Thermodynamics of lattice QCD with 2 flavours of colour-sextet quarks: A model of walking/conformal Technicolor

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QCD with two flavours of massless colour-sextet quarks is considered as a model for conformal/walking Technicolor. If this theory possess an infrared fixed point, as indicated by 2-loop perturbation theory, it is a conformal(unparticle) field theory. If, on the other hand, a chiral condensate forms on the weak-coupling side of this would-be fixed point, the theory remains confining. The only difference between such a theory and regular QCD is that there is a range of momentum scales over which the coupling constant runs very slowly (walks). In this first analysis, we simulate the lattice version of QCD with two flavours of staggered quarks at finite temperatures on lattices of temporal extent $N_t = 4$ and 6. The deconfinement and chiral-symmetry restoration couplings give us a measure of the scales associated with confinement and chiral-symmetry breaking. We find that, in contrast to what is seen with fundamental quarks, these transition couplings are very different. $\beta = 6/g^2$ for each of these transitions increases significantly from $N_t = 4$ and $N_t = 6$ as expected for the finite temperature transitions of an asymptotically-free theory. This suggests a walking rather than a conformal behaviour, in contrast to what is observed with Wilson quarks. In contrast to what is found for fundamental quarks, the deconfined phase exhibits states in which the Polyakov loop is oriented in the directions of all three cube roots of unity. At very weak coupling the states with complex Polyakov loops undergo a transition to a state with a real, negative Polyakov loop.
I. INTRODUCTION

The advent of the LHC era has led to renewed interest in extensions of the Standard Model which have a strongly-coupled symmetry-breaking (Higgs) sector. The most promising of these theories are the so-called Technicolor theories [1, 2]. These are QCD-like theories with massless fermions (techni-quarks) in which the Goldstone bosons of spontaneously-broken chiral symmetry (techni-pions) play the rôle of the Higgs field, giving masses to the $W$ and $Z$. Such models need to be extended in order to also give masses to the quarks and leptons. Phenomenological difficulties with such models, such as flavour-changing neutral currents which are too large, can be avoided if the fermion content of a candidate (extended) Technicolor theory is such that the running gauge coupling constant evolves very slowly. Such theories are referred to as Walking Technicolor models [3–6].

Let us consider a Yang-Mills gauge theory with $N_f$ fermions in a specified (not-too-large) representation of the “colour” group. The evolution of the coupling constant $g$ in such a theory is described by the Callan-Symanzik beta function $\beta(g)$ defined by

$$\beta(g) = \mu \frac{\partial g}{\partial \mu} = -\beta_0 \frac{g^3}{(4\pi)^2} - \beta_1 \frac{g^5}{(4\pi)^4} \ldots$$

where $\mu$ is the momentum scale at which the running coupling constant $g(\mu)$ is defined. $\beta_0, \beta_1, \ldots$ are given by perturbation theory. For $N_f$ sufficiently small, $\beta_0$ and $\beta_1$ are both positive, the theory is asymptotically free and confining, and chiral symmetry is spontaneously broken. There exists some value of $N_f$ above which $\beta_0$ (and $\beta_1$) are negative and asymptotic freedom is lost. Between these two regimes is a range of $N_f$ over which $\beta_0$ is positive and $\beta_1$ is negative. In this range the theory remains asymptotically free, but if this 2-loop beta function describes the physics, it has a second zero which is an infrared (IR) fixed point and the theory is a conformal (unparticle) field theory. There is, however, another possibility. If the coupling becomes strong enough that a chiral condensate forms before this would-be IR fixed point can be reached, the fermions will become less effective at screening technicolor. $\beta$ will then start to decrease again, and the theory will be confining in the infrared. There will, however, be a range of $\mu$ over which $\beta$ is small and $g$ evolves only slowly, i.e. it walks. Since the formation of a chiral condensate which spontaneously breaks chiral symmetry is a non-perturbative process, the boundary between conformal and walking behaviour cannot be determined perturbatively. Lattice gauge theory simulations can enable one to decide
between these two options for a theory with a specified gauge group, fermion technicolor representation and $N_f$.

