Throughput Maximization for Multi-Cluster WPCN: User Cooperation, Non-linearity, and Channel Uncertainty

Omid Rezaei, Maryam Masjedi, and Mohammad Mahdi Naghsh

Abstract

In this paper, we study a general multi-cluster wireless powered communication network (WPCN) with user cooperation under harvest-then-transmit (HTT) protocol where the hybrid access point (HAP) as well as each user is equipped with multiple antennas. In the downlink phase of HTT, the HAP employs beamforming to transfer energy to the users. In the uplink phase, users in each cluster transmit their signals to the HAP and to their cluster heads (CHs). Afterward, the CHs first relay the signals of their cluster users and then transmit their own information signals to the HAP. The aim is to design the energy beamforming (EB) matrix, transmit covariance matrices of the users and time allocations among energy transfer and cooperation phases in order to optimize the max-min and sum throughputs of the network. The corresponding maximization problems are non-convex and NP-hard in general. We devise iterative algorithm based on alternating optimization (AO) and then the minorization-maximization (MM) technique is used to deal with the non-convex sub-problems with respect to (w.r.t.) the EB and covariance matrices in each iteration. We recast the resulting sub-problems as a convex second order cone programming (SOCP) and quadratic constraint quadratic programming (QCQP) for the max-min and sum throughput maximization problems, respectively. We also consider imperfect channel state information (CSI) at the HAP and CHs and non-linearity in energy harvesting (EH) circuits. Numerical examples show that the proposed cooperative method can effectively improve the achievable throughput in the multi-cluster wireless powered communication under various setups.

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Index Terms

Cluster-based communication, minorization-maximization, user cooperation, user fairness, wireless powered communication.

I. INTRODUCTION

The lifetime of the conventional energy-constrained wireless networks is limited by the operational time of wireless devices batteries. The network lifetime can be enhanced by employing recent advances in wireless power transfer (WPT) technology in which the wireless devices are powered by continuous (uninterrupted) energy from dedicated wireless power transmitters [1]. One of the important applications of WPT is wireless powered communication network (WPCN), where the wireless devices are powered by the wireless energy in the downlink in order to transfer information in the uplink. In a WPCN, the energy transmitter and the information receiver are placed either at separate locations or at the same location as a hybrid access point (HAP) [2]. The authors of [3] have proposed a harvest-then-transmit (HTT) protocol for a WPCN consisting of $K$ single-antenna users and a single-antenna HAP. They have shown that in a WPCN which employs a HAP, there is an unfair “doubly near-far” phenomenon causing the users far from the HAP to harvest less energy in the downlink and to require more energy to transmit their information in the uplink. To overcome the doubly near-far problem in WPCNs, several cooperative methods have been proposed in the literature.

A. Related Works

In [4]–[9], the user cooperation is considered for a three-node WPCN in which all nodes are equipped with single-antenna. In the model proposed in [4] and [5], the user nearer to the HAP transmits the signal of the distant user along with its own signal to the HAP. The sum throughput of the network is maximized by optimal design of time and power allocations. In [6], the max-min throughput fairness is considered for a single-pair WPCN, where both users first harvest the energy transmitted from an energy source and then help each other to send their information to a destination node.

In [7] and [8], outage performance was studied for a single-user WPCN, where the user sent its information to the HAP with help of a dedicated relay node. The authors of [9] consider a
single-pair WPCN with user cooperation, where the cooperation is developed in both downlink and uplink phases.

Several references employ multiple antennas energy transmitter and design the energy beam-former (EB) to focus the transmitted energy toward the specific receivers and maximize the received energy [10]–[14]. In [10]–[12], the throughput maximization problem are investigated in a WPCN with user cooperation for multiple antennas HAP and a pair of single-antenna users. The authors in [13] calculate the throughput of a single-user WPCN with a dedicated single-antenna relay, where a multiple antennas energy source first powers the single antenna user and the relay. Then, the source transmits its information to a single-antenna destination node with the help of the relay. The work in [14] considers a three-node cooperative wireless powered scenario consists of a source, destination and dedicated relay that each are equipped with multiple antennas. The energy and information precoders are designed to maximize the throughput of the system.

The above scenarios consider the WPCN with user cooperation for only single-pair of users. The multi-user cluster-based WPCN and simultaneous wireless information and power transfer (SWIPT) with user cooperation are studied in [15] and [16], respectively, for the case of single-cluster. In [15], all single-antenna users transmit their signals to the single-antenna cluster head (CH) and then the CH transmits the signals of the users as well as its own signal to the multiple antennas HAP via time-division-multiple-access (TDMA) protocol. The transmit time allocations and EB matrix are designed to improve the throughput fairness among the users. The problem of maximizing the sensing data rate in a cluster is considered in [16] for a scenario with single-antenna HAP, CH and cluster members\(^1\).

### B. Contributions

To the best of our knowledge, the design of cluster-based WPCN with multiple antennas users, the multi-cluster WPCN with user cooperation (and therefore the intra-cluster interference), and also, the effect of non-linear energy harvesting (EH) circuit as well as imperfect channel state information (CSI) in the multi-user cooperative WPCN design problems have not been addressed in the literature. Therefore, in this paper, we consider a new multi-cluster WPCN under HTT

\(^1\)Note that cluster-based WPCN have been studied in [17], [18] from performance analysis point of view.
protocol with user cooperation in which, the HAP as well as each user is equipped with multiple antennas and all of them operate in the same frequency band\(^2\). As shown in Fig. 1, CH of each cluster helps its cluster members to transmit their informations to the HAP. Indeed, in the downlink phase, the HAP employs beamforming technique to transfer energy to the users. In the uplink phase, the users in each cluster transmit their signals to the HAP and to their CH. Finally, the CHs first relay the signals of their cluster members and then transmit their own information signals to the HAP. Herein, the aim is to design the EB matrix, transmit covariance matrices of the users and time allocations among energy transfer and cooperation phases in order to maximize max-min and sum throughputs of the network. The main contributions of this manuscript are summarized as the following:

- To deal with the doubly near-far problem in WPCNs, we propose a general multi-cluster WPCN with user cooperation where all devices are equipped with multiple antennas and work in the same frequency band. To transfer the energy signal to the users more efficiently, we employ EB in the downlink. Also, to enable multi-user transmission in the uplink phase, we use space-division-multiple-access (SDMA) and employ beamforming techniques at the nodes.

- To improve the network throughput, we formulate the max-min and sum throughput optimization problems for the proposed system which are shown to be non-convex and consequently, are hard to solve. In light of minorizer derived in [19] for the first time, we develop algorithms based on alternating optimization (AO) and minorization-maximization (MM) techniques to dealing with these problems.

- We also consider the impact of imperfect CSI as well as the non-linearity in the EH circuits in our design problem and extend the proposed design method to these non-ideal scenarios.

C. Organization

The rest of this paper is organized as follows. In section II we present the general multi-cluster cooperative model for WPCN. In section III the max-min throughput optimization problem is analyzed and the steps of the proposed design method are derived. In section IV, the sum

\(^2\)Precisely, in our proposed model, we extend the work in [15] to multi-cluster cooperative WPCN with multiple antennas users.
Fig. 1. A $K N$-user multi-cluster WPCN with user cooperation.

throughput maximization problem is investigated. Also, the proposed method under deterministic energy signal assumption is considered in this section. Numerical results are provided in section V and finally the conclusions are drawn in section VI.

**Notation:** Bold lowercase (uppercase) letters are used for vectors (matrices). The notations $\mathbb{E}[\cdot]$, $\Re\{\cdot\}$, $\|\cdot\|_2$, $(\cdot)^T$, $(\cdot)^H$, $\tr\{\cdot\}$, $\lambda_{\text{max}}(\cdot)$, $\lambda_{\text{min}}(\cdot)$, $\text{vec}(\cdot)$ and $\nabla^2_{x} f(\cdot)$ indicate statistical expectation, real-part, $l_2$-norm of a vector, vector/matrix transpose, vector/matrix Hermitian, trace of a square matrix, principal eigenvalue of a Hermitian matrix, minimum eigenvalue of a Hermitian matrix, column-wise stacking of the elements of a matrix and the Hessian of the twice-differentiable function with respect to (w.r.t.) $x$, respectively. The symbol $\otimes$ stands for the Kronecker product of two matrices/vectors. $\mathcal{CN}(\omega, \Sigma)$ denotes the circularly symmetric complex Gaussian (CSCG) distribution with mean $\omega$ and covariance $\Sigma$. $\mathbb{R}_+$ represents non-negative real numbers. $\mathbb{C}^{N\times N}$ ($\mathbb{C}^N$) indicates the set of $N \times N$ ($N \times 1$) complex matrices (vectors). $\mathbf{I}_N$ represents the identity matrix in $\mathbb{C}^{N\times N}$. $\mathbb{S}_+^N$ denotes the positive definite matrices in $\mathbb{C}^{N\times N}$. Finally, the notation $\mathbf{A} \succeq \mathbf{B}$ ($\mathbf{A} \succeq \mathbf{B}$) implies $\mathbf{A} - \mathbf{B}$ is positive (semi)definite.
II. System Model

As shown in Fig. 1, we consider a multi-cluster multiple-input multiple-output (MIMO) WPCN with user cooperation consisting of a HAP and \( KN \) users (\( K \) clusters) denoted by \( U_k^n, 1 \leq k \leq K, 1 \leq n \leq N \). The HAP and each user \( U_k^n \) are equipped with \( M_h \) and \( M_{k,n} \) antennas, respectively, and all of them operate in the same frequency band. It is assumed that the HAP has a stable power supply, whereas the users have no fixed batteries [4]. Therefore, the HAP first broadcasts radio frequency (RF) energy signals to all users and then the users transmit their information to the HAP using the energy collected in the downlink phase. It is assumed that CH of each cluster, i.e., \( U_k^n \), could relay the information signals of its cluster members \( U_k^n, 1 \leq n \leq N-1 \). We further assume that the channels are reciprocal and follow flat block fading model. The channel matrix from HAP to the user \( U_k^n \) and the channel matrix between the user \( U_k^n \) and CH \( k \) (i.e., \( U_k^N \)) are denoted by \( H_{k,n} \in \mathbb{C}^{M_{k,n} \times M_h}, 1 \leq k \leq K, 1 \leq n \leq N \) and \( G_{k,n,i} \in \mathbb{C}^{M_{k,N} \times M_{i,n}}, 1 \leq k, i \leq K, 1 \leq n \leq N-1 \), respectively. We assume that the entries of \( H_{k,n} \) and \( G_{k,n,i} \) are i.i.d random variables with zero mean and variances \( \sigma^2_{h_{k,n}} \) and \( \sigma^2_{g_{k,n,i}} \), respectively.

