The study of the lightest charmed baryon $\Lambda_c^+$ is important for the understanding of the whole charmed baryon sector. In recent years, there has been significant progress in studying the $\Lambda_c^+$, both experimentally and theoretically [1, 2]. This provides crucial information in detailed explorations of the singly charmed baryons ($\Sigma_c$, $\Xi_c$ and $\Omega_c$) [3, 4], and further searches or discoveries of the doubly charmed baryons ($\Xi_{cc}$ and $\Omega_{cc}$) [5, 6]. Moreover, as the charmed baryon is the favored weak decay final state of $b$-baryons and its properties are inputs to study $b$-baryons, improved knowledge in the charm sector can contribute substantially to understanding the properties of $b$-baryons.

Some QCD-inspired charmed baryon models that have been developed [7] are the flavor symmetry model [8], factorization model [9], pole model [10], and current algebra framework [11]. As shown in Refs. [2, 7], many of these models calculate $\Lambda_c^+$ decay rates in good agreement with experimental results. But the decay asymmetry predictions by these models for $\Lambda_c^+$ two-body hadronic weak decays do not agree very well.

The decay asymmetry parameter, $\alpha_{BP}$, in a weak decay $\Lambda_c^+ \rightarrow BP$ ($B$ denotes a $J^P = 1^+$ baryon and $P$ denotes a $J^P = 0^-$ pseudoscalar meson) is defined as $\alpha_{BP} = \frac{2Re(s + p)}{|s|^2 + |p|^2}$, where $s$ and $p$ stand for the parity-violating $s$-wave and parity-conserving $p$-wave amplitudes in the decay, respectively. Model calculations of $\alpha_{BP}$ in $\Lambda_c^+ \rightarrow pK_S^0$, $\Lambda^+\pi^0$, $\Sigma^+\pi^0$, and $\Sigma^0\pi^+$ are listed in Table I, which shows large variations among the different models. As predictions of $\alpha_{BP}$ rely on the relative phase between the two amplitudes, the experimental measurements of the decay asymmetry parameters serve as very sensitive probes to test different theoretical models.

Experimentally, only $\alpha_{\Lambda^+\pi^+}$ and $\alpha_{\Sigma^+\pi^0}$ have been measured previously [12–15]. The measured value for $\alpha_{\Sigma^+\pi^0}$ is $-0.45 \pm 0.32$, in contradiction with the predicted values in many theoretical models [10, 11, 16–19]. Therefore, it is important to carry out independent measurements of $\alpha_{\Sigma^+\pi^0}$ to confirm the sign of $\alpha_{\Sigma^+\pi^+}$ and test these models. Moreover, $\alpha_{\Sigma^+\pi^0}$ and $\alpha_{\Sigma^+\pi^+}$ should have the same value according to hyperon isospin symmetry [20], and any deviation from this expectation provides critical information on final state interactions in $\Lambda_c^+$ hadronic decays. All the models predict $\alpha_{\Lambda^+\pi^+}$ consistent with the measured values, and it is necessary to further improve the experimental precision to discriminate between them.

In previous experiments, $\Lambda_c^+$ was assumed to be unpolarized, and the decay asymmetry parameter $\alpha_{BP}$ was obtained by analyzing the longitudinal polarization from the weak two-body decay of the produced baryon $B$, such as $\Lambda \rightarrow p\pi^−$ and $\Sigma^+ \rightarrow p\pi^0$ for $\alpha_{\Lambda^+\pi^+}$ and $\alpha_{\Sigma^+\pi^0}$, respectively. However, the hypothesis of unpolarized $\Lambda_c^+$ may not be valid. There have been observations of transverse $\Lambda$ polarization in inclusive $\Lambda$ production in $e^+e^−$ collisions at 10.58 GeV [21] and in $e^+e^− \rightarrow \Lambda\Lambda$ at $J/\psi$ mass position [22], and it has been postulated that the produced $\Lambda_c^+$ could be polarized [23]. Further, as the polarization of the proton in the decay $\Lambda_c^+ \rightarrow pK_S^0$ is not accessible with the above method, a non-zero transverse polarization of the $\Lambda_c^+$ provides an alternative way to measure $\alpha_{\Lambda^+\pi^+}$ [24].

In this Letter, we investigate for the first time the transverse polarization of the $\Lambda_c^+$ baryon in unpolarized $e^+e^−$ annihilations. We present for the first time measurements of the decay asymmetry parameters in $\Lambda_c^+$ decays into $pK_S^0$, $\Lambda\pi^+$, $\Sigma^+\pi^0$, and $\Sigma^0\pi^+$ based on a multi-dimensional angular analysis of the cascade-decay final states, which greatly improves the resulting precision. Data sample used in this analysis corresponds to an integrated luminosity of 567 pb$^{-1}$ collected with the BESIII
In the context, unless otherwise stated explicitly.

The detector at BEPCII at center-of-mass (CM) energy of 4.6 GeV.

Since the close proximity of the CM energy to the $\Lambda_c^+\Lambda_c^-$ mass threshold does not allow an additional hadron to be produced, $\Lambda_c^+\Lambda_c^-$ are always generated in pairs, which provides a clean environment to study their decays. When one $\Lambda_c^+$ is detected, another $\Lambda_c^-$ partner is inferred. Hence, to increase signal yields, we adopt a partial reconstruction method, in which only one $\Lambda_c^+$ is reconstructed out of all the final-state particles in an event. The charge conjugation modes are always implied in the context, unless otherwise stated explicitly.

