MATHEMATICAL MODEL OF DIABETES AND ITS
COMPLICATION INVOLVING FRACTIONAL OPERATOR
WITHOUT SINGULAR KERNEL

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Abstract. Diabetes is one of the burning issues of the whole world. It effected
the world population rapidly. According to the WHO approx 415 million peo-
ple are living with diabetes in the world and this figure is expected to rise up
to 642 million by 2040. World various organizations raise their voice against
the dire facts about the increasing graph of diabetes and its complicated pa-
tients. In this paper authors define the fractional model of diabetes and its
complications involving to fractional operator with exponential kernel. The
authors discuss the existence of the solution by using fixed point theorem and
also show the uniqueness of the solution. To validate the models efficiency
the authors provided numerical simulation by using HPM. To strengthen the
model the results have been presented in the form of graphs.

1. Background. Diabetes is a group of metabolic disorders which is also known
as Diabetes mellitus (DM). In this metabolic disorder the patient has high blood
sugar levels over a prolonged period. Blood glucose is the main source of energy
of human body and it comes from the food. Blood glucose is also known as Blood
Sugar. To control and adjust the quantity of blood glucose in human body, pancreas
excretes a hormone known as Insulin [1]. Insulin played a very important role to
control Blood Sugar. Sometimes your body is unable make enough or any insulin
or is unable to use insulin well. Glucose then stays in your blood and doesn’t reach
your cells. This disorder has been seen worldwide. And the growth of patients is a
serious issue. Generally Dramatic is considered of two types.

Type I: In type I we have considered those patients who are dependent on insulin
injection for survival. People with type I diabetes require daily administration of
insulin to normalize the quantity of glucose in their body. If they are not tacking
insulin at time it caused series problems. There is no medicine in the world which
can prevent this problem. The symptoms of this type are excessive urination as well
as thirst, vision changes, constant hunger, weight loss, and fatigue etc. Generally

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it is seen that Type I affecting the people at the age of 40. About 10 to 15% of diabetic patient lie in this type.

Type II: In type II we have considered those patients who are not dependent on insulin injection for survival. The advice of doctor for those patients is to follow the prescribed diet chart, regular exercise, Yoga etc. The majority of Diabetes Mellitus is of type II in the whole world [1]. The symptoms may be similar to those of type I diabetes, but are often less marked or absent. That’s why the patients are not able to diagnose the problem for several years until complications have already arisen. In past era it was seen that type II diabetes affecting only youngster but it has begun to occur in children also. Also it has absorbed and mention in WHO report type II is moving very rapidly in the direction of type I. Diabetes is progressively rising everywhere in the world, most strikingly in the world’s middle-income countries [5]. It is seen that type II diabetes is frequently highest among the poor people [34].

By the IDF (International Diabetes Federation) report the top five countries having highest diabetes patients are, China (109.6 million), India (69.2 million), USA (30.3 million), Brazil (14.2 million), Russian Federation (12.5 million). There are lots of countries having this problem in highest proportion of people for detail see [5]. In 1980 approx 108 million adults were living with diabetes but the report of WHO it is approx 422 million in 2014. According to global prevalence it is just double from 1980 that is, rising from 4.7% to 8.5% in the adult population [8]. It also reflects enhance in connected risk factors such as being overweight or obese [7]. Overweight or obese is stoutly linked to diabetes. According to report out of 3 adults more than 1 was overweight and out of 10, one is obese ([9], [30]). In comparison with male, females are more overweight or obese. Although the global intentional aim to stop the rise in obesity by 2025 ([3], [6]). Over the past decade, diabetes occurrence has risen faster in low- and middle-income countries than in high-income countries. According to the data about 1.5 million deaths main cause is diabetes in 2012. Additional 2.2 million deaths main cause is cardiovascular and other diseases, by increasing the optimal blood glucose in body. One more important thing to be notice that these 3.7 million deaths occur before the age of 70 years. As well as high blood glucose causes about 7% of deaths among men aged 20–69 and 8% among women aged 20–69.

1.1. Diabetes effect on health and wealth: Diabetes which may be I type or II type both are very dangerous to human life. It is caused many types of complications of varies parts of the body and can increase the overall risk. Few of them are heart attack, kidney failure, stroke, nerve damage, lower limb amputation and blindness. In pregnancy, poorly controlled diabetes increases the risk of fetal death and other complications (see [2],[24],[25],[28],[33] and [35]). Diabetes of type I or type II and their complications always bring substantial financial loss to the patient and his family. A huge amount of his wealth is spending in hospitals and outpatient care. One of the most important drug insulin for type I patient is also make an economic impact to the patient and nation as well. It also affects the national economies system, which may be by medical costs and loss of work and wages.

