Evidence for an anisotropy of the speed of light

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Abstract

By comparing with the most recent experimental results, we point out the model dependence of the present bounds on the anisotropy of the speed of light. In fact, by replacing the CMB with a class of preferred frames that can better account for the experimental data, one obtains values of the RMS anisotropy parameter \((1/2 - \beta + \delta)\) that are one order of magnitude larger than the presently quoted ones. The resulting non-zero anisotropy can be understood starting from the observation that the speed of light in the Earth’s gravitational field is \textit{not} the basic parameter \(c = 1\) entering Lorentz transformations. In this sense, light can propagate isotropically only in one ‘preferred’ frame.

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1. Introduction

The idea of a preferred reference frame dates back to the origin of the Theory of Relativity, i.e. to the basic differences between Einstein’s Special Relativity and the Lorentzian point of view. Today the former approach is generally accepted and with it the interpretation of the relativistic effects in terms of the relative motion between any pair S’ and S” of observers. However, in spite of the deep conceptual differences, as emphasized by Bell [1, 2], it is not so simple to distinguish experimentally between the two alternatives. In fact, relativistic effects might be interpreted, equally well, as arising from the individual motions of each observer with respect to a preferred frame Σ. In this case, the basic Lorentz transformations would be associated with the velocity parameters \( \beta' = v'/c \) and \( \beta'' = v''/c \), \( v' \) and \( v'' \) being the velocities of S’ and S” with respect to Σ (we restrict for simplicity to the one-dimensional case). The equivalence of the two formulations is then a simple consequence of the basic group property of Lorentz transformations where the relation between S’ and S” is also a Lorentz transformation with relative velocity parameter

\[
\beta_{\text{rel}} = \frac{\beta' - \beta''}{1 - \beta' \beta''}.
\]

It would be possible to distinguish the two formulations if the individual parameters \( \beta' \) and \( \beta'' \) could be experimentally determined through ether-drift experiments. At the same time, it is clear that this possibility crucially depends on having a speed of light in the vacuum, say \( c_\gamma \), that is \textit{not} the same parameter \( c \equiv 1 \) entering Lorentz transformations. In this case, light would be seen to propagate isotropically with a speed \( c_\gamma \neq c \equiv 1 \) in Σ. However, in any other moving frame S’ the speed of light would exhibit a non-trivial angular dependence \( c_\gamma(\theta) \) induced by the Lorentz transformation connecting S’ to Σ.

Notice that, once the space-time transformations are taken to be Lorentz transformations, the basic isotropy and homogeneity of space and time (assumed to be valid in Σ) hold true in any other moving frame S’ as well. Thus for S’ the length of a rod, at rest in S’, does not depend on its orientation. This means that for S’ the length \( L \) of a resonating cavity (at rest in S’) cannot depend on the angle \( \theta \) between the cavity axis and the velocity of S’ with respect to Σ. Therefore, in the general relation between the cavity frequency \( \nu = \nu(\theta) \), the cavity length \( L = L(\theta) \) and the two-way speed of light \( \bar{c}_\gamma(\theta) \)

\[
\nu(\theta) \sim \frac{\bar{c}_\gamma(\theta)}{L(\theta)} \tag{2}
\]

one can take \( L(\theta) = L = \text{constant} \) if Lorentz transformations are valid. In this way, the
relative frequency shift of two orthogonal optical resonators
\[
\frac{\delta \nu(\theta)}{\nu_0} = \frac{\nu(\pi/2 + \theta) - \nu(\theta)}{\nu_0} = \bar{c}_\gamma(\pi/2 + \theta) - \bar{c}_\gamma(\theta)
\]
(3)
provides a direct measurement of the anisotropy of the speed of light in the rest frame S’ of the apparatus.

As a possible scenario for a value \( c_\gamma \neq c \equiv 1 \) one can consider, for instance, models with extra space-time dimensions [3]. These represent an interesting approach toward a consistent quantum theory of gravity and predict typically a speed of gravity \( c_g \neq c \). This leads, in the 4D effective theory, to a version of relativity where there is a preferred frame \( \Sigma \), the one associated with the isotropic value of \( c_g \). At the same time, through the coupling to gravitons, the induced Lorentz-violations [4] will extend to the other sectors of the theory. Namely, through the effect of graviton loops the photon energy spectrum becomes
\[
E_\gamma(|p|) = c_\gamma |p|,
\]
(4)
where \( c_\gamma \) differs from the basic parameter \( c \equiv 1 \) entering Lorentz transformations.

Therefore, assuming the correct space-time transformations to be Lorentz transformations, the isotropic relation Eq. (4) cannot hold true in more than one frame. In this sense, the vacuum can be viewed as a physical medium with a non-trivial refractive index \( N_{\text{vacuum}} \equiv \frac{1}{c_\gamma} \neq 1 \). Thus, if light is seen to propagate isotropically by the observer in \( \Sigma \), on the Earth there would be a small anisotropy of the two-way speed of light
\[
\frac{\delta \bar{c}_\gamma}{\bar{c}_\gamma} \sim (N_{\text{vacuum}} - 1)\frac{v_{\text{earth}}^2}{c^2},
\]
(5)
\( v_{\text{earth}} \) being the Earth’s velocity with respect to \( \Sigma \).

