Hardon-quark hybrid stars constructed by
the nonlinear $\sigma$-$\omega$-$\rho$ mean-field model and MIT-bag model

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Abstract

Density-dependent relations among saturation properties of symmetric nuclear matter and hyperonic matter, properties of hadron-(strange) quark hybrid stars are discussed by applying the conserving nonlinear $\sigma$-$\omega$-$\rho$ hadronic mean-field theory. Nonlinear interactions that will be renormalized as effective coupling constants, effective masses and sources of equations of motion are constructed self-consistently by maintaining thermodynamic consistency to the mean-field approximation. The coupling constants expected from the hadronic mean-field model and SU(6) quark model for the vector coupling constants are compared; the coupling constants exhibit different density-dependent results for effective masses and binding energies of hyperons, properties of hadron and hadron-quark stars. The nonlinear $\sigma$-$\omega$-$\rho$ hadronic mean-field approximation with or without vacuum fluctuation corrections and strange quark matter defined by MIT-bag model are employed to examine properties of hadron-(strange) quark hybrid stars. We have found that hadron-(strange) quark hybrid stars become more stable in high density compared to pure hadronic and strange quark stars.

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1 Introduction

The symmetric nuclear matter is a self-bound matter whose binding energy exhibits a characteristic concave curve at saturation density, where pressure of nuclear matter vanishes ($p = 0$). It has been known as a constraint to examine self-consistency to nuclear many-body approximations \cite{[1] – [3]}. The self-consistency has been fundamental for many-body approximations in terms of Landau’s Fermi-liquid theory \cite{[4] – [6]}, Kadanoff-Baym’s theory of conserving approximations \cite{[7] – [9]} and the density functional theory \cite{[10] – [12]}. The relativistic, field-theoretical, linear $\sigma$-$\omega$ mean-field approximations that maintain conditions of conserving approximations have been applied to finite nuclei, nuclear matter and neutron stars \cite{[13] [14]}. These nuclear mean-field approximations reveal an important relation among self-consistent single particle energy, energy density and particle density, which is denoted as thermodynamic consistency \cite{[15] [16]}. The conserving nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation which maintains thermodynamic consistency has been applied to investigate density-dependent correlations among properties of nuclear, neutron and hyperonic matter \cite{[17] – [19]}. Thermodynamic consistency is important to derive consistent results for high energy and high density phenomena, such as heavy-ion collision, hypernuclei-formation experiments \cite{[20]} and astrophysical phenomena \cite{[21] – [23]}. The nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation has several nonlinear coefficients whose values are determined by reproducing binding energy, symmetry energy and simultaneously searching the minimum value of incompressibility at saturation density, as well as reproducing a maximum mass of neutron stars at high density. The binding energy at saturation is taken as $-15.75$ MeV at $k_F = 1.30$ fm$^{-1}$ ($\rho_0 = 0.148$ fm$^{-3}$) with the symmetry energy, $a_4 = 30.0$ MeV in the current calculation \cite{[13]}. The values of nonlinear coefficients which will produce effective masses of hadrons $M_N^*/M_N \sim 0.70$, $m_\pi/m_\pi \sim 1.02$, $m_\rho/m_\rho \sim 1.01$ and incompressibility, $K \sim 320$ MeV, with symmetry energy $a_4 \sim 30$ MeV, the maximum mass of neutron stars in $\beta$-equilibrium.
matter, $M_{\text{max}}(n,p,e) \sim 2.50 \, M_\odot$ (the solar mass: $M_\odot \sim 1.989 \times 10^{30}$ kg), are found reasonable and admissible in the nonlinear $\sigma$-$\omega$-$\rho$ Hartree approximation [18, 19].

The maximum masses of neutron stars are expected to be below $2.5 \, M_\odot$ [21], or $2.1 \pm 0.2 \, M_\odot$ [25], which depends on the hadronic equation of state (EOS) for isospin asymmetric, hyperonic matter in the density range, $2 \rho_0 \lesssim \rho_\text{n} \lesssim 5 \rho_0$. The hadronic liquid-gas phase transition at the surface of neutron stars and determination of the radius of smeared, gas-phase surface are not directly relevant to properties of hadronic matter and maximum masses of neutron stars. The nonlinear interactions exhibit significant density-dependent effects on incompressibility, $K$, and symmetry energy, $a_4$, in high densities; these Fermi liquid properties monotonically increase about saturation density, but they are piecewise continuously softened at an hyperon onset density. These phenomena of piecewise continuous change of Fermi-liquid properties will be important for the analysis of Landau parameters, heavy-ion collision, high energy and high density experiments. At a hyperon onset density from $(n,p,e)$ to $(n,p,H,e)$, the EOS suddenly becomes softer. This is because the nucleon Fermi energy, $E_N(k_F)$ in the phase $(n,p,e)$, will be redistributed to the hyperon Fermi energy, $E_H(k_F)$ in the phase $(n,p,H,e)$; consequently, the Fermi energies become relatively small in the phase $(n,p,H,e)$ compared with those of $(n,p,e)$. The redistribution and slow increase of Fermi energies appear whenever hyperons are generated, resulting in a softer equation of state and discontinuous changes of $K$ and $a_4$. This is numerically checked as discrete changes of physical quantities, such as effective masses of hadrons, incompressibility, symmetry energy and energy density [18, 19].

Since the hyperon-onset will confine Fermi-energies of baryons as explained above, single and double hyperon generations exhibit different density-dependent phenomena. For example, the $\Lambda$-hyperon onset density such as in $(n,p,e)-(n,p,\Lambda,e)$ is $\rho_\Lambda/\rho_0 \sim 2.2$; however, it gives $\rho_\Lambda/\rho_0 \sim 4.2$ when $\Lambda$ is generated in $(n,p,e)-(n,p,\Sigma^-,e)-(n,p,\Sigma^-,\Lambda)$. The same phenomena are observed with other hyperons, and generally the onset-density of a hyperon is pushed up to a higher density, which is denoted as the push-up phenomena of hyperon onset-densities in many-fold hyperon generations [18]. Because of the push-up phenomena of the hyperon onset density, we have found that hyperon generations are suppressed in high densities and hyperons relevant to determine the maximum mass of neutron stars are $\Sigma^-$ and $\Lambda$. The similar results are discussed in nonrelativistic Brueckner-Hartree-Fock [26, 27] and quark matter calculations [28]. The hyperon onset densities are related to hyperon single particle energies by way of phase equilibrium conditions. Hence, the Hugenholtz-Van Hove theorem and thermodynamic consistency at saturation of hyperon binding energy are essential to define self-consistent approximation of hyperon matter. The single particle energies of hyperons are also important to study $K$ and $a_4$ for magic nuclei [29] and hypernuclei [30–32].

