Prospects of measuring the leptonic CP phase with atmospheric neutrinos

Abhijit Samanta

Harish-Chandra Research Institute, Chhatnag Road, Jhusi, Allahabad 211 019, India

We have studied the prospects of measuring the CP violating phase with atmospheric neutrinos at a large magnetized iron calorimeter detector considering the muons (directly measurable) of the neutrino events generated by a MonteCarlo event generator Nuance. The effect of $\theta_{13}$ and $\delta_{CP}$ appears dominantly neither in atmospheric neutrino oscillation nor in solar neutrino oscillation, but appears as subleading in both cases. These are observable in range of $E \sim 1$ GeV for atmospheric neutrino, where solar and atmospheric oscillation couple. In this regime, the quasi-elastic events dominate and the energy resolution is very good, but the angular resolution is very poor. Unlike beam experiments this poor angular resolution acts against its measurements. However, we find that one can be able to distinguish $\delta_{CP} \approx 0^\circ$ and $180^\circ$ at 90% confidence level. We find no significant sensitivity for $\delta_{CP} \approx 90^\circ$ or $270^\circ$.

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I. INTRODUCTION

The evidence of neutrino masses and their mixing through neutrino oscillation \cite{1} is the first footprint onto the physics beyond the standard model (SM). However, the first hint was obtained observing the anomaly by IMB in 1986 and then confirmed by Kamiokande in 1988 \cite{2, 3}. The oscillation experiments provide the measurements of mass squared differences and mixing angles. At present 1(3)$\sigma$ ranges are \cite{4}: $\Delta m^2_{21} = 7.67^{+0.16}_{-0.19} (0.53)$, $|\Delta m^2_{31}| = 2.39^{+0.11}_{-0.33}$, $\sin^2 \theta_{12} = 0.312^{+0.019}_{-0.063}$, $\sin^2 \theta_{23} = 0.466^{+0.058}_{-0.073}$, and $\sin^2 \theta_{13} = 0.016^{+0.010}_{-0.010}(<0.046)$. Here $\theta_{ij}$ are the mixing angles in Pontecorvo, Maki, Nakagawa, Sakata (PMNS) mixing matrix \cite{5} and $\Delta m^2_{ij} = m^2_j - m^2_i$. Currently, there is no constraint on the CP violating phase $\delta_{CP}$ or on the sign of $\Delta m^2_{31}$. The origin of CP violation is still an open problem that needs both theoretical and experimental exploration. Till now the best tests come from only neutral kaon oscillations.

The mixing angle $\theta_{13}$ tells how strongly the atmospheric and solar oscillations couple and therefore also determines the CP violation effects in neutrino oscillation. Both $\theta_{13}$ and $\delta_{CP}$ are observable in solar and atmospheric neutrino oscillation experiments only as subleading. The three flavor effects are masked mainly by wide resolutions and systematic uncertainties. There are enormous efforts to probe CP violation and to measure $\theta_{13}$ in long baseline reactor experiments Double Chooz \cite{6}, Daya bay \cite{7}, RENO \cite{8} and accelerator experiments T2K \cite{9} and NOvA \cite{10}. In a recent paper \cite{11}, it is shown that there will be only a hint of CP-violation at 90% confidence level for $\sin^2 2\theta_{13} \gtrsim 0.05$ for most values of $\delta_{CP}$ if one considers only the upgraded beams for T2K and NOvA combined with reactor data.

The CP violating phase $\delta_{CP}$ (Dirac) has been studied for a magnetized Iron CALorimeter (ICAL) detector with atmospheric neutrinos in \cite{12}, but there is no significant sensitivity. In this paper, we have re-examined the sensitivity of $\delta_{CP}$. Then we point out the difference from the past analysis. The main difference is that they considered a high threshold (2 GeV) in their analysis. For events beyond this energy, the sensitivity almost vanishes. In this work, we consider only muons (directly measurable) of the events generated by a MonteCarlo event generator Nuance-v3 \cite{13} and the muon threshold energy of 0.8 GeV. The main advantage of a magnetized detector is that it can measure the oscillation effect separately for muon neutrinos and anti-neutrinos. This type of detector has been proposed by India-based Neutrino Observatory (INO) \cite{14}.

