Maximum baryon masses for static neutron stars in f(R) gravity

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Abstract – We investigate the upper mass limit predictions of the baryonic mass for static neutron stars in the context of f(R) gravity. We use the most popular f(R) gravity model, namely the $R^2$ gravity, and calculate the maximum baryon mass of static neutron stars adopting several realistic equations of state and one ideal equation of state, namely that of causal limit. Our motivation is based on the fact that neutron stars with baryon masses larger than the maximum mass for static neutron star configurations inevitably collapse to black holes. Thus with our analysis, we want further to enlighten the predictions for the maximum baryon masses of static neutron stars in $R^2$ gravity, which, in turn, further strengthens our understanding of the mysterious mass gap region.

As we show, the baryon masses of most of the equations of states studied in this paper lie in the lower limits of the mass gap region $M\sim$2.5–5M⊙, but intriguingly enough, the highest value of the maximum baryon masses we found is of the order of $M\sim$3M⊙. This upper mass limit also appears as a maximum static neutron star gravitational mass limit in other contexts. Combining the two results which refer to baryon and gravitational masses, we point out that the gravitational mass of static neutron stars cannot be larger than three solar masses, while based on maximum baryon masses results of the present work, we can conspicuously state that it is highly likely that the lower mass limits of astrophysical black holes in the range of $M\sim$2.5–3M⊙. This, in turn, implies that maximum neutron star masses in the context of $R^2$ gravity are likely to be in the lower limits of the range of $M\sim$2.4–3M⊙. Hence our work further supports the General Relativity claim that neutron stars cannot have gravitational masses larger than 3M⊙ and then, to explain observations comparable or over this limit, we need alternative extensions of General Relativity, other than f(R) gravity.

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Introduction. – Currently and for the next 10–15 years, gravitational waves and neutron stars are in the focus of the scientific community. As it seems, the Large Hadron Collider at CERN indicates that new physics may lie well above 15 TeV center of mass, thus contemporary science is focused on neutron star (NS) physics (for an important stream of reviews and textbooks see for example [1–5]) and astrophysical objects merging, which may provide insights to fundamental physics problems. Indeed, neutron stars (NSs) have multiple correlations with various physics research areas, like nuclear physics [6–15], high-energy physics [16–20], modified gravity [21–33] and relativistic astrophysics [34–41]. Apart from the physical implications of isolated neutron stars, surprises for fundamental physics may arise from the merging of astrophysical objects, the mysteries of which are analyzed by the LIGO-Virgo Collaboration. Already the GW170817 event [42] has changed the way of thinking in theoretical cosmology indicating that the gravitational waves propagate with a speed equal to that of light. The physics of astrophysical gravitational waves, and more importantly that of primordial gravitational waves is expected to change the way of thinking, or to verify many theoretical proposals in theoretical cosmology. Future

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collaborations like the Einstein Telescope Hz kHz frequencies [43], the LISA Space-borne Laser Interferometer Space Antenna [44,45] the BBO [46,47], DECIGO [48,49] and finally the SKA (Square Kilometer Array) Pulsar Timing Arrays at frequencies $10^{-8}$ Hz [50] are expected to shed new light on fundamental high-energy physics problems, with most of these telescopes revealing the physics of the radiation domination era. As we already stated, in the next 10–15 years, particle physics, theoretical cosmology and theoretical astrophysics will heavily rely on gravitational wave and NSs observations. Although it seems that things are more or less settled with theoretical astrophysics, a recent report [51] has cast doubt on the maximum mass issue of NSs, and, in parallel, it indicated that alternative astrophysical objects, like strange stars, may come into the play in the near future. Although it is quite early phenomenologically speaking, for exotic stars to be discovered, it is a realistic possibility. Then, if exotic objects are not yet fully phenomenologically supported, the problem with the observation [51] is that it is probable to find NSs with masses in the mass gap region $M\sim 2.5–5M_\odot$. This possibility is sensational and it raises the fundamental question inherent to the maximum mass problem of NSs, which is, what is the lowest mass of astrophysical black holes. In the context of General Relativity (GR), non-rotating neutron stars with masses in the mass gap region can only be described by ultra-stiff equations of state (EoS), thus it is rather hard to describe them without being in conflict with the GW170817 results. Modified and extended gravity in its various forms [52–59] can provide a clear cut description for large mass NSs [31–33,60–66] see also refs. [21–23] for recent descriptions of the GW190814 event, and thus serves as a cutting edge probable description of nature in limits where GR needs to be supplemented by a Occam’s razor compatible theory.

