Theoretical and Numerical Analysis of the Fatigue Strength of Mechanical Material and Part under Uniaxial Stress

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Abstract. To characterize the fatigue strength of mechanical material and part, a methodology combining the fatigue damage theory with the numerical simulation is proposed under three types of uniaxial loads. The equal-life fatigue curve of a shaft is used for the theoretical and numerical analysis of fatigue damage. Its theoretical fatigue strengths under different loadings are gained through theory equations. Utilizing the mean stress correction method, the stress distribution and fatigue life of the shaft are obtained through the collaborative simulation of ANSYS and nCode software. Consequently, the resultant fatigue lives are all within the value band of basic cycle number as demanded in practice, implying the correction and validation of theory analysis. The proposed strategy provides a new pathway and perspective for the analysis of fatigue damage.

Keywords. Fatigue strength and life, uniaxial stress, mechanical part and material, nCode.

1. Introduction

The structural failure of part and component occurs under the cyclical changeable loading even if its stress amplitude is below the fatigue strength of material, which is called the fatigue phenomenon [1, 2]. In general, the fatigue refers to metal fatigue and the stress within the shaft, gear, bearing, blade, spring, etc., is cyclically changeable, and the stress with cyclically changeable direction and amplitude is called alternating stress [3]. Under the alternating stress, initiation of new crack and propagation of old cracks often take place within mechanical parts, leading to structural failures [4, 5]. The structure suffering fatigue damage reaches a collapse without an obvious deformation [6, 7].

In order to illustrate the application of fatigue damage theory to durability design engineering, the fatigue characteristics of mechanical part and material is investigated under three types of uniaxial loadings, utilizing the theoretical analysis and the numerical simulation. The equal-life fatigue curve for material is described to lay the theory foundation of fatigue analysis. The theoretical calculation of fatigue strength under different conditions is then performed. Based on the mean stress correction method, the numerical analysis of stress distribution and predication of fatigue life are conducted with ANSYS and nCode software. Consequently, the simulated fatigue life conforms to the practical engineering criterion, verifying the theory analysis and deduction. The proposed method shields a new light on the durability design of mechanical part and the application of fatigue damage theory.
2. Analysis of Fatigue Strength of Mechanical Part

The fatigue failure of material correlates with the maximum stress ($\sigma_{\text{max}}$), cyclic number of stress ($N$) and the stress ratio ($r$) ($r=\sigma_{\text{min}}/\sigma_{\text{max}}$, $\sigma_{\text{min}}$ is the minimum stress). The utmost value of applied stress is called the fatigue strength of material ($\sigma_{\text{lim}}$) under a certain $r$. Furthermore, the stress is called symmetric circulating stress ($\sigma_1$) when the $r$ is equal to minus one, namely the $\sigma_{\text{max}}$ and $\sigma_{\text{min}}$ have the same amplitudes but opposite directions. And the stress is called fluctuating circulating stress ($\sigma_0$) when the $r$ is equal to zero, namely the $\sigma_{\text{min}}$ is equal to zero [8].

Two types of fatigue curves are gained to illustrate the fatigue damage: i) the fatigue limit ($\sigma_{\text{lim}}$) versus number of cycle ($N$) curve, namely $\sigma$-$N$ curve; ii) the stress amplitude ($\sigma_a$) ($\sigma_a$, half of the difference of maximum and minimum stresses) of $\sigma_{\text{lim}}$ versus mean stress ($\sigma_m$) ($\sigma_m$, half of the sum of maximum and minimum stresses) of $\sigma_{\text{lim}}$ curve, namely equal-life curve [9, 10].

