Co-evolutionary Models for Reconstructing Ancestral Genomic Sequences: Computational Issues and Biological Examples

Tamir Tuller\textsuperscript{1,2,3,4,*}, Hadas Birin\textsuperscript{1,*}, Martin Kupiec\textsuperscript{2}, and Eytan Ruppin\textsuperscript{1,3}

\textsuperscript{1} School of Computer Science
\textsuperscript{2} Department of Molecular Microbiology and Biotechnology
\textsuperscript{3} School of Medicine, Tel Aviv University
\{tamirtul,birinhad,martin,ruppin\}@post.tau.ac.il
\textsuperscript{4} Faculty of Mathematics and Computer Science, Weizmann Institute of Science

Abstract. The inference of ancestral genomes is a fundamental problem in molecular evolution. Due to the statistical nature of this problem, the most likely or the most parsimonious ancestral genomes usually include considerable error rates. In general, these errors cannot be abolished by utilizing more exhaustive computational approaches, by using longer genomic sequences, or by analyzing more taxa. In recent studies we showed that co-evolution is an important force that can be used for significantly improving the inference of ancestral genome content.

In this work we formally define a computational problem for the inference of ancestral genome content by co-evolution. We show that this problem is NP-hard and present both a Fixed Parameter Tractable (FPT) algorithm, and heuristic approximation algorithms for solving it. The running time of these algorithms on simulated inputs with hundreds of protein families and hundreds of co-evolutionary relations was fast (up to four minutes) and it achieved an approximation ratio $< 1.3$.

We use our approach to study the ancestral genome content of the Fungi. To this end, we implement our approach on a dataset of 33,931 protein families and 20,317 co-evolutionary relations. Our algorithm added and removed hundreds of proteins from the ancestral genomes inferred by maximum likelihood (ML) or maximum parsimony (MP) while slightly affecting the likelihood/parsimony score of the results. A biological analysis revealed various pieces of evidence that support the biological plausibility of the new solutions.

Keywords: Co-evolution, reconstruction of ancestral genomes, maximum parsimony, maximum likelihood.

1 Introduction

The problem of reconstructing ancestral genomic sequences is as old as the field of molecular evolution. The first approach for inferring ancestral genomic sequences

\* TT and HB contributed equally to this work.

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was suggested by Fitch around 40 years ago [18]. This first algorithm assumed a binary alphabet, and was based on the Maximum Parsimony (MP) criteria, i.e. find the labels to the internal nodes of a tree that minimize the number of mutations along the tree edges. Over the years this basic algorithm was generalized in many ways. Sankoff showed how to efficiently solve versions of the maximum parsimony problem with a non-binary alphabet and with multiple edge weights [43]. Similar algorithms for inferring ancestral sequences based on maximum likelihood (ML; instead of maximum parsimony) were suggested more than 15 years later [1,40,8,17,30,38]. Recently, similar approaches were used to infer ancestral sequences in phylogenetic networks [25].

Dozens of biological studies have dealt with the reconstruction of ancestral genomic sequences and ancestral genomes. For example, reconstruction of ancestral sequences was used for understanding the origins of genes and proteins [54,46,45,24,22,21,4,3,37], and for aligning genomic sequences [23], and for inferring ancestral enzymes and genomes [51,29,41,34,7,6].

The main problem related to reconstructing ancestral sequences and genomes is that in practice many times the reconstructed sequences contain a large number of errors. A major source of this phenomenon is the existence of multiple local and/or global maxima in the solution space searched by both the maximum likelihood and the maximum parsimony approaches (see e.g. [5,48]). Thus, many times our confidence in the most likely or most parsimonious reconstructed ancestral state is not too high. As the above mentioned algorithms assume that different sites and different genes/proteins evolve independently, this problem cannot be solved by adding more samples or more taxa [32].

Several studies demonstrated that functionally and physically interacting proteins tend to co-evolve [26,44,12,49,16], and that co-evolutionary relations between proteins are quite ubiquitous [10,9,13,15,11]. Some of these previous investigations used the fact that interacting proteins have correlative evolution in order to successfully predict physical interactions (for example, see [26,44,12]).

