Stability of vortex in a two-component superconductor

Jun-Ping Wang

Department of Physics, Yantai University, Yantai 264005, P. R. China

Abstract

Thermodynamic stability of composite vortex in a two-component superconductor is investigated by the Ginzburg-Landau theory. The predicted nature of these vortices has recently attracted much attention. Here we consider axially symmetric quantized vortex and show that the stability of vortex depends on three independent dimensionless parameters: \( \kappa_1, \kappa_2, \kappa_\xi \), where \( \kappa_i (i = 1, 2) \) is the Ginzburg-Landau parameter of individual component, \( \kappa_\xi = \xi_1/\xi_2 \) is the ratio of two coherence lengths. We also show that there exists thermodynamically stable vortex in type-1+type-2 or type-2+type-2 materials over a range of these three parameters.

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The existence of quantized vortex in a type-2 superconductor (SC) is one of the most striking phenomena in condensed matter physics \[1\]. The criterion for stability of vortex in a conventional SC is the Ginzburg-Landau parameter $\kappa$, which is defined as the ratio of penetration depth to coherence length: $\kappa = \lambda/\xi$ \[2\]. Vortex can exist as a thermodynamically favorable state under external field in a type-2 material with $\kappa > 1/\sqrt{2}$, while the penetration of vortex is not thermodynamically favorable in a type-1 SC with $\kappa < 1/\sqrt{2}$.

Recently, there has been growing interest in investigating the vortex in multi-component SCs \[3–9\]. The predicted nature of vortex in these materials is quite different from that of vortex in a conventional type-2 SC. Babaev and Speight showed that interaction potential between two vortices in a two-component SC can be non-monotonic: intervortex force is attractive at long range and repulsive at short range \[5\]. The key question regarding Babaev and Speight’s work is whether vortices in a two-component SC are thermodynamically stable or not. In this paper, we revisit this issue addressed in Ref. \[5\]. The idea is that, vortex can survive as a thermodynamically favorable state if the Gibbs energy of the vortex state under the thermodynamic critical field is smaller than that of the fully superconducting state (Meissner state). We find that, the stability of vortex depends on three independent dimensionless parameters: $\kappa_1, \kappa_2, \kappa_\xi$, where $\kappa_i (i = 1, 2)$ is the Ginzburg-Landau parameter of individual component, $\kappa_\xi = \xi_1/\xi_2$ is the ratio of two coherence lengths. We also find that vortex is thermodynamically stable in a type-1+type-2 or type-2+type-2 SC over a range of these three parameters.

Based on the Ginzburg-Landau model, we use free energy density in the two-component SC as follows:

$$f = f_{n0} + \sum_{i=1}^{2} \frac{\hbar^2}{2m_i}(|\nabla - \frac{ie_i^*}{\hbar c} A|^2 + V(|\Psi_i|^2) + \eta(\Psi_i^*\Psi_2 + \Psi_1^*\Psi_2^*) + \frac{1}{8\pi} (\nabla \times A)^2, \quad (1)$$

where $f_{n0}$ is the free energy density of the body in the normal state in the absence of the magnetic field, $V(|\Psi_i|^2) = a_i|\Psi_i|^2 + b_i|\Psi_i|^4/2$ ($i = 1, 2$). $\eta$ is a coefficient characterizes Josephson coupling between two superconducting components. In the following we consider in particular weak Josephson coupling limit and set $\eta = 0$. We also assume that the effective mass $m_i^*$ and charge $e_i^*$ of two components are equal: $m_i^* = m^*$, $e_i^* = e^*$. There are four characteristic lengths: the penetration depth $\lambda_i$ and coherence length $\xi_i$ for each component are given by: $\lambda_i = (m^*c^2/4\pi e^2\Psi_i^0)^{1/2}$, $\xi_i = \hbar/(2m^*|a_i|)^{1/2}$, where $\Psi_i^0 = (-a_i/b_i)^{1/2}$. The thermodynamic critical magnetic field of the
individual component is $H_{ct(i)} = \Phi_0 / (2 \sqrt{2\pi} \lambda_i \xi_i)$, where $\Phi_0 = hc/e^* \pi$ is the flux quantum. The magnetic field penetration depth and the thermodynamic critical magnetic field of the system are:

\[
\lambda = \left(1/\lambda_1^2 + 1/\lambda_2^2\right)^{-1/2}, \quad H_{ct} = (H_{ct(1)}^2 + H_{ct(2)}^2)^{1/2}.
\]

Note that $\lambda < \min(\lambda_1, \lambda_2)$, $H_{ct} > \max(H_{ct(1)}, H_{ct(2)})$.

