Type I X-ray bursts (XRBs) arise from thermonuclear runaways within the accreted envelopes of neutron stars in close binary systems \[1,2\]. About one hundred bursting systems have been identified in the Galaxy, with light curves of about 10–100 s in duration, recurrence periods of ~ hours to days, and peak luminosity \( L_{\text{peak}} \approx 10^{44}–10^{45} L_\odot \) (similar, e.g., to \( L_{\text{peak}} \) of classical novae). During the thermonuclear runaway, an accreted envelope enriched in H and He may be transformed to matter strongly enriched in heavier species (up to \( A \approx 100 \)) via the \( \alpha \)-process and the rapid proton capture process (rp-process) \[5–7\]. Current XRB models do not predict the ejection of any appreciable amounts of synthesized material during the burst. Nonetheless, calculations indicate that radiative winds generated during some bursts may eject material. Studies are ongoing to examine the visibility of detecting any associated absorption features. For reviews on aspects of type I X-ray bursts, see, e.g., Refs. \[8–10\].

The rp-process is largely characterized by localized \((p,\gamma)\) and \((\gamma,p)\) equilibrium within particular isotonic chains near the proton drip-line. Slower \( \beta \)-decays (followed by fast \((p,\gamma)\) reactions) connect these isotonic chains and set the timescale for processing towards heavier nuclei.

In such an equilibrium situation the abundance distribution within an isotonic chain depends exponentially on nuclear mass differences as the abundance ratio between two neighboring isotones is proportional to \( \exp[\lambda S_p/kT] \), where \( S_p \) is the proton separation energy and \( T \) the temperature of the stellar environment. In particular, those isotonic chains with sufficiently small \( S_p \) values (relative to XRB temperatures - at 1 GK, \( kT \approx 100 \) keV) need to be known with a precision of at least 50–100 keV \[9,11\]. These include, among others, \( S_p(26\text{P}) \), \( S_p(43\text{V}) \), \( S_p(46,47\text{Mn}) \), \( S_p(64\text{Ga}) \), and \( S_p(65\text{As}) \) \[11\]. As well, reliable nuclear physics input (including precise mass values and nuclear structure information) is needed for those nuclei along the rp-process path to calculate the thermonuclear reaction rates required for XRB models. Model predictions can then be compared with, e.g., observations of XRB light curves to extract quantitative information about the stellar environments \[12\].

The level structure of \( 43\text{V} \) is not experimentally known. The thermonuclear rate of the \( 42\text{Ti}(p,\gamma)43\text{V} \) reaction was first estimated by Wormer \[13\] based entirely on the properties of four states in the mirror nucleus \( 42\text{Ca} \) \[14,15\]. Later, this rate was recalculated by Herndl et al. \[16\] using two states determined through a shell model calculation of \( 43\text{V} \). Theoretical rates calculated using statistical models are available \[17\]: however, due to the low density of excited states expected in \( 43\text{V} \) near the proton threshold, such calculations are not ideal for this reaction \[18,20\]. A theoretical value of \( S_p = 90 \pm 200 \) keV from the atomic mass evaluation (AME85) \[21\] was utilized in the above rate calculations. Another theoretical value of \( S_p = 190 \pm 230 \) keV was adopted in later AME95 \[22\] and AME03 \[23\] compilations.

Recently, precise mass measurements of nuclei along the rp-process path have become available. These measurements were made at the HIRFL-CSR (Cooler-Storage Ring at the Heavy Ion Research Facility in...
Lanzhou) \cite{24}, in an IMS (Isochronous Mass Spectrometry) mode. Masses measured include those of a series of \( T_z = -1/2 \) nuclei \(^{66}\text{Ge}, \, ^{65}\text{As}, \, ^{67}\text{Se}, \, \text{and} \, ^{71}\text{Kr} \) \cite{25, 26} and \( T_z = -3/2 \) nuclei \(^{41}\text{Ti}, \, ^{43}\text{V}, \, ^{45}\text{Cr}, \, ^{47}\text{Mn}, \, ^{49}\text{Fe}, \, ^{53}\text{Ni}, \, \text{and} \, ^{55}\text{Cu} \) \cite{27}. The proton separation energy of \(^{43}\text{V} \) has been experimentally determined to be \( S_p \approx 83 \pm 43 \text{ keV} \) for the first time \cite{27}. Although the predicted values in the previous compilations (AME85, AME95 and AME03) agree with the experimental value within \( 1 \sigma \) uncertainties, the latter is significantly more precise. This allows the uncertainty in the rate of the \(^{42}\text{Ti}(p,\gamma)^{43}\text{V} \) reaction to be dramatically reduced. In this work, the thermonuclear resonant and direct capture (DC) rates. The astrophysical impact of our new rates has been investigated through one-zone postprocessing x-ray burst calculations.