Based on these results, we recently suggested a different approach, the Ancestral Co-Evolver (ACE), for improving the accuracy of reconstructed ancestral genomes [48]. Our approach was based on utilizing information embedded in the co-evolution of functionally/physically interacting proteins. We used this approach to study the genome content of the Last Universal Common Ancestor (LUCA). In this work we give a formal description of the ancestral co-evolution problem, we analyze its computational complexity and describe algorithms for solving it; the performances of these algorithms are demonstrated by a simulation study. Additionally, we use the ACE for studying a new biological example, the ancestral genome content of the Fungi.

2 Definitions and Preliminaries

For simplicity, we assume a binary alphabet. However, all the results here can be easily generalized to models with more than two characters. In this work we assume that in general, if they do not have co-evolutionary relation, neighbor
sites in the input sequences evolve independently. Thus, the basic components in the model and algorithms are single characters.

Our goal is to reconstruct the ancestral states for a set of organisms \( T \) of size \(|T| = n\). A phylogenetic tree is a rooted binary tree \( T = (V(T), E(T)) \) together with a leaf labeling function \( \lambda \), where \( V(T) \) is the set of vertices and \( E(T) \) the set of edges. In our context, a weight table is attributed to each edge \((u, v) = e \in E(T)\). This weight table includes a weight (a positive real number) for each pair of labels of the two vertices \((u, v) = e\).

In this work, we assume that each node in a phylogenetic tree corresponds to a different organism. The leaves in a phylogenetic tree correspond to organisms that exist today (\( T \)), while the internal nodes correspond to organisms that have become extinct (\( T' \)). Thus, the leaf labeling function is a bijection between the leaf set \( L(T) \) and the set of organisms that exist today, \( T \).

In our binary case, each label is a binary sequence; all the sequences have the same length. In the case of conventional ML/MP, as we assume an i.i.d. case where different characters in a sequence evolve independently, we can describe the algorithm for sequences of length one: i.e. either '1' or '0'. A full labeling of a phylogeny \( \hat{\lambda}(T) \) is a labeling of all nodes of the tree such that the labels at the leaves are the same as in \( \lambda \), i.e., for all \( l \in L(T) \) \( \lambda(l) = \hat{\lambda}(l) \).

We can name each node after its corresponding organism. Let \( O_T(\cdot) \) denote a function that returns the index of the organisms corresponding to each node in \( T \), i.e. for every \( v \in V(T) \), \( O_T(v) \) is the index of the organism (from \( T \cup T' \)) corresponding to \( v \).

A co-evolving forest \( F = (S_F = \{T_1, T_2, \ldots \}, E_c(S_F)) \) is a set of phylogenetic trees \( S_F \) with identical topology that correspond to the same organisms [i.e. each tree has the same \( O(\cdot) \)], and an additional set of edges, \( E_c(S_F) \), that connect pairs of nodes in different trees. This set of edges and represent the co-evolutionary relations between pairs of protein families. Edges in \( E_c(S_F) \) must connect pairs of nodes that correspond to the same organism (i.e. \((v, u) \in E_c(S_F), v \in V(T_1), u \in V(T_2) \implies O_{T_1}(v) = O_{T_2}(u)\); Figure 1); we call such pairs of nodes legal co-evolutionary pairs.

The edges in \( E_c(S_F) \) are named co-evolution edges while edges that are part of the evolutionary trees are named tree edges. For example, Figure A includes a co-evolving forest with two trees (the co-evolution edges are dashed with arrows while the tree edges are continuous). As co-evolutionary relations. In this work we assume that new co-evolutionary edges do not appear/dissapear during evolution. Namely, we assume that if there is a co-evolutionary edge between a legal co-evolutionary pair of nodes in two trees than all the legal co-evolutionary pairs of nodes in the two trees are connected by co-evolutionary edge.

A full labeling of a co-evolving forest \( \hat{\lambda}(S_F) \) is a full labeling, \( \{\hat{\lambda}(T_1), \hat{\lambda}(T_2), \ldots \} \), of all the nodes of the trees in \( S_F \). The roots of a co-evolving forest are the set of roots of the phylogenetic trees in the co-evolving forest.

