Analysis and Research on Multi-layer Docking Error of Complex Interlayer Column Module Based on Point Cloud

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Abstract. Modular design of large offshore oil and gas module reduces the construction difficulty of the whole building, but increases the cumulative error between modules. For large-scale steel structure buildings, the error between the inter-story column modules will affect the overall building stability. It is a feasible method to use point cloud to monitor the errors in real time, so as to trim the columns in time. The goal of this paper is to put forward a set of error analysis method which is suitable for point cloud pre-matching of interlayer integral column. This method includes the calculation and analysis of the center line angle error of each butt joint and the position error of butt joint center of gravity after transformation based on interlayer optimal matching. In practical work, the column structure with large error can be trimmed according to the calculated data.

1. Introduction

Aiming at the problems of heavy monitoring workload and low matching accuracy in traditional steel structure construction and installation, this paper proposes a registration model based on contact plane key information. After noise reduction, the point cloud is divided into regional growth planes to obtain key plane information, and the three-dimensional centerline equation is extracted according to the boundary line of the point cloud segment, and an appropriate transformation matrix is calculated to carry out matching and docking of interlayer integral modules to obtain error data, which provides a numerical reference for subsequent column adjustment.

2. Materials and Methods

2.1. Plane Information Acquisition Method

Firstly, the plane point cloud to be solved is extracted by the plane segmentation method of region growth. A plane fitting method based on singular value decomposition is adopted, which is an optimization process and requires the distance between each point of the point cloud and the fitting plane to be minimum. Let the coordinates of each point of the extracted point cloud be \((x_i, y_i, z_i)\)(\(i = 1, 2, \cdots, n\)), the average coordinates of all points be \((\bar{x}, \bar{y}, \bar{z})\), and let the fitting plane equation be

\[ ax + by + cz + d = 0 \]
The singular value decomposition method is adopted to satisfy the prior condition: the plane crosses the average value of many scatter points [1]. Let matrix
$$A = \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} & z_1 - \bar{z} \\ \vdots & \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} & z_n - \bar{z} \end{bmatrix}$$
column matrix
$$X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$, in actual case some points are out of plane, in order to minimize the sum of distances of all points, let the objective function be \(\min \|AX\|\), and the constraint conditions is
$$\|X\| = 1$$ (2)

Next, singular value decomposition is performed on matrix A
$$A = UDV^T$$ (3)

In equation (3), D is a diagonal matrix, and U and V are unitary matrices. According to the properties of unitary matrix, equation (4) is satisfied
$$\|AX\| = \|UDV^TX\| = \|DV^TX\|$$ (4)

Where \(V^TX\) is a column matrix, satisfying \(\|V^TX\| = \|X\| = 1\). If and only if equation (5) is satisfied, the last diagonal element of D is the minimum singular value, so that \(\|AX\|\) obtains the minimum value and the result satisfies equation (6)
$$V^TX = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$ (5)

$$X = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = V \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$ (6)

\((a,b,c)\) is the normal vector of plane \(ax + by + cz + d = 0\).

2.2. Fitting of Column Centerline and Calculation of Angle Error.

According to the symmetry of standard I-steel and square steel in the interlayer module, the projection method is adopted to fit them. When projecting on XOZ plane, set Z coordinate fitting range as \([Z_n, Z_{n+1}]\), and select fitting reference point as n, then search interval step size is
$$m = \frac{Z_n - Z_{n+1}}{n}$$ (7)

Traversing each division interval to find appropriate boundary reference points, such as a certain step interval \([Z_i, Z_{i+1}]\), finding the minimum and maximum values of x coordinates, noting that the
selected reference boundary points within the interval range are \(X_{\text{max}}, \frac{Z_{\text{max}}+Z_{\text{min}}}{2}\) and \(X_{\text{min}}, \frac{Z_{\text{max}}+Z_{\text{min}}}{2}\), then the center line of the interval is fitted with the selected point \(\left(X_{\text{max}}+X_{\text{min}}, \frac{Z_{\text{max}}+Z_{\text{min}}}{2}\right)\). Traversing all intervals, the fitting point set \(\{Q_i\}\) on the XOZ plane is obtained, and the least square method is used to fit the equation of the point set [3]. Let \(Q_i\) coordinate of the i-th point be \((X_i, Z_i)\), then the two-dimensional fitting equation is

\[
X = aZ + b
\]  

