Consensus-based adaptive guaranteed-performance formation control for second-order multi-agent systems

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Abstract
Adaptive guaranteed-performance formation analysis and design problems for second-order multi-agent systems are studied, where the global information is not required, which means the main results of this article are fully distributed. First, an adaptive guaranteed-performance formation control protocol is presented for second-order multi-agent systems, where the control input is constructed using neighboring state errors and adaptively adjustable interaction weights. Then, an adaptive guaranteed-performance formation control is proposed based on Riccati inequalities. Furthermore, the guaranteed-performance cost is determined and the adjusting approach of the formation control gain is presented in terms of linear matrix inequalities. Finally, a numerical simulation is provided to demonstrate the effectiveness of the theoretical results.

Keywords
Guaranteed-performance control, multi-agent systems, consensus approach, formation control, adaptive control

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Introduction
In the past few years, distributed cooperative control of multi-agent systems has aroused great attention from both scientific and engineering communities.\(^1\)–\(^6\) Multi-agent cooperative control problems in previous works\(^7\)–\(^17\) were solved by consensus-based approach, which requires that the specific state variables of all agents reach an agreement. As an important research topic of cooperative control of multi-agent systems, formation control has been widely applied in transportation, surveillance and rescue, and so on. As a point of fact, formation control problems have been extensively studied during the past decades, and some typical formation control approaches have been proposed, such as leader–follower, virtual structure, and behavior-based ones.\(^18\)–\(^20\) Recently, motivated by the development of consensus theory, the formation control problems were solved by the consensus-based approach as in previous works.\(^21\)–\(^25\)

Multi-agent systems in previous works\(^26\)–\(^28\) achieved formation without considering the control performance, however, multi-agent systems in practical tasks are usually required to achieve certain performance, such as the shortest time and the least energy consumption. For example, Xu et al.\(^29\) optimized the acceleration control parameters using the particle swarm optimization algorithm to reduce the energy

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consumption in the multiple robotic fishes’ formation process. The formation performance usually can be modeled as certain cost functions. A quadratic performance function is constructed to achieve guaranteed-performance time-varying formation in Wang et al.\textsuperscript{30} A cost function was constructed by synchronization errors and control input to describe the formation performance of the multi-agent system in Cheng and Ugrinovskii.\textsuperscript{31} As a matter of fact, the global information of the multi-agent system is required in Wang et al.\textsuperscript{30} and Cheng and Ugrinovskii,\textsuperscript{31} which means their main results are not fully distributed.

A distributed formation control algorithm is proposed for multi-vehicle nonlinear system model with simple kinetics characteristics coupling time-delay and jointly connected topologies in Xue et al.\textsuperscript{32} and Savkin et al.\textsuperscript{33} proposed a distributed motion control algorithm for groups of robots. It should be pointed that the formation control results in Xue et al.\textsuperscript{32} and Savkin et al.\textsuperscript{33} are fully distributed, while the formation performance was not considered. To the best of our knowledge, fully distributed guaranteed-performance formation control problems for second-order multi-agent systems with adaptive control scheme are still open.

Motivated by the facts stated above, this article investigated adaptive guaranteed-performance formation control for second-order multi-agent systems. First, an adaptive guaranteed-performance formation control protocol is proposed. Then, an adaptive guaranteed-performance formation control criterion is presented based on Riccati inequalities, and the guaranteed-performance cost is determined. Moreover, a formation control gain regulation approach is proposed.

Compared with the related works on formation control of second-order multi-agent systems, the new features of this article are threefold. First, a translation-adaptive strategy is proposed to deal with the fully distributed formation control problems for second-order multi-agent systems. However, the global information is required in Wang et al.\textsuperscript{30} and Cheng and Ugrinovskii.\textsuperscript{31} Second, the multi-agent systems in this article can achieve guaranteed-performance formation control, while the formation performance was not considered in Xue et al.\textsuperscript{32} and Savkin et al.\textsuperscript{33} Third, the formation control gain regulation strategy is proposed based on linear matrix inequality (LMI) techniques, while the formation control gains in previous works\textsuperscript{36–28} cannot be adjusted.

The remainder of this article is organized as follows. In section “Problem description,” the problem description and the formation control protocol are proposed. The main results are given under section “Main results.” Section “Numerical simulation” gives numerical simulations. Concluding remarks are presented in the final section.