We emphasize that the idea of extra space-time dimensions is just an example of theoretical framework that can produce a vacuum value \( c_\gamma \neq c \equiv 1 \). In fact, the same conclusion applies equally well to the more conventional case of a background gravitational field [6]. To better appreciate this remark, let us observe that the universality of free fall, at the base of the Equivalence Principle and of local Lorentz invariance, guarantees the identity of the local speed of light with the basic parameter \( c \equiv 1 \) entering Lorentz transformations.

However, for an observer sitting at rest on the Earth’s surface, this identification does not account for the Earth’s gravitational field which is equivalent to an effective refractive index. In fact, from the Earth’s gravitational potential
\[
\varphi = -\frac{G_N M_{\text{earth}}}{c^2 R_{\text{earth}}} \sim -7 \cdot 10^{-10}
\]
(6)
and the weak-field isotropic form of the metric

\[ ds^2 = (1 + 2\varphi)dt^2 - (1 - 2\varphi)(dx^2 + dy^2 + dz^2) \]  

(7)

one obtains an energy as in Eq. (4) with a value

\[ c_\gamma = \frac{1}{N_{\text{vacuum}}} \sim 1 + 2\varphi. \]

(8)

Again, since \( c_\gamma \neq c \equiv 1 \), Eq. (4) cannot be valid in more than one frame.

The aim of this paper is to explore the observable consequences of such a scenario where \( c_\gamma \neq c \equiv 1 \) by comparing with the ether-drift experiments and, in particular, with the new generation where vacuum cryogenic optical resonators are maintained under active rotation. The existence of a preferred frame \( \Sigma \) should produce periodic modulations of the signal as those associated with the typical angular frequency defined by the Earth’s rotation.

Comparing with the results of two recent experiments [7, 8], we shall show that the present interpretation of the data is not unambiguous but strongly depends on restricting the hypothetical preferred frame to coincide with the CMB. For this reason, we shall report model-independent relations that can be used to restrict from experiment the class of possible Earth’s cosmic motions. At the same time, such a model-independent analysis of the present data provides substantially different indications on the anisotropy of the speed of light.

2. General formalism

Within the framework outlined in the Introduction, we shall follow the authors of Ref. [9] by introducing a set of effective Minkowski tensors \( \hat{\eta}(i)_{\mu\nu} \)

\[ \hat{\eta}(i)_{\mu\nu} = \eta_{\mu\nu} - \kappa_i v_\mu v_\nu. \]

(9)

Here \( \eta_{\mu\nu} = \text{diag}(-1,1,1,1) \), \( v_\mu \) is the 4-velocity of \( S' \) with respect to the hypothetical preferred frame \( \Sigma \) while \( \kappa_i \) represent generalized Fresnel’s drag coefficients for particles of type \( i \). In the context of the models with extra space-time dimensions considered in Ref. [9] they originate from the interactions of the various particles with the gravitons. In general, Eqs. (9) represent a convenient framework to parameterize the dependence of the results on the motion of the observer with respect to a preferred frame.

In this way, the energy-momentum relation in a given frame \( S' \) can be expressed as

\[ p^\mu p^\nu \hat{\eta}(i)_{\mu\nu} + m^2(i) = 0. \]

(10)
For photons this becomes

$$p^\mu p'^\nu \eta(\gamma)_{\mu\nu} = 0,$$

(11)

with $\eta(\gamma)_{\mu\nu} = \eta_{\mu\nu} - \kappa_\gamma v_\mu v_\nu$ and with a photon energy that, in the $S'$ frame, depends on the direction between the photon momentum and the $S'$ velocity $v$ with respect to $\Sigma$.

To obtain the photon energy spectrum we shall follow Jauch and Watson [10] who also derived Eq. (11) working out the quantization of the electromagnetic field in a moving medium. They noticed that the procedure introduces unavoidably a preferred frame, the one where the photon energy does not depend on the direction of propagation, and which is “usually taken as the system for which the medium is at rest”. However, such an identification reflects the point of view of Special Relativity with no preferred frame. Therefore, we shall adapt their results to our case where the photon energy does not depend on the angle in some frame $\Sigma$. In this way, in a moving frame $S'$, we get the radiation field Hamiltonian

$$H_0 = \sum_{r=1,2} \int d^3p \left[ \hat{n}_r(p) + \frac{1}{2} \right] E(|p|, \theta),$$

(12)

where $\hat{n}_r(p)$ is the photon number operator and

$$E(|p|, \theta) = \frac{\kappa_\gamma v_0 \zeta + \sqrt{|p|^2(1 + \kappa_\gamma v_0^2) - \kappa_\gamma \zeta^2}}{1 + \kappa_\gamma v_0^2}$$

(13)

with

$$\zeta = p \cdot v = |p||v| \cos \theta,$$

(14)

$\theta \equiv \theta_{\text{lab}}$ being the angle defined, in the $S'$ frame, between the photon momentum and the $S'$ velocity $v$ with respect to $\Sigma$. Notice that only one of the two roots of Eq. (11) appears and the energy is not positive definite in connection with the critical velocity $1/\sqrt{1 + \kappa_\gamma}$ defined by the occurrence of the Cherenkov radiation.