*E-mail address: abhijit@hri.res.in
II. ATMOSPHERIC NEUTRINOS

The atmospheric neutrinos are produced by the interactions of cosmic rays with the atmosphere. These are primarily protons and Heliums and some heavy ions. They produce mainly muons, pions and kaons, which decay into neutrinos. Here we discuss briefly the uncertainties which arise in flux calculation. For calculation of the neutrino flux, one needs the detailed information on (i) the primary cosmic-ray spectra at the top of the atmosphere, (ii) the hadronic interactions between cosmic rays and atmospheric nuclei, (iii) the propagation in the atmosphere, (ii) the hadronic interactions between cosmic rays and atmospheric nuclei, (iii) the propagation of cosmic-ray particles inside the atmosphere, and (iv) the decay of the secondary particles. The uncertainty on primary cosmic-ray spectra has been improved by the experiments \[15, 16, 17, 18\]. The hadronic interactions are measured from accelerator experiments. However, the available data do not cover all required phase space for calculation of the neutrino flux. The propagation of cosmic-ray particles inside the atmosphere, and the decay of the secondary particles are handled accurately by simulation \[19\].

Then the total uncertainty can be expressed as \[19\]

\[
\delta_{\text{total}}^2 = \delta_{\pi}^2 + \delta_{K}^2 + \delta_{\pi}^2 + \delta_{\text{air}}^2 + \ldots,
\]

where \(\delta_{\pi}\) is the uncertainty due to uncertainty of \(\pi\) production in the hadronic interaction model, \(\delta_{K}\) due to \(K\) production, \(\delta_{\pi}\) due to hadronic interaction cross section and \(\delta_{\text{air}}\) due to atmospheric density profile.

For calculating atmospheric neutrino flux from primary cosmic ray flux, the measured atmospheric muon flux is compared with the calculation \[19\]. The error in \(\mu\) and \(\nu\) fluxes comes mainly in the \(\pi\) production in the hadronic interaction model. It has been shown that in case of pions above 1 GeV \[19\],

\[
\frac{\Delta \phi_\mu}{\phi_\mu} \simeq \frac{\Delta \phi_{\nu_\mu}}{\phi_{\nu_\mu}} \simeq \frac{\Delta \phi_{\nu_e}}{\phi_{\nu_e}}.
\]

However, this equation does not hold for \(\Delta E \lesssim 1\) GeV. The present estimated uncertainty of atmospheric neutrino flux is discussed later in section \[IV.A\].

III. THE \(\delta_{\text{CP}}\) DEPENDENCE OF OSCILLATION PROBABILITY

The algebraic expression for full three flavor oscillation probability with CP phase \(\delta_{\text{CP}}\) is very long and complicated to understand its change with the oscillation parameters. We have plotted the oscillogram of \(\Delta P = [P_{\mu\mu}(\delta_{\text{CP}} = x\degree) - P_{\mu\mu}(\delta_{\text{CP}} = 0\degree)]\) in \(E - L\) plane with \(x = 90\degree\) and \(180\degree\) for \(\theta_{13} = 10\degree\). We set \(|\Delta m_{32}^2| = 2.5 \times 10^{-3}\) eV\(^2\), \(\theta_{23} = 45\degree\) and the solar parameters at their best-fit values. Since \(\delta_{\text{CP}}\) appears together with \(\sin\theta_{13}\) in PMNS matrix, the effect \(\delta_{\text{CP}}\) in \(P_{\mu\mu}\) increases with increase in \(\sin\theta_{13}\). The solar and atmospheric oscillation are coupled through \(\theta_{13}\). The solar neutrino oscillation is dominant at \(E \sim a\) few tens of MeV, while the atmospheric neutrino oscillation is dominant at \(E \sim GeV\). These are determined from the corresponding mass squared differences. For this reason we see that the \(\delta_{\text{CP}}\) effect is prominent in sub-GeV region and decreases rapidly with increase in energy. Another interesting feature of the plots is that over a large regions of \(E - L\) plane, \(\Delta P\) is positive for anti-neutrino and negative for neutrino in both cases for inverted hierarchy (IH) and normal hierarchy (NH). This is why the poor resolutions cannot wash out fully the CP sensitivity. It is also expected that the effect of the CP phase is not fully nullified due to the systematic uncertainties with pull method of chi-square analysis. The reason is following. The systematic uncertainties are high compared to the difference \(\Delta P\). But, they are constant over a large region of \(E - L\) plane, while \(\Delta P\) changes with \(L\) and \(E\). However, from these plots one can also find the optimized CP sensitive \(E\) and \(L\) values for the experiments with neutrino beams.

