KIC 2856960: the impossible triple star

T. R. Marsh,1*D. J. Armstrong1 and P. J. Carter1,2

1Department of Physics, University of Warwick, Gibbet Hill Road, Coventry CV4 7AL, UK
2School of Physics, H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, UK

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ABSTRACT

KIC 2856960 is a star in the Kepler field which was observed by Kepler for four years. It shows the primary and secondary eclipses of a close binary of period 0.258 d as well as complex dipping events that last for about 1.5 d at a time and recur on a 204 d period. The dips are thought to result when the close binary passes across the face of a third star. In this paper, we present an attempt to model the dips. Despite the apparent simplicity of the system and strenuous efforts to find a solution, we find that we cannot match the dips with a triple star while satisfying Kepler’s laws. The problem is that to match the dips, the separation of the close binary has to be larger than possible relative to the outer orbit given the orbital periods. Quadruple star models can get round this problem but require the addition of a so-far undetected intermediate period of the order of 5–20 d that has been a near-perfect integer divisor of the outer 204 d period. Although we have no good explanation for KIC 2856960, using the full set of Kepler data we are able to update several of its parameters. We also present a spectrum showing that KIC 2856960 is dominated by light from a K3- or K4-type star.

Key words: binaries: close – binaries: eclipsing.

1 INTRODUCTION

A large fraction of stars are found in binary systems, and a significant number of binary stars reside in triple systems. Triple stars add complexity to the dynamics and evolution of stars (Eggleton & Kiseleva-Eggleton 2001; Naoz et al. 2013). Even though binary stars offer many outcomes closed to single star evolution, there are systems where evolution within a triple is the simplest explanation for otherwise puzzling data (O’Brien, Bond & Sion 2001), and location within triple systems has been suggested as a way to speed the merger of compact objects, which might help drive Type Ia supernovae and other exotic transients (Thompson 2011). Triple stars are mini clusters, with three co-eval stars in orbits which, in favourable circumstances, may allow us to determine precision fundamental parameters for all three objects.

Eclipsing systems are a well-travelled route to precision stellar parameters. In the case of triple systems, there are three different pairs of stars that can eclipse, but the chances of suitably aligned systems are low, given the hierarchical structure of triples which contain binary stars in much longer period and therefore wider orbits with third stars. Fortunately, the nearly uninterrupted coverage provided by the Kepler satellite has uncovered a significant number of triples (Gies et al. 2012; Rappaport et al. 2013; Conroy et al. 2014), and a number of these are multiply eclipsing. Examples are KOI-126, which has a 1.77 d close binary in a 33.9 d orbit with a third star (Carter et al. 2011), and HD 181068, which contains a 0.90 d binary in a 45 d orbit with a red giant (Derekas et al. 2011). KOI-126 in particular led to precise masses and radii of all three component stars.

KIC 2856960 is another eclipsing triple star observed by Kepler. Listed as an eclipsing binary by Prša et al. (2011), KIC 2856960 was subsequently found to be a triple system after the discovery of a second set of eclipses in addition to those of the binary (Armstrong et al. 2012). The binary in KIC 2856960 reveals itself through ~1 per cent deep eclipses (primary and secondary) on a period of 0.258 d. Its triple nature is apparent from complex clusters of dips in flux up to 8 per cent deep, which last for a little over 1 d at each appearance, and recur on a period of ~204 d (Armstrong et al. 2012; Lee et al. 2013). Armstrong et al. (2012) suggested two very different models for the system. In the first, the dip clusters are produced when a dim circumbinary object, possibly a planet, passes in front of the close binary, with multiple eclipses taking place as the binary completes its orbits. In the second, it is the close binary passing across the face of a third star that produces the dips. The second model was proven correct by Lee et al. (2013) who found variations in the times of the eclipses of the close binary consistent with light travel time variations as it orbited a third star. Thus, KIC 2856960 has all the characteristics of a hierarchical triple, with a close binary of a period ~0.258 d in a 204 d period orbit with a third star.

There has been no analysis of KIC 2856960 to see if, like KOI-126, it can yield precision parameters of its component stars. Here we document our efforts to do this, efforts which ended in failure. Not only do we not find a precision set of masses and radii for
the component stars, we do not find any physically consistent set of masses and radii that comes close to explaining KIC 2856960’s light curve. The nature of the disagreement leads us to conclude that, despite appearances, KIC 2856960 cannot be modelled as a triple star. Here we describe why we are led to this conclusion, and tentatively propose quadrupole star models as a possible escape route.

2 OBSERVATIONS

The NASA Kepler satellite is a mission producing extremely high precision, near-continuous light curves of ~155 000 stars on the level of 20 ppm (Batalha et al. 2010; Koch et al. 2010). The mission began science operations on 2009 May 13.

Kepler added significant new data on KIC 2856960 to those published by Armstrong et al. (2012) and Lee et al. (2013). Particularly significant are somewhat more than two quarters of short-cadence (1 min) data which fully resolve both the binary eclipses and one set of dips, compared to the majority of the data which were taken in long-cadence mode (30 min). Sadly, Kepler suffered a failure of one of its reaction wheels shortly before a second set of dips were to be observed in short cadence, terminating its coverage of KIC 2856960. This left 1500 d of data publicly available on the NASA Data Archive, from which we sourced the light curve of KIC 2856960. Detrending of the data was performed by the Kepler science team using the Pre-search Data Conditioning pipeline (PDC-MAP; see Stumpe et al. (2012) for an overview and examples, and Smith et al. (2012) for a full description of the detrending process).

