Parameter estimation for a simplified model of an electrolytic capacitor in transient regimes

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Abstract. The real capacitors' behaviour in electric circuits modelled by a single capacity deviates from the ideal one. In order to find better compromises between precision and simplicity, different C-R-L models are used. In these models, C, R, L are called equivalent parameters and take constant values. Under these assumptions, the capacitors are modelled as lumped parameter subsystems although it is well known that the real capacitors are essentially distributed parameter systems. As highlighted in this paper, the capacitors are also time-variant subsystems. To prove this, we use two types of experimental data: data measured during the capacitor's discharge process and data obtained from frequency characteristics. The article proposes two estimation methods of equivalent values for the model parameters C and R based on their time variance highlighted by the experimental data. The estimation methods use a system of equations associated with the discharging of capacitors, respectively, with the frequency characteristics via polynomial regression. The experiments were carried out with an electrolytic polymer capacitor rated 220 μF, 25 V, 2.5 A rms, 85 °C, designed mainly for energy storage and filtering, the results being confirmed by experiments performed on other similar capacitors.

1. Introduction
The capacitor is one of the most used physical components of the electrical circuits and a key component with high breakdown rates in power electronics. Usually, the electrical circuits are designed considering the capacitor as a lumped parameters subsystem corresponding to a time-invariant R-L-C model. The real capacitors deviate from such models, the invariance assumption being breached. From a temporal perspective, effects should be grouped into three categories: long-term effects (LTE), medium-term effects (MTE) and short-term effects (STE).

The LTE category includes effects such as aging ([1], [2]) which is due, in particular, to the irreversible changes of physicochemical nature during lifetime combined with the mission profile. MTEs are non-permanent effects specific to regimes and operating conditions and can have multiple causes, most often the influences of working temperature and signal frequency.

STE are the effects consisting of the variance of the equivalent parameters of the capacitor due to the variation in time of voltages and currents during the operation of the circuits, i.e. causes that act on the short time period, including transient regimes. We are indirectly aware of their existence through the frequency characteristics of the various parameters that appear in the presentation of products by manufacturing companies and the attention paid to these characteristics in the literature [3], [4], [5], [6], [7]. The variations of the frequency characteristics of the equivalent series resistance (ESR) and of the equivalent capacitance (EC) of a capacitor suggest by their variations, that the two parameters are time-
variable. This is usually masked by the integration of the two parameters into the equivalent impedance (Z) that explicitly depends on the frequency.

A known effect that can be partially included in the STE category is the so-called „voltage coefficient“. It consists of changing the capacitance of the multilayer ceramic capacitors depending on the voltage at the capacitor terminals. The attribute „partial“ is due to the fact that the experiments reported in the literature refer to sinusoidal or constant steady-state regimes and not to states characterized by signals with spread spectrum [8], [9], [10], [11].

The present study focuses on the investigation of STE for another type of capacitor, an electrolytic one, and illustrates the possibility of identifying the breach of the capacitance value invariance assumption in transient regimes.

The capacitor participates in transient regimes with the equivalent C-R lumped-element model in figure 1. This model is used in practice when the equivalent series inductance is neglected [12], [2], [6]. From this point on, the resistances are noted only by the symbol $R$, including the ESR which is denoted by $R_s$.

![C-R model of an electrolytic capacitor.](image)

The analytical investigation in this paper aims mainly to highlight the variation of the capacitor capacitance of the model (2) during the transient discharge process and to propose an estimation method of the equivalent parameters $C$ and $R_s$ considering this variation. The paper does not address electrochemical explanations for the observed behaviour.

The article contains 6 sections. Section 2 presents the experimental support of the study. At the same time, the variance of the time constant ($T$) of an RC circuit, based on an electrolytic capacitor, is qualitatively highlighted. Section 3 defines the equivalent values in transient regimes for $T$, $C$ and $R_s$. In section 4 equivalent values in the hypothesis of the quasi-stationarity of the parameters $T$, $C$ and $R_s$ are determined, while section 5 introduces equivalent values suitable to illustrate the time-variance of $C$ and $R_s$ in transient processes. The last section summarizes the conclusions of the paper.
2. Experimental characteristics and correlation curves used

Because for the model (2) the variations of $C$ and $R_s$ cannot be separated through a single experiment, investigating the variation of the capacitor's parameters in transient regimes requires two different experiments. The characteristics we have used were: capacitor discharge characteristics, $u_{xy}(t)$, and frequency characteristics $C(f)$ and $R_s(f)$.

