Uniqueness of Rotating Charged Black Holes in
Five-Dimensional Minimal Gauged Supergravity

Haji Ahmedov and Alikram N. Aliev

Feza Gürsey Institute, Çengelköy, 34684 Istanbul, Turkey

(Dated: August 13, 2009)

Abstract

We study a five-dimensional spacetime admitting, in the presence of torsion, a non-degenerate conformal Killing-Yano 2-form which is closed with respect to both the usual exterior differentiation and the exterior differentiation with torsion. Furthermore, assuming that the torsion is closed and co-closed with respect to the exterior differentiation with torsion, we prove that such a spacetime is the only spacetime given by the Chong-Cvetiċ-Lü-Pope solution for stationary, rotating charged black holes with two independent angular momenta in five-dimensional minimal gauged supergravity.

Dedicated to Nihat Berker on the occasion of his 60th birthday
I. INTRODUCTION

Uniqueness is one of the most striking features of “the truth and beauty” of black holes in all spacetime dimensions. In four dimensions, general relativity describes the final equilibrium state of black holes in terms of stationary asymptotically flat exact solutions of spherical topology to the Einstein field equations. The fundamental property of these solutions is their uniqueness: In most general case, a stationary and asymptotically flat black hole is uniquely characterized by the mass, angular momentum and the electric charge \([1, 2, 3]\) (see also \([4]\) and references therein). The uniqueness has been a crucial basis for studying many remarkable properties of black holes, thereby constituting a firm ground for their search in the real universe. However, it turns out that the uniqueness property is fundamental to black holes only in four dimensions and it does not survive in higher dimensions.

For static and asymptotically flat vacuum black holes, the uniqueness of the Schwarzschild solution can still be extended to higher spacetime dimensions \([5]\), but it is not the case for rotating black holes. For instance, in five dimensions there exist rotating black hole solutions with different horizon topologies: The Myers-Perry solution with spherical horizon topology \([6]\) and the Emparan-Reall black ring solution \([7]\) with the horizon topology of \(S^2 \times S^1\). These solutions may have the same mass and angular momenta. Clearly, this fact breaches the black hole uniqueness in five dimensions. The lack of black hole uniqueness is also supported by a recent generalization of Hawking’s theorem \([2]\) to higher dimensions \([8]\). This generalization guarantees the existence of a higher-dimensional stationary black hole with a single rotational Killing symmetry, unlike the Myers-Perry solution, which possesses multi-rotational Killing symmetries. Thus, in general the higher-dimensional black holes are not uniquely characterized by their physical parameters, such as the mass and angular momenta. However, to classify these black holes one can still look for the uniqueness of each black hole solution separately. In particular, a uniqueness result along this line was achieved for a Myers-Perry black hole in five dimensions. Namely, it was proved that in five dimensions, the only stationary, asymptotically flat black hole solution with two rotational symmetries and spherical topology of the horizon is given by the Myers-Perry metric \([9, 10]\).

It is also a remarkable fact that the uniqueness results for stationary black holes in four and higher dimensions are intimately related to the hidden symmetries of these black holes. As is known, stationary black holes in four dimensions admit a closed conformal Killing-Yano
(CCKY) 2-form which encodes all hidden symmetries generated by 2-rank Killing-Yano and Killing tensors of these spacetimes [11, 12, 13]. Using this fact, it was shown that the most general solution of the Einstein field equations with a cosmological constant which admits the CCKY 2-form is given only by the Kerr-NUT-(A)dS metric [14, 15]. Recently, it was demonstrated that the higher-dimensional generalization of the Kerr-NUT-(A)dS spacetime constructed in [16] also admits a CCKY 2-form which generates the tower of hidden symmetries in higher dimensions [13, 17]. With this CCKY 2-form, the authors of [18] managed to prove that the higher-dimensional Kerr-NUT-(A)dS metric of [16] is a unique solution (see also [19]).

