Mechanical Stability of Cylindrical Thin-Shell Wormholes

M. Sharif\textsuperscript{1} * and M. Azam\textsuperscript{1,2} †

\textsuperscript{1} Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore-54590, Pakistan.
\textsuperscript{2} Division of Science and Technology, University of Education, Township Campus, Lahore-54590, Pakistan.

Abstract

In this paper, we apply the cut and paste procedure to charged black string for the construction of thin-shell wormhole. We consider the Darmois-Israel formalism to determine the surface stresses of the shell. We take Chaplygin gas to deal with the matter distribution on shell. The radial perturbation approach (preserving the symmetry) is used to investigate the stability of static solutions. We conclude that stable static solutions exist both for uncharged and charged black string thin-shell wormholes for particular values of the parameters.

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1 Introduction

The study of thin-shell wormholes is of great interest as it links two same or different universes by a tunnel (throat) [1]. The throat is threaded by the unavoidable amount of exotic matter (which violates the null energy condition). It is necessary to minimize the violation of energy conditions.

* msharif.math@pu.edu.pk
† azammath@gmail.com
for the physically viability of wormholes. For this purpose, cut and paste procedure was considered by various authors, for instance [2]-[4], to construct a theoretical wormhole (thin-shell). Moreover, the required exotic matter to support wormhole can be minimized with the appropriate choice of geometry [5].

The key issue of thin-shell wormhole is to explored its mechanical stability under radial perturbations in order to understand its dynamical aspects. In this scenario, many people worked for the stability analysis of thin-shell wormholes. Poisson and Visser [6] have constructed the Schwarzschild thin-shell wormhole and explored its stability regions under linear perturbations. Visser [7] analyzed stability of thin-shell wormholes with specific equations of state. It was found that charge [8] and positive cosmological constant [9] increase the stability regions of spherically symmetric thin-shell wormholes. Thibeault et al. [10] explored stability of thin-shell wormhole in Einstein-Maxwell theory with a Gauss-Bonnet term.

Cylindrical thin-shell wormholes associated with and without cosmic strings have been widely discussed in literature [11]-[14]. These cosmic strings have many astrophysical phenomena like structure formation in the early universe and gravitational lensing effects [15]. Eiroa and Simeone [16] have discussed the cylindrical thin-shell wormholes associated with local and global cosmic strings and found that the wormhole configurations are unstable under velocity perturbations. Bejarano et al. [17] discussed stability of static configurations of cylindrical thin-shell wormholes under perturbations and found that the throat will expand or collapse depending upon the sign of the velocity perturbations.

Richarte and Simeone [18] found more configurations which are not stable under radial velocity perturbations, consistent with the conjecture discussed in a paper [17]. Eiroa and Simeone [19] summarized the above results and found that stable configurations are not possible for the cylindrical thin-shell wormholes. Recently, we have explored stability of spherical and cylindrical geometries at Newtonian and post-Newtonian approximations and also spherically symmetric thin-shell wormholes [20].

Several candidates have been proposed like quintessence, K-essence, phantom, quintom, tachyon, family of Chaplygin gas, holographic and new agegraphic DE [21] for the explanation of accelerated universe. Besides all these candidates, Chaplygin gas (an exotic matter) has been proposed widely to explain the expansion of the universe. Kamenshchik et al. [22] described the feature of Chaplygin gas and explored cosmology of FRW universe filled with
a Chaplygin gas. The Chaplygin cosmological models support the observational evidence [23, 24].

Models of exotic matter like phantom energy with equation of state \( p = \omega \sigma, \omega < -1 \) [25] and Chaplygin gas \( p \sigma = -A \) [26], where \( A \) is a positive constant, have been of interest in wormhole construction. Wormholes have also been studied in dilaton gravity [27] and Einstein-Gauss-Bonnet theory [28]. Eiroa [29] found stable static solutions of spherically symmetric thin-shell wormholes with Chaplygin equation of state.

