Excited Baryon Decay Widths in Large $N_c$ QCD

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We study excited baryon decay widths in large $N_c$ QCD. It was suggested previously that some spin-flavor mixed-symmetric baryon states have strong couplings of $O(N_c^{-1/2})$ to nucleons [implying narrow widths of $O(1/N_c)$], as opposed to the generic expectation based on Witten’s counting rules of an $O(N_c^0)$ coupling. The calculation obtaining these narrow widths was performed in the context of a simple quark-shell model. This paper addresses the question of whether the existence of such narrow states is a general property of large $N_c$ QCD. We show that a general large $N_c$ QCD analysis does not predict such narrow states; rather they are a consequence of the extreme simplicity of the quark model.

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I. INTRODUCTION

Large $N_c$ QCD has proved useful in predicting hadronic properties. This paper explores the important issue of excited hadron lifetimes. The behaviors of mesons and baryons appear to differ radically at large $N_c$. For example, Witten’s well-known large $N_c$ power counting rules specify that the decay widths of mesons scale like $N_c^{-1}$, but the decay widths of baryons above the ground-state band scale like $N_c^0$. Mesons are clearly narrow at large $N_c$; this helps explain why they are visible in the spectrum. However, generic baryons are not narrow at large $N_c$. Why can one still resolve excited baryons? Perhaps the answer is simply that the baryons actually dominating the low-lying spectrum are exceptional states whose decay widths are narrow in the large $N_c$ sense: They vanish in the large $N_c$ limit. In fact, it has been suggested that those baryons transforming under the mixed-symmetric (MS) representation of the spin-flavor group SU($2N_f$) are narrow. If this is true, it is an exciting result; the observed narrow baryons are generally assigned to such a representation in simple quark models. The purpose of the present paper is to investigate whether the narrowness of such states is, in fact, a consequence of large $N_c$ scaling in QCD.

We begin with a brief discussion of the relevant background. A seminal development in the study of large $N_c$ ground-state baryons was the derivation of consistency conditions that constrained their coupling to pions. These consistency conditions were found by examining pion-baryon scattering. A contracted SU($2N_f$) spin-flavor symmetry emerges from this analysis, in which the ground-state band of baryons fills a completely symmetric (S) representation; it links various quantities such as baryon axial-vector couplings, magnetic moments, and masses. Recent interest has focused on its application to the study of the $\Delta$ and the $N-\Delta$ transition.

The success of the consistency condition method for describing ground-state baryons naturally led to the question of whether excited baryons could be understood in an analogous fashion. Pirjol and Yan (PY) developed just such an approach. However, there is an important distinction between the treatments of ground-state baryons and their excited cousins. The scattering of pions off ground-state baryons is physically realizable at large $N_c$. On the other hand, the excited baryons are resonances, so it is not immediately obvious how one can formulate a pion-excited baryon scattering problem: The excited states decay and cannot be used as targets in scattering experiments. Thus, the consistency condition approach is only applicable if there exist excited baryons in the large $N_c$ world that are indeed narrow. PY tacitly assumed that such baryons exist. As such, their general model-independent predictions are not strictly valid. However, if there turns out to be a class of narrow excited baryons, then the model-independent arguments can be applied to these in a legitimate way. Thus, the existence of a class of narrow baryon states at large...
$N_c$ is also of importance in providing a theoretical justification for the elegant model-independent analysis of Ref. \cite{3}, as applied to at least some states.

Interestingly, Ref. \cite{3} itself provided an argument that there exists a class of narrow excited baryons at large $N_c$. It finds that the baryons in the MS representation of the spin-flavor group have widths of $O(N_c^{-1})$, which thus vanish at large $N_c$. This is in contrast to the generic large $N_c$ counting rule prediction, in which the widths are $O(N_c^0)$. Unfortunately, the prediction of narrow decay widths for the MS states in Ref. \cite{3} was not based directly on large $N_c$ consistency rules. Rather, it arose from calculations done in the context of a simple, nonrelativistic quark model. The particular model employed was the quark-shell model to be described in Section II. Of course, quark models have a long and distinguished history of successful phenomenology. In the large $N_c$ context, quark models have been used to describe the lowest-lying excited baryon states \cite{20, 21, 22, 23, 24, 25, 26}, and have revealed interesting mass degeneracy patterns \cite{8, 15, 27}. Recently, it was shown that the quark-shell model is compatible with the more realistic picture of excited baryons as resonances in meson-baryon scattering, in the sense that both describe the same mass and width degeneracy patterns \cite{15}.

