General Relativistic Magnetohydrodynamic Bondi–Hoyle Accretion

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In this paper we present a fully relativistic study of axisymmetric magnetohydrodynamic Bondi–Hoyle accretion onto a moving Kerr black hole. The equations of general relativistic magnetohydrodynamics are solved using high resolution shock capturing methods. In this treatment we consider the ideal MHD limit. The parameters of interest in this study are the adiabatic constant $\Gamma$, the asymptotic speed of sound $c_s^\infty$, and the plasma beta parameter $\beta_P$. We focus the investigation on the parameter regime in which the flow is supersonic, or when $v_\infty \geq c_s^\infty$. In some cases, subsonic asymptotic flows are considered for comparison purposes. We study the accretion rates of the total energy and momenta, as well as the hydrodynamic energy and momentum accretion rates. The models presented in this study exhibit a matter density depletion in the downstream region of the black hole which tends to vacuum ($\rho_0 = 0$) in convergence tests. This feature is due to the presence of the magnetic field, more specifically the magnetic pressure, and is not seen in previous purely hydrodynamic studies.

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I. INTRODUCTION

The Bondi–Hoyle–Lyttleton accretion process was originally studied in 1942 [4]. This model of the accretion process assumes a massive point particle travels through a perfect fluid background (no heat transport) with a Newtonian treatment of gravity. Such a configuration is thought to approximate the dynamics of two bodies in a common envelope, or the dynamics of a body traveling through the medium accreting onto an active galactic nuclei [9]. In such astrophysical systems we are able to safely neglect viscosity in the fluid treatment due to the length and velocity scales involved in the dynamics. However, past investigations of this phenomenon rarely account for the presence of magnetic fields. In many cases this is due to the relative simplicity of the hydrodynamic models. Astrophysically, it is expected that material accreting onto, say, an AGN, will be highly ionized, and consequently a magnetic field will be generated by the accreting material [7]. Due to the high conductivity of the accretion disk, the magnetic field is bound to the disk, by an effect known as flux freezing [23]. The flux freezing is a characteristic of ideal magnetohydrodynamics. The typical magnetic field strengths for AGN have a wide range, from $10^9$G [12] to $10^{10}$G [22] (as cited by Tyul’bashev [37]). The magnetic fields observed in common envelopes are on the order of $10^7 - 10^8$G [34].

We investigate a modified version of this problem where the massive point particle is replaced with a black hole with a non-trivial radius. Consequently, we must consider a relativistic treatment of gravity. This line of research began with Petrich et al. [31] where they found a closed form solution of an ultrarelativistic spherically symmetric black hole and found that the flows will be steady. Further work by the same authors, [30], solved the fluid equations of motion using numerical methods. In [30] they studied both the point mass model with Newtonian gravity, as well as a model in which the point mass is replaced with a spherically symmetric black hole and relativistic treatment of gravity. Both the Newtonian and relativistic models were studied using axisymmetry. In both works they found that the flows were steady to long term evolution. Font et al. [15, 16, 29] extended this research to include an axisymmetric black hole, and in [17] they modelled a infinitely thin disk model, where accretion is assumed to occur only in the equatorial plane. In all previous research only a hydrodynamic background fluid was studied, and all relativistic flows were determined to reach a steady state. This problem continues to be of interest in modern research such as in the work by Farris et al. [11] who found an application of the three dimensional relativistic Bondi–Hoyle accretion in binary neutron star mergers, in perturbations of fully three dimensional hydrodynamic models for Bondi–Hoyle–Lyttleton accretion [3], and most recently in the search for QPO’s once again using the infinitely thin disk approximation [35].

Our contribution to this research determines the phenomenology of the same system when including a background magnetic field. We extend the original analysis by Font et al. [15, 16, 29] by introducing an asymptotically uniform magnetic field as described by Wald [38]. In this paper, we use the ideal magnetohydrodynamic stress-energy tensor which reflects the presence of an embedded magnetic field. To parameterize the magnetic field, we use a method from plasma physics, which introduces the plasma beta parameter, $\beta_P$ [18, 19, 25]. The plasma beta parameter is a ratio of the magnetic pressure to the thermodynamic pressure. The ideal MHD approx-
imation is only valid for systems in which the plasma beta parameter obeys the condition $\beta_p \lesssim 1$. We use initial parameters for our system such that the plasma beta parameter is less than or approximately one over the entire domain of integration. As reported in [16], the interesting dynamics are in what is referred to as the supersonic regime, where the asymptotic velocity $v_\infty$ of the central body is greater than the asymptotic speed of sound $c_s^\infty$ in the fluid.

During this study we restrict our attention to the “hot” relativistic equation of state, $P = (\Gamma - 1)\rho v$. In particular, we focus on two different values of the adiabatic constant $\Gamma$, one in the non-relativistic regime $\Gamma = 5/3$, and the other the relativistic equation of state $\Gamma = 4/3$. Although other values in between these are possible and considered physically valid, we feel that the interesting dynamics for a first approach to solving the GRMHD Bondi–Hoyle problem are captured by studying the extremes. In a future study we will study a wider range of the parameters.

One of the difficulties in studying magnetized fluids involves the enforcement of the $\nabla \cdot B = 0$ constraint. The numerical treatment of magnetic fields has only become tractable with the techniques developed such as the constraint transport method [10], the flux transport method [36], or the hyperbolic-divergence cleaning method [8]. In this study we implement the hyperbolic divergence cleaning method as described by Palenzuela et al. [28].

The outline of the paper is as follows: section II will discuss the coordinates used in the problem. Section III will discuss the equations of motion used for the relativistic ideal magnetohydrodynamic system. Section IV will discuss the initial conditions and boundary conditions used to perform the simulations. Section V will briefly cover the numerical methods developed and used for the simulations. Section VI will discuss the flow morphology and in section VII we draw our conclusions.

In the rest of this paper we use geometric units, where $G = c = 1$ with $c$ the speed of light in vacuum, and $G$ being Newton’s gravitational constant. Further we will use the notation that Greek scripts run over the entire spacetime, i.e. $\mu = 0, 1, 2, 3$, and Latin scripts run over the spatial coordinates, i.e. $i = 1, 2, 3$. All variables are assumed to be functions of both the radial and polar coordinates.

