A novel Technique for An Integrated Optical Wavelength Demultiplexer

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Abstract— In this paper we propose a new technique for optical wavelength demultiplexing (DEMUX) relaying on two phenomena: Goos–Hänchen (GH) shift and continuous refraction at a graded-index medium interface. In the first case, two light beams are totally reflected at a plane interface separating two dielectric lossless media. The reflected beams suffer different lateral shifts (GH shifts) which depend on the wavelength; thus accomplishing the required spatial beam separation. In the second case, the two light beams have different “turning points” inside the graded index medium; hence the “back-refracted” beams are spatially separated. In this paper, we optimized the conditions of operation of such demultiplexing technique. This makes possible the integration of such technique in “planer integrated–optics” structures which can be used reliably in optical fiber communication networks.

Keywords— Wavelength Demultiplexing, Beam Propagation Method, Graded-Index Interface.

I. INTRODUCTION

Wavelength division multiplexing (WDM) is the optical equivalent of Frequency Division Multiplexing (FDM) to electrical signaling. WDM transmits two or more different light wavelengths in the same optical fiber. Though the process of multiplexing is reasonably straight forward, demultiplexing is much more difficult in an optical system. These are many methods [1-4] to demultiplex optical signals: prisms, diffraction grating, arrayed waveguide grating (AWG), thin-film filters and interferometers. These methods vary in their relative complexities, reliability and performance. This is why optical networks components manufactures try to find out simpler and more reliable techniques for demultiplexing, especially dedicated to integrated–optics architecture. Because that architecture can be integrated very easily in fiber optics networks (the dream of all optical networking engineers). Consequently, we propose in this paper a novel technique which can be realized by two different methods as shown in Fig. 1.
In the first method: the GH shift depends on the wavelength and hence when two light beams with different wavelengths are incident at the critical angle on a dielectric interface separating two lossless dielectric media with refractive indices \( n_1 \) and \( n_2 \) (\( n_1 > n_2 \)), the reflected and the transmitted beams also suffer from two different lateral shifts. Thus the separation of these wavelengths is achieved, and we are interested to optimize the separation angle \( \Delta \theta \) or \( \Delta \theta_T \) of the transmitted beam by varying the angle of incidence around the critical value (\( \sin^{-1}(n_2/n_1) \)).

In the second method: the rarer dielectric medium \( n_2 \) is made graded-index (i.e. \( n_2(x) \)). Accordingly, the two light beams incident from the medium \( n_1 \), vary their penetration depth and hence reach two different “turning points” in the graded-index medium \( n_2(x) \). We tried different graded “profiles” to maximize the separation angle \( \Delta \theta \).

**II. THE BEAM PROPAGATION METHOD BPM:**

To assess and evaluate the performance of the proposed novel demultiplexer we have to solve a major problem: how to study the propagation of almost “realistic” light beams? That is to say: optical beams with finite “spatial extension” like gaussian beams.

Obviously, the interest in gaussian beams relies on many facts; because the laser beams radiated from laser sources are well approximated by gaussian beams. Also the fundamental mode in a single – mode fiber is well approximated by a gaussian field distribution. One of the most powerful methods used to study the propagation of light beams in complex media is the BPM [5-8].

The literature on the BPM is so extensive [8-10], and hence, we shall not expose the details of that method; but we shall give a brief exposition of that method.

Referring to fig. 2, the unity amplitude \( y \)-polarized Gaussian beam at \( z = 0 \) in the interface coordinate system \((x, z)\) can be written as [11]:

\[
E_y(x,0) = \exp\left[-(X - X_d)\cos\theta_0/w^2\right] \cdot \exp\left[jk \left(x - x_d\right)\sin\theta_0\right]
\] (1)

Where, the time dependence \( e^{-j\omega t} \) is suppressed. The BPM, first introduced by Fleck et.al. [12], relies on the expansion of \( E_y(x, 0) \) as a continuous spectrum of plane waves (i.e. a “spatial” Fourier transform). Each component of the spectrum is made to propagate a small distance \( \Delta z \) in a “homogeneous” (reference) medium having a refractive index \( n_0 \). The reference medium is usually chosen arbitrarily in the range \( n_2 \leq n_0 \leq n_1 \).