As mentioned, a co-evolving forest also includes a weight table for each co-evolution edge and each tree edge. These weight tables are cost functions that return a real positive number for each pairs of labels at the two ends of the edge.
Fig. 1. A. A simple example of a co-evolving forest with three trees, and one co-evolution edge connecting node $x_2$ in tree $T_1$ and node $y_2$ in tree $T_2$; the weight table corresponding to this co-evolution edge is in red. The weight table corresponding to the tree edge $(x_1, x_2)$ in $T_1$ is in green. B. The co-evolutionary graph corresponding to the co-evolving forest in A.

In the case of tree edges, these weights reflect the probability of a mutation along the edge. In the case of co-evolution edges, these weights reflect the distribution of mutual occurrence of the labels of the nodes at the ends of the edge.

This leads us to the formal definition of the problem we are concerned with, the Ancestral co-evolution problem:

**Problem 1.** Ancestral co-evolution (Ancest − co − evol)

**Input:** A co-evolving forest, $F = (S_F, E_c(S_F))$, and a real number, $B$.

**Question:** Are there labels for the internal nodes of all the trees in the co-evolving forest such that the sum of the corresponding weights along all the tree edges and the co-evolution edges is less than $B$?

The following example demonstrates the advantages of the ancestral co-evolver compared to the simple i.i.d parsimony approach.

**Example 1.** Consider the co-evolving forest that appears in Figure 1 and assume that all the weight tables of the tree edges are identical to the table that appears in the figure where $[v_1, v_2, v_3, v_4] = [0, 1, 1, 0]$. It is easy to see that there are two MP solutions for the labels of the internal nodes in the phylogenetic tree $T_1$: either $[x_1, x_2, x_3, x_4] = [0, 0, 0, 0]$ or $[x_1, x_2, x_3, x_4] = [0, 1, 1, 1]$ gives the same score (2). In the case of the tree $T_2$, it is easy to see that there is one MP solution: all the labels of the internal nodes are '0'. Now, suppose that in the weight table corresponding to the co-evolution edge $(x_2, y_2)$, $w1$ is the smallest
entry. Thus, by co-evolution study, the solution \(x_1, x_2, x_3, x_4 = 0, 0, 0, 0\) is more plausible and the ambiguity in the labels of \(T_1\) is solved.

Note that in general it is not necessarily required that the solution of each tree separately will be the most parsimonious (see the next sections). The minimal sum of edge weights corresponding to a co-evolving forest, \(F\) (problem 1) is named the cost of \(F\). A co-evolutionary graph is an undirected graph that describes the co-evolution edges in the co-evolving forest. In such a graph, each node corresponds to a tree in the co-evolving forest, and two nodes are connected by an edge if there is at least one co-evolution edge between their corresponding trees. A connected component in the co-evolving forest is a sub-set of trees whose corresponding nodes in the co-evolutionary graph induce a connected component (see an example in Figure 1B.)

We finish this section with an observation that will be used in the next sections (the proof is omitted due to lack in space and will appear in the full version of the paper).

**Observation 1.** The optimization problem of inferring the ancestral states of a phylogenetic tree when the optimization criteria is maximum likelihood (see, for example, [40]) under i.i.d models such as Jukes Cantor (JC) [27], Neyman [36], or the model of Yang et al. [51] can be formalized as a maximum parsimony problem for non-binary alphabet and with multiple edge weights [43].

Observation 1 teaches us that the Ancestral co-evolution problem without co-evolution edges (\(|E_c(S_F)| = 0\)) can describe a Maximum Likelihood (ML) problem. In this work, indeed the weights of the tree edges correspond to the probabilities to gain/lose proteins and thus we describe and solve a generalization of both the ML and the MP problems on trees.

3 Hardness Issues

In this section we show that the Ancestral co-evolution problem is NP-hard. We will show it by reduction from the max − 2 − sat problem which is known to be NP-hard [20] (the reduction appears in Figure 2).

**Problem 2.** Maximum 2-Satisfiability (max − 2 − sat)

**Input:** Set \(U\) of variables, collection \(C\) of clauses over \(U\) such that each clause \(c \in C\) has \(|c| = 2\), and a positive integer \(K \leq |C|\).

**Question:** Is there a truth assignment for \(U\) that simultaneously satisfies at least \(K\) of the clauses in \(C\)?

