Negative Sampling for Recommendation

Bin Liu, Bang Wang*
School of Electronic Information and Communications, Huazhong University of Science and Technology (HUST), Wuhan, China
liubin0606@hust.edu.cn, wangbang@hust.edu.cn

ABSTRACT
How to effectively sample high-quality negative instances is important for well training a recommendation model. We argue that a high-quality negative should be both informativeness and unbiasedness. Although previous studies have proposed some approaches to address the informativeness in negative sampling, few has been done to discriminating false negative from true negative for unbiased negative sampling, not to mention taking both into consideration. This paper first adopts a parameter learning perspective to analyze negative informativeness and unbiasedness in loss gradient-based model training. We argue that both negative sampling and collaborative filtering include an implicit task of negative classification, from which we report an insightful yet beneficial finding about the order relation in predicted negatives’ scores. Based on our finding and by regarding negatives as random variables, we next derive the class condition density of true negatives and that of false negatives. We also design a Bayesian classifier for negative classification, from which we define a quantitative unbiasedness measure for negatives. Finally, we propose to use a harmonic mean of informativeness and unbiasedness to sample high-quality negatives. Experimental studies validate the superiority of our negative sampling algorithm over the peers in terms of better sampling quality and better recommendation performance.

KEYWORDS
negative sampling, pairwise learning, implicit CF

1 INTRODUCTION
Negative sampling originates from the Positive-Unlabel (PU) problem [11, 23]: A training dataset, called PU-dataset, contains both positively labelled and unlabeled instances, yet an unlabeled instance could belong to either the positive or negative class. Negative sampling is to determine a policy for guiding how to sample an unlabeled instance from a PU-dataset, so as to effectively train downstream task models. Negative sampling can find many applications in diverse tasks, such as natural language processing [16, 25], computer vision [19, 40], as well as recommendation systems [7, 21, 38].

We focus on negative sampling for recommendation. Many recommendation tasks can be formulated as how to rank unlabeled items for users, yet an unlabeled item is specific to one user, called his negative instance, but may be a positive instance to another users. Most recommendation algorithms adopt a pairwise learning framework to train a recommendation model with learnable users’ and items’ representations for the ranking computation [22, 28, 32, 33]. A pairwise comparison is to first form a training triple \((u, i, j)\) consisting of his positive instance \(i\) and negative instance \(j\) for a user \(u\), and the pairwise loss over all users is optimized for training a recommendation model.

The dilemma of such pairwise learning for recommendation lies in that a training triple consists of a negative instance of a user, yet such a negative instance could be potentially interested by the same user and should be recommended by the trained model. This motivates the problem of negative sampling for recommendation, that is, how to effectively sample negative instances for training a recommendation model. Many studies have shown that negative sampling is important to improving recommendation performance [5, 7, 10, 18, 21, 34, 38].

Recently, some negative sampling algorithms have been proposed for recommendation. We group them into the following categories. Static negative sampling [3, 16, 17, 22, 28, 32, 36]. Algorithms of this kind are to sample a negative instance according to a fixed sampling distribution, such as uniform sampling. Hard negative sampling [21, 30, 38]. Algorithms of this kind favor those negative instances with representations more similar to that of positive instances in the embedding space, for example, selecting higher scored or higher ranked instances [21, 38, 42]. A recent algorithm SRNS [7] of this kind advocates sampling a negative with a large variance of its predicted scores in the training precess.

For a user \(i\), his negative instances can be classified into two kinds. True Negative (TN): an unlabeled item is truly not liked by the user; and False Negative (FN): an unlabeled item is potentially liked by the user but not interacted before. It is desirable for a negative sampling algorithm to mainly sample his true negatives, without or less sampling his false negatives. Those static negative sampling algorithms ignore the issue of false negatives for their using a fixed sampling distribution. Those hard negative algorithms are more likely to suffer from the negative problem, as reported in some recent studies [7, 19, 40].

Negative sampling for recommendation needs to answer the following questions:

(a) Can we distinguish true negatives from false negatives?
(b) How to measure the quality of a negative instance?
(c) How to effectively sample high-quality negatives?

Note that the challenges to answer these questions mainly come from the fact that except the PU-dataset, no other prior information are available for describing items. Nonetheless, this paper provides answers to the three questions and a new solution of negative sampling for recommendation.

For question (a), we provide an affirmative answer from a Bayesian classifier viewpoint. We argue that both negative sampling and collaborative filtering include an implicit negative classification task. We also observe an insightful finding about the order relation in
predicted negatives’ scores. That is, the distribution of false negatives’ predicted scores is concentrated at a high score compared with that of true negatives’ predicted scores.

For question (b), we conduct systematic analysis on the pairwise loss function, and define informativeness and unbiasedness based on the loss gradient. The informativeness is defined as the gradient magnitude, while the unbiasedness as the gradient direction. As the answer to question (b), we argue that the quality of a negative instance should be measured from both of its informativeness and unbiasedness.

For question (c), we derive the class condition density of true negatives’ predicted scores and that of false negatives’ predicted scores, from which we quantitatively measure the unbiasedness for a negative instance. Finally, we use a harmonic mean of informativeness and unbiasedness to sample high-quality negatives. Experiment studies validate our analysis and solution in terms of sampling quality and recommendation performance.

2 NEGATIVE SAMPLING ANALYSIS

In this section, we use a general formulation of the personalized recommendation task to analyze the properties of negative sampling for training a recommendation model. We consider the following personalized recommendation problem, which has been intensively studied in the field [9, 21, 22, 32]. Let $M$ denote a recommendation model. Its input is an user-item interaction matrix $X = [x_{ui}] \in \mathbb{R}^{M \times N}$, consisting of $M$ users and $N$ items. An element $x_{ui} = 1$ indicates a user $u$ has interacted with an item $i$; Otherwise, $x_{ui} = 0$. The output is for each user his recommendation list, consisting of his un-interacted items ranked according to their predicted scores.

To train the recommendation model $M$, the widely used optimization objective is the following pairwise loss:

$$L_{loss} = \max \sum_{(u,i,j)} \ln \sigma(\hat{x}_{ui} - \hat{x}_{uj}).$$

where for a user $u$, $\hat{x}_{ui}$ and $\hat{x}_{uj}$ is the predicted score for his already interacted item $i$ and un-interacted item $j$, respectively. $\Theta$ contains model trainable parameters, and $\lambda$ is a hyper-parameter in the learning process. In order to compute $\hat{x}_{ui}$, some representation learning techniques such as MF [12] and LightGCN [32] can be used to learn a user representation $w_u$ and an item representation $h_i$, such that $\hat{x}_{ui} = \text{sim}(w_u, h_i)$ with a similarity function $\text{sim}$, e.g., a cosine or dot-product function.

