Images and Spectra of Time Dependent Two Component Advective Flow in Presence of Outflows

Arka Chatterjee\textsuperscript{1} *, Sandip K. Chakrabarti\textsuperscript{2,1,†}, Himadri Ghosh\textsuperscript{4,3,1} & Sudip K. Garain\textsuperscript{4,§}

\textsuperscript{1}Indian Centre for Space Physics, Chalantika 43, Garra Station Rd., Kolkata, 700084, India
\textsuperscript{2}S. N. Bose National Centre for Basic Sciences, Salt Lake, Kolkata, 700098, India
\textsuperscript{3}Heritage Institute of Technology, Kolkata, 700107, India
\textsuperscript{4}Department of Physics, University of Notre Dame, Notre Dame, IN 46556, USA

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
Two Component Advective Flow (TCAF) successfully explains the spectral and temporal properties of outbursting or persistent sources. Images of static TCAF with Compton cloud or CENtrifugal pressure supported Boundary Layer (CENBOL) due to gravitational bending of photons have been studied before. In this paper, we study time dependent images of advective flows around a Schwarzschild black hole which include cooling effects due to Comptonization of soft photons from a Keplerian disks as well as the self-consistently produced jets and outflows. We show the overall image of the disk-jet system after convolving with a typical beamwidth. A long exposure image with time dependent system need not show the black hole horizon conspicuously, unless one is looking at a soft state with no jet or the system along the jet axis. Assuming these disk-jet configurations are relevant to radio emitting systems also, our results would be useful to look for event horizons in high accretion rate Supermassive Black Holes in Seyfert galaxies, RL Quasars.

Key words: black hole physics – accretion, accretion discs – hydrodynamics – radiative transfer – relativistic processes

1 INTRODUCTION
Spectral and temporal properties of galactic and extragalactic black holes suggest time dependence of various degrees. Imaging a black hole accretion disk would therefore not be accurate without inclusion of the time dependence of various disk components. There is a class of stellar mass black hole candidates, known as the outbursting sources, which totally changes its disk configurations in a matter of few days, when it goes through transitions of its spectral states from hard to hard intermediate to soft intermediate and then soft states during their rising phase. Recently, through fits of data with Two Component Advective Flow (TCAF) solution as prescribed by Chakrabarti & Titarchuk (1995; hereafter, CT95), the accretion rates of the halo and disk components, the sizes of the hot electron cloud (‘Compton cloud’) etc. were all shown to be highly variable. The Compton cloud in this solution is the inner part of the advective halo, puffed up due to slowing down by the centrifugal barrier, which not only inverse Comptonizes the soft photons to produce high energy X-rays, but also produces outflowing matter just as a normal boundary layer. The disk component on the equatorial plane supplies soft photons. In the absence of the CENBOL as in the soft state, the flow behaves as a standard thin disk with the inner edge close to the inner stable circular orbit or ISCO (Shakura & Sunyaev, 1973) and no jets or outflows are seen. In class-variable sources, such as GRS 1915+105, temporal variation in significant count rates and state transitions such as (burst-on and burst-off states) are seen in a matter of few seconds (Muno et al. 1999). This source also exhibits strong relativistic radio jets (Fender et al. 1999). Along with the monotonic changes in the CENBOL sizes, the frequency of the Quasi-Periodic Oscillations (QPOs) is also seen to change monotonically (Chakrabarti et al. 2005). Theoretical work which forms the basis of TCAF is in Chakrabarti (1996) and references therein.

The time dependent simulations (Giri, Garain & Chakrabarti 2013 and references therein) show the formation of two components in the flow with the Keplerian disk on the equatorial plane and advective halo surrounding it formed the shock and outflows exactly as envisaged in CT95. Outflows are intrinsically associated with accretion mechanism (see C90 and Chakrabarti 1996 for further details) in this solution. The explicit ratio of mass outflow rate to the
inflow rate was first computed in Chakrabarti (1999) and was found to strongly depend on the compression ratio at the shock. From Fig. 4 of Giri & Chakrabarti (2012), it is clear that the ratio of the outflow to the inflow rate increases with decreasing viscosity parameter and the inviscid flow produces maximum outflows. In outbursts or class variable sources, outflows are found to be sporadic and highly variable. Thus, it is very important to have the time dependent spectra and images of TCAF where outflows are self-consistently generated from the CENBOL itself. Cooling via Comptonization is an essential mechanism by which the CENBOL reduces its size making the spectrum softer. While the CENBOL lasts, rough agreement between the cooling and compressional heating timescales causes it to oscillate and produce the so-called QPOs (Chakrabarti et al. 2015 and references therein). Detailed hydrodynamic simulations in presence of Compton cooling are in Ghosh, Garain & Chakrabarti (2011) and Garain, Ghosh & Chakrabarti (2014) where effects of cooling on the spectra, shock location and the formation of QPOs are described.

Theoretical efforts for imaging of accretion disks started with the pioneering work by Luminet (1978) who drew the image of a standard Keplerian disk around a Schwarzschild black hole (Shakura-Sunyaev, 1973). The method of image construction was done from the observers end. This technique is very useful as photons were not lost while tracking them from the observer. Following this, Fukue & Yokohama (1988) published the colored version of the image. With that, they also studied the occultation effect on accretion disk light curve. A few years later, Viergutz (1992) generalized the transfer functions for Kerr geometry. Mark (1996), for the first time, produced the Keplerian disk image using the actual Ray-Tracing mechanism. Bromley et al. (2001) added polarization determination. Dexter & Agol (2009) first published ‘goekerr’, a public code to compute images in Kerr geometry. ‘GYOTO’ (Vincent et al., 2011), ‘YNOGK’ (Yang & Wang, 2013), ‘GRTRANS’ (Dexter, 2016) came into public domains. The focus of these works was to prepare one to interpret results from the Event Horizon Telescope (EHT) whose objective is to identify the horizon of the supermassive black hole at the centre of our Galaxy.

Armitage & Reynolds (2003), first performed MHD simulation of Keplerian disk and produced the images where the disk structure remained the same at all the time. More recently, Broderick & Loeb (2009) presented results where simulated images of force free jets (Tekhkovskoy et al. 2008) launched from M87 are shown for various model parameters and polarization maps of the magnetically driven jets are obtained. However, under TCAF paradigm, one does not have to introduce magnetic field to generate outflows. The hydrodynamic inflow produces time varying outflows self-consistently. Magnetic field may be useful for collimation and acceleration purposes further out.

