Analysis of pion electromagnetic form factor by the dispersion relation with QCD constraint

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Abstract

The pion electromagnetic form factor is investigated by using the dispersion relation with the superconvergence condition, which was proposed to synthesize the vector meson dominance model and QCD. The absorptive part is given as an addition of the resonance and the QCD terms, the latter being parametrized so as to realize the prediction of perturbative QCD.

The experimental data are analyzed and the parameters for the QCD part, the resonance part as well, are determined. Existing experiments for the form factor are reproduced very well both for the space-like and time-like momenta.

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1 Introduction

According to the perturbative QCD hadron electromagnetic form factors decrease more rapidly than an inverse polynomial for large squared momentum transfer [1, 2]. The boson electromagnetic form factor, \( F(t) \), decreases as

\[
F(t) \to -32\pi^2 f_B^2 / \beta_0 t \ln |t|
\]

for \(|t| \to \infty\), where \( f_B \) is the boson decay constant, \( t \) is the squared momentum transfer and \( \beta_0 \) is the lowest order of the \( \beta \) function in the renormalization group, namely \( \beta_0 = 11 - 2n_f/3 \) with \( n_f \) being the number of flavor. Experimentally, the charge and magnetic form factors of nucleons decrease more rapidly for large \( t \) than the well-known dipole formula as is predicted by the perturbative QCD.

The QCD property for the form factor is realized by using the dispersion relation in which the absorptive part, \( \text{Im}F(t) \), decreases as some power of \( \ln t \) and satisfies the superconvergence constraint [3, 4]. We break up the absorptive part as a simple summation of the resonance and the QCD part.

The former is given as addition of the Breit-Wigner formula with possible mixing among resonances...
and the latter as a power series in the running coupling constant which is analytically extrapolated to the time-like momentum. We impose the superconvergence condition on $\text{Im} F(t)$. For the form factor the low momentum part is rather easy to deal with by the dispersion relation. It is, therefore, possible to treat whole range, from the low to the high momentum regions, systematically; the vector meson dominance model (VMD) is realized for low momentum and the QCD condition is satisfied for large $|t|$. The dispersion relation works as an interpolation.

In this paper we improve the result of Ref.\cite{6}, by investigating correlations among QCD parameters. We search much wider range of parameters than in Ref.\cite{6}. We show that QCD part of $\text{Im} F(t)$ can be determined from the experiments on the pion electromagnetic form factor although the data are restricted to low momentum. At least the second order in the running coupling constant is necessary to fit the experimental data. The QCD scale parameter $\Lambda$ is determined to be in the range $\Lambda = 0.3 \sim 2$ GeV. The calculated form factor agrees with the existing experimental results very well both for the time-like and space-like momenta.

\section{Superconvergence dispersion relation and QCD}

As the superconvergent dispersion relation has already been discussed \cite{3}-\cite{7}, we summarize our prescription to make the paper self-contained. To realize the asymptotic form of QCD we make use of the following property of the Hilbert transformation. Let $F(t')$ be given as

$$F(t) = \int_{s_0}^{\infty} dt' \frac{\rho(t')}{t' - t},$$

where $s_0 > 0$ is a constant and the function $\rho(t')$ decreases as

$$\rho(t') \to a/(\ln(t'))^{\gamma} \quad \text{for} \quad t' \to \infty,$$

with $a$ and $\gamma$ being constants, then

$$F(t) = \int_{s_0}^{\infty} dt' \frac{\rho(t')}{t' - t} \to \frac{a}{(\gamma - 1)(\ln(|t|))^{\gamma - 1}} \quad \text{for} \quad t \to \infty.$$  

Here $\gamma > 1$. In the case that the spectral function $\rho(t') \to a/t' \ln(t')$ for $t' \to \infty$ and satisfies the superconvergence condition $\int_{s_0}^{\infty} \rho(t') dt' = 0$, $F(t)$ given by \cite{3} decreases asymptotically as $F(t) \to a/(\gamma - 1)t \ln^{\gamma - 1}(|t|)$ for $t \to \infty$.