**Theorem 1.** The Ancest − co − evol problem is NP-hard.

**Proof.** By reduction from max − 2 − sat. Given an input < \(U, C, K\) > to the max − 2 − sat problem reconstruct the following input to the Ancest − co − evol problem (See Figure 2): \(|S_F| = |U|\), and each tree in \(S_F\) corresponds to one variable in \(U\); each tree has the same structure and leaf labels as described in
Figure 2. Each co-evolution edge in $E_C$ corresponds to one clause in $C$, and connects the right-most internal nodes in two trees (see Figure 2). The translation of the four types of possible clauses (true-true, true-false, false-false) to the weight matrix of its corresponding co-evolution edge appears in Figure 2.

Finally, choose $B = 2 \cdot |U| + |C| - |K|$. It is easy to see that the optimal parsimony score for each tree in the Ancest−co−evol problem (excluding the co-evolution edges) is 2; either a solution that labels all the internal nodes with ‘0’ or a solution that labels all the internal nodes with ‘1’ gives this score.

$\Rightarrow$ Suppose the answer to the max$-2$−sat problem is YES (i.e. there is a truth assignment for $U$ that simultaneously satisfies at least $K$ of the clauses in $C$). In this case, we choose the labeling of all the internal nodes of each tree to be identical to the assignment of its corresponding variable in $U$. Thus, the contribution of all the tree edges to the parsimony score (i.e. excluding the co-evolution edges) is $2 \cdot |U|$.

By the construction of the weight tables (Figure 2), the contribution to the parsimony score due to co-evolution edges that correspond to one of the $K$ satisfied clauses is 0, and the contribution to the parsimony score due to each of the other co-evolution edges is at most 1. Thus, the contribution of all the co-evolution edges to the parsimony score is at most $|C| - |K|$, and the answer to the Ancest−co−evol problem is YES.

$\Leftarrow$ Suppose the answer to the Ancest−co−evol problem is YES (i.e. the parsimony score along all the edges is no more than $2 \cdot |U| + |C| - |K|$). As mentioned earlier, for optimal parsimony score, in each tree all the internal nodes should be labeled with the same label as that of the internal node that is connected by co-evolution edges. Thus, the contribution of all the tree edges to the parsimony score is $2 \cdot |U|$, and the contribution of all co-evolution edges to the parsimony score is at most $|C| - |K|$. Thus, the contribution to the parsimony score for $K$ of the co-evolution edges is 0. By the reconstruction of the co-evolution edges, they correspond to $K$ clauses in $C$ that are be satisfied when the assignment to each variable in $U$ is identical to the labeling of the internal nodes of its corresponding tree. Thus, the answer to the max$-2$−sat problem is YES.
4 Methods

4.1 An FPT Algorithm and Approximation Heuristics

As we have shown in the previous section, the Ancestral co-evolution problem is NP-hard. In this section, we describe an FPT algorithm and an approximation algorithm for the Ancestral co-evolution problem. The approximation algorithm is described in Figure 3. It has three main steps: 1) The input co-evolving forest is partitioned into smaller co-evolving forests; 2) optimal labels are assigned to the internal nodes of each of these co-evolving forests by an FPT algorithm; in total, these labels are an approximation of the solution for the input co-evolving forest; 3) finally, the solution is further improved greedily.

We will start by describing an FPT algorithm that finds the optimal solution for the Ancestral co-evolution problem in time complexity that is exponential with the number of trees in $S_F$ but polynomial with the other properties of the input. This algorithm is used in step 2) of the approximation algorithm where it is implemented on subsets of $S_F$. Similarly to many algorithms for computing the labels of internal tree nodes [19,43], our algorithm has two phases (in the first phase, it traverses the co-evolving forest from the leaves to the root; in the second phase, it traverses the co-evolving forest from the root to the leaves). However, our algorithm is performed jointly for all the trees in each connected component.

