The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR16 sample: a multi-tracer analysis in Fourier space for measuring the cosmic structure growth and expansion rate

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ABSTRACT
We perform a joint BAO and RSD analysis using the eBOSS DR16 LRG and ELG samples in the redshift range of \( z \in [0.6, 1.1] \), and detect a RSD signal from the cross power spectrum at a \( \sim 4\sigma \) confidence level, \( f \sigma_8 = 0.317 \pm 0.080 \) at \( z_{\text{eff}} = 0.77 \). Based on the chained power spectrum, which is a new development in this work to mitigate the angular systematics, we measure the BAO distances and growth rate simultaneously at two effective redshifts, namely, \( D_M/r_d (z = 0.70) = 17.96 \pm 0.51 \), \( D_M/r_d (z = 0.70) = 21.22 \pm 1.20 \), \( f \sigma_8 (z = 0.70) = 0.43 \pm 0.05 \), and \( D_M/r_d (z = 0.845) = 18.90 \pm 0.78 \), \( D_M/r_d (z = 0.845) = 20.91 \pm 2.86 \), \( f \sigma_8 (z = 0.845) = 0.30 \pm 0.08 \). Combined with BAO measurements including those from the eBOSS DR16 QSO and Lyman-\( \alpha \) sample, our measurement has raised the significance level of a nonzero \( \Omega_M \) to \( \sim 11\sigma \). The data product of this work is publicly available at https://github.com/icosmology/eBOSS_DR16_LRGxELG

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1 INTRODUCTION
Large spectroscopic galaxy surveys are one of the key probes of both the expansion history and structure growth of the Universe, thus can in principle break the ‘dark degeneracy’ between scenarios of dark energy (DE) (Weinberg et al. 2013) and modified gravity (MG) (Koyama 2016), which are proposed as possible physical origins of the cosmic acceleration (Riess et al. 1998; Perlmutter et al. 1999).

Being almost not clustered, dark energy primarily affects the background expansion of the Universe, which can be probed by Baryonic acoustic oscillations (BAO), a special three-dimensional clustering pattern of galaxies, formed in the early Universe due to interactions between photons and baryons. The BAO feature was first detected by the Sloan Digital Sky Survey (SDSS) collaboration in 2005 (Eisenstein et al. 2005), and has been extensively investigated by a large number of studies since then.

Modified gravity, on the other hand, can dictate both the expansion and the structure formation of the Universe. After the required tuning for the cosmic acceleration, MG leaves imprints at the perturbation level, i.e., it can alter the history of the structure growth on linear and nonlinear scales. On such scales where the peculiar motion of galaxies is relevant, the redshift space distortions (RSD) can be directly mapped by redshift surveys, as reported by its first detection in 2001 by the 2 degree Field Galaxy Redshift Survey (2dFGRS) collaboration (Peacock et al. 2001).

Measurements of BAO and RSD from redshift surveys and cosmological implications have been extensively performed (Perivolaropoulou et al. 2010; Beutler et al. 2011, 2012; Contreras et al. 2013; Blake et al. 2013; Kazin et al. 2014; Ross et al. 2015; The Dark Energy Survey Collaboration et al. 2017; Alam et al. 2017; Beutler et al. 2017; Zhao et al. 2017b; Wang et al. 2017, 2018; Bautista et al. 2018; Ata et al. 2018; Gil-Marín et al. 2018; Zarrouk et al. 2018; Zheng et al. 2019; Zhao et al. 2019), but most of studies focus on the clustering of a single type of galaxies. This is, however, largely due to the fact that most finished galaxy surveys, including 2dFGRS and SDSS-III-BOSS, only target at a single tracer in the same cosmic volume.

The statistical error budget of RSD measurements is dominated by the shot noise and the cosmic variance on small and large scales, respectively. While the former can be in principle reduced by increasing the number densities of the observed tracers, the latter is difficult to suppress, due to fact that the number of large-scale modes is limited by the survey volume. One possible way to tackle the cosmic variance, however, is to combine multiple tracers with different biases covering the same footprint and redshift range (Seljak 2009; McDonald & Seljak 2009). The idea is that by contrasting different tracers of the same underlying density field, the uncertainty of statistics of the density field, which is dominated by the cosmic variance on large scales, can be cancelled out if the shot noise of all the concerning tracers is negligible, yielding a measurement of $f_{NL}$, the ratio of effective biases, $b_{\text{eff}} = \frac{b_X}{b_Y}$, between tracers $X$ and $Y$ without cosmic variance. The measured bias is effective because it includes the RSD term, namely, $b_{\text{eff}} \approx b (1 + \beta \mu^2)$, where $b$, $\beta$ and $\mu$ are the linear bias, the RSD parameter and the cosine of the angle between the line-of-sight and the pair of tracers, respectively. The effective bias also receives a contribution from the primordial non-Gaussianity parametrized by $f_{NL}$, if $f_{NL} \neq 0$. By combining measurements of $b_X$ using various $\mu$ modes, parameters of $\beta$, or $f_{NL}$ can be determined to an arbitrary precision in the ideal case, where the shot-noise is negligible.

It is challenging to run a multi-tracer survey, as different tracers may require different methods of target selection, different treatments of observational systematics, and different tracers have to be observed separately, making it expensive to build and perform. Fortunately, the extended Baryon Oscillation Spectroscopic Survey (eBOSS) project has provided such an opportunity for a multi-tracer analysis. Targeted for both Luminous Red Galaxies (LRG) and Emission Line Galaxies (ELG) at $z \in [0.6, 1.1]$ in a large overlapping patch of sky, the eBOSS Data Release (DR) 16 provided a total of ~ 550,000 spectra for the multi-tracer analysis, which is the largest sample for such an analysis to date. This is the natural motivation for this work. In this analysis, we develop new methods for a joint BAO and RSD analysis using the DR16 LRG and ELG sample, and pay particular attention to the mitigation of possible systematics.

The paper is structured as follows. In Section 2, we describe the observational and simulated datasets used in this analysis, and in Section 3, we present the method, followed by mock tests and main result of this work in Section 4, before conclusion and discussion in Section 5.

This work is one of a series of papers presenting results based on the final eBOSS DR16 samples. The multi-tracer analysis of the same galaxy sample is performed in configuration space to complement this work (Wang et al. 2020). For the LRG sample, produced by Ross et al. (2020), the correlation function is used to measure BAO and RSD in Bautista et al. (2020), and the analyses of BAO and RSD from power spectrum are discussed in Gil-Marin et al. (2020). The LRG mock challenge for assessing the modelling systematics is described in Rossi et al. (2020). The ELG catalogs are presented in Raichoor et al. (2020), and analyzed in Fourier space (de Mattia et al. 2020) and in configuration space (Tamone et al. 2020), respectively. The clustering catalogue of quasar is generated by Lyke et al. (2020); Ross et al. (2020). The quasar mock challenge for assessing the modelling systematics is described in Smith et al. (2020). The quasar clustering analysis in Fourier space is discussed in Neveux et al. (2020), and in configuration space in Hou et al. (2020). Finally, the cosmological implication from the clustering analyses is presented in eBOSS Collaboration (2020).

2 THE DATASETS
In this section, we briefly describe the observational and simulated datasets used in this analysis.

2.1 The eBOSS DR16 LRG and ELG samples
Being part of the Sloan Digital Sky Survey-IV (SDSS-IV) project (Blanton et al. 2017), the eBOSS survey (Dawson et al. 2016; Zhao et al. 2016) started in 2014 using the 2.5-metre Sloan telescope (Gunn et al. 2006) at the Apache Point Observatory in New Mexico. After the eBOSS target selection, which is described in Prakash et al. (2016) and Raichoor et al. (2017) for the LRG and ELG,
respectively, the spectra are taken using the double-armed spectrographs (Smee et al. 2013), which were used for the Baryon Oscillation Spectroscopic Survey (BOSS) mission, as part of the SDSS-III project (Eisenstein et al. 2011).

The footprint of the LRG and ELG samples is shown in Fig. 1, with statistics in Table 2. The eBOSS LRG sample used in this work is a combination of the eBOSS LRG with those observed by the BOSS program at $z > 0.6$, and it is denoted as ‘LRGpCMASS’ in other companion papers. This sample covers the redshift range of $z \in [0.6, 1.0]$ with a sky coverage of $\sim 9500 \text{ deg}^2$, and consists of approximately 255 K and 121 K galaxies in the northern galactic cap (NGC) and southern galactic cap (SGC), respectively. Details of this LRG catalog is described in Rosset al. (2020). The ELG are selected to cover $z \in [0.6, 1.1]$, covering $\sim 730 \text{ deg}^2$, with $\sim 174 \text{ K}$ redshifts in total, and the ELG catalog is detailed in Raichoor et al. (2020).

Figs. 1 and 2 show that almost all the ELG are in the footprint of the LRG, but the overlapping region is only $8\%$ of the LRG coverage. This means that the cross-correlation of these samples is much more relevant to the auto-correlation of the ELG sample, while the auto-correlation of the LRG sample, which is dominated by LRG that do not overlap with the ELG, is much less related.

