Study of the $\eta\pi$ and $\eta'\pi$ spectra and interpretation of possible exotic $J^{PC} = 1^{-+}$ mesons

Adam P. Szczepaniak and Maciej Swat
Physics Department and Nuclear Theory Center, Indiana University, Bloomington, Indiana 47405

Alex R. Dzierba and Scott Teige
Physics Department, Indiana University, Bloomington, Indiana 47405

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We discuss a coupled channel analysis of the $\eta\pi$ and $\eta'\pi$ systems produced in $\pi^-p$ interactions at 18 GeV/c. We show that known QQ resonances, together with residual soft meson-meson rescattering, saturate the spectra including the exotic $J^{PC} = 1^{-+}$ channel. A possibility of a narrow exotic resonance at a mass near 1.6 GeV/c$^2$ cannot, however, be ruled out.

Gluons play a central role in the structure of matter as they mediate the strong interactions leading to quark and gluon confinement. The quantitative description of the dynamics of low energy gluons, however, is still unavailable and the direct manifestation of gluons in the current spectroscopy of hadrons, for the most part, seems to be elusive. Any theoretical approach is complicated by the strong character of the interactions of soft gluons in QCD. Furthermore, the phenomenology of gluonic excitations is challenging because gluons do not carry electromagnetic or weak charge to which probes can directly couple. The spectroscopy of hadrons with excited gluonic degrees of freedom remains as the only avenue to obtain information about the nature of the soft gluonic modes. In this context the spectroscopy of exotic mesons is of fundamental importance. Exotic mesons have quantum numbers that cannot be attributed to valence quarks alone. Lattice gauge simulations indicate the gluonic field confining quarks forms flux tubes and the quantum numbers of the excited flux tubes couple with those of the quarks leading to exotic quantum numbers. The spin ($J$), parity ($P$) and charge conjugation ($C$) quantum numbers of the lightest exotic isovector should be $J^{PC} = 1^{-+}$ and have mass around 1.9 GeV/c$^2$ \cite{1,2,3}. It is possible, however, that corrections due to extrapolation of the lattice mass predictions to the physical light quark masses could introduce a 100 - 200 MeV/c$^2$ downward shift \cite{4}. Based on the large $N_C$ expansion it has been shown that exotic mesons ought to have hadronic decay widths comparable to the other mesonic resonances \cite{4} i.e. to be of the order of $\Gamma = 100 - 200$ MeV/c$^2$. Lattice simulations also indicate that at least one exotic meson multiplet should exist below the threshold energy for string breaking and quark production \cite{4}.

Finding a $J^{PC} = 1^{-+}$ wave is not equivalent to establishing the existence of a QCD exotic. A partial wave can have many poles as a function of complex energy in the unphysical domain of the complex energy plane (Riemann sheets). With an incomplete knowledge of the underlying strong interaction dynamics it is impossible to unambiguously determine positions of distant poles. Similarly limited experimental data are unable to isolate and interpret such poles. Only poles relatively close to the real axis can potentially be unambiguously identified, however, even in this case theoretical input is needed to discriminate between a preexisting bound state and a dynamical resonance.

The current experimental situation appears to be puzzling. Recently three exotic $J^{PC} = 1^{-+}$ meson candidates have been reported. The $\pi_1(1400)$, which has an unusually light mass, $M = 1370 \pm 16^{+50}_{-30}$ MeV/c$^2$, and a large width, $\Gamma = 385 \pm 40^{+65}_{-105}$ MeV/c$^2$, was reported by the E852 collaboration in $\pi^-p \rightarrow \pi_1(1400)p \rightarrow \pi^-\eta\pi'\pi$ \cite{5} at 18 GeV/c. This state was confirmed in the $\eta\pi$ channels in $pd$ annihilations to $\pi^-\pi^0np$ by the Crystal Barrel collaboration \cite{6}. Two other $\pi_2(1600)$ exotics were also reported by the E852 collaboration, one with $M = 1597 \pm 10^{+45}_{-20}$ MeV/c$^2$ and a large width of $\Gamma = 340 \pm 40 \pm 50$ MeV/c$^2$ observed in the $\eta'\pi^-\pi^0$ channel \cite{7}, and the other with $M = 1593 \pm 8^{+29}_{-47}$ MeV/c$^2$ and a normal width of $\Gamma = 168 \pm 20^{+150}_{-47}$ MeV/c$^2$ decaying into $\rho^0\pi^-\pi^0$. The $\eta\pi$ and $\eta'\pi$ channels are well suited for $J^{PC} = 1^{-+}$ exotic meson searches, since a neutral $P$-wave resonance has exotic quantum numbers.