Let $e = (i, j)$ denote a co-evolution edge. Let $W_{e:(a,b)}^c = W_{(i,j):(a,b)}^c$ denote the cost of assigning $a$ to node $i$, and $b$ to node $j$ where $(i,j)$ is a co-evolution edge. Similarly, if $(i,j)$ is a tree edge we use $W_{e:(a,b)}^b = W_{(i,j):(a,b)}^b$ to denote the cost of assigning $a$ and $b$ to the two nodes of $e$ respectively. Let $S_t(\bar{v})$ denote the cost of a sub-forest in the co-evolving forest whose roots are $\bar{t}$, when assigning $\bar{v}$ to these roots.

Let $t_j$ denote the $j$-th node in the vector of nodes $\bar{t}$. Let $\bar{t}$ and $\bar{v}$ denote the corresponding vectors of children of $\bar{t}$ in the co-evolving forest. In the first phase, all $S_t(\bar{v})$ are computed by the following dynamic programming formula (see Figure 3C.):

$$S_t(\bar{v}) = \min_{\pi} \{ \sum_j W_{(t_j,k_j):(v_j,w_j)}^b + S_{\bar{t}}(\overline{v}) \} + \min_{\pi} \{ \sum_j W_{(t_j,k_j):(v_j,w_j)}^b + S_{\bar{t}}(\overline{v}) \} + \sum_{j_1,j_2:(j_1,j_2) \in E_t} W_{(t_{j_1},t_{j_2}):(v_{j_1},v_{j_2})}$$

In the second phase, the algorithm traverses the sub-forest from the roots to the leaves, and optimal values are assigned to the internal nodes of the co-evolving forest by the following dynamic programming formula (see Figure 3C.):

For the roots of the co-evolving forest:

$$\bar{v}^* = \arg\min_{\pi} \{ S_{\bar{t}}(\overline{v}) \}$$

For a general vector of internal nodes $\bar{k}$ corresponding to the same organism in the co-evolving forest (after the values $\bar{t}^*$ were assigned to its parents, $\bar{t}$):

$$\bar{v}^* = \arg\min_{\pi} \{ \sum_j W_{(t_j,k_j):(v_j,w_j)}^b + S_{\bar{t}}(\overline{v}) \}$$

The running time of this algorithm on a co-evolving forest with $m'$ trees of size $n$ is $O(n \cdot 2^{2n+m'})$ since, in each of the $O(n)$ it checks all the $2^{m'} \times 2^{m'}$ possible
simultaneous labels to all nodes corresponding to the direct descendant of all nodes corresponding to the same organism.

As the running time of the algorithm described above is exponential with the size of co-evolving forest, the general algorithm (see Figure 3 A.) has an initial stage (stage 1) where the input graph is partitioned into small enough connected components.

As the running time of the algorithm described above is exponential with the size of the largest connected component in the co-evolutionary graph, the general algorithm (see Figure 3 A.) has an initial stage where the input graph is partitioned into small enough connected components. The input to the general algorithm (ACE) includes the maximal size of a connected component after this stage. Let $K$ denote this parameter.

This step is described in Figure 3 B. and is performed by an algorithm that recursively implements k-means [35] on the co-evolutionary graph. On the first iteration, the number of clusters is $|V|/K$ where $|V|$ is the number of vertices in the co-evolutionary graph. If the size of some cluster is larger than $K$, the algorithm is executed recursively on this cluster to further partite it to smaller connected components. The algorithm stops when all parts of the graph (connected components) are smaller than $K$. Though the problem of clustering is NP-hard, in practice, and as reported in the next section, the k-means algorithm is very fast.

The input to this step is a weighted graph whose edges correspond to the edges in the co-evolutionary graph. The weights of the graph edges can be any measure that represents the strength of the co-evolution between the corresponding trees (for example, the correlation between the phyletic pattern of the corresponding proteins).

The final step of the Ancestral-Co-Evolver algorithm is a greedy stage (3 D.). In each iteration, the greedy algorithm searches for an edge and labels its ends in a way that improves the cost of the co-evolving forest. As demonstrated in the simulations in section 5.2 this algorithm converge to a local optimum faster than the running time of the dynamic programming stage with $K = 7$. Note that the greedy algorithm can be stopped after a certain number of iteration if it does not converge to a local optimum. The greedy stage can be used as an independent algorithm when running it from various initial points (e.g., one of the initial points can be the ML solution).