Notations: let $\mathbb{R}^d$ stand for the $d$-dimension real column vector space and $\mathbb{R}^{d \times d}$ represent the set of $d \times d$ dimensional real matrices. For simplicity, let 0 denote zero matrices of any size with zero vector and zero number as special cases. $I_N$ stands for the identity matrix of dimension $N$. $1_N$ represents a column vector of dimension $N$, whose entries are equal to 1. $P^T$ stands for the transpose of the symmetric matrix $P$. $Q^T = Q > 0$ and $Q^T = Q < 0$ mean that $Q$ is positive semidefinite and negative semidefinite, respectively. The symbol $\otimes$ denotes the Kronecker product. The notation $*$ stands for the symmetric terms of a symmetric matrix.

**Problem description**

Consider a second-order multi-agent system consisting of $N$ homogeneous agents, whose dynamics is described as follows

$$\begin{cases}
\dot{x}_i(t) = v_i(t) \\
\dot{v}_i(t) = u_i(t)
\end{cases}$$

where $i \in \mathcal{I}_N$ with $\mathcal{I}_N = \{1, 2, \ldots, N\}$. $x_i(t) \in \mathbb{R}^d$, $v_i(t) \in \mathbb{R}^d$ and $u_i(t) \in \mathbb{R}^d$ are the position state, the velocity state, and the control input of agent $i$, respectively.

The interaction topology of multi-agent system (1) can be described by a connected undirected graph $G = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{V_1, V_2, \ldots, V_N\}$ denotes the set of nodes, $\mathcal{E} = \{(V_i, V_j), i \neq j, V_i, V_j \in \mathcal{V}\}$ denotes the set of edges. In the graph $G$, each node represents an agent in the system. The edge between two nodes represents the interaction relationship between agents, and the length of edge represents the strength of the interaction between agents. $\mathcal{N}_i$ stands for the neighboring set of agent $i$. The weight matrix is represented by a symmetric adjacency matrix $W(t) = [w_{ij}(t)]_{N \times N}$, where $l_{ii} = 0$ and $l_{ij} = 0$ if agent $j$ is not a neighbor of agent $i$ and $l_{ij} = 1$ otherwise, $w_{ij}(t)$ with $w_{ij}(t) > 0$ denotes the interaction weight from agent $j$ to agent $i$. If $l_{ii} = 0$, then $w_{ii}(t)$ denotes a virtual interaction weight from agent $j$ to agent $i$. $D(t) = \text{diag}(d_1(t), d_2(t), \ldots, d_N(t))$ with $d_i(t) = \sum_{j = 1, j \neq i}^N l_{ij} w_{ij}(t)$ stands for the in-degree matrix of $G$. $L(t) = D(t) - W(t)$ denotes the Laplacian matrix of $G$. The properties of the Laplacian matrix for an undirected graph are shown in the following lemma.

**Lemma 1.** Let $G$ denote an undirected graph, then its Laplacian matrix $L \in \mathbb{R}^{N \times N}$ has at least nonzero eigenvalue, and the corresponding eigenvector is $1_N$. If $G$ is connected, 0 is a simple eigenvalue of $L$ and all the other $N - 1$ eigenvalues are positive.\textsuperscript{34}

**Definition 1.** Multi-agent system (1) is said to achieve consensus if the following conditions are satisfied
\[
\lim_{t \to \infty} (x_i(t) - x_j(t)) = 0
\]
\[
\lim_{t \to \infty} (v_i(t) - v_j(t)) = 0
\]
where \(i, j \in \mathcal{I}_N\).

**Definition 2.** For a given vector \(\epsilon = [\epsilon_1^T, \epsilon_2^T, \ldots, \epsilon_N^T]^T\) with \(\epsilon_i \in \mathbb{R}^d\), multi-agent system (1) achieves formation \(\epsilon\) if the following conditions are satisfied
\[
\lim_{t \to \infty} (x_i(t) - x_j(t)) = \epsilon_i - \epsilon_j
\]
\[
\lim_{t \to \infty} (v_i(t) - v_j(t)) = 0
\]
where \(i, j \in \mathcal{I}_N\), the vector \(\epsilon\) is called a formation vector.

