1. Introduction

At present, given the rapid development of transport technologies, the issue of economic feasibility and reliability of different transportation vehicles becomes more and more important.

The durability of the rolling stock operation, as well as the upper structure of the rail way, depends on the patterns of interaction between elements, which is affected by mechanical, structural and geometrical characteristics. The functions of work of the rail track under rolling stock depend on the type and kind of fastening of rails, the characteristics of rigidity of the components of the upper structure of the track, conditions of service and operation. Reducing parameters of the interaction between a railroad car and a track, especially in the area of joints, ensures the transition to durable, highly-reliable, fast rail transport. In practice, this also contributes to the implementation of resource-saving technologies.

Analysis of studies into mechanical interaction between a four-axle railroad car and the rail track shows [1] that the weakest link in the examined system is the isolated zones of rail joints. In addition, it is appropriate to take into account the phases of a railroad car motion over the rail joint unevenness, as well as the characteristics of rigidity of the ballast layer of a track [2].

It is also relevant to consider combined work of the rolling stock and a rail track that determines the patterns of their static and dynamic interaction [3] when a railroad car passes over a rail joint.

The most informative indicator of mechanical interaction in the system “railroad car wheel – rail” is the settlement of the ballast layer under the sleepers of the track in the butt zones of rails [4]. In such places a rail is typically exposed to the largest impact loads. This leads to the deflection and sagging of rails, the emergence of blind and elongated joints. Under actual conditions, the loads of assemblies, nodes and parts of the railroad car and the upper structure of a rail define parameters of durability in operation [5], strength and rigidity [6], and, ultimately, the generalized technical resource and service life. Durability of the interacting rolling stock and rails also affects the frequency of replacement, that is, financial costs of acquisition.

It should be noted that during rail track operation there occurs the nonstationarity of geometrical parameters of the rail track unevenness. This is due to climatic factors, as well as mechanical impact loads, which occur in the area of the joint and are repeated over time.
This also leads to a significant influence of the geometrical and mechanical parameters of the processes of interaction between a railroad car and a rail track in the places of joints, the description of which necessitates application of a multifactor model.

Therefore, it is a relevant and important task to conduct the study aimed at improving existing models of interaction between a railroad car and the upper structure of a track.

2. Literature review and problem statement

Experience in the operation of rail transport shows [1] that for the mechanical complex “railroad car — rail track” the indicators of reliability and durability greatly depend on the features of processes of interaction between a track and the rolling stock and on the conditions of operation of the examined system. In addition, this interaction affects the ability of the system to withstand destructive action of the emerging impact and vibration loads [7], which have a cyclically repeating character [8].

The largest level in the ballast layer settlement occurs under the first sleeper of the receiving rail [4]. This is due to the fact that the rail in these places is typically exposed to the largest force interaction between the rolling stock and the upper structure of a track.

In order to analyze interaction between the rolling stock and the upper structure of a track, it is necessary to solve several interrelated problems. Specifically, static, dynamic and contact. These issues have recently been in the focus of attention with a sufficient number of new studies in this field.

In particular, authors of [6] reported a scheme for the implementation of a vibro-impact system with a flat spring and a locally centered mass. The paper considers techniques that involve a piecewise-linear elastic characteristic in the form of two intermediate cylindrical supports. The authors took into account a parametrical dependence of contact rigidity between the cylindrical support and the flat spring. However, as regards the standard rails on 22 intermediate supports, this study requires substantial revision. The authors also considered contact stresses, which take into account the asymmetry of change in the elastic characteristic. Contact rigidity is accounted for through a change in the own frequency of string oscillations. The dynamic problem was solved numerically with obtained correlations between kinematic characteristics and parameters of the stressed state, specifically contact stresses, which are nonlinearly depend on displacements. The combined impact of asymmetric and contact stresses is accounted for through the introduction of the equivalent force factor.

