Fixed Time Disturbance Observer Based Sliding Mode Control for a Miniature Unmanned Helicopter Hover Operations in Presence of External Disturbances

IHSAN ULLAH AND HAI-LONG PEI, (Member, IEEE)

1School of Automation Science and Engineering, South China University of Technology, Guangzhou 510640, China
2Key Lab of Autonomous Systems and Networked Control, Ministry of Education, Guangzhou 510640, China
3Unmanned System Engineering Center of Guangdong Province, Guangzhou 510640, China

Corresponding author: Hai-Long Pei (auhlpei@scut.edu.cn)

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ABSTRACT This paper presents a novel fixed time sliding mode disturbance observer (FTSMDO) based second-order fixed time sliding mode control (FTSMC) for small scale unmanned helicopter to do hover operations in the presence of external disturbances. Sliding mode control is insensitive to matched uncertainties but sensitive to mismatched uncertainties. The novel FTSMDO exactly estimates the total mismatched uncertainties effecting the sliding mode control performance. To counter the mismatched uncertainties a new sliding surface augmented by the total mismatched disturbance approximated by the FTSMDO is defined. The second-order FTSMC forces the system states to reach the sliding surface in fixed time and then on the sliding surface the system states exponentially converge to the desired equilibrium point. The control performance is compared with the disturbance observer based sliding mode controller (DOB-SMC) and shows superior performance.

INDEX TERMS Unmanned helicopter, external disturbances, fixed time Sliding mode control, fixed time disturbance observer, mismatched uncertainty.

I. INTRODUCTION

Miniature helicopters are highly unstable, agile, nonlinear under-actuated systems with significant inter-axis dynamic coupling. They are considered to be much more unstable than fixed-wing unmanned air vehicles, and constant control action is required at all times. However, helicopters are highly flexible aircraft, having the ability to hover, maneuvers accurately and carry heavy loads relative to their own weight [1]. Fixed-wing aircraft are used for application in favorable non-hostile conditions but in adverse conditions, agile miniature helicopters become a necessity. The conditions where a helicopter can perform better than fixed-wing UAVs include military investigation, bad weather, firefighting, search and rescue, accessing remote locations and ship operations. In such conditions, helicopters are subjected to unknown external disturbances such as wind and ground effect. These external disturbances have a significant opposing effect on the helicopter stability and can have disastrous results in extreme cases. So it is essential to design a controller for the helicopter which can effectively reject the effect of these unknown external disturbances.

In the last two decades, there is substantial research about helicopter control problems. Early results showed that classical control methods using Single-Input Single-Output feedback loops for each input exhibit moderate performance since they are unable to couple the highly multivariable dynamics of the helicopter [2]. Control schemes typically used to maintain stable control of helicopters include PID [3], Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG) [4], H2 [5], H∞ [6], [7]. The majority of linear controllers designed for
unmanned helicopters are based on the $H_{\infty}$ method. An $H_{\infty}$ static output feedback control design method [8] was proposed for the stabilization of a miniature unmanned helicopter at hover. An interesting comparative study between several control methods is given in [9], [10]. There are lots of results about Disturbance Observer-based control techniques [11]–[14]. A direct feed-through simultaneous state and disturbance observer [14] is used where the control and observer gains are obtained using $H_{\infty}$ synthesis but in presence of external disturbances, there is a steady-state error in helicopter translational dynamics. Back-stepping control design techniques are used for linear tracking control of a miniature helicopter without considering external disturbances [15], the control design is based on the linearized model of the helicopter and shows good results in X-plane flight simulator. DOB-SMC [16], [17] is used for controlling magnetic levitation suspension system and small scale unmanned helicopter hover operations respectively. The experimental results showed that the proposed method has excellent robustness in the presence of both matched and mismatched uncertainties. In [18], [19] second-order sliding mode control law is used which provides finite time convergence but convergence time depends on the initial condition. Fixed time consensus control of multi agent and leader follower system is presented in [20], [21]. A novel multi-variable FTSMC method is proposed in [22] where a formula is derived to estimate the fixed convergence time of the controller. The fixed convergence time of the FTSMC does not depend on the initial condition.

