Nonlinear Finite Element Analysis of Shear Strength for Steel Fiber Reinforced Concrete I-Section Beams

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Abstract. This research presents a numerical model of I-section fiber reinforced concrete beams to evaluate the shear strength and complete behavior of the beams using the numerical formulation, which compared with the results of (twenty-three) tested I-section beams taken from previous investigations which are varying in shear span to effective depth ratio and volume fraction of steel fibers. Models using finite element method having the same dimensions and properties as the experimental test specimens. It was shown that the behavior of SFRC I-section beams could be modeled well using the proposed material constitutive relationships and there was a good agreement between the experimental and numerical failure load, crack pattern load deflection curves obtained from this study. The finite element model shows slightly more stiffness results than the test data ranges. The average ratio of the experimental to the calculated results is (1.027).

Keywords: Finite element modeling, I-section beams, Shear strength, Steel Fiber Reinforced Concrete (SFRC).

1. Introduction

The steel fibers addition in concrete develops the mechanical properties of the plain matrix as they decelerate the growth of crack and creates compressive forces at the top of cracks thus, the energy absorption for SFRC materials increases, moreover it would be the excellent choice for enhancing the seismic-resistance and blast-resistance for structures [1]

It has been established that steel fibers can offer significant improvement in the properties of plain concrete, mainly directed towards flexure strength and much less is known about shear strength of RC beams, especially I-beams [2].

Awoyera [3] used ADINA software to model a SFRC beams with dimensions of 100mm×100mm×500mm. The steel fiber was added in three different percentages (0, 0.75, and 1) % by volume. It was concluded that the existence of steel fibers delays the expansion of micro cracks due to the bond between the fibers and matrix and the failure occurs due to the fiber cutoff.

Min Sun et al. [4] analyzed two T-section beams, one of them made of SFRC, and the other one was made of NC. The effect of various volume fractions of steel fibers on ultimate shear capacity and crack spreading using ANSYS software was studied. The main conclusions were that the steel fiber improved the rigidity and ductility and the finite element results indicated a perfect conformity with test results.
and a perfect emulation of T-section beams employing smeared crack model.

Aisyahira [5] made a nonlinear analysis using LUSUS software to model three beams with three types of concrete, Normal Concrete (NC), Steel Fiber Self Compacting Concrete (SFSCC) and Self-Compacting Concrete (SCC).

The SFSCC beams with 50% of traditional shear reinforcement were analyzed to investigate the steel fibers contribution to the shear reinforcement with percentage of (1%) by volume. It was established that the ultimate shear strength improved by 37.1% as compared as normal concrete, the deflection also improved between (15.6-35) percent. The difference between the theoretical using finite element method and experimental results was (8-18) percentage, which indicates that LUSAS software gives a good representation.

Jing et. al. [6] used ANSYS workbench 16.1 to model three categories of beams. All of them contain the same percentage of steel fiber of (0, 1, 2, 3, 4) % by volume, in the second category of beams the basalt fiber reinforced polymer (BFRP) bars were used, while the third one contained carbon fiber reinforced polymer (CFRP) bars. It was determined that when the percentage of fiber increases from 0% to 4% the failure load increases and the deflection decreases.

Studies by Mansur, Ong, and Paramasivam [7] suggest that the ultimate shear capacity predictions for SFRC beams can be made based on the post cracking tensile strength of fiber along with some empirical formulas for reinforced concrete beams, while Narayanan and Kareem Palanjian [8] and Sharma [9] recommend using tensile strength obtained using split cylinder tests. Also, there are studies by Lim, Paramasivam and Lee [10] indicating that, for ultimate load predictions for SFRC beams, the effect of steel fibers can be expressed in terms of an equivalent shear reinforcement.

The goals of this work are to study the efficiency of steel fiber on the shear strength of SFRC I-sections. Two-dimensional nonlinear finite element models are used to study the tested I-beams performance.

2. Finite Element Formulations

Plane stress state were used to represent the I-section beams, and eight nodded isoperimetric elements were used to exemplify the concrete. Cook [11] displays the formulation of this element and the appropriate matrices.

The embedded bar model was used to exemplify the conventional reinforcing bars. The concrete and the reinforcement are simulated so that a perfect bond between them; which indicates that the strain is the same for concrete and steel at any point.

