Color-octet mechanism in the inclusive $D$--wave charmonium productions in $B$ decays

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Abstract

The inclusive $D$--wave charmonium production rates in $B$ decays are considered in the Bodwin-Braaten-Lepage (BBL) approach. We find that the color-octet subprocesses $B \to \bar{c}c(3S_1^{(8)}$ or $3P_J^{(8)}) + s, d$, followed by the transition $\bar{c}c(3S_1^{(8)}$, or $3P_J^{(8)}) \to 3D_J$, strongly dominate over any other subprocess, due to the large Wilson coefficient for the $\Delta B = 1$ effective lagrangian. Assuming that the numerical values of the matrix elements $\langle 0|O^{3D_J(3S_1^{(8)})}|0 \rangle$ and $\langle 0|O^{3D_J(3P_J^{(8)})}|0 \rangle$ are the same order of magnitudes with the $\langle 0|O^{\psi'}(3S_1^{(8)})|0 \rangle$, we find that the $1^{3}D_2$ can be observed at future B-factories.
1. Estimates of $S$– and $P$–wave charmonium production rates in $B$ decays were one of the earliest applications of the heavy quarkonium physics in the framework of perturbative QCD (PQCD) \[1\]. In the case of $P$–wave charmonia, it had long been known that their decay and production rates in the color-singlet model have infrared singularities, which signaled the failure of the factorization procedure in terms of a single wavefunction of the $c\bar{c}$ pair in the color-singlet $P$–wave state. Recent development in Nonrelativistic QCD (NRQCD) as an effective field theory of QCD \[2\] helped such inconsistency be resolved including higher Fock states of the $P$–wave charmonia in a systematic manner in NRQCD \[3\] \[4\].

On the contrary, the $S$–wave charmonium productions in $B$ decays have no such infrared divergence problems. However, the prediction based on the color-singlet model falls short of the data by a factor of $\sim 3$. This can be partly overcome by including the color-octet mechanism, since the relevant Wilson coefficient is a factor of $\sim 30$ larger than that of the color-singlet case \[5\]. Also the polarization of $J/\psi$ produced in $B$ decays depend sensitively on the color-octet contributions \[6\].

In this work, we extend previous works \[4\] \[5\] \[6\] to the case of the $D$–wave charmonium productions in $B$ decays. In various potential models for the charmonium family, $D$–wave charmonium states are predicted in the mass range of $3.81 - 3.84$ GeV \[7\]. Although the masses are above the $D\bar{D}$ threshold, they are below the $D\bar{D}^*$ threshold. Also, two states $^3D_2(J^{PC} = 2^{--})$ and $^1D_2(J^{PC} = 2^{-+})$ are forbidden to decay into $D\bar{D}$ because of parity conservation. So these two states are predicted to have narrow decay widths, and we can easily tag these two states through dominant decay channels (to be discussed below). Therefore, $D$–wave charmonium states (at least for $J = 2$) is certainly accessible at $B$-factories where one expects to have $\sim 10^8$ $B$ decays per year. Thus $B$ factories can serve as a place for studying new charmonium states below/above the $D\bar{D}$ threshold, including some hypothetical states like hybrids or $\chi_{cJ}(2P)$ state \[8\], in addition to its main role for measurements of CP asymmetries in $B$ decays.

2. The $\Delta B = 1$ effective Hamiltonian for $b \to c\bar{q}$ (with $q = d, s$) is written as \[9\]
\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^* \left[ 2C_+ - C_- \right] \bar{c} \gamma_\mu (1 - \gamma_5)c \, \bar{q} \gamma^\mu (1 - \gamma_5)b \\
+ \left( C_+ + C_- \right) \bar{c} \gamma_\mu (1 - \gamma_5)T^a c \, \bar{q} \gamma^\mu (1 - \gamma_5)T^a b \right], \]  

where \( C_\pm \)'s are the Wilson coefficients at the scale \( \mu \approx M_b \). We have neglected penguin operators, since their Wilson coefficients are small and thus they are irrelevant to our case.