In Fig. 2, we show the proposed WPCN with user cooperation protocol which is a modification of the HTT protocol presented in [3]. According to Fig. 2, the system operates in four phases/time slots. At the first phase, the channel estimation is performed within a (known) fixed duration \( \tau_1 \) [15]. In this phase, users transmit their pilot signals to the HAP and CHs. By employing the linear minimum mean square error (LMMSE) channel estimator, the channel matrices \( H_{k,n} \) (at HAP) and \( G_{k,n,i} \) (at CHs) can be modeled as [20]

\[
H_{k,n} = \tilde{H}_{k,n} + \Delta H_{k,n}, \quad 1 \leq k \leq K, 1 \leq n \leq N, \tag{1}
\]

\[
G_{k,n,i} = \tilde{G}_{k,n,i} + \Delta G_{k,n,i}, \quad 1 \leq k, i \leq K, 1 \leq n \leq N-1, \tag{2}
\]

where \( \tilde{H}_{k,n} \) and \( \tilde{G}_{k,n,i} \) are the estimates of the channel matrices \( H_{k,n} \) and \( G_{k,n,i} \), respectively. Also, \( \Delta H_{k,n} \) and \( \Delta G_{k,n,i} \) are the channel estimation errors which are uncorrelated with \( \tilde{H}_{k,n} \) and \( \tilde{G}_{k,n,i} \). We assume that the elements of \( \tilde{H}_{k,n}, \tilde{G}_{k,n,i} \), \( \Delta H_{k,n} \) and \( \Delta G_{k,n,i} \) are also i.i.d random variables with variance \( \sigma^2_{h_{k,n}}, \sigma^2_{g_{k,n,i}}, \sigma^2_{h_{k,n}}, \sigma^2_{g_{k,n,i}} \). Using LMMSE estimator properties, we can write the following expressions

\[
\sigma^2_{h_{k,n}} = (1 - \rho^2_{h_{k,n}}) \sigma^2_{h_{k,n}}, \quad \sigma^2_{h_{k,n}} = \rho^2_{h_{k,n}} \sigma^2_{h_{k,n}}, 1 \leq k \leq K, 1 \leq n \leq N, \tag{3}
\]

\[
\sigma^2_{g_{k,n,i}} = (1 - \rho^2_{g_{k,n,i}}) \sigma^2_{g_{k,n,i}}, \quad \sigma^2_{g_{k,n,i}} = \rho^2_{g_{k,n,i}} \sigma^2_{g_{k,n,i}}, 1 \leq k, i \leq K, 1 \leq n \leq N-1. \tag{4}
\]
Fig. 2. The proposed multi-cluster cooperation protocol in WPCN.

\[ T_{\text{Channel Estimation}} = \begin{array}{c|c|c|c|c|c|c|c} 
\text{HAP} & U_k^1 & \cdots & U_k^{N-1} & U_k^N & \cdots & U_k^N & \text{HAP} \\
\hline
k = 1, \ldots, K & k = 1, \ldots, K & \vdots & k = 1, \ldots, K & (U_k^1 \text{'s signal}) & \vdots & (U_k^{N-1} \text{'s signal}) & (U_k^N \text{'s own signal}) \\
\end{array} \]

\begin{align*}
\tau_1 + \tau_2 + \sum_{n=1}^{N-1} \tau_{3,n} + \sum_{n=1}^{N} \tau_{4,n} & \leq T. 
\end{align*}

in which the parameters \(0 \leq \rho_{b_{k,n}} \leq 1\) and \(0 \leq \rho_{g_{k,n,i}} \leq 1\) indicate the estimation accuracy. Then, the CHs send their estimations to the HAP and therefore HAP has the full knowledge of CSI in the network [15].

In the second phase (i.e., downlink phase) with time duration \(\tau_2\), the HAP transmits the energy signal to the users. In the next tow phases (i.e., uplink phases), the users transmit the information signals to the HAP using their harvested energy at the downlink phase. In the third phase, the users of each cluster transmit their information signals in turn to the HAP and their CHs in time slot \(\tau_3 = \sum_{n=1}^{N-1} \tau_{3,n}\). More precisely, in time slot \(\tau_{3,n}\), the \(n^{th}\) users of all clusters transmit to the HAP and their CH, i.e., \(U_k^N\), simultaneously. In the final phase with duration \(\tau_4 = \sum_{n=1}^{N} \tau_{4,n}\), the CHs transmit the decoded signals of \(N-1\) cluster members along with their own signals to the HAP in time slots \(\sum_{n=1}^{N-1} \tau_{4,n}\) and \(\tau_{4,N}\), respectively\(^3\). By introducing \(T\) as the total operation time of one block, we can write the following constraint for the time duration of the phases [15]

\[ \tau_1 + \tau_2 + \sum_{n=1}^{N-1} \tau_{3,n} + \sum_{n=1}^{N} \tau_{4,n} \leq T. \]

In the following subsections, we describe the signal and system models in more details and obtain the links throughput.

\(^3\)Notice that all users of each cluster, i.e., \(U_k^n\), \(1 \leq n \leq N\) could benefit from the cooperation. This is obvious for \(U_k^n\), \(1 \leq n \leq N-1\) but for the CH, i.e., \(U_k^N\), this claim can be verified using the fact that this cooperation allows the HAP to allocate more time for information transmission (i.e., \(\tau_3 + \tau_4\)) instead of energy transmission (i.e., \(\tau_2\)) and therefore the CH’s throughput loss (due to cooperation) could be compensated by longer uplink time [4].
A. Downlink Phase

In the downlink phase of the protocol, the HAP transfers its energy signal \( x_0 \in \mathbb{C}^{M_k} \) with the EB matrix \( Q = \mathbb{E}[x_0 x_0^H] \geq 0 \) to the users\(^4\). The transmit power of the HAP is constrained by \( p_0 \), i.e.,

\[
\mathbb{E}[\|x_0\|_2^2] = \text{tr}\{Q\} \leq p_0. \tag{6}
\]

The received signal at \( U_{k,n} \) during \( \tau_2 \) is expressed as

\[
y_{k,n}^{(\text{HAP-U})} = (\bar{H}_{k,n} + \Delta_{k,n}) x_0 + z_{k,n}^{(\text{HAP-U})}, \quad \forall k, n, \tag{7}
\]

where \( z_{k,n}^{(\text{HAP-U})} \sim \mathcal{CN}(0, L_{k,n}^{(\text{HAP-U})}) \) is the receiver noise with \( L_{k,n}^{(\text{HAP-U})} \in \mathbb{S}^{M_k}_{++} \). We consider a non-linear circuit model from [21] for the EH circuit in which the curve fitting procedure is performed in logarithmic scale to obtain more accurate results in low power regimes [22]. Therefore, the energy harvested by \( U_{k,n} \) (by ignoring the harvested energy due to the receiver noise [15]) for the non-linear EH model (eq. (21) of [22] in linear scale) becomes

\[
E_{k,n} = \tau_2 \exp\left( a (\log( P_{k,n}))^2 \right) P_{k,n}^b \exp(c), \quad \forall k, n, \tag{8}
\]

where

\[
P_{k,n} = \mathbb{E}\left[ (y_{k,n}^{(\text{HAP-U})})^H y_{k,n}^{(\text{HAP-U})} \right| \bar{H}_{k,n}] = \text{tr}\{(\bar{H}_{k,n} Q \bar{H}_{k,n}^H + \sigma^2_{h,k,n} \text{tr}\{Q\} I_{M_k,n})\}, \quad \forall k, n, \tag{9}
\]

and \( a, b \) and \( c \) are the curve fitting parameters.

B. Uplink Phase

In the third phase, the users \( U_{k,n} \), \( 1 \leq n \leq N - 1 \) of each cluster use the energy harvested in the previous phase and transmit their independent information signals in turn to the HAP and their CH. Let \( V_{k,n} \in \mathbb{C}^{M_k \times d_{k,n}} \) denote the linear precoder matrix of \( U_{k,n} \) that converts the symbol stream \( m_{k,n} \in \mathbb{C}^{d_{k,n}} \), viz. a Gaussian random vector with zero mean and covariance matrix \( I_{d_{k,n}} \), to the vector \( x_{k,n} \in \mathbb{C}^{M_k} \) with \( x_{k,n} = V_{k,n} m_{k,n} \). The total consumed energy at \( U_{k,n} \) in time slot \( \tau_{3,n} \) is constrained by its harvested energy, i.e.,

\[
P_{ck,n} T + \eta_{k,n} \tau_{3,n} \text{tr}\{S_{k,n}\} \leq E_{k,n}, \quad \forall k, 1 \leq n \leq N - 1, \tag{10}
\]

\(^4\)Note that we consider \( x_0 \) as a random vector and design its EB matrix. The special case of deterministic \( x_0 \) is discussed in subsection IV-B.
where $P_{k,n}$ is the constant circuit power consumption, $\eta_{k,n}$ accounts for power amplifier inefficiency and $S_{k,n} = \mathbb{E}[x_{k,n}x_{k,n}^H] = V_{k,n}V_{k,n}^H \in \mathbb{C}^{M_{k,n} \times M_{k,n}}$, $1 \leq n \leq N - 1$ is the information transmit covariance matrix of the $U_k^n$. The received signals at the CH$_k$, i.e., $U_k^n$ and also at the HAP in time slot $\tau_{3,n}$ are respectively given by
\[
y_{k,n}^{(U-CH)} = \mathbf{G}_{k,n}^H x_{k,n} + \sum_{j=1,j\neq k}^{K} \mathbf{G}_{k,n,j}^H x_{j,n} + \sum_{j=1}^{K} \Delta \mathbf{G}_{k,n,j}^H x_{j,n} + z_{k,n}^{(U-CH)} , \quad \forall k, 1 \leq n \leq N - 1 ,
\]
\[
y_{n}^{(U-HAP)} = \sum_{k=1}^{K} \mathbf{H}_{k,n}^H x_{k,n} + \sum_{j=1}^{K} \Delta \mathbf{H}_{k,n,j}^H x_{j,n} + z_{n}^{(U-HAP)} , \quad 1 \leq n \leq N - 1 ,
\]
where $z_{k,n}^{(U-CH)} \sim \mathcal{CN}(0, L_{k,n}^{(U-CH)})$ and $z_{n}^{(U-HAP)} \sim \mathcal{CN}(0, L_{n}^{(U-HAP)})$ denote the additive noises at the CH$_k$ and the HAP, respectively, with $L_{k,n}^{(U-CH)} \in \mathbb{S}_{++}^M$ and $L_{n}^{(U-HAP)} \in \mathbb{S}_{++}^M$. CH$_k$ uses the linear decoder matrix $W_{k,n} \in \mathbb{C}^{d_{k,n} \times M_{k,n}}$ to obtain an estimate of the transmitted symbol vector $\mathbf{m}_{k,n}$:
\[
\hat{\mathbf{m}}_{k,n} = W_{k,n} y_{k,n}^{(U-CH)} , \quad \forall k, 1 \leq n \leq N - 1 .
\]