For $SU(N)$ gauge theories, the most promising candidates for walking behaviour have fermions in the fundamental, adjoint, symmetric 2-index tensor or antisymmetric 2-index tensor representations of the gauge group. A good summary of what is known is given in reference [4]. Estimates of the value of $N_f$ below which a gauge theory walks and above which it is conformal have been made using various methods, none of which can be guaranteed to capture the full non-perturbative behaviour of QCD-like theories [8–13]. This has led people to use lattice gauge theory simulations to study this boundary. There have been a number of simulations of QCD with $N_f$ fundamental quarks, with $N_f$ large enough that conformal or walking behaviour might be expected [14–29]. While progress has been made, the boundary value for $N_f$ is still uncertain. Some simulations have been made of $SU(2)$ gauge theory with 2 Dirac flavours of adjoint fermions [30–37]. While early indications are that this is a conformal field theory, there is still a great deal of uncertainty.

For QCD with colour-sextet (symmetric tensor) quarks, $\beta_1$ changes sign at $N_f = \frac{28}{125}$, and asymptotic freedom is lost at $N_f = 3\frac{3}{10}$. Hence only $N_f = 2$ and $N_f = 3$ lie in the domain of interest. $N_f = 3$ is just below the value where asymptotic freedom is lost and is thus expected to be conformal. This leaves $N_f = 2$ as the most likely candidate for walking behaviour. DeGrand, Shamir and Svetitsky have simulated lattice QCD with 2 flavours of colour-sextet Wilson quarks [38–40]. Their initial results suggest that this is a conformal field theory. Fodor et al. have performed some preliminary simulations of lattice QCD with 2 sextet quarks using domain-wall quarks [41].

We are performing simulations of lattice QCD with 2 colour-sextet staggered quarks. Staggered quarks have the advantage over Wilson quarks in having a simple chiral order parameter, and no parameter tuning is needed to find the chiral limit. We are performing simulations at finite temperatures, where the deconfinement and chiral-symmetry restoration temperatures can be used as measures of the scales of confinement and chiral-symmetry breaking respectively. By varying the number of time slices $N_t$ we can determine whether these transitions scale as finite temperature transitions or whether they are bulk transitions. Preliminary results of these simulations were presented at Lattice 2009 [42].

Our simulations indicate that, whereas for fundamental quarks the deconfinement and chiral-symmetry restoration transitions appear coincident, for colour-sextet quarks these
transitions are well separated. Chiral-symmetry restoration occurs at a much weaker coupling than deconfinement. This differs from what is seen with Wilson quarks by DeGrand, Shamir and Svetitsky [39] where the two transitions appear coincident. Such a separation has been reported in earlier simulations with colour-adjoint quarks [43, 44], and in SU(2) gauge theory with colour-adjoint quarks [45]. Both transitions move to significantly weaker couplings when \( N_t \) is increased from 4 to 6, which is what would be expected for finite temperature transitions governed by asymptotic freedom. This in turn favours the walking scenario.

In the deconfined region, just above the deconfinement transition, we find 3 states, where the Wilson Line (Polyakov Loop) is oriented in the directions of the 3 cube roots of unity, similar to what occurs for quenched QCD or QCD with adjoint quarks. For \( N_t = 4 \) only the state with a real positive Polyakov Loop appears stable. The other two states, while long-lived, appear to be only metastable. For \( N_t = 6 \), all 3 states appear to be stable. Between the deconfinement and chiral-symmetry restoration transitions there is another transition where the 2 states with complex Polyakov Loops disorder into a state with a real, negative Polyakov Loop. Machtey and Svetitsky have argued that such additional states where the Polyakov Loop has arguments \( \pm 2\pi/3 \) and \( \pi \) are to be expected, and have presented evidence for their existence and metastability in simulations using Wilson quarks [46].

In section 2 we discuss our simulations. Our results are described in section 3. Section 4 presents discussions and conclusions and identifies directions for ongoing and future investigation.