### II. Reaction Rate Calculation

#### A. Resonant rate

We begin by estimating the \(^{42}\text{Ti}(p,\gamma)^{43}\text{V} \) resonant rate using exactly the level energies, half-lives and single-particle spectroscopic factors from the mirror nucleus \(^{43}\text{Ca} \) \cite{14}. A similar approach was used in Ref. \cite{13}. The resonant rate is calculated by the well-known narrow resonance formalism \cite{13, 16, 28}.

\[
N_A \langle \sigma v \rangle_{\text{res}} = 1.54 \times 10^{11} (AT_9)^{-3/2} \omega_7\gamma [\text{MeV}] \exp \left( -\frac{11.605 E_x [\text{MeV}]}{T_9} \right) [\text{cm}^3\text{s}^{-1}\text{mol}^{-1}] . \tag{1}
\]

Here, the resonant energy \( E_\gamma \) and strength \( \omega_\gamma \) are in units of MeV. For the proton capture reaction, the reduced mass \( A \) is defined by \( A_T/(1+A_T) \) where \( A_T \) is the target mass. The resonant strength \( \omega_\gamma \) is defined by

\[
\omega_\gamma = \frac{2J + 1}{2(2J + 1)} \frac{\Gamma_p \times \Gamma_\gamma}{\Gamma_{\text{tot}}} . \tag{2}
\]

Here, \( J_T \) and \( J \) are the spins of the target and resonant state, respectively. \( \Gamma_p \) is the partial width for the entrance channel, and \( \Gamma_\gamma \) is that for the exit channel. In the excitation energy range considered in this work, other decay channels are closed \cite{23}, and hence the total width \( \Gamma_{\text{tot}} = \Gamma_p + \Gamma_\gamma \). Similar to the approach used by Wormer \textit{et al.}, the gamma partial widths of the unbound states in \(^{43}\text{V} \) were estimated by the life-times (\( \tau \)) of the corresponding bound states in the mirror \(^{43}\text{Ca} \) via \( \Gamma^\gamma = \hbar/\tau \); the proton partial widths were calculated by the following equation,

\[
\Gamma_p = \frac{3h^2}{4R^2} P_t(E) C^2 S_p . \tag{3}
\]

Here, \( R = 1.26 \times (1 + 42^2) \) fm is the nuclear channel radius \cite{13}, \( P_t \) the Coulomb penetrability factor, and \( C^2 S_p \) the proton spectroscopic factor of the resonance.

For this reaction, a temperature of 2 GK corresponds to a Gamow peak \( E_\gamma \approx 1.5 \text{ MeV} \) with a width of \( \Delta \approx 1.2 \text{ MeV} \) \cite{28}. Therefore, its resonant rate is determined by the excited states of \(^{43}\text{V} \) up to \( \sim 2.1 \text{ MeV} \). This first estimate of the resonant rate shows that the first excited state \( (E_\gamma = 0.373 \text{ MeV}) \) dominates the resonant contribution below 0.2 GK, the second excited state \( (E_\gamma = 0.593 \text{ MeV}) \) dominates around 0.2–1.7 GK, and the high-lying 2.067 MeV state (with much shorter life-time \( \tau = 30 \text{ fs} \)) dominates at even higher temperature. It shows that the contribution owing to those high-lying states above 0.26 MeV is negligible at temperatures of interest in XRBs.

We then improved upon this first estimate of the resonant rate. The simplest model for calculating the isobaric-multiplet-mass-equation (IMME) is the \( 0f_7/2 \) shell model used in \cite{29} where the displacement energies in the mass region \( A = 41-55 \) were used to deduce the effective isovector and isotensor two-body matrix elements. The root-mean-square difference between experiment and theory for 60 \( \Delta Z = 1 \) displacement energies was 12 keV. With this model the \( \Delta Z = 3 \) displacement energy difference between \(^{43}\text{Ca} \) and \(^{43}\text{V} \) (7/2–) state is predicted to be 22.854(36) MeV compared to the new experimental value of 22.857(43) MeV. The agreement is impressive.

In the framework of an OXBASH \cite{30} shell model, the resonant parameters of the three states discussed above have been recalculated and summarized in Table I. These calculations are discussed in detail in Appendix A.

#### B. Direct capture rate

The nonresonant direct capture (DC) rate can be estimated using methods presented in Refs. \cite{16, 28},

\[
N_A \langle \sigma v \rangle_{\text{DC}} = \frac{1}{2} \frac{C^2 S_p \sigma_{\text{DC}}(E_T)}{E_T} .
\]
TABLE I: Parameters for the present \( ^{42}\text{Ti}(p,\gamma)^{43}\text{V} \) resonant rate calculation. The uncertainties quoted for strengths \((\omega\gamma)\) arise from the energy dependence of the widths \(\Gamma_{\gamma} \) and \(\Gamma_{p} \), as well as the assumed uncertainties of spectroscopic factors \((C^2S_p)\) (a factor of 2).