2.2 The simulated mock samples

A large number of mock samples, each of which has the same clustering property of the eBOSS DR16 sample, are required to estimate the data covariance matrix. In this analysis, we use the Extended Zel’dovich (EZ) mocks, which consist of 1000 realisations, produced following the prescription in Zhao et al. (2020); Chuang et al. (2015). The cosmological parameters used for the EZ mocks are listed in Eq (1), where the parameters are: the physical energy density of cold dark matter and baryons, the sum of neutrino masses, the amplitude of the linear matter power spectrum within $8h^{-1}$ Mpc, the power index of the primordial power spectrum, and the (derived) scale of the sound horizon at recombination respectively.

$$
\Theta = \left\{ \Omega_m h^2, \Omega_b h^2, \sum M_i / eV, \sigma_8, n_s, r_g / \text{Mpc} \right\} = \{0.1190, 0.022, 0.8288, 0.96, 147.74\} \left|_{\text{LRG}} \right.
= \{0.1190, 0.022, 0.8285, 0.96, 147.66\} \left|_{\text{EZ}} \right.
$$

We list another set of parameters in Eq (1), which is the fiducial cosmology we adopt for this analysis.

Note that the EZmocks for different tracers are produced using the same set of random seeds, thus the clustering of different tracers are intrinsically correlated. This is crucial for the multi-tracer analysis in this work and Wang et al. (2020), which is a complementary analyse using the same data sample in the configuration space.

3 METHODOLOGY

We describe the method used in this work, including a brief review of the multi-tracer method, a development of the chained power spectrum to mitigate the angular systematics, and prescriptions of creating the power spectrum template, measuring the power spectrum multipoles with the survey window function, handling the mismatch of $z_{\text{eff}}$ between different tracers, and performing parameter estimations. For the ease of presentation, we include a mini-dictionary in Table 1 for abbreviations used in this paper.

### Table 1. Abbreviations used in this work with meanings.

| Abbreviation | Meaning |
|--------------|---------|
| LRG          | Luminous Red Galaxies |
| ELG          | Emission Line Galaxies |
| LRG (P)      | $P_k(k)$ for LRG |
| LRG (Q)      | $Q_k(k)$ for LRG |
| ELG (P)      | $P_k(k)$ for ELG |
| ELG (Q)      | $Q_k(k)$ for ELG |
| X            | The cross power between LRG and ELG |
| QQP          | LRG (Q) + ELG (Q) + X (P) |
| PPP          | LRG (P) + ELG (Q) + X (P) |
| zL           | $z_{\text{eff}}(\text{LRG}) = 0.70$ |
| zL           | $z_{\text{eff}}(\text{ELG}) = 0.845$ |
| zX           | $z_{\text{eff}}(\text{LRG} \times \text{ELG}) = 0.77$ |
| FoM          | Figure of Merit |
| NGC          | Northern Galactic Cap |
| SGC          | Southern Galactic Cap |
| LoS          | Line of Sight |

### Table 2. Statistics of the galaxy sample used in this work. Quantities $P_{\text{shot}}$ and $I$, as defined in Eqs (12) and (14), are the shot noise subtracted from the measured monopole, and the normalisation factor for the power spectrum measurement, respectively.

| $P_{\text{LRG}}(N)$ | $P_{\text{LRG}}(S)$ | $P_{\text{ELG}}(N)$ | $P_{\text{ELG}}(S)$ | $X(N)$ | $X(S)$ |
|---------------------|---------------------|---------------------|---------------------|--------|--------|
| Area                | $6,934$             | $2,560$             | $370$               | $358$  | $369$  |
| $N_c$               | $255,741$           | $121,717$           | $83,769$            | $89,967$ | -     | -     |
| $P_{\text{shot}}$   | $12,641$            | $11,995$            | $5,318$             | $4,498$ | -     | -     |
| $I$                 | $6.18$              | $3.00$              | $5.42$              | $5.93$  | $0.88$ | $1.54$ |

3.1 The multi-tracer method

The clustering of galaxies, as biased tracers of the underlying dark matter field, is subject to the cosmic variance (CV) on large scales. The CV is an intrinsic source of uncertainty for surveys probing a single type of galaxies, but can be significantly suppressed by contrasting the clustering of multiple types of galaxies covering the same range of redshifts and footprints, if the number density of the overlapping tracers are sufficiently high so that the shot noise is negligible on large scales (Seljak 2009; McDonald & Seljak 2009).

As described previously, the eBOSS DR16 sample consists of two types of tracers partially overlapping in cosmic volume at $z < 1.1$, allowing for a multi-tracer analysis to probe the BAO and RSD jointly.

Under the assumption of Gaussianity, the covariance matrix for power spectrum multipole of DR16 tracers for a given $k$ mode can be modelled as,

$$
C = \begin{pmatrix}
LLLL & LLEE & LLLE \\
SYM & EEEE & EEEL \\
& LELE &
\end{pmatrix}
$$

---

1 Throughout the paper, the subscript or superscript ‘f’ denotes the fiducial value.
Figure 1. The footprint of the DR16 LRG (larger red region) and ELG (smaller blue) samples in the NGC (left) and SGC (right), respectively, used in this analysis.

Figure 2. The redshift distribution of the number density of eBOSS DR16 ELG and LEG samples, as illustrated in the legend. For each tracer, the solid and dashed lines represent the NGC and SGC, respectively.

where

\[ \begin{align*}
\text{AAAA} &= \left( P_A + \frac{1}{n_A} \right) ^2 ; \\
\text{ABAB} &= \frac{1}{2} \left[ P^2_{AB} + \left( P_A + \frac{1}{n_A} \right) \left( P_B + \frac{1}{n_B} \right) \right] ; \\
\text{AABB} &= P^2_{AB}; \\
\text{AAAB} &= P_{AB} \left( P_A + \frac{1}{n_A} \right),
\end{align*} \]

for \( \{A, B\} \in \{L, E\} \) (White et al. 2008). In the low noise limit, the cumulative signal-to-noise (S/N),

\[ (S/N)^2 = P^T C^{-1} P, \tag{4} \]

is immune to the cosmic variance, thanks to the multi-tracer nature of the sample.

3.2 The effective redshifts

The measured galaxy cross power spectrum between tracers A and B \(^2\) in a redshift slice is actually a combination of power spectra at multiple redshifts (Zhao et al. 2019), i.e.,

\[ P(z) = \sum P(\bar{z}_i) w_A^i w_B^i, \tag{5} \]

where \( \bar{z}_i \) is the average redshift for the \( i \)th galaxy pair made of galaxies \( w_A^i \) and \( w_B^i \), and the summation is over all galaxy pairs in the catalog. Traditionally, the clustering analysis is performed at a single effective redshift, \( z_{\text{eff}} \), for simplicity. This is an approximation, which can be understood from the following Taylor expansion,

\[ P(z) = P(z_{\text{eff}}) + P'(z - z_{\text{eff}}) + \frac{1}{2} P''(z - z_{\text{eff}})^2 + O(P''') \tag{6} \]

Combining Eqs. (5) and (6) yields,

\[ P = P(z_{\text{eff}}) + P'\Delta_1 + \frac{1}{2} P''\Delta_2 + O(P''') \tag{7} \]

where

\[ \Delta_1 = \frac{\sum \bar{z}_i w_A^i w_B^i}{\sum w_A^{i} w_B^{i}} - z_{\text{eff}}, \]

\[ \Delta_2 = \frac{\sum \bar{z}_i^2 w_A^i w_B^i}{\sum w_A^{i} w_B^{i}} - 2z_{\text{eff}} \frac{\sum \bar{z}_i w_A^i w_B^i}{\sum w_A^{i} w_B^{i}} + z_{\text{eff}}^2. \tag{8} \]

\(^2\) It is the auto power spectrum if A is identical to B.
Table 3. The quantities $\Delta_1$ and $\Delta_2$ for various power spectrum types at different effective redshifts.

| $z_{\text{eff}}$ | $P_{1L}$ | $P_{1E}$ | $P_{2E}$ |
|------------------|--------|--------|--------|
| 0.700            | 0.007  | 0.145  | 0.031  |
| 0.845            | -0.145 | 0.028  | 0.011  |
| 0.770            | -0.070 | 0.012  | 0.075  |

Where $\gamma$ is the growth index introduced in Linder (2005), $z_p$ is the pivot redshift, $x = \chi(z)/\chi(z_p) - 1$ and $\chi(z)$ and $H(z)$ are the co-moving distance and the Hubble function at redshift $z$, respectively. This set of parametrisation has been proven to be sufficiently general to cover a broad class of cosmologies in a wide redshift range (Zhu et al. 2015; Wang et al. 2019). In this work, we use this framework to relate BAO and RSD parameters at $z = 0.77$ and $z = 0.845$, which is well within the validity of this parametrisation, given the uncertainty of the eBOSS DR16 sample.

