Sterile neutrinos produced near the EW scale I: mixing angles, MSW resonances and production rates.

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We study the production of sterile neutrinos in the region $T \sim M_W$ in an extension beyond the standard model with the see-saw mass matrix originating in Yukawa couplings to Higgs-like scalars with masses and vev’s of the order of the electroweak scale. Sterile neutrinos are produced by the decay of scalars and standard model vector bosons. We obtain the index of refraction, dispersion relations, mixing angles in the medium and production rates including those for right-handed sterile neutrinos, from the standard model and beyond the standard model self-energies. For $1 \lesssim M_W/T \lesssim 3$ we find narrow MSW resonances with $k \lesssim T$ for both left and right handed neutrinos even in absence of a lepton asymmetry in the (active) neutrino sector, as well as very low energy ($k/T \ll |\xi|$) narrow MSW resonances in the presence of a lepton asymmetry consistent with the bounds from WMAP and BBN. For small vacuum mixing angle, consistent with observational bounds, the absorptive part of the self-energies lead to a strong damping regime very near the resonances resulting in the exact degeneracy of the propagating modes with a concomitant breakdown of adiabaticity.

We argue that cosmological expansion sweeps through the resonances, resonant and non-resonant sterile neutrino production results in a highly non-thermal distribution function enhanced at small momentum $k < T$, with potentially important consequences for their free streaming length and transfer function at small scales.

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I. INTRODUCTION

In the concordance ΛCDM standard cosmological model, the Universe today is composed approximately by 70% of a dark energy component responsible for the acceleration, about 25% of dark matter (DM) and about 5% of ordinary matter (baryons). In this scenario the (DM) component is cold and collisionless, and structure formation proceeds in a hierarchical “bottom-up” manner: small scales become non-linear and collapse first and their merger and accretion lead to structures on larger scales [1]. This is a consequence of the fact that cold dark matter (CDM) particles feature negligible velocity dispersion leading to a power spectrum that favors small scales. In this hierarchical scenario dense clumps that survive the merger process form satellite galaxies. Numerical simulations of structure formation with (CDM) predict many orders of magnitude more (DM) sub-haloes than observed low luminosity dwarf galaxies [2, 3, 4, 5, 6, 7]. These simulations also yield a density profile that increases monotonically towards the center [3, 5, 6, 8, 9, 10, 11]. $\rho(r) \sim r^{-\gamma}$, $\gamma = 1$ corresponds to the Navarro-Frenk-White (NFW) profile, but steeper profiles with $\gamma \sim 1.2$ have been found recently in numerical simulations [5]. These density profiles accurately describe clusters of galaxies but there has been recent observational evidence that seems to indicate a shallow cored profile instead of cusps in dwarf spheroidal galaxies (dShps) which are deemed to be (DM) dominated [12, 13, 14, 15, 16, 17].

This core vs. cusp controversy is still being debated, and recent arguments suggest that the interpretation of the data is subject to assumptions and modelling [18]. Recently yet another discrepancy between the predictions of ΛCDM and observations has been revealed, the “emptiness of voids”, possibly related to the overabundance problem [19].

Warm dark matter particles (WDM) were invoked [20, 21, 22, 23] as possible solutions to the core vs. cusps and the overabundance problems in satellite galaxies. (WDM) particles feature a non-vanishing velocity dispersion with a range in between (CDM) and hot dark matter (HDM) leading to a free streaming scale that cutoffs power at small scales thereby smoothing out small scale structure. If the free streaming scale of the (WDM) particles is smaller than the scale of galaxy clusters, the large scale structure properties are indistinguishable from those of (CDM), but may affect structure at small scales [24], thereby providing an explanation of the smoother inner profiles and the fewer

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satellites. A small scale cutoff in the (DM) power spectrum may also explain the apparent smallness of galaxies at $z \sim 3$ found in ref.[25].

Although the interpretation of cores in (dSphs) may be challenged by alternative explanations, and the missing satellite problem could be resolved by astrophysical mechanisms such as complex “gastrophysics”, and recent simulations suggest that the dynamics of subhalos is not too different in (WDM) and (CDM) models[26], there is an intrinsic interest in studying alternatives to the standard (CDM) paradigm.

Any particle physics explanation of (DM) involves extensions beyond the Standard Model (SM), allowing quite generally, both (CDM) and (WDM) candidates.

Sterile neutrinos, namely SU(2) singlets, with masses in the $\sim$ keV range may be suitable (WDM) candidates[27, 28, 29, 30, 31, 32, 33, 34, 35] and may provide possible solutions to other astrophysical problems[31]. The main property that is relevant for structure formation of any (DM) candidate is its distribution function after decoupling[34, 35] which depends on the production mechanism and the quantum kinetic evolution from production to decoupling. Sterile neutrinos may be produced by various different mechanisms[27, 28, 29, 30, 31, 32, 33], among them non-resonant mixing, or Dodelson-Widrow (DW)[27, 28, 29] has been invoked often. However, there seems to be some tension between the X-ray[38] and the Lyman-α forest data[39, 40] leading to the suggestion[41] that (DW)-produced sterile neutrinos cannot be the dominant (WDM) component.

A phenomenologically appealing extension of the (SM) with only one scale has been recently proposed[30, 31, 32]. In this model sterile neutrinos may be produced by the decay of a gauge-singlet scalar with a mass of the order of the electroweak scale[31, 32, 33, 42]. In this scenario sterile neutrinos are produced and decouple at a temperature of the order of the mass of the scalar[32, 33, 43].

Recently[43] the quantum kinetics of production and decoupling of $\sim$ keV sterile neutrinos in these models was studied with the result that production via the decay of the gauge-singlet scalar leads to a non-thermal distribution function that favors small momentum. This result was combined with an analytic method to obtain the transfer function during matter domination recently introduced in ref.[44]. This method reveals the influence of the distribution function of the decoupled particle upon the power spectrum and free streaming length[45]. The results of ref.[43] point out that sterile neutrinos produced via the decay of gauge-singlet scalars in the model advocated in refs.[30, 31, 32, 33, 42] yield smaller free streaming lengths and an enhancement of power at small scales as compared to those produced by the (DW) mechanism. Combining the results for the distribution function of sterile neutrinos produced via scalar decay with abundance and phase space constraints from dwarf spheroidal galaxies (dSphs)[44] yields a narrow window for the mass of sterile neutrinos[43]: $0.56$ keV $\lesssim M_\nu \lesssim 1.3$ keV. The robustness of this bound has been confirmed in ref.[46] but there may be some tension with recent analysis of the Lyman-α forest with non-thermal populations[47], although the results in this reference relate mainly to resonant production.

Recent observations of the X-ray spectra from the Ursa Minor dwarf spheroidal galaxy with the Suzaku satellite[48] suggest that sterile neutrinos with masses in the keV range with mixing angles $\theta \sim 10^{-5}$ remain viable candidates as main dark matter constituents, a result that seems to be confirmed by those of ref.[47].

Motivation and objectives: The clustering properties of dark matter candidates depend on the free streaming length which determines the scale below which power is suppressed. When the (DM) particle of mass $M_\nu$ becomes non-relativistic, the free streaming length is approximately given by

$$\lambda_{fs} \simeq \left[\frac{\rho^2}{G M^4_\nu}\right]^{1/2},$$

where $\rho$ is the (DM) density and the average is with the distribution function of the decoupled (DM) particle. Distribution functions that favor small momenta lead to smaller free streaming lengths and more power at small scales[43, 44, 45].

The study in ref.[43] revealed that the non-resonant mechanism of sterile neutrino production by scalar decay advocated in ref.[30, 31, 32, 33, 42] leads to a non-thermal distribution function that favors small momenta with important consequences for structure formation and remarkable differences with sterile neutrinos produced by the (DW) mechanism[27], whose distribution function is that of a thermal relic that decoupled while relativistic, but multiplied by an overall factor$^{27}$. This overall factor in the (DW) distribution function only affects the abundance, but for a fixed (DM) density $\rho$ the resulting free streaming length is that of a neutrino of mass $M_\nu$ decoupled at the sterile neutrino decoupling temperature. For a fixed mass and relic density the non-thermal distribution function from the production mechanism studied in ref.[32, 33, 42] yields a smaller $\lambda_{fs}$ and more power at small scales than in the (DW) mechanism without modifying the large scale power spectrum.

In the extension beyond the standard model (bsm) advocated in ref.[30, 31, 32, 33, 42] sterile neutrinos mix with active neutrinos via a Yukawa coupling to the standard model Higgs[30] whose expectation value yields a see-saw mass matrix. The diagonalization of this see-saw mass matrix yields interaction vertices between vector bosons and the sterile-like neutrino. This is important because the distribution functions being a function of the energy, are necessarily associated with mass or energy eigenstates, not flavor eigenstates.
The study in ref.\textsuperscript{32, 33, 34} reveals that sterile neutrinos are produced and decouple at a temperature of the order of the mass of the scalar, which in the model of refs.\textsuperscript{30, 32, 33, 42} is of the order of the Higgs mass.

At this temperature the charged and neutral vector bosons are present in the medium with large abundance, comparable to that of the scalar. Their decay into the sterile-like neutrinos will therefore contribute to their total abundance and distribution function. This is one of the main observations in this article. The coupling of the charged and neutral vector bosons to the sterile-like neutrino is suppressed by the (small) mixing angle, but since the standard model couplings are much larger than the Yukawa couplings of the scalar to the sterile neutrino, the question is whether the decay of vector bosons may lead to a substantial contribution to the production rate of the sterile-like neutrinos. For $M_s \sim \text{keV}$ and the expectation value of the Higgs-like scalar in the range $\sim 100\,\text{GeV}$ the Yukawa coupling $Y \sim 10^{-8}$, the production rate via this process $\propto Y^2$, whereas the contribution from $Z, W$ decay would be expected to be $\propto \alpha_w \sin^2(\theta)$, with $\theta$ the mixing angle. For $\theta \sim 10^{-5}$\textsuperscript{14, 15} the production rate of sterile-like neutrinos via vector boson decay can be of the same order of or larger than that from scalar decay. This observation suggests that sterile neutrino production via the decay of vector bosons in the medium may be competitive with the production via scalar decay.

At high temperature and or density the mixing angle is modified by medium effects\textsuperscript{40, 50, 51, 52, 53, 54, 55}, therefore the first step towards understanding whether vector boson decay contributes to the production of sterile-like neutrinos is to obtain the in-medium correction to the mixing angles.

A more general aspect of sterile neutrino production via vector boson decay at $T \sim M_W$ is that both the index of refraction (real part of the self-energy) and the production rate determined by the absorptive part (imaginary part of the self-energy) are of $O(G_F)$. This is in contrast to the usual situation at temperatures much smaller than the electroweak scale when the index of refraction is of $O(G_F)$, but the absorptive part is of $O(G_F^2)$.

Although the finite temperature and density corrections to the index of refraction have been obtained for $T \ll M_{W,Z}$, to the best of our knowledge the study of the self-energy, the index of refraction (real part) and absorptive part (width) at $T \approx M_{W,Z}$ has not been carried out.

To be sure, upon the diagonalization of the mass matrix, standard model interaction vertices with the sterile-like neutrino lead to production processes via both charged and neutral current interactions such as $T \rightarrow \overline{\nu}_1 \nu_2; f \rightarrow \overline{\nu}_1 \nu_2$ with charged leptons (l) or quarks (f) and $\nu_1 \sim \nu_a; \nu_2 \sim \nu_s$ (active and sterile respectively) for small mixing angle. These processes are of $O(\alpha_w^2 \sin^2 \theta)$ and while they will eventually become important for $T \ll M_{W,Z}$ when the population of vector bosons in the medium becomes $\ll \alpha_w$, these are formally subleading in the weak coupling at $T \sim M_{W,Z}$.

Therefore at $T \sim M_{W,Z}$ vector boson decay is the \textit{leading} production mechanism from weak interactions.

Our objective is to provide a comprehensive assessment of sterile neutrinos as potential (DM) candidates implementing the following program:

- Obtain the production rates and mixing angle in the medium from the quantum field theory model at $T \sim M_W$, studying the possibility of MSW resonances to determine whether sterile neutrinos are produced resonantly or non-resonantly.
- Obtain and solve the kinetic equations describing the production and decoupling of sterile neutrinos using the rates and mixing angles obtained from previous step.
- The asymptotic solution of the kinetic equation yields the distribution function after freeze out, which determines the abundance and the free streaming length. This distribution function is input in the program described in refs.\textsuperscript{43, 44} to obtain the transfer function and power spectrum.

In this article we carry out the first step of this program. We implement methods of field theory at finite temperature and density developed in refs.\textsuperscript{54, 56, 57, 58, 59} to obtain the mixing angles in the medium and production rates both from scalar and vector boson decay.

**Results:** We study a simple extension of the standard model with one active and one sterile neutrino to extract the robust features in a simpler setting. Both active and sterile neutrinos are considered to be Dirac, this is to include the possibility of a lepton asymmetry hidden in the (active) neutrino sector (Majorana neutrinos cannot be assigned a chemical potential), and to allow us to study the production of left and right-handed neutrinos.

We obtain the dispersion relations, index of refraction, mixing angles and production rates in the medium from the self-energy contributions from standard model (sm) and beyond the standard model (bsm) interactions. The see-saw mass matrix that mixes them emerges from the Yukawa couplings to Higgs-like scalars with masses of the order of $M_{W,Z}$ that acquire expectation values also of this order. We focus on the temperature region $T \sim M_W$ where vector and scalar bosons are present in the medium with large thermal populations. The decay of both the scalar and vector bosons contribute to the production of sterile neutrinos. Our main results are:
• We find one MSW resonance even in absence of a lepton asymmetry. For \( 1 \lesssim M_W/T \lesssim 3 \) this resonance is in the low momentum region \( 0.2 \lesssim k/T \lesssim 1 \) and well within the regime of validity of the perturbative expansion. Including a lepton asymmetry in the active neutrino sector consistent with the data from Wilkinson Microwave Anisotropy Probe (WMAP) and Big Bang Nucleosynthesis (BBN), we find two low energy MSW resonances, the lowest one is a consequence of the lepton asymmetry that occurs at \( k/T \ll \xi \) with \( |\xi| \) being the lepton asymmetry parameter. In the region of interest for this study for small vacuum mixing matrix consistent with the observational bounds from X-ray data, these resonances are very narrow. We find resonances also for positive energy, positive helicity, namely nearly right-handed neutrinos.

• At the resonances the propagating frequencies become exactly degenerate in striking contrast with the quantum mechanics of neutrino mixing wherein there is level repulsion at the resonance. This exact degeneracy at the resonance entails the breakdown of adiabaticity. It is a distinct consequence of the absorptive part of the self-energy and leads to a strong damping regime.

• The form of the standard model contribution to the production rate is similar to that from scalar decay found in ref.\(^{43}\). We argue that cosmological expansion will lead to a rapid crossing of the narrow resonances resulting in both resonant and non-resonant sterile neutrino production. In particular nearly right handed sterile neutrinos are produced by the decay of \( Z^0, W^\pm \) vector bosons. Their distribution functions after freeze-out will be highly non-thermal with a distinct enhancement at small momentum \( k < T \) and perturbatively small population. This low momentum enhancement of the non-thermal distribution function is expected\(^{43, 44}\) to have important consequences: a shortening of the free-streaming length (smaller velocity dispersion) and an increase of the transfer function and power spectrum at small scales.

• We find a consistent range of parameters for which there is a resonance for positive helicity, positive energy neutrinos, namely nearly right-handed at \( T \sim M_W \). The general field theory framework allows a systematic study of the properties for both helicity states, including the helicity dependence of mixing angles and production rates.

II. THE MODEL

The extension of the standard model presented in ref.\(^{32, 33, 42}\) generalizes the proposal of the \( \nu \)-MSM of ref.\(^{30, 37}\) and is also a generalization of the model presented in ref.\(^{60}\). These models include three SU(2) singlet (sterile) neutrinos which couple to the active neutrinos via a see-saw mass matrix. The generalization of ref.\(^{32, 33, 42}\) gives a mass to the sterile neutrino via a Yukawa coupling to a Higgs-like scalar field which could be the neutral Higgs component, or another scalar whose expectation value is of the same order as that of the (sm) Higgs boson, therefore this type of extension features only one scale.

