The effect of dephasing on superadiabatic single-qubit rotation gates

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Abstract. To implement quantum gates stimulated Raman adiabatic passage (STIRAP) can be used. This STIRAP requires high Rabi frequencies and to overcome this problem we use superadiabatic approach. Our model is a tripod consisting of four-level system driven by three resonant fields. These fields are modulated by Gaussian pulses with different amplitudes, phases and time delays. We investigate the robustness of our model against dephasing which are caused by collisions or phase fluctuations of the fields.

1. Introduction
The process of stimulated Raman adiabatic passage (STIRAP) is well known technique in quantum information. It is used to transfer adiabatically the population from one state to another state and it is based on the adiabatic theorem [1]. This theorem states that if the system is initially prepared in an instantaneous eigenstate of a time-depend Hamiltonian which varies slowly, the state of the system remain all the time in this instantaneous eigenstate.

Recently, STIRAP has been used to generate quantum gates [2,3]. Single-qubit gates are quantum rotation gates which are given in matrix form by

\[ R(a, \phi) = \begin{bmatrix} \cos a & e^{i\phi} \sin a \\ -e^{-i\phi} \sin a & \cos a \end{bmatrix}, \]

where \( a \) is the angle of rotation and \( \phi \) is the phase of the gate. These rotation gates can be implemented efficiently in a three-level lambda system using a reversed STIRAP followed by a standard STIRAP [2]. In both STIRAPs, strong fields are required which is a disadvantage in many practical realizations. This technical problem can be reduced by superadiabatic or transitionless approach which has been recently proposed [4–7].

We have recently discussed the generation of single-qubit rotation gates using superadiabatic [3]. Our model is robust against decay rate of the excited state, but not against the dephasing of the ground state which caused by collisions or phase fluctuations of the driving fields. We will extend our previous paper to include other dephasing rate.
2. STIRAP model

The model is a tripod consisting of four-level system driven by three resonant coherent fields with Rabi frequencies $\Omega_0$, $\Omega_1$, and $\Omega_2$. These coherent fields couple the excited level $|e\rangle$ to the three lower levels $|0\rangle$, $|1\rangle$, and $|2\rangle$ as in Figure 1. The fields are modulated by Gaussian pulses with width $\delta$, amplitudes $A_j$, phase $\phi_j$, and time delay $\tau_j$

$$\Omega_j(t - \tau_j) = A_j e^{i\phi_j} e^{-\frac{(t - \tau_j)^2}{\delta^2}}.$$  \hspace{1cm} (2)

The three Rabi frequencies are given by

$$\Omega_0 = \Omega(t + T - \tau) + \Omega(t + T + \tau) \cos \theta + \Omega(t - T - \tau) \sin \theta,$$

$$\Omega_1 = \Omega(t + T + \tau) \sin \theta + \Omega(t - T + \tau) + \Omega(t - T - \tau) \cos \theta$$

$$\Omega_2 = \Omega(t + T + 3\tau) + \Omega(t - T - 3\tau).$$ \hspace{1cm} (3)

They represent two STIRAPs separated by $T$ and each STIRAP has two pulses separated by $\tau$ (see Figure 2). The Hamiltonian of the system is given in the basis $\{|0\rangle, |1\rangle, |e\rangle, |2\rangle\}$ as follows.
where we have assumed that the Rabi frequencies $\Omega_i$ are all real numbers. The Hamiltonian $H_0$ has four instantaneous adiabatic eigenvalues

$$\lambda_{\pm} = \pm \frac{1}{2} \sqrt{\Omega_0^2 + \Omega_1^2 + \Omega_2^2} \quad \text{and} \quad \lambda_1 = \lambda_2 = 0.$$ (5)

The eigenstates that correspond to zero eigenvalue are called dark states. They are given by

$$|D_1\rangle = -\cos \theta_1 \sin \theta_0 |0\rangle + \cos \theta_1 \cos \theta_0 |1\rangle + \sin \theta_1 |2\rangle,$$

$$|D_2\rangle = \cos \theta_0 |0\rangle + \sin \theta_0 |1\rangle,$$ (6)

where

$$\tan \theta_0 = \frac{\Omega_0}{\Omega_1}, \quad \text{and} \quad \tan \theta_1 = \frac{\sqrt{\Omega_0^2 + \Omega_1^2}}{\Omega_2}. $$ (7)

So, according to the adiabatic theorem [1], if the system starts in the superposition of the two dark states it evolution will remain in this superposition.

