A Modified Proximity Approach in the Fusion of Heavy Ions

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By using a suitable set of the surface energy coefficient, nuclear radius, and universal function, the original proximity potential 1977 is modified. The overestimate of the data by 4% reported in the literature is significantly reduced. Our modified proximity potential reproduces the experimental data nicely compared to its older versions.

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Recently, great theoretical and experimental efforts have been taken to study the fusion of heavy nuclei leading to several new phenomena including the understanding of the formation of neutron-rich and super heavy elements. The precise knowledge of the interaction potential between two nuclei is a difficult task, and continuing efforts are needed in this direction. This problem has been of very active research over the last three decades and remains one of the most widely studied subject in low-energy heavy-ion physics.\(^{ [1-10]} \)

The total interaction potential is sum of the long range Coulomb repulsive force and short range nuclear attractive force. The Coulomb part of the interaction potential is well known, whereas nuclear part is not clearly understood. A large number of efforts have been made to give simple and accurate forms of the nuclear interaction potentials.\(^{ [1-10]} \) Among such efforts, proximity potential is well known for its simplicity and numerous applications. Based upon the proximity force theorem,\(^{ [4,5]} \) a simple formula for ion-ion interaction potential as a function of the separation between the surfaces of two approaching nuclei was presented\(^ { [4,5]} \).

As pointed out by many authors,\(^ { [7]} \) the original form of the proximity potential 1977 overestimates the experimental data by 4% for fusion barrier heights. In a recent study involving the comparison of 16 proximity potentials, we pointed out that proximity potential 1977 overestimates the experimental data by 6.7% for symmetric colliding nuclei.\(^{ [1]} \)\(^{ [1]} \) Similar results were obtained for asymmetric colliding nuclei.\(^ { [1]} \)

With the passage of time, several improvement modifications were made over the original proximity potential 1977 to remove the gray part of the potential. It includes either the better form of the surface energy coefficient\(^ { [4]} \) or the universal function and/or nuclear radius.\(^ { [7]} \) A careful look reveals that these modifications/improvements are not able to explain the experimental data.\(^ { [1,4]} \) A deep survey also pointed out that these technical parameters (i.e. surface energy coefficient, nuclear radius, and universal function) were chosen quite arbitrarily in the literature. Among them, the surface energy coefficient is available in a large variety of forms from time to time.\(^ { [1,4]} \)

It affects the fusion barrier heights and cross sections significantly.\(^ { [1,2]} \) Also, nuclear radius is available in large variety of forms.\(^ { [1,2]} \) These forms vary either in terms of its coefficients or either different mass or isospin dependence. The third technical parameter, i.e., the universal function, is also parametrized in different forms.\(^ { [1,4,5,7]} \) Unfortunately, no systematic study is available in the literature, where one can explore the role of these technical parameters in fusion barrier positions, heights, and cross sections. Alternatively, a best set of the above-mentioned parameters is still missing.

In the present study, our aim is to modify the original proximity potential 1977 by using a suitable set of the above-stated technical parameters available in the literature. The present systematic study includes the reactions with combine mass between \( A = 19 \) and \( A = 294 \) units. In total, 390 experimentally studied reactions with symmetric as well as asymmetric colliding partners are taken into consideration.

The total ion-ion interaction potential \( V_T(r) \) between two colliding nuclei with charges \( Z_1 \) and \( Z_2 \), center separation \( r \), and density distribution assumed spherical, and frozen, is approximated as\(^ [7] \)

\[
V_T(r) = V_N(r) + \frac{Z_1 Z_2 e^2}{r},
\]

where \( e \) is the charge unit. The nuclear part of the potential \( V_N(r) \) is calculated in the framework of the proximity potential 1977\(^ [4] \) as

\[
V_N(r) = 4\pi \tilde{R} \gamma b^2 \Phi \left( \frac{r - C_1 - C_2}{b} \right) \text{MeV},
\]

where \( \tilde{R} = \frac{C_1 C_2}{C_1^2 + C_2^2} \) is the reduced radius. Here \( C_i \) denotes the matter radius and is calculated using relation\(^ [7] \)

\[
C_i = c_i + \frac{N_i}{A_i} t_i, \quad (i = 1, 2),
\]

where \( c_i \) denotes the half-density radii of the charge distribution and \( t_i \) is the neutron skin of the nucleus. To calculate \( c_i \), we used the relation given in Ref.\(^ [7] \)
\[
\Phi(\xi) = \begin{cases}
-\frac{1}{2}(\xi - 2.54)^2 - 0.0852(\xi - 2.54)^3, & \text{for } \xi \leq 1.2511, \\
-3.437 \exp \left(-\frac{\xi}{0.75}\right), & \text{for } \xi \geq 1.2511,
\end{cases}
\]

with \(\xi = (r - C_1 - C_2)/b\). The surface width \(b\) (i.e. \(b = 3.7758a\) with \(a = 0.55\) fm) has been evaluated close to unity. We labeled this universal function as \(\Phi\)-1977.