Let $\hat{\lambda}(S_F)$ denote the labels found by the Ancestral-Co-Evolver algorithm. Let $MP(\hat{\lambda}(S_F))$ denote the parsimony score induced by these labels. Let $MP^- (\hat{\lambda}(S_F))$ denote the parsimony score induced by these labels when not considering the co-evolution edges between the connected components found by the Partite algorithm; let $E^-$ denote this set of co-evolution edges. An upper bound on the approximation ratio of the general algorithm is given in the following observation (the proof is omitted due to lack in space and will appear in the full version of the paper):
Fig. 3. A. The general algorithm for inferring ancestral states under co-evolution. B. The algorithm for partitioning the co-evolutionary graph. C. The algorithm for finding the ancestral states of a connected component (output of the Partite algorithm). D. The Greedy stage of the algorithm.

Observation 2. The approximation ratio of the Ancestral-Co-Evolver algorithm is
\[ \frac{MP(\hat{\lambda}(S_F))}{MP(\lambda(S_F))} \leq \frac{MP(\lambda(S_F))}{MP(\lambda(S_F))}. \]

We used Observation 2 in order to estimate the approximation ratios of the different algorithms in the simulations.

One important property of the algorithm is that it enables a trade-off between accuracy and speed. A larger \(K\) (see Figure 3) increases the running time exponentially but at the same time increases the accuracy of the solution; \(K = |S_F|\) will give the optimal solution for the Ancestral co-evolution problem.

Finally, it is important to note that by weighting the co-evolution edges relatively to the tree edges we can control the relative influence of these two sources of information (co-evolution vs. the evolutionary tree) on the resulting labels. Thus, for example, it is easy to see that (and a very similar proof can be outlined for the case where tree edges are rational numbers):

Let \(\hat{\lambda}(S_F/E_C)\) denote the set of the optimal labels of \(S_F\) when not considering the co-evolution edges (i.e. \(E_C = 0\)).
Observation 3. If the weight tables of the tree edges are natural numbers and all the entries in the weight tables of the co-evolution edges \(< \frac{1}{|E_c(S_F)|}\) then the optimal labeling of the co-evolving forest is one of the optimal labels in \(\hat{\lambda}(S_F/E_C)\).

Due to lack in space the proof of Observation 3 is omitted and will appear in the full version of this paper. By Observation 1 the ancestral co-evolution problem without co-evolution edges describes a conventional maximum likelihood problem. Thus, by Observation 3 if we choose small enough weights for the co-evolution edges our method can be used for choosing one of the optima (or a point very close to one of the optima) of the maximum likelihood function – the one that is supported by co-evolutionary relations.

4.2 Weighting of the Co-evolution Edges

We tested several values for the weights of the co-evolution edges compared to the tree edges. At one extreme, the entries of the tree edges weight tables are multiplied by a very large constant. In this case, the tree edge weight tables are dominant compared to the weights of the co-evolution edges (the solution is one of the ML/MP solutions). At the second extreme, the fifth weighting, the co-evolution edge weights are dominant compared to the tree edge weights. In this case, the entries of the tree edges weight tables are multiplied by a very large constant.

Let \(MP^b(S_F, W)\) denote the parsimony score when solving the ancestral co-evolution problem with weighting \(W\) and when considering only tree edges. Let \(MP^c(S_F, W)\) denote the parsimony score when solving the ancestral co-evolution problem with weighting \(W\) and when considering only co-evolution edges. In this work, we used the weighting, \(W^*\), that optimizes the sum of the two sources of information (co-evolution, and the evolutionary trees); i.e. we used \(W^* = \arg\min_W (\frac{MP^c(S_F,W)}{\min_W (MP^c(S_F,W))} + \frac{MP^b(S_F,W)}{\min_W (MP^b(S_F,W))})\).

5 Experimental Results

5.1 Simulated Evolution

To analyze the performances of the algorithms described in the previous section we generate a probabilistic process that describes the evolution of a co-evolving forest. In the simulation, each character evolves along the branches of the evolutionary trees, but also has correlations with the other characters that interact with it. The simulation was performed as in [48]; due to lack in space, all the details about the simulations are deferred to the full version of this paper.

