TESTING THE BLACK HOLE NO-HAIR THEOREM WITH OJ287

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ABSTRACT

We examine the ability to test the black hole no-hair theorem at the 10% level in this decade using the binary black hole in OJ287. In the test we constrain the value of the dimensionless parameter \( q \) that relates the scaled quadrupole moment and spin of the primary black hole: \( q_2 = -q \chi^2 \). At the present we can say that \( q = 1 \pm 0.3 \) (1\( \sigma \)), in agreement with general relativity and the no-hair theorems. We demonstrate that this result can be improved if more observational data are found in historical plate archives for the 1959 and 1971 outbursts. We also show that the predicted 2015 and 2019 outbursts will be crucial in improving the accuracy of the test. Space-based photometry is required in 2019 July due the proximity of OJ287 to the Sun at the time of the outburst. The best situation would be to carry out the photometry far from the Earth, from quite a different vantage point, in order to avoid the influence of the nearby Sun. We have considered in particular the STEREO space mission, which would be ideal if it has a continuation in 2019, or the Long Range Reconnaissance Imager on board the New Horizons mission to Pluto.

Key words: black hole physics – BL Lacertae objects: individual (OJ287) – gravitation – quasars: general – quasars: individual (OJ287) – relativistic processes

1. INTRODUCTION

Astronomical observations and detailed astrophysical considerations strongly support the existence of black hole candidates having masses in the range from a few 10\(^{10} \) \( M_\odot \) to a few 10\(^{10} \) \( M_\odot \). In order to make sure that they actually are black holes as postulated in general relativity (GR), we should prove that in at least one case the black hole no-hair theorems are satisfied.

According to the black hole no-hair theorems, an electrically neutral rotating black hole in GR is completely described by its mass \( M \) and angular momentum \( S \) (Israel 1967, 1968; Carter 1970; Hawking 1971, 1972; see Misner et al. 1973 for discussions). This implies that the multipole moments, required to specify the external metric of a black hole, are fully expressible in terms of \( M \) and \( S \). In the case of a Kerr black hole, characterized by the Kerr parameter \( \chi \), its dimensionless quadrupole parameter \( q_2 \) is uniquely defined by

\[
q_2 = -\chi^2,
\]

where \( q_2 = c^4 Q_2/G^2 M^3 \) and \( \chi = cS/GM^2 \), and \( Q_2 \) is the quadrupole moment of the black hole (Thorne 1980; Thorne et al. 1986).

Recently, the first attempt to probe the black hole no-hair theorems was made by Valtonen et al. (2010b) using the available optical observations of the BL Lacertae object OJ287. The quasi-periodic optical light curve of this quasar (Sillanpää et al. 1988) displays temporal variations having 12 and 60 year cycles (Figure 1).

A simple model for OJ287 involves a secondary black hole orbiting a more massive primary black hole in an eccentric orbit having a periodicity of about 12 years, while the 60 year period arises from the associated periastron precession (Lehto & Valtonen 1996). In principle, one could imagine many different binary models that would satisfy these requirements. However, there exists a third requirement that nails down the model. This is related to the observed double peak structure in the light curve of OJ287, with the two peaks separated by one to two years and the pair occurring approximately every 12 years. These observations are interpreted as being due to the double impact of the secondary black hole on the accretion disk of the primary (Lehto & Valtonen 1996). The model was able to predict the 2007 September 13 sharp outburst within one day (Valtonen 2007, 2008; Valtonen et al. 2008).

In describing the full light curve of OJ287, one must also calculate the indirect effects of the binary action on the accretion flow. After the orbit is fully determined by the sharp impacts and the related short but bright outbursts, calculating the more gentle rise and fall of the light curve arising from variations in the accretion flow is straightforward. Thus, the rest of the optical light curve was also predicted with fair accuracy (Sundelius et al. 1997; Valtonen et al. 2009). It is a relatively simple matter to separate these two kinds of flux variations by their quite different timescales (Valtonen et al. 2011; see Figure 2). These investigations give us the confidence to employ the binary black hole model of OJ287 to test GR.

The first orbit model of Lehto & Valtonen (1996) made use of five outbursts, giving four independent intervals of time. They allow a unique solution of four orbital parameters, the mass of the primary (1.71 \( \pm 0.15 \) \( \times 10^{10} \) \( M_\odot \)), the eccentricity of the orbit \( e = 0.678 \pm 0.004 \), the precession rate of the major axis of the orbit 33.3 \( \pm 2.7 \)\( \circ \), and a constant \( \phi_0 \) specifying the orientation of the orbit at some initial moment of time.

After the predicted 2005 outburst was observed, a new solution was calculated using six outbursts, allowing the determination of five parameters (Valtonen 2007). The new additional parameter is the thickness of the accretion disk (scale height \( \sim 150 \) AU), while the precession rate was updated to 37.5–39.1. The timing of the 2007 outburst together with some new historical data allowed a solution using nine outbursts, solving for eight parameters (Valtonen et al. 2010b). These nine outbursts all follow the basic light curve shape of Figure 3, with a rapid
Figure 1. Optical light curve of OJ287 from 1891 to 2010. The light curve includes previously unpublished data obtained at Harvard by R. Hudec and M. Basta. The line represents the binary black hole model.

