RENORMALIZED QUARK-ANTI-QUARK FREE ENERGY *

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We present results on the renormalized quark-anti-quark free energy in $SU(3)$ gauge theory at finite temperatures. We discuss results for the singlet, octet and colour averaged free energies and comment on thermal relations which allow to extract separately the potential energy and entropy from the free energy.

1. Introduction

It is believed that strongly interacting matter undergoes a confinement-deconfinement phase transition at very high temperature $T$ and/or baryon density and it is well-established that QCD is a valid theory for both phases. The thermal modification of meson and hadron properties is often discussed in terms of $n$-point Polyakov loop correlation functions which describe the free energy $F_{nq,\bar{n}q}$ of a gluonic heat bath including static colour charges$^1$. It is well-known, however, that Polyakov loop correlation functions need to be fixed by renormalization. On a lattice a renormalization prescription has recently been suggested$^2$. Applying this renormalization scheme to the $n$-point Polyakov loop correlation functions, one fixes the free energy which consists of potential energy ($V_{nq,\bar{n}q}$) and entropy ($S_{nq,\bar{n}q}$): $F_{nq,\bar{n}q} = V_{nq,\bar{n}q} - TS_{nq,\bar{n}q}$. The calculation of the potential energy from Polyakov loop correlation functions at finite $T$ is still an open problem - although the potential energy is expected to be an essential quantity for heavy-quarkonium physics.

In this paper we refer to results on the renormalization scheme$^{2,6}$ applied to 2-point Polyakov loop correlation functions in $SU(3)$ gauge theory on a

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2. A new look at quark-anti-quark free energy

The renormalized quark-anti-quark free energies in each colour channel are shown in Fig. 1 exemplarily for one temperature below and one above $T_c$. Also included is the heavy quark potential at zero temperature which we
take to be the standard Cornell potential. In a rather rough way we may distinguish three different distance regions with different behaviour of the free energies at fixed temperature:

At large quark anti-quark separations the colour singlet ($F_1$), colour octet ($F_8$) and colour averaged ($F_{q\bar{q}}$) quark-anti-quark free energies coincide at both temperatures. The same feature can be observed at all temperatures analyzed by us. In fact, if the separation between the quark anti-quark sources becomes large, the relative orientation of the colour charges in the colour space will not influence the free energy: For large distances we therefore expect $F_1(R \gg 1, T) = F_8(R \gg 1, T) = F_{q\bar{q}}(R \gg 1, T)$ in both phases. For $T < T_c$, this picture suggests that the quark-anti-quark free energy is dominated by a single, temperature dependent string tension for all color channels at large separations - including the color octet channel.

At intermediate distances, i.e. for distances $1 \lesssim R\sqrt{\sigma} \lesssim 4$ and temperatures below $T_c$, we note that all color channels lead to an enhancement of the free energies compared to the potential at zero temperature. In this intermediate distance regime the different colour structures of the free energies become visible. Both features, the confining colour octet free energy below $T_c$ and the enhancement of the colour singlet free energy were first observed in SU(2) - and recent lattice studies of Polyakov loop correlation functions in SU(2) indicate similar properties.

At small quark anti-quark separations the different colour structures of the free energies dominate the picture: The color singlet free energy coincides with the potential at zero temperature while the colour octet free energy behaves repulsive as one may expect from leading order perturbation theory. The color averaged free energy respects the relation $F_{q\bar{q}} - F_1 = T \ln 9$ at short distances. It is worth noting that the temperature dependence of the free energy becomes less important with decreasing quark anti-quark separations in all colour channels.

3. From quark-anti-quark free energies to the QCD-force at finite $T$?

The investigation of medium effects on free energies is essential in heavy-quark physics and has been subject of many lattice studies so far. Of even greater relevance for heavy-quark physics then the free energy, however, is the potential energy at finite temperature as it appears in fundamental quantum mechanical and field theoretical relations. Thus the knowledge of potential energy would lead to a better understanding of the basic forces
Figure 2. The renormalized free energy at infinite quark separation for $T > T_c$. The filled symbols describe $F^\infty/T_c$ and the open symbols $F^\infty/T$. According to our discussion in section 2 the data points reflect the free energy for all colour channels. In combination with (1) the magnitude of the potential energy can be read off from the open symbols while the magnitude of the entropy follows from the filled symbols. The same study can be done at any finite quark anti-quark separation and will lead to the force at finite temperature.

...in particle physics but its calculation on a lattice is still an outstanding problem at finite temperatures.

Having fixed the free energy, however, opens the opportunity to refer to thermal relations in order to study, for instance, the potential energy and entropy, at arbitrary distance $R_0$

$$S(R_0, T) \bigg|_{R_0} = -\frac{\partial F}{\partial T} \bigg|_{R_0} \quad \text{and} \quad V(R_0, T) \bigg|_{R_0} = -T^2 \frac{\partial F/T}{\partial T} \bigg|_{R_0}; \quad (1)$$

When analyzing these relations at a set of distances, the QCD force at finite $T$ follows from $-dV/dR$. In order to analyze the quantities in (1) we have plotted $F^\infty = \lim_{R \to \infty} F_1(R, T)$ for temperatures above $T_c$ (see Fig. 2). We only note here two interesting features that follow from this figure: Firstly, since $F^\infty/T_c$ is a monotonicly decreasing function with increasing temperature it follows that at large quark anti-quark separation the entropy contribution to the free energy is non-zero (and positive). On the other hand the free energies become temperature independent in the limit of vanishing quark anti-quark separation. This implies that the entropy...
contribution to the free energy is $R$-dependent,

$$F_i(R, T) = V_i(R, T) - T S_i(R, T) \quad \text{with } i = 1, 8, q\bar{q}. \quad (2)$$

Moreover, the behaviour of $F^\infty/T$ in Fig. 2 tells us that also the potential energy is non-zero at large quark anti-quark separations and temperatures near $T_c$. Indeed, in the temperature regime analyzed by us the potential energy at large quark anti-quark separations is a positive and monotonically decreasing function with increasing temperature. Moreover, from high temperature perturbation theory, one expects $F^\infty(T) \sim -T g^2(T)$ which implies $V^\infty(T) \sim -2\beta_0 T g^4(T)$ with $\beta_0$ being the first coefficient of the perturbative $\beta$-function.

It should be obvious that the outlined program - including the renormalization prescription - is neither bound to a temperature nor to a distance regime and equally well should hold for free energies in full QCD. It will be of even greater importance in this case as the potential energy reflects the string breaking energy.

4. Conclusion

We have presented a new look at the renormalized quark-anti-quark free energy including the colour singlet, colour octet and colour averaged free energy in the confined and deconfined phase. The renormalization scheme also allows to have a new interpretation of the heavy quark potential at finite temperatures. In particular, we have shown that the $R$-dependence of the potential energy is not only given by that of the free energy. A detailed analysis of the potential energy, the entropy and the QCD-force at finite temperature will be presented elsewhere.

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