In this paper, a novel fixed time sliding mode disturbance observer based second-order fixed time sliding mode control FTSMDO-FTSMC design technique is presented for small scale unmanned helicopter to do hover operations in the presence of external disturbances. The controller design is based on the linearized state-space model of the helicopter. As in [14], [15]and [17] the linearized model of the helicopter can be divided into two subsystems, such as the longitudinal-lateral subsystem and the heading-heave subsystem. As there is no strong coupling between the two subsystems at hover and limited by the scope of the paper, for hovering only the longitudinal-lateral dynamics are considered for designing the control law. To counteract both matched and mismatched uncertainties a new sliding surface is designed based on the total disturbance estimated by the FTSMDO. The model mismatch and external disturbances are estimated as lumped disturbances and are compensated in the controller design. In [16], [17] the proposed disturbance observer (DOB) asymptotically estimates the model mismatch and external disturbance at each and every channel separately. But in this paper using FTSMDO, the total mismatched disturbance (caused by uncertainties and external disturbances acting through all mismatched channels) effecting the sliding mode control performance is estimated as lumped disturbance, exactly in the fixed time. The rotor flapping dynamics are approximated by the steady-state dynamics of the main rotor which helps reducing controller order. Simulink simulations have demonstrated superior performance of the FTSMDO-FTSMC compared to the DOB-SMC.

The proposed control method has two attractive features:
- It is a simple first order fixed time disturbance observer which approximate the total mismatch disturbance acting on an $n^{th}$ order system through all $n$ channels provided it satisfy some assumption. Other disturbance observers are all $n^{th}$ order for an $n^{th}$ order system.
- The chattering problem is substantially reduced due to the use of second-order FTSMC.

The rest of the paper is organized as follows: The linearized model of the helicopter is derived from the nonlinear model in section 2. FTSMDO is designed in section 3. The proposed controller is derived in detail in section 4. Simulation results are given in section 5 and finally concluding remarks are given in section 6.

II. HELICOPTER MODEL
A. NONLINEAR DYNAMICS OF THE HELICOPTER
The general 11th state nonlinear model [23] of the miniature unmanned helicopter is given as

$$
\begin{align*}
\dot{u} &= vr - wq - g \sin \theta + X_{mr}/m + dw_1 \\
\dot{v} &= wp - ur + g \sin \phi \cos \theta + Y_{mr}/m + dw_2 \\
\dot{w} &= uq - vp + g \cos \phi \cos \theta + Z_{mr}/m + dw_3 \\
\dot{p} &= \phi + (\sin \phi \tan \theta) q + (\cos \phi \tan \theta) r \\
\dot{q} &= (\cos \phi) q - (\sin \phi) r \\
\dot{\psi} &= (\sin \phi / \cos \phi) q + (\cos \phi / \cos \phi) r \\
\dot{\phi} &= pr (I_{yy} - I_{zz}) / I_{xx} + L_{mr} / I_{xx} + dw_4 \\
\dot{\theta} &= q r (I_{zz} - I_{xx}) / I_{yy} + M_{mr} / I_{yy} + dw_5 \\
\dot{\omega} &= N_1 v + N_2 p + N_3 w + N_r r + N_{ped} \cdot u_{ped} + N_{col} \cdot u_{col} + dl + dm + dw_6 \\
\dot{a} &= -q - 1/tf \cdot a + A_b \cdot b + A_lom \cdot u_{lon} + A_lat \cdot u_{lat} \\
\dot{b} &= -p - 1/tf \cdot b + B_a \cdot a + B_lom \cdot u_{lon} + B_lat \cdot u_{lat}
\end{align*}
$$

where $\mathbf{x} = [u \ v \ w \ \phi \ \theta \ \psi \ p \ q \ r \ a \ b]^T$ is the vector of the state variables all available for measurement except $a$ and $b$; $u, v$ and $w$ represents linear velocities in longitudinal, lateral and vertical direction respectively; $m$ is mass of helicopter; $g$ represents acceleration due to gravity; $p, q$ and $r$ represents angular velocities in roll, pitch and yaw axis respectively; $\phi, \theta$ and $\psi$ are Euler angles of roll, pitch and yaw axes; $\mathbf{u}_e(t) = [u_{lon} \ u_{lat} \ u_{col} \ u_{ped}]^T$ is the control input vector; $dw_i \ \forall \ i = 1, 2, \ldots 6$ are unknown external wind disturbances effecting translational as well as rotational dynamics of helicopter; $I_{xx}, I_{yy}$ and $I_{zz}$ are the rolling moment of inertia, pitching moment of inertia and yawing moment of inertia respectively; $a$ and $b$ are flapping angles of tip-path-plane(TPP) in longitudinal and lateral direction respectively; $X_{mr}, Y_{mr}$ and $Z_{mr}$ are the force components of main rotor trust along $x$, $y$ and $z$ axis; $L_{mr}$ and $M_{mr}$ are roll and pitch moments.
generated by main rotor; \( N_c \), \( N_p \), \( N_w \) and \( N_f \) are helicopter stability derivatives and \( N_{ped} \) and \( N_{col} \) are input derivatives of yaw dynamics identified as in [15]; \( T_f \) is flapping time constant; \( B_{\text{a}}, B_{\text{lat}} \) and \( B_{\text{lon}} \) are lateral flapping derivatives; \( A_b, A_{\text{lon}} \) and \( A_{\text{lat}} \) are longitudinal flapping derivatives. A diagram showing the directions of the helicopter body fixed coordinate system is given in Fig.1.