Let \(\delta_\epsilon(t) = [\delta v_1^T(t), \delta v_2^T(t), \ldots, \delta v_N^T(t)]^T\), where \(\delta v_i(t) = \epsilon_i(t) - \epsilon_j(t)\) with \(\epsilon_i(t) = x_i(t) - u_i\), and \(\delta v_j(t) = \delta v_j(t) - \delta v_i(t)\). Since \(\dot{\epsilon}_i(t) = \dot{x}_i(t) = v_i(t)\), it can be obtained that
\[
\begin{cases}
\dot{\epsilon}_i(t) = v_i(t) \\
\dot{v}_i(t) = u_i(t)
\end{cases}
\]
and formation control criterion (3) can be rewritten as
\[
\lim_{t \to \infty} (\epsilon_i(t) - \epsilon_j(t)) = 0
\]
\[
\lim_{t \to \infty} (v_i(t) - v_j(t)) = 0
\]

**Remark 1.** When the formation is achieved, it can be obtained by Definition 2 that \(\lim_{t \to \infty} (x_i(t) - x_j(t)) = \epsilon_i - \epsilon_j\), which does not mean that the position states of the agents accurately converge to the corresponding formation vector under the formation control protocol. Moreover, if \(\epsilon = 0_N \times N\), the formation control criterion (3) can be rewritten as the consensus criterion (2). Therefore, the consensus problem is a special case of formation problem. More generally, by equations (4) and (5), the formation control criterion (3) is written in a form similar to the consensus criterion (2); that is, a formation control problem can be described as a consensus problem.

**Definition 3.** Multi-agent system (1) is said to achieve adaptive guaranteed-performance formation by protocol (10) if there exist \(K_u\) and \(K_w\) such that
\[
\lim_{t \to \infty} \delta \epsilon_j(t) = 0 \quad (i, j = 1, 2, \ldots, N)
\]
and \(J_i < J_x^c\) for any bounded initial states \(x_i(0)|i = 1, 2, \ldots, N\), where \(J_x^c\) is said to be the guaranteed-performance cost.

The states of second-order multi-agent system (1) include position state and velocity state, so a control input \(u_i(t)\) is proposed as follows
\[
u_i(t) = u_{ic}(t) + u_{nv}(t)
\]
with
\[
u_{ic}(t) = K_{a1} \sum_{j \in \mathcal{I}_N} w_{ij} (x_j(t) - x_i(t))
\]
\[
u_{nv}(t) = K_{a2} \sum_{j \in \mathcal{I}_N} w_{ij} (v_j(t) - v_i(t))
\]
where \(K_{a1} \in \mathbb{R}^{d \times d}\) and \(K_{a2} \in \mathbb{R}^{d \times d}\) are the control gain matrices. Let \(K_u = [K_{a1}, K_{a2}]\), then the control input \(u_i(t)\) can be rewritten as
\[
u_i(t) = K_u \sum_{j \in \mathcal{I}_N} w_{ij} \delta \epsilon_j(t)
\]

Based on equation (9), a guaranteed-performance formation control protocol of second-order multi-agent system (1) is proposed as follows
\[
\begin{aligned}
u_i(t) &= K_u \sum_{j \in \mathcal{I}_N} w_{ij} \delta \epsilon_j(t) \\
\dot{w}_j(t) &= \delta \epsilon_j(t) K_u \delta \epsilon_j(t) \\
J_r &= \frac{1}{N} \sum_{i,j = 1}^{N} \int_0^\infty \delta \epsilon_j^T(t) Q \delta \epsilon_j(t) dt
\end{aligned}
\]
where \(i \in \mathcal{I}_N, K_u\) with \(K_u = K_u^T \geq 0\) is a gain matrix and \(Q = Q^T > 0\). It should be pointed out that \(w_j(t)\) is constantly positive and adaptively adjusted, and it is consistently increasing unless the states of agent \(i\) and agent \(j\) achieve consensus. Let \(\gamma_{ij}\) be the upper bound of \(w_j(t)\).

**Main results**

Let \(\chi(t) = [\chi_1^T(t), \chi_2^T(t), \ldots, \chi_N^T(t)]^T\) with \(\chi_i^T(t) = [x_i^T(t), v_i^T(t)]^T\), then multi-agent system (1) can be described in global form as
\[
\dot{\chi}(t) = (I_N \otimes A) \chi(t) - (I_N \otimes B) u(t)
\]
where
\[
A = \begin{bmatrix} 0 & I_d \\
0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\
I_d \end{bmatrix}
\]

Let \(l = [l_1^T, l_2^T, \ldots, l_N^T]\) with \(l_i = [l_i^T, 0]^T\) represent the formation vector of system (11) and \(\omega(t) = [\omega_1^T(t), \omega_2^T(t), \ldots, \omega_N^T(t)]^T\) with \(\omega_i^T(t) = [x_i^T(t) - \epsilon_i^T]\), then \(\omega(t) = \chi(t) - l\).