The square error function of all fitting points is expressed as

\[
Q = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (X_i - aZ_i - b)^2
\]  

The Q function in equation (9) is a function of \(a\) and \(b\), the partial derivative of the Q function is obtained, and two quantities to be calculated are solved

\[
\frac{\partial Q}{\partial b} = 2\sum_{i=1}^{n} (X_i - aZ_i - b)(-1) = 0
\]

\[
\frac{\partial Q}{\partial a} = 2\sum_{i=1}^{n} (X_i - aZ_i - b)(-Z_i) = 0
\]

Similarly, reference point selection and least square fitting are carried out on the YOZ plane to obtain a two-dimensional fitting equation on the YOZ projection

\[
Y = cZ + d
\]

Combining the two projection equations, the fitted centerline space equation is obtained

\[
\begin{cases}
X = aZ + b \\
Y = cZ + d
\end{cases}
\]

The linear vector of the three-dimensional centerline is \((a, c, 1)\).

2.3. Registration of Center of Gravity Points Based on Minimum Error.

Set the center of gravity point set to be docked \(\{M_i\}\), the target center of gravity point set \(\{N_i\}\), the rotation matrix is R, the translation matrix is T, and the difference between the center of gravity point set to be docked and the target center of gravity point set M is expressed by \(f(R, T)\).
\[
\min f(R,T) = \min \sum_{i=1}^{n} \| M_i - (RN_i + T) \|^2
\] (14)

The coordinates of \( n \) barycenter points of the two-point set are barycentrized

\[
M'_i = M_i - \sum_{k=1}^{n} M_k / n, \quad N'_i = N_i - \sum_{k=1}^{n} N_k / n \quad (i, k = 1, 2, \ldots)
\] (15)

In equation (15), \( M_i \) and \( M_k \) represent the \( i \)-th and \( k \)-th barycenter coordinate vectors in the point set \( \{M_i\} \), \( N_i \) and \( N_k \) represent the \( i \)-th and \( k \)-th barycenter coordinate vectors in the point set \( \{N_i\} \). According to the coordinates after barycentrization, the rotation matrix in the process of spatial similarity transformation can be solved according to the direct solution based on orthogonal Procrustes analysis [3]

\[
Q = N'^T M'
\] (16)

And singular value decomposition is carried out on the matrix \( Q \)

\[
Q = S \Sigma B^T
\] (17)

In equation (17), \( \Sigma \) is a diagonal matrix, \( S \) and \( B \) are left and right eigenvectors obtained by singular value decomposition of matrix \( Q \). The rotation matrix from coordinates \( N' \) to \( M' \) can be obtained as follows

\[
R = SOB^T
\] (18)

In equation (18), when the rank of matrix \( Q \) is 3 and the determinant of matrix \( Q \) is not less than 0, or the rank of matrix \( Q \) is 2 and the product of determinant of matrix \( S \) and \( B \) is 1, matrix \( O \) is the third-order identity matrix; When the rank of matrix \( Q \) is 3 and the determinant of matrix \( Q \) is less than 0, or the rank of matrix \( Q \) is 2, and the product of determinant of matrix \( S \) and \( B \) is -1, matrix \( O \) is \( \text{diag}(1,1,-1) \). Calculating translation parameters of spatial similarity transformation

\[
T = \sum_{i=1}^{n} M_i / n - R \sum_{k=1}^{n} N_k / n
\] (19)

3. Results & Discussion

Calculating the integral transformation matrix of the interlayer column module and the offset of the center of gravity after butt joint. Using the fitted centerline equation, the center-of-gravity point equation of the surface to be butted is obtained, and the integral rotation equation and translation equation of the interlayer integral column module are calculated using the algorithm based on the minimum error iteration. The transformation equation is of formula (20). The butt-joint effect is shown in the following fig.1
The deviation of butt joint center of gravity of each column and the angle error of center line are shown in Table 1.

