Rashba Spin-Orbital Splitting in Kane Type Quantum Anti-Wires

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Abstract. In the present study, using three-band Kane’s model including the conduction band, light and spin-orbital hole bands, the electronic states of a semiconductor quantum anti-wire is studied in presence of Rashba spin-orbital interaction and external magnetic field and compared with those of a quantum anti-wire of the same size. It’s calculated the radii, height, coupling strength and magnetic field dependence of Rashba splitting for carriers respectively. It has been found for the InSb quantum anti-wire that Rashba splitting of electrons are decreased with the increasing of radius.

1. Introduction

The study of semiconductor nanocrystals in recent years has been of explosive interest both experimental and theoretical points [1]. The interest concentrates on size quantization in solids in those objects. The electron energy spectrum consists of a set of discrete levels for an ideal quantum dot. So the semiconductors become very attractive for possible applications in micro and nano opto-electronics [2]. The electron spin plays an important role in the quantum nanostructure design. Spin orbital quantum interaction can arise in quantum dots by various mechanisms related to electron confinement and are generally introduced into Hamiltonian via Rashba [3] and Dresselhaus terms [4]. Dresselhaus showed that in bulk semiconductors of zinc blend symmetry the \( \Gamma_6 \) band (conduction band of III-V compounds) is characterized by an anisotropic spin splitting proportional to \( k^3 \) for small \( k \) values. As demonstrated by a Dyakonov and Perel [5] the splitting leads to electron spin relaxation. Rashba interaction was used firstly to controllably rotate the electron spin by Datta and Das [6]. The earliest theoretical study in the spin splitting of conduction band in semiconductor heterostructures was done by Okava and Uemura [7]. Lassnig was the first to recognize the essential point that the spin inversion asymmetry splitting in the conduction band is related to the gradient of the potential including the valance offsets at the interfaces [8]. Pfeffer and Zawadski used the complete five level \( k.p \) approach to the band structure of III-V compounds, which allowed to treat bulk inversion asymmetry and structure inversion asymmetry mechanism of spin-splitting within the same model [9]. Analytical solutions to the problem of the Rashba spin-orbit coupling were done in semiconductor quantum dots taking into account the parabolic band model [10,11]. Hashimzade \textit{et al} were studied theoretically the electronic states of electrons in Kane type semiconductor quantum disk in presence of Rashba spin orbital interaction [12]. Kudryashov were calculated the Rashba spin orbit interaction, wave functions and energy spectrum in a two dimensional quantum dot [13]. In the study of [14], Babayev \textit{et al} calculated the energy spectrum of a quantum anti-wire taking into account the real band

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structure of Kane type materials. Babayev and collaborators calculated the electron eigenstates in a quantum anti-wire in presence of Rashba spin-orbital splitting using parabolic band model [15]. However, the experimental advantages of using narrow-gap semiconductors for the low-dimensional systems make it necessary to account for the real band structure of these materials. In the present study, it’s aimed to take into account the coupling of the conduction and valence bands for considering the non-parabolicity of the electron dispersion in narrow and medium gap semiconductors. It’s calculated the total spin-splitting energy in an InSb quantum anti-wire with hard walls both without and with an applied constant axial magnetic field. The system used in this study might simply be a very long cylindrical cavity of radius $r_0$ etched out of a bulk composite semiconductor material. It’s shown an example of an anti-wire in Figure 1. Every cavity in Figure 1 corresponds to an anti-wire. The whole figure is called as photonic crystals which are now envisaged as an essential building of future photonic devices [16]. We describe the confinement of electrons in quantum anti-wire by a separable potential $V(r)=V(z)+V(x, y)$, where $V(x, y)$ is the confinement potential in the xy plane and $V(z)$ is the potential in the z direction. The confining potential in the xy plane is assumed to be symmetric, $V(x, y)=V(\rho)$. Here we will mainly consider a hard-wall confining potential in the xy plane, $V(\rho)=0$ for $\rho > r_0$ and $V(\rho)=\infty$ for $\rho = r_0$, $r_0$ being the radius of the quantum anti-wire. The perpendicular confinement $V(z)$ is assumed strong enough that only the lowest $z$-sub-band is occupied. Here $V(z)$ is assumed to be asymmetric.

