ON THE CORRECT USE OF GRAVITY DARKENING COEFFICIENTS IN THE JKTEBOP ECLIPSING BINARY CODE

GUILLERMO TORRES

Center for Astrophysics | Harvard & Smithsonian, 60 Garden St., Cambridge, MA 02138, USA; gtorres@cfa.harvard.edu
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ABSTRACT

Users of the JKTEBOP code to solve the light curves of eclipsing binaries often confuse the gravity darkening coefficients, $y(\lambda)$, with the bolometric gravity darkening exponents, $\beta$. JKTEBOP requires the wavelength-dependent coefficients. I show that the numerical values of $y(\lambda)$ and $\beta$ can be rather different, leading to potential biases in the solution if the wrong quantities are used.

1. INTRODUCTION

von Zeipel (1924) showed that under conditions of radiative equilibrium, the emergent bolometric flux at any point on the surface of a tidally or rotationally distorted star is proportional to the local gravity. A rotating star will be flattened at the poles, which will then be brighter (hotter) than the equatorial regions. This “gravity darkening” (or gravity brightening) effect can have a non-negligible impact on the light curve of an eclipsing binary.

More generally, the gravity darkening law may be expressed as $T_{\text{eff}}^4 \propto g^\beta$, where $g$ is the local gravity and $T_{\text{eff}}$ the effective temperature. For stars in radiative equilibrium $\beta \approx 1.0$ (von Zeipel 1924), whereas Lucy (1967) showed that stars with deep convective envelopes have $\beta \approx 0.32$, on average. More recently Claret (1998) published extensive tabulations giving $\beta$ as a function of mass, age, and effective temperature.

Some light curve modeling programs including the Wilson-Devvinney code (Wilson & Devinney 1971; Wilson 1979), WINK (Wood 1971, 1973), ELC (Orosz & Hauschildt 2001), and PHOEBE (Prsa 2018) expect the user to supply a value for the exponent $\beta$ of the bolometric gravity darkening law. Application of the effect to the specific bandpass of the light curve is then dealt with internally, using either theoretical model atmospheres or a blackbody approximation.

Other light curve modeling programs adopt a different formulation to deal with gravity darkening directly at the wavelength of the photometric observations. Examples include CILLC (Maxted 2016), as well as the EBOP code (Etzel 1981, Popper & Etzel 1981) and its descendants, EB (Irwin et al. 2011) and JKTEBOP (Southworth et al. 2014, Southworth 2013), which are most suitable for well-detached systems. These programs use a Taylor expansion of the wavelength-dependent flux as a function of local gravity, retaining only the linear term, consistent with other approximations in the binary model (see, e.g., Kopal 1959; Kitamura & Nakamura 1983). The coefficient of the linear term of these codes requires $y(\lambda)$, which is specific to the bandpass under consideration. Tables of $y(\lambda)$ based on model atmospheres, calculated for a wide range of stellar properties, have been published by Claret & Bloemen (2011) for several of the most common photometric systems, and more recently by Claret (2017) for the photometric band of NASA’s planet-finding TESS mission (Ricker et al. 2014). The latter source of coefficients is particularly useful, as TESS has already collected high-precision light curves for many thousands of eclipsing binaries over much of the sky.

Among the simpler light curve programs using the linearized, wavelength-specific formulation of gravity darkening, JKTEBOP is one of the more popular ones for its speed and ease of use. This Note is motivated by the observation that some eclipsing binary studies that use JKTEBOP confuse the gravity darkening coefficient with the exponent of the bolometric law, and adopt numerical values for $\beta$ instead of $y(\lambda)$. The author himself has made this mistake once or twice in the past. Other papers using JKTEBOP give no indication of what was assumed about gravity darkening, so it is not possible to tell whether or not the same mistake was made.

The use of the linearized prescription for gravity darkening is clearly described in the original documentation for EBOP, although admittedly that unpublished document is difficult to obtain. As of this writing, full documentation for JKTEBOP itself is not yet available, aside from some notes on the website1 and in the sample input file, but the source code does correctly indicate that the input for gravity darkening should be the coefficient $y(\lambda)$. This was also explained in detail by Torres et al. (2017), although mostly only in a footnote. It is also described correctly in the documentation for the eb code of Irwin et al. (2011), which relies on the same underlying binary model as JKTEBOP, although JKTEBOP users would probably have little reason to consult that document, as it pertains to a different program2. It is perhaps not surprising, therefore, that many authors are unaware of this detail, and proceed with their JKTEBOP light curve solution adopting values for $\beta$ instead of $y(\lambda)$.