II. LATTICE SIMULATIONS WITH Sextet QUARKS

For our simulations we use the simple Wilson gauge action

\[
S_g = \beta \sum_U \left[ 1 - \frac{1}{3} \text{Re}(\text{Tr}UUUU) \right].
\] (2)

The fermion action is based on the unimproved staggered-quark action written formally as

\[
S_f = \sum_{\text{sites}} \frac{N_t/4}{N_f/4} \sum_{f=1}^{N_f/4} \psi_f^\dagger [\slash{D} + m] \psi_f
\] (3)

where \( \slash{D} = \sum_\mu \eta_\mu D_\mu \) with

\[
D_\mu \psi(x) = \frac{1}{2} [U^{(6)}_\mu(x) \psi(x + \hat{\mu}) - U^{(6)\dagger}_\mu(x - \hat{\mu}) \psi(x - \hat{\mu})].
\] (4)
To allow tuning the number of flavours to values of $N_f$ which are not multiples of 4, and to use a positive-definite operator for the transition to pseudofermions, this is replaced with

$$S_f = \sum_{\text{sites}} \chi^\dagger \{(\mathcal{D} + m)[-\mathcal{D} + m]\}^{N_f/8} \chi.$$

We use the RHMC algorithm for our simulations in which the fractional powers of the positive-definite Dirac operator are approximated to any desired accuracy by a rational approximation, and each trajectory is subjected to a global Metropolis accept/reject step, thus removing all dependence on the updating increment.

We perform simulations on $8^3 \times 4$, $12^3 \times 4$ and $12^3 \times 6$ lattices, over a range of values of $\beta = 6/g^2$ which covers all transitions. For each lattice size we perform runs at quark mass $m = 0.005$, $m = 0.01$ and $m = 0.02$ in lattice units, to allow us to access the chiral limit. Away from the transitions our run lengths are typically 10,000 length-1 trajectories for a given $m$ and $\beta$. Near deconfinement transitions run lengths of 50,000 to 200,000 trajectories are used for each $\beta$ and $m$. Some runs of 50,000 trajectories have also been used near the transitions from negative Polyakov Loop states to complex Polyakov Loop states.

We create deconfined states with positive Polyakov Loops by starting a run at $\beta = 7.0$ (weak coupling) from a completely ordered state (all $U$s equal to the identity matrix). The configurations from these runs are used to start runs at progressively smaller $\beta$s (and masses). The states with negative Polyakov Loops are obtained by starting a run at $\beta = 7.0$ with all $U$s equal to the identity matrix, except for the timelike $U$s on a single time slice, which are set to the matrix $\text{diag}(1, -1, -1)$.

The triplet Wilson Line (Polyakov Loop) is used to identify the position of the deconfinement transition. The chiral-symmetry restoring phase transition occurs at that $\beta$ above which the chiral condensate $\langle \bar{\psi} \psi \rangle$ vanishes in the chiral ($m \to 0$) limit. Since the chiral extrapolation is difficult to perform with the masses we use, we estimate the position of the chiral transition from the positions of the peaks in the chiral susceptibilities $\chi_{\bar{\psi}\psi}$ as functions of mass.

$$\chi_{\bar{\psi}\psi} = V \left[ \langle (\bar{\psi}\psi)^2 \rangle - \langle \bar{\psi}\psi \rangle^2 \right]$$

where the $\bar{\psi}\psi$s in this formula are lattice averaged quantities and $V$ is the space-time volume of the lattice. Since we use stochastic estimators for $\bar{\psi}\psi$, we obtain an unbiased estimator for this quantity by using several independent estimates for each configuration (5, in fact). Our
estimate of \((\bar{\psi}\psi)^2\) is then given by the average of the (10) estimates which are ‘off diagonal’ in the noise.

