological process makes self simplification an urgent task [9-19].

In the paper [20], this problem was solved by obtaining a paraboloid mirror from the original spherical one with an accurately executed surface using elastic deflection of the workpiece surface. But the process of obtaining such a workpiece, in the case of its significant size, also is difficult. In this paper, the same idea of directional elastic deformation will be applied to a more general problem of synthesizing a surface being asymptotically equivalent to a paraboloid while the polar radius increasing. The initial workpiece is supposed to be arbitrary.
More definitely we will try to develop the methodology of the reflecting surface synthesis with the ability to transform a flat wavefront to a converging spherical one from one class flexible shell with a reflecting surface, the diagram of the fixing and the diagram of control force actions application.

2. Controlled deflection of the round shell
Let us suppose that the shell is concave and has a constant thickness. Let us define its generating curve in the free state as $y(r)$ and in the loaded state as $y_1(r)$. In accordance with the theory the next asymptotic formula exists

$$\lim_{y(r) \to 0} y_1(r) = y(r) + w(r),$$

(1)

where $w(r)$ is the deflected surface generating curve which is asymptotically equal to generating curve of deflected round plate. Differential equation of the deflection of a shell that little differs from a round flat plate takes form

$$\frac{\partial^4 w(r)}{\partial r^4} + 2 \cdot \frac{\partial^3 w(r)}{\partial r^3} - \frac{1}{r^2} \cdot \frac{\partial^2 w(r)}{\partial r^2} + \frac{1}{r^3} \cdot \frac{\partial w(r)}{\partial r} = \frac{P(r)}{D(r)},$$

(2)

where $P$ is the pressure, $D$ is cylindrical rigidity, both the values can be variable along the polar radius $r$ and besides

$$D = \frac{E h(r)^3}{12(1-\mu^2)},$$

(3)

where $E$ is the Young’s modulus of the shell material, $\mu$ is the Poisson ratio of the shell material, $h(r)$ is shell thickness at the polar radius $r$.

The influence of the supports balancing the pressure forces by reactions and moments is taken into account by the boundary conditions to equation (2), which can be varied.

Let us assume that for the given loading scheme and the initial unstressed shell we have integrated equation (2) so that the formula for the summary generating function (1) can be expanded in the next power series

$$y_1(r) = \sum_{n=0}^{\infty} c_n \cdot r^n.$$

(4)

The aim of the synthesis task for the shell given is the choice of the support system defined by the generalized coordinate vector $X$ and the load intensity defined by the load vector $P$, so that the solution (4) in the form $y_1(r, X, P) = \sum_{n=0}^{\infty} c_n (X, P) \cdot r^n$ would satisfy some properties that we will formulate below.

One of the formal criteria for assessing the suitability of the resulting product is the principle of constant focal length while parallel incidence of beams, which can be formulated as

$$OF(r) = y_1(r) + \frac{r \left[ 1 - \left( \frac{dy_1}{dr} \right)^2 \right]}{2 \cdot \frac{dy_1}{dr}} = F = \text{const}.$$  

(5)

The fulfillment of (5) is equivalent to the statement that the resulting surface is a paraboloid of revolution with the generating function

$$y_1(r) = \frac{r^2}{4F}.$$

(6)
Any physically obtained surface can satisfy a given constraint only within tolerances. Nevertheless, not satisfying criterion (5), the resulting solution can satisfy a wider class of functions given by the relation

\[ \lim_{r \to \infty} OF(r) = F. \]  

Physically, criterion (7) requires that the accuracy of the wavefront transformation increases while one moves towards the edges of the shell. As the elementary ring zones squares increase during such a movement, the part of the correctly transformed radiation for all the surface also increases.

When checking the obtained (and physically implemented using a system of supports and loads) solution of equation (2) for compliance with criterion (7), the problem of research the properties of (5) right part function while an unlimited increase in the argument arises.

Taking into account that the solution is given only by its analytical element - series (4), this problem requires additional consideration. Thus, in the case when series (4) diverges from some \( r \), one cannot obtain the (5) right part function by the immediate summation.

At the same time, using the rules of algebraic operations with power series and substitution of a series in a series, it is possible to expand the right side of (5) into a series obtaining the dependence of the focal length of the resulting surface on the radius of the parallel, in the form of a series

\[ OF(r, \mathbf{X}, \mathbf{P}) = \sum_{n=0}^{\infty} q_n(\mathbf{X}, \mathbf{P}) \cdot r^n, \]  

with coefficients determined uniquely. Then, for fixed vectors \( \mathbf{X} \) and \( \mathbf{P} \), the following problem arises: using the obtained vector of coefficients of series (8), it is required to check the existence of a finite limit (7) of the resulting surface focal length with an unlimited increase in the radius of its parallel.

