The effect of nonmagnetic impurities on the local density of states in s-wave superconductors

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We study the effect of nonmagnetic impurities on the local density of states (LDOS) in s-wave superconductors. The quasicalequations of superconductivity are solved selfconsistently to show how LDOS evolves with impurity concentration. The spatially averaged zero-energy LDOS is a linear function of magnetic induction in low fields, $N(E = 0) = cB/Hc_2$, for all impurity concentration. The constant of proportionality $c$ depends weakly on the electron mean free path. We present numerical data for differential conductance and spatial profile of zero-energy LDOS which can help in estimating the mean free path through the LDOS measurement.

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I. INTRODUCTION

Since Hess and co-workers succeeded in measuring the LDOS in the superconducting NbSe$_2$ there have been many reports and theoretical studies on the electronic structure of the superconductor in the mixed state. The novel experimental technique introduced, scanning tunneling spectroscopy, enables one to measure differential conductivity (DC) $\sigma(r, V)$ at various positions $r$ and bias-voltages $V$. DC is closely related to LDOS of the superconductor ($k_B = 1$),

$$\frac{\sigma(r, V)}{\sigma_N} = \frac{1}{4T} \frac{N(r, E)}{N_0 \cosh^2 \left( \frac{E + eV}{2T} \right)} \int_{-\infty}^{\infty} \frac{dE}{N_0}. \tag{1}$$

where $\sigma_N$ is DC in the normal state, $e$ is electron charge, $N(r, E)$ is LDOS at the position $r$ and energy $E$ relative to the Fermi level, and $N_0$ is DOS at the Fermi level in the normal state. Only in the limit of zero temperature $T \to 0$ DC and LDOS are proportional: $\sigma(r, V)/\sigma_N = N(r, |eV|/N_0).$ At finite temperature, DC is actually thermally broadened LDOS.

It is clear that at low temperatures DC should follow the spatial structure of LDOS. Two prominent features should be mentioned. DC measured at the vortex center revealed a peak at the Fermi level (zero-bias peak) that well exceeds $\sigma_N$. This indicates that vortex core can not be viewed as being “normal” at least in clean superconductors. The zero-bias peak in DC originates from the zero energy peak of LDOS at the vortex center, which is due to the low lying bound states inside the vortex core. The other remarkable feature revealed in Ref. [1] is a star-shaped DC around the vortex core measured at fixed bias-voltage, with star orientation depending on the bias-voltage value. The six-fold structure of DC in NbSe$_2$ is coming either from the effect of the hexagonal vortex lattice, anisotropic s-wave pairing or anisotropic Fermi surface, and most probably it is coming from each effect simultaneously. Again, the star-shaped DC originates from the star shaped LDOS in the vortex lattice.

However, the measured DC does not follow the sharp features of the corresponding, theoretically calculated, LDOS even if the experiment is performed at very low temperature. The height and width of the zero-bias peak was found to be sample dependent indicating impurities as a plausible explanation for the discrepancy. Indeed, impurities are inevitably present in superconducting samples on which the experiments are performed. Therefore, it is important to quantitatively study how LDOS is changing with impurity concentration. This is the purpose of our paper.

There is another one topic that we analyze in this paper: the effect of impurities on the specific heat field dependence. In s-wave superconductors low energy quasiparticles are trapped inside the vortex core. Therefore, zero-energy LDOS, spatially averaged, is proportional to the number of vortexes: $N(E = 0) \propto N_0 \xi^2 B$, with $B$ being the magnetic induction and $\xi$ the size of the vortex core. This translates into the linear field dependence of low temperature specific heat given by $C_v/T = 2\pi^2 N(E = 0)/3$. The nonlinearity in low field $C_v(B)$ curve should be related to the gap anisotropy. In the case of anisotropic s-wave pairing, addition of nonmagnetic impurities can smear out the gap anisotropy, which can be tracked by examining $C_v(B)$ curves. This kind of measurement has been performed on Nb$_{1-x}$Ta$_x$Se$_2$ and Y(Ni$_{1-x}$Pt$_x$)$_2$B$_2$C. The intention was to make the gap isotropic by adding impurities, but with the price to have rather dirty s-wave superconductor. The effect of impurities on LDOS notwithstanding it would be of value to study how field dependence of spatially averaged LDOS evolves with impurity concentration in the most simple case of s-wave superconductors.