The force components generated by the main rotor trust in \( x, y \) and \( z \) direction are given as

\[
X_{\text{mr}} = -T \sin a \\
Y_{\text{mr}} = T \sin b \\
Z_{\text{mr}} = -T \cos a \cos b
\]

where \( T \) is the total trust generated by the main rotor. The moments generated by the main rotor along the \( x \) and \( y \) direction are calculated as

\[
L_{\text{mr}} = (k_b + T \cdot h_{\text{mr}}) \sin b \\
M_{\text{mr}} = (k_b + T \cdot h_{\text{mr}}) \sin a
\]

where \( k_b \) is the torsional stiffness of the main rotor hub; \( h_{\text{mr}} \) is the main rotor hub height above the center of gravity of the helicopter.

The trust of the main rotor is calculated by iteratively solving the equations of trust and the induced inflow velocity [24].

\[
T = (w_b - v_i) \frac{\rho \Omega R^2 C_{\text{lb}}^m b_m c_m}{4} \\
v_i^2 = \sqrt{\left(\frac{\dot{v}_i^2}{2}\right)^2 + \left(\frac{T}{2\rho \pi R^2}\right)^2 - \frac{\dot{v}_i^2}{2}} \\
\dot{v}_i^2 = u^2 + v^2 + w(w - 2v_i) \\
w_b = w + \frac{2}{3} \Omega R k_{\text{col}} u_{\text{col}}
\]

where \( v_i \) is the induced inflow velocity; \( \Omega \) is rotational speed of the main rotors; \( \rho \) is air density; \( R \) is main rotor radius; \( b_m \) is the number of main rotor blades; \( c_m \) is the chord length of the main rotor; \( C_{\text{lb}}^m \) is coefficient of lift curve slope of the main rotor; \( k_{\text{col}} \) is link gain from the collective actuator to the main blade.

**B. LINEARIZED STATE SPACE MODEL OF THE HELICOPTER**

To derive the control law, the nonlinear model (1) of the helicopter is linearized at hover condition as

\[
\dot{x} = Ax + Bu + Ed_t, \\
y = Cx
\]

At hover condition, the longitudinal-lateral and heading-heave dynamics of the helicopter are weakly coupled with each other and are expressed as two separate sub-systems [15].

\[
\dot{x}_1 = A_1 x_1 + B_1 u_{\text{col}} + E_{\text{d}1}, \\
y_1 = C_1 x_1
\]

\[
\dot{x}_2 = A_2 x_2 + B_2 u_{\text{col}} + E_{\text{d}2}, \\
y_2 = C_2 x_2
\]

where (6) represents longitudinal-lateral subsystem and (7) represents the heading-heave subsystem. For hover operation, the FTSMDO-FTSMC control law is only derived for the longitudinal-lateral subsystem and heading-heave dynamics are regulated at hover condition using PID controllers.

The subsystem (6) is expanded as

\[
\dot{u} = X_u u - g \theta + d_1 \\
\dot{v} = Y_{\text{u}} v + g \phi + d_2 \\
\dot{\theta} = q + d_{13} \\
\dot{\phi} = p + d_{14} \\
\dot{q} = M_{\text{u}} u + M_{\text{v}} v + M_{\text{a}} a + d_{15} \\
\dot{p} = L_{\text{u}} u + L_{\text{v}} v + L_{\text{a}} a + d_{16} \\
\dot{a} = -q - 1/T_f \cdot a + A_b \cdot b + A_{\text{lon}} \cdot u_{\text{lon}} + A_{\text{lat}} \cdot u_{\text{lat}} \\
\dot{b} = -p - 1/T_f \cdot b + B_a \cdot a + B_{\text{lon}} \cdot u_{\text{lon}} + B_{\text{lat}} \cdot u_{\text{lat}}
\]

where \( X_u, Y_{\text{u}}, M_{\text{u}}, M_{\text{v}}, M_{\text{a}}, L_{\text{u}}, L_{\text{v}} \) and \( L_{\text{a}} \) are stability derivatives; \( A_{\text{lon}}, A_{\text{lat}}, B_{\text{lon}} \) and \( B_{\text{lat}} \) are input derivatives; \( d_i \) \( \forall \) \( i = 1, 2, \cdots, 6 \) is the total disturbance including both model mismatch and external disturbances acting at channel \( i \).

