Recent progress in *ab-initio* studies of nuclear reactions of astrophysical interest with $A \leq 3$

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Abstract. We review the most recent theoretical studies of nuclear reactions of astrophysical interest involving few-nucleon systems. In particular, we focus on the consequences for the solar neutrino fluxes of the recent determination for the astrophysical $S$-factor of the proton weak capture by proton, and on the radiative capture of protons by deuterons in the energy range of interest for Big Bang Nucleosynthesis.

1. Introduction
The weak proton capture on protons, *i.e.*, the reaction $p + p \rightarrow d + e^+ + \nu_e$ (hereafter labelled $pp$), is the most fundamental process in stellar nucleosynthesis: it is the first reaction in the $pp$ chain, which converts hydrogen into helium in main sequence stars like the Sun. Also the proton radiative capture by deuteron, *i.e.*, the reaction $p + d \rightarrow ^3\text{He} + \gamma$ (hereafter labelled $pd$), is among the nuclear fusion which are part of the $pp$ chain. However, being the only possible reaction after the initial step, it has no interest for the solar neutrino fluxes. Nevertheless, this reaction is of interest for the theory of Big Bang Nucleosynthesis (BBN), since it is the main process through which deuterons are destroyed, and therefore it strongly affects the primordial deuterium abundance.

For both reaction, it is customary to express the reaction rates in terms of the astrophysical $S$-factor, $S(E)$, where $E$ is the initial center-of-mass (c.m.) energy of the two particles ($pp$ or $pd$), by the relation

$$S(E) = E \exp(2\pi \eta) \sigma(E). \quad (1)$$

Here $\eta = \alpha/v_{\text{rel}}$ is the Sommerfeld parameter, $\alpha$ being the fine structure constant and $v_{\text{rel}}$ the $pp$ or $pd$ relative velocity, and $\sigma(E)$ is the capture cross section.

The most recent studies of the $pp$ reaction are those of Refs. [1, 2]. The study of Ref. [1] is the first one performed within the so-called chiral effective field theory ($\chi$EFT) approach. The goal of this work was to apply $\chi$EFT to the $pp$ fusion, comparing the results for the $S$-factor with what was obtained within the conventional phenomenological approach, and studying the effects to the $S$-factor arising from higher order electromagnetic contributions in the Hamiltonian and the $P$-wave components of the initial $pp$ scattering state. A first attempt to estimate the theoretical uncertainty was also performed, basically varying consistently in the nuclear Hamiltonian and nuclear currents the $\Lambda$ cutoff, present in the regularization momentum-cutoff function needed to take care of the power-law behavior for large momenta of potentials and currents. It should be noticed, though, that some inconsistencies in the study of Ref. [3] were present, as, for example,
the different chiral order for the two- and three-nucleon potential and also the axial current. These inconsistencies have been overcome in the study of Ref. [2], where the same chiral order is kept in the nuclear potentials and currents. This allows to quantify the theoretical uncertainty for the \( pp \) fusion in a more consistent way. However, the contributions beyond \( S \)-waves in the initial \( pp \) scattering state and the higher order terms in the electromagnetic part of the Hamiltonian were not included. The conclusions of that study are that the theoretical uncertainty for the zero-energy astrophysical \( S \)-factor cannot be reduced more than 0.7 \%. Furthermore, the central values of \( S(0) \) is in very nice agreement with the results of Ref. [1].

We will review in Section 2 the \( \chi \)EFT framework used to perform the \( pp \) fusion calculation, and we will consider the consequences of these most recent determinations of \( S(E) \) on the solar neutrino fluxes, as they have been investigated in Ref. [4].