We obtained spectra of KIC 2856960 in service mode on the night of 2012 May 21. The spectra were acquired with the ISIS spectrograph on the 4.2 m William Herschel Telescope at the Roque de Los Muchachos on the island of La Palma in the Canary Islands. ISIS uses a dichroic and two separate arms to cover blue and red wavelengths simultaneously. We used the 600 lines mm$^{-1}$ gratings to cover the wavelength ranges 370–530 and 565–735 nm at resolutions of 0.20 and 0.18 nm, respectively, with around 4 pixels per resolution element. We took two spectra in each arm with 1200 s exposures, separated by 3 h in time. There was no difference or radial velocity shift between the spectra so we combined them into one with 2400 s total exposure in each arm.

3 ANALYSIS

3.1 An overview of the light curve of KIC 2856960

Fig. 1 displays an overview of the Kepler light curve of KIC 2856960. For most of the time, the light curve displays ~1 per cent deep eclipses which repeat every 0.129 d which are the primary and secondary eclipses of a $P = 0.258$ d binary star. KIC 2856960 would be unremarkable were it not for seven brief intervals during which the flux dips up to 8 per cent below its normal level. These are the ‘dips’, previously referred to, which recur every 204 d. As the insets of two of these dips show, they have a complex structure in which the flux sometimes returns to its normal level between dips and each cluster contains up to nine minima. Our aim is to try to elucidate how these structures come about.

3.2 The close binary and its orbit within the triple

We begin our analysis by looking at the close binary light curve and the light travel time variations. These are the most secure aspect of the system, and given the difficulties we encounter understanding the dips, it is desirable in the first instance to understand as much as possible about the light curve away from the dips. This is a repeat of the work of Lee et al. (2013) but with the considerable advantage of the short-cadence data which allows us to resolve the binary eclipses. Before starting, we first define all the parameters that define the triple-star model (Table 1). We adopt the convention that the $P = 0.258$ d close binary and its orbit will be referred to as ‘the binary’, while the $P = 204$ d long-period outer orbit will be called ‘the triple’.

We will exclusively use the symbols defined in Table 1. These differ in some respects from those defined by Lee et al. (2013) and care should be taken when comparing our values to theirs. When we quote their values, we have translated the symbols they used
### Table 1. Physical parameters defining the triple-star model.

| Name         | Unit       | Description                                      | Comment       |
|--------------|------------|--------------------------------------------------|---------------|
| $R_1$        | R$_\odot$ | Radius of star 1 of the binary                   |               |
| $R_2$        | R$_\odot$ | Radius of star 2 of the binary                   |               |
| $R_3$        | R$_\odot$ | Radius of star 3, the tertiary component of the system |               |
| $a_1$        |           | Semimajor axis of star 1 within the binary       |               |
| $a_2$        |           | Semimajor axis of star 2 within the binary       |               |
| $a_3$        |           | Semimajor axis of star 3 within the triple       |               |
| $\omega_b$  | deg       | Longitude of ascending node of the binary        | Fixed to 270° |
| $\omega_t$  | deg       | Longitude of ascending node of the triple        | Fixed to 0°   |
| $e_b$        |           | Eccentricity of the binary                       |               |
| $e_t$        |           | Eccentricity of the triple                       |               |
| $\omega_p$  | deg       | Longitude of periastron of the binary            |               |
| $\omega_\theta$ | deg | Longitude of periastron of the triple          |               |
| $S_1$        | R$_\odot$ | Central surface brightness, star 1               |               |
| $S_2$        | R$_\odot$ | Central surface brightness, star 2               |               |
| $S_3$        | R$_\odot$ | Central surface brightness, star 3               |               |
| $\mu_1$      |           | Linear limb-darkening coefficient, star 1        | Fixed to 0.5  |
| $\mu_2$      |           | Linear limb-darkening coefficient, star 2        | Fixed to 0.5  |
| $\mu_3$      |           | Linear limb-darkening coefficient, star 3        | Fixed to 0.5  |
| $l_3$        |           | ‘Third light’ as a fraction of total flux         |               |

starting from the premise that we understand the binary better than the dips, we adopt the approach of letting the binary data fix as many parameters as possible, before attempting to model the dips. We used the same code as we later use in fitting the dips and will defer a description of the methods until then.
As part of fitting the binary's light curve, we had to allow for the timing variations caused by its orbit within the triple; therefore, the parameters associated with the triple orbit are a natural by-product of the light-curve modelling. In order to show the effect of the triple star's orbit, we measured the epoch of the binary over small intervals (roughly 5 d) of time throughout the *Kepler* observations. The results, along with the fit derived from the binary light-curve analysis, are shown in Fig. 4.

The variations seen are highly significant. The dips occur close to the time when the binary is closest to us, so that we see its eclipses arrive early. There can be no doubt over the Lee et al. (2013) conclusions that the binary is in an eccentric orbit with another object and that the dips occur when it passes in front of that object.