The known mechanical model of contact interaction cannot, however, be applied to study the stressed state in the system “wheel — rail” given its dependence on the phases of railroad car motion over a joint unevenness. This provides the basis for the development of further research in the field of modeling a contact between the rolling stock and a track as the interaction between a discrete system and a functional system.

Currently, significant financial resources are invested in transportation infrastructure in many countries of the world. This is especially true of ecologically clean electric transport. Improving the quality and potential of existing services, as well as development of the new infrastructure, are required to meet the growing demand for high-quality and reliable organization of goods and people. Here the efficiency and reliability of rail track design is crucial for successful operation. Many contemporary studies into rail track focus on specific aspects of design and operation, for example, fatigue [1], destruction of the ballast [3], ride comfort [2], noise or vibration [6]. In order to ensure a comprehensive approach, it is necessary to devise a generalized tool, or several computational tools, which would integrally examine certain aspects of the design. Such a technique could be used to comprehensively assess performance of railroads [9]. Traditionally, designing a rail track employs conservative assumptions and thus only small changes are introduced to the concepts. At the same time, certain estimation and analysis methods were developed to describe the model and to predict behavior of the structure. Specifically, a model is analyzed using the method of finite elements that involves commercially available software complexes [10], which requires substantial financial costs at the design stage.

The method of finite elements is also applied for the model of a ballast layer [11], which is an important element of the track design. A rail transmits the load from a wheel to the soil, above which a track is built [12].

Thus, some countries currently build reinforced concrete road ways [13], but their construction cost turns out to be significantly higher compared with a standard ballast track design.

Another technique implies increasing the thickness of the ballast layer, which leads to a decrease in deflection under load and causes lower stresses in the soil base. This improves performance efficiency and length of the track service [14].

This requires and predetermines further search for the adequate model of a rail track as a totality of different elements of the track — rails, sleepers, and a ballast layer.

Thus, it was proven the multiparametric dependence of mechanical interaction between a railroad car and a rail track in the area of a joint on the load, design and mechanical characteristics of a railroad car, rails, and a ballast layer. Correlation between the considered elements requires the construction of an adequate model, which would consider a car of the rail transport in the form of a multidimensional discrete system, and the upper structure of a rail as a continuum system.

The above analysis of the available research results shows that the modern trend in the development of the theory of mechanical interaction between a four-axle railroad car and a rail track is the use of a multifactor systems analysis, which takes into account phases of the railroad car motion over the track joint unevenness, and rigidity characteristics of the ballast layer. Therefore, we can argue about a need to develop an adequate and a convenient-to-use model of interaction between the rolling stock and a rail track, as well as an appropriate procedure in order to analyze their force interaction.

3. The aim and objectives of the study

The aim of present study is to examine mechanical interaction between a railroad car and the upper structure of a rail in order to improve parameters of a discrete-continuum system through rational choice and the optimization of parameters of its components. This would provide additional effect on the characteristics of operational
reliability via components that depend on the parameters of mechanical interaction in the transport mechanical complex “railroad car – rail track in the zone with isolated joint unevenness.”

To accomplish the aim, the following tasks have been set:
- to devise a comprehensive method for calculating parameters of interaction between a four-axle railroad car and a rail track based on a comprehensive approach and general correlations of mechanics with respect to phases of the railroad car motion over a joint unevenness;
- by employing the methods of numerical analysis, to establish and analyze interaction between the elements of the transport systemic discrete-continuum mechanical complex “railroad car – rail track, in the place of joint connection” considering operational parameters;
- to identify new patterns in the mechanical interaction between a four-axle railroad car and a rail track on the road when a four-axle railroad car passes over a joint unevenness with respect to the motion phases.

4. Materials and methods to study the interaction between a railroad car and a rail track at the time of moving over a joint unevenness

The essence of a static interaction between a railroad car and a rail track implies determining the height of a joint unevenness using the method of finite parameters based on the elastic lines of the dispatching and receiving rails whose load corresponds to four phases of the railroad car motion and includes the following.

