Improved Non-Fragile State Feedback Control for Stochastic Jump Systems With Uncertain Parameters and Mode-Dependent Time-Varying Delays

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ABSTRACT This paper reports the investigation on non-fragile state feedback control for stochastic Markovian jump systems with uncertain parameters and mode-dependent time-varying delays. The resulted closed-loop system is stochastic stabilization by virtue of an improved L-K functional. By free-weight-matrix technique, the non-fragile state feedback controller is designed and novel conditions for robust stochastic stabilization are acquired in the form of linear matrix inequalities. Two examples including a take-off and landing guidance system are employed to show the effectiveness and validity of the proposed approach.

INDEX TERMS Non-fragile controller, Itô stochastic system, parametric uncertain Markovian jump system, stochastic stabilization, mode-dependent time-varying delays.

I. INTRODUCTION As a very significant type of hybrid systems, Markovian jump systems (MJSs) have been paid more and more concern for many years. MJSs have always been investigated due to which can describe the dynamic systems with abrupt variations, as well as the important significance of practice and application in control and communication fields; see, [1]–[7]. MJSs are also deemed to be a special type of switched systems with continuous time and discrete modes. Jump switching phenomenon among the subsystems of MJSs are based on the modes, which are decided by Markovian process [8]–[13].

The stability analysis and control synthesis for MJSs have got extensive attention for a long time. A mass of satisfactory results have been obtained with the further study of MJSs. For example, by using the stochastic linear copositive Lyapunov function method, [14] studied the stochastic stability and stabilization of autonomous and non-autonomous MJSs, respectively; [15] addressed the stability and stabilization for stochastic MJSs with random switching signals in the form of linear matrix inequalities (LMIs), and then extended the results to uncertain and partially unknown transition rate matrices case.

Recently, many researchers focus on MJSs with uncertain parameters, which exist in the state or input. For instance, the passivity of uncertain MJSs was analyzed in [4]; improved delay-dependent stochastic stability for MJSs was proved by applying slack matrix variable method in [16]. In addition, time-delay phenomenon is almost ubiquitous in dynamic systems, which often leads to poor performance and instability of the systems; see, [17]–[22]. Specifically, [1] designed a controller of the MJSs with uncertain parameters and multiple time delays; a new criteria was established to show the stability of uncertain MJSs with polytopic parameter uncertainties and delays in [23]; the issue of stability analysis for uncertain time-delay MJSs was taken into account in [24].

It is worth pointing out that many practical application systems are affected by stochastic disturbance, such as the
amplitude and frequency of the power supply voltage, the ambient temperature, humidity and air pressure and the variation of load [25]–[27]. Therefore, it is essential to study the issues related to stochastic systems. The problems of filters for some stochastic systems were considered and some novel stability criteria were obtained in [28]. Beyond that, the topic of controllers of stochastic systems is often mentioned, as pointed out in [29]–[31]. MJSs combined with stochastic phenomenon, which called the stochastic MJSs, were considered in [32], [33]. It is worth mentioning that the $H_\infty$ controller of uncertain stochastic systems was designed by LMI approach in [34].

Noting that feedback control can effectively realize the stabilization of various kinds of dynamic systems [35]–[46]. In fact, there exists quite small uncertainties in controller implementation, which may cause the concerned closed-loop system efficiency reduction even unstability. Therefore, so far, numerous results about non-fragile controller designing were reported [47]–[50]. To mention a few, non-fragile controller was discussed for repeated scalar non-linearities time-varying delays systems in [51]; non-fragile stochastic stabilization for uncertain stochastic time-delay systems with time-dependent parameters was solved in [52]; non-fragile stabilization for MJSs were considered in [53]–[57]; non-fragile $H_\infty$ state feedback controller was mentioned for singular fuzzy MJS with an interval delay in [58]. According to our limited knowledge, the issue of non-fragile control for stochastic MJSs with uncertain parameters and mode-dependent time-varying delays is rarely studied, which stimulates the current research.

This paper will address the issue of the non-fragile state feedback controller for stochastic MJSs with uncertain parameters and time-varying mode-dependent delays. The parameter uncertainty will be assumed to be norm bounded. Improved L-K functional will be constructed to demonstrate the closed-loop system is robust stochastic stabilization. By free-weight-matrix (FWM) technique, novel sufficient conditions for stochastic stability will be acquired in terms of LMIs. Two examples will be presented to attest the validity of the proposed approach.

The main contributions of this paper are summed up as follows:

1) the mode-dependent and time-varying delays $\tau_r(t)$ are simultaneously considered for stochastic MJSs;

2) time-varying delays derivative constraint condition of $\dot{\tau}_r(t) \leq h < 1$ is extended to $\dot{\tau}_r(t) \leq h$ based on FWM technique, thus, the constraint condition is more general than the most existed results;

3) non-fragile controller is considered and improved mode-dependent and delay-dependent L-K functional is constructed in this paper.

**Notation.** $\mathcal{E}(\cdot)$ represents the expectation operator; $|\cdot|$ represents the Euclidean vector norm; Matrix $Q > 0 (Q \geq 0)$ refers to $Q$ is positive definite( positive semidefinite ). * expresses a term that is induced by symmetry in symmetric block matrices or long matrix expressions.

II. PROBLEM FORMULATION AND PRELIMINARIES

We consider the following stochastic MJSs with uncertainties and time-varying mode-dependent delays for giving a probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$
\begin{align*}
\dot{x}(t) &= ((A_{\gamma} + \Delta A_{\gamma}(t))x(t) + (A_{dr} + \Delta A_{dr}(t))x(t - \tau_r(t)) + (B_{1r} + \Delta B_{1r}(t))u(t))dt \\
&\quad + ([\Delta E_r + \Delta E_r(t)]x(t) + (E_{dr} + \Delta E_{dr}(t))x(t - \tau_r(t)) + (B_{2r} + \Delta B_{2r}(t))u(t))dw(t), \\
z(t) &= C_r x(t), \\
x(0) &= \psi(t), \quad \forall t \in [-\tau, 0],
\end{align*}
$$

where $x(t) \in \mathbb{R}^n$ is the state; $\psi(t)$ is a initial function; $u(t) \in \mathbb{R}^m$ is the control input; $z(t) \in \mathbb{R}^q$ is control output; $\omega(t)$ is a scalar Brownian motion. $\tau_r(t)$ denotes the time-varying mode-dependent delay satisfying

$$
0 < \tau_r(t) \leq \tau < +\infty, \quad \dot{\tau}_r(t) \leq h, \tag{2}
$$

where scalars $\tau > 0$ and $h$ are known. Continuous time Markovian process $r_t$ takes values in a finite set $S := \{1, 2, \ldots, s\}$, $\Pi := [\pi_{ij}]_{k \times k}$ is the transition rate matrix which satisfies the common conditions in [16], [53], [54], [56], etc.

