Anisotropic spin transport in GaAs quantum wells in the presence of competing Dresselhaus and Rashba spin-orbit coupling

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Aiming at the optimization of the spin diffusion length in (001) GaAs quantum wells, we explore the effect of the anisotropy of the spin-orbit coupling on the competition between the Rashba and the Dresselhaus spin-orbit couplings by solving the kinetic spin Bloch equations with the electron-phonon and the electron-electron scattering explicitly included. For identical strengths of the Rashba and the Dresselhaus spin-orbit couplings, the spin diffusion length shows strong anisotropy not only for the spin polarization direction but also for the spin diffusion direction. Two special directions are used seeking for the large diffusion length: (110) and (110). Without the cubic term of the Dresselhaus spin-orbit coupling and with the identical Dresselhaus and Rashba strengths, infinite diffusion lengths can be obtained either for the spin diffusion/injection direction along (110), regardless of the direction of spin polarization, or for the spin polarization along (110), regardless of the direction of the spin diffusion/injection. However, the cubic Dresselhaus term cannot be neglected, resulting in a finite spin diffusion length which decreases with the temperature and the electron density. The anisotropy for the spin diffusion direction and spin polarization direction is maintained. For the spin diffusion/injection direction along (110), the spin diffusion length increases first with the increase of the Rashba strength (from 0) which can be tuned by the external gate voltage; when the Rashba strength is slightly smaller than (in stead of equal to) the Dresselhaus strength, the diffusion length reaches its maximum, followed by a decrease with further increase of the Rashba strength.

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I. INTRODUCTION

The control of the electron spin in nanoscale is an object of great interest in the field of spintronics.1,2 The knowledge of how the spin evolves as the electron goes through a nano-device is a key element for the purpose of using the spin degree of freedom as a mechanism for information transfer and processing. Randomization of the electron spin, however, occurs due to the spin-orbit coupling as the electron wave vector changes during and after scattering processes. Between scattering events, the precession frequency of the spin about the local magnetic field also changes from electron to electron depending on the wave vector. To overcome these detrimental effects, many efforts have been made to understand the spin diffusion and the spin relaxation,3,4,5,6,7,8,9,10,11,12,13,14,15,16 in particular after the proposal of the spin-field-effect transistor (SFET) by Datta and Das more than sixteen years ago.17 In such a device, ferromagnetic material is used as source and drain for the injection and the detection of spin-polarized electrons. The electron spins, moving through a quasi-one-dimensional channel, precess about an effective magnetic field tuned by a gate voltage. In n-type zinc-blende semiconductors the local magnetic field gives origin to a Zeeman-like splitting which, combined with the scattering, can cause a spin relaxation/dephasing known as the D’yakonov-Perel’ (DP) mechanism.16,18 In GaAs, the Dresselhaus term19 is dominant in the DP wave vector dependent magnetic field, giving rise to a spin splitting due to bulk inversion asymmetry. Furthermore in low-dimensional semiconductor structures with asymmetric confining potential, another term known as the Rashba term20 contributes to the DP mechanism, giving rise to a spin splitting based on the structure inversion asymmetry. While the Rashba term is linear, the Dresselhaus term is cubic in the wave vector components.

A few years ago Schliemann et al.21 proposed a non-ballistic spin-field-effect transistor based on the competition of the Dresselhaus and the Rashba terms. In such a transistor a gate voltage is tuned to give equal strengths to both terms, leading to a very long spin dephasing time for the spin polarization along (110) direction.21 In their work the cubic term is argued to be unimportant. Cheng and Wu studied the effect of the cubic term on the spin relaxation time of spins along the (110) direction by solving the kinetic spin Bloch equations, and obtained a finite spin relaxation time.22 Actually, in spin transport, when spins are injected into a sample, in addition to the direction of spin polarization, the direction of spin diffusion/injection is also important. The spin relaxation in transport depends on both directions. In the present work, the anisotropy of the spin-orbit coupling is taken into account when we study the ideal balance between the Rashba and the Dresselhaus terms in a quasi-two-dimensional channel of a GaAs quantum well. Therefore, both the direction of the spin polarization and the direction of the spin injection are analyzed. We further show that the cubic term appearing in the Dresselhaus effective magnetic field cannot be neglected in the competition

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with the Rashba field. Our treatment goes beyond the single particle method in that we use a fully microscopic many-body approach by solving the kinetic spin Bloch equations (KSBE). As shown by Weng and Wu, the correlations between the spin-up and spin-down states, i.e., the off-diagonal terms of the density matrix in spin space, play an essential role in the spin diffusion/transport. 

