\( B \to X_s \tau^+ \tau^- \) in a two Higgs doublet model

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Abstract

The inclusive rate and forward-backward asymmetry of dilepton angular distribution for a B-meson to decay to strange hadronic final states and a \( \tau^+ \tau^- \) pair in a two Higgs doublet model are computed. In particular, contributions of neutral Higgs bosons to the decay are included. QCD corrections to the effective Hamiltonian for \( B \to X_s \tau^+ \tau^- \) are calculated using the leading logarithmic approximation.
Flavor changing neutral current (FCNC) transitions $B \to X_s \gamma$ and $B \to X_s l^+ l^-$ provide testing grounds for the standard model (SM) at the loop level and sensitivity to new physics. Rare decays $B \to X_s l^+ l^-(l = e, \mu)$ have been extensively investigated in both SM and the two Higgs doublet models (2HDM) [1-13]. In these processes contributions from exchanging neutral Higgs bosons can be safely neglected because of smallness of $\frac{m_{l\mu}}{m_W}(l = e, \mu)$. Recently, $\tau$ polarization asymmetry in $B \to X_s \tau^+ \tau^-$ in SM has been studied [14] by using the method same as that for $B \to X_s l^+ l^-(l = e, \mu)$. The author of ref.[14] did not consider the role played by the neutral Higgs boson because the contributions due to exchanging the neutral Higgs boson in SM can also be neglected, compared with those due to $\gamma, Z$.

In this note we investigate the inclusive decay $B \to X_s \tau^+ \tau^-$ in a 2HDM. We consider the 2HDM in which the up-type quarks get masses from Yukawa couplings to the one Higgs doublet $H_2$ (with the vacuum expectation value $v_2$) and down-type quarks and leptons get masses from Yukawa couplings to the another Higgs doublet $H_1$ (with the vacuum expectation value $v_1$). Such a model occurs as a natural feature in supersymmetric theories. The Higgs boson couplings to quarks and leptons depend on the ratio $tg\beta = \frac{v_2}{v_1}$ which is a free parameter in the model. Constraints on $tg\beta$ from $K - \bar{K}$ and $B - \bar{B}$ mixing, $\Gamma(b \to s\gamma), \Gamma(b \to c\tau \bar{\nu}_\tau)$ and $R_b$ have been given [15, 16]

$$0.7 \leq tg\beta \leq 0.6(\frac{m_{H^\pm}}{1GeV})$$

(and the lower limit $m_{H^\pm} \geq 200$Gev has also been given in the ref. [16]). It is obvious that the contributions from exchanging neutral Higgs bosons now is enhanced roughly by a factor of $tg^2\beta$ and can compete with those from exchanging $\gamma$, $Z$ when $tg\beta$ is large enough.

Inclusive decay rates of heavy hadrons can be calculated in heavy quark effective theory (HQET) [17] and it has been shown that the leading terms in $1/m_Q$ expansion turns out to be the decay of a free (heavy) quark and corrections stem from the order $1/m_Q^2$ [18]. In what follows we shall calculate the leading term (the $1/m_Q^2$ correction can be easily added if needed). The transition rate for $b \to s\tau^+ \tau^-$ can be computed in the framework of the QCD corrected effective weak hamiltonian, obtained by integrating out the top quark, Higgs bosons and $W^\pm, Z$ bosons

$$H_{eff} = \frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*(\sum_{i=1}^{10} C_i(\mu)O_i(\mu) + \sum_{i=1}^{10} C_Q_i(\mu)Q_i(\mu))$$

where $O_i(i = 1, \cdots, 10)$ is the same as that given in the ref.[4], $Q_i$’s come from exchanging the
neutral Higgs bosons and are defined by

\[ Q_1 = \frac{e^2}{16\pi^2} (s_L^0 b_R^0)(\bar{\tau}\tau) \]

\[ Q_2 = \frac{e^2}{16\pi^2} (s_L^0 b_R^0)(\bar{\gamma}_5\tau) \]

\[ Q_3 = \frac{g^2}{16\pi^2} (s_L^0 b_R^0)(\sum_q \bar{q}_L q_R^\beta) \]