| \(E_x^{(43}\text{V})\) (MeV) | \(E_r\) (MeV)
a | \(\tau\) (ps) | \(J^\pi\) | \(C^2S_p\) | \(\Gamma_{\gamma}\) (eV) | \(\Gamma_{p}\) (eV) | \(\omega\gamma\) (eV) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0.436(0.050)² | 0.355(0.066)   | 22(2)          | 5/2            | 3              | 0.15³         | 3.04×10⁻³      | 5.10×10⁻⁴      | 1.5×10⁻⁸  |
| 0.537(0.050)² | 0.454(0.066)   | 117(6)         | 3/2            | 1              | 0.046⁴        | 3.42×10⁻⁶      | 6.27×10⁻⁵      | 6.5×10⁻⁶  |
| 2.067(0.100)² | 1.984(0.109)   | 0.03(0.01)     | 7/2            | 3              | 0.0003⁵       | 2.19×10⁻²      | 3.45×10⁻²      | 5.4×10⁻²  |

\(a\) Resonance energies calculated using \(E_r=E_x^{(43}\text{V})-S_p^{\text{exp}}\), where \(S_p^{\text{exp}}=83±43\) keV. \(b\) Estimated theoretical uncertainties in the parenthesis. \(c\) Value from the previous \((p,d)\) \([31]\) and \((d,t)\) \([32]\) experiments. \(d\) Averaged value from the \((d,p)\) experiments \([33,34]\). \(e\) Value calculated by the OXBASH code with same model-space and interactions as in Ref.\([16]\).

\[
N_A(\sigma v) = N_A \left(\frac{8}{\pi A}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S_{\text{dc}}(E) \exp \left[-\frac{E}{kT} - \frac{b}{E^{1/2}}\right] dE
\]  

(4)

If \(S_{\text{dc}}(E)\) factor is nearly a constant over the Gamow window, the nonresonant reaction rate can be approximated in a form of \([16,28]\)

\[
N_A(\sigma v)_{\text{dc}} = 7.83 \times 10^9 \left(\frac{Z}{A}\right)^{1/3} T_9^{-2/3} S_{\text{dc}}(E_0)[\text{MeV b}] \times \exp \left[-4.249 \left(\frac{Z^2 A}{T_9}\right)^{1/3}\right] [\text{cm}^3\text{s}^{-1}\text{mol}^{-1}].
\]  

(5)

The critical parameter is \(S_{\text{dc}}(E_0)\), the astrophysical S-factor at the Gamow energy \(E_0\). Herndl et al. listed an effective \(S_{\text{dc}}(E_0)\) factor of \(4.91 \times 10^{-20} [\text{MeV b}]\) in their Table XIII. We have recalculated this factor and found that the above number is actually \(4.91 \times 10^{-22} [\text{MeV b}]\).

In this work, the \(^{42}\text{Ti}(p,\gamma)^{43}\text{V} \) reaction rate from direct capture into ground state of \(^{43}\text{V}\) has been calculated with a RADCAP code \([35,36]\) by using a Woods-Saxon nuclear potential (central + spin orbit) and a Coulomb potential of a uniform charge distribution. The nuclear potential parameters were determined by matching the bound-state energy \((E_b=83\) keV). A spectroscopic factor of \(C^2S=0.75\) \([16]\), which agrees with the \((d,p)\) experimental values of 0.68 \([33]\) and 0.55 \([34]\), was adopted in the present calculations. The DC rate contributes to the total rate only by 10–20% in the temperature region of 2–3 GK, and dominates the rate below 0.07 GK. The RADCAP calculations are described in detail in Appendix B.

C. Total reaction rate

The total reaction rate of \(^{42}\text{Ti}(p,\gamma)^{43}\text{V} \) has been calculated by simply summing up the resonant and DC contributions. Our new rate is tabulated in Table III and plotted in Fig. II. The uncertainty in the present rate arises from uncertainties in our adopted \(E_r\) (which also lead to uncertainties in the strengths since the values of \(\Gamma_{p} \) and \(\Gamma_{\gamma} \) have been scaled using the values of \(E_r\) - see Appendix A) and the uncertainty in the DC contribution (±40% - see Appendix B). In addition, we have assumed a factor of two uncertainty in the adopted spectroscopic factors. The uncertainty of the total rate is dominated by the uncertainty of the \(S_p\) value due to the
exponential dependence of the rate on $S_p$. The rate based upon calculations in Herndl et al., where only two states (at $E_p=0.36, 0.55$ MeV) were assumed, is also shown in Fig. 1 for comparison. Because the uncertainty of the DC contribution was not determined in Herndl et al., the uncertainty of Herndl et al. rate shown originates only from those of the calculated resonant rates (i.e., the error of $S_p$ propagating into the strengths). It shows our new rate calculated with the precise experimental $S_p$ value has much smaller uncertainties than the previous ones. This clearly demonstrates the importance of precise mass measurements.

Figure 2 compares five different rates for the $^{42}$Ti($p,\gamma$)$^{43}$V reaction: (a) present rate (Fig. 1(a)); (b) the rate from Herndl et al. [18]; (c) the rate from Wormer et al. [13]; (d) the statistical model rate ths8_v4 available in the JINA REACLIB (with $S_p=-0.0189$ MeV [37]); (e) the statistical model rate rath_v2 in the REACLIB [17] (with $S_p=-0.411$ MeV based on the FRDM mass model [38]). Because of the rather similar $S_p$ value used, our new rate does not deviate significantly from those of Herndl et al. and Wormer et al. in the temperature region of interest in XRBs. Our new rate, however, is very well constrained with the precise mass measurement as shown in Fig. 1. The statistical-model calculations deviate from our new rate considerably over the entire temperature region of interest. This demonstrates again that the statistical-model is not ideally applicable for this reaction mainly owing to the low density of low-lying excited states in $^{43}$V.

