Modelling of the education quality of a high schools in Sumenep Regency using spatial structural equation modelling

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Abstract. In some cases, education research often involves the latent variables that have a causal relationship as well as a spatial effect. Therefore, it requires a statistical analysis technique called spatial structural equation modelling (spatial SEM). In this research, a spatial SEM was developed to model the quality of education in high schools in Sumenep Regency. This model was improved after the evaluation of an outer and inner model of the model scheme centroid, factor and path since some indicators were not valid. The path scheme model showed better results compared to the other schemes since all of its indicators were valid and its value of R-square increased. Furthermore, only the model of path scheme was tested for spatial effects. The result of the identification test of spatial effects on the inner model using a robust Lagrange multiplier test (using queen contiguity) showed that the education quality model leads to a spatial autoregressive model (SAR in SEM) with a significance level α of 5%, while the model of school infrastructure has no significant spatial effects. The improved model of SAR in SEM, the R² value obtained was 47.33%, so that it is clear that data variation can be explained by the model of SAR in SEM for the quality of education in high schools. Keywords: Spatial SEM, SAR in SEM, Queen Contiguity

1. Introduction

Educational research is included in the category of social research, which often involves the latent variables. One of the statistical analysis technique that has the ability to analyze the pattern of relationship between latent variables as well as latent variables with its indicator is structural equation modeling (SEM). The analysis of SEM aims to estimate the causal relationship among latent variables illustrated in the path diagram and also to empirically confirm the suitability of the construct seen according to the indicator that conceptualized as measurable variables of the construct [5].

The educational research also often involves spatial dependence. Observation at a particular location is influenced by observations in other locations as stated in the Law of Tobler I proposed by W Tobler [1], which reads: "Everything is related to everything else, but near thing are more related than a distant thing". Some estimation procedures of the linear model with spatial dependence have been developed. An iterative two-stage procedure to maximize the log-likelihood of a spatial error model was proposed by [1]. [6], which have combined spatial information in regression relation showed spatial autocorrelation.
However, some cases of educational research involving latent variable that has a causal relationship as well as a spatial influence. To overcome these problems, spatial element needs to be included in the model SEM. A common spatial factor model with parameter estimation using Bayesian approach was proposed by [11]. This research was applied to the cases of death due to cancer. The study was followed by [7]. They proposed a common spatial structural equation model, namely by combining the spatial factor model belongs [11] and the Bayesian approach. The Bayesian approach is used to measure latent variable and for modeling the regression relationship between latent variables. A hierarchy model for factor analysis of multivariate data with a single spatial correlation for some social indicators and using the Bayesian approach was described by [4]. A spatial SEM with nonlinear constructs effects and using spline regression was investigated by [2]. The spatial lag and spatial errors in the SEM using the estimation method of full information maximum likelihood (FIML) was examined by [8].

The aim of this work was to develop a modeling of high school education quality in Sumenep using a spatial structural equation modelling and the maximum likelihood estimation (MLE) approach. The latent variable estimated into the score of the latent variable using the iterative process performed by [10]. Thus, the score of the latent variable represents the latent variable. The latent variable in the inner model replaced by the score of the latent variable and given the spatial weight, that is the continuity queen. The continuity queen is the intersection of sides and angles. The weighted inner model would be estimated using MLE

2. Structural equation modelling based on variance

In general, methods of estimation in the SEM is MLE. The covariance-based SEM is based on a large sample size theory and it must fulfill the requirements of multivariate normal distribution. As an alternative solution to this problem, a variant-based SEM or partial least square (PLS) has been developed. The inner model in PLS describes relationships among latent variables. The model equation can be written as follows [10]:

\[ \eta = \beta^\circ \xi + \Gamma \zeta, \]

where \( \eta \) is the vector of the endogenous random variable, \( \xi \) is the vector of the exogenous random variable, \( \beta^\circ \) is the coefficient matrix showing the relationship influence the endogenous latent variable to the others and \( \Gamma \) is a coefficient matrix that shows the influence of the exogenous latent variable on the endogenous latent variable, whereas \( \zeta \) is the vector of random error, with an expected value of zero.