2.1 Constitutive Relationships for Materials Used in Fibrous Concrete:

2.1.1 Compressive Stress-Strain Model

In the current investigation a strain–hardening approach was implemented, which consists of a linear part that begins from zero to 30 % of the compressive strength trailed by stress–strain curve with a parabolic shape, that sustains until reaching the compressive strength of fibrous concrete, then the behavior is assumed to be a perfect plastic till crushing happens as shown, Figure (1). Soroushian and Lee [12] provide the strain $\varepsilon_{pf}$ at peak stress $f_{cf}$, which is adopted in this study:

$$\varepsilon_{pf} = \frac{2f'}{E_c} + 0.0007\frac{V_I}{d_f}$$

(1)
where $f'_c = \text{compressive strength of standard cylinder for normal concrete}$, $V_f = \text{steel fiber percentage by volume}$ and $d_f$ and $l_f$ = diameter and length of the steel fibers correspondingly.

To estimate the compressive strength for fibrous concrete, the equation suggested by Soroushian and Lee [12] and adopted by Abdul Razzak [13], can be used:

$$f'_c = f'_c + 3.60v_f l_f / d_f \quad (2)$$

The result of ultimate compressive strain obtained by Abdul-Razzak [13] is adopted in this analysis, which was obtained using the experimental test results by shah and Rangan [14]

$$\varepsilon_{cu} = 3011.0 + 2295.0v_f (x10^{-6}) \quad (3)$$

### 2.1.2 Tensile Stress–Strain Model

Carreira and Chu [15] proposed a continuous function for the ascending and descending portions of the tensile stress-strain curve of plain concrete which is adopted in references [16 and 17] for steel fiber concrete, see Figure (1), in Equation 4 that is adopted in this study:

$$\frac{f'_t}{f'_t} = \frac{\beta (\varepsilon / \varepsilon'_t)}{\beta - 1.0 + (\varepsilon / \varepsilon'_t)^\beta} \quad (4)$$

where $\varepsilon = \text{the tensile strain at a stress equal to } f'_t$, $f'_t$ = tensile strength of steel fiber concrete, $\varepsilon'_t = \text{strain corresponding to peak stress}$ and $\beta$ = a fibers parameter predicted in references [18, 19] as represented below:

$$\beta = 1.093 + 0.7132 R.I.^{-0.926} \quad \text{Fiber type is hooked (Eq. 5a)}$$

$$\beta = 1.093 + 7.4848 R.I.^{-1.387} \quad \text{Fiber type is smooth (Eq.5b)}$$

$$\beta = 0.5811 + 1.93 R.I.^{-0.7406} \quad \text{Fiber type is crimped (Eq.5c)}$$

$R.I.$ is an index of reinforcing $= w_f l_f / d_f$ and $w_f$ is the ratio of the weight of the fibers to that of concrete. If there was no available experimental data, the strain $\varepsilon'_t$ at ultimate stress $f'_t$ for fibrous concrete can be represented as suggested by Soroushian and Lee [12] as follow:

$$\varepsilon'_t = \varepsilon_t \left(1.0 + 0.35 N_f d_f l_f \right) \quad (6)$$
\[ f'_f = f'_f \left( 1.0 + 0.016 N_f \left( \eta \right) + 0.05 \pi d_f J_f \right) \] (7)

where, \( e_c \) = the strain at which cracking is occurred in plain concrete = \( f'_c / E_c \), \( f'_f \) = tensile strength of normal concrete, \( E_c \) = modulus of elasticity of normal concrete and \( N_f \) = Number of fibers for each unit area and it is equal to \( \eta \left( 4.0 V_f / \pi d_f^2 \right) \). \( \eta \) is the factor of orientation, which is equal to 0.41 as proposed by Ezeldin [19] or 0.5 as suggested by Hannant [21] for three dimensional distribution.