Studying the spin polarizations and the oscillations in the direction of injection one can calculate the injection length as a function of the direction of injection. In Sec. IV we apply the result injection directions in the quantum well. In Sec. III we construct the KSBE and we discuss how they change with the injection direction. In Sec. IV we apply the KSBE to our model and present the results for different parameters. We conclude in Sec. V.

II. SPIN-ORBIT COUPLING

The DP mechanism introduces a spin-orbit term in the Hamiltonian which can be expressed by a Zeeman-like term with a wave-vector-dependent effective magnetic field:

\[ H_{so} = \Omega(k) \cdot \sigma. \tag{1} \]

For convenience, the effective \( \gamma \)-factor and the Bohr magneton are absorbed into the definition of \( \Omega(k) \). For GaAs, which lacks an inversion symmetry center, the effective magnetic field contains the Dresselhaus term. For electrons in infinite-barrier-height quantum wells with a small well width \( a \) in the \( \Gamma_6 \) band, this term is\(^{22}\)

\[ \Omega_D(k) = \gamma \begin{pmatrix} k_x (k_x^2 - (k_z^2)) \\ k_y ((k_z^2) - k_x^2) \\ 0 \end{pmatrix}, \tag{2} \]

where \( \gamma \) represents the spin splitting parameter and \( (k_z^2) = (\pi/a)^2 \).

If a space gradient is applied to the sample, another effect shows up, the Rashba effect. The Rashba term in the Hamiltonian for electrons in the \( \Gamma_6 \) band is

\[ H_R = r_{41}^g \sigma \cdot k \times \varepsilon, \tag{3} \]

where \( r_{41}^g \) is a material parameter, and \( \varepsilon \) is the electric field determined by the asymmetry.\(^{22}\)

Taking the \( z \)-axis in the (001) direction (\( z \) will be considered throughout this paper as the growth direction of the quantum well) and \( \varepsilon = \varepsilon \hat{z}, \) the contribution of the Rashba term to the effective magnetic field may be written as

\[ \Omega_R(k) = \alpha \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}, \tag{4} \]

where \( \alpha \) is the Rashba parameter. Next, we consider the influence on the effective magnetic field when we change the injection direction to \( \cos \theta, \sin \theta, 0 \) and choose this direction as the \( x \)-axis (A schematic of the configuration is shown in Fig. I). The effective magnetic field changes according to \( \Omega'(k) = U^{-1} \Omega(Uk) \), with

\[ U = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5} \]

Therefore, we obtain for the Dresselhaus field:

\[ \Omega'_D(k) = \gamma \langle k_z^2 \rangle \begin{pmatrix} -k_x \cos 2\theta + k_y \sin 2\theta \\ k_x \sin 2\theta + k_y \cos 2\theta \\ 0 \end{pmatrix} + \gamma \langle k_z^2 \rangle \begin{pmatrix} k_y - \frac{k_x^2}{2} \sin 2\theta + k_x k_y \cos 2\theta \\ \frac{k_x^2}{2} \sin 2\theta - k_x k_y \cos 2\theta \\ 0 \end{pmatrix}, \tag{6} \]

where the wave vector components are written in the rotated coordinate system. For the Rashba field we obtain:

\[ \Omega'_R(k) = \alpha \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}. \tag{7} \]

Notice that the Rashba field is expressed in the new coordinates in the same way as in Eq. (4).

In the following, we discuss the diffusion in two different ways: (i) We take the identical strengths of \( \alpha = \beta \) with \( \beta = \gamma \langle k_z^2 \rangle \) to study the diffusion direction dependence and the spin polarization dependence of the diffu-
sion. In this case, the spin orbit coupling \( \Omega' = \Omega'_{\text{D}} + \Omega'_{\text{R}} \)

is written as

\[
\Omega'(k) = 2\beta \left( \sin(\theta - \pi/4)k_x + \cos(\theta - \pi/4)k_y \right) \mathbf{n}_0 + \gamma \left( \frac{k_x^2 - k_y^2}{2} \sin 2\theta + k_xk_y \cos 2\theta \right) \begin{pmatrix} k_y \\ -k_x \end{pmatrix} \tag{8}
\]

with the special direction \( \mathbf{n}_0 = \begin{pmatrix} \cos(\pi/4 - \theta) \\ \sin(\pi/4 - \theta) \end{pmatrix} \) representing the crystal direction (110) in the rotated coordinates. (ii) We study the effect of the gate voltage on the spin diffusion. We treat this issue in Sec. IV C.

