Control design using state observation and model reference adaptive systems for a dc motor

A E Florian-Villa\textsuperscript{1}, and J A Patiño-Murillo\textsuperscript{1,2}
\textsuperscript{1} Red Tecnoparque Nodo Medellín, Servicio Nacional de Aprendizaje, Medellín, Colombia
\textsuperscript{2} Facultad de Ingeniería, Institución Universitaria Pascual Bravo, Medellín, Colombia

E-mail: aflorian@sena.edu.co

Abstract. This paper proposes an adaptive control structure involving the integration of a Luenberger observer with a model reference adaptive system. The Luenberger state observation algorithm describes a dynamic system that estimates the state vector of a system under observation. Model reference adaptive system is a technique providing an automatic adjustment of a controller in real-time. The integrated control structure was implemented in the simulation of a first-order system that describes the dynamics of a direct-current motor. The results obtained show the performance improvement to other classic control techniques for these direct-current systems. Simulation results also demonstrate the acceptable performance achieved by the proposed control for systems involving variables without direct measurement. For these cases, it is mandatory to have a good observation of the unmeasured variable and a suitable control structure developed for the overall process.

1. Introduction
In many industrial operations, there are process variables in need of constant supervision for monitoring and control tasks. Often, the operation of these tasks deals with systems with highly sensitive parameters or elements with continually changing characteristics [1]. Moreover, some industrial variables neither can be measured directly, or the use or existence of the appropriate sensor for that magnitude is not feasible. This situation involves a significant challenge for designers of control systems, who must somehow have measurement and manipulation of system variables [2].

Traditional control methodologies use feedback strategies measuring the control variable and then reacting to deviations from the desired reference value. Adaptive control schemes arise as an alternative to classical methods, where the controller parameters and even the control structure are adjusted according to process variations. Another alternative when direct measurement is unfeasible is the estimation of the non-measurable states of the system through a state observer [3]. The observer is a dynamic representation whose states converge to those of the observed system, based on the system model and the observation of the inputs and outputs. This information is used to estimate variables that are difficult to measure to make possible the full description of the behavior of the dynamic system.

In general, several researchers have been interested in designing state-observers, and there are many solutions reported in the literature applied to problems of estimation, model identification, and controller design, among others [4,5]. Also, the design of controllers using model reference adaptive systems (MRAS) is widely explored in control applications such as chemical processes, power systems, and other industrial activities [6-9]
The main objective of this work is to control process variables when direct measurement is unfeasible, requiring the use of state observation. The latter is a necessary condition to perform adequate control actions of the unmeasured process variable. In this case, the control variable is the stator current of a direct current motor, and the control action is applied to an estimated measurement. The control methodology is a model reference adaptive control (MRAC) based on Lyapunov stability analysis to guarantee the adaptation to changes in process dynamics and unknown inputs [10].

2. Design of state observers

Observability is the possibility of determination of the states of a system from measurements of inputs and outputs during a finite time. If a system is observable, then the states of the system could be rebuilt from a state observer [11]. However, the problem relies upon finding conditions where knowledge of the input-output data reconstructs the unique form of the state of the system. Consider the linear system below, where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$, Equation (1).

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

Matrices $A$, $B$, and $C$ are known. The state Equation (1) is observable if, for any initial state $x(0)$ (unknown), there exists a finite time $t_1$ such that knowledge of the input $u$ and the output $y$ over the interval $[0; t_1]$ is sufficient to determine the initial state $x(0)$ uniquely; otherwise, the system is deemed unobservable.

The initial state must be able to be determined at any final time [6]. Kalman showed that observability could be easily analyzed using a test that is now denominated as Kalman observability test, which relies on the so-called observability matrix $M_0$, Equation (2).

$$M_0 = \begin{bmatrix}
C \\
CA \\
\vdots \\
CA^{n-1}
\end{bmatrix}$$

If the observability matrix is of rank $n$, the dimension of the system, the system is observable [7]. Let $\hat{x}$ and $\hat{y}$ represent the estimated vectors of system states and outputs, respectively. The availability of system inputs $u(t)$, outputs $y(t)$ and the observer gain $L$ is essential to construct an estimate of the system, as shown in the next equations and depicted in Figure 1 (also known as a Luenberger observer): Equation (3).