We study a simplified version of these models by considering only one sterile and one active neutrino. In the usual see-saw mechanism an off-diagonal Dirac mass matrix for the active species is considered along with a diagonal Majorana mass for the sterile neutrino\(^{30, 37, 50, 51, 52}\). However, instead of considering a Majorana sterile neutrino, we allow for Dirac mass terms for all species. This generalization allows to study simultaneously the possibility of a lepton asymmetry in the (active) neutrino sector for which a Dirac field is required, along with the possibility of a right-handed component leading to potentially relevant degrees of freedom within the same simple model.

Our goal is to extract generic and robust features of the production rates and mixing angles in the medium along with a reliable estimate of sterile production rates. The generalization to three species can be done relatively straightforwardly (but for the complications associated with dealing with larger mixing matrices), and the case of a Majorana neutrino is regained straightforwardly by projection.

We consider a model with one active (\( \nu_a \)) and one sterile (\( \nu_s \)) (an SU(2) singlet) Dirac neutrinos, described by the Lagrangian density

\[
\mathcal{L} = \mathcal{L}_{SM} + \overline{\nu}_a i \not{\partial} \nu_a - Y_1 \overline{\nu}_a \tilde{H}^\dagger l - Y_2 \overline{\nu}_s \Phi \nu_s + \mathcal{L}[\Phi] + h.c.,
\]

where

\[
l = \begin{pmatrix} \nu_a \\ f \end{pmatrix} ; \quad \tilde{H} = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}.
\]

\(f\) is the charged lepton associated with \( \nu_a \) and \( H^0, H^- \) are the components of the standard model Higgs doublet, and \( \Phi \) is a real scalar singlet field whose expectation value gives a Dirac mass to the sterile neutrino. The Lagrangian density \( \mathcal{L}[\Phi] \) describes the kinetic and potential terms of \( \Phi \).
In unitary gauge we write

\[ H^0 = \langle H^0 \rangle + \sigma \quad ; \quad \Phi = \langle \Phi \rangle + \varphi \]  

(II.3)

and consistently with the single scale assumption of the \( \nu \)-MSM: \( \langle H^0 \rangle \sim \langle \Phi \rangle \) are of the same order of magnitude (the weak scale) and that their masses are also of the same scale. In fact our analysis is quite general, and this assumption will only be invoked for a quantitative assessment. The Lagrangian density (II.1) becomes

\[
\mathcal{L} = \mathcal{L}_{SM} + \bar{\nu}_a i \dot{\nu}_a - \bar{\nu}_a m_{a\beta} \nu_\beta - Y_1 \bar{\nu}_a \sigma_{\nu_a} - Y_2 \bar{\nu}_a \varphi_{\nu_a} + \mathcal{L}[\langle \Phi \rangle + \varphi] + \text{h.c} ; \quad \alpha, \beta = a, s ,
\]

(II.4)

where

\[
\mathbb{M} = \begin{pmatrix} 0 & m \\ m & M_s \end{pmatrix} ; \quad m = Y_1 \langle H^0 \rangle ; \quad M_s = Y_2 \langle \Phi \rangle.
\]

(II.5)

Introducing the “flavor” doublet \( (\nu_a, \nu_s) \) the diagonalization of the mass term \( \mathbb{M} \) is achieved by a unitary transformation to the mass basis \( (\nu_1, \nu_2) \), namely

\[
\begin{pmatrix} \nu_a \\ \nu_s \end{pmatrix} = U(\theta) \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} ; \quad U(\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix},
\]

(II.6)

where

\[
\cos(2\theta) = \frac{M_s}{[M_s^2 + 4m^2]^{\frac{1}{2}}} ; \quad \sin(2\theta) = \frac{2m}{[M_s^2 + 4m^2]^{\frac{1}{2}}}.
\]

(II.7)

In the mass basis

\[
\mathbb{M}_m = U^{-1}(\theta) \mathbb{M} U(\theta) = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} ; \quad M_1 = \frac{1}{2} \left[ M_s - [M_s^2 + 4m^2]^{\frac{1}{2}} \right] ; \quad M_2 = \frac{1}{2} \left[ M_s + [M_s^2 + 4m^2]^{\frac{1}{2}} \right].
\]

(II.8)

We focus on a see-saw with \( M_s \sim \text{keV} \gg m \) therefore

\[
M_1 \simeq \frac{m^2}{M_s} ; \quad M_2 \simeq M_s ; \quad \sin(2\theta) \simeq \frac{2m}{M_s} \sim \left| \frac{M_1}{M_2} \right|^{\frac{1}{2}} \ll 1.
\]

(II.9)

Taking \( \langle H^0 \rangle \sim \langle \Phi \rangle \) the small mixing angle entails that \( Y_1 \ll Y_2 \) which results in self-energy corrections from the \( \sigma \) exchange are subleading as compared to those from the \( \varphi \) exchange. For example taking \( \langle \Phi \rangle \sim \langle H^0 \rangle \), and for a \( \sim \text{keV} \) sterile neutrino it follows that

\[
Y_2 \sim 10^{-8} \gg Y_1 ; \quad \sin(2\theta) \sim Y_1/Y_2.
\]

(II.10)

However, we can alternatively consider a pre-determined see-saw mass matrix and set \( Y_1 = Y_2 = 0 \) which corresponding to a simpler extension of the standard model that posits a mass matrix that originates beyond the standard model.

Our goal is to obtain the dynamical aspects of sterile neutrinos in the medium, mixing angles, dispersion relations and damping rates, which determine the production rates. These are obtained directly from the solution of the equations of motion including the self-energy corrections in the medium. The one-loop self-energies require the neutrino propagators in the medium in the mass basis, since the mass eigenstates are the true propagating states. For \( \theta \ll 1 \) the mass eigenstates \( \nu_1 \sim \nu_a; \nu_2 \sim \nu_s \), and the active neutrino reaches equilibrium at \( T \gtrsim 1 \text{ MeV} \) via the weak interactions, whereas the sterile neutrinos are not expected to equilibrate.

This argument, however, hinges on the smallness of the vacuum mixing angle, but in a medium the mixing angle can become very large, and if there are MSW resonances the roles of the medium eigenstates may be reversed. Whether there are MSW resonances and the medium mixing angle becomes large can only be answered \textit{a posteriori}.

Therefore we assume that the mass eigenstate \( \nu_1 \) is active-like, and features a Fermi-Dirac distribution function, whereas for \( \nu_2 \) the propagators are the vacuum ones. Furthermore, it is possible that if there is a large lepton asymmetry it may be stored in the neutrino sector, whereas the asymmetry in the charged leptons equals the baryon asymmetry and can be neglected. Hence the Fermi-Dirac distribution functions in the \( \nu_1 \) propagator includes a chemical potential.

In our study we explicitly separate the fermionic and bosonic contributions to the self-energies to assess the consistency of the assumption that the eigenstate “1” is active-like.
III. EQUATIONS OF MOTION

The effective Dirac equation in the medium is derived with the methods of non-equilibrium quantum field theory described in \[56, 57, 59\]. We follow the approach presented in refs. \[56, 57\] and introduce an external Grassmann-valued source that couples linearly to the neutrino field via the Lagrangian density

\[
\mathcal{L}_S = \overline{\nu}_\alpha \eta_\alpha + \overline{\eta}_\alpha \nu_\alpha, \tag{III.1}
\]

whence the total lagrangian density is given by \( \mathcal{L} + \mathcal{L}_S \). The external source induces an expectation value for the neutrino field which obeys the effective equation of motion with self-energy corrections from the medium \[59\].

The equation of motion is derived by shifting the field \( \nu_\alpha^\pm = \psi_\alpha + \Psi_\alpha \), \( \psi_\alpha = \langle \nu_\alpha^\pm \rangle \) imposing \( \langle \Psi_\alpha \rangle = 0 \) order in the perturbation theory \[56, 57, 59\]. Since the self-energy corrections to the equations of motion require the neutrino propagators, we obtain the equation of motion in the mass basis.

Implementing this program up to one loop order, we find the following equation of motion for the doublet in the mass basis \( \psi^T \equiv (\psi_1, \psi_2) \), it is given by

\[
(i \not{\partial} - \mathbb{I}_m + \Sigma^{tad}_m) \psi(\vec{x}, t) + \int d^3x' dt' \left[ \Sigma^{rel}_m(\vec{x} - \vec{x}', t - t') L + \Sigma^{ret}_m(\vec{x} - \vec{x}', t - t') \right] \psi(\vec{x}', t') = -\eta(\vec{x}, t), \tag{III.2}
\]

where \( \mathbb{I} \) is the identity matrix, \( \mathbb{I}_m = \text{diag}(M_1, M_2) \) is the mass matrix in the mass basis, \( L = (1 - \gamma^5)/2 \) is the left-handed chiral projection operator, \( \Sigma^{tad}_m \) is the (local) tadpole contribution from the (sm) neutral current interaction, (see Fig. (1)) \( \Sigma^{rel}_m(\vec{x} - \vec{x}', t - t') \) and \( \Sigma^{ret}_m(\vec{x} - \vec{x}', t - t') \) are respectively the real-time retarded self-energies from (sm) and (bsm) (scalar) interactions. Introducing the space-time Fourier transform in a spatial volume \( V \)

\[
\psi(\vec{x}, t) = \frac{1}{\sqrt{V}} \sum_k \int dk_0 e^{i\vec{k} \cdot \vec{x}} e^{-ik_0 t} \tilde{\psi}(k_0, \vec{k}) \tag{III.3}
\]

and similarly for the self-energy kernels and the source term, the equation of motion in the mass basis becomes

\[
\left[ (\gamma_0 k_0 - \vec{\gamma} \cdot \vec{k}) \mathbb{I} - \mathbb{I}_m + \Sigma^{tad}_m L + \Sigma^{rel}_m(k_0, \vec{k}) L + \Sigma^{bsm}_m(k_0, \vec{k}) \right] \tilde{\psi}(k_0, \vec{k}) = -\eta(k_0, \vec{k}). \tag{III.4}
\]

The space-time Fourier transform of the retarded self-energies (not the tadpole) feature a dispersive representation

\[
\Sigma(k_0, k) = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega \frac{\text{Im}\Sigma(\omega, \vec{k})}{\omega - k_0 - i 0^+}. \tag{III.5}
\]

A. One-Loop Self-Energy

We focus on the temperature region \( M_{Z,W,\sigma,\varphi} \gtrsim T \), in which using the unperturbed thermal propagators for the scalar and vector bosons is valid \[61\]. In section (V D) we show that perturbation theory is valid for \( k \gtrsim \alpha_{\mu} T \) for \( M_{W,\sigma,\varphi} \gtrsim T \), furthermore for \( k \ll M_W \) our results reproduce those found in the literature for \( T \ll M_W \) \[53, 54\] and the perturbative expansion is reliable for \( M_W \gtrsim 2T \).

The (sm) charged and neutral current contributions to the self-energy in the mass basis are depicted in Fig. (1). The charged current diagram in Fig. (1) refers to the charged lepton associated with the active neutrino. The contributions from scalar exchange (bsm) in the mass basis are depicted in Fig. (2).

(SM) neutral currents: The tadpole contribution in the mass basis is given by

\[
\Sigma^{tad}_m = \Sigma^t \ U^{-1}(\theta) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} U(\theta), \tag{III.6}
\]

where\(^1\)

\[
\Sigma^t = -\gamma^0 \frac{g^2}{4M_W^2} \int \frac{d^3q}{(2\pi)^3} (n_\nu - \bar{n}_\nu) = -\gamma^0 \frac{g^2 T^3}{24M_W^4} \xi \left[ 1 + \frac{\xi^2}{\pi^2} \right] ; \quad \xi = \frac{\mu}{T}. \tag{III.7}
\]

\(^1\) This expression corrects a typographic error in ref.\[56\).
FIG. 1: Standard model contributions to the self-energy $\Sigma_{sm}$. The indices $i, k, j = 1, 2$ corresponding to mass eigenstates, the index $f$ for the intermediate fermion line in the charged-current self-energy refers to the charged lepton associated with the active neutrino.

In this expression $n_{\nu}, \overline{n}_{\nu}$ are the Fermi-Dirac distribution functions for neutrinos and antineutrinos respectively, and we have neglected the contribution from the asymmetry of the charged lepton and quark sectors since these are proportional to the negligible baryon asymmetry. We allow for a lepton asymmetry stored in the neutrino sector. A recent analysis\[62\] from the latest WMAP and BBN data suggests that $|\xi| \lesssim 10^{-2}$.

The neutral current diagrams that contribute to the one-loop self energy feature two different terms corresponding to the intermediate neutrino line being either $\nu_1$ or $\nu_2$. As argued above, for small mixing angles $\nu_1 \sim \nu_0$ and weak interactions equilibrate these mass eigenstates with the medium, therefore their finite temperature propagator features the Fermi-Dirac distribution function (with a chemical potential allowing for a lepton asymmetry). However, $\nu_2 \sim \nu_s$ will not equilibrate with the medium since their coupling to the environmental degrees of freedom is suppressed by at least two powers of the (small) mixing angle, therefore $\nu_2$ features a vacuum propagator. The one loop diagrams are shown in Fig. (3) where the superscripts (1) and (2) are used to specify the intermediate neutrino propagator $\nu_1$ and $\nu_2$ respectively.

FIG. 2: Beyond the standard model contributions to the self-energy $\Sigma_{bsm}$. The indices $i, k, j = 1, 2$ corresponding to mass eigenstates. The dashed line is a scalar propagator either for $\sigma$ or $\varphi$.

FIG. 3: Neutral currents contribution to the one-loop retarded self-energy $\Sigma_{sm}$. The indices $i, j = 1, 2$ and the indices 1, 2 denote the corresponding mass eigenstate in the intermediate state.
In the mass basis we find for the neutral current contributions shown in Fig. 3
\[
\Sigma_{nc}(k_0, \vec{k}) = \left[ \cos^2(\theta) \Sigma_{nc}^{(1)}(k_0, \vec{k}) + \sin^2(\theta) \Sigma_{nc}^{(2)}(k_0, \vec{k}) \right] U^{-1}(\theta) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} U(\theta) .
\] (III.8)

**(sm) charged currents:** the charged current one-loop self energy is shown in Fig. 1, since the intermediate state is a charged lepton we find in the mass basis
\[
\Sigma_{cc}(k_0, \vec{k}) = \Sigma_{cc,sm}(k_0, \vec{k}) U^{-1}(\theta) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} U(\theta) ,
\] (III.9)

where \(\Sigma_{cc,sm}(k_0, \vec{k})\) is the usual standard model one-loop self-energy in thermal equilibrium.

