Measuring Smuon-Selectron Mass Splitting at the LHC and Patterns of Supersymmetry Breaking

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With sufficient data, Large Hadron Collider (LHC) experiments can constrain the selectron-smuon mass splitting through differences in the di-electron and di-muon edges from supersymmetry (SUSY) cascade decays. We study the sensitivity of the LHC to this mass splitting, which within mSUGRA may be constrained down to $O(10^{-4})$ for 30 fb$^{-1}$ of integrated luminosity. Over substantial regions of SUSY breaking parameter space the fractional edge splitting can be significantly enhanced over the fractional mass splitting. Within models where the selectron and smuon are constrained to be universal at a high scale, edge splittings up to a few percent may be induced by renormalisation group effects and may be significantly discriminated from zero. The edge splitting provides important information about high-scale SUSY breaking terms and should be included in any fit of LHC data to high-scale models.

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TeV-scale supersymmetry is one of the most promising solutions to the weak hierarchy problem of the Standard Model. If TeV-scale supersymmetry is realized in nature, the LHC will be able to discover it by the production of sparticles and measure some of the parameters of the minimal supersymmetric standard model (MSSM). Through the use of renormalisation group equations (RGEs) to connect the low and high scale Lagrangians, the measured MSSM parameters can be used to test hypotheses, such as flavor universality (or non-universality) of the high-scale soft SUSY breaking terms. To test hypotheses, such as flavor universality (or non-universality) of the high-scale soft SUSY breaking terms.

The flavor problem is one of the most pressing questions of supersymmetric phenomenology. The Lagrangian of a TeV-scale MSSM is highly constrained by the absence of new contributions to low energy flavor changing neutral currents (FCNCs). Classic examples are $K^0 \bar{K}^0$ mixing or in the leptonic sector the $\mu \rightarrow e\gamma$ branching ratio. These strongly suggest that new physics present at the TeV scale should obey to high accuracy the principle of MFV (Minimal Flavor Violation), at least in the first two generations. The $\mu \rightarrow e\gamma$ branching ratio, which is currently bounded at $BR(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ and in the future will be measured to at least $10^{-13}$, constrains off-diagonal propagator mixing between the smuon and selectron LL flavor eigenstates to $\delta_{12,LL} \lesssim 6 \times 10^{-4}$.

The RR constraints are similar over most of parameter space, but potential cancellations exist and so $\delta_{12,RR}$ has a weaker bound than $\delta_{12,LL}$.

Nonetheless, exact low-scale flavor universality is not expected; some non-universality will be automatically induced by RGEs and in the context of string theory models of supersymmetry breaking loop effects are expected to violate flavor universality even if leading order physics preserves it \cite{RGEs}. It is therefore important to investigate the sensitivity of the LHC to the mass splitting between the right-handed selectron and smuon:

$$\Delta m^2 \equiv m_{\chi_{12}}^2 - m_{\tilde{\mu}}^2$$

$\Delta m^2$ is not directly constrained by the $BR(\mu \rightarrow e\gamma)$ measurement as it does not give lepton flavor violation (LFV). LFV effects at the LHC have been studied in various papers \cite{LFV} but are not our focus here.

