Slave-boson Keldysh field theory for the Kondo effect in quantum dots

Sergey Smirnov and Milena Grifoni
Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany
(Dated: January 19, 2013)

We present a nonequilibrium nonperturbative field theory for the Kondo effect in strongly interacting quantum dots at finite temperatures. Unifying the slave-boson representation with the Keldysh field integral an effective Keldysh action is derived and explored in the vicinity of the zero slave-bosonic field configuration. The theory properly reflects the essential features of the Kondo physics and at the same time significantly simplifies a field-theoretic treatment of the phenomenon, avoiding complicated saddle point analysis or $1/N$ expansions, used so far. Importantly, our theory admits a closed analytical solution which explains the mechanism of the Kondo effect in terms of an interplay between the real and imaginary parts of the slave-bosonic self-energy. It thus provides a convenient nonperturbative building block, playing the role of a "free propagator", for more advanced theories. We finally demonstrate that already this simplest possible field theory is able to correctly reproduce experimental data on the Kondo peak observed in the differential conductance, correctly predicts the Kondo temperature and, within its applicability range, has the same universal temperature dependence of the conductance as the one obtained in numerical renormalization group calculations.

PACS numbers: 72.15.Qm, 73.63.-b, 72.10.Fk

I. INTRODUCTION

Experimentally discovered almost eighty years ago\cite{1} and qualitatively explained thirty years after\cite{2}, the Kondo effect, a minimum in the temperature dependence of the resistance in magnetic alloys, was later anew brought into the world and garbed with physics of quantum dots (QD)\cite{3,4}. That time this complicated phenomenon was supplemented by physics of nonequilibrium due to the dot coupling to contacts biased by an external voltage. In this setup the Kondo effect, a nonperturbative phenomenon induced by both the electron-electron interactions and the QD-contacts coupling, appears as a sharp many-particle resonance in the tunneling density of states (TDOS) at the Fermi energy. An immediate consequence of this resonance is the zero-bias maximum observed experimentally in the QD differential conductance at low temperatures.

Theoretical predictions\cite{5,6} of this behavior made before the actual experiments use the single-impurity Anderson model (SIAM)\cite{8} with the on-dot interaction $U$. Those early works utilized either a quasi-particle transformation\cite{5} to analytically predict resonant transmission through QDs for arbitrary $U$ or the noncrossing approximation (NCA), equations of motion and perturbation theory\cite{6,9,10} to numerically and analytically describe several important stationary and nonstationary features of the Kondo effect in strongly interacting QDs modeled by the infinite-$U$ SIAM, in particular, in a slave-boson representation\cite{12,13}.

Later, as an alternative to the slave-boson representation, diagrammatic expansions within the reduced density matrix formalism\cite{14} were applied to SIAM for the infinite-$U$ case. Extracting a certain infinite subset of diagrams, an analytical nonperturbative expression for the TDOS was obtained at finite temperatures. Here the absence of the double occupancy played a crucial role in identifying the relevant diagrams.

For small, intermediate and large $U$ the Kondo problem in QDs was widely explored numerically\cite{15,16,17}.

In the present paper we develop an analytical nonequilibrium real-time field theory of the Kondo effect in strongly interacting QDs at finite temperatures using the infinite-$U$ SIAM. The field-theoretic approach is based on the slave-boson representation and the Keldysh field
Importantly, our theory is nonperturbative in both the electron-electron interaction and QD-contacts coupling. The necessity in such a theory is obvious from its advantages. Firstly, it is a field theory and, thus, it has a clear systematic generalization from the present relatively simple basic model to more involved setups such as the ones with ferromagnetic contacts, superconducting contacts, finite $U$ systems, etc. This is especially important since there is a limited number of analytical theories nonperturbative in both the electron-electron interaction and QD-contacts coupling and which at the same time have a straightforward generalization scheme. Secondly, it is an analytical theory and, thus, it will help to reveal a relevant physical picture behind new and more complicated physical systems as, e.g., those mentioned above. This is definitely an advantage over numerical methods such as, e.g., NCA, which provide good quantitative description but hide the essence of the physics, giving only an indirect access to it.

