A Firefly Algorithm-Based Spectral Fitting Technique for Wavelength Modulation Spectroscopy Systems

Tingting Zhang, Yongjie Sun, Pengpeng Wang, Yufeng Qiu, Chenxi Wang, Xiaohui Du, Shaokai Li, Haixu Liu, Tongwei Chu, and Cunguang Zhu

Abstract—This article proposes a novel calibration-free wavelength modulation spectroscopy (WMS) spectral fitting technique based on the firefly algorithm (FA). The technique simulates the behavior of information interaction between fireflies to accurately retrieve gas concentrations and laser parameters. Compared to the spectral fitting technique based on the classical Levenberg–Marquardt (LM) algorithm, the proposed technique exhibits weak dependence on the precharacterization of laser tuning parameters during gas concentration retrieval. We select the P(13) absorption line of C$_2$H$_2$ at 1532.82 nm as the target spectra and compare the performance of two optimization methods (LM and firefly) on gas concentration and laser tuning parameters retrieval by simulation. The simulation results demonstrate that the FA-based spectral fitting technique exhibits superior performance in terms of both convergence speed and fitting accuracy for multiparameter models without exact characterization.

Index Terms—Calibration-free, firefly algorithm (FA), spectral fitting, wavelength modulation spectroscopy (WMS).

I. INTRODUCTION

With the rapid advancement in infrared diode laser technology, tunable diode laser absorption spectroscopy (TDLAS) has gained widespread usage in various domains, including greenhouse gas and air pollution monitoring, combustion monitoring, industrial process control, and related areas [1], [2], [3], [4], [5], [6]. Wavelength modulation spectroscopy (WMS), a branch of TDLAS, has gained significant popularity for conducting in situ measurements in challenging environments characterized by strong turbulence, high temperature, or high pressure due to its high sensitivity and robustness to background noise [7], [8], [9], [10], [11], [12], [13], [14], [15], [16].

Although the technique of WMS presents numerous advantages, the accurate quantitative measurement of WMS signals remains challenging owing to their inherent complexity. The WMS measurement is performed using the harmonics of the gas absorption spectrum, which depends on the laser intensity and the line shape. Laser intensity is susceptible to interference from beam steering, scattered particles, and window fouling. Moreover, the line shape strongly depends on temperature (Doppler broadening) and pressure (Collision broadening). The traditional approach chosen by the researchers was to perform in situ calibration with a known gas mixture [17]. However, this is impractical when gas conditions of interest are poorly understood or highly transient.

Consequently, numerous scholars have devised calibration-free methods for WMS. Li et al. [18] proposed an analytical model that simulates WMS signals as a function...
of the well-characterized laser parameters and absorption spectra. Rieker et al. [19] employed the second harmonic (2f) normalized by the first harmonic (1f) to infer gas parameters, which eliminated the effect of light intensity fluctuations. Sun et al. [20] devised a calibration-free approach by leveraging the measured non-absorption laser intensity, thereby obviating the necessity to construct an analytical model for characterizing the temporal laser intensity. Based on Sun’s work, Goldenstein et al. [21] proposed a novel approach to achieve precise calibration-free measurements of gas properties using WMS. This strategy effectively eliminates the requirement for prior knowledge regarding the transition line shape parameters. In their work, Yang et al. [22] introduced a gas concentration retrieval method employing the first harmonic phase angle (θ1f) technique, which displays insensitivity to variations in laser intensity and demodulation phase. Building upon this concept, a second harmonic phase angle method (θ2f) was later developed based on WMS [23]. This innovative approach allows for trace gas detection with the advantages of background-free analysis and immunity to fluctuations in light intensity. In the realm of calibration-free WMS method development over the past few decades, the Levenberg–Marquardt (LM) algorithm has received significant traction as a means to address nonlinear least-squares problems in the context of spectral fitting for gas concentration retrieval. For instance, Yang and Zhang [24] used the LM algorithm to fit the 2f spectrum of known concentration methane to measured spectrum to accurately measure low concentrations of methane. Li et al. [25] developed a single continuous wave room-temperature quantum cascade laser sensor that used the LM algorithm for data processing to achieve simultaneous concentration measurements of water vapor, nitrous oxide, and atmospheric carbon monoxide. Cui et al. [30] developed a ppm-level CO sensor system for a sulfur hexafluoride decomposition analysis in power systems by fitting the 2f spectral information using the LM algorithm. However, the LM algorithm is highly sensitive to initial values and requires appropriate initial values to obtain accurate optimization results, thus requiring accurate precharacterization support. When multiple free parameters are included in the model, a matrix of high-order coefficients is generated, leading to high computational dimensionality, so its optimization results often do not match expectations.