2.1. Obtaining capacitor discharge characteristics

The discharge characteristics were obtained with the circuit of figure 2. It is based on a capacitor considered by the diagram in figure 1, a resistor with nominal resistance $R = 10.1 \Omega$ and two synchronous electronic keys $K_a$ and $K_b$ working in anti-phase. The keys ensure the switching of the circuit between the State 1 and the State 2. In State 1, the capacitor is charged at constant DC voltage of value $U_a$ through the key $K_a$ and the resistor of resistance $R$. In State 2 the capacitor is discharged through the same resistor connected to ground by the key $K_b$. The significances of the remaining notations are the following: $R_K$ - resistance of the keys $K_a$ and $K_b$, $R_K = (30-50) \text{ m}\Omega$; $u_{xy}$ - voltage at the terminals of the capacitor; $u_C$ - voltage on capacitance $C$. The main data of the polarized polymer electrolytic capacitor studied are: 220 $\mu\text{F}$ nominal capacitance (tolerance +/- 20%), $V_{max} = 25 \text{ V}$ and maximum residual current of 117.5 $\mu\text{A}$. All experimental data in the paper refer only to this capacitor. The results related in this paper obtained by processing them have been validated on other capacitors of the same type.

![Figure 2](image.png)

Figure 2. The experimental circuit with the capacitor connected between terminals x and y: State 1 - Capacitor charging scheme from DC voltage source of value $U_a$. State 2 - Capacitor discharge scheme to ground.

The $u_{xy}(t)$ voltage waveforms were recorded with a LeCroy Waverunner digital oscilloscope. It offers the possibility to extract the results in a file format compatible with the MATLAB application. The measurements were done with a resolution of 10,000 points, i.e. with a sampling frequency $f_s = 10,000/\tau_w$ - Hz, where $\tau_w$ is the duration of the waveform studied. Figure 3 shows the $u_{xy}(t)$ waveform corresponding to charge-discharge cycles obtained by switching between State 1 and State 2 with a frequency of 5 Hz at a duty cycle of 50% and at a supplying voltage of $U_a = 20.23 \text{ V}$. The measured points, $\mathcal{M} = \{(t_i, u_{xy}(t_i)) | i = 1, 2, ..., 10000\}$, were stored in a file for further processing.
In order to derive the necessary calculation formulas, the cycle in figure 3 is transposed in the form of figure 4. In State 1, the voltage $u_C$ would reach asymptotically the value $U_a$, and in State 2 the value 0 V. Since in the discharging process of the capacitor, the DC voltage source is not connected to the circuit, and since the final portion of the discharge curve has a higher reproducibility potential in relation to different values of $U_a$, we chose to take into account only the discharge process. Also, our choice was determined by the large scale use of one of the main applications in which capacitors are used for storing energy during charging cycles and releasing it by a discharging cycles [13], [14]. Thus, this study focuses exclusively on the transient process corresponding to State 2 to which the equivalent circuit in figure 5 corresponds.
In relation to the terminals of the internal capacitance, the discharge circuit in figure 5 has the equivalent resistance:

\[ R_\sigma = \frac{R_p \cdot (R_S + R + R_K)}{R_p + R_S + R + R_K}. \]

Taking into consideration the range values for \( R_p \) (very high) and \( R_K \) (very low) the following approximation holds:

\[ R_\sigma = R_S + R, \quad (4) \]

For the time \( t_0 = 0 \), of switching from State 1 to State 2, the \( u_c \) voltage is:

\[ u_C(0) = U_b, \quad (5) \]

At \( t = 0+ \), when the capacitor discharge begins, due to the discharge current the voltage distribution on the capacitor is:

\[ u_C(t_0+) = U_b, \quad u_{xy}(t_0+) = \frac{R}{R_x + R} U_b = U_{xyo}. \quad (6) \]