Figure 2. Optical light curve of OJ287 during 2006–2008. Only low-polarization (less than 10%) data points are shown. There is a big “hump” lasting about one year and a “spike” at 2007 September 13 lasting only a few days. It is the “spikes” of the light curve that are used to determine the times of impact on the accretion disk, and then to calculate the orbit of the secondary.

rise to the maximum and then a slower decay to pre-outburst level. The timescales of the outbursts follow the dependence on the impact distance established by Lehto & Valtonen (1996). They form a very well-defined sequence. There are no cases in which an outburst in this sequence was expected but was not observed. All missing members are at times when there were no observations. There are no extra unexplained members of this sequence.

The new parameters are the spin of the primary black hole, with $\chi_1 = 0.28 \pm 0.08$, the mass of the secondary $(1.4 \pm 0.1) \times 10^8 M_\odot$, and $q$, which is described below.

Parallel to the increase in the number of outbursts in the solution, the number of post-Newtonian (PN) terms was increased in calculating the acceleration between the binary components. Valtonen et al. (2010b) include the dominant order general relativistic and classical spin–orbit coupling, which is required in order to relate the dimensionless quadrupole parameter $q_2$ of the primary to its Kerr parameter. They write

$$q_2 = -q\chi^2,$$

and let $q$ be among the eight parameters of the solution. Its value was determined as $q = 1 \pm 0.3$. Valtonen et al. (2010b) noted that the timing of the next outburst in 2015 should help to improve the accuracy of the $\chi$ estimate to about $\pm 5\%$. This conclusion along with the fact that the mass of the primary is determined with the accuracy of $\pm 1\%$ prompted us to explore the ways of testing the no-hair theorem at the $\pm 10\%$ level in the current decade by measuring $q$ more accurately. In the literature, there exist a number of proposals to test the black hole no-hair theorems,
plausible in the next decade with the help of electromagnetic and gravitational wave observations. The scenarios include radio timing of eccentric millisecond binary pulsars having an extreme Kerr black hole as a companion (Wex & Kopeikin 1999) and observing several stars orbiting the massive galactic center black hole at milliarcsecond distances with infrared telescopes capable of doing astrometry at the ∼10 mas level (Will 2008). Further, LISA observations of gravitational waves from extreme mass ratio in-spirals (Glampedakis & Babak 2006) and quasi-normal ringdown phases associated with massive black hole mergers (Berti et al. 2006) will also be used to try to validate black hole no-hair theorems. It has also been argued that the imaging of accretion flow around Sgr A*, if its Kerr parameter is not close to one, may allow the testing of the no-hair theorems in the near future (Johannsen & Psaltis 2010). The test relies on the argument that a bright emission ring characterizing the flow image will be elliptical and asymmetric if the theorems are violated. Johannsen & Psaltis (2011) further explore the possibility of detecting modes of quasi-periodic variability in accretion disks as test cases for the no-hair theorems.

In what follows, we briefly summarize our approach, detailed in Valtonen et al. (2010b), and list the improvements desirable for the test. We then identify by timing experiments those impact outbursts (historical and future) that will be crucial for constraining the q value. Finally, we discuss observational requirements for achieving it.

2. ADDITIONAL THEORETICAL INPUTS AND NEW TIMING EXPERIMENTS

In order to obtain the best timing, we construct a “standard” outburst light curve (Figure 3). This is based on the observations of the 2007 outburst that was monitored throughout the outburst both in the total optical brightness and polarization (Valtonen et al. 2008). No such light curve was previously available. These data allow us to separate the underlying bremsstrahlung emission (unpolarized) from the polarized flares. We model the underlying light curve in three separate sections using analytical functions. From the time 0 to time 2.59 days we use a rising power-law form,

\[ F_V = 2.9(t/3)^{1.5} f^{-1.5}; \]  

from day 2.59 to day 3.35 we use a linear rise,

\[ F_V = 2.33 + 4.7(t - 2.59 f) f^{-1}; \]  

and beyond day 3.35 we assume a decaying power-law form,

\[ F_V = 5.6((t + 1.5 f)/(5 f))^{-1.5}. \]

The flux is in mJy and t is the time measured from the beginning of the outburst.

Even though the standard light curve is adopted from observations, the three sections may be justified as follows: At time zero the optical depth of the radiating bubble equals unity and we start seeing to the interior of the bubble. The optical depth decreases and larger and larger volumes of the bubble come into view. At day 2.59 approximately half of the volume is visible; thereafter, the rest of the volume produces emission as quickly as the visibility front advances into the bubble. This stage happens quickly in the light-travel time of the bubble. At the third stage the flux from the bubble decreases as the radiating plasma cools adiabatically. There is a free parameter f in the formulae, which contracts or stretches the outburst timescale. This is necessary since the timescale is a function of the impact distance, measured from the primary black hole. This function is given in Equation (12) and in Table 3 of Lehto & Valtonen (1996).

