Dynamic Positioning Control for Accommodation Vessels With Input Time Delay

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ABSTRACT Offshore operations have been moving towards ultra-deep waters where rich resources are detected and mined in a challenging environment. Dynamic Positioning (DP) control remains a challenge due to time delay in the input control and unknown wave or wind disturbance. In this paper, robust control is proposed for the DP of an accommodation vessel in the presence of the input time delay. In order to overcome the unknown dynamics and external disturbances, a robust controller with predefined tracking constraints is developed to guarantee a safe operation. Time delay caused by the control input is also considered on the predictor-based method. The boundedness of all closed-loop signals is demonstrated via rigorous Lyapunov analysis. Simulation studies help demonstrate outstanding performance and feasibility of the proposed control structure.

INDEX TERMS Dynamic positioning, robust control, prescribed performance, input delay.

I. INTRODUCTION

Due to global energy sources continuing to grow in the past few decades, there is a great concern for offshore resources development. The technology of Floating Production Storage and Offloading (FPSO) will allow floating plants to be positioned in a deep-sea environment while storing oil in one position. The maintenance related to work offshore needs to be handled from a remote offshore area. Accommodation Vessels (AV) are designed to support FPSO connected in a side-by-side configuration. AV are used in the field to provide offshore accommodation and living facilities, deck space and workshops for fabrication, and cranes for supply and topside lifts. Continuous personnel and equipment transferred from AV to an FPSO unit are handled through a gangway. One of the unavoidable challenges we have to cope with the positioning system of AV is the complexity and unpredictability of ultra-deep water. The control system should be adaptive to changing variables, and robust to uncertain factors. This design directly addresses the technical issues industry faces. It is expected to increase productivity and smoothness of operation in a harsh environment that is not supported by current technology, and at the same time help the industry remain competitive. We are not only interested in designing control systems for the floatel system to be highly robust and adaptive to environmental impacts but also to maintain and increase the uptime of operations between the floatel and FPSO [1].

The technical challenge of offshore resources development remains an open problem in spite of significant advancements from universities and technology companies. A great deal of attention has been paid to the research of DP control system, such as the nonlinear adaptive control, sliding mode control, backstepping control technique, even mixed two or more control methods to deal with the uncertain disturbances, parameterizing uncertainties et al. [2]–[6]. For example, an adaptive fuzzy design procedure has been developed for unknown parameters and environmental disturbances in the DP backstepping control. A DP control system with environmental disturbances has been solved by combining with optimal \( H_{\infty} \) and a robust fuzzy technique. Experimental tests have confirmed the proposed fuzzy controller can offer satisfactory control performance compared with a PID controller. The fuzzy controller and the neural network controller are approximation-based approaches. The parameters involved in the control system require complicated design and adjustment [7].

The dynamic positioning considering the constraints on the transient and steady-state position errors is another important role in various marine applications. With traditional nonlinear
control designs, the operation envelopes can converge to a residual set. The design parameters and some unknown terms determine the set size. However, no prior selection of controller parameters exists to satisfy certain steady-state behavior. As one of the noticeable methods to deal with constraints, the Barrier Lyapunov function is proposed to ensure the performance requirements [8]. By handling the output constraints in active suspension systems, the Barrier Lyapunov function was employed in backstepping control. Although the technique used is based on the Barrier Lyapunov function instead of quadratic Lyapunov function and thus a less conservative control design, maximum overshoot and convergence rate cannot be retained in a priori bound [9]. Utilizing a transformation function incorporating the desirable performance characteristics, prescribed performance provides a systematic procedure to accurately guarantee the critical properties of the convergence rate and maximum overshoot. The main idea of prescribed performance is to formulate the predefined constraints on tracking errors into a relatively easier unconstrained problem by using an appropriate function. Both the errors and constraints are incorporated into the transformation function, so that the stabilization of the transformed system is sufficient to achieve tracking control of the original system with guaranteed performance [10]–[13]. The authors in [14] designed adaptive neural network control of underactuated surface vessels with guaranteed transient and steady-state tracking performances. The boundary functions of the predefined regions provide the preselected specifications on the tracking errors. Predefined tracking error bound for a marine surface vessel was presented in [15]. Exponentially decaying time functions are used as the transformation function to solve the prescribed transient and steady-state tracking control performances. Then, the research result is extended to the formation control problem for unmanned surface vehicles in [16]. Prescribed performance guarantees transient performance specifications on formation errors. However, prescribed performance has rarely been applied to dynamic positioning systems.