The propagation process is accomplished in the Fourier-domain by a single multiplication of the spatial spectrum with a phase function (propagator) which will be explained later on. After the propagation over \( \Delta z \) is performed, we Fourier-invert the “propagated spectrum” to recover the “field” after a small distance \( \Delta z \). Finally, to take into account for the deviation \( \delta n(x) \) of the actual refractive index distribution \( n(x) \) from the value \( n_0 \), we correct the “phase” of the “propagated” field (after a small distance \( \Delta z \)) through a simple multiplication by \( \exp\left[jk_0 \delta n (x) \cdot \Delta z \right] \), to get finally the field after a propagation distance \( \Delta z \).

This procedure is summarized as follows:

1- Calculate \( f \left[E_y(x, 0)\right] \), where “\( f \)” stands for the “spatial Fourier transform operation”.

2- Multiply \( f \left[E_y(x, 0)\right] \) by a propagator operator \( P \).
3- Calculate $f^{1+}$ of the propagated spectrum obtained in step 2.

4- Apply “Q” on the field obtained in step 3, where Q represents an operator which takes into account for the deviation of the actual refractive distribution $n(x)$ from the “reference value $n_0$”.

Hence, the procedure described by the previous four steps allows the calculation of $E_y(x,\Delta z)$ once $E_y(x,0)$ is known. This means that this procedure can be repeated to calculate the total field at certain distance $Z$ once the initial field at $z = 0$ is known. The details of the procedure are outlined in the appendix.

![Diagram](image)

Fig. 2 Gaussian beam incident at an angle $(\vec{z} - \theta_0)$ where $\theta_0$ is the beam tilt angle.

### III. RESULTS FOR PLANE INTERFACE:

The situation depicted in Fig. 1(a) is considered. Two Gaussian beams at wavelength 1.55 $\mu$m and 1.33 $\mu$m are incident at the angle $(\vec{z} - \theta_0)$ where $\theta_0$ is the tilt angle of the beams as shown in figure (3). The angle of incidence is varied in close vicinity around the critical value $\sin^{-1}(n_2/n_1)$ where we take $n_1 = 1.5$ and $n_2 = 1$. The peaks of the transmitted beams at the end of the propagation distance $z_0$ are separated by a distance $\Delta X_0$ due to the wavelength dependence of the G H shift, and hence its impact on the transmitted beams can be explained with reference to Fig. 3 as follows:

![Diagram](image)

Fig. 3 Two Gaussian beams at $\lambda_1$ and $\lambda_2$ incident at angles around the critical value on the interface $x=0$

Fig. 4 Shows that the peak of $\Delta X_0 = 34$ $\mu$m occurs at a tilt angle $\Theta_0 = 47.95^\circ$, hence the optimal transverse separation distance $\Delta X_0$ is as expected close to $\Theta_0 = 48.19^\circ$ (which corresponding to $\Theta_c = \sin^{-1}(1/1.5) = 41.81^\circ$). This agrees with the explanation given above.

![Diagram](image)

Fig. 4 Transverse separation of the transmitted beams as function of beam tilt angle $\Theta_0$
IV. RESULTS FOR GRADED – INDEX INTERFACE:

The situation depicted in fig. 1(b) is considered where $n_2(x)$ is a graded–index distribution as shown in fig. 5. The beam width $W$ is taken to be equal to 10 µm and $x_d = 20W$ (c.f. fig. (1b) and fig. 2).

The exponential profile $n_2(x)$ is taken as:

$$n_2(x) = n_1 e^{0.0028 x} \quad -2000 \mu m \leq x < 0$$

Where $n_1 = 1.5$ and $X$ is in µm and extends in the -ve direction.

The linear profile is taken as:

$$n_2(x) = mx + n_1 \quad -1500 \mu m \leq x < 0$$

Where:

$$m = \frac{n_1 - n_2}{1500 \mu m}$$

is the straight line slope ($n_1 = 1.5$, $n_2 = 1$)

And finally, the quadratic profile is taken as:

$$n_2(x) = n_1 [1-(\alpha x)^2] \quad -2000 \mu m \leq x < 0$$

Where:

$X$ is in µm and $\alpha = 385 \times 10^{-6}$ to guarantee that $n_2(x)$ varies from 1.5 at the interface $x = 0$ to 1 at $x = -1500 \mu m$ (the half width of the computational window).

Fig. 6 shows a sample of the BPM calculations corresponding to the linear profile. The figure reveals the “continuous refraction” of the two beams as they propagate in the graded index medium until the “turning points” are reached, where the two beams propagate back to the homogeneous medium $n_1$ where they are clearly separated.
As we mentioned before, we searched for the optimum angle of incidence which results in a maximum transverse separation distance $\Delta x_0$. Fig. 7 shows that there is always an optimum angle of incidence which results in maximum transverse separation distance $\Delta x_0$ as expected earlier.