The achievable throughput of the transmission from $U_k^n$ to CH$_k$ in time slot $\tau_{3,n}$ becomes [23]
\[
R_{k,n}^{(U-CH)}(\tau_{3,n}, S_{k,n}, W_{k,n}) = \tau_{3,n} \log_2 \det \left( I_{d_{k,n}} + W_{k,n} \mathbf{G}_{k,n,k}^H S_{k,n} \mathbf{G}_{k,n,k}^H W_{k,n}^H \left( L_{k,n}^{(U-CH)} \right) \right)
+ \sum_{j=1,j\neq k}^{K} \mathbf{G}_{k,n,j} S_{j,n} \mathbf{G}_{k,n,j}^H + \sum_{j=1}^{K} \sigma_{g,\Delta_{k,n,j}}^2 \text{tr} \left\{ S_{j,n} \mathbf{I}_{M_{k,n}} \right\} \left( W_{k,n}^H \right)^{-1} , \quad \forall k, 1 \leq n \leq N - 1 .
\]

It has been shown that among all linear precoders, LMMSE decoder is optimal for interference suppression [23]. Thus, we assume CH$_k$ employs LMMSE decoder which is given by (see Appendix A)
\[
W_{k,n} = V_{k,n}^H \mathbf{G}_{k,n,k}^H \left( L_{k,n}^{(U-CH)} + \sum_{j=1}^{K} \mathbf{G}_{k,n,j} S_{j,n} \mathbf{G}_{k,n,j}^H \right)^{-1} \left( \sum_{j=1}^{K} \sigma_{g,\Delta_{k,n,j}}^2 \text{tr} \left\{ S_{j,n} \mathbf{I}_{M_{k,n}} \right\} \right)^{-1} , \quad \forall k, 1 \leq n \leq N - 1 .
\]

Next, by using matrix inversion lemma and Sylvester’s determinant property i.e., $\det(I + AB) = \det(I + BA)$ and by substituting (15) in (14), (14) can be rewritten as [24]
\[
R_{k,n}^{(U-CH)}(\tau_{3,n}, S_{k,n}) = \tau_{3,n} \log_2 \det \left( I_{M_{k,n}} + \mathbf{G}_{k,n,k} S_{k,n} \mathbf{G}_{k,n,k}^H \left( L_{k,n}^{(U-CH)} \right) \right)
+ \sum_{j=1,j\neq k}^{K} \mathbf{G}_{k,n,j} S_{j,n} \mathbf{G}_{k,n,j}^H + \sum_{j=1}^{K} \sigma_{g,\Delta_{k,n,j}}^2 \text{tr} \left\{ S_{j,n} \mathbf{I}_{M_{k,n}} \right\} \left( \sum_{j=1}^{K} \sigma_{g,\Delta_{k,n,j}}^2 \text{tr} \left\{ S_{j,n} \mathbf{I}_{M_{k,n}} \right\} \right)^{-1} , \quad \forall k, 1 \leq n \leq N - 1 .
\]
The transmitted signal of $U^n_k$ is also received by the HAP (see (12)). HAP can employ the LMMSE decoder along with successive interference cancellation (SIC) technique to decode the symbol stream of each user. In this case, the achievable throughput of transmission from $U^n_k$ to the HAP for $1 \leq n \leq N - 1$ can be written as [25]

$$R_{k,n}^{(U-HAP)}(\tau_{3,n}, S_{k,n}) = \begin{cases} 
\tau_{3,n} \log_2 \det \left( I_{M_{h}} + \hat{\mathbf{H}}_{k,n}^H S_{k,n} \hat{\mathbf{H}}_{k,n} \left( \mathbf{L}_{n}^{(U-HAP)} \right) + \sum_{j=1}^{K} \sigma_{h,j,n}^2 \text{tr} \left( S_{j,n} I_{M_{h}} \right)^{-1} \right), & 1 \leq k \leq K - 1, \\
\tau_{3,n} \log_2 \det \left( I_{M_{h}} + \hat{\mathbf{H}}_{k,n}^H S_{k,n} \hat{\mathbf{H}}_{k,n} \left( \mathbf{L}_{n}^{(U-HAP)} \right) + \sum_{j=1}^{K} \sigma_{h,j,n}^2 \text{tr} \left( S_{j,n} I_{M_{h}} \right)^{-1} \right), & k = K.
\end{cases} \quad (17)$$

During the fourth phase, CHs relay the signal of their cluster members and transmit their own signals to the HAP. Let $\vec{x}_{k,n} \in \mathbb{C}^{M_k \times N}$ denote the transmitted vectors of CH$_k$ in time slot $\tau_{4,n}$ as follows

$$\vec{x}_{k,n} = \vec{V}_{k,n} \vec{m}_{k,n}, \quad \forall k, n,$$  

(18)

where $\vec{m}_{k,n} \in \mathbb{C}^{d_k,n}$ and $\vec{V}_{k,n} \in \mathbb{C}^{M_k \times N \times d_k,n}$ are respectively the symbol stream and the precoder that CH$_k$ employed for relaying the signals of its cluster members ($1 \leq n \leq N - 1$) along with transmitting its own signal ($n = N$) in the fourth phase. We further assume that the elements of information stream vectors are independent Gaussian random variables with zero mean and unit variance. The total energy consumed by CH$_k$ in time slot $\tau_4 = \sum_{n=1}^{N} \tau_{4,n}$ is constrained by its harvested energy in the downlink phase:

$$P_{c,k,N} T + \eta_{k,N} \sum_{n=1}^{N} \tau_{4,n} \text{tr} \{ \overline{S}_{k,n} \} \leq E_{k,N}, \quad \forall k,$$  

(19)

where $\overline{S}_{k,n} = \mathbb{E}[\vec{x}_{k,n} \vec{x}_{k,n}^H] = \vec{V}_{k,n} \vec{V}_{k,n}^H \in \mathbb{C}^{M_k \times N \times M_k \times N}$ is the transmit covariance matrix of CH$_k$. Afterward, the received signal at the HAP during time slot $\tau_{4,n}$ can be expressed as

$$y_n^{(CH-HAP)} = \sum_{j=1}^{K} \mathbf{H}_{j,n}^H \vec{x}_{j,n} + z_n^{(CH-HAP)}, \quad \forall n,$$  

(20)

where $z_n^{(CH-HAP)} \sim \mathcal{CN} \left( \mathbf{0}, \mathbf{L}_{n}^{(CH-HAP)} \right)$ denotes the receiver additive noise with $\mathbf{L}_{n}^{(CH-HAP)} \in \mathbb{S}_{+}^{M_h}$. Similar to the third phase, the HAP uses the LMMSE decoder along with SIC technique to decode the corresponding symbol streams, in the last phase. Therefore, the achievable throughput for
decoding $U_k^n$, $1 \leq n \leq N - 1$ and $CH_k$ (i.e., $U_k^N$) streams in time slot $\tau_{4,n}$ at HAP is expressed as

$$R_{k,n}^{(CH-HAP)}(\tau_{4,n}, \tilde{S}_{k,n}) = \left\{ \begin{array}{ll} \tau_{4,n} \log_2 \det \left( I_{M_k} + \tilde{H}_{k,N}^H \tilde{S}_{k,n} \tilde{H}_{k,N} \left( L_n^{(CH-HAP)} \right) + \sum_{j=1}^{K} \tilde{H}_{j,N}^H \tilde{S}_{j,n} \tilde{H}_{j,N} \right) \\
+ \sum_{j=1}^{K} \sigma_{h,j,n}^2 \text{tr} \left( \tilde{S}_{j,n} \right) I_{M_k}^{-1}, & 1 \leq k \leq K - 1, \\
\tau_{4,n} \log_2 \det \left( I_{M_k} + \tilde{H}_{k,N}^H \tilde{S}_{k,n} \tilde{H}_{k,N} \left( L_n^{(CH-HAP)} \right) \right) + \sum_{j=1}^{K} \sigma_{h,j,n}^2 \text{tr} \left( \tilde{S}_{j,n} \right) I_{M_k}^{-1}, & k = K. \end{array} \right. \quad (21)$$

By combining (16), (17) and (21), the achievable throughput for $U_k^n$ becomes [4]:

$$R_{k,n}^{(U)} = \min \left( R_{k,n}^{(CH-HAP)} + R_{k,n}^{(U-CH)} \right), \quad \forall k, 1 \leq n \leq N - 1. \quad (22)$$

Also, the achievable throughput for $CH_k$ is given by (21), i.e.,

$$R_{k,N}^{(U)} = R_{k,N}^{(CH-HAP)}, \quad \forall k. \quad (23)$$

III. THE PROPOSED MAX-MIN METHOD

In this section, we cast the max-min fairness throughput problem in which we aim to maximize the minimum throughputs of the proposed cooperative model by designing time allocations as well as energy and information transmit covariance matrices of the users and HAP in different phases. Using (5), (6), (10), (19), (22) and (23), the max-min throughput problem can be formulated as

$$\max_{\tau, Q, S, \tilde{S}} \min_{1 \leq k \leq K} \min_{1 \leq n \leq N} R_{k,n}^{(U)} \quad (24)$$

s. t. $C_1: 0 \leq \tau \leq 1, \quad \tau_1 + \tau_2 + \sum_{n=1}^{N-1} \tau_{3,n} + \sum_{n=1}^{N} \tau_{4,n} \leq 1,$

$C_2: Q \succeq 0, \quad \text{tr} \{ Q \} \leq p_0,$

$C_3: S_{k,n} \succeq 0, \quad P_{k,n} + \eta_{k,n} \tau_{3,n} \text{tr} \{ S_{k,n} \} \leq E_{k,n}, \quad \forall k, 1 \leq n \leq N - 1,$

$C_4: \tilde{S}_{k,n} \succeq 0, \quad P_{k,n} + \eta_{k,n} \sum_{n=1}^{N} \tau_{4,n} \text{tr} \{ \tilde{S}_{k,n} \} \leq E_{k,n}, \quad \forall k,$

where $S = [S_{k,n}, \forall k, 1 \leq n \leq N - 1], \tilde{S} = [\tilde{S}_{k,n}, \forall k, n], \quad \tau = [\tau_2, \tau_{3,1}, ..., \tau_{3,N-1}, \tau_{4,1}, ..., \tau_{4,N}]^T$ and without loss of generality we set $T = 1$. Note that the problem in (24) is non-convex due to the coupled design variables in the objective function and in the constraints $C_3$ and $C_4$. To deal with this problem, we devise a method based on AO with partitioning $[\tau : Q, S, \tilde{S}]$. Indeed, we first consider the problem w.r.t. $\tau$ for fixed $[Q, S, \tilde{S}]$, i.e.,

$$\max_{\tau} \min_{1 \leq k \leq K} \min_{1 \leq n \leq N} R_{k,n}^{(U)} \quad (25)$$

s. t. $C_1, C_3, C_4,$
and then we consider the problem w.r.t. \([Q, S, \tilde{S}]\) for fixed \(\tau\):

\[
\max_{Q, S, \tilde{S}} \min_{1 \leq k \leq K, 1 \leq n \leq N} R_{k,n}^{(U)} \quad \text{s.t.} \quad C_2 - C_4.
\] (26)

These two steps will continue till a predefined stop criterion is satisfied. In what follows, we tackle the sub-problems (25) and (26).

