Why is the current seeding regime of the UEFA Champions League unfair?

László Csató*

Institute for Computer Science and Control (SZTAKI)
Laboratory on Engineering and Management Intelligence
Research Group of Operations Research and Decision Systems

Corvinus University of Budapest (BCE)
Department of Operations Research and Actuarial Sciences

Budapest, Hungary

17th December 2019

Abstract

Fairness has several interpretations in sports, one of them being that the rules should guarantee incentive compatibility, namely, a team cannot be worse off due to better results in any feasible scenario. The current seeding regime of the most prestigious annual European club football tournament, the UEFA (Union of European Football Associations) Champions League, is shown to violate this requirement since the 2015/16 season. In particular, if the titleholder qualifies for the first pot by being a champion in a high-ranked league, its slot is given to a team from a lower-ranked association, which can harm a top club from the domestic championship of the titleholder. However, filling all vacancies through the national leagues excludes the presence of perverse incentives. UEFA is encouraged to introduce this policy from the 2021-24 cycle onwards.

Keywords: OR in sports; football; UEFA Champions League; seeding; incentive compatibility

MSC class: 62F07, 91B14

JEL classification number: C44, D71, Z20

* E-mail: csato.laszlo@sztaki.hu

1 “When once the mind has taken a decided direction towards an object, or turned back towards a harbour of refuge, it may easily happen that the motives which in the one base naturally serve to restrain, and those which in the other as naturally excite to enterprise, are not felt at once in their full force; and as the progress of action in the mean time continues, one is carried along by the stream of movement beyond the line of equilibrium, beyond the culminating point, without being aware of it.” (Source: Carl von Clausewitz: *Vom Kriege*, Book 7, Chapter 21 – Invasion. Translated by Colonel James John Graham, London, N. Trübner, 1873. [http://clausewitz.com/readings/OnWar1873/TOC.htm](http://clausewitz.com/readings/OnWar1873/TOC.htm))

Hat die Seele einmal eine bestimmte Richtung fort zum Ziele oder zurückgewendet nach einem Rettungshafen, so geschieht es leicht, daß die Gründe, welche den einen zum Innehalten nötigen, den anderen zum Unternehmen berechtigen, nicht leicht in ihrer ganzen Stärke gefühlt werden, und da die Handlung indes fortschreitet, so kommt man im Strom der Bewegung über die Grenze des Gleichgewichts, über die Kulminationslinie hinaus, ohne es gewahr zu werden.¹

(Carl von Clausewitz: *Vom Kriege*)
1 Introduction

It is widely accepted that every sporting contest should provide the appropriate incentives to perform (Szymanski, 2003). However, this simple requirement does not always hold as several historical cases attest (Preston and Szymanski, 2003; Kendall and Lenten, 2017). A classical example is presented by designs where the lowest-ranked team receives the first draft pick in the following season, which makes losing profitable after a team is eliminated from the later rounds (e.g. play-offs) of the tournament (Taylor and Trogdon, 2002; Balsdon et al., 2007). Since managers are found to apply concrete tanking strategies (Fornwagner, 2019), it would be important to adopt a policy that ensures integrity (Lenten, 2016; Lenten et al., 2018).

According to recent game-theoretical studies (Pauly, 2014; Vong, 2017), incentive incompatibility sometimes cannot be avoided because the unique theoretical solution would be too harsh to implement in practice, for example, by allowing only the top team to proceed from a round-robin tournament. On the other hand, in certain cases there exists an (almost) costless guarantee of fairness, and there is even some development towards this outcome in the real-world.

To mention some illustrative cases, Durán et al. (2017) demonstrate the openness of the governing bodies in football to improve fairness by rescheduling the FIFA World Cup South American qualifiers. The drawing procedure of the 2018 FIFA World Cup was reformed to resemble one of the suggestions in Guyon (2015b). UEFA used the results of Guyon (2018b) to modify the knockout bracket in the UEFA European Championship 2020 to minimise group advantage.

There is also an evolution in the direction of incentive compatibility. For instance, tournament systems, consisting of one round-robin and multiple knockout tournaments with noncumulative prizes, are proved to satisfy strategy-proofness only if all vacant slots are awarded based on the results of the round-robin tournament (Dagaev and Sonin, 2018). The qualification to the two annual European club football tournament organised by the UEFA (Union of European Football Associations) was incentive incompatible due to this result: the entry of the more prestigious UEFA Champions League between 2015 and 2018 (Csató, 2019b), and the entry of the second-tier competition UEFA Europa League before 2016 (Dagaev and Sonin, 2018). However, these mistakes have been corrected, and now no team can be strictly better off by losing in both championships.