Using the above relation, the one-way speed of light in the $S'$ frame depends on $\theta$ (we replace $v = |v|$ and $v_0^2 = 1 + v^2$)

$$\frac{E(|p|, \theta)}{|p|} = c_\gamma(\theta) = \frac{\kappa_\gamma v \sqrt{1 + v^2} \cos \theta + \sqrt{1 + \kappa_\gamma + \kappa_\gamma v^2 \sin^2 \theta}}{1 + \kappa_\gamma (1 + v^2)}.$$  

(15)

This is different from the $v = 0$ result, in the $\Sigma$ frame, where the energy does not depend on the angle

$$\frac{E(\Sigma)(|p|)}{|p|} = c_\gamma = \frac{1}{N_{\text{vacuum}}}$$

(16)
and, as in Eq. (4), the speed of light is simply rescaled by the inverse of the vacuum refractive index

\[ N_{\text{vacuum}} = \sqrt{1 + \kappa \gamma}. \]  

(17)

Working to \( O(\kappa \gamma) \) and \( O(v^2) \), one finds in the \( S' \) frame

\[ c_\gamma(\theta) = \frac{1 + \kappa \gamma v \cos \theta - \frac{\kappa^2 \gamma^2}{2} v^2 (1 + \cos^2 \theta)}{\sqrt{1 + \kappa \gamma}}. \]  

(18)

This expression differs from Eq. (6) of Ref. [11], for the replacement \( \cos \theta \rightarrow -\cos \theta \) and for the relativistic aberration of the angles. In Ref. [11], in fact, the one-way speed of light in the \( S' \) frame was parameterized in terms of the angle \( \theta \equiv \theta_\Sigma \), between the velocity of \( S' \) and the direction of propagation of light, as defined in the \( \Sigma \) frame. In this way, starting from Eq. (18), replacing \( \cos \theta \rightarrow -\cos \theta \) and using the aberration relation

\[ \cos(\theta_{\text{lab}}) = \frac{-v + \cos \theta_\Sigma}{1 - v \cos \theta_\Sigma}, \]  

(19)

one re-obtains Eq. (6) of Ref. [11] in terms of \( \theta = \theta_\Sigma \).

Further, using Eq. (18), the two-way speed of light (in terms of \( \theta = \theta_{\text{lab}} \)) is

\[ \bar{c}_\gamma(\theta) = \frac{2c_\gamma(\theta)c_\gamma(\pi + \theta)}{c_\gamma(\theta) + c_\gamma(\pi + \theta)} \sim 1 - \left[ \kappa \gamma - \frac{\kappa \gamma^2}{2} \sin^2 \theta \right] v^2. \]  

(20)

Now, re-introducing, for sake of clarity, the speed of light entering Lorentz transformations, \( c = 2.997 \cdot 10^{10} \) cm/s, one can define the RMS [12, 13] parameter \((1/2 - \beta + \delta)\). This is used to parameterize the anisotropy of the speed of light in the vacuum, through the relation

\[ \frac{\bar{c}_\gamma(\pi/2 + \theta) - \bar{c}_\gamma(\theta)}{\langle \bar{c}_\gamma \rangle} \sim (1/2 - \beta + \delta) \frac{v^2}{c^2} \cos(2\theta) \]  

(21)

so that one can relate \( \kappa \gamma \) to \((1/2 - \beta + \delta)\) through

\[ (1/2 - \beta + \delta) = \frac{\kappa \gamma}{2}. \]  

(22)

These results can be easily applied to the propagation of photons in a background gravitational field, such as on the Earth’s surface. In this case, if there were a preferred frame, the \( S' \) energy would not be given by Eq. (4) but would rather be given by Eq. (13) with a value of \( \kappa \gamma \) obtained from Eqs. (8) and (17)

\[ \kappa \gamma = N_{\text{vacuum}}^2 - 1 \sim 28 \cdot 10^{-10}. \]  

(23)

This corresponds to a RMS parameter

\[ (1/2 - \beta + \delta) \sim N_{\text{vacuum}}^2 - 1 \sim 14 \cdot 10^{-10} \]  

(24)

and should produce an anisotropy of the two-way speed of light in modern ether-drift experiments.
3. Cosmic motions and ether-drift experiments

In modern ether-drift experiments, one measures the relative frequency shift $\delta \nu$ of two vacuum cryogenic optical resonators under the Earth’s rotation [7,8]. If there is a preferred frame $\Sigma$, using Eqs. (20) and (21), the frequency shift of two orthogonal optical resonators to $O(\frac{v^2}{c^2})$ can be expressed as

$$\frac{\delta \nu(\theta)}{\nu_0} = \bar{c}_\gamma(\pi/2 + \theta) - \bar{c}_\gamma \langle \bar{c}_\gamma \rangle = A \cos(2\theta),$$

where $\theta = 0$ indicates the direction of the ether-drift and the amplitude of the signal is given by

$$A = \left(\frac{1}{2} - \beta + \delta \right) \frac{v^2}{c^2},$$

$v$ denoting the projection of the Earth’s velocity with respect to $\Sigma$ in the plane of the interferometer.

