Geometric scaling in ultrahigh energy neutrinos and nonlinear perturbative QCD

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It is shown that in ultrahigh energy inelastic neutrino-nucleon(nucleus) scattering the cross sections for the boson-hadron(nucleon) reactions should exhibit geometric scaling on the single variable $\tau_F = Q^2/Q_{\text{sat}}^2$. The dependence on energy and atomic number of the charged/neutral current cross sections are encoded in the saturation momentum $Q_{\text{sat}}$. This fact allows an analytical computation of the neutrino scattering on nucleon/nucleus at high energies, providing a theoretical parameterization based on the scaling property.

PACS numbers: 13.15.+g, 13.60.Hb, 12.38.Bx

I. INTRODUCTION

One important property of the nonlinear perturbative QCD approaches for high energy deep inelastic $ep(A)$ scattering is the prediction of the geometric scaling. Namely, the total $\gamma^* p(A)$ cross section at large energies is not a function of the two independent variables $x$ and $Q^2$, but is rather a function of the single variable $\tau_F = Q^2/Q_{\text{sat}}^2$. As usual, $Q^2$ is the photon virtuality and $x$ the Bjorken variable. It was demonstrated \(\text{[1]}\) that geometric scaling is the exact asymptotic solution of a general class of nonlinear evolution equations \(\text{[2, 3]}\) and it appears as a universal property of these kind of equations. The specific scaling solutions correspond to traveling wave solutions of those equations. The saturation momentum $Q_{\text{sat}}^2(x; A) \propto xG_A(x, Q_{\text{sat}}^2) \sim A^\alpha x^{-\lambda}$ ($\alpha \approx 1/3$, $\lambda \approx 0.3$) is connected with the phenomenon of gluon saturation. It is expected that the rise of the gluon distribution function at small values of Bjorken $x$ be naturally tamed by saturation and circumvent the subsequent violation of unitarity. In principle, geometric scaling is predicted to be present only on process dominated by low-$x$. However, it was theoretically demonstrated \(\text{[4]}\) that geometric scaling is preserved by the QCD evolution up to relatively large virtualities, within the kinematical window $Q_{\text{sat}}^2(x) \approx Q^2 \lesssim Q_{\text{sat}}^4(x)/\Lambda_{\text{QCD}}^2$. That is, the scaling property extends towards very large virtualities provided one stays in low-$x$. This kinematical window is further enlarged due to the nuclear enhancement of the saturation scale. In Ref. \(\text{[5]}\), the observation that the DESY-HERA $e^p$ collider data on the proton structure function $F_2$ is consistent with scaling at $x \lesssim 0.01$ and $Q^2 \lesssim 400$ GeV$^2$ was for the first time presented. Similar behavior was further observed on electron-nuclei processes \(\text{[6]}\). Geometric scaling has been also observed in inclusive charm production \(\text{[7]}\) and predicted to be present also in heavy quark production in lepton-nuclei scattering \(\text{[8]}\).

Recently, the high energy lepton-hadron, proton-nucleus and nucleus-nucleus collisions have been related through geometric scaling \(\text{[9]}\). Within the color dipole picture and making use of a rescaling of the impact parameter of the $\gamma^* h$ cross section in terms of hadronic tar-
\( x \simeq m_{W,Z}^2/E_\nu \sim 10^{-8} \) at \( E_\nu \sim 10^{12} \) GeV and virtualities \( Q^2 \sim m_{W,Z}^2 \approx 10^4 \) GeV \(^2\), where \( m_{W,Z} \) are the boson masses. Therefore, accurate predictions for UHE neutrinos require precise extrapolation of the structure functions to the small-\( x \). Recently, the approaches containing saturation effects and/or scaling property have been compared and their main features were identified and investigated \([11, 12]\). A very important feature is that, within the color dipole framework \([13]\), the charged (CC) and neutral (NC) current structure functions are described by the same mathematical expressions as the proton structure function up to a different coupling of the electroweak bosons. Therefore, the geometric scaling property should be present also in neutrino scattering on hadron targets and allows to obtain the dependences on energy and atomic number of CC/NC cross sections, which are encoded in the nuclear saturation momentum. In fact, geometric scaling holds for a large region of phase space and thus it contributes for an important part of the integrated cross section. Furthermore, the upper limit for scaling window should be enhanced by a factor \( \sim 10-100 \) in the nuclear case. In what follows, the weak boson-hadron/nucleus cross section is computed and it is shown to exhibit geometric scaling on the scaling variable \( \tau_A \). The CC and NC structure function are further studied and compared with the available high energy approaches. The scaling property will allow to obtain simple analytical expressions for the CC/NC neutrino cross sections at high energies. They will be used to provide theoretical parameterizations for the UHE neutrino scattering on nucleons and nuclei.