In the following, we will present that the seeding regime applied in the group stage of the UEFA Champions League from the 2015/16 season leads to another form of incentive incompatibility: the rules may punish a team for better results in its domestic championship by seeding it in a weaker pot. Naturally, a straightforward solution is also provided.

This is probably the first paper analysing the draw systems of sports tournaments with respect to strategy-proofness, which is our main contribution. On the other hand, the effects of the seeding reforms in the UEFA Champions League have been recently evaluated via Monte-Carlo simulations in Dagaev and Rudyak (2019) and Corona et al. (2019). The current article is strongly connected to the works investigating the draw of round-robin groups under some geographical and/or seeding restrictions, too (Guyon, 2015b; Laliéna and López, 2019; Cea et al., 2019). The procedure of the UEFA Champions League Round of 16 draw has been considered by Klößner and Becker (2013), as well as by Boczoń and Wilson (2018). The importance of our analysis is reinforced by the fact that the draws of the UEFA Champions League are regularly discussed in the mainstream media (Guyon, 2015a, 2017a,b,c, 2018a, 2019b).
The structure of the paper is as follows. Section 2 presents a hypothetical example with a slight modification of real-world match results to motivate our approach. Its implications are discussed in Section 3 together with a suggestion for guaranteeing incentive compatibility. Finally, Section 4 summarises the main message.

2 An illustrative example

The participants of the 2015/16 UEFA Champions League were determined by the previous season of the national leagues across the continent, as well as by the two European club competitions, the UEFA Champions League (shortly Champions League or simply CL) and the UEFA Europa League (shortly Europa League or simply EL). We start from the real results except for the following modifications:

- Sevilla FC defeated FC Barcelona in the Spanish La Liga on 11 April 2015 (the real result was 2-2);
- SK Rapid Wien advanced to the Champions League group stage from the play-off round of the League Route in the qualifying against FC Shakhtar Donetsk (in fact, FC Shakhtar Donetsk won 3-2 on aggregate).

In this case, the Spanish national league would have been won by Real Madrid CF as it would have 92 points similarly to FC Barcelona but better head-to-head results, which was the first tie-breaking criterion (the outcomes of their clashes were 3-1 in Madrid and 2-1 in Barcelona, see https://en.wikipedia.org/wiki/2014%E2%80%9315_%La_Liga). Nonetheless, the CL titleholder, FC Barcelona, would have also qualified through its domestic championship, thus the vacant slot would have been filled by the EL titleholder Sevilla FC from Spain, despite being only the fifth in La Liga (UEFA, 2015, Article 3.04).

The draw of the Champions League group stage was regulated by UEFA (2015, Article 13.05) as follows:

“For the purpose of the draw, the 32 clubs involved in the group stage are seeded into four groups of eight. The first group comprises the titleholder (top seed) and the domestic champions of the seven top-ranked associations in accordance with the access list (see Annex A). If the titleholder is one of the top seven associations’ domestic champions, the group is completed with the champion of the association ranked eight. The other three groups are composed in accordance with the club coefficient rankings established at the beginning of the season (see Annex D).”

Table 1 shows the composition of the pots in the scenario above. Note that the CL titleholder is not a domestic champion of the top seven associations, and Sevilla FC is the lowest-ranked team of Pot 2.

Consider what happens if Sevilla FC would have scored at least one point against Real Madrid CF in the 2014/15 season of the Spanish national league (the real results of their matches were 2-3 in Sevilla and 2-1 in Madrid). Then FC Barcelona would have won La Liga, and the teams in the Champions League would have remained the same. However, Pot 1 contains the champion of the association ranked eight, PSV Eindhoven from the Netherlands, instead of Real Madrid CF. Hence the latter club is drawn from Pot 2, while Sevilla FC is relegated to Pot 3.