To the leading order in \( \alpha_s(M_b) \) and to all orders in \( \alpha_s(M_b) \ln(M_W/M_b) \), the above Wilson coefficients are

\[ C_+(M_b) \approx 0.87, \quad C_-(M_b) \approx 1.34. \]

At the parton level, the \( B \) decay into \( ^{2S+1}D_J \) charmonium state occurs through

\[ b \to (c\bar{c})^{2S+1}L_j^{(1,8)} + s, d \to ^{2S+1}D_J + X. \]

Let us first estimate the velocity scaling of the various subprocesses. From the effective Hamiltonian given in Eq. (1), one can get the amplitude for \( b \to (c\bar{c})^{2S+1}L_j^{(1,8)} + s, d \) by the Taylor expansion of the amplitude with respect to the relative velocity \( v \) of the heavy quark inside the heavy quarkonium and projection over the specific partial wave state of \((c\bar{c})\).

Expanding the physical \( ^{2S+1}D_J \) state in terms of higher Fock states as following,

\[ |^3D_j \rangle = O(1)|c\bar{c}(^3D_j^{(1)}) \rangle + O(v^1)|c\bar{c}(^3P_j^{(8)})g \rangle + O(v^2)|c\bar{c}(^3P_j^{(8)})g \rangle + O(v^2)|c\bar{c}(^3S_j^{(1,8)}g)g \rangle + O(v^2)|c\bar{c}(^3S_j^{(1,8)}g)g \rangle + \ldots, \]

\[ |^1D_2 \rangle = O(1)|c\bar{c}(^1D_2^{(1)}) \rangle + O(v^1)|c\bar{c}(^1P_1^{(8)})g \rangle + O(v^1)|c\bar{c}(^1P_1^{(8)})g \rangle + O(v^2)|c\bar{c}(^1S_0^{(1,8)}g)g \rangle + O(v^2)|c\bar{c}(^1S_0^{(1,8)}g)g \rangle + \ldots, \]

we can find the velocity scaling of the soft process. Note that the chromoelectric E1 and the chromomagnetic M1 transitions accompany with the extra factors of \( v \) and \( v^2 \), respectively, in the amplitude for the soft transition, \((c\bar{c})^{(2S+1)}L_j^{(1,8)} \to ^{2S+1}D_J\). In Table I, we present the velocity scalings of the amplitude in NRQCD for various channels. According to Table I, the following processes are leading order in velocity scaling:
\[
\begin{align*}
  b &\to (c\bar{c})^3D_J^{(1)} + s, d \\
  b &\to (c\bar{c})^3P_J^{(8)} + s, d \\
  b &\to (c\bar{c})^3S_1^{(1,8)} + s, d \\
  b &\to (c\bar{c})^1D_J^{(1)} + s, d \\
  b &\to (c\bar{c})^1S_0^{(1,8)} + s, d \\
\end{align*}
\]

Note that in the hard process amplitude for \( B \to (c\bar{c})^{2S+1}L_J + s, d \), the spectroscopic states \((c\bar{c})^1P_1^{(1,8)}\) do not contribute in the leading order. Since the Wilson coefficient for the color-octet subprocess is about 30 times larger than that of the color-singlet subprocess in the (amplitude)\(^2\) level [5], we find that the following subprocesses (which are \(O(u^2)\) in the amplitude level) dominate over all the others:

\[
\begin{align*}
  b &\to (c\bar{c})^3P_J^{(8)} + s, d \quad \rightarrow \quad 3D_J + X, \\
  b &\to (c\bar{c})^3S_1^{(8)} + s, d \quad \rightarrow \quad 1D_2 + X. \\
\end{align*}
\]