The question of interest in this paper is whether the prediction in Ref. \cite{3}, that there exist narrow excited baryon states, is a direct consequence of large $N_c$ QCD. It is useful to note that a large number of the model-independent relations found \cite{4, 5, 6, 8, 9, 10, 11} were originally seen in the context of soliton models \cite{28}, and indeed all of these hold for quark models. Thus, the issue is whether the existence of narrow excited baryon states found in the quark-shell model of Ref. \cite{3} similarly indicates a general large $N_c$ result. If so, this is an important general result in understanding excited baryons. In contrast, if the prediction is merely an artifact of the particular choice of model, then it is of far less import. In this context, it is important to note that the model used in Ref. \cite{3} is not completely general. It is limited in that it does not include admixtures of different single-particle descriptions (i.e., it neglects configuration mixing).

The issue of whether the MS excited baryons described in the simple quark-shell model correspond to physical states in large $N_c$ QCD is not addressed in Ref. \cite{3}. If the states are physical, then a previously unrecognized symmetry in large $N_c$ QCD is manifesting itself and prevents the states from decaying rapidly. In particular, the narrowness depends upon a symmetry beyond the contracted SU(2$N_f$) spin-flavor symmetry deduced in Refs. \cite{4, 5, 6, 8, 9, 10, 11}. In those works the “spin” in “spin-flavor” corresponds to the total angular momentum of the baryon state in its rest frame. As shown below, the narrow states predicted in Ref. \cite{3} depend on two distinct spin-flavor symmetries: one as before, in which the spin corresponds to the total angular momentum of the baryon state in its rest frame, and a second one in which the spin is purely associated with the spin of the quarks. While these two symmetries are identical for a quark-shell model emerging in the large $N_c$ limit, they differ for more general models. The prediction of a new symmetry emerging in the large $N_c$ limit for excited states is certainly exciting; the issue, however, is whether it is real.

In this paper, we show that the seemingly narrow excited baryons are in fact not a feature of large $N_c$ QCD; they are merely artifacts of the simple quark model used. Our paper is organized as follows: We review the construction of baryon states in the quark-shell model and the significance of key matrix elements in Ref. \cite{3}. Next, we use a quark-shell model Hamiltonian to show that the states used in Ref. \cite{3} are not physical in large $N_c$ QCD. In fact, the physical states are superpositions of both S and MS representations of spin-flavor.

II. CONSISTENCY CONDITIONS AND THE QUARK-SHELL MODEL

Pirjol and Yan’s analysis \cite{3} of excited baryons proceeds in a manner that is formally similar to that of Dashen, Jenkins, and Manohar \cite{3}. Pions are scattered off excited baryons, and the large $N_c$ counting rules are enforced for the total scattering amplitude. Any analysis of a conceivable scattering event presupposes that the target is stable; in Ref. \cite{3}, the target was a p-wave baryon, and it was taken to be narrow at large $N_c$. This is a tenuous assumption unless the quark mass is taken to be large, in which case the baryon has no possible decay channels due to phase space limitations. For the sake of argument, we implicitly work in such a world in the following and assume that results obtained in such a world can be safely extrapolated back to the physical world of light quarks. As noted in Section I, it is by no means clear that such a procedure is legitimate, since the consistency conditions cannot really be formulated for states that are unstable at large $N_c$. However, for the present purpose this procedure is adequate.

States may be narrow for one of two reasons: Either the phase space for decay is small (or zero), or the meson-baryon coupling is small. The claim of Ref. \cite{3} is that MS states have a meson-baryon coupling that goes as $N_c^{-1/2}$, and thus even when the phase space is of $O(N_c^0)$, it still produces narrow resonances. If the coupling turned out to be truly $O(N_c^{-1/2})$ as a result of general large $N_c$ physics arguments, this counting would be expected to hold regardless of whether the quark mass were taken to be light or heavy enough to suppress the phase space for decay. Conversely, if one can show even in this world of heavy quarks that the coupling to would-be decay channels (which are now phase
space suppressed) is generally $O(N^2)$ and not $O(N^{-1/2})$ for all low-lying states in the spectrum, then it is clear that large $N_c$ arguments by themselves do not predict a class of weakly-coupled states.

The analysis of PY is of two parts: i) A set of consistency conditions for couplings of excited baryons to mesons (analogous to the consistency conditions for ground-state baryons) is derived. Functional forms of relations that solve these conditions are proposed and then verified. ii) A simple nonrelativistic quark-shell model is used to motivate (analogous to the consistency conditions for ground-state baryons) is derived. Functional forms of relations that solve these conditions are proposed and then verified.

In the model-independent section, the aforementioned consistency conditions were determined by imposing Witten’s counting rules on the following scattering processes at large $N_c$:

\[
\pi^a + B(\text{s-wave}) \rightarrow \pi^b + B'(\text{s-wave}),
\]

\[
\pi^a + B(\text{p-wave}) \rightarrow \pi^b + B'(\text{p-wave}),
\]

\[
\pi^a + B(\text{p-wave}) \rightarrow \pi^b + B'(\text{s-wave}),
\]

where s-wave refers to the ground-state band of baryons modeled as having all quarks in a spatial s-wave; p-wave refers to excited states that have quantum numbers consistent with a single quark excited into a p-wave orbital. The $a$ and $b$ are isospin indices.

