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Parametric Study and Optimization of a Piezoelectric Energy Harvester from Flow Induced Vibration

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Abstract. Self-powered systems have become the need of the hour and several devices and
techniques were proposed in favour of this crisis. Among the various sources, vibrations, being
the most practical scenario, is chosen in the present study to investigate for the possibility of
harvesting energy. Various methods were devised to trap the energy generated by vibrating
bodies, which would otherwise be wasted. One such concept is termed as flow-induced vibration
which involves the flow of a fluid across a bluff body that oscillates due to a phenomenon known
as vortex shedding. These oscillations can be converted into electrical energy by the use of
piezoelectric patches. A two degree of freedom system containing a cylinder as the primary mass
and a cantilever beam as the secondary mass attached with a piezoelectric circuit, was considered
to model the problem. Three wake oscillator models were studied in order to determine the one
which can generate results with high accuracy. It was found that Facchinetti model produced
better results than the other two and hence a parametric study was performed to determine the
favourable range of the controllable variables of the system. A fitness function was formulated
and optimization of the selected parameters was done using genetic algorithm. The parametric
optimization led to a considerable improvement in the harvested voltage from the system owing
to the high displacement of secondary mass.

1. Introduction

Energy has been a crisis for quite a long time now and researchers are battling it out to find new sources
and techniques to produce power which is renewable and harmless. There has absolutely been no pause
when it comes to technological advancement and research and these sophisticated systems require
round-the-clock power supply which would not be feasible if energy is supplied through batteries, as
periodic replacement or recharge from an external source is necessary. These issues led to the necessity
of self-energizing systems which are capable of producing uninterrupted and eco-friendly power which
can be renewed over time. Depending upon the application, one such emerging field of science to
produce renewable energy which the present study deals with, is vibrations. Energy harvesting from
vibrations gained attention in the previous year [1-3]. There are quite a few techniques by which the
energy trapped in a vibrating system could be extracted effectively include the bistable systems,
magnetic levitation and energy harvesting from vibrations are a few to mention [4-6].
The vibrations can be converted to electric power through different transduction mechanisms including
electrostatic, electromagnetic, magnetostrictive and piezoelectric transduction [7,8]. Piezoelectric
transduction stands out among other techniques due to its feasibility of manufacturing, large power
density and ease of application. Steven Anton et al. [9] conducted an extensive review on energy
harvesting using piezoelectric materials and concluded that piezoelectric materials can be easily incorporated into many systems of vibration energy. It has a lot of variables which can be manipulated to obtain electrical energy from ambient vibration such that wireless electronics can be operated in a self-powered manner. In addition to the benefit of being lighter and smaller, the piezo electrics have three times higher energy density as compared to their counterparts electrostatic and electromagnetics [10].

The usual means to tap flow energy is by installing turbines in the path of the flow but is a costly affair. Therefore, flow-induced vibration energy harvester would operate at any scale and wind velocity and overcome all the limitations caused by miniaturized turbines. The flow past a bluff body such as a cylinder or prism generates vortices, known as Von Kármán Vortex Street, in the wake region which shed periodically. A study was conducted by Gao on flow-induced piezoelectric cantilever (PC) energy harvesters [11]. It was found that PC’s with unequal piezoelectric and non-piezoelectric lengths produced better voltage, current and power. Also a light-weight hollow bluff extension was attached to the tip of the PC and it was found that the power output increased with increasing size of a rectangular bluff extension than a circular one. The forces generated by the vortices may cause the bluff body to oscillate and this phenomenon is referred to as vortex-induced vibration (VIV). In cases where the vortices are shed at a frequency near the natural frequency of the cylinder, it undergoes high-amplitude oscillations. This phenomenon is called “lock-in” or “synchronization”. The dimensionless quantity relating the vortex shedding frequency with the free stream velocity is defined as the Strouhal number.

In the present study, a beam is attached with piezoelectric patches which would be excited upon interaction with fluid flow. The effect of vortex shedding near the wake region of the bluff body is also taken into account with the help of different wake oscillator models. A complete parametric study of the system is performed and optimized using genetic algorithm for maximum output power.

