Relative facts do not exist. Relational Quantum Mechanics is Incompatible with Quantum Mechanics. Response to the critique by Aurélien Drezet.

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In this comment we answer to the recent critique of our article [arXiv:2208.11793] about Relational Quantum Mechanics (RQM) by Aurélien Drezet [arXiv:2209.01237]. Here we point out that our critical analysis of RQM was precisely based on the most recent formulation of RQM published in 2021 by its founder Carlo Rovelli (Rovelli, 2021). This presentation has recently been incorporated as chapter 43 in “The Oxford Handbook of the History of Quantum Interpretations” published on 7 June 2022 by Oxford University Press.

A critique of our article (Lawrence et al., 2022) was posted shortly thereafter by Aurélien Drezet, entitled “In defense of Relational Quantum Mechanics: A note on (above title)” (Drezet, 2022). Far from defending RQM, however, Drezet misrepresents it, as well as our paper. Based on his misunderstandings, the critique of our paper is unfounded. Here we discuss several specific instances, in order of appearance in his paper:

1. A repeated misgiving regarding RQM is first revealed in paragraph 2 of Drezet’s paper, where he “reminds us” that:

"... in RQM the main issue concerns the interpretation of the full wavefunction \( |\Psi_{SO}\rangle\) involving observer (O) and observed system (S). In RQM the fundamental object relative to (O) is not \( |\Psi_{SO}\rangle\) but the reduced density matrix

\[
\hat{\rho}_{SO}^{\text{red}} = \text{Tr}_O[\hat{\rho}_{SO}] = \text{Tr}_O[|\Psi_{SO}\rangle\langle\Psi_{SO}|].
\]

This statement is wrong for at least two reasons: First, the reduced density matrix is not the fundamental object relative to (O). Quoting Rovelli (Rovelli, 2021), [RQM has] “an ontology based on facts (or events), not quantum states.” Second, Drezet’s formula (1) provides an incorrect description of what the observer sees in a measurement. Not only in RQM, but also in ordinary quantum mechanics, the observer sees a single unambiguous outcome (in RQM, this is called a “fact”). The reduced density matrix, in contrast, represents an ensemble average. This is not what the observer sees in a single measurement.

2. Based on the above (inappropriate) use of the reduced density matrix, Drezet compares our GHZ correlation equations with his own, which of course show less correlation and do not pose a contradiction. The two cases are Eqs. (14) and (15) for the Friend (portrayed here by Alice), and later, Eqs. (19) and (27) for Wigner (played here by Bob). To be more explicit, he states [immediately prior to Eq. (14)] “Due to decoherence i.e., entanglement with the environment (Alice) we have lost coherence and correlations between spins.” This flatly contradicts the description of Wigner-Friend scenarios provided by Rovelli, dating back to the original 1996 work (Rovelli, 1996), and more comprehensively in his recent article (Rovelli, 2021), in which relative facts are realized by Alice and, by definition, have not yet been converted into stable facts by decoherence. To illustrate this point, let us again cite Rovelli’s description of the Wigner-Friend scenario ((Rovelli, 2021), page 3):

• For instance, in the Wigner’s friend scenario, the friend interacts with a system and a fact is realised with respect to the friend. But this fact is not realised with respect to Wigner, who was
not involved in the interaction, and the probability for facts with respect to Wigner (realised in interactions with Wigner) still includes interference effects.

3. Drezet concludes his commentary on Alice’s RQM measurements with the sentence (following Eq. 15): “The actualization of measurements in RQM is a debatable issue and we will not consider this problem here.” This statement further misrepresents RQM: first, the term “actualization” is not defined or used in RQM. Instead, referring to the above quote of Rovelli, “a fact is *realized* with respect to the friend” (here Alice). This is conventional usage in RQM, and it’s meaning is not debatable. The subject of measurements in RQM is discussed more broadly by Di Biagio and Rovelli (Di Biagio and Rovelli, 2022), and, as far as we know, this presentation is not generally considered to be debatable.

It may be worth noting that we reviewed the “rules of RQM” in Sec. II of our paper, before implementing them in Sec. III to set up the Wigner-Friend scenario. We did not make up our own rules.

4. In criticizing the treatment of Bob’s RQM measurements on the compound system $S \otimes A$, Drezet again introduces the reduced density matrix (as mentioned in item 1) to (erroneously) describe the situation relative to Bob. Using this, he finds Eq. (19) on p. 5, which shows no (GHZ) correlations in three of the four cases where we find them. What is more, he misrepresents our argumentation by saying, that ”they consider that Bob only measures one of the 3 qubits belonging to $SA$”. In fact, in our scenario, Bob performs unitary interactions, which in RQM lead to relative facts, on each of the three subsystems $S_i \otimes A_i$ (see top paragraph, right column of p. 4 in (Lawrence et al., 2022)). The ordering of these RQM measurements is immaterial, since the entangling processes commute, so that Bob’s RQM measurements may be regarded as simultaneous. We then consider the state (13) [or (20) in (Drezet, 2022)], not to describe further RQM measurements, but only to find deterministic relations between the relative facts established in previous measurements, as exhibited by eigenvalues of the involved observables.

5. Drezet challenges our assertion that relative facts are non-contextual. He argues that we introduce non-contextualism arbitrarily, misrepresenting us with the statement (p. 6, ten lines from the bottom), “since the 3 operations $\ldots$ are acting ‘locally’ on only one of the subsystems $SA_m$ (at a time) their meaning should be non-contextual and absolute.” We do not say this, nor do we assume it. In brief, our paper demonstrates that Alice’s relative facts, which (according to RQM) come into existence with Alice’s RQM measurements, are non-contextual hidden variables with respect to the context of Bob’s future measurements. In contradiction with other hidden variables theories, they do not exist in RQM at the stage of the preparation of the experiment, or equivalently, of the quantum state of the system ($S$) in question. Recall that, as mentioned in our paper, we regard a hidden variable as a notion, variable, value, or whatever which is not an element of quantum formalism. Relative facts are definitely not a part of quantum formalism.

As a final remark let us emphasize once more, that in our work we do not compare values of relative facts which are realised with respect to different observers. This no comparison rule, or “relativity of comparisons” axiom, is one of the main features postulated in (Di Biagio and Rovelli, 2022). Instead we utilise constraints on products of them, which are demanded by ordinary quantum mechanics. If one rejects the assumption, that relative facts should follow the same constraints as corresponding eigenvalues of quantum observables, one immediately loses any relation with quantum mechanics.

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REFERENCES

Adlam, E., and C. Rovelli (2022), arXiv:2203.13342.
Di Biagio, A., and C. Rovelli (2022), Foundations of Physics 52 (3), 62.
Drezet, A. (2022), arXiv:2209.01237 [quant-ph].
Lawrence, J., M. Markiewicz, and M. Żukowski (2022), arXiv:2208.11793 [quant-ph].

1 On the contrary, local measurements can be used to demonstrate contextuality.

2 However, please note that this axiom is withdrawn, and replaced by “cross-perspective links” axiom, which does not rule out comparisons of values... (Adlam and Rovelli, 2022). This effectively puts relative facts as counterfactuals, exactly of the form discussed in "step two" of our letter (Żukowski and Markiewicz, 2021)
Rovelli, C. (1996), *International Journal of Theoretical Physics* **35** (8), 1637.

Rovelli, C. (2021), arXiv:2109.09170 [quant-ph].

Żukowski, M., and M. Markiewicz (2021), *Phys. Rev. Lett.* **126**, 130402.