So far, the only systematic experimental study of the effect of disorder on LDOS is by Renner et al. In particular they measured the zero-bias DC at the vortex center in the alloy system Nb$_{1-x}$Ta$_x$Se$_2$. Substitution of Nb by Ta leads to systematic decrease of the electron mean free path. On the other hand small changes in the electronic spectrum is expected since Nb and Ta are isoelectronic and with similar atomic radii. Zero-energy DC is found to be very sensitive to the impurity concentration. It grad-
ually disappears and for \( x = 0.2 \) the zero-energy LDOS in the vortex center is the same as that of normal phase \( N_0 \). It was even proposed that DC spectra can serve as a measure of quasiparticle scattering time.

In this paper the problem of LDOS in presence of nonmagnetic impurities will be studied within the quasiclassical equations of superconductivity. Quasiclassical approximation is adequate in superconductors where coherence length \( \xi \) is much larger than the atomic length \( k_F^{-1} \). LDOS is studied within the quasiclassical approximation by Ullah et al.\footnote{5,10} and Klein\footnote{6} for the case of isolated vortex in the isotropic s-wave superconductor. The full self-consistent analysis of LDOS in the case of vortex lattice is performed by Ichioka et al.\footnote{12,13} All these studies assume clean superconductor without impurities. Dirtiness of the superconductor is only roughly estimated by Klein.\footnote{6} As for the single vortex case, the effect of impurities was studied in Refs. 14,15. Those phenomena are beyond the scope of this text. As for the study of the extreme case, dirty superconductor, the reader is referred to Refs. 14,15.

The paper is organized as follows. In section II the method of solving Eilenberger equations is presented. The readers not interested in technical details may skip that section. In section III spatial and energy dependence of LDOS and DC for various impurity concentration is shown. In section IV the effect of impurities on the specific heat field dependence is discussed.

II. METHOD OF SOLUTION

There are various methods to solve Eilenberger equations for the vortex lattice. The main problem in the numerical procedure is that the initial conditions for the differential equations are unknown. One way to overcome that problem is to use special, divergent, gauge in which Green’s functions are periodic, and solve equations in Fourier space (periodic boundary condition).\footnote{14,15} The other method is based on the fact that during the integration process Green’s functions exponentially grow (explode). Fortunately, the exponentially growing unphysical solutions can be manipulated to form the physical one. This is the essence of the so-called “explosion method”\footnote{14,15,16}. Here we will use a different approach. It is interesting that if one parameterizes quasiclassical Eqs. in the form of Riccati’s differential equation, then during the numerical integration the physical solution is stabilized regardless of the initial condition. Here we will give more details.

For the s-wave superconductor in presence of nonmagnetic impurities Eilenberger equations are

\[
[\omega - \mathbf{u} \left( \nabla - i \mathbf{A} \right)] f = \Psi g + F^* g - G f,
\]

These are supplemented by the self-consistency equations for the gap function \( \Psi \) and vector-potential \( \mathbf{A} \)

\[
\Psi \ln t = 2t \sum_{\omega > 0} \left[ \left( f - \frac{\Psi}{\omega} \right) i \right],
\]

\[
\nabla \times \nabla \times \mathbf{A} = -\frac{2t}{\kappa^2} \text{Im} \sum_{\omega > 0} \left( \mathbf{u} g \right),
\]

as well as for the impurity potentials

\[
F = \frac{1}{\tau} \langle f \rangle, \quad G = \frac{1}{\tau} \langle g \rangle.
\]

