Maximum Throughput for a Cognitive Radio Multi-Antenna User with Multiple Primary Users

Ahmed El Shafie† and Tamer Khattab∗
†Wireless Intelligent Networks Center (WINC), Nile University, Giza, Egypt.
∗Electrical Engineering, Qatar University, Doha, Qatar.

Abstract—We investigate a cognitive radio scenario involving a single cognitive transmitter equipped with \(K\) antennas sharing the spectrum with \(M\) primary users (PUs) communicating with their respective receivers. Each terminal has a queue to store its incoming traffic. We propose a novel protocol where the cognitive user transmits its packet over a channel formed by the aggregate of the inactive (empty) primary bands. We study the impact of the number of the PUs, the sensing errors, and the number of antennas on the maximum secondary stable throughput.

Index Terms—Cognitive radio, stable throughput, stability region, queue stability, multiple access.

I. INTRODUCTION

One challenge in cognitive radio networks is designing an optimal channel access for the secondary users (SUs) under certain quality of service for the primary users (PUs). This opens the research area for the invention of novel medium access protocols which enhance the performance of the network.

Considering a single cognitive transmitter-receiver pair in presence of multiple primary transmitter-receiver pairs has got extensive attention in the literature [1]–[4]. The authors of [2] developed the optimal policy under the finite-horizon partially observable Markov decision processes (POMDP) formulation that has a complexity growing exponentially with the duration of the transmission. In that work, the overall state of the network is partially observable due to the fact that a secondary transmitter senses only some of the available channels. Under the assumption of having the same transmission structure for both primary and secondary users, the authors derived the optimal and suboptimal spectrum sensing and access strategies under the formulation of finite-horizon POMDPs.

In [3], the authors considered an infinite-horizon optimization where the complexity does not grow with the length of the transmission in contrast to [2]. The authors assumed that the multiple PU channels evolve independently as continuous-time Markov chains. Based on such assumption, the authors proposed an access scheme referred to as periodic sensing opportunistic spectrum access (PS-OSA). The essence of PS-OSA is to remove the partial observability by sensing the available channels periodically. In general, restricting to periodic sensing is suboptimal, but the proposed scheme significantly reduces the complexity required by the optimal OSA proposed in [2] under the POMDP framework. Based on partial observability of the Markov process and all past sensing results, the SU takes an action of either transmitting on one of the channels or remaining silent (not transmitting at all). The authors showed that when the constraints on interference are tight, the performance loss of PS-OSA is negligible.

In [1], the PUs are modeled as independent continuous-time Markovian on-off processes. The secondary transmitter aims at maximizing its throughput subject to collision constraints. The authors investigated some access policies for the SU: sensing one primary channel, sensing all primary channels simultaneously, and memoryless with periodic sensing. The case of multiple SUs is also discussed. In [4], the authors designed an opportunistic spectrum access (OSA) in the presence of reactive PUs, where PU’s access probability in a given channel is related to SU’s past access decisions. The channel occupancy of the reactive PU is modeled as a four state discrete-time Markov chain and the optimal OSA design for SU throughput maximization is formulated as a constrained finite-horizon POMDP problem.

In a cognitive setting with buffered nodes, the authors in [5] considered multiple PUs with a common destination and one cognitive radio user with relaying capability. When all primary packets being served in all primary and relaying queues, the cognitive radio user switches to the best idle band which has the maximum channel gain, based on the channel conditions of the time, for the transmission of its own packets. In [6], krikidis et. al. considered a simple configuration composed of one cognitive transmitter-receiver pair and two primary transmitter-receiver pair wishing to deliver their packets to a single receiver in a multi-access channel (MAC). The secondary transmitter is capable of relaying the undelivered packets of the PUs. If a primary packet is correctly decoded at either the secondary transmitter or the primary destination, it is dropped from the relevant primary queue. A priority of transmission is given to the relaying packets over the secondary own packets when the primary queues are empty. In addition, the secondary transmitter sends its own packets in two ways: 1) when all the primary and relaying queues are empty or 2) simultaneously with the PUs via a superposition technique when the primary queues are nonempty.

In this work, we investigate a cognitive scenario with one cognitive radio user (SU) possessing \(K\) antennas and \(M\) single antenna PUs. Each PU is assigned to a primary licensed band with certain bandwidth. The SU senses the primary bands and merges the free bands into an aggregate channel used to send one of its packets. The analysis of the proposed protocol is carried out from the datalink layer standpoint. Specifically, we characterize the maximum secondary stable throughput. The proposed protocol is simple and doesn’t require the knowledge
of channel state information at the transmitters.