2.1. Stress Ratio of the Changeable Stress Being a Constant

When the $r$ of changeable stress is a constant, such as $r=$constant1, an ultimate stress needs to be figured out as the fatigue limit strength of part. A definition is introduced as follows:

$$K = \frac{\sigma_a}{\sigma_m} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{\sigma_{\text{max}} + \sigma_{\text{min}}} = \frac{1 - r}{1 + r}$$ (1)

Accordingly, the formulas can be deduced as follows:

$$\sigma_{\text{max}} = \sigma_a + \sigma_m = (1 + K)\sigma_m = \frac{2}{1 + r}\sigma_m$$ (2)

$$\sigma_{\text{min}} = \sigma_m - \sigma_a = (1 - K)\sigma_m = \frac{2r}{1 + r}\sigma_m$$ (3)

The $\sigma_{\text{max}}$ and $\sigma_{\text{min}}$ are both of positive proportion to the $\sigma_m$ and the described curves of $\sigma_{\text{max}}$, $\sigma_{\text{min}}$, $\sigma_m$ can be drawn as the red, blue and green lines in figure 1(a), respectively. Accordingly, the points A$_1$(150, 200), B$_1$(150, 150) and C$_1$(150, 100) have the same abscissas and their ordinates confirm that the $\sigma_m$ is half of the sum of the $\sigma_{\text{max}}$ and $\sigma_{\text{min}}$, indicating the $r$ is a constant. The equal-life fatigue curve of mechanical part is depicted as the figure 1(b), each point on HP$_{00}$ line has the same stress ratio and cyclic features, and its slope can be calculated by equation (1).

![Figure 1](image)

**Figure 1.** Stress state and equal-life fatigue curve when $r$ is a constant. a. The stress state curve with a certain $r$. b. The equal-life fatigue curve with a certain $r$.

The coordinate of point $G(\sigma_{\text{mg}}, \sigma_{ag})$ can be described as follows:

$$G(\sigma_{\text{mg}}, \sigma_{ag}) = G\left(\frac{K_\sigma \sigma_s - \sigma_{-1}}{K_\sigma - \phi_\sigma}, \frac{\sigma_{-1} - \phi_\sigma \sigma_s}{K_\sigma - \phi_\sigma}\right)$$ (4)
\[ K_{OG} = \frac{\sigma_{AG}}{\sigma_{mG}} = \frac{\sigma_m - \varphi \sigma_s}{K_\sigma \sigma_s - \sigma_m - 1} \] (5)

where the \( K_\sigma \) is comprehensive coefficient of material, \( \varphi \) is the material factor, and \( \sigma_s \) is the yield limit strength of material. The stresses on the line of AG are seen as fatigue limit strengths as follows:

\[ K_{OG} \leq \frac{1 - r}{1 + r} < \infty \Rightarrow -1 < r \leq r_1 = \frac{1 - K_{OG}}{1 + K_{OG}} \] (6)

Furthermore, the coordinate of \( P_{00} \) can be gained through the following correlation:

\[ k_1 = \frac{\sigma_{a0}}{\sigma_{m0}} = \frac{\sigma_{ap00}}{\sigma_{mp00}} \Rightarrow P_{00}(\sigma_{mP_{00}}, \sigma_{aP_{00}}) = P_{00}\left(\frac{\sigma_m - 1}{k_1 K_\sigma + \varphi \sigma}, k_1 K_\sigma + \varphi \sigma_1\right) \] (7)

The fatigue limit strengths are equivalent to the \( \sigma_s \) as follows:

\[ 0 \leq \frac{1 - r}{1 + r} < K_{OG} \Rightarrow r_1 = \frac{1 - K_{OG}}{1 + K_{OG}} < r \leq 1 \] (8)

Therefore, the fatigue limit strength of part can be calculated through the line AD or line CG based on the value of \( r \) when being a constant.

2.2. Mean Stress of the Changeable Stress Being a Constant

When the \( \sigma_m \) is a constant, such as \( \sigma_m = \text{constant}_2 \), the curves of \( \sigma_{\text{max}} \) and \( \sigma_{\text{min}} \) can be described as the red and blue lines in figure 2(a) when the \( \sigma_m \) is chosen to be 50 MPa. The points \( A_2 (-0.52, 208) \), \( B_1 (-0.52, 50) \), and \( C_1 (-0.52, -108) \) have the same abscissas.

![Figure 2](image-url)

**Figure 2.** Stress state and equal-life fatigue curve when \( \sigma_m \) is a constant. a. The stress state curve with a certain \( \sigma_m \). b. The equal-life fatigue curve with a certain \( \sigma_m \).

