Ultracold fermion cooling cycle using heteronuclear Feshbach resonances

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New Journal of Physics 7 (2005) 87
Received 16 December 2004
Published 4 April 2005
Online at http://www.njp.org/
doi:10.1088/1367-2630/7/1/087

Abstract. We consider an ideal gas of Bose and Fermi atoms in a harmonic trap, with a Feshbach resonance in the interspecies atomic scattering that can lead to the formation of fermionic molecules. We map out the phase diagram for this three-component mixture in chemical and thermal equilibrium. Considering adiabatic association and dissociation of the molecules, we identify a possible cooling cycle, which in ideal circumstances can yield an exponential increase of the phase-space density.

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1. Introduction

Feshbach resonances [1] have become a powerful tool for controlling atomic interactions in ultracold gases [2], utilized to create bright matter wave solitons [3, 4], and to induce a collapse of a bosonic gas [5]–[7]. Recently, experiments have begun to probe the crossover between weak coupling Bardeen, Cooper, Schrieffer pairing and the strong coupling limit of Bose–Einstein condensation (BEC) of diatomic molecules in a degenerate Fermi gas [8]–[13]. By tuning an external magnetic field, the energy of a molecular bound state can be adjusted to be on resonance with a scattering state of two free atoms, thus facilitating external control of the atomic interactions. Tuning the molecular state below threshold can lead to the production of stable ultracold diatomic molecules. This has been accomplished both in bosonic [14]–[17] and fermionic gases [18]–[22]. In all of these cases the diatomic molecules are homonuclear bosons. Two recent experiments, at MIT [23] and at JILA [24] have observed Feshbach resonances in Bose–Fermi atomic mixtures. The molecular species in this case is a heteronuclear fermion.

In an earlier paper we analysed the equilibrium phase diagram of a two-component Fermi mixture within an ideal gas approximation [25]. We found that the noninteracting model captured the essential features of the thermodynamics of the system, contributing to a deeper understanding of the experiments. The Bose–Fermi mixture is a more complicated system, due to the possibility of an interaction induced mechanical collapse, absent in a Fermi–Fermi mixture [26]. Nevertheless, a simple analysis again elucidates the important physics of the equilibrium system, and allows us to identify a potential source of further cooling of a degenerate Fermi gas.

Specifically, we construct the phase diagram for an ideal trapped gas of bosonic and fermionic atoms, and the heteronuclear dimer formed from them. In contrast to previous work on Fermi gases [25, 28], we find that adiabatic passage from free atoms to molecules can lead to cooling of the system under conditions comparable to those of current experiments. Based on this observation we propose a thermodynamic cooling cycle, consisting of adiabatic interconversion between atoms and molecules in conjunction with selective removal of atoms. This cycle is capable of reducing the absolute temperature of the system while increasing the atomic phase–space density.

The proposed cycle is operational deep in the degenerate regime, where the efficiency of evaporative cooling is known to be reduced, limiting the lowest attainable temperatures to about 0.05T_F. The thermodynamic cycle suggested here could potentially extend the experimentally accessible regime to lower temperatures.

2. Equilibrium theory

We consider an ideal gas mixture composed of fermionic (f) and bosonic (b) atoms and heteronuclear fermionic molecules (bf) trapped in an anisotropic harmonic optical trap. While the assumption of a noninteracting mixture certainly breaks down in the strongly interacting regime, it is our view, based on the previous analysis of the Fermi–Fermi mixture [25], that one can qualitatively account for the thermodynamics of the system without considering interaction effects.

