Low $x$ Behaviour of the Isovector Nucleon Polarized Structure Function and the Bjorken Sum Rule

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Abstract

The combination $g_1^p(x) - g_1^n(x)$ is derived from SLAC data on polarized proton and deuteron targets, evaluated at $Q^2 = 10\text{GeV}^2$, and compared with the results of SMC experiment. The agreement is satisfactory except for the points at the three lowest $x$, which have an important role in the SMC evaluation of the l.h.s. of the Bjorken sum rule.
The measurement by EMC \cite{1} of
\[ I_p = \int_0^1 g_1^p(x) dx = 0.126 \pm 0.010 \pm 0.015, \]
smaller than the prediction of the Ellis and Jaffe (EJ) sum rule \cite{2}, \( F/2 - D/18 \) (\( F = 0.46 \pm 0.04, \quad D = 0.79 \pm 0.04 \) \cite{3}), stimulated experimental and theoretical work on the spin structure of the nucleon.

The data on \( g_1^p(x) \) were in fair agreement with a previous calculation by the Bari group \cite{4} in the framework of a model where the Bjorken (Bj) sum rule \cite{5} is not obeyed. To explain the low value of \( I_p \) it was assumed \cite{6} that the strange quark carries a large negative component of the proton (and neutron) spin. However, a phenomenological study \cite{7} of the strange sea (from charm production) shows that its non-diffractive part is very small and, for positivity reasons, cannot account for the polarization of the strange sea needed to explain the EMC result.

The defect in the EJ sum rule for the proton is in part explained by the QCD corrections \cite{8}, and what is left may be accounted by a large positive projection \cite{9} (\( \sim 2 \)) of the gluon spin along the proton spin (to be compensated by a negative contribution of the orbital angular momentum). In this framework the Bj sum rule, apart from \( Q^2 \)-dependent QCD corrections \cite{8}, should be valid:
\[ I_p - I_n = \frac{1}{6} \frac{G_A}{G_V} \left[ 1 - \frac{\alpha_s(Q^2)}{\pi} - 3.58 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.22 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 \right]. \] (2)

A different explanation \cite{10}, which relates the defects in the EJ sum rule to the defect in the Gottfried sum rule \cite{11}, implies a violation of the Bj sum rule of \( (-0.025 \pm 0.007) \).

Experimentally, one has measurements on \( g_1^p(x) \) and \( g_1^d(x) \) at SLAC \cite{12} (at \( Q^2 = 3 GeV^2 \)) and CERN \cite{13} (at \( Q^2 = 10 GeV^2 \)): within their experimental errors they do not show evidence for a violation of the Bj sum rule. A more severe test is coming from the forthcoming measurements \cite{14} with \( He^3 \) polarized targets, which practically give directly \( g_1^n(x) \); the experiment already performed with this target at \( Q^2 = 2 GeV^2 \) \cite{15} shows the good precision one can reach. It is worth stressing that the errors in measuring \( g_1^p(x) \) and \( g_1^n(x) \) count twice less in testing the Bj sum rule with respect to measurements on \( g_1^p(x) \) and \( g_1^d(x) \).

It is also worth observing that an important contribution to the l.h.s of the Bj sum rule at \( Q^2 = 10 GeV^2 \) comes from the low \( x \) region, where the lowest point for \( xg_1^p(x) \) is higher than the subsequent points and \( xg_1^d(x) \) becomes negative. This behaviour is not reproduced with simple parameterizations of the polarized structure functions in terms of the contributions of the valence quarks and of the gluons, either by taking as input the SMC data \cite{16} or by getting predictions at \( Q^2 = 10 GeV^2 \) from the evolution of the result of a fit of the E143 data at \( Q^2 = 3 GeV^2 \) \cite{17}.
Here we want to focus our attention on the isovector part of $g_1^N(x)$, the one relevant for the Bj sum rule and not influenced in its evolution by the isoscalar terms $\Delta G(x)$ and $\Delta s(x) + \Delta \bar{s}(x)$. Our strategy will be to get $g_1^p(x) - g_1^n(x)$ from SLAC data and to compare the evolved structure function with CERN data.

To construct the isovector combination $g_1^p(x) - g_1^n(x)$ from SLAC data we have the technical problem that only the first lowest fourteen values of $x$ coincide in the two experiments which measure $g_1^p(x)$ and $g_1^n(x)$, while at higher $x$ one has other fourteen values of $x$ for the proton and seven different values for the deuteron. To get a reliable interpolation of proton data in correspondence of the seven highest values of $x$ measured for the deuteron, we take the following parameterization for $g_1^p(x)$,

$$g_1^p(x) = A_1 x^{A_2} (1 - x)^{A_3} \exp\{A_4 x\},$$  \hspace{1cm} (3)

and find, from the fit to the values of $g_1^p(x)$,

$$A_1 = 0.37 \pm 0.01, \quad A_2 = 0.046 \pm 0.013, \quad A_3 = 1.32 \pm 0.07, \quad A_4 = -0.41 \pm 0.09.$$  \hspace{1cm} (4)

