Thickness dependence of electron-electron interactions in topological $p$-$n$ junctions

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(Dated: December 13, 2018)

Abstract

Electron-electron interactions in topological $p$-$n$ junctions consisting of vertically stacked topological insulators are investigated. $n$-type Bi$_2$Te$_3$ and $p$-type Sb$_2$Te$_3$ of varying relative thicknesses are deposited using molecular beam epitaxy and their electronic properties measured using low-temperature transport. The screening factor is observed to decrease with increasing sample thickness, a finding which is corroborated by semi-classical Boltzmann theory. The number of two-dimensional states determined from electron-electron interactions is larger compared to the number obtained from weak-antilocalization, in line with earlier experiments using single layers.

PACS numbers: 73.20.-r, 73.25.+i, 73.50.-h
I. INTRODUCTION

Topological insulators are fascinating materials with conducting surfaces, harboring electronic states with a Dirac-like bandstructure. Large spin-orbit interaction together with time reversal symmetry cause the topological nature of these surface states (TSS), manifesting itself in the suppression of backscattering and leading to the weak-antilocalization effect (WAL) and to spin-momentum coupling. Furthermore, magnetic topological insulators exhibit the quantum anomalous Hall (QAH) effect, characterized by dissipationless chiral currents. These properties of topological insulators have attracted great attention because of their potential applications in energy-efficient electronics and quantum computing.

The analysis of the topological properties is complicated by the non-zero conductivity of the bulk, which often dominates the overall transport characteristics. Several methods have been devised to suppress the bulk contribution, such as doping, gating, and reducing the thickness of the layer. A relatively unexplored but elegant method is to combine an electron and hole dominated material to form a p-n junction, and thus creating a depletion layer at the interface.

The π-Berry phase of the Dirac fermions gives rise to quantum corrections of the conductivity, with a magnetic field and temperature dependence resembling the WAL effect. By analyzing of the WAL in topological p-n junctions the transport through TSS and bulk states was disentangled. Additional modifications of the conductivity are caused by electron-electron interactions (EEI), originating from an effective decrease of the electron density at the Fermi level. The combined study of both WAL and EEI can reveal information about spin (EEI) and orbital (WAL) part of the electron wave function to transport.

Especially the number of 2D states $n$ is of utmost interest, since it can provide evidence of the topological nature of a TI. By careful observation of either the WAL or EEI, a value for $n$ can be gained. It turns out that in single layer TI, $n_{EEI}$ tends to be larger than $n_{WAL}$ (see Fig. 1 and Tab. 1). It seems that surface states on the top and bottom contribute independently to EEI but that, under certain circumstances, they appear to be coupled when the WAL effect is concerned. The physical origin of this coupling effect remains elusive. Predominantly in very thin layers only one 2D state contributes to WAL. Thicker films tend to be decoupled when WAL is concerned and therefore exhibit a higher number of 2D-channels. Microflakes and hot wall epitaxy deposited...
layers\textsuperscript{27} are exceptions where coupling effects can be observed even at thicknesses > 60 nm. A combined study of the WAL and EEI in TI-multilayers is entirely missing.

In the following, we present the first investigation of the interplay of WAL and EEI in topological $p$-$n$ junctions. Conductivity corrections are measured at temperatures < 10 K as a function of temperature, magnetic field and sample thickness. The conductivity correction are used to find the number of 2D channels contributing to either EEI or WAL. Finally, a semiclassical Boltzmann theory is derived to understand the thickness dependence of the conductivity corrections due to EEI.

II. EXPERIMENT

The Bi$_2$Te$_3$/Sb$_2$Te$_3$-bilayers (BST) were grown using molecular beam epitaxy (MBE). Details of the MBE sample preparation can be found in Ref.\textsuperscript{17}. The bottom Bi$_2$Te$_3$-layer was $t_{\text{BiTe}} = 6$ nm and the top Sb$_2$Te$_3$-layers was 6.6 nm (BST6), 7.5 nm (BST7), 15 nm (BST15), and 25 nm (BST25) thick, respectively. The films were patterned into Hall bars which were 200 $\mu$m wide and 1000 $\mu$m long. Transport in these samples was measured in a He-3 cryostat at temperature down to 300 mK while a perpendicular magnetic field could be applied using a superconductive magnet.

III. RESULTS

In Fig. 2 the sheet resistance $R_s$ during cooldown is shown for all sample thicknesses. Metallic behavior is dominant, except for the thinnest samples, BST6 and 7, which are insulating between room temperature and 200 K, where they become metallic. At base temperature (300 mK) all samples are insulating, with the transition temperature between the metallic and insulating phase, $T^*$, found to be between 7 to 11 K, depending on the sample thickness (see insert in Fig.2(a)).

The temperature range below $T^*$ is explored in more detail in Fig.3 for each sample thickness. The temperature was increased in small steps starting at base temperature of 300 mK, taking care for the temperature to stabilize. An external magnetic field was swept between 0 and 0.5 T at each temperature step. Both longitudinal and transverse resistance were recorded from which the conductivity could be calculated. Only one field loop needed
TABLE I. Sample details of experiments reporting both on WAL and EEI. Most results are reported on thin films grown by molecular beam epitaxy (MBE) and sputtering (sp) and a few by hot wall epitaxy (HTW) and on microflakes.

to be taken since the noise level was low.

IV. DISCUSSION

EEI originate from pairing of electrons at the Fermi energy and lead to a decrease in the carrier density, which in turn leads to a reduction of the conductivity. As can be seen in Fig. 3, the correction to conductivity due to EEI sets in below a transition temperature and exhibits a well-defined temperature dependence, given by

$$\delta \sigma(T) = -\frac{e}{\pi \hbar} n \left(1 - \frac{3}{4} F\right) \ln\left(\frac{T}{T^*}\right)$$

where $n$ is the number of 2D channels, $F$ the screening factor, and $T^*$ the transition temperature. By applying Eq. 1 to the measured conductivity in Fig. 3 using $T^*$ (see insert in Fig. 2(a)), we obtain $f = n(1 - 3/4 \times F)$ from the slope of the temperature dependence.

The overall change of the conductivity correction between base and transition temperature, $\delta \sigma_{5K} - \delta \sigma_{300mK}$, increases with sample thickness (see Fig. 4(a)).

Fig. 4(b) shows the change of $f$ when a magnetic field is applied perpendicular to the sample. The value of $f$ is smaller than 1 without magnetic field but rises to values close or
above 1 at fields $\approx 0.2$ T. This abrupt change reflects the disruption of phase coherence due to the magnetic field, impacting WAL. At fields $> 0.2$ T, where WAL has disappeared\textsuperscript{18}, any change in conductivity can be attributed to EEI. $f$ saturates above this field (see Fig. 4(b)) and is employed to investigate the underlying EEI it originates from. The screening parameter $F$ can be inferred from $f$ if $n$, the number of 2D states is known. $F$ can attain values between 0 (no screening) and 1 (strong screening). This condition cannot be fulfilled when $f$ is larger than 1 and $n = 1$. Thus, to obtain an $F$ within the allowed range from our experimental results\textsuperscript{27} we assume that $n > 1$ (see Fig. 4(c)).

For $n = 2$ the screening factor $F$ decreases with thickness, from 0.73 for BST6 to 0.5 for BST25 (see Fig. 4(c)). It cannot be excluded that $n > 2$ but although the values of $F$ differ, the thickness dependence remains unchanged. This goes hand-in-hand with a similar thickness-dependent increase of the conductivity correction, since weaker screening means
FIG. 2. (a)-(d) Sheet resistance $R_s$ dependance on temperature for four different samples. The arrows indicate the transition temperature $T^*$. Insert in (a) Transition temperature $T^*$ dependence on thickness of the Sb$_2$Te$_3$-layer

stronger EEI, hence larger $\delta \sigma$. In single layers, both a decrease$^{35,37}$ as well as an increase$^{34}$ of $F$ with increasing thickness have been reported. The increase was attributed to a stronger screening due to the bulk states in thicker samples$^{32}$.

To explain our results in light of these contradicting earlier observations, we derived a semi-classical Boltzmann theory for the topological $p$-$n$ junctions. The total conductivity (see Eqns. C18 and C19 in the Supplement$^{38}$ for its derivation) is given by
FIG. 3. (a) - (d) Conductivity of four different samples at low temperature for three different perpendicular magnetic fields. Using a logarithmic scale for the temperature, the linear regions are fitted using Eqn. 1 (straight lines). The magnetic field leads to a change of slope, from which the screening and number of 2D channels can be derived.

\[
\hat{\sigma}_{\text{tot}}(B) = e\hat{\mu}_v(B)N_A A_h \left[ (L_A - W_p) + \int_0^{W_p} dz \exp \left( -\frac{\beta e\bar{\mu}_h N_A}{2\epsilon_0\epsilon_r D_h} z^2 \right) \right] - e\hat{\mu}_c(B)N_D A_e \\
\times \left[ (L_D - W_n) + \int_0^{W_n} dz \exp \left( -\frac{\beta e\mu_e N_D}{2\epsilon_0\epsilon_r D_e} z^2 \right) \right] + e\hat{\mu}_{\sigma}^\pm(B) \left( \frac{\alpha_0 \Delta_0}{2\pi h^2 v_F^2} \right) (L_A - L_0) A_s
\]

(2)

where \( A_s = \tau_s/\tau_{sp} \) and \( A_{e,h} = \tau_{e,h}/\tau_{p(e,h)} \). \( \tau_s \) and \( \tau_{e,h} \) are the energy relaxation and \( \tau_{sp} \) and
the momentum relaxation time of the surface and bulk, respectively. $L_{A,D}$, $N_{A,D}$, $\tilde{\rho}_{h,e}$, $W_{p,n}$, and $D_{h,e}$ are thickness, electron density, mobility, range of depletion zone and diffusion coefficient of the acceptor (donator) layer, respectively. $v_F$ is the Fermi velocity of the surface states which are allowed to have a small band gap $\Delta_0$ due to hybridization at small thicknesses. $\alpha_0$ and $L_0$ are constants to be determined experimentally. The surface mobility is

$$
\mu_s^*(B) = \frac{\mu_1}{1 + \mu_1^2 B^2} \begin{bmatrix} 1 & \mu_1 B \\ -\mu_1 B & 1 \end{bmatrix},
$$

(3)

with $\mu_1 = e\tau_{sp} v_F^2 / \Delta_0 = e\tau_{sp} v_F^2 / 2k_B T_0$. For weak magnetic field, we have $\mu_1 B \ll 1$, $\mu_{xx} = \mu_{yy} = \mu_1$ and $\mu_{xy} = -\mu_{yx} = \mu_2 B$.

When $B \to 0$ the conductance correction (see Eq. C20 in the Supplement) is given by

$$
\delta \sigma(T_e, u_s) \equiv \sigma_{\text{tot}}(T_e, u_s) - \sigma_{\text{tot}}^{(0)}(T_e, u_s)
$$

$$
= -\mu_0^T \left( \frac{\alpha_0 \Delta_0}{2\pi \hbar^2 v_F^2} \right) (L_A - L_0) \left[ \frac{\tau_{s0}^T(T_e, u_s)}{\tau_{s0}^T(T_e, u_s) + \tau_{\text{pair}}^T(T_e, u_s)} \right]
$$

$$
\approx -\sigma_0^s \left[ \frac{\tau_{s0}^T(T_e, u_s)}{\tau_{\text{pair}}^T(T_e, u_s)} \right],
$$

(4)

where $\mu_0^s = e\tau_{s0}^0 v_F^2 / \Delta_0 = e\tau_{s0}^0 v_F^2 / 2k_B T^*$, $\sigma_0^s$ and $\tau_0^s$ are the mobility, conductivity and energy-relaxation time, respectively, of surface electrons in the absence of EEI.

