Parallel robots modelling and optimization

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Abstract. Parallel robots are mechanical systems with numerous applications in manufacturing, medicine, space industry, etc. Such robots are characterized by closed kinematic chains ensuring robustness and maneuverability. The design of a fully functional parallel robot is a time-consuming task as an engineer needs to enumerate and compare many variants in order to find one with the best characteristics. Typical characteristics are the area of the workspace, the number of singularity-free regions, dexterity index. The design of a parallel robot is thus a multi-objective constraint optimization problem. We propose to use interval analysis to reliably compute characteristics and multiobjective optimization methods to find the Pareto-optimal set of robot parameters. The proposed approach is illustrated on a set of several realistic parallel robotic systems.

1. Introduction

Nowadays the robots are used in many areas of our lives. Their appearance has brought benefits in various fields of engineering, medicine, construction, conveyor systems and many others. Usually, multi-link lever manipulators or flexible manipulators are used. A serious disadvantage of such manipulators is their insufficient rigidity, which complicates their control and makes it difficult to achieve high accuracy. To a large extent, manipulators, which are constructed on the basis of multi-section mechanisms with a parallel structure, can overcome these disadvantages. Such mechanisms have good controllability and rigidity. To evaluate such characteristics, special indices are proposed based on the Jacobi matrix of the studied robot. In this work, two such indices are used: GDI (global dexterity index) and GRI (global resistivity index), which reflecting the controllability and rigidity respectively of the robot throughout its working area. The working area is also an important characteristic of the robot, which serves for constructing the track of the working tool movement and its determination is the problem of approximating an implicitly given set. This paper presents such an approximation method based on interval methods, in particular, on the Krawczyk method and its modifications. The main goal of the work is to optimize the robot according to the two above-mentioned criteria and the area of its workspace with the considered constraint: the configuration of the robot mustn’t contain singularities.

2. Related works

Since the appearance of the parallel robots, their kinematic, dynamic and static analysis have been studied \cite{1, 2, 3}. One of the most comprehensive work is the the book \cite{4}, which contains a
fundamental information about parallel robots, methods for singularity analysis, algorithms for solving the direct and inverse kinematic problems and basic workspace determination examples. Detailed information about singularity positions and its determination, analytical calculating of Jacobi matrices and the GDI index, their mathematical interpretation and basic definitions are provided in Gosselin and Angeles articles [5, 6]. Problem of the robot optimization has already been considered in article [7]. The authors also calculate several indexes (Global Resistivity Index (GRI), Global Conditioning Index (GCI), Space Utilization Index (SUI)), but multi-criteria problem wasn’t considered and only composite index was constructed and optimized. In this paper, the robot’s design optimization problem is treated as a multi-objective problem. As there are three design objectives the Pareto-frontier region can be efficiently visualized. Such visualization provides useful feedback to the decision-maker, i.e. the engineer in our case.

3. Kinematic and singularity analysis works

The 2-RPR planar robot is controlled by two linear actuators, which can change the bar lengths $\theta_1$, $\theta_2$, thereby controlling the position of the robot’s tool fixed at the point P (Fig. 1). We use the following notation throughout the paper: input parameter vector (bar lengths) $\bar{\theta} = (\theta_1, \theta_2)$, and output parameter vector (coordinates of the point P) $\bar{X} = (X_1, X_2)$. Let $d$ is the distance between the actuators. Assume that the lower $l$ and upper $L$ bounds of changing lengths are the same for both bars: $l \leq \theta_1 \leq L, l \leq \theta_2 \leq L$. Obviously, the 2-RPR robot has 2 degrees of freedom. Based on the simplest geometric expressions, we obtain a system of kinematic equations that connects the input parameters and the output parameters of the robot:

$$\begin{align*}
\theta_1^2 - X_1^2 &= X_2^2 = 0, \\
\theta_2^2 - (X_1 - d)^2 - X_2^2 &= 0.
\end{align*}$$

System 1 can also be rewritten as $F(\theta, X) = 0$. Now let’s find the time derivatives of the equations $F(\theta, X)$, to get a system that determines the velocity ratio of the input and output coordinates of the robot: $J_\theta \dot{\theta} - J_X \dot{X} = 0$, where $J_X = \frac{\partial F}{\partial X}$ and $J_\theta = \frac{\partial F}{\partial \theta}$ are Jacobi square matrices. As a result, we obtain the system of instant kinematics equations:

$$\begin{align*}
\theta_1 \cdot \dot{\theta}_1 - X_1 \cdot \dot{X}_1 - X_2 \cdot \dot{X}_2 &= 0, \\
\theta_2 \cdot \dot{\theta}_2 - (X_1 - d) \cdot \dot{X}_1 - X_2 \cdot \dot{X}_2 &= 0.
\end{align*}$$

