The effect of geographical distance on epidemic spreading

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Abstract

In this paper, we introduced the geographical effect into susceptible-infected-susceptible (SIS) model on a growing network proposed by Xie et al. [Phys. Rev. E 75 (2007) 036106] to investigate the effect of geographical distance on epidemic spreading. In our SIS model, the susceptible individual $i$ can be infected by infectious neighbors with probability $\theta_i$: $\theta_i = 1 - \prod_{j \in N(i)} (1 - \lambda_j)$, where $\lambda_j = \min(1, \lambda_0 \times (d_{ij}/\min_{k \in N(i)}(d_{ik}))^{-\beta})$, where $\lambda_0$ is the average transmission rate of the virus, $j$ is one of the infected neighbors ($N(i)$) of susceptible individual $i$, $\lambda_j$ is transmission rate of individual $j$, $d_{ij}$ is Euclidean distance between individual $i$ and $j$, and $\beta$ is the tunable parameter. Simulation results show that the steady density of infected individuals monotonously decreases with the increment of $\beta$ and the epidemic threshold emerges in the scale-free networks when the effect of geographical distance is taken into account. Moreover, when the network is star-like, the density of the infected individuals shows large amplitude oscillations.

Keywords: geographical effect, SIS, growing network, scale-free network, epidemic threshold

1. INTRODUCTION

There have been many investigations on the properties of complex network since the pioneering small-world network model [1] and scale-free network model [2] were proposed at the end of the last century, such as degree distribution [3, 4, 5], clustering coefficient [6, 7], average shortest path length between nodes [8, 9], and so on. Recent years, the dynamical processes on complex network draw even more attentions, such as synchronization [10, 11], information traffic [12, 13], cascading failures [14, 15], evolutionary games [16, 17, 18, 19] and especially the epidemic spreading [20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. As epidemic spreading is a meaningful job for world-wide infectious disease, there are a lot of works on it. Some pioneering works give

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several valuable results: there is no existence of any epidemic threshold in Barabási-Albert (BA) scale-free network with adequately large size [20, 21, 22, 23]; on the contrary, on the WS small-world model in the BA networks, Barthélémy et al. found that there has an infinite spreading velocity of epidemic process in the infinite population limit [28, 29]. Yan et al. [25] investigated the effects of weight measure on SI models and found that epidemic spreads slower on weighted scale-free networks than that on unweighted scale-free networks. Zhou et al. [26] studied the SI model with identical infectivity on scale-free networks and found that there exists a finite threshold for the spreading of epidemic no matter the size of networks.

Recently, it has been revealed that geographical measures [30, 31, 32, 33, 34, 35, 36] play an important role in many real-world networks, such as transportation networks [37], power grids [38] and the Internet [39, 40] etc. Consequently, several network models considering the geographical measures are proposed [41, 42, 43]. In Ref. [41], Xu et al. studied SIS model [44] on an evolutionary network which considered both the topological and geographical measures, and they found that when only the geographical measure is considered, the network is random-like and the epidemic threshold emerges; when only the topological measure is considered, the epidemic threshold also emerges except for some certain parameters which can induce scale-free topology. However, they only considered the process of epidemic on geographical networks, an important factor, which was omitted in their work, is that the ability of transmission is closely-related to the distance of the individuals. For instance, an infected person who lives in China can hardly infect his far away friends who live in the United States; oppositely, his family members can easily be infected. In Ref. [45], Wang et al. investigated the epidemic spreading and immunizations in geographically embedded scale-free (SF) and Watts-Strogatz (WS) networks and implemented two different immunization strategies: one is the connection neighbors (CN) strategy based on network distances, the other is spatial neighbors (SN) strategy based on geographical distances. They found that for both SF networks and WS networks, the SN strategy always performs better than the CN strategy. Therefore, we focus on the effect of geographical distance on transmission rate. It is found that the steady density of the infected individuals monotonously decreases with the increment of $\beta$ and the epidemic threshold emerges in the scale-free networks when the effect of geographical distance is taken into account. Moreover, when the network is star-like, the density of the infected individuals shows large amplitude oscillations.