3.4 Measuring the auto and cross power spectrum multipoles

The measurement of the power spectrum multipoles can be performed efficiently using the Fast Fourier Transformation (FFT) (Bianchi et al. 2015; Scoccimarro 2015), based on the Yamamoto estimator (Yamamoto et al. 2006),

$$\tilde{P}_\ell(k) = \frac{2\ell + 1}{4\pi} \int \frac{d\Omega_k}{\ell} \left| \int dr_1 F(r_1) \hat{e}^{i\ell \cdot r_1} \right| \int \frac{d\Omega_{r_2}}{\ell} F(r_2) \hat{e}^{i\ell \cdot r_2} \mathcal{L}_\ell \left( \frac{k \cdot \hat{r}_2}{\ell} \right) - P_{\text{shot}} \right|$$

This line-of-sight (LOS) of pairs is approximated as the LOS of one of the galaxies in the pair, i.e., $\mathcal{L}_\ell \left( \frac{k \cdot \hat{r}}{\ell} \right) \approx \mathcal{L}_\ell \left( \frac{k \cdot \hat{r}_2}{\ell} \right)$, and the overdensity field is estimated as Feldman et al. (1994),

$$F(r) = \frac{w(r)}{\ell + 1} \left[ n(r) - \alpha n_s(r) \right]$$

where $w$ is the total weight of each galaxy, and $n, n_s$ denotes the number density of the data and random samples, respectively. The quantity $\alpha$ is the ratio of the weighted numbers of the data and random, and the normalisation $I$ is evaluated as,

$$I = \int dr w^2(r) \approx \alpha \sum_i w^2_i$$

Note that the above approximation using sums over the randoms is only valid for the auto-power. For the cross power, one has to take the overlapping geometry into account. A practical way is to assign random galaxies of both tracers onto a grid, and for each tracer, compute $w^2_n$ for each grid cell, and compute the product $\sqrt{(w^2_n) n_h \sqrt{(w^2_n) n_h}}$ for each grid cell, and sum over the cells. The final result for $I$ and $P_{\text{shot}}$ for each tracer is summarised in Table 2.

We use a 1024$^3$ grid for evaluating $F$ and $w^2$, use a fourth-order B-spline for interpolation, and correct for the aliasing effect following Jing (2005). We use the following estimator to measure the cross power between tracers A and B, which makes use of the Addition Theorem to factor the Legendre polynomial into a product of spherical harmonics,

$$\tilde{P}_\ell(k) = \frac{2\ell + 1}{2\ell} \int \frac{d\Omega_k}{\ell} \left[ F_{0A}(k) F_{\ell,B}(-k) + F_{0B}(k) F_{\ell,A}(-k) \right]$$

where

$$F_{\ell}(k) = \int dr F(r) e^{i\ell \cdot r} \mathcal{L}_\ell \left( \frac{k \cdot \hat{r}}{\ell} \right)$$

$$= \frac{4\pi}{2\ell + 1} \sum_{m=-\ell}^\ell Y_{\ell m}(k) \int dr F(r) Y_{\ell m}^*(\hat{r}) e^{i\ell \cdot r}$$

Diminishing $\Delta_1$ by properly defining $z_{\text{eff}}$ as,

$$z_{\text{eff}} = \frac{\sum z_i w_i^A w_i^B}{\sum w_i^A w_i^B}$$

where $w_i$ is the total weight of each sample, leaves a residual $\Delta_2$ term,

$$\Delta_2 = \frac{\sum z_i^2 w_i^A w_i^B}{\sum w_i^A w_i^B} - \left( \frac{\sum z_i w_i^A w_i^B}{\sum w_i^A w_i^B} \right)^2$$

Thus one has to make sure that $\Delta_2$ (and higher order residuals) is sufficiently small to be ignored for the redshift distribution of the concerning galaxy sample, when using a fixed power spectrum template, otherwise the analysis may be subject to systematics. We explicitly evaluate $\Delta$ defined in Eq. (8) at $z_{\text{eff}}$ shown in Eq. (9) for our samples, and summarise the result in Table 3. By construction, $\Delta_1$ vanishes for each power spectrum at its own effective redshift, e.g., $z_{\text{eff}} = 0.700, 0.845$ and 0.770 for $P_{1L}$, $P_{1E}$ and $P_{2E}$, respectively, and $\Delta_2$ gets minimised in this case. We have numerically confirmed that, in this case, the second-order correction term, $P_{\ell}^\alpha \Delta_2/2$, is safely negligible compared to the leading term. However, this may not hold if the analysis is performed at a redshift that is significantly different from the effective redshift. For example, analysing the LRG sample at $z = 0.845$ would require $\Delta_2 = 0.028$ to compensate, which is four times larger than that at its own $z_{\text{eff}}$, thus the second or higher order correction terms may have to be included in the template to avoid theoretical systematics.

3.3 The time dependence of the BAO and RSD parameters

Care must be taken when cross-correlating galaxy samples, because different samples may have different effective redshifts, even if they perfectly overlap. One could, in principle, use different $z_{\text{eff}}$ to generate templates for auto-correlation of each tracer, and for their cross-correlation respectively, but this inevitably requires additional parameters for BAO and RSD, which may degrade the efficacy of the multi-tracer technique. One way out is to relate the BAO and RSD parameters at different redshifts by a general parametrisation. For this purpose, we follow Zhao et al. (2019) to use the parametrisation for evaluating the optimal redshift weights, when necessary.

$$a_\perp(z) = a_\perp(z_p) + \left[ a_{\parallel}(z_p) - a_\perp(z_p) \right] x,$$

$$a_{\parallel}(z) = a_{\parallel}(z_p) + 2 \left[ a_{\parallel}(z_p) - a_\perp(z_p) \right] x,$$

$$f(z) = f(z_p) \left( \frac{1 + z}{1 + z_p} \right)^{\gamma y} \left( \frac{a_{\parallel}(z) H(z)}{a_{\parallel}(z_p) H(z_p)} \right)^{2y}$$

where $\gamma$ is the growth index introduced in Linder (2005), $z_p$ is the pivot redshift, $x = \chi(z)/\chi(z_p) - 1$ and $\chi(z)$ and $H(z)$ are the co-moving distance and the Hubble function at redshift $z$, respectively. This set of parametrisation has been proven to be sufficiently general to cover a broad class of cosmologies in a wide redshift range (Zhu et al. 2015; Wang et al. 2019). In this work, we use this framework to relate BAO and RSD parameters at $z = 0.77$ and $z = 0.845$, which is well within the validity of this parametrisation, given the uncertainty of the eBOSS DR16 sample.

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Unlike the observed \( P_\ell \), the observed \( Q_\ell \) is immune to the angular systematics, as demonstrated by Eq. (18), thus is a better quantity to use for data analysis.

For the first three multipoles of \( Q \), Eq. (19) means,

\[
\begin{bmatrix}
Q_0 \\
Q_2 \\
Q_4
\end{bmatrix} = \begin{bmatrix}
1 & -A_0 & 0 & 0 \\
0 & 1 & -A_2 & 0 \\
0 & 0 & 1 & -A_4
\end{bmatrix} \begin{bmatrix}
P_0 \\
P_2 \\
P_4 \\
P_6
\end{bmatrix}
\]

To reconstruct \( P \) from \( Q \), a truncation in \( P_\ell \) is necessary, otherwise the above matrix equation is not invertible. As \( P_\ell = 0 \ (\ell > 4) \) in linear theory, we show an example in which \( P_k \) is set to zero after finding \( Q_4 \) from data. An matrix inversion of the first 3 \times 3 block of the transformation matrix in Eq. (21) yields the cleaned \( P \), denoted as \( P^c \),

\[
\begin{bmatrix}
P_0^c \\
P_2^c \\
P_4^c
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & -A_0 & A_2 & 0 \\
0 & 1 & 0 & -A_2 & 0 & 0 \\
0 & 0 & 1 & -A_4 & 0 & 0
\end{bmatrix} \begin{bmatrix}
P_0 \\
P_2 \\
P_4 \\
P_6
\end{bmatrix}
\]

This equation is physically transparent: the role of measured \( P_\ell \), which is supposed to be zero as a theoretical prior chosen in this example, is to provide an estimate of the transverse contamination, \( X(k) \).

We caution that the window function of galaxy surveys can complicate the above formalism, because \( P^{\text{obs}}_\ell \) receives contributions from not only \( P^{\text{true}}_\ell \), but also \( P^{\text{true}}_L \), which are the true \( P(k) \) multipoles with similar orders, due to the convolution with the anisotropic survey window function. A complete prescription for mitigating the angular systematics with the window function effect is beyond the scope of this paper, but we argue that the chained power spectrum method developed here can remove the primary angular systematics, because \( P^{\text{true}}_\ell \) dominates \( P^{\text{obs}}_\ell \), even with the window function effect.

For the eBOSS DR16 sample, we find that \( Q_4 \) is rather noisy, as it involves the \( P_6 \) component, which is barely informative on linear scales. We thus choose not to use \( Q_4 \) for this work. Admittedly, we learn less of the galaxy clustering from \( Q_0 \) and \( Q_2 \) than from \( P_0 \), \( P_2 \) and \( P_4 \), but the information loss can be largely compensated by adding \( P_6^c \), multipoles of cross power spectrum between LRG and ELG, to the analysis. As LRG and ELG are selected using different photometry, we assume that the angular systematics of these tracers are uncorrelated, i.e., \( P^{\text{N}}_6 \) is immune to angular systematics.