The outer models defined of how each block of indicators associated with the block's latent variable. The equation of outer model for the reflective indicator block such as simple regression equation as follows [10]:

\[ jk^\circ \lambda^\circ x^\circ = + \]

where \( jk^\circ \lambda^\circ \) is the loading coefficient of correlation between the \( j \)-th latent variable and the \( k \)-th indicator. Notation \( jk^\circ \varepsilon^\circ \) is an error of each measurement variable.

The most important part in SEM PLS is the weight estimated. In the process of weight estimation, there are three iterative procedures of the simple regression or the multiple regression by considering the relationship among the inner model, the outer model and the weight relations. The results of the iterative procedure are a set of stable weights used to calculate the value of the score of the latent variable, which is a linear combination of indicators [9].

The stages to estimate the score of the latent variable are as follows: estimation of the outer model, estimation of the inner model, updating weights of the outer model and checking the convergence. Component of score estimate for each of the latent variables obtained through two ways, namely the outside approximation and inside approximation. The outside approximation describes the aggregate weight of the indicator of the construct. The inside approximation describes the aggregate weight of the other component score that is related to construct in the theoretical model [3].

The iteration process of the inside approximation estimation calculates the relationship among the latent variables in the inner models. The results of these estimates is to obtain the initialization of each latent variable calculated as a weighted aggregate of the adjacent latent variables, namely...
\[ Z_j = \sum_{i,j,l} e_{ijl} Y_{ij} \], where \( e_{ijl} \) is the inner weight that can be selected from the three schemes, namely centroid, factor or path, where each scheme is defined by [10] as follows:

i. Inner weight (\( e_{ijl} \)) at the centroid scheme is the mark of the correlation between the \( l_i \) and \( l_j \)

ii. Inner weight (\( e_{ijl} \)) at the factor scheme is the correlation between the \( l_i \) and \( l_j \) consider not only the direction mark but also the strength in the inner model

iii. Inner weight (\( e_{ijl} \)) at the path scheme is the weighting of a latent variable’s neighbor depending on whether the variable neighbor is an antecedent or consequent of the latent variable that will be estimated. Path scheme is defined as follows: \( e_{ijl} = \text{cor}(l_i, l_j) \) if \( \xi_j \) is explained by \( \xi_i \) and \( l_j = \sum_i e_{jil} l_i \), where \( e_{ijl} \) is coefficient in the regression equation of \( l_i \) on \( l_j \).

3. Model of linear regression spatial

A spatial model using cross-sectional spatial data was developed by [1], which is generally expressed as follows:

\[ y^* = X^* \beta^* + \lambda^* W y^* + u^* \] (1)

where \( u^* = \rho^* M u^* + \varepsilon^* \), with \( \varepsilon^* \sim N(0, \sigma^2 I) \). \( y^* \) is the vector of the endogenous variable that has a spatial dependence, \( X^* \) is the matrix of the exogenous variable, \( \beta^* \) is the vector of the parameter at regression model, \( \lambda^* \) is the coefficient of a spatial autoregressive with \( |\lambda^*| < 1 \), \( \rho^* \) is the coefficient of a spatial error with \( |\rho^*| < 1 \), \( W \) and \( M \) are the matrix of a spatial weight with the diagonal elements are 0, \( u^* \) is the vector of a regression error which is assumed has the effect of region random and also error which has autocorrelation spatially, \( \varepsilon^* \) is the vector of an error.

The model of derivative from the equation (1), namely: (1). If \( \rho^* = 0 \) and \( \lambda^* = 0 \), obtained linear regression model OLS, namely the regression that has no spatial effect with the following model: \( y^* = X^* \beta^* + \varepsilon^* \); (2). If \( \rho^* = 0 \) and \( \lambda^* \neq 0 \), obtained Spatial Autoregressive Model (SAR) with the following model: \( y^* = X^* \beta^* + \lambda^* W y^* + \varepsilon^* \). This model assumes the process of autoregressive only on the endogenous variables; (3). If \( \rho^* \neq 0 \) and \( \lambda^* = 0 \), obtained Spatial Error Model (SERM) with the following model: \( y^* = X^* \beta^* + u^* \) or \( y^* = \rho^* M y^* + X^* \beta^* - \rho^* M X^* \beta^* + \varepsilon^* \); (4). If \( \rho^* \neq 0 \) and \( \lambda^* \neq 0 \), obtained Spatial Autoregressive Moving Average Model (SARMA) with model as equation (1).