We consider axially symmetric quantized vortex in the model (1):

\[
\Psi_1 = |\Psi_1| e^{i\theta}, \quad \Psi_2 = |\Psi_2| e^{i\theta}, \quad A = A(r)e_y.
\]

In order to study the stability of this vortex, we consider the Gibbs energy difference between the vortex state under the thermodynamic critical field $H_{ct}$ and the fully superconducting state (Meissner state, or fully normal state under the thermodynamic critical field since these must be equal)

\[
\Delta G = G_{vortex}(H_{ct}) - G_M = G_{vortex}(H_{ct}) - G_n(H_{ct}).
\]

Let us show that if $\Delta G < 0$ then the isolated vortex can appear as a thermodynamically favorable state under external field $H < H_{ct}$. There are three possible states: fully superconducting state (Meissner state), vortex state and fully normal state. And we note both the Gibbs energies of vortex state and normal state are decreasing with increasing field. If $\Delta G > 0$, the normal state becomes energetically favorable when the field exceeds the thermodynamic critical value and vortex state is energetically unfavorable. If $\Delta G < 0$, vortex state becomes favorable under certain value of the external field which is smaller than the thermodynamic critical field $H < H_{ct}$.

The Gibbs energies of the vortex state and the Meissner state can be written as

\[
G_{vortex}(H_{ct}) = \int d\mathbf{r}g_v, \quad G_M = G_n(H_{ct}) = \int d\mathbf{r}g_M.
\]

Here $g_v = f_v - H_{ct} \cdot (\nabla \times \mathbf{A})/4\pi, g_M = f_M - H_{ct}^2/8\pi$ are the Gibbs energy densities of the vortex state and the Meissner state, respectively. To investigate the vortex stability in the considered case, one can write down

\[
\Delta G = \int d\mathbf{r} \left\{ \sum_{i=1}^{2} \frac{\hbar^2}{2m_i} \left[ (\nabla - \frac{ie^*}{\hbar c} \mathbf{A}) \Psi_i \right]^2 + V(|\Psi_{1,2}|^2) + \frac{1}{8\pi} (H_{ct} - \nabla \times \mathbf{A})^2 \right\}.
\]

We shall use instead of the variable $r$, the functions $\Psi_i$ and $\mathbf{A}$ the dimensionless quantities
\[ \rho = \frac{r}{\lambda}, \quad \psi_1 = \frac{|\Psi_1|}{\Psi_{10}}, \quad \psi_2 = \frac{|\Psi_2|}{\Psi_{20}}, \quad A = \frac{|A|}{H_c / \lambda}. \]  \hfill (6)

In the following we calculate \( \Delta G \) and find

\[ \Delta G = \frac{H_c^2 \lambda^2}{4} \int_0^\infty \rho d\rho \left\{ \sum_{i=1}^2 \frac{C_i}{B_i} (\frac{d^2 \psi_i}{d\rho^2})^2 + (\sqrt{B_i} \psi_i - \sqrt{2A_i} \frac{\psi_i}{\rho})^2 - 2\psi_i^2 + \psi_i^4 + \left( 1 - \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A) \right)^2 \right\}. \] \hfill (7)

Here \( A_1 = 1/\kappa_1^2 + \kappa_2^2/\kappa_1^2 \), \( B_1 = (\kappa_2^2 + \kappa_1^2 \kappa_2^2)/(\kappa_2^2 + \kappa_1^2 \kappa_2^2) \), \( C_1 = \kappa_1^2/(\kappa_2^2 + \kappa_1^2 \kappa_2^2) \), \( A_2 = 1/\kappa_2^2 + 1/\kappa_1^2 \kappa_2^2 \), \( B_2 = (\kappa_2^2 + \kappa_1^2 \kappa_2^2)/(\kappa_2^2 + \kappa_1^2 \kappa_2^2) \), \( C_2 = \kappa_1^2 \kappa_2^2/(\kappa_2^2 + \kappa_1^2 \kappa_2^2) \), \( \kappa_i = \lambda_i / \xi_i (i = 1, 2) \) is the Ginzburg-Landau parameter of individual component, \( \kappa_\xi = \xi_1 / \xi_2 \) is the ratio of two coherence lengths. The Ginzburg-Landau equations which determine the profile of the vortex solution are determined by minimizing the \( \Delta G \) with respect to functions \( \psi_i(i = 1, 2) \) and \( A \)