For simplicity, $X_r(t)$ will be expressed as $X_r(t)$ for each $r_t = i \in S$ in the sequel. $A_{\gamma}, A_{dr}, E_r, E_{dr}, B_{1r}, B_{2r}$ and $C_r$ are known real matrices, $\Delta A_{\gamma}(t), \Delta E_r(t), \Delta A_{dr}(t), \Delta E_{dr}(t), \Delta B_{1r}(t)$ and $\Delta B_{2r}(t)$ stand for the uncertain parameters, which are unknown matrices and listed below:

$$
\begin{align*}
\Delta A_{\gamma}(t) &= M_{1r}F_r(t)\left[N_{1i} \quad N_{12} \quad N_{13}\right], \\
\Delta E_r(t) &= M_{1r}F_r(t)\left[N_{4i} \quad N_{5i} \quad N_{6i}\right].
\end{align*}
$$

(3)

Now, the mode-dependent non-fragile state feedback controller is designed as:

$$
u(t) = (K_i + \Delta K_i(t))x(t), \tag{5}
$$

the following form of controller gain perturbations $\Delta K_i(t)$ is considered:

$$
\Delta K_i(t) = H_{1i}F_{1i}(t)E_{1i}, \tag{6}
$$

where $H_{1i}$ and $E_{1i}$ are known and real matrices. Unknown matrix function $F_{1i}(t)$ satisfying

$$
F_{1i}^T(t)F_{1i}(t) \leq I. \tag{7}
$$

**Remark 1:** It is noted that the time-delay $\tau_r(t)$ of system (1) is time-varying and mode-dependent, the derivative constraint condition of $\dot{\tau}_r(t) \leq h < 1$ is extended to $\dot{\tau}_r(t) \leq h$ based on FWM technique. To the best of our knowledge,
time-varying delays associated with the modes are seldom taken into account and the derivative constraint condition of delays are usually written as \( \dot{r}(t) \leq h < 1 \) in the existed literature. This paper gives a more general condition than the most existed results.

**Definition 1 [16]:** The unforced stochastic MJS (1) is said to be stochastically stable, if the following inequality holds for the initial condition \( \psi(t) \in \mathbb{R}^n \), \( r_0 \in \mathcal{S} \) and \( t \in [-\tau, 0] \):

\[
\lim_{t \to \infty} \mathbb{E} \left\{ \int_0^t |x(s, \psi, r_0)|^2 ds \right\} < \infty, \tag{8}
\]

where \( x(s, \psi, r_0) \) is the solution to unforced system (1).

**Lemma 1 [25]:** Let \( \varphi \in \mathcal{M}^2(\{\alpha, \beta\}; \mathbb{R}) \),

\[
\mathbb{E} \left( \int_\alpha^\beta \varphi(t)dB(t)\middle|\mathcal{F}_a \right) = 0,
\]

\[
\mathbb{E} \left( \int_\alpha^\beta \varphi(t)dB(t) \middle| \mathcal{F}_a \right)^2 = \mathbb{E} \left( \int_\alpha^\beta |\varphi(t)|^2 dt \middle| \mathcal{F}_a \right) = \int_\alpha^\beta \mathbb{E}(\varphi(t)^2) \mathcal{F}_a dt. \tag{9}
\]

**III. MAIN RESULTS**

For the sake of simplicity, we define the drift term \( f(t) \) of the unforced system as

\[
f(t) = (A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t - \tau_i(t)), \tag{10}
\]

and the diffusion term \( g(t) \) as

\[
g(t) = (E_i + \Delta E_i(t))x(t) + (E_{di} + \Delta E_{di}(t))x(t - \tau_i(t)). \tag{11}
\]

In the sequel, we put forward a stochastic stability criterion for the mentioned system (1).

**Theorem 1:** Unforced stochastic MJS (1) is stochastically stable, if for all \( i \in \mathcal{S} \) and \( j = 1, \ldots, 6 \), there exist matrices \( Q_i > 0, R_i > 0, S_i > 0, Z_i > 0, P_i > 0, T, U \), and scalars \( \epsilon_j > 0, \tau > 0 \) and \( h \), such that:

\[
\Pi_{1i} = \begin{bmatrix} \Theta_i & \sqrt{T} \\ * & -Z \end{bmatrix} < 0, \tag{12}
\]

\[
\Pi_{2i} = \begin{bmatrix} \Theta_i & \sqrt{T}U \\ * & -Z \end{bmatrix} < 0, \tag{13}
\]

where

\[
\Theta_i = \begin{bmatrix} \Phi_i & W_i^T \\ * & -(P_i + \tau S)^{-1} + \epsilon_i M_i M_i^T \end{bmatrix} W_i^T \begin{bmatrix} 0 \\ -\epsilon_i I \\ * \\ * \\ * \end{bmatrix}.
\]

where

\[
W_{ti} = \begin{bmatrix} W_{ti} \\ T \\ U \end{bmatrix},
\]

\[
T = \begin{bmatrix} T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},
\]

\[
U = \begin{bmatrix} U \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},
\]

\[
\Phi_i = \begin{bmatrix} \Phi_i & 0 \\ * & -(1 + \eta\tau)Q + R + A_i T P_i \end{bmatrix} + \Phi_2 + \Phi_3,
\]

\[
\Phi_2 = \begin{bmatrix} T \\ T - T + U \\ -U \end{bmatrix},
\]

\[
W_i = \begin{bmatrix} E_i \\ E_{di} \\ 0 \end{bmatrix},
\]

\[
W_{2i} = \begin{bmatrix} N_{3i} \\ N_{4i} \\ 0 \end{bmatrix},
\]

\[
W_{3i} = \begin{bmatrix} A_i \\ A_{di} \\ 0 \end{bmatrix},
\]

\[
W_{4i} = \begin{bmatrix} N_{3i} \\ N_{2i} \\ 0 \end{bmatrix},
\]

\[
V(x_i, r_i, t) = \sum_{i=1}^{6} V_i(x_i, r_i, t), \tag{15}
\]

where

\[
V_1(x_i, r_i) = x_i^T(t)P(r_i)x_i(t),
\]

\[
V_2(x_i, r_i) = \int_{t-\tau_i(t)}^t x_i^T(s)Qx(s)ds,
\]

\[
V_3(x_i, r_i) = \eta \int_{t-\tau_i(t)}^t \int_{t-\tau_i(t)}^t x_i^T(s)Qx(s)\alpha d\alpha d\theta,
\]

\[
V_4(x_i, r_i) = \int_{t-\tau_i(t)}^t x_i^T(s)Rx(s)ds,
\]

\[
V_5(x_i, r_i) = \int_{t-\tau_i(t)}^t \int_{t-\tau_i(t)}^t g_i^T(s)Sg(s)dsd\theta,
\]

\[
V_6(x_i, r_i) = \int_{t-\tau_i(t)}^t \int_{t-\tau_i(t)}^t f_i^T(s)Zg(s)dsd\theta.
\]