4. Spatial structural equation modelling (Spatial SEM)

Spatial autoregressive model (SAR) in SEM in this study is the spatial dependence on the latent variables (inner model) and not on the observation variables. Matrix \( W \) sized of \( n \times n \) shows spatial dependence among observations or locations. A spatial autoregressive model (SAR) in the SEM in the form of Multiple Indicators Multiple Causes (MIMIC) was described as follows [8]:

\[ \tilde{y} = \rho^{\theta} W \tilde{y} + X^{\theta} \gamma + \tilde{\varepsilon} \] (2)

where \( \tilde{y} \) is the vector of observations on the dependent variable \( y \), \( W \) is the contiguity matrix, \( X^{\theta} \) is the matrix of observation of the explanatory variables, \( \tilde{\varepsilon} \) is vector of an error, \( \rho^{\theta} \) is the coefficient of spatial autoregressive and \( \gamma \) is vector of regression coefficient of explanatory variables.

The model of SAR in the SEM has a latent variable that cannot be measured directly as a sample unit. Therefore, to represent the latent variable is replaced by the score of a latent variable as a sample unit. In this study, the model does not use MIMIC model because there are no exogenous or endogenous variables as an observed variable. So, the model of SAR in the SEM in equation (2) changed as follows:

\[ l = \lambda W l + X \beta + \varepsilon \] (3)
The spatial error model (SERM) in SEM in the form of Multiple Indicators Multiple Causes (MIMIC) was described as follows [8]:

\[ \hat{y} = X^\gamma + \hat{\epsilon} \]  

with \( \hat{\epsilon} = \lambda^\omega W\hat{\epsilon} + \zeta \) or written as follows:

\[ \hat{y} = \hat{\lambda}^\omega W\hat{y} + X^\gamma - \lambda^\omega WX^\gamma + \zeta \]

This research does not use MIMIC model and the latent variables replaced by the score of the latent variable. Thus, the model of SEM in SERM as equation (4) and (5) can be rewritten as follows:

\[ l = X\beta + u \]  

\[ l = \rho Wl + K\beta - \rho Wk\beta + \epsilon \]

with \( \rho \) is the coefficient of the structure of spatial autoregressive in the error \( \epsilon \).

so, overall SEM spatial model can be written as follows:

\[ l = \lambda Wl + X\beta + u \]

with \( u = \rho Mu + \epsilon \). The information on the equation (3), (6), (7) and (8) are as follows: \( l \) is the vector of a score of an endogenous variable that sized \( nx1 \), \( K \) is the matrix of a score of an exogenous variable that sized \( n \times f \), \( \lambda \) is the coefficient of a spatial lag of ascore of an endogenous variable, \( \beta \) is the vector of the parameter at regression that sized \( k \times 1 \), \( \epsilon \) is the vector of an error that sized \( nx1 \) and has the normal distribution with mean equal zero and variance \( \sigma^2 I \), and \( I \) is the matrix of identity that sized \( n \times n \), \( u \) is the vector of a regression error which is assumed has the effect of region random and also error which has autocorrelation spatially, \( n \) is the number of observations or locations \( (i = 1,2,3...n) \), \( W \) and \( M \) are the matrix of a spatial weight that sized \( n \times n \) with the diagonal elements are 0.

5. Application

The observation units in this research were the districts in Sumenep which consists of 27 districts. Additionally, there were 2 endogenous variables that are the education quality of high school \( (Y_1) \) and the school infrastructure \( (Y_2) \), and 1 exogenous variable that is socio-economic condition \( (X_1) \). \( Y_1 \) have 4 indicators, \( Y_2 \) have 4 indicators, and \( X_1 \) have 6 indicators.