In principle, we can use the following data vectors for analysis,

\[
\begin{align*}
\text{PPP} & = \left(p_0^{\ell}, p_2^{\ell}, p_4^{\ell}, P_0^{\ell}, P_2^{\ell}, P_4^{\ell}, p_0^{\alpha}, p_2^{\alpha}, p_4^{\alpha}\right)^T; \\
\text{PQP} & = \left(p_0^{\ell}, p_2^{\ell}, p_4^{\ell}, Q_0^{\ell}, Q_2^{\ell}, Q_4^{\ell}, p_0^{\beta}, p_2^{\beta}, p_4^{\beta}\right)^T; \\
\text{QQP} & = \left(Q_0^{\ell}, Q_2^{\ell}, Q_4^{\ell}, p_0^{\gamma}, p_2^{\gamma}, p_4^{\gamma}\right)^T,
\end{align*}
\]

where ‘\( L \)’, ‘\( E \)’ and ‘\( X \)’ denote observables for the LRG, ELG and their cross correlation, respectively. Apparently, PPP and QQP are the most aggressive and most conservative combinations, respectively, and QQP is in between. We shall make the choice in Sec. 4, after validating our pipeline by performing analyses on the mocks using all these combinations.

### 3.6 The power spectrum template

The TNS model (Taruya et al. 2010) has been widely used as a theoretical template for analyses using the auto-power spectrum
with the linear and nonlocal bias terms included (McDonald & Roy 2009; Beutler et al. 2017). For multiple tracers, the TNS model can be generalised as follows,

$$P_g^{AB}(k, \mu) = D_{fgG}(k, \mu) \left[ P_{g00}^{AB}(k) + 2f\mu^2 P_{g00}^{AB}(k) + f^2 \mu^4 P_{00}^{AB}(k) + A^{AB}(k, \mu) + B^{AB}(k, \mu) \right],$$

where

$$P_{g00}^{AB}(k) = \left( b_1^A + b_1^B \right) P_{00}(k) + \left( b_2^A + b_2^B \right) P_{02,0}(k) + \left( b_3^A + b_3^B \right) P_{02,0}(k) + \left( b_4^A + b_4^B \right) P_{02,0}(k),$$

$$P_{gbo}(k, \mu) = P_{g00}(k),$$

$$D_{fgG}(k, \mu) = \left[ 1 + k^2 \mu \right]^{-1/2}.$$  

with a full derivation of the $A^{AB}$ and $B^{AB}$ terms for the multi-tracer case included in Appendix A 4. This template restores the form for the auto-power if $A = B$.

The subscripts $\delta$ and $\theta$ denote the overdensity and velocity divergence fields, respectively, and $P_{\delta \delta}, P_{\delta \theta}$ and $P_{\theta \theta}$ are the corresponding nonlinear auto- or cross-power spectrum, evaluated using the regularised perturbation theory (RegPT) up to second order (Taruya et al. 2012). The linear matter power spectrum $P_{00}^{AB}$ is calculated using CAMB (Lewis et al. 2000). Terms $b_1$ and $b_2$ stand for the linear bias and the second-order local bias respectively. We have eliminated the second-order non-local bias $b_3$ and the third-order non-local bias $b_4$ using the following relation (Chan et al. 2012; Baldauf et al. 2012; Saito et al. 2014).

$$b_2 = -4 \left( b_1 - 1 \right),$$

$$b_3 = -32 \left( b_1 - 1 \right).$$

Note that the template of the cross power cannot be represented using that for the auto-power by redefining a single set of bias parameters in the framework of the TNS model, as explicitly shown in Appendix B, therefore we choose not to introduce an additional set of bias parameters for the cross power for theoretical consistency, although this approach is taken for the analysis in the configuration space (Ross et al. 2014; Wang et al. 2020).

3.7 The Alcock-Paczynski effect

The Alcock-Paczynski (AP) effect (Alcock & Paczynski 1979) distorts the observed power spectrum due to a possible mismatch between the input cosmology, which is used to convert redshifts to distances, and the true cosmology hidden in the observations. This effect creates anisotropy at the background level, via the following dilution parameters,

$$a_\parallel = \frac{D_M(z)r_{d}}{D_M(z)r_{d}}, \quad a_\parallel = \frac{D_H(z)r_s}{D_H(z)r_s},$$

with

$$D_M(z) = (1 + z)D_A(z); \quad D_H(z) = c/H(z),$$

where $D_A(z), H(z)$ are the angular diameter distance and the Hubble function at redshift $z$, respectively, and $c$ is the speed of light. The $\alpha$ parameters then distort the wavenumber $k$ and $\mu$, which is the cosine of the angle between the LoS and the galaxy pair, in the following way,

$$k' = \frac{k}{a_\parallel} \left[ 1 + \mu^2 \left( \frac{1}{F^2 - 1} \right) \right]^{1/2} ; \quad \mu' = \frac{\mu}{F} + \mu^2 \left( \frac{1}{F^2 - 1} \right)^{-1/2},$$

where $F = \alpha_{\parallel}/\alpha_{\perp}$ (Ballinger et al. 1996), and the resultant power spectrum multipole with order $\ell$ reads,

$$P^{AB}_\ell(k) = \left( \frac{2\ell + 1}{2\pi}\alpha_\parallel \alpha_\perp \right) \int_{-1}^{1} d\mu' P^{AB}(k', \mu', \mu) L_\ell(\mu).$$  

3.8 The survey window function

To account for the geometry of the survey, we follow Wilson et al. (2017) to compute the survey window functions for the auto-power spectrum of all tracers, and the cross-power spectrum between LRG and ELG, using the pair-count approach,

$$W_{\ell}^{AB}(s) = \frac{(2\ell + 1)}{\pi a_s^2} \sum_{i,j} w_{tot}^{A}(s_i) w_{tot}^{B}(s_j) + s \right) \log s \right) L_\ell(\mathbf{s_{los}} \cdot \hat{s}).$$

where superscripts $A, B$ denote different types of tracers, and again, $A = B$ is the limit for the auto-correlation. This is a multi-tracer generalisation of the formalism in Gil-Marín et al. (2020), and note that, the factor $I$ appears in the denominator, as suggested by de Mattia & Ruhlmann-Kleider (2019), to match the normalisation in the measurement of power spectrum, so that the final BAO and RSD measurement does not depend on how exactly the power spectrum is normalised.

3.9 The radial integral constraint

Due to the ignorance of the true selection function of the galaxy survey, the redshift distribution of actual observations, $n(z)$, is used instead for clustering analysis, which can bias the final measurement of BAO and RSD parameters due to this radial integral constraint (RIC). This effect is investigated recently in de Mattia & Ruhlmann-Kleider (2019), and corrected for theoretically for the eBOSS DR16 ELG sample (de Mattia et al. 2020). In this work, we take a different approach, namely, subtracting the RIC component from the data directly. In practice, we analyse two sets of EZmocks, with different treatments of the randoms so that one has the RIC effect, while the other does not. Then a comparison of the measured power spectrum from these two sets provides an estimate of the RIC component.
Figure 4. The power spectrum multipoles measured from 1000 realisations of the EZmocks (filled bands) and from the DR16 data (data points with error bars) for the LRG (left), ELG (right) and the cross correlation between LRG and ELG (middle).

Figure 5. Same as Fig. 4, but for the SGC.
of the power spectrum. Under the assumption that the RIC effect weakly depends on cosmology, which is proven to be true using mock tests, the RIC component measured from the EZmocks is used to correct for the RIC in the DR16 data for this work.

3.10 Parameter estimation

With a modified version of CosmoMC (Lewis & Bridle 2002), we used the Markov Chain Monte Carlo (MCMC) algorithm to sample the following general parameter space,

$$ P = \{\alpha, \alpha, f, \sigma_8, \{b_1 \sigma_8\}, \{b_2 \sigma_8\}, \{\sigma_v\}, \{N\} \}. $$

(34)

where quantities in the inner bracket denote a collection of parameters for each tracer in each galactic cap, e.g.,

$$ \{b_1 \sigma_8\} = \{b_{1,N} \sigma_8, b_{1,S} \sigma_8, b_{1,L} \sigma_8, b_{1,E} \sigma_8, b_{1,S} \sigma_8\}. $$

and $N$ is fixed to zero for the cross power.

By default, we assign a full set of the above parameters for a joint analysis at three effective redshift, resulting in joint BAO
and RSD measurements at $z_\text{L} = 0.70$, $z_\text{X} = 0.77$ and $z_\text{E} = 0.845$, dubbed as the ‘3z’ measurement. Alternatively, we also perform a ‘2z’ measurement, by relating parameters at $z_\text{X}$ with those at $z_\text{E}$ using the parametrisation introduced in Eq. (11) with $z_p = z_\text{E} = 0.845$. This essentially spends the information of the cross power for measuring parameters at the effective redshift of the ELG sample, which yields a joint measurement at $z_\text{L}$ and $z_\text{E}$. The reason for combining the autopower of the ELG with the cross power is the following,

- As we shall present in Sec. 4, the power spectrum of the ELG, or the cross power on their own, struggles to constrain the BAO parameter well, which results in loose and highly non-Gaussian constraints without combining with each other;
- The ELG sample is known to be much more contaminated by the systematics than the LRG sample (de Mattia et al. 2020), thus combining with the cross power is an efficient way to mitigate the systematics, in addition to using the $Q_\ell$’s as observables;
- The LRG sample, on the other hand, is much less subjective to systematics (Gil-Marin et al. 2020), and it can provide a decent measurement on its own, making it unnecessary to combine it with the cross power;
- Tomographic information on the lightcone is key for probing physics including the nature of dark energy (Zhao et al. 2017a), thus we choose not to compress all the power spectra into a measurement at a single redshift.