Motivated by the fundamental and inherently related questions, which is the maximum mass of NSs and what is the lowest mass of astrophysical black holes, in this work we shall approach these issues by calculating the maximum baryonic mass of NSs in the context of $f(R)$ gravity and specifically for the $R^2$ model in the Jordan frame. We shall use several different phenomenological EoSs and our main aim is to pave the way towards answering the question as to what the maximum gravitational mass that neutron stars can have is. At the same time, if this question is answered, one may also have a hint on the question as to which the lowest mass that astrophysical black holes can have is. Before getting to the details of our analysis, we provide here an overview of the treatment of spherically symmetric compact objects in the context of Jordan frame $f(R)$ gravity, and the Tolman-Oppenheimer-Volkoff (TOV) equations.

The $f(R)$ gravity action in the Jordan frame is the following:

$$A = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left[ f(R) + L_{\text{matter}} \right],$$  \hspace{1cm} (1)

with $g$ denoting the metric tensor determinant and $L_{\text{matter}}$ denoting the Lagrangian of the perfect matter fluids that are present. Upon variation of the action (1) with respect to the metric tensor $g_{\mu\nu}$, the field equations are obtained [55],

$$\frac{df(R)}{dR} R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - [\nabla_\mu \nabla_\nu - g_{\mu\nu}] \frac{df(R)}{dR} = \frac{8\pi G}{c^4} T_{\mu\nu},$$  \hspace{1cm} (2)

for a general metric $g_{\mu\nu}$, where $T_{\mu\nu} = \frac{-\delta}{\sqrt{-g}} \delta g_{\mu\nu}$ stands for the energy-momentum tensor of the perfect matter fluids present. We shall consider static NSs, which are described by the following spherically symmetric metric:

$$ds^2 = e^{2\psi} c^2 dt^2 - e^{2\lambda} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$  \hspace{1cm} (3)

where $\psi$ and $\lambda$ are arbitrary functions with radial dependence only. The energy momentum tensor for the perfect matter fluid describing the NS is $T_{\mu\nu} = \text{diag}(e^{2\psi} \rho c^2, e^{2\lambda} p, r^2 p, r^2 p \sin^2 \theta)$, where $\rho$ denotes the energy-matter density and $p$ stands for the pressure [68]. By using the contracted Bianchi identities, one can obtain the equations for the stellar object, by also implementing the hydrostatic equilibrium condition,

$$\nabla^\mu T_{\mu\nu} = 0,$$  \hspace{1cm} (4)

which, in turn, yields the Euler conservation equation,

$$\frac{dp}{dr} = -(\rho + p) \frac{d\psi}{dr}.$$  \hspace{1cm} (5)

Upon combining the metric (3) and the field equations (2), we obtain the equations governing the behavior of the functions $\lambda$ and $\psi$ inside and outside the compact object, which are [69]

see eqs. (6), (7) on top of the next page.
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\[
\frac{d\lambda}{dr} = \frac{e^{2\lambda} [r^2 (16\pi p + f(R)) - f'(R)(r^2 R + 2)] + 2 R^2 f''(R) r^2 + 2 r f''(R) [r R_{\gamma \gamma} + 2 R_{\gamma}]}{2 r [2 f'(R) + r R_{\gamma} f''(R)]},
\]

(6)

\[
\frac{d\psi}{dr} = \frac{e^{2\lambda} [r^2 (16\pi p - f(R)) + f'(R)(r^2 R + 2)] - 2 [2 r f''(R) R_{\gamma} + f'(R)]}{2 r [2 f'(R) + r R_{\gamma} f''(R)]},
\]

(7)

with the prime in eqs. (6) and (7) denoting differentiation with respect to the function \( R(r) \), that is \( f'(R) = \frac{df}{dR} \). The above set of differential equations constitute the \( f(R) \) gravity TOV equations, and by using \( f(R) = R \) one obtains the standard TOV equations of GR \([70,71]\). In addition to the above TOV equations, for \( f(R) \) gravity, the TOV equations are also supplemented by the following differential equation:

\[
\frac{d^2 R}{dr^2} = R_{\gamma} \left( \lambda_{\gamma} + \frac{1}{r} \right) + f'(R) \left( \frac{1}{r} \right) \left[ 3 \psi_{\gamma} - \lambda_{\gamma} + \frac{2}{r} \right] - e^{2\lambda} \left( \frac{R}{2} - \frac{2}{r} \right) - \frac{R_{\gamma}^2 f''(R)}{f'(R)},
\]

(8)

which is obtained from the trace of eqs. (2) by replacing the metric (3). The differential equation (8) basically expresses the fact that the Ricci scalar dynamically evolves in the context of \( f(R) \) gravity, as the radial coordinate \( r \) changes.