In this paper, we construct black string thin-shell wormholes with and without charge supported by the Chaplygin gas. We study the mechanical stability of these constructed wormholes under radial perturbations preserving the cylindrical symmetry. The format of the paper is as follows. In section 2, we formulate surface stresses of the matter localized on the shell through Darmois-Israel junction conditions. Section 3 provides the mechanical stability analysis of the static configuration of thin-shell wormholes. In the last section 4, we conclude our results.

## 2 Thin-Shell Wormhole Construction

In this section, we build a thin-shell wormhole from charged black string through cut and paste technique and discuss its dynamics through the Darmois-Israel formalism. We consider the Einstein-Hilbert action with electromagnetic field as [30]

\[
\mathcal{A} + \mathcal{A}_{em} = \frac{1}{16\pi G} \int \sqrt{-g} (R - 2\Lambda) d^4 x - \frac{1}{16\pi} \int \sqrt{-g} F_{\mu \nu} F^{\mu \nu} d^4 x, \quad (1)
\]

where \( \mathcal{A} \) is defined by

\[
\mathcal{A} = \frac{1}{16\pi G} \int \sqrt{-g} (R - 2\Lambda) d^4 x. \quad (2)
\]

Here, \( g, R, \Lambda, F_{\mu \nu} \) are the metric determinant, the Ricci scalar, negative cosmological constant and the Maxwell field tensor \( (F_{\mu \nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu) \), respectively and \( \phi_\nu = -\lambda(r) \delta_\nu^0 \) is the electromagnetic four potential with an arbitrary function \( \lambda(r) \).

The Einstein-Maxwell equations from the above action yields a cylindrically symmetric vacuum solution, i.e., charged black string given as

\[
ds^2 = -g(r) dt^2 + g^{-1}(r) dr^2 + h(r)(d\phi^2 + \alpha^2 dz^2), \quad (3)
\]
where \( g(r) = \left( \alpha^2 r^2 - \frac{4M}{\alpha r} + \frac{4Q^2}{\alpha^2 r^2} \right) \) is a positive function for the given radius and \( h(r) = r^2 \). We have following restrictions on coordinates to preserve the cylindrical geometry

\[-\infty < t < \infty, \quad 0 \leq r < \infty, \quad -\infty < z < \infty, \quad 0 \leq \phi \leq 2\pi.\]

The parameters \( M, Q \) are the ADM mass and charge density, respectively and \( \alpha^2 = -\frac{\Lambda}{3} > 0 \). We remove the region with \( r < a \) of the given cylindrical black string and take two identical 4D geometries \( V^\pm \) with \( r \geq a \) as

\[ V^\pm = \{ x^\gamma = (t, r, \phi, z) / r \geq a \}, \quad (4) \]

where “\( a \)” is the throat radius and glue these geometries at the timelike hypersurface \( \Sigma = \Sigma^\pm = \{ r-a = 0 \} \) to get a new manifold \( V = V^+ \cup V^- \). This manifold is geodesically complete exhibiting a wormhole with two regions connected by a throat satisfying the radial flare-out condition, i.e., \( h'(a) = 2a > 0 \) [14]. If ‘\( r_h \)’ is the event horizon of the black string given in Eq.(3), then we assume \( a > r_h \) in order to prevent the occurrence of horizons and singularities in wormhole configuration.

We use the standard Darmois-Israel formalism [31, 32] to analyze the dynamics of the wormhole. The two sides of the shell are matched through the extrinsic curvature defined on \( \Sigma \) as

\[ K_{ij}^\pm = -n_\gamma^\pm \frac{\partial^2 x^\gamma}{\partial \xi^i \partial \xi^j} + \Gamma^\gamma_{\mu\nu} \frac{\partial x^\mu_{\pm}}{\partial \xi^i} \frac{\partial x^\nu_{\pm}}{\partial \xi^j}, \quad (i, j = 0, 2, 3), \quad (5) \]

where \( \xi^i = (\tau, \theta, \phi) \) are the coordinates on \( \Sigma \) and \( n^\pm_i \) are the unit normals obtained to \( \Sigma \) as

\[ n^\pm_\gamma = \left( -\ddot{a}, \frac{\sqrt{g(r)} + \dot{a}^2}{g(r)}, 0, 0 \right), \quad (6) \]