To illustrate the procedure, let us take the light curve of the 1983 outburst. Figure 4 shows the observed points overlaid by the standard light curve. The value of f is 6, the best-fitting value that is also in agreement with Lehto & Valtonen (1996) timescale. The goodness of fit is judged by minimizing the \( \chi^2 \). The standard light curve is shifted left to right in order to identify the beginning of the outburst by minimizing the \( \chi^2 \) of the difference between observations and the standard curve. This produces a single value for \( t_0 \), the beginning of the outburst.
Table 1

| Time    | Uncertainty |
|---------|-------------|
| 1912.970| ± 0.010     |
| 1947.282| ± 0.0005    |
| 1957.080| ± 0.020     |
| 1972.94 | ± 0.005     |
| 1982.964| ± 0.0005    |
| 1984.130| ± 0.002     |
| 1995.843| ± 0.0005    |
| 2005.74 | ± 0.005     |
| 2007.692| ± 0.0005    |

Note. These are the starting times of the outbursts.

In order to see how uncertain this value is, we have varied the observed flux values by ±1 mJy in a random uniform way. We have also varied the number of points included in the fit, starting from 40 points and adding up to 48 more points to the tail. In this way we get 48 values of $t_0$. Their distribution is centered on 1982.964, and it fits reasonably well with a Gaussian of $\sigma = 0.0004$ year.

The same procedure was applied to the other eight outburst light curves. The results are shown in Table 1. Notice that the error limits are generally a little narrower than in Valtonen et al. (2010b). In that paper only the first section of the standard light curve was used. Thus, the use of the full light curve produces some improvement in timing.

We may also ask how accurately the theoretical model can determine the outburst timing. In the impact model a typical disk crossing time of the secondary black hole is one week. However, events at much shorter timescale can make a difference in the outburst timing, as illustrated by Ivanov et al. (1998). They show that there is a factor of two pressure change over the distance of 1/30 of the disk width ahead of the secondary black hole. Thus, we may consider time steps of ~1/30 of a week as physically meaningful, i.e., the relevant time step is ~0.0005 year. Also, in the gas bubble bursting out of the disk, this same timescale produces a significant amount of evolution, as shown in their Figure 4. We consider this the minimum time step that has an astrophysical relevance.

Another limitation to the timing accuracy comes from the influence of the secondary on the level of the accretion disk. The approaching secondary lifts the disk up and causes an impact earlier than the one predicted in a rigid disk model (Ivanov et al. 1998). The calculation of this effect by particle disk simulations was carried out by Valtonen (2007). In Figure 5, we show the profile of the accretion disk immediately after the impact in the summer of 1912. We note that the disk is lifted toward the approaching secondary and is bent. In the model we need to know the raised level of the accretion disk at the time of the impact. The orbit of the secondary is marked by ticks at intervals of 0.01 year. We estimate that the timing accuracy in this occasion is ±0.005 year. This is the typical accuracy that we may use for impacts at the outer disk (i.e., the 1913, 1957, 1973, 2005, 2015, and 2022 outbursts). For the inner disk the effect is negligible. For example, the 2005 outburst may be timed within ±0.001 year from observations, but such accuracy is not justified by theoretical considerations.

The PN approximation provides the equations of motion of a compact binary as corrections to the Newtonian equations of motion in powers of $(v/c)^2 \sim GM/(c^2 R)$, where $v$, $M$, and $R$ are the characteristic orbital velocity, the total mass, and the typical orbital separation of the binary, respectively. In Valtonen et al. (2010b), the binary black hole was modeled using a spinning primary black hole with an accretion disk and a non-spinning companion. The calculation of the orbit included all the 2PN-accurate non-spinning finite mass contributions as well as the leading order general relativistic (1.5PN order) and classical spin–orbit (2PN order) spinning contributions, and radiation reaction effects (2.5PN order). Here, the terminology 2PN, for example, refers to corrections to Newtonian dynamics in powers of $(v/c)^4$.

For the present work, we incorporated the following three new features in orbit calculation.

The non-spinning finite mass contributions to the binary black hole dynamics are now fully 3PN accurate. This is achieved by adding the 3PN contributions to $d^2x/dt^2$, where $x$ is the
relative separation vector in the center-of-mass frame, as given by Equation (1) in Valtonen et al. (2010a). These 3PN contributions are available in Mora & Will (2004). Second, we let the smaller black hole also spin. Therefore, we add the leading order spin–spin contributions to \( \ddot{x} \), given by Equation (54) in Barker & O’Connell (1979). These contributions appear at the 2PN order. This is a desirable addition due to the fact that the classical spin–orbit coupling, crucial to constraining the value of \( q \), also enters orbital dynamics at the 2PN order. Furthermore, we have also included the contributions due to classical spin–orbit and general relativistic spin–spin interactions in the precession equation for the unit spin vector \( s_1 \) in the present analysis. The compact binary dynamics, employed in the present investigation, schematically reads

\[
\ddot{x} \equiv \frac{d^2x}{dt^2} = \dot{x}_0 + \ddot{x}_{1PN} + \ddot{x}_{SO} + \ddot{x}_{SS} + \ddot{x}_{Q} + \ddot{x}_{2PN} + \ddot{x}_{2.5PN} + \ddot{x}_{3PN},
\]

\[
\frac{ds_1}{dt} = (\Omega_{SO} + \Omega_{SS} + \Omega_{Q}) \times s_1.
\]