At each pion-baryon vertex, the baryon axial-vector current is derivatively coupled to the pion field:

\[
\langle B'|q\gamma^\mu\gamma_5\tau^a q|B \rangle \frac{\partial^\mu \pi^a}{f_\pi} = N_c \langle B'|X^{ia}|B \rangle \frac{\partial^\mu \pi^a}{f_\pi},
\]

where the current matrix element is parameterized in terms of an irreducible tensor operator $X^{ia}$ and an explicit power of $N_c$. An $O(N^0)$ coupling $g(X)$ usually included on the right-hand side of this equation is absorbed into the definition of $X$ for convenience. The scattering amplitude for the two leading-order tree-level diagrams for Eq. (2.1) is $A \sim (N_c/f_\pi)^2[X^{jb}, X^{ia}]$ and thus is naively $O(N_c)$ since $f_\pi \sim O(N_c^{1/2})$. Such scaling contradicts the Witten $N_c$ power-counting prediction (as well as unitarity at large $N_c$), which require it to be $O(N^0)$; one is led to the conclusion that $[X^{jb}, X^{ia}] = 0$ in the large $N_c$ limit. The vanishing of the commutator is the leading-order consistency condition for ground-state baryons and is the key to the contracted SU(4) algebra.

This procedure can be extended to the process in Eq. (2.2) if the current matrix element is parameterized in terms of a new operator $Z^{ia}$:

\[
\langle B'|q\gamma^\mu\gamma_5\tau^a q|B \rangle = N_c \langle B'|Z^{ia}|B \rangle.
\]

As in the above case, the scattering amplitude apparently diverges at large $N_c$ in the absence of cancellations, and thus consistency requires that

\[
[Z^{jb}, Z^{ia}] = 0.
\]

in the large $N_c$ limit. This condition is analogous to that for $X^{ia}$, meaning that solutions for $Z^{ia}$ also fill irreducible representations of a contracted SU(4) algebra. These representations can be labeled by the magnitude of a spin vector $\vec{\Delta}$ such that $\vec{\Delta} = \vec{I} + \vec{J}$ (but only in the sense that allowed eigenvalues of $\vec{\Delta}$ are determined by the vector addition rule; indeed, Ref. uses a relative minus sign in this definition). Note that this operator (denoted $\vec{K}$ in Ref. ) has a very simple interpretation in terms of chiral soliton models, in which case $\vec{\Delta} = \vec{I} + \vec{J}$ in a true vector operator sense, as one has from studies of a canonical hedgehog configuration, where the combined operator is called the “grand spin.”

To extend this procedure to the process in Eq. (2.3), one must introduce two new operators, $Y^a$ and $Q^{ij, a}$, in order to parameterize the current matrix elements between an s-wave and p-wave baryon:

\[
\langle B'|q\gamma^\mu\gamma_5\tau^a q|B \rangle = N_c^{1/2} \langle B'|Y^a|B \rangle,
\]

\[
\langle B'|q\gamma^\mu\gamma_5\tau^a q|B \rangle = N_c^{1/2} q^i \langle B'|Q^{ij, a}|B \rangle,
\]

where $q^i$ is the momentum of the current and $|B\rangle$ indicates the ground-state baryon. These expressions differ from those in Ref. by the absorption of possible additional $N_c$ powers and coefficients $g(Y, Q)$ into the right-hand sides, which can be accommodated by explicit rescaling of $Y$ and $Q$. The scattering amplitude for Eq. (2.2) still violates the Witten power-counting prediction if the $Y$ and $Q$ matrix elements scale as $N_c^{-1/2}$ or larger (note that generic Witten counting rules suggest that $Y$ and $Q$ scale as $N_c^0$), and in these cases consistency requires that

\[
X^{ia}Y^{jb} - Y^{jb}Z^{ia} = 0 \quad \text{and} \quad X^{bj}Y^{a} - Y^{a}Z^{bj} = 0,
\]

\[
X^{ia}Q^{jk, b}| - Q^{jk, b}Z^{ia} = 0 \quad \text{and} \quad X^{bk}Q^{ij, a} - Q^{ij, a}Z^{bk} = 0.
\]

The set of consistency conditions in Eqs. (2.5), (2.8), (2.9) form the basis of Pirjol and Yan’s model-independent analysis. Matrix elements of the operators $Z, Y,$ and $Q$ between baryon states $|J, J_3, I, I_3, \Delta\rangle$ can be found by solving
where the fact that \(|B'\rangle\) is a ground-state baryon imposes the condition \(J' = J'\). The constant \(g_Y\) encodes the overall strength, including any overall nontrivial \(N_c\) scaling. If the system scales according to the generic Witten rule, then \(g_Y \sim N_c^0\). If for some special class of states the coupling is characteristically smaller, then \(g_Y\) is smaller than \(O(N_c^0)\).

