Abstract

$T$-parity in the Little Higgs model could be violated by anomalies that allow the lightest $T$-odd $A_H$ to decay into $ZZ$ and $W^+W^-$. We analyze these anomaly induced decays and the two-particle and the three-particle decay modes of other heavy quarks and bosons in this model which yield unique Large Hadron Collider (LHC) signals with fully reconstructable events. $T$-odd quarks in the Little Higgs model are nearly degenerate in mass and they decay by almost identical processes; however, members of the heavy Higgs triplet follow distinct decay modes. The branching fractions of three-body decays increase with the global symmetry-breaking energy scale $f$ and are found to be at the level of a few percent in heavy quark decays while they can reach up to 10% for heavy bosons.
The Higgs mass in the Standard Model (SM) receives large radiative corrections from the short-distance physics at the cutoff scale. Fine-tuning in the Higgs sector becomes an eminent problem, especially when the SM predictions are confronted with precision electroweak data.\[1\] In order to naturally alleviate the quadratic divergent contributions, new particles are expected to exist with TeV scale masses.

The Little Higgs mechanism\[2\] makes use of the light mass property of the pseudo-Nambu–Goldstone boson (pNGB) to protect the Higgs mass from the one-loop quadratic divergence: its mass receives one-loop radiative corrections from the new TeV scale particles, which cancel the corrections from Standard Model fermion and boson loops.

**Little Higgs with T Parity**

One of the simplest implementations of such a mechanism is the Littlest Higgs (LH) model\[3\] based on

\[
G = SU(5) \quad \text{and} \quad G_1 \otimes G_2 = [SU(2)_1 \otimes U(1)_1] \otimes [SU(2)_2 \otimes U(1)_2].
\]

At \(f \sim 1\) TeV the initial \(SU(5)\) global symmetry spontaneously breaks down to an \(SO(5)\) subgroup in the direction

\[
\Sigma_0 = \begin{pmatrix}
\mathbb{1} \\
1 \\
\mathbb{1}
\end{pmatrix}
\]

where \(\mathbb{1}\) is the identity matrix. After symmetry breaking at the energy scale \(f\), the dynamics near \(\Sigma_0\) is described by the non-linear sigma field \(\Sigma = e^{\frac{2i}{f}X_a t_a \Sigma_0}\), where \(t_a\) are the pseudo-Nambu-Goldstone bosons (pNGB) associated with the 14 generators \(X_a\) of the broken symmetry. An \([SU(2) \times U(1)]^2\) subgroup of the \(SU(5)\) is weakly gauged. Gauging each of the two \(SU(2) \times U(1)\) leaves a different \(SU(3)\) subgroup unbroken, i.e. unless both \(SU(2) \times U(1)\) are broken there will be a preserved \(SU(3)\) symmetry and the Higgs field will be an exact massless Nambu-Goldstone field. Thus any loop contribution to Higgs mass must involve couplings from both copies of \(SU(2) \times SU(1)\). At one loop level the leading contribution is only logarithmically divergent under this requirement. This mechanism that protects the Higgs mass from quadratic divergences is often referred as ”collective symmetry breaking”.

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\( \Sigma_0 \) breaks the full gauge group to the diagonal SM electroweak group \( SU(2) \times U(1) \) at energy scale \( f \). Four pNGB’s give TeV scale masses to \( W_{H}^{\pm}, W_{H}^{3}, B_{H} \). \( W_{H}^{3} \) and \( B_{H} \) mix and form mass eigenstates \( A_{H} \) and \( Z_{H} \) in analog to the SM photon and \( Z \) boson. Other pNGB fields group into a doublet identified as the SM Higgs and a weak triplet \( \Phi \)

\[
\Pi = \begin{pmatrix}
0 & \frac{H}{\sqrt{2}} & \phi \\
\frac{H^\dagger}{\sqrt{2}} & 0 & \frac{H^\dagger}{\sqrt{2}} \\
\phi^\dagger & \frac{H^\dagger}{\sqrt{2}} & 0
\end{pmatrix}, \quad \text{with} \quad H = \begin{pmatrix}
-\frac{i}{\sqrt{2}} \pi^+ \\
h + i \pi^0 \frac{1}{\sqrt{2}} \\
-\frac{i}{\sqrt{2}} \pi^0
\end{pmatrix}, \quad \Phi = \begin{pmatrix}
-\frac{i}{\sqrt{2}} \phi^{++} & -\frac{i}{\sqrt{2}} \phi^+ \\
-\frac{i}{\sqrt{2}} \phi^- & -\frac{i}{\sqrt{2}} \phi^0 + i \phi^p
\end{pmatrix},
\]

where \( \pi^+ \) and \( \pi^0 \) in the Higgs doublet are eaten by SM weak bosons. All the new particles are massive. The new TeV scale particles \( W_{H}^{\pm}, Z_{H}, A_{H}, \phi^{\pm}, \phi^{\pm\pm}, \phi^0, \phi^p \) couple to the Higgs field and cancel the quadratic radiative corrections to the Higgs mass arising from their SM counterparts.