The explicit expressions for the non-spinning contributions to \( \ddot{x} \) are listed in Mora & Will (2004). The spin-related contributions to \( \ddot{x} \) and \( \frac{ds_1}{dt} \) are from Barker & O’Connell (1979). The additional spin-related contributions to the dynamics, namely, \( \ddot{x}_{SS}, \Omega_{SS}, \) and \( \Omega_{Q} \), that are not listed in Valtonen et al. (2010b), are given by

\[
\ddot{x}_{SS} = - \left( \frac{3G^3m_1^3}{c^4r^4} \right) \chi_1 \chi_2 \eta \left( (s_1 \cdot n) s_2 - (s_2 \cdot n) s_1 - 5 (s_1 \cdot n) (s_2 \cdot n) n + (s_1 \cdot s_2) n \right),
\]

\[
\Omega_{SS} = \left( \frac{G^3m_2^3}{c^3r^3} \right) \chi_2 \left( 3 (s_2 \cdot n) n - s_2 \right),
\]

\[
\Omega_{Q} = \left( \frac{G^2m_2^2}{c^3r^3} \right) q \chi_1 \left[ \{3(s_1 \cdot n) n - s_1 \} \right].
\]

where the Kerr parameter \( \chi_1 \) and the unit vector \( s_1 \) define the spin of the primary black hole by the relation \( S_1 = Gm_1^2 \chi_1 / c \), while \( \chi_1 \) is allowed to take values between 0 and 1 in GR. A similar rule applies to \( \chi_2 \) in \( S_2 = Gm_2^2 \chi_2 s_2 / c \). The vector \( n \) is defined as \( n \equiv \chi / r \), where \( r = |\chi| \), while \( m = m_1 + m_2 \) and \( \eta = m_1 m_2 / m^2 \).

The effect of including the 3PN corrections to the orbital dynamics is roughly a 1% increase in the estimated mass of the bigger black hole, demonstrating that the employed PN dynamics is in the convergent regime (Valtonen et al. 2010a). Therefore, its influence on the \( q \) estimate is within our desirable error limits.

The fact that the primary black hole is spinning slowly, in other words \( \chi_1 \) is much less than 1, indicates that the spin–spin contributions to \( \ddot{x} \) enter the binary dynamics at the 3PN order. Note that it is the definition of the spin of a compact object, namely, \( S \sim Gm_2^2v^{\text{spin}} / c^2 \), where \( m_2 \) and \( v^{\text{spin}} \) are the typical mass and rotational velocity of the spinning compact object, that makes the spin–spin contributions appear at the 3PN order in our model. Further, the presence of the symmetric mass ratio \( \eta \) as a common factor in the spin–spin corrections (\( \eta \sim 10^{-3} \) in our binary black hole model) ensures that these corrections have only minor effects on the orbits.

The combined effect of higher PN order and the presence of \( \eta \) as a common factor ensures that the leading order spin–spin and classical spin–orbit couplings make negligible contributions to \( s_1 \). The timing experiments also reveal that the change in the orientation of the secondary spin axis does not affect the \( q \) estimates.

In this investigation, we make a third improvement based on astrophysical considerations. In Valtonen et al. (2010b), the black hole spin of the primary black hole was parallel to the accretion disk spin at the initial epoch, which was the year 1856. Due to PN effects, the black hole spin wanders about 9° off
from this direction during its precession cycle that lasts around 1300 years. In the present model, the precession cone axis coincides with the mean accretion disk axis. After performing a number of numerical experiments, we found that it is possible to choose a suitable initial direction for the spin such that the angle between the spin and the disk axes remains constant ($\sim 8^\circ$) during the precessional motion of $s_1$.

It is reasonable to expect such a situation due to the Bardeen-Petterson effect. Because the timescale of the Bardeen-Petterson effect is much longer than the black hole spin precession timescale (Lodato & Pringle 2006), the two directions do not coincide. The timescale of the Bardeen-Petterson effect is of the order of one million years (Natarajan & Pringle 1998, Equation (2.16)) which is intermediate between the spin precession timescale of $10^3$ years and the binary merger evolution timescale of about $10^8$ years (Iwasawa et al. 2011). Thus, we expect that in $10^8$ years the Bardeen-Petterson effect is important up to the distance of about $10^2$ Schwarzschild radii in the disk (Natarajan & Pringle 1998, Equation (2.8)), but the disk can follow only the mean direction of the spin. It cannot keep up with the $10^3$ year evolution of the actual spin.

With the above-mentioned additional features, we have searched for solutions. As before, an automatic search algorithm is used. It takes a trial orbit, then improves it until all nine outbursts happen within their allotted time intervals. Typically, one solution is found in 3 minutes of computing time with a modern PC. We have used sets of 1080 orbits with given standard parameters. However, the convergence was not always found in a reasonable amount of time. Then, the attempt to find a solution was discarded and the next trial was started. For this reason the number of orbits in a set is always fewer than 1080.

