1. Introduction

Gamma-ray bursts (GRBs) are the most energetic phenomenon that occurred in the cosmological distance. The luminosity of GRBs can reach the order of around $10^{50}$ erg s$^{-1}$. There are various models to explain the energy source of GRBs. The popular model for the generation of short GRBs is the mergers of compact binaries (e.g., Narayan et al. 1992; Nakar 2007; Berger 2014), either black hole–neutron star binaries or double neutron stars. Furthermore, it is commonly believed that the collapse of a massive star can account for long GRBs (e.g., Woosley 1993; Paczynski 1998; Woosley & Bloom 2006). In both cases, a dense accretion disk is expected to form, with extremely high temperature and accretion rates. The neutrino–antineutrino annihilation process is a possible mechanism to provide energy for the bursts. The other mechanism based on hyper-accretion disks around black holes is the well-known Blandford–Znajek (BZ) process (e.g., Blandford & Znajek 1977; Lee et al. 2000). Alternatively, a rapidly rotating neutron star with strong magnetic fields can also be responsible for GRBs (e.g., Usov 1992; Dai & Lu 1998; Dai et al. 2006; Gao & Fan 2006; Metzger et al. 2011; Lü et al. 2015).

In this paper, we will focus on the hyper-accreting disks with extremely high accretion rates ($10^{-3} M_\odot$ s$^{-1}$ $\lesssim M \lesssim 10 M_\odot$ s$^{-1}$), where the neutrino radiation may be a dominant cooling mechanism, and therefore such flows are usually named as neutrino-dominated accretion flows (NDAFs). Popham et al. (1999) pioneered the detailed study of NDAFs, and proposed that the neutrino annihilation of a hyper-accreting black hole system can explain GRBs up to $10^{52}$ erg s$^{-1}$. The hyper-accreting black hole system typically has accretion rates around $0.01 \sim 10 M_\odot$ s$^{-1}$. The NDAFs have been widely studied on the radial structure and neutrino radiation (e.g., Popham et al. 1999; Narayan et al. 2001; Di Matteo et al. 2002; Kohri & Mineshige 2002; Kohri et al. 2005; Gu et al. 2006; Chen & Beloborodov 2007; Liu et al. 2007; Zalamea & Beloborodov 2011; Kawanaka et al. 2013; Xue et al. 2013; Cao et al. 2014; Xie et al. 2016), vertical structure and convection (e.g., Liu et al. 2010, 2014, 2015a; Pan & Yuan 2012; Kawanaka & Kohri 2012), and time-dependent variation (Janiuk et al. 2004, 2013). The density can reach $10^8 \sim 10^{12}$ g cm$^{-3}$ in the inner region of the disk, and the temperature can be up to $10^{10} \sim 10^{11}$ K. Such disks can be extremely optically thick, leading to the trapping of large portion of photons, which are carried along the radial direction and are eventually absorbed by the central black hole. Thus, the neutrino cooling is the dominant cooling process in the inner part.

Recently, whether the neutrino annihilation process can work as the central engine for GRBs has been investigated in more detail (Liu et al. 2015b; Song et al. 2015, 2016). Such works revealed that the neutrino annihilation process can account for most GRBs. For some long-duration GRBs, the central engine is more likely to be the BZ mechanism rather than the neutrino annihilation since the former is much more efficient. On the other hand, if the X-ray flares after the prompt gamma-ray emission are regarded as the reactivity of the central engine, the neutrino annihilation mechanism may encounter difficulty in interpreting the X-ray flares. Even including a possible magnetic coupling (Lei et al. 2009) between the inner disk and the central black hole, Luo et al. (2013) showed that the annihilation mechanism can work for the X-ray flares with durations of $\lesssim$100 s. However, the annihilation mechanism is unlikely to be responsible for those long flares with durations of $\gtrsim$1000 s, even though the role of magnetic coupling is included. More recently, Mu et al. (2016) investigated the central engine of the extremely late-time X-ray flares with peak time larger than $10^4$ s, and suggested that neither the neutrino annihilation nor the BZ process seem to...
work well. Instead, a fast rotating neutron star with strong bipolar magnetic fields may account for such flares.

