Correlations in gravitational-wave reconstructions from eccentric binaries: a case study with GW151226 & GW170608

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The eccentricity of binary black hole mergers is predicted to be an indicator of the history of their formation. In particular, eccentricity is a strong signature of dynamical formation rather than formation by stellar evolution in isolated stellar systems. We investigate the efficacy of the existing quasi-circular parameter estimation pipelines to determine the source parameters of such eccentric systems. We create a set of simulated signals with eccentricity up to 0.3 and find that as the eccentricity increases, the recovered mass parameters are consistent with those of a binary with up to a \( \approx 10\% \) higher chirp mass and mass ratio closer to unity. We also employ a full inspiral-merger-ringdown waveform model to perform parameter estimation on two gravitational wave events, GW151226 and GW170608, to investigate this bias on real data. We find that the correlation between the masses and eccentricity persists in real data, but that there is also a correlation between the measured eccentricity and effective spin. In particular, using a non-spinning prior results in a spurious eccentricity measurement for GW151226 as it exhibits signs of non-zero black-hole spin. Performing parameter estimation with an aligned spin, eccentric model, we constrain the eccentricities of GW151226 and GW170608 to be \(< 0.15\) and \(< 0.12\) respectively.

I. INTRODUCTION

The Laser Interferometer Gravitational-wave Observatory (LIGO) and the Virgo observatory have detected gravitational wave (GW) signals from dozens of binary black hole (BBH) mergers during their first three observing runs \([1,5]\). As the field of gravitational wave astronomy moves forward, the network of Earth-based detectors will be joined by KAGRA \([4]\) and LIGO India \([5]\) in the coming few years, and possibly followed up by third-generation instruments such as the Cosmic Explorer (CE) \([6]\) and the Einstein Telescope (ET) \([7]\). Expanding the network to include more geographical locations will improve the sky localization of events, and the increased sensitivity of future detectors (CE and ET) will increase our detection rates, particularly at lower frequencies.

The mechanisms of formation and merging of these binary systems are the subject of active research \([5,11]\). One possibility is that of isolated binary formation, either through formation of a common envelope phase \([12,15]\), or through chemically homogeneous evolution \([16,17]\). This formation channel generally results in black holes with lower masses, \( \lesssim 50 \, M_\odot \), and aligned spins \([18,19]\).

Another promising formation channel that can account for the rate of mergers seen is that of dynamical capture binary mergers in dense globular clusters \([10,25]\) or in active galactic nuclei (AGN) disks \([26,31]\). In the case of dense environments, binary black holes merge after undergoing many interactions with third bodies or other binaries \([25]\). Dynamical interactions can produce multiple generations of mergers. Such formation channels lead to binaries with higher mass \([32,33]\) that can lie in the supernova pair-instability mass gap \([19,31,35]\), have isotropic spins \([22,36]\), and eccentricity \([37,39]\). In such cases the eccentricity of a binary can be driven to non-zero values through the Lidov-Kozai effect \([40,41]\). By performing Monte Carlo simulations of such dense clusters, it has been shown \([42,43]\) that on the order of 5\% of mergers in globular clusters can have eccentricity \(> 0.1\) when the frequency of their orbits enters the sensitivity band of terrestrial GW detectors at 10 Hz. Interactions within AGN disks can also lead to binaries with measurable eccentricity \([44,45]\). Orbital eccentricity is therefore striking evidence that can conclusively establish dense clusters as breeding grounds for compact binary mergers \([40]\).

The presence of eccentricity in any of the gravitational-wave signals seen thus far has yet to be conclusively established. Searches for eccentric compact binary candidates in GW data is an active area of research \([47,51]\), although the first three observing runs of LIGO-Virgo have yielded no extra GW detections \([48,52,54]\) beyond those already found by non-eccentric searches \([3]\). There have been various studies \([55,59]\) that attempt to measure orbital eccentricities of binary black hole merger events, as well the neutron star merger events in GW transient catalogs \([61]\). While they agree that most observed GW signals are produced by quasi-circular binary mergers, some of these studies find evidence of residual orbital eccentricity in a few events \([56,57,60]\). There is strong statistical evidence that at least one merger in GWTC-2 is the result of hierarchical mergers \([11]\).

Of special note is the event GW190521 that is favoured to have been formed from the merger of second gener-
Our quantitative findings are consistent with similarly significant bias in the measurement of binary mass ratio. Our ability to accurately determine the eccentricity of mergers such as GW190521 [52] will improve as the network of GW detectors expands its sensitive frequency band to include deciHertz detectors [65]. It has been shown that the addition of detectors such as Cosmic Explorer and the Einstein Telescope can substantially increase the signal-to-noise ratio (SNR) of eccentric signals, in some cases, to higher values than similar quasi-circular systems [68].

There has been previous work to investigate the effect that orbital eccentricity can have on the detectability of GW signals when the quasi-circular template banks that LIGO-Virgo currently employs are used [66, 69, 70]. Mildly eccentric systems can still be detected by search pipelines, but with a loss of SNR. However, moderately and highly eccentric systems (with $e_{15\text{Hz}} > 0.275$) would likely be missed by search pipelines [69, 71] (although sufficiently loud eccentric events can still be detected by unmodelled transient GW searches that make minimal assumptions about the gravitational waveform [15, 52–54]). Once detected, ignoring eccentricity could still lead us to infer with biases other intrinsic parameters of the source.

In this paper we pursue two related goals. The first is to understand how our lack of inclusion of orbital eccentricity in waveform templates alters our inference on the source parameters of GW signals. To this end we simulate a set of eccentric inspiral-merger-ringdown (IMR) signals and perform Bayesian parameter estimation on them using waveform templates that represent binary mergers on quasi-circular orbits. We restrict these injections to non-spinning systems in order to keep the dimensionality of the parameter space being sampled small while studying parameter space degeneracies. This is a reasonable approximation as most GW events observed so far (more than half) are consistent with small black hole spins [72]. We find that as we increase the orbital eccentricity of GW sources, our quasi-circular templates furnish larger chirp mass values and less asymmetric mass ratios than the those simulated. Although eccentricity alters the rate of inspiral in the same way a larger chirp mass does [72], it also introduces modulations in the orbital frequency as a function of time to make the phase evolution qualitatively very different from what a shift in binary masses does. This bias is therefore not obvious, especially when considering the full inspiral-merger-ringdown. We quantify the bias to find an $O(10\%)$ drift in chirp mass measurement alone in the case of low-mass binaries with moderate SNRs (20-30), which is significant given that the measurement precision on the chirp mass is typically smaller than $O(1\%)$. We also find a similarly significant bias in the measurement of binary mass ratio. Our quantitative findings are consistent with previous results [61, 74].