The paper is organized as follows. In the next section, a growing network model is presented. SIS model with geographical constraints on this network is given in Section 3. And the paper is concluded by the last section.

2. NETWORK MODEL

The growing network model which considered both the topological and geographical measures is constructed as follows. Considering a $40 \times 40$ square in Euclidean space, initially, $m_0$ fully connected individuals are randomly embedded in the square and each of them has a two-dimension coordinate $(x, y)$. At each time step, a new individual $i$ with $m (m \leq m_0)$ edges is randomly located in $(x_i, y_i)$ of the square. Each previously existing individual has measure:

$$F_j(t) = \frac{k_i(t) k_j(t)}{d_{ij}^2},$$

(1)
where $k_i(t)(k_j(t))$ is the degree of individual $i(j)$ at time $t$, $\alpha$ is the tunable parameter, and $d_{ij}$ is the Euclidean distance between $i$ and $j$.

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, \quad (2)$$

We assume that the probability $\prod$ that individual $i$ will be connected to individual $j$ depends on the measure $F_j(t)$ of that individual, so that $\prod(F_j(t)) = F_j(t)/\sum_k F_k(t)$. After $t$ time steps, the model forms a network with $t + m_0$ individuals. In this paper, all the results shown are restricted to the case $m_0 = m = 3$. We also examined the results of different size of the square and different values of $m$ and $m_0$, and the results are alike.

The degree distribution for different values of $\alpha$ is shown in Figure 1. When $\alpha = 0$, only the effect of geographical measure was taken into account, and the individuals are more likely to connect to nearby individuals. Therefore, the individuals which are distributed closely will connect densely with each other, the degree distribution is very narrow and exhibits an exponential form [see Figure. 1(a)]. When $\alpha = 1$, the degree distribution the scale-free structure emerges and exhibits a power-law form [see Figure. 1(b)]: $p(k) \sim k^{-3.05}$ as the topological measure takes effect. When $\alpha = 2$, the topological measure plays a major role, and then a newly added individual is more likely to connect to individuals with larger degrees. Thus, the network becomes a star-like network and the degree distribution becomes broader than others [see Figure. 1(c)].

In Figure 2, we plot the simulation results of edge length distribution for the case of $\alpha = 0$, $\alpha = 1$ and $\alpha = 2$. When $\alpha = 0$ and $\alpha = 1$, most edges concentrate within the area of $0 \sim 10$ [see Figure. 2(a)and(b)]. When $\alpha = 2$, these edges length are longer [see Figure. 2(c)] as the
Figure 2: The edge length distribution of the networks with $m_0 = m = 3$ in three cases $\alpha = 0$(a),1(b),2(c). The size of the network is $N=1600$.

topological measure is enhanced. According to Eq.(1), the new individual is more likely to be linked to the hub individuals even if they are far away. So, there will be more long-distance edges when $\alpha = 2$.

3. EPIDEMIC DYNAMIC ON NETWORKS

In present work, we adopt the well-known SIS model [44]. Firstly, we give a simple instruction of SIS model. In this model, each node of the network represents an individual and each link is a connection along which the infection can spread; these individuals have two states: susceptible and infected; each susceptible individual is infected with probability $\lambda_0$ if at least one of its neighbors is infected; each infected individual, on the other hand, recovers and becomes susceptible with probability $\mu = 1$. $\mu$ can only affects the time scale of the virus spreading and is usually set as 1 [41].

In our SIS model, initially, there is only one individual is infected. At one time, each susceptible individual $i$ can be infected by infectious neighbors with probability $\theta_i$:

$$\theta_i = 1 - \prod_{j \in N(i)} (1 - \lambda_j), \tag{3}$$

$$\lambda_j = \min(1, \lambda_0 \times \left(\frac{d_{ij}}{\min_{k \in N(i)}(d_{ik})}\right)^{-\beta}), \tag{4}$$
Figure 3: The density of the infected individuals $i$ as a function of time $t$ in the network with $\alpha=0(a), 1(b), 2(c)$, respectively. The spreading rate is $\lambda_0=0.15$. The network is $N=1600$ and $m_0=m=3$.

