Economic-Statistical Design of Integrated Model of VSI Control Chart and Maintenance Incorporating Multiple Dependent State Sampling

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ABSTRACT The mutualistic relationship between quality control and maintenance of production systems has contributed to the development of the integration model of statistical process control (SPC) and maintenance. Such models are not only beneficial to the improvement of production quality, but also to the reduction of maintenance cost. This paper presents an integrated model of SPC and maintenance that applies a VSI & MDSS control chart designing based on the variable sampling interval (VSI) scheme and multiple dependent/deferred state sampling (MDSS) policy. A Markov chain approach is employed to model the integrated process, and the average total cost per unit time is derivated and solved by a developed genetic algorithm subject to statistical quality constraints. A shrimp canning production process example is given to illustrate the application of integrated model. Finally, a comparative study using the design of experiments approach is conducted to demonstrate the effectiveness of the suggested model.

INDEX TERMS Statistical process control, maintenance, multiple dependent state sampling, variable sampling interval.

I. INTRODUCTION
In today’s fiercely competitive global market, quality plays a crucial role in the development of business enterprise. As it is pointed out by a well-known modern definition of quality, quality is inversely proportional to variability. This definition indicates that if variability in the critical-to-quality characteristics of a product decreases, there will be an increase in quality of the product [1]. SPC is now one of the key monitoring tools, which is widely used to improve process capability and achieve process stability through the reduction of variability. Meanwhile, owing to the changing markets and the increased variety of products, production equipment becomes more and more complex and automated. Undoubtedly, the lack of proper maintenance of production equipment will ultimately affect the quality of products and result in increased process variation. It is clear that the product quality may be affected directly by the equipment operating condition, which can be real-time monitored by SPC tools (e.g., control charts), and, correspondingly, the connected maintenance can be activated. Maintenance actions, on the other hand, can improve the performance of production equipment, which ensure the quality of products. Therefore, SPC and maintenance have a mutualistic relationship and that forces organizations to establish integrated systems [2]. However, in the past for a long time, SPC and maintenance have been treated as two separate topics and studied separately in the literature.

This paper presents an economic-statistical design of the integrated model of maintenance and SPC, which takes the VSI scheme and MDSS policy into account. We consider a production system consisting of a single equipment and producing a single-product. The production process begins from the statistically controlled state. An improved $\bar{X}$ control chart is used to monitor one critical-to-quality characteristic of the product and determine if the process is under control. When the process shifts to the out-of-control state, the control chart triggers the alert signal which implies that the deterioration of equipment may have occurred. Once the process is in the out-of-control condition, compared with the case of in-control condition, there will be an increase in equipment stability.
failure rate and production cost. For improving the performance of the production system, the reactive maintenance (RM) and corrective maintenance (CM) are planned to handle the true signal and the failure, respectively. Obviously, an efficient control-chart scheme for the detection of process shifts is pretty necessary for the integrated system. For this reason, a VSI & MDSS X control chart designing based on the VSI scheme and MDSS policy is developed and applied to construct an effective integration model.

The remainder of this article is organized as follows. Section II presents a brief review of relevant studies. Section III describes the basic assumptions and gives the model description. In Section IV, a Markov chain approach is employed to construct the suggested integrated process, while Section V presents the derivation of the expected cost function. In Section VI, an illustrative example and a comparative study are presented to justify the effectiveness of the developed model. Finally, Section VII summarizes the conclusions.

II. LITERATURE REVIEW

**NOTATIONS**

- $\mu_0$ mean value of the CTQ characteristic in state 0
- $\delta$ magnitude of the process mean shift
- $\lambda_i$ failure rate of the system in state $i$, $i = 0, 1$
- $R$ index that indicates relaxed sampling scheme
- $T$ index that indicates tightened sampling scheme
- $S$ index that indicates signal
- $k(w)$ control (warning) limit coefficient
- $F$ index that indicates facility failure
- $h_1(h_2)$ sampling interval when the relaxed (tightened) sampling scheme is used
- $m$ supplementary runs rules
- $C_k$ the cost of restoring the process to the in-control state
- $G(t), G_i(t)$ cdf of the time of quality shift, of failure when the process is in state $i$, respectively, $i = 0, 1$
- $g(t), g_i(t)$ pdf of the time of quality shift, of failure when the process is in state $i$, respectively, $i = 0, 1$
- $R(t)$ complement of $G(t)$
- $p_{00}(h_i)$ probability that the process remains in control within time interval $h_i$
- $p_{01}(h_i)$ probability that the process shifts to the out-of-control state within time interval $h_i$, yet surviving until the next sampling
- $p_{11}(h_i)$ probability that the process starts from the out-of-control state and always remains out-of-control within time interval $h_i$
- $p_{1F}(h_i)$ probability that the process suffers a failure within time interval $h_i$, given that the process is in state $j(i = 0, 1)$ at the beginning of the interval
- $\sigma_0$ standard deviation of the CTQ characteristic
- $\lambda$ rate of occurrence of the assignable cause
- $n$ sample size
- $a$ fixed cost per sample
- $b$ variable sampling cost per unit
- $T_i$ expected time to investigate a signal
- $T_r$ expected time to remove the assignable cause
- $T_{cm}$ expected time to repair a process failure
- $C_O$ cost per time unit of operating in the out-of-control state
- $C_S$ cost of investigating the process in case of alarm
- $C_{CM}$ the cost of repairing in case of failure
- $C_I$ cost per time unit of operating in the in-control state
- $C_{CPM}$ the compensatory cost due to a false signal
- $R_i(t)$ complement of $G_i(t)$, $i = 0, 1$
- $\pi_{ij}$ transition probabilities, $i, j = 0, 1$; $k, l = R, T^0, S$, where $q = 1, 2, \ldots , m - 1$
- $\eta_{0\rightarrow 0}(h_i)$ expected time of operating in control given that the process is in control at the beginning of time interval $h_i$
- $\eta_{0\rightarrow 1}(h_i)$ expected time of operating out of control given that the process is in control at the beginning of time interval $h_i$
- $\eta_{1\rightarrow 1}(h_i)$ expected time of operating out of control given that the process is out of control at the beginning of time interval $h_i$

This work is closely related to two research streams, i.e. integration study on quality control and maintenance and the application of MDSS policy on the design of control chart. Each research stream is briefly reviewed in the subsequent sections.