However, the tight constraints from precision electroweak data disfavor the LH model at a natural symmetry breaking scale \( f \sim 1 \) TeV. Phenomenological constraints on LH parameters push the lower boundary of new physics up to about 10 TeV, but the naturalness principle sets all dimensionless couplings to \( \sim 1 \) and requires the energy scale to be around 1 TeV. Thus the LH model needs an energy scale higher than the natural value to stay consistent with electroweak results. This tension is often referred to as the ‘little hierarchy’ problem. To address this issue, Cheng and Low proposed that an additional discrete \( T \)-Parity can be imposed to relax the confrontation between theory and experimental constraints.

Similar to the matter parity in supersymmetry, \( T \)-parity is introduced as a global discrete parameter. It exchanges \( [SU(2)_1 \times U(1)_1] \) and \( [SU(2)_2 \times U(1)_2] \). \( \Sigma \) transforms under \( T \)-parity as \( \Sigma \rightarrow \tilde{\Sigma} = \Sigma_0 \Omega \Sigma^\dagger \Omega \Sigma_0 \) with \( \Omega = \text{diag}(1, 1, -1, 1, 1) \).

All SM particles and the LH heavy top quark \( T_+ \) are assigned \( T \)-even in the Little Higgs model with \( T \)-parity (LHT). All other heavy particles are assigned \( T \)-odd. In the fermion sector, each SM fermion is extended into a pair of \( SU(2) \) doublets \( q_1 \) and \( q_2 \) that transform under \( SU(2)_1 \) and \( SU(2)_2 \). \( T \)-parity interchanges \( q_1 \) and \( q_2 \). Their \( T \)-even combination is associated with the SM fermion, while the \( T \)-odd combination is the heavy partner to the SM particle. Interaction terms \( -\kappa f (\bar{\Psi}_2 e^{iH} \Psi_c + \bar{\Psi}_1 \Sigma_0 \Omega e^{-iH} \Omega \Psi_c) + \text{h.c.} \) give mass to \( T \)-odd
fermions

\[ M_{d_\pm} \simeq \sqrt{2}\kappa f, \quad M_{u_\pm} \simeq \sqrt{2}\kappa f \left( 1 - \frac{v_{SM}^2}{8f^2} + \cdots \right), \]  

(2)

where \( v_{SM} = 246 \text{ GeV} \) is the Higgs vacuum expectation value in the SM and \( \kappa \) is a free parameter. For illustration, we take \( \kappa = 1 \) throughout this paper. In the interaction term \( \Psi \) is the SU(2) fermion doublet embedded into the SU(5) multiplet: \( \Psi_1 = (q_1, 0, 0_2)^T \), \( \Psi_2 = (0_2, 0, q_2)^T \) and \( \Psi_c = (q_c, \chi_c, \tilde{q}_c)^T \). Details are given in Ref.[8].

There is a special treatment in the top sector besides the heavy \( T \)-odd weak doublet \( (b_-, t_-) = (u_3, d_3) \) of the third generation. In LH the large top quark loop correction demands an additional vector-like weak singlet \( T_+ \) quark to stabilize the Higgs mass. It is assigned even \( T \)-parity in LHT and its \( T \)-odd partner is introduced as another top-like heavy quark \( T_- \).

In the bosonic sector, the doublet (triplet) Higgs \( H (\Phi) \) is even (odd) under \( T \)-parity. The \( T \)-even combinations of the gauge fields are the SM \( SU(2)_L \) gauge bosons \( (W^a_\mu) \) and \( U(1)_Y \) hypercharge gauge boson \( (B_\mu) \); the \( T \)-odd combinations are \( T \)-parity partners \( (A_H, W^\pm_H, Z_H) \) of the SM gauge bosons. The masses are given as

\[ M_{A_H} = \frac{g'f}{\sqrt{5}} \left[ 1 - \frac{5v_{SM}^2}{8f^2} + \cdots \right], \quad M_{Z_H} \simeq M_{W_H} = gf \left[ 1 - \frac{v_{SM}^2}{8f^2} + \cdots \right]. \]  

(3)

The lightest \( T \)-odd particle is \( A_H \), which could be a dark matter candidate if \( T \)-parity was strictly conserved.