In this article, a calibration-free WMS spectral fitting technique based on the firefly algorithm (FA) is proposed, which simulates the information interaction behavior between fireflies to retrieve laser parameters and gas concentration. In contrast to the spectral fitting technique, which is based on the classical LM algorithm, the use of this technique to retrieve gas concentrations weakens the dependence of gas concentration retrieval on precharacterization. The simulation results demonstrate that the FA-based spectral fitting technique outperforms in both fitting accuracy and convergence speed, particularly in cases involving multiparameter models where exact characterization is not available.

II. THEORY AND METHODOLOGY

A. Theory

In a scanned WMS-2f/1f system, the laser is usually controlled by a combination of low-frequency scanning and high-frequency modulated current. Both the frequency and intensity of the outgoing laser will change, with a phase delay between them, which can be expressed as

\[ I_0(t) = T_0(v_c)[1 + i_0 \cos(\omega t + \varphi_1) + i_2 \cos(2\omega t + \varphi_2)] \]

\[ v(t) = v_c + \Delta v \cdot \cos(\omega t) \]

where \( T_0(v_c) \) denotes the average laser intensity at the center laser frequency \( v_c \), \( i_0 \), and \( i_2 \) are the linear and nonlinear intensity modulation (IM) depth (normalized by the average laser intensity), modulation angular frequency \( \omega = 2\pi f_m \), \( f_m \) is laser modulation frequency, \( \varphi_1 \) and \( \varphi_2 \) are the phase shift between frequency modulation (FM) and linear and nonlinear IM, respectively, and \( \Delta v \) represents the modulation depth of the laser frequency.

According to the Beer-Lambert law, the transmitted laser intensity at frequency \( v \) through a gas cell filled with an absorbing gas can be expressed as follows:

\[ I_1(t) = I_0(t) \cdot \exp[-\alpha(v(t))] \]

where \( I_0(t) \) is incident laser intensity and \( \alpha(v(t)) \) is the spectral absorbance

\[ \alpha(v(t)) = I_0(t) \cdot [1 - PS(T)CLg(v, v_0)] \]

where \( P \) denotes the total pressure of the mixed gas species, \( S(T) \) is the line strength of the absorption spectrum at temperature \( T \), \( C \) represents the concentration of the gas under measurement, \( L \) denotes the effective absorption path length, \( g(v, v_0) \) is the line shape function at optical frequency \( v \) of the absorption feature, and \( v_0 \) is the line-center frequency of the absorption spectrum.

At atmospheric pressure, \( g(v, v_0) \) can be given in terms of a Lorentzian function

\[ g(v, v_0) = \frac{2}{\pi} \frac{1}{\Delta v_c \left[ x + m \cos(\omega t) \right]^2} \]

\[ x = \frac{2v_c - v_0}{\Delta v_c} \]

\[ m = \frac{2\Delta v}{\Delta v_c} \]

where \( \Delta v_c \), \( x \), and \( m \) denote the half-maximum full width, the normalized frequency, and the modulation index, respectively.

When the laser is modulated by a sinusoidal injection current, the expression \( \exp[-\alpha(v(t))] \) can be represented through expansion in a Fourier cosine series as follows:

\[ \exp[-\alpha(v(t))] = \sum_{k=0}^{\infty} H_i(v_c, \Delta v) \cdot \cos(i\omega t) \]

where \( H_i \) stands for the \( i \)th Fourier coefficient, it could be expressed as follows:

\[ H_i(v_c, \Delta v) = \begin{cases} \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp[-\alpha(v(t))] d\theta, & i = 0 \\ \frac{1}{\pi} \int_{-\pi}^{\pi} \exp[-\alpha(v(t))] \cdot \cos(k\theta) d\theta, & i = k. \end{cases} \]
The transmitted intensity $I_t(t)$ is multiplied by the quadrature reference signal $\cos(wt)$ and $\cos(wt + \pi/2)$ in the lock-in amplifier (LIA), and low-pass filtered to obtain the $X$ and $Y$ components of the $1f$ signal as given below

\[
X_{1f} = \frac{I_0}{2} \left[ H_1 + i_0 \left( H_0 + \frac{H_2}{2} \right) \cos \varphi_1 + i_2 \left( H_1 + H_3 \right) \cos \varphi_2 \right]
\]