Apart from position error constraints, long response time of the thruster driver leads to time-delay for dynamic positioning. Input delay existing in the thrusters can degrade the control performance such as larger overshoot or longer adjustment time and can even lead to instability of the floating plants [17]. The time-delay problem poses great challenges and difficulties in control systems. As a result, the control design of time-delay systems has been an active research topic for several decades, and many remarkable approaches have been established and applied to various control systems [18]–[20]. Linear uncertain time-delay systems have been investigated in [21]. It should be pointed out that linear matrix inequalities can hardly be applied to nonlinear time-delay systems. The Teel’s forwarding approach was utilized for multiple integrators systems by using delayed and bounded control [22]. Later on, the research results were applied to a family of nonlinear systems [23]. Feedforward nonlinear systems have the property of upper triangular structures. DP control system is usually modeled as a low triangular form. The Backstepping control technique has been regarded as a promising approach for feedback nonlinear systems with time delay. The systems with unknown parameters and nonlinear functions are more complicated to deal with the delay. Model predictive control approaches of the time-delay system have been proposed to achieve high performances in terms of output response [24], [25]. Note that the constrained optimal control action suffers from the required computational cost. Only a few results on the problem of time-delay in DP control systems are available in the literature.

In this paper, it is natural for us to solve the bounded problem of DP control systems with both complex unknown parameters and input time delay. DP control systems are a complex multi-input multi-output nonlinear system. This study is aimed at developing a methodology based on the constraint function to handle prescribed performance. We also focus on the actuator design for generating forces and moments to eliminate the effect of input time delay and environment disturbances. This paper is dedicated to developing a solution for floatel working systems under shielding effects with the following main contributions:

1) Robust adaptive learning control system design is developed for a floatel exposed to uncertain shielding effects to increase the uptime between the floatel and FPSO. The developed algorithm is robust to uncertainties associated with unknown system parameters and environment.

2) We impose the prescribed tracking performance on a side-by-side dynamic position system. Constructive results are presented to stabilize the nonlinear systems for the case of uncertain parameters and bounded time-delay.

3) Due to the actuator’s mechanical properties, the Lyapunov Krasovskii function is used to help with the input time delay. Artstein model was employed to reduce the input delay of the DP control system to delay-free system. The controller is constructed using an augmented model. Finally, the nonlinear system controller renders the closed-loop system semi-globally uniformly ultimately bounded (SGUUB) stable.

A. PROBLEM FORMATION AND PRELIMINARIES

Three degrees of freedom (DOF) model of a typical supply vessel is considered for the station keeping task. The problem addressed in this paper can be formulated with the coordinate system shown in Figure 1. The earth-fixed reference frame adopts the North-East Down (NED) coordinate frame. \( O_X \) is the longitudinal axis which points to north. \( O_Y \) is the transversal axis which directs to the east. \( O_Z \) are directed to the fore and starboard, respectively. Kinematics and dynamics describe the movement of the vessel in planar space as follows:

\[
\dot{\eta} = J(\eta)\nu, \quad (1)
\]
\( (M_v + \alpha M_v)\ddot{v}_v + [C_v(v_v) + \beta C_v(v_v)]v_v + D_v(v_v) + g_v(\eta) = u(t - t_d) + f_v(t) \) (2)

