Computational analysis of thermally stratified mixed convective non-Newtonian fluid flow with radiation and chemical reaction impacts

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Abstract: The important goal of this article is to examine the thermally stratified flow of Oldroyd-B liquid induced by a stretch sheet. Mathematical model of physical problem is situated with radiation and chemical reaction impacts. By help of similarity transformation, PDE’s are converted into ODE’s with dimensionless variables. Homotopy technique is employed for solving nonlinear ODE’s. Graphs are represented to notify the change of flow parameters, temperature and concentration parameters. Also mass and heat transfer rates are demonstrated through graphically. We observe that Thermal stratification constant reduces the heat transfer rate. On the other hand, thermophoretic and chemical reaction constants boost up the mass transfer rate.

1. Introduction
The study of heat transport in non-Newtonian fluids is growing enormously due to the highest number of engineering and industrial applications. The uses of non-Newtonian liquids are friction reduction oil pipeline, cooling process in electronic devices, flow trackers, etc. The non-Newtonian liquids are dissimilar from the Newtonian liquids. Oldroyd-B liquid is one of the types of non-Newtonian liquids which involves retardation and relaxation time effects. Hayat et al. [1]. Exposes the magnetohydrodynamic flow of Oldroyd-B liquid caused by a stretchy plate in addition of Christov-Cattaneo heat flux model. The study of radiation and chemical reaction over a stretchable sheet was attracted by numerous scientists because of its industrial importance. Few more studies are seen in Ref [2]-[5]. Shehzad et al. [6] observe the thermophoresis effects of the magnetohydrodynamic flow bounded by a stretching paper with mass and heat transfer effects. In addition, heat and mass transfer in magneto hydrodynamic flow with Joule heating was studied by Muhammed et al. [7]. HAM is the one of the powerful analytical method for solving nonlinear equations. Some other important studies about HAM is found in [8]-[10].

Inspired by the above literatures we attempt this study to make the impacts of thermal stratification, thermophoresis, radiation and, chemical reaction in mixed convection of Oldroyd-B liquid flow through a stretching paper.

2. Development of the physical problem:
2-D incompressible Oldroyd-B liquid flow influenced by a stretchy sheet is considered. The stretching velocity \( \tilde{u}_w = \tilde{a} \tilde{x} \) is taken. Where \( \tilde{a} > 0 \) is assigned as a stretching velocity. The temperatures divided into two different segments are pointed by \( T_w^\infty \) and \( T_w^\odot \). \( B_m \) is denoted the magnetic field strength and assigned to the stretchy surface. Variable concentration and variable temperature are noted by \( C_w(x) \) and \( T_w(x) \), while the liquid has a uniform ambient temperature.

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T∞, and uniform ambient concentration C∞. From the above details we construct a governing equation,

$$\frac{\partial \hat{u}}{\partial x} + \frac{\hat{\nu}}{\partial y} = 0$$

(1)

\[\hat{u} \frac{\partial \hat{u}}{\partial x} + \hat{\nu} \frac{\partial \hat{u}}{\partial y} + E_f \left( \hat{u} \frac{\partial^2 \hat{u}}{\partial x^2} + \hat{\nu} \frac{\partial^2 \hat{u}}{\partial y^2} + 2 \hat{u} \hat{\nu} \right) = \mu \frac{\partial^2 \hat{u}}{\partial y^2} + \mu E_f \left( \hat{u} \frac{\partial^2 \hat{u}}{\partial x^2} + \hat{\nu} \frac{\partial^2 \hat{u}}{\partial y^2} \right) - \frac{\sigma \theta_{in}^2}{\rho} \left( \hat{u} + E_f \hat{\nu} \frac{\partial \hat{u}}{\partial y} \right) + g \left[ \beta_T(T - T_\infty) + \beta_c(C - C_\infty) \right]
\]

(2)

\[\hat{u} \frac{\partial \hat{t}}{\partial x} + \hat{\nu} \frac{\partial \hat{t}}{\partial y} = \frac{\lambda_g}{\rho c_p} \frac{\partial^2 \hat{t}}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \left( T - T_\infty \right)
\]

(3)

\[\hat{u} \frac{\partial \hat{c}}{\partial x} + \hat{\nu} \frac{\partial \hat{c}}{\partial y} = D_m \frac{\partial^2 \hat{c}}{\partial y^2} - \frac{\partial V_T \hat{c}}{\partial y} - K_m \hat{c}
\]

(4)

The correlated boundary conditions are

\[\hat{u} = \hat{u}_w(x) = \hat{a}x, \hat{\nu} = 0, T = T_w = T_\infty + bx, C = C_w = C_\infty + cy \text{ at } y = 0\]