Approximating the flapping angles \( a \) and \( b \) by the steady state dynamics of the main rotor [25] as

\[
a = -T_f q + T_f (A_b \cdot b + A_{\text{lon}} \cdot u_{\text{lon}} + A_{\text{lat}} \cdot u_{\text{lat}}) \\
b = -T_f p + T_f (B_a \cdot a + B_{\text{lon}} \cdot u_{\text{lon}} + B_{\text{lat}} \cdot u_{\text{lat}})
\]

Substituting (10) in (8) gives the reduced order linearized model, written in state space form as following

\[
\dot{x}_r = A_r x_r + B_r u_{\text{col}} + E_r d_{\text{tr}}
\]

\[
y_1 = C_r x_r
\]

where \( x_r = [u \ v \ \theta \ \phi \ q \ p]^T; u_{\text{col}} = [u_{\text{lon}} \ u_{\text{lat}}]^T; d_{\text{tr}} = [d_1 \ d_2 \ d_3 \ d_4 \ d_5 \ d_6]^T; E_r \) is 6 \times 6 identity matrix; \( y_r \) is
To derive the proposed control law for hover operation, first and satisfies

\[ K = K_2K_3 \left[ u_{lon} \quad u_{iat} \right]^T + K_2K_4 \left[ u \quad v \quad q \quad p \right]^T + K_1^2d_{tr1} + K_1K_2d_{tr2} + K_2d_{tr3} \]  

where

\[ d_{tr3} = \begin{bmatrix} d_{15} \\ d_{16} \end{bmatrix}; \quad K_3 = \begin{bmatrix} M_{lon} & M_{lat} \\ L_{lon} & L_{lat} \end{bmatrix}; \]

\[ K_4 = \begin{bmatrix} M_u & M_v & -M_q & -M_p \\ L_u & L_v & -L_q & -L_p \end{bmatrix} \]

Combining (15), (16) and (17) gives

\[ \dot{y}_r = f_1(x) \quad \dot{y}_r = f_2(x) + D_1 \quad \ddot{y}_r = f_3(x) + D_2 \quad \dddot{y}_r = f_4(x) + Ku + D_3 \]

where

\[ f_1(x) = \left[ u \quad v \right]^T; \]

\[ f_2(x) = K_1 \left[ u \quad v \right]^T + K_2 \left[ \theta \quad \phi \right]^T; \]

\[ f_3(x) = K_2^2 \left[ u \quad v \right]^T + K_1K_2 \left[ \theta \quad \phi \right]^T + K_2 \left[ q \quad p \right]^T; \]

\[ f_4(x) = K_1^2 \left[ u \quad v \right]^T + K_1K_2 \left[ \theta \quad \phi \right]^T + K_1K_2 \left[ q \quad p \right]^T + K_2K_3 \left[ u_{lon} \quad u_{iat} \right]^T + K_2K_4 \left[ u \quad v \quad q \quad p \right]^T \]

\[ K = K_2K_3; \]

\[ u = \left[ u_{lon} \quad u_{iat} \right]^T, \]

\[ D_1 = d_{tr1}; \]

\[ D_2 = K_1d_{tr1} + K_2d_{tr2} \]

and

\[ D_3 = K_1^2d_{tr1} + K_1K_2d_{tr2} + K_2d_{tr3}. \]

From assumption 4 it is concluded

\[ |D_1| \leq \delta_{ij} \quad \forall \quad i = 1, 2, 3 \]

\[ 0 < |D_1| \leq \delta_{ij} \quad (19) \]

where all \( \delta_{ij} \) are positive bounded unknown constants.

III. FIXED TIME DISTURBANCE OBSERVER

In this section FTSMDO is designed to estimate the total mismatched disturbance effecting the performance of FTSMC. In order to design FTSMC for system (11) in presence of mismatched uncertainties the sliding surface is defined as

\[ \sigma = C_1y_r + C_2\dot{y}_r + \dddot{y}_r \]

where

\[ \sigma = \left[ \sigma_1 \quad \sigma_2 \right]^T; \quad C_1 = \text{diag}(c_1, c_2); \quad C_2 = \text{diag}(c_3, c_4). \]

\( C_1 \) and \( C_2 \) are designed such that \( \sigma_{1,2} = 0 \) is Hurwitz.

