Long-Range Rapidity Correlations in Heavy Ion Collisions at Strong Coupling from AdS/CFT

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Abstract: We use AdS/CFT correspondence to study two-particle correlations in heavy ion collisions at strong coupling. Modeling the colliding heavy ions by shock waves on the gravity side, we observe that at early times after the collision there are long-range rapidity correlations present in the two-point functions for the glueball and the energy-momentum tensor operators. We estimate rapidity correlations at later times by assuming that the evolution of the system is governed by ideal Bjorken hydrodynamics, and find that glueball correlations in this state are suppressed at large rapidity intervals, suggesting that late-time medium dynamics can not “wash out” the long-range rapidity correlations that were formed at early times. These results may provide an insight on the nature of the “ridge” correlations observed in heavy ion collision experiments at RHIC and LHC, and in proton-proton collisions at LHC.

Keywords: AdS/CFT, Heavy Ion Collisions, Rapidity Correlations, Shock Waves.
1. Introduction

In recent years it has been suggested that the medium of quarks and gluons produced in heavy ion collisions at RHIC goes through a strongly-coupled phase at least during some period of its evolution [1–4]. The Anti-de Sitter space/Conformal Field Theory (AdS/CFT) correspondence [5–7] is often used to study the dynamics of this strongly-coupled medium [8–29]: while it is valid only for $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory, there is a possibility that the qualitative (and some of the quantitative) results obtained from AdS/CFT correspondence may be applied to the real-world case of QCD.

The main thrust of the efforts to study the dynamics of the medium produced in heavy ion collisions using AdS/CFT correspondence has been directed toward understanding how (and when) the medium isotropizes and thermalizes [8–25, 27, 28]. The existing approaches can be divided into two categories: while some studies concentrated on the dynamics of the produced medium in the forward light-cone without analyzing the production mechanism for the medium [8–12, 25, 28], a
large amount of work has been concentrated on studying the collisions by modeling the heavy ions with shock waves in AdS, and attempting to solve Einstein equations in the bulk for a collision of two AdS shock waves [13–24,27]. Many of the existing calculations strive to obtain the expectation value of the energy-momentum tensor $\langle T_{\mu\nu} \rangle$ of the produced medium in the boundary gauge theory [8–14, 16, 17, 23–25, 27, 28], since this is the quantity most relevant for addressing the question of the isotropization of the medium. Other works address the general question of thermalization by noticing that it corresponds to creation of a black hole in the AdS bulk, and by constructing a physical proof of the black hole formation with the help of a trapped surface analysis [15,18–22].

In this work we concentrate on a different observable characterizing heavy ion collisions: we study correlation functions in the produced expanding strongly-coupled medium. Correlation functions have become a powerful tool for the analysis of data coming out of heavy ion collisions, allowing for a quantitative measure of a wide range of phenomena, from Hanbury-Brown–Twiss (HBT) interferometry [30], to jet quenching [31] and Color Glass Condensate (CGC) [32]. In recent years a new puzzling phenomenon was discovered in the two-particle correlation functions measured in $Au + Au$ collisions at Relativistic Heavy Ion Collider (RHIC) [33–35]: the experiments see correlations with a rather small azimuthal angle spread, but with a rather broad (up to several units) distribution in rapidity. This type of correlation is referred to as “the ridge”. More recently the ridge correlations have been seen in high-multiplicity proton-proton collisions at the Large Hadron Collider (LHC) [36], as well as in the preliminary data on $Pb + Pb$ collisions at LHC.

Several theoretical explanations have been put forward to account for the ridge correlations. They can be sub-divided into two classes: perturbative and non-perturbative. Perturbative explanations, put forward in the CGC framework in [37–42], are based on the long-range rapidity correlations present in the initial state of a heavy ion collision due to CGC classical gluon fields [43–46] (see [47–49] for reviews of CGC physics). In [37,38] the authors invoke causality to argue that long-range rapidity correlation can only arise in the early times after the collision, since at later times the regions at different rapidities become causally disconnected. This is illustrated in Fig. 1, where one can see that the gray-shaded causal pasts of two particles produced in the collision (labeled by arrows with momenta $k_1$ and $k_2$) overlap only at very early time (the red-shaded region). The authors of [38] then suggest that the late-time radial flow due to hydrodynamic evolution would lead to azimuthal correlations characteristic of the “ridge”. Alternatively, the authors of [37,39–41] have identified a class of Feynman diagrams which generate azimuthal correlations in nucleus–nucleus collisions.

The CGC correlations found in [37, 39–41] are based on purely perturbative small-coupling physics: however, it remains to be shown whether such perturbative dynamics contains large enough azimuthal correlations to account for all of the observed “ridge” phenomenon. In the scenario of [37, 38] CGC dynamics provides rapidity correlations, while azimuthal correlations are generated by hydrodynamic evolution. As we have already mentioned, it is possible that the medium created at RHIC is strongly-coupled [1–4]: if so, hydrodynamic evolution would then be a non-
perturbative effect, making the scenario proposed in [37, 38] implicitly non-perturbative. Purely non-perturbative explanations of the “ridge” include parton cascade models [50], hadronic string models [51], and event-by-event hydrodynamic simulations [52]. The causality argument of [37, 38] is valid in the non-perturbative case as well: one needs correlations in the initial state, either due to soft pomeron/hadronic strings interactions [50,51], or due to initial-state fluctuations [52], in order to obtain long-range rapidity correlations. In this work we will use AdS/CFT to address the theoretical question whether long-range rapidity correlations are present in the non-perturbative picture of heavy ion collisions. At the same time we recognize that a complete understanding of whether the “ridge” correlations observed at RHIC and LHC are perturbative (CGC) or non-perturbative in nature is still an open problem left for future studies.

Figure 1: Space-time picture of a heavy ion collision demonstrating how long-range rapidity correlations can be formed only in the initial stages of the collision, as originally pointed out in [37, 38]. Gray shaded regions denote causal pasts of the two produced particles with four-momenta \( k_1 \) and \( k_2 \), with their overlap region highlighted in red. We have drawn the lines of constant proper time \( \tau \) and constant space-time rapidity \( \eta \) to guide the eye and to underscore that late-time emission events for the two particles are likely to be causally disconnected.

The goal of the present work is to study long-range rapidity correlations in heavy ion collisions in the strongly-coupled AdS/CFT framework. In order to test for the long range rapidity correlations observed in heavy ion collisions, we would like to study the two-point function \( \langle \text{tr} F_{\mu\nu}^2(x) \text{tr} F_{\rho\sigma}^2(y) \rangle \) of glueball operators \( \text{tr} (F_{\mu\nu}^2) \) right after the collision but before the thermalization. According to causality arguments of [37,38], one expects that the long range correlations in rapidity should occur at such early times. The choice of observable is mainly governed by calculational simplicity. The metric for the early times after the collision of two shock waves in AdS\(_5\) was obtained in [14,16,17]:

\[
\eta = \text{const} \quad \tau = \text{const}
\]
after formulating the problem in Sec. 2 and presenting our general expectation for the answer in Sec. 3, we use this metric to calculate the correlation function of two glueball operators in Sec. 4. (Since the glueball operator corresponds to the massless scalar field in the bulk, we compute the two-point function of the scalar field in the background of the colliding shock waves metric.) Our main result is that we do find long-range rapidity correlations in the strongly-coupled initial state, albeit with a rather peculiar rapidity dependence: the two-glueball correlation function scales as

\[ C(k_1, k_2) \sim \cosh(4 \Delta y) \]  

with the (large) rapidity interval \( \Delta y \) between them. We also show in Sec. 4 that the correlator of two energy-momentum tensors \( \langle T^1_1(x) T^2_1(y) \rangle \) (with 1, 2 transverse directions) exhibits the same long-range rapidity correlations. This should be contrasted with the CGC result, in which the correlations are at most flat in rapidity [37, 38, 40, 41]. Indeed the growth of correlations with rapidity interval in Eq. (1.1) also contradicts experimental data [33–36]. Although we should not \textit{a priori} expect an agreement between AdS/CFT calculations and experimental QCD data, we argue in Sec. 6 that inclusion of higher-order corrections in the AdS calculation along the lines of [17] should help to flatten out such growth, though it is a very difficult problem to demonstrate this explicitly.

Using the causality argument of [37, 38] illustrated in Fig. 1 we also expect that after thermalization the rapidity correlations should only be short-ranged. As a result, due to causality, the initial long ranged correlations can not be “washed away” and will be observed at later times. This explanation is analogous to the resolution of the ‘horizon problem’ in the cosmic microwave background radiation (CMB), where the observed near-homogeneity of the CMB suggests that the universe was extremely homogeneous at the time of the last scattering even over distance scales that could not have been in causal contact in the past. This problem was solved by assuming that the universe, when it was still young and extremely homogeneous, went through a very rapid period of expansion (inflation). As a consequence of inflation, different regions of the universe became causally disconnected, while preserving the initial homogeneity. The idea that we pursue here for the heavy ion collisions seems to be of similar nature. To verify the statement that late-time dynamics can not generate (or otherwise affect) long-range rapidity correlations we study glueball correlation again in Sec. 5 now using the metric found by Janik and Peschanski [8], which is dual to Bjorken hydrodynamics [53]. (This is done in the absence an analytic solution of the problem of colliding shock waves: despite some recent progress [14, 16, 17] the late-time metric is unknown at present.) Performing a perturbative estimate, we find that, indeed, only short-range rapidity correlations result from the gauge theory dynamics dual to the Janik and Peschanski metric.