### III. Astrophysical Implications

The impact of our new $^{42}$Ti($p,\gamma$)$^{43}$V rate was examined in the framework of one-zone XRB models. Using the representative K04 thermodynamic history ($T_{peak}=1.4$ GK [39]), we performed a series of postprocessing calculations to explore the role of different $^{42}$Ti($p,\gamma$)$^{43}$V rates and $S_p$ values on the nuclear energy generation rate ($E_{nuc}$) and XRB yields. Rates of all other reactions in the network were left unchanged during these calculations. To be clear, in the discussion below we will refer explicitly to $^{42}$Ti($p,\gamma$)$^{43}$V forward rates (e.g., as shown in Fig 1) and to the $S_p$ value used to determine the corresponding reverse rates through the principle of detailed balance (see, e.g., Table II).

No significant differences in the respective nuclear energy generation rates were found by comparing XRB calculations with the (a) present forward rate ($S_p=83$ keV for the reverse rate); (b) Herndl et al. forward rate ($S_p=83$ keV); (c) Wormer et al. forward rate ($S_p=88$ keV); and (d) ths8_v4 forward rate ($S_p=-19$ keV). $E_{nuc}$ determined using the rath_v2 forward rate (e), $S_p=-111$ keV), however, was up to 10% lower than that from the above cases (a–d) during the burst. This (minor) dif-
The effects on XRB yields by using different \( {^{42}\text{Ti}}(p,\gamma)^{43}\text{V} \) forward rates and \( S_p \) values have been investigated. Fig. 3 shows representative yields in this mass range for the different cases discussed above, as determined immediately following the respective XRB calculations. No significant differences in yields were observed for cases (a–d) above. The two cases (d,e) with reverse rates determined using negative \( S_p \) values gave somewhat different yields for species with \( \Lambda =42-44 \). For example, the negative \( S_p \) values produce relatively more \( ^{42}\text{Ti} \) but less \( ^{43}\text{V} \).

The dominant role of the \( S_p \) value used in the reverse rate in determining the yields is clearly seen in Fig. 3 from the comparison of cases (a) (labeled as “Present, IMP \( \Delta S_p \))”, (d) (labeled as “ths8\_v4”), (e) (labeled as “rath\_v2, FRDM \( S_p \))”, and (f) (labeled as “rath\_v2, IMP \( S_p \))”. It shows the yields calculated with the new experimental \( S_p \) value (for the reverse rate) significantly differ from those yields with other theoretical \( S_p \) values. In addition, to demonstrate the impact of the uncertainty in \( S_p \), we performed additional XRB calculations using the present forward rate, along with reverse rates that reflect the one sigma uncertainties in \( S_p \) from AME03 (\( \Delta S_p =233 \text{ keV} \)) and the IMP mass measurement (\( \Delta S_p =43 \text{ keV} \)). As shown in Fig. 3 the reduced uncertainty in \( S_p \) directly influences the possible ranges of mass fractions for the affected species. Indeed, the uncertainty from the IMP mass measurement leads to variations, by less than a factor of three, in the yields of the most produced isotopes in this mass region, such as \( ^{42,43}\text{Ti}, ^{42}\text{Sc}, ^{43}\text{V} \) and \( ^{43,44}\text{Cr} \).

IV. SUMMARY

The thermonuclear rate of the \( {^{42}\text{Ti}}(p,\gamma)^{43}\text{V} \) reaction has been recalculated using the recent precise proton separation energy of \( S_p =83\pm43 \text{ keV} \) measured at the HIRFL-CSR facility in Lanzhou, China. We have also used new, updated calculations of the direct capture and resonant contributions to the rate. Our new rate deviates significantly from other rates found in the literature. We confirm that statistical model calculations are not ideally applicable for this reaction primarily because of the low density of low-lying excited states in \( ^{43}\text{V} \). We recommend that our new rate be incorporated in future astrophysical network calculations.

The astrophysical impact of our new rate has been investigated through one-zone postprocessing Type I x-ray burst calculations. Even when using dramatically different rates, we find no significant changes to the calculated nuclear energy generation rate during a representative burst. This is because equilibrium between the forward \( {^{42}\text{Ti}}(p,\gamma)^{43}\text{V} \) and reverse \( ^{43}\text{V}(\gamma,p){^{42}\text{Ti} \) processes rapidly develops at XRB temperatures. As such it is the reac-

![FIG. 2: (Color online) Ratios between the present rate (see Table II) and other available ones (Herndl 1995 [16], Wormer 1994 [17], rath\_v2 [18] and ths8\_v4 [19]).](image)