III. RESULTS FROM SIMULATIONS

A. \(N_t = 4\)

We perform simulations at a selection of \(\beta\) values in the range \(5.0 \leq \beta \leq 7.0\) on \(8^3 \times 4\) and \(12^3 \times 4\) lattices. For each of the 3 chosen masses (0.005, 0.01, 0.02) the values of the Wilson Line and chiral condensate on the \(8^3 \times 4\) and \(12^3 \times 4\) lattices are so close that we conclude that finite size effects are negligible. Figure 1 shows the Wilson Line and chiral condensates as functions of \(\beta\) for each of the 3 masses (0.005, 0.01, 0.02) on a \(12^3 \times 4\) lattice. In the deconfined phase, we have included only those states where the Wilson Line is real and positive, noting that runs which start in a state with a positive Wilson Line remain in this state for the duration of the run. We have not included the results for the \(8^3 \times 4\) lattice, since at the resolution of figure 1 the points for the 2 lattice sizes would be virtually indistinguishable.

It is clear that the Wilson Line is very small below \(\beta \approx 5.4\), and rises rapidly shortly after this value for all 3 quark masses. This is taken as a signal for the deconfinement transition. It is also clear that chiral symmetry remains broken well beyond this point. Thus the deconfinement and chiral-symmetry restoration transitions are far apart, unlike what is observed for Wilson quarks, where they appear to be coincident [39].

Figure 2 shows a histogram of the Wilson Line at \(\beta = 5.42\), \(m = 0.02\), showing a clear two-state signal, suggesting that this \(\beta\) is very close to the transition. The separation of the two states suggests that this transition is a first-order phase transition. We conclude that at \(m = 0.02\) the deconfinement transition is at \(\beta = \beta_d = 5.420(5)\). For \(m = 0.01\), two-state signals are seen at \(\beta = 5.411\) and \(\beta = 5.412\) leading to an estimate \(\beta_d = 5.4115(5)\). Finally we note that for \(m = 0.005\), \(\beta = 5.4\) appears to lie below the transition while \(\beta = 5.41\) appears to be above the transition leading to an estimate \(\beta_d = 5.405(5)\). Thus the mass dependence of the deconfinement \(\beta, \beta_d\), is very weak.

Now let us turn to the chiral transition. Because this is only expected to be a phase transition at \(m = 0\), and the crossover becomes smoother as the quark mass is increased,
it is difficult to determine its position directly from the chiral condensate at the masses we use. This is made more difficult since it is clear from the measured condensates as functions of mass, that the mass dependence is far from linear, making it extrapolating to $m = 0$ difficult if not impossible. We therefore use the peak in the chiral susceptibility defined in equation 6 as an estimate of the position of the crossover for finite mass. This is shown in figure 3 for the two smallest masses. From these graphs we estimate that the transition occurs at $\beta = 6.30(5)$ for both $\beta$s. We thus estimate that the phase transition at $m = 0$ occurs at $\beta = \beta_\chi = 6.3(1)$. Note that the spacing of the $\beta$s in this range is too large to allow us to use Ferrenberg-Swendsen reweighting to determine the positions of these transitions with more resolution. (The distributions of values of the plaquette action for adjacent $\beta$s

\[ 12^3 \times 4 \text{ lattice} \]

**FIG. 1:** Wilson line (Polyakov Loop) and $\langle \bar{\psi} \psi \rangle$ as functions of $\beta$ on a $12^3 \times 4$ lattice.
FIG. 2: Histogram of the Wilson Line at $\beta = 5.42$, $m = 0.02$ on a $12^3 \times 4$ lattice.

