Pairing Correlations Near a Kondo-Destruction Quantum Critical Point

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Motivated by the unconventional superconductivity observed in heavy-fermion metals, we investigate pairing susceptibilities near a continuous quantum phase transition of the Kondo-destruction type. We solve two-impurity Bose-Fermi Anderson models with Ising and Heisenberg forms of the interimpurity exchange interaction using a continuous-time quantum Monte Carlo and numerical renormalization-group methods. Each model exhibits a Kondo-destruction quantum critical point separating Kondo-screened and local-moment phases. For antiferromagnetic interimpurity exchange interactions, singlet pairing is found to be enhanced in the vicinity of the transition. Implications of this result for heavy-fermion superconductivity are discussed.

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Quantum criticality is accompanied by enhanced entropy, which tends to nucleate new phases. An important opportunity for detailed exploration of this general issue is provided by heavy-fermion metals, in which many quantum critical points (QCPs) have been explicitly identified. Theoretical studies have shown that antiferromagnetic QCPs in a Kondo lattice system fall into two classes. Spin-density-wave (SDW) QCPs are characterized by a vanishing Kondo energy scale, which in turn gives rise to $\omega/T$ scaling in the local spin susceptibility and a linear-in-temperature single-particle relaxation rate. Such properties are reminiscent of the aforementioned experiments near the antiferromagnetic QCPs of heavy-fermion metals.

We solve the two-impurity Bose-Fermi Anderson models using a continuous-time quantum Monte Carlo (CTQMC) approach in the form applied in Refs. and as well as the numerical renormalization group (NRG). We determine the magnetic quantum critical properties and compute pairing susceptibilities across the phase diagram. We find that pairing correlations are in general enhanced near the Kondo-destruction QCP. This suggests a new mechanism for superconductivity near antiferromagnetic quantum phase transitions (QPTs).

The two-impurity Bose-Fermi Anderson models, illustrated in Fig. 4, are defined by Hamiltonians of the form

$$\begin{align}
H &= \sum_{i=1,2} \left( \epsilon_d \sum_{\sigma} n_{d_i \sigma} + U n_{d_i \uparrow} n_{d_i \downarrow} + H_{12} \right) \\
&\quad + \sum_{k,\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{V}{\sqrt{N_b}} \sum_{i,k,\sigma} \left( e^{ikr_i} d_{i\sigma}^\dagger c_{k\sigma} + \text{H.c.} \right) \\
&\quad + \sum_q \omega_q \phi_q \phi_q^\dagger + g(S_1^z - S_2^z) \sum_q \left( \phi_q^\dagger \phi_q + \phi_{-q}^\dagger \phi_{-q} \right). \tag{1}
\end{align}$$

Here, $d_{i\sigma}$ destroys an electron on impurity site $i = 1$ or $2$ with spin $\sigma = \uparrow$ or $\downarrow$, energy $\epsilon_d$, and on-site Coulomb repulsion $U$. $n_{d_i \sigma} = d_{i\sigma}^\dagger d_{i\sigma}$, and $S_i = \frac{1}{2} \sum_{\alpha,\beta} d_{i\alpha \sigma}^\dagger d_{i\beta \sigma}$ where $\sigma_{x,y,z}$ are the Pauli matrices. The operator $c_{k\sigma}$ destroys a conduction electron with wave vector $k$, spin $\sigma$, and energy $\epsilon_k$ that has a hybridization $V$ with each impurity, while $\phi_q$ destroys a boson with energy $\omega_q$ that couples with strength $g$ to the difference of impurity spin $z$ components. $N_b$ is the number of $k$ values.
To control the interimpurity exchange interaction, we take the limit of infinite impurity separation \(|r_1 - r_2|\) to ensure the vanishing of the indirect Ruderman-Kittel-Kasuya-Yosida exchange interaction between \(S_1\) and \(S_2\). Then impurities 1 and 2 hybridize with linearly independent combinations of band states, and interact only through their coupling to the bosonic bath and via a direct exchange term \(H_{12}\), either of the Ising form
\[
H_{12} = I_z S_1^z S_2^z \quad \text{or} \quad H_{12} = IS_1 \cdot S_2.
\]
We note that integrating out the bosonic bath will induce a retarded antiferromagnetic exchange of Ising symmetry.

