The shock waves in relativistic superfluid

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Abstract

We consider the discontinuities in a two-constituent relativistic superfluid. In the acoustic limit they degenerate into the first and second sound which are independent up to the second-order linear approximation. Inclusion of the quadratic deviations relates to the small-amplitude shock. Particularly we consider a plane shock at low temperature when the phonon excitations contribute to the normal constituent. So we found the generalization of the temperature increment and acoustic wave velocity in relativistic superfluid. The fourth sound speed is also calculated.

1 INTRODUCTION

The first approach to the relativistic superfluid mechanics proposed by Israel [1] and Dixon [2] concerns with perfect fluids. The method is useful for particular calculations, and as a general model can be applied to relativistic superfluidity. For, strictly speaking, the coupled constituents are not perfect fluids: any coupling results to deviation from ideality. Or: the absence of coupling on microscopic level implies thermo-isolation of the constituents [15]. Nevertheless, the attempt of taking into account the deviation from perfect fluid is not senseless. The further development of Khalatnikov and Lebedev [3] includes the interaction between the superfluid and normal constituent. This principle developed independently by Carter [6] was

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gained in recent works \[6, 7\]. Although the main attention was directed to the general formalism, including the equations of motion, rather than applied problems, Carter and Langlois \[8\] have recently derived the first and second sound speed in a superfluid with a phonon equation of state of the normal constituent. This inspires us to discuss the shock wave propagation in a two-constituent relativistic superfluid. While the relativistic shock waves first considered by Taub \[10\] have been under detailed discussion \[11, 12, 13, 14\] and the shock waves in superfluid helium \[9\] were also investigated, the present task bears a qualitatively new feature: we work not in the frames of perfect fluid hydrodynamics, for it is impossible to split the conserving particle number current into a conserving ”superfluid” and ”normal” parts \[8\] as it is in the Newtonian limit \[8\].

We use the natural system of units (\(\hbar = c = 1\)) and the metric corresponding to the Minkowsky space with a metric tensor \(\text{diag}(-+++)\).

2 THE CONSERVATION LAWS

The equations of relativistic superfluid mechanics are determined by the Lagrangian \(L\), whose infinitesimal variation is given by formula \[6, 7, 8\]

\[
\delta L = \Theta \delta s^e - n^e \delta \mu^e \tag{1}
\]

where the particle number vector \(n^e\) conjugated to the momentum co-vector \(\mu^e\) obeys the conservation law

\[
\nabla^e n^e = 0 \tag{2}
\]

and the entropy vector \(s^e\) conjugated to the thermal momentum 1-form \(\Theta^e\) is also conserved

\[
\nabla^e s^e = 0 \tag{3}
\]

till a shock wave appears. The energy-momentum tensor corresponding to the Lagrangian \(L\) has the form

\[
T^e_{\nu} = n^e \mu^\nu + s^e \Theta^\nu + \Psi g^e_{\nu} \tag{4}
\]

with the pressure function

\[
\Psi = L - \Theta^e s^e \tag{5}
\]
The conservation law of the energy-momentum tensor [4]

$$\nabla^\mu T^\mu_v = 0$$ (6)

leads to the equation of motion of the normal constituent

$$s^e (\nabla^\mu \Theta_v - \nabla_v \Theta^\mu) = 0$$ (7)

and the irrotationality condition

$$\nabla^\mu \mu_v - \nabla_v \mu^\mu = 0$$ (8)

Vectors $n^e$ and $\mu^e$ (also $s^e$ and $\Theta^e$) are not colinear. For the particle number vector $n^e$ determines the Eckart rest frame, while $\mu^e$ determines similarly the superfluid rest frame and that does not coincide with the former. The Lagrangian $L(\mu, y, s)$ depends on three invariants, them being

$$s^2 = -s^e s_e$$ (9)

for the normal rest frame entropy density; for the cross product given by

$$y^2 = -s^e \mu_e$$ (10)

and for the effective mass variable (i.e. the chemical potential in the superfluid rest frame)

$$\mu^2 = -\mu^e \mu_e$$ (11)