We may relate $\Delta m^2$ to the SUSY breaking terms defined at a high scale $M_X$ through the MSSM RGEs. At
one-loop order \(10\),
\[
16\pi^2 \frac{d\Delta m^2}{d\ln \mu} = 4 \left[ Y^2_{\mu}(m^2_{\mu_R} + m^2_{\mu_L}) - Y^2_e (m^2_{\mu_R} + m^2_{\mu_L}) + m^2_{H_1}(Y^2_{\mu} - Y^2_e) + Y^2_{\mu} h^2_\mu - Y^2_e h^2_\mu \right],
\]
where \(\mu\) is the \(\overline{\text{MS}}\) renormalisation scale, and \(m_{\mu_R}, m_{\mu_L}, m_{\tau_R}, m_{\tau_L}\) and \(m_{H_1}\) are the soft SUSY breaking masses for right- and left-handed smuons and selectrons and the Higgs field, evaluated at a renormalisation scale \(\mu\). \(h_\mu, h_\tau\) are the trilinear selector and smuon soft SUSY breaking couplings evaluated at a renormalisation scale \(\mu\). At \(M_Z\), we have the boundary condition \(Y_\mu(M_Z) = \sqrt{2} m_\mu(M_Z)/(v \cos \beta)\), where \(v = \sqrt{v^2_1(M_Z) + v^2_2(M_Z)}\), \(v_{1,2}\) being the MSSM Higgs vacuum expectation values. We may solve the RGE for \(Y_\mu, Y_\mu(M_X) = Y_\mu(M_Z) + \mathcal{O}(\ln(M_X/M_Z)/16\pi^2)\). We neglect the ratio of the electron Yukawa coupling to the muon Yukawa coupling (\(Y_e/Y_\mu\)) and solve eq. (3) to first order in \(\ln(M_X/M_Z)/(4\pi)^2\) to obtain
\[
\Delta m^2(M_Z) = \Delta m^2(M_X) + \frac{8m^2_\mu}{16\pi^2 v^2} \left[ m^2_{\mu_R}(M_X) + m^2_{\mu_L}(M_X) + A^2_\mu(M_X) \right] \tan^2 \beta \ln \left( \frac{M_X}{M_Z} \right),
\]
where \(\tan \beta = v_2/v_1\). We have allowed for the presence of a primordial mass splitting \(\Delta m^2(M_X)\) and used the large tan \(\beta\) limit in eq. (3) so that \(\cos \beta \approx 1/\tan \beta\). We have also neglected the QED running of the muon mass below the electroweak scale. We see from eq. (3) that the magnitude of \(\Delta m^2\) is always enhanced as \(\tan^2 \beta \ln(M_X/M_Z)\). The evaluation of \(\Delta m^2(m_Z)\) is complicated by the unknown boundary conditions on \(m^2_{\mu_R}, m^2_{\mu_L}, m^2_{H_1}\), and \(A^2_\mu\) at \(M_X\), but within constrained models other edge measurements are expected to help pin them down. For example, in the mSUGRA model, the SUSY breaking scalar masses have a universal value of \(m_0\) at \(M_X\) and the trilinear SUSY breaking scalar couplings have a universal value of \(A_0\). In this case, \(\Delta m^2(M_X) = 0\) and the quantity in square brackets is simply \(3m^2_{\chi_0} + A^2_\mu\), which may be bounded from other measurements.

From eq. (3) the variation of the edge position with the slepton mass is given by
\[
\frac{dm^2_{\chi_i}}{dm^2_{\chi_i}} = \frac{m^2_{\chi_i}m^2_{\chi_0}}{m^2_{\chi_i}} - 1,
\]
with a fractional shift in the invariant mass edge of
\[
\frac{\Delta m_{ll}}{m_{ll}} = \frac{\Delta m_{l}}{m_{l}} \left( \frac{m^2_{\chi_i}m^2_{\chi_0} - m^2_{l}}{(m^2_{\chi_i} - m^2_{\chi_0})(m^2_{l} - m^2_{\chi_0})} \right),
\]
up to terms of \(\mathcal{O}(\Delta m_{l}/m_{l})^2\).

We define the enhancement factor by \((\Delta m_{mu}/m_{mu})/(\Delta m_{l}/m_{l})\). Fig. 1 shows the enhancement factor as a function of the slepton mass. As well as the mSUGRA bino-dominated lightest supersymmetric particle (LSP) case \(n \equiv m_{\chi_0}^{-1}/m_{\chi_0} = 2\) we also allow for a more compressed spectrum with different values of \(n\). There are several notable points. When \(m_{l} = \sqrt{m_{\chi_1}m_{\chi_2}}\) the shift in the edge vanishes to leading order in \(\Delta m_{l}/m_{l}\). Conversely, large splittings of the di-lepton and di-muon edges can be achieved for relatively small splittings of the slepton and smuon masses. The enhancement factor is larger for more degenerate masses between sparticles in the chain and can easily be \(\mathcal{O}(10)\) depending upon the value of \(m_{\chi_i}/m_{\chi_0}\). The enhancement diverges as the slepton mass approaches either neutralino mass. The benefits of the enhancement may be diluted for highly degenerate spectra by the fact that leptons coming from such chains will tend to be softer and thus harder to identify and measure experimentally, and also by any change in precision due to rate changes from differing branching ratios.

The muon energy calibration will depend heavily on their energies, but we may expect particularly accurate calibration when the \(m_{l}\) edge is close to, but not on, the \(Z^0\) pole. Electrons and muons from the \(Z^0\) decay may then be used to calibrate energies and efficiencies, performing a small extrapolation to the relevant energy. For instance, the scale of the \(m_{ll}\) endpoint is 80, 118, 99, 122, 343 GeV for the Snowmass benchmark points SPS1a, 3, 5, 8 and 9 respectively [18]. Thus 4 of the 5 relevant endpoints are rather close in invariant mass to the \(Z^0\).