GRMHD simulations in the context of our Galactic centre were mostly performed (Ohsuga et al. 2005; Noble et al. 2007; Moscibrodzka et al. 2009, 2011; Dexter et al. 2010; Hibburn et al. 2010; Dexter & Fragile 2012; Dolence et al. 2012; Shecherbakov, Penna & McKinney 2012, Sadowski et al. 2013, Roelofs et al. 2017) considering a very low accretion rate as applicable. Cooling term was neglected as appropriate for Sgr A*. In fact, Dibi et al. (2012) added the cooling term and found the effect to be negligible in case Sgr A*. Drappeau et al. (2013) constrained the mass accretion rate to be $\sim 10^{-7} M_\odot$/yr with a highly spinning central black hole. This accretion rate is much lower than that of the earlier result ($\sim 10^{-5} M_\odot$/yr) predicted by Coker & Melia (1997). A detailed review on the simulations and observational possibilities of the black hole shadow of Sgr A* was presented by Falcke & Markoff (2013). But, most of the earlier studies were based on the RIAF model to simulate spectra and images of Sgr A* only. However, there exists a large number of SMBHs in Radio Loud Quasars, Seyfert galaxies, where the accretion rate could be substantially high (Bian & Zhao, 2003) and thus formation of disk and cooling becomes important. Another situation which might induce a sudden high mass inflow is from tidal disruption of neighboring object (Gillessen et al. 2012).

Chatterjee, Chakrabarti & Ghosh, (2017a, hereafter CCG17a) showed the static images of TCAF with general relativistic thick disks acting as the Compton cloud and relativistic Keplerian disk (Page & Thorne, 1974) acting as the source of seed photons. Images for various disk rates and inclination angle are shown with their observable spectral counterparts. The variation of optical depth of the CENBOL medium is achieved by changing the halo rate ($\dot{m}_h$). They have shown that by increasing the optical depth, the Keplerian disk from the other side is completely blocked by the CENBOL medium. Also, with increasing inclination angle, the spectral hardening can be seen in their work. Following the same geometry, Chatterjee, Chakrabarti & Ghosh, 2017b, (hereafter CCG17b) simulated the time lags of various energy bins and compare the simulated results with the observed counterparts.

In this paper, for the first time, we present time dependent images of TCAF with Comptonization and outflows. Basic simulation process by hydrodynamic TVD code is explained §2. Thermodynamic properties of CENBOL and Keplerian disk are presented in §3. Section deals with Monte-Carlo simulation of Comptonization and cooling mechanism. The spectra, time dependent images with and without cooling effects are presented in the results section. Convolved images of accretion disk at different spectral states are presented. We discuss our results and conclude this paper in the final section.

2 TIME DEPENDENT SIMULATIONS OF TCAF

Hydrodynamic simulation procedure is similar to what has been earlier reported in Ryu et al. (1997), Giri et al. (2010), Giri & Chakrabarti (2012, 2013), Ghosh, Garain & Chakrabarti (2015). Governing equations of TVD are explicitly presented by Ryu et al. 1996; Molteni et al. 1996; Giri & Chakrabarti 2012. The conservation equation of mass, momentum and energy for an axisymmetric, inviscid, non-magnetic flow takes the following form

$$\frac{\partial q}{\partial t} + \frac{1}{r} \frac{\partial F_1}{\partial \theta} + \frac{\partial F_2}{\partial r} + \frac{\partial G}{\partial z} = S,$$

(1)
where \( q = \left( \begin{array}{c} \rho \\ \rho v_r \\ \rho v_\theta \\ E 
\end{array} \right) \) is the state vector,

\[
F_1 = \left( \begin{array}{c} \rho v_r \\ \rho v_\theta \\ \rho v_r v_\theta \\ (E + \rho)v_r 
\end{array} \right), \quad F_2 = \left( \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 
\end{array} \right),
\]

are the flux functions.

The source function is defined as

\[
S = \left( \begin{array}{c} \rho \frac{v_r^2}{r} - 2\sqrt{r^2 + Z^2} p \\ -2\sqrt{r^2 + Z^2} (v_r v_\theta + v_\theta v_r) \\ -2\sqrt{r^2 + Z^2} v_r v_\theta \\ -2\sqrt{r^2 + Z^2} v_r v_\theta 
\end{array} \right).
\]

The energy density \( E = \rho/(\gamma - 1) + \rho(v_r^2 + v_\theta^2 + v_z^2)/2 \) is the sum of thermal and kinetic energy of the infalling matter. The equation corresponding to energy density can be extracted as

\[
\frac{\partial E}{\partial t} + \frac{1}{r} \left( \frac{\partial (r\rho v_r E)}{\partial r} + \frac{\partial (r\rho v_\theta E)}{\partial \theta} \right) = \frac{\rho (r v_r + z v_z)}{2\sqrt{r^2 + Z^2} + \sqrt{r^2 + Z^2}}.
\]

During cooling, change in \( E \) directly modifies the flow configurations following Eqn (1).

The gravitational field, as far as the hydrodynamics is concerned, is assumed to follow the Paczyński & Wiita (1980) pseudo-Newtonian potential which is given by,

\[
\phi(r, Z) = \frac{GM_\bullet}{r - r_g}
\]

where \( G \) is the universal gravitational constant, \( M_\bullet \) is the mass of the black hole kept at 10 \( M_\odot \) throughout the simulation and \( r_g = GM_\bullet/c^2 = 1 \) is the Schwarzschild radius and \( r = \sqrt{r^2 + Z^2} \). Velocity of light is denoted by \( c \). The flow is considered to be adiabatic and axisymmetric with the black hole sitting at centre of the cylindrical co-ordinate system defined by \( (r, \theta, Z) \). In our current simulation, we have ignored viscosity to have the strong and hot shock and consequently highest outflow rate. Thus, the specific angular momentum \( \lambda \) is constant everywhere. Also, from C89 and C90, it is known that for a steady state situation the specific energy \( \varepsilon \) is also constant. Matter is injected at the outer boundary with specific energy \( \varepsilon = 0.003 \) (in units of \( c^2 \)) and specific angular momentum \( \lambda = 1.8 \) in units of \( cr_g \).

TVD scheme was originally developed by Harten (1983) to solve the hyperbolic equations present in the hydrodynamic conservation equations. This method works on the second order accuracy by modifying first order flux function to use the non-oscillatory results in the second order solutions. The details of the code were presented in Ryu et al. (1995, 1997) where the code is modified to suit black hole accretion.