Recent simulations have made significant progress on the super-Eddington accretion process, including the presence of strong outflows (Ohsuga et al. 2005; Ohsuga & Mineshige 2011; Yang et al. 2014) and the radiation-powered baryonic jet (Sadowski & Narayan 2015). In addition, some simulations (e.g., Sadowski & Narayan 2016) show the anisotropic feature of radiation. More importantly, the simulation of Jiang et al. (2014) revealed a new energy transport mechanism in addition to the diffusion, which is named the vertical advection. Their simulation results show that, for the super-Eddington accretion rate $\dot{M} = 220L_{\text{Edd}}/c^2$, the radiative efficiency is around 4.5%, which is comparable to the value in a standard thin disk model. The physical reason is that a large fraction of photons can escape from the disk before being advected into the black hole, through the vertical advection process based on the magnetic buoyancy, which dominates over the photon diffusion process.

In this work, following the spirit of the vertical advection, we incorporate the vertical advection process into NDAFs and revisit the structure and radiation of hyper-accretion disks around stellar-mass black holes. The remainder is organized as follows. The basic physics and equations for our model are described in Section 2. Numerical results and analyses are presented in Section 3. Conclusions and discussion are provided in Section 4.

### 2. Basic Equations

In this section, we describe the basic equations of our model. We consider a steady state, axisymmetric hyper-accretion disk around a stellar-mass black hole. The well-known Paczyński–Wiita potential (Paczyński & Wiita 1980) is adopted, i.e., $\psi = -GM_{\text{BH}}/(R - R_g)$, where $M_{\text{BH}}$ is the black hole mass and $R_g = 2GM_{\text{BH}}/c^2$ is the Schwarzschild radius. The Keplerian angular velocity can be expressed as $\Omega = (GM_{\text{BH}}/R)^{1/2}/(R - R_g)$. We use the usual convention to describe the accretion disk: the half-thickness of the disk is $H = c_s/\Omega_\kappa$, where $c_s = (P/\rho)^{1/2}$ is the isothermal sound speed, with $P$ being the pressure, and $\rho$ the density. We adopt the standard Shakura–Sunyaev prescription for the kinematic viscosity coefficient, i.e., $\nu = \alpha c_s H$.

The basic equations that describe the accretion disk are the continuity, azimuthal momentum, energy equation, and the equation of state. The continuity equation is

$$\dot{M} = -4\pi \rho HR v_R,$$

where $v_R$ is the radial velocity. With the Keplerian rotation assumption $\Omega = \Omega_\kappa$, the azimuthal momentum equation can be simplified as (e.g., Gu et al. 2006; Liu et al. 2007)

$$v_R = -\alpha c_s \frac{H}{R} f^{-1} g,$$

where $f = 1 - j/\Omega_\kappa R^2$, $g = -d \ln \Omega_\kappa / d \ln R$, and $j$ represents the specific angular momentum per unit mass accreted by the black hole. The equation of state takes the form (e.g., Di Matteo et al. 2002)

$$P = \frac{\rho k_B T}{m_p} \left( 1 + 3X_{\text{nuc}} \right) \frac{1}{4}$$

$$+ \frac{11}{12} \frac{\alpha T^4}{3} + \frac{2\pi \hbar c}{3} \frac{\left( \frac{3}{8\pi m_p \mu_e} \right)^{1/2} + \frac{u_e}{3}}{g}$$

where the four terms on the right-hand side are the gas pressure, radiation pressure of photons, degeneracy pressure of electrons, and radiation pressure of neutrinos, respectively.