**(bsm) scalar exchange:** The scalar exchange contributions to the self-energy are shown in Fig. 4. For \(\sin^2(\theta) \ll 1\) we find
\[
\Sigma_{bsm}(k_0, \vec{k}) = \left[ \cos^2(\theta) \Sigma_{\sigma}^{(1)}(k_0, \vec{k}) + \sin^2(\theta) \Sigma_{\phi}^{(1)}(k_0, \vec{k}) + \cos^2(\theta) \Sigma_{\phi}^{(2)}(k_0, \vec{k}) \right] U^{-1}(\theta) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} U(\theta) + \\
\cos^2(\theta) \Sigma_{\sigma}^{(2)}(k_0, \vec{k}) U^{-1}(\theta) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} U(\theta) .
\] (III.10)

**Summary of self-energies in the flavor basis:** The structure of the self-energies (to leading order in \(\sin^2(\theta)\)) indicates that they are diagonal in the flavor basis. In this basis the total self-energy is given by
\[
\Sigma(k_0, \vec{k}) = \begin{pmatrix} \Sigma_{aa}(k_0, \vec{k}) & 0 \\ 0 & \Sigma_{ss}(k_0, \vec{k}) \end{pmatrix} ,
\] (III.11)

where
\[
\Sigma_{aa}(k_0, \vec{k}) = \left[ \Sigma^t + \cos^2(\theta) \Sigma_{nc}^{(1)}(k_0, \vec{k}) + \sin^2(\theta) \Sigma_{nc}^{(2)}(k_0, \vec{k}) + \Sigma_{cc,sm}(k_0, \vec{k}) \right] L + \cos^2(\theta) \Sigma_{\phi}^{(2)}(k_0, \vec{k}) ,
\] (III.12)
\[
\Sigma_{ss}(k_0, \vec{k}) = \cos^2(\theta) \Sigma_{\phi}^{(1)}(k_0, \vec{k}) + \sin^2(\theta) \Sigma_{\phi}^{(1)}(k_0, \vec{k}) + \cos^2(\theta) \Sigma_{\phi}^{(2)}(k_0, \vec{k}) .
\] (III.13)

Since in the (sm) contributions we have explicitly factored out the left-handed projector \(L\), the remainder contributions to the (sm) self-energies \(\Sigma_{nc,cc}\) are those of a vector-like theory. The (bsm) contributions feature both chiralities since we have considered a Dirac mass term for the sterile neutrino, a left-handed Majorana mass term can be obtained by neglecting the right-handed contribution. We consider the regime \(T \gg M_{1,2}, m_f\) and \(k_0, k \gg M_{1,2}, m_f\), where \(m_f\) stand for the charged lepton masses, therefore we can safely neglect the mass terms and consider the propagators of massless fermionic fields.
In this regime the general form of the \((\text{sm})\) self-energies with vector boson exchange, either charged or neutral currents is written in dispersive form as in eqn. \((\text{III.5})\) with \([56, 57]\).

\[
\text{Im} \Sigma_{\text{sm}}(\omega, \vec{k}) = \pi g_{\text{sm}}^2 \int \frac{d^3 q}{(2\pi)^3} \int dp_0 \, dq_0 \, \delta(\omega - p_0 - q_0) \left[ 1 - n_F(p_0) + N_B(q_0) \right] \gamma^\mu \rho_F(p_0, \vec{p}) \rho_B(q_0, \vec{q}) \gamma^\nu P_{\mu\nu}(q_0, \vec{q}) ,
\]

\((\text{III.14})\)

where \(F\) stands for the fermionic species in the intermediate state. For \(\nu_1\) and charged lepton \(n_F\) is the Fermi-Dirac distribution function, whereas for \(\nu_2\) it is \(n_F = 0\) since the “sterile” neutrino does not thermalize with the medium. For the \((\text{bsm})\) contributions, the general form for scalar exchange is

\[
\text{Im} \Sigma_{\text{bsm}}(\omega, \vec{k}) = \pi Y^2 \int \frac{d^3 q}{(2\pi)^3} \int dp_0 \, dq_0 \, \delta(\omega - p_0 - q_0) \left[ 1 - n_F(p_0) + N_B(q_0) \right] \rho_F(p_0, \vec{p}) \rho_B(q_0, \vec{q}) .
\]

\((\text{III.15})\)

where

\[
g_{\text{sm}} = \begin{cases} 
\frac{g}{\sqrt{2}} & \text{CC} \\
\frac{g}{2 \cos(\theta_W)} & \text{NC}
\end{cases}
\]

\((\text{III.16})\)

and \(Y = Y_1, Y_2\) for \(\sigma\) and \(\varphi\) exchange respectively. The spectral densities are respectively (for massless fermions)

\[
\rho_F(p_0, \vec{p}) = \frac{1}{2} \left( \frac{\gamma^0 - \gamma^i \cdot \vec{p}}{p} \right) \delta(p_0 - p) + \frac{1}{2} \left( \frac{\gamma^0 + \gamma^i \cdot \vec{p}}{p} \right) \delta(p_0 + p) ,
\]

\((\text{III.17})\)

\[
\rho_B(q_0, \vec{q}) = \frac{1}{2W_q} \left[ \delta(q_0 - W_q) - \delta(q_0 + W_q) \right] ; \quad W_q = \sqrt{q^2 + M^2}.
\]

\((\text{III.18})\)

The projection operator

\[
P_{\mu\nu}(q_0, \vec{q}) = - \left[ g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2} \right] ; \quad q^\mu = (q^0, \vec{q})
\]

\((\text{III.19})\)

and

\[
n_F(p_0) = \frac{1}{e^{(p_0 - \mu)/T} + 1} ; \quad n_F(p_0) = 1 - n_F(-p_0) ; \quad N_B(q_0) = \frac{1}{e^{q_0/T} - 1}.
\]

\((\text{III.20})\)

We have allowed a chemical potential for the neutrinos (only for \(\nu_1 \sim \nu_\alpha\)) to include the possibility of a lepton asymmetry in the (active) neutrino sector.

In the expressions above, the masses for the scalars or vector bosons are \(M_{\sigma, \varphi, M_{Z, W}}\) as appropriate for each contribution. All the self-energies share the general form

\[
\Sigma(k_0, \vec{k}) \equiv \gamma^0 A(k_0, k) - \gamma^i \cdot \vec{k} \, B(k_0, k) ,
\]

\((\text{III.21})\)

the detailed expressions for the imaginary parts of the \((\text{sm})\) and \((\text{bsm})\) contributions are given in the appendices.

In particular, for the neutral current tadpole \(B(k_0, k) = 0\) and \(A(k_0, k)\) can be recognized from eqn. \((\text{III.7})\). Combining \((\text{III.11})\) with this form we write the self-energy matrix in the flavor basis as

\[
\Sigma_{\text{sm}}^{\text{flavor}} L + \Sigma_{\text{sm}}(k_0, \vec{k}) L + \Sigma_{\text{bsm}}(k_0, \vec{k}) = \begin{bmatrix} \gamma^0 \, A_L(k_0, k) - \gamma^i \cdot \vec{k} \, B_L(k_0, k) \end{bmatrix} L + \begin{bmatrix} \gamma^0 \, A_R(k_0, k) - \gamma^i \cdot \vec{k} \, B_R(k_0, k) \end{bmatrix} R .
\]

\((\text{III.22})\)

In the flavor basis these matrices are of the form

\[
A(k_0, k) = \begin{pmatrix} A_{aa}(k_0, k) & 0 \\ 0 & A_{ss}(k_0, k) \end{pmatrix} ; \quad B(k_0, k) = \begin{pmatrix} B_{aa}(k_0, k) & 0 \\ 0 & B_{ss}(k_0, k) \end{pmatrix}.
\]

\((\text{III.23})\)

where the matrix elements are obtained from the expressions \((\text{III.22})\).

The equations of motion for the left (L) and right (R) handed components are obtained by multiplying the equation of motion \((\text{III.4})\) on the left by the projectors \(R\) and \(L\) respectively.
It proves convenient at this stage to separate the Dirac spinors into the left $\psi_L$ and right $\psi_R$ handed components and to expand them into helicity eigenstates\footnote{55}, namely

$$\psi_L = \sum_{h=\pm 1} v^h \otimes \varphi^h ; \quad \varphi^h = \begin{pmatrix} \varphi^h_L \\ \varphi^h_R \end{pmatrix}, \tag{III.24}$$

and

$$\psi_R = \sum_{h=\pm 1} v^h \otimes \zeta^h ; \quad \zeta^h = \begin{pmatrix} \zeta^h_L \\ \zeta^h_R \end{pmatrix}, \tag{III.25}$$

where the left $\varphi$ and right $\zeta$ handed doublets are written in the flavor basis, and $v^h$ are eigenstates of the helicity operator

$$\hat{h}(\mathbf{k}) = \gamma^0 \gamma^5 \cdot \mathbf{k} \gamma^5 = \sigma \cdot \mathbf{k} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{III.26}$$

namely,

$$\sigma \cdot \mathbf{k} v^h = h v^h ; \quad h = \pm 1. \tag{III.27}$$

To leading order in weak and Yukawa couplings, and neglecting a commutator $[\mathbf{M}, \Sigma]$ because it is higher order in these couplings, we find in the flavor basis for both the left and right-handed component doublets

$$\left[(k_0^2 - k^2) \mathbb{I} + (k_0 - h k) (A_L + h B_L) + (k_0 + h k) (A_R - h B_R) - \mathbf{M}^2 \right] \begin{pmatrix} \varphi^h \\ \zeta^h \end{pmatrix} = \begin{pmatrix} I^h_L \\ I^h_R \end{pmatrix}, \tag{III.28}$$

where $\mathbf{M}$ is the mass matrix in the flavor basis and the inhomogeneities in these equations are obtained by projection and using the corresponding equations, we need not specify them as they are no longer used in our study.

In absence of interactions, for the left-handed component a positive energy solution corresponds to $h = -1$ and a negative energy solution to $h = +1$ with the opposite assignment for the right-handed component.

In the flavor basis

$$\mathbf{M}^2 = \mathbf{M}^2 \mathbb{I} + \frac{\delta \mathbf{M}^2}{2} \begin{pmatrix} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}, \tag{III.29}$$

where

$$\mathbf{M}^2 \equiv \frac{1}{2} (M^2_1 + M^2_2) ; \quad \delta \mathbf{M}^2 \equiv M^2_2 - M^2_1, \tag{III.30}$$

and $M_{1,2}$ are given by eqn. (II.8).

It proves convenient to define the combinations

$$S_h(k_0, k) = (k_0 + h k) \left[ (A_R - h B_R)_{aa} + (A_R - h B_R)_{ss} \right]$$

$$+ (k_0 - h k) \left[ (A_L + h B_L)_{aa} + (A_L + h B_L)_{ss} \right], \tag{III.31}$$

and

$$\Delta_h(k_0, k) = \frac{(k_0 + h k)}{\delta \mathbf{M}^2} \left[ (A_R - h B_R)_{aa} - (A_R - h B_R)_{ss} \right]$$

$$+ \frac{(k_0 - h k)}{\delta \mathbf{M}^2} \left[ (A_L + h B_L)_{aa} - (A_L + h B_L)_{ss} \right], \tag{III.32}$$

where we have suppressed the arguments. The equation of motion (III.28) can now be written as

$$\mathbf{G}_h^{-1}(k_0, k) \begin{pmatrix} \varphi^h \\ \zeta^h \end{pmatrix} = \begin{pmatrix} I^h_L \\ I^h_R \end{pmatrix}, \tag{III.33}$$
where the inverse propagator is given by

\[
G_h^{-1}(k_0, k) = \left( k^2_0 - k^2 + \frac{1}{2} S_h(k_0, k) - \overline{M}^2 \right) I - \frac{1}{2} \delta M^2 \rho_h(k_0, k) \begin{pmatrix}
-C_h(k_0, k) & D_h(k_0, k) \\
D_h(k_0, k) & C_h(k_0, k)
\end{pmatrix},
\]  

(III.34)

where

\[
\rho_h(k_0, k) = \left( \cos(2\theta) + \Delta_h(k_0, k) \right)^2 + \sin^2(2\theta)
\]

(III.35)

and

\[
C_h(k_0, k) = \frac{\left( \cos(2\theta) + \Delta_h(k_0, k) \right)}{\rho_h(k_0, k)},
\]

(III.36)
\[
D_h(k_0, k) = \frac{\sin(2\theta)}{\rho_h(k_0, k)}.
\]

(III.37)

We note that if \( \Delta_h(k_0, k) \) were real, then \( C_h(k_0, k) = \cos(2\theta_m(h_k, k)) \) and \( D_h(k_0, k) = \sin(2\theta_m(h_k, k)) \) with \( \theta_m(h_k, k) \) the mixing angle in the medium for the different helicity projections and as a function of frequency and momentum.

**B. Propagator: complex poles and propagating modes in the medium**

From (III.34) we read off the propagator projected onto helicity eigenstates

\[
G_h(k_0, k) = \frac{I + T_h(k_0, k)}{2\left( \alpha_h(k_0, k) - \beta_h(k_0, k) \right)} + \frac{I - T_h(k_0, k)}{2\left( \alpha_h(k_0, k) + \beta_h(k_0, k) \right)},
\]

(III.38)

where

\[
T_h(k_0, k) = \begin{pmatrix}
-C_h(k_0, k) & D_h(k_0, k) \\
D_h(k_0, k) & C_h(k_0, k)
\end{pmatrix},
\]

(III.39)

\[
\alpha_h(k_0, k) = k^2_0 - k^2 + \frac{1}{2} S_h(k_0, k) - \overline{M}^2,
\]

(III.40)
\[
\beta_h(k_0, k) = \frac{1}{2} \delta M^2 \rho_h(k_0, k).
\]

(III.41)

If \( \Delta_h(k_0, k) \) given by eqn. (III.32) were real, the propagator (III.38) would be diagonalized by the unitary transformation

\[
U_h(\theta_m^h(k_0, k)) = \begin{pmatrix}
\cos(\theta_m^h(k_0, k)) & \sin(\theta_m^h(k_0, k)) \\
-\sin(\theta_m^h(k_0, k)) & \cos(\theta_m^h(k_0, k))
\end{pmatrix},
\]

(III.42)

leading to

\[
U^{-1}(\theta_m)G(k_0, k)U(\theta_m) = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix},
\]

(III.43)

where we have suppressed the helicity argument for simplicity. However, because \( \Delta_h(k_0, k) \) features an imaginary part determined by the absorptive part of the self-energies, there is no unitary transformation that diagonalizes the propagator. However, since the imaginary part is perturbatively small the expression (III.43) clearly indicates that the pole for \( \alpha = \beta \) corresponds to the mass eigenstate 2, namely a sterile-like neutrino state, and the pole for \( \alpha = -\beta \) corresponds to the mass eigenstate 1, namely an active-like state.
We note that in absence of interactions, namely $S_h = 0$; $\Delta_h = 0$ it follows that

$$\alpha + \beta = k_0^2 - k^2 - M_1^2, \quad (\text{III.44})$$

$$\alpha - \beta = k_0^2 - k^2 - M_2^2. \quad (\text{III.45})$$

The propagating eigenstates in the medium are determined by the (complex) poles of the propagator (III.38), which again correspond to $\alpha_h(k_0, k) = \pm \beta_h(k_0, k)$.

Before we analyze the complex poles, it proves convenient to separate the real and imaginary parts of $\alpha, \beta$. For this purpose and to simplify notation, we suppress the label $h$ and the arguments $k_0, k$ in these quantities, and we write

$$S = S_R + iS_I; \quad \Delta = \Delta_R + i\Delta_I, \quad (\text{III.46})$$

where the subscripts $R, I$ stand for real and imaginary parts respectively. Furthermore, we define the mixing angles in the medium solely in terms of the real parts of the self-energy (index of refraction), namely

$$\cos(2\theta_m) = \frac{\cos(2\theta) + \Delta_R}{\rho_0}; \quad \sin(2\theta_m) = \frac{\sin(2\theta)}{\rho_0}, \quad (\text{III.47})$$

where

$$\rho_0 = \left[ \left( \cos(2\theta) + \Delta_R \right)^2 + \sin^2(2\theta) \right]^{\frac{1}{2}}. \quad (\text{III.48})$$

An MSW resonance occurs whenever $\cos(2\theta_m) = 0$[49, 50, 51, 52], namely when

$$\Delta_R = -\cos(2\theta). \quad (\text{III.49})$$

We emphasize that the mixing angle in the medium $\theta_m$ and $\rho_0$ depend on helicity, $k_0, k$. In terms of these quantities we find

$$\beta = \frac{\delta M^2}{2} \rho_0 r \left[ \cos(\phi) + i \sin(\phi) \right] \equiv \beta_R + i\beta_I, \quad (\text{III.50})$$

where

$$r = \left[ \left( 1 - \gamma^2 \right)^2 + \left( 2\gamma \cos(2\theta_m) \right)^2 \right]^{\frac{1}{2}}; \quad \gamma = \frac{\Delta_I}{\rho_0}, \quad (\text{III.51})$$

and

$$\phi = \text{sign}(\gamma \cos(2\theta_m)) \left\{ \frac{1}{2} \arctg \left( \frac{2\gamma \cos(2\theta_m)}{1 - \gamma^2} \right) \Theta(1 - \gamma^2) + \left( \frac{\pi}{2} - \frac{1}{2} \arctg \frac{2\gamma \cos(2\theta_m)}{1 - \gamma^2} \right) \Theta(\gamma^2 - 1) \right\}. \quad (\text{III.52})$$

This form is similar to that obtained in a model of oscillations and damping with mixed neutrinos studied in ref.[63], and suggests two distinct situations: a weak damping case for $|\gamma| < 1$ and a strong damping case for $|\gamma| > 1$. These cases will be analyzed below.

