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The development of grade 11 students’ mathematical literacy on sequences and series using mathematical modelling

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Abstract. Mathematical literacy is one of the necessary performances to apply mathematical knowledge and skill in real-life. The objective of this research is to develop grade 11 student’s mathematical literacy through mathematical modelling in the topic of sequences and series. The mathematical modelling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena. The participants are 36 11th grade students along with the classroom action research methodology. The results of this study demonstrate that the competency of mathematical literacy can be enhanced through the mathematical modelling. The research shows the improvement of mathematical literacy revealing in processes of mathematical modelling, which are formulating situations mathematically and interpreting, applying and evaluating mathematical outcomes. However, employing mathematical concepts, facts, procedures, and reasoning does not show noticeable improvement. It is necessary for teachers to provide familiar situations to make more meaningful and tangible lesson for students.

1. Introduction

There are many competencies that citizens of the world should possess in order to survive in the 21st globalization. One of the most important skills is applying knowledge in real-life scenarios. On the other hand, the Organization for Economic Co-operation and Development (OECD) developed the assessment named PISA which was designed to assesses “literacy”, one of the competencies. Literacy in this context is categorized into 3 processes; mathematical literacy, scientific literacy, and readying literacy. Literacy is traditionally meant as the ability to read and write [1]. The modern term's meaning has been expanded to include the ability to use language, numbers, images, computers, and other basic means to understand, to communicate, to gain useful knowledge, to solve mathematical problems and to use the dominant symbol systems of a culture [2]. In this research, we refer to the mathematical literacy that consists of 3 processes; (i) formulating situations mathematically, (ii) employing mathematical concepts, facts, procedures, and (iii) reasoning, interpreting, applying and evaluating mathematical outcomes [3]. There are specific characters to explain each process that shall be delineated in latter section. The result of PISA in Thailand is below the average standards defined by OECD. Mean score of PISA mathematics in 2015 is 490, whereas the mean score in Thailand is 415. Therefore, mathematical literacy in Thailand is critical with the decreasing statistics. In addition, PISA assessment shows that Thai students on average are weak at formulating situations mathematically [4]. This agrees with the results of classroom action research that show some students were not able to formulate mathematical situations and solve
them. The approach that aims to develop mathematical literacy is to support students to use mathematics in real-life situation. With mathematical models, this approach can help students to gain the insight, in obtaining solutions or patterns related to real-life problems.

Mathematical modelling has been defined in many ways. In this research we follow the mathematical modelling definition of GAIMME stating that it is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena [5]. The process consists of 6 components, which are; (i) identifying the problem to be solved, (ii) making assumptions and defining variables, (iii) doing the math to get a solution, (iv) analyzing and assessing the solution, (v) iterating to refine and extend the model, and (vi) implementing the model and reporting the results. In the process, students will develop the models to solve the problem of their interests. Teachers will be facilitators, giving a suggestion or guiding direction to solve the models.

In this research, GAIMME procedure and process of mathematical modelling stated above were employed to use 11th grade students of the school in Phitsanulok province of Thailand in the topic of sequences and series [5].

2. Method
The research was conducted through classroom action research procedure, consisting of 4 cycles that have 4 stages; plan, action, observation, and reflection. It is a qualitative research over lessons of sequences and series topic.

2.1. The participants
The participants were selected based on purposive sampling method. There were 36 11th grade students, 16 boys and 20 girls, from the high school in Phitsanulok province of Thailand. On regular basis, they studied mathematics 6 hours a week; 3 hours in basic mathematics subject and 3 hours in additional mathematics subject. The topic sequences and series was taught in basic mathematics subject. Students never had experience with mathematical modelling process prior to the research implementation. All students have their personal smartphones, computers with internet connection.

2.2. Research instruments
This study has two main instruments, which are lesson plans and evaluation of mathematical literacy. Design of lessons includes four lesson plans; arithmetic sequences, geometric sequences, arithmetic series, and geometric series. Each lesson plan has situation sheets and worksheets. The worksheets were designed with the same template partitioned into group members name and five mathematics questions. The 1st question corresponds to the objective in seeing students’ intention to use data to identify the problem and to formulate the given situation mathematically. The 2nd question asks to translate problem into mathematical representation. Students will be assessed in the ability to find necessary information or assumptions and variables identification. The 3rd question would like to know student’s process of mathematical problem solving. The 4th question gives students the opportunities to investigate the appropriation of their processes, models, and the results. In this question, students can fix or renew the process, model and result until they meet students’ satisfaction and understanding. The 5th question asks students to explain their process and application of the model to other situations—which situations could be applied by using this model.

The evaluation tool to assess the levels of mathematical literacy of students is the form of subjective writing. Students will be assessed after gone through mathematical modelling process. The evaluation form comprises four questions; each has two sub-questions. Serving the assessment regarding to mathematical literacy process, one sub-question intends to assess the ability to formulate situation mathematically and to employ mathematical concepts, facts, procedures. The other one will be designed to assess the reasoning and interpreting, applying and evaluating mathematical outcomes.