To address the problem in a model-independent way, let us introduce the time-dependent amplitude of the ether-drift effect

$$A(t) = v^2(t) X$$

in terms of the time-dependent Earth’s velocity and of the normalization of the experiment $X$. The main point is that the relative variations of the signal depend only on the kinematic details of the given cosmic motion and, as such, can be predicted independently of the knowledge of $X$. To describe the variations of $v(t)$, we shall use the expressions given by Nassau and Morse [16]. These have the advantage of being fully model-independent and extremely easy to handle. Their simplicity depends on the introduction of a cosmic Earth’s velocity

$$\mathbf{V} = \mathbf{V}_{\text{sun}} + \mathbf{v}_{\text{orb}}$$

that, in addition to the genuine cosmic motion of the solar system defined by $\mathbf{V}_{\text{sun}}$, includes the effect of the Earth’s orbital motion around the Sun described by $\mathbf{v}_{\text{orb}}$. To a very good approximation, $\mathbf{V}$ can be taken to be constant within short observation periods of 2-3 days. Therefore, by introducing the latitude of the laboratory $\phi$, the right ascension $\tilde{\Phi}$ and the declination $\tilde{\Theta}$ associated with the vector $\mathbf{V}$, the magnitude of the Earth’s velocity in the plane of the interferometer is defined by the two equations [16]

$$\cos z(t) = \sin \tilde{\Theta} \sin \phi + \cos \tilde{\Theta} \cos \phi \cos(\lambda)$$

and

$$v(t) = V \sin z(t),$$

6
$z = z(t)$ being the zenithal distance of $V$. Here, we have introduced the time $\lambda \equiv \tau - \tau_0 - \tilde{\Phi}$, where $\tau = \omega_{\text{sid}} t$ is the sidereal time of the observation in degrees and $\omega_{\text{sid}} \sim \frac{2\pi}{23^h56'}$. Also, $\tau_0$ is an offset that, in general, has to be introduced to compare with the definition of sidereal time adopted in Refs. [7, 8].

Now, operation of the interferometer provides the minimum and maximum daily values of the amplitude and, as such, the values $v_{\text{min}}$ and $v_{\text{max}}$ corresponding to $|\cos(\lambda)| = 1$. In this way, using the above relations one can determine the pair of values $(\tilde{\Phi}_i, \tilde{\Theta}_i)$, $i = 1, 2, \ldots, n$, for each of the $n$ short periods of observations taken during the year, and thus plot the direction of the vectors $V_i$ on the celestial sphere.

Actually, since the ether-drift is a second-harmonic effect in the rotation angle of the interferometer, a single observation is unable to distinguish the pair $(\tilde{\Phi}_i, \tilde{\Theta}_i)$ from the pair $(\tilde{\Phi}_i + 180^\circ, -\tilde{\Theta}_i)$. Only repeating the observations in different epochs of the year one can resolve the ambiguity. Any meaningful ether-drift, in fact, has to correspond to pairs $(\tilde{\Phi}_i, \tilde{\Theta}_i)$ lying on an ‘aberration circle’, defined by the Earth’s orbital motion, whose center $(\Phi, \Theta)$ defines the right ascension and the declination of the genuine cosmic motion of the solar system associated with $V_{\text{sun}}$. If such a consistency is found, using the triangle law, one can finally determine the magnitude $|V_{\text{sun}}|$ starting from the known values of $(\tilde{\Phi}_i, \tilde{\Theta}_i)$, $(\Phi, \Theta)$ and the value $|v_{\text{orb}}| \sim 30 \text{ km/s}$. In simple terms, for $|v_{\text{orb}}| \ll |V_{\text{sun}}|$, the opening angle $\Delta \varphi$ defined by the aberration circle can be approximated as $\Delta \varphi \sim \frac{|v_{\text{orb}}|}{|V_{\text{sun}}|}$.

We emphasize that the kinematical solution of the Earth’s cosmic motion, as obtained from the basic pairs of values $(\tilde{\Phi}_i, \tilde{\Theta}_i)$, only depends on the relative magnitude of the ether-drift effect, namely on the ratio $\frac{v_{\text{min}}}{v_{\text{max}}}$, in the various periods. As such, it is insensitive to any possible theoretical and/or experimental uncertainty that can affect multiplicatively the absolute normalization of the signal.

For instance, suppose one measures a relative frequency shift $\delta \nu/\nu \sim 10^{-15}$. Assuming a value $(1/2 - \beta + \delta) \sim 10 \cdot 10^{-10}$ in Eq. (20), this would be interpreted in terms of a velocity $v \sim 300 \text{ km/s}$. Within Galilean relativity, where one predicts the same expressions by simply replacing $(1/2 - \beta + \delta) \rightarrow 1/2$, the same frequency shift would be interpreted in terms of a velocity $v \sim 14 \text{ m/s}$. Nevertheless, from the relative variations of the ether-drift effect one would deduce the same pairs $(\tilde{\Phi}_i, \tilde{\Theta}_i)$ and, as such, exactly the same type of cosmic motion. Just for this reason, Miller’s determinations with this method, namely \[ V_{\text{sun}} \sim 210 \text{ km/s}, \quad \Phi \sim 74^\circ \text{ and } \Theta \sim -70^\circ, \] should be taken seriously.