In Fig. 1 we show a comparative layout of the slave-bosonic theories for the Kondo effect in QDs. A basic nonperturbative analytical slave-bosonic field theory of the Kondo effect in QDs within the range from below $T_K$ to higher temperatures, i.e., within the most relevant experimental temperature range (blue (uppermost) class in Fig. 1), is formulated in this work.

So far, using the slave-boson method in the context of QDs, the results were obtained nonperturbatively numerically[21,22] below and/or above $T_K$, perturbatively semi-analytically[10] above $T_K$ and nonperturbatively analytically[24] below $T_K$.

The slave-boson approach excludes the double occupancy of a QD restricting its Hilbert space to zero and single occupancy. This restriction was taken into account exactly in the numerical solutions[21,22] using an additional integral removing the constraint[23]. However, since the mean field[23] and $1/N$ expansion[23] theories, using the same integral trick, impose the constraint only approximately, they fail at higher temperatures[31].

Here our goal is the development of an analytical ground for the slave-bosonic nonperturbative field theories which could be placed within the blue (uppermost) class in Fig. 1. To avoid problems with high temperatures we do not use the integral trick to take into account the restriction of the Hilbert space but instead use an alternative method based on taking a certain limit with respect to a real parameter.[23] This alternative method has not been applied so far in conjunction with the Keldysh field integral. We demonstrate that the corresponding limit can be exactly performed analytically after the Keldysh field integral for the TDOS has been analytically calculated using a certain approximation.

Specifically, the approximation concerns the effective Keldysh action obtained after integrating out all electronic degrees of freedom. This action being a nonlinear functional of the slave-bosonic field is expanded up to second order in this field. Note, that this does not imply any perturbation since the action itself is the argument of an exponent. According to the general concept of the condensed matter field theory[23] the physics of our model is the physics in the vicinity of the zero slave-bosonic field configuration. We demonstrate that this physics contains the Kondo effect in QDs at finite temperatures. This scenario complements[23] the saddle point analysis[23] valid deep below the mean field theory slave-bosonic phase transition, i.e., at low temperatures when the slave-bosonic field in the mean field theory is condensed. Surprisingly, our simple or "bare" theory, which might play a role of a nonperturbative "free propagator" for more advanced nonperturbative theories of the blue class in Fig. 1 already provides a good description of the Kondo peak observed[3] in the differential conductance at temperatures close to $T_K$.

The paper is organized as follows. In Section III we formulate the problem on the operator level. Then in Section III we translate this formulation into a field-theoretic framework using the Keldysh field integral and derive an analytic expression for the TDOS. Finally, we discuss our results and make conclusions in Sections IV and V, respectively.

II. OPERATOR FORMULATION: HAMILTONIAN AND OBSERVABLES

We start with the infinite-$U$ Anderson Hamiltonian in a slave-boson representation. As is well known[23] in the case when $U = \infty$, the Anderson Hamiltonian, $H_\text{A} = \sum_\sigma \epsilon_\text{d} \hat{n}_\text{d,} \sigma + U \hat{n}_\text{d,} \uparrow \hat{n}_\text{d,} \downarrow$, where $\hat{n}_\text{d,} \sigma = \hat{c}_\text{d,} \sigma \hat{c}_\text{d,} \sigma ^\dagger$, takes the form $H_\text{d} = \sum_\sigma \epsilon_\text{d} f_\sigma ^\dagger f_\sigma$, where the new fermionic operators are related to the original ones as $d_\sigma = f_\sigma b_\uparrow$, $d_\sigma ^\dagger = f_\sigma ^\dagger b_\downarrow$. Using these new fermionic and slave-bosonic operators the tunneling Hamiltonian, describing the coupling of the infinite-$U$ Anderson QD to contacts, can be written as

$$\hat{H}_\text{T} = \sum_{\sigma a} (T_{a\sigma} c_\sigma ^\dagger f_\sigma b_\uparrow + T_{a\sigma} ^* f_\sigma ^\dagger c_\sigma b),$$

where $c_\sigma$, $c_\sigma ^\dagger$ are the annihilation and creation operators of the contacts fermions, $a$ includes the contacts complete set of quantum numbers and the contacts labels, left (L) or right (R), and $T_{a\sigma}$ are the tunneling matrix elements. The contacts are described by the Hamiltonian $H_\text{C} = \sum_\sigma \epsilon_\text{a} c_\sigma ^\dagger c_\sigma$ and are assumed to be in equilibrium with the chemical potentials $\mu_{L,R} = \mu_0 - eV_{L,R}$ with $V \equiv V_L - V_R$ being an external voltage. The total Hamiltonian is $\hat{H} = H_\text{A} + \hat{H}_\text{T} + H_\text{C}$.

