Optimal Harmonic Measuring Device Placement in Distribution Networks in Consideration of Topology Changes

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ABSTRACT As effective measures of energy savings and emission reduction, non-linear loads, such as variable-frequency drives, are widely used. However, the problem of harmonic pollution in power systems is becoming increasingly severe. A harmonic on-line monitoring system is required for statistically analyzing the harmonic distribution rule in a power network and for taking harmonic suppression measures. For solving the frequent topology adjustment issue in practical distribution networks, a new observability analysis method is proposed. The probability of a switch having a closed status is used to express the nodal observability degree in consideration of network topology changes. The condition number of the measurement matrix is used as the index for estimating the precision of the harmonic state. An optimal placement model of harmonic measuring devices is presented, which considers the system observability and the condition number of the measurement matrix. The genetic algorithm (GA) is improved to solve the optimal model. A simulation model is developed based on the improved IEEE33-node system. The simulation results demonstrate the validity of the optimal harmonic measuring device placement model in distribution networks.

INDEX TERMS Distribution network, harmonic state estimation, observability analysis, phasor measurement units.

I. INTRODUCTION

To resolve the energy crisis, power-electronic-technology-based non-linear equipment, such as variable-frequency drives, has been used widely in recent years. However, harmonic pollution in distribution networks has become more severe. Harmonics can influence the power quality of customers and might also lead to resonance, which harms the stable operation of the whole system. Thus, harmonics has become the common focus of power supply enterprises and customers. Typically, the topology of a distribution network is highly complex [1], and it changes frequently according to the load variation. Equipping all the nodes in a distribution system with harmonic measuring devices for the realization of complete observability is impossible due to the high cost of this approach. A limited number of harmonic measuring devices can be installed in key nodes in a distribution network to construct a harmonic online monitoring system. Then, harmonic state estimation technology can be used to calculate the harmonic content for the other nodes. This is an effective method for analyzing the characteristics of harmonic distribution rules and for proposing suppression measures. However, the observable area and state estimation accuracy of the harmonics are determined directly by the placement positions of the harmonic measuring devices. Therefore, research has focused on optimizing the installation positions of measuring devices.

The wide-area measurement system (WAMS), which is based on phasor measurement units (PMUs), has been widely used in power systems [2], [3]. The optimal placement of PMUs has become a research area of interest, especially in distribution networks [4], [5]. Observability analysis is the basis for determining PMU placement. Spanning trees of the power system graph and a tree search technique were
used to analyze the system observability [6]. The concept of unobservability depth was introduced, and its impact on the number of equipped PMUs was explained [7]. This method guarantees a dispersed placement of PMUs around the system and ensures that the distances between the unobserved buses and the observed buses are not too large. The concept of a spanning tree was extended to that of a spanning measurement subgraph with an actual or a pseudo-measurement assigned to each of its branches. A minimum variance criterion was introduced to establish a mathematical index that could reflect the likelihood of harmonic sources being present in each bus line [8]. The objective is to select the measurement locations that will minimize the expected value of the sum of the squares of the differences between estimated and true values. A network topology analysis method with suspected harmonic injection nodes was introduced for analyzing the network observability [9]. For non-harmonic current injection nodes, the observability can be extended by Kirchhoff's current law.