In the three bands Kane’s Hamiltonian; the valence and conduction band interaction is taken into account via the only matrix element $P$ (so called Kane’s parameter). The system of Kane equations including the non-dispersional heavy hole bands have the form [17,18,19].

$$-\varepsilon \varphi_1 - \frac{P_k}{\sqrt{2}} \varphi_3 + \sqrt{\frac{2}{3}} P_k \varphi_4 + \frac{P_k}{\sqrt{6}} \varphi_5 - \frac{P_k}{\sqrt{3}} \varphi_7 = 0$$

(1)

$$-\varepsilon \varphi_2 - \frac{P_k}{\sqrt{6}} \varphi_4 + \sqrt{\frac{2}{3}} P_k \varphi_3 + \frac{P_k}{\sqrt{2}} \varphi_5 - \frac{P_k}{\sqrt{3}} \varphi_7 - \frac{P_k}{\sqrt{3}} \varphi_8 = 0$$

(2)

$$\frac{P_k}{\sqrt{2}} \varphi_1 - (\varepsilon + \varepsilon_s) \varphi_3 = 0$$

(3)

$$\sqrt{\frac{2}{3}} P_k \varphi_1 - \frac{P_k}{\sqrt{6}} \varphi_2 - (\varepsilon + \varepsilon_s) \varphi_4 = 0$$

(4)

$$\frac{P_k}{\sqrt{6}} \varphi_1 + \sqrt{\frac{2}{3}} P_k \varphi_2 - (\varepsilon + \varepsilon_s) \varphi_3 = 0$$

(5)

$$\frac{P_k}{\sqrt{2}} \varphi_2 - (\varepsilon + \varepsilon_s) \varphi_6 = 0$$

(6)
\[-\frac{P}{\sqrt{3}} \phi_i - \frac{P}{\sqrt{3}} \phi_2 = (\Delta + \varepsilon + \varepsilon_0) \phi_i = 0 \quad (7)\]

\[\frac{P}{\sqrt{3}} \phi_i - \frac{P}{\sqrt{3}} \phi_2 = (\Delta + \varepsilon + \varepsilon_0) \phi_k = 0 \quad (8)\]

Here \(P\) is Kane parameter, \(E_g\) is the band gap energy, \(\Delta\) is the value of spin-orbital splitting and \(k_z = k_x \pm ik_y, \quad \hat{k} = -i \nabla\). The energy origin has been chosen at the conduction-band minima.

2. Zero Magnetic Field

In the first part, the solution of Kane equations is investigated in presence of Rashba spin-orbital splitting in zero magnetic field.

Firstly, if it’s substituted the expression (3-8) into formulas (1) and (2), it’s obtained;

\[\left( -\varepsilon + \frac{P^2}{2} k_z k_z + \frac{2}{3} \varepsilon + \varepsilon_0 + \frac{P^2}{6} k_x k_x + \frac{1}{\Delta + \varepsilon + \varepsilon_0} + \frac{P^2 k_z k_z}{3(\Delta + \varepsilon + \varepsilon_0)} \right) \phi_i = \left( -\varepsilon \right) \phi_i = 0 \quad (9)\]

Applying the perturbation approach it’ presented the Hamiltonian \(H\) as;

\[H = H_0 + H_R \quad (10)\]

where the Hamiltonian \(H_0\) describes Kane Hamiltonian, and the Hamiltonian \(H_R\) describes the effect of a weak Rashba spin-orbital interaction. \(H_0\) and \(H_R\) are;

\[H_0 = \begin{bmatrix} H_{11} - \varepsilon & 0 \\ 0 & H_{22} - \varepsilon \end{bmatrix}, \quad H_R = \begin{bmatrix} 0 & -i Rk_z \\ i Rk_z & 0 \end{bmatrix} \quad (11)\]

where

\[H_{11} = \frac{P^2}{2} k_z k_z + \frac{2}{3} \varepsilon + \varepsilon_0 + \frac{P^2}{6} k_x k_x + \frac{1}{\Delta + \varepsilon + \varepsilon_0} + \frac{P^2 k_z k_z}{3(\Delta + \varepsilon + \varepsilon_0)} \quad (12)\]

\[H_{22} = \frac{P^2}{6} k_z k_z + \frac{2}{3} \varepsilon + \varepsilon_0 + \frac{P^2}{2} k_x k_x + \frac{P^2 k_z k_z}{3(\Delta + \varepsilon + \varepsilon_0)} + \frac{1}{\Delta + \varepsilon + \varepsilon_0} \quad (13)\]

if the above Equations are used, equation (10) can be written as;