The restriction of the QD Hilbert space to zero and single occupancy requires the total number of the new fermions and slave-bosons, $\hat{Q} \equiv b_\uparrow b + \sum_\sigma f_\sigma b_\sigma$, to be equal to one, $\hat{Q} = \hat{I}$. This restriction must be taken into account in a QD observable ($\hat{O}$). There are two ways of doing that[23]. In the QD context only the first one,
i.e., the integral way was used so far. Here we employ the second way from Ref. \(25\)
\[
\langle \hat{O} \rangle (t) = \frac{\lim_{\mu \to -\infty} e^{\beta \mu} \text{Tr}[\hat{U}_{-\infty, t} \hat{O} \hat{U}_{t, -\infty} \hat{\rho}_0 e^{-\beta \mu \hat{Q}}]}{\lim_{\mu \to -\infty} e^{\beta \mu} \text{Tr}[\hat{\rho}_0 e^{-\beta \mu \hat{Q}}]}, \tag{2}
\]
where \(\hat{U}_{t, t'}\) is the evolution operator with respect to the Hamiltonian \(\hat{H}, \hat{\rho}_0 = \exp[-\beta(\hat{H}_d - \mu_0 \hat{N}_d)] \otimes \exp[-\beta(\hat{H}_C - \sum_x \mu_x \hat{N}_x)]\), \((x = L, R)\) is the initial statistical operator with \(\hat{N}_d\) and \(\hat{N}_x\) being the number operators of the QD and contacts and \(\beta\) is the inverse temperature. Eq. \(2\) may be rewritten in two equivalent forms,
\[
\langle \hat{O} \rangle (t) = \frac{\lim_{\mu \to -\infty} e^{\beta \mu} \text{Tr}[\hat{U}_{-\infty, \infty} \hat{U}_{t, -\infty} \hat{O} \hat{U}_{t, -\infty} \hat{\rho}_0 e^{-\beta \mu \hat{Q}}]}{\lim_{\mu \to -\infty} e^{\beta \mu} \text{Tr}[\hat{\rho}_0 e^{-\beta \mu \hat{Q}}]}, \tag{3}
\]
and
\[
\langle \hat{O} \rangle (t) = \frac{\lim_{\mu \to -\infty} e^{\beta \mu} \text{Tr}[\hat{U}_{-\infty, 0} \hat{O} \hat{U}_{0, \infty} \hat{\rho}_0 e^{-\beta \mu \hat{Q}}]}{\lim_{\mu \to -\infty} e^{\beta \mu} \text{Tr}[\hat{\rho}_0 e^{-\beta \mu \hat{Q}}]} \tag{4}.
\]
These two forms have different interpretation: in Eq. \(3\) the observable is taken within the evolution from \(-\infty\) to \(\infty\) while in Eq. \(4\) it is taken within the evolution from \(\infty\) to \(-\infty\).