Density-dependent effective masses and effective coupling constants, saturation properties for nucleons and hyperons are discussed in the nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation [18, 19]. The results show: (1) coupling constants of hyperons are related to those of nucleons by effective masses, effective coupling constants, binding energy at onset-density of respective hyperons. Hence, it shows that binding energies of symmetric nuclear matter and hyperon matter are self-consistently related to each other. (2) Self-consistency suppresses hyperon generations in high densities, denoted as the push-up phenomena of hyperon onset densities. The suppression of hyperon generations is also discussed in different calculations [26–28]. (3) Coupling ratios of hyperons are expected to be $g_{\sigma H}/g_{\sigma N}, g_{\omega H}/g_{\omega N} \gtrsim 1$, in order to be consistent with conditions of thermodynamic consistency, empirical values of nuclear matter and neutron stars. In the current calculations, we have included vacuum fluctuation corrections (VFC) into the nonlinear $\sigma$-$\omega$-$\rho$ approximation and examined properties of $(n,p,e)$, $(n,p,H_1,e)$, $(n,p,H_1,H_2,e)$ hyperonic matter, constrained by transitions to strange quark matter and properties of hadron-quark hybrid stars. The hadron-quark phase transitions are assumed to be a first order and computed by Maxwell construction [33].

The conserving nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation [17–19] and quark-based effective nuclear models [34–36] have been applied to finite nuclei, nuclear and isospin asymmetric, high-density matter. We have compared hyperon coupling constants required by the nonlinear $\sigma$-$\omega$-$\rho$ mean-field model with those required by SU(6) quark model for the vector coupling constants [37, 38]. The hyperon coupling constants required by hadronic and SU(6) quark models exhibit quite different results for effective masses, binding energies of hyperons [18, 19] and
properties of hadron-quark hybrid stars. The coupling ratios required by SU(6) quark model cannot reproduce hyperon saturation properties, which will be discussed in terms of conditions of thermodynamic consistency in sec. 6. The analysis of discrepancies of predictions by hadronic and quark-based models may provide us with insight for constructing self-consistent nuclear many-body problems [13 – 19]. The effective masses of hyperons depend on coupling ratios \( g_{\omega H}/g_{\omega N} = r_{H N}^\omega \), \( g_{\omega H}/g_{\pi N} = r_{H N}^\pi \), and decrease analogous to effective masses of nucleons in high densities, which shows strong density-dependency of hadronic interactions. However, the effective masses of hyperons exhibit weak density-dependent results with \( r_{H N}^\omega \leq 2/3 \) required by SU(6) quark model for the vector coupling constants. In general, hadrons exhibit strong density-dependent interactions and correlations among properties of nuclear matter, hyperonic matter and neutron stars. In the hadronic mean-field approximation, \( r_{H N}^\omega \geq 1.0 \) is preferred in order to be consistent with properties of nuclear matter and the maximum mass of isospin asymmetric neutron stars, \( M_{\text{max}}(n, p, e) = 2.00 \sim 2.50 \, M_\odot \). The value, \( r_{H N}^\omega = 2/3 \), results in different effective masses and binding energies of hyperons; the discrepancies originate from density-dependent interactions of hadrons [19].

The density-dependent many-body effects produced by the conserving nonlinear \( \sigma-\omega-\rho \) mean-field approximation should be compared to chiral hadronic mean-filed approximations [39 – 42]. The effective quark-based chiral lagrangian approach suggests that the appropriate in-medium scaling law, \( m^*_\pi/m_\pi \approx m^*_N/m_N \approx m^*_\omega/m_\omega \approx f_\pi/f_\pi \approx m^*_\rho/m_\rho \), be expected [43, 44]. Although it is ambiguous whether \( m^*_N/m_N \approx m^*_\omega/m_\omega \), means that \( m^*_\pi/m_\pi \approx 1.0 \) or \( m^*_\rho/m_\rho \approx 1.0 \) at saturation density, they are certainly decreasing above saturation density, since \( m^*_N/m_N \) is model-independently expected to be decreasing. This is significantly different from nonlinear mean-field approximations, since self-consistency of hadronic mean-field approximation demands \( m^*_\pi/m_\pi \geq 1.0 \) [17, 15] and \( m^*_\rho/m_\rho \approx m^*_\pi/m_\pi \leq 1.0 \) [16] in high densities. In other words, since effective masses and coupling constants are self-consistently related to each other, if some values decrease, the others have to counterbalance the variations. Therefore, all correlated effective masses and coupling constants must decrease or increase simultaneously in order to compensate for variations among others. At saturation density, effective masses of mesons are density-dependent and their ratios are approximately equal: \( m^*_\pi/m_\pi \approx m^*_\omega/m_\omega \approx m^*_\rho/m_\rho \approx m^*_\pi/m_\pi \approx 1.0 \). However, mesons exhibit different behavior above saturation densities as explained. The discrepancy has been discussed in terms of thermodynamic consistency in the hadronic mean-field approximations [17 – 19] and should be investigated in other hadronic models to extract consistent model-independent results.

We have applied the nonlinear \( \sigma-\omega-\rho \) mean-field approximation and MIT-bag model upon hadron-quark hybrid stars [23 – 47]. The numerical analysis exhibits new results that the hadron-(strange) quark hybrid stars are more stable in high density than pure hadronic and strange quark stars. It suggests a relation between bag constant and QCD strong coupling constant, \( (B, \alpha_s) \) [48, 49], to the central density and maximum mass of hadron-(strange) quark hybrid stars, \( (\epsilon_c, M_{\text{max}}) \). The results obtained in the current calculations should be examined from astronomical data whether or not hadron-quark stars are possible and the values of bag constant and strong coupling constant, \( (B, \alpha_s) \), could be consistent with astronomical data for neutron stars.

Self-consistent relations among saturation properties of nucleons and hyperons are briefly reviewed in sec. 2. Quantitative numerical calculations for effective masses, onset densities and conditions of hyperon saturation are discussed in the articles [18, 19]. The MIT-bag quark matter and vacuum fluctuation correation to nonlinear mean-field approximation are explained in sec. 3. Results of pure hadron and hadron-strange quark hybrid stars are discussed in sec. 4; concluding remarks are in sec. 5.

2 Self-consistent effective masses and coupling constants in the nonlinear \( \sigma-\omega-\rho \) mean-field approximation

The hadronic lagrangian with nonlinear \( \sigma-\omega-\rho \) interactions which yields density-dependent effective masses and coupling constants is given by [17].
where \( \psi_B \) \((B = n, p, \Lambda, \Sigma, \ldots)\) and \( \psi_l \) \((l = e^-, \mu^-)\) denote the field of baryons and leptons, respectively. The meson-fields operators are: \( \phi \) for the \( \sigma \)-field, \( V \) for the vector-isoscalar \( \omega \)-meson, \( V_\mu V^\mu = V_0^2 - V^2 \), \((\mu = 0, 1, 2, 3)\) and \( R_\mu \) for \( p \)-meson. The vector field strengths, \( F_{\mu\nu} \) and \( L_{\mu\nu} \), are defined as, \( F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \) and \( L_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu + g_\rho R_\mu \times R_\nu \).

