No strong coupling regime in the fermion-Higgs sector of the standard model

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We present results for the renormalized quartic self-coupling λ_R and the renormalized Yukawa coupling y_R in a fermion-Higgs model with two SU(2) doublets, indicating that these couplings are not very strong.

1. INTRODUCTION

It is an important issue to investigate within a non-perturbative regularization scheme whether the quartic self-coupling and Yukawa coupling of the fermion-Higgs sector of the Standard model (StM) remain relatively small when increasing the bare couplings to very large values. For this it is desirable to construct a lattice fermion-Higgs model with a realistic fermion content. A naive transcription of the continuum lagrangian with one SU(2) doublet leads to the large number of 16 doublets on the lattice because of the species doubling phenomenon. There are two proposals which allow to reduce this large number of mass-degenerate SU(2) doublets to one: The mirror fermion model [1] and the reduced staggered fermion model [2]. The mirror fermion model is discussed in ref. [3]. In this contribution we use the reduced staggered formalism [4,5]. The basic idea here is to couple the two reduced staggered flavors to the Higgs field. For this purpose it is convenient to introduce the 4 × 4 matrix fields

\[ \Psi_x = \frac{1}{8} \sum_b (\gamma^{x+b}) \chi^{x+b} \]
\[ \overline{\Psi}_x = \frac{1}{8} \sum_b (\gamma^{x+b})^\dagger (1 + \varepsilon_{x+b}) \chi^{x+b} \] (1)

with \( \gamma^x = \gamma_1 \cdots \gamma_4 \), \( \varepsilon_x = (-1)^{x_1+x_2+x_3+x_4} \) and the sum running over the corners of a hypercube, \( b_{\mu} = 0, 1 \). The one-component staggered fermion field \( \chi \) is a real Grassmann variable. In contrast to usual staggered fermions (s. eq. (1) of ref. [6]) we inserted here the factors \( \frac{1}{2}(1-\varepsilon_x) \) and \( \frac{1}{2}(1+\varepsilon_x) \) which are used in the reduced staggered formalism to project the usual staggered fields \( \chi \) and \( \chi^\dagger \) to the odd and even sites of the hypercubic lattice. The form (1) implies the following structure for the matrix fields

\[ \Psi = \left( \begin{array}{cc} \psi_L & 0 \\ 0 & \psi_R \end{array} \right), \quad \overline{\Psi} = \left( \begin{array}{c} 0 \\ \psi_L \end{array} \begin{array}{c} \psi_R \\ 0 \end{array} \right) \] (2)

where \( \overline{\psi}_L, \overline{\psi}_R, \psi_L \) and \( \psi_R \) are 2 × 2 matrices. The row (column) indices of the \( \psi_L \) and \( \psi_R \) fields act as Weyl-spinor (flavor) labels, and vice versa for \( \overline{\psi}_L \) and \( \overline{\psi}_R \). When using the relation (2) and introducing the 4 × 4 matrix for the O(4) Higgs field \( \phi \),

\[ \Phi = \left( \begin{array}{cc} 0 & \phi \\ \phi^\dagger & 0 \end{array} \right) = -\sum_{\mu} \varphi_{\mu} \gamma_{\mu} \] (3)

one can show that the action

\[ S_F = -\sum_x \sum_{\mu} \frac{1}{2} \text{Tr}(\overline{\Psi}_x \gamma_{\mu} \Psi_{x+\mu} - \overline{\Psi}_{x+\mu} \gamma_{\mu} \Psi_x) \]
\[ + y \text{Tr}(\overline{\Psi}_x \Psi_x \Phi^T) \] (4)

reduces in the classical continuum limit to the action of the fermion-Higgs sector of the StM with
Figure 1. Upper figure: Phase diagram at \( \lambda = \infty \) with \( N_D = 2 \). Lower figure: \( m_\sigma/v_R \) as a function of \( m_F/v_R \).