5.2 Simulation Results

We compared the performances of the following algorithms: 1) The Partitioning algorithm (Figure 3 B.) with the Dynamic Programming algorithm (Figure 3
C.). 2) The greedy algorithm (Figure 3 D.) with a few initial points (one of them is the the ML solution). 3) The ACE algorithm (all the stages in Figure 3). 4) The ML and MP algorithms that do not consider the co-evolution edges. Let \( DP_X \) denotes a Dynamic Programming algorithm with \( K = X \); let \( ACE_X \) denotes an ACE algorithm with \( K = X \).

A summary of the simulation results appears in Figure 4; sub-figures A – C depict the running times (log scale) and sub-figures D – F describe the approximation ratios as functions of the size of the evolutionary trees, the number of evolutionary trees, the number of co-evolution edges per node in the co-evolutionary graph. All the running finished in less than a four minutes. As can be seen, the running time increases exponentially with \( K \) (see \( ACE_7, DP_7 \) vs. \( ACE_2, DP_2 \) sub-figures A – C) while the running time of the greedy algorithm alone is larger than \( DP_2 \) but exponentially smaller than \( DP_7 \). As can be seen, in the case of running time, the most influential parameter is the number of evolutions trees in the input.

In the case of the approximation ratio (the upper bound from Observation 2), the most influential parameter is number of co-evolution edges per node in the co-evolutionary graph. As can be seen, \( ACE_7 \) performs better than all the other algorithms and always has approximation ratio < 1.3. Interestingly, the greedy algorithm is only a few percentages worse. These results support using the greedy algorithm if running time matters. The fact that the upper bound of the \( ACE_7 \) < 1.3 demonstrates that our approach can find solutions that are very close to the optimal ones. As we used here an upper bound on the approximation ratio the actual ratio can be significantly lower.

In the simulation, as in the case of biological data (see the next section), the ML solutions (when ignoring co-evolution) are relatively similar to the solutions found by our approach. Thus, it is not surprising that approximation ratio of ML is bound to be < 1.7. On the other hand, the margin between the approximation ratio of the ML and that of the ACE is significant: up to 30%. This result demonstrates the essentiality of our approach.

5.3 A Biological example: Reconstruction of the Ancestral Genome Content the Fungi

Using the method outlined above we set to reconstruct the ancestral genome content of 17 Fungi whose evolutionary tree appears in Figure 5. The input included 33,931 families of Fungi orthologs (downloaded from [11]) and a total of 20,317 co-evolution edges.

We represented each of the 17 genomes by a binary string of length 33931 where ‘1’ in the \( x \) position of a string means that there is a gene/protein from the \( x \) group of orthologs in this genome, and ‘0’ means that there is no gene/protein from the \( x \) group of orthologs in this genome. We used Neyman’s two state model [36], a version of Jukes Cantor (JC) model [27] for inferring the edge lengths of the tree by maximum likelihood. This was done by PAML [50]. These edge lengths correspond to the probabilities that a protein will appear/vanish along the corresponding lineage.
Fig. 4. Running time and accuracy of the partitioning algorithm with the Dynamic Programming algorithm, the greedy algorithm, the ACE, and the ML/MP algorithms. A – C.: The log running time (in ms) of the algorithms as function of the size of the evolutionary trees (A.; 200 trees and 2.5 co-evolution edges per node in the co-evolutionary graph), number of co-evolutionary trees (B.; 2.5 co-evolution edges per node in the co-evolutionary graph, each evolutionary tree with 70 leaves), and the number of co-evolution edges per node in the co-evolutionary graph (C.; 200 evolutionary trees, each with 70 leaves). D – F.: The upper bound on the approximation ratio (Observation 2) of the solution found by each of the algorithms as function of the number of leaves in each evolutionary tree (D.), the number of evolutionary trees (E.), and the number of co-evolution edges per node in the co-evolutionary graph (F.).
As co-evolution edges we use various physical and functional interactions that were downloaded from String [14] (http://string.embl.de/; for example, [15,49] reported the relation between co-evolution and similar functionality). We filtered co-evolutionary edges whose ratio between the highest and lowest probability in the co-occurrence distribution table was less than 4.25. The weights in the tables were computed according to the co-occurrence probabilities of the corresponding pairs of orthologs. We used the weighting whose corresponding solution optimizes both the score of the co-evolution edges and the score of the tree edges (see more details in subsection 4.2). The annotation of ancestral proteins was based on the GO annotations of \textit{S. cerevisiae}.