where \( \eta = [x, y, \psi]^T, v_v = [u, v, \psi]^T \) denotes earth-fixed frame for positions \( x, y \) and body-fix frame for velocities \( u, v, \psi \) representing heading angle and \( r \) is the corresponding angular rate. \( (M_v + \alpha M_v) \in \mathbb{R}^{3 \times 3} \) describes the inertia matrix which is the sum of rigid body inertia and added mass. \( C_v(v_v) + \beta C_v(v_v) \in \mathbb{R}^{3 \times 3} \) denotes the matrix of Coriolis and centripetal. \( D_v(v_v) \in \mathbb{R}^{3 \times 3} \) and \( g_v(\eta) \in \mathbb{R}^{3 \times 1} \) are damping matrix and restoring force respectively. \( u(t - t_d) \in \mathbb{R}^3 \) denotes control input with known constant time delay \( t_d \in \mathbb{R}, d_v(t) \in \mathbb{R}^{3 \times 1} \) is the low-frequency external environmental forces and disturbance such as shielding, gap resonance and interaction effects between FPSO and AV. In particular, \( \alpha M_v, \beta C_v(v_v) \) are multiplicative error. The terms \( \alpha M_v, \beta C_v(v_v) \) and \( d_v(t) \) would vary in a large scale along with the changing of the sea state.

The traditional dynamic model of (1)-(2) is built according to the availability of the motion sensors. For designing the controller conveniently and eliminating \( J(\eta) \) in (1), the model is transferred to a unified earth-fixed frame in following subsection. Equation (1) described by

\[
\dot{v}_v = J^{-1}(\eta)[\ddot{\eta} - J(\eta)J^{-1}(\eta)\dot{\eta}] \tag{3}
\]

Substituting (3) into (2) and multiply \( J^{-T}(\eta) \), we obtain

\[
J^{-T}(\eta)(M_v + \alpha M_v)J^{-1}(\eta)\dot{\eta} - J^{-T}(\eta)(C_v(v_v) + \beta C_v(v_v))J^{-1}(\eta)\dot{\eta} + J^{-T}(\eta)D_v(v_v)J^{-1}(\eta)\dot{\eta} + J^{-T}(\eta)g_v(\eta)J^{-1}(\eta)\dot{\eta} = J^{-T}(\eta)[u(t - d) + f_v(t)] \tag{4}
\]

Separating \( M_v, \alpha M_v, C_v(v_v) \) and \( \beta C_v(v_v) \) yields

\[
J^{-T}(\eta)M_vJ^{-1}(\eta)\dot{\eta} + J^{-T}(\eta)C_v(v_v)J^{-1}(\eta)\dot{\eta} + J^{-T}(\eta)D_v(v_v)J^{-1}(\eta)\dot{\eta} + J^{-T}(\eta)g_v(\eta)J^{-1}(\eta)\dot{\eta} = J^{-T}(\eta)[u(t - d) + f_v(t) - \alpha M_vJ^{-1}(\eta)\dot{\eta} - \beta C_v(v_v)J^{-1}(\eta)\dot{\eta}] \tag{5}
\]

Define

\[
M(\eta) = J^{-T}(\eta)M_vJ^{-1}(\eta) \tag{6}
\]
\[
C(\eta, v_v) = J^{-T}(\eta)[C_v(v_v) - M_vJ^{-1}(\eta)J(\eta)]J^{-1}(\eta) \tag{7}
\]
\[
D(\eta, v_v) = J^{-T}(\eta)D_v(v_v)J^{-1}(\eta) \tag{8}
\]
\[
g(\eta) = J^{-T}(\eta)g_v(\eta) \tag{9}
\]
\[
J^{-T}(\eta)u(t - t_d) = \tau(t - t_d) \tag{10}
\]

and

\[
d(\ddot{\eta}, \dot{\eta}, \eta, t) = J^{-T}(\eta)[d_v(t) - \alpha M_vJ^{-1}(\eta)\dot{\eta}] \tag{11}
\]
\[
+ \alpha M_vJ^{-1}(\eta)\dot{\eta}J^{-1}(\eta)\dot{\eta} - \beta C_v(v_v)J^{-1}(\eta)\dot{\eta} \tag{12}
\]

Then the original model with unmodelled dynamics can be transformed into a class of cascaded systems

\[
\dot{\eta} = v \tag{13}
\]
\[
M(\eta)\dot{v} + C(\eta, v_v)v + D(\eta, v_v)v + g(\eta) = \tau(t - t_d) + d(\ddot{\eta}, \dot{\eta}, \eta, t) \tag{14}
\]

To achieve the prescribed performance, the following assumptions are considered.

Assumption 1 The reference trajectory and its time derivative are known and bounded.

Assumption 2 The newly defined term \( d \) is bounded and can be rationally limited as \( d_1 \) with \( \|d\| \leq d_1 \) where \( d_1 \) is a positive constant.