Reference [3] calculates the matrix elements in the quark-shell model and finds the same spin-flavor structure as given by Eq. (2.10), regardless of the symmetry of the state \(|B'\rangle\). However, the \(N_c\) dependence of the coefficient \(g_Y\) in the quark-shell model is found to depend upon the symmetry of the excited states, with \(g_Y \sim N_c^0\) for spin-flavor S states and \(g_Y \sim N_c^{-1/2}\) for MS states. It is this scaling for the MS states that leads to the prediction of narrow baryon resonances.

In the simple quark-shell model of Ref. [3], each excited baryon is treated as a single orbitally excited quark with angular momentum \(\ell\) acting on top of a spin-flavor symmetric core of \(N_c - 1\) quarks. The Pauli principle requires that the complete wave function describing the baryon is antisymmetric under the exchange of any two quarks. The singlet color wave function is fully antisymmetric; accordingly, the space and spin-flavor wave functions together must be symmetric. Only symmetric or mixed-symmetric spatial wave functions can be constructed when a single quark is excited. Therefore, the spin-flavor wave functions of the quark-shell model states are either symmetric or mixed-symmetric under exchange. It is worth noting that in the present context “spin-flavor” refers to the spin and flavor of the quarks and not of the baryons.

We focus on the nonstrange states of SU(4), in which the distinction between the MS and the S representations is clean: Spin and isospin are related by \(S = I\) for the S case and \(|S - I| \leq 1\) for the MS case. This distinction may be neatly encoded by introducing the the concept of P-spin [3], with \(P = 0\) for the S case and \(P = 1\) for the MS case. Thus, treating \(\bar{B}\) as though it were a true angular momentum, one sees that the single triangle rule \(\delta(SIP)\) characterizes both permutational symmetries.

The matrix elements of \(Z, Y,\) and \(Q\) in the quark model are obtained by defining the currents and constructing the baryon states in quark model language. Up to overall multiplicative constants of order unity, the currents in the quark model are:

\[
N_c Z^a = \sigma^i \otimes \tau^a, \quad (2.11)
\]

\[
N_c^{1/2} Y^a = \frac{1}{\sqrt{3}} \sum_{j=-1}^{+1} (-1)^{1-j} \sigma^j r^{-j} \otimes \tau^a, \quad (2.12)
\]

\[
N_c^{1/2} Q^{ka} = \sum_{i,j=-1}^{+1} \langle 2, k|1, j; i \rangle \sigma^j r^{-1} \otimes \tau^a. \quad (2.13)
\]

The \(\sigma, \tau,\) and \(r\) operators act on the quark’s spin, isospin, and orbital degrees of freedom, respectively.

Reference [3] constructs quark model states of good (quark) spin \((S, m_S)\), isospin \((I, m_I)\), and orbital angular momentum \((\ell, m_\ell)\) in such a manner that each is either symmetric or mixed-symmetric under spin-flavor. It is not clear at the outset how such states should be interpreted. Either these states may be taken to be eigenstates of some unspecified Hamiltonian \(\mathcal{H}\) that is assumed to model QCD, or they may be taken to be merely a convenient basis encoding the overall \(N_c\) symmetry that makes these states stable. This ostensibly symmetry arises because \(\mathcal{H}\) commutes separately with the (quark) spin and with the total angular momentum.

Note that if the states are merely used to form a basis, then one is faced with the issue of determining the scaling of mixing between the basis states. If each physical state is predominantly a single basis state (with admixtures of other states characteristically suppressed in the large \(N_c\) limit), the system then acts much as it would for the case where the states are treated as eigenstates of a Hamiltonian that mocks up QCD: The physical eigenstates that are predominantly mixed-symmetric are then narrow. In contrast, if the mixing is \(O(N_c^0)\), then the concept of a state that is predominantly mixed-symmetric is ill-defined, and all states allowed by phase space have widths that go as \(N_c^0\).

It should also be observed that the quantum numbers \(\{P, S, m_S, I, I_3, \ell, m_\ell\}\) denoting these states are different from those used in the (model-independent) consistency condition method, \(\{J, J_3, I, I_3, \Delta\}\). In particular, it should
be noted that the there is no analog for the $P$ quantum number (which specifies the nature of the spin-flavor symmetry of the quark model state) in the model-independent analysis. This raises the obvious question of whether or not the concept of the $P$-spin has a well-defined meaning at large $N_c$ outside the context of the simple quark model.