The new \( T \)-odd bosons have masses around a few hundred GeV. The new fermions have higher masses near 1 TeV. A typical mass spectrum of heavy particles in LHT is shown in Table [I].

The addition of \( T \)-parity forbids \( T \)-odd particles from mixing with their SM counterparts and leaves low-energy observables unaffected by heavy particles at tree level. This significantly loosens precision electroweak constraints on the symmetry breaking scale \( f \), permitting \( f \) to be as low as 500 GeV at the expense of a high Higgs mass.[9] For instance, \( f = 1 \text{ TeV} \) requires \( 280 < m_h < 625 \text{ GeV} \).[10]
| Particle | Mass (TeV) | $T$ parity |
|----------|-----------|------------|
| $A_H$    | 0.24      | $-$        |
| $Z_H$ ($W_H$) | 0.97  | $-$        |
| $\phi$  | 1.7       | $-$        |
| $T_-$    | 1.5       | $-$        |
| $u_-, c_-, t_-, d_-, s_-, b_-$ | 2.1 | $-$        |
| $T_+$    | 2.1       | $+$        |
| $e_H^-, \mu_H^-, \tau_H^-, \nu_e H, \nu_{\mu H}, \nu_{\tau H}$ | 2.1 | $-$        |

**TABLE I**: Characteristic masses of the heavy partners of the SM particles. Here we take the scale $f = 1.5$ TeV, $\kappa = 1$, and the top quark and Higgs boson masses to be 175 and 200 GeV, respectively. The dependences of particle masses on $f$ are plotted in Fig. 1.

**$T$ Parity Violation in LHT**

A recent topological study by C. Hill and R. Hill finds that $T$-parity is violated by anomalies \cite{11}, in which case the 4D spacetime is a membrane embedded in a 5D bulk. The Little Higgs (LH) lagrangian is reconstructed from a more general 5D bulk lagrangian. The $T$-parity plays a role similar to that of the KK-mode parity: Symmetric under reflection in the 5th dimension, the zeroth mode of a 5D SU(3) gauge field is assigned $T$-even and identified with the vector field. The first mode of the 5D gauge field transforms antisymmetrically under reflection in the 5th dimension, and is identified with the $T$-odd axial vector field.

In the Little Higgs model with $T$-parity (LHT) pseudo-Nambu-Goldstone bosons introduce anomalous topological interactions at the global symmetry breaking scale $\sim \Lambda = 4\pi f$. Consequently the Wess–Zumino–Witten (WZW) term \cite{12} that contains these topological effects must be included into the full LHT Lagrangian and is essential for the UV completion of the theory. Ref. \cite{11} showed that $T$-parity is generally violated by anomaly; therefore the WZW term violates $T$-parity as well.

The leading order anomaly terms containing $B_H W \partial W$ and $B_H B \partial B$ cancel in the sum of WZW terms, and the remaining $T$-parity-violating terms have the forms $H^\dagger H B H W \partial W$ and
FIG. 1: Masses dependency on symmetry breaking scale $f$ with $m_H=200$ GeV. Note that most heavy fermions are very degenerate in mass. In this figure $f_H$ denotes the heavy fermions in the LHT model except for $T_-$, and $\phi$ denotes the heavy Higgs fields.

$H^\dagger H B_H B \partial B$.\cite{11} The $B_H$ field is the $T$-odd partner of the SM axial $B$ field with parameters

$$m_{B_H} \simeq g' f / \sqrt{5}, \quad \tilde{g} = g' / \sqrt{5}.$$ \hspace{1cm} (4)

The WZW term allows the $T$-odd $B_H$ field to couple to $T$-even SM gauge fields. The leading relevant interaction is

$$\mathcal{L}_{WZW} \supset - \frac{K \tilde{g} g^2 N_{WZ} v_{SM}^2}{48 \sqrt{3} \pi^2 f^2} \epsilon^{\mu\nu\rho\sigma} B_{H\mu} \left[ \sec^2 \theta_W \partial_\nu Z_\rho \partial_\sigma Z_\mu + (D^A_{\nu} W^+_{\rho}) W^-_\sigma + (D^A_{\nu} W^-_{\rho}) W^+_\sigma \right]$$ \hspace{1cm} (5)