\subsection{Vector meson dominance model and QCD}

For the pion electromagnetic form factor, $F(t)$, we assume the unsubtracted dispersion relation with respect to the squared momentum transfer $t$.

$$F(t) = \frac{1}{\pi} \int_{s_0}^{\infty} dt' \frac{\text{Im} F(t')}{t' - t},$$

(3)
where $s_0$ is the threshold. To satisfy the asymptotic condition of perturbative QCD (1), the absorptive part should satisfy the following conditions:

$$\text{Im} F(t) \to -\frac{32\pi^3 f_\pi^2}{t[\beta_0 \ln^2(t/\Lambda^2)]} \quad (t \to \infty)$$

and the superconvergence condition

$$\int_{s_0}^{\infty} dt' \text{Im} F(t') = 0.$$  

We take $\text{Im} F$ so as to realize the result of the vector meson dominance model for low momentum region and the QCD prediction asymptotically; we express it as the summation of the Breit-Wigner formula with mixing among resonances [10] and the QCD term.

$$\text{Im} F(t) = \text{Im} F^{BW}(t) + \text{Im} F^{QCD}(t),$$

where the Breit-Wigner term $\text{Im} F^{BW}(t)$ is given as

$$\text{Im} F^{BW}(t) = \sum_j \frac{c_j M_j^2 \gamma_j}{(M_j^2 - t)^2 + \gamma_j^2}$$

$$+ \sum_{j<k} c_{jk} \left[ \frac{\alpha_{jk}^I (M_j^2 - t) + \alpha_{jk}^R \gamma_j}{M_j^2 - t)^2 + \gamma_j^2} - \frac{\alpha_{jk}^I (M_k^2 - t) + \alpha_{jk}^R \gamma_k}{(M_k^2 - t)^2 + \gamma_k^2} \right].$$

Here the suffixes denote numbering of resonances and $\alpha_{jk}^R$ and $\alpha_{jk}^I$ are

$$\alpha_{jk}^R = -\frac{\gamma_j \gamma_k (M_j^2 - M_k^2)}{(M_j^2 - M_k^2)^2 + (\gamma_j - \gamma_k)^2},$$

$$\alpha_{jk}^I = -\frac{\gamma_j \gamma_k (\gamma_j^2 - \gamma_k^2)}{(M_j^2 - M_k^2)^2 + (\gamma_j - \gamma_k)^2}.$$  

$\text{Im} F^{QCD}(t)$ is given as a power series in the running coupling constant multiplied by a function $h(t)$, which is introduced to assure the threshold behavior and the convergence of the superconvergence integral [3]. In fact,

$$\text{Im} F^{QCD}(t) = \text{Im} \left[ \sum_{j \geq 1} c_j^{QCD} \{ \alpha_R(t) \}^j \right] h(t).$$

Here $\alpha_R$ is the running coupling constant in QCD analytically regularized [3, 4, 5]. We shall discuss on $\alpha_R$ in the next subsection. For the function $h(t)$ we assume the following formula [3], motivated by Kroll et al. [11]:

$$h(t) = \left[ \frac{t - t_0}{t + t_1} \right]^{3/2} \frac{t_2}{t + t_2},$$

with $t_i \ (i = 0, 1, 2)$ being parameters. It must be noticed that the result is not sensitive to the form of the function $h(t)$.
For the form factor we impose the normalization condition at \( t = 0 \),

\[
F(0) = 1.
\] (12)

We have two conditions on \( F(t) \), the superconvergence and the normalization condition \( (5) \) and \( (12) \), respectively.

### 2.2 Running coupling constant for the time-like momentum

It is known that the running coupling constant \( \alpha_S \) has an unphysical pole for the space-like momentum if it is calculated by the perturbative QCD. The removal of the singularity was proposed by Dokshitzer et al.\cite{8,9} via analytic regularization. The regularized running coupling constant, being denoted as \( \alpha_R \), is given by the spectral representation

\[
\alpha_R = \frac{1}{\pi} \int_0^\infty \frac{dt'}{t' - t} \sigma(t').
\] (13)

Here the function \( \sigma \) is given in terms of the \( \alpha_S \), which is obtained for the space-like momentum via the renormalization group. We have

\[
\sigma(t) = \frac{1}{2i}[\alpha_S(e^{-\pi t}) - \alpha_S(e^{\pi t})].
\] (14)

