HELIUM ABUNDANCE IN GALAXY CLUSTERS AND SUNYAEV-ZELDOVICH EFFECT

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ABSTRACT

It has long been suggested that helium nuclei in the intracluster plasma can sediment in the cluster gravitational potential well. Some theoretical estimates for the cores of relaxed clusters predict an excess of helium abundance by up to a factor of a few over its primordial value. The intracluster helium abundance cannot be measured directly. This presents a significant source of uncertainty for cosmological tests based on the X-ray derived cluster quantities, such as the gas mass, total mass, and gas mass fraction, all of which depend on the assumed helium abundance. We point out that cluster distances derived by combining the Sunyaev-Zeldovich (SZ) and X-ray data also depend on the helium abundance. This dependence can be used to measure the abundance, provided the distance is known independently. For example, if one adopts the WMAP H0 value, then the recent H0 measurement by Bonamente and collaborators, derived from SZ data on 38 clusters assuming a primordial helium abundance, corresponds to an abundance excess by a factor of 1.9 ± 0.8 within r ~ 1 Mpc (using only their statistical errors). This shows that interesting accuracy is within reach. We also briefly discuss how the SZ and X-ray cluster data can be combined to resolve the helium abundance dependence for the d(R) cosmological test.

Subject headings: cosmic microwave background — distance scale — intergalactic medium — X-rays: galaxies: clusters

1. INTRODUCTION

The majority of helium in the universe, predominantly in the form of 4He, was produced during the Big Bang. For the WMAP value of Ω_bh^2 = 0.0223 ± 0.0007 (Spergel et al. 2007), the standard hot big bang nucleosynthesis model predicts a primordial fraction of helium in the total baryonic mass density of Y = 0.2482 ± 0.0007 (Walker et al. 1991; Kneller & Steigman 2004). Recent spectral measurements in metal-poor extragalactic HII regions give a value within 1% of this theoretical prediction, with similarly small uncertainties (Izotov et al. 2007; Peimbert et al. 2007). Thus, the primordial helium abundance appears to be known quite accurately.

Helium abundance in the hot intracluster medium (ICM) may differ significantly from the primordial one. First, additional helium comes from the stars. The ratio of star mass to ICM mass is higher in the cluster centers, so stellar enrichment will be stronger there. However, the mass of the primordial helium in the ICM is comparable to the total stellar mass in a cluster, so helium enrichment by stars should not be significant (unlike stellar contribution of heavier elements, which are present in the ICM in trace amounts). A much greater increase of helium abundance in the central regions of clusters may be caused by sedimentation of heavy nuclei of the ICM in the cluster gravitational potential (Fabian & Pringle 1977; Rephaeli 1978; Abramopoulos, Chanan, & Ku 1981; Gilfanov & Sunyaev 1984; Qin & Wu 2000; Chuzhoy & Nusser 2003; Chuzhoy & Loeb 2004; Ettori & Fabian 2006). The consensus of the recent works is that if sedimentation is not suppressed, then in a hot cluster undisturbed for several gigayears, the relative helium abundance can increase by a factor of 2 or more within r < 0.2 – 0.3r200, and even more at smaller radii (Chuzhoy & Loeb 2004). However, sedimentation can be inhibited by several mechanisms, including tangled magnetic fields (which should also suppress diffusion and thermal conduction in the ICM, as seems to be observed, e.g., Ettori & Fabian 2000; Vikhlinin et al. 2001; Markevitch et al. 2003), gas mixing by cluster mergers and turbulence, and the formation of a cluster cool core (Ettori & Fabian 2006), because the diffusion rate is a strong function of the temperature.

While mergers and turbulence should inhibit any contemporary sedimentation, they are unlikely to permanently erase a large-scale abundance gradient already present by the time of the disturbance. The reason is the ICM in relaxed clusters is stratified, with low-entropy gas at the bottom of the gravitational well and higher-entropy gas in the outskirts. Such a stable gas distribution should restore itself, and any radial abundance gradient with it, shortly after a disturbance, provided that small-scale ICM mixing during a merger is inefficient. Indeed, we do observe radially declining iron abundance profiles in all relaxed clusters with sufficiently detailed X-ray data, with the decline traced from the cluster cores to at least r ~ 0.5r200 (e.g., Fukazawa et al. 1994; Tamura et al. 2004; Vikhlinin et al. 2005). While the iron abundance gradients are probably caused by enrichment rather than sedimentation, they should be old enough to have survived a merger or two, suggesting that it is difficult to erase an abundance gradient. Thus, given the uncertainties in the processes that inhibit sedimentation in the ICM, it is unclear how significant it would be in a typical cluster.