where $\theta_W$ is the electroweak mixing angle. $K$ is an overall factor for the littlest $SU(5)/SO(5)$ model. The WZW quantized integer $N_{WZ}$ is taken to be 3. The leading anomaly induced decays of $B_H$ in the LHT model are $B_H \rightarrow ZZ$ and $B_H \rightarrow W^+ W^-$. Their partial widths are

$$\Gamma(B_H \rightarrow ZZ) = \frac{1}{2\pi} \left( \frac{K \tilde{g}^3 N_{WZ}}{144 \pi^2} \right)^2 \frac{m_Z^2}{m_{B_H}} \left( 1 - \frac{4m_Z^2}{m_{B_H}^2} \right)^{\frac{3}{2}}$$ \hspace{1cm} (6)

$$\Gamma(B_H \rightarrow W^+ W^-) = \frac{1}{\pi} \left( \frac{K \tilde{g}^3 N_{WZ}}{144 \pi^2} \right)^2 \frac{m_W^2}{m_{B_H}} \left( 1 - \frac{4m_W^2}{m_{B_H}^2} \right)^{\frac{3}{2}}$$ \hspace{1cm} (7)

Details of the calculation are given in the Appendix. $B_H$ is a combination of the $T$-odd $A_H$ and $Z_H$ fields

$$B_H = A_H \cos \theta_H + Z_H \sin \theta_H,$$ \hspace{1cm} (8)
where $\theta_H$ is the mixing angle of the neutral heavy gauge bosons at electroweak symmetry breaking, with its value given in Ref.[8]

$$\sin \theta_H = \frac{5gg'}{4(5g^2 - g'^2)} \frac{v_{SM}^2}{f^2}.$$  

Numerically the coefficient of the $Z_H$ term is negligible compared to the coefficient of the $A_H$. i.e., $B_H \approx A_H$. The branching fractions of $A_H$ decay modes are shown versus $f$ in Fig[2]. The $A_H \rightarrow ZZh, W^-W^+h$ processes are kinematically forbidden at natural $f$ values near 1 TeV. In contrast $Z_H$ has many other decay senarios available will readily decay through dominant $T$-preserving modes discussed in the next section.

$A_H$ is not a viable dark matter candidate due to these $T$-violating decays. The total decay width of $A_H$ is found to be $\sim 10^{-1}$eV; the dependence of the $A_H$ rest decay length ($\lambda = c \hbar / \Gamma$) versus $f$ is plotted in the right panel of Fig[2]. The typical width of $\sim$ eV corresponds to a short track of micrometers, which is practically an instantaneous decay.

$A_H$ is always a daughter particle of the decays of all other heavy particles in LHT, as illustrated in Fig[3]. Thus the decays of $A_H$ greatly enhance the number of final state gauge bosons, instead of contributing to missing energy as expected in a strictly $T$-parity conserving model.
FIG. 3: Diagrams of the leading decay modes that produce $A_H$. $f$ stands for a fermion and $f_H$ for the heavy counterpart. Note that the $T$-even $T_0$ decays into $A_H$ and $T_-$. 

Masses and Decay Widths of Heavy Particles in LHT

At the LHC the new heavy particles in the Little Higgs model with $T$-parity (LHT) can be copiously produced. For an enumeration of the different production channels see [13]. Table II gives representative cross sections of the leading heavy quark and gauge boson production processes at LHC.

| Final state                      | $\sigma$ [fb] |
|----------------------------------|---------------|
| $q^+ q^-$                        | 5.2           |
| $q^+ q^+$                        | 2.6           |
| $T_+ T_-$                        | 1.5           |
| $q_- W_H^+ + q_- W_H^-$          | 1.8           |
| $q_- Z_H$                        | 0.90          |
| $Z_H W_H^+ + Z_H W_H^-$          | 1.6           |
| $W_H^+ W_H^-$                    | 1.0           |