To develop a Keldysh field theory one first equivalently rewrites Eqs. \(3\) and \(4\) on the Keldysh contour \(C_K\). To this end one gives the creation and annihilation operators a formal temporal argument to allow the time-ordering operator to appropriately interlace operators. Afterwards one may take the half-sum of the two equivalent expressions to get the following symmetric form:
\[
\langle \hat{O} \rangle (t) = \frac{1}{N_0} \lim_{\mu \to -\infty} \frac{e^{\beta \mu} \text{Tr}[(\hat{U}_{-\infty, \infty} \hat{U}_{t, -\infty} \hat{O} \hat{U}_{t, -\infty} \hat{\rho}_0 e^{-\beta \mu \hat{Q}})]}{\lim_{\mu \to -\infty} e^{\beta \mu} \text{Tr}[\hat{\rho}_0 e^{-\beta \mu \hat{Q}}]} \times \frac{O(t_+) + O(t_-)}{2}, \tag{5}
\]
where \(t_+\) and \(t_-\) are the projections of \(t\) onto the forward and backward branches of \(C_K\),
\[
\hat{\rho}_0(\mu) = \hat{\rho}_0 \exp(-\beta \mu \hat{Q}), \quad \hat{H} = \hat{H} + \mu \hat{Q},
\]
\[
\lim_{\mu \to -\infty} \{\exp(\beta \mu) \text{Tr}[\hat{Q} \hat{\rho}_0(\mu)]\} = 1 / N_0 = \text{Tr}(\hat{\rho}_C), \tag{6}
\]
and \(\hat{\rho}_C\) is the statistical operator of the contacts. The expression under the limit in Eq. \(5\) can be written as the Keldysh field integral \(21,22\) for a fixed value of \(\mu\). The basic steps in the construction of the Keldysh field integral are identical to the ones presented in Ref. \(22\). The details specific to our application of this field integral are given in the next section.

We would like to emphasize the absence in Eq. \(5\) of any prefactor depending on the QD-contacts coupling. This is a great advantage of the Keldysh field theory over non-field-theoretic approaches \(22\) and imaginary-time field theories \(25\). Indeed, in our approach there is no need for an independent calculation of such prefactors. This greatly simplifies the analytical exact projection onto the physical subspace. This fact was not realized in the first attempt \(23\) to combine the Keldysh field theory and slave-boson approach and, as a result, this attempt was reduced to a nonequilibrium analog of equilibrium imaginary-time field theories with only an approximate projection onto the physical subspace.

### III. FIELD-THEORETIC SOLUTION: KELDYSH FIELD INTEGRAL

We obtain the effective Keldysh field theory in a way similar to the one used in Refs. \(22,26\) for Coulomb-blockaded QDs. Namely, we first integrate out the QD and contacts Grassmann fields. After this step the field-theoretic description is given in terms of the effective Keldysh action, \(S_{\text{eff}}[\chi^d(t), \chi^q(t)] = S_B^{\text{cl}}[\chi^d(t), \chi^a(t)] + S_{\text{tun}}[\chi^d(t), \chi^q(t)],\) where \(\chi^d, \chi^q\) are the classical and quantum components of the slave-bosonic complex field, which is just a bosonic coherent state \(22\). \(S_{\text{tun}}[\chi^d(t), \chi^q(t)]\) is the free slave-bosonic action with the standard matrix form in the Keldysh space and \(S_{\text{tun}}[\chi^d(t), \chi^q(t)]\) is the slave-bosonic tunneling action,
\[
S_{\text{tun}}[\chi^d(t), \chi^q(t)] = -i\hbar \text{tr} \ln[I + \mathcal{T} G^{(0)}], \tag{7}
\]
where the trace and matrix product are taken with respect to the temporal arguments and both single-particle and Keldysh indices. In Eq. \(7\) the matrices \(G^{(0)}\) and \(\mathcal{T}\) have the block form in the dot-contacts space,
\[
G^{(0)} = \begin{pmatrix} G_d^{(0)}(\sigma|\sigma') & 0 \\ 0 & G_C^{(0)}(at|at') \end{pmatrix}, \tag{8}
\]
\[
\mathcal{T} = \begin{pmatrix} 0 & M^{(0)}_T(\sigma|\sigma') \\ M_T(\sigma|\sigma') & 0 \end{pmatrix}, \tag{9}
\]
where the blocks \(G_d^{(0)}(at|at')\) are the standard fermionic Keldysh Green’s function matrices \((2 \times 2)\) matrices in the Keldysh space) of the free QD \((\alpha = \sigma)\) and contacts \((\alpha = a)\),
\[
G_{d,c}^{(0)}(at|at') = \begin{pmatrix} G_{d,c}^{(0)}(at|at') & G_{d,c}^{(0)K}(at|at') \\ 0 & G_{d,c}^{(0)}(at|at') \end{pmatrix}, \tag{10}
\]
with \(G_{d,c}^{(0)++} (at|at')\) being the retarded, advanced and Keldysh components \(22\) respectively, and the block \(M_T(\sigma|\sigma')\) is the tunneling matrix \((2 \times 2)\) matrix in the Keldysh space,
\[
M_T(\sigma|\sigma') = \frac{1}{\hbar} \delta(t - t') T_{a\sigma} \frac{1}{\sqrt{2}} \begin{pmatrix} \chi^d(t) & \chi^q(t) \\ \chi^q(t) & \chi^d(t) \end{pmatrix}. \tag{11}
\]
and Keldysh slave-bosonic self-energies are
given by