Our second goal is to investigate whether this bias appears with real GW events [75]. We investigate two events: GW151226 and GW170608, that have been debated in literature for being possibly eccentric [55, 56]. It is useful to note that as Refs. [55, 56] use different waveform models to study these events, their eccentricity measurements are not quantitatively comparable [76, 78]. However the claim of being eccentric/non-eccentric could still be qualitatively contrasted between both studies, as finite eccentricity measured by one waveform model is unlikely to be measured as nearly zero by another. We find that if we use a non-spinning, eccentric waveform model to perform parameter estimation, GW151226 is consistent with being a moderately eccentric binary merger, with a chirp mass that is an underestimate of that found by LIGO-Virgo [11]. However, if we use a waveform model that includes black hole spins (aligned) in addition to orbital eccentricity, we find that GW151226 is well described as a non-eccentric spinning binary merger with component masses and spins consistent with the original LVC analysis [11] that generally ignored eccentricity. Inclusion of BH spins significantly weakens the evidence of orbital eccentricity in GW151226. The implication that the eccentricity measurement can be correlated with the measured masses and spins implies that non-spinning eccentric waveform models may be insufficient for the measurement of eccentricities of binary black holes if any of the binary components possess spin angular momentum.

In Section II we describe our methods, including a description of the waveform models we use in II A. In Section II B we describe the setup for our parameter estimation runs using BILBY. Section II C describes the eccentric simulated signals and results of their recovery with quasi-circular templates. In section III B we employ a full IMR, eccentric waveform to perform parameter estimation on the two GW events described above, with and without spins included in our prior. We follow up our studies on real events in Section III C by simulating the best fit quasi-circular parameters for these events, and performing inference with the same eccentric IMR model to determine how their parameters would be recovered if they were truly non-eccentric.

Throughout, we refer to mass as measured in the detector frame. We denote eccentricity as $e_y$, meaning eccentricity at a reference frequency of $y$Hz.

II. METHODS

A. Waveform models

Detection and characterization of the source properties of eccentric signals require accurate and efficient waveform models. Some of the existing waveform models use a post-Newtonian description and only predict the inspiral portion of the signal, such as the EccentricFD [79] and EccentricTD [80], which are both implemented in LALSuite [81]. Recently, models have been developed to produce full inspiral-merger-ringdown (IMR) wave-
form for eccentric binaries, by hybridizing eccentric inspiral signals with a quasi-circular merger-ringdown portion (ENIGMA) \cite{71}, through an effective one-body (EOB) formalism \cite{82, 83}, or by surrogate modelling of numerical relativity waveforms \cite{84}.

The first eccentric model we use is the inspiral portion of ENIGMA \cite{71}. ENIGMA is a time-domain model designed to produce full inspiral-merger-ringdown waveforms up to moderate eccentricities. ENIGMA incorporates corrections for the energy flux of quasi-circular binaries and gravitational self-force corrections to the binding energy of compact binaries up to 6PN order. Higher-order corrections to the effect of the inclination angle on the inspiral waveform were introduced in \cite{68}. The framework coincides with the TaylorT4 approximant at 3PN order. We opt to use the inspiral portion of the model alone in our analyses, for reasons elaborated in Appendix A. In order to produce the inspiral waveform, we integrate the equations of motion from \cite{71} until the expansion parameter \( x = (M\omega)^{2/3} \) reaches the Schwarzschild innermost stable circular orbit (ISCO), at \( x_{\text{ISCO}} = \frac{1}{6} \). Here, \( M \) is the total mass of the binary and \( \omega \) is the mean orbital frequency. We refer the reader to \cite{68, 71} for more details.

The second waveform model we use is an extension of the full IMR quasi-circular model TEOBiResumS_SM \cite{85} to include eccentric effects, which was introduced in \cite{86}. This model, which we will refer to as TEOBResumE, alters the angular momentum flux portion of TEOBiResumS_SM with a Newtonian-like prefactor, which generalizes the quasi-circular model to moderate eccentricities \((e \approx 0.3)\). We have used the publicly available implementation at \cite{87} without any changes to the core routines, and use the dominant waveform modes in our analyses. However, we have loosened the absolute and relative error

1 Using the eccentric branch with commit 10f6110
tolerances of its ODE integrator from their default values of $10^{-13}$ and $10^{-11}$ to $10^{-8}$ and $10^{-7}$. We find that this allows the likelihood function to be evaluated on the $O(10^{-2} \text{ s})$ timescale, which makes full parameter estimation analyses viable. To ensure that loosening this tolerance does not significantly degrade the produced waveforms, we calculate the mismatch,

$$M = 1 - \frac{\langle h_{\text{strict}} | h_{\text{loose}} \rangle}{\langle h_{\text{strict}} | h_{\text{strict}} \rangle}$$

(1)

between the stricter waveform $h_{\text{strict}}$ and the looser waveform $h_{\text{loose}}$, where we use the straightforward inner product between two timeseries

$$\langle a | b \rangle = \int a(t)b(t)dt.$$  

(2)

We find that waveform generation is robust to such changes with the ODE integrator, and the mismatches differ by no more than $10^{-3}$ over the parameter space.

Although this waveform model can produce waveforms with higher-order modes, we have opted to only use the dominant (2, 2) mode in this study. We also note that TEOBResumE does not allow for variation of the argument of periapsis in initial conditions, and consequently the same was kept fixed in the parameter estimation analysis conducted with the model. This restriction can have a small effect on our results here [84, 88], and we defer a detailed quantification of the same to future work.