As pointed out by Qin & Wu (2000) and in later works (and summarized in [2] below), helium abundance affects cluster quantities derived from X-ray observations, such as the cluster total mass, gas mass, and gas mass fraction (see also Belmont et al. 2005 for an application to the hot gas in the center of our Galaxy). The assumed helium abundance also affects abundances of heavier elements derived from their X-ray emission lines (e.g., Drake 1998; Ettori & Fabian 2006). Most of the current X-ray cluster analyses are restricted to bright central regions — precisely those regions that may be affected by sedimentation. Unfortunately, helium in the ICM is fully ionized and not directly observable by spectroscopic means. For this reason, its abundance is unknown and has to

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be adopted from unrelated measurements, e.g., helioseismology (for a review see, e.g., Lodders 2003). The widely used X-ray spectral fitting package XSPEC offers a choice of abundance models with the number density of helium relative to hydrogen,

$$X \equiv \frac{n_{\text{He}}}{n_p},$$  

(1)

spanning a range between 0.0792 (Lodders 2003) to 0.0977 (Anders & Grevesse 1989 and some others). For comparison, if one takes the abundances of heavier elements to be 0.3 – 0.5 solar (as in clusters), the CMB-based primordial helium abundance (Spergel et al. 2007) corresponds to $X = 0.083$.

The unknown cluster helium abundance is a source of uncertainty for X-ray cluster-based cosmology studies. In this Letter, I propose a way to measure it.

2. X-RAY DERIVED QUANTITIES

2.1. Gas mass fraction

As mentioned above, a helium abundance in the ICM is implicitly assumed in most X-ray derived cluster quantities, such as gas mass, total mass from the hydrostatic equilibrium equation, and their ratio $f_{\text{gas}}$. All of these are being used for cosmological tests (see, e.g., Vikhlinin et al. 2003, Henry 1997, and Allen et al. 2004 for the three quantities, respectively), with projects underway to use them for “precision cosmology”. Below, their dependences on the assumed helium abundance are written in a form relevant for the X-ray analysis (that is, fixing the X-ray and other observables). For simplicity, I assume a uniform abundance over the region involved. Current cosmological tests use relatively small central cluster regions ($r < r_{500}$ or even $r < r_{2500}$), for which the effects of sedimentation and enrichment can be significant.

In a fully ionized intracluster plasma, the number of electrons per proton is

$$\frac{n_e}{n_p} = 1 + 2X + x_{\text{ch}} \approx 1 + 2X,$$  

(2)

where $x_{\text{ch}}$ represents electrons from elements heavier than helium; $x_{\text{ch}} \approx 0.005$ for the intracluster chemical abundances (0.3 – 0.5 solar), and we will neglect it. The mean molecular weight of the ICM is

$$\mu = \frac{1 + 4X + m_h}{2 + 3X + x_{\text{ch}} + x_h} \approx \frac{1 + 4X}{2 + 3X},$$  

(3)

where $m_h$ and $x_h$ are the mass and number density contribution from elements heavier than helium. For clusters, $m_h \approx 0.01$ and can be neglected; $x_h \ll x_{\text{ch}}$ and is certainly negligible as well. Thus, for clarity, we will ignore heavy elements below.

The total mass of a cluster within a certain radius, derived under the assumption of hydrostatic equilibrium (e.g., Sarazin 1988) and (for our illustrative purposes) isothermality, is

$$M_{\text{tot}} \propto \frac{T_e}{\mu} \frac{d \log \rho_{\text{gas}}}{d \log r} \propto \frac{1}{\mu} = \frac{2 + 3X}{1 + 4X}.$$  

(4)

Here we used the fact that the logarithmic density gradient does not depend on $X$ under our assumption that helium abundance is spatially uniform; this relation will be more complicated if one uses a range of radii where this assumption does not hold. The electron temperature $T_e$, derived from the shape of the X-ray spectrum, is practically independent of the helium abundance.