The aim of this Letter is to prove the similar uniqueness result for rotating charged black holes in five-dimensional minimal gauged supergravity. The general solution with two independent rotational symmetries that describes these black holes was found by Chong, Cvetič, Lü and Pope (CCLP) [20]. In a recent paper [21], it was shown that this solution can be put in a Kerr-Schild type framework with two independent scalar functions, that provides its simple derivation. The gyromagnetic ratios of these black holes were studied in [22]. The CCLP metric also admits hidden symmetries generated by a 2-rank Killing tensor. This results in a complete separability of variables for the Hamilton-Jacobi and Klein-Gordon equations [23, 24]. The separability properties of the equation of motion for a stationary string in this metric were examined in [25]. However, the CCLP metric does not admit the usual Killing-Yano tensor and therefore the separation of variables for the Dirac equation [26], unlike its uncharged counterpart [27, 28, 29, 30]. On the other hand, the author of [31] showed that such a separability can be achieved by adding a counter-term into the usual Dirac equation. The hidden symmetries underlying the separability of variables in the modified Dirac equation are governed by the generalized (“non-vacuum”) Killing-Yano equation [31]. A nice geometrical interpretation of this result was given in [32]. The authors introduced a torsion 3-form, defining it as the Hodge dual of the Maxwell 2-form. They showed that the CCLP metric admits a CCKY 2-form in the presence of the torsion and the associated 3-rank Killing-Yano tensor which ensures the separability of variables in the modified Dirac equation.

We prove that the only spacetime admitting a closed (with respect to both the usual differential operator and the differential operator with torsion) conformal Killing-Yano 2-form in the presence of torsion is given by the CCLP metric, provided that the torsion is
closed and co-closed with respect to differential operators with torsion. We note that in the asymptotically flat case, the uniqueness of rotating charged black holes in minimal ungauged supergravity was proved in [33] by extending the boundary value analysis of [9].

II. THE METRIC AND ITS HIDDEN SYMMETRIES

The five-dimensional minimal gauged supergravity is governed by the Lagrangian

\[ \mathcal{L} = (R + \Lambda) * 1 - \frac{1}{2} * F \wedge F + \frac{1}{3\sqrt{3}} F \wedge F \wedge A , \]

which results in the following system of Einstein-Maxwell-Chern-Simons field equations

\[ R_{\mu}^{\nu} = \frac{1}{2} \left( F_{\mu\lambda} F^{\nu\lambda} - \frac{1}{6} \delta_{\mu}^{\nu} F_{\alpha\beta} F^{\alpha\beta} \right) - \frac{1}{3} \Lambda \delta_{\mu}^{\nu} , \]

\[ dF = 0 \quad d * F - \frac{1}{\sqrt{3}} F \wedge F = 0 . \]

As we have mentioned above, the general rotating charged black hole solution subject to these equations was constructed in [20]. It is interesting that this solution can be written in the most simple form [34] (see also [32]) by using the “canonical” basis 1-forms given by

\[ e^1 = \sqrt{\frac{x - y}{4X}} dx, \quad e^2 = \sqrt{\frac{y - x}{4Y}} dy, \]

\[ e^\tilde{1} = \sqrt{\frac{X}{x(y - x)}} (dt + yd\phi), \quad e^\tilde{2} = \sqrt{\frac{Y}{y(x - y)}} (dt + xd\phi), \]

\[ e^0 = \frac{1}{\sqrt{xy}} \left[ \mu dt + \mu(x + y)d\phi + xyd\psi - yA_q - xA_p \right] , \]

where

\[ A_q = \frac{q}{x - y}(dt + yd\phi), \quad A_p = \frac{p}{y - x}(dt + xd\phi) , \]

such that \( Q = q - p \) and the electromagnetic potential 1-form have the form

\[ A = \sqrt{3}(A_q + A_p), \quad F = dA . \]

The functions \( X \) and \( Y \) are given by

\[ X = (\mu + q)^2 + a_1 x + a_3 x^2 + \frac{\Lambda}{12} x^3 , \]

\[ Y = (\mu + p)^2 + a_2 y + a_3 y^2 + \frac{\Lambda}{12} y^3 . \]
Thus, we have the metric in the form\(^1\)

\[
g = \sum_{a=1}^{2} (e^a e^a + e^{\tilde{a}} e^{\tilde{a}}) + e^0 e^0. \tag{9}\]

This metric involves four free parameters related to the mass, electric charge and two angular momenta of the black hole. We note that the parameter \(a_3\) in (7) and (8) can be eliminated using the translations in the directions of \(x\) and \(y\).