We compared our results of the ACE to those obtained by using only tree edges (by using MP and ML).

The total running time of the ACE algorithm on this large biological dataset was about 1.5 hours on a conventional PC. Figure\textsuperscript{5} summarizes the results of the analysis by ACE. As can be seen, ACE removed/added hundreds of proteins to ML/MP labels of the internal nodes of the evolutionary trees. A major fraction of the discrepancies between the results of the ACE algorithm and those obtained with the conventional methods (ML and MP) appears in the \textit{Euascomycota} subtree (internal nodes 0 – 2 in the Fungi tree; see Figure\textsuperscript{5} A.). In general, ACE
mainly added proteins to these nodes, implying that ML/MP underestimated the size of these ancestral genomes. Additionally, both in the case of MP and ML, the ACE added/removed many proteins from internal nodes 14 and 15. It is important to note that the likelihood score of the ACE solution was only 0.25% lower than the ML score, demonstrating that this solution was very close to the ML point. Similarly, when the ACE was implemented with MP the parsimony score of its solution was only 2.2% higher than that of the MP solution. These solutions, however, are supported by the co-evolutionary information and thus are more biologically plausible.

We further analyzed the proteins added by the ACE to the ML solution for the ancestors of the Euascomycota (internal nodes 0 – 2): the nodes with the largest number of discrepancies between ML and ACE. The groups of proteins added to each of these nodes were very similar (around 95% similarity); thus we report only the results for node 2, the ancestor of the Euascomycota.

ACE added 89 proteins to the ML solution of internal node 2. Various pieces of evidence support the biological plausibility of the addition of these proteins by ACE: First, the group of proteins added to this node was enriched with proteins that take part in basic and essential metabolic processes. Specifically, it was enriched with the cellular process: protein amino acid phosphorylation (p-value = 0.00054), amine transport (p-value = 0.00153), and amino acid transport (p-value = 0.00518); all p-values passed the False Discovery Rate (FDR) control for multiple hypothesis testing [2]. Second, all these proteins have orthologs in most of the analyzed Fungi (on average in 76% of the Fungi); this fact also supports the essentiality of these proteins. Third, in S. cerevisiae, many of these proteins are part of the same complex with proteins inferred by the standard ML (note that co-evolutionary relations used by ACE do not necessarily imply association in the same complex).

The following is a typical example that demonstrates the three points mentioned above:

**Example 1:** Three of the proteins added by the ACE are orthologs of S. cerevisiae TPK1, TPK2, and TPK3. The presence of at least one of these genes is required for normal growth in S. cerevisiae [47]. These genes are part of the complex cAMP-dependent protein kinase complex that includes another protein (BCY1), which was inferred by ML.

### 6 Concluding Remarks

In this study we formally described a new computational approach for reconstructing ancestral genomic sequences using information about co-evolution. Our model captures co-evolutionary dependencies between different proteins and uses this information to disambiguate the labels of the reconstructed ancestral genomes.

In the future we intend to generalize this work in several ways. First, currently, our approach is presented in the setup of ancestral genome reconstruction, due to the importance of this problem and because co-evolutionary information can
be readily obtained on the gene/protein level. However, it is important to note that the potential scope of our approach goes beyond the ancestral genome reconstruction problem, to tackle the more general problem of ancestral sequence reconstruction: that is, the reconstruction of different sites or domains in proteins, and even (provided that sufficient information is available) the reconstruction of single sites in DNA or RNA sequences (see [52,53,33,39,28,42]). In this setup, the success of such future applications depends on the existence of reliable co-evolutionary information on the individual site level. Second, it is also clear that our approach can be generalized in the future to more complex reconstruction models (for example, using non-binary alphabets, dependency between close sites, and various versions of maximum likelihood). Third, we intend to design algorithms that may compete with those described in this work. Specifically, we intend to check algorithms that are based on the belief propagation approach. Finally, we believe that a generalization of the approach described in this work can be used for joint inference of ancestral genomes and protein interactions or for joint inference of ancestral genomes and metabolic networks of ancestral and extant organisms.

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