The coupled nonlinear quantum-field lagrangian (2.1) is interpreted as baryon quantum-field lagrangian in mean fields of mesons [13]. All dynamics generated by baryon fields are mediated by mean fields of mesons which will be self-consistently defined in an approximation. The effective coupling constants, \( g^*_B \) \((i = \sigma, \omega, \rho)\), denote renormalized, density-dependent coupling constants defined by self-consistent mean-field of \( \sigma \)-meson. The nonlinear \( \sigma-\omega-\rho \) mean-field model maintains the structure of Serot and Walecka’s linear \( \sigma-\omega \) mean-field approximation [13]. Lorentz-invariance and renormalizability, the Hugenboltz-Van Hove theorem [2], conditions of conserving approximations [7 – 12], the virial theorem [15, 16] and Landau’s hypothesis of quasiparticles [4 – 6]. As we proved in the ref. [17], nonlinear mean-field approximations are equivalent to Hartree approximation when nonlinear interactions are properly renormalized, and consequently, the concepts of effective masses and effective coupling constants are naturally generated by nonlinear mean-field interactions. Self-consistent relations among single particle energy, effective masses and coupling constants will restrict empirical values of low-density nuclear matter and high-density hadronic matter. The admissible values of effective coupling constants and masses are confined in certain values due to strong density-dependent correlations among physical quantities of nuclear matter and neutron stars [17 – 19].

Meson-fields operators are replaced by mean-fields denoted by \( \phi_0, V_0 \), and \( R_0 \). The equations of motion for baryons are given by,

\[
\left[(i\gamma_\mu \partial^\mu - g^*_\omega B \gamma_0 V_0 - \frac{g^*_B}{2} \gamma_0 \tau_3 R_0) - (M_B - g^*_B B_0)\right] \psi_B = 0 ,
\]

where \( g^*_B \), \( g^*_\omega B \) and \( g^*_B \) are effective coupling constants for \( \sigma, \omega \) and \( \rho \) mesons. One should notice that effective coupling constants cannot be simply introduced as experimental, external inputs to an employed approximation, since density-dependent coupling constants will modify equations of motion for mesons. We will assume that only nucleon-meson coupling constants are density-dependent since we are interested in the density-dependent correlations among properties of symmetric nuclear matter. Nonlinear interactions are not assumed in the coupling constants for hyperons; effective masses of hyperons are defined by \( M^*_H = M^*_H - g_\sigma H \phi_0 \). The density-dependent nucleon-meson coupling constants that maintain thermodynamic consistency are,

\[
\begin{align*}
g^*_\sigma N &= g_\sigma + g_\sigma N \phi_0 / 2 m_\sigma , \\
g^*_\omega N &= g_\omega + g_\omega N \phi_0 / m_\sigma , \\
g^*_\rho N / 2 &= g_\rho N / 2 + g_\sigma N \phi_0 / m_\sigma .
\end{align*}
\]
The equations of motion for mesons are given by,

\[ m_{\sigma}^2 \phi_0 + \frac{g_{\sigma\omega}}{2} \phi_0 + \frac{g_{\sigma\omega}}{3!} \phi_0^3 - \frac{g_{\sigma\omega}}{2} V_0^2 \phi_0 - \frac{g_{\sigma\rho}}{2} R_0^2 \phi_0 - \frac{g_{\sigma N}}{2 m_\sigma} \rho_s \phi_0 = \frac{g_{\sigma N}^*}{m_\sigma} \rho_s \frac{g_{\omega N}}{m_\sigma} V_0 \rho_\omega - \frac{g_{\rho N}}{m_\sigma} R_0 \rho_3 , \]

\[ m_{\omega}^2 V_0 + \frac{g_{\omega\omega}}{3!} V_0^3 + \frac{g_{\omega\rho}}{2} V_0 \phi_0^2 + \frac{g_{\omega N}}{2} R_0 V_0 = \frac{g_{\omega N}^*}{m_\omega} \rho_\omega , \]

\[ m_{\rho}^2 R_0 + \frac{g_{\rho\omega}}{3!} R_0^3 + \frac{g_{\rho\rho}}{2} R_0 \phi_0^2 + \frac{g_{\rho N}}{2} V_0^2 R_0 = \frac{g_{\rho N}^*}{m_\rho} \rho_3 . \]

(2.4)

where \( \rho_s = \sum_B \rho_{sB} \) is the total scalar source, \( \rho_\omega \) the isoscalar density, and \( \rho_3 = (k_F^3 - k_{F_n}^3)/3\pi^2 \), the isovector density. The density-dependent coupling constants will modify the equation of motion for \( \omega \)-meson which acquires a mass term, \( \frac{g_{\omega N}}{2 m_\sigma} \rho_s \phi_0 \), and new source terms, \( -g_{\sigma N} V_0 \rho_\omega / m_\sigma - g_{\rho N} R_0 \rho_3 / m_\sigma \) from density-dependency of effective coupling constants.

The introduction of nonlinear \( \sigma N \)-vertex interaction leads to the effective mass of nucleon:

\[ M_N^* = M_N - \frac{g_{\sigma N}^*}{g_{\sigma N}} \phi_0 = M_N - g_{\sigma N} \phi_0 - (g_{\sigma N}/2m_\sigma) \phi_0^2 , \]

(2.5)

and effective masses of nucleons and hyperons are related to each other as,

\[ M_H - M_H^* = \frac{g_{\sigma N}}{g_{\sigma N}^*} (M_N - M_N^*) . \]

(2.6)

The total scalar source is obtained by the requirement of self-consistency,

\[ \Sigma^s = \Sigma_N^s + \Sigma_H^s = -\frac{g_{\sigma N}^2}{m_\sigma^2} (\rho_{sN}^* + \rho_{sH}) , \]

(2.7)

where the scalar sources are respectively given by

\[ \rho_{sB} = \frac{g_{\sigma N}^*}{g_{\sigma N}} \frac{k_{FB}}{\pi^2} \int_0^{k_{FB}} dq q^2 \frac{M_B^*}{E_B(q)} , \]

(2.8)

and \( \rho_{sN}^* \) is the modified scalar density defined by \( g_{\sigma N}^* \rho_{sN}^* = g_{\sigma N} \rho_{sN} - g_{\sigma N} V_0 \rho_B / m_\sigma - g_{\rho N} R_0 \rho_3 / m_\sigma ; \)

\( N \) is used to denote proton and neutron, \( N = (p, n) \); hyperons are denoted as, \( H = \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \cdots \).