### B. Parameter estimation

We use the python package BILBY [90] to simulate the signals and perform parameter estimation. BILBY calculates the posterior distribution $p(\theta | y)$ for the set of source parameters $\theta$ given the data $y$, according to Bayes’ theorem, $p(\theta | y) \propto p(y | \theta)p(\theta)$ where $p(y | \theta)$ is the likelihood and $p(\theta)$ is the prior. The likelihood function for a set of $N$ detectors is the standard used for gravitational wave astronomy:

$$p(y | \theta) = \exp \left( -\frac{1}{2} \sum_{i=1}^{N} (\hat{y}_i(f) - \hat{s}_i(f, \theta))|\hat{y}_i(f) - \hat{s}_i(f, \theta)) \right),$$

(3)

where $\hat{y}_i(f)$ and $\hat{s}_i(f, \theta)$ are the frequency-domain representations of the data and the model waveform. The inner product $\langle \cdot | \cdot \rangle$ is given by

$$\langle \hat{a}_i(f) | \hat{b}_i(f) \rangle = 4\text{Re} \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{\hat{a}_i(f)\hat{b}_i(f)}{S_i^0(f)} df,$$

(4)

with the weight factor $S_i^0(f)$ being the advanced LIGO power spectral density (PSD). Superscript $i$ labels the detector, in our case either LIGO Hanford or LIGO Livingston. We use the zero-detuning high-power noise curve for LIGO detectors (labeled ZERO_DET_highP in [90]). The interpretation of this likelihood function is that a given candidate waveform is likely to be present in the data, if subtracting it from the data resembles colored Gaussian noise.

We sample from the posterior distribution using a nested sampling algorithm implemented via the dynesty package [91] in BILBY. We use 1,000 live points, and a stopping criterion of $\Delta \log Z < 0.1$ where $Z$ is the estimated Bayesian evidence. To improve efficiency, we use the option in BILBY to analytically marginalize over the coalescence time $t_c$, coalescence phase $\psi$, and coalescence time $t_c$.

### III. RESULTS

#### A. Simulated signals

We have already observed binary black hole mergers with masses ranging from $14 M_\odot$ to $160 M_\odot$ [72]. In this study, we focus on binaries at the lower end of this spectrum, as a longer inspiral signal will have a stronger

| Parameter | Prior | Range |
|-----------|-------|-------|
| $q$       | Uniform | 0.25–1. |
| $M_c$     | Uniform | 5–10 $M_\odot$ |
| $D_L$     | Uniform Comoving | 10–1000 Mpc |
| $\iota$   | Uniform sin | 0–$\pi$ |
| $\alpha$  | Uniform | 0–2$\pi$ |
| $\delta$  | Uniform cos | $-\pi/2–\pi/2$ |
| $\psi$    | Uniform | 0–$\pi$ |
| $\phi_c$  | Uniform | 0–2$\pi$ |
| $t_c$     | Uniform | 0–8 |

**TABLE II.** Priors used in the parameter estimation for the mass ratio $q$, chirp mass $M_c$, luminosity distance $D_L$, inclination $\iota$, right ascension $\alpha$, declination $\delta$, polarization $\psi$, coalescence phase $\phi_c$, and coalescence time $t_c$.
eccentric signature. Also, at lower masses the inspiral of the signal dominates the SNR as compared to the merger portion, which allows us to draw reliable conclusions using inspiral-only waveform models.

Since our goal is to understand the biases of existing GW parameter estimation pipelines that use quasi-circular waveforms as filter templates, we limit our waveform models in the likelihood evaluation to SEOBNRv4_ROM [54] and IMRPhenomD [90]. We note that while more recent EOB and Phenom models have become available more recently [81, 99], we do not employ them here for two reasons: (i) for the binary mass and spin parameter space we consider, the SEOBNRv4_ROM and IMRPhenomD models have good agreement with numerical relativity waveforms [95–97], and (ii) the newer EOB models were too computationally expensive for Bayesian calibration relativity waveforms [95–97], and (ii) the newer EOB models were too computationally expensive for Bayesian parameter estimation. Since the waveforms produced with our modified ENIGMA model are inspiral-only, we also utilise the PN quasi-circular model TaylorF2 to rule out any biases that may arise compared to inspiral-merger-ringdown templates. PE results with TaylorF2 also serve to nicely illustrate the kind of biases that result from ignoring the merger-ringdown in template waveforms. Using 3 waveform models for the likelihood, 2 waveform models for the signals, and the direct product of the parameters described in Table I gives us 126 signals on which to perform parameter estimation. As a control, we also study a further set of six signals using ENIGMA-Inspiral as the waveform approximant for the signal and likelihood evaluation, with \( q = 0.5 \) and the eccentricity taking the values in Table I. The priors for all the parameter estimation analyses are shown in Table I. We use a non-spinning prior for our simulated signals since the majority of the black holes detected thus far have had negligible spins.

Each of these signals is generated from a starting frequency 15 Hz using the given waveform model, and then injected into zero noise. As described in Appendix C of [100], using zero noise eliminates any biases in parameter estimation that would be caused by a particular noise realization. The data is sampled at 2048 Hz, and we use the two-detector network of LIGO Hanford and Livingston. In the calculation of the likelihood, we use \( f_{\text{min}} = 20 \text{ Hz} \). We sample in chirp mass and mass ratio, rather than component masses. We use sampling priors shown in Table I. At a luminosity distance of 400 Mpc, the signals are relatively loud, and have a two-network SNR ranging from 21 to 33 at design sensitivity. We deliberately choose for our signals to be loud in order to unambiguously quantify systematic correlations in the intrinsic parameters of non-spinning eccentric binaries. Low SNRs would lead the statistical uncertainties to dominate and obscure the systematic degeneracies. In comparison, the GW events we investigate in Sec. III B are also located at a similar luminosity distance (350–450 Mpc), but have SNRs \( \sim 15 \) due to lower sensitivity of the detectors at the time of their observation. In a sense, our simulated signals represent the future versions of GW151226 and GW170608 if they were seen during the fourth observing run of LIGO-Virgo or beyond. It is worth noting that at SNRs \( \sim 15 \), we expect statistical uncertainties in parameter measurements that are \( \sim \sqrt{2} \) times (i.e. 40%) larger than those of the injections discussed here.