### III. KSBE AND INHOMOGENEOUS BROADENING

Our fully microscopic treatment concerns the calculation of the spin density matrix for the electron with momentum \( k \) at position \( r = (x, y) \):

\[
\rho_k(r, t) = \begin{pmatrix} f_{k\uparrow} \\ \rho_{k\uparrow\downarrow} \\ \rho_{k\downarrow\uparrow} \\ f_{k\downarrow} \end{pmatrix} . \tag{9}
\]

By using the non-equilibrium Green function method with gradient expansion as well as the generalized Kadanoff-Baym ansatz, we construct the KSBE as follows, assuming the diffusion to take place in the \( x \)-direction:

\[
\hbar \frac{\partial \rho_k(x, t)}{\partial t} + e \frac{\partial \Psi(x, t)}{\partial x} \frac{\partial \rho_k(x, t)}{\partial k_x} + \frac{\hbar^2 k_x}{m^*} \frac{\partial \rho_k(x, t)}{\partial x} + i \left( [g\mu_B B + \Omega'(k)] \cdot \frac{\sigma}{2} + \mathcal{E}_\text{HF}(x, t), \rho_k(x, t) \right) = \hbar \frac{\partial \rho_k(x, t)}{\partial t} \bigg|_z . \tag{10}
\]

Here \( \Psi(x, t) \) is the electric potential satisfying the Poisson equation

\[
\nabla^2 \Psi(r) = e [n(r) - N_0(r)]/(a\kappa_0) , \tag{11}
\]

with \( n(r) \) standing for the electron density at position \( r \), \( N_0(r) \) representing the background positive charge density and \( \kappa_0 \) being the static dielectric function. \( \mathcal{E}_\text{HF}(x, t) \) is the Hartree-Fock term from the Coulomb interaction. The scattering terms \( \frac{\partial \rho_k(x, t)}{\partial t} \bigg|_z \) include the electron-electron and electron-phonon scattering. Their expressions can be found in Ref. 8. Details of the numerical scheme as well as the material parameters are laid out in Ref. 24. It has been shown that Coulomb scattering plays a very important role in spin dephasing\(^8,\)\(^21,\)\(^25,\)\(^26\) and spin diffusion/transport\(^24\) and therefore cannot be neglected in calculating the spin diffusion length.

When the system reaches its steady state, in the absence of an external field, it can be described by the equation

\[
\hbar \frac{\partial \rho_k(x, t)}{\partial t} + e \frac{\partial \Psi(x, t)}{\partial x} \frac{\partial \rho_k(x, t)}{\partial k_x} + \frac{\hbar^2 k_x}{m^*} \frac{\partial \rho_k(x, t)}{\partial x} + i \left( [g\mu_B B + \Omega'(k)] \cdot \frac{\sigma}{2} + \mathcal{E}_\text{HF}(x, t), \rho_k(x, t) \right) = \hbar \frac{\partial \rho_k(x, t)}{\partial t} \bigg|_z . \tag{12}
\]

By neglecting the Hartree-Fock and the scattering terms, Eq. (12) can be solved analytically\(^24\). After dividing both
sides of the equation by $k_z$, we see that along with the diffusion, the spin polarization for each wave vector $k$ precesses along $\mathbf{\omega}_k = \frac{m^*}{2\hbar k} \Omega(k)$. The fact that spins with different momentums precess with different frequencies is referred to as inhomogeneous broadening. With any spin-conserving scattering included, the inhomogeneous broadening results in spin dephasing.

For $\alpha = \beta$, $\mathbf{\omega}_k$ reads

$$
\mathbf{\omega}_k = \frac{m^*}{2\hbar^2} \left\{ 2\beta \left( \sin(\theta - \frac{\pi}{4}) + \cos(\theta - \frac{\pi}{4}) \frac{k_y}{k_x} \right) \mathbf{n}_0 + \gamma \left( \frac{k_x^2 - k_y^2}{2} \sin 2\theta + k_x k_y \cos 2\theta \right) \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix} \right\}. \quad (13)
$$

We find that $\mathbf{\omega}_k$ includes two parts: the zeroth-order term (on $k$) which is always along the same direction $\mathbf{n}_0$, and the second-order term. The zeroth-order term includes both the $k$-dependent and the $k$-independent terms, while the second-order term is always $k$-dependent. According to the previous works, the $k$-dependent term contributes to the inhomogeneous broadening.