If we know this limit exists we can adjust the loads and supports so that the surface tends to be a parabolical one while movement from its center to its edges.

A sufficient condition for the existence of such a limit is given by the following theorem (we give it in general form for the case of a complex-valued argument, a particular case of which is the real radius of the parallel)

3. Asymptotic value of a function defined by its analytical element

Let the next power series be defined

\[ \sum_{n=0}^{\infty} c_n \cdot (z - z_0)^n, \]  

where \( c_n \in \mathbb{C} \).

**Theorem 1**

If there exists the finite value \( S \) so that

\[ S = \lim_{m \to \infty} \sum_{n=0}^{m} \binom{m}{n} \cdot c_n, \quad S \in \mathbb{C} \]  

then the function \( f(z) \), the analytical element of which is the series (9) within its disc of convergence, can be analytically continued into the domain \( M \subset \mathbb{C} \), \( M : z \in M \Rightarrow \text{Re}(z) \in (\text{Re}(z_0) - 1, \infty), \text{Im}(z) \in (-\infty, \infty) \), and the next formula takes place

\[ \lim_{z \to z_0^{+}} f(z) = S. \]  

**Proof.** For any positive integer \( m \) the next is true
\begin{equation}
\sum_{n=0}^{m} \binom{m}{n} \cdot c_n = \sum_{n=0}^{m} g_n,
\end{equation}

where \( g_0 = c_0, g_1 = c_1, g_2 = c_1 + c_2, g_n = \sum_{k=1}^{n} \binom{n-1}{k-1} \cdot c_k, n \in \mathbb{N} \). We can prove this statement by induction. Let us use the immediate substitution and see that \( (1.1) \) is correct for \( (m=0) \) and for \( (m=1) \).

Let the identity \( (12) \) be true for some \( (m=s, s \in \mathbb{Z}) \), so that
\begin{equation}
\sum_{n=0}^{s+1} \binom{s+1}{n} \cdot c_n = \sum_{n=0}^{s} \binom{s}{n} \cdot c_n + \sum_{k=1}^{s+1} \binom{s}{k-1} \cdot c_k,
\end{equation}

the right side of which can be represented as \( c_0 + c_{s+1} + \sum_{n=1}^{s} \binom{s}{n} \cdot c_n \cdot c_k \). The Pascal triangle identity \( \binom{s}{n} + \binom{s}{n-1} = \binom{s+1}{n} \) turns \( (15) \) also into an identity, so the statement is proved.

So the sequence of partial sums \( \sum_{n=0}^{m} \binom{m}{n} \cdot c_n \) coincides with the sequence of partial sums of the numeral series \( \sum_{n=0}^{\infty} g_n \). Then the statement that the first of these sequences has the limit \( (10) \) is equal to the statements that the series \( \sum_{n=0}^{\infty} g_n \) converges and its sum is equal to \( S \).

Then in accordance with Abel’s theorem the series
\begin{equation}
\sum_{n=0}^{\infty} g_n \cdot z^n,
\end{equation}
converges uniformly to some analytical function within unitary open disc with the center in \( (z = 0) \). For boundary point \( (z = 1) \) of this disc the value of this function is equal to the sum of the numeral series \( \sum_{n=0}^{\infty} g_n \), what is equal to \( S \) as it was proved above.

The series
\begin{equation}
u(z) = \sum_{n=0}^{\infty} g_n \cdot (z - z_0)^n
\end{equation}
is the expansion of the composed function
\begin{equation}
u(z) = f(w(z)),
\end{equation}
where
\begin{equation}
w(z) = z_0 + \frac{z - z_0}{1 + z_0 - z},
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in powers of \((z-z_0)\) that converges for all \(z \in D\). This statement can be proved by the next way.

In accordance with (9) let us write the expansion

\[
f \left( z_0 + \frac{z-z_0}{1-z_0+z} \right) = \sum_{n=0}^{\infty} c_n \cdot \left( \frac{z-z_0}{1-(z-z_0)} \right)^n \tag{20}
\]

the right-hand side of which will converge for all \(z: |z-z_0/(1-z_0+z)|<R\), where \(R\) is the convergence radius of the series (9). As \(R \geq 1\), the domain \(A \in D\) exists for all \(z\) in which (20) is true.

