Research Article

Graphical Structures of Cubic Intuitionistic Fuzzy Information

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1. Introduction

Jun et al. [1] proposed cubic set (CS) and started a new research area. A CS is a mixture of two concepts known as fuzzy set (FS) and interval-valued fuzzy set (IVFS). The concept of CS draws the attentions of researchers and some potential works in this direction have been done; for example, the idea of CS was proposed in semigroup theory by Khan et al. [2], as well as some KU-ideal by Yaqoob et al. [3], and KU-algebras are developed for CS by Lu and Ye [4]; the similarity measures of CSs have been proposed and applied in decision-making problem. The framework of cubic neutrosophic sets is proposed by Jun et al. [5], while some pattern recognition problems are solved using neutrosophic sets by Ali et al. [6]. The concept of cubic soft sets was proposed by Muhiuddin and Al-roqi [7], which was further utilized by Muhiuddin et al. [8]. The theory of G-algebras is studied by Jun and Khan in [9] and by Jana and Senapatı [10] along with the concepts of ideal in semigroups. Some other works in this direction are given in [11–14].

The theory of intuitionistic fuzzy set (IFS) was developed by Atanassov [15] as a generalization of FS by Rosenfeld [16]. An IFS described the membership and nonmembership degree of an element by two characteristic functions and can model phenomena of yes or no type easily. Garg and Kaur [17] initiated the concept of cubic intuitionistic fuzzy sets (CIFSs) and discussed their properties. Atanassov model of IFS provided a motivation for the concept of intuitionistic fuzzy graphs (IFGs) proposed by Kauffman and Rosenfeld [19, 20] after Zadeh’s exemplary work in [16]. FG theory has a potential role in application point of view as described by Chan and Cheung [21] who studied an approach to clustering algorithm using the concepts of FGs. Some FG problems are solved by a novel technique in [22, 23] by discussing the domination of FGs in pattern recognitions. Mathew and Sunitha [24] worked on fuzzy attribute graphs applied to Chinese character recognitions, and Bhattacharya [25] used FGs in image classifications and so forth. For some other works on FG, one may refer to [26–31].

The theory of IFG received great attention as Parvathi and Thamizhendhi [32] introduced the concept of strong IFGs; Akram and Dudek [33] discussed the order, degree,
and size of IFGs; Akram and Alshehri [34] developed operations for IFGs; Karunambigai [35] worked on the domination of IFGs; Pasi et al. [36] developed the theory of intuitionistic fuzzy hypergraphs; Karunambigai et al. [37] studied the concepts of trees and cycles for IFGs; Parvathi [38] developed the idea of balanced IFGs, a multicriteria and multiperson decision-making based on IFGs was discussed by Chountas [39]; Akram and Dudek [40] studied constant mappings on a universe of discourse and \( M_1 \) is a map on \([0, 1]\).

Definition 1 (see [13]). A FS on \( X \) is defined as 
\[ A = \{ u, (M_A (u)/u \in X) \}, \]where \( M_A (1/2) \) is a map on \([0, 1]\).

Definition 2 (see [20]). A pair \( \hat{G}^* = (\mathcal{Y}^*, E) \) is known as FG if

(i) \( \mathcal{Y}^* = \{M_i; i \in I\} \) and \( M_i: \mathcal{Y}^* \rightarrow [0, 1] \) is the association degree of \( M_i \in \mathcal{Y}^* \)

(ii) \( E = \{(u_i, u_j); (u_i, u_j) \in \mathcal{Y}^* \times \mathcal{Y}^* \} \quad \text{and} \quad M_2: \mathcal{Y}^* \times \mathcal{Y}^* \rightarrow [0, 1] \) where \( M_2 (u_i, u_j) \leq \min \{ M_1 (u_i), M_1 (u_j) \} \) for all \( (u_i, u_j) \in E \).

Definition 3 (see [15]). An IFS \( A \) on \( X \) is defined as 
\[ A = \{ \langle u, M_A (u), \Pi_A (u) \rangle/u \in X \} \], where \( M_A \) and \( \Pi_A \) are mappings on \([0, 1]\) interval such that \( 0 \leq M_A + \Pi_A \leq 1 \).

Definition 4 (see [18]). A pair \( \tilde{G}^* = (V, \tilde{E}) \) is known as IFG if

(i) \( V \) is the collection of nodes such that \( M_1 \) and \( \Pi_1 \) are mappings on unit intervals from \( V \) with a condition \( 0 \leq M_1 (u) + \Pi_1 (u) \leq 1 \) for all \( u \in V, i \in I \)

(ii) \( E \subseteq \mathcal{Y}^* \times \mathcal{Y}^* \), where \( M_2 \) and \( \Pi_2 \) are mappings that associate some grade to each \( (u_i, u_j) \in E \) from \([0, 1]\) interval such that \( M_2 (u_i, u_j) \leq \min \{ M_1 (u_i), M_1 (u_j) \} \) and \( \Pi_2 (u_i, u_j) \leq \max \{ \Pi_1 (u_i), \Pi_1 ((u_i)) \} \) with a condition \( 0 \leq M_2 + \Pi_2 \leq 1 \).

Example 1. The graph in Figure 1 is an IFG having four vertices and four edges.

Definition 5 (see [33]). The complement of an IFG \( \hat{G}^* = (\mathcal{Y}^*, E) \) is \( \hat{G}^{c*} = (\mathcal{Y}^{c*}, E^{c*}) \), where

(i) \( \mathcal{Y}^{c*} = V \)

(ii) \( M_A (u)c = M_A (u), \Pi_A (u)c = \Pi_A (u) \forall u \in V \)

(iii) \( M_B (u_i, u_j)^{c*} = \min \{ M_B (u_i), M_B (u_j) \} - M_B (u_i, u_j), \Pi_B (u_i, u_j)^{c*} = \max \{ \Pi_B (u_i), \Pi_B (u_j) \} - \Pi_B (u_i, u_j), \)

for all \( (u_i, u_j) \in E \).

Here \( (u_i, M_A, \Pi_A) \) represent the vertices and \( (e_{ij}, M_B, \Pi_B) \) represent the edges.

Definition 6 (see [32]). A pair \( \tilde{G}^* = (\mathcal{Y}, E) \) is known as strong IFG if

(i) \( \mathcal{Y} \) is the collection of nodes such that \( M_1 \) and \( \Pi_1 \) are mappings on unit intervals from \( \mathcal{Y} \) with a condition \( 0 \leq M_1 (u) + \Pi_1 (u) \leq 1 \) for all \( u \in \mathcal{Y} \) \((i \in I)\)

(ii) \( E \subseteq \mathcal{Y} \times \mathcal{Y} \), where \( M_2 \) and \( \Pi_2 \) are mappings that associate some grade to each \( (u_i, u_j) \in E \) from \([0, 1]\) interval such that \( M_2 (u_i, u_j) = \min \{ M_1 (u_i), M_1 (u_j) \} \) and \( \Pi_2 (u_i, u_j) = \max \{ \Pi_1 (u_i), \Pi_1 ((u_i)) \} \) with a condition \( 0 \leq M_2 + \Pi_2 \leq 1 \).

Remark 1 (see [32]). If \( \hat{G}^* = (\mathcal{Y}, E) \) is an IFG, then by the above definition \( (\hat{G}^{c*})^c = \hat{G}^* \) and it is called self-complementary.

Proposition 1 (see [32]). If \( \hat{G}^* \) is strong IFG, then it preserves self-complementary law.