(10)

\[
Y_{1f} = \frac{I_0}{2} \left[ i_0 \left( H_0 - \frac{H_2}{2} \right) \sin \varphi_1 + i_2 \left( H_1 - H_3 \right) \sin \varphi_2 \right]
\]

(11)

the $X$ and $Y$ components of the $2f$ signal are given by

\[
X_{2f} = \frac{I_0}{2} \left[ H_2 + \frac{i_0}{2} (H_1 + H_3) \cos \varphi_1 + i_2 H_0 \cos \varphi_2 \right]
\]

(12)

\[
Y_{2f} = \frac{I_0}{2} \left[ \frac{i_0}{2} (H_1 - H_3) \sin \varphi_1 + i_2 H_0 \sin \varphi_2 \right].
\]

(13)

Then the $1f$ normalized $2f$ $S_{2f/1f}$ are defined as

\[
S_{2f/1f} = \sqrt{\frac{X_{2f}^2}{X_{1f}^2} + \frac{Y_{2f}^2}{Y_{1f}^2}}
\]

(14)

which states that $S_{2f/1f}$ signal is a function of multiple parameters, including gas concentration $C$, gas absorption line shape $g(v, v_0)$, Fourier expansion coefficient $H_k$, the linear and nonlinear IM depth $i_0$ and $i_2$, and the phases shift $\varphi_1$ and $\varphi_2$ between FM and linear and nonlinear IM, respectively. Therefore, both gas concentrations and laser parameters are then inferred from the spectral fitting of the $S_{2f/1f}$ signal.

B. Objective Function

In spectral line fitting, the objective function is used to define the optimization problem and evaluate the fitting effect of the model. The optimal combination of parameters can be found by minimizing the objective function, which is generally defined as

\[
\text{Given } F : R^D \rightarrow R \\
\text{Find } \min(F(\beta_i))
\]

\[
F(\beta_i) = \sum_{k=1}^{N} z_k^2
\]

\[
z_k = y_k - f_i(x_k, \beta_i)
\]

(15)

where $R^D$ denotes the D-dimensional solution space, the value of $D$ is contingent upon the number of free parameters, $y$ is the measured spectrum, $f_i(x, \beta_i)$ is the simulated spectrum, $z_k$ is the residual between the $k$th sampling points of measured and simulated spectrum, and $x_k$ is the normalized frequency corresponding to the $k$th sampling point and the objective function $F(\beta_i)$ is obtained by accumulating the squares of all the residuals. $\beta$ is the potential solution expressed as a vector form of free parameters. The expression of $\beta$ is related to the number of free parameters, and the following equations give the representation when there are one and six free parameters:

\[
\beta_1^1 = [C]
\]

(16)

\[
\beta_2^1 = [m, C, i_0, i_2, \varphi_1, \varphi_2].
\]

(17)

The use of $\beta_1^1$ implies the existence of a single concentration $C$ of free parameters within the algorithm, while the use of $\beta_2^1$ indicates that the free parameters contain $m, C, i_0, i_2, \varphi_1$, and $\varphi_2$.

C. FA-Based Calibration-Free WMS Spectral Fitting Technique

In this text, we have developed an FA-based spectral fitting technique, which emulates the interaction behavior among fireflies for calibration-free measurements of gas concentration and laser parameters. The pseudocode for the FA optimal procedure is shown in Fig. 1.

Firefly individuals search for potential partners within their visual range based on their luminous behaviors and actively move toward fireflies, emitting brighter flashes (indicating advantageous positioning). The luminosity of the firefly is related to the objective function. When the termination condition is satisfied, the highest luminosity value of the firefly is the optimal solution for the objective function. The flowchart of the FA-based calibration-free WMS spectral fitting technique is illustrated in Fig. 2. The specific processes are as follows.

