Numerical analysis of the effect of torispherical head on the buckling of pressure vessels

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Abstract. Effect of torispherical head on the buckling of pressure vessels was investigated by finite element (FE) method. The FE method with use of nonlinear buckling analysis was applied to predict the critical buckling Load. The influences of geometrical parameter such as thickness, knuckle radius and diameter of cylindrical part, on the buckling of heads have been studied. The Arc Length method which can control the load level, the length of the displacement increment and the maximum displacement was been used. By verification performed with the European Convention for Constructional Steelwork (ECCS) code, it was confirmed that the nonlinear buckling analysis could assure accurate results for buckling strength. It was shown that geometrical imperfections had little effect on buckling strength.

1. Introduction
Theories of thin-walled structures applied on pressure vessels were reviewed by Teng et al [1]. Results of numerical evaluation of buckling by using linear and non-linear theories of thin-walled shells of revolution have been presented.

Regarding to existence of non-continue stress in cylinder-head intersection, the choice of head considering the geometrical limitation and production facilities is the most important point in designing of pressure vessels. Torispherical heads are used commonly in pressure vessels because of their simple manufacturing and good strength in high pressure condition (Fig. 1). The buckling strength is one of the most important points in design of pressure vessels [2]. Internal pressurization is often an important loading condition for pressure vessels. Finite element method is often used in the buckling analysis of pressure vessels due to its capability.

The hydrostatic buckling of shells under different boundary conditions (B.C.) has been investigated by using energy method [3]. Results showed that in shells with medium height, under different B.C., buckling load is obtained by applying a scalar coefficient to the buckling load of the pin ended case, but this method is not applicable for the long shells for which the occurred circumferential waves from buckling, are higher than 3.

Teng et al [4] have introduced a numerical model, aided by the method of Eigenmode-affine, in the non-linear analysis of elastic shells. As the shells are sensitive to initial geometrical imperfections, predicting of their buckling resistance would be precise if those imperfections are taken into account.

In torispherical heads by increasing the ratio of knuckle radius per vessel diameter (r/D), dimension of spherical part decreases. Thus, the spherical part as part of the head becomes weaker and in a defined r/D a notable fall in buckling resistance is occurred [5].
European recommendation ECCS [6] had introduced several experimental relations for design of spherical shells. In following we will discuss about the buckling load and influence of different parameters on it. We will try to bring some propositions for limitation of buckling.

2. Numerical simulation method

Eigenvalue buckling analysis predicts the theoretical buckling strength of an ideal linear elastic structure. Bifurcation buckling refers to the unbounded growth of a new deformation pattern. Imperfection and material non-linearity can not be included in this analysis. Thus, the buckling strength obtained by Eigenvalue buckling analysis may differ from that of a real structure and often yield non conservative results. Therefore, care is needed when using this method in actual evaluation of buckling strength.

Non-linear buckling analysis including geometric and material non-linearity is usually the more accurate approach and is therefore recommended for design or evaluation of actual structures. There are two methods for obtaining buckling strength by non-linear buckling analysis. One basic approach is to constantly increment the applied loads until the solution begins to diverge, which can be obtained from load-controlled buckling analysis. In this approach, a simple static analysis will be done with large deflection extended to a point where the structure reaches its limit load.

3. Modelling

Non-linear finite element method with large deflection analysis was performed using commercial Ansys software. A three dimensional finite element model was generated using Ansys 9.0. For studying of buckling of pressure vessel with torispherical head; we modeled intersection of cylinder-head. The influence of welding and forming on material property were neglected while the effect of welding can be accounted for by modifying the yield stresses. The length of the cylinder was kept at $4\lambda$ ($\lambda$ is the linear elastic meridian bending of half-wave length given by $2.44 \sqrt{Rt}$) to ensure that the boundary effects at the far end of the cylinder do not interfere with the behavior of the intersection [7]. The model was meshed with shell 93 element. SHELL93 is particularly well suited to model curved shells. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y and z-axes. The element has plasticity, stress stiffening, large deflection, and large strain capabilities. The material of the intersections was assumed to have typical properties of steel: an elastic modulus of $1.9 \times 10^5 \text{ MPa}$; a Poisson’s ratio of 0.26, and yield stress of 206 MPa, and exhibit an elastic-perfectly plastic behavior.

For modeling of the geometrical imperfections in ANSYS package, we applied them in the form of initial deformations on the model [8]. For this reason, first we analyzed the model by using the "Update Geom" order, we give values to the magnificent factor. In fact by the resulted displacement of different buckling resolution, a new model with

![Fig. 1. Geometry of torispherical head [5].](image)
geometrical imperfection was obtained. This factor of geometrical imperfection, which is in fact deviation from the perfect model or the initial deformation, was presented by \( W_0 \). The buckled model is illustrated in the Fig. 2.

