Influence of the modulation format on the nonlinear impairments in space division multiplexed systems

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Abstract. This paper is focused on the analysis of nonlinear mode coupling in space division multiplexed communication systems with linear mode coupling. Simultaneous propagation of modulated mode channels is simulated by numerical solving of the system of generalized coupled nonlinear Schroedinger equations. Different types of modulation format are simulated and corresponding nonlinear impairments are presented. We shown that QAM-modulated signal is more robust for increasing of nonlinear impairments due to the linear mode coupling than NRZ-coded signal.

1. Introduction

Since the capacity crunch of optical communications using single-mode fibers was predicted in 2009 [1], space division multiplexed (SDM) communication systems became a subject of attraction for many studies aimed on sufficient increasing of link capacity [2 – 5]. Concept of SDM was proved experimentally using different technical approaches: SDM was implemented in the system based on multi-core fiber, and in the system based on single-core fiber providing propagation of finite number of spatial modes [6 – 9]. Such few modes fibers nowadays may be considered as a leading candidate for physical layer of future transport optical networks. It is well known from Shannon theorem that the nonlinear effects are the strong limiting factor of system capacity for the single-fiber based systems [10]. Few modes fiber commonly has larger cross-section than single-mode fiber therefore problem of nonlinear impairments is addressed for higher signal power. Capacity scaling with the number of modes was estimated in [10].

One of the challenging tasks for implementation of SDM systems is the linear mode coupling [11]. This process is stochastic therefore such system requires linear compensator for proper signal receiving. Note that modes which have closed values of propagation constant are coupling much more intensive than modes with different propagation constants [10]. For quasi-degenerate spatial modes length-scale of coupling ranges between tens and a hundreds of meters [12]. For non-degenerate modes length-scale of linear coupling may achieve hundreds of kilometers. The length-scale of nonlinear impairments is quietly large and depends on peak power of signal and pulse duration [13]. For useful communication values of power it ranges between tens and hundreds kilometers. The large
difference between length scales of simultaneous processes is the basis of averaging of the fast changes within the Manakov generalization approach [14].

Nonlinear propagation of space division multiplexed signals is the hot topic in recent years and many theoretical and calculation tools developed for single-mode fibers were adapted for the multimode case [12, 15, 16]. Generalized coupled nonlinear Schroedinger equations were obtained for multimode transmission in [17]. Strict numerical solving of these equations is very complex for long haul lines therefore averaged Manakov-type equation is useful for many kinds of simulation.

It is evident that nonlinear impairments reduce effectiveness of linear mode coupling compensation therefore interaction of linear mode coupling and nonlinear effects has sufficient importance for high-capacity communications. Previously we have shown that the presence of strong linear mode coupling leads to the increasing of nonlinear impairments of NRZ-modulated signal [18]. In this paper we investigate nonlinear propagation of QAM-modulated mode multiplexed signal in the fiber with linear mode coupling. During simulations we do not use Manakov-type equations because strict consideration of linear coupling causes the phenomena of nonlinear impairments enhancement because of fast power transition between coupling modes. As a basis of spatial modes we consider fundamental mode LP01 and modes of the first and second orders with helicoidal phase front (OAM-modes).

2. Theoretical model

The electric field of the signal propagating along the fiber may be written in frequency domain as

\[ \mathbf{E}(r, \phi, z, \omega) = \sum_{p=1}^{N} \exp(i\beta_p(\omega)z) \mathbf{\hat{A}}_p(z, \omega) F_p(r) \exp(il_p\phi). \]  

(1)

where \( N \) is the total number of spatial modes, \( p \) is the mode number, \( \beta_p \) is the propagation constant of the \( p \)-th spatial mode, \( F_p(r, \phi) = F_p(r) \exp(il_p\phi) \) is the transverse distribution of the \( p \)-th mode field, \( l_p \) is the integer number defining azimuthal order of the \( p \)-th mode and \( A_p \) is the slowly varying amplitude of the \( p \)-th mode. The pulse amplitude \( A_p \) is assumed to be normalized such that \( |A|^2 \) represents the optical power.

Note that exponential multiplier in (1) provide that every mode has linear density of \( z \)-projection of orbital angular momentum, that is proportional to the \( l_p \).

In the linear case propagation of the mode may be described by the following equation:

\[ \frac{d\mathbf{A}_p}{dz} = -i\beta_p A_p + \sum_m q_{mp} \mathbf{A}_m, \]  

(2)

where \( q_{mp} \) is the linear coupling coefficients, in the general case defined as [19]

\[ q_{mp} = \frac{\omega \epsilon_0}{2P(\beta_p - \beta_m)} \left( \int \frac{\epsilon_n(r, z)}{\epsilon_z} F_p^*(r) E_m(r) d^2r \right). \]  

(3)

