Sensor selection for target tracking based on single dimension information gain

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Abstract: In sensor management for target tracking, the existing sensor selection methods based on information entropy generally select the sensor that brings the most information gain for target; however, for multi-dimension state vector of target, information gain for target in the specified dimension cannot be guaranteed. For the ambiguity problem in the measure for multi-dimension state vector using information entropy, this study proposes a sensor selection method based on single dimension information gain of the target, which uses particle filter to compute the marginal probability density of target state in the specified dimension and exploits the Kullback–Leibler divergence to measure the information gain of target in the specified dimension, in such a way, by selecting the sensor that brings the most information gain for target in the specified dimension. The proposed method can guarantee higher tracking accuracy gain of target in the specified dimension and obtain more reliable tracking results. Experiment results illustrate that the proposed method has a higher track accuracy of position and velocity in the specified dimension than sensor selection method based on target state entropy.

1 Introduction

In multi-sensor cooperative tracking, for the limitation of communication and computational complexity, optimal sensor is selected by a certain optimal criterion to obtain optimal tracking accuracy at each moment, which is called sensor selection problem.

In recent years, information-driven sensor selection has been widely researched [1–4]. This approach uses Shannon entropy or more general Rényi entropy to measure the uncertainty of target state and selects the sensor that makes the target state has minimum Shannon entropy or Rényi entropy to track the target. Another popular information-theoretic measure function is the Kullback–Leibler (KL) divergence or more general the Rényi divergence. The literature [3] derives the relations of different information-theoretic measure functions, and shows that the expected KL divergence is equivalent to both the mutual information and the expected change in differential entropy for Bayesian updating problem. In 1991, information theory was first applied in a problem related to sensor management and state estimation [5, 6]. Subsequently, expected information gain was applied in sensor management and data-fusion problems [1, 7]. Sensor selection method by computing the expected Rényi divergence between the prior and posterior probability density was presented [1, 8], and the KL divergence has been applied in various sensor management scenes [3, 4, 9, 10].

Information entropy is used to measure the uncertainty of random variable; however, for multi-dimension random variable, information entropy only measures the integral uncertainty. Based on information entropy of multi-dimension random variable, the uncertainty of the random variable in the specified dimension cannot be obtained. In other words, for multi-dimension random variable, the decrease of the integral uncertainty cannot guarantee the decrease of the uncertainty in the specified dimension. In sensor management for target tracking, the existing sensor selection methods based on information entropy generally select the sensor that makes the target state uncertainty has a maximum reduction; however, for multi-dimension state vector of target, the reduction of the state uncertainty in the specified dimension cannot be guaranteed.

For the ambiguity problem in the measure for multi-dimension state vector using information entropy, in the background of target tracking, this paper proposes a sensor selection method based on single dimension information gain of the target. In the proposed method, particle filter is used to compute the marginal probability density of multi-dimension target state in the specified dimension, and based on the prior and posterior marginal probability density of target state in the specified dimension, the KL divergence is exploited to measure the information gain of target in the specified dimension, then the sensor that brings the most information gain for target in the specified dimension is selected. Compared with sensor selection method based on target state entropy, the proposed method can guarantee higher tracking accuracy gain of target in the specified dimension, and eliminate the ambiguity of the information gain in the specified dimension, which increases the reliability of target tracking results in real applications.

2 Particle filter implementation of marginal probability density of target state

Suppose the target state in the Cartesian coordinate system is \( x = [x_x, x_y, x_z, x_v, x_v'] \), where \( x_x \) and \( x_y \) respectively, is the position and velocity in the \( x \)-direction, and \( x_z \) and \( x_v \), respectively, is the position and velocity in the \( y \)-direction. In Bayesian inference, a posterior (i.e. after observation) distribution of the target state is obtained using a likelihood model for sensor observation and a prior distribution on the target state. More specifically, given the received observation \( z \), let \( p(x|z) \) represent the posterior probability density of the target state \( x \). According to Bayes' theorem, the posterior probability density \( p(x|z) \) is

\[
p(x|z) = \frac{L(z|x)p(x)}{\int L(z|x')p(x')dx'}
\]

where \( L(z|x) \) is the likelihood of observation \( z \) given the true state \( x \), and \( p(x) \) is the prior probability density of the state \( x \). Then the posterior marginal probability density \( p(x_z|z) \) of state \( x \) in the \( x \)-direction is

\[
p(x_z|z) = \int \int p(x|z)dx_xdx_y
\]

Analogously, we can obtain the posterior marginal probability density \( p(x_v|z) \) of state \( x \) in the \( y \)-direction.

