On three-dimensional boundary layer stability analysis on curved aerodynamic surfaces

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Abstract. Numerical simulation of three-dimensional flow past a prolate spheroid located at an angle of attack of $10^\circ$ accompanied by the laminar-turbulent transition was performed for subsonic ($M_\infty = 0.136$) and transonic ($M_\infty = 0.75$) velocities. The general-purpose gas-dynamic package ANSYS Fluent in integration with a laminar-turbulent transition module based on software bundle LOTRAN 3 was used. The principles of numerical simulation of the main flow are discussed: requirements for the near-wall region allocation and independence of the results from various methods of presetting the position of forced turbulence. It was shown that two instability mechanisms take place in the boundary layer of the spheroid: the Tollmien–Schlichting instability and the cross-flow instability with a predominance of the last on side surfaces. A comparison of predicted position of the transition for different flow regimes was performed.

1. Introduction
The problem of determining and predicting the position of laminar-turbulent transition (LTT) in boundary layers (BL) of various elements of a transport aircraft is important, as the flow laminarization is a promising approach to reduce the drag caused by turbulent BL. To use this approach, engineers need appropriate tools to predict the LTT with an acceptable practical accuracy. The so-called $e^X$-method is a popular engineering method for predicting the LTT, the method being based on criteria of certain threshold amplitudes of disturbances propagating in the BL. The popularity of this method in respect to the aerodynamic applications is motivated by the use of physically justified linear theory of hydrodynamic stability to compute the amplitudes, which is valid for both two-dimensional and three-dimensional incompressible and compressible (up to hypersonic) flows, if the level of freestream turbulence is sufficiently low (e.g. flight conditions, low noise wind tunnels). This approach is used, e.g., in proprietary computer codes of leading aerospace companies [1-2].

At present time, this method for determining the LTT position is absent in general-purpose gas-dynamic packages [3], but has been implemented as an additional extension module integrated into the ANSYS Fluent package [4-6]. This LTT module was created on the basis of the LOTRAN 3 software package, which is being developed at ITAM SB RAS, and is intended for basic scientific research of the stability of three-dimensional boundary layers of viscous compressible fluids and determining the LTT positions based on the $e^X$-method.

In this study the LOTRAN 3 software package, converted into a module that operates in integration with the ANSYS Fluent gas dynamic package, is used to calculate the position of an LTT in the three-dimensional boundary layer of a prolate spheroid [7] exposed to a subsonic and transonic air flows.
This model is of particular interest as on its curved surfaces such aspects of the LTT as transverse flow with pressure gradient and flow separation take place, therefore the LTT can occur both due to the Tollmien–Schlichting (TS) and cross-flow (CF) instabilities. In this paper computational problems of modeling the basic flow with the practical accuracy are discussed. The computed basic flow is used later in the LTT module to determine the position of the LTT.

2. Numerical simulation

The subsonic and transonic flows past the prolate spheroid [7] with a semi-axis ratio of 6:1 and length \( L = 2.4 \) m aligned at an angle of attack \( \alpha = 10^\circ \) is considered. The computational domain is a hemisphere with a radius of 6 m, in the center of which the prolate spheroid is placed.

The computational grid was constructed as follows. At the beginning, a blocking regular computational grid with a refinement towards the surface of the body was constructed in a two-dimensional section of the computational domain passing through the axis of symmetry. To accurately resolve the boundary layer on the spheroid surface, a dedicated subzone of the near-wall region was created with a height of 45 mm that corresponds to several units of the boundary-layer thickness. In the subzone, 80 cells were specified in the direction normal to the surface. In the longitudinal direction, 200 cells along the body was used, the total number of cells in a two-dimensional section being 40,000. At this stage, preliminary axisymmetric calculations were performed using this computational grid, in order to determine the flow characteristics, such as \( y^+ \) and the BL thickness.

