Energy Flow Observables$^1$

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Abstract

A few comments are made on the role of nonperturbative and perturbative power corrections. This is followed by a description of energy flow observables and correlations that may provide a flexible approach to rapidity distributions in hadronic scattering.

1 From Factorized Cross Sections to Classical Fields

This short talk describes a modest attempt to synthesize one or two of the ideas exchanged at this stimulating workshop. I will try to connect two themes relevant to the very highest energy collisions, the saturation scale and rapidity distributions, with perturbative QCD for hard-scattering cross sections.

The very high energies observed in cosmic ray collisions make it natural to think of perturbative QCD, but of course momentum transfers are by no means asymptotically large in most inelastic collisions in the upper atmosphere [1], or at accelerators for that matter. At extremely high energies, however, the same Lorentz contractions of the target and projectile that are at the basis of factorization may also result in very high effective field strengths, which can provide a new dynamical energy, referred to as the saturation scale [2]. The saturation scale acts in a sense as a mean momentum transfer for all partons in the dense medium. For high enough densities and energies, it can be well into the perturbative region. We can ask how

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such a phenomenon can arise in the language of perturbative QCD for hard scattering.

In perturbative QCD, the cross section for $A + B \rightarrow E(Q) + X$, with $A$ and $B$ hadrons and $E(Q)$ an object in the final state of mass $Q$ (heavy particle, jet pair, etc.) can be factorized. In terms of parton distributions $f_{a/A}$ and a hard scattering function $H_{ab\rightarrow E}$, we have schematically

$$\sigma_{AB\rightarrow E} = \sum_{ab} f_{a/A} \otimes H_{ab\rightarrow E} \otimes f_{b/B}, \quad (1)$$

up to corrections that are suppressed by a relative power of $1/Q$. The convolution represented by $\otimes$ is an integral in momentum fraction ($x$), and possibly also transverse momentum [4]. For “minimum bias” events, or events with one or more moderate momentum transfers between partons, the power corrections are not highly suppressed no matter how high the energy. It is therefore worthwhile to review the origins of power corrections as they appear in perturbative QCD.

An essentially exhaustive list of power corrections derived from perturbation theory for unpolarized scattering is:

- **Strong coupling (renormalon) and/or vacuum corrections** can appear in the perturbative hard-scattering function through nonconvergent behavior at high orders: $\alpha_s^n(Q) b_0^n n!$, with $b_0$ the first coefficient of the QCD beta function. These corrections often begin at $1/Q$ in semi-inclusive cross sections [5], but at $1/Q^2$ in single-particle inclusive cross sections [6].

- **Multiparton corrections**, including parton transverse momentum effects involve two rather than one partonic degree of freedom from one or both of the hadrons that participate in the hard scattering [7]. These corrections may be put in a form analogous to (1), and typically begin at $1/Q^2$ in unpolarized cross sections,

$$\frac{1}{Q^2} \sum_{aa',b} F_{aa'/A} \otimes H_{aa',b\rightarrow E} \otimes f_{b/B}, \quad (2)$$

where $F_{aa'/A}$ is a double distribution for degrees of freedom $a$ and $a'$, while $f_{b/B}$ is a standard single-parton distribution.
• Multiple scattering processes, with distinguishable components to the
final state: \( E = E_1 + E_2 \), can result from independent hard scatterings
of different partons from both incoming hadrons \(^9\). These again begin
at \( 1/Q^2 \),

\[
\frac{1}{Q^2} \sum_{aa',bb'} \bar{F}_{aa'/A} \otimes h_{ab \to E_1} \otimes h_{a'b' \to E_2} \otimes \bar{F}_{bb'/B} .
\]

(3)

These are the corrections that correspond to multiple parton interac-
tions, as incorporated into models based on multiple minijet produc-
tion. The \( \bar{F}'s \) in (3) are generally not the same as the \( F's \) in (2).

• Certain initial-state interactions beginning at \( 1/Q^4 \) cannot be written
as the product of separate parton distributions for the two hadrons at
all \(^8\). Such corrections still enjoy a factorization of the hard scat-
tering, which is initiated by two partons collinear to the incoming hadrons, at
leading relative power,

\[
\frac{1}{Q^4} \sum_{ab} \mathcal{F}_{ab/AB} \otimes \bar{H}_{ab \to E} .
\]

(4)

These corrections, which do not allow a mutual factorization in terms
of universal parton distributions in individual hadrons, are associated
with nonperturbative soft gluons. Such soft gluons can couple to hard
partons originating from the colliding hadrons via the QCD field strength.
In quantum perturbation theory \(^10\), as in solutions of the classical
equations of motion \(^11\), the Lorentz contraction properties of the field
strength play an essential role. It is at this level that a classical color
field enters the perturbative factorization-based picture, as a necessary
completion of it.