In Table 2, we give the set number, the orbit number, and the value of the dimensionless spin of the primary $\chi_1$ in the first three columns, respectively. The spin value was generally taken as $\chi_1 = 0.275$, except in two sets (3 and 9) where a range of $\chi_1$ values was used. The next column in Table 2 gives the value of the secondary spin. The spin $\chi_2$ components are either $-0.5, -0.5, -0.5$ (standard case); 0, 0, 0 (set 6); or $+0.5, +0.5, +0.5$ (set 5). Smaller sets were calculated to ascertain that these three $\chi_2$ values are representative, in a statistical sense, of the different orientations and magnitudes of $s_2$. The last column in Table 2 gives the range of the parameter $q_0$, which is initially uniformly distributed between the limits. The solutions converge to a distribution of $q$ that is narrower than this range. Only in set 3 was a fixed value of $q_0 = 1$ used.

| Set | No. | $\chi_1$ | $\chi_2$ | $q_0$ |
|-----|-----|---------|---------|-----|
| 1   | 1012 | 0.275   | -0.87   | 0.6–1.4 |
| 2   | 864  | 0.275   | -0.87   | 0.0–2.0 |
| 3   | 901  | 0.26 ±0.04 | -0.87 | 1.0 |
| 4   | 362  | 0.275   | -0.87   | 0.6–1.4 |
| 5   | 1009 | 0.275   | +0.87   | 0.6–1.4 |
| 6   | 1017 | 0.275   | 0.0     | 0.6–1.4 |
| 7   | 598  | 0.275   | -0.87   | 0.6–1.4 |
| 8   | 454  | 0.275   | -0.87   | 0.0–2.0 |
| 9   | 914  | 0.26 ±0.03 | -0.87 | 0.6–1.4 |
| 10  | 283  | 0.27    | -0.87   | 0.6–1.4 |
| 11  | 658  | 0.275   | -0.87   | 0.6–1.4 |
| 12  | 1015 | 0.275   | -0.87   | 0.6–1.4 |

Even though $q_0$ is not a physical parameter, but rather an ingredient of the orbit finding algorithm, its proper choice is still important. We initially tried setting $q_0$ far from the value $q_0 = 1$ using either $q_0 = 0$ or $q_0 = 2$ but found that our code was not able to find enough solutions to justify these choices. For example, in the latter case only 23 solutions were found, concentrated around $q_{center} = 1.16$ with a standard deviation of 0.15. Taking the distribution uniformly between these two limits produces more solutions, but mostly in the range between $q_0 = 0.6$ and $q_0 = 1.4$. Therefore, we decided to carry out most of the experiments using this range of $q_0$. However, since it is possible to add some solutions using the wider range of $q_0$, we have sometimes added two sets together, one with the narrower range and the other with the wider range. The distribution of $q_0$ then mimics a Gaussian with a standard deviation of 0.42. The resulting $q$-distribution is always narrower than this, demonstrating that we are not biasing the final $q$-distribution to be unduly compact by our choice of $q_0$.

There were additional conditions in some sets that are not listed in Table 2. In set 4 the outburst uncertainty limits were taken from Valtonen et al. (2010b). They are generally somewhat wider, and some of them are also centered a little differently from the ranges listed in Table 1. (Note that Valtonen et al. 2010b has a misprint; one of the central values they use is 1995.843, not 1995.841). On the other hand, sets 7 and 8 explore the solutions in which one of the intervals is made narrower, i.e., the range of the timing of the 2005 outburst is 2005.74 ± 0.015, five times narrower than in our standard case. In general the initial angle between the disk and the primary spin $\chi_1$ is $8^\circ$, but in set 10 we also tested the case of an initial zero angle. In sets 11 and 12, the $t_0$ in 1995 is shifted down and up by 3.5 hr, respectively.

| Set | $q_{center}$ | Error | $\sigma$ | Error |
|-----|--------------|-------|---------|-------|
| 1   | 1.00         | 0.01  | 0.26    | 0.01  |
| 2   | 1.01         | 0.05  | 0.59    | 0.04  |
| 3   | 1.03         | 0.01  | 0.08    | 0.01  |
| 4   | 1.01         | 0.02  | 0.31    | 0.02  |
| 5   | 0.96         | 0.01  | 0.24    | 0.01  |
| 6   | 0.98         | 0.01  | 0.25    | 0.01  |
| 7   | 0.98         | 0.01  | 0.25    | 0.01  |
| 8   | 0.98         | 0.02  | 0.42    | 0.02  |
| 9   | 1.00         | 0.01  | 0.27    | 0.01  |
| 10  | 0.58         | 0.01  | 0.19    | 0.01  |
| 11  | 0.74         | 0.01  | 0.19    | 0.01  |
| 12  | 1.06         | 0.01  | 0.30    | 0.01  |
| 1+2 | 1.02         | 0.01  | 0.33    | 0.01  |
| 1+2+5–9 | 0.99       | 0.01  | 0.12    | 0.01  |

3. RESULTS

For every set we have constructed the distribution of $q$ values, and since these distributions resemble a Gaussian, we have determined the best-fitting Gaussian parameters, the central value $q_{center}$, and the standard deviation $\sigma$ for each distribution. These are listed in Table 3, together with the errors in each parameter. Figure 6 illustrates one such distribution, a combination of sets 7 and 8.