The weak infinitesimal generator \( \mathcal{L} \) of the random process \( \{x_i, r_i, t \geq \tau_i\} \) is given by:

\[
\mathcal{L}V_i(x_i, t) = \sum_{l=1}^{6} \mathcal{L}V_i(x_i, i, t), \text{ for each } r_i = i, \text{ } i \in \mathcal{S}, \tag{16}
\]

where

\[
\mathcal{L}V_1(x_i, t) = x_i^T(t) \sum_{j=1}^{N} \pi_{ij} P_j x_i(t) + 2x_i^T(t)P_I r_f(t) + g_i^T(t)P_i g(t),
\]

\[
\mathcal{L}V_2(x_i, t) = \sum_{j=1}^{N} \pi_{ij} \int_{t-\tau_j(t)}^t x_i^T(s)Qx(s)ds
\]
By the Newton-Leibnitz formula, we have

\[ 2\xi^T(t)T[x(t) - x(t - \tau(t))] - \int_{t-\tau(t)}^t dx(s) = 0, \]  
(17)

\[ 2\xi^T(t)U[x(t - \tau(t)) - x(t - \tau) - \int_{t-\tau(t)}^t dx(s)] = 0. \]  
(18)

It is easy to get from (10):

\[ 2x^T(t)P_i[(A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t - \tau(t)) - f(t)] = 0. \]  
(19)

Let \( \xi^T(t) = [x^T(t), x^T(t - \tau(t)), x^T(t - \tau)] \), together with (16)-(18) to (14), which signify that

\[ LV(x_i, t, i) = 2x^T(t)Pf(t) + g^T(t)P_i(g(t) + \sum_{j=1}^N \pi_{ij}x^T(t)P_jx(t) \]
\[- (1 - \hat{\xi}(t))x^T(t - \tau(t))Qx(t - \tau(t)) + x^T(t)Qx(t), \]
\[- \eta \int_{t-\tau}^t x^T(s)Qx(s)ds \]
\[- x^T(t - \tau)Rx(t - \tau) + x^T(t)Rx(t) \]
\[- \int_{t-\tau}^t g^T(s)Sg(s)ds + \tau g^T(t)Sg(t) + \tau f^T(t)Zf(t) \]
\[- \int_{t-\tau}^t f^T(s)Zf(s)ds \]
\[ + 2\xi^T(t)T[x(t) - x(t - \tau(t))] - \int_{t-\tau(t)}^t dx(s) \]
\[ + 2\xi^T(t)U[x(t - \tau(t)) - x(t - \tau) - \int_{t-\tau(t)}^t dx(s)] \]
\[ + 2x^T(t)P_i[(A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t - \tau(t)) - f(t)] \]
\[ = g^T(t)(P_i + \tau S)g(t) + \tau f^T(t)Zf(t) \]
\[ + x^T(t) \left( (1 + \eta \tau)Q + \sum_{j=1}^N \pi_{ij}P_j \right) x(t) \]
\[- (1 - \hat{\xi}(t))x^T(t - \tau(t))Qx(t - \tau(t)) \]
\[- x^T(t - \tau)Rx(t - \tau) \]
\[ + \sum_{j=1}^N \pi_{ij} \int_{t-\tau(t)}^t x^T(s)Qx(s)ds - \eta \int_{t-\tau(t)}^t x^T(s)Qx(s)ds \]
\[- \int_{t-\tau(t)}^t f^T(s)Zf(s)ds - \int_{t-\tau(t)}^t g^T(s)Sg(s)ds \]
\[ + 2\xi^T(t)T \left[ x(t) - x(t - \tau(t)) \right] - \int_{t-\tau(t)}^t dx(s) \]
\[ + 2\xi^T(t)U \left[ x(t - \tau(t)) - x(t - \tau) - \int_{t-\tau(t)}^t dx(s) \right] \]
\[ + 2x^T(t)P_i[(A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t - \tau(t)) - f(t)] \]  
(20)

Now, let \( \max |\pi_{ij}| = \eta \), we deduce that

\[ \sum_{j=1}^N \pi_{ij} \int_{t-\tau(t)}^t x^T(s)Qx(s)ds \leq \eta \int_{t-\tau(t)}^t x^T(s)Qx(s)ds. \]  
(22)

From Lemma 1 and lemmas in [26], there exists a matrix \( S > 0 \) such that

\[ -2\xi^T(t)T \int_{t-\tau(t)}^t g(s)do\( s \)
\[- 2\xi^T(t)U \int_{t-\tau(t)}^t g(s)do\( s \)
\[ \leq \xi^T(t)TS^{-1}T^T \xi(t) + \xi^T(t)US^{-1}U^T \xi(t) \]
\[ + \left( \int_{t-\tau(t)}^t g(s)do\( s \) \right)^T S \left( \int_{t-\tau(t)}^t g(s)do\( s \) \right) \]
\[ + \left( \int_{t-\tau(t)}^t g(s)do\( s \) \right)^T S \left( \int_{t-\tau(t)}^t g(s)do\( s \) \right) \]  
(23)

and

\[ E \left[ \left( \int_{t-\tau(t)}^t g(s)do\( s \) \right)^T S \left( \int_{t-\tau(t)}^t g(s)do\( s \) \right) \right] \]
\[ = \int_{t-\tau(t)}^t g^T(s)Sg(s)ds, \]  
(24)

\[ E \left[ \left( \int_{t-\tau(t)}^t g(s)do\( s \) \right)^T S \left( \int_{t-\tau(t)}^t g(s)do\( s \) \right) \right] \]
\[ = \int_{t-\tau(t)}^t g^T(s)Sg(s)ds. \]  
(25)