We summarize our results in Sec. 6.
2. Generalities and Problem Setup

2.1 AdS/CFT Tools

We start with a metric for a single shock wave moving along a light cone in the \(x^+\) direction \([8]\) in Fefferman–Graham coordinates \([54]\):

\[
ds^2 = \frac{L^2}{z^2} \left\{ -2 \, dx^+ \, dx^- + t_1(x^-) \, z^4 \, dx^- \, dz + dx_{\perp}^2 + d\tilde{z}^2 \right\}
\]  

(2.1)

where

\[
t_1(x^-) \equiv \frac{2 \pi^2}{N_c^2} \langle T_{1-}(x^-) \rangle.
\]  

(2.2)

Here \(x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}\), \(x = (x^1, x^2)\), \(dx_{\perp}^2 = (dx^1)^2 + (dx^2)^2\), \(z\) is the coordinate describing the 5th dimension such that the ultraviolet (UV) boundary of the AdS space is at \(z = 0\), and \(L\) is the radius of the AdS space. According to holographic renormalization \([55]\), \(\langle T_{-}(x^-) \rangle\) is the expectation value of the energy-momentum tensor for a single ultrarelativistic nucleus moving along the light-cone in the \(x^+\)-direction in the gauge theory. We assume that the nucleus is made out of nucleons consisting of \(N_c^2\) “valence gluons” each, such that \(\langle T_{-}(x^-) \rangle \propto N_c^2\), and the metric \((2.1)\) has no \(N_c^2\)-suppressed terms in it. The metric in Eq. \((2.1)\) is a solution of Einstein equations in AdS5:

\[
R_{\mu\nu} + \frac{4}{L^2} g_{\mu\nu} = 0.
\]  

(2.3)

Imagine a collision of the shock wave \((2.1)\) with another similar shock wave moving in the light cone \(x^-\) direction described by the metric

\[
ds^2 = \frac{L^2}{z^2} \left\{ -2 \, dx^+ \, dx^- + t_2(x^+) \, z^4 \, dx^+ \, dz + dx_{\perp}^2 + d\tilde{z}^2 \right\}
\]  

(2.4)

with

\[
t_2(x^+) \equiv \frac{2 \pi^2}{N_c^2} \langle T_{2++}(x^+) \rangle.
\]  

(2.5)

Here we will consider the high-energy approximation, in which the shock waves’ profiles are given by delta-functions,

\[
t_1(x^-) = \mu_1 \, \delta(x^-), \quad t_2(x^+) = \mu_2 \, \delta(x^+).
\]  

(2.6)

The two scales \(\mu_1\) and \(\mu_2\) can be expressed in terms of the physical parameters in the problem since we picture the shock waves as dual to the ultrarelativistic heavy ions in the boundary gauge theory \([16, 56]\) :

\[
\mu_1 \sim p_1^+ \, \Lambda_1^2 \, A_1^{1/3}, \quad \mu_2 \sim p_-^2 \, \Lambda_2^2 \, A_2^{1/3}.
\]  

(2.7)
Here \( p_1^\perp, p_2^\perp \) are the large light-cone momenta per nucleon, \( A_1 \) and \( A_2 \) are atomic numbers, and \( \Lambda_1 \) and \( \Lambda_2 \) are the typical transverse momentum scales in the two nuclei [16]. Note that \( \mu_1 \) and \( \mu_2 \) are independent of \( N_c \).

The exact analytical solution of Einstein equations (2.3) starting with the superposition of the metrics (2.1) and (2.4) before the collision, and generating the resulting non-trivial metric after the collisions, is not known. Instead one constructs perturbative expansion of the solution of Einstein equations in powers of \( t_1 \) and \( t_2 \), or, equivalently, \( \mu_1 \) and \( \mu_2 \) [14,16,17,23,24]. At present the metric is known up to the fourth order in \( \mu 's \) [14,16,17], and also a resummation to all-orders in \( \mu_2 (\mu_1) \) while keeping \( \mu_1 (\mu_2) \) at the lowest order has been performed in [17]. The validity of the perturbatively obtained metric is limited to early proper times \( \tau = \sqrt{2x^+ x^-} \), see e.g. [17] (though indeed the fully-resummed series in powers of \( \mu_1, \mu_2 \) would be valid everywhere). Since here we are interested in the early-time correlations (and due to complexity of the \( \mu_2 \)-resummed metric obtained in [17]), we limit ourselves to the \( O(\mu_1 \mu_2) \) metric obtained in [14,16] in the Fefferman–Graham coordinates:

\[
\begin{align*}
\frac{ds}{\lambda^2} = & \left\{ - \left[ 2 + G(x^+, x^-, z) \right] dx^+ dx^- + \left[ t_1(x^-) z^4 + F(x^+, x^-, z) \right] dx^{-2} \\
& + \left[ t_2(x^+) z^4 + \tilde{F}(x^+, x^-, z) \right] dx^{2+} + \left[ 1 + H(x^+, x^-, z) \right] dx^2_{\perp} + dz^2 \right\}. \\
\end{align*}
\tag{2.8}
\]

The components of the metric at the order-\( \mu_1 \mu_2 \) are

\[
\begin{align*}
F(x^+, x^-, z) &= -\lambda_1(x^+, x^-) z^4 - \frac{1}{6} \partial_-^2 h_0(x^+, x^-) z^6 - \frac{1}{16} \partial_-^2 h_1(x^+, x^-) z^8 \\
\tilde{F}(x^+, x^-, z) &= -\lambda_2(x^+, x^-) z^4 - \frac{1}{6} \partial_-^2 h_0(x^+, x^-) z^6 - \frac{1}{16} \partial_-^2 h_1(x^+, x^-) z^8 \\
G(x^+, x^-, z) &= -2 h_0(x^+, x^-) z^4 - 2 h_1(x^+, x^-) z^6 + \frac{2}{3} t_1(x^-) t_2(x^+) z^8 \\
H(x^+, x^-, z) &= h_0(x^+, x^-) z^4 + h_1(x^+, x^-) z^6,
\end{align*}
\tag{2.9}
\]

where we defined [16]

\[
\begin{align*}
h_0(x^+, x^-) &= \frac{8}{\partial_+^2 \partial_-^2} t_1(x^-) t_2(x^+), \quad h_1(x^+, x^-) = \frac{4}{3 \partial_+ \partial_-} t_1(x^-) t_2(x^+) \\
\lambda_1(x^+, x^-) &= \frac{\partial_+}{\partial_-} h_0(x^+, x^-), \quad \lambda_2(x^+, x^-) = \frac{\partial_+}{\partial_-} h_0(x^+, x^-)
\end{align*}
\tag{2.10}
\]

along with the definition of the causal integrations

\[
\begin{align*}
\frac{1}{\partial_+} [\ldots](x^+) \equiv \int_{-\infty}^{x^+} dx^+ [\ldots](x^+), \quad \frac{1}{\partial_-} [\ldots](x^-) \equiv \int_{-\infty}^{x^-} dx^- [\ldots](x^-).
\end{align*}
\tag{2.11}
\]
Below we will calculate correlation functions of the glueball operators

$$J(x) \equiv \frac{1}{2} \text{tr}[F_{\mu\nu}F^{\mu\nu}]$$

(2.12)

in the boundary gauge theory.\(^1\) According to the standard AdS/CFT prescription,\(^2\) the glueball operator is dual to the massless scalar (dilaton) field \(\phi\) in the AdS\(_5\) bulk \([57]\) with the action

$$S^\phi = -\frac{N_c^2}{16\pi^2L^3} \int d^4x \, dz \sqrt{-g} g^{MN} \partial_M \phi(x, z) \partial_N \phi(x, z),$$

(2.13)

where \(M, N = (\mu, z)\), \(\mu = (0, 1, 2, 3)\) and \(x^\mu\) correspond to 4D field theory coordinates, while \(z\) is the coordinate along the extra fifth (holographic) dimension. (As usual \(g = \det g_{MN}\).)

The equation of motion (EOM) for the scalar field is

$$\frac{1}{\sqrt{-g}} \partial_M \left[ \sqrt{-g} g^{MN} \partial_N \phi(x, z) \right] = 0.$$  

(2.14)

Using Eq. (2.14), the dilaton action evaluated on the classical solution can be cast in the following form convenient for the calculation of correlation functions:

$$S^\phi_{cl} = \frac{N_c^2}{16\pi^2L^3} \int d^4x \left[ \sqrt{-g} g^{zz} \phi(x, z) \partial_z \phi(x, z) \right] \bigg|_{z=0} = \frac{N_c^2}{16\pi^2} \int d^4x \phi_B(x) \left[ \frac{1}{z^3} \partial_z \phi(x, z) \right] \bigg|_{z=0}.$$  

(2.15)

In arriving at the expression on the right of Eq. (2.13) we have used the metric in Eqs. (2.8), (2.9), and (2.10), along with the standard assumption that the fields \(\phi\) have the following boundary condition (BC) at the UV boundary, \(\phi(x, z \to 0) = \phi_B(x)\), which allowed us to approximate near \(z = 0\)

$$g = -\frac{L^{10}}{z^{10}} \left( 1 - \frac{1}{3} z^8 t_1(x^-) t_2(x^+) \right) \approx -\frac{L^{10}}{z^{10}}.$$  

(2.16)

In arriving at Eq. (2.15) we have also demanded that\(^3\)

$$\sqrt{-g} g^{zz} \phi(x, z) \partial_z \phi(x, z) \to 0 \quad \text{as} \quad z \to \infty.$$  

(2.17)

Define the retarded Green function of the glueball operator (2.12) (averaged in the heavy ion collision background),

$$G_R(x_1, z_2) = -i \theta(x_1^0 - x_2^0) \langle [J(x_1), J(x_2)] \rangle.$$  

(2.18)

---

\(^1\)When defining the glueball operator we assume that in the boundary theory the gluon field \(A_\mu^a\) is defined without absorbing the gauge coupling \(g_{YM}\) in it, such that the field strength tensor \(F_{\mu\nu}^a\) contains the coupling \(g_{YM}\).

\(^2\)Since \(\Delta = 4\), with \(\Delta\) the conformal dimension of \(J(x)\), the mass of the dual scalar field, \(m^2 = \Delta(\Delta - 4)\), is zero.

\(^3\)As one can see later, our classical solutions satisfy this condition.
According to the AdS/CFT correspondence the contribution to the retarded Green function coming from the medium produced in the collision is given by [58] \(^4\)

\[
G_R(x_1, x_2) = \frac{\delta^2[S_0^2 - S_0]}{\delta \phi_B(x_1) \delta \phi_B(x_2)}, \tag{2.19}
\]

where we subtract the action \(S_0\) of the scalar field in the empty AdS\(_5\) space to remove the contribution of the retarded Green function in the vacuum. The latter has nothing to do with the properties of the medium produced in the collision and has to be discarded.