![FIG. 3: Abundances following one-zone XRB calculations using the K04 thermodynamic history [39]. Abundance variations determined using the present \( 42\text{Ti}(p,\gamma)^{43}\text{V} \) forward rate with reverse rates calculated using \( \Delta S_p =43 \text{ keV} \) (IMP [27], solid black line), and \( \Delta S_p =233 \text{ keV} \) (AME03 [23], dotted grey line) are indicated. As well, abundances determined using the rath\_v2 forward rate [18] along with reverse rates calculated with \( S_p =411 \text{ keV} \) (FRDM [28], open squares) and \( S_p =83 \text{ keV} \) (IMP [27], open circles) are shown. Abundances determined with the ths8\_v4 rate [19] (\( S_p =19 \text{ keV} \) [28], open triangles) are also shown.](image)
TABLE III: Mass fractions following one-zone XRB calculations using the K04 thermodynamic history. These values are plotted in Fig. 3. The first two columns give ranges of mass fractions as determined using the one sigma uncertainties for $S_p(^{43}\text{V})$ from the recent measurement and other theoretical estimates.

| Species | IMP $\Delta S_p$ | AME03 $\Delta S_p$ | rath$_{v2}$ (FRDM $S_p$) | rath$_{v2}$ (IMP $S_p$) | ths8$_{v4}$ |
|---------|-----------------|-------------------|-----------------|-----------------|-------------|
| $^{42}\text{Ti}$ | (2.3–5.3) $\times 10^{-5}$ | (1.6–75) $\times 10^{-10}$ | 8.7 $\times 10^{-5}$ | 3.9 $\times 10^{-4}$ | 7.1 $\times 10^{-5}$ |
| $^{42}\text{Sc}$ | (1.0–2.3) $\times 10^{-5}$ | (6.9–33) $\times 10^{-10}$ | 3.9 $\times 10^{-9}$ | 1.7 $\times 10^{-9}$ | 3.1 $\times 10^{-9}$ |
| $^{43}\text{Ti}$ | (1.1–2.2) $\times 10^{-5}$ | (3.2–31) $\times 10^{-9}$ | 3.5 $\times 10^{-8}$ | 1.7 $\times 10^{-8}$ | 2.9 $\times 10^{-8}$ |
| $^{43}\text{V}$ | (2.3–4.2) $\times 10^{-5}$ | (8.0–55) $\times 10^{-9}$ | 1.0 $\times 10^{-9}$ | 3.2 $\times 10^{-8}$ | 1.1 $\times 10^{-8}$ |
| $^{44}\text{Cr}$ | (1.0–1.9) $\times 10^{-5}$ | (3.6–25) $\times 10^{-6}$ | 1.6 $\times 10^{-6}$ | 1.4 $\times 10^{-5}$ | 5.0 $\times 10^{-6}$ |

The $Q$-value (or $S_p$) that mainly characterizes the equilbrium abundances of $^{42}\text{Ti}$ and $^{43}\text{V}$. In this respect, the present $^{42}\text{Ti}(p,\gamma)^{43}\text{V}$ rate and $S_p(^{43}\text{V})$ value are sufficiently well known to determine the nuclear energy generation rate within the framework of the adopted XRB model. In addition, we find that the new experimental value of $S_p$ affects significantly the yields of a limited number of species with $A=42–44$, such as $^{42,43}\text{Ti}$, $^{42,43}\text{Sc}$, $^{43}\text{V}$ and $^{43,44}\text{Cr}$. The precision in $S_p$ achieved from the IMP mass measurement restricts the variation of these yields to better than a factor of three. It demonstrates clearly the importance of precise mass measurements for these key nuclei (especially those waiting-point nuclei) along the rp-process occurring in x-ray bursts.

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Appendix A: Calculation of resonant parameters

Below we summarize our calculations of the resonant parameters of the three states in $^{43}\text{V}$ around 0.373 MeV (Resonance 1), 0.593 MeV (Resonance 2) and 2.067 MeV (Resonance 3).

1. Resonance 1

In the $0f_{7/2}$ model, the energy of the first excited $5/2^-$ state is predicted to be 63 keV higher in $^{43}\text{V}$ compared to $^{43}\text{Ca}$. Thus, with the experimental energy of 0.370 keV for $^{43}\text{Ca}$ we obtain a predicted excitation energy of 0.436 MeV ($E_x=0.353$ MeV) for $^{43}\text{V}$. The $5/2^-$ to $7/2^-$ transition in $^{43}\text{Ca}$ has an experimental $B(M1)$ value of 0.023(2) $\mu^2$. In the $pf$ model space with the FPD6 interaction the $B(M1)$ values are predicted to be 0.018, 0.025 $\mu^2$ for the excited states in $^{43}\text{Ca}$ and $^{43}\text{V}$, respectively. Therefore, a value of $B(M1)=0.032\mu^2$ is derived for the predicted 0.436 MeV state, and $\Gamma_1$ is thus calculated to be about 3.04$\times 10^{-5}$ eV. This implies that the resonant strength for this state is determined by the much smaller $\Gamma_p$.