All the above arguments support for performing the ‘2z’ measurement, which will be presented in Sec. 4 as the primary result of this paper.

In this work, we use data points in the range of $k \in [0.02, 0.15] h \text{ Mpc}^{-1}$ for all spectra, as motivated by the LRG analysis (Gil-Marin et al. 2020), and have confirmed that this is an appropriate choice for the multi-tracer analysis, based on analyses using the mocks. In all cases, we combine the likelihoods for the NGC and SGC using a direct sum, and properly correct for the (inverse) data covariance matrix with relevant correction factors suggested by Hartlap et al. (2007); Percival et al. (2014).

We analyse the chains using GetDist (Lewis 2019), after the chains are fully converged.

4 RESULTS

This section is devoted to the main result of this work. We show our measurement of power spectrum multipoles from the EZmocks and from the DR16 galaxy sample, respectively, from which we derive a joint constraint on BAO and RSD parameters at multiple effective redshifts, after validating our pipeline using the EZmocks.

The data product of this work is publicly available at https://github.com/icosmology/eBOSS_DR16_LRGxELG

4.1 The power spectrum multipoles

Figures 4 and 5 show the measurement of power spectrum multipoles $P_\ell$ and $Q_\ell$ for the LRG and ELG samples in the NGC and SGC, respectively. The shaded bands illustrate the measurements (68% CL uncertainty around the averaged power spectra) from 1000 realisations of the EZmocks, and the data points with error bars are from the DR16 galaxy sample. Although measurements of the auto power spectrum in $P_\ell$ are presented and extensively discussed in Gil-Marin et al. (2020) and de Mattia et al. (2020) for the LRG and ELG samples, respectively, they are included here for completeness, which is helpful for presenting and discussing the measurement of $Q_\ell$ and the cross power spectrum.
Figure 9. The one dimensional (1D) posterior distribution (panels on the diagonal) and 68 and 95% CL contour plots for BAO and RSD parameters derived from the eBOSS DR16 (panels in the lower triangle) and EZmock catalogs (upper triangle), respectively. For measurements from the DR16 catalog, results are derived at three (blue) and two (red) effective redshifts, respectively, denoted as DR16 ($3z$) and DR16 ($2z$) in the legend. For results derived from the mocks, measurements are performed at three effective redshifts, from three different data combinations: QQP (unfilled solid contours); PQP (filled) and PPP (unfilled dashed). The dashed horizontal and vertical lines illustrate the fiducial model used to produce the EZmocks, which is identical to that used for this work.

As expected, we see that $Q_{\ell}$ generally has larger uncertainties compared to $P_{\ell}$, because $P_{\ell+2}$, which is less well determined than $P_{\ell}$, is involved in $Q_{\ell}$. However, as claimed earlier, the unknown systematics in the data, if exists and couples to the $\mu = 0$ mode, should be largely suppressed by using $Q_{\ell}$ instead. Interestingly, for $P_{\ell}$ measured from the ELG (NGC) sample, which is believed to be contaminated more by systematics than the NGC (de Mattia et al. 2020), an offset between the DR16 sample and the EZmock in the quadrupole is clearly visible on scales at $k \lesssim 0.06 h$ Mpc$^{-1}$, which might signal a component of unknown systematics. However, this glitch vanishes completely in the corresponding $Q_{\ell}$. For the LRG sample, the qualitative difference between $P_{\ell}$ and $Q_{\ell}$ is less obvious, but still visible: the offset between data and mock in the monopole, which is extensively discussed in Gil-Marín et al. (2020); Zhao et al. (2020), is suppressed in $Q_{\ell}$.

The cross-power spectra in both galactic caps are successfully detected and well measured, although the signal to noise ratio is less than that of the auto-power spectra. We find that there is almost no
The correlation coefficient between the LRG sample, so that the majority of the ELG contributes to what we see that caps.

P from the measured showsthe anisotropic cross powerspectrum, which is reconstructed as systematics for different tracers should be uncorrelated. Figure 6 reinforcing that the cross power is least affected by the systematics, marked in the figure for the ease of reading.

\[
\begin{array}{ccc}
\alpha_1(z = 0.70) & 1.028 \pm 0.031 & 1.028 \pm 0.029 \\
\alpha_1(z = 0.70) & 1.047 \pm 0.063 & 1.052 \pm 0.059 \\
f_{\sigma M}(z = 0.70) & 0.450 \pm 0.051 & 0.434 \pm 0.050 \\
b_{1,N,0}(z = 0.70) & 1.146 \pm 0.052 & 1.186 \pm 0.058 \\
b_{1,0}(z = 0.70) & 1.204 \pm 0.053 & 1.233 \pm 0.044 \\
a_1(z = 0.77) & 0.961 \pm 0.041 & - \\
a_2(z = 0.77) & 1.161 \pm 0.122 & - \\
f_{\sigma M}(z = 0.77) & 0.317 \pm 0.080 & -
\end{array}
\]

\[
\chi^2/DoF = 205/(208 - 27) = 208/(208 - 25)
\]

Fig. 11. A comparison of BAO and RSD measurements between this work (x-axis) and the DR16 LRG analysis in k-space (Gil-Marin et al. 2020) (first three data points on y-axis) and the DR16 ELG analysis in k-space (de Mattia et al. 2020) (last three data points). The data points are offset (as shown in the figure) for the ease of illustration. The diagonal dashed line shows \( y = x \) for reference.

4.2 Demonstration using the EZmocks

We perform a joint ‘3z’ fit on the averaged power spectra of 1000 realisations of the contaminated EZmocks using data vectors of PPP, PQP and QQP, respectively, for a validation and demonstration, and present the result in the upper triangle part of Fig. 9.

To start with, we notice that PPP and QQP provide the tightest and weakest constraint, respectively, and PQP is in between, as expected. The constraint from QQP and PQP are in excellent agreement with the expected values for all parameters, but the constraint from PPP can deviate by a noticeable amount, e.g., the constraints on \( a_1(z_E) \) and \( f_{\sigma M}(z_E) \) are higher or lower than the expected value by \( \sim 1\sigma \), due to the systematics in the ELG mock sample. This makes us decide not to use PPP for this work, although it provides the tightest constraint. QQP on the other hand, unnecessarily trashes information of the LRG sample, which significantly dilutes the constraint at \( z = 0.70 \). Due to these reasons, we choose to use PQP for presenting the primary result of this paper, as it is a reasonable compromise between retaining the constraining power of the data, and mitigating the systematics in the ELG sample. One point worth noting is that, the cross power, almost on its own, is able to provide a decent measurement at \( z_{\text{eff}} = 0.77 \) with nearly no bias at all, which once again shows the robustness of the cross power against the systematics.

Mock tests with other data combinations and parametrisations, e.g., POP with ‘2z’, are performed but not shown here for brevity. Our pipeline is well validated, i.e., the constraint derived in all cases are consistent with the expected ones well within the uncertainty, from all the tests. These mock tests also demonstrate that the cross power spectrum is informative, and more robust against systematics than the auto power spectrum.
4.3 Measurements from the DR16 sample

The ‘3z’ and ‘2z’ measurements from the DR16 galaxy sample (using PQP) are presented in Table 4 and in the lower triangle and the diagonal part of Fig. 9.

Measurement at $z_{\text{eff}} = 0.70$ is well performed, thanks to the robust LRG observations. However, the ‘3z’ measurement at $z_{\text{eff}} = 0.77$ and 0.845 are rather weak for some parameters, including all parameters for the ELG and $\alpha_{\parallel}(z_{X})$, compared to those measured from the mean of mocks. This is largely because the ELG sample is subject to systematics due to redshift failures, and unfortunately this kind of systematics affect both auto- and cross-correlations, so that the BAO feature gets distorted in the ELG auto- and cross-correlation functions (Tamone et al. 2020; Wang et al. 2020).
cross power can constrain $f\sigma_8(z_\parallel)$ fairly well, namely, $f\sigma_8(z_\parallel) = 0.317 \pm 0.080$, which is a $\sim 4\sigma$ detection of the RSD signal, as visually illustrated in Fig. 6.

Due to the large correlation between $Q^E$ and $P^X$ as shown in Fig. 7, the BAO and RSD parameters measured at $z_\parallel = 0.77$ and $z_\perp = 0.845$ are correlated. For example, $\text{corr}[\alpha(z_\parallel), \alpha(z_\perp)] = 0.50$, $\text{corr}[\sigma(z_\parallel), \sigma(z_\perp)] = 0.45$ and $\text{corr}[f\sigma_8(z_\parallel), f\sigma_8(z_\perp)] = -0.33$. This means that the weak constraints at $z_\perp$ can be improved, if the cross power spectrum is used to constrain parameters at $z_\perp$, which is designed as the ‘$2z$’ measurement, as described in Sec. 3.10.

Comparing the ‘$2z$’ (top red layer in Fig. 9) with ‘$3z$’ (bottom blue) measurements, we see that the constraint on all parameters at $z_\perp$ is significantly improved, primarily due to the contribution of the cross power spectrum. Specifically, $\alpha(z_\parallel)$, which is almost unconstrained in ‘$3z$’ (it has no upper bound given the wide flat prior), is now measured at a precision of 4% with a perfectly Gaussian likelihood distribution. However, the posterior measured in Mattia et al. (2020) are highly asymmetric, because of the non-Gaussian likelihood distribution. Nevertheless, the posterior measured in this work is much closer to Gaussian, due to the contribution from the cross power.