\[ A_1 (\psi_1'' + \frac{\psi_1'}{\rho} - \frac{\psi_1}{\rho^2}) + \sqrt{2A_1 B_1} \psi_1 A - \frac{1}{2} B_1 A^2 \psi_1 + \psi_1 - \psi_1^3 = 0, \]

\[ A_2 (\psi_2'' + \frac{\psi_2'}{\rho} - \frac{\psi_2}{\rho^2}) + \sqrt{2A_2 B_2} \psi_2 A - \frac{1}{2} B_2 A^2 \psi_2 + \psi_2 - \psi_2^3 = 0, \]

\[ A'' + \frac{A'}{\rho} - \frac{A}{\rho^2} = (C_1 \psi_1^2 + C_2 \psi_2^2) A - (C_1 \sqrt{2A_1 B_1} \frac{\psi_1}{\rho}) + C_2 \sqrt{2A_2 B_2} \psi_2^2. \] \hfill (8)

The order parameters vanish in the vortex core which is the phase singularity of both components. And it can be expected that the potential \( A \) correlates linearly with \( \rho \) in the core region due to the constant value of the field inside the core. Then the boundary conditions at \( \rho = 0 \) are

\[ \psi_1(0) = \psi_2(0) = A(0) = 0. \] \hfill (9)

We assume the following power series solutions to Eqs. (8) in the region of the core: \( \psi_1(\rho) = \sum_{n=0}^\infty b_n \rho^n \), \( \psi_2(\rho) = \sum_{n=0}^\infty c_n \rho^n \), \( A = \sum_{n=0}^\infty a_n \rho^n \) and can prove that

\[ \psi_1 = b_1 \rho - \frac{1}{8A_1} (\sqrt{2A_1 B_1} a_1 + 1) b_1 \rho^3 + O(\rho^5), \quad \psi_2 = c_1 \rho - \frac{1}{8A_2} (\sqrt{2A_2 B_2} a_1 + 1) c_1 \rho^3 + O(\rho^5), \]

\[ A = a_1 \rho - \frac{1}{8} (C_1 \sqrt{2A_1 B_1} b_1 + C_2 \sqrt{2A_2 B_2} c_1^2) \rho^3 + O(\rho^5). \] \hfill (10)

Here \( a_1, b_1, c_1 \) are three constants which will be deduced from the solutions far from the vortex core [10]:
FIG. 1: Stable vortex solutions in a two-component SC. (a) is a vortex solution in a type-1+type-2 SC, while (b) and (c) are vortex solutions in the type-2+type-2 SCs. Note the similarity of the vortex configurations in (a) and (b).

TABLE I: Stable vortex solutions in a two-component SC

| $\kappa_1$ | $\kappa_2$ | $\kappa_\xi$ | $b_1$    | $c_1$    | $a_1$    | $\Delta G/(H^2_{c1}\lambda^2/4)$ |
|-----------|-----------|-----------|---------|---------|---------|----------------------------------|
| 3         | 0.5       | 1/12      | 1.7253  | 0.4111  | 0.2830  | -0.9044                         |
| 3         | 3         | 1/6       | 1.8973  | 0.5843  | 0.2762  | -0.5721                         |
| 3         | 3         | 0.7       | 1.6275  | 1.2383  | 0.3425  | -1.1159                         |

$\psi_1(\infty) = \psi_2(\infty) = 1$, $B(\infty) = \left(\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A)\right)|_{\infty} = 0$, \hspace{1cm} (11)

where $B = (1/\rho)\partial(A\rho)/\partial \rho$ is the scaled magnetic field: $B = |\nabla \times A|/H_{c1}$. It is clear from (7), (8), (9) and (11) that the sign of $\Delta G$, and the stability of vortex in the model are determined by the three independent dimensionless parameters: $\kappa_1, \kappa_2, \kappa_\xi$.