The parameters derived from the fits to the binary and its orbit within the triple are listed in Table 2. We did not determine the triple ephemeris from the binary data as it is more precisely pinned down by the dips (see Section 3.6). Where possible we list the equivalent values from Lee et al. (2013), although our two models are not precisely the same since they used a more sophisticated model accounting for tidal deformation, gravity darkening and star-spots, which, for reasons of compatibility with the triple-star models to be described later, we do not apply. We believe that in any case the degeneracy associated with the spots limits the accuracy with which some parameters can be determined, as we will detail shortly. It should be noted that we truncated the distribution of $i_t$ at 90° where it peaks. We distinguish between those parameters which describe the binary’s ephemeris and the orbit of its centre of mass within the triple ($T_b, P_b, a_b, e_b$ and $ω_b$) and those which control the shape of the light curve [the scaled radii $r_1 = R_1/(a_1 + a_2)$ and $r_2 = R_2/(a_1 + a_2)$, $i_b, S_2/S_1$ and $I_3$]. The first set depends upon timing information, and can only be fixed by a fit to the whole light curve. When we come later to fit the dips, we will hold these fixed since any individual set of dips contains little information to constrain them (with the possible exception of $T_b$ and $P_b$ — see Section 3.6). In contrast, the dips are highly sensitive to $r_1, r_2$, etc., which we will call the shape parameters.

The shape parameters are not nearly as well constrained as the purely statistical uncertainties listed in Table 2 suggest because of distortion of the light curve, which like Lee et al. (2013), we put down to star-spots. The effect of these is obvious outside eclipse in Figs 2 and 3. It can be seen quantitatively in the poor agreement between our radius values and those of Lee et al. (2013). We obtained an overall radius ratio (Table 2) of $R_1/R_2 = 1.10$, but when fitting to the short-cadence data alone, we find $R_1/R_2 = 0.88$ (Fig. 2).

We focus on the radius ratio in particular because it turns out that to fit the dips, we need much more extreme radius ratios than the
The same as Fig. 5, except now the times have been adjusted to the nearest binary cycle. Dashed lines mark binary phase 0; dotted lines mark binary phase 0.5.

Then a final single dip. The pattern of dips of one set can be seen to arrive slightly earlier at the next set, as indicated by the dashed line in Fig. 5. This cannot happen while maintaining a lock to the triple orbit without the dips evolving, and this can be seen as early arrivals fade away on the left-hand side of Fig. 5 just as new dips grow in strength on the right-hand side. (The situation is reminiscent of the behaviour of wave crests in groups of ripples on the surface of a pond.) The earliest any dip is seen is in the fourth event at around $-0.72 \text{ d}$; the latest is also the fourth event at around $+0.72 \text{ d}$, so the total duration of the dips exceeds $1.4 \text{ d}$.

In Fig. 6, we show the dips again but now with the times adjusted to the binary phase. This plot shows some very interesting features. First, the dips are stable relative to the binary phase. Secondly, we see that the first dip in a given set always occurs during the binary phase interval 0.0–0.5, while the last always occurs in the interval 0.5–1.0. Moreover, no dip is seen in the half cycle following the first dip or in the half cycle preceding the last dip. This is extremely odd. In the previous section, we showed that the two components of the binary have similar radii, so it should look very similar at any two phases 0.5 cycles apart. However, it seems instead that in one configuration obscuration occurs while it does not in the other. It is as if only one of the two stars in the binary occults the third object.

In contrast to the half cycle following the first dip in a set, which seems to be free of any obscuration, the half cycle following the double dip (covering around $-0.22$ to $-0.1$ d in Fig. 6) is usually not entirely clean, but shows some slight slopes. The half cycle preceding the final double dip behaves similarly. These are perhaps small signs of the presence of star 2, although the contrast between successive half cycles is still much stronger than the binary model would suggest.

### 3.3 The dips

#### 3.3.1 Overview of the dip light curves

We start with a qualitative, model-independent assessment of the light curves during the appearance of the dips. Fig. 5 shows all seven of the occurrences of the dips observed by *Kepler*. No two sets of dips are identical, but many bear strong similarities to each other. For instance, counting from the bottom, the second, third and sixth sets are very similar to each other, with each showing first a single dip, followed by a closely spaced double dip, then by three somewhat more widely spaced dips, then another double dip and then a final single dip. The pattern of dips of one set can be seen to arrive slightly earlier at the next set, as indicated by the dashed line in Fig. 5. This cannot happen while maintaining a lock to the triple orbit without the dips evolving, and this can be seen as early arrivals fade away on the left-hand side of Fig. 5 just as new dips grow in strength on the right-hand side. (The situation is reminiscent of the behaviour of wave crests in groups of ripples on the surface of a pond.) The earliest any dip is seen is in the fourth event at around $-0.72 \text{ d}$; the latest is also the fourth event at around $+0.72 \text{ d}$, so the total duration of the dips exceeds $1.4 \text{ d}$.

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#### 3.3.2 Modelling the dips