The secondary variables $n^e$ and $\Theta^e$ can be expressed through the primary variables $\mu^e$ and $s^e$ according to the formula

$$\begin{pmatrix} n^e \\ \Theta^e \end{pmatrix} = \begin{pmatrix} B & -A \\ A & C \end{pmatrix} \begin{pmatrix} \mu^e \\ s^e \end{pmatrix}$$ (12)

with the coefficients obtained immediately by differentiation of the Lagrangian

$$B = 2 \frac{\partial L}{\partial \mu^2} \quad C = -2 \frac{\partial L}{\partial s^2} \quad A = -\frac{\partial L}{\partial y^2}$$ (13)

On the other hand we can write

$$\begin{pmatrix} n^e \\ s^e \end{pmatrix} = \begin{pmatrix} F & Q \\ Q & G \end{pmatrix} \begin{pmatrix} \mu^e \\ \Theta^e \end{pmatrix} = \begin{pmatrix} B + A^2/C & -A/C \\ -A/C & 1/C \end{pmatrix} \begin{pmatrix} \mu^e \\ \Theta^e \end{pmatrix}$$ (14)
and calculate new coefficients through the pressure function \[ F = 2 \frac{\partial \Psi}{\partial \mu^2} \quad G = 2 \frac{\partial \Psi}{\partial \Theta^2} \quad Q = \frac{\partial \Psi}{\partial \varepsilon^2} \] (15)

However, a more convenient way is to obtain all parameters (13) in terms of (9), (10) and (11) and express them in terms of the relative translation speed between the superfluid and normal reference frames

\[ w^2 = 1 - \frac{\mu^2 s^2}{y^4} \] (16)

and the effective temperature

\[ \Theta^2 = -\Theta^0 \Theta^4 \] (17)

instead of invariants \( y \) and \( s \). At low temperature the phonon-like excitations with energetic spectrum \( \omega = cp \), where

\[ c^2 = \frac{n d\mu}{\mu d\nu} \] (18)

and the latter is the sound speed contribute to thermodynamical functions of the normal constituent. The ”phonon” Lagrangian has the form \[ L = P - 3\psi \quad \psi = k \frac{c^2}{c^2 - w^2} \left[ s^2 + \left( c^2 - 1 \right) \frac{y^4}{\mu^2} \right]^{2/3} = \frac{\pi^2}{90c^3} \frac{\Theta^4}{\left[ 1 - w^2/c^2 \right]^2} \] (19)

where the pressure of excitations is

\[ \psi = \frac{s_s \Theta}{4} \] (20)

and

\[ s_s = \frac{s}{\sqrt{1 - w^2}} = \frac{\pi^2}{15c^3} \frac{\Theta^4}{\left[ 1 - w^2/c^2 \right]^2} \] (21)

is the entropy \( s_s \) in the superfluid reference frame, while \( k \) is a definite constant. The coefficients \[ L \] then are

\[ A = \frac{1 - c^2}{\mu} \quad C = -\frac{1}{s_s c^2 - w^2} \] (22)

For an ultra-relativistic spectrum of excitations \( (c \to 1) \) the term \( A \) disappears.
3 THE DISCONTINUITIES

The discontinuities in superfluid helium have been first discovered by Khala-
latnikov [9]. Discussing the discontinuities in a relativistic superfluid we shall
follow the standard formalism of the relativistic shock waves [11]. However,
since we deal with a strongly self-interacting medium and two-constituent at
that, a standard perfect fluid theory is impossible. So we have to think
of a new extended formalism, combining the theory of relativistic shock
waves [11, 12, 13, 14] and the relativistic superfluid mechanics [6, 7, 8]. In the
Newtonian limit this method must be reduced to the non-relativistic theory
of discontinuities in helium [H], and in the acoustic limit this method must
give the first and the second sound in a relativistic superfluid [8]. The am-
plitude of discontinuities is assumed to be not very large, since superfluidty
is expected to take place on both sides of the front with no phase transition.