This outlined (hypothetical) scenario reveals that the seeding rules of the Champions League group stage can punish Sevilla FC for having more favourable results in its domestic championship as it should face a team from the stronger Pot 2 instead of the weaker Pot 3.
Table 1: Pot composition in the (hypothetical) 2015/16 UEFA Champions League

**Bold** teams qualified directly for the group stage. Coefficient stands for the UEFA club coefficient (source: [https://www.footballseeding.com/club-ranking/a2014-2015/](https://www.footballseeding.com/club-ranking/a2014-2015/)).

(a) Pot 1

| Club                          | Association (position) | Coefficient |
|-------------------------------|------------------------|-------------|
| **Real Madrid CF**            | Spain (champion)       | 171.999     |
| **FC Barcelona**              | Spain (runner-up, CL titleholder) | 164.999     |
| Chelsea FC                    | England (champion)     | 142.078     |
| **FC Bayern Munchen**         | Germany (champion)     | 154.833     |
| Juventus                      | Italy (champion)       | 95.102      |
| SL Benfica                    | Portugal (champion)    | 118.276     |
| Paris Saint-German FC         | France (champion)      | 100.483     |
| FC Zenit St Petersburg        | Russia (champion)      | 90.099      |

(b) Pot 2

| Club                          | Association (position) | Coefficient |
|-------------------------------|------------------------|-------------|
| Club Atlético de Madrid       | Spain (3rd)            | 120.999     |
| **FC Porto**                  | Portugal (runner-up)   | 111.276     |
| Arsenal                       | England (3rd)          | 110.078     |
| Manchester United FC          | England (4th)          | 103.078     |
| Valencia CF                   | Spain (4th)            | 99.999      |
| Bayer 04 Leverkusen            | Germany (4th)          | 87.883      |
| Manchester City FC            | England (runner-up)    | 87.078      |
| **Sevilla FC**                | Spain (5th, EL titleholder) | 80.499      |

(c) Pot 3

| Club                          | Association (position) | Coefficient |
|-------------------------------|------------------------|-------------|
| **Olympique Lyonnais**        | France (runner-up)     | 72.983      |
| FC Dinamo Kyiv                | Ukraine (champion)     | 65.033      |
| **Olympiacos FC**             | Greece (champion)      | 62.380      |
| PSV Eindhoven                 | Netherlands (champion) | 58.195      |
| PFC CSKA Moskva               | Russia (runner-up)     | 55.199      |
| Galatasaray AŞ                | Turkey (champion)      | 50.020      |
| **AS Roma**                   | Italy (runner-up)      | 43.602      |
| FC BATE Borisov               | Belarus (champion)     | 35.150      |

(d) Pot 4

| Club                          | Association (position) | Coefficient |
|-------------------------------|------------------------|-------------|
| **VfL Borussia Mönchengladbach** | Germany (3rd)        | 33.883      |
| **VfL Wolfsburg**             | Germany (runner-up)   | 31.883      |
| GNK Dinamo Zagreb             | Croatia (champion)    | 24.700      |
| Maccabi Tel-Aviv FC           | Israel (champion)     | 18.200      |
| SK Rapid Wien                 | Austria (runner-up)   | 15.635      |
| **KAA Gent**                  | Belgium (champion)    | 13.440      |
| Malmö FF                      | Sweden (champion)     | 12.545      |
| FC Astana                     | Kazakhstan (champion) | 3.825       |
3 Discussion

What is the price of seeded in Pot 3 rather than in Pot 2? Corona et al. (2019) analyse the effects of the new seeding regime for the teams participating in the 2015/16 Champions League. Compared to the original seeding based exclusively on the UEFA club coefficients, FC Shakhtar Donetsk was lowered to Pot 3, which lead to a substantial reduction in the probability of qualifying to the first knockout round, from 0.633 to 0.483.

We have also attempted to quantify how Sevilla FC suffers from the unfair rules. For this purpose, the club Elo ratings from http://clubelo.com/ have been used. It measures the strength of each club on the basis of its past results such that winning against a stronger team is more valuable, while the influence of a game decreases when new matches are played. Besides that, club Elo also reflects home advantage and goal difference. Elo-inspired methods provide good predictive performance (Lasek et al., 2013), and have been extensively applied in the scientific literature (Hvattum and Arntzen, 2010; Lasek et al., 2016; Cea et al., 2019; Csató, 2019c). In particular, Csató (2019a) uses the same dataset to illuminate the impact of reforming the Champions League qualification in 2018.