1.2. Preventing ways of diabetes: According to the current knowledge type I diabetes cannot be prevent. But there are lots of ways to prevent the type II diabetes, diabetes complications and premature death due to diabetes. To prevent diabetes we have to make polices and practice to avoid or control diabetes. The whole social system will tack it as prime member in this fight against deviates
The incidence of Diabetes Mellitus

Number of person having diabetics without complications

Number of person having diabetics with complications

Size of population of diabetics at time \( t \)

The probability of a person having diabetic and developing complications

Natural rate of mortality

Rate of complications are recovered

Rate of diabetic patients having complication and become severely disabled

Rate of mortality due to diabetic complications

**Table 1.** Table-1

which may be home, school or workplace. We should be very careful for good health, regular exercise, healthy food, blood pressure, and avoid smoking etc. The habits which we develop in childhood will be very helpful to avoiding diabetes of type II, so school, social atmosphere will be very helpful in that concern. No single policy can prevent it to happen. So we should work together against the diabetes. Government should make some guidelines and the whole society will follow the same then we can achieve our goal to prevent the diabetes. In that concern WHO and FAO make available the guidance for diet to prevent Diabetes of type II. [2]. WHO also provide guidelines on physical activity for different age groups [4]. Because they both (diet & physical activity) are more effective rather than medication.

2. **Mathematical model.** In this section we describe the mathematical model of the patient having diabetes and it’s complication which is given by Boutayeb’s et al in 2004 [14]. This model is capable to study the patient of diabetes with and without complication. In this model \( C(t) \) and \( B(t) \) are representing the patients with and without complications. In this module we consider that total number of diabetic at a time \( t \) is \( E(t) \), i.e. \( E(t) = B(t) + C(t) \). To better understand about the module we define the flow chart.

![Figure 1. Flow Chart](image)

In this module we use some parameters which are define bellow

In the flow chart we consider \( A(t) \) patients of diabetes and diagnosed for some time interval \( t \) and consider no complications in diagnosis. In above flow chart, \( \delta B(t) \)
represent the Persons have developed complications, $\lambda C(t)$ Persons whose complications are recover, $\nu C(t)$ represent to those patients who become strictly disabled and the their disabilities cannot be recover, $\mu C(t)$ represent to those patients who die from their complications , $C(t)$ The number of sufferers with complications. All other parameter which are essential are define in above flow chart.

To define the model, Boutayeb et al. uses definition of ordinary differential equation and gave the following the mathematical model

$$\frac{dB(t)}{dt} = -(\delta + \varepsilon)B(t) + \lambda C(t)$$

$$\frac{dC(t)}{dt} = A(t) - (\mu + \lambda + \varepsilon + \nu)C(t) + \delta B(t).$$

Let

$$E(t) = B(t) + C(t),$$

where $E(t)$ represent the size of those persons who having diabetics at time $t$. By using the relation (3), (1) and (2), becomes

$$\frac{dC(t)}{dt} = -(\delta + \varphi)C(t) + \delta E(t), \quad t > 0,$$

$$\frac{dE(t)}{dt} = A(t) - (v + \mu)C(t) - \varepsilon E(t), \quad t > 0,$$

where $\varphi = \mu + \lambda + \varepsilon + \nu$. With

$$C(0) = C_0 \text{ and } E(0) = E_0$$

for the detail see [14].

3. Fractional calculus and its application. Fractional calculus means fractional derivatives and fractional integrals which become a very important tool for researcher due to its lots of application in various dimension of the science and technology. Many researchers like Caputo, Miller, Ross, Kilbas, Srivastava, Trujillo, etc give lots of knowledge to know more about the fractional calculus and its application. In the path of knowledge they have contributed a lot (see [13],[16],[17],[18],[23],[26],[29] and[31]). Fractional calculus is also having a great importance in mathematical modeling to define the real world problems. Many mathematician and researchers are working in this field, like Atangana and Alkahtani [12], Yang, Machado, Cattani, Gao [36], Dubey, Atangana and Alkahtani [20], Debbouche and Nieto [19], Alkahtani and Goswami [10], Belgacem and Dubey [22], Yang, Zhang, Cattani, Xie, Rashidi, Zhou, Yang [37] Srivastava and Jain [32] Dubey and Goswami[21]etc., introduce various fractional module.