Determination of the model of the education quality at the high school level is associated with many dimensions, including means of infrastructures and socio-economic conditions of these regions. The conceptual model is formed based on these dimensions. Meanwhile, the structural model is formed based on a conceptual model as shown in the image below:

![Figure 1. The initial structural model.](image-url)
The model is repaired based on the evaluation of outer and inner models as well as by test of significance. The evaluation was done on three schemes estimated of the latent variables, namely centroid, factor and path, so that three new models are formed.

All three new models are re-evaluated. The evaluation results of the outer models for 3 schemes are as follows: (a). all indicators have a good validity convergence because the values of a loading factor were above 0.5; (b). The discriminant value is good because the correlation of the latent variable with these indicators was higher than the correlation of indicators with other variables; (c). The reliability composite value of more than 0.7. It proves the accuracy, consistency, and accuracy of instruments to measure the latent variables; (d). The result of the significance test of outer models demonstrate all indicators valid, except X_{11} in centroid scheme.

The results of significance test at the inner model are as follows: (a). The model of centroid scheme: X_1 effected on Y_2, Y_2 effected on Y_1, and X_1 has no effect on Y_1; (b). The model of factor scheme: X_1 effected on Y_2, Y_2 effected on Y_1, and X_1 has no effect on Y_1; (c). The model of factor scheme path: X_1 has no effect on Y_2, Y_2 effected on Y_1, and X_1 has no effect on Y_1.

The changes of a R^2 value in the initial model and an improved model used also as a consideration for choosing the best model. The changes of an R^2 value are summarized in Table 1.

| Model                      | Scheme | Centroid | Factor | Path |
|----------------------------|--------|----------|--------|------|
| Initial Model              |        | 0.393    | 0.399  | 0.411|
| Improved Model             |        | 0.366    | 0.384  | 0.418|

The changes of a R^2 value in the initial model and an improved model used also as a consideration for choosing the best model. The changes of an R^2 value are summarized in Table 1.

The model of the centroid scheme has 1 invalid indicator and the R^2 value has decreased. The model of the factor scheme has all valid indicators, but the R^2 value has decreased. The model of the path scheme has all valid indicators and the R^2 value has increased. Therefore, the model that will be used next is the model of path scheme. Significance test results showed Y_2 is not significantly affected by X_1.

The model equation of spatial SEM of the education quality based equation (8) can be written as follows:

\[ \hat{l}_i = a + \lambda \sum_{j=1, j \neq i}^n W_{ij} l_j + \beta_1 k_1 + \beta_2 k_2 + \rho \sum_{j=1, j \neq i}^n W_{ij} u_j \]  \hspace{1cm} (9)

The spatial dependence of the model in equation (9) is tested using a weighted matrix, namely the queen contiguity. The test uses the robust lagrange multiplier test. The test results of spatial dependence on the model of the education quality in high school summarized in Table 2.

| Test of Spatial Dependence | Chi-Sq     | P-Value | Conclusion | Inference |
|----------------------------|------------|---------|------------|-----------|
| LM (lag) Robust            | 3.8415     | 0.05    | Reject of H_0 | effected |
| LM (error) Robust          | 3.8415     | 0.2363  | Failed Reject of H_0 | Not effected |

The identification results of the spatial effect in models of the education quality in high school leads to a model of spatial autoregressive (SAR in SEM) with a significance level at \( \alpha \) of 5%. Furthermore, the model parameter of SAR in the SEM was estimated using MLE method. The results of parameter estimation in the model are summarized in Table 3.
Table 3. The Results of Parameter Estimation.

| Variabel                      | Coefficient | P-Value |
|-------------------------------|-------------|---------|
| Spasial Autoregressive Coefficient | -0.4347    | 0.0216  |
| Constant                      | 0.0223      | 0.8733  |
| K1 (the school infrastructure) | 0.6101      | 0.000   |
| K2 (the socio-economic condition) | -0.0080    | 0.9557  |