Branch PMUs, which monitor a single branch by measuring the associated current and terminal voltage phasors, are considered [10]. Then, the optimal locations of these PMUs for making the entire network observable are determined. The optimization objective function of PMU placement has also been considered recently. An observability analysis method that includes a vulnerability analysis was proposed [11]. The cost and the extent to which the PMU placement benefited the system observability in the presence of network vulnerabilities were considered. The condition number of the measurement matrix and the PMU cost were considered in the optimal configuration objective function for the realization of high estimation accuracy under minimal measurement cost [12]. The phase angle mismatch of the PMU measurement was considered, and the Cramer-Rao boundary that is based on the state estimation error was deduced [13]. Then, a PMU placement model that uses the Cramer-Rao boundary was proposed. The optimal PMU-communication link placement (OPLP) problem was proposed for investigating PMU placement and communication links (CLs) for full observability [14]. Compared with the traditional optimal PMU placement model, OPLP can reduce the total installation cost significantly. A novel integrated model was presented, which considers the effects of zero-injection buses and conventional measurements for PMU placement [15]. The zero-injection buses and the conventional measurement methods are used as pseudo measurements to increase the observability of the system. The use of an information-theoretic approach to address the PMU placement problem was discussed [16]. The observability was regarded as a special case of the proposed criterion, and the uncertainty reduction on power system states from PMU measurements was modeled rigorously. A new approach for the optimal placement of PMUs that considers the phasor data concentrator and the required communication infrastructure was proposed for minimizing the cost of the WAMS [17]. A PMU optimal placement method that considers comprehensively the measurement redundancy and observability was proposed [18]. Meanwhile, a new methodology was presented for the valuation of the observability under contingencies such as line outages and losses of PMUs. It is assumed that each PMU has unlimited channel capacities. However, the number of channels in available PMUs is limited, and the cost varies accordingly. A PMU optimal placement method that considers the information channel limit was proposed [19], [20]. The impacts of the PMU quantity and number of channels on the system observability were illustrated. The influence of network sparsity on the channel limitations of PMUs is analyzed [21]. The results demonstrate that densely connected systems will enable the efficient utilization of PMUs with many channels.

The topological structure of a distribution network is highly complex and flexible. If the network topology is adjusted while the measuring device placement is fixed, some nodes might lose observability, and the accuracy of harmonic state estimation might change. However, the influence of topology changes on PMU placement is not considered in previous studies.

This paper makes the following original contributions:

1) It proposes an observability analysis method that considers network topology changes. The influence of the closed or opened state of the connection switch on the observability of each node is analyzed.

2) The optimal placement model of harmonic measuring devices considers the observability of the system, and the estimation accuracy of the harmonic state is established.

3) The optimal placement model is identified using an improved genetic algorithm.

The remainder of the paper is organized as follows: Section II describes the foundations of harmonic state estimation and observability analysis. Section III presents the observability analysis method, the optimal harmonic measuring device placement model that considers distribution network topology changes and an improved genetic algorithm. Section IV demonstrates the proposed method on the improved IEEE33-node system as a case study and presents the conclusions of this study.

II. HARMONIC STATE ESTIMATION AND OBSERVABILITY ANALYSIS FOUNDATION

A. HARMONIC STATE ESTIMATION BASIC TECHNOLOGY

The mathematical model for harmonic state estimation is expressed as follows [22], [23]:

\[ Z = HX + \varepsilon \]  

(1)

where \( Z \) is the measurement vector, which consists of the nodal harmonic voltage, the branch harmonic current and the nodal injection current; \( X \) is the state variable vector, which is typically the nodal harmonic voltage to be estimated; \( H \) is the measurement matrix, of which the elements relate the measurement vector to the state variable vector; and \( \varepsilon \) is the measurement noise matrix.
By dividing the vector $Z$ into the voltage subvectors $Z_V$ and the current subvectors $Z_I$ and dividing the vector $X$ into the measured subvectors $V_M$ and the non-measured subvectors $V_C$, equation (1) can be expressed as follows [7], [24]:

$$
\begin{bmatrix}
Z_V \\
Z_I
\end{bmatrix} =
\begin{bmatrix}
I & Y_{IM} & Y_{IC} \\
0 & V_M & V_C
\end{bmatrix}
+ 
\begin{bmatrix}
\epsilon_V \\
\epsilon_I
\end{bmatrix}
$$

(2)

where $I$ is the identity matrix and $Y_{IM}$ and $Y_{IC}$ are submatrices of which the entries are the series and shunt admittances of the network branches, namely, $H$ in (1) is

$$
H =
\begin{bmatrix}
I & 0 \\
Y_{IM} & Y_{IC}
\end{bmatrix}
$$

The system state vectors, namely, $Y_{IM}$ and $Y_{IC}$ in (2), can be determined via the least-square method.

$$
X = (H^T H)^{-1} H^T Z
$$

(3)

B. GENERAL RULES OF NODAL OBSERVABILITY

The voltages and currents of all the branches at measurement points can be measured directly by harmonic measuring devices. From (2), the general rules of nodal observability can be obtained:

Rule 1: The node state at the measurement point, along with its related nodes’ states by branches, is observable. In this article, such nodes are called as directly observable nodes.