Therefore, the measured voltage \( u(t) \) has at time \( t = 0+ \) a falling edge which in figure 4 corresponds to the portion Q1-Q2. Its value is \( U_{xyo} - U_b < 0 \). For the purpose of better highlighting the magnitude of the fall, the falling edge was exaggerated in the figure. The falling edge is followed by an exponential decrease of the measured voltage according to the relationship:

\[ u_{xy}(t) = \frac{R}{R_x + R} U_b \cdot u_C(t) = U_{xyo} \cdot e^{-\frac{t}{R_\sigma C}}. \quad (7) \]

When the product \( R_\sigma C = \text{const.} \), it is called the time constant of the discharge circuit and is denoted by \( T \):

\[ T = R_\sigma \cdot C, \quad (8) \]

Assuming \( T = \text{constant} \), the statements in Appendix are valid.

As shown below, for the studied electrolytic capacitors \( T \neq \text{const.} \), and consequently that the statements in Appendix are checked only locally, on a point-by-point basis. As mentioned before, we investigate only the Q1-Q4 segment of the discharge curve (figure 4). It should be underlined that the duration of the Q1-Q2 segment is very short, and the weight of measurement noises in this portion as well as in the quasi-stationary portion Q3-Q4 of the discharge characteristic is significant. Hence, we appreciate that the modifications of the capacitance \( C \) and the resistance \( R_\sigma \) of the capacitor can be best highlighted in the Q2-Q3 portion of the discharge curve. In order to exclude the jump portion Q1-Q2,
point Q2 has always been taken by approx. 0.5 V below $U_a$ level. Thus, further we analyse only discharge curves as the one in figure 6.

Let $\mathcal{M} = \{n, u_k(t_i)\}_{i=nQ2}^{nQ2+1} \ldots_{nQ3}$ be the set of points in the Q2-Q3 portion selected for processing the sampled acquisition made with the digital oscilloscope. We denoted the serial numbers of the first and last point on the portion Q2-Q3 by $nQ2$ and $nQ3$, respectively. We found that the values of $T$ calculated based on these points with the formulas (A-2), (A-3) and (A-4) are not constant. In the figure this is highlighted by the fact that $T_A \neq T_B$. Repeated experiments with the same capacitor as well as with other capacitors led to the same observation. Thus, the meaning of $T$ becomes that of „momentary time constant of the discharge circuit”. The modification of its values must be attributed primarily to the variation of the values of $C$ and $R_s$ during the discharging process.

2.2. Obtaining the frequency characteristics of the capacitors

In the second experiment we determined the variations of capacitance $C$ and resistance $R_s$ as functions of frequency with the measurement scheme in figure 7. We call the resulting characteristics, $C(f)$ and $R_s(f)$, frequency characteristics of the capacitor. The type of the RLC-bridge used was BK Precision. This bridge offers, in addition to visualizing the results in graphic mode, the possibility of extracting the results in a file format compatible with the MATLAB environment.

Figure 8 presents the semi-logarithmic frequency characteristics $C(f)$ and $R_s(f)$ measured in 151 points, with 1V rms sinusoidal signals. The figure shows only the frequency range of interest for the studied discharge waves.
Figure 8. Frequency characteristics of a polymer electrolytic capacitor: (a) $C(f)$; (b) $R_s(f)$.

The fact that $C$ and $R_s$ change with the frequency not only reinforces the observation made in section 2.1 based on discharge curves, but leads also to an even more general conclusion, in the sense that the values of $C$ and $R_s$ generally vary during the transient processes. These variations may have complex explanations that must include, as a main argument, the fact that the electrochemical processes active inside the electrolytic capacitor when the current “passes” through it are time-varying inertial processes. The inconsistency of values of the $T$ in figure 6 reflects this time-varying inertial character due to the variation of the capacitor’s currents and voltages.

