Overview of the CKM Matrix

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With thanks to numerous contributing experiments, theorists, fitting groups, and especially working group conveners from

CKM2010
http://ckm2010.warwick.ac.uk
The Cabibbo-Kobayashi-Maskawa
Quark Mixing Matrix

\[
V_{\text{CKM}} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

Dirac medal 2010

Nobel prize 2008
The Cabibbo-Kobayashi-Maskawa Quark Mixing Matrix

\[ V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

- A 3x3 unitary matrix
- Described by 4 parameters – **allows CP violation**
  - PDG (Chau-Keung) parametrisation: \( \theta_{12}, \theta_{23}, \theta_{13}, \delta \)
  - Wolfenstein parametrisation: \( \lambda, A, \rho, \eta \)
- Highly predictive

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CKM Matrix Overview
Range of CKM phenomena

- nuclear transitions
  - pion decays
  - kaons
    - hyperon decays
    - tau decays
    - neutrino interactions
    - charm
  - bottom

- $W$ decays

- top
Range of CKM phenomena

- nuclear transitions
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- charm
- bottom
- W decays
- top

- PIBETA
- NA48, KTeV, KLOE, ISTRA
- CHORUS
- KEDR, FOCUS, CLEO, BES
- BABAR, BELLE, LHCb
- ALEPH, DELPHI, L3, OPAL
- CDF, D0, ATLAS, CMS

- dispersion relations
- hadronic matrix elements
- chiral perturbation theory
- lattice QCD
- flavour symmetries
- heavy quark effective theories
- operator product expansion
- perturbative QCD

- apologies for omissions

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Outline

• CKM phenomenology

• **Measurements of magnitudes of CKM matrix elements through tree-level processes**
  - $|V_{ud}|, |V_{us}|, |V_{cd}|, |V_{cs}|, |V_{cb}|, |V_{ub}|$
    - tree-level measurements of $|V_{tx}|$ covered in top session on Tuesday
    - loop-level level measurements covered in following talks

• **Measurements of CP violation in the quark sector**
  • Direct CP violation in D & B systems
  • Unitarity Triangle angles: $\alpha, \beta, \gamma$
    - CP violation in $D^0$ and $B_s^0$ oscillations covered in followed talks

• **Summary**

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apologies for omissions
CKM phenomenology

- CKM theory is highly predictive
  - huge range of phenomena over a massive energy scale predicted by only 4 independent parameters

- CKM matrix is hierarchical
  - theorised connections to quark mass hierarchies, or (dis-)similar patterns in the lepton sector
    - origin of CKM matrix from diagonalisation of Yakuwa (mass) matrices after electroweak symmetry breaking
  - distinctive flavour sector of Standard Model not necessarily replicated in extended theories → strong constraints on models

- CKM mechanism introduces CP violation
  - only source of CP violation in the Standard Model ($m_{\nu} = \theta_{\text{QCD}} = 0$)
Wolfenstein parametrisation

\[ V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta) \\ -\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\ A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \]

Expansion parameter
\[ \lambda = \sin(\theta_c) \sim 0.22 \]

Source of CP violation

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Magnitudes of CKM matrix elements (starting with a digression)
The Fermi constant $w^{(*)}$

$$\frac{1}{\tau_{\mu}} = \frac{G_F^2 m_{\mu}^5}{192 \pi^3} (1 + \Delta q)$$

**MuLan experiment**
PRL 106 (2011) 079901

World's best measurement of the muon lifetime:

$$\tau_{\mu^+} = \left( 2196980.3 \pm 2.2 \right) \text{ps}$$

$$G_F = \left( 1.1663788 \pm 7 \right) \times 10^{-5} \text{GeV}^{-2}$$

< 1 part per million precision! (PDG 2010: 9 ppm)
\[ |V_{ud}| \text{ determination} \]

From \(0^+ \rightarrow 0^+\) nuclear beta decays

Measure

- energy gap \(Q\) \(\rightarrow f\)
- half-life \(\rightarrow t\)
- branching fraction \(\rightarrow f_t\)

\[ f_t = \frac{K}{2G_F^2|V_{ud}|^2} \]