Final data product of this work is summarised in Eq. (35) and Fig. 10, which are data vectors and covariance matrices for the BAO distances and $f\sigma_8$ at two redshifts,
\begin{align}
V(0.70) &= \{17,954 \pm 0.509, 21,221 \pm 1,198, 0.434 \pm 0.050\} \\
V(0.845) &= \{18,897 \pm 0.776, 20,910 \pm 2,862, 0.297 \pm 0.081\}
\end{align}

where $V = \{D_M/r_d, D_H/r_d, f/\sigma_8\}$. These measurements are over-plotted with external measurements published in recent years, including one from the Planck2018 observations (Planck Collaboration et al. 2018), based on a $\Lambda$CDM model. Compared with the Planck result, our measurement at $z_\parallel$ shows a deviation, especially on $D_M/r_d$ and $f/\sigma_8$. The same trend is independently found in de Mattia et al. (2020) in the RSD measurement, which used a completely different scheme to mitigate the angular systematics. This may suggest interesting new physics beyond $\Lambda$CDM, although it may be subject to unknown residual systematics in the ELG sample, even after the mitigation by using the chained power spectrum and the cross power.

Projecting our BAO measurement onto the $\Omega_M, \Omega_\Lambda$ plane with $H_0/\sigma_8$ marginalised over as performed in Zhao et al. (2019), we find that the constraint is largely improved by combining our measurement with the that derived from the BOSS DR12 sample, namely, the error on $\Omega_\Lambda$ is reduced by 22%, and the correlation with $\Omega_M$ is lowered from 0.85 to 0.75, which raises the significance of $\Omega_\Lambda > 0$ from 2.95σ to 4.65σ. Combining other BAO data to date, including the DR16 QSO and Lyman-α measurements, the nonzero $\Omega_\Lambda$ is now favoured at a $\sim 11\sigma$ confidence level, which is consistent with the multi-tracer analysis in the configuration space in a complementary paper (Wang et al. 2020).

## 5 CONCLUSION AND DISCUSSIONS

eBOSS is a first galaxy survey to observe multiple tracers with a large overlap in the cosmic volume, which naturally motivated this work, as a study of BAO and RSD using multiple tracers in Fourier space.

This work is based on the eBOSS DR16 LRG and ELG samples in redshift range of $z \in [0.6, 1.1]$, with more than 550,000 galaxies in total. Being a first ELG sample for cosmological analysis in history, the DR16 ELG sample is analysed with particular care, to mitigate the systematics in the observations. For this purpose, we develop a new method using the chained power spectrum multipoles ($Q_{\ell}$), and has demonstrated using EZmocks that it can efficiently remove angular systematics. Being simply the algebraic difference between the normal power spectrum multipoles ($P_{\ell}$) with different orders, $Q_{\ell}$ is less well measured. Fortunately, the information loss in using $Q_{\ell}$ can be compensated by the cross power spectrum between the LRG and ELG samples, which itself is least affected by angular systematics.

We measure both $P_{\ell}$ and $Q_{\ell}$ for each tracer, as well as their cross power spectrum, and perform a joint BAO and RSD analysis at multiple redshifts after validating our pipeline using the EZmocks with systematics built in. For the first time, we have detected the RSD signal from the cross power spectrum alone, at a $\sim 4\sigma$ confidence level, i.e., $f/\sigma_8 = 0.317 \pm 0.080$, and find that adding cross-correlation in the analysis to the ELG sample can significantly boost the precision of the BAO and RSD measurement at $z = 0.845$. Our final data product is summarised in Eq. (11) and Fig. 10, which is a joint BAO and RSD measurement at $z = 0.70$ and $z = 0.845$, with the associated covariance matrix. Our measurement, combined with those measured from the eBOSS DR16 QSO (Hou et al. 2020; Neveux et al. 2020) and Lyman-α sample (du Mas des Bourboux et al. 2020) and other galaxy catalogs at low redshifts including the MGS (Ross et al. 2015) and 6dFGS (Beutler et al. 2011) samples, has raised the significance level of $\Omega_\Lambda > 0$ to $\sim 11\sigma$.

Methods developed in this work is directly applicable to forthcoming multi-tracer surveys including Dark Energy Spectroscopic Instrument (DESI) (DESI Collaboration et al. 2016), and in fact, measurement of the cross power spectrum from DESI with a high S/N can better reduce the information loss when using the chained power spectrum, which makes it possible for mitigating angular systematics without degrading the statistical precision.

\begin{table}
\centering
\caption{The constraints on $\Omega_M, \Omega_\Lambda$ from BAO datasets, with $H_0/\sigma_8$ marginalised over.}
\begin{tabular}{lccc}
\hline
 & BOSS & BOSS + this work & Full BAO \\
$\Omega_\Lambda$ & $0.706 \pm 0.239$ & $0.864 \pm 0.186$ & $0.752 \pm 0.069$ \\
$\Omega_M$ & $0.443 \pm 0.204$ & $0.480 \pm 0.172$ & $0.302 \pm 0.021$ \\
$\text{corr}(\Omega_M, \Omega_\Lambda)$ & 0.85 & 0.75 & 0.55 \\
S/N & 2.95 & 4.65 & 10.95 \\
\hline
\end{tabular}
\end{table}
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APPENDIX A: THE EXTENDED TNS MODEL FOR THE CROSS POWER SPECTRUM

A1 Preliminaries
Throughout the report, we work with the distant-observer limit, and assume that the line-of-sight direction is parallel to the z-axis. Then the observed redshift space may be written as

$$s = r - f u_z(r)z,$$

(A1)

where the quantity $u_z$ is the normalized velocity field along the line-of-sight, defined by $u_z = -v_z/(cH f)$. The density field in observed redshift space, $\delta^{(S)}$, is expressed in Fourier space as

$$\delta^{(S)}(k) = \int d^3r \left[ \delta(r) + f \nabla_z u_z(r) \right] e^{i(k \cdot r - k \cdot f u_z)}$$

(A2)

with $\mu \equiv k_z/k$.

We are particularly interested in the cross correlation between the different samples (with different bias parameter). We denote the number density as $N$. Then, with a help of cumulant expansion theorem, we obtain

$$P^{(S)}(k) = \int d^3x e^{i k \cdot x} \left[ \langle e^{i A_1} \rangle_c \right] \left[ \langle e^{i A_2} \rangle_c + \langle e^{i A_1} \rangle_c \langle e^{i A_2} \rangle_c \right].$$

(A5)

with the quantities $j_i, A_i$ given by

$$j_1 = -ik \mu,$n
$$A_1 = f \Delta u_z,$n
$$A_2 = \delta_A(r) + f \nabla_z u_{A_z}(r),$$n
$$A_3 = \delta_B(r') + f \nabla_{z'} u_{B_z}(r').$$

Then, with a help of cumulant expansion theorem, we obtain

$$P^{(S)}(k) = \int d^3x e^{i k \cdot x} \left[ \langle e^{i A_1} \rangle_c \right] \left[ \langle e^{i A_2} \rangle_c + j_1 \langle A_1 A_2 \rangle_c + j_1^2 \langle A_1 \rangle_c \langle A_2 \rangle_c \right].$$

(A6)

Here, $(\cdots)_c$ indicates the cumulant.

As it is clear from the expression, the exponential prefactor $\exp \left\{ \langle e^{i A_1} \rangle_c \right\}$ can be non-perturbative, and it lead to a strong damping even at large scales. We thus keep it untouched. But, at weakly nonlinear scales, we may expand the rest of the terms regarding $j_1$ as a small expansion parameter. Up to the order of $O(j_1^2)$, we obtain

$$P^{(S)}(k) = \int d^3x e^{i k \cdot x} \left[ \langle e^{i A_1} \rangle_c \right] \left[ \langle A_2 \rangle_c + j_1 \langle A_1 A_2 \rangle_c + j_1^2 \langle A_1 \rangle_c \langle A_2 \rangle_c \right].$$

(A7)

Here, the term $\frac{1}{2} j_1^2 \langle A_1 A_2 \rangle_c$ is ignored according to Ref. Taruya et al. (2010). For more simplification, we shall assume that $\exp \left\{ \langle e^{i A_1} \rangle_c \right\}$ is independent of separation $x$, and is expressed as (even) function of $k \mu$. With this assumption/ansatz, the model of redshift-space cross power spectrum, $P^{(S)}_{AB}$, is given by

$$P^{(S)}_{AB}(k) = D_{AB}(k \mu \sigma_v) \left[ \tilde{P}_{kmo}(k) + \tilde{A}(k) + \tilde{B}(k) \right].$$

(A8)

with

$$\tilde{P}_{kmo}(k) = \int d^3x e^{i k \cdot x} \langle A_2 A_3 \rangle_c,$n
$$\tilde{A}(k) = j_1 \int d^3x e^{i k \cdot x} \langle A_1 A_2 A_3 \rangle_c,$n
$$\tilde{B}(k) = j_1^2 \int d^3x e^{i k \cdot x} \langle A_3 A_4 \rangle_c.$$n

(A9)