The \( \omega \)-meson and \( \rho \)-meson contributions to self-energies are given by

\[ \Sigma^\mu_\omega = \frac{g_{\omega N}^2}{m_\omega^2} \rho_\omega \delta_{\mu, 0} \quad \text{and} \quad \Sigma^\mu_\rho (\rho_{\Sigma}) = \frac{g_{\rho N}^2}{4m_\rho^2} \rho_3 \delta_{\mu, 0} , \]

(2.9)

where the isoscalar density, \( \rho_\omega \), is given by

\[ \rho_\omega = \rho_p + \rho_n + \sum_H \rho_{H N}^\omega \rho_H , \]

(2.10)

and \( \rho_{H N}^\omega = g_{\omega N}/g_{\sigma N}^* \) is the density-dependent ratio of hyperon-nucleon coupling constants. The self-energies, \( \Sigma^\mu_\rho (\rho_{\Sigma}) \), and \( \Sigma^\mu_\rho (\rho_{\Sigma}) \), are briefly denoted as \( \Sigma^\mu_{\rho (\Sigma)} \); the isovector density is denoted as \( \rho_3 = (k_F^3 - k_{F_n}^3)/3\pi^2 \) where the Fermi momentum \( k_{F_p} \) is for proton and \( k_{F_n} \) for neutron [17] [18].

The baryon-isovector density, \( \rho_{3B} \), and the ratios of sigma-nucleon coupling constants on \( \rho \)-meson are also defined; for example, \( \rho_{3B} = \rho_3 + \rho_{\Sigma N}^\rho \rho_{\Sigma} \), where \( \rho_{\Sigma N}^\rho = g_{\rho N}^\Sigma / g_{\rho N}^\Sigma \) and \( \rho_{\Sigma} = \rho_{\Sigma^+} - \rho_{\Sigma^-} \).
Thermodynamically consistent effective masses of mesons compatible with effective coupling constants (2.3) are required to be:

\[
m_{\sigma}^2 = m_{\sigma}^2 \left(1 + \frac{g_{\sigma\lambda}}{2m_{\sigma}^2} \phi_0 + \frac{g_{\sigma\pi}}{3!m_{\sigma}^2} \phi_0^2 - \frac{g_{\sigma\omega}}{2m_{\sigma}^2} V_0^2 - \frac{g_{\sigma\rho}}{2m_{\sigma}^2} R_0^2 - \frac{g_{\sigma\piN}}{2m_{\sigma}^2} \rho_{\piN} \right),
\]

\[
m_{\omega}^2 = m_{\omega}^2 \left(1 + \frac{g_{\omega\lambda}}{2m_{\omega}^2} \phi_0 + \frac{g_{\omega\pi}}{2m_{\omega}^2} \phi_0^2 + \frac{g_{\omega\rho}}{2m_{\omega}^2} R_0^2 \right),
\]

\[
m_{\rho}^2 = m_{\rho}^2 \left(1 + \frac{g_{\rho\lambda}}{2m_{\rho}^2} R_0^2 + \frac{g_{\rho\pi}}{2m_{\rho}^2} \phi_0^2 + \frac{g_{\omega\rho}}{2m_{\rho}^2} R_0^2 \right).
\]

(2.11)

Since effective masses of mesons and coupling constants depend on mean fields of mesons, they are density-dependent through meson fields. Note that the effective mass of \(\sigma\)-meson depends on the \((n, p)\) scalar source of nucleons, \(\rho_{\piN}\). The modifications to equations of motion, propagators and self-energies produced by density-dependent effective coupling constants and masses have to be carefully discussed.

The energy density, pressure of isospin-symmetric, asymmetric and charge-neutral hadronic matter are calculated by way of the energy-momentum tensor:

\[
E_{NHA} = \frac{1}{\pi^2} \sum_B \int_0^{k_{FB}} dk k^2 E_B(k) \left(\frac{m_{\sigma}^2}{2} \phi_0^2 + \frac{g_{\sigma\lambda}}{3!m_{\sigma}^2} \phi_0^3 - \frac{g_{\sigma\pi}}{4!m_{\sigma}^2} \phi_0^4 - \frac{m_{\omega}^2}{2} V_0^2 - \frac{g_{\omega\pi}}{4!m_{\omega}^2} V_0^4 - \frac{g_{\omega\rho}}{4!m_{\omega}^2} \phi_0^2 V_0^2 \right)
\]

\[
- \left(\frac{m_{\rho}^2}{2} + \frac{g_{\rho\pi}}{4!m_{\rho}^2} R_0^2 + \frac{g_{\omega\rho}}{4!m_{\rho}^2} V_0^2 \right)R_0^2 + \sum_{l=e^-} d \kappa \int_0^{k_{FL}} dk k^2 E_l(k),
\]

(2.12)

\[
p_{NHA} = \frac{1}{3\pi^2} \sum_B \int_0^{k_{FB}} dk \frac{k^4}{E_B(k)} \left(\frac{m_{\sigma}^2}{2} \phi_0^2 - \frac{g_{\sigma\lambda}}{3!m_{\sigma}^2} \phi_0^3 - \frac{g_{\sigma\pi}}{4!m_{\sigma}^2} \phi_0^4 + \frac{m_{\omega}^2}{2} V_0^2 + \frac{g_{\omega\pi}}{4!m_{\omega}^2} V_0^4 + \frac{g_{\omega\rho}}{4!m_{\omega}^2} \phi_0^2 V_0^2 \right)
\]

\[
+ \left(\frac{m_{\rho}^2}{2} + \frac{g_{\rho\pi}}{4!m_{\rho}^2} R_0^2 + \frac{g_{\omega\rho}}{4!m_{\rho}^2} V_0^2 \right)R_0^2 + \sum_{l=e^-} d \kappa \int_0^{k_{FL}} dk \frac{k^4}{E_l(k)} ,
\]

(2.13)

where \(k_{FB}\) is the Fermi-momentum; \(E_B(k)\) and \(E_l(k)\) are single particle energies for baryons and leptons, respectively. One can check that the thermodynamic relations, such as \(\delta E_{NHA} + \delta p_{NHA} = \rho_{\mu}E_n(k_{F\mu})\) and the chemical potential, \(\mu = \delta E_{NHA}/\rho_{\mu} = E_n(k_{F\mu}) = E^*(k_{F\mu}) - \Sigma^0(k_{F\mu})\), are exactly satisfied with a given baryon density, \(\rho_{\mu} = 2k_{F\mu}^3/3\pi^2\).