Here, $\tau_{\text{pair}}^s(T_e, u_s)$ is the additional electron-electron pair scattering contribution to the inverse energy relaxation time (see Eq. C16 and C17 in the Supplement), given by

$$
\frac{1}{\tau_{\text{pair}}^s(T_e, u_s)} = \frac{1}{n_0 A} \sum_{\mathbf{k}_\parallel} \frac{f_{k_\parallel}}{\tau_{\text{pair}}^s(\mathbf{k}_\parallel)} \approx \frac{1}{16\pi^4 \hbar n_0} \left( \frac{e^2}{2\varepsilon_0 e_b} \right)^2
$$

$$
\times \int_{q_0}^{1/\delta} dq \| q_\parallel \left\{ 1 - \left( \frac{e^2q_\parallel}{2\varepsilon_0 e_b} \right) \frac{32k_B T^*}{\pi \hbar^2 \Gamma_0^2} T^* \left( \frac{T^*}{T_e} \right) D \right\} \int d^2k_\parallel f_{k_\parallel}^s
$$

$$
\times \int d^2k_\parallel' \left[ f_{k_\parallel}^s (1 - f_{k_\parallel}^s) (1 - f_{k_\parallel'}^s) + f_{k_\parallel}^s f_{k_\parallel'}^s (1 - f_{k_\parallel}^s) \right]
$$

$$
\times \frac{\Gamma_0 / \pi}{(\varepsilon_{k_\parallel} + \varepsilon_{k_\parallel}' - \varepsilon_{k_\parallel}^s - \varepsilon_{k_\parallel}^{s^+})^2 + \Gamma_0^2},
$$

(5)

where
\[ f_{k ||}^n \approx \frac{2 \pi \hbar^2 v_F^2 n_0}{(k_B T_e)^2(1 + \Delta_0/k_B T_e)} \exp \left( -\frac{\varepsilon_{k ||} - \Delta_0}{k_B T_e} \right), \]

and \( n_0 = (m^* / 2 \pi \hbar^2) E_F^a = (\Delta_0 / 2 \pi \hbar^2 v_F^2) E_F^a = (k_B T^*/\pi \hbar^2 v_F^2) E_F^a \sim \alpha_0 (L_{A} - L_{0}). \) We use \( \gamma = +1 \) and \( q_0 = \Gamma_0 / \hbar v_F \) as a cutoff for \( q_{\parallel} \to 0. \) \( k_{\parallel}^\pm \) stands for \( k_{\parallel} \pm q_{\parallel} \) and \( D = C_0 + \ln (T_e/T^*) - 1/2 \ln 2 (T_e/T^*)^2. \) Here, pair scattering of bulk electrons will lead to reduction of total conductivity.

Important conclusions can be drawn from these theoretical results. Firstly, for a weak magnetic field \( B, \) the longitudinal conductivity becomes independent of \( B, \) although the Hall conductivity depends on \( B \) (see Eqns. 2 and 3). Furthermore, Eqn. 5 for the energy relaxation time indicates that both pair scattering and screening effects from EEI do not depend on \( B. \) This is a strong argument in favor analyzing EEI by applying a weak magnetic field, in order to separate quantum corrections due to WAL from \( \delta \sigma \) (see Eqn. 4 and Fig. 4(b)).

Secondly, the experimentally found strong increase of EEI with the sample thickness (see Fig. 4(a)) can be directly derived from the theory. Eqn. 4 gives the dominant EEI-induced change in surface longitudinal conductivity at low \( B \) fields and reveals its thickness dependence. On the one hand, we know that \( \delta \sigma \propto \sigma_0^s \sim (L_{A} - L_{0}). \) On the other hand, we find that the ratio \( \tau_0^s / \tau_{\text{pair}}^s \propto n_0 \sim (L_{A} - L_{0}). \) Overall, \( \delta \sigma \propto (L_{A} - L_{0})^2 \) which for \( (L_{A} - L_{0})/L_{0} \ll 1 \) leads to \( \delta \sigma \propto L_{A}. \) This linear relationship describes our experimental findings remarkably well (see Fig. 4(a)). Finally, bulk electrons can also screen impurity scattering of surface electrons, but it becomes insignificant due to the large separation between the surface layer and the center of film.

Lastly, we determine the WAL contribution form the difference between the saturated and zero field amplitude \( \Delta f. \) We have shown already that EEI is independent of the magnetic...
FIG. 4. (a) Difference of conductivity correction $\delta \sigma$ between 5 K and base temperature as a function of the Sb$_2$Te$_3$-thickness. (b) Change of the slope $f$ with an external, perpendicular magnetic field, as shown in Fig. 3. (c) The screening factor $F$ calculated from $f = n(1 - 3/4 \times F)$, assuming the number of 2D states $n$ is 1 (black squares) or 2 (red circles). The screening is negative for $n = 1$ and between 0 and 1 for $n = 2$, supporting the presence of more than one 2D channel. (d) Number of 2D channels $\alpha$ from WAL, obtained as described in the text. A value of 0.5 corresponds to one 2D channel.
field, and thus the change of the slope of the $\delta \sigma$ with and without applied field can be attributed to WAL alone. The number of 2D states can be calculated using $\Delta f = p \times \alpha$ with $p = 1$, which characterizes the temperature dependence of the coherence length (see Ref.\textsuperscript{15}). We obtain $\alpha \approx 0.5$, i.e. that only one TSS is present at all thicknesses\textsuperscript{18,33,34} (see Fig. 4(d)). Since a TSS on the top surface has been confirmed in ARPES experiments\textsuperscript{17}, we conclude that the TSS at the bottom must be disrupted.

In summary, topological $p$-$n$ junctions exhibit a rich set of transport characteristics related to their topological surfaces states. At low temperature, WAL and EEI compete in reducing the conductivity. The fact that EEI are unaffected by an external magnetic field was taken advantage of to determine the number of 2D channels. While exactly one was found from WAL, at least two are contributing to EEI. The growing presence of bulk states does not lead to stronger screening. On the contrary, conductivity corrections due to EEI are getting stronger with increase thickness. This effect could be understood within a semiclassical Boltzmann theory.

**ACKNOWLEDGMENTS**

D.B., D.R. and V.N. acknowledge funding from the Leverhulme Trust, UK, D.B., R.M., D.R., and V.N. acknowledge funding from EPSRC (UK). D.H. would like to thank the support from the Air Force Office of Scientific Research (AFOSR). G.M., M.L., J.K. and D.G. acknowledge financial support from the DFG-funded priority programme SPP1666.
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38 see Supplementary Material
Appendix A: Bulk Boltzmann Moment Equation

For an \( n \)-doped semiconductor bulk material, we will start with the standard semiclassical Boltzmann transport equation for electrons within a conduction band \( \varepsilon_c(k) \) of a bulk. For this case, the electron distribution function \( f_c(r, k; t) \) satisfies

\[
\frac{\partial f_c(r, k; t)}{\partial t} + \left\langle \frac{dR_0(t)}{dt} \right\rangle_{av} \cdot \nabla_r f_c(r, k; t) + \left\langle \frac{dK_0(t)}{dt} \right\rangle_{av} \cdot \nabla_k f_c(r, k; t) = \left. \frac{\partial f_c(r, k; t)}{\partial t} \right|_{coll}, \tag{A1}
\]

where \( r \) is a three-dimensional position vector, \( k \) is a three-dimensional wave vector, and the term on the right-hand side of this equation corresponds to the collision contribution of electrons with other electrons, impurities, and phonons. Moreover, for conduction-band electrons, we can define, in a semiclassical way\(^2\), their group velocity through \( v_c(k) = \nabla_k \varepsilon_c(k) / \hbar \equiv \langle dR_0(t)/dt \rangle_{av} \), where \( R_0(t) \) is the center-of-mass position vector. Furthermore, we introduce the semiclassical Newton-like force equation\(^2\) for the wave vector of miniband electrons, yielding

\[
\hbar \left\langle \frac{dK_0(t)}{dt} \right\rangle_{av} = F_c(k, t) = -e \left[ \mathbf{F}(t) + \mathbf{v}_c(k) \times \mathbf{B}(t) \right], \tag{A2}
\]

where \( K_0(t) \) is the center-of-mass wave vector, \( \mathbf{E}(t) \) and \( \mathbf{B}(t) \) are the external electric and magnetic fields, respectively, and \( F_c(k, t) \) is the electromagnetic force acting on an electron in the \( k \) state.

Based on Eq. (A1), the zeroth-order Boltzmann moment equation can be obtained by summing over all the \( k \) states on both sides of this equation. This gives rise to the electron number conservation equation, i.e.,

\[
\frac{\partial \rho_c}{\partial t} + \nabla_r \cdot \mathbf{J}_c = 0, \tag{A3}
\]

where the electron number volume density \( \rho_c(r, t) \) and particle-number current density \( \mathbf{J}_c(r, t) \) (per area) are defined by

\[
\rho_c(r, t) \equiv \frac{2}{V} \sum_k f_c(r, k; t), \tag{A4}
\]
\[ J_e(r, t) \equiv \frac{2}{V} \sum_k v_c(k) f_e(r, k; t), \quad (A5) \]

and \( V \) is the sample volume.

For the first-order Boltzmann moment equation, we have to employ the so-called Fermi kinetics. Therefore, we first introduce the relaxation-time approximation for the electron collision, given by

\[
\frac{\partial f_c(r, k; t)}{\partial t} \bigg|_{\text{coll}} = -\frac{f_c(r, k; t) - f_0[\varepsilon_c(k), T; u_c]}{\tau_c(k)}, \quad (A6)
\]

where \( f_0[\varepsilon_c(k), T; u_c] = \{\exp[\varepsilon_c(k) - u_c/k_BT] + 1\}^{-1} \) is the Fermi function for electrons in thermal-equilibrium states, \( T \) is the lattice temperature, \( u_c \) is the chemical potential of electrons in the system, and \( \tau_c(k) \) is the energy-relaxation time for electrons in the \( k \) state.

