1 Introduction

Event shapes are sensitive to the strong coupling, $\alpha_s$, and the gauge structure of the strong force. However, the observables investigated (mean values, higher moments and normalised distributions) don’t depend on the production rate nor on the event orientation and are therefore independent of the electroweak production process. They are thus directly connected to fundamental properties of the strong force.

QCD predictions for these observables exist in NLO and NLLA, but their combination suffers from ambiguities in avoiding double counting. Of the available matching schemes $\log R$ is the most popular. All calculations depend on the unphysical renormalisation scale, which is varied in order to estimate theoretical uncertainties.

The conversion of perturbatively accessible partons into hadrons may have a significant impact on the final value of an event shape observable. In order to match perturbative predictions with data these non-perturbative hadronisation effects thus have to be taken into account. Traditionally the only way to correct for these non-perturbative effects was the application of Monte Carlo models, which suffer from a large number of free parameters that need to be tuned. Since a few years the analytical ansatz of power corrections with only one free parameter is used as an alternative.
2 Measurements of the strong coupling

To give an overview Fig. 1 compares measurements of the strong coupling from several methods used at LEP to the current PDG world average. Beside the results from the LEP QCD working group, a combination of pure NLLA results, an application of \( \mathcal{O}(\alpha_s^2) \) and \( \mathcal{O}(\alpha_s^3) \) with MC models and optimised scales and results from \( \mathcal{O}(\alpha_s^2) \) with power corrections are shown. The error contribution from the renormalisation scale was adapted to use a consistent variation of \( \mu/Q \) by a factor 2 around the central value. The increase, if any, is shown as extra (red) error bars.

These direct methods show a good consistency within the quoted errors which are in all cases dominated by the contribution from the renormalisation scale variation.

The indirect determinations of the strong coupling stem from the investigation of \( \tau \) hadronic branching ratio \( R_\tau \), from the hadronic width of the \( Z \) and from a five parameter fit to electroweak precision data. They show a good consistency with the direct results and with the world average within the given errors.

3 Mass effects in event shape observables

In order to improve the consistency in the description of event shape observables it is important to take mass effects into account. Mass effects arise from heavy quarks as well as from non-zero hadron masses. The size of these effects depend on the observable and the energy.

Depending on the observable mass effects from stable hadrons enter either indirectly via energy-momentum-conservation (e.g. for thrust) or directly into the observable (e.g. jet masses). The direct dependence on the hadron masses can be avoided by an appropriate redefinition of the four-momenta used to calculate the observable: \( p = (\vec{p}, E) \rightarrow (\vec{p}, |\vec{p}|) \) (\( p \)-scheme) or \( p = (\vec{p}, E) \rightarrow (\hat{p}, E) \) (\( E \)-scheme).

This redefinition doesn’t influence the power correction term that is calculated for massless particles. The difference between the schemes is therefore an indicator for the size of the effect of hadron masses. As shown in Fig. 2 it can be of significant size. Of the different schemes the \( E \)-scheme is singled out, because in this scheme the mass correction is universal and can thus to a good approximation be absorbed into the standard power correction.
Extra transverse momentum from the decay of hadrons, especially $B$-hadrons, does similarly change the observables, but can’t be significantly reduced by the redefined scheme. Their influence including the varying initial rate of $b$-quarks need to be accounted for otherwise.

The DELPHI analysis presented in the following section takes the $B$-mass effects into account by applying a MC correction and uses the $E$-scheme definition for jet masses to reduce the influence of stable hadrons.

### 4 RGI

The renormalisation group invariant perturbation theory (RGI) uses the observable itself as expansion parameter. Thus it has no dependence on the renormalisation scale.

For a mean event shape $R = \langle f \rangle / A_f$ RGI connects the energy dependence of an observable with the observables $\beta$-function:

$$Q \frac{dR}{dQ} = \beta_R(R) = -\frac{\beta_0}{2\pi} R^2 \left(1 + \frac{\beta_1}{2\pi \beta_0} R + \rho_2 R^2 + \ldots\right)$$

As usual, this can be solved introducing an integration constant, $\Lambda_R$. $\Lambda_R$ can be connected to $\Lambda^{\text{QCD}}_{\text{MS}}$ and thus be used to measure the strong coupling.

The $\alpha_s$-results obtained by the DELPHI collaboration with this method show good consistency of $\alpha_s = 0.119 \pm 0.004$ when including RGI power corrections. (The quoted variation indicates the spread of 6 observables.) As the power corrections are consistent with zero, the data can be described without power correction leading to an even improved consistency of $\alpha_s = 0.117 \pm 0.002$. That data can be described without power corrections or hadronisation.
corrections raises the question, whether power corrections which usually have a size of around 10% at \(M_Z\) are to a large part perturbative.

In fact RGI is able to 'predict' a value for \(\alpha_0\) as function of \(\alpha_s\), from setting the RGI prediction equal to the power correction formula. Fig. 2 shows \(\alpha_0\), as obtained after choosing the PDG average of \(\alpha_s = 0.118\), compared to those obtained with the 2-parameter power corrections fits. The 'predictions' (blue bands) agree with the fit results (symbols) much better than any universal \(\alpha_0\)-value around the expected 0.5.

5 Measurement of the \(\beta\)-function

The RGI formula Eq. (1) allows to deduce the \(\beta\)-function of an observable directly from the energy evolution of its measured mean values. Because the \(\beta\)-functions are universal to NLO this is equivalent to a measurement of the \(\beta\)-function of QCD.

Experimentally the energy dependence is measured as slope of the inverse mean value as function of the logarithm of the energy, which measures \(-R^2\beta_R(R)\). From DELPHI thrust data one finds \(dR^{-1}/d\log Q = 1.35 \pm 0.16\); including data from low energy experiments the fit results in \(dR^{-1}/d\log Q = 1.38 \pm 0.05\). Fig. 3 (left) picturates the excellent data description of this fit and the good agreement with the QCD expecation of 1.32. When interpreted within QCD these obtained slopes correspond to a number of active flavours of \(n_f = 4.7 \pm 0.7\) and \(n_f = 4.75 \pm 0.44\), respectively.

As the \(\beta\)-function is a function of the structure constants, it is interesting to compare the constraints of this measurements with other measurements of the structure constants, which are usually plotted in the \(n_fT_R/C_F\) vs. \(C_A/C_F\) plane. In Fig. 3 (right) the result for the \(\beta\)-function is compared to results from angular distributions of ALEPH and OPAL, and to a result from gluon jet multiplicity of DELPHI.

It shows that the structure is by now very well confirmed by measurements and that it poses strict constraints on new physics, excluding e.g. light gluinos.
6 Conclusions

Event shapes in $e^+e^-$ collisions directly probe properties of hard QCD. At LEP they can be measured with a precision that exceeds the precision of the current theoretical understanding. The investigation of the consistency of $\alpha_s$-results from several such event shapes gives hints in which areas problems still hide or which renormalisation scheme may be singled out.

As (most of) the calculations don’t take mass effects into account, the influence of the $B$-hadron mass must be corrected for and the influence of stable hadron masses should be minimised by using observables without explicit mass dependence.

Compared to the standard method ($O(\alpha_s^2) + NLLA$ in $log R$-scheme) the RGI method shows a dramatically reduced spread in $\alpha_s$ results. This improved consistency awaits theoretical explanation. RGI can further be used for a measurement of the $\beta$-function from mean thrust with a remarkable precision, excluding recently discussed light gluinos.

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