Non-Fermi liquid behavior and superconductivity in the dissipative model

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We study an exactly-solvable model which shows a zero-temperature transition from a non-Fermi liquid to a Fermi liquid as a function of particle density. The quantum critical point separating these two states is not associated with the usual ordering transition of a bosonic degree of freedom. We analyze the properties of this critical point in terms of an effective density of states, and we use this concept to generalize this transition to a class of related quantum critical points. We study the superconducting instability of the low-density (non-Fermi liquid) regime and we show that it has an enhanced pair susceptibility relative to the Fermi liquid. We comment on the applicability of our results to low-carrier density superconductors.

In recent years there has been great interest in understanding the physics of metallic states which are not Fermi liquids—motivated largely by a host of experimental observations [1]. One route to non-Fermi liquid behavior stems from proximity to a quantum critical point [2, 3], in other words a zero-temperature phase transition which occurs as a parameter such as pressure or chemical composition is varied. Examples include CePd$_2$Si$_2$ [4], and CeCu$_6$-xAu$_x$ [5] which show quantum critical behavior as pressure and gold concentration are varied respectively. In both of these materials the critical point separates a magnetically ordered state from a paramagnetic one.

The metallic state of the cuprates also shows non-Fermi liquid properties. Furthermore there is evidence of a pseudogap energy scale [6], $T^*$, which is suppressed to zero temperature at some critical doping $p_{\text{crit}}$ [7], and it has been suggested [8] that this forms a quantum critical point albeit masked by superconductivity. However, if this quantum critical point is responsible for the non-Fermi liquid physics of the metallic state, it differs from the above examples in that it does not appear to have an associated ordered phase: thermodynamic measurements [9] show that $T^*$ corresponds to a crossover and not a phase transition. It has been argued [10] that there may be a hidden order parameter, but an alternative scenario is that the cuprates show quantum criticality without associated order at a single dimension where thermal fluctuations destroy finite-temperature order, but this motivates the search for examples of zero-temperature phase transitions in $d > 1$ without any associated order.

In this Letter we consider a model of dissipative fermions, first studied by Wheatley [11], which possesses a zero-temperature transition but with no order other than the Fermi surface itself. At low carrier density, this model is a non-Fermi liquid metal but with a zero temperature transition to a Fermi liquid at a critical density.

We characterize the model’s behavior at the critical point by the Sommerfeld coefficient and the paramagnetic susceptibility as $T \to 0$. We present an interpretation of the phase diagram by introducing an effective density of states. This allows us to consider more general metal to metal transitions of this type. We also consider the superconducting instability of this model. Although our starting model is artificial, we speculate on the consequences of more general spectral functions.

Our starting point is to consider a single particle of mass $M$ coupled to a bath of harmonic oscillators [12]. This bath has a spectral density $J(\omega)$ and was originally introduced to mimic the dissipative effects of a real-world environment [13]. However, our philosophy here is to introduce the oscillator bath to give the single particle a non-trivial (i.e. non-local) dynamics in imaginary time while retaining the exact solvability of the system. This may be seen on integrating out the oscillators, which leads to a single-particle effective action $S_{\text{eff}} = S_{\text{KE}} + S_{\text{diss}}$, where $S_{\text{KE}} = \frac{1}{2}M\beta \sum n r_n r_n \omega_n^2$ is the kinetic energy and

$$S_{\text{diss}} = \frac{2\hbar M\beta}{\pi} \sum_{n=-\infty}^{\infty} r_n r_n \int_0^{\infty} d\omega \, J(\omega) \frac{\omega_n^2}{\omega(\omega^2 + \omega_n^2)} .$$

Here $r_n$ is the Fourier component of the path in imaginary time and $\omega_n = 2\pi \hbar n$ is a Matsubara frequency. One usually chooses $J(\omega) = \eta \omega$ with a high-frequency cut-off $\sim \omega_c$, so that $S_{\text{diss}}$ corresponds classically to a frictional force proportional to velocity, $F = \eta v$. In the first part of this paper we also make this choice, but later we consider the consequences of more general spectral functions.