Substituting (18) in (20) gives

\[ \sigma = C_1f_1(x) + C_2f_2(x) + f_3(x) + D \]

where \( D = C_2D_1 + D_2 \) is the total mismatched uncertainty effecting the performance of the FTSMC and FTSMDO is designed to estimate it.
The first step in designing FTSMDO is to define an auxiliary sliding surface given as
\[
\sigma_1 = Z_1 - C_2 f_1(x) - f_2(x)
\]
(22)
where
\[
\dot{Z}_1 = -K_{11} |\sigma_1|^{1/2} \text{sgn}(\sigma_1) - K_{12} |\sigma_1|^{p_1} \text{sgn}(\sigma_1) + C_2 f_2(x) + f_3(x) + Z_2
\]
(23)
\[
\dot{Z}_2 = -K_{13} \text{sgn}(\sigma_1)
\]
(24)
where \( p_1 > 1 \) and \( K_{11} = \text{diag}(k_1, k_2), K_{12} = \text{diag}(k_3, k_4), K_{13} = \text{diag}(k_5, k_6) \).
All \( k_1, k_2, k_3, k_4, k_5 \) and \( k_6 \) are positive bounded constant.
Differentiating (22) and substituting (23) and (18) gives
\[
\dot{\sigma}_1 = -K_{11} |\sigma_1|^{1/2} \text{sgn}(\sigma_1) - K_{12} |\sigma_1|^{p_1} \text{sgn}(\sigma_1) + Z_2 - D
\]
(25)
Let
\[
\sigma_2 = Z_2 - D
\]
(26)
Differentiating (26) gives
\[
\dot{\sigma}_2 = -K_{13} \text{sgn}(\sigma_1)
\]
(27)
Combining (25) and (27) gives
\[
\dot{\sigma}_1 = -K_{11} |\sigma_1|^{1/2} \text{sgn}(\sigma_1) - K_{12} |\sigma_1|^{p_1} \text{sgn}(\sigma_1) + \sigma_2
\]
\[
\dot{\sigma}_2 = -K_{13} \text{sgn}(\sigma_1)
\]
(28)
Definition [26]: The origin of the system (28) is said to be fixed-time stable equilibrium point if it is globally finite-time stable with bounded convergence time \( T(\sigma_0) \). That is, there exists a bounded positive constant \( T_{\text{max}} \) such that \( T(\sigma_0) < T_{\text{max}} \) is satisfied, where \( \sigma_0 = [\sigma_1(0) \, \sigma_2(0)] \).

Theorem 1: Considering the 2nd order system (28), both \( \sigma_1 \) and \( \sigma_2 \) converge uniformly to the origin in fixed time
\[
T_f \leq \left( \frac{1}{K_{12} (p_1 - 1) \varepsilon^{n-1} + \frac{2\varepsilon^{1/2}}{K_{11}}} \right) \left( 1 + \frac{1}{1 - H(K_{11})/K_{11}} \right)
\]
(29)
where \( T_f = [tf_1 \, tf_2]^T; \varepsilon > 0; K(K_{11}) = 1/K_{11} + (2\varepsilon/K_{13} K_{11})^{1/3} \); \( e \) is the base of natural logarithms, provided the following conditions holds:
\[
\begin{align*}
[1 \quad 0]K_{13} [1 \quad 0]^T & > 0; \\
[0 \quad 1]K_{13} [0 \quad 1]^T & > 0; \\
[1 \quad 0]K_{11} H^{-1}(K_{11}) [1 \quad 0]^T & > [1 \quad 0]K_{13} [1 \quad 0]^T; \\
[0 \quad 1]K_{11} H^{-1}(K_{11}) [0 \quad 1]^T & > [0 \quad 1]K_{13} [0 \quad 1]^T.
\end{align*}
\]
Proof: Considering the following second order system in presence of bounded disturbance \( \xi \)
\[
\dot{\xi}(t) = -k_{11} |x(t)|^{1/2} \text{sgn} (x(t))
\]
\[
-k_{12} |x(t)|^{p_2} \text{sgn} (x(t)) + y(t)
\]
(30)
the fixed time in which the states \( x(t) \) and \( y(t) \) converge uniformly to the origin is given by [22]
\[
T_f \leq \left( \frac{1}{k_{12} (p_1 - 1) \varepsilon^{n-1} + \frac{2\varepsilon^{1/2}}{k_{11}}} \right) \left( 1 + \frac{1}{m_1 (1/M_1) - h(k_{11})/k_{11}} \right)
\]
(31)
where \( \varepsilon > 0; M_1 = k_{13} + \zeta; m_1 = k_{13} - \zeta; h(k_{11}) = 1/k_{11} + (2\varepsilon/m_1 k_{11})^{1/3}; e \) is the base of natural logarithms; \( k_{13} > \zeta \) and \( k_{11} h^{-1}(k_{11}) > M_1 \).

Dynamics of (28) is similar to (30) except \( \zeta \) is zero. Therefore, by substituting \( \zeta = 0 \) in (31) the convergence time \( T_f \) of \( \sigma_1 \) and \( \sigma_2 \) to the origin is simplified as (29).
According to theorem 1 as \( \sigma_1 \) and \( \sigma_2 \) goes to zero in fixed time \( T_f \), then (26) implies that
\[
Z_2 = D
\]
(32)

IV. CONTROLLER DESIGN
In this section FTSMC is designed to stabilize helicopter to do hover operations in presence of external wind disturbances. The sliding surface augmented with the total mismatched disturbance approximated by FTSMDO is designed as
\[
\dot{S}_1 = C_1 f_1(x) + C_2 f_2(x) + f_3(x) + \dot{D}
\]
(33)
where \( \dot{D} \) is estimation of \( D \) given by the disturbance observer and parameters \( C_1 \) and \( C_2 \) are same as in (20).