Quality control and maintenance are two very important elements of a manufacturing system. In today’s market place, effective integration of these two components can help an industry get a competitive edge [3]. In recent years, the two interconnected areas of research have been studied extensively due to the development of integrated model in which the theory of quality control and reliability engineering are applied simultaneously. Among these works, [4] may be the earliest scholars who began to remark the strong interrelationship between quality control and maintenance. In their work, two kinds of approaches for modelling the relation between quality control and maintenance have been proposed: one uses the concept of imperfect maintenance, the other applies the Taguchi approach to maintenance and quality. Usually, quality control is executed by a control chart for detecting shifts in the manufacturing process. The integration of SPC and maintenance is preliminary proposed by [5], [6] and [7], and the X control chart together with a preventive maintenance (PM) policy is tentatively introduced in their integrated model. References [8] and [9] further discussed the effect of maintenance on the economic design of X control chart. Reference [10] extended the work of [8] and
integrated Shewhart-residual ($Z_X - Z_e$) joint control chart and maintenance for two-stage dependent process. Reference [11] employed an $\bar{X}$ control chart to monitor the output of a two-identical unit series system and determine when to perform condition-based maintenance using economic and economic-statistical design methods. Reference [12] developed an integrated model of SPC and maintenance, and took the delay monitoring policy into consideration. More recently, [13] integrated $\chi^2$ control chart and maintenance planning for a two-stage dependent process. Reference [14] proposed a reliability-oriented integration model for joint PM and SPC with time-between-events (TBE) control chart. Reference [15] also established a condition-based maintenance model for service facility by integrating a TBE control chart and maintenance policy. Reference [16] investigated the effects of non-normally distributed data on the economic design of $\bar{X}$ control charts for production systems under imperfect maintenance. Reference [17] developed an integrated approach for maintenance and $\bar{X}$ control chart for batch manufacturing systems.

It is worth noting that, as the key tool in SPC, control chart is indispensable in the integration model. Thus, an efficient control-chart scheme for the detection of process shifts is pretty necessary for the integrated system. Although traditional control chart (also known as Shewhart chart) has the advantage of simple implementation and detecting large shifts quickly, it is insensitive in detecting small process shifts [18]. In order to overcome this shortcoming, different kinds of control charts, such as the exponentially weighted moving average chart, cumulative sum chart, synthetic chart etc, have been proposed. Beyond that, researchers also come up with the concept of adaptive control charts in which the control chart design parameters (sampling interval, sample size and control limits) can be varied depending on the current state of the process. Control charts with adaptive schemes greatly improve the detection power in detecting small process shifts as compared with the corresponding static control charts. Considering how to build the integrated model of SPC and maintenance based on adaptive schemes attracts more and more attention from researchers. There are far too few studies of the integration of adaptive control chart and maintenance. Reference [19] recently considered an integrated model in which a variable-parameter (VP) control chart and maintenance policy are coordinated to monitor the process mean. Reference [20] also presented an integration model of VP Shewhart control chart and maintenance based on economic-statistical design approach. More recently, [21] presented an integrated model of maintenance and quality control with an adaptive synthetic $\bar{X}$ control chart. Inspired by the work of [19] and [11], [22] integrated the economic-statistical design of a VP-Shewhart control chart with condition-based maintenance for two-unit series systems. Research in this field still needs to be strengthened.

MDSS, proposed by [23], is one of the extensively used sampling methods in the field of acceptance sampling schemes. Reference [24] concluded that, with regard to the average sample number needed, the application of MDSS in acceptance sampling schemes is superior to the single sampling scheme. In view of its good performance, some researchers attempt to utilize the concept of MDSS in the design of control charts. Recently, [25] proposed a new $S^2$ control chart by integrating the MDSS scheme and repetitive group sampling scheme in the design of traditional $S^2$ chart. Reference [26] introduced a new chart based on MDSS to monitor processes with exponentially distributed quality characteristics. Reference [27] proposed a double sampling and variable sampling interval (DSVSI) np chart by integrating the DSVSI and MDSS policy in the design of traditional np chart. All the above-mentioned papers indicate that the MDSS policy is helpful in improving the statistical/economic performance of the initial control chart. However, to our best knowledge, there is still no research on the integration of maintenance and the MDSS-based control chart.

Table 1 summarizes the aforementioned contributions indicating for each work: (1) The control chart used ($\bar{X}$ chart, other charts) (2) Parameter type of control chart (fixed, adaptive) (3) The maintenance type: corrective maintenance, preventive maintenance (4) Type of design control chart.
(economic, statistical, economic-statistical) (5) Whether the MDSS policy is used (yes, no).

This research presents an integration model of an adaptive $\bar{X}$ control chart and maintenance based on economic-statistical design approach. Therefore, the main contributions of this paper are as follows:

1) The proposed model designs an adaptive $\bar{X}$ control chart based on VSI and MDSS policy to accelerate shift detection in comparison with fixed parameter $\bar{X}$ control chart.

2) The proposed model integrates the proposed control chart and maintenance. In contrast to the most of the existing models that have been focused on the economic design of the integrated model, the proposed model gives the economic-statistical design of the integration model.

3) The proposed method aims to minimize the expected cost of production process subject to statistical constraints.

III. MODEL DESCRIPTION

In this paper, a production system consisting of one-component is considered and assumed to start from the statistically controlled state. At run-time, the system process may shift from an in-control (denoted as state 0) to an out-of-control state (denoted as state 1) due to the occurrence of an assignable cause. When the process goes out of control, the equipment degrades and that affects the quality of produced items. Assume also that the equipment failure can occur either in state 0 or state 1, but the failure rate increases when the process is in state 1. Whenever a failure occurs, the system stops running immediately, that is to say, the failure is self-announced, and then a CM action will be executed to bring the equipment back to state 0.