Rule 2: If only one branch current is unknown at a directly observable node without a measuring device, the unknown branch current can be solved according to Kirchhoff’s current law (KCL). The related nodes of this directly observable node are still observable. Such nodes are called as indirectly observable nodes.

Rule 3: According to the Rule 1 and Rule 2, the system observable range will be the largest if the harmonic measuring device is installed on the node with the maximum number of branches.

III. OBSERVABILITY ANALYSIS AND OPTIMAL HARMONIC MEASURING DEVICE PLACEMENT METHOD THAT CONSIDERS DISTRIBUTION NETWORK TOPOLOGY CHANGES

The distribution network observability under a topology can be obtained by directly observable nodes and indirectly observable nodes. However, after it is reconfigured, observability of each node may change, and the measuring device placement scheme should be adjusted accordingly. However, this is impossible in practice. Therefore, the influence of topology changes on the nodal observability should be considered.

In this article, the probability of the switch having closed status is introduced. It describes the observability probabilities of nodes at the interconnection switch. The nodal observability degree and the system observability degree can be calculated from this probability. The numbers of directly and indirectly observable nodes can be determined via observability analysis. It is the observable range of the system. The observability degree is divided into the nodal observability degree and the system observability degree. The nodal observability degree is 0 or 1 if the nodal observability is not affected by topology changes. The observability degree of an observable node is 1 and that of an unobservable node is 0. The nodal observability degree is equal to the probability of the switch having closed status if the nodal observability is affected by a topology change. The system observability degree is defined as the sum of the nodal observability degrees divided by the total number of nodes. The system is completely observable if the system observability degree is 1, and the system is completely unobservable if the system observability degree is 0. In practice, the distribution system is partially observable, and the system observability degree is 0 to 1. The cost of installing sufficient measuring devices for making the distribution system completely observable is too high.

A. DISTRIBUTION NETWORK OBSERVABILITY ANALYSIS METHOD WITHOUT THE CONSIDERATION OF TOPOLOGY CHANGES

It is highly difficult to judge the observable range of a distribution system with a complex network topology and many nodes. A symbolized admittance matrix is used to express the correlations of network topology nodes. Symbolized admittance matrix $A$ is an $n$ by-$n$ symmetric matrix, where $n$ is the number of nodes. If $A_{ij} = 1$, node $i$ and node $j$ are related by a branch. If $A_{ij} = 0$, there is no branch between them.

Determination of the node types, namely, directly observable or indirectly observable, is the basis of the system observability analysis.

1) The judgment matrix of directly observable nodes is

$$
F^T = AM^T
$$

(4)

where $F$ is a $l$-by-$n$ matrix and $M$ is a $l$-by-$n$ matrix that specifies the harmonic measuring device placement. If $M_{ij} = 1$, node $i$ is equipped with a harmonic measuring device. If $M_{ij} = 0$, node $i$ is not equipped with a harmonic measuring device. If $F_{ij} \geq 1$, node $i$ is directly observable. If $F_{ij} = 0$, node $i$ is unobservable. From (4), whether a node without a measuring device is directly observable or not can be determined.

2) Based on a directly observable node $m$ without a measuring device, the judgment matrix of indirectly observable nodes is expressed in (5).

$$
f = A_m - M
$$

(5)

where $A_m$ is row $m$ of the symbolized admittance matrix.

If only one element in $f$ is 1 and it is not an element of column $m$, the node that corresponds to the column is an indirectly observable node. Then, the corresponding column element in $F$ is incremented by 1. All the indirectly observable nodes can be determined via (5).
$F_i$ is the observability degree of node $i$. After all the directly and indirectly observable nodes have been determined via (4) and (5), the elements in matrix $F$ must be adjusted. If $F_i \geq 1$, then $F_i = 1$. If $F_i = 0$, then $F_i$ will remain unchanged.