Fig. 7 Variation of the transverse separation distance as function of the incidence angle for the (a) linear, (b) quadratic and exponential (c) profiles.

V. CONCLUSION

In this paper we demonstrated theoretically the feasibility of a novel technique for optical wavelength Division Multiplexing (WDM). To our knowledge, we think that this is the simplest technique in the existing literature on WDM. The technique can be realized reliably in planar integrated optics structures, and this will be more preponderant than many other existing techniques in such structures, since the integration of our proposed method in semiconductor laser diode technologies and high speed optical detectors is straight forward. The realization of the proposed method can be extended to low loss active substrates and this can open the door to many devices extremely useful in optical switching, optical computing and optical memories.
Appendix

The problem under consideration is invariant with respect to the y-coordinate, consequently \[ \frac{\delta^2}{\delta y^2} = 0 \] and the scalar wave equation for \( E_y \) takes the form:

\[
\left( \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta z^2} + k_0^2 n^2(x) \right) E_y = 0 \tag{A1}
\]

Where \( n(x) \) is the refractive index distribution which is function of the transverse coordinate: \( x \). Equation (A1) can be written as:

\[
\frac{\delta^2 E_y}{\delta z^2} = -\left[ \nabla^2_t + k_0^2 n^2(x) \right] E_y \tag{A2}
\]

Where \( \nabla^2_t \) is the transverse laplacian. The coefficient of \( E_y \) in the right hand member of (A2) is an operator which depends only on the transverse coordinate "x" (transverse to the direction of propagation "z") and hence, a formal operator solution of (A2) for the forward propagation field at \( z = \Delta z \) in terms of its value at \( z = 0 \) is:

\[
E_y(x, \Delta z) = \{ \exp(i\Delta z R) \} . E_y(x, 0) \tag{A3}
\]

Where a time dependence \( \exp(-i\omega t) \) is assumed and \( R \) is the operator:

\[
R = \left[ \nabla^2_t + k_0^2 n^2(x) \right]^\frac{1}{2} \tag{A4}
\]

If \( n(x) \) is denoted shortly by \( n \), then the operator \( R \) can be written as:

\[
R = \left[ \frac{\nabla^2_t + k_0^2 n^2}{(\nabla^2_t + k_0^2 n^2)^{\frac{1}{2}} + k_0 n} \right] = k_0 n \tag{A5}
\]

If \( n \) in the dominator of the first term in the right–hand member of (A5) is replaced by a certain reference value \( n_0 \) where \( n_2 \leq n_0 \leq n_1 \) then the last equation can be written as:

\[
\left[ \nabla^2_t + k_0^2 n^2 \right]^\frac{1}{2} \approx \frac{\nabla^2_t}{(\nabla^2_t + k_0^2 n^2)^{\frac{1}{2}} + k_0 n} = k_0 n \tag{A6}
\]

Where \( k = k_0 n \). The approximation in (A6) is valid if the maximum deviation \( \Delta n_{\text{max}}(x) \) of \( n(x) \) from the reference value \( n_0 \) satisfies the following criterion:

\[
|\Delta n_{\text{max}}| (\frac{\Delta z}{\lambda_0}) \sin^2 \theta_{\text{max}} \ll 1 \tag{A7}
\]

Where \( \theta_{\text{max}} \) is the angle between the direction of the highest significant plane wave component in the spatial spectrum of the total propagation field and z-axis.

If \( E_y(x, z) \) is written as:

\[
E_y(x, z) = e_y(x, z) . \exp(ikz) \tag{A8}
\]

Then, apart from a constant phase factor \( \exp(ik\Delta z) \), direct substitution from (A8) into (A3) gives:

\[
e_y(x, \Delta z) = \{ \exp[i\Delta z(S + k_0 \delta n)] \} e_y(x, 0) \tag{A9}
\]

Where \( \delta n = n(x) - n_0 \) and \( e_y(x, 0) \) is the initial field distribution at \( z = 0 \). The operator \( S \) is defined as:

\[
S = \frac{\nabla^2_t}{(\nabla^2_t + k_0^2 n^2)^{\frac{1}{2}} + k} \tag{A10}
\]