\[ Q_4 = \frac{g^2}{16\pi^2} (s_L^0 b_R^0)(\sum_q \bar{q}_R q_L^\beta) \]

\[ Q_5 = \frac{g^2}{16\pi^2} (s_L^0 b_R^0)(\sum_q \bar{q}_L q_R^\alpha) \]

\[ Q_6 = \frac{g^2}{16\pi^2} (s_L^0 b_R^0)(\sum_q \bar{q}_R q_L^\alpha) \]

\[ Q_7 = \frac{g^2}{16\pi^2} (s_L^0 \sigma^{\mu\nu} b_R^0)(\sum_q \bar{q}_L^{\beta\alpha} q_R^\beta) \]

\[ Q_8 = \frac{g^2}{16\pi^2} (s_L^0 \sigma^{\mu\nu} b_R^0)(\sum_q \bar{q}_R^{\beta\alpha} q_L^\beta) \]

\[ Q_9 = \frac{g^2}{16\pi^2} (s_L^0 \sigma^{\mu\nu} b_R^0)(\sum_q \bar{q}_L^{\beta\alpha} q_R^\alpha) \]

\[ Q_{10} = \frac{g^2}{16\pi^2} (s_L^0 \sigma^{\mu\nu} b_R^0)(\sum_q \bar{q}_R^{\beta\alpha} q_L^\alpha) \]

with \( e \) and \( g \) being the electromagnetic and strong coupling constants respectively.

At the renormalization point \( \mu = m_W \) the coefficients \( C_i \)'s in the effective hamiltonian have been given in the ref.\[4\] and \( C_{Q_i} \)'s are (neglecting the \( O(tg\beta) \) term)\[4\].

\[ C_{Q_1}(m_W) = \frac{m_b m_{\tau}}{m_{h_0}^2} g^2 \frac{\beta}{2} \frac{1}{\sin^2 \theta_W} \frac{x}{4} \{(\sin^2 \alpha + h\cos^2 \alpha) f_1(x, y) \}
\]

\[ + \left[ m_{h_0}^2 / m_{h_0}^2 + (\sin^2 \alpha + h\cos^2 \alpha)(1 - z) \right] f_2(x, y) \]

\[ + \frac{\sin^2 \alpha}{2m_{H^+}^2 - \left( m_{h_0}^2 + m_{h_0}^2 \right)} f_3(y) \] (3)

\[ \text{We use the Feynman rules given in ref.\[19\].} \]

\[ 1 \text{We use the Feynman rules given in ref.\[19\].} \]
\[ C_Q_2(m_W) = -\frac{m_b m_\tau}{m_W} g^2 \beta \{ f_1(x, y) + (1 + \frac{m^2_{H^\pm} - m_W^2}{m_W^2}) f_2(x, y) \}, \quad (5) \]

\[ C_Q_3(m_W) = \frac{m_b e^2}{m_W g^2} (C_Q_1(m_W) + C_Q_2(m_W)), \quad (6) \]

\[ C_Q_4(m_W) = \frac{m_b e^2}{m_W g^2} (C_Q_1(m_W) - C_Q_2(m_W)), \quad (7) \]

\[ C_Q_i(m_W) = 0, \quad i = 5, \ldots, 10 \quad (8) \]

where

\[ x = \frac{m_t^2}{m_W}, \quad y = \frac{m_t^2}{m_{H^\pm}^2}, \]

\[ z = \frac{x}{y}, \quad h = \frac{m_b^2}{m_{H^0}^2}, \]

\[ f_1(x, y) = \frac{x \ln x}{x - 1} - \frac{y \ln y}{y - 1}, \]

\[ f_2(x, y) = \frac{x \ln y}{(z - x)(x - 1)} + \frac{y \ln z}{(z - 1)(x - 1)}, \]

\[ f_3(y) = \frac{1 - y + y \ln y}{(y - 1)^2}. \]