Due to formation vector \(l\) being a constant vector, \(\dot{l} = 0\). Since \(\omega_0(t) = \chi(t) - l\), it can be obtained by equations (9) and (11) that
\[
\dot{\omega}(t) = (I_N \otimes A - L_n(t) \otimes BK_u) \omega(t) + (I_N \otimes A) l
\]
By the special structure of \(I_N, A, l\), one has
\[(I_N \otimes A) l = 0 \quad (14)\]

So, equation (13) can be converted into
\[\dot{\omega}(t) = (I_N \otimes A - L_{w(t)} \otimes B_k) \omega(t) \quad (15)\]

where \(L_{w(t)}\) denotes the Laplacian matrix of \(G\).

From Lemma 1, one has \(L_{w(0)I_N} = 0\). Because \(G\) is connected, there exists an orthonormal matrix \(U = [1_N / \sqrt{N}, \bar{U}]\) satisfies that
\[U^T L_{w(0)} U = \begin{bmatrix} 0 & 0^T \\ 0 & A_\lambda \end{bmatrix} \quad (16)\]

where \(A_\lambda = \text{diag}(\lambda_2, \lambda_3, \ldots, \lambda_N)\) with \(0 < \lambda_2 \leq \ldots \leq \lambda_N\). Let \(\kappa(t) = (U^T \otimes I_{2d}) \omega(t) = [\kappa^T_1(t), \kappa^T_2(t), \ldots, \kappa^T_N(t)]^T\), where \(\kappa_1(t) \in \mathbb{R}^{2d}, \ k_2(t) = [\kappa^T_2(t), \ldots, \kappa^T_N(t)]^T \in \mathbb{R}^{2d(N-1)}\), then equation (15) can be rewritten as
\[\dot{\kappa}_1(t) = A \kappa_1(t) \quad (17)\]
\[\dot{\kappa}_e(t) = (I_{N-1} \otimes A - A_\lambda \otimes B_k) \kappa_e(t) = (I_{N-1} \otimes A - \bar{U}^T L_{w(t)} \bar{U} \otimes B_k) \kappa_e(t) \quad (18)\]

Define
\[\tilde{\kappa}_1(t) = U e_1 \otimes \kappa_1(t) = \frac{1}{\sqrt{N}} 1_N \otimes \kappa_1(t) \quad (19)\]
\[\tilde{\kappa}_e(t) = \sum_{i=2}^{N} U e_i \otimes \kappa_e(t) \quad (20)\]

where \(e_i(i \in \mathcal{I}_N)\) denotes \(N\)-dimensional column vectors with the \(i\)-th element 1 and 0 elsewhere.

It can be derived from equations (19) and (20) that
\[\tilde{\kappa}_1(t) = (U \otimes I_{2d}) [\kappa^T_1(t), 0^T]^T \quad (21)\]
\[\tilde{\kappa}_e(t) = (U \otimes I_{2d}) [0^T, \kappa^T_e(t)]^T \quad (22)\]

Since \(U \otimes I_{2d}\) is nonsingular, one can obtain that \(\tilde{\kappa}_1(t)\) and \(\tilde{\kappa}_e(t)\) are linearly independent. Due to
\[(U^T \otimes I_{2d}) \tilde{\kappa}(t) = [\kappa^T_1(t), \kappa^T_e(t)]^T \quad (23)\]

and
\[(U^T \otimes I_{2d}) \omega(t) = [\kappa^T_1(t), \kappa^T_e(t)]^T \quad (24)\]

one can see that \(\omega(t) = \tilde{\kappa}_1(t) + \tilde{\kappa}_e(t)\). Then, it can be obtained that the multi-agent system (1) achieves formation control if and only if \(\lim_{t \to \infty} \kappa_e(t) = 0\).