The contributions of this paper can be summarized as follows. A new cognitive medium access control protocol is proposed which enables the SU to merge (or aggregate) the sensed free bands for a single packet transmission. The proposed protocol is simple as it doesn’t require the instantaneous tracking and estimation of the channel state information (CSI) at the transmitters. We characterize the maximum stable throughput of a secondary node coexisting with multiple primary nodes. We include sensing errors to the analysis and study the impact of the secondary number of antennas, $K$, and the number of primary bands, $M$, on the secondary throughput.

The rest of this paper is organized as follows: Next we describe the system model adopted in this paper. In Section III we provide the stability analysis of the proposed system. We present the numerical results and discussions in Section IV. Finally, we draw our conclusions in Section V.

II. SYSTEM MODEL

We consider a cognitive scenario with one SU equipped with $K$ antennas and $M$ PUs as shown in Fig. 1. The PUs are communicating to their respective receivers using frequency division multiple-access technique. Specifically, we assume the existence of $M$ PUs each of which is assigned a unique orthogonal band. The $m$th PU, $p_m$, owns band $m$. The channel is slotted and the length of one time slot is $T$. Each user has an infinite capacity buffer (queue) to store the incoming fixed-length packets, denoted by $Q_i$ (see [6] for a similar assumption). The arrivals at queue $Q_i$ are independent and identically distributed (i.i.d.) Bernoulli random variables. Arrivals from slot to slot with mean $\lambda_i \in [0, 1]$ packets of size $b$ bits per time slot, $i$ reads ‘$p_m$’ for the PU assigned to band $m$, and $i$ reads ‘s’ for the secondary queue. Arrival processes are independent from queue to queue and terminal to another [6, 7].

All wireless links exhibit fading and additive white Gaussian noise (AWGN). The fading is assumed to be stationary, with frequency non-selective Rayleigh block fading. This means that the fading coefficient $h_i$ (for the link connecting node $i$ and its respective receiver) remains constant during one slot and over all frequency bands, but changes independently from one slot to another according to a circularly symmetric complex Gaussian distribution with zero mean and variance $\sigma_i^2$. Furthermore, the receivers are modeled as AWGN with zero mean and with power spectral density $N_0$ Watts/Hz. The channel state information are known at the receivers only [6]. The primary node $j$ has bandwidth $W_j = W$, where $j \in \{p_1, p_2, \ldots, p_M\}$. The cognitive radio user senses all bands simultaneously for $\tau$ seconds relative to the beginning of the time slot whose length is $T$ seconds. If the SU has $K \leq M$ antennas, the time needed to sense $M$ bands is $\tau = [M/K]\tau_B$, where $[\cdot]$ denotes the ceil of the argument and $\tau_B$ is the time spent in sensing one primary band [1]. Since the cognitive radio user spends $\tau$ seconds in spectrums sensing, the remaining time for data transmission is $T - \tau$. An immediate observation is that as the sensing duration increases, the remaining time for data transmission decreases. Hence, the outage probability of the secondary link. All packets are assumed to be of the same length, and each contains $b$ bits.

As will be explained later, the cognitive radio user emerges the available bands. If $\eta$ bands are free, the transmission bandwidth is $\eta W$ and with transmit power $\eta W P_s$ in Watts, where $P_s$ is the power spectral density of the SU in Watts/Hz. The outage event occurs when the instantaneous capacity of the link is lower than the transmitted spectral efficiency rate. Let $\sigma_p^2$ denote the channel variance of user $p_m$ and $P_p$ denote the transmit power per unit frequency of user $p_m$. The packet correct reception of the $m$th PU is characterized by the success probability [6].

$$T_{p_m} = \Pr \left\{ \log_2 \left( 1 + \gamma p_m \right)^2 \right\} > R = \exp \left( - N_0 \frac{2 R - 1}{\sigma_p^2 P_p} \right),$$

where $R = b/(\eta W T)$.

For simplicity of presentation, we assume symmetric PUs, which implies that all primary queues have the same arrival rate and channel parameters [5]. Hence, $\lambda_{p_m} = \lambda_p$ and the sensing error probabilities are equal for all primary bands. The essential difference is that for the general asymmetric case, the analysis has to keep track of the different possible primary

---

Footnote:

1 If $K > M$, the time needed for sensing all bands is $\tau_B$, and the SU can assign some antennas to the same band to enhance the quality of the sensing process.
transmitting and channels sets which clutters the notation whereas for the symmetric case, only the number of the PUs matters. However, the approach in both cases is similar.