The interest in the \( pd \) radiative capture has received a boost recently within the BBN context, since, in order to achieve agreement between the measured primordial deuterium abundance [5] and the BBN prediction, it has been questioned the accuracy of the only available experimental data in the BBN region [6]. Therefore, an \( ab\text{-initio} \) calculation of the astrophysical \( S \)-factor, as well as a more precise direct measurement, have become very urgent. The most recent theoretical study has been performed in Ref. [3], within the so-called conventional approach, which uses realistic phenomenological models for nuclear potentials and currents. Before, the \( S \)-factor was studied in Refs. [7, 8], still within the conventional approach, and it was found an excellent agreement between the theoretical results and the available experimental data [9] around the solar Gamow peak [8, 10]. In the energy range of interest for BBN, on the other hand, the theoretical predictions of Ref. [8] were found to be 2–10 \% higher than the central value of the polynomial best fit of Ref. [10]. Furthermore, the only available experimental data in the BBN energy range are those of Ref. [6], which are quite in disagreement with the polynomial best fit of Ref. [10]. Therefore, a new experimental determination is highly necessary and this is the main motivation behind the experiment recently proposed by the LUNA Collaboration at the Gran Sasso National Laboratories (Italy), with the goal of measuring the \( pd \) \( S \)-factor in the BBN energy range with a 3 \% accuracy.

We will review in Section 3 the conventional phenomenological framework used to study the \( pd \) astrophysical \( S \)-factor, as performed in Ref. [3], and we will review its implications for BBN in Subsection 3.2.

2. The \( \chi \)EFT framework and the \( pp \) fusion
We consider here the study of the \( pp \) fusion within the \( \chi \)EFT framework of Ref. [1]. We review in the following subsection the main ingredients of the calculation, and in Subsection 2.2 we discuss the results for the astrophysical \( S \)-factor and their consequences on the solar neutrino fluxes.

2.1. The \( \chi \)EFT theoretical framework
In a very schematic view, the \( \chi \)EFT framework can be seen as a formulation of Quantum Chromodynamics (QCD) in terms of effective degrees of freedom suitable for low-energy nuclear physics: pions and nucleons. The symmetries of QCD, in particular its (spontaneously broken) chiral symmetry, severely restrict the form of the interactions of nucleons and pions among themselves and with external electroweak fields, and make it possible to expand the Lagrangian describing these interactions in powers of \( Q/\Lambda_{\chi} \), \( Q \) being the pion momentum and \( \Lambda_{\chi} \) being the chiral-symmetry-breaking scale (here \( \Lambda_{\chi} \sim 700 \text{ MeV} \)). As a consequence, classes of Lagrangians emerge, each of the order \( (Q/\Lambda_{\chi})^{n} \) and each involving a certain number of unknown coefficients, the so-called low-energy constants (LECs), which arise when the high-energy degrees of freedom are integrated out. These LECs are in practice constrained by fits to experimental data. The potentials and currents derived within this framework have power-law behavior for large
momenta, and are regularized by introducing a momentum-cutoff function. In Ref. [1], this function is taken of the type \( \exp(-q^4/\Lambda^4) \) (\( q \) is the three-momentum transfer), and it is expected that increasing the chiral order \( n \), the dependence on \( \Lambda \) will become weaker. In \( \chi \text{EFT} \), two- and three-nucleon interactions (TNIs) arise naturally and are used to describe the few-nucleon systems. Of course, in the case of the \( pp \) fusion, TNIs are not present. However, they are needed to fix the LECs entering the axial current (see below). In Ref. [1], the two-nucleon \((NN)\) interaction is the one derived by Entem and Machleidt in Ref. [11] at next-to-next-to-leading order (N3LO) and the TNI is derived at next-to-next-to leading order (N2LO), in the local form of Ref. [12]. To be noticed that, in the \( pp \) sector, the nuclear \( NN \) potential has been augmented by the Coulomb interaction and the higher-order electromagnetic (EM) terms, due to two-photon exchange and vacuum polarization. These higher-order terms are the same as those of the Argonne \( v_{18} \) (AV18) \( NN \) potential [13], and therefore also retain short-range corrections associated with the finite size of the proton charge distribution. The effect of the additional distortion of the \( pp \) wave function, induced primarily by vacuum polarization, on the astrophysical \( S \)-factor will be discussed below.