The detailed calculation of \( \tau_c(k) \) has been presented in Appendix [D]. The chemical potential \( u_c \) of the system is determined self-consistently by

\[
2 \sum_k f_0[\varepsilon_c(k), T; u_c] = \int d^3r \rho_c(r, t) \equiv \frac{2}{V} \sum_k \int d^3r f_c(r, k; t) = N_e, \quad (A7)
\]

where \( N_e \) represents the total number of electrons in the system. Finally, by applying this relaxation-time approximation to the standard Boltzmann transport equation in Eq. (A1), we obtain

\[
f_c(r, k; t) + \tau_e(T, u_c) \frac{\partial f_c(r, k; t)}{\partial t} \approx f_0[\varepsilon_c(k), T; u_c]
\]

\[
-\frac{\tau_e(T, u_c)}{\hbar} F_c(k, t) \cdot \nabla_k f_0[\varepsilon_c(k), T; u_c] - \tau_e(T, u_c) \nabla_r \cdot \{v_c(k) f_0[\varepsilon_c(k), T; u_c] \}
\]

\[
= f_0[\varepsilon_c(k), T; u_c] - \frac{\tau_e(T, u_c)}{\hbar} F_c(k, t) \cdot \nabla_k f_0[\varepsilon_c(k), T; u_c], \quad (A8)
\]

where we have used the fact that \( T \) is spatially uniform throughout the system and equals the lattice temperature, and the statistically-averaged energy-relaxation time \( \tau_e(T, u_c) \) is defined by

\[
\frac{1}{\tau_e(T, u_c)} = \frac{2}{N_e} \sum_k f_0[\varepsilon_c(k), T; u_c] \frac{1}{\tau_c(k)}. \quad (A9)
\]
By introducing another inverse momentum-relaxation time tensor $\tau_p^{-1}$ and using Eq. (A2), we can further write the force-balance equation for a macroscopic drift velocity $v_d(t)$, which yields

$$
\frac{dv_d(t)}{dt} = -\tau_p^{-1} \cdot v_d(t) + \mathcal{M}^{-1} \cdot F_e(t)
$$

$$
= -\tau_p^{-1} \cdot v_d(t) - e\mathcal{M}^{-1} \cdot [E(t) + v_d(t) \times B(t)] = 0 ,
$$

(A10)

where $F_e(t) = -e [E(t) + v_d(t) \times B(t)]$ is the macroscopic electromagnetic force, and the statistically-averaged inverse effective-mass tensor $\mathcal{M}^{-1}$ for conduction-band electrons is given by

$$
\mathcal{M}^{-1}_{ij}(T, u_c) = \frac{2}{N_e} \sum_k \left[ \frac{1}{\hbar^2} \frac{\partial^2 \varepsilon_c(k)}{\partial k_i \partial k_j} \right] f_0[\varepsilon_c(k), T; u_c] ,
$$

(A11)

and $i, j = x, y, z$. The detailed calculations for the inverse momentum-relaxation time tensor $\tau_p^{-1}$ in our system can be found in Appendix E. Moreover, the internal Coulomb force between a pair of electrons will not contribute to this force-balance equation. The solution of Eq. (A10) can be formally expressed as

$$
v_d(t) = \mathcal{\mu}[B(t)] \cdot E(t) ,
$$

(A12)

where $\mathcal{\mu}[B(t)]$ is the so-called mobility tensor for conduction-band electrons, which also depends on $\tau_p^{-1}$ and $\mathcal{M}^{-1}$ in addition to $B(t)$. The details for calculating the mobility tensor $\mathcal{\mu}[B(t)]$ are presented in Appendix F. Using Eqs. (A10) and (A12), we can rewrite $F_e(t) = \left( \mathcal{M} \otimes \tau_p^{-1} \right) \cdot [\mathcal{\mu}[B(t)] \cdot E(t)]$, where $\mathcal{M}$ represents the inverse of $\mathcal{\mu}^{-1}$.

In a similar way, multiplying both sides of Eq. (A8) by $v_c(k)$ and summing over all the $k$ states afterwards, we get

$$
J_c(t) + \tau_e(T, u_c) \frac{\partial J_c(t)}{\partial t} = -\tau_e(T, u_c) \frac{2}{V} \sum_k v_c(k) \cdot F_e(t) \cdot v_c(k) \frac{\partial f_0[\varepsilon_c(k), T; u_c]}{\partial \varepsilon_c}
$$

$$
= -e\tau_e(T, u_c) \frac{2}{V} \sum_k v_c(k) \left\{ \left( \mathcal{M} \otimes \tau_p^{-1} \right) \cdot [\mathcal{\mu}[B(t)] \cdot E(t)] \right\} \cdot v_c(k) \left[ -\frac{\partial f_0[\varepsilon_c(k), T; u_c]}{\partial \varepsilon_c} \right].
$$

(A13)
From Eq. (A13) we know the particle-number current density $J_c$ is independent of $r$. Consequently, from Eq. (A3) we find that the number volume density $\rho_c$ becomes a constant $\rho_0$, determined by

$$\rho_0 = \frac{2}{V} \sum_k f_0[\varepsilon_c(k), T; u_c], \quad (A14)$$

which determines the chemical potential $u_c$ of the system for fixed $T$. If the external fields are static ones, i.e., $E_0$ and $B_0$, we get the charge current density $J_0$ from Eq. (A13)

$$J_0 = -e^2 \tau_e \left(\frac{2}{V}\right) \sum_k v_c(k) \left\{ \left( \mathcal{M} \otimes \mathcal{T}_p^{-1} \right) \cdot [\hat{\mu}(B_0) \cdot E_0] \right\} \cdot v_c(k) \left[ -\frac{\partial f_0[\varepsilon_c(k), T; u_c]}{\partial \varepsilon_c} \right]. \quad (A15)$$

In this case, the elements of the conductivity tensor $\bar{\sigma}(B_0)$ can be obtained through

$$\sigma_{ij}(B_0) = J_0 \cdot \hat{e}_i/(E_0 \cdot \hat{e}_j),$$

where $i, j = x, y, z$ and $\hat{e}_x, \hat{e}_y, \hat{e}_z$ are three unit vectors in a position space. From Eq. (A15), we know that the conductivity tensor depends not only on the mobility tensor, but also on how electrons are distributed within an anisotropic conduction band.

As a special case, we consider an isotropic parabolic band structure given by $\varepsilon_c(k) = \hbar^2 k^2/2m^*$, we find from Eq. (A11) that $\mathcal{M}^{-1} = (1/m^*) \delta_{ij}$ and $\mathcal{M}_ij = m^* \delta_{ij}$, as well as $(\mathcal{T}_p^{-1})_{ij} = (1/\tau_p) \delta_{ij}$. In this case, from Eq. (F15) we obtain the mobility tensor as

$$\hat{\mu}[B(t)] = -\frac{\mu_0}{1 + \mu_0^2 B^2} \left[ \begin{array}{ccc} 1 + \mu_0^2 B_x^2 & -\mu_0 B_z + \mu_0^2 B_x B_y & \mu_0 B_y + \mu_0^2 B_x B_z \\ \mu_0 B_z + \mu_0^2 B_x B_y & 1 + \mu_0^2 B_y^2 & -\mu_0 B_x + \mu_0^2 B_y B_z \\ -\mu_0 B_y + \mu_0^2 B_x B_z & \mu_0 B_z + \mu_0^2 B_x B_y & 1 + \mu_0^2 B_z^2 \end{array} \right], \quad (A16)$$

where $\mu_0 = e\tau_p/m^*$, $B = \{B_x, B_y, B_z\}$, and $B^2 = B_x^2 + B_y^2 + B_z^2$. If we further assume $B = 0$, Eq. (A16) simply leads to $\mu_{ij} = -\mu_0 \delta_{ij}$. In this case, from Eq. (A15) we get the well-known result $J_0 = (\rho_0 e^2 \tau_e/m^*) E_0$, which implies $\sigma_{ij} = (\rho_0 e^2 \tau_e/m^*) \delta_{ij}$. For a $p$-doped semiconductor bulk material, similar equations can be derived for $f_v(r, k; t), \rho_v(r, t)$ and $J_v(t)$ with replacements of $\varepsilon_c(k), r_c(t), F_c(t), \tau_c(k), u_c, -e$ by $\varepsilon_v(k), r_v(t), F_h(t), \tau_v(k), u_v, +e$, respectively.
Appendix B: Surface Boltzmann Moment Equation

For a semiconductor sheet, we will also start with the standard semiclassical Boltzmann transport equation for electrons within conduction subbands $\varepsilon_n(k_\parallel)$ of a sheet, where $n = 1, 2$ for two spin-resolved conduction subbands within the bulk semiconductor bandgap. For this case, the electron distribution function $f_n(r_\parallel, k_\parallel; t)$ satisfies

\[
\begin{align*}
\frac{\partial f_n(r_\parallel, k_\parallel; t)}{\partial t} + \left\langle \frac{d\mathbf{R}_\parallel(t)}{dt} \right\rangle_{av} \cdot \nabla_{r_\parallel} f_n(r_\parallel, k_\parallel; t) \\
+ \left\langle \frac{d\mathbf{K}_\parallel(t)}{dt} \right\rangle_{av} \cdot \nabla_{k_\parallel} f_n(r_\parallel, k_\parallel; t) = \left. \frac{\partial f_n(r_\parallel, k_\parallel; t)}{\partial t} \right|_{coll},
\end{align*}
\]

(B1)

where $r_\parallel$ is a two-dimensional position vector on the bulk surface, $k_\parallel$ is a two-dimensional wave vector within the surface plane, and the term at the right-hand side of this equation corresponds to the collision contribution of electrons with other electrons, impurities, and phonons. Moreover, for conduction-subband electrons, we can define, in a semiclassical way, their group velocity through $v_n(k_\parallel) = \nabla_{k_\parallel} \varepsilon_n(k_\parallel)/\hbar \equiv \left\langle \frac{d\mathbf{R}_\parallel(t)}{dt} \right\rangle_{av}$. Furthermore, we introduce the semiclassical Newton-like force equation for the wave vector of miniband electrons, yielding

\[
\hbar \left\langle \frac{d\mathbf{K}_\parallel(t)}{dt} \right\rangle_{av} = F_n(k_\parallel, t) = -e \left[ E(t) + v_n(k_\parallel) \times B(t) \right],
\]

(B2)

where $E(t)$ and $B(t)$ are the external electric and magnetic fields, respectively, and $F_n(k_\parallel, t)$ is the electromagnetic force acted on an electron in the $k_\parallel$ state of the $n$th subband.

Based on Eq. (B1), the zeroth-order Boltzmann moment equation can be obtained by summing over all the $k_\parallel$ states and all the subbands on both sides of this equation. This gives rise to the electron number conservation equation, i.e.,

\[
\frac{\partial n_s}{\partial t} + \nabla_{r_\parallel} \cdot j_s = 0,
\]

(B3)

where the surface density of electron number $n_s(r_\parallel, t)$ and surface particle-number current density $j_s(r_\parallel, t)$ (per length) are defined by

\[
n_s(r_\parallel, t) \equiv \frac{1}{A} \sum_{n,k_\parallel} f_n(r_\parallel, k_\parallel; t),
\]

(B4)
\[ j_s(r_\parallel, t) \equiv \frac{1}{A} \sum_{n,k_\parallel} v_n(k_\parallel) f_n(r_\parallel, k_\parallel; t) , \quad (B5) \]

\( A \) is the surface area.