We are aware that Miller’s observations have been considered spurious by the authors of Ref. [18] as partly due to statistical fluctuations and/or thermal fluctuations. However,
to a closer look (see the discussion given in Ref. [19]) the arguments of Ref. [18] are not so solid as they appear by reading the abstract of that paper. Moreover, Miller’s solution is 

*doubly* internally consistent since the aberration circle due to the Earth’s orbital motion was obtained in two different and independent ways (see Fig. 23 of Ref. [17]). In fact, one can
determine the basic pairs \((\tilde{\Phi}_i, \tilde{\Theta}_i)\) either using the daily variations of the magnitude of the ether-drift effect or using the daily variations of its apparent direction \(\theta_0(t)\) (the ‘azimuth’) in the plane of the interferometer. Since the two methods were found to give consistent results, in addition to the standard choice of preferred frame represented by the CMB, we shall also compare with the cosmic motion deduced by Miller.

Replacing Eq. (30) into Eq. (26) and adopting a notation of the type introduced in Ref. [15], we can express the theoretical amplitude of the signal as

\[ A(t) = A_0 + A_1 \sin \tau + A_2 \cos \tau + A_3 \sin(2\tau) + A_4 \cos(2\tau), \]

where \((\chi = 90^\circ - \phi)\)

\[ A_0 = (1/2 - \beta + \delta) \frac{V^2}{c^2} \left(1 - \sin^2 \tilde{\Theta} \cos^2 \chi - \frac{1}{2} \cos^2 \tilde{\Theta} \sin^2 \chi \right), \]

\[ A_1 = \frac{1}{2}(1/2 - \beta + \delta) \frac{V^2}{c^2} \sin 2\tilde{\Theta} \sin(\tilde{\Phi} + \tau_0) \sin 2\chi, \]

\[ A_2 = \frac{1}{2}(1/2 - \beta + \delta) \frac{V^2}{c^2} \sin 2\tilde{\Theta} \cos(\tilde{\Phi} + \tau_0) \sin 2\chi, \]

\[ A_3 = \frac{1}{2}(1/2 - \beta + \delta) \frac{V^2}{c^2} \cos^2 \tilde{\Theta} \sin[2(\tilde{\Phi} + \tau_0)] \sin^2 \chi, \]

\[ A_4 = \frac{1}{2}(1/2 - \beta + \delta) \frac{V^2}{c^2} \cos^2 \tilde{\Theta} \cos[2(\tilde{\Phi} + \tau_0)] \sin^2 \chi. \]

Recall that \(V, \tilde{\Theta}\) and \(\tilde{\Phi}\) indicate respectively the magnitude, the declination and the right ascension of the velocity defined in Eq. (28). As such, they change during the year.

To compare with the experiments of Refs. [7, 8], however, it will be more convenient to re-write Eq. (30) in the form of Ref. [7] where the frequency shift at a given time \(t\) is expressed as

\[ \frac{\delta \nu[\theta(t)]}{\nu_0} = \check{B}(t) \sin 2\theta(t) + \check{C}(t) \cos 2\theta(t) \]

\(\theta(t)\) being the angle of rotation of the apparatus, \(\check{B}(t) \equiv 2B(t)\) and \(\check{C}(t) \equiv 2C(t)\) so that one finds an experimental amplitude

\[ A(t) = \sqrt{\check{B}^2(t) + \check{C}^2(t)}. \]
In this case, using now Eqs. (21-22) of Ref. [16], one finds

\[ \dot{C}(t) = \dot{C}_0 + \dot{C}_1 \sin \tau + \dot{C}_2 \cos \tau + \dot{C}_3 \sin(2\tau) + \dot{C}_4 \cos(2\tau), \]  

(39)

where

\[ \dot{C}_0 = \frac{1}{2}(1/2 - \beta + \delta) \frac{V^2 \sin^2 \chi}{c^2} (3 \cos^2 \tilde{\Theta} - 2), \]  

(40)

\[ \dot{C}_1 = \frac{1}{2}(1/2 - \beta + \delta) \frac{V^2}{c^2} \sin 2\tilde{\Theta} \sin(\tilde{\Phi} + \tau_o) \sin 2\chi, \]  

(41)

\[ \dot{C}_2 = \frac{1}{2}(1/2 - \beta + \delta) \frac{V^2}{c^2} \sin 2\tilde{\Theta} \cos(\tilde{\Phi} + \tau_o) \sin 2\chi, \]  

(42)

\[ \dot{C}_3 = -\frac{1}{2}(1/2 - \beta + \delta) \frac{V^2}{c^2} \cos^2 \tilde{\Theta} \sin[2(\tilde{\Phi} + \tau_o)](1 + \cos^2 \chi), \]  

(43)

\[ \dot{C}_4 = -\frac{1}{2}(1/2 - \beta + \delta) \frac{V^2}{c^2} \cos^2 \tilde{\Theta} \cos[2(\tilde{\Phi} + \tau_o)](1 + \cos^2 \chi). \]  

(44)

Analogously, we find

\[ \dot{B}(t) = \dot{B}_1 \sin \tau + \dot{B}_2 \cos \tau + \dot{B}_3 \sin(2\tau) + \dot{B}_4 \cos(2\tau), \]  

(45)

with \( \dot{B}_1 = -\dot{C}_2 / \cos \chi, \dot{B}_2 = \dot{C}_1 / \cos \chi, \dot{B}_3 = -\frac{2 \cos \chi}{1 + \cos^2 \chi} \dot{C}_4 \) and \( \dot{B}_4 = \frac{2 \cos \chi}{1 + \cos^2 \chi} \dot{C}_3 \). These expressions are in full agreement with the results reported in Table I of Ref. [8] (for \( \tau_o = 180^o \)).