When training $M$, negative sampling is used to construct training triples. For a user $u$, let $I^+_u$ and $I^-_u$ denote the set of his already interacted items, called positive instances and the set of his un-interacted items, called negative instances. A training triple $(u, i, j)$ is constructed as follows: For a user $u$ and one of his positive instance $i \in I^+_u$, sample one of his negative instances $j \in I^-_u$, viz., negative sampling. On the one hand, although the instance $j$ is sampled from $I^-_u$, it could be the case that the user $u$ actually likes it, but the un-interaction is simply due to that he had not seen it before, that is, the item $j$ is a false negative with respect to the user $u$. On the other hand, the item $j$ is called a true negative, if the user $u$ truly dislikes it.

The stochastic gradient descent (SGD) is often used to iteratively optimize the loss function for each training triple $(u, i, j)$. For a sampled instance $j \in I^-_u$, if it is a true negative to $u$, the loss gradient with respect to its estimated score $\hat{x}_{uj}$ is computed by

$$\frac{\partial L_{loss}}{\partial \hat{x}_{uj}} = [1 - \sigma(\hat{x}_{ui} - \hat{x}_{uj})](−1), \quad (2)$$

where $\sigma(\cdot)$ is the sigmoid function. However, if the sampled instance $j$ is actually a false negative, we do not expect that such an incorrect sampling impacts much on the model training, especially causing an opposite gradient direction. As we have no prior knowledge about the sampled instance $j$, we argue to replace the last term $−1$ in Eq. (2) by a $\text{sgn}(\cdot)$ function, that is,

$$\frac{\partial L_{loss}}{\partial \hat{x}_{uj}} = [1 - \sigma(\hat{x}_{ui} - \hat{x}_{uj})] \cdot \text{sgn}(j), \quad (3)$$

where $\text{sgn}(j) = −1$, if $j$ is a true negative to $u$, and $\text{sgn}(j) = 1$ is a false negative to $u$.

The loss gradient Eq. (3) can be decomposed into two parts, i.e., gradient magnitude and gradient direction. This motivates our negative sampling analysis on what is a high quality negative: A sampled instance $j$ in a training triple $(u, i, j)$ is called a high-quality negative, if it is both informative and unbiased.

- **Informativeness**: The informativeness of a negative $j$ in a high-quality negative $\text{info}(j)$ is defined as the loss gradient magnitude, i.e.,

  $$\text{info}(j) = [1 - \sigma(\hat{x}_{ui} - \hat{x}_{uj})]. \quad (4)$$

- **Unbiasedness**: The unbiasedness of a negative $j$ is defined as the probability that it is a true negative to user $u$, i.e.,

  $$\text{unbias}(j) = P(\text{sgn}(j) = −1). \quad (5)$$

The informativeness is directly defined as how much the negative $j$ can help updating the parameters of a recommendation model, in terms of its predicted score $\hat{x}_{uj}$. Given the predicted score $\hat{x}_{ui}$ of a positive instance, an excessively small value of $\hat{x}_{uj}$ leads to $\sigma(\hat{x}_{ui} - \hat{x}_{uj}) \rightarrow 1$ and $\text{info}(j) \rightarrow 0$, i.e., the gradient vanishes, and little can be learned from $j$.

The unbiasedness is actually defined as the probability of $j$ being true positive to user $u$. We notice that the so-called uniform negative sampling [22] directly set $\text{sgn}(j) = −1$ for all negatives, which introduces some sampling bias in model training for $j$ being actually a false negative. We can understand its adverse effects in two aspects. On the one hand, a recommendation model aims at maximizing the likelihood probability of pairwise comparisons between positive instances and negative instances by assigning higher scores to positives and lower scores to negatives. By assigning $\text{sgn}(j) = −1$, the false negative $j$’s score will be decreased when performing stochastic gradient descent, since its gradient is directed to the negative direction. On the other hand, it treats a false negative $j$ that a user $u$ may be potentially interacted as a true negative instance, causing incorrect preference learning for a recommendation model.

The multiplicative form in Eq. (3) indicates that a high-quality negative should have large gradient magnitude (informational) with negative direction (unbiased) for maximizing (1). Only being informational or unbiased cannot ensure a high-quality negative. The objective of negative sampling is to select high-quality negatives for training a recommendation model.
3 THE PROPOSED ALGORITHM

The informativeness has been well measured by Eq (4), and the key is how to measure the unbiasedness for unlabeled instances.

3.1 Order analysis on sampled instances

From un-interacted instances, negative sampling aims to select true negatives for model training; While the recommendation model aims to rank false negatives. Let us define negative classification as the task of classifying an un-interacted instance as either a true negative or a false negative. Like the two sides of a coin, both negative sampling and recommendation include an implicit task of negative classification, that is, how to effectively classify a sampled negative instance?

Although prior knowledge is not available for negative classification, we would like to expect that a functional recommendation model can rank false negatives higher than true negatives. This suggests that the following order relation of predicted scores might hold in general

\[ \hat{x}_{tn} \leq \hat{x}_{fn} \]

(6)

where \( \hat{x}_{tn} \) and \( \hat{x}_{fn} \) is the predicted score of a true negative and a false negative, respectively.

To verify the order relation, we use the ground-truth labels for instances in the test set to obtain the false negatives that are positively labeled but unobserved during the training process. And the rest of un-interacted items are true negatives. We adopt the classic matrix factorization recommendation model with random negative sampling to train the model on the training set, and record their predicted scores at each training epoch. By counting the predicted scores of true negatives and false negatives, we report their distribution densities in the model training by Fig.1.

Fig. 1 provides two insightful findings: (a) The higher the predicted score of a negative instance, the higher the probability density that it is a false negative and the lower the probability density that it is a true negative; (b) As the training continues, the distinction between two distributions gradually becomes clearer: Compared to that of true negatives, the distribution of false negatives is centered on a larger score. This suggests that the score function of a recommendation model is capable of rating the false negatives higher than true negatives.

3.2 Distribution Analysis

Since the scores of true negatives and false negative are predicted using the same score function in a recommendation model, it is safe to assume that \( \hat{x}_{fn} \) and \( \hat{x}_{tn} \) are identically and independently distributed with a same probability density function \( f(x) \) and a same cumulative distribution function \( F(x) = P(X \leq x) \).