At each of our two larger masses, we have performed a series of runs starting from $\beta = 7.0$ with a negative Wilson Line. From $\beta = 7.0$ down to $\beta = 6.0$, the Wilson Lines remain negative over the length of the 10,000 trajectory run for each $(\beta, m)$. By $\beta = 5.8$, these states have transitioned to states in which the Wilson Line is oriented in the direction of one of the complex cube roots of unity. Hence we deduce that there is a transition at $\beta \approx 5.9$. More discussion of this transition is to be found in the $N_t = 6$ subsection. On the $12^3 \times 4$ lattice with $m = 0.02$ these states with complex Wilson Lines persist down to $\beta = 5.48$, and appear stable over at least 50,000 trajectories. For $m = 0.01$ these persist down to $\beta = 5.46$. For $\beta \leq 5.46$ at $m = 0.02$ or $\beta \leq 5.45$ at $m = 0.01$ but above the deconfinement transition, these states with complex Wilson Lines appear to be metastable and eventually decay into states with positive Wilson Lines. Figure [4] shows an example of
FIG. 3: Chiral susceptibilities $\chi_{\bar{\psi}\psi}$ as functions of $\beta$ on a $12^3 \times 4$ lattice for $m = 0.005, 0.01$, for a $\beta$ range which includes the chiral transition.

such metastability in our $12^3 \times 4$ simulations. As is to be expected, this metastability starts at larger $\beta$ values on an $8^3 \times 4$ lattice. We have observed no cases where configurations with positive Wilson Lines evolve to configurations with complex Wilson Lines for $\beta$ values above the deconfinement transition.

B. $N_t = 6$

We perform simulations on a $12^3 \times 6$ lattice at quark masses $m = 0.005, 0.01$ and $m = 0.02$ for values of $\beta = 6/g^2$ in the range $5.0 \leq \beta \leq 7.2$. We perform two series of runs starting at $\beta = 7.0$, for $m = 0.01, 0.02$. The first starts with the Wilson Line real and positive, and the second with the Wilson Line negative. For the lowest quark mass
FIG. 4: Time evolution of the argument of the Wilson Line (Polyakov Loop) at $m = 0.02$, $\beta = 5.45$ showing a decay of a state with a complex Wilson Line (argument close to $2\pi/3$) to a state with a real positive Wilson Line (argument close to 0).

$m = 0.005$ we only perform one set of runs starting with a positive Wilson Line at $\beta = 7.0$.

Above the deconfinement transition we again see a 3-state signal where the Wilson Lines orient themselves in the directions of one of the 3 cube roots of unity. A scatterplot showing such a 3-state signal is shown in figure 5a. Unlike the $N_t = 4$ case, there is no sign of metastability and transitions between all 3 states are seen over the duration of our runs, up to $\beta$ values far enough above the deconfinement transition that the mean relaxation time between tunnelings exceeds the lengths of our runs (50,000 trajectories). Figure 5b shows an example of such tunnelings. Thus to make meaningful measurements of the Wilson Line, we need to separate these 3 states. This we do by binning the Wilson Lines measured at the end of each trajectory according to their phase $\phi$ into bins $-\pi < \phi < -\pi/3$, $-\pi/3 < \phi < \pi/3$, $\pi/3 < \phi < \pi$. To increase our statistics, we use symmetry to include the complex conjugates.
FIG. 5: a) Scatterplot of Wilson Lines for $m = 0.02$ and $\beta = 5.58$ on a $12^3 \times 6$ lattice showing the 3-state signal.

b) ‘Time’ evolution of the argument (phase) of the Wilson Lines for one of the 2 runs included in part (a).
of the Wilson Lines in the first of these bins, with the Wilson Lines in the last of these bins. Other observables are binned according to the values of the corresponding Wilson Lines.

Figure 6a shows the Wilson Lines (Polyakov Loops) and chiral condensates as functions of \( \beta \) for all 3 masses from our simulations on a \( 12^3 \times 6 \) lattice for the states with positive Wilson Lines \((-\pi/3 < \phi < \pi/3)\). Figure 6b shows the magnitudes of the average Wilson Lines and the chiral condensates for the states with complex or negative Wilson Lines. The deconfinement transition manifests itself as a rapid increase in the Wilson Line, which is clearly seen at \( \beta \) just above 5.5. The fact that the \( Z_3 \) centre symmetry is broken manifests itself in the difference between the magnitudes of the Wilson Lines in the positive Wilson Line and complex Wilson Line states. The rapid change in the magnitude of the complex/negative Wilson line between \( \beta = 6.2 \) and \( \beta = 6.6 \) marks the transition between a state whose Wilson Line is oriented in the direction of one of the complex cube roots of unity and one where the Wilson Line is real and negative. The chiral transition, above which \( \langle \bar{\psi}\psi \rangle \) vanishes in the chiral \((m \to 0)\) limit, clearly occurs at a \( \beta \) appreciably larger \( \beta_d \).