In order to take into account the dynamic nature of this estimation of strength, the average of the Elo ratings on the day of the group stage draw (27 August 2015, see http://clubelo.com/2015-08-27/Data) and one day after the last match of the group stage (10 December 2015, http://clubelo.com/2015-12-10/Data) have been considered. Furthermore, clubs from the same association could not be drawn against each other in the group stage of the Champions League, therefore Sevilla FC is not allowed to play against Real Madrid, Club Atlético de Madrid, and Valencia CF if it would be drawn from Pot 3.

Figure 1: The average Elo rating of the possible opponents of Sevilla FC in the group stage of the 2015/16 UEFA Champions League

Figure 1 highlights that Sevilla FC is certainly better off in our hypothetical scenario if it would be drawn from Pot 2 as all of the eight possible opponents are weaker than any
possible opponent if the club would be drawn from Pot 3. The expected Elo rating of the eight teams from Pot 3 is 1689.625, while this value is 1830.9 for the five teams from Pot 2. Since Sevilla FC has an Elo rating of 1871.5, it wins with a 74.02% probability against the former teams, but only with a 55.82% probability against the latter clubs according to the standard formula $\frac{1}{1 + 10^{-\Delta/400}}$, where $\Delta$ is the difference between the Elo ratings of the two teams. This is a robust difference, especially because it can be crucial with respect to qualifying for the knockout stage as only the first two teams from each group advance, while the third is relegated to the Round of 32 in the Europa League.

To summarise, the ill-designed seeding regime from the 2015/16 season of the UEFA Champions League can severely harm an innocent team merely for scoring more points in its domestic championship. The Champions League is regulated in three-year cycles, thus the problem caused by UEFA (2015, Article 13.05) could have emerged in the three seasons played between 2015 and 2018.

Since the 2018/19 season, the titleholder of the Europa League from the previous year also automatically qualifies for the group stage of the Champions League, where it is seeded in Pot 1. Consequently, the top pot consists of the two titleholders and the champions of the six highest-ranked associations. Furthermore, all vacancies are awarded to the champion(s) of the next highest-ranked association(s) as UEFA (2019, Article 13.06) describes for the 2019/20 season. It means that the problem has probably become worse because the scenario presented in Section 2 can occur in two national leagues, although there were no such vacancies in the 2018/19 and 2019/20 seasons.

The policy of guaranteeing a place in the top pot to certain champions can be criticised for creating unbalanced groups, too (Guyon, 2019a, Section 3.4). For example, FC Lokomotiv Moskva was in Pot 1 in the 2018/19 season as the champion of Russia (the sixth-ranked UEFA association), while its UEFA club coefficient would have placed the team only in Pot 4. Unsurprisingly, FC Lokomotiv Moskva was the fourth in its group, and two “lucky” teams, the Portuguese FC Porto from Pot 2 and the German FC Schalke 04 from Pot 3 had an easy path to the Round of 16. In addition, forming Pot 1 on the basis of national leagues and ignoring this principle for the other pots is inconsistent, difficult to justify, and unfair to the champion of the next best league (Guyon, 2015a). Interestingly, there is no such differentiation in the UEFA Europa League, where all pots are created based on the UEFA club coefficients.

Nonetheless, while rewarding league champions in a tournament called Champions League seems to be a reasonable principle – even though it is moving farther from its original concept of being a “league of champions” (Csató, 2019a) –, the current definition of Pot 1 remains unfair. It is a shame because there exists a straightforward solution, revealed by Dagaev and Sonin (2018, Proposition 3): all vacancies should be filled through the round-robin tournament, i.e. the national leagues. That is, UEFA (2019, Article 13.06) should be modified in the following way:

“For the purpose of the draw, the 32 clubs involved in the group stage are seeded into four groups of eight. The first group comprises the titleholder (top seed), the UEFA Europa League titleholder and the domestic champions of the six associations ranked highest in the access list (see Annex A). If either or both titleholders are the domestic champions of one of the top six associations, the group is completed with the runner(s)-up (and the third-placed club) of the same association(s). The other three groups are composed in accordance with the club coefficient rankings established at the beginning of the season (see Annex D).”

Our proposal immediately guarantees incentive compatibility because of no champion
can gain a slot in Pot 1 due to the identity of the titleholder(s), hence no team would be relegated to a lower pot merely by having better results in its domestic league. With this policy, Sevilla FC would have been placed into Pot 2 regardless of which team would have won La Liga in our hypothetical example of Section 2. UEFA is encouraged to introduce the suggested amendment in the Champions League from the 2021-24 cycle onwards.