Since the functions $C(f)$ and $R_s(f)$ depend on the same variable, the frequency $f$, their variations can be correlated by removing the common variable $f$. A possible of correlation consists of associating a polynomial approximation function $R_s = F(C)$ for the points measured with the RLC-bridge (figure 8). We obtain the best results with a 6th degree polynomial using the “polyfit” regression function of the MATLAB environment:

$$R_s = p_6 C^6 + p_5 C^5 + p_4 C^4 + p_3 C^3 + p_2 C^2 + p_1 C + p_0.$$  \hspace{1cm} (9)

The coefficients in (9) have the values: $p_6 = 6.1461 \times 10^{-7} \Omega F^{-6}$, $p_5 = -5.7553 \times 10^{-7} \Omega F^{-5}$, $p_4 = 2.2440 \times 10^{3} \Omega F^{-4}$, $p_3 = -4.6628 \times 10^{4} \Omega F^{-3}$, $p_2 = 5.4455 \times 10^{5} \Omega F^{-2}$, $p_1 = 3.3891 \times 10^{7} \Omega F$, $p_0 = 8.7808 \times 10^{5} \Omega$. The result shown in figure 9 illustrates that the polynomial (9) approximates the experimental data very well.

Figure 9. The graph of dependences $R_s(C)$: blue - the experimental dependence, red - the dependence (9).
3. Equivalent values of the parameters $T$, $C$ and $R$, in a transient regime

From a systemic point of view, the electrochemical processes in a capacitor corresponds to a time-variant distributed parameter system. $C$ and $R$ represent the parameters of a considerably simplified approximation model with lumped parameters. According to section 2, for this simplified model the values of $T$, $C$ and $R$, change during the transient processes.

Admitting that we have a mathematical model of such time-variant system, we have every reason to doubt the possibility of its operative use to monitor the variation of the three parameters in transient regimes. In this context, for practical purposes, we consider that a realistic alternative to the STE approach is to assume the working hypothesis of the quasi-stationarity of the parameters $T$, $C$ and $R$.

This means accepting that:

- the discharge curves and/or the frequency characteristics are repeatedly measured at discrete moments of time $\tau_i$, $\tau_i$ being the moments at which the experiments begin;
- the value of $T$ at the time $\tau_i$, denoted as $T_i$, is determined as the equivalent value in a transient regime from the discharge curve;
- the values of $C$ and $R$, for the time $\tau_i$, denoted as $C_i$ and $R_i$, are determined as equivalent values corresponding to $T_i$ by solving the 7th degree polynomial equation resulting from the system of equations (9) and (10):

$$T_i = \left( R_i + R \right) C_i \quad ; \quad (10)$$

- between $\tau_i$ and $\tau_{i+1}$ the values of $T$, $C$ and $R$ are constant:

$$T = T_i, \quad C = C_i \quad \text{and} \quad R = R_i.$$

We note that the term "equivalent", with the meaning "equivalent value in a transient regime (EVTR)", refers to a constant value of $T$ that theoretically leads to a discharge curve that approximates very well the experimental curve, and that was extended to the values of $C$ and $R$. The EVTR of $T$, $C$ and $R$, are obtained from the points corresponding to the segment Q2 - Q3 of the discharge curve. We discuss further the approach envisaged in Chapter 4. Then, to highlight the time-variance of $C$ and $R$, in Chapter 5 we use the same method on subsegments of Q2 - Q3, and we name the values obtained for $T$, $C$ and $R$, "equivalent values (EV)".

4. Determining the EVTR of $T$, $C$ and $R$

Mathematically, an EVTR of $T$, denoted by $T_{EVTR}$, can be determined based on the area under the experimental discharge curve in figure 6. The idea of determination is illustrated in figure 10. The blue curve is the measured characteristic $u_{xy}(t)$, the same as in figure 6, and the red curve $\bar{u}_{xy}(t)$ is an exponential characteristic corresponding to the $T_{EVTR}$ calculated in such a way that the areas between each of the curves and their projections on the time axis are equal.

$$\int_{t_a}^{t_a} u_{xy}(t) \cdot dt = \int_{t_a}^{t_a} \bar{u}_{xy}(t) \cdot dt \quad , \quad u_{xy}(t_a) = \bar{u}_{xy}(t_a) \quad . \quad (11)$$
Figure 10. Relative to the principle of determining the value of the EVTR of the time constant.