Correct for nuclear medium related effects

- radiative and isospin breaking corrections
  \(\rightarrow\) nucleus-independent quantity \(F_t\)
  confirmed to be constant to \(3 \times 10^{-4}\)

\[ |V_{ud}| = 0.97425 \pm 0.00022 \]
Alternative approaches to $|V_{ud}|$

- Can also measure $|V_{ud}|$ from
  - alternative nuclear decays ("nuclear mirrors")
  - neutron and pion $\beta$ decay
    - do not require nucleus dependent or isospin breaking corrections
    - pion $\beta$ decay is a pure vector transition (like $0^+ \rightarrow 0^+$)
  - potential for more precise future measurements
\[ |V_{us}| \text{ from semileptonic kaon decays} \]

\[
|V_{us}| = 0.2254 \pm 0.0013
\]

Comparison with
- \(|V_{us}|/|V_{ud}|\) from leptonic kaon and pion decays (using lattice input on \(f_K/f_\pi\))
- \(|V_{ud}|\)

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Unitarity holds to better than \(10^{-3}\)
Alternative approaches to $|V_{us}|$

- Can also measure $|V_{us}|$ from
  - hyperon decays
  - strange vs. non-strange hadronic tau branching fractions

$$|V_{us}| = 0.2166 \pm 0.0019 \text{(exp)} \pm 0.0005 \text{(th)}$$

- discrepancy from $|V_{us}|$ from kaons: $3.7\sigma$
  - also discrepant with $|V_{us}|$ from $B(\tau \to K\nu)/B(\tau \to \pi\nu) + f_K/f_\pi$ from lattice
  - several multibody tau decays not measured yet

- improved measurements urgently needed
$|V_{cd}|$ and $|V_{cs}|$ from charm decays

- Benchmark measurement of $|V_{cd}|$ from charm production in nuclear interactions $|V_{cd}| = 0.230 \pm 0.011$
- Measurements from semileptonic charm decays suffer form-factor uncertainties
  - further improvement in lattice calculations needed

$|V_{cd}| = 0.234 \pm 0.007 \pm 0.002 \pm 0.025$ $|V_{cs}| = 0.961 \pm 0.011 \pm 0.024$

CLEOc experiment
PRD 80 (2009) 032005

Lattice input from
PRD 82 (2010) 114506
Alternative approaches to $|V_{cd}|$ and $|V_{cs}|$

- Leptonic $D^+$ and $D_s^+$ decays probe $f_D |V_{cx}|$, e.g.

$$\Gamma (D_s^+ \rightarrow l^+ \nu) = \frac{G_F^2}{8\pi} f_{D_s}^2 m_l^2 M_{D_s^+} \left(1 - \frac{m_l^2}{M_{D_s^+}^2}\right)^2 |V_{cs}|^2$$

![CLEOc experiment PRD 79 (2009) 052001](image)

|                | $f_D$ (MeV)     | $f_{D_s}$ (MeV) |
|----------------|-----------------|-----------------|
| CLEOc          | 206.7 ± 8.5 ± 2.5 | 259.0 ± 6.2 ± 3.0 |
| BaBar          | 275 ± 16 ± 12    |                 |
| Belle          | 258.6 ± 6.4 ± 7.5 |                 |
| Lattice average| 213.9 ± 4.2      | 248.9 ± 3.9     |

$|V_{cs}| = 1.005 \pm 0.026 \pm 0.016$

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$D_s^+ \rightarrow \mu^+ \nu$
$|V_{cb}|$ from semileptonic B decays

- Both **exclusive** and **inclusive** approaches

**Belle experiment**
PRD 82 (2010) 112007

**PDG 2010 quotes**

$|V_{cb}|^{(excl)} = (38.7 \pm 1.1) \times 10^{-3}$

$|V_{cb}|^{(incl)} = (41.5 \pm 0.7) \times 10^{-3}$

$|V_{cb}| = (37.5 \pm 0.2 \pm 1.1 \pm 1.0) \times 10^{-3}$

**2σ tension**

Lattice uncertainty – reduced to 0.7 in arXiv:1011.2166

Tension reduced (~1.6σ)
Searches for charged Higgs in $B \to D^{(*)}\tau\nu$

Branching fraction ratio ($R^{(*)}$) relative to $B \to D^{(*)}\ell\nu$ predicted in the Standard Model with reduced form-factor uncertainty

$B^{-} \to D^{0}\tau\nu$

$R(D) = 0.456 \pm 0.053 \pm 0.056$

$R^{SM}(D) = 0.31 \pm 0.02$

$R(D^*) = 0.325 \pm 0.023 \pm 0.027$

$R^{SM}(D^*) = 0.25 \pm 0.07$

$\bar{B}^{0} \to D^{+}\tau\nu$

BaBar experiment
EPS 2011 preliminary

$\bar{B}^{0} \to D^{*+}\tau\nu$

See also
Belle experiment
PRD 82 (2010) 072005

1.8σ excess over the Standard Model – more in Rare Decays talk
$|V_{ub}|$ from semileptonic $B$ decays