Using the results in Ref. [5], we immediately obtain the inclusive \(2S+1D_J\) charmonium production rate in \(B\) decay as

\[
\Gamma(B \to 3D_J + X) \simeq \Gamma(B \to (c\bar{c})^3S_1^{(8)} + s, d \rightarrow 3D_J + X) \\
+ \Gamma(B \to (c\bar{c})^3P_J^{(8)} + s, d \rightarrow 3D_J + X) \\
= (C_+ + C_-)^2 \left( 1 + \frac{8M_c^2}{M_b^2} \right) \hat{\Gamma}_0 \\
\times \left[ \frac{\langle 0|O^{3D_J}3S_1^{(8)}|0 \rangle}{2M_c^2} + \frac{3\langle 0|O^{3D_J}3P_0^{(8)}|0 \rangle}{M_c^4} \right],
\]

\[
\Gamma(B \to 1D_2 + X) \simeq \Gamma(B \to (c\bar{c})^1S_0^{(8)} + s, d \rightarrow 1D_2 + X) \\
= (C_+ + C_-)^2 \hat{\Gamma}_0 \frac{3\langle 0|O^{1D_2}1S_0^{(8)}|0 \rangle}{2M_c^2},
\]

where

\[
\hat{\Gamma}_0 \equiv |V_{cb}|^2 \left( \frac{G_F^2}{144\pi} \right) M_b^3M_c \left( 1 - \frac{4M_c^2}{M_b^2} \right)^2.
\]
In order to evaluate the branching ratios numerically, we need to know three nonperturbative matrix elements, $\langle 0| O^{3D_J(3S_1^{(8)})} |0 \rangle$, $\langle 0| O^{3D_J(3P_0^{(8)})} |0 \rangle$ and $\langle 0| O^{1D_2(1S_0^{(8)})} |0 \rangle$.

In connection with this, we note that Qiao et al. [9] have recently shown that the color-octet production processes $Z^0 \rightarrow ^3D_J(c\bar{c})q\bar{q}$ have distinctively large branching ratios and it is of the same order as that of the $J/\psi$ production in $Z^0$ decays, assuming $\langle 0| O^{3D_J(3S_1^{(8)})} |0 \rangle = (2J + 1)^5 \times 4.6 \times 10^{-3}$ GeV$^3$, for $J = 1, 2, 3$. (13)

Similarly, we can estimate the numerical value of the matrix element $\langle 0| O^{3D_J(3P_0^{(8)})} |0 \rangle$ as

$$\langle 0| O^{3D_J(3P_0^{(8)})} |0 \rangle = (2J + 1)^5 \times 1.3 \times 10^{-2} \text{ GeV}^5 \quad \text{for } J = 1, 2, 3, (14)$$

using the numerical value which was maximally allowed in Ref. [10]. This value is about an order of magnitude larger than that deduced from the velocity scaling. For the case of $^1D_2$, we use the numerical value of $\langle 0| O^{1D_2(1S_0^{(8)})} |0 \rangle$ by imposing the approximate heavy quark spin symmetry as

$$\langle 0| O^{1D_2(1S_0^{(8)})} |0 \rangle = \langle 0| O^{3D_J(3S_1^{(8)})} |0 \rangle. \quad (15)$$

With the numerical values as

$$M_b = 5.3 \text{ GeV}, \quad M_c = 1.5 \text{ GeV}, \quad (16)$$

$$\alpha_s(2M_c) = 0.253, \quad (17)$$

$$\langle 0| O^{3D_J(3S_1^{(8)})} |0 \rangle = 4.6 \times 10^{-3} \text{ GeV}^3, \quad (18)$$

we estimate the branching ratios as

$$B(B \rightarrow ^3D_J + X) = \frac{2J + 1}{5} \times \left\{ \begin{array}{l}
5.2\% \text{ when } \langle 0| O^{3D_J(3P_0^{(8)})} |0 \rangle = 1.3 \times 10^{-2} \text{ GeV}^5 \\
1.1\% \text{ when } \langle 0| O^{3D_J(3P_0^{(8)})} |0 \rangle = 1.3 \times 10^{-3} \text{ GeV}^5
\end{array} \right., (19)$$