If we use the following conditions

$$\lim_{t \to -\infty} \tan \theta_0 = \cot \theta, \quad \lim_{t \to -\infty} \tan \theta_1 = 0,$$

$$\lim_{t \to \infty} \tan \theta_0 = \tan \theta, \quad \lim_{t \to \infty} \tan \theta_1 = 0,$$

and the initial state is in a superposition of the two dark states, then we obtain the rotation gate

$$R(a, 0) = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix} = \begin{bmatrix} \sin 2\theta & \cos 2\theta \\ -\cos 2\theta & \sin 2\theta \end{bmatrix}.$$

So, $R(\pi/2, 0)$ corresponds to $\theta = 0$.

3. Superadiabatic approach

The process of superadiabatic or transitionless adiabatic passage was proposed to overcome the technical problem of high Rabi frequencies required by STIRAP. Additional pulse is required to couple between the two lower states. This coupling can be implemented using magnetic field with $\pi$-area or near $\pi$-area [7]. Thus, the total Hamiltonian becomes

$$H = H_0 + H_1,$$ (9)

where $H_0$ is given by eq. (4) and $H_1$ is the correction due to the superadiabatic process [4,6,7]

$$H_1 = i \sum_n \left| \partial_t n \right\rangle \left\langle n \right| - \left\langle n \right| \partial_t n \left| n \right\rangle \left\langle n \right|. $$ (10)

The sum in eq. (10) is taken over all the instantaneous eigenstates. The Hamiltonian $H_1$, in the basis $\{|0\rangle, |1\rangle, |e\rangle, |2\rangle\}$, takes the form [3].

$$H_1 = \begin{bmatrix} 0 & h_{0,1} & 0 & h_{0,2} \\ h_{0,1}^* & 0 & 0 & h_{1,2} \\ 0 & 0 & 0 & 0 \\ h_{0,2}^* & h_{1,2}^* & 0 & 0 \end{bmatrix}. $$ (11)
where

\[ h_{0,1} = i \frac{\Omega_0 \dot{\Omega}_1}{\Omega_0^2 + \Omega_1^2}, \]
\[ h_{0,2} = i \frac{\Omega_1 (\Omega_0 \dot{\Omega}_0 + \Omega_1 \dot{\Omega}_1) \Omega_2 - (\Omega_0^2 + \Omega_1^2) \dot{\Omega}_2}{(\Omega_0^2 + \Omega_1^2)(\Omega_0^2 + \Omega_1^2 + \Omega_2^2)}, \]
\[ h_{1,2} = i \frac{\Omega_1 (\Omega_0 \dot{\Omega}_0 + \Omega_1 \dot{\Omega}_1) \Omega_2 - (\Omega_0^2 + \Omega_1^2) \dot{\Omega}_2}{(\Omega_0^2 + \Omega_1^2)(\Omega_0^2 + \Omega_1^2 + \Omega_2^2)}. \]

With the Gaussian pulses given in eq. (2), the term \( h_{0,1} = 0 \). Thus, the Hamiltonian \( H_1 \) becomes

\[ H_1 = \begin{bmatrix}
0 & 0 & 0 & h_{0,2} \\
0 & 0 & 0 & h_{1,2} \\
0 & 0 & 0 & 0 \\
h_{0,2}^* & h_{1,2}^* & 0 & 0
\end{bmatrix}. \] (12)

which is equivalent to an additional driving field which couples the two lower states \(|0\rangle\) and \(|1\rangle\) to the level \(|2\rangle\). So, this Hamiltonian does not induce any effects on the excited state \(|e\rangle\).

**Figure 3.** The population of the tripod in the absence of dephasing for the rotation gate \( R(\pi/2, 0) \).

In Figure 3 we plot the population of the four states of the tripod for the rotation gate \( R(\pi/2, 0) \). The left figure, the initial state is at \(|0\rangle\) and the right figure, the initial state is \(|1\rangle\). It is clear from the figure that the state \(|2\rangle\) and the excited state \(|e\rangle\) are not populated during the evolution of the system.