To get $g_1^p(x) - g_1^n(x)$ from $g_1^p(x)$ and $g_1^n(x)$, we keep into account the small D-wave component in the deuteron ground state, which implies, with $\omega_D = 0.058$,

$$g_1^d(x) = \left(1 - \frac{3}{2} \omega_D\right) \frac{g_1^p(x) + g_1^n(x)}{2}.$$  \hspace{1cm} (5)

We write $g_1^p(x) - g_1^n(x)$ at $Q_0^2 = 3 \text{GeV}^2$ as the sum of the contributions of the valence quarks and of a possible term coming from the sea, $\Delta S(x)$ (indeed the sea is responsible of the defect in the Gottfried sum rule),

$$g_1^p(x) - g_1^n(x) = \frac{1}{6} (\Delta u(x) - \Delta d(x) + \Delta S(x)),$$ \hspace{1cm} (6)

with the standard parameterization,

\begin{align*}
x \Delta u(x) &= \eta_u A_u x^\alpha (1 - x)^{\beta_u} (1 + \gamma x), \\
x \Delta d(x) &= \eta_d A_d x^\alpha (1 - x)^{\beta_d} (1 + \gamma x). \hspace{1cm} (7)
\end{align*}

Since in the quark parton model $\Delta S$ should correspond to $\Delta \bar{u} - \Delta \bar{d}$, we adopt for it the same shape of the unpolarized sea, $\bar{u} - \bar{d}$ [18], which gives

$$x \Delta S(x) = \eta_S A_S x^{0.3} (1 - x)^{10.1} (1 + 64.9 x).$$  \hspace{1cm} (8)

The normalization factors $A_q$ and $A_S$ are defined in such a way that the first moments are:

$$\Delta q = \eta_q,$$

$$\Delta S = \eta_S.$$  \hspace{1cm} (9)
To remove the ambiguity coming from describing one function as a combination of two or three functions, we require that, for $x \geq 0.2$,

$$g_1^p(x) = \frac{2}{9} \Delta u(x) + \frac{1}{18} \Delta d(x).$$

(10)

This constraint finds its motivations in the fact that the region considered is dominated by the valence partons. Finally, to keep into account that the $u^\uparrow$ quark is dominating at high $x$ \cite{19, 20}, to get $\beta_d$ larger than $\beta_u$ we require

$$\beta_d \geq 2.5.$$  

(11)

We have considered several options for $\eta_u$, $\eta_d$ and $\eta_S$, but, since the main conclusion is the same, we present in Table 1 the results for three particular choices, which are good examples of what happens also for the other options. For the three options we fix $\eta_d$ to its QCD value, with the corrections up to the third order in $\alpha_s$,

$$\eta_d = -0.26 \pm 0.02.$$  

(12)

For the first option (1) we also fix $\eta_u$ to its QCD value, again up to $O(\alpha_s^3)$,

$$\eta_u = 0.76 \pm 0.04.$$  

(13)

Instead, for the second (2) and the third (3) option $\eta_u$ is left free and for the second one $\eta_S$ is fixed in such a way that the Bj sum rule is obeyed. In conclusion, only in the third option we leave open the possibility that the Bj sum rule is violated.

The comparison with the data on $x(g_1^p(x) - g_1^n(x))$ and $xg_1^p(x)$ (for $x \geq 0.2$), and the distributions $x \Delta u(x)$, $x \Delta d(x)$ and $x \Delta S(x)$ for the three fits are reported in Figures 1, 2 and 3 respectively. The fits do not differ so much and have satisfactory $\chi^2$.

The third option supplies a favourable test to the Bj sum rule with a defect of only $2\%$ well consistent with the experimental errors.

The intercept for the first option, $\alpha = 0.63 \pm 0.12$, is slightly less than the value $0.77 \pm 0.01$ mostly dependent on the non-singlet (NS) unpolarized structure function, $F_2^p(x) - F_2^n(x)$ and $xF_3(x)$, obtained in Ref. \cite{20}; the agreement is better for the other two options. The difference $\beta_d - \beta_u$ is around 1 and $\gamma$ is consistent with zero.

To get the polarized NS distribution at $Q^2 = 10 \, \text{GeV}^2$, one has the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations \cite{21} in the variable $t \equiv \ln Q^2/\Lambda^2_{\text{QCD}}$ ($\Delta q_{\text{NS}} \equiv x \Delta q_{\text{NS}}$):

$$\frac{d}{dt} \Delta q_{\text{NS}}(x,t) = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dz}{z} \Delta \bar{P}_{qq}(z) \Delta q_{\text{NS}} \left( \frac{x}{z}, t \right),$$

(14)
with, at next to leading order (NLO) in $\alpha_s$ \textsuperscript{[22]},
\begin{equation}
\Delta \tilde{P}_{qq}(z) = \Delta \tilde{P}_{qq}^{(0)} + \frac{\alpha_s}{2\pi} \Delta \tilde{P}_{qq}^{(1)}, \tag{15}
\end{equation}
where $\Delta \tilde{P}_{qq}^{(0)}$ is the NS polarized splitting function at leading order \textsuperscript{[21]} and $\Delta \tilde{P}_{qq}^{(1)}$ is the NLO one \textsuperscript{[23]}. We fix $\Lambda_{QCD}^{(4)} = 359 \text{ MeV}$ to get at NLO, with $n_f = 4$, $\alpha_s(3 \text{ GeV}^2) = 0.35 \pm 0.05$.