We use the three loop approximation for \( \alpha_S \), which is expressed by the formula

\[
\alpha_S(Q^2) = \frac{4\pi}{\beta_0} \left[ \ln(Q^2/\Lambda^2) + a_1 \ln(\ln(Q^2/\Lambda^2)) \right. \\
+ \left. a_2 \frac{\ln(\ln(Q^2/\Lambda^2))}{\ln(Q^2/\Lambda^2)} + a_3 \ln(\ln(Q^2/\Lambda^2)) + \cdots \right]^{-1}
\] (15)

where

\[
a_1 = 2\beta_1/\beta_0^2, \quad a_2 = 4\beta_2^2/\beta_0^4, \quad a_3 = (4\beta_2^2/\beta_0^4)(1 - \beta_0\beta_2/8\beta_1^2),
\] (16)

and \( \beta_i (i = 0, 1, 2) \) are the \( \beta \) functions in QCD with \( \beta_0 = 11 - 2n_f/3, \beta_1 = 51 - 19n_f/3 \) and \( \beta_2 = 2857 - 5033n_f/9 + 325n_f^2/27, n_f \) being the number of flavor. Expanding \( (15) \) in terms of \( \ln(Q^2/\Lambda^2) \) and \( \ln(\ln(Q^2/\Lambda^2)) \), we get the usual expression for the running coupling constant \( (12) \). We perform the analytic continuation of the effective coupling constant \( \alpha_S \) to the time-like momentum by the replacement \( Q^2 \rightarrow e^{-\pi t} \) and decompose to the real and imaginary parts \( \alpha_S = \text{Re}[\alpha_S(t)] + i \text{Im}[\alpha_S(t)] \). Substituting \( \text{Im}[\alpha_S(t)] \) in \( (14) \), we obtain the running coupling constant for the time-like momentum. We write it by the same notation as \( \alpha_R \). We showed in Ref.\cite{7} that \( \alpha_R \) given by \( (13) \) approximately coincides with that which is given by \( (15) \) with the ghost pole subtracted, namely, for the space-like momentum

\[
\alpha_R(Q^2) \approx \alpha_S(Q^2) - A^*/(Q^2 - Q^{*2}),
\] (17)

where \( Q^{*2} \) is the pole of \( \alpha_S(Q^2) \) and \( A^* \) is the residue at the pole. Writing \( Q^{*2}/\Lambda^2 = e^{u^*} \), we have \( u^* = 0.7910487 \) for \( n_f = 2 \) with the three loop approximation and

\[
A^* = 4\pi\Lambda^2 e^{u^*} \left\{ \beta_0 \left( 1 + \frac{a_1}{u^*} - \frac{a_2}{u^*^2} + \frac{a_3}{u^*^3} \right) \right\},
\]
where $a_i$ are defined by (16). To extend to time-like momentum we replace $Q^2 \rightarrow e^{-i\pi t}$ as is mentioned above. The difference between $\alpha_R$ given by the spectral representation and the approximate one is less than 0.4% for $|t| \gtrsim 3$ GeV$^2$. The approximation becomes better as $|t|$ becomes larger [7]. We shall use the approximate one for the regularized running coupling constant hereafter; as we have multiplied the function $h(t)$ to define $\text{Im} F_{QCD}$, the contribution from the low momentum part is considerably suppressed so that the approximation turns out to be very good.

For the time-like momentum $\alpha_R$, being obtained by the replacement $Q^2 \rightarrow e^{-i\pi t}$ in (17), is given as follows:

\[
\text{Re}[\alpha_R(t)] = \frac{4\pi u}{\beta_0 D(t)} + \frac{A^*}{t + Q^2},
\]

(18)

\[
\text{Im}[\alpha_R(t)] = \frac{4\pi v}{\beta_0 D(t)},
\]

(19)

where

\[
u = \ln(t/\Lambda^2) + \frac{a_1}{2} \ln\{\ln(t/\Lambda^2)\}
\]

\[+ \frac{a_2}{\ln^2(t/\Lambda^2) + \pi^2} \left\{ \frac{1}{2} \ln(t/\Lambda^2)\ln\{\ln(t/\Lambda^2)\} + \pi \theta \right\}
\]

\[+ \frac{a_3 \ln(t/\Lambda^2)}{\ln^2(t/\Lambda^2) + \pi^2},
\]

(20)

\[
u = \pi + a_1 \theta - \frac{a_2}{\ln^2(t/\Lambda^2) + \pi^2} \left\{ \frac{\pi}{2} \ln\{\ln(t/\Lambda^2)\} - \theta \ln(t/\Lambda^2) \right\}
\]

\[- \frac{\pi a_3}{\ln^2(t/\Lambda^2) + \pi^2},
\]

(21)

and

\[
D = u^2 + v^2.
\]

(22)

Here

\[
\theta = \tan^{-1} \left\{ \frac{\pi}{\ln(t/\Lambda^2)} \right\}.
\]

(23)

The spectral function $\sigma$ is obtained by taking imaginary part of $\alpha_S$, that is,

\[
\sigma(t) = 4\pi v/\beta_0 D,
\]

(24)

where $v$ and $D$ are defined by (21) and (22), respectively.