one mass-degenerate isospin doublet. After inserting (4) and (3) into eq. (4) the final form of the fermionic action in terms of the \( \chi \) fields reads

\[ S_F = -\frac{1}{2} \sum_{x \mu} \chi_x \chi_{x+\mu} \left( \eta_{\mu x} + y \xi_x \zeta_{\mu x} \overline{\varphi}_{\mu x} \right), \]  

where \( \overline{\varphi}_{\mu x} = \frac{1}{16} \sum_b \varphi_{\mu,x-b} \) is the average of the scalar field over a lattice hypercube and \( \eta_{\mu x} = (-1)^{x_1+\cdots+x_{\mu-1}}, \zeta_{\mu x} = (-1)^{x_{\mu+1}+\cdots+x_4} \) are the usual staggered sign factors. The total form of the action is given by

\[ S = S_F + S_H, \]  

where \( S_H = \sum_x \left[ 2\kappa \sum_{\alpha} \varphi_{\alpha,x+\mu} \varphi_{\alpha,x+\mu} - \varphi_{\alpha x} \varphi_{\alpha x} - \lambda (\varphi_{\alpha x} \varphi_{\alpha x} - 1)^2 \right] \) is the pure scalar field action. The action \( S \) is invariant under the so-called staggered fermion (SF) symmetry group which includes shifts by one lattice distance, 90° rotations, lattice parity and the global U(1) symmetry, \( \chi_x \to e^{i\alpha_x} \chi_x \). This invariance of \( S \) ensures the staggered flavor interpretation in the scaling region.

\[ O^{(1)} = \sum_{x \mu} \varphi_{\mu x}^4, \quad O^{(2)} = \frac{1}{2} \sum_{x \mu} (\varphi_{\mu,x+\mu} - \varphi_{\mu x})^2. \]  

In order to recover the full O(4) symmetry one has in principle to add these operators as counterterms to the action \( S \to S + \epsilon_0 O^{(1)} + \delta_0 O^{(2)} \) and tune the coefficients \( \epsilon_0 \) and \( \delta_0 \) as a function of the bare parameters such that the O(4) invariance gets restored in the scaling region. Here we shall not add these counterterms to the action. However, we will show in the next section that the effect of the symmetry breaking is small in the parameter region of interest.

Since we are interested in the largest possible renormalized couplings we have fixed in the numerical simulation \( \lambda = \infty \). For the use of the Hybrid Monte Carlo algorithm it is necessary to use two mass-degenerate doublets, \( N_D = 2 \). The \( \kappa-y \) phase diagram is shown in fig. 1. There are four different phases, a paramagnetic (PM), a broken or ferromagnetic (FM), an antiferromagnetic (AM) and a ferrimagnetic (FI) phase. The various symbols mark the points in the FM phase where we carried out numerical simulations on lattices ranging in size from \( 6^3 24 \) to \( 16^3 24 \).

2. O(4) SYMMETRY BREAKING

To estimate the amount of O(4) symmetry breaking we have computed the one- and two-point functions in the FM phase using renormalized perturbation theory. We can decompose the scalar field in the FM phase in a Higgs mode, \( \sigma \), and three Goldstone modes, \( \pi^a, a = 1, 2, 3 \), according to \( \varphi_{\alpha\mu} = (v_R + \sigma_R) e^\alpha_{\mu} + \pi_R^a e^a_{\mu} \), where \( \{e^\alpha_{\mu}\} \) form an orthogonal set of O(4) unit vectors, which is arbitrary when neglecting fermion loop effects. This arbitrariness is removed after taking into account the one fermion loop contri-
malized perturbation theory, ref. [5], can be computed numerically, and coefficients estimate for the Goldstone mass, malized propagator we can read off the following

Explicit expressions for $\Sigma_{\mu\nu}$ tree level effective action with fluctuations and should therefore be included in the terms in eq. (6) are generated by quantum fluctuations and should therefore be included in the

equation gives rise to the non-zero value of the Goldstone mass. Fig. 2 shows that the numerical values (squares) for $m_\pi$ are very small in the scaling region with $y \approx 3.6 - 4.0$. Moreover the numerical results for $m_\pi$ are in good agreement with the analytic prediction (diamonds) after inserting the measured values for $y_R$ and $m_F$. This motivates us to take also the corrections for the renormalized field expectation value and Higgs mass in eq. (6) seriously and to define corrected couplings, $\lambda_R = m_F^2 + \frac{1}{m_F^2}(1 + \delta_R)^2$, and $\lambda'_R = m_F^2 + \frac{1}{m_F^2}(1 + \delta_R)^2$. A measure for the $O(4)$ symmetry breaking corrections is given by the ratios $R_y = (y_R - y'_R)/y'_R$ and $R_\lambda = (\sqrt{2}\lambda_R - \sqrt{2}\lambda'_R)/\sqrt{2}\lambda'_R$. A numerical calculation of these ratios gives $|R_y| < 5\%$ and $|R_\lambda| < 7\%$, in a parameter region with $m_F < 0.5$ and $m_\pi < 0.7$, which shows that the symmetry breaking effects are small.