Consider two IID random variables \( X_{tn} \) and \( X_{fn} \) with their corresponding realizations \( \hat{x}_{tn} \) and \( \hat{x}_{fn} \) that are sorted in an ascending

\[ dx \frac{\hat{x}_{fn} - \hat{x}_{tn}}{\hat{x}_{fn} + dx + \infty} \]

Figure 2: The probability differential of \( X_{tn} \)
order:  
\[ X_{tn} \leq X_{fn} \]  
(7)

For the two random variables \( X_{tn} \) and \( X_{fn} \), there exists a sufficient small interval \( dx \), where one and only one random variable \( X_{tn} \) with its realization \( \hat{x}_{tn} \) \( \in \) \([\hat{x}_{tn}, \hat{x}_{tn} + dx]\). Since \( \hat{x}_{tn} \leq \hat{x}_{fn} \), the random variables \( X_{fn} \) with its realization \( \hat{x}_{fn} \) \( \in \) \([\hat{x}_{tn} + dx, \infty)\). The probability differential (Fig. 2) of \( X_{tn} \) can be computed by:

\[
g(\hat{x}_{tn}) = \lim_{dx \to 0} \frac{2f(\hat{x}_{tn})dx[1 - F(\hat{x}_{tn} + dx)] + o(dx)}{dx} \tag{9}
\]

where \( g(\hat{x}_{tn}) \) is the class conditional density of true negatives. \( f(\hat{x}_{tn})dx \) evaluates the probability of \( X_{tn} \in [\hat{x}_{tn}, \hat{x}_{tn} + dx] \), and \([1 - F(\hat{x}_{tn} + dx)] \) evaluates the probability of the rest random variables \( X_{fn} \in (\hat{x}_{tn} + dx, \infty) \). \( o(dx) \) is the high-order infinitesimal of \( dx \).

Dividing both sides of the equation by \( dx \), the class conditional density of true negatives can be written as:

\[
g(\hat{x}_{tn}) = \lim_{dx \to 0} \frac{2f(\hat{x}_{tn})dx[1 - F(\hat{x}_{tn} + dx)] + o(dx)}{dx} \tag{9}
\]

\[
= 2f(\hat{x}_{tn})[1 - F(\hat{x}_{tn})] \tag{9}
\]

Likewise, the probability differential of \( X_{fn} \) can be computed by:

\[
h(\hat{x}_{fn}) = 2F(\hat{x}_{fn})f(\hat{x}_{fn})dx + o(dx), \tag{10}
\]

and the distribution of false negatives is given by:

\[
h(\hat{x}_{fn}) = 2F(\hat{x}_{fn})f(\hat{x}_{fn}). \tag{11}
\]

**Proposition 3.1.** If \( f(x) \) is a probability density function, \( F(x) = \int_{-\infty}^{x} f(t)dt \) is the corresponding cumulative distribution function, then

(a) \( g(x) = 2f(x)[1 - F(x)] \) is a probability density function that satisfies \( g(x) \geq 0 \) and \( \int_{-\infty}^{\infty} g(x)dx = 1 \).

(b) \( h(x) = 2f(x)F(x) \) is a probability density function that satisfies \( h(x) \geq 0 \) and \( \int_{-\infty}^{\infty} h(x)dx = 1 \).

**Proof.** Since \( f(x) \geq 0 \), \( F(x) = \int_{-\infty}^{x} f(t)dt \in [0, 1] \), so \( g(x) = 2f(x)[1 - F(x)] \geq 0 \), \( h(x) = 2f(x)F(x) \geq 0 \).

\[
\int_{-\infty}^{\infty} g(x)dx = \int_{-\infty}^{\infty} 2f(x)[1 - F(x)]dx
\]

\[
= 2 \int_{-\infty}^{\infty} f(x)dx - 2 \int_{-\infty}^{\infty} f(x)F(x)dx
\]

\[
= 2 - 2 \int_{-\infty}^{\infty} F(x)dF(x)
\]

\[
= 1.
\]

\[
\int_{-\infty}^{\infty} h(x)dx = \int_{-\infty}^{\infty} 2f(x)F(x)dx
\]

\[
= 2 \int_{-\infty}^{\infty} F(x)dF(x)
\]

\[
= 1.
\]

Fig 3 exhibits the distribution morphology of false negatives and true negatives derived from the ordinal relation with different types of \( f(x) \): Gaussian distribution \( x \sim N(\mu, \sigma) \) (symmetrical), student distribution \( x \sim t(n) \) (symmetrical), and Gamma distribution \( x \sim Ga(\alpha, \lambda) \) (asymmetrical). As we can see, during the training process, the actual distribution of true/false negatives in Fig 1 gradually exhibit the same structure as depicted in Fig 3.

For negative sampling, we do not need to concern about the specific expression of \( f(x) \); Instead, the separated structure of the distributions of true negative and false negative suffices for performing negative classification. We put the negative classification against the Bayesian framework.

### 3.3 Bayesian Negative Classification

For an un-interacted item \( j \) with its predicted score \( \hat{x} \), the posterior probability of the item \( j \) being a true negative can be computed using the Bayesian formula:

\[
P(tn|\hat{x}) \propto P(\hat{x}|tn)P(tn) = 2f(\hat{x})[1 - F(\hat{x})]P(tn), \tag{12}
\]

where \( P(\hat{x}|tn) \) is the class conditional density of true negatives given by \( g(\hat{x}) \), \( P(tn) = 1 - P(fn) \) is the prior probability of item \( j \) being true negative sample. \( f(\hat{x}) \) is the score distribution of un-interacted items, \( F(\hat{x}) = \int_{\hat{x}}^{\infty} f(t)dt \) is the corresponding cumulative distribution function.  