The Kerr parameter of the primary black hole $\chi_1$ will be constrained by the timing of the outburst in 2015 (Valtonen et al. 2010b). Figure 7 shows the correlation of $\chi_1$ with $t_0$, the
zero point of the 2015 outburst, using set 3. The accuracy of the $\chi_1$ determination, after the 2015 outburst time is known, is $\pm 0.005$ ($1\sigma$). It is likely that OJ287 becomes the “Christmas star” of 2015.

We will now discuss those historical outbursts not employed in finding the orbital solution. We will look for a correlation between the starting times of these outbursts and $q$. We will also ask whether the distribution of $q$ can be narrowed down by future observations.

The first outburst that we studied is the 1934 outburst, with the expected starting time of 1934.3439 if $\chi_1 = 0.275$ and $q = 1$. The correlation of the starting time with $q$ is so weak that this outburst is of no interest in determining $q$. Moreover, there are no data in the historical light curve yet to verify this outburst. The 1935 outburst is no better in this respect. It is expected at 1935.3939, with little correlation with $q$. This outburst has not been verified in the observational record.

The next interesting outburst should have taken place in 1959. Here, we expect quite a strong correlation between the start of the outburst and $q$ (Figure 8).

We see from Figure 8 that the difference in the starting time of the outburst by 15 units corresponds to the range of 0.8 in $q$. Since 15 units in the figure corresponds to 13 hr, with good timing and with 5 hr accuracy, it should be possible to determine the $q$ value at the level of $1 \pm 0.20$ if a fair number of detections are found in the historical plate collections. The current observational situation is depicted in Figure 9.

The next outburst of interest is the 1971 outburst; at present, there exists only one observing point (Figure 10). Figure 11 shows the expected correlation between $q$ and $t_0$. If enough observations are found, it should be possible to determine the $q$ value with accuracy as high as $1 \pm 0.16$ units.

The 1995 outburst was already used in our solution. There was an intensive monitoring campaign of OJ287 (called OJ94)
Figure 8. Correlation between $q$ and the starting time of the 1959 outburst $t_0$ (labeled $t(1959.21..)$) when $\chi_1 = 0.275$. The functional form of the correlation is given inside the figure. There is also a correlation with the $\chi_1$ such that the line of regression is shifted to the right in the figure by one unit for a decrease of $\chi_1$ by 0.015 units. Five units in the time axis correspond to 4.4 hr.

Figure 9. Observation of the brightness of OJ287 at the expected 1959 outburst time. The dashed line is the template from the 2007 outburst, while the squares represent the three observations. The base level has been arbitrarily normalized, as there are not enough data to determine it.

during this outburst season, but unfortunately there exists a gap in these observations at the crucial time (Figure 12). It may still be possible that there are measurements somewhere that are not recorded in the OJ94 campaign light curve, and which would be valuable in narrowing down $q$ even from these data. The line in Figure 12 is drawn using the standard light curve of Figure 3 as a template to compare with the 1995 observations. In set 11 the value of $t_0$ has been shifted down by 3.5 hr, and in set 12 it is shifted up by the same amount. The shifts lead to shifting $q_{\text{center}}$ up or down by $\sim 15\%$. It should be noted that even a few more measurements of 1995 could narrow down the range of $q$.

Let us now turn our attention to the expected future outbursts in our binary black hole model. We expect three more outbursts during the next two decades, occurring in 2015, 2019, and 2022.

As previously mentioned, the 2015 outburst should be easy to detect, as it is expected in December of that year. The exact date will in fact give us a good spin value. The dependence on $q$ is secondary; thus, it is of no use by itself for the testing of the no-hair theorems.

The 2019 outburst is sensitive to $q$, and with $\chi = 0.275$ it should begin at 2019.53175 if $q = 1$ (Figure 13). With good timing the $q$ value is determined with the accuracy of $1 \pm 0.16$ (Figure 14). If by good luck we will find the necessary historical data to time both the 1959 and 1971 outbursts in addition to observing the 2019 outburst, we will get close to the 10% accuracy in $q$ (see the combined sets 1+2+5−9 in Table 3). Improvements in understanding the astrophysical processes in OJ287 may then bring the accuracy to below 10%. Without
accurate timing of 2019, the $q$ value cannot be determined better than to $1 \pm 0.3$ even if the $\chi_1$ determination in 2015 is a success (Figure 6).

However, observing OJ287 during the expected 2019 outburst window, namely, around 2019 July 21, is practically impossible from the ground. This is because the angular distance between the Sun and OJ287 in the sky is only about $12^\circ$ at the beginning of this event, and it goes down to $8^\circ$ by the time of the peak flux. Carrying out the measurement may require space observations. The 2022 outburst is scheduled at practically the same time of year as the 2019 outburst. Obviously, observing this event would also be of interest as it would more narrowly tie down the parameters of the general model. However, it will not give any further information on $q$. The 1922 outburst follows an impact on the outer disk, and these impact timings are not sensitive to $q$.

