Critical behaviour at the metal-insulator transition in 3-dimensional disordered systems in a strong magnetic field.

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The critical behaviour of 3-dimensional disordered systems in a strong magnetic field is investigated by analysing the spectral fluctuations of the energy spectrum. The level spacing distribution $P(s)$ as well the Dyson-Mehta statistics $\Delta_3(L)$ are considered. Except for the small $s$ behaviour of $P(s)$, which depends on the presence or absence of a magnetic field but not on its strength, the other quantities, large $s$ behaviour of $P(s)$ and $\Delta_3(L)$, turn out to be independent of its presence or absence and therefore of the time reversal symmetry. This suggests that the metal-insulator transition, which is defined by the symmetry of the system at the critical point, is independent of the presence or absence of the time reversal symmetry.

This has been checked in the case of a system subject to an Aharonov-Bohm (AB)-flux $B$. Despite the fact there is no magnetic field inside the system the AB-flux breaks the time reversal symmetry and one observes a transition this time from the Gaussian unitary ensemble (GUE) to the PE. But the CE, the critical exponents, does not depend on the time reversal symmetry and so a reservation which has to be considered with this kind of system is that a change of the vector potential is, by means of a gauge transformation, equivalent to a change of the boundary conditions and therefore should have no influence in the localised regime. Despite this it must be stressed that, when using periodic boundary conditions, the vector potential cannot be gauged away and the system is not equivalent to a system invariant under orthogonal transformation. Nevertheless, in this paper we would like to address the problem now with a magnetic field inside the system and to show that the conclusions derived from the case with the AB-flux still remain valid.

In order to investigate the MIT with a magnetic field we consider the usual Anderson Hamiltonian,

$$H = \sum_\lambda \epsilon_\lambda |\lambda| + \sum_{\lambda \lambda'} V_{\lambda \lambda'} |\lambda| |\lambda'|,$$  \hspace{1cm} (1)

where the sites $i=lmn$ are distributed regularly in 3D space, e.g. on a simple cubic lattice. Only interactions with the nearest neighbours are considered. The site energy $\epsilon_{lmn}$ is described by a stochastic variable. In the present investigation we use a box distribution with variance $\sigma^2 = W^2/12$. $W$ represents the disorder and is the critical parameter. Introducing a homogeneous magnetic field $B = B(1,1,1)$ leads to complex hopping elements (Peierls factors) $\bar{B}$. Using the following vector potential $\Lambda = B(z,x,y)$ and imposing $|V_{lmn,l'm'n'}| = 1$, which defines the energy scale, yields for $V_{lmn,l'm'n'}$: 

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where $a$ is the step of the lattice. $\alpha = \frac{2\pi a^2 B}{\hbar}$ is the number of flux quanta $\hbar/e$ per plaquette. The boundary conditions are taken to be periodic.

Based on this Hamiltonian, the MIT in the presence of a magnetic field will be studied by the ELS method, i.e. via the fluctuations of the energy spectrum \cite{8}. Before giving the results we shortly review the ELS method. Starting from Eq.\(\text{(1)}\) the energy spectrum was computed by means of the Lanczos algorithm (which is suited to diagonalise such very sparse secular matrices) for systems of size $M \times M \times M$ with $M = 13, 17,$ and $21$, disorder $W$ ranging from 3 to 40 and phase $\alpha$ from 0.005 to 0.250. The number of different realizations of the random site energies $\varepsilon_{imn}$ was chosen so that about $10^5$ eigenvalues were obtained for every pair of parameters $(M, W)$ which means between 25 and 90 realizations, for which only half of the spectrum around the band centre is considered so that the results do not deteriorate due to the strongly localised states near the band edges. After unfolding the spectrum obtained, the fluctuations can be appropriately characterised \cite{11} by means of the spacing distribution $P(s)$ and the Dyson-Mehta statistics $\Delta_3(L)$. $P(s)$ measures the level repulsion, it is normalised, as is its first moment, because the spectrum is unfolded. $\Delta_3(L)$ measures the spectral rigidity.