Particle filter uses finite particles to approximate the posterior distribution on the target state, where the particles are obtained by
sampling in the state space according to the proposal distribution. Each particle consists of a state vector and a weight, and the weight is used to describe the difference between the proposal distribution and the true distribution. Suppose the sampling number of particles in state space is \( N \), and the state vector of the \( k \)th particle is \( x^k = [x^k_1, x^k_2, \ldots, x^k_x] \). Let \( w_k \) be the weight before receiving observation, and \( w_k^\prime \) be the weight after receiving observation \( z \), then the prior probability density \( p(x) \) of the target state is

\[
p(x) = \sum_{k=1}^{N} w_k \delta(x - x^k)
\]

(3)

and the posterior probability density \( p(x|z) \) of the target state is

\[
p(x|z) = \sum_{k=1}^{N} w_k^\prime \delta(x - x^k)
\]

(4)

For the above \( N \) particles, different particles may have same positions in the \( x \)-direction. In order to compute the marginal probability density of state \( x \) in the \( x \)-direction using the above \( N \) particles, assume particles can have same positions in the \( x \)-direction. Then the prior marginal probability density \( p(x_i) \) of state \( x \) in the \( x \)-direction is

\[
p(x_i) = \sum_{k=1}^{N} w_k \delta(x_i - x_i^k)
\]

(5)

and the posterior marginal probability density \( p(x_i|z) \) of state \( x \) in the \( x \)-direction is

\[
p(x_i|z) = \sum_{k=1}^{N} w_k^\prime \delta(x_i - x_i^k)
\]

(6)

Analogously, we can obtain the marginal probability density \( p(x_i) \) and \( p(x_i|z) \) of state \( x \) in the \( y \)-direction.

3 Particle filter implementation of single dimension information gain

Given the prior distribution of target state, the posterior distribution is obtained by updating target state using the selected sensor observation. Existing sensor selection criteria based on information entropy usually selects the sensor observation that makes the updated target state has minimum information entropy, namely, the selected sensor observation makes the target has maximum information gain. In information theory, the KL divergence is often used to measure the difference between two probability distributions. For the prior and posterior marginal probability density of target state in the specified dimension, the particle filter implementation of the expected KL divergence is given as follows.

For the given prior and posterior probability density, the KL divergence is defined as

\[
D_{KL}(p(x|z) \| p(x)) = \int p(x|z) \log \frac{p(x|z)}{p(x)} \, dx
\]

(7)

However, before a sensor is used to track the target, the true observation \( z \) of the sensor is unknown. Therefore, it is necessary to consider all the possible observations of the sensor to compute expected information gain of the target, namely

\[
E[D_{KL}(p(x|z) \| p(x)) ] = \int p(z) \left( \int p(x|z) \log \frac{p(x|z)}{p(x)} \, dx \right) \, dz
\]

(8)

As \( p(x, z) = p(z|x)p(x) \), (8) can be written as

\[
E[D_{KL}(p(x|z) \| p(x)) ] = \int \int p(x, z) \log \frac{p(x|z)}{p(x)p(z)} \, dx \, dz
\]

(9)

where \( p(z) \) can be estimated using the prior probability density and the likelihood of the observation

\[
p(z) = \int p(z|x)p(x) \, dx
\]

(10)

In order to compute the information gain of target by numerical calculation, the observation space needs to be discretised. Assume we get \( M \) observations by sampling in the observation space, and the sampling interval is \( \Delta z \), then

\[
E[D_{KL}(p(x|z) \| p(x)) ] = \sum_{i=1}^{M} \frac{1}{N} \sum_{k=1}^{N} \left( p(z_i|x^k) \cdot w_k \cdot \log \frac{p(z_i|x^k)}{p(z_i)} \right) \Delta z
\]

(11)

where \( z_i \) is the \( i \)th observation, and

\[
p(z_i) = \sum_{k=1}^{N} p(z_i|x^k) w_k
\]

(12)

Based on the marginal probability density of target state in the \( x \)-direction, we can get the corresponding expected KL divergence as

\[
E[D_{KL}(p(x_i|z) \| p(x_i)) ] = \sum_{i=1}^{M} \frac{1}{N} \sum_{k=1}^{N} \left( p(x_i|z_i^k) \cdot w_k \cdot \log \frac{p(x_i|z_i^k)}{p(x_i)} \right) \Delta z
\]

(13)

Analogously, we can obtain the expected KL divergence \( E[D_{KL}(p(x_i|z) \| p(x_i)) ] \) of target state in the \( y \)-direction.

4 Sensor selection criterion based on single dimension information gain

Assume the prior distribution of target state is Gaussian distribution, and the corresponding covariance matrix is \( \Sigma_x \). Consider that both sensors T1 and T2 have linear observation model. Suppose covariance matrix \( \Sigma_x \) corresponds to the posterior distribution of target state updated by observation of T1, and covariance matrix \( \Sigma_z \) corresponds to the posterior distribution of target state updated by observation of T2. Let \( \Sigma_z = 0.5 \Sigma_x \) and let \( \Sigma_z \) be a 90° rotation of \( \Sigma_x \) such that \( \Sigma_z = \Sigma_x' \Sigma_x \), where

\[
M = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \Sigma_x = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}
\]

(14)

The corresponding covariance ellipses are shown in Fig. 1.