Below, we explicitly write down the expression of each term in the bracket. In what follows, we assume the linear bias for $\delta_A$ and $\delta_B$, and rewrite them with $b_A \delta$ and $b_B \delta$, respectively. Similarly, assuming the linear relation, we may write biased velocity field as $u_{A,B} = c_{A,B} u$. With the velocity-divergence field $\theta$ defined by $\theta = \nabla \cdot u = \nabla \cdot v/(cH)$, we then have:

$$\tilde{P}_{kmo}(k, \mu) = b_A b_B P_{\theta A B}(k) + f \mu^2 (b_A c_B + b_B c_A) P_{\theta A B}(k) + f^2 \mu^2 c_A c_B P_{\theta A B}(k),$$

(A10)

$$\tilde{A}(k, \mu) = k \mu f \int d^3p \frac{p_z}{(2\pi)^3 c_A} \left[ c_B \tilde{B}_\mu(p, k - p, -k) - c_B \tilde{B}_\mu(p, k, -k - p) \right],$$

(A11)

$$\tilde{B}(k, \mu) = (k \mu f)^2 c_A c_B \int d^3p d^3q \frac{\delta_0(k - p - q)}{(2\pi)^3} F_A(p) F_B(q),$$

(A12)
where the quantities $\tilde{B}_{\sigma r}$, $F_X (X = A$ or $B$) are the cross bispectrum and power spectrum, respectively, defined by

$$(2\pi)^3 \delta \varphi (k_1 + k_2 + k_3) \tilde{B}_{\sigma r} (k_1, k_2, k_3) = \left\langle \delta \varphi (k_1) \delta \varphi (k_2) + c_A f \left( k_{1, 2}^2 \right) \delta \varphi (k_3) \right\rangle.$$

(A13)

We will derive below the explicit expressions for $\tilde{A}$ and $\tilde{B}$, which are given in powers of $\mu$ and $f$.

### A2.1 $\tilde{A}$ term

The bispectrum $\tilde{B}_{\sigma r}$ given in Eq. (A13) is related to the real-space matter bispectra, $B_{\sigma A R}$, defined by $(\Phi_{\sigma A} (k_1) \Phi_{\sigma A} (k_2) \Phi_{\sigma A} (k_3)) = (2\pi)^3 \delta \varphi (k_1 + k_2 + k_3) B_{\sigma A R} (k_1, k_2, k_3)$ with doublet $\Phi_{\sigma A} = (\delta, \theta)$. It is given by

$$\tilde{B}_{\sigma r} (k_1, k_2, k_3) = b_3 b_{11} B_{211} (k_1, k_2, k_3) + c_A c_B f \left( k_{1, 2}^2 \right) \tilde{B}_{\sigma r} (k_1, k_2, k_3)$$

NotethatthefirstlineatRHSor

$$\tilde{B}_{\sigma r} (k_1, k_2, k_3) = b_3 b_{11} B_{211} (k_1, k_2, k_3) + c_A c_B f \left( k_{1, 2}^2 \right) \tilde{B}_{\sigma r} (k_1, k_2, k_3)$$

(A15)

Note that the first line at RHS or $B_{\sigma r}^{(sym)}$ is symmetric under $k_2 \rightarrow k_3$, but the second line or $B_{\sigma r}^{(non-sym)}$ is not, and can become symmetric only in the auto-power spectrum (i.e., $b_A = b_B$ and $c_A = c_B$). This asymmetry gives rise to non-trivial contribution, which makes the $\tilde{A}$ term different from that in the auto-power spectrum case.

To derive the explicit expressions of the $\tilde{A}$ term in powers of $\mu$ and $f$, we rewrite Eq. (A11) as

$$\tilde{A} (k, \mu) = k \mu f \int \frac{d^3 p}{(2\pi)^3} \left[ \frac{p \sigma}{p^2} \tilde{B}_{\sigma r}^{(sym)} (p, k - p, k) + c_B k \frac{p \sigma}{p^2} \tilde{B}_{\sigma r}^{(non-sym)} (p, k - p, k) \right]$$

(A16)

where $\tilde{B}_{\sigma r}^{(sym)}$ and $\tilde{B}_{\sigma r}^{(non-sym)}$ are defined below:

$$\tilde{B}_{\sigma r}^{(sym)} (k_1, k_2, k_3) = b_A b_{11} B_{211} (k_1, k_2, k_3) + c_A c_B f \left( k_{1, 2}^2 \right) \tilde{B}_{\sigma r}^{(sym)} (k_1, k_2, k_3)$$

$$\tilde{B}_{\sigma r}^{(non-sym)} (k_1, k_2, k_3) = b_A c_B f \left( k_{1, 2}^2 \right) \tilde{B}_{\sigma r}^{(non-sym)} (k_1, k_2, k_3)$$

(A17)

With the form given above, the $\tilde{A}$ is expanded as

$$\tilde{A} (k, \mu) = \frac{k^3}{(2\pi)^3} \sum_{n=0}^{\infty} \sum_{a, b} \mu^{n+1} f^{a+b-1} \int_0^\infty \! d\tau \int_0^1 \! dx \left\{ A_{n, a}^{(r, x)} (r, x) \right\} B_{211} (k, k - p, k)$$

(A18)

where we define $\tau = p/k$ and $x = (k - p)/(kp)$. Then, according to Appendix B of Taruya et al. (2010), the coefficients $A_{n, a}^{\sigma}$, $\tilde{A}_{n, a}^{\sigma}$, and $\tilde{A}_{n, a}^{\sigma}$ are derived, and the non-vanishing coefficients are expressed as follows:

$$A_{11}^{11} = \frac{k \sigma}{(2\pi)^3} \sum_{n=0}^{\infty} \sum_{a, b} \mu^{n+1} f^{a+b-1} \int_0^\infty \! d\tau \int_0^1 \! dx \left\{ A_{n, a}^{(r, x)} (r, x) \right\} B_{211} (k, k - p, k)$$

(A19)

The contributions coming from the symmetric bispectrum $\tilde{B}_{\sigma r}^{(sym)}$, i.e., $A_{11}^{11}$, $\tilde{A}_{11}^{11}$, and $\tilde{A}_{11}^{11}$, coincide with those obtained in the auto-power spectrum case Taruya et al. (2013), but others do not necessarily reproduce the previous results.

### A2.2 $\tilde{B}$ term

We first rewrite Eq. (A12) with

$$\tilde{B} (k, \mu) = \frac{(k \mu f)^2}{2} c_A c_B \int \frac{d^3 p}{(2\pi)^3} \left[ F_X (p) F_R (k - p) + F_A (k - p) F_R (p) \right].$$

(A20)

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The integrand of this expression is symmetric under $p \leftrightarrow k - p$. Then, as similarly done in the auto-power spectrum case (Taruya et al. 2010), we expand the $B$ term in powers of $f$ and $\mu$,:

$$\bar{B}(k, \mu) = \frac{k^3}{(2\pi)^2} \sum_{n=1}^{\infty} \sum_{a,b=1}^{\infty} \mu^{2n} (-f)^{a+b} \int_0^\infty dr \int_1^1 dx \frac{P_{AB}(k \sqrt{1 + r^2 - 2rx}) \bar{P}_{AB}(kr)}{(1 + r^2 - 2rx)^n}. \quad (A21)$$

Note again that $r \equiv p/k$ and $x = (p \cdot k)/(pk)$. With the symmetric form of Eq. (A20), the integral over $r$ and $x$ can be replaced with

$$\int_0^\infty dr \int_1^1 dx \rightarrow 2 \int_0^\infty dr \int_1^1 \Theta[1/2r] \frac{dr}{dx}. \quad (A22)$$

This would help to improve the convergence of numerical integration, avoiding poles. The coefficient $\bar{B}_{ab}^n$ is derived based on Appendix B of Taruya et al. (2010), and the results are summarized below:

$$\begin{align*}
\bar{B}_{11}^1 &= \frac{r^2}{2}(x^2 - 1) b_A b_B c_A c_B, \quad \bar{B}_{12}^1 = \frac{3r^2}{16}(x^2 - 1)^2 c_A c_B (b_A c_B + b_B c_A), \quad \bar{B}_{21}^1 = \frac{3r^2}{16}(x^2 - 1)^2 c_A c_B (b_A c_B + b_B c_A), \\
\bar{B}_{12}^2 &= \frac{5r^4}{16}(x^2 - 1) c_A c_B^2 b_A b_B, \quad \bar{B}_{11}^2 = \frac{r}{2}(r + 2x - 5r x^2) c_A c_B b_A b_B, \quad \bar{B}_{12}^2 = \frac{3r}{8} (x^2 - 1)(r + 2x - 5r x^2) c_A c_B (b_A c_B + b_B c_A), \\
\bar{B}_{21}^2 &= \frac{3r^2}{8}(x^2 - 1)(-2 + r^2 + 6r x - 5r x^2) c_A c_B (b_A c_B + b_B c_A), \quad \bar{B}_{22}^2 = \frac{3r^2}{16}(x^2 - 1)^2(-6 + 5r^2 + 30r x - 35r x^2)c_A^2 c_B^2, \\
\bar{B}_{11}^3 &= 0, \quad \bar{B}_{12}^3 = \frac{r}{16}(4x(3 - 5x^2) + r(3 - 30x^2 + 35x^4))c_A c_B (b_A c_B + b_B c_A), \\
\bar{B}_{21}^3 &= \frac{r}{16}(-8x + r(-12 + 36x^2 + 12r x(3 - 5x^2) + r^2(3 - 30x^2 + 35x^4)))c_A c_B (b_A c_B + b_B c_A), \\
\bar{B}_{22}^3 &= \frac{3r}{16}(x^2 - 1)(-8x + r(-12 + 60x^2 + 20r x(3 - 7x^2) + 5r^2(1 - 14x^2 + 21x^4)))c_A^2 c_B^2, \\
\bar{B}_{11}^4 &= \frac{r}{16}(8x(-3 - 5x^2) - 6r(3 - 30x^2 + 35x^4) + 6r x(15 - 70x^2 + 63x^4) + r^2(5 - 21x^2(5 - 15x^2 + 11x^4)))c_A^2 c_B^2. \quad (A23)
\end{align*}$$