Details of the BESIII apparatus, the software framework and the Monte Carlo (MC) simulation sample have been given in Ref. [25]. The $\Lambda_c^+$ signal candidates are reconstructed through the decays into $pK_S^0$, $\Lambda \pi^+$, $\Sigma^+ \pi^0$ and $\Sigma^0 \pi^+$. Here, the intermediate particles $K_S^0$, $\Lambda$, $\Sigma^+$, $\Sigma^0$ and $\pi^0$ are reconstructed via the decays $K_S^0 \rightarrow \pi^+ \pi^-$, $\Lambda \rightarrow p\pi^-$, $\Sigma^+ \rightarrow p\pi^0$, $\Sigma^0 \rightarrow \gamma \Lambda$, and $\pi^0 \rightarrow \gamma\gamma$. The event selection criteria follow those described in Ref. [25], unless otherwise stated explicitly. To suppress the $\Lambda_c^+ \rightarrow pK_S^0$, $K_S^0 \rightarrow \pi^0\pi^0$ events in the $\Sigma^+\pi^0$ candidate samples, the invariant mass of the $\pi^+\pi^0$ system is required to be outside the range $[400,550]$ GeV/c$^2$.

For each signal decay mode, the yields are obtained from a fit to the beam-constrained mass ($M_{BC}$) distribution, $M_{BC} = \sqrt{E_{beam}^2 - p_{\Lambda_c^+}^2}$, where $E_{beam}$ is the average beam energy and $p_{\Lambda_c^+}$ is the measured $\Lambda_c^+$ momentum in the CM system of the $e^+e^-$ collisions. If more than one candidate is reconstructed in the event, the one with the smallest energy difference ($|\Delta E|$) is kept, where $\Delta E \equiv E_{\Lambda_c^+} - E_{beam}$, and $E_{\Lambda_c^+}$ is the measured total energy of the $\Lambda_c^+$ candidate.

Figure 1 shows the $M_{BC}$ distributions for the signal candidates, where the $\Lambda_c^+$ signal peak is evident at the nominal $\Lambda_c^+$ mass. The backgrounds can be classified into two types. The Type-I backgrounds are from the true $\Lambda_c^+$ signal decays, where at least one of the final state particle candidates is wrongly assigned in reconstruction. The Type-II backgrounds correspond to combinatorial backgrounds mostly from $e^+e^- \rightarrow q\bar{q}$ $(q = u,d,s)$ processes. To evaluate the Type-I and Type-II background level, unbinned maximum likelihood fits (shown in Fig. 1) are applied to the $M_{BC}$ spectra. The signal and Type-I background shapes, as well as the ratio of their yields, are derived from the signal MC simulation samples. These two shapes are convolved with a common Gaussian function, whose width is left free and represents the difference in resolution between data and MC simulations. The Type-II background shape is modeled by an ARGUS function [26]. The $\Lambda_c^+$ signal and sideband regions are chosen as $[2.278,2.294]$ GeV/c$^2$ and $[2.250,2.270]$ GeV/c$^2$, respectively.

The decay asymmetry parameters are determined by analyzing the multi-dimensional angular distributions, where the full cascade decay chains are considered. The full angular dependence formulae (4), (6), and (10) in Ref. [24], constructed under the helicity basis, are used in the fit. To illustrate the helicity system defined in this analysis, we take as an example the two-level cascade decay process $\Lambda_c^+ \rightarrow \Lambda \pi^+, \Lambda \rightarrow p\pi^-$ following the level-0 process $e^+e^- \rightarrow \gamma^*, \gamma^* \rightarrow \Lambda_c^+\Lambda_c^-$. An analogous formalism is applied to the other $\Lambda_c^\pm \rightarrow BP$ decays.

![Figure 1](image1.png)

**FIG. 1.** (color online) Fits to the $M_{BC}$ spectra of the signal candidates of (a) $\Lambda_c^+ \rightarrow pK_S^0$, (b) $\Lambda_c^+ \rightarrow \Lambda\pi^+$, (c) $\Lambda_c^+ \rightarrow \Sigma^+\pi^0$, and (d) $\Lambda_c^+ \rightarrow \Sigma^0\pi^+$. Points with error bars correspond to data, solid lines are the fitting curves, dashed lines describe the signal events distribution, dash-dotted lines show the Type-II backgrounds and shadowed areas correspond to Type-I backgrounds. Dashed and solid arrows show the sideband and signal regions, respectively.

![Figure 2](image2.png)

**FIG. 2.** (color online) Definition of the helicity frame for $e^+e^- \rightarrow \Lambda_c^\pm\Lambda_c^\mp, \Lambda_c^\pm \rightarrow \Lambda\pi^+, \Lambda \rightarrow p\pi^-$. 

\[ E_{beam}^2 - p_{\Lambda_c^+}^2 \]
Figure 2 illustrates the definitions of the full system of helicity angles for the $\Lambda^+_c \rightarrow \Lambda \pi^+$ mode. In the helicity frame of $e^+e^- \rightarrow \Lambda^+_c \bar{\Lambda}^+_c$, $\theta_0$ is the polar angle of the $\Lambda^+_c$ with respect to the $e^+e^-$ beam axis in the $e^+e^-$ CM system. For the helicity angles of the $\Lambda^+_c \rightarrow \Lambda \pi^+$ decay, $\phi_1$ is the angle between the $e^+\Lambda^+_c$ and $\Lambda\pi^+$ planes, and $\theta_1$ is the polar angle of the $\Lambda$ momentum in the rest frame of the $\Lambda^+_c$ with respect to the $\Lambda^+_c$ momentum in the CM frame. The angle subscript represents the level numbering of the cascade signal decays. For the helicity angles describing the $\Lambda \rightarrow p\pi^+$ decay, $\phi_2$ is the angle between the $\Lambda\pi^+$ plane and $p\pi^-$ plane and $\theta_2$ is the polar angle of the proton momentum with respect to opposite direction of $\pi^+$ momentum in the rest frame of $\Lambda$. For the three-level cascade decays $\Lambda^+_c \rightarrow \Sigma^0 \pi^+, \Sigma^0 \rightarrow \Lambda \gamma, \Lambda \rightarrow p\pi^-$ process, $\phi_3$ is the angle between the $\Lambda\gamma$ and $p\pi^-$ planes, while $\theta_3$ is the polar angle of the proton with respect to the opposite direction of the photon momentum (from $\Sigma^0 \rightarrow \Lambda \gamma$) in the rest frame of $\Lambda$.