Our computational domain is defined as: 0 < \( r < 100 \) and 0 < \( Z < 100 \) in \( r - Z \) plane with a grid resolution of 512 \( \times \) 512. So, each grid size is \( \delta r = 0.195r_g \) and \( \delta Z = 0.195r_g \). The black hole absorption boundary is kept at 1.5\( r_g \) after which all informations about the matter as well as photons are lost. Before starting the simulation, entire grid is filled with a low floor density (10\(^{-8}\) where the injected density is 1 in code unit) to avoid any numerical singularities. At the outer boundary sub-Keplerian matter is allowed to flow through. During the accretion, the matter follows the inviscid hybrid accretion mechanism (Chakrabarti 1990) and forms a shock at a specified position. We assume axisymmetry and perform our hydro-simulation on the first quadrant of the \( r-Z \) plane. The black hole is located at the origin of the \( r-Z \) coordinate system. The initial pressure is chosen such that the sound speed as of the interior material is same as the incoming matter. Also, inside the computational domain, all the velocity components are set to zero initially. The simulation result does not depend on the chosen initial condition as the initial matter is washed away by the incoming matter in a few dynamical time. On the right radial boundary, we use an inflow boundary condition. Incoming matter enters the computational domain through this boundary. Since the density of the incoming matter is chosen to be unity, it has 10\(^5\) times higher pressure and it rushes towards the central black hole practically through vacuum until it hits the centrifugal barrier.

Total dynamical time scale for this simulation is 4000 which in physical units (2\(GM_\bullet M_\odot \times f_1 \), where \( f_1 \) contains informations of each time) is about 40b. In the present context, the hydrodynamical simulation has been performed for Galactic black hole candidates. However, these results will also hold for supermassive or intermediate mass black holes.

3 DENSITY AND TEMPERATURE PROFILE OF THE ELECTRON CLOUD

CENBOL is a region of hot electron cloud where inverse Comptonization of soft photons originated from the Keplerian disk occurs. The electrons in this region are ultrarelativistic. So, \( \gamma = 4/3 \) is a good assumption for this region. Number density (per \( cm^3 \)) and temperature (in the kilo electron Volt) of the electrons are supplied for each grid. The pressure and density is related via polytropic equation of state \( p = K \rho^{\gamma} \), where \( p \) is the pressure, \( K \) the entropy constant and \( \rho \) is the density of the flow. From this, we can write \( T \propto p^{1/3} \), where \( T \) is the temperature of the electron cloud.

In our studies, the maximum number density of electron is \( \sim 10^{19} \) per cm\(^3\) and temperature is \( \sim 250 \) keV. Hydrodynamical simulation changes the configuration of CENBOL region for each time stamp. The variation of electron cloud geometry with number density and temperature as given in the color bar are presented in Figs. 1.

From Fig. 1, the region of outflows can be distinctly seen as a region where temperature and electron number density drops rapidly. Over the time, the geometry of the CENBOL region changes as hydrodynamical simulation progresses. One can easily differentiate between Fig. 1a and Fig. 1f as the amount of outflow increased substantially in the time passed. The inner isobaric contours of the CENBOL region (Figs. 1a-f) behave similar to thick disks (Abramowicz et al. 1978; Kozlowski et al. 1978; Paczyński & Wiita 1980; Begelman, Blandford & Rees 1982; Chakrabarti, 1985a) as
Electron Density (in the units of number per \( \text{cm}^{-3} \) shown in log scale, left panel) and temperature (in the units of keV, right panel) distribution in the post-shock region at (a) \( t = 14.25 \text{s} \), (b) \( t = 14.35 \text{s} \), (c) \( t = 14.45 \text{s} \), (d) \( t = 14.55 \text{s} \), (e) \( t = 14.65 \text{s} \) and (f) \( t = 14.75 \text{s} \).

**Figure 1.**

Fraction of mass absorption (circle-black), outflows (triangle-red) and the ratio of outflowing matter to that of a b-absorption (square-green) through the inner boundary with time are shown. Injected halo rate (\( \dot{m}_h \)) is kept fixed at 0.1.

**Figure 2.**

pointed out by Molteni, Lanzafame & Chakrabarti (1994, hereafter MLC94).

The TVD method simulates a true accretion mechanism. The injected matter washes away the initial low floor matter and attains a quasi steady state. Inflowing matter undergoes a shock due to the centrifugal barrier. After the shock, matter starts to flow towards black hole. The mass absorption rate through the black hole boundary \( 1.5 r_g \) is represented in the Fig. 2. It can be seen that the mass absorption rate after the transient phase saturates at \( \sim 75\% \) of the injected mass.

First simulation of this kind was performed by MLC94 followed by Molteni, Ryu & Chakrabarti 1996 (MRC96), Giri & Chakrabarti 2012, 2013 (GC12, GC13). The outflows are generated out of inflowing sub-Keplerian matter forming a shock due to centrifugal barrier. The oscillation of the shock front after attaining a steady state and the consequence of that on light curves are well studied in MLC94 and Giri et al. (2010) and Garain et al. (2013). This formation of shock induces a temperature rise in the post-shock region which acts as the base of the outflows or Jets. Due to the increase of temperature in the post shock region, force due to gradient of pressure pushes matter outward along the vertical direction. This creates the outflows. In HD simulations, this is not well collimated. One may require magnetic effects for collimation (Chakrabarti, 2013). To show details, we have shown the post-shock region only. The sub-Keplerian flow of the pre-shock region has much lower density and optical depth as compared to the post-shock region (GC13). Maximum number of scatterings occur in the post-shock region and for an observer trying to capture the image of an accretion disk in presence of the outflows, the last scattering surface of the CENBOL will appear as the shape of the Compton cloud.

### 3.1 Keplerian Disk Acting as the Soft Photon Source

In order not to include a time dependence in the Keplerian disk, we put it on the equatorial plane, whose purpose is to supply soft photons as per Page & Thorne (1974) prescription. We employ the Monte-Carlo simulations to compute the resulting spectrum which includes the original source photons from this Keplerian disk as well as those scattered from the CENBOL and the outflows. The Keplerian disk is truncated at the inner edge at the CENBOL surface and the outer edge is extended up to 100 \( r_g \) for simulation purpose.

\[
F(r) = \frac{F_c(\dot{m}_d)}{(r - 3/2)^{5/2}} \times \sqrt{\frac{3}{2}} \left[ \sqrt{\frac{3}{2}} + 2 \log \left( \frac{\sqrt{3} - \sqrt{3/2}}{\sqrt{3} - \sqrt{3/2}} \right) \right] \\
\text{and} \\
T(r) = \left( \frac{F(r)}{\sigma} \right)^{1/4} (3)
\]
where, $F_γ(\dot{m}_d) = \frac{3\dot{m}_d}{4\pi\sigma}$, $\dot{m}_d$ is the Keplerian disk accretion rate in Eddington unit, $\sigma = \frac{2\pi^2h^2}{15k_Bc^3}$ is the Stefan-Boltzmann constant.