The matrix elements of the above currents can be calculated with the quark model states described above. A lengthy calculation in Ref. 3 revealed the $N_c$ scaling of the matrix elements of $Y$ and $Q$ as well as the detailed spin-flavor structure; it was found that

$$\langle P' = 0, \ell' = 0 | Y, Q | P = 0, \ell \neq 0 \rangle \sim N_c^0,$$  \hspace{1cm} (2.14)

while

$$\langle P' = 0, \ell' = 0 | Y, Q | P = 1, \ell \neq 0 \rangle \sim N_c^{-1/2},$$  \hspace{1cm} (2.15)

where the states are labeled by their $P$-spin and orbital angular momentum.

The excited baryon decay widths are determined by squaring the matrix elements of the physical states and dividing by the pion decay constant $f_\pi^2$ and including the appropriate phase space factor. Using that $f_\pi^2 \sim O(N_c)$ and the phase space is $O(N_c^0)$, it is straightforward to determine the scaling of the decay widths. If one assumes that quark model assignments of states correspond to physical states (with only small admixtures of states of different quark model symmetries), then the decay width of an MS baryon ($P = 1$) is $O(N_c^{-1})$, while the decay width of an S baryon ($P = 0$) is of order $N_c^0$. In the large $N_c$ limit, the former vanishes. This result is of import; it implies that MS excited baryon states are narrow. This would neatly explain the phenomenological fact that certain baryons, like mesons, are narrow enough to discern and would render the consistency argument of Ref. 3 valid. However, these desirable results depend on the physical baryon states corresponding to the quark model states in terms of their quantum numbers. Thus, one must face the question of whether they do.

III. SPIN-FLAVOR SYMMETRY BREAKING AND BARYON WIDTHS

In this section we focus on the issue of whether the physical states truly correspond to the simple quark model states of Ref. 3, which in effect is the question of whether narrow excited baryons are realized in large $N_c$ QCD. If this result is generic to large $N_c$ QCD, one would expect it to be seen in all models that correctly encode large $N_c$ physics. Thus, to disprove it we need only find some model that encodes the correct large $N_c$ scaling rules for which it is untrue. Here we consider a fairly general quark-shell model Hamiltonian that shares some essential properties with the QCD Hamiltonian. In particular, we consider the most general quark model for which the number of quarks in a given orbital is well defined. In practice, this restriction means one excludes operators that remove quarks from one orbital and place them in different orbitals. We impose this restriction to keep the model tractable. Note, however, that even with this restriction, this Hamiltonian is considerably more general than the Hamiltonian implicitly used to construct the states in the previous section.

Large $N_c$ scaling rules greatly restrict the number of possible operators that contribute at $O(N_c^0)$ or larger (and hence that can contribute in the large $N_c$ limit $\text{(2.6)-(2.11)}$). For example, as shown in Ref. 20 21, the only operators that contribute at $O(N_c^0)$ for states with a single excited quark are given by:

$$\mathcal{H} = c_1 I + c_2 \ell \cdot s + c_3 \ell(2) g G_c / N_c,$$  \hspace{1cm} (3.1)

where the $\ell$, $s$, $\ell(2)$, and $g$ are the orbital, spin, $\Delta \ell = 2$ tensor, and combined spin-flavor (Gamow-Teller) operators, respectively, acting only on the excited quark, while $G_c$ is the combined spin-flavor operator acting only on the core of unexcited quarks. The coefficients have the following scaling rules:

$$c_1 \sim N_c^0, \hspace{1cm} c_2 \sim N_c^0, \hspace{1cm} c_3 \sim N_c^0.$$  \hspace{1cm} (3.2)

The scaling of $c_1$ is a bit subtle. Most of the contribution to $c_1$ comes from the unexcited quarks in the core. Thus $c_1 = M_N + \delta c_1$ with $\delta c_1 \sim N_c^0$. In general, each coefficient contains corrections at all subleading powers of $N_c$.

Consider, as an example, the operator associated with $c_2$. If this operator induces significant $[O(N_c^0)]$ mixing between the $S$ and MS states of the basis described above, then this model—which correctly encodes the large $N_c$ scaling rules—does not automatically give excited baryons that are weakly coupled at large $N_c$. To begin, note that the $\ell \cdot s$ term does not commute with the spin operator $S$: i.e., $m_S$ is not generally a good quantum number for the Hamiltonian eigenstates. Thus, the operator does induce mixing between the states enumerated above. The central question becomes the scale of this mixing.