4. **Comparison with the experimental results**

Let us now compare the above theoretical predictions with the experimental results of Ref. [7] (referring to the short period February 6th- February 8th 2005) and with those of Ref. [8] (summarizing the observations from December 2004 to April 2005). To this end, we shall concentrate on the observed time modulation of the signal, i.e. on the quantities \( \dot{C}_1, \dot{C}_2, \dot{C}_3, \dot{C}_4 \) (and on their \( \dot{B} \)-counterparts). In fact, the average values \( \langle \dot{C} \rangle \) and \( \langle \dot{B} \rangle \) are most likely affected by spurious systematic effects as thermal drift (see in particular the discussion in Ref. [7] and the corresponding one in Ref. [8] about the non-zero value of \( B_0 \), there called \( S_0 \)).

We shall report in Table 1 the experimental values for the combinations

\[ \dot{C}(\omega_{\text{sid}}) \equiv \sqrt{\dot{C}_1^2 + \dot{C}_2^2} \]  

(46)

and

\[ \dot{C}(2\omega_{\text{sid}}) \equiv \sqrt{\dot{C}_3^2 + \dot{C}_4^2} \]  

(47)
Table 1: The experimental data and the values of the RMS parameter obtained by constraining the hypothetical preferred frame to coincide with the CMB.

| Experiment | Observable | \((1/2 - \beta + \delta)\) |
|------------|------------|--------------------------|
| Ref. [7]   | \(\hat{C}(\omega_{\text{sid}})\) = \((11 \pm 2) \cdot 10^{-16}\) | \((71 \pm 13) \cdot 10^{-10}\) |
| Ref. [8]   | \(\hat{C}(\omega_{\text{sid}})\) = \((3.0 \pm 2.4) \cdot 10^{-16}\) | \((20 \pm 16) \cdot 10^{-10}\) |
| Ref. [8]   | \(\hat{B}(\omega_{\text{sid}})\) = \((8.4 \pm 4.4) \cdot 10^{-16}\) | \((43 \pm 23) \cdot 10^{-10}\) |
| Ref. [7]   | \(\hat{C}(2\omega_{\text{sid}})\) = \((1 \pm 2) \cdot 10^{-16}\) | \((0.8 \pm 1.6) \cdot 10^{-10}\) |
| Ref. [8]   | \(\hat{C}(2\omega_{\text{sid}})\) = \((2.3 \pm 3.4) \cdot 10^{-16}\) | \((1.9 \pm 2.8) \cdot 10^{-10}\) |
| Ref. [8]   | \(\hat{B}(2\omega_{\text{sid}})\) = \((4.8 \pm 2.6) \cdot 10^{-16}\) | \((4.0 \pm 2.2) \cdot 10^{-10}\) |

and for their \(\hat{B}\)-counterparts. This is useful to reduce the model dependence in the data analysis. In this way, in fact, the right ascension \(\hat{\Phi}\) and the offset \(\tau_o\) drop out in the theoretical predictions that will only depend on \(\hat{\Theta}, V\) and \((1/2 - \beta + \delta)\).

At the same time, since Ref. [8] provides data that have been averaged over various periods of the year, we shall parameterize the predictions in terms of the average declination \langle \hat{\Theta} \rangle = \Theta\) and of the average velocity \langle V\rangle = V_{\text{sun}} obtaining the relations Ref. [15]

\[
\hat{C}(\omega_{\text{sid}}) \sim \frac{1}{2}(1/2 - \beta + \delta)\frac{V_{\text{sun}}^2}{c^2} |\sin 2\Theta| \sin 2\chi
\]

and

\[
\hat{C}(2\omega_{\text{sid}}) \sim \frac{1}{2}(1/2 - \beta + \delta)\frac{V_{\text{sun}}^2}{c^2} \cos^2 \Theta (1 + \cos^2 \chi).
\]