Akaike Info Criterion: 68.89  R-squared: 0.473

The estimation results of the model of SAR in SEM for the education quality of the high school in Sumenep Regency can be interpreted as follows: (1) The school infrastructure has a significant effect on the education quality at a significance level of 5%; (2) The socio-economic condition has a significant effect on the education quality at a significance level of 5%; (3) The rho value shows a significant effect at the significance level of 5%, which means that there is an education quality relationship between the high school in a district and other neighboring districts; (4) The R-square value ($R^2$) in a model of education quality in high school amounted to 47.3%. It explains that the data variation of the education quality of the high school can be explained by the school infrastructure and socio-economic condition amounted to 47.3%.

In general, the model of SAR in SEM for the latent variables of education quality at the high school is:

$$\hat{l}_i = 0.022 - 0.435 \sum_{j=1}^{n} W_{ij} l_j + 0.610 k_1 - 0.008 k_2$$

The models of education quality in the form of SAR in SEM for some districts are:

a. Districts of Pragaan has 3 neighboring districts
$$\hat{l}_{pragaan} = 0.022 - 0.145 l_{gulak} - 0.145 l_{blato} - 0.145 l_{ganding} + 0.610 k_1 - 0.008 k_2$$

b. Districts of Nonggunong has 1 neighboring district
$$\hat{l}_{nonggunong} = 0.022 - 0.435 l_{gunan} + 0.610 k_1 - 0.008 k_2$$

c. Districts of Dungkek has 2 neighboring districts
$$\hat{l}_{dungkek} = 0.022 - 0.2175 l_{gapura} - 0.2175 l_{batang-batang} + 0.610 k_1 - 0.008 k_2$$

d. Districts of Sumenep city has 6 neighboring districts
$$\hat{l}_{kota} = 0.022 - 0.072 l_{rubaru} - 0.072 l_{kalianget} - 0.072 l_{manding} - 0.072 l_{batan} - 0.072 l_{gapura} + 0.610 k_1 - 0.008 k_2$$

Table 4 is a comparison summary of the sufficiency of the inner model using the model of initial SEM, the model of improvement SEM, and the model of spatial SEM. The model of education quality of high school using the model of SAR in SEM has the R-square value 15% higher than the model of improvement SEM. It shows that the model of education quality of the high school in Sumenep regency has a spatial effect. This is supported by the real conditions, that coordination, development, and evaluation in the field of education are interrelated among districts within a regency.
Table 4. The Comparison of the Model Sufficiency.

| Model                     | R-square |
|---------------------------|----------|
| Model of initial SEM      | 0.411    |
| Model of Improved SEM     | 0.418    |
| Model of Spatial SEM      | 0.473    |

6. Summary and discussion

The rho value significantly affected at the level of significance of 5%. It means that there is a relationship of education quality at the high school in a district with other districts are contiguous. However, the value is negative so that further study related to the weighting was required. A customized study of spatial weightings is necessary, especially in the islands. For example, in Sumenep, there are 9 districts in the islands. Some districts do not even intersect with other districts. According to the weighting of queen contiguity, these districts were considered unrelated to other districts. However, in reality, it was impossible not to relate them to the other districts. These districts always coordinate with other districts, especially district of sumenep city as the center of administration at regency level. In the education field, the coordination must be conducted in terms of the evaluation, development, monitoring and others. Therefore, a customized weighting was required for districts that do not intersect with other districts. This customized weight considers aspects such as coordination, evaluation, development, supervision and other aspects undertaken in the field of education.

The R-square value ($R^2$) is 47.3% that means the data variation of the education quality of the high school can be explained by the school infrastructure and socio-economic condition amounted to 47.3%. The comparison result of model showed that the model of spatial SEM is better than the model of the improvement SEM, since the model of spatial SEM has the R-square 15% higher than the model of the improvement SEM. The increasing value of $R^2$ is not so high that further studies are necessary that are related to the methods of parameter estimation on the model of a spatial SEM, customized spatial weighting, consider the addition of variables, such as the quality of teachers, etc.

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