It should be noted that the above resonance parameters come from a Breit-Wigner (BW) parameterization thus assuming a resonance interpretation of the data. But a BW parameterization is only applicable if amplitudes have singularities near the real axis, which in the case of a 300 – 400 MeV/c$^2$ wide resonances is not reasonable. Furthermore, in the original analysis of the E852 data, only selected $J_z$ states of the $\eta\pi$ and $\eta'\pi$ systems were considered in the mass dependent analysis that gave the resonance parameters. If an exotic $J = 1$ resonance is produced, however, all $2J + 1$ spin components should be included in the analysis. Even though these spin projections can have different production characteristics they should all display similar resonance behavior as a function of the invariant mass of the decay products. In a recent analysis of the $\eta^{0}$ E852 data \cite{8} it was shown that an exotic $P$-wave is indeed present, and very similar to the one found in the $\eta\pi^-$ mode, but when all information was used, including all spin components and production modes, no self-consistent set of BW parameters...
describing the observed $P$-wave could be found.

Within QCD, the quark structure of a typical narrow meson resonance, e.g. the $\rho(770)$ or the $a_2(1320)$, is not much different from a bound state. The quark wave functions are compact and the small width arises from coupling to a few open channels. In contrast, broad structures like the $\pi_1(1400)$ or the $\pi_1(1600)$, as observed in the $\eta\pi$ and $\eta'\pi$ channels, respectively, are likely to be of a different origin. In this paper we present a new analysis of the E852 data and explain the dynamical origin of the $P$-wave enhancement seen in the $\eta\pi$ and $\eta'\pi$ channels. In particular, we argue that these might be very similar to the $\sigma$ meson which is used to parameterize the low energy $S$-wave, isoscalar $\pi\pi$ spectrum. It is generally accepted that the $\sigma$ meson does not originate from a quark bound state but is instead a dynamical resonance arising from residual, low energy $\pi\pi$ interactions. 

The low energy interactions of the pseudoscalar meson nonet can be constrained using standard low energy expansion methods, e.g. effective range theory. Chiral symmetry provides a natural scale for such an expansion, $f \sim 100$ MeV/c. In particular, for a two-body potential acting between two-meson channels, $i,j$ in $L$-th partial wave one has,

$$V_{ij}^L(p,q) = \langle p,i|V_L|q,j\rangle = \langle pq \rangle \left[ c_{0,ij} + c_{2,ij} \frac{p^2+q^2}{f^2} \cdots \right]$$

with, $p$ and $q$ denoting the relative momenta between mesons. It is expected that the coefficients, $c_{ij}$ are $O(1)$, and that these coefficients determine the low momentum expansion of the scattering amplitude. The real part of the scattering amplitude, however, arises from iteration of the potential leading to important non-analytical contributions from integration over low momentum components of the virtual states. These non-analytical contributions have, in many cases, been found to be sufficient to extend the range of applicability of the low momentum expansion to regions well above the momentum scale set by $f$, even up to $1$ GeV/c. Furthermore, it is expected that higher terms in the expansion can be approximated by pole terms corresponding to genuine QCD bound states. The approach can also be verified a posteriori by studying how sensitive are the parameters of these bound states and intensity of the coherent backgrounds introduced by $V_L$ to the details of the approximations. In particular, we have studied the $N/D$ approach with $V_L$ determining the numerator and the poles determining the CDD terms. We also examined the Lippmann-Schwinger formalism with and without relativistic kinematics. Finally, in all cases we studied various methods for removing the large momentum contributions from loop integrals, e.g. with or without explicit subtraction of power divergences. Below we present results which we have found to be robust with respect the various implementations.