The authors of [32] suggested a modification of the conformal Killing-Yano equation, introducing a torsion into the spacetime. In particular, a “closed” conformal 2-rank Killing-Yano (CCKY) tensor in this spacetime obeys the equation

\[
\nabla^T h_{\mu \rho} = g_{\mu \nu} \xi_{\rho} - g_{\mu \rho} \xi_{\nu}, \tag{10}\]

which implies that

\[
d^T h = 0, \quad \xi = -\frac{1}{4} \delta^T h. \tag{11}\]

Here the covariant derivative operator with torsion acting on a vector field \(V\) is defined as follows

\[
\nabla^T V_{\nu} = \nabla_{\nu} V_{\nu} - \frac{1}{2} T_{\mu \nu} V^\mu, \tag{12}\]

where \(T\) is the torsion 3-form and \(\nabla_{\mu}\) is the usual covariant derivative operator. Moreover, we have the metricity condition \(\nabla^T g_{\mu \rho} = 0\). Similarly, for a 3-form field \(\Psi\) in five dimensions, we have

\[
d^T \Psi = d\Psi - (\ast T) \wedge (\ast \Psi). \tag{13}\]

We note that \(\delta^T\) is the adjoint of the exterior derivative operator with torsion \(d^T\). Further details of the differential operations with torsion can be found in [32].

Next, defining the torsion by the Hodge dual of the Maxwell 2-form \(F = dA\) through the relation

\[
T = \frac{1}{\sqrt{3}} \ast F, \tag{14}\]

and using the Maxwell-Chern-Simons equations [3] along with (13) and the fact that \(\delta^T T = \delta T\), it is easy to show that the torsion is “harmonic” with respect to \(d^T\) and \(\delta^T\) operations. That is, we have

\[
d^T T = 0, \quad \delta^T T = 0. \tag{15}\]

\(^1\) We adopt the positive-definite signature for convenience.
Remarkably, the spacetime (9) admits a non-degenerate CCKY tensor \((d^T h = 0)\) \([32]\), which is given by
\[
h = \sqrt{-x} e^1 \wedge e^\bar{1} + \sqrt{-y} e^2 \wedge e^\bar{2}.
\] (16)

It is straightforward to verify that this tensor is also \(d\)-closed, \(dh = 0\). It is important to note that the Hodge dual of this tensor is a 3-rank Killing-Yano tensor that explains the separability of variables for the Dirac equation \([31]\) in the metric (9). Moreover, this tensor also results in a 2-rank Killing tensor of this metric \([23, 24, 32]\).

III. THE UNIQUENESS

In this section, we wish to prove the uniqueness of the general rotating charged black hole solution of five-dimensional minimal gauged supergravity, constructed by Chong, Cvetič, Lü and Pope in \([20]\). Namely, we prove the following

**Theorem:** Suppose a five-dimensional spacetime admits, in the presence of torsion, a non-degenerate conformal Killing-Yano (CKY) 2-form \(h\) which is both \(dT\) and \(d\)-closed and the torsion is harmonic, satisfying the conditions \(d^T T = 0\) and \(\delta^T T = 0\). Then, this spacetime is the only spacetime given by the Chong-Cvetič-Lü-Pope solution for stationary, rotating charged black holes with two independent angular momenta in five-dimensional minimal gauged supergravity.