The retransmission mechanism of lost packets is based on the feedback acknowledgement/negative-acknowledgement (ACK/NACK) signals. In particular, at the end of the time slot, each receiver broadcasts a feedback signal to inform the respective transmitter about the decodability status of the transmitted packet. The overhead for transmitting the ACK and NACK messages is assumed to be very small compared to packet sizes [7]. As usual, we assume that errors in the feedback messages are negligible, which is reasonable for short length packets as strong and low rate codes can be implemented in the feedback channel [6], [7].

Assume that the SU detects \( \eta \) bands to be free, if all these bands are truly free of any primary transmissions, the probability of complement channel outage of the secondary channel is then given by

\[
\mathcal{P}_{s,\eta W} = \exp \left( - N_0 \frac{2 R_s - 1}{\sigma_s^2 \mu_s} \right),
\]

with

\[
R_s = \frac{b}{\eta W (T - \frac{\eta}{K} \tau_B)^+} = \frac{R}{\eta (1 - \frac{\eta}{K} \tau_B)^+}
\]

where \((V)^+\) denotes \(\max\{V, 0\}\). If there are two concurrent transmissions over any band, both packets under transmission will be lost. Increasing the number of antennas allows the SU to invest more time in data transmission, hence increases the secondary throughput. Whereas, the huge increase of the number of primary bands may cause time slot consumption, hence successful transmission with probability zero. We assume that the cognitive radio user cannot send more than one packet at any slot. The medium access control operation can be described as follows.

- The PUs access the channel at the beginning of the time slot.
- The SU senses all the primary bands from the beginning of the time slot to \( \tau \) seconds to detect the possible activity of the PUs.
- The SU emerges (aggregates) the available bandwidths of the sensed free bands, and sends exactly one packet.
- At the end of each time slot, a feedback signal from the respective receiver is sent to inform the transmitter about the status of its packet decoding. A correctly received packet is removed from the respective transmitter’s queue. In the case of packets lost due to concurrent transmission or channel impairments, re-transmission of the lost data is required.

### III. Stability Analysis

A fundamental performance measure of a communication network is the stability of the queues. Stability can be defined rigorously as follows. Denote by \( Q(t) \) the length of queue \( Q \) at the beginning of time slot \( t \). Queue \( Q \) is said to be stable if \( \lim_{x \to \infty} \lim_{t \to \infty} \Pr\{Q(t) < x\} = 1 \). A system is said to be stable, if every queue belongs to the system is stable. We can apply Løynes’ theorem to check the stability of a queue \( Q \). This theorem states that if the arrival process and the service process of a queue are strictly stationary, and the average service rate is greater than the average arrival rate of the queue, then the queue is stable, otherwise it is unstable.

Let \( X^t_j \) denote the arrivals to \( Q_j \) at an arbitrary time slot \( t \), and \( Y^t_j \) denote the number of departures of \( Q_j \) at an arbitrary time slot \( t \). Based on the late arrival model, which means that an arriving packet will be blocked during its arrival time slot even if the queue is empty, the evolution of \( Q_j \) is given by

\[
Q_j^{t+1} = (Q_j^t - Y_j^t)^+ + X_j^t
\]

where \((V)^+\) denotes \(\max\{V, 0\}\) and \(\max\{\cdot\}\) returns the maximum among the values in the argument.

Due to imperfect sensing, the SU may interfere with the PUs due to sensing errors, hence cause packet collision and throughput loss. As will be discussed later, the SU will suffer throughput degradation due to sensing errors. Let \( P_{EA} \) denote the probability that the SU sensor generates false alarm, and \( P_{MD} \) denote the probability that the SU misdetects the primary activity [4].

Since the queues service rates are coupled, i.e., interacting queues, we resort to the idea of the dominant system (or saturated/backlogged SU) [7], [8]. In the dominant system, the behavior of nodes and channels realization are the same, but the secondary node, if it is emptied, sends a dummy packet. This dummy packet may interfere with the PUs, but it doesn’t contribute to the secondary throughput. This idea has been used in a lot of works (see for example [6]–[8] and the references therein). It has been shown that the stability of the dominant system and the original system are indistinguishable at the boundary points. Hence, the stability of the dominant system is necessary and sufficient for the stability of the original system.