We assume a flat electronic density of states \(\rho_0(\epsilon) = \rho_0 \Theta(D - |\epsilon|)\) and a sub-Ohmic bosonic density of states
\[
\rho_\omega(\omega) = K^2_0 \omega_0^{1-s} \Theta(\omega) f(\omega/\omega_0). \quad (2)
\]
For the CT-QMC calculations we have used a cutoff function \(f(x) = \exp(-|x|)\) and chosen \(K^2_0 = \omega_0^2 \Gamma(s + 1)\) so that the density of states is normalized to unity. Within the NRG, we use \(f(x) = \Theta(1 - |x|)\) with \(K_0 = 1\). In this work we restrict ourselves to the range \(1/2 < s < 1\).

In the absence of the bosonic bath, the pure-fermionic two-impurity Anderson model can be mapped via a Schrieffer-Wolff transformation to a two-impurity Kondo model with a direct exchange interaction \[19\]. In the case of Heisenberg exchange, both the Anderson and Kondo formulations are well studied \[20, 21\], displaying a critical point at an antiferromagnetic exchange \(I_z > 0\) in the presence of particle-hole symmetry; at this point, the static singlet-pairing susceptibility diverges \[22\]. For an Ising \(H_{12}\), the model possesses a Kosterlitz-Thouless (KT) QPT at \(|I_z| > 0\) between a Kondo-screened phase and an interimpurity Ising-ordered phase \[23, 24\]. Without the conduction band, Eq. \[1\] reduces to a two-spin boson model; studies of this model with \(S_1^z + S_2^z\) coupled to a spin bath found a QCP separating a delocalized phase and a ferromagnetically localized phase \[25, 26\].

We have solved Eq. \[1\] with \(H_{12}\) of Ising form by extending the CT-QMC approach \[14, 16, 17\]. After a generalized Lang-Firsov transformation \[14, 16\], the CT-QMC performs time-dependent perturbation theory in the hybridization and stochastically sums the resulting series via a Monte-Carlo algorithm. In order to locate the \(T = 0\) transition via calculations performed at \(T \equiv 1/\beta > 0\), we compute the staggered Binder cumulant \[15, 16, 27\]
\[
U_s^2(\beta, I_z, g) = \left(\langle M_s^2 \rangle/\langle M_s^2 \rangle^2\right),
\]
where the staggered magnetization \(M_s = \beta^{-1} \int_0^\beta d\tau S_3^z(\tau)\) with \(S_3^z = \frac{1}{2}(S_1^z - S_2^z)\). We also calculate the staggered static spin susceptibility \(\chi_s(T) = \beta \langle M_s^2 \rangle\). To measure the pairing correlation between the \(d\)-electrons at different impurity sites, we study dynamic singlet \((d-wave)\) and triplet \((p-wave)\) pairing susceptibilities
\[
\chi_\alpha(\tau, \beta) = \langle T_\tau \Delta_\alpha(\tau) \Delta_\alpha^\dagger \rangle, \quad \alpha = d \text{ or } p,
\]
where \(\Delta_\alpha^d = \frac{1}{\sqrt{2}}(d_{1\uparrow}^\dagger d_{2\downarrow} - d_{1\downarrow}^\dagger d_{2\uparrow}),\Delta_\alpha^p = \frac{1}{\sqrt{2}}((d_{1\uparrow}^\dagger d_{2\downarrow}^\dagger + d_{1\downarrow}^\dagger d_{2\uparrow}^\dagger)),\) and \(T_\tau\) orders in imaginary time. The static pairing susceptibilities follow via \(\chi_\alpha(T) = \int_0^\beta d\tau \chi_\alpha(\tau, \beta)\). In the presence of a bosonic coupling to \(S_3^z\), the Heisenberg form of \(H_{12}\) is beyond the reach of the CT-QMC.

Both forms of \(H_{12}\) can be solved using the Bose-Fermi extension \[11\] of the NRG \[18\]. The staggered spin susceptibility is calculated as \(\chi_s(T) = -\lim_{H_s \to 0} \langle S_3^z \rangle / H_s\) with an additional Hamiltonian term \(H_s S_3^z\). The static pairing correlations are obtained by Hilbert transformation of the imaginary part of the dynamical susceptibilities, computed on the real frequency axis in the usual manner \[28\]. The NRG results presented below were obtained using discretization parameter \(\Lambda = 9\), allowing up to 4 bosons per site of the Wilson chain, and keeping up to 1300 many-body eigenstates after each iteration. Further details of each numerical technique as applied to the models studied here will be given elsewhere \[29\].