Figure 1. 2-RPR robot scheme
Obviously

\[ J_\theta = \begin{pmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{pmatrix} \]

\[ J_X = \begin{pmatrix} -X_1 & -X_2 \\ -X_1 + d & -X_2 \end{pmatrix} \]

Solving the linear system 2, we obtain the solution of the inverse kinematics problem

\[ \dot{\theta} = -J_\theta^{-1} \cdot J_X \dot{X} = J \dot{X}, \]

where

\[ J = -J_\theta^{-1} \cdot J_X \quad (3) \]

4. Workspace approximation

![Figure 2](image-url)

**Figure 2.** The results produced by usual (a) and bicentered (b) Krawczyk methods for uniform grids of sizes 30x30, 60x60 for 2-RPR robot 1

We consider the system of equations

\[ F(u, v) = 0, \quad (4) \]

where \( u, v \) are vectors in the spaces \( \mathbb{R}^m, \mathbb{R}^n \), respectively, and \( F : \mathbb{R}^{m+n} \to \mathbb{R}^n \) is a continuously differentiable mapping. Given a box \( U = [\underline{u}_1, \bar{u}_1] \times \cdots \times [\underline{u}_m, \bar{u}_m] \subseteq \mathbb{R}^m \) and a box \( V = [\underline{v}_1, \bar{v}_1] \times \cdots \times [\underline{v}_n, \bar{v}_n] \subseteq \mathbb{R}^n \), let us define the solution set \( \Omega \) as

\[ \Omega = \{ u \in U \subseteq \mathbb{R}^m \mid \exists v \in V \subseteq \mathbb{R}^n \text{ such that } F(u, v) = 0 \}, \quad (5) \]
i.e. as the set formed by all such parameters \( u \in U \) that there exists a solution \( v \) for the equations system \( F(u, v) = 0 \).

It is required to construct inner and outer approximations of the solution set \( \Omega \) by a collection of boxes, i.e. find such a finite sets of boxes \( S_n \) and \( S_b \) that

\[
\bigcup_{u \in S_n} \subseteq \Omega \subseteq \bigcup_{u \in S_n \cup S_b} \subseteq U. \tag{6}
\]

We use the so-called differential centered form (also known as mean-value form, see [8, 9]):

\[
G_m(u, v, c) = \hat{z}G(u, c) + \hat{z}G'(u, v)(v - c), \tag{7}
\]

where \( c \in v \) is the point from \( v \), \( G'(u, v) = \frac{\partial G}{\partial v}(u, v) \) is the gradient of the mapping \( G \) with the respect to variables from \( v \), and \( \hat{z} \) is notation for the interval natural extension.

Taking \( c = \text{mid} v \), we obtain the classical Krawczyk operator [10]:

\[
Kr(u, v) \subseteq [G](u, \text{mid} v) + [G'](u, v)(v - \text{mid} v). \tag{8}
\]

The modification of Krawczyk operator based on the Baumann theorem [11] can give tighter bounds, which was proposed in [12]. Denote \( d^{(i)} = \hat{z}g'_i(u, v), \ d^{(i)}(i) \in \mathbb{R}^n \). According to the Baumann theorem,

\[
g_i(u, v) \subseteq \tilde{g}_i \left( u, v, \tilde{c}^{(i)} \right) \cap \tilde{g}_i \left( u, v, \hat{c}^{(i)} \right), \tag{9}
\]

where for \( j = 1, 2, \ldots, n \)

\[
\tilde{c}^{(i)}_j = \begin{cases} v_j, & \text{if } d^{(i)}_j \leq 0, \\ \tilde{v}_j, & \text{if } d^{(i)}_j \geq 0, \\ \frac{d^{(i)}_j v_j - d^{(i)}_j \tilde{v}_j}{d^{(i)}_j - d^{(i)}_j}, & \text{if } d^{(i)}_j < 0 < d^{(i)}_j, \\ \frac{d^{(i)}_j \tilde{v}_j - d^{(i)}_j \tilde{v}_j}{d^{(i)}_j - d^{(i)}_j}, & \text{if } d^{(i)}_j < 0 < d^{(i)}_j. 
\end{cases}
\]

At the result we visualise all inner boxes with green colour and all border boxes with yellow colour (Fig 2) for usual and bicentered Krawczyk operator for different grid sizes. It’s obvious, that the bicentered Krawczyk method generates tight approximations, while those produced by the usual Krawczyk method contain redundant boxes.

