Chiral symmetry in lattice QCD

A.A.Slavnov

Steklov Mathematical Institute, Russian Academy of Sciences, Gubkina st. 8, GSP-1, 117966,Moscow, Russia

A chiral invariant effective action for lattice QCD is proposed. Its connection to the multifermion model is established. A possibility of using this action for computer simulations is discussed.

1. Introduction.

Approximate chiral symmetry is known to play an essential role in QCD. Hence a faithful transcription of chiral symmetry in lattice regularization of QCD is of prime importance. Recently an important progress was achieved in this direction (see e.g. [1], [2], [3], [4], [5], [6], [7], [8]). In particular it was understood that Nielsen-Ninomiya "no-go" theorem [9] may be bypassed by avoiding some of its assumptions. Nevertheless all models used so far for numerical simulations do not possess exact chiral invariance.

The most important problem at present is to formulate a lattice version of QCD which allows efficient numerical simulations producing a "minimal" breaking of chiral symmetry. By a "minimal" breaking I mean that the model does not require chiral noninvariant counterterms in the continuum limit and for a finite lattice spacing symmetry breaking effects can be done negligible in a reasonable computer time.

In my paper [10] an effective action for lattice QCD, based on the idea of multifield formalism [10], was proposed, which provides exponential suppression of chirality breaking effects. In the present talk I show that this model may be described by a manifestly chiral invariant single field action. This action is formally nonlocal but possible nonlocal effects are suppressed.

2. The model.

The effective action for lattice QCD may be taken in the form

\[ I = I_W + \sum_{x,\mu} \bar{\psi}(x) \hat{D} \coth\left( \frac{\pi}{2} |(i\hat{D})^{1/2}D^{-2}(i\hat{D})^{1/2}| \right)\psi(x), \]

where \( I_W \) is the usual Wilson action for gluons,

\[ \hat{D} = \gamma_\mu(D_\mu + D_\mu^*) \]

\[ D^2 = \kappa aD_\mu^*D_\mu \]

(2)

and \( D_\mu \) is the lattice covariant derivative

\[ D_\mu \psi = \frac{1}{a}[U_\mu(x)\psi(x + a_\mu) - \psi(x)] \]

(3)

In the formal continuum limit \( a \to 0 \), the path integral of the exponent of this action obviously reproduces the usual QCD partition function.

The action [10] is manifestly chiral invariant, but formally nonlocal. We shall show however that this nonlocality is harmless as nonlocal effects disappear in the continuum limit and are suppressed for a finite lattice spacing. No spectrum doubling is generated by the action [10].

However this action is too complicated to be used for computer simulations. Luckily, the corresponding path integral may be also written in terms of a local multifield action which is suitable for numerical simulations. This local action was introduced before in the paper [10]

\[ \tilde{I} = I_W + \sum_{n=-\infty, n \neq 0}^{+\infty} \sum_x \bar{\psi}^n(x)[\hat{D} - nD^2]\psi^n(x) \]

(4)
Here the spinor fields $\psi^n$ have the Grassmanian parity $(-1)^{n+1}$. Integrating the exponent of the action \([1]\) over $\tilde{\psi}_n, \psi_n$, we get

$$Z = \prod_{n=\infty}^{+\infty} \det(\hat{D} - nD^2)^{(-1)^n}$$

$$= \exp \{ \sum_n (-1)^n \Tr \ln [\hat{D} - nD^2] \}$$

(5)

The exponent in the eq.(5) may be presented in the form

$$\sum_n (-1)^n \Tr \ln [\hat{D} - nD^2] =

\lim_{\Lambda \to \infty} \{ \Tr [\hat{D}(\alpha \hat{D} - nD^2)^{-1}] + \sum_n (-1)^n \Tr \ln (\Lambda \hat{D} - nD^2) \}$$

(6)

The limit of the second term at the r.h.s. is equal to

$$\Tr \ln (\hat{D}) + \text{const}$$

and in the first term we use the following relation

$$\Tr [\hat{D}(\alpha \hat{D} - nD^2)^{-1}] = \Tr [\alpha \hat{D} - n\hat{D}^{-1/2}D^2\hat{D}^{-1/2}]^{-1} = \sum_k (\alpha + inB_k)^{-1}$$

(7)

where $B_k$ are eigenvalues of the operator $B = (i\hat{D})^{-1/2}D^2(i\hat{D})^{-1/2}$

(8)

Using this representation one can perform the summation over $n$ in the eq.(3) explicitly with the result:

$$\lim_{\Lambda \to \infty} \int_1^{\Lambda} \frac{d\alpha}{\pi} \sum_k \left[ \frac{\pi |B_k^{-1}|}{\sinh(\pi \alpha |B_k^{-1}|)} \right] + \frac{1}{\alpha} =$$

$$\lim_{\Lambda \to \infty} \sum_k \ln \coth(\frac{\pi |B_k^{-1}|}{2}) -$$

$$- \ln \coth(\frac{\pi |B_k^{-1}|}{2})$$

(9)

where a nonessential constant was omitted. So we proved that

$$\prod_{n=\infty}^{+\infty} \det(\sum_{\mu} \gamma_\mu (D_\mu + D_\mu^\dagger)) -$$

$$- \text{na} \arctan(D_\mu^\dagger D_\mu)^{-1} =$$

$$\det(\frac{\pi}{2} \left\{ (i\hat{D})^{1/2}D^{-2}(i\hat{D})^{1/2} \right\})$$

(10)

which coincides with the expression obtained by integration the exponent of the action \([1]\) over $\psi, \bar{\psi}$.