In order to model the dips, we developed a model of a triple star involving three limb-darkened spheres, in hierarchical Keplerian orbits, specified by the parameters listed in Table 1. Our concern here is to capture the main features of the data, and we do not include effects of secondary importance (for KIC 2856960 at least) such as tidal distortion and gravity darkening. This considerably speeds the computations, which are a limiting factor in much of the modelling. We do not account for N-body corrections to the Keplerian orbits because we expect these to be small given the...
Figure 7. A face-on view of the three types of orbits (one triple, two quadruple) we consider for KIC 2856960, all drawn to scale with tilts removed so that there is no distortion by projection. The observer is assumed to lie in the plane of the figure and view the orbits edge-on from the bottom of the page. The horizontal dashed line represents the plane of the sky, with the ‘+’ signs marking the system’s centre of mass. The two ellipses show the centre of mass of the close binary in the left two panels (solid) and star 3 in the left- and right-hand panels (dashed). The lower panels show factor of 10 magnified views of the regions delineated by small dotted squares centred near the close binary. In the central panel, the dashed ellipse shows the centre of mass of a second binary formed from star 3, the star that is eclipsed during the dips, and an extra star, ‘star 4’, introduced in order to alter the dynamics for reasons explained in the text. The upper circle shows the orbit of star 3 around a fixed point on the dashed ellipse. All orbits are traversed counter-clockwise in this figure, so that in the configuration shown, which matches the time of the dips, the relative transverse speed between star 3 and the close binary is slowed by star 3’s motion within its binary with star 4. In the right-hand panel, the solid-lined ellipse shows the track of the centre of mass of an inner triple made up of the close binary and ‘star 4’, once more introduced to alter the dynamics of the system, but in a different configuration. The close binary thus moves on a circle around this guiding centre, as indicated by the large circle in the right-hand magnified view. The small circles, which can only be seen in the magnified views, represent the orbit of star 1 (again with its guiding centre held stationary at the time of the dips). Dots in each panel show the centre of mass of the close binary and star 3. Additionally, in the centre panel the dot on the dashed ellipse represents the centre of mass of the second binary, while in the right-hand panel the dot on the solid ellipse marks the centre of mass of the inner triple. The orbits of stars 2 and 4 are suppressed for clarity.

The ~800-fold ratio of the outer and inner orbital periods. To compute the flux from each star, its circular projected face was split into a set of concentric annuli of constant radial increment but variable surface brightness because of limb darkening. The task then breaks down to working out how much of each annulus is visible given the locations and sizes of the other two stars. The number of annuli determines the extent of numerical noise. We used 80 for each star, verifying that the resultant numerical noise was less than the noise level in the data. The only notable feature of the parameters chosen is our choice of zero-points, $T_b$ and $T_i$, which mark central times of the binary star’s primary eclipses on the one hand and the ‘dips’ on the other, as opposed to the more usual time of periastron passage. We made this choice because the eclipse and event times are more or less directly fixed by the data. This means that the resulting epochs are much less correlated with other parameters than they would have been had we used the periastron times instead. We tested the code by verifying that it correctly reproduced the light curve of the triple system KOI-126 at the epoch closest to the epoch of the orbital elements quoted by Carter et al. (2011).

Later on we will examine quadruple star models for KIC 2856960, making the geometry harder to visualize. Therefore in Fig. 7, we give schematic pictures of the three types of orbits that we will consider. The observer is assumed to be in the plane of the figure, looking from below, and the system is shown at the time of the dips.

We varied or fixed parameters according to whether the data significantly constrained them. For instance, we set $\Omega_b = 270^\circ$ (ascending node due west on the sky, in the usual direction of the $x$-axis) from the start since there is no information upon the absolute orientation of the system. On the other hand, the data strongly constrain the relative orientation of the binary and triple, so $\Omega_b$ was allowed to vary. Similarly, the limb-darkening coefficients were uniformly set equal to 0.5 since they have a relatively minor effect upon the light curves. In the case of the binary, the nature of its light curve and its short period very much suggest that it has a circular orbit, and so we fixed $e_1 = 0$ and $a_1 = 0$ for all models. Finally, when fitting to data, we scaled the fluxes to minimize $\chi^2$ as this is a fast operation. This meant that one of the surface brightness parameters could be fixed (since otherwise there would be degeneracy between the surface brightnesses and the scaling factor), leaving us with a maximum of 18 parameters that could be varied.

As previously explained, owing to clear differences between the parameter space favoured by the binary light curve compared to the dips, we first fitted those parameters which could be determined from the binary alone. Thus, we fixed $T_b$ and $P_b$, which define the binary’s ephemeris, and $a_3$, $e_1$ and $a_1$, which define the triple orbit, to the values listed in the top section of Table 2. We will see later that all lengths in the system scale with the value of $a_2 + a_3$, the semimajor axis of the triple, and masses therefore scale as $(a_1 + a_2 + a_3)^3$. While $a_2$ is fixed by the light travel times (given that $\sin \phi = 1$ to a good approximation), we have no direct information upon $a_3$, although it can be estimated by seeking a consistent set of masses and radii for the binary star, assuming it to be composed of a pair of M dwarfs. This is because the binary light curve fixes the radii scaled by the total separation, the masses scale as the total separation cubed, while the mass and radius of low-mass stars are nearly linearly related (Torres 2013). Assuming a precise linear relation, $M/M_\odot = R/R_\odot$, and starting from the values listed in...
Table 2 leads to an estimate for \( a_1 \approx 75 \, R_\odot \). We therefore adopt a round number of similar magnitude, and henceforth will assume that \( a_3 = 100 \, R_\odot \). Where relevant later, we point out aspects that depend upon the particular value chosen for \( a_1 \).

We carried out the fitting through a combination of standard minimization methods (Powell 1964; Nelder & Mead 1965) and (mainly) Markov chain Monte Carlo (MCMC) iteration. MCMC takes a Bayesian point of view whereby one constructs models that are distributed with the posterior probability distribution of the parameters, given the data. The posterior probability has prior probabilities representing one’s knowledge before any data are taken times the probability of the data, given the model. The latter is encapsulated by \( \chi^2 \) in our case since we assume independent Gaussian uncertainties on the data. The prior provides a flexible way to impose physical constraints without requiring that they hold precisely at all times during minimization. The most important such constraint comes from Kepler’s laws. We implemented our model with a combination of \( c \) and PYTHON, and used the EMCEE package (Foreman-Mackey et al. 2013) to manage the MCMC computations.