(5)

\[\hat{u} \rightarrow 0, \hat{\nu} \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty,
\]

(6)

where (\(\hat{u}, \hat{\nu}\)) – velocity components, \(\mu\) -kinematic viscosity, \(\rho\) -liquid density, \(E_f\) –relaxation time, \(E_f\) –retardation time, \(\sigma\) -thermal expansion coefficients of temperature, \(\beta_c\) –thermal expansion coefficients of concentration, \(g\)-acceleration due to gravity, \(\lambda_g\) –liquid thermal conductivity, \(c_p\) - specific heat at constant pressure, \(q_r\) –radiative heat flux, \(\mu^*\) –dynamic viscosity, \(D_m\)-molecular diffusivity of the species concentration, \(V_T\)-thermophoretic velocity and \(K_m\)-chemical reaction constant.

The similarity transformations are,

\[\eta = \frac{\sqrt{\hat{a} \mu}}{y}, \hat{u} = \hat{a} x f(\eta), \hat{\nu} = -\sqrt{\hat{a} \mu} f'(\eta)
\]

(7)

\[g(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, h(\eta) = \frac{C - C_\infty}{C_w - C_\infty}
\]

Using equation (7) in (2), (3) and (4) we get:

\[f'''' - f''' + ff'' - M(\hat{f}' - \alpha ff') + \lambda(g + Nh) + \alpha(2ff\hat{f}'' - f^2f''') + \beta(f^2 - ff'') = 0
\]

(8)

\[(1 + \frac{4}{3}Rd)g'' + Pr f(g' + \hat{f}g') + Pr Sf' + Pr Qg = 0
\]

(9)

\[h'' + Sc[f\hat{h}' + f\hat{h} - \tau(g'\hat{h}' + g''\hat{h})] - Kh = 0
\]

(10)

Subject to boundary conditions

\[f(0) = 0, f'(0) = 1, g(0) = 1 - S, h(0) = 1 \text{ at } \eta = 0
\]

(11)

\[f' \rightarrow 0, f'' \rightarrow 0, g \rightarrow 0, h \rightarrow 0 \text{ as } \eta \rightarrow \infty
\]

(12)

Now we define the non-dimensional values are known as a relaxation time, retardation time constants, Magnetic field parameter, Grashof number, Reynolds number, local buoyancy constant, Mixed convection parameter, Radiation constant, Prandtl number, Eckert number, chemical reaction constant and Schmidt number.
\[ \alpha = E_1 \alpha, \quad \beta = E_2 \alpha, \quad M = \sigma B_m^2 / \rho, \quad \text{Gr}_x = g \beta \beta \left( T_w - T_e \right) \alpha^3 / \mu^2, \quad \text{Re}_x = u_w \alpha / \mu, \quad N = \beta \left( C_w - C_e \right) / \beta \left( T_w - T_e \right), \quad \lambda = \text{Gr}_x / \text{Re}_x^2, \quad \text{Rd} = (4 \pi \sigma T_e^3) / (k * \lambda g), \quad \text{Pr} = \rho c_p \mu / \lambda g, \quad \text{E}_C = u_0 \alpha / \left( c_p (T_w - T_e) \right), \quad \text{K} = k_m \sigma c / \alpha, \quad \text{Sc} = \mu / D_m \]

The interesting engineering quantities are defined below:

\[ Nu_x = - \left( 1 + \frac{4}{3} \text{Rd} \right) g \left( 0 \right) \text{Re}^{0.5} \]

\[ Sh_x = -h \left( 0 \right) \text{Re}^{0.5} \]

3. HAM results

The initial guesses \( \{ f_0, g_0, h_0 \} \) are \( f_0 = 1 - e^n, g_0 = (1 - S) e^n \) and \( h_0 = e^{-n} \).

The auxiliary linear operators \( \{ L_1, L_2, L_3 \} \) are \( L_1 = f''''(\eta) - f'(\eta), \quad L_2 = g''''(\eta) - g(\eta) \) and \( L_3 = h''''(\eta) - h(\eta) \).

Subject to \( L_1 \left( B_1 + B e^n + B e^{-n} \right) = 0, \quad L_2 \left( B_4 e^n + B_5 e^{-n} \right) = 0, \quad \) and \( L_3 \left( B_6 e^n + B_7 e^{-n} \right) = 0 \).

where \( B_i, (i = 1 \ldots, 7) \) are the arbitrary parameters.