Let us write the expansion

\[
\left( \frac{z-z_0}{1-(z-z_0)} \right)^k = \sum_{n=k}^{\infty} \binom{n-1}{k-1} (z-z_0)^n \cdot |z-z_0| < 1. \tag{21}
\]

Taking into account (21) and collecting the coefficients at equal powers of \((z-z_0)\) on the right-hand side of (20), for all \(z \in A\) the identity is true

\[
f \left( z_0 + \frac{z-z_0}{1-z_0+z} \right) = \sum_{n=0}^{\infty} \sum_{k=1}^{n} \binom{n-1}{k-1} c_k \cdot (z-z_0)^n, \tag{22}
\]

coefficients of which coincide with definition of \(g_n\). From the convergence (2.4) for all \(z \in D\) and the theorem about the power series sum uniqueness it follows the truth of the statement to be proved.

The function \(w(z)\) performs a conform mapping of the open unitary disc \(D\) with the center \(z_0\) into considered domain \(M\), so that the boundary point \((z = z_0+1)\) of the disc \(D\) transits into the point at infinity. Let us prove this statement. Function \(w(z)\) is the fractional linear function defined on the whole extended complex plane, except for the point \((z = z_0+1)\) \(D \notin\).

Hence \(w(z)\) is defined on \(D\). A fractional linear function realizes a univalent conformal mapping of the extended complex plane to the extended complex plane, while the point at infinity of the image corresponds to the zero of the denominator of the inverse.

Consequently, (15) realizes an univalent conformal mapping of the domain \(D\) onto some domain \(M \subset C\). The domain \(D\) as a simply connected domain can be conformally mapped only onto a simply connected domain; therefore, the resulting domain \(M\) is simply connected. By immediate substitution one can check that the border of \(D\) transits into the line \(w = z_0 - 1\), and the inner point \(z = z_0\) stays fixed, \(w = z_0\). Hence,

\[
\forall w \in M \quad \text{Re}(w) \in (\text{Re}(z_0)-1, \infty), \text{Im}(w) \in (-\infty, \infty). \tag{23}
\]

The statement is proved.

The boundary point \((z = z_0+1)\) and the center of the disc \((z = z_0)\) are the fixed points of the mapping.

The function (19) is a fractional linear function conform mapping with the aid of which can have two fixed points or less.

Let us defined the required properties of \(f(z)\) in the domain \(M\).

1. The function is defined.

The function (19) as a fractional linear function one-to-one maps the domain \(D\) into the domain \(M\), hence the inverse function to (19)

\[
z(w) = \frac{(w-z_0)(1+z_0)}{1+w-z_0} \tag{24}
\]

one-to-one maps the domain \(M\) into the domain \(D\).

Hence,

\[
\forall w \in M \exists z(w) \in D. \tag{25}
\]

From the convergence of series (16) in the unitary disc and definition (18) of the function \(u(z)\) it follows

\[
\sum_{n=0}^{\infty} \frac{1}{n+1} t^n < \sum_{n=0}^{\infty} \frac{1}{n+1} (z-z_0)^n
\]

is a convergent series for all \(z \in A\) and the series converges to the function (16) for all \(z \in A\).
∀z ∈ D ∃ u(z) ∈ C.

From (18) and the uniqueness of (19) it follows
∀w ∈ M ∃ f(w) = u(z(w)) ∈ C.

The statement is proved.

2. The function is analytic.

A necessary and sufficient condition for f(z) to be analytic in the domain M is the existence of the complex derivative
∀z ∈ M ∃ df(z)
dz ∈ C.

Let us differentiate (18) as a composed function and obtain
\frac{du(z)}{dz} = \frac{df(w(z))}{dw(z)} \cdot \frac{dw(z)}{dz},

moreover, from the proved analyticity of the function given by series (16) in the unitary disk, it follows
∀z ∈ D ∃ \frac{du(z)}{dz} ∈ C.

Using the formula (29) let us obtain
∀z ∈ D \frac{df(w(z))}{dw(z)} = \frac{du(z)}{dz}.

Let us differentiate (19) and obtain
∀z ∈ D \frac{dw(z)}{dz} = \frac{1}{(1 + z_0 - z)^2}.

Considering w as an independent variable, and z as its single-valued function and taking into account (31) - (32), let us obtain
∀w ∈ M \frac{df(w)}{dw} = (1 + z_0 - z(w))^2 \cdot \frac{du(z(w))}{dz(w)}.

From (25) and (30) it follows
∀w ∈ M ∃ \frac{du(z(w))}{dz(w)} ∈ C.

From (33) and (34) it follows the required justice of (28).

The statement is proved.

