Statistics of the individual-pulse polarization based on propagation effects in the pulsar magnetosphere

S. A. Petrova

Institute of Radio Astronomy, 4, Chervonopraporna St., Kharkov 61002, Ukraine

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ABSTRACT

Pulsar radio emission is modelled as a sum of two completely polarized non-orthogonal modes with the randomly varying Stokes parameters and intensity ratio. The modes are the result of polarization evolution of the original natural waves in the hot, magnetized, weakly inhomogeneous plasma of the pulsar magnetosphere. In the course of the wavemode coupling, the linearly polarized natural waves acquire purely orthogonal elliptical polarizations. Further on, as the waves pass through the cyclotron resonance, they become non-orthogonal. The pulse-to-pulse fluctuations of the final polarization characteristics and the intensity ratio of the modes are attributed to the temporal fluctuations in the plasma flow.

The model suggested allows one to reproduce the basic features of the one-dimensional distributions of the individual-pulse polarization characteristics. Besides that, the propagation origin of the pulsar polarization implies a certain correlation between the mode ellipticity and position angle. On a qualitative level, for different sets of parameters, the expected correlations appear compatible with the observed ones. Further theoretical studies are necessary to establish the quantitative correspondence of the model to the observational results and to develop a technique of diagnostics of the pulsar plasma on this basis.

Key words: plasmas – polarization – waves – pulsars: general.

1 INTRODUCTION

The pulsar magnetosphere contains the ultrarelativistic electron–positron plasma which streams along the open magnetic lines. The radio emission observed from pulsars is undoubtedly associated with the processes in the plasma flow, and its characteristics are believed to reflect the properties of the ambient plasma. Whatever the radio emission mechanism, it should give rise to the natural waves of the plasma. The ultrarelativistic, strongly magnetized plasma of the pulsars allows two types of non-damping natural waves, the ordinary and extraordinary ones, which are linearly polarized in orthogonal directions, in the plane of the ambient magnetic field and perpendicularly to this plane, respectively. The pulsar beam is believed to be an incoherent mixture of the two polarization modes. Since the electric vector of each mode is related to the magnetic field direction, the position angle (PA) of linear polarization should vary smoothly across the pulse as the pulsar beam rotates with respect to an observer (Radhakrishnan & Cooke 1969). The expected S-shaped swing of the PA is indeed observed in a number of the pulsars. In other cases, abrupt, nearly orthogonal jumps of the PA can break the smooth swing, testifying to the presence of both polarization modes.

In general, these modes can be markedly non-orthogonal (e.g. Gil, Snikowski & Stinebring 1991; Gil et al. 1992; Gil & Lyne 1995).

An extensive analysis of observational data has led to the conclusion that the two polarization modes are simultaneously present in pulsar radiation (McKinnon & Stinebring 1998, 2000). From the theoretical point of view, this is the only way to account for the partial depolarization of the pulsar radiation, since the natural waves are completely polarized by definition. Thus, the model of superposed polarization modes with randomly varying intensities has weighty observational and theoretical grounds. At the same time, it faces serious difficulties and seems too simplified to explain diverse and complicated behaviour of the individual pulse polarization. Generally, the observed modes have elliptical rather than linear polarization and can be non-orthogonal (Karastergiou et al. 2001; McKinnon 2003). Besides that, the observed pulse-to-pulse polarization fluctuations cannot be solely attributed to the variations of mode intensities – the polarization characteristics of the modes should fluctuate as well (e.g. Karastergiou, Johnston & Kramer 2003). These difficulties can be avoided if one take into account the propagation effects during the wave passage through the flow of the pulsar plasma. Then the fluctuations in the observed polarization characteristics can be attributed to the fluctuations in the parameters of the plasma. It should be noted that the propagation origin of the pulsar polarization implies a certain correlation between the final polarization characteristics, as they are determined by the instantaneous state.
of the plasma in the regions of significant polarization evolution inside the pulsar magnetosphere. The indications of such a correlation are indeed discovered in observations (Edwards & Stappers 2004; Edwards 2004).

Deep inside the magnetosphere, the plasma number density is so large that the natural waves propagate in the regime of geometrical optics, with the polarization planes being adjusted to the local magnetic field direction. Further along the trajectory, as the plasma density decreases considerably, the wave mode coupling starts: each of the incident waves becomes a coherent mixture of the two natural waves peculiar to the ambient plasma. As a result, the waves acquire some circular polarization and a shift in PA occurs, so that it no longer reflects the magnetic field geometry. The analytical and numerical tracings of the polarization evolution caused by the wave mode coupling have proved that, at the conditions relevant to the magnetosphere of a pulsar, this process can be efficient enough to account for the typically observed circular polarization and PA shift (Lyubarskii & Petrova 1999; Petrova & Lyubarskii 2000; Petrova 2003). For given plasma parameters, both types of waves acquire the same shift in PA and the same fractional circular polarization of opposite senses. Thus, the outgoing waves have an elliptical polarization that is purely orthogonal at the Poincaré sphere. This representation is compatible with the bulk of the observational data. However, sometimes the observed polarization modes are markedly non-orthogonal (e.g. McKinnon 2003).