A. The Sub-problem in (25): Maximization w.r.t. \(\tau\)

By introducing an auxiliary variable \(\beta_a\), the problem in (25) can be equivalently transformed into its epigraphic form

\[
\max_{\tau, \beta_a} \beta_a \quad \text{s.t.} \quad C_5^a : R_{k,n}^{(U)}(\tau) \geq \beta_a, \quad \forall k, 1 \leq n \leq N - 1, \quad C_6^a : R_{k,n}^{(U)}(\tau) \geq \beta_a, \quad \forall k, \quad C_1, C_3, C_4.
\] (27)

Now, by substituting (22) and (23) in (27), the problem above can be rewritten as

\[
\max_{\tau, \beta_a} \beta_a \quad \text{s.t.} \quad C_5^{a,1} : R_{k,n}^{(U-CH)}(\tau) \geq \beta_a, \quad \forall k, \quad C_5^{a,2} : R_{k,n}^{(U-HAP)}(\tau) + R_{k,n}^{(CH-HAP)}(\tau) \geq \beta_a, \quad \forall k, 1 \leq n \leq N - 1, \quad C_6^a : R_{k,N}^{(CH-HAP)}(\tau) \geq \beta_a, \quad \forall k, \quad C_1, C_3, C_4.
\] (28)

It is observed that (28) is a linear programming (LP) and can be solved efficiently by e.g. interior point methods.

B. The Sub-problem in (26): Maximization w.r.t. Transmit Covariance Matrices

By introducing an auxiliary variable \(\beta_b\) and using (22) and (23), the problem in (26) can be recast as the following

\[
\max_{Q, S, \tilde{S}, \beta_b} \beta_b \quad \text{s.t.} \quad C_{5,1}^b : R_{k,n}^{(U-CH)}(S) \geq \beta_b, \quad C_{5,2}^b : R_{k,n}^{(U-HAP)}(S) + R_{k,n}^{(CH-HAP)}(\tilde{S}) \geq \beta_b, \quad \forall k, 1 \leq n \leq N - 1, \quad C_6^b : R_{k,N}^{(CH-HAP)}(\tilde{S}) \geq \beta_b, \quad \forall k, \quad C_2 - C_4.
\] (29)

It is observed that the constraints \(C_{5,1}^b, C_{5,2}^b\) and \(C_6^b\) in (29) are not convex and so the problem above. Thus, in what follows, we develop an algorithm based on MM technique to deal with this non-convex problem.
MM is an iterative method that can be used to obtain a solution to the general optimization problem:

\[
P_0 : \begin{cases} 
\max_x \tilde{f}(x) \\
\text{s.t. } \tilde{g}(x) \leq 0.
\end{cases}
\] (30)

In general, \(\tilde{f}(x)\) and \(\tilde{g}(x)\) are non-convex functions. To apply MM to \(P_0\), we should obtain two functions at the \(\kappa^{th}\) iteration; let say \(\tilde{h}(\kappa)(x)\) and \(\tilde{q}(\kappa)(x)\) such that \(\tilde{q}(\kappa)(x)\) minorizes \(\tilde{f}(x)\):

\[
\tilde{f}(x) \geq \tilde{q}(\kappa)(x), \ \forall x, \ \tilde{f}(x^{(\kappa-1)}) = \tilde{q}(\kappa)(x^{(\kappa-1)}),
\] (31)

and \(\tilde{h}(\kappa)(x)\) majorizes \(\tilde{g}(x)\) according to the rule

\[
\tilde{h}(\kappa)(x) \geq \tilde{g}(x), \ \forall x, \ \tilde{h}(\kappa)(x^{(\kappa-1)}) = \tilde{g}(x^{(\kappa-1)}),
\] (32)

where \(x^{(\kappa-1)}\) is the value of \(x\) at the \((\kappa-1)^{th}\) iteration. Next, the following optimization problem is solved at the \(\kappa^{th}\) iteration (which is simpler than the original problem):

\[
P_\kappa : \begin{cases} 
\max_x \tilde{q}(\kappa)(x) \\
\text{s.t. } \tilde{h}(\kappa)(x) \leq 0.
\end{cases}
\] (33)

We begin by applying MM technique on \(C^{b}_{5,1}\). For this purpose, by using the matrix inversion lemma, we rewrite the throughput \(R^{(U-CH)}_{k,n}\) in (16) as (see Appendix B)

\[
R^{(U-CH)}_{k,n} = \tau_{3,n}\log_2 \det \left( A_{k,n}^H D_{k,n}^{-1} A_{k,n} \right), \ \forall k, 1 \leq n \leq N - 1,
\] (34)

where

\[
A_{k,n} = \begin{bmatrix} I_{M_{k,n}} & 0_{M_{k,n} \times M_{k,N}} \end{bmatrix}^T, \ \forall k, 1 \leq n \leq N - 1,
\] (35)

\[
D_{k,n} = \begin{bmatrix} I_{M_{k,n}} & (S_{k,n}^2)^{1/2} \tilde{G}_{k,n,k}^H \\
\tilde{G}_{k,n,k} S_{k,n}^2 & L^{(U-CH)}_{k,n} + \sum_{j=1}^{K} \left\{ \tilde{G}_{k,n,j} S_{j,n} \tilde{G}_{k,n,j}^H + \sigma^2_{g,\Delta_{k,n,j}} \text{tr} \{ S_{j,n} \} I_{M_{k,N}} \right\} \end{bmatrix}, \ \forall k, 1 \leq n \leq N - 1.
\] (36)

We include below lemma from [19] to move forward with finding a minorizer for \(R^{(U-CH)}_{k,n}\) in (34).

**Lemma 1.** The function \(f(X) = \log_2 \det (A^H X^{-1} A) : \mathbb{S}_+^N \rightarrow \mathbb{R}_+\) is convex for any full column rank matrix \(A\).

\[\square\]
Since $D_{k,n} > 0^1$, according to Lemma 1, $R_{k,n}^{(U-CH)}$ is convex w.r.t. $D_{k,n}$. Therefore, it can be minorized at $\kappa^{th}$ iteration of the algorithm as follows [19], [27]:

$$
\tau_{3,n} \log_2 \det \left( A_{k,n}^H D_{k,n}^{-1} A_{k,n} \right) \geq \tau_{3,n} \log_2 \det \left( A_{k,n}^H \left( D_{k,n}^{(\kappa-1)} \right)^{-1} A_{k,n} \right) - \tau_{3,n} \text{tr} \left\{ F_{k,n}^{(\kappa-1)} \left( D_{k,n} - D_{k,n}^{(\kappa-1)} \right) \right\},
$$

where $F_{k,n}^{(\kappa-1)} = \left( D_{k,n}^{(\kappa-1)} \right)^{-1} A_{k,n} \left( A_{k,n}^H \left( D_{k,n}^{(\kappa-1)} \right)^{-1} A_{k,n} \right)^{-1} \left( A_{k,n}^H D_{k,n}^{(\kappa-1)} \right)^{-1} \geq 0, \forall k, 1 \leq n \leq N-1.

To derive an explicit expression for the constraint $C_{5,1}^b$ in terms of the design variables $S_{k,n}$, we define

$$
F_{k,n} = \begin{bmatrix}
(F_{k,n})_{11} \in \mathbb{C}^{M_{k,n} \times M_{k,n}} & (F_{k,n})_{12} \in \mathbb{C}^{M_{k,n} \times M_{k,n}} \\
(F_{k,n})_{21} \in \mathbb{C}^{M_{k,n} \times M_{k,n}} & (F_{k,n})_{22} \in \mathbb{C}^{M_{k,n} \times M_{k,n}}
\end{bmatrix}.
$$

Next, by using (36), we can write the expression

$$
\text{tr} \left\{ F_{k,n} D_{k,n} \right\} = 2\Re \left\{ \text{tr} \left\{ \left( F_{k,n} \right)_{12} \tilde{G}_{k,n,k} S_{k,n} \right\} + \text{tr} \left\{ \left( F_{k,n} \right)_{11} \right\} + \text{tr} \left\{ \left( F_{k,n} \right)_{22} I_{k,n}^{(U-CH)} \right\} \right\}
$$

$$
+ \text{tr} \left\{ \left( F_{k,n} \right)_{22} \sum_{j=1}^{K} \left\{ \tilde{G}_{k,n,j} S_{j,n} \tilde{G}_{k,n,j}^H + \sigma_{g,j,k,n,j}^2 \text{tr} \left\{ S_{j,n} \right\} I_{M_{j,n}} \right\} \right\}, \forall k, 1 \leq n \leq N-1.
$$