Later we will be interested in the Fourier transform of the retarded Green function

\[
G_R(k_1, k_2) = \int d^4x_1 d^4x_2 e^{-ik_1 \cdot x_1 - ik_2 \cdot x_2} G_R(x_1, x_2). \tag{2.20}
\]

(We are working in the \((-\, +\, +\, +\, +)\) metric in the boundary four dimensions.)

### 2.2 Kinematics

We have defined above \(k^\pm = (k^0 \pm k^3)/\sqrt{2}, \, k = (k^1, k^2), \, k_\perp = |k|\) and \(k^2 = k_\perp^2 - 2 k^+ k^- = -m^2\). The particle rapidity, defined as, \(y = \frac{1}{2} \ln \frac{k^+}{k^-}\), is a useful variable, since the rapidity difference between any pair of particles remains unchanged if we go from the center of mass frame to any other frame by performing a boost along the longitudinal direction, \(x^3\). On the other hand, when \(k^0 \gg m, \, y \approx y_p = \ln \cot(\theta/2)\), where \(y_p\) is pseudorapidity, and \(\theta\) is the angle at which the particle emerges in the center of mass frame. Furthermore, defining \(m_\perp \equiv \sqrt{k_\perp^2 + m^2}\), we can rewrite the light-cone components of the momentum as: \(k^+ = m_\perp e^{y}/\sqrt{2}\) and \(k^- = m_\perp e^{-y}/\sqrt{2}\). In the case when \(k_\perp^2 \gg m^2\) one has \(k^+ k^- \approx k_\perp^2/2\).

Consider two identical on-mass-shell particles with momenta \(k_1 = (k_1^+, k_1^- , k_1)\) and \(k_2 = (k_2^+, k_2^- , k_2)\). Assuming \(k_1^2 = k_2^2 = -m^2\) and \(k_1 = k_2 = k\), we obtain

\[
q^2 \equiv (k_2 - k_1)^2 = -2 m^2 - 2 k_1 \cdot k_2 = 4 m_\perp^2 \sinh^2 \frac{\Delta y}{2} > 0, \tag{2.21}
\]

where \(\Delta y = y_2 - y_1\) with \(y_1\) and \(y_2\) the rapidities of the two particles. In case when \(k_\perp^2 \gg m^2\) and \(\Delta y \gg 1\), we have \(q^2 \approx 2 k_\perp^2 \cosh \Delta y \approx k_\perp^2 e^{\Delta y}\). It is worth noting that the momentum difference is space-like, since \(q^2 \equiv Q^2 > 0\).

\(^4\)As was shown in [59,60] the right-hand side of Eq. (2.19) contains contributions of both the retarded and advanced Green functions \(G_R\) and \(G_A\). In the lowest-order calculation we are going to perform here the Green functions are real, and, since \(\text{Re}G_R = \text{Re}G_F = \text{Re}G_A\) (with \(G_F\) the Feynman Green function defined below in Eq. (2.34)), we do not need to address the question of disentangling the contributions of different wave functions to Eq. (2.19) and will adopt the convention of [61,62] by calling the object in Eq. (2.19) a retarded Green function.
2.3 Defining the observable in the boundary gauge theory

Let us now specify the observable we want to calculate in the boundary gauge theory. Our primary goal is to study rapidity correlations using AdS/CFT. Ideally one would like to find correlations between produced particles. However, $\mathcal{N} = 4$ SYM theory has no bound states, and, at strong coupling, it does not make sense to talk about individual supersymmetric particles. Therefore we will study correlators of operators, starting with the glueball operator defined in Eq. (2.12).

One can think of the glueballs as external probes to $\mathcal{N} = 4$ SYM theory (in the sense of being particles from some other theory in four dimensions), which couple to the gluons in $\mathcal{N} = 4$ SYM, and therefore can be produced in the collision. Later on we will also consider correlators of the energy-momentum tensor $T_{\mu\nu}$, which should be also thought of as an operator coupling to a particle (in four dimensions) external to the $\mathcal{N} = 4$ SYM theory.

We start with the glueball production. To study two-particle correlations we need to find the two-particle multiplicity distribution

\[
\frac{d^6N}{d^2k_1 dy_1 d^2k_2 dy_2} \quad (2.22)
\]

where $k_1^\perp$, $y_1$ and $k_2^\perp$, $y_2$ are the transverse momenta of the produced particles (glueballs) and their rapidities, and $d^2k \equiv dk^1 dk^2$. As usual we can decompose the two-particle multiplicity distribution into the uncorrelated and correlated pieces

\[
\frac{d^6N}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{d^3N}{d^2k_1 dy_1} \frac{d^3N}{d^2k_2 dy_2} + \frac{d^6N_{\text{corr}}}{d^2k_1 dy_1 d^2k_2 dy_2}. \quad (2.23)
\]

We are interested in computing the second (correlated) term on the right hand side of (2.23). We begin by writing it as

\[
\frac{d^6N_{\text{corr}}}{d^2k_1 dy_1 d^2k_2 dy_2} \propto \langle |M(k_1, k_2)|^2 \rangle \quad (2.24)
\]

where $M(k_1, k_2)$ is the two-particle production amplitude. (Note that since we are primarily interested in rapidity dependence of correlators, we are not keeping track of prefactors and other coefficients not containing two-particle correlations.)

For the correlated term in Eq. (2.23) the amplitude of inclusive two-glueball production in a heavy ion collision is

\[
M(k_1, k_2) \propto \int d^4x_1 d^4x_2 e^{-ik_1 \cdot x_1 - ik_2 \cdot x_2} \langle n | T \{ J(x_1) J(x_2) \} | A_1, A_2 \rangle, \quad (2.25)
\]

which is a consequence of the LSZ reduction formula with $T$ denoting time-ordering. Here $|n\rangle$ denotes an arbitrary state of the gauge theory which describes other particles which may be produced in a collision apart from the two glueballs.
The state \( |A_1, A_2\rangle \) can be thought of as the vacuum in the presence of a source, with the source being the two nuclei with atomic numbers \( A_1 \) and \( A_2 \). Consider first the expectation value of the energy-momentum operator \( \langle T_{\mu\nu} \rangle \) in a nuclear collision. According to the standard prescription we can write it as

\[
\langle T_{\mu\nu}(x) \rangle = \frac{\int DA_\mu e^{iS[A]} W_+[A] W_-[A] T_{\mu\nu}(x)}{\int DA_\mu e^{iS[A]} W_+[A] W_-[A]}
\]  

(2.26)

where \( S[A] \) is the action of the gauge theory. For simplicity we only explicitly show the integrals over gauge fields in Eq. (2.26), implying the integrals over all other fields in the theory. The objects \( W_+[A] \) and \( W_-[A] \) are some functionals of the fields in the theory describing the two colliding nuclei. For instance, in the perturbative QCD approaches such as CGC, these operators are Wilson lines along \( x^- = 0 \) and \( x^+ = 0 \) light cone directions [43–46].

Using operators and states in Heisenberg picture one can rewrite Eq. (2.26) as

\[
\langle T_{\mu\nu}(x) \rangle = \langle A_1, A_2 | T_{\mu\nu}(x) | A_1, A_2 \rangle.
\]  

(2.27)

Comparing Eq. (2.27) to Eq. (2.26) clarifies the meaning of the \( |A_1, A_2\rangle \) state by demonstrating that the averaging in Eq. (2.27) is over a state of vacuum in the presence of nuclear sources (which of course strongly disturb the vacuum).

Using Eq. (2.25) in Eq. (2.24) we obtain

\[
\frac{d^6N_{corr}}{d^2k_1 dy_1 d^2k_2 dy_2} \propto \int d^4x_1 d^4x_2 d^4x_1' d^4x_2' e^{i\mathbf{k}_1 \cdot (x_1 - x_1') - i\mathbf{k}_2 \cdot (x_2 - x_2')} \times \sum_n \langle A_1, A_2 | \overline{T} \{ J(x_1') J(x_2') \} | n \rangle \langle n | T \{ J(x_1) J(x_2) \} | A_1, A_2 \rangle
\]  

(2.28)

where \( \overline{T} \) denotes the inverse time-ordering and we have used the fact that \( J(x) \) is a hermitean operator. Summing over a complete set of states \( |n\rangle \) yields

\[
\frac{d^6N_{corr}}{d^2k_1 dy_1 d^2k_2 dy_2} \propto \int d^4x_1 d^4x_2 d^4x_1' d^4x_2' e^{-i\mathbf{k}_1 \cdot (x_1 - x_1') - i\mathbf{k}_2 \cdot (x_2 - x_2')} \times \langle A_1, A_2 | \overline{T} \{ J(x_1') J(x_2') \} T \{ J(x_1) J(x_2) \} | A_1, A_2 \rangle.
\]  

(2.29)

As one could have expected, in order to calculate two-particle production, we need to calculate a 4-point function given in Eq. (2.29). This is, in general, a difficult task: instead we will use the following simplification. Begin by replacing the complete set of states \( |n\rangle \) by states \( \mathcal{O}_n(x) | A_1, A_2 \rangle \) obtained by acting on our “vacuum” state \( |A_1, A_2\rangle \) by a complete orthonormal set of gauge theory operators \( \mathcal{O}_n(x) \), such that

\[
1 = \sum_n \langle n | \langle n | = \sum_n \int d^4x \mathcal{O}_n(x) | A_1, A_2 \rangle \langle A_1, A_2 | \mathcal{O}_n^\dagger(x)
\]  

(2.30)

\(^5\text{Calculation of the expectation value of } T_{\mu\nu} \text{ in CGC is reduced to perturbative evaluation/resummation of Eq. (2.26) (see e.g. [63] for an example of such calculation).} \)
with the normalization condition
\[
\langle A_1, A_2 | O_m^\dagger(y) O_n(x) | A_1, A_2 \rangle = \delta_{nm} \delta^{(4)}(x - y).
\] (2.31)

Using Eq. (2.30) in Eq. (2.28) we write
\[
\frac{d^6 N_{\text{corr}}}{d^2 k_1 dy_1 d^2 k_2 dy_2} \propto \int d^4 x_1 d^4 x_2 d^4 x'_1 d^4 x'_2 e^{-i k_1 \cdot (x_1 - x'_1) - i k_2 \cdot (x_2 - x'_2)} \sum_n \int d^4 x \times \langle A_1, A_2 | \mathcal{T} \{ J(x'_1) J(x'_2) O_n(x) \} | A_1, A_2 \rangle \langle A_1, A_2 | T \{ O_n^\dagger(x) J(x_1) J(x_2) \} | A_1, A_2 \rangle.
\] (2.32)

To evaluate Eq. (2.32) we have to insert all possible operators $O_n(x)$ from the orthonormal set in it. Noting that $J(x)$ is a gauge-invariant color-singlet operator, we conclude that only color-singlet $O_n(x)$ would contribute. Also, since the final state in a scattering problem should be an observable, the operators $O_n$ should be hermitean. The set of contributing $O_n(x)$’s should therefore include the identity operator, $J(x)$, $T_{\mu\nu}(x)$, etc.