Wormer et al. estimated a resonant strength value of $\omega_\gamma=1.0\times 10^{-10}$ eV for the $E_x=0.28$ MeV state. A spectroscopic factor of 0.014 would reproduce their proton width of $\Gamma_p\approx 3.3\times 10^{-11}$ eV based on Eq. 3. Later, Herndl et al. estimated a value of $\Gamma_p=1.1\times 10^{-12}$ eV with a spectroscopic factor of 0.008, by the equation of

$$\Gamma_p = C^2 S_p \times \Gamma_{sp},$$

where the single-particle width $\Gamma_{sp}$ was calculated from the scattering phase shifts in a Woods-Saxon potential whose depth was determined by matching the resonant energy. Based on the method introduced in Ref. [43], we have recalculated this proton width using Eq. [A1] with potential parameters of $E_x=0.27$ MeV, $\eta_0=1.17$ fm, $a=0.69$ fm and $r_1=1.28$ fm. The justification is that the choice of the parameters can be found in [43].

We have obtained a single-particle width of $\Gamma_{sp}=1.76\times 10^{-10}$ eV, which roughly agrees with the value of $1.38\times 10^{-10}$ eV calculated by Herndl et al. [43].

Neutron spectroscopic factor measurements imply values of $S_{np}\approx 0.15$ [31, 32], in disagreement with the values assumed by Wormer et al. and Herndl et al. We assumed $S_{np}=S_p$ in the following proton width calculations. With Eq. [A1], a width of $\Gamma_p=5.10\times 10^{-9}$ eV was obtained with the above parameters of $\eta_0=1.17$ fm, $a=0.69$ fm and $r_1=1.28$ fm (parameter Set 3 in Table IV). The proton width calculated by Eq. 3 is always larger than that by Eq. [A1] because the former equation does not take the dimensionless single-particle reduced width $\theta_{sp}^2$ into account. $\theta_{sp}^2$ is usually assumed to be unity, although this is not appropriate for many cases [43]. Here, $\theta_{sp}^2$ is
calculated to be 0.24.

2. Resonance 2

A description of the 3/2− excited state requires the full $pf$ shell-model basis. With the empirically determined isospin-nonconserving interactions for the $pf$ shell [44], the second excited 3/2− state in $^{43}$V is estimated to be located at $E_x=0.537$ MeV ($E_x=0.454$ MeV).

A spectroscopic factor of $C^2 S=0.046$ averaged from the $(d,p)$ experiments [33, 34] was used for this state, as adopted in Ref. [13]. The proton width is calculated to be 6.27×10−5 eV ($\theta_{np}=0.56$) with the same parameter Set 3 (Table I). Since it is difficult to make a reliable life-time calculation for this state, we estimated this $\Gamma_\gamma$ based on the mirror life-time. In the mirror $^{43}$Ca, this state decays either to the ground state ($J^\pi=7/2^−$) or to the first excited state ($J^\pi=5/2^−$) with branching ratios 15 of 70.2% and 29.8%, respectively. The ground-state transition is a pure $E2$, whose width can be estimated by the relation of $\Gamma_\gamma (E2)=5\times10^{-10} S(E2) \times BR$ [45]. Here, $S$ is the strength of the transition in Wiesskopf units, and $BR$ is the branching ratio (70.2%). The Weisskopf-unit gamma width (in eV) for an $E2$ transition is $\Gamma_W^W(E2)=4.9\times10^{-8} A^{1/3} E_\gamma^2$ [45, 46] with $A=43$. This results in a ground-state-transition width of $\Gamma_\gamma (E2)=1.7\times10^{-6}$ eV with $S=7.2 \times 10^{-3}$ [13]. The first-excited-state transition is a mixture of $M1$ and $E2$, where the dominant $M1$ width can be calculated by the relation of $\Gamma_\gamma (M1)=S\times\Gamma_W^W(M1) \times BR$. Here, a value of $S=7.6\times10^{-3}$ [45] was adopted in the calculation. The Weisskopf-unit gamma width (in eV) for an $M1$ transition is $\Gamma_W^W(M1)=2.1\times10^{-4} E_\gamma^2$ [45, 46]. $\Gamma_\gamma (M1)$ is estimated to be about $4.9\times10^{-7}$ eV with a branching ratio of 29.9%. Therefore, only the ground-state-transition dominates the actual total $\Gamma_\gamma$ width, and the energy dependence of $\Gamma_\gamma$ can be accounted for by using the scale factor $E_\gamma^3$. For the 0.593-MeV state in $^{43}$Ca, $\Gamma_\gamma$ is about 5.62×10−6 eV (as estimated from the lifetime of 117 ps) In this work, we have adopted a value of $\Gamma_\gamma (\gamma)=3.42\times10^{-18}$ eV for the 0.537-MeV state in $^{43}$V by correcting for the energy difference between $^{43}$V ($E_x=0.537$ MeV) and $^{43}$Ca ($E_x=0.593$ MeV).

3. Resonance 3

The higher-lying 2.067-MeV 7/2− state in $^{43}$Ca is not described in the $pf$ model space, and requires nucleons to be excited from the $sd$ shell for its description. We do not have a good model for its displacement energy and simply use the same value for its excitation energy in $^{43}$V with an estimated error of 100 keV.