II. COORDINATES

In our study we are interested in the flow around a rotating black hole. Thus the line element defined by our black hole will be described using the Kerr spacetime. The original formulation of this line element contains coordinate singularities as is described in for example [6]. To circumvent the coordinate singularity, we use what are know as Kerr–Schild coordinates, as described by Font et al. [29]. In the Kerr–Schild coordinate system, the rotat-

ing black hole line element is written

$$ds^2 = -(1 - \frac{2Mr}{\Delta})dt^2 + \frac{4Mr}{\Delta}dt dr + (1 + \frac{2Mr}{\Delta})dr^2 + (r^2 + a^2\cos^2\theta)d\theta^2 + (\Delta + a^2\left(1 + \frac{2Mr}{\Delta}\right)\sin\theta^2)\sin\theta^2 d\phi^2,$$

with

$$\Delta = r^2 + a^2\cos^2\theta.$$

$a$ is the dimensionless measure of the rotation rate of the black hole and is related to the angular momentum of the black hole via $J = Ma^2$, where $M$ is the mass of the black hole. For the present study we set $M = 1$ without loss of generality.

In this work, we consider an axisymmetric spacetime geometry. As a result of this symmetry we restrict our study to an asymptotically uniform magnetic field which is aligned with the axis of rotation of the axisymmetric black hole. This particular magnetic field configuration is the only configuration that is compatible with the symmetries imposed. For more general magnetic field configurations, such as those presented in Bicák et al. [5], we will require the use of a three dimensional code, currently under investigation.

III. EQUATIONS OF MOTION

To obtain the equations of motion for the ideal relativistic magnetohydrodynamic (MHD) system, we consider the conservation of the baryon density, the stress energy tensor and the Maxwell equations as seen in [6]. To close the system of equations, we use an equation of state to relate the internal energy density to the thermodynamic pressure. To decompose our system of equations into the $3 + 1$ formalism we use the ADM $3 + 1$ variables with the spacetime metric

$$ds^2 = -(\alpha^2 - \beta^i\beta_i)dt^2 + 2\xi_i dx^i + \gamma_{ij} dx^i dx^j$$

where $\alpha$ is the lapse function, $\beta^i$ is the shift vector, and $\gamma_{ij}$ is the induced metric on the spacelike hypersurfaces.

A. Hyperbolic Divergence Cleaning

The evolution of the magnetic field has a physical constraint $\nabla \cdot B = 0$, we have three evolution equations and one constraint equation. This leaves us with an over determined set of equations. Traditionally, the numerical treatment of a constrained system uses free evolution, where the evolution equations are used to evolve the system of equations and there is an implicit assumption that
the constraint will be maintained. However, when using free evolution, any numerical errors that arise are often linked to constraint violations, thus we need a method that enforces the constraint as the flow evolves.

To maintain the magnetic field constraint we use the hyperbolic divergence cleaning method as originally proposed by Neilsen et al. [11, 2], and used by other groups such as Palenzuela et al. [28].

To implement the diffusive hyperbolic method, we add a divergence term acting on an auxiliary field \( \psi \) of the form \( \nabla_\mu (g^{\mu \nu} \psi) \) to the Maxwell equations \( \nabla_\mu F^{\mu \nu} = 0 \),

\[
\nabla_\mu (F^{\mu \nu} \mu_\nu \psi) = \kappa B^\mu \psi
\]

We also add a diffusive term, \( \kappa B^\mu \psi \) to [4] to damp out any \( \nabla \cdot B = 0 \) violations [28], where \( \kappa \) is a tunable parameter. Our final expression for the Maxwell equations becomes

\[
\nabla_\mu (F^{\mu \nu} \mu_\nu + g^{\mu \nu} \psi) = \kappa B^\mu \psi
\]

In the absence of any \( \nabla \cdot B = 0 \) violations we expect that the extra parameter \( \psi \) will reduce to zero and thus we recover the original formulation of the ideal MHD equations as described in [2] for example.

B. Stress-Energy Tensor

We use the now standard form of the ideal magnetohydrodynamic stress-energy tensor as described in, for example, Noble et al. [27]

\[
T^{\mu \nu} = (\rho v^\mu v^\nu + (P + b^2/2)g^{\mu \nu} - b^\mu b^\nu)
\]

where \( h \) is the enthalpy of the system

\[
h = 1 + \epsilon + P/\rho_0,
\]

and \( b^2 \) is

\[
b^2 = g_{\mu \nu} b^\mu b^\nu,
\]

where

\[
b^\nu = (W^{\gamma j} B^j v^j, B^i/W + \dot{v}^i).
\]

C. Plasma Beta Parameter

We introduce an expression to allow us to parameterize the magnetic field relative to the hydrodynamic contributions. The relativistic definition of the plasma beta parameter [18, 19] is

\[
\beta_p = \frac{2P}{b^2}.
\]

When the magnetic field strength increases this parameter decreases, and when the magnetic field strength decreases this parameter increases. The magnetic fields in our simulations will be initialized using the asymptotic plasma beta parameter, \( \beta^P \), which will fully specify the magnetic field strength.

D. Equations of Motion

We use the Valencia formulation for ideal MHD. These are most readily found in [14, 21] and thus we will not restate them here. We introduce 4 new variables relating to the divergence cleaning we implement, which are explained below.

New Variables

Considering our hyperbolic divergence cleaning method we have two additional conserved quantities, the conserved divergence cleaned magnetic density, \( \Pi^i \), and the divergence violation field, \( \Psi \),

\[
\Pi^i = B^i + \beta^i \Psi
\]

\[
\Psi = \psi/\alpha.
\]

We also have the corresponding primitive variables,

\[
B^i = \Pi^i - \beta^i \Psi
\]

\[
\psi = \alpha \Psi.
\]

Just as in the standard GRMHD model, there are no known closed form solutions to the inverse relations, so we use numerical methods to perform this conversion when necessary. Primitive variable recovery is performed using a modified version of Mignone’s one parameter inversion scheme [24]. This was chosen due to its simplicity, and when we compared this inversion scheme against the two variable solver promoted by Noble et al. [27] there was no performance difference. The scheme is readily found in [21] and is not repeated here. The primitive recovery for variables \( B^i \) and \( \Psi \) are trivial and are seen in Eqn. [14].