In the following, we work with fixed \(\Gamma_0 = 0.25D\) and...
FIG. 3: (color online). Staggered spin susceptibility $\chi_s(T)$ in the Kondo phase (circles), at the QCP (triangles), and in the LM phase (squares), for (a) an Ising $H_{12}$ with $s = 0.8$, and (b) a Heisenberg $H_{12}$ with $s = 0.6$. At the QCP, $\chi_s \sim T^{-s}$.

$U = -2e_d = 0.001D$. This choice places the Anderson impurities at mixed valence with a high Kondo temperature $T_K \approx 1.39D$ (for $g, I_z = 0$), ensuring a correspondingly high temperature of entry into the quantum critical regime \cite{14}. We also take $\omega_c = D$ and focus on sub-Ohmic bath exponents [see Eq. (2)] $s = 0.8$ for Ising exchange and $s = 0.6$ for the Heisenberg case.

Ising $H_{12}$: Figure 2(a) shows the $T = 0$ phase diagram for the case of Ising exchange, as obtained using CT-QMC. For $0 \leq g, I_z \ll D$, each impurity spin is locked into a Kondo singlet with the conduction band and $\chi_s(T)$ approaches a constant at low temperatures [e.g., Fig. 3(a)]. Upon increasing $g$ and/or $I_z$, the system passes through a QPT into an Ising-antiferromagnetic local-moment phase (LM) in which the impurity spins are anti-aligned and decoupled from the conduction band, as seen through a Curie-Weiss behavior of the staggered spin susceptibility: $\chi_s(T) \sim T^{-1}$ [Fig. 3(a)]. The Kondo energy scale vanishes continuously on the Kondo side of the QPT, characteristic of a Kondo-destruction QCP. The staggered Binder cumulant $U^*_{\nu}(\beta, I_z, g)$ varies from 3 deep in the Kondo phase to 1 far into the LM phase. For fixed $I_z$, the cumulant near the QCP has a scaling form

$$U^*_{\nu}(\beta, I_z, g) = U^*_{\nu}(\beta I_z^s (g/g_c - 1); I_z),$$

identifying $g_c$ as the point of temperature independence of $U^*_\nu$ vs $g$ [Fig. 3(a)]. Optimizing the scaling collapse according to Eq. (4) gives a correlation-length exponent $\nu(s = 0.8) = 0.458(8)$ [Fig. 3(b)].

For $g = 0$, the Ising critical point is KT-like, characterized by a divergence $\chi_s(T, I_z = I_z^c, g = 0) \sim T^{-1}$. Consequently, the coupling $g$ has a scaling dimension $[g] = (1 - s)/2$ and is relevant for $s < 1$. This dictates a flow away from the KT fixed point along the phase boundary in Fig. 2(a) toward the $I_z = 0$ critical point \cite{30}. Tuning $g$ to the boundary at fixed $I_z > 0$, we find that the staggered spin susceptibility diverges as

$$\chi_s(T, I_z, g = g_c(I_z)) \sim T^{-x}$$

with $x = 0.79(3), 0.78(3), 0.80(3), 0.82(3), 0.82(3), 0.83(4)$ for increasing $I_z$. These values are consistent with

$$x = s,$$

suggesting that the staggered channel exhibits the same critical properties as the single-impurity Ising-symmetric Bose-Fermi Kondo model \cite{11}.

Heisenberg $H_{12}$: For Heisenberg exchange, the NRG gives the phase diagram shown in Fig. 2(b). For small $g$ and $I$, the model is in the Kondo phase. Tuning $I$ for $g = 0$, we pass through a critical point into an interimpurity singlet (IS) phase, in which the impurity spins are locked into a singlet and decoupled from the conduction band. At the particle-hole-symmetric critical point \cite{21}, \cite{21}, the staggered spin susceptibility diverges as $\chi_s(T, I = I_c, g = 0) \sim \ln(T_K/T)$. Using the corresponding scaling dimension of the staggered impurity spin, along with the scaling dimension of the bosonic operator, we determine that the bosonic coupling has scaling dimension $[g] = -s/2$ and is irrelevant for $s > 0$. Indeed, we find that the NRG spectrum along the phase boundary is independent of $g$ for small values of $g$ \cite{30}, indicating that the critical behavior is governed by the $g = 0$ QCP.