TABLE II: Cross sections at the LHC of leading production processes from $p p \rightarrow XX'$. We take $f = 1.5$ TeV, $\kappa = 1$ and $m_h = 200$ GeV. In the left column $q^+ = (u_-, c_-, \bar{d}_-, \bar{s}_-)$, $q^- = (\bar{u}_-, \bar{c}_-, d_-, s_-)$, $q_+ = (u_+, c_+, t_-, \bar{d}_-, \bar{s}_-, b_-)$ and the cross section is the sum of contributions from all heavy quarks in the corresponding set.
Fig. 4 gives the total decay widths of the massive quarks and gauge bosons. Besides the dominant two-body decay modes, many three-body decay channels may not be negligible for the reason that the interactions of the longitudinal polarization of the gauge bosons are enhanced and the bosonic couplings are large. Due to the large mass gap between the SM particles and the heavy partners, and among the heavy particles themselves, three-body decays are usually accessible (for instance $T$ at TeV mass) in strong contrast to the restrictive three-body decay channels $t \rightarrow bWZ$ [14] and $t \rightarrow bWH^0$ [15] of the top quark in SM. As the energy scale $f$ increases, the phase space of many three-body channels opens up, and their branching fractions become experimentally relevant at higher $f$ values. The $f$ dependence of significant three-body decays is plotted in Fig. 6 and Fig. 7. The three-body channels can provide a good testing ground for the detailed structure of the LHT interactions.

The particle table, Feynman rules, tables of parameters, and event simulations of the LHT model have been coded in a public package CalcHEP LHT [13] available at http://hep.pa.msu.edu/LHT for the phenomenology of the LHT model. We make use of this convenient tool to calculate various $T$-parity preserving multi-body decay channels. In our analysis we fix the parameter $\kappa = 1$, the SM Higgs mass $m_h = 200$ GeV, and take $1.5 < f < 2.5$ TeV as a compromise between naturalness and the electroweak precision constraints. [9] We do not include photons among the daughter particles because of their suppressed coupling.

**Multiple Body Decays of Heavy Bosons**

The lightest $T$-odd particle is the heavy photon $A_H$. Exact $T$-parity conservation would require that normally $A_H$ be a final decay product from any $T$-odd heavy particle. On the other hand, when $T$-parity is violated by the anomaly interaction [11] the $A_H$ will decay rapidly into $ZZ$ and $W^+W^-$. The final products of the $A_H$ decay can be detected, and the event kinematics can thereby be fully reconstructed. The identification of three-particle decay modes becomes feasible.

Branching fractions of the leading decay channels are plotted versus $f$ in Fig. 8. We retain the channels with a fraction above 0.1%. It is interesting to see what new channels are available:
FIG. 4: Total widths of different parent heavy particles in LHT. The calculation assumes $m_H=200$ GeV for heavy Higgs boson widths. In the figure the symbol $*$ denotes $T$-odd partners of the SM quarks and leptons.

(i) We start with the $T$-odd neutral $Z_H$. The two-body decay mode $Z_H \to A_H h$ dominates. A fermionic final state is not kinematically allowed. However, the three-boson phase space is open and there is a substantial branching fraction at the level of 10% for $Z_H \to W^+W^-A_H$.

(ii) $W_H^+ \to W^+A_H$ is the dominating two-body mode in $W_H^+$ decay. Similar to $Z_H$ decay, $W_H^+$ is less massive than heavy fermions and any fermionic final state is kinematically disallowed. The heavy mass $M_{W_H}$ allows both $W^+A_H h$ and $W^+A_H Z$ decays at the level of a few percent.

(iii) The decay of the singly charged $\phi^+$ is mainly dominated by the two-body mode $\phi^+ \to W^+A_H$. Three-body channels $W^+A_H h$ and $W^+A_H Z$ also give significant contribution to the total width.

(iv) The neutral component $\phi$ of the triplet scalar boson is a complex field. It is decomposed into the real part (a scalar $\phi^0$) and the imaginary part (a pseudoscalar $\phi^p$) of
approximately equal masses. The different spatial parities imply different decay modes.
\[
\phi^0 \rightarrow ZA_H, \phi^0 \rightarrow ZhA_H; \\
\phi^+ \rightarrow hA_H, \phi^+ \rightarrow ZZA_H.
\]
Note the role swap \( Z \leftrightarrow h \) when \( \phi^0 \leftrightarrow \phi^p \). Both \( \phi^0 \) and \( \phi^p \) decay to \( W^+W^-A_H \) as well.

(v) The doubly charged scalar boson \( \phi^{++} \) cannot decay into \( \phi^+W^+ \) because of the common \( \phi^{++}, \phi^{+} \) mass. At low Higgs mass \( \sim 120 \) GeV there are no two-body decays of \( \phi^{++} \); however \( \phi^{++} \rightarrow W_H^+W^+ \) emerges at higher Higgs mass when \( \phi \) becomes much more massive than heavy weak gauge bosons. The virtual process \( \phi^{+}_{\text{virtual}} \rightarrow W^+A_H \) gives the overall leading three-body decay \( \phi^{++} \rightarrow W^+W^+A_H \). The four-body decay channels \( (\phi^{++} \rightarrow W^+W^+A_Hh \) or \( W^+W^+A_HZ) \) are smaller but can still be of comparable size to the two/three body-decay modes because of the limited width of leading decay modes.

