Analysis of Damage Tolerance of the Stiffened Structure Based on XFEM

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Abstract. A calculation method of damage tolerance of stiffened structure based on extended finite element method (XFEM) is proposed. The crack propagation process is divided into small intervals to calculate the amplitude of stress intensity factor of the structure under different crack lengths. Each small interval is integrated to calculate the crack growth life. The accuracy of the method is verified by calculating the stress intensity factor, the load at both ends and crack growth life. The flat slab structure model with different numbers of stiffeners is established. The above calculation method shows that the stiffeners can share the load for the structure, reduce the amplitude of stress intensity factor, increase the fatigue life. The more the number of stiffeners, the more obvious the improvement effect. The equivalent treatment of the stiffened structure was conducted, and the equal area widening model, unfolding model and thickening model were established. The calculation results show that the calculation results of expansion model is very close to the stiffened structure. It shows that the stiffeners share the load for the structure by increasing the local thickness of the area, reduce the amplitude of stress intensity factor and improve the fatigue intensity.

Keywords. Damage tolerance; fatigue crack growth; fatigue life; fatigue intensity; XFEM.

1. Introduction
With the gradual development of modern engineering technology, the engineering structures are developing in both directions of lightness and thinness. The use of stiffened structure is an effective method to make the structure lighter and thinner. Stiffeners [1-5] can not only improve the durability and reliability of the structure, but also reduce the weight of the structure.

In 1999, Professors Belyschko and Professors Black [6] at Northwestern University proposed the concept of extended finite element method which was officially named the extended finite element method in 2000. In 2001, Daux [7] successfully simulated the crack bifurcation phenomenon by using the extended finite element method. Stolarska [8] introduced the level set method in the extended finite element to realize the description of discontinuous geometric surfaces. In 2013, M. C. Baiett [9] successfully simulated the fatigue crack growth of the contact surfaces through the idea of combining experience and finite element. In 2007, Fang Xiujun [10] simulated three-point bending specimens by using XFEM of ABAQUS software. Yang Jun [11] wrote subroutines in Fortran language and simulated the crack growth of three-point bending specimens by using XFEM. In 2010, Yu Tiantang [12] combined the linear complementary method with XFEM, calculated the stress intensity factor of the example of three-dimensional elastostatic, and described the contact problem of crack surfaces.
Based on XFEM and Python language, Gao Mingxing [13] developed a new algorithm of fatigue crack growth that considers the effect of overload hysteresis. The prediction of hybrid crack growth path and the life evaluation with constant amplitude and overload fatigue crack were realized.

This paper proposes a calculation method of damage tolerance of stiffened structures based on the extended finite element method (XFEM). The main idea is that the crack growth process is divided into sufficiently small intervals to calculate the amplitude of stress intensity factor of the structure under different crack lengths. Each small interval is integrated based on the limit idea to calculate the crack growth life.

2. The Model of Analysis of Fatigue Damage Based on ABAQUS-XFEM

2.1. Process of the Program of the Method of Analysis of Fatigue Damage Based on ABAQUS-XFEM

The extended finite element method (XFEM) is an emerging numerical method to deal with discontinuous problems based on the idea of element decomposition and developed on the basis of finite element. Compared with the theoretical equation, the value of stress intensity factor obtained based on the extended finite element method has a smaller error, which is a more accurate and simple method. The simulation of fatigue crack growth is realized through the secondary development of ABAQUS using Python language and the calculation of stress intensity factors based on the extended finite element method. The program block diagram of this method is shown in figure 1.

![Figure 1. Analysis process of the program of fatigue damage tolerance.](image)

2.2. Life Calculation of Flat Sheet with Edge Cracks

The following will calculate the tension problem of a flat sheet with edge cracks in order to verify the feasibility and accuracy of the method and program.