Photon flux emitted from the Keplerian disk surface of radius $r$ to $r + \delta r$ is written as,

$$n_\gamma(r) = \frac{4\pi}{c^2} \left( \frac{k_bT(r)}{h} \right)^3 \times \zeta(3) \text{ cm}^{-2}\text{s}^{-1},$$

(4)

where, $\zeta(3) = \sum_{i}^{\infty} \frac{1}{i^3} = 1.202$ is the Riemann zeta function. So, the rate of photons emitted from the radius $r$ to $r + \delta r$ is given by,

$$dN(r) = 4\pi\delta rn_\gamma(r),$$

(5)

where the Keplerian disk height $H(r)$ is considered as 0.1 (since $H(r) << 1$ for Keplerian disk region).

The disk is divided into several annuli of radial width

$$D(r) = 0.1 \text{ and of mean temperature } T(r).$$

The exact number of photons come to be around $\sim 10^{39} - 10^{40}$ per second for a disk rate of $\dot{m}_d = 0.1$. With increasing disk rate this number increases rapidly. Using a weightage factor $F_w = \frac{N_w}{N_{cmp}(r)}$, where $N_{cmp}(r) = 10^9$, we bundle these photons to save computational time.

Soft photon energy is calculated using Planck distribution law for $T(r)$. Photon number density ($n_\gamma(E)$) which corresponds to an energy $E$ is given by,

$$n_\gamma(E) = \frac{1}{2\zeta(3)} \frac{y^3}{E^2} (\nu^E - 1)^{-1}.$$  

(6)

The process is similar to the earlier works presented by Ghosh, Chakrabarti & Laurent (2009); Chatterjee, Chakrabarti & Ghosh (2017a, 2017b).

4 MONTE-CARLO SIMULATION OF COMPTONIZATION

In Monte-Carlo process, the photons inside the CENBOL region are assumed to follow straight line trajectories in between two scatterings. This enhances the Computational efficiency without sacrificing science. This has been pointed out by Laurent & Titarchuk (1999). The Keplerian disk emits seed photons with six random number associated with position and velocity. In reality, the Keplerian disk should have the maximum flux along Z-axis and minimum along the equatorial plane. This has been implemented by assuming $F_v = \int I_\nu \cos \theta d\Omega$. Inside the CENBOL region, a critical optical depth ($\tau_c$) is set up using random number corresponding to each scattering. The next scattering would occur if the optical depth of a particular photon crosses $\tau_c$.

We choose Klein-Nishina scattering cross section $\sigma$ which is given by:

$$\sigma = \frac{2\pi r_e^2}{x} \left( \frac{3}{4} - \frac{8}{x^4} \right) \ln(1 + x) + \frac{1}{2} + \frac{8}{x} - \frac{1}{2(1 + x)^2},$$

(7)

where, $x$ is given by,

$$x = \frac{2E}{mc^2} \left( \frac{1 - \nu}{c} \right),$$

(8)

$r_e = e^2/mc^2$ is the classical electron radius and $m$ is the mass of the electron. This yields Thomson scattering cross section ($\sigma_T$) in low frequencies and Compton scattering cross section ($\sigma_C$) in higher frequencies like X-rays and $\gamma$-Rays. Between each pair of scatterings, the gravitational red-shift modifies the energy of the photon. This process continues until it leaves the CENBOL region or get sucked by the black hole. The process is similar to that presented in GCL09; Ghosh, Garain, Chakrabarti, Laurent, 2010; Ghosh, Garain, Giri, Chakrabarti, 2011 (here-after GGGC11); CCG17a, CCG17b.

After Comptonization, the spectra at various times are presented in Fig. 3. Fig. 3a shows the Comptonized spectra of TCAF at $t = 14.25s$ (Black-Solid), $t = 14.45s$ (Red-Dotted) and $t = 14.75s$ (Green-Dashed) and injected (Blue-Dot-Dash-Dash) spectrum. (b) Injected spectrum (Black-Solid), composite spectrum (Red-Dashed), Reflected Spectrum (Blue-Dot-Dash-Dash), Outflow spectrum (Green-Dotted-Dashed). (c) Inclination dependence of reflected spectra. The inclination bins are $0^\circ$ — $20^\circ$ (Black-Solid), $40^\circ$ — $60^\circ$ (Red-Dotted), $70^\circ$ — $90^\circ$ (Green-Dot-Dashed). (d) Outflow spectra at $t = 14.25s$ (Black-Solid), $t = 14.45s$ (Red-Dotted) and $t = 14.75s$ (Green-Dotted-Dashed). Note that the outflow spectra changed significantly due to the increase in the outflow rate. Disk accretion rate $\dot{m}_d = 0.1$ is kept fixed throughout the simulations.

Figure 3. Spectra obtained after Comptonization. (a) The spectra at $t = 14.25s$ (Black-Solid), $t = 14.45s$ (Red-Dotted) and $t = 14.75s$ (Green-Dotted-Dashed) and injected (Blue-Dot-Dash-Dash) spectrum. (b) Injected spectrum (Black-Solid), composite spectrum (Red-Dashed), Reflected Spectrum (Blue-Dot-Dash-Dash), Outflow spectrum (Green-Dotted-Dashed). (c) Inclination dependence of reflected spectra. The inclination bins are $0^\circ$ — $20^\circ$ (Black-Solid), $40^\circ$ — $60^\circ$ (Red-Dotted), $70^\circ$ — $90^\circ$ (Green-Dot-Dashed). (d) Outflow spectra at $t = 14.25s$ (Black-Solid), $t = 14.45s$ (Red-Dotted) and $t = 14.75s$ (Green-Dotted-Dashed). Note that the outflow spectra changed significantly due to the increase in the outflow rate. Disk accretion rate $\dot{m}_d = 0.1$ is kept fixed throughout the simulations.
of the outflow rate, the spectral contribution of the outflow also increases. In Fig. 3d, the outflow spectra have been plotted for $t = 14.25s$ (black-solid), $t = 14.45s$ (red-dotted) and $t = 14.75s$ (green-dot-dashed).

