Dynamic numerical simulation of pre-cracked concrete samples under different mechanical parameters

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Abstract. Numerical simulations of pre-cracked concrete subjected to dynamic impact compressive stress wave are carried out via DRFPA (Dynamic Realistic Failure Process Analysis). This code is validated firstly by comparing experimental results. Then, the initiation, propagation and coalescence of wing cracks and secondary cracks are numerically investigated by varying elastic modulus, uniaxial compressive strength, and homogeneity index of some phases (bond and aggregate) of concrete. The numerically simulated concrete failure mode (tension/shear failure) and its development in both quasi-coplanar plane and concrete bridge are analyzed and discussed. Numerical results show that the tension failure occurs firstly whereas shear failure occurs shortly thereafter with continuous dynamic loadings and its reflected wave. Meanwhile, the evolution of wing cracks and secondary cracks are numerically investigated. In addition, the effect of elastic modulus, uniaxial compressive strength, and homogeneity index of concrete phases on the crack evolutions are also studied.

1. Introduction

Cracks generally occur in many structures such as rock [1,2], masonry [3] and concrete [4], which can cause the instability or failure of these structures. Many researchers have studied the two parallel pre-existing cracks or other single/multi-cracks problems to understand the failure mechanism and predict stability of structure/material by investigating the initiation, propagation, and coalescence of cracks [5-7].

In terms of the pre-cracked concrete model, two sorts of cracks are significantly studied under uniaxial compressive stress. Such cracks are called wing cracks and secondary cracks, respectively [6]. In addition, secondary cracks could be divided into quasi-coplanar secondary cracks and oblique second cracks. Many papers have reported that both wing cracks and secondary cracks were more likely to initiate firstly from tips of pre-cracks [4,6]. In particular, wing cracks initiated from tips of flaw (pre-existing cracks) when specimens were under uniaxial compressive stress at the very beginning. Generally, the characteristic of this kind of cracks is that it will propagate along the direction of uniaxial compressive stress. Furthermore, the secondary crack is caused by shearing. Its propagation is steady from tips of flaw to opposite direction of wing cracks (i.e., oblique second crack) or along the direction of pre-existing crack (i.e., quasi-coplanar secondary crack) [5]. Haeri indicated that wing cracks would be generated in the first stage of loadings (tension stage) and continued to propagate in the direction of the shear loadings [4]. The crack propagation in bridge (the area between two pre-existing cracks) was mainly affected by the angles of dip and crack length, and the shear strength was closely related to the failure mode. In addition, according to previous study, it was found that vertical pre-crack could greatly
increase the ultimate shear capacity due to the pre-crack preventing the propagation of diagonal cracks [8]. However, such studies related to pre-cracked concrete under external loadings were few involved previously, especially for specimens under dynamic loadings.

Furthermore, studies on strain rate of concrete have been carried out for decades. Ross et al. studied the effect of strain rate related to both tensile and compressive aspects through experimental and numerical approaches [9]. A comprehensive finite element method was used, and it was found that both tensile and compressive strength increase with strain rate. Meanwhile, such strength could increase dramatically when strain rate was beyond its critical value. According to this study, Cusatis extended it to a mesoscopic model to study the effect of loading rate on concrete strength and fracture behavior via constrained shear lattice method [10]. However, there were also few studies on strain rate based on pre-cracked concrete under dynamic loadings.

In addition, some numerical methods related to pre-cracked concrete were proposed in previous papers. Vecchio and Collins proposed a theory in modified compression field [11]. Softened truss model theory was developed by Hsu [12]. Ozbolt and Bazant considerably evolved the nonlocal micro-plane model based on their own previous works [13,14]. However, some of above methods could not apply multiple cracks into specimens. Meanwhile, the function of model setup was also defective for some models to assign different mechanical parameters of some concrete phases, such as aggregate and bond phases.