The corresponding \(\hat{B}\)-quantities are also given by \(\hat{B}(\omega_{\text{sid}}) = \hat{C}(\omega_{\text{sid}})/\cos \chi\) and \(\hat{B}(2\omega_{\text{sid}}) = \frac{2\cos \chi}{1+\cos^2 \chi} \hat{C}(2\omega_{\text{sid}})\). Notice that \(V_{\text{sun}}\) and \((1/2 - \beta + \delta)\) are completely correlated in each single measurement. Therefore, an unambiguous extraction of the RMS parameter in ether-drift experiments cannot be done without a preliminary determination of the cosmic velocity. In turn, as explained in Sect. 3, this depends on the possibility to use the triangle laws after observation of the Earth’s aberration circle. Since the present data just cover a small portion of the Earth’s orbit, we shall first compare with the experimental data of Refs. [7, 8] assuming in the above relations the fixed values \(V_{\text{sun}} \sim 370\) km/s and \(\Theta \sim -6^o\) that correspond to the Earth’s motion relatively to the CMB. In this way, we obtain the values of the RMS parameter reported in the third column of Table 1 (the colatitude of the laboratory has been taken \(\chi \sim 39^o\) for Ref. [7] and \(\chi \sim 38^o\) for Ref. [8]).
As one can see, the experimental data at the frequency \( \omega = \omega_{\text{sid}} \) produce systematically higher estimates of the RMS parameter. In fact, averaging only the determinations from observables at \( \omega = \omega_{\text{sid}} \) gives \( (1/2 - \beta + \delta) \sim (45 \pm 10) \cdot 10^{-10} \). This should be compared with the value \( (1/2 - \beta + \delta) \sim (2 \pm 2) \cdot 10^{-10} \) obtained from the data at \( \omega = 2\omega_{\text{sid}} \). On the other hand, averaging all determinations gives again the smaller value \( (1/2 - \beta + \delta) \sim (2 \pm 2) \cdot 10^{-10} \) but the chi-square of the mean is unacceptably large.

It is not difficult to understand the reason for such a discrepancy. It originates from the inadequacy of the CMB type of motion to describe some basic features of the data. In fact, using the theoretical prediction

\[
R = \frac{\hat{C}(2\omega_{\text{sid}})}{\hat{C}(\omega_{\text{sid}})} \sim \frac{0.82}{|\tan \Theta|},
\]

for the latitude of the two experiments, one would expect a value

\[
R_{\text{CMB}} \sim 7.8.
\]

This is very far from the precise experimental value of Ref. [\text{7}]

\[
R_{\text{EXP}}(\text{Ref. [\text{7}]}) \sim 0.09^{+0.18}_{-0.09}
\]

that would rather require \(|\Theta| \sim 83^\circ \pm 7^\circ\).

Analogously, introducing the other ratio

\[
R' = \frac{\hat{B}(2\omega_{\text{sid}})}{\hat{B}(\omega_{\text{sid}})} \sim \frac{0.62}{|\tan \Theta|},
\]

the theoretical prediction for \( |\Theta| \sim 6^\circ \)

\[
R'_{\text{CMB}} \sim 5.9
\]

does not agree with the experimental result from Ref. [\text{8}]

\[
R'_{\text{EXP}}(\text{Ref. [\text{8}]}) \sim 0.57^{+0.63}_{-0.31}
\]

that would rather require \(|\Theta| \sim (47^\circ \pm 21^\circ)\) [\text{20}].

Starting from these observations, one can check the RMS parameters that would rather be obtained from the various observables replacing the CMB with a class of preferred frames in better consistency with Eq. [\text{52}]. For instance, replacing \(|\Theta| \sim 6^\circ \) with the value \(|\Theta| \sim 70^\circ \) (as in Miller’s solution) obtained by averaging the \( \Theta \)-values from Eqs. [\text{52}] and [\text{55}], and
Table 2: The experimental data and the values of the RMS parameter obtained by replacing the CMB with a class of preferred frames with declination |Θ| = 70°. k accounts for the possible values of the Sun velocity as discussed in the text.

| Experiment | Observable | $(1/2 - \beta + \delta)$ |
|------------|------------|--------------------------|
| Ref.[7]    | $\hat{C}(\omega_{\text{sid}})$ | $(11 \pm 2) \cdot 10^{-16}$ $k(23 \pm 4) \cdot 10^{-10}$ |
| Ref.[8]    | $\hat{C}(\omega_{\text{sid}})$ | $(3.0 \pm 2.4) \cdot 10^{-16}$ $k(6 \pm 5) \cdot 10^{-10}$ |
| Ref.[8]    | $\hat{B}(\omega_{\text{sid}})$ | $(8.4 \pm 4.4) \cdot 10^{-16}$ $k(14 \pm 7) \cdot 10^{-10}$ |
| Ref.[7]    | $\hat{C}(2\omega_{\text{sid}})$ | $(1 \pm 2) \cdot 10^{-16}$ $k(7 \pm 14) \cdot 10^{-10}$ |
| Ref.[8]    | $\hat{C}(2\omega_{\text{sid}})$ | $(2.3 \pm 3.4) \cdot 10^{-16}$ $k(16 \pm 24) \cdot 10^{-10}$ |
| Ref.[8]    | $\hat{B}(2\omega_{\text{sid}})$ | $(4.8 \pm 2.6) \cdot 10^{-16}$ $k(34 \pm 19) \cdot 10^{-10}$ |

leaving out $V_{\text{sun}}$ as a free parameter, the results change substantially. In fact, the values of $(1/2 - \beta + \delta)$ from the observables at $\omega = \omega_{\text{sid}}$ are now decreased by a factor $\frac{\sin(12^\circ)}{\sin(140^\circ)} \sim 0.32$ while the values of $(1/2 - \beta + \delta)$ from the observables at $\omega = 2\omega_{\text{sid}}$ are now increased by a factor $\frac{\cos^2(6^\circ)}{\cos^2(70^\circ)} \sim 8.4$. These new results are shown in Table 2 where we have also introduced the additional rescaling $k$, defined through the relation $\sqrt{k} = \frac{370 \text{ km/s}}{V_{\text{sun}}}$, to treat $V_{\text{sun}}$ as a free parameter.