Example 2. Figures 2(a) and 2(b) provide a verification of Proposition 1. Clearly \( (\hat{G}^{c*})^c = \hat{G}^* \) is self-complementary.
Definition 7 (see [55]). A pair \( \mathcal{G} = (\mathcal{A}, \mathcal{B}) \) of a graph \( \mathcal{G}^* = (\mathcal{V}, E) \) is known as IVIFG, where \( \mathcal{A} = \{[\mathcal{M}_{\mathcal{AL}}, \mathcal{M}_{\mathcal{AU}}], [\mathcal{N}_{\mathcal{AL}}, \mathcal{N}_{\mathcal{AU}}]\} \) is IVFS on \( \mathcal{V} \), and \( \mathcal{B} = \{[\mathcal{M}_{\mathcal{BL}}, \mathcal{M}_{\mathcal{BU}}], [\mathcal{N}_{\mathcal{BL}}, \mathcal{N}_{\mathcal{BU}}]\} \) is the IVF relation on \( E \) satisfying the following conditions:

(i) \( \mathcal{V} = \{u_1, u_2, u_3, \ldots, u_n\} \) such that \( \mathcal{M}_{\mathcal{AL}}: \mathcal{V} \to [0,1], \mathcal{M}_{\mathcal{AU}}: \mathcal{V} \to [0,1] \) and \( \mathcal{N}_{\mathcal{AL}}: \mathcal{V} \to [0,1], \mathcal{N}_{\mathcal{AU}}: \mathcal{V} \to [0,1] \) represent the degrees of membership and nonmembership of the element \( u \in \mathcal{V} \), respectively, and \( 0 \leq \mathcal{M}_{\mathcal{AL}} + \mathcal{M}_{\mathcal{AU}} \leq 1 \) for all \( u_i \in \mathcal{V} \) (i = 1, 2, \ldots, n)

(ii) The functions \( \mathcal{M}_{\mathcal{BL}}: \mathcal{V} \times \mathcal{V} \to [0,1], \mathcal{M}_{\mathcal{BU}}: \mathcal{V} \times \mathcal{V} \to [0,1], \mathcal{N}_{\mathcal{BL}}: \mathcal{V} \times \mathcal{V} \to [0,1], \mathcal{N}_{\mathcal{BU}}: \mathcal{V} \times \mathcal{V} \to [0,1] \) are such that \( \mathcal{M}_{\mathcal{BL}}(u, y) \leq \min(\mathcal{M}_{\mathcal{AL}}(u), \mathcal{M}_{\mathcal{AU}}(y)), \mathcal{N}_{\mathcal{BL}}(u, y) \leq \max(\mathcal{N}_{\mathcal{AL}}(u), \mathcal{N}_{\mathcal{AU}}(y)) \) \( \mathcal{M}_{\mathcal{BU}}(u, y) \leq \min(\mathcal{M}_{\mathcal{AL}}(u), \mathcal{M}_{\mathcal{AU}}(y)), \mathcal{N}_{\mathcal{BU}}(u, y) \leq \max(\mathcal{N}_{\mathcal{AL}}(u), \mathcal{N}_{\mathcal{AU}}(u)) \) for all \( (u_i, y_j) \in E \) (i = 1, 2, \ldots, n)

Example 3. Let \( \mathcal{G}^* = (\mathcal{V}, E) \) be a graph, where \( \mathcal{V} = \{u_1, u_2, u_3\} \) is the set of vertices and \( E = \{u_1u_2, u_2u_3, u_3u_1\} \) is the set of edges.

3. Cubic Intuitionistic Fuzzy Graphs

In this section, we discussed the basic concept of CIFG-like complement of CIFG, degree of CIFG, and bridge and cut vertex of CIFG with the help of examples and several results (Figures 3 and 4).

Definition 8. A pair \( \mathcal{G} = (\mathcal{A}, \mathcal{B}) \) of a graph \( \mathcal{G}^* = (\mathcal{V}, E) \) is known as cubic IFG, where \( \mathcal{A} = \{[\mathcal{M}_{\mathcal{AL}}, \mathcal{M}_{\mathcal{AU}}], [\mathcal{N}_{\mathcal{AL}}, \mathcal{N}_{\mathcal{AU}}]\} \) is a cubic IFS on \( \mathcal{V} \), and \( \mathcal{B} = \{[\mathcal{M}_{\mathcal{BL}}, \mathcal{M}_{\mathcal{BU}}], [\mathcal{N}_{\mathcal{BL}}, \mathcal{N}_{\mathcal{BU}}]\} \) is the cubic IF relation on \( E \) satisfying the following conditions:

(iii) \( \mathcal{V} = \{u_1, u_2, u_3, \ldots, u_n\} \) such that \( \mathcal{M}_{\mathcal{AL}}: \mathcal{V} \to [0,1], \mathcal{M}_{\mathcal{AU}}: \mathcal{V} \to [0,1] \) and \( \mathcal{N}_{\mathcal{AL}}: \mathcal{V} \to [0,1], \mathcal{N}_{\mathcal{AU}}: \mathcal{V} \to [0,1] \) are such that \( \mathcal{M}_{\mathcal{BL}}(u, y) \leq \min(\mathcal{M}_{\mathcal{AL}}(u), \mathcal{M}_{\mathcal{AU}}(y)), \mathcal{N}_{\mathcal{BL}}(u, y) \leq \max(\mathcal{N}_{\mathcal{AL}}(u), \mathcal{N}_{\mathcal{AU}}(y)) \) for all \( u_i \in \mathcal{V} \) (i = 1, 2, \ldots, n)

(iv) The functions \( \mathcal{M}_{\mathcal{BL}}: \mathcal{V} \times \mathcal{V} \to [0,1], \mathcal{M}_{\mathcal{BU}}: \mathcal{V} \times \mathcal{V} \to [0,1], \mathcal{N}_{\mathcal{BL}}: \mathcal{V} \times \mathcal{V} \to [0,1], \mathcal{N}_{\mathcal{BU}}: \mathcal{V} \times \mathcal{V} \to [0,1] \) are such that \( \mathcal{M}_{\mathcal{BL}}(u, y) \leq \min(\mathcal{M}_{\mathcal{AL}}(u), \mathcal{M}_{\mathcal{AU}}(y)), \mathcal{N}_{\mathcal{BL}}(u, y) \leq \max(\mathcal{N}_{\mathcal{AL}}(u), \mathcal{N}_{\mathcal{AU}}(u)) \) for all \( (u_i, y_j) \in E \) (i = 1, 2, \ldots, n)
Definition 9. A pair $\mathcal{G} = (A, \mathcal{B})$ of a graph $\mathcal{G}^* = (\mathcal{V}, E)$ is known as strong cubic IFG, where $A = \{ (M_{AL}, M_{AU}), (\mathcal{F}_{AL}, \mathcal{F}_{AU}) \}$ and $A_{\mathcal{B}} = \{ (M_{AL\mathcal{B}}, M_{AU\mathcal{B}}), (\mathcal{F}_{AL\mathcal{B}}, \mathcal{F}_{AU\mathcal{B}}) \}$ is a cubic IFS on $\mathcal{V}$, and $A = \{ (M_{AL}, M_{AU}), (\mathcal{F}_{AL}, \mathcal{F}_{AU}) \}$ is a cubic IF relation on $E$ satisfying the following conditions:

(i) $\mathcal{V} = \{ u_i, u_j, \ldots, u_n \}$ such that $M_{AL}: \mathcal{V} \rightarrow [0, 1], M_{AU}: \mathcal{V} \rightarrow [0, 1]$ and $\mathcal{F}_{AL}: \mathcal{V} \rightarrow [0, 1]$ and $\mathcal{F}_{AU}: \mathcal{V} \rightarrow [0, 1]$ represent the degrees of membership and nonmembership of the element $u \in \mathcal{V}$, respectively, and $0 \leq M_{A} + \mathcal{F}_{A} \leq 1$ for all $u_i \in \mathcal{V}$ ($i = 1, 2, \ldots, n$)

(ii) The functions $M_{AL}: \mathcal{V} \times \mathcal{V} \rightarrow [0, 1], M_{AU}: \mathcal{V} \times \mathcal{V} \rightarrow [0, 1], \mathcal{F}_{AL}: \mathcal{V} \times \mathcal{V} \rightarrow [0, 1], \mathcal{F}_{AU}: \mathcal{V} \times \mathcal{V} \rightarrow [0, 1]$ and $M_{\mathcal{B}}: \mathcal{V} \times \mathcal{V} \rightarrow [0, 1], \mathcal{F}_{\mathcal{B}}: \mathcal{V} \times \mathcal{V} \rightarrow [0, 1]$ are such that $M_{\mathcal{B}L}(u, y) = \min (M_{AL}(u), M_{AL}(y)), M_{\mathcal{B}U}(u, y) = \max (\mathcal{F}_{AL}(u), \mathcal{F}_{AL}(y)), M_{\mathcal{B}U}(u, y) = \min (M_{AU}(u), M_{AU}(y))$, and $\mathcal{F}_{\mathcal{B}}(u, y) = \max (\mathcal{F}_{AU}(u), \mathcal{F}_{AU}(y))$ and $M_{\mathcal{B}}(u, y) = \min (M_{A}(u), M_{A}(y))$ and $\mathcal{F}_{\mathcal{B}}(u, y) = \max (M_{A}(u), M_{A}(y))$ such that $0 \leq M_{\mathcal{B}}(u, y) + \mathcal{F}_{\mathcal{B}}(u, y) \leq 1$ for all $(u_i, y_j) \in E$ ($i, j = 1, 2, \ldots, n$)

Definition 10. A cubic IFG $H = (\mathcal{V}', \mathcal{E}')$ is said to be cubic IFG subgraph of $\mathcal{G} = (\mathcal{V}, E)$ if $\mathcal{V} \subseteq \mathcal{V}'$ and $E' \subseteq E$. In other words, $M_{AL}', M_{AU}', \mathcal{F}_{AL}', \mathcal{F}_{AU}' \leq M_{AL}, M_{AU}, \mathcal{F}_{AL}, \mathcal{F}_{AU}$ and $(M_{AL}, \mathcal{F}_{AL}) \leq (M_{AL}, \mathcal{F}_{AL})$ and $(M_{AU}, \mathcal{F}_{AU}) \leq (M_{AU}, \mathcal{F}_{AU})$ for $i, j = 1, 2, \ldots, n$.