II. UHE NEUTRINO CROSS SECTION

Deep inelastic neutrino scattering can proceed via \( W^\pm \) (charged current interactions - CC) or \( Z^0 \) (neutral current interactions - NC) exchanges. The standard kinematical variables describing them are given by \( s = 2m_NE_\nu \) (center-of-mass energy squared), \( Q^2 \) (boson virtuality), Bjorken \( x \) and \( y = Q^2/xs \) (inelasticity variable). Here, \( m_N \) is the nucleon mass and \( E_\nu \) labels the neutrino energy. At small-\( x \), a successful framework describing QCD interactions is provided by the color dipole formalism \([12]\), which allows an all-twist computation of the structure functions. The physical picture of the interaction is the deep inelastic scattering (DIS) at low \( x \) viewed as the result of the interaction of a color \( q\bar{q} \) dipoles, which are fluctuations of the electroweak gauge bosons, with the hadron target. The interaction is modeled via the dipole-target cross section, whereas the boson fluctuation in a color dipole is given by the corresponding wave function. The DIS structure functions for neutrino scattering read as \([11]\),

\[
F_{T,L}^{CC,NC}(x,Q^2) = \frac{Q^2}{4\pi^2} \sigma_{tot}(W^\pm(Z^0)\ N \to X),
\]

\[
\sigma_{tot}(W^\pm(Z^0)\ N) = \int d^2r \int_0^1 dz |\psi_{W^\pm,Z}^{\ d,\ \tau_A}(z)|^2 \sigma_{dip},
\]

where \( r \) denotes the transverse size of the color dipole, \( z \) the longitudinal momentum fraction carried by a quark and \( \psi_{W^\pm,Z}^{\ d,\ \tau_A}(z) \) are the sum over dipoles of the wave functions of the charged or neutral gauge bosons, respectively. Their explicit expressions can be found for instance in Refs. \([11, 12, 14]\). Here one considers only four flavors \( (u,d,s,c) \) assumed to be massless. Heavy quarks \( (b,t) \) give relatively small contribution and will be disregarded. Color dipoles contributing to Cabibbo favored transitions are \( ud(\bar{d}u), \ c\bar{s}(\bar{s}c) \) for CC interactions and \( \bar{u}d, \bar{d}s, \bar{s}c \) for NC interactions. The dipole cross section \( \sigma_{dip}(x,r;A) \), describing the dipole-target interaction, is substantially affected by saturation effects at dipole sizes \( r \gtrsim 1/Q_{sat} \).

Based on the fact that the expressions for the photon wavefunction and the electroweak gauge bosons, appearing on Eq. \([6]\), are exactly the same up to the different coupling of the bosons to the quark color dipole, we can simply write,

\[
\sigma_{tot}^{(W^\pm)}(x,Q^2, A) = \frac{4}{\alpha_{em} \sum_f e_f^2} \sigma_{tot}^{(\gamma^*N)}(x,Q^2; A),
\]

\[
\sigma_{tot}^{(Z^0)}(x,Q^2, A) = \frac{K_{chiral}}{\alpha_{em} \sum_f e_f^2} \sigma_{tot}^{(\gamma^*N)}(x,Q^2; A),
\]

where \( \alpha_{em} \) is the QED constant coupling, \( e_f \) is the electric charge of the quark of flavor \( f \). The constant \( K_{chiral} = (L_0^2 + L_2^2 + R_0^2 + R_2^2) \) is the sum of the chiral couplings expressed as functions of the Weinberg angle \( \theta_W \).

The scaling present in the lepton-hadron cross section at high energies, as quantified by Eq. \([6]\) and further experimentally demonstrated, is automatically translated to the UHE neutrino scattering on nucleon or nucleus. In Fig. \(1\) one presents the boson-hadron cross sections, Eqs. \([6,7]\), as a function of the scaling variable \( \tau_A \) for distinct nuclei as well as for the nucleon. They are normalized...
to the nucleon. The cross sections exhibit geometric scaling, verifying a transition in the behavior on $\tau_A$ of the cross section from a smooth dependence at small $\tau_A$ and an approximated $1/\tau$ behavior at large $\tau_A$. Similarly to the lepton-hadron case, the transition point is placed at $\tau_A = 1$. The asymptotic $1/\tau^b$ dependence reflects the fact that the cross section scales as $Q^2_{\text{sat}}/Q^2$ modulo logarithmic corrections, with energy dependence driven by the saturation scale. The mild dependence at $\tau_A \lesssim 1$ corresponds to the fact that the cross section scales as $\propto \sigma_0 \log(Q^2_{\text{sat}}/m_W^2)$ towards the photoproduction limit. The main features present in the cross sections, which are driven by the scaling function Eq. (3), can be qualitatively reproduced in the phenomenological saturation models (see discussion in Ref. [7]). The expressions get simplified to $\sigma_{\text{boson}} \propto \tau_A^{-1} (1 + \log(\tau_A))$ ($\gamma \simeq 1$).