The functional derivative of energy density, \(\delta E_{NHA}/\delta n_i\), with respect to the baryon number distribution, \(n_i\), is given by:

\[
\delta E_{NHA}/\delta n_i = E(k_i) + \sum_i \left(\frac{\delta E_{NHA}}{\delta \phi_0} \frac{\delta \phi_0}{\delta n_i} + \frac{\delta E_{NHA}}{\delta V_0} \frac{\delta V_0}{\delta n_i} + \frac{\delta E_{NHA}}{\delta R_0} \frac{\delta R_0}{\delta n_i} \right).
\]

(2.14)

Thermodynamic consistency requires:

\[
\frac{\delta E_{NHA}}{\delta \phi_0} = 0, \quad \frac{\delta E_{NHA}}{\delta V_0} = 0 \quad \text{and} \quad \frac{\delta E_{NHA}}{\delta R_0} = 0. \quad \text{[12]}
\]

Now, one can directly prove that the self-energies calculated by propagators and the conditions of conserving approximations become equivalent, only if the effective masses and effective coupling constants of mesons are given by (2.3) and (2.11) [17] – [19].

In order to start self-consistent hadronic matter calculations, nonlinear coupling constants, \(g_{\sigmaN}, g_{\omegaN}, g_{\rhoN}\) and other 9 nonlinear coefficients should be supplied; but admissible values of nonlinear coefficients are determined to satisfy properties of symmetric nuclear matter at saturation (–15.75 MeV at \(k_F = 1.30 \text{ fm}^{-1}\), \(a_4 = 30.0 \text{ MeV}\)), simultaneously searching for the
minimum value of incompressibility and reproducing the maximum mass of isospin-asymmetric neutron stars \((M_{\text{max}}(n,p,e) = 2.50 \, M_\odot)\). In addition, hadronic phase transitions from \((n, p, e)\) to \((n, p, H_1, H_2, \cdots , e)\) should be carefully incorporated in EOS. The binding energies of hyperons required by hadronic model and SU(6) quark model for vector coupling constants \([37, 38]\) are compared and discussed quantitatively in the articles \([18, 19]\).

3 The nonlinear \(\sigma-\omega-\rho\) mean-field approximation with vacuum fluctuation correction (VFC)

We have included vacuum fluctuation correction (VFC) into the nonlinear \(\sigma-\omega-\rho\) mean-field approximation and applied the EOS with or without VFC to hadronic stars and hadron-quark hybrid stars. The EOSs for hadrons with VFC and quark matter generated by MIT-bag model are briefly discussed; based on the formalism, the pure hadronic and quark stars, stability of hadron-quark stars, are discussed in sec. 6. The vacuum fluctuation corrections are explicitly performed with counterterms required by power-counting and the method of dimensional regularization \([13]\).

The self-energies with VFC in the conserving nonlinear \(\sigma-\omega-\rho\) approximation are given with \((2.7)\) by,

\[
\Sigma^s_B = M^*_B(k_F) - M_B = \Sigma_N + \Sigma^s_H + \frac{1}{2 \pi^2 m^2} \sum_B g^2_{\sigma B} \left[ M^3_B \ln \left( \frac{M^*_B}{M_B} \right) - M^2_B(M_B - M^*_B) - \frac{5}{2} M_B(M^*_B - M_B)^2 - \frac{11}{6} (M^*_B - M_B)^3 \right],
\]

(3.1)

where \(B = n, p, \Lambda, \Sigma^-, \cdots\), and \(g^2_{\sigma B} \equiv g^2_{\sigma B}\) for \(B = \Lambda, \Sigma^-, \cdots\), since we are investigating effects of density-dependent interactions of nucleons; the density-dependent nonlinear self and mixing interactions among hyperons will be studied in the future. The self-energies \(\Sigma^v\) and \(\Sigma^0\) obtain no VFC in the mean-field approximation, which will be proven directly by dimensional regularization method. Note that it is also essential for an approximation with VFC to maintain conditions of conserving approximations in order to obtain \((3.1)\).

The energy density with VFC is derived as,

\[
\Delta E_{VFC} = - \frac{1}{8 \pi^2} \sum_B \left[ M^4_B \ln \left( \frac{M^*_B}{M_B} \right) + M^3_B(M_B - M^*_B) - \frac{7}{2} M^2_B(M_B - M^*_B)^2 + \frac{13}{3} M_B(M_B - M^*_B)^3 - \frac{25}{12} (M_B - M^*_B)^4 \right].
\]

(3.2)

Then, the total energy density is given with eq. \((2.12)\) as,

\[
E_{VFC} = E_{\text{NHA}} + \Delta E_{VFC}.
\]

(3.3)

The self-energies and pressure are also evaluated by dimensional regularization and thermodynamic consistency can be proved including VFC. As discussed in sec. 2, all coupling constants have to be evaluated by maintaining properties at nuclear matter saturation, searching the minimum value of incompressibility. Incompressibility is \(K \sim 350\) MeV and \(M_{\text{max}} = 2.33 \, M_\odot\) for \((n, p, e)+\text{VFC}\) matter. The vacuum fluctuation corrections to nonlinear \(\sigma-\omega-\rho\) mean-field approximation are not significant in low and high densities compared with those of nonlinear \(\sigma-\omega-\rho\) self and mixing interactions.
The energy density and pressure of MIT-bag model are derived [47 – 49]:

\[
\mathcal{E}_f = \frac{3}{8\pi^2} \sum_f \left[ 2k_f E_f(k_f) - m_f^2 E_f(k_f) - m_f^4 \log \left( \frac{k_f + E_f(k_f)}{m_f} \right) \right] + B , \tag{3.4}
\]

\[
P_q = \frac{1}{8\pi^2} \sum_f \left[ (2k_f^3 - 3k_f m_f^2) E_f(k_f) + 3m_f^4 \log \left( \frac{k_f + E_f(k_f)}{m_f} \right) \right] - B , \tag{3.5}
\]

where \( E_f(k_f) = (k_f^2 + m_f^2)^{1/2} \), \((f = u, d, s)\), and \(B\) is the bag-constant (MeV/fm³); \(k_f\) is the Fermi-momentum of flavor \(f\). The energy density and pressure with the following given baryon density, charge neutrality and phase equilibrium conditions:

\[
\rho_B = \frac{2k_f^3}{3\pi^2} = \frac{1}{3}(\rho_u + \rho_d + \rho_s) , \tag{3.6}
\]

\[
0 = 2\rho_u - \rho_d - \rho_s , \tag{3.7}
\]

\[
E_d(k_d) = E_s(k_s) , \tag{3.8}
\]

determine \((u, d, s)\)-quark matter uniquely (note: \(\rho_f = k_f^3/\pi^2\)). The thermodynamic potential for \(f\) to first order in the strong coupling constant is given by [48, 49, 23]:

\[
\Omega_f = -\frac{\gamma_f}{24\pi^2} \left\{ k_f E_f(k_f)(k_f^2 - \frac{3}{2}m_f^2) + \frac{3}{2}m_f^4 \log \left( \frac{k_f + E_f(k_f)}{m_f} \right) \right. \\
- \left. \frac{2\alpha_s}{\pi} \left[ 3(\frac{E_f(k_f)k_f - m_f^2 \log \left( \frac{k_f + E_f(k_f)}{E_f(k_f)} \right)}{E_f(k_f)})^2 - 2k_f^4 - 3m_f^4 \log^2 \left( \frac{m_f}{E_f(k_f)} \right) \right] \right. \\
+ \left. 6 \log \left( \frac{\sigma}{E_f(k_f)} \right) \left( k_f E_f(k_f)m_f^2 - m_f^4 \log \left( \frac{k_f + E_f(k_f)}{m_f} \right) \right) \right\} , \tag{3.9}
\]

where \(\gamma_f = \text{spin} \times \text{color} \) degeneracy. \(\alpha_s\) is the strong interaction coupling constant, and \(\sigma\) is the renormalization scale constant which is considered as a typical chemical potential of the problem and chosen as \(\sim 300\) MeV [23]. However, the EOS of hadron-quark compact stars is not sensitive to the parameter \(\sigma\), which is numerically checked and theoretically expected from [3.9] in high density region. It is suggested that the EOS of quark matter interconnected with hadronic matter be mainly sensitive to parameters, \((B, \alpha_c)\) in the current analysis. The expected value and restriction to \((B, \alpha_c)\) can be numerically extracted from properties of hadron-quark matter and compact stars in the sec. 6.

The energy density and pressure are now expressed as,

\[
\mathcal{E}_q = B + \sum_f (\Omega_f + \mu_f n_f) ,
\]

\[
P_q = -B - \sum_f \Omega_f , \tag{3.10}
\]

where \(\mu_f = E_f(k_f)\) and \(n_f\) are chemical potential and particle distribution for the flavor \(f\). The equations of state for hadronic and quark matter, phase transition conditions and TOV equation [21] are employed to calculate pure-hadron, pure-quark and hadron-quark compact stars.
We have applied the conserving nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation with a VFC to examine properties of hadronic neutron stars and H-Q hybrid stars.

The properties of pure hadronic stars are produced by employing hadronic equations of state, phase transition conditions, self-consistent effective masses and coupling constants, and the TOV equation. The masses v.s. central densities of hadronic stars produced by EOSs for $(n, p, e)$-$(n, p, \Sigma^-, e)$-$(n, p, \Sigma^-, \Lambda)$ matter with the coupling ratios $(r_{\Lambda N}^\sigma = 0.677, r_{\Lambda N}^\omega = 2/3)$ and $(r_{\Sigma^+ N}^\sigma = 0.632, r_{\Sigma^+ N}^\omega = 3/2, r_{\Sigma^+ N}^\rho = 1.00)$ give $M_{\text{max}} = 1.40$ $M_\odot$.

4 The properties of hadronic and hadron-quark (H-Q) hybrid stars

We have applied the conserving nonlinear $\sigma$-$\omega$-$\rho$ mean-field approximation with a VFC to examine properties of hadronic neutron stars and H-Q hybrid stars. The properties of pure hadronic stars are produced by employing hadronic equations of state, phase transition conditions, self-consistent effective masses and coupling constants, and the TOV equation. The masses v.s. central densities of hadronic stars produced by EOSs for $(n, p, e)$-$(n, p, \Sigma^-, e)$-$(n, p, \Sigma^-, \Lambda)$ matter are shown in Figs. 1a and 1b. The solid line shows $(n, p, e)$ matter which produces the maximum mass of hadronic neutron stars, $M_{\text{max}} = 2.50$ $M_\odot$ [13]. LHA(n) is the result of the linear $\sigma$-$\omega$ mean-field approximation [13] $(M_{\text{max}} = 3.06$ $M_\odot$). The arrows respectively indicate phase transitions from $(n, p, e)$-$(n, p, \Sigma^-, e)$ and $(n, p, \Sigma^-, e)$-$(n, p, \Sigma^-, \Lambda)$ matter. The EOS of hadronic matter with $r_{\Lambda N}^\sigma = 1.0, r_{\Sigma N}^\sigma = 1.0$ generates the maximum mass $M_{\text{max}}(n, p, \Sigma^-, \Lambda) = 1.98$ $M_\odot$ (Fig. 1a). However, the EOS with $r_{\Lambda N}^\sigma = 2/3, r_{\Sigma N}^\sigma = 2/3$ as suggested by the SU(6) quark model, produces the small maximum mass of neutron stars, $M_{\text{max}}(n, p, \Sigma^-, \Lambda) = 1.33$ $M_\odot$ (Fig. 1b). This is unable to support the observed masses of neutron stars. The discrepancy calculated by $r_{\Lambda N}^\sigma = 1.0$ and $r_{\Sigma N}^\sigma = 2/3$ is clearly recognized by comparing the EOS and mass of neutron stars.

All curves of $E_c - M_{\text{star}}$ in Fig. 1a and 1b have simple positive slopes, $dM/dE_c > 0$, and one maximum point which is typical for lines of one critical (inflection) point, $d^2M/dE_c^2 < 0$. In this case, stability can be examined for these simple saturating curves by the condition $dM_{\text{star}}/dE_c > 0$ [23] [54], respectively. Hence, values of positive slopes of $E_c - M_{\text{star}}$ are solutions of stable neutron stars, but the decreasing curve, $dM_{\text{star}}/dE_c > 0$, after the maximum point indicates unstable neutron stars. These characters of $E_c - M_{\text{star}}$ are important to examine stability of hadron-quark stars.

