ANALYSIS OF TWO QUINTESSENCE MODELS WITH TYPE Ia SUPERNOVA DATA

M. Pavlov
Osservatorio Astronomico di Capodimonte, Via Moiariello 16, I-80131 Napoli, Italy

C. Rubano
Dipartimento di Scienze Fisiche, Università Federico II di Napoli, INFN Sezione di Napoli, Complesso Universitario di Monte S. Angelo, Via Cintia, ed. G, I-80126 Naples, Italy; rubano@na.infn.it

M. Sazhin
Sternberg Astronomical Institute, Universitetskij Prospect 13, Moscow 119899, Russia

AND

P. Scudellaro
Dipartimento di Scienze Fisiche, Università Federico II di Napoli, INFN Sezione di Napoli, Complesso Universitario di Monte S. Angelo, Via Cintia, ed. G, I-80126 Naples, Italy

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ABSTRACT

The Type Ia supernovae data are used to analyze two general exact solutions for quintessence models. The best-fit values for \( \Omega_m \) are smaller than in the \( \Lambda \)-term model, but still acceptable. With present-day data, it is not possible to discriminate among the various situations.

Subject headings: cosmology: theory — supernovae: general

1. INTRODUCTION

Recently, astronomers discovered an accelerated expansion of our universe. It is well known that all known types of matter generate attraction, which leads to a decelerated expansion of the universe. That discovery then reveals a new type of matter, which is now called quintessence, or sometimes dark energy (Ostriker & Steinhardt 1995; Turner & White 1997; Chiba, Sugiyama, & Nakamura 1997; Caldwell, Dave, & Steinhardt 1998; Zlatev, Wang, & Steinhardt 1999; Perlmutter, Turner, & White 1999b).

The discovery of the presence of dark energy became possible when astronomers recognized that Type Ia supernovae (SNe Ia) can be the long-expected standard candle for cosmological investigations. Two main features provide the use of SN Ia as a standard candle (Filippenko & Riess 2000): (i) they are exceedingly luminous, comparable with luminosity of a whole galaxy; they can, thus, be detected and observed with high signal-to-noise ratio (S/N) even at cosmological distances; (ii) “normal” SNe Ia have small variations among their peak absolute magnitudes (around 0.3).

The accelerated expansion of our universe was discovered as a result of two projects: the High-z SN Search (Schmidt et al. 1998; Riess et al. 1998) and the Supernova Cosmology Project (Perlmutter et al. 1999a).

In fact, a new type of matter was predicted many years ago by A. Einstein, who included a \( \Lambda \)-term into his considerations (Einstein 1917). At the beginning of the past century, the \( \Lambda \)-term was just a new fundamental constant, and only much later was it really considered as a formidable challenge by both observational and theoretical cosmologists (Weinberg 1988; Dolgov, Sazhin, & Zel’’dovich 1990; Zel’dovich 1992; Carroll, Press, & Turner 1992; Carroll 2001; Sahni & Starobinsky 2000; Rubakov 2000). Moreover, during the last 20 years cosmologists understood that this constant can be replaced with a scalar field, which induces the repulsive gravitational force dynamically. Accordingly, several models were proposed (Ratra & Peebles 1988; Peebles & Ratra 1988; Wetterich 1995, 1998; Copeland, Liddle, & Wands 1998; Ferreira & Joyce 1998; Liddle & Scherrer 1999; Steinhardt, Wang, & Zlatev 1999; Brax & Martin 1999; Sahni & Wang 2000; Binetruy 2000; Rubano & Scudellaro 2001) in order to explain the observed present acceleration of our universe.

Two of these models continued to be developed after the discovery of acceleration and were also roughly elaborated and adapted for present-day data (Rubano & Scudellaro 2001). Here, we use these models again to such an end, but in a much more refined way: the goal is now to fit the observed data of apparent magnitude and redshift of the SNe Ia and test the models themselves.

2. MODEL DESCRIPTION

As said above, in this paper we discuss two models for quintessence, both based on a scalar field with a special type of potential. The field is minimally coupled with pressureless matter, and the total density parameter \( \Omega \) of the universe is fixed to be 1. A detailed discussion of the consequences of assuming such models in cosmology is given in Rubano & Scudellaro (2001), so we limit ourselves here to only a short summary of the results we need for our purpose.

The main attractive feature of these models is that they allow a general exact solution of the field equations obtained through a suitable transformation of variables. Anyway, independently of the fact that this is an exact solution, we also find that this solution reflects many properties of the real universe correctly.

The first model considers a potential of the form

\[
V(\phi) = B^2 e^{-\sigma \phi},
\]

where \( B^2 \) is a generic positive constant and \( \sigma^2 \) is some fixed combination of universal constants,

\[
\sigma^2 = \frac{12\pi G}{c^2}.
\]

Actually, this kind of potential has already been widely
discussed in the literature, but without any particular assumption on the value of $\sigma$ (Ratra & Peebles 1988; Peebles & Ratra 1988; Wetterich 1995, 1998; Copeland et al. 1998; Ferreira & Joyce 1998; Brax & Martin 1999; Sahni & Wang 2000; Binetruy 2000; Fabris, Goncalves, & Tomimura 2000). We stress that it is the particular choice of this constant given above that allows the exact integration of the field equations (see also Barrow 1987; Burd & Barrow 1988).