In Ref. [24], we define $\Delta_0$ as the phase angle difference between two individual helicity amplitudes, $H_{\lambda_1,\lambda_2}$, for the $\Lambda^+_c$ production process $\gamma^* \rightarrow \Lambda^+_c(\lambda_1)\bar{\Lambda}^+_c(\lambda_2)$ with total helicities $|\lambda_1 - \lambda_2| = 0$ and 1, respectively. In the case where one-photon exchange dominates the production process, $\Delta_0$ is also the phase between the electric and magnetic form factors of the $\Lambda^+_c$ [23, 27]. The transverse polarization observable of the produced $\Lambda^+_c$ can be defined as

$$P_T(\cos \theta_0) \equiv \sqrt{1 - \alpha_0^2 \cos \theta_0 \sin \theta_0 \sin \Delta_0},$$

where the magnitude varies as a function of $\cos \theta_0$. Similarly, two parameters, $\alpha_{BP}$ and $\Delta_{BP}$, describe the level-1 decays $\Lambda^+_c \rightarrow \Lambda \pi^+, \Sigma^+ \pi^0$, and $\Sigma^0 \pi^+$, where $\Delta_{BP}$ is the phase angle difference between the two helicity amplitudes in the $BP$ mode. The Lee-Yang parameters [24, 28] can be obtained with the relations

$$\beta_{BP} = \sqrt{1 - (\alpha_{BP}^2)^2 \sin^2 \Delta_{BP}},$$

$$\gamma_{BP} = \sqrt{1 - (\alpha_{BP}^2)^2 \cos^2 \Delta_{BP}}.$$ (2)

In the angular analysis, the free parameters describing the angular distributions for the four data sets are determined from a simultaneous unbinned maximum likelihood fit, as $\alpha_0$ and $\Delta_0$ are common. The likelihood function is constructed from the probability density function (PDF) jointly by

$$L_{\text{data}} = \prod_{i=1}^{N_{\text{data}}} f_S(\vec{\xi}).$$ (3)

Here, $f_S(\vec{\xi})$ is the PDF of the signal process, $N_{\text{data}}$ is the number of the events in data and $i$ is event index. Signal PDF $f_S(\vec{\xi})$ is formulated as

$$f_S(\vec{\xi}) = \frac{\epsilon(\vec{\xi})|M(\vec{\xi}; \vec{\eta})|^2}{\int \epsilon(\vec{\xi})|M(\vec{\xi}; \vec{\eta})|^2 d\vec{\xi}},$$ (4)

where the variable $\vec{\xi}$ denotes the kinematic angular observables, and $\vec{\eta}$ denotes the free parameters to be determined. $M(\vec{\xi})$ is the total decay amplitude [24] and $\epsilon(\vec{\xi})$ is the detection efficiency parameterized in terms of the kinematic variables $\vec{\xi}$. The background contribution to the joint likelihood is subtracted according to the calculated likelihoods for the Type-I background based on inclusive MC simulations and for the Type-II background according to the $M_{BC}$ sideband. The MC-integration technique is adopted to compute the normalization factor as follows

$$\int \epsilon(\vec{\xi})|M(\vec{\xi}; \vec{\eta})|^2 d\vec{\xi} = \frac{1}{N_{\text{gen}}} \sum_{k_{\text{MC}}} |M(\vec{\xi}_k; \vec{\eta})|^2,$$ (5)

where $N_{\text{gen}}$ is the total number of MC-simulated signal events. $N_{\text{MC}}$ is the number of the MC signal events survived from the full selection criteria and $k_{\text{MC}}$ is its signal index.

Minimization of the negative logarithmic likelihood with background subtraction over all the four signal processes is carried out using the MINUIT package [29]. Here, $\alpha_0$ is fixed to the known value $-0.20$ [27]. For the charge-conjugation $\Lambda^+_c$ decays, under the assumption of $CP$ conservation, $\Delta_0 = 0$, $\alpha_{BP} = -\alpha_{\bar{BP}}$, and $\Delta_{BP} = -\Delta_{\bar{BP}}$. The decay asymmetry parameter $\alpha_\Lambda$ for $\Lambda \rightarrow p\pi^-$ is taken from the recent BESIII measurement [22] and $\alpha_{\Sigma^+}$ for $\Sigma^+ \rightarrow p\pi^0$ from the Particle Data Group (PDG) [2]. From the fit, we obtain $\sin \Delta_0 = -0.28 \pm 0.13$(stat.) which differs from zero with a statistical significance of 2.1$\sigma$ according to a likelihood ratio test. This indicates that transverse polarization $P_T$ of the $\Lambda^+_c$ is non-zero when $\sin(2\theta_0) \neq 0$. The numerical fit results are given in Table I, together with the calculated $\gamma_{BP}$ and $\beta_{BP}$.

In Fig. 3, the fit results are illustrated using several projection variables. The real data are compared with the MC generated events re-weighted according to the fit.

For the $\Lambda^+_c \rightarrow \Lambda \pi^+$ and $\Sigma^+ \pi^0$ decays, if all angles are integrated over except for the angle $\theta_2$, the decay rate becomes [32]

$$dN \propto 1 + \alpha_{\Lambda\pi^+}(\Sigma^+ \pi^0)\alpha_{\Lambda}(\Sigma^+) \cos \theta_2.$$ (6)

Equation (6) shows a characteristic longitudinally polarized distribution of the produced $\Lambda(\Sigma^+)$ from the $\Lambda^+_c$ decays, and the asymmetry of $\cos \theta_2$ distribution reflects the product of the decay asymmetries $\alpha_{\Lambda\pi^+} \alpha_{\Lambda}(\Sigma^+) \alpha_{\Sigma^+} \alpha_{\Sigma^+ \pi^0}$ [33]. The distributions of $\cos \theta_2$ in the $\Lambda^+_c \rightarrow \Lambda \pi^+$ and $\Sigma^+ \pi^0$ modes are shown in Figs. 3(a) and (b), respectively. The drop at the right side in Fig. 3(b) is due to the $K^0_S \rightarrow \pi^0 \pi^0$ veto.