Assumption 3 The inertia matrix \( M \) is invertible and bounded, which satisfies \( \|M^{-1}\| \leq M^{-1} \).

Lemma 1 [26, 27] For bounded initial conditions, if there exists a \( C^1 \) continuous and positive definite Lyapunov function \( V(x) \) satisfying \( k_1(\|x\|) \leq V(x) \leq k_2(\|x\|) \), such that \( \dot{V} \leq -aV(x) + \beta \), where \( k_1, k_2 : \mathbb{R}^n \rightarrow \mathbb{R} \) are class \( K \) functions and \( a, \beta > 0 \), then the solution \( x(t) \) is uniformly bounded.

In this context, the control objective is to design a control approach to ensure the vessels operate in a predefined area even with the problem of input time-delay and the other states are kept bounded.

B. ROBUST CONTROL DESIGN AND STABILITY ANALYSIS

In this section, input time-delay control with prescribed performance is considered for AV. On the one hand, to ensure the position of the vessel fixed in a predefined operation area, a prescribed performance control approach is involved for the robust controller. On the other hand, the Artstein Model is one approach to deal with time-delay control for linear systems. Actually, it is a predictor-like controller which can convert the original system into a delay-free system. Uncertain floatal dynamics are highly nonlinear. Therefore, inspired by [28] and prescribed performance, a model-based robust controller is developed combining the predefined tracking performance constraint with input time delay.

Based on the backstepping method, the design process is shown as follows.

Step I: The tracking error is defined as

\[
\ddot{\eta} = \eta - \eta_d \tag{15}
\]

with initially limited range

\[
-\rho < \ddot{\eta}(0) < \rho \tag{16}
\]

where \( \rho = \text{diag}([\rho_{01}, \ldots, \rho_{0n}], \rho_{\infty}, \ldots]) \), \( i = 1, 2, 3 \) denotes the performance function with the initial error bound \( \rho_{0i} \), maximum steady error \( \rho_{\infty} \) and convergence rate \( \eta_i \). An unconstrained system is equivalently derived through the error transformation operation as

\[
\epsilon = S^{-1}(\rho^{-1}\ddot{\eta}), \quad S^{-1}(\epsilon) = [\ldots, \frac{1}{\epsilon_{i}+1}, \ldots]^T, \quad \epsilon_i \text{ is the } i\text{-th element of } \epsilon. \text{ If } \epsilon \text{ approaches to zero in the presence of}
\]
disturbance $d$, the prescribed control performance is able to be achieved. The derivative of $\epsilon$ is

$$
\dot{\epsilon} = \frac{\partial S^{-1}}{\partial (\rho^{-1}\dot{\eta})} [\rho^{-1}\dot{\eta} - \dot{\rho}^{-1}\dot{\eta}]
= \frac{\partial S^{-1}}{\partial (\rho^{-1}\dot{\eta})} [\dot{\eta} - \dot{\eta}_d - \dot{\rho}^{-1}\dot{\eta}]
= \Phi [\dot{\eta} - \dot{\eta}_d - \dot{\rho}^{-1}\dot{\eta}]
$$

(17)

Select the Lyapunov candidate as follow

$$
V_1 = \frac{1}{2} \epsilon^T \epsilon
$$

(18)

The time derivative of $V_1$ yields

$$
\dot{V}_1 = \epsilon^T \Phi [\dot{\eta} - \dot{\eta}_d - \dot{\rho}^{-1}\dot{\eta}]
$$

(19)

Design the virtual control as

$$
\alpha_c = \dot{\eta}_d + \rho \dot{\rho}^{-1}\dot{\eta} - K_1 \Phi^{-1} \epsilon
$$

then we have

$$
\dot{V}_1 = -\epsilon^T K_1 \epsilon + \epsilon^T \Phi \nu
$$

(21)

with a positive definite matrix $K_1 \in \mathbb{R}^{3 \times 3}$.