As shown in figure 2, the thin sheet is 200 mm long, 100 mm wide, and 1 mm thick. Both ends are subjected to a cyclic tensile load. The maximum load is 4000N, that is, the maximum value $\Delta \sigma$ is 40Mpa, the stress ratio $R=0$, and the minimum value $\Delta \sigma$ is 0. The material is LY12 aluminum alloy, and the mechanical performance parameters are shown in the table 1.
Engineering algorithm of stress intensity factor:

\[
K = \beta \sigma \sqrt{\pi a}
\]  

(1)

\[
\beta = 1.12 - 0.231 \frac{a}{W} + 10.55 \left( \frac{a}{W} \right)^2 - 21.72 \left( \frac{a}{W} \right)^3 + 30.39 \left( \frac{a}{W} \right)^4
\]  

(2)

The threshold value of the stress intensity factor of the aluminum alloy thin sheet is 4.64, which is 0.2 times of the fracture toughness. The initial crack length is set to 10mm. At this time, \( K_{\text{max}} \) is calculated as 8.392 \( \text{MPa} \sqrt{\text{m}} \) according to the engineering experience equation.

After the program runs, the amplitude of stress intensity factor \( \Delta K \) of the aluminum alloy sheet corresponding to each crack length is output. It has a comparison with the amplitude of the stress intensity factor \( \Delta K \) of each crack length obtained by the engineering algorithm. As shown in figure 3.

As shown in the figure, the amplitude of stress intensity factor obtained by this method and the engineering empirical equation is basically same, as the crack length continues to increase. \( \Delta K \) is gradually increasing. In the process of edge crack propagation from 10 mm to 33 mm (stable crack propagation stage), the result obtained by this method program is slightly larger than obtained by the engineering experience equation. It shows that the calculation result is conservative, and the maximum relative error between the two data is 3.33%. The uniform load at both ends of the aluminum alloy sheet can be calculated by equations (2)-(5):

\[
\Delta \sigma = \Delta K / \beta \sqrt{\pi a}
\]  

(3)

And compare it with the equivalent uniform load actually applied on both ends of the model, as shown in figure 4:
As shown in Figure 4, the equivalent uniform load calculated according to the program output is basically same with the actual load, which is slightly larger, and fluctuates with the change of crack length. The average relative error of the two data is 2.33%. Since the equivalent uniform load on both ends of the model estimated based on the program output is not a fixed value, this paper proposes an approximate method for estimating the fatigue life based on the limit idea.

When the fatigue crack growth increment \( \Delta a \) is sufficiently small, assuming that the cyclic load remains unchanged in this small interval \([a, a + \Delta a]\) (the interval length used in all models in this article is 1 mm), the growth life \( N_i \) in this interval can be calculated by the Paris equation. The curve of fatigue crack growth life of aluminum alloy sheet can be obtained by accumulating step by step. The cumulative equation of fatigue crack growth life is:

\[
\sum_{i=1}^{L} N_i = N
\]  

(4)

The estimation equation of fatigue crack growth life based on the Paris equation in each individual interval is:

\[
N_i = \int_{a_i}^{a_i + \Delta a} \frac{da}{C(\beta \Delta \sigma \sqrt{\pi a})^n}
\]

(5)

In the equation, \( a_i \) is the lower limit of the interval, that is, the initial length of the crack. \( \Delta a \) is the interval length, that is, the increment of fatigue crack growth, which is taken as 1mm in this paper. \( C \) and \( n \) are the Paris equation parameters, which are the test constants related to the material. \( \beta \) is the geometric modifying factor, which is related to the ratio of the crack length \( a \) to the width \( W \) of the aluminum alloy sheet. \( \Delta \sigma \) is the uniform load of the model, which is the average value of the equivalent uniform load calculated according to the secondary development program at both ends of each interval. As shown in Figure 5, the fatigue crack growth life estimated based on the program output is about 92880 times during the process of crack growth from 10mm to 33mm of the aluminum alloy plate under tensile load. The fatigue crack growth life calculated based on the engineering experience equation is 100000 times. The relative error both two is 7.12%, which is less than 10%. It shows that the accuracy of this method meets engineering needs.
3. Analysis of Crack Growth of Stiffened Structure

A finite element model with one stiffener and two stiffeners has been established, as shown in figures 6 and 7. The thickness of the stiffener is 1mm, the cross-sectional area is 10mm², the length is 200mm, and the material is LY12 aluminum alloy. The connection relationship between the stiffeners and the flat plate selects Tie connection. The pressure load of total force type is added to the upper section of the flat plate and the stiffeners. The load is 4400N and 4800N respectively for the finite element model containing one stiffener and two stiffeners, that is, both $\Delta \sigma$ is 40 Mpa. Fixed constraints are imposed on the lower section of the flat plate and the stiffeners.