By defining $s_{k,n} = \text{vec}(S_{k,n}^{1/2})$ and using the vectorization properties $\text{tr}\{AB\} = (\text{vec}(A^T))^T \text{vec}(B)$ and $\text{vec}(ABC) = (C^T \otimes A) \text{vec}(B)$, the right-hand side of expression in (37) can be rewritten as

$$
-\tau_{3,n} \left(T_{k,n}^{(\kappa-1)} + 2\Re \left\{ \left( v_{k,n}^{(\kappa-1)} \right)^H s_{k,n} \right\} + \sum_{j=1}^{K} s_{j,n}^H \gamma_{k,n,j}^{(\kappa-1)} s_{j,n} \right), \forall k, 1 \leq n \leq N-1,
$$

where by considering $\text{tr} \left\{ \left( F_{k,n} \right)_{22} \sigma_{g,j,k,n,j}^2 \text{tr} \left\{ S_{j,n} \right\} I_{M_{j,n}} \right\} = \text{tr} \left\{ \left( F_{k,n} \right)_{22} \right\} \sigma_{g,j,k,n,j}^2 S_{j,n} \right\},$ we let

$$
T_{k,n}^{(\kappa-1)} = -\log_2 \det \left( A_{k,n}^H \left( D_{k,n}^{(\kappa-1)} \right)^{-1} A_{k,n} \right) - \text{tr} \left\{ F_{k,n}^{(\kappa-1)} D_{k,n}^{(\kappa-1)} \right\}
$$

$$
+ \text{tr} \left\{ \left( F_{k,n}^{(\kappa-1)} \right)_{11} \right\} + \text{tr} \left\{ \left( F_{k,n}^{(\kappa-1)} \right)_{22} I_{k,n}^{(U-CH)} \right\}, \forall k, 1 \leq n \leq N-1,
$$

$$
v_{k,n}^{(\kappa-1)} = \text{vec} \left( \tilde{G}_{k,n,k}^H \left( F_{k,n}^{(\kappa-1)} \right)_{22} \tilde{G}_{k,n,j} \right), \forall k, 1 \leq n \leq N-1,
$$

$$
\gamma_{k,n,j}^{(\kappa-1)} = I_{M_{j,n}} \otimes \left( \tilde{G}_{k,n,j}^H \left( F_{k,n}^{(\kappa-1)} \right)_{22} \tilde{G}_{k,n,j} + \sigma_{g,j,k,n,j}^2 \text{tr} \left\{ \left( F_{k,n} \right)_{22} \right\} I_{M_{j,n}} \right), \forall k, j, 1 \leq n \leq N-1.
$$

Note that $\left( F_{k,n}^{(\kappa-1)} \right)_{22} \geq 0$ because $F_{k,n}^{(\kappa-1)} \geq 0$, and as a result, $\tilde{G}_{k,n,j}^H \left( F_{k,n}^{(\kappa-1)} \right)_{22} \tilde{G}_{k,n,j} \geq 0$. Consequently, according to the Kronecker product properties, $\gamma_{k,n,j}^{(\kappa-1)} \geq 0$ since $\tilde{G}_{k,n,j}^H \left( F_{k,n}^{(\kappa-1)} \right)_{22} \tilde{G}_{k,n,j}^+$

---

1 Positive definiteness of $D_{k,n}$, $\forall k, 1 \leq n \leq N-1$ can be proved straightforwardly by using Schur complement [26].
\[ \sigma_{g_i \Delta_{k,n}}^2 \text{tr} \left\{ (F_{k,n})_{22} \right\} I_{M_{j,n}} \geq 0. \] Therefore, (40) is a quadratic and concave expression w.r.t. design variable \( s_{k,n} \).

The constraints \( C_{6,2}^b \) and \( C_6^b \) of the problem (29) can be managed similar to that of \( C_{5,1}^b \) via applying MM technique. Precisely, considering SIC (see (17) and (21)), the left-hand side of the constraints \( C_{5,2}^b \) and \( C_6^b \) will be substituted with the following expressions at the \( \kappa^{th} \) iteration respectively

\[ -\tau_{3,n} \left( \widetilde{T}_{k,n}^{(\kappa-1)} + 2\Re \left\{ (\overline{v}_{k,n}^{(\kappa-1)})^H \overline{s}_{k,n} \right\} + \sum_{j=k}^{K} \overline{s}_{j,n} \overline{Y}_{j,n}^{(\kappa-1)} \overline{s}_{j,n} \right) \] (44)

\[ -\tau_{4,n} \left( \widetilde{T}_{k,n}^{(\kappa-1)} + 2\Re \left\{ (\overline{v}_{k,n}^{(\kappa-1)})^H \overline{s}_{k,n} \right\} + \sum_{j=k}^{K} \overline{s}_{j,n} \overline{Y}_{j,n}^{(\kappa-1)} \overline{s}_{j,n} \right), \quad \forall k, 1 \leq n \leq N - 1, \] (45)

where \( \overline{s}_{k,n} \equiv \text{vec}(\overline{S}_{k,n}^{[1]}) \). Also, the matrices/vectors \( \overline{T}_{k,n}^{(\kappa-1)}, \overline{T}_{k,n}^{(\kappa-1)}, \overline{v}_{k,n}^{(\kappa-1)}, \overline{v}_{k,n}^{(\kappa-1)}, \overline{Y}_{j,n}^{(\kappa-1)}, \overline{Y}_{j,n}^{(\kappa-1)}, \overline{\beta}_{k,n}^{(\kappa-1)}, \overline{\beta}_{k,n}^{(\kappa-1)}, \overline{A}_{k,n}, \overline{A}_{k,n}, \overline{D}_{k,n} \) and \( \overline{D}_{k,n} \) are defined in Appendix C.

Finally, the \( \kappa^{th} \) iteration of the proposed method is handled via solving the following problem

\[ \max_{Q,s,\beta} \beta^b \] (46)

s. t. \( C_{0,1}^b : (40) \geq \beta^b, \quad C_{0,2}^b : (44) \geq \beta^b, \quad C_6^b : (45) \geq \beta^b, \quad C_2, \]

\[ C_3^b : P_{c_{k,n}} + \eta_{k,n} \tau_{3,n} \|s_{k,n}\|_2^2 \leq E_{k,n}, \quad \forall k, 1 \leq n \leq N - 1, \]

\[ C_4^b : P_{c_{k,N}} + \eta_{k,N} \sum_{n=1}^{N} \tau_{4,n} \|s_{k,n}\|_2^2 \leq E_{k,N}, \quad \forall k, \]

where \( s = [s_{k,n}, \forall k, 1 \leq n \leq N - 1] \) and \( \overline{s} = [\overline{s}_{k,n}, \forall k, n] \). It is observed that the problem in (46) is convex, i.e., a second order cone programming (SOCP) w.r.t. \([s,\overline{s}]\) with linear constraints on matrix \( Q \) and \( \beta^b \). The steps of the proposed algorithm are summarized in Table I. The method consists of a nested loop. In the outer loop, denoted by superscript \( l \), we have alternation between partitioned variables as \([\tau : Q,S,\overline{S}]\); indeed, at step 1, \( \tau \) is optimized for fixed \([Q,S,\overline{S}]\). At step 2, \([Q,S,\overline{S}]\) are optimized for fixed \( \tau \). Note that optimizing \([Q,S,\overline{S}]\) for fixed \( \tau \) itself is performed with inner iteration, denoted by superscript \( \kappa \), associated with applying MM to the constraint set of problem (29). Note that the LP in (28) and SOCP in (46) can be efficiently solved by interior point methods in polynomial time.
TABLE I
THE PROPOSED METHOD FOR MAX-MIN THROUGHPUT OPTIMIZATION IN COOPERATIVE MULTI-CLUSTER WPCN

| Step | Description |
|------|-------------|
| Step 0 | Initialize $Q$, $s$ and $\tilde{s}$ such that satisfy the constraints $C_2$, $C_3^0$ and $C_4^0$. |
| Step 1 | Compute $\tau^{(l)}$ by solving the LP in (28). |
| Step 2 | Compute $Q^{(l)}$, $s^{(l)}$ and $\tilde{s}^{(l)}$. |
| Step 2-1 | Solve the convex problem in (46). |
| Step 2-2 | Update $T_{k,n}$, $\tilde{T}_{k,n}$, $\bar{T}_{k,n}$, $v_{k,n}$, $\tilde{v}_{k,n}$, $v_{k,n}$, $\Upsilon_{k,n,j}$, $\tilde{\Upsilon}_{j,n}$ and $\bar{\Upsilon}_{j,n}$ according to (41), (42), (43) and Appendix C. |
| Step 2-3 | Repeat steps 2-1 and 2-2 till the stop criterion is satisfied. |
| Step 3 | Repeat steps 1 and 2 until a pre-defined stop criterion is satisfied, e.g. $|g^{(l+1)} - g^{(l)}| \leq \xi$ (where $g$ denotes the objective function of the problem (24)) for some $\xi > 0$. |

**Remark 1** (Convergence). The sequence of objective values of problem (29) obtained by applying MM in step 2 has a ascent property and converges to a finite value [28]. Also, the sequence of the objective values of (24), the design problem, has the same property and converges to a finite value [28]. The interested reader may see [28] for the conditions in which MM and AO converge to stationary points.

IV. EXTENSIONS

A. Sum Throughput Maximization

The proposed max-min throughput method can be straightforwardly extended to the sum throughput maximization problem. Considering the sum throughput of the network, i.e., $\sum_{k=1}^{K} \sum_{n=1}^{N} R_{k,n}^{(U)}$, we cast the following sum throughput optimization problem

$$
\max_{\tau, Q, S, \tilde{S}} \sum_{k=1}^{K} \sum_{n=1}^{N} R_{k,n}^{(U)}
\text{ s. t. } C_1 - C_4.
$$

(47)

Similar to the problem in (24), the problem in (47) is non-convex due to the coupled design variables in the objective function and in the constraints $C_3$ and $C_4$. Therefore, we employ AO with partitioning $[\tau : Q, S, \tilde{S}]$. The resulting sub-problem w.r.t. the parameter $\tau$ is an LP and can be solved via e.g. interior point methods (similar to the subsection III-A). Then, by introducing
the auxiliary variables $\phi_{k,n}$, $\forall k, 1 \leq n \leq N - 1$ and using (22) and (23), the problem (47) w.r.t. $[Q, S, \tilde{S}]$ can be recast as

$$
\max_{Q, s, \tilde{s}, \Phi} \sum_{k=1}^{K} \left\{ R_{k,n}^{(U-CH)}(S) + \sum_{n=1}^{N-1} \phi_{k,n} \right\} \\
\text{s. t. } C_{5,1}^{\text{sum}} : R_{k,n}^{(U-HAP)}(S) \geq \phi_{k,n}, \quad C_{5,2}^{\text{sum}} : R_{k,n}^{(CH-HAP)}(\tilde{S}) \geq \phi_{k,n}, \forall k, 1 \leq n \leq N - 1,
$$

$$
C_2 - C_4,
$$

where $\Phi = \left[ \phi_{k,n}, \forall k, 1 \leq n \leq N - 1 \right]$. The constraints $C_{5,1}^{\text{sum}}$ and $C_{5,2}^{\text{sum}}$ can be tackled similar to the constraints $C_{5,1}^b$ and $C_{5,2}^b$ in subsection III-B. Also, similar to the minorization process in (34)-(43), the objective function of the problem above (considering SIC) can be dealt with by neglecting the constant terms. Consequently, the sum throughput maximization problem in (48) can be handled at the $\kappa^{th}$ MM iteration by the following problem iteratively:

$$
\min_{Q, s, \tilde{s}, \Phi} \sum_{k=1}^{K} \left\{ \sum_{j=k}^{K} \sum_{n=0}^{(k-1)N} \tilde{s}_{j,n}^{(k-1)} - 2\Re \left\{ \left( S_{k,n}^{(k-1)} \right)^{\mathcal{H}} \tilde{s}_{k,n} \right\} - \sum_{n=1}^{N-1} \phi_{k,n} \right\} \\
\text{s. t. } C_{5,1}^{\text{sum}} : (40) \geq \phi_{k,n}, \quad C_{5,2}^{\text{sum}} : (44) \geq \phi_{k,n}, \forall k, \ 1 \leq n \leq N - 1,
$$

$$
C_2, \quad C_3^b - C_4^b.
$$

It can be seen that the problem in (49) is convex, i.e., a quadratic constraint quadratic programming (QCQP) w.r.t. $[s, \tilde{s}]$ with linear constraints on matrix $Q$ and $\Phi$. Therefore, the above problem can be solved efficiently by interior point methods.