The distribution of events, $N(M_X,\Omega,t)$, at fixed energy in the reaction $\pi^- p \rightarrow X^- p$ with $X = \eta\pi^-$ or $\eta'\pi^-$, is proportional to $|\sum_{LM} a_{LM}(t,M_X)|^2$. The partial wave amplitudes, $a_{LM}$, determine the production strengths of a two-pseudoscalar meson system, with spin $J = L$, as a function of their invariant mass ($M_X$) and the momentum transferred squared, $t$, to the target. The peripheral nature of the high-$s$, low-$t$ production implies that the $M_X$ and $t$ dependence is expected to factorize. Verification of the assumption can be found in. The dependence on the solid angle, $\Omega$, is defined with respect to the Gottfried-Jackson frame. For a mass dependent analysis we bin the data in $M_X$ and integrate over all $t$ (the limited statistics prevents us from considering more than two $t$-bins). The number of events, $N(M_X,\Omega)$, in a given mass bin is decomposed into moments defined as $H_{LM}(M_X) = \int d\Omega N(M_X,\Omega)$ which are corrected for detector acceptance. The partial wave amplitudes are calculated from two-meson interactions as described above and fitted to the moments. In addition, it is necessary to specify the production amplitudes, $P^L_L$, whose mass dependence can, in principle, be constrained by duality. Instead, we use the requirement that they should have an expansion similar to the potential,

$$P^L_L(q) = \langle q,ip|V_L|\pi^- p\rangle = A^L_L q^L \left( 1 + d_{2,L}\frac{q^2}{f^2} \cdots \right)$$

with the relative strengths satisfying, $A^L_L/A^L' = O(1)$. In the mass range from threshold to $M_X < 2$ GeV/c$^2$ we find that only the $L = S, P, D$ waves are needed to describe the data. The partial waves are further classified by the magnitude of the spin projection, $M = 0, 1, \cdots L$ onto the beam axis and parity under reflection in the production plane. We find that only amplitudes with $M < 1$ are relevant, leading to a set of seven partial waves, $S, P_0, P_1, D_0, D_\pm$. Finally the positive and negative parity waves do not interfere and arise from $t$-channel natural and unnatural parity exchange, respectively.

For completeness, in addition to the $\eta\pi$ and $\eta'\pi$ channels, we also include the isovector $K\bar{K}$ channel. In the $S$-wave the $K\bar{K}$ attraction near threshold produces a cusp at $M_X \sim 1$ GeV/c$^2$ which corresponds to the $a_0(980)$ meson. A QCD resonance is expected at $M_X \sim 1.3 - 1.4$ GeV/c$^2$ and could be associated with the $a_0(1400)$ meson. This is similar to the scalar-isoscalar channel where the low energy $\pi\pi$ attraction results in the initial growth of the phase shift from threshold to $700$ MeV/c$^2$ which is commonly attributed to an effective $\sigma$ meson. The opening of the attractive $K\bar{K}$ channel responsible for the $f_0(980)$ meson leads to the rapid phase increase at $1$ GeV/c$^2$. Finally further growth of the phase in the $1.3 - 1.5$ GeV/c$^2$ can be attributed to presence of QCD resonances. In the $S$-wave we thus include all three coupled two-meson channels and a single pole term with a mass expected around $1.3 - 1.4$ GeV/c$^2$. The $D$ wave is dominated by the narrow $a_2(1320)$ meson.
discrepancy between data and theory in the $\eta\pi$ and $\eta'\pi$ (right) mass spectra compared with the data. Also shown are intensities of some of the partial waves, the $D_+$ (dashed), the $S$ (dashed-dotted) and the $P_+$ wave (dotted). In the $\eta\pi$ channel the $S$ and $P$ (larger then $S$ at higher mass) are a small fraction of the dominant $D$-wave. For $\eta'\pi$ the $D$ and $P$ waves are comparable.

which we introduce as a pole term. The soft interactions in the $D$-wave correspond to $O(p^4/f^4)$ in the chiral expansion and thus cannot be unambiguously disentangled from pole contributions, also assumed to be $O(p^4/f^4)$. This introduces a theoretical uncertainty in the coherent $D$-wave background. Thus the only constraint imposed on the $D$-wave is that coefficients are of $O(1)$. The ability to correctly describe the known spectrum in the $S$ and $D$-wave in an important check on both the theoretical method and data selection.