As we will see below, since we are using the metric (2.8), which is a perturbative solution of Einstein equations to order $\mu_1 \mu_2$, we can only calculate correlators to order $\mu_1 \mu_2$ as well. Moreover, correlators which are independent of $\mu_1$ and $\mu_2$ are vacuum correlators that we are not interested in. Correlators of order $\mu_1$ or $\mu_2$ correspond to performing deep inelastic scattering (DIS) on a single shock wave similar to [61, 62, 64], and are thus not directly relevant to the problem of heavy ion collisions at hand. Thus in this paper we are only interested in correlators exactly at the order $\mu_1 \mu_2$ in the expansion in the two shock waves. Using such power counting it is easy to see that inserting the identity operator (normalized to one to satisfy Eq. (2.31)) into Eq. (2.32) in place of $O_n$’s would give us a contribution of the order of $\mu_1^2 \mu_2^2$, which is the lowest order contribution to double glueball production. Inserting $J(x)$ or $T_{\mu\nu}(x)$ into Eq. (2.32) instead of $O_n$’s would give zero. One can also see that replacing $O_n$’s by higher (even) powers of $J(x)$ or $T_{\mu\nu}(x)$ (properly orthogonalized) in Eq. (2.32) would generate non-zero contributions, which are either higher order in $\mu_1$ and $\mu_2$ or $N_c$-suppressed. We therefore insert the identity operator into Eq. (2.32), which in the color space can be written as $1 = \delta^{ab}/N_c$ to satisfy normalization in Eq. (2.31), and write
\[
\frac{d^6 N_{\text{corr}}}{d^2 k_1 dy_1 d^2 k_2 dy_2} \propto \int d^4 x_1 d^4 x_2 d^4 x'_1 d^4 x'_2 e^{-i k_1 \cdot (x_1 - x'_1) - i k_2 \cdot (x_2 - x'_2)}
\]
\[
\times \frac{1}{N_c^2} \langle A_1, A_2 | \mathcal{T} \{ J(x'_1) J(x'_2) \} | A_1, A_2 \rangle \langle A_1, A_2 | T \{ J(x_1) J(x_2) \} | A_1, A_2 \rangle [1 + O(1/N_c^2)] .
\] (2.33)

We have thus reduced the problem of two-gluon production to calculation of two-point correlation functions! Note that the prefactor of $1/N_c^2$ makes the $N_c$ counting right: since each connected correlator is order-$N_c^2$, we see from Eq. (2.33) that the correlated two-particle multiplicity scales as $N_c^2$ as well, in agreement with perturbative calculations [37–41].

Defining Feynman Green function
\[
G_F(k_1, k_2) = \int d^4 x_1 d^4 x_2 e^{-i k_1 \cdot x_1 - i k_2 \cdot x_2} \langle A_1, A_2 | T \{ J(x_1) J(x_2) \} | A_1, A_2 \rangle
\] (2.34)
we can summarize Eq. (2.33) as
\[ \frac{d^6 N_{\text{corr}}}{d^2 k_1 dy_1 d^2 k_2 dy_2} \propto \frac{1}{N_c^2} |G_F(k_1, k_2)|^2. \] (2.35)

With the help of the retarded Green function
\[ G_R(k_1, k_2) = -i \int d^4 x_1 d^4 x_2 e^{-i k_1 \cdot x_1 - i k_2 \cdot x_2} \theta(x_1^0 - x_2^0) \langle A_1, A_2 | J(x_1), J(x_2) | A_1, A_2 \rangle \] (2.36)
and using the fact that at zero temperature $|G_F|^2 = |G_R|^2$ [58], we rewrite Eq. (2.35) as
\[ \frac{d^6 N_{\text{corr}}}{d^2 k_1 dy_1 d^2 k_2 dy_2} \propto \frac{1}{N_c^2} |G_R(k_1, k_2)|^2. \] (2.37)

Therefore we need to calculate the two-point retarded Green function at the order $\mu_1 \mu_2$. This is exactly the kind of Green function one can calculate using the AdS/CFT techniques of Eqs. (2.19) and (2.20).

3. A Simple Physical Argument

Before we present the full calculation of the two-particle correlations in AdS, we would like to give a simple heuristic argument of what one may expect from such a calculation. First of all, as we have noted already, we are going to expand the Green function, and, therefore, the bulk field $\phi$ into powers of $\mu_1$ and $\mu_2$, stopping at the order-$\mu_1 \mu_2$. To find the field $\phi$ at the order-$\mu_1 \mu_2$ one has to solve Eq. (2.14) with the metric taken up to the order $\mu_1 \mu_2$. Since we are interested in the long-range rapidity correlations, our goal is to obtain the leading rapidity contribution from the calculation. Analyzing Eqs. (2.20), (2.15), and (2.19), one can conclude that the leading large-rapidity contribution comes from terms with the highest number of factors of light-cone momenta, i.e., from terms like $k_1^+ k_2^-$ and $k_1^- k_2^+$ (but clearly not from $k_1^+ k_1^- = m_1^2 / 2$ which is rapidity-independent). Taking $M = N = -$ in Eq. (2.14) one obtains, among other terms, the following (leading-rapidity) contribution:
\[ g_{(2)}^{-+} \partial_-^2 \phi_0, \] (3.1)
where $\phi_0$ is the field at the order $(\mu_1)^0 (\mu_2)^0$ and $g_{MN}^{(2)}$ is the metric at order-$\mu_1 \mu_2$. Concentrating on order-$z^4$ terms in the metric, which, according to holographic renormalization [55], are proportional to the energy-momentum tensor in the boundary theory, and remembering that the latter is rapidity-independent at order-$\mu_1 \mu_2$ [14, 16], we use energy-momentum conservation, $\partial_\mu T^{\mu \nu} = 0$, which, in particular, implies that $\partial_- T^{--} + \partial_+ T^{++} = 0$, to write
\[ g_{(2)}^{--} = -\frac{\partial_+}{\partial_-} g_{(2)}^{+-}. \] (3.2)
Therefore Eq. (3.1) contains the term

\[ - \left( \frac{\partial_+}{\partial_-} g_{(2)}^+ \right) \partial_2^2 \phi_0, \quad (3.3) \]

which contributes to the field \( \phi \) at order-\( \mu_1 \mu_2 \), and, as follows from Eq. (2.20), resulting in a contribution to the retarded Green function in momentum space proportional to

\[ G_R \sim \frac{k^-}{k^+} \tilde{g}_{(2)}^{+-} (k_2^+)^2 \quad (3.4) \]

with \( \tilde{g}^{+-} \) the Fourier transform of \( g^{+-} \) into momentum space. Since metric component \( \tilde{g}^{+-} \) at the order-\( \mu_1 \mu_2 \) can not be rapidity-dependent [14, 16], we see that Eq. (3.4) gives

\[ G_R \big|_{|\Delta y| \gg 1} \sim e^2 (y_2 - y_1) = e^2 \Delta y. \quad (3.5) \]

Adding the \( k_1 \leftrightarrow k_2 \) term, arising from the \( g^{++} \) component of the metric in Eq. (2.14), we get

\[ G_R \big|_{|\Delta y| \gg 1} \sim \cosh(2 \Delta y). \quad (3.6) \]

Defining the correlation function

\[ C(k_1, k_2) \equiv \frac{d^6 N_{cor}}{d^2 k_1 dy_1 d^2 k_2 dy_2} \quad (3.7) \]

and using Eqs. (3.6) and (2.37) to evaluate it we observe that at large rapidity intervals it scales as

\[ C(k_1, k_2) \big|_{|\Delta y| \gg 1} \sim \cosh(4 \Delta y). \quad (3.8) \]

Indeed the argument we have just presented relies on several assumptions: in particular it assumes that no other term in the metric would cancel correlations arising from the terms we have considered. To make sure that this is indeed the case we will now present the full calculation. The result of our simplistic argument given in Eq. (3.8) would still turn out to be valid at the end of this calculation.

4. Two-Point Correlation Function at Early Times

4.1 Glueball correlator

We now proceed to the calculation of the retarded Green function in the background of the metric (2.8), following the AdS/CFT prescription outlined in Eqs. (2.20), (2.15), and (2.19).
4.1.1 Bulk scalar field

First we have to find the classical scalar field $\phi$. Similar to the way the metric (2.8) was constructed in [16], we will build the scalar field $\phi$ order-by-order in the powers of $\mu_1$ and $\mu_2$, assuming $\mu_1$ and $\mu_2$ are small perturbations. We would like to find the solution of Eq. (2.14) up to order $\mathcal{O}(\mu_1 \mu_2)$. For this we use the following expansion,

$$\phi(x, z) = \phi_0(x, z) + \phi_a(x, z) + \phi_b(x, z) + \phi_2(x, z) + \ldots ,$$  \hspace{1cm} \text{(4.1)}$$

where $\phi_0 \sim \mathcal{O}(\mu_{1,2}^0)$, $\phi_{a,b} \sim \mathcal{O}(\mu_{1,2})$ and $\phi_2 \sim \mathcal{O}(\mu_1 \mu_2)$. We will use the standard method (see e.g. [61, 62, 64]) and demand that the boundary conditions at $z \to 0$ are as follows:

$$\phi_0(x, z \to 0) = \phi_B(x), \quad \phi_a(x, z \to 0) = \phi_b(x, z \to 0) = \phi_2(x, z \to 0) = \ldots = 0. \hspace{1cm} \text{(4.2)}$$

In this case the variation of the classical action with respect to boundary value of the field $\phi_B$ required in Eq. (2.13) is straightforward.