The $\Gamma_\gamma$ for this state was calculated to be 2.19×10−2 eV with a mirror lifetime of $\tau=0.03$ ps. In the mirror $^{43}$Ca, this state mainly decays to the ground state ($J^\pi=7/2^−$) and to the first excited state ($J^\pi=5/2^−$) with branching ratios 12 of 78% and 22%, respectively; both $\gamma$ transitions have $M1(E2)$ characters. By using the same strength $S$ value for the above 0.537 MeV state with respect to $E2$ and $M1$ transitions, $\gamma$ widths of the ground-state and first-excited transitions were calculated. It is found that the ground-state $E2$ transition dominates the total $\Gamma_\gamma$ for this state. Therefore, the factor $E_\gamma^3$ was again used to account for the energy dependence of $\Gamma_\gamma$. The proton width $\Gamma_p$ was calculated to be 3.45×10−2 eV with parameter Set 3 (Table IV). We have used a spectroscopic factor of 0.0003 as determined with the OXBASH code (using the same model-space and interactions as in Ref. [13]). This factor may be larger in nature, and should be determined experimentally.

Appendix B: Calculation of direct capture rate

The astrophysical $S$ factor of the direct-capture $^{42}$Ti$(p,\gamma)^{43}$V reaction has been calculated by the RADCAP code. The calculated $S_{dc}$ factors are shown in Fig. 4 with three parameter sets listed in Table IV. With a spin-orbit potential of $V_{so}=-10$ MeV, the $S_{dc}(E)$ factors calculated using three sets of parameters (Table IV) vary by no more than 15% over the energy range of 0–3 MeV. This energy range covers the Gamow window for a temperature up to 3 GK. The above changes can not be regarded as substantial. Since Huang et al. [33] reproduced successfully the $S$ factors for a series of radiative capture reactions, we have adopted their potential parameters (Set 1 in Table IV) in the final DC rate calculation. The present $S_{dc}$ factors can be well parameterized in a Taylor-series form [28] of $S_{dc}(E)=\sum_{k=0}^{\infty} \frac{c_k(E_0)}{E} E^k$, where $S_{dc}$ factors are in units of MeV b and $E$ in MeV. The fitted parameters are $S(0)=3.97\times10^{-2}$ MeV b for the $S$ factor at zero energy, and the derivatives with respect to energy are $S(1)(0)=3.37\times10^{-2}$, $S(2)(0)=1.31\times10^{-2}$, $S(3)(0)=9.72\times10^{-3}$, and $S(4)(0)=1.18\times10^{-2}$, respectively.

In addition, the parameter dependence on $S_{dc}(E)$ has been studied and the results are shown in Fig. 5. It shows that $S_{dc}$ factor is insensitive to the parameters $V_{so}$ and $R_e$ (or $r_e$), but rather sensitive to the parameters $R_0$ (or $r_0$) and $a$. The choice of parameter ranges is based on the literature values [33, 47, 48]. The error of the present DC rate is estimated simply by adding in quadrature the uncertainties originating from the potential parameters discussed above; it is about ~40% in the energy range of 0–3 MeV. The DC rate as a function of temperature is calculated by numerical integration of our calculated $S$ factors using an EXP2RATE code [49].
TABLE IV: Potential parameter lists used in the $S_{dc}$ factor calculations.

| Parameters | Set 1 [36] | Set 2 [47] | Set 3 [43] |
|------------|------------|------------|------------|
| $R_0=R_{so}$ (fm)$^a$ | $1.25\times(1+42)^+ \ [r_0=r_{so}=1.26]$ | $1.25\times42^+ \ [r_0=r_{so}=1.25]$ | $1.25\times42^+0.23 \ [r_0=r_{so}=1.17]$ |
| $R_c$ (fm)$^a$ | $1.25\times(1+42)^+ \ [r_c=1.26]$ | $1.25\times42^+ \ [r_c=1.25]$ | $1.24\times42^+0.12 \ [r_c=1.28]$ |
| $a_0=a_c$ (fm) | 0.65 | 0.65 | 0.69 |
| $V_{so}$ (MeV) | -10.0 | -10.0 | -10.0 |
| $V_0$ (MeV)$^c$ | -100.48 | -101.73 | -111.22 |
| $S_{dc}(0)$ (MeV b) | $3.98\times10^{-2}$ | $3.84\times10^{-2}$ | $3.48\times10^{-2}$ |

$a$ $r_0$, $r_{so}$ and $r_c$ are commonly defined as $r=R/A^{1/3}$ for comparison. $^b$ The choice of parameters can be found in Ref. [43]. $^c$ $V_0$ is varied to match the bound-state energy $E_0=83$ keV.

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FIG. 4: (Color online) Direct-capture $S_{dc}$ factors calculated with three parameter sets listed in Table IV. A previous constant value of $S_{dc}(E_0)=4.91\times10^{-2}$ [MeV b] (Herndl 1995 [10]) is shown for comparison.