Then, the following inequalities can hold:

\[ g^T(t)(P_i + \tau S)g(t) \leq \xi^T(t)\Gamma_1\xi(t), \]
\[ f^T(t)(\tau Z)f(t) \leq \xi^T(t)\Gamma_2\xi(t), \]  
(26)

where

\[ \Gamma_{1i} = \left[ \begin{array}{cc} E_i & E_{di} \\ M_i F_i(t) & [N_{3i} N_{4i}] \end{array} \right]^T (P_i + \tau S)^{-1} \times \left[ \begin{array}{cc} E_i & E_{di} \\ M_i F_i(t) & [N_{3i} N_{4i}] \end{array} \right], \]
\[ \leq \left[ E_i E_{di} 0 \right]^T (P_i + \tau S)^{-1} \]
\[ \begin{align*}
& -\epsilon_2 M M_1^T (E_i E_{di} 0) + \epsilon_2^{-1} [N_{3i} N_{4i}] 0^T [N_{3i} N_{4i}] 0, \\
& \Gamma_{2i} \leq [A_i A_{di} 0]^T (\tau Z)^{-1} [A_i A_{di} 0] + \epsilon_1^{-1} [N_{1i} N_{2i}] 0^T [N_{1i} N_{2i}] 0. \quad (27)
\end{align*} \]

Thus, using (20)-(25) to (19):

\[ \begin{align*}
& \mathcal{L} V (x_s, i, t) \\
& \leq x^T (t) \left( (1 + \eta \tau) Q + R + \sum_{j=1}^{N_s} \pi_j P_j \right) x(t) \\
& \quad - (1 - h) x^T (t - \tau (t)) Q x(t - \tau (t)) \\
& \quad - x^T (t - \tau) \mathcal{R} x(t - \tau) \\
& \quad - \int_{t - \tau}^{t} f^T (s) Z f(s) ds + 2x^T (t) P_i A_i x(t) \\
& \quad + A_{di} x(t - \tau) - f(t) \\
& \quad + 2 \xi^T (t) T (x(t) - x(t - \tau(t))) + \xi^T (t) T S^{-1} T \xi (t) \\
& \quad - 2 \xi^T (t) T \int_{t - \tau}^{t} f(s) ds \\
& \quad + 2 \xi^T (t) U (x(t - \tau(t)) - x(t - \tau)) \\
& \quad + \xi^T (t) U S^{-1} U^T \xi (t) - 2 \xi^T (t) U \int_{t - \tau}^{t - \tau} f(s) ds \\
& \quad + \xi^T (t) \left[ E_i E_{di} 0 \right]^T ((P_i + \tau S)^{-1} - \epsilon_2 M M_1^T)^{-1} \left[ E_i E_{di} 0 \right] \xi (t) \\
& \quad + \epsilon_2^{-1} \xi^T (t) \left[ N_{3i} N_{4i} 0 \right]^T \left[ N_{3i} N_{4i} 0 \right] \xi (t) \\
& \quad + \xi^T (t) \left[ A_i A_{di} 0 \right]^T ((\tau Z)^{-1} - \epsilon_1 M M_1^T)^{-1} \left[ A_i A_{di} 0 \right] \xi (t) \\
& \quad + \epsilon_1^{-1} \xi^T (t) \left[ N_{1i} N_{2i} 0 \right]^T \left[ N_{1i} N_{2i} 0 \right] \xi (t).
\end{align*} \]

Consequently,

\[ \begin{align*}
& \mathcal{L} V (x_i, i, t) \leq \tau^{-1} \int_{t - \tau (t)}^{t} \xi^T (s) \left[ \begin{array}{cc}
\Upsilon_i & -\tau T \\
* & -\tau Z
\end{array} \right] \xi (s) ds \\
& \quad + \tau^{-1} \int_{t - \tau}^{t - \tau (t)} \xi^T (s) \left[ \begin{array}{cc}
\Upsilon_i & -\tau U \\
* & -\tau Z
\end{array} \right] \xi (s) ds. \quad (28)
\end{align*} \]

where

\[ \begin{align*}
\xi^T (s) &= [\xi^T (s), f^T (s)]^T, \\
\Upsilon_i &= \Phi_i + \Gamma_i + \Gamma_{2i} \\
& \quad + \left[ T^T 0 0 \right] S^{-1} \left[ T^T 0 0 \right] \\
& \quad + \left[ U^T 0 0 \right] S^{-1} \left[ U^T 0 0 \right]. \quad (30)
\end{align*} \]

By Schur complement, imply that \( \Upsilon_i \geq 0 \), thus

\[ \mathcal{L} V (x_i, i, t) < 0. \quad (31) \]

Moreover, there exists scalar \( \gamma > 0 \), then

\[ \mathcal{L} V (x_i, i, t) < -\gamma x^T (t) x(t). \quad (32) \]

Applying Dynkin’s formula, we obtain

\[ E \left\{ \int_{t_0}^{t} \mathcal{L} V (x(s), s) ds \right\} < -\epsilon \gamma E \left\{ \int_{t_0}^{t} x^T (s) x(s) ds \right\}, \quad (33) \]

and

\[ E \{ V (x(t), t) \} - E \{ V (x(t_0), t_0) \} < -\epsilon \gamma E \left\{ \int_{t_0}^{t} x^T (s) x(s) ds \right\}, \quad (34) \]

which means

\[ E \left\{ \int_{t_0}^{t} x^T (s) x(s) ds \right\} < \frac{1}{\epsilon} E \{ V (x(t_0), t_0) \}. \quad (35) \]

Thus, by Definition 1, closed-loop system (1) is stochastically stable.

**Remark 2:** Significantly, the improved delay-dependent and mode-dependent Lyapunov-Krasovskii functional are constructed for the design of the desired controller of the stochastic MJS with the time-varying mode-dependent delays.

We focus on the closed-loop stochastic MJSs (1) with (5). The drift term \( \hat{f}(t) \) and the diffusion term \( \hat{g}(t) \) are represented as:

\[ \hat{f}(t) = (A_i + \Delta A_i(t)) x(t) + (A_{di} + \Delta A_{di}(t)) x(t - \tau(t)) + (B_{1i} + \Delta B_{1i}(t))(K_i + \Delta K_i) x(t), \quad (36) \]

\[ \hat{g}(t) = (E_i + \Delta E_i(t)) x(t) + (E_{di} + \Delta E_{di}(t)) x(t - \tau(t)) + (B_{2i} + \Delta B_{2i}(t))(K_i + \Delta K_i) x(t). \quad (37) \]