2. NUMERICAL DIFFERENCES BETWEEN COEFFICIENTS AND EXPONENTS

To illustrate the numerical differences between the bandpass specific coefficients and the exponents of the gravity darkening law, I adopt a representative solar-metallicity model isochrone from Chen et al. (2014), shown in the top panel of Figure 1. For each point along this isochrone, I extracted the $y(\lambda)$ values from the tables

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1 https://www.astro.keele.ac.uk/jkt/codes/jktebop.html
2 The paper describing the eb code of Maxted (2016), which also uses the linearized gravity darkening law, explicitly mentions the wavelength-dependent $y(\lambda)$ quantities, but confusingly calls them exponents.
Figure 1. Top: Representative PARSEC isochrone by Chen et al. (2014). Bottom: Gravity darkening coefficients $y(\lambda)$ (for microturbulence $\xi = 2 \text{ km s}^{-1}$) calculated along the isochrone in the top panel, for the Johnson $B$ and $V$ bands, and TESS $T$ (Claret & Bloemen 2011; Claret 2017). The dashed lines indicate the numerical values for the gravity darkening exponent $\beta$ for stars with radiative and convective envelopes, which are sometimes adopted erroneously instead of the correct gravity darkening coefficients $y(\lambda)$.

of Claret & Bloemen (2011) and Claret (2017), shown in the bottom panel, which vary considerably with temperature and not in a monotonic way. I also indicate the numerical values of the bolometric exponent often adopted for radiative stars ($\beta = 1.0$; von Zeipel 1924) and convective stars ($\beta = 0.32$; Lucy 1967).

For stars of spectral type K ($\sim 4000$–$5000$ K) the numerical values of $y(\lambda)$ for the bluer wavelengths ($B$, $V$) are rather different from the $\beta$ value typically adopted for such stars, by up to a factor of two or more. Differences are smaller but still significant in the TESS band, depending on the temperature. For hotter stars, mistakenly adopting a value of $\beta = 1.0$ in using JKTEBOP, instead of the proper wavelength-dependent coefficient $y(\lambda)$, will be a poor approximation for all except the mid A stars in the bluer bandpasses. Errors will be larger at longer wavelengths.

It is difficult to predict how this mistake might bias the results of a particular light curve solution with JKTEBOP, or with other programs that expect $y(\lambda)$ values instead of $\beta$. In general the impact will depend on the binary configuration, the stellar properties of the components, and the wavelength and photometric precision of the observations. For well-detached systems with nearly spherical stars (for which JKTEBOP is most suitable), the gravity darkening effect is small so using the wrong number may not be serious. On the other hand, the TESS light curves now available for large numbers of eclipsing binaries are much more precise than ground-based photometry, so the impact may be more noticeable. In any case, the correct gravity darkening values to use with JKTEBOP are the wavelength-dependent coefficients $y(\lambda)$, and not the exponents $\beta$.

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REFERENCES

Chen, Y., Girardi, L., Bressan, A., et al. 2014, MNRAS, 444, 2525
Claret, A. 1998, A&AS, 131, 395
Claret, A. 2017, A&A, 609, A30
Claret, A., & Bloemen, S. 2011, A&A, 529, A75
Etzel, P. B. 1981, Photometric and Spectroscopic Binary Systems, Proc. NATO Adv. Study Inst., ed. E. B. Carling & Z. Kopal (Dordrecht: Reidel), p. 111
Irwin, J. M., Quinn, S. N., Berta, Z. K., et al. 2011, ApJ, 742, 123
Kitamura, M., & Nakamura, Y. 1983, AnTok, 19, 413
Kopal, Z. 1959, Close Binary Systems, The International Astrophysics Series, Vol. 5, (New York, John Wiley & Sons, Inc.)
Lucy, L. B. 1967, Zeitschrift für Astrophysik, 65, 89
Maxted, P. F. L. 2016, A&A, 591, A111
Orosz, J. A. & Hauschildt, P. H. 2000, A&A, 364, 265
Popper, D. M., & Etzel, P. B. 1981, AJ, 86, 102
Prša, A. 2018, Modeling and Analysis of Eclipsing Binary Stars: The theory and design principles of PHOEBE, ISBN: 978-0-7503-1288-2, IOP ebooks (Bristol, UK: IOP Publishing), doi:10.1088/978-0-7503-1287-5
Ricker, G. R., Winn, J. N., Vanderspek, R., et al. 2014, Proc. SPIE, 9143, 914320
Southworth, J. 2013, A&A, 557, A119
Southworth, J., Maxted, P. F. L., & Smalley, B. 2004, MNRAS, 351, 1277
Torres, G., McGruder, C. D., Siverd, R. J., et al. 2017, ApJ, 836, 177.
von Zeipel, H. 1924, MNRAS, 84, 702
Wilson, R. E. & Devinney, E. J. 1971, ApJ, 166, 605
Wilson, R. E. 1979, ApJ, 234, 1054
Wood, D. B. 1971, AJ, 76, 701
Wood, D. B. 1973, PASP, 85, 253