**Theorem 1.** For any given translation factor \(\gamma > 0\), multi-agent system (1) is said to achieve adaptive guaranteed-performance formation by protocol (10) if there exists a matrix \(P\) with \(P = P^T > 0\) such that \(2Q + PA + A^T P - \gamma PBB^T P = 0\). In this case, \(K_w = B^T P, K_w = PBB^T P\). The guaranteed-performance cost is
\[J^* = \omega^T(0) \left( \left( I_N - \frac{1}{N} 1_N \otimes I_N^T \right) \otimes P \right) \omega(0) + \gamma \int_0^\infty \omega^T(t) \left( \left( I_N - \frac{1}{N} 1_N \otimes I_N^T \right) \otimes PBB^T P \right) \omega(t) dt \quad (25)\]

**Proof.** Design a Lyapunov function candidate as follows
\[V(t) = \kappa_e^T(t) (I_{N-1} \otimes P) \kappa_e(t) + \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} (w_{ij}(t) - w_{ij}(0))^2 / 2 \quad (26)\]
\[+ \frac{\gamma}{2N} \sum_{i=1}^{N} \sum_{j \neq i} \sum_{j \neq i} \sum_{j \neq \hat{j}} \left( y_{ij} - w_{ij}(t) \right) \quad (27)\]

Due to \(P = P^T > 0\) and \(y_{ij} = w_{ij}(t)\), it can be shown that \(V(t) = 0\). Since, the topologic graph \(G\) is undirected, one can obtain that \(L_{w(t)}\) is a symmetric matrix. By equation (18), one has
\[\dot{V}(t) = \kappa_e^T(t) (I_{N-1} \otimes (PA + A^T P)) \kappa_e(t) - \kappa_e^T(t) \left( 2U^T L_{w(t)} \bar{U} \otimes PBB^T P \right) \kappa_e(t) \quad (28)\]

Since \(U\) is an orthonormal matrix, it can be obtained that \(\dot{U} U^T = L_N\), where \(L_N\) denotes the Laplacian matrix of a topologic graph \(G^*\) and the weight of all the edges of \(G^*\) is \(1/N\). From protocol (10) and equation (16), one can show that
\[\frac{N}{2N} \sum_{i=1}^{N} \sum_{j \neq i} \sum_{j \neq \hat{j}} \left( w_{ij}(t) - w_{ij}(0) \right) \dot{w}_{ij}(t) - \frac{\gamma}{2N} \sum_{i=1}^{N} \sum_{j \neq i} \sum_{j \neq \hat{j}} \dot{w}_{ij}(t) = \kappa_e^T \left( 2U^T L_{w(t)} \bar{U} - 2U^T L_{w0(t)} \bar{U} - \gamma I_{N-1} \otimes K_w \right) \kappa_e \quad (29)\]

Due to \(K_w = PBB^T P\), it can be derived that
\[\dot{V}(t) = \kappa_e^T \left( I_{N-1} \otimes (PA + A^T P) \right) \kappa_e - \kappa_e^T \left( 2U^T L_{w0(t)} \bar{U} - \gamma I_{N-1} \otimes K_w \right) \kappa_e \quad (29)\]
Then, it means that
\[
\dot{V}(t) \leq \sum_{i=2}^{N} \kappa_i^T(t) \left( PA + A^T P - (2\lambda_i + \gamma)PBB^T P \right) \kappa_i(t)
\] (30)

If \( 2Q + PA + A^T P - \gamma PBB^T P \leq 0 \), then one has \( PA + A^T P - \gamma PBB^T P < 0 \). Hence, one can obtain that \( \dot{V}(t) \leq -\varepsilon \| \kappa \|_2^2 \), where \( \varepsilon \) is a positive constant.

Thus, \( \lim_{t \to +\infty} \kappa_i(t) = 0 \); that is, multi-agent system (1) achieves adaptively guaranteed-performance formation control by protocol (10). In the following, the guaranteed-performance cost \( J^* \) is determined.