Definition 11. The order of cubic IFG $\mathcal{G}^* = (\mathcal{V}, \mathcal{E})$ is denoted and defined by

$$O(\mathcal{G}^*) = \left( \sum_{u \in \mathcal{V}} M_{AL}(u), \sum_{u \in \mathcal{V}} M_{AU}(u), \sum_{u \in \mathcal{V}} \mathcal{F}_{AL}(u), \sum_{u \in \mathcal{V}} \mathcal{F}_{AU}(u) \right), \left( \sum_{u \in \mathcal{V}} M_{AL}(u), \sum_{u \in \mathcal{V}} M_{AU}(u), \sum_{u \in \mathcal{V}} \mathcal{F}_{AL}(u), \sum_{u \in \mathcal{V}} \mathcal{F}_{AU}(u) \right),$$

and the size of cubic IFG is

$$S(\mathcal{G}) = \left( \sum_{u, y \in \mathcal{V}} M_{AL}(u, y), \sum_{u, y \in \mathcal{V}} M_{AU}(u, y), \sum_{u, y \in \mathcal{V}} \mathcal{F}_{AL}(u, y), \sum_{u, y \in \mathcal{V}} \mathcal{F}_{AU}(u, y) \right), \left( \sum_{u, y \in \mathcal{V}} M_{AL}(u, y), \sum_{u, y \in \mathcal{V}} M_{AU}(u, y), \sum_{u, y \in \mathcal{V}} \mathcal{F}_{AL}(u, y), \sum_{u, y \in \mathcal{V}} \mathcal{F}_{AU}(u, y) \right).$$
Definition 12. The degree of a vertex in a cubic IFG $\hat{G}^* = (\mathcal{V}, E)$ is defined and denoted by

$$d(u) = \left( (dM_{AL}(u), dM_{AU}(u), d\bar{\Gamma}_{AL}(u), d\bar{\Gamma}_{AU}(u)), (d(M_{\bar{\Lambda}}(u), d(\Gamma_{\bar{\Lambda}}(u))) \right),$$

where

$$dM_{AL}(u) = \sum_{u \neq y \atop y \in V} M_{\bar{\Lambda}L}(u, y),$$

$$dM_{AU}(u) = \sum_{u \neq y \atop y \in V} M_{\bar{\Lambda}U}(u, y),$$

$$d\bar{\Gamma}_{AL}(u) = \sum_{u \neq y \atop y \in V} \bar{\Gamma}_{\Lambda L}(u, y),$$

$$d\bar{\Gamma}_{AU}(u) = \sum_{u \neq y \atop y \in V} \bar{\Gamma}_{\Lambda U}(u, y),$$

$$d(M_{\bar{\Lambda}}(u)) = \sum_{u \neq y \atop y \in V} M_{\bar{\Lambda}}(u, y),$$

$$d(\Gamma_{\bar{\Lambda}}(u)) = \sum_{u \neq y \atop y \in V} \Gamma_{\bar{\Lambda}}(u, y).$$

Example 5. Let Figure 5 be a graph $\hat{G}^* = (\mathcal{V}, E)$, where $\mathcal{V} = \{u_1, u_2, u_3, u_4\}$ is the set of vertices and $E = \{u_1u_2, u_2u_3, u_3u_4, u_4u_1\}$ is the set of edges.

The degrees of vertices are

$$d(u_1) = ([0.3, 0.6], [0.5, 0.8], (0.3, 0.8)),$$

$$d(u_2) = ([0.4, 0.7], [0.5, 0.8], (0.3, 0.8)),$$

$$d(u_3) = ([0.3, 0.7], [0.4, 0.8], (0.2, 0.8)),$$

$$d(u_4) = ([0.2, 0.6], [0.4, 0.8], (0.2, 0.9)).$$

Definition 13. The complement of a cubic IFG $\hat{G} = (A, \mathcal{R})$ on $G^* = (\mathcal{V}, E)$ is defined as follows.

(i) $\mathcal{X} = A$

(ii) $\mathcal{M} = \mathbf{M}$

(iii) $\mathbf{E} = \mathbf{E}$

iv) $\mathbf{M} = \mathbf{M}$

Proposition 2. $G = \overline{\hat{G}}$ if and if $\hat{G}$ is strong cubic IF graph.

Proof. The proof is straightforward.

Definition 14. A strong IFG is said to be self-complementary if $\hat{G} \equiv \overline{\hat{G}}$, where $\overline{\hat{G}}$ is the complement of IFG $\hat{G}$.

Example 6. Let Figures 6 and 7 be two graphs of $\hat{G}^* = (\mathcal{V}, E)$, where $\mathcal{V} = \{u_1, u_2, u_3, u_4\}$ is the set of vertices and $E = \{u_1u_2, u_2u_3, u_3u_4, u_4u_1\}$ is the set of edges.

Clearly $\hat{G} = \overline{\hat{G}}$; hence, $\hat{G}$ is self-complementary.

Definition 15. The power of edge relation in a cubic IFG is defined as

$$e_{ij}^1 = e_{ij}(\{M_{\bar{\Lambda}L}(u, v), M_{\bar{\Lambda}U}(u, v)\}, \{\bar{\Gamma}_{\bar{\Lambda}L}(u, v), \bar{\Gamma}_{\bar{\Lambda}U}(u, v)\})$$

$$e_{ij}^2 = e_{ij}^1 e_{ij}^1 = e_{ij}(\{M_{\bar{\Lambda}L}(u, v), M_{\bar{\Lambda}L}(u, v)\}, \{\bar{\Gamma}_{\bar{\Lambda}L}(u, v), \bar{\Gamma}_{\bar{\Lambda}L}(u, v)\})$$

$$e_{ij}^3 = e_{ij}^2 e_{ij}^1 = e_{ij}(\{M_{\bar{\Lambda}L}(u, v), M_{\bar{\Lambda}L}(u, v)\}, \{\bar{\Gamma}_{\bar{\Lambda}L}(u, v), \bar{\Gamma}_{\bar{\Lambda}U}(u, v)\}).$$

Also,

$$e_{ij}^0 = e_{ij}(\{M_{\bar{\Lambda}L}(u, v), M_{\bar{\Lambda}L}(u, v)\}, \{\bar{\Gamma}_{\bar{\Lambda}L}(u, v), \bar{\Gamma}_{\bar{\Lambda}L}(u, v)\}).$$

Here, $M_{\bar{\Lambda}L}(u, v) = \max\{M_{\bar{\Lambda}L}(u, v), M_{\bar{\Lambda}L}(u, v)\}$ and $\bar{\Gamma}_{\bar{\Lambda}L}(u, v) = \min\{\bar{\Gamma}_{\bar{\Lambda}L}(u, v), \bar{\Gamma}_{\bar{\Lambda}L}(u, v)\}$.
Definition 16. An edge in a cubic IFG $\tilde{G} = (\mathcal{V}, E)$ is said to be a bridge, if deleting that edge reduces the strength of connectedness between some pair of vertices.

Example 7. Let Figure 8 be a graph $\tilde{G} = (\mathcal{V}, E)$, where $\mathcal{V} = \{u_1, u_2, u_3, u_4\}$ is the set of vertices and $E = \{u_1u_2, u_2u_3, u_3u_4, u_4u_1\}$ is the set of edges.