Further, to create a three-dimensional computational grid, the two-dimensional grid was rotated around the \( x \) axis by 180 degrees, while maintaining the blocking structure. In the azimuthal direction the size of one cell was 2 degrees. Thus, the size of the three-dimensional computational grid was 3.5 million cells. Figure 1a shows a three-dimensional computational domain with the computational grid blocking structure. Figure 1b shows a fragment of the computational grid near the prolate spheroid.

![Figure 1](image1.png)

**Figure 1.** Three-dimensional computational domain with the computational grid blocking structure (a) and a fragment of the computational grid near the prolate spheroid (each 16 cells) (b): the border of the selected subzone is marked with a white dotted line.

It should be noted that the selection of the subzone near the surface with thicknesses of several BL is one of the important principles for basic flow calculation, as only the data from this region are transferred to the transition module in order to save computational resources. The LTT module, developed on the basis of the LOTRAN 3, assimilates the data to use them in calculating the stability of the BL to TS waves and CF vortices, and then estimates the transition positions.

It is known that hexahedral computational grids, in which the grid lines are practically aligned with the streamlines around the body, are preferred from the point of view of both the accuracy of flow prediction and the saving of computing resources. Therefore, another important principle of the basic flow calculation is the construction of a detailed regular hexahedral computational grid in the selected subzone. However, when the two-dimensional computational grid is rotated, prismatic cells with a triangular base are formed near the axis of symmetry on the nose and tail of the body, so the regions
containing prismatic cells are excluded from the selected subzone. In figure 2 the computational grid on the spheroid surface, which is included in selected subzone, is highlighted in green. The blue color indicates the region containing prismatic cells that is excluded from the selected subzone.

![Figure 2](image.png)

**Figure 2.** The computational grid on the prolate spheroid surface: the green cells are included in and the blue cells are excluded from the selected subzone.

During the numerical simulation of the initial laminar flow with pressure gradients, a flow separation occurs in the rear part of the prolate spheroid. Disturbances generated in that part could distort the upstream laminar boundary layer, which, in turn, can lead to incorrect determination of the LTT position. One of the ways to prevent the laminar flow separation and to suppress the disturbances is to perform a stationary calculation with splitting the flow domain into two regions: the laminar flow and the turbulent flow in the rear part downstream of the expected physical transition to turbulence. For that purpose, a home-made UDF module is used, which allows zeroing the turbulence kinetic energy in the laminar flow region. In the other region, the numerical simulation of turbulent flow is carrying out using the $k-w$ SST model. The numerical simulation of the basic flow is performed using the three-dimensional Reynolds-averaged Navier–Stokes equations in ANSYS Fluent package.

The split position was determined by trials and errors in preliminary calculations of the basic flow. However, the question of robustness of the split remained open, as it is necessary to clarify the influence of the method of setting the position of the split on the physical LTT position.

In this paper, five different ways of the forced flow turbulization for the subsonic flow past the prolate spheroid at an angle of attack $\alpha=10^\circ$ is considered. The first two methods (vert1, vert2) specify turbulization in a vertical line so that the flow is turbulent after $x = 2.3$ m and $x = 2.1$ m from the body nose, respectively (the model length is 2.4 m). The other three methods (skew1, skew2, skew3) specify the split along oblique lines so that the forced turbulization on the leeward (upper) side of the body comes closer to the prolate spheroid nose and on the windward (lower) side downstream. For skew3, the entire lower, windward side remains laminar. Figure 3 (a-d) shows the intermittency values for all cases under consideration. The blue and the red regions correspond to the laminar and turbulent flows, respectively. Also, figure 3 (e-h) shows the wall shear stress fields on the body surface. As seen, a characteristic feature of the flow is the presence of a flow separation on the leeward side of the prolate spheroid (above), where the wall shear stress drops virtually to zero. Inclined turbulization methods take this feature into account, therefore, for them, on the leeward side of the body, the intermittency value equal to 1 is assigned closer to the model nose. However, it could be seen that for all turbulization methods, the laminar flow regions are almost identical in terms of the wall shear stress.