To determine the position of the deconfinement transition we examine histograms of the magnitudes of the Wilson Line close to the transition for each mass. Although the explicit breaking of the \( Z_3 \) centre symmetry means that the magnitudes of the positive and complex Wilson Lines are not identical, they are close enough in the vicinity of the deconfinement transition that this fact can be ignored. Such histograms are presented in figure 7. For the lower 2 masses the histogram for each beta represents 50,000 trajectories. At \( m = 0.02 \) the histograms for \( \beta = 5.55 \) and \( \beta = 5.57 \) represent 100,000 trajectories each, while that at \( \beta = 5.56 \) is for 200,000 trajectories. In each case the histogram peaks at a low value below the transition and an appreciably higher value above the transition. Very close to the transition the peak of the histogram is relatively flat, with some hint of a double peaked structure. From these graphs we estimate that the transition \( \beta s (\beta_d) \) are \( \beta_d(m = 0.005) = 5.545(5) \), \( \beta_d(m = 0.01) = 5.550(5) \) and \( \beta_d(m = 0.02) = 5.560(5) \), respectively. As in the \( N_t = 4 \) case, the mass dependence is weak.

Again we estimate the position of the chiral-symmetry restoration transition by examining the peaks in the chiral susceptibilities for \( m = 0.005, 0.01 \). As for \( N_t = 4 \) we obtain estimates of the chiral condensate from 5 independent noise vectors, which yields an unbiased estimate for the chiral susceptibility. These chiral susceptibilities are plotted versus \( \beta \) in figure 8 for each of these lowest two masses. Again, the \( \beta s \) we use in this region are two sparse to allow
FIG. 6: a) Wilson Line and chiral condensate for the state with a real positive Wilson Line as functions of $\beta$ for each of the 3 masses on a $12^3 \times 6$ lattice.

b) Magnitude of the Wilson Line and chiral condensate for the states with a complex or a real negative Wilson Line as functions of $\beta$ for each of the 3 masses on a $12^3 \times 6$ lattice.

the use of Ferrenberg-Swendsen reweighting to interpolate between them. Since our estimate
FIG. 7: Histograms of the magnitudes of Wilson Lines for $\beta$ values bracketing the deconfinement transition on a $12^3 \times 6$ lattice for a) $m = 0.005$, b) $m = 0.01$, c) $m = 0.02$. 

for the peak of the susceptibility plots for each mass is $\beta = 6.60(5)$, we estimate for the position of the chiral transition at $m = 0$ is $\beta = \beta_\chi = 6.6(1)$. Note that this estimate is for the states with positive real Wilson Lines only.

Starting at $\beta = 7.0$, $m = 0.02$ with a real negative Wilson Line, we find that this state is stable for runs of up to 50,000 trajectories for $\beta$s down to $\beta = 6.6$. Decreasing this to $\beta = 6.4$, we find that the system has transitioned to a state with a Wilson Line oriented in
FIG. 8: Chiral susceptibilities $\chi_{\bar{\psi}\psi}$ as functions of $\beta$ on a $12^3 \times 6$ lattice for $m = 0.005, 0.01$, for a $\beta$ range which includes the chiral transition.