**Theorem 2:** Stochastic MJSs (1) is stochastically stable, if for all \( i \in S \) and \( j = 1, \ldots, 6 \), there exist matrices \( Q > 0 \), \( R > 0, S > 0 \), \( Z > 0, P_i > 0 \), \( T \) and \( U \), and scalars \( \epsilon_j > 0 \), \( \epsilon > 0 \), \( \tau > 0 \) and \( h \), such that:

\[ \hat{\Pi}_{1i} = \left[ \begin{array}{cc}
\hat{\Theta}_{1i} & \sqrt{\tau T} \\
* & -Z
\end{array} \right] < 0, \quad (38) \]

\[ \hat{\Pi}_{2i} = \left[ \begin{array}{cc}
\hat{\Theta}_{1i} & \sqrt{\tau U} \\
* & -Z
\end{array} \right] < 0, \quad (39) \]

where

\[ \hat{\Theta}_{1i} = \left[ \begin{array}{cccc}
\hat{\Theta}_{11i} & \hat{\Theta}_{12i} & \hat{\Theta}_{13i} \\
* & \hat{\Theta}_{22i} & 0 \\
* & * & \hat{\Theta}_{33i}
\end{array} \right], \]

\[ \hat{T} = \left[ \begin{array}{cc}
T & 0 \\
U & 0
\end{array} \right], \quad (40) \]

\[ \hat{\Upsilon} = \left[ \begin{array}{cc}
\hat{\Theta}_{11i} & \hat{W}_{T_i} \\
* & \hat{\Theta}_{11i}
\end{array} \right], \]

\[ \hat{\Theta}_{11i} = \left[ \begin{array}{cc}
\Phi_i & \hat{W}_{T_i} \\
* & \hat{\Theta}_{11i}
\end{array} \right], \quad (41) \]

\[ \hat{\Theta}_{11i} = \epsilon_1 M M_1^T + \epsilon_2 B_2 H_{11i} H_{12i} T - (P_i + \tau S)^{-1}, \quad (42) \]

\[ \hat{\Theta}_{12i} = \hat{W}_{T_i} \hat{W}_{T_i}^T - \left[ \begin{array}{cc}
T & U \\
0 & 0
\end{array} \right], \quad (43) \]

\[ \hat{\Theta}_{13i} = \left[ \begin{array}{cc}
T & U \\
0 & 0
\end{array} \right] \hat{W}_{T_i} \hat{W}_{T_i}^T - \left[ \begin{array}{cc}
\hat{W}_{T_i} & \hat{W}_{T_i} \\
\hat{W}_{T_i} & \hat{W}_{T_i}
\end{array} \right]. \quad (44) \]
\[\hat{\Theta}_{22i} = \text{diag}(\epsilon_5 N_0 H_1 H_1^T N_0^T - \epsilon_4 I, \epsilon_1 M_i M_i^T + \epsilon_2 B_1 H_1 H_1^T B_1^T - (\tau Z)^{-1}, \epsilon_3 N_0 H_1 H_1^T N_0^T),\]
\[\hat{\Theta}_{33i} = \text{diag}(-S, -S, -\epsilon_2 I, -\epsilon_5 I, -\epsilon_5 I, -\epsilon_6 I, -\epsilon_6 I, -I),\]
\[\hat{\Phi}_i = \begin{bmatrix} \hat{\Phi}_{i1} & P_i A_{di} \\ * & -(1-h)Q \\ * & -R \end{bmatrix} + \Phi_2 + \Phi_2^T.\]

\[\hat{\Phi}_i = \sum_{j=1}^{N} \pi_j P_j + (1 + \eta\tau)Q + R + P_i(A_i + B_1 K_i) + (A_i + B_1 K_i)^T P_i,\]
\[\hat{W}_{1i} = \begin{bmatrix} E_{1i} + B_2 K_i & E_{di} \\ 0 & 0 \end{bmatrix},\]
\[\hat{W}_{2i} = \begin{bmatrix} N_{3i} + N_0 K_i & N_{4i} \\ 0 & 0 \end{bmatrix},\]
\[\hat{W}_{3i} = \begin{bmatrix} A_i + B_1 K_i & A_{di} \\ 0 & 0 \end{bmatrix},\]
\[\hat{W}_{4i} = \begin{bmatrix} N_{1i} + N_0 K_i & N_{2i} \\ 0 & 0 \end{bmatrix},\]
\[\hat{W}_{5i} = \begin{bmatrix} E_{1i} & 0 \\ 0 & 0 \end{bmatrix},\]
\[\hat{W}_{6i} = \begin{bmatrix} C_i \\ 0 & 0 \end{bmatrix}.\]

**Proof:** From (36), we know that
\[
\begin{align*}
2x^T(t)P_i [(A_i + \Delta A_i(t))x(t) \\
+ (A_{di} + \Delta A_{di}(t)) x(t - \tau_i(t)) \\
+ (B_i + \Delta B_1(t))(K_i + \Delta K_i(t))x(t) - \hat{f}(t)]
\end{align*}
= 0. \tag{40}
\]

Now, together with (15)-(17) and (37), we find that
\[
\mathcal{L}V(x_i, i, t) = \sum_{j=1}^{N} \pi_j x^T(t)P_j x(t) + 2x^T(t)P_i \hat{f}(t) + \hat{g}^T(t)P_i \hat{g}(t) + \tau \hat{f}^T(t)\hat{z}(t) + \tau \hat{g}^T(t)S\hat{g}(t) + \sum_{j=1}^{N} \pi_j Q \int_{t-j\tau}^{t} x^T(s)sx(s)ds - (1 - \xi(t))x^T(t - \tau_i(t))Qx(t - \tau_i(t)) + x^T(t)Qx(t) - \eta \int_{t-\tau}^{t} x^T(s)Qx(s)ds + \eta \tau x^T(t)Qx(t) - x^T(t - \tau)Rxx(t - \tau) + \tau x^T(t)Rxx(t) - \int_{t-\tau}^{t} \hat{f}^T(s)S\hat{g}(s)ds - \int_{t-\tau}^{t} \hat{g}^T(s)S\hat{g}(s)ds + 2\xi^T(t)T \left[ x(t) - x(t - \tau_i(t)) - \int_{t-\tau_i(t)}^{t} dx(s) \right] + 2\xi^T(t)U \left[ x(t - \tau_i(t)) - x(t - \tau) - \int_{t-\tau_i(t)}^{t} dx(s) \right] + 2x^T(t)P_i [(A_i + \Delta A_i(t))x(t) \\
+ (A_{di} + \Delta A_{di}(t)) x(t - \tau_i(t)) \\
+ (B_i + \Delta B_1(t))(K_i + \Delta K_i(t))x(t) - \hat{f}(t)].
\]