We first analyze the spin diffusion without the third-order term of the Dresselhaus term (and hence the second-order term of $\mathbf{\omega}_k$). In this case, the effective magnetic field for each wave vector $k$ points to the same direction $\mathbf{n}_0$. From the previous works, we know that the spin dephasing in the time domain, in the spatial homogeneous case, shows strong anisotropy with respect to the direction of spin polarization: The spin polarization along the direction $\mathbf{n}_0$ has an infinite dephasing time, while the ones vertical to this direction have a very short spin dephasing time due to the large spin-orbit coupling. This is because when the spin polarization direction is the same as the direction of the effective magnetic field, the spin polarization cannot precess and hence there is no inhomogeneous broadening. Consequently the spin polarization has an infinite dephasing time. The same is true for the spin diffusion: When the spin polarization is along the same direction of $\mathbf{\omega}_k$, i.e., $\mathbf{n}_0$ (the crystal direction (110)), the spin polarization will not decay regardless of the direction of the spin injection, even in the presence of scattering, as there is no spin precession and hence no inhomogeneous broadening; However, when the spin polarization is perpendicular to $\mathbf{n}_0$, the spin diffusion length is very short due to the large inhomogeneous broadening.

Nevertheless, in contrast to the spin dephasing in a spatially homogeneous system, the spin diffusion has an extra degree of freedom, i.e., the spin diffusion/injection direction (which should not be confused with the spin polarization direction). This degree of freedom introduces an additional level of anisotropy to the spin diffusion/transport. When the spin diffusion/injection direction is along $\theta = 3\pi/4$, i.e., (110) direction, the precession frequency $\mathbf{\omega}_k = \frac{m^*}{2\hbar} \mathbf{n}_0$ becomes $k$-independent if the cubic term of the Dresselhaus term is neglected. Moreover, even with the scattering included, Eq. (12) can also be satisfied by the following solution:

$$
\rho_k(x) = e^{im^* \gamma(k_x^2) \sigma_x / \hbar^2} \rho_k(x = 0) e^{-im^* \gamma(k_x^2) \sigma_x / \hbar^2}. \quad (14)
$$

where $\rho_k(x = 0)$ is the Fermi distribution with the spin polarization at the left boundary. We see from Eq. (12) that in this special case the oscillation period is the same for all $k$'s, with the value $\frac{2\pi}{m^* \gamma(k_x^2)}$. So, there is no inhomogeneous broadening and the spin has an infinite diffusion length, regardless of the direction of the initial spin polarization. This result is mainly induced by the peculiar precession frequency $\mathbf{\omega}_k$ which is determined by the spin-orbit coupling and the diffusion velocity $k_x$, so it cannot be obtained in the spatially homogeneous case where the precession frequency is only determined by the spin-orbit coupling. Of course, when we include the cubic term in the spin-orbit coupling, the oscillation period becomes wave-vector dependent. This inhomogeneous broadening, then, results in a finite diffusion length.

In brief, we stress the importance of the inhomogeneous broadening caused by the diffusion term as compared to the spin dephasing in the spatially homogeneous case. Consequently the spin polarization shows strong anisotropy not only for the direction of the initial spin polarization but also for the direction of the spin diffusion. Moreover, the cubic Dresselhaus term cannot be neglected. In what follows we solve the problem by using the fully microscopic many-body approach of the KSBE with the cubic Dresselhaus term and the scattering explicitly included.