The general solution of the cosmological equations (both for the metric and for the scalar field) has five free parameters (including $B^2$) (Rubano & Scudellaro 2001). We fix two of them and keep three as free. One of these three parameters, however, determines only the present value of the scale factor of our universe, which, in a spatially flat geometry, is not observable. It is not included in a statistical analysis and does not affect the degrees of freedom of our analysis.

We list below only the three cosmological functions that we need in our analysis (the other ones can be found in Rubano & Scudellaro 2001, of course):

$$\begin{align*}
(1 + z)^3 &= \frac{\tau_0 (1 + t^2)}{t^2 (1 + t^2)} , \qquad (3) \\
H &= \frac{2(1 + 2t^2)}{3t_0 \tau_0 (1 + t^2)} , \qquad (4) \\
\Omega_m &= \frac{1 + t^2}{(1 + 2t^2)^2} . \qquad (5)
\end{align*}$$

They are the redshift of the epoch, the Hubble parameter, and the $\Omega_m$ parameter of pressureless matter and are expressed in terms of the dimensionless time $\tau \equiv t/t_0$. The free parameters are then the timescale $t_0$ and the present value of the dimensionless time $\tau_0$. Let us remark that $\tau_0$ is of the same order of magnitude (but not necessarily equal to) as the age of the universe.

As for the second model, it considers a potential of the form

$$V(\phi) = A^2 e^{\sigma \phi} + B^2 e^{-\sigma \phi} ,$$

with $\sigma^2 = 12 G \pi / c^2$ as before and $A^2$ and $B^2$ free parameters.

We have, now, one additional free parameter; therefore, according to the same considerations as above, we have to deal with three of them.

The equations that describe the Hubble parameter, the redshift, and the density parameter in the second model are

$$\begin{align*}
(1 + z)^3 &= \frac{\lambda^2 \sin^2 \tau_0 \sin^2 \tau_0}{\lambda^2 \sin^2 \tau - \sin^2 \tau} , \quad (7) \\
H(\tau) &= \frac{\omega \sin (2\tau) - \lambda^2 \sinh (2\tau)}{3(\sin^2 \tau - \lambda^2 \sinh^2 \tau)} , \quad (8) \\
\Omega_m(\tau) &= \frac{2 (\lambda^2 - 1) \left[ \cos (2\tau) + \lambda^2 \cosh (2\tau) - 1 - \lambda^2 \right]}{[\sin (2\tau) - \lambda^2 \sinh (2\tau)]^2} . \quad (9)
\end{align*}$$

The dimensionless time in this case is $\tau = \omega t$. Following Rubano & Scudellaro (2001), we use $\omega$ here instead of $t_0$ because of the fact that it is directly connected with the parameters in the potential of the scalar field and has the meaning of a mass factor in theoretical considerations. So, the free parameters are $\omega$, $\lambda$, and $\tau_0$.

In the analysis of the SNe data, we use the bolometric distance. As explained better below, it can be expressed in terms of the “Hubble-free” luminosity distance and of a parameter, $m_0$, connected with the absolute magnitude and the Hubble parameter. The parameters of the first model ($\tau_0$ and $t_0$) can be recast into $\tau_0$ and $m_0$. The parameters of the second model ($\lambda$, $\tau_0$, and $\omega$) can be recast into $\lambda$, $\tau_0$, and $m_0$. Once the best fit is made, it is easy to compute the relevant physical quantities $H_0$ and $\Omega_m$. In all the considerations below, $H_0$ turns out to have the same value as in Perlmutter et al. (1999a). So, we concentrate on $\Omega_m$.

3. SNe Ia DATA

The published data of the supernovae consist of 60 SNe Ia (Perlmutter et al. 1999a). The data analysis and the determination of cosmological parameters can be considered in two steps. The first one is the measurement of the Hubble parameter for close SNe (Calan-Tololo survey; Hamuy et al. 1996), to be compared with the absolute magnitude $M$ of an SN Ia. The second step is the comparison of the high-redshift supernovae with the theoretical prediction of bolometric distance:

$$m = 5 \log (D_b) + m_0 ;$$

here, $D_b$ is the Hubble-free bolometric distance

$$D_b = H_0 (1 + z) \int_0^z \frac{dz'}{H(z')} ,$$

and $m_0$ is a parameter connected to the absolute magnitude and the Hubble parameter.

In data presented in Perlmutter et al. (1999a), there are several values for corrected apparent magnitude. The authors consider $m^\text{eff}$ and stretch luminosity corrected effective $B$-band magnitude $m^\text{eff}$. For the analysis of cosmological parameters only $m^\text{eff}$ is used, together with its errors $\sigma_m$.