3.1. The Caputo and Fabrizio fractional order derivative.

Definition 3.1. [15] Let $h \in H^1(a_1,b_1)$, $b_1 > a_1$, $\beta \in [0, 1]$ then the new Caputo-Fabrizio derivative of fractional order $\alpha$

$$\begin{equation}
\mathcal{C}_0^F D_t^\beta (h(t)) = \frac{M(\beta)}{1-\beta} \int_a^t h'(x) e^{-\beta \frac{t-x}{1-\beta}} dx
\end{equation}$$
Definition 3.2. [15] The fractional integral of the function \( h(t) \) of order \( \beta \) \((0 < \beta < 1)\) is

\[
I_0^\beta \left[ h(t) \right] = \frac{2 (1 - \beta)}{2 - \beta} C(t) + \frac{2 \beta}{2 - \beta} \int_0^t h(s) \, ds, \quad t \geq 0,
\]

For more detail see ([11],[15] and [27]).

4. Fractional mathematical model. In this section we describe the fractional mathematical model of the patient diabetes and it’s complication.

\[
C^F D_t^\alpha (C(t)) = -(\delta + \varphi)C(t) + \delta E(t), \quad t > 0, \quad 0 < \alpha \leq 1,
\]

\[
C^F D_t^\beta (E(t)) = A(t) - (v + \mu)C(t) - \varepsilon E(t), \quad t > 0, \quad 0 < \beta \leq 1,
\]

with

\[
C(0) = C_0 \quad \text{and} \quad E(0) = E_0
\]

\[
E(t) = B(t) + C(t),
\]

where \( E(t) \) represent the size of population of those persons who having diabetics at time \( t \), and \( \varphi = \mu + \lambda + \varepsilon + v, \) these variables are define in Table 1. In this research work \( A(t) \) the incidence of Diabetes Mellitus are considered as a constant.

5. Existence of the coupled solutions. To define the existence of the coupled solution we use the Fixed-Point theorem. In this way first we transform the equations (10) and (11) in to integral equation as follows

\[
C(t) - C(0) = C^F I_0^\alpha \left[ -(\delta + \varphi)C(t) + \delta E(t) \right] \quad t > 0, \quad 0 < \alpha \leq 1,
\]

\[
E(t) - E(0) = C^F I_0^\beta \left[ A(t) - (v + \mu)C(s) - \varepsilon E(s) \right] \quad t > 0, \quad 0 < \beta \leq 1.
\]

Up on using the Nieto’s definition, we obtain

\[
C(t) = C(0) + \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \left\{ -(\delta + \varphi)C(t) + \delta E(t) \right\} + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \left[-(\delta + \varphi)C(s) + \delta E(s)\right] ds,
\]

and

\[
E(t) = E(0) + \frac{2(1-\beta)}{(2-\beta)M(\beta)} \left\{ A(t) - (v + \mu)C(t) - \varepsilon E(t) \right\} + \frac{2\beta}{(2-\beta)M(\beta)} \int_0^t \left[A(t) - (v + \mu)C(s) - \varepsilon E(s)\right] ds.
\]

Let us consider the following kernels:

\[
K_1(t, C) = -(\delta + \varphi)C(t) + \delta E(t), \quad (18)
\]

\[
K_2(t, E) = A(t) - (v + \mu)C(t) - \varepsilon E(t). \quad (19)
\]
Theorem 5.1. Show that $K_1$ and $K_2$ satisfy Lipschitz condition and contraction if $0 \leq L < 1$ and $0 \leq \Omega < 1$.

Proof. First we prove the condition for $K_1$. Assume $C$ and $C_1$ be two functions, then

$$
\|K_1(t, C) - K_1(t, C_1)\| = \|(-(\delta + \varphi)C(t) + \delta E(t)) - (-(\delta + \varphi)C_1(t) + \delta E(t))\|,
$$

(20)

using the Cauchy’s inequality, we attain

$$
\|K_1(t, C) - K_1(t, C_1)\| \leq \|(-\delta \varphi)\| \|C(t) - C_1(t)\|,
$$

(21)

or

$$
\|K_1(t, C) - K_1(t, C_1)\| \leq \|(C(t) - C_1(t))\|,
$$

(22)

where

$$
\|(-\delta \varphi)\| \leq L.
$$

(23)

Also similarly we can prove the condition for $K_2$

$$
\|K_2(t, E) - K_2(t, E_1)\| = \|(A - (v + \mu)C(t) - \varepsilon E(t)) - (A - (v + \mu)C(t) - \varepsilon E_1(t))\|,
$$