IV. THE CHI-SQUARE ANALYSIS

Due to the low statistics at high energy, the \(\chi^2\) is calculated according to the Poisson probability distribution defined by the expression:

\[
\chi^2 = \sum_{i,j=1}^{n_L,n_E} \left[ 2 \left( N_{ij}^p \left( 1 + \sum_{k=1}^{n_s} f_{ij}^k \cdot \xi_k^k \right) - N_{ij}^o \right) - 2N_{ij}^o \ln \left( \frac{N_{ij}^p \left( 1 + \sum_{k=1}^{n_s} f_{ij}^k \cdot \xi_k^k \right)}{N_{ij}^o} \right) \right] + \sum_{k=1}^{n_s} \xi_k^2.
\]

The \(N_{ij}^o\) is considered as the number of observed events.
FIG. 1: The oscillogram of the difference $\Delta P = P_{\mu\mu}(\delta_{CP} = x^\circ) - P_{\mu\mu}(\delta_{CP} = 0^\circ)$ in $E - L$ plane for $x = 90^\circ$ and $180^\circ$ with $\theta_{13} = 10^\circ$ for both neutrino and anti-neutrino with NH. We set other parameter same as fixed for the analysis and given in the text.

The events are binned in the grid of $\log E - L^{0.4}$ plane. We use total number of $\log E$ bins $n_E = 35$ ($0.8 - 40$ generate by Nuance for a set of oscillation parameters with an exposure of 1 Mton.year of ICAL. The $N_{ij}^P$ is the corresponding number of predicted events (discussed later). These are obtained in a 2-dimensional grids in the plane of $\log E - L^{0.4}$. The $f_{ij}^k$ is the systematic uncertainty of $N_{ij}^P$ due to the $k$th uncertainty (discussed later). The $\xi_k$ is the pull variable for the $k$th systematic uncertainty.
GeV) and the number of $L^{0.4}$ bins $n_L$ as a function of the energy. We consider $n_L = 2 \times 25$, $2 \times 27$, $2 \times 29$, $2 \times 31$, and $2 \times 33$, when $E = 0.8 - 1$, $1 - 2$, $2 - 3$, $3 - 4$, and $> 4$ GeV, respectively. For the down-going events, the binning is done by replacing `$L^{0.4}$' by `$-L^{0.4}$'. We choose this type of binning to reflect oscillation effect in a better way in the chi-square analysis [20, 21]. The binning has been optimized in [20, 21] for the precision study of $\Delta m^2_{32}$ and $\theta_{23}$. We consider the mirror $L$ for down going events, which is same if the neutrino comes from exactly opposite direction. The factor ‘2’ is to consider both up and down going cases. We consider number of events
in a bin $> 4$ to maintain $\chi^2/d.o.f \approx 1$. We have checked that this always keeps it $\lesssim 1.08$ at the minima in all cases.