Therefore, in the current study, a new software called DRFPA is applied to make up above mentioned shortcomings [15]. The tensile/shear failure model can be numerically simulated and be analyzed. The evolutions of wing cracks and secondary cracks are also investigated. In addition, the influence of elastic modulus, uniaxial compressive strength, and homogeneity index of concrete phases (bond and aggregate) are discussed. The structure of this paper is: after the introduction, the basic mechanism of DRFPA is introduced in Section 2; In Section 3, the numerical model is verified by experimental results. Section 4 presents the numerical model setup. The numerical results and analysis are given in Section 5. The last section is the conclusion.

2. Brief introduction of DRFPA

DRFPA is an abbreviation of Dynamic Realistic Failure Process Analysis. Based on finite element theory and statistical damage theory, current software can simulate progressive fracture and instability of quasi-brittle rock-like materials under either dynamic or static loadings as a numerical regime at a mesoscopic level. In contrast to experimental specimens and other numerical models, the inhomogeneity of material properties via current software can be taken into account to simulate the material nonlinearity, and a continuum mechanics method is applied to solve the mechanical problems of discontinuous media. The mechanical properties (e.g., elastic modulus, strength, and Poisson’s ratio) of matrix, bond and aggregates related to concrete specimens are assumed to follow the Weibull distribution. It is defined as the following probability density function [16]:

\[
f(u) = \frac{m}{\omega_0} \left( \frac{u}{\omega_0} \right)^{m-1} \exp \left[ - \left( \frac{u}{\omega_0} \right)^m \right]
\]  

(1)

Where, \( u \) is a given mechanical property, \( \omega_0 \) is a scale parameter, \( m \) is another parameter associated with shape which can define the shape of distribution function. Specifically, \( m \) is referred to homogeneity index, the assigned material will become more homogeneous with such index increases.

2.1. Constitutive law of DRFPA

According to constitutive relationship of the elements, the design of the DRFPA must consider elastic damage mechanics firstly. It has been generally accepted that elastic modulus of an element may decrease as the damage progresses. The elastic modulus of damaged material can be expressed as below [16]:

\[
E = \begin{cases} 
(1 - D^+)E_0 & \text{for tensile damage} \\
(1 - D^-)E_0 & \text{for shear damage} 
\end{cases}
\]  

(2)

Where, \( D^+ \) and \( D^- \) are tensile and shear damage, respectively. \( E \) and \( E_0 \) illustrate the damaged and
undamaged materials related to elastic modulus, respectively.

More importantly, it is assumed that damage of element is isotropic elasticity. Therefore, $E$, $E_0$ and $D$ are scalars. Meanwhile, at the outset of simulation, no initial damage is contained, and the stress-strain curve is linear elastic without damage occurring in this model, i.e., $D^*$ (or $D$) = 0.

According to previous works, the constitutive relationship associated with meso-level elements under uniaxial tension can be expressed as below when considering damage evolution of element itself [17]:

$$ D = \begin{cases} 
0 & \varepsilon < \varepsilon_{t0} \\
1 - \frac{\lambda \varepsilon_0}{\varepsilon} & \varepsilon_{t0} < \varepsilon < \varepsilon_{tu} \\
1 & \varepsilon \geq \varepsilon_{tu}
\end{cases} \tag{3} $$

Where, $\lambda$ is residual strength coefficient defined by $f_0 = \lambda f_{00}$, $f_0$ is residual tensile strength. $f_{00}$ is uniaxial tensile strength, $\varepsilon_0$ is threshold strain which represents that the strain is at elastic limit. Meanwhile, ultimate tensile strain is defined as $\varepsilon_{m0} = \eta \varepsilon_0$, $\eta$ is coefficient of ultimate strain, $\varepsilon_{m0}$ is ultimate tensile strain of elements, that is, the elements will be damaged when tensile strain of elements reaches the ultimate value.