For the $\Lambda^+_c \rightarrow \Sigma^0 \pi^+$ decay, the correlations of $\cos \theta_2$ and $\cos \theta_3$ in the subsequent level-2 decay $\Sigma^0 \rightarrow \gamma \Lambda$ and level-3 decay $\Lambda \rightarrow p\pi^-$ are shown in Figs. 3(c) and (d), respectively. The correlation of the average value of $\cos \theta_i$
TABLE I. Comparisons between different theoretical calculations and experimental measurements.

| $\Lambda^+_CP \rightarrow pK^0_S$ | $\Lambda^{0+}$ | $\Sigma^{0+}$ | $\Sigma^0\pi^+$ | $\Sigma^0\pi^+$ |
|-----------------------------|--------|--------|--------|--------|
| Predicted                   | $-1.0$ | $-0.70$ | $0.71$ | $0.70$ |
| $\alpha_{BP}$               | $-0.49$ | $-0.95$ | $0.79$ | $0.78$ |
| This work                   | $-0.66$ | $-0.90$ | $0.39$ | $0.39$ |
| $\Delta\beta_P$ (rad)       | $0.18 \pm 0.14$ | $-0.80 \pm 0.02$ | $4.1 \pm 0.6$ | $0.8 \pm 0.2$ |
| $\beta_{BP}$                | $3.0 \pm 2.4$ | $-0.66 \pm 0.22$ | $0.48 \pm 0.07$ |
| $\gamma_{BP}$               | $0.66 \pm 0.17$ | $-0.48 \pm 0.21$ | $-0.56 \pm 0.12$ |
| PDG [2]                     | $0.8 \pm 0.0$ | $0.0 \pm 0.0$ | $0.0 \pm 0.0$ |

TABLE II. Summary of the systematic uncertainties. $A$, $B$, $C$ and $D$ stand for the modes of $pK^0_S$, $\Lambda^{0+}$, $\Sigma^{0+}$, and $\Sigma^0\pi^+$, respectively.

| Source            | $\alpha_{BP}$ | $\alpha_{BP}$ | $\alpha_{BP}$ | $\alpha_{BP}$ | $\sin\Delta\phi_A$ | $\Delta\Delta\phi_B$ | $\Delta\Delta\phi_C$ |
|------------------|---------------|---------------|---------------|---------------|---------------------|---------------------|---------------------|
| Reconstruction   | 0.00          | 0.00          | 0.00          | 0.00          | 0.00                | 0.00                | 0.00                |
| $\pi^0\pi^0$ veto | 0.01          | 0.01          | 0.01          | 0.01          | 0.00                | 0.00                | 0.00                |
| $\Delta E$ signal | 0.07          | 0.01          | 0.02          | 0.05          | 0.02                | 0.3                | 0.1                |
| $M_{BC}$ signal   | 0.12          | 0.01          | 0.05          | 0.02          | 0.02                | 0.5                | 0.4                |
| $Bkg$ subtraction | 0.03          | 0.01          | 0.05          | 0.04          | 0.02                | 0.3                | 0.0                |
| **Total**        | 0.14          | 0.02          | 0.07          | 0.07          | 0.03                | 1.0                | 0.6                |

Therefore, in a given $\cos \theta_0$ interval,

$$\langle \sin \theta_1 \sin \phi_1 \rangle = \int_0^{2\pi} \int_{-1}^1 \sin \theta_1 \sin \phi_1 W \cos \theta_1 d\phi_1$$

is directly proportional to $\alpha_{BP} P_T(\cos \theta_0)/(1+\alpha_0 \cos^2 \theta_0)$ for the acceptance corrected data. In Fig. 3(e), the effect of the transverse polarization $P_T(\cos \theta_0)$ is illustrated by plotting the average value $\langle \sin(\alpha_{BP}) \rangle \sin \theta_1 \sin \phi_1$ from all four decay modes and including both particles and antiparticles. The sign function of the measured decay asymmetry parameter, $\text{sign}(\alpha_{BP})$, is used to avoid the cancellation of contributions from the opposite charge modes.

The systematic uncertainties arise mainly from the reconstruction of final state tracks, $K^0_S \rightarrow \pi^0\pi^0$, $\Delta E$ requirement, signal $M_{BC}$ selections and background subtraction. The contributions are summarized in Table II. The uncertainty of the input $\alpha_0$ is found to be negligible, after considering the experimental uncertainty [27].

Systematic uncertainties from different sources are combined in quadrature to obtain the total systematic uncertainties.

To understand the reconstruction efficiencies in data and MC simulations, a series of control samples are used for different final states. The proton and charged pion are studied based on the channel $J/\psi \rightarrow p\pi^+\pi^-$, photon on $e^+e^- \rightarrow \gamma \mu^+\mu^-$ [34], $\pi^0$ on $\psi(3086) \rightarrow \pi^0\pi^0 J/\psi$ and $e^+e^- \rightarrow \omega\pi^0$, $\Lambda$ on $J/\psi \rightarrow p\bar{K}^-\Lambda$ and $J/\psi \rightarrow \Lambda\Lambda$ [35], and $K^0_S$ on $J/\psi \rightarrow K^+(892)^+K^-, K^*(892)^+ K^-, K^*(892)^+ \rightarrow K^0_S \pi^+$ and $J/\psi \rightarrow \phi K^0_S K^+\pi^-$ [36]. The efficiency differences between data and MC simulations are used to reweight...

satisfies the relation

$$\langle \cos \theta_i \rangle = -\frac{1}{6} \alpha_{\Sigma^0\pi^+} \alpha_{\Lambda} \cos \theta_i,$$

with $(i, j) = (2, 3)$ or $(3, 2)$.