The out-of-control state is undesirable but usually cannot be observed directly. In order to detect the out-of-control process in time, a critical-to-quality characteristic (denoted by $\bar{X}$) of produced items is monitored. Without loss of generality, we assume that when the process is in an in-control condition, the critical-to-quality characteristic is normal with mean $\mu_0$ and variance $\sigma^2_0$, i.e., $\bar{X} \sim N(\mu_0, \sigma^2_0)$. In the literature, the vast majority of works are designed to monitor a single process parameter, such as the variance or the mean. In this work, without loss of generality, we only focus on the process mean monitoring. When a process shift occurs, the value of $\bar{X}$ changes from $\mu_0$ to $\mu_1 = \mu_0 + \delta\sigma_0$ ($\delta$ is the magnitude of the shift), but assuming the variance keeps unchanged. Assuming that only the occurrence of assignable cause can result in a process shift, and the time to process shift and the times to failure for states 0 and 1 follow the exponential distribution with densities $g(t)$, $g_0(t)$, $g_1(t)$ and parameters $\lambda$, $\lambda_0$, $\lambda_1 (\lambda_1 > \lambda_0)$, respectively. $G(t)$, $G_0(t)$ and $G_1(t)$ are the corresponding cumulative functions with complements $R(t)$, $R_0(t)$ and $R_1(t)$.

As mentioned, when the system operates in an out-of-control condition, the system suffers a higher failure rate and also poor-quality output which can lead to a higher operational cost. Therefore, a monitor scheme is necessary for the operational status monitoring of the process so that when a process shift occurs, it can be found in time. In this paper, a VSI & MDSS $\bar{X}$ control chart (see Section III(A)) is developed and employed to monitor the process. At run-time, if the chart sends an out-of-control signal, the process is stopped and an action is then executed immediately to find and eliminate the possible assignable cause. Specifically, if the signal is true, a RM action is implemented to bring the process back to the in-control state; otherwise, the process will be restarted to the in-control state after a compensatory maintenance (CPM) action. Assuming that the process always stops its operation during investigation and repair. Figure 1 gives the framework of the developed integrated system.

A. THE VSI & MDSS $\bar{X}$ CONTROL CHART

The $\bar{X}$ control chart with VSI scheme is comprised of two sets of limits, that is, the warning limits and the control limits, where the warning limits lie between the target mean value and the control limits [28]. The idea of the VSI $\bar{X}$ chart are: (a) When a sample point lies between the warning and control limits, there exists a higher chance that the next sample may fall outside the control limits, and that indicates the occurrence of a possible mean shift. Therefore, a short sampling interval before the next sample should be employed so as to have a quick detection of the process shift. (b) When a sample lies between the target mean value and warning limits, there exists a higher probability for the process to be under control. Thus, a long sampling interval before the next sample should be employed.

A unified VSI $\bar{X}$ control-chart model can be developed as follows. Assume that a finite number of interval lengths, denoted as $h_1, h_2, \ldots, h_N$, where $h_1 > h_2 > \cdots > h_N$ and $\theta \geq 2$, are used for designing the VSI $\bar{X}$ chart. A function of $\bar{X}_i$, i.e. $h(\bar{X}_i)$, is used to interpret the choice of a sampling interval. Let the interval between the lower and upper control limits be divided into $I_1, I_2, \ldots, I_N$ sub-intervals, such that

$$h(\bar{X}_i) = h_j \text{ if } \bar{X}_i \in I_j, \quad \text{for } j = 1, 2, \ldots, N.$$  

Then, the interval length used between samples $\bar{X}_i$ and $\bar{X}_{i+1}$ is $h(\bar{X}_i)$. 

![FIGURE 1. The framework of the developed integrated model.](image-url)
Assume without loss of generality that at the start of the process monitoring, the relaxed inspection is employed. Then the rules of using the two levels of inspection with the MDSS policy are:

1. If a sample produces a value within zone 1, no action takes place and the decision concerning the next sampling is still to use the relaxed inspection $h_1$.

2. If the current statistic lies within zone 2 and not all $m - 1 (m \in N^+)$ successive preceding statistics lie within zone 2, the process is again not interrupted but the tightened scheme $h_2$ is applied for the next sampling.

3. If the statistic lies within zone 3, or $m$ consecutive samples fall in zone 2, the chart issues an alarm, then an investigation takes place. If the signal is true, the RM is executed to remove the assignable cause; if the signal is a false alarm, the CPM action takes place.

It is noteworthy that [19] proposed an integrated model of VP $\bar{X}$ control chart and maintenance. Their SPC-maintenance integration scheme is remarkable and enlightening. But our work is different from theirs in that: (a) in our model, a VSI&MDSS $\bar{X}$ chart is employed to monitor the process mean. Although the VP $\bar{X}$ chart is more sensitive than the VSI $\bar{X}$ chart for detecting small shifts in the process mean, it is overcomplicated and idealistic and can be hardly used in practice. The VSI scheme is used most often due to its simplicity and good usability. Moreover, in our paper, the MDSS policy is adopted to further enhance the detection power of the VSI $\bar{X}$ chart; (b) In their model, the economic design method is used, but in our paper, the economic-statistical method is employed. Reference [30] concluded that, in many economic designs, the probability of Type I error of the control chart is markedly higher than it would be in a statistical design. With statistical design, economic factors are usually not considered explicitly. However, the determination of control chart parameters can greatly impact the costs linked to operation of the control chart. The economic-statistical design method not only ensures the effectiveness of economic designs, but also can simultaneously maintain the required statistical performance of the control charts.

IV. MODEL DEVELOPMENT

According to the description given above, at each sampling instance, the process state can be fully described by (a) the actual process state (state 0 or 1) and (b) the sampling level ($h_0$ or $h_1$) selected for next sampling based on the statistic produced by the present sample. For mathematical convenience, the relaxed sampling inspection and the tightened sampling inspection are marked as $R$ and $T$, respectively. Then if the statistic falls in zone 1, the $R$ inspection level for next sampling is used; if the statistic falls in zone 2, then the $T^i$ inspection level for next sampling is used (Note: $T^i = T$ where $i (= 1, \ldots, m - 1)$ only represents that there are $i$ consecutive samples fall in zone 2); the process is in $S$ state if a signal is issued. (c) Apart from the above $2m + 2$ states, the failure state $F$ is also included.