An exact expression can be found for the single-particle partition function $z(\beta)$ [11]. When the temperature is low compared with the damping rate, $\tau_0^{-1} = 2M/\eta$, so $\hbar\beta/\tau_0 \gg 1$, this may be simplified to

$$z(\beta) = n_0 e^{-\varepsilon_0 d\beta} \left[ 1 + \frac{d\pi \tau_0}{6\hbar \beta} + O(\beta^{-2}) \right]$$

in $d$-dimensions, where $n_0 = (M/2\pi \hbar \tau_0)^{d/2}$ has the dimensions of density and $\varepsilon_0$ depends on the details of the cut-off in $J(\omega)$ and will be absorbed as a shift in the band-edge. The dissipation defines an energy scale $\varepsilon_d = 6\hbar/d\pi \tau_0$. 


Having defined the properties of the single particle we now consider a gas of such particles (i.e., fermions coupled to their own independent oscillator baths). Anti-symmetrizing appropriately for fermions is most easily done using a path integral approach [11] which gives the grand thermodynamic potential for the dissipative gas

$$\Omega = \sum_{m=1}^{\infty} (-1)^m \frac{e^{\beta \mu}}{m\beta} z(m\beta) .$$  

(3)

Wheatley [1] solved this equation numerically to determine the chemical potential $\mu$ and the susceptibility $\chi$, finding a $T = 0$ phase transition from a low density ($n < n_0$) phase with Curie susceptibility to a high density ($n > n_0$) phase with Pauli susceptibility. Here we consider the system properties in the vicinity of the critical point ($n \sim n_0$) and find a generalization of the model.

The chemical potential is given by the solution of

$$n = -\left( \frac{\partial \Omega}{\partial \mu} \right)_T = \frac{n_0}{1 + e^{-\beta\mu}} + \frac{n_0}{\varepsilon_d \beta} \ln(1 + e^{\beta\mu}) ;$$  

(4)

once we have determined $\mu$, the formulas for $\chi$ and $\gamma$ follow straightforwardly. The solution of Eq. (4) in the limit $T \to 0$ depends on whether we approach the critical density from higher or lower densities.

Approaching the critical density from above ($\delta_n = \frac{n}{n_0} - 1 > 0$) we find

$$\chi = \frac{n_0 \mu^2_B}{\varepsilon_d} \left( 1 + [\varepsilon_d \beta - 1] e^{-\beta \varepsilon_d \delta_n} \right) ,$$  

(5)

$$\gamma = \frac{n_0 \mu^2_B}{\varepsilon_d} \left( \frac{\pi^2}{3} + (\beta \varepsilon_d)^3 \delta_n^2 e^{-\beta \varepsilon_d \delta_n} \right) .$$  

(6)

We see that proximity to the critical density sets a new scale $\varepsilon_d \delta_n$. For low temperatures, $\beta \varepsilon_d \delta_n \gg 1$, the properties recover those of the free Fermi gas: $\chi$ and $\gamma$ are constant. We call this the Fermi liquid regime, although there are no Landau parameters since the particles do not interact with each other.

Approaching the critical density from below, $\delta_n < 0$, we find $\mu = k_B T \ln(-\delta_n^{-1} - 1)$ and hence for small $\delta_n$

$$\chi = \frac{n_0 \mu^2_B}{\varepsilon_d} \frac{\beta \delta_n}{1 - \delta_n} ,$$  

(7)

$$\gamma = \frac{n_0 \mu^2_B}{\varepsilon_d} \left[ \frac{\pi^2}{3} - \left( 1 - \frac{4}{\beta \varepsilon_d \delta_n} \right) \ln^2 |\delta_n| \right] .$$  

(8)

There is also a residual ground state entropy in the low density phase which is given by $S(T = 0) = k_B n_0 (\delta_n - |\delta_n| \ln |\delta_n|)$.

It is rather straightforward to see how this new phase of matter comes about. Usually the transition to a quantum fluid occurs when the thermal wavepacket of a particle starts to overlap with the wavepacket of neighboring particles. In this model dissipation cuts off the de Broglie wavelength such that for particle densities lower than $n_0$ the quantum liquid regime is not entered at any temperatures. For the case of a dissipative Bose gas one has the possibility of a Bose metal phase [4]—the bosonic analogue of the non-Fermi liquid state that results here for dissipative fermions.