We consider a generalized mechanical scheme of the four-axle railroad car, which can be a tram carriage, or a passenger or a freight car of the railway transport. When modeling, we take into account structural parameters and load of a transportation vehicle, connection conditions of the dispatching and receiving rails through a joint rail coupling, as well as rigidity of the track ballast layer. Given the generalized character of mechanical scheme of the discrete-continuum system “railroad car – rail track in the zone with an isolated joint unevenness”, the research in present paper is based on a tram car. The results obtained are universal and apply to railroad cars of any purpose. We examined the motion of a four-axle car over a joint unevenness of the track during four phases. At the first phase, all wheel-sets of a car are positioned on the dispatching rail; at the second phase, three wheel-sets remain there; at the third phase, two wheel-sets remain; at the fourth phase, only one wheel-set remains there.

The mechanical scheme, the study procedure, the equation for the deflected axis of rails, the block-diagram of numerical calculation of the rail track deflections, are given in the present paper using the first motion phase as an example. However, the results of research regarding other motion phases are given in this work through graphical dependences and the resulting analysis.

A schematic of the motion of the first wheel-set of a car over a rail junction is shown in Fig. 1. The following denotations are used: 1 – a railroad car of the transportation vehicle, 2–5 – corresponding wheel of the wheel-set; 6–7 – central suspension of the car; 8 – receiving rail; 9 – dispatching rail; 10 – elastic elements of the ballast layer under sleepers; 11 – elastic element that simulates rigidity of the dispatching rail at the end.

When performing static calculation of rail deflections, one uses a model of a multispan beam on 23 elastic supports (22 sleepers and a support that models the connection to the adjacent rail through a working joint bar). To calculate the value of rigidity for a rail \( \varepsilon_r \), it is necessary to determine its deflection at end \( \delta_e \) under the action of a single force, then: \( \varepsilon_r = 1/\delta_e \). Considering the connection with a working joint bar with rigidity \( \varepsilon_p \) of the dispatching and receiving rails, we shall obtain rigidity of the rail at its end:

\[
\varepsilon_{r,e} = \frac{\varepsilon_r \cdot \varepsilon_p}{\varepsilon_r + \varepsilon_p}.
\]

This allows us to estimate a characteristic of the elastic support upon which the rail is based, taking into account the features of connection between the rails. Then we consider the deformation of a rail on 23 elastic supports. One assigns four focused forces on the dispatching rail, which matches the number of railroad car wheels on it.

We shall consider a statement of the statically undefined problem on the deformation of a multispan beam (continuous beam) on elastic supports [15].

The main force factors that determine static deflection of the receiving rail under the first elastic support are the external forces \( P_0 \), constant by their magnitude. They correspond to the current number of wheel-sets on rails, applied to the dispatching rail (Fig. 2), with coordinates \( X_{Gj} \), where \( j = 1, 2, 3, 4 \) is the number of wheel-set of the car.