Step 1 (Parameter Initialization): Generate $W$ vectors $\beta_i$ for the fitting procedure by randomly assigning them within a predefined range. $\beta_i$ represents the initial position of the $i$th firefly, $i = 1, 2, \ldots, W$.

Step 2 (Acquisition of Simulated Spectrum): The simulated spectrum $f_i(x, \beta_i)$ is acquired by substituting the initialized parameters $\beta_i$ into (14).

Step 3 (Objective Function Updating): Update the value of the objective function $F$. The objective function is used to evaluate the luminosity of each firefly, which is negatively correlated with each other.

Step 4 (Judgment): If the $F$, [i.e., the sum of squares of the residuals between the measured spectrum $S_{2f/1f}$ and

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the simulated spectrum $S^{2/11}$ generated by substituting the parameter $\beta_i$ into (14) meets the accuracy requirement or the number of iterations reaches the maximum number of iterations, or the convergence time exceeds the set maximum convergence time, then skip to Step 6, otherwise, perform Step 5.

**Step 5 (Firefly Movement):** Fireflies utilize position and luminosity updates in order to accomplish optimization, thereby determining the spatial search trajectory of fireflies. The firefly $i$ with less luminosity value is attracted to and approached by the brighter firefly $j$, and the attractiveness function of the firefly is established by

$$h(r) = h_0e^{-\gamma r^2}$$

where $r_{ij}$ is the distance from $\beta_i$ and $\beta_j$, $h_0$ is the firefly attractiveness value at $r = 0$, and $\gamma$ is the light absorption coefficient, generally set as a constant, $r_{ij}$ can be defined as

$$r_{ij} = \|\beta_i - \beta_j\| = \sum_{d=1}^{D} (\beta_{i,d} - \beta_{j,d})^2$$

where $D$ indicates the $D$-dimensional search space in firefly optimization, which is determined by the free parameters. The firefly optimization needs to be searched under the dimension of every free parameter. Firefly $i$ will be moving toward firefly $j$ and the position of firefly $i$ is updated by the following equation:

$$\beta_{i,d}^{t+1} = \beta_{i,d}^{t} + h_0e^{-\gamma r^2} (\beta_{i,d}^{t} - \beta_{j,d}^{t}) + \lambda [\text{rand} - \frac{1}{2}]$$

where $t$ is the iteration number, $\lambda \in [0, 1]$ is the step factor, and $\text{rand} \in [0, 1]$ is a random value.

The brightest firefly moves randomly, which is defined as

$$\beta_{i,d}^{t+1} = \beta_{i,d}^{t} + \lambda \text{rand}()$$

After updates of individual firefly luminance and positions in steps 3 and 5, the firefly population will progressively approach the area with the greatest luminance.

**Step 6 (Termination Condition):** If the optimization termination condition is satisfied, the optimal predicted values of the free parameters, as well as the laser tuning parameters and the gas concentration $C$, can be obtained.

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**TABLE I**

| Parameters | Value |
|------------|-------|
| Light absorption coefficient ($\gamma$) | 1 |
| firefly attractiveness ($h_0$) | 2 |
| Step factor ($\lambda$) | 0.3 |
| Maximum number of iterations | 20 |
| Number of firefly | 40 |

**TABLE II**

| Parameters | Value |
|------------|-------|
| isotopologue | $^{12}$C$_2$H$_2$ |
| $\nu_i$ | 6523.8792 cm$^{-1}$ |
| $\Delta\nu/2$ | 0.0777 cm$^{-1}$ |
| $S$ | 1.035 |
| $T$ | $10^{15}$ cm/mol |
| $P$ | 290 K |
| $L$ | 1 atm |

**TABLE III**

| Parameters | Initial values |
|------------|---------------|
| $m$ (cm$^{-1}$) | 1.4 |
| $c$ (ppmv) | 390.6 |
| $k_0$ | 0.15 |
| $k_2$ | 0.003 |
| $\sigma_1$ ($\mu$ rad) | 0.6 |
| $\sigma_2$ ($\mu$ rad) | 0.5 |

**III. SIMULATION**

The effectiveness of the FA-based WMS spectral fitting technique is verified in the subsequent analysis. To mitigate potential characterization errors caused by metrology instruments, which could impact the evaluation of algorithm performance, we opted for conducting simulation validation on the MATLAB R2019b platform. By comparing the results obtained from fitting the same spectra using both the LM algorithm and the FA, we assessed the respective effects. The specific constant parameters used in the FA optimization are outlined in Table I.