A. Systematic uncertainties

We divide the systematic uncertainties into two categories: I) overall uncertainties (which are independent of energy and zenith angle), and II) tilt uncertainties (which are dependent of energy and/or zenith angle). The energy dependent flux uncertainty which arises due to the uncertainty in spectral indices, can be expressed as

$$
\Phi_{\delta E}(E) = \Phi_0(E) \left( \frac{E}{E_0} \right)^{\delta E} \approx \Phi_0(E) \left[ 1 + \delta E \log_{10} \frac{E}{E_0} \right]
$$

b) Similarly, the flux uncertainty as a function of zenith angle can be expressed as

$$
\Phi_{\delta \theta_z}(\cos \theta_z) \approx \Phi_0(\cos \theta_z) \left[ 1 + \delta_z \cos \theta_{\text{zenith}} - \cos \theta_{0\text{zenith}} \right]
$$

c) overall flux normalization uncertainty $\delta_{f_N}$, d) overall neutrino cross section uncertainty $\delta_{\sigma}$. We consider following values of the above systematic uncertainties: $\delta_E = 5\%$ with $E_0 = 1$ GeV for $E < 1$ GeV, and $\delta_E = 5\%$ with $E_0 = 10$ GeV for $E > 10$ GeV, $\delta_z = 4\%$ with $\cos \theta_{0\text{zenith}} = 0.5$, $\delta_{f_N} = 10\%$, and $\delta_{\sigma} = 15\%$. These are derived from the latest calculation of atmospheric neutrino flux [19]. For each set of oscillation parameters we minimize the $\chi^2$ with respect to the all pull variables and then use these values to calculate the $\chi^2$ for that set of parameters.

We consider all uncertainties as a function of reconstructed neutrino energy and zenith angle. Here we assumed that the tilt uncertainties will not be changed too much due to the reconstruction. However, on the other hand, if any tilt uncertainty arises in reconstructed neutrino events from the reconstruction method, it is then accommodated in $\chi^2$.

To generate the theoretical data for chi-square analysis, we first generate 500 years un-oscillated data for 1 Mton detector. From this data we find the energy-angle correlated resolutions separately for $\nu_\mu$ and $\bar{\nu}_\mu$ (see Fig. 3 of [21]) in 35 $E_\nu$ bins (in log scale for the range of 0.8–40 GeV) and 17 $\cos \theta_{\text{zenith}}$ bins ($-1$ to $+1$). For a given grid of $(E_\nu - \cos \theta_{\text{zenith}})$, we calculate the efficiency of having $E_\mu \geq 0.8$ GeV (threshold of the detector). For each set of oscillation parameters, we integrate the oscillated atmospheric neutrino flux folding the cross section, exposure time, target mass, efficiency and resolution function to obtain the predicted data in the reconstructed log $E - L^{0.4}$ grid [20]. We use the Charge Current (CC) cross section of Nuance-v3 [13] and the Honda flux of 3-dimensional scheme [10]. This method of obtaining the theoretical data in bins of muon energy and zenith angle has been used and discussed in our previous works [20, 21, 22, 23].

We have done this study considering only the muons produced in the CC interactions. It is highly expected that the addition of hadron energy to the muon energy of an event will improve the result. However, since the hadron energy resolution depends significantly on the thickness of the iron plates, this addition of hadron energy will be more reliable in case of GEANT-based studies with realistic backgrounds.

From GEANT simulation of ICAL detector it is found that the energy resolution varies 4-10% and angular resolution 4-12% depending on the energy and the direction for our considered range of energy. The width of these resolutions are very negligible compared to that obtained from kinematics of the scattering processes. We have also checked from GEANT simulation that the wrong charge identification possibility of ICAL detector is also almost zero when the magnetic field is $\gtrsim 1$ Tesla for our considered range of muon energy.

The iron plates are stacked horizontally and the muons produced in the horizontal direction or very near to it cannot be detected. So we put a selection criteria in our analysis. The muons for a given energy must be with a zenith angle such that $|90^\circ - \theta_{\text{zenith}}|$ is greater than the half width at half maxima of the scattering angle distribution with that energy. This is discussed in detail in [20].