2. Mathematical Model

The physical model consists of an elastically restrained bluff cylinder kept in a fluid flow. A cantilever beam is attached to the fluid flow as shown in figure 1 (a). Piezo patches are placed along the surface of the cantilever beam.

The system shown in figure 1 (a) can be modelled as a discrete two degree of freedom model as shown in figure 1 (b) with $M_1$ the mass of the cylinder, $M_2$ the mass of the cantilever beam, $K_1$, $K_2$, $C_1$ and $C_2$ are the associated stiffness and damping in the system. $U$ is the free stream velocity. $F_{lift}$ is the lift force generated due to vortex shedding. $Y_1$ and $Y_2$ are the displacement of mass $M_1$ and $M_2$. The piezoelectric circuit is represented by its internal capacitance $C_p$, Resistance $R$. $\theta$ is the electromechanical coupling coefficient and $V$ is the voltage generated across the resistor.

The mathematical model of the flow induced vibration energy harvester consists of three components which are listed below in sections 2.1 – 2.3.
2.1 Structural Equation
The equation of motion of the primary mass can be written as,

\[ M_1 \ddot{Y}_1 + (C_1 + C_2) \dot{Y}_1 + (K_1 + K_2)Y_1 - C_2 \dot{Y}_2 - K_2 Y_2 = F_{lift} \quad (2.1) \]

Where, \( M_1 = m_s + m_f \) is the mass accounting to primary mass and fluid added mass, \( C_1 \) is the viscous damping accounting to primary mass and the support and fluid added damping, \( C_2 \) is the damping due to piezo-electric material and secondary mass \( K_1 \) and \( K_2 \) is the stiffness of primary and secondary system, \( F_{lift} = \frac{1}{2} \rho U^2 D C_{L_0} q \) is the Lift force per unit length, \( \rho \) is the fluid density, \( U \) is the free stream velocity, \( D \) is the Diameter, \( q = \frac{2C_L}{C_{L_0}} \) is the wake variable, \( C_{L_0} \) is the reference lift coefficient and \( C_L \) is the fluctuating lift coefficient.

Similarly, the equation of motion of the secondary mass can be written as

\[ M_2 \ddot{Y}_2 - C_2 \dot{Y}_1 + C_2 \ddot{Y}_2 - K_2 \dot{Y}_2 = \theta \dot{V} = 0 \quad (2.2) \]

Where, \( \theta \) is electro mechanical coupling coefficient, \( V \) is the voltage induced across the load resistance and \( M_2 \) is the mass accounting to secondary mass and fluid added mass. In order to develop a 2-D model for vortex induced vibration, all mass, damping and stiffness parameters are defined per unit length.

2.2 Fluid Flow Equations
Bluff bodies when exposed to a fluid flow experience vortex induced vibration. The fluid flow is modelled using nonlinear oscillators because of their potential to describe most of the characteristics of the system. Three different wake oscillator models namely Harmonic lift force model, Hartlen & curie lift oscillator model, Fachinetti’s wake oscillator model are considered in this study to model the fluid flow. In all these models, the model parameters are determined by curve fitting experimental results for stationary and forced cylinders in the Reynolds number range between 10^2-10^5.

2.2.1 Harmonic Lift Force Model. For small value of Reynolds’ number, the lift coefficient is almost constant and does not fluctuate. This allows us to model the lift force as function of sine or cosine given by the relation below,

\[ F_{lift} = F_0 \sin(\omega_s t) \quad (2.3) \]

where \( F_0 = \frac{1}{2} \rho U^2 D C_{L_0} \) is the magnitude of lift force, \( \rho \) is the fluid density, \( U \) is the fluid velocity, \( D \) is the diameter of the cylinder, \( C_{L_0} \) is the reference lift coefficient, \( \omega_s = \frac{2 \pi S_t U}{D} \) is the vortex shedding frequency and \( S_t \) is the Strouhal number. In general, \( C_{L_0} \) is taken as 0.3 and strouhal number is taken as 0.2 in the sub-critical range, 300 < Re < 5x10^5