Since both curves of figure 10 start from the same point and have the same horizontal asymptote, we have \( \bar{u}_{xy}(t_{a}) - \bar{u}_{xy}(t_{o}) = u_{xy}(t_{a}) - u_{xy}(t_{o}) \). As the integral on the right-hand side of (11) is that of the formula (A-3), we obtain the equation (12). Then, taking into consideration (11), we obtain the formula (13). It allows for the calculation of \( TEVTR \) from the discharge curve on the interval \([t_{a}, t_{o}]\).

\[
TEVTR = \frac{1}{\bar{u}_{xy}(t_{o}) - \bar{u}_{xy}(t_{a})} \int_{t_{a}}^{t_{o}} \bar{u}_{xy}(t) \, dt, \quad (12)
\]

\[
TEVTR \equiv \frac{1}{u_{xy}(t_{a}) - u_{xy}(t_{o})} \int_{t_{a}}^{t_{o}} u_{xy}(t) \, dt. \quad (13)
\]

Table 1 shows values of \( TEVTR \) calculated for several values of the difference \( t_{a} - t_{o} \). It should be observed that the values depend on the length of the interval \([t_{a}, t_{o}]\): the value of \( TEVTR \) increases with the increase of the interval by 3% - 4%. At the same time the stabilizing tendency of the \( TEVTR \) is obvious. Thus, we retain the last value from the table 1:

\[
TEVTR = 1.91446 \times 10^{-3} \text{ s.} \quad (14)
\]

The first method to estimate the EVTR for capacity and serial resistance is based on equation (10) and correlation formula (9). If in (10) we adopt \( T_{e} = TEVTR \), the EVTRs of \( C \) and \( R_{s}, CEVTR \), \( Rs,EVTR \), are obtained as solutions of the systems of equation (15), for \( R = 10.1 \Omega \):

\[
(R_{s,EVTR} + R) CEVTR = T_{EVTR}, \quad R_{s}(CEVTR) = R_{s,EVTR}. \quad (15)
\]

After solving the system, we obtain the values (16):

\[
CEVTR = 172.903 \times 10^{-6} \text{ F,} \quad R_{s,EVTR} = 0.9725 \Omega. \quad (16)
\]

The estimated value for \( CEVTR \) differs from the nominal value of \( 220 \times 10^{-6} \text{ F} \). The result must be seen in the following context: i) the initial capacity value \( C=233 \times 10^{-6} \text{ F} \) of an off-the-shelf capacitor was measured with a RLC bridge at 120 Hz according with the datasheet conditions. This value is within the
tolerance limits ±20%. ii) The measurements described in this article were performed after an accelerated ageing of the capacitor under high voltage across the device terminals combined with high ripple current (for several days). Before performing the measurements in section 3 the measured value of capacity was $C = 169.2 \times 10^{-6}$ F. The estimated value (16) differs from this value by 2.18% which is the effect of the transient process.

5. Highlighting the variation of EV of $T$, $C$ and $R_s$ during the transient processes

To highlight the variation of $T$, $C$ and $R_s$ during the transient processes we used their equivalent values determined on short time intervals of discharge characteristic. The approach presented in the sequel was applied. Let $(t_1, u_{xy}(t_1))$ and $(t_2, u_{xy}(t_2))$ be the coordinates of two sliding points on the discharge characteristic $u_{xy}(t)$ arranged at a voltage interval $\Delta$, so that $\Delta = \text{abs} (u_{xy}(t_1) - u_{xy}(t_2)) \equiv \text{const}$. The length $t_2 - t_1$ of the subinterval $[t_1, t_2] \subseteq [t_0, I_{so}]$ is set low enough to consider that on this subinterval the discharge is exponential. Then, to calculate the equivalent value of $T$, we will use the formula (17) resulting from equation (A-4):

$$
T = \frac{t_2 - t_1}{\ln \frac{u_{xy}(t_1)}{u_{xy}(t_2)}}.
$$

As we associate the values of $T$ calculated with (17) with the values of $t_1$, assuming $t_1 < t_2$, or with the values of $u_{xy}(t_1)$, we obtain the dependencies $T(t)$ from figure 11 or $T(u_{xy})$ in figure 12. For simplicity, the notations in abscissa are $t$ and $u_{xy}$. Figures 11 and 12 show that when we use the formula (17) to calculate $T$ values, the higher the $\Delta$, the lower the download time or discharge voltage intervals for which $T$ obtains values close to those in table 1. Taking into consideration the shape of the discharge curve in figure 10 these observations are consistent: they correspond to its initial portion up to approximate 3 ms or up to approximate 3 V. As an indication, this means a subsegment from Q2 - Q3 of reduced extension (figure 3).