In the general case nonlinear effects in the quartz fiber include third-order Kerr-nonlinearities, Raman scattering and Brillouin scattering. Taking into account conditions corresponding to the real communication lines using SDM technology we neglect scattering. Generalized coupled nonlinear Schroedinger equations for this case may be written as [16]:

\[ \frac{\partial A_p}{\partial z} = \left( \beta_{up} - \beta_r \right) A_p - \left( \beta_{up} - \frac{1}{v_g} \right) \frac{\partial A_p}{\partial t} - i \frac{\partial}{\partial t} A_p + i \sum_m f_{mp} A_m A_n A_m + i \sum_m q_{mp} A_m, \]  

(4)

where \( A_p \) is the slowly varying envelope expressed in a reference moving frame at a group velocity \( v_g \), \( \beta_r \) is the reference propagation constant and \( \gamma \) is the nonlinear parameter.

\[ \beta_{up} = \beta_p(\omega), \quad \beta_r = \partial \beta_p(\omega) |_{\omega, \omega}, \quad \beta_{up} = \partial^2 \beta_p(\omega) |_{\omega, \omega}, \]  

are respectively the propagation constant, inverse group velocity and group-velocity dispersion (GVD) of the \( p \)-th spatial mode.

Linear mode coupling in equation(4) is defined by the coefficients [20]
\[ q_{\mu \nu}^{(k)} = \frac{\gamma_{\mu} \gamma_{\nu}}{2n_1 k_0 a e_\mu e_\nu} \left( \frac{\varepsilon^{(1)}}{\beta_\mu - \beta_\nu} \right)^3 \gamma_{\mu \nu}, \]  

where \( n_1 \) is the refractive index of the fiber core in the point \( z = 0 \), \( \varepsilon(k) \) is the curvature of the \( k \)-th section, \( p \) and \( m \) are the mode azimuthal indices, \( \delta \) is the Kronecker delta function, \( e_m = 2 \) for \( m = 0 \) and \( e_m = 1 \) in other cases,

\[ \gamma_{\mu \nu} = \frac{J_{p+1}(\kappa_\mu a)}{J_{m+1}(\kappa_\mu a) J_{p+1}(\kappa_\nu a) J_{m+1}(\kappa_\nu a) J_{m+1}(\kappa_\mu a)} \left( \frac{\varepsilon^{(1)}}{\beta_\mu - \beta_\nu} \right)^3. \]  

In equation (7) \( J(\kappa_\mu a) \) is the Bessel function with \( \kappa_\mu \) defined as

\[ \kappa_\mu = \sqrt{(k_0 n_1)^2 - \beta_\mu^2}. \]  

Nonlinear mode coupling in equation (2) is defined by coefficients [16]

\[ f_{lmnp} = \frac{A_{eff}}{\left( I_{lm} I_{m} I_{nP} I_{p} \right)^{1/2}} \iint F_l^* F_m F_n F_p dx dy, \]  

where the intensity normalization terms are defined as

\[ I_m = \bar{\eta}_{lm} \iint F_m^2(x, y) dx dy. \]  

Here \( n_{eff} \) and \( n_m \) are the effective refractive indices for the fundamental mode and for the \( m \)-th mode respectively.

Considering equation (4) one can see that nonlinear term describe self-phase modulation (SPM) which is present to all the mode channels, inter-modal cross-phase modulation (XPM) and inter-modal four-wave mixing (FWM). Because these equations are written only for the one wavelength, all these effects are intra-channel in the terms of WDM-systems.

Wavelengths in the regions of 850 nm and 1550 nm are the most interest for telecommunication applications. Near 850 nm standard step-index single-mode fiber provide propagation of three spatial modes: fundamental mode LP01 and conjugate first-order modes. In the case of vortex modes it is LP11+ and LP11-. In the wavelength region near 1550 nm for implementation of few-modes transmission one needs special fiber providing propagation of finite number of modes. In our simulation we consider step-index fiber with enlarged core cross-section. Chosen orthogonal basis of the spatial modes defines nonlinear coupling coefficients. We have calculated these values for mentioned type of fiber, and the results are presented in the table 1.

Note that effectiveness of SPM for different modes are defined by the coefficients given in the table 1. It is evident that this process is the most effective for the fundamental mode, and SPM of the first- and second-order modes have lower efficiencies.

Efficiency of the XPM is defined by the nonlinear coupling coefficient and by the difference between propagation constants of interacting modes [10]. Therefore interaction through XPM is much more efficient between quasi-degenerate modes than between modes of the different orders.

Model described in (4) do not consider polarization modes of the same spatial mode because fiber birefringence is not included in fiber description. Previously we have studied interaction of the polarization mode dispersion (PMD) and Kerr-nonlinearity in long haul fiber communications and we have shown that interaction between PMD and nonlinear effects in WDM-systems may lead to reducing of the nonlinear impairments [21]. Length scale of PMD is differ from length scale of linear coupling of spatial modes therefore simultaneous consideration of these effects is the future task and such model have to be verified experimentally.
Table 1. Non-zero nonlinear coupling coefficients for a few-mode fiber with 5 spatial modes at 1550 nm (absolute values).