Figure 4: The steady density of the infected individuals $i$ as a function of $\beta$ in three network. The spreading rate is $\lambda_0=0.15$. The network is $N=1600$ and $m_0=m=3$. 
where $\lambda_0$ is the average transmission rate of the virus, $j$ is one of the infected neighbors ($N(i)$) of susceptible individual $i$, $\lambda_j$ is transmission rate of individual $j$, $d_{ij}$ is Euclidean distance between individual $i$ and $j$, and $\beta$ is the tunable parameter. Obviously, $\beta$ can adjust the intensity of geographical measure to affect the epidemic spreading: when $\beta = 0$, the model degrades to the standard SIS model where the geographical measure is not considered; when $\beta > 0$, the longer distance between two individuals, the more difficult to infect each other; when $\beta < 0$, just the opposite.

Figure 3 shows the relationship of the density of infected individuals and time $t$ with the spreading rate $\lambda_0 = 0.15$. $i(t)$ is the density of infected individuals at time $t$. In Figure 3(a) and (b), the density of infected individuals for the different values of $\beta$ reaches steady condition after several generations. However, as shown in Figure 3(c), in the case of $\alpha = 2$, we found that the density of infected individuals shows large amplitude oscillations where the underlying network is star-like. Here, the individuals are infected as follows: initially, an individual is infected and the hubs can be easily infected due to the star-like topology; in the next generation, the hubs are cured and many individuals which are connected with the hubs are infected, thus the density of infected individuals becomes high; in the next generation, the hubs may be infected again and their numerous neighbors recover, so the density of infected individuals becomes low; iterating this process, the oscillation appears. Interestingly, when $\beta > 2$, the oscillations of infected individuals' density disappear (see the curves of $\beta = 3, \beta = 4$ of Figure 3(c)): the individual's
transmission rates are reduced when the effect of geographical measure is taken into account and thus fewer individuals change their state in one time step. Figure 4 shows the relationship of the steady density of infected individuals and the value of $\beta$ with $\lambda_0 = 0.15$. The steady density of infected individuals is the fraction of infected individuals in the steady state. One can find that the steady density of infected individuals monotonously decreases with the increment of $\beta$.

Figure 5 shows the relationship of $i(t)$ and $\lambda_0$. Obviously, $i(t)$ becomes larger as $\lambda_0$ increases. When $\beta < 0$, the epidemic threshold doesn’t exist in the networks if $\alpha = 0, \alpha = 1, \alpha = 2$. Interestingly, it is found that the epidemic threshold exists when $\beta > 0$ no matter in the exponential network (see Figure 5(a)), power-law network (see Figure 5(b)) or star-like network (see Figure 5(c)). When $\alpha = 0$ and $\alpha = 1$, the epidemic threshold is 0.1 when $\beta > 0$. When $\alpha = 2$, the epidemic threshold is 0.05 when $\beta > 0$. When $\beta = 0$ the epidemic threshold still exists in homogeneous network of $\alpha = 0$ and the epidemic threshold disappears in the heterogeneous network of $\alpha = 1$ and $\alpha = 2$, which is the same as the results gained by Pastor et al. [20, 21, 22, 23]. However, in the scale-free network the epidemic threshold occurs in the case of $\beta > 0$. Namely, if the geographical distance is introduced, the epidemic threshold will occur.

4. CONCLUSION

To summarize, we investigated the effect of geographical distance on the epidemic spreading by introducing a tunable parameter $\beta$ into SIS model on a growing network embedded in the Euclidean space. When $\beta > 0$, the infected individuals are more difficult to infect their far-away neighbors; when $\beta < 0$, just the opposite. It is found that the steady density of infected individuals monotonously decreases with the increment of $\beta$. Moreover, when the network is scale-free, the epidemic threshold emerges when $\beta > 0$; when the network is star-like, the density of infected individuals shows large amplitude oscillations.

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