## IV. NUMERICAL RESULTS

The KSBE are solved numerically for (001) GaAs quantum wells of width $a = 5$ nm including the electron-electron and the electron–LO-phonon scattering. The value of the Dresselhaus coefficient is as $\gamma = 25 \text{ eV} \cdot \text{Å}^3$. The electron density and the temperature are taken as $N_e = 4.0 \times 10^{11} \text{ cm}^{-2}$ and $T = 200$ K separately unless otherwise specified. All matrix elements of the interactions are given in Ref. [8]. For the spin polarization along...
direction $\mathbf{n}$, the boundary conditions are given by \(21\)

$$
\begin{align*}
\rho_k(x = 0, t)|_{k_x > 0} &= \frac{F_{k,\uparrow} + F_{k,\downarrow}}{2} n \cdot \sigma, \\
\rho_k(x = L, t)|_{k_x < 0} &= \frac{F_{k,\uparrow} + F_{k,\downarrow}}{2} n \cdot \sigma,
\end{align*}
$$

with the Fermi distribution $F_{k,\sigma} = [e(k^2/2m^* - \mu_\sigma)/k_B T + 1]^{-1}$ ($\sigma = \uparrow, \downarrow$) and the chemical potential $\mu_\sigma$ determined by the polarized electron density. The diffusion length and the oscillation period are extracted from the spatial evolution of the spin polarization along $\mathbf{n}$ direction $\Delta N(x) = \sum_k \text{Tr}[\rho_k(x) n \cdot \sigma]$.

### A. Spin-diffusion/injection–direction and spin-polarization–direction dependence at $\alpha = \beta$

We first fix the spin polarization along the (110) direction ($\mathbf{n}_0$) and study the spin diffusion length as a function of the spin injection direction in the presence of the cubic Dresselhaus term. For comparison, we also study the cases when the spin polarizations are perpendicular to the (110) direction, i.e., $\mathbf{z}$ and $\mathbf{n}_1 = \mathbf{z} \times \mathbf{n}_0$ and show how the spin injection lengths change as a function of spin diffusion/injection direction. Subsequently we explore the special case when the spin diffusion/injection direction is along (110), i.e., $\theta = 3\pi/4$ and show how the spin injection length changes with the spin polarization in the presence of the cubic Dresselhaus term. The spin polarization calculated from the KSBE Eq. (10) and the Poisson Eq. (11) along these three spin polarizations can be well fitted by

$$
\Delta N(x) = C \exp\left(-\frac{x}{L_d}\right) \cos\left(\frac{2\pi x}{L_0} + \phi\right),
$$

where $L_d$ is the spin diffusion length and $L_0$ represents the oscillation period.

In Fig. 2 the spin diffusion length $L_d$ and the inverse of the spin oscillation period $L_0^{-1}$ are plotted against the spin diffusion/injection angle $\theta$ for spin polarizations along $\mathbf{n}_0$, $\mathbf{z}$ and $\mathbf{n}_1$, respectively. It is interesting to see that with the inclusion of the cubic Dresselhaus term, the spin injection length becomes finite but still independent on the direction of the spin diffusion/injection if the spin polarization is along (110) ($\mathbf{n}_0$). This is in agreement with the case without the cubic term where $L_d$ becomes infinite. At the same time, the spin polarization along this direction has an infinite oscillation period $L_0 = \infty$. This can be understood because the spin polarization is in the same direction as the effective magnetic field given by the zeroth-order term in $\omega_k$ and, thus, results in no oscillations. It is noted that the effective magnetic field from the second-order term in $\omega_k$ is $k$-dependent and therefore cannot lead to any oscillation at high temperature due to the scattering $21,25,30$.

However, this second-order term causes an inhomogeneous broadening which leads to a finite spin diffusion length. By rewriting the second-order term of $\omega_k$ into

$$
\frac{m^*}{2\hbar^2} \frac{d^2}{d\theta^2} \sin(2\theta_k + \theta) \begin{pmatrix} ky/kx & -1 \\ 0 & 0 \end{pmatrix}
$$

with $k$ and $\theta_k$ denoting the magnitude and the direction of the wave vector $k$ separately, one finds that the magnitude of the inhomogeneous broadening does not change with the spin diffusion/injection direction $\theta$. Consequently the spin diffusion length does not change with the spin diffusion/injection direction.

Strong anisotropy is again found for spin polarizations along the directions perpendicular to $\mathbf{n}_0$ except for the special case when $\theta = 3\pi/4$. Moreover, it is found in the figure that, for the perpendicular directions, both $L_d$ and $L_0$ depend sensitively on the spin diffusion/injection direction (additional anisotropy). It is further noticed that $L_d$ and $L_0$ are almost identical for both the perpendicular directions $\mathbf{z}$ and $\mathbf{n}_1$. These results can be understood again from the inhomogeneous broadening induced by $\omega_k$. As $\gamma$ in Eq. (13) is much smaller than $\beta$, the inhomogeneous broadening for spin polarizations $\mathbf{z}$ and $\mathbf{n}_1$ are therefore defined by the zero-th order term of $\omega_k$, which is the same for both perpendicular directions and is spin diffusion/injection direction $\theta$ sensitive. The oscillation period in the diffusion $L_0$ is determined by the $k$-independent component of $\omega_k$, i.e., $m^*\beta \sin(\theta - \pi/4)/\hbar^2$ which is in good agreement with the results in Fig. 2.