**Zeroes of $\alpha + \beta$:** We are concerned with the ultrarelativistic limit $k \gg M_2^2 \gg M_1^2$. Just as in the usual case[50, 51, 52] it is convenient to introduce the average or reference frequency

$$\om(k) = \sqrt{k^2 + M^2}. \quad (\text{III.53})$$

The poles are near $\om(k)$, therefore write

$$k_0 = \om(k) + (k_0 - \om(k)), \quad (\text{III.54})$$

keeping only the linear term in $(k_0 - \om(k))$ we find

$$\alpha + \beta \sim 2\om(k) \left[ k_0 - \Omega_1(k) + i\Gamma_1(k) \right] \quad (\text{III.55})$$
with
\[ \Omega_1(k) = \Omega(k) - \frac{1}{4\overline{\omega}(k)} \left[ S_R + \delta M^2 \rho_0 r \cos(\phi) \right] \bigg|_{k_0 = \overline{\omega}(k)}, \quad (III.56) \]
\[ \Gamma_1(k) = \frac{1}{4\overline{\omega}(k)} \left[ S_I + \delta M^2 \rho_0 r \sin(\phi) \right] \bigg|_{k_0 = \overline{\omega}(k)}. \quad (III.57) \]

**Zeroes of \( \alpha - \beta \):** Proceeding in the same manner, we find
\[ \alpha - \beta \sim 2\overline{\omega}(k) \left[ k_0 - \Omega_2(k) + i \Gamma_2(k) \right] \quad (III.58) \]
with
\[ \Omega_2(k) = \Omega(k) - \frac{1}{4\overline{\omega}(k)} \left[ S_R - \delta M^2 \rho_0 r \cos(\phi) \right] \bigg|_{k_0 = \overline{\omega}(k)}, \quad (III.59) \]
\[ \Gamma_2(k) = \frac{1}{4\overline{\omega}(k)} \left[ S_I - \delta M^2 \rho_0 r \sin(\phi) \right] \bigg|_{k_0 = \overline{\omega}(k)}. \quad (III.60) \]

From (III.55,III.58) it is clear that the propagator in the medium features two Breit-Wigner complex poles corresponding to the two propagating modes in the medium.

In the expressions above we have only focused on the positive energy modes. The expressions for the negative energy modes may be obtained from the following relations which are consequences of the imaginary parts of the self-energies and the dispersive representation valid both for scalar and vector boson exchange (III.5),
\[ \text{Im} \mathcal{A}(-k_0, k, \mu) = \text{Im} \mathcal{A}(k_0, k, -\mu); \quad \text{Re} \mathcal{A}(-k_0, k, \mu) = -\text{Re} \mathcal{A}(k_0, k, -\mu), \quad (III.61) \]
\[ \text{Im} \mathcal{B}(-k_0, k, \mu) = -\text{Im} \mathcal{B}(k_0, k, -\mu); \quad \text{Re} \mathcal{B}(-k_0, k, \mu) = \text{Re} \mathcal{B}(k_0, k, -\mu). \quad (III.62) \]

These properties can be read-off the explicit expressions for the imaginary parts of the self-energies given in the appendix equations (A.1,A.3) for the standard model contributions and equations (B.1,B.3) for the scalar exchange contributions. The matrices \( \mathcal{A} \) are extracted from the coefficient of \( \gamma^0 \) and \( \mathcal{B} \) from the coefficients of \( \gamma \cdot \hat{k} \) in the self-energies respectively. The relations for the real parts follow from the dispersive representation (III.5).

In what follows we use the ultrarelativistic approximation
\[ \overline{\omega}(k) \simeq k + \frac{m^2}{2k}. \quad (III.63) \]

In the limit of interest \( k/T \lesssim 1 \) with \( M_1 \ll M_2 \sim M_s \sim \mathcal{O}(keV) \), the region \( k < T \sim \mathcal{O}(100 GeV) \) corresponds to a wide window in which the ultrarelativistic approximation is reliable.

We note that the difference in the real part of the pole position in the ultrarelativistic limit becomes
\[ \Omega_2(k) - \Omega_1(k) \simeq \frac{\delta M^2}{2k} \rho_0 r \cos(\phi). \quad (III.64) \]

From the expression (III.52) for \( |\tilde{\gamma}| > 1 \) it follows that when an MSW resonance occurs, namely for \( \theta_m = \pi/4 \) resulting in \( \cos(\phi) = 0 \) and the real part of the poles become *degenerate*. This is in striking contrast with the quantum mechanical description of mixed neutrinos where no level crossing (or complete degeneracy) can occur. Indeed the degeneracy is a consequence of the fact that the self-energy is complex and only occurs when damping is strong in the sense that \( |\tilde{\gamma}| > 1 \).

The degeneracy near an MSW resonance for strong damping will necessarily result in a breakdown of adiabaticity during cosmological evolution. We analyze below the conditions required for this phenomenon to occur.

Furthermore, as discussed in refs.\[^{[32,43]}\] decoupling and freeze-out of sterile neutrinos of neutrinos produced via scalar decay occurs near the electroweak scale, and it will be seen consistently that vector boson decay yields a production rate with a similar structure as for scalar decay therefore a similar range of temperatures in which sterile neutrino production by this mechanism is effective.

Perturbation theory is reliable when the change in the dispersion relations (positions of the poles in the propagators) is small. In the relativistic limit the (bare) poles correspond to \( k_0 = k \) (for positive energy particles), therefore perturbation theory is valid for \( k \gg (\Omega_{1,2} - k) ; \Gamma_{1,2} \) namely \( k \gg \Sigma(k, k) \) where \( \Sigma \) is any of the self-energies. In the next section we obtain explicitly the self energies and in section (V,D) we assess the regime of validity of the perturbative expansion.
C. Helicity dependence: right-handed sterile neutrinos and standard model interactions

We have purposely kept the general form of the self-energies and propagators in terms of the helicity projections \( h = \pm 1 \). In the non-interacting massless case, positive energy left-handed particles correspond to \( h = -1 \) and negative energy left-handed correspond to \( h = 1 \), with the opposite assignment for right-handed particles. For the massive but ultrarelativistic case the mass term yields corrections to the handedness-helicity assignment of \( O(M^2/k^2) \).

**\( h = -1 \):** Neglecting subleading terms of \( O(M^2/k^2) \) that multiply (bsm) right-handed contributions in the ultrarelativistic limit, we obtain

\[
S(k) = 2k \left[ (A_L - B_L)_{aa} + (A_L - B_L)_{ss} \right],
\]

\[
\Delta(k) = \frac{2k}{\delta M^2} \left[ (A_L - B_L)_{aa} - (A_L - B_L)_{ss} \right].
\]

**\( h = 1 \):** In this case the corrections of \( O(M^2/k^2) \) multiply (sm) left-handed contributions, which may be of the same order of the (bsm) right-handed contributions. We find,

\[
S(k) = 2k \left[ (A_R - B_R)_{aa} + (A_R - B_R)_{ss} + \frac{M^2}{4k^2} (A_L + B_L)_{aa} \right],
\]

\[
\Delta(k) = \frac{2k}{\delta M^2} \left[ (A_R - B_R)_{aa} - (A_R - B_R)_{ss} + \frac{M^2}{4k^2} (A_L + B_L)_{aa} \right].
\]

The terms proportional to \( \frac{M^2}{4k^2} \) only receive contribution from the standard model self-energies, whereas the right-handed components only originate in the contributions beyond the standard model which are suppressed by much smaller Yukawa couplings. However the last contribution in (III.68) from (sm) interactions may be of the same order as the (bsm) contributions for a relevant range of \( k \). To see this note that \( A_R, B_R \sim Y_f^2 \sim 10^{-10} \), whereas \( A_L, B_L \sim g^2 \sim 0.4 \) therefore with \( M \sim \text{KeV} \) and \( k \lesssim 100 \text{GeV} \), it is clear that both contributions (bsm) and (sm) are of the same order.

The point of maintaining the helicity dependence throughout is that for the case of sterile neutrinos, namely the propagating modes “2” in the medium, the exchange of standard model vector bosons yields a contribution to the positive helicity and positive energy components, namely the right-handed component, which could be of the same order of the (bsm) contributions for small \( k \) which is a region of interest for sterile neutrino production.

IV. REAL PARTS: MIXING ANGLES AND MSW RESONANCES

The dispersion relations (real parts of the poles) and the mixing angles in the medium are determined by the real parts of the self-energy, namely the “index of refraction”. Whereas the neutral current tadpole contribution (III.7) is real, the real part of the other contributions is obtained from the dispersive form (III.5), namely

\[
\text{Re} \Sigma(0, k) = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega \mathcal{P} \left( \frac{\text{Im} \Sigma(\omega, \vec{k})}{\omega - k_0} \right).
\]

In general the real part must be obtained numerically and is a function of three parameters \( k_0, k, \mu \) which makes its exploration a daunting numerical task. However, progress can be made by focusing on the “on-shell” contribution, namely setting \( k_0 \simeq k \), and neglecting the dependence on \( \mu \), which is warranted in the whole region of \( k, T \) of interest, but for \( k/T, |\mu|/T \ll M/T \), in which case we provide below an accurate approximate form.

In obtaining the real parts we consider only the finite temperature contribution, because the zero temperature part is absorbed in the renormalization of the parameters in the Lagrangian.

**Scalars (bsm):** For the real part of the scalar (bsm) self-energy we find for \( k_0 = k \); \( \mu = 0 \)

\[
\text{Re} \Sigma_{bsm}(k, k) = \frac{Y_f^2 T}{16\pi^2} \left\{ \gamma^0 \left[ Af \left( \frac{k}{T} ; \frac{M}{T} \right) + Ab \left( \frac{k}{T} ; \frac{M}{T} \right) \right] - \gamma \cdot \vec{k} \left[ Bf \left( \frac{k}{T} ; \frac{M}{T} \right) + Bb \left( \frac{k}{T} ; \frac{M}{T} \right) \right] \right\},
\]
where $A_f, B_f$ and $A_b, B_b$ are the fermionic and bosonic contributions respectively and $Y = Y_{1,2}$ for $\sigma, \varphi$ exchange. Figs. 5 show $A_f, B_f$ and $A_b, B_b$ for $M/T = 1, 2, 3$ as a function of $k/T$.

For $\Sigma^{(2)}_{\sigma,\varphi}$ the intermediate fermion line corresponds to a sterile-like neutrino, therefore for these contributions we must set $A_f = 0; B_f = 0$, under the assumption that the sterile neutrino population can be neglected and the propagator for the internal line is the vacuum one. For the mixing angle the relevant contribution is $A - B$. Figs. 6 display $A_f - B_f$ and $A_b - B_b$ for $M/T = 1, 2, 3$ as a function of $k/T$.

We note that the fermionic and bosonic contributions $A_f, A_b$ are qualitatively very similar and the same property holds for $B_f, B_b$. Therefore neglecting the fermionic contributions both for $\Sigma^{(1)}$ does not affect the results and the conclusions in a substantial manner.
Similarly, for the real part of the (sm) self-energy we find for:

\[ \text{Re}\Sigma^{(1)}_{\sigma}(k, 0) = \frac{Y^{2}T^{2}}{M_{\sigma}^{2}} \left\{ \gamma^{0} \left\{ -\frac{T \xi}{12} \left( 1 + \frac{\xi^{2}}{4\pi^{2}} \right) + \frac{7\pi^{2}}{120} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \right\} \right\} \left\{ \frac{T}{12} \left( 1 + \frac{\xi^{2}}{4\pi^{2}} \right) + \frac{7\pi^{2}}{120} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \left[ 1 + F[M_{\sigma}/T] \right] \right\} \right\} , \quad \text{(IV.3)}

\[ \text{Re}\Sigma^{(1)}_{\sigma}(k, 0) = \frac{Y^{2}T^{2}}{M_{\sigma}^{2}} \left\{ \gamma^{0} \left\{ -\frac{T \xi}{12} \left( 1 + \frac{\xi^{2}}{4\pi^{2}} \right) + \frac{7\pi^{2}}{120} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \right\} \right\} \left\{ \frac{T}{12} \left( 1 + \frac{\xi^{2}}{4\pi^{2}} \right) + \frac{7\pi^{2}}{120} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \left[ 1 + F[M_{\sigma}/T] \right] \right\} \right\} \left\{ -\frac{7\pi^{2}}{360} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \left[ 1 + J[M_{\sigma}/T] \right] \right\} \right\} , \quad \text{(IV.4)}

\[ \text{Re}\Sigma^{(2)}_{\sigma}(k, 0) = \frac{Y^{2}T^{2}}{M_{\sigma}^{2}} \left\{ \gamma^{0} \left\{ \frac{7\pi^{2}}{120} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \right\} \left\{ -\frac{7\pi^{2}}{360} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \right\} \right\} \left\{ \frac{T}{12} \left( 1 + \frac{\xi^{2}}{4\pi^{2}} \right) + \frac{7\pi^{2}}{120} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \left[ 1 + F[M_{\sigma}/T] \right] \right\} \right\} \left\{ -\frac{7\pi^{2}}{360} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \left[ 1 + J[M_{\sigma}/T] \right] \right\} \right\} , \quad \text{(IV.5)}

\[ \text{Re}\Sigma^{(2)}_{\sigma}(k, 0) = \frac{Y^{2}T^{2}}{M_{\sigma}^{2}} \left\{ \gamma^{0} \left\{ \frac{7\pi^{2}}{120} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \right\} \left\{ -\frac{7\pi^{2}}{360} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \right\} \right\} \left\{ -\frac{7\pi^{2}}{360} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \right\} \left\{ \frac{T}{12} \left( 1 + \frac{\xi^{2}}{4\pi^{2}} \right) + \frac{7\pi^{2}}{120} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \left[ 1 + F[M_{\sigma}/T] \right] \right\} \right\} \left\{ -\frac{7\pi^{2}}{360} \frac{k_{0} T^{2}}{M_{\sigma}^{2}} \left[ 1 + J[M_{\sigma}/T] \right] \right\} \right\} , \quad \text{(IV.6)}

where

\[ J(m) = \frac{1}{120} \frac{1}{7\pi^{4}} \int_{0}^{\infty} dq \frac{q^{2}}{W_{q}} N_{B}(W_{q}) \left[ W_{q}^{2} + \frac{m^{2}}{2} \right] ; \quad F(m) = \frac{1}{120} \frac{1}{7\pi^{4}} \int_{0}^{\infty} dq \frac{q^{2}}{W_{q}} N_{B}(W_{q}) \left[ W_{q}^{2} - \frac{m^{2}}{2} \right] . \quad \text{(IV.7)}

These functions are displayed in Fig. (9), they are O(1) in the region of interest \( M_{\sigma,\varphi} \sim T \).

A comprehensive numerical study of Af, Ab, Bf, Bb confirms the validity of the above approximations for \( k_{0} = k, \mu = 0 \) for \( k/T \ll 1 \).

**Vector bosons (sm):** Similarly, for the real part of the (sm) self-energy we find for \( k_{0} = k ; \mu = 0 \)

\[ \text{Re}\Sigma_{\text{sn}}(k, 0) = \frac{g^{2}_{\text{sn}} T^{2}}{16\pi^{2}} \left\{ \gamma^{0} \left\{ \frac{1}{4} T \xi \left( 1 + \frac{\xi^{2}}{4\pi^{2}} \right) + \frac{7\pi^{2}}{60} \frac{k_{0} T^{2}}{M_{W}^{2}} \left[ 1 + G[M_{W}/T] \right] \right\} \right\} \left\{ -\frac{7\pi^{2}}{180} \frac{k_{0} T^{2}}{M_{Z}^{2}} \left[ 1 + G[M_{Z}/T] \right] \right\} \right\} , \quad \text{(IV.8)}

where we use the same definition, namely Af, Bf and Ab, Bb are the fermionic and bosonic contributions respectively.