Modified Equations of Motion

To determine the equations of motion for the new variables we take the 3 + 1 projection of Eqn. [4], to get

\[
\partial_t \sqrt{\gamma} \Pi^i + \frac{\partial}{\partial x^j} \sqrt{-g} \left( B^j \dot{v}^j - B^j \dot{v}^j + \alpha g^{ij} \psi \right)
\]

\[
= -\alpha \sqrt{-g} g^{\alpha \mu} \Gamma^j_{\alpha \mu} \psi + \kappa \beta^j \psi
\]

\[
\partial_t \sqrt{\gamma} \Psi + \frac{\partial}{\partial x^j} \sqrt{-g} \left( \frac{\dot{\Pi}^i}{\alpha} - \beta^i \right)
\]

\[
= -\alpha \sqrt{-g} g^{\alpha \mu} \Gamma^j_{\alpha \mu} \psi + \kappa \Psi.
\]

The remaining equations of motion are as presented in [14].

IV. Initialization and Boundary Conditions

We use the method described by Font et al. [15, 16] to initialize the hydrodynamic variables. Since the characteristics of the system are independent of the choice of
initial pressure, we use $P_\infty = 1$ and note that this choice is entirely arbitrary. The velocity fields are initialized using the following form \cite{32}:

\[
\begin{align*}
    v^r &= \frac{1}{\sqrt{g_{rr}}} v_\infty \cos \theta \\
    v^\theta &= -\frac{1}{\sqrt{g_{\theta\theta}}} v_\infty \sin \theta \\
    v^{\phi} &= 0.
\end{align*}
\]  

\tag{17}

One may easily verify that $v^2 = v_\infty^2$.

The magnetic field is initialized using Wald’s solution \cite{35}.

\[
F_{\mu\nu} = B_0 \left( (^{(\phi)} \xi_{[\mu,\nu]} + 2^{(t)} \xi_{[\mu,\nu]} \right).
\]  

\tag{18}

$(^{(\phi)} \xi_\mu$ and $(^{t)} \xi_\mu$ are the azimuthal and temporal Killing vectors respectively, the square brackets denote antisymmetrization, and $a$ is the spin parameter of the black hole. $B_0$ is a scaling factor that determines the magnitude of the magnetic field. This reduces to the initial conditions for the magnetic field components;

\[
\begin{align*}
    B^r &= -\frac{B_0}{2\sqrt{\gamma}} (\gamma_{\phi\phi,\theta} + 2ag_{\phi t,\theta}) \\
    B^\theta &= -\frac{B_0}{2\sqrt{\gamma}} (\gamma_{\phi\phi,\theta} + 2ag_{\phi t,\theta}) \\
    B^{\phi} &= -\frac{B_0}{2\sqrt{\gamma}} (\gamma_{\phi\phi,\theta} + 2ag_{\phi t,\theta})
\end{align*}
\]  

\tag{19, 20, 21}

This configuration is divergence free, and is uniform as $r \to \infty$. Due to the symmetry of our configuration we align the magnetic field with the rotation axis of the black hole.

The domain of integration for this study is defined by $r_{\text{min}} \leq r \leq r_{\text{max}}$ and $0 \leq \theta \leq \pi$. $r_{\text{min}}$ is determined in such a way that it will always fall inside the event horizon.

For some simulations, if the radial domain is not set to be large enough, unphysical waves travel back towards the black hole, ultimately destabilizing the system.

The boundary conditions for this problem are broken into three regions, two for the radial coordinate, and one for the angular.

(i) In the radial direction, near the event horizon, the flow is strictly absorbing, also known as outflow. In our implementation all scalar quantities are copied to the ghost cell regions and all vector quantities in the radial direction are linearly extrapolated to the ghost cells. This mixing of methods prevented unphysical backward traveling waves exiting the event horizon into the rest of the domain from destroying the setup.

(ii) For the boundary at the outer radial domain, we use prescribed boundaries “upstream” of the black hole, and outflow boundaries “downstream” of the black hole. Upstream was determined to be any region with angular coordinate in the range $\pi/2 \leq \theta \leq \pi$, while the remaining domain is considered downstream. The inflow condition is a prescribed boundary condition where we use the same asymptotic values as defined in the initialization routine.

(iii) In the angular direction, due to the symmetries considered in this study, reflective boundary conditions are used, which copy all scalar variables, and all non-$\theta$-component variables to ghost cells. This procedure inverts the sign of the $\theta$-component of all vector quantities.

V. NUMERICAL METHODS

The equations of motion for our system take on the general form,

\[
\frac{\partial}{\partial t} \sqrt{-g} Q + \frac{\partial}{\partial x^i} \sqrt{-g} F^i(Q) = \sqrt{-g} S(Q),
\]  

\tag{22}

where $Q$ are the conservative variables, $F^i$ denotes the flux, and $S$ are the geometric source terms. To solve this system, we developed our own high resolution shock capturing MHD code based on algorithms presented in the works \cite{16, 20, 26–29}. These are all based on an integral solution of Eqn. \cite{22} which allows for discontinuities in each fluid variable. In the presence of a discontinuity these methods solve a one dimensional Riemann problem.

We use the second order variable reconstruction MC limiter, with the HLL (Harten, Lax, and van Leer) approximate Riemann solver. The system was time evolved using second order Runge–Kutta integration. It is noted that, like all codes using approximate Riemann solvers, near extremum and discontinuities the order of accuracy reduces to first order. For the results presented we used regular grid spacing with a $400 \times 160$ numerical grid.

Parallelization was performed using the PAMR infrastructure developed by Frans Pretorius \cite{32} as well as direct use of the Message Passing Interface (MPI). Simulations were performed using the woodhen cluster at Princeton, USA, and the WestGrid cluster in Canada.