To solve the DGLAP equations, we use a method which, by expanding $\Delta \tilde{q}_{NS}$ into a truncated series of Chebyshev polynomials \textsuperscript{[24]}, gives rise to a system of coupled differential equations for the values of $\Delta \tilde{q}_{NS}$ in the points corresponding to the nodes of the Chebyshev polynomials.

With the initial conditions, given by the fits to SLAC data reported in Table \textsuperscript{[1]}, we get the corresponding evolved distributions at $Q^2 = 10 \text{ GeV}^2$, which are compared in Figure 4 with the values of $x (g_1^n(x) - g_1^q(x))$ deduced by the SMC measurements on $_{1}^{1}(x)$ and $g_1^{d}(x)$ (for the deuteron, instead of the value of Ref. \textsuperscript{[13]}, we take the more recent ones presented at Morillon and reported to us in Ref. \textsuperscript{[14]}).

In Table \textsuperscript{[1]} we report also the corresponding $\chi^2$ for the comparison between the evolved distribution and SMC data. The large experimental errors help to get $\chi^2_{ISOVEC}$ in the range (11,13), but for all the three options, as well as for the ones we did not report in Table \textsuperscript{[1]}, the curves obtained from the evolution of SLAC data pass below the points corresponding to the three lowest values of $x$, implying a behaviour in the limit $x \to 0$ different from what could be derived from SMC data. Since the low $x$ region gives an important contribution to the l.h.s. of the Bj sum rule evaluated by SMC Collaboration, the fact that, even allowing for isovector polarization in the sea, one is not able to recover the low $x$ behaviour suggested by the three points at the lowest $x$, does not allow to be confident on conclusions about the validity of the Bj sum rule based on these three points in a range of $x$ not explored by SLAC.

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### Table I

|       | 1               | 2               | 3               |
|-------|-----------------|-----------------|-----------------|
| $\alpha$ | $0.63 \pm 0.12$ | $0.71 \pm 0.19$ | $0.70 \pm 0.23$ |
| $\beta_u$ | $1.5 \pm 0.4$ | $1.6 \pm 0.4$ | $1.4 \pm 0.4$ |
| $\beta_d$ | $2.5 \pm 0.9$ | $2.5 \pm 1.3$ | $2.5 \pm 1.3$ |
| $\gamma$ | $1.6 \pm 2.1$ | $1.9 \pm 2.7$ | $1.1 \pm 2.3$ |
| $\eta_u$ | $0.76 \pm 0.04$ | $0.70 \pm 0.08$ | $0.74 \pm 0.07$ |
| $\eta_d$ | $-0.26 \pm 0.02$ | $-0.26 \pm 0.02$ | $-0.26 \pm 0.02$ |
| $\eta_S$ | 0               | $0.06 \pm 0.07$ | 0               |
| $\chi^2/dof$ | $32.06/29 = 1.11$ | $31.31/28 = 1.12$ | $31.96/28 = 1.14$ |
| $\chi^2_{ISOVEC}$ | 11.7           | 12.7           | 12.7           |

Table 1: The results of the options 1, 2 and 3 for the values of the parameters of the fits at $Q^2 = 3 GeV^2$. In the last row we show the $\chi^2$ for the comparison of the three options, evolved to $Q^2 = 10 GeV^2$, with the SMC data (see text).
Figure Captions

Fig. 1 The results corresponding to fits 1 (solid line), 2 (dashed line), and 3 (dotted line) are compared with the SLAC data on \( x(g_1^p(x) - g_1^n(x)) \) at \( <Q^2> = 3\text{GeV}^2 \) from ref. [12].

Fig. 2 Same as Fig. 1 for the proton SLAC data for \( xg_1^p(x) \) from ref. [12]. Note that only the experimental data used for the fits are plotted.

Fig. 3 The distributions \( x\Delta u \), \( x\Delta d \), and \( x\Delta S \) are plotted for fit 1 (solid line), fit 2 (dashed line) and fit 3 (dotted line). Note that for fit 1 and 3 \( x\Delta S \) is zero.

Fig. 4 The evolution to \( Q^2 = 10\text{GeV}^2 \) of the results of fits 1 (solid line), 2 (dashed line), and 3 (dotted line) are compared with the SMC data on \( x(g_1^p(x) - g_1^n(x)) \) at \( <Q^2> = 10\text{GeV}^2 \) from Ref. [13] (the deuteron data are from Ref. [14]).
Fig. 1

\[ \langle Q^2 \rangle = 3 \text{GeV}^2 \]
\[ \langle Q^2 \rangle = 3 \text{GeV}^2 \]
\[ <Q^2> = 3 \text{GeV}^2 \]
Fig. 4

$\langle Q^2 \rangle = 10 \text{GeV}^2$