Figs. (7) show Af, Bf and Ab, Bb and Fig. (8) shows Af \( (k/T; M/T) - Bf(k/T; M/T) \); Ab \( (k/T; M/T) - Bf(k/T; M/T) \) for \( M/T = 1, 2, 3 \) as a function of \( k/T \).

Just as in the (bsm) case analyzed above, we note that the fermionic and bosonic contributions Af, Ab are qualitatively similar and the same holds for Bf, Bb. Again this observation confirms that our results are robust, independently of whether any of the neutrino modes is thermalized.

We also obtain the analytic forms for \( \text{Re}\Sigma_{\text{sn}}(k_{0}; 0) \) for \( k_{0}/T, k/T, \mu/T \ll M_{W,Z}/T \sim 1 \). To leading order in these small ratios we find

\[ \text{Re}\Sigma^{(1)}_{\text{nc}}(k_{0}, 0) = \frac{g^{2} T^{2}}{4M_{W}^{2}} \left\{ \gamma^{0} \left\{ -\frac{T \xi}{4} \left( 1 + \frac{\xi^{2}}{4\pi^{2}} \right) + \frac{7\pi^{2}}{60} \frac{k_{0} T^{2}}{M_{W}^{2}} \left[ 1 + G[M_{Z}/T] \right] \right\} \right\} \left\{ -\frac{7\pi^{2}}{180} \frac{k_{0} T^{2}}{M_{Z}^{2}} \left[ 1 + G[M_{Z}/T] \right] \right\} \right\} , \quad \text{(IV.9)}

\[ \text{Re}\Sigma^{(2)}_{\text{nc}}(k_{0}, 0) = \frac{g^{2} T^{2}}{4M_{W}^{2}} \left\{ \gamma^{0} \left\{ \frac{7\pi^{2}}{60} \frac{k_{0} T^{2}}{M_{Z}^{2}} G[M_{Z}/T] \right\} \right\} \left\{ -\frac{7\pi^{2}}{180} \frac{k_{0} T^{2}}{M_{Z}^{2}} G[M_{Z}/T] \right\} \right\} , \quad \text{(IV.10)}

\[ \text{Re}\Sigma_{\text{cc},\text{sn}}(k_{0}, 0) = \frac{g^{2} T^{2}}{2M_{W}^{2}} \left\{ \gamma^{0} \left\{ \frac{7\pi^{2}}{60} \frac{k_{0} T^{2}}{M_{W}^{2}} \left[ 1 + G[M_{W}/T] \right] \right\} \right\} \left\{ -\frac{7\pi^{2}}{180} \frac{k_{0} T^{2}}{M_{W}^{2}} \left[ 1 + G[M_{W}/T] \right] \right\} \right\} . \quad \text{(IV.11)}

This observation confirms that the general results presented below are robust even when the neutrinos “1” are not thermalized and their propagators are the vacuum ones.

Although an analytic form for the full range of \( k_{0}; k; \mu \) is not available, we obtain an analytic expression for the relevant case \( k_{0}/T, k/T, \mu/T \ll M/T \sim 1 \). We find to leading order in the small ratios \( k_{0}/T; k/T; \xi = \mu/T \),
FIG. 7: The functions $A_f(k/T; M/T); A_b(k/T; M/T); B_f(k/T; M/T); B_b(k/T; M/T)$ as a function of $k/T$ for $M/T = 1, 2, 3$ for the standard model contributions (sm) with $\mu = 0$.

FIG. 8: The functions $A_f(k/T; M/T) - B_f(k/T; M/T); A_b(k/T; M/T) - B_b(k/T; M/T)$ as a function of $k/T$ for $M/T = 1, 2, 3$ for the standard model contributions (sm) with $\mu = 0$.

In the charged current contribution we have neglected the asymmetry of the charged lepton because it is of the order of the baryon asymmetry. In the above expressions

$$G[m] = \frac{120}{\pi^4} \int_0^\infty dq \, \frac{q^2}{W_q} N_B(W_q) \left[ W_q^2 - \frac{m^2}{4} \right] ; \quad N_B(W_q) = \frac{1}{e^{W_q} - 1} ; \quad W_q = \sqrt{q^2 + m^2}.$$  \hspace{1cm} (IV.12)

This function is depicted in Fig. 8, it is $O(1)$ in the region of interest $T \sim M_{Z,W}$.

The validity of these approximations for $k_0 = k, \mu = 0$ is confirmed by the numerical analysis of $A_f, A_b, B_f, B_b$ for $k/T \ll 1$.

It is remarkable that the leading order in $k_0/T, k/T$ but for $M_{W,Z} \sim T$ reproduce the results of references [53, 54] which were obtained in the low energy limit $T, \mu \ll M_{W,Z}$. The numerical analysis carried out for $k_0 = k; \mu = 0$
In this case $\Delta$ is given by eqn. (III.66), furthermore from eqn. (III.13) it follows that $H_M/T$ depends on the ratio sign, even in absence of a lepton asymmetry. This fact entails that there are MSW resonances near the momentum regions where the coefficient functions change the sign persists until $M/T \sim k$ mass shell the figures above. We find that for $H_M/T$ change sign. In particular the combinations $Af, Ab$ in the expressions for the real part of the matrices $\text{Re}A_L : \text{Re}B$ for $\Delta_R$, namely the real part of eqns. (III.66,III.68) for $h = \mp 1$ respectively. For $\mu = 0; k_0 = k$ and general $k, M$ the fermionic and bosonic contributions to the real parts of the (bsm) self-energies are given by eqn. (IV.2) where the fermionic ($Af, Bf$) and bosonic ($Ab, Bb$) contributions are depicted in figs. (5-6). The real parts of the (sm) self-energies are given by eqn. (IV.8) and the fermionic and bosonic contributions depicted in figs. (7-8).

These figures distinctly show that the contributions $Af, Ab$ and $Bf, Bb$ for (bsm) and (sm) self-energies are qualitatively the same, with only a quantitative difference in the amplitudes. A remarkable result is that these functions $\Delta_R$ are determined by the (bsm) contributions which are suppressed by small Yukawa couplings $Y \lesssim 10^{-8}$ as compared to the (sm) contributions. Therefore the (bsm) contribution can be neglected and $\Delta_R$ is determined by the (sm) contributions given by eqns. (IV.8), furthermore approximating $\cos(\theta) \sim 1; \sin(\theta) \sim 0$ in eqn. (IV.12) and $\delta M^2 \simeq M^2_s$, we find (for $\mu = 0; h = -1$)

$$\Delta_R(k) \simeq \frac{g^2}{16\pi^2} \left( \frac{T}{M_s} \right)^2 \left\{ [A_f \left( k \right) M_W \left( T \right)] + A_b \left( k \right) M_W \left( T \right) - B_f \left( k \right) M_W \left( T \right) - B_b \left( k \right) M_W \left( T \right) \right\}$$

$$+ \frac{1}{2\cos(\theta_w)} \left\{ [A_f \left( k \right) M_Z \left( T \right)] + A_b \left( k \right) M_Z \left( T \right) - B_f \left( k \right) M_Z \left( T \right) - B_b \left( k \right) M_Z \left( T \right) \right\} .$$

Taking as representative $T \sim 100\text{ GeV}; M_s \sim \text{keV}$ it follows that

$$\frac{g^2}{16\pi^2} \left( \frac{T}{M_s} \right)^2 \simeq 2.7 \times 10^{13} .$$

(A. Mixing angles and MSW resonances:

As shown in detail in the previous sections, the mixing angle in the medium $\theta_m$ determined by the relations (III.47) depends on $k_0, k$ and the helicity $h$. On the mass shell of the propagating modes in the medium we can replace $k_0 \simeq k$ in the expressions for the real part of the matrices $\text{Re}A_L : \text{Re}B$ for $\Delta_R$, namely the real part of eqns. (III.66,III.68) for $h = \mp 1$ respectively. For $\mu = 0; k_0 = k$ and general $k, M$ the fermionic and bosonic contributions to the real parts of the (bsm) self-energies are given by eqn. (IV.2) where the fermionic ($Af, Bf$) and bosonic ($Ab, Bb$) contributions are depicted in figs. (5-6). The real parts of the (sm) self-energies are given by eqn. (IV.8) and the fermionic and bosonic contributions depicted in figs. (7-8).

These figures distinctly show that the contributions $Af, Ab$ and $Bf, Bb$ for (bsm) and (sm) self-energies are qualitatively the same, with only a quantitative difference in the amplitudes. A remarkable result is that these functions change sign. In particular the combinations $Af - Bf, Ab - Bb$ which enter in $\Delta_R$ change sign at a value of $k/T$ that depends on the ratio $M/T$. For $M/T \sim 1$ these differences vanish at $k/T \sim 0.2$. A numerical exploration reveals that the sign change persists until $M/T \sim 3$ but occurs at monotonically larger values of $k/T$. This behavior is shown in the figures above. We find that for $M/T \gtrsim 3$ the change in sign occurs for $k >> T$ or does not occur at all. On the mass shell $k_0 \sim k$ and for $\mu = 0$ this study reveals that $\Delta_R$ is negative in a wide region of momentum for $M/T \lesssim 1$. This fact entails that there are MSW resonances near the momentum regions where the coefficient functions change sign, even in absence of a lepton asymmetry. To understand this important point more clearly let us study the case $h = \mp 1$ separately.

$h = -1$: In this case $\Delta$ is given by eqn. (III.66), furthermore from eqn. (III.13) it follows that $(A_L - B_L)_{ss}$ is determined by the (bsm) contributions which are suppressed by small Yukawa couplings $Y \lesssim 10^{-8}$ as compared to the (sm) contributions. Therefore the (bsm) contribution can be neglected and $\Delta_R$ is determined by the (sm) contributions given by eqns. (IV.8), furthermore approximating $\cos(\theta) \sim 1; \sin(\theta) \sim 0$ in eqn. (IV.12) and $\delta M^2 \simeq M^2_s$, we find (for $\mu = 0; h = -1$)

$$\Delta_R(k) \simeq \frac{g^2}{16\pi^2} \left( \frac{T}{M_s} \right)^2 \left\{ [A_f \left( k \right) M_W \left( T \right)] + A_b \left( k \right) M_W \left( T \right) - B_f \left( k \right) M_W \left( T \right) - B_b \left( k \right) M_W \left( T \right) \right\}$$

$$+ \frac{1}{2\cos(\theta_w)} \left\{ [A_f \left( k \right) M_Z \left( T \right)] + A_b \left( k \right) M_Z \left( T \right) - B_f \left( k \right) M_Z \left( T \right) - B_b \left( k \right) M_Z \left( T \right) \right\} .$$

Taking as representative $T \sim 100\text{ GeV}; M_s \sim \text{keV}$ it follows that

$$\frac{g^2}{16\pi^2} \left( \frac{T}{M_s} \right)^2 \simeq 2.7 \times 10^{13} .$$

Figures (8) show that for $M_{W,Z}/T \lesssim 3$ there is a region in $k/T$ in which the bracket in (IV.13) is negative and there is a value $(k/T)_c$ that increases with $M/T$ at which the bracket vanishes, for example from the Fig. (8) we find

![Graph](image-url)
(k/T)_c \sim 0.2; 0.45; 1 for M/T \sim 1, 2, 3 respectively. For k/T < (k/T)_c the bracket is positive (for \( \mu = 0 \)) whereas for k/T > (k/T)_c it is negative, therefore there is a value of k/T at which the resonance condition (III.19) is fulfilled. Since the coefficient of the bracket is \( \approx 10^{13} \) (eqn. [IV.14]) and the terms inside the bracket are of \( \mathcal{O}(1) \) for k/T \( \lesssim 1 \), and \( \cos(\theta) \sim 1 \) it follows that the MSW resonance occurs for a value of k/T such that the bracket \( \sim 10^{-13} \) namely for k/T \( \sim (k/T)_c \). The large coefficient [IV.14] results in a very narrow MSW resonance as can be seen as follows, expanding \( \Delta_R \) near (k/T)_c as

\[
\Delta_R(k) \simeq -\kappa \left( \left( \frac{k}{T} \right) - \left( \frac{k}{T} \right)_c \right) + \cdots \;; \kappa > 0,
\]

where \( \kappa \approx 10^{13} \) for \( M_{W,Z}/T \lesssim 3 \) (see Fig. 8) and approximating \( \cos(2\theta) \sim 1 \) we find

\[
\sin^2(2\theta_m) \approx \frac{c^2}{\left( \frac{\pi}{2} - \frac{\phi}{\kappa} - \frac{\pi}{2} \right)^2 + c^2} \;; \quad \epsilon = \sin(2\theta)/\kappa.
\]

For example taking \( \sin(2\theta) \sim 10^{-5} \) it follows that \( \epsilon \ll 10^{-18} \) which makes the resonance very narrow. During cosmological expansion the ratio M/T(t) increases with the scale factor, while the ratio k/T (with k the physical momentum) is fixed. Therefore, for a fixed value of k/T < 1 as M/T increases the resonance is crossed very sharply.

**h = 1:** To assess the possibility of MSW resonances for h = 1 we need the real part of (III.08). From (III.12) and (IV.2) it follows that \( (A_R - B_R)_{aa} \propto Y^2_1 \); \( (A_R - B_R)_{ss} \propto Y^2_2 \), since \( Y^2_2 \gg Y_1 \) we can neglect the first term (corresponding to \( \sigma \)-exchange). Similarly in the term \( (A_R - B_R)_{ss} \) we neglect the contribution from \( \sigma \)-exchange and approximate \( \cos(\theta) \sim 1 \); \( \sin(\theta) \sim 0 \) in (III.13), hence only \( \Sigma_{s(2)} \) contributes to \( \Sigma_{ss} \). Furthermore, approximating \( \delta M^2 \sim M^2 \sim M_s^2 \) we finally find for h = 1; \( \mu = 0 \),

\[
\Delta_R(k) \simeq -\left( \frac{Y^2_2}{\sqrt{8\pi M_s}} \right)^2 \left( \frac{k}{T} \right)^2 \left[ AbkT, M^2 \right] - BbkT, M^2 \right] \\
+ \frac{g^2}{128\pi^2} \left( \frac{T}{k} \right)^2 \left[ AfkT, M^2 \right] + AbkT, M^2 \right] + BfkT, M^2 \right] + BbkT, M^2 \right] \\
+ \frac{1}{2\cos(\theta_m)} \left[ AfkT, M^2 \right] + AbkT, M^2 \right] + BfkT, M^2 \right] + BbkT, M^2 \right] 
\]

where in the first line the Ab; Bb are (bsm) displayed in figs. 5.

We note that with T \( \sim 100 \text{GeV}, M_s \sim \text{KeV} \) the value of the \( Y_2 \) (see eqn. (III.10) is such that \( Y^2_2 T/M_s \sim \mathcal{O}(1) \)), therefore fig 5 (right panel) suggests that the (bsm) contribution may yield an MSW resonance in the region k/T \( \lesssim 0.15 \); \( M_s \sim T \), where the (bsm) contribution \( Ab - Bb \) is positive and large. Since \( g^2/128\pi^2 \sim 3.4 \times 10^{-4} \) and \( Af + Bf; Ab + Bb \sim \mathcal{O}(1) \) for k/T \( \lesssim 1 \) it follows that the (sm) contribution to \( \Delta_R \) is subleading for k/T \( \lesssim 1 \) and the (bsm) contribution may lead to an MSW resonance in this region depending on the parameters of the extension (bsm).