By (40), (41), we can get the following inequalities:
\[
\hat{g}^T(t)(P_i + \tau S)\hat{g}(t) \leq \xi^T(t)\hat{f}_1\xi(t), \tag{42}
\]
\[
\hat{f}^T(t)(\tau Z)\hat{f}(t) \leq \xi^T(t)\hat{f}_2\xi(t), \tag{43}
\]

where
\[
\hat{f}_1 = \begin{bmatrix} E_i + B_2 K_i + B_2 \Delta K_i & E_{di} \\ 0 & M_i F_i(t) \end{bmatrix} \begin{bmatrix} N_{3i} + N_{6i} K_i & N_{6i} H_1 F_1 E_{1i} \\ N_{4i} & 0 \end{bmatrix}^T \times (P_i + \tau S)^{-1} \times \begin{bmatrix} E_i + B_2 K_i + B_2 \Delta K_i & E_{di} \\ 0 & M_i F_i(t) \end{bmatrix} \begin{bmatrix} N_{3i} + N_{6i} K_i & N_{6i} H_1 F_1 E_{1i} \\ N_{4i} & 0 \end{bmatrix},
\]

and
\[
\hat{f}_2 = \begin{bmatrix} A_i + B_1 K_i & A_{di} \\ 0 & 0 \end{bmatrix}^T ((P_i + \tau S)^{-1} - \epsilon_4 M_i M_i^T - \epsilon_2 B_2 H_1 H_1^T B_2^T)^{-1} \times \begin{bmatrix} A_i + B_1 K_i & A_{di} \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} N_{1i} + N_{2i} K_i & N_{2i} \\ 0 & 0 \end{bmatrix}^T - \epsilon_4 I - \epsilon_3 N_0 H_1 H_1^T N_0^T)^{-1} \times \begin{bmatrix} N_{1i} + N_{2i} K_i & N_{2i} \\ 0 & 0 \end{bmatrix} + \epsilon^{-1}_5 \begin{bmatrix} E_{1i} & 0 \\ 0 & 0 \end{bmatrix}^T \begin{bmatrix} E_{1i} & 0 \\ 0 & 0 \end{bmatrix}.
\]

Thus,
\[
\mathcal{L}V(x_i, i, t) \leq \sum_{j=1}^{N} \pi_j x^T(t)P_j x(t) - (1 - h)\tau x^T(t - \tau_i)Qx(t - \tau_i) - \int_{t-\tau}^{t} \hat{f}^T(s)\hat{z}(s)ds - x^T(t - \tau)Rx(t - \tau) + x^T(t)R(x(t) - x(t - \tau)) + 2\xi^T(t)T(x(t) - x(t - \tau)) + 2\xi^T(t)U(x(t - \tau_i(t)) - x(t - \tau)) + 2x^T(t)P_i [(A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t)) x(t - \tau_i(t)) + (B_i + \Delta B_1(t))(K_i + \Delta K_i(t))x(t) - \hat{f}(t)].
\]

By (40), (41), we can get the following inequalities:
\[
\hat{g}^T(t)(P_i + \tau S)\hat{g}(t) \leq \xi^T(t)\hat{f}_1\xi(t), \tag{42}
\]
\[
\hat{f}^T(t)(\tau Z)\hat{f}(t) \leq \xi^T(t)\hat{f}_2\xi(t), \tag{43}
\]
Similar to Theorem 1, the considered system (1) is stochastically stable if the following LMIs are satisfied

\[ \begin{align*}
&N_{3i} + N_{6i}K_i - N_{4i} \preceq 0, \\
&\xi(t) + \sum_{i=1}^{n} e_i N_{6i} H_i^T N_{6i} X_i^{-1} - \xi(t) = 0,
\end{align*} \]

where

\[ \begin{align*}
&\xi(t) = \begin{bmatrix}
E_{ii} & 0 \\
0 & E_{ii}
\end{bmatrix} \xi(t), \\
&T = T^T, \\
&U = U^T.
\end{align*} \]

Consequently,

\[ \begin{align*}
\mathcal{L}(V(x_i, t), t) &\leq \tau^{-1} \int_{t-\tau}^{t} \xi(s) \begin{bmatrix}
\hat{T}_i & \tau T \\
* & -\tau Z
\end{bmatrix} \xi(s) ds + \tau^{-1} \int_{t-\tau}^{t} \xi(s) \begin{bmatrix}
\hat{T}_i & -\tau U \\
* & -\tau Z
\end{bmatrix} \xi(s) ds,
\end{align*} \]

where

\[ \begin{align*}
\hat{T}_i &= \Phi_i + \hat{T}_{1i} + \hat{T}_{2i} + \hat{T}_{3i} + \hat{T}_{4i}, \\
&+ \begin{bmatrix} T^T & 0 & 0 \end{bmatrix} S^{-1} \begin{bmatrix} T^T & 0 & 0 \end{bmatrix}, \\
&+ \begin{bmatrix} U^T & 0 & 0 \end{bmatrix} S^{-1} \begin{bmatrix} U^T & 0 & 0 \end{bmatrix}.
\end{align*} \]

By Schur complement, we acquire \( \hat{T}_i < 0 \), which means

\[ \mathcal{L}(V(x_i, t), t) < 0. \]

Similar to Theorem 1, the considered system (1) is stochastically stable.

**Theorem 3:** Stochastic MJS (1) is stochastically stable via the non-fragile state feedback controller (5), if for all \( i \in S \) and \( j = 1, \ldots, 6 \), there exist matrices \( Q > 0, R > 0, S > 0 \), \( Z > 0 \), \( P_i > 0 \), \( \tilde{T}_i \) and \( \tilde{U}_i \), and scalars \( \epsilon_j > 0 \) such that the following LMIs are satisfied:

\[ \begin{align*}
\hat{\Theta}_{1i} &= \begin{bmatrix}
\hat{\Theta}_{1i} & \sqrt{T^T} \\
* & -\tau T
\end{bmatrix} < 0, \\
\hat{\Theta}_{2i} &= \begin{bmatrix}
\hat{\Theta}_{2i} & \sqrt{T^T} \\
* & -\tau T
\end{bmatrix} < 0,
\end{align*} \]

where

\[ \begin{align*}
\hat{\Theta}_{i} &= \begin{bmatrix}
\hat{\Theta}_{1i} & \hat{\Theta}_{12i} & \hat{\Theta}_{13i} & \hat{\Theta}_{14i} \\
* & \hat{\Theta}_{22i} & 0 & 0 \\
* & 0 & \hat{\Theta}_{33} & 0 \\
* & 0 & 0 & \hat{\Theta}_{44i}
\end{bmatrix},
\end{align*} \]