For the first-order Boltzmann moment equation, we again have to employ the so-called Fermi kinetics. Therefore, we first introduce the relaxation-time approximation for the electron collision, given by

\[ \frac{\partial f_n(r_\parallel, k_\parallel; t)}{\partial t} \bigg|_{\text{col}} = -\frac{f_n(r_\parallel, k_\parallel; t) - f_0[\varepsilon_n(k_\parallel), T; u_s]}{\tau_n(k_\parallel)} , \quad (B6) \]

where \( f_0[\varepsilon_n(k_\parallel), T; u_s] = \{\exp[\varepsilon_n(k_\parallel) - u_s]/k_B T]\} + 1 \}^{-1} \) is the Fermi function for electrons in thermal-equilibrium states, \( T \) is the lattice temperature, \( u_s \) is the chemical potential of surface electrons and \( \tau_n(k_\parallel) \) is the energy-relaxation time for electrons in the \( k_\parallel \) state of the \( n \)th subband. The surface chemical potential \( u_s \) is determined self-consistently by

\[ \sum_{n,k_\parallel} f_0[\varepsilon_n(k_\parallel), T; u_s] = \int d^2r_\parallel n_s(r_\parallel, t) \equiv \frac{1}{A} \sum_{n,k_\parallel} \int d^2r_\parallel f_n(r_\parallel, k_\parallel; t) = N_s , \quad (B7) \]

where \( N_s = n_0 A \) represents the total number of surface electrons for each spin and \( n_0 \) is the areal density for surface electrons. Finally, by applying this relaxation-time approximation to the standard Boltzmann transport equation in Eq. (B1), we obtain

\[ f_n(r_\parallel, k_\parallel; t) + \tau_s(T, u_s) \frac{\partial f_n(r_\parallel, k_\parallel; t)}{\partial t} \approx f_0[\varepsilon_n(k_\parallel), T; u_s] - \tau_s(T, u_s) \frac{\tau_s(T, u_s)}{\hbar} F_n(k_\parallel, t) \cdot \nabla_{k_\parallel} f_0[\varepsilon_n(k_\parallel), T; u_s] \cdot \{v_n(k_\parallel) f_0[\varepsilon_n(k_\parallel), T; u_s]\} \]

\[ = f_0[\varepsilon_n(k_\parallel), T; u_s] - \tau_s(T, u_s) \frac{\tau_s(T, u_s)}{\hbar} F_n(k_\parallel, t) \cdot \nabla_{k_\parallel} f_0[\varepsilon_n(k_\parallel), T; u_s] , \quad (B8) \]

where we have used the fact that \( T \) is uniform throughout the system and equals the lattice temperature, and the statistically-averaged surface energy-relaxation time \( \tau_s(T, u_s) \) is defined by

\[ \frac{1}{\tau_s(T, u_s)} = \frac{1}{N_s} \sum_{n,k_\parallel} \frac{f_0[\varepsilon_n(k_\parallel), T; u_s]}{\tau_n(k_\parallel)} , \quad (B9) \]
By introducing another inverse surface momentum-relaxation time tensor \( \hat{\tau}_{sp}^{-1} \) and using Eq. (B2), we can further write the force-balance equation for the macroscopic surface drift velocity \( v_s(t) \), which yields

\[
\frac{dv_s(t)}{dt} = -\hat{\tau}_{sp}^{-1} \cdot v_s(t) + \hat{M}_s^{-1} \cdot F_s(t)
\]

where the macroscopic surface electromagnetic force is \( F_s(t) = -e [E(t) + v_s(t) \times B(t)] \), and the statistically-averaged inverse effective-mass tensor for surface electrons is given by

\[
(M_s^{-1})_{ij} = \frac{1}{N_s} \sum_{n,k} \left[ \frac{1}{\hbar^2} \frac{\partial^2 \varepsilon_n(k)}{\partial k_i \partial k_j} \right] f_0[\varepsilon_n(k), T; u_s],
\]

and \( i, j = x, y \). The solution of Eq. (B10) can be formally written as

\[
v_s(t) = \hat{\mu}_s[B(t)] \cdot E(t),
\]

where \( \hat{\mu}_s[B(t)] \) is the so-called mobility tensor for surface electrons, which also depends on \( \hat{\tau}_{sp}^{-1} \) and \( \hat{M}_s^{-1} \) in addition to \( B(t) \). Using Eqs. (B10) and (B12), we can rewrite \( F_s(t) = (\hat{M}_s \otimes \hat{\tau}_p^{-1}) \cdot [\hat{\mu}_s[B(t)] \cdot E(t)] \), where \( \hat{M}_s \) represents the inverse of \( \hat{M}_s^{-1} \).

In a similar way, multiplying both sides of Eq. (B8) by \( v_n(k) \) and summing over all the \( k \) states and all the subbands afterwards, we get

\[
\dot{j}_s(T, u_s) - \tau_s(T, u_s) \frac{\partial j_s(t)}{\partial t} = -\tau_s(T, u_s) \frac{1}{A} \sum_{n,k} v_n(k) \left[ F_s(t) \cdot v_n(k) \right] \frac{\partial f_0[\varepsilon_n(k), T; u_s]}{\partial \varepsilon_n}
\]

\[
= e\tau_s(T, u_s) \frac{1}{A} \sum_{n,k} v_n(k) \left\{ (\hat{M}_s \otimes \tau_{sp}^{-1}) \cdot [\hat{\mu}_s[B(t)] \cdot E(t)] \right\} \cdot v_n(k) \left[ -\frac{\partial f_0[\varepsilon_n(k), T; u_s]}{\partial \varepsilon_n} \right].
\]

From Eq. (B13) we know the surface particle-number current density \( j_s \) is independent of \( r \). As a result, from Eq. (B3) we find the surface number areal density \( n_s \) becomes a constant \( n_0 \), determined by

\[
n_0 = \frac{1}{A} \sum_{n,k} f_0[\varepsilon_n(k), T; u_s],
\]
which determines the surface chemical potential \( u_s \) for fixed \( T \). If the external fields are static ones, i.e., \( E_0 \) and \( B_0 \), we get the surface charge-current density \( j_0 \) from Eq. (B13)

\[
j_0 = e^2 \tau_s \left( \frac{1}{A} \right) \sum_{n, k_i} v_n(k_i) \left\{ \left( \mathcal{M}_s \otimes \mathcal{T}_{sp}^{-1} \right) \cdot [\mu_s(B_0) \cdot E_0] \right\} \cdot v_n(k_i) \left[ - \frac{\partial f_0[\varepsilon_n(k_i), T; u_s]}{\partial \varepsilon_n} \right].
\]

(B15)

In this case, the elements of the conductivity tensor \( \hat{\sigma}(B_0) \) can be obtained through

\[
\sigma_{ij}(B_0) = j_0 \cdot \hat{e}_i / (E_0 \cdot \hat{e}_j),
\]

where \( i, j = x, y \) and \( \hat{e}_x, \hat{e}_y \) are the unit vectors. From Eq. (B15), we know that the conductivity tensor not only depends on the mobility tensor, but also depends on how electrons are distributed within anisotropic conduction subbands.

Appendix C: Coulomb Effect on Surface Conductivity

From Eqs. (E6) and (E14), we find the total inverse momentum-relaxation-time tensor \( \mathcal{T}_{sp}^{-1} = \mathcal{T}_{s,i}^{-1} + \mathcal{T}_{s,ph}^{-1} \) in Eq. (B15) through

\[
\mathcal{T}_{s,i}^{-1} = - \frac{2\sigma_i}{n_0 A} \sum_{q} |U_i(q)|^2 \left\{ \frac{\partial \text{Im} \left[ \Pi_s(q || \omega) \right]}{\partial \omega} \right\}_{\omega = \Gamma_0} \left\{ \mathcal{M}_s^{-1} \otimes [q || q]^T \right\},
\]

(C1)

and

\[
\mathcal{T}_{s,ph}^{-1} = - \frac{4}{n_0 A} \sum_{q || \lambda} \left| C_{q || \lambda} \right|^2 \left\{ \frac{\partial \text{Im} \left[ \Pi_s(q || \omega) \right]}{\partial \omega} \right\}_{\omega = \omega_{\mathcal{LO}}} \left\{ \mathcal{M}_s^{-1} \otimes [q || q]^T \right\}
\]

\[
\times \left[ N_0(\omega_{q || \lambda}, T) - N_0(\omega_{q || \lambda} + q || v_d, T_e) \right] \left\{ \mathcal{M}_s^{-1} \otimes [q || q]^T \right\}
\]

\[
- \frac{4}{n_0 A} \sum_{q} \left| C_{q} \right|^2 \left\{ \frac{\partial \text{Im} \left[ \Pi_s(q || \omega) \right]}{\partial \omega} \right\}_{\omega = \omega_{\mathcal{LO}}} \left\{ \mathcal{M}_s^{-1} \otimes [q || q]^T \right\}
\]

\[
\times \left[ N_0(\omega_{\mathcal{LO}}, T) - N_0(\omega_{\mathcal{LO}} + q || v_d, T_e) \right] \left\{ \mathcal{M}_s^{-1} \otimes [q || q]^T \right\}
\]

(C2)

with

\[
[q || q]^T = \begin{bmatrix}
q_x^2 & q_x q_y \\
q_y q_x & q_y^2
\end{bmatrix},
\]
where $\Gamma_0$ is the inverse of particle lifetime due to vertex correction, $\sigma_i$ is the areal density of impurities, $\omega_{q||\lambda}$ and $\omega_{LO}$ are the frequencies of acoustic and longitudinal-optical phonons, $N_0(\omega, T) = [\exp(h\omega/k_BT) - 1]^{-1}$ is the Bose function for thermal-equilibrium phonons, and $T_e$ is the hot-electron temperature due to inelastic phonon scatterings. Here, we assume that only the lowest subband of surface electrons is occupied, and the imaginary part of the screened polarization function, $\text{Im} \left[ \Pi_s(q||, \omega) \right]$, is given by

$$
\text{Im} \left[ \Pi_s(q||, \omega) \right] = \frac{\text{Im} \left[ \Pi_s^{(0)}(q||, \omega) \right]}{\left\{ 1 - v_s(q||) \text{Re} \left[ \Pi_s^{(0)}(q||, \omega) \right] \right\}^2 + \left\{ v_s(q||) \text{Im} \left[ \Pi_s^{(0)}(q||, \omega) \right] \right\}^2} , \quad \text{(C3)}
$$

where the denominator represents the screening effect, $v_s(q||) = (e^2/2\epsilon_0\epsilon_b) \exp(-q||\delta_s)$ is the two-dimensional Fourier transform of a bare Coulomb potential, $\epsilon_b$ is the dielectric constant of the host material and $\delta_s$ is the thickness of the surface layer. Moreover, the bare polarization function $\Pi_s^{(0)}(q||, \omega)$ in Eq. (C3) is calculated within the random-phase approximation as

$$
\Pi_s^{(0)}(q||, \omega) = \frac{2}{A} \sum_{\gamma, \gamma'} \sum_{k||} \mathcal{F}_{\gamma, \gamma'}(k||, q||) \frac{f_{\gamma, k||}^s - f_{\gamma', k|| + q||}^s}{\hbar\omega + i0^+ - \varepsilon_{\gamma, k|| + q||}^s + \varepsilon_{\gamma, k||}^s} , \quad \text{(C4)}
$$

where the overlapping factor for zero-bandgap is given by

$$
\mathcal{F}_{\gamma, \gamma'}(k||, q||) = \frac{1}{2} \left[ 1 + \gamma \gamma' \frac{k|| \cdot (k|| + q||)}{|k|| (k|| + q||)|} \right] ,
$$

$\varepsilon_{\gamma, k||}^s = \gamma\varepsilon_{k||}^s$ and $f_{\gamma, k||}^s = f_0[\varepsilon_{\gamma, k||}^s, T_e; u_s] = \left\{ 1 + \exp \left[ (\gamma\varepsilon_{k||}^s - u_s)/k_BT_e \right] \right\}^{-1}$ is the Fermi-Dirac function for thermal-equilibrium surface electrons at an elevated temperature $T_e$.