\( \mu \neq 0 \); k/T \( \ll M/T \sim 1 \):

The above results are valid for \( \mu = 0 \), for \( \mu \neq 0 \) a full numerical evaluation of the real parts of the kernel is not available, however, the bounds on the lepton asymmetry from ref.[62] suggest that |\( \mu/T | \lesssim 0.02 \ll 1 \) and we can obtain a reliable understanding of the influence of the lepton asymmetry (in the neutrino sector) by focusing on the region of k/T \( \ll 1 \), in which we can use the results (IV.3,IV.3) for (bsm) and (III.7) along with (IV.9,IV.11) for (sm) and approximate \( \cos(\theta) \sim 1 ; \sin(\theta) \sim 0 \) in (III.12,III.13) and \( \delta M^2 \sim M_s^2 \).

For h = -1 again we neglect the (bsm) contributions to \( \Delta_R(k) \) in (III.66), and for \( \mu/T; k/T \ll 1 \) we obtain,

\[
\Delta_R(k) \simeq \frac{g^2T^3k}{M_W^2 M_s^2} \left\{ -\frac{5 \xi}{24} + \frac{7\pi^2}{90} \left( \frac{k}{T} \right)^2 \left[ 1 + G(\frac{M_s}{T}) \right] + 2 \left( \frac{T}{M_s} \right)^2 \right\} .
\]

We note that for T \( \sim M_W; M_s \sim \text{KeV} \) the prefactor

\[
\frac{g^2T^3k}{M_W^2 M_s^2} \sim 10^{16} \left( \frac{k}{T} \right)
\]

(IV.19)
and the resonance condition \((\text{III}.49)\) can be fulfilled for \(\xi > 0\) when the bracket in \((\text{IV}.18)\) approximately vanishes, namely for

\[
\left( \frac{k}{T} \right) \sim \frac{25 \xi}{56 \pi^2} \Rightarrow k \sim 0.05 \mu ,
\]

(IV.20)

where we have used \(G(M_{W,Z}/T) \sim 1\) for \(T \sim M_W\), a result that can be gleaned from Fig. \((9)\). For \(\xi > 0\) this MSW resonance occurs for antineutrinos (namely \(k_0 = -k\)), a result that follows from the relations \((\text{III}.61,\text{III}.62)\).

Similarly, for \(h = 1\) and \(\mu/T, k/T < 1\), we obtain

\[
\Delta_R(k) \approx -\left( \frac{Y_2^2 T^2}{M_z^2} \left( \frac{T^4}{M_z^2} \right) \left( \frac{k^2}{T^2} \right) \frac{7\pi^2}{180} \left[ J(M_\varphi T) + 3F(M_\varphi T) \right] \right)
+ \frac{g^2 T^2}{M_W^2} \left( \frac{T}{k} \right) \left\{ -\frac{5\xi}{192} + \frac{7\pi^2}{1440} \left( \frac{k}{T} \right) \left[ \left( \frac{T}{M_z} \right)^2 \left( 1 + G(M_z T) \right) + 2 \left( \frac{T}{M_W} \right)^2 \left( 1 + G(M_W T) \right) \right] \right\}
\]

(IV.21)

Obviously, there is a competition between (sm) and (bsm) contributions in eqn. \((\text{IV}.21)\). When \(T \sim M_{W,Z,\varphi}\), \(J(1), F(1), G(1) \sim 1\) and \(\left( \frac{Y_2^2 T^2}{M_z^2} \right) / M_z^2 \sim 1\). Therefore, the (bsm) contribution to \(\Delta_R(k)\) is

\[
\Delta_R^{(\text{bsm})} \sim -\frac{7\pi^2}{45} \left( \frac{k}{T} \right)^2 = -1.54 \left( \frac{k}{T} \right)^2 ,
\]

(IV.22)

and the (sm) contribution to \(\Delta_R(k)\) reads

\[
\Delta_R^{(\text{sm})} \sim 0.1 \left( \frac{T}{k} \right) \left[ -\frac{5\xi}{192} + \left( \frac{k}{T} \right) \frac{7\pi^2}{240} \right] \sim 0.029 - 3 \times 10^{-3} \left( \frac{T}{k} \right) \xi .
\]

(IV.23)

The resonance happens for \(\Delta_R(k) \sim -1\), namely

\[
3 \times 10^{-3} \left( \frac{T}{k} \right) \xi \sim 1.029 .
\]

(IV.24)

Obviously, one is always able to find a value of \(k/T\) to satisfy eqn. \((\text{IV}.24)\) for any given positive lepton asymmetry \(\xi\). For \(|\xi| \sim 10^{-2}\) consistent with the WMAP and BBN data \([62]\), we obtain

\[
\frac{k}{T} \sim 3 \times 10^{-3} \xi \sim 3 \times 10^{-5} .
\]

(IV.25)

Note that the asymmetry term from (sm) contribution dominates over the (bsm) contribution, which is different from \(\mu = 0\) case where (bsm) contribution would dominate as shown in \((\text{IV}.17)\). This analysis leads us to conclude that for a lepton asymmetry hidden in the neutrino sector compatible with the bounds from ref. \([62]\) there is the possibility of two MSW resonances.

V. IMAGINARY PARTS: WIDTHS FROM VECTOR AND SCALAR BOSON DECAY.

The quasiparticle widths \(\Gamma_{1,2}(k)\) are given by eqns. \((\text{III}.57, \text{III}.60)\). Analyzing the explicit expressions for the imaginary parts of the (sm) and (bsm) contributions given in the appendix, eqns. \((\text{A}.1, \text{A}.3)\) and \((\text{B}.1, \text{B}.3)\) respectively, the “on-shell” contributions are obtained from those whose \(\delta\) function constraints can be satisfied for \(\omega \sim k\). It is straightforward to find that only the terms with \(\delta(\omega + p - W_{\bar{\nu}_e})\) have non-vanishing support for \(\omega \simeq k\). These terms are given in the last lines of \((\text{A}.2)\) and \((\text{A}.3)\) for (sm) and the last lines of \((\text{B}.2)\) and \((\text{B}.3)\) for (bsm) contributions.

These contributions to the quasiparticle widths in the medium arise from the decay of the intermediate boson, either the vector bosons in the (sm) contributions or the scalars in the (bsm) contributions. This is depicted in Fig. \((10)\), the Cutkosky cut through the intermediate boson (vector or scalar) yields the imaginary part. The process that contributes on shell \(\omega \simeq k\) is the decay of the boson into the fermions (neutrinos and or charged leptons) depicted in this figure.

The fact that the decay of a heavy intermediate state leads to a width was recognized in ref. \([58]\).
FIG. 10: The Cutkosky cut for imaginary part of the (sm) and (bsm) contributions, and the contribution on the mass shell $\omega \simeq k$. 

The analysis of the different cases is simplified by introducing

$$\Gamma_{aa}(k_0, k) = \text{Im} \left[ \frac{(k_0 + h k)}{2k} (A_R - h B_R)_{aa} + \frac{(k_0 - h k)}{2k} (A_L + h B_L)_{aa} \right],$$  \hspace{1cm} (V.1)$$

$$\Gamma_{ss}(k_0, k) = \text{Im} \left[ \frac{(k_0 + h k)}{2k} (A_R - h B_R)_{ss} + \frac{(k_0 - h k)}{2k} (A_L + h B_L)_{ss} \right],$$  \hspace{1cm} (V.2)$$

in terms of which (see eqn. (III.32))

$$S_I = 2k \left[ \Gamma_{aa}(k_0, k) + \Gamma_{ss}(k_0, k) \right]; \ \Delta_I(k_0, k) = \frac{2k}{\delta M^2} \left[ \Gamma_{aa}(k_0, k) - \Gamma_{ss}(k_0, k) \right].$$  \hspace{1cm} (V.3)$$

We need these quantities evaluated on the “mass shell”, namely for positive energy $k_0 = \omega(k) \sim k + M^2/2k$. We find:

$h = -1:\n
$$\Gamma_{aa}(k) \simeq \text{Im}(A_L - B_L)_{aa}; \ \Gamma_{ss}(k) \simeq \text{Im}(A_L - B_L)_{ss}.$$

$h = 1:\n
$$\Gamma_{aa}(k) \simeq \text{Im} \left[ (A_R - B_R)_{aa} + \frac{M^2}{4k^2} (A_L + B_L)_{aa} \right]; \ \Gamma_{ss}(k) \simeq \text{Im} (A_R - B_R)_{ss}.$$

In the above expressions we have used $Y_{1,2} \ll g$ and $M^2/4k^2 \ll 1$ and neglected terms accordingly, we have suppressed the arguments on $A, B$, however, these matrix elements depend on $k$. The term with $A_L + B_L$ in (V.5) is noteworthy: the leading contribution to this term is from (sm) interactions, even setting the Yukawa couplings in the (bsm) sector to zero a nearly right-handed sterile neutrino is produced via the decay of the vector bosons.

The expression for the imaginary parts (III.57,III.60) simplify in two relevant limits [63]:

a) weak damping: $|\tilde{\gamma}| \ll 1$: in this limit we find

$$r \sin(\phi) \simeq \tilde{\gamma} \cos 2\theta_m,$$

leading to the following results for the poles with positive energy

$$\Gamma_1(k) = \Gamma_{aa}(k) \cos^2 \theta_m + \sin^2 \theta_m \Gamma_{ss}(k),$$

$$\Gamma_2(k) = \Gamma_{aa}(k) \sin^2 \theta_m + \cos^2 \theta_m \Gamma_{ss}(k).$$

Furthermore the difference in the dispersion relations becomes

$$\Delta \Omega_{wd} \equiv \Omega_2(k) - \Omega_1(k) \simeq \frac{\delta M^2}{2k} \beta_0,$$
which is the usual result for neutrino mixing.

b) strong damping: \(|\tilde{\gamma}| \gg 1\), in this limit we find

\[ r \sin(\phi) \simeq \tilde{\gamma} \, \text{sign}(\cos(2\theta_m)) \left[ 1 - \frac{\sin^2(2\theta_m)}{2\tilde{\gamma}^2} \right] \]  

(V.10)

leading to the following results

\[ \Gamma_1(k) = \frac{1}{2} (\Gamma_{aa}(k) + \Gamma_{ss}(k)) + \frac{1}{2} (\Gamma_{aa}(k) - \Gamma_{ss}(k)) \left( \text{sign}(\cos(2\theta_m)) - \frac{\sin^2(2\theta_m)}{2\tilde{\gamma}^2} \right), \]  

(V.11)

\[ \Gamma_2(k) = \frac{1}{2} (\Gamma_{aa}(k) + \Gamma_{ss}(k)) - \frac{1}{2} (\Gamma_{aa}(k) - \Gamma_{ss}(k)) \left( \text{sign}(\cos(2\theta_m)) - \frac{\sin^2(2\theta_m)}{2\tilde{\gamma}^2} \right). \]  

(V.12)

In this case the frequency difference between the propagating states becomes

\[ \Delta \Omega_{sd} = \Omega_2(k) - \Omega_1(k) \simeq \frac{\delta M^2}{2k} \rho_0 |\cos(2\theta_m)| \simeq \frac{\delta M^2}{2k} |\cos(2\theta) + \Delta R(k)|. \]  

(V.13)

This is a remarkable result, the frequency difference vanishes at a MSW resonance in striking contrast with the usual quantum mechanics description of neutrino mixing and oscillations wherein there is a “level repulsion” at an MSW resonance that prevents level crossing.

In all the expressions above \( \Gamma_{aa}(k); \Gamma_{ss}(k) \) are given by (V.4, V.5) in the respective cases \( h = \mp 1 \), and the mixing angle \( \theta_m \) is obtained from eqns. (III.47) evaluating \( \Delta R \) at \( k_0 = k \).

The widths for negative energies and \( h = \mp 1 \) are obtained from the expressions above by the replacement \( \mu \rightarrow -\mu \), this is a consequence of the relations (III.61,III.62) and the fact that the chemical potential is CP-odd, therefore the particle and antiparticle widths only differ because of the chemical potential.

We emphasize that the results (V.7,V.8) and (V.11,V.12) are general and hold to all orders in perturbation theory as they follow from the general form of the self-energies. In particular these relations are valid beyond the one-loop order studied here and hold for any processes that contributes to the absorptive parts of the self-energy at one-loop or higher order.

A. Widths from scalar and vector boson decay:

As discussed above, the imaginary parts of the self-energy are given in the appendix, both for (sm) and (bsm) contributions. Inspection of the different delta functions shows that the only contribution “on-shell”, namely \( \omega \simeq k \) arises from the terms with \( \delta(\omega + p - W_{\vec{p}+\vec{k}}) \) in the expressions for the imaginary parts (B.2,B.3).

This delta function corresponds to a Cutkosky cut that describes the process of a scalar (in (bsm)) or a vector (in (sm)) boson decay into a neutrino and another lepton, displayed in Fig. 10.

Scalars (bsm): For scalars the (R) and (L) components are the same. We find for \( Y = Y_1,2; M = M_{\sigma,\varphi} \) for \( \sigma,\varphi \) exchange respectively

\[ \text{Im}(A_R - B_R) = \text{Im}(A_L - B_L) = \frac{Y^2 T}{32 \pi} \frac{M^2}{k^2} \ln \left[ \frac{1 + C_1 e^{-x^* - \xi}}{1 - e^{-x^* - y}} \right], \]  

(V.14)

where

\[ x^* = \frac{M^2}{4kT} ; \; \xi = \frac{\mu}{T} ; \; y = \frac{k}{T}, \]  

(V.15)

and

\[ C_1 = \begin{cases} 1 & \text{for } \Sigma^{(1)}_{\sigma,\varphi}, \\ 0 & \text{for } \Sigma^{(2)}_{\sigma,\varphi}. \end{cases} \]  

(V.16)
In the relevant region $k < M_{\sigma,c} \sim T$ we can safely neglect the contribution from the leptonic chemical potential in \(\text{(V.14)}\) and set $\xi = 0$, since the bounds from ref.\[42\] suggest that $|\xi| \lesssim 0.02$. The result \(\text{(V.14)}\) agrees with that found in ref.\[43\] for the decay of the scalar boson into sterile neutrinos \(2\) \((C_1 = 0)\) for vanishing chemical potential.

For $k/T \ll M/T \sim 1$ we can approximate

$$\text{Im}(A_R - B_R) = \text{Im}(A_L - B_L) = \frac{Y^2 T M^2}{32\pi} \frac{e^{-x^*}}{k^2} e^{x^*} \left(C_1 + e^{-y}\right). \quad \text{(V.17)}$$

**Vector bosons (sm):** For (sm) vector boson exchange (only L), the imaginary parts “on-shell” are obtained from the terms with $\delta(\omega + p - W\bar{p} + \xi)$ in the imaginary parts \(\text{(A.2,A.3)}\) setting $\omega \approx k$. We find

$$\text{Im}(A_L - B_L) = \frac{g_{\text{sm}}^2 T M^2}{16\pi} \ln \left[\frac{1 + C_2 e^{-x^* - \xi}}{1 - e^{-x^* - y}}\right], \quad \text{(V.18)}$$

where $g_{\text{sm}}$ is given by eqn. \(\text{(III.16)}\), $M = M_{Z,W}$ for neutral and charged current contributions respectively, and

$$C_2 = \begin{cases} 1 & \text{for } \Sigma^{(1)}_{cc}, \Sigma_{sc}, \\ 0 & \text{for } \Sigma^{(2)}_{nc}. \end{cases} \quad \text{(V.19)}$$

For positive energy, and positive helicity (right-handed), we also need (see eqn. \(\text{(V.5)}\))

$$\text{Im}(A_L + B_L) = \frac{g_{\text{sm}}^2 T^2}{4\pi k} e^{-x^*} \left(C_2 e^{-y}\right). \quad \text{(V.20)}$$

where $Li_2$ is the dilogarithm or Spence’s function and we have set $\mu = 0$. This expression simplifies in the limit $k/T \ll M/T \sim 1$ with the result

$$\text{Im}(A_L + B_L) \approx \frac{g_{\text{sm}}^2 T^2}{4\pi k} e^{-x^*} \left(C_2 + e^{-y}\right). \quad \text{(V.21)}$$

In the above results for vector bosons $M = M_{Z,W}$ respectively.