In this paper, a Markov chain with $2m + 3$ states is employed to describe the process of operation. The Markov chain can be developed with the transition probability matrix $P$, as shown at the bottom of the next page.

A. THE PROBABILITIES INDICATING THE PROCESS STATE CHANGE WITHIN A SAMPLING INTERVAL

For computing those transition probabilities of the matrix $P$, the following probabilities need to be calculated firstly:

1) The probability that the process remains in state 0 for a whole interval of duration $h_i$ ($i = l, s$), which is denoted by $p_{00}(h_i)$. This is the probability that within a sampling interval, neither a process shift nor an equipment failure occurs. Thus, $p_{00}(h_i)$ can be given as

$$p_{00}(h_i) = R(h_i)R_0(h_i)$$  \hspace{1cm} (6)

2) The probability that the process shifts from state 0 to state 1 within $h_i$ time units, yet surviving until the next sampling instance, which is denoted by $p_{01}(h_i)$; $p_{01}(h_i)$ represents the probability that the occurrence
of a process shift during the sampling interval \( h_i \) and no occurrence of failure until the end of that sampling interval. Thus, \( p_01(h_i) \) can be calculated as

\[
p_01(h_i) = \int_0^{h_i} g(t)R_0(t) \frac{R_1(h_i)}{R_1(t)} dt \quad (7)
\]

3) The probability that the process remains in state 1 for a whole interval of duration \( h_i \), which is denoted by \( p_{11}(h_i) \), can be calculated as following

\[
p_{11}(h_i) = R_1(h_i) \quad (8)
\]

4) The probability that the system suffers a failure in the \( h_i \) time interval, given that the process was in state 0 or 1 at the beginning of the interval, which is respectively denoted by \( p_{0F}(h_i) \) and \( p_{1F}(h_i) \). With regard to the calculation of \( p_{0F}(h_i) \), two scenarios should be considered: one is that the failure occurs while the process is in control; the other is that before the arrival of the failure, a process shift occurs first. Therefore, \( p_{0F}(h_i) \) should be calculated by the following form which contains the two scenarios mentioned above:

\[
p_{0F}(h_i) = \int_0^{h_i} g(t)R(t)dt + \int_0^{h_i} g(t)R_0(t) \int_t^{h_i} \frac{g_1(t')}{R_1(t)} dt' dt \quad (9)
\]

\( p_{1F}(h_i) \) is the probability of the case that a failure occurs within the sampling interval while the process has gotten out of control at the beginning of that interval, which can be calculated as

\[
p_{1F}(h_i) = \int_0^{h_i} g_1(t) dt \quad (10)
\]

**B. DERIVATION OF TRANSITION PROBABILITIES \( p_{ij}^{il} \)**

The \( p_{ij}^{il} \) probabilities \((i, j = 0, 1 \text{ and } k, l = R, T, S, \text{ where } q = 1, 2, \ldots, m - 1)\) in the transition probability matrix \( P \) are decided based on the value of the statistic \( \bar{X} \) as well as on the process state at each sampling instance.

Let \( P_{z1}^{ij}, P_{z2}^{ij} \) and \( P_{z3}^{ij} \) be the probability that the chart statistic falls in zone 1, zone 2 and zone 3, respectively, when the process is in control. Then

\[
P_{z1}^{0} = \Phi(w) - \Phi(-w), \quad P_{z2}^{0} = 2(\Phi(k) - \Phi(w)), \quad P_{z3}^{0} = 2\Phi(-k), \quad (11, 12, 13)
\]

where \( \Phi(\cdot) \) is the cumulative density function of the standard normal distribution.

In this sense, based on the discussion in Section IV(A), the probabilities of the upper left part of \( P \) can be computed by

\[
\begin{align*}
P_{RR}^{00} &= P_{RR}^{0} = P_{00}^{0} = P_{00}^{0}, \\
P_{RT}^{01} &= P_{RT}^{0} = P_{01}^{0}, \\
P_{RS}^{0S} &= P_{RS}^{0} = P_{0S}^{0}, \quad (0, R) \\
P_{TS}^{01} &= P_{TS}^{0} = P_{10}^{0}, \quad (0, T) \\
P_{SS}^{0S} &= P_{SS}^{0} = P_{S0}^{0}, \quad (0, S)
\end{align*}
\]

The transition probabilities of the middle left part of \( P \) are given by

\[
\begin{align*}
P_{RR}^{10} &= P_{RR}^{R} = P_{01}^{R} = P_{10}^{R}, \quad (1, R) \quad \quad & P_{RT}^{11} &= P_{RT}^{T} = P_{11}^{T}, \\
P_{RS}^{1S} &= P_{RS}^{R} = P_{1S}^{S}, \quad (1, R) \\
P_{TS}^{11} &= P_{TS}^{T} = P_{11}^{S} \quad \quad & P_{SS}^{1S} &= P_{SS}^{S}
\end{align*}
\]

The transition probabilities of the bottom right part of \( P \) are given by

\[
\begin{align*}
P_{RR}^{1R} &= P_{RR}^{R} = P_{01}^{R} = P_{10}^{R}, \\
P_{RT}^{1T} &= P_{RT}^{T} = P_{11}^{T}, \\
P_{RS}^{1S} &= P_{RS}^{R} = P_{1S}^{S}, \\
P_{SS}^{1S} &= P_{SS}^{S}
\end{align*}
\]
Let $P_{z1}^1$, $P_{z2}^1$, and $P_{z3}^1$ denote the probability that the chart statistic falls in zone 1, zone 2 and zone 3, respectively, when the process is in an out-of-control state. Then

$$p_{z1}^1 = \Phi(w - \delta \sqrt{n}) - \Phi(-w - \delta \sqrt{n}), \quad (14)$$

$$p_{z2}^1 = \Phi(k - \delta \sqrt{n}) - \Phi(-w - \delta \sqrt{n}) + \Phi(-w - \delta \sqrt{n}) - \Phi(w - \delta \sqrt{n}), \quad (15)$$

$$p_{z3}^1 = 1 - \Phi(k - \delta \sqrt{n}) + \Phi(-k - \delta \sqrt{n}). \quad (16)$$