\[\begin{bmatrix} H_{11} - \varepsilon & -i Rk_z \\ i Rk_z & H_{22} - \varepsilon \end{bmatrix} \begin{bmatrix} \phi_i \\ \phi_2 \end{bmatrix} = 0 \quad (14)\]

for the lowest subband we obtain the following set of two equations for the envelope functions of \(\phi_1(r)\) and \(\phi_2(r)\):

\[-\varepsilon + \frac{P^2}{2} k_z k_z + \frac{2}{3} \varepsilon + \varepsilon_0 + \frac{P^2}{6} k_x k_x + \frac{1}{\Delta + \varepsilon + \varepsilon_0} + \frac{P^2 k_z k_z}{3(\Delta + \varepsilon + \varepsilon_0)} \phi_i - i Rk_z \phi_2 = 0 \quad (15)\]

\[-\varepsilon + \frac{P^2}{6} k_z k_z + \frac{2}{3} \varepsilon + \varepsilon_0 + \frac{P^2}{2} k_z k_z + \frac{P^2 k_z k_z}{3(\Delta + \varepsilon + \varepsilon_0)} + \frac{1}{3(\Delta + \varepsilon + \varepsilon_0)} \phi_2 + i Rk_z \phi_1 = 0 \quad (16)\]

is obtained, and operators \(\hat{k}_z\) in cylindrical coordinates take the form;

\[k_z = -ie^{z\varphi} \left( \frac{d}{d\rho} \pm \frac{i}{\rho} \frac{d\varphi}{d\rho} \right) \quad (17)\]
We consider a quantum anti-wire modeled as a quantum cylindrical cavity of radius \( r_0 \) etched out of a bulk composite semiconductor material and the height \( d \). Because of the symmetry of the problem it is convenient to use cylindrical coordinates \((\rho, \varphi)\).

The eigenfunctions of the total momentum operator with a half integer eigenvalue \( m \) are of the following form:

\[
\begin{pmatrix}
\varphi_1 \\
\varphi_2
\end{pmatrix}
= \begin{pmatrix}
e^{i\rho} e^{i\varphi} \\
e^{i(m+1)\varphi} e^{-i\rho}
\end{pmatrix}
\]  

(18)

where \( m = 0, \pm 1, \pm 2, \pm 3, \ldots \) are the magnetic quantum numbers through the \( z \)-axis. The radial function \( f(\rho) \) and \( g(\rho) \) is found to satisfy the following differential equation;

\[
\left\{-\varepsilon + \frac{p^2}{3} \left( \frac{1}{\varepsilon + \varepsilon_g} + \frac{1}{\Delta + \varepsilon + \varepsilon_g} \right)(-\frac{d^2}{d\rho^2} - \frac{1}{\rho} \frac{d}{d\rho} + \frac{(m+1)^2}{\rho^2} + k_z^2) f(\rho) + R\left(\frac{d}{d\rho} - \frac{m}{\rho}\right) g(\rho) \right\} = 0
\]  

(19)

\[
\left\{-\varepsilon + \frac{p^2}{3} \left( \frac{1}{\varepsilon + \varepsilon_g} + \frac{1}{\Delta + \varepsilon + \varepsilon_g} \right)(-\frac{d^2}{d\rho^2} - \frac{1}{\rho} \frac{d}{d\rho} + \frac{(m+1)^2}{\rho^2} + k_z^2) g(\rho) + R\left(\frac{d}{d\rho} - \frac{m}{\rho}\right) f(\rho) \right\} = 0
\]  

(20)

The perturbation theory can be applied by accepting the Rashba spin-orbital interaction term as weak perturbation term. Then the wave functions are taken as Bessel function;

\[
\begin{bmatrix}
f(\rho) \\
g(\rho)
\end{bmatrix}
= \begin{bmatrix}
d_1 Y_{\nu}(v\rho) \\
d_2 Y_{\nu+1}(v\rho)
\end{bmatrix}
\]  

(21)

If it’s used the wave functions in equation (21) in the equations (19) and (20) respectively, the following differential equations are obtained;

\[
x^2 \frac{d^2}{dx^2} Y_{\nu}(x) + x \frac{d}{dx} Y_{\nu}(x) + x^2 \left( \frac{\varepsilon_0}{v^2} + \frac{d_1 R_0}{d_2 v} \right) Y_{\nu}(x) - m^2 Y_{\nu}(x) = 0
\]  