In Eq. (12) the distribution $n_d(\epsilon)$ is double
the noninteracting nature of the new fermions.

Since the effective Keldysh action $S_{\text{act}}[\chi^c(t), \chi^q(t)]$ is quadratic, the functional integral in Eq. (16) may be performed exactly for any real $\mu > 0$. The limit $\mu \to \infty$ is readily taken afterwards. The final nonperturbative result for the QD TDOS is

$$\nu_\sigma(\epsilon) = \frac{\mathcal{P}(\epsilon)}{[\epsilon_d - \epsilon + (g/h)\Sigma_R(\epsilon)]^2 + [(g/h)\Sigma_I(\epsilon)]^2},$$

with $n_d(\epsilon)$ being the QD distribution of the new fermions.

The QD TDOS is defined through the imaginary part of the QD retarded Green’s function, $\nu_\sigma(\epsilon) \equiv -\frac{1}{\pi\hbar}\text{Im}[G_{\text{R}}^+(\epsilon)]$.

The Keldysh field integral expression for $\nu_\sigma(\epsilon)$ is

$$\nu_\sigma(\epsilon) = \frac{\mathcal{P}(\epsilon)}{[\epsilon_d - \epsilon + (g/h)\Sigma_R(\epsilon)]^2 + [(g/h)\Sigma_I(\epsilon)]^2},$$

where $\Sigma_R(\epsilon), \Sigma_I(\epsilon)$ are the real and imaginary parts of the retarded slave-bosonic self-energy for which we find the following analytical expressions,

$$\Sigma_R(\epsilon) = \hbar[D(\epsilon)/\pi]\{\epsilon/2W + (1/2\pi)\Re\sum_x [(1 + i\epsilon/W)\psi[1/2 - i\beta(iW + \mu_x)]/2\pi] - \psi[1/2 - i\beta(\mu_x - \epsilon)/2\pi]\},$$

$$\Sigma_I(\epsilon) = D(\epsilon)/2\pi\sum_x n_x(\epsilon),$$

where $\psi(x)$ is the digamma function and $n_x(\epsilon)$ are the contacts Fermi-Dirac distributions, and the numerator is

$$\mathcal{P}(\epsilon) = \frac{gD(\epsilon)n_L(\epsilon_d) + n_R(\epsilon_d) - 2n_L(\epsilon_d)n_R(\epsilon_d)}{1 - n_L(\epsilon_d)n_R(\epsilon_d)}.$$
It is important to emphasize that the QD TDOS, Eq. (18), is finite at any finite temperature but logarithmically diverges for \( T = 0 \) at the chemical potentials. Thus, one expects that for very low temperatures the simple quadratic theory is not valid. This is in accordance with what one generally expects on purely theoretical grounds\(^\text{22}\) from expansions around zero field configurations. Zero field expansions complement low temperature theories, being expansions around non-zero field configurations, like mean field theories\(^\text{23,24}\). These non-zero field expansions fail to describe the high-temperature behavior of the Kondo effect in QDs as we have demonstrated in Fig. 1. Therefore, one concludes that for a comprehensive description of the Kondo effect in the whole temperature range it is desirable to have in one’s disposal both types of field theories.