Using Eq. (2.8) in Eq. (2.14), and expanding the linear operator in the latter in powers of $\mu_1$ and $\mu_2$ up to order-$\mu_1 \mu_2$ with the help of (2.9) and (2.10), the EOM can be written explicitly in the form

$$\left[ \Box_5 + z^4 t_1 \partial_+^2 + z^4 t_2 \partial_-^2 + \frac{1}{12} z^4 \hat{M} \right] \phi(x, z) = 0 .$$  \hspace{1cm} \text{(4.3)}$$

Taking into account that $t_1 = t_1(x^-)$ and $t_2 = t_2(x^+)$, we give the following list of definitions:

$$\Box_5 \equiv -\partial_+^2 + \frac{3}{z} \partial_z + \Box_4 , \quad \Box_4 \equiv 2 \partial_+ \partial_- - \nabla_1^2 , \quad \frac{1}{\partial_{\pm}} \equiv \int_{-\infty}^{x_{\pm}} dx' \pm ,$$  \hspace{1cm} \text{(4.4)}$$

$$\hat{M} \equiv \left( \hat{D} + z^4 \right) t_1 t_2 \nabla_1^2 - \frac{\partial_+}{\partial_-} \hat{D} t_1 t_2 \partial_+^2 - \frac{\partial_-}{\partial_+} \hat{D} t_1 t_2 \partial_-^2 + 2 \left( \hat{D} + 5 z^4 \right) t_1 t_2 \partial_+ \partial_- + 5 z^4 t_1 (\partial_+ t_2) \partial_- + 5 z^4 t_2 (\partial_- t_1) \partial_+ + 10 z^3 t_1 t_2 \partial_z + 2 z^4 t_1 t_2 \partial_+^2 ,$$

$$\hat{D} \equiv 96 \frac{1}{\partial_+^2} \frac{1}{\partial_-^2} + 16 z^2 \frac{1}{\partial_+} \frac{1}{\partial_-} + z^4 .$$

Substituting expansion (4.1) into (4.3), and grouping different powers of $\mu_1$ and $\mu_2$ together we end up with the following set of equations, listed here along with their boundary conditions:

$$\Box_5 \phi_0(x, z) = 0 , \quad \phi_0(x, z \to 0) = \phi_B(x) ,$$  \hspace{1cm} \text{(4.5a)}$$

$$\Box_5 \phi_a(x, z) = -z^4 t_1 (x^-) \partial_+^2 \phi_0(x, z) , \quad \phi_a(x, z \to 0) = 0 ,$$  \hspace{1cm} \text{(4.5b)}$$

$$\Box_5 \phi_b(x, z) = -z^4 t_2 (x^+) \partial_-^2 \phi_0(x, z) , \quad \phi_b(x, z \to 0) = 0 ,$$  \hspace{1cm} \text{(4.5c)}$$

$$\Box_5 \phi_2(x, z) = -z^4 t_1 (x^-) \partial_+^2 \phi_b(x, z) - z^4 t_2 (x^+) \partial_-^2 \phi_a(x, z) - \frac{z^4}{12} \hat{M} \phi_0(x, z) , \quad \phi_2(x, z \to 0) = 0 ,$$  \hspace{1cm} \text{(4.5d)}$$
where we also imply that all the solutions should be regular at $z \to \infty$. To solve equations (4.5) it is convenient to introduce a Green function $G(x, z, z')$ satisfying the equation

$$\square_5 G(x, z, z') = z^2 \delta(z - z').$$  \hfill (4.6)

The Green function can be written as

$$G(x, z, z') = \frac{z^2}{z'} \beta_4 K_2(z \sqrt{\beta_4}) \tag{4.7}$$

where $z_{\{<,>\}} = \{\min, \max\} \{z, z'\}$. We can rewrite the inverse of $\square_5$ operator as

$$\frac{1}{\square_5} f(x, z) \equiv \int_0^{\infty} \frac{dz'}{z^3} G(x, z, z') f(x, z').$$  \hfill (4.8)

Solving the first equation in (4.5) we find

$$\phi_0(x, z) = \frac{1}{2} z^2 \beta_4 K_2(z \sqrt{\beta_4}) \phi_B(x).$$  \hfill (4.9)

From Eqs. (4.5b), (4.5c), and Eq. (4.5d) we have

$$\phi_a(x, z) = -\frac{1}{\square_5} \left[ z^4 t_1 \partial^2_+ \phi_0 \right], \quad \phi_b(x, z) = -\frac{1}{\square_5} \left[ z^4 t_2 \partial^2_\phi \phi_0 \right],$$  \hfill (4.10)

$$\phi_2(x, z) = \frac{1}{\square_5} z^4 t_1 \partial^2_+ \frac{1}{\square_5} z^4 t_2 \partial^2_- \phi_0 + \frac{1}{\square_5} z^4 t_2 \partial^2_+ \frac{1}{\square_5} z^4 t_1 \partial^2_- \phi_0 - \frac{1}{\square_5} z^4 \hat{M} \phi_0.$$  \hfill (4.11)

We have constructed the bulk scalar field which we need to find the correlation function.

### 4.1.2 Glueball correlation function

We can now calculate the retarded glueball correlation function using Eq. (4.11) in Eqs. (2.15), (2.19), and (2.20). It is straightforward to check that

$$\left[ \frac{1}{z^3} \partial_z G(x, z, z') \right]_{z \to 0} = \frac{1}{2} z^2 \beta_4 K_2(z' \sqrt{\beta_4}).$$  \hfill (4.12)

Using Eq. (4.12), along with Eqs. (2.13), (2.19), and (2.20), we obtain

$$G_R(k_1, k_2) = \frac{N_c^2}{16} \mu_1 \mu_2 \delta^{(2)}(k_1 + k_2) k_1^2 k_2^2 \left[ F(k_1, k_2) + F(k_2, k_1) \right],$$  \hfill (4.13)

where

$$F(k_1, k_2) \equiv F_1(k_1, k_2) + F_1(k_2, k_1)$$  \hfill (4.14)
with
\[
F_1(k_1, k_2) = \int_0^\infty dz \ z^5 K_2 \left( z \sqrt{k_1^2} \right) \int_0^\infty dz' \ z'^5 K_2 \left( z' \sqrt{k_2^2} \right) \\
\times \left[ (k_1^- k_2^+)^2 I_2 (Q_1 z_<) K_2 (Q_1 z_>) + (k_1^+ k_2^-)^2 I_2 (Q_2 z_<) K_2 (Q_2 z_>) \right]
\] (4.15)
and
\[
F_{11}(k_1, k_2) = \frac{k_1^2}{12} \int_0^\infty dz \ z^5 K_2 \left( z \sqrt{k_1^2} \right) \left[ \frac{96}{(k_1^+ k_1^-)^2} - \frac{16 z^2}{k_1^+ k_1^-} \right] K_2 \left( z \sqrt{k_2^2} \right) \\
- \frac{1}{12} \left[ k_1^- k_2^+ + k_1^+ k_2^- \right] \int_0^\infty dz \ z^5 K_2 \left( z \sqrt{k_1^2} \right) \left[ \frac{96}{(k_1^+ k_1^-)^2} - \frac{16 z^2}{k_1^+ k_1^-} + z^4 \right] K_2 \left( z \sqrt{k_2^2} \right) \\
+ \frac{1}{6} k_1^2 k_2^- \int_0^\infty dz \ z^5 K_2 \left( z \sqrt{k_1^2} \right) \left[ \frac{96}{(k_1^+ k_1^-)^2} - \frac{16 z^2}{k_1^+ k_1^-} + 8 z^4 \right] K_2 \left( z \sqrt{k_2^2} \right) \\
- \frac{5}{12} \left[ 2 k_2^+ k_2^- + k_2^+ k_1^- + k_2^- k_1^+ \right] \int_0^\infty dz \ z^5 K_2 \left( z \sqrt{k_1^2} \right) K_2 \left( z \sqrt{k_2^2} \right) \\
+ \frac{4 k_2^2}{3} \int_0^\infty dz \ z^8 K_2 \left( z \sqrt{k_1^2} \right) K_1 \left( z \sqrt{k_2^2} \right).
\] (4.16)

We have defined
\[
Q_1^2 = 2 k_1^- k_2^+ + k_1^2, \quad Q_2^2 = 2 k_1^+ k_2^- + k_2^2,
\] (4.17)
with \(k_{1,\perp} = k_{2,\perp} = k_\perp\).

Before evaluating the obtained expressions further, let us comment on some of their features. First one may note that Eq. (4.13) contains a delta-function of transverse momenta of the two glueballs \(\delta^2 (\vec{k}_1 + \vec{k}_2)\). This demonstrates that at the lowest non-trivial order in \(\mu_1 \) and \(\mu_2\) expansion (order-\(\mu_1 \mu_2\)) there will be nothing else produced in the shock wave collision apart from the two glueballs. Note that indeed a non-zero \(\langle T_{\mu\nu} \rangle\) in the forward light-cone at the order-\(\mu_1 \mu_2\) found in [14,16] indicates that a medium is created: however this strongly-coupled medium in the \(\mathcal{N} = 4\) SYM theory without bound states and confinement does not fragment into individual particles, and at late times simply results in a very low (and decreasing) energy density created in the collision, similar to the asymptotic future of Bjorken hydrodynamics dual found in [8]. Since in our calculation we have explicitly projected out two glueballs with fixed momenta in the final state, those two glueballs are all that is left carrying transverse momentum in the forward lightcone. (Leftovers of the original shock waves may also be present, though they would not carry any transverse momentum.) This picture is in agreement with the dominance of elastic processes in high energy scattering in the AdS/CFT framework suggested in [65].