In Fig. 2 one shows the charged and neutral current structure functions, given by Eq. (4), rescaled by the factor $R_y^2/R_A^2$, as a function of $x$ and distinct nuclei, including the proton case. They are computed at scale $Q^2 = m_W^2 (m_Z^2)$ and plotted down to extremely small $x \simeq 10^{-8}$. For the proton case, the results are similar to the saturation model, whereas stay a factor about 1.7 smaller than the unified BFKL-DGLAP approach with screening effects and a factor 3 in the case without screening (for more detail on these results, see Refs. [11, 13]). This has consequences in the reduction of the magnitude of the neutrino cross section due to the nonlinear perturbative QCD effects. The saturation effects slow down the rise of the structure functions as $x$ decreases. These effects are sizeable even at larger values of order $Q^2 \simeq m_W^2$ because they also contribute to the leading twist part of the structure functions due to the geometric scaling window. Concerning the nuclear structure functions, the nuclear effects at the scale $Q^2 \simeq m_W^2$ are relatively smaller than the usual expectation $F_2^A \propto A^{1/3} F_2^N$. As in the present calculation $Q^2_{\text{sat}, A} \propto A^{4/9}$, a smaller nuclear shadowing should be expected. Therefore, this implies a smaller reduction in the magnitude of the neutrino-nucleus inelastic scattering due to nuclear shadowing in contrast with recent approaches using the strength of the nonlinear term proportional to $A^{1/3}$.

The total CC (NC) neutrino-nucleon cross sections as a function of the neutrino energy and atomic number are given by the integration over available phase space and read as,

$$\sigma_{CC,NC}^{NC,NC}(E_{\nu}; A) = \int_{Q_{\text{min}}^2}^s dQ^2 \int_{Q_{\text{min}}^2/s}^1 dx \frac{1}{xs} \frac{Q^2}{\partial x \partial y} \frac{\partial^2 \sigma_{CC,NC}^{CC,NC}}{\partial x \partial y}(x, Q^2) \times \left[ 1 + \frac{(1 - y)^2}{2} F_{CC,NC}^C(x, Q^2) - \frac{y^2}{2} F_{CC,NC}^N(x, Q^2) \right].$$

where a minimum $Q_{\text{min}}^2 \propto O(1) \text{ GeV}^2$ is introduced in order to stay in the DIS region and $G_F$ is the Fermi constant. Here, one considers UHE neutrinos, where the valence quark contribution stays constant and physics is driven by sea quark contributions. Hence, the $x F_{CC,NC}^C$ contribution should be negligible and it will be disregarded. In Eq. (9), the nuclear dependence on the structure functions is implicit.

In Fig. 3 one shows CC and NC neutrino cross sections, given by Eqs. (3), rescaled by the factor $R_y^2/R_A^2$ as a function of neutrino energy $E_{\nu}$ and distinct nuclei, including the nucleon. Based on the color dipole picture, one used the simple relation $F_L \approx (2/11) F_2$ in Eq. (9), which gives a very small contribution. It is worth mentioning that the calculations are also shown for either low neutrino energies ($E_{\nu} \lesssim 10^{11}$ GeV), where a part of the contribution to the integrated cross section would be out the scaling window. For the nucleon case, the results
are close to the saturation models \cite{11}. For the nuclear case, the results indicate a weak nuclear shadowing, as discussed above. A comparison with other high energy approaches is presented in Fig. 4 for the nucleon case (see Ref. \cite{11} for detail on those results). At energies $E_\nu \gtrsim 10^{12}$ GeV, the current calculation produces a reduction by a factor 2 in relation to both NLO DGLAP and unified BFKL-DGLAP approaches and a milder energy dependence. On the other hand, the result has similar behavior as the CGC model. The reason is that the CGC model takes the geometric scaling property and its extrapolation to the saturation region in the dipole cross section.