We will present the proof of this theorem in several steps: (i) We begin by noting that a 2-rank antisymmetric tensor \(h_{\mu\nu}\) on a metric space defines the linear map
\[
H \cdot v^\mu \equiv h^\mu_{\nu} v^\nu
\] (17)
and the “eigenfunctions” of this operator given by
\[
H \cdot e_a^\mu = -x_a e_a^\mu, \quad H \cdot e_{\bar{a}}^\mu = x_{\bar{a}} e_{\bar{a}}^\mu, \quad H \cdot e_0^\mu = 0, \quad a = 1, 2.
\] (18)
form a Darboux basis \([18]\). The CKY 2-form \(h\) determines an orthonormal Darboux basis, in which one can diagonalize the metric \(g\) and “skew”-diagonalize the 2-form \(h\). We have
\[
g = \sum_{a=1}^{2} (e^a e^a + e^{\bar{a}} e^{\bar{a}}) + e^0 e^0, \quad h = \sum_{a=1}^{2} x_a e^a \wedge e^{\bar{a}}.
\] (19)
Clearly, there still exists a freedom with respect to \( SO(2) \) rotations in \((e^a, e^\bar{a})\) 2-planes and we can use this freedom to choose the vector filed \( \xi \) in equation (10) as follows

\[
\xi^\mu = \sum_{a=1}^{2} \sqrt{Q_a} e^\mu_a + \sqrt{S} e^\mu_0 ,
\]

where \( Q_a \) and \( S \) are unknown scalar functions. For further convenience, it is also useful to use the dual Darboux basis \( e_A \) with \( A = a, \bar{a}, 0 \). In this notation, equations (18) reduce to the form

\[
H \cdot e^\mu_A = Z_A e^\mu_{\bar{A}} ,
\]

where the eigenvalues

\[
Z_a = -x_a , \quad Z_{\bar{a}} = x_a , \quad Z_0 = 0 .
\]

Using now the closedness conditions for CKY 2-form \( h \),

\[
d^T h = 0 , \quad dh = 0 ,
\]

we find that the torsion obeys the following algebraic equations

\[
T_{A[B} h^A_{C]} = 0 ,
\]

where square brackets denote antisymmetrization. These equations are solved by

\[
T = T_1 e^0 \wedge e^1 \wedge e^\bar{1} + T_2 e^0 \wedge e^2 \wedge e^\bar{2} .
\]

Later, we shall also present the explicit expressions for the components \( T_1 \) and \( T_2 \).

(ii) Next, using equation (10) along with (17), we arrive at the equation

\[
d^T H \cdot e^\mu_A = \xi_A e^B e^\mu_B - e_A \xi_B e^\mu_B .
\]

Combining this equation with (21) and taking into account the orthogonality condition \((e_A, e_B) = \delta_{AB}\), we find that

\[
dZ_A = \xi_A e^\bar{A} - \xi_{\bar{A}} e^A .
\]

This equation along with (20) determines the gradient of the eigenvalues \( x_a \). We have

\[
dx_a = \sqrt{Q_a} e^a .
\]

(iii) We shall now show that the CKY 2-form \( h \) under consideration is constant along the associated vector field \( \xi \). We note that

\[
\mathcal{L}_\xi h = d(\iota_\xi h) + \iota_\xi dh ,
\]
where $\iota_\xi$ is the interior product operator. Since $h$ is $d$-closed as well, the second term on the right-hand side vanishes. Using (27), we have

$$\iota_\xi h = -\sum_{a=1}^{2} x_a \sqrt{Q_a} e^a = -\frac{1}{2} d \left( \sum_{a=1}^{2} x_a^2 \right),$$

(29)

which shows the first term on the right-hand side of (28) is of an exact differential. Thus, we obtain that

$$\mathcal{L}_\xi h = 0.$$  

(30)

Let us now assume that $\xi$ is the Killing vector. Then applying the Lie derivative to equation (21), we obtain

$$\mathcal{L}_\xi e^\mu_A = -P_A e^\mu_A,$$

(31)

where

$$P_a = P_a = i \xi d \log \sqrt{Q_a}, \quad P_0 = i \xi d \log \sqrt{S}. $$

(32)