It should be noted that the above-mentioned constitutive relationship is based on the element under uniaxial tensile stress which is one dimensional. According to the method proposed by Mazars and Pijaudier-Cabot, one dimensional damage constitutive relationship can be extended to three dimensional when the elements satisfy the maximum tensile strain criterion to produce tensile damage [18]. Thus, when it is under three-dimensional condition, the equivalent strain can be defined as Equation (4). The $\varepsilon$ in Equation (3) is replaced with the equivalent strain $\bar{\varepsilon}$:

$$ \bar{\varepsilon} = \sqrt{\left(\varepsilon_1\right)^2 + \left(\varepsilon_2\right)^2 + \left(\varepsilon_3\right)^2} \tag{4} $$

Here, $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ are the main strains, respectively, $< >$ is a function which is defined as follows:

$$ \langle x \rangle = \begin{cases} 
x & x \geq 0 \\
0 & x < 0
\end{cases} \tag{5} $$

It should be noted that above descriptions only cover the elastic damage constitutive relationship when meso-level elements produce tensile damage. In fact, the softening and damage of elements also exist under uniaxial compression or shear stress. However, there were few studies involving in this area so far. Therefore, in current paper, it is assumed that there is only microscopic shear damage to reflect the damage of meso-level elements under compression and shear stress. Mohr-Coulomb criterion is used to be the second damage threshold criterion:

$$ F = \frac{1+\sin \phi}{1-\sin \phi} \sigma_1 - \sigma_3 \geq f_{c0} \tag{6} $$

Where, $\phi$ is the internal friction angle of meso-level elements and $f_{c0}$ is the uniaxial compressive strength of meso-level elements as a positive number. $\sigma_1$ and $\sigma_3$ are maximum and minimum principal stress of meso-level elements, respectively.

Similarly, according to Mohr-Coulomb criterion, another constitutive relationship curve related to uniaxial compressive stress can be expressed to describe the damage variable [17]:

$$ D = \begin{cases} 
0 & \varepsilon > \varepsilon_{c0} \\
1 - \frac{\lambda \varepsilon_0}{\varepsilon} & \varepsilon \leq \varepsilon_{c0}
\end{cases} \tag{7} $$

Where, $\lambda$ is the residual strength coefficient of elements. Meanwhile, its value is assumed to be consistent with that under uniaxial tension, i.e., $f_{c0} = f_{00} - \lambda f_{00} = \lambda$ is satisfied. $\varepsilon_{c0}$ is maximum compressive principal strain when maximum compressive principal stress reaches its uniaxial compressive strength.

Meanwhile, it satisfies the Mohr-Coulomb criterion when elements are under multi-axial stress state. Thus, maximum compressive principal strain $\varepsilon_{c0}$ of elements can be expressed as follows:

$$ \varepsilon_{c0} = \frac{1}{\varepsilon_0} \left[ -f_{c0} + \frac{1+\sin \phi}{1-\sin \phi} \sigma_1 - \mu (\sigma_1 + \sigma_2) \right] \tag{8} $$

Where, $\mu$ is Poisson’s ratio.

The maximum compressive principal strain $\varepsilon_3$ can replace the uniaxial compressive strain of Equation (7) when elements are under multi-axial stress state and satisfy Mohr-Coulomb criterion. Thus,
one dimensional condition can be extended to three dimensional and the expression of damage variable $D$ is shown in below:

$$D = \begin{cases} 
0 & \varepsilon_3 > \varepsilon_{c0} \\
1 - \frac{\Delta \varepsilon_0}{\varepsilon_3} & \varepsilon_3 \leq \varepsilon_{c0}
\end{cases}$$ (9)

In addition, the constitutive relationship of elements is expressed in elastic damage mechanics. That is, when elements are unloaded or reloaded, no damage will occur. Such elastic modulus of material can be constant, and elements will keep residual deformation when all elements are unloaded (all stresses are released). Although the constitutive relationship in current paper is only a constitutive relationship of elastic damage, elements are constantly damaged, and material properties of elements are continuously weakened. Meanwhile, when an element fails in a loading step, it is necessary to repeat the calculation under the condition of constant external load until there is no element damage under load condition (step-in-step function), which is an iterative process.

2.2. Strain-Rate damage threshold
According to Mohr-Coulomb criterion, such criterion can be also applied to dynamic loading condition when increment of cohesion with strain rate is counted [19]. The relationship between dynamic uniaxial compressive strength and loading rate can be expressed as a semi-log formula:

$$f_{cd} = A \log \left( \frac{f_{cd}}{f_{c0}} \right) + f_{c0}$$ (10)

Where, $f_{cd}$ is dynamic uniaxial compressive strength (MPa), $f_{cd}$ is dynamic loading rate (MPa/s), $f_{c0}$ is quasi-brittle loading rate which approximately equals $5 \times 10^2$ MPa/s, $f_{c0}$ is uniaxial compressive strength at quasi-static loading rate. $A$ is a parameter related to strain rate which can influence dynamic strength. The values of $A$ based on different materials are detailed described in previous paper [20].