The lesson plans together with the corresponding worksheets and evaluation of mathematical literacy were complete in the plan stage of the cycle. They were done for all 4 cycles with different situation coving the topic.
2.3. Data collection Data analysis

The data was collected from students’ work and the evaluation of mathematical literacy after learning four cycles had completed. In the first cycle, the teacher and students worked together. For the next cycles, students worked in groups of 3-4 students and the teacher acted as an advisor, facilitating the class. The implementation was divided into 4 stages; plan stage where the lessons and worksheets were created, action stage for class conduction, observation stage for observing students’ behaviors, and reflection stage for analyzing students’ worksheets and evaluation.

In the action stage, the worksheets were handed out to students with different situations. Data collection was designed to follow 6 components of mathematical modelling processes of GAIMME [5]. The (i) identifying the problem to be solved component is embedded in the 1st question of the worksheet. The (ii) making assumptions and defining variables assumptions component can be interpreted from the answering in 2nd question. The action of mathematics, (iii) doing the math to get a solution, corresponds how the 3rd question is answered. The 4th question combines component (iv) analyzing and assessing the solution and (v) iterating to refine and extend the model. Lastly, the 5th question corresponds the las component, (vi) implementing the model and reporting the results, where students show the use and application of the model. This 5-question worksheet was applied in all 4 cycles.

In the observation stage, the author observed the behaviors of students. Information found was the problems during the process of learning, types of suggestions students needed in order to get working models and to be able to solve them. This stage also satisfies with the role of a teacher in the mathematical modelling process that is meant to guide and to facilitate the class.

In the reflection stage, the measurement of the worksheets were analyzed according to chosen rubrics, score criteria which correspond to the processes of mathematical literacy. When the problem found and reflected, teacher would take as an input to prepare the next cycle. In the next cycle, some teaching and learning problems found in previous cycle were fixed to improve mathematical literacy of students in classes. Upon completing of 4 cycles, the assessment of students’ mathematical literacy in the form of evaluation tool of mathematical literacy was used and students had 1 hours to complete the form. Data and information were then collected and analyzed.

3. Result and Discussion

The results show that mathematical literacy could be developed through mathematical modelling with the 3 processes consisting: (1) formulating situations mathematically: identifying the mathematical aspects of a problem situated in a real-world context and identifying the significant variables, simplifying a situation or problem in order to make it amenable to mathematical analysis, and translating a problem into mathematical language or a representation; (2) employing mathematical concepts, facts, procedures: devising and implementing strategies for finding mathematical solutions; using mathematical tools, including technology, to help find exact or approximate solutions; applying mathematical facts, rules, algorithms, and structures when finding solutions; (3) reasoning, interpreting, applying and evaluating mathematical outcomes: interpreting a mathematical result back into the real-world context, evaluating the reasonableness of a mathematical solution in the context of a real-world problem, and explaining why a mathematical result or conclusion does, or does not, make sense given the context of a problem. The formulating situations mathematically and reasoning, interpreting, applying and evaluating mathematical outcomes show through students’ worksheets and evaluation. However, the ability of employing mathematical concepts is not noticeably presented. The details of the processes that showed the improvement are described as follows.

3.1. Formulating situations mathematically

Formulating situations mathematically is the first process of mathematical modelling. It contains 3 activities to describe student’s behaviors, indicating that this process has been performed. The 3 activities were denoted by: A represents identifying the mathematical aspects of a problem situated in a real-world context and identifying the significant variables, B represents simplifying a situation or problem to make it amenable to mathematical analysis, and C represents translating a problem into mathematical language or a representation. Since students were arranged to work in a group of 3-4 members, there were 10 groups in the class. And the results shows in Table 1.
Table 1. The numbers of groups performing the process of formulating situations mathematically in any activities of 4 cycles

| Points | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 |
|--------|---------|---------|---------|---------|
| Activities | Number of groups | Number of groups | Number of groups | Number of groups |
| A      | 3       | 7       | 1       | 9       | 1       | 9       | 10      |
| B      | 3       | 6       | 1       | 3       | 7       | 2       | 7       | 1       | 1       | 5       | 4       |
| C      | 2       | 8       | 1       | 3       | 6       | 1       | 7       | 2       | 1       | 4       | 5       |
| total  | 3       | 11      | 4       | 16      | 1       | 15      | 4       | 14      | 12      | 2       | 9       | 19      |

Table 1 shows the score ranged from 1 to 3 points and number of student groups that performed the activities A, B or C according to the scores each group received. In cycle 2 the first activity A, identifying the mathematical aspects of a problem situated in a real-world context and identifying the significant variables, 9 group of students got 3 points and in the last cycle all 10 groups got full 3 points. Next, the second activity B, most students got 2 points. And, in the last cycle, there were 4 groups that got 3 points. For the activity C, there were some uncertainty on the performance of students.

3.2. Employing mathematical concepts, facts, procedures

In the second process, it contains 3 activities, which are: A represents devising and implementing strategies for finding mathematical solutions, B represents using mathematical tools, including technology, to help find exact or approximate solutions, and C represents applying mathematical facts, rules, algorithms, and structures when finding solutions.