**B. OBSERVABILITY ANALYSIS METHOD BY CONSIDERING NETWORK TOPOLOGY CHANGES**

Fig. 1 shows the variation of the node observability with a topology change. Node $d$ is a measurement point, and it has four branches. Node $d$ and node $l$ are connected by interconnection switch $TS_{d-l}$. Node $d$ and node $q$ are connected by interconnection switch $TS_{d-q}$. Typically, the distribution network operates in radial mode. The statuses of $TS_{d-l}$ and $TS_{d-q}$ are determined by the reconfiguration result of the distribution network.

**FIGURE 1. Relationship between the node observability and the network topology.**

Based on Section III-A, the observability degrees of both node $g$ and node $e$ are 1. If $TS_{d-l}$ is open, the observability degree of node $l$ is 0. If $TS_{d-l}$ is closed, the observability degree of node $l$ will change from 0 to 1. It is determined by the status of $TS_{d-l}$. The probability of $TS_{d-l}$ having closed status is defined as $P_{d-l}$. Similarly, the probability of $TS_{d-q}$ having closed status is defined as $P_{d-q}$. That probability can be determined by the time that the interconnection switch is closed in unit time, as shown in (6)

$$P_{l-j} = \frac{t_{l-j}}{T} \tag{6}$$

where $P_{l-j}$ is the probability of $TS_{l-j}$ having closed status. $T$ is the selected unit time, such as one year. $t_{l-j}$ is the time that the $TS_{l-j}$ is closed in unit time. $P_{d-l}$ and $P_{d-q}$ can be used to describe the observability degree of nodes such as $l$ and $q$. $P_l$ is defined as a parameter that is equal to the sum of probabilities of all the switches that are connected to node $i$ having closed status. For example, $P_d$ is equal to the sum of $P_{d-l}$ and $P_{d-q}$.

Matrix $P$ can be expressed as follows:

$$P = \begin{pmatrix} P_1 & P_2 & \cdots & P_l & \cdots & P_n \end{pmatrix} \tag{7}$$

where $P$ is a $1$-by-$n$ matrix.

Considering matrix $P$, the system observability degree can be expressed as follows:

$$K = \frac{\sum_{i=1}^{n} (F_i + M_i \times P_i)}{n} \tag{8}$$

where $K$ is the system observability degree.

From (4)-(8), the nodal and system observability degrees can be analyzed in consideration of topology changes.

**C. OPTIMAL MODEL OF HARMONIC MEASURING DEVICE PLACEMENT THAT CONSIDERS DISTRIBUTION NETWORK TOPOLOGY CHANGES AND THE CONDITION NUMBER**

The condition number of matrix $H$ in equation (1) can be determined using the function $\text{Cond}(H)$ [25].

$$\text{Cond}(H) = \|H\| \cdot \|H^{-1}\| \tag{9}$$

where $\|\cdot\|$ denotes the matrix norm.

Assume that $x$ is the true value or exact solution of $X$ and $\Delta x$ is the estimation error. Equation (1) can be expressed as follows:

$$Z - \varepsilon = H(x + \Delta x) \tag{10}$$

From (10), the relationship between $\varepsilon$ and $\Delta x$ can be expressed as follows:

$$\Delta x = -H^{-1}\varepsilon \tag{11}$$

From (11), the following relation can be obtained:

$$\|\Delta x\| \leq \|H^{-1}\| \cdot \|\varepsilon\| \tag{12}$$

From (10), the relationship between $Z$ and $x$ can be expressed as follows:

$$x = H^{-1}Z \tag{13}$$

From (13), the following relation can be obtained:

$$\|x\| \leq \|H^{-1}\| \cdot \|Z\| \tag{14}$$

From (12) and (14), the following relation can be obtained:

$$\frac{\|\Delta x\|}{\|x\|} \leq \|H^{-1}\| \cdot \frac{\|\varepsilon\|}{\|Z\|} \tag{15}$$

From (9), equation (15) can be expressed as follows:

$$\frac{\|\Delta x\|}{\|x\|} \leq \text{Cond}(H) \cdot \frac{\|\varepsilon\|}{\|Z\|} \tag{16}$$

According to Equation (16), the condition number of matrix $H$ can influence the harmonic state estimation error $\Delta x$. It can make $\Delta x$ large even if the measurement error is small. Therefore, when determining the harmonic measurement device placement optimization, the measurement matrix condition number should be minimized.