![Figure 1 Uniaxial Stress-Strain in Compression Relationships for steel Fiber Concrete.](image)

2.2 **Biaxial Performance of Steel Fiber Concrete**

In the state of compression-compression, plastic behavior of concrete supposed to begin when the effective stress transcends \( 0.3 f'_c \) and perfect plasticity supposed to take place when the effective stress arrives at \( f'_c \). A yield criterion was proposed to follow the plastic performance:

\[ f(I_1, J_2) = \left[ \beta_f (3J_2) + \alpha_f I_1 \right]^{0.5} = f'_c \] (8)

where \( I_1 \) and \( J_2 \) are the 1st stress invariant and 2nd stress invariant and \( \alpha_f, \beta_f \) are materials factors attained in reference [13] using fibrous concrete mixes obtained from previous experimental work:

\[ \alpha_f = \frac{1 - \omega^2}{\omega^2 - 2 \omega} f'_c \quad \beta_f = \frac{1 - 2 \omega}{\omega^2 - 2 \omega} \] (9)

and \( \omega = e^x \) where:

\[ x = \frac{1}{3.339 - 0.9772 \ln(v_f l_f / d_f)} \] (10)

An isotropic rule of hardening is assumed to express the uniform expansion for the yield surface.

In state of the tension-compression, the parabolic curve described below that has been proposed by Mahmood [22] and adopted by Reference [13] is adopted in this study:
\[
\sigma_2 / f'_{cf} + S^2 \left[ \sigma_1 / f'_{cf} \right]^2 = 1
\]

where \( S = f'_{cf} / f'_{gf} \).

In the state of tension-tension, the assumption is there was no reciprocal interaction, i.e., the tensile strength of concrete in the two orthogonal directions is the same as that in the uniaxial state.

2.3 Shear Modulus of Cracked Concrete

A stress principle is used for cracks beginning and when the concrete crack occurs in direction 1; the formula proposed by Al-Mahaidi [23] for predicting the shear modulus of the cracked plain concrete and was used soon after by Abdul-Razzak and AL-Hasan [13and16] for fibrous concrete is adopted in the present study:

\[
\bar{G} = 0.40G / (\varepsilon_1 / \varepsilon_{cf})
\]

If the cracking is occurred in direction 2, \( \bar{G} \) turn into Eq. 13:

\[
\bar{G} = 0.40G / (\varepsilon_2 / \varepsilon_{cf})
\]

In this region, the relationship proposed by Vecchio and Collins [23] treated the negative impact of cracks on the strength of concrete in the normal direction and was used by Abdul-Razzak [13] for steel fiber concrete.

\[
\frac{f_{cf}^{\max}}{f'_{cf}} = \frac{1.0}{0.80 + 0.34 \left( \frac{\varepsilon_1}{\varepsilon_0} \right)} \leq 1.0
\]

Where:

- \( \varepsilon_1 \): Principal tensile strain in concrete.
- \( \varepsilon_0 \): Concrete cylinder strain at peak stress.

2.4 Reinforcement Stress-Strain Relationship

A bilinear relationship for tension and compression was proposed from zero stress up to the yielding strength with a slope \( E_s = 200 \text{GPa} \) trailed by a straight line with a suggested hardening modulus \( E_s' \).

3. Numerical applications:

An interesting series of researches by Muhidin and Tan [24 and 25] have encompassed a wide-ranging study on fiber concrete I-section beams failing in shear and flexure, discussing the influence of several parameters including percentage of fibers, concrete compressive strength, a/d ratio with the same boundary restraint. Twenty-three SFRC I-section beams have been analyzed by using computer program described previously. Finite element formulation was employed to obtain the behavior of SFRC I-section beams without stirrups as shear reinforcement. The predictions obtained using the numerical model has been computed with the test results.
Table 1. Beams details with the experimental and calculated results