Later on, Blocki \textit{et al.} \cite{5} modified the above form as
\[
\Phi(\xi) = \begin{cases}
-1.7817 + 0.9270\xi + 0.1433^2 - 0.09\xi^3, & \text{for } \xi \leq 0.0, \\
-1.7817 + 0.9270\xi + 0.01696\xi^2 - 0.05148\xi^3, & \text{for } 0.0 \leq \xi \leq 1.9475, \\
-4.41\exp\left(-\frac{\xi}{0.77}\right), & \text{for } \xi \geq 1.9475.
\end{cases}
\]

The surface energy coefficient \(\gamma\) was taken from the work of Myers and Świątek\cite{12} and has the form
\[
\gamma = \gamma_0 \left[1 - k_s \left(\frac{N - Z}{A}\right)^2\right],
\]

where \(N\) and \(Z\) refer to the total neutrons and protons content, \(\gamma_0\) is the surface energy constant and \(k_s\) is the surface-asymmetry constant. Both constants were first parameterized by Myers and Świątek\cite{12} by fitting the experimental binding energies. The first set of these constants yielded values \(\gamma_0\) and \(k_s\) to be 1.01734 MeV/fm\(^2\) and 1.79, respectively. In original proximity version, \(\gamma_0\) and \(k_s\) were taken to be 0.9517 MeV/fm\(^2\) and 1.7826,\cite{13} respectively. Later on, these values were revised in a large variety of forms depending upon the advancement in the theory as well in experiments.\cite{1,2}

In total, 14 such surface energy coefficients are highlighted in Ref.\cite{2} and the role of extreme 4 sets is analyzed deeply. Out of them, two best sets of surface energy coefficients are stressed. In the present study, we shall restrict to the latest set of \(\gamma\) values, i.e. \(\gamma_0 = 1.25284\) MeV/fm\(^2\) and \(k_s = 2.345\) presented in Ref.\cite{2}. This particular set of values were obtained directly from a least-squares adjustment to the ground-state masses of 1654 nuclei ranging from \(^{16}\)O to \(^{263}\)106 and fission-barrier heights.\cite{4,14}

The universal function \(\Phi(\frac{r - C_1 - C_2}{b})\) used in Eq.\,(1) has been derived by several authors in different forms.\cite{4,5,7} In original proximity potential, \(\Phi(\frac{r - C_1 - C_2}{b})\) was parametrized in the cubic-exponential form\cite{4}

\[
\Phi(\xi) = \begin{cases}
-\frac{1}{2}(\xi - 2.54)^2 - 0.0852(\xi - 2.54)^3, & \text{for } \xi \leq 1.2511, \\
-3.437 \exp\left(-\frac{\xi}{0.75}\right), & \text{for } \xi \geq 1.2511,
\end{cases}
\]

with \(\xi = (r - C_1 - C_2)/b\). The surface width \(b\) (i.e. \(b = 3.7758a\) with \(a = 0.55\) fm) has been evaluated close to unity. We labeled this universal function as \(\Phi\)-1977.

The universal function \(\Phi(\xi)\) is calculated according for \(\Phi\)-1977 and its recently modified form proximity potential 2000.\cite{7} We labeled them as \(\Phi\) Prox 1977 and Prox 2000, respectively.

In the present study, we use this form of universal function and marked it as \(\Phi\)-1981. By using the above stated parameters, we construct a new proximity potential and labeled as Prox 2010. Along with the above modified form, we shall also use the original proximity potential 1977\cite{4} and its recently modified form proximity potential 2000.\cite{7} We labeled them as Prox 1977 and Prox 2000, respectively.

By using the above new version of the proximity potential (Prox 2010) along with its older versions (i.e. Prox 1977 and Prox 2000), fusion barriers are calculated for 390 reactions by using the conditions:

\[
\frac{dV_T(r)}{dr}|_{r=R_B} = 0, \quad \frac{d^2V_T(r)}{dr^2}|_{r=R_B} \leq 0.
\]

The height of the barrier and position is marked, respectively, as \(V_B\) and \(R_B\).

As one can see from the preceding section, three factors govern the success of proximity potential are (i) the surface energy coefficient, (ii) the universal function, and (iii) nuclear radius. We analyzed the literature very carefully and found that the latest information on these three factors can shape the new proximity potential. Recently, the role of surface energy coefficient stated above is studied in detail in Ref.\cite{2}. As far as radius is concern, we shall restrict to its latest form given in Ref.\cite{11}. However, the role of the third parameter, i.e., the universal function in fusion barriers, is analyzed in Fig.\,1. Here we display \(\Delta V_B(\%)\) and \(\Delta R_B(\%)\) defined as

\[
\Delta V_B(\%) = \frac{V_B^\text{theor} - V_B^\text{expt}}{V_B^\text{expt}} \times 100,
\]

and

\[
\Delta R_B(\%) = \frac{R_B^\text{theor} - R_B^\text{expt}}{R_B^\text{expt}} \times 100,
\]

as a function of \(Z_1Z_2\) using two sets of the above-mentioned universal functions [Eqs.\,(7) and (8)]. It is clear from the figure that deviations are significantly reduced by using \(\Phi\)-1981 compared to its original form \(\Phi\)-1977. The universal function \(\Phi\)-1981 reduces the average deviation over 390 reactions by 1% for fusion barriers.