Having characterized the properties in the vicinity of the critical density, we show in Fig. 1 the results of a numerical calculation of the Sommerfeld coefficient as the critical density is approached. This clearly shows the divergence in $\gamma$. It also shows how the divergence is cut off at higher densities by the crossover into the Fermi liquid regime. We have calculated the form of the divergence at the critical point itself and find that $\chi \propto T \ln T$ and $\gamma \propto T \ln^2 T$. At a given $T$ the properties of the high-density regime below a crossover scale $\delta_n \sim T/\varepsilon_d$ qualitatively resemble those of the low-density regime, which we illustrate by a dashed line on the phase diagram of Fig. 1b [4]. There is a second, high-temperature crossover which is set by the highest scale in the oscillator bath $\beta \omega_c \sim 1$, above which no dissipation is felt by the particles.

Further insight into the nature of the phase diagram and the route to generalizing this model can be found by considering the “effective” density of states. We use this term advisedly for the energy eigenstates of a dissipative particle are not well defined—it continuously interacts with the oscillator bath. However, since we have a single-particle partition function, Eq. (2), we can use it to define an effective single-particle density of states $g_{\text{eff}}$ via

$$z(\beta) = \int_0^\infty dE g_{\text{eff}}(E) e^{-\beta E} .$$

Thus the effective density of states is formally the inverse Laplace transform of the single-particle partition function. From here it is easy to show that the thermodynamics can be obtained directly from $g_{\text{eff}}$ by direct substitution into the usual definition of the non-interacting thermodynamic potential $\Omega = -T \int_0^\infty dE g(E) \ln [1 + e^{\beta(E - \mu)}]$. The effective density of states therefore captures the thermodynamics of the dissipative gas, although it does not contain information about correlations.

![FIG. 1: (a) $\gamma(T)$ for a range of densities. (b) $T - n$ phase diagram of the dissipative gas. Solid lines are phase transitions, dashed lines are crossovers, SC = superconductor and the circle is a quantum critical point.](image-url)
For the partition function of Eq. (4) we find an effective density of states near the band-edge
\[ g_{\text{eff}}(E + \varepsilon_0) = \frac{n_0}{\varepsilon_d} \left\{ \delta \left( \frac{E}{\varepsilon_d} \right) + \Theta \left( \frac{E}{\varepsilon_d} \right) \left[ 1 + O \left( \frac{E}{\varepsilon_d} \right) \right] \right\}. \] (9)

It is the initial delta function in the effective density of states that leads to the non-Fermi liquid to Fermi liquid transition. The delta function can accommodate \( n_0 \) fermions so for \( n < n_0 \) the Fermi energy is pinned there and the entropy is clearly finite at \( T = 0 \); this is the non-Fermi liquid regime. When \( n > n_0 \) the Fermi energy moves a distance \( \delta_n \varepsilon_d \) above the delta function, to the region where \( g_{\text{eff}} \) is weakly energy-dependent, and the system behaves like a degenerate Fermi gas. However, when \( T \sim \delta_n \varepsilon_d \) the Fermi function is sufficiently broad that the occupancy of the delta function fluctuates and there is a crossover to dissipative behavior.

We now consider extensions of this approach to consider a wider range of Fermi liquid to non-Fermi liquid transitions that can be found for dissipative fermions. The obvious extension is to go beyond Ohmic dissipation and consider more general oscillator baths with power-law spectral densities \( J(\omega) = n_0 \omega^\alpha \) up to a high-frequency cut-off \( \sim \omega_c \), where \( 0 < \alpha < 2 \). Integrating out the oscillators gives an effective partition function for \( N = 1 \) the function
\[ z(\beta) = \left( \frac{M}{2\pi \beta} \right)^{1/2} \prod_{n=1}^{\beta^{1/2}} \frac{\omega_n^2}{\omega_n^2 + \omega_n^\alpha / T^{2-\alpha}}. \] (10)

At low temperatures, \( k_B T \ll \hbar \tau_0^{-1} \), this is approximately
\[ z(\beta) = n_0 \left( \frac{\beta}{\tau_0} \right)^{1-\alpha} e^{-\varepsilon_0 \beta} \left[ 1 + \frac{2 - \alpha \pi \tau_0}{6} \hbar \beta \right] + O(\beta^{-2}). \] (11)