For small $I > 0$, tuning the bosonic coupling $g$ yields a QPT from the Kondo phase to the LM phase [Fig. 3(c)]. The Kondo energy scale vanishes continuously on approach from the small-$g$ side of this Kondo-destruction QCP. At the QCP, the staggered spin susceptibility obeys Eq. (5) with $I_z$ replaced by $I$ and $x = 0.61(2)$ [Fig. 3(b)], again consistent with Eq. (6). Nearby, the correlation-
length exponent is $\nu(s = 0.6)^{-1} = 0.40(2)$ [Fig. 4(d)]. Within numerical accuracy, we find that the Kondo-destruction QCPs for Ising and Heisenberg exchange fall within the same universality class. In both cases, the Kondo destruction QCPs are insensitive to breaking of particle-hole symmetry via setting $U \neq -2\epsilon_d$, as well as to a finite impurity separation $\epsilon_d$.

We turn next to the transition between the IS and LM phases. Fixing $I$ at a large value and tuning $g$, the bosonic bath decoheres and destroys the interimpurity singlet state at a QCP, where we find similar critical properties to those on the Kondo-LM boundary: $\chi_s$ diverging according to Eq. (5) with $x = 0.61(3)$, and a correlation length exponent $\nu(s = 0.6)^{-1} = 0.40(2)$.

In the particle-hole-symmetric case that is the focus of this paper, the Kondo, IS, and LM phases all meet at a tricritical point, as shown in Fig. 2(b). Generic particle-hole asymmetry is known to turn the Kondo-to-IS transition into a crossover $\chi_d \propto \epsilon_d^z$, leaving only a single line of Kondo-destruction QPTs.

**Pairing susceptibilities:** We now consider the singlet and triplet pairing susceptibilities defined in Eq. (4). For both the Ising and Heisenberg forms of the interimpurity exchange, the static triplet pairing susceptibility $\chi_p$ (not shown) is reduced by any nonzero value of $g$, $I_z$, or $I$.

More interesting is the singlet pairing susceptibility, which we illustrate along paths on the $g$-$I_z$ and $g$-$I$ phase diagrams that start from $g = I_z = I = 0$ and cross the Kondo-LM boundary. Figure 5(a) plots $\chi_d$ vs Ising exchange coupling at a sequence of temperatures along the cut $g = 0.372I_z$. The pairing susceptibility grows as $I_z$ increases from zero, is peaked for $I_z$ slightly below $I^{c}_z$, and then falls off within the LM phase as the $d$ electrons localize and decouple from the conduction band. The singlet pairing susceptibility saturates at temperatures $T \lesssim 0.004D \approx 0.003T_K$.

Figure 5(b) illustrates the Heisenberg form of $H_{12}$, plotting the $T = 0$ singlet pairing susceptibility vs $I$ at $T = 0$ along a path $g = 2.54I$ that crosses the Kondo-LM boundary. Very much as in the Ising case, $\chi_d$ rises from $I = 0$ and peaks just below $I = I^{c}_z$.

The enhancement of the static singlet pairing susceptibility near a Kondo-destruction QCP is one of the principal results of this work. Although $\chi_d$ peaks just inside the Kondo phase, the pairing correlation at the QCP is significantly higher than at $g = I_z = I = 0$. We stress that these results are associated with the critical destruction of the Kondo effect. They differ from those for $g = 0$, where for Heisenberg exchange $\chi_d(T = 0)$ diverges at the Kondo-IS QPT [22]. We have found (by following the path $g = 0.717I$, not shown) that the singlet pairing susceptibility also diverges on crossing the Kondo-IS boundary at some $g > 0$, consistent with the picture that this boundary is governed by the $g = 0$ critical point.

The models considered here have both a dynamic (induced by $g$) and a static ($I_z$ or $I$) exchange interaction between the impurities. The combination of dynamic and static antiferromagnetic interactions enhances the singlet pairing susceptibility between the impurities and gives rise to a peak near the Kondo-destruction QCP. This behavior is likely to have significant effects in lattice systems. Within an extended dynamical mean field approach [31], for example, the enhanced pairing susceptibility in the impurity model may give rise to a pairing instability near a Fermi-surface-collapsing QCP of a Kondo lattice [43]. This would represent a new mechanism for superconductivity in the vicinity of antiferromagnetic order, and would be of considerable interest in connection with the superconductivity observed in the Ce-115 materials [32] and related heavy-fermion superconductors [43].