\begin{equation}
\begin{cases}
\exp\left(\pm i \frac{2\pi a^2 B m}{\hbar} \right) & \text{if } l' = l \pm 1, m' = m, n' = n \\
\exp\left(\pm i \frac{2\pi a^2 B n}{\hbar} \right) & \text{if } l' = l, m' = m \pm 1, n' = n \\
\exp\left(\pm i \frac{2\pi a^2 B l}{\hbar} \right) & \text{if } l' = l, m' = m, n' = n \pm 1,
\end{cases}
\end{equation}

In Fig. 1 the results for the spacing distribution $P(s)$ are reported. One finds, as expected, the GUE and the PE regimes for small and large disorder respectively as well as the transition between them as a function of $W$. But less expected is the fact that, in contrast to the case with time reversal symmetry \cite{20}, there is no longer one point where all the curves intersect but rather one which seems to appear only for the curves in the metallic regime.

The next step is now to find where the MIT takes place. For this one uses the fact that the quantities we are considering here, $P(s)$ and $\Delta_3(L)$, depend on the system size except at the critical point where they are scale invariant. This is directly related to the fact that the MIT is a second order transition and that finite-size scaling laws will apply close to the transition. From a numerical point view it has been shown that $\Delta_3(L)$ gives more accurate results. So we calculate $\delta(M, W) = \frac{1}{W} \int_0^3 \Delta_3(L) dL$ as a function of $M$ and $W$, the critical disorder, $W_c$, being given by $W$ for which $\delta(M, W)$ is independent of $M$ \cite{2}. The results are shown in Fig. 2.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Spacing distribution, $P(s)$, for $M = 21$ and different disorders $W$. The full curves give the asymptotic regimes namely, GUE for the diffusive regime and PE for the localised regime. We see the transition between GUE and PE as a function of the disorder $W$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{$\delta$ as a function of $M$ and $W$. $\circ : M = 13, \triangle : M = 17$ and $\diamond : M = 21$. The critical disorder is given by the point for which $\delta$ is independent of $M$. $W_c$ ranges from 16.5 without a magnetic field up to 18.75 where $W_c$ seems to become independent of the strength of the magnetic field.}
\end{figure}

In contrast to the results obtained with an AB-flux \cite{10}, $W_c$ depends on the vector potential. This is due to the fact that now we have a magnetic field inside the system which breaks the weak localisation phenomena \cite{1}. It is then more difficult to localise the states which means an upwards shift of $W_c$. $W_c$ ranges now from 16.5 without magnetic field up to 18.75 where $W_c$ seems to become independent of the strength of the magnetic field. A similar behaviour has already been observed by Henneke \textit{et al} \cite{8}. With $W_c$ it is now possible to study $P(s)$ and $\Delta_3(L)$ at the critical point. The first feature we would like to consider is the small $s$ behaviour of $P(s)$. In fact for numerical reasons what we calculate is the cumulative level-spacing distribution \(I(s) = \int_0^s P(s') ds'\) which allows a better study of the small $s$ behaviour of $P(s)$. The results are plotted in Fig. 3. As we can see the curves depend on the absence or presence of a magnetic
field but are independent of its strength.

In Fig. 4 the other limit, namely the large $s$ behaviour of $P(s)$, is reported. The curves are no longer only independent of the strength of the magnetic field, as for small $s$, but are even independent of the presence or absence of the magnetic field. The question of the shape, for large $s$, of $P(s)$ is rather controversial. It was first claimed \[ \text{[9]} \] that $P(s)$ should be $\propto \exp(-As)$. Then following some new analytical calculations at the critical point Aronov et al. \[ \text{[14]} \] proposed that $P(s) = Bs^\delta \exp(-A\delta s^{2-\gamma})$ with $\gamma = 1 - 1/\nu d$ where $\nu$ is the critical exponent of the correlation length $\xi_\infty$. It has to be noted that some doubts have been recently raised concerning the validity of these results \[ \text{[12]} \]. Numerically the situation is not really clearer. When one tries to fit a curve like $P(s) = Bs \exp(-As^\delta)$ for the case without magnetic field one finds an $\eta$ between 1.20 and 1.30 \[ \text{[10,11]} \] in a good agreement with the results obtained by Aronov et al. \[ \text{[4]} \]. But when one looks just at the tail of $P(s)$ one can see some discrepancies between the fit and the numerical results. Following some other numerical calculations it was then claimed \[ \text{[17]} \] that $P(s)$ was indeed $\propto \exp(-As)$ as first suggested \[ \text{[9]} \]. But the fit was done only for the large $s$ and not on the whole curve as we did previously \[ \text{[8,16]} \].