In obtaining these expressions we have used equations (26) and (27). We note that equation (31) can also be written in the alternative form

$$\partial_A \xi^C + (\omega^C_A - \omega^C_B) \xi^B = \delta^C_A P_A$$

(33)

and the connection 1-forms

$$\omega^C_A = \omega^C_B e^B$$

(34)

are defined by the equation

$$de_A = \omega^C_A \wedge e_C.$$  

(35)

Next, we define a symmetric operator $H^2 = H \cdot H$, for which we have

$$-H^2 e^\mu_A = Z^2_A e^\mu_A, \quad -H^2 e^\mu_A = Z^2_A e^\mu_A.$$  

(36)

Taking the usual covariant derivative of this equation, it is easy to show that

$$\sum_B (Z^2_B - Z^2_A) \omega^B_A e_B = dH^2 \cdot e_A + d \log Z^2_A e_A.$$  

(37)

Combining this equation with (25), we find that the connection 1-forms are given by

$$\omega^B_A = -\frac{1}{2} T^B_A + \frac{Z_A(\xi^B e^A - \xi^A e^B) + Z_B(\xi^A e^B - \xi^B e^A)}{Z^2_B - Z^2_A}, \quad A \neq B,$$

(38)
where $T_{AB} = e^C T_{CAB}$. Using this expression in (33) we see that for $b \neq a$

$$\partial_a Q_b = 0, \quad \partial_b Q_b = 0, \quad \partial_a S = 0 \quad (39)$$

and

$$\omega^a_a = -\frac{\partial \sqrt{Q_a}}{\partial x_a} e^a + \sqrt{S} \frac{x_a}{x_b} e^b + \sum_{b \neq a} \frac{x_a \sqrt{Q_b}}{x_a^2 - x_b^2} e^b - \frac{1}{2} T_a^a. \quad (40)$$

These results enable one to calculate explicitly the corresponding Lie derivatives of all basis 1-forms. We have

$$\mathcal{L}_{e_a} e_b = 0, \quad \mathcal{L}_{e_b} e_b = 0, \quad \mathcal{L}_{e_0} e_0 = 0. \quad (41)$$

With this in mind and for $\xi \neq 0$, it follows from equation (31) that $P_A = 0$. This justifies the assumption made above that $\xi$ is the Killing vector. That is,

$$\mathcal{L}_\xi g = 0. \quad (42)$$

(iv) Substituting the quantities (38) in equation (33), we obtain the following equations

$$\frac{\partial \sqrt{Q_a}}{\partial x_b} = \frac{x_b \sqrt{Q_a}}{x_a^2 - x_b^2}, \quad a \neq b, \quad (43)$$

$$\frac{\partial \sqrt{S}}{\partial x_a} + \frac{\sqrt{S}}{x_a} = T_a. \quad (44)$$

From equations in (43) we easily find that

$$Q_1 = \frac{X_1(x_1)}{x_1^2 - x_2^2}, \quad Q_2 = \frac{X_2(x_2)}{x_2^2 - x_1^2}, \quad (45)$$

where $X_a(x_a)$ is an arbitrary function. In order to solve equation (44) we need the components of the torsion tensor. From the condition $d^T T = 0$ we obtain that

$$T_1 x_1 + T_2 x_2 = 0, \quad (46)$$

and

$$\frac{\partial T_a}{\partial x_b} + \frac{T_a}{x_a} x_b = 2 \frac{x_b T_a - x_a T_b}{x_a^2 - x_b^2}, \quad a \neq b. \quad (47)$$

The solution to these equations is given by

$$T_1 = \frac{2 Q x_2}{(x_1^2 - x_2^2)^2}, \quad T_2 = -\frac{2 Q x_1}{(x_1^2 - x_2^2)^2}. \quad (48)$$
where $Q$ is an arbitrary constant. It is easy to check that with this solution the condition $\delta^T T = 0$ is fulfilled as well. Using (48) in equation (44) we find its solution in the form

$$\sqrt{S} = \frac{\mu}{x_1 x_2} + \frac{1}{x_1^2 - x_2^2} \left( p \frac{x_1}{x_2} - q \frac{x_2}{x_1} \right),$$

(49)

where $\mu$, $p$ and $q$ are constants parameters and $q - p = Q$.