5. Criteria indexes

5.1. Global dexterity index

Before defining this index, we introduce several additional relations. Let us determine the dexterity coefficient \( \eta = 1/k \), where \( k = \|J\| \cdot \|J^{-1}\| \) is the condition number of the matrix \( J \). The coefficient \( \eta \) reflects the controllability of the robot at a given point on the workspace. The closer \( \eta \) to 1, the better the controllability. Indeed, from the System 3 it follows that the larger the condition number, the greater the errors that can arise when solving System 3. Moreover, with small inaccuracies in the determination of the output coordinates, significant errors can appear in the corresponding input coordinates, causing inadequate control.

Now we can move to the definition of the \( GDI = A/B \), where \( A = \int g \eta(\theta_1, \theta_2) d\theta_1 d\theta_2 \) and \( B = \int g 1 d\theta_1 d\theta_2 \) is the workspace area computed in the input coordinates. This index shows the controllability of the robot over its workspace.

For 2-RPR robot the integrals are computed over a square with sides \( L - l \). Thse calculation of the integral can be carried out on a uniform two-dimensional grid. We approximate the integral over a uniform grid:

\[
\int g \eta(\theta_1, \theta_2) d\theta_1 d\theta_2 \simeq \sum_{i=1}^{n} \sum_{j=1}^{n} \eta_{ij} \cdot S_n,
\]
where \( S_n = \left( \frac{L-l}{n} \right)^2 \), \( n \) is the number of nodes of the discrete grid, \( \eta_{ij} \) is the value of the dexterity coefficient in the center of the grid’s cell. Since the workspace area in the input coordinates is equal to \( (L-l)^2 \):

\[
GDI \simeq \frac{\sum_{i=1}^{L_1} \sum_{j=1}^{L_2} \eta_{ij}}{n^2}
\]

5.2. Global resistivity index

We also should define some equations before considering second index: \( \omega = \frac{1}{|\det J|} = |\det(J^{-1})| \), where \( \omega \) is the resistivity coefficient, which displays the rigidity of the robot in current position. The GRI is defined as \( GRI = C/B \), where \( C = \int \int \omega(\theta_1, \theta_2) \ d\theta_1 \ d\theta_2 \) and workspace \( B \) has already been defined before. We use same technique to approximate integral on a uniform grid and at the result we get the equation for calculating GRI:

\[
GRI \simeq \frac{\sum_{i=1}^{L_1} \sum_{j=1}^{L_2} \omega_{ij}}{n^2}
\]

6. Numerical experiments

![Figure 3. The Pareto-frontier for different sets of criteria: three-criteria problem (a), the Pareto-frontier for pairs of criteria (b, c, d)](image)

Consider optimizing the design of the robot for given ranges of the length variation of the bars relative to the changing distance \( d \) between the points of their attachment \( A \) and \( B \). We consider three criteria: the workspace area, global dexterity index, global resistivity index and
one constraint: we restrict the robot’s design to cases where the configurations don’t contain singularities (see [2]). Let’s fix the values of the ranges of lower and upper limits for bars lengths to 6 and 15, respectively. The only parameter – distance varies discretely from 1 to 20 with a step one. Such values of \( d \) constitute the set of non-singular configurations. Other values form so-called set of singular configurations. Then compute three criteria for all non-singular configurations to evaluate and plot feasible solutions. All feasible solutions are divided into two subsets: Pareto points (blue) and remaining ones (orange). As we use three criteria in our work, we visualize such sets for all pairs of criteria and the final three-criteria plot. The commonly adopted approach to select a particular solution form the Pareto set is to find the ideal point: a Pareto-optimal point belonging to the Pareto-frontier and closest (has minimum distance) to utopian point. The utopian point is the point in the criteria space where all criteria reach maximum.

7. Results
We considered a problem of the modelling parallel robot 2-RPR and its optimal design. Before the main goal of this research we also solve the direct and inverse kinematic problems and the problem of the workspace approximation using new interval technique. We formulate and solve the constrained three-criteria design optimization problem for this mechanism and find the best configuration of this robot for used parameter. In the future, we plan to apply this approach to robots of a more complex design [13]. Evaluation of the design objectives is a computationally demanding task. The computational complexity can be mitigated by applying high-performance computing methods and tools [14, 15].

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