The exponent in the right–hand member of (A9) is in fact the product of two operators:

\[
\left[ \exp(i\Delta z S) \right] \left[ \exp(i\Delta z k_0 \delta n) \right] \tag{A11}
\]
These operators do not commute; hence an approximation is indispensable to evaluate the right-hand side of (A9). It can be shown that to second order in \( \Delta z \), equation (A9) can be written in a symmetric split-operator form as:

\[
e_y (x, \Delta z) = \{P.Q.P\}.e_y(x, 0) + O(\Delta z)^3
\]  

(A12)

Where \( O(\Delta z)^3 \) is a negligible term of the order of \( (\Delta z)^3 \) and \( P \) and \( Q \) are the two operators:

\[
P = e^{x p \left\{ i (\Delta z/2) S \right\}}
\]  

(A13)

And:

\[
Q = e^{x p \left( i \Delta z k_0 \delta n \right)}
\]  

(A14)

the operation \( \{P\}.e_y(x,0) \) represents the propagation of the initial field \( e_y(x,0) \) for a distance equal to half the step size \( \Delta z/2 \) in a homogeneous medium having a constant refractive index \( n_0 \), i.e. It is equivalent to solving the Helmholtz wave equation:

\[
\left( \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta z^2} + k^2 \right) E_y = 0
\]  

(A15)

With \( E_y(x,0) \) as an initial condition at \( z = 0 \). Therefore advancing \( E_y(x,0) \) by repeated application of (A12) allows us to obtain the total propagating field \( E_y(x,0) \) at any distance \( z \) once the initial field is known. The operation \( \{P\}.e_y(x,0) \) is easily performed in Fourier space because the spatial Fourier transform of \( \{P\}.e_y(x,0) \) can be written as:

\[
\mathcal{F}\{P.e_y(x,0)\} = \Psi(k_x,0) \exp\left\{ i(\Delta x/2)k_x^2/[(k_x^2-k_y^2)^{1/2}]+l \right\}
\]  

(A16)

where \( \Psi(k_x,0) \) is the spatial Fourier transform of the initial field \( e_y(x,0) \), i.e:

\[
\Psi(k_x,0) = \int_{-\infty}^{\infty} e_y(x,0) \exp(-ik_x x) dx
\]  

(A17)

Thus, advancing the initial field for a distance equal to half of the propagation step \( \Delta z/2 \) by performing the first operation \( \{P\}.e_y(x,0) \) in Fourier space (via (A16)), then returning back to the ordinary \( (x,z) \) plane by fourier inversion to take into account the deviation of the actual refractive index distribution \( n(x) \) from the reference value \( n_0 \), we multiply the propagated field by the correcting operator \( Q \) defined in (A14). Then, performing again the propagation process over the other half \( \Delta z/2 \) of the propagation step. Repeated application of these processes allows us to calculate the total propagation field at any distance \( z \). The Fourier transform is calculated numerically from the sampled field values at “N” discrete points \( x_m \) where \( m = 1, 2, ..., N \), i.e. a Discrete Fourier transform (DFT) which is calculated by the Fast Fourier Transform algorithm (FFT). According to the discretized version of (A17) is written as:

\[
\Psi(k_{xm},0) = \sum_{j=-(N/2)+1}^{N/2} e_y(j\Delta x,0) \exp(-i k_{xm} j \Delta x)
\]  

(A18)

Where the spacing \( \Delta x \) between the samples of the field values is calculated from:

\[
\Delta x = L/N
\]  

(A19)

“L” being the length of the computational region along the \( x \)-axis. The variable of the DFT (the transverse wavenumber) \( k_{xm} \) is given by:

\[
k_{xm} = 2\pi m / L
\]  

(A20)

From (A19) and (A20), we can write (A18) as:

\[
\Psi(k_{xm},0) = \sum_{j=-(N/2)+1}^{N/2} e_y(j\Delta x,0) \exp(-i 2\pi m j / N)
\]  

(A21)

The propagation process between \( z = 0 \) and \( z = \Delta z \) can be summarized as follows:

1. Calculate the initial spectrum \( \Psi(k_{xm},0) \) from field values \( e_y(j\Delta x,0) \) at \( N \) discrete points using the FFT algorithm.
2. Propagating the initial spectrum over a half step \( \Delta z/2 \) in the Fourier domain using (A16).
3- Fourier inverting the propagated spectrum using the inverse FFT algorithm to recover the uncorrected field after a half step.
4- Making the phase correction by multiplying the uncorrected field with the operator $Q$ .
5- Repeating the propagation process over the half of the propagation step as described in the first two steps to obtain finally the field at $z = \Delta z$.