Suppose one considers only states for which the excited quark is in an orbital with $\ell \neq 0$ (The case of $\ell = 0$ is special and is discussed below). This implies that the $\ell \cdot s$ operator mixes states of different spin-flavor symmetry.
Consider a state labeled by total angular momentum \((J, J_3)\), total isospin \((I, I_3)\), total (quark) spin \((S)\), and \(P\)-spin: \([JJ_3; II_3(S, I = I + \rho)](P)\). The \(\rho\) (introduced in Refs. \[21\]) plays a role similar to the \(P\)-spin of Sec. \[11\]. It is a number that equals either \(+1\) or \(-1\) for the mixed-symmetric case \((P = 1)\), or \(0\) for the symmetric case \((P = 0)\). The states so labeled are identical to the ones enumerated in Sec. \[11\].

The matrix element of \(\ell\cdot s\) that connects two states in this basis of equal \(JJ_3\), \(II_3\), and \(\ell\) but different \(P\)-spin \((\text{i.e.,} \, \text{symmetry})\) is written as \(\langle \ell \cdot s \rangle_\rho = \langle JJ_3; II_3(\ell, S' = I + \rho')|\ell \cdot s| JJ_3; II_3(\ell, S = I)|0\rangle\). We follow the methods of Ref. \[21\], but note a more concise expression than their Eq. (A7):

\[
\langle \ell \cdot s \rangle = (-1)^{J-I+\ell} \sqrt{\frac{3}{2}} \sqrt{\ell(\ell+1)(2\ell+1)(2S+1)(2S'+1)} \left\{ \begin{array}{ccc} \ell & S & 1 \\ \ell' & S' & I \end{array} \right\} \sum_{s=\pm 1} c_{\rho \rho'} (\ell(1-s^{1-\eta})/2 \left\{ \begin{array}{ccc} 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & S' & S \end{array} \right\} + \left\{ \begin{array}{ccc} 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & S & S \end{array} \right\}).
\]

From here, we set \(\rho = 0\) (meaning that \(S = I\) for the \(S\) state in the ket) and calculate this matrix element for each value of \(\rho'\) in the mixed-symmetric bra. For \(\rho' = 0\), we have:

\[
\langle \ell \cdot s \rangle_0 = (-1)^{J-I+\ell+1} \sqrt{\frac{3}{2}} \sqrt{\ell(\ell+1)(2\ell+1)(2S+1)} \left\{ \begin{array}{ccc} \ell & S & 1 \\ \ell' & S & I \end{array} \right\} \left\{ \begin{array}{ccc} 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & S & S \end{array} \right\} + \left\{ \begin{array}{ccc} 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & S & S \end{array} \right\}. \tag{3.3}
\]

The coefficients \(c_{0-}^{MS}\) and \(c_{0+}^{MS}\) are given by

\[
c_{0-}^{MS} = -\sqrt{\frac{(S+1)(N_c - 2S)}{N_c(2S + 1)}},
\]

\[
c_{0+}^{MS} = +\sqrt{\frac{S(N_c + 2(2S+1))}{N_c(2S+1)}}. \tag{3.5}
\]

For \(\rho' = \pm 1\), we have:

\[
\langle \ell \cdot s \rangle_{\pm 1} = (-1)^{J-I+\ell+1} \sqrt{\frac{3}{2}} \sqrt{\ell(\ell+1)(2\ell+1)(2S+1)(2S+1 \pm 2)} \left\{ \begin{array}{ccc} \ell & S & 1 \\ \ell' & S & I \end{array} \right\} \left\{ \begin{array}{ccc} 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & S \pm 1 & S \end{array} \right\}. \tag{3.6}
\]

In general, evaluation of these formulas for appropriate values of \(S, I, J,\) and \(\ell\) yields nonzero values. The nonvanishing of MS \(\rightarrow\) MS matrix elements is confirmed in Refs. \[21\]. Moreover, in all of these cases the matrix element of \(\ell \cdot s\) is \(O(N_c^0)\). Combining this with the scaling of Eq. (4.2) for the strength of the operator, implies that the \(\ell \cdot s\) term in the Hamiltonian connects states of different spin-flavor symmetry with strength of \(O(N_c^0)\). Due to this mixing, an energy eigenstate of the quark-shell model cannot be described as having a well-defined spin-flavor symmetry. Thus, for this class of models there is no special set of weakly-coupled excited baryons at large \(N_c\) (at least for \(\ell \neq 0\)). Since these models encode generic large \(N_c\) scaling, one concludes that generic large \(N_c\) scaling rules by themselves do not imply that a set of weakly coupled states exists. Returning now to a world where the quarks are light enough that the decays are permitted by phase space \((\text{which is } O(N_c^0))\), one concludes that there is no generic argument why such states should be narrow.

Note that an analogous argument could be made using the \(\ell^{(2)} g G_c /N_c\) term in the Hamiltonian, which also leads to mixing of \(O(N_c^0)\). One might wonder whether there is some way to evade this conclusion by having some type of cancellation between the \(\ell^{(2)} g G_c /N_c\) and \(\ell\cdot s\) terms. However, the two terms generically have nothing to do with each other; their ratio is not fixed by large \(N_c\) arguments. Moreover, although the operators commute at leading order, they are distinct—their matrix elements are not proportional to each other, even at leading order \[12\ 20\ 21\ 27\].