3 Previous work has discussed the phase diagram of a homogeneous system at T = 0 only [27].
The trapping frequencies for the different components are taken to be
\[
\omega_i^f = \sqrt{\alpha_i^f / m_f}, \quad \omega_i^b = \sqrt{\alpha_i^b / m_b}, \quad \omega_i^{bf} = \sqrt{\alpha_i^f + \alpha_i^b / (m_f + m_b)},
\]
where \( i = \{x, y, z\} \), \( \alpha_i^k \) is proportional to the atomic polarizability, and \( m_k \) are the particle masses, with \( k = \{f, b, bf\} \). The expression for the molecular frequency \( \omega_i^{bf} \) in terms of the atomic frequencies is appropriate when the molecular internuclear separation is large compared to the mean atomic radii. It is possible to change the ratio \( \omega_i^f / \omega_i^b \) over a wide range by varying the wavelength of the trapping laser and specific prescriptions for doing this is given in [29]. Note, however, that for some experiments the axial trapping frequency may include an additional magnetic field dependent contribution arising from the curvature of the field used to access the Feshbach resonance. Hence, our calculated value of \( \omega_z^{bf} \) is only a rough estimate in this case. We stress that the results derived below depend on the axial frequency only through \( \bar{\omega}_k = (\omega_x^k \omega_y^k \omega_z^k)^{1/3} \), the mean harmonic frequency, and for both experiments \( \omega_k^x = \omega_k^y \gg \omega_k^z \) [30, 31]. In addition, we mention that our conclusions are independent of a particular value of \( \omega_z^{bf} \).

We assume chemical and thermal equilibrium: \( \mu_f + \mu_b = \mu_{bf} \) and \( T_f = T_b = T_{bf} = T \), where \( \mu_k \) are the chemical potentials of the three components and \( T \) is the temperature. The energy of the molecular state as measured from the atomic dissociation continuum is \( \epsilon_{res} \), which is a function of an applied magnetic field. The results of this paper are based on considering the phase diagram of this three-component system as a function of \( T, \epsilon_{res} \) and the atomic populations.

We employ a semi-classical approximation, such that populations of each component are given by
\[
N_f(T, \mu_f) = \left( \frac{k_B T}{\hbar \bar{\omega}_f} \right)^3 \mathcal{F}_3(z_f),
\]
\[
\tilde{N}_b(T, \mu_b) = \left( \frac{k_B T}{\hbar \bar{\omega}_b} \right)^3 \mathcal{G}_3(z_b),
\]
\[
N_{bf}(T, \mu_{bf}, \epsilon_{res}) = \left( \frac{k_B T}{\hbar \bar{\omega}_{bf}} \right)^3 \mathcal{F}_3(z_{bf}),
\]
where the fugacities \( z_k \) are
\[
z_f = e^{\mu_f/k_B T}, \quad z_b = e^{\mu_b/k_B T}, \quad z_{bf} = e^{(\mu_{bf} - \epsilon_{res})/k_B T}.
\]

Here, \( k_B \) is Boltzmann’s constant, and \( \mathcal{F}_n(z) \) and \( \mathcal{G}_n(z) \) are the Fermi–Dirac and Bose–Einstein integrals, respectively [25, 32]. This approximation is valid provided \( k_B T \gg \hbar \bar{\omega}_k \). \( \tilde{N}_b(T, \mu_b) \)

\( r_f = \bar{\omega}_f / \bar{\omega}_b = 2.09 \) and \( r_{bf} = \bar{\omega}_f / \bar{\omega}_{bf} = 1.60 \).

The MIT frequencies are \( \omega_b^f = \omega_b^y = 2\pi \times 810 \text{ s}^{-1}, \omega_b^x = 2\pi \times \sqrt{59.4 + 0.0370B} \text{ s}^{-1}, \) for \(^{23}\text{Na} \), and \( \omega_f^x = \omega_f^y = 2\pi \times 1700 \text{ s}^{-1}, \omega_f^z = 2\pi \times \sqrt{262 + 0.142B} \text{ s}^{-1} \) for \(^{6}\text{Li} \), with the magnetic field strength \( B \) expressed in Gauss [30]. If we take \( B = 750 \text{ G} \) we then have \( \bar{\omega}_b = 2\pi \times 183 \text{ s}^{-1} \) and \( \bar{\omega}_f = 2\pi \times 381 \text{ s}^{-1} \). Using (1) this gives \( r_b = \bar{\omega}_f / \bar{\omega}_b = 2.09 \) and \( r_{bf} = \bar{\omega}_f / \bar{\omega}_{bf} = 1.60 \).
represents the number of non-condensed Bose atoms; we denote the number of Bose condensed atoms by \( N_c \) and \( N_b = N_c + \tilde{N}_b \).