We start the discussion with Figure 1 where we show the posterior distributions recovered for four representative simulated binary mergers. The signal waveform model is TEObResumE, which we expect to be the best example since it is a full IMR model, with zero and nonzero eccentricity, and the mass ratios \( q = 1 \) and \( q = 0.33 \). We opt to plot the symmetric mass ratio \( \eta = \frac{q}{1+q} \) instead of \( q \) in this section as it is the parameter that appears to leading order in a PN expansion.

In the \( e = 0 \) cases, IMRPhenomD well recovers all of the component parameters. Since these signals are high SNR and have a long inspiral, the chirp mass is extremely precisely and accurately estimated, with the 90% confidence interval within less than 0.1% of the simulated value.

The \( e = 0.3 \) cases, however, show a very precise but inaccurate inference of the chirp mass, with the quasi-circular template drastically overestimating its value. The median recovered value deviates from the simulated value by \( \approx 3\% \), whereas the 90% confidence interval is about 0.2% of the median estimated value. \( \eta \) is well determined to be \( \frac{1}{4} \) in both the eccentric and non-eccentric cases. Moreover, in the asymmetric mass, non-eccentric case, the correct value of \( \eta \) is inferred within our error-estimates. However, when the simulated signal is asymmetric and eccentric, the value of \( \eta \) is confidently and incorrectly inferred to be that of a symmetric binary.
FIG. 3. Estimated values of the chirp mass for signals at different initial eccentricities \( e_0 \) (defined at waveform frequency of 15Hz). Dashed lines indicate the value of the simulated signal. The colors blue, green and purple correspond to the \( q = 1, 0.5, 0.33 \) signals respectively. The bars represent the 90% confidence intervals, with the diamond representing the median value. The shaded connects the ends of the error bars to illustrate the upward trend. The labels on top specify which model was used in the parameter estimation, and the labels on the right side of the figure specify which model was used to simulate the signal. Note the different scales between \([0, 0.01]\) and \([0.01, 0.3]\) on the x-axes.

Figure 3 illustrates these trends for the case of using ENIGMA-Inspiral for simulating the signal and performing parameter estimation. At low eccentricities, the mass parameters are correctly recovered, but at higher eccentricities, the recovered parameters are those of a heavier, symmetric binary.

1. Chirp Mass Recovery

Figure 3 summarizes the chirp mass estimates for the rest of our simulated signals, showing the six combinations of signal and recovery waveform models. The top row shows the case where the simulated signal is generated with the IMR model, TEOBResumE, and the bottom row with the inspiral model ENIGMA-Inspiral.

Let us consider the top row, where TEOBResumE is used for the signal. No matter the template model, we find a systematic bias in chirp mass recovery that increases in proportion to orbital eccentricity. When TaylorF2 is used as the waveform model for templates, the error bars are wider due to the missing merger-ringdown portion. Also in the \( e_{15} = 0 \) case, there is a slight bias in the TaylorF2 recovery due the missing merger. But since we see very similar results in the case of SEOBNRv4_ROM and IMRPhenomD, we conclude that this result is invariant under a change of waveform family.

We now consider the bottom row, using ENIGMA-Inspiral for the signal. In the left and middle panels, when using IMR models for inference, we see a small bias in chirp mass recovery even at \( e_{15} = 0 \). This comes from the merger-ringdown portion of the IMR templates fitting to the last few cycles of the ENIGMA-Inspiral signal, resulting in a lower chirp mass. We see no such bias in the TaylorF2 case at \( e_{15} = 0 \).

For each combination of signal and recovery model there is a clear trend: as the eccentricity of the simulated signal increases, the circular templates strongly overestimate the chirp mass beyond the values within the 90% credible intervals. The effect becomes more pronounced at higher eccentricities, and the maximum fractional deviation over the dataset is \( \sim 9\% \).

2. Mass Ratio Recovery

Figure 4 shows the effects of increasing eccentricity on the estimation of the symmetric mass ratio, \( \eta \). Overall, the trend is less systematic than in the case of the chirp mass.

Let us consider the top row with TEOBResumE signals. At \( e_{15} = 0 \), the recovered \( \eta \) agrees well with the injected value for the two IMR waveform templates. For TaylorF2 there is a systematic shift of the posterior toward higher \( \eta \) values due to the missing merger-ringdown cycles in templates. This illustrates that we need to use IMR waveform templates for measurably more precise parameter recovery from GW signals. Another thing we notice is that there is a jump in the recovered \( \eta \) posterior when the eccentricity changes from \( e_{15} = 0 \) to \( e_{15} = 10^{-3} \). This is due to a slight discontinuity in the
FIG. 4. Estimated values of the symmetric mass ratio for signals at different eccentricities. Dashed lines indicate the value of the simulated signal. The colors blue, green, and purple correspond to the $q = 1, 0.5, 0.33$ signals respectively. The bars represent the 90% confidence intervals, with diamonds representing the median value. The shaded region is to illustrate the upward trend. The labels on top specify which model was used in the parameter estimation, and the labels on the right side of the figure specify which model was used to simulate the signal. Note the different scales between $[0.01, 0.01]$ and $[0.01, 0.3]$ on the $x$-axis.

FIG. 5. Estimated values of the luminosity distance for varying eccentricity using TEOBResumE as the signal waveform and IMRPhenomD in the likelihood model. The dashed line is the signal's distance of 400 Mpc, with the bars represent the 90% confidence intervals, and diamonds the median value. Note the different scales between $[0.01, 0.01]$ and $[0.01, 0.3]$ on the $x$-axis.

TEOBResumE waveform as the eccentricity goes to 0. For small eccentricities, $e_{15} \leq 0.01$, there is good agreement between TEOBResumE and SEOBNRv4_ROM. There is a slight bias when $e_{15} \leq 0.01$ in the IMRPhenomD case, since the Phenom family of waveforms is constructed differently from the EOB family. In the TaylorF2 case, the recovered $\eta$ is overestimated as compared to the simulated value except for $\eta = \frac{1}{3}$. This systematic bias is due to the lack of the merger-ringdown portion in the templates.