The charge-changing weak current has been first derived in \( \chi \text{EFT} \) by Park et al. in the late nineties, using the so-called heavy-baryon chiral perturbation theory (HB\( \chi \text{PT} \) approach, where the baryons are treated as heavy static sources, and the perturbative expansion is performed in terms of the involved momenta over the baryon mass (see Refs. [14, 15] and references therein). In particular, the polar-vector part is related, via the conserved-vector-current constraint, to the (isovector) EM current, and at N3LO includes in the model of Park et al. [16], apart from one- (OPE) and two-pion-exchange (TPE) terms, two contact terms—one isoscalar and the other isovector—whose strengths are parametrized by the LECs \( g_{AS} \) and \( g_{AV} \). Few years ago, the problem of deriving the EM current and charge operators in \( \chi \text{EFT} \) has been revisited by Pastore et al. [17] and, in parallel, by Kölling et al. [18]. Pastore et al. used time-ordered perturbation theory (TOPT) to calculate the EM transition amplitudes, which allows for an easier treatment of the so-called reducible diagrams than the HB\( \chi \text{PT} \) approach. On the other hand, Kölling et al. used the method of unitary transformation, the same one used to derive the chiral potentials mentioned above. The EM operators of Pastore et al. have been found in good agreement with those of Kölling et al. but significantly different from those of Park et al., especially for the structure of the contact terms. This might give rise to significant differences in the study of those reactions where the polar-vector current gives significant contributions, as muon capture on light nuclei. Studies to verify whether this is really the case are currently underway.

The axial-vector current are diagrammatically represented in figure 1 up to N3LO, where they are listed according to their scaling in \( Q \), the pions’ and nucleons’ momenta. The leading-order (LO) contribution consists of the well known single-nucleon axial current, and is of order \( Q^{-3} \). At order \( Q^{-2} \) it turns out that there are no contributions, and therefore the next-to-leading (NLO) contribution is of order \( Q^{-1} \), and arises from the \((Q/m)^2\) relativistic corrections to the LO contribution (\( m \) is the nucleon mass). The N2LO currents, of order \( Q^0 \), consist of the OPE term and a contact term with one LEC, denoted with \( d_R \). The N3LO contributions arise from loop and TPE terms, and they have been calculated in Refs. [19, 20] using the TOPT approach mentioned above. They are not all represented in figure 1. The diagrammatic representation and a full discussion of these contributions can be found in the original references. These N3LO contributions at the moment have been used only to calculate the Gamow-Teller matrix element of tritium \( \beta \)-decay [20]. In fact, in the study of the \( pp \) fusion of Ref. [1], as well as Ref. [2], the N3LO axial currents are not retained. Therefore, in principle, a new study of this reaction with the N3LO terms in the axial current is required. However, considering that the two-body contributions to the astrophysical \( S \)-factor are of the order of a 1–2 \%, we do not expect to see a significant change in the \( S \)-factor, going to N3LO.
\[ \mathcal{O}(Q^{-3}) \]
\[ \mathcal{O}(Q^{-1}) \]
\[ \mathcal{O}(Q^0) \]
\[ \mathcal{O}(Q^1) \]

\textbf{Figure 1.} Diagrams illustrating one- and two-body \(\chi\)EFT axial currents entering at LO \((Q^{-3})\), NLO \((Q^{-1})\), and N2LO \((Q^0)\), and N3LO \((Q^1)\). Nucleons, pions, and weak probes are denoted by solid, dashed, and wavy lines, respectively. The solid square represents the relativistic corrections to the one-body current, while the solid circles in the N2LO contribution represent the contact terms. Only three of all possible N3LO contributions are shown.

Some remarks on the fitting procedure of the LEC \(d_R\) are in order. As first shown in Ref. [21], \(d_R\) can be related to the LEC \(c_D\) entering one of the two contact terms present in the TNI at N2LO, via the relation

\[ d_R = \frac{m}{\Lambda_\chi g_A} c_D + \frac{1}{3} m(c_3 + 2c_4) + \frac{1}{6}, \]  

(2)