Multiple Body Decays of Heavy Quarks

There are many \( T \)-odd fermions \( f_- \) of different flavors \( (u_, d_, c_, s_, t_, b_, T_\) with TeV scale masses. \( T \)-parity demands at least one heavy boson in the final state.

The general pattern of decay channels according to descending branching fractions are
\[
f_- \rightarrow W_H + f' , \quad Z_H + f , \quad A_H + f \quad \text{or}, \\
f_- \rightarrow W + W_H + f , \quad W + Z_H + f' , \quad W_H + Z + f' , \quad W + A_H + f' , \quad Z + Z_H + f
\]
where \( f, f' \) are the SM doublet. The relevant Feynman diagrams are shown in Fig. 5.

![Feynman diagrams for the three-body decay processes of a \( T \)-odd fermion. \( q, q' \) refer to different flavors of a SM doublet.](image)

FIG. 5: Feynman diagrams for the three-body decay processes of a \( T \)-odd fermion. \( q, q' \) refer to different flavors of a SM doublet.

The \( T_\pm \) quark of even \( T \)-parity decays readily through \( t - T_\pm \) mixing into \( W^+b, Zt, ht \) as dominating two-body modes. The three-body mode \( Zht \) occurs as a rare process, in analog
to the rare decay modes of the fourth-generation quark as studied in Ref. [14]. In addition, being $T$-even $T_+$ has a rare mode of decaying into two heavy photons.

Note that some three-body channels that can cascade from the primary on-shell two-body decay modes are not shown in our plots, mainly because their rates depend very much on the mass cuts in separating out the resonance components; for example, $u_- \rightarrow d(W^+A_H)$ where $(W^+A_H)$ can be the resonance of $W_H^+$. These channels with intermediate resonance would nevertheless be very important in determining the resonance masses.

**Detection**

Little Higgs phenomenology at the LHC has been investigated recently in a number of studies [16] but not for the situation where $A_H$ decays. The key feature is that the new heavy quarks can also be produced in proton-proton collisions, either from gluon fusion or quark interactions. Most $T$-odd quarks are pair produced with the exception that the $T$-even $T_+$ particle can be produced along with a SM quark.

The produced heavy quarks decay quickly into less massive SM particles and $T$-odd bosons that subsequently decay into SM counterparts and heavy photons. The dominant two-body decay channels $f_- \rightarrow W_H^\pm + f', Z_H + f$, $A_H + f$ transform each heavy quark into a SM quark that may form a jet and one $T$-odd gauge boson. $W_H^\pm$ and $Z_H$ decay into SM gauge/Higgs bosons and $A_H$. $A_H$ decays through $T$-violating WZW interaction into $ZZ$, $W^+W^-$, resulting in an overall $T$-even final state.

As shown in Fig., the leading three-body decay processes of $T$-odd quarks will either add a $W^\pm$ or $Z$ boson to the daughter particles, while $T_\pm$ gives an additional $Z$ boson in the final state. In the bosonic sector, the heavy gauge bosons have significant decay rates to three-body final states.

The contributions of the three-body decays visibly depend on the energy scale $f$. As $f$ increases to higher $f$ values, the three-body phase space opens up faster than the two-body phase space, and the three-body branching fractions steadily increase as the mass gap widens between the heavy particles and SM particles.
**Conclusion**

The Little Higgs model with $\mathcal{T}$-parity (LHT) is an interesting extension of the Little Higgs framework. It alleviates the tension of the “little hierarchy” problem and it is also a phenomenologically rich model, giving rise to testable new physics at the TeV scale.

In the LHT model one can expect that the LHC will produce a large amount of heavy quarks beyond the SM via the strong interaction, and also substantial numbers of new heavy leptons and new heavy gauge bosons by Drell-Yan-like processes. Their decay patterns can go beyond the usual dominant two-body modes and include contributions from various measurable three-body modes.

Since $\mathcal{T}$-parity is broken by anomaly, the lightest $\mathcal{T}$-odd particle $A_H$ will decay into detectable $ZZ$ or $W^+W^-$. As $A_H$ appears as a decay product of all LHT processes, the $\mathcal{T}$-parity-violating decays allow reconstruction of the full event configurations and thereby comprehensive physics tests of the Little Higgs model at the LHC.

We have studied the multi-body decays of the heavy particles in the LHT model that can be produced at the LHC. Detailed analyses of these multi-body channels may be useful in revealing the new symmetry and its interactions at the TeV scale.