Due to \( \omega^T(t)(L_N \otimes I)\omega(t) = \kappa_i^T(t)\kappa_i(t) \), one gets
\[
\omega^T(t)(L_N \otimes Q)\omega(t) = \sum_{i=2}^{N} \kappa_i^T(t)Q\kappa_i(t)
\] (31)

Let \( \nabla = \sum_{i=2}^{N} \sum_{j=2}^{N} (\omega_j(t) - \omega_i(t))^T Q(\omega_j(t) - \omega_i(t)) \) and \( h > 0 \), then one can obtain that
\[
J^*_r \triangleq \frac{1}{N} \int_0^h \nabla dt = \sum_{i=2}^{N} \int_0^h 2\kappa_i^T(t)Q\kappa_i(t) dt
\] (32)

Due to \( \lim_{t \to +\infty} (\gamma_{ik} - w_{ik}(t)) = 0 \), one can infer that
\[
\lim_{h \to +\infty} \sum_{i=2}^{N} \sum_{j=2}^{N} (\gamma_{ik} - w_{ik}(h)) = 0
\] (33)

Due to \( \lambda_i > 0 \), by equation (30), it holds that
\[
\dot{V}(t) \leq \sum_{i=2}^{N} \kappa_i^T(t) \left( PA + A^T P - \gamma PBB^T P \right) \kappa_i(t)
\] (34)

Hence, it can be derived that
\[
J^*_r \leq \sum_{i=2}^{N} \int_0^h 2\kappa_i^T(t)Q\kappa_i(t) dt
\] (35)
\[
\leq -\int \dot{V}(t) dt = V(0) - V(h)
\]

Due to
\[
\lim_{h \to +\infty} V(h) = \lim_{h \to +\infty} \kappa_i^T(t)(I_{N-1} \otimes P)\kappa_i(h)
\]
\[
+ \lim_{h \to +\infty} \sum_{i=2}^{N} \sum_{j \in N_i} \frac{(w_j(h) - w_j(0))^2}{2} > 0
\] (36)

Then, it can be shown that
\[
\lim_{h \to +\infty} J^*_r \leq \lim_{h \to +\infty} V(0) = \kappa_e^T(0)(I_{N-1} \otimes P)\kappa_e(0)
\]
\[
+ \frac{\gamma}{2N} \sum_{i=2}^{N} \sum_{j \in N_i} (\gamma_{ij} - w_{ij}(0))
\] (37)

Since \( UU^T = L_N \) and \( \kappa_e(t) = [0_{2(N-1)d \times 2d}, I_{2(N-1)d}] \) \( (U^T \otimes I_{2d})\omega(t) \), it can be obtained that
\[
\kappa_e^T(0)(I_{N-1} \otimes P)\kappa_e(0) = \omega^T(0) \left( \left( I_N - \frac{1}{N} I_N^T \right) \otimes P \right) \omega(0)
\] (38)

Due to \( \lim_{t \to +\infty} (\gamma_{ik} - w_{ik}(t)) = 0 \), one can conclude that
\[
\sum_{i=2}^{N} \sum_{j=2}^{N} \sum_{j \neq i} (\gamma_{ik} - w_{ik}(t)) = \sum_{i=2}^{N} \sum_{j=2}^{N} \int_0^\infty \dot{w}_{ik}(t) dt = 2N \int_0^\infty \omega^T(t)(L_N \otimes K_u)\omega(t) dt
\] (39)

From Theorem 1, \( K_u = B^T P \) and \( K_w = PBB^T P \). \( P \) can be determined by LMI techniques if \( \gamma \) and \( Q \) are given.

Introduce a positive constant \( \mu \) such that \( \mu I \geq P \), then one can obtain that \( PBB^T P \leq \mu^2 BB^T \) if \( \lambda_{\text{max}}(BB^T) \leq 1 \), where \( \lambda_{\text{max}}(BB^T) \) denotes maximum eigenvalues of \( BB^T \). By LMI techniques, an adaptive guaranteed-performance formation control criterion based on Schur complement Lemma in Boyd et al.35 is proposed as follows.

**Corollary 1.** For any given \( \mu > 0 \), multi-agent system (1) achieves adaptive guaranteed-performance formation by protocol (10) if \( \lambda_{\text{max}}(BB^T) \leq 1 \) and there exist \( \gamma > 0 \) and \( \hat{P} = P \geq \mu^{-1} I \) such that
\[
\begin{bmatrix}
A\hat{P} + \hat{P}A^T - \gamma PBB^T \\
-2\hat{P}Q
\end{bmatrix} \leq 0
\] (40)

In this case, \( K_u = B^TP^{-1} \) and \( K_w = P^{-1}BB^TP^{-1} \) and
\[
J^*_r = \sum_{i=2}^{N} \left( \mu ||\kappa_i(0)||^2 + \mu \gamma \int_0^\infty ||B^T\kappa_i(t)||^2 dt \right)
\] (41)