V. RESULTS

We have marginalized the $\chi^2 = \chi^2_{\nu} + \chi^2_{\bar{\nu}}$ over all oscillation parameters $\Delta m^2_{21}$, $\theta_{23}$, $\theta_{13}$, $\delta_{\text{CP}}$, $\Delta m^2_{32}$, and $\theta_{12}$ choosing the range of $\Delta m^2_{32} = 2.0 - 3.0 \times 10^{-3}eV^2$, $\theta_{23} = 37^\circ - 54^\circ$, $\theta_{13} = 0^\circ - 12.5^\circ$ and $\delta_{\text{CP}} = 0^\circ - 360^\circ$. We set the range of $\Delta m^2_{21} = 7.06 - 8.34 \times 10^{-5}eV^2$ and
\[ \theta_{12} = 30^\circ - 40^\circ. \] However, the effect of \( \Delta m^2_{31} \) comes in subleading order in the oscillation probability when \( E \sim \text{GeV} \). The 2-dimensional 68\%, 90\%, 99\% confidence level (CL) allowed parameter spaces (APSs) are obtained by considering \( \chi^2 = \chi^2_{\text{min}} + 2.48, 4.83, 9.43 \), respectively. We show the contours in \( \theta_{13} - \delta_{CP} \) plane in Fig. 3 for the inputs of IH and \( \theta_{13} = 10^\circ \) with \( \delta_{CP} = 0^\circ, 90^\circ \) and \( 180^\circ \), respectively. We see that both upper and lower bounds can be obtained for \( \delta_{CP} = 180^\circ \) at 90\% CL. In Fig. 4 we show the contours for \( \theta_{13} = 7.5^\circ \) with \( \delta_{CP} = 180^\circ \). The contours with inputs of NH and \( \delta_{CP} = 180^\circ \) are also shown in Fig. 5. Here, we find that both upper and lower bounds can be obtained with atmospheric neutrinos only when \( \delta_{CP} \approx 0^\circ \) and \( 180^\circ \). However, we find no bounds if \( \delta_{CP} = 90^\circ \).

The absolute bounds on \( \delta_{CP} \) for its different inputs are shown in Fig. 6. This is obtained after marginalization over all oscillation parameters except \( \delta_{CP} \). In Fig. 7 we see that the sensitivity falls drastically if one changes the threshold of the detector from 0.8 GeV to 2 GeV. This result can be understood from Figs. 1 and 2. Now we can conclude that the events at energy range \( E \approx 1 - 2 \text{ GeV} \) contribute mainly in CP phase sensitivity in atmospheric neutrino oscillation.

It should be notable here that the binning of the data to obtain the results in Fig. 7 is different from all other analyses in this paper. Here, we have used linear binning in \( E \) and \( \cos \theta_{\text{zenith}} \) with bin size of 1 GeV and 0.066, respectively. However, we have enlarged the bin size of \( E \) by adding the nearest bin in \( E \) if number of events in a bin is \( \leq 4 \). This type of binning has been considered to compare our results with that obtained in [12]. If we use the same threshold of 2 GeV as used for the analysis in [12], the results are very similar in nature with [12]. However, here we have considered the energy range 2-35 GeV, while the range 2-8 GeV is used for analysis in [12]. From this study, one extra point we gain is that results does not change significantly for different types of
VI. THE CASE WITH $\delta_{CP} \approx 90^\circ$ OR $270^\circ$

At $\delta_{CP} = 90^\circ$ or $270^\circ$, there is practically no $\delta_{CP}$ dependence in the muon neutrino survival probability $P(\nu_\mu \rightarrow \nu_\mu)$ in symmetric matter profile [24]. This can be understood in the following way. The oscillation probabilities can be expressed as linear combinations of cosine and sine functions of $\delta_{CP}$ [24, 25]. The probability $P(\nu_\mu \rightarrow \nu_\mu)$ in symmetric matter profile can be expressed as

$$P(\nu_\mu \rightarrow \nu_\mu) = A \cos \delta_{CP} + D \cos 2\delta_{CP} + C,$$