We can now gather all the results needed for $\Gamma_{\alpha\alpha}(k)$; $\Gamma_{ss}(k)$ \(\text{(V.4,V.5)}\) and the quasiparticle widths $\Gamma_{1,2}(k)$ obtained from them. Approximating $\cos(\theta) \sim 1; \sin(\theta) \sim 0$ we find,

$$\text{Im}(A_R - B_R)_{\alpha\alpha} = \frac{Y^2 T}{32\pi} \left(\frac{M^2}{k^2}\right) \ln \left[\frac{1}{1 - e^{-x^* - e^{-y}}}\right]; \quad \text{(V.22)}$$

$$\text{Im}(A_R - B_R)_{ss} = \text{Im}(A_L - B_L)_{ss} = \frac{Y^2 T}{32\pi} \left(\frac{M^2}{k^2}\right) \ln \left[\frac{1 + e^{-x^*}}{1 - e^{-x^* - e^{-y}}}\right] + \frac{Y^2 T}{32\pi} \left(\frac{M^2}{k^2}\right) \ln \left[\frac{1}{1 - e^{-x^* - e^{-y}}}\right]; \quad \text{(V.23)}$$

$$\text{Im}(A_L - B_L)_{\alpha\alpha} = \frac{g^2 T}{32\pi} \left(\frac{1}{2 \cos^2(\theta_w)}\right) \left(\frac{M^2}{k^2}\right) \ln \left[\frac{1 + e^{-x^*}}{1 - e^{-x^* - e^{-y}}}\right] + \frac{g^2 T}{32\pi} \left(\frac{M^2}{k^2}\right) \ln \left[\frac{1 + e^{-x^*}}{1 - e^{-x^* - e^{-y}}}\right]; \quad \text{(V.24)}$$

$$\text{Im}(A_L + B_L)_{\alpha\alpha} = \frac{g^2 T}{16\pi} \left(\frac{1}{2 \cos^2(\theta_w)}\right) \left[\ln \left(\frac{1 + e^{-x^*}}{1 - e^{-x^* - e^{-y}}}\right) + \frac{2T}{k} \left[Li_2(e^{-x^* - e^{-y}} - Li_2(-e^{-x^*})\right] \right] + \frac{g^2 T}{16\pi} \left(\frac{1}{2 \cos^2(\theta_w)}\right) \left[Li_2(e^{-x^* - e^{-y}} - Li_2(-e^{-x^*})\right] \right] + \frac{Y^2 T}{16\pi} \left(\frac{2T}{k}\right) Li_2(e^{-x^* - e^{-y}}). \quad \text{(V.25)}$$
In the expressions above we have defined
\[ x_\alpha^* = \frac{M_\alpha^2}{4kT}; \quad \alpha = \sigma, \phi, Z, W. \] (V.26)

For small values of the arguments \( L \log(z) \sim z \) which may be used appropriately whenever \( x_\alpha^* > 1 \), a situation which describes the relevant range \( M_\alpha \sim T; k < T \).

Eqs. (V.24) combined with (V.25) yield the complete expressions for the quasiparticle widths \( \Gamma_{1,2} \) in all cases, and as per the discussion below, the production rates.

### B. Imaginary parts: from the width to the production rates.

The connection between the quasiparticle widths (imaginary part of the self-energy “on-shell”) and the production rate is established via the Boltzmann equation for the production of a given species, in this case that of a “sterile” neutrino. Consider as an example the scalar vertex \( Y_\nu \sigma \nu_\alpha \), the analysis is similar for the other, including (sm) vertices. The Boltzmann equation is of the form (gain)–(loss) (see for example the appendix in ref. [43]). The gain term corresponds to the decay process \( \sigma \to \nu_\alpha + \nu_\sigma \) and is of the form [43]

\[ \frac{dn_\alpha(k)}{dt} \bigg|_{\text{gain}} = \int \frac{d^3p}{(2\pi)^3} |M_{fi}|^2 \delta(W_{p'k} - p - k) N_B(W_{p'+k})(1 - \overline{\pi}_F(p))(1 - n_\sigma(k)), \] (V.27)

where \( N_B, n_F \) are the bosonic and fermionic distribution functions respectively. The loss term describes the inverse process, namely the recombination \( \nu_\alpha + \nu_\sigma \to \phi \) with

\[ \frac{dn_\alpha(k)}{dt} \bigg|_{\text{loss}} = \int \frac{d^3p}{(2\pi)^3} |M_{fi}|^2 \delta(W_{p'k} - p - k) \left[ 1 + N_B(W_{p+k}) \right] \overline{\pi}_F(p)n_\sigma(k). \] (V.28)

Therefore the Boltzmann equation is of the form

\[ \frac{dn_\alpha(k)}{dt} = \int \frac{d^3p}{(2\pi)^3} |M_{fi}|^2 \delta(W_{p'k} - p - k) \left[ N_B(W_{p+k})(1 - \overline{\pi}_F(p))(1 - n_\sigma(k)) - \left[ 1 + N_B(W_{p+k}) \right] \overline{\pi}_F(p)n_\sigma(k) \right]. \] (V.29)

If the distribution function of the particle in question is slightly perturbed off equilibrium, the relaxation rate of the distribution function towards equilibrium is obtained by writing \( n_\sigma(k) = n_\sigma^0(k) + \delta n_\sigma(k) \) and linearizing the Boltzmann equation in \( \delta n_\sigma(k) \) [59]. The linearized Boltzmann equation reads

\[ \frac{d\delta n_\sigma(k)}{dt} = -\Gamma_{\text{rel}} \delta n_\sigma(k), \] (V.30)

where

\[ \Gamma_{\text{rel}} = \int \frac{d^3p}{(2\pi)^3} |M_{fi}|^2 \delta(W_{p'k} - p - k) \left[ \overline{\pi}_F(p) + N_B(W_{p+k}) \right]. \] (V.31)

As discussed in ref. [59], the relaxation rate \( \Gamma_{\text{rel}} \) is twice the quasiparticle width [59], since the distribution function is bilinear in the fields. The relation between \( \Gamma_{\text{rel}} \) and the on-shell width becomes evident comparing the expression (V.31) with the “on-shell” imaginary parts, namely the last lines in eqns. (A.2, A.3, B.2, B.3) with \( \omega \approx k \). The production rate of the sterile species is obtained by neglecting the inverse process and neglecting the sterile population buildup in the Boltzmann equation (V.29), namely,

\[ \left. \frac{dn_\alpha(k)}{dt} \right|_{\text{prod}} = \int \frac{d^3p}{(2\pi)^3} |M_{fi}|^2 \delta(W_{p'k} - p - k) N_B(W_{p+k})(1 - \overline{\pi}_F(p)). \] (V.32)

Therefore by obtaining the bosonic and fermionic contributions to the quasiparticle widths as in the previous section, we can obtain the production rates. Although the term with the product \( N_B\overline{\pi}_F \) is not included in the width, such term is smaller than the term with \( N_B \) only, since \( p^2\overline{\pi}_F(p) \) features a maximum at \( p/T \sim 2.3 \) for which \( \overline{\pi}_F(p) \sim 0.09 \) (for \( \mu/T \ll 1 \)). Therefore in the region of importance in the integral \( p \gtrsim T \), the production and relaxation rates only differ by a few percent, and the results for the relaxation rates yield a reliable approximation to the production rate.
An important bonus of obtaining the production rate from the quasiparticle decay width as carried out here is the correct dependence on the mixing angle in the medium, which would be missed by a naive perturbative calculation.

Therefore the quasiparticle width yields an excellent approximation to the production rate, in particular it describes correctly the dependence on the mixing angles in the medium, its magnitude and $k$-dependence.

In particular, the result (V.23) confirms the result of ref. \cite{43} for $Y_1 = 0$. For the scalar contribution (bsm) the right and left-handed yield the same result (multiplying (V.23) by a factor 2 in the total rate) and as discussed above the production rate is twice the width, which restores the factor 4 between (V.23) and the result in ref. \cite{43} which corresponds to the case $Y_1 = 0$.

Thus we conclude that the results of eqns. (V.4,V.5,V.7,V.8,V.11,V.12) along with the explicit forms (V.22,V.25) provide a complete and reliable assessment of the production rates ready to be input in the kinetic equations that include the cosmological expansion\cite{43}.

C. Weak or strong damping?

We have now all the ingredients to assess under which circumstances the weak ($|\tilde{\gamma}| \ll 1$) or strong ($|\tilde{\gamma}| \gg 1$) damping conditions are fulfilled. In terms of the widths and real parts it follows that

$$\tilde{\gamma} \simeq \frac{2k}{M_s^2} \left[ \Gamma_{aa}(k) - \Gamma_{ss}(k) \right] \left( \cos(\theta) + \Delta_R(k) \right)^2 + \sin^2(2\theta)^2 \right]^{1/2} \right] \right].$$

(V.33)

For $h = -1$, $\Gamma_{aa} - \Gamma_{ss}$ and $\Delta_R$ are dominated by the (sm) contributions, therefore from eqns. (V.13) and (V.24) we find

$$\Delta_R(k) \sim \frac{g^2}{16\pi^2 M_s^2} A(k),$$

(V.34)

$$\Delta_I(k) \sim \frac{g^2}{32\pi M_s^2} \left( \frac{M_Z}{k} \right) B(k),$$

(V.35)

where $A(k), B(k)$ can be read off (V.13,V.24). In the region of parameters where $\Delta_R(k) \gg \cos(\theta) \sim 1$, it follows that $\tilde{\gamma} \simeq \Delta_I(k)/\Delta_R(k)$, furthermore, for $k < T \sim M_{Z,W}$ the function $B(k) \sim e^{-\frac{2\Delta_I}{\gamma}} \ll 1$, leading to $\Delta_I/\Delta_R \ll 1$ corresponding to the weak damping case in which the widths (production rates) are given by (V.4,V.5).

Far away from the MSW resonances but in the region where $\cos(2\theta) \sim 1 \gg \Delta_R(k)$ it also follows that $\Delta_I/\Delta_R \ll 1$, corresponding again to the weak damping regime. Therefore the parameter region far away from MSW resonances (either above or below) corresponds to the weak damping regime.

Very near MSW resonances $\cos(2\theta) + \Delta_R \sim 0$ and $\tilde{\gamma} \sim \Delta_I/|\sin(2\theta)|$, in the region of relevance for our analysis $T \sim M_{Z,W}$ with $M_s \sim$ KeV it follows that

$$\frac{\Delta_I(k)}{|\sin(2\theta)|} \sim \frac{4 \times 10^{13}}{|\sin(2\theta)|} \left( \frac{k}{T} \right) \left( \frac{M_Z}{k} \right) B(k),$$

(V.36)

therefore, since the resonance occurs at $k/T < 1$ for $M_{Z,W} \sim T$ we conclude that the strong damping condition $\tilde{\gamma} \gg 1$ is fulfilled near MSW resonances. Because the MSW resonance(s) are very narrow for $T \sim M_{Z,W}$ as discussed above (see the discussion leading to eqn. (IV.16), we conclude that in most of the regime of temperatures and momenta the weak damping results (V.7,V.8) are valid and only in a very narrow region near MSW resonances the strong damping results (V.11,V.12) are valid.

An identical analysis confirms a similar conclusion for the case $h = 1$, namely the weak damping condition holds in most of the relevant range of $M/T; k/T$ but for a narrow region near the MSW resonances in which the strong damping condition holds.

An alternative interpretation of the weak and strong damping regime is obtained using eqn. (V.9) to write

$$\tilde{\gamma} \simeq \frac{\Gamma_{aa} - \Gamma_{ss}}{\Delta\Omega_{ud}}.$$

(V.37)

Since $\Delta\Omega_{ud} \leq \Delta\Omega_{wd}$ the denominator gives an upper bound to the oscillation frequency between the active and sterile neutrinos. The weak damping regime $|\tilde{\gamma}| \ll 1$ describes the case in which there are many oscillations before the overlap amplitude is suppressed, whereas the strong damping regime describes the case in which damping occurs before oscillations take place. For a similar discussion see the second ref. in \cite{50}. 

D. Regime of validity of perturbation theory.

In the relativistic approximation the validity of the perturbative expansion requires that $k \gg \Sigma_{bsm}; \Sigma_{sm}$. Since the weak interaction coupling constant $g_{sm}$ is much larger than $Y_{1, 2}$ we focus on the standard model contributions.

From the expression (IV.3) and the results displayed in fig. (1) we see that for $M_W/T \gtrsim 1$, it follows that $\text{Re} \Sigma_{sm} \propto \alpha_w T$ since the coefficient functions $A, B \lesssim 12$. Therefore perturbation theory is valid for $k \gg T/30$, hence for $M_W/T \gtrsim 1$ the resonances in absence of lepton asymmetry at $0.2 \lesssim k/T \lesssim 1$ for $1 \lesssim M_W/T \lesssim 3$ are comfortably within the regime of validity of the perturbative expansion. The lepton-asymmetry induced resonance for $k/T < M_W/T$ is the usual resonance and for $T < M_W$ the expressions (IV.9)-(IV.11) reduce to the results available in the literature [53, 54]. In the regime $k \ll T < M_W$ the on-shell self-energies are linear in $k$. We see that, for $g^2 \sim 0.4$, the terms proportional to $k$ are $\ll 1$ for $M_W \gtrsim 2T$, hence perturbation theory is reliable within the regime of interest in this article. The imaginary parts are always perturbatively small because of the exponential suppression factors $e^{-M^2/kT}$.

Perturbation theory breaks down for $M \lesssim T$ for the small $k/T$ region and requires a hard thermal loop resummation program [61] akin to the one presented in ref. [57] in the standard model without mixing. This is well known in gauge theories where the gauge bosons are nearly massless on the scale $T$ [61]. Such program is well beyond the realm of this study, however for $M/T \gtrsim 3$ our results are reliable for $k/T \gg \alpha_w$, as analyzed above. For example for the case $M/T \sim 1$ although the peak in the coefficients $A, B$ in the self-energy occur for $k/T \approx 0.07$ which is not too large compared to $\alpha_w \sim 0.03$, the position of the resonance at $k/T \sim 0.2$ is well within the regime of validity of the perturbative expansion. The validity of perturbation theory improves dramatically for $M/T > 1$ even in the low momentum region as discussed above. Therefore, we conclude that for $M/T > 1$ the perturbative results are reliable for $k/T > \alpha_w$, in particular the new resonances are well within the regime of validity of the perturbative expansion. The results for the production rates are always perturbatively small and reliable because of the exponential suppression factor.

VI. DISCUSSION

Our goal is to study the production of sterile neutrinos in cosmology near the electroweak scale when the universe is radiation dominated. To include the effects of cosmological expansion in the production rates and mixing angles, one must first replace the momentum $k \rightarrow k_p(t) = k/a(t)$ and temperature $T \rightarrow T(t) = T_i a_i/a(t)$ where $k$ is the comoving momentum, $a(t)$ the scale factor and $T_i; a_i$ correspond to the initial temperature and scale factor at which the kinetic equations are initialized. Whereas the ratio $k_p(t)/T(t) = k/(T_i a_i)$ is constant $M/T(t) = M a(t)/(T_i a_i)$ grows during the expansion. Consider setting initial conditions at $T_i \lesssim M_W$, so that $M/T_i \sim 1$, the analysis of section (IV) shows that there exists at least one very narrow MSW resonance even for nearly right-handed sterile neutrinos (two if a lepton asymmetry in the neutrino sector is included) at a value $(k_p(t)/T(t))_c < 1$. For $(k_p(t)/T(t)) < (k_p(t)/T(t))_c$ the analysis shows that $\Delta_R \gg 1$ and

$$\theta_m \sim \frac{\theta}{\Delta_R} \ll \theta, \quad (VI.1)$$

therefore for $(k_p(t)/T(t)) < (k_p(t)/T(t))_c$ we find

$$\Gamma_1 \sim \Gamma_{aa},$$

$$\Gamma_2 \sim \Gamma_{ss} + \left(\frac{\theta}{\Delta_R}\right)^2 \Gamma_{aa}. \quad (VI.2)$$

For these values of $k_p(t)/T(t)$ the mode “1” is active-like and it is produced with a weak interaction rate, whereas the mode “2” is sterile like and is produced with the rate similar to that of ref. [43] plus small corrections from standard model interaction rates suppressed by the mixing angle in the medium $\sim \theta/\Delta_R$. On the other hand for $(k_p(t)/T(t)) > (k_p(t)/T(t))_c$ we found above that $\Delta_R \ll -1$ leading to $\theta_m \sim \pi/2$, namely the mode “1” is sterile-like and the mode “2” is active like, with the production rates

$$\Gamma_1 \sim \left(\frac{\theta}{2\Delta_R}\right)^2 \Gamma_{aa} + \Gamma_{ss},$$

$$\Gamma_2 \sim \Gamma_{aa}. \quad (VI.3)$$

As the cosmological expansion proceeds eventually $M/T(t) \gg 1$ and the resonances disappear (in absence of lepton asymmetry the MSW resonances for $k_p(t)/T(t) < 1$ disappear for $M/T(t) \geq 3$), $\Delta_R$ remains large but positive and
the mixing angle in the medium is given by $\theta_{\alpha\alpha}$ and the production rates are given by $\Gamma_{\alpha\alpha}$ for all values of $k_p(t)/T(t)$, namely the mode “1” remains the active-like and the mode “2” the sterile-like.