If the full expressions for the joint angular distributions (Ref. [24]) are integrated over the angles of the level 2 and 3 decay products, the remaining partial decay rate $W$ is

$$W \propto 1 + \alpha_0 \cos^2 \theta_0 + P_T \alpha_{BP} \sin \theta_1 \sin \phi_1.$$
the summed likelihood values. The changes of the fit results after likelihood minimization are taken as systematic uncertainties. The uncertainties due to the $K_S^0 \rightarrow \pi^0\pi^0$ veto in $\Sigma^+\pi^0$ candidate events are evaluated by taking the maximum changes with respect to the nominal results when varying the $\pi^0\pi^0$ veto range. A similar method is applied when estimating the systematic uncertainties from the signal $\Delta E$ and $M_{BC}$ selection criteria. In the likelihood construction, the subtraction of the background contributions are modeled with the sideband control samples and the inclusive MC samples. The associated uncertainties are studied by varying the sideband range and adjusting the scaling factors of the two background components. The altered scaling factors are obtained by changing the background line shapes within their 1σ uncertainties from the fits to the $M_{BC}$ distribution. The resultant maximum changes of the fit results are taken as corresponding systematic uncertainties.

To summarize, based on the 567 pb$^{-1}$ data sample collected from $e^+e^-$ collisions at a CM energy of 4.6 GeV, a simultaneous full angular analysis of four decay modes of $\Lambda_c^+ \rightarrow pK_S^0$, $\Lambda_c^+\Sigma^+\pi^0$, and $\Sigma^0\pi^+$ from the $e^+e^- \rightarrow \Lambda_c^+\Lambda_c^-$ production is carried out. We study the $\Lambda_c^+$ transverse polarization in unpolarized $e^+e^-$ collisions for the first time, which gives $\sin \Delta_0 = -0.28 \pm 0.13 \pm 0.03$ with a statistical significance of 2.1σ. This information will help in understanding the production mechanism of the charmed baryons in $e^+e^-$ annihilations. With the transverse polarization measurement, the decay asymmetry parameter in $\Lambda_c^+ \rightarrow pK_S^0$ becomes accessible experimentally. Moreover, this improves the precision in determining the decay asymmetry parameters in $\Lambda_c^+ \rightarrow \Lambda\pi^+, \Sigma^+\pi^0$, and $\Sigma^0\pi^+$, as listed in Table I.

The parameters $\alpha_{pK_S^0}^+$ and $\alpha_{\Sigma^0\pi^+}^+$ are measured for the first time. The measured $\alpha_{\Lambda\pi^+}^+$ and $\alpha_{\Sigma^+\pi^0}^+$ parameters are consistent with previous measurements, but with much improved precisions (by a factor of 3 for $\alpha_{\Sigma^+\pi^0}^+$). The negative sign of the $\alpha_{\Sigma^0\pi^+}^+$ parameter is confirmed and differs from the previous predictions [10, 11, 16–19] by at least 8σ, which rules out those model calculations. The measured $\alpha_{\Sigma^0\pi^+}^+$ and $\alpha_{\Sigma^0\pi^+}^+$ values agree well, which supports hyperon isospin symmetry in $\Lambda_c^+$ decay. For the results on $\alpha_{pK_S^0}^+$, $\alpha_{\Sigma^0\pi^+}^+$, and $\alpha_{\Sigma^0\pi^+}^+$ listed in Table I, at present no model gives predictions fully consistent with all the measurements. These improved results in $\Lambda_c^+$ decay asymmetries provide essential inputs for the $b$-baryon decay asymmetry measurements to be performed in the future.

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Supplemental Material for "Measurements of Weak Decay Asymmetries of $\Lambda_c^+ \to pK^-\pi^0$, $\Lambda^+_c$, $\Sigma^{+\prime 0}$, and $\Omega^{++0}$"
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| Shanghai Normal University, Taiyuan 030006, People’s Republic of China     | People’s Republic of China  | Taiyuan        | 030006        |
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For the process $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$, $\Lambda_c^+ \rightarrow BP$ and $\bar{\Lambda}_c^- \rightarrow inclusive$, where $B$ and $P$ denote a $J^P = \frac{1}{2}^+$ baryon and a pseudoscalar meson, respectively, the amplitude can be constructed under the helicity basis. For the weak non-leptonic decay $\Lambda_c^+ \rightarrow BP$, the Lee-Yang variables$^{[1]} \alpha_{BP}$, $\beta_{BP}$, and $\gamma_{BP}$ are defined with respect to the $s$-wave and $p$-wave amplitudes, such as

$$
\alpha_{BP} = \frac{2\text{Re}(s \cdot p)}{|s|^2 + |p|^2}, \quad \beta_{BP} = \frac{2\text{Im}(s \cdot p)}{|s|^2 + |p|^2}, \quad \gamma_{BP} = \frac{|s|^2 - |p|^2}{|s|^2 + |p|^2},
$$

(1)

and with equality $\alpha_{BP}^2 + \beta_{BP}^2 + \gamma_{BP} = 1$.

We work with helicity amplitudes. For $\Lambda_c^+ \rightarrow B(\frac{1}{2}^+)P(0^-)$ decay, we have two helicity amplitudes, $H_{1/2}$ and $H_{-1/2}$. Using relations $s = \frac{1}{\sqrt{2}}(H_{1/2} + H_{-1/2})$, $p = \frac{1}{\sqrt{2}}(H_{1/2} - H_{-1/2})$, we have the asymmetry parameters defined with helicity amplitudes as

$$
\alpha_{BP} = |H_{1/2}|^2 - |H_{-1/2}|^2, \quad \beta_{BP} = \sqrt{1 - \alpha_{BP}^2} \sin \Delta_{BP}^1, \quad \gamma_{BP} = \sqrt{1 - \alpha_{BP}^2 \cos \Delta_{BP}^1},
$$

(2)

here we have taken the normalization $|H_{1/2}|^2 + |H_{-1/2}|^2 = 1$, and $\Delta_{BP}^1$ is the phase angle difference between two helicity amplitudes $H_{1/2}$ and $H_{-1/2}$.

If $\Lambda_c^+$ and $\bar{\Lambda}_c^-$ decays conserve the CP transformation, we have relations for the $\bar{\Lambda}_c^-$ asymmetry parameters as

$$
\bar{\alpha}_{BP} = -\alpha_{BP}, \quad \bar{\beta}_{BP} = -\beta_{BP}, \quad \bar{\gamma}_{BP} = \gamma_{BP}.
$$

(3)

In the context, for the helicity frame of $\Lambda_c^+$ production process $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$, $\theta_0$ is defined as the polar angle of the $\Lambda_c^+$ with respect to the $e^+e^-$ center-of-mass (CM) system, as illustrated in Fig. 1.

