Nuclear Spin Relaxation in Rashba Nanowires

Alexander A. Zyuzin, Tobias Meng, Viktoria Kornich, and Daniel Loss

Department of Physics, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland

We study the nuclear spin relaxation in a ballistic nanowire with hyperfine and Rashba spin-orbit interactions (SOI) and in the presence of magnetic field and electron interactions. The relaxation rate shows pronounced peaks as function of magnetic field and chemical potential due to van Hove singularities in the Rashba bands. As a result, the regimes of weak and strong SOIs can be distinguished by the number of peaks in the rate. The relaxation rate increases with increasing magnetic field if both Rashba subbands are occupied, whereas it decreases if only the lowest one is occupied.

PACS numbers: 76.60.-k, 71.10.Pm, 73.21.Hb, 71.70.Ej

Introduction. Low-dimensional condensed matter systems with strong spin-orbit interaction (SOI) have attracted much attention both theoretically [1-5] and experimentally, [6-9] for their realization of nontrivial momentum space topology [5]. A particular example of such systems are semiconducting Rashba nanowires in a helical state, in which the Rashba SOI [10] locks the spin of the electron to its direction of motion. In a Rashba nanowire, the helical state can be obtained by tuning the chemical potential into the partial gap at zero momentum induced by a magnetic field. Insulating and superconducting states of helical Rashba wires can host Jackiw-Rebbi [11,12] and Majorana bound states around topological defects [13].

Not only as a prerequisite for the creation and identification of these exotic bound states, but also on its own right, it is important to gain information about the strength of the SOI, and to detect signatures of the helical state in nanowires. For example, the drop of the conductance of a ballistic conduction channel from $2e^2/h$ to $e^2/h$ as a function of Fermi level can serve as an experimental probe of the helical state [14-17]. Signatures of the helical state can also be found in the electron spin susceptibility [18]. So far, the SOI in nanowires has been measured only in quantum dots (via transport) [19-21], where, however, the Rashba SOI of interest is masked by the one that is induced by the dot confinement potential.

In this work, we propose an alternative and non-invasive way to access information about the SOI and the helical regime in a Rashba nanowire, namely via the nuclear spins. These are sensitive to the electronic state due to the hyperfine interaction present in III-V semiconductors such as GaAs or InAs. A main motivation for our proposal stems from striking experimental progress in the field of nanoscale magnetometry. In particular, it has recently been demonstrated that cantilever-based magnetic sensing enables the nuclear spin magnetometry of nanostructures, in particular of InP and GaP nanowires [22]. In the remainder, we show how such ultra-sensitive techniques can be used to probe the strength of the SOI, and to detect the helical states via the Korringa nuclear spin relaxation mechanism, i.e. the change of the nuclear spin state due to the spin-flip scattering of itinerant electrons of energies within the thermally broadened region close to the Fermi level [23]. We evaluate the nuclear spin relaxation rate in a one-dimensional ballistic electron gas in the presence of Rashba SOI and magnetic field. We first derive an explicit dependence of the nuclear relaxation rate on the parameters of the electronic spectrum for non-interacting electrons, in which the relaxation rate is proportional to the electronic temperature. We then discuss how electron interactions modify this temperature dependence to an interaction-dependent power law.

We find that the relaxation rate shows distinct peaks as function of magnetic field and chemical potential due to van Hove singularities in the Rashba bands. Remarkably, the regime of weak SOI is characterized by one peak while the one of strong SOI by three peaks. The relaxation rate for weak SOI vanishes as a function of chemical potential. For strong SOI, the Zeeman energy exceeds $\mu$ if the Fermi level crosses both Rashba bands, while the rate decreases if only the lowest Rashba subband is occupied.

Relaxation in non-interacting electron gas. We consider nuclear spins coupled by hyperfine interaction to itinerant electrons in a semiconducting quantum or nanowire with Rashba SOI. Our goal is to calculate the nu-
clear spin relaxation rate, first without electron-electron interactions. We assume the electrons to occupy the lowest transverse subband of the wire with cross-sectional area $d_x \times d_y$, see Fig. [1], described by the wave function $\Psi(x, y) = \frac{2}{\sqrt{d_x d_y}} \cos(\pi x/d_x) \cos(\pi y/d_y)$. Taking into account that the nuclear density in the wire is much larger than the electron density we can approximate the wave function $|\Psi(x, y)| \approx 2/\sqrt{d_x d_y}$ by its value at the centre of the wire and sum over the nuclear spin density in the transverse direction.