### 3 Numerical results

We analyzed the experimental data of the pion electromagnetic form factor for the space-like and the time-like momenta. The parameters appearing in our formulas are determined so as to minimize the $\chi^2$, addition of the chi square for the time-like and space-like momenta. We fix the parameters which appear in the function $h(t)$ at $t_0 = \Lambda^2$ and $t_1 = t_2 = 16$ GeV$^2$ as the result is not sensitive to these values [7]. The number of flavor is taken as $n_f = 2$. 

5
For large $t$, $\text{Im} F$ given by (10), approaches to $\text{Im} F(t) \to c_1^{QCD} \text{Im} \alpha_R(t_2/t)$ and $\text{Im} \alpha_R$ given by (19) to $4\pi^2/\beta_0 \ln^2(t/\Lambda)$. Therefore, comparing (4) and (10), we have

\[
c_1^{QCD} = -8\pi f_\pi^2/t_2,
\]

which is fixed at $c_1^{QCD} = -0.02683$ by taking $t_2 = 16$ GeV and the pion decay constant $f_\pi = 0.1307$ GeV.

To realize the data for the time-like momentum we take the masses and widths of vector bosons as parameters, which are determined as follows: $m_\rho = 0.760$ GeV, $\Gamma_\rho = 0.138$ GeV; $m_\omega = 1.40$ GeV, $\Gamma_\omega = 0.24$ GeV. We investigated the effect of the unconfirmed vector boson whose mass and width are taken as $m_{\rho''} = 2.11$ GeV and $\Gamma_{\rho''} = 0.368$ GeV. The mass and width of $\omega$ meson are kept at the experimental values [12]. We leave out the $\phi$ meson in this calculation as the contribution is estimated to be very small. In the chi square analysis the following experimental data are used: Amendolia et al. [14] and Bebek et al. [13] for the space-like momentum and Barkov et al. [13] for the time-like momentum. The number of data points are 101 for the space-like and 157 for the time-like regions; totally 258 data are used in the analysis. Degrees of freedom (DOF) is 241 when $\rho''$ is omitted. If $\rho''$ is taken into account, DOF = 238.

First, we investigate the QCD scale parameter $\Lambda$. We restrict to the second order approximation for the QCD running coupling constant in $\text{Im} F^{QCD}$, namely, $c_3^{QCD} = 0$. We illustrate in Fig.1 the contour lines for $\chi^2 = 400, 500$ and 600 to give correlation between the pion decay constant $f_\pi$, being taken as a parameter, and $c_2^{QCD}$. Here $\rho''$ is left out. To draw the contour lines, $f_\pi$ and $c_2^{QCD}$ is kept fixed and the other parameters are determined so as to make $\chi^2$ minimum. The QCD scale parameter is fixed at $\Lambda = 0.8$ GeV. $\chi^2$ is then given as a function of $f_\pi$ and $c_2^{QCD}$. The horizontal lines in Fig.1 denote the experimental value for the pion decay constant $f_\pi = 0.1307 \pm 0.00037$ GeV. In the same way, we examined correlation between $\Lambda$ and $c_2^{QCD}$. In Fig.2 we show the contour lines which give correlation between $\Lambda$, being taken as a parameter, and $c_2^{QCD}$, where the pion decay constant is fixed at the experimental value, $f_\pi = 0.1307$ GeV. Taking $\chi^2 \leq 500$ in Fig.1 and the experimental value for $f_\pi$, we obtain $c_2^{QCD} = -0.87 \sim -0.49$. The one-loop approximation for the QCD part, namely $c_2^{QCD} = 0$, leads to poor result. The QCD scale parameter is determined from the correlation of $c_2^{QCD}$ and $\Lambda$ which is illustrated in Fig.2. The condition $\chi^2 \leq 500$ leads to $0.3 \leq \Lambda \leq 2$ GeV, where we take the value of $c_2^{QCD}$ determined above, namely, $c_2^{QCD} = -0.87 \sim -0.49$. The best fit is obtained for $\Lambda \approx 0.8$ GeV.
Fig. 1. Contour lines for the parameters $c_2^{QCD}$ and $f_\pi$. $\chi^2$ is obtained as a function of $f_\pi$ and $c_2^{QCD}$ with the other parameters determined so as to make $\chi^2$ minimum. $\Lambda$ is fixed at 0.8 GeV. Solid, dotted and dashed lines denote the contours with $\chi^2 = 400, 500$ and 600, respectively. Data for the space-like momentum [14], [15] and the time-like momentum [13] are used.