**I. JOINT ANGULAR DISTRIBUTION FOR THE DECAY $\Lambda_c^+ \rightarrow pK^0_S$**

Figure 1 illustrates the definitions of the helicity angles for a 1-level decay $\Lambda_c^+ \rightarrow pK^0_S$. In the helicity system describing the $\Lambda_c^+ \rightarrow pK^0_S$ decay, the angle $\phi_1$ is the angle between the $e^+\Lambda_c^+$ plane and $pK^0_S$ plane, and $\theta_1$ is the polar angle of the $p$ momentum in the rest frame of the $\Lambda_c^+$ with respect to the $\Lambda_c^+$ momentum in the CM frame.

**TABLE I. Definition of decays, helicity angles and amplitudes of $\Lambda_c^+ \rightarrow pK^0_S$, where $\lambda_i$ indicates the helicity for the corresponding hadron.**

| level | reaction | helicity angle | helicity amplitude |
|-------|----------|----------------|-------------------|
| 0     | $e^+e^- \rightarrow \gamma \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$ | $\theta_0$ | $A_{\lambda_1,\lambda_2}$ |
| 1     | $\Lambda_c^+ \rightarrow p(\lambda_1)K^0_S$ | ($\theta_1, \phi_1$) | $B_{\lambda_3}$ |

As listed in Table I, $\lambda_1$, $\lambda_2$, and $\lambda_3$ denote the helicities of $\Lambda_c^+$, $\bar{\Lambda}_c^-$ and $p$. $A_{\lambda_1,\lambda_2}$ and $B_{\lambda_3}$ are the helicity amplitudes.

The differential decay rate is defined as

$$
M_{\lambda_i} = D_{\lambda_i,\lambda_1,\lambda_2}^{1}\left(\theta_0\right)A_{\lambda_1,\lambda_2}B_{\lambda_3}^{1}\left(\Omega_1\right),
$$

(4)
TABLE II. Definition of decays, helicity angles and amplitudes in $\Lambda^{+}_c \rightarrow \Lambda \pi^{+}$, where $\lambda_i$ indicates the helicity values for the corresponding hadron.

| level | reaction | helicity angle | helicity amplitude |
|-------|----------|----------------|-------------------|
| 0     | $e^+e^- \rightarrow \gamma^+ \rightarrow \Lambda^{+}_c (\lambda_1)\Lambda^{0}_c (\lambda_2)$ | $\theta_0$ | $A_{\lambda_1, \lambda_2}$ |
| 1     | $\Lambda^{+}_c \rightarrow \Lambda (\lambda_3)\pi^+$ | $(\theta_1, \phi_1)$ | $B_{\lambda_3}$ |
| 2     | $\Lambda \rightarrow p (\lambda_4)\pi^-$ | $(\theta_2, \phi_2)$ | $C_{\lambda_4}$ |

where $m$ is the helicity of virtual photon, $D_{m,\lambda_1-\lambda_2}^1(\theta_0)$ and $D_{\lambda_1,\lambda_2}^2(\Omega_i) \equiv D_{\lambda_1,\lambda_2}^2(\phi_i, \theta_i, 0)$ is Wigner-D function [2].

The total helicity amplitudes is calculated by

$$|M|^2 = \sum_m |M_m|^2 = \sum_m |\sum_{\lambda_i} M_{\lambda_i}|^2 = \sum_m (\sum_{\lambda_i} M_{\lambda_i})(\sum_{\lambda_i} M_{\lambda_i}^*) .$$

(5)

If we define the $\gamma^+$ spin density matrix $\rho^{(\lambda_1, -\lambda_2, \lambda_1', -\lambda_2')}_0 = \sum_{m, \lambda_1-\lambda_2} d_{m, \lambda_1-\lambda_2}^1(\theta_0) d_{m, \lambda_1'-\lambda_2'}^1(\theta_0)$ we can get

$$\frac{d\Gamma}{d \cos \theta d \cos \phi d \phi} \propto \sum_{m, \lambda_1-\lambda_2, \lambda_1' \lambda_2'} D_{m, \lambda_1-\lambda_2}^1(0, \theta_0, 0) D_{m, \lambda_1'-\lambda_2'}^1(0, \theta_0, 0) A_{\lambda_1, \lambda_2}^* A_{\lambda_1', \lambda_2'} \times D_{\lambda_1', \lambda_2'}^1(\phi_1, \theta_1, 0) D_{\lambda_1', \lambda_2'}^{1/2}(\phi_1, \theta_1, 0) |B_{\lambda_3}|^2 ,$$

(6)

Helicity amplitude $A_{\lambda_1, \lambda_2}$ is related to the angular distribution parameters $\alpha_0 = \frac{|A_{1,1}^{1/2} - |A_{1,1}^{1/2}|^2|}{|A_{1,1}^{1/2}|^2 + 2|A_{1,1}^{1/2}|^2}$, and helicity amplitude $B_{\lambda_3}$ is related to the decay asymmetry parameter $\alpha_{PK}^{+} = \frac{|B_{1}^{1/2} - |B_{1}^{1/2}|^2|}{|B_{1}^{1/2}|^2 + 2|B_{1}^{1/2}|^2}$. In helicity frame, conventional $s$–wave amplitude can be expressed by $\frac{1}{\sqrt{2}} (B_{1}^{1/2} + B_{-1}^{1/2})$ and $p$–wave by $\frac{1}{\sqrt{2}} (B_{1}^{1/2} - B_{-1}^{1/2})$. The joint angular dependence of the decay rate is written as

$$\frac{d\Gamma}{d \cos \theta d \cos \phi d \phi} \propto 1 + \alpha_0 \cos^2 \theta_0 + \mathcal{P}_T \alpha_{PK}^{+} \sin \theta_0 \sin \phi_1 ,$$

(7)

$$\mathcal{P}_T = \sqrt{1 - \alpha_{PK}^{+} \sin \theta_0 \sin \phi_1 }$$

(8)

where $\Delta_0 = \phi_{1/2} - \phi_{-1/2}$ is the difference of phase angle for the helicity amplitudes $A_{1/2}$ and $A_{-1/2}$, and $\mathcal{P}_T$ corresponds to a transverse polarization observable of the produced $\Lambda^{+}_c$. For the charge conjugation mode $\bar{\Lambda}^{+}_c \rightarrow \bar{p}K^{0}_S$, the formula of angular distribution is same, but with the parameter relations of $\Delta_0 = \Delta_0$ and $\alpha_{PK}^{+} = -\alpha_{PK}^{-}$, when neglecting $CP$ violation.