There’s a comparison with the output result of the amplitude of stress intensity factor $\Delta K$ of the structure without stiffeners. As shown in figure 8, the amplitude of stress intensity factor output by the model with stiffeners is smaller than that of the model without stiffeners under the same crack length. At the same time, the more stiffeners in the model, the smaller the output result of the secondary development program. It has been strongly proved that the stiffeners can enhance the fatigue resistance of the structure in engineering practice. Also, as the fatigue cracks continue to expand, the closer the crack is to the stiffeners, the more significant the decrease in the amplitude of stress intensity factor. This indicates that the closer the crack is to the stiffeners, the more significant the improvement of the fatigue resistance of the structure by the stiffeners is.
Figure 8. Comparison of the amplitude of stress intensity factor output by different models.

Figure 9. Comparison of loads at both ends of different stiffened flat slab models.

The distribution of the equivalent uniform load at both ends of the aluminum alloy flat slab structure can be calculated by the equation

$$\Delta \sigma = \Delta K / \beta \sqrt{\pi a}$$  \hspace{1cm} (6)

Figure 9 shows that the stiffener can reduce the equivalent load of the structure to improve the ability of the fatigue crack resistance from the load level. The closer the crack is to the stiffeners, the more significant the effect. At the same time, the more the number of stiffeners, the larger the reduction value of the equivalent uniform load of the flat slab structure.

As shown in the figure 10, the stress intensity factor of the edge-cracked flat slab structure without stiffeners exceeds the fracture toughness when the crack expands from the initial length of 10mm to 33mm. We can see, the more the number of stiffeners, the larger the reduction value of the equivalent uniform load of the flat slab structure.

Figure 10. The curve of fatigue crack growth life of different stiffened structures.

4. Equivalent Comparison of Stiffened Structure

The section 3 verified the reinforcement effect of stiffeners on fatigue strength of structure. In section 4, the stiffened structure is equivalently treated, and the equal-area widening model, the unfolding model and the thickening model are established. The mechanism of how stiffeners to improve the fatigue life of the structure is analyzed by comparing the amplitude of stress intensity factor, uniform load, fatigue life, etc.
As shown in figure 11 and figure 12, it can be seen that under the same crack length, the output result of the thickening equivalent model is obviously larger, and the output result of the widening equivalent model is obviously smaller. The output results of the unfolding model are more consistent with the stiffened model, and the relative error is small.

It can be seen from figures 13-14 that the load estimated from the output result of the equivalent thickening model is the largest, which is located at the top of the graph. The calculation result of the equivalent widening model is also larger. Although the amplitude of stress intensity factor output is the smallest. The width value $W$ of the model is larger than that of other models when the load is calculated using the equation (6). So it has some difference. The load error at both ends of unfolding model and the stiffened model are smaller. The trend of the curve changing with the crack length is also completely consistent.
As shown in figures 15 and 16, we compare the fatigue life of each equivalent model with a stiffener and two stiffers. Compared with the fatigue life calculated by the stiffened structure, the calculation result of the widening model is too large, the calculation result of the thickening model is too small, and the error of the calculation result of the unfolding model is very small. It shows that the stiffener increases the fatigue life by increasing the thickness of the area and enhances the ability to resist fatigue crack propagation.

5. Conclusions
1) The stress intensity factor of the structure can be accurately calculated based on the ABAQUS-XFEM secondary development program. The error of the uniformly distribute load and the crack growth life calculated are within 10% of the engineering algorithm. It can provide a useful reference for analysis of fatigue crack growth in engineering practice.

2) The mode of flat slab structure with different amounts of stiffeners has been established. And the secondary development program is called. The results show that the stiffeners can share the load for the structure, reduce the amplitude of the stress intensity factor, and improve the fatigue life. The more the number of stiffeners, the more obvious the lifting effect.

3) The equal-area widening, unfolding, and thickening model of the stiffeners is established. The calculated results of the unfolding model are very close to that of the stiffened structure. It shows that the stiffeners share the load for the structure, reduce the amplitude of the stress intensity factor, and increase the fatigue strength by increasing the local thickness of the area.

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