2.2.2 Hartlen and Currie Lift Oscillator Model. In this model [12], the aerodynamic lift force is considered to be proportional to the instantaneous value of fluctuating lift coefficient. The instantaneous lift coefficient was derived from the response of the proposed lift oscillator model, whose characteristic were deduced from experimental data. The oscillator is self-excited, its natural frequency is proportional to wind speed, and it is subjected to a coupling term proportional to the transverse cylinder velocity. The dimensional form for the variation of the instantaneous lift coefficient, \( C_L \) is given by

\[ \ddot{C}_L - \delta \omega_s \dot{C}_L + \frac{\gamma \dot{C}_L^3}{\omega_s^3} + \omega_s^3 C_L = F_c \quad (2.4) \]

Where \( F_c = b \omega_n \dot{Y}_1 \) is the force coupling term that gives the influence of structural oscillator on the wake oscillator and \( b, \gamma, \delta \) are experimentally determined constants. The parameter values, obtained from Hartlen and Currie (1970), are \( \delta = 0.02, \gamma = 0.6667, b = 0.4, C_{L_0} = 0.2 \).
2.2.3 Fachinetti’s Wake Oscillator Model [13]. The fluctuating nature of the vortex street is modelled by a self-excited and self-limiting nonlinear oscillator satisfying the van der Pol equation given by,
\[ \ddot{q} + \epsilon \omega_s (q^2 - 1) \dot{q} + \omega_s^2 q = F_c \] (2.5)

The fluid variable q is interpreted as a reduced vortex lift coefficient given by \( q = \frac{2c_l}{c_{L0}} \) where the reference lift coefficient \( c_{L0} \) is that observed on a fixed structure subjected to vortex shedding.

\( F_c \) is the force coupling term and is given by the relation \( F_c = \frac{2A}{D} \), where \( A=12, \epsilon =0.3 \) (A and \( \epsilon \) are experimentally determined constants).

2.3 Piezoelectric Equations [14]

The governing equation for the voltage generated across the piezoelectric material is obtained and is given by
\[ c_p \dot{V} + \frac{V}{R_t} + \theta \dot{V} = 0 \] (2.6)

Where \( c_p \) = capacitance of piezo electric material.
\( R_t \) = load resistance and \( \theta = \) electromechanical coupling coefficient

2.4 Non-Dimensional Form of the equations

The equations of motion can be expressed in a non-dimensional form as,
\[ y_1'' + (2\zeta_1 \Omega_1 + 2\zeta_2 \Omega_2 \mu) y_1' - 2\mu \zeta_2 \Omega_2 y_2' + (\Omega_1^2 + \Omega_2^2 \mu) y_1 - \mu \Omega_2^2 y_2 = Bq \] (2.7)
\[ y_2'' - 2\zeta_2 \Omega_2 y_1' + 2\zeta_2 \Omega_2 y_2' - \Omega_2^2 y_1 + \Omega_2^2 y_2 - \chi v = 0 \] (2.8)

Where, \( B = \frac{c_{L0} \rho D^2}{16\pi^2 S_t^2 M_1} \)
\( c_{L0} \) is the reference lift coefficient, \( \rho \) is the fluid density, \( D \) is the diameter of the cylinder, \( S_t \) is the Strouhal number, \( M_1 \) is the mass of the cylinder and the non-dimensional parameters are as follows, non dimensional displacement, \( y_1 = \frac{y_1}{D}, y_2 = \frac{y_2}{D} ; D= \) Diameter of cylinder, Non-dimensional time (\( \tau \)) = \( \omega_s t \); where \( \omega_s \) = vortex shedding frequency. Non-dimensional flow velocity \( u_0 = 2\pi St \frac{u}{\omega_n D} = \frac{\omega_s}{\omega_n} ; St= \) Strouhal number, \( u= \) fluid flow velocity; \( \omega_n = \) undamped first natural frequency of the system. Non-dimensional voltage \( V = \frac{V}{V_0} ; V_0 = \frac{\sqrt{M_1} \omega_n D^2}{\phi} \) is the standard reference voltage. \( \theta = \frac{\theta}{\sqrt{M_2 c_p}} \) = non dimensional electro-mechanical coupling coefficient. \( \alpha = \frac{1}{c_{pR_t}} \) is the reciprocal of time constant. Mass ratio (\( \mu \)) = \( \frac{M_2}{M_1} ; \) Damping ratio \( \zeta_1 = \frac{c_1}{2M_1 \omega_1} ; \zeta_2 = \frac{c_2}{2M_2 \omega_2} ; \omega_1 = \sqrt{\frac{K_1}{M_1}} ; \omega_2 = \sqrt{\frac{K_2}{M_2}} ; \Omega_1 = \frac{\omega_1}{\omega_s} ; \Omega_2 = \frac{\omega_2}{\omega_s} ; \lambda = \frac{1}{c_{pR_t \omega_s}} \) = dimensionless time constant;