To estimate the experimental sensitivity to differences in the positions of the di-muon and di-electron edge, we generate 400,000 \(R\)-parity conserving supersymet-
ric events at SUGRA Point 5 from proton-proton collisions at a 14 TeV centre of mass energy. We use HERWIG 6.510 which reports the two-to-two (Standard Model to Supersymmetric Particle) cross section to be $\sim 24 \text{ pb}$. Our sample of 400,000 events therefore corresponds to an integrated luminosity of $\sim 16 \text{ fb}^{-1}$. We pass the generated events through AcerDET [14] which represents a fast simulation of a generic general-purpose LHC-type experiment. The main feature that AcerDET provides for our purposes is a reasonable model of the electron and muon momentum resolution for a typical LHC-type detector. AcerDET accomplishes this by ‘smearing’ the generated momenta by appropriate amounts dependent on $p_T$, $\eta$ and $\phi$. AcerDET provides only minimal simulation of reconstruction efficiencies. For example, electrons will always be reconstructed unless they are out of acceptance or are too close to other particles. Neither does AcerDET model any uncertainty in the absolute energy scale for either electrons or muons. For this we use an estimate of 0.1% [1, 13]. Absolute energy scale calibration at this level might appear to be a tall order – however the standard candles used in this calibration (Z-bosons) will be produced in such large numbers at the LHC that the estimate may even turn out to be conservative. To evaluate the sensitivity with which the endpoints of the di-electron and di-muon invariant mass spectra can be measured, we generate their distributions, fit them in the vicinity of the endpoint, and report the endpoint fit uncertainties. To select our event we require two opposite sign same family (OSSF) isolated leptons of greater than 10 GeV and missing energy greater than 100 GeV. At this point, the di-lepton invariant mass distribution should in principle end at 97.48 GeV. We fit over the range 60 GeV to 140 GeV. For our (log likelihood) fit we use a function which takes the form of a triangle distribution (i.e. $f(x; e) \propto x \Theta(e - x) \Theta(x)$ where $\Theta(x)$ is the Heaviside step-function and $e$ is the notional endpoint) sitting on top of a constant background, and with the entire distribution convolved with a Gaussian resolution of width $\sigma$. In the fit, the three free parameters were (1) the endpoint position $e$, (2) its resolution $\sigma$, and (3) the ratio of the number of signal and SUSY background events. The overall normalisation was fixed analytically to allow a log-likelihood fit to be performed with narrow bins (0.25 GeV). Example fits are shown in figures 2 and 3. Note that we neither generate nor fit Standard Model (SM) events. In reality, irreducible sources of SM di-leptons from Z-bosons would constitute an important background if the di-lepton endpoint were to occur on the Z resonance. We ignore this special case for the moment because it is our primary aim to make statements about the “generic” di-lepton edge sensitivity. Away from the Z-peak, the missing energy cut will reduce the SM backgrounds. The results of the endpoint fits are summarised in Table I.

Accordingly we parametrise the LHC di-lepton edge sensitivity $\Sigma$ (which we define to be the expected frac-

![FIG. 2: Log likelihood fit to di-electron edge in AcerDET as described in the text. $m_{ee}$ has been placed in 0.25 GeV bins on abscissa. The ordinate shows events per bin per 16 $\text{fb}^{-1}$.](image1)

![FIG. 3: Log likelihood fit to di-muon edge in AcerDET as described in the text. $m_{\mu\mu}$ has been placed in 0.25 GeV bins on abscissa. The ordinate shows events per bin per 16 $\text{fb}^{-1}$.](image2)

| Luminosity ($\text{fb}^{-1}$) | Events below 100 GeV | Electron Endpoint (GeV) | Muon Endpoint (GeV) |
|-------------------------------|----------------------|-------------------------|---------------------|
| 16.0                          | 22145                | 97.47 ± 0.09            | 97.56 ± 0.18        |
| 8.0                           | 11131                | 97.41 ± 0.13            | 97.83 ± 0.23        |
| 4.0                           | 5520                 | 97.54 ± 0.19            | 97.63 ± 0.35        |
| 2.0                           | 2707                 | 97.52 ± 0.28            | 97.56 ± 0.50        |