Let us now consider the cluster gas mass. Provided the cluster absolute size is known, it is proportional to the plasma density. In X-ray analysis, the plasma hydrogen density $n_p$ is derived from the normalization of the cluster continuum spectrum (assuming that emission lines from heavy elements are detected and properly modeled). The X-ray continuum luminosity is

$$L_X \propto n_p n_{e_p} (1 + 4X) = n_p^2 e_p (1 + 4X)(1 + 2X)$$  

(5)

where $e_p$ is bremsstrahlung emissivity for a pure electron-proton plasma, and the factor $(1 + 4X)$ accounts for the additional bremsstrahlung on helium nuclei with charge 2. Again, heavier elements add very little to the continuum emission. $L_X$ is the observable quantity that one obtains directly from spectral fitting (and a known distance). Fixing it, we obtain the dependence of the derived $n_p$ on the assumed helium abundance:

$$n_p \propto [(1 + 4X)(1 + 2X)]^{-1/2}.$$  

(6)

The gas mass then depends on $X$ as follows:

$$M_{\text{gas}} \propto n_p + 4n_{\text{He}} = n_p (1 + 4X) \propto \frac{(1 + 4X)}{1 + 2X}.$$  

(7)

Note that $M_{\text{gas}}$ and $M_{\text{tot}}$ change with $X$ in the opposite directions, so their ratio, the gas mass fraction, depends on $X$ stronger than either of these quantities:

$$f_{\text{gas}} = \frac{M_{\text{gas}}}{M_{\text{tot}}} \propto \frac{(1 + 4X)^{3/2}}{(2 + 3X)(1 + 2X)^{1/2}}.$$  

(8)

We do not know the true value of $X$ and have to assume one to calculate $f_{\text{gas}}$. Fig. 1 shows the resulting relative error. Note that the above $f_{\text{gas}}(X)$ dependence is weaker than that derived by Ettori & Fabian (2006); the difference is due to our fixing of the observable quantity $L_X$ when deriving $n_p$ in order to mimic the X-ray data analysis.

Fig. 1 shows that an error by a factor of 2 in the assumed helium abundance corresponds to a ~15% error in $f_{\text{gas}}$. This is comparable to the expected difference between the apparent $f_{\text{gas}}$ at $z = 1$ for open and flat $\Omega_m = 0.3$ cosmologies. Allen et al. (2004) derived $f_{\text{gas}}$ values within $r < r_{2500} = 0.25 r_{200}$ for a sample of hot clusters, detected such a difference, and used it
as evidence for $\Omega_\Lambda > 0$. Of course, this cosmological test is based on a comparison of $f_{\text{gas}}$ at high and low redshifts, so the value of helium abundance does not matter as long as it does not evolve between those redshifts. Furthermore, the sign of the error arising from wrongly assuming a primordial helium abundance in the presence of sedimentation is such that the derived $f_{\text{gas}}$ would be closer to its “pre-sedimentation” value, which is what one would ideally want to use for such a test (A. Vikhlinin, private communication). A discussion of this error-cancellation effect is beyond the scope of this paper (it would require a more accurate calculation than that used for eq. [4]). However, it is clear that any tests relying on an even higher accuracy of $f_{\text{gas}}$ (such as deriving the dark energy equation of state, which would need a few percent accuracy on $f_{\text{gas}}$) will require the knowledge of the intracluster helium abundance at different redshifts. We will return to this in §4.1.

### 2.2. Hubble constant

It has long been suggested (Cavaliere, Danese, & de Zotti 1979; Silk & White 1978) that combining the X-ray and Sunyaev-Zeldovich (SZ) observations of a cluster can be used to measure the absolute distance to the cluster. This method uses the fact that the SZ decrement and the X-ray brightness depend on different powers of the intracluster electron density, whose value can thus be determined and converted to the distance. Below I show how this measurement depends on the assumed helium abundance. The underlying reason for this dependence is that the SZ effect is caused only by electrons, while the X-ray emission is caused by scattering of electrons on protons and helium nuclei. The fact that helium sedimentation would affect the cluster SZ decrement was mentioned by Gilfanov & Sunyaev (1984) but, to our knowledge, never considered in any SZ data analyses.