The Hamiltonian of the system becomes ($\hbar = 1$)

$$H = -\int dz c_d^\dagger(z) \left[ \left( \frac{\partial^2}{2m} + \mu \right) \delta_{ss'} + i\alpha \sigma_z^s \partial_z + h\sigma_z^{ss'} \right] c_{s'}(z) + \int dz \left[ \frac{\sqrt{d_x d_y}}{2} \hat{A}(z) \cdot \frac{\sigma_{ss'}}{2} c_s(z) c_{s'}(z) - \omega_N I^z(z) \right]$$

(1)

Here, $\mu$ is the chemical potential of the electrons with effective mass $m$ and spin projection $s$, where summation over repeated spin indexes is implied, $\alpha$ is the SOI constant, $h = g_e \mu_B B/2$ the Zeeman energy of the electrons due to the external magnetic field, $B$, applied along the $z$ axis, where $g_e$ is the electron $g$-factor and $\mu_B$ the Bohr magneton, $\sigma_z$ the Pauli matrices, and $A = A_3 d_2/(d_x d_y)$, where $A_3 d_2$ is the bulk value of the hyperfine interaction constant between a nuclear spin and the spin of an electron. We assume the dominant contribution to the hyperfine interaction is given by Fermi contact interaction.

The nuclear spin density is given by $I(z) = N_2 \sum_j I_j \delta(z - z_j)$, where $I_j$ is the spin operator of the $j$th nuclei, $N_2$ is the number of nuclear spins in the plane transverse to the wire axis, and the sum runs over the nuclear spins the wire axis. We assume that the Zeeman energy of the nuclear spins induced by the external magnetic field, $\omega_N = g_N \mu_B B$, where $g_N$ and $g_e$ are the nuclear magneton and the effective g-factor, respectively, is small compared to the temperature $T$. We also assume temperature to be larger than the Kondo temperature associated with the localized spin.

It is convenient to express the single-particle Green function in the Rashba eigenbasis (for $A = 0$),

$$G_{ss'}(\epsilon_n, p) = \frac{1}{2} \sum_{\lambda = \pm 1} \delta_{ss'} - \lambda \frac{\epsilon_n - \epsilon_\lambda(p)}{\alpha^2 p^2 + h^2},$$

(2)

where $\epsilon_\lambda(p) = p^2/2m - \mu - \lambda \sqrt{\alpha^2 p^2 + h^2}$ is the electron spectrum and $p$ the momentum in the Rashba spin-split subband defined by $\lambda = \pm 1$; $\epsilon_n = (2n + 1)\pi T$ is the fermionic Matsubara frequency.

We can now calculate the nuclear spin relaxation rate $1/T_1$ in a one-dimensional electron gas with SOI and Zeeman energy in second order perturbation theory in the hyperfine interaction between the nuclear spin and electron spin density. The relaxation rate is determined by the dynamical spin susceptibility of the conduction electrons at the nuclear site as $\chi_{xx}(q, \omega) = \frac{1}{\omega_N} \int \frac{dq}{2\pi} \Im[\chi_{xx}(q, \omega) |_\omega = \omega_N + i\alpha^+].$ (3)
FIG. 2. The nuclear spin relaxation rate $1/T_1 = T_1(0)/(2T_1(\mu/h))$ as function of $\mu/h$ for small (a) and large (b) SOI strengths $m\lambda^2/h = 0.2$ and $m\lambda^2/h = 10$, respectively, as numerically obtained from Eq. (5). The rate is normalized by its value at $\mu = 0$ (we set $\mu = 0$ at the middle of the Zeeman gap between the Rashba subbands at $p = 0$). The pronounced peaks are due to Van Hove singularities at the edges of the Rashba subbands. (a) Curves from top to bottom are for $T/h = 0.07, 0.1, 0.2, 0.3$, respectively. (b) Curves from top to bottom are for $T/h = 0.05, 0.07, 0.1$, respectively. Inset (a): The Rashba spectrum with possible Fermi levels (lines). Strong SOI gives rise to regions in the spectrum (dashed-dotted line) with negative group velocity and non-monotonic relaxation rate as shown in (b). The strong increase of the rate for $\mu/h$ approaching the band bottom $E_0/h = -5$ (see also inset (b)) signals the breakdown of perturbation theory.

Physically, this expresses the fact that the nuclear spin polarization cannot decay via flip-flop processes with the electrons if the latter are spin polarized due to the presence of a large Zeeman field. We note that the present calculation does not take into account competing nuclear spin relaxation mechanisms.

In the vicinity of the van Hove singularity, $\mu \gtrsim h > 0$, see Fig. 2, the relaxation rate scales with the chemical potential as $T_1^{-1} = mA^2T h + m\alpha^2 \sqrt{2h/\mu - h}$. 