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$
$$\hat{y} = C\hat{x}$$

The dynamics of the error are given by Equation (4).

$$\dot{e} = \dot{\hat{x}} - \hat{x}$$

Replacing terms from Equation (3), we get Equation (5).

$$\dot{e} = (Ax + Bu) - (A\hat{x} + Bu + L(y - \hat{y}))$$
$$\dot{e} = A(x - \hat{x}) - L(Cx - C\hat{x})$$
$$\dot{e} = (A - LC)(x - \hat{x})$$
$$\dot{e} = (A - LC)e$$
3. Control using state observation and model reference adaptive systems

The objective of MRAC is to make the control loop have the same dynamic behavior of another model representing the desired dynamic behavior of the closed-loop system. The response of the reference model to input is the desired closed-loop response of the plant. Accordingly, the controller must be designed making the response of the control loop equal to that of the reference model, i.e., the difference between the actual system output and the reference model output tends to zero. That is, the adaptation mechanism adjusts control parameters in such a way that the system follows the reference model.

3.1. Design of model reference controllers using Lyapunov theory

Let a system be represented by the transfer function G(s) as shown in Figure 2. Commonly, a first-order system is selected as the reference model, represented by the Equation (6):

\[
\frac{dy_m}{dt} = -a_m y_m + b_m u_c
\]  

(6)

A first-order differential equation (Equation (6)) also describes the original system, Equation (7).

\[
\frac{dy}{dt} = -ay + bu
\]  

(7)

The control law u and the error signal e are described by Equation (8).

\[
u = \theta_1 u_c - \theta_2 y \\
e = y - y_m
\]  

(8)

After replacing some terms and performing derivation of the error function, we get Equation (9).

\[
\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)u_c
\]  

(9)

The error goes to zero for a given set of parameters \(\theta_1\) and \(\theta_2\). To assess the Lyapunov stability, we introduce the next quadratic function to adjust the parameters \(\theta_1\) and \(\theta_2\), Equation (10).

\[
V(e, \theta_1, \theta_2) = \frac{1}{2} (e^2 + \frac{1}{by} (b\theta_2 + a - a_m)^2 + \frac{1}{by} (b\theta_1 - b_m)^2)
\]  

(10)

To qualify as a Lyapunov function, the derivative of the function in Equation (10) must be negative, Equation (11).

\[
\frac{dV}{dt} = e \frac{de}{dt} + \frac{1}{y} (b\theta_2 + a - a_m) \frac{d\theta_2}{dt} + \frac{1}{y} (b\theta_1 - b_m) \frac{d\theta_1}{dt}
\]  

(11)

Reordering parameters in Equation (12).
\[
d\frac{dV}{dt} = -a_m e^2 + \frac{1}{\gamma} (b\theta_2 + a - a_m) \left( \frac{d\theta_2}{dt} - \gamma ye \right) + \frac{1}{\gamma} (b\theta_1 - b_m) \left( \frac{d\theta_1}{dt} + \gamma u_c e \right)
\]  

(12)

If the parameters are updated as \( \frac{d\theta_1}{dt} = -\gamma u_c e \) and \( \frac{d\theta_2}{dt} = \gamma ye \), then in the Equation (13).

\[
d\frac{dV}{dt} = -a_m e^2
\]

(13)

The derivative of V with respect to time is thus negative semidefinite but not negative definite. In this case, \( V(t) \leq V(0) \) and thus \( e, \theta_1 \) and \( \theta_2 \) must be bounded [3]. This implies that the system output \( y = e + y_m \) is also bounded. The application of this Lyapunov stability design methodology prevents the stability difficulties commonly associated with gradient-based methods.