- Both exclusive and inclusive approaches

$$|V_{ub}| = (3.09 \pm 0.08 \pm 0.12 \pm 0.29) \times 10^{-3}$$  $|V_{ub}| = (3.43 \pm 0.33) \times 10^{-3}$

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Belle experiment  
PRD 83 (2011) 071101(R)

BaBar experiment  
PRD 83 (2011) 052011  
PRD 83 (2011) 032007

$B^0 \to \pi^- \nu$
$|V_{ub}|$ from semileptonic B decays

- Another tension between exclusive and inclusive
  - PDG2010 quotes
    \[
    |V_{ub}|(excl) = (3.38 \pm 0.36) \times 10^{-3}
    \]
    \[
    |V_{ub}|(incl) = (4.27 \pm 0.38) \times 10^{-3}
    \]
  - A distinguished theorist recently said:
    "... this tension may be due to the fact that over the last 30 years hundreds of theory papers have been devoted to the determination of $V_{ub}$ with each author claiming that his/her work led to a decrease of the theoretical error ..."
  - In my view more, not less, theoretical attention is required
    - e.g. SIMBA collaboration to improve understanding of inclusive decays
      - N.B. $|V_{ub}|$ from leptonic decays covered in rare decays talk

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CP violation
CP violation and the matter-antimatter asymmetry

• Two widely known facts

1) CP violation is one of 3 “Sakharov conditions” necessary for the evolution of a baryon asymmetry in the Universe

2) The Standard Model (CKM) CP violation is not sufficient to explain the observed asymmetry

• Therefore, there must be more sources of CP violation in nature … but where?

• extended quark sector, lepton sector (leptogenesis), supersymmetry, anomalous gauge couplings, extended Higgs sector, quark-gluon plasma, flavour-diagonal phases, …

• Testing the consistency of the CKM mechanism provides the best chance to find new sources of CP violation today
Observations of CP violation

- Still a rare phenomenon:
  - only seen (>5σ) in $K^0$ and $B^0$ systems
- In $B$ system, only
  - $\sin(2\beta)$ in $B^0 \to J/\psi K_{S,L}^0$ (etc.) – BaBar & Belle
  - $S(B^0 \to \eta'K_{S,L}^0)$ (etc.) – BaBar & Belle
  - $S(B^0 \to \pi^+\pi^-)$ – BaBar & Belle
  - $C(B^0 \to \pi^+\pi^-)$ – Belle
  - $A_{CP}(B^0 \to K^+\pi^-)$ – BaBar, Belle & LHCb

$A_{CP}(B^0 \to K^+\pi^-) = -0.088 \pm 0.011 \pm 0.008$
Unitarity Triangles

Build matrix of phases between pairs of CKM matrix elements

$\Phi_{ij} = \text{phase between remaining elements when row } i \text{ and column } j \text{ removed}$

unitarity implies sum of phases in any row or column $= 180^\circ \rightarrow 6 \text{ unitarity triangles}$

\[
\Phi = \begin{pmatrix}
d & s & b \\
u & \Phi_{ud} & \Phi_{us} & \Phi_{ub} \\
t & \Phi_{td} & \Phi_{ts} & \Phi_{tb}
\end{pmatrix}
\approx
\begin{pmatrix}
\beta_s \\
u & 1^\circ & 22^\circ & 22^\circ \\
t & 67^\circ & 90^\circ & 68^\circ
\end{pmatrix}
\]

$\beta \equiv \varphi_1$

$\alpha \equiv \varphi_2$

$\gamma \equiv \varphi_3$

$\varphi_D / 2$

"The Unitarity Triangle"
CP violation null tests: charm decays

- All (almost) CP violation effects in the charm system expected to be negligible
- searches for direct CP violation (see also talk on mixing)

Belle experiment
PRL 104 (2010) 181602

BaBar experiment
arXiv:1105.4410 (PRD(R))

LHCb experiment
EPS 2011 preliminary

All consistent with zero and with the Standard Model
\[ \sin(2\beta) \text{ from } B^0 \rightarrow J/\psi K_{s,L} \text{ (etc.)} \]

BaBar experiment
PRD 79 (2009) 072009

Belle experiment
Moriond EW 2011 preliminary

\[ \sin(2\beta) = \sin(2\phi) \]

Final Belle dataset (772M BB pairs) with reprocessed data

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Hagfa
Checking the quality of gold