Thus, the probabilities of the upper middle part of $P$ are computed by

$$p_{RR}^{01} = p_{SS}^{01} = P_{z1}^1 p_{01}(h_t), \quad (b)$$

$$p_{RT}^{01} = p_{ST}^{01} = P_{z2}^1 p_{01}(h_t), \quad (c)$$

$$p_{RS}^{01} = p_{SS}^{01} = P_{z3}^1 p_{01}(h_t), \quad (d)$$

$$p_{RI}^{01} = P_{z1}^1 p_{10}(h_t), \quad i = 1, 2, \ldots, m - 2,$$

$$p_{TI}^{01} = P_{z2}^1 p_{10}(h_t), \quad i = 1, 2, \ldots, m - 2,$$

$$p_{RI}^{01} = p_{II}^{01} = P_{z3}^1 p_{10}(h_t), \quad i = 1, 2, \ldots, m - 2.$$ (17)

The transition probabilities of the central part of $P$ are computed by

$$p_{RR}^{01} = p_{SS}^{01} = P_{z1}^1 p_{11}(h_t), \quad (a)$$

$$p_{RT}^{01} = p_{ST}^{01} = P_{z2}^1 p_{11}(h_t), \quad (b)$$

$$p_{RS}^{01} = p_{SS}^{01} = P_{z3}^1 p_{11}(h_t), \quad (c)$$

$$p_{RI}^{01} = P_{z1}^1 p_{10}(h_t), \quad i = 1, 2, \ldots, m - 2,$$

$$p_{RI}^{10} = P_{z2}^1 p_{10}(h_t), \quad i = 1, 2, \ldots, m - 2,$$

$$p_{II}^{10} = P_{z3}^1 p_{10}(h_t), \quad i = 1, 2, \ldots, m - 2.$$ (18)

$p_{SF}^{10}$, $p_{RS}^{10}$ and $p_{IF}^{10}$ are respectively the probabilities that a system failure occurs within a sampling interval while the process starts from the $(0, R)$, $(0, S)$ and $(1, S)$ state. Because the process will be reset as in-control and the relaxed sampling scheme will be employed when the system is in all these states, the values of all these probabilities are equal to $p_{0F}(h_t)$, i.e.,

$$p_{SF}^{0F} = p_{RS}^{0F} = p_{IF}^{0F} = p_{0F}(h_t)$$

while the probability of a failure given that the process starts from the $(0, T')$ $(i = 1, 2, \ldots, m - 1)$ state is

$$p_{TF}^{0F} = p_{0F}(h_t), \quad i = 1, 2, \ldots, m - 1.$$ (19)

That’s because the process will be restarted as in-control but the tightened sampling scheme will be employed in such a situation.

In a similar way, the probabilities of a system failure given that the process starts from the $(1, R)$ or $(1, T')$ $(i = 1, 2, \ldots, m - 1)$ state can be respectively calculated as

$$p_{RF}^{1F} = p_{1F}(h_t), \quad i = 1, 2, \ldots, m - 1.$$ (20)

Finally, after a failure the process is immediately repaired to an “as-good-as-new” state; in other words state $F$ reduces to state $(0, R)$. Thus,

$$p_{FR}^{F} = p_{00}^{00}, \quad p_{FT}^{F} = p_{11}^{01}, \quad p_{FS}^{F} = p_{10}^{10}, \quad p_{FF}^{F} = p_{0F}(h_t).$$

V. EXPECTED COST PER TIME UNIT

In this section, the expected quality and maintenance cost per unit of time of the integrated model is derived. To this end, firstly, the expressions of the average durations are derived, as well as the costs of the transition steps (when a planned inspection or a failure occurs, the present transition step terminates); secondly, the explanation of the derivation for the expected cost of each transition step is introduced; finally, the long-run expected cost per unit of time is provided.

A. DERIVATION OF THE EXPECTED OPERATIONAL TIMES

When the process starts its operation from an in-control state within a time interval, four following possible scenarios may occur:

(a) The process keeps its in-control state throughout the entire interval.

(b) A process shift occurs within the interval and no system failure happens till next sampling.

(c) A failure occurs with the process in the in-control state.

(d) A failure occurs with the process in the out-of-control state; that is a process shift precedes failure.

On the other hand, two possible scenarios may occur when the process starts its operation from an out-of-control state with a time interval:

(a) The process remains its out-of-control state throughout the entire interval.

(b) A failure occurs within the interval.

According to the discussion above, the expected time of in-control operation in an interval of planned duration $h_t(i = l, s)$, which starts in control, can then be given by

$$\eta_{0\to0}(h_t) = h_t R(h_t) R_0(h_t) + \int_0^{h_t} t g(t) R_0(t) \frac{R_1(h_t)}{R_1(t)} dt \quad + \int_0^{h_t} t g(t) R(t) dt \quad + \int_0^{h_t} t g(t) R(t) \frac{g_1(t)'}{R_1(t)} dt'.$$ (17)

Similarly, the expected time of out-of-control operation in an interval of planned duration $h_t$, which starts again in
TABLE 2. The selection of the next sampling scheme when the present process state is determined.

| The process state | The time interval | \( h_1 \) | \( h_s \) |
|-------------------|-------------------|------------|------------|
| \( 0, R \)       | \( (0, T_x) \)     | \( T_{IG} \) | \( T_{OC} \) | \( T_{IR} \) |
| \( (0, T_x) \)   | \( (0, T_x) \)     | \( \eta_{0 \to h_1} \) | \( \eta_{0 \to h_1} \) | \( \eta_{0 \to h_1} \) |
| \( 0, S \)       | \( 0, S \)         | \( 0 \)    | \( 0 \)    | \( 0 \)    |
| \( 1, S \)       | \( 1, S \)         | \( 0 \)    | \( 0 \)    | \( 0 \)    |
| \( 1, T_x \)     | \( 1, T_x \)       | \( 0 \)    | \( 0 \)    | \( 0 \)    |