Kepler’s third law applied to the triple orbit gives us the total system mass in terms of the controlling scale factor, \( a_1 + a_2 \):

\[
G(m_1 + m_2 + m_3) = n^2 \left( a_1 + a_2 \right)^3,
\]

where \( n = 2\pi/P \). The centre-of-mass condition

\[
m_3 a_3 = (m_1 + m_2) a_0,
\]

then allows us to deduce the mass of each component of the triple. A second application of Kepler’s third law then fixes the total separation of the binary

\[
G(m_1 + m_2) = n^2 \left( a_1 + a_2 \right)^3,
\]

where \( n = 2\pi/P_0 \). Thus, the value of \( a_1 + a_2 \) is fixed once \( a_1 + a_0 \) is fixed and cannot be allowed to vary independently of it. This we ensured through the prior probability by demanding near-equality between the value of \( a_1 + a_2 \) computed as above starting from \( a_1 + a_0 \) and the value derived from the models proposed during the MCMC process. Inequality was punished through low prior probability. This method allows great flexibility in terms of what is allowed to vary, whilst ensuring physical consistency. The degree of equality demanded, which has some impact upon the MCMC efficiency, could be tuned at will.

Fixing what parameters we could from the binary model, setting \( a_1 = 100 \, R_\odot \), we went ahead and optimized the remaining 12 parameters which were the stellar radii \( R_1, R_2, R_3 \), the binary semimajor axes \( a_1, a_2, a_3 \), the orbital inclinations \( i_1, i_2, i_3 \), the epoch of the triple \( T_0 \), the surface brightness parameters \( S_1, S_2 \) and \( S_3 \) along with the ‘third light’ \( I_3 \) and the orientation of the binary orbit, \( \Omega_c \). We applied Kepler’s laws via the prior as just outlined, and the constraint upon \( r_1, r_2 \) and \( i_0 \) to match the eclipse width that we described in Section 3.2. The best fit to the short-cadence dips resulting from this procedure is shown in the left-hand panel of Fig. 8. Clearly, ‘best’ is very much a relative term here as the fit is extremely poor with \( \chi^2 \approx 60\,000 \) for 4589 points. In particular, the level of modulation in the central part of the dips is much weaker in the model than the data. This is because in the model \( R_1 \approx 3.9 \, R_\odot \) is large compared to \( R_1 = 1.29 \, R_\odot \) and \( a_1 = 1.85 \, R_\odot \). Once the binary starts to cross star 3, one part of it is always in an occulting position. Besides providing a poor fit, the parameters that lead to the fit shown in Fig. 8 can be ruled out on astrophysical grounds. For instance, star 1 ends up with almost zero mass \( (10^{-3} \, M_\odot) \) but a radius of \( 1.29 \, R_\odot \) (and thus overfills its Roche lobe).

Despite the poor fit, the model does show some similarities to the data indicating that it contains elements of truth. A much better although still imperfect fit \( (\chi^2 = 13\,900) \) is obtained if the Keplerian constraint is removed, as shown in the right-hand panel of Fig. 8. While physically impossible, it is useful to understand how this model manages to match the dips as well as it does. To facilitate our discussion of this, we label specific features of the light curve using the letters A to F as shown in the right-hand panel of Fig. 8. Further, we concentrate upon how the largest star of the binary (star 1) transits star 3 because star 2 in this model plays almost no part in the dips. This is forced by the peculiar absence of any obscuration between minima A and B that we noted in Section 3.3.1. With this simplification, the geometry equivalent to the model of the right-hand panel of Fig. 8 is shown schematically in Fig. 9.

As time progresses, the centre of mass of the binary moves from left to right in this figure. Minima in the light curve occur at the points of closest approach (minimum impact parameter) between the centres of stars 1 and 3. The first of these (with significant
The geometry of stars 1 (small and shaded) and 3 (large and outlined) at the times labelled in Fig. 8, according to the model shown in the right-hand panel of that figure. Star 1 is closer to the observer than star 3 at all times, and is blocking flux from it. The circular dots connected by dotted lines show the centres of mass of star 1 and the binary at the time in question; the cross marks the centre of mass of star 3. The solid curves indicate the path of the centre of mass of star 1 over the range $-0.1$ to $+0.1$ d relative to the particular time in question. They can be thought of as small sections of the projection of a squashed helix. The orbital inclination of the binary in this model $i_b > 90^\circ$ so that the paths are travelled in a clockwise manner. For clarity, we have shrunk the radius of star 1 (solid) by a factor of 2; this does not affect the times of minima/maxima which depend upon the impact parameter only. The precise radius ratio, which controls the depth of the dips, depends also upon the amount of ‘third light’ and is therefore not well defined.

Figure 9. The geometry that leads to the double dips. The ellipse shows the projected motion of the centre of star 1 around the binary’s centre of mass (lower-left cross) which for simplicity we have assumed to be stationary. The circle shows the outline of star 3, and the upper cross its centre. A double dip in the light curve occurs if star 1 crosses the diameter of star 3 marked with the vertical dashed line from left to right and then back again. B and D mark times of minimum flux while C marks a maximum. The binary inclination used here is $i_b = 87^\circ$, and we have aligned the ascending nodes of the binary and triple orbits. The horizontal dashed line indicates the path of the binary’s centre of mass.

process was well behaved in that very different starting models would eventually reach the same fit with the same parameters. Thus, we are convinced that the left-hand panel of Fig. 8 is the best that a physically consistent triple-star model can do. For this reason, we do not think that KIC 2856960 can be as simple as a triple. In the next section, we back up this conclusion with a more analytical treatment of the problem.