Since the SU is saturated (backlogged), the probability of \( m \)th PU correct transmission is given by the event that the link between \( p_m \) and its respective receiver is not in outage and that the SU detects the primary active correctly. Hence, the mean service rate of the \( m \)th PU is

\[
\mu_{p_m} = \mathcal{P}_p (1 - P_{MD})
\]

Let \( \pi = 1 - \lambda_p / \mu_p \) denote the probability of the \( m \)th PU being empty [5]–[7]. When \( \eta \leq M \) of the primary bands are free, the SU must detect one of them to be free correctly. Also, it must detect the activity of all the active users correctly to avoid concurrent transmission and packets loss. Hence, the mean service rate of the secondary queue is given by

\[
\mu_s = \sum_{n=1}^{M} \binom{M}{n} \pi^n (1 - \pi)^{M-n} \mathcal{P}_{MD} P_{FA} P_{MD} \mathcal{P}_{s,nW}
\]

where the term \((1 - P_{MD})^{(M-n)}\) means that the SU must not

\(^2\)Since the decision of PUs activity is taken separately, i.e., the SU senses and decides on the sensing outcome of each band independently, the probability of misdetecting the \( m \)th PU is \( P_{MD} \) regardless of other PUs’ state.
use the band of an active PU, which will cause packet loss for the SU and the that PU; and the term $\left( \frac{\eta}{\eta_n} \right) ^{M_p} \left( 1 - p_{\text{MD}} \right)$ represents the probability of generating false alarm over $\eta - n$ bands out of the $\eta$ empty bands.

Since the PUs are symmetric, the instability/stability of one of the PUs is sufficient and necessary for instability/stability of the system. Based on this, we can assume that the primary queues stability as a single queue stability. Hence, the system stability converges to stability of a two queues system, i.e., a system with one primary queue and one secondary queue. We can then establish the argument of indistinguishability between the dominant system and the original system at the boundary points.

Based on the construction of the dominant system, the data queues of the dominant system are always larger in size than those of the original system, provided the queues start with identical initial conditions in both systems. Therefore, for a given $\lambda_p \leq \overline{P}_p(1 - P_{\text{MD}})$, if for some $\lambda_s$, the queue $Q_s$ is stable in the dominant system then it must be stable also in the original system. Conversely, if for some $\lambda_s$ in the dominant system, the queue $Q_s$ saturates, then it will send a physical packet instead of transmitting dummy packets and thus the behavior of the dominant system becomes identical to that of the original system. Therefore, the original system and the dominant system are indistinguishable at the boundary points and thus have the same stability region.

As the number of PUs increases, the secondary throughput increases due to the possibility of exploiting more free bands. However, due to sensing errors, for specific value of $M$, the secondary throughput will start to degrade as the number of PUs increases, due to the increases of the probability of misdetecting one of the active PUs, as will be shown in the numerical results. One can think about seeking for the optimal number of bands, $M_0 \leq M$, which should be selected by the SU from the available primary bands such that its throughput is maximized under the stability of the primary queues. Given that all the PUs are stable, i.e., $\pi > 0$, the optimization problem which describe the optimal secondary maximum stable throughput for each $\lambda_p$ can be stated as follows:

$$\max_{M_0 \leq M} \mu_s, \; \pi \geq 0$$ \hspace{1cm} (7)

It should be noted that if the SU is a power-limited device, i.e., if the transmit power per time slot is constrained by a certain value, i.e., $P_s = P_{\text{W}}$, where $P_{\text{W}}$ is the total allowable power per time slot in Watts, the sole variation in the above analysis is in the value of the complement of channel outage. Specifically, when there are $\eta$ free bands and the SU detects sensed them to be inactive, the packet correct reception is given by

$$\overline{P}_{s,\text{W}} = \exp \left( -\eta N_c \frac{2R_s - 1}{\eta_n \sigma_{\text{W}}^2} \right).$$ \hspace{1cm} (8)

For the multiple SU with single destination, we can assume that the SUs share the spectrum using random multi-access channel system [6] (such as standard MAC [6], where users can simultaneously use the spectrum) or time division multiple-access schemes. Also, they can cooperatively detect the available band[4] and split them such that each SU assigned bunch of the sensed free bands for the transmission of its packet.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present some numerical results for the presented optimization problems in this paper. Let $\sigma_{\text{w}}^2 P_s / N_0 = 1$, $K = 8$ and $\tau_B = 0.01T$. In Fig. 2 we plot the secondary throughput versus $M$. As shown in the figure, the secondary throughput increases with $M$ until specific $M = M_c$, where the behavior is reversed. It is noted that if the SU chooses only $M_c = 13$ bands to exploit when some of them become empty, the stable throughput is maximized. The parameters used to generate the figure are: $R = b/(TW) = 2$ bits/sec/Hz, $\overline{P}_p = 0.9$, $P_{\text{MD}} = 0.05$, $P_{\text{FA}} = 0.05$, and $\lambda_p = 0.5$ packets per time slot. Fig. 3 shows the maximum secondary stable throughput of the proposed system. The plot is generated for many values of $M$, as shown in the figure, the secondary stable throughput for each $\lambda_p$ expands as the number of the secondary bands increases until specific $M$. For $M = 100$, $\mu_s$ for some $\lambda_p$ is higher than $\mu_s$ for the lower $M$ curves and for other values the behavior is reversed. The parameters used to generate the figure are: $R = b/(TW) = 2$ bits/sec/Hz, $\overline{P}_p = 0.9$, $P_{\text{MD}} = 0.01$, and $P_{\text{FA}} = 0.05$.