Also, the posterior probability of item \( j \) being a false negative can be computed by:

\[
P(fn|\hat{x}) \propto P(\hat{x}|fn)P(fn) = 2F(\hat{x})f(\hat{x})P(fn) \tag{13}
\]

Generally, the Bayesian classifier can be obtained by maximizing the posterior probability:

\[
\arg \max_{c \in \{fn, tn\}} P(c|\hat{x}). \tag{14}
\]

However, directly computing the score density function \( f(\cdot) \) in Eq. (12) and Eq. (13) is complicated. It depends on how the preference modeling is performed and its corresponding scoring function.
We note that using such an interaction ratio $s(\cdot)$. To overcome this problem, we define unbiasedness of an un-interacted item $j$ as:

$$\text{unbias}(j) = \frac{P(tn|x)}{P(tn|x) + P(fn|x) f(x)[1 - F(x)]P(tn)$$

$$\approx \frac{f(x)[1 - F(x)]P(tn) + F(x)f(x)P(fn)}{[1 - F(x)]P(tn) + F(x)P(fn)}$$

$$= \frac{[1 - F(x)]P(tn) + F(x)P(fn)}{[1 - F(x)]P(tn) + F(x)P(fn)}$$

$$= \frac{1 - F(x) - P(fn) + 2F(x)P(fn)}{1 - F(x) - P(fn) + 2F(x)P(fn)}.$$  

In Eq. (16), $F(x)$ can be calculated using the percentage of $x_{ul} \leq x$ given $u$’s un-interacted items’ predicted scores.

$$F(x) = \frac{\#(x_{ul} \leq x | l \in I_u^-)}{\#(I_u^-)},$$

and $P(fn)$ is the prior probability of $j$ being false negative. Notice that an item $j$, though having not been interacted by one user, could have been interacted by other users. Since a false negative is such an item $j$ that a user $u$ not interacted before but may be potentially interested, the probability of $j$ being interacted in future by $u$ can be approximated by its interaction ratio $r_j$ in the training dataset, that is, the number of users having interacted with $j$ divided by the number of all users.

$$P(fn) = r_j = \frac{\#(\text{interacted users})}{\#(\text{all users})}.$$  

We note that using such an interaction ratio $r_j$ is one of many possible choices for the prior distribution $P(fn)$. Some other additional information, such as item tags, categories and etc., can also be tried for describing the prior distribution.

The definition of unbiasedness can be interpreted as the normalized posterior probability of $j$ being true negative. We are more concerned with the relative value of the posterior probability unbiased($j$) than its absolute value, because the absolute value $P(tn|x)$ varies from different expressions of $f(x)$; While the relative value is uninfluenced by $f(x)$ when used for guiding negative sampling. The density function $F(x)$ is eliminated due to the fractional expression of Eq. (16). This suggests that the metric unbiased($j$) is model-independent and can be generalized to different recommendation models.

Eq. (16) implies that the unbiasedness of an un-interacted item $j$ is formally affected by two factors: (a) Preference information determined by the $j$’s own features. Also, the higher the prior probability of $j$ being preferred, the lower probability of $j$ being a true negative. (b) Ranking position determined by a recommendation model. A larger $F(x)$, i.e., a higher ranking position, indicates that to a large extent a recommendation model classifies $j$ as a false negative, and the probability of $j$ being true negative is low. We note that this helps to explain why those hard negative sampling algorithms [21, 38] for over-sampling higher scored or higher ranked items are more likely to suffer from the false negative problem.

Fig 4 plots unbiased($j$) as a function of $F(x) \in [0, 1]$ and $P(fn) \in [0, 1]$. We observe that unbiased($j$) is a decreasing function w.r.t both $F(x)$ and $P(fn)$. The monotonicity of unbiased($j$) is consistent with our analysis, and the value domain of unbiased($j$) $\in [0, 1]$ conforms to the probability form.

### 3.4 Negative Sampling Algorithm

In order to sample high-quality negatives, we use the harmonic mean of unbiased($j$) and info($j$) to take into account of both informativeness and unbiasedness, and select a negative instance $j$ by:

$$j = \arg \max_{l \in M_u} \frac{(1 + \beta^2) \times \text{unbias}(l) \times \text{info}(l)}{\text{unbias}(l) + \beta^2 \times \text{info}(l)}.$$  

where $M_u \in I_u^-$ is a small candidate set containing randomly selected $u$’s negative instances in $I_u^-$. $\beta$ is a weight that controls the contribution of info($\cdot$) when guiding negative sampling. $\beta \leq 1$ emphasizes more on $l$’s informativeness in negative sampling.

We next discuss the reason of using the harmonic mean. On the one hand, a negative instance $j$ with small unbiased($j$) might lead to the model learning from inaccurate negative preference. On the other hand, a negative instance $j$ with small info($j$) is of small contribution to model training. As such, only using one of two cannot necessarily guarantee that $j$ is a high-quality negative. By using harmonic mean, the proposed method is more likely to select high-quality negatives with large informativeness and unbiasedness.

**Complexity:** Algorithm 1 provides the pseudo-codes of our proposed negative sampling algorithm. The number of total items is $N$. Sample $m$ negative instances (line 6) needs $O(N)$, calculate info($j$) (line 8) needs $O(1)$, calculate $F(x)$ (line 9) needs $O(N)$, calculate $p(fn)$ (line 10) needs $O(1)$, calculate unbiased($j$) (line 11) needs $O(1)$, and select item $j$ is $O(1)$. So the time complexity for sampling one negative is $O(mN)$, where $m$ is a predefined small constant. We note that the time complexity of proposed algorithm for sampling one negative is the same as that of the uniform sampling.
Algorithm 1: The proposed negative sampling algorithm

Input: Training set $\mathcal{R} = \{(u, i), \text{ score function } s(\cdot), m \text{ (size of } \mathcal{M}_u), \text{ embedding size } d, \text{ weight } \beta \}.$

Output: User embedding $\{w_u | u \in U\} \in \mathbb{R}^d$, item embedding $\{h_i | i \in I\} \in \mathbb{R}^d$

for epoch = 1, 2, ..., $T$ do
   Sample a mini-batch $\mathcal{R}_{\text{batch}} \in \mathcal{R}$
   for each $(u, i) \in \mathcal{R}_{\text{batch}}$ do
      Get rating vector $\hat{x}_u$.
      \text{\\ \textbf{Starting Negative Sampling}}
      Uniformly sample candidate set $\mathcal{M}_u \in \mathcal{I}_u$.
      for each $(u, l) \in \mathcal{M}_u$ do
         Calculate info$(l)$ by Eq. (4).
         Calculate $F(x)$ by Eq. (17).
         Calculate $P(fn)$ by Eq. (18).
         Calculate unbiased$(l)$ by Eq. (16).
      end
      Sampling negative $j$ based on strategy Eq. (19).
      Update embeddings $w_u, h_i, h_j$ based on $(u, i, j)$.
   end
end

Result: Final embeddings.

Table 1: Dataset Statistics

|                | users | items | training set | test set |
|----------------|-------|-------|--------------|----------|
| MovieLens-100k| 943   | 1,682 | 80k          | 20k      |
| MovieLens-1M  | 6,040 | 3,952 | 800k         | 200k     |
| Yahoo!-R3     | 5,400 | 1,000 | 146k         | 36k      |

4 EXPERIMENT

We first examine the quality of sampled negatives, and next compare recommendation performance with competitors.