Let us first consider the case with a zero bandgap, i.e., $\Delta_0 = 0$. For $T_e = 0$ and in the long-wavelength limit ($q|| \rightarrow 0$), we obtain from Eq. (C4)

$$
\text{Re} \left[ \Pi_s^{(0)}(q||, \omega) \right] = \frac{q||^2}{4\pi\hbar\omega} \left[ \frac{4E_F^s}{\hbar\omega} + \ln \left| \frac{2E_F^s - \hbar\omega}{2E_F^s + \hbar\omega} \right| \right] , \quad \text{(C5)}
$$

$$
\text{Im} \left[ \Pi_s^{(0)}(q||, \omega) \right] = -\frac{q||^2}{4\hbar\omega} \Theta(\hbar\omega - 2E_F^s) , \quad \text{(C6)}
$$

where $\Theta(x)$ is a unit-step function, $E_F^s = \hbar\nu_Fk_F^s$ and $k_F^s = \sqrt{4\pi n_0}$. On the other hand, in the high-temperature $k_BT_e \gg E_F^s$, $\hbar\omega$ and long-wavelength $q|| \rightarrow 0$ limits, we arrive at
\[
\text{Re} \left[ \Pi_s^{(0)}(q, \omega) \right] = \frac{2 \ln 2}{\pi} \left( \frac{q^2}{\hbar^2 \omega^2} \right) k_B T_e \left[ 1 + \frac{(E_F^s)^4}{128 (2 \pi^3 (k_B T_e)^4)} \right], \quad (C7)
\]

\[
\text{Im} \left[ \Pi_s^{(0)}(q, \omega) \right] = -\frac{q^2}{16 k_B T_e} \left[ 1 - \frac{\hbar^2 \omega^2}{48 (k_B T_e)^2} \right]. \quad (C8)
\]

For \( T_e \neq 0 \) but \( k_B T_e \ll E_F^s, \hbar \omega \), the zero-temperature results in Eqs. (C5) and (C6) can be formally generalized to:

\[
\text{Re} \left[ \Pi_s^{(0)}(q, \omega) \right] = \frac{q^2}{4 \pi \hbar \omega} \left[ 4 u_s(T_e) + \ln \left| \frac{2 u_s(T_e) - \hbar \omega}{2 u_s(T_e) + \hbar \omega} \right| \right], \quad (C9)
\]

\[
\text{Im} \left[ \Pi_s^{(0)}(q, \omega) \right] = -\frac{q^2}{8 \hbar \omega} \left[ 1 + \tanh \left( \frac{\hbar \omega - 2 u_s(T_e)}{4 k_B T_e} \right) \right] \quad (C10)
\]

with a chemical potential at finite temperatures

\[
u_s(T_e) \approx E_F^s \left[ 1 - \frac{\pi^2}{6} \left( \frac{k_B T_e}{E_F^s} \right)^2 \right].
\]

If we further consider a gaped and undoped subband for surface electrons with an energy gap \( \Delta_0 \) and \( E_F^s \to 0 \), then we acquire the generalized overlapping factor

\[
\mathcal{F}_{\gamma, \gamma'}(k, q) = \frac{1}{2} \left[ 1 + \gamma \gamma' \frac{(\hbar v_F)^2 k \cdot (k + q) + \Delta_0^2}{\sqrt{\hbar^2 v_F^2 k^2 + \Delta_0^2} \sqrt{\hbar^2 v_F^2 k^2 + \Delta_0^2} + \Delta_0^2} \right].
\]

Moreover, Eq. (C4) under the condition of \( k_B T_e \gg \Delta_0, \hbar \omega \) turns into:

\[
\text{Re} \left[ \Pi_s^{(0)}(q, \omega) \right] = \frac{4 k_B T_e q^2}{\pi \hbar^2 \omega^2} \left\{ 2 \ln 2 - \left( \frac{\Delta_0}{k_B T_e} \right)^2 \left[ C_0 - \ln \left( \frac{\Delta_0}{2 k_B T_e} \right) \right] \right\}, \quad (C11)
\]

\[
\text{Im} \left[ \Pi_s^{(0)}(q, \omega) \right] = -\frac{q^2}{16 k_B T_e} \left( 1 - \frac{\Delta_0}{\hbar \omega} \right), \quad (C12)
\]

where \( C_0 \approx 0.79 \) is a constant.

Firstly, let us consider only the impurity scattering at low temperatures. We know from Eq. (C1) that \( \tau_{sp}^{-1} \) becomes diagonal and its identical diagonal element \( 1/\tau_{sp} \) is given by
leads to

\[
\frac{1}{\tau_{sp}} = -\frac{\sigma_i}{2\pi n_0 m^*} \left( \frac{Z^* e^2}{2\epsilon_0 \epsilon_b} \right)^2 \int_0^{1/\delta_s} dq_{||} q_{||} \left\{ \frac{\partial \text{Im} \left[ \Pi_s(q_{||}, \omega) \right]}{\partial \omega} \right\}_{\omega = \Gamma_0}.
\]

(C13)

By making use of the results in Eqs. (C11) and (C12), Eq. (C13) for impurity scattering leads to

\[
\frac{1}{\tau_{sp}} \approx \frac{v_F^2}{128\pi \hbar \Gamma_0^2 \delta_s k_B T^*} \left( \frac{\sigma_i}{n_0} \right) \left( \frac{Z^* e^2}{2\epsilon_0 \epsilon_b \delta_s} \right)^2 \left( \frac{T^*}{T_e} \right)
\]

\[
\times \left\{ 1 - \frac{128}{5\pi \hbar \Gamma_0} \left( \frac{e^2}{2\epsilon_0 \epsilon_b \delta_s} \right) \left( 1 - \frac{3k_B T^*}{\hbar \Gamma_0} \right) \left( \frac{T^*}{T_e} \right) \left[ C_0 \ln \left( \frac{T_e}{T^*} \right) - \frac{\ln 2}{2} \left( \frac{T_e}{T^*} \right)^2 \right] \right\},
\]

(C14)

which satisfies \(1/\tau_{e,h} \propto 1/n_0 \sim (L_A - L_0)^{-1}\), where we have used \(1/m^*_s = v_F^2/\Delta_0\) and defined \(T^* = \Delta_0/2k_B\).

The results for phonon scattering can be obtained in a similar way. Furthermore, we find from Eq. (B15) that

\[
\dot{j}_0 = \left( \frac{e^2 v_F^2 \tau_s}{\Delta_0 \tau_{sp}} \right) \sum_{k_{||}} v^s_{k_{||}} \left\{ \left[ \mathbf{I}_0 \cdot [\mathbf{E}_0, B_0] \right] \cdot v^s_{k_{||}} \right\} \cdot v^s_{k_{||}} \left[ -\frac{\partial f_0[\epsilon^s_{k_{||}}, T_e; u_s]}{\partial \epsilon^s_{k_{||}}} \right],
\]

(C15)

where \(v^s_{k_{||}} = \hbar v_F^2 k_{||}/\Delta_0 = \hbar v_F^2 k_{||}/2k_B T^*\) is the group velocity of surface electrons and \(\tau_s(T_e, u_s)\) is the statistically averaged energy-relaxation time \(\tau_s(k_{||})\).

Secondly, including the screened pair scattering of surface electrons, we get \(1/\tau_{\text{pair}}(T, u_s)\) from Eqs. (D1)-(D3), as well as from Eqs. (C11) and (C12), yielding

\[
\frac{1}{\tau_{\text{pair}}(T_e, u_s)} = \frac{1}{n_0 A} \sum_{k_{||}} \frac{f^s_{k_{||}}}{\tau_{\text{pair}}^s(k_{||})} \approx \frac{1}{16\pi^4 \hbar n_0} \left( \frac{e^2}{2\epsilon_0 \epsilon_b} \right)^2
\]

\[
\times \int_0^{1/\delta_s} dq_{||} q_{||} \left\{ 1 - q_{||} \left( \frac{e^2}{2\epsilon_0 \epsilon_b} \right) \frac{32k_B T^*}{\pi \hbar^2 \Gamma_0^2} \left( \frac{T^*}{T_e} \right) \left[ C_0 \ln \left( \frac{T_e}{T^*} \right) - \frac{\ln 2}{2} \left( \frac{T_e}{T^*} \right)^2 \right] \right\}
\]

\[
\times \int d^2 k_{||} f^s_{k_{||}} \int d^2 k'_{||} \left[ f^s_{k_{||}} (1 - f^s_{k_{||}-q_{||}})(1 - f^s_{k'_{||}+q_{||}}) + f^s_{k_{||}-q_{||}} f^s_{k'_{||}+q_{||}} (1 - f^s_{k'_{||}}) \right]
\]

\[
\times \frac{\Gamma_0/\pi}{\left( \epsilon^s_{k_{||}} + \epsilon^s_{k'_{||}} - \epsilon^s_{k_{||}-q_{||}} - \epsilon^s_{k'_{||}+q_{||}} \right)^2 + \Gamma_0^2},
\]

(C16)

where

\[
f^s_{k_{||}} \approx \frac{2\pi \hbar^2 v_F^2 n_0}{(k_B T_e)^2(1 + \Delta_0/k_B T_e)} \exp \left( -\frac{\epsilon^s_{k_{||}} - \Delta_0}{k_B T_e} \right),
\]

(C17)
\[ n_0 = (m^*_e/2\pi\hbar^2)E^*_F = (\Delta_0/2\pi\hbar^2v_F^2)E^*_F = (k_BT^*/\pi\hbar^2v_F^2)E^*_F \sim \alpha_0(L_A - L_0) \] with \( L_A \) as the acceptor-layer thickness, \( \gamma \equiv 1 \) is taken and \( q_0 = \Gamma_0/hv_F \) is a cutoff for \( q_\parallel \to 0 \). Here, pair scattering of bulk electrons will lead to reduction of total conductivity. Furthermore, the relaxation time, respectively, of surface electrons in the absence of electron-electron interaction of the total conductivity due to the screened pair scattering of surface electrons is \( \tau_\parallel e,h = \tau_\parallel \). Even for two-dimensional electron gases in a quantum well, where they can also screen impurity scattering of surface electrons, but it becomes insignificant due to large separation between the surface layer and the center of the film.

Finally, by using Eq. (C5) the total conductivity is calculated as

\[ \sigma_{\text{tot}}(B) = e^2 \mu_s(B)N_A A_k \left[ (L_A - W_p) + \int_0^{W_p} dz \exp \left( -\frac{\beta e\hbar N_A}{2\epsilon_0 \Delta h} z^2 \right) \right] - e^2 \mu^0_s(B)N_D A_e \]

\[ \times \left[ (L_D - W_n) + \int_0^{W_n} dz \exp \left( -\frac{\beta e\hbar N_D}{2\epsilon_0 \Delta h} z^2 \right) \right] + e^2 \mu^\pm_s(B) \left( \frac{\alpha_0 \Delta_0}{2\pi\hbar^2v_F^2} \right) (L_A - L_0) A_s , \] (C18)

where \( A_s = \tau_s/\tau_{sp} \) and \( A_{e,h} = \tau_{e,h}/\tau_{p(e,h)} \). Here, the surface mobility is given by

\[ \mu_s(B) = \frac{\mu_1}{1 + \mu_1 B^2} \begin{bmatrix} 1 & \mu_1 B \\ -\mu_1 B & 1 \end{bmatrix} , \] (C19)

and \( \mu_1 = \epsilon \tau_{sp}v^2_F/\Delta_0 = \epsilon \tau_{sp}v^2_F/2k_BT^* \). For weak magnetic field, we have \( \mu_1 B \ll 1, \mu_{xx} = \mu_{yy} = \mu_1 \) and \( \mu_{xy} = -\mu_{yx} = \mu^2_1 B \). As \( B \to 0 \), we find from Eqs. (C18) and (C19) that the change of the total conductivity due to the screened pair scattering of surface electrons is given by

\[ \delta \sigma_{\text{tot}}(T_e, u_s) \equiv \sigma_{\text{tot}}(T_e, u_s) - \sigma_{\text{tot}}(0) = -\mu^*_0 \left( \frac{\alpha_0 \Delta_0}{2\pi\hbar^2v_F^2} \right) (L_A - L_0) \frac{\tau_0^*(T_e, u_s)}{\tau_0^*(T_e, u_s) + \tau_{\text{pair}}^*(T_e, u_s)} \approx -\sigma^*_0 \left[ \frac{\tau_0^*(T_e, u_s)}{\tau_{\text{pair}}^*(T_e, u_s)} \right] , \] (C20)

where \( \mu^*_0 = \epsilon \tau_{sp}v^2_F/\Delta_0 = \epsilon \tau_{sp}v^2_F/2k_BT^* \), \( \sigma^*_0 \) and \( \tau_0^* \) are the mobility, conductivity and energy-relaxation time, respectively, of surface electrons in the absence of electron-electron interaction (EEI).