Let the hypersurface \( \Sigma \) be the front of the discontinuity and vector \( \lambda^{\rho} \)
be a unit (space-like) normal to it. The conservation laws (2), (6) en-
tail the conditions

\[
[\lambda_{\rho} n^{\rho}] = 0 \quad \quad \quad [\lambda_{\rho} T_{\nu}^{\rho}] = 0
\]
i.e.

\[
\begin{align*}
    n_{\perp} &= \lambda_{\rho} n_{\rho}^{\perp} = \lambda_{\rho} n_{\rho}^{\perp} \\
    \lambda_{\rho} T_{\nu}^{\rho} &= \lambda_{\rho} T_{\nu}^{\rho}
\end{align*}
\]

where indexes + and - relate to the quantities ahead of and behind the
discontinuity, respectively, and the square brackets imply the change across
the front of a discontinuity. Eq. (23) conveys the continuity of the orthogonal
part (marked always by index \( \perp \)) of the particle number current.

We cannot make use of the entropy conservation law (3) since it does not
take place in the shock waves. For that reason the results of Ref. [15]
pertain for a most general two-constituent system (but not a superfluid itself), while
the discussion of small perturbations suffers no restrictions concerning eq. (3).
Indeed, the results for shock waves in a two-constituent system and in a
superfluid will coincide when the magnitude of the relevant discontinuities
tends to zero.

For a two-constituent superfluid the additional equation is the irrota-
 tionality condition (8) which yields

\[
\lambda_{\rho} \mu_{+\nu} - \lambda_{\nu} \mu_{+\rho} = \lambda_{\rho} \mu_{-\nu} - \lambda_{\nu} \mu_{-\rho}
\]
Multiplying it by $\lambda^e$ and, then, by a unit vector $\eta^e$ orthogonal to $\lambda^e$, we get an important relation

$$\mu^\parallel_+ = \mu^\parallel_-$$

(where index $\parallel$ denotes the tangential part of any quantity, particularly $\mu^\parallel = \eta^e_\mu^e$). This is the very condition for conservation of irrotational motion passing a plane shock wave \cite{16}. It is clear that for a multi-constituent system the irrotational motion conserves at both sides of the shock wave if condition (3) takes place for each constituent.

Substituting the expression (1) and (23) in (24), we obtain

$$n^\perp (\mu^\perp_+ - \mu^\perp_-) + s^\perp_+ \Theta^\perp_+ - s^\perp_- \Theta^\perp_- = -(\Psi^+ - \Psi^-) \lambda^\nu$$

Multiplying this equation by $\lambda^\nu$, we get

$$n^\perp (\mu^\perp_+ - \mu^\perp_-) + s^\perp_+ \Theta^\perp_+ - s^\perp_- \Theta^\perp_- = -(\Psi^+ - \Psi^-)$$

Then, multiplying eq. (26) by the unit vector $\eta^\nu$, in light of (25), we get

$$n^\perp (\mu^\parallel_+ - \mu^\parallel_-) + s^\perp_+ \Theta^\parallel_+ - s^\perp_- \Theta^\parallel_- = 0$$

This important relation determines the types of discontinuities. Then, the irrotationality condition (27) than coincides with the condition of strong discontinuity in the superfluid constituent which occurs when $n^\perp \neq 0$. As for the discontinuity in the normal component, in light of (25), it is determined merely by the single relation

$$s^\perp_+ \Theta^\parallel_+ = s^\perp_- \Theta^\parallel_-$$

Thus the constraint (23) corresponds to ordinary shock waves, while (24) beseeems to a “temperature” discontinuity of the second sound type.