The strength of $(u_1, u_4)$ is $[0.1, 0.4), [0.3, 0.5), (0.1, 0.4)$, so $(u_1, u_4)$ is a bridge because when deleting $(u_1, u_4)$ the strength of the connectedness between $u_1$ and $u_4$ is decreased.

Theorem 1. If $\tilde{G} = (\mathcal{V}, E)$ is a cubic IFG, then for any two vertices $y_i$ and $y_j$, the following are equivalent:

(i) $(y_i, y_j)$ is a bridge

(ii) $[M_{\tilde{G}ijL}, M_{\tilde{G}ijU}]^\infty < [M_{\tilde{G}ijL}, M_{\tilde{G}ijU}], M'_{\tilde{G}ij} < M_{\tilde{G}ij}$ and $[\Pi_{\tilde{G}ijU}]^\infty > [\Pi_{\tilde{G}ijL}, \Pi_{\tilde{G}ijU}], \Pi'_{\tilde{G}ij} > \Pi_{\tilde{G}ij}$

(iii) $(y_i, y_j)$ is not an edge of any cycle

Proof. (ii) $\implies$ (i).

Consider $[M_{\tilde{G}ijL}, M_{\tilde{G}ijU}]^\infty < [M_{\tilde{G}ijL}, M_{\tilde{G}ijU}], M'_{\tilde{G}ij} < M_{\tilde{G}ij}$ and $[\Pi_{\tilde{G}ijU}]^\infty > [\Pi_{\tilde{G}ijL}, \Pi_{\tilde{G}ijU}], \Pi'_{\tilde{G}ij} > \Pi_{\tilde{G}ij}$ to show that $(y_i, y_j)$ is a bridge; then $[M_{\tilde{G}ijL}, M_{\tilde{G}ijU}]^\infty = [M_{\tilde{G}ijL}, M_{\tilde{G}ijU}]^\infty \geq [M_{\tilde{G}ijL}, M_{\tilde{G}ijU}], M'_{\tilde{G}ij} = M'_{\tilde{G}ij} \geq M_{\tilde{G}ij}$ and $[\Pi_{\tilde{G}ijU}]^\infty \leq [\Pi_{\tilde{G}ijL}, \Pi_{\tilde{G}ijU}], \Pi'_{\tilde{G}ij} \leq \Pi_{\tilde{G}ij}$, which is a contradiction. Hence, $(y_i, y_j)$ is a bridge.

(i) $\implies$ (iii).

Suppose that $(y_i, y_j)$ is a bridge to show that $(y_i, y_j)$ is not an edge of any cycle. If $(y_i, y_j)$ is an edge of cycle, then any path involving the edge $(y_i, y_j)$ can be converted into a path not involving $(y_i, y_j)$ by using the rest of the cycle as a path from $y_i$ to $y_j$. This implies that $(y_i, y_j)$ cannot be a bridge, which is a contradiction to our supposition. Hence, $(y_i, y_j)$ is not an edge of any cycle.

(iii) $\implies$ (i).

The proof is straightforward. $\square$

Definition 17. A vertex $u_i$ in a cubic IFG $\tilde{G}$ is said to be a cut-vertex if deleting a vertex $u_i$ reduces the strength of connectedness between some pair of vertices.

Example 8. Consider a graph $\tilde{G} = (\mathcal{V}, E)$, where $\mathcal{V} = \{u_1, u_2, u_3, u_4, u_5\}$ is the set of vertices and $E = \{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_1\}$ is the set of edges.

In Figure 9, $u_1$ is a cut-vertex.

4. Operations on Cubic IFG

In this section, the operations of CIFG-like Cartesian product of CIFG, union of CIFG, joint operation of CIFG, and so forth with the help of examples are discussed and some interesting results related to these operations are proved.
Definition 18. The Cartesian product $\tilde{G} = \tilde{G}_1 \times \tilde{G}_2 = (A_1 \times A_2, \mathcal{B}_1 \times \mathcal{B}_2)$ of two cubic IFGs $\tilde{G}_1 = (A_1, \mathcal{B}_1)$ and $\tilde{G}_2 = (A_2, \mathcal{B}_2)$ of the graphs $\tilde{G}_1^* = (\mathcal{V}_1, E_1)$ and $\tilde{G}_2^* = (\mathcal{V}_2, E_2)$ is defined as follows:

\[
\begin{align*}
(M_{AIL} \times M_{A2L})(u_1, u_2) &= \min(M_{A1L}(u_1), M_{A2L}(u_2)), \\
(M_{AIU} \times M_{A2U})(u_1, u_2) &= \min(M_{A1U}(u_1), M_{A2U}(u_2)), \\
(\lceil \mathcal{A} \rceil_{AIL} \times \lceil \mathcal{A} \rceil_{A2L})(u_1, u_2) &= \max(\lceil \mathcal{A} \rceil_{AIL}(u_1), \lceil \mathcal{A} \rceil_{A2L}(u_2)), \\
(\lceil \mathcal{A} \rceil_{AIU} \times \lceil \mathcal{A} \rceil_{A2U})(u_1, u_2) &= \max(\lceil \mathcal{A} \rceil_{AIU}(u_1), \lceil \mathcal{A} \rceil_{A2U}(u_2)), \\
(M_{A1} \times M_{A2})(u_1, u_2) &= \min(M_{A1}(u_1), M_{A2}(u_2)), \\
(\lceil \mathcal{A} \rceil_{AI} \times \lceil \mathcal{A} \rceil_{A2})(u_1, u_2) &= \max(\lceil \mathcal{A} \rceil_{AI}(u_1), \lceil \mathcal{A} \rceil_{A2}(u_2)), \quad \text{for all } u_1, u_2 \in \mathcal{V}.
\end{align*}
\]
Example 9. Let $\bar{G}^* = (\mathcal{V}, E)$ be a graph, where $\mathcal{V}$ is the set of vertices and $E$ is the set of edges; then the product of two cubic IFGs in Figures 10–12 is given below.

Consider $E = \{(u, u_2) \mid u, u_2 \in E_1, u \neq u_2\} \cup \{(u_1, y_2) \mid (y_2, z) \in E_2, u_1 y_1 \in E_1\}$.

Let $(u, u_2) \in E$; then

**Proposition 3.** If $\bar{G}_1$ and $\bar{G}_2$ are strong cubic IFGs, then the Cartesian product $\bar{G}_1 \times \bar{G}_2$ is also strong cubic IFG.

**Proof.** Suppose that $\bar{G}_1$ and $\bar{G}_2$ are strong cubic IFGs; then there exist $u_1, y_1 \in E_1$ such that

\[
\begin{align*}
M_{\bar{G}_1} (u_1, y_1) &= \min \{ M_{\bar{G}_1} (u_1), M_{\bar{G}_1} (y_1) \}, \\
\Omega_{\bar{G}_1} (u_1, y_1) &= \max \{ \Omega_{\bar{G}_1} (u_1), \Omega_{\bar{G}_1} (y_1) \}, \\
M_{\bar{G}_2} (u_1, y_1) &= \min \{ M_{\bar{G}_2} (u_1), M_{\bar{G}_2} (y_1) \}, \\
\Omega_{\bar{G}_2} (u_1, y_1) &= \max \{ \Omega_{\bar{G}_2} (u_1), \Omega_{\bar{G}_2} (y_1) \}, \\
M_{\bar{G}_1} (u_1, y_1) &= \min \{ M_{\bar{G}_1} (u_1), M_{\bar{G}_1} (y_1) \}, \\
\Omega_{\bar{G}_1} (u_1, y_1) &= \max \{ \Omega_{\bar{G}_1} (u_1), \Omega_{\bar{G}_1} (y_1) \}.
\end{align*}
\]

Example 9. Let $\bar{G}^* = (\mathcal{V}, E)$ be a graph, where $\mathcal{V}$ is the set of vertices and $E$ is the set of edges; then the product of two cubic IFGs in Figures 10–12 is given below.

Consider $E = \{(u, u_2) \mid (u, u_2) \in \mathcal{V}_1, u \neq u_2\} \cup \{(u_1, y_2) \mid (y_2, z) \in E_2, u_1 y_1 \in E_1\}$.

Let $(u, u_2) \in E$; then

**Proposition 3.** If $\bar{G}_1$ and $\bar{G}_2$ are strong cubic IFGs, then the Cartesian product $\bar{G}_1 \times \bar{G}_2$ is also strong cubic IFG.