We note that (see section V.A)

$$\Gamma_{ss}, \Gamma_{aa} \propto \left(\frac{M^2}{k^2}\right) \ln \left[ \frac{1}{1 - e^{-x - y}} \right],$$  \hspace{1cm} (VI.4)

this is precisely the form of the production rate that leads to a distribution function after freeze-out that is enhanced at small momentum, a feature that leads to a larger free streaming length and transfer function at small scales.

During the time when $M/T(t) \sim 1$ the MSW resonance for $k_p(t)/T(t) < 1$ leads to a non-thermal population of neutrinos: for $M_p(t)/T(t) < (k_p(t)/T(t))_c$, there is a large production of mode “1” leading to large populations and a small production of “2” (sterile-like) leading to small populations, whereas for $M_p(t)/T(t) > (k_p(t)/T(t))_c$ there is a “population inversion” in the sense that mode “1” is slightly populated whereas mode “2” will be substantially populated, however, without the small momentum enhancement. Consider a fixed value of $k_p(t)/T(t) < 1$ during the cosmological expansion the ratio $M/T(t) \propto a(t)$ increases sweeping through the MSW resonance, when this happens the mixing angle in the medium vanishes very rapidly because the resonance is very narrow and the mode “2” becomes sterile like. As the expansion continues the MSW resonances (in absence of lepton asymmetry) disappear altogether and the mixing angles and production rates are given by $\Gamma_{ss} \propto \Gamma_{aa}$ respectively for all values of $k_p(t)/T(t)$. The population of the active-like neutrino (mode “1”) continues to build up via weak interaction processes, including those that become dominant at $T \ll M_W$ and eventually thermalizes, whereas the population of the sterile-like neutrino will be frozen-out as the production rate $\Gamma_2$ shuts-off as $\Gamma_{ss}$ vanishes rapidly for $M/T(t) \ll 1$ (see ref.[43]) and $\theta_{\alpha\alpha} \to 0$ as $M/T(t) \gg 1$ even when $\Gamma_{aa}$ (weak interaction rates) remain large down to the decoupling temperature of weak interactions $\sim 1$ MeV.

This analysis indicates that sterile neutrino production via the decay of scalar or vector bosons will be effective only in a region for $M_W/T(t) \sim 1$ and the distribution function at freeze-out will be strongly non-thermal with very small population but with an enhancement at small momentum as found in ref.[43]. However, the weak interaction contribution will freeze out much later, depending on the temperature dependence of the mixing angle in the medium and will eventually merge with the non-resonant (DW) production mechanism at $T \sim 150$MeV.

However the non-thermal distribution built up during the stage when scalar and vector boson decay dominate the production will remain.

At this stage it is important to understand the self-consistency of the analysis. In obtaining the self-energies we had assumed that the eigenstate “1” is active like with a thermal distribution function. We have learned, however, that there are resonances and the eigenstates “1” and “2” are either active-like or sterile-like depending on $k$, namely on which side of the MSW resonance the wavevector lies. This finding calls into question the thermal nature of the neutrino propagator in the intermediate state (of course there is no such ambiguity in the charged lepton propagator that enters in the charged current self-energy). This issue notwithstanding, we have found that the fermionic and bosonic contributions to the real parts of the self-energies are qualitatively the same with a rather small quantitative difference, both for (sm) and (bsm) contributions. Therefore replacing the thermal fermion propagator for a vacuum one leads to a minor quantitative modification of our arguments. However because of the enormous pre-factors the conclusions about the sharpness of the resonance and the resonance positions do not change and the general analysis remains the same. Therefore, we conclude that the results obtained above are very robust not depending on whether the intermediate fermion line features a thermal or vacuum propagator or non-thermal propagator interpolating between these two cases.

VII. CONCLUSIONS AND COSMOLOGICAL CONSEQUENCES

A comprehensive program to assess the viability of any potential (DM) candidate begins with the microphysics of the production and freeze-out process of the particle physics candidate. This initial step determines the distribution function at freeze-out which in turn determines, along with the mass, its abundance, free streaming length, phase space density at decoupling and the transfer function and power spectrum in the linear regime. Our objective is to carry out this program for sterile neutrinos with mass in the KeV range which seems to be the range favored not only as a (DM) candidate but also provide potential solutions to a host of astrophysical problems.

In this article we focus on the first step of the program and study the production of sterile neutrinos in a temperature regime near the electroweak scale in an extension beyond the standard model in which the see-saw mass matrix emerges from expectation values of Higgs-like scalars with masses of the order of the electroweak scale. This simple and compelling extension which features only one scale yields rich phenomenology. The main observation in this article is that in this temperature range sterile neutrinos are produced by the decay not only of the Higgs-like...
scalar as explored in refs. [32, 43] but also of the charged and neutral vector bosons of the standard model. We consider active and sterile species to be Dirac fermions to allow the possibility to include a lepton asymmetry hidden in the (active) neutrino sector consistent with recent bounds from WMAP and BBN [62].

The assessment of the contribution from standard model vertices to sterile neutrino production requires an analysis of the mixing angles in the medium and production rates. We obtain both from the study of the full equation of motion of the active and sterile neutrinos that input the self-energies in the medium. The real part of the self-energy (index of refraction) determines the dispersion relations and mixing angles in the medium, and the imaginary (absorptive) part determines the production rates.

We provide a detailed analysis of the contributions from “beyond the standard model” and standard model interactions to the mixing angles, dispersion relations and production rates, thereby facilitating the analysis of different situations. The study of the “index of refraction” in the temperature regime near the electroweak scale has not been performed before and yields a wealth of remarkable phenomena.

Our study reveals the presence of narrow MSW resonances even in the absence of a lepton asymmetry, in the temperature regime $T \gtrsim M_W$ for $k/T \lesssim 1$. For vanishing lepton asymmetry the resonance occurs at a value $(k/T)_c$ that depends on the ratio $M_W/T$ with $0.15 \lesssim (k/T)_c \lesssim 1$ for $1 \lesssim (M_W/T) \lesssim 3$. The position of the resonance $(k/T)_c$ increases with $M_W/T$, the resonance eventually disappear for $M_W \gg T$ recovering the result valid in the Fermi limit of the weak interactions [53, 54].

Including the possibility of a (small) lepton asymmetry in the neutrino sector with a value compatible with the bounds from WMAP and BBN [62] yields two narrow MSW resonances in these regions, with the resonance associated with the lepton asymmetry occurring at $k < \mu \ll T$ where $\mu$ is the chemical potential for the active species that determines the lepton asymmetry.

A remarkable aspect of these results is that near these resonances the contribution of the imaginary part of the self-energies leads to a strong damping regime, and the difference in the propagating frequencies vanishes exactly at the position of the resonance, with a concomitant breakdown of adiabaticity. For $M_W \gg T$ the MSW resonances that are independent of the lepton asymmetry disappear leaving only the low energy resonances associated with the lepton asymmetry.

Furthermore, we have found that it is quite possible that the region of parameters of the extension (bsm) allow for MSW resonance for positive energy, positive helicity, namely nearly right-handed states both with and without lepton asymmetry. We also find that the decay of the $Z^0, W^\pm$ vector bosons leads to the production of nearly right-handed sterile-like neutrinos.

Because the resonances are very narrow, we obtain a simple expression for the production rates (see section VI) that is valid in a wide range of temperatures and clearly displays the contribution from standard model and beyond standard model interactions.

We have argued that in the early universe the cosmological expansion leads to a highly non-thermal distribution function for sterile neutrinos with an enhancement of the low momentum region $k < T$ both as a consequence of the MSW resonances and the vanishing of the mixing angle and production rates as the temperature falls well below the electroweak scale. Furthermore, we expect that because the MSW resonances are very narrow, the cosmological expansion will lead to sterile neutrino production resulting in a highly non-thermal distribution with low momentum enhancement. The form of the production rates via scalar and vector boson decay are similar to that in ref. [43], which leads us to conjecture that the distribution function after freeze-out will be enhanced in the low momentum region, leading to a smaller free streaming length and larger power spectrum at small scales as compared to the (DW) mechanism [27, 43].

The next step of the program will input these results into the kinetic equations that describe the production and freeze-out of the sterile species from which the distribution function at decoupling is obtained. We expect to report on these studies in a forthcoming article.

An important remaining question is the extrapolation of these results to $T \gg M_{W,Z}$. At temperatures above the electroweak symmetry breaking scale the $SU(2) \times U(1)$ symmetry is restored and the vector bosons become massless at tree level, therefore the production channel described here shuts off. However, vector bosons acquire electric screening masses of order $g^2 T$ [61] and scalar bosons may also acquire thermal mass corrections of $O(Y_{1,2} T)$.

Furthermore the see-saw mass matrix also vanishes at tree level if all the mass terms arise from the expectation value of the Higgs-like scalar field. This high temperature regime requires a deeper understanding of radiative corrections to the propagators of the vector bosons, in particular the hard-thermal loop corrections [61].

Understanding the possibility of sterile neutrino production in this high temperature regime entails a non-perturbative resummation program also for neutrinos, akin to the study in ref. [57]. This program although clearly interesting in its own right is far beyond the realm of our goals here and deserves a deeper study.
APPENDIX A: VECTOR BOSON EXCHANGE (SM)

The (SM) self-energy contributions with the exchange of a vector boson are given by the spectral representation \( \Pi_{\text{bsm}}^{\omega, p, k} \) with the imaginary part given by eqn. \( \Pi_{\text{bsm}}^{\omega, p, k} \) which is of the form

\[
Im\Sigma_{\text{sm}}(\omega, \vec{k}) = \frac{\pi g_{\text{sm}}^2}{4} \int \frac{d^3p}{(2\pi)^3} W_{\vec{p}+\vec{k}} \left[ \gamma^0 \Pi_{\text{sm}}^0(\omega, \vec{p}, \vec{k}) - \gamma^\dagger \cdot \vec{k} \Pi_{\text{sm}}^1(\omega, \vec{p}, \vec{k}) \right]. \tag{A.1}
\]

Neglecting the mass of the neutrinos and charged leptons we find

\[
\Pi_{\text{sm}}^0(\omega, \vec{p}, \vec{k}) = \left[ 1 - n_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \left[ p(1 + \frac{2W_{\vec{p}+\vec{k}}^2}{M^2}) + \frac{2W_{\vec{p}+\vec{k}}^2}{M^2} (p^2 + \vec{k} \cdot \vec{p}) \right] \delta(\omega - p - W_{\vec{p}+\vec{k}}) \\
+ \left[ 1 - \bar{n}_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \left[ p(1 + \frac{2W_{\vec{p}+\vec{k}}^2}{M^2}) - \frac{2W_{\vec{p}+\vec{k}}^2}{M^2} (p^2 + \vec{k} \cdot \vec{p}) \right] \delta(\omega + p - W_{\vec{p}+\vec{k}}) \\
+ \left[ n_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \left[ p(1 + \frac{2W_{\vec{p}+\vec{k}}^2}{M^2}) - \frac{2W_{\vec{p}+\vec{k}}^2}{M^2} (p^2 + \vec{k} \cdot \vec{p}) \right] \delta(\omega - p - W_{\vec{p}+\vec{k}}) \\
+ \left[ \bar{n}_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \left[ p(1 + \frac{2W_{\vec{p}+\vec{k}}^2}{M^2}) + \frac{2W_{\vec{p}+\vec{k}}^2}{M^2} (p^2 + \vec{k} \cdot \vec{p}) \right] \delta(\omega + p - W_{\vec{p}+\vec{k}}) \tag{A.2}
\]

and

\[
\Pi_{\text{sm}}^1(\omega, \vec{p}, \vec{k}) = \left[ 1 - n_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \left[ - \vec{k} \cdot \vec{p} + \frac{2(\vec{k} + \vec{k} \cdot \vec{p})}{M^2} \left( p W_{\vec{p}+\vec{k}} + p^2 + \vec{k} \cdot \vec{p} \right) \right] \delta(\omega - p - W_{\vec{p}+\vec{k}}) \\
- \left[ 1 - \bar{n}_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \left[ - \vec{k} \cdot \vec{p} + \frac{2(\vec{k} + \vec{k} \cdot \vec{p})}{M^2} \left( p W_{\vec{p}+\vec{k}} + p^2 + \vec{k} \cdot \vec{p} \right) \right] \delta(\omega + p - W_{\vec{p}+\vec{k}}) \\
+ \left[ n_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \left[ - \vec{k} \cdot \vec{p} + \frac{2(\vec{k} + \vec{k} \cdot \vec{p})}{M^2} \left( - p W_{\vec{p}+\vec{k}} + p^2 + \vec{k} \cdot \vec{p} \right) \right] \delta(\omega - p + W_{\vec{p}+\vec{k}}) \\
- \left[ \bar{n}_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \left[ - \vec{k} \cdot \vec{p} + \frac{2(\vec{k} + \vec{k} \cdot \vec{p})}{M^2} \left( - p W_{\vec{p}+\vec{k}} + p^2 + \vec{k} \cdot \vec{p} \right) \right] \delta(\omega + p - W_{\vec{p}+\vec{k}}). \tag{A.3}
\]

APPENDIX B: SCALAR EXCHANGE (BSM)

For scalar boson exchange we find

\[
\text{Im}\Sigma_{\text{bsm}}(\omega, \vec{k}) = \frac{\pi Y^2}{4} \int \frac{d^3p}{(2\pi)^3} W_{\vec{p}+\vec{k}} \left[ \gamma^0 \Pi_{\text{bsm}}^0(\omega, \vec{p}, \vec{k}) - \gamma^\dagger \cdot \vec{k} \left( \vec{k} \cdot \vec{p} \right) \Pi_{\text{bsm}}^1(\omega, \vec{p}, \vec{k}) \right], \tag{B.1}
\]

where

\[
\Pi_{\text{bsm}}^0(\omega, \vec{p}, \vec{k}) = \left[ 1 - n_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \delta(\omega - p - W_{\vec{p}+\vec{k}}) + \left[ 1 - \bar{n}_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \delta(\omega + p + W_{\vec{p}+\vec{k}}) \\
+ \left[ n_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \delta(\omega - p + W_{\vec{p}+\vec{k}}) + \left[ \bar{n}_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \delta(\omega + p - W_{\vec{p}+\vec{k}}), \tag{B.2}
\]

\[
\Pi_{\text{bsm}}^1(\omega, \vec{p}, \vec{k}) = \left[ 1 - n_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \delta(\omega - p - W_{\vec{p}+\vec{k}}) - \left[ 1 - \bar{n}_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \delta(\omega + p + W_{\vec{p}+\vec{k}}) \\
+ \left[ n_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \delta(\omega - p + W_{\vec{p}+\vec{k}}) - \left[ \bar{n}_F(p) + N_B(W_{\vec{p}+\vec{k}}) \right] \delta(\omega + p - W_{\vec{p}+\vec{k}}). \tag{B.3}
\]
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