Theorem 2: Considering system (11) under the fixed time sliding mode control law
\[
u = -k \left( C_1 f_2(x) + C_2 f_3(x) + f_4(x) + K_2 |S_1|^{1/2} \times \text{sgn}(S_1) + K_{22} |S_1|^{p_2} \text{sgn}(S_1) + K_{23} \int_{0}^{t} \text{sgn}(S_1)dS_1 \right)
\]
(34)
The closed loop system is exponentially stable and the system output \( y_r \) exactly converge to the desired equilibrium point, provided the following conditions holds:
\[
\begin{align*}
& \begin{bmatrix} 1 & 0 \end{bmatrix} K_{23} [1 \quad 0]^T > [1 \quad 0] \dot{\mathbf{D}}; \\
& \begin{bmatrix} 0 & 1 \end{bmatrix} K_{23} [0 \quad 1]^T > [0 \quad 1] \dot{\mathbf{D}}; \\
& \begin{bmatrix} 1 & 0 \end{bmatrix} K_{21} H^{-1}(K_{21}) [1 \quad 0]^T > [1 \quad 0] \mathbf{M}_2; \\
& \begin{bmatrix} 0 & 1 \end{bmatrix} K_{21} H^{-1}(K_{21}) [0 \quad 1]^T > [0 \quad 1] \mathbf{M}_2.
\end{align*}
\]
where
\[
\mathbf{M}_2 = K_{23} [1 \quad 1]^T + \dot{\mathbf{D}};
\]
\[
\mathbf{m}_2 = \begin{bmatrix} m_{21} & m_{22} \end{bmatrix}^T = K_{23} [1 \quad 1]^T - \dot{\mathbf{D}};
\]
\[
H(K_{21}) = 1/K_{21} + (2\varepsilon/\text{diag}(m_{21}, m_{22}) K_{21})^{1/3};
\]
K_{21} = \text{diag} \ (k_7, k_8); \ K_{22} = \text{diag} \ (k_9, k_{10});
K_{23} = \text{diag} \ (k_{11}, k_{12})
and all k_7, k_8, k_9, k_{10}, k_{11}, k_{12} are positive bounded constant.

Proof: Differentiating the sliding surface (33) results
\[ \dot{S}_1 = C_1(f_2(x) + D_1) + C_2(f_3(x) + D_2) + f_4(x) + D_3 + \dot{D} + Ku \]  
(35)

Substituting (34) in (35) gives
\[ \dot{S}_1 = -K_{21}|S_1|/2 \text{sgn} (S_1) - K_{22}|S_1|/2 \text{sgn} (S_1)
- K_{23} \int_{0}^{t} \text{sgn}(S_1) dS_1 + C_1D_1 + C_2D_2 + D_3 + \dot{\hat{D}}. \]  
(36)

Let
\[ S_2 = -K_{23} \int_{0}^{t} \text{sgn}(S_1) dS_1 + C_1D_1 + C_2D_2 + D_3 + \dot{\hat{D}} \]  
(37)

Differentiating (37) and combining with (36) gives
\[ \dot{S}_1 = -K_{21}|S_1|/2 \text{sgn} (S_1) - K_{22}|S_1|/2 \text{sgn} (S_1) + S_2
\dot{S}_2 = -K_{23} \text{sgn} (S_1) + \dot{\hat{D}} \]  
(38)
where
\[ \begin{bmatrix} 1 & 0 \end{bmatrix} K_{23} \begin{bmatrix} 1 & 0 \end{bmatrix}^T > \begin{bmatrix} 1 & 0 \end{bmatrix} \dot{\hat{D}}; \\
\begin{bmatrix} 0 & 1 \end{bmatrix} K_{23} \begin{bmatrix} 0 & 1 \end{bmatrix}^T > \begin{bmatrix} 0 & 1 \end{bmatrix} \dot{\hat{D}} \]

Dynamics of (38) is similar to (30) and according to theorem1 S_1 and S_2 goes uniformly to zero in fixed time. The fixed convergence time depends on the parameters of control law (34) and can be calculated in a similar fashion as in theorem1. At condition S_1 = 0, (33) gives
\[ \dot{y}_r = -C_1y_r - C_2\dot{y}_r + D - \dot{\hat{D}} \]  
(39)

As FTSMDO is designed such that after time T_f the disturbance approximation error converge to zero so
\[ D - \dot{\hat{D}} = 0 \]  
(40)
and after time T_f, (39) reduces to
\[ \dot{y}_r = -C_1y_r - C_2\dot{y}_r \]  
(41)
Now (41) is Hurwitz and the system output y_r is uniformly ultimately exponentially convergent to the desired equilibrium point.