3. RESULTS OF THE SIMULATION

Since the effect of the symmetry breaking is small we have computed the renormalized couplings from the usual tree level relations $y_R = m_F/v_R$ and $\lambda_R = m_F^2/2v_R^2$, where the renormalized field expectation value is defined as $v_R = v/\sqrt{Z_\pi}$. Here $v$ is the unrenormalized scalar field expectation value and $Z_\pi$ the wave-function renormalization constant of the Goldstone propagator. For the determination of the quantities $m_F, m_\pi$ and $Z_\pi$ we have measured the fermion, $\sigma$ particle and Goldstone propagators in momentum space. The fermion propagator could be well described for all momenta by a one pole Ansatz, which is characteristic for weakly interacting fermions. For the Goldstone propagator we have displayed a typical example in fig. 3. The inverse propagator $G^{-1}_{\pi} (p)$ is plotted here as a function of $\tilde{p}^2$, the square of the lattice momentum. The numerical data (crosses) exhibit a significant curvature at small $\tilde{p}^2$ which can be described by the one fermion loop contribution to the self-energy. This non-linear $\tilde{p}^2$ dependence can be parametrized by the Ansatz $G^{-1}_{\sigma,\pi}(p) = (\tilde{p}^2 + m_\pi^2 + \Sigma_{\text{sub}}(p))/Z_{\sigma,\pi}$, where $\Sigma_{\text{sub}}$ is the subtracted one fermion loop self-energy. The circles in fig. 3 were obtained by fitting this Ansatz to the numerical data. The fact that the one loop
Figure 3. $G_\pi^{-1}(p)$ as a function of $\hat{p}^2$.

Ansatz is sufficient to describe the numerical results perfectly over a large momentum interval indicates already that the renormalized couplings are small. This fitting method allows us to determine $m_\sigma$ and $Z_\pi$ accurately, also on small volumes.

As a next step we have to extrapolate the finite volume results for $y_R$ and $\lambda_R$ to the infinite volume. We carried out simulations on lattices of size $L^3\times 24$ with $L$ ranging from 6 to 16. If the spectrum contains massless Goldstone bosons, this gives rise to a $1/L^2$ dependence of the finite volume quantities. Since the Goldstone particles are massive in our model we expect deviations from the linear $1/L^2$ dependence when the volume increases beyond the Goldstone correlation length, $L > O(1/m_\pi)$. The fact that we did not observe significant deviations gives further evidence that the symmetry breaking effects are small.

In the lower graph of fig. 1 we display the infinite volume results for the ratios $m_\sigma/v_R = \sqrt{2\lambda_R}$ and $m_F/v_R = y_R$. The symbols in the upper and lower diagrams of fig. 1 match, so that one can see where in the phase diagram the results for the ratios have been obtained. It can be seen that the numerical values for neither ratio change when lowering $\kappa$ beyond $\kappa = 0$, while keeping the cut-off roughly constant. The $v_R$ values of these points vary from 0.08 to 0.27. The arrows in fig. 1 mark the tree level unitarity bounds for $\lambda_R$ and $y_R$. The graph shows that the points obtained in the regions (II) and (III) of the phase diagram (see fig. 1) are still very close to these values, which indicates that the renormalized couplings are not very strong. The solid line encloses the allowed regions obtained by integrating the one loop $\beta$ functions from infinite couplings at the cut-off downward to the renormalization scale. The cut-off was adjusted such that the agreement with the numerical data is best. It is remarkable that the shape is in reasonable agreement with our data. Fig. 1 shows that the Yukawa interaction gives a slight increase in $\lambda_R$. From fig. 1 we can read off an upper bound for $m_\sigma/v_R$ and $m_F/v_R$: For $m_\sigma < 0.7/a$, we find $m_\sigma/v_R \lesssim 4$ and $m_F/v_R \lesssim 2.6$. From experience in the O(4) model with various regularizations, we expect that these numbers for the upper bounds may be stretched by perhaps 20-30%.

All in all we conclude that the renormalized quartic and Yukawa couplings are in accordance with triviality and that they cannot be strong, unless the cut-off is unacceptably low.

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