where \(g_A\) is the single-nucleon axial coupling constant, \(c_3\) and \(c_4\) are LECs of the \(\pi N\) Lagrangian, already part of the chiral \(NN\) potential at NLO, and \(\Lambda_\chi\) is the the chiral-symmetry-breaking scale. Therefore, it has become common practice since the work of Ref. [22], to fit \(c_D\) (and \(c_E\)—the other LEC entering the N2LO TNI) to the triton binding energy and the Gamow-Teller (GT) matrix element in tritium \(\beta\)-decay. The values obtained in this way for \(c_D\) and \(c_E\) are listed, for different values of the cutoff \(\Lambda\), in Refs. [15, 23, 24, 25], where they have been used to study muon capture on deuteron and \(^3\)He, the \(A = 3\) and \(4\) elastic scattering observables, and the nuclear matter equation of state in many-body perturbation theory, respectively. In the study of the \(pp\) fusion of Ref. [1], the values obtained in Ref. [15] were used. However, it should be noticed that these values are strongly dependent on the chiral order retained in the calculation, as shown in Ref. [20]. Such a dependence should not be considered a problem of the theory, since LECs are not observables. It is expected from the theory, though, that the physical observables do not significantly change, if the calculation is performed consistently. Whether this is the case is currently under investigation.

\textbf{2.2. Results for the \(pp\) fusion}

We recall the main characteristics of the calculation of the \(pp\) fusion performed in Ref. [1]: (i) the astrophysical \(S\)-factor is studied in the wide energy range, from 2 to 100 keV. The solar Gamow peak is at \(\simeq 6\) keV, but in larger-mass stars the Gamow peak becomes 15 keV. (ii) The \(\chi\)EFT N3LO \(NN\) potential of Ref. [11] is augmented not only of the Coulomb interaction but also of the higher-order EM interaction contributions, like those due to two-photon exchange,
Darwin-Foldy and vacuum polarization. (iii) All the $L \leq 1$ $pp$ partial waves are considered. To our knowledge, this is the first time that the $P$-wave contributions are retained. The results for the $pp$ astrophysical $S$-factor can be summarized as follows: (i) the zero-energy $S$-factor $S(0)$ obtained retaining all the $L \leq 1$ partial waves and the full one- plus two-body weak current contributions is $S(0) = \left( 4.030 \pm 0.006 \right) \times 10^{-23}$ MeV fm$^2$, where the theoretical uncertainty is due to the fitting procedure (the uncertainty on the value for the experimental GT matrix element reflects on an uncertainty for the LECs). The theoretical uncertainty arising from the cutoff dependence is extremely small. (ii) The $P$-wave contribution is $\sim 0.02 \times 10^{-23}$ MeV fm$^2$. Therefore, the $S$-factor obtained retaining only the $^1S_0$ contributions in the initial $pp$ wave function results in $S(0) \simeq 4.01$ MeV fm$^2$, in very nice agreement with the value quoted in Ref. [10]. (iii) The higher-order EM contributions to the initial $pp$ wave functions are of the order of 1% or less, as already obtained in Ref. [26]. (iv) The study of Ref. [1] has shown that the results obtained in $\chi$EFT or within the conventional phenomenological approach are in excellent agreement.

To be noticed that the most recent study for the $pp$ astrophysical $S$-factor at zero energy is given in Ref. [2], where $S(0) = \left( 4.047^{+0.024}_{-0.021} \right) \times 10^{-23}$ MeV fm$^2$, in nice agreement with the results of Ref. [1]. As already mentioned above, the 7 % theoretical uncertainty has been determined within a more robust approach. See Ref. [2] for more details.

From the astrophysical $S$-factor, it is possible to obtain the $pp$ reaction rate $R$ via the relation

$$R = N_A \langle \sigma v \rangle = \frac{3.73 \times 10^{10}}{\sqrt{\hat{\mu} T_9^3}} \times$$

$$\times \int_0^\infty S(E) \exp \left( -2\pi \eta - 11.605 \frac{E}{T_9} \right) dE,$$