where, the coefficient $A$, $B$ and $C$ are functions of oscillation parameters, but independent of $\delta_{CP}$. The magnitude of $D$ is $O(\Delta m^2_{21} \sin^2 \theta_{13})$ and hence it is difficult to observe the effect of the term proportional to $\cos 2\delta_{CP}$. So, effectively, $P(\nu_\mu \rightarrow \nu_\mu)$ is proportional to $\cos \delta_{CP}$ and there is no $\delta_{CP}$ dependence when $\delta_{CP} = 90^\circ$ or $270^\circ$. The dependency grows very slowly as one goes away from these particular values and hence it is very difficult to measure $\delta_{CP}$ if its true value is around $90^\circ$ or $270^\circ$. However, due to slow increase of its dependence, one can expect the possibility of its measurement at these values. In Fig. 3 we have shown the case for $\delta_{CP} = 90^\circ$. We see there is almost no sensitivity.
FIG. 6: The variation of $\Delta \chi^2 [\chi^2 - \chi^2_{\text{min}}]$ with $\delta_{\text{CP}}$ for different inputs of $\delta_{\text{CP}}$. The $\chi^2$ is marginalized over all oscillation parameters except $\delta_{\text{CP}}$.

FIG. 7: The variation of $\Delta \chi^2 [= \chi^2 - \chi^2_{\text{min}}]$ with $\delta_{\text{CP}}$ keeping other oscillation parameters fixed. We use linear binning in $E$ and $\cos \theta_{\text{zenith}}$ with bin size of 1 GeV and 0.066, respectively for the chi-square analysis of this plot. This binning is similar with that in [12]. The lower solid curve represents the result with threshold of 2 GeV used in [12] and dashed one with threshold of 0.8 GeV used in our analysis.

We have studied further the possibilities of improving the sensitivity in different optimistic ways. First, we try to see if any improvement of the systematic uncertainties can make it possible to observe the CP violation. For this purpose we have changed the uncertainties and see the impact of them on APSs. We see from the discussion in section II that the flux is less known for $E \leq 1$ GeV. We have assumed a tilt uncertainty $\delta_E = 5\%$ with energy for this region. We have checked by improving the energy tilt uncertainty from 5% to 2% that there is no much significant change in CP sensitivity, except a little hump around $90^\circ$. To check the effect of horizontal/vertical flux uncertainty, we have changed the value of $\delta_z$ from 4% to 2%. Here also, we find no such significant change.

If we decrease the threshold from 0.8 GeV to 0.6 GeV, both CP sensitivity in oscillation probability (see Fig. 1 and 2) and flux increases. But, we have checked that there is also no improvement in APSs with decreasing the threshold. The fact is that with decrease in energy the angular resolution worsens very rapidly and this kills the above prospects.

At $\delta_{\text{CP}} \approx 90^\circ$ or $270^\circ$, we see from the discussion at the beginning of this section that the CP sensitivity vanishes over whole $L - E$ space. It grows gradually as one goes
away from these values. In Fig. 1 and 2 this feature is reflected and the difference $\Delta P$ appears relatively in the lower energy zone compared with $\delta_{CP} = 180^\circ$ case. In case of atmospheric neutrinos the direction is not fixed. The scattering angle between the muon and the neutrino is very large at this low energy and this mainly acts against the CP phase measurements for $E < 1$ GeV. In case of the experiments with neutrino beams, the direction is known. This gives the main advantage to measure the CP phase there.

VII. CONCLUSION

In this paper we have studied the sensitivity of a magnetized ICAL detector in measuring the CP phase with atmospheric neutrino oscillation. We have presented the results for 1 Mton.year exposure of ICAL, which is 10 years running of the proposed 100 kTon detector. Here, we have considered the muons (directly measurable) of the events, which are produced by the charge current interactions generated by neutrino event generator Nuance-v3. We performed a marginalized chi-square study considering all possible systematic uncertainties. We find that one can be able to distinguish $\delta_{CP} \approx 0^\circ$ and $180^\circ$ at 90% confidence level. However, there is no significant sensitivity for $\delta_{CP} \approx 90^\circ$ or $\delta_{CP} \approx 270^\circ$.

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However, an exactly equivalent result is obtained here using the energy-angle correlated resolution functions.