Step 2: Let

$$
z_2 = \alpha_c - \nu
$$

(22)

In order to compensate for the input delay, an auxiliary state $S \in \mathbb{R}^{3 \times 1}$ is defined as following

$$
S = z_2 - M^{-1} \int_{t-t_d}^t \tau(\theta)d\theta - z_f
$$

(23)

In (23), $z_f \in \mathbb{R}^{3 \times 1}$ satisfies the following adaptive law.

$$
\dot{z}_f = K_2 S - \Gamma_1 z_2 - \Theta z_f
$$

(24)

where $K_2, \Gamma_1, \Theta \in \mathbb{R}^{3 \times 3}$ are positive parameters to be determined later. Multiply both sides of equation (23) by $M$, the term of $M \dot{S}$ gives

$$
M \dot{S} = M \dot{z}_2 - \tau(\theta) + \tau(t) - \dot{\tau}(t) - \dot{z}_f
= M \dot{\alpha}_c + C(\nu)v + D(v)\nu + g(\theta) - d_1 - \tau(\theta)
- K_2 S + \Theta \tau(\theta)
= M \dot{\alpha}_c + M_\nu - d_1 + N_\nu - \tau(t) - K_2 S - K_2 z_2
- (S^T)^+ \dot{S}^T z_2
$$

(25)

where $M_\nu = C(\nu)v + D(v)\nu + g(\theta)$, the term $N_\nu$ is defined as the following expression.

$$
N_\nu = \Theta z_f + \Gamma_1 z_2 + K_2 z_2 + (S^T)^+ \dot{S}^T z_2
$$

(26)

As the auxiliary state $S$ is introduced, the delayed system is converted into a delay-free one shown in (25). Additionally, no limitation of the velocity for the vessel is needed. Therefore, a quadratic form Lyapunov-Krasovskii candidate is chosen as [30]

$$
V_2 = V_1 + \frac{1}{2} \epsilon^T \epsilon + \frac{1}{2} S^T M S + \frac{1}{2} \tilde{z}_f^T \tilde{z}_f
+ \int_{t-t_d}^{t} \dot{\epsilon}(\theta)^2 d\theta d\tau
$$

(27)

Differentiating $V_2$ and according to (19), (23), (24) and (25), we have

$$
\dot{V}_2 = \dot{V}_1 + \epsilon^T \Phi \nu + \frac{1}{2} \epsilon^T \epsilon + \frac{1}{2} S^T M S + \frac{1}{2} \tilde{z}_f^T \tilde{z}_f
+ \int_{t-t_d}^{t} \dot{\epsilon}(\theta)^2 d\theta d\tau
\leq -\epsilon^T K_1 \epsilon + \epsilon^T \Phi \nu
+ \frac{1}{2} \epsilon^T \epsilon + \frac{1}{2} S^T M S + \frac{1}{2} \tilde{z}_f^T \tilde{z}_f
+ \int_{t-t_d}^{t} \dot{\epsilon}(\theta)^2 d\theta d\tau
$$

(28)

Design the following control law

$$
\tau(t) = M \dot{\alpha}_c + M_\nu + K_2 \dot{z}_f + (S^T)^+ \epsilon^T \Phi \nu
$$

(29)

where $z_\tau \in \mathbb{R}^{3 \times 1}$ denotes

$$
z_\tau = \tau(t) - \tau(t) - \dot{\tau}(t)
= \int_{t-t_d}^{t} \dot{\epsilon}(\theta)d\theta
$$

(30)

Substitute (29) into (28) and considering Assumption 3, we obtain

$$
\dot{V}_2 = -\epsilon^T K_1 \epsilon - \epsilon^T \Theta \epsilon + \frac{1}{2} \epsilon^T \epsilon + \frac{1}{2} S^T M S + \frac{1}{2} \tilde{z}_f^T \tilde{z}_f
+ \int_{t-t_d}^{t} \dot{\epsilon}(\theta)^2 d\theta d\tau
\leq -\epsilon^T K_1 \epsilon - \epsilon^T \Theta \epsilon + \frac{1}{2} \epsilon^T \epsilon + \frac{1}{2} S^T M S + \frac{1}{2} \tilde{z}_f^T \tilde{z}_f
+ \int_{t-t_d}^{t} \dot{\epsilon}(\theta)^2 d\theta d\tau
$$

