MAPPING STRING STATES INTO PARTONS:
FORM FACTORS AND THE HADRON SPECTRUM IN
AdS/QCD

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New developments in holographic QCD are described in this talk in the context of the correspondence between string states in AdS and light-front wavefunctions of hadronic states in physical space-time.

The AdS/CFT correspondence gives unexpected connections between seemingly different theories which represent the same observables. On the bulk side it describes the propagation of weakly coupled strings, where physical quantities are computed using an effective gravity approximation. The duality provides a non-perturbative definition of quantum gravity in a (d+1)-dimensional AdS spacetime in terms of a d-dimensional conformally-invariant quantum field theory at the anti–de Sitter (AdS) boundary.

The AdS/CFT duality has the potential for understanding fundamental properties of quantum chromodynamics such as confinement and chiral symmetry breaking which are inherently non-perturbative. As shown by Polchinski and Strassler, the AdS/CFT duality, modified to incorporate a mass scale, provides a non-perturbative derivation of dimensional counting rules for the leading power-law fall-off of hard scattering. The modified theory generates the hard behavior expected from QCD, instead of the soft behavior characteristic of strings.

In its original formulation, a correspondence was established between the supergravity approximation to Type IIB string theory on a curved background asymptotic to the product space of AdS5 × S5 and the large $N_c$, $N = 4$, super Yang-Mills (SYM) gauge theory in four dimensional space-time. The group of conformal transformations $SO(2,4)$ which acts at the AdS boundary, is isomorphic with the group of isometries of AdS space,
and $S^5$ corresponds to the $SU(4) \sim SO(6)$ global symmetry which rotates the particles present in the SYM supermultiplet. The supergravity duality requires a large AdS radius $R$ corresponding to a large value of the 't Hooft parameter $g_s N_C$, where $R = (4\pi g_s N_C)^{1/4} \alpha_s^{1/2}$ and $\alpha_s^{1/2}$ is the string scale. The classical approximation corresponds to the stiff limit where the string tension $T = R^2/2\pi\alpha' \to \infty$, effectively suppressing string fluctuations.

QCD is fundamentally different from SYM theories where all the matter fields transform in adjoint multiplets of $SU(N_C)$. QCD is also a confining theory in the infrared with a mass gap $\Lambda_{QCD}$ and a well-defined spectrum of color singlet states. Its fundamental string dual is unknown. The duality should be extended to include different boundary conditions and non conformal quantum field theories. We may expect that a dual gravitational description would emerge in the strong coupling regime of QCD. Indeed, the string dual should remain well defined also in a highly curved space where the AdS radius become small compared to the string size.

In practice, we can deduce salient properties of the QCD dual theory by studying its general behavior, such as its ultraviolet limit at the conformal AdS boundary $z \to 0$, as well as the large-$z$ infrared region, characteristic of strings dual to confining gauge theories. The fifth dimension in the anti-de Sitter metric corresponds to the scale transformations of the quantum field theory, thus incorporating the renormalization group flow of the boundary theory. This approach, which can be described as a bottom-up approach, has been successful in obtaining general properties of scattering processes of QCD bound states, the low-lying hadron spectra, hadron couplings and chiral symmetry breaking, quark potentials in confining backgrounds and pomeron physics.

In contrast to the simple bottom-up approach described above, a top-bottom approach consists in studying the full supergravity equations to compute the glueball spectrum or the introduction of additional higher dimensional branes to the $AdS_5 \times S^5$ background, as a prescription for the introduction of flavor with quarks in the fundamental representation and the computation of the meson spectrum.

It has been shown recently that the string amplitude $\Phi(z)$ describing hadronic modes in $AdS_5$ can be precisely mapped to the light-front wavefunctions $\psi_{n/h}$ of hadrons in physical space-time. Indeed, there is an exact correspondence between the holographic variable $z$ and an impact variable $\zeta$ which represents the measure of the transverse separation of the constituents within the hadrons. This remarkable holographic feature follows from the fact that current matrix element in AdS space can be mapped
to the exact Drell-Yan-West formula at the asymptotic AdS boundary\textsuperscript{14}. It was also found that effective Schrödinger equations describing hadronic bound states can be expressed as $3 + 1$ QCD light-front wave equations\textsuperscript{14}.