TABLE I: Results of the endpoint fits for various integrated luminosities. The number of OSSF di-lepton events passing cuts and having di-lepton invariant mass below 100 GeV is also recorded.
tional uncertainty in the di-muon di-electron edge difference as estimated by the fit error on the less well determined endpoint added in quadrature to the energy scale uncertainty) by

$$\Sigma = \sqrt{(0.002\sqrt{22100/N})^2 + 0.001^2},$$

(6)

for $N$ expected signal events in the di-lepton channel. In this expression, the “0.001 term” represents the 0.1% absolute energy scale error described earlier. Note that the di-lepton edge sensitivity $\Sigma$ is not to be confused with the slepton mass sensitivity $E$ defined later.

By the above definition, the edge sensitivity $\Sigma$ is a measure of the scale down to which fractional differences in endpoint positions $\Delta m_{\ell\ell}/m_\ell$ can be measured. More precisely, assuming that the endpoint fit error is approximately Gaussian distributed, it should be possible to make an “$S_1$-sigma” discovery of selectron-smuon pole mass non-universality (i.e. rule out a null hypothesis of “no splitting in the di-electron and di-muon endpoints” at the $S_1$-sigma level) for a real endpoint splitting of size $\Delta m_{\ell\ell}$ according to

$$S_1 = \frac{\Delta m_{\ell\ell}}{m_\ell} \div \Sigma.$$  

(7)

In what follows we will therefore refer to $S_1$ as the “discovery significance” for selectron-smuon mass non-universality. We note that, when calculating $S_1$, the numbers of events $N_{ee}$ contributing to the $e^+e^-$ di-lepton signal may differ from the number of $\mu^+\mu^-$ pairs $N_{\mu\mu}$ due to phase-space differences induced by the mass differences. If systematic uncertainties on trigger and reconstruction efficiencies can be controlled, this could provide an additional means of testing selectron-smuon mass universality by looking at significant differences from zero in the statistic

$$S_2 = \frac{N_{ee} - N_{\mu\mu}}{\sqrt{N}}$$

(8)

which, like $S_1$, will be approximately normally distributed. We do not use $S_2$ ourselves.

In the examples which follow we calculate the $N$ in $S_1$ for $30 \, \text{fb}^{-1}$ of integrated luminosity at the LHC using WIGISASUGRA1.200 and HERWIG6.5. Performing a Markov Chain Monte Carlo maximisation on $S_1$ in mSUGRA, we find a maximum value $S_1 = 0.52$ after direct search constraints have been applied. Thus we find that the smuon-selectron splitting cannot be discriminated from zero in mSUGRA at the LHC. On the other hand, any significant measured difference in the end points at the LHC will discriminate against mSUGRA. A future international linear collider would achieve much improved accuracy 19 upon the mass splitting and could be combined with other constraints to help bound $\tan \beta$ assuming mSUGRA.

We wish to emphasise that the reason mSUGRA fails to generate an observable edge splitting is not the one often suggested. It is not true that the muon and electron Yukawa couplings are too small to play any role in the RGEs. Indeed, at large $\tan \beta$ the RGEs, combined with the enhancement factor, can generate slepton spectra giving edge splittings at the per cent level. The real reason is that in this case $\tilde{\tau}_R$ is driven light and it dominates the $\chi^0_2$ decay modes, with $\text{BR}(\chi^0_2 \rightarrow \tilde{\tau}_R \tau) \sim 1$. In models where there is extra third family physics that would lead to the weak scale mass ordering $m_{\tilde{\tau}_R} > m_{\chi^0_2} > m_{\tilde{\tau}_L}$, the selectron and smuon RGEs can be sufficient to generate electron-muon edge splittings that can be significantly discriminated from zero. As an example, we consider the point $m_0 = 148 \, \text{GeV}$, $M_{1/2} = 250 \, \text{GeV}$, $A_0 = -600 \, \text{GeV}$, $\tan \beta = 40$ with $m_{\tilde{\tau}_R}(M_X) = 950 \, \text{GeV}$. At this point, $\Delta m_{\tilde{\tau}/m_\tau} = 2.3 \times 10^{-5}$ and $\Delta m_{\ell\ell}/m_\ell = 1.5\%$ whereas $\Sigma = 0.27\%$, allowing an ($S_1 > 5\sigma$) discovery significance for smuon-selectron pole mass non-universality.