The direction of one of the complex cube roots of unity. Figure 9 shows the time evolution of the phases of the Wilson Lines at $\beta = 6.4$ and $\beta = 6.6$ for $m = 0.02$. We see that the way the transition proceeds is that the fluctuations in the phase become large as $\beta$ approaches the transition from above until eventually they reach $2\pi/3$ and $4\pi/3(−2\pi/3)$. When this occurs the system spends appreciably more time at these two values that at intermediate values. We can then consider the system as being in a state with its Wilson Line oriented in the direction of one of the two complex cube roots of unity. On the finite lattice it tunnels between these 2 states. Approaching this transition from below we could describe it as the disordering of the 2 states with Wilson Lines in the directions of the complex cube roots of unity, as suggested by Machtey and Svetitsky. The position of this transition for $m = 0.02$
FIG. 9: The ‘time’ evolution of the argument (phase) of the Wilson Line for $m = 0.02$ on a $12^3 \times 6$ lattice, a) for $\beta = 6.4$ and b) for $\beta = 6.6$. The horizontal lines are at $2\pi/3$, $\pi$ and $4\pi/3$.

is at $\beta \approx 6.5$, while for $m = 0.01$ it occurs at $\beta \approx 6.4$. 
IV. DISCUSSION AND CONCLUSIONS

We simulated QCD with two flavours of staggered quarks at finite temperatures using the RHMC method. Our simulations were performed on $8^3 \times 4$, $12^3 \times 4$ and $12^3 \times 6$ lattices with 3 quark masses, to allow for chiral extrapolations. We find widely separated deconfinement and chiral-symmetry restoration transitions. Both the deconfinement and chiral transitions move to significantly lower couplings as $N_t$ is increased from 4 to 6, which is the expected behaviour for finite temperature transitions in an asymptotically free theory. This suggests that the theory is confining with spontaneously-broken chiral symmetry, while being under the control of the weak-coupling asymptotically-free ultraviolet fixed point, i.e. that it walks.

The deconfinement transition occurs at $\beta = \beta_d$ where for $N_t = 4$, $\beta_d(m = 0.02) = 5.420(5)$, $\beta_d(m = 0.01) = 5.4115(5)$ and $\beta_d(m = 0.005) = 5.405(5)$ while for $N_t = 6$, $\beta_d(m = 0.02) = 5.560(5)$, $\beta_d(m = 0.01) = 5.550(5)$ and $\beta_d(m = 0.005) = 5.545(5)$. The chiral-symmetry restoration transition occurs at $\beta = \beta_\chi$, where $\beta_\chi = 6.3(1)$ at $N_t = 4$ and $\beta_\chi = 6.6(1)$ at $N_t = 6$.

The large separation of the deconfinement and chiral transitions indicates that the enhanced attraction between quark-antiquark pairs over that for fundamental quarks due to the larger quadratic Casimir operator for the sextet representation ($10/3$ versus $4/3$), causes a chiral condensate to form at a distance much shorter than the scale of confinement. This effectively removes the quarks from consideration at longer distances where the theory will behave more like a pure glue (quenched) theory. This is presumably why, in the deconfined phase, we see a three state system, where the Wilson Lines align in the directions of one of the cube roots of unity, a relic of the now-broken $Z_3$ symmetry. The breaking of $Z_3$ is seen in the difference in magnitudes of the real positive Wilson Lines versus those with phases close to $\pm 2\pi/3$. At $N_t = 4$, the states with complex Wilson Lines are only metastable, while at $N_t = 6$ all 3 states appear to be stable. The existence of states with all 3 $Z_3$ Wilson Line orientations has been predicted by Machtey and Svetitsky [46] who observed them in their simulations with 2 flavours of colour-sextet Wilson quarks. They also observed the metastability of the states with complex sextet Wilson quarks. Earlier simulations by DeGrand, Shamir and Svetitsky had reported deconfined states with Wilson Lines oriented in the directions of the complex cube roots of unity [39].

More work needs to be done to determine whether the chiral-symmetry restoring transi-
tion of these complex/negative Wilson Line states is coincident with that of the state with a positive Wilson Line. In addition we would like to know whether this chiral transition shows the scaling properties of the expected $O(2)/O(4)$ universality class. The fact that this transition occurs at relatively weak coupling should help us in this endeavour.