V. SIMULATION RESULTS
In this section evaluation of the proposed controller (34) is presented. Performance of (34) is compared with a DOB-SMC. The sliding surface of the traditional DOB-SMC method is designed as
\[ S_3 = C_1f_1(x) + C_2\left(f_2(x) + \hat{D}_1\right) + f_3(x) + \hat{D}_2 \]  
(42)

| TABLE 1. Parameters of Raptor 90SE RC helicopter [27]. |
|------------------------|------------------------|------------------------|
| Nonlinear parameter    | Value                  | Nonlinear parameter    | Value                  |
|------------------------|------------------------|------------------------|------------------------|
| m                      | 7.495 kg               | f_0                    | 172.788 rad/s          |
| h_0                    | 2                      | c_m                    | 0.606 m                |
| g                      | 9.81 m/s^2             | C_{mz}                 | 20.0734                |
| k_2                     | 0.3813                 | k_3                    | 167.6592 m/s/|rad |
| I_x                     | 0.1855 kg.m^2          | N_{x}                  | 2.982                  |
| N_x                     | -10.71                 | N_{y}                  | 0                       |
| t_f                     | 0.0285 sec             | A_{x1}                 | 4.059                  |
| B_{total}              | 4.085                  | A_{x2}                 | -0.01610               |

where \( \dot{D}_1 = \hat{d}_{tr1} \) and \( \dot{D}_2 = K_1\hat{d}_{tr1} + K_2\hat{d}_{tr2} \). The disturbance observer used to estimate the unknown disturbance vector \( \hat{d}_{tr} \) is designed as
\[ \dot{P} = -LE_r (P + Lx_r) - L(A_r x_r + B_r u_c1) \]
(43)

where \( \hat{d}_{tr} = \left[ \dot{d}_{t1} \dot{d}_{t2} \dot{d}_{t3} \dot{d}_{t4} \dot{d}_{t5} \dot{d}_{t6} \right]^T \) is the disturbance estimation vector, \( P \) is a 6 \times 1 auxiliary vector and \( L = \text{diag} \ (l_1, l_2, l_3, l_4, l_5, l_6) \) is the observer gain.

Using DOB there is initial peaking at time \( t_0 \) in disturbance approximation which causes higher control gain and even can takes the control input to saturation. So to avoid the initial peaking phenomena the observer gain \( L \) is designed as follows
\[ L = \begin{cases} \sin \left( \pi t / 2 \right) Q I, & 0 \leq t \leq 1 \\ Q I, & t > 1 \end{cases} \]  
(44)
where \( Q \) is any positive number and \( I \) is 6 \times 6 identity matrix. So \( L \) is zero at \( t_0 \) and positive elsewhere. It satisfies the condition that \(- L \) is Hurwitz.

Finally DOB-SMC is designed as
\[ u = -K^{-1} \left[ C_1 (f_2(x) + \hat{D}_1) + C_2 (f_3(x) + \hat{D}_2) + f_4(x) + \hat{D}_3 + \beta \text{sgn}(S_3) \right] \]  
(45)

where \( E_{D1} = D_1 - \hat{D}_1 \) and \( E_{D2} = D_2 - \hat{D}_2 \)
\[ \beta = \text{diag} \ (\beta_1, \beta_2) \]
\[ \beta_1 > \left( \left[ 1 \ 0 \right] \left[ C_1 E_{D1} + C_2 E_{D2} + E_{D3} + C_2 \hat{D}_1 + \hat{D}_2 \right] \right) \]
\[ \beta_2 > \left( \left[ 0 \ 1 \right] \left[ C_1 E_{D1} + C_2 E_{D2} + E_{D3} + C_2 \hat{D}_1 + \hat{D}_2 \right] \right) \]

Raptor 90 SE radio controlled helicopter is used in these simulations. The Simulink model is established using the nonlinear model of helicopter defined in (1) and then the proposed FTSMDO-FTSMC (34) and the DOB-SMC (45) based on the linearized model (11) are applied on it to check the hovering performance of the helicopter in presence of wind disturbance. Parameters of the nonlinear model of the helicopter are given in table 1 and parameters of the reduced order linearized model (11) are given in table 2.
### A. CASE1 PERFORMANCE COMPARISON