Let us clarify the nature of the approximation used for the tunneling action, Eq. (12), and the applicability range of the result for the TDOS, Eq. (18). Formally the small parameter of the expansion is the tunneling matrix element \( T \). Hence, \( \Gamma = 2g/\pi \) should be small. However, the tunneling matrix elements always enter in product with the slave-bosonic fields. Thus our theory will be valid in physical situations where the large values of the slave-bosonic amplitude are not important, \( i.e. \), when the probability of the QD empty state is not large. On one side this happens when \( \mu_0 - \epsilon_d \) is large and on the other side when the temperature is not too low and thus the Kondo resonance is not too strong so that the fluctuations of the QD empty state are not too large. Therefore, we assume that our theory should be applicable for

\[
\mu_0 - \epsilon_d \gtrsim \Gamma \quad \text{and} \quad T \gtrsim T_K. \tag{21}
\]

One should keep in mind that this is a crude estimate and our theory may still work qualitatively and perhaps semiquantitatively outside those inequalities. As demonstrated in the next section, our simple quadratic approximation, accounting for the physics in the vicinity of the zero slave-bosonic field configuration, provides a reasonable description of the Kondo physics. It also gives a finite single-particle resonance. This situation is definitely better than the one taking place in perturbative approaches\(^\text{19}\) which have divergences at the single-particle resonance. Our theory being nonperturbative avoids this problem but requires the next order term, \( i.e. \), the one which is quartic in the slave-bosonic field, in the expansion of the tunneling action (7) for a better quantitative description. This will be the subject of a more advanced theory. Here we only note that this advanced theory may be constructed in terms of the quadratic theory presented in this work. Namely, the present theory will play a role of a nonperturbative “free propagator” for the quartic theory.

It is interesting to look at the result for the QD TDOS, Eq. (18), from the physical point of view. Within the range of its applicability it suggests that the quasiparticle state in the QD represents a superposition of the bare electron state and the empty state of the QD coupled to contacts. Thus in this range the life-time of the QD quasiparticles can be estimated as the life-time of the QD empty state. This may be very attractive for the experiments measuring the quasiparticle life-times since observing QD states is easier than a direct observation of its quasiparticles.

Finally, we would like to emphasize an additional fundamental advantage of our method: our field-theoretic approach is a truly field theory in contrast with mean field theories\(^\text{23,24}\). The point is that here we work only with a physical field, being the slave-bosonic field, and not with artificial fields like the Lagrange multipliers fields used in mean field theories. Thus our theory has more transparent access to physics avoiding such artefacts of mean field theories as slave-bosonic condensation.

### IV. DISCUSSION OF THE RESULTS

In Fig. 2 the QD TDOS (18) is shown. In addition to the single-particle resonance at the renormalized QD noninteracting energy level it reveals a many-particle peak at the Fermi energy. In nonequilibrium, \( V \neq 0 \), the peak is reduced and split into two lower peaks (see inset). The total spectral weights of the equilibrium and nonequilibrium situations are almost the same differing by 1% which is due to the accuracy of the numerical integration involved in the total spectral weight.
Keldysh field theory

2

Kondo temperature,

T

self-energies,

Σ

R

(ε /Γ ) /Γ ,

Σ

I

(ε /Γ ) /Γ

differential conductance maximum

\[ \frac{e}{h} \]

differential conductance

\[ \frac{e}{h} \]

[\text{mK}]

we have argued that our theory must be valid for temper-

atures \( T \gtrsim T_K \), within this temperature range the differ-

ential conductance maximum in our theory must have the

same temperature dependence as the one in Refs. [28,29]

(see, e.g., Eq. (2) in Ref. [28] where we take \( s = 0.21 \))

for the case of a symmetric coupling. From this high-

temperature comparison, for each value of \( \mu_0 - \epsilon_d \) used

to calculate the differential conductance maximum in the

Keldysh field theory, we can fix the value of \( T_K \) to be used

in the empirical form of Ref. [28]. For example, for the

parameters used in Fig. 4 we get the temperature depen-

dence of the differential conductance maximum shown in

Fig. 5. The Kondo temperature dependence on the QD single-particle

energy level \( (\mu_0 - \epsilon_d) / \Gamma \).