Born approximation is assumed in treating scattering on impurity. For convenience, equations are written in following dimensionless units: order parameter is measured in units \( \pi T_c \), length in units \( R_0 = v/(2\pi T_c) \), \( v \) is Fermi velocity, magnetic field in units \( H_0 = \Phi_0/2\pi R^2 \), where \( \Phi_0 \) is flux quantum. Vector-potential is in units \( A_0 = \Phi_0/2\pi R_0 \), energy in units \( E_0 = (\pi T_c)^2 N_0 R^2 \). Scattering time \( \tau \) is in units \( 1/(2\pi T_c) \). It can be expressed via electron mean free path \( l \), \( \tau = l/0.882\xi_0 \), with \( \xi_0 \) being BCS coherence length. Eilenberger parameter \( \kappa \) is the only material constant that enters the equations

\[
\kappa^{-2} = 2\pi N_0 \left( \frac{\pi}{\Phi_0} \right)^2 \frac{v^4}{(\pi T_c)^2}.
\]

It is related to GL parameter \( \kappa \) via \( \kappa^2 = (7\zeta(3)/18)\kappa^2 \) in 3D case and

\[
\kappa^2 = \frac{7\zeta(3)}{8} \kappa^2.
\]

in 2D case. Here \( \zeta \) is Riemann’s zeta function. \( \omega = t(2n+1) \) is Matsubara frequency with integer \( n \), \( t = T/T_c \) is reduced temperature, \( \mathbf{u} \) is unit vector directed along Fermi velocity. Eilenberger Green’s functions \( f, f^\dagger \) and \( g \) are normalized so that \( g = \sqrt{1 - ff^\dagger} \). Fermi surface is assumed to be isotropic and two-dimensional.\footnote{17} Average over the isotropic cylindrical Fermi surface reduces to \( \langle \cdots \rangle = \langle 1/2\pi \rangle \int \cdots d\varphi \), average over polar angle \( \varphi \).

The quantity of our interest, LDOS as a function of position \( \mathbf{r} \) and quasiparticle excitation energy \( E \), is defined as

\[
N(\mathbf{r}, E) = N_0 \text{Re} \left( \text{g}(\mathbf{r}, \mathbf{u}, \omega \rightarrow \delta - iE) \right),
\]

where Eilenberger function \( g \) describes normal excitations and \( \delta \) is a small number. It is very convenient to introduce auxiliary functions \( a \) and \( b \) through the following transformation\footnote{18,19}

\[
f = \frac{2a}{1 + ab}, \quad f^\dagger = \frac{2b}{1 + ab}, \quad g = \frac{1 - ab}{1 + ab}.
\]
Equations for auxiliary functions $a$ and $b$ are decoupled and have the form of Riccati’s differential equation

$$u\nabla a = - (\omega + G + iuA) a + \frac{\Psi + F}{2} - \frac{a^2}{2} (\Psi^* + F^*),$$

(11)

$$u\nabla b = (\omega + G + iuA) b - \frac{\Psi^* + F^*}{2} + \frac{b^2}{2} (\Psi + F).$$

(12)

Auxiliary functions $a(r, u, \omega)$ and $b(r, u, \omega)$ are not independent. Once we solve the Eq. (11), function $b$ can be readily calculated:

$$b(r, u, \omega) = - a^*(r, u, \omega).$$

(13)

In the coordinate system $(\rho, \eta)$, where Fermi velocity direction $u$ coincides with $\rho$-axis,

$$\rho = x \cos \phi + y \sin \phi,$$

$$\eta = y \cos \phi - x \sin \phi,$$

(14)

Eq. (11) reduces to

$$\frac{\partial a}{\partial \rho} = - (\omega + G + iuA) a + \frac{\Psi'}{2} - \frac{a^2 \Psi^*}{2},$$

(15)

where $\Psi' = \Psi + F$. Integrating along the direction $\rho$ from $\rho' = \rho_\infty$ to the desired point $\rho'$, the physical solution $a_\infty$ is stabilized. Note that integrating in the opposite direction, toward decreasing $\rho$, one will get solution $a_- = -1/a_\infty$. How long integration path $\rho_\infty$ should be taken depends on $\omega$ (is it real or complex) and on impurity concentration.

Vector-potential is written as

$$A(r) = \frac{B \times r}{2} + A'(r),$$

(16)

where $B$ is magnetic induction, and $A'$ is periodic with $\nabla \cdot A' = 0$. Therefore, the selfconsistent equation for vector-potential can be written as

$$\nabla^2 A' = \frac{2t}{k^2} \text{Im} \sum_{\omega > 0} \langle u| g \rangle.$$  

(17)

It can be easily solved in the Fourier space.