Finally, we comment on the effect of additional features introduced in the present study. First, the improvement in the timing of the outbursts with respect to Valtonen et al. (2010b) does improve the accuracy of the $q$ determination. Comparing set 4 (wider uncertainty limits of timing) with sets 1, 5, and 6 (all with standard limits) we can see that the standard deviation of the distribution is greater by $\sim 10\%$ when the timing intervals are wider. If the angle between the disk axis and the primary spin axis varies as in Valtonen et al. (2010b), the distribution is not centered on $q = 1$ (set 10), unlike in our standard case. The solutions in Valtonen et al. (2010b) were not numerous enough to reliably detect this effect.

The effect of the $\chi_2$ on the $q$-distribution was calculated in three cases: first with the “normal” direction of the secondary spin $\chi_2$, then with the opposite spin, and finally with zero spin. The three distributions are different, showing that the spin–spin
interaction has influenced the orbits (sets 1, 5, and 6), but the Gaussian parameters of the distributions are only marginally different from one another.

4. THE METHODS OF OBSERVATION OF STARS NEAR THE SUN AND THE POSSIBILITIES OF OBSERVING OJ287 IN 2019

The objects at small angular distances from the Sun (the estimated value for OJ287 is $8^\circ$–$12^\circ$) are difficult to observe due to high background caused by intense sunlight. The vignetting of the highly luminous solar disk enables a reduction of the background and observation of stars near the Sun. Recently, two different methods have been used to observe stars at small angular distances from the Sun, namely the coronagraph method and the helioscopic imager method.

4.1. Coronagraph Method

The coronagraph has already been in use for a long time both in ground-based as well as in space-based observations. The Solar and Heliospheric Observatory’s (SOHO) Large Angle and Spectrometric Coronagraph Experiment (LASCO) C3 can serve as an example of a recent coronagraph in space (Morrill et al. 2006). The SOHO spacecraft has three coronagraphs (LASCO) on board, two of which are still working (C2 and C3). We have used the publicly available images from these experiments to estimate the expected limiting magnitudes for this method. Estimating limiting magnitude for C2 is difficult because most of the LASCO C2 image is obscured by the solar corona, and only a few stars are visible. LASCO C3 covers an area of 32 diameters of the Sun (i.e., about 16°), hence the OJ287 position during the 2019 predicted flare

Figure 12. Observations of OJ287 at the beginning of 1995 November, transformed to optical V band. Altogether, 50 observations have been binned to seven points. Overlaid is the theoretical light curve profile from Figure 5. The zero point of time is at Julian Day (JD) 2450026.65, i.e., at 3:36 hr GMT on 1995 November 6.

Figure 13. Correlation between $q$ and the starting time of the 2019 outburst $t_0$ (labeled $t_{(2019.53)}$) when $\chi_1 = 0.275$. The functional form of the correlation is given inside the figure. There is also a correlation with $\chi_1$ such that the line of regression is shifted to the right in the figure by one unit for a decrease of $\chi_1$ by 0.0125 units. Five units in the time axis correspond to 4.4 hr.
would be covered) and stars are clearly visible in the images, although a large part of the image is also obscured by the solar corona.

There are several problems that complicate the estimation of the limiting magnitude. The dominant one is the stray light, which can be mistaken as background stars. There are also several problems in determining the position and rotation of the spacecraft. One has to align the stars on the image from LASCO C3 and a star chart for the position (AAVSO charts can be used). For the elimination of the stray light, the video that is provided on the SOHO Web sites was used. If there is a point source in five images in a row, we consider it a star. The rotation also complicates determination of the background stars. The best method here is to find a noticeably bright star and to align the image with the chart containing this star and its surroundings.

Pleiades are possibly the best objects to use, because the star cluster clearly defines the rotation. For our work we have used an image of Pleiades where stars show the limiting magnitude of up to 10. This is compared with an image from LASCO C3 (14.05.2010, Sun approaching Pleiades). The faintest stars of Pleiades detectable in the LASCO C3 image are of magnitude 8. At best, we can obtain a limiting magnitude around 8 at the edges of the field where the corona is faint. The result also strongly depends on the state of the corona, as during strong coronal mass ejections (CMEs) the limiting magnitude will be lower.

A deeper magnitude could likely be achieved for a space-based coronagraph with a larger aperture, but this would require an independent feasibility study, as the previous coronagraphs were designed for solar studies, not for photometry of nearby stars.

In an independent study of the C3 limiting magnitude by Andrews (2000) using the same target (Pleiades cluster), deeper limits of magnitudes between 10 and 14 were achieved. As we would require a limiting magnitude of 15 or better for our timing measurement in 2019 July, it is clear that a LASCO C3 type instrument will not be able to do the job.