VI. ACCRETION PROFILES

In this section we describe the new and major features of each flow studied. This is done by presenting cross sections of the pressure accretion profiles.

In Fig. \ref{fig:accretion} we see the cross section of the final state for model M3. The upstream profile at the boundary smoothly agrees with the the profile, indicating that boundary effects will not be an issue in these simulations. The effects due to the presence of a magnetic field are minimal in the upstream region, and the magnetic pressure is uniform except in the region closest to the black
As the black hole accretes, the magnetic field builds up, the effects are largest near the black hole. However, the effects of the magnetic field are noticeable well outside of the hydrodynamic accretion radius, \( r_{\text{acc}} \),

\[
r_{\text{acc}} \equiv \frac{M}{v_\infty^2 + (c_s^\infty)^2}.
\]

\( c_s^\infty \) is the speed of sound in the fluid as measured by an asymptotic observer, \( M \) is the mass of the accretor, and \( v_\infty \) is the velocity of the accretor as measured by an observer at asymptotic infinity. This radius is the relativistic extension to the Newtonian Bondi radius as explained in detail by Petrich et al. [30]. Matter outside this radius is subsonic, whereas matter within this radius has a supersonic flow and will inevitably fall into the black hole. We refer the reader to [30] for details of the Newtonian calculations and more history. For the relativistic flows, this radius is fairly small, typically \( 0.5 \leq r_{\text{acc}} \leq 1.5 \).

For the sake of consistency, we choose to measure the accretion rate at the event horizon, where we can focus on the material as it falls into the black hole. The radial location of the event horizon only depends on the mass and the angular momentum of the black hole, so this is constant for each choice of fluid parameters.

The profiles presented in Figs. 1, 2 and 3 have similar general appearance. The upstream pressure profile takes on the appearance of a stationary Bondi accretor, as was originally presented in Font et al. [16]. We find that the similarity continues when comparing the upstream profile of our models to the magnetic Bondi accretor. Just as in [16] the downstream region is significantly different from the stationary accretor. In all cases the downstream profile indicates that the magnetic pressure builds up significantly in this region, and dominates the total pressure close to the event horizon of the black hole. An interesting trend occurs when modeling M1 and M3, the upstream profiles have the exact same trends; however, for model M3 in the downstream region the magnetic pressure is more than three times larger, and the effect has a shorter range. The profile for model M1 shows that the magnetic pressure extends to approximately \( r = 20M \), while the same profile for model M3 shows that the magnetic pressure only extends to approximately \( r = 15M \).

In Fig. 4 we present a pseudocolour plot for the thermal pressure for model M1 with a contour plot of the magnetic pressure. The magnetic field wraps around the black hole, as is seen by the magnetic pressure contour in the upstream region that traces the edge of the Mach cone in the downstream region. This forces the matter to decrease there. The magnetic reconnection in the downstream region is sufficiently weak that the magnetic field lines pile up in the downstream region. The magnetic pressure forces the downstream matter away from the axis of symmetry creating an evacuated region, which is similar to the effect as seen in the plasma depletion region associated with the earth’s magnetosphere.

In Fig. 5 we see the pseudocolour plot of the thermal pressure, with a contour plot of the magnetic pressure for model M2. In addition we present the magnetic field vectors. Outside of the Mach cone the magnetic field points in the same direction as the asymptotic magnetic field, however the field reduces magnitude, and reverses direction when it crosses into the tail shock. The magnitude of the magnetic field is represented by the size of the head of the vector. The magnitude is the largest closest to the black hole.

Figures 6, 7, and 8 present evidence that there is a balance between the thermal pressure and the magnetic pressure, in particular the total pressure has a similar profile to the hydrodynamic models.

A general feature of all relativistic Bondi–Hoyle accretion is the presence of a Mach cone [15, 17, 35]. This cone attaches to the downstream side of the black hole with what is known as the opening angle. We observe the opening angle of the Mach cone, seen in Figs. 6, 7, and 8. The opening angle is known to be a function of the parameters of the fluid for Newtonian systems [13], and is seen in the study performed by Font et al. [16].

In Figs. 9, 10, and 11 we present the cross section of the pressure accretion profiles on \( r = 2M \) for models M1, M2, and M3, as well as models H1, H2, and H3 for comparison. The balance between the two sources of pressure, magnetic and thermal, is clear in these cross sections since the total pressure exhibits a similar form as those seen in purely hydrodynamic flows [16]. The effect of the magnetic field increases the overall pressure in this cross section, especially along the axis of symmetry, and widens the Mach cone opening angle. The presence of the magnetic field amplifies the downstream pressure along the axis of symmetry and widens the Mach cone angle of attachment. Since the region downstream, inside the Mach cone, is the location of the maximum accretion, we expect that the larger cross sectional area leads to an increased accretion rate, as is seen in Fig. 13. In Fig. 11 we see that model H3 presents a maximum pressure on

| Model | \( v_\infty \) | \( c_s^\infty \) | \( O(\beta P^c) \) |
|-------|------------|-------------|----------------|
| M1    | 4/3        | 0.5         | 1              |
| M2    | 5/3        | 0.5         | 1              |
| M3    | 4/3        | 0.9         | 2              |
| M4    | 4/3        | 0.5         | 3              |
| M5    | 4/3        | 0.5         | 4              |
| M6    | 5/3        | 0.5         | 1              |
| M7    | 5/3        | 0.5         | 1              |
| M8    | 5/3        | 0.5         | 1              |
| M9    | 4/3        | 0.5         | 1              |
| M10   | 4/3        | 0.5         | 1              |
| H1    | 4/3        | 0.5         | \( \infty \)   |
| H2    | 5/3        | 0.5         | \( \infty \)   |
| H3    | 4/3        | 0.5         | \( \infty \)   |

TABLE I: Table of parameters used for the axisymmetric systems studied in this paper. We perform a small sampling of the available parameter space. Only supersonic flows, \( v_\infty > c_s^\infty \), are investigated. The \(^H\) models are purely hydrodynamic models.
FIG. 1: We show the pressure cross-section in the upstream region (left) and downstream (right) along the axis of symmetry plotted from $r_{EH} \leq r \leq 50$ for model M1. We see that the asymptotic behaviour of our system agrees with our boundary conditions, indicating that boundary effects are negligible for these simulations. We see a small effect due to the presence of the magnetic pressure in the upstream region; however, in the downstream region the magnetic effects are far more dominant in the region $r_{EH} \leq r \lesssim 10$ which is outside of the expected hydrodynamic accretion radius as defined in Eqn. (23). The upstream thermal pressure agrees with the hydrodynamic pressure from model H1, and the downstream total pressure profile agrees with the pressure profile of the same model. The magnetic pressure profile was shifted by one to fit the scale presented here. As is described in [16] the upstream pressure cross section takes on the appearance of a Bondi accretor while the downstream region is far more extreme.