### 4.1 Cooling Process

During Comptonization, the photons either gain or lose energy. In the context of accretion physics, the electrons in the CENBOL region is at much higher temperature than that of the photons coming from a Keplerian disk. So, mostly inverse Compton scattering dominates and the signature of the photons coming from a Keplerian disk. So, mostly.

In Fig. 4, we see that the shock location moves inward due to loss of post-shock pressure. Disk rates $\dot{m}_d = 0.1$ and $\dot{m}_h = 0.1$ are kept constant throughout.

almost the same electron density and temperature as Fig. 1 (a). We have considered this particular data file to see the effect of Compton cooling. Fig 4(a) is presented when cooling just started to work. Then, after a few iterations cooling starts to reduce the CENBOL size. Spectra softens as the size of hotter Compton cloud becomes smaller. Finally, the Compton cloud reduces more which makes the accretion disk spectra to be in soft state. The corresponding spectra of Panel (b) and (d) suggest the CENBOL region is so small that a very few numbers of soft photons are intercepted by the cloud. Thus, the spectrum softens and spectral slope increases. The effect of cooling on the outflows are also visible from Fig. 4. With the inclusion of cooling, the amount of matter outflow rate ($\dot{m}_o$) reduces drastically.

Fig. 6 represents the outflows spectra for two different CENBOLs. Black-solid one is same as the spectra presented for $t = 14.25s$ in Fig. 3d. This is the input TVD file on which Compton cooling started to act. The CENBOL reduces the its size due to cooling. And outflows component vanishes with cooling. The outflow spectra shrinks more and more and the it softens. Basically, all photons in the outflows spectra for the to particular case comes from the base of the jet which is CENBOL itself. In this paper, our goal is to show the variation of images and spectra when cooling is included. A very detailed study of the effect of cooling on outflows has already been reported in GGC12.

### 5 Ray-Tracking Process

In Schwarzschild geometry, the non vanishing components Christoffel symbol yield geodesic equations from the field equation,

$$\frac{d^2x^\mu}{dp^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{dp} \frac{dx^\lambda}{dp} = 0,$$

where $\mu = 0, 1, 2, 3$; $x^0 = t$, $x^1 = r$, $x^2 = \theta$ and $x^3 = \phi$, $p$ is the affine parameter. The four coupled, second order differential equations for photons can be reduced to three using energy $P_t = E = \left(1 - \frac{1}{2} \frac{dx}{dp}\right)$ and angular momentum $P_\phi = L = r^2 \sin^2 \theta \frac{dx}{dp}$ definitions (Chandrasekhar, 1985). So,
three geodesic equations which dictate the trajectory of photons can be written as

\[
\frac{d^2r}{dt^2} + \frac{3}{2r(r-1)} \left( \frac{dr}{dt} \right)^2 - \left( r - 1 \right) \frac{\partial \tau}{\partial r}^2 - (r-1) \sin^2 \theta \left( \frac{d\phi}{dt} \right)^2 + \frac{r-1}{2r^2} = 0,
\]

\[
\frac{d^2\theta}{dt^2} + \frac{2r-3}{r(r-1)} \left( \frac{d\theta}{dt} \right)^2 - \sin \theta \cos \theta \left( \frac{d\phi}{dt} \right)^2 = 0 \quad \text{and}
\]

\[
\frac{d^2\phi}{dt^2} + \frac{2r-3}{r(r-1)} \left( \frac{d\phi}{dt} \right)^2 + 2\cot \theta \left( \frac{d\theta}{dt} \right) \left( \frac{d\phi}{dt} \right) = 0.
\]

Velocity components are derived using Tetrad formalism (Park 2006) and are given by,

\[
v^r = \frac{dr}{dt}, \quad \frac{r}{(r-1)} \frac{d\theta}{dt}, \quad \frac{r}{(r-1)} \frac{d\phi}{dt} = \frac{\sqrt{r\tau}}{(r-1)} \frac{d\theta}{dt}
\]

(12)

and \( v^\phi = \frac{\sqrt{r\tau} \sin \theta}{(r-1)} \frac{d\theta}{dt} \).

The redshift factor added to connect the source and the observer frame is given by the relation

\[
1 + z = \frac{E_{\text{em}}}{E_{\text{obs}}} = \left( \frac{P_{\text{em}}}{P_{\text{obs}}} \right)^{\frac{2}{3}},
\]

(13)

where, \( E_{\text{em}} \) and \( E_{\text{obs}} \) are the energy of emitted and observed photons respectively. The observed and emitted fluxes are related via the fourth power of redshift factor

\[
F_k^{\text{obs}} = \left( \frac{E_k^{\text{em}}}{(1+z)^2} \right)^{\frac{2}{3}}.
\]

(14)

This relationship between the observed flux and the observed temperature can be expressed as,

\[
T_k^{\text{obs}} = \left( \frac{F_k^{\text{obs}}}{\sigma} \right)^{1/4}.
\]

(15)

Doppler boosting is added during the emitter-observer frame transformation. The Ray-Tracing process and formation of images are done in the same way as reported earlier in CCG17a, CCG17b.

6 RESULTS

6.1 Time Dependent Images without Cooling

The static images of TCAF in presence of Comptonization were presented in CCG17a. The Compton cloud was modelled using natural angular momentum description of General Relativistic thick disks given by Chakrabarti (1985a). The variations of images and spectra were presented for various inclination angles and disk accretion rates. They show that due to steep variation of density and temperature inside the CENBOL, the image does not have sharp edges. On the top of that, for an increasing optical depth of the CENBOL region, the photons from the Keplerian disk region located opposite to the observer are blocked. In the present case, where TVD code was used to create the CENBOL and the outflow self-consistently, the optical depth of the CENBOL medium allows a fraction of photons from the Keplerian disk to pass through the Compton cloud without getting scattered. This can be seen in each panels presented in Fig. 7 & Fig. 8.

Figure 7 is drawn at (a) \( t = 14.25s \) and (b) \( t = 14.75s \). In panel (a), the amount of outflow is much less as compared to the panel (b). The photons emitted by the base of the lower jet are mostly blocked by the optically thick Keplerian disk. In Fig. 7a, a very low number of photons is visible from the lower jet. Due to reflection symmetry of the outflows, these photons are also visible in the upper part of the outflows. However, due to Lorentz boosting in the upper jet, its observed temperature is enhanced and it is overall brighter. The sonic surface of the outflow is the cap-like curved bright surface. The flow is subsonic and hotter below this surface. We include the photons in this plot which are emitted from the regions with optical depth greater than unity. This creates a special shape for the base of the outflow.