(22)

and

\[
x^2 \frac{d^2}{dx^2} Y_{\nu+1}(x) + x \frac{d}{dx} Y_{\nu+1}(x) + x^2 \left( \frac{\varepsilon_0}{v^2} + \frac{d_1 R_0}{d_2 v} \right) Y_{\nu+1}(x) - (m + 1)^2 Y_{\nu+1}(x) = 0
\]  

(23)

where;

\[
\varepsilon_0 = \frac{3\varepsilon}{p^2 \left( \frac{2}{\varepsilon + \varepsilon_g} + \frac{1}{\varepsilon + \varepsilon_g + \Delta} \right)} - k_z^2 \\
R_0 = \frac{3R}{p^2 \left( \frac{2}{\varepsilon + \varepsilon_g} + \frac{1}{\varepsilon + \varepsilon_g + \Delta} \right)}
\]

Let’s accept that; the solutions of the equation (22) and equation (23) are;

\[
\frac{\varepsilon_0}{v^2} + \frac{d_1}{d_2} \frac{R_0}{v} = 1 \quad \text{and} \quad \frac{\varepsilon_0}{v^2} + \frac{d_1}{d_2} \frac{R_0}{v} = 1
\]  

(24)

If the two expressions in the equation (24) are multiplied by each other, then it’s obtained;

\[
\left( \frac{R_0}{v} \right)^2 = \left( 1 - \frac{\varepsilon_0}{v^2} \right)^2
\]  

(25)

where

\[
v^2 \pm R_0 v - \varepsilon_0 = 0
\]  

(26)

If the equation (26) is solved for the \( v \) then

\[
v_{1,2} = \pm \frac{R_0}{2} \pm \sqrt{\frac{R_0^2}{4} + \varepsilon_0}
\]  

(27)

is found. In addition to this, if the expressions in equation (24) is subtracted each other;
\[
\frac{R_0}{\nu} \left( \frac{d_2}{d_1} - \frac{d_1}{d_2} \right) = 0
\]  

(28)

is found. It’s clearly seen in equation (28) that \(\frac{R_0}{\nu} \neq 0\), so \(\frac{d_2}{d_1} - \frac{d_1}{d_2} = 0\). This implies two possibilities which are \(d_1 = d_2\) and \(d_1 = -d_2\). Hence there are two degenerate solutions for \(\nu\) which are combined in the general solution;

\[
\begin{align*}
  f(\rho) &= A \left[ d_1^+ Y_m(\nu, \rho) \right] + B \left[ d_1^- Y_m(\nu, \rho) \right] \\
  g(\rho) &= A \left[ d_2^+ Y_{m+1}(\nu, \rho) \right] + B \left[ d_2^- Y_{m+1}(\nu, \rho) \right]
\end{align*}
\]

(29)

where

\[
\nu_\pm = \pm \frac{R_0}{2} + \sqrt{\frac{R_0^2}{4} + \epsilon_0}
\]

It’s implied the Dirichlet boundary condition as \(f(\rho = r_0) = g(\rho = r_0) = 0\) for \(\nu\) linear combination of solution (29).

\[
\begin{bmatrix}
  d_1^+ Y_m(\nu, \rho) & d_1^- Y_m(\nu, \rho) \\
  d_2^+ Y_{m+1}(\nu, \rho) & d_2^- Y_{m+1}(\nu, \rho)
\end{bmatrix}
\begin{bmatrix}
  A \\
  B
\end{bmatrix} = 0
\]

(30)

It gives us the following exact equation for energy spectrum of the quantum anti-wire with spin orbital interacting

\[
\frac{d_1^+}{d_2^-} - Y_m(\nu, r_0)Y_{m+1}(\nu, r_0) - Y_m(\nu, r_0)Y_{m+1}(\nu, r_0) = 0
\]

(31)

For value of \(\nu_\pm\)

\[
\frac{d_1^+}{d_2^-} = -1
\]

Equation (31) simplifies to

\[
Y_m(\nu, r_0)Y_{m+1}(\nu, r_0) + Y_m(\nu, r_0)Y_{m+1}(\nu, r_0) = 0
\]

(32)

Not that for the quantum wire with the finite height \(d\) \(k_z = \frac{\pi}{d} n, n = 1, 2, 3, \ldots\) and the minimal value for \(k_z\) must be taken as \(k_z = \frac{\pi}{d}\).