The previous scheme is repeated until we reach any desired propagation distance $Z_{tot}$. A crucial question regarding the spatial sampling interval $\Delta x$: how to choose it? It is known that as the sampling interval $\Delta x$ decreases, the resolution of the spatial Fourier spectrum is enhanced. This means that higher spatial frequencies in the spectrum can be “viewed”, i.e. the “fine details” of the field are enhanced. The spectrum of the incident field is centered around $k_{xi} = \sin \theta_i$, and its maximum significant width is $\Delta k_{xi} = 4 \cos \theta_i / W$ and hence the maximum deviation from $k_{xi}$ is $\pm 2 \cos \theta_i / W$.

From (A20) the maximum value of the transverse wavenumber $K_{x_{max}}$ in the DFT corresponds to $m = N/2$, from (A19) we have:

$$k_{x_{max}} = \pi / \Delta x$$  \hspace{1cm} (A22)

Taking into account for the maximum deviation of $K_x$ around $K_{xi}$, an acceptable for the maximum value of the transverse wavenumber is:

$$k_{x_{max}} = k_{xi} + (2 \cos \theta_i / W) .$$

From (A22), we deduce:

$$\left( \pi / \Delta x \right) = k_{xi} + (2 \cos \theta_i / W)$$  \hspace{1cm} (A23)

This means that the sampling interval $\Delta x$ should not exceed the upper limit $\pi /[k_{xi} + (2 \cos \theta_i / W)]$ otherwise the high spatial frequencies in the spectrum would not be “viewed”, i.e. the “fine details” of the field would be lost. Thus an acceptable upper limit on the sampling interval $\Delta x$ is:

$$\Delta x_{\text{max}} \leq \pi /[k_{xi} + (2 \cos \theta_i / W)]$$  \hspace{1cm} (A24)

Thus the actual sampling interval $\Delta x$ must be less than $\Delta x_{\text{max}}$, for example 0.5 to 0.25 that value.

Finally, it is worthy to point out that the propagating field which reaches the boundary of the computational window whose width is “L”, will appear as a fictitious field reflected from the boundary of that window and causes aliasing. To prevent this numerical problem, an “absorber” is put near the edges of the computational window. A wide variety of absorbers exist and are extensively used. We used a “Hanning” truncation function as an absorber, which is defined as:

$$A(x) = 0.5 \left[ 1 - \cos \left( 2 \pi (x - x_f) / L \right) \right], 0 \leq x \leq L$$  \hspace{1cm} (A25)

References:

[1] Robert C. Elsenpeter and Toby J.Vete, “optical Networking”, McGraw-Hill/Osborne, 2002.
[2] Cedric F. Lan, “Passive Optical Networks”, Academic press, 2007.
[3] Makkerjee B., “Optical WDM Networks”, Springer, Berlin, 2006.
[4] Ellinas G., Antoniades N. and Roudas I., “WDM System and Networks”, Springer, New York, 2012.
[5] Katsunari Okamoto, “Fundamentals of Optical Waveguides.2nd Edition”, Academic Press, 2006.
[6] Salah Obaya, “Computational Photonics”, John Wiley &Sons, Ltd. publication, 2011.
[7] M. D. Feit and J. A. Fleck, Jr, “Light propagation in graded-index optical fibers”, Applied Optics vol 17, No 24, pp. 3990 – 3998, 15 December 1978.
[8] C. L. Xu and W. P. Huang, “Finite difference beam propagation methods for guide-wave optics”, Progress In Electromagnetics Research, PIER 11, pp. 1-49, 1995.

[9] J. van Roey, J. van der Donk, and P. E. Lagasse, “Beam propagation method: analysis and assessment”, Journal of Optics Society of America, Vol. 71, No. 7, pp. 803-810, 1981.

[10] L. Thylen, “The beam propagation method: an analysis of its applicability”, Optical and Quantum Electronics, Vol. 15, pp. 433-439, 1983.

[11] B. R. Horowitz and T. Tamir, “Lateral displacement of a light beam at a dielectric interface”, Journal of the Optical Society of America, Vol. 61, No. 5, pp. 586-594, 1971.

[12] J. A. Fleck, J. R. Morris and M. D. Feit, “Time-dependent propagation of high energy laser beams through the atmosphere”, Applied Physics, Vol. 10, pp. 129-160, 1976.

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