For a given \( T, \epsilon_{\text{res}} \), total number of atoms \( N \) and boson–fermion population difference \( \Delta N \) the atomic chemical potentials are determined by the conditions

\[
N = N_f(T, \mu_f) + N_b(T, \mu_b) + 2N_{bf}(T, \mu_{bf}, \epsilon_{\text{res}}),
\]

\[
\Delta N = N_f(T, \mu_f) - N_b(T, \mu_b).
\]

For \( T \) above a critical temperature \( T_c \) all bosons are thermal and \( N_c = 0 \), whereas for \( T < T_c \) a Bose–Einstein condensate (BEC) emerges: \( N_c > 0, \mu_b = 0 (\mu_{bf} = \mu_f) \), and

\[
\tilde{N}_b(T, \mu_b = 0) = \zeta(3) \left( \frac{k_B T}{\hbar \omega_b} \right)^3,
\]

where \( \zeta(n) \) is the Riemann zeta function.

### 3. Phase diagrams

We now discuss the phase diagrams resulting from the numerical solutions to equations (6) and (7). Defining the molecular and condensate fraction as \( \eta_{bf} \equiv 2N_{bf}/N \) and \( \eta_c \equiv N_c/N_b \), respectively, we plot these quantities in figure 1 versus temperature and \( \epsilon_{\text{res}} \) for a system where \( \Delta N = 0 \), using trapping parameters for the JILA experiment.\(^5\) The \( T \) and \( \epsilon_{\text{res}} \) axes are normalized by \( T_F/r_b \) and \( k_B T_F/r_{bf} \), respectively, where \( T_F \) is the Fermi temperature of the fermionic atoms, \( k_B T_F = \hbar \omega_f (6xN)^{1/3}, \) \( x = (N_f + N_{bf})/N \) is the total fermionic fraction, \( r_{bf} = \tilde{\omega}_f/\tilde{\omega}_{bf} \) and \( r_b = \tilde{\omega}_f/\tilde{\omega}_b \). For a finite population difference the maximum number of molecules is \( N_{bf}^{\text{max}} = (N - |\Delta N|)/2 \).

When \( \Delta N < 0 \), there will always be free bosons in the system and condensation is possible for any value of \( \epsilon_{\text{res}} \). When \( \Delta N \geq 0 \), there is a critical resonance energy, \( \epsilon_{\text{res}}^{(c)} \), at which the condensate fraction vanishes at \( T = 0 \). This can be seen in figure 1. Setting \( N_b = 0 \) in equations (6) and (7) and \( \mu_{bf} = \epsilon_{\text{res}}^{(c)} + (6N_{bf})^{1/3} \hbar \tilde{\omega}_{bf} \) at \( T = 0 \), we find that

\[
\frac{\epsilon_{\text{res}}^{(c)}}{k_B T_F} = \left( 2 - \frac{1}{x} \right)^{1/3} - \frac{1}{r_{bf}} \left( \frac{1}{x} - 1 \right)^{1/3}.
\]

In a similar manner we can solve for the critical temperature \( T_c \) for BEC in the limit \( \epsilon_{\text{res}}/k_B T_F \gg 1 \), where \( N_{bf} = 0 \). From equation (8), we find

\[
\frac{T_c}{T_F} = [6\zeta(3)]^{-1/3} \frac{1}{r_b} \left( \frac{1}{x} - 1 \right)^{1/3}.
\]

\(^5\) The JILA frequencies are \( \tilde{\omega}_b = 2\pi \times 519 \text{ s}^{-1} \) and \( \tilde{\omega}_f = 2\pi \times 722 \text{ s}^{-1} \) [31]. Using (1) this gives \( r_b = \tilde{\omega}_f/\tilde{\omega}_b = 1.39 \) and \( r_{bf} = \tilde{\omega}_f/\tilde{\omega}_{bf} = 1.22 \).
Figure 1. Phase diagram for trapping parameters of the JILA system (see footnote 5), with $\Delta N = 0$; this representation is universal in $N$ subject to validity of the semi-classical approximation. Left frame: molecule fraction, $\eta_{bf}$, as a function of $\epsilon_{res}$ and $T$; right frame: condensate fraction, $\eta_c$. White lines indicate adiabatic trajectories, i.e. curves of constant entropy.