II. JOINT ANGULAR DISTRIBUTION FOR THE DECAYS $\Lambda^{+}_c \rightarrow \Lambda \pi^{+}$ AND $\Sigma^{+} \pi^{0}$

Figure 2 illustrates the definitions of the helicity angles for a 2-level cascade decay $\Lambda^{+}_c \rightarrow \Lambda \pi^{+}$, $\Lambda \rightarrow p\pi^-$. In the helicity system describing the $\Lambda^{+}_c \rightarrow \Lambda \pi^{+}$ decay, the angle $\phi_1$ is the angle between the $e^+\Lambda^{+}_c$ plane and $\Lambda \pi^{+}$ plane, and $\theta_1$ is the polar angle of the $\Lambda$ momentum in the rest frame of the $\Lambda^{+}_c$ with respect to the $\Lambda^{+}_c$ momentum in the CM frame. In the helicity system describing the $\Lambda \rightarrow p\pi^-$ decay, the angle $\phi_2$ is the angle between the $\Lambda \pi^{+}$ plane and $p\pi^-$ plane, and $\theta_2$ is the polar angle of the proton momentum with respect to the opposite direction of $\pi^{+}$ momentum in the rest frame of $\Lambda$.

As listed in Table II, $B_{\lambda_3}$ and $C_{\lambda_4}$ are the helicity amplitudes of the $\Lambda^{+}_c \rightarrow \Lambda \pi^{+}$ and $\Lambda \rightarrow p\pi^-$ decays, respectively.
The joint angular dependence of the decay rate is written as

\[
\frac{d\Gamma}{d\cos\theta_0\cos\theta_1 d\cos\theta_2 d\phi_1 d\phi_2} \propto 2 + 2\alpha_0 \cos^2 \theta_0 \\
+ \sqrt{1 - \alpha_0^2} \alpha_L \sin \Delta_0 \sin(2\theta_0) \sin \theta_1 \cos \theta_2 \sin \phi_1 \\
+ \sqrt{1 - \alpha_0^2} \alpha_L \sin \Delta_0 \sin(2\theta_0) \cos \theta_1 \sin \theta_2 \sin \phi_1 \sqrt{1 - (\alpha_{\Lambda \pi^+})^2} \cos(\Delta_{\Lambda \pi^+} + \phi_2) \\
+ \sqrt{1 - \alpha_0^2} \alpha_L \sin \Delta_0 \sin(2\theta_0) \sin \theta_2 \cos \phi_1 \sqrt{1 - (\alpha_{\Lambda \pi^+})^2} \sin(\Delta_{\Lambda \pi^+} + \phi_2) \\
+ \sqrt{1 - \alpha_0^2} \Delta_0 \sin(2\theta_0) \sin \theta_1 \sin \phi_2 \cos \theta_2 \alpha_{\Lambda \pi^+} \\
+ 2\alpha_0 \alpha_L \cos^2 \theta_0 \cos \theta_2 \alpha_{\Lambda \pi^+} + 2\alpha_L \cos \theta_2 \alpha_{\Lambda \pi^+},
\]

where $\alpha_L$ denotes the decay asymmetry parameter in the weak hadronic decay $\Lambda \rightarrow p\pi^-$, $\Delta_0 = \delta_0 - \frac{1}{2} - \frac{1}{2}$ is the difference of phase angle for the helicity amplitudes $A_{\Lambda_+ \Lambda_2}$ and $\Delta_{\Lambda \pi^+}$ is the difference of the phase angle between the helicity amplitudes $B_{\Lambda_+ \Lambda_2}$ and $B_{\Lambda \pi}$. For the case of the charge conjugation mode $\Lambda^+ \rightarrow \Lambda^- \pi^-$, the formula is the same, but with the parameter relations $\alpha_L = -\alpha_L$, $\alpha_{\Lambda \pi^+} = -\alpha_{\Lambda \pi^+}$, $\Delta_0 = \Delta_0$, $\Delta_{\Lambda \pi^+} = -\Delta_{\Lambda \pi^+}$ on the basis of no $CP$ violation.

If the phase space of level-2 decay $\Omega_2 = (\theta_2, \phi_2)$ is integrated out, one has

\[
\frac{d\Gamma}{d\cos\theta_0 d\cos\theta_1 d\phi_1} \propto 1 + \alpha_0 \cos^2 \theta_0 + P_T \alpha_{\Lambda \pi^+} \sin \theta_1 \sin \phi_1,
\]

\[
P_T = \sqrt{1 - \alpha_0^2} \cos \theta_0 \sin \theta_0 \sin \Delta_0.
\]

Equation (7) becomes in the same form of Eq. (4). If the proton helicity angular $\theta_2$ is only measured, one has

\[
\frac{dN}{d\cos\theta_2} \propto 1 + \alpha_{\Lambda \pi^+} \alpha_L \cos \theta_2.
\]

This equation indicates that even without information of $P_T$, the decay asymmetry parameter $\alpha_{\Lambda \pi^+}$ can be accessed from the distribution of $\cos \theta_2$. 