Now we turn to the case with special fixed spin diffusion/injection direction $\theta = 3\pi/4$, and investigate the spin diffusion length with different spin polarizations. It is interesting to see from Fig. 2 that similar to the case without the cubic Dresselhaus term, the spin diffusion lengths for the three spin polarizations are almost identical. Therefore the anisotropy disappears for this particular spin diffusion/injection direction. This is because...
Hence we choose the spin polarization to be along the isotropic with respect to the spin polarization direction. As shown in the previous section the spin diffusion is density dependence of the spin diffusion length at the special hand side of the frame. Note the scale of the oscillation period is on the right hand side of the frame.

FIG. 3: Spin diffusion length $L_d$ and spin oscillation period $L_0$ at $\theta = 3\pi/4$ and $T = 200$ K as functions of the electron density. Note the scale of the oscillation period is on the right hand side of the frame.

We now tune the gate voltage in order to change the relative importance of the Dresselhaus and the Rashba terms. The spin diffusion/injection direction is again fixed at $\theta = 3\pi/4$. The calculated spin diffusion length $L_d$ and the spin oscillation length $L_0$ are plotted against $\alpha/\beta$ in Fig. 5 at $T = 200$ K, with the spin polarizations being along $\hat{n}_0$, $\hat{z}$ and $\hat{n}_1$ respectively. It is noted that since $L_0 = \infty$ when the spin polarization is along $\hat{n}_0$ [((110)], we only plot $L_0$ for the two perpendicular directions in the figure.

Three main results are obtained: (i) Differing from the case without the cubic Dresselhaus term, the maximum diffusion length occurs interestingly at $\alpha/\beta \sim 0.95$ (instead of 1) for each spin polarization direction; (ii) The isotropy of the spin polarization direction at $\alpha/\beta \neq 1$ is suppressed except at $\alpha/\beta \sim 0.9$, and the diffusion lengths

C. Gate-voltage dependence with injection direction along (110)

When the spin diffusion/injection direction is $\theta = 3\pi/4$, the $\mathbf{k}$-dependent component in the zero-th order term of $\omega_k$ disappears. This results in the same inhomogeneous broadening for any spin polarization.

B. Temperature and electron-density dependence with injection direction along (110) and $\alpha = \beta$

Now we investigate the temperature and electron density dependence of the spin diffusion length at the special spin diffusion/injection direction $\theta = 3\pi/4$ with $\alpha = \beta$. As shown in the previous section the spin diffusion is isotropic with respect to the spin polarization direction. Hence we choose the spin polarization to be along the $z$-axis. In Fig. 3 the oscillation period and the diffusion length are plotted as functions of the electron density. One finds that the diffusion length $L_d$ decreases with the density, whereas the oscillation period $L_0$ increases with it. However, it is noted that the change observed in the oscillation period is almost negligible (1 %) in contrast to the pronounced changes observed in the diffusion length (of the order of 40 %). We further investigate the temperature dependence of the spin diffusion length and the oscillation period in Fig. 4 by taking two typical electron densities, i.e., $N = 4 \times 10^{10}$ cm$^{-2}$ and $4 \times 10^{11}$ cm$^{-2}$. Similar behavior is found here compared to the density dependence: the spin diffusion length $L_d$ decreases markedly with the temperature whereas the oscillation period $L_0$ only increases slightly with it.