Therefore, from Eq. (C20) we know \( \delta \sigma_{\text{tot}}(T_e, u_s) \propto \sigma^*_0 \sim (L_A - L_0) \). Meanwhile, we also find the ratio \( \tau_0^{*}/\tau_{sp}^{*} \propto n_0 \sim (L_A - L_0) \), as can be seen from Eqs. (C16) and (D2). Although the screening due to electron-electron interaction can weaken the impurity scattering and increases the mobility, the conductivity is not affected by the momentum-relaxation time \( \tau_{sp} \) of surface electrons. Even for two-dimensional electron gases in a quantum well, where they
acquire a static dielectric function\textsuperscript{[4]} $\epsilon_s(q_\parallel) \equiv \epsilon(q_\parallel, \omega = 0) = 1 + q_s/q_\parallel$ with a Thomas-Fermi screening length $1/q_s$, the screened impurity scattering can also increase their conductivity.
Appendix D: Energy-Relaxation Time

By using the detailed-balance condition, the energy-relaxation time $\tau_e(k)$ initially introduced in Eq. (A6) can be calculated according to

$$\frac{1}{\tau_e(k)} = \mathcal{W}_{\text{in}}(k) + \mathcal{W}_{\text{out}}(k), \quad (D1)$$

where the scattering-in rate for electrons in the final $k$-state is

$$\mathcal{W}_{\text{in}}(k) = n_i \frac{2\pi}{\hbar V} \sum_q |U_i(q)|^2 \left[ f_{k-q} \delta(\varepsilon_k - \varepsilon_{k-q}) + f_{k+q} \delta(\varepsilon_k - \varepsilon_{k+q}) \right] + \frac{2\pi}{\hbar V} \sum_{q,\lambda} |C_{q\lambda}|^2 \left\{ f_{k-q} N_0(\omega_{q\lambda}) \delta(\varepsilon_k - \varepsilon_{k-q} - \hbar \omega_{q\lambda}) + f_{k+q} [N_0(\omega_{q\lambda}) + 1] \delta(\varepsilon_k - \varepsilon_{k+q} + \hbar \omega_{q\lambda}) \right\} + \frac{2\pi}{\hbar V} \sum_q |C_q|^2 \left\{ f_{k-q} N_0(\omega_{LO}) \delta(\varepsilon_k - \varepsilon_{k-q} - \hbar \omega_{LO}) + f_{k+q} [N_0(\omega_{LO}) + 1] \delta(\varepsilon_k - \varepsilon_{k+q} + \hbar \omega_{LO}) \right\} + \frac{2\pi}{\hbar V^2} \sum_{k',q} |V_c(q)|^2 (1 - f_{k'}) f_{k-q} f_{k'+q} \delta(\varepsilon_k + \varepsilon_{k'} - \varepsilon_{k-q} - \varepsilon_{k'+q}), \quad (D2)$$

and the scattering-out rate for electrons in the initial $k$-state is

$$\mathcal{W}_{\text{out}}(k) = n_i \frac{2\pi}{\hbar V} \sum_q |U_i(q)|^2 \left[ (1 - f_{k+q}) \delta(\varepsilon_{k+q} - \varepsilon_k) + (1 - f_{k-q}) \delta(\varepsilon_{k-q} - \varepsilon_k) \right] + \frac{2\pi}{\hbar V} \sum_{q,\lambda} |C_{q\lambda}|^2 \left\{ (1 - f_{k+q}) N_0(\omega_{q\lambda}) \delta(\varepsilon_{k+q} - \varepsilon_k - \hbar \omega_{q\lambda}) + (1 - f_{k-q}) [N_0(\omega_{q\lambda}) + 1] \delta(\varepsilon_{k-q} - \varepsilon_k + \hbar \omega_{q\lambda}) \right\} + \frac{2\pi}{\hbar V} \sum_q |C_q|^2 \left\{ (1 - f_{k+q}) N_0(\omega_{LO}) \delta(\varepsilon_{k+q} - \varepsilon_k - \hbar \omega_{LO}) + (1 - f_{k-q}) [N_0(\omega_{LO}) + 1] \delta(\varepsilon_{k-q} - \varepsilon_k + \hbar \omega_{LO}) \right\} + \frac{2\pi}{\hbar V^2} \sum_{k',q} |V_c(q)|^2 f_{k'} (1 - f_{k-q}) (1 - f_{k'+q}) \delta(\varepsilon_{k-q} + \varepsilon_{k'+q} - \varepsilon_k - \varepsilon_{k'}). \quad (D3)$$

Here, $n_i$ is the volume density of ionized impurities. For simplicity, we have introduced the notations, $f_k \equiv f_0[\varepsilon_c(k), T; u_c]$, $\varepsilon_k \equiv \varepsilon_c(k)$, and $N_0(x) = [\exp(\hbar x/k_B T) - 1]^{-1}$ is the
Bose function for thermal-equilibrium phonons, and $\hbar \omega_{q\lambda}$ ($\hbar \omega_{LO}$) is the energy of acoustic (longitudinal-optical) phonons, respectively.

For the electron-impurity scattering, $N_i = n_i \mathcal{V}$ represents the total number of impurities in the system, and

$$|U_i(q)| = \frac{Z^* e^2}{\epsilon_0 \epsilon_b (q^2 + Q_c^2)} , \quad (D4)$$

where $Z^*$ is the charge number of fully-ionized impurity atoms.

For the scattering of electrons with acoustic phonons, we have

$$|C_{q\ell}|^2 = \frac{\hbar}{2 \rho_i \omega_{q\ell}} \left[ D_0^2 q^2 + \frac{9}{32} (eh_{14})^2 \right] \frac{q^2}{(q^2 + Q_c^2)} , \quad (D5)$$

$$|C_{qt}|^2 = \frac{\hbar}{2 \rho_i \omega_{qt}} \frac{13}{64} (eh_{14})^2 \frac{q^2}{(q^2 + Q_c^2)} , \quad (D6)$$

where $\lambda = \ell, t$ represents the longitudinal ($\ell$) and transverse ($t$) acoustic phonons, respectively, $\rho_i$ is the ion mass density, $D_0$ is the deformation potential, and $h_{14}$ is the piezoelectric constant.

For the scattering of electrons with longitudinal-optical phonons, on the other hand, we find

$$|C_q|^2 = \frac{\hbar \omega_{LO}}{2} \left( \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_s} \right) \frac{e^2}{\epsilon_0 (q^2 + Q_c^2)} , \quad (D7)$$

where $\epsilon_s$ and $\epsilon_\infty$ are the static and high-frequency dielectric constants of the host semiconductors.

Finally, for the scattering between two electrons, we require

$$V_c(q) = \frac{e^2}{\epsilon_0 \epsilon_b (q^2 + Q_c^2)} . \quad (D8)$$

For the surface case, the wave vector $k$ should be replaced by $k_\parallel$, and the Coulomb potential $1/[(q^2 + Q_c^2)]$ should be replaced by $\exp(-q_\parallel \delta_s)/2[(q_\parallel + q_c)]$, where $\delta_s$ represents the thickness of the surface layer.
Appendix E: Inverse Momentum-Relaxation-Time Tensor

The inverse momentum-relaxation-time tensor \( \tau_p^{-1} \) initially introduced in Eq. (A10) comes from the statistically-averaged frictional forces \( \mathbf{F}_x = \mathbf{F}_x^i + \mathbf{F}_x^{ph} \) due to scattering of electrons with impurities (i) and phonons (ph).

For electrons moving with a drift velocity \( \mathbf{v}_d \), the frictional force \( \mathbf{F}_x^i \) from the impurity scattering is calculated as

\[
\mathbf{F}_x^i = -n_i \frac{2\pi}{\hbar V} \sum_{k,q} \hbar \mathbf{q} (\hbar \mathbf{q} \cdot \mathbf{v}_d) \left| U_i(q) \right|^2 \left( -\frac{\partial f_k}{\partial \varepsilon_k} \right) \delta(\varepsilon_{k+q} - \varepsilon_k),
\]

and we have \( \tau_p^{-1} \cdot \mathbf{v}_d = -\frac{2}{N_v} \mathbf{M}^{-1} \cdot \mathbf{F}_x^i \), where

\[
\tau_p^{-1} = \frac{4\pi \hbar n_i}{\rho_0 V^2} \sum_{k,q} \left| U_i(q) \right|^2 \left( -\frac{\partial f_k}{\partial \varepsilon_k} \right) \delta(\varepsilon_{k+q} - \varepsilon_k) \left\{ \mathbf{M}^{-1} \otimes [\mathbf{q} \otimes \mathbf{q}^T] \right\},
\]

\( \rho_0 \) and \( n_i \) are the volume densities of electrons and impurities, and

\[
[q \otimes q^T] \equiv \begin{pmatrix}
q_x^2 & q_x q_y & q_x q_z \\
q_x q_y & q_y^2 & q_y q_z \\
q_x q_z & q_y q_z & q_z^2
\end{pmatrix}.
\]

Physically, we can rewrite Eq. (E1) as

\[
\mathbf{F}_x^i = n_i \sum_q \mathbf{q} |U_i(q)|^2 \text{Im} \left[ \Pi(q, \mathbf{q} \cdot \mathbf{v}_d) \right],
\]

where

\[
\text{Im} \left[ \Pi(q, \omega) \right] = \frac{\text{Im} \left[ \Pi^{(0)}(q, \omega) \right]}{\{1 - v_c(q) \text{Re} \left[ \Pi^{(0)}(q, \omega) \right]\}^2 + \{v_c(q) \text{Im} \left[ \Pi^{(0)}(q, \omega) \right]\}^2},
\]

and \( v_c(q) = e^2/\epsilon_0 \epsilon_k q^2 \) is the Fourier transform of a bare Coulomb potential. Moreover, the bare polarization function \( \Pi^{(0)}(q, \omega) \) introduced in Eq. (E4) is calculated within the random-phase approximation as

\[
\Pi^{(0)}(q, \omega) = \frac{2}{V} \sum_k \frac{f_k - f_{k+q}}{\hbar \omega + i0^+ - \varepsilon_{k+q} - \varepsilon_k}.
\]
Therefore, by using Eq. (E3), the inverse momentum-relaxation-time tensor $\hat{\tau}^{-1}_i$ given by Eq. (E2) can be rewritten into the form

$$\hat{\tau}^{-1}_i = -\frac{2n_i}{\rho_0 V} \sum_q |U_i(q)|^2 \left\{ \frac{\partial \text{Im}[\Pi(q, \omega)]}{\partial \omega} \right\}_{\omega=\Gamma_0} \left\{ \hat{M}^{-1} \otimes [q \otimes q^T] \right\}, \quad (E6)$$

where $\Gamma_0$ is the inverse of particle lifetime.