**Corollary 2.** If multi-agent system (1) achieves adaptive guaranteed-performance formation by protocol (10), then
\[
\lim_{t \to \infty} (x_i(t) - u_i) = \lim_{t \to \infty} \left( e^{At} \left( \frac{1}{N} \sum_{i=1}^{N} (x_i(0) - u_i) \right) \right) \quad (40)
\]

Remark 2. The translation-adaptive strategy is proposed in Theorem 1 to solve the guaranteed-performance constraints of the adaptive consensus strategy in Li et al.\textsuperscript{36} We use the special properties of the Laplacian matrix of the entire topology to achieve eigenvalue translation, which means all eigenvalues equivalent.

Numerical simulation

The following is a numerical simulation example to demonstrate the effectiveness of the theoretical results.

Consider a second-order mobile robot swarms with five agents, which are labeled from 1 to 5. The dynamics of each agent can be described by multi-agent system (1) and

\[
A = \begin{bmatrix} 0 & I_2 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I_2 \end{bmatrix}
\]

Let \( Q = 0.8I_4 \), \( \gamma = 5 \), then it can be obtained that

\[
K_u = \begin{bmatrix} 1.8861 & 0 & 2.0304 & 0 \\ 0 & 1.8861 & 0 & 2.0304 \\ 3.5574 & 0 & 3.8295 & 0 \\ 0 & 3.5574 & 0 & 3.8295 \\ 3.8295 & 0 & 4.1225 & 0 \\ 0 & 3.8295 & 0 & 4.1225 \end{bmatrix}
\]

and \( J^* = 21.7675 \)

Let \( C_i = (x_i(t), y_i(t), v_{x_i}(t), v_{y_i}(t)) \) with \( i \in \{1, 2, 3, 4, 5\} \) denote the state vector of the \( i \)th agent. The initial state vector \( (x_0i, y_0i, v_{x0i}, v_{y0i}) \) and formation vector \( \ell = [\xi^*_x, \xi^*_y, \xi^*_z, \xi^*_r, \xi^*_s] \) with \( \ell_i = (\ell_x, \ell_y) \) are shown in Table 1. The interaction topology of the mobile robot swarms is shown in Figure 1. From Definition 2, the second-order mobile robot swarms is said to achieve formation \( \ell \) if and only if

\[
\lim_{t \to \infty} \left( (x_i(t), y_i(t)) - (x_j(t), y_j(t)) \right) = (\ell_x - \ell_y) \quad (43)
\]

Table 1. Initial state vector and formation vector.

| Serial number | \((x_0, y_0, v_{x0}, v_{y0})\) | \((\ell_x, \ell_y)\) |
|---------------|-------------------------------|-------------------|
| 1             | \((-2, 1, 0, 0)\)             | \((-4, 3)\)       |
| 2             | \((-1, 2, 0, 0)\)             | \((-2, 3)\)       |
| 3             | \((0, 3, 0, 0)\)              | \((0, 3)\)        |
| 4             | \((1, 2, 0, 0)\)              | \((2, 3)\)        |
| 5             | \((2, 3, 0, 0)\)              | \((4, 3)\)        |

The position state trajectories and velocity state trajectories of the mobile robot swarms are shown in Figures 2 and 3, respectively. One can see that position state trajectory of each agent converges to the corresponding value and the velocity states achieve consensus. Figure 4 shows that the interaction weights among agents.
neighboring agents converge to a finite value. Figure 5 shows that the performance function converges to a value less than $J^*/C^r$. Figure 6 depicts the movement tracks of the robot system, one can see that the expected formation of the system is formed by the formation control protocol designed; that is, the system achieves adaptive guaranteed-performance formation control.

**Conclusion**

Adaptive guaranteed-performance formation control problems for second-order multi-agent systems were studied. An adaptive guaranteed-performance formation control strategy is proposed in the current article. The interaction weights of the system can be adaptively adjusted by the designed formation control protocol and the formation control gain can be regulated by choosing some proper translation factors.

Based on this result, it is meaningful to further study the formation control of practical multi-agent systems. Another important research topic is to address the adaptive guaranteed-performance formation control of multi-agent systems with leader-follower structure. For future work, we will focus on the time-varying guaranteed-performance formation control problem with switching topology.

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