- \( B^0 \rightarrow J/\psi K_S \) is a golden mode for \( \sin(2\beta) \)
  - Can check purity using flavour symmetries
    - \( B^0 \rightarrow J/\psi \pi^0 \) (related by SU(3))
    - \( B_s^0 \rightarrow J/\psi K_S \) (related by U spin)

CDF experiment
PRD 83 (2011) 052012

LHCb experiment
LHCb-CONF-2011-048

\[
\frac{B(B_s^0 \rightarrow J/\psi K_S)}{B(B^0 \rightarrow J/\psi K_S)} = 0.041 \pm 0.007 \text{ (stat)} \\
\pm 0.004 \text{ (syst)} \pm 0.005 \left( f_s / f_d \right)
\]

\[
\frac{B(B_s^0 \rightarrow J/\psi K_S)}{B(B^0 \rightarrow J/\psi K_S)} = 0.0378 \pm 0.0058 \text{ (stat)} \\
\pm 0.0020 \text{ (syst)} \pm 0.0030 \left( f_s / f_d \right)
\]
Other approaches to $\sin(2\beta)$

- Compare $b \to c\bar{c}s$ transitions (e.g. $B^0 \to J/\psi K_S$) with $b \to s\bar{s}s$ (e.g. $B^0 \to \eta'K_S$), $b \to c\bar{c}d$ (e.g. $B^0 \to D^+D^-$), or $b \to c\bar{u}d$ (e.g. $B^0 \to D_{CP}\pi^0$)

\[
\sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}})
\]

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Hints of deviations in $b \to s\bar{s}s$ diminished
Belle update on $B^0 \rightarrow D^+D^-$

Belle experiment

EPS 2011 preliminary

$$S(D^+D^-) = -1.06 \pm 0.21 \pm 0.07$$
$$C(D^+D^-) = -0.43 \pm 0.17 \pm 0.04$$

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Belle – BaBar – Standard Model discrepancy diminished
\( \alpha \) from \( B \to \pi \pi, \rho \pi, \rho \rho \) systems

- Awaiting final results from both BaBar and Belle on
  - \( B^0 \to \pi^+ \pi^- \)
  - \( B^0 \to (\rho \pi)^0 \)
  - \( B^0 \to \rho^+ \rho^- \)
- World average
  \[
  \alpha = \left( 89.0 \pm 4.4 \right)^\circ
  \]
  - dominated by \( B \to \rho \rho \)
  - strong influence of single (BaBar) measurement of \( B(B^+ \to \rho^+ \rho^0) \)
- Is \( \alpha = 90^\circ \)?
y from $B \to D^{(*)}K$ decays

Tree-level determination of $y$ from interference of $B \to DK (b \to c\bar{u}s)$ and $B \to \bar{D}K (b \to u\bar{c}s)$ amplitudes

- need $D$ and $\bar{D}$ to decay to common final state

\[
\begin{align*}
B^- & \xrightarrow{b} K^- \\
& \xrightarrow{w} D^0 \\
& \propto V_{cb} V_{us}^* \\
& \xrightarrow{u} \bar{D}^0 \\
& \xrightarrow{\bar{c}} K^- \\
& \propto V_{ub} V_{cs}^* \end{align*}
\]

- colour allowed
- final state contains $D^0$

- colour suppressed
- final state contains $\bar{D}^0$
y from $B \to DK, \ D \to CP$ eigenstate (GLW)

Belle experiment
BELLE-CONF-1112

CP violation clearly established

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γ from $B \to DK$, $D \to$ suppressed states (ADS)

LHCb experiment
LHCb-CONF-2011-044

LHCb Preliminary

B$^+$(πK)$^*_D$, DLL$_D$ > 4

B$^-$

B$^+$(πK)$^*_D$, DLL$_D$ > 4

LHCb Preliminary

D$^-$Kπ K R$_{ADS}$

HFAG

BaBar:
PRD 82 (2010) 072006

Belle:
PRL 106 (2011) 231803

CDF:
PLHC2011 preliminary

LHCb:
EPS 2011 preliminary

Average

HFAG

0.0110 ± 0.0060 ± 0.0020

0.0163 ± 0.0060 ± 0.0020

0.0221 ± 0.0096 ± 0.0026

0.0166 ± 0.0039 ± 0.0024

0.0160 ± 0.0027

D$^-$Kπ K A$_{ADS}$

HFAG

BaBar:
PRD 82 (2010) 072006

Belle:
PRL 106 (2011) 231803

CDF:
PLHC2011 preliminary

LHCb:
EPS 2011 preliminary

Average

HFAG

-0.86 ± 0.47 ± 0.12

-0.39 ± 0.28 ± 0.04

-0.82 ± 0.44 ± 0.09

-0.39 ± 0.17 ± 0.02

-0.46 ± 0.13

All new results in last 2 years

ADS suppressed mode now clearly established ...

