Dilepton Production in High Luminosity Multi-GeV Electron Scattering

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Abstract

We consider the production of a 300GeV dilepton in very intense 4GeV electron scattering off of a lead target. The production cross-section and angular distribution of the resulting muons are calculated. There occur several such events per year, and their detection is rendered feasible by measurement of $e^+\mu^-\bar{\mu}^-$ angular correlations.
It is generally assumed that to produce a very massive particle with a mass of, say, 300 GeV the best means is to employ a collider with center of mass energy well above the particle’s mass and to search for its production as an on-shell state. While this is generally true, we find it worthwhile to examine whether virtual effects of the heavy particle can be detected in accelerators with much lower energy. This is motivated in part by the temporary absence of a 10 TeV hadron or 1 TeV lepton collider. The production cross-section will be suppressed at lower energy but this suppression can be compensated by higher luminosity. In the present paper we consider, as an example, the production of the dilepton (a doubly-charged gauge boson) predicted by the 331 model \[1\]. Our analysis is a generalization of one performed earlier \[2\] at high energies. First we shall evaluate the Feynman diagrams for the process \(e^- p \rightarrow e^+ p \mu^- \mu^-\) at general energies then we shall consider production in a multi-GeV highly-luminous electron beam scattering off a stationary nuclear target.

The basic process that we consider is \(e^- p \rightarrow e^+ p \mu^- \mu^-\). The individual family lepton number violation is the reason this signal is so clear; there is no Standard Model background. In order to compute \(e^- q \rightarrow e^+ q \mu^- \mu^-\), we must first compute \(e^- q \rightarrow e^+ q \mu^- \mu^-\). This cross-section is computed from the Feynman diagrams shown in Fig.(1), the same as were considered in reference \[4\]. This cross section is then folded into the proton cross section by using the EHLQ \[3\] proton structure functions. Using the Feynman rules in reference \[4\], the amplitudes for \(e^- q \rightarrow e^+ q \mu^- \mu^-\) are

\[
Amp(a) = \left( \frac{g_3 L}{\sqrt{2}} \right) e^2 Q_q \frac{(-1)}{(p_2 - p_1)^2} \frac{(-1)}{(p_5 + p_6)^2} \frac{1}{M^2 + iM \Gamma_Y} M(a),
\]

\[
Amp(b) = \left( \frac{g_3 L}{\sqrt{2}} \right) e^2 Q_q \frac{(-1)}{(p_2 - p_1)^2} \frac{(-1)}{(p_5 + p_6)^2} \frac{1}{(-p_1 + p_5 + p_6)^2} M(b),
\]

\[
Amp(c) = 2 \left( \frac{g_3 L}{\sqrt{2}} \right) e^2 Q_q \frac{(-1)}{(p_2 - p_4)^2} \frac{(-1)}{(p_1 - p_3)^2} \frac{1}{M^2 + iM \Gamma_Y} M(c),
\]
\[
Amp(d) = \left( \frac{g_3 \alpha}{\sqrt{2}} \right) e^2 \frac{Q_q}{(p_2 - p_4)^2 (p_1 - p_3)^2 - M_V^2 + iM_V \Gamma_V} \\
\times \frac{1}{(p_1 - p_3 - p_5)^2} M(d),
\]

(1d)

\[
Amp(e) = \left( \frac{g_3 \alpha}{\sqrt{2}} \right) e^2 \frac{Q_q}{(p_2 - p_4)^2 (p_1 - p_3)^2 - M_V^2 + iM_V \Gamma_V} \\
\times \frac{1}{(p_1 - p_3 - p_6)^2} M(e),
\]

(1e)

with \( M(a), M(b), M(c), M(d) \) and \( M(e) \) defined as

\[
M(a) = \bar{u}(p_4) \gamma_\alpha u(p_2) \bar{u}(p_6) \gamma_\mu \gamma_5 \bar{C} \bar{u}\gamma^T(p_5) \\
\times v^T(p_3) C \gamma^\alpha \gamma^\beta (p_3 + p_5 + p_6) \gamma^\gamma_\alpha u(p_1),
\]

(2a)

\[
M(b) = \bar{u}(p_4) \gamma_\alpha u(p_2) \bar{u}(p_6) \gamma_\mu \gamma_5 \bar{C} \bar{u}\gamma^T(p_5) \\
\times v^T(p_3) C \gamma^\alpha \gamma^\beta (-p_1 + p_5 + p_6) \gamma^\gamma_\alpha u(p_1),
\]