where $N_A$ is the Avogadro number, $\langle \sigma v \rangle$ is the Maxwellian-average rate, $\hat{\mu} = 0.504$ is the $pp$ reduced mass in atomic mass units, $T_9$ is the temperature in units of $10^9$ K, and $\eta$ is the Sommerfeld parameter as defined above. The integration over the center-of-mass energy $E$ in Eq. (3) can be performed numerically with standard techniques. This has been done in Ref. [4], where we have analyzed the effect of the adoption of different $pp$ reaction rates on stellar models, focusing, in particular, on the age of mid and old stellar clusters (1-12 Gyr) and on standard solar model predictions. Here we consider only the standard solar model predictions, and, in particular, the results for the neutrino fluxes. In table 1 we report the values for the central solar temperature and the neutrino fluxes obtained using the astrophysical $S$-factor of Ref. [1], and that of Refs. [27, 10, 28]. By inspection of the table we can conclude that: (i) the largest difference in the central temperature (for the standard solar model with the $pp$ rate from Ref. [27]) is of the order of 3%. (ii) Due to the high temperature dependence for all the neutrino fluxes except for the $pp$ one, the effect on solar neutrinos is not totally negligible, even if small, and it can reach the order of few %. (iii) The maximum difference for the solar neutrino fluxes corresponds to the adoption of the $pp$ reaction rate of Ref. [27], reaching in some cases a maximum of about 8%. (iv) The $\sim 1\%$ effect of the $P$-partial waves to the $pp$ astrophysical $S$-factor, and therefore on the reaction rate, turns out into a maximum of 3% effect on the $^8B$, $^{15}O$ and $^{17}F$ neutrino fluxes.

3. The conventional approach and the $pd$ fusion
We consider here the $pd$ fusion reaction, studied within the conventional approach. We review this approach in the next Subsection, and we summarize the results for the $pd$ fusion and their implications for BBN in Subsection 3.2.
Table 1. The central solar temperature (K) and neutrino fluxes ($s^{-1} cm^{-2}$) as obtained with the $pp$ S-factor of Ref. [1] with both $S$- and $P$-waves included, considered as our reference model and labelled $(S + P)$, are given in the first column. The others columns list the relative difference between our reference model and the results obtained from the $S$-factor of Ref. [1], but with only the $1S_0$ component, labelled $(S)$, Ref. [27], Ref. [10], and Ref. [28].

| $T_C [10^7 K]$ | $\Phi_{pp} [10^{10}]$ | $\Phi_{pep} [10^8]$ | $\Phi_{lep} [10^8]$ | $\Phi_{Be-7} [10^6]$ | $\Phi_{B-8} [10^6]$ | $\Phi_{N-13} [10^8]$ | $\Phi_{O-15} [10^8]$ | $\Phi_{F-17} [10^6]$ |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Ref. [1]($S + P$) | Ref. [1]($S$) | Ref. [27] | Ref. [10] | Ref. [28] |
| $1.54794$ | $-1\%$ | $-3\%$ | $< 1\%$ | $-1\%$ |
| $6.020$ | $1\%$ | $2\%$ | $< 1\%$ | $1\%$ |
| $1.446$ | $-2\%$ | $-6\%$ | $1\%$ | $-1\%$ |
| $8.584$ | $-1\%$ | $-3\%$ | $2\%$ | $2\%$ |
| $4.503$ | $-1\%$ | $-3\%$ | $< 1\%$ | $-9\%$ |
| $3.694$ | $-3\%$ | $-7\%$ | $4\%$ | $-2\%$ |
| $2.417$ | $-2\%$ | $-6\%$ | $6\%$ | $-1\%$ |
| $1.811$ | $-3\%$ | $-8\%$ | $7\%$ | $-2\%$ |
| $3.373$ | $-3\%$ | $-8\%$ | $7\%$ | $-2\%$ |

3.1. The conventional approach

The conventional approach has been developed well before the $\chi$EFT approach described above, and, although very successful in reproducing a large body of experimental data, it is affected by the “original sin” of having no connection (or no clear connection) with QCD. In fact, this approach is purely phenomenological, and is based on realistic models for the nuclear interactions and currents. The nuclear interaction includes both two- and three-nucleon potentials. These are constructed to reproduce the $A = 2$ large body of experimental data with a $\chi^2/datum \sim 1$, and the $A = 3$ binding energies. In our study of the $pd$ fusion, the Argonne $v_{18}$ (AV18) model [13] for the $NN$ interaction, and the Urbana IX (UIX) model [29] for the TNI are used. These models are quite complex, and therefore it is necessary to have at hand an $ab$-initio technique to solve the $A = 3$ bound and scattering problem. To this aim, we use the hyperspherical harmonics (HH) technique, a variational approach which adopts a basis of HH functions to expand the bound or the scattering wave functions (in this second case, for relative interparticle distances small compared with the interaction range). We will not describe this method in details and refer the reader to the review of Ref. [30]. Here, we only remark that the HH method is the only one which can calculate the nuclear wave function for the $pd$ scattering state at low relative energies, as the ones of interest for BBN, including the Coulomb interaction between the charged initial particles. The $A = 3$ bound and scattering properties, as the triton and $^3$He binding energies, the $nd$ scattering length and elastic scattering observables as cross sections and analyzing powers, are in general nicely reproduced by the AV18/UIX potential model, used in conjunction with the HH technique. Some discrepancies appear only for few delicate polarization observables (see for instance Ref. [31]).