Now consider the bottom row of ENIGMA-Inspiral signals. In both the SEOBNRv4_ROM and IMRPhenomD case, we see the recovered $\eta$ is underestimated compared to that of the simulated signal. Again this is because the full IMR templates fit their merger-ringdown portion to the last few cycles of the inspiral-only signal. In these cases the bias in recovery between IMR templates with an inspiral signal is greater than the bias caused by using different waveform families. The bias at $e_{15} = 0$ is much reduced in the case of TaylorF2, since both the signal and templates are inspiral-only. However there is a slight bias in the recovery of the $\eta = \frac{1}{3}$ signal, so the two waveforms models do not entirely agree in this limit.

Despite these discrepancies, the trend we have already seen in Figure 2 clearly persists in all cases: as the eccentricity of the signal increases, the recovered mass ratio becomes more consistent with that of a symmetric binary system. This is quite striking, particularly at $e_{15} = 0.3$, the recovered mass ratio is consistent with an equal-mass binary, regardless of the signal's mass ratio for all com-
3. Distance Estimates

From the representative corner plots of Figure [1] we can see that in eccentric case there is also a slight bias in the recovery of the luminosity distance. Figure [2] shows the trend for the case of a TEOBResumE signal with IMRPhenomD template. This overestimation of luminosity distance is consistent with the templates not capturing the entire signal SNR because of missing physical effects, i.e., orbital eccentricity. Having said that, even though there is a clear trend upward in the distance recovery, we find it to broadly remain within the statistical errors of the parameter estimation. We expect this bias to become significant only at even higher SNR than we consider. In the worst case, the median value is ≈ 50% higher than that of the simulated value. We also note that the behavior is largely identical for each of the different simulations’ mass ratio.

4. Maximum a-posteriori Estimated Waveforms

To help understand what features of the simulated and recovered best-fit waveforms contribute to the biases in parameter estimation, it is useful to look at the recovered maximum a posteriori (MAP) waveforms for some of our signals.

Figure [3] shows the maximum likelihood estimated IMRPhenomD waveform from four representative runs with \( q = 0.33 \), with TEOBResumE and ENIGMA-Inspiral used for the signals. There is strong agreement in the zero-eccentricity limit between the template and both signal waveform models, as is to be expected. When the eccentricity is increased to \( e = 0.3 \), the quasi-circular model tries to best fit to the eccentric signal. In the case of the full IMR signal with TEOBResumE, the best quasi-circular template fits to the frequency of the last few cycles along with the merger-ringdown. In the inspiral-only case, the best template tries to fit to the lower frequency cycles of the last portion of the inspiral.

B. Gravitational-wave Events

Recently, Ref. [102] has posited that events GW151226 and G170608 in the first GW transients catalog [11] are mergers of eccentric binary black holes. This contrasts the conclusion of Ref. [55], that these two events are likely quasicircular binary black hole mergers. It is important to note that of these two studies, the first used an inspiral-only, non-spinning eccentric waveform model [79], while the latter used a complete inspiral-merger-ringdown model from the effective-one-body family: SEOBNRE [82]. Because they use different waveform models, their eccentricity measurements are not directly comparable in a precise sense [77]. But the claim of an event being eccentric at all could qualitatively withstand this difference, as its unlikely that any waveform model would assign zero eccentricity to a waveform that has modulations manifested by eccentric binaries. In addition, Ref. [55] does not sample from the posterior distribution directly as SEOBNRE is too expensive to evaluate, instead their eccentric posteriors were found by re-weighting posteriors from non-eccentric parameter estimation runs.

In this section we study these two events with a two-fold purpose. We use the full IMR eccentric waveform model to attempt to better understand the nature of these two events. And we use them to further confirm the measurement degeneracy between binary chirp mass and initial orbital eccentricity that we have so far shed light on using simulated signals. Our application of TEOBResumE is sufficiently fast to allow its direct use in parameter estimation runs with eccentricity and spins.

We use the same Bayesian inferencing setup as described in Sec. [113] We use the open-source data available from the Gravitational-Wave Open-Science Centre [72]. The estimated parameters with confidence intervals for this section are summarised in Table [IV].

1. Non-spinning, Eccentric Inference

First we perform parameter estimation using TEOBResumE with a non-spinning prior, but now we include eccentricity in the analysis. Our prior for the eccentricity is uniform in the range \( 0 - 0.3 \), and the waveforms are generated from a lower frequency of 10 Hz, rather than the 15 Hz used in our simulated signals. This is why we denote the measured initial eccentricity as \( e_{10} \). This is to allow us to compare our eccentricity measurements with Refs [55] and [50] who use this as their reference frequency. We again use \( f_{\text{min}} = 20 \) Hz in the evaluation of the likelihood integral in Eq. [3].

Panels (a) and (b) in Figure [7] show the parameter estimation results of these two events. In the case of GW151226 we measure an eccentricity of \( e_{10} \approx 0.2 \), with \( e_{10} = 0 \) strongly excluded from the posterior by our non-spinning eccentric waveform model (this measurement will change when we include the effect of BH spins in the following section). Furthermore, in agreement with our study of simulated eccentric signals, the recovered chirp mass differs remarkably from that measured by the LVK in [72], by an amount greater than the statistical error of the measurement. In the case of GW170608, the recovered eccentricity is very much consistent with \( e_{10} = 0 \), with an upper 90% estimate of \( e_{10} = 0.16 \). Its chirp mass is measured to be somewhat lower than the LVC estimate, albeit consistent with 90% credible intervals. In both cases the symmetric mass ratio is determined to be closer to \( q = 1 \) with greater certainty than the LVC estimate [11].

In both cases, the 2-dimensional marginalized poste-
rrior for the chirp mass and eccentricity corroborate what our simulated signal study found along with [61]: the measurement of the chirp mass and eccentricity are correlated. These results show that GW151226 would be interpreted as an eccentric system with a lower chirp mass than previously thought [1], if we disregard component spins. The same effect is observed to a lesser degree in GW170608, which we find to be consistent with having a non-eccentric origin. We also note that allowing for eccentricity as a degree of freedom helps make the measurement of both chirp mass and mass ratio more precise for this event.

2. The importance of including spin

It is well known that the measurement of binary spins is correlated with the measurement of the mass parameters when determining the source parameters of gravitational-wave signals. As explained by [103], the error in the estimated chirp mass and mass ratio increase when one includes the spins of compact objects in their analysis, even if the system is non-spinning. This makes it reasonable to expect our confident mass estimates to be influenced by the exclusion of spin in our prior. Also, as we have found previously, the chirp mass measurement is correlated with orbital eccentricity, and so it is also possible that the chirp mass measurement is correlated with the spin measurement.