(2b)

\[
M(c) = \bar{u}(p_4) \gamma_\alpha u(p_2) \bar{u}(p_6) \gamma_\mu \gamma_5 \bar{C} \bar{u}\gamma^T(p_5) v^T(p_3) C \gamma^\beta \gamma_5 u(p_1) \\
\times [(p_2 - p_4 + p_5 + p_6) \gamma g_{\mu \alpha} + (p_3 - p_5 - p_6 - p_1) \gamma g_{\mu \beta} \\
+ (p_1 + p_4 - p_3 - p_2) \gamma g_{\alpha \beta}],
\]

(2c)

\[
M(d) = \bar{u}(p_4) \gamma_\alpha u(p_2) \bar{u}(p_6) \gamma^\alpha \gamma^\beta (p_1 - p_3 - p_5) \gamma^\gamma_\alpha \bar{C} \bar{u}\gamma^T(p_5) \\
\times v^T(p_3) C \gamma_\mu \gamma_5 u(p_1),
\]

(2d)

\[
M(e) = \bar{u}(p_4) \gamma_\alpha u(p_2) \bar{u}(p_6) \gamma_\mu \gamma_5 \gamma^\beta (p_1 - p_3 - p_6) \gamma^\gamma_\alpha \bar{C} \bar{u}\gamma^T(p_5) \\
\times v^T(p_3) C \gamma_\mu \gamma_5 u(p_1).
\]

(2e)

These amplitudes are most easily evaluated by using the method of helicity amplitudes [5], since it is a good approximation that all of the relevant fundamental particles are nearly massless even at \( \sqrt{s} = 2.9 \text{ GeV} \). The helicity amplitudes are explicitly shown in reference
The $e^- p \to e^+ p \mu^- \mu^-$ is computed with the EHLQ structure functions (set 1), $F_q(x, Q^2)$.

The relevant cross section is

$$
\sigma(s, M_Y) = \int_0^1 dx \sum_q F_q(x, Q^2) \hat{\sigma}(\hat{s} = xs, M_Y).
$$

Now we consider several relevant values for $\sqrt{s}$ and $M_Y$. We first consider scattering off an individual proton of an electron with beam energy on a stationary target ranging from 2 GeV up to $10^7$ GeV; this covers everything from CEBAF (Continuous Electron Beam Accelerator Facility) at the low end through HERA to LEPII-LHC at the high end. The result is depicted in Fig.(2) for a dilepton mass of 300 GeV. We see that for the cross-section increases by some nine orders of magnitude between CEBAF center-of-mass energies and LEPII-LHC center-of-mass energies.

A comparison of the number of events expected per year at different colliders is as follows. At $\sqrt{s} = 1790$ GeV (LEPII-LHC) equivalent to $E_e = 1.7 \times 10^6$ GeV, the cross-section for a proton cut-off $p \leq 100$ MeV is 0.015pb. The projected luminosity is $2 \times 10^{32} cm^{-2} s^{-1} = 6,000 pb^{-1} y^{-1}$. This translates into 90 events/year. At $\sqrt{s} = 314$ GeV (HERA) equivalent to $E_e = 5.2 \times 10^4$ GeV, the cross-section is $8 \times 10^{-6} pb$. The luminosity is $1.6 \times 10^{31} cm^{-2} s^{-1} = 500 pb^{-1} y^{-1}$. This translates into less than $10^{-3}$ events/year. At HERA we confirm the conclusion of [3] that the event-rate is too small to have a realistic chance of discovering the dilepton. This is because the center-of-mass energy was unfortunately chosen too low.

At $\sqrt{s} = 2.9$ GeV (CEBAF) equivalent to $E_e = 4$ GeV, the cross-section is $9 \times 10^{-11} pb$. At CEBAF the relevant process is not $e^- p \to e^+ p \mu^- \mu^-$, but $e^- (Pb) \to e^+ (Pb) \mu^- \mu^-$. We relate these two processes by conservatively assuming an incoherent superposition of protons in the lead nucleus. The contribution of neutrons to the cross section only increases the estimate of the total dilepton production; accurate estimates of this correction and the effects of coherent enhancement in the nucleus are not justified by the accuracy we employ in this paper but may become so if unexpected events are detected. With these approximations the projected $e^- p$ luminosity for a lead (Pb) target is $3.4 \times 10^{39} cm^{-2} s^{-1} = 1.1 \times 10^{11} pb^{-1} y^{-1}$. This translates into between 1 and 100 events/year depending on the value of $M_Y$. 