**Proof.** Suppose that $\bar{G}_1$ and $\bar{G}_2$ are strong cubic IFGs; then there exist $u_1, y_1 \in E_1$ such that

\[
\begin{align*}
M_{\bar{G}_1} (u_1, y_1) &= \min \{ M_{\bar{G}_1} (u_1), M_{\bar{G}_1} (y_1) \}, \\
\Omega_{\bar{G}_1} (u_1, y_1) &= \max \{ \Omega_{\bar{G}_1} (u_1), \Omega_{\bar{G}_1} (y_1) \}, \\
M_{\bar{G}_2} (u_1, y_1) &= \min \{ M_{\bar{G}_2} (u_1), M_{\bar{G}_2} (y_1) \}, \\
\Omega_{\bar{G}_2} (u_1, y_1) &= \max \{ \Omega_{\bar{G}_2} (u_1), \Omega_{\bar{G}_2} (y_1) \}, \\
M_{\bar{G}_1} (u_1, y_1) &= \min \{ M_{\bar{G}_1} (u_1), M_{\bar{G}_1} (y_1) \}, \\
\Omega_{\bar{G}_1} (u_1, y_1) &= \max \{ \Omega_{\bar{G}_1} (u_1), \Omega_{\bar{G}_1} (y_1) \}.
\end{align*}
\]

Example 9. Let $\bar{G}^* = (\mathcal{V}, E)$ be a graph, where $\mathcal{V}$ is the set of vertices and $E$ is the set of edges; then the product of two cubic IFGs in Figures 10–12 is given below.
Similarly,

\[
(M_{\mathcal{I}L} \times M_{\mathcal{I}L})(u, u_2)(u, y_2) = \min(M_{\mathcal{I}L}(u), M_{\mathcal{I}L}(u_2)) = \min(M_{\mathcal{I}U}(u), M_{\mathcal{I}U}(u_2)), \\
(M_{\mathcal{I}L} \times M_{\mathcal{A}L})(u_1, u_2) = \min(M_{\mathcal{I}L}(u_1), M_{\mathcal{A}L}(u_2)), \\
(M_{\mathcal{I}L} \times M_{\mathcal{A}L})(u_1, u_2) = \min(M_{\mathcal{I}L}(u_1), M_{\mathcal{A}L}(u_2)), \\
(M_{\mathcal{A}L} \times M_{\mathcal{A}L})(u_1, y_2) = \min(M_{\mathcal{A}L}(u_1), M_{\mathcal{A}L}(y_2)), \\
(M_{\mathcal{A}L} \times M_{\mathcal{A}U})(u_1, y_2) = \min(M_{\mathcal{A}L}(u_1), M_{\mathcal{A}U}(y_2)), \\
(M_{\mathcal{A}L} \times M_{\mathcal{A}U})(u_1, y_2) = \min(M_{\mathcal{A}L}(u_1), M_{\mathcal{A}U}(y_2)),
\]

(13)

\[\min(M_{\mathcal{I}L}(u), M_{\mathcal{I}L}(u_2), M_{\mathcal{I}U}(u), M_{\mathcal{I}U}(u_2), M_{\mathcal{A}L}(u), M_{\mathcal{A}L}(u_2), M_{\mathcal{A}U}(u), M_{\mathcal{A}U}(u_2)).\]
Proposition 4. If $\tilde{G}_1 \times \tilde{G}_2$ is a strong cubic IFG, then at least $\tilde{G}_1$ or $\tilde{G}_2$ must be strong.

Proof. Suppose that $\tilde{G}_1$ and $\tilde{G}_2$ are not strong cubic IFGs, then there exist $u_i, y_i \in E_i$ such that
\begin{align*}
M_{\tilde{G}_1}(u_i, y_i) &< \min(M_{\tilde{G}_1}(u_i), M_{\tilde{G}_1}(y_i)), \\
\Omega_{\tilde{G}_1}(u_i, y_i) &> \max(\Omega_{\tilde{G}_1}(u_i), \Omega_{\tilde{G}_1}(y_i)), \\
M_{\tilde{G}_2}(u_i, y_i) &< \min(M_{\tilde{G}_2}(u_i), M_{\tilde{G}_2}(y_i)), \\
\Omega_{\tilde{G}_2}(u_i, y_i) &> \max(\Omega_{\tilde{G}_2}(u_i), \Omega_{\tilde{G}_2}(y_i)).
\end{align*}

(16)

Consider $E = \{(u, u_2)(u, y_2)/u_2 \in \mathcal{V}'_2, u_2 y_2 \in E_2\} \cup \{(u_1, z)(y_1, z)/z \in \mathcal{V}'_2, u_1, y_1 \in E_1\}$

Let $(u, u_2)(u, y_2) \in E$, then
\begin{align*}
(M_{\tilde{G}_1} \times M_{\tilde{G}_2})(u, u_2)(u, y_2) &= \min((M_{\tilde{G}_1} \times M_{\tilde{G}_2})(u, u_2), (M_{\tilde{G}_1} \times M_{\tilde{G}_2})(u, y_2)), \\
&M_{\tilde{G}_1}(u, u_2) = \min(M_{\tilde{G}_1}(u), M_{\tilde{G}_1}(y)) \\
&< \min(M_{\tilde{G}_1}(u), M_{\tilde{G}_1}(y_2)),
\end{align*}

(17)

Similarly,
\[(M_{\mathcal{G}_1} \times M_{\mathcal{G}_2}')(u, u_2)(u, y_2) = \min(M_{\mathcal{G}_1}(u), M_{\mathcal{G}_2}(u_2)), \]
\[(M_{\mathcal{G}_1} \times M_{\mathcal{G}_2})(u_1, u_2) = \min(M_{\mathcal{G}_1}(u_1), M_{\mathcal{G}_2}(u_2)), \]
\[(M_{\mathcal{G}_1} \times M_{A_{\mathcal{G}_2}})(u_1, u_2) = \min(M_{\mathcal{G}_1}(u_1), M_{A_{\mathcal{G}_2}}(u_2)).\]
Proposition 5. The composition $\tilde{G}_1 [\tilde{G}_2]$ of cubic IFG for the graphs $G_1$ and $G_2$ of the graphs $G_1^*$ and $G_2^*$ is a cubic IFG of $G_1 [G_2^*]$.

\begin{align}
(M_{A1L \cup A2L}) (u_1, u_2) (y_1, y_2) &= \min(M_{A1L} (u_1), M_{A2L} (y_1)), \\
(M_{A1U \cup A2U}) (u_1, u_2) (y_1, y_2) &= \min(M_{A1U} (u_1), M_{A2U} (y_1)), \\
(\Delta_{A1L} \cup \Delta_{A2L}) (u_1, u_2) (y_1, y_2) &= \max(\Delta_{A1L} (u_1), \Delta_{A2L} (y_1)), \\
(\Delta_{A1U} \cup \Delta_{A2U}) (u_1, u_2) (y_1, y_2) &= \max(\Delta_{A1U} (u_1), \Delta_{A2U} (y_1)), \\
(\Delta_{A1} \cap \Delta_{A2}) (u_1, u_2) (y_1, y_2) &= \min(\Delta_{A1} (u_1), \Delta_{A2} (y_1)),
\end{align}

for all $(u_1, u_2) (y_1, y_2) \in E^* - E.$

Example 10. Let $\tilde{G}^* = (\bar{V}, E)$ be a graph; then the compositions of two cubic IFGs in Figures 13–15 are given as follows.