An optimal method for measurement device placement under a static harmonic state and in consideration of topology changes is presented. The placement optimization objective function is presented in (17).

$$\begin{align*}
\text{max } K(M) \\
\text{min } \text{Cond}(H) \\
s.t. \sum_{i=1}^{n} M_i = C
\end{align*} \tag{17}$$

where $H$ is the measurement matrix that corresponds to $M$, $\text{Cond}(H)$ is the conditional number of $H$, and $C$ is the number of harmonic measuring devices.
D. IMPROVED GA-BASED OPTIMAL MODEL SOLUTION METHOD

To solve the optimal model of (17), the genetic algorithm (GA) with binary coding is improved. Each chromosome represents a measurement placement scheme. The length of each chromosome is equal to the number of nodes in the distribution network. In each chromosome, active genes are assigned a value of 1. They represent the measurement points, and their number equals that of the harmonic measuring devices. Inactive genes are assigned a value of 0 and represent nodes without measurement points [12]. According to the locations of the active genes in each chromosome, the measurement matrix \( H \) that corresponds to each chromosome can be constituted. Then, the condition number and system observability degree can be calculated. For the 0-1 integer programming problem, the location of placement will change abruptly after the application of the crossover operator, which is undesirable.

The improved mutation operator is applied in the GA. The random mutation method is commonly used in the conventional GA. One active gene and one inactive gene are selected randomly in the chromosome, and their contents are reversed. They can be changed from 1 to 0 or 0 to 1. The random mutation method can increase the species diversity and avoid premature convergence. However, it causes GA to converge slowly and the locations of placement to change abruptly. Therefore, the improved mutation method is proposed. Genes that differ between the best chromosome in this generation and the other chromosomes can be identified via comparison. One such gene in the chromosome can be selected randomly, and its content can be reversed. If an active gene is selected, one of the inactive genes will be reversed. If an inactive gene is selected, one of the active genes will be reversed. The best chromosome does not mutate.

In each iteration, every chromosome will choose to mutate. The random mutation probability function \( P_r \) is expressed as

\[
P_r = e^{-\frac{n_g+1}{n_S}}
\]

where \( n_g \) is the generation number.

If a chromosome does not undergo a random mutation, an improved mutation will occur. In the early algorithm iterative process, \( P_r \) is large. The chromosomal mutations are based mainly on random mutations, and it is beneficial to increase the species diversity. In the late algorithm iteration stage, chromosomal mutations are based mainly on improved mutations, which is conducive to improving the ability to search for the best chromosome. When applying the selection operator, the best chromosome will be completely copied to the next generation. Other chromosomes will be selected to the next generation via the roulette method. The selection operator can also improve the ability to search for the best chromosome. The procedure of the proposed optimal measurement placement method, which uses improved GA, is illustrated as Fig. 2.

IV. CASE ANALYSIS

In this article, the improved IEEE 33-node system, as illustrated in Fig. 3, is used to simulate the presented optimal model of harmonic measuring device placement.

The improved mutation operator is applied in the GA. The random mutation method is commonly used in the conventional GA. One active gene and one inactive gene are selected randomly in the chromosome, and their contents are reversed. They can be changed from 1 to 0 or 0 to 1. The random mutation method can increase the species diversity and avoid premature convergence. However, it causes GA to converge slowly and the locations of placement to change abruptly. Therefore, the improved mutation method is proposed. Genes that differ between the best chromosome in this generation and the other chromosomes can be identified via comparison. One such gene in the chromosome can be selected randomly, and its content can be reversed. If an active gene is selected, one of the inactive genes will be reversed. If an inactive gene is selected, one of the active genes will be reversed. The best chromosome does not mutate.