As a particular instance of another type of discontinuity we consider a vortex sheet in superfluid \cite{17}. Since the vortex sheet separates the whole space into domains where the superfluidity takes place, the irrotationality condition (3) does not hold in the global sense and we cannot establish the constraint (23) at both sides of the sheet. Therefore, the tangential discontinuities are possible. The conservation law (23) then yields $n^\perp_+ = n^\perp_- = 0$ that determining weak or slip-stream discontinuity: the particle number flow across the front of discontinuity equals zero, indeed, no matter crosses the hypersurface of the discontinuity, i.e. this hypersurface is made up of streamlines of the fluid.
4 A PLANE SHOCK IN FLAT SPACE

Let the discontinuity propagates along the axis $x^1$. We choose the unit normal $\lambda^\nu = (0, 1, 0, 0)$ - and the medium at rest before the front. As a rule one used to practice with the rest frame co-moving the front, so that the fluid flows in the front with the velocity which is equal to that of a shock wave. Hereby, the relevant vectors and co-vectors may be presented as

$$n^\varphi = n \left( \sqrt{1 + \varphi^2}, \varphi, 0, 0 \right) \quad \mu_\varphi = \mu \left( -\sqrt{1 + \xi^2}, \xi, 0, 0 \right)$$

$$s^\varphi = s \left( \sqrt{1 + \alpha^2}, \alpha, 0, 0 \right) \quad \Theta_\varphi = \Theta \left( -\sqrt{1 + \beta^2}, \beta, 0, 0 \right)$$

(30)

Since the medium ahead of front is at rest, the relative velocity $w_-$ equals zero, $y_-^2 = \mu_- s_-$, while behind the shock

$$y_+^2 = \frac{\mu_+ s_+}{\sqrt{1 - w^2}}$$

Also

$$\alpha_\varphi = \beta_\varphi = \varphi_\varphi = \xi_\varphi \equiv x$$

(32)

Hence, the velocity of the shock is determined as

$$u = \frac{x}{\sqrt{1 + x^2}}$$

(33)

Substituting our definitions (30), (31) and (32) in eqs. (23), (25), (29) and (27) we, firstly obtain

$$n_+ \varphi = n_- x$$

(34)

The rest equations, in view of (34), will be

$$\mu_+ \sqrt{1 + \xi^2} = \mu_- \sqrt{1 + x^2}$$

$$\alpha_\varphi \Theta_+ \sqrt{1 + \beta^2} = x \Theta_- \sqrt{1 + x^2}$$

$$n_- x (\xi \mu_+ - x \mu_-) + \alpha \beta s_+ \Theta_+ - x \Theta_- = -\Psi_+ + \Psi_-$$

(37)

The parameter $\varphi$ incorporates only in eq. (34) and it can be calculated as soon as the rest unknowns are found. Thus, in eqs. (33), (35) and (36)
the unknowns are: the four parameters $\alpha$, $\beta$, $\xi$, $x$ and three invariants (9), (10), (11) behind the shock on which the pressure $\Psi_+$ depends. The pressure behind the shock can be expressed through $\mu_+$, $\Theta_+$, and through the relative velocity for which we use the notation $w$. The formula

$$\frac{1}{\sqrt{1 - w^2}} = \sqrt{1 + \alpha^2} \sqrt{1 + \xi^2} - \alpha \xi$$

relates the later quantity with $\alpha$ and $\xi$. Our goal is to find the velocity of the shock wave $u$ for a single parameter given behind the shock. Without the loss of generality $\mu_+$ can be chosen for this parameter. Thus, there are six unknowns in four equations (35), (36), (35), (38). The rest two relations follow from (12) or (14)

$$n_+ x = F_+ \mu_+ \xi + Q_+ \Theta_+ \beta$$

$$s_+ \alpha = Q_+ \mu_+ \xi + G_+ \Theta_+ \beta$$

with the coefficients (13) calculated for the state behind the front.

The knowledge of the equation of state in explicit form is necessary for calculation of the right-hand side of eq. (37) and the coefficients in eqs. (39) and (40).