3.4 The dynamical–geometrical paradox

Why does the triple model do such a poor job of explaining the dips? In the previous section, we found that we could only get somewhere near the data by ‘relaxing’ Kepler’s law, so that the binary separation could become larger (than physically allowable). Obviously, this is not acceptable, but the reason for the problem perhaps contains pointers to solving it, and so it is of interest to come to an analytical understanding of it. This also serves as a reassurance that the problems of the previous section are real, and not simply coding errors.

To begin, consider the geometry of Fig. 10. Since the centre of mass of the binary is slowly moving from left to right in this diagram, it is almost inevitable that the circumstance shown will occur at some point during a set of dips. The hard part is to ensure that there are two sets of double dips, as widely spaced in time as observed, with the second set mirroring the first during the egress phase of the dips. From Fig. 10, ignoring possible tilts of the orbit relative to the horizontal, this can only happen if the semimajor axis of star 1, $a_1$, exceeds half the distance traversed by the centre of mass of the binary between the occurrence of the double dips, which we define to take time $\Delta t$. Quantitatively, we find that we require

$$\frac{a_1}{a_1 + a_0} < \frac{1 + e_1 \cos \nu_1}{\sqrt{1 - e_1^2}} \cdot \frac{n_1 \Delta t}{2},$$

where $\nu_1$ is the true anomaly of the triple orbit at the time of the dips, which is related to theperiastron angle via $\nu_1 = 3\pi/2 - \omega_3$. From our fits to the triple’s orbit (Section 3.2), we calculate the first term on the right-hand side to be $1.449 \pm 0.018$. Using the 4$\sigma$
lower bound on this factor and setting $\Delta t = 0.7 \, \text{d}$, as measured from the outermost minima of the double dips, one then finds that
\[
\frac{a_1}{a_1 + a_b} > 0.0148.
\]
(6)
Projection factors that arise if the binary’s orbit is tilted with respect to the triple or problems of exact timing only serve to increase this lower limit.

An alternative limit upon the same quantity is derivable from Kepler’s third law. Eliminating the masses from equations (2)–(4) gives
\[
\frac{(a_1 + a_2)^3}{P_h^2} = \frac{(a_3 + a_b)^2}{P_1^2} a_3,
\]
which can also be written as
\[
\frac{a_1 + a_2}{a_1 + a_b} = \left( \frac{P_h}{P_1} \right)^{2/3} \left( \frac{a_3}{a_1 + a_b} \right)^{1/3}.
\]
(7)
The final term on the right-hand side is $\leq 1$, while $a_2 \geq 0$, so we deduce that
\[
\frac{a_3}{a_1 + a_b} \leq \frac{a_1 + a_2}{a_1 + a_b} \leq \left( \frac{P_h}{P_1} \right)^{2/3} = 0.0117.
\]
(8)
The upper limit from equation (9) (dynamics) is lower than the lower limit from equation (6) (geometry). This is the ‘paradox’ of the triple-star model of KIC 2856960. There is no room to escape this conflict. Indeed, the equalities in equation (9) can only be met if $a_2 \ll a_1$, and $a_b \ll a_1$. We do not expect either of these to hold true, so even the 0.0117 is likely to be a significant overestimate. Thus, just as we found with the numerical models, in triple-star models one can fit the data (roughly) or Kepler’s laws, but not both with the same model.

A less rigorous constraint, which points in the same direction, but does not rely on our interpretation of the double dips, can be deduced from the total width of the events which was noted in Section 3.3.1 to be $\Delta w \approx 1.4 \, \text{d}$. The maximum width is obtained when all orbits are aligned and star 1 is exactly at quadrature when it first contacts star 3. In that case, a similar argument to the constraint that led to equation (6) implies that
\[
\frac{a_1}{a_1 + a_b} \geq 0.0296.
\]
(9)
It becomes difficult to satisfy this and Kepler’s laws without making the ratio of $R_i/R_1$ so small that one cannot match the dips. For example, if we assume that the two stars in the binary are M dwarfs following the typical mass–radius relation of M stars, then $a_1$, $R_1$ and $a_3$ are determined, and we find that the maximum depth of dips should be $\approx 1$ per cent compared to the $8$ per cent observed. This is easier to escape than the problem with Kepler’s laws, since an increase in $R_1$ implies a decrease in $R_1$, and thus a significant increase in the maximum dip depth, $(R_i/R_3)^2$, but it is suggestive of a similar problem, i.e. that the dips last too long for the triple model to accommodate.

### 3.5 Quadruple models

Both the problems described in the previous section can be ascribed to a high relative transverse speed between the centre of mass of the binary and star 3. In a triple system, this is a simple function of the orbital parameters and, given the light travel time constraints, we have no freedom to alter it. The only plausible way we have thought of to change the transverse speed significantly is adding a fourth star. If such a star is coupled either to star 3 to form a pair of binaries (which we label ‘mode 1’, see Fig. 7) or to the close binary to form a hierarchical quadruple (mode 2, Fig. 7), the resultant orbital motion may slow the relative speed between the centre of mass of the close binary and star 3, thereby alleviating the problems of the previous section.

We implemented quadruple models along the lines of the triple-star model, and for the first time were able to find solutions that qualitatively agree with the data while obeying Kepler’s laws. The fits that result are visually indistinguishable from the right-hand panel of Fig. 8, so we do not show them. The quadruple model is no panacea, but it is the closest we have come to explaining KIC 2856960. The new orbit comes at the cost of a fine-tuning problem since we require its period to be close to an integer divisor of the 204 d period (within $\approx 0.001$ of an exact integer ratio). This is needed to ensure that the binary occults star 3 at the same part of the new orbit at each set of dips so that the relative speed is always slowed down. If this ratio is not perfect, then at some point in the future one can anticipate considerable changes in the dips, which could change duration or disappear altogether.