[4]
Of the planned machines, the LEPII-LHC (which is, presumably, at least a decade in the future) would likely be the first $e^-N$ device which would be comparable to CEBAF for dilepton discovery. Thus, for the remainder of this article we shall focus specifically on the latter case. We shall consider: (i) background estimation, (ii) angular dependence of the muon and positron decay products, (iii) the effects of increasing the beam energy from 4 GeV to 8 or 10 GeV, and (iv) the dependence of the total cross section on the dilepton mass $M_Y$.

(i) Background Estimates.

We are proposing to detect the process $e^-p \rightarrow e^+ \mu^- \mu^- X$. From QCD background, there will be a large number of muons arising from pion decay. A typical pion multiplicity at CEBAF energy (4 GeV $e^-$ on nucleons) is $\sim 2 \ [7]$ and the cross-section is a little above 10$\mu$b. With the design luminosity of $10^{39}/cm^2/s$ $[8]$ this implies on the order of $10^{17}$ pions per year produced, about two-thirds of which are charged according to isotopic spin invariance. Essentially all charged pions decay into muons.

Nevertheless, the process of interest, although it is at the level of only 1 to 100 events per year has several specific signatures, particularly the angular dependence of the produced muons and positron. With a triple coincidence of $e^+\mu^-\mu^-$ there is a realistic chance of detecting the dilepton signal. The key to the identification lies in the angular dependence of the products to which we now turn.

(ii) Angular Dependence.

Let the polar angles of the positron be $(\theta, \phi)$ and those for the two muons be $(\alpha_1, \phi_1)$ and $(\alpha_2, \phi_2)$. One azimuthal angle is provided by the definition of a plane containing the initial beam direction, hence there are five independent angles describing the correlations of $e^+\mu^-\mu^-$. Let us first focus on $\theta$. The positron in $e^-p \rightarrow e^+\mu^-\mu^-X$ is preferentially produced forward in the center-of-mass frame as shown in Fig.(3). So the first of our three coincidence triggers is on such a forward $e^+$ particle.

The relative angle between the two muons $\beta$ depends on the polar angles according to:
Each $\mu^-$ is produced approximately as $1 + \cos^2(\alpha_i)$ in the center-of-mass frame, as indicated in Fig.(4), and as intuitively expected by the slowness of the spin 1 dilepton in the center-of-mass frame at this low a beam energy. Finally, we need to see two $\mu^-$ and the dependence on $\beta$ is dependent on the relative azimuth of the muons. We find that the muons are preferentially emitted with $\beta < \pi/2$. This leads us to look at the relative angle between the positron and an emitted muon; this dependence is shown in Fig.(5). Fig.(5) shows the probability that a muon is produced at the corresponding $\cos(\alpha_i)$ with the relative azimuth between the positron and the muon of pi (solid line) or a relative azimuth of zero (dashed line), assuming that the positron was detected with $0.6 < \cos(\theta) < 0.8$ (Its most probable values).

The physical interpretation of a typical event is as follows: the virtual dilepton is not quite at rest in the center-of-mass frame. The dilepton transverse momentum is approximately opposite to the positron transverse momentum because the proton final momentum tends to be nearly parallel to its initial momentum. When the highly virtual dilepton decays into the two muons they carry off its momentum. The dilepton momentum is nonrelativistic with respect to its mass, $p^2 \ll M_Y^2$, but this momentum is relativistic for all other particles concerned. These calculations were all performed with $M_Y = 300$ GeV.

To confirm the consistancy of Figs.(3)-(5) we have also computed the energy distributions of the final state particles in the $e^-q$ center-of-mass frame. If we let $\hat{s}$ be the squared $e^-q$ center-of-mass energy we find the most likely values the final state positron and quark energies are $E_{e^+} \sim 0.45\sqrt{\hat{s}}$ and $E_q \sim 0.3\sqrt{\hat{s}}$. As can be seen from Fig. (3) the most likely angle for the $e^+$ is $\cos(\theta) \sim 0.7$. A crude estimate of $\hat{s}$ is obtainable from the center-of-mass electron and proton four-momenta as $\hat{s} = (p_{e^-} + xp_p)^2$ where $x$ is the fraction of proton momentum carried by the quark. If we assume the positron and the quark are emitted near their most likely angle and near their most likely energy, independent of $x$ this gives the $(\mu^-\mu^-)$ 3-momentum at approximately 90° in the $e^-q$ CM frame. This simple picture can
help clarify the complex angular correlations presented in Figs. (3)-(5).