Neglecting the strange quark mass, the effective Hamiltonian (2) leads to the following matrix element for \( b \to s \tau^+ \tau^- \)

\[ M = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* \left[ C_8^{\text{eff}} \bar{s}_L \gamma_\mu b_L \bar{\tau}_\gamma^\mu \tau + C_Q \bar{s}_L \gamma_\mu b_L \bar{\tau}_\gamma^5 \tau + 2C_1 m_b \bar{s}_L i \sigma^{\mu\nu} \frac{q^\nu}{q^2} b_R \bar{\tau}_\gamma^\mu \tau + C_Q_1 \bar{s}_L b_R \bar{\tau}_\gamma^5 \tau + C_Q_2 \bar{s}_L b_R \bar{\tau}_\gamma^5 \tau \right], \quad (9) \]

where

\[ C_8^{\text{eff}} = C_8 + \left\{ g(\frac{m_e}{m_b}, \hat{s}) \right\} + \frac{3}{\alpha^2} k \sum_{V_i = \psi'} \frac{\pi M_{V_i} \Gamma(V_i \to \tau^+ \tau^-)}{M_{V_i}^2 - q^2 - i M_{V_i} \Gamma(V_i)} (3C_1 + C_2), \quad (10) \]

with \( \hat{s} = q^2/m_b^2, \quad q = (p_{\tau^+} + p_{\tau^-})^2. \) In [10] \( g(\frac{m_e}{m_b}, \hat{s}) \) arises from the one-loop matrix element of the four-quark operators \( O_i \)’s and has been given in [4]. The final term in (10) estimates the long-distance contribution from the intermediate \( \psi' \) [10].
The QCD corrections to coefficients $C_i$ and $C_{Q_i}$ can be incorporated in the standard way by using the renormalization group equations. The mixing of the operators $O_i (i = 1, 2, \cdots, 10)$ at $\alpha_s$ order has been studied and the corresponding anomalous dimension matrix has been given [1, 13]. (The mixing of $O_i$ at the next-to-leading order has also been studied in ref. [13].) We studied the one-loop mixing of the new set of operators $Q_i$ and found that the corrections to coefficients due to mixing with $Q_i$ are small and can be neglected provided that $\tan\beta < 50$. For example, including the mixing between $O_7$ and $Q_3$ (and neglecting the mixing of the other $Q_i$ with $O_7$ and the mixing of $Q_3$ with the other $Q_i$), we have

$$C_7(m_b) = \eta^{-16/23} \{ C_7(m_W) - \left[ \frac{58}{135} (\eta^{10/23} - 1) + \frac{29}{189} (\eta^{28/23} - 1) \right] C_2(m_W) - \frac{1}{34} (\eta^{17/23} - 1) C_{Q_3}(m_W) \}$$

(11)

where $\eta = \alpha_s(m_b)/\alpha_s(m_W)$. The additional term is less than 5% correction to the value of $C_7(m_b)$ provided $\tan\beta < 50$. If we include the mixing of $Q_i (i = 3, 5, 7, 9)$ with $O_7$ and also consider the mixing among $Q_3, Q_5, Q_7, Q_9$ which can be derived from Eq.(39) of ref.[20] and is given by

$$\gamma^{(0)} = \begin{bmatrix}
Q_3 & 1/6 \\
Q_5 & 1/2 \\
Q_7 & -1/6 \\
Q_9 & -1/2 
\end{bmatrix}$$

(12)

(13)

with $\beta_0 = 11 - \frac{2}{3}n_f$, then we have

$$C_7(m_b) = \eta^{-16/23} \{ C_7(m_W) - \left[ \frac{58}{135} (\eta^{10/23} - 1) + \frac{29}{189} (\eta^{28/23} - 1) \right] C_2(m_W) - 0.012 C_{Q_3}(m_W) \}$$

(14)

The final term in (14) is still small (3% correction) when $\tan\beta < 50$. Using (11) or (14), the limit on $\tan\beta$ obtained from $\Gamma(b \to s\gamma)$ is $\frac{\tan\beta}{m_{A^0}} < 0.25 Gev^{-1}$. 