FIG. 2: We show the pressure cross-section in the upstream region (left) and downstream (right) along the axis of symmetry plotted from $r_{EH} \leq r \leq 50$ for model M2. We see a similar profile as seen in Fig. 1. In this model the magnetic pressure has a much larger amplitude upstream than for model M1, and shifting the data was not necessary. Upstream we also see a deviation between the hydrodynamic model and the thermal pressure. In the downstream region the magnetic effects are far more dominant in the region $r_{EH} \leq r \lesssim 20$ well outside the hydrodynamic accretion radius. The downstream the total pressure from model M2, and the hydrodynamic pressure from model H2 are again very similar.
FIG. 3: We show the pressure cross-section in the upstream region (left) and downstream (right) along the axis of symmetry plotted from $r_{\text{EH}} \leq r \leq 50$ for model M3. The profile in this model is very similar to that seen in Fig. 1 including the upstream similarities between the hydrodynamic pressure and the thermal pressure. The effect of the rotating black hole is seen by the shortened magnetic pressure profile, which decreases to $r_{\text{EH}} \leq r \lesssim 6$ which, as in Fig. 2, is outside of the expected hydrodynamic accretion radius. Careful inspection also shows that the downstream thermal pressure profile is dramatically different between models M1 and M3. The thermal pressure maximum occurs around $r = 3M$ here whereas in model M1 the maximum is on the event horizon. Like in Fig. 1 the magnetic pressure was shifted by one to fit the scale.
We see that there is a balance between the two sources of pressure. This is made more apparent in figure 6. We also see that the magnetic field wraps around the black hole as shown by the dark blue contour connecting the magnetic pressure downstream to the magnetic pressure upstream. The magnetic reconnection downstream is sufficiently weak that the magnetic field lines pile up on the downstream side.

In models M1 and M2 we see that the magnetic pressure in the centre of the Mach cone has a much larger amplitude than the surrounding thermal pressure, while in model M3 the magnetic pressure has a smaller amplitude than the surrounding thermal pressure. Clearly, by allowing the magnetic field to “wind” with the black hole the magnetic pressure decreases which reduces the opening angle of the Mach cone. Since we also see that when we turn on the rotation, that there is very little change in the opening angle between the hydrodynamic model and the magnetohydrodynamic model. However, we do need to perform a larger parameter survey to determine how the magnetic fields are affected by the rotation of the black hole.

VII. ACCRETION RATES

In this section, we discuss the measured quantities that we observe for each run. The first several are simply the same quantities as developed by Petrich et al. [30] and later used by Font et al. [16, 17]. The accretion rates come from conservation laws at asymptotic infinity. We calculate the mass accretion rate from the mass of the accreted matter,

\[ m = \int d^3 x \rho \]  

(24)

which when we take the time derivative and substitute the mass flux into the integral becomes,

\[ \dot{m} = \int d^3 x \partial_t (\rho \dot{\upsilon}) = 4\pi \int \sqrt{-g} \rho \dot{\upsilon}^r \, d\theta. \]  

(25)

We calculate the energy and momentum accretion rates in a similar fashion,

\[ Q^{(i)} = -4\pi \int d^3 x T^{\mu \nu} n_\mu (^{(i)} \xi_\nu \]  

(26)
FIG. 5: The thermodynamic pressure (colour plot) and the magnetic pressure (contours) on a logarithmic scale for model M2. We superimpose the magnetic field vectors. We see that the downstream side has a distinct region where the magnetic field vectors diminish and switch direction. The thermal pressure balances the magnetic pressure in this region. This is a similar effect as seen in the plasma depletion region associated with the earth’s magnetosphere.

where \((\iota)\xi_\nu\) is the \(\iota\)-th Killing vector of the system, and \(n_\mu\) is a normal vector to the 3 + 1 hypersurface. With the given 3 + 1 description, we use \(n_\mu = (-\alpha, 0, 0, 0)\) so the above equation reduces to,

\[
Q^{(i)} = 4\pi \int d^3x T^{\mu \nu} \alpha^{(i)}\xi_\nu. \tag{27}
\]

For the axisymmetric black hole there are two Killing vectors, one temporal and one azimuthal. These lead to conservation of energy and azimuthal angular momentum. At asymptotic infinity, the metric takes the form of Minkowski space which is maximally symmetric \[17\]. Thus we may consider the radial Killing vector. By the same logic we are able to use the polar Killing vector as well; however, due to the symmetries involved in this study, no relevant physics can be extracted from this value.

To obtain the momentum accretion rates, we take the time derivative of Eqn. \[27\], where the time derivative commutes with the spatial integral. Subsequently, we replace the time derivative quantities with their flux counterpart to get an expression for the accretion rates \[16, 35\]. Since our HRSC scheme is at most second order, we use a second order numerical integration scheme to solve the resulting expressions.

