Prim’s algorithm to model the pipe network at the water supply company

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Abstract. It is the network in the water distribution already optimal, or does it form circuit instead? The circuit on the network will increase production costs which will have a direct impact on the price of the service. In this paper, the optimal model was further analyzed to detect the circuit — the minimum spanning tree as a mathematical model, with vertex end pipe and edge length pipe. Modelling with the Prim’s Algorithm produces a length of pipe distribution which more optimal than a pipe installed in PDAM Tirtanadi Medan. The result shows the required optimally of the distribution network by applying Prim’s Algorithm.

1. Introduction
Perusahaan Daerah Air Minum (PDAM) Tirtanadi Medan is a group of regional companies which provides clean water services [1]. Population growth and level of clean water needs are increasing every year. PDAM must improve the services by adding new pipe network [2]. The problem is investigated whether the network in the water distribution has been optimal, and it consists of the circuit. The network consisting of the circuit hasn’t been optimal, and it can decrease the operational cost. In this research the pipe network in PDAM Tirtanadi will be drawn as a connected weighted graph G where the end of the pipe is as a vertex and length of pipe is as the edge. Based on [3] described how graph theory to a network problem.

On the graph G, a circuit known as a cycle. A cycle is a path with a sequence of n edges e1, ..., en where c1 = (v0, v1) and vn = v0. Graph G (V, E) through some iterations will be formed as a spanning tree T (V1, E1), which V1 = V and E1 ⊆ E. The minimum spanning tree is a spanning tree with the least weight among others spanning tree in graph G [4]. A weighted graph can consist of more than one minimum spanning tree. There are some techniques to generate a minimum spanning tree from weighted graph G. A lot of scientists and mathematicians have learned some graph algorithms.

Some previous study [5] and [6] creates the minimum spanning tree to solve the travelling salesman problem. Meanwhile [7] improves local access network in rural areas and compares the solution obtained before and after using the Prim’s Algorithm. The objective function value using Prim Algorithm is found to be more optimal than without applying the algorithm as reported in [7]. Furthermore, [8] creates set A minimum spanning tree of G to minimize the installation cost of LAN university by using the algorithm as the greedy method. The application of Prim’s Algorithm can be valuable to optimize the greedy problem [9].

In this work, Prim’s Algorithm will be used to model the pipe network where the pipe distribution network of PDAM Tirtanadi Medan is a very complex and containing so many vertices and edges as a...
greedy problem. Therefore, the area of Letda Sujono from the Tuasan branch was taken as a sample. This is an optimization problem by minimizes weight. The solution will be constructed in sequences of iteration and must be feasible, and it cannot be replaced to effectively searching the shortest path in the graph [10]. This paper is organized into four parts, the main idea on the Prim Algorithm is described in Section 2. Next, Section 3 presents the results obtained by application of Prim’s Algorithm and offers some discussion of the water pipeline network problem. Lastly, Section 4 deals with the conclusion.

2. Prim’s Algorithm
Prim’s Algorithm constructed in 1957 by Robert Clay Prim [11]. The Prim’s Algorithm produces a minimum spanning tree in a weighted graph. The main minimum spanning problem is modelled on a graph [12]. The weight of T is the sum of all $w(e_i)$ for some integer $i$ with $1 \leq i < n-1$. Spanning tree $T$ of a connected weighted graph $G$ is designed as follows. First, the first edge $e_1$ of $T$, an edge of minimum weight consisting with $u$, is selected from an arbitrary vertex $u$ for $G$. For subsequent edges $e_2, e_3,...,e_n$, then select an edge of minimum weight among those edges having which one of its vertices incident with an edge already selected [13].

Prim’s Algorithm modification chooses only minimum weight edges. From forest $G$ formed, selected root vertex with minimum edge weight. Choose a starting location to get the shortest path distance, the Prim’s Algorithm will get a minimum spanning tree which not forming a cycle in the tree for an undirected connected weighted graph $G(V, E)$ as [10] did. Repeat the process until n-1 edges have been added.