4
\( Q_i(i = 1, \cdots, 10) \) does not mix with \( O_8, O_9 \) so that \( C_8 \) and \( C_9 \) remain unchanged and are given in ref.\[4\]

\[
C_8(m_b) = C_8(m_W) + \frac{4\pi}{\alpha_s(m_W)} \left[ -\frac{4}{33}(1 - \eta^{-11/23}) + \frac{8}{87}(1 - \eta^{-29/23}) \right] C_2(m_W), \\
C_9(m_b) = C_9(m_W).
\]

It is obvious that operators \( O_i(i = 1, \cdots, 10) \) and \( Q_i(i = 3, \cdots, 10) \) do not mix into \( Q_1 \) and \( Q_2 \) and also there is no mixing between \( Q_1 \) and \( Q_2 \). Therefore, the evolution of \( C_{Q_i}, C_{Q_2} \) is controlled by the anomalous dimensions of \( Q_1, Q_2 \) respectively.

\[
C_{Q_i}(m_b) = \eta^{-\gamma_Q/\beta_0} C_{Q_i}(m_W), \quad i = 1, 2,
\]

where \( \gamma_Q = -4 \) is the anomalous dimension of \( \bar{s}_L b_R \).

From Eq.(9), it is easy to derive the double differential distribution \( \frac{d^2\Gamma}{d\hat{s}dz} \) as follows

\[
\frac{d^2\Gamma(B \rightarrow X_s \tau^+ \tau^-)}{d\hat{s}dz} = B(B \rightarrow X_c \ell \bar{\nu})(2\pi)^2 f^2(m_b) (1 - \hat{s})^2 (1 - \frac{4t^2}{\hat{s}})^{1/2} \frac{|V_{tb}V_{ls}^*|^2}{|V_{cb}|^2} \\
\left\{ \frac{3}{2} |C_{8}^{eff}|^2 [(1 + \hat{s}) - (1 - \hat{s})(1 - \frac{4t^2}{\hat{s}}) z^2 + 4t^2] \\
+ 6 |C_1|^2 [(1 + \frac{1}{\hat{s}}) - (1 - \frac{4t^2}{\hat{s}}) (1 - \frac{1}{\hat{s}}) z^2 + 4t^2] \\
+ \frac{3}{2} |C_9|^2 [(1 + \hat{s}) - (1 - \hat{s})(1 - \frac{4t^2}{\hat{s}}) z^2 - 4t^2] \\
+ 12 Re(C_{8}^{eff}C_{7}^{*})(1 + \frac{2t^2}{\hat{s}}) + 6 Re(C_{8}C_{5}^{*})(1 - \frac{4t^2}{\hat{s}})^{1/2} \hat{s} z \\
+ 12 Re(C_{7}C_{5}^{*})(1 - \frac{4t^2}{\hat{s}})^{1/2} \hat{s} + \frac{3}{2} |C_{Q_1}|^2 (\hat{s} - 4t^2) + \frac{3}{2} |C_{Q_2}|^2 \hat{s} \\
+ 6 Re(C_{8}C_{Q_1}^{*})(1 - \frac{4t^2}{\hat{s}})^{1/2} \hat{t} z + 12 Re(C_{7}C_{Q_1}^{*})(1 - \frac{4t^2}{\hat{s}})^{1/2} \hat{t} z \\
+ 6 Re(C_{9}C_{Q_2}^{*})),
\]

where \( z = \cos\theta, \ t = \frac{m_\tau}{m_b} \) and \( \theta \) is the angle between the momentum of the B-meson and that of \( l^+ \) in the center of mass frame of the dileptons \( \tau^+ \tau^- \). Integrating the angle variable, we obtain the
invariant dilepton mass distribution

\[
\frac{d\Gamma(B \to X_s\tau^+\tau^-)}{ds} = B(B \to X_c\ell\bar{\nu}) \frac{\alpha^2}{4\pi^2 f(m_c/m_b)} (1 - s)^2 (1 - \frac{4t^2}{s}) \frac{1}{2} |V_{tb}V_{ts}^*|^2
\]