Each of the extensions of leading-power factorized cross sections appears
in perturbation theory, in general requiring new sets of nonperturbative de-
grees of freedom, and the introduction of new distributions for multiple par-
tons. It therefore makes sense to ask whether we can develop phono-
menological tools to connect perturbative and nonperturbative dynamics in a con-
trollable and continuous fashion. We may seek observables with adjustable
parameters, such that for some range of values perturbation theory is ac-
curate, while as we vary these parameters outside the perturbative range,
failures of perturbation theory may provide insight into the relevant dynamics. In the second section, I will suggest a class of observables that are sensitive to energy flow in hadronic collisions to illustrate this approach.

2 Energy Flow in Hadronic Collisions

Plans for forward coverage at the LHC \[12, 13\], and the need to test shower event generators \[1\] at cosmic energies both suggest the usefulness of observables that are sensitive to the global structure of particle production in hadronic collisions. Such observables are familiar from $e^+e^-$ annihilation as event shapes \[14\]. The classic set of event shapes includes the thrust, heavy jet mass, and others. Modified versions of these shapes can be adapted to high-$p_T$ jets in hadronic collisions \[15\]. Our goal here is to point out how event shapes can interpolate between high $p_T$ and the forward direction.

It is generally not possible to take event shapes over unchanged from leptonic annihilation to the forward jets in hadronic scattering cross sections. This is because initial-state radiation and forward parton-parton scattering are highly singular in the forward directions, due to the $1/\sin^4(\theta/2)$ behavior of the ‘Rutherford’ cross section associated with the exchange of gluons.

Nevertheless, it is possible that event shapes with adjustable parameters may be adapted to hadronic scattering, so that for some range of values they can be written in factorized form, similar to Eq. \(1\) above. One such set of event shapes with an adjustable parameter is the set of “angularities” \[16\],

$$
\tau_a(n) = \frac{1}{\sqrt{s}} \sum_{i \in n} \omega_i \sin^a \theta_i \left(1 - |\cos \theta_i|\right)^{1-a}
$$

For the case at hand, the angle $\theta$ and the rapidity $\eta$ are defined relative to the collision axis in the center of mass frame. Defined this way, $0 < \tau_a < 1$.

We can construct a set of dimensionless global energy flow observables for the collisions of hadrons $A$ and $B$ in terms of angularities,

$$
M_{AB}(N, \zeta) = S^{1-N/2} \sum_{\text{states} \ n} \sigma_{AB}(n) e_\zeta^N(n),
$$

where for each final state $n$ we define weighted transverse energies as

$$
e_\zeta(n) = \sum_{i \in n} k_{i\perp} e^{-\zeta|\eta_i|},
$$

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e_\zeta(n) = \sum_{i \in n} k_{i\perp} e^{-\zeta|\eta_i|},
$$
with parameter $\zeta \equiv 1 - a$.

The angularities, and hence the energy flow variables $e_\zeta$ are defined so that as $\zeta$ increases or decreases, the contributions of particles in the forward directions $|\eta| \to \infty$ are weighted less or more. To show how this works, consider the lowest order contribution to $M_{AB}$ from a single parton-parton scattering, with two high-$p_T$ particles (or jets) in the final state, one of which is at fixed rapidity $\eta$ in the hadronic center of mass. If the momentum transfer, and therefore transverse momentum is large ($\eta$ is fixed), this contribution is simply the value of $e^N_\zeta$ times the lowest order jet cross section, where $e_\zeta$ gets contributions from both final-state particles. The cross section depends in the usual way on parton distributions and hard scattering functions. For simplicity and to study the most singular behavior, we show only the lowest-order $t$-channel exchange contribution,

$$|A_{ij}(s, t)|^2 \to C_{ij}(\alpha_s/\pi)^2 \left(\frac{s}{t}\right)^2 = C_{ij}(\alpha_s/\pi)^2 e^{2\eta^*} \cosh^2 \eta^*, \quad (8)$$

with $C_{ij}$ a constant, for parton flavors $i$ and $j$, and with $\eta^*$ the rapidity of either outgoing parton in the partonic center of mass.

We can write the full lowest-order contribution to $M_{AB}$ as an integral over hadronic c.m. rapidity $\eta$ and $x_T \equiv 2p_T/\sqrt{S}$,

$$M_{AB}^{t \text{ channel}}(N, \zeta) = \sum_{ij} C_{ij} \int_{-\infty}^{\infty} d\eta \int_0^\infty dx_T \frac{x_T^{-1}}{x_T} \times \left(\frac{\alpha_s (x_T \sqrt{S})}{\pi}\right)^2 \left[ x_T e^{-\zeta |\eta|} + x_T e^{-\zeta |\eta-2\eta^*|} \right]^N \times f_{i/A} \left( x_T e^{\eta^*-\eta^*} \cosh \eta^*, \frac{x_T \sqrt{S}}{2} \right) f_{j/B} \left( x_T e^{\eta^*+\eta^*} \cosh \eta^*, \frac{x_T \sqrt{S}}{2} \right). \quad (9)$$