$$B(B \rightarrow ^1D_2 + X) = 0.67\%. \quad (20)$$
Before closing this subsection, we have to mention on theoretical uncertainties in our estimates for \( B(B \to D + X) \). The first comes from the assumptions, (13)–(15), which originate from our ignorance of nonperturbative physics in QCD. Up to now, there are no known processes which depend on these three parameters. So we relied upon the naive guess based on the velocity scaling rule and heavy quark spin symmetry. Another intrinsic uncertainty comes from the approach adopted here and in Ref. \[4\]. Namely, the parton picture adopted here for the case of \( D \)–wave charmonium productions in \( B \) decays may not be so good as in the case of the \( S \)–wave charmonium productions in Ref. \[5\] because the phase space available in the former is much less than the latter. There are also uncertainties coming from the quark masses \( M_b \), \( M_c \) and the relation between \( M_c \) and the mass of the \( D \)–wave charmonium state. In order to minimize these last uncertainties, we have used the prescriptions made in Ref. \[4\]. Despite of all of these uncertainties, we still anticipate our results to be correct estimates of \textit{an order of magnitude} for the branching ratios for the decays \( B \to D + X \).

3. In order to identify the desired \( D \)–wave charmonium state produced in \( B \) decays, one has to know its decay channels. As mentioned earlier, \( J = 2 \) states \((2S+1D_2)\) are predicted to have narrow decay widths of order of \( 300 \)–\( 400 \) keV. So these states are easier to reconstruct in the experiments, and hereafter we consider only \( J = 2 \) \( D \)–wave states, and try to find out which mode is best to identify the \( J = 2 \) \( D \)–wave charmonium state in \( B \) decays.

First consider the spin-triplet \( D \)–wave state \((J^{PC} = 2^{--})\). The main decay modes of this state are the E1 transitions into \( \chi_{cJ=1,2}^{J} (1P) \) states, the hadronic transition into \( J/\psi + \pi\pi \), and the decay into light hadrons via \( c\bar{c} \) annihilation into three gluons. The decay widths for each process is rather model dependent, but it is enough to quote some nominal value for each mode, in order to have some idea about the branching ratio for each channel. We adopt the numbers given in Ref. \[3\] : 

\[
B(3D_2 \to \chi_{c1} + \gamma) = 0.64, \\
B(3D_2 \to \chi_{c2} + \gamma) = 0.15, 
\]

(21)
\[ B(3D_2 \rightarrow J/\psi + \pi^+\pi^-) = 0.12. \]

Combining the branching ratios for \( \chi_{cJ=1,2}(1P) \rightarrow J/\psi + \gamma \) and \( J/\psi \rightarrow \mu^+\mu^- \), we are led to

\[
\begin{align*}
B(3D_2 \rightarrow J/\psi + \pi^+\pi^-) \times B(J/\psi \rightarrow \mu^+\mu^-) &= 7 \times 10^{-3}, \\
B(3D_2 \rightarrow \chi_{c1} + \gamma) \times B(\chi_{c1} \rightarrow J/\psi + \gamma) \times B(J/\psi \rightarrow \mu^+\mu^-) &= 10.4 \times 10^{-3}, \\
B(3D_2 \rightarrow \chi_{c2} + \gamma) \times B(\chi_{c2} \rightarrow J/\psi + \gamma) \times B(J/\psi \rightarrow \mu^+\mu^-) &= 1.2 \times 10^{-3}.
\end{align*}
\] (22)