| Beam No. | Beam Designation | Ref. | Vt, % | l₁ (mm) | d₁ (mm) | fₑu (N/mm²) | fₛ (MPa) | bₜ (mm) | Experimental Ultimate Load (kN) | Theoretical Ultimate Load (kN) | Exp./Theo. Ratio |
|----------|-----------------|------|-------|---------|----------|--------------|----------|---------|---------------------------------|--------------------------|-----------------|
| 1        | A1              | 24   | 2.13  | 60      | 0.64     | 45.7         | 50       | 71.3    | 65.0                            | 1.09                     |                 |
| 2        | A2              | 24   | 2.25  | 25      | 0.4      | 49.8         | 4.7      | 50      | 81.0                            | 78.0                     | 1.037           |
| 3        | A3              | 24   | 0.75  | 25      | 0.4      | 66.1         | 4.9      | 50      | 64.5                            | 64.8                     | 0.99            |
| 4        | A4              | 24   | 1.5   | 25      | 0.4      | 54.1         | 5.1      | 50      | 61.1                            | 60.7                     | 1.00            |
| 5        | A5              | 24   | 3.0   | 25      | 0.4      | 52.5         | 4.9      | 50      | 78.9                            | 76.3                     | 1.034           |
| 6        | A6              | 24   | 2.25  | 40      | 0.5      | 42.3         | 4.7      | 50      | 59.9                            | 59.7                     | 1.00            |
| 7        | A7              | 24   | 2.25  | 60      | 0.64     | 42.3         | 5.3      | 50      | 76.4                            | 73.6                     | 1.037           |
| 8        | A8              | 24   | 1.5   | 40      | 0.5      | 52.2         | 4.7      | 50      | 65.0                            | 65.5                     | 0.99            |
| 9        | A9              | 24   | 1.5   | 40      | 0.5      | 24.3         | 3.2      | 50      | 54.9                            | 49.5                     | 1.11            |
| 10       | A10             | 24   | 1.5   | 40      | 0.5      | 76.3         | 6.5      | 50      | 69.5                            | 83.7                     | 0.83            |
| 11       | A11             | 24   | 1.5   | 40      | 0.5      | 55.0         | 5.2      | 70      | 93.8                            | 96.0                     | 0.97            |
| 12       | A12             | 24   | 1.5   | 40      | 0.5      | 55.0         | 5.1      | 30      | 55.4                            | 51.4                     | 1.07            |
| 13       | B1              | 24   | 1.5   | 40      | 0.5      | 61.1         | 6.3      | 50      | 87.4                            | 85.4                     | 1.02            |
| 14       | B2              | 24   | 1.5   | 40      | 0.5      | 49.0         | 4.5      | 50      | 81.4                            | 80.0                     | 1.01            |
| 15       | B3              | 24   | 0.3   | 40      | 0.5      | 49.1         | 3.6      | 50      | 39.4                            | 39.1                     | 1.00            |
| 16       | B4              | 24   | 0.75  | 40      | 0.5      | 42.1         | 4.2      | 50      | 44.9                            | 45.3                     | 0.99            |
| 17       | B5              | 24   | 1.13  | 40      | 0.5      | 52.8         | 5.3      | 50      | 57.4                            | 57.7                     | 0.99            |
| 18       | B6              | 24   | 2.25  | 40      | 0.5      | 54.1         | 7.1      | 50      | 60.0                            | 60.8                     | 0.98            |
| 19       | C1**            | 25   | 0.5   | 30      | 0.5      | 42.0         | 3.32     | 60      | 218.0                           | 160.0                     | 1.36            |
| 20       | C2**            | 25   | 0.75  | 30      | 0.5      | 39.6         | 3.3      | 60      | 180.9                           | 182.0                     | 0.99            |
| 21       | C3**            | 25   | 1.0   | 30      | 0.5      | 43.2         | 4.03     | 60      | 210.3                           | 210.0                     | 1.00            |
| 22       | C4***           | 25   | 1.0   | 30      | 0.5      | 43.2         | 4.03     | 60      | 154.2                           | 184.0                     | 0.83            |
| 23       | C5***           | 25   | 1.0   | 30      | 0.5      | 43.2         | 4.03     | 60      | 307.0                           | 252.0                     | 1.21            |

* hf = 100 mm, d = 410 mm
** a/d = 2
*** a/d = 2.5
**** a/d = 1.5

3.1 Type (A) Beams:

Due to symmetry of the beam, only half of the beam is analyzed. The properties of the material for the tested beams are abbreviated in Table 1. dimensions; reinforcement’s details and loading are shown in Figure 2. The beam is modeled by eight-nodded isoperimetric elements are adopted in the present study. For all beams the concrete mix proportions was 1: 2: 2 /0.8 and 1:1:1.5/0.35 respectively and the steel fiber type is duoform.

![Figure 2. Beam details and Finite Element Mesh of Type (A) Beams.](image)

Figure 3 represents the experimental and numerical load versus compressive strain curves for beam A5 near mid span. It can be noticed from this figure that the numerical results have an excellent approval with the experimental results at strain. Figure 4 shows the analytical load deflection curve for beam A5.