![Figure 11](image-url). The variations of EV of $T$ as functions of time for different $\Delta$. 


Both figures, 11 and 12, show that the values of \( T \) rise as the discharge curves tend asymptotically towards the steady state. However, due to the fact that in the asymptotic area, the measured voltage variations reach the insensitivity threshold of the oscilloscope, only \( T \)'s increases below 3 ms are relevant.

Figures 11 and 12 highlight the following aspect: the average value of \( T \) in the interval 2-3 ms in figure 11 and interval 4-6 V in figure 13 is very close to the value in relation (14), i.e. 1.91446 ms. This finding was confirmed in all cases experienced in the form of the existence of time or voltage intervals where the average \( T \) approximates very well the corresponding value \( T_{LVTB} \).

On this base, the calculation of \( T \) on such time or voltage intervals can be considered as a second estimation method.

The above specification, correlated with the observation regarding the relevance of the recorded samples during the first 3ms or until the voltage drops to about 3 V, opens the perspective of the calculation of capacitor’s parameters in real time.

Figure 13 presents the variations of the EV of \( C \) and \( R \) corresponding to the characteristics in figures 11 and 12. It is obvious that during the transient discharge process, the values of \( C \) and \( R \) change. Without intending to deepen the analysis of experimental data, we note that the EV variations of \( C \) and \( R \) can be correlated not only with time, and with the decrease of the \( u_{xy} \) voltage value, but also with the \( u_{xy} \) voltage gradient and/or with the spectrum of the discharge characteristics.
6. Conclusions
In dynamic processes, i.e. in the presence of time-varying currents and voltages, the capacitance and the serial resistance of the C-R model of a capacitor are time-variant. This behaviour can be highlighted and studied using discharge waves and the frequency characteristics of the capacitor.

In this article the variations of the capacitor’s parameters are investigated by conducting repetitive charge-discharge cycles with a polymer electrolytic capacitor and by analyzing the discharge curves in correlation with the frequency characteristics of the capacitance and resistance. The changes to the values of the capacitor’s parameter in transient regimes can be approximated by computing the appropriate equivalent values. To this end, the article proposes a first estimation method based on both experimental characteristics. From this method a second estimation approach is developed based on a reduced number of measured values on the discharge curve.

The article opens a research niche for applications in which the variation of the parameters of the components of electrical circuits in transient regimes can generate critical situations.

Appendix
For the time constant $T$ of the exponential function (A.1), represented in figure A-1,

$$x(t) = a \cdot e^{-\frac{t}{T}},$$

(A.1)

with $a > 0$, $T > 0$, $t \geq 0$, the following three statements are valid:
Figure A-1. Regarding the possibilities of interpretation of the time constant $T$

1. Whatever the point M on the graph of $x(t)$, the length of the subtangent AB is exactly:

   $$T = AB.$$  

   (A.2)

2. Whatever the points M and N on the graph of $x(t)$, the ratio $\frac{AAMNB}{MA-NB} = \text{const.} = T \cdot \frac{AAMNB}{AB}$ represents the hatched area of the curvilinear trapezium formed by points A, M, N, B. This property is rewritten as:

   $$T = \frac{1}{x(t_2)-x(t_1)} \int_{t_1}^{t_2} x(t) \cdot dt, \forall t_1, t_2.$$  

   (A.3)

3. Whatever the moments $t_1$ and $t_2$ we have:

   $$T = \frac{t_2-t_1}{\ln \frac{x(t_2)}{x(t_1)}}.$$  

   (A.4)

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