Differentiation with respect to non-dimensional time is given by, \( \dot{f} = \omega_s f' \) and \( \ddot{f} = \omega_s^2 f'' \)

The non-dimensional form of wake oscillator models are given by
(i) Hartlen and Currie model
\[ q'' - \delta q' + \frac{\chi}{4c_{L0} u_0^2} q^3 + q' = \frac{2b}{c_{L0} u_0^2} y_1' \] (2.9)
(ii) Fachinetti model
\[ q'' + \epsilon(q^2 - 1)q' + q = A y_1'' \] (2.10)

The non-dimensional form of piezoelectric equation is given by
\[ v' + \lambda v + \frac{\theta}{\omega_n} y_2' = 0 \]  

(2.11)

Where, \( v = \) dimensionless voltage \( \lambda = \frac{1}{c_p R_t \omega_s} \) = dimensionless time constant and \( \theta \) is electro mechanical coupling coefficient.

3. Results and discussions

In this section, the frequency response plots of the system was generated using three wake oscillator models as mentioned in section 2.3 and the results were analysed to choose the best suited model. The equations of motion are integrated numerically including the wake oscillator models for the following parameter values which are kept constant - \( \mu=0.3, \zeta_1=0.01, \zeta_2=0.06, \alpha=0.05, \theta=0.5, \omega_1=3, \beta=0.7692 \).

3.1 Harvester with Harmonic Lift Force Model

The equations of motion for the harvester with harmonic lift force model is given in equations 2.1, 2.2 and 2.3. The equations of motion are integrated numerically to plot the variation of \( y_1, y_2 \) and \( V \) with non-dimensional flow velocity \( U_0 \). In order to fulfil the objective of this study, the region of operation of the system where maximum voltage can be harvested is to be determined. To serve this purpose, the non-dimensional displacement of primary mass and secondary mass and the non-dimensional voltage are plotted against flow velocity. The non-dimensional flow velocity \( U_0 \) is varied from 0 to 4. The plots reveal that the first and second resonant peaks occur at non-dimensional flow velocities of 1.03 and 1.733, respectively. Of the two resonant regions, the first resonant region would prove to be efficient as it produces higher peaks of secondary displacement and voltage with minimum primary displacement. The variation of the primary mass displacement \( (y_1) \) and secondary mass \( (y_2) \) with \( U_0 \) is shown in figure 2a and the variation of non-dimensional voltage with \( U_0 \) is shown in figure 2b.

![Figure 2. Variation of Non-dimensional a) Displacement b) voltage with non-dimensional velocity](image)

3.2 Harvester with Hartlen & Currie Model

The equations of motion for the harvester with Hartlen and Currie model is given in equations 2.1, 2.2 and 2.4. The equations of motion are integrated numerically to plot the variation of \( y_1, y_2, V \) and \( q \) with \( U_0 \). The same set of parametric values were considered and the simulation was carried out for Hartlen and Currie model. From figure 3a, it can be seen that the first peak occurs in and around the first resonant frequency which gives the minimum primary mass displacement and maximum secondary mass displacement. Figure 3b also shows that this resonant frequency produces maximum possible voltage from the harvester. This model supports the previous model in terms of the region of operation which is about the first resonant frequency.
3.3 Harvester with Facchinetti Model