In addition to the fine tuning, which is at least not an impossibility, some astrophysical problems remain. Table 3 lists the masses and radii of the three types of models we have considered. The triple model listed is the one forced to obey Kepler’s laws, so that we can define masses consistently, which means that it corresponds to the poor fit of the left-hand panel of Fig. 8. When viewing this table, the unknown value of $a_1$ which enters into the length that defines the scale, $a_0 + a_1$, should be recalled (Section 3.3.2), so that all radii are subject to an unknown scale factor $s$ relative to the values listed, and all masses to its cube, $s^3$. Since $a_3 > 0$ and we used $a_b = 99.4 \, R_\odot$ and $a_1 = 100 \, R_\odot$ in the table, then we can assert that $s > 0.5$, with $s = 1$ for the values listed in the table. Of the three models, the triple model can be ruled out astrophysically as well as from its poor fit to the data, as we noted earlier, since it implies almost zero mass for star 1 ($10^{-5} \, M_\odot$ to be exact). This comes about from a vain effort to make $a_1$ as large as possible, at the expense of $a_2$ to match Kepler’s laws. The small size of star 2 is probably the thorniest issue for the quadruple models. For the mode 1 model, the large mass of star 4 is also a potential problem, since it might end up dominating the light from the system, even though it does not participate in creating the variations seen. However, both quadratic models appear to need significant ($\approx 70$ per cent) ‘third light’ contributions in addition to the light contributed by stars 1, 2 and 3, so this may not be an impossibility. Otherwise, apart from star 2, the mode 1 mass–radius values look slightly preferable to those of mode 2, although there is not a great deal to choose between them. A final point against mode 2 is that we expect an $\approx 20$ s semi-amplitude variations in the light travel times on the intermediate period (which in our fits lies in the range $\approx 20$ d). We searched for such a signal in the light travel times of Fig. 4 and were sensitive to semi-amplitudes of $10$s or more, but did not find anything.

### Table 3. Masses and radii in solar units of the three types of models for $a_3 = 100 \, R_\odot$ (the values are taken from the best-fitting models for each case). The radius of star 4 is not listed as it is not constrained by any of the models.

| Mode     | $M_1$ | $R_1$ | $M_2$ | $R_2$ | $M_3$ | $R_3$ | $M_4$ | $R_4$ |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|
| Triple   | 0.00  | 1.29  | 1.28  | 0.16  | 1.27  | 3.89  | –     | –     |
| Quad, mode 1 | 0.50  | 0.60  | 0.78  | 0.16  | 0.36  | 0.51  | 0.91  | –     |
| Quad, mode 2 | 0.23  | 0.53  | 0.42  | 0.13  | 1.27  | 0.40  | 0.63  | –     |
In summary, the quadruple model is shaky, but it does at least not violate basic physics in the same fashion as the triple model when getting close to the data. In implementing the quadruple model, we continued to use hierarchical Keplerian two-body orbits, but it is now much less clear that Newtonian effects can be still neglected. Indeed if the quadruple model is correct, Newtonian perturbations will need consideration to establish the dynamical stability of the system. However, we did not try to add them because the number of parameters of the quadruple model outstrips our ability to constrain them, and because the poor fit to just the short-cadence set of dips alone (during which there would be no significant dynamical evolution) suggests that our model is currently lacking important ingredients beyond N-body perturbations.

3.6 Nagging problems

During our attempts to fit KIC 2856960, two other peculiar problems surfaced that we never resolved. The elusive second component of the binary is one of these. In all the models that provide a reasonable fit to the dips, star 2 is 2.5 to 4 times smaller than star 1. We mentioned this as the chief drawback of the quadruple models in the previous section, because there star 2’s small radius is out of kilter with its mass. Such unequal ratios are also not favoured by the binary light curve, although in Section 3.2 we argued that starspots made the true ratio uncertain. However, there can be no doubt that the two eclipses in the binary light curve have a similar depth, implying a similar surface brightness for each component of the binary. It is then very hard to understand how these two stars, which are most likely unevolved, low-mass main-sequence stars, can differ so much in radius.

Another very puzzling problem concerns the binary ephemeris. None of the models described so far provide a particularly good fit to the data, with, at best, \( \chi^2 \) values around 13 600 for the 4589 points covering the short-cadence dips. A very odd feature is that this can be greatly improved, with \( \chi^2 \) decreasing to \( \approx 9000 \), simply by letting the close binary epoch \( T_b \) vary (Fig. 11).

This comes at the significant cost of a misalignment between the observed and model binary eclipses. We examined this further by freeing up both \( T_b \) and \( P_b \) to fit all seven sets of dips simultaneously. Fig. 12 shows the result. This fit returns a significantly longer binary period \( P_b \) than obtained from fitting the binary only data (Table 2), with \( \Delta P = (3.05 \pm 0.03) \times 10^{-6} \) d, corresponding to 1250 s difference over the time between the first and last dips. This suggests that the offset to the binary ephemeris that best fits the dips changes with time. We have no solution to this curious problem which is perhaps another clue to finding an improved model for KIC 2856960.