The obtained characteristics of the basic flow are converted into an internal representation of the LTT module using a specially created data import module, which reads the cas and dat files. Within the framework of this module, the evolution of small perturbations in the boundary layer is described on the basis of the complete linearized with respect to the basic laminar flow heat and mass transfer equations of a compressible fluid. The LTT module performs the construction of streamlines in the BL. Normals to the surface are built along the streamlines, and all necessary flow characteristics are
interpolated to these normals. Further, on each selected streamline, the problems of temporal (to find the neutral curves) and spatial stability (to find the $N$-factors) are solved. This allows determining the LTT position from the given threshold values of the $N$-factors. In this study, the threshold $N$-factor is taken equal to 6.58 for both the TS waves and the CF vortices that correspond to a degree of free stream turbulence of 0.2%, as in experiments [7].

Figure 3. The intermittency (a–e) and the wall shear stress field (f–j) on the surface of prolate spheroid: $U_\infty = 45$ m/s ($M_\infty = 0.136$), $\alpha = 10^\circ$.

The estimated positions of the beginning of LTT for all methods of turbulization are compared in figure 4. Here, the black solid line corresponds to the position of the transition beginning for the TS instability waves, and the dashed line for the CF instability. As seen, the difference in the method of forced turbulization did not affect the position of the transition beginning caused by the CF instability. However, for the TS waves, a change in the position of turbulization leads to a variation in the position of the transition beginning within 4% of the body length $L = 2.4$ m. This is explained by a higher sensitivity of the TS waves to the influence of pressure gradient changes on the model surface, in contrast to the CF instability associated with inflections in the velocity profiles. It is also worth noting that for the TS waves, a certain discrepancy in the position of the LTT beginning is observed only between the vertical-line and inclined-line groups of methods of turbulization, while the methods inside each group show almost the same results.

Figure 4. The position of the LTT beginning caused by the TS waves instability (solid lines) and the CF instability (dashed lines) for different methods of turbulization: $U_\infty = 45$ m/s ($M_\infty = 0.136$), $\alpha = 10^\circ$. 
3. Results

The subsonic $M_\infty = 0.136$ and transonic ($M_\infty = 0.75$) flows around the prolate spheroid at $Re_L = 7.2 \times 10^6$ were considered. For the case of subsonic flow (figure 5a), the velocity-inlet boundary condition was set at the external boundary of the computational domain with follow parameters $U_\infty = 45 \text{m/s}, p_\infty = 105984.7 \text{Pa}, T_\infty = 300 \text{K}$. For the case of transonic flow (figure 5e), the pressure-far-field boundary condition was set with the parameters $M_\infty = 0.75, p_\infty = 15977.25 \text{Pa}, T_\infty = 269.66 \text{K}$. The position of forced turbulization corresponded to vert1 for both cases. The streamlines colored with the values of $N$-factors for the TS instability waves and stationary CF instability vortices are shown in figure 5bf and figure 5cg, respectively. The black solid line with dots indicates the position of the transition beginning corresponding to the threshold value $N = 6.58$ for the TS instability waves, and the dashed line for the CF instability vortices. As seen, in the larger portion of the prolate spheroid, the LTT is caused by the CF instability for both subsonic and transonic flow regimes. The TS instability dominates only in the windward part of the model at $U_\infty = 45 \text{m/s} (M_\infty = 0.136)$, but at $M_\infty = 0.75$ the transition does not occur. The flow regions are given in figure 5dh presenting a general view of the transition. It should be noted that the region position for the subsonic case (figure 5d) is consistent with the experimental data obtained in [7].

Thus, the numerical simulation of three-dimensional flow on a prolate spheroid was carried out for subsonic and transonic flow regimes using the LTT module based on the $e^N$-method in integration with the general-purpose gas-dynamic package ANSYS Fluent. The main principles of the numerical simulation of the basic flow are discussed: the requirement of the near-wall region allocation for data transfer to the LTT module and the independence of results from various methods of setting the position of forced turbulization. It was shown that for the flow over the model at an angle of attack of
10°, two instability mechanisms are active in the BL: the TS instability waves and the stationary CF instability vortices with the prevalence of CF instability on the larger portion of the surface.

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