5 A LOW TEMPERATURE CASE

The low-temperature equation of state was derived by Carter and Langlois [8]. In view of (20), (21), (22) the expressions (36), (37), (39), (40) take the form

$$\bar{\psi} \sqrt{1 - w^2} \alpha \sqrt{1 + \beta^2} = x \sqrt{1 + x^2}$$

$$x (\xi \bar{\mu} - x) + \tau \left(4 \alpha \beta \bar{\psi} \sqrt{1 - w^2} - 4 x^2 + \bar{\psi} - 1\right) = -\frac{1}{\Gamma} (\bar{P} - 1)$$

$$\bar{\mu} \bar{F} \xi + \left(1 - c_+^2\right) \frac{\bar{\psi}}{\bar{\mu}} \beta = x$$

$$\sqrt{1 - w^2} \alpha = \left(1 - c_+^2\right) \xi + \left(c_+^2 - w^2\right) \beta$$
where
\[ \tau = \frac{\psi_-}{\mu_- n_-} \quad \Gamma = \frac{\mu_- n_-}{P_-} \quad \bar{F} = F_+/\left(\frac{n_-}{\mu_-}\right) \]

At low temperature we have the estimations [8, 15]

\[ F = \frac{n}{\mu} + O(\Theta^4) \quad G \sim \Theta^2 \quad Q \sim \Theta^3 \]

implying that \( \tau = O(\Theta^4) \) and, hence, equations (37) and (39) approximately (up to the terms \( O(\Theta^4) \)) coincide with their zero-temperature version.

6 THE SOUND, STRONG AND SMALL-AMPLITUDE SHOCK WAVES

If all parameters behind the front tend to their values ahead, the shock becomes a sound wave. Since the entropy in the sound wave is conserved, we can apply formalism [13] achieved for a two-constituent relativistic medium with the conserved particle currents of both constituents. In the linear approximation both methods lead to the same result, namely from the system (35), (36), (37), (38), (39), (40) we obtain an equation for two branches of sound at arbitrary temperature which is analogous to that derived by Carter [5] and, under assumption \( A^2 = o(C) \), splits into

\[ u^2_I = -\frac{B}{B + \mu B_\mu + s A_\mu} \quad (46) \]

\[ u^2_{II} = 1 + \frac{s B_s + \mu A_s}{C} \quad (47) \]

and reduces, in the low temperature limit, to the first and the second sound speed, respectively [8]: \( u_I = c, \quad u_{II} = c \sqrt{3} \). Here for any variable \( V \) we used the notation

\[ V_s = \frac{\partial V}{\partial s} + \frac{\mu}{2y} \frac{\partial V}{\partial y} \quad V_\mu = \frac{\partial V}{\partial \mu} + \frac{s}{2y} \frac{\partial V}{\partial y} \quad (48) \]

However, if we omit \( A \), the second sound speed calculated by formula (47) with the phonon Lagrangian (19) of Carter and Langlois [8] will be \( u_{II} = 1/3 \) instead of obvious \( u_{II} = c/3 \). Because the Lagrangian (19) is derived for the two-fluid theory with non-zero cross term; while the Lagrangian of thermal
excitations of the Israel theory \[12\] differs from \(13\); although the relative translation speed \(w\) between the constituents is presented in both approaches. So each Lagrangian is useful in the theory to which it does belong.

In order to find the velocity of a small-amplitude shock we rewrite eqs. (35), (41), (42), (38), (43), (44) in the second-order approximation. After tedious calculations we find the velocity increment

\[
\Delta u_{II} = \frac{1 - c^2}{1 + c^2} \Theta \Delta \Theta
\]

of the shock corresponding to the second sound when the temperature increment \(\Delta \Theta\) tends to zero. In turn, the latter gives rise to a finite relative speed \(w\) behind the shock, since

\[
\frac{\Delta \Theta}{\Theta} = \frac{w}{c \sqrt{3} \sqrt{1 - c^2/3}}
\]

In the non-relativistic limit the equations (49) and (50) yield well known expressions \[9\]. The shock occurs ahead of the second sound, as is in superfluid helium at low temperature.