4.1 Experiment Settings

4.1.1 Dataset. We conduct experiments on three public datasets, including MovieLens-100k, MovieLens-1M, and Yahoo!-R3\(^3\) [33]. They contain users’ ratings on items according to a discrete five-point grading system on an interacted item. In the research of personalized ranking from implicit feedbacks [17, 21, 22, 33, 36, 38], a commonly used technique for data preprocessing is to convert all rated items to implicit feedbacks. Following their approaches, our training sets only consist of interacted items but not with their rating details. For each dataset, we randomly select 20% as test data, and the rest 80% as training data. Table 1 summarizes the dataset statistics.

4.1.2 Competitors. We compare the proposed algorithm with three types of negative sampling methods: (a) Fixed negative sampling distribution, including RNS and PNS, (b) hard negative sampling with dynamic sampling distribution, including OABPR and DNS, and hard negative sampling based on priori statistical information to oversample high-variance negatives, including SRNS. We note that all these competitors also only use the positive and unlabeled data but without additional information for negative sampling.

4.1.3 Experimental setup. We use the classic matrix factorization (MF) [12] and the recent light graph neural network (LightGCN) [32] as two recommendation models. The MF model optimizes a pairwise learning loss function to learn user representation $w_u$ and item representation $h_i$. The LightGCN treats X as a graph and learn user and item representation via a graph neural model. For a fair comparison, we set the identical parameters of the recommendation model for all comparing negative sampling algorithms.

(a) MF [9]: embedding dimension $d = 32$, learning rate $\alpha = 0.01$, regulation constant $\lambda = 0.01$ and training epoch $T = 100$, batch size $b = 1$.

(b) Light GCN [32]: embedding dimension $d = 32$, the initial learning rate $\alpha = 0.01$ and decays every 20 epochs with the decay rate=0.1, regulation constant $\text{reg} = 10^{-5}$, number of LightGCN layers $l = 1$, training epoch $T = 100$, batch size $b = 128$ for MovieLens-100K and Yahoo!-R3 datasets, $b = 1024$ for MovieLens-1M dataset.

4.1.4 Evaluation Metrics. To examine sampling quality, it is interesting know whether an algorithm is capable of sampling informational and unbiased negatives. By flipping labels of ground-truth records in the test set, we are able to obtain the false negatives (FN) that are positive labeled but unobserved during the negative sampling process. And the rest of un-interacted items are true negatives (TN). For each epoch, we record each sampled instance’s label and info$(j)$, then define the unbiasedness and informativeness epoch-wisely by:

$$TNR = \frac{\#TN}{\#TN + \#FN}, \quad (20)$$

$$INF = \frac{\sum_j \text{info}(j) \cdot \text{sgn}(j)}{\#TN + \#FN}, \quad (21)$$

where $\#TN(\#FN)$ is the number of sampled true (false) negatives in each training epoch. Eq. (20) evaluates the proportion of sampled true negatives in each training epoch, i.e., true negative rate (TNR). sgn$(j)$ is the indicator function: sgn$(j) = 1$ if the sampled item’s label is TN; Otherwise, sgn$(j) = -1$ as a penalty \(^4\) for sampling the FN instance. The informativeness (INF) defined Eq. (21) can

\(^3\)Source codes will be released after the anonymous review.

\(^4\)We implicitly assume that the gain of sampling a TN equals to the cost of sampling a FN. The penalty weights can be adjusted as needed.
be interpreted as the average gradient magnitude with respect to selected training triple \((u, i, j)\) in each training epoch.

To evaluate recommendation performance, the widely used metrics are adopted, including \(P\) (precision), \(R\) (recall), NDCG (normalized discounted cumulative gain), to evaluate the Top-K recommendation. For their common usage, we do not provide their definitions here.

4.2 Experiment Results

4.2.1 Negative Sampling Quality. We examine whether the proposed posterior probability criterion in terms of Eq. (16) is capable of selecting unbiased negatives, and whether the sampling strategy of Eq. (19) is capable of selecting high quality negatives that are both informational and unbiased. Fig. 5 presents the experiment results, from which we have the following observations:

(a) Unbiasedness: Fixed distribution sampling (RNS and PNS) achieves relatively moderate performance. Their TNRs fluctuate around the probability of a random sample being a true negative. Hard negative sampling (AOBPR and DNS) has the worst performance. They adopt a greedy strategy to emphasize higher ranked negatives, also bringing higher risk of sampling false negatives per our discussion in Section 3.1. The SRNS uses simple prior statistic information of variance of predicted scores, which limits the potentials of negative classification, because this prior variance may result in the sampling distribution to be overly concentrated. The proposed Posterior probability negative sampling (PPNS) achieves the best performance for its TNR closer to 1, owing to our Bayesian negative classification.

(b) Informativeness: The INF decreases with the increase of training epoches. This is because the trained recommendation model can rank the false negatives potentially interested by users higher than true negatives (cf. Fig. 1). Our PPNS achieves the best performance after enough training epoches. The hard negative sampling (AOBPR and DNS) suffer more penalties due to its highest sampling bias. The SRNS adopts a linearly weighted average to combine informativeness and variance, which may not guarantee sampling unbiased and informative instances.

4.2.2 Recommendation Performance. Table 2 compares the recommendation performance for the negative sampling algorithms, where the boldface and underline are used to indicate the best and the second best in each comparing group.

The proposed PPNS algorithm achieves the best performance in almost all cases (except two second best) of the two recommendation models, three testing datasets and three performance metrics. The results validate that our algorithm can sample high-quality negatives measured from both informativeness and unbiasedness.

Take a close look on the results. It is interested to find that among the two static negative sampling algorithms, the RNS generally outperforms the PNS. This suggests that the popularity-based sampling distribution favoring popular items may actually introduce more biases in negative sampling. Among the three hard negative sampling algorithms, viz., AOBPR, DNS and SRNS, it is noted that the DNS often outperforms the other two. The AOBPR prioritizes those global higher ranked items, while the DNS first randomly selects a few negatives, among which favors those local relatively higher ranked items. Since the DNS balances between informativeness and unbiasedness to some extent, it can achieve the second best in many cases. The SRNS exploits the empirical observation that a negative with high-variance of its predicted scores could be a true negative. Although it is an interesting approach to deal with unbiasedness, the linear average operation of SRNS may weaken the effectiveness of its negative sampling.