Table 2. The numbers of groups performing the process of employing mathematical concepts, facts, procedures in any activities of 4 cycles

| Points Activities | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 |
|-------------------|---------|---------|---------|---------|
|                   | Number of groups | Number of groups | Number of groups | Number of groups |
| A                 | 10      | 1       | 1       | 8       | 1       | 1       | 8       | 1       | 9       |
| B                 | 10      | 1       | 9       | 1       | 9       | 1       | 9       | 1       | 9       |
| C                 | 1       | 9       | 1       | 8       | 1       | 9       | 1       | 9       | 1       |
| Total             | 1       | 9       | 1       | 8       | 1       | 9       | 1       | 9       | 1       |

Table 2 demonstrates that the process of employing mathematical concepts, facts. It is a procedure where students began to develop their critical thinking, showing how to employ mathematical concepts to solve the problems. From the data collected in this study, it shows a little improvement across the 4 cycles. From the mathematical literacy assessment report show that the competency of students employing mathematical concepts, facts, procedures in Thailand get high score [3]. Consequently, the development of this competency could not say to be improved.

3.3. Reasoning and interpreting, applying and evaluating mathematical outcomes

For the last process, it contains 3 activities: A represents interpreting a mathematical result back into the real-world context, B represents evaluating the reasonableness of a mathematical solution in the context of a real-world problem, and C represents explaining why a mathematical result or conclusion does, or does not, make sense given the context of a problem. Students were arranged into 10 groups. Table 3 shows the results of this process and score (1-3 points) according to the rubric that assessed how well students performing the process. In the last process, Table 3. shows that the development of this process was not quite improved. Most of student groups got 1 point across the 3 activities.
First and foremost, we would like to thank the Faculty of Education, Naresuan University and the Institute for the Promotion of Teaching Science and Technology (IPST) for support with founding and the observation and reflection steps are important to really train students to get acquainted in mathematical modelling and gain mathematical literacy from the activities. The process of reasoning, interpreting, applying and evaluating mathematical outcomes in any activities of 4 cycles:

| Activities | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 |
|------------|---------|---------|---------|---------|
| A          | 2       | 8       | 9       | 1       | 8       | 1       | 8       | 1       | 1 |
| B          | 1       | 8       | 1       | 9       | 1       | 7       | 3       | 8       | 1 |
| C          | 7       | 3       | 9       | 1       | 9       | 1       | 10      |         |   |
| total      | 10      | 19      | 27      | 2       | 1       | 24      | 5       | 1       | 26 |

3.4 The result of the evaluation of mathematical literacy

The evaluation form comprises four questions; each has two sub-questions. Each contain 2 sub-questions. The first sub-question is intended to assess process (i), formulating situations mathematically, and process (ii), employing mathematical concepts, facts, procedures was represented. The second sub-question is intended to assess process (iii), reasoning, interpreting, applying and evaluating mathematical outcomes. The questions were ordered based on difficulties and familiarity from obvious or easy to hard or new to students' experience. The familiar question to unseen question and also the difficulty respectively. This evaluation form was used to assess 36 students individually. The results of 4 questions corresponding to the number of students receiving scores (ranged from 0-2 in the first sub-question and 0-1 in the second sub-question) are shown in Table 4.

| Question 1 | Question 2 | Question 3 | Question 4 |
|------------|------------|------------|------------|
| points     | 0          | 1          | 2          | 0          | 1          | 2          | 0          | 1 |
| Process (i)| 3          | 29         | 4          | 3          | 24         | 9          | 25         | 7          | 4 |
| Process (ii)| 3         | 29         | 4          | 3          | 24         | 9          | 25         | 7          | 4 |
| points     | 0          | 1          | 0          | 1          | 0          | 1          | 0          | 1 |
| Process (iii)| 2       | 34         | 17         | 19         | 28         | 8          | 27         | 9 |

Table 4 shows that students were able to answer Question 1 and 2 for which they have seen and been familiar before in classes. However, for Question 3 and 4, most students got 0 point. This could imply that the mathematical modelling processes required time to develop. Then, 12-hour lesson may not be sufficient to make students perceive the processes with true understanding and integrating into their habits of thinking. Each sub-question that has embedded the assessment of the ability to understand and perform process (i), (ii) and (iii) of mathematical modelling could be used to confirm the development of mathematical literacy for each individual.

4. Conclusion

Mathematical modelling can be the means to encourage students to build their critical thinking and perform problem solving in real-life situations. The process of formulating situations mathematically was enhanced significantly and better than other processes in mathematical modelling according to GAIMME procedure [5]. By achieving that, students have to understand the situations, be able to identify the problems and hence to make assumptions and variables. Later, they can apply mathematical procedure to solve the models. In addition, students could interpret the model back to the real-world phenomena. Therefore, it is important to select the situations close to students’ experience. Teachers would have to collaborate with students and collect feedbacks from all students. The observation and reflection steps are important to really train students to get acquainted in mathematical modelling and gain mathematical literacy from the activities.

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