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TS\(_{12-22}\) is closed. The 5th harmonic is considered as an analysis example. There is a 5th harmonic source at node 7, and its current magnitude 10A. The values of \( P \) are presented in TABLE 1.

| \( P_1 \) | \( P_2 \) | \( P_3 \) | \( P_{12} \) | \( P_{13} \) | \( P_{21} \) | \( P_{22} \) | \( P_{23} \) | \( P_{31} \) |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Value    | 0.4      | 0.7      | 0.9      | 0.7      | 0.5      | 0.4      | 0.9      | 0.6      | 0.6      | 0.5      |

The remaining elements of matrix \( P \) are 0. The number of measurement points is 5. Equation (17) is solved by using the
TABLE 2. One of the schemes that provide near-optimal values are considered.

| Measurement points | Observable nodes (directly and indirectly) | System observability degree |
|--------------------|--------------------------------------------|----------------------------|
| 3, 6, 12, 29, 32   | 2, 3, 4, 5, 6, 7, 11, 12, 13, 22, 23, 26, 28, 29, 30, 31, 32, 33 | 54.55%                     |

TABLE 3. Optimal placement results with consideration of topology changes.

| Measurement points | Observable nodes (directly and indirectly) | System observability degree |
|--------------------|--------------------------------------------|----------------------------|
| 3, 6, 12, 15, 29   | 2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 22, 23, 25, 26, 28, 29, 30 | 58.18%                     |
| 2, 6, 9, 12, 29    | 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 19, 22, 25, 26, 28, 29, 30 | 58.18%                     |

TABLE 4. Condition number differences in consideration of only the system observability.

| Scheme name | Measurement points | Measurement matrix condition number |
|-------------|--------------------|-------------------------------------|
| S1          | 3, 6, 12, 15, 29   | 16.16                               |
| S2          | 2, 6, 9, 12, 29    | 34.72                               |

TABLE 5. Estimated voltage amplitude error without the current measurement error when S1 is adopted.

| Node | 5th harmonic voltage amplitude/V | Error% |
|------|---------------------------------|--------|
|      | Simulation value                | Calculation value |
| 3    | 10.97                           | 10.97   | 0.00   |
| 6    | 48.67                           | 48.67   | 0.00   |
| 12   | 46.42                           | 46.42   | 0.00   |
| 15   | 44.32                           | 44.32   | 0.00   |
| 29   | 43.15                           | 43.15   | 0.00   |
| 2    | 2.16                            | 3.47    | 60.76  |
| 4    | 17.62                           | 16.92   | -3.95  |
| 23   | 10.83                           | 10.81   | -0.19  |
| 5    | 24.58                           | 24.65   | 0.29   |
| 7    | 70.88                           | 70.83   | -0.07  |
| 26   | 48.23                           | 48.22   | -0.02  |
| 11   | 48.12                           | 48.12   | 0.01   |
| 13   | 45.08                           | 45.08   | -0.03  |
| 22   | 25.68                           | 25.79   | 0.41   |
| 14   | 44.60                           | 44.74   | 0.30   |
| 16   | 44.07                           | 44.04   | -0.07  |
| 28   | 44.91                           | 44.93   | 0.05   |
| 30   | 42.45                           | 42.45   | 0.00   |

A. SYSTEM OBSERVABILITY DEGREE SIMULATION ANALYSIS

When topology changes are not considered, many solutions are obtained. One of the schemes that provide near-optimal values are considered is presented in Table 2. In this case, $P$ is a zero matrix.

Considering topology changes, only two results of optimal measurement placement are obtained, as presented in Table 3. Based on the minimum condition number, the final optimal result can be determined, and it will be discussed in the next section. Comparing Table 2 with Table 3, the system observability degree is smaller when the topology changes are not considered.