The line \( H P'_00 \) (MP'\(_{11} \)) can be figured out in figure 2(b) by drawing a line through point \( P_0(P_1) \), parallel to Y-axis, and intersecting broken line AGC at point \( P'_00(P'_{11}) \). The stresses on AG can be regarded as fatigue limit strengths when the \( \sigma_m \) is less than \( \sigma_{mG} \) but more than or equal to zero, while the ones on GC can be seen as fatigue limit strengths and the fatigue limit strengths are all equivalent to yield strength of material.

With the abscissas of \( P'_00 \) and \( P_0(\sigma_{m0}, \sigma_{a0}) \), the coordinate of \( P'_00 \) can be obtained as follows:

\[ P'_00(\sigma_{mP_{00}}, \sigma_{aP_{00}}) = P_{00}\left(\sigma_{m0}, \frac{\sigma_m - \varphi \sigma \sigma_{m0}}{K_\sigma}\right) \] (9)

Therefore, when \( \sigma_m \) is a constant, the fatigue limit strength of part can be calculated through the AD or CG lines based on the difference between \( \sigma_m \) and \( \sigma_{mG} \).
2.3. Minimum Stress of the Changeable Stress Being a Constant

When the $\sigma_{\text{min}}$ of changeable stress is a constant, such as $\sigma_{\text{min}} = \text{constant}^3$, such as $\sigma_{\text{min}} = C$:

$$\sigma_{\text{min}} = \sigma_m - \sigma_a = C$$

(10)

$$\sigma_{\text{max}} = \sigma_m + \sigma_a = 2\sigma_m - C$$

(11)

The $\sigma_{\text{max}}$ and $\sigma_m$ can be illustrated as the red and green lines in figure 3(a) respectively when the $\sigma_{\text{max}}$ is 20 MPa. The points of $A_3(150, 280)$, $B_3(150, 150)$ and $C_3(150, 20)$ have the same abscissas. The fatigue analysis is only to discuss the case in which the $\sigma_m$ is larger than the $\sigma_{\text{min}}$.

According to equation (10), the HP"$_{00}$MP"$_{11}$ can be depicted in figure 3(b) by passing through point $P_0(P_1)$ and reclining to $45^\circ$ away from X-axial as well as intersecting the line of AGC at point $P"_{00}(P"_{11})$. The stresses on AG can be seen as the fatigue limit strengths when the $\sigma_m$ is less than $\sigma_{mN}$ but more than or equal to zero.

![Figure 3](image)

**Figure 3.** Stress state and equal-life fatigue curve when $\sigma_{\text{min}}$ is a constant. a. The stress state curve with a certain $\sigma_{\text{min}}$. b. The equal-life fatigue curve with a certain $\sigma_{\text{min}}$.

Furthermore, based on the coordinate of G ($\sigma_{mG}$, $\sigma_{aG}$) and equivalent lengths of NB and BG lines, the ordinate of N can be given as follows:

$$\sigma_{mN} = \sigma_{mG} - \sigma_{aG} = \frac{(K_\sigma + \varphi_\sigma)\sigma_s - 2\sigma_{s1}}{K_\sigma - \varphi_\sigma}$$

(12)

Therefore, when $\sigma_{\text{min}}$ is a constant, the fatigue limit strength of part can be calculated through the AD or CG line based on the comparison between $\sigma_m$ and $\sigma_{mN}$.

3. Analysis of Fatigue Strength of Mechanical Part

The mechanical specimen adopted is a shaft of SAE1045 used for an extensive round-robin testing program. And its yield stress is 355 MPa and ultimate stress is 600 MPa. In addition, its 3D structure and 2D geometry are shown as figure 4(a) and figure 4(b), respectively.