**B. Case Study: Deterministic Energy Signal**

In this case study, the energy signal $x_0$ is considered as a deterministic signal where the EB matrix $Q = x_0 x_0^{\mathcal{H}}$, viz. a rank-1 matrix. Hereby, the instantaneous received energy by $U_k^n$ in downlink phase can be used more reliably for information transmission in the next phases [28]. Therefore, in light of (9), the RF input power of user $U_k^n$ can be formulated as

$$
\tilde{p}_{k,n}(x_0) = \mathbb{E} \left[ \left( y_{k,n}^{(HAP-U)} \right)^{\mathcal{H}} y_{k,n}^{(HAP-U)} \right] = x_0^{\mathcal{H}} (B_{k,n}) x_0, \forall k, n,
$$

(50)

where $B_{k,n} = \tilde{H}_{k,n}^{H} \tilde{H}_{k,n} + \sigma_n^2 \Delta_{k,n} \mathcal{M}_{k,n} \mathcal{I} \mathcal{M}_{k,n}$. Thus, the constraints $C_2$ and $C_3^b - C_4^b$ of the problems (46) and (49) will update as the following:

$$
C_2^{\text{det}} : \| x_0 \|^2 \leq p_0,
$$
where $E_{k,n}(\tilde{P}_{k,n})$, $\forall k$, $n$ is defined in (8). It is observed that the constraint $C_2^{\text{det}}$ is convex. Next, we consider the constraints $C_3^{\text{det}}$ and $C_4^{\text{det}}$. The energy expression $E_{k,n}(x_0)$ is neither convex nor concave w.r.t. $x_0$ due to the facts that $E_{k,n}(\tilde{P}_{k,n})$ is a non-decreasing concave function w.r.t. $\tilde{P}_{k,n}$ and $\tilde{P}_{k,n}(x_0)$ is a convex function w.r.t. $x_0$. Therefore, $C_3^{\text{det}}$ and $C_4^{\text{det}}$ represent non-convex sets. To deal with the aforementioned non-convexity, first we define the sufficiently large parameter $^5 \xi_{k,n}$ such that $\nabla_{x_0}^2 E_{k,n}(x_0) + \xi_{k,n} I_{M_h} \succeq 0$, $\forall k$, $n$ as sum of a convex and a concave function:

$$E_{k,n}(x_0) = E_{k,n}(x_0) + \frac{1}{2} \xi_{k,n} x_0^H x_0 + \frac{1}{2} \xi_{k,n} x_0^H x_0^H x_0, \quad \forall k, n.$$  \hspace{1cm} (51)

Then, we apply MM technique on $C_3^{\text{det}}$ and $C_4^{\text{det}}$ to obtain convex constraints. To this end, we keep the concave part and minimize the convex part of $E_{k,n}(x_0)$ as follows

$$E_{k,n}(x_0^{(\kappa-1)}) + \frac{1}{2} \xi_{k,n} (x_0^{(\kappa-1)})^H x_0^{(\kappa-1)} + \mathfrak{R}\{u_{k,n}^{(\kappa)}(x_0 - x_0^{(\kappa-1)})\} - \frac{1}{2} \xi_{k,n} x_0^H x_0,$$ \hspace{1cm} (52)

where

\begin{equation}
\begin{aligned}
u_{k,n}^{(\kappa)} &= \xi_{k,n} (x_0^{(\kappa-1)})^H \\
&+ 2\tau_1 \exp(c) \exp\left(a \left(\log\left(\omega_{k,n}^{(\kappa)}\right)\right)\right) \left(\omega_{k,n}^{(\kappa)}\right)^{b-1} \left(2a\log\left(\omega_{k,n}^{(\kappa)}\right) + b\right) \left(x_0^{(\kappa-1)}\right)^H H_{k,n}^H H_{k,n},
\end{aligned}
\end{equation}

with $\omega_{k,n}^{(\kappa)} = \left(x_0^{(\kappa-1)}\right)^H B_{k,n} x_0^{(\kappa-1)}$. Now, we can rewrite the constraints $C_3^{\text{det}}$ and $C_4^{\text{det}}$ as follows

$$C_3^{\text{det}} : P_{c_{k,n}} + \eta_{k,n} \tau_{3,n} \|s_{k,n}\|^2_2 \leq (52), \quad \forall k, 1 \leq n \leq N - 1,$$ \hspace{1cm} (54)

$$C_4^{\text{det}} : P_{c_{k,N}} + \eta_{k,N} \sum_{n=1}^{N} \tau_{4,n} \|\tilde{s}_{k,n}\|^2_2 \leq (52), \quad \forall k.$$ \hspace{1cm} (55)

Finally, to deal with the new design problem, we can modify the algorithm in Table I by replacing the constraints $C_2$ and $C_0 - C_4^b$ in (46) with $C_2^{\text{det}} - C_4^{\text{det}}$ above.

\textsuperscript{5}See Appendix D for a selection of $\xi_{k,n}$. 

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DRAFT
V. Numerical Examples

In this section, the performance of the proposed method is investigated in different scenarios via Monte-Carlo simulations. The channels from HAP to users and the channels between user $U_i^n$ and CH$_k$ (i.e., $U_k^N$) are modeled as $H_{k,n} = 0.1 \left( \frac{d_{k,n}}{d_0} \right) ^{\frac{\alpha}{2}} \tilde{H}_{k,n} \forall k, n$ and $G_{k,n,i} = 0.1 \left( \frac{d_{k,n,i}}{d_0} \right) ^{\frac{\alpha}{2}} \tilde{G}_{k,n,i} \forall k, i, 1 \leq n \leq N - 1$, respectively, where $d_0 = 1$ meters (m) is a reference distance, $d_{k,n}$ is the distance between HAP and users, $d_{k,n,i}$ is the distance between user $U_i^n$ and CH$_k$ (see Fig. 3) and $\alpha$ is the path-loss exponent. It is assumed that the elements of $\tilde{H}_{k,n}$ and $\tilde{G}_{k,n,i}$ are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unit variance. We set $N = 2$ and also $2d_{k,2}^{(U-HAP)} = 2d_{k,1,k}^{(U-CH)} = d_{k,1}^{(U-HAP)} = 10 \text{m}, \forall k, n$, $\theta = 30^\circ$ (as shown in Fig. 3) and $\alpha = 3$, unless otherwise stated. We also consider HAP and the users equipped with $M_h = M_{k,n} = M = 4$, $\forall k, n$ antennas and the maximum power of HAP is set to $p_0 = 3$ watts (w), unless otherwise specified. The receiver noise vectors are assumed to be white with $L_{n}^{(U-HAP)} = L_{k,n}^{(U-CH)} = \sigma^2 I_M, \forall k, 1 \leq n \leq N - 1$, where $\sigma^2 = -80$ dBm. We further assume the number of pairs $K = 4$, circuit power consumption $P_{c_{k,n}} = -23$ dBm, $\forall k, n$ and power amplifier inefficiency factor $\eta_{k,n} = 1, \forall k, n$. Also, the bandwidth is set to 1 MHz and the curve fitting parameters are set to $a = -0.0977$, $b = -0.9151$ and $c = -11.1648$ [21, Fig. 5.a]. Finally, we consider $\rho_{h_{k,n}} = \rho_{g_{k,n,i}} = \rho = 0.95, \forall k, n, i$ and solve the convex optimization problems using CVX package [29].

A. Convergence of the Proposed Algorithm

Fig. 4 shows the values of the max-min throughput, i.e., equivalently the objective function of (24), versus the number of outer iterations (see Table I), for different numbers of antennas at the HAP and the users ($M$). As expected, the max-min throughput increases at each iteration. Furthermore, we observe that the max-min throughput increases as the number of antennas $M$ increases. This is because increasing the number of antennas can enhance the spatial diversity in both downlink and uplink phases and consequently, improves the throughput. This can also explained as larger $M$ leads to more degrees of freedom for the design problem.

B. The Effect of the CH Locations

In this scenario, we study the effect of the distance between CH$_k$ (i.e., $U_k^2$) and HAP as well as distance between CH$_k$ and $U_k^1$ on the max-min throughput performance. For this purpose,
we define a parameter called distance ratio $0 \leq \gamma \leq 1$ which satisfies $d_{k,2}^{(U-HAP)} = \gamma d_{k,1}^{(U-HAP)}$ and $d_{k,k,1}^{(U-CH)} = (1 - \gamma) d_{k,1}^{(U-HAP)}$, $\forall k$ (see Fig. 3). The max-min throughput versus $\gamma$ is plotted in Fig. 5 for different values of $\alpha$. In this figure, we also compare the proposed cooperative resource allocation method with a non-cooperative method. In the non-cooperative method, CHs do not perform their relaying duty in the fourth phase; indeed, for the time duration of the fourth phase we can write $\tau_{4,n} = 0$, $1 \leq n \leq N - 1$ and therefore $\tau_4 = \tau_{4,N}$ (see Fig. 2). We observe that the
method with cooperation has larger minimum throughput than that of non-cooperation, for all values of \( \gamma \). Also, the minimum throughput of the method with cooperation first increases until \( \gamma \leq \gamma_{opt} \), and then decreases by increasing \( \gamma \), i.e., when \( \gamma \geq \gamma_{opt} \). It is worth noting that, since the doubly near-far problem is more severe for larger values of \( \alpha \), the value of \( \gamma_{opt} \) increases w.r.t. \( \alpha \) for the method with cooperation.