(v) In the vacuum case with zero torsion, one can construct all Killing vectors admitted by the spacetime, using only the fact of the existence of a closed conformal Killing-Yano tensor in this spacetime [18]. For instance, in five dimensions in addition to the primary Killing vector $\xi$, we have two other Killing vectors given by

$$\varphi^A = K^A_B \xi^B, \quad \chi^A = \frac{1}{8} \varepsilon^{ABCD} h_{BC} h_{DE},$$

(50)

where the Killing tensor

$$K_{AB} = h_{AC} h^C_B - \frac{1}{2} \delta_{AB} h^2.$$  

(51)

However in the presence of torsion only $\chi$ appears to be the Killing vector. Indeed, using the identity

$$\nabla_{(A \chi B)} = \nabla^T_{(A \chi B)}$$

(52)

and (10) we find that

$$\nabla^T_{(A \chi B)} = \frac{1}{4} \varepsilon_{(AB)CDE} \xi^C h^{DE} = 0,$$

(53)

where round brackets stand for symmetrization. On the other hand, using (30) it is straightforward to show that

$$\nabla_{(A \varphi B)} = -\xi^C \nabla_C K_{AB}.$$  

(54)

Equations (10) and (51) enable us to put this equation in the form

$$\nabla_{(A \varphi B)} = \frac{1}{2} \xi^C (T_{AC}^D K_{DB} + T_{BC}^D K_{DA}).$$

(55)

Thus, it follows that in the presence of torsion, the information encoded in $h$ is not enough to construct the whole set of Killing vectors. Therefore, to construct the third Killing vector one needs to invoke the torsion as well. We assume that the putative third Killing vector has the form

$$\xi_A = \varphi_A + f \chi_A,$$

(56)
where \( f = f(x_1, x_2) \) is a scalar function. Then, from the associated Killing equations we find that

\[
\frac{\partial f}{\partial x_1} + \frac{x_1}{x_2} T_1 = 0, \tag{57}
\]

\[
\frac{\partial f}{\partial x_2} + \frac{x_2}{x_1} T_2 = 0. \tag{58}
\]

Substituting in these equations the expressions in (48), we find the simple solution

\[
f = \frac{Q}{x_1^2 - x_2^2}. \tag{59}
\]

Thus, the desired Killing vector is given by

\[
\eta = x_2^2 \sqrt{Q_1} e_1 + x_1^2 \sqrt{Q_2} e_2 + \left[ \sqrt{S(x_1^2 + x_2^2)} + \frac{Q x_1 x_2}{x_1^2 - x_2^2} \right] e_0. \tag{60}
\]

We can now choose the coordinate system \((t, \phi, \psi)\), such that

\[
\xi = \partial_t, \quad \eta = \partial_\phi, \quad \chi = \partial_\psi. \tag{61}
\]

and using equations (20) and (60) together with \( \chi = x_1 x_2 e_0 \), we find that

\[
e^1 = \sqrt{Q_1} (dt + x_2^2 d\phi), \quad e^2 = \sqrt{Q_2} (dt + x_1^2 d\phi) \tag{62}
\]

\[
e^0 = x_1 x_2 d\psi + \sqrt{S} dt + \left[ \sqrt{S(x_1^2 + x_2^2)} + \frac{Q x_1 x_2}{x_1^2 - x_2^2} \right] d\phi \tag{63}
\]

With these basis 1-forms and those given by (27) and (45), the metric in (19) satisfies the field equations (2) and (3) of five-dimensional minimal gauged supergravity, if one takes

\[
x = -x_1^2, \quad y = -x_2^2, \quad X_1 = -\frac{X}{x_1^2}, \quad X_2 = -\frac{Y}{x_2^2}, \quad \phi \rightarrow -\phi \tag{64}
\]

and

\[
F = \sqrt{3} \ast T. \tag{65}
\]

That is, it becomes precisely the same as the CCLP metric (9) with the canonical basis (4). This completes the proof of the theorem.