So, in the acoustic limit the solution splits into two independent branches \(u_I\) and \(u_{II}\) corresponding to the first and the second sound. The first branch describes wave propagation through the medium which behaves as a perfect fluid composed of two constituents whose pressure and enthalpy are \(P + \psi\) and \(\mu n + 4\psi\) respectively. While the constituents in the waves of the second branch move independently, a counterflow appears: \(w \neq 0\). In general, a "mixed" solution occurs, and the temperature increases together with the chemical potential.

The estimations (45) imply that the first sound and the relevant shock wave coincide roughly with usual discontinuity in the cold constituent. A great pressure jump \(\Delta \Psi\) is produced inevitably by the change in pressure of the superfluid constituent \(P\), since the contribution of the normal constituent \(\psi\) is small. Hence, in view of eqs. (12) and (13) we conclude that strong shock waves at low temperature propagate with the speed \(u = u_0 + O(\Theta^4)\) which approximately equals to the speed of a usual shock wave in cold constituent \(u_0\), but always \(u_I > u_{II}\). A more precise result is

\[
u_I^2 = u_0^2 \left(1 - \tau u_0^2 (4 - \Gamma)\right)
\]
Table 1. The gamma-factor $\gamma_w = w/\sqrt{1 - w^2}$ of the relative translation speed vs the Mach number $M = u/c$ of the shock wave and the pressure change

| $\Psi_+ / \Psi_- - 1$ | 1.1 | 1.5 | 2   | 5   | 10   |
|-----------------------|-----|-----|-----|-----|------|
| $M$                   | 1.024 | 1.10 | 1.17 | 1.35 | 1.46 |
| $\gamma_w$           | $6.7 \cdot 10^{-3}$ | 0.031 | 0.065 | 0.256 | 0.476 |

\[
u_{II}^2 = \nu_0^2 \left\{ 1 - \tau \nu_0^2 \left( 4 - \Gamma \sqrt{1 - w^2} \right) \right\}
\]

(52)

The second-sound discontinuities should be regarded as "moderate" for intermediate values of $u_{II}$. This takes place if relative changes in the superfluid and normal variables are of the same order and they can be of the same order if they do not access $\tau$ greatly. For a superfluid matter of neutron stars and phonon equation of state it is easy to estimate $\tau \sim 10^{-9}$. For this particular example we performed calculations with an ultra-relativistic superfluid matter. The sound speed in this medium equals exactly to $c = 1/\sqrt{3}$, and approximately it is the first sound, while the speed of the second sound $c_{II} = c/\sqrt{3}$. For a not very small pressure change the velocity of the shock wave will be merely $\nu_0$. The dependence of the relative translation speed $w$ on $\nu_0$ is given in table 1.

Although the second sound velocity attains to the saturation value $\nu_0$, the relative translation speed $w$ grows with the growth of the shock wave intensity.

7 THE FOURTH SOUND

The fourth sound takes place when the normal constituent is restrained by some external agent, while the sound propagates through the superfluid constituent. We cannot use the equation of motion (4), but the conservation laws (2), (3) and the irrotationality condition (8) will determine the fourth sound speed. If the sound wave propagates in the direction determined by vector $\lambda^\nu = (u, 1, 0, 0)$ the change of gradient of arbitrary quantity $V$ is proportional to its infinitesimal change $\hat{V}$ \[8\]: $[\nabla_\nu \hat{V}] = \hat{V} \lambda_\nu$. Thereby, we write

\[
- u \hat{n}^0 + \hat{n}^1 = 0
\]

(53)

\[
- u \hat{\nu}_1 - \hat{\nu}_0 = 0
\]

(54)
\[ -u\hat{s}^0 + \hat{s}^1 = 0 \]  \hspace{1cm} (55)

instead of (2), (3), (8) and (7). In the reference frame co-moving with the normal constituent we put

\[ s^\theta = s(1, 0, 0, 0) \quad \mu_\theta = \mu_n(-1, w, 0, 0) \]  \hspace{1cm} (56)

where \( \mu_n = \mu/\sqrt{1-w^2} \) is the chemical potential in the "normal" reference frame. Since the discontinuities propagate through the superfluid constituent, there must be

\[ \hat{s}_\theta = 0 \]  \hspace{1cm} (57)