Another important aspect of the result in Eqs. (4.15) and (4.16) above is that the integrals over \(z\) and \(z'\) diverge for time-like momenta \(k_1\) and \(k_2\), i.e., for \(k_1^2 = -m^2\) and \(k_2^2 = -m^2\) corresponding
to production of physical glueballs of mass $m$. This result should be expected in $\mathcal{N} = 4$ SYM theory: since there are no bound states in this theory, we conclude that there are no glueballs. Thinking of Bessel functions $K_2(z\sqrt{k^2_{1,2}})$ in Eqs. (4.13) and (4.16) as contributing to the wave functions of glueballs in AdS$_5$ space [66–68], we conclude that the lack of glueball bound states in the theory manifests itself through de-localization of these wave functions, resulting in “bound states” of infinite radii, both in the bulk and in the boundary theory (if we identify the holographic coordinate $z$ with the inverse momentum scale on the UV boundary). Since the glueballs for us have always been some external probes of the $\mathcal{N} = 4$ SYM theory, we conclude that one has to define the probes by re-defining their wavefunctions. This can be accomplished, for instance, by introducing confinement in the theory, by using either the “hard-wall” or “soft-wall” models [66,69–77]. The inverse confinement scale would define the typical size of the bound states. Indeed such procedure would introduce a model-dependent uncertainty associated with mimicking confinement in AdS/CFT, but is unavoidable in order to define glueball probes. Besides, our main goal here is to calculate long-range rapidity correlations, which are not affected (apart from a prefactor) by the exact shape of the glueball AdS$_5$ wave functions. We therefore model confinement by modeling the glueball (external source) AdS wave functions by simply replacing $K_2(z\sqrt{k^2_{1,2}}) \rightarrow K_2(z\Lambda)$ in Eqs. (4.15) and (4.16)

$$F_1(k_1, k_2) = \int_0^\infty dz \ z^5 K_2(z\Lambda) \int_0^\infty dz' \ z'^5 K_2(z'\Lambda) \times \left[ (k_1^- k_2^+)^2 I_2(Q_1 z_<) K_2(Q_1 z_>) + (k_1^+ k_2^-)^2 I_2(Q_2 z_<) K_2(Q_2 z_>) \right], \quad (4.18)$$

$$F_{II}(k_1, k_2) = \frac{k_1^2}{12} \int_0^\infty dz \ z^5 [K_2(z\Lambda)]^2 \left[ \frac{384}{m_1^4} - \frac{32 z^2}{m_2^2} \right]$$

$$- \frac{1}{12} \left[ \frac{k_1^+ k_2^+}{k_1^-} + \frac{k_1^+ k_2^-}{k_1^-} \right] \int_0^\infty dz \ z^5 [K_2(z\Lambda)]^2 \left[ \frac{384}{m_1^4} - \frac{32 z^2}{m_2^2} + z^4 \right]$$

$$+ \frac{1}{12} m_1^2 \int_0^\infty dz \ z^5 [K_2(z\Lambda)]^2 \left[ \frac{384}{m_1^4} - \frac{32 z^2}{m_2^2} + 8 z^4 \right]$$

$$- \frac{5}{12} [m_1^2 + k_2^+ k_1^- + k_2^- k_1^+] \int_0^\infty dz \ z^8 K_2(z\Lambda) K_1(z\Lambda) \right], \quad (4.19)$$

where we have also replaced all rapidity-independent factors with powers of either glueball mass $m$ or $m_\perp = \sqrt{k_\perp^2 + m^2}$.

The contributions in Eqs. (4.18) and (4.19) (or those in Eqs. (4.13) and (4.16)) to the retarded Green function (2.20) are shown diagrammatically in Fig. 2 in terms of Witten diagrams. There the
wiggly lines represent gravitons, while the dashed line denotes the scalar field. Crosses represent insertions of the boundary energy-momentum tensors of the two shock waves ($\mu_1$ and $\mu_2$). $F_I$ from Eq. (4.18) corresponds to the diagram on the left of Fig. 2, while $F_{II}$ from Eq. (4.19) is given by the term on the right of Fig. 2.

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{fig2.png}
\caption{Diagrammatic representation of the correlation function calculated in this Section.}
\end{figure}

It is important to note that the Green function given by Eqs. (4.13), (4.14), (4.18), and (4.19) is indeed real, justifying the assumption we employed in stating that Eq. (2.19) provides us a retarded Green function. This can also be seen from the diagrams in Fig. 2 in which one can not cut the scalar propagator. The imaginary part of $G_R$ appears at higher order in $\mu_1 \mu_2$, when one has more graviton insertions in the scalar propagator, allowing for non-zero cuts of the latter.

Let us now study the large-rapidity interval asymptotics of the obtained correlation function (4.13). One can deduce from the kinematics described in Section 2.2 that

$$k_1^+ k_2^- = \frac{m_1^2}{2} e^{-\Delta y} , \quad k_1^- k_2^+ = \frac{m_2^2}{2} e^{\Delta y} ,$$

such that when $\Delta y = y_2 - y_1 \gg 1$ we have

$$Q_1^2 = k_1^2 + m_1^2 e^{\Delta y} \approx m_1^2 e^{\Delta y} , \quad Q_2^2 = k_1^2 + 2 m_1^2 e^{-\Delta y} \approx k_1^2 .$$

Therefore, the contribution from Eq. (4.18) becomes

$$F_I(k_1, k_2)|_{\Delta y \gg 1} \approx \int_0^\infty dz \; z^5 K_2(z \Lambda) \int_0^\infty dz' \; z'^5 K_2(z' \Lambda) \; (k_1^- k_2^+) I_2(Q_1 z_<) \; K_2(Q_1 z_>) .$$

(4.22)
To determine the large-$Q_1$ asymptotics of $I_2(Q_1z_<) K_2(Q_1z_>)$ note that, according to Eqs. (4.4) and (4.7), $z^2 z'^2 I_2(Q_1z_<) K_2(Q_1z_>)$ satisfies

$$\left[-i\partial_z^2 + \frac{3}{2} \partial_z + Q_1^2\right] z^2 z'^2 I_2(Q_1z_<) K_2(Q_1z_>) = z'^3 \delta(z - z').$$

(4.23)

Hence, for $Q_1$ larger than the inverse of the typical variation in $z$ we have

$$z^2 z'^2 I_2(Q_1z_<) K_2(Q_1z_>) \bigg|_{\text{large } Q_1} \approx \frac{z'^3}{Q_1} \delta(z - z'),$$

(4.24)

which, when used in Eq. (4.22) yields

$$F_I(k_1, k_2) \bigg|_{\Delta y \gg 1} \approx \frac{2048}{7} \frac{(k_<^0 k_>^2)^2}{Q_1^2 \Lambda^{10}} \approx \frac{512}{7} \frac{m_1^2}{\Lambda^{10}} e^{\Delta y}.$$  

(4.25)

This result implies that the rapidity correlations coming from this term grow as $e^{\Delta y}$ at the early stages after the collision.

On the other hand, the dominant contributions from the second term, $F_{II}(k_1, k_2)$, are coming from the expressions in the second and the fourth lines of Eq. (4.19). They give

$$F_{II}(k_1, k_2) \bigg|_{\Delta y \gg 1} \approx -\frac{1}{12} \left[ k_<^0 k_>^2 k_< k_1^2 \right] \int_0^\infty dz \ z^5 \left[ K_2(z \Lambda) \right]^2 \left[ \frac{384}{m_1^4} - \frac{32 z^2}{m_2^2} + z^4 \right]$$

$$- \frac{5}{12} \left[ k_<^2 k_1 k_>^2 + k_< k_1^2 k_>^2 \right] \int_0^\infty dz \ z^9 \left[ K_2(z \Lambda) \right]^2$$

$$\approx - \frac{256}{21} \frac{m_1^2}{\Lambda^{10}} e^{2 \Delta y} \left[ 1 - 3 \frac{\Lambda^2}{m_2^2} + \frac{42}{5} \frac{\Lambda^4}{m_1^4} \right] - \frac{1280}{21} \frac{m_1^2}{\Lambda^{10}} e^{\Delta y}.$$  

(4.26)

Combining Eqs. (4.25) and (4.26) in Eqs. (4.13) and (4.14) we obtain

$$G_R(k_1, k_2) \bigg|_{\Delta y \gg 1} \approx - \frac{64}{21} N_c^2 \frac{\mu_1 \mu_2 m_1^4 m_2^2}{\Lambda^{10}} \delta^{(2)}(k_1 + k_2) \left\{ e^{2 \Delta y} \left[ 1 - 3 \frac{\Lambda^2}{m_2^2} + \frac{42}{5} \frac{\Lambda^4}{m_1^4} \right] + e^{\Delta y} \right\},$$

(4.27)

which, dropping the second term in the curly brackets and using the $+ \leftrightarrow -$ symmetry of the problem can be generalized to

$$G_R(k_1, k_2) \bigg|_{\Delta y \gg 1} \approx - \frac{128}{21} N_c^2 \frac{\mu_1 \mu_2 m_1^4 m_2^2}{\Lambda^{10}} \delta^{(2)}(k_1 + k_2) \cosh(2 \Delta y) \left[ 1 - 3 \frac{\Lambda^2}{m_2^2} + \frac{42}{5} \frac{\Lambda^4}{m_1^4} \right].$$

(4.28)

We thus conclude that at large rapidity separations

$$G_R(k_1, k_2) \bigg|_{\Delta y \gg 1} \sim \cosh(2 \Delta y)$$

(4.29)
in agreement with our estimate in Eq. (3.6).

Using Eq. (2.37) we conclude that

\[
\frac{d^6 N_{\text{corr}}}{d^2 k_1 \, dy_1 \, d^2 k_2 \, dy_2} \bigg|_{\Delta y \gg 1} \sim \cosh(4 \Delta y)
\]  

(4.30)

such that the two-glueball correlation function defined in Eq. (3.7) scales as

\[
C(k_1, k_2) \big|_{\Delta y \gg 1} \sim \cosh(4 \Delta y),
\]

(4.31)

just like in Eq. (3.8). We have demonstrated the presence of long-range rapidity correlations in case of strongly-coupled high-energy heavy ion collisions. The rapidity shape of the obtained correlations is very different from the “ridge” correlation observed experimentally at RHIC and at LHC [33–36]. It is possible that higher order in \(\mu_1\) and \(\mu_2\) corrections would modify the rapidity shape of the correlation, putting it more in-line with experiments. We will return to this point in Sec. 6.