Having presented the TOV equations, the focus is now on solving numerically them, namely eqs. (5), (6) and (7) together with (8), for the \( R^2 \) model,

\[
f(R) = R + \alpha R^2,
\]

(9)

where the parameter \( \alpha \) is expressed in units of \( r_s^2 = 4 G^2 M_B^2 / c^4 \) and \( r_s \) is the Sun gravitational radius. With regard to the EoS, we shall consider five phenomenological and one ideal limiting case EoSs, specifically: a) the APR4 which is a \( \beta \)-equilibrium EoS proposed by Akmal, Pandharipande and Ravenhall \([72]\), b) The BHF which is a microscopic EoS of dense \( \beta \)-stable nuclear matter obtained using realistic two-body and three-body nuclear interactions denoted as \( N3LO\Delta + N2LO\Delta_1 \) \([73]\) derived in the framework of chiral perturbation theory and including the \( \Delta(1232) \) isobar intermediate state. This EoS has been derived using the Brueckner-Bethe-Goldstone quantum many-body theory in the Brueckner-Hartree-Fock approximation. c) The GM1 EoS, which is the classical relativistic mean field parametrization GM1 \([74]\) for cold neutron star matter in \( \beta \)-equilibrium containing nucleons and electrons. d) The QHC18, which is a phenomenological unified EoS proposed in \([75]\) and describes the crust, nuclear liquid, hadron-quark crossover, and quark matter domains. e) The SLy \([76]\), which is a well-known and phenomenologically successful EoS. f) Finally, the limiting case ideal EoS, called the causal limit EoS, in which case,

\[
P(\rho) = P_u(\rho_u) + (\rho - \rho_u) v_u^2,
\]

with \( P_u \) and \( \rho_u \) corresponding to the pressure and density of the well-known segment of a low-density EoS at \( \rho_u \approx \rho_0 \) where \( \rho_0 \) denotes the saturation density. We shall assume that the low-density EoS is the SLy EoS and consider the case when \( v_u^2 = \epsilon^2 / 3 \). It is conceivable that the causal EoS is an ideal limit, thus the resulting baryonic mass for static NSs that will be obtained for this EoS will serve as a true upper bound for the baryonic masses NSs in \( R^2 \).

Now let us get into the core of our analysis. The clue point is that NSs with baryons masses larger than \( M_{B_{\text{max}}} \), that is,

\[
M_B > M_{B_{\text{max}}},
\]

(10)

will inevitably collapse to black hole. Thus, in principle, knowing the maximum baryonic mass for a specific EoS and a specific theory can yield a first hint on where to find black holes and what is, for sure, the upper limit of NSs, indirectly though. Let us explain in detail these two syllogisms in some detail, considering firstly the black hole syllogism although the two are inherently related. If one knows the maximum baryonic mass for a specific EoS and theory, this can provide a hint on the lowest mass of astrophysical black holes. Basically, we can indirectly know where to start seeking for the lower mass limit of astrophysical black holes, since \( M_{B_{\text{max}}} \) is an ideal upper limit that the gravitational masses of NSs cannot reach for sure. Thus the lower limit of astrophysical black holes could be \( M_{B_{\text{max}}} \) because it is not possible to find NSs with such large gravitational masses. On the other hand, and in the same line of research, the gravitational masses of NSs can never be as large as the maximum baryonic masses. If one knows the maximum baryonic mass for a specific EoS and theory, this can provide a hint on the lowest mass limit of astrophysical black holes, since \( M_{B_{\text{max}}} \) is an ideal upper limit that the gravitational masses of NSs cannot reach for sure. Thus the lower limit of astrophysical black holes could be \( M_{B_{\text{max}}} \) because it is not possible to find NSs with such large gravitational masses. On the other hand, and in the same line of research, the gravitational masses of NSs can never be as large as the maximum baryonic masses. Thus, in the NSs case, we know where not to find NSs and seek them in quite lower values. In both cases, the analysis would be perfectly supplemented by knowing for a large number of EoSs and a large number of theories, the theoretical universal relation between the baryonic and gravitational NSs masses, as in \([67]\). However this task is quite complicated and it will be addressed in more detail in a future focused work. In this work, we aim to find hints on where to start finding the lowest limit of astrophysical black holes, and also to discover where not to find NSs, thus aiming at providing another theoretical upper bound on static NSs masses. This work could be considered as a theoretical complement of our work on causal EoSs developed in ref. \([22]\). Remarkably, the two results seem to provide a quite interesting result and may lead to an interesting conjecture.