The P(13) absorption line of C$_2$H$_2$ at 1532.82 nm was designated as the target spectra for subsequent simulations. The spectral parameters are listed in Table II. The actual measured spectrum that was supposed to be obtained during the experimental procedure were substituted with a simulated dataset derived from the calculation by employing (14), which can be formulated as follows:

$$[X_{%4000}, Y_{%4000}] = [(x_1, y_1), (x_2, y_2), \ldots, (x_{%4000}, y_{%4000})].$$

The virtual spectra described above were fit utilizing the LM algorithm and FA for comparison. The initial values of the LM algorithm and FA were set as shown in Tables III and IV. Considering the dependence of the LM algorithm on the
TABLE IV
INITIAL RANGE OF PARAMETERS IN THE FA

| Free parameters | Initial ranges |
|-----------------|----------------|
| $m$ [cm$^{-1}$]  | 1.0-2.0        |
| $C$ [ppmv]      | 195.3-390.6    |
| $i_0$           | 0.08-0.55      |
| $i_2$           | 0-0.0007       |
| $\phi_1$ [rad]  | 0-1            |
| $\phi_2$ [rad]  | 0-1            |

Fig. 3. Fitting of the WMS-2/1f spectral line at the free parameter vector $\beta^1 = [C]$ using (a) LM algorithm with a convergence time of 150 s and (b) FA with a convergence time of 30 s.

We presented a comparison of the fitting of the LM and FA algorithms in both cases. In the first case, the only free parameter was the gas concentration $C$, while all other parameters were assumed to be known, and the results are presented in Fig. 3. For an accurately precharacterized multi-parameter model, it is clear that the LM algorithm provides a better fit than the FA. Fig. 4 displays the fitting results for the second case, i.e., more parameters are set as free parameters, including optical FM index $m$, gas concentration $C$, the linear and nonlinear IM depth $i_0$ and $i_2$, and the phases shift $\phi_1$ and $\phi_2$ between FM and linear and nonlinear IM, respectively. It is clear that the residual accuracy of the FA is two orders of magnitude higher than that of the LM algorithm ($10^{-3}$ to $10^{-5}$), even though the initial values of the LM algorithm are closer to the expected values. It is worthwhile to note that the convergence time of the FA is set to only one-fifth of that of the LM algorithm. In the second case, we treat the laser parameters as free parameters, which reduces the workload associated with the precharacterization of the laser parameters and avoids any potential errors due to failure of the precharacterization. Compared to the spectral fitting technique based on the classical LM algorithm, the FA-based spectral fitting technique weakens the dependence on precharacterization.

Tables V and VI show the final predicted values of the free parameters in Fig. 4. It is apparent that the relative errors of free parameters predicted by the LM algorithm all exceed 5%. However, the effect of the FA fit is significantly better.

Fig. 4. Fitting of the WMS-2/1f spectral line at the free parameter vector $\beta^2 = [m, C, i_0, i_2, \phi_1, \phi_2]$ using (a) LM algorithm with a convergence time of 150 s and (b) FA with a convergence time of 30 s.

TABLE V
PREDICTED VALUES OF THE LM ALGORITHM FOR FREE PARAMETERS WITH A CONVERGENCE TIME OF 150 s

| Free parameters | Expected values | Predicted values | Relative Errors |
|-----------------|-----------------|-----------------|-----------------|
| $m$ [cm$^{-1}$] | 1.5000          | 1.5904          | 6.03%           |
| $C$ [ppmv]     | 355.2           | 408.6           | 15.03%          |
| $i_0$          | 0.1500          | 0.1386          | 7.60%           |
| $i_2$          | 0.0030          | 0.0042          | 6.67%           |
| $\phi_1$ [rad] | 0.6000          | 0.6309          | 5.15%           |
| $\phi_2$ [rad] | 0.5000          | 0.4747          | 5.06%           |