The energy equation is written as

$$Q_{\text{vis}} = Q_{\text{adv}} + Q_z + Q_{\nu}.$$

The above equation shows the balance between the viscous heating and the cooling by radial advection, vertical advection, and neutrino radiation. The viscous heating rate $Q_{\text{vis}}$ and the advective cooling rate $Q_{\text{adv}}$ for a half-disk above or below the equator are expressed as (e.g., Di Matteo et al. 2002; Gu et al. 2006)

$$Q_{\text{vis}} = \frac{1}{4\pi} M \Omega^2 f g,$$

$$Q_{\text{adv}} = -\xi v_R^2 H \frac{1}{R} \left( 3 - \frac{3}{4} \frac{\rho}{m_p} + \frac{3}{4} \frac{X_{\text{nuc}}}{T} + \frac{4}{3} \frac{u_e}{T} \right),$$

where $T$ is the temperature, $s$ is the specific entropy, $u_e$ is the neutrino energy density, and $\xi$ is taken to be 1. $X_{\text{nuc}}$ is the mass fraction of free nucleons (e.g., Kohri et al. 2005):

$$X_{\text{nuc}} = \text{Min} \left[ 1, 20.13\rho_{10}^{3/4} T_{11}^{9/8} \exp(-0.61/T_{11}) \right],$$

where $\rho_{10} = \rho/10^{10}$ g cm$^{-3}$ and $T_{11} = T/10^{11}$ K. The quantity $Q_z$ is the cooling rate due to the neutrino radiation. We adopt a bridging formula for calculating $Q_{\nu}$ as shown in Di Matteo et al. (2002) and Liu et al. (2007).

The main difference from previous works is that the vertical advection term $Q_z$ is taken into account in our work, which is written as

$$Q_z = \nabla_z (u_{\phi} + u_e + u_{\text{gas}}),$$

where $u_{\phi}$ is the energy density of photons, $u_e$ is the energy density of neutrinos, $u_{\text{gas}}$ is the energy density of the gas. In our calculation, the third term $\nabla_z u_{\text{gas}}$ is dropped since the escaped gas through the magnetic buoyancy can be negligible. The quantity $\nabla_z$ is the averaged velocity of the vertical advection process, which can be simply written as

$$\nabla_z = \lambda c_s,$$

where $\lambda$ is a dimensionless parameter. By comparing the typical vertical velocity $\nabla_z$ in the simulation results (Figure 14 of Jiang et al. 2014) and the theoretical estimate of sound speed, we take $\lambda = 0.1$ for our numerical calculations. The vertical advection term describes the released photons and neutrinos due to the magnetic buoyancy, which can dominate over the normal diffusion process.

Equations (1)–(4) can be solved if the parameters $M_{\text{BH}}, \dot{M}, \alpha, \kappa$, and $j$ are given. We consider a stellar-mass black hole with $M_{\text{BH}} = 3M_\odot$. The viscous parameter $\alpha = 0.02$ is taken from the simulation results (Hirose et al. 2009). The specific angular momentum $j = 1.83 c R_g$ is just a little less than the Keplerian
angular momentum at the marginally stable orbit, i.e., $l_{\text{K}} = 1.837 R_k$. Our study focuses on the solutions in the range of $3 \sim 10^3 R_g$.

3. Numerical Results

In this section, we present our numerical results and the analyses of the physics behind these results. The calculation reveals that the vertical advection process has essential effects on the structure and radiation of the disk. First, the radial profiles of mass density and temperature are investigated for two cases, i.e., with and without the vertical advection process. The radial profiles of density $\rho$ are shown in Figure 1, where the solid (dashed) lines correspond to the results with (without) the vertical advection process. The five typical mass accretion rates are $\dot{M} = 10^{-3}, 10^{-2}, 0.1, 1$, and $10 M_\odot$ s$^{-1}$, which are shown by different colors. It is seen from Figure 1 that, for a certain radius $R$ and a given $\dot{M}$, the density $\rho$ in the disk with vertical advection process is significantly higher than that without such a process, particularly for low accretion rates such as $\dot{M} = 10^{-3} M_\odot$ s$^{-1}$. The radial profiles of temperature $T$ are shown in Figure 2, where the explanation of different color and types of lines is the same as in Figure 1. It is seen that the temperature of the disk with the vertical advection is generally lower than that without the advection.