The appropriate outcomes \( \{ f_m^n, g_m^n, h_m^n \} \) are

\[ f_m(\eta) = f_m^n(\eta) + B_1 + B_2 e^n + B_3 e^{-n}, \]

\[ g_m(\eta) = g_m^n(\eta) + B_4 e^n + B_5 e^{-n}, \]

\[ h_m(\eta) = h_m^n(\eta) + B_6 e^n + B_7 e^{-n}. \]

4. Results and Discussion

The impacts of various constants on velocity, temperature and concentration profiles for the stable values of \( M = 0.5, \alpha = 0.3, \beta = 0.2, \lambda = 0.1, \quad S = 0.2, \quad \text{Sc} = 0.9, \quad \text{Pr} = 1.2, \quad N = 0.5, \quad \text{Rd} = 0.3, \quad Q = 0.1, \quad \tau = 0.2 \) and \( \text{K} = 1.0 \). The effect of \( \alpha \) on \( f'(\eta) \) is presented in Figure. 1(a).

Evidently, velocity and momentum boundary layer thickness are decreasing function of greater Deborah number \( \alpha \) because \( \alpha \) rises due to the relaxation time factor. A higher \( \alpha \) corresponds to a stretched relaxation time aspect, and this aspect reduces the liquid flow by the reason of the lesser velocity and thinner momentum layer. Moreover, an improvement in \( \beta \) produce a decrement in \( f'(\eta) \), and in the thickness of the momentum boundary layer (for details, see Figure. 1(b)). Essentially, the Deborah number \( \beta \) is directly combine with the retardation time factor, this factor has the capability to increase the liquid velocity. Figure 1(c) displays the impact of \( \lambda \) on \( f'(\eta) \). Here, both \( f'(\eta) \) and the associated thickness layer thickness rises when \( \lambda \) increases as the result of \( \lambda \) produces developed buoyancy forces, which leading to the larger velocity. The impact of Hartmann number \( M \) on \( f'(\eta) \) is illustrated in Figure. 1(d). The magnetic field has the trend of reduce the speed of the liquid which diminishes the velocity. Figure 2(a) is sketched for various values of relaxation time constant \( \alpha \) on temperature profile. Here temperature profile rises for higher values of \( \alpha \). When thermal relaxation time increased so production of heat causes the temperature to enhance due to viscous forces. As elasticity enhances for higher retardation time constant \( \beta \) so decrement of temperature profile is noted in Figure. 2(b). From Figure. 2(c), we identify that Lorentz force opposing the liquid motion, so heat is produced and therefore thermal boundary layer thickness enhances for rising the value of Hartmann number \( M \). Figure. 2(d) illustrated that temperature and its affiliated boundary layer thickness is reduces for the higher mixed
convection constant. Figure 3 depicts the influence of $S$ on $g(\eta)$. Here temperature profile is lower for higher values of $S$. In fact, the temperature variance consistently drops among the surface of the sheet and the ambient liquid this decrement generated a drop-in temperature. From Figure 4 shows that the concentration diminishes for higher thermophoretic constant $\tau$.

Figures (5)-(6) present the numerical results for local Nusselt number $\text{Re}^{-0.5} \text{Nu}_h$ and the local Sherwood number $(\text{Re}^{-0.5} \text{Sh}_c)$ for wide range of physical constants. Figure 5(a) & 5(b) depicts that the heat transfer rate reduces while enhancing the value of thermal stratification constant combined with magnetic field $(M)$ and mixed convection $(\lambda)$. Heat transfer rate increase while rising the values of thermal stratification $(S)$. Figure 6(a) & 6(b) drawn for calculating the mass transfer rates of chemical reaction constant $(K)$ and thermophoretic constant $(\tau)$. We conclude that mass transfer rate boosts for greater values of $K$ and $\tau$.

![Figure 1](image1.png)

**Figure. 1. Effects of $\alpha, \beta, \lambda$ and $M$ on $f'(\eta)$**

![Figure 2](image2.png)

**Figure 2. Influence of $\alpha$ and $\beta$ on $g(\eta)$**

![Figure 3](image3.png)

**Figure 3. Influence of $S$ on $g(\eta)$**
Figure 2. Effects of $\alpha, \beta, \lambda$ and $M$ on $\theta(\eta)$.

Figure 3. Effects of $S$ on $\theta(\eta)$.

Figure 4. Effects of $\tau$ on $\phi(\eta)$.

Figure 5. Effects of ($S$ & $M$) and ($S$ & $\lambda$) on $\left(\text{Re}^{-1}\text{Nu}_x\right)$.
5. Conclusion

The mixed convection of an Oldroyd-B fluid flow upon a stretching sheet with the appearance of thermal stratification, radiation, thermophoresis and chemical reaction is inspected. The final observations are,

1. Velocity falling via magnetic higher magnetic field constant.
2. Thermal stratification constant reduces the heat transfer rate.
3. Thermophoretic and chemical reaction constants boost up the mass transfer rate.
4. Thermophoretic constant act as a key role in reducing concentration profile.

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