We used the triple model without Keplerian constraint to fit all seven sets of dips in order to establish the ephemeris for the outer orbit which we used when fitting the binary-only data in Section 3.2. We found

\[
\text{BMJD} = 56018.5661(18) + 204.2723(9)E, \tag{11}
\]

where \( E \) is an integer, giving the mid-point of the dips, with \( E = 0 \) coinciding with the set of dips observed in short cadence. We used the triple model because the quadruple model introduces an extra degree of freedom which renders these values much more uncertain, but the possibility of such uncertainty should be borne in mind. Testing for this is a strong reason to attempt ground-based observations of the dips. In using this ephemeris to predict future occurrences of dips, note the use of modified Julian days (MJD = JD – 2400000.5).

3.7 A spectral type for KIC 2856960

We obtained spectra of KIC 2856960 on the night of 2012 May 21. The spectra were taken during service time, in two sets, three hours apart. The spectra did not change significantly in this time, and so in Fig. 13 we present the normalized average of the two spectra. We compare this to stars of known spectral type taken from the ELODIE.3.1 library of stellar spectra which contains 1962 spectra of 1388 stars taken with the ELODIE spectrograph at the Observatoire de Haute-Provence 193 cm telescope in the wavelength range 390–680 nm (Prugniel & Soubiran 2001). We used the \( R = 42 \) 000 spectra from this library, which we blurred and rebinned to match our data. Fig. 13 shows that KIC 2856960 is dominated by light from a K3 or K4 star. In comparing this with the masses listed in Table 3, the unknown scale factor should be recalled.
The spectrum of KIC 2856960 (centre) observed with the ISIS 13 bandpass, but this is poorly α and thus the total system mass. We expect radial and Na J I 445, I Kepler ˇ E 2001 309–319 (2014) a period of 204 d. The light curve of the binary is consistent with a pair of dips in the light curve that last for a little over 1 d and recur on a passes in front of its companion causing the appearance of a series orbit with another object. Its orientation is such that the close binary KIC 2856960 is an apparent triple star containing a close binary in 5 CONCLUSIONS dips can be expected.

be of interest for dynamical studies, and significant evolution of the eclipses and dips and highly eccentric outer orbit, suggest that it may short, long and intermediate orbital periods, together with the binary time. If the system is truly a quadruple star, then the simultaneous will also be of value to see whether they evolve significantly with possible from the ground given the 8 per cent maximum depth, the binary in the of Fig. 13. Figure 13. The spectrum of KIC 2856960 (centre) observed with the ISIS spectrograph on the William Herschel Telescope on 2012 May 21, compared to the spectra of main-sequence K stars (Prugniel & Soubiran 2001). Some mismatches are caused by missing data in the templates (at Hr and Na I D), and by the dichroic cut between the blue and red arms of ISIS, but otherwise mismatches are caused by missing data in the templates (at H D), and by the dichroic cut between the blue and red arms of ISIS, but otherwise the dichroic cut between the blue and red arms of ISIS, but otherwise measurements of temperature-sensitive features suggests either a K3 or K4 type for KIC 2856960. We normalized the spectra by spline division and blurred and rebinned the template spectra to match the resolution of the ISIS data.

4 DISCUSSION
What at first appears to be a fortuitously aligned, but essentially straightforward, triple system, raises a host of problems when one tries to fit the dips that occur when the binary transits its tertiary companion. In some despair, we turned to quadruple star models for the system, although our faith in them is limited as they feel contrived – epicycles spring to mind. Quadruple models have their own set of problems, although they are less show-stopping than those which afflict the triple model.

Further observations are essential to guide future modelling of KIC 2856960. Spectroscopy over several days could test the binary nature of star 3. Spectroscopy over the 204 d cycle can provide a measurement of e3 and thus the total system mass. We expect radial velocity variations of several tens of kilometres per second, and the spectrum (Fig. 13) has plenty of sharp line features which should allow precise radial velocities. Spectroscopy at long wavelengths might reveal the close binary. The latter contributes a minimum of ~3 per cent of the light in the Kepler bandpass, but this is poorly constrained and could be larger. Even at the minimum contribution, if the stars in the binary are M stars, then given the mid-K star of Fig. 13, we can expect a significantly higher contribution from the binary in the I and J bands. Further monitoring of the dips, possible from the ground given the 8 per cent maximum depth, will also be of value to see whether they evolve significantly with time. If the system is truly a quadruple star, then the simultaneous short, long and intermediate orbital periods, together with the binary eclipses and dips and highly eccentric outer orbit, suggest that it may be of interest for dynamical studies, and significant evolution of the dips can be expected.

5 CONCLUSIONS
KIC 2856960 is an apparent triple star containing a close binary in orbit with another object. Its orientation is such that the close binary passes in front of its companion causing the appearance of a series of dips in the light curve that last for a little over 1 d and recur on a period of 204 d. The light curve of the binary is consistent with a pair of fairly well-detached and similar low-mass M dwarfs. While we expected the system to be straightforward to understand, we were entirely unable to model it as a triple star. Under triple models, the dips can only be modelled with separations of the binary which violate Kepler’s laws. Quadruple star models, involving either two binaries in orbit around each other or a binary orbited by another star with another star orbiting the three of them, can match the data without straining Kepler’s laws, but require a very near integral ratio between the 204 d period that the dips recur on and the period of the additional orbit. There are moreover significant remaining mismatches between the model and data even with the extra freedom provided by quadruple systems, and the derived stellar parameters do not seem astrophysically plausible. KIC 2856960 thus defies easy explanation. We urge further observations to uncover the true nature of this remarkable object.

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