The physical understanding of the above difference in density and temperature is as follows. Since a large fraction of the trapped photons can be released through the vertical advection process, the radiative cooling through such a process is efficient. As a consequence, the temperature $T$ together with the total pressure $P$, the vertical height $H/R$, and the radial velocity $v_R$ will decrease, whereas the mass density $\rho$ will increase. For relatively low accretion rates such as $\dot{M} = 10^{-3} M_\odot$ s$^{-1}$, the radiative cooling of neutrinos is quite inefficient, so the effects of vertical advection may be more significant, as indicated by Equation (4).

Figure 3 shows the radial profiles of vertical scale height of the disk. It is seen that the relative height $H/R$ of the disk with vertical advection is significantly thinner than that without the vertical advection, particularly for low accretion rates. The physical reason is mentioned above, which is related to the decrease of temperature and pressure. The decrease of vertical height also implies the decrease of sound speed and therefore the decrease of radial velocity, as inferred by Equation (2).

For a typical mass accretion rate $\dot{M} = 0.1 M_\odot$ s$^{-1}$, Figure 4 shows the radial profiles of energy components, where the three dimensionless factors are $f_e = Q_e/Q_{\text{vis}}$ (red line), $f_{\text{adv}} = Q_{\text{adv}}/Q_{\text{vis}}$ (green line), and $f_0 = Q_0/Q_{\text{vis}}$ (blue line). It is seen that in the most inner region ($R \lesssim 4R_g$) the radial advection is
the dominant cooling mechanism. For the outer part of the disk ($R \gtrsim 20R_g$), the energy transport through the vertical advection becomes dominant. For the region $4R_g \lesssim R \lesssim 20R_g$, the neutrino cooling may dominate over the other two mechanisms. The red line implies that the photon radiation through the vertical advection process can reach a large fraction of the total released gravitational energy, and therefore the photon luminosity can be extremely super-Eddington. We will investigate the corresponding photon luminosity in Figure 6.

Our main focus is the energy transport through the vertical advection process. Figure 5 shows the radial profiles of the dimensionless cooling rate due to the vertical advection, i.e., $f_z = Q_z/Q_{vis}$, where five typical accretion rates are adopted. It is seen that $f_z$ generally decreases with increasing $\dot{M}$. The physical reason is that the neutrino cooling is less important for relatively low accretion rates. For the highest accretion rate with $\dot{M} = 10 M_\odot$ s$^{-1}$, the red line shows that there exists a big bump in the inner region ($\lesssim 20R_g$). The physics for this bump is that the neutrino cooling is again less significant since the inner disk is optically thick to the neutrinos.

Finally, we calculate the neutrino luminosity and the photon luminosity of the accretion disk. The neutrino luminosity is derived by the integration of the whole disk:

$$L_\nu = \int_{R_{in}}^{R_{out}} 4\pi R \cdot (Q_\nu + V_{\nu}u_\nu) \, dR,$$

where $L_\nu$ includes the contributions from the direct neutrino radiation $Q_\nu$ and the vertical advection process on neutrinos $V_{\nu}U_\nu$. Actually, the latter is negligible except for extremely high accretion rates $\dot{M} \gg 1 M_\odot$ s$^{-1}$. The variation of neutrino luminosity with mass accretion rates is shown in Figure 6, where the red solid line corresponds to the neutrino luminosity with the vertical advection process, whereas the red dashed line corresponds to the neutrino luminosity without the process. It is seen that the red solid line is under the red dashed line, which means that the neutrino luminosity with the vertical advection is lower. The physical reason is as follows. The neutrino radiation is more sensitive to the temperature than the density. As shown by Figure 2, the disk temperature is lower for the case with the vertical advection. Thus, the corresponding $Q_\nu$ and $L_\nu$ in our cases are lower than those without the vertical advection.
The variation of photon luminosity $L_{\text{ph}}$ is shown by the blue solid line in Figure 6, where $L_{\text{ph}}$ is calculated by

$$L_{\text{ph}} = \int_{R_{\text{in}}}^{R_{\text{out}}} 4\pi R \cdot \nabla_{\text{ph}} u_{\text{ph}} \, dR.$$  \hspace{1cm} (11)

It is seen that $L_{\text{ph}}$ is in the range of $10^{50} \sim 10^{53} \text{erg s}^{-1}$ for $0.001 \leq n_t \leq 10$, which is more than 10 orders of magnitude higher than the Eddington luminosity. The released photons are mainly in the gamma-ray band according to the thermal radiation of the inner disk with $10^{10} \text{K} < T < 10^{11} \text{K}$, as shown by Figure 2. The huge amount of gamma-ray photons escape from the optically thick disk through the vertical advection process, which is much more efficient than the diffusion process. Such an extremely high photon radiation should have observational effects. We will have a discussion on that in the next section.