The EOS of pure-quark matter is used to calculate masses of pure-quark stars, as shown in Fig. 1a. Masses of hadronic neutron stars. Pure-neutron matter in linear $\sigma$-$\omega$: $M(n)_{\text{max}} = 3.06$ $M_\odot$. Isospin asymmetric $\beta$-equilibrium matter: $M(n, p)_{\text{max}} = 2.50$ $M_\odot$. The properties of hadronic and hadron-quark (H-Q) hybrid neutron stars and H-Q hybrid stars. (The TOV equation. The masses v.s. central densities of hadronic neutron stars produced by EOSs for state, phase transition conditions, self-consistent effective masses and coupling constants, and $n, p, e$ phase transitions from neutron matter in linear $\sigma$-$\omega$-$\rho$ mean-field approximation [13] $(n, p, \Sigma^-, \Lambda)$ matter with the coupling ratios $(r_{\Lambda N}^\sigma = 0.964, r_{\Lambda N}^\omega = 1.00)$ and $(r_{\Sigma N}^\sigma = 0.925, r_{\Sigma N}^\omega = 1.00)$ give $M_{\text{max}} = 1.98$ $M_\odot$. LHA(n) (dot-dashed line) is the result of the linear $\sigma$-$\omega$-$\rho$ mean-field approximation with a VFC to examine properties of hadronic neutron stars and H-Q hybrid stars. The properties of pure hadronic stars are produced by employing hadronic equations of state, phase transition conditions, self-consistent effective masses and coupling constants, and the TOV equation. The masses v.s. central densities of hadronic stars produced by EOSs for $(n, p, e)$-$(n, p, \Sigma^-, e)$-$(n, p, \Sigma^-, \Lambda)$ matter are shown in Figs. 1a and 1b. The solid line shows $(n, p, e)$ matter which produces the maximum mass of hadronic neutron stars, $M_{\text{max}} = 2.50$ $M_\odot$ [13]. LHA(n) is the result of the linear $\sigma$-$\omega$ mean-field approximation [13] $(M_{\text{max}} = 3.06$ $M_\odot$). The arrows respectively indicate phase transitions from $(n, p, e)$-$(n, p, \Sigma^-, e)$ and $(n, p, \Sigma^-, e)$-$(n, p, \Sigma^-, \Lambda)$ matter. The EOS of hadronic matter with $r_{\Lambda N}^\sigma = 1.0, r_{\Sigma N}^\sigma = 1.0$ generates the maximum mass $M_{\text{max}}(n, p, \Sigma^-, \Lambda) = 1.98$ $M_\odot$ (Fig. 1a). However, the EOS with $r_{\Lambda N}^\sigma = 2/3, r_{\Sigma N}^\sigma = 2/3$ as suggested by the SU(6) quark model, produces the small maximum mass of neutron stars, $M_{\text{max}}(n, p, \Sigma^-, \Lambda) = 1.33$ $M_\odot$ (Fig. 1b). This is unable to support the observed masses of neutron stars. The discrepancy calculated by $r_{\Lambda N}^\sigma = 1.0$ and $r_{\Sigma N}^\sigma = 2/3$ is clearly recognized by comparing the EOS and mass of neutron stars.

All curves of $E_c - M_{\text{star}}$ in Fig. 1a and 1b have simple positive slopes, $dM/dE_c > 0$, and one maximum point which is typical for lines of one critical (inflection) point, $d^2M/dE_c^2 < 0$. In this case, stability can be examined for these simple saturating curves by the condition $dM_{\text{star}}/dE_c > 0$ [23] [54], respectively. Hence, values of positive slopes of $E_c - M_{\text{star}}$ are solutions of stable neutron stars, but the decreasing curve, $dM_{\text{star}}/dE_c > 0$, after the maximum point indicates unstable neutron stars. These characters of $E_c - M_{\text{star}}$ are important to examine stability of hadron-quark stars.

The EOS of pure-quark matter is used to calculate masses of pure-quark stars, as shown in
Fig. 2. Masses of pure-quark stars with $B = 80, 120, 150$ MeV/fm$^3$, ($\alpha_c = 0$). The stable pure-quark stars with $M_{\text{max}} \gtrsim 1.0$ are in $10^{15} \sim E_c \sim 5.0 \times 10^{15}$ g/cm$^3$.

Fig. 3. Hadron stars with VFC and pure-quark stars. The stable hadronic stars are in $5.0 \times 10^{14} \sim E_c \sim 10^{15}$ g/cm$^3$.

Fig. 2. The bag constant should be $B \gtrsim 80$ MeV/fm$^3$, as normal nuclear matter becomes quark matter in the case of $B \simeq 80$ MeV/fm$^3$. The maximum masses of quark stars are $M_{\text{max}} = 1.59\,M_\odot$ ($B = 80$ MeV/fm$^3$), $M_{\text{max}} = 1.31\,M_\odot$ ($B = 120$ MeV/fm$^3$) and $M_{\text{max}} = 1.18\,M_\odot$ ($B = 150$ MeV/fm$^3$). The $E_c - M_{\text{star}}$ curves in Fig. 2 and 3 for the current pure hadron and quark stars are smoothly increasing and saturating. These curves are classified as those of one critical (inflection) point. Therefore, stability of neutron stars in pure-hadron and pure-quark matter can be checked by the condition: $dM_{\text{star}}/dE_c > 0$, though one needs to check other conditions if $E_c - M_{\text{star}}$ curves exhibit complicated phase transitions so that critical points are larger than 2 [54].

Stable quark stars with $M_{\text{star}} \gtrsim 1.0\,M_\odot$ are limited in the range of central energy densities: $10^{15} \lesssim E_c \lesssim 5.0 \times 10^{15}$ g/cm$^3$, as shown in bold, solid lines in Fig. 2. Stable hadronic neutron stars with the VFC and pure quark stars are shown in Fig. 4, to remark on the energy-density regions of stable stars. The results of hadronic and quark stars in Fig. 3 suggest that H-Q hybrid stars are possible about the central energy density, $E_c \sim 10^{15}$ g/cm$^3$, as the EOS of quark matter could be energetically preferable at high densities, compared with that of hadrons. However, the overlap in central energy density between stable hadronic stars ($M_{\text{max}}^H \gtrsim 1.0$) and pure-quark
Hadron-quark stable stars

\[ \text{Hadron-quark stars based on isospin asymmetric } \langle n,p,e \rangle + \text{VFC matter. Note that the quark-core of H-Q stars is stable.} \]

\[ M_{\text{Q}}^{\text{max}} \gtrsim 1.0 \]

The stability of pure hadronic and quark matter is examined, respectively, by the condition \( dM_{\text{star}}/d\varepsilon_c > 0 \) [23, 54]. One should note that the stability criterion, \( dM_{\text{star}}/d\varepsilon_c > 0 \), is for single phase compact stars, such as pure hadronic and quark stars. As H-Q stars are 2-phase compact stars (i.e., the quark phase for a star’s core and hadron phase for a mantle), the stability of H-Q stars should be reconsidered.

The H-Q stars in Fig. 4 show that the stable hadronic stars \( (dM_H/d\varepsilon_c > 0) \) in the central energy range, \( \varepsilon_c = 10^{14} \sim 10^{15} \text{ g/cm}^3 \), will undergo a phase transition (dotted horizontal line), reaching H-Q stars \( (B = 150 \text{ MeV/fm}^3) \). The Fig. 4 indicates that the total mass of H-Q stars decreases, but a stable quark-core \( (dM_Q/d\varepsilon_c > 0) \) develops. Therefore, this suggests that the H-Q star is stable, even though the total mass of the star becomes smaller. Moreover, by comparing stable energy densities of the quark phase in Fig. 3 with those of H-Q stars, the central energy density of stable H-Q stars is found to be more extended for higher densities than that of single phase stars. This suggests that compact stars consisting of a mantle and a high density core are more stable than stars in a homogeneous single phase structure [49, 55]. When the QCD coupling constant, \( \alpha_c \), increases, the H-Q phase transition density and quark-core will shift to higher densities, but the quark-core is stable and extends to higher densities. If the bag constant
Fig. 5a. Hadron-quark stars based on \((n,p,e)-(n,p,\Sigma^-,e)-(n,p,\Sigma^-,\Lambda) + VFC\) matter. The coupling ratios: \((r_\sigma^{\Lambda N} = 0.964, r_\omega^{\Lambda N} = 1.00)\) and \((r_\sigma^{\Sigma^- N} = 0.925, r_\omega^{\Sigma^- N} = 1.00, r_\rho^{\Sigma^- N} = 1.00)\).

is small, such as \(B \sim 100\) MeV/fm\(^3\), one will obtain a saturation curve (an inflection point for stability) within \(10^{15} \sim 10^{16}\) g/cm\(^3\), as in Fig. 3.