satisfying the relation \( n^\gamma n_\gamma = 1 \). The induced metric on \( \Sigma \) is defined as

\[ ds^2 = -d\tau^2 + a^2(\tau)(d\phi^2 + \alpha^2 dz^2), \quad (7) \]

where \( \tau \) is the proper time on the hypersurface. The non-vanishing components of the extrinsic curvature turns out to be

\[ K^\pm_{\tau\tau} = \mp \frac{g'(a) + 2\ddot{a}}{2\sqrt{g(a)} + \dot{a}^2}, \quad K^\pm_{\phi\phi} = \pm a \sqrt{g(a)} + \dot{a}^2, \quad K^\pm_{zz} = \alpha^2 K^\pm_{\phi\phi}, \quad (8) \]
Here dot and prime mean derivative with respect to $\tau$ and $r$ respectively.

The surface stress-energy tensor $S_{ij} = \text{diag}(-\sigma, p, p)$ provides surface energy density $\sigma$ and surface tensions $p$ of the shell. The Lanczos equations are defined on the shell as

$$S^i_j = -\frac{1}{8\pi} [\kappa^i_j - \delta^i_j \kappa^k_k], \quad (9)$$

where $\kappa_{ij} = K_{ij}^+ - K_{ij}^-$ and due to the simplicity of cylindrical symmetry, we have $\kappa^i_j = \text{diag}(\kappa_{\tau\tau}, \kappa_{\phi\phi}, \kappa_{\phi\phi})$. The Lanczos equations with surface stress-energy tensor provides

$$\sigma = -\frac{1}{4\pi} \kappa^\phi = -\frac{1}{2\pi a} \sqrt{g(a) + \dot{a}^2}, \quad (10)$$

$$p = \frac{1}{8\pi} (\kappa^\tau_{\tau} + \kappa^\phi_{\phi}) = \frac{1}{8\pi a} \frac{2a\ddot{a} + 2\dot{a}^2 + 2g(a) + ag'(a)}{\sqrt{g(a) + \dot{a}^2}}. \quad (11)$$

*Matter that violates the null energy condition is known as exotic matter.* We see from Eq. $(10)$ that the surface energy density is negative indicating the existence of exotic matter at the throat. For the explanation of such matter, the Chaplygin equation of state is defined on the shell as

$$p = -\frac{A}{\sigma}, \quad (12)$$

where $A > 0$. Using Eqs. $(10)$ and $(11)$ in the above equation, we obtain a second order differential equation satisfied by the throat radius $'a'$

$$2a\ddot{a} + 2\dot{a}^2 - 16\pi^2 Aa^2 + 2g(a) + ag'(a) = 0. \quad (13)$$

### 3 Stability Analysis

In this section, we have adapted and applied the criteria introduced in Ref. [29] for the stability analysis of static solutions. The existence of static solutions is subject to the condition $a_0 > r_h$. For such solutions, we consider static configuration of Eq. $(13)$ as

$$-16\pi^2 Aa_0^2 + 2g(a_0) + a_0 g'(a_0) = 0, \quad (14)$$
and the corresponding static configuration of surface energy density and pressure are

\[ \sigma_0 = -\frac{1}{2\pi a_0} \sqrt{g(a_0)}, \quad p_0 = \frac{2A\pi a_0}{\sqrt{g(a_0)}}. \] (15)

The perturbed form of the throat radius is given by

\[ a(\tau) = a_0[1 + \epsilon(\tau)], \] (16)

where \( \epsilon(\tau) \ll 1 \) is a small perturbation preserving the symmetry. Using the above equation, the corresponding perturbed configuration of Eq.(13) can be written as

\[ (1 + \epsilon)\ddot{\epsilon} + \ddot{\epsilon}^2 - 8\pi^2 A(2 + \epsilon)\epsilon + D(a_0, \epsilon) = 0, \] (17)

where

\[ D(a_0, \epsilon) = \frac{2g(a_0 + a_0\epsilon) + a_0(1 + \epsilon)g'(a_0 + a_0\epsilon) - 2g(a_0) - a_0g'(a_0)}{2a_0^2}. \]