4. Conclusions and Discussion

In this work, we have studied the structure and radiation of hyper-accretion flows around stellar-mass black holes by considering the role of the vertical advection process. Through the comparison of our results with the classic NDAF solutions, we have shown that the density is higher, the temperature is lower, and the vertical height is thinner in our solutions. The physical reason is that a large fraction of photons can escape from the optically thick disk through the vertical advection process. As a consequence, the neutrino luminosity from the disk is decreased. Thus, even without calculating the neutrino annihilation luminosity, we can conclude that the annihilation mechanism cannot be responsible for the long-duration GRBs and X-ray flares.

We would point out that outflows are not taken into consideration in the present work. However, outflows are believed to generally exist in accretion flows. Recent MHD simulations have shown that outflows exist both in optically thin flows (Yuan et al. 2012a, 2012b) and optically thick flows (Jiang et al. 2014; Sadowski & Narayan 2015, 2016). From the observational view, Wang et al. (2013) reveals that more than 99% of the accreted mass escapes from the accretion flow by outflows in our Galactic center. Based on the energy balance argument, Gu (2015) shows that the outflow is inevitable for the accretion flows where the radiative cooling is far below the viscous heating, no matter whether the flow is optically thin or thick. Thus, outflows may work as another process to help the trapped photons to escape (Shen et al. 2015), and will also have effects on the structure and neutrino radiation of the accretion flow. Such a mechanism is not included in the current work.

Our calculations are based on the relation $\nabla_{\text{c}} = \lambda c_s$ with $\lambda = 0.1$. As mentioned in Section 2, the value for $\lambda$ is adopted following the simulation results for $M = 220L_{\text{edd}} / c^2$ (Jiang et al. 2014). In our case, the mass accretion rate is higher for more than 10 orders of magnitude. Then, a key question may exist whether the vertical advection due to the magnetic buoyancy can also work for such hyper-accretion systems. In our opinion, the radiation pressure is always dominant up to $M \lesssim 0.1 M_\odot \text{s}^{-1}$ or for the outer part of even higher accretion rates. Thus, such a mechanism seems to be an efficient process. On the other hand, even for the case in which the parameter $\lambda$ is significantly smaller than 0.1 in the hyper-accretion case, such as several orders of magnitude smaller, the released gamma-ray photons may still be extremely super-Eddington and the potential application is significant.

In this work, we have assumed $\alpha = 0.02$ according to the simulation results of Hirose et al. (2009). However, other simulations may provide different values for $\alpha$. As shown by Yuan & Narayan (2014), such a value may be related to the magnitude of net magnetic flux in the simulations. The values of $\alpha$ may also have significant effects on the energy transport of the vertical advection. As Equations (2) and (6) imply, $v_R$ is proportional to $\alpha$ and $Q_{\text{adv}}$ is proportional to $v_R$ and therefore $\alpha$. Thus, we can expect that, for a larger value of $\alpha$, the advective cooling rate $Q_{\text{adv}}$ can significantly increase, and therefore the cooling rate due to the vertical advection $Q_z$ will decrease according to the energy balance of Equation (4). Nevertheless, the luminosity related to the radial integration of $Q_z$ will still be hyper-Eddington even though $Q_z$ may be lower than $Q_{\text{adv}}$ for a large range of radii.