**TABLE 3. Sojourn times of in-control state, out-of-control state and restoring of failure with a given initial process state.**

| State \( \Theta \) | \( T_{IG} \) | \( T_{OC} \) | \( T_{IR} \) |
|-------------------|-------------|-------------|-------------|
| \( (0, R) \)      | \( \eta_{0 \to h_1} \) | \( \eta_{0 \to h_1} \) | \( 0 \)      |
| \( 0, T_x \)     | \( 0 \)    | \( 0 \)    | \( 0 \)    |
| \( 0, S \)       | \( 0 \)    | \( 0 \)    | \( 0 \)    |
| \( 1, S \)       | \( 0 \)    | \( 0 \)    | \( 0 \)    |
| \( 1, T_x \)     | \( 0 \)    | \( 0 \)    | \( 0 \)    |
| \( F \)           | \( \eta_{0 \to h_1} \) | \( \eta_{0 \to h_1} \) | \( T_{CM} \) |

\( T_{IG} \) and \( T_{OC} \) respectively denote the expected in-control and out-of-control operating time when the process runs under state \( \Theta \). \( T_{IR} \) be the investigation and restoration time when an assignable cause or a failure occurs.

The average cost related with the process leaving from state \( (0, R) \) is: (a) the cost of sampling \( a + bn \) indicated that there is no occurrence of failure in present sampling interval (probability \( 1 - p_{0R}(h_i) \)); (b) the out-of-control operation cost \( C_O \), for an expected time \( \eta_{0 \to h_1} \); and (c) the in-control operation cost \( C_I \), for an expected time \( \eta_{0 \to h_1} \). Similarly, the average cost related with the process leaving from state \( (0, T_x) (i = 1, 2, \ldots, m - 1) \) will be \( (a + bn)(1 - p_{0R}(h_i)) \), \( C_O \eta_{0 \to h_1} \) and \( \eta_{0 \to h_1} \).

When the considered control chart sends a false alarm (i.e. type I error, denoted by state \( (0, S) \)), a cost of \( C_S \) and a complementary cost \( C_{CPM} \) are used to investigate and improve the process and the expected cost thereafter is equal to that of state \( (0, R) \).

When the process departs from state \( (1, R) \), a sampling cost of \( a + bn \) plus the out-of-control operation cost \( C_O \eta_{1 \to h_1} \) is incurred if there is no occurrence of failure within \( h_1 \) (probability \( 1 - p_{1R}(h_i) \)). In the same way, when the process departs from state \( (1, T_x) (i = 1, 2, \ldots, m - 1) \), a cost of \( (a + bn)(1 - p_{1R}(h_i)) + C_O \eta_{1 \to h_1} \) will be incurred.

Finally, we can find the expected average cost of two sampling schemes between states \( (1, S) \) and \( F \), a cost of \( C_S + C_R \) and \( C_{CM} \) will be respectively incurred, and the average cost for both cases thereafter will be equal to the average cost related with the process leaving from state \( (0, R) \). Denote \( \pi_{0R}, \pi_{0T}, \pi_{0S}, \pi_{1R}, \pi_{1T}, \pi_{1S} \) \( (i = 1, 2, \ldots, m - 1) \) and \( \pi_F \) as the steady-state probabilities of each of the \( 2m + 3 \) states of the Markov chain.

Then we have

\[
ET = \pi_{0R} \eta_{0 \to h_1} + \pi_{0T} \eta_{0 \to h_1} + \sum_{i=1}^{m-1} \pi_{0T} \eta_{0 \to h_1} + \pi_{1R} \eta_{1 \to h_1} + \sum_{i=1}^{m-1} \pi_{1T} \eta_{1 \to h_1} + \pi_{1S} \eta_{1 \to h_1} + \pi_F \eta_{0 \to h_1} + \eta_{0 \to h_1}.
\]

Similarly, the average cost of a transition step can be given by

\[
EC = \pi_{0R} \eta_{0 \to h_1} + \pi_{0S} \eta_{0 \to h_1} + \pi_{0T} \eta_{0 \to h_1} + \pi_{1R} \eta_{0 \to h_1} + \pi_{1S} \eta_{0 \to h_1} + \pi_F \eta_{0 \to h_1} + \eta_{0 \to h_1}.
\]

The long-run average cost per unit of time \( ECT \) is then calculated by the ratio of \( EC \) over \( ET \):

\[
ECT = \frac{EC}{ET}.
\]

With regard to the production costs described above, the objective function and constraints associated with the developed model can be explained as follows:

Objective function: \( \min ECT \)

Constraints: \( ATS_0 = \tau \)

\[
\begin{align*}
\eta_{\max} & \geq h_1 \geq h_1 \geq \eta_{\min} \\
n_{\max} & \geq n \geq 1, n \in \mathbb{Z}^+ \\\nk & > w
\end{align*}
\]

Design variable: \( k, w, h_1, h_s, n, m. \) (Obj.F.)
where $\tau$ is the allowed minimum $ARL_0$ to signal, which means the probability of Type-I error must be equal to a predetermined value. $h_{\text{min}}$ and $h_{\text{max}}$ represent the maximum and minimum sampling intervals, respectively. $n_{\text{max}}$ is the maximum sampling size.

**B. A GENETIC ALGORITHM FOR SOLVING THE OBJECTIVE FUNCTION**

The following three characteristics of the integrated model determine the fact that it is almost impossible to obtain the exact optimal solution of the Obj.F. by using the traditional exact methods: (a) Both continuous and discrete decision variables are involved in the developed model; (b) The solution space has the characteristic of discontinuity and non-convexity. (c) Some decision variables lie in the integration limits. Meta-heuristic methods, especially genetic algorithms (GAs), are commonly employed to solve these non-linear programming model due to its effectiveness and adaptiveness. Many studies can be found in the literature that used GA to solve models similar to our model. For instance, [31] used GA to optimize the parameters of their proposed $G_\alpha$ control chart. Reference [32] considered the economic-statistical design of VSI $\bar{X}$ chart, and a Multi-Objective GA is applied to solve the model. In this work, a GA is developed for solving the optimal designs of the suggested integrated model.