Hence, the only way for them to cancel generally is if they are both zero.

The conclusion that S and MS configurations are mixed in models of this type is valid for all cases except where the excited quark is in an \(\ell = 0\) orbital. However, it is clear that all of the matrix elements of \(\ell \cdot s\) \((\Delta \ell = 1)\) and \(\ell^{(2)} g G_c /N_c\) \((\Delta \ell = 2)\) between states with \(\ell = 0\) are zero. Thus, the argument presented above does not exclude the possibility of weakly coupled, and hence narrow, \(\ell = 0\) MS excited states. We note, however, that the quark model considered in this paper, though more general than that implicitly used to construct the basis states, is by no means the most general one that one can consider. In particular, one can consider models with configuration mixing—\(\text{that is, in which the physical states are admixtures of different single-particle descriptions}\). Such operators can induce admixtures between the S and MS states at \(O(N_c^0)\). It is easy to see how this can come about. An allowable operator can mix a state with a quark in an excited \(\ell = 0\) orbital and a state with a quark in an \(\ell = 1\) orbital that has total angular momentum (spin plus orbital) equal to \(1/2\). Such mixing violates no symmetries of the system and is allowable at \(O(N_c^0)\). Once such a state admixes with the \(\ell = 1\) orbitals, the previously considered operators induce mixing between the S and MS spin-flavor components.
The case of $\ell = 1$ presents its own subtlety, the well-known problem in many-body physics of spurious modes associated with broken symmetries [29]. In fact, the existence of spurious modes does not alter the conclusions drawn above for the $\ell = 1$ case, as discussed in Appendix A.

IV. CONCLUSION

In this paper we have explored the issue of whether excited baryons with mixed-symmetric spin-flavor wave functions have decay widths that vanish in the large $N_c$ limit. As noted earlier, if previous claims that a set of states are automatically narrow at large $N_c$ were in fact generic, it would be of real significance both theoretically and phenomenologically. On the other hand, the assertion that such states are narrow is based on calculations with a very simple quark model. In this work we showed that the purported narrowness of these states is an artifact of the simple quark model used in the calculations and not a generic feature of large $N_c$ QCD. This was shown in the context of a slightly more general class of quark models that encode generic large $N_c$ scaling rules by a demonstration that excited baryons cannot be assigned a well-defined, fixed spin-flavor symmetry; the symmetry configurations are admixed at $O(N_0^0)$. This implies that the relative narrowness of baryon states observed in nature cannot be simply attributed to large $N_c$ scaling behavior. It also implies that the general model-independent analysis of Ref. [2] is not strictly correct: Without a scattering target of narrow states, the large $N_c$ consistency condition analysis is not applicable. Fortunately, many of the conclusions of this analysis remain correct despite this, such as the predicted pattern of degeneracies [18].

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APPENDIX A: THE ROLE OF SPURIOUS MODES

The quark-shell model possesses degrees of freedom associated with the motion of all of the particles. The total motion may always be separated into center-of-mass (c.m.) motion and internal motion. Moreover, since the underlying dynamics is translationally invariant, the c.m. motion may always be quantized in states of good total momentum. Thus, in principle for a quark model with $N_c$ quarks, the internal dynamics is only associated with the $N_c - 1$ displacements from c.m. Since the model for the excitation spectrum as written includes the positions of all $N_c$ quarks as explicit degrees of freedom, there is in principle redundant information. This raises the obvious question of how one can separate the motion of the c.m., which is spurious from the point of view of internal dynamics, from the internal dynamics of interest. Since the spurious c.m. motion is vectorial in nature—it is associated with the total momentum $\vec{P}$—one expects that it manifests itself only in channels that transform vectorially, which means the $\ell = 1$ channels.

The issue of how the spurious motion is dealt with depends in part on how the model under consideration is derived. One may view the quark-shell model used in this paper as being obtained from some underlying translationally-invariant quark model with $N_c$ quarks undergoing mutual interactions. Traditionally, one approximates this model with some type of self-consistent single-particle potential model state such as the Hartree-Fock state [29]. The idea is to choose a single-particle description that is optimal in the sense of capturing the maximum amount of the underlying physics. One then attempts to include systematically the physics excluded by the single-particle description. Note that this single-particle description necessarily breaks translational invariance, since one cannot have a single-particle description with nontrivial internal dynamics that simultaneously is an eigenstate of the c.m. momentum (since the remaining degrees of freedom in this approach remain inert). From the perspective of such a model state, the spurious c.m. motion is now associated with the fact that the approximation scheme breaks translational symmetry.