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**Appendix**

The decay amplitude $B_H(\epsilon') \to Z(k_1, \epsilon_1) + Z(k_2, \epsilon_2)$ can be derived from Eq. (5)

$$\mathcal{M} = -\frac{K g^3 N_{WZ}}{12 \sqrt{3} \pi^2 M_B^2} L , \quad L = \epsilon^{\mu\nu\rho\sigma} \epsilon_{\mu}'(k_1 - k_2)_{\nu}(\epsilon_1)_{\rho}(\epsilon_2)_{\sigma} , \quad (9)$$

where the Levi-Civita symbol is contracted with vectors. The momentum term $k_1 - k_2$ comes from two ways of contracting the $Z$ field. It antisymmetrizes the momentum part.
and the Levi–Civita antisymmetrizes the polarization part. The combined product is overall symmetric as expected for the boson decay. We choose \( \varepsilon' \) in the rest frame of \( B_H \) along the \( z \) direction, and \( k_1 = -k_2 = k = |k| (\sin \theta \mathbf{x} + \cos \theta \mathbf{z}) \). Notice that transverse-transverse (TT) modes vanish, as well as the longitudinal-longitudinal (LL) mode. The only surviving modes are LT or TL. The relevant vectors in the LT mode are

\[
\varepsilon'_1 = (0, 0, 0, 1) \\
k_1 - k_2 = (0, \sin \theta, 0, \cos \theta) \\
\varepsilon_1(L) = (|k|, E_Z \sin \theta, 0, E_Z \cos \theta) / m_Z \\
\varepsilon_2(T) = (0, 0, 1, 0)
\]

The Levi–Civita symbol becomes the determinant of the above arrays, \( L = 2|k|^2 \sin \theta / m_Z \).

\[
\sum_{\text{final}} |M|^2 = \left( \frac{K g^3 N_{WZ}}{144 \pi^2} \right)^2 \frac{m_Z^4}{m_{B_H}^2} \frac{m_{B_H}^4}{m_Z^2} \left( 1 - \frac{4m_Z^2}{m_{B_H}^2} \right)^2 \sin^2 \theta \times 2
\]

The last factor two counts both LT and TL modes. Thus we obtain

\[
\Gamma(B_H \rightarrow ZZ) = \frac{1}{2m_{B_H}} \frac{1}{8 \pi} \left( 1 - \frac{4m_Z^2}{m_{B_H}^2} \right)^{\frac{1}{2}} \sum_{\text{final}} |M|^2 \left( \frac{d\Omega}{4\pi} \right) \frac{1}{2!}
\]

The factor \( \frac{1}{2!} \) in the above decay width comes from the combinatorics of the two identical \( Z \) bosons. After some algebra, we derive the final expressions Eqs. (6, 7) of \( \Gamma(B_H \rightarrow ZZ) \), and the similar one \( \Gamma(B_H \rightarrow W^+W^-) \). The threshold dependence agrees with that in Ref. [17]. Note that the overall factor of 2 difference between \( WW \) and \( ZZ \) comes from identical particle effect.

[1] R. Barbieri and A. Strumia, [arXiv:hep-ph/0007265](http://arxiv.org/abs/hep-ph/0007265).

[2] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 513, 232 (2001) [arXiv:hep-ph/0105239](http://arxiv.org/abs/hep-ph/0105239). For reviews, see, for example, M. Schmaltz and D. Tucker-Smith, [arXiv:hep-ph/0502182](http://arxiv.org/abs/hep-ph/0502182).

[3] N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 0207, 034 (2002) [arXiv:hep-ph/0206021](http://arxiv.org/abs/hep-ph/0206021).

[4] T. Han, H. E. Logan, B. McElrath and L. T. Wang, Phys. Rev. D 67, 095004 (2003) [arXiv:hep-ph/0301040](http://arxiv.org/abs/hep-ph/0301040).
[5] C. Csaki, J. Hubisz, G. D. Kribs, P. Meade and J. Terning, Phys. Rev. D 67, 115002 (2003) [arXiv:hep-ph/0211124]. J. L. Hewett, F. J. Petriello and T. G. Rizzo, JHEP 0310, 062 (2003) [arXiv:hep-ph/0211218]. C. Csaki, J. Hubisz, G. D. Kribs, P. Meade and J. Terning, Phys. Rev. D 68, 035009 (2003) [arXiv:hep-ph/0303236]. M. C. Chen and S. Dawson, Phys. Rev. D 70, 015003 (2004) [arXiv:hep-ph/0311032]. W. Kilian and J. Reuter, Phys. Rev. D 70, 015004 (2004) [arXiv:hep-ph/0311095]. Z. Han and W. Skiba, Phys. Rev. D 71, 075009 (2005) [arXiv:hep-ph/0412166].