The interaction between the considered nuclear systems and the external electromagnetic probe implies the construction of the electromagnetic current and charge operators. Here we consider only the current operator. A recent review of also the electromagnetic charge operator can be found in Ref. [32]. The nuclear electromagnetic current operator is constructed so as to satisfy the current conservation relation (CCR) exactly with the adopted Hamiltonian. The one-body term, the so-called impulse approximation (IA), obtained performing a $1/m$ expansion ($m$ is the nucleon mass) of the single-nucleon covariant current, satisfies CCR with the kinetic energy operator. The two-body electromagnetic current consists of two contributions, the so-
called model-independent (MI) and model-dependent (MD) terms. The MI contributions have been constructed in Ref. [8], where particular care has been put to verify CCR with the AV18 potential. In particular, the two-body currents related to the momentum-space components of the two-body potential (spin-orbit $L \cdot S$, $L^2$ and $(L \cdot S)^2$ operators) are difficult to treat in the meson-exchange scheme and a new procedure based on minimal substitution has been devised. In addition, in order to satisfy CCR with the “full” Hamiltonian operator, which also includes a three-body interaction term (here the UIX model), three-body electromagnetic currents need to be considered. They have been constructed in Ref. [8] both in the meson-exchange and minimal substitution scheme. The three-body currents, although they have found to give small contribution, are essential to describe correctly those observables sensitive to current conservation, as the $T_{20}$ and $T_{21}$ polarization observables in radiative capture of polarized deuterons on protons. The MD terms of the electromagnetic current are due to the $p\pi\gamma$ and $\omega\pi\gamma$ transitions and to the current associated with the excitation of one intermediate $\Delta$ resonance. These terms are transverse, and therefore they do not affect CCR. In the calculation for the $pd$ radiative capture of Ref. [3], a new one-body term is included in the nuclear electromagnetic current, which is a relativistic correction of the order $1/m^3$ to the leading $1/m$ term. It was first derived in Refs. [17, 16] in the context of $\chi$EFT, and it was found in Ref. [33] that it reduces the $nd$ radiative capture total cross section at thermal energies of about 4.5\%, bringing the theoretical prediction in a much better agreement with the experimental datum (within 4\%).

3.2. Results for the $pd$ fusion

The astrophysical $S$-factor for the $pd$ radiative capture is crucial to determine the consistency of BBN theoretical prediction for deuterium abundance, the new Planck results, and the most recent experimental determination of such abundance. In the absence of an accurate experimental determination in the energy range of interest for BBN, 30-300 keV, the $pd$ reaction has been studied within the conventional approach described above in Ref. [3]. The predicted astrophysical $S$-factor in the energy range of interest for BBN is shown in figure 2, where it is compared with the previous ab-initio calculation of Ref. [8], with the best fit of Ref. [10], and with the available experimental data of Refs. [34, 35, 6, 9]. By inspection of the figure we can conclude that the results of Ref. [3] are systematically larger than those of Ref. [8] as well as the polynomial fit of Ref. [10]. The origin of this difference can be traced back in part to the one-body $1/m^3$ term, responsible for an increase of 1–3\% over the whole energy range, but mostly to the more accurate new solutions for the $A = 3$ scattering problem. The difference with the available experimental data in the BBN region, i.e. those of Ref. [6], is even larger, which is the main motivation for a new experimental determination, as the one under progress by the LUNA Collaboration. Finally, the numerical uncertainty relative to the solution of the $A = 3$ scattering problem is lower than 1\%.