Let us proceed by briefly recalling how to calculate the baryonic mass for a static NS. The central values of the
Table 1: Parameters fo various EOSs. Maximum baryonic mass for stars in $R^2$ gravity for some equation of states and various values of the parameter $\alpha$.

| EoS         | $\alpha$ | $M_{B,\text{max}}^{\text{max}}$ | $M_{\text{max}}$ |
|-------------|----------|----------------------------------|------------------|
|             | 0        | 2.65                             | 2.17             |
| APR         | 0.25     | 2.67                             | 2.18             |
|             | 2.5      | 2.76                             | 2.24             |
|             | 10       | 2.85                             | 2.30             |
| BHF         | 0.25     | 2.49                             | 2.09             |
|             | 2.5      | 2.58                             | 2.15             |
|             | 10       | 2.65                             | 2.21             |
| GM1         | 0.25     | 2.87                             | 2.40             |
|             | 2.5      | 3.01                             | 2.49             |
|             | 10       | 3.11                             | 2.56             |
| QHC18       | 0.25     | 2.44                             | 2.04             |
|             | 2.5      | 2.61                             | 2.15             |
|             | 10       | 2.70                             | 2.22             |
| SLY         | 0.25     | 2.44                             | 2.05             |
|             | 2.5      | 2.54                             | 2.11             |
|             | 10       | 2.63                             | 2.17             |
| SLY +       | 0.25     | 2.98                             | 2.52             |
|             | 2.5      | 3.13                             | 2.63             |

pressure and of the mass of the NS are

$$P(0) = P_c, \ m(0) = 0,$$

and near the center, the pressure and the mass of the NS behave as

$$P(r) \simeq P_c - (2\pi)(\epsilon_c + P_c) \left( P_c + \frac{1}{3} \epsilon_c \right) r^2 + O(r^4),$$

$$m(r) \simeq \frac{4}{3} \pi \epsilon_c r^3 + O(r^4).$$

Considering the spherically symmetric spacetime (3), the gravitational mass of the NS is

$$M = \int_0^R 4\pi r^2 c^2 dr,$$

or equivalently,

$$M = \int_0^R 4\pi r^2 e^{(\psi + \lambda)/2} (\epsilon + 3P) dr,$$

while the baryon mass of the static NS is

$$M_B = \int_0^R 4\pi r^2 e^{\lambda/2} \rho dr.$$

For the numerical calculation, we use a length scale of $M_\odot = 1$, and the results of our numerical analysis are presented in table 1. Specifically, in table 1, we present the maximum baryonic mass, and the maximum gravitational mass for all the EOS we mentioned earlier, for various values of the parameter $\alpha$ which is the coupling of the $R^2$ term in the gravitational action. With regard to the values of the free parameter $\alpha$, it is expressed in units $r_\odot^2 = 4G^2 M_\odot^2 / c^4$, see below eq. (9). Making the correspondence with the standard cosmological $R^2$ model, the small values of $\alpha$ are more compatible with the cosmological scenarios. However, at this point one must be cautious since the cosmological $R^2$ model constraints are usually imposed on the Einstein frame theory. Specifically the constraints on the parameter $\alpha$ come from the amplitude of the scalar curvature primordial perturbations, thus yielding a small $\alpha$. If however the theory is considered in the Jordan frame directly, the expression of the amplitude of the scalar perturbations is different compared to the Einstein frame expression, resulting in different constraints on the parameter $\alpha$. This is why we chose the parameter $\alpha$ to vary in the range $0 < \alpha < 10$, in order to investigate the physics of NSs for a wide range of values in order to cover the constraints from both frames.