The band edge shift \( \varepsilon_0 \) is again cut-off dependent. In terms of the energy scale \( \varepsilon_\alpha = 6\hbar^2/(2 - \alpha)\pi \tau_0 \), the inverse Laplace transform gives, to leading order in \( E/\varepsilon_\alpha \),
\[ g_{\text{eff}}(E + \varepsilon_0) = \begin{cases} n_0 \left( \frac{E}{\varepsilon_d} \right)^{\alpha/2} & \text{for } \alpha > 1, \\ n_0 \left( \frac{\delta_n(E/\varepsilon_d)}{\varepsilon_d} \right) + E & \text{for } \alpha = 1, \\ \text{non-integrably divergent} & \text{for } \alpha < 1. \end{cases} \] (12)

So, for \( \alpha < 1 \) an infinite number of particles can be accommodated in the ground state and the gas is a non-Fermi liquid for all particle densities. For \( \alpha > 1 \) we always have conventional Fermi gas behavior. Thus only Ohmic dissipation gives rise to a quantum phase transition between a Fermi liquid and a non-Fermi liquid.

Nevertheless, the notion of an effective density of states gives us an alternative route to explore other models. Since a quantum phase transition will always be present when there is a delta function in \( g_{\text{eff}} \), we will work directly with a generalized \( g_{\text{eff}}(E) = n_0 / \varepsilon_d \left[ \delta(E/\varepsilon_d) + \Theta(E/\varepsilon_d) \right] \), rather than explicitly choose the spectral function of the oscillator bath. We must have \( \nu > -1 \) for \( g_{\text{eff}} \) to be integrable. This gives the partition function
\[ z(\beta) = n_0 \left[ 1 + \Gamma(\nu + 1) (\varepsilon_d T)^{\nu+1} \right]. \] (13)

Substituting this into Eq. (3) and using the properties of the polylog \( \text{Li}_\nu(x) = \sum_n \frac{x^n}{n^\nu} \), we find at \( n = n_0 \)
\[ \chi = \frac{n_0 k_B^2 (\nu + 1)^\nu}{\varepsilon_d} \left( 1 + \frac{1}{\varepsilon_d} \right)^\nu \ln^{\nu+1} \varepsilon_d \beta, \] (14)
\[ \gamma = \frac{n_0 k_B^2 (\nu + 1)^{\nu+2}}{\varepsilon_d} \left( 1 + \frac{1}{\varepsilon_d} \right)^{\nu+1} \ln^{\nu+2} \varepsilon_d \beta. \] (15)

Thus only for \( \alpha < 0 \) is the quantum critical point characterized by divergences in \( \chi \) and \( \gamma \). Note that for all \( \alpha \) the Wilson ratio \( \gamma/\chi \) is proportional to \( \ln T \).

We now address the applicability of this model to real physical systems. Our method of including all non-trivial effects via a bath of oscillators has parallels with dynamical mean field theory where interactions are included via a self-consistent frequency-dependent self-energy \cite{17, 18}. We have not sought a self-consistent approach since our goal was to categorize all the possible transitions in the model. Nevertheless our starting model may be directly relevant to physical systems such as low density Fermi gases which couple primarily to low energy bosonic modes rather than each other. In particular, this necessitates short range interactions. Examples could include neutral \(^6\text{Li} \) ions in atom traps where the bosonic modes would be the “optical molasses” of laser cooling giving a dissipative environment \cite{13}.

In a condensed matter context we apply this to very low carrier density metals with strong electron-phonon coupling but in a temperature window where the Coulomb repulsion does not dominate (above any tendency to Wigner crystallize for example). One possible case is SrTiO\(_3\). This is a band insulator \cite{20} but becomes a superconductor with \( T_c < 1 \text{ K} \) \cite{21} on electron doping to a concentration of about \( 10^2 \)\text{ cm}^{-3}. The mechanism of superconductivity in this material has long been in question but because superconductivity only appears below a critical density. It is therefore natural to ask whether the non-Fermi liquid state we have considered is unstable to pairing.