\[ \begin{align*}
\hat{\Theta}_{1i} &= \begin{bmatrix}
\hat{\Theta}_{11i} & \hat{\Theta}_{12i} & \hat{\Theta}_{13i} & \hat{\Theta}_{14i} \\
* & \hat{\Theta}_{22i} & 0 & 0 \\
* & 0 & \hat{\Theta}_{33} & 0 \\
* & 0 & 0 & \hat{\Theta}_{44i}
\end{bmatrix}
\end{align*} \]

\[ \begin{align*}
\hat{\Theta}_{11i} &= \begin{bmatrix}
\hat{\Theta}_{11i} & \hat{\Theta}_{12i} & \hat{\Theta}_{13i} & \hat{\Theta}_{14i} \\
* & \hat{\Theta}_{22i} & 0 & 0 \\
* & 0 & \hat{\Theta}_{33} & 0 \\
* & 0 & 0 & \hat{\Theta}_{44i}
\end{bmatrix},
\end{align*} \]

\[ \begin{align*}
\hat{\Theta}_{12i} &= \begin{bmatrix}
\hat{\Theta}_{11i} & \hat{\Theta}_{12i} & \hat{\Theta}_{13i} & \hat{\Theta}_{14i} \\
* & \hat{\Theta}_{22i} & 0 & 0 \\
* & 0 & \hat{\Theta}_{33} & 0 \\
* & 0 & 0 & \hat{\Theta}_{44i}
\end{bmatrix}
\end{align*} \]

Then, the desired controller (5) can be realized by

\[ u(t) = K_i x(t), \quad K_i = Y_i^{-1} X_i^{-1}. \]

**Proof:** Along the line of the proof of Theorem 2, we pre-multiply and post-multiply \( \text{diag}(P_i, I, I) \) to \( \hat{\Theta}_i \), the inequalities in Theorem 3 are deduced via Schur complement, where

\[ \begin{align*}
\hat{T}_i &= X_i T_i X_i, \\
\hat{U}_i &= U_i X_i, \\
\hat{R}_i &= R_i X_i, \\
\hat{Z}_i &= (\tau Z_i^{-1}).
\end{align*} \]

Therefore, by Theorem 2, we can easily finish the proof.

**Remark 3:** Noticing that the appropriate dimensions matrices \( \hat{T}_i > 0 \), \( \hat{U}_i > 0 \), \( \hat{R}_i > 0 \), \( \hat{Z}_i > 0 \) are employed to transform the MJs to LMSs. The proof procedures of Theorem 3 are similar to Theorem 2.

**IV. SIMULATION EXAMPLES**

**Example 1:** System (1) is considered with parameters

\[ \begin{align*}
A_1 &= \begin{bmatrix}
-1 & 0 \\
0 & 1
\end{bmatrix}, \\
A_2 &= \begin{bmatrix}
-1.5 & 0 \\
0.7 & -0.5
\end{bmatrix}.
\end{align*} \]
FIGURE 1. The operation modes in Example 1.

\[ A_3 = \begin{bmatrix} -0.5 & 0.5 & 0 \\ 0.7 & 0.5 & -1.5 \\ 0.2 & -0.2 & 1.2 \end{bmatrix}, \]

\[ A_{d1} = \begin{bmatrix} -0.2 & 0.1 & 0 \\ 0.1 & -0.8 & 0.2 \\ 1 & -0.3 & -1 \end{bmatrix}, \]

\[ A_{d2} = \begin{bmatrix} -1.1 & 0.6 & 0 \\ 0 & -1 & 0.8 \\ 0.5 & -1 & -1 \end{bmatrix}, \]

\[ A_{d3} = \begin{bmatrix} -0.5 & 0.7 & 0 \\ 0 & -0.6 & 1 \\ 0.5 & -1 & -1 \end{bmatrix}, \]

\[ E_1 = \begin{bmatrix} 1.5 & 0.1 & 0.2 \\ 0 & 1.5 & -0.1 \\ 0 & 0 & 0.2 \end{bmatrix}, \]

\[ E_2 = \begin{bmatrix} 1 & 0.1 & 0.2 \\ 0 & 0.5 & -0.1 \\ 0 & 1.3 & 0 \end{bmatrix}, \]

\[ E_3 = \begin{bmatrix} -0.5 & 0 & 0.2 \\ 0.1 & 0.4 & 0.1 \\ 0.2 & -0.1 & 0.5 \end{bmatrix}, \]

\[ E_{d1} = \begin{bmatrix} -1.5 & 0 & 0.2 \\ 0.1 & 0.9 & 0.1 \\ 0.2 & -0.6 & 0.5 \end{bmatrix}, \]

\[ E_{d2} = \begin{bmatrix} -1 & 0 & 0.2 \\ 0.1 & 1.4 & 0.1 \\ 0.2 & -1.1 & 0.5 \end{bmatrix}, \]

\[ B_{11} = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 \end{bmatrix}^T, \quad B_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T, \]

\[ B_{13} = \begin{bmatrix} 1.5 & 0 & 1.5 \\ 0 & 1.5 & 0 \end{bmatrix}^T, \]

\[ B_{21} = \begin{bmatrix} 0.5 & 0 & 0.5 \end{bmatrix}^T, \]

\[ B_{22} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_{23} = \begin{bmatrix} 1.5 & 0 & 1.5 \\ 0 & 1.5 & 0 \end{bmatrix}^T, \]

\[ E_{11} = E_{12} = E_{13} = \begin{bmatrix} 0.1 & -0.1 & 0.1 \end{bmatrix}, \]

\[ M_1 = M_2 = M_3 = \begin{bmatrix} -0.1 & 0.1 & -0.1 \end{bmatrix}^T, \]

\[ N_{11} = N_{12} = N_{13} = [0.2 \, 0.01 \, 0.02], \]

\[ N_{21} = N_{22} = N_{23} = [0.2 \, 0 \, 0.01], \]

\[ N_{31} = N_{32} = N_{33} = [0.2 \, 0.01 \, 0], \]

\[ N_{41} = N_{42} = N_{43} = [0.2 \, 0 \, 0.02], \]

\[ N_{51} = N_{52} = N_{53} = [0.2 \, 0.02], \]

\[ N_{61} = N_{62} = N_{63} = [0.2 \, 0.01], \]

\[ H_{11} = H_{12} = H_{13} = [0.1 \, 1]^T, \]

\[ C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}. \]

Let \( h = 1.3 \) and \( \tau = 0.8 \), and

\[ \Pi = \begin{bmatrix} -6.7 & 5.7 & 1 \\ 2 & -5.5 & 3.5 \\ 1 & 0.8 & -1.8 \end{bmatrix}. \]