Similarly, for electrons moving with a drift velocity $\mathbf{v}_d$, the frictional force $\mathbf{F}^{ph}_x$ from the acoustic and optical phonon scattering is found to be

$$\mathbf{F}^{ph}_x = -\frac{1}{V} \sum_{k,q,\lambda} \hbar q \left\{ \Theta_{q,\lambda}^{em}(k) [N_0(\omega_{q\lambda}) + 1] - \Theta_{q,\lambda}^{abs}(k) N_0(\omega_{q\lambda}) \right\}$$

$$- \frac{1}{V} \sum_{k,q} \hbar q \left\{ \Theta_{q}^{em}(k) [N_0(\omega_{LO}) + 1] - \Theta_{q}^{abs}(k) N_0(\omega_{LO}) \right\}, \quad (E7)$$

where the emission and absorption rates for acoustic phonons are

$$\Theta_{q,\lambda}^{em}(k) = -\frac{4\pi}{\hbar} |C_{q\lambda}|^2 (\hbar \mathbf{q} \cdot \mathbf{v}_d) \left( -\frac{\partial f\k}{\partial \varepsilon_k} \right) \delta(\varepsilon_k - \varepsilon_{k+q} + \hbar \omega_{q\lambda}), \quad (E8)$$

$$\Theta_{q,\lambda}^{abs}(k) = -\frac{4\pi}{\hbar} |C_{q\lambda}|^2 (\hbar \mathbf{q} \cdot \mathbf{v}_d) \left( -\frac{\partial f\k}{\partial \varepsilon_k} \right) \delta(\varepsilon_k - \varepsilon_{k-q} - \hbar \omega_{q\lambda}). \quad (E9)$$

In a similar way, the emission and absorption rates for longitudinal-optical phonons are calculated as

$$\Theta_{q}^{em}(k) = -\frac{4\pi}{\hbar} |C_{q}|^2 (\hbar \mathbf{q} \cdot \mathbf{v}_d) \left( -\frac{\partial f\k}{\partial \varepsilon_k} \right) \delta(\varepsilon_k - \varepsilon_{k+q} + \hbar \omega_{LO}), \quad (E10)$$

$$\Theta_{q}^{abs}(k) = -\frac{4\pi}{\hbar} |C_{q}|^2 (\hbar \mathbf{q} \cdot \mathbf{v}_d) \left( -\frac{\partial f\k}{\partial \varepsilon_k} \right) \delta(\varepsilon_k - \varepsilon_{k-q} - \hbar \omega_{LO}). \quad (E11)$$

Therefore, from Eq. (E7) we get $\hat{\tau}^{-1}_i \cdot \mathbf{v}_d = -(2/N_e) \hat{M}^{-1} \cdot \mathbf{F}^{ph}_x$, where

$$\hat{\tau}^{-1}_i = \frac{8\pi\hbar}{\rho_0 V^2} \sum_{k,q,\lambda} |C_{q\lambda}|^2 \left\{ \frac{\partial f\k}{\partial \varepsilon_k} \right\} \left\{ \hat{M}^{-1} \otimes [q \otimes q^T] \right\}$$

$$\times \left\{ [N_0(\omega_{q\lambda}) + 1] \delta(\varepsilon_k - \varepsilon_{k+q} + \hbar \omega_{q\lambda}) + N_0(\omega_{q\lambda}) \delta(\varepsilon_k - \varepsilon_{k-q} - \hbar \omega_{q\lambda}) \right\}$$

$$+ \frac{8\pi\hbar}{\rho_0 V^2} \sum_{k,q} |C_q|^2 \left\{ \frac{\partial f\k}{\partial \varepsilon_k} \right\} \left\{ \hat{M}^{-1} \otimes [q \otimes q^T] \right\}$$

$$\times \left\{ [N_0(\omega_{LO}) + 1] \delta(\varepsilon_k - \varepsilon_{k+q} + \hbar \omega_{LO}) + N_0(\omega_{LO}) \delta(\varepsilon_k - \varepsilon_{k-q} - \hbar \omega_{LO}) \right\}. \quad (E12)$$
Again, we can rewrite Eq. (E7) as

\[ F_{x}^{ph} = 2 \sum_{q, \lambda} q |C_{q\lambda}|^2 \text{Im} [\Pi(q, \omega_{q\lambda} + q \cdot v_d)] [N_0(\omega_{q\lambda}, T) - N_0(\omega_{q\lambda} + q \cdot v_d, T_e)] \]

\[
+ 2 \sum_{q} q |C_q|^2 \text{Im} [\Pi(q, \omega_{LO} + q \cdot v_d)] [N_0(\omega_{LO}, T) - N_0(\omega_{LO} + q \cdot v_d, T_e)] ,
\]

and thus Eq. (E12) becomes

\[
\tau_{ph}^{-1} = -\frac{4}{\rho_0 V} \sum_{q, \lambda} |C_{q\lambda}|^2 \left\{ \frac{\partial \text{Im} [\Pi(q, \omega)]}{\partial \omega} \right\}_{\omega=\omega_{q\lambda}}
\times [N_0(\omega_{q\lambda}, T) - N_0(\omega_{q\lambda} + q \cdot v_d, T_e)] \left\{ \tilde{\mathbf{M}}^{-1} \otimes [q \otimes q^T] \right\}
\]

\[-\frac{4}{\rho_0 V} \sum_{q} |C_q|^2 \left\{ \frac{\partial \text{Im} [\Pi(q, \omega)]}{\partial \omega} \right\}_{\omega=\omega_{LO}}
\times [N_0(\omega_{LO}, T) - N_0(\omega_{LO} + q \cdot v_d, T_e)] \left\{ \tilde{\mathbf{M}}^{-1} \otimes [q \otimes q^T] \right\} ,
\]

where \( T_e \) is the temperature of hot electrons, determined from the energy-conservation equation\(^6\).

\[
(F_i^i + F_{x}^{ph}) \cdot v_d + 2 \sum_{q, \lambda} |C_{q\lambda}|^2 \omega_{q\lambda} \text{Im} [\Pi(q, \omega_{q\lambda} + q \cdot v_d)] [N_0(\omega_{q\lambda}, T) - N_0(\omega_{q\lambda} + q \cdot v_d, T_e)]
\]

\[
+ 2 \sum_{q} |C_q|^2 \omega_{LO} \text{Im} [\Pi(q, \omega_{LO} + q \cdot v_d)] [N_0(\omega_{LO}, T) - N_0(\omega_{LO} + q \cdot v_d, T_e)] = 0 .
\]

Finally, the inverse momentum-relaxation-time tensor is simply given by

\[
\tau_{p}^{-1} = \tau_i^{-1} + \tau_{ph}^{-1}.
\]

For the surface case, the wave vector \( q \) should be replaced by \( q_{||}, q_z = 0 \) and both \( \tilde{\mathbf{M}}_{s}^{-1} \) and \( \tilde{\tau}^{s-1}_p \) reduce to \( 2 \times 2 \) tensors.

**Appendix F: Mobility Tensor**

From the force-balance equation in Eq. (A10), by using the approximation \( \tau_p^{-1} \approx (1/\tau_j) \delta_{ij} \) for simplicity, we get the following group of linear equations for \( v_d = \{v_1, v_2, v_3\} \)
where we have used the notations $B = \{B_1, B_2, B_3\}$, $E = \{E_1, E_2, E_3\}$, $q = -e$ and $\mathbf{M}^{-1} = \{r_{ij}\}$. By defining the coefficient matrix $\hat{\mathbf{C}}$ for the above linear equations, i.e.,

$$\hat{\mathbf{C}} = \begin{bmatrix}
1 + q\tau_1(r_{12}B_3 - r_{13}B_2) & q\tau_1(r_{13}B_1 - r_{11}B_3) & q\tau_1(r_{11}B_2 - r_{12}B_1) \\
q\tau_2(r_{22}B_3 - r_{23}B_2) & 1 + q\tau_2(r_{23}B_1 - r_{21}B_3) & q\tau_2(r_{21}B_2 - r_{22}B_1) \\
q\tau_3(r_{32}B_3 - r_{33}B_2) & q\tau_3(r_{33}B_1 - r_{31}B_3) & 1 + q\tau_3(r_{31}B_2 - r_{32}B_1)
\end{bmatrix}, \quad (F4)$$

as well as the source vector $s$, given by

$$s = \begin{bmatrix}
q\tau_1(r_{11}E_1 + r_{12}E_2 + r_{13}E_3) \\
q\tau_2(r_{21}E_1 + r_{22}E_2 + r_{23}E_3) \\
q\tau_3(r_{31}E_1 + r_{32}E_2 + r_{33}E_3)
\end{bmatrix}, \quad (F5)$$

we can reduce the linear equations to a matrix equation $\hat{\mathbf{C}} \cdot \mathbf{v}_d = s$ with the formal solution $\mathbf{v}_d = \hat{\mathbf{C}}^{-1} \cdot s$. Explicitly, we find the solution $\mathbf{v}_d = \{v_1, v_2, v_3\}$ for $j = 1, 2, 3$ by

$$v_j = \frac{\text{Det}\{\hat{\Delta}_j\}}{\text{Det}\{\hat{\mathbf{C}}\}}, \quad (F6)$$

where

$$\hat{\Delta}_1 = \begin{bmatrix}
q\tau_1(r_{11}E_1 + r_{12}E_2 + r_{13}E_3) & q\tau_1(r_{13}B_1 - r_{11}B_3) & q\tau_1(r_{11}B_2 - r_{12}B_1) \\
q\tau_2(r_{21}E_1 + r_{22}E_2 + r_{23}E_3) & 1 + q\tau_2(r_{23}B_1 - r_{21}B_3) & q\tau_2(r_{21}B_2 - r_{22}B_1) \\
q\tau_3(r_{31}E_1 + r_{32}E_2 + r_{33}E_3) & q\tau_3(r_{33}B_1 - r_{31}B_3) & 1 + q\tau_3(r_{31}B_2 - r_{32}B_1)
\end{bmatrix}, \quad (F7)$$

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By assuming $r_{ij} = 0$ for $i \neq j$, $r_{jj} = 1/m_j^*$ and introducing the notation $\mu_j = q\tau_j/m_j^*$, we find

$$
\mathbf{\hat{C}} = \begin{bmatrix}
1 & -\mu_1 B_3 & \mu_1 B_2 \\
\mu_2 B_3 & 1 & -\mu_2 B_1 \\
-\mu_3 B_2 & \mu_3 B_1 & 1
\end{bmatrix}, \quad \text{(F10)}
$$

$$
\mathbf{\hat{\Delta}}_1 = \begin{bmatrix}
\mu_1 E_1 & -\mu_1 B_3 & \mu_1 B_2 \\
\mu_2 E_2 & 1 & -\mu_2 B_1 \\
\mu_3 E_3 & \mu_3 B_1 & 1
\end{bmatrix}, \quad \text{(F11)}
$$

$$
\mathbf{\hat{\Delta}}_2 = \begin{bmatrix}
1 & \mu_1 E_1 & \mu_1 B_2 \\
\mu_2 B_3 & \mu_2 E_2 & -\mu_2 B_1 \\
-\mu_3 B_2 & \mu_3 E_3 & 1
\end{bmatrix}, \quad \text{(F12)}
$$

$$
\mathbf{\hat{\Delta}}_3 = \begin{bmatrix}
1 & -\mu_1 B_3 & \mu_1 E_1 \\
\mu_2 B_3 & 1 & \mu_2 E_2 \\
-\mu_3 B_2 & \mu_3 B_1 & \mu_3 E_3
\end{bmatrix}, \quad \text{(F13)}
$$

and

$$
\text{Det}\{\mathbf{\hat{C}}\} = 1 + (B_1^2 \mu_2 \mu_3 + B_2^2 \mu_3 \mu_1 + B_3^2 \mu_1 \mu_2),
$$

$$
\text{Det}\{\mathbf{\hat{\Delta}}_1\} = \mu_1 E_1 + \mu_1 (B_3 E_2 \mu_2 - B_2 E_3 \mu_3) + \mu_1 \mu_2 \mu_3 B_1 (E \cdot B),
$$

$$
\text{Det}\{\mathbf{\hat{\Delta}}_2\} = \mu_2 E_2 + \mu_2 (B_1 E_3 \mu_3 - B_3 E_1 \mu_1) + \mu_1 \mu_2 \mu_3 B_2 (E \cdot B),
$$