Figure 8 shows the dynamical evolution of outflows at \( t = 14.25s, t = 14.35s, t = 14.45s, t = 14.55s, t = 14.65s \) and \( t = 14.75s \). The mass outflow increases with time in this case. The images with gravitationally bent rays have been drawn from the perspective of an observer placed at an inclination angle of 80°. The asymmetry of outflows become prominent in this Figure also. The energy range is chosen in such a way that the observer temperature variation of the Keplerian disk can be found easily.

Figure 9 shows energy dependent images of TCAF. With a higher energy detector which works best in the en-
energy range of 1.0 – 100 keV, the Keplerian disk might not be seen for disk rates lower than 0.1. With increasing \( \dot{m}_d \), the disk may reappear. So, for an outbursting source, whose accretion rates change on a daily basis, the images will change for any particular energy range of detector. These pictorial changes will be due to the physical variations of accretion disk geometry which will be triggered by the thermodynamical properties (see, CT95 for details) of the disk.

In Panel (b), we see the top part of the outflows is also missing in a high energy observation. This is due to the velocity profile of the outflowing electrons. Most of the photons suffer downscattering during their collision in the outflowing electrons. In Panel 3d, we see that the Jet spectra extends upto 1000 keV. But, in reality, in our model of non-magnetic jets, maximum hard photons in the Jet spectra are contributed only from the base of the jet. The photons that are coming from below the equatorial plane are mostly reflected or absorbed by optically thick Keplerian disk. So, up to 70°, visibility of the base of a lower jet is screened by the Keplerian disk itself. But, at 80°, as in Fig. 8, both sides become visible to the observer.

6.2 Observed Spectra

The source spectrum as presented in the Fig. 3a has a peak at 0.6 keV. This has been modified due to photon bending and shifted to a lower value for an observer at higher inclination angle. In the observed spectrum of Keplerian disk has a peak at 0.3 keV for 80° inclination angle (Fig. 10). The corresponding characteristic temperature ranges from \( 10^{6.0} \) – \( 10^{6.6} \) Kelvin and this is visible in Fig. 9a for 70°. The observed spectral variation of the black body part with an inclination angle remains almost the same as reported in CCG17a.

From Fig. 10, we see that the contribution of hard photons increases with inclination angle. But, the spectral slope is less for low inclination angle. This is to be contrasted with our earlier studies (see CCG17a) where the outflow component has not been added. Without the outflows, the hard photons in powerlaw component increases and the slope of the spectrum decreases with increasing inclination angle. With the inclusion of outflows, the spectral slope increases with increasing inclination angle. But, the contribution of hard photons increases more significantly when the observer moves to a higher inclination angle.

Figure 8. Images of TCAF (in \( \log(T_{obs}) \) scale) in presence of outflows at \( t = 14.25 \) s, \( t = 14.35 \) s, \( t = 14.45 \) s, \( t = 14.55 \) s, \( t = 14.65 \) s and \( t = 14.75 \) s. Colorbar is in the range \( 10^{6.0} \) – \( 10^{6.5} \) Kelvin and inclination of the observer is at 80°. Asymmetry in the shape is due to difference in Doppler shifts in upper and lower jet components.

Figure 9. Grid averaged images of TCAF (in \( \log(T_{obs}) \) scale) in presence of outflows at time \( t = 14.65 \) s. Colorbar is in the range \( 10^{5.4} \) – \( 10^{6.7} \) Kelvin for panel (a) and \( 10^{6.7} \) – \( 10^{10.3} \) Kelvin for panel (b). The observer is placed at 70°. For higher energy range detectors, the Keplerian disk and upper part of jet becomes completely invisible to an observer.
The intensity variation due to the disk and the black hole can lead to a soft spectrum (see CT95 and SS73 for further details). The dark central region of the black hole just starts to appear to an observer at 50°, and becomes clearer if the onlooker moves to an inclination angle of 75° away from our line of sight. Assuming this is along the perpendicular direction with respect to the accretion plane, and assuming radio intensity roughly tracks the disk/jet as discussed above, the Galactic centre should show similar images to the panels Fig. 12(a-f). Our conclusion is that we may not be able to discern the horizon itself, but some features as in our Fig. 12(a-f) should be visible. Similarly the presence and absence of accretion disk with change in accretion rates should be seen.

The convolved images presented in Fig. 12 suggest a low spatial resolution of about 5 $r_g$ will be able to distinguish features in the hole-disk system, unless the object has a very low inclination. Integrated counts from the central region is plotted in Fig. 13b for the Keplerian disk only. A spatial resolution of 5 $r_g$ shows double-peaked curve in just resolved condition. Figure 13a shows unresolved count distribution when 20$r_g$ is used instead. Considering the possibilities of high mass inflow onto Sgr A*, in its flaring state, we expect an X-Ray image similar to Fig. 8 which is difficult to resolve at a high inclination. Thus, probing inner region to capture the event horizon can be difficult in this state. Future projects which aim at capturing the image of an event horizon will not be able to detect the difference in the intensity of disk to that of the hole unless the object is in the soft state. Otherwise, only low inclination disks would show the even horizon in any spectral states, even in presence of outflows. Of course, this conclusion is valid only if the radio emission roughly tracks the inflow-outflow features as observed in our numerical simulations.

7 DISCUSSIONS AND CONCLUSION

Recent study of the effects of the photon bending on standard and two component accretion disk spectra and images was made by CCG17a. Earlier, static images of Two component advective flows were presented where we discussed the Keplerian disk (valid for soft states only) and the two component disks with its intrinsic Compton cloud at the event horizon can be difficult in this state. Future studies similar to the panels Fig. 12(a-f). Our conclusion is that we may not be able to discern the horizon itself, but some features as in our Fig. 12(a-f) should be visible. Similarly the presence and absence of accretion disk with change in accretion rates should be seen.

The convolved images presented in Fig. 12 suggest a low spatial resolution of about 5 $r_g$ will be able to distinguish features in the hole-disk system, unless the object has a very low inclination. Integrated counts from the central region is plotted in Fig. 13b for the Keplerian disk only. A spatial resolution of 5 $r_g$ shows double-peaked curve in just resolved condition. Figure 13a shows unresolved count distribution when 20$r_g$ is used instead. Considering the possibilities of high mass inflow onto Sgr A*, in its flaring state, we expect an X-Ray image similar to Fig. 8 which is difficult to resolve at a high inclination. Thus, probing inner region to capture the event horizon can be difficult in this state. Future projects which aim at capturing the image of an event horizon will not be able to detect the difference in the intensity of disk to that of the hole unless the object is in the soft state. Otherwise, only low inclination disks would show the even horizon in any spectral states, even in presence of outflows. Of course, this conclusion is valid only if the radio emission roughly tracks the inflow-outflow features as observed in our numerical simulations.