The boost invariant light-front wavefunctions (LFWFs) in the Fock expansion at fixed light-cone time $x^+ = x^0 + x^3$ of any hadronic system $\psi_{n/h}(x_i, b_{i\perp}, \lambda_i)$, encode all its bound-state quark and gluon properties and their behavior in high-momentum transfer reactions\textsuperscript{15}. The light-cone momentum fractions $x_i = k^+_i/P^+_\perp$ and the impact position variables $b_{i\perp}$ represent the relative coordinates of constituent $i$ in Fock state $n$, and $\lambda_i$ the helicity along the $z$ axis.

In the case of a two-parton constituent bound state the correspondence between the string amplitude $\Phi(z)$ and the light-front wave function $\tilde{\psi}(x, b)$ is expressed in closed form\textsuperscript{14}

$$|\tilde{\psi}(x, \zeta)|^2 = \frac{R^3}{2\pi} \frac{x(1-x)}{\zeta^4} e^{3A(\zeta)} |\Phi(\zeta)|^2,$$

(1)

where $\zeta^2 = x(1-x)b_{\perp}^2$. The variable $\zeta$, $0 \leq \zeta \leq \Lambda_{\text{QCD}}^{-1}$, represents the invariant separation between point-like constituents, and it is also the holographic variable $z$ in AdS; i.e., we can identify $\zeta = z$. In the “hard wall” approximation\textsuperscript{3} the nonconformal metric factor $e^{3A(z)}$ is a step function.

The short-distance behavior of a hadronic state is characterized by its twist (dimension minus spin) $\tau = \Delta - \sigma$, where $\sigma$ is the sum over the constituent’s spin $\sigma = \sum_{i=1}^n \sigma_i$. Twist is also equal to the number of partons $\tau = n$. Matching the boundary behavior of string modes $\phi(z)$ with the twist of the boundary interpolating operators we find, upon the substitution $\phi(z) = z^{-\frac{3}{2}} \Phi(z)$ in the wave equations in AdS space, an effective Schrödinger equation as a function of the weighted impact variable $\zeta$

$$\left[ -\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = M^2 \phi(\zeta),$$

(2)

with the effective potential $V(\zeta) \to -(1-4L^2)/4\zeta^2$ in the conformal limit\textsuperscript{14}. The solution to (2) is $\phi(z) = z^{-\frac{3}{2}} \Phi(z) = C z^\frac{3}{2} J_L(zM)$. Its lowest stable state is determined by the Breitenlohner-Freedman bound\textsuperscript{16}. Its eigenvalues are obtained from the boundary conditions at $\phi(z = 1/\Lambda_{\text{QCD}}) = 0$, and are given in terms of the roots of the Bessel functions: $M_{L,k} = \beta_{L,k} A_{\text{QCD}}$. Normalized LFWFs $\tilde{\psi}_{L,k}(x, \zeta)$ follow from (1)

$$\tilde{\psi}_{L,k}(x, \zeta) = B_{L,k} \sqrt{x(1-x)} J_L(\zeta \beta_{L,k} A_{\text{QCD}}) \theta(z \leq \Lambda_{\text{QCD}}^{-1}),$$

(3)

where $B_{L,k} = \pi^{-\frac{3}{4}} A_{\text{QCD}}/J_{1+L}(\beta_{L,k})$. The spectrum of the light mesons is compared in Figure 1 with the data listed by the PDG\textsuperscript{17}. 
A different approach consists on matching AdS results following Migdal procedure for the regularization of UV conformal correlators, using Padé approximants to build the spectrum with poles of zeros of Bessel functions. This has been discussed recently for two- and three-point functions.

Consider the twist-three, dimension $2\frac{1}{2} + L$, baryon operators $O(9/2)^+_{L} = \psi D_{i_{1}} \ldots D_{i_{q}} \psi D_{i_{q+1}} \ldots D_{i_{m}} \psi$. Since we are taking a product of operators at the same point, we match the dependence of the corresponding AdS spin-$1\frac{1}{2}$ or $3\frac{1}{2}$ modes to the boundary operators at the ultraviolet $Q \to \infty$ or $z \to 0$ limit. A three-quark baryon is described by wave equation

$$[z^2 \partial_z^2 - 3z \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4] \psi_\pm(z) = 0 \quad (4)$$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = C e^{-iP \cdot x} [\psi(z)_+ u_+(P) + \psi(z)_- u_-(P)], \quad (5)$$