Setting $b_A, b_B, c_A$ and $c_B$ to unity, the above expressions exactly coincide with those presented in Ref. Taruya et al. (2010).
APPENDIX B: THE RELATION BETWEEN AUTO AND CROSS POWER SPECTRUM TEMPLATES

To show the relation and difference between the auto- and cross power spectrum templates in an explicit way, here we rewrite Eq.(24) by introducing two sets of bias parameters

\[ b_1 = \frac{b_A^A + b_B^B}{2}; \Delta b_1 = \frac{b_B^B - b_A^A}{2}. \] (B1)

and

\[ b_2 = \frac{b_A^A + b_B^B}{2}; \Delta b_2 = \frac{b_B^B - b_A^A}{2}. \] (B2)

With Eq. (29), \( b_{A2}^A \) and \( b_{A3}^A \) can be written as

\[ b_{A2}^A = -\frac{4}{7} \left( b_{A1}^A - 1 \right) = b_{A2} + \frac{4}{7} \Delta b_1; \quad b_{A3}^A = -\frac{4}{7} \left( b_{A1}^A - 1 \right) = b_{A3} - \frac{4}{7} \Delta b_1. \] (B3)

\[ b_{A3}^A = b_{A3} + \frac{32}{315} \left( b_{A1}^A - 1 \right) = b_{A3} + \frac{32}{315} \Delta b_1; \quad b_{A3}^B = b_{A3} + \frac{32}{315} \left( b_{B1}^B - 1 \right) = b_{A3} + \frac{32}{315} \Delta b_1. \] (B4)

Substituting these new parameters into Eq. (25) gives

\[ P_{g,bb}^{AB}(k) = P_{g,bb}(k) + \Delta P_{g,bb}(k), \] (B5)

where \( P_{g,bb} \) takes the form of the auto-power, i.e.,

\[ P_{g,bb}(k) = b_{A2}^A P_{gb}(k) + b_{A3}^A P_{gb}(k) + b_{B2}^B P_{gb}(k) + b_{A2}^B P_{gb}(k) + 2 b_{A3}^A b_{B2}^B P_{gb}(k) + 2 b_{A3}^A b_{B2}^B P_{gb}(k) + b_{A3}^A P_{gb}(k) + b_{B3}^B P_{gb}(k) + N_{AB}. \] (B6)

and

\[ \Delta P_{g,bb}(k) = -\left( b_{A1}^A - 1 \right)^2 P_{gb}(k) - 2 \Delta b_1 \Delta b_2 P_{gb}(k) + \frac{8}{7} \left( b_{A1}^B - 1 \right)^2 P_{gb}(k) + \frac{8}{7} \Delta b_1 \Delta b_2 P_{gb}(k) \]

\[ \quad - \frac{64}{315} \left( \Delta b_1 \right)^2 \sigma_b^2(k) P_{gb}(k) - \left( \Delta b_2 \right)^2 P_{gb}(k) - \frac{16}{9} \left( \Delta b_1 \right)^2 \sigma_b^2(k). \] (B7)

For \( P_{g,bb}^{AB} \), we find that both \( \Delta b_1 \) and \( \Delta b_2 \) vanish, so

\[ P_{g,bb}^{AB}(k) = P_{g,bb}(k). \] (B8)

To see how the A and B terms change under transformation of bias parameters, we first rewrite them in the following form,

\[ A^{AB}(k, \mu) = \mu^2 f \left[ A_{11a}(k) b_A b_B c_A + A_{11b}(k) b_A b_B c_B + \mu^2 f^2 \left[ A_{12a}(k) b_A^2 c_A^2 + A_{12b}(k) b_A^2 c_B^2 \right] \right. \]

\[ + \mu^2 f^2 \left[ A_{22a}(k) b_A^2 c_A^2 + A_{22b}(k) b_A^2 c_B^2 + A_{22a}(k) b_A c_A c_B + A_{22b}(k) b_A c_A c_B \right] \]

\[ + \mu^2 f^3 \left[ A_{23a}(k) c_A^2 c_B + A_{23b}(k) c_A c_B^2 \right] + \mu^2 f^3 \left[ A_{33a}(k) c_A^2 c_B + A_{33b}(k) c_A c_B^2 \right] \] \quad \quad \quad (B9)

\[ B^{AB}(k, \mu) = \mu^2 \left[ f^2 B_{12a}(k) b_A^2 c_A c_B + f^2 B_{12b}(k) b_A c_A c_B + f^2 B_{13a}(k) c_A c_B (b_A c_B + b_B c_A) + f^2 B_{14}(k) c_A^2 c_B^2 \right. \]

\[ + \mu^2 \left[ f^2 B_{22a}(k) b_A^2 c_A c_B + f^2 B_{22b}(k) b_A^2 c_A c_B + f^2 B_{23a}(k) c_A c_B (b_A c_B + b_B c_A) + f^2 B_{24}(k) c_A^2 c_B^2 \right] \]

\[ + \mu^2 \left[ f^2 B_{31a}(k) c_A c_B (b_A c_B + b_B c_A) + f^2 B_{31b}(k) c_A c_B (b_A c_B + b_B c_A) \right] \]

\[ + \mu^2 \left[ f^2 B_{44}(k) c_A^2 c_B^2 \right]. \] \quad \quad \quad (B10)

Setting \( c_A = c_B = 1 \) as assumed in this paper, and eliminating \( b_A, b_B \) using Eq.(B1), we obtain,

\[ A^{AB}(k, \mu) = A(k, \mu) + \Delta A(k, \mu), \] \quad \quad \quad (B11)

with

\[ A(k, \mu) = f \mu^2 \left[ A_{11a}(k) + A_{11b}(k) \right] b_1^2 \]

\[ + f^2 \left[ \mu^2 A_{12a}(k) + \mu^2 A_{12b}(k) + f^2 A_{22a}(k) + \mu^2 A_{22b}(k) + \mu^2 A_{22a}(k) + \mu^2 A_{22b}(k) + \mu^2 A_{22a}(k) + \mu^2 A_{22b}(k) \right] b_1 \]

\[ + f^3 \left[ \mu^2 A_{23a}(k) + \mu^2 A_{23b}(k) + f^2 A_{33a}(k) + \mu^2 A_{33b}(k) + \mu^2 A_{33a}(k) + \mu^2 A_{33b}(k) \right], \] \quad \quad \quad (B12)

\[ \Delta A(k, \mu) = -f \mu^2 \left[ A_{11a}(k) + A_{11b}(k) \right] \Delta b_1 \]

\[ - f^2 \left[ \mu^2 A_{12a}(k) + \mu^2 A_{12b}(k) + \mu^2 A_{22a}(k) + \mu^2 A_{22b}(k) + \mu^2 A_{22a}(k) + \mu^2 A_{22b}(k) \right] \Delta b_1, \] \quad \quad \quad (B13)

\[ B^{AB}(k, \mu) = B(k, \mu) + \Delta B(k, \mu), \] \quad \quad \quad (B14)
\[ B(k, \mu) = f^2 \left[ \mu^2 B_{12}(k) + \mu^4 B_{22}(k) \right] b_1^2 + f^3 \left[ \mu^2 B_{13}(k) + \mu^4 B_{23}(k) + \mu^6 B_{13}(k) \right] \times 2b_1 + f^4 \left[ \mu^2 B_{14}(k) + \mu^4 B_{24}(k) + \mu^6 B_{34}(k) + f^6 \mu^8 B_{44}(k) \right]. \] (B15)

\[ \Delta B(k, \mu) = -f^2 \left[ \mu^2 B_{12}(k) + \mu^4 B_{22}(k) \right] (\Delta b_1)^2. \] (B16)

Finally, the relation between the auto- and cross power spectrum templates is,

\[ P^{AB}_g(k, \mu) = P_g(k, \mu) + \Delta P_g(k, \mu), \] (B17)

where

\[ P_g(k, \mu) = D_{FGG}(k, \mu) \left[ P_{g,\delta\delta}(k) + 2f\mu^2 P_{g,\delta\theta}(k) + f^2 \mu^4 P_{\theta\theta}(k) + A(k, \mu) + B(k, \mu) \right] \] (B18)

is the auto-power spectrum, and

\[ \Delta P_g(k, \mu) = D_{FGG}(k, \mu) \left[ \Delta P_{g,\delta\delta}(k) + \Delta A(k, \mu) + \Delta B(k, \mu) \right] \] (B19)

gives the difference.

The above calculation explicitly shows that the template of the cross power cannot be represented using that for the auto-power by redefining a single set of bias parameters.