Fig. 4 shows the maximum secondary stable throughput for the system with and without power constraint per time slot. For comparison purposes, the system in which the SU selects one of the empty bands, when there is more than one band free, and transmit with the whole available power. The figures reveal the gains of the proposed protocols in case of limited power and unlimited power over the one band transmission protocol. Fig. 5 is generated with $M = 15$, $P_{\text{MD}} = P_{\text{FA}} = 0$, $\lambda_p = 0.2$, and $\sigma_{\text{w}}^2 P_s / N_0 = 4$.

Finally, we present the impact of number of antennas on the stability region in Fig. 5. The fact that the throughput increases with increasing the number of antennas is shown. However, this increasing is limited by the factor $[M/K] \tau_B / T$. If this factor is negligible, i.e., $[M/K] \tau_B / T \approx 0$, the increasing of $K$ will not boost the secondary throughput. The parameters used to generate the figure are: $R = b/(TW) = 2$ bits/sec/Hz, $\overline{P}_p = 0.9$, $P_{\text{MD}} = 0.01$, $P_{\text{FA}} = 0.05$, $\tau_B = 0.05T$, and $M = 40$.

V. CONCLUSIONS

In this paper, we have proposed a new medium access scheme for a multi-antenna SU coexisting with multiple PUs. The SU merges the available primary bandwidth for its packet transmission. The proposed protocol is studied for both limited and unlimited power SU. The SU may select a set of PUs to be used for employing its protocol. We have characterized the secondary stable throughput for the proposed system. We have investigated the impact of the number of secondary antennas

3 Which could enhance the quality of channel sensing probabilities as an appropriate fusion and combination of the individual sensing data improves the ability of the system and decreases the probability of detection error (misdetction) and false alarm.
Fig. 2. Maximum mean secondary service rate versus the number of PUs.

Fig. 3. Maximum mean secondary service rate for the proposed protocol versus $\lambda_p$. The figure is plotted for $M = 5, 10, 20, 100$.

Fig. 4. Maximum throughput of the proposed protocol with and without power constraint. We also plotted the case in which the SU, when there is more than one band, selects one of the free bands.

and number of PUs on the secondary throughput. One possible extension of this work is to consider a generalized multipacket reception channel with the possibility of estimating the CSI at the secondary transmitter.

REFERENCES

[1] X. Li, Q. Zhao, X. Guan, and L. Tong, “Optimal cognitive access of markovian channels under tight collision constraints,” IEEE J. Sel. Areas Commun., vol. 29, no. 4, pp. 746–756, 2011.

[2] Q. Zhao, L. Tong, A. Swami, and Y. Chen, “Decentralized cognitive mac for opportunistic spectrum access in ad hoc networks: A pomdp framework,” IEEE J. Sel. Areas Commun., vol. 25, no. 3, pp. 589–600, 2007.

[3] Q. Zhao, S. Geirhofer, L. Tong, and B. M. Sadler, “Opportunistic spectrum access via periodic channel sensing,” IEEE Trans. Signal Process., vol. 56, no. 2, pp. 785–796, 2008.

[4] Y. L. Che, R. Zhang, and Y. Gong, “On design of opportunistic spectrum access in the presence of reactive primary users,” IEEE Trans. on Comm., vol. 61, no. 7, pp. 2678–2691, July 2013.

[5] X. Bao, P. Martins, T. Song, and L. Shen, “Stable throughput analysis of multi-user cognitive cooperative systems,” in IEEE GLOBECOM, Dec. 2010, pp. 1–5.

[6] I. Krikidis, N. Devroye, and J. Thompson, “Stability analysis for cognitive radio with multi-access primary transmission,” IEEE Trans. on Wire. Commu., vol. 9, no. 1, pp. 72–77, Jan. 2010.

[7] A. Sadek, K. Liu, and A. Ephremides, “Cognitive multiple access via cooperation: protocol design and performance analysis,” IEEE Trans. Inf. Theory, vol. 53, no. 10, pp. 3677–3696, Oct. 2007.

[8] R. Rao and A. Ephremides, “On the stability of interacting queues in a multiple-access system,” IEEE Trans. Inf. Theory, vol. 34, no. 5, pp. 918–930, Sept. 1988.