In the inset of Fig. 4 we show that, when considering only the tail of $P(s)$, $\exp(-As)$ with $A \approx 1.8$ (Fit 2) gives indeed a good result but so does $\exp(-(A's+C)s^{1.25})$ (Fit 1). It is therefore very difficult to draw a definitive conclusion about the problem just based on the fit of the tail of $P(s)$. In the case with a magnetic field it turned out to be very difficult to fit a whole curve using the result of Aronov et al. which could be related, somehow, to the absence of a fixed point in $P(s)$ as noticed above. It clearly shows that more work needs to be done in order to understand properly the shape of $P(s)$ at the critical point.

FIG. 4. Large $s$ behaviour of $P(s)$. The curves are no longer only independent of the strength of the magnetic field as for small $s$ but are independent of the presence or absence of the magnetic field. In the inset $\exp(-As)$, with $A \approx 1.8$, (Fit 2) and $\exp(-(A's+C)s^{1.25})$, with $A' \approx 1.37$ and $C \approx 0.26$, (Fit 1) have been fitted to the tail of $P(s)$.

FIG. 5. Dyson-Mehta statistics ($\Delta_3(L)$) at the critical point for various magnetic fields. As for the large $s$ behaviour of $P(s)$, $\Delta_3(L)$ is not only independent of the strength of the magnetic field but of its presence or absence too.
Finally we compare $\Delta_3(L)$ for the different magnetic fields at the critical point in Fig. 3. As for the large $s$ behaviour of $P(s)$, $\Delta_3(L)$ is not only independent of the strength of the magnetic field but seems to be independent of its presence or absence too. The shape of $\Delta_3(L)$ contains a term linear in $L$ as well as a non-linear term $L^\omega$, with $\omega < 1$ and has already been studied in a previous work \cite{10}. In the inset of Fig. 3 we show how $\Delta_3(L)$ varies, when we slightly change $W$, to give an idea about the accuracy of the curves keeping in mind that the uncertainty for $W_c$ is $\pm 0.25$. These results suggest that, although the magnetic field brings some modifications at the critical point, as for the critical disorder $W_c$ or the small $s$ behaviour of $P(s)$, the symmetry of the system is indeed independent of the magnetic field and then of the time reversal symmetry. What happens is, when one considers the localised regime. If the matrix would be $O(N)$ breaking of the $O(N)$ symmetry appears in the large $L$ behaviour of $\Delta_3(L)$ explaining why one can have a symmetry at the critical point which is independent of the magnetic field whereas the small $s$ behaviour of $P(s)$ is not. It has to be noted that the breaking of the $O(N)$ or $U(N)$ invariance is at the origin of some interesting new results related with the lifting of the sum rule prohibition that is observed when studying the two-level correlation function \cite{13}.

In summary, by means of the ELS method, we have studied the statistical properties of the energy spectrum of the Anderson model in a strong magnetic field at the MIT. For this we have considered the level spacing distribution $P(s)$ as well as the Dyson-Mehta statistics, $\Delta_3(L)$. Except for the small $s$ behaviour of $P(s)$, which depends on the presence or absence of a magnetic field but not on its strength, the other quantities studied, the large $s$ behaviour of $P(s)$ and $\Delta_3(L)$ turn out to be independent of its presence or absence and thus of the time reversal symmetry. This suggests that the MIT, which is defined by the symmetry of the system at the critical point, is defined by a new universality (CE) class which is independent of the presence or absence of time reversal symmetry.

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