First performance of the two controllers FTSMDO-FTSMC and DOB-SMC is done in absence of external disturbance. The initial states of helicopter system (1) are set as $u = 1$ and $v = -1$, the rest of the states are zero initially. The parameters of the FTSMDO-FTSMC are:

$$
C_1 = \text{diag} (10, 10) ; \quad C_2 = \text{diag} (25, 25) ; \\
K_{11} = \text{diag} (300, 300) ; \quad K_{12} = \text{diag} (10, 10) ; \\
K_{13} = \text{diag} (25, 25) ; \quad K_{21} = \text{diag} (20, 20) ; \\
K_{22} = \text{diag} (1, 1) ; \quad K_{23} = \text{diag} (160, 160) ; \\
p_1 = 3 ; \quad p_2 = 1.1.
$$

while parameters of the DOB-SMC are:

$$
C_1 = \text{diag} (10, 10) ; \quad C_2 = \text{diag} (25, 25) ; \\
\beta_1 = 30 ; \quad \beta_2 = 30 ; \quad Q = 10.
$$

It is observed from Fig.2 that DOB-SMC and FTSMDO-FTSMC have almost the same settling time but FTSMDO-FTSMC have reduced control chattering compare to DOB-SMC as shown in Fig.3. Fig.4 shows that the mismatched disturbances ($d_{t1}, d_{t2}, d_{t3}, d_{t4}$) approximated by DOB goes to zero as soon as all the states of the system reach zero as there was no external disturbance applied.
on the system. There is small scale matched disturbances \((d_{t5}, d_{t6})\) in steady state due to the model mismatch caused by order reduction. Fig.5 shows that the total mismatched disturbances approximated by FTSMDO. It goes to zero after initial settling time. Calculating the total mismatched disturbance \(C_{21}D_{1} + D_{2}\) using DOB output \((\hat{d}_{t1}, \hat{d}_{t2}, \hat{d}_{t3}, \hat{d}_{t4})\) and comparing it with FTSMDO output it is clear that contrary to DOB, FTSMDO has low peaking during initial settling time.

**B. CASE2 HANDLING MISMATCH UNCERTAINTIES**

In second case to compare the mismatched uncertainty handling capacity of both controllers, external wind disturbances \(d_{w1}\) and \(d_{w2}\) are applied on the helicopter system (1) defined as

\[
d_{w1} = d_{w2} = d_{w} = \begin{cases} 0, & 0 \leq t \leq 1 \\ 1, & t > 0.5 \end{cases} \tag{46}
\]

Control parameters are same as case 1. All initial states are zero. In Fig.6 it is observed that both FTSMDO-FTSMC and DOB-SMC suppress the external wind disturbances and bring back \(u\) and \(v\) to the desired equilibrium point. Fig.7 shows control inputs and confirm that FTSMDO-FTSMC have reduced control chattering. Fig8 shows external wind disturbances approximated by DOB and Fig9 shows the total mismatched disturbance approximated by FTSMDO.

**VI. CONCLUSION**

This paper presented a novel FTSMDO based FTSMC for miniature unmanned helicopter to do hover operations in presence of external disturbance. The main contribution is design of FTSMDO which can estimate the total mismatch disturbance (caused by uncertainties and external disturbances acting through all mismatched channels) effecting the sliding mode control performance and defining a new sliding surface (augmented by this total disturbance) to design the FTSMDC. The FTSMDC forces the system states to
reach the defined sliding surface and then slides towards the desired equilibrium point in presence of mismatched uncertainties. Simulation results showed superior performance of FTSMDO-FTSMC compared to DOB-SMC.

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IHSAN ULLAH received the B.Sc. degree in mechatronics engineering from the University of Engineering and Technology, Pakistan, in 2011, and the M.Sc. degree in mechatronics engineering from Beihang University, Beijing, China, in 2014. He is currently pursuing the Ph.D. degree with the School of Automation Sciences and Engineering, South China University of Technology, Guangzhou, China. His main research interests are in the fields of UAVs, helicopter control, nonlinear control, robust control, sliding mode control, uncertainty, and disturbance observer.

HAI-LONG PEI (Member, IEEE) received the bachelor’s and master’s degrees from Northwestern Polytechnical University, China, in 1986 and 1989, respectively, and the Ph.D. degree from the South China University of Technology, China, in 1992. He is currently a Professor at the School of Automation Science and Engineering, South China University of Technology, the Director of the Key Lab of Autonomous Systems and Networked Control, Ministry of Education, and the Director of the Unmanned System Engineering Center of Guangdong Province. He works on unmanned aerial systems and robotic control. He currently serves as the Deputy Editor-in-Chief of *Journal of Control Theory and Applications*, an Associate Editor of *Journal of Intelligent & Robotic Systems*, and an Associate Editor of *Acta Automatica Sinica*. 

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