The \(n\)th solution of equation (32), is shown the \(n\)th energy level of the electron which have a momentum of \(m+1\). Equation (32) is invariant under the change \(m \rightarrow -m\) reflecting the Kramer’s degeneracy. According to the standard analysis the spin-orbit coupling splits all the \(l=0\) states into two Kramer’s doublets with \(m=\pm 1/2\) and \(m=\pm 1/2\), while \(l=0\) states naturally remain Kramer’s doublets. We have analyzed Equation (32) numerically, labeling the energy eigenstates as \((m, n)\) where \(n\) is a non-negative integer such that \(E_{m,n} < E_{m,n+1}\) at \(R=0\). In Figure 2, it’s shown the radii dependence of Rashba spin-orbit splitting in quantum anti-wire including both the parabolic and Kane models using the band structure parameters for InSb; \(m=0, 0.014 m_0, E_g=0.24\) eV, \(R=2.5 \times 10^{-9}\) eV.cm [12]. As it’s seen from the Figure 3, the value of the Rashba spin-splitting is decreasing with the increasing of the radius for both models. According to this Figure 2, with increase of \(r_0\), the electron energy levels in both cases are close to each other. At rather small sizes of \(r_0\), the variance electron dispersion laws becomes more and more important and, therefore, the curves for \(\Delta E / E_g(R)\) \(\Delta E=E(J=1/2,m=1)-E(J=3/2,m=0)\) separate from each other.
Figure 2. The change of Rashba spin-orbit splitting in InSb quantum anti-wire (B=0).

In Figure 3 and 4, the radius and height dependence of electron energies in InSb quantum anti-wire, are shown where Rashba parameter is constant again comparing the parabolic band model and Kane model. It’s seen from the Figure 2 and 3 that the energies are decreasing with the increasing of radius and height. It can be seen from the Figure 4 that the curves for the parabolic band model and Kane model becomes closer in the early height values.

Figure 3. The radii dependence of electron energies in InSb quantum anti-wire (B=0).
The evolution of the first few energy levels with the Rashba parameter plotted in Figure 5. The curves are labeled by quantum numbers \((n, m)\).

3. Applied Magnetic Field
In the second case the problem is solved under an applied magnetic field. For a uniform magnetic field \(H\), directed along the \(z\)-axis, the vector potential may be chosen in the form;
the operators \( k_\pm \) in Eqs. (15) and (16) become cylindrical coordinates and take the form

\[
k_\pm = i \exp(\pm i \varphi) \left( \frac{\partial}{\partial \rho} \pm \frac{1}{\rho} \frac{\partial}{\partial \varphi} \pm \frac{i \rho}{2\alpha^2} \right)
\]

where \( \alpha = \sqrt{\frac{hc}{eH}} \) is the magnetic length. Since the system is cylindrically symmetric, the wave function can be represented as [19]:

\[
\varphi(x, y, z) = \begin{cases} 
\exp(\pm i \varphi) \exp \left( -\frac{x}{2} \right) x^{1/2} Y(x) 
\end{cases} \exp(\pm i \varphi) \exp \left( -\frac{x}{2} \right) x^{1/2} Y(x)
\]

Where; \( x = \frac{\rho^2}{2\alpha^2} \).

Substituting expressions (3)-(8) into formulas (1) and (2) and using the formulas of (34) and (35), we obtain second order differential equations for the radial functions:

\[
x \frac{d^2 Y(x)}{dx^2} + \left( -x + 1 + |m| \right) \frac{dY(x)}{dx} - \frac{1}{2} \left( 1 + |m| + m + h \alpha^2 - \alpha^2 \right) Y(x) = 0
\]

\[
x \frac{d^2 Z(x)}{dx^2} + \left( -x + 1 + |m| \right) \frac{dZ(x)}{dx} - \frac{1}{2} \left( 1 + |m| + m - 2 \alpha^2 \right) Z(x) = 0
\]

In Equations (37) and (38) the following notations are used:

\[
\lambda = \frac{1}{E + E_e - \left( E + E_e + \Delta \right)}
\]

\[
k_\pm^2 = \frac{1}{E + E_e - \left( E + E_e + \Delta \right)} - k_\pm^2
\]

\[
\beta = \frac{3R}{p^2 \left( \frac{2}{E + E_e - \left( E + E_e + \Delta \right)} \right)}
\]