Let us now pause to determine the parameter of our approximation. Until now we have, somewhat loosely, referred to our approximation as to an expansion in \(\mu_1\) and \(\mu_2\). However, these parameters have dimensions of mass cubed, and cannot be expanded in. From Eq. (4.28) we may suggest that the dimensionless expansion parameters are \(\mu_1/\Lambda^3\) and \(\mu_2/\Lambda^3\), where \(\Lambda\) is the inverse glueball size. Thus our result in Eq. (4.28) dominates the correlation function only for

\[
\frac{\mu_1}{\Lambda^3} \ll 1, \quad \frac{\mu_2}{\Lambda^3} \ll 1.
\]

(4.32)

Since, as can be seen from Eq. (2.4), \(\mu_1\) and \(\mu_2\) are energy-dependent, these conditions limit the energy range of applicability of Eq. (1.28). Eq. (1.32) also makes clear physical sense: since the metric (2.8) with the coefficients given by Eqs. (2.9) and Eq. (2.10) is valid only for early proper times \(\tau\) satisfying \(\mu_{1,2} \tau^3 \ll 1\) [16,17], we see that the glueballs have to be small enough, \(1/\Lambda \approx \tau \approx \mu_{1,2}^{-1/3}\), to be able to resolve (and be sensitive to) the metric at such early times.

Note also that the obtained Green function (4.28) is not a monotonic function of \(m_\perp\): for \(m_\perp \ll \Lambda\) it grows with \(m_\perp\) as \(m_\perp^2\), but, for \(m_\perp \gg \Lambda\) it falls off as \(1/m_\perp^4\), peaking at \(m_\perp^2 = (28/5) \Lambda^2\). This translates into correlation function \(C(k_1, k_2)\) first growing with \(m_\perp\) (and, therefore, \(k_\perp\)) as \(m_\perp^4\) for \(m_\perp \ll \Lambda\), and then decreasing as \(1/m_\perp^4\) for \(m_\perp \ll \Lambda\). Similar non-monotonic behavior has been observed for “ridge” correlation experimentally [33–36]. While in CGC-based approaches [37–42] the maximum of the correlation function is given by the saturation scale \(Q_s\), and happens at \(k_\perp \approx Q_s\), in our AdS/CFT case the maximum appears to be related to the inverse size of the produced bound state and its mass, such that it takes place at \(k_\perp \approx \sqrt{\Lambda^2 - m^2}\). At this point it is not clear though whether such conclusion is a physical prediction or an artifact of the perturbative solution of the problem in the AdS space.

In order to make a more detailed comparison with experiment one needs to improve on our AdS/CFT approach both by calculating higher-order corrections in \(\mu_1\) and \(\mu_2\), and, possibly, by
implementing non-conformal QCD features, such as confinement, along the lines of the AdS/QCD models [66, 69–77]. The latter modification would certainly change our glueball wave functions in the bulk, modifying the Bessel functions in Eqs. (4.18) and (4.19). However, while the use of AdS/QCD geometry may affect the $m_1$-dependence of the correlation function (1.28), one may see from Eqs. (4.18) and (4.19) that such modification would not affect our main conclusion about the rapidity-dependence of the correlations shown in Eq. (4.31). The leading large-rapidity asymptotics of the correlation function (4.31) results from the second term on the right-hand-side of Eq. (4.19); modifying the glueball wave function would only change the coefficient in front of the rapidity-dependent part. Since the growth of correlations with rapidity does not reproduce experimental data [33–36], our conclusion is that the inclusion of higher-order corrections in $\mu_1$ and $\mu_2$ is the only possibility for AdS/CFT (or AdS/QCD) calculations to get in line with the data.

### 4.2 Energy-momentum tensor correlator

We have shown that there are long-range rapidity correlations in the glueball operator of Eq. (2.12) in the strong-coupling heavy ion collisions. At the same time we would like to extend this statement to correlations of other operators. Energy-momentum tensor is a natural next candidate. Indeed the glueball operator (2.12) is a part of the energy-momentum tensor: hence correlations in $J(x)$ probably imply correlations in $\langle T_{\mu\nu}(x) T_{\mu\nu}(y)\rangle$ as well. To show this is true we will present an argument below, largely following [78, 79].

Consider a field theory whose dual holographic description is given by the metric of the general form

$$ds^2 = g^{(0)}_{MN} dx^M dx^N = f(x^+, x^-, z) dx_\perp^2 + g_{\mu\nu}(x^+, x^-, z) d\xi^\mu d\xi^\nu ,$$  

(4.33)

where $x = (x^1, x^2)$, $dx_\perp^2 = (dx^1)^2 + (dx^2)^2$, and $\xi^\mu = (x^+, x^-, z)$. Now, consider small perturbations around the metric independent of $x^1, x^2$, $g_{MN} = g^{(0)}_{MN} + h_{MN}(x^+, x^-, z)$. We will work in the $h_{Mz} = 0$ gauge. The metric (1.33) has a rotational $O(2)$ symmetry in the transverse plane. Under the transverse rotations one may naively expect $\{h_{11}, h_{12}, h_{22}\}$ components to transform as tensors, $\{h_{01}, h_{31}, h_{02}, h_{32}\}$ components to transform as vectors, and $\{h_{00}, h_{03}, h_{33}\}$ components to be scalars under rotations. However, rewriting the transverse part of the metric as

$$\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} (h_{11} + h_{22})/2 & 0 \\ 0 & (h_{11} + h_{22})/2 \end{pmatrix} + \begin{pmatrix} (h_{11} - h_{22})/2 & h_{12} \\ h_{21} & -(h_{11} - h_{22})/2 \end{pmatrix}$$  

(4.34)

we see that $h_{11} + h_{22}$ is also invariant under $O(2)$ transverse plane rotations. Hence the final classification of the metric components under $O(2)$ rotations is: $\{h_{11} - h_{22}, h_{12}\}$ are in the tensor representation, $\{h_{01}, h_{31}, h_{02}, h_{32}\}$ are vectors, and $\{h_{00}, h_{03}, h_{33}, h_{11} + h_{22}\}$ are scalars [78, 79].

Using the above classification we see that we can assume that the only non-vanishing component of $h_{MN}$ is $h_{12} = h_{21} = h_{12}(x^+, x^-, z)$. It is in the tensor representation and, as can be seen

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6As the integrand in that term is positive-definite for any glueball wave function, the coefficient can not vanish.
with the help of Eq. (4.34), by rotating in the transverse plane we can always find a coordinate system in which $h_{11} - h_{22} = 0$ and $h_{12} = h_{21}$ remains the only non-zero metric component in the tensor representation. Since all other components of the metric are in other representations of the $O(2)$ symmetry group, they do not mix with $h_{12}$ in Einstein equations, and can be safely put to zero \[78, 79\].

Substituting the metric $g_{MN} = g^{(0)}_{MN} + h_{MN}(x^+, x^-, z)$ with $g^{(0)}_{MN}$ given by (4.33) into Einstein equations (2.3), and expanding the result to linear order in $h_{12}$ we get \[78, 79\]

$$\Box h_{12} - 2 \frac{\partial f}{f} \partial \mu h_{12} + 2 \frac{(\partial f)^2}{f^2} h_{12} - \frac{\Box f}{f} h_{12} = 0,$$

(4.35)

where

$$\Box = \frac{1}{\sqrt{-g}} \partial_M \left[ \sqrt{-g} g^{MN} \partial_N \ldots \right]$$

(4.36)

and $(\partial f)^2 = g^{MN} \partial_M f \partial_N f$. Changing the variable from $h_{12}$ to $h^1_{12} = h_{12}/f$, one can see that $h^1_{12}$ indeed satisfies the equation for a minimally coupled massless scalar \[78, 79\]:

$$\Box h^1_{12} = 0.$$

(4.37)

Therefore, since our metric (2.8) falls into the category of Eq. (4.33), the analysis of Sec 4.1 applies to the metric component $h^1_{12}$. Defining the retarded Green function for the $T^1_{2}$ components of the energy-momentum tensor (EMT) by

$$G^\text{EMT}_{R}(k_1, k_2) = -i \int d^4x_1 d^4x_2 e^{-i k_1 \cdot x_1 - i k_2 \cdot x_2} \theta(x^0_1 - x^0_2) \langle A_1, A_2 | [T^1_{2}(x_1), T^1_{2}(x_2)] | A_1, A_2 \rangle$$

(4.38)

we conclude that, similar to the glueball operator,

$$G^\text{EMT}_{R}(k_1, k_2)|_{\Delta y \gg 1} \sim \cosh(2 \Delta y).$$

(4.39)

Hence we have shown that the correlators of EMT operators exhibit the same long-range rapidity correlations as the glueball correlators. It is therefore very likely that such correlations are universal and are also present in correlators of other operators.

5. Estimate of the Two-Point Correlation Function at Late Times

Our conclusion about long-range rapidity correlations was derived using the metric (2.8) which is valid only at very early times after a shock wave collision. As discussed in the Introduction, we do not expect the interactions at later times to affect these correlations, since different-rapidity regions of the produced medium become causally disconnected at late times. To check that no long-range rapidity correlations can arise from the late-time dynamics one would have to calculate
the correlation function (3.7) in the full metric produced in a shock wave collision including all powers of $\mu_1$ and $\mu_2$. Since no such analytical solution exists, instead we will use the metric dual to Bjorken hydrodynamics [53] constructed in [8]. One has to be careful in interpreting the result we obtain in this Section: Bjorken hydrodynamics [53] is rapidity-independent, while there are reasons to believe that the medium produced in a shock wave collision would exhibit rapidity dependence, as indicated by perturbative solutions of Einstein equations done in [14,16,17]. Nonetheless, we expect that our calculation below would be a good initial estimate of the late-time rapidity correlations.