| Mode indices | $f_{non}$ |
|--------------|-----------|
| LP01, LP01, LP01, LP01 | 1.0000 |
| LP01, LP01, LP01, LP01, LP01+ | 0.6965 |
| LP01, LP01, LP01, LP01, LP01+ | 0.6589 |
| LP01, LP01, LP01, LP01, LP01+ | 0.6096 |
| LP01, LP01, LP01, LP01, LP01+ | 0.6035 |
| LP01, LP01, LP01, LP01, LP01+ | 0.4514 |

3. Simulation model and results

Commonly numerical solving of nonlinear Schrödinger equations is implemented using Split-step Fourier method [13]. Equations (4) are written for the fiber with constant coupling parameters therefore stepping of the simulation have to be associated with dividing of the fiber on sections with constant parameters (curvature of the fiber in the presented model). Linear mode coupling is considered by multiplying the field column vector $A(t)$ at each step by the matrix

$$V^{(k)} = \exp[i\theta^{(k)}],$$

where $B_0$ is the square matrix with main diagonal elements defined by the first term of the right side of equation (4), $Q(z)$ is the square matrix with elements defined from equation (6) and $\Delta z$ is the simulation step. Additionally after each section column vector $A(t)$ was multiplied on the rotation diagonal matrix $V^{(k)}$ to simulate stochastic changing of the fiber curvature plain:

$$V = \begin{pmatrix}
    e^{-i\theta^{(k)}} & 0 & 0 & 0 \\
    0 & e^{-i2\theta^{(k)}} & 0 & 0 \\
    0 & 0 & ... & 0 \\
    0 & 0 & 0 & e^{-iN\theta^{(k)}}
\end{pmatrix},$$

where $\theta^{(k)}$ is the rotation angle of $k$-th section.
For estimation of the nonlinear impairments, equations (4) were solved twice for each set of random parameters defining sections of simulated fiber: taking into account nonlinear terms of equations, and leaving them out of account (i.e., solving equation (2) in the time-domain). Thus, two envelopes of the field were found using this method. First of them takes into account both dispersion and nonlinear equation terms, while the second one takes into account only dispersion terms. Then rms variance of these two envelopes was calculated.

We have simulated 2.5 km-length step-index fiber with enlarged cross-section (8.6 mkm). For strong-coupling simulation we have divided fiber on 10000 sections with mean length equal to 25 cm. For weak-coupling simulation we have divided fiber on 500 sections with mean length equal to 5 m. In figures 1–2 we present results of simulation for NRZ-coded mode channels with pulse duration set to 100 ps [22].

It is evident that strong linear coupling leads to sufficient increasing of nonlinear impairments, but in the case of weak coupling we have more complex stochastic picture: nonlinear impairments might be enlarged for not all the modes channels. Similar results were obtained previously for the 3-modes case in standard single-mode fiber with 850 nm signal [18]. Presented results show that influence of linear coupling is sufficiently different for mode channels.

![Figure 1. Nonlinear distortions in NRZ mode channels of few-mode fiber in strong linear coupling (LC) regime with 25 cm coupling sections at 1550 nm.](image1)

![Figure 2. Nonlinear distortions in NRZ mode channels of few-mode fiber in strong linear coupling (LC) regime with 5 m coupling sections at 1550 nm.](image2)

In figures 3–8 we present results of simulation for QAM-signals of different orders. Symbol duration for all the simulations was set to 100 ps. All the channels were modulated by independent random bit sequences.
Figure 3. Nonlinear distortions in QAM-16 mode channels of few-mode fiber in strong linear coupling (LC) regime with 25 cm coupling sections at 1550 nm.

Figure 4. Nonlinear distortions in QAM-16 mode channels of few-mode fiber in weak linear coupling (LC) regime with 5 m coupling sections at 1550 nm.

Figure 5. Nonlinear distortions in QAM-64 mode channels of few-mode fiber in strong linear coupling (LC) regime with 25 cm coupling sections at 1550 nm.

Figure 6. Nonlinear distortions in QAM-64 mode channels of few-mode fiber in weak linear coupling (LC) regime with 5 m coupling sections at 1550 nm.

Figure 7. Nonlinear distortions in QAM-256 mode channels of few-mode fiber in strong linear coupling (LC) regime with 25 cm coupling sections at 1550 nm.

Figure 8. Nonlinear distortions in QAM-256 mode channels of few-mode fiber in weak linear coupling (LC) regime with 5 m coupling sections at 1550 nm.
Comparing these results with figures 1–2 one can see that for QAM-signal linear coupling influence on the different mode channels in the same manner. Moreover QAM-signals are more robust to the increasing of nonlinear impairments due to linear coupling. Increasing of QAM order leads to further alignment of interaction effects between different modes.

4. Conclusion
We have simulated propagation of five mode channels through step-index few-mode fiber. We have compared numerical estimation of nonlinear impairments of NRZ-modulated SDM-signal and QAM-modulated SDM-signal caused by linear mode coupling. Simulation results shows that NRZ-modulated signal demonstrate strong dependence of nonlinear impairments enhancement on the carrier mode. For QAM-signal there is no evident dependence. From these results we conclude that QAM-signal is more robust for the effect of nonlinear enhancement than OOK-signal.

5. References
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