Finally, we note that for fair comparison, we perform the proposed PPNS algorithms from the beginning epoch of model training. Indeed, as discovered from Fig. 1, we can first use some other sampling algorithm, say for example the RNS, to train a recommendation model for some epoches, and then resume the training by replacing it with our PPNS. This kind of warm-start training...
Table 2: Comparison of recommendation performance on the three datasets.

| Dataset | CF Model | Method | Top-5 Precision | Top-5 Recall | Top-5 NDCG | Top-10 Precision | Top-10 Recall | Top-10 NDCG | Top-20 Precision | Top-20 Recall | Top-20 NDCG |
|---------|----------|--------|-----------------|-------------|------------|-----------------|---------------|-------------|-----------------|---------------|-------------|
| 100K    | MF       | RNS    | 0.3971          | 0.1353      | 0.4236     | 0.3401          | 0.2207        | 0.4044      | 0.2778          | 0.3386        | 0.4071      |
|         |          | PNS    | 0.2647          | 0.0864      | 0.2694     | 0.2329          | 0.1475        | 0.2637      | 0.1949          | 0.2374        | 0.2709      |
|         |          | AOBPR  | 0.2521          | 0.0843      | 0.1084     | 0.2059          | 0.1352        | 0.2496      | 0.1671          | 0.2097        | 0.2495      |
|         |          | DNS    | 0.4053          | 0.1414      | 0.4314     | 0.3348          | 0.2214        | 0.4042      | 0.2734          | 0.3413        | 0.4069      |
|         |          | SRNS   | 0.3951          | 0.1342      | 0.4176     | 0.3394          | 0.2174        | 0.3998      | 0.2747          | 0.3374        | 0.4013      |
|         |          | Proposed | 0.4197          | 0.1431      | 0.4492     | 0.3503          | 0.2278        | 0.4218      | 0.2806          | 0.3458        | 0.4198      |
| LightGCN| MF       | RNS    | 0.4261          | 0.1453      | 0.4544     | 0.3571          | 0.2319        | 0.4275      | 0.2867          | 0.3490        | 0.4248      |
|         |          | PNS    | 0.3527          | 0.0735      | 0.3634     | 0.3004          | 0.1250        | 0.3356      | 0.2502          | 0.3306        | 0.3742      |
|         |          | AOBPR  | 0.3946          | 0.0954      | 0.4135     | 0.3416          | 0.1549        | 0.3837      | 0.2857          | 0.3422        | 0.4064      |
|         |          | DNS    | 0.4066          | 0.0991      | 0.4272     | 0.3521          | 0.2620        | 0.3965      | 0.2945          | 0.3537        | 0.4335      |
|         |          | SRNS   | 0.3955          | 0.0934      | 0.4225     | 0.3408          | 0.1609        | 0.3953      | 0.2779          | 0.3520        | 0.4244      |
|         |          | Proposed | 0.4318          | 0.1518      | 0.4640     | 0.3671          | 0.2140        | 0.4368      | 0.3057          | 0.3608        | 0.4383      |
| 1M      | MF       | RNS    | 0.4095          | 0.0953      | 0.4305     | 0.3512          | 0.1547        | 0.3985      | 0.2915          | 0.2405        | 0.3781      |
|         |          | PNS    | 0.3658          | 0.0907      | 0.3855     | 0.3152          | 0.1486        | 0.3564      | 0.2608          | 0.2314        | 0.3440      |
|         |          | AOBPR  | 0.4073          | 0.0997      | 0.4286     | 0.3535          | 0.1620        | 0.3965      | 0.2945          | 0.2537        | 0.3838      |
|         |          | DNS    | 0.4130          | 0.0972      | 0.4242     | 0.3552          | 0.1577        | 0.4002      | 0.2958          | 0.2468        | 0.3840      |
|         |          | SRNS   | 0.4026          | 0.0973      | 0.4239     | 0.3515          | 0.1526        | 0.3953      | 0.2922          | 0.2524        | 0.3815      |
|         |          | Proposed | 0.4228          | 0.1087      | 0.4438     | 0.3639          | 0.1612        | 0.4088      | 0.3025          | 0.2527        | 0.3917      |
| LightGCN| MF       | RNS    | 0.4195          | 0.1440      | 0.4509     | 0.3564          | 0.2333        | 0.4275      | 0.2834          | 0.3520        | 0.4244      |
|         |          | PNS    | 0.4095          | 0.0953      | 0.4305     | 0.3512          | 0.1547        | 0.3985      | 0.2915          | 0.2405        | 0.3781      |
|         |          | AOBPR  | 0.3946          | 0.0954      | 0.4135     | 0.3416          | 0.1549        | 0.3837      | 0.2857          | 0.2422        | 0.3714      |
|         |          | DNS    | 0.4066          | 0.0991      | 0.4272     | 0.3521          | 0.1620        | 0.3965      | 0.2945          | 0.2537        | 0.3838      |
|         |          | SRNS   | 0.3955          | 0.0934      | 0.4225     | 0.3408          | 0.1609        | 0.3953      | 0.2779          | 0.2431        | 0.3974      |
|         |          | Proposed | 0.4207          | 0.1062      | 0.4324     | 0.3518          | 0.1703        | 0.4191      | 0.3045          | 0.2614        | 0.4002      |
| Yahoo  | Uniform |       |                 |             |            |                 |               |             |                 |               |             |
|         | NNCF    |       |                 |             |            |                 |               |             |                 |               |             |
|         | AOBPR   |       |                 |             |            |                 |               |             |                 |               |             |
|         | DNS     |       |                 |             |            |                 |               |             |                 |               |             |
|         | SRNS    |       |                 |             |            |                 |               |             |                 |               |             |
|         | Proposed |       |                 |             |            |                 |               |             |                 |               |             |

5 RELATED WORK

Pairwise learning and pairwise loss have been widely applied in recommendation systems [17, 21, 22, 36]. Pairwise comparisons of positive instances and negative instances are first constructed to train a recommendation model. How to select negatives for pairwise comparisons, i.e., negative sampling, is a key to model training [7, 21, 38]. We review the related work of negative sampling for recommendation from two categories according to whether the sampling policy is fixed during the model training process.

The first category is the static negative sampling. This kind of methods adopt a fixed sampling distribution for negative sampling during the whole training process. The most widely used is the random negative sampling (RNS) [22, 32, 33, 36], which uniformly samples negatives from un-labeled instances. Some have proposed to set sampling probability of a negative instance according to its popularity (interaction frequency), as so-called popularity-biased negative sampling (PNS) [1, 3, 8, 16, 25]. Among them, the most widely used sampling distribution is $p(j) \propto r_j^{0.75}$, where $r_j$ is the interaction frequency of an item in the training dataset.