In summary, we have introduced and solved two variants of the two-impurity Bose-Fermi Anderson model using robust numerical methods. We have mapped out the phase diagrams for these models and shown that each possesses a line of Kondo-destruction QCPs. The QCPs in the two models belong to the same universality class despite the differing symmetries of the interimpurity exchange interaction. Just as importantly, we have shown that the Kondo-destruction quantum criticality in these models enhances singlet pairing correlations. Our results hold promise for elucidating the superconductivity observed in heavy-fermion metals whose normal state shows characteristics of Kondo-destruction quantum criticality.

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[1] Q. Si and F. Steglich, Science 329, 1161 (2010)
[2] H. v. Löhneysen, A. Rosch, M. Vojta and P. Wölfle, Rev. Mod. Phys., 79, 1015 (2007)
[3] J. A. Hertz, Phys. Rev. B 14, 1165, (1976); A. J. Millis, ibid. 48, 7183, (1993).
[4] Q. Si, S. Rabello, K. Ingersent and J. L. Smith, Nature 413, 804 (2001); Phys. Rev. B 68, 115103 (2003).
[5] P. Coleman, C. Pépin, Q. Si and R. Ramazashvili, J. Phys. Cond. Matt. 13, R723 (2001).
[6] A. Schröder, G. Aeppli, R. Coldea, M. Adams, O. Stockert, H. v. Löhneysen, E. Bucher R. Ramazashvili and P. Coleman, 2000 Nature 407, 351 (2000); M. C. Aronson, R. Osborn, R. A. Robinson, J. W. Lynn, R. Chau, C. L. Seaman and M. B. Maple, Phys. Rev. Lett. 75, 725 (1995).
[7] P. Aynajian, E. H. da Silva Neto, A. Gyenis, R. E. Baumbach, J. D. Thompson, Z. Fisk, E. D. Bauer, and A. Yazdani, Nature 486, 201 (2012).
[8] S. Paschen, T. Lühmann, S. Wirth, P. Gegenwart, O. Trovarelli, C. Geibel, F. Steglich, P. Coleman and Q. Si, Nature 432, 881 (2004); P. Gegenwart, T. Westerkamp, C. Krellner, Y. Tokiwa, S. Paschen, C. Geibel, F. Steglich, E. Abrahams, Q. Si, Science, 315, 969 (2007); S. Friedemann, N. Oeschler, S. Wirth, C. Krellner, C. Geibel, F. Steglich, S. Paschen, S. Kirchner and Q. Si, Proc. Natl. Acad. Sci. USA 107, 14547 (2010).
[9] H. Shishido, R. Settai, H. Harima, and Y. Ônuki, J. Phys. Soc. Jpn. 74, 1103 (2005).
[10] P. Gegenwart, Q. Si and F. Steglich, Nat. Phys. 4, 186 (2008).
[11] M. T. Glossop and K. Ingersent Phys. Rev. Lett. 95, 067202 (2005); Phys. Rev. B 75, 104410 (2007).
[12] K. Ingersent and Q. Si Phys. Rev. Lett. 89, 076403 (2002).
[13] S. Kirchner and Q. Si Phys. Rev. Lett. 100, 026403 (2008).
[14] J. H. Pixley S. Kirchner M. T. Glossop and Q. Si, J. Phys.: Conf. Ser. 273, 012050 (2011).
[15] M. T. Glossop, S. Kirchner, J. H. Pixley and Q. Si, Phys. Rev. Lett. 107, 076404 (2011); J. H. Pixley, S. Kirchner, K. Ingersent and Q. Si, ibid. 109, 086403 (2012).
[16] J. H. Pixley S. Kirchner K. Ingersent and Q. Si, arXiv:1306.2352 (2013).
[17] E. Gull, A. J. Millis, A. I. Lichtenstein, A. N. Rubtsov, M. Troyer, and P. Werner, Rev. Mod. Phys. 83, 349 (2011).
[18] K. G. Wilson, Rev. Mod. Phys. 47, 773 (1975).
[19] L. J. Zhou and Q. Q. Zheng, J. Magn. & Magn. Mater. 109, 237 (1992); T. T. Ong and B. A. Jones, Euro. Phys. Lett. 93, 57004 (2011).
[20] B. A. Jones and C. M. Varma, Phys. Rev. Lett. 58, 843 (1987); B. A. Jones, C. M. Varma and J. W. Wilkins, ibid. 61, 125 (1988); B. A. Jones, B. G. Kotliar and A. J. Millis, Phys. Rev. B 39, 3415 (1989); R. M. Fye, Phys. Rev. Lett. 72, 916 (1994); L. Zhu and J. X. Zhu, Phys. Rev. B 83, 195103 (2011); L. Zhu and J. X. Zhu, ibid. 87, 085120 (2013).