4.2. The Imager Method

Another method recently used in a space experiment is the method of heliospheric imager, which is part of the SECCHI experiment (Howard et al. 2008; Eyles et al. 2009). The SECCHI experiment is on board the STEREO space mission and consists of five telescopes, which together image the solar corona from the solar disk to beyond 1 AU. These telescopes are an extreme ultraviolet imager (EUVI: 1–1.7 solar radii), two traditional Lyot coronagraphs (COR1: 1.5–4 solar radii and COR2: 2.5–15 solar radii), and two new designs of heliospheric imagers (HI-1: 15–84 solar radii and HI-2: 66–318 solar radii). All the instruments use 2048 × 2048 pixel CCD arrays in a backside-in mode. The EUVI backside surface has been specially processed for EUV sensitivity, while the others have an anti-reflection coating applied.

The HI objectives, like the rest of the SECCHI suite, make visible light observations of CMEs and other structures as they transit from the corona and into the heliosphere. The HI package consists of two small, wide-angle telescope systems (HI-1 and HI-2) mounted on the side of each STEREO spacecraft, which together view the region between the Sun and the Earth. HI has no shutter mechanism, other than a one-shot door that protects the instrument from contamination during ground operations and the launch. Thus, an image is collected in a shutterless mode, in which the intensity at each pixel is an accumulation of the static scene and a smearing of the image during readout. This smearing can be removed on the ground.

The HI instrument concept was derived from the laboratory measurements of Buffington et al. (1996), who determined the scattering rejection as a function of the number of occulters and the angle below the occulting edge. The concept is not unlike observing the night sky after the Sun has gone below the horizon. While the specific concept used here has not been flown before, two other instruments that have flown have validated the ability to measure the electron scattered component against the strong zodiacal light and stellar background. The Zodiacal Light Photometer (Pitz et al. 1976) on the Helios spacecraft, launched...
in 1974, and the Solar Mass Ejection Imager instrument (Eyles et al. 2003), on the Coriolis spacecraft, launched in 2003 have demonstrated that a properly baffled instrument can detect CMEs (Tappin et al. 2003).

The HI-1 and HI-2 telescopes are directed to angles of about 13° and 53° from the principal axis of the instrument, which in turn is tilted upward by 0:33 to ensure that the Sun is sufficiently below the baffle horizon.

The novel heliospheric imagers achieve magnitude limits for stars of about 13–14 in a 40 minute exposure. The HI-1 imager covers the region of 7°:5–24° from the Sun and hence is well suited to observe the OJ287 during the predicted 2019 event (OJ287 is expected to brighten from ~14.3 to ~13 in V magnitude). A specific design for a dedicated space experiment to observe OJ287, optimizing the performance for stellar photometry at angular distances around 10° from the Sun, may lead to a limiting magnitude increase to about 15.

Another possibility involves pointing Long Range Reconciliation Imager in the New Horizons mission to Pluto, which consists of a telescope with a 20.8 cm aperture, at OJ287 for one week in 2019 July. In the 4 × 4 pixel binning mode the limiting magnitude in V is expected to be greater than 17 (Cheng et al. 2008).

4.3. Secondary Science

The OJ287 heliospheric imager is expected to provide, in addition to the OJ287 photometry, valuable data for other scientific fields, such as monitoring of astrophysical targets near the Sun and optical searches for optical counterparts to gamma-ray bursts occurring at small angular distances from the Sun.

5. DISCUSSIONS AND CONCLUSIONS

We have shown that it is possible to test GR at the second PN order using the binary black hole system in OJ287. We find that GR can be confirmed with 1σ accuracy of 30% using the correct observations and theoretical understanding of the system. One of the theoretical conditions is that the rotation axis of the accretion disk is at a constant angle with respect to the precessing spin axis of the primary black hole. It may be possible to verify this in the future by studying the structure of the radio/X-ray jet in OJ287, and by theoretical studies of how the jet direction is determined when the two axes are not parallel to each other. With this proviso, we argue that it should be possible to test, in principle, the black hole no-hair theorem at the 10% level in the current decade by employing the binary black hole model of OJ287.

We have also shown that the third-order PN terms are too small to be detected in the OJ287 system. They depend on the exact value of the primary mass, at the 1% level, as well as on the spin of the secondary, for which it is difficult to find independent measurements at the required level of accuracy.

We demonstrate that the testing at the above precision will require a certain amount of good luck in the sense that there should exist some yet unknown observations in the historical records at certain crucial time windows. Also, it is highly desirable to have space-based optical observations to monitor the impact outburst of OJ287 in 2019. One possibility is to use the New Horizons mission to Pluto, which by 2019 will already be past Pluto.

Employing a suitable solar observing mission to monitor OJ287 could be another option. We have looked at the SOHO coronagraph images and find that the limiting magnitude there is ~8. We would need to get to magnitudes of ~15 in blue or UV, and thus one would need to cover the innermost 5° of the solar image instead of just the solar disk. A better case for the no-hair test would be a continuation or follow-up mission of STEREO, which would for one week concentrate on OJ287 instead of solar flares.

Due to the fundamental nature of the test and the fact that the astrophysical systems associated with the other proposed tests are yet to be observed and likely to be plausible only in the next decade, it may not be out of question to plan a small space mission to monitor the 2019 outburst and hence to test the black hole no-hair theorems.

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Note added in proof. Since this paper was submitted, Bambi (2011) has constrained the q parameter to an interval consistent with our work.

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