4 Conclusions

Regulations that allow for a successful tanking strategy or a punishment of a team when it scores more points threaten the integrity of sports and are against the spirit of the game. Therefore, the design of a sports tournament remains an important field of analysis for game theory and operations research. In our opinion, the scientific community has a responsibility to present all possible cases of incentive incompatibility, regardless of the frequency of dubious situations.

We have revealed that the seeding of the clubs into pots in the group stage of the UEFA Champions League, the most prestigious annual club football competition in Europe, suffers from perverse incentives since the 2015/16 season because vacancies in the top pot are filled through an ill-constructed policy. Hopefully, this work will persuade the administrators to implement our straightforward proposal for solving the problem before it causes controversy.

Acknowledgements

Discussions with Julien Guyon provided an important motivation for the paper. We are indebted to the Wikipedia community for contributing to our work by collecting and structuring useful information on the tournaments discussed. The research was supported by the MTA Premium Postdoctoral Research Program grant PPD2019-9/2019.

References

Balsdon, E., Fong, L., and Thayer, M. A. (2007). Corruption in college basketball? Evidence of tanking in postseason conference tournaments. *Journal of Sports Economics*, 8(1):19–38.

Boczoń, M. and Wilson, A. J. (2018). Goals, constraints, and public assignment: A field study of the UEFA Champions League. Technical Report 18/016, University of Pittsburgh, Kenneth P. Dietrich School of Arts and Sciences, Department of Economics. [https://www.econ.pitt.edu/sites/default/files/working_papers/Working%20Paper.18.16.pdf](https://www.econ.pitt.edu/sites/default/files/working_papers/Working%20Paper.18.16.pdf).

Cea, S., Durán, G., Guajardo, M., Sauré, D., Siebert, J., and Zamorano, G. (2019). An analytics approach to the FIFA ranking procedure and the World Cup final draw. *Annals of Operations Research*, in press. DOI: 10.1007/s10479-019-03261-8.

Corona, F., Forrest, D., Tena, J. D., and Wiper, M. (2019). Bayesian forecasting of UEFA Champions League under alternative seeding regimes. *International Journal of Forecasting*, 35(2):722–732.
Csató, L. (2019a). UEFA against the champions? An evaluation of the recent reform of the Champions League qualification. Manuscript. arXiv: 1912.02076.

Csató, L. (2019b). UEFA Champions League entry has not satisfied strategyproofness in three seasons. *Journal of Sports Economics*, 20(7):975–981.

Csató, L. (2019c). The unfairness of the qualification for the UEFA European Championship 2020. Manuscript. arXiv: 1905.03325.

Dagaev, D. and Rudyak, V. (2019). Seeding the UEFA Champions League participants: Evaluation of the reform. *Journal of Quantitative Analysis in Sports*, 15(2):129–140.

Dagaev, D. and Sonin, K. (2018). Winning by losing: Incentive incompatibility in multiple qualifiers. *Journal of Sports Economics*, 19(8):1122–1146.

Durán, G., Guajardo, M., and Sauré, D. (2017). Scheduling the South American Qualifiers to the 2018 FIFA World Cup by integer programming. *European Journal of Operational Research*, 262(3):1109–1115.

Fornwagner, H. (2019). Incentives to lose revisited: The NHL and its tournament incentives. *Journal of Economic Psychology*, 75(Part B):102088.

Guyon, J. (2015a). Champions League: How to Solve the Seeding Problem. *The New York Times*. 21 January. https://www.nytimes.com/2015/01/22/upshot/champions-league-how-to-solve-the-seeding-problem.html.

Guyon, J. (2015b). Rethinking the FIFA World Cup™ final draw. *Journal of Quantitative Analysis in Sports*, 11(3):169–182.

Guyon, J. (2017a). Barcelona x Chelsea é o confronto com mais chances de acontecer na Champions; veja outras probabilidades (in Portuguese). *El País Brasil*. 11 December. https://brasil.elpais.com/brasil/2017/12/07/deportes/1512643772_372491.html.