3.2. Design of model reference controllers using Lyapunov theory and Luenberger observer

Here we have two theories working jointly to get the best results for controlling a variable that cannot be measured directly: these are the Luenberger state observation algorithm, and the design of MRAC using Lyapunov theory. The goal of this mixed approach is to obtain an improved controller in both performance and stability. Figure 3 depicts a block diagram of the proposed control scheme with a state-feedback observer and model reference adaptive system. From Figure 3, the control law for the estimated system is Equation (14).

\[
u = \theta_1 u_c - \theta_2 \hat{y}
\]

\[
e = \hat{y} - y_m
\]

(14)

After replacing some terms and performing derivation of the error function, Equation (15).

\[
d\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m) \hat{y} + (b\theta_1 - b_m) u_c
\]

(15)

Following the same procedure described in section 3.1, we introduce the same quadratic function to assess the Lyapunov stability of the system. After the respective operations, if the change in parameters \( \theta_1 \) and \( \theta_2 \) is adjusted with the expressions \( \frac{d\theta_1}{dt} = -\gamma u_c e \) and \( \frac{d\theta_2}{dt} = \gamma \hat{y} e \), then, Equation (16).
With the same analysis as before, Equation (16) implies that the system output \( \dot{y} = e + y_m \) is also bounded. With this result, the error sensitivity to the controller parameters \( \frac{\partial e}{\partial \theta} \) is equal to the current parameter value (\( \theta \)). The derived adaptation scheme is often employed for the analysis of first or second-order systems, although the structure is also applied in a broad range of processes.

A key result of this is that a different adaptation law need not be calculated when changing to a different plant or model unless the performance of the adaptation law is proven to be insufficient.

![Block diagram of the implemented control system with a state-feedback observer.](image)

**Figure 3.** Block diagram of the implemented control system with a state-feedback observer.

### 4. Simulation results

This section presents the simulation results for the MRAC with Luenberger observer described in section 3. A linearized model for a direct current (DC) motor was selected as simulation benchmark. For space restrictions, the complete modeling of the DC motor is omitted; see reference [12] for verification. For the sake of comparison, a classic proportional-integral-derivative (PID) controller was also implemented for the simulation benchmark.

#### 4.1. Direct current motor model

The state matrices representing the system dynamics are Equation (17).

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
-\frac{R_a}{L_a} & -\frac{K_v}{J} \\
\frac{K_v}{J} & -\beta
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
1 \\
0
\end{bmatrix} \begin{bmatrix}
V_a \\
T_L
\end{bmatrix}
\]

(17)

\[
y = \begin{bmatrix}
1 \\
0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = \begin{bmatrix}
i_a \\
\omega_a
\end{bmatrix}
\]

In Equation (17), sub-index a denotes stator variables. Ra is resistance, ia is the current, La is the inductance, Va is the voltage, and \( \omega_a \) the angular speed. Also, \( K_v \) denotes the electro-motive force, J is inertia moment of the motor, \( \beta \) is the friction coefficient and \( T_L \) represents load torque.
4.2. Model reference adaptive control with Luenberger observer

The mathematical model in section 3 shows how two theories can work together to adapt the system parameters for the given control system. Figure 4 depicts the disturbance response of the proposed control system for the DC motor. A step disturbance was applied at the first second. The control scheme presents a satisfactory response without overshoot and a smooth behavior of the curve. Even for a linearized version of the system, simulation verifies the applicability of the MRAC for controlling variables that are difficult to access or cannot be measured directly.

![Figure 4. Response with a Luenberger state observation algorithm and MRAS using Lyapunov theory.](image)

4.3. Proportional-integral-derivative with Luenberger observer

A classic PID control was implemented together with a Luenberger state observation algorithm. This configuration allows the description of the system behavior when a common control law is applied to variables that cannot be measured directly [13]. The PID control system was designed using error criteria for a stable linear time-continuous invariant system [14], and the index applied in the process integrates the absolute error (ITAE) multiplied by the time over time [15], Equation (18).

\[ ITAE = \int_0^\infty t|e(t)|dt \quad (18) \]

The selection of ITAE index obeys to its rapid action because the integration works as soon as disturbance has occurred [16]. However, that capability is not enough to make the system work correctly with a good response speed. Despite the long establishment time, PID control is unable to meet the performance of the mixed Lyapunov and MRAC system. Figure 5 shows how the PID control takes almost ten times more than the MRAC scheme to compensate for the disturbance effects.