Fig. 2. Contour lines for $c_2^{QCD}$ and $\Lambda$. The pion decay constant is fixed at the experimental value. See the caption of Fig. 1.
The parameters which minimize $\chi^2$ are summarized in Table I, where $\Lambda$ is fixed at $\Lambda = 0.8 \text{ GeV}$ and the pion decay constant at the experimental value for the cases with and without $\rho'''$.

| Parameter | without $\rho'''$ (2.11) | with $\rho'''$ (2.11) |
|-----------|--------------------------|----------------------|
| $c_\rho$  | 2.723                    | 2.701                |
| $c_\omega$ | -0.0030                  | -0.0030              |
| $c_{\rho'}$ | -0.2701                  | -0.0924              |
| $c_{\rho''}$ | -0.1956                  | -0.2830              |
| $c_{\rho'''}$ | —                       | -0.0260              |
| $c_{\omega - \rho}$ | 1.069                    | 1.068                |
| $c_{\rho - \rho'}$ | 1.832                    | -0.089               |
| $c_{\rho - \rho''}$ | 0.562                    | -0.136               |
| $c_{\rho - \rho'''}$ | -51.92                   | -49.99               |
| $c_1^{QCD}$ | -0.02683                 | -0.02683             |
| $c_2^{QCD}$ | -0.645                   | -0.573               |
| $\chi^2$  | 379.1                    | 355.1                |

$\dagger$ $c_{1}^{QCD}$ is calculated by (25).

Table I Residues at the resonance poles, mixing parameters among resonances and the QCD parameters $c_i^{QCD}$ ($i = 1, 2$). We take $\Lambda = 0.8 \text{ GeV}$ and $c_1^{QCD} = 0$. The parameters in the function $h(t)$ are $t_0 = \Lambda^2$, $t_1 = t_2 = 16 \text{ GeV}^2$. Two cases are investigated without the unconfirmed resonance $\rho'''$ and with $\rho'''$. The value of $\chi^2$ is the summation of that for the space-like and time-like momenta. In Figs.1-5 the same parameters are used.

Let us compare our calculated results with the experimental data of the pion electromagnetic form factor. In Fig.3 we give our results for $|F(t)|^2$ for the space-like momentum. The experimental data are taken from Amendolia et al.\cite{14}. In Fig.4 $|t| F(t)$ is compared with the experiments. Our result agrees with the recent experiment by J.Volmer et al.\cite{10}. In the figure the data by Bebek et al.\cite{13} are included.
Fig. 3. Electromagnetic form factor $|F(t)|^2$ for the space-like momentum. Low momentum region. Data are taken from Amendolia et al. Two cases with and without $\rho^{\prime\prime\prime}$ almost coincide. The parameters given in Table I are used.

Fig. 4. $t F(t)$ for the space-like momentum. The data denoted as $\times$ are taken from Bebek et al., the open circles and closed circles are taken from Volmer at al. Solid line stands for the calculation without $\rho^{\prime\prime\prime}$ and the dashed one with $\rho^{\prime\prime\prime}$. The parameters given in Table I are used.
The solid lines stand for the calculation without assuming $\rho'''$ and the dashed ones with $\rho'''$.

Fig. 5 $|F(t)|^2$ for the time-like momentum. Data are taken from Barkov et al. [13]. The solid lines stand for the calculation without $\rho'''$ and the dashed ones with $\rho'''$. (a) $t = 0.3 - 0.75$ GeV$^2$, (b) $t = 0.75 - 0.85$ GeV$^2$, (c) $t = 0.85 - 1$ GeV$^2$ and (d) $t = 1 - 2$ GeV$^2$. The parameters given in Table I are used. We compare in Fig. 5 (a) - (d) the calculated results for $|F(t)|^2$ for the time-like momentum with the experimental data. It is necessary to take account of the $\omega$ boson and $\rho$-$\omega$ mixing to explain the dip in the form factor near the $\rho$ meson mass. In Fig. 4 and Fig. 5 (a) - (d) the dashed curves denote the result obtained by taking account of $\rho'''$. We find that the assumption of $\rho'''$ improves the result especially for the time-like momentum above 1.5 GeV$^2$. The chi square is obtained as $\chi^2 = 379.1$ for the case without $\rho'''$ and $\chi^2 = 355.1$ for the case with $\rho'''$. 
4 Concluding remarks