We repeat our analysis of the two events GW151226 and GW170608, with the same set of priors as in the previous section, but now with an aligned-spin prior. We use the “z-prior” implemented in Bilby and described in [104] for the aligned spins. We perform several runs with different values of the maximum spin magnitude for the prior to investigate how the resulting posterior depends on the spin prior. We use $|\chi_{\text{max}}| < 0.3, 0.7, 0.99$. The value of 0.7 is chosen since the model TEOBResumE is only validated against NR waveforms with spin up to this value. In the results we will quote only the results of the effective spin, which is the best recovered spin parameter [105] and is defined as

$$\chi_{\text{eff}} = \frac{m_1\chi_1 + m_2\chi_2}{M}$$

where $M$ is the total mass and $\chi_{1,2}$ are the components of the spin aligned with the binary’s orbital angular momentum.

Panels (c) and (d) in Figure 7 show the results for the two events with $|\chi_{1,2}| < 0.3$. Panel (c) shows the results from GW151226. The posterior for eccentricity now has support for $e_{10} = 0$, in contrast with the non-spinning prior, although the $e_{10}$ posterior peaks at about $e_{10} = 0.16$. From the $M_c - e_{10}$ plane, we can clearly see the negative correlation between the chirp mass and eccentricity. The chirp mass estimate agrees better with that of the LVC than in the non-spinning case, except the posterior is slightly broader due to the higher eccentricity contribution of the posterior. Most interesting for this event is the $\chi_{\text{eff}} - e_{10}$ plane: we see a negative correlation between the spin and eccentricity measurements, with $e_{10} = \chi_{\text{eff}} = 0$ excluded.

Panel (d) of Figure 7 shows the results for GW170608. We find that including spins has much improved the agreement with the LVC chirp mass estimate. Also the posterior for orbital eccentricity has become narrower, with the upper 90% credible limit decreasing from $e_{10} = 0.18$ to 0.13. We can conclude that even when the event being analysed is likely non-eccentric, including spin in the analysis improves the confidence in the
FIG. 7. *Red:* Posterior probability distribution using TEOBResumE for GW151226 and GW170608. Panel captions indicate the maximum value of the \( z \)-component of BH spins, \( \chi_{1,2} \), allowed in the parameter estimation prior. Median estimates with 90\% confidence intervals are shown for this data. *Blue:* GWTC-2 data from [101]. Contours show the regions of 50\% and 90\% confidence. See Table IV for a comparison with the exact medians and error estimates of the GWTC data. See section III B for discussion.
eccentricity constraints.

Panels (e) and (f) show the results from expanding the maximum spin in the prior to 0.99. Panel (e) shows the event GW151226. We can immediately see that the constraint on the eccentricity is improved, and the recovery of the masses and effective spin are in better agreement with the GWTC-2 values. The correlation between the effective spin and eccentricity is still present in the 2D $\chi_\text{eff} \sim e_{10}$ posterior but appears weaker than in the case of the restricted spin prior. This demonstrates further that the measurement of the eccentricity is correlated with that of the spin, and that to accurately measure the eccentricity one should sample from as much of the spin prior as possible, which requires waveform models that are reliable for moderate to large spin values.

Panel (f) shows the event GW170608. In this case the posteriors are largely unchanged from the $|\chi_{1,2}| < 0.3$ case, with the eccentricity constraint becoming $e_{10} \lesssim 0.12$. Since this event does not have appreciable spin, the more restrictive spin prior is well able to account for the full range of physics contributing to the posterior.

Table IV summarises the results from each of our runs. We opt not to plot the posteriors for the case of $|\chi_{1,2}| < 0.7$ but the results are largely unchanged between this case and that of $|\chi_{1,2}| < 0.99$, despite the TEOBResumE model not being validated to spins of that magnitude.

To evaluate the preference for the eccentric model over the non-eccentric one, we can calculate the Bayes factor:

$$B = \frac{Z_{\text{eccentric}}}{Z_{\text{non-eccentric}}}.$$  \hspace{1cm} (6)

where $Z$ is the Bayesian evidence. Since the set of parameters for the non-eccentric model is nested within the full eccentric model, this ratio becomes the Savage-Dickey factor [106]:s

$$B = \frac{\pi(e_{10} = 0)}{p(e_{10} = 0)}$$  \hspace{1cm} (7)

which is simply the ratio of value of the prior weight $\pi(e_{10} = 0)$ to the posterior weight $p(e_{10} = 0)$.

Table III shows the estimated Bayes’ factors for the two events as a function of increasing the maximum spin in the prior. In all cases we have $B < 1$, indicating that the non-eccentric model is preferred. In the case of GW151226, the preference for the non-eccentric model increases as the maximum spin is increased in the prior. As expected from our previous results, increasing the spin has little effect on the results in the case of GW170608.

### C. Circular Simulated Signals for GW151226 and GW17060

We reaffirm our findings for GW151226 and GW170608 by follow-up analyses with a set of simulated signal studies. The purpose of this is to investigate how binary source parameters would be recovered using TEOBResumE, if the signals were truly non-eccentric. We use the MAP source parameters from the GWTC posteriors for GW170608 and GW151226, and again inject the signals into zero-noise, and use the LIGO Hanford and Livingston detectors noise curves.

We use the TEOBResumE model with aligned spins to simulate the signals, and then repeat our parameter estimation using the same model, with the sampler setup from our previous runs. We wish for the signal to have $e_{10} = 0$, but from Figure [4] we saw that TEOBResumE has a discontinuity at $e_{10} = 0$ so we set the value to $10^{-10}$. We generate our signals from a lower frequency of 10 Hz, and use 20 Hz as the lower frequency cutoff in the evaluation of the likelihood integral. We use the same eccentricity prior as before in all cases, and again perform two runs for each event’s parameters: with a non-spinning prior and with an aligned spin prior.