We calculate the pair susceptibility in the dissipative model via the pair propagator \cite{22}. In \( d \) dimensions the real-space density matrix \( \rho(r, \beta) \) for a dissipative particle is given approximately by
\[ \rho \approx \left( \frac{m}{\pi \hbar} \right)^{d/2} \left\{ \frac{1}{(\beta)^d/2} + \frac{1}{r_0^d/2} \right\} \exp \left[ -r^2 \left( \frac{m}{\hbar^2 \beta} + \frac{1}{\beta \epsilon_r^2} \right) \right]. \] (16)
where \( \xi = \sqrt{\pi \hbar |t_0|/m \ln 2} \) is a dissipation length. The Matsubara Green’s function can then be calculated using
\[
G_{\omega_n}(r) = \int_0^\infty \rho(r, x) e^{(\mu + i\hbar \omega_n)x} dx,
\]
which follows from the definitions of \( G_{\omega_n} \) and \( \rho \). This integral is only well defined for \( \mu < 0 \), which is the case for densities \( n < \frac{n_0}{c^2} \), but it can be analytically continued to all \( \mu \), as can be checked by evaluating it for free fermions. Substituting Eq. (16) with \( d=3 \) yields
\[
G_{\omega_n}(r) \simeq \left( \frac{m}{2\pi \hbar^2} \right)^{3/2} e^{-r^2/2\sigma^2} e^{-r\sqrt{\hbar/(\mu + i\hbar \omega_n)}},
\]
\[
\times \left[ \frac{2\pi \hbar^2}{m r^2} \right]^{1/2} - \left( \frac{\hbar}{\hbar_0} \right)^{3/2} \frac{1}{\mu + i\hbar \omega_n} \right].
\]
(17)

From the definition of the pair propagator \( K(r) = T \sum \omega_n G_{\omega_n}(r) G_{-\omega_n}(r) \) we can determine \( T_c \) in the usual way as the solution of \( 1 = \int dr V(r) K(r, \beta) \). We consider a BCS-like attractive potential with small finite width \( r_0 \). For \( n < n_0 \) this leads to
\[
T_c = V_{eff} \left( \frac{m}{\hbar^2} \right)^{3/2} \frac{1 - 2n/n_0}{\ln((2n/n_0 - 1)}.
\]
(18)

where the normalized potential \( V_{eff} \) depends on the details of \( V(r) \) at \( r < r_0 \). For \( n > n_0 \), on the other hand, \( T_c \) is heavily suppressed and superconductivity does not occur for arbitrarily small \( V_{eff} \). The dependence of \( T_c \) on \( n \) is shown in Fig. 1b. The pairing enhancement can be traced back to the delta function in \( g_{eff}(E) \).

However, we can expect a different field dependence of the upper critical field, \( B_c2(T) \), for superconductivity in the dissipative model because of the appearance of the dissipation length scale. A magnetic field destroys superconductivity by dephasing the two electrons that make up a Cooper pair; in a dissipative gas the electrons are already dephased due to the oscillator bath and so we expect a field to have negligible effect at small \( T \). \( B_c2(T) \) is given by the solution of \( 1 = \int_{r_0}^\infty dr K(r, \beta) e^{-r^2/2\hbar} \).
\]
(24)

For \( n < n_0 \) we find just below \( T_c \) that \( B_c2 = 3\pi \sigma / (m^2 c^2 (1 - T/\phi_c)) \), and at low temperatures \( B_c2(T) \sim \ln T \). In practice this divergence in \( B_c2(T) \) would be cut off by Landau quantization effects which we have not included. So we expect a significant enhancement in the upper critical field in the dissipative model compared with \( B_c2(0)/T_c^2 \) for which a Fermi liquid is \( 2\sigma^2/\phi_c (m^* \hbar^2) \). In thin films of SrTiO\textsubscript{3} \( B_c2(0)/T_c^2 \) this ratio is enhanced by about an order of magnitude relative to the Fermi liquid expression, where \( m^* = 5.3 m_0 \) is determined from \( \gamma_2 \).

Thus SrTiO\textsubscript{3} qualitatively resembles the dissipative gas both in its phase diagram and its relative insensitivity to applied magnetic fields.

To summarize, we have considered the Fermi liquid to non-Fermi liquid zero-temperature transition in a model of dissipative fermions. This critical point occurs without any associated ordering and may be generalized via the concept of an effective density of states. At low temperatures the non-Fermi liquid shows an enhanced pairing susceptibility and a phase diagram similar to the low density superconductor SrTiO\textsubscript{3}. More generally, this type of non-Fermi liquid state could occur during laser cooling in fermionic atom traps.