The last decay channel through the intermediate \( \chi_{c2}(1P) \) state is negligible. For the radiative decay channels, one needs a good photon detector. Otherwise the first option through \( 3D_2 \rightarrow J/\psi + \pi^+\pi^- \) would be the best to reconstruct \( 3D_2 \) state at \( B \) factories. Combining the branching ratios for the cascade decays, we get

\[
\begin{align*}
B(B \rightarrow 3D_2 \rightarrow J/\psi(\rightarrow \mu^+\mu^-) + \pi^+\pi^-) &
\sim \begin{cases} 
3.7 \times 10^{-4} & \text{when } \langle 0|O^{3D_2}(3P_0^{(8)})|0 \rangle = 1.3 \times 10^{-2}\text{GeV}^5, \\
7.5 \times 10^{-5} & \text{when } \langle 0|O^{3D_2}(3P_0^{(8)})|0 \rangle = 1.3 \times 10^{-3}\text{GeV}^5.
\end{cases}
\end{align*}
\] (23)

Comparing these numbers with those for \( B \rightarrow J/\psi(\text{or } \psi') + X \),

\[
\begin{align*}
B(B \rightarrow J/\psi + X \rightarrow \mu^+\mu^- + X) &= 4.7 \times 10^{-4}, \\
B(B \rightarrow \psi' + X \rightarrow \mu^+\mu^- + X) &= 2.0 \times 10^{-4},
\end{align*}
\] (24) (25)

we find that the \( D^- \)–wave charmonium productions in \( B \) decays are not so rare, and can be within the reach of \( B \)-factories. There would be several tens to a few hundreds’ \( 3D_2 \)'s in \( 10^6 \) \( B \) meson decays into \( B \rightarrow 3D_2 \rightarrow J/\psi\pi^+\pi^- \) with \( J/\psi \rightarrow \mu^+\mu^- \), depending on the numerical values of \( \langle 0|O^{3D_2}(3S_1^{(8)})|0 \rangle \) and \( \langle 0|O^{3D_2}(3P_0^{(8)})|0 \rangle \).

Next consider the spin-singlet \( D^- \)–wave state \( (J^{PC} = 2^{-+}) \). One possible decay channel for this state is \[ 11 \]

\[
\begin{align*}
B(1D_2 \rightarrow h_c(1P_1) + \gamma) &= 0.80 \\
B(h_c(1P_1) \rightarrow J/\psi + \pi^0) &= 0.005 \\
B(J/\psi \rightarrow \mu^+\mu^-) &= 0.06.
\end{align*}
\] (26)
Here one has to tag three photons, one from the first chain, and the other two from the \( \pi^0 \) decay. The product of three branching ratios in this channel is \( 2.4 \times 10^{-4} \), so that we expect

\[
B(B \rightarrow ^3D_2 \rightarrow \gamma + h_c(\rightarrow \pi^0 + J/\psi(\rightarrow \mu^+\mu^-))) \sim 1.6 \times 10^{-6},
\]

(27)
corresponding to \( \sim 160 \) such cascade events in \( 10^8B \) decays.

4. In conclusion, we have considered the \( D \)-wave charmonium productions in \( B \) decays. The dominant contributions come from the color-octet (\( c\bar{c} \))\(^3S_1^{(8)} \) and (\( c\bar{c} \))\(^3P_0^{(8)} \) states for \( ^3D_J \) and (\( c\bar{c} \))\(^1S_0^{(8)} \) state for \( ^1D_2 \) production, respectively, because of the large Wilson coefficient, \( (C_+ + C_-) \) in Eq. (1). Assuming \( \langle 0|O^{3D_J}(3S_1^{(8)})|0 \rangle = \langle 0|O^{J\psi}(3S_1^{(8)})|0 \rangle \times (2J + 1)/5 \) and \( \langle 0|O^{3D_J}(3P_0^{(8)})|0 \rangle = \langle 0|O^{J\psi}(3P_0^{(8)})|0 \rangle \times (2J + 1)/5 \), we obtain \( B(B \rightarrow ^3D_2 + X) \approx 1.1\% - 5.1\% \). Assuming \( B(^3D_2 \rightarrow J/\psi + \pi^+\pi^-) \approx 10\% \) and using \( B(J/\psi \rightarrow \mu^+\mu^-) = 0.06 \), one can find 75 to 370 \(^3D_2 \)'s in the \( 10^6B \) decays into \( B \rightarrow ^3D_2 + X \) followed by \( ^3D_2 \rightarrow J/\psi + \pi^+\pi^- \) and \( J/\psi \rightarrow l^+l^- \) with \( l = \mu \). This is quite accessible in \( B \)-factories where \( 10^8B \) meson decays will be accumulated per year \([2]\). Since the \( D \)-wave charmonium states are not easy to find at other colliders, it is recommended to search for these states as well as others such as \( \chi_{cJ}(2P) \) and hybrid charmonia from \( B \) decays. Finding such states at the level predicted in this work will be another test of the factorization based on the NRQCD, including the color-octet mechanism for the heavy quarkonium productions. Also it will deepen our understanding of the heavy quarkonium spectroscopy based on the color-singlet pair and exotic hybrid states.