\begin{align}
(M_{A1L} \cdot M_{A2L}) (u_1, z) (y_1, z) &= \min(M_{A1L} (u_1), M_{A2L} (z)), \\
(M_{A1U} \cdot M_{A2U}) (u_1, z) (y_1, z) &= \min(M_{A1U} (u_1), M_{A2U} (z)), \\
(\bar{\Delta}_{A1L} \cdot \bar{\Delta}_{A2L}) (u_1, z) (y_1, z) &= \max(\bar{\Delta}_{A1L} (u_1), \bar{\Delta}_{A2L} (z)), \\
(\bar{\Delta}_{A1U} \cdot \bar{\Delta}_{A2U}) (u_1, z) (y_1, z) &= \max(\bar{\Delta}_{A1U} (u_1), \bar{\Delta}_{A2U} (z)), \\
((M_{A1} \cdot M_{A2}) (u_1, z) (y_1, z) &= \min(M_{A1} (u_1), M_{A2} (z)), \\
(\bar{\Delta}_{A1} \cdot \bar{\Delta}_{A2}) (u_1, z) (y_1, z) &= \max(\bar{\Delta}_{A1} (u_1), \bar{\Delta}_{A2} (z)),
\end{align}

for all $z \in \bar{V}$ and $u_1 y_1 \in E_1.$

Definition 20. The union $\tilde{G}_1 \cdot \tilde{G}_2 = (A_1 \cup A_2, B_1 \cup B_2)$ of two cubic IFGs $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ of the graphs $G_1^* = (\bar{V}_1, E_1)$ and $G_2^* = (\bar{V}_2, E_2)$ is defined as follows:

\begin{align}
(M_{A1L \cup B1L}) (u) &= M_{A1L} (u), \\
(M_{A1U \cup B1U}) (u) &= M_{A1U} (u), \\
(M_{A1L \cup B1L}) (u) &= \max(M_{A1L} (u), M_{A2L} (u)), \\
(M_{A1U \cup B1U}) (u) &= \max(M_{A1U} (u), M_{A2U} (u)), \\
(\Delta_{A1L \cap B1L}) (u) &= \Delta_{A1L} (u), \\
(\Delta_{A1U \cap B1U}) (u) &= \Delta_{A1U} (u), \\
(\Delta_{A1L \cap B1L}) (u) &= \min(\Delta_{A1L} (u), \Delta_{A2L} (u)),
\end{align}

for all $u \in \bar{V}_1 \cap \bar{V}_2.$

Proof. The proof is straightforward. \hfill \Box
\((\{0.2,0.5\},\{0.3,0.6\},(0.3,0.6))\)

\((\{0.1,0.4\},\{0.3,0.6\},(0.3,0.6))\)

\((\{0.1,0.4\},\{0.2,0.5\},(0.4,0.2))\)

Figure 13: Cubic intuitionistic fuzzy graph.

\((\{0.3,0.2\},\{0.4,0.2\},(0.4,0.2))\)

\((\{0.1,0.2\},\{0.4,0.5\},(0.3,0.4))\)

\((\{0.1,0.4\},\{0.2,0.5\},(0.3,0.4))\)

Figure 14: Cubic intuitionistic fuzzy graph.

(iv)

\[
\begin{align*}
\bigl(\cap_{A1 U} \cap_{A2 U}\bigr)(u) &= \cap_{A1 U}(u), & \text{if } u \in V_1 - V_2, \\
\bigl(\cap_{A1 U} \cap_{A2 U}\bigr)(u) &= \cap_{A2 U}(u), & \text{if } u \in V_2 - V_1, \\
\bigl(\cap_{A1 U} \cap_{A2 U}\bigr)(u) &= \min\left(\cap_{A1 U}(u), \cap_{A2 U}(u)\right), & \text{if } u \in V_1 \cap V_2.
\end{align*}
\]

(v)

\[
\begin{align*}
\bigl(M_{A1} \cup M_{A2}\bigr)(u) &= M_{A1}(u), & \text{if } u \in V_1 - V_2, \\
\bigl(M_{A1} \cup M_{A2}\bigr)(u) &= M_{A2}(u), & \text{if } u \in V_2 - V_1, \\
\bigl(M_{A1} \cup M_{A2}\bigr)(u) &= \max\left(M_{A1}(u), M_{A2}(u)\right), & \text{if } u \in V_1 \cap V_2.
\end{align*}
\]
(vi) \[
\begin{align*}
\bigcap_{A1} \cap \bigcap_{A2} (u) &= \bigcap_{A1} (u), & \text{if } u \in \mathcal{V}_1 - \mathcal{V}_2, \\
\bigcap_{A1} \cap \bigcap_{A2} (u) &= \bigcap_{A2} (u), & \text{if } u \in \mathcal{V}_2 - \mathcal{V}_1, \\
\bigcap_{A1} \cap \bigcap_{A2} (u) &= \min(\bigcap_{A1} (u), \bigcap_{A2} (u)), & \text{if } u \in \mathcal{V}_1 \cap \mathcal{V}_2.
\end{align*}
\]

(vii) \[
\begin{align*}
(M_{\mathcal{G}1} \cup M_{\mathcal{G}2L})(u) &= M_{\mathcal{G}1L}(u), & \text{if } uy \in E_1 - E_2, \\
(M_{\mathcal{G}1} \cup M_{\mathcal{G}2L})(u) &= M_{\mathcal{G}2L}(u), & \text{if } uy \in E_2 - E_1, \\
(M_{\mathcal{G}1} \cup M_{\mathcal{G}2L})(u) &= \max(M_{\mathcal{G}1L}(u), M_{\mathcal{G}2L}(u)), & \text{if } y \in E_1 \cap E_2.
\end{align*}
\]

(viii) \[
\begin{align*}
(M_{\mathcal{G}1U} \cup M_{\mathcal{G}2U})(u) &= M_{\mathcal{G}1U}(u), & \text{if } uy \in E_1 - E_2, \\
(M_{\mathcal{G}1U} \cup M_{\mathcal{G}2U})(u) &= M_{\mathcal{G}2U}(u), & \text{if } uy \in E_2 - E_1, \\
(M_{\mathcal{G}1U} \cup M_{\mathcal{G}2U})(u) &= \max(M_{\mathcal{G}1U}(u), M_{\mathcal{G}2U}(u)), & \text{if } uy \in E_1 \cap E_2.
\end{align*}
\]

(ix) \[
\begin{align*}
\bigcap_{\mathcal{G}1L} \cap \bigcap_{\mathcal{G}2L}(u) &= \bigcap_{\mathcal{G}1L}(u), & \text{if } uy \in E_1 - E_2, \\
\bigcap_{\mathcal{G}1L} \cap \bigcap_{\mathcal{G}2L}(u) &= \bigcap_{\mathcal{G}2L}(u), & \text{if } uy \in E_2 - E_1, \\
\bigcap_{\mathcal{G}1L} \cap \bigcap_{\mathcal{G}2L}(u) &= \min(\bigcap_{\mathcal{G}1L}(u), \bigcap_{\mathcal{G}2L}(u)), & \text{if } uy \in E_1 \cap E_2.
\end{align*}
\]

(x) \[
\begin{align*}
\bigcap_{\mathcal{G}1U} \cap \bigcap_{\mathcal{G}2U}(u) &= \bigcap_{\mathcal{G}1U}(u), & \text{if } uy \in E_1 - E_2, \\
\bigcap_{\mathcal{G}1U} \cap \bigcap_{\mathcal{G}2U}(u) &= \bigcap_{\mathcal{G}2U}(u), & \text{if } uy \in E_2 - E_1, \\
\bigcap_{\mathcal{G}1U} \cap \bigcap_{\mathcal{G}2U}(u) &= \min(\bigcap_{\mathcal{G}1U}(u), \bigcap_{\mathcal{G}2U}(u)), & \text{if } uy \in E_1 \cap E_2.
\end{align*}
\]

(xi) \[
\begin{align*}
(M_{\mathcal{G}1} \cup M_{\mathcal{G}2})(u) &= M_{\mathcal{G}1}(u), & \text{if } uy \in E_1 - E_2, \\
(M_{\mathcal{G}1} \cup M_{\mathcal{G}2})(u) &= M_{\mathcal{G}2}(u), & \text{if } uy \in E_2 - E_1, \\
(M_{\mathcal{G}1} \cup M_{\mathcal{G}2})(u) &= \max(M_{\mathcal{G}1}(u), M_{\mathcal{G}2}(u)), & \text{if } uy \in E_1 \cap E_2.
\end{align*}
\]
Proposition 6. The union of two cubic IFGs is a cubic IFG.

Example 11. Let $\mathcal{G}^* = (\mathcal{V}, E)$ be a graph; then the union of two cubic IFGs is given below.

In Figures 16–18 the union of two CIFGs is defined.

Proof. Let $\tilde{G}_1 = (A_1, B_1)$ and $\tilde{G}_2 = (A_2, B_2)$ be the cubic IFGs $\mathcal{G}^*_1$ and $\mathcal{G}^*_2$, respectively. Then, we have to prove $\tilde{G}_1 \cup \tilde{G}_2 = (A_1 \cup A_2, B_1 \cup B_2)$ is a cubic IFG and of the graphs $\mathcal{G}^*_1 \cup \mathcal{G}^*_2$. As all the conditions of $A_1 \cup A_2$ are satisfied, we only have to verify the conditions of $B_1 \cup B_2$.