Fig. 2 shows: \( l_{i} (i = 1..22) \), \( l_{e,r} \) – geometrical coordinates of elastic supports; \( P_0 = P/8 \) – the load at the side of the car that falls on one wheel; \( P \) is the weight of the car with respect to its load; \( F_{r,e} = F_{r,e} \), \( c_t \) – force of elasticity that is applied to the end of the receiving rail from the side of the dispatching rail at the end at \( x = l_{e,r} - 12.5 \text{ m} \); \( Q_0 \), \( M_0 \) are the transverse force and the bending moment in the coordinate origin; \( c \) – rigidity of the ballast layer under the sleeper of the upper structure of the track. Here \( X_{G1} = 1.9 \text{ m} \); \( X_{G3} = 3.8 \text{ m} \); \( X_{G7} = 10.6 \text{ m} \); \( X_{G1} = 12.5 \text{ m} \); \( l_1 = 0.28 \text{ m} \); \( l_2 = 0.85 \text{ m} \); \( l_3 = 1.42 \text{ m} \); \( l_4 = 1.99 \text{ m} \); \( l_5 = 2.56 \text{ m} \); \( l_6 = 3.13 \text{ m} \); \( l_7 = 3.69 \text{ m} \); \( l_8 = 4.26 \text{ m} \); \( l_9 = 4.83 \text{ m} \); \( l_{10} = 5.40 \text{ m} \); \( l_{11} = 5.97 \text{ m} \); \( l_{12} = 6.53 \text{ m} \); \( l_{13} = 7.10 \text{ m} \); \( l_{14} = 7.67 \text{ m} \); \( l_{15} = 8.24 \text{ m} \); \( l_{16} = 8.81 \text{ m} \); \( l_{17} = 9.38 \text{ m} \); \( l_{18} = 9.94 \text{ m} \); \( l_{19} = 10.51 \text{ m} \); \( l_{20} = 11.08 \text{ m} \); \( l_{21} = 11.65 \text{ m} \); \( l_{22} = 12.22 \text{ m} \); \( l_{e,r} = 12.5 \text{ m} \).
The equation for a curved axis of rails at the first motion phase will be written using the method of initial parameters, taking into consideration the conditions for fastening ($y_0-y_0^0=0$).

We shall obtain for the dispatching rail:

$$y(x) = \frac{1}{EJ} \times \left[ Q_0 \frac{x^3}{6} + M_0 \frac{x^2}{2} \sum_{j=1}^{i} \frac{P_j}{6} (x-X_j) + \sum_{i=1}^{22} c_i y_i (x-L_i)^3 + c_{h_e} h_e (x-L)^3 \right].$$ \[(2)\]

For the receiving rail:

$$y(x) = \frac{1}{EJ} \times \left[ Q_0 \frac{x^3}{6} + M_0 \frac{x^2}{2} - F_a (x-L)^3 + \sum_{i=1}^{22} c_i y_i (x-L_i)^3 + c_{h_r} h_r (x-L)^3 \right].$$ \[(3)\]

Here, $J$ is the moment of inertia of the rail cross section relative to the neutral axis; $E$ is the modulus of elasticity of the rail's material; $L=l_e$, the length of the rail.

For expressions (2) and (3) include additions in the right side, which in turn depend on deflections, we solved these equations numerically.

A procedure for calculating rail deflections at the end due to a single force $F = 1$ N, dispatching and receiving rails at their ends, as well as the current height of joint $h$ includes 4 stages.

At stage 1, using expression (1), we calculate rigidity $c_{e,r}$ of the rail at the end [1] at $c_p=3.2 \times 10^6$ N/m, where $c_{e,r}/\delta$, and $\delta$ is the deflection of the rail at the end due to a single force $F = 1$ without considering the joint bars, that is, at $c_p=0$.

According to formula (2), deflection $\delta$ is determined from:

$$y(x) = \frac{1}{EJ} \left[ Q_0 \frac{x^3}{6} + M_0 \frac{x^2}{2} - F_a (x-L_e)^3 + \sum_{i=1}^{22} c_i y_i (x-L_i)^3 + c_{h_r} h_r (x-L)^3 \right].$$ \[(5)\]

A block diagram for the calculation of deflection of the dispatching rail at the end at the first phase of the car motion, according to the estimated mechanical scheme in Fig. 2, a, is shown in Fig. 4.

Here block 2 is the block for introduction of mechanical, geometrical and boundary parameters of the system. In block 3, at step 1 ($j=1$), we calculate initial parameters $Q_{01}$ and $M_{01}$ when the rail is loaded only with effort $F = 1$ and in the absence of all elastic supports (Fig. 2, a) under conditions of static equilibrium:

$$Q_{01} = F; \quad M_{01} = -F l_{e,r}.$$ \[(6)\]

Next, using expression (5), we calculate deflections $y_i$ ($i=1,23$) of the rail under respective elastic supports in Fig. 2, a, which is loaded with force $F$ and defined reactions $Q_{02}$ and $M_{02}$.