To verify our field-theoretic description of the Kondo

physics we first compare it with experimental data. The

presence of the Kondo resonance in the QD TDOS has an

impact on the other QD observables. In particular, experi-

ments [3] show a peak in the differential conductance at \( V = 0 \).

Using the expression for the current (see Eq. (3) in Ref. [7] through a QD together with Eqs. [18] and

[19]) we obtain this behavior of the differential conduc-

tance shown in Fig. 4. To get Fig. 4 we have taken the

values of the parameters, \( \Gamma = 2.6875 \text{ meV}, W = 5 \text{ eV}, \)

\( T = 50 \text{ mK}, \Gamma / [\pi (\mu_0 - \epsilon_d)] = 0.1224 \), close to the ones

which were estimated in Ref. [3].

We would like further to compare our Keldysh field

theory with existing theoretical approaches, in particu-

lar, with the numerical renormalization group theory

from Ref. [27] which was successfully employed to de-

scribe experiments [28,29] on the Kondo effect in QDs. As

we have argued that our theory must be valid for temper-

atures \( T \gtrsim T_K \), within this temperature range the differ-

ential conductance maximum in our theory must have the

same temperature dependence as the one in Refs. [28,29]

(see, e.g., Eq. (2) in Ref. [28] where we take \( s = 0.21 \))

for the case of a symmetric coupling. From this high-

temperature comparison, for each value of \( \mu_0 - \epsilon_d \) used

to calculate the differential conductance maximum in the

Keldysh field theory, we can fix the value of \( T_K \) to be used

in the empirical form of Ref. [28]. For example, for the

parameters used in Fig. 4 we get the temperature depen-

dence of the differential conductance maximum shown in

Fig. 5. The Kondo temperature dependence of the QD single-particle

energy level \( (\mu_0 - \epsilon_d) / \Gamma \).

FIG. 3: (Color online) The mechanism of the Kondo reso-

nance formation in the nonperturbative Keldysh field theory

in the vicinity of the zero slave-bosonic field configuration. Here \( V = 0, kT = 0.008\Gamma, \mu_0 - \epsilon_d = 1.95\Gamma \).

FIG. 4: (Color online) The Kondo peak in the differential

conductance. Inset shows the experimental data of Ref. 3.

FIG. 5: (Color online) Comparison of our field theory

with the numerical renormalization group theory. The solid

line is obtained from the numerical renormalization group

calculations [27–29]. The circles show the differential conduc-
tance maximum obtained from our field theory for the values

of the parameters used in Fig. 4.

FIG. 6: (Color online) Keldysh field-theoretic prediction of

the Kondo temperature dependence on the QD single-particle

energy level \( (\mu_0 - \epsilon_d) / \Gamma \).
The differential conductance maximum has a universal temperature dependence $T_K$, which perfectly agrees with the quadratic approximation and this will be done in a subsequent study. This universal temperature dependence is shown in Fig. 7 and additionally proves that our Keldysh field theory, within its applicability range, correctly predicts the Kondo temperature. Moreover, our theory, within its applicability range, also predicts that the differential conductance maximum has a universal temperature dependence with the scaling given by the Kondo temperature, Eq. \[ T_K \approx \exp \left[ -2\pi \frac{\mu_0 - \epsilon_d}{\Gamma} \right], \] (22)

where our definition of $\Gamma$ is twice that of Ref. 6 (see also the caption of Fig. 1). This proves that our theory for the Kondo effect in QDs correctly predicts the Kondo temperature. Moreover, our theory, within its applicability range, correctly describes the Kondo physics in QDs. As one can see from Fig. 7 the Keldysh field-theoretic description is quantitatively reliable for temperatures $T \gg 2T_K$, which perfectly agrees with our theoretical prediction made above in Section 3.