Both dependences above can be again understood by the inhomogeneous broadening. Unlike the results in Ref. [24] where only the Dresselhaus term is considered and a very weak temperature dependence is obtained, the diffusion length here has a strong temperature dependence. This can be understood by the difference in the behavior of the inhomogeneous broadening, which is determined by the anisotropy of the precession frequency $\omega_k$. In the previous case, the zero-th order term dominates the inhomogeneous broadening which changes little with temperature and electron density. However, in the present case with $\alpha = \beta$ and $\theta = 3\pi/4$, the $\mathbf{k}$-dependent zeroth-order term is always zero and the second-order term alone determines the inhomogeneous broadening. This term increases effectively with the temperature and with the electron density due to the increase of the average value of $k^2$. This is the reason for the obtained marked decrease of the spin diffusion length. The oscillation period $L_0$ is determined by the $\mathbf{k}$-independent zeroth-order term of $\omega_k$, which is independent of the electron density and the temperature. Therefore one observes only a slight change in the oscillation period, originating from the second-order term and the scattering.
by cession frequency along the diffusion direction is given. These results can also be understood from the in-
ternal; (iii) The spin oscillation period
homogeneous broadening. For
for the spin polarization along \( \hat{z} \) and \( \hat{n}_1 \) are always iden-
tical; (iii) The spin oscillation period \( L_0 \) decreases with \( \alpha/\beta \). These results can also be understood from the in-
homogeneous broadening. For \( \theta = 3\pi/4 \), the spin pre-
cession frequency along the diffusion direction is given by

\[
\omega_k = \left( (-\beta + \alpha + \frac{\gamma k^2}{2}) \frac{k_y}{k_x} - \gamma k_x k_y \right) \hat{n}_1 \\
- \left( \beta + \alpha - \frac{\gamma}{2}(k_x^2 - k_y^2) \right) \hat{n}_0.
\]

At \( T = 200 \) K, \( \langle k^2 \rangle / 2 \sim 0.11 (\pi/\alpha)^2 \) with \( \langle \cdot \rangle \) representing
the average over the imbalance of the spin-up and -down electrons. Therefore, \(-\beta + \alpha + \frac{\gamma k^2}{2} \hat{n}_1 \) is almost zero
at \( \alpha/\beta \sim 0.95 \) and does not contribute to the inhomo-
geneous broadening. This results in a maximum of the spin diffusion length. It is further noted that the inho-
mogeneous broadening term \( \frac{1}{2}(k_x^2 - k_y^2) \hat{n}_0 \) gives a larger effect when the spin polarization is along \( \hat{z} \) and \( \hat{n}_1 \) than that when the spin polarization is along \( \hat{n}_0 \) as the latter coincides with the precession axis. This explains why
one obtains a larger diffusion length when the spin po-
larization is along \( \hat{n}_0 \) at \( \alpha/\beta \sim 0.95 \). Finally as the spin
oscillation period \( L_0 \) is determined by the magnitude of the \( k \)-independent term of \( \omega_k \), i.e., \( \beta(\alpha/\beta + 1) \hat{n}_0 \), which increases with \( \alpha/\beta \). Therefore, \( L_0 \) decreases with \( \alpha/\beta \).

V. CONCLUSIONS

In conclusion, we study the spin diffusion in (001)
GaAs quantum wells with competing Dresselhaus and
Rashba spin-orbit coupling strengths by solving the
KSBE with the electron-phonon and the electron-
electron Coulomb scattering explicitly included. It is
shown that unlike the spin dephasing in the time do-
main where strong anisotropy is determined by different
spin polarization, for spin diffusion it is also determined
by the spin diffusion/injection direction. By neglecting
the cubic term of the Dresselhaus spin-orbit coupling and
with \( \alpha = \beta \), the ideal case of an infinite diffusion length
is obtained either for the spin polarization along (110)
regardless of the spin diffusion direction or for the spin
diffusion direction along (110) regardless of the spin po-
larization direction. However, the cubic term cannot be
neglected, resulting in a finite diffusion length, which in
fact is small, about 2 \( \mu \)m at the temperature of 200 K
when the electron density lies between \( 4 \times 10^{10} \) cm\(^{-2} \) and
\( 4 \times 10^{11} \) cm\(^{-2} \). It is then shown that when \( \alpha = \beta \), for the
spin polarization along (110), the spin diffusion length
changes little for the spin diffusion direction, whereas for
the other two perpendicular spin polarization directions
(110) and (001), the spin diffusion length shows strong
anisotropy for the spin diffusion/injection direction and
has a peak when the spin diffusion/injection direction is
along (110). When the spin diffusion/injection direction
is along (101), the spin diffusion length is isotropic for
spin polarization. The electron density and temperature
dependence of the spin diffusion is closely investigated.

We also tune the gate voltage to show the competing
effect of the Dresselhaus and the Rashba strengths. It is
found that with the cubic Dresselhaus term included, the
maximum spin diffusion length appears at \( \alpha/\beta \sim 0.95 \) in-
stead of 1 where the cubic term is neglected. The spin dif-
fusion length decreases when the ratio of the Rashba and
Dresselhaus strengths deviates from 0.95. These results
can be well understood by the inhomogeneous broaden-
ing.