In a nonrelativistic approximation, the SZ signal in the direction of a cluster is proportional to the comptonization parameter

$$y \equiv \int \frac{kT_e}{m_e c^2} \sigma_T n_e(l) dl \propto T_e n_{e0} d_a$$  \hspace{1cm} (9)

(Sunyaev & Zeldovich 1972), where the integral is along the line of sight, $n_{e0}$ is some characteristic electron density (e.g., near the cluster center), and $d_a$ is the angular distance to the cluster. Here for the rightmost part of the equation, the plasma cloud is assumed isothermal and spherically symmetric. The latter assumption is important (the line-of-sight distribution of the gas density is taken to be the same as that in the plane of the sky) but cannot be tested directly, so in practice, a cluster sample has to be used to average out any possible ellipticities. The surface brightness of the X-ray continuum emission from the same line of sight is given by

$$S_X = \int n_e(l) n_p(l) c_{\text{sp}} (1 + 4x) dl \propto n_{e0}^2 d_a \left( \frac{1 + 4x}{1 + 2x} \right)$$  \hspace{1cm} (10)

The distance can be determined by combining the above equations as follows:

$$d_a \propto \frac{y^2}{S_X f_{\text{e}}^2} \frac{1 + 4x}{1 + 2x}$$  \hspace{1cm} (11)

The quantities $y$, $S_X$, and $T_e$ are directly measured, but helium abundance $x$ has to be assumed. Fig. 2 shows the effect of this assumption on the derived $d_a$ (or $H_0 \propto d_a^{-1}$), based on eq. (11). For example, a factor of 2 error in helium abundance results in a 10% distance error.

### 3. Measuring Helium Abundance in ICM

The above dependence of the SZ–X-ray distances on the helium abundance can be turned around and used to derive the cluster helium abundance — provided the distance scale is known independently, for example, from Cepheids and supernovae (Freedman et al. 2001; Sandage et al. 2006) or from the CMB fluctuations (Spergel et al. 2007). At present, the latter two methods yield similar or smaller uncertainties on distances than those from state of the art SZ–X-ray studies (e.g., Bonamente et al. 2006). This is because measurement errors on the SZ signal are still quite large, while this observable ($y$) enters squared in eq. (11). Neither the supernovae distances nor those from CMB fluctuations have any significant dependence on primordial helium abundance (Ichikawa & Takahashi 2006).

Fig. 2 shows the value of $H_0 = 77.6^{+4.8}_{-4.4}$ km s$^{-1}$ Mpc$^{-1}$ from Bonamente et al. (2006), derived from the SZ and X-ray data on 38 clusters at different redshifts. Of their 3 reported values, I chose the least model-dependent one, without the hydrostatic equilibrium assumption and excluding the central cool regions. The above error bars are 68% statistical-only, to illustrate an accuracy not quite achieved yet but within immediate reach (once the instruments have been better calibrated); their current systematic uncertainties are twice as big.

According to M. Bonamante (private communication), they assumed the Anders & Grevesse (1989) helium abundance, which is a factor of 1.17 higher than the primordial value (11). Had they used the primordial abundance, their result would be $H_0 \approx 79$ km s$^{-1}$ Mpc$^{-1}$. If, for the sake of argument, we take the WMAP value of $H_0$ as “true”, and attribute the difference between these values to a helium abundance error (Fig. 2), we conclude that it should be a factor 1.9 $\pm$ 0.8 higher than the primordial value. Of course, at this accuracy, it is consistent with the primordial value, but we can already exclude some of the more extreme predictions for helium sedimentation.
This and many other current SZ experiments use interferometric mapping of the radio brightness. For typical clusters, this experimental design effectively “subtracts” the signal from the shell outside \( r \sim 1 \) Mpc, so the above constraint corresponds to this approximate central region. This is about the same radius as used for the \( f_{\text{gas}} \) test by Allen et al. (2004).