TABLE VI
PREDICTED VALUES OF THE FA FOR FREE PARAMETERS WITH A CONVERGENCE TIME OF 150 s

| Free parameters | Expected values | Predicted values | Relative Errors |
|-----------------|-----------------|-----------------|-----------------|
| $m$ [cm$^{-1}$] | 1.5000          | 1.5026          | 0.17%           |
| $C$ [ppmv]     | 355.2           | 354.8           | 0.11%           |
| $i_0$          | 0.1500          | 0.1526          | 1.73%           |
| $i_2$          | 0.0030          | 0.0029          | 3.33%           |
| $\phi_1$ [rad] | 0.6000          | 0.6045          | 0.75%           |
| $\phi_2$ [rad] | 0.5000          | 0.5017          | 0.34%           |
Fig. 5. Convergence analysis of free parameters for the FA optimization: (a) modulation index \( m \); (b) concentration \( C \); (c) linear IM depth \( i_0 \); (d) nonlinear IM depth \( i_2 \); (e) phases shift \( \phi_1 \); and (f) phases shift \( \phi_2 \).

Fig. 6. Comparison of the fitting effect of the LM algorithm and FA for the same number of iterations.

TABLE VII

| Free parameters | Expected values | Predicted values | Relative Errors |
|-----------------|-----------------|-----------------|-----------------|
| \( m \) [cm\(^{-1}\)] | 1.5000          | 1.8904          | 26.03%          |
| \( C \) [ppmv]  | 355.2           | 508.7           | 43.2%           |
| \( i_0 \)       | 0.1500          | 0.1386          | 7.60%           |
| \( i_2 \)       | 0.0030          | 0.0039          | 30.0%           |
| \( \phi_1 [\pi \text{ rad}] \) | 0.6000 | 0.8301 | 38.3% |
| \( \phi_2 [\pi \text{ rad}] \) | 0.5000 | 0.6747 | 34.9% |

Fig. 7. Fitting effect of FA and LM on virtual data containing noise.

TABLE VIII

| Free parameters | Expected values | Predicted values | Relative Errors |
|-----------------|-----------------|-----------------|-----------------|
| \( m \) [cm\(^{-1}\)] | 1.5000          | 1.4326          | 4.49%           |
| \( C \) [ppmv]  | 355.2           | 415.8           | 17.1%           |
| \( i_0 \)       | 0.1500          | 0.1426          | 4.93%           |
| \( i_2 \)       | 0.0030          | 0.0033          | 10.0%           |
| \( \phi_1 [\pi \text{ rad}] \) | 0.6000 | 0.6535 | 8.91% |
| \( \phi_2 [\pi \text{ rad}] \) | 0.5000 | 0.5312 | 6.24% |

Compared to the LM algorithm. For instance, the linear and nonlinear IM depth \( i_0 \) and \( i_2 \) are consistent with predicted values within 5%; the rest of the parameters are consistent with predicted values within 1%.

The fitting residuals are directly reflected by the objective function and are closely related to the prediction errors of the free parameters. There is a positive correlation between the prediction errors of the free parameters and the fit residuals. We analyzed the convergence of the six free parameters of the FA throughout the iterative fitting process, as depicted in Fig. 5. Within a convergence time of 30 s, all six free parameters gradually converge to the expected values with increasing number of iterations.

We further extended the number of iterations of the LM algorithm to match the FA (the FA completed 20 iterations in 30 s). As shown in Fig. 6, after 20 iterations, the fitting residuals of the LM algorithm are significantly reduced, but it should be noted that the convergence time has been extended to 782 s, which is unsatisfactory in terms of speed. Noise is extremely harmful in real scenarios, as it is always mixed with useful signals and thus affects the accurate detection of signals, especially weak signals. Noise that often occurs in WMS is the shot noise and thermal noise in optoelectronic detection systems, both of which are white noise. For the possible noise in WMS, we added the same type of white noise to the virtual data. The fitting results of FA-based and LM-based spectral fitting techniques are shown in Fig. 7. Tables VII and VIII demonstrate their free parameter predictions. It is worth noting that even when subjected to Gaussian white noise, the FA technique demonstrates satisfactory performance for virtual spectral fitting compared to LM.