2.3. Finite element implementation
According to previous paper, the equilibrium equation governs the linear dynamic response of the finite element system can be expressed [21]:

$$M \ddot{U} + C \dot{U} + KU = R$$ (11)

Where, $M$, $C$ and $K$ are mass, damping and stiffness, respectively, $R$ is vector of externally applied loads or unbalanced force. $U$, $\dot{U}$ and $\ddot{U}$ are nodal displacement, velocity, and acceleration vectors, respectively. For the description of some other assumptions, such as Rayleigh damping, Wilson $\theta$ method of implicit time integration has been discussed in detail in the previous paper [20].

In terms of this dynamic response problem, the maximum time step is related to the wave speed in the material and the size of the finite element. The maximum time step is selected so that the stress wave cannot propagate beyond the distance between the integration points of the elements in the time increment.

2.4. Acoustic Emission (AE) principle
It is necessary to clarify the principle of acoustic emission (AE) since it is a crucial approach to capture the energy release during the development of concrete failure processes [22]. In current software, it is assumed that the AE events will occur when element fails owing to stored elastic energy in element must be released when deformed. Based on it, AE activities can be simulated and therefore the accumulative damage $D$ can be calculated as follows [23]:

$$D = \frac{1}{N} \sum_{i=1}^{N} n_i$$ (12)

Where $s$ is the number of calculated steps, $n_i$ is the damaged elements in the $n$th step, $N$ is the total elements of simulated model. The released energy can be obtained as follows when element fails [24]:

$$W_i = \frac{1}{2E} (\sigma_1^2 + \sigma_2^2 - 2\nu \sigma_1 \sigma_2) V$$ (13)

Where $i$ is the number of elements, $W_i$ is released elastic strain energy, $E$ is elastic modulus, $\sigma_1$ and $\sigma_2$ are maximum and minimum principal stress, respectively. $\nu$ is Poisson ratio; $V$ is the element volume [24].
3. Verification of numerical model
In this section, the numerical model is verified by comparing the numerically simulated results with the corresponding experimental results. The mechanical parameters of experimental specimen are shown in Table 1 [25]. The dimension of this specimen is both 100mm in length and height. Moreover, it is simulated the numerical specimen with consistent dimension and mechanical parameters via DRFPA.

Table 1. Mechanical parameters of concrete specimen under uniaxial compression [25].

| Phase     | Elastic modulus (GPa) | Uniaxial compressive strength (MPa) | Homogeneity index m |
|-----------|-----------------------|------------------------------------|---------------------|
| Matrix    | 28.6                  | 175                                | 3                   |
| Bond      | 15                    | 150                                | 1.5                 |
| Aggregate | 80                    | 500                                | 6                   |

Figure 1 shows the crack distribution and propagation patterns of both experimental and numerical results, respectively. It indicates that the numerically simulated failure mode of concrete specimen is in agreement with the experimental results. Nevertheless, the cracks evolutions are not exactly the same, because it is difficult to determine the heterogeneity of concrete three phases (matrix, bond, and aggregate) in the experimental model. In the following section, a series of numerical simulations will be conducted to study the failure mode of concrete cracks, evolutions of wing cracks as well as secondary cracks and influence of mechanical parameters (elastic modulus, uniaxial compressive strength, and homogeneity index) of concrete three phases.