Auxiliary function $a$ has the same symmetry properties as Eilenberger function $f$, which are described in Ref. 8. The equilibrium vortex lattice structure is assumed to be hexagonal. Therefore, it is sufficient to solve equation (11) in the whole vortex lattice cell and only for velocity directions $0 < \varphi < \pi/6$. With the help of symmetry properties, $a(r, u, \omega)$ can be obtained for all velocity directions.

A. Iteration procedure

Iterative procedure for solving Eq. (15) is the following. We start from some potentials $\Psi(r)$, $A'(r)$, $F$ and $G$. It is usual to start from the Abrikosov solution for $\Psi(r)$, $A'(r) = 0$, and local values of impurity potentials:

$$F = \frac{1}{\tau} \sqrt{\omega^2 + |\Psi(r)|^2}, \quad G = \frac{1}{\tau} \sqrt{\omega^2 + |\Psi(r)|^2}.$$  

(18)

After solving the Eq. (15) the new values of potentials are obtained from the self-consistency eqs. (11), (12), and the new potentials we plug again into the Eq. (15) and solve it. This iterative procedure is repeated until the selfconsistency is achieved. The maximum frequency $\omega_{cut} = t(2N_{cut} + 1)$ should be chosen so the result does not depend on the number of Matsubara frequencies. On the other hand the number of iteration cycles needed to stabilize pair potential increase with the $N_{cut}$. We followed Klein and choose $\omega_{cut} = 20nT_c$ (in physical units) as appropriate for various temperatures. This gives the number of Matsubara frequencies

$$N_{cut} \approx \text{Int} \left( \frac{10}{\tau} \right).$$

(19)

Fortunately it is not necessary to solve Eq. (15) for all $\omega$. For high frequencies the solution can be well approximated by:

$$a \approx \frac{1}{2} \left( \frac{1}{\omega'} - \frac{u \Pi}{\omega'^2} + \frac{(u \Pi)^2}{\omega'^3} \right) (\Psi + F),$$

(20)

where $\omega' = \omega + 1/\tau$ and $\Pi = \nabla + iA$. For all $n > N_{cut}/2$ we use the equation (20). Solution is quasi-periodic. Translation by $R_{nm} = nr_1 + nr_2$ will amount in phase factor $\exp(i\chi(r, R_{nm}))$,

$$a(r + R_{nm}, v, \omega) = a(r, v, \omega)e^{i\chi(r, R_{nm})},$$

(21)

where $r_1$ and $r_2$ are primitive vectors of vortex lattice, $n, m$ are integers, and

$$\chi = \frac{\pi}{a_0} \left[ \frac{m \eta x}{a_0} - \frac{ny(m + m \cos \beta)}{a_0 \sin \beta} + nm + n - m \right].$$

(22)

The angle between primitive vectors is denoted as $\beta$ ($\beta = \pi/3$ in our case). Once the selfconsistent potentials $\Psi(r)$ and $A(r)$ are calculated, the Eilenberger Eqqs. are solved again but this time for $\omega = \delta - iE$ where $\delta$ is small number and $E$ is quasiparticle energy. Note that in the presence of impurities the Eq. (15) has to be solved selfconsistently with respect to impurity potentials $F$ and $G$. As for the choice of $\delta$ one should be very careful. It was already noted that density of states $N(E = 0)$ is very sensitive to the absolute value of $\delta$. Finite $\delta$ has roughly the effect of impurities and suppresses the peak in DOS at the vortex center. For small values of $\delta$, $N(E = 0)$ is spiked at the vortex centers and very fine mesh is needed to evaluate average LDOS. We find that $\delta = 0.001$ suffice for our calculation.
III. LOCAL DENSITY OF STATES AND DIFFERENTIAL CONDUCTANCE