The property of the geometric scaling allows to analytically calculate the UHE neutrino cross section given by Eq. (5). The integration on Bjorken-$x$ can be carried out by making a change of variables. The integration on $x$ is then replaced by an integration on $\tau_A$ of the scaling function, Eq. (6). This produces the explicit expression $d\sigma/dQ^2 \propto b \beta 3 \tau_A [1, 1, 1, 2, 2, 2, -\beta]_{\tau_{\text{max}}}^{\tau_{\text{min}}}$, where $\tau_{\text{min}} = Q^2/Q_{\text{sat}}^2(x = Q^2/s)$, $\tau_{\text{max}} = Q^2/Q_{\text{sat}}^2(x = 1)$ and $3\beta F_3$ is a hypergeometric function. The further integration on $Q^2$ can be performed, using the remaining leading term on virtuality and supposing $y = Q^2/\nu s \ll 1$. The result using the leading term represents with good accuracy the full numerical calculation. The ratio between the full calculation and the leading term is independent of energy and takes a constant value for $E_\nu \gtrsim 10^8$ GeV, giving $R_{\text{cor}} = \sigma_\nu(\text{exact})/\sigma_\nu(\text{approx.}) = 0.82$. Therefore, the analytical calculation provides a theoretical parameterization for the UHE neutrino-nucleus cross sections based on the geometric scaling property. They are as follows,

$$\sigma_{(\nu, \bar{\nu})}^{\text{CC, NC}} = N_i^{(i)} A^n \left( \frac{R_A^2}{R_B^2} \right) \nu^{1-\alpha} \left[ C_1^{(i)} E_\nu^{\omega_{\text{scal}}} - C_2^{(i)} \right],$$

where $N_i^{(i)}$ are overall normalizations, $C_1^{(i)}$ are numerical constants with $i = \text{CC, NC}$, $\omega_{\text{scal}} = b\lambda$ and $\alpha = b/\delta$. This implies in a mild power-like rise $\omega_{\text{scal}} \approx 0.2$ for the neutrino cross section in contrast with other theoretical approaches. The nuclear dependence is approximately linear, $\sigma^{\text{nuclei}}_{(\nu, \bar{\nu})} \propto A \sigma^{\text{nucleon}}_{(\nu, \bar{\nu})}$, once $b \simeq \delta$ and hence $\alpha \approx 1$. The remaining constants are given by,

$$N_i^{(i)} = B_{\text{cor}} \left( \frac{\sigma_0 \Gamma_{W,Z}^2}{8\pi^3\lambda} \right) \left( \frac{\alpha_{\text{em}}}{\alpha_{\text{em}}} \right) \sum_{f} e_f^{(i)} (11),$$

where one uses the notation $B_{\text{CC}} = 4$, $B_{\text{NC}} = K_{\text{chiral}}$ and $\nu_{\text{scal}} = b - \omega_{\text{scal}}$. Numerically, this gives a total cross section $\sigma^{\text{tot}}_{(\nu, \bar{\nu})} = 1.48 \times 10^{-34} A^n (E_\nu/\text{GeV})^{0.277}$ cm$^2$. The cross section above can have implications for neutrino observatories because experiments are planned to detect UHE by observation of the nearly horizontal air showers in Earth coming from neutrino-air interactions \cite{10}. A reduced cross section produces a smaller event rate for such neutrino-induced showers and could compromise the detection signal. However, the rate of up-going air showers initiated by muon and tau leptons produced in neutrino-nucleon reactions just below the surface would increase, being possibly larger than the horizontal air shower rate.

In summary, it is demonstrated that the cross sections for $W^\pm (Z^0)$-nucleus processes in UHE inelastic neutrino scattering should exhibit geometric scaling on the single scaling variable $\tau_A = Q^2/Q_{\text{sat}}^2$. This implies that such dimensionless scale absorbs their energy and atomic number dependences. Based on this property, an analytical calculation at high energies is made possible. This allows to propose a theoretical parameterization for the UHE neutrino cross sections supposed to be valid at energies $E_\nu \gtrsim 10^8$ GeV. The resulting cross section has a mild energy behavior on energy in comparison with the usual QCD calculations based on linear evolution equations. Moreover, the nuclear dependence is approximately linear on the atomic number.

Acknowledgments

The author is grateful for the warm hospitality and financial support of Departamento de Física de Partículas, Universidade de Santiago de Compostela, where this work was accomplished. Special thanks go to Elena Ferreiro and Nestor Armesto for useful remarks and comments on the manuscript. The FORTRAN codes are available at http://www.if.uff.br/~magnus/neutrinos.html.
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