(24)

by the Cauchy’s inequality, we obtain

$$
\|K_2(t, E) - K_2(t, E_1)\| \leq \|\varepsilon\| \|(E(t) - E_1(t))\|,
$$

(25)

or

$$
\|K_2(t, E) - K_2(t, E_1)\| \leq \Omega \|(E(t) - E_1(t))\|,
$$

(26)

where

$$
\|\varepsilon\| \leq \Omega.
$$

(27)

Let the following recursive formula

$$
C_n(t) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}K_1(t, C_{n-1}) + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t K_1(s, C_{n-2}) ds,
$$

(28)

the difference of two consecutive terms is

$$
U_n(t) = C_n(t) - C_{n-1}(t) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}K_1(t, C_{n-1}) - \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}K_1(t, C_{n-2}) + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \{K_1(s, C_{n-1}) - K_1(s, C_{n-2})\} ds,
$$

on taking norm on both sides of equation (28) we get

$$
\|U_n(t)\| = \|C_n(t) - C_{n-1}(t)\| = \bigg\| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}K_1(t, C_{n-1}) - \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}K_1(t, C_{n-2}) + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \{K_1(s, C_{n-1}) - K_1(s, C_{n-2})\} ds \bigg\|,
$$

(29)

or

$$
\|U_n(t)\| = \|C_n(t) - C_{n-1}(t)\| \leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\|K_1(t, C_{n-1}) - K_1(t, C_{n-2})\| + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \{K_1(s, C_{n-1}) - K_1(s, C_{n-2})\} ds.
$$

(30)
On the same manner we can prove the result
\[\|V_n(t)\| = \|E_n(t) - E_{n-1}(t)\| \leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \|K_2(t, E_{n-1}) - K_2(t, E_{n-2})\| + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \|K_2(s, E_{n-1}) - K_2(s, E_{n-2})\| \, ds \] (31)

**Theorem 5.2.** The fractional model of diabetic and its complications have exact coupled solutions under the condition as
\[\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \omega + \frac{2\alpha}{(2-\alpha)M(\alpha)} \omega a < 1. \] (32)

**Proof.** We have already proved that, the above equations (30) and (31) are bounded. Also we have established that both kernels satisfy the Lipschiz condition, therefore we have obtained the following result using the recursive technique,
\[\|U_n(t)\| \leq \|C(0)\| \left[ \left\{ \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \omega \right\} + \left\{ \frac{2\alpha}{(2-\alpha)M(\alpha)} \omega \right\} \right]^n, \] (33)

Hence the above solution exists and is continuous. Now to prove that the above defined relation is a solution of equation (10), we consider
\[C(t) - C(0) = C_n(t) - P_n(t) \] (34)

thus, we have
\[\|P_n(t)\| = \left\| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} (K(t, C) - K(t, C_{n-1})) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t (K_1(s, C) - K(s, C_{n-1})) \, ds \right\|, \] (35)

on using this method recursively, we get
\[\|P_n(t)\| \leq \left( \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} + \frac{2\alpha}{(2-\alpha)M(\alpha)} t \right)^{n+1} \omega^{n+1} \kappa. \] (36)

then at \( t = a \)
\[\|P_n(t)\| \leq \left( \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} + \frac{2\alpha}{(2-\alpha)M(\alpha)} a \right)^{n+1} \omega^{n+1} \kappa. \] (37)

it is clear from (37) as \( n \to \infty \)
\[\|P_n(t)\| \to 0. \] (38)

On the same way we can prove another result. Hence from above result we can say that solution exists.

6. **Uniqueness of the coupled solutions.** In this section, we show that the solutions of equation (10) and (11) are unique. To prove the uniqueness, we have considered that there is an another solution for equation (10), then
\[C(t) - C_1(t) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \left\{ K_1(t, C) - K_1(t, C_1) \right\} + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \left\{ K_1(s, C) - K_1(s, C_1) \right\} \, ds, \] (39)
tacking norm on the both sides of equation (39),
\[ \|C(t) - C_1(t)\| \leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \left\{ \|K_1(t, C) - K_1(t, C_1)\| \right\} + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \|K_1(s, C) - K_1(s, C_1)\| \, ds. \] (40)

On using the Lipchitz condition, we get
\[ \|C(t) - C_1(t)\| < \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \omega \|C(t) - C_1(t)\| t \] (41)

or
\[ \|C(t) - C_1(t)\| \left( 1 - \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \omega \frac{2\alpha}{(2-\alpha)M(\alpha)} \omega t \right) \leq 0. \] (42)

If we have the condition
\[ \left( 1 - \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \omega \frac{2\alpha}{(2-\alpha)M(\alpha)} \omega t \right) > 0. \] (43)

then this is true, that
\[ \|C(t) - C_1(t)\| = 0. \] (44)

Hence
\[ \|C(t) - C_1(t)\| = 0. \] (45)

Similarly we can find
\[ \|E(t) - E_1(t)\| = 0. \] (46)

Hence on the behalf of equation (45) and (46) we can say that equation (10) and (11) have unique solution.