with $\psi_+(z) = z^2 J_{1+L}(z \mathcal{M})$ and $\psi_-(z) = z^2 J_{2+L}(z \mathcal{M})$. The constant $C$ in (5) is determined by the normalization $R^3 \int \frac{dz}{z^2} \left( |\psi_+(z)|^2 + |\psi_-(z)|^2 \right) = 1$ and is given by $C = \sqrt{2R^{-\frac{3}{2}} \Lambda_{QCD} / J_0(\beta_{1,1})}$. The physical string solutions have plane waves and chiral spinors $u_\pm(P)$ along the Poincaré coordinates and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$. Similar solutions follow from the Rarita-Schwinger AdS modes $\Psi^\mu$ in the $\Psi_z = 0$ gauge. In the large $P^+$ limit $\psi_\pm$ are the light-cone $\pm$ components along the $z$ axis: $\psi_+ = \psi^\uparrow$, $\psi_- = \psi^\downarrow$. The four-dimensional spectrum follows from $\psi_\pm(z = 1/\Lambda_{QCD}) = 0$: $\mathcal{M}_+^{\alpha, k} = \beta_{\alpha, k} \Lambda_{QCD}$, $\mathcal{M}_-^{\alpha, k} = \beta_{\alpha+1, k} \Lambda_{QCD}$, with a scale independent mass ratio. Figure 2(a) shows the predicted orbital spectrum of the nucleon states and Fig. 2(b) the $\Delta$ orbital resonances. The data is from [17]. The internal parity of states is determined from the SU(6) spin-flavor symmetry.
The predictions for the lightest hadrons are improved relative to the results of [7] using the boundary conditions determined by twist instead of conformal dimensions. The model is remarkably successful in organizing the hadron spectrum, although it underestimates the spin-orbit splittings of the $L = 1$ states. A better understanding of the relation between chiral symmetry breaking and confinement is required to describe successfully the pion. This would probably need a description of quark spin-flip mechanisms at the wall.

We now consider the spin non-flip nucleon form factors in the hard wall model. The effective charges are determined from the spin-flavor structure of the theory. We choose the struck quark to have $s^z = +1/2$. The two AdS solutions $\psi_+$ and $\psi_-$ correspond to nucleons with $J^z = +1/2$ and $-1/2$. For $SU(6)$ spin-flavor symmetry\(^\text{20}\)

\[
F^p_1(Q^2) = R^3 \int \frac{dz}{z^3} J(Q,z) |\psi_+(z)|^2, \tag{6}
\]

\[
F^n_1(Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3} J(Q,z) \left[ |\psi_+(z)|^2 - |\psi_-(z)|^2 \right], \tag{7}
\]

where $J(Q,z)$ is a solution to the AdS wave equation for the external electromagnetic current polarized along the Minkowski coordinates, $A_\mu = \epsilon_\mu e^{-iQ \cdot x} J(Q,z)$, $A_z = 0$, subject to the boundary conditions $J(Q = 0, z) = J(Q, z = 0) = 0$ and is given by $J(Q,z) = zQ K_1(zQ)^3$. The conditions $F^p_1(0) = 1$ and $F^n_1(0) = 0$ follow from the identity

\[
\int_0^1 dx \left[ J_0^2(x\beta) - J_{n+1}^2(x\beta) \right] = J_n(\beta) J_{n+1}(\beta)/\beta.
\]

Figure 3 compares the predictions for the Dirac nucleon form factors with the experimental data\(^\text{21}\).
We have shown how the string amplitude $\Phi(z)$ defined on the fifth dimension in $\text{AdS}_5$ space can be precisely mapped to the light-front wavefunctions of hadrons in physical spacetime\(^{14}\). This specific correspondence provides an exact holographic mapping in the conformal limit at all energy scales between string modes in AdS and boundary states with well-defined number of partons. Consequently, the AdS string mode $\Phi(z)$ can be regarded as the probability amplitude to find $n$-partons at transverse impact separation $\zeta = z$. Its eigenmodes determine the hadronic mass spectrum. Although major dynamical questions remain to be solved for extending the duality from large to small 't Hooft coupling, the string-parton correspondence described in [14] suggests that basic features of QCD can be understood in terms of a higher dimensional dual gravity theory which holographically encodes multi-parton boundary states into string modes and allows the computation of physical observables at strong coupling.

**Acknowledgements**

This work was done in collaboration with Stan Brodsky. We thank Misha Shifman for his invitation to CAQCD 2006 and Joe Polchinski for encouraging comments.

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