Substituting \( v(\tau) = \dot{\epsilon}(\tau) \) in Eq.(17), we have a first order differential equation in \( v \)

\[ \dot{v} = \frac{8\pi^2 A(2 + \epsilon)\epsilon - D(a_0, \epsilon) - v^2}{1 + \epsilon}. \] (18)

Using Taylor expansion to first order in \( \epsilon \) and \( v \), we obtain a set of equations

\[ \dot{\epsilon} = v, \quad \dot{v} = \Delta \epsilon, \] (19)

where

\[ \Delta = 16\pi^2 A - \frac{3g'(a_0) + a_0g''(a_0)}{2a_0}. \] (20)

This set of equations can be written in matrix form as

\[ \dot{\eta} = L\eta, \]

where \( \eta \) and \( L \) are

\[ \eta = \begin{bmatrix} \epsilon \\ v \end{bmatrix}, \quad L = \begin{bmatrix} 0 & 1 \\ \Delta & 0 \end{bmatrix}. \]

The matrix ‘L’ has two eigenvalues \( \pm\sqrt{\Delta} \).
The stability analysis of static solutions on the basis of $\Delta$ can be formulated as follows [29]:

**i.** When $\Delta > 0$, the given matrix has two real eigenvalues $\lambda_1 = -\sqrt{\Delta} < 0$ and $\lambda_2 = \sqrt{\Delta} > 0$. The negative eigenvalue has no physical significance, while the positive eigenvalue indicates the presence of unstable static solution.

**ii.** When $\Delta = 0$, the corresponding eigenvalues are $\lambda_1 = \lambda_2 = 0$ and Eq. (19) leads to $v = v_0 = constant$ and $\epsilon = \epsilon_0 + v_0(\tau - \tau_0)$. In this case, the static solution is unstable.

**iii.** When $\Delta < 0$, we have two imaginary eigenvalues $\lambda_1 = -i\sqrt{|\Delta|}$ and $\lambda_2 = i\sqrt{|\Delta|}$. Eiroa [29] discussed this case in detail and found that the only stable static solution with throat radius $a_0$ exists for $\Delta < 0$ which is not asymptotically stable.

### 3.1 Uncharged Black String Thin-Shell Wormhole

Here, we investigate the stability of thin-shell wormhole static solution constructed from the asymptotically anti-de Sitter uncharged black string [30]. For the uncharged black string, we have

$$g(r) = \alpha^2 r^2 - \frac{4M}{\alpha r}.$$  \hfill (21)

The event horizon of the black string is given by ($g_{tt} = 0$)

$$r_h = \left(\frac{4M}{\alpha}\right)^{\frac{1}{3}}.$$  \hfill (22)

The corresponding surface energy density and pressure of black string for static configuration leads to

$$\sigma_0 = -\frac{\sqrt{\alpha^3 a_0^3 - 4M}}{2\pi a_0^\frac{3}{2} \sqrt{\alpha}}, \quad p_0 = \frac{2A\pi a_0^\frac{3}{2} \sqrt{\alpha}}{\sqrt{\alpha^3 a_0^3 - 4M}}.$$  \hfill (23)

Using Eq. (21) in (14), we get a cubic equation satisfied by the throat radius with $A > 0$ and $M > 0$

$$(4A\pi^2 \alpha - \alpha^3)a_0^3 + M = 0.$$  \hfill (24)
The corresponding roots \( a_0^1, a_0^2, a_0^3 \), of the cubic equation are

\[
a_0^1 = \frac{1}{\alpha} \left( \frac{M}{1 - 4A\pi^2\alpha^{-2}} \right)^{\frac{1}{3}},
\]
\[ (25) \]

\[
a_0^2 = \frac{(-1 + i\sqrt{3})}{2\alpha} \left( \frac{M}{1 - 4A\pi^2\alpha^{-2}} \right)^{\frac{1}{3}},
\]
\[ (26) \]

\[
a_0^3 = \frac{(-1 - i\sqrt{3})}{2\alpha} \left( \frac{M}{1 - 4A\pi^2\alpha^{-2}} \right)^{\frac{1}{3}}.
\]
\[ (27) \]