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\[\text{footnote 1} \] Here, we have neglected the detector efficiencies for simplicity. Typically, the detector efficiency for \( J/\psi \rightarrow \mu^+\mu^- \) is less than \( \sim 50\% \), and that for \( \pi^0 \) is even less. So our numbers of events should be reduced by such efficiencies. Also, one can use \( J/\psi \rightarrow e^+e^- \) for \( J/\psi \) tagging, which would help double the number of events from our estimates.
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TABLE I. The velocity scalings of the amplitudes for the various subprocesses relevant to $B \to ^3D_J + X$ and $B \to ^1D_2 + X$. $L$ is the number of derivatives in the NRQCD 4-quark operators ($O_H^{(2S+1)L_J}$), and the $E$ and $M$ are the numbers of the chromoelectric $E1$ and the chromomagnetic $M1$ transitions in order to reach the color-singlet physical $D$–wave state.

| subprocess                                      | $L$ | $E$ | $M$ | total $(v^{L+E+2M})$ |
|------------------------------------------------|-----|-----|-----|----------------------|
| $b \to (c\bar{c})^3D_J^{(1)} + s, d \to ^3D_J + X$ | 2   | 0   | 0   | $v^2$                |
| $b \to (c\bar{c})^3D_J^{(8)} + s, d \to ^3D_J + X$ | 2   | 2   | 0   | $v^4$                |
| $b \to (c\bar{c})^1D_2^{(8)} + s, d \to ^3D_J + X$ | 2   | 0   | 1   | $v^4$                |
| $b \to (c\bar{c})^1P_1^{(1,8)} + s, d \to ^3D_J + X$ | 1   | 1   | 1   | $v^4$                |
| $b \to (c\bar{c})^3P_J^{(8)} + s, d \to ^3D_J + X$ | 1   | 1   | 0   | $v^2$                |
| $b \to (c\bar{c})^1S_0^{(1,8)} + s, d \to ^3D_J + X$ | 0   | 2   | 1   | $v^4$                |
| $b \to (c\bar{c})^3S_1^{(1,8)} + s, d \to ^3D_J + X$ | 0   | 2   | 0   | $v^2$                |
| $b \to (c\bar{c})^1D_2^{(1)} + s, d \to ^1D_2 + X$ | 2   | 0   | 0   | $v^2$                |
| $b \to (c\bar{c})^1D_2^{(8)} + s, d \to ^1D_2 + X$ | 2   | 2   | 0   | $v^4$                |
| $b \to (c\bar{c})^3D_J^{(8)} + s, d \to ^1D_2 + X$ | 2   | 0   | 1   | $v^4$                |
| $b \to (c\bar{c})^3P_J^{(1,8)} + s, d \to ^1D_2 + X$ | 1   | 1   | 1   | $v^4$                |
| $b \to (c\bar{c})^1S_0^{(1,8)} + s, d \to ^1D_2 + X$ | 0   | 2   | 0   | $v^2$                |
| $b \to (c\bar{c})^3S_1^{(1,8)} + s, d \to ^1D_2 + X$ | 0   | 2   | 1   | $v^4$                |