(31)

where the function $z_s$ is defined as $z_s = [\epsilon^T, \tilde{z}_f^T, z_\tau^T, \dot{z}_f^T]^T$, and $N_\nu$ satisfies the Mean Value Theorem [31]

$$
\|\dot{N}_\nu\| \leq N_\nu(\|\dot{z}_s\|)\|z_s\|
$$

(32)

and the bounds of $N_\nu(\|\dot{z}_s\|), M^{-1}, (-\Gamma_1 - I)$ are globally positive. Based on the Young’s inequality, the term
Then the time derivative of $V_2$ yields

$$
\dot{V}_2 \leq - \left[ \lambda_{\min}(K_1) - \frac{\sigma_4}{4} N_c^2 (\|z_s\|) \right] \varepsilon^T \varepsilon
- \left[ \lambda_{\min}(\Gamma_1) - \frac{\sigma_3}{4} N_c^2 (\|z_s\|) \right] z^T z_2
- \left[ \lambda_{\min}(K_2) - \frac{\sigma_3}{4} N_c^2 (\|z_s\|) \right] S^T S
- \left[ \lambda_{\min}(\Theta_1) - \frac{\sigma_3}{4} N_c^2 (\|z_s\|) \right] z^T z_f
+ \frac{\sigma_1 M_1}{4} \|z_2\|^2 + \frac{\sigma_4}{4} (\Gamma_1 - I)^T \|z_2\|^2
+ \frac{1}{\sigma_2} \|z_f\|^2 + \frac{1}{\sigma_3} \|S\|^2 + \frac{\sigma_4}{4} \|T\|^2 + \frac{1}{\sigma_4} \|S\|^2
+ u_1 \|\dot{\theta}\|^2 - u \int_{t_1}^{t} \|\dot{\theta}(t)\|^2 d\theta
$$

Cauchy-Schwarz inequality gives the upper bound of $\|z_r\|$ as

$$
\|z_r\|^2 \leq t_d \int_{t_1}^{t} \|\dot{\theta}(t)\|^2 d\theta
$$

Moreover, it can be proven that

$$
\int_{t_1}^{t} \int_{w} \|\dot{\theta}(t)\|^2 d\theta d\theta \leq t_d \int_{t_1}^{t} \|\dot{\theta}(t)\|^2 d\theta
$$

According to (35) and (36), the inequality (34) becomes

$$
\dot{V}_2 \leq - \left[ \lambda_{\min}(K_1) - \frac{\sigma_4}{4} N_c^2 (\|z_s\|) \right] \varepsilon^T \varepsilon
- \left[ \lambda_{\min}(\Gamma_1) - \frac{\sigma_3}{4} N_c^2 (\|z_s\|) \right] z^T z_2
- \left[ \lambda_{\min}(K_2) - \frac{\sigma_3}{4} N_c^2 (\|z_s\|) \right] S^T S
- \left[ \lambda_{\min}(\Theta_1) - \frac{\sigma_3}{4} N_c^2 (\|z_s\|) \right] z^T z_f
+ \frac{\sigma_1 M_1}{4} \|z_2\|^2 + \frac{\sigma_4}{4} (\Gamma_1 - I)^T \|z_2\|^2
+ \frac{1}{\sigma_2} \|z_f\|^2 + \frac{1}{\sigma_3} \|S\|^2 + \frac{\sigma_4}{4} \|T\|^2 + \frac{1}{\sigma_4} \|S\|^2
+ u_1 \|\dot{\theta}\|^2 - u \int_{t_1}^{t} \|\dot{\theta}(t)\|^2 d\theta
$$

$$
\leq - \left[ \lambda_{\min}(K_1) - \frac{\sigma_4}{4} N_c^2 (\|z_s\|) \right] \varepsilon^T \varepsilon
- \left[ \lambda_{\min}(\Gamma_1) - \frac{\sigma_3}{4} N_c^2 (\|z_s\|) \right] z^T z_2
$$

where $\rho_0 = \min\left[ 2(\lambda_{\min}(K_1) - \frac{\sigma_4}{4} N_c^2 (\|z_s\|)), 2(\lambda_{\min}(\Gamma_1) - \frac{\sigma_3}{4} N_c^2 (\|z_s\|)) \right] - \frac{\sigma_4}{4} (\Gamma_1 - I)^T \|z_2\|^2 - \frac{1}{\sigma_3} \|S\|^2 + \frac{\sigma_4}{4} \|T\|^2 + \frac{1}{\sigma_4} \|S\|^2 > 0$. If the tuning parameters are selected as