At step 2 ($j=2$), with respect to load $F$, we determine new values for the initial parameters $Q_{02}$ and $M_{02}$:

$$Q_{e,i} = F - \sum_{i=1}^{22} c_{y_i} y_{i2};$$

$$M_{e,i} = -F l_{e,i} + \sum_{i=1}^{22} c_{y_i} y_{i2} l_{e,i},$$ \[(7)\]

where $c_{y_i}$ and $y_{i2}$ are forces of elasticity of the corresponding rail support.
4. Calculation of initial conditions $Q_{0i}, M_{0i}$

5. Calculation of deflections $y_{ij}$ ($i = 1-23$)

6. Calculation of $c_{e.r.}$ using expression (5)

7. Calculation of $c_{e.r.}$ using expression (2)

8. Finish

These initial parameters are the starting data for the cycle at $j = 2$.

If error

$$\Delta = \text{Max} \left| y_{ij} - y_{ij-1} \right| (i = 1, 23)$$

appears to be greater than the assigned value $\varepsilon$, then the next cycle takes place for $j$. We accept $\varepsilon = 1 \times 10^{-6}$ m.

Upon satisfying condition $\varepsilon \Delta$, the calculation for cycle $j$ is terminated, we determine the final value of deflection $y_{i e.r.}$ of the rail at the end, which is used in the calculation in accordance with $c_{e.r.} = 1/6$ and $c_{e.r.}$ from expression (1).

At stage 2, using the algorithm of stage 1, we calculate the magnitude of deflection $h_{01}$ of the dispatching rail at the end. In this case, block 1 of the algorithm takes the following form: the introduction of static load $P_{0}$; geometrical coordinates $l_{i} (i = 1, 22), l_{23}, l_{e.r.}$; mechanical characteristics of the rail $E, J$; allowable deflection $\varepsilon$; rigidity $c_{e.r.}$.

Then the equilibrium equations (6) and (7) will take the form:

$$Q_{01} = 4P_{0}; \quad M_{01} = \sum_{j=1}^{2} P_{0}x_{c_{e.r.}};$$

$$Q_{02} = 4P_{0} - \sum_{j=1}^{22} c_{y_{i}} - c_{e.r.} y_{23};$$

$$M_{02} = - \sum_{j=1}^{2} P_{0} x_{c_{e.r.}} + \sum_{j=1}^{22} c_{y_{i}} l_{i} + c_{e.r.} y_{23} l_{e.r.}.$$

A deflection of the dispatching rail at the end is determined from expression (2) as $h_{01} - y_{i e.r.}$ ($l_{e.r.}$) upon meeting condition $\varepsilon \Delta$.

At stage 3, when calculating $h_{i} = h_{01} - y_{i e.r.}$, the introduction of static load $P_{0}$; geometrical coordinates $l_{i} (i = 1, 22), l_{23}, l_{e.r.}$; mechanical characteristics of the rail $E, J$; allowable deflection $\varepsilon$; rigidity $c_{e.r.}$.

In this case, the equilibrium equations (6) and (7) will take the form:

$$Q_{i1} = 0; \quad M_{i1} = 0;$$

$$Q_{i2} = \sum_{j=1}^{22} c_{y_{i}} y_{j} - c_{e.r.} y_{23};$$

$$M_{i2} = \sum_{j=1}^{22} c_{y_{i}} l_{ij} + c_{e.r.} y_{23} l_{e.r.}.$$

Next, using formula (3), we determine a deflection of the receiving rail at the end: $h_{i} - y_{i e.r.}$ ($l_{e.r.}$) at meeting condition $\varepsilon \Delta$.

At stage 4 of our research technique, using expression (4), we calculate height $h = h_{i} - h_{r}$ of the upward step at the first phase of the car motion.

Mechanical schemes, procedures of research, equations of rail deflections and block-diagrams of the numerical computation of the height of joint for other phases, correspond to the similar components of the first phase of the motion.