4. SUMMARY AND DISCUSSION

The usual assumption of a primordial helium abundance for the ICM is not necessarily correct, especially in the cluster central regions where helium may concentrate via several mechanisms, such as sedimentation. We cannot measure the cluster helium abundance spectroscopically. As pointed out by many authors, an incorrect helium abundance assumption will result in incorrect cluster gas masses, total masses and gas mass fractions derived from the X-ray data.

In this paper, we point out that cluster distances derived using the SZ–X-ray combination also depend on helium abundance. If one gives up on the original purpose of this method and combines it with an independent distance scale estimator (such as supernovae or CMB fluctuations), one can take advantage of this dependence and derive the cluster helium abundance. At present, this seems to be the only practical way of measuring it. The accuracy may already be interesting — if one compares the best SZ–X-ray value for \( H_0 \) (Bonamente et al. 2006) with the CMB value (Spergel et al. 2007), the difference corresponds to a helium abundance \( 1 \pm 9\% \) within the primordial value within the cluster central 1 Mpc region (68% statistical-only uncertainty). Increasing the number of clusters in the sample and improving the SZ data accuracy will reduce this uncertainty.

Chuzhoy & Loeb (2004) proposed another way of constraining the cluster helium abundance — by comparing the cluster total masses derived using the hydrostatic equilibrium assumption (eq. 4 above) and any other technique independent of helium abundance, such as gravitational lensing. However, this approach appears less practical at present, because the expected apparent mass difference (\( \sim 10\% \) for a factor of 2 error in helium abundance) is much smaller than persistent discrepancies between these mass estimators that are likely to be caused by substructure in the dark matter distribution and deviations from hydrostatic equilibrium in the ICM (e.g., Gavazzi 2005; Meneghetti et al. 2007). In comparison, the SZ–X-ray method uses the same object — the ICM — at both wavelengths and does not rely on hydrostatic equilibrium. It does, however, assume that the ICM is not clumpy (which appears to be supported by Chandra imaging), and requires accurate mapping of the ICM temperature structure. It also needs spherical symmetry, which has always been an issue for the SZ method of \( H_0 \) determination. It can be overcome by proper (i.e., X-ray) selection of a sample of relaxed clusters that is also big enough to average out the asymmetries.

Chuzhoy & Loeb (2004) also pointed out that a higher helium abundance in the ICM would affect stars that forms out of this ICM. Thus, evidence of helium sedimentation may also be found in the spectra of the central cluster galaxies.

4.1. The \( d_a(z) \) cosmological test and helium abundance

The main reason why we want to know the cluster helium abundances is to remove the related uncertainty from the cluster-based cosmological tests, such as the growth of structure tests that use cluster mass functions and the \( d_a(z) \) tests that use \( f_{\text{gas}} \) or the SZ–X-ray distances. Obviously, using a competing distance estimator to measure helium abundances will introduce degeneracies into the resulting cosmological constraints, the detailed analysis of which is beyond the scope of this paper. In principle, the above two distance tests can be combined to solve for the dependence on helium abundance, because, as seen from eqs. (8) and (11), the distances derived from \( f_{\text{gas}} (d_a \sim f_{\text{gas}}^2) \) and from the SZ–X-ray method depend on \( x \) differently. One complication is that if one allows for helium sedimentation, the basic assumption of the \( f_{\text{gas}} \)-based test, that \( f_{\text{gas}} \) within a certain central region does not evolve with \( z \), may be violated. So in practice, one would have to fit together the X-ray and SZ data for a sample of relaxed clusters spanning a range of \( z \), parameterize and fit any systematic change of helium abundance with redshift (hopefully small or negligible), and use modeling to deduce “pre-sedimentation” \( f_{\text{gas}} \) values (to within a scaling factor), which would be the ones proportional to the universal baryon fraction. To derive cosmological parameters other than \( H_0 \), the absolute values of \( d_a \) (and so the absolute values of \( x \)) are not needed, only its change with \( z \). However, independent distances for at least a few clusters will be required to determine if any helium sedimentation occurs at all, and if so, to model it.

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