If $TS_{12-22}$ is opened after the distribution network reconfiguration, $TS_{15-9}$ will be closed with higher probability because $P_{15}$ and $P_9$ are relatively large. In this case, node 22 will lose observability. When the topology change is not considered, the system observability degree might remain unchanged or decrease from 54.55%, as presented in Table 2. When the topology change is considered, node 9 will be added to the observable nodes, and the system observability degree will remain unchanged at 58.18%, as presented in Table 2. Therefore, with a specified number of harmonic measuring devices, the maximum system observability degree can be guaranteed by placing the measuring devices at nodes with high probability $P_i$, as discussed at Section III-B. Hence, the optimal system observability degree can be realized when the measuring device placement is determined in consideration of distribution network topology changes.
TABLE 7. Estimated voltage phase error without the current measurement error when S1 is adopted.

| Node | 5th harmonic voltage phase /degree | Error/% |
|------|----------------------------------|---------|
| 3    | -33.99                          | 0.00    |
| 6    | -27.96                          | 0.00    |
| 12   | -24.93                          | 0.00    |
| 15   | -31.49                          | 0.00    |
| 29   | -37.90                          | 0.00    |
| 2    | -34.77                          | 1.44    |
| 4    | -33.24                          | 0.00    |
| 23   | -35.68                          | -0.10   |
| 5    | -32.79                          | 0.2     |
| 7    | -21.02                          | -0.15   |
| 26   | -28.49                          | 0.03    |
| 11   | -25.06                          | -0.02   |
| 13   | -28.46                          | -0.11   |
| 22   | -27.63                          | 0.16    |
| 14   | -30.46                          | 0.05    |
| 16   | -32.33                          | 0.03    |
| 28   | -34.15                          | 0.04    |
| 30   | -39.05                          | -0.05   |

TABLE 8. Estimated voltage phase error with the current measurement error when S1 is adopted.

| Node | 5th harmonic voltage phase /degree | Error/% |
|------|----------------------------------|---------|
| 3    | -33.99                          | 0.00    |
| 6    | -27.96                          | 0.00    |
| 12   | -24.93                          | 0.00    |
| 15   | -31.49                          | 0.00    |
| 29   | -37.90                          | 0.00    |
| 2    | -34.77                          | 1.44    |
| 4    | -33.24                          | 0.00    |
| 23   | -35.68                          | -0.10   |
| 5    | -32.79                          | 0.2     |
| 7    | -21.02                          | -0.15   |
| 26   | -28.49                          | 0.03    |
| 11   | -25.06                          | -0.02   |
| 13   | -28.46                          | -0.11   |
| 22   | -27.63                          | 0.16    |
| 14   | -30.46                          | 0.05    |
| 16   | -32.33                          | 0.03    |
| 28   | -34.15                          | 0.04    |
| 30   | -39.05                          | -0.05   |

B. MAXIMUM-SYSTEM-OBSERVABILITY-DEGREE-BASED CONDITION NUMBER SIMULATION ANALYSIS

Cond($H$) is considered on the premise of maximum system observability degree. There are two measurement points placement schemes that have maximum system observability degree. Their Cond($H$) values are presented in Table 4.

The current measurement errors of node 3 and node 15 are 10% in the simulation model. When scheme S1 is adopted, the measurement error influences on the estimated harmonic voltage amplitude and phase are presented in TABLE 5, TABLE 6, TABLE 7 and TABLE 8. When scheme S2 is adopted, measurement error influences on the estimated
TABLE 11. Estimated voltage phase error without the current measurement error when S2 is adopted.

| Node | 5th harmonic voltage phase/degree | Error/% |
|------|----------------------------------|---------|
|      | Simulation value | Calculation value |         |
| 2    | -34.76            | -34.76    | 0.00    |
| 6    | -27.95            | -27.95    | 0.00    |
| 9    | -22.81            | -22.81    | 0.00    |
| 12   | -24.93            | -24.93    | 0.00    |
| 29   | -37.90            | -37.90    | 0.00    |
| 1    | -92.99            | -88.24    | -5.11   |
| 3    | -33.98            | -33.91    | -0.21   |
| 19   | -30.94            | -30.91    | -0.11   |
| 5    | -32.79            | -32.87    | 0.26    |
| 7    | -21.02            | -20.98    | -0.18   |
| 26   | -28.49            | -28.49    | -0.01   |
| 11   | -20.87            | -20.85    | -0.09   |
| 13   | -25.10            | -25.11    | 0.03    |
| 22   | -25.06            | -25.06    | -0.02   |
| 14   | -28.46            | -28.43    | -0.11   |
| 16   | -27.62            | -27.71    | 0.31    |
| 28   | -34.15            | -34.16    | 0.04    |
| 30   | -39.05            | -39.03    | -0.05   |