---

[1] B. Carter, Phys. Rev. Lett. 26 (1971) 331.
[2] S. W. Hawking, Commun. Math. Phys. 25 (1972) 152.
[3] D. C. Robinson, Phys. Rev. Lett. 34 (1975) 905.
[4] M. Heusler, Black Hole Uniqueness Theorems (Cambridge University Press, Cambridge, England, 1996)
[5] G. W. Gibbons, D. Ida and T. Shiromizu, Phys. Rev. Lett. 89 (2002) 041101.
[6] R. C. Myers and M. J. Perry, Ann. Phys. (N.Y.) 172 (1986) 304.
[7] R. Emparan and H. S. Reall, Phys. Rev. Lett. 88 (2002) 101101.
[8] S. Hollands, A. Ishibashi and R. M. Wald, Commun. Math. Phys. 271 (2007) 699.
[9] Y. Morisawa and D. Ida, Phys. Rev. D 69 (2004) 124005.
[10] S. Hollands and S. Yazadjiev, Commun. Math. Phys. 283 (2008) 749.
[11] M. Walker and R. Penrose, Commun. Math. Phys. 18 (1970) 265.
[12] R. Penrose, Ann. N.Y. Acad. Sci. 224, 125 (1973); R. Floyd, Ph.D. thesis, London University, 1973.
[13] V. P. Frolov and D. Kubizňák, Class. Quant. Grav. 25, 154005 (2008).
[14] W. Dietz and R. Rüdiger, Proc. R. Soc. Lond. Ser. A 375 (1981) 361.
[15] P. Taxiahris, Gen. Relat. Grav. 17 (1985) 149.
[16] W. Chen, H. Lü and C. N. Pope, Class. Quant. Grav. 23 (2006) 5323.
[17] D. Kubizňák and V. P. Frolov, Class. Quant. Grav. 24 (2007) F1.
[18] P. Krtouš, V. P. Frolov and D. Kubizňák, Phys. Rev. D 78 (2008) 064022.
[19] T. Houri, T. Oota and Y. Yasui, Phys. Lett B 656 (2007) 214.
[20] Z-W. Chong, M. Cvetic, H. Lü and C. N. Pope, Phys. Rev. Lett. 95 (2005) 161301.
[21] A. N. Aliev and D. K. Çiftçi, Phys. Rev. D 79 (2009) 044004.
[22] A. N. Aliev, Phys. Rev. D 77 (2008) 044038.
[23] P. Davis, H. K. Kunduri and J. Lucietti, Phys. Lett. B 628 (2005) 275.
[24] A. N. Aliev and O. Delice, Phys. Rev. D 79 (2009) 024013.
[25] H. Ahmedov and A. N. Aliev, Phys. Rev. D 78 (2008) 064023.
[26] A. N. Aliev and O. Delice, unpublished work (2008).
[27] T. Oota and Y. Yasui, Phys. Lett. B 659 (2008) 688.
[28] S. Q. Wu, Phys. Rev. D 78 (2008) 064052.
[29] S. Q. Wu, Class. Quant. Grav. 26 (2009) 055001.
[30] H. Ahmedov and A. N. Aliev, Phys. Rev. D 79 (2009) 084019.
[31] S. Q. Wu, arXiv:0902.2823 [hep-th]; arXiv:0906.2049 [hep-th].

[32] D. Kubizňák, H. K. Kunduri and Y. Yasui, Phys. Lett B 678 (2009) 240.

[33] S. Tomizawa, Y. Yasui and A. Ishibashi, arXiv:0901.4724 [hep-th].

[34] H. Lü, J. Mei and C. N. Pope, arXiv:0806.2204 [hep-th].