Minimum spanning tree modified Prim ($G, w, r$) based on [7 - 9] illustrated with the pseudo-code of the algorithm is as follows.

The set $T$ as the minimum spanning tree $T$ for $G$ is $T = \{ (v, \Pi[v]) : v \in V \setminus \{r\} \}$

MST-PRIM ($G, w, r$); $G$ is an undirected graph, $w$ is the weight of edges and $r$ is root node.

1. $n \leftarrow V[G]$
2. For each $u \in V[G]$
3. do key $[\text{Min } u] \leftarrow \infty$
4. set $[r] \leftarrow 0$
5. $\Pi[r] \leftarrow \text{Nil}$
6. do $u \leftarrow \text{Extract}_\text{Min}(L)$
7. while $n \neq \emptyset$
8. for each $v \in \text{Adj}[u]$
9. do if $v \in L$ and $T_w(u,v) < \text{min } [v]$
10. then $\Pi[v] \leftarrow u$
11. set $\text{min } [v] \leftarrow T_w(u,v)$
12. endif
13. endfor

Iteration of an algorithm MST-PRIM to build a minimum spanning tree of a connected weighted graph $G$ can be illustrated as described in Figure 1. Iterations start from (a) to (b) then to (c) and finally construct (d) as the minimum spanning tree of graph $G$.
3. Results and Discussion

3.1. Problem Pipe Water Distribution Defined

The primary focus of this work is to construct an efficient algorithmic implementation as a model of the problem with a brief description of the algorithm. The model of a minimum length path must be determined and can be formulated as \( \text{Min } L(T) = \sum_{e \in E_T} L(e) \). \( T = (VT, ET) \) of \( G \), and \( VT \subseteq V \), \( ET \subseteq E \), for \( G = (V, E, P, L) \) minimized the objective function.

The network set is a vertex as the end of a pipe water connection and long pipe as an edge in meters as described in the introduction. Sample data are taken from the branch of PDAM Tirtanadi Medan in Tuasan Branch. The number of length of pipe water distribution installed in the branches of Tuasan is 118.712 meters, 216 vertices and 322 edges. The Tuasan branch is divided into three regions, Pancing area, Krakatau area, and Letda Sujono area.

The map of the pipe water distribution network is constructed as a weighted and non-directed connected graph. The area of Letda Sujono with a total length of 40,507 meters as weight, 94 vertices and 130 edges (See Figure 2). It can be seen that the network hasn’t been optimal and containing circuits. All networks are closed with cycles except only for vertex \( V_5 \), \( V_6 \), and \( V_{15} \). Such networks are not optimal and will increase operational costs.

Figure 1. Constructing a minimum spanning tree (d) from graph \( G \) by Prim’s Algorithm
Figure 2. The pipe water distribution of PDAM Tirtanadi Medan in Letda Sujono area
3.2. Application of Prim’s Algorithm
The length of pipe installation was measured to minimize the objective function. Assume that the diameter of the pipe all the same, and weights in meter is a length of pipe on each junction of the pipe. A length of pipe water distribution installed in a distribution pipe doesn’t reach to the residents’ houses directly. However, it only reaches the front road of residents’ houses.

Starting the iteration for the whole Tuasan Branch, V is chosen as root vertex, and the end shall be at V75, until n-1 edges have been added in the 215th iteration. A sample of the result iteration using MST-PRIM shows in Table 1. The iterations 1st to 4th, 65th to 73th, 106th to 161th, and 211th are Pancing areas, while the iterations 5th to 64th, and 74th to 105th are Letda Sujono's area, and the iterations 162th to 210th, 212th to 215th are Krakatau area.