3. The limit of the function while z → ∞ is equal to S.

When mapped by the function (19), the interior of a circle of radius ρD centered at (z_0 + 1), belonging to D, is mapped into the exterior of a circle of radius ρM = 1/ρD centered at (z_0 - 1), belonging to M. Moreover, if for two different circles in D ρD1 < ρD2, then ρM1 > ρM2. Let us prove this statement. Let us take an arbitrary point z ∈ D and draw a circle through it with center at (z_0 + 1), the radius of the circle let us note by ρ. The equation of the circle resulted will be

\[(x - x_0)^2 + (y - y_0)^2 = \rho^2.\]

When mapping with the function (19) the point z ∈ D transits to the point w ∈ M. If the image of the circle (35) is the circle with radius 1/ρ centered at (z_0 - 1) then the next equation should be true
\[(x_w - x_{z0} + 1)^2 + (y_w - y_{z0})^2 = \frac{1}{\rho^2}. \]  (36)

Point \(w\) has the next coordinates:

\[x_w = x_{z0} + \frac{(1 + x_{z0} - x_z)(1 + x_{z0} - x_z) - (y_{z0} - y_z)^2}{(1 + x_{z0} - x_z)^2 + (y_{z0} - y_z)^2}; \]  (37)

\[y_w = y_{z0} + \frac{y_{z0} - y_z}{(1 + x_{z0} - x_z)^2 + (y_{z0} - y_z)^2}. \]  (38)

In accordance with (35) let us explain these coordinates through the radius \(\rho\)

\[x_w = x_{z0} + \frac{(1 + x_{z0} - x_z)}{\rho^2} - 1; \]  (39)

\[y_w = y_{z0} + \frac{y_{z0} - y_z}{\rho^2}. \]  (40)

If one substitute (39) and (40) in (35) and take into account (35) he will get an identity.

Let us take the circle with radius \(\rho_1 < \rho\) centered at \((z_0 + 1)\). This circle will lie completely within the circle with radius \(\rho\) with the same center. From what has been proved, the image of this circle when mapped using function (19) is a circle of radius \(1/\rho_1\) centered at \((z_0 - 1)\). Since \(\rho_1 < \rho\), then \(1/\rho_1 > 1/\rho\) and the resulting circle will lie completely outside the circle of radius \(1/\rho\) centered at \((z_0 - 1)\).

The statement is proved.

Therefore, the mapping by the function \(w(z)\) of any path in \(D\) ending in the \(\varepsilon -\) neighborhood of the point \((z_0 + 1)\) is a path in \(M\) ending in the \(\varepsilon -\) neighborhood of the point at infinity, so that

\[\lim_{z \to z_0 + 1, z \in D, w \to \infty, w \in M} u(z) = \lim_{w \to \infty} f(w), \]  (41)

where it follows from what has been proved that the left-hand side limit exists and is equal to \(S\).

The theorem is proved.

4. Application of the Theorem 1 to the problem of the reflective surface synthesis

With regard to the obtained sequence of real-valued coefficients of the series (4), the existence of a finite limit

\[F = \lim_{m \to \infty} \sum_{n=0}^{m} \left(\frac{m}{n}\right) q_n \]  (42)

in accordance with Theorem 1 will be sufficient condition of criterion (7) fulfillment and the existence of the next asymptotic equality for the solution (4)

\[y_1(r) \approx \frac{r^2}{4F}, r \to \infty. \]  (43)

Taking into account the accepted designations, for a given focal length \(F\) of the final surface, the synthesis problem can be mathematically represented as the problem of finding vectors \(X\) and \(P\) such that

\[\lim_{m \to \infty} \sum_{n=0}^{m} \left(\frac{m}{n}\right) q_n (X, P) = F. \]  (44)

This allows to replace a mechanical finishing of the selected shell (that can in general be very large and thin) by its controlled deflection with application of these supports \(P\) and loads \(X\). In particulars, we can optimize only one of these vectors and let another to be constant.
5. Conclusion

The paper considers the synthesis of a concave reflective surface from an elastic shell of the form of a body of revolution that already has a mirror surface, a scheme for fixing it and a scheme for applying control force actions. A condition is formulated for the coefficients of the power series expansion of the solution of the differential equation of bending, under which the resulting surface will (from the center to the edges) asymptotically approach the surface of a paraboloid of revolution, which gives an exact solution.

We formulate this condition by Theorem 1 about the limit of a complex-valued function given by the vector of coefficients of its analytic element. As applied to the real variable and the problem under consideration, the existence of limit (42) turns out to be equivalent to the existence of limit (7), which, in turn, gives an affirmative answer to the question of the applicability of the resulting surface to the problem of converting a wavefront from flat to spherical according to the criterion of asymptotic focal length consistency. The choice of vectors of generalized coordinates of supports $X$ and generalized loads $P$ at a given focal length is made in accordance with formal criterion (44).

The physical basis of the proposed method can be the use of pulling-pushing elements that create an adjustable force, in particular on the basis of commercially available actuators. Sequential application of the criterion in automatic mode allows to optimize the location of supports and loads acting on the shell.

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