In this case, a method for investigating parameters of the mechanical interaction between a four-axle railroad car and a rail track includes several stages. First, this is the stage of building the scheme for the case when the wheel-set passes over a rail joint with respect to the phase of car motion. The second phase implies the construction of estimated schemes for elastic lines of the dispatching and receiving track rails, as well as equations for determining, which take into account the phases of motion. The last stage is building a block-diagram for determining the height of the joint, which emerges between the rails, depending on the phases of motion and a car load.

5. Results of research into static interaction between a railroad car and a rail track

By applying the proposed model, we performed numerical analysis of parameters of the static interaction between a four-axle railroad car and a rail track in the zone of an isolated joint unevenness of the type “gap” [1]. The calculations were conducted based on the variation of mechanical and operational factors: a car load, the rigidity of a ballast layer. Using a method of numerical experiment, we obtained dependences of mechanical interaction in the system “railroad car – rail track in the zone of butt joint”. In this case, we took into consideration boundary conditions for fatening the dispatching and receiving rails, geometrical and mechanical characteristics of rails and joint bars, sleepers, and a ballast layer.

The analysis was carried out in accordance with the block diagram for calculating the deflections of dispatching and receiving rails at the end, which is shown in Fig. 4. This allowed us to determine the height of the joint unevenness according to relations (4), (5).

The calculations were performed relative to the mechanical scheme of a railroad car and a rail track, which is shown in Fig. 1, as well as the geometrical and mechanical characteristics of rail R-65 used for the tram T-3 [15]. Modulus of elasticity of the rail’s material is $E = 2.6 \times 10^{11}$ N/m$^2$; moment of inertia of the rail’s cross section relative to the
neutral axis is $h=3.573 \text{ cm}^3$; rigidity of the track ballast layer is $c=0.5 \times 10^9 \text{ N/m}$; the magnitude of gap between the rails $\Delta=9 \text{ mm}$. Mass of the empty car, reduced to one wheel, is $m=2.125 \text{ kg}$; maximum mass of the loaded car (with 193 passengers) is $m=3.814 \text{ kg}$.

These data correspond to the results of analysis of the research into structural characteristics of the rolling stock, rail track and the joint bars of actual objects. We also took into account the status of the issue [1] regarding the transportation mechanical complex “four-axle railroad car – rail track in the zone with an isolated joint unevenness.”

Table 1 gives heights of the rail track joints (where index number for $h$ corresponds to the number of phase of the car motion) when a four-axle railroad car passes over a zone of joint unevenness, which are calculated using formula (4).

It was established that for each load the height of the joint is dependent on the phase of the motion. In addition, the results indicate that increasing a car load leads to a growth of the magnitude of height of the joint unevenness. In this case, the largest height value is observed at the first phase of the car motion.

| $m$, kg | 2.125 | 2.294 | 2.463 | 2.632 | 2.800 | 3.138 | 3.307 | 3.476 | 3.645 | 3.814 |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $h_1$, mm | 2.60 | 2.68 | 2.77 | 2.88 | 3.03 | 3.21 | 3.39 | 3.55 | 3.72 | 3.89 |
| $h_2$, mm | 0.67 | 0.69 | 0.71 | 0.75 | 0.78 | 0.83 | 0.88 | 0.92 | 0.96 | 1.01 |
| $h_3$, mm | 1.30 | 1.34 | 1.39 | 1.44 | 1.52 | 1.61 | 1.70 | 1.78 | 1.86 | 1.95 |
| $h_4$, mm | 0.45 | 0.46 | 0.48 | 0.50 | 0.52 | 0.55 | 0.59 | 0.61 | 0.64 | 0.67 |

The data derived from calculations are shown in Fig. 5 in the form of integrated graphical dependences $h (mm)$ that represent the process of static interaction when a vehicle carrying different loads passes over a joint unevenness. They allow us to ensure that the data obtained with respect to height $h$ could be applied to run high-quality comparative and visual analyses.