... very promising for γ determination

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y from $B \to D^*K$, $D \to$ suppressed states (ADS)

**Belle experiment**  
BELLE-CONF-1112

Suppressed modes also appearing in $D^*K$?
γ from $B \to DK$, $D \to$ multibody states (GGSZ)

Study of $D \to K_S \pi^+ \pi^-$ Dalitz plot distribution provides good statistical sensitivity to γ but with model dependence

$BaBar$ experiment
PRL 105 (2010) 121801

$γ = \left( 68^{+15}_{-14} \pm 4 \pm 3 \right)^°$

$Belle$ experiment
PRD 81 (2010) 112002

$γ = \left( 78^{+11}_{-12} \pm 4 \pm 9 \right)^°$

Model independent (binned) approach exploiting $Ψ(3770) \to D\overline{D}$ data

$CLEOc$ experiment
PRD 82 (2010) 112006

Belle experiment
arXiv:1106.4046

$γ = \left( 77^{+15}_{-12} \pm 4 \pm 4 \right)^°$
$\gamma$ from $B_s \rightarrow D_s^- K^\pm$

$\gamma$ can be extracted from time-evolution of $B_s \rightarrow D_s^- K$ decays

first stage: establish signals & measure branching fraction

yields split by magnet polarity

\[
B \left( B_s \rightarrow D_s^- K^\pm \right) = \left( 1.97 \pm 0.18 \text{ (stat)} +0.19 -0.20 \text{ (syst)} +0.11 -0.10 (f_s/f_d) \right) \times 10^{-4}
\]

Promising for future $\gamma$ measurement
Alternative ways to measure $\gamma$

- Test Standard Model by comparing $\gamma$ from tree-level processes to $\gamma$ from loop-dominated amplitudes
- various approaches exploiting flavour symmetries
  - $B^0 \rightarrow K^+\pi^-$ (see rare decays talk)
  - $B_s^0 \rightarrow K^+K^-$ & $B^0 \rightarrow \pi^+\pi^-$ (see LHCb talk)
  - $B^0 \rightarrow K_S^+\pi^-\pi^-$ & $B^0 \rightarrow K^+\pi^-\pi^0$

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BaBar experiment
PRD 83 (2011) 112010
Global CKM fits

http://ckmfitter.in2p3.fr

http://www.utfit.org

\[ \overline{\rho} = 0.144 \pm 0.018 \quad (\text{CKMfitter}) = 0.132 \pm 0.020 \quad (\text{UTfit}) \]

\[ \overline{\eta} = 0.343 \pm 0.014 \quad (\text{CKMfitter}) = 0.353 \pm 0.014 \quad (\text{UTfit}) \]

Different statistical approaches – similar results
Overall good consistency with the Standard Model

Does not include new results on $\gamma$ shown today
Future projects

- Nuclear transitions
- Pion decays
- Kaons
- Hyperon decays
- Tau decays
- Neutrino interactions
- Charm
- Bottom
- Top
- Hadronic matrix elements
- Chiral perturbation theory
- Lattice QCD
- Dispersion relations
- Great progress in theory anticipated

Apologies for omissions

- KLOE-2, NA62, KOTO
- NA48, Kfiev, KEK-ETRA
- Project X
- Tau-charm factory
- BABAR, BELLE, LHCb
- Belle-2, SuperB, LHCb upgrade
- W decays
- Top
- CDF, D0, ATLAS, CMS

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Summary

- CKM paradigm continues its unreasonable success
- Current and future projects promise significant improvements
  - short term: BESIII, LHCb, lattice
- Look forward to discovering the destiny of our hopes and hints
  - one certainty: new sources of CP violation exist, somewhere
Summary

- CKM paradigm continues its unreasonable success
- Current and future projects promise significant improvements
  - short term: BESIII, LHCb, lattice
- Look forward to discovering the destiny of our hopes and hints
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- Will we be top of the world … ?

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Summary

- CKM paradigm continues its unreasonable success
- Current and future projects promise significant improvements
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- Look forward to discovering the destiny of our hopes and hints
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- Will we be top of the world … ?

... or do we have to wait for the historic achievement?