The quantities we are specifically interested in presenting are the radial momentum accretion rate, the energy accretion rate, and the mass accretion rate.
The first quantity considered is the energy accretion rate. The equation is stated here and is easily derived using Eqn. (26).

\[ Q(t) = E = - \int \sqrt{-g} \alpha T^{tt} \sqrt{\gamma} d^3x = - \int \sqrt{-g} \alpha T^{tt} \sqrt{\gamma} d^3x \]

Likewise we have the momentum accretion rates

\[ \dot{Q}(r) = \dot{P} = \oint \sqrt{-g} T^{rr} \sqrt{\gamma} d\theta + \int_{r_{EH}}^{r_m} \sqrt{-g} T^{\mu \nu} \Gamma_{\mu \nu}^{rr} dr d\theta. \]  

(28)

where \( r_{EH} \) is the radial location of the black hole event horizon and \( r_m \) denotes the radius that we measure the accretion rate. In our measurements, we are interested in capturing dynamics on the event horizon, so \( r_m = r_{EH} \), thus all volume integrals above are zero.

To normalize the mass accretion rate, previous authors used the mass accretion rates determined at the sonic point for a relativistic Bondi (stationary) accretor \[16, 30, \]

\[ \dot{M} = \frac{4\pi \lambda M^2 \rho_\infty m_B}{(v_\infty^2 + (c_\infty^2) \lambda)^{3/2}}. \]  

(30)

\( \lambda \) is a dimensionless form factor as described in Shapiro and Teukolsky \[33\]. When normalizing the radial momentum accretion rate, they scaled the solution by both the mass accretion rate and the asymptotic velocity.

\[ \dot{P} = \frac{4\pi \lambda M^2 \rho_\infty m_B v_\infty}{(v_\infty^2 + (c_\infty^2) \lambda)^{3/2}} = \dot{M} v_\infty. \]  

(31)

We do not use these factors for two reasons. First, we measure the accretion rates at the event horizon not the accretion radius; second, we are measuring the accretion rates for a magnetohydrodynamic system, and the above simple forms do not take into account magnetic effects. Instead we time evolve the relativistic Bondi–Hoyle system using \( v_\infty = 0 \), therein solving the relativistic magnetized Bondi problem, starting from a uniform density background. When the flow reaches a steady state, we
FIG. 8: We present the thermodynamic pressure (left), the magnetic pressure (middle) and the total pressure (right) for model M3. We see that the total pressure takes on a familiar form, exhibiting a tail shock as seen in the work by Font et al. [16].

FIG. 9: A cross section at $r = 2M$ for model M1 of the thermodynamic pressure, magnetic pressure, and the total pressure, as well as the pressure for model H1. The balance between the two sources of pressure is more clear in this cross section since the total pressure exhibits a similar form as those seen in purely hydrodynamic flows [16]. The effect of the magnetic field increases the overall pressure in this cross section, especially along the axis of symmetry, and widens the Mach cone angle of attachment.

use those values of the accretion rates to normalize the accretion rate measurements for the relativistic Bondi–Hoyle systems.

Due to the symmetry of the system the azimuthal velocity field is exactly zero and, consequently, is not plotted.

We see that in all accretion plots that the flows reach a steady state. We also see the general trend that all magnetic flows have a wider Mach cone opening angle and consequently all magnetic flows experience a larger mass and energy accretion rate. This trend also extends to the radial momentum accretion rate for all models with $\Gamma = 4/3$; however, for models with $\Gamma = 5/3$ we find that this accretion rate decreases for larger amplitudes of the magnetic field. This is attributed to the relative build up of material in the downstream region. For all models studied we have an evacuated region downstream; however, it is observed that the models with $\Gamma = 4/3$ have

FIG. 10: A cross section at $r = 2M$ for model M2 of the thermodynamic pressure, magnetic pressure, and the total pressure. We also plot the same cross section of the pressure for model H2. The balance between the two sources of pressure clearer in this cross section since the total pressure exhibits a similar form as those seen in purely hydrodynamic flows [16]. As was the case in model M1, the presence of the magnetic field amplifies the downstream pressure along the axis of symmetry and widens the Mach cone angle of attachment. Since the region downstream, inside the Mach cone, is the location of the maximum accretion, we expect that the larger cross sectional area leads to an increased accretion rate, as is seen in Fig. [13].
FIG. 11: A cross section at \( r = 2M \) for model M3 of the thermodynamic pressure, magnetic pressure, and the total pressure, and the pressure profile for model H3. The balance between the two sources of pressure is more clear in this cross section since the total pressure exhibits a similar form as those seen in purely hydrodynamic flows [10]. As in the case for model M1, we find that the total pressure exceeds that for model H3, where no magnetic field is present; however, model H3 presents a maximum pressure on the axis of symmetry/rotation, whereas the total pressure for model M3 decreases on the axis. Further investigation is necessary to relate the spin of the black hole to the pressure decrease in the magnetic field cases.

The energy and mass accretion rates decrease as a function of the magnetic field strength, and converge to the hydrodynamic accretion rate in the zero magnetic field limit. Although we do note that in Fig. 15 the presence of even a weak magnetic field damps out the oscillatory accretion rate present in the hydrodynamic study. The oscillatory behaviour of the hydrodynamic solution is discussed in [16, 35]. The radial momentum accretion rates also increase as a function of the magnetic field strength.

In Figs. 12, 13, and 14 we compare the accretion rates for the radial momentum, the energy and the mass. Just as presented in [10], the accretion rates are dependent on the choice of adiabatic constant.

In Figs. 15, 16, and 17 we display the mass, energy and radial momentum accretion rates for models M1, M4, M5, and H1. As we expect, as the magnetic field reduces in magnitude, the accretion rates approach the hydrodynamic rates. In the blown up region within each plot we see the accretion rate as a function of the plasma beta parameter.

The simulations presented here all had the property that they converge to the asymptotic solution shortly after 600M.
When comparing the Mach cone opening angles between models M1 and M2, we see that the accretion rates are related to the opening angle, the larger the cross section in the shock cone, the more energy in accreted. This finding is in agreement with the results of the purely hydrodynamic models in [16].

Just as in the case of drag, the mass accretion rate is significantly affected by the value of the adiabatic constant used.

VIII. FLOW MORPHOLOGY

The results of the simulations using the parameters found in table 1 are discussed here. We present the final state of a sampling of the parameters surveyed in the spherically symmetric evolution. We see that the thermal energy accretion rates increase as a function of the magnitude of the plasma beta parameter. What is of particular interest is the oscillatory behaviour of the purely hydrodynamic model, investigated by [35] is damped out quickly by the presence of even a weak magnetic field as presented in model M5.