Fig. 1 Covariance ellipses
Table 1 Observation error of sensors

| Sensor | x-direction, m | y-direction, m |
|--------|----------------|----------------|
| 1      | 50             | 50             |
| 2      | 20             | 80             |
| 3      | 80             | 20             |

From Fig. 1, compared with the y-direction, the uncertainty of the prior distribution in the x-direction is much higher, and thus the sensor that makes more decrease of the uncertainty in the x-direction is expected to be selected. According to the posterior distribution of target state, sensor T2 is preferred. However, the expected KL divergence of sensor T1 is 0.69, which is equal to the expected KL divergence of sensor T2. Therefore, sensor selection criterion based on information entropy still has ambiguity.

For the ambiguity problem in sensor selection criterion based on information entropy, this paper proposes a sensor selection method based on single dimension information gain of the target. Particle filter is used to compute the prior and posterior marginal probability density of target state in the specified dimension, and information gain of target in the specified dimension is measured by the KL divergence, then the sensor that brings the most information gain for target in the specified dimension is selected. Compared with sensor selection method based on target state entropy, the proposed method can guarantee higher information gain of target in the specified dimension, and improve tracking accuracy in the specified dimension.

Let the number of sensors is S, in order to guarantee much higher tracking accuracy gain in the x-direction, sensor selection criterion is

\[
j = \arg \max_{1 \leq i \leq S} E[D_{KL}(p(x_t | z, s_i) || p(x_t))]
\]

(15)

where \(p(x_t | z, s_i)\) is the x-direction posterior probability density of target state updated by the observation \(z\) from the \(i\)-th sensor, \(s_i\), and \(j\) is the index of the selected sensor. Similarly, in order to guarantee much higher tracking accuracy gain in the y-direction, sensor selection criterion is

\[
j = \arg \max_{1 \leq i \leq S} E[D_{KL}(p(x_t | z, s_i) || p(x_t))]
\]

(16)

5 Simulation results

In our simulation, there are three sensors for tracking one target. The target moves with constant speed. The state vector consists of position and velocity for each coordinate axis. The initial state vector of the target is \((40\, \text{m}, 200\, \text{m/s}, 30\, \text{m}, 100\, \text{m/s})\). Particle filter is used for target tracking, and 2500 particles are uniformly selected in the square region \([0\, \text{m}, 100\, \text{m}] \times [0\, \text{m}, 100\, \text{m}]\). The initial velocity of each particle is 0 m/s. Assume the target can be observed by the three sensors, and only one sensor is selected to track the target. In order to show that different sensor selection methods result in different tracking results, this paper, respectively, uses sensor selection based on target state entropy and the method in this paper for sensor management. The observation error of each sensor is shown in Table 1, and the sampling interval for each direction in observation space is 1 m. The number of scans is 50, and the scan period is 0.1 s.

The simulation results are based on 50 Monte Carlo trials. Using (15) as sensor selection criterion, Figs. 2 and 3, respectively, show root mean square error (RMSE) of the position and velocity in the x-direction. Using (16) as a sensor selection criterion, Figs. 4 and 5, respectively, display RMSE of the position and velocity in the y-direction.

Form Figs. 2 and 3, compared with sensor selection method based on target state entropy, the method based on information gain in the x-direction has much lower RMSE of the position and velocity in the x-direction, in other words, the method based on information gain in the x-direction can guarantee much higher information gain of the target in the x-direction. From Figs. 4 and 5, sensor selection method based on information gain in the y-direction has much higher tracking accuracy of the position and velocity in the y-direction than the method based on target state entropy, namely, higher information gain of the target in the y-direction can be guaranteed by the method based on information gain in the y-direction.

The simulation results show that, compared with the method based on target state entropy, the method in this paper can improve tracking accuracy in the specified dimension, and eliminate the ambiguity of the information gain in the specified dimension, which increases the reliability of tracking results in real applications.

6 Conclusion

For the ambiguity problem in sensor selection criterion based on information entropy, a sensor selection method based on single dimension information gain of the target was proposed. In this new method, the marginal probability density of target state in the
specified dimension was computed using particle filter, and the KL divergence was used for computing information gain in the specified dimension. According to the proposed method, the sensor that has most information gain in the specified dimension is selected for target tracking, which guarantees higher information gain of the target in the specified dimension and is more reliable in applications. Experiment results show that new method has higher tracking accuracy of the position and velocity in the specified dimension than the method based on target state entropy.

7 References

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