Figure 5 represents the crack pattern for beam A7 from the current analysis with the experimental one. Initial damage in the beam happened in the form of tension cracking in the vertical direction. With
increasing load, crack spread widely through the depth and in vertical direction and in the inclined direction in region close to the support, such distribution of cracks forms a mechanism of shear failure. The experimental pattern of failure of the beam is shown in Figure 5. A good agreement was obtained between the numerical and experimental crack pattern.

![Load-Strain Curve](image1)

**Figure 3.** Experimental and Theoretical Load-Strain Curve for Beam A5.

![Load-Deflection Curve](image2)

**Figure 4** Analytical Load Deflection Curve for Beam A5.
The effect of web-width of the beams on the response of the load versus deflection curve is shown in Figure 6. From this figure, it can be observed predictable influences of the web breath; there was a remarkable growth of ultimate load with the increase of beam web width.

Figure 5 Theoretical and Experimental Crack Pattern for Beam (A7) at failure Load Respectively.

Figure 6 Load versus Deflection Relationship for beams A8, A10 and A11.

In Figure 7, there was a noticeable increase in load-deflection curve with the growth of concrete compressive strength. There was reduction in web deflection with the increase of compressive strength are employed. In addition, as the compressive strength increases the
failure of the beam close to the brittle failure. Figure 8 displays the relationship between the load and deflection of beam A5 at different distance from the support.

![Figure 7](image1)

**Figure 7.** The Effect of Compressive Strength on the Load Deflection Curve.

![Figure 8](image2)

**Figure 8.** Load versus Deflection Curves at Different Distance from Support for Beam (A5)
3.2 Type (B) Beams

For this type of beams the geometry and beams cross section were described in Figure (9) with finite element mesh. Six beams were tested by Muhidin [24] were analyzed in the present study. The experimental results and beams properties are summarized in Table [1]. The concrete mix components ratios by weight for type (B) beams were 1:2:2 /0.5 and the steel fiber type is duoform.

![Figure 9. Beam Details and Finite Element Mesh of Type(B) Beams.](image)

3.3 Type (C) Beams

Tan and Murugappan [25] tested five simply supported I-section beams under two points loading. Details of cross section are shown if Figure 10. The experimental failure load and the predicted values are given in Table [1].

The steel reinforcement bars had yield strength of 460 MPa. Hook-ended steel fibers 30mm long 0.5mm diameter were used, ordinary Portland cement, crushed granite and natural sand in the ratio 1:1.2:2.1 by weight, the water to cement ratio was (0.5). All beams are simply supported with a constant moment zone of 550 mm length.

In this study each beam is modeled by (8-node) element, Figure 9 shows the finite element mesh and the cross-section details. The effect of fiber volume fraction on the load versus deflection relationship was described in Figure 11. It is remarked that when the $V_f$ increase the results become stiff and higher in ultimate load. The experimental versus the calculated loads for the 23 beams are shown in Figure 12.

![Figure 10. Beam Details and Finite Element Mesh of Type (C) Beams.](image)
Figure 11. The Effect of Fiber Percentage by Volume on the Load Deflection

Figure 12. Experimental Versus the Calculated Values of the Beams Tested in the Present Study.
4. Conclusions and Recommendations:
The ultimate load for the SFRC I-section beams are well predicted using the present solution, algorithms and materials constitutive relationships for steel fibers concrete, which provided a secure and representative estimate of the behavior and strength of the studied beams.

The finite element model shows slightly more stiffness results than the test data ranges. Excluding the effects of bond slip in the finite element models leads to increasing the stiffness of the finite element models.

The current finite element model can be used to perform a parametric study rather than carrying out expensive experimental test programs. Executed relationships can be replaced in order to obtain better results.

The material model of the fiber reinforced concrete ascertained to give acceptable results for the analysis of SFRC I-section beams exposed to progressive loading till reaching the failure load. The numerical compressive strain results using the present model, show excellent agreement with experimental results. From the theoretical results obtained in this study, the addition of steel fiber increased the ultimate shear strength by about 17.7% for beams A3 and A5 when the steel fiber volume fraction increased from 0.75 to 3% respectively. The increase in strength reaches 55.5% for beams B3 and B6 when the fiber volume fraction increased from 0.3 to 2.25%.

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