Just as in Fig. 15, we see that the mass accretion rate is only significantly impacted by the presence of a large magnetic field as simulated in model M1. As we expect, the mass accretion rates decrease as a function of the magnitude of the plasma beta parameter. Indicating that the increase in the Mach cone opening angle allows more fluid to accrete.
FIG. 17: The radial momentum accretion rates for models M1, M4, M5, and H1. We see that like all the previous measurements for these models that the larger opening angle leads to an increased drag experienced by the star.

FIG. 18: The mass accretion rates for models M2, M6, M7, M8, and H2. We see that the mass accretion rate is only significantly impacted by the presence of a large magnetic field as simulated in model M2. As we expect, the mass accretion rates decrease as a function of the magnitude of the plasma beta parameter. What is of particular interest is the oscillatory behaviour of the purely hydrodynamic model, investigated by [35] is damped out quickly by the presence of even a weak magnetic field as presented in model M5. We note that for this adiabatic constant the oscillations in the hydrodynamic simulation do not exist. The mass accretion rate increases monotonically as a function of the plasma beta parameter.

FIG. 19: The energy accretion rates for models M2, M6, M7, M8, and H2. Just as in Fig. 18, the energy accretion rate is only significantly impacted by the presence of a large magnetic field, indicating that the increase in the Mach cone opening angle allows more fluid to accrete. As is expected, the energy accretion rate increases in a similar fashion as the mass accretion rate [30].

Hydrodynamic quantities establish a steady state solution. As in the general hydrodynamic models, we find that in the upstream region the contours are smooth, but in the downstream region there is the presence of a Mach cone. As with the original hydrodynamic case we also look at the profile along the $\phi = 0$ and $\phi = \pi$ lines.

The simulations result in a rest mass evacuation immediately downstream of the black hole. Upon further analysis, this is the result of a buildup of the magnetic pressure downstream in the same region. The baryonic particles are transported along the magnetic field lines away from this region. As we see in figures 6 and 7, the convergence tests as seen in Figs. 26 and 27 indicate that the region is tending to zero rest mass density. This phenomenon is similar to the behaviour of the particles in the solar wind as they interact with the earth’s magnetopause [39]. In that setup, [39], a region known as a plasma depletion layer develops in the upstream side of the earth moving through the solar wind from the sun. In said region the magnetic field strength increases and as a result the plasma density decreases. Although this process is most dominant in the upstream side of the earth’s trajectory there is evidence that it also exists for the downstream side when the magnetic reconnection is insufficient to relieve the pile-up of magnetic field lines.

In our system, the pile-up occurs primarily in the downstream side, which is likely due to the fact that the fluid itself is magnetic, and that the black hole is not assumed to have its own magnetosphere in our model. Future models may include such features, but such a study would
FIG. 20: The radial momentum accretion rates for models M2, M6, M7, M8, and H2. We see that unlike all the previous measurements for these models that the larger opening angle leads to a decreased drag experienced by the star.

FIG. 21: The mass accretion rates for models M3, M9, M10, and H3. We see that the mass accretion rate is only significantly impacted by the presence of a large magnetic field as simulated in model M3. As we expect, the mass accretion rate decreases as a function of the magnitude of the plasma beta parameter. First, we see no evidence of oscillatory mass accretion as seen in model H1. We note that for this adiabatic constant the oscillations in the hydrodynamic simulation do not exist. The mass accretion rate increases monotonically as a function of the plasma beta parameter. The mass accretion rate quickly diverges towards the end of the simulation. This is determined to be a result of the resolution of the simulation.

FIG. 22: The energy accretion rates for models M3, M9, M10, and H3. Just as in Fig. [21] the energy accretion rate is only significantly impacted by the presence of a large magnetic field, indicating that the increase in the Mach cone opening angle allows more fluid to accrete. As is expected, the energy accretion rate increases in a similar fashion as the mass accretion rate [30].

be purely academic, since uncharged black holes are not expected to have their own magnetic fields.

Following the flow from upstream to downstream within a few Schwarzschild radii of the black hole reveals a very similar morphology as the hydrodynamic models. The flow is drawn to the black hole via gravitational attraction. As the material flows past the black hole, it is attracted to the hole by gravitational forces. The angular momentum of the fluid prevents the direct inflow of all the gravitationally bound material. This angular momentum is lost as the material moves downstream and begins to converge on the axis of symmetry. Here the fluid increases in density and pressure. This is where the similarities end. Closer to the black hole, we see a noticeable difference. While the same flow process occurs the effects of the magnetic field are far more obvious. The downstream pressure increases, including the magnetic pressure. As more fluid accumulates the frozen flux tubes also accumulate, which increases the local magnetic field strength. Eventually the magnetic pressure begins to dominate the local flow, and the density in that region begins to decrease as the fluid pressure outside of this region deflects new fluid from entering, but at the same time the magnetic pressure decelerates the fluid and directs it into the black hole. A balance between magnetic pressure and thermodynamic pressure is established and the flow reaches a steady state.

In Fig. [9] we have a cross section of model M1. We see that the total pressure of the system appears continuous; however, when we look at the contributions from
the different pressures the story is very different. In the evacuated region, we see that the magnetic pressure is dominant, showing that the force due to the magnetic field is strongest in this region. The radial velocity of the fluid in that region becomes negative indicating flow into the black hole, which is being forced faster than a hydrodynamic model due to the pressure from the magnetic field. As the oncoming flow reaches the tail shock, it first interacts with a region dominated by the thermodynamic pressure. If the fluid velocity is great enough the fluid will pass the first shock point, along the way interacting with a relatively minor build-up of magnetic field within the thermal pressure wall. Once past the wall, the fluid reaches a region where the thermal pressure suddenly drops and the magnetic pressure dramatically increases. It is in this region that the magnetic field is at its strongest, which forces the fluid to follow the path of least resistance, either the fluid accretes onto the black hole or flowing faster upstream, either way causing a density depletion. The low point in the magnetic pressure denotes the location where the magnetic field changes direction and points towards the black hole.