Finally, we would like to say a few words about the numerical consistency of our theory. To do this, we employ the sum rule given by Eq. (39) of Ref. 7. In NCA this sum rule is always satisfied within 0.5%. In our theory this depends on how well the applicability criteria, Eq. (21), of the Keldysh field theory are satisfied. For example, for the parameters presented in Fig. 2 the sum rule is satisfied within 15% while for $\mu_0 - \epsilon_d = 2.5\Gamma$ it is 8.5% and for $\mu_0 - \epsilon_d = 4.0\Gamma$ it is 4.4%. One should note that the sum rule is an integral estimate over the whole energy range. Thus, the error is gained over the whole range of energies. At the same time for a given energy the QD TDOS may have higher accuracy. Since the only approximation was the truncation of all the terms of higher orders than the terms quadratic in the slave-bosonic field, to improve the consistency of the method and extend its applicability criteria one should go beyond the quadratic approximation and this will be done in a subsequent study.

V. CONCLUSION

We have developed a basic slave-boson nonperturbative Keldysh field theory for the Kondo effect in quantum dots. The theory deals with the physics in the vicinity of the zero slave-bosonic field configuration, as we have shown, the main fraction of the Kondo physics is located at experimentally relevant temperatures. The presented theory has a closed analytical solution for the quantum dot tunneling density of states and, despite being relatively simple, properly describes experimental data on the Kondo peak observed in the differential conductance, correctly predicts the Kondo temperature and, within its applicability range, has the same universal temperature dependence of the conductance as the one obtained in numerical renormalization group calculations. Therefore, it represents a convenient basis, as a free nonperturbative propagator, for more advanced theories which could extend the applicability of our approach to larger values of the slave-bosonic amplitude and, thus, to temperatures much lower than the Kondo temperature.

VI. ACKNOWLEDGMENTS

The authors thank Alexander Altland and Dmitry Ryndyk for fruitful discussions. Support from the DFG under the program SFB 689 is acknowledged.
(1994).
8 P. W. Anderson, Phys. Rev. 124, 41 (1961).
9 M. H. Hettler and H. Schoeller, Phys. Rev. Lett. 74, 4907 (1995).
10 N. Sivan and N. S. Wingreen, Phys. Rev. B 54, 11622 (1996).
11 O. Entin-Wohlman, A. Aharony, and Y. Meir, Phys. Rev. B 71, 035333 (2005).
12 P. Coleman, Phys. Rev. B 29, 3035 (1984).
13 P. Coleman, Phys. Rev. B 35, 5072 (1987).
14 A. C. Hewson, The Kondo Problem to Heavy Fermions (Cambridge University Press, Cambridge, 1997).
15 J. König, H. Schoeller, and G. Schön, Phil. Mag. B 77, 1219 (1998).
16 R. Gezzi, T. Pruschke, and V. Meden, Phys. Rev. B 75, 045324 (2007).
17 F. B. Anders, Phys. Rev. Lett. 101, 066804 (2008).
18 F. Heidrich-Meisner, A. E. Feiguin, and E. Dagotto, Phys. Rev. B 79, 235336 (2009).
19 J. Eckel, F. Heidrich-Meisner, S. G. Jakobs, M. Thorwart, M. Pletyukhov, and R. Egger, New J. Phys. 12, 043042 (2010).
20 L. Mühlbacher, D. F. Urban, and A. Komnik, Phys. Rev. B 83, 075107 (2011).
21 A. Kamenev and A. Andreev, Phys. Rev. B 60, 2218 (1999).
22 A. Altland and B. Simons, Condensed Matter Field Theory (Cambridge University Press, Cambridge, 2010), 2nd ed.
23 R. Aguado and D. C. Langreth, Phys. Rev. Lett. 85, 1946 (2000).
24 Z. Ratiani and A. Mitra, Phys. Rev. B 79, 245111 (2009).
25 N. E. Bickers, Rev. Mod. Phys. 59, 845 (1987).
26 A. Altland and R. Egger, Phys. Rev. Lett. 102, 026805 (2009).
27 T. A. Costi and A. C. Hewson, J. Phys. Condens. Matter 6, 2519 (1994).
28 D. Goldhaber-Gordon, J. Göres, M. A. Kastner, H. Shtrikman, D. Mahalu, and U. Meirav, Phys. Rev. Lett. 81, 5225 (1998).
29 M. Grobis, I. G. Rau, R. M. Potok, H. Shtrikman, and D. Goldhaber-Gordon, Phys. Rev. Lett. 100, 246601 (2008).