As one can see, independently of $k$, the values of the RMS parameter obtained from all observables are now in good consistency with each other. In fact, the average value is $(1/2 - \beta + \delta) \sim k(17 \pm 3) \cdot 10^{-10}$ with a good chi-square per degree of freedom. Notice also that, for $k \sim 1$ there would also be a good consistency with the prediction $(1/2 - \beta + \delta) \sim 14 \cdot 10^{-10}$ expected on the base of Eq. [24].

5. Summary and outlook

In this paper we have explored some phenomenological consequences of assuming the existence of a preferred frame. This scenario, that on the one hand leads us back to the old Lorentzian version of relativity, is also favoured by present models with extra space-time dimensions where the interactions with the gravitons change the vacuum into a physical medium with a non-trivial refractive index where the speed of light

$$c_\gamma = \frac{1}{\mathcal{N}_{\text{vacuum}}}$$  \hspace{1cm} (56)
differs from the basic parameter \( c \equiv 1 \) entering Lorentz transformations. In this case, where light can propagate isotropically in just one (preferred) frame \( \Sigma \), in any other frame there would be an anisotropy that could be detected through ether-drift experiments.

Our main point is that there is another simpler mechanism accounting for \( c_\gamma \neq c \equiv 1 \): the Earth’s background gravitational field. This introduces an effective vacuum refractive index

\[
N_{\text{vacuum}} \sim 1 - 2\varphi,
\]

with

\[
\varphi = -\frac{G_N M_{\text{Earth}}}{c^2 R_{\text{Earth}}} \sim -7 \cdot 10^{-10}
\]

and corresponds to a RMS parameter

\[
(1/2 - \beta + \delta) \sim N_{\text{vacuum}} - 1 \sim 14 \cdot 10^{-10}.
\]

Thus, if there were a preferred frame \( \Sigma \) where light is seen isotropic, one should be able to detect some effect with the new generation of precise ether-drift experiments using rotating cryogenic optical resonators. In particular, one should look for periodic modulations of the signal that might be associated with the Earth’s rotation and its orbital motion around the Sun.

When comparing with the experimental results of Ref. [7, 8] we can draw the following conclusions. Assuming the preferred frame to coincide with the CMB (\( V_{\text{sun}} \sim 370 \ \text{km/s} \) and an average declination \( \Theta \sim -6^o \)) the observables at the Earth’s rotation frequency \( \omega = \omega_{\text{sid}} \) are consistent with rather a large value of the RMS parameter \( (1/2 - \beta + \delta) \sim (45 \pm 10) \cdot 10^{-10} \) (see Table 1). At the same time, the signal at \( \omega = 2\omega_{\text{sid}} \) is comparably weaker yielding the much smaller value \( (1/2 - \beta + \delta) \sim (2 \pm 2) \cdot 10^{-10} \).

In our opinion, both estimates are likely affected by a systematic uncertainty of theoretical nature. In fact, as explained in Sect. 4, the average declination favoured by the experimental data is far from the value \( |\Theta| \sim 6^o \) defined by the Earth’s motion relatively to the CMB. For this reason, and since the physical nature of the preferred frame is unknown, one should check the stability of \( (1/2 - \beta + \delta) \) against some other type of cosmic motion. In this case, adopting an average declination in better consistency with the data, say \( |\Theta| \sim 70^o \), and an arbitrary value of the Sun velocity, as embodied in the rescaling \( \sqrt{k} = \frac{370 \ \text{km/s}}{V_{\text{sun}}} \), one obtains good consistency among all observables (see Table 2) with an average RMS parameter \( (1/2 - \beta + \delta) \sim k(17 \pm 3) \cdot 10^{-10} \). For \( k \) of order unity, this is consistent with the prediction in Eq. (59).
On the other hand, eliminating such $k$–dependence of the RMS parameter requires a preliminary determination of $V_{\text{sun}}$. As explained in Sect. 3, this can only be done after observing the effect of the Earth’s orbital motion through the relation $\Delta \varphi \sim \frac{30 \text{ km/s}}{|V_{\text{sun}}|}$, $\Delta \varphi$ being the opening angle of the ‘aberration circle’ induced by the Earth’s revolution around the Sun. In turn, this requires more data, covering larger parts of the Earth’s orbit, that exhibit seasonal modulations of the signal. For instance, Miller’s observations were indicating a variation of the declination from $|\tilde{\Theta}| \sim 77^\circ$ in February-April to $|\tilde{\Theta}| \sim 62^\circ$ in August-September. This should correspond to a $\sim +70\%$ increase of the daily variations in the two periods. Such kind of confirmations would represent clean experimental evidence for the existence of a preferred frame, a result with far-reaching implications for both particle physics and cosmology.

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