When investigating axisymmetric Bondi–Hoyle accretion onto a rotating black hole we found that the general flow morphology was substantially different to the non-rotating black hole with the same equation of state. Which is most dramatic when observing the plasma beta parameter as a function of the computational domain in Fig. 24.

Finally, in Fig. 24 we present the plasma beta parameter through the domain, after each simulation has reached a steady state. We see that a black hole traveling well above supersonic speeds produces a readily apparent tail shock. Also note the peaks that form along the inside edge of the tail shock for model M3. This is a feature only seen in simulations with a rotating black hole. The plasma beta parameter in the tail clearly follows a similar form as the tail shock near the black hole; however, for the spherically symmetric black holes, as we move further away downstream from the black hole the plasma beta parameter begins to smoothly reach asymptotic values. The rotation of the black hole clearly winds the magnetic field which may be seen in the maximum of \( \beta_P \) in model M3 being an order of magnitude larger than that seen in model M1.

IX. CONCLUSIONS

In this paper we presented a preliminary survey of the magnetized Bondi–Hoyle accretion onto both spherically symmetric and rapidly rotating axisymmetric black holes with the rotation axis aligned with the asymptotic uniform magnetic field. We used a high-resolution shock-capturing scheme with an approximate Riemann solver. To maintain the \( \nabla \cdot B = 0 \) constraint, we used a hyperbolic-divergence cleaning method and used the auxiliary field as more than just a means to remove divergence violations, but also as a well behaved function that may be used to determine the convergence rates for magnetized systems.

We have established that when magnetic fields are introduced to the typical hydrodynamic calculations, the evolution of an axisymmetric system exhibits behaviour that is qualitatively broadly similar to the purely hydrodynamic counterpart in [15, 16] but differs quantitatively in a range of crucial ways. Ultimately, we have shown that all parameters surveyed resulted in a steady state solution with an evacuated region downstream of the black hole. This evacuation is a result of a build-up of magnetic pressure in the same region, which forces all matter out of this region. Future studies will investigate a wider range of parameters and will attempt to match the parameters investigated to astrophysical observations. We have also seen evidence that although a rotating black hole in a hydrodynamic simulation aligned in a configuration similar to ours yields very little in the way of different morphology, the introduction of an ideal magnetic field is sufficiently different to make the dynamics much more interesting.

The presence of the magnetic field is seen to increase the Mach cone opening angle, and consequently increase the accretion rates for both the mass and energy. However, we see that the radial momentum accretion rates, or drag, decreases as a function of the magnetic field strength for cold plasmas, while hot plasmas experience more drag as a function of the magnetic field strength. This is attributed to the fact that the magnetic pressure of a \( \Gamma = 5/3 \) fluid downstream of the black hole is wider (relative to the \( \Gamma = 4/3 \) models), and creates a wider
FIG. 24: Plasma beta parameter, $\beta_P(r,\theta)$ for the axisymmetric evolution for model M1 (left, $t = 2500M$), M2 (centre, $t = 1500M$), and M3 (right, $t = 2500M$). These snapshots were taken well after the flow reaches its steady state. We see that a black hole travelling well above supersonic speeds produces a readily apparent tail shock. Also note the peaks that form along the inside edge of the tail shock for model M3. This is a feature only seen in simulations with a rotating black hole. The plasma beta parameter in the tail clearly follows a similar form as the tail shock near the black hole; however, for the spherically symmetric black holes, as we move further away downstream from the black hole the plasma beta parameter begins to smoothly reach asymptotic values. The rotation of the black hole clearly winds the magnetic field which may be seen in the maximum of $\beta_P$ in model M3 being an order of magnitude larger than that seen in model M1.

This was a phenomenological study, a detailed analysis will require the use of less diffusive Riemann approximations such as the Roe or Marquina solvers as used in recent hydrodynamic studies of the Bondi–Hoyle accretion problem in [35]. Further, a detailed study of the parameters needed in the hyperbolic diffusion cleaning method will need to be refined. Future studies will also involve a full three dimensional code which will allow a general asymptotic magnetic field configuration as prescribed by Bičák [5], in order to capture more astrophysically justifiable configurations. This line of study is far from complete, as this configuration will provide an excellent and simple testbed for more general fluid models as the field grows increasingly more complicated.

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**X. APPENDIX: CODE VERIFICATION**

To verify the numerical accuracy of the code we performed convergence testing of two different quantities. First we used a global quantity, the $L^2$-norm of the auxiliary variable $\Psi$, and second we used a cross section of the pressure accretion profile. We considered a cross section of the data at a constant radial coordinate, and independently a slice through a constant value of the angular coordinate. Different techniques for convergence testing HRSC schemes is an active research avenue and there is no universally accepted scheme at the time of writing. Convergence tests for models M1 and M3 may be seen in Figs. 25, 26 and 27. While we see that the depletion region has not converged by our selected resolution, the various accretion rates are within convergence. Future work will investigate the different parameters using higher resolution to work in the convergent regime of the depletion region.

To determine the convergence of the divergence cleaning variable $\psi$ to zero, we calculate the $L^2$-norm of this scalar field. While this would at best indicate a first order convergence rate, the fact that this term converges to zero is critical in showing the validity of our selected method for handling divergence violations. A sample convergence test for the $\psi$ variable is seen in Fig. 25.

The disagreement between the true second order convergence value and our system comes from the special treatment of shock regions and local maxima where the system reduces to first order convergence as mentioned.
FIG. 25: The convergence of the $L_2$ norm of the auxiliary field $\psi$ for model M3. It is clear that the system is convergent. Level 1 uses a grid resolution $200 \times 80$, level 2 denotes $400 \times 160$, and level 3 denotes $800 \times 320$.

FIG. 26: The convergence of the rest mass density density $\rho_0$ cross section at $r = 2M$ for model M1. We see that the evacuated region near the axis of symmetry is converging to zero. Level 1 uses a grid resolution $200 \times 80$, level 2 denotes $400 \times 160$, and level 3 denotes $800 \times 320$.

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