7. Stability analysis. In this section we describe the fractional mathematical model of the patient diabetes and it’s complication defined in equation (10) and (11). As we talk about the fractional model defined in equation (10) and (11). In these equation $C(t)$ is the number of patient of diabetics with complications and $E(t)$ represent the population of those patient who having diabetics at time $t$ (of two forms X and Y) at time $t$ are given by the nonlinear difference equations. Also If we discuss about their solution, we want the positive solutions. Hence the Jacobian matrix $\Delta$ of equation (10) and (11) at any point is
\[ \Delta = \begin{bmatrix} - (\delta + \varphi) & \delta \\ - (\nu + \mu) & - \varepsilon \end{bmatrix} \] (47)
so we have the characteristic roots of the above defined matrix $\Delta$, are
\[ \lambda_1 = \frac{1}{2} \left( \sqrt{(\delta + \varphi + \varepsilon)^2 - 4(\delta \nu + \delta \varphi + \delta \varepsilon + \varphi \varepsilon)} - (\delta + \varphi + \varepsilon) \right) \] (48)
and
\[ \lambda_2 = \frac{1}{2} \left( -\sqrt{(\delta + \varphi + \varepsilon)^2 - 4(\delta \nu + \delta \varphi + \delta \varepsilon + \varphi \varepsilon)} - (\delta + \varphi + \varepsilon) \right) \] (49)
also the trace of $\Delta = - (\delta + \varphi + \varepsilon)$.

As we know that the system is stable if trace of $\Delta < 0$, or
\[ (\delta + \varphi + \varepsilon) > 0. \] (50)

As we have $\delta, \varphi,$ and $\varepsilon$ are positive constants hence (50) always exist. So it is clear from (50) that the system defined in (10) and (11) is always stable.
8. **Numerical outcome.** In this section, we discuss some numerical simulations of our solution. We solve the system of equations defined in (10-11) for some particular sample data and plot some curves for different value of with respect to time $t$. To solve these equations we use HPM which is a very efficient approach. To draw the curves we consider the number of incidence Diabetes Mellitus $A=6000000$, initially the number of person having diabetics with complications $C_0 = 47000000$, initially the size of population of diabetics $E_0 = 61100000$. Also for the numerical results, we considered the define values of respective parameters $\varepsilon = 0.02$, $\mu = 0.05$, $\lambda = 0.08$, $\nu = 0.05$, and $\delta = 0.66$. In the Figure 2 we define the behavior of $C$: number of persons having diabetics with complications with respect to time for different values of $\alpha$ and $\beta$. It is observe that with respect to time the graph of $\alpha = \beta = 1$, decreases rapidly, but as in current scenario it is not to be seen the persons having diabetics with complications rapidly decreased. By the Figure 2 it has been observe that the fractional order is better to describe the module. This model gave free hand to scientist, as medical science resists the diabetic complications and decreases the number of patients with complications the order of derivative will change accordingly. On the same manner the size of population of diabetics in the world increases rapidly. To describe that population our model is more compatible. It is observe that after some time period the graph of $\alpha = \beta = 1$, falls its values of persons having diabetics speedily but as in present scenario it is not to be seen. By the Figure 3 it has been observe that the fractional order is better to describe the module, and gave free hand to adjust and control accordingly

![Figure 2](image.png)

**Figure 2.** Represents for the behavior of the solution $C(t)$, with respect to $t$ for different values of other perimeter defined above.

9. **Conclusion.** In this article, we have discussed the very serious problem of the whole world. We have defined the fractional model of diabetes and its complications. We have study it with Caputo-Fabrizio derivative and extends to the scope of fractional calculus. We use the fixed point theorem to find the existence and uniqueness of the coupled solution. We also use perturbation technique to derive the solution for the problem. To characterize the outcome of fractional order we plot some graphs. By these it is clear that when the order of differential equations tends to one the Caputo-Fabrizio derivative demonstrates more remarkable behavior. Thus, it is clear that Caputo-Fabrizio derivative become very useful in modeling the real word problems.
Figure 3. Represents for the behavior of the solution E(t), with respect to t for different values of other perimeter defined above.

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