Figure 8 shows the results for the two events’ parameters using both a non-spinning and an aligned-spin prior with $\chi_{1,2} < 0.3$ and $< 0.99$. Panels (a) and (b) show the non-spinning case. We see the same general trends as in Figure [4] with the real event data: the chirp mass for both events is underestimated, the mass ratio is overestimated to be more consistent with that of a symmetric binary, and the presence of spin in the case of GW151226 leads to a spurious eccentricity measurement.

Panels (c) and (d) shows the results when the prior is expanded to include aligned spins up to 0.3. Since we are using the same waveform model for the signal and the parameter estimation, we now see a much better recovery of parameters as expected. Again the non-zero eccentricity measurement for GW151226 disappears.

In particular, the correlation between spin and eccentricity seen in panels (c) and (e) in Figure 7 is not seen, and the signal is well recovered as a spinning non-eccentric signal.

Panels (e) and (f) show the results when the maximum spin is increased to 0.99. The results are qualitatively similar, but the recovery of the effective spin is improved.

These results also further illustrate the fact the mass parameters, spin and eccentricity are all correlated and it is important to include each of these effects when performing parameter estimation. We defer a more systematic investigation of this degeneracy to future work.

| Prior | GW151226 | GW170608 |
|-------|----------|----------|
| $|\chi_{1,2}| < 0.3 \ | 0.81 | 0.28 |
| $|\chi_{1,2}| < 0.7 \ | 0.45 | 0.28 |
| $|\chi_{1,2}| < 0.99 \ | 0.40 | 0.28 |

### TABLE III. Bayes factors for the eccentric versus non-eccentric model, with increasing value of the maximum spin in the prior.
FIG. 8. Posterior probability distribution using TEOBResumE for GW151226-like and GW170608-like simulated signals, with an eccentric prior. Shown are the recovered distributions for source chirp mass, $\mathcal{M}$, mass ratio $q$, effective spin $\chi_{\text{eff}}$ and eccentricity $e_{10}$. Red marks the simulated signal’s values, which correspond to the MAP values given by the GWTC posteriors for the corresponding event. Shaded contours correspond to the 90% and 50% confidence regions. See section III C for discussion.
TABLE IV. Median estimates of source parameters from using TEOBResumE for parameter estimation. The left-hand column specifies the prior on the aligned spin, or whether the data is from GWTC-2. Upper and lower error estimates are the 90% confidence intervals of the 1D marginalized posterior.

| $\mathcal{M}_c$ | $q$ | $\chi_{\text{eff}}$ | $\epsilon_{10}$ |
|----------------|-----|---------------------|----------------|
| **GW151226** |
| | | | |
| $|\chi_{1,2}| = 0$ | 9.38$^{+0.04}_{-0.06}$ | 0.90$^{+0.09}_{-0.15}$ | 0.19$^{+0.02}_{-0.02}$ |
| $|\chi_{1,2}| \leq 0.3$ | 9.64$^{+0.09}_{-0.16}$ | 0.84$^{+0.14}_{-0.26}$ | 0.14$^{+0.07}_{-0.08}$ |
| $|\chi_{1,2}| \leq 0.7$ | 9.76$^{+0.08}_{-0.12}$ | 0.68$^{+0.29}_{-0.37}$ | 0.20$^{+0.15}_{-0.09}$ |
| $|\chi_{1,2}| \leq 0.99$ | 9.71$^{+0.08}_{-0.12}$ | 0.64$^{+0.33}_{-0.34}$ | 0.21$^{+0.06}_{-0.09}$ |
| **GWTC-2:** | 9.69$^{+0.08}_{-0.06}$ | 0.56$^{+0.38}_{-0.23}$ | 0.18$^{+0.20}_{-0.12}$ |
| **GW170608** |
| | | | |
| $|\chi_{1,2}| = 0$ | 8.44$^{+0.02}_{-0.03}$ | 0.91$^{+0.08}_{-0.15}$ | 0.08$^{+0.08}_{-0.07}$ |
| $|\chi_{1,2}| \leq 0.3$ | 8.56$^{+0.05}_{-0.06}$ | 0.83$^{+0.15}_{-0.26}$ | 0.05$^{+0.06}_{-0.04}$ |
| $|\chi_{1,2}| \leq 0.7$ | 8.56$^{+0.05}_{-0.05}$ | 0.79$^{+0.19}_{-0.34}$ | 0.06$^{+0.11}_{-0.05}$ |
| $|\chi_{1,2}| \leq 0.99$ | 8.51$^{+0.05}_{-0.06}$ | 0.79$^{+0.19}_{-0.34}$ | 0.06$^{+0.12}_{-0.05}$ |
| **GWTC-2:** | 8.49$^{+0.05}_{-0.05}$ | 0.69$^{+0.28}_{-0.36}$ | 0.03$^{+0.18}_{-0.06}$ |

IV. DISCUSSION

Let’s consider a single case from our simulated signal recovery, with $M^\text{signal}_{\text{total}} = 20M_\odot$ and $q^\text{signal} = 0.33$. This corresponds to $m^\text{signal}_1 = 15M_\odot$ and $m^\text{signal}_2 = 5M_\odot$, but the recovered source masses would be $m_1 \approx m_2 \approx 9.2M_\odot$. Correct inference of the source masses is important for our understanding of the population of such objects, and in extreme cases such biases could be enough to incorrectly identify certain objects to inside or outside of the NS-BH mass gap of $\approx 2 - 5M_\odot$ or the supernova pair-instability mass gap of $\gtrsim 50M_\odot$. Thus, determining whether a signal is eccentric is important for the determination of the rest of its source parameters. As more detectors join the array of ground based detectors, the SNR of such eccentric signals will increase and systematic biases will become more pronounced when compared to the estimated error from the parameter estimation.

In section III B and III C we could see the $\mathcal{M}_c - \epsilon_{10}$ correlation manifest in the 2-D marginalized posterior for the two gravitational wave events GW151226 and GW170608. Although performing inference on these events with a non-spinning, eccentric prior corroborated this correlation, neglecting to include spin in the analysis leads to over-confident and biased estimates of the mass parameters. Clearly a more systematic investigation of the relation effects of eccentricity on the spin measurement and vice-versa are needed, which we defer to a future study.