**Images and Spectra of TCAF with outflows**

**Figure 10.** Observed spectra of TCAF seen from inclination angles 10°, 50° and 80° in presence of Outflows. Increase in the hard photon contribution with inclination angle is visible. However, with the inclusion of outflows, the spectral slope increases with increasing the inclination angle.

**Figure 11.** Images of TCAF (in log($T_{\text{obs}}$) scale) as seen from the inclination angle of 50°, 70° in presence of Compton cooling. Colorbar is in the range 10^{1.5} – 10^{6.6} K. If the outflows are absent, dark central region of black hole just starts to appear to an observer at 50° and becomes clearer if the onlooker moves to a lower inclination angle.

**6.3 Images with Cooling**

To demonstrate the effects of Compton cooling, we consider first the data file from hydrodynamic simulations at $t = 14.25$s. The electron number density and temperature profile of the post-shock region after Compton cooling is presented in Fig. 4. Corresponding spectrum in Fig. 5 shows the spectral softening due to cooling. From Fig. 11, we see that the size of the CENBOL region is substantially reduced due to cooling effect and the optical depth of CENBOL region is so low that very few soft photons from Keplerian disk are intercepted in the Compton cloud. This is the reason of spectral softening. With the inclusion of Compton cooling the outflow drastically reduces its size. This effect can also be visible in the images of TCAF. As the CENBOL size is reduced, the inner edge of the truncated Keplerian disk moves radially inwards.

**6.4 Realistic Observed Image with a finite beamwidth**

Figure 12, panel (a) shows a convolved image of a Keplerian disk with the inner edge at 3 and outer edge at 50. Convolution is over a beam width of (a) $\sim 20r_g$ and (d) $\sim 5r_g$. Corresponding spectrum for this image gives a pure soft spectrum (see CT95 and SS73 for further details). The intensity variation due to the disk and the black hole can
In the present paper, we have employed numerical simulations of hydrodynamic process to generate time dependent accretion disk configurations. At every time step, Monte-Carlo simulation was done to compute the effects of Comptonization and to obtain the spectrum. We produced images of the CENBOL with the jet as well as the composite spectra when the effects of cooling due to Comptonization was turned off or turned on. We find that with cooling effects the CENBOL collapses to a smaller size and the outflow is also reduced as in a soft state. In presence of outflows, we draw images of an accretion disk where the Compton cloud or CENBOL is showing temporal variation of its physical shape and internal properties. With these, the images depict the outflows which are self-consistently produced from the inflowing matter. We showed the combined effects of the disk and the jet and particularly how they would look like in different X-ray energies. Doppler shift was seen to break the symmetry as expected, but the photon bending made it even more asymmetric.

Though our simulations are carried out in regions which emit X-rays, and this will remain true for higher mass black holes also, but the efforts are on to observe the event horizon in radio waves which have higher spatial resolutions. On the other hand, both the radio and the Comptonization track the high energy electron distribution. So we believe that our simulations will have relevance for radio observations as well. However, intensity of synchrotron photon in radio band depends on the magnetic field strength and are expected to be primarily originated from the post shock region. Thus, our X-ray images without the cooler Keplerian disk, can be treated as radio images. We showed, using two different spatial resolutions, that one requires a resolution of $\sim 5r_g$ in order to separate distinct features in disk-jet systems when high inclination objects are observed, unless the object is in a soft state and only has the Keplerian disk. However, the

earlier Sections. The outflow could be time varying and the image of the disk would, in general, vary with time.

In the present paper, we have employed numerical simulations of hydrodynamic process to generate time dependent accretion disk configurations. At every time step, Monte-Carlo simulation was done to compute the effects of Comptonization and to obtain the spectrum. We produced images of the CENBOL with the jet as well as the composite spectra when the effects of cooling due to Comptonization was turned off or turned on. We find that with cooling effects the CENBOL collapses to a smaller size and the outflow is also reduced as in a soft state. In presence of outflows, we draw images of an accretion disk where the Compton cloud or CENBOL is showing temporal variation of its physical shape and internal properties. With these, the images depict the outflows which are self-consistently produced from the inflowing matter. We showed the combined effects of the disk and the jet and particularly how they would look like in different X-ray energies. Doppler shift was seen to break the symmetry as expected, but the photon bending made it even more asymmetric.

Though our simulations are carried out in regions which emit X-rays, and this will remain true for higher mass black holes also, but the efforts are on to observe the event horizon in radio waves which have higher spatial resolutions. On the other hand, both the radio and the Comptonization track the high energy electron distribution. So we believe that our simulations will have relevance for radio observations as well. However, intensity of synchrotron photon in radio band depends on the magnetic field strength and are expected to be primarily originated from the post shock region. Thus, our X-ray images without the cooler Keplerian disk, can be treated as radio images. We showed, using two different spatial resolutions, that one requires a resolution of $\sim 5r_g$ in order to separate distinct features in disk-jet systems when high inclination objects are observed, unless the object is in a soft state and only has the Keplerian disk. However, the
obscuration of the central region is not done when viewed at low inclination angle and the event horizon should be observable at any spectral state if the beam width is low enough. The images are of particular interest where the mass observable at any spectral state if the beam width is low at low inclination angle and the event horizon should be discerned. The peak was seen at a lower energy due to including the effects of synchrotron radiation, Compton cooling is included, the CENBOL itself shrinks and consequently the spectra become softer. Further simulations including the effects of synchrotron radiation, Compton cooling and viscosity are being carried out and will be reported elsewhere.

8 ACKNOWLEDGEMENTS

This research was possible in part due to a grant from Ministry of Earth Sciences with Indian Centre for Space Physics.