The effect of this new ab-initio determination of the astrophysical $S$-factor on the primordial deuterium abundance has been studied in Ref. [3] using the numerical code PArthENoPE (see Ref. [36]). In particular, the deuterium to hydrogen density ratio $^2H/H_\text{tot}$ has been computed as function of two parameters, the baryon density $\Omega_b h^2$ and the effective neutrino number $N_{\text{eff}}$, and has been compared with the experimental determination $^2H/H_{\text{exp}}$ of Ref. [5]. It has been found in Ref. [3] that for the Planck 2015 value of $\Omega_b h^2$ [37], and for standard value for $N_{\text{eff}}$, the ratio between the deuterium and hydrogen abundances is $^2H/H_{\text{tot}} = (2.46 \pm 0.03 \pm 0.03) \times 10^{-5}$, where the two errors are due to nuclear rate and $\Omega_b h^2$ uncertainties, respectively. This result is indeed in nice agreement with the experimental value of Ref. [5], $(2.53 \pm 0.04) \times 10^{-5}$. Of course, these results ought to be confirmed by direct measurement of the $pd$ $S$-factor.
4. Summary and Conclusions
The two low-energy reactions $p + p \rightarrow d + e^+ + \nu_e$ and $p + d \rightarrow ^3\text{He} + \gamma$ have been studied within an \textit{ab-initio} approach, the former using $\chi$EFT, the latter within the conventional approach. The observable of interest, the astrophysical $S$-factor, has been calculated, and the implications for the astrophysical environments where these reactions are important have been discussed. In particular, the $S$-factor for the $pp$ fusion is by now known with such an accuracy that it does not represent a source of uncertainty for stellar evolution studies. In fact, the $\chi$EFT framework allows for a robust determination of the theoretical uncertainty, quantified in Ref. [2] at the 7–8 \%/ level. The determination for the $pp$ $S$-factor of Ref. [1] implies a change in the central solar temperature of at most 3 \%/ , which however reflects to relative differences in the solar neutrino fluxes from the $^8\text{B}$, $^{13}\text{N}$, $^{15}\text{O}$ and $^{17}\text{F}$ of 2–8 \%, depending on which older database is used. To be noticed that the most commonly used database of Ref. [27] is the one which gives rise to the largest differences.

The $pd$ fusion has been studied in the energy range relevant for BBN, 30–300 keV, within the conventional approach, since it allows to derive electromagnetic currents which do satisfy \textit{exactly} the current conservation relation. The $pd$ $S$-factor has been found systematically larger than the available experimental data in the BBN range [6]. Furthermore, the $S$-factor predictions are
larger also than the older \textit{ab-initio} determination of Ref. [8] and than the polynomial best fit of Ref. [10]. The origin of this increase respect to the results of Ref. [8] can be mostly traced back to the use of more accurate solutions for the $A=3$ scattering problem. The primordial deuterium abundance which derives from this new determination of the $pd$ $S$-factor is in very nice agreement with the most recent experimental determination of Ref. [5], when the Planck 2015 value for $\Omega_b h^2$ [37] and the standard value for $N_{\text{eff}}$ are used.

Further studies in this sector can move into two directions: first, we would like to apply the $\chi$EFT framework also to the electromagnetic processes. This would allow to determine the theoretical uncertainty to be assigned to our predictions. In order to do this, it is necessary to construct a set of $\chi$EFT potential and currents, such that the current conservation relation is \textit{exactly} satisfied. A first step is represented by the new set of $\chi$EFT potentials derived in Ref. [38] and first used in Ref. [39]. These new potentials include $\Delta$-isobar degrees of freedom in the two-pion exchange term up to N3LO. Furthermore, being minimally non-local and having an operatorial structure similar to the one of the AV18 potential, they are simpler to be used than the other $\chi$EFT potentials. Finally, it is of interest to extend the \textit{ab-initio} framework also to the electromagnetic processes. This would allow to determine the $\chi$EFT potential and currents, such that the current conservation relation is satisfied. A first step is represented by the new set of $\chi$EFT potentials derived in Ref. [38] and first used in Ref. [39]. These new potentials include $\Delta$-isobar degrees of freedom in the two-pion exchange term up to N3LO. Furthermore, being minimally non-local and having an operatorial structure similar to the one of the AV18 potential, they are simpler to be used than the other $\chi$EFT potentials. Finally, it is of interest to extend the \textit{ab-initio} framework also to the electromagnetic processes.

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