$$
\lambda_{\min}(K_1) > \frac{\sigma_4}{4} N_c^2 (\|z_s\|), \lambda_{\min}(\Gamma_1) > \frac{\sigma_4}{4} N_c^2 (\|z_s\|), \lambda_{\min}(K_2) > \frac{\sigma_4}{4} N_c^2 (\|z_s\|), \lambda_{\min}(\Theta_1) > \frac{\sigma_4}{4} N_c^2 (\|z_s\|),
$$

then $\rho_0 > 0$. It is obvious that $V(t)$ is SGUUB for $V(0) \leq B_0$, where $B_0 = V(e_1, z_s, z_r, z_f)$ is positive constant.

**Theorem 1:** Suppose system (1) and (2) satisfy Assumption 1, 2 and 3, the closed-loop system consists of vessel model with input delay, and the proposed controller (29) based on the intermediate control vector (20). Given the initial conditions $V(0) \leq B_0$, the prescribed position performance of the vessel is guaranteed and the other signals in the DP system is SGUUB.

**II. SIMULATION**

In this section, the simulations on Cybership II are carried out in two cases to evaluate the performance of the proposed control scheme. Cybership II is considered as the case study with the vessel parameters listed in Tab.1 of this section.

**TABLE 1. Parameters of the target vessel.**

| Parameter  | Description                          | Value       |
|-----------|--------------------------------------|-------------|
| $m$       | Total mass                           | 23.8 kg     |
| $L_v$     | Length of vessel                     | 1.255 m     |
| $B_v$     | Breadth of vessel                    | 0.290 m     |
| $z_g$     | Position along Z of the center of gravity | 0.046 m |
| $I_{oz}$  | Moments of inertia along Z            | 1.76 kg·m²  |
| $\rho_{air}$ | Density of air                      | 1.29 kg/m³  |
| $\rho_{water}$ | Density of water                | 1025 kg/m³  |
| $g$       | Gravitational acceleration           | 9.8 m/s²    |

Case 1: The desired vessel position and heading are set as $[4.2 m, 0.8 m, 1.29^T]$. The initial states are $\eta(0) = [4.5 m, 1 m, 90^T]^T$, $\nu(0) = [0 m/s, 0 m/s, 0^T]^T$, time delay $t_d = 2 s$. The parameters in exponential performance function
are taken as $\rho_{10} = 0.07$, $\rho_{20} = 0.07$, $\rho_{30} = 0.1744$, $\rho_{\infty i} = 0.5$ and $l_i = 0.05$, $i = 1, 2, 3$. The design parameters are chosen as $K_1 = K_2 = K_3 = [0.27, 0, 0; 0, 0.32, 0; 0, 0, 2.62]^T$. Fig.1 illustrates the developed control scheme can force the vessel to the desired position. Velocities of the AV is shown in Fig.2. From Fig.3, the vessel’s position and heading is regulated in around 40s based on the proposed DP control law. It can be seen control input of the AV in Fig.4. The position errors are within the top and bottom boundaries in Fig.5.
Case 2: the simulation is carried out to show the good robustness in the disturbance case. The time-varying disturbance \( d = \sin(t) \) is considered when \( 50s < t < 100s \). The simulation results are depicted in Fig.6-Fig.10. The initial conditions and the control design parameters are chosen same as case 1. It can be observed that from Fig.6, the vessel can be kept around the desired position in the presence of different disturbances corresponding to different sea states. From the simulations, it is obvious that the proposed control can achieve satisfactory control performance under the influence of the disturbances.

### III. CONCLUSION

Research has been carried out on combining robust control and input delay together in a systematic manner in order to handle unknown disturbances and parametric uncertainties while capturing the dominant dynamic behaviours with guaranteed stability and performance. The control system has been designed with prescribed performance and a predictor-based approach to deal with bounded tracking error, the input delay, and the shielding effects. The stability of the proposed controller has been proven by Lyapunov-Krosovski analysis. Efficient simulation for this control system design provides an understanding of actual implementation.

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