The equations of motion for the harvester with Facchinetti model is given in equations 2.1, 2.2 and 2.5. The equations of motion are integrated numerically to plot the variation of $y_1$, $y_2$ and $V$ with non-dimensional flow velocity $U_0$. The plots that follow the explanation reveal the non-dimensional displacements of the primary mass (cylinder) and secondary mass (cantilever beam) as shown in figure 4a. As mentioned earlier, the point of interest in the plot lies at the resonant region where there exists maximum displacement of the secondary mass with a relatively less response in the primary mass. The non-dimensional frequencies at which resonance occurs are 1.03 and 1.733 and the non-dimensional displacement of the secondary mass at these resonant frequencies are 0.004172 and 0.003668, respectively.

The resonant characteristics are influenced by the choice of tuning parameter ‘β’ which could be because of the internal resonance where the natural frequencies are much closer to each other. The present set of parameter values were selected based on previous research and literature study and also a parametric study was conducted and included in later part of this paper.

Another response of interest is the non-dimensional voltage of which the first and second natural frequencies are almost equally dominant as shown in figure 4b. The maximum non-dimensional voltage occur in and around the first and second natural frequencies, 1.03 and 1.733, respectively.

3.4 Comparison among different wake oscillator models

A comparative study among the different models was conducted to verify the obtained results and responses. Figure 5 show the plot of the first and second resonant frequency region, respectively, of the wake models under study with same parametric values. It can be observed that there is only a slight negligible variation of result among the three models in and around the first resonant frequency whereas there is a drastic difference in response about the second resonant frequency using the Harmonic lift
force model when compared with the other two models. Hence, the harmonic lift force model turns out to be inadequate to model the system at higher velocities of the fluid.

![Figure 5. Wake model comparison around a) First resonance region b) second resonance region](image)

**Figure 5.** Wake model comparison around a) First resonance region b) second resonance region

### 3.5 Facchinetti model with the inclusion of an inductor

The dimensional form of the electrical equation, with the addition of an inductor, is modified as,

$$c_p \dot{V} + \frac{V}{R_t} + \frac{V}{L} + \theta \dot{Y}_2 = 0$$

(3.1)

Where \( L \) is the inductance of the circuit.

Non-dimensioning the above equation using \( \tau = \omega_s t \) along with non-dimensional displacement and voltage terms, we arrive at,

$$\ddot{U} + \frac{\alpha}{\omega_s} \dot{U} + \frac{\Phi}{\omega_s^2} U + \frac{\theta \beta_0}{\omega_s} \dot{Y} = 0$$

(3.2)

Where \( \Phi = \frac{1}{c_p L} \)

The equations of motion of motion for facchinetti’s wake oscillator model with inclusion of inductor is given by equations 2.1, 2.2, 2.5 and 3.1. The equations of motion are integrated numerically to determine the variation of \( y_1, y_2 \) and \( V \) with non-dimensional flow velocity \( U_0 \). The same set of parametric values as in section 4.1.3 are considered for the simulation. The figure 6 shows that the inclusion of an inductor into the circuit considered in the Facchinetti model has shown a significant improvement in the voltage response of the system while the other responses have shown a negligible difference.

![Figure 6. Variation of Non-dimensional a) Displacement of primary mass b) voltage with \( U_0 \) with inductor](image)

**Figure 6.** Variation of Non-dimensional a) Displacement of primary mass b) voltage with \( U_0 \) with inductor

### 3.6 Parametric Study

The parameters involved in modelling the system influence the response of the system. Hence, a study of the effect of various controllable parameters would help in improving the response of the system. The variables which can be altered and optimized to produce maximum voltage are Electrical parameter (\( \propto \)),
Secondary damping ratio ($\zeta_2$), Piezoelectric coupling parameter ($\theta$), Mass ratio ($\mu$), Inductance parameter ($\phi$).