We analyzed the experimental data for the pion electromagnetic form factor by using the dispersion relation in which the QCD constraint is satisfied. The imaginary part of the form factor is given as the summation of the Breit-Wigner formula with mixing among resonances and the QCD term. The latter is expressed as a power series in the running coupling constant multiplied by a function that assures the convergence of the superconvergence constraint. The function may be interpreted as representing the quark-photon and quark-hadron vertices. In order that the form factor satisfies the QCD prediction we imposed the asymptotic condition on the absorptive part, by which the coefficient of $\alpha_R$ in $\text{Im}F^{\text{QCD}}$, that is $c_1^{\text{QCD}}$, is given in terms of the pion decay constant. The QCD constraint is very strong so that at least $O(\alpha_R^2)$ approximation is required to realize the experimental data. By taking the second order approximation for the QCD running coupling constant we are able to reproduce the experimental data of the pion electromagnetic form factor very well. The result is improved by taking account of the third order term, namely, $c_2^{\text{QCD}}$ and $c_3^{\text{QCD}}$ as free parameters. However, the experimental data are not sufficient to determine them accurately.

We assumed the mixing among vector bosons. Especially mixing between $\omega$ and $\rho(760)$ is necessary to realize the structure of form factor near the $\rho$ meson mass as in Ref.[10]. The structure is explained by the mixing very well as is illustrated in Fig.5 (b). An enhancement is observed in the pion electromagnetic form factor for $t = 1.1 \sim 1.3$ GeV [13] (see Fig.5(d)), although there is no isovector vector boson with the mass between 1.2 and 1.7 GeV. To explain the enhancement, without assuming resonance with the mass about 1.2 GeV, large mixing between $\rho'$ and $\rho''$ is necessary.

Analysis is performed by using the data of Amendolia et al.[14] and Bebek et al.[15] for the space-like momentum. As we have shown in Fig.3 our result agrees with the recent experiments for the space-like momentum [16].

We examined the effect of unconfirmed vector boson $\rho'''$ with the mass above 2 GeV. It is shown that $\rho'''$ improves the result, especially for the time-like momentum with $t$ above 1.5 GeV$^2$ (see Fig.5 (d)).

The QCD scale parameter $\Lambda$ is determined to be in the range $0.3 \sim 2$ GeV from the experiments of the pion electromagnetic form factor; the best fit is attained for $\Lambda \sim 0.8$ GeV. $\Lambda$, obtained from the analysis of pion form factor, seems to be larger than that reported in the deep inelastic scattering. This implies that $\Lambda$ becomes larger when the number of effective flavor is decreased [18].
Appendix  A functional realization of theoretical results for the space-like momentum

For the convenience of reproducing the theoretical form factor for the space-like momentum we express our calculated results by a functional approximation similar to the formula proposed by Gari and Krümperman [17],

\[
F(t) = \frac{M^2}{|t| + M^2} \frac{1 + N_1 x + N_2 x^2}{1 + D_1 x + D_2 x^2 + D_3 x^3},
\]

(26)

where \( x = \ln(1 + |t|) \), with \( t \) being expressed in the unit of GeV\(^2\). \( M \), a constant with the dimension of mass, is fixed at the \( \rho \) meson mass; \( M = 0.76 \) GeV. The parameters \( N_i \) and \( D_i \) are determined so as to realize the calculated results given in Fig.4.

For the case without \( \rho'''' \), the parameters are determined as follows: \( N_1 = 4.5919 \), \( N_2 = -0.62524 \), \( D_1 = 4.9604 \), \( D_2 = -0.7049 \), \( D_3 = 0.0540 \). For the case with \( \rho'''' \) the parameters are \( N_1 = 4.7597 \), \( N_2 = 0.30520 \), \( D_1 = 5.1071 \), \( D_2 = 0.3598 \), \( D_3 = 0.1904 \). The formula is valid up to \( |t| \leq 10 \) GeV\(^2\); the theoretical result is reproduced within the accuracy of 0.1% for the squared momentum \( |t| \leq 10 \) GeV\(^2\) and for \( |t| \leq 6 \) GeV\(^2\) the error is less than 0.02%.

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