The second category is the hard negative sampling that adopts an adaptive sampling distribution targeting on sampling hard negative instances. The so-called hard negatives refer to those unlabeled instances that are similar to those positive instances in the embedding space [20, 24, 39]. Many hard negative sampling strategies have been proposed for personalized recommendation [5, 7, 10, 18, 21, 38].
For example, Zhang et al [38] and Steffen et al [21] propose to oversample higher scored thus higher ranked negatives that are argued to be more similar to positive instances. Some have proposed to exploit graph-based information for boosting negative sampling [2, 13, 26, 31, 35]. For example, Wang et al [30] and Wang et al [31] propose to leverage the types of relations on a knowledge graph to filter hard negatives. Another approach is to use the random walk on a graph for selecting hard negatives that are structurally similar to positive instances [2, 13, 26, 35].

Additional or prior information can be exploited for identifying and sampling hard negatives. Such side information is intuitive for distilling users’ negative preferences, such as users’ connections in social networks [29, 41], geographical locations of users [15, 37], and additional interaction data such as viewed but non-clicked [4, 6]. Beside only sampling negatives from unlabeled instances, a novel class of methods is to generate a kind of virtual hard negatives from multiple negative instances or by using some generative method. For example, Huang et al. [10] propose to synthesize virtual hard negatives by hop mixing embeddings. Jun et al. [27] and Park et al. [18] design generative adversarial neural networks for generating virtual hard negatives.

6 CONCLUSION

This paper has provided a comprehensive analysis on negative sampling for recommendation. We have reported an insightful yet beneficial finding about the order relation of predicted negatives’ scores, and class derived conditional density for true negatives’ predicted scores and that of false negatives’ predicted scores. In addition, this paper has defined the posterior probability estimate as a quantitative unbiasedness measure, which together with the informativeness measure is used in our negative sampling algorithm to select high-quality negatives for training a recommendation model. Experiment studies have validated our arguments and findings. We note that if some additional information are available to assist in evaluating prior distribution 𝑃(𝑛) and/or 𝑃(𝑓𝑛), negative sampling indeed faces an exploration-and-exploitation trade-off: Exploration suggests to prioritize those higher ranked informational instances that have been classified as false negative by a ranking model; Exploitation indicates to favor those lower ranked unbiased instances that have been classified as true negative by the same ranking model. That is, the ranking position of a negative instance can be further utilized to improve negative sampling quality. With the help of posteriori estimation of unbiasedness, future work can go further to investigate the optimal trade-off between such exploration and exploitation.

REFERENCES

[1] Hugo Caselles-Dupré, Florian Lesaint, and Jimena Royo-Letelier. 2018. Word2vec applied to recommendation: Hyperparameters matter. In Proceedings of the 12th ACM Conference on Recommender Systems. 352–356.
[2] Jiawei Chen, Can Wang, Sheng Zhou, Qihao Shi, Yan Feng, and Chun Chen. 2019. SamWalker: Social recommendation with informative sampling strategy. In WWW. 228–239.
[3] Ting Chen, Yizhou Sun, Yue Shi, and Liangjie Hong. 2017. On Sampling Strategies for Neural Network-Based Collaborative Filtering. In Proceedings of the 23rd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining. 767–776.
[4] Jingtao Ding, Fuli Feng, Xiangnan He, Guanghui Yu, Yong Li, and Depeng Jin. 2018. An Improved Sampler for Bayesian Personalized Ranking by Leveraging View Data. In Companion Proceedings of the The Web Conference 2018. 13–14.
[5] Jingtao Ding, Yuhuan Quan, Xiangnan He, Yong Li, and Depeng Jin. 2019. Reinforced Negative Sampling for Recommendation with Exposure Data. In IJCAI. 2230–2236.
[6] Jingtao Ding, Yuhuan Quan, Xiangnan He, Yong Li, and Depeng Jin. 2019. Reinforced Negative Sampling for Recommendation with Exposure Data. In Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI-19. International Joint Conferences on Artificial Intelligence Organization, 2230–2236.
[7] Jingtao Ding, Yuhuan Quan, Quanming Yao, Yong Li, and Depeng. 2020. Simplify and Robustify Negative Sampling for Implicit Collaborative Filtering. In NeurIPS.
[8] Mihailo Grbovic, Nemania Djuric, Vladan Radonjavliev, Fabrizio Silvestri, and Narayan Bhamidipati. 2015. Context- and Content-aware Embeddings for Query Rewriting in Sponsored Search. In Proceedings of the 35th International ACM SIGIR Conference on Research and Development in Information Retrieval, Santiago, Chile, August 9-15, 2015. 383–392.
[9] Xiangnan He, Hanwang Zhang, Min-Yen Kan, and Tat-Seng Chua. 2016. 549–558. Fast matrix factorization for online recommendation with implicit feedback. In Proceedings of the 39th International ACM SIGIR Conference on Research and Development in Information Retrieval. 2016.
[10] Tinglin Huang, Xuyiao Dong, Ming Ding, Zhen Yang, Wenzheng Feng, Xinyu Wang, and Jie Tang. 2021. MaxGCF: An Improved Training Method for Graph Neural Network-Based Recommender Systems. In Proceedings of the 27th ACM SIGKDD Conference on Knowledge Discovery & Data Mining. 665–674.
[11] Bekker Jessa and Davist Jessi. 2020. Learning from positive and unlabeled data: a survey. Machine Learning 109 (2020), 719–760.
[12] Yehuda Koren, Robert Bell, and Chris Volinsky. 2009. Matrix factorization techniques for recommender systems. Computer 42, 8 (2009), 30–37.
[13] Jing Li, Feng Xia, Wei Wang, Zhen Chen, Nana Yaw Asahere, and Huizhen Jiang, 1209–1214. Acre: a co-authorship based random walk model for academic collaboration recommendation. In Proceedings of the 23rd international conference on World Wide Web. 2014.
[14] Shenghao Liu, Bang Wang, and Minghua Xu. 2017. Event Recommendation Based on Graph Random Walking and History Preference Reranking. In Proceedings of the 40th International ACM SIGIR Conference on Research and Development in Information Retrieval. 861–864.
[15] Wei Liu, Zhi-Jie Wang, Bin Yao, and Jian Yin. 2019. Geo-ALM: POI Recommendation by Fusing Geographical Information and Adversarial Learning Mechanism. In IJCAI. 1807–1813.
[16] Tomas Mikolov, Bya Sutskever, Kai Chen, Greg Corrado, and Jeffrey Dean. 2013. Distributed Representations of Words and Phrases and Their Compositionality. In Proceedings of the 26th International Conference on Neural Information Processing Systems. 3111–3119.
[17] Weike Pan and Li Chen. 2013. Gbpr: Group preference based bayesian personalized ranking for one-class collaborative filterin. In Twenty-Third International Joint Conference on Artificial Intelligence.
[18] Dae Hoon Park and Yi Chang. 2019. Adversarial Sampling and Training for Semi-Supervised Information Retrieval. In WWW. 1443–1453.
[19] Xiao Qin, Nasrullah Sheikh, Berthold Reinwald, and Lingfei Wu. 2021. Relation-aware Graph Attention Model With Adaptive Self-adversarial Training. In Proceedings of the AAAI Conference on Artificial Intelligence. 9368–9376.
[20] Jinhong Rao, Hua He, and Jimmy Lin. 2013–1916. Noise-contrastive estimation for answer selection with dense neural networks. In Proceedings of the 25th ACM International Conference on Information and Knowledge Management. 2016.
[21] Steffen Rendle and Christoph Freudenthaler. 2014. Improving pairwise learning for item recommendation from implicit feedback. In Proceedings of the 7th ACM international conference on Web Search and Data Mining. 273–282.
[22] Steffen Rendle, Christoph Freudenthaler, Zeno Gantner, and Lars Schmidt-Thieme. 2009. BPR: Bayesian Personalized Ranking from Implicit Feedback. In UAI 2009. Proceedings of the Twenty-Fifth Conference on Uncertainty in Artificial Intelligence, Montreal, QC, Canada, June 18-21, 2009. 452–461.
[23] Guangxin Su, Weitong Chen, and Miao Xu. 2021. Positive-unlabeled learning from imbalanced data. In Proceedings of the 83th International Joint Conference on Artificial Intelligence, Virtual Event. 2995–3001.
[24] Zequn Sun, Wei Hu, Qingsheng Zhang, and Yuzhong Qu. 2018. Bootstrapstrapping Entity Alignment with Knowledge Graph Embedding. In IJCAI. 4396–4402.
[25] Jian Tang, Meng Qu, Minghui Wang, Ming Zhang, Jun Yan, and Qiaozhu Mei. 2008. Line: Large-Scale Information Network Embedding. In IJCAI. 4396–4402.
[26] Can Wang, Jiawei Chen, Sheng Zhou, Qihao Shi, Yan Feng, and Chun Chen. 2021. SamWalker++: recommendation with informative sampling strategy. IEEE Transactions on Knowledge and Data Engineering (2021).
[27] Jun Wang, Lantao Yu, Weinan Zhang, Yu Gong, Yinghui Xu, Benyou Wang, Peng Zhang, and Delli Zhang. 2017. IRGAN: A Minimax Game for Unifying Generative and Discriminative Information Retrieval Models. In SIGIR. 515–524.
[28] X. Wang, X. He, M. Wang, F. Feng, and T. S. Chua. 2014. Proceedings of the 42nd International ACM SIGIR Conference. Neural Graph Collaborative Filtering. In SIGIR. 2019. 2344–2353.
[29] Xin Wang, Wei Lu, Martin Ester, Can Wang, and Chun Chen. 2016. Social recommendation with strong and weak ties. In Proceedings of the 25th ACM International on Conference on Information and Knowledge Management. 5–14.