4. Adiabatic sweep

During an adiabatic sweep, the resonant energy is varied on a time scale much longer than the average relaxation time of the system [33]. Then entropy is conserved and the system follows an adiabatic path through the phase diagram. We will find below that the final temperature after such an adiabatic ramp depends both on the direction of the sweep and on the initial state of the gas. In particular, both heating and cooling of the gas can occur in either of the sweep directions, depending on the initial preparation of the system. This is in contrast to earlier studies of time-dependent Feshbach ramps in atomic Bose–Einstein condensates in a Landau–Zener picture [34, 35]. Within that description, the ramp is fast compared with the relaxation rate, and sweeping the field from the molecular to the atomic side of the resonance, will leave the dissociation products in an excited vibrational levels in the trap, effectively heating the system. The fast quantum dynamics of the present system of a Feshbach coupled Bose–Fermi mixture has been investigated by Wouters et al [36] and Dannenberg et al [37].

The total entropy of the system, $S(T)$, is given by the sum of the entropies of each of the three components:

\[
S_f(T, \mu_f) = k_B N_f \left[ 4 \frac{\mathcal{F}_4(z_f)}{\mathcal{F}_3(z_f)} - \frac{\mu_f}{k_B T} \right],
\]

\[
S_b(T, \mu_b) = k_B \tilde{N}_b \left[ 4 \frac{\mathcal{G}_4(z_b)}{\mathcal{G}_3(z_b)} - \frac{\mu_b}{k_B T} \right],
\]

\[
S_{bf}(T, \mu_{bf}, \epsilon_{res}) = k_B N_{bf} \left[ 4 \frac{\mathcal{F}_4(z_{bf})}{\mathcal{F}_3(z_{bf})} - \frac{\mu_{bf} - \epsilon_{res}}{k_B T} \right].
\]
The final temperature and chemical potentials at the end of the sweep are determined from the condition \( S(T_i) = S(T_f) \) together with the constraints (6) and (7). Figure 1 shows the contours of constant entropy in the phase diagrams for the case of equal populations. Starting with a pure atomic system at \( \epsilon_{\text{res}} \gg k_B T_F \), we observe that there is a temperature, \( T_{\text{iso}} \), such that for \( T^i > T_{\text{iso}} \) the final temperature after complete association of the gas is higher than the initial temperature. However, for \( T^i < T_{\text{iso}} \), we find the interesting result that the temperature of the system decreases after the atoms have been converted into molecules. This can be understood from the fact that at low temperatures, the majority of the free bosons are condensed and do not contribute to \( S \). By sweeping the value of \( \epsilon_{\text{res}} \) across the resonance, we go from an almost perfect Fermi sea of atoms to an almost perfect Fermi sea of molecules. But since \( \bar{\omega}_{bf} < \bar{\omega}_f \), the final temperature must be lower if \( S(T_i) = S(T_f) \).

An analytical expression for the relation between the initial and final temperatures can be obtained in high- and low-\( T \) limits. For \( T^i \gg (T_F, T_c) \), the entropy of each component is of the Boltzmann form [32]

\[
S_k(T) = k_B N_k \left\{ 4 - \ln \left( \frac{\hbar \bar{\omega}_k}{k_B T} \right) N_k \right\}.
\]  

(14)

One obtains \( T^f \) from \( S(T^f) = S(T^i) \); in the case \( x = 1/2 \) the result is

\[
\frac{T^f}{T_F} = 6^{1/3} e^{\Delta/3} \left( \frac{\bar{\omega}_{bf}}{\bar{\omega}_b} \right) \left( \frac{T^i}{T_F} \right)^2.
\]  

(15)

The bosonic entropy for \( T \ll (T_F, T_c) \) is

\[
S_b(T) = \frac{2\pi^4}{45} k_B \left( \frac{k_B T^i}{\hbar \bar{\omega}_b} \right)^3,
\]  

(16)

while for the fermionic atoms the low-temperature expansion of \( F_n(z) \) [32] yields

\[
S(T) = \pi^2 k_B N_f \left( \frac{T}{T_F} \right) \left[ 1 - \frac{\pi^2}{S} \left( \frac{T}{T_F} \right)^2 \right].
\]  

(17)

The entropy of the molecules is given by a similar expression. The adiabatic constraint \( S(T^i) = S(T^f) \) in general does not lead to a simple compact expression. However, by setting \( T^i = T^f = T_{\text{iso}} \), we find the initial temperature for which an adiabatic sweep is also isothermal (in the sense that the temperature endpoints are equal):

\[
\frac{T_{\text{iso}}}{T_F} = \frac{15}{\pi x^{1/3}} \sqrt{\frac{r_{bf}(1-x)^{2/3} + (2x-1)^{2/3} - x^{2/3}}{4r_b + 3r_{bf}}}. 
\]  