This serves as a perturbative description of energy flow. The two rapidity-dependent terms in the square brackets give the contributions of the two final-state particles, with equal $x_T$, to the weight at lowest order. For small $N$ there is a manifest singularity at $x_T = 0$, which is strengthened by the exponential growth of the $t$-channel amplitude in $\eta^*$. Choosing as above the factorization scale as $\mu_F = p_T = x_T/2\sqrt{S}$, the precise perturbative predictions will also depend on the low-$x$ behavior of the parton distributions at low $\mu_F$, but for...
$N$ and $\zeta$ large enough, $M_{AB}$ is perturbatively calculable, and insensitive to this behavior up to power corrections. (This is similar to the situation for transverse momentum distributions of Drell-Yan pairs.) Such moments of the energy flow distribution are calculable for any fixed rapidity $\eta$.

The observables described above are not limited to perturbative calculation based on collinear factorization. They can in principle be used to test predictions of any model of high energy scattering that provides detailed predictions for energy flow in the final state. By varying the two parameters, $N$ and $\zeta$, and exploiting the forward coverage planned for the LHC, it may be possible, for example, to quantify correlations of activity in the forward region with hard scattering [17]. Other examples may include predictions based on $k_T$-factorized cross sections, as have been advocated especially for nuclear collisions, in addition to those of event generator models. Angularity-based analyses of the global properties of final states in hadron-hadron collisions may be useful, but they are only a first proposal. The main message of this talk is that the large rapidity coverage planned for the LHC will open the way to varied studies and invite the development of new analyses that probe the formation of QCD final states.

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References

[1] L. Anchordoqui et al. *Annals Phys.* **314**, 145 (2004) [hep-ph/0407020]

[2] N. Armesto, *Acta Phys. Polon.* **B35**, 213 (2004) [hep-ph/0311182]

[3] J.C. Collins, D.E. Soper and G. Sterman, in *Perturbative Quantum Chromodynamics*, A.H. Mueller, ed. (World Scientific, Singapore, 1989).

[4] J.C. Collins and R.K. Ellis, *Nucl. Phys.* **B360**, 3 (1991); S. Catani, M. Ciafaloni and F. Hautmann, *Nucl. Phys.* **B366**, 135 (1991).

[5] M. M. Beneke and V.M. Braun, in the Boris Ioffe Festschrift *At the Frontier of Particle Physics / Handbook of QCD*, edited by M. Shifman (World
Scientific, Singapore, 2001) vol. 3, 1719, hep-ph/0010208.
L. Magnea, at 14th Conference on High-Energy Physics, Parma, Italy, 3-5 Apr 2002, hep-ph/0211013.

[6] G. Sterman and W. Vogelsang, hep-ph/0409234

[7] R.K. Ellis, W. Furmanski and R. Petronzio, *Nucl. Phys.* B207, 1 (1982); R.L. Jaffe, *Nucl. Phys.* B229, 205 (1983); J.W. Qiu, *Phys. Rev.* D42, 30 (1990).

[8] J.W. Qiu and G. Sterman, *Nucl. Phys.* B353, 105 (1991), ibid. 137 (1991).

[9] G. Calluci and D. Treleani, *Nucl. Phys. Proc. Suppl.* 71, 392 (1999) hep-ph/9711225.
M. Strikman and D. Treleani, *Phys. Rev. Lett.* 88, 031801 (2002) hep-ph/0111468.
CDF Collaboration (F. Abe et al.), *Phys. Rev.* D56, 3811 (1997).

[10] R. Basu, A.J. Ramalho and G. Sterman, *Nucl. Phys.* B244, 221 (1984).

[11] L. D. McLerran and R. Venugopalan, *Phys. Rev.* D50, 2225 (1994) hep-ph/9402335.

[12] TOTEM Collaboration (Mario Deile for the collaboration), at the 12th International Workshop on Deep Inelastic Scattering (DIS 2004), Strbske Pleso, Slovakia, 14-18 Apr 2004, hep-ex/0410084.

[13] J.-P. Baud, et al., at the 2003 Conference for Computing in High-Energy and Nuclear Physics (CHEP 03), La Jolla, California, 24-28 Mar 2003, in eConf C0303241:TUDT007 (2003) e-Print Archive: cs.oh/0305047.

[14] M. Dasgupta and G.P. Salam, *J. Phys.* G30, R143 (2004) hep-ph/0312283.

[15] A. Banfi, G.P. Salam and G. Zanderighi, *JHEP* 0408, 062 (2004) hep-ph/040728.

[16] C.F. Berger, T. Kucs and G. Sterman, *Phys. Rev.* D68, 014012 (2003) hep-ph/0303051.
C.F. Berger and L. Magnea, *Phys. Rev.* D70, 094010 (2004) hep-ph/0407024.
[17] L. Frankfurt, M. Strikman and C. Weiss, Phys. Rev. D\textbf{69}, 114010 (2004) \texttt{hep-ph/0311231}