We have also observed an additional transition. At $\beta \approx 5.9$ on an $N_t = 4$ lattice with $m = 0.02$ or $m = 0.01$, the states with complex Wilson Lines disorder to a state with a negative Wilson Line. Such a transition is also observed for $N_t = 6$ at $\beta \approx 6.5$ for $m = 0.02$ and $\beta \approx 6.4$ for $m = 0.01$. The existence of states with negative Wilson Lines and of such a transition is predicted and observed by Machtey and Svetitsky. Such a transition would be expected to be either a second-order phase transition in the universality class of the 3-dimensional Ising model, or a first-order phase transition. The large increase in the $\beta$ for this transition between $N_t = 4$ and $N_t = 6$ makes us suspect that this is a lattice artifact.

We also note that by $N_t = 6$, it is close to the chiral transition and could well merge with it at larger $N_t$. Since the negative Wilson Line state (phase $\pi$) comes from disordering the two states with phases $\pm 2\pi/3$ (a fact also predicted by Machtey and Svetitsky), the magnitude of the Wilson Line above the transition is approximately half what it is below the transition. Just below the transition, the magnitude of Wilson Line in the complex Wilson Line states is approximately $2/3$ that of the positive Wilson Line state, so that after the transition the magnitude of the negative Wilson Line is approximately $1/3$ of that of the positive Wilson Line. This would suggest that the transition might be associated with the symmetry breaking $SU(3) \rightarrow SU(2) \times U(1)$.

We need to understand why we see well-separated deconfinement and chiral transitions with staggered fermions, whereas DeGrand, Shamir and Svetitsky observed these transitions to be coincident with Wilson fermions. Of course, since we are far from the weak-coupling limit, different fermion actions do not have to have the same physics. This is especially true, if we happen to be in the strong-coupling domain, beyond the infrared fixed point of a conformal field theory. Of course, the observation from our simulations that the coupling at each of the transitions is decreasing as the lattice spacing is decreasing would appear to exclude this possibility, since the $\beta$ function becomes positive above this fixed point. However, it has recently been suggested that there could be two non-trivial fixed points in such theories. In this case, if we are beyond the second non-trivial fixed point (which would be an ultraviolet fixed point), the coupling would decrease at short distances. Of course,
if we are beyond the region where the ultraviolet behaviour of the theory is controlled by asymptotic freedom, drawing any conclusions is pure speculation.

To better understand our results, and to help determine whether the theory is indeed walking, rather than conformal as indicated by the work of DeGrand, Shamir and Svetitsky using Wilson fermions, we are now extending our simulations to $N_t = 8$, where finite lattice spacing effects should be reduced, to see if the deconfinement and chiral-symmetry restoration transitions remain consistent with being finite-temperature transitions. We also plan to perform simulations at zero temperature, measuring the chiral condensate, the hadron spectrum, $f_\pi$, etc., to test other aspects of the theory which should help us determine whether this theory is conformal or walking. In addition we will try to determine the running of a suitably-defined renormalized coupling. Having two different spatial lattice sizes at $N_t = 4$ showed us that finite size effects are small. We need a second spatial lattice size for $N_t = 6$. We have learned that the authors of reference [41] are now starting simulations of this theory using improved staggered fermions, which should help resolve these issues [48].

We are now performing simulations of lattice QCD with 3 flavours of staggered quarks at finite temperature. Since this theory is almost certainly conformal, it is interesting to determine whether its behaviour is qualitatively different from that of the $N_f = 2$ case, and whether our simulations can see this conformality.

Acknowledgements

DKS is supported in part by the U.S. Department of Energy, Division of High Energy Physics, Contract DE-AC02-06CH11357, and in part by the Argonne/University of Chicago Joint Theory Institute. JBK is supported in part by NSF grant NSF PHY03-04252. These simulations were performed on the Cray XT4, Franklin at NERSC under an ERCAP allocation, and on the Cray XT5, Kraken at NICS under an LRAC/TRAC allocation.

DKS thanks J. Kuti, D. Nogradi and F. Sannino for helpful discussions.

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