Now, we analyze all the roots numerically and find which of them represent static solution. The condition for the existence of static solution is subject to \( a_0^1 > \frac{(4M)^{\frac{1}{3}}}{\alpha} \). For \((A\alpha^{-2}) > (4\pi^2)^{-1}\), we have one negative real root \( a_0^3 \) and two non-real roots \( a_0^1, a_0^2 \). All the three roots have no physical significance. Thus, there are no static solutions corresponding to \((A\alpha^{-2}) > (4\pi^2)^{-1}\). For \((A\alpha^{-2}) < (4\pi^2)^{-1}\), there is one positive real root \( a_0^1 \) and two non-real roots \( a_0^2, a_0^3 \). The non-real roots are discarded being of no physical significance. We check numerically that for \((A\alpha^{-2}) < (4\pi^2)^{-1}\), the only static solution corresponding to positive real root is \( a_0^1 \geq \frac{(4M)^{\frac{1}{3}}}{\alpha} \). The critical value for which the throat radius greater than event horizon is \( \beta = 1.9 \times 10^{-2} < (4\pi^2)^{-1} \). It turns out that the static solution will exist if \( \beta < (A\alpha^{-2}) \leq (4\pi^2)^{-1} \).

For the stability of \( a_0^1 \), we work out \( \Delta \) for the uncharged black string. From Eq. (20), we have

\[
\Delta = 16A\pi^2 - 4\alpha^2 - \frac{2M}{\alpha a_0^3}.
\]
\[ (28) \]

The above equation with Eq. (24) leads to

\[
\Delta = -\frac{6M}{\alpha a_0^3}.
\]
\[ (29) \]

This shows that \( \Delta \) is always negative for \( a_0^1 > \frac{(4M)^{\frac{1}{3}}}{\alpha} \). Thus, we have one stable solution for the uncharged black string corresponding to \((A\alpha^{-2}) < (4\pi^2)^{-1}\) with throat radius \( a_0^1 \) as shown in Figure 1. There does not exist any other static solution corresponding to \((A\alpha^{-2}) > (4\pi^2)^{-1}\).
Figure 1: Plot of stable static solution of $a_0^1$ for $\beta < (A\alpha^{-2}) \leq (4\pi^2)^{-1}$.

### 3.2 Charged Black String Thin-Shell Wormhole

For the case of charged black string, we have considered $g(r)$ from Eq. (3)

$$g(r) = \alpha^2 r^2 - \frac{4M}{\alpha r} + \frac{4Q^2}{\alpha^2 r^2}. \quad (30)$$

The event horizons (30) for the charged black string are obtained from the equation

$$\alpha^2 r^2 - \frac{4M}{\alpha r} + \frac{4Q^2}{\alpha^2 r^2} = 0. \quad (31)$$

The above quartic equation has two real and two complex roots. We discard the complex roots being of unphysical and the real roots are taken as inner and outer event horizons of the charged black string

$$r_{\pm} = \left(\frac{4M}{\alpha}\right)^{\frac{1}{4}} \left[\sqrt{s} \pm \sqrt{2\sqrt{s^2 - Q^2 \left(\frac{2}{M}\right)^{\frac{4}{3}}} - s}\right] = \frac{(4M)^{\frac{1}{4}}}{\alpha} \chi, \quad (32)$$

provided that the inequality $Q^2 \leq \frac{3}{4}M^{\frac{4}{3}}$ holds, where $s$ and $\chi$ are given by

$$s = \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{64Q^6}{27M^4}}\right)^{\frac{1}{4}} + \left(\frac{1}{2} - \frac{1}{2} \sqrt{1 - \frac{64Q^6}{27M^4}}\right)^{\frac{1}{4}}, \quad (33)$$

$$\chi = \frac{1}{2} \left[\sqrt{s} \pm \sqrt{2\sqrt{s^2 - Q^2 \left(\frac{2}{M}\right)^{\frac{4}{3}}} - s}\right]. \quad (34)$$
Figure 2: Plot of $\chi$ vs $Q$ shows that $\chi$ is a decreasing function of $Q$.