**4. The effect of dephasing**

In the absence of any sort of decoherence the evolution is given by the Shrödinger equation

\[ i \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle. \] (13)

However, in the presence of decoherence, this equation (13) is replaced by the Lindblad master equation

\[ \dot{\rho} = -i [H, \rho] + \frac{1}{2} \sum_i \left( 2 C_i \rho C_i^\dagger - C_i^\dagger C_i \rho - \rho C_i^\dagger C_i \right), \] (14)
where $\rho$ is the atomic reduced density operator, $H$ is the total Hamiltonian operator $H = H_0 + H_1$, and $C_i$ are the Lindblad operators associated with the decoherence.

The effect of dephasing on quantum rotation gates has been investigated by various authors [3,8,9]. In this paper we restrict ourselves to the dephasing of the form $C_i = \sqrt{2\Gamma} \sigma_{zi}$, where $\Gamma$ is the dephasing rate and $\sigma_{zi} = |i\rangle\langle i| - |e\rangle\langle e|$ is the Pauli operator. The measure of the performance of the rotation gate is given by the fidelity

$$F = |\langle \psi(t_f) | R(a,\phi) | \psi(t_i) \rangle|,$$

where the initial state is $|\psi(t_i)\rangle$ and the final state is $|\psi(t_f)\rangle$. The final state is obtained by solving the master equation (14). In our numerical computations of fidelity, we focus on the generation of the rotation gate $R(\pi/2,0)$.

**Figure 4.** The Maximum, minimum and average fidelities for the rotation gate $R(\pi/2,0)$ as a function of the dephasing rate $\Gamma$.

If the dephasing rate is small enough we can use the quantum trajectory approach [10–14]. This approach unravels the Lindblad master equation (14) as trajectories and each trajectory consists of the non jump evolution processes, and the quantum jump processes. The non jump evolution is a continuous coherent evolution and takes the form

$$i \frac{d|\psi(t)\rangle}{dt} = H_{\text{eff}}(t)|\psi(t)\rangle,$$

where the operator $H_{\text{eff}}$ is non Hermitian operator

$$H_{\text{eff}}(t) = H(t) - \frac{i}{2} \sum_i C_i^\dagger C_i.$$

The quantum jump is a discrete random jump resulted from emission or absorption of energy quanta [15]. These jumps are described by the operators $C_i$ which act on the state according to the following rule [15]

$$|\psi\rangle \rightarrow C_i |\psi\rangle.$$

The maximum, minimum, and average fidelities are shown in Figure 4 as a function of $\Gamma$ for the rotation gate corresponds to $a = \pi/2$ and phase $\phi = 0$. These fidelities are computed numerically for 1000 initial random states uniformly distributed on the Bloch Sphere as follows.

$$|\psi(t_i)\rangle = \sqrt{u}|0\rangle + e^{i2\pi v}\sqrt{1-u}|1\rangle,$$
where the two random variable $u$ and $v$ are uniformly distributed on the unit interval $[0, 1]$. We have used quantum jump approach which is valid for very small dephasing rates. As the figure shows, the fidelity does not depend on the initial state. We should note that the quantum jump has two processes, the coherent evolution and the quantum jump. We have neglected the quantum jump due to the fact that the dephasing is very small. By increasing the dephasing rate it is better to use the master equation which is the average over many trajectories, coherent evolution and random quantum jumps.

5. Conclusion

Quantum rotation gates are single-qubit gates. To implement these gates we can use STIRAP. However, this scheme require high Rabi frequencies which is a disadvantage in many experiments. In this study we have shown that using superadiabatic approach can overcome this technical problem. Furthermore, we have investigated the effect of a realistic dephasing on the performance of the rotation gate $R(\pi/2, 0)$. We have shown that it is robust to the dephasing rate of the excited state due to the fact that the excited state is not populated during the evolution of the system. However, the dephasing which cause by collisions or phase fluctuations of the fields can leads to imperfect gate. In our study we have focused on the dephasing proportional to the $\sigma_z$ and we have used the quantum trajectory approach. We have found that for small values of the dephasing rate, the fidelity decreases linearly. So, to obtain a practical useful gate one must keep the dephasing as small as possible.

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