It is obvious that if one treats the internal dynamics exactly, then any “excitation” of the spurious c.m. motion on top of some internal state does not alter the internal dynamics and necessarily leads to a state whose internal energy is degenerate with the original state. There are certain approximation schemes that automatically give zero-energy excitations for motion associated with the spurious c.m. motion [29]. Such approximations are referred to as...
"conserving approximations" if they provide such an order-by-order decoupling in the approximation scheme. An example of such an approximation is an RPA treatment above a Hartree-Fock trial state [20].

Unfortunately, the quark-shell model does not correspond to a truncation of a conserving approximation of an underlying translationally invariant model at some consistent order. Rather, it is the form one obtains via the truncation of a Tamm-Dancoff type expansion; such expansions are not conserving in an order-by-order sense. A true separation of the spurious motion from the internal motion occurs if one includes all possible spatial orbitals (including the continuum) and all possible N-body forces (and if the coefficients of all of these are obtained in a consistent way from the underlying translationally-invariant system). This is not done for practical reasons; the system so obtained is not computationally tractable. The effect of the ad hoc truncation is the contamination of the physically interesting physics with unphysical spurious motion. To the extent that the truncation is not too severe, such contamination is modest in that the physically interesting mode only has small admixtures of the spurious motion, and the predicted physical motion is accurate to good approximation.

There is one obvious drawback to such a scheme, apart from the necessary numerical inaccuracy induced by truncation. The Hilbert space contains both physical and spurious motion, and the final answers contain mixtures of both. One hopes that one set of modes is mostly physical (and can be identified with the physical result), while the other set is mostly spurious and can be discarded. One therefore needs some method to discern which set is which. The obvious approach is to perform the calculation and a posteriori identify the modes that are largely spurious. One natural way to identify them is to pick out anomalously low-lying modes, recalling that true spurious modes correspond to zero-energy excitations.

The problem of spurious modes as discussed here is generic in quark-shell models, and in principle is totally divorced from the issue of relevance to this paper, that of whether the S and MS states mix strongly in forming the physical states. One should follow the strategy given above: Using the model, calculate all of the \( \ell = 1 \) modes and discard the ones that are mostly spurious. The general arguments given in Sec. III show that the states so generated have strong mixing, and this is sufficient for our purpose. One expects the physical \( \ell = 1 \) state to exhibit strong mixing between states of different spin-flavor symmetries, and hence one expects the states to have widths of \( O(N_c^0) \).

However, there is one exceptional situation in which these two issues appear to be related. Consider a simple single-particle quark model with orbitals given by harmonic oscillator states. Since this model has no spin-dependent interactions, the quark spin is a good quantum number, and states may be labeled according to spin-flavor. It is a straightforward exercise to show that the spin-flavor S state associated with a single quark in the lowest p-wave orbital is entirely associated with spurious c.m. motion [20]. The underlying mathematical reason is simply that the lowest harmonic oscillator p-wave state is proportional to \( \vec{r} \) times the ground state. Based on this experience, one might be under the impression that the spurious mode is generally the lowest p-wave S state. This is not the case. In fact, even for the simple case with harmonic oscillator orbitals it is clear that separation into spurious and physical modes is not dynamically correct. The spurious \( \ell = 1 \) modes in such a model are degenerate with the physical \( \ell = 1 \) modes rather than, as one might expect, with the ground state. Indeed, the spurious modes in this model correspond to the c.m. oscillating harmonically with the excitation frequency. Physically, of course, the c.m. moves with a constant velocity.

Thus, we conclude that our previous arguments are valid even for \( \ell = 1 \) states. We note, however, that the entire issue is moot. In the end, the problem of spurious states is a disease of quark-shell models. They appear as translationally-invariant quark models with ad hoc truncations, and the disease is associated with the truncations. Of course, we introduced these models following the treatment of Ref. 3, and then generalized the models to show that even quark-shell models have \( O(N_c^0) \) mixing between different spin-flavor symmetry classes. Such models are computationally tractable. Of course, in principle one may use any model consistent with large \( N_c \) scaling to establish this point. Ideally, one should consider models that do not suffer from the spurious mode problem, in order to avoid the issue entirely. One might consider, for example, a translationally invariant model of \( N_c \) quarks interacting among themselves. Such a model obviously suffers the drawback that it is computationally extremely hard to solve. However, for our purposes the only issue of relevance is whether quark spin is a good quantum number. If the model has tensor interactions between quarks at leading order [which is \( 1/N_c \), leading to \( O(N_c^0) \) matrix elements when quark combinatorics are included], then quark spin is not a good quantum number, and even in the absence of explicit computation one expects generic mixing of \( O(N_c^0) \) between states of different spin-flavor symmetry.

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