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Semiconductor Spintronics and Quantum Computation, edited by D. D. Awschalom, D. Loss, and N. Samarth (Springer-Verlag, Berlin, 2002).

I. Žutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. 76, 323 (2004).

N. S. Averkiev and L. E. Golub, Phys. Rev. B 60, 15582 (1999).

M. W. Wu, M. Q. Weng, and J. L. Cheng, to be published in Physics, Chemistry and Application of Nanostructures (World Scientific, Singapore, 2007); cond-mat/0611649, and references therein.

M. Q. Weng and M. W. Wu, Phys. Rev. B 66, 235109 (2002); J. Appl. Phys. 93, 410 (2003).

M. W. Wu and H. Metiu, Phys. Rev. B 61, 2945 (2000); M. Q. Weng and M. W. Wu, ibid. 68, 075312 (2003).

M. W. Wu and L. Jiang, Phys. Rev. B 69, 245320 (2004).

N. S. Averkiev, L. E. Golub, and M. Willander, J. Phys.: Condens. Matter 14, R271 (2002).

J. Schliemann, J. C. Egues, and D. Loss, Phys. Rev. Lett. 90, 146801 (2003).

J. Schliemann and D. Loss, Phys. Rev. B 68, 165311 (2003).

S. I. Erlingsson, J. Schliemann, and D. Loss, Phys. Rev. B 71, 035319 (2005).

R. Winkler, Phys. Rev. B, 69, 045317 (2004).

C. Lechner and U. Rössler, Phys. Rev. B 72, 045311 (2005).

X. Cartoixà, D. Z.-Y. Ting, and Y.-C. Chang, Phys. Rev. B 71, 045313 (2005).

E. Shafir, M. Shen, and S. Saikin, Phys. Rev. B 70, 241302(R) (2004).

S. Datta and B. Das, Appl. Phys. Lett. 56, 665 (1990).

M. I. D'yakonov and V. I. Perel', Zh. Eksp. Teor. Fiz. 60, 1954 (1971) [Sov. Phys. JETP 33, 1053 (1971)]; M. I. D'yakonov and V. I. Perel', Fiz. Tverd. Tela 13, 3581 (1971) [Sov. Phys. Solid State 13, 3023 (1972)]; M. I. D'yakonov and V. Yu. Kachorovskii, Fiz. Tekh. Poluprovodn. 20, 178 (1986).

G. Dresselhaus, Phys. Rev. 100, 580 (1955).

Y. A. Bychkov and E. I. Rashba, J. Phys. C: Solid State Phys. 17, 6039 (1984); Pis'ma Zh. Eksp. Teor. Fiz. 39, 66 (1984) [JETP Lett. 39, 78 (1984)].

J. L. Cheng and M. W. Wu, J. Appl. Phys. 99, 083704 (2006).

R. Winkler, Spin-Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems (Springer, Berlin, 2003).

H. Haug and A.P. Jauho, Quantum Kinetics in Transport and Optics of Semiconductors (Springer, Berlin, 1996).

J. L. Cheng and M. W. Wu, J. Appl. Phys. 101, 2007, in press; cond-mat/0608413.

C. Li, J. L. Cheng, and M. W. Wu, Phys. Rev. B 73, 125314 (2006).

J. Zhou, J. L. Cheng, and M. W. Wu, Phys. Rev. B 75, 045305 (2007).

M. W. Wu and C. Z. Ning, Eur. Phys. J. B 18, 373 (2000); M. W. Wu, J. Phys. Soc. Jpn. 70, 2195 (2001).

L. Allen and J. H. Eberly, Optical Resonance and Two-level Atoms (Dover, New York, 1975).

B. Jusserand, D. Richards, H. Peric, and B. Etienne, Phys. Rev. Lett. 69, 848 (1992); B. Jusserand, D. Richards, G. Allan, C. Priester, and B. Etienne, Phys. Rev. B 51, 4707 (1995).

M. A. Brand, A. Malinowski, O. Z. Karimov, P. A. Marden, R. T. Harley, A. J. Shields, D. Sanvitto, D. A. Ritchie, and M. Y. Simmons, Phys. Rev. Lett. 89, 236601 (2002).