[30] Xiang Wang, Yaokun Xu, Xiangnan He, Yixin Cao, Meng Wang, and Chua. 2020. Reinforced negative sampling over knowledge graph for recommendation. In WWW. 99–109.

[31] Yu Wang, Zhiwei Liu, Ziwei Fan, Lichao Sun, and Philip S Yu. 2021. DokReg: Differentiable sampling on knowledge graph for recommendation with relational gnn. In Proceedings of the 30th ACM International Conference on Information & Knowledge Management. 3513–3517.

[32] He Xiangnan, Deng Kuan, Wang Xiang, Li Yan, Zhang Yongdong, and Wang Meng. 2020. LightGCN: Simplifying and Powering Graph Convolution Network for Recommendation. In SIGIR. 10.

[33] X. Yang and B. Wang. 2020. Local ranking and global fusion for personalized recommendation. Applied Soft Computing 96, 1 (2020), 166636.

[34] Zhen Yang, Ming Ding, Chang Zhou, Hongxia Yang, Jingren Zhou, and Jie Tang. 2020. Understanding negative sampling in graph representation learning. In Proceedings of the 26th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining. 1666–1676.

[35] R. Ying, R. He, K. Chen, P. Ekombatchai, William L Hamilton, and J. Leskovec. 2018. Graph Convolutional Neural Networks for Web-Scale Recommender Systems. In KDD. 974–983.

[36] R. Yu, Y. Zhang, Y. Ye, L. Wu, C. Wang, Q. Liu, and E. Chen. 2018. Multiple pairwise ranking with implicit feedback. In Proceedings of the 27th ACM Int. Conference on Information and Knowledge Management. ACM. 1727–1730.

[37] Fajie Yuan, Joemon M Jose, Guibing Guo, Long Chen, Haitao Yu, and Rami S Alkhawaldeh. 2016. Joint geo-spatial preference and pairwise ranking for point-of-interest recommendation. In 2016 IEEE 28th International Conference on Tools with Artificial Intelligence (ICTAI). 46–53.

[38] Weinan Zhang, Tianqi Chen, Jun Wang, and Yong Yu. 2013. Optimizing Top-n Collaborative Filtering via Dynamic Negative Item Sampling. In Proceedings of the 36th International ACM SIGIR Conference on Research and Development in Information Retrieval. 785–788.

[39] Yongqi Zhang, Quanming Yao, Yingxia Shao, and Lei Chen. 2019. NSCaching: simple and efficient negative sampling for knowledge graph embedding. In 2019 IEEE 35th International Conference on Data Engineering (ICDE). 614–625.

[40] Han Zhao, Xu Yang, Zhenru Wang, Erkun Yang, and Cheng Deng. 2021. Graph debiased contrastive learning with joint representation clustering. In IJCAI. 3434–3440.

[41] Tong Zhao, Julian McAuley, and Irwin King. 2014. Leveraging social connections to improve personalized ranking for collaborative filtering. In Proceedings of the 23rd ACM international conference on conference on information and knowledge management. 261–270.

[42] Tong Zhao, Julian McAuley, and Irwin King. 2015. Improving Latent Factor Models via Personalized Feature Projection for One Class Recommendation. In Proceedings of the 24th ACM International on Conference on Information and Knowledge Management. 821–830.