On the other hand, in the presence of both SOI and magnetic field, tuning the chemical potential to the middle of the gap of the spectrum at $p = 0$, $\mu = 0$, we obtain,

$$\frac{1}{T_1} = \frac{mA^2T}{4\pi} \frac{m\alpha^2}{h^2 + m^2\alpha^4}. \quad (8)$$

We note that the relaxation rate diverges at the van Hove singularities of the spectrum occurring at zeroes of $\partial \chi_\alpha(p)/\partial p$. For instance, $1/T_1 \sim |h(\mu - h)|^{-1/2}$ for weak SOI and $\mu > h$, and $1/T_1 \sim |\mu - E_0|^{-1}$ for strong SOI, where $E_0$ denotes the band bottom. Formally, the perturbation expansion in $A$ for the rate breaks down at these singularities. However, these singularities turn into well-defined peaks by finite-temperature effects, as we confirmed by evaluating Eq. (5) numerically for various temperatures, see Figs. 2 and 3. The peak at $E_0$, however, remains large also for $T > 0$ and thus is outside the perturbative regime considered here. As an important result, we see that the relaxation rate behaves qualitatively very different for weak and for strong SOIs: in the former case, there is only one peak, while in the latter there are three peaks in $1/T_1$ as function of $\mu$, see Figs. 2(a) and 2(b).

Finally, let us discuss the dependence of the relaxation rate on the magnetic field $h$. The relaxation rate increases with the increase of the Zeeman energy as $\sim h^2/\mu^2$ for $\mu > m\alpha^2$ if the Fermi level crosses both Rashba subbands, see Fig. 3(a) and inset in Fig. 2(a), where the position of the Fermi level is shown by the solid line. On the other hand, the relaxation rate decreases with increase of the Zeeman energy as $\sim -h^2/m^2\alpha^4$ for $|\mu| < m\alpha^2$ if the Fermi level crosses only the lowest Rashba subband, see Fig. 3(b) and inset in Fig. 2(a), where the position of the Fermi level is shown by the dashed-dotted line. The singularity of the relaxation rate shown in Figs. 3(a) and 3(b) corresponds to the condition $h = |\mu|$.

Relaxation in an interacting electron system. In a one-dimensional system, electron-electron interactions modify the temperature dependence of the nuclear spin relaxation rate from a linear scaling to interaction dependent power laws. We derive these in a Luttinger liquid calculation valid when the chemical potential is sufficiently far from the van Hove singularities, so that the dispersion can be linearized. For $|\mu| \gg |h|, m\alpha^2$, the system can be understood as a spinful Luttinger liquid with subleading corrections due to the SOI and the magnetic field. Neglecting these subleading terms, we first decompose the fermionic operators into their right and left moving parts. Next, we bosonize these operators using standard techniques [25]. The effect of electron-electron interactions is captured by a Luttinger liquid parameter $1 \geq K_\epsilon \geq 0$ for the charge fluctuations (we take the Luttinger liquid parameter in the spin sector to be $K_\alpha \approx 1$).

In order to calculate the relaxation rate $T_1^{-1}$ for interacting electrons, we proceed by bosonizing the real space, imaginary time susceptibility $\chi_{xx}(z, \tau)$ according to the above prescription. From this expression, the retarded spin susceptibility as a function of frequency can be calculated through a Wick rotation to real time, and a subsequent Fourier transformation [25]. Eq. (6) then
small, while the contribution of the gapless helical modes in Eq. (4) involving the gapped modes (i.e. right and left directions and have antiparallel spins). In this case, all terms depend on the short distance cutoff $T$.

Here, $\chi_{xx}(0, \tau)$ is given by 

$$
\chi_{xx}(0, \tau) = -\frac{1}{8(\pi a_0^2)} \frac{(\pi a_0 T/u_{hel})^{2K_{hel}}}{\sinh(i\pi T\tau) \sinh(-i\pi T\tau)}^{K_{hel}}.
$$

This implies that the relaxation rate takes the form

$$
\frac{1}{\tilde{T}_1} \approx T_{hel}(\alpha, K_{hel}) \frac{T^{2K_{hel}}}{h} - 1,
$$

where $T_{hel}(\alpha, K_{hel})$ is again a cutoff-dependent prefactor that reduces to $T_{hel}(\alpha, 1) = \frac{\pi^2}{4\alpha^2}$ in the non-interacting limit. This power law complies with the fact that in the helical regime the nuclear spin relaxation results from electronic backscattering processes only.

For $ma^2 \gg |\mu| \gg |h|$, the system is gapless, and thus shows a scaling of the type given in Eq. (9). For $|h| \gg |\mu|, ma^2$, finally, the system essentially behaves like a spinful wire with a Zeeman splitting between spin up and spin down in which only the lower of the two Zeeman-split bands is occupied. In this case, we obtain $\frac{1}{\tilde{T}_1} \approx 0$, which follows from Eq. (7), as well as from Eq. (8) in the limit $ma^2/h \to 0$.