When a simple GA works, firstly an initial population of strings is randomly generated, which is known as gene pool and then using three operators to create new, and expectantly, better populations as successive generations. Reproduction is the first operator. During its operation, based on the objective function, strings are probably copied to the next generation. The other two operators, that is, crossover operator and mutation operator, are essential and most crucial for the using of GA. For crossover operation, randomly pairs of strings are selected firstly and then mated, creating new strings finally. Mutation refers to the occasional random alteration of the value at a string position. In our developed GA, the GA parameters are selected by trial and error. In detail, the number of population and the rates of crossover and mutation are set up to 500, 0.8, and 0.2, respectively. As we know, GA is a meta-heuristic algorithm, it cannot give the accurate optimal solution. To address this drawback, for each instance, the GA are run three times and only the best obtained value is marked as the optimal solution.

**VI. NUMERICAL INVESTIGATION**

**A. AN ILLUSTRATIVE EXAMPLE**

In this section, a step-by-step illustrative example is provided to illustrate the application of the proposed model.

The case is about a shrimp canning manufacturer, where the fillweight, $X$, is determined as the critical-to-quality characteristic of the product. Based on a set of historical data obtained on the fillweights from the shrimp canning factory, the following information can be estimated: 1) The target mean and standard deviation of $X$ when the process is in the healthy (in-control) state are usually set to be $\mu_0 = 6.6$ and $\sigma_0 = 0.5$, respectively. 2) The average sojourn time in the in-control state for the production process is 100 hours. In other words, the rate of occurrence of process shifts raised by the occurrence of assignable causes is 0.01. The average time intervals from the in-control and out-of-control state to failure for the system are 100 hours and 20 hours, respectively. 3) The cost per unit time of operating in the in-control state and out-of-control state are $30$ and $100$, respectively. 4) The cost of investigating an alarm is $50$. The cost of executing CM action is $1000$, while if the process is classified as out of control, the cost of executing RM action will be $200$. The cost of executing CPM action due to the false alarm is $100$. 5) The variable cost and fixed cost of sampling are $0.5$ and $2$, respectively.

| Parameter | Value |
|-----------|-------|
| $T_f$     | 0.1   |
| $T_c$     | 1     |
| $\mu_0$  | 6.6   |
| $\sigma_0$ | 0.5   |
| $C_{GM}$  | 1000  |
| $C_R$     | 200   |
| $C_S$     | 50    |
| $C_{GM,M}$| 100   |
| $\delta$  | 0.01  |
| $\lambda$ | 0.01 |
| $\lambda_1$ | 0.05 |
| $C_T$     | 30    |
| $C_O$     | 100   |

According to the specified variable values and constraints above-mentioned, the model applied to the illustrative example can be written as

Objective function: $\min ECT$

Constraints: $ATS_0 = 370.4$

\begin{align*}
5 &\geq h_l > h_s \geq 0.01 \\
20 &\geq n \geq 1, n \in \mathbb{Z}^+ \\
k &> w > 0
\end{align*}

By using the developed GA, the optimal design variables of the example are obtained through minimizing the objective function (Obj.F.) in Equation (6.1), which are listed as follows:

$\{k^*, w^*, h_l^*, h_s^*, n^*, m^*\} = \{3.43, 1.42, 3.7, 0.68, 14, 9\}$

$ETC^* = $543.76.

These results indicate that, during production, the control limit and the warning limit should be set at 3.43$\bar{x}$ and 1.42$\sigma_{\bar{x}}$, respectively. When the tightened sampling plan is activated, authorised workers should take a sample of size...
TABLE 5. The produced 27 instances with the Taguchi L_{27} design.

| Instance | δ | λ | λ₀ | λ₁ | a | b | C_O | C_I | C_S | C_{GM} | C_R | C_{GP} |
|----------|---|---|----|----|---|---|-----|-----|-----|--------|-----|--------|
| 1        | 0.5| 0.01| 0.01| 0.05| 0.5| 0.1| 50  | 30  | 50  | 500    | 100 | 100    |
| 2        | 0.5| 0.01| 0.01| 0.05| 2.0| 0.5| 100 | 60  | 100 | 1000   | 200 | 200    |
| 3        | 0.5| 0.01| 0.01| 0.05| 4.0| 1.0| 200 | 100 | 200 | 2000   | 400 | 500    |
| 4        | 0.5| 0.03| 0.03| 0.06| 0.5| 0.1| 50  | 60  | 100 | 1000   | 200 | 400    |
| 5        | 0.5| 0.03| 0.03| 0.06| 2.0| 0.5| 100 | 100 | 200 | 2000   | 100 | 100    |
| 6        | 0.5| 0.03| 0.03| 0.06| 4.0| 1.0| 200 | 50  | 50  | 500    | 200 | 200    |
| 7        | 0.5| 0.05| 0.05| 0.07| 0.5| 0.1| 50  | 100 | 200 | 2000   | 200 | 200    |
| 8        | 0.5| 0.05| 0.05| 0.07| 2.0| 0.5| 100 | 30  | 50  | 500    | 400 | 500    |
| 9        | 0.5| 0.05| 0.05| 0.07| 4.0| 1.0| 200 | 60  | 100 | 1000   | 100 | 100    |
| 10       | 1.0| 0.01| 0.03| 0.07| 0.5| 0.5| 200 | 30  | 100 | 2000   | 200 | 200    |
| 11       | 1.0| 0.01| 0.03| 0.07| 2.0| 1.0| 50  | 60  | 100 | 1000   | 500 | 500    |
| 12       | 1.0| 0.01| 0.03| 0.07| 4.0| 1.0| 100 | 100 | 50  | 1000   | 400 | 100    |
| 13       | 1.0| 0.03| 0.05| 0.05| 0.5| 0.5| 200 | 60  | 200 | 500    | 400 | 100    |
| 14       | 1.0| 0.03| 0.05| 0.05| 2.0| 1.0| 50  | 100 | 50  | 1000   | 100 | 200    |
| 15       | 1.0| 0.03| 0.05| 0.05| 4.0| 1.0| 100 | 30  | 100 | 2000   | 500 | 200    |
| 16       | 1.0| 0.05| 0.01| 0.06| 0.5| 0.5| 200 | 100 | 50  | 1000   | 500 | 200    |
| 17       | 1.0| 0.05| 0.01| 0.06| 2.0| 1.0| 50  | 30  | 100 | 2000   | 100 | 100    |
| 18       | 1.0| 0.05| 0.01| 0.06| 4.0| 1.0| 100 | 100 | 50  | 1000   | 500 | 500    |
| 19       | 2.0| 0.01| 0.05| 0.06| 0.5| 1.0| 100 | 30  | 200 | 1000   | 500 | 100    |
| 20       | 2.0| 0.01| 0.05| 0.06| 2.0| 0.1| 200 | 60  | 50  | 2000   | 500 | 500    |
| 21       | 2.0| 0.01| 0.05| 0.06| 4.0| 0.5| 50  | 100 | 100 | 500    | 400 | 100    |
| 22       | 2.0| 0.03| 0.01| 0.07| 0.5| 1.0| 100 | 60  | 50  | 2000   | 400 | 100    |
| 23       | 2.0| 0.03| 0.01| 0.07| 2.0| 0.1| 200 | 100 | 100 | 1000   | 500 | 100    |
| 24       | 2.0| 0.03| 0.01| 0.07| 4.0| 0.5| 50  | 30  | 200 | 1000   | 200 | 200    |
| 25       | 2.0| 0.05| 0.03| 0.05| 0.5| 1.0| 100 | 100 | 100 | 500    | 200 | 200    |
| 26       | 2.0| 0.05| 0.03| 0.05| 2.0| 1.0| 200 | 30  | 200 | 1000   | 400 | 200    |
| 27       | 2.0| 0.05| 0.03| 0.05| 4.0| 0.5| 50  | 60  | 50  | 2000   | 100 | 500    |