We have two alternative representations for the determinant in the r.h.s. of equation \([10]\). The first one is given by the integral of the exponent of the manifestly chiral invariant but formally nonlocal action \([3]\), and the second one is the integral of the multifield action \([4]\). The last action is formally local, but includes infinite series of auxiliary fields. One may suspect that this infinite summation may generate some nonlocal effects. Such a possibility indeed exists, however due to exponential convergence of the series in the eq.(10) one may cut the series by some finite number $N$ and for $N$ big enough the correction term is small. For a finite $N$ the action \([3]\) is local. It proves that both representations define an "almost" local theory, which becomes exactly local in the continuum limit. The representation in terms of the effective action \([3]\) makes chiral symmetry manifest. Possible counterterms must respect chiral symmetry. In principle these counterterms could be nonlocal, as it happens in SLAC model \([11]\), however the alternative representation of quark determinant as the path integral of the local multifield action \([4]\) shows that it does not happen. The explicit proof of the exponential suppression of chirality breaking effects in perturbation theory for the model described by the multifield action \([4]\) was given in the paper \([8]\).

The nonlocal action \([1]\) is not suitable for numerical simulations. In the alternative multifield formulation one has to calculate the product of the usual Wilson fermion determinants. However the effective action \([3]\) includes an infinite number of fields and to make simulations one has to cut the series. The crucial question is how sensitive is the result to the cutting the series by some finite $N$. The estimate given in the paper \([8]\) in the framework of perturbation theory shows that for external momenta $q$ satisfying the relation $|q| \ll 1$ a small number of auxiliary fields is sufficient. This conclusion was also supported by numerical simulations in two-dimensional models. However only real four-dimensional simulations may check the efficiency of the method. The estimates show that the convergence may slow down
for small eigenvalues of the operator $B$ (eq.3). A nonperturbative study of the spectrum of this operator would be important for the estimate of convergence rate.

3. Discussion

It was shown in the previous section that the lattice QCD quark determinant may be presented in two alternative forms. In one form it is given by the path integral of the exponent of the chiral invariant but formally nonlocal effective action (1). Another form is given by the path integral of the local multifield action (4), which is not manifestly chiral invariant. We proved that both these forms represent the same quark determinant which enjoys therefore both locality and chiral invariance. In the framework of perturbation theory it was checked before that our model in the continuum limit reproduces exactly the quark determinant of massless QCD without any chiral noninvariant counterterms. For a finite lattice spacing chirality breaking corrections are exponentially small

$$\sim O(\exp\{-(\kappa \epsilon)^{-1}\}), \quad \epsilon \sim |q|a$$

, where $q$ is a maximal external momentum.

Finite quark masses may be easily incorporated into our scheme. One should modify the effective action (1) as follows

$$I = I_W + \sum_{x,\mu} \bar{\psi}(x) \hat{D} \coth(\frac{\pi}{2}(i\hat{D} + m)^{1/2}D^{-2} \times$$

$$\times (i\hat{D} + m)^{1/2})\psi(x) \quad (11)$$

Of course in this case the chiral invariance is explicitly broken by the bare quark mass.

Acknowledgements.
I am grateful to the organizers of the Conference "Lattice-00" in particular Apoorva Patel for hospitality and financial support.

REFERENCES
1. D.B.Kaplan, Phys.Lett. B288 (1992) 342.
2. S.A.Frolov, A.A.Slavnov, Nucl.Phys. B411 (1994) 647
3. R.Narayanan, H.Neuberger, Nucl.Phys. B412 (1994) 574.
4. Y.Shamir, Nucl.Phys. B406 (1993) 90.
5. P.Hasenfratz, Nucl.Phys. B525 (1998) 401.
6. H.Neuberger, Phys.Lett. B417 (1998) 141; B427 (1998) 353.
7. M.Lüscher, Phys.Lett. B428 (1998) 342.
8. A.A.Slavnov, Nucl.Phys. B544 (1999) 759.
9. H.B.Nielsen, M.Ninomiya, Nucl.Phys. B105 (1981) 219.
10. S.A.Frolov, A.A.Slavnov, Phys.Lett. B309 (1993) 344.
11. H.Karsten, J.Smit, Phys.Lett. B85 (1979) 100.