![Figure 1](image1.png)

4. Numerical model setup
In these numerical simulations, thirteen specimens are considered. First of all, the dimension of all specimens is 200mm high and 100mm wide with a thickness of 1mm. It is set two parallel pre-cracks with a width of 2mm and a length of 30mm in all specimens. The α and β of two parallel cracks are 45° and 90°, respectively. Thus, the ligament length is fixed to 40mm. Secondly, all concrete specimens include three phases, they are matrix, aggregate, and bond. Detailed specimen model is shown in Figure 2. Where, matrix is the cementitious material of concrete, it constitutes the basic part of concrete, whereas aggregates are composite materials added to matrix, which can be varied according to different function. Bond can be considered as the adhesive materials that connects aggregates and matrix. Thirdly, such phases are assigned different values in terms of elastic modulus, uniaxial compressive strength, and homogeneity index, respectively. Specimen I is a reference term whose data comes from previous paper [17]. Mechanical parameters of from Specimen II to Specimen XIII vary in proportion to that of Specimen I, as shown in Table 2.
In addition, it is necessary to clarify some basic assumptions and parameters contained in both numerical codes and specimens before simulation through DRFPA. First of all, above mentioned mechanical parameters all follow Weibull distribution, and the element constitutive parameters should also be covered such as softening scheme of the constitutive curve, residual strength coefficient, ultimate tensile strain coefficient and friction angle. It is considered that the compressive strength and tensile strength of elements are to be closely related. Therefore, both compressive and tensile strength will follow the same distribution in current paper. Secondly, the value of the tensile strength will be obtained from multiplying the compressive strength by the coefficient of tensile strength. Thirdly, the scale of the element will also have a certain effect on the calculation results. In order to facilitate the discussion of the influence of other important parameters, it is assumed that the elastic modulus and uniaxial compressive strength of material will meet the Weibull distribution of the same homogeneity in the subsequent calculations. Fourthly, the tensile strength ratio is 10, and the internal friction angle is fixed at 30°. Fifthly, the dispersion of Poisson's ratio is small and therefore its homogeneity and mean value are assumed to be 100 and 0.2, respectively. Furthermore, aggregates distribution and bond thickness are set randomly through all specimens. Finally, according to previous paper, a dynamic impact compressive stress wave is applied to all specimens to achieve dynamic simulation and the detailed frequency and magnitude of such wave is shown in Figure 3 [20]. It should be noted that the total step

| Specimen | Phase     | Elastic Modulus (GPa) | Uniaxial Compressive Strength (MPa) | Homogeneity Index m |
|----------|-----------|----------------------|-------------------------------------|---------------------|
| I        | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 80.00                | 500                                 | 6.0                 |
|          | Bond      | 11.44                | 70                                  | 3.0                 |
| II       | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 80.00                | 500                                 | 6.0                 |
|          | Bond      | 16.38                | 70                                  | 3.0                 |
| III      | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 80.00                | 500                                 | 6.0                 |
|          | Bond      | 24.57                | 70                                  | 3.0                 |
| IV       | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 80.00                | 500                                 | 6.0                 |
|          | Bond      | 11.44                | 99                                  | 3.0                 |
| V        | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 80.00                | 500                                 | 6.0                 |
|          | Bond      | 11.44                | 148.5                               | 3.0                 |
| VI       | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 80.00                | 500                                 | 6.0                 |
|          | Bond      | 11.44                | 60                                  | 1.5                 |
| VII      | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 80.00                | 500                                 | 6.0                 |
|          | Bond      | 11.44                | 70                                  | 6.0                 |
| VIII     | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 30.00                | 500                                 | 6.0                 |
|          | Bond      | 11.44                | 70                                  | 3.0                 |
| IX       | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 13.00                | 500                                 | 6.0                 |
|          | Bond      | 11.44                | 70                                  | 3.0                 |
| X        | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 8.000                | 330                                 | 6.0                 |
|          | Bond      | 11.44                | 70                                  | 3.0                 |
| XI       | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 80.00                | 83                                  | 6.0                 |
|          | Bond      | 11.44                | 70                                  | 3.0                 |
| XII      | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 80.00                | 500                                 | 3.0                 |
|          | Bond      | 11.44                | 70                                  | 3.0                 |
| XIII     | Matrix    | 27.30                | 165                                 | 3.0                 |
|          | Aggregate | 80.00                | 500                                 | 1.5                 |
|          | Bond      | 11.44                | 70                                  | 3.0                 |
of dynamic numerical model is 1500, in other word, it is 150 microseconds. Step-in-step function is used to identify the difference between tension and shear failure.