The physics of the vortex core in the clean limit is very different from the physics of the vortex core in dirty superconductors. Properties of the vortex core are governed by Andreev bound states in the clean limit, while in the dirty limit properties of the core are governed by normal electrons. To understand the role of impurities we briefly explain the formation of bound states. Andreev scattering from the pair potential (order parameter) inside the vortex core is caused by electron-like excitation into hole-like excitation and vice versa. States inside the vortex core are coherent superposition of particle and hole states. At certain energies the coherent superposition of particle and hole states is constructive and the bound state is formed. The lowest bound state has the energy $E = \Delta/k_F\xi$. In the quasiclassical limit $k_F\xi \gg 1$, the lowest bound state energy is pushed to zero. Zero-energy bound state inside the vortex core will manifest as a peak in zero-energy LDOS at the vortex center. Scattering on impurities will randomize the motion of electron, and the coherency is lost. Thus, the impurities will smear out the sharp structure of LDOS. To illustrate this we focus on the spatial structure of zero-energy LDOS $N(r, E = 0)$.

In Fig. 1 spatial variation of LDOS along the line connecting two nearest neighbor vortexes is shown. Data for a clean superconductor ($\xi_0/\ell = 0.0$), for a relatively large mean free path $\xi_0/\ell = 0.1$ and for impure case $\xi_0/\ell = 4.0$ are presented. Distance between vortexes is normalized so that 0 and 1 on the abscissa are position of vortexes. To remind the reader again, in the clean limit the height and width of the LDOS peak depend on the small parameter $\delta$, which measures how far we are from the pole of the Green function $g$. In this sense height and width of the peak in the clean limit are arbitrary.

At the vortex core zero energy DOS (ZEDOS) in the clean limit highly exceeds the normal state value $N_0$. This was in the beginning at odds with generally accepted naive picture of vortex core as being “normal”. Analyzing the ZEDOS in the impure case, it is clear that coherency is crucial in forming the main peak at the vortex. In the dirty limit $\xi_0/\ell \rightarrow \infty$ ZEDOS within the vortex core approaches normal state value $N_0$, and only in this limit one can view vortex core as being “normal”. Even a small impurity concentration has a great impact on ZEDOS profile. The comparison of the ideal case of a clean superconductor $\xi_0/\ell = 0$ with rather pure superconductor $\xi_0/\ell = 0.1$ reveals a change of the vortex core size by a factor 2. The change of the vortex core size is compensated by the reduction of the peak height, so the ZEDOS averaged over vortex lattice cell is approximately the same in all cases.

It is instructive to see how the spatial structure of ZEDOS within the vortex lattice evolves by adding impurities. In the clean limit ZEDOS around the single vortex is cylindrically symmetric. As soon as vortex lattice is formed cylindrically symmetric ZEDOS transforms into the star-shaped structure within the hexagonal vortex lattice. This is presented in Fig. 2. The effect of vortex lattice notwithstanding, the other effects such as the anisotropy of the pairing function and the anisotropy of the Fermi surface in hexagonal crystal can also contribute to the specific star-shaped structure of ZEDOS. By reducing the mean free path, star-shaped structure gradually disappears and is completely absent in the dirty limit even at relatively high fields. This indicates that periodicity of the order parameter is not the key element to explain the structure of $N(r, 0)$ in Fig. 2. Only coherent superposition of electron and hole states in the periodic vortex lattice can account for the star-shaped ZEDOS.

In Fig. 3) LDOS at the vortex center is plotted as a function of quasiparticle excitation energy $E$ (in units $\pi T_c$) for the clean case. LDOS oscillates with energy, the result previously reported in

![FIG. 1: Spatial variation of zero energy DOS along the nearest neighbor vortex direction. Full line corresponds to the clean limit and dashed lines correspond to the superconductors with $\xi_0/\ell = 0.1$ and $\xi_0/\ell = 4.0$. The calculation is performed at approximately the same relative field $B = 0.1H_c$.](image1)

![FIG. 2: ZEDOS within the vortex lattice for superconductors with $\xi_0/\ell = 0.0$, $\xi_0/\ell = 0.1$ and $\xi_0/\ell = 4.0$ (in order from left to right). Only data points $N(E = 0)/N_0 < 1$ are presented. Small parameter $\delta = 0.03$ is used for clean limit data to clarify the spatial distribution.](image2)
Ref. [24]. This phenomenon has the same origin as oscillation of DOS in superconducting-normal proximity systems,\textsuperscript{25,26,27} interference of quasiparticles reflected at the superconducting-normal barrier. The mixed state can be viewed as periodically arranged infinite number of “normal”-superconducting boundaries. Here the vortex cores play the role of normal region in the sense that tex cores play the role of normal region in the sense that gap drops to zero at the vortex axes. In Fig. 3b) DC at \(T = 0.1T_c\), calculated according to Eq. (1), is presented. At this temperature DC is thermally broadened LDOS, but the oscillating pattern is still visible.