Guyon, J. (2017b). Ligue des champions : pourquoi le PSG a presque une chance sur trois de rencontrer Chelsea (in French). *Le Monde*. 10 December. https://www.lemonde.fr/football/article/2017/12/10/ligue-des-champions-pourquoi-le-psg-a-une-chance-sur-trois-de-rencontrer-chelsea_5227638_1616944.html.

Guyon, J. (2017c). Por qué el Barcelona tiene un 41.3% de probabilidades de emparejarse con el Chelsea en octavos (in Spanish). *El País*. 7 December. https://elpais.com/deportes/2017/12/07/actualidad/1512643772_372491.html.

Guyon, J. (2018a). Ligue des champions : Atléticão Madrid ? Dortmundo ? Qui seront les adversaires du PSG et de Lyon en 8e de finale ? (in French). *Le Monde*. 13 December. https://www.lemonde.fr/football/article/2018/12/13/ligue-des-champions-les-adversaires-les-plus-probables-pour-le-psg-et-lyon-en-8e-de-finale_5396808_1616938.html.

Guyon, J. (2018b). What a fairer 24 team UEFA Euro could look like. *Journal of Sports Analytics*, 4(4):297–317.
Guyon, J. (2019a). “Choose your opponent”: A new knockout format for sports tournaments. Application to the Round of 16 of the UEFA Champions League and to maximizing the number of home games during the UEFA Euro 2020. Manuscript. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3488832.

Guyon, J. (2019b). Ligue des champions : pourquoi les <<TV pairings>> ont augmenté la probabilité d’un PSG-Real (in French). Le Monde. 30 August. https://www.lemonde.fr/sport/article/2019/08/30/ligue-des-champions-pourquoi-les-tv-pairings-ont-augmente-la-probabilite-d-un-psg-real_5504652_3242.html.

Hvattum, L. M. and Arntzen, H. (2010). Using ELO ratings for match result prediction in association football. International Journal of Forecasting, 26(3):460–470.

Kendall, G. and Lenten, L. J. A. (2017). When sports rules go awry. European Journal of Operational Research, 257(2):377–394.

Klößner, S. and Becker, M. (2013). Odd odds: The UEFA Champions League Round of 16 draw. Journal of Quantitative Analysis in Sports, 9(3):249–270.

Laliena, P. and López, F. J. (2019). Fair draws for group rounds in sport tournaments. International Transactions in Operational Research, 26(2):439–457.

Lasek, J., Szlávik, Z., and Bhulai, S. (2013). The predictive power of ranking systems in association football. International Journal of Applied Pattern Recognition, 1(1):27–46.

Lasek, J., Szlávik, Z., Gagolewski, M., and Bhulai, S. (2016). How to improve a team’s position in the FIFA ranking? A simulation study. Journal of Applied Statistics, 43(7):1349–1368.

Lenten, L. J. A. (2016). Mitigation of perverse incentives in professional sports leagues with reverse-order drafts. Review of Industrial Organization, 49(1):25–41.

Lenten, L. J. A., Smith, A. C. T., and Boys, N. (2018). Evaluating an alternative draft pick allocation policy to reduce ‘tanking’ in the Australian Football League. European Journal of Operational Research, 267(1):315–320.

Pauly, M. (2014). Can strategizing in round-robin subtournaments be avoided? Social Choice and Welfare, 43(1):29–46.

Preston, I. and Szymanski, S. (2003). Cheating in contests. Oxford Review of Economic Policy, 19(4):612–624.

Szymanski, S. (2003). The economic design of sporting contests. Journal of Economic Literature, 41(4):1137–1187.

Taylor, B. A. and Trogdon, J. G. (2002). Losing to win: Tournament incentives in the national basketball association. Journal of Labor Economics, 20(1):23–41.

UEFA (2015). Regulations of the UEFA Champions League 2015-18 Cycle. 2015/16 Season. http://www.uefa.com/MultimediaFiles/Download/Regulations/uefaorg/Regulations/02/23/57/51/2235751_DOWN.pdf.
UEFA (2019). *Regulations of the UEFA Champions League 2018-21 Cycle. 2019/20 Season*.  
https://www.uefa.com/MultimediaFiles/Download/Regulations/uefaorg/Regulations/02/60/37/12/2603712_DOWNLOAD.pdf.

Vong, A. I. K. (2017). Strategic manipulation in tournament games. *Games and Economic Behavior*, 102:562–567.