(18)

If \( T^i > T_{\text{iso}} \), the gas is heated by the adiabatic sweep, while a starting temperature \( T^i < T_{\text{iso}} \) leads to cooling. Assuming extremely low temperatures, such that the entropy can be taken to be linear in \( T \), a simple relation can be obtained for \( \Delta N > 0 \),

\[
\frac{T^f}{T^i} = \frac{x^{2/3}}{(2x-1)^{2/3} + r_{bf}(1-x)^{2/3}}, \quad x \geq 0.5.
\]  

(19)
Figure 2. Final temperature after adiabatic sweep from atomic to molecular configuration as a function of the initial temperature (······). The frequency ratios correspond to those of the JILA experiment (see footnote 5), and $x = 1/2$. The analytical relation (20) is shown for comparison (——), along with the line $T^f = T^i$ (---). The inset shows the same numerical results compared with the analytical expression (15) in the high temperature limit (——).

In the case of equal atomic populations, the relation becomes

$T^f = T^i / r_{bf}$.  

The same expression holds in the case of $\Delta N < 0$, where the ratio $T^f / T^i$ is independent of $|\Delta N|$, as long as there is a condensate in both initial and final states.

Figure 2 plots the dependence of the final temperature on the initial temperature for $x = 1/2$ for the JILA trap parameters (see footnote 5) in comparison to the low- and high-$T$ limits (20) and (15), respectively. For these parameters, equation (18) gives $T_{iso} = 0.20T_F$, which is seen to provide a good estimate of the value of $T^i$ for which $T^f = T^i$.

5. Cooling cycle

We plot equation (19) in figure 3. As is evident, $T^f / T^i$ depends strongly on the fermionic fraction $x$. In particular, there is an optimal fraction, $x_{opt} = [1 + r_{bf}^3] / [1 + 2r_{bf}^3]$, which maximizes cooling; it corresponds to the minimum of the $T^f / T^i$ curve.

Since $x_{opt} > 1/2$, the following cooling cycle is possible.

1. Start with an atomic gas at $\epsilon_{res} / k_B T_F \gg 1$. Cool the gas to $T < T_{iso}$ through standard cooling techniques.
2. Alter the fermionic fraction to $x_{opt}$ (see below).
3. Sweep $\epsilon_{\text{res}}$ adiabatically to $-\epsilon_{\text{res}}/k_BT_F \gg 1$, forming the maximum number of molecules possible.

4. Remove the free atoms from the system (see below).

5. Adiabatically increase $\epsilon_{\text{res}}$ into the dissociated regime.

6. Continue the cycle by repeating steps 2–5 until the desired final temperature is reached.

Efficient selective removal of an atomic component of a mixed atom–molecule gas has been demonstrated by optical resonance techniques \cite{38}. A similar selective removal from an atomic Bose–Fermi mixture has also been demonstrated \cite{19}. Assuming $T \ll T_F$, we can use \eqref{eq:19} and \eqref{eq:20} to calculate the final temperature of the system after $n$ cycles:

$$T_n = T_0 \left( \frac{rbf \left( \frac{x^2}{3} \right)}{(2x - 1)^{2/3} + rbf (1 - x)^{2/3}} \right)^n, \quad (21)$$

where we have neglected a small decrease in temperature after suddenly decreasing the populations in steps 2 and 4. Notice that values of $rbf$ closer to unity produce better cooling; this can be attained by an appropriate choice of wavelength of the trapping laser \cite{29}.