TABLE 6. Optimal ETC values for our model and the comparison models A, B and C.

| Instance | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| ETC of our model | 234.12 | 316.47 | 463.12 | 517.23 | 652.47 | 630.54 | 924.51 | 783.64 | 684.78 |
| ETC of model A  | 238.07 | 323.21 | 459.37 | 520.35 | 653.14 | 635.13 | 931.46 | 790.30 | 691.18 |
| ETC of model B  | 243.10 | 349.52 | 486.46 | 548.63 | 640.20 | 640.75 | 987.54 | 813.56 | 704.30 |
| ETC of model C  | 249.15 | 368.04 | 503.12 | 554.18 | 675.32 | 642.84 | 1021.25 | 819.68 | 705.48 |
| Cost savings (%) | 6.03 | 7.95 | 6.67 | 3.83 | 2.28 | 1.97 | 0.40 | 2.93 | 3.71 |

Due to the limitation of the design of experiments approach and for mathematical tractability, 12 main input parameters are considered and then 27 instances are generated, demonstrated in Table 5. It is worthy note that the other input parameters take the same values used in Table 4. The optimal ECT values of model A, B, C, and our model are obtained by using the proposed GA, which are given in Table 6.

Define that

\[
\text{Cost savings (\%)} = \frac{\text{max}(\text{ECT values of model A, B and C}) - \text{ECT of our model}}{\text{max}(\text{ECT values of model A, B and C})}
\]

(21)

According to Table 6, from a point of view of cost saving, the results obtained from all 27 instances come to the expected optimal total cost per unit time.
conclusion that the proposed model is superior to the other three models. In more details, according to those cost savings in Table 6, it can be found that the optimal ETC in these 27 instances has been improved between 2.28 and 14.01 percent with the average of 5.86 percent by using the developed model. 5.86 percent cost saving can be viewed as a significant value for the producer due to the fact that the production costs are usually very high in practice. Moreover, it can be found that the model A generally achieves less ECT than the model C, showing that the VSI policy is helpful in improving the economic performance of the $\bar{X}$ chart. Obviously, Table 6 also provides the conclusion that the MDSS policy is useful to improve the economic performance of the $\bar{X}$ chart.

Finally, the main effects of the main input parameters above-mentioned in Table 7 on cost savings are discussed. To this end, the values of the cost of saving are computed and stored in Table 7. In Table 7, we define that, for a specified parameter,

$$D.V. = \max(\text{level 1}, \text{level 2}, \text{level 3}) - \min(\text{level 1}, \text{level 2}, \text{level 3}).$$

According to Table 7, we find that the failure rate of the system in state 0, $\lambda_0$, has the most significant effect on the cost savings. Furthermore, we also graphically illustrate the main effects of the 12 input parameters on the cost savings in Figure 3. Similarly, as illustrated in Figure 3, the values of $C_O$ and $C_I$ have minimal effects on cost of saving.

VII. CONCLUSION

The aim of this paper is to develop a new model which integrates statistical process control (SPC) and maintenance using both adaptive control-chart scheme and multiple dependent/deferred state sampling (MDSS) policy. In this paper, not only the process shift but also the equipment failures in two process states (healthy state and unhealthy state) are considered as well. A cost model is developed for the simultaneous optimization of SPC and MDSS policy with the purpose of minimizing the total expected cost per unit time of the system. A specified GA is developed to optimize the decision variables and obtain the minimal expected total cost. The numerical investigation concludes that: 1) the proposed integration model is sensitive to small process shift due to the application of MDSS policy in which the historical sampling information cooperative to the diagnosis of process is contained. 2) the proposed integration model is more cost effective than the comparative model in which the corresponding FSI chart without adopting the MDSS policy is employed.
This study can be extended in the following directions: (1) considering the problem of simultaneously monitoring the variance and mean of a production process, (2) considering the imperfect maintenance and assuming the system is degenerative after maintenance. (3) considering multiple quality characteristics and multiple assignable causes to make the model more applicable to real-world production environments. We leave these potential directions for future research.

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