TABLE 12. Estimated voltage phase error with the current measurement error when S2 is adopted.

| Node | 5th harmonic voltage phase/degree | Error/% |
|------|----------------------------------|---------|
|      | Simulation value | Calculation value |         |
| 2    | -34.76            | -34.76    | 0.00    |
| 6    | -27.95            | -27.95    | 0.00    |
| 9    | -22.81            | -22.81    | 0.00    |
| 12   | -24.93            | -24.93    | 0.00    |
| 29   | -37.90            | -37.90    | 0.00    |
| 1    | -92.99            | 16.01     | -117.22 |
| 3    | -33.98            | -40.03    | 17.80   |
| 19   | -30.94            | -34.16    | 10.42   |
| 5    | -32.79            | -32.87    | 0.26    |
| 7    | -21.02            | -20.98    | -0.18   |
| 26   | -28.49            | -28.49    | -0.01   |
| 11   | -20.87            | -19.82    | -5.02   |
| 13   | -25.10            | -26.18    | 4.32    |
| 22   | -25.06            | -25.06    | -0.02   |
| 14   | -28.46            | -28.43    | -0.11   |
| 16   | -27.62            | -27.71    | 0.31    |
| 28   | -34.15            | -34.16    | 0.04    |
| 30   | -39.05            | -39.03    | -0.05   |

According to the simulation results that are presented in Table 5 -Table 12, when S1 is adopted, the estimated voltage amplitude and phase errors that are caused by the measurement error are small. However, when S2 is adopted, the estimated voltage amplitude and phase errors that are caused by the measurement error are large. Therefore, measurement placement schemes that have the same system observability range may differ in terms of the accuracy of harmonic state estimation. A measurement placement scheme for which the condition number of the measurement matrix is small has higher accuracy in harmonic state estimation.

C. COMPARISON OF SYSTEM OBSERVABILITY DEGREE CURVES BETWEEN IMPROVED GA AND STANDARD GA

System observability degree curves of the improved GA and standard GA are plotted in Fig. 4. It can be demonstrated via the enumeration method that both the improved GA and the standard GA converge to the global optimum. The improved GA converges at the 4th generation and takes about 2.3 seconds. The standard GA converges at the 11th generation and takes about 5.4 seconds. The algorithm can get global optimal solution and the improved GA converges faster than the standard GA.

FIGURE 4. System observability degree evolution curve comparison.

V. CONCLUSIONS

In this article, for the optimal harmonic measuring device placement in a distribution network, observability analysis in consideration of the probability of the switch having closed status is studied. The condition number of the measurement matrix is selected as the index for estimating the precision of the harmonic state. Finally, the optimization target of the harmonic measuring device placement in a power distribution network in consideration of topology changes is established. The improved genetic algorithm is used to solve the problem. The results of simulations that were based on the IEEE 33 distribution network support the following conclusions:

1) With a suitable number of harmonic measuring devices, it is easy for the system to be in a state of high observability when measuring devices are installed on nodes that have high probability of the switch being in the closed status and have more branches.
2) Measurement placement schemes that have the same system observability range may differ in terms of the accuracy of harmonic state estimation. A measurement placement scheme for which the condition number of the measurement matrix is small has high accuracy of harmonic state estimation.

3) In the improved GA, the improved mutation method is combined with traditional mutation. When the improved mutation method is adopted, mutation of the chromosomes is determined by the best chromosome in the generation. The simulation results demonstrate that the improved GA can get global optimal solution and converge faster than the standard GA.

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