As is the case for evaporation, this cooling scheme leads to particle loss from the system in steps 2 and 4. This results in a reduction of the Fermi temperature $T_F$. Hence, the figure of merit for increasing phase-space density is the decrease of the ratio of the temperature $T$ to the Fermi temperature $T_F$ at the end of the cycle. A simple calculation shows that after $n$ cycles $T^n_F = T^0_F \left( \frac{(1 - x)}{x} \right)^{n/3}$. The expression for $T/T_F$ as a function of the number of cycles performed

**Figure 3.** Ratio $T^f/T^i$ after adiabatic conversion of atoms to molecules (equation \eqref{eq:19}; ---), and ratio $T^i/T^i_F$ after one cooling cycle, steps 1–6, normalized by its initial value $T^0_i/T^0_F$ (equation \eqref{eq:22}; - - - -), both as a function of the fermionic fraction $x$. The colours correspond to the JILA parameters (see footnote 5) (red), the MIT parameters (see footnote 4) (blue) and the best case $rbf = 1$ (black).
Figure 4. Ratio of absolute to Fermi temperature after \( n \) cycles operated at \( x = \tilde{x}_{\text{opt}} \). The numerical results (symbols) are compared with the analytical estimate (22), which neglects a small temperature decrease that occurs in steps 2 and 4 (lines). Results are shown for the JILA parameters (see footnote 5) (* and ---) and for the best case \( r_{bf} = 1 \) (○ and ——). Inset: \( \tilde{x}_{\text{opt}} \) as a function of the frequency ratio \( r_{bf} \).

\[
\frac{T^n}{T_F^n} = \frac{T_F^0}{T_F^0} \left[ \frac{r_{bf} x}{(2x - 1)^{2/3}(1 - x)^{1/3} + r_{bf}(1 - x)} \right]^n. \tag{22}
\]

Figure 3 shows this expression for \( n = 1 \) and figure 4 shows an example of the change in \( T/T_F \), as a function of the number of cycles performed. For \( x > x_{\text{opt}} \), the cycle results in a net increase of \( T/T_F \) and for \( x = x_{\text{opt}} \) the phase–space density remains unchanged. We find that there is a fermionic fraction, \( \tilde{x}_{\text{opt}} < x_{\text{opt}} \), which maximizes the reduction of \( T/T_F \); this corresponds to the minimum of the respective curve depicted in figure 3. We plot it as a function of \( r_{bf} \) in the inset of figure 4.

The efficacy of the proposed cooling scheme is contingent on losses from three-body collisions being minimal. At present, no estimates of their rates exist for the systems under consideration. Additionally, the system may become unstable during the passage through the resonance region. The large increase in the interspecies atomic scattering length on resonance could result in a collapse of the cloud, as observed in an atomic Bose–Fermi mixture with attractive interspecies interactions [26]. For a large boson–fermion repulsion, phase separation of the components may limit the conversion efficiency [39, 40].

6. Conclusion

We have presented the phase diagram of an ideal mixture of bosonic and fermionic atoms in thermal and chemical equilibrium with heteronuclear fermionic molecules. A cooling cycle has
been identified that exploits the mechanism of adiabatic atom–molecule interconversion. This cycle could provide a useful complement to existing cooling techniques. Its practical efficiency will be limited by effects of atom–atom and atom–molecule interactions, which lie outside the scope of the present treatment and must be determined by experiment.

Acknowledgments

MAM acknowledges support by an NIST SURF Summer Undergraduate Research Fellowship supported by the National Science Foundation under grant no 0097450; NN and CWC acknowledge support by the National Science Foundation under grant PHY-0100767.