Let us discuss the results presented in table 1 in some detail. As a general comment for all the EOSs, the baryonic mass is significantly larger than the maximum gravitational mass, for all the values of the parameter $\alpha$, and this is a general expected result. With regard to the APR EoS, the baryonic mass takes values in the range $2.65 M_\odot - 2.85 M_\odot$, and the maximum gravitational mass in the range $2.17 M_\odot - 2.30 M_\odot$. Thus, for the APR EoS, it is apparent that NSs with baryonic masses larger than $2.85 M_\odot$ at most will collapse into black holes. The value $2.85 M_\odot$ is indicative of the maximum limit of the baryon mass in the context of $R^2$ gravity and the corresponding gravitational mass is $2.30 M_\odot$, which means that astrophysical black holes will be larger than $2.30 M_\odot$.

Thus, for the BHF EoS, it is apparent that NSs with baryonic masses larger than $2.65 M_\odot$ at most will collapse into black holes. The corresponding gravitational mass is $2.21 M_\odot$, which means that astrophysical black holes will be larger than $2.30 M_\odot$, in the case of $R^2$ gravity and for the APR EoS. With regard to the GM1 EoS, the baryonic mass takes values in the range $2.47 M_\odot - 2.65 M_\odot$, and the maximum gravitational mass in the range $2.08 M_\odot - 2.21 M_\odot$. Thus, for the GM1 EoS, it is apparent that NSs with baryonic masses larger than $2.65 M_\odot$ at most will collapse into black holes. The corresponding gravitational mass is $2.21 M_\odot$, which means that astrophysical black holes will be larger than $2.30 M_\odot$ in the case of $R^2$ gravity and for the BHF EoS. With regard to the QHC18 EoS, the baryonic mass takes values in the range $2.84 M_\odot - 3.11 M_\odot$, and the maximum gravitational mass in the range $2.38 M_\odot - 2.56 M_\odot$. Thus, for the QHC18 EoS, it is apparent that NSs with baryonic masses larger than $2.65 M_\odot$ at most will collapse into black holes. The corresponding gravitational mass is $2.21 M_\odot$, which means that astrophysical black holes will be larger than $2.30 M_\odot$ in the case of $R^2$ gravity and for the QHC18 EoS.
larger than $3.11M_\odot$ at most, will collapse to black holes. The corresponding gravitational mass is $2.56M_\odot$, which means that astrophysical black holes will be larger than $2.56M_\odot$ in the case of $R^2$ gravity and for the GM1 EoS. With regard to the QHC18 EoS, the baryonic mass takes values in the range $2.44M_\odot$–$2.70M_\odot$, and the maximum gravitational mass in the range $2.04M_\odot$–$2.22M_\odot$. Thus, for the QHC18 EoS, it is apparent that NSs with baryonic masses larger than $2.7M_\odot$ at most, will collapse to black holes. The corresponding gravitational mass is $2.22M_\odot$, which means that astrophysical black holes will be larger than $2.22M_\odot$ in the case of $R^2$ gravity and for the QHC18 EoS. With regard to the SLy EoS, the baryonic mass takes values in the range $2.44M_\odot$–$2.63M_\odot$, and the maximum gravitational mass in the range $2.05M_\odot$–$2.17M_\odot$. Thus, for the SLy EoS, it is apparent that NSs with baryonic masses larger than $2.63M_\odot$ at most, will collapse to black holes. The corresponding gravitational mass is $2.17M_\odot$, which means that astrophysical black holes will be larger than $2.17M_\odot$ in the case of $R^2$ gravity and for the SLy EoS. Finally, for the theoretical ideal EoS, namely the causal EoS, the baryonic mass takes values in the range $2.98M_\odot$–$3.24M_\odot$, and the maximum gravitational mass in the range $2.52M_\odot$–$2.74M_\odot$. Thus, for the causal EoS, it is apparent that NSs with baryonic masses larger than $3.24M_\odot$ at most, will collapse to black holes. The corresponding gravitational mass is $2.74M_\odot$, which means that astrophysical black holes will be larger than $2.74M_\odot$ in the case of $R^2$ gravity and for the causal EoS.