The coherency of quasiparticles is essential both for zero-bias peak and oscillation of LDOS with energy at the vortex axis. In Fig. 4 LDOS at the vortex center as a function of energy is plotted for various values of impurity concentration. Oscillation amplitude is very sensitive to the presence of impurities and is almost lost even in very clean samples with \(\xi_0/\ell = 0.1\). Proliferating impurity concentration will manifest as a flattening of LDOS at the vortex center: disappearance of zero-energy peak of LDOS, as well as disappearance of deep minima for \(E < \Psi(B = 0)\). In the dirty limit \(\xi_0/\ell \rightarrow \infty\), LDOS at the vortex center is equal to \(N_0\) for all quasiparticle energies \(N(r = 0, E) = N_0\).\textsuperscript{12,13}

**IV. LOCAL DENSITY OF STATES AND SPECIFIC HEAT**

The low energy quasiparticle excitations play the important role in the low temperature thermodynamics. Specific heat \(C_s(T)\) of a superconductor is given by

\[
\frac{C_s}{T} = 2 \int_{-\infty}^{\infty} dE \frac{\partial N(E)}{\partial T} \left\{ \ln \left[ 2 \cosh \left( \frac{E}{2T} \right) \right] - \frac{E}{2T} \tan \left( \frac{E}{2T} \right) \right\} + 2 \int_{-\infty}^{\infty} \frac{E^2}{4T^3} N(E) dE \cosh^2 \left( \frac{E}{2T} \right) \cosh \left( \frac{E}{2T} \right).
\]

One can utilize this expression only if the energy dependent, spatially averaged, LDOS \(N(E)\) is provided. However, in the limit \(T \rightarrow 0\) the first integral is zero. For small \(T\) the function to be integrated in the second integral is nonzero only in the small vicinity of \(E = 0\). Therefore we can replace \(N(E)\) by \(N(E = 0)\)

\[
\lim_{T \rightarrow 0} \frac{C_s}{T} = 2 \int_{-\infty}^{\infty} \frac{E^2}{4T^3} \frac{N(E = 0)}{\cosh^2 \left( \frac{E}{2T} \right)} = \frac{2\pi^2 N(E = 0)}{3}.
\]

In the normal phase \(C_n/T = 2\pi^2 N_0/3\) which gives us the well known result

\[
\lim_{T \rightarrow 0} \frac{C_s}{C_n} = \frac{N(E = 0)}{N_0}.
\]

If the low energy quasiparticles are localized in the vortex cores, which is true for \(s\)-wave superconductors at least in the limit of very small fields, then \(N(E = 0) \sim \rho^2/S_{cell}\). Here \(S_{cell} = \Phi_0/B\) is vortex lattice cell area and \(\rho\) is the size of the vortex core. If we further assume that \(\rho^2 \sim \Phi_0/H_{c2}\) then we arrive at the following scaling relationship \(N(E = 0) \sim B/H_{c2}\), for \(s\)-wave superconductors. However there is a number of reports of nonlinear field dependence of \(\gamma_s(H)\) in \(s\)-wave superconductors. One of the offered explanations is that vortex core size \(\rho\) itself is field dependent which in turn lead
to the nonlinear field dependence of zero-energy DOS. The shrinking of the vortex core with increasing field is detected in NbSe$_2$ and YBa$_2$Cu$_3$O$_{6.60}$.$^{28}$ This is further supported by numerical calculations in dirty and clean limit. Such an explanation brings out another puzzle. Experimental study on influence of non-magnetic impurities on the $\gamma(H)$ in Y(Ni$_{1-x}$Pt$_x$)$_2$B$_2$C and Nb$_{1-x}$Ta$_x$Se$_2$ revealed that linear $\gamma(H)$ is achieved only in dirty samples.$^{28}$ This result suggest that the vortex core size in the dirty superconductors is field independent. Numerical calculation by Golubov and Hartman$^{13,28}$ as well as Sonnier et al$^{28}$ shows quite contrary, that even in the dirty limit $\rho$ should shrink with increasing field.