| Iteration | w (u,v)       | Weight (Meter) |
|-----------|---------------|----------------|
| 1         | (V1, V2)      | 215            |
| 2         | (V2, V3)      | 270            |
| 10        | (V165, V166)  | 224            |
| 20        | (V202, V201)  | 514            |
| 30        | (V205, V175)  | 221            |
| 40        | (V158, V159)  | 245            |
| 50        | (V126, V154)  | 110            |
| 100       | (V181, V181)  | 313            |
| 150       | (V62, V63)    | 210            |
| 200       | (V121, V126)  | 313            |
| 215       | (V73, V74)    | 515            |
| Sum       |               | 53,794         |

When MST-PRIM algorithm completes, the entire length of the required pipe will be minimized. Figure 3 shows the result after application of Prim’s Algorithm to model the existing network at Letda Sujono Area. In the 5th iteration selected area of Letda Sujono with V5 as the root vertex which has the minimum weight. Furthermore, from v5 selected v124 which has a minimum weight of 65. The iteration repeats until it ends at 105th iteration which results a minimum spanning tree.
Figure 3. The model of water distribution network at Letda Sujono area using Prim’s Algorithm
The water modelling distribution network is seen in Figure 3. In the next research to get the more optimal solution, we can add more varying values such as time, the size of the pipe, the water debit, and area condition — the parameters are not real number as stochastic or fuzzy minimum spanning tree problem.

4. Conclusion
The model of water distribution network using a Prim’s Algorithm obtained 53,794 meters the length of pipe distribution, 216 vertices and 215 edges. The result is more optimal than it doesn’t use a Prim’s Algorithm. The same iteration process can be done for Pancing area and Krakatatau area. The entire area in the Tuasan branch can be done in a unified iteration process. Thus Figure 3 can be a model to be applied to all branches of PDAM Tirtanadi Medan with the same iterative process stage.

This paper can be recommended as a guide to explore another pipe water distribution in North Sumatera to effective distribution in improving service to society. The Prim’s Algorithm method can also be explored by other organizations based on the company’s goal to reduce cost and effectiveness their resources especially electrical installation and telecommunication company.

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References
[1] Naibaho Y 2016 PDAM Tirtanadi harus layani seluruh warga Medan (Medan: Medan Bisnis Daily)
[2] Rahayu L 2018 Anggota DPRD Medan minta Penko dan PDAM Tirtanadi harus tindaklanjuti keluhan warga (Medan: Tribun News)
[3] Bassey U N, Bassey K N and Bassey E N 2013 Synthesis and performance analysis of network topology using graph theory The IISTE Network and Complex Systems 3 45
[4] Taha H A 2006 Operations Research: An Introduction (India: Prentice-Hall)
[5] Favaretto D, Moretti E and Pellegrini P 2006 An ant colony system approach for variants of the travelling salesman problem with time windows J. of Info. and Op. Sci. 27 35
[6] Pallavi P L and Lakshmi R A 2015 A mat lab oriented approch to solve the transportation problem Int. J. of Adv. Res. Found. 50 6000
[7] Arogundade O T, Sobowale B and Akinwale A T 2011 Prim algorithm approach to improving local access network in rural areas Int. J. of Comp. Theory and Eng. 3 413
[8] Gitonga C K 2015 Prims algorithm and its application in the design of university LAN networks Int. J. of Adv. Res. in Comp. Sci. and Manag. Stud. 3 131
[9] Chu C H, Premkumar G and Chou H 2000 Digital data networks design using genetic algorithms Euro. J. of Op. Res. 127 140
[10] Kalpanadave D 2013 Effective searching shortest path in graph using Prim’s Algorithm Int. J. of Comp. & Org. Trends 3 310
[11] Prim R C 1957 Shortest connection and some generalizations Bell Syst Tech J 36
[12] Dei A and Pal A 2016 Prim’s Algorithm for solving minimum spanning tree problem in fuzzy environment Annals of Fuzzy Math. and Inf. 12 419
[13] Chartrand G and Zhang P 2005 Introduction to Graph Theory (New York: McGraw-Hill Int. Edition) pp 94-9