Fig. 2. Finite element model after buckling.

4. Choice of resolving method

For the solution of a nonlinear problem, the choice of solution method and load step (referred to as the time step in the ANSYS software), is very important. It should be made taking into account the anticipated structural behavior and the characteristics of the specific solution method.

Prior to carrying out a nonlinear buckling analysis, it is often beneficial to undertake a linear buckling analysis in order to obtain some appreciation of the buckling behavior. It may also help to identify those regions in the model that will first exhibit nonlinear response and at what load levels these nonlinearities develop.

There are several methods available in ANSYS for the solution of the nonlinear buckling equations. They include the Newton–Raphson and Arc Length methods. For geometrically nonlinear analysis, the Newton–Raphson method has been shown to be one of the best methods available. The most important characteristic of this method is its ability to converge even when the behavior is highly nonlinear. The method is also extremely accurate and generally converges quite rapidly, provided a realistic initial estimate of the displacement vector is used. With this method too, it is possible to control the solution error and estimate the rate of convergence, since for any particular load step the iterations continue until the specified solution error is achieved. Preliminary ANSYS FE analyses of the columns, in which compression loads were applied, showed that the Newton–Raphson method converged quite rapidly. For nonlinear buckling analysis, coarse time steps may be used in the pre-buckling regime, but fine steps are required close to the buckling load and in the post buckling regime. Different time steps may be used in the pre- and post buckling regimes through the multiple ‘load steps’ option within ANSYS. However, it is not easy to choose an appropriate maximum load level in load controlled analysis using the Newton–Raphson method.

Moreover, the Newton–Raphson method fails when snap-through occurs. The Arc Length method does not have this drawback and allows one to control the load level, the length of the displacement increment and the maximum displacement. Therefore, in the nonlinear buckling analyses, the Arc Length method was used [9].

5. Determination of buckling pressure

In our study we used a similar method as Theng`s study on cone-cylinder intersection [7]. In this approach the curves of load-displacement for nodes in one circumferential path near cylinder-head intersection were plotted. In the initial stage of loading, the curves for all nodes were similar,
indicating dominantly axis-symmetric behavior. As the pressure reached a certain value, the curves of nodes at different locations started to diverge from each other. The divergence of these curves is an indication of the growth of non-symmetric buckling deformations. The load corresponded to the divergence point is critical buckling load (Fig. 3).

![Graph](image)

Fig. 3. Determination of buckling pressure.
(Curve of load-radial displacement for torispherical head \(t/L=0.002, r/L=0.06\))

6. Parametrical study

The geometry of torispherical head was introduced with the \(t/L, L/D\) and \(r/L\) parameters in which \(t\) is the thickness of vessel which has identical values in heads and cylindrical part, \(L\) is the radius of spherical section, \(r\) is the Knuckle radius and \(D\) is the diameter of cylindrical section. The common heads are used for pressure vessels have radius of sphere equal to diameter of cylinder \((L/D=1)\). Our study was limited to heads with \(t/L \leq 500\) and \(r/L \geq 0.06\).

| \(t/L\) | \(r/L\) | \(P_{Ansys}\) (MPa) | \(P_{ECCS}\) (MPa) | Error % |
|---|---|---|---|---|
| 0.002 | 0.06 | 0.20872 | 0.179 | 16 |
| 0.08 | 0.22807 | 0.276 | 9 |
| 0.1 | 0.34149 | 0.332 | 2.8 |
| 0.14 | 0.43883 | 0.438 | 0.9 |
| 0.17 | 0.48337 | 0.514 | 5.9 |
| 0.2 | 0.542663 | 0.588 | 7 |
| 0.003 | 0.06 | 0.397296 | 0.400 | 0.67 |
| 0.08 | 0.46551 | 0.507 | 8 |
| 0.1 | 0.53235 | 0.610 | 12.7 |
| 0.14 | 0.8196 | 0.805 | 1.7 |
| 0.17 | 0.95637 | 0.944 | 1.2 |
| 0.2 | 1.058 | 1.08 | 1.9 |

The data was compared with the design rules given in European Convention for Construtional Steelwork (ECCS) [6]. The ECCS rules are based on buckling of the knuckle and the limit pressure is as given in Eq. (1):
\[ P_{cr} = \frac{120c \left( \frac{r}{D} \right)^{0.425}}{(D/t)^{1.5} \left( \frac{L}{D} \right)^{1.15}} \]  

Where \( c = 1.0 \) for crown and segment steel heads and \( c = 1.6 \) for cold-spun steel heads.