**C. Minimum Throughput Versus Maximum Power of HAP**

In Fig. 6, we plot the max-min throughput as a function of the maximum power budget at HAP i.e., \( p_0 \), by considering distance ratio \( \gamma = 0.8 \) (see Fig. 3) for different values of the path-loss exponent \( \alpha \) and for the cooperative and non-cooperative methods. We observe that the proposed cooperative resource allocation method achieves higher throughput than the non-cooperative method. Also, as illustrated in Fig. 6, we have three zones for this scenario: in zone 1, the non-linear EH circuits of both \( \text{CH}_k \) and \( U_k^2 \) in each cluster work in linear region, in zone 2, the non-linear EH circuit of \( U_k^2 \) works in linear region but the non-linear EH circuit of \( \text{CH}_k \) goes to saturation and in zone 3, the non-linear EH circuits of both \( \text{CH}_k \) and \( U_k^2 \) go to saturation. Note that minimum throughput improvement for the scenario with \( \alpha = 3 \) is less than that of the scenario with \( \alpha = 2.5 \) in zone 1 (linear region of the non-linear EH) due to the larger channel attenuation.
D. The Max-min Versus Sum Throughput Optimizations

Fig. 7 compares the throughput obtained from the max-min and sum throughput optimization problems, for the proposed (cooperative) method. It is observed that in sum throughput optimization, the values of the minimum throughput and the sum throughput of all pairs are equal to \( \min_{1 \leq k \leq 4} R_{k,n}^{(U)} = 0.012 \text{ Mbps} \) and \( \sum_{k=1}^{4} \sum_{n=1}^{2} R_{k,n}^{(U)} = 0.911 \text{ Mbps} \), respectively; whereas for the max-min case we have \( \min_{1 \leq k \leq 4} R_{k,n}^{(U)} = 0.083 \text{ Mbps} \) and \( \sum_{k=1}^{4} \sum_{n=1}^{2} R_{k,n}^{(U)} = 0.664 \text{ Mbps} \). Therefore, we can conclude that the max-min throughput method achieves fairness among users by compromising sum throughput\(^1\).

We also illustrate the effect of the initialization (see step 0 of Table I) on the performance of the both methods. Considering red parts, it can be seen that the quality of the obtained solution has a small sensitivity w.r.t. various initializations. Precisely, for 10 different initial points, the maximum level of relative error (i.e., the maximum deviation from average value divided by average value) for the minimum throughput of the max-min problem and sum throughput of the sum throughput maximization problem are respectively 2.37% and 3.32%.

\(^1\)Note that the objective in the sum throughput maximization method is to maximize the sum throughput of all users, and as a result, we numerically observed that the time allocated to the time slots \( \tau_{4,n}, 1 \leq n \leq N - 1 \), i.e., relaying time slots of the sum throughput problem is very small.
E. The Effect of Intra-Cluster Interference

In Fig. 8, we investigate the minimum throughput of the cooperative method versus the parameter $\theta$ (see Fig. 3). We observe that by decreasing $\theta$, the minimum throughput decreases for both scenarios with $\alpha = 2.5$ and $\alpha = 3$. This is due to the fact that decreasing $\theta$ increases the interference between clusters which leads to a reduction in $R_{k,n}^{(U-CH)}, \forall k, 1 \leq n \leq N - 1$ and thus a reduction in $R_{k,n}^{(U)}, \forall k, 1 \leq n \leq N - 1$ (see equation (22)). Note that the reduction in throughput for the case with $\alpha = 3$ is less than that of the case with $\alpha = 2.5$, because the interference among the users is more in the latter case.

F. The Effect of Channel Estimation Error

In this subsection, we study the effect of channel estimation error by comparing the robust and the non-robust schemes for the cooperative max-min method in Fig. 9. The robust scheme considers the errors of channel estimation in the resource allocation stage; whereas, in the non-robust scheme, the channel estimation errors are not considered in the design stage. In this case, we fit the results of both methods (in a least-squares sense) using a polynomial degree 1 as follows

$$R_{dB}^{(U)} = a_\rho \rho + b_\rho$$ (56)
where $R_{dB}^{(U)} = \log(R^{(U)})$, $R^{(U)} = \min_{k,n} R_{k,n}^{(U)}$ and $a_\rho$ as well as $b_\rho$ are the coefficients of the polynomial. As observed in Fig. 9.a, the minimum throughput increases by increasing the estimation quality $\rho$. Note that in Fig. 9.a, we consider the performance evaluation of the robust and the non-robust schemes in the average sense. However, there are some cases in which the non-robust scheme has a much worse performance than the robust one. Therefore, we define the loss parameter as

$$\chi(\rho) = 1 - \frac{R^{(nr)}(\rho)}{R^{(r)}(\rho)},$$

(57)

where $R^{(nr)}$ and $R^{(r)}$ represent the value of the minimum throughput for the robust and the non-robust schemes, respectively. Then, we plot the maximum value of the loss parameter for 100 realizations of the channel matrices in Fig. 9.b. As expected, the loss decreases as the estimation quality increases, and it will be zero in case of perfect CSI, i.e., when $\rho = 1$. We also observe that the proposed (robust) scheme provides larger minimum throughput than the non-robust one even for relatively large values of the estimation quality.

G. The Impact of CH Selection in the Case of $N > 2$

Herein, the aim is to analyze the impact of CH selection to the minimum throughput performance for the cooperative max-min throughput optimization problem\(^6\). In this scenario, it

\(^6\)Note that for the setup with $N = 2$ (that is used for the previous subsections), it was obvious that the user nearer to the HAP should be selected as CH in each cluster.

![Fig. 8. Minimum throughput versus the parameter $\theta$ (in degrees).](image-url)
Fig. 9. The effect of channel estimation error: (a) curve fitting for minimum throughput (in logarithmic scale) versus the estimation quality $\rho$ by using a polynomial degree 1; proposed (robust) method: $a_\rho = 4.1123$, $b_\rho = 7.4633$, non-robust method: $a_\rho = 5.6956$, $b_\rho = 5.7844$, (b) maximum value of the loss parameter $\chi(\rho)$ versus the estimation quality $\rho$.

is assumed that the users of each cluster are uniformly distributed in a circle with radius 5m. Furthermore, the distance between the center of the clusters are set to 10m and also each cluster center is 10m away from the HAP. In Fig. 10, we compare the performance of the two CH selection methods in which the nearest user to the HAP/cluster center is selected as CH for each cluster. Precisely, Fig. 10 shows the max-min throughput of two mentioned methods versus number of clusters $K$ for the case of $N = 4$. The throughput of the both methods decrease as the number of clusters increases. This behavior can be explained by using the fact that the intra-cluster interferences become stronger when the number of clusters increases. Also, the superior performance of the second method (where the CH is the nearest user to the cluster center) compared to the first method can be seen from Fig. 10. This is because for the method in which CH is the nearest user to the HAP, the maximum distance between users and CH in each cluster is larger and as a result the minimum throughput will be lower.

VI. CONCLUSION

In this paper, we have proposed a cooperative multi-cluster method in a WPCN where the HAP and all users are equipped with multiple antennas. In the downlink phase the HAP transfers the RF energy to the users and then the users transmit their information to the HAP with cooperation in the uplink phase. We considered the minimum and sum throughputs of the users as the design metrics. The maximization problems were non-convex w.r.t. the time allocation vector, EB matrix
and covariance matrices of the users. Therefore, we devised an AO approach to deal with the problems. The resulting sub-problems w.r.t. the time allocation vector were convex but the sub-problems w.r.t. the EB and covariance matrices were non-convex and hence, the MM technique was employed to handle them. We further extended the proposed design methodology to the case of imperfect CSI and non-linear EH circuit. Various numerical examples illustrated the effectiveness of the proposed max-min scheme to achieve fairness among users. In future work, we will propose a distributed algorithm for cooperative multi-cluster WPCN to reduce signaling overhead especially in systems with large number of users.

APPENDIX A
PROOF OF THE LMMSE DECODERS IN (15)

Without loss of generality, we assume that the information stream of the $U_k^n$, $1 \leq n \leq N - 1$ and the received signal at the CH$_k$ (i.e., $U_k^N$) in (11) satisfy $E[m_{k,n}] = E[y_{k,n}^{(U-CH)}] = 0$. Then, for the LMMSE decoder $W_{k,n}$ we have [20]

$$W_{k,n} = E\left[m_{k,n} \left(y_{k,n}^{(U-CH)}\right)^H\right]\left(E\left[y_{k,n}^{(U-CH)} \left(y_{k,n}^{(U-CH)}\right)^H\right]\right)^{-1}$$

$$= V_{k,n}^H \bar{G}_{k,n,k}^H \left(L_{k,n}^{(U-CH)} + \sum_{j=1}^{K} \bar{G}_{k,n,j} S_{j,n} \bar{G}_{k,n,j}^H + E\left[\sum_{j=1}^{K} \sum_{j'=1}^{K} \Delta G_{k,n,j} V_{j,n} m_{j,n} m_{j',n}^H V_{j',n}^H \Delta G_{k,n,j}^H\right]\right)^{-1}$$

$$= V_{k,n}^H \bar{G}_{k,n,k}^H \left(L_{k,n}^{(U-CH)} + \sum_{j=1}^{K} \bar{G}_{k,n,j} S_{j,n} \bar{G}_{k,n,j}^H + \sum_{j=1}^{K} \sigma_{g,j}^2 \Delta G_{k,n,j} \text{tr}\left\{S_{j,n}\right\} I_{M_k,N}\right)^{-1}.$$


APPENDIX B

THE DERIVATION OF THE THROUGHPUT EXPRESSION IN (34)

Let denote $D_{k,n}^{-1}$ as inversion of the matrix $D_{k,n}$, with
\[
D_{k,n}^{-1} = \begin{bmatrix}
D_{k,n,11}^{-1} & D_{k,n,12}^{-1} \\
D_{k,n,21}^{-1} & D_{k,n,22}^{-1}
\end{bmatrix} \in \mathbb{C}^{M_{k,n} \times M_{k,n}}, \quad \forall k, 1 \leq n \leq N - 1.
\] (59)