For $Q^2 > \frac{2}{3}M^2$, the given metric has no event horizon and represents a naked singularity. If $Q^2 = \frac{2}{3}M^2$, the inner and outer horizons coincide, which corresponds to extremal black strings. For the static wormhole associated to the charged black string, the surface energy density and pressure turn out to be

$$
\sigma_0 = -\frac{\sqrt{\alpha^4 a_0^4 - 4M \alpha a_0 + 4Q^2}}{2\pi a_0^2 \alpha}, \quad (35)
$$

$$
p_0 = \frac{2A\pi a_0^3 \alpha}{\sqrt{\alpha^4 a_0^4 - 4M \alpha a_0 + 4Q^2}}. \quad (36)
$$

We would like to find an equation satisfied by the throat radius. We substitute Eq. (30) in (14), it turns out that the charge terms cancel out and again we get the same equation (24) satisfied by the throat radius of the wormhole with the roots given in Eqs. (25)-(27). For the existence of static solutions, the throat radius $a_0$ should be greater than $r_h = r_+$. It is noted that the event horizon (32) is a decreasing function of $Q$ for positive values of $M$ and $\alpha$ as shown in Figure 2. Again, we check all the roots numerically and find that there exists only one static solution, $a_0^1$, subject to large values of $Q$. Thus in the case of charged black string, we have one static solution for $(Aa_0^{-2}) < (4\pi^2)^{-1}$ with $Q < M$ and large values of $Q$ and no static solution for $(Aa_0^{-2}) > (4\pi^2)^{-1}$. The stability of static solutions, whether they are stable or unstable, depends upon the sign of $\Delta$ which is again negative for $a_0^1 > r_h$. Hence, the obtained static solution is of stable type for the static wormhole associated with the charged black string.
4 Summary and Discussion

In this paper, we have constructed black string thin-shell wormholes and investigated their mechanical stability under radial perturbations preserving the cylindrical symmetry. We have formulated the Darmois Israel junction conditions and imposed Chaplygin equation of state on the matter shell. We have constructed wormholes using cut and paste procedure such that the throat radius should be greater than the event horizon of the given metric, i.e., $a_0 > r_h$. A dynamical equation with the Chaplygin gas of the shell has been obtained by considering throat radius as a function of proper time. The proposed criteria for the stability analysis of static solutions has been extended to uncharged and charged constructed wormholes.

It was shown in Refs. [16]-[19] that cylindrical thin-shell wormholes with equations of state depending upon the metric functions are always mechanically unstable. On the other hand, we have constructed cylindrical thin-shell wormholes from charged black string solution (whose causal structure is similar to Reissner-Nordström solution) with Chaplygin equation of state. We have found stable configurations depending upon the parameters involving in the model. This shows that the choice of bulk solution as well as equation of state plays a significant role in the stability of wormhole configurations.

For the case of static wormhole associated with uncharged black string, we have found one negative real root and two non-real roots corresponding to $(A\alpha^{-2}) > (4\pi^2)^{-1}$, which implies no static solution. For $(A\alpha^{-2}) < (4\pi^2)^{-1}$, we have two complex roots and one positive real root which is stable for $\beta < (A\alpha^{-2}) \leq (4\pi^2)^{-1}$, where, $\beta = 1.9 \times 10^{-2} < (4\pi^2)^{-1}$. In the case of static wormhole associated to charged black string, we see that event horizon $r_h$ is a decreasing function of $Q$. Also, we have found the same three roots as in the case of uncharged black string. Thus, there is one stable static solution corresponding to $(A\alpha^{-2}) < (4\pi^2)^{-1}$ depending upon the values of the parameters $Q < M$ and $\alpha > 0$. For $(A\alpha^{-2}) > (4\pi^2)^{-1}$, there is no static solution. It is mentioned here that there always exists one stable static solution for $(A\alpha^{-2}) < (4\pi^2)^{-1}$ and no static solution for $(A\alpha^{-2}) > (4\pi^2)^{-1}$ in both uncharged as well as charged black string.

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