First assume that $uv \in E_1 \cap E_2$. Then,

\[
\begin{align*}
(M_{\#1L} \cup M_{\#2L})(uv) &= \max(M_{\#1L}(uv), M_{\#2L}(uv)) \\
&\leq \max(\min(M_{A1L}(u), M_{A2L}(y)), \min(M_{A2L}(u), M_{A1L}(y))) \\
&= \min(M_{A1L}(u), M_{A2L}(u)), \max(M_{A1L}(y), M_{A2L}(y)), \\
&= \min(M_{A1L} \cup M_{A2L})(u), (M_{A1L} \cup M_{A2L})(y), \\
(M_{\#1U} \cup M_{\#2U})(uv) &= \max(M_{\#1U}(uv), M_{\#2U}(uv)) \\
&\leq \max(\min(M_{A1U}(u), M_{A2U}(y)), \min(M_{A2U}(u), M_{A1U}(y))) \\
&= \min(M_{A1U}(u), M_{A2U}(u)), \max(M_{A1U}(y), M_{A2U}(y)) \\
&= \min(M_{A1U} \cup M_{A2U})(u), (M_{A1U} \cup M_{A2U})(y), \\
(\Omega_{\#1L} \cup \Omega_{\#2L})(uv) &= \min(\Omega_{\#1L}(uv), \Omega_{\#2L}(uv)) \\
&\leq \min(M_{A1L} \cup \Omega_{\#1L})(u), (M_{A1L} \cup M_{\#2L})(y), \\
&= \min(M_{A1L} \cup M_{\#2L})(u), (M_{A1L} \cup M_{A2L})(y), \\
(M_{\#1} \cup M_{\#2})(uv) &= \max(M_{\#1}(uv), M_{\#2}(uv)) \\
&\leq \max(\min(M_{A1}(u), M_{A2}(y)), \min(M_{A2}(u), M_{A1}(y))) \\
&= \min(M_{A1}(u), M_{A2}(u)), \max(M_{A1}(y), M_{A2}(y)) \\
&= \min(M_{A1} \cup M_{A2})(u), (M_{A1} \cup M_{A2})(y), \\
(\Omega_{\#1} \cup \Omega_{\#2})(uv) &= \min(\Omega_{\#1}(uv), \Omega_{\#2}(uv)) \\
&\leq \min(M_{A1} \cup \Omega_{\#1})(u), (M_{A1} \cup M_{\#2})(y), \\
&= \min(M_{A1} \cup M_{\#2})(u), (M_{A1} \cup M_{A2})(y), \\
&= \min(M_{A1} \cup M_{A2})(u), (M_{A1} \cup M_{A2})(y).
\end{align*}
\]
If \( uy \in E_1 \) and \( uy \notin E_2 \), then

\[
\begin{align*}
(M_{\mathcal{A}1} \cup M_{\mathcal{A}2})(uy) & \leq \min((M_{\mathcal{A}1L} \cup M_{\mathcal{A}2L})(u), (M_{\mathcal{A}1L} \cup M_{\mathcal{A}2L})(y)), \\
(M_{\mathcal{A}1} \cup M_{\mathcal{A}2})(uy) & \leq \min((M_{\mathcal{A}1U} \cup M_{\mathcal{A}2U})(u), (M_{\mathcal{A}1U} \cup M_{\mathcal{A}2U})(y)), \\
(\mathfrak{P}_{\mathcal{A}1L} \cup \mathfrak{P}_{\mathcal{A}2L})(uy) & \leq \max((\mathfrak{P}_{\mathcal{A}1L} \cup \mathfrak{P}_{\mathcal{A}2L})(u), (\mathfrak{P}_{\mathcal{A}1L} \cup \mathfrak{P}_{\mathcal{A}2L})(y)), \\
(\mathfrak{P}_{\mathcal{A}1U} \cup \mathfrak{P}_{\mathcal{A}2U})(uy) & \leq \max((\mathfrak{P}_{\mathcal{A}1U} \cup \mathfrak{P}_{\mathcal{A}2U})(u), (\mathfrak{P}_{\mathcal{A}1U} \cup \mathfrak{P}_{\mathcal{A}2U})(y)), \\
(M_{\mathcal{B}1} \cup M_{\mathcal{B}2})(uy) & \leq \min((M_{\mathcal{A}1} \cup M_{\mathcal{A}2})(u), (M_{\mathcal{A}1} \cup M_{\mathcal{A}2})(y)), \\
(\mathfrak{N}_{\mathcal{B}1} \cup \mathfrak{N}_{\mathcal{B}2})(uy) & \leq \max((\mathfrak{N}_{\mathcal{A}1} \cup \mathfrak{N}_{\mathcal{A}2})(u), (\mathfrak{N}_{\mathcal{A}1} \cup \mathfrak{N}_{\mathcal{A}2})(y)).
\end{align*}
\]  

(38)
Figure 18: Union of cubic intuitionistic fuzzy graphs.

If \( u y \notin E_1 \) and \( u y \in E_2 \), then

\[
(M_{\mathcal{A}L} \cup M_{\mathcal{A}2})(u y) \leq \min((M_{\mathcal{A}1L} \cup M_{\mathcal{A}2L})(u), (M_{\mathcal{A}1U} \cup M_{\mathcal{A}2U})(y)),
\]

\[
(M_{\mathcal{A}1U} \cup M_{\mathcal{A}2U})(u y) \leq \min((M_{\mathcal{A}1U} \cup M_{\mathcal{A}2U})(u), (M_{\mathcal{A}1U} \cup M_{\mathcal{A}2U})(y)),
\]

\[
(\mathcal{A}1 \cup \mathcal{A}2)(u y) \leq \max((\mathcal{A}1 \cup \mathcal{A}2)(u), (\mathcal{A}1 \cup \mathcal{A}2)(y)),
\]

\[
(M_{\mathcal{A}1L} \cup M_{\mathcal{A}2L})(u y) \leq \min((M_{\mathcal{A}1L} \cup M_{\mathcal{A}2L})(u), (M_{\mathcal{A}1L} \cup M_{\mathcal{A}2L})(y)),
\]

\[
(\mathcal{A}1 \cup \mathcal{A}2)(u y) \leq \max((\mathcal{A}1 \cup \mathcal{A}2)(u), (\mathcal{A}1 \cup \mathcal{A}2)(y)).
\]

(39)

This completes the proof. \( \square \)

Definition 21. The joint \( \tilde{G}_1 \times \mathcal{G}_2 = (A_1 + A_2, \tilde{S}_1 + \tilde{S}_2) \) of two cubic IFGs \( \mathcal{G}_1 = (A_1, \tilde{S}_1) \) and \( \mathcal{G}_2 = (A_2, \tilde{S}_2) \) of the graphs \( \mathcal{G}_1^* = (\tilde{V}_1, E_1) \) and \( \mathcal{G}_2^* = (\tilde{V}_2, E_2) \) is defined as follows:

(i) \[
(M_{\mathcal{A}1L} + M_{\mathcal{A}2L})(u) = (M_{\mathcal{A}1L} \cup M_{\mathcal{A}2L})(u),
\]

\[
(M_{\mathcal{A}1U} + M_{\mathcal{A}2U})(u) = (M_{\mathcal{A}1U} \cup M_{\mathcal{A}2U})(u),
\]

\[
(\mathcal{A}1 + \mathcal{A}2)(u) = (\mathcal{A}1 \cup \mathcal{A}2)(u).
\]

(40)

If \( u \in \tilde{V}_1 \cup \tilde{V}_2 \),

(ii) \[
(M_{\mathcal{A}1L} + M_{\mathcal{A}2L})(u y) = (M_{\mathcal{A}1L} \cup M_{\mathcal{A}2L})(u y),
\]

\[
(M_{\mathcal{A}1U} + M_{\mathcal{A}2U})(u y) = (M_{\mathcal{A}1U} \cup M_{\mathcal{A}2U})(u y),
\]

\[
(\mathcal{A}1 \ cup \mathcal{A}2)(u y) = (\mathcal{A}1 \cup \mathcal{A}2)(u y).
\]

\[
(\mathcal{A}1 \cup \mathcal{A}2)(u y) = (\mathcal{A}1 \cup \mathcal{A}2)(u y),
\]

(41)

\[
(u y \in E_1 \cap E_2, \quad \text{and then}
\]

\[
(M_{\mathcal{A}1L} + M_{\mathcal{A}2L})(u y) = \min(M_{\mathcal{A}1L}(u), M_{\mathcal{A}2L}(y)),
\]

\[
(M_{\mathcal{A}1U} + M_{\mathcal{A}2U})(u y) = \min(M_{\mathcal{A}1U}(u), M_{\mathcal{A}2U}(y)),
\]

\[
(\mathcal{A}1 \cup \mathcal{A}2)(u y) = \max(\mathcal{A}1 \cup \mathcal{A}2)(u y),
\]

(42)

\[
(\mathcal{A}1 \cup \mathcal{A}2)(u y) = \max(\mathcal{A}1 \cup \mathcal{A}2)(u y),
\]

\( u y \in E' \), where \( E' \) is the set of all edges joining the nodes of \( \tilde{V}_1 \) and \( \tilde{V}_2 \).
The joint of two cubic IFGs is a cubic IFG.