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\[ \text{det} \{ \tilde{\Delta}_3 \} = \mu_3 E_3 + \mu_3 (B_2 E_1 \mu_1 - B_1 E_2 \mu_2) + \mu_1 \mu_2 \mu_3 B_3 (\mathbf{E} \cdot \mathbf{B}) . \]

For a special case with \( \mathbf{B} = \{0, 0, B_3\} \), we get

\[
\tilde{\mu}(B_3) = \frac{1}{1 + \mu_1 \mu_2 B_3^2} \begin{bmatrix}
\mu_1 & \mu_1 \mu_2 B_3 & 0 \\
-\mu_2 \mu_1 B_3 & \mu_2 & 0 \\
0 & 0 & \mu_3 (1 + \mu_1 \mu_2 B_3^2)
\end{bmatrix} . \quad (F14)
\]

If we further assume \( m_1^* = m_2^* = m_3^* = m^* \) and \( \tau_1 = \tau_2 = \tau_3 = \tau_p \), we obtain \( \text{det} \{ \tilde{\mathbf{C}} \} = 1 + \mu_0^2 B^2 \), \( \text{det} \{ \tilde{\Delta}_1 \} = -\mu_0 E_1 + \mu_0^2 (B_3 E_2 - B_2 E_3) - \mu_0^3 B_1 (\mathbf{E} \cdot \mathbf{B}) \), \( \text{det} \{ \tilde{\Delta}_2 \} = -\mu_0 E_2 + \mu_0^2 (B_3 E_1 - B_3 E_2) - \mu_0^3 B_2 (\mathbf{E} \cdot \mathbf{B}) \), and \( \text{det} \{ \tilde{\Delta}_3 \} = -\mu_0 E_3 + \mu_0^2 (B_2 E_1 - B_1 E_2) - \mu_0^3 B_3 (\mathbf{E} \cdot \mathbf{B}) \), where \( \mu_0 = e \tau_p / m^* \). As a result, the mobility tensor \( \tilde{\mu}(\mathbf{B}) \), which is defined through \( \mathbf{v}_d = \tilde{\mu}(\mathbf{B}) \cdot \mathbf{E} \), can be written as

\[
\tilde{\mu}(\mathbf{B}) = -\frac{\mu_0}{1 + \mu_0^2 B^2} \begin{bmatrix}
1 + \mu_0^2 B^2 & -\mu_0 B_3 + \mu_0^2 B_1 B_2 & \mu_0 B_2 + \mu_0^2 B_1 B_3 \\
\mu_0 B_3 + \mu_0^2 B_2 B_1 & 1 + \mu_0^2 B^2 & -\mu_0 B_1 + \mu_0^2 B_2 B_3 \\
-\mu_0 B_2 + \mu_0^2 B_3 B_1 & \mu_0 B_1 + \mu_0^2 B_1 B_2 & 1 + \mu_0^2 B_3^2
\end{bmatrix} . \quad (F15)
\]

where \( B^2 = B_1^2 + B_2^2 + B_3^2 \).

For the surface case with \( E_3 = 0 \) and \( v_3 = 0 \), \( \tilde{\mathbf{M}}_{s, \perp}^{-1} \), \( \tilde{\tau}_{s, \perp}^{-1} \) and \( \tilde{\mu}_s(\mathbf{B}) \) all reduce to \( 2 \times 2 \) tensors.

**Appendix G: Bulk and Surface Conductivity Tensors**

Under a parallel external electric field \( \mathbf{E} = (E_x, E_y, 0) \) and a perpendicular magnetic field \( \mathbf{B} = (0, 0, B) \), the total parallel current per length in a \( p-n \) junction structure is given by

\[
\int_{-L_A}^{L_D} dz \left[ j^{\parallel}_c(z) + j^{\parallel}_v(z) + j^{\pm}_c(z) \right] + j^{\pm}_s(z), \quad \text{where } L_D \text{ and } L_A \text{ are the distribution ranges for donors and acceptors, respectively. Here, by using the second-order Boltzmann moment equation, the bulk current densities are found to be } \]
where $D_{c,v}[u_{c,v}(z)] = (\sqrt{u_{c,v}(z)/4\pi^2}) (2m_{e,h}^*/\hbar^2)^{3/2}$ is the electron and hole density-of-states per spin, $u_{c,v}(z) = (\hbar k_F^{c,h})^2/2m_{e,h}^*$ and $k_F^{c,h}$ are Fermi energies and wave vectors in a bulk, $m_{e,h}^*$ are effective masses of electrons and holes, $\tau_{e,h}(z)$ and $\tau_{p(e,h)}(z)$ are bulk energy- and momentum-relaxation times, $v_{c,v}(k) = -\gamma_{e,h} \hbar k_||/m_{e,h}^*$, and $\gamma_{e,h} = -1$ (electrons) and $+1$ (holes), respectively. Similarly, the surface current per length is

$$j_s^\pm = \frac{e\tau_s m_s^*}{\tau_{sp}} v_s^\pm(u_s) \left\{ [\mu_s^\pm(B) \cdot E] \right\} \cdot v_s^\pm(u_s) \rho_s(u_s), \quad \text{(G2)}$$

where $\rho_s(u_s) = \Delta_0/(2\pi\hbar^2 v_F^2)$ and $u_s = (\hbar k_F^{c,h}v_F^2)^2/2\Delta_0$ are the surface density-of-states and Fermi energy, $k_F^* = \sqrt{4\pi\sigma_s}$, $v_F$ is the Fermi velocity of a Dirac cone, $\tau_s$ and $\tau_{sp}$ are surface energy- and momentum relaxation times, and $v_s^\pm(k_||) = \pm \hbar v_F^2 k_||/\Delta_0$.

From Eq. (G1), we find the bulk conductivity tensor as

$$\sigma_{c,v}^\parallel(B) = e\gamma_{e,h} \int_{-L_A}^{L_D} dz n_{e,h}(z) \left[ \frac{\tau_{e,h}(z)}{\tau_{p(e,h)}(z)} \right] [\mu_{c,v}^\parallel(B,z)]. \quad \text{(G3)}$$

On the other hand, from Eq. (G2) we get the surface conductivity tensor, given by

$$\sigma_s^\pm(B) = e\sigma_s \left( \frac{\tau_s}{\tau_{sp}} \right) \mu_s^\pm(B). \quad \text{(G4)}$$

Therefore, the total conductivity tensor $\sigma_{tot}(B) = \sigma_c^\parallel(B) + \sigma_v^\parallel(B) + \sigma_s^\pm(B)$ can be obtained from

$$\sigma_{tot}(B) = e\mu_v^\parallel(B) N_A A_h \left[ (L_A - W_p) + \int_0^{W_p} dz \exp \left( -\frac{\beta e\mu_h N_A}{2\epsilon_0 e_r D_h} z^2 \right) \right] - e\mu_v^\parallel(B) N_{D,A} e \times \left[ (L_D - W_n) + \int_0^{W_n} dz \exp \left( -\frac{\beta e\mu_{D,A}}{2\epsilon_0 e_r D_e} z^2 \right) \right] + e\mu_s^\pm(B) \left( \frac{\alpha_0 L_0}{2\pi\hbar^2 v_F^2} \right) (L_A - L_0) A_s , \quad \text{(G5)}$$

where $\alpha_0$ and $L_0$ are constants to be determined experimentally, $N_{D,A}$ are doping concentrations, $W_n$ and $W_p$ are depletion ranges for donors and acceptors in a $p$-$n$ junction, $\mu_{e,h}$ are $\mu_0(z)$ evaluated at $n_{e,h}(z) = N_{D,A}$, $D_{e,h}$ are diffusion coefficients, and $\beta = 4/3$ ($\beta = 7/3$) for longitudinal (Hall) conductivity. In addition, the averaged mobilities $\mu_{c,v}^\parallel(B)$ are defined by their values of $\tau_{p(e,h)}(z)$ at $n_{e,h}(z) = N_{D,A}$, and three introduced coefficients are

$$A_{e,h} = \frac{\tau_{e,h}(z)}{\tau_{p(e,h)}(z)} \bigg|_{n_{e,h}(z)=N_{D,A}}$$

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\[
\frac{1}{Q_c} = \frac{1}{6} \left( \frac{Q_c}{k_F^{e,h}} \right)^2 \left[ 2 \ln \left( \frac{2k_F^{e,h}}{Q_c} \right) - 1 \right] = \frac{Q_c^2}{6(3\pi^2 N_{D,A})^{2/3}} \left\{ 2 \ln \left[ \frac{2(3\pi^2 N_{D,A})^{1/3}}{Q_c} \right] - 1 \right\}, \quad (G6)
\]
where \(1/Q_c\) is the Thomas-Fermi screening length.

In addition, the bulk energy-relaxation times \(\tau_{e,h}(z)\) are calculated as

\[
\frac{1}{\tau_{e,h}(z)} = \left[ \frac{2n_i}{n_{e,h}(z)\pi\hbar Q_c^2} \right] \left( \frac{e^2}{\epsilon_0 \epsilon_r} \right)^2 \int_0^{k_F^{e,h}(z)} dk D_{e,v}(\varepsilon_{k,v}) \left( \frac{4k^2}{4k^2 + Q_c^2} \right)
\]

\[
= \left[ \frac{n_i m^*_{e,h}}{8n_{e,h}(z)\pi^3 \hbar^3 Q_c^2} \right] \left( \frac{e^2}{\epsilon_0 \epsilon_r} \right)^2 \left\{ \frac{[2k_F^{e,h}(z)]^2 - Q_c^2 \ln \left( \frac{[2k_F^{e,h}(z)]^2 + Q_c^2}{Q_c^2} \right)}{Q_c^2} \right\}, \quad (G7)
\]

and the surface energy-relaxation time \(\tau_s\) is found to be

\[
\frac{1}{\tau_s} = \frac{2\sigma_i}{\pi^2 \sigma_i \hbar^2 v_F} \left( \frac{e^2}{2\epsilon_0 \epsilon_r} \right)^2 \int_0^\pi d\phi \int_0^{k_F} \frac{k^2 dk}{(q_c + 2k|| \cos \phi)^2},
\]

where \(n_i\) and \(\sigma_i\) are the concentration and surface density of impurities, respectively.

Finally, the bulk chemical potentials for electrons \([u_e(z)]\) and holes \([u_v(z)]\) are calculated as

\[
[u_{e,v}(z)]^{3/2} = 3\pi^2 \left( \frac{\hbar^2}{2m^*_{e,h}} \right)^{3/2} n_{e,h}(z), \quad (G9)
\]

and the carrier density functions are

\[
n_{e,h}(z) = N_{D,A} \exp \left\{ -\gamma_{e,h} \left( \frac{\bar{\mu}_{e,h}}{D_{e,h}} \right) \left[ \Phi(z) + \gamma_{e,h} \left( E_F^{e,h}/e \right) \right] \right\}. \quad (G10)
\]

Here, the expression for the introduced potential function \(\Phi(z)\) is given by

\[
\Phi(z) = \begin{cases} 
-E_F^{h}/e, & z < -W_p \\
-E_F^{h}/e + (eN_A/2\epsilon_0 \epsilon_r)(Z + W_p)^2, & -W_p < z < 0 \\
\frac{E_F^e}{e} - (eN_D/2\epsilon_0 \epsilon_r)(W_n - Z)^2, & 0 < z < W_n \\
\frac{E_F^e}{e}, & z > W_n
\end{cases}, \quad (G11)
\]

and \(E_F^e, E_F^{h}\) is the Fermi energy of electrons (holes) at zero temperature and defined far away from the depletion region.

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