![Figure 5. Response with a Luenberger state observation algorithm and PID control.](image)
5. Conclusions

In this work, a model reference adaptive control strategy has been developed for a DC motor, involving the necessary relationships to adapt its parameters according to the system behavior. Simulations have shown the applicability of the proposed scheme to control variables that are difficult to access or cannot be measured directly by using a state observation structure. Once we have an estimated signal measurement, it is possible to implement a control law to get a correct response for the plant when there is either a change of load or a sudden disturbance.

Performance of the MRAC was compared with a classic PID controller using ITAE for stable linear time-invariant continuous systems, both including a Luenberger observer. After a step disturbance, settling time was noticeably worse for the PID system. The MRAC designed by Lyapunov stability with Luenberger Observer offers better performance when compared to classic control structures.

References

[1] Syed M, Usha M, Orman Z, Arik S 2019 Improved result on state estimation for complex dynamical networks with time varying delays and stochastic sampling via sampled-data control Neural Networks 114 28
[2] Coman S, Boldisor C 2013 Model reference adaptive control for a dc electrical drive Bulletin of the Transilvania University of Brașov 6 33
[3] Anbu S, Jaya N 2014 Design of adaptive controller based on lyapunov stability for a CSTR Int. Journal of Electronics and Communication Engineering 8 176
[4] Osorio B, Castro H, Torres J 2011 State and unknown input estimation in a CSTR using higher-order sliding mode observer IX Latin American Robotics Symposium and IEEE Colombian Conference on Automatic Control (Bogotá: IEEE)
[5] Ruiz S, Patiño J, Espinosa J 2018 PI and LQR controllers for frequency regulation including wind generation JIECE 8 3711
[6] Zhang S, Dian S 2018 Controller design for compound pendulum with PID and MRAC switch control IOP Conf. Series: Materials Science and Engineering 428 012036
[7] Nuñez R 2014 Control adaptativo por modelo de referencia con predictor Smith a partir de la regla MIT para una mesa vibratoria de dos grados de libertad Ing. Solidaria 9 89
[8] Patiño J, López J, Espinosa J 2019 Sensitivity analysis of frequency regulation parameters in power systems with wind generation Advanced Control and Optimization Paradigms for Wind Energy Systems ed R Precup (Singapore: Springer)
[9] Duque E, Patiño J 2013 El mercado de bonos de carbono y su aplicación para proyectos hidroeléctricos Rev. CINTEX 18 131
[10] Souza J, Takamoto L 2019 Lyapunov stability for impulsive control affine systems Journal of Differential Equations 266 4232
[11] Bernat J, Kolota J 2018 Adaptive observer-based control for an IPMC actuator under varying humidity conditions Smart Materials and Structures 27 055004
[12] Florián A 2018 Comparación de desempeño de observadores de estado en sistemas lineales con aplicación a un motor de corriente continua Rev. CINTEX 23 51
[13] Patino J, Ramirez C, Espinosa J 2019 Modal Analysis for a power system benchmark with topological changes Workshop on Engineering Applications 1 628
[14] Ang K, Chong G, Li Y 2005 PID control system analysis, design, and technology IEEE Trans. on Control Systems Technology 13 559
[15] Srdanovic V, Botero O, Hernando S 2017 Modelado e identificación de un sistema electromecánico y diseño del control PID para gobernar inalámbricamente el desplazamiento de un objeto móvil Rev. CINTEX 22 25
[16] Merigo L, Padula F, Latronico N, Paltenghi M, Visioli A 2019 Optimized PID control of propofol and remifentanil coadministration for general anesthesia Communications in Nonlinear Science and Numerical Simulation 72 194