3.6.1 Effect of secondary damping ratio. The damping value of the secondary mass has an inverse effect on the resonant peak value i.e., the resonant peaks reduce when the damping is increased. A similar trend can be for the displacement of primary mass, non-dimensional voltage as well as for the fluctuating lift coefficient. Another observation that can be made is that the system behaves as a single degree of freedom model for higher secondary mass damping values. It can also be observed that the fixed point theory is applicable as the plots in each figure pass through two invariant points which also opens up the possibility to find analytical solution using Den Hartog’s fixed point theory. The parametric study is performed for four values of secondary damping ranging from 0.06 to 0.3. The plot is shown in figure 7.

3.6.2 Effect of electrical parameter. The study of the influence of ‘$\alpha$’ on the displacement of secondary mass is crucial as it in turn affects the voltage that can be harvested. Also a significant shift in peaks of non-dimensional voltage is observed in figure 8 for different ‘$\alpha$’ values and it can be concluded from the plots that a choice of the lowest possible value of ‘$\alpha$’ would prove favourable in harvesting the maximum voltage.

Similarly, variation of voltage for different $\theta$, $\mu$, $\phi$ values were studied and appropriate inferences were made.

4. Parameter optimization

From the parametric study done in section 3.6, the effect of different parameter values on the output voltage was found out. For instance, in case of electrical parameter alpha, a low value of alpha results in high voltage generation. Therefore an optimization procedure is necessary in order to calculate the optimized parameters which satisfy the above requirement. The parameters to be optimized are non-dimensional flow velocity ($U_0$), mass ratio ($\mu$), electrical inductance parameter ($\phi$), damping ratio of the secondary system ($\zeta$), non-dimensional electrical coupling coefficient ($\theta$) and electrical capacitance parameter ($\alpha$). The objective of the optimization is to maximize the energy harvested. The problem can thus be converted as a maximization problem with objective function $f$.

$$f = \max v^2(U_0, \mu, \phi, \zeta, \theta, \alpha)$$

The constrained are defined with respect to the parameter as

$$0.1 < \mu < 0.3, 1 < \phi < 3, 0.05 < \zeta < 0.3, 0.5 < \theta < 2, 0.01 < \alpha < 0.1, 1.2 < U_0 < 2.6$$

The range of constraints are selected with respect to the parametric study conducted in section 3.6. The fitness function is obtained using response surface methodology. The fitness function is validated by...
comparing the voltage obtained through fitness function and numerical integration. Figure 9 shows that the results obtained using fitness function is fairly close to that obtained using numerical integration.

Figure 9. Validation of fitness function (Fachinetti model)

The optimization is done using Genetic Algorithm which optimize the parameters $\mu$, $\phi$, $\zeta$, $\theta$, $\alpha$, $U_0$ bounded as given earlier in equation (4.2).

The optimum parameter value for maximum energy harvesting subject to the constraints given by equation (4.2) is given in table 1.

| S.No. | $\mu$  | $\phi$  | $\zeta$ | $\theta$ | $\alpha$ |
|-------|--------|----------|---------|----------|----------|
| 1.    | 0.102  | 2.995    | 0.0511  | 0.298    | 0.015    |

The non-dimensional Voltage for the initial parameter values and for the optimized parameter values as a function of $U_0$ is shown in figure 10.

Figure 10. Graph comparing Voltage for initial and optimized parameter values for Fachinetti model

The parametric influence on the vibration absorption capacity and the amount of energy harvested is clearly visible in figure 10. The optimized parameters results in an increase in harvested voltage and thus results in an increase in harvested power.
5. Conclusion
In this work, the feasibility of harvesting energy from flow-induced vibration is tested and analysed. The system is modelled using a cylinder as the primary mass and a cantilever beam having piezoelectric patches as the secondary mass and exposed to a fluid flow. A 2 DOF model with a piezoelectric circuit is considered as a discrete approximation of the system under study. The different wake models are considered and analysed to model the system and the Facchinetti model has been observed to produce accurate results when compared to other models. A parametric study of the Facchinetti model containing an inductor is performed to select a proper range of values for the parameters involved. An optimization problem based on genetic algorithm is formulated for the purpose of optimizing parameters. The objective function is formulated using response surface methodology and the algorithm provided optimized values of parameters which enhanced the voltage that can be harvested from the system.

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