By verification performed with ECCS code, as Table 1, Fig. 4 and Fig. 5 illustrate, it was confirmed that the nonlinear buckling analysis could assure accurate results for buckling strength. The discrepancy between numerical analysis (FE) and ECCS [6] corresponded to the geometrical imperfection and residual stress which were taken into account by ECCS code and not by the FE.

6.1. Influence of knuckle radius on buckling pressure

In the internal pressure vessels, due to the existence of circumferential tensile stresses in both cylindrical and spherical parts, the intersection is deformed to internal side. Thus, both of the spherical and cylindrical part near the intersection was subjected to circumferential compressive stresses, and so buckling deflection occurred in both of them [10].

The growths of buckles can now be clearly seen in Fig. 6. The number of periodical waves on the ring can be counted from this plot to be 39. It should be noted that this counted number is a rough indication, as the buckling waves are not so uniform.

Fig. 6. Influence of knuckle radius 
\( t/L = 0.002 \) and \( r/L = 0.06 \).
Numerical results for the all points of intersection between the head and cylinder have shown clearly that the post-buckling behavior of internally pressurized sphere–cylinder intersections is stable (Fig. 6).

The curves of Fig. 7 show the influence of knuckle radius on the pressure buckling with different thicknesses. In the analysis the value of the radius of spherical part (L) was kept constant (L=0.5 m). For varying the ratio r/L, only the knuckle radius (r) was varied. We observe that for the whole thicknesses, increasing of radius leads to increasing of buckling pressure. So the knuckle radius is an influent parameter for increasing of buckling resistance.

![Fig. 7. Influence of knuckle radius on the buckling behavior of the torispherical head.](image)

### 6.2. Influence of thickness on buckling pressure

Fig. 8 illustrates the influence of t/L on the buckling pressure for different ratios of r/L. Increasing of t/L leads to increasing of buckling pressure. Also the rate of increase, comparing to increasing resulted from the ratio r/L, is higher. The slope of the curves of Fig. 8 comparing to those of the Fig. 7 shows it. Thus the buckling pressure is more sensitive to the thickness.

![Fig. 8. Influence of t/L on the buckling pressure of torispherical head.](image)
7. Result and discussion

7.1. Analysis of development of circumferential wrinkles
Due to the buckling, circumferential wrinkles are developed. The number and amplitude of theses wrinkles increase by increasing of internal pressure. The number and amplitude of developed wrinkles are a criterion to evaluate the buckling resistance [11]. Creation and development of wrinkles could be followed by the curve of radial displacement versus distance in a circumferential path in the vicinity of intersection of cylinder and head of the vessel (Fig. 9). Although by accounting of the number of wrinkles in this curve, the number of circumferential wrinkles in the head could be obtained.

![Fig. 9. Development of buckling wrinkle](image)

7.2. Comparing of buckling pressure with limit pressure
The limit pressure of vessel (P_L) with torispherical head is computed with Eq. (2) [12] derived by Shield and Drucker [13].

\[
\frac{P_{SD}}{F_y} = (0.33 + 5.5 \frac{r}{D}) \left( \frac{t}{D} \right) + 28(1 - 2.2 \frac{r}{D}) \left( \frac{t}{L} \right)^2 - 0.0006
\]

(2)

In which P_{SD} is the limit pressure (psi), P_{SD} = P_L and F_y is the yield stress (ksi).

Fig. 10 shows the critical buckling pressure (P_{cr}) versus the pressure obtained from relation (2). The curve shows that the buckling pressure is most often higher than the limit pressure. It means that the limit pressure is more critical than the buckling pressure. Thus for vessels with torispherical head the buckling pressure as a design criterion will not be sufficient.

![Fig. 10. Curve of buckling pressure versus limit pressure.](image)
Conclusion
In this research, we modeled intersection of cylinder-head of a pressure vessel to analyze buckling using the finite element approach. A three dimensional finite element model was generated. Non-linear finite element method with large deflection analysis including geometric and material non-linearity was performed. By resulted displacement of different buckling resolution, a new model with geometrical imperfection was obtained. In result following points were detected:
- The non-linear FE analysis brings the numerical results in the vicinity of experimental. Scatters are generally due the geometrical imperfections and residual stresses in vessel.
- Buckling pressure is influenced by the thickness and height of the vessel. Higher thickness and height lead to a better buckling resistance.
- The influence of knuckle radius in decreasing of compression stresses and also increasing of the buckling pressure is underlined.
- Using of the buckling pressure criterion in design of torispherical heads would be conservative.

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