The $P$-wave is of the prime interest here. We have studied cases with and without a pole term. The soft interactions in the $P$-wave channel are less constrained than, for example, the $S$-wave since they require mapping between the physical, $\eta, \eta'$ and the $SU(3)$ flavor, $\eta_0, \eta_8$ states [18]. In general it is predicted however that both $\eta\pi$ and $\eta'\pi$ interactions are attractive with the former weaker by $\sin^2(\theta) \sim 0.01$ where $\theta$ is the $\eta_0 - \eta_8$ mixing angle. With these general assumptions we have found that the predictions for the spectrum of the $P$-wave is very insensitive to the details of the channel interactions.

The results of the combined analysis of the $\eta\pi^-$ and $\eta'\pi$ channels are shown in figures 1 and 2. The $\eta\pi$ channel is dominated by the $a_2(1320)$ resonance. The contribution from the coherent $D$-wave background is small leading to a pole mass within 10% of the resonance mass listed by the PDG. The $S$-wave production is very weak and does not constrain parameters of the $S$-wave pole, the $a_0(1400)$ not the dynamically generated, $a_0(980)$. The discrepancy between data and theory in the $\eta\pi$ intensity

at $M_X \sim 1$ GeV/c$^2$ is within systematical errors of the $S$-wave parameters.

We did find, however, that the $H_{23}$ moment which is sensitive to the interference between the unnatural exchange, $D_-$ wave and the $S$ wave, has nontrivial structure at $M_X = 1.4$ GeV/c$^2$ which could be related to the the $a_0(1400)$ pole. The $P$ wave intensity in the $\eta\pi$ channel is weak, $\leq 5\%$ of the $D$ wave, and leads to a broad structure consistent with the absence of nearby resonance in this channel. As discussed in [1] the peak-like structure at $M_X = 1.3$ GeV/c$^2$ is most likely due to the dominant $a_2(1320)$ leaking into the $P$-wave due to imperfect acceptance corrections. The phase of the natural exchange $P_+$ wave is found to be very weakly mass dependent, $|\delta\phi_P| < 10^\circ$ over the mass range considered, and as shown in figure 2(d) consistent with the data. The variations in the $\phi_P - \phi_P$ phase difference at $M_X > 1.6$ GeV/c$^2$ is due entirely to the weak coherent $D$-wave background. Similarly the sharp variation of this phase difference at low mass can be fully accounted for by the $D_+$ coherent background. Even though the channel interaction in the $D$ wave are not well constrained it seems clear that both phase and magnitude of the $P$-intensity in the $\eta\pi$ channel can be well accounted for by $\eta\pi$ rescattering. In contrast, in the $\eta'\pi$ channel the contribution from the $P$ wave is significant and consistent.
with the theoretical expectations discussed above. As shown in figure 1b, the $P$ is as large as the $D$-wave while the $S$-wave is small. Since the $D$ and $P$ waves are comparable there is no danger of one leaking into another and the partial wave analysis is quite unambiguous. It is seen from figure 2 that the soft channel interactions describe both the intensity and the phase motion of the $P$ wave very well.

We have found, however, that there is a structure near $M_X = 1.6$ GeV/$c^2$ in several moments which could be accounted for by a pole term in the $P$-wave. Since the bulk of the $P$-wave intensity comes from soft re-scattering, the resonance associated with the pole term becomes narrow. If parameterized as a BW resonance it has $\Gamma \sim 200$ MeV/$c^2$ and is thus consistent with the narrow resonance observed in the $\rho\pi$ channel.

In conclusion, we find that the broad $P$ wave in the $\eta\pi$ and $\eta'\pi$ spectra can be accounted for by low energy re-scattering effects. Comparing this to the $\pi\pi$ channel, we interpret this enhancement as the equivalent of the $\sigma$ meson, i.e. arising from correlated meson-meson state and not from a QCD bound state. The applicability of low momentum expansion used here is well justified since, after subtracting the threshold energies, the relevant momentum range is comparable to the $\pi\pi$ case and the two analyses are completely consistent. In the non-exotic $P$-wave $\pi\pi$ channel there are, in addition to residual meson-meson interactions, narrow resonances like the $\rho$ and $\phi$ mesons that are believed to be true QCD bound states. We have found that the $\eta'\pi$ data does not rule out a narrow exotic $P$-wave resonance which might be the $\pi_1(1600)$ that is found to decay into $\rho\pi$ but from no final conclusions can be drawn at this time.

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