Referring to the manuals, the $k_\sigma$, $\varepsilon_\sigma$, $\beta_\sigma$ and $\beta_\varphi$ are equal to 1.63, 0.79, 0.89 and 1.51, respectively. The $K_\sigma$ can be then calculated to 1.45 and the $P_0(90, 120)$ is found. The fatigue limit strength can be calculated in table 1, laying the foundations of following numerical simulation.
**Figure 4.** Three-dimensional structure and two-dimensional geometry of the shaft. a. The three-dimensional structure of the shaft. b. The two-dimensional geometry of the shaft.

| Table 1. Theoretically calculated results derived from the whole theory equations above. |
|---|---|---|---|---|---|---|---|---|---|---|
| Items | $K_\sigma$ | $\varphi_\sigma$ | $K_{OG}$ | $r_1$ | $\sigma_mG$ | $\sigma_aG$ | $C$ | $\sigma_{mP00}$ | $\sigma_{aP00}$ | ... | $\sigma_{mP0}$ | $\sigma_{aP0}$ | $\sigma_{mP'}$ | $\sigma_{aP'}$ |
| Value | 1.45 | 0.4 | 0.8 | 1.33 | 197.58 | 157.42 | -30 | 131.71 | 175.61 | ... | 90 | 187.13 | 142.6 | 172.6 | 40.16 |

**4. Fatigue Simulation Analysis of Mechanical Part Specimen**

**4.1. Minimum Stress of the Changeable Stress Being a Constant**

The model of shaft, formed on the platform of Solidworks software, is transformed into another graphic format as a FE model in the environment of ANSYS Workbench software for the finite element analysis.

The local refinement meshing is performed on the surface of neck and two transition fillets as shown in figure 5(a), aiming to gain a better meshing quality and then a more precise analytical result. The resultant stress $\sigma_m$ and $\sigma_a$ are obtained as 131.71 MPa and 175.61 MPa in the figure 5(b) and figure 5(c) respectively as the fatigue limit stress of point $P_{00}$.

| Table 2. Three types of the applied loadings and the resultant stresses. |
|---|---|---|
| Items | The $r$ is a constant, $P_{00}$ | The $\sigma_m$ is a constant, $P'_0$ | The $\sigma_{m\text{in}}$ is a constant, $P''_{00}$ |
| Stress (MPa) | $\sigma_{mP00}$ | $\sigma_{mP'0}$ | $\sigma_{mP''0}$ |
| Force (N) | $\sigma_{aP00}$ | $\sigma_{aP'0}$ | $\sigma_{aP''0}$ |
| Value | 131.71 | 90 | 142.6 |
| | 175.61 | 187.13 | 172.6 |
| | 2654.15 | 5518.49 | 5090 |
| 3884.27 | 4205.3 |
4.2. The Fatigue Analysis of Mechanical Specimen

The essence of methodology is the linear superposition of damages caused by the $\sigma_a$ and $\sigma_m$. More specifically, two stress distributions are simultaneously adopted as FE model inputs: the one deriving from $\sigma_m$ while the other from $\sigma_a$ [11, 12].

![Figure 6. Fatigue life predictions of the $P_{00}$, $P'_{00}$, $P''_{00}$ and G using the finite element method. a. b. c. The fatigue lives under three loadings respectively. d. The fatigue life in the static stress state.](image)

The chosen mean stress correction method is the Gerber Correction Methodology appropriate for the plastic material [13], e.g. SAE1045 steel, and the Stress-Life approach is selected as the solving kernel suitable for high cycle fatigue strength (HCFS). Consequently, the fatigue results are the predicted fatigue lives of the $P_{00}$, $P'_{00}$, $P''_{00}$ and G as shown in figure 6(a)-6(d), respectively. In order to explicitly describe the results of fatigue analysis, the basic cycle number ($N_0$) is introduced and the $N_0$ is to be $(1~10) \times 10^6$ for the cast and steel.

Not only does the figure 6 illustrate the region distribution with a finite fatigue life, but also it quantitatively shows the exact value of it so that comparisons among the fatigue lives of different districts can be made with more persuasive evidences. The resultant fatigue lives of different stress points are collected in table 3.