[6] H. C. Cheng and I. Low, JHEP 0309, 051 (2003) [arXiv:hep-ph/0308199].

[7] I. Low, JHEP 0410, 067 (2004) [arXiv:hep-ph/0409025].

[8] J. Hubisz and P. Meade, Phys. Rev. D 71, 035016 (2005) [arXiv:hep-ph/0411264].

[9] J. Hubisz, P. Meade, A. Noble and M. Perelstein, JHEP 0601, 135 (2006) [arXiv:hep-ph/0506042].

[10] J. A. Casas, J. R. Espinosa and I. Hidalgo, JHEP 0503, 038 (2005) [arXiv:hep-ph/0502066].

[11] C. T. Hill and R. J. Hill, Phys. Rev. D 75, 115009 (2007) [arXiv:hep-ph/0701044]; C. T. Hill and R. J. Hill, arXiv:0705.0697 [hep-ph].

[12] J. Wess and B. Zumino, Phys. Lett. B 37, 95 (1971). E. Witten, Nucl. Phys. B 223, 422 (1983).

[13] A. Belyaev, C. R. Chen, K. Tobe and C. P. Yuan, effects of Phys. Rev. D 74, 115020 (2006) [arXiv:hep-ph/0609179].

[14] V. D. Barger, W. Y. Keung and T. G. Rizzo, Phys. Rev. D 40, 2274 (1989). G. Mahlon and S. J. Parke, Phys. Lett. B 347, 394 (1995) arXiv:hep-ph/9412250. E. Jenkins, Phys. Rev. D 56, 458 (1997) arXiv:hep-ph/9612211.

[15] V. D. Barger and W. Y. Keung, Phys. Lett. B 202, 393 (1988).

[16] Q. H. Cao and C. R. Chen, arXiv:0707.0877 [hep-ph]. K. Kong and S. C. Park, arXiv:hep-ph/0703057. D. Choudhury and D. K. Ghosh, arXiv:hep-ph/0612299. S. Matsumoto, M. M. Nojiri and D. Nomura, Phys. Rev. D 75, 055006 (2007) [arXiv:hep-ph/0612249]. M. Carena, J. Hubisz, M. Perelstein and P. Verdier, arXiv:hep-ph/0610156. F. Ledroit, arXiv:hep-ex/0610005. A. Freitas and D. Wyler, JHEP 0611, 061 (2006) [arXiv:hep-ph/0609103]. K. Cheung, C. S. Kim, K. Y. Lee and J. Song, Phys. Rev. D 74, 115013 (2006) [arXiv:hep-ph/0608259]. C. S. Chen, K. Cheung and T. C. Yuan, Phys. Lett. B
M. Perelstein, arXiv:hep-ph/0512128; H. C. Cheng, I. Low and L. T. Wang, Phys. Rev. D 74, 055001 (2006) [arXiv:hep-ph/0510225]. A. Hektor, M. Kadastik, M. Muntel, M. Raidal and L. Rebane, arXiv:0705.1495 [hep-ph]. T. Han, H. E. Logan and L. T. Wang, JHEP 0601, 099 (2006) [arXiv:hep-ph/0506313]. T. Han, H. E. Logan, B. McElrath and L. T. Wang, Phys. Lett. B 563, 191 (2003) [Erratum-ibid. B 603, 257 (2004)] [arXiv:hep-ph/0302188].

[17] D. Chang, W. Y. Keung and S. C. Lee, Phys. Rev. D 38 (1988) 850.
FIG. 6: Branching fractions of heavy boson decay modes in LH are plotted versus the global symmetry breaking scale $f$ (GeV). Solid lines (Blue) and long dashed lines (Red) show two- and three-body channels, respectively. Due to the limited width of the two-body mode of $\phi^{++}$, the leading four-body modes also reach high branching fractions, as shown in short dashed lines (Grey).
FIG. 7: Branching fractions of heavy fermions in LH are plotted versus $f$. Cases of parent particles $u_-$ (or similarly $c_-$), $d_-$ (or $s_-$), $b_-$, $T_+$, $t_-$, are shown in the composite graphs. The branching fractions are calculated at $m_h=200$ GeV when Higgs bosons are involved.