FIG. 2. Definition of the helicity frame for $e^+e^- \rightarrow \Lambda_c^+ \Lambda_c^-$, $\Lambda_c^+ \rightarrow \Lambda^0 \pi^+$ and $\Lambda \rightarrow p\pi^-$. 

\[\frac{dN}{d\cos\theta_2} \propto 1 + \alpha_{\Lambda \pi^+} \alpha_L \cos \theta_2.\]
TABLE III. Definition of decays, helicity angles and amplitudes in $\Lambda^+_c \to \Sigma^+\pi^0$, where $\lambda_i$ indicates the helicity values for the corresponding hadron.

| level | reaction | helicity angle | helicity amplitude |
|-------|----------|----------------|-------------------|
| 0     | $e^+e^- \to \gamma \to \Lambda^+_c (\lambda_1)\bar{\Lambda}^- (\lambda_2)$ | $\theta_0$ | $A_{\lambda_1\lambda_2}$ |
| 1     | $\Lambda^+_c \to \Sigma^+ (\lambda_3)\pi^0$ | $(\theta_1, \phi_1)$ | $B_{\lambda_3}$ |
| 2     | $\Sigma^+ \to p (\lambda_4)\pi^0$ | $(\theta_2, \phi_2)$ | $C_{\lambda_4}$ |

For the 2-level cascade decays $\Lambda^+_c \to \Sigma^+\pi^0$, $\Sigma^+ \to \pi^+\pi^-$, the formalism is analogous to that of $\Lambda^+_c \to \Lambda\pi^+$ as listed in Table III, but replacing the symbols of $\Lambda$ and $\pi^+$ with $\Sigma^+$ and $\pi^0$ in the level-1 decay and replacing $\pi^-$ with $\pi^+$ in the level-2 decay, respectively.

III. JOINT ANGULAR DISTRIBUTION FOR $\Lambda^+_c \to \Sigma^0\pi^+$

Figure 3 illustrates the definitions of the helicity angles for a 3-level cascade decay $\Lambda^+_c \to \Sigma^+\pi^0$, $\Sigma^0 \to \gamma\Lambda$, $\Lambda \to p\pi^-$. In the helicity system describing the $\Lambda^+_c \to \Sigma^0\pi^+$ decay, the angle $\phi_1$ is the angle between the $e^+\Lambda^+_c$ plane and $\Sigma^0\pi^+$ plane, and $\theta_1$ is the polar angle of the $\Sigma^0\pi^+$ momentum in the rest frame of the $\Lambda^+_c$ with respect to the $\Lambda^+_c$ momentum in the CM frame. In the helicity system describing the $\Sigma^0 \to \gamma\Lambda$ decay, the angle $\phi_2$ is the angle between the $\Sigma^0\pi^+$ plane and $\gamma\Lambda$ plane, and $\theta_2$ is the polar angle of the $\Lambda$ momentum with respect to the opposite direction of $\pi^+$ momentum in the rest frame of $\Sigma^0$. In the helicity system describing the $\Lambda \to p\pi^-$ process, $\phi_3$ is the angle between the $\Lambda\gamma$ and
TABLE IV. Definition of decays, helicity angles and amplitudes in $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$, where $\lambda_i$ indicates the helicity values for the corresponding hadron.

| level | decay | helicity angle | helicity amplitude |
|-------|-------|----------------|-------------------|
| 0     | $e^+ e^- \rightarrow \Lambda_c^+(\lambda_1)\Lambda_c^-(\lambda_1)\bar{\Lambda}_c$ | $\theta_0$ | $A_{\lambda_1 \lambda_2}$ |
| 1     | $\Sigma^0 \rightarrow \Lambda(\lambda_1)\gamma(\lambda_3)$ | $\theta_1, \phi_1$ | $B_{\lambda_3}$ |
| 2     | $\Sigma^0 \rightarrow \Lambda(\lambda_1)\gamma(\lambda_2)$ | $\theta_2, \phi_2$ | $C_{\lambda_1 \lambda_3}$ |
| 3     | $\Lambda \rightarrow p(\lambda_1)\gamma(\lambda_3)$ | $\theta_3$ | $F_{\lambda_3}$ |

$p\pi^-$ planes, while $\theta_4$ is the polar angle of the proton with respect to the opposite direction of the photon momentum (from $\Sigma^0 \rightarrow \Lambda \gamma$) in the rest frame of $\Lambda$.

The helicity angles and amplitudes are defined in Table IV. The joint angular dependence of the decay rate is expressed as

$$d\Gamma \propto 2 \cos^2 \theta_0$$

$$- \sqrt{1 - \alpha_0^2} \sin^2 \theta_0 
- \sqrt{1 - \alpha_0^2} \sin^2 \theta_1 \sin \phi_1 \sin \Delta_0$$

$$+ \sqrt{1 - \alpha_0^2} \sin^2 \theta_1 \sin \phi_1 \sin \Delta_0$$

$$\propto 1 + \alpha_0 \cos^2 \theta_0$$

where $\Delta_{\pi^+}^{\Sigma^0}$ is the phase angle difference for the helicity amplitudes $B_{\lambda_3}$ and $B_{-\lambda_3}$. For the corresponding charge-conjugate $\bar{\Lambda}_c$ decays, one has a similar formula, but with replacements $\bar{\alpha}_0 = -\alpha_0$, $\bar{\alpha}_0^\Sigma = -\alpha_0^\Sigma_\pi^+$, $\Delta_0 = \Delta_{\pi^+}^{\Sigma^0}$.

If the phase spaces of level-2 and level-3 decays $\Omega_2 = (\theta_2, \phi_2)$ and $\Omega_3 = (\theta_3, \phi_3)$ are integrated out, one get the angular distribution

$$d\Gamma \propto 1 + \alpha_0 \cos^2 \theta_0$$

Equation (11) becomes in the same forms of Eqs. (4) and (7). If the $\theta_2$ and $\theta_3$ angles are only measured, one has

$$dN \propto 1 - \alpha_0^\Sigma \alpha_0 \cos \theta_2 \cos \theta_3$$

This formula provides a way to measure the decay asymmetry parameter $\alpha_0^\Sigma$ with no information of $P_T$.

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[1] T. D. Lee and C. N. Yang, Phys. Rev., 108, 1645 (1957).

[2] Group Theory, Academic Press, New York, 1959.