Here we emphasize the necessity to perform the calculation at low temperature in order to analyze the specific heat data through the ZEDOS. In Ref. 3 calculation performed at $T = 0.5T_c$ revealed that $\overline{N(E = 0)} \sim \xi^2(B)B$, where $\xi(B)$ is independently calculated vortex core radius. At lower temperatures, due to the Kramer-Pesch effect, the core radius is smaller and it might have different field dependence.

The result for the field dependence of ZEDOS in the clean limit, for $T = 0.1T_c$, is shown in Fig. 5. In the inset we plot the field dependence of the core radius at the same temperature. We define the core radius $\xi$ as $1/\xi = (\partial \Psi(r)/\partial r)/|\Psi_{NN}|$ where $|\Psi_{NN}|$ is the maximum value of the order parameter along the nearest neighbor direction, and derivative is taken along the same direction. Compared to the previously reported result at higher temperature $T = 0.5T_c$, where $\xi(B)$ decreases with field at $T = 0.1T_c$, vortex core radius is rather constant at low fields. As a consequence, zero-energy LDOS is also linear function of magnetic induction.

In the clean limit ZEDOS in between vortexes is negligible in fields as large as $B = 0.4H_{c2}$. In other words, the main contributions to ZEDOS is coming from the vortex cores. On the other hand, in the dirty limit, ZEDOS is not confined to the vortex cores, but it is spread throughout the vortex vortex cores. Thus, the scaling relation $\overline{N(E = 0)} \sim \xi^2(B)B$ is of no use in the dirty limit. This is the reason why we do not attempt to correlate vortex core size $\xi(B)$ and field dependence of LDOS in the impure case. However, $\overline{N(E = 0, B)}$ is a linear function of magnetic induction at low fields for any impurity concentration: $\overline{N(E = 0, B)/N_0} = c(\tau)B/H_{c2}$. Constant of proportionality $c(\tau)$ weakly depends on the electron mean free path and saturates to $c \approx 0.8$ in the dirty limit. Numerical calculation of $\overline{N(E = 0, B)/N_0}$ as a function of mean-free path value is presented in Fig. 6. We note the concave curves for dirtier cases. This behaviors coincide with the analysis near $H_{c2}$ by Kita.$^{30}$

In Fig. 5 is shown the field dependence of the core radius as calculated from the pair potential profile $\Psi(r)$. For a fixed relative field $B/H_{c2}$ core radius $\xi$ is a nonmonotonic function of mean free path, first sharply increases and then slowly decreases with increasing of ratio $\xi_0/c$. In the dirty limit vortex core shrinks with increasing field, which is consistent with the previous calculations,$^{13,28}$ in sharp contrast with vortex core enlargement with increasing field in the clean limit.

The experimental data, however, revealed that constant $c = 1$ in the dirty limit.$^{2}$ It also shows that scaling $\overline{N(E = 0, B)/N_0} = c(\tau)B/H_{c2}$ for all field values, which is a remarkable feature that still lacks the explanation. Worth is mentioning that in Ref. 2 specific heat is a nonlinear function of field in samples Y(Ni$_{1-x}$Pt$_x$)$_2$B$_2$C for all $0 < x < 1$. In these materials, we need to consider also the effect of gap anisotropy.
FIG. 7: Field dependence of vortex core size for various mean free path.

V. SUMMARY

In this paper we examined the effect of impurities on LDOS in isotropic s-wave superconductors. We showed that coherency is crucial in forming the spatial structure of LDOS. As soon as impurities are introduced into the superconductor, scattered electrons lose the information on their initial state, the coherency is lost and sharp LDOS structure is flattened. It is calculated how DC spectra evolve with electron mean-free path. Although the impurities have a great impact on LDOS, spatially averaged LDOS shows weak dependence on relative field \( B/H_{c2} \). We hope that present calculation can be helpful to roughly estimate the electron mean free path through the LDOS measurement.

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