Equations (53), (54), (57) analogous to the relevant non-relativistic set [9] determine the speed of the fourth sound. Requiring the vanishing determinant of the system (53), (54) we get a quadratic equation for \( u \). While the speed of the first and the second sound is determined by a 4-order system [8], the fourth sound speed follows from two equations. At zero temperature the speed of the first and the fourth sound are obtained by the same equations (53) and (54) and coincide exactly with the sound speed \( c \) in the superfluid constituent. The difference appears at finite temperature on account of the relationship [8]

\[ \hat{n}_\theta = B^{\theta\nu} \hat{\mu}_\nu + C^{\theta\nu} \hat{s}_\nu \]

between the infinitesimal discontinuities in (53), (54) and the temperature dependence of matrices in (58). For the phonon equation of state we get the explicit formula

\[ u_{IV} = c + c \frac{\rho_n}{\rho_s} \left( - \frac{1}{3} + \frac{17}{6} c^2 - 3c^4 - \frac{c \mu}{1 + c^2 \partial_c / \partial \mu} + \frac{2}{3} c^2 \partial^2 c / \partial \mu^2 \right) \]  \hspace{1cm} (58)

which generalizes the relevant non-relativistic relation [9], where \( \rho_n \) and \( \rho_s \) is the normal and the superfluid energy-density respectively [8].

8 CONCLUSION

Summarizing the results obtained in the present study, we emphasize the formulae (23), (25), (29), (27) which determine the propagation of discontinuities through a two-constituent relativistic superfluid in the general case. For a plane shock wave seven equations (34), (35), (36), (37), (38), (39) and
with seven unknowns must be solved. In the acoustic limit these equations reduce to formulae (46), (47) for the first and the second sound. At low temperature the system (34), (35), (36), (37), (38), (39) and (40) reduces to (34), (35), (41), (42), (38), (43) and (44). The velocity of strong shock waves is given by (51) and (52), while (49) and (50) describe the change of parameters in a weak shock wave. As for perspectives and applications, the shock waves and spin-isospin sound in the nuclear matter are worth to be discussed in future.

References

[1] W. Israel, Phys. Lett. A86, 79 (1981).
[2] W.G. Dixon, Arch. Rat. Mech. Anal. 80, 159, (1982).
[3] I.M. Khalatnikov and V.V. Lebedev, Phys. Lett. A91, 70 (1982).
[4] G.V. Vlasev, Phys. Lett. A231, 149 (1997).
[5] B. Carter, in Relativistic fluid dynamics, (eds. A. Anile and Y. Choquet-Bruhat, Springer, Berlin, 1989), p. 1.
[6] B. Carter and I.M. Khalatnikov, Phys. Rev. D45, 4536 (1992).
[7] B. Carter and I.M. Khalatnikov, Rev. Math. Phys. 6, 277 (1992).
[8] B. Carter and D. Langlois, Phys. Rev. D51, 4536 (1995).
[9] I.M. Khalatnikov, Introduction to the theory of superfluidity, (Addison-Wesley, Redwood City, 1989). See also his original paper in Sov. JETP (1952).
[10] A.H. Taub, Phys. Rev. 74, 328 (1948).
[11] H. Taub, Ann. Rev. Fluid Mech. 10, 301 (1978).
[12] W. Israel, in Relativistic fluid dynamics, (eds. A. Anile and Y. Choquet-Bruhat, Springer, Berlin, 1989), p. 152.
[13] M. Cissoko Phys. Rev. D45, 1045 (1992).
[14] M. Cissoko, Phys. Rev. D55, 4555 (1997).

[15] G.V. Vlasov, Sov. JETP 84, 729 (1997).

[16] G.V. Vlasov and A.J. Khalfin, Teor. Math. Phys. 110, 378 (1997).

[17] Ü. Parts, E.V. Thuneberg, G.E. Volovik et al, Phys. Rev. Lett. 72, 3839 (1994).