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[1] S.E. Woosley and R.E. Taam, Nature 263, 101 (1976).
[2] P.C. Joss, Nature 270, 310 (1977).
[3] H. Schatz et al., Phys. Rev. Lett. 86, 3471 (2001).
[4] V.-V. Elomaa et al., Phys. Rev. Lett. 102, 252501 (2009).
[5] R.K. Wallace and S.E. Woosley, Astrophys. J. Suppl. 45, 389 (1981).
[6] H. Schatz et al., Phys. Rep. 294, 167 (1998).
[7] S.E. Woosley et al., Astrophys. J. Suppl. 151, 75 (2004).
[8] W. Lewin et al., Space Sci. Rev. 62, 223 (1993).
[9] T. Strohmayer, L. Bildsten, in: W. Lewin, M. van der Klis (Eds.), Compact Stellar X-Ray Sources, (Cambridge Univ. Press, Cambridge, 2006).
[10] A. Parikh et al., Prog. Part. Nucl. Phys. 69, 225 (2013).
[11] A. Parikh et al., Phys. Rev. C 79, 045802 (2009).
[12] H. Schatz and K. E. Rehm, Nucl. Phys. A777, 601 (2006).
[13] L. Van Wormer et al., Astrophys. J. 432, 326 (1994).
[14] P.M. Endt and C. Van Der Leun, Nucl. Phys. A310, 1 (1978).
[15] P.M. Endt, Nucl. Phys. A521, 1 (1990).
[16] H. Herndl et al., Phys. Rev. C 52, 1078 (1995).
[17] JINA Reaclib Database, please see, https://groups.nscl.msu.edu/jina/reaclib/db/.
[18] T. Rauscher et al., Phys. Rev. C 56, 1613 (1997).
[19] T. Rauscher and F.-K. Thielemann, At. Data Nucl. Data Tables 75, 1 (2000).
[20] T. Rauscher and F.-K. Thielemann, At. Data Nucl. Data Tables 79, 47 (2001).
[21] G. Audi and A.H. Wapstra, Nucl. Phys. A432, 1 (1985).
[22] G. Audi and A.H. Wapstra, Nucl. Phys. A595, 409 (1995).
[23] G. Audi et al., Nucl. Phys. A729, 337 (2003).
[24] J. W. Xia et al., Nucl. Instr. Meth. A 488, 11 (2002).
[25] X.L. Tu et al., Phys. Rev. Lett. 106, 112501 (2011).
[26] Y.H. Zhang et al., Phys. Rev. Lett. 109, 102501 (2012).
[27] X.L. Yan et al., Astrophys. J. Lett. 766, L8 (2013).
[28] C.E. Rolfs and W.S. Rodney, Cauldrons in the Cosmos, (University of Chicago Press, Chicago, 1988)
[29] B.A. Brown and R. Sherr, Nucl. Phys. A322, 61 (1979).
[30] B.A. Brown, E. Etchegoyen, W.D.M. Rae, and N.S. Godwin, OXBASH, 1984 (unpublished).
[31] S.M. Smith et al., Nucl. Phys. A113, 303 (1968).
FIG. 5: (Color online) Dependence of $S_{dc}$ factors on parameters (a) spin-orbit potential $V_{so}$, (b) Coulomb radius parameter $r_c$, (c) diffuseness $a$, and (d) optical-model (real) potential radius parameter $r_0$. Here, radius $r$ is defined as $r=R/A^{1/3}$, with $r_0=r_{so}, a_0=a_{so}$ in all calculations.

[32] P. Doll et al., Nucl. Phys. A263, 210 (1976).
[33] W.E. Dorenbusch, T.A. Belote, and O. Hansen, Phys. Rev 146, 734 (1966).
[34] G. Brown, A. Denning, J.G.B. Haigh, Nucl. Phys. A225, 267 (1974).
[35] C.A. Bertulani, Comput. Phys. Commun. 156, 123 (2003).
[36] J.T. Huang et al., Atom. Data Nucl. Data Tables 96, 824 (2010).
[37] R.H. Cyburt et al., Astrophys. J. Suppl. 189, 240 (2010).
[38] P. Möller et al., Atom. Data Nucl. Data Tables 59, 185 (1995).
[39] A. Parikh et al., Astrophys. J. Suppl. 178, 110 (2008).
[40] W.A. Richter, M.G. van der Merwe, R.E. Julies, and B.A. Brown, Nucl. Phys. A523, 925 (1991).
[41] B.A. Brown et al., Phys. Rev. C 48, 1456 (1993).
[42] A.E. Champagne et al., Nucl. Phys. A556, 123 (1993).
[43] C. Iliadis, Nucl. Phys. A618, 166 (1997).
[44] W.E. Ormand and B.A. Brown, Nucl. Phys. A491, 1 (1989).
[45] P.M. Endt, Atom. Data Nucl. Data Tables 23, 3 (1979).
[46] D.H. Wilkinson, in Nuclear Spectroscopy, edited by F. Ajzenberg-Selove (Academic Press, New York, 1960), Vol. B.
[47] F. G. Perey, Phys. Rev. 131, 745 (1963).
[48] R.L. Varner et al., Phys. Rep. 201, 57 (1991).
[49] T. Rauscher, EXP2RATE v2.1, http://nucastro.org/codes.html.