In future we will want to accurately measure the eccentricities of newly detected signals, as well as those already present in GWTC2. It is clear that we will not only need accurate and fast-to-evaluate waveform models that can incorporate eccentricity, but also spins, and most likely precession. By using TEOBResumE we have constrained the eccentricities of GW170608 and GW151226 to be $< 0.12$ and $< 0.15$ at 90% confidence respectively. Our constraints are slightly tighter than the values of $< 0.166$ and $< 0.181$ found by [54] which used a non-spinning, inspiral-only model. However, we find a much looser constraint than those set by Ref. [55], which found $\epsilon_{10} < 0.04$ for each of these events by re-weighting samples from a non-eccentric parameter estimation run using the model SEOBNRE. From Fig 8 we found that our method could only constrain a zero eccentricity to about $\epsilon_{10} \approx 0.06$. In a similar light Ref [51] also found that their measurement of the eccentricity of GW190425 is a much looser constraint than that of Ref. [55], and explains that this could be due to the use of a log-uniform prior on $\epsilon_{10}$, but also that the application of posterior re-weighting scheme might have failed to capture the correlation between the chirp mass and eccentricity, leading to overconfident measurements as compared to their full MCMC calculation (although one does find a hint of the correlation between eccentricity in the supplementary figures [107, 108] of Ref. [55], making the difference in priors a more likely explanation). One way to compare posteriors which use different priors is to use rejection sampling to reweight the posterior from one prior to another. However, we found that the reweighting efficiency from a uniform to log-uniform prior is too low and results in only $\approx 40$ samples from the initial $\approx 20000$ samples. To fully investigate the choice of prior thus requires performing the parameter estimation with a log-uniform prior, which we defer to a future study.

In this paper we chose to focus on the two lower mass black hole events from the GWTC1 data. It would be interesting to continue this analysis by applying TEOBResumE to the parameter estimation of the rest of GW transient catalog, in particular to see whether the eccentricity of events such as GW190521 can be established or further constrained, and also whether the presence of eccentricity has biased the existing LVC estimates of their source parameters.

Finally, we list some of the limitations and caveats for the results presented in this paper. First: our investigation of intrinsic parameter space degeneracies focuses on non-spinning black hole binaries. This was chosen in order to reduce computational time and simplify the parameter space. We plan to extend this to include black hole spins in the future. It is important to note though that in dynamical environments where eccentric binaries are expected to form, there is not much reason for component BH spins to align with the orbit, and therefore a
thorough investigation including spins would need some consideration of orbital precession as well, and no waveform model exists yet that can model precession as well as eccentricity. Second: our investigations do not include sub-dominant waveform modes in templates. We also plan to extend this study to include them, once suitable waveform models become available. Third: the waveform model we use to study GW events, i.e. TEOBResumE, does not allow for variation of mean anomaly and we therefore kept it fixed in our parameter estimation analyses. This can lead to small biases that are not expected to qualitatively change this paper’s results. However, we also plan to address this limitation in future work.

V. SUMMARY

In section III A we showed that there is correlation between the measurement of the mass parameters of a binary compact object system and its eccentricity. If one uses quasi-circular templates in their parameter estimation, this correlation manifests as a bias in the measured chirp mass and mass ratio. In the moderately eccentric case of $e = 0.3$, a bias in the chirp mass of up to $\approx 4\%$ can occur, but the bias in the mass ratio can be much more, since it tends to make the binary system appear more symmetric.

In section III B we sought to further establish this correlation on real gravitational wave data. However, we found that in order to correctly estimate the masses along with the eccentricity of such systems, we must employ a waveform model that can account for eccentricity and spins simultaneously. Using the eccentric, aligned spin model TEOBResumE, we measure the eccentricity of GW170608 and GW151226 to be $<0.12$ and 0.15 at 90% confidence respectively.

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Appendix A: ENIGMA Merger-ringdown

The merger-ringdown portion of the ENIGMA waveform is constructed under the assumption that a moderately eccentric compact binary has circularized by the time it has reached merger, which has been shown to occur in numerical relativity simulations of eccentric binaries [110]. Under this assumption, the parameter space for the merger-ringdown portion of the waveform becomes one-dimensional, depending only on the mass ratio $q$. A one-dimensional Gaussian-process-regression surrogate model is constructed to interpolate between a training set of quasi-circular numerical relativity waveforms to any mass ratio up to $q = 10$.

The final step in generating an ENIGMA waveform is finding the optimal attachment time between the PN inspiral waveform and the surrogate merger-ringdown waveform. The optimal attachment frequency is determined to be that which optimizes the overlap between a circular ENIGMA waveform and the corresponding SEOBNRv4 waveform. This overlap is calculated as in equation 3 with $f_{\text{min}} = 15\text{ Hz}$ and $f_{\text{max}} = 4096\text{ Hz}$. Due to the instantaneous monochromaticity of these circular waveforms, the attachment frequency can be converted to a corresponding attachment time.

In our studies, we found that ENIGMA sometimes furnishes qualitatively incorrect waveforms at smaller values of total mass, and mass ratios $<0.5$. This is because the optimal matching time is found by optimizing the overlap of two waveforms, but with respect to the LIGO PSD. At lower masses, which correspond to higher gravitational wave frequencies, the LIGO PSD pushes the optimum attachment frequency lower than it should. The attachment time is then many cycles before the merger, and the assumption that the eccentricity has radiated before attaching the merger-ringdown waveform is no longer valid. This manifests as a cusp in the amplitude of the waveform. Figure 9 illustrates the difference between ENIGMA-generated waveforms with $M_{\text{total}} = 20M_{\odot}$ and $M_{\text{total}} = 80M_{\odot}$. The rescaled amplitudes should be coincident, but in the low mass case the eccentricity “switches off” in a non-smooth manner many cycles before the merger. We find this same behavior regardless

![FIG. 9. Green: Amplitude of ENIGMA generated waveform with $m_1 = 14M_{\odot}$, $m_2 = 6M_{\odot}$, $e_{10} = 0.3$. Blue: The ENIGMA generated waveform with the masses scaled by a factor of 4. The amplitude and samples times of the waveform are scaled by the same factor of 4 so they can be compared.](image-url)
of the eccentricity value, but have chosen $e_{10} = 0.3$ to accentuate the modulation of the amplitude and make the difference more visible.

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