We have numerically solved Ginzburg-Landau eqs. (8) with boundary conditions (10), (11) and identified thermodynamically stable vortex solutions in the type-1+type-2 and type-2+type-2 SCs, as predicted previously by the surface energy calculations [11]. Figure 1 illustrates several examples of stable vortex solutions. We found that the vortex stability in a two-component SC depends not only on the Ginzburg-Landau parameter of individual component, but also on the third parameter $\kappa_\xi = \xi_1/\xi_2$.

Here we want to relate works in Ref. [5] to the results of the present work. In their paper, Babaev and Speight identified three characteristic lengths in the model (1): penetration depth $\lambda$
FIG. 2: Examples of stable vortex solutions with $\xi_1/\lambda = \sqrt{2}/3$ and $\xi_2/\lambda = 4\sqrt{2}$. (a) is a vortex solution in a type-1+type-2 SC, while (b) and (c) are vortex solutions in the type-2+type-2 SCs. Detailed parameters can be found in the table 2 below.

TABLE II: Stable vortex solutions with $\xi_1/\lambda = \sqrt{2}/3$ and $\xi_2/\lambda = 4\sqrt{2}$

| $\kappa_1$ | $\kappa_2$ | $\kappa_\xi$ | $\xi_1/\lambda$ | $\xi_2/\lambda$ | $b_1$ | $c_1$ | $a_1$ | $\Delta G/(H_c^2\lambda^2/4)$ |
|------|------|------|----------------|----------------|------|------|------|-------------------|
| 2.3  | 0.4574 | 1/12 | $\sqrt{2}/3$ | $4\sqrt{2}$  | 1.4096 | 0.3753 | 0.3161 | -0.9122          |
| 2.15 | 1.0859 | 1/12 | $\sqrt{2}/3$ | $4\sqrt{2}$  | 1.4194 | 0.3813 | 0.3197 | -0.6696          |
| 2.13 | 1.9602 | 1/12 | $\sqrt{2}/3$ | $4\sqrt{2}$  | 1.4208 | 0.3821 | 0.3203 | -0.6387          |

and coherence lengths of two components $\xi_1, \xi_2$. They presented an example of vortex solution with $\xi_1/\lambda = \sqrt{2}/3, \xi_2/\lambda = 4\sqrt{2}$. Our results reveal that the intrinsic parameters which determine the magnetic properties of a two-component SC are three independent dimensionless parameters: $\kappa_1, \kappa_2, \kappa_\xi$. It is easy to verify that the values of these three parameters are not unique for given $\xi_1/\lambda$ and $\xi_2/\lambda$. This means that, the vortex solution presented in Ref. [5] is not the unique solution. In figure 2 we present several examples in which $\kappa_1, \kappa_2$ are different while $\xi_1/\lambda = \sqrt{2}/3, \xi_2/\lambda = 4\sqrt{2}$. It is clear from table 2 that the slopes of $\psi_1, \psi_2$ and $A$ near the center of vortex core are slightly different. This proved that these vortex solutions are different solutions of Ginzburg-Landau eqs. (8). In particular, there are cases in which two component are both of type-2 ((b) and (c) in the figure 2), which were not mentioned in Ref. [5].

A notable aspect of our results is that in a type-1+type-2 SC, stable vortex solution always has a extended core associated with the type-1 component (see fig.1(a),fig.2(a)). It is pointed that non-monotonic interaction between vortices originates from this exceptional vortex configuration.
However, in a type-2+type-2 SC, the situation is much more complicated. We found that, stable vortex solutions in a type-2+type-2 SC may have a extended core (whose range is much larger than that of penetration depth of the system, see fig. 1(b),fig.2(b),2(c)), or may have contracting core (whose range is smaller than the penetration depth, see fig. 1(c)). This means that, in a type-2+type-2 material, intervortex fore may be attractive at long range and repulsive at short range, as that of vortices in a type-1+type-2 SC. Alternatively, intervortex force in a type-2+type-2 SC may be repulsive at all range, as that of vortices in a conventional type-2 SC.

In conclusion, we have identified the intrinsic parameters which determine the stability of vortex in a two-component SC. Isolated vortex can appear as a thermodynamically stable state in a type-1+type-2 or type-2+type-2 SC over a range of these three parameters: $\kappa_1, \kappa_2, \kappa_\xi$.

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