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\[\text{[1]}\text{See for example J. Phys. Cond. Matt. 8, #48 (1996).}\]
\[\text{[2]}\text{J. A. Hertz, Phys. Rev. B 14, 1165 (1976).}\]
\[\text{[3]}\text{J. A. Millis, Phys. Rev. B 48, 7183 (1993).}\]
\[\text{[4]}\text{S. R. Julian et al., J. Phys. Cond. Matt. 8, 9675 (1996); N. D. Mathur et al., Nature 394, 39 (1998).}\]
\[\text{[5]}\text{H. von Lohneysen et al., Phys. Rev. Lett. 72, 3262 (1994); H. von Lohneysen, J. Phys. Cond. Matt. 8, 9689 (1996).}\]
\[\text{[6]}\text{T. Timusk and B. W. Statt, Rep. Prog. Phys. 62, 61 (1999).}\]
\[\text{[7]}\text{J. L. Tallon and J. W. Loram, Physica C 349, 53 (2001).}\]
\[\text{[8]}\text{J. L. Tallon et al., Phys. Stat. Solidi 215, 531 (1999).}\]
\[\text{[9]}\text{J. W. Loram et al., Physica C 235, 134 (1994).}\]
\[\text{[10]}\text{S. Chakravarty, R. B. Laughlin, D. K. Morr, and C. Nayak, Phys. Rev. B 63, 094503 (2001).}\]
\[\text{[11]}\text{J. M. Wheatley, Phys. Rev. Lett. 67, 1181 (1991).}\]
\[\text{[12]}\text{A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. 46, 211 (1981); Ann. Phys. 149, 374 (1983).}\]
\[\text{[13]}\text{In Ref. [2] the oscillator bath was introduced phenomenologically. More careful derivations exist for special cases [see I. S. Beloborodov et al., cond-mat/0006337 for example], and these find that long-range interactions cut off the dissipation as \( T \to 0 \).}\]
\[\text{[14]}\text{J. M. Wheatley and A. J. Schofield, Int. J. Mod. Phys. B 6, 655 (1992).}\]
\[\text{[15]}\text{The phase diagram in Fig. 1b is reminiscent of the \( T - \mu \) phase diagram of a gas of free fermions [S. Sachdev, Quantum Phase Transitions, (Cambridge University Press, Cambridge, 1999), Ch. 1]. In the latter case a dilute classical gas is separated from the Fermi liquid regime by a \( \mu = 0 \) quantum critical point; however, in the \( T = 0 \) and \( \mu < 0 \) part of the phase diagram there are no fermions present at all. The dissipative model allows this type of non-Fermi liquid to Fermi-liquid transition to occur for finite particle concentration.}\]
\[\text{[16]}\text{L. Lewin, Polylogarithms and Associated Functions (North Holland, New York, 1981).}\]
\[\text{[17]}\text{A. Georges, G. Kotliar, W. Krauth, and M. J. Rozenberg, Rev. Mod. Phys. 68, 113 (1996).}\]
\[\text{[18]}\text{Q. Si, J. Llewellyn Smith, and K. Ingersent, Int. J. Mod. Phys. B 13, 2331 (1999).}\]
\[\text{[19]}\text{S. Chu et al., Phys. Rev. Lett. 55, 48 (1985).}\]
\[\text{[20]}\text{L. F. Mattheiss, Phys. Rev. B 6, 4718 (1972).}\]
\[\text{[21]}\text{J. F. Schooley et al., Phys. Rev. Lett. 14, 305 (1965); E. Pfeiffer and J. F. Schooley, Phys. Lett. A 29, 589 (1969).}\]
\[\text{[22]}\text{A. J. Schofield, Phys. Rev. B 51, 11733 (1995).}\]
\[\text{[23]}\text{E. Helland and N. R. Werthamer, Phys. Rev. 147, 288 (1966).}\]
[24] A. Leitner, D. Olaya, C. T. Rogers, and J. C. Price, Phys. Rev. B 62, 1408 (2000).
[25] E. Ambler, J. H. Colwell, W. R. Hosler, and J. F. Schooley, Phys. Rev. 148, 280 (1966).