Let us introduce \( a_1 \) and \( a_2 \) parameters which are related to the equation in the following form:

\[
x \frac{d^2 Y(x)}{dx^2} + \left( -x + 1 + |m| \right) \frac{dY(x)}{dx} - a_1 Y(x) = 0
\]

\[
x \frac{d^2 Z(x)}{dx^2} + \left( -x + 1 + |m| + 1 \right) \frac{dZ(x)}{dx} - a_2 Z(x) = 0
\]

The equations are canonical form of Kummer’s equations for the confluent hypergeometric function [20]. Equation (36) and (37) can be written as follows taking into account the equations of (41) and (42).

\[
a_1 = \frac{1}{2} \left( 1 + |m| + m + h \alpha^2 - \alpha^2 k_\pm^2 \right) Y(x) + \frac{\alpha \beta}{\sqrt{2}} x^{1/2} \left( - \frac{dZ(x)}{dx} + \frac{1}{2} \left( 1 + |m| + m \right) Z(x) \right) = 0
\]
The solutions of Equations (43) and (44) that is bounded is
\[ Y(x) = d_1 U(a_1, b_1, x) \] and
\[ Z(x) = d_2 U(a_2, b_2, x), \]
where
\[ a_1 = \frac{m + |m|}{2} - \epsilon_0, \quad b = |m| + 1 \] (45)
\[ a_2 = \frac{m + 1 + |m + 1|}{2} - 1 - \epsilon_0, \quad b = |m + 1| + 1 \] (46)
\[ \epsilon_{\alpha \beta} = \frac{1}{2} (2 + \frac{1}{2} \alpha^2 \beta_k^2 + \alpha^2 k_i^2) \pm \sqrt{1 - 4 s + 4 s^2 + \frac{1}{4} \alpha^4 \beta_k^2 + \alpha^4 k_i^2}, \quad h \alpha^2 = -2 s \] (47)

The wave function can be written:
\[ R^+(x) = c_1 d_1 U \left( \frac{m + |m|}{2} - 1 - \epsilon_{0\alpha}, b_1, x \right) + c_2 d_2 U \left( \frac{m + 1 + |m + 1|}{2} - 1 - \epsilon_{0\beta}, b_2, x \right) \] (48)
\[ R^-(x) = c_1 d - U \left( \frac{m + |m|}{2} - 1 - \epsilon_{0\alpha}, b_1, x \right) + c_2 d_2 U \left( \frac{m + 1 + |m + 1|}{2} - 1 - \epsilon_{0\beta}, b_2, x \right) \] (49)

In a nano anti-wire with an infinite potential barrier, the wave function must vanish at the quantum anti-wire’s surface giving the dispersion equation for the electron quantum size levels:
\[ \frac{d}{dx} \left( \frac{m + |m|}{2} - 1 - \epsilon_{0\alpha}, b_1, x \right) + U \left( \frac{m + 1 + |m + 1|}{2} - 1 - \epsilon_{0\beta}, b_2, x \right) - U \left( \frac{m + |m|}{2} - 1 - \epsilon_{0\alpha}, b_1, x \right) + U \left( \frac{m + 1 + |m + 1|}{2} - 1 - \epsilon_{0\beta}, b_2, x \right) \] (50)

The above equation provides all information about the energy spectrum of electrons. The energy is a complicated function of the anti-wire parameters \( \rho, \phi \) and the electron angular momentum. Equation (50) can be useful for analyzing the influence of nonparabolicity on the energy spectrum of electrons near a quantum anti-wire.
It’s seen the magnetic field dependence of the sub-bands for different \( n \) and different \( m \) values for electrons in InSb type quantum anti-wire with a constant radius 200 Å and the height 20 Å. It can be from the Figure 6 that seen that the lowest sub-band corresponds to the state having \((m=-1, n=0)\) which differs from the same graph in the quantum-wire problem [11]. As it’s seen from Figure 6, the electron energies in Kane model is smaller than that of parabolic band model.

4. Conclusion
It’s presented the analytical solutions of Kane equations for a quantum anti-wire in presence of Rashba spin-orbit interaction with and without an external magnetic field. The calculations are based on the eight-band \( \mathbf{k}\cdot\mathbf{p} \) theory, which takes into account the nonparabolicity of the electron energy levels. A nonzero magnetic field breaks the time-inversion symmetry and removes the corresponding degeneracy. It’s shown that Rashba splitting is increasing with the increasing of the radius of the quantum anti-wire in Kane model.

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