The H-Q stars calculated with the EOS for \((n,p,e)-(n,p,\Sigma^-,e)-(n,p,\Sigma^-,\Lambda) + VFC\) to quark matter \((B = 150\) MeV/fm\(^3\)) are shown in Fig. 5a \((r_\omega^{HN} = 1.0)\) and 5b \((r_\omega^{HN} = 2/3)\), respectively.

The maximum masses in Fig. 5a are \(M_{\text{max}} = 1.31 M_\odot\) \((\alpha_c = 0.0)\) and \(M_{\text{max}} = 1.77 M_\odot\) \((\alpha_c = 0.1)\); whereas, in Fig. 5b, they are \(M_{\text{max}} = 1.37 M_\odot\) \((\alpha_c = 0.0)\) and \(M_{\text{max}} = 1.48 M_\odot\) \((\alpha_c = 0.1)\). A remarkable feature of H-Q stars in Fig. 5a \((r_\omega^{HN} = 1.0)\) is that they can reasonably explain central energy densities and maximum mass configurations \[25\] for stable compact stars, ranging from \(10^{14} \sim 10^{16}\) g/cm\(^3\).

In the case of the softer hadronic EOS in Fig. 5b \((r_\omega^{HN} = 2/3)\) with \(B = 100 \sim 150\) and \(\alpha_c\)-correction, phase transitions occur in relatively low densities. This results in smaller maximum masses of H-Q stars, \(M_{\text{max}} \lesssim 1.4 M_\odot\). Thus, the EOS with the coupling constants \((r_\omega^{HN} = 2/3)\) may not be appropriate to explain the observed masses of neutron stars. A physical reason from the hadronic sector is clear in terms of the EOS; however, it is very interesting to investigate how hadronic and quark models reconcile the problems pointed out in the paper.

If the QCD coupling constant is set large \((\alpha_c \gtrsim 0.1)\) in the soft EOS of Fig. 5b \((r_\omega^{HN} = 2/3)\), generation of the quark-phase will move up to a very high density, separating the energy densities of hadron and quark phases completely. In this case, no stable hadron-phase can exist or coexist even in a crust of the surface of quark stars, resulting in literally pure-quark stars. Although more precise and detailed analyses are needed for many-body interactions of hadronic and quark matter, the coupling constants for hadrons \((r_\omega^{HN} = 2/3\) or \(r_\omega^{HN} = 1/3)\), as suggested by the SU(6) quark model, would not be appropriate to examine the properties of nuclear and hyperonic matter, or the maximum masses of high density compact stars.

5  Concluding remarks

The quantum hadrodynamics (QHD) and mean-field approximations are interesting for describing hadronic many-body systems at large distances, but they must ultimately break down at short distances where QCD is valid; hence, it is stimulating to investigate how properties of hadron and quark dynamics will interconnect and influence each other dynamically \[13, 14\]. We have applied the conserving nonlinear \(\sigma-\omega-\rho\) mean-field approximation to study self-consistent, density-dependent interactions of hadrons and examined hadronic dynamics by employing cou-
pling constants required from the QCD degrees of freedom. Discrepancies between hadronic and QCD predictions are shown in terms of effective masses, effective coupling constants, incompressibility and symmetry energy, saturation properties of hyperon binding energies and maximum masses of high density compact stars. One of our purposes is to compare predictions indicated by effective hadronic and quark models so as to clarify interrelations and distinctions between them.

As saturation properties of symmetric nuclear matter are self-consistently related to those of hyperons, reproducing saturation properties of hyperons should be one of fundamental problems of nuclear many-body approximations \[1\] − \[7\]. Although more quantitative and model-independent analyses are needed, it is concluded that hyperon coupling ratios required by SU(6) quark model for the vector coupling constants \[37, 38\] are not appropriate in order to generate saturation properties of hyperons. The coupling ratios of hyperons, \(r_{\omega HN} \sim 1.0\), are appropriate to explain Fermi-liquid properties, saturation of hyperons \[18, 19\] and maximum masses of neutron stars. Discrepancies of hyperon coupling ratios required by hadronic and SU(6) quark models should indicate that both effective approaches be improved with each other as consistent theories for hadronic physics.

Heavy-ion collision experiments as well as neutron stars are useful to examine applications of quark models and constraints for hadronic calculations \[56, 57\], since conditions of hadron-quark phase transition depend on both equations of state for hadronic and quark matter. The appropriate values of \((B, \alpha_c)\) can be independently suggested from properties of infinite matter, which will qualitatively support properties discussed in the paper \[49\]. The stable pure-hadronic stars and pure-quark stars based on MIT-bag model exist in different energy densities respectively, but if a first-order phase transition is assumed, the energy density of stable hadron-quark stars expands from \(10^{14}\) to \(10^{16}\) g/cm\(^3\). In the range of energy density, pure-hadron stars are less than \(M(n, p, e)_{max} \lesssim 2.5\), but H-Q stars are \(M_{max}^{H-Q} \gtrsim 1.0 \, M_\odot\). It reasonably explains expected masses and energy density relations, \(M - E_c\) \[25\]. Therefore, existing high density compact stars are more likely to be regarded as hadron-quark stars rather than pure-hadron and pure-quark stars. These results should be further examined quantitatively from empirical data and effective theories of hadrons.

It is noteworthy that density-dependent many-body effects simulated by nonlinear \(\sigma-\omega-\rho\) interactions are more important than those of VFC in the nonlinear \(\sigma-\omega-\rho\) mean-field approximation. The contributions of VFC should be investigated further in more complicated nonlinear \(\sigma-\omega-\rho\) HF, BHF approximations in order to study vacuum-fluctuation and density-dependent corrections. The role of chiral symmetry in hadronic models should be examined by extending the current nonlinear \(\sigma-\omega-\rho\) mean-field approximation by including \(\pi\)-meson, which would reveal physical meaning and significance of chirality for hadronic and quark models.

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