Proof. Assume that $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ are two cubic IFGs of the graphs $G_1 = (\mathcal{V}_1, E_1)$ and $G_2 = (\mathcal{V}_2, E_2)$. Then, we have to prove $G_1 + G_2 = (A_1 + A_2, B_1 + B_2)$ is a cubic IFG. In view of proposition 6 it is sufficient to verify the case when $uy \in E_t$. In this case, we have

\[
(M_{u1} \cup M_{u2})(uy) = \min((M_{A1} (u), (M_{A2} (y)))
\]

\[
(M_{u1} \cup M_{u2})(uy) = \min((M_{A1} + M_{A2}) (u), (M_{A1} + M_{A2}) (y)),
\]

\[
(M_{u1} \cup M_{u2})(uy) = \min((M_{A1}(u), (M_{A2}(y)))
\]

\[
(M_{u1} \cup M_{u2})(uy) \leq \min((M_{A1} \cup M_{A2})(u), (M_{A1} \cup M_{A2})(y))
\]

\[
(M_{u1} \cup M_{u2})(uy) \leq \min((M_{A1} + M_{A2})(u), (M_{A1} + M_{A2})(y))
\]

\[
(\eta_{u1} \cup \eta_{u2})(uy) = \max((\eta_{A1}(u), (\eta_{A2}(y)))
\]

\[
(\eta_{u1} \cup \eta_{u2})(uy) \leq \max((\eta_{A1} \cup \eta_{A2})(u), (\eta_{A1} \cup \eta_{A2})(y))
\]

\[
(\eta_{u1} \cup \eta_{u2})(uy) \leq \max((\eta_{A1} + \eta_{A2})(u), (\eta_{A1} + \eta_{A2})(y))
\]

This completes the proof.

5. Application

In this section, we apply the concept of CIFGs in multi-attribute decision-making problem, where the selection of suitable subjects has been carried out.

There are many career options for the students of present times. Moreover, some of the courses are usually chosen where all the available choices remain superior and best choices until a single student has to choose a field of his interest by keeping in view his preferences. At the finishing of college level education requires selecting their first choice of career planning. During this time, pupils must be given enough information about choosing career according to their interest. According to the survey of random sample of 100 pupils of class X carried out in this part, pupils with favour of interests and no favouring of choices of a specific subject up to class X are measured and given below. Based on the data, cubic nonrational fuzzy graph is used as a tool as it makes the level of membership (interval-valued membership) (percentage of students who favour a subject or a pair of subjects) and level of nonmembership (interval-valued nonmembership) (percentage of students who disfavour a subject or a pair of subjects). Employing CIFS, the best subject’s combination may be evaluated that are the class having subjects that could be productive to most students and have best academic performance of most of the students.

Let $S = \{\text{English (E)}, \text{Language (L)}, \text{Maths (M)}, \text{Science (S)}, \text{Social Sciences (SS)}\}$ be the set of vertices. Tables 1 and 2 illustrate the percentages of students with interest/disinterest towards a subject or a pair of subjects.

Based on the above information, we generate an CIFG as follows (Figure 19).

In every vertex of the graph, the degree of membership shows the percentage of students with zeal for a specific subject and the degree of nonmembership is the percentage of students with no zeal in subject from a random sample of 100 students of class X chosen for survey. Also, the corners of graph of both membership and nonmembership show the favour and disfavour of students to study the combined subjects at higher secondary corner. From the given graph, the corner $(L - SS)$ possesses high degree of nonmembership, which shows that majority of pupils do not like to study the combined subjects Language and Social Science, and the corner $(M - S)$ possesses high degree of membership, which shows that majority of pupils have zeal for studying the
combined subjects of Math and Science. There is disfavour to study the combined subjects of Tamil and Math, which indicates that these subjects do not require to be combined. Therefore, a high (low) level of membership of any corner shows the high (low) weightage of combined subjects at higher studies.

Table 1: Subject combination.

| Subject combination | Interest percentage | Disinterest percentage |
|---------------------|---------------------|------------------------|
| E                   | [0.3, 0.4], 0.3     | [0.4, 0.5], 0.7        |
| L                   | [0.2, 0.4], 0.4     | [0.55, 0.6], 0.6       |
| M                   | [0.2, 0.3], 0.3     | [0.6, 0.7], 0.5        |
| S                   | [0.1, 0.4], 0.5     | [0.5, 0.6], 0.4        |
| SS                  | [0.2, 0.3], 0.7     | [0.3, 0.6], 0.3        |

Table 2: Subjects combinations.

| Subjects combination | Interest percentage | Disinterest percentage |
|----------------------|---------------------|------------------------|
| E – M                | [0.2, 0.3], 0.3     | [0.6, 0.7], 0.7        |
| E – L                | [0.2, 0.4], 0.3     | [0.55, 0.6], 0.7       |
| E – S                | [0.1, 0.4], 0.3     | [0.5, 0.6], 0.7        |
| E – SS               | [0.2, 0.3], 0.3     | [0.4, 0.6], 0.7        |
| L – M                | [0.2, 0.3], 0.3     | [0.6, 0.7], 0.6        |
| L – S                | [0.1, 0.4], 0.4     | [0.55, 0.6], 0.6       |
| L – SS               | [0.2, 0.3], 0.4     | [0.55, 0.6], 0.6       |
| M – S                | [0.1, 0.3], 0.3     | [0.6, 0.7], 0.5        |
| M – SS               | [0.2, 0.3], 0.3     | [0.6, 0.7], 0.5        |
| S – SS               | [0.1, 0.3], 0.5     | [0.5, 0.6], 0.4        |

Figure 19: Cubic intuitionistic fuzzy graph.

6. Comparison

Proposition 8. A cubic IFG is a generalization of cubic FG.

Proof. Let $G^* = (\mathcal{V}', E)$ be a cubic IFG. Then if we put the value of nonmembership of the vertex set and edge set as
zero in the IVFS and FS, then the cubic IFG reduces to cubic FG.

**Proposition 9.** An IVIFG is a generalization of IVFG.

*Proof.* Let \( \tilde{G} = (\mathcal{V},E) \) be an IVIFG. If we put the value of nonmembership of the vertex set and edge set as zero, then the IVIFG reduces to IVFG.

**Proposition 10.** An IFG is a generalization of FG.

*Proof.* Let \( \tilde{G} = (\mathcal{V},E) \) be an IFG. If we put the value of nonmembership of the vertex set and edge set as zero, then the IFG reduces to FG.

### 7. Conclusion

In this article, we developed a novel concept of CIFG as a generalization of IFGs. The graph theoretic terms like subgraphs, complements, degree of vertices, strength of graphs, paths, and cycle are briefly presented with the help of examples. Some related results and properties of the defined concepts are discussed. The generalization of CIFG is proved by some examples and remarks. A comparison of CIFG with IFG and other related concepts is given. The theory of CIFG is a generalization of IFG and can be applied to many real-life problems such as shortest path problem, communication problem, cluster analysis, and traffic signal problems. In the future, the graphs of the cubic Pythagorean fuzzy sets, cubic q-rung orthopair fuzzy sets, and cubic spherical fuzzy sets can be developed and different aggregation operators are defined for better decision-making.

### Data Availability

No data were used in this study.

### Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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