In conclusion, the FA algorithm surpasses the LM algorithm in terms of convergence time and error when applied to models with multiple free parameters. Adequate optimization of these parameters eliminates the reliance on precharacterization and further aids in mitigating measurement errors arising from precharacterization failures.

Finally, it is worth noting that the LIA used to extract the harmonic signals contains key components such as integrators and filters, and the resulting transfer functions complicate the
analytical formulas followed for the actual harmonic signals.
In the next step of the study, it is necessary to explore an ana-
litical formula that is more compatible with the experimental
data.

IV. CONCLUSION
In this article, a novel FA-based novel calibration-free
WMS spectral fitting technique is developed to retrieve gas
concentration and laser tuning parameters by simulating the
information interaction behavior between fireflies. Compared
to the spectral fitting technique using the classical LM
algorithm, the proposed technique demonstrates a less sig-
nificant reliance on the precharacterization of laser tuning
parameters for gas concentration retrieval. Our simulation
results demonstrate that the LM algorithm predicts relative
errors of free parameters that exceed 5%, while the FA predicts
errors mostly within 1%. These findings conclusively validate
the superiority of the FA over the LM algorithm in terms of
convergence time and accuracy for models involving multiple
free parameters. This technique weakens the dependence of
gas concentration retrieval on precharacterization. The influ-
ence of precharacterization failure caused by temperature
change and laser aging on the measurement is effectively
avoided.

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Dec. 2022.
Tingting Zhang was born in Dezhou, Shandong, China, in 1997. She is currently pursuing the M.S. degree with the School of Physics Science and Information Technology, Liaocheng University, Liaocheng, China. Her current research interests include optical gas sensing devices, optical fiber sensor fabrication, and intelligent algorithms.

Yongjie Sun was born in Zibo, Shandong, China, in 1997. He received the M.S. degree from the University of Jinan, Jinan, China, in 2023. His current research interests include optical fiber sensor fabrication and intelligent algorithms.

Pengpeng Wang was born in Jinan, Shandong, China, in 1985. She received the Ph.D. degree from Shandong University, Jinan, in 2014. She is currently with the School of Physics Science and Information Technology, Liaocheng University, Liaocheng, China. Her current research interests include optical fiber sensors and fiber lasers.

Yufeng Qiu was born in Zhangjiakou, Hebei, China, in 1996. He is currently pursuing the M.S. degree with the School of Physics Science and Information Technology, Liaocheng University, Liaocheng, China. His current research interests include optical fiber sensors and fiber lasers.

Chenxi Wang was born in Liaocheng, Shandong, China, in 1999. She is currently pursuing the M.S. degree with the School of Physics Science and Information Technology, Liaocheng University, Liaocheng, China. Her current research interests include optical fiber sensor fabrication.

Xiaohui Du was born in Weifang, Shandong, China, in 1997. She is currently pursuing the M.S. degree with the School of Physics Science and Information Technology, Liaocheng University, Liaocheng, China. Her current research interests include optical fiber sensors and fiber lasers.

Shaokai Li was born in Jinan, Shandong, China, in 1999. He is currently pursuing the M.S. degree with the School of Physics Science and Information Technology, Liaocheng University, Liaocheng, China. His current research interests include optical fiber sensors.

Haixu Liu was born in Weifang, Shandong, China, in 1999. He is currently pursuing the M.S. degree in electronic engineering with Duke University, Durham, NC, USA. His current research interests include optical gas sensing devices, optical sensor fabrication, and engineering applications.

Tongwei Chu was born in Liaocheng, Shandong, China, in 1998. He is currently pursuing the Ph.D. degree in electronic engineering with Duke University, Durham, NC, USA. His current research interests include optical gas sensing devices, optical sensor fabrication, and engineering applications.

Cunguang Zhu was born in Liaocheng, Shandong, China, in 1985. He received the Ph.D. degree in optoelectronic engineering from Shandong University, Jinan, China, in 2015. He is currently with the School of Physics Science and Information Technology, Liaocheng University, Liaocheng. His current research interests include optical gas sensing devices, optical fiber sensor fabrication, and engineering applications.