![Fig. 5. Dependence of the joint height on the railroad car load](image)

According to the diagram shown in Fig. 5, we observe a monotonous growth of the height of the joint with increasing the load of a car at all phases of motion. It is possible to determine that a change in the car load, from empty to maximally loaded ($m=[2.125; 3.814]$ kg per wheel) leads to an increase in the height of the joint by 1.49 times at the first phase of the motion, by 1.51 times at the second phase, and by 1.5 times and 1.48 times at the third and fourth stages, respectively.

Regardless of the car load, the lowest value of the height of the joint will be observed at the fourth phase of the car motion, followed in ascending order by the second phase, the third phase, with the largest value for the height of the joint at the first phase. However, for an empty car, the difference between the lowest value of the height of the joint (at the fourth phase of the motion) and the largest value (first phase) is 5.77 times. At a maximum load – a given difference (between the first and fourth phases) is 3.8 times. That is, one should admit that it almost does not depend on the railroad car load.

6. Discussion of results of studying the interaction while passing over a joint unevenness

The paradigm of research into the patterns of interaction between a four-axle railroad car and a rail track was built on the basis of a multifactor analysis. This allowed us to establish unknown regularities in the processes of joints interaction between the rolling stock and a rail track in the zone of a joint unevenness with respect to the phases of motion. The new approach was devised to solving the problem about mechanical interaction between a rail transport car and the track in the zone of butt joints. This makes it possible to consider various types of joints in a rail track with respect to the structural and operational features of a transportation vehicle.

Using the proposed method of research shows that we have developed an effective technique for determining the parameters of mechanical interaction in the transportation systems complex “railroad car – rail track in the zone of a joint unevenness”. In this case, the operational, mechanical and geometrical parameters of a transportation vehicle, of the rail, joint bars, sleepers and a ballast layer, are accounted for, as well as boundary conditions of fastening and deflections of the track dispatching and receiving rails.

However, it should be noted that the considered sequence of stages implied by a given method of calculation, as well as its components, is applicable for standard rails with a length of 12.5 m on 22 sleepers. That is why its adaptation for other types of rails would need additional research. It refers to the mechanical scheme of the discrete-continuum system in Fig. 1, as well as the block-diagram of the deflection estimation in Fig. 4.

In the course of present research we established the character of impact of the car load on the height of a joint unevenness when passing over the joint. It was proven that it increases at a change in the load, increasing with positive dynamics, and the dependences $h (m)$, shown in Fig. 5, are parabolic in nature. An analysis of charts in Fig. 5 also showed that ignoring the multifactorial influence on the deflections of a rail track leads to false conclusions, given a sequence of curves.

The results obtained could be used in practice when improving operational characteristics of the rolling stock and a rail track through the rational selection and optimization.

The established values for the degree of a joint unevenness allow us to determine the magnitude of the after-impact speed of the receiving rail when solving the problem about dynamic deflections in the zone of the first sleeper’s position.
Further comprehensive research into impact and dynamic processes at all phases of the railroad car passing over a junction would make it possible to construct a closed model for a given approach.

7. Conclusions

1. The new method for studying the parameters of mechanical interaction between a four-axle railroad car and a rail track is proposed, built on the basis of the scheme of a multispian (continuous) beam resting on 23 elastic supports. It allows determining the height of the track joint with respect to the phases of the car motion, as well as structural and operational features of a transportation vehicle and the track.

2. We give tabular and graphical results of numerical calculations of parameters of the static interaction between a four-axle railroad car with a rail track in the zone of a butt joint, depending on the load at all phases of passing over a joint unevenness.

3. By using the method of comparative analysis, we determined the character of influence of the phases of motion and a car load on the height of a joint unevenness. This enables the implementation of the devised research method and the model of a multispian rail in order to solve a problem on improving the operational parameters of a railroad car and the upper structure of the track in the course of their design.

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