Table 1: The deviation of butt joint center of gravity of each column and the angle error of center line, and the port number refer to fig. 10 from left to right from bottom to top.

| Port number | X     | Y     | X     | Y     | Centerline angle error[°] |
|-------------|-------|-------|-------|-------|---------------------------|
| Literature [4] method | This paper method |
| X       | Y     | X     | Y     |       |                           |
| DK001    | 1.33800 | -9.29452 | -0.47064 | -1.46924 | 0.84741                  |
| DK002    | -0.33504 | 0.76092  | -0.25070 | -0.35281 | 0.17360                  |
| DK003    | -1.79989 | -8.94311 | -3.42848 | 1.55462 | 0.96778                  |
| DK004    | 1.31094  | -8.25147 | -0.37109 | -0.32975 | 0.94274                  |
| DK005    | 4.18265  | -2.43417 | 4.17811  | -3.54802 | 0.10074                  |
| DK006    | -3.34312 | -4.50835 | -5.45117 | 3.31880 | 1.73448                  |
| DK007    | -1.49247 | 0.39915  | -0.03179 | -0.17013 | 0.64602                  |
| DK008    | 1.79981  | 4.98663  | 2.99630  | 2.95651 | 0.30285                  |
| DK009    | -1.14340 | -1.83847 | 0.98792  | -0.57996 | 0.10617                  |
| DK010    | 1.34499  | 1.84582  | -0.28267 | 0.86337 | 0.24011                  |
| DK011    | 2.54735  | -9.02781 | 0.54527  | 0.42482 | 0.30785                  |
| DK012    | 0.88560  | 0.86925  | 2.57491  | -0.93831 | 0.38704                  |
| DK013    | 1.45700  | -9.17135 | -0.53993 | 0.83719 | 1.08958                  |
| DK014    | 1.75846  | -8.85282 | -0.26131 | -1.03601 | 0.61789                  |
| DK015    | 0.85281  | -5.01961 | -0.37130 | -1.23909 | 0.72057                  |
| DK016    | 2.21506  | -9.44168 | 0.17657  | -0.29201 | 1.40890                  |

It can be seen from the data in Table 1 that the deviation data of the center of gravity calculated in this paper is quite obvious compared with that in Literature [4]. The deviation data calculated in Literature [4] has many large values, especially in Y coordinate, and there are many ports larger than 5 mm. After hoisting on site by using the transformation matrix calculated in Literature [4], a large number of ports will be trimmed, which shows that there are some problems in this way. The first
reason is that the whole point cloud module contains multiple structural data, and the data points of each structure can not be homogenized, which will have the influence of weight in the actual iteration. On the other hand, there may be some manufacturing errors in the actual steel structure, and there may be dimensional deviations in the characteristic contours of the two butt bodies. The nearest distance points calculated by each point may not be the closest matching points in theory. It can be seen from the data calculated by this method that some ports have achieved good matching results. In the subsequent hoisting operation, considering the deviation of center of gravity and the angle error of centerline, the ports with better quality should be docked first, and the ports with larger errors should be corrected before installation.

4. Conclusions
By adopting the method described in this paper, virtual butt joint can be carried out on the lifting column modules and butt joint errors can be obtained, so that the defects of long time consumption and difficult trimming of the traditional lifting error measurement method are avoided, and the method is suitable for error monitoring of steel structure modules of complex steel structure buildings with high stability requirements and multi-layer column modules, so as to ensure that the building platform meets appropriate performance requirements.

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