\[
\{ |C_{\text{eff}}^s|^2 (1 + \frac{2t^2}{s}) (1 + 2s) + 4|C_7|^2 (1 + \frac{2t^2}{s}) (1 + \frac{2}{s}) \}
\]

\[
+ \quad |C_9|^2 [(1 + 2s) + \frac{2t^2}{s} (1 - 4s)] + 12Re(C_7 C_{\text{eff}}^s) (1 + \frac{2t^2}{s})
\]

\[
+ \quad \frac{3}{2} |C_{Q_1}|^2 (s - 4t^2) + \frac{3}{2} |C_{Q_2}|^2 \hat{s} + 6Re(C_9 C_{Q_2}^*) t \}.
\]

(19)

Switching off the channel exchanging neutral Higgs bosons, Eq.(18) turns out to be the same as that given in Ref.[14]. We also give the forward-backward asymmetry in \( B \to X_s\tau^+\tau^- \)

\[
A(\hat{s}) = \int_{0}^{1} dz \frac{d^2\Gamma}{dsdz} - \int_{-1}^{0} dz \frac{d^2\Gamma}{dsdz} = B(B \to X_c\ell\bar{\nu}) \frac{3\alpha^2}{2\pi^2 f(m_c/m_b)} (1 - s)^2 (1 - \frac{4t^2}{s}) \frac{1}{2} |V_{tb}V_{ts}^*|^2
\]

\[
\{ Re(C_8 C_9^*) \hat{s} + 2Re(C_7 C_{\text{eff}}^s) + Re(C_8 C_{Q_1}^*) t + 2Re(C_7 C_{Q_2}^*) t \}.
\]

(20)

The following parameters have been used in the numerical calculations:

\[ m_t = 175\text{Gev}, \quad m_b = 5.0\text{Gev}, \quad m_c = 1.6\text{Gev}, \quad m_\tau = 1.7\text{Gev}, \quad \Lambda_{\overline{MS}} = 0.1\text{Gev}. \]

Numerical results are summarized in Fig.1-2.

It is obvious that there are sharp peaks on the curves of differential branching ratio \( \frac{d\Gamma}{ds} \) and asymmetry \( A(\hat{s}) \) around the mass of resonance \( \psi' \). One can see from the figures that the contributions of neutral Higgs bosons is significant in 2HDM when \( tg\beta \) is larger than 25 and the values of the masses of the Higgs bosons are in the reasonable range, and the forward-backward asymmetry of dilepton angular distribution is more sensitive to \( tg\beta \) than the invariant mass distribution. We expect that the more precise experiments on rare decays of B mesons will shed light on the two Higgs doublet model.

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Figure 1: The differential branching ratio \( \frac{d\Gamma}{d\hat{s}} \) (normalized to \( \Gamma(B \rightarrow X_c e\bar{\nu}_e) \)) as a function of the scaled invariant dilepton mass \( \hat{s} = s/m_b^2 \) in the decay \( B \rightarrow X_s \tau^+ \tau^- \). We have taken \( m_{H^\pm} = 200\text{GeV} \), \( m_{A^0} = 80\text{GeV} \), \( m_{H^0} = 150\text{GeV} \), and \( m_{A^0} = 100\text{GeV} \). Assumed \( \tan\beta = 30,50 \) are indicated on the curves. The solid line corresponds to the case of switching off the channel exchanging the neutral Higgs bosons.

Figure 2: Forward-backward asymmetry of the dileptons on the decay \( B \rightarrow X_s \tau^+ \tau^- \), \( A(\hat{s}) \) (normalized to \( \Gamma(B \rightarrow X_c e\bar{\nu}_e) \)), as a function of the scaled invariant dilepton mass \( \hat{s} = s/m_b^2 \). We have taken \( m_{H^\pm} = 200\text{GeV} \), \( m_{A^0} = 80\text{GeV} \), \( m_{H^0} = 150\text{GeV} \), and \( m_{A^0} = 100\text{GeV} \). Assumed \( \tan\beta = 30,50 \) are indicated on the curves. The solid line corresponds to the case of switching off the channel exchanging the neutral Higgs bosons.
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