[21] I. Affleck and A. W. W. Ludwig, Phys. Rev. Lett. 68, 1046 (1992); I. Affleck, A. W. W. Ludwig and B. A. Jones Phys. Rev. B 52, 9528 (1995).
[22] L. Zhu and J. X. Zhu, J. Phys.: Conf. Ser. 273, 012068 (2011).
[23] N. Andrei, G. T. Zimanyi, and G. Schön, Phys. Rev. B 60, R5125 (1999).
[24] M. Garst, S. Kehrein, T. Pruschke, A. Rosch, and M. Vojta, Phys. Rev. B 69, 214413 (2004).
[25] P. Orth, D. Roosen, W. Hofstetter, and K. Le Hur, Phys. Rev. B 82, 144423 (2010).
[26] D. P. S. McCutcheon, A. Roosen, S. Bose, and A. J. Fisher, Phys. Rev. B 81, 235321 (2010).
[27] K. Binder Z. Phys. B 43, 119 (1981).
[28] R. Bulla, T. A. Costi, and Th. Pruschke, Rev. Mod. Phys. 80, 395 (2008).
[29] J. H. Pixley, L. Deng, K. Ingersent and Q. Si, in preparation.
[30] See supplementary information at http://
[31] J. L. Smith and Q. Si, Phys. Rev. B 61, 5184 (2000); R. Chitra and G. Kotliar, Phys. Rev. Lett. 84, 3678 (2000).
[32] H. Hegger, C. Petrovic, E. G. Moshopoulou, M. F. Hundley, J. L. Sarrao, Z. Fisk, and J. D. Thompson, Phys. Rev. Lett. 84, 4986 (2000).
[33] Q. Si and S. Paschen, Phys. Status Solidi B 250, 425 (2013).
In this supplementary material, we present the renormalization-group (RG) flow of the two impurity Bose-Fermi Anderson models. Figure S1 corresponds to the case with Ising interimpurity exchange $H_{12}$. There are two stable fixed points, indicated by filled circles, which govern the Kondo-screened (Kondo) phase and the local-moment (LM) phase. The Kondo fixed point is at $(g, I_z) = (0, 0)$, while the LM fixed point is at $(g, I_z) = (\infty, 0)$. On the phase boundary, the RG flow is from the Kosterlitz-Thouless (KT) fixed point toward the Kondo-destruction (KD) fixed point. The KT and KD fixed points are both unstable and are shown by open circles. The direction of the RG flow on the phase boundary reflects the discussion in the main text, i.e., all the points for nonzero $g$ on the phase boundary have the same critical behavior as the KD QCP on the $g$ axis.

Figure S2 shows the RG flow for the case with Heisenberg interimpurity exchange $H_{12}$. In this case, there are three stable fixed points corresponding to the three phases. The Kondo-screened (Kondo), local-moment (LM) and interimpurity-singlet (IS) phases are respectively located at $(g, I) = (0, 0)$, $(g, I) = (\infty, 0)$ and $(g, I) = (0, \infty)$, and are marked by filled circles. On the Kondo-LM phase boundary, the RG flow is from the triple-point (where the three phases meet) to the Kondo-destruction (KD) fixed point. Likewise, the RG flow on the Kondo-IS phase boundary is from the triple-point toward the fixed point KI, which separates the Kondo and IS phases on the $I$-axis.

**FIG. S1**: (color online). Schematic RG flow on the $g$-$I_z$ plane for the two-impurity Bose-Fermi Anderson model with Ising interimpurity exchange $H_{12}$. Trajectories with arrows represent the flows of the couplings $(g$ and $I_z$) with the decrease of energy.

**FIG. S2**: (color online). Schematic RG flow on the $g$-$I$ plane for the two-impurity Bose-Fermi Anderson model with Heisenberg inter-impurity exchange $H_{12}$.