Conclusions. Let us now comment on the experimental observability of the predicted behavior of the nuclear spin relaxation rates in InAs nanowires. For an InAs nanowire with a cross-sectional area of $d_x \times d_y = 50 \times 50$ nm$^2$, and with Fermi velocity $v_F = 10^7$ cm/s, we obtain for the one-dimensional density of states $\nu = 1/\pi v_F h \approx 5$ (eV nm)$^{-1}$. For an electron $g$-factor $g = 8$, the Zeeman energy $h \approx 4$ K at a magnetic field of 1 Tesla, requires correspondingly low temperatures, $T \lesssim 4$ K. The dominant hyperfine coupling comes from In with nuclear spin $I = 9/2$ and bulk constant $A_{3D} = 4.7$ $\mu$eVnm$^3$. With this, we estimate the nuclear spin relaxation time

$$
T_K \approx \left(\frac{A_{3D}}{v_F h}\right)^2 \frac{h^2}{\pi^2 K_{hel}} \approx 20 \text{ s, at } T = 1 \text{ K.}
$$

This estimate for $T_K$ is consistent with recent measurements performed on InP nanowires with cantilever techniques [22]. Thus, it seems worthwhile to search for the predicted signatures of the SOI in the relaxation rate as a function of magnetic field, chemical potential, and temperature.

Acknowledgements. We thank M. Poggio and D. Becker for discussions and acknowledge support from the Swiss NSF and NCCR QSIT.

---

1. L. Fu and C. L. Kane, Phys. Rev. Lett. 100, 096407 (2008).
2. S. Satoh and S. Fujiwara, Phys. Rev. B 79, 094504 (2009).
3. R. M. Lutchyn, J. D. Sau, and S. Das Sarma, Phys. Rev. Lett. 105, 077001 (2010).
4. Y. Oreg, G. Refael, and F. von Oppen, Phys. Rev. Lett. 105, 177002 (2010).
5. G. Volovik, The Universe in a Helium Droplet (Oxford University Press, Oxford, 2003).
[6] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Science 336, 1003 (2012).
[7] M. T. Deng, C. L. Yu, G. Y. Huang, M. Larsson, P. Caro, and H. Q. Xu, Nano Lett. 12, 6414 (2012).
[8] A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum, and H. Shtrikman, Nat. Phys. 8, 887 (2012).
[9] L. P. Rokhinson, X. Liu, and J. K. Furdyna, Nature Physics 8, 795 (2012).
[10] E. I. Rashba, Sov. Phys. Solid State 2, 1109 (1960).
[11] R. Jackiw and C. Rebbi, Phys. Rev. D 13, 3398 (1976).
[12] J. Klinovaja, P. Stano, and D. Loss, Phys. Rev. Lett. 109, 236801 (2012).
[13] J. Alicea, Reports on Progress in Physics 75, 076501 (2012).
[14] P. Streda and P. Seba, Phys. Rev. Lett. 90, 256601 (2003).
[15] C. H. L. Quay, T. L. Hughes, J. A. Sulpizio, L. N. Pfeiffer, K. W. Baldwin, K. W. West, D. Goldhaber-Gordon, and R. de Picciotto, Nature Physics 6, 336 (2010).
[16] B. Braunecker, P. Simon, and D. Loss, Phys. Rev. Lett. 102, 116403 (2009).
[17] C. Scheller, T.-M. Liu, G. Barak, A. Yacoby, L. Pfeiffer, K. West, and D. Zumbuhl, Phys. Rev. Lett. 112, 066801 (2014).
[18] T. Meng and D. Loss, Phys. Rev. B 88, 035437 (2013).
[19] C. Fasth, A. Fuhrer, L. Samuelson, V. N. Golovach, and D. Loss, Phys. Rev. Lett. 98, 266801 (2007).
[20] Y. Kanai, R. S. Deacon, S. Takahashi, A. Oiwa, K. Yoshida, K. Shibata, K. Hirakawa, Y. Tokura, and S. Tarucha, Nat. Nano 6, 511 (2011).
[21] S. Nadj-Perge, V. S. Pribiag, J. W. G. van den Berg, K. Zuo, S. R. Plissard, E. P. A. M. Bakkers, S. M. Frolov, and L. P. Kouwenhoven, Phys. Rev. Lett. 108, 166801 (2012).
[22] P. Peddibhotla, F. Xue, H. I. T. Hauge, S. Assali, E. P. A. M. Bakkers, and M. Poggio, Nature Phys. 9, 631 (2013).
[23] C. P. Slichter, Principles of Magnetic Resonance (Springer-Verlag, Berlin, 1980).
[24] R. White, Quantum Theory of Magnetism (Springer-Verlag, Berlin, 2007).
[25] T. Giamarchi, Quantum Physics in One Dimension (Oxford University Press, New York, 2003).