There are several methods for SN Ia data analysis. Two of them are used in Riess et al. (1998). The first one is the multicolor light-curve shape (MLCS) method and the second one is a template-fitting method. In Perlmutter et al. (1999a) another method is used. The data of both groups have statistical errors of approximately $\sigma_m \sim 0.25$.

We follow Perlmutter et al. (1999a) to analyze the models described in Rubano & Scudellaro (2001). First of all, as a check of the procedure we apply the flat cosmological model with a $\Lambda$-term to fit the data. The standard $\chi^2$ algorithm of data analysis reveals a good agreement of our analysis with published statistical values (Perlmutter et al. 1999a). We use the complete set of data of 60 SNe Ia. It results in $\chi^2 = 1.75$ per degree of freedom, not significantly different from $\chi^2 = 1.76$ found in Perlmutter et al. (1999a).

The same is true for the $\Omega_m$ parameter. Since four points in the data are outliers, we can proceed with the analysis and exclude these data from our considerations. The total number of SN Ia data then drops to 56. The $\chi^2$ per degree of freedom in this case becomes 1.16, which is in good agreement with previously published results (Perlmutter et al. 1999a) and is within the 1 $\sigma$ level.

4. DATA ANALYSIS AND FITTING

In our analysis we use the standard $\chi^2$ method. The analysis is done by minimizing the value of weighted $\chi^2$:

$$\chi^2 = \sum \omega_m (m - m^\text{model})^2 ,$$

(12)
where $w_i$ is the weight of the $i$th SN Ia, $m_i$ is its $B$-band effective apparent magnitude, and $m^\text{model}_i$ is its magnitude as predicted with the models introduced before and thoroughly discussed in Rubano & Scudellaro (2001).

4.1. The First Model

In the first model, it is possible to eliminate $\tau$ from equations (3) and (4) and to obtain an analytical expression for $H(z)$. Thus, it is possible to compute $m$ from equations (10) and (11) and $\chi^2$ as a function of $\tau$ and $m_0$.

First, we use 60 SNe Ia and get the $\chi^2$ minimum at $m_0 = 24.01$, $\tau_0 = 1.04$, with $\chi^2 = 1.77$ per degree of freedom. As it is unsatisfactory, we reject data that are outside $3\sigma$ level, as done in Perlmutter et al. (1999a).

After data rejection, the $\chi^2$ minimum drops down to $\chi^2 = 1.195$ per degree of freedom. It is definitely within $1\sigma$ of the expected value of $\chi^2$. The minimum now has values other than $m_0 = 23.985$ and $\tau_0 = 1.268$.

If we accept the value of this minimum, we obtain, from equations (4) and (5), $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$) and $m_0 = 0.15$. The situation is illustrated in Figure 1.

4.2. The Second Model

The second model has been tested and fitted with data of only 56 SNe Ia. The number of parameters in this case is equal to three. The true minimum of the $\chi^2$ is at $m_0 = 23.98$, $\tau_0 = 0.8$, and $\lambda = 1.182$. We find a value of $\chi^2 = 1.1906$, which is definitely within $1\sigma$ of the expected value. From such value we obtain from equations (8) and (9) that $\Omega_{m0} = 0.17$.

The $\chi^2$ value is a function of three arbitrary values: $m_0$, $\tau_0$, and $\lambda$. Therefore, the $\chi^2$ as a function of all parameters is impossible to plot, but we can nonetheless plot several slits. The situation is illustrated in Figures 2 and 3.

5. CONCLUSIONS

In a quintessential universe we have analyzed the same data as in Perlmutter et al. (1999a), where only a cosmological constant is present, and we have found good values for $\chi^2$ in both cases. The values of $\Omega_{m0}$ found are rather different from the one found in Perlmutter et al. (1999a), but it is impossible to say if this results from differences in the models or from the influence of measurement errors on the final values.

In fact, in both models we have degeneracy in the parameters, particularly large in $\lambda$ (model II). This makes it impossible to give significant confidence limits for the values of $\Omega_{m0}$ that we found. Only very rough estimates can be given. Our main results are summarized in Table 1.

| TABLE 1 |
| --- |
| RESULTS |
| Parameter | Model I | Model II |
| $\chi^2$ | 1.195 | 1.19 |
| $m_0$ | 23.985 | 23.98 |
| $\tau_0$ | 1.268 | 0.8 |
| $\lambda$ | ... | 1.182 |
| $\Omega_{m0}$ | 0.15 | 0.17 |
| $\tau_0$ range | 0.82–1.40 | ... |
| $\Omega_{m0}$ range | 0.12–0.30 | 0.14–0.22 |
We observe that our results are in very good agreement with the one found in Bahcall et al. (2000) in a completely independent way and that the high degeneracy we get for the model parameters seems to support the opinion of those who claim that it is very difficult to discriminate among theories on the basis of observational data only (Maor, Brunstein, & Steinhardt 2001; Barger & Marfatia 2000). The fact that some parameters are left completely free and others are found with large degeneracy also seems to indicate that there is no fine tuning with regard to initial conditions. Instead, the value of $\sigma$ is a priori fixed, and we are working on the possibility of a generalization.

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