REFERENCES

Abramowicz M. A., Jaroszynski, M., Sikora M., 1978, A&A, 63, 221
Armitage P. J. & Reynolds C. S., 2003, MNRAS, 341, 1041
Bian W. H, Zhao Y. H., 2003, PASJ, 55, 599
Bromley B. C., Melia F. & Liu S., 2001, ApJ, 555, 83
Broderick, A. E. & Loeb A., 2009, ApJ, 697, 1164
Chakrabarti S. K., 1990, MNRAS, 245, 747
Chakrabarti S. K., 1985, ApJ, 288, 1, (Paper 1)
Chakrabarti S. K., 1990b, MNRAS, 243, 610 (C90)
Chakrabarti S. K. 1996, ApJ, 464, 664(C96)
Chakrabarti S. K., Titarchuk L. G., 1995, ApJ, 455, 623 (CT95)
Chakrabarti S. K., 2013, BASL, 8, 1
Chakrabarti S. K., Nandi A., Debnath D., Sarkar R. & Dutta B. G., 2005, IJP, 79, 841
Chandrasekhar S., 1985, The Mathematical Theory of Black Holes, Oxford University press
Chatterjee A., Chakrabarti S. K., Ghosh H., 2017, MNRAS, 465, 3902
Chatterjee A., Chakrabarti S. K., Ghosh H., 2017, MNRAS, 472, 1842
Coker, R., & Melia, F. 1997, ApJ, 488, L149
Cui W., Zhang S. N., Chen W., 2000, ApJL, 531, L45
Dexter J. & Agol E., 2009, ApJ, 696, 1166
Dexter J., Agol E., Fragile P. C. & McKinney J. C., 2010, ApJ, 717, 1092
Dexter J., Fragile P. C., 2013, MNRAS, 432, 2252
Dexter J., 2016, MNRAS, 462, 115

Images and Spectra of TCAF with outflows

Dibi S., Drappeau S., Fragile P. C., Markoff S., & Dexter J. 2012, MNRAS, 426, 1928
Dolence J. C., Gamme C. F., Shiokawa H. & Noble S. C., 2012, ApJ, 746, L10
Draphaul S., Dibi, S., Dexter, J.; Markoff, S., Fragile, P. C., 2013, MNRAS, 431, 2872
Fender R. P. et al., 1999, MNRAS, 304, 865
Fuerst S. V. & Wu K., 2004, A&A, 424, 733
Fukue J., Yokoyama T., 1988, PASJ, 40, 15
Garain S. K., Ghosh H., Chakrabarti S. K., 2012, ApJ, 758, 114
Garain S. K., Ghosh H., Chakrabarti S. K., 2012, ApJ, 437, 1329
Ghosh H., Garain S. K., Giri K., Chakrabarti S. K. 2011, MNRAS, 416, 959G
Ghosh H., Chakrabarti S. K., Laurent P., IJMPD, 18, 2009, 1693
Ghosh H., Garain S. K., Chakrabarti S. K., Laurent P., 2010, IJMPD, 19, 607
Gillies et al., 2012, Nature, 481, 51
Giri K., Chakrabarti S. K., Samanta M. M. & Ryu D., 2010, MNRAS, 403, 516
Giri K. & Chakrabarti S. K., 2012, MNRAS, 421, 666
Giri K. & Chakrabarti S. K., 2013, MNRAS, 430, 2836
Giri K., Garain S. K. & Chakrabarti S. K., 2015, MNRAS, 448, 3221
Harten, A., 1983, Journal of Computational Physics, 49, 357
Hilburn G., Liang E., Liu S., Li H., 2010, MNRAS, 401, 1620
Johnson M. D. et al., 2015, Science, 350, 6265, 1242
Kozlowski M., Jaroszynski, M., Abramowicz M. A., Jaroszynski, M., 1978, A&A, 63, 209
Landau L. D. & Lifshitz E. D., Fluid Mechanics, 1959, Pergamon, New York
Laurent P., Titarchuk L. G., 1999, ApJ, 511, 289
Luminet J. P., 1979, A&A, 75, 228
Marck J. A., 1995, Classical and Quantum Gravity, 13, 393
Markoff S., Bowler G. C. & Falcke H., 2007, MNRAS, 379, 1519
Molteni D., Lanzaafage G., & Chakrabarti, S. K, 1994, ApJ, 425, 161
Molteni D., Sponholz H., & Chakrabarti, S. K., 1996, ApJ, 457, 805
Moscibrodzka M., Gamme C. F., Dolen I C, Shiokawa H. & Leung P. K., 2009, ApJ, 706, 497
Moscibrodzka M., Gamme C. F., Dolen I C, Shiokawa H., Leung P. K., 2011, in Morris M. R., Wang Q. D., Yuan F., eds, ASP Conf. Ser. Vol. 439, The Galactic Center: A Window to the Nuclear Environment of Disk Galaxies, Astron. Soc. Pac., San Francisco, p. 358
Moscibrodzka M., Falcke H. & Shiokawa H., 2016, A&A, arXiv:1510.07243v3
Muno M. M., Morgan E. H. & Remillard R. A., 1999, ApJ, 527, 321
Müller T. & Fraueniener J., 2012, Eur. J. Phys, 33, 955
Noble S. C., Leung P. K., Gamme C. F., & Book L. G. 2007, Classical Quant. Grav., 24, 259
Nowak M. A., Wilms J. & Dove J. B., 2001, ApJ, 547, 355
Ohsuga K., Kato Y., & Mineshige S. 2005, ApJ, 627, 782
Paczyski B., Wiita P. J., 1980, A&A, 88, 23
Paczyski, B., Harten, A., 1983, Journal of Computational Physics, 49, 357
Paczyński B., Wiita P. J., 1980, A&A, 88, 23
Page D. N., Thorne K. S., 1974, Astrophys. J., 191, 499
Pozdnyakov A., Sobol I. M., Sunyaev R. A., 1983, Astrophys. Space Sci. Rev., 2, 189
Rees M. J., Begelman M. C., Blandford R. D., Phinney, E. S., 1982, Nature, 295, 17
Roelofs F., Johnson, M. D., Shiokawa H., Doeleman S. E. & Falcke H., 2017, ApJ, 817arXiv:170801056R
Ryu, D., Ostriker, J. P., Kang, H., & Cen, R., 1993, ApJ, 441, 1
Ryu D., Chakrabarti S. K. & Molteni D., 1997, ApJ, 378, 388
Shakura N. I. & Sunyaev R. A., 1973, A&A, 24, 337 (SS73)
Sadowski A., Narayan R., Penna R., & Zhu Y, 2013, MNRAS,436, 3856

MNRAS 000, 1–77 (2018)
Shcherbakov R. V., Penna R. F., McKinney J. C., 2012, ApJ, 755, 133
Viergutz S. U., 1992, Astron. Astrophysics, 272, 355
Vincent F. H., Paumard T., Gourgoulhon E., Perrin G., 2011, CQG, 28, 22501
Weinberg S., 1972, Gravitation and Cosmology, John Wiley & Sons, UK
Wu K. et al., 2002, ApJ, 565, 1161
Yang X. & Wang J., 2013, ApJS, 207, 6