5. Numerical results and analysis

Table 3 and Table 4 illustrate the shear stress distribution patterns of all specimens under dynamic impact compressive stress waves. The bright spots in figures indicate the location of shear stress concentration whereas the dark patterns are cracks. In addition, Table 5 and Table 6 show the acoustic emission (AE) data captured by software which can be utilized to assist analysis of released energy of those specimens subjected to dynamic loadings to study the relationship between cracks and energy release.

5.1. Wing cracks

5.1.1. Initiation and propagation of external tip wing cracks.

It is found that near the external tips of pre-existing cracks, there are some micro-cracks initiating firstly when dynamic impact compressive stress wave passes through such areas. Then, the capacity of concrete shear-resistant significantly weakened, those micro-cracks then form wing cracks. It is in agreement with the previous findings which reported that similar weakened cut-resistant capacity occurred in rock bridge area when expansion of wing cracks forming at the pre-cracks in compression [6]. In terms of wing cracks initiated from external tip of upper pre-existing cracks, they propagate along the direction perpendicular to the pre-cracks at the very beginning. With the constantly dynamic compressive loadings subjecting to the specimens, such propagation transforms to develop parallel to the direction of loadings. This phenomenon is similar as the findings reported in Haeri’s paper which considered the direction of wing cracks propagation is along the shear loadings [4]. When the fracture of the specimens caused by the wing cracks reaches a considerable level, the wing cracks will propagate along the most fragile phase of specimen (bond), forming transverse fracture trajectories above the upper pre-cracks, and has a tendency to penetrate the entire cross section of the specimen, as shown in Table 3(a)(c) and Table 4(d)(f)(g).

In addition, for wing cracks initiated from external tip of lower pre-cracks, the propagation of it is straightly along the direction parallel to the dynamic loadings. Unlike upper external wing cracks, it is not found that lower external wing cracks produce fracture trajectories across the specimens. Meanwhile, all specimens indicate that fracture damage of upper wing cracks is more serious than that of lower ones, as shown in Table 3 and Table 4.

![Figure 2. Specimen model.](image2)

![Figure 3. Applied dynamic impact compressive stress wave](image3)

In general, the wing cracks are mainly concentrated on both upper and lower left corner of specimens, a reasonable explanation is that the geometry of pre-cracks can induce the propagation of wing cracks. Pimanmas and Maekawa indicated that the shear behavior could be significantly affected by the width
and inclination of pre-cracks in terms of reinforced concrete structure [26]. Moreover, it is well accepted that the shear behavior has the dominant influence on reinforced/non-reinforced concrete performance [27, 28]. However, in term of the concrete structure, it still needs to be proved in subsequent studies by varying the inclination of pre-cracks.

5.1.2. Coalescence of internal tip wing cracks.
Wing cracks initiated from internal tip of pre-cracks propagate perpendicular to the direction of dynamic compressive stress waves. The micro-cracks firstly form around the internal tip of upper pre-cracks when dynamic waves reaching this area. After that, micro-cracks initiate around the internal tip of lower pre-cracks as well when upper micro-cracks transform into wing cracks. After such formation related to both upper and lower micro-cracks completes, the coalescence of wing cracks within concrete bridge area occurs and causes serious failure to bond phase in this area, as shown in Table 3(b)(c)(f) and Table 4(b)(c)(d)(e).

5.2. Secondary cracks
Secondary cracks involve oblique second cracks and quasi-coplanar secondary cracks. Overall, the evolutions of secondary cracks from upper pre-cracks are much more significant than that from lower pre-cracks.

5.2.1. Initiation and propagation of secondary cracks related to upper pre-cracks.
Compared to the initiation of wing cracks from tips of pre-cracks, the quasi-coplanar secondary cracks not initiate and propagate obviously from both internal and external tips of upper pre-cracks under dynamic compressive stress waves except Specimen IV, Specimen VII and Specimen IX (Table 3(d)(g) and Table 4(e)). On the contrary, oblique second cracks obviously initiate and propagate perpendicular to the direction of upper pre-cracks along the bond phase. Such second cracks from internal tips form “crack channels” with those wing cracks initiated from external tips of upper pre-cracks, as shown in Table 3(b)(c)(f) and Table 4(b)(c)(d)(e)(f)(g). In terms of those oblique second cracks from external tips of upper pre-cracks, they form serious fracture/damage along the bond phase below upper pre-cracks, as shown in Table 3(a)(b)(c) and Table 4(a)(b)(c)(d)(e)(f)(g).