The dual geometry corresponding to the perfect fluid was obtained by Janik and Peschanski in [8]. It can be written as

$$ds^2 = \ell^2 \left\{ -\frac{1}{z^2} \frac{(1 - z^4/z_h^4(\tau))^2}{1 + z^4/z_h^4(\tau)} d\tau^2 + \frac{1 + z^4/z_h^4(\tau)}{z^2} (\tau^2 d\eta^2 + dx_\perp^2) + \frac{dz^2}{z^2} \right\} ,$$

(5.1)

where $\tau = \sqrt{2 x^+ x^-}$ is proper time, $\eta = \frac{1}{2} \ln(x^+/x^-)$ is space-time rapidity, and $z_h(\tau) = \left(\frac{3}{E_0}\right)^{1/4} \tau^{1/3}$ (with $E_0$ some dimensionful quantity) determines the position of the dynamical horizon in $\text{AdS}_5$ such that the Hawking temperature is

$$T(\tau) = \frac{\sqrt{2}}{\pi z_h(\tau)} = \frac{\sqrt{2}}{\pi} \left(\frac{E_0}{3}\right)^{1/4} \tau^{-1/3} .$$

(5.2)

Unfortunately finding the glueball correlation function in Bjorken hydrodynamic state is equivalent to finding boundary-to-boundary scalar propagator in the background of the Janik-Peschanski metric (5.1), which is a daunting task: such propagator has not yet been found even for the static $\text{AdS}$ Schwarzschild black hole metric. Instead, to estimate the correlations we will perform a perturbative calculation.

At late times, when $\tau \gg E_0^{-3/8}$, assuming either that $z$ is fixed or is bounded from the above (by let us say an infrared (IR) cutoff coming from the definition of the glueball wave function), we can consider the ratio $u(\tau) \equiv z/z_h(\tau) \ll 1$ to be a small quantity. If so, we can expand the EOM for the scalar field (2.14) up to $O(u^4)$ obtaining

$$\Box_5 \phi(\tau, \eta, x_\perp, z) + u^4 \left[ 4 \partial_\tau^2 - \Box_4 \right] \phi(\tau, \eta, x_\perp, z) = 0 ,$$

(5.3)

$$\Box_5 \phi \equiv -z^3 \partial_z \left( \frac{1}{z^3} \partial_z \phi \right) + \Box_4 \phi , \quad \Box_4 \phi \equiv \frac{1}{\tau} \partial_\tau \left( \tau \partial_\tau \phi \right) - \frac{1}{\tau^2} \partial_\eta^2 \phi - \nabla_\perp^2 \phi = (2 \partial_+ \partial_- - \nabla_\perp^2) \phi .$$

Expanding the scalar field in the powers of $u$ we write

$$\phi = \phi_0 + \phi_1 + \ldots$$

(5.4)

where $\phi_0 \sim O(u^0)$ and $\phi_1 \sim O(u^4)$. Substituting this back into Eq. (5.3), we get

$$\Box_5 \phi_0 = 0 , \quad \Box_5 \phi_1 = -\frac{\mathcal{E}_0}{3} \frac{z^4}{\tau^{4/3}} \left[ 4 \partial_\tau^2 - \Box_4 \right] \phi_0 .$$

(5.5)
The solution for \( \phi_0 \) was found above and is given in Eq. (4.9). We write the solution for \( \phi_1 \) as

\[
\phi_1 = -\frac{\mathcal{E}_0}{3} \frac{1}{\Box^5} \frac{z^4}{\tau^{4/3}} \left[ 4 \partial^2_{\tau} - \Box_{4} \right] \phi_0 \approx \frac{\mathcal{E}_0}{3} \frac{1}{\Box^5} \frac{z^4}{\tau^{4/3}} \Box_{4} \phi_0
\]

(5.6)

where in the last step we neglected \( \partial^2_{\tau} \), since a derivative like this generates \( O(1/\tau^2) \) corrections (at fixed \( u \)), which were neglected in constructing the original metric (5.1) and are thus outside of the precision of our approximation. We are now ready to calculate the retarded Green function. Using Eq. (5.6) in Eqs. (2.19), (2.15), and (2.20), and employing Eq. (4.12) yields

\[
G_{Bj}^{Bj}(k_1, k_2) \big|_{O(1/z^4_\Lambda)} = -\frac{N_\mathcal{E}_0 m^6}{24} \left( k_1 + k_2 \right) \int_{z=0}^{\infty} \frac{dz}{z^5} K_2 \left( z \sqrt{k_{1}^2} \right) K_2 \left( z \sqrt{k_2^2} \right) \times \int_{0}^{\infty} dx^+ dx^- e^{i x^+ (k_1^- + k_2^-) + i x^- (k_1^+ + k_2^+)} \frac{1}{\tau^{4/3}}
\]

(5.7)

where we have replaced \( k_1^2 \) and \( k_2^2 \) with \( -m^2 \) everywhere except for the arguments of the Bessel functions. The integrals over \( x^+ \) and \( x^- \) in Eq. (5.7) run from 0 to \( \infty \) since the matter only exists in the forward light-cone. (On top of that the metric (5.1) is valid at late times only, for \( u \ll 1 \), such that the actual \( x^+ \) and \( x^- \) integration region should be even more restricted, possibly suppressing the correlations we are about to obtain even more.)

Just like in the case of the early times considered in Sec. 4.1, the integral over \( z \) in Eq. (5.7) is divergent for time-like momenta \( k_1 \) and \( k_2 \). Similar to what we did in Sec. 4.1, we recognize the Bessel functions in Eq. (5.7) as the glueball wave functions in the bulk, which need to be modified to reflect the finite size of glueballs, which do not exist in \( \mathcal{N} = 4 \) SYM theory. Replacing \( K_2(z \sqrt{k_{1,2}^2}) \rightarrow K_2(z \Lambda) \) in Eq. (5.7) and integrating over \( z \) yields

\[
G_{Bj}^{Bj}(k_1, k_2) \big|_{O(1/z^4_\Lambda)} = -\frac{4 N_\mathcal{E}_0 m^6}{15 \Lambda^6} \delta^2(k_1 + k_2) \int_{0}^{\infty} dx^+ dx^- e^{i x^+ (k_1^- + k_2^-) + i x^- (k_1^+ + k_2^+)} \frac{1}{\tau^{4/3}}.
\]

(5.8)

Evaluating the integrals left in Eq. (5.8),

\[
\int_{0}^{\infty} dx^+ dx^- e^{i x^+ (k_1^- + k_2^-) + i x^- (k_1^+ + k_2^+)} \frac{1}{(2 x^+ x^-)^{2/3}} = \frac{N}{(k_1^+ + k_2^-)^{1/3}(k_1^- + k_2^+)^{1/3}},
\]

(5.9)

where

\[
N = \frac{\Gamma^2 (1/3) e^{i \pi/3}}{2^{2/3}},
\]

(5.10)

we obtain

\[
G_{Bj}^{Bj}(k_1, k_2) \big|_{O(1/z^4_\Lambda)} = -\frac{4 N_\mathcal{E}_0 m^6}{15 \Lambda^6} \delta^2(k_1 + k_2) \frac{N}{m_{\perp}^{2/3} (1 + \cosh \Delta y)^{1/3}}.
\]

(5.11)
The corresponding two-glueball correlation function scales as

$$C^{Bj}(k_1, k_2)\big|_{\Delta y \gg 1} \sim \frac{1}{m^4 \cosh^2 \Delta y^{2/3}}.$$  (5.12)

We conclude that rapidity correlations coming from the AdS dual of Bjorken hydrodynamics are suppressed at large rapidity interval, at least in the perturbative estimate we have performed. This result appears to agree with the causality argument [37,38] making appearance of long-range rapidity correlations unlikely at late times. Moreover, the locality of $C^{Bj}$ in rapidity suggests that late-time dynamics is not likely to affect long-range rapidity correlations coming from the early stages of the collision: hydrodynamic evolution can not “wash out” such long-range rapidity correlations.

Note that the complete momentum space two-glueball correlation function receives contributions from all regions of coordinate space, i.e., from all $x_1$ and $x_2$. In Sec. 4 we have calculated the contribution arising from early proper times, while here we have estimated the late-time contribution. One may expect that in the complete result the two contributions coming from different integration regions would simply add together: in such case clearly the early-time contribution in Eq. (4.31) would dominate for large rapidity intervals, leading to long-range rapidity correlations arising in the collision.

6. Summary

Let us summarize by first restating that we have found long-range rapidity correlations in the initial stages of strongly-coupled heavy ion collisions as described by AdS/CFT correspondence. We expect that due to causality the correlations would survive the late-time evolution of the produced medium, though one needs to have a full solution of the shock wave collision problem to be able to verify this assertion. The long-range rapidity correlations may be relevant for the description of the “ridge” correlation observed in heavy ion and proton-proton collisions [33–36]. Indeed “ridge” correlation is characterized not only by the long-range rapidity correlation, but also by a narrow zero-angle azimuthal correlation between the triggered and associated particles. As was suggested in [37,38] such azimuthal correlation may be due to the radial flow of the produced medium. The advantage of the AdS/CFT approach to the problem is that the full solution to the problem for a collision of two shock waves with some non-trivial transverse profiles would have radial flow included in the evolution of the dual metric, and would be able to demonstrate whether radial flow is sufficient to lead to the “ridge” phenomenon. Indeed such calculation appears to be prohibitively complicated to do analytically at the moment.

The correlations we found grow very fast with rapidity interval, as one can see from Eq. (3.8), while the experimentally observed correlation [33–36] is at most flat in rapidity. This result may lead to the conclusion that the initial stages of heavy ion collisions can not be strongly-coupled, since this contradicts existing observations. At the same time, it may happen that higher-order
corrections in $\mu_1$ and $\mu_2$ would affect this rapidity dependence, flattening the resulting distribution. On yet another hand, such higher-order corrections become important at later times, and eventually causality may prohibit further late-time modification to the long-range rapidity correlations. More work is needed to clarify this important question about the rapidity-shape of the correlations coming from the solution of the full problem in AdS.

Assuming that the issue of rapidity shape would be resolved, we would also like to point out that $k_T$-dependence of obtained correlator (4.28) closely resembles that reported in the data [33–36]: it starts out growing with $k_T$ at low-$k_T$, and, at higher $k_T$, it falls off with $k_T$. The location of the maximum of the correlator in our case was determined by the mass and size of the produced particles, and was thus energy-independent. It is possible that the solution of the full problem, resumming all powers of $\mu_1$ and $\mu_2$ would lead to the maximum of the correlation function given by $\mu_{1,2}^{1/3}$, which in turn would be inversely proportional to the thermalization time [14, 22], thus providing an independent way of measuring this quantity. Again more research is needed to explore this possibility.

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