Our results hold true for the $R^2$ gravity and for the specific EoSs which we studied, thus it is conceivable that these are model dependent. However, it seems that there is a tendency from these data, that NSs masses cannot be larger than 3 solar masses, and this combined with the result of ref. [22], further supports the GR claim about finding NSs with masses not larger than 3 solar masses. Also, astrophysical black holes can be found or created when NSs with baryon masses larger than the corresponding maximum baryon masses collapse to black holes. In general, astrophysical black holes can be found even in the lower limit of the mass gap region, specifically at $2.5M_\odot$–$3M_\odot$, however, our data are model dependent. Thus our claims are somewhat model dependent and it is vital to find a universal relation between the maximum baryon masses and the corresponding maximum gravitational masses in order to be more accurate. The universal baryon-gravitational masses relation for $R^2$ gravity can be obtained using the techniques in ref. [67]. The work is in progress along this research line. This future study will yield a more robust result and may further indicate, in a refined way, where to find the maximum masses of NSs and where the corresponding lower masses of astrophysical black holes.

**Concluding remarks.** – In this work we focused on the calculation of the maximum baryonic mass for static NSs in the context of extended gravity. We specified our analysis for one of the most important extended gravity candidate theories, namely $f(R)$ gravity, and we chose one of the most important models of $f(R)$ gravity, namely the $R^2$ model. We derived the TOV equations for $f(R)$ gravity and numerically integrated these for the following EoSs, the APR4, the BHF, the GM1, the QHC18, the SLy, and finally, the limiting case ideal EoS, called the causal limit EoS. The calculations of the baryonic mass yielded quite interesting results, with a general characteristic being that the maximum baryonic mass was higher than the maximum gravitational mass for the same set of the model’s parameters and the same EoS. The latter characteristic was expected. However the results tend to indicate some interesting features for static neutron stars in the context of $R^2$ gravity. Specifically, the upper limit of all maximum baryon masses for all the EoSs we studied, and for all the values of the model free parameters, seems to be of the order of $\sim3M_\odot$. This feature clearly shows that the static NSs maximum gravitational mass is certainly significantly lower than this limit, thus the maximum gravitational mass of static NSs is expected to be found somewhere in the lower limits of the mass range $2.5M_\odot$–$3M_\odot$. At the same time, one may have hints on where to find the lower mass limit of astrophysical black holes, since NSs have baryonic masses larger than the maximum baryonic mass for the same range of values of the model’s parameters and for the same EoS. Thus one may say that the lower masses of astrophysical black holes may be found in the same range $2.5M_\odot$–$3M_\odot$, which is basically the range where to find the maximum gravitational masses of static NSs. However, our analysis is model dependent and also strongly depends on the underlying EoS. Thus, what is needed is to find a universal and EoS-independent relation for the baryonic and gravitational masses of static NSs, at least for the $R^2$ model at hand. The motivation for this is strong, since we may reach more rigid answers on the questions as to which the maximum NSs masses are and which the minimum astrophysical black holes masses are. This issue will be thoroughly addressed in the near future. As a final comment, let us note that it is remarkable that the magic number of 3 solar masses seems to appear in the context of maximum baryonic masses. Basically for the baryonic masses, the $3M_\odot$ limit is a mass limit that NSs will never reach for sure. Hence this is an ideal number, not a true limit. On the same line of research, the causal EoS maximum gravitational mass for NSs studied in [22] also involves the “magic” number of $3M_\odot$. Thus the GR claim that neutron stars cannot have gravitational masses larger than $3M_\odot$ is also verified from the perspective of baryonic masses calculations. This conclusion is expected to hold true even if the universal and EoS-independent relation between baryonic and gravitational masses for static NSs is found. However, in order to be accurate, one must perform similar calculations for spherical symmetric spacetimes in other modified gravities, like $f(T)$ gravity [77,78] or even Einstein-Gauss-Bonnet gravity [79].

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Finally, let us discuss an interesting question, specifically, whether the analysis we performed would help to break the degeneracies between the NS EoS and modified gravity forms. Indeed this would be the ideal scenario and it is generally not easy to answer. One general answer can be obtained if the following occurs: if a future observation yields a large mass of a static or nearly static NSs, which cannot be explained by a stiff EoS, since the latter is constrained by the GW170817 event. This seems to be the case in the GW190814 event, and this is an upper bound in NSs masses. However, we obtained this result in a model-approxi- mately 3 solar masses, and this is an upper bound in NSs masses. However, we obtained this result in a model- and EoS-dependent way, so we need to extend our analysis in a more universal way. Work is in progress toward this research line.

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