5.2.2. Initiation and propagation of secondary cracks related to lower pre-cracks.
It is not found obvious secondary cracks initiating and propagating from neither internal nor external tips of lower pre-cracks. Only a few micro quasi-coplanar secondary cracks can be observed that initiating at the external tips of lower pre-cracks and steadily propagate along the bond phase, as shown in Table 3(d)(f) and Table 4(g).

5.3. Tension failure and shear failure development.
From Table 3(a)(b)(d)(e)(f)(g) and Table 4(a)(d), it is found that bright spots can be left near the tips of pre-cracks when dynamic compressive stress waves pass through the specimens. Moreover, according to the constitutive relationship of DRFPA, the shear strength of concrete is set to be ten times its tensile strength. That is, tension failure will occur earlier than shear failure. Therefore, based on bright spots distribution patterns, tension failure occurs firstly at the tips of pre-cracks when specimens are subjected to dynamic loadings. After that, some micro-cracks steadily coalesce and propagate as dynamic waves reach the edges of specimens forming single or multiple reflected waves, causing more damage/fracture to specimens. At this moment, the original tension failure transforms into shear failure from tips of pre-cracks to entire specimens along the propagation of wing cracks and secondary cracks. Overall, the shear damage plays the dominant role during the entire process of specimen fracture when comparing the tensile damage. This agrees well with the previous findings published by Wang et al [29]. They conducted the study on rock specimens under conventional triaxial compression conditions and summarized the similar analysis. Both rock and concrete are considered as quasi-brittle material and therefore they have similar stress-strain properties and crack evolutions when such specimens subjecting
to external loads.

5.4. Effect of elastic modulus, uniaxial compressive strength and homogeneity index of concrete three phases

First, it is found that the mutative trend of captured AE energy and AE counts is consistent along the time axes. Meanwhile, when there is a leap increment of both AE energy and counts occurring, new cracks will initiate, or existing cracks will coalesce each other at corresponding time axes. Compared those curves and histograms as shown in Table 5 and Table 6 to the shear stress movement patterns(Table 3 and Table 4), when a period of 30µs dynamic wave completely reaches the bottom ed-

| Time (µs) | 30 | 60 | 90 | 120 | 150 |
|----------|----|----|----|-----|-----|
| (a)      | ![Specimen I](image1) | ![Specimen I](image2) | ![Specimen I](image3) | ![Specimen I](image4) | ![Specimen I](image5) |
| (b)      | ![Specimen II](image6) | ![Specimen II](image7) | ![Specimen II](image8) | ![Specimen II](image9) | ![Specimen II](image10) |
| (c)      | ![Specimen III](image11) | ![Specimen III](image12) | ![Specimen III](image13) | ![Specimen III](image14) | ![Specimen III](image15) |
| (d)      | ![Specimen IV](image16) | ![Specimen IV](image17) | ![Specimen IV](image18) | ![Specimen IV](image19) | ![Specimen IV](image20) |
| (e)      | ![Specimen V](image21) | ![Specimen V](image22) | ![Specimen V](image23) | ![Specimen V](image24) | ![Specimen V](image25) |
Specimen VI

Specimen VII

g. Specimen VI

Specimen VII

g. Specimen VII

ge of specimens to form a reflected wave, it is captured the first peak value of AE energy and counts, along with some cracks occurring near the bottom of specimens due to the reflected wave. Compared to previous AE events caused by the formation of wing cracks and secondary cracks, the reflected wave can cause more fracture/damage to specimens. The second reflected wave will occur when the first reflected wave is back to top edge of specimens, forming another peak value of AE event, and so on. It is clearly found that under consistent geometric relationship of pre-cracks, different timelines of AE events to different specimens are captured. That is, the inhibition related to dynamic waves is different due to different mechanical parameters (elastic modulus, uniaxial compressive strength, and homogenei-