![Table 3. Fatigue life predictions of the fatigue limit stress points.](table)

The basic cycle number $N_0$ is $(1~10) \times 10^6$, and the fatigue lives of $P_{00}$, $P'_{00}$, $P''_{00}$ and G is $4.150 \times 10^6$, $3.023 \times 10^6$, $4.428 \times 10^6$ and $5.374 \times 10^6$, respectively. Conspicuously, all of the fatigue lives are within the value band of $N_0$, suggesting that the stresses may be regarded as the fatigue limit strengths and the theoretically calculated mean stresses and stress amplitudes on equal-life fatigue curves could be regarded as the correct limit stress of mechanical specimens. In other words, the methodology in this work is appropriate for the analysis and calculation of fatigue strength of mechanical material and part.
5. Conclusions
To characterize the fatigue damage theory, a methodology combining the theoretical analysis with the numerical simulation is put forward under three kinds of loadings. The validated parameters from the theoretical deduction are used as the inputs of numerical simulation. The obtained stress distribution illustrates the fatigue damage. Through the stress cloud chart, the located position with maximum damage is observed on the shaft due to stress concentration. The shortest fatigue life occurs on the place where the stress is the maximum. Meanwhile, the predicted fatigue lives all fall into the value band of basic cycle number, meeting the demand in engineering practice. Those results indicate that the calculated mean stress and stress amplitude are suitably regarded as the fatigue limit strength and then validate the method of fatigue analysis, shedding a new light on the fatigue damage analysis.

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References
[1] Schijve J 2003 Fatigue of structures and materials in the 20th century and the state of the art Int. J. Fatigue 25 679-702
[2] Googarchin H, Sharifi S, Forouzesh F, Hosseinpour G, Etesami S and Zade S 2017 Comparative study on the fatigue criteria for the prediction of failure in engine structure Eng. Fail. Anal. 79 714-25
[3] Puigoriol-Forcada J, Alsina A, Salazar-Martín A, Gomez-Gras G and Pérez M 2018 Flexural fatigue properties of polycarbonate fused-deposition modelling specimens Mater. Des. 155 414-21
[4] Shiozawa K, Morii Y, Nishino S and Lu L 2006 Subsurface crack initiation and propagation mechanism in high-strength steel in a very high cycle fatigue regime Int. J. Fatigue 28 1521-32
[5] Yuan R, Li H, Huang H, Zhu S and Gao H 2015 A nonlinear fatigue damage accumulation model considering strength degradation and its applications to fatigue reliability analysis Int. J. Damage Mech. 24 646-62
[6] Wang X, Liu J, Jin T, Sun X, Zhou Y, Hu Z, Do J, Choi B, Kim I and Jo C 2015 Deformation mechanisms of a nickel-based single-crystal superalloy during low-cycle fatigue at different temperatures Scripta. Mater. 99 57-60
[7] Chen X and Zhao S 2005 Evaluation of fatigue damage at welded tube joint under cyclic pressure using surface hardness measurement Eng. Fail. Anal. 12 616-22
[8] Hsu T and Wang Z 2014 Cyclic stress–strain response and microstructure evolution of polycrystalline Cu under pure compressive cyclic loading condition Mater. Sci. Eng. A-Struct. 615 302-12
[9] Kassapoglou C 2007 Fatigue life prediction of composite structures under constant amplitude loading J. Compos. Mater. 41 2737-54
[10] Zhu S, Lei Q and Wang Q 2017 Mean stress and ratcheting corrections in fatigue life prediction of metals Fatigue Fract. Eng. Mater. Struct. 40 1343-54
[11] Liang J, Nie X, Masud M, Li J and Mo Y A 2017 study on the simulation method for fatigue damage behavior of reinforced concrete structures Eng. Struct. 150 25-38
[12] Kumar J, Rao A and Kumar V 2015 Simulation of elevated temperature fatigue damage evolution using the finite element method for near alpha titanium alloy Fatigue Fract. Eng. Mater. Struct. 38 466-74
[13] Ince A 2017 A mean stress correction model for tensile and compressive mean stress fatigue loadings Fatigue Fract. Eng. Mater. Struct. 40 939-48