References

[1] Tiesinga E, Verhaar B J and Stoof H T C 1993 Phys. Rev. A 47 4114
[2] Inouye S, Andrews M R, Stenger J, Miesner H J, Stamper-Kurn D M and Ketterle W 1998 Nature 392 151
[3] Streeker K E, Partridge G B, Truscott A G and Hulet R G 2003 Nature 417 150
[4] Khaykovich L, Schreck F, Ferrari G, Bourdel T, Cubizolles J, Carr L D, Castin Y and Salomon C 2002 Science 296 1290
[5] Cornish S L, Claussen N R, Roberts J L, Cornell E A and Wieman C E 2000 Phys. Rev. Lett. 85 1795
[6] Roberts J L, Claussen N R, Cornish S L, Donley E A, Cornell E A and Wieman C E 2001 Phys. Rev. Lett. 86 4211
[7] Claussen N R, Donley E A, Thompson S T and Wieman C E 2002 Phys. Rev. Lett. 89 010401
[8] O’Hara K M, Hemmer S L, Gehm M E, Granade S R and Thomas J E 2002 Science 298 2179
[9] Regal C A, Greiner M and Jin D S 2004 Phys. Rev. Lett. 92 040403
[10] Zwierlein M W, Stan C A, Schunck C H, Raupach S M F, Kerman A J and Ketterle W 2004 Phys. Rev. Lett. 92 120403
[11] Bartenstein M, Altmeyer A, Riedls S, Jochim S, Chin C, Hecker Denschlag J and Grimm R 2004 Phys. Rev. Lett. 92 203201
[12] Bourdel T, Kaykovich L, Cubizolles J, Zhang J, Chevy F, Teichmann M, Tarruell L, Kokkelmans S J J M F and Salomon C 2004 Phys. Rev. Lett. 93 050401
[13] Kinas J, Hemmer S L, Gehm M E, Turpalov A and Thomas J E 2004 Phys. Rev. Lett. 93 150402
[14] Donley E A, Claussen N R, Thompson S T and Wieman C E 2002 Nature 417 529
[15] Herbig J, Kraemer T, Mark M, Weber T, Chin C, Nägerl H-C and Grimm R 2003 Science 301 1510
[16] Dürr S, Volz T, Marte A and Rempe G 2004 Phys. Rev. Lett. 92 020406
[17] Xu K, Mukaiyama T, Abo-Shaeer J R, Chin J K, Miller D E and Ketterle W 2003 Phys. Rev. Lett. 91 210402
[18] Regal C A, Ticknor C, Bohn J L and Jin D S 2003 Nature 424 47
[19] Streeker K E, Partridge G B and Hulet R G 2003 Phys. Rev. Lett. 91 080406
[20] Cubizolles J, Bourdel T, Kokkelmans S J J M F, Shlyapnikov G V and Salomon C 2003 Phys. Rev. Lett. 91 240401
[21] Jochim S, Bartenstein M, Altmeyer A, Hendl G, Chin C, Hecker Denschlag J and Grimm R 2003 Phys. Rev. Lett. 91 240402
[22] Zwierlein M W, Stan C A, Schunck C H, Raupach S M F, Gupta S, Hadzibabic Z and Ketterle W 2003 Phys. Rev. Lett. 91 250401
[23] Stan C A, Zwierlein M W, Schunck C H, Raupach S M F and Ketterle W 2004 Phys. Rev. Lett. 93 143001
[24] Inouye S, Goldwin J, Olsen M L, Ticknor C, Bohn J L and Jin D S 2004 Phys. Rev. Lett. 93 183201
[25] Williams J E, Nygaard N and Clark C W 2004 New J. Phys. 6 123
[26] Modugno G, Roati G, Riboli F, Ferlaino F, Brecha R J and Inguscio M 2002 Science 297 2240
[27] Yabu H, Takayama Y and Suzuki T 2003 Physica B 329–333 25

Yabu H, Takayama Y, Suzuki T and Schuck P 2004 Nucl. Phys. A 738 273

New Journal of Physics 7 (2005) 87 (http://www.njp.org/)
[28] Carr L D, Shlyapnikov G V and Castin Y 2004 Phys. Rev. Lett. 92 150404
    Carr L D, Chiaramonte R and Holland M J 2004 Phys. Rev. A 70 043609
[29] Safronova M S and Clark C W 2005, to be submitted
[30] Zwierlein M, private communication
[31] Goldwin J, private communication
[32] Pathria R K 1972 Statistical Mechanics (Oxford: Pergamon)
[33] Williams J E, Nikuni T, Nygaard N and Clark C W 2004 J. Phys. B: At. Mol. Opt. Phys. 37 L351
[34] Mies F H, Tiesinga E and Julienne P S 2000 Phys. Rev. A 61 022721
[35] Yurovsky V A and Ben-Reuven A 2003 Phys. Rev. A 67 043611
[36] Wouters M, Tempere J and Devreese J T 2003 Phys. Rev. A 67 063609
[37] Dannenberg O, Mackie M and Suominen K-A 2003 Phys. Rev. Lett. 91 210404
[38] Xu K, Mukaiyama T, Abo-Shaeer J, Chin J K, Miller D E and Ketterle W 2003 Phys. Rev. Lett. 91 210402
[39] Mølmer K 1998 Phys. Rev. Lett. 80 1804
[40] Nygaard N and Mølmer K 1999 Phys. Rev. A 59 2974