Our results of extremely super-Eddington luminosity of gamma-ray emission can also be generally applied to short GRBs. It is commonly believed that short GRBs originate from the merger of compact objects, i.e., the black hole–neutron star binary or the binary with double neutron stars. More importantly, the merger is a significant source of gravitational wave event. Thus, the released high-energy photons may have significant contribution to an electromagnetic counterpart for the gravitational wave event, such as kilonovae. Kilonovae have been widely studied in recent years (e.g., Li & Paczyński 1998; Metzger & Berger 2012; Yu et al. 2013; Gao et al. 2015; Kasen et al. 2015; Fernández et al. 2016; Jin et al. 2016; Kawaguchi et al. 2016; Metzger 2016). In our case, a new picture is shown by Figure 7. It is seen from this figure that either the merger of a black hole and a neutron star or the merger of two neutron stars may result in a gravitational wave event and a black hole hyper-accretion disk. According to our study in this work, most gamma-ray photons escaping from the direction perpendicular to the equatorial plane together with the neutrino annihilation contribute to the thermal fireball, and

![Figure 7: Illustration of the association of short GRBs, kilonovae, and gravitational wave events.](image-url)
the remnant escaped photons diverge from other directions to trigger non-thermal emission due to diffusion into the ambient environment. Obviously, such a progenitor of kilonovae is quite different from the origin from disk wind (e.g., Metzger & Berger 2012; Kasen et al. 2015) or magnetar wind (e.g., Yu et al. 2013; Gao et al. 2015). In addition, a faint gamma-ray thermal component may exist owing to the large amount of escaped thermal gamma-ray photons from the disk, which may have potential contribution to the thermal component of the prompt gamma-ray emission (Abdo et al. 2009; Zhang et al. 2016). Moreover, the neutrino annihilation mechanism or the BZ mechanism based on the accumulation of magnetic fields through the hyper-accretion process will work as the main central engine for the GRB. In summary, in our scenario of black hole hyper-accretion with vertical advection process, short GRBs, kilonovae, and gravitational wave events can be naturally blended together (e.g., Fernández & Metzger 2016).

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References

Abdo, A. A., Ackermann, M., Ajello, M., et al. 2009, ApJL, 706, L138
Berger, E. 2014, ARA&A, 52, 43
Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433
Cao, X., Liang, E.-W., & Yuan, Y.-F. 2014, ApJ, 789, 129
Chen, W.-X., & Beloborodov, A. M. 2007, ApJ, 657, 383
Dai, Z. G., & Lu, T. 1998, PhRvL, 81, 4301
Dai, Z. G., Wang, X. Y., Wu, X. F., & Zhang, B. 2006, Sci, 311, 1127
Di Matteo, T., Perna, R., & Narayan, R. 2002, ApJ, 579, 706
Fernández, R., Foucart, F., Kasen, D., et al. 2016, arXiv:1612.04829
Fernández, R., & Metzger, B. D. 2016, ARNPS, 66, 23
Gao, H., Ding, X., Wu, X.-F., Dai, Z.-G., & Zhang, B. 2015, ApJ, 807, 163
Gu, W.-M., & Fan, Y.-Z. 2006, CHJAA&A, 6, 513
Yu, Y.-F., & Gu, W.-M. 2015, ApJL, 797, 71
Yu, Y.-F., Gu, W.-M., & Fan, Y.-Z. 2012a, ApJL, 748, L87
Xie, W., Lei, W.-H., & Wang, D.-X. 2016, arXiv:1609.09183
Xue, L., Liu, T., Gu, W.-M., & Lu, J.-F. 2013, ApJS, 207, 23
Yang, X.-H., Yuan, F., Ohsuga, K., & Bu, D.-F. 2014, ApJ, 780, 79
Yu, Y.-W., Zhang, B., & Gao, H. 2013, ApJL, 776, L40
Yuan, F., Bu, D., & Wu, M. 2012, ApJL, 751, L20
Yuan, F., & Narayan, R. 2014, ARA&A, 52, 529
Yuan, F., Wu, M., & Bu, D. 2012b, ApJL, 761, 129
Zalamea, I., & Beloborodov, A. M. 2011, MNRAS, 410, 2302
Zhang, B.-B., Zhang, B., Castro-Tirado, A. J., et al. 2016, arXiv:1612.03089