Solving the LMIs (50)-(51) in Theorem 3, we obtain the desired state feedback controller realization by Matlab LMI toolbox:

\[ K_1 = \begin{bmatrix} -0.0584 & -0.5477 & -0.5012 \\ -0.1470 & -1.3357 & 0.7172 \end{bmatrix}, \]

\[ K_2 = \begin{bmatrix} -0.0596 & -0.5464 & -0.0633 \\ -0.4068 & -1.3731 & 0.2626 \end{bmatrix}, \]

\[ K_3 = \begin{bmatrix} -0.0656 & -0.2337 & -0.3173 \\ -0.2672 & -0.4210 & 0.7483 \end{bmatrix}. \]

Figure 1 shows the jump modes of Example 1. Figure 2 depicts the considered system is unstable without
Example 2: Consider the vertical take-off and landing helicopter system (VTOLHS). In this practical example, the system parameters are changed due to time-varying environment such as wind speed, so the model belongs to Markovian jump system. This system can be described by (1) and $x_1(t)$ is the velocity of horizontal, $x_2(t)$ is velocity of vertical, $x_3(t)$ is velocity of pitch, $x_4(t)$ is angle of pitch and $\omega(t)$ is airflow interference. Consult the parameters in [56] as follows:

$$A_i = \begin{bmatrix} 0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & 1.01 & 0.0024 & -4.0208 \\ 0.1002 & \alpha_{32}(i) & 0.707 & \alpha_{34}(i) \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_{1i} = \begin{bmatrix} 0.4422 & 0.1761 \\ \beta_{21}(i) & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -0.001 & 0 & -0.002 & 0 \\ 0 & 0.005 & 0.004 & -0.003 \\ -0.007 & 0.003 & 0.002 & 0 \\ 0 & 0.005 & 0 & -0.003 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 0 & 0.002 & -0.001 & 0 \\ 0 & 0.003 & 0 & -0.003 \\ 0 & 0.003 & 0.002 & 0 \\ 0 & 0.004 & 0 & -0.003 \end{bmatrix},$$

$$A_{d3} = \begin{bmatrix} -0.001 & 0.001 & 0 & 0 \\ 0 & 0.003 & 0.002 & -0.003 \\ -0.004 & 0.003 & 0.002 & 0 \\ 0 & 0.002 & 0.003 & 0 \end{bmatrix},$$

$$B_{21} = B_{22} = B_{23} = \begin{bmatrix} 1.0 & 0 & 1.0 & 0 \\ 0 & 1.0 & 0 & 1.0 \end{bmatrix}^T,$$

$$M_1 = M_2 = M_3 = \begin{bmatrix} -0.1 & 0.1 & -0.1 & 0 \end{bmatrix}^T,$$

$$E_{11} = E_{12} = E_{13} = \begin{bmatrix} 0.1 & -0.1 & 0.1 & 0 \end{bmatrix},$$

$$N_{11} = N_{12} = N_{13} = \begin{bmatrix} 0.01 & 0 & 0 & 0.01 \end{bmatrix},$$

$\overrightarrow{\text{FIGURE 2.}}$ Trajectories of the system states without controller in Example 1.

$\overrightarrow{\text{FIGURE 3.}}$ Trajectories of the system states with non-fragile controller in Example 1.
\[ N_{21} = N_{22} = N_{23} = \begin{bmatrix} 0.01 & 0 & 0.01 & 0 \end{bmatrix}, \]
\[ N_{31} = N_{32} = N_{33} = \begin{bmatrix} 0.01 & 0.01 & 0 & 0 \end{bmatrix}, \]
\[ N_{41} = N_{42} = N_{43} = \begin{bmatrix} 0.01 & 0 & 0 & 0.01 \end{bmatrix}, \]
\[ N_{51} = N_{52} = N_{53} = \begin{bmatrix} 0.01 & 0.01 \end{bmatrix}, \]
\[ N_{61} = N_{62} = N_{63} = \begin{bmatrix} 0.01 & 0.02 \end{bmatrix}. \]
\[ H_{11} = H_{12} = H_{13} = \begin{bmatrix} -0.1 & -1 \end{bmatrix}. \]
\[ C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \]
\[ C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \]
\[ C_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}. \]

where \( \alpha_{32}(i), \alpha_{34}(i) \) and \( \beta_{21}(i) \) describe the most significant characteristics impacted by the airspeed changes.

The switching is assumed to follow a Markovian process between three modes with the transition rate matrix \( \Pi \) given as follows:

\[ \Pi = \begin{bmatrix} -6.7057 & 5.7057 & 1.0000 \\ 2.0000 & -5.5631 & 3.5631 \\ 1.0000 & 0.8406 & -1.8406 \end{bmatrix}. \]

Figure 4 describes the jump modes of Example 2. First, the unstable trajectories of the VTOLHS states without controller are shown in Figure 5.

Let \( \tau = 0.8 \) and \( h = 1.5 \), solve the LMIs (50)-(51) in Theorem 3, the desired state feedback controller parameters can be acquired as follows:

\[ K_1 = \begin{bmatrix} -0.0463 & 0.0164 & -0.0266 & -0.4156 \\ -0.0191 & 0.0877 & -0.0406 & -0.5634 \end{bmatrix}, \]
\[ K_2 = \begin{bmatrix} -0.0685 & 0.1063 & 0.0289 & -0.1877 \\ -0.0551 & 0.1463 & -0.0361 & -0.5894 \end{bmatrix}, \]
\[ K_3 = \begin{bmatrix} -0.0590 & 0.2077 & 0.1191 & 0.1090 \\ -0.0506 & 0.2445 & 0.0430 & -0.4513 \end{bmatrix}. \]

The trajectories of the VTOLHS state with the non-fragile controller (5) are given in Figure 6. These simulation figures...
demonstrate that the designed controller can effectively guarantee the stochastic stability.

V. CONCLUSION

This paper has reported the investigation on non-fragile control for stochastic MJSs with uncertain parameters and mode-dependent time-varying delays. The robust stochastic stabilization of closed-loop stochastic MJSs has been realized by virtue of the improved Lyapunov-Krasovskii functional. By FWM technique, the non-fragile state feedback controller has been designed and novel robust stochastic stabilization conditions have been acquired in terms of LMIs. Two examples including the VTOLHS have proved the effectiveness and validity of the proposed approach. In the future works, we will find more effective ways to solve the problem of non-fragile feedback control for fuzzy systems, switching systems, singular systems, etc.

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