This unusual decay of two like-sign muons in the same hemisphere with transverse momentum approximately balancing that of a positron in the other hemisphere is unique. The realistic hope is then that this triple coincidence of $e^+\mu^-\mu^-$ will be sufficiently enhancing (even to the extent of $10^{15} \sim 10^{17}$) to compete successfully with QCD background.

(iii) Effects of Increasing Beam Energy.

If the beam energy at CEBAF were increased to 8 GeV ($\sqrt{s} = 4.0$ GeV) or 10 GeV ($\sqrt{s} = 4.4$ GeV) the cross-section increases to $2 \times 10^{-10}$pb or $3 \times 10^{-10}$pb, increasing the number of events per year from 10 to 22 or 33 events/year respectively, with uncertainties of an order of magnitude. At these energies the $e^-p \rightarrow e^+p\tau^-\tau^-$ cross section becomes kinematically possible and hence provides another detectable family lepton number violating process.

(iv) Dilepton Mass Dependence.

For any $\sqrt{s} \ll M_Y$ the total cross section goes as $M_Y^{-4}$. This is easily seen as $\text{Amp}(c)$ in equation (1) is suppressed because it is approximately proportional to $M_Y^{-4}$ (for $\sqrt{s} \ll M_Y$), due to the second dilepton propagator, while all of the other amplitudes are approximately proportional to $M_Y^{-2}$, and hence $\text{Amp}(c)$ is suppressed by $M_Y^{-2}$ relative to the other amplitudes. The total cross section goes as $M_Y^{-4}$. If 2 events per year are required for detection then dileptons of up to mass 450 GeV can be detected at current CEBAF energies.

The general conclusions of this paper are twofold: (i) to describe an exotic event due to physics beyond the standard model, and more importantly (ii) these events can occur at a significant rate in low energy high luminosity $e^-N$ colliders and there exists the possibility of detecting new physics at e.g. CEBAF. Although we have, as an example, focused on a 300-450 GeV dilepton as predicted in reference [1], our general conclusion (ii) has a wider applicability which may merit further study.

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REFERENCES

[1] P. H. Frampton, Phys. Rev. Lett. 69, 2889 (1992)

[2] J. Agrawal, P. H. Frampton and D. Ng, Nucl. Phys. B386, 267 (1992)

[3] E. Eichten et al., Rev. Mod. Phys. 56, 579 (1984)

[4] P. H. Frampton and D. Ng, Phys. Rev. D45, 4240 (1992)

[5] R. Kleiss and W. J. Stirling, Nucl. Phys. B262, 235 (1985)

[6] D. B. Day et al., Phys. Rev. C48, 1849 (1993)
    A. S. Rinat and M. F. Taragin, Nucl. Phys. A571, 733 (1994).

[7] D. J. Dean et al., Phys. Rev. C46, 2066 (1992)

[8] CEBAF Conceptual Design Report, Basic Experimental Equipment, April 13, 1990
Figure Captions

**Figure 1.**
Feynman diagrams for $e^- q \rightarrow e^+ \mu^- \mu^- q$

**Figure 2.**
Cross-section for $e^- p \rightarrow e^+ \mu^- \mu^- p$ for beam energies ranging from 2GeV to $10^7$ GeV. The solid line is for transverse momentum $p_T \geq 100$ MeV. The dashed line is for $p_T \geq 5$ GeV.

**Figure 3.**
Angular distribution of $e^+$ in center-of-mass frame.

**Figure 4.**
Angular distribution of $\mu^-$ in center-of-mass frame (solid line) and $1 + cos^2(\alpha)$ normalized to the real differential cross section at $cos(\alpha) = 0$ (dashed line).

**Figure 5.**
Angular correlation between the positron and a $\mu^-$ with a relative azimuth of $\Delta \phi = \pi$ (solid line) and $\Delta \phi = 0$ (dashed line); the positron is assumed detected with $.6 < cos(\theta) < .8$. 
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