By using blockwise matrix inversion lemma (see, e.g., [30, Appendix A]), we can write:
\[
D_{k,n,11}^{-1} = \left( I_{M_{k,n}} - \left( S_{k,n}^{1/2} \right) H G_{k,n}^{H} \left( L_{k,n}^{U-CH} + \sum_{j=1}^{K} G_{k,n,j} S_{j,n} G_{k,n,j}^{H} \right) \right)
\]
\[+ \sum_{j=1}^{K} \sigma_{g,j}^{2} \Delta_{k,n,j} \text{tr} \{ S_{j,n} \} I_{M_{k,n}} \}
\left( G_{k,n,k} S_{k,n}^{1/2} \right)^{-1}, \quad \forall k, 1 \leq n \leq N - 1.
\] (60)

Then, by using Woodbury matrix identity, i.e., $(F + UCV)^{-1} = F^{-1} - F^{-1}U(C^{-1} + VF^{-1}U)^{-1}VF^{-1}$, we can rewrite $D_{k,n,11}^{-1}$ as
\[
D_{k,n,11}^{-1} = I_{M_{k,n}} + \left( S_{k,n}^{1/2} \right) H G_{k,n}^{H} \left( L_{k,n}^{U-CH} + \sum_{j=1,j \neq k}^{K} G_{k,n,j} S_{j,n} G_{k,n,j}^{H} \right)
\]
\[+ \sum_{j=1}^{K} \sigma_{g,j}^{2} \Delta_{k,n,j} \text{tr} \{ S_{j,n} \} I_{M_{k,n}} \}
\left( G_{k,n,k} S_{k,n}^{1/2} \right)^{-1}, \quad \forall k, 1 \leq n \leq N - 1.
\] (61)

Finally, by substituting (61) in (34) and using Sylvester’s determinant property i.e., det$(I + AB) = \text{det}(I + BA)$, (16) is obtained.

APPENDIX C

DETAILS ON VARIABLES IN (44) AND (45)

Note that variables that are used for the constraints $C_{5,2}^{b}$ and $C_{6}^{b}$ of the problem (29) in (44) and (45) can be written as
\[
\tilde{A}_{k,n} = [I_{M_{k,n}}, 0_{M_{k,n} \times M_{h}}]^{T}, \quad \forall k, 1 \leq n \leq N - 1,
\]
\[
\tilde{A}_{k,n} = [I_{M_{k,N}}, 0_{M_{k,N} \times M_{h}}]^{T}, \quad \forall k, n,
\]
\[
\tilde{D}_{k,n} = \begin{bmatrix}
I_{M_{k,n}} \\
H_{k,n}^{H} S_{k,n}^{1/2} L_{n}^{U-HAP} + \sum_{j=k}^{K} \{ H_{j,n}^{H} S_{j,n} H_{j,n} + \sigma_{h,j}^{2} \Delta_{j,n} \text{tr} \{ S_{j,n} \} I_{M_{h}} \}
\end{bmatrix}, \quad \forall k, 1 \leq n \leq N - 1,
\]
\[ \vec{D}_{k,n} = \begin{bmatrix} I_{M,k,n} & \left( S^+_{k,n} \right)^H \tilde{H}_{k,N} \
 \tilde{H}_{k,N}^H S^+_{k,n} & L_n^{(CH-HAP)} + \sum_{j=k}^{K} \left\{ \tilde{H}_{j,N}^H \tilde{S}_{j,n} \tilde{H}_{j,N} + \sigma^2_{h,\Delta,j,n} \right\} \text{tr} \{ \tilde{S}_{j,n} \} I_{M,n} \end{bmatrix} \], \ \forall k, n.

\[
\vec{F}^{(\kappa-1)}_{k,n} = \left( \vec{D}^{(\kappa-1)}_{k,n} \right)^{-1} \vec{A}_{k,n} \left( \vec{A}^H_{k,n} \left( \vec{D}^{(\kappa-1)}_{k,n} \right)^{-1} \vec{A}_{k,n} \right)^{-1} \vec{A}^H_{k,n} \left( \vec{D}^{(\kappa-1)}_{k,n} \right)^{-1}, \ \forall k, 1 \leq n \leq N - 1,
\]

\[
\vec{F}^{(\kappa-1)}_{k,n} = \left( \vec{D}^{(\kappa-1)}_{k,n} \right)^{-1} \vec{A}_{k,n} \left( \vec{A}^H_{k,n} \left( \vec{D}^{(\kappa-1)}_{k,n} \right)^{-1} \vec{A}_{k,n} \right)^{-1} \vec{A}^H_{k,n} \left( \vec{D}^{(\kappa-1)}_{k,n} \right)^{-1}, \ \forall k, n,
\]

\[
\vec{T}^{(\kappa-1)}_{k,n} = -\log_2 \det \left( \vec{A}^H_{k,n} \left( \vec{D}^{(\kappa-1)}_{k,n} \right)^{-1} \vec{A}_{k,n} \right) - \text{tr} \{ \vec{F}^{(\kappa-1)}_{k,n} \vec{D}^{(\kappa-1)}_{k,n} \} + \text{tr} \left\{ \left( \vec{F}^{(\kappa-1)}_{k,n} \right)^{11} \right\}
\]

\[
\vec{v}^{(\kappa-1)}_{k,n} = \text{vec} \left( \tilde{H}_{k,n} \left( \vec{F}^{(\kappa-1)}_{k,n} \right)^{12} \right), \ \forall k, 1 \leq n \leq N - 1, \ \vec{v}^{(\kappa-1)}_{k,n} = \text{vec} \left( \tilde{H}_{k,n} \left( \vec{F}^{(\kappa-1)}_{k,n} \right)^{12} \right), \ \forall k, n,
\]

\[
\vec{\gamma}^{(\kappa-1)}_{j,n} = I_{M,j,n} \otimes \left( \tilde{H}_{j,n} \left( \vec{F}^{(\kappa-1)}_{k,n} \right)^{12} \tilde{H}^H_{j,n} \right) + \sigma^2_{h,\Delta,j,n} \text{tr} \left\{ \left( \vec{F}^{(\kappa-1)}_{k,n} \right)^{12} \right\} I_{M,j,n}, \ \forall k, 1 \leq n \leq N - 1,
\]

\[
\vec{\gamma}^{(\kappa-1)}_{j,n} = I_{M,j,n} \otimes \left( \tilde{H}_{j,n} \left( \vec{F}^{(\kappa-1)}_{k,n} \right)^{12} \tilde{H}^H_{j,n} \right) + \sigma^2_{h,\Delta,j,n} \text{tr} \left\{ \left( \vec{F}^{(\kappa-1)}_{k,n} \right)^{12} \right\} I_{M,j,n}, \ \forall k, n.
\]

**APPENDIX D**

**A CHOICE OF \( \xi_{k,n} \) IN SUBSECTION IV-B**

The parameter \( \xi_{k,n} \) can be selected in such a way that it satisfies the following condition:

\[
\nabla^2_{x_0} E_{k,n} (x_0) + \xi_{k,n} I_{M,n} = 0, \ \forall k, n,
\]

where \( \nabla^2_{x_0} E_{k,n} (x_0) \) is straightforwardly calculated as

\[
\nabla^2_{x_0} E_{k,n} (x_0) = \varrho_{k,n} B_{k,n} + \varpi_{k,n} B_{k,n} x_0 x_0^H B_{k,n}, \ \forall k, n,
\]

with

\[
\varrho_{k,n} = 2 \tau_2 \exp(c) \exp \left( a \left( \log \left( \tilde{P}_{k,n} \right) \right)^2 \right) \tilde{P}_{k,n}^{b-1} \left( 2a \log \left( \tilde{P}_{k,n} \right) + b \right),
\]

\[
\varpi_{k,n} = 4 \tau_2 \exp(c) \exp \left( a \left( \log \left( \tilde{P}_{k,n} \right) \right)^2 \right) \tilde{P}_{k,n}^{b-2} \left( 4a^2 \left( \log \left( \tilde{P}_{k,n} \right) \right)^2 + \log \left( \tilde{P}_{k,n} \right) (4ab - 2a) + b^2 - b + 2a \right).
\]
As $a < 0, b < 0, B_{k,n} \geq 0$, and $B_{k,n} x_0^H B_{k,n} \geq 0$, it suffices to choose $\xi_{k,n}$ such that

$$\xi_{k,n} I_{M_h} \geq -\overline{\gamma}_{k,n} B_{k,n} - \overline{\omega}_{k,n} B_{k,n} x_0^H B_{k,n},$$

(66)

where

$$\overline{\gamma}_{k,n} = 2\tau_2 b \exp(c) \exp\left( a \left( \log \left( \overline{P}_{k,n} \right) \right)^2 \right) \overline{P}_{k,n}^{b-1},$$

(67)

$$\overline{\omega}_{k,n} = 4\tau_2 \exp(c) \exp\left( a \left( \log \left( \overline{P}_{k,n} \right) \right)^2 \right) \overline{P}_{k,n}^{b-2} \left( \log \left( \overline{P}_{k,n} \right) \left( 4ab - 2a \right) + 2a \right).$$

(68)

Then, using $C_{2}^{t}$, (66), (67), (68) and considering the fact that $x_0^H B_{k,n} x_0 \leq \|x_0\|_{2}^{2} \lambda_{\max} \left( B_{k,n} \right)$ as well as $\|x_0\|_{2}^{2} \lambda_{\min} \left( B_{k,n} \right) \leq \overline{P}_{k,n} = x_0^H B_{k,n} x_0 \leq \|x_0\|_{2}^{2} \lambda_{\max} \left( B_{k,n} \right)$, the Parameter $\xi_{k,n} > \xi_{k,n}^{t}$ can be selected as

$$\xi_{k,n}^{t} = -2\tau_2 \exp(c) \exp\left( a \left( \log \left( p_0 \lambda_{\max} \left( B_{k,n} \right) \right) \right)^2 \right) \left( p_0 \lambda_{\min} \left( B_{k,n} \right) \right) \lambda_{\max} \left( B_{k,n} \right) b-1 \left( b p_0 \lambda_{\min} \left( B_{k,n} \right) \lambda_{\max} \left( B_{k,n} \right) \right) + 2 \left( \log \left( p_0 \lambda_{\min} \left( B_{k,n} \right) \right) \left( 4ab - 2a \right) + 2a \right) \lambda_{\max} \left( B_{k,n} \right)$$

$$\forall k, n,$$

where, we should remark on the fact that $\lambda_{\min} \left( B_{k,n} \right) = \lambda_{\min} \left( \overline{H}_{k,n} \overline{H}_{k,n} \right) + \sigma_{h,\Delta_{k,n}}^{2} M_{k,n} > 0, \forall k, n.$

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