We now focus directly on the sensitivity to slepton mass splittings and analyse how degenerate the selectron and smuon masses can be while still allowing a 1-sigma sensitivity to a non-zero mass difference. We do this assuming $30 \, \text{fb}^{-1}$ in perturbed mSUGRA around SPS1a. In perturbed mSUGRA, we take mSUGRA boundary conditions but we allow $m_{\tilde{\mu}_R}$ to float away from the mSUGRA prediction, as could be derived from $\Delta m_\chi^2(M_X) \neq 0$ in Eq. 3. By using Eqs. 4, 5, and 7 setting $S_1 = 1$ we obtain the fractional slepton mass splitting which might be discriminated from zero at the 1-sigma level:

$$E \equiv \left. \frac{\Delta m_{\ell}}{m_{\ell}} \right|_{S_1=1} = \left( \frac{m_{\chi^0_2}^2 - m_{\chi^0_1}^2}{m_{\chi^0_1}^2 m_{\chi^0_2}^2 - m_{\chi^0_1}^4} \right) \Sigma_{30 \, \text{fb}^{-1}},$$

(9)

valid in the limit $\Delta m_{\ell}/m_{\ell} \ll 1$. Fig. 4 displays $E$ for

**Fig. 4:** Expected $30 \, \text{fb}^{-1}$ 1-sigma sensitivity, $E$, to selectron-smuon mass splitting in perturbed mSUGRA around SPS1a.
FIG. 5: Probability distribution difference in GeV$^{-1}$ of SUSY cascade chain electrons and muons as a function of their invariant mass $m_{ll}$ (in GeV) for an endpoint of 70 GeV and a relative splitting of 2%. The solid line shows the distribution without any energy resolution taken into account, whereas the dashed line displays the effect of smearing due to energy resolution. The dotted line shows a smeared distribution with no mass splitting.

a scan of perturbed mSUGRA around SPS1a. The expected sensitivity at SPS1a itself is $E = 2.8 \times 10^{-3}$. The strict mSUGRA prediction for the smuon-selectron mass splitting is $\Delta m_{\tilde{L}}/m_{\tilde{e}} = 5.9 \times 10^{-5}$. We see that sensitivities down to $O(10^{-4})$ are possible while restricting to the region where $m_{\tilde{e}_L} - m_{\tilde{\chi}_1^0} > 10$ GeV to ensure sufficiently hard leptons.

It is tempting to display the difference in $\mu^+\mu^-$ and $e^+e^-$ probability distributions ($\Delta P$) in order to look for a spike, see the solid curve in Fig. 5. In practice, energy resolution effects would smear out this curve. In the dashed curve, $m_{\mu\mu}$ and $m_{ee}$ have been Gaussian smeared by assumed fractional resolutions of 1% and 3% respectively. We have assumed $N_{ee} = N_{\mu\mu}$ and simulated no backgrounds. We have also plotted $\Delta P$ for equal endpoints while taking the energy resolution into account in the dotted line. Simply having a worse energy resolution for muons still leads to a similar feature in the difference near the end-point and could mislead us into thinking there is a splitting when in fact there is none. We conclude that, in practice, the best approach will be to fit the di-electron and di-muon endpoints separately, and then examine the difference, as was done in Figs. 2 and 3. We note here that one could still employ the technique of subtracting opposite sign different flavor di-leptons from the di-muon or di-electron decay chain samples in order to subtract backgrounds (e.g. from top pairs or W pairs), as was recommended in the case 1 where the two samples are summed.

In this paper we have studied the sensitivity of the LHC to the $\tilde{\mu}-\tilde{e}$ mass splitting through endpoint differences in SUSY cascade decays. Enhancement factors mean that measured differences in the endpoints can be a factor of ten more sensitive to the mass splitting. In the large $\tan{\beta}$ limit RGE can induce edge splittings up to the per cent level. However, in mSUGRA the $\chi_2^0 \rightarrow \tilde{\tau}_1\tau$ branching ratio increases, significantly reducing the $\chi_2^0 \rightarrow l_l l_l$ branching ratio. A significant rejection of the universal smuon-selectron pole mass hypothesis would discriminate against mSUGRA. If additional mass terms were to cause $m_{\tilde{e}_L} > m_{\tilde{\chi}_1^0}$, then the RGE-induced edge splitting may be significantly discriminated from zero. The di-lepton edge splitting may be a powerful measurement and discrimination tool in SUSY data analysis, and should not be forgotten in global fits to data.

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