Table 4. Shear stress distribution patterns under dynamic waves (variable: aggregate).

| Time (µs) | 30   | 60   | 90   | 120  | 150  |
|-----------|------|------|------|------|------|
| (a) Specimen I |     |      |      |      |      |
| (b) Specimen VIII |    |      |      |      |      |
| (c) Specimen IX |    |      |      |      |      |
| (d) Specimen X |    |      |      |      |      |
ty index) of concrete three phases. Generally, the speed of dynamic waves will decelerate when the uniaxial compressive strength and homogeneity index of bond phase increase, respectively, causing the total number of AE events decreases as well, as shown in Table 5(d)(e)(f)(g). Meanwhile, with the increase of elastic modulus of bond phase, the energy of dynamic waves is absorbed effectively to let less AE events occur caused by reflected waves, resulting in minor damage of specimens, as shown in Table 5(c).

Furthermore, in terms of those mechanical parameters associated with aggregates, the speed of dynamic waves will accelerate with decrease of elastic modulus, uniaxial compressive strength, and homogeneity index, respectively. Simultaneously, the total number of AE events increases. However, when comparing the increased ratio of AE events caused by varying in aggregates’ mechanical parameters to those by bond’s, generally, the most effective way to strengthen the concrete performance is to increase those mechanical parameters of bond. Meanwhile, compared with another two mechanical parameters, it is considered that the uniaxial compressive strength plays a more important role when resisting dynamic loadings.

Table 5. AE energy and counts distribution patterns and their accumulative curves (variable: bond).
(b) Specimen II

(c) Specimen III

(d) Specimen IV

(e) Specimen V

(f) Specimen VI

(g) Specimen VII
Table 6. AE energy and counts distribution patterns and their accumulative curves (variable: aggregate).

| Specimen | (i) AE Energy (J) | (ii) AE Counts |
|----------|-------------------|---------------|
| (a) Specimen I | ![Graph](image1) | ![Graph](image1) |
| (b) Specimen VIII | ![Graph](image2) | ![Graph](image2) |
| (c) Specimen IX | ![Graph](image3) | ![Graph](image3) |
| (d) Specimen X | ![Graph](image4) | ![Graph](image4) |
| (e) Specimen XI | ![Graph](image5) | ![Graph](image5) |
6. Conclusions

Thirteen concrete specimens with two parallel pre-existing cracks were numerically simulated by varying their mechanical parameters via DRFPA. The initiation, propagation, and coalescence of wing cracks as well as secondary cracks were numerically investigated. In addition, it is also clarified the development of tension failure and shear failure. Finally, the influence of mechanical parameters (elastic modulus, uniaxial compressive stress, and homogeneity index of concrete three phases) related to specimens was discussed. Some conclusions are as follows:

1. The crack initiation is mainly concentrated in tips of pre-existing cracks owing to tensile damage occurring such areas. In addition, dynamic compressive stress waves can cause micro-cracks occurring near tips of pre-cracks firstly, whereas reflected waves initiate more cracks located at both edges of specimens to cause serious fracture/damage.

2. The upper external wing cracks propagate perpendicular to the direction of upper pre-cracks firstly and then propagate to the entire upper left corner along the bond phase. The lower external wing cracks straightly propagate along the direction of dynamic loadings. The two internal wing cracks also propagate in the direction of dynamic loadings and then coalesce each other within concrete bridge areas.

3. Tension failure occurs at tips of pre-cracks at the very beginning when specimens are subjected to dynamic loadings. As flawed concrete specimens are under constantly dynamic waves and reflected waves, such tension failure transforms into shear failure.

4. The influence of mechanical parameters on performance of pre-cracked concrete under dynamic waves is obvious. It is a more effective way to strengthen the concrete performance by promoting the mechanical parameters of bond than that of aggregate. Moreover, the uniaxial compressive strength of both bond and aggregates plays a more important role than elastic modulus and homogeneity index when resisting dynamic loadings.
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