In Fig.\,2, we display the theoretical fusion barrier heights \(V_B^\text{theor}\) (MeV) and positions \(R_B^\text{theor}\) (fm) versus the corresponding experimental values. We note from the figure that Prox 2010 potential reproduces the experimental fusion barrier heights within 1.4%. This result is in close agreement with other recently
parametrized potentials presented in Ref. [1]. However, the original form of the proximity potential 1977 presented in Ref. [1] overestimates the data by 6.7% for symmetric colliding nuclei. However, the fusion barrier positions show some scattering from the central line (marked by shaded area). This scattering may be due to the variation in the experimental setups and theoretical method one used to extract these values.\(^{1,5,10}\)

\[
\Delta V_B (\%) = \frac{V_B^{\text{theor}} - V_B^{\text{expt}}}{V_B^{\text{expt}}} \\
\Delta R_B (\%) = \frac{R_B^{\text{theor}} - R_B^{\text{expt}}}{R_B^{\text{expt}}}
\]

Fig. 1. The percentage deviation $\Delta V_B (\%)$ and $\Delta R_B (\%)$ as a function of $Z_1Z_2$ using two different sets of universal functions [Eqs. (7) and (8)] implemented in the original proximity potential Prox 1977. The experimental values are taken from Refs. [1,2,7].

Fig. 2. The fusion barrier heights $V_B$ (MeV) and positions barriers $R_B$ (fm) as a function of the corresponding experimental values using our modified proximity potentials Prox 2010. The experimental values are taken from Refs. [1,2,7].

We quantify our outcome in Figs. 3 and 4. In Fig. 3, the percentage deviations between the theoretical and experimental values are presented. The original proximity potential 1977 (Prox 1977) along with its recently modified form (Prox 2000) are also displayed. We note from the upper panel of Fig. 3 that Prox 2010 potential on average gives better results compared to its older versions for fusion barrier heights. However, slight deviations are visible for fusion barrier positions. This may be due to the fact that in the proximity potential Prox 2010, we use the value of surface energy coefficient that gives stronger attraction compared to the one used in Prox 1977 and Prox 2000 potentials. Therefore, in Prox 2010 potential, the counterbalance between the repulsive Coulomb and attractive nuclear part of the interaction potential occurs at larger distances, and hence it pushes the barrier outwards. The fusion barrier heights are reproduced within $\pm 5\%$ on average. On the other hand, fusion barrier positions reproduced the experimental values within $\pm 10\%$. Especially for the heavier colliding nuclei, we see that Prox 2010 potential reproduces the data much better on the average compared to other versions. For lighter nuclei, however, small scattering is visible. This could also be due to the uncertainty in the radius of the lighter colliding nuclei. In Figs. 1–4, only 155 reactions are displayed to maintain the clarity. The average deviation for the fusion barrier heights over 390 reactions is 0.77% using our modified potential Prox 2010, whereas Prox 1977 and Prox 2000 give 3.99% and 4.45%, respectively. This shows that our modified proximity explains the experimental data nicely.

Fig. 3. The same as Fig. 1, but for different older proximity potentials along with our modified form, i.e., Prox 1977, Prox 2000, and Prox 2010, respectively.

In Fig. 4, we display the difference between the theoretical and experimentally extracted fusion barriers. We further note that Prox 2010 potential gives better results. The difference especially for the heavy
systems is significantly reduced. This was the problem with original as well as its recently modified form as pointed out by several authors.\cite{7,8} It is clear from Figs. 3 and 4, that Prox 2010 potential is able to reproduce the experimental data much better than its older versions. The small difference is not significant because of the uncertainties in the analysis of the experimental data.

Finally, we test our newly modified proximity potential Prox 2010 on fusion probabilities. In Fig. 5, we display the fusion cross sections $\sigma_{\text{fus}}$ (in mb) as a function of the center-of-mass energy $E_{c.m.}$ (MeV) for the reactions of $^{26}\text{Mg} + ^{30}\text{Si}$, \cite{17} $^{18}\text{O} + ^{40}\text{Ti}$, \cite{18} $^{48}\text{Ca} + ^{48}\text{Ca}$, \cite{19} $^{12}\text{C} + ^{52}\text{Zr}$, \cite{20} $^{40}\text{Ca} + ^{58}\text{Ni}$, \cite{21} and $^{16}\text{O} + ^{144}\text{Sm}$.\cite{22} The fusion cross sections are calculated using the well-known Wong model.\cite{23} The older versions of proximity potentials, that is, Prox 1977 and Prox 2000, are also displayed. It is clearly visible from the figure that Prox 1977 and Prox 2010 potentials are in good agreement. However, its older forms are far from the experimental data. We further note that Prox 1977 and Prox 2000 potentials show similar results. This means that no improvement is seen in Prox 2000 potential as was claimed in Ref.\cite{7}.

In the present study, we present a best set of the surface energy coefficient, the nuclear radius, and the universal function available in the literature. Our newly constructed proximity potential Prox 2010 reproduces the fusion barriers and cross sections better than its earlier versions.

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