The non-orthogonality of the polarization modes can be attributed to the cyclotron absorption in pulsar magnetosphere, if this process follows the wave mode coupling (Petrova 2005, 2006). A similar idea has recently been discussed in Melrose et al. (2006). Typically the region of cyclotron resonance lies beyond the region of efficient wave mode coupling, but they can be quite close to each other, in which case the plasma number density is still large enough for the resonance to affect wave polarization considerably. For the two types of natural waves, the rate of cyclotron absorption is slightly different. Because the waves entering the resonance region already present a coherent mixture of the two natural waves and these constituents are absorbed to slightly different extents, the polarization state of the waves changes. The original ordinary and extraordinary waves suffer different polarization evolution and become non-orthogonal.

The aim of the present paper is to study statistics of the propagation-induced polarization of the pulsars resulting from the fluctuations in the plasma flow. Special attention is paid to studying the correlation between the mode ellipticity and PA because of propagation effects. Section 2 contains the general equations of polarization evolution in pulsar plasma and an example of numerical tracings of the ellipticity and PA. The statistics of the final polarization parameters in the plasma with randomly varying parameters are examined in Section 3. The numerically simulated histograms of the PA and ellipticity, as well as the two-dimensional scatter plots of the Stokes parameters are given. The results of the paper are discussed and summarized in Section 4.

2 GENERAL THEORY OF THE POLARIZATION EVOLUTION IN PULSAR PLASMA

Let the radio waves propagate through the ultrarelativistic electron–positron plasma, which streams along the open field lines of the dipolar magnetic field of a pulsar. Because of the continuity of the plasma flow within the open field line tube, the plasma number density decreases with distance from the neutron star just as the magnetic field strength does; \( N \propto B \propto z^{-3} \). Thus, the radio waves propagate in the weakly inhomogeneous medium, with the scale length for change in the parameters much larger than the wavelength.

The plasma is assumed to be hot, so that the region of cyclotron resonance is also much larger than the wavelength.

The waves are believed to originate deep inside the magnetosphere, where the plasma number density is large enough to provide the conditions of geometrical optics and the magnetic field is strong enough for the electron gyrofrequency to be much larger than the radio frequency in the plasma rest frame. As both \( N \) and \( B \) decrease rapidly along the trajectory, both these conditions are ultimately broken. First of all, the scale length for beats between the natural waves, \( L_b \sim c/(\omega \Delta n) \), where \( \Delta n \) is the difference in their refractive indices, becomes comparable to the scale length for change in the medium parameters, \( L_p \sim z \), so that geometrical optics is violated and wave mode coupling holds. In the case of the cold, ultrarelativistic, strongly magnetized plasma,

\[
\Delta n = \frac{2\omega_p^2}{\gamma \omega^2 \delta^2},
\]

where \( \omega_p \equiv \sqrt{(4\pi n e^2)/m} \) is the plasma frequency, \( \gamma \) is the Lorentz factor, \( \theta \) is the wavevector tilt to the ambient magnetic field, and it is taken into account that the wave propagation is generally quasi-transverse, \( \theta \gg 1/\gamma \). Then the characteristic radius of the wave mode coupling is determined by the relation

\[
\frac{2\omega_p^2(r_p)}{\gamma^2 \omega^2 \delta^2(r_p)} = 1.
\]

In the case of hot plasma, equations (1) and (2) remain the same, but \( \gamma \) means some characteristic Lorentz factor of the plasma particle distribution. For the conditions relevant to pulsar magnetosphere, the radius of the wave mode coupling, \( r_p \), can be estimated as (equation 16 in Petrova 2006):

\[
\frac{r_p}{r_L} = 0.18 P^{3/2} \gamma_{1.5}^{3/2} \kappa_2^{1/2} B_{12}^{1/2} v_9^{1/2} \psi^{-1/2}.
\]

Here \( r_L \) is the light cylinder radius, \( P \) is the pulsar period, \( \kappa \) is the plasma multiplicity, \( \kappa_2 \equiv \kappa/10^2 \). \( B_\star \) is the magnetic field strength at the surface of the neutron star, \( B_{12} \equiv B_\star /10^{12} \) G, \( \nu \) is the radio frequency, \( v_9 \equiv \nu/10^9 \) Hz, \( \gamma_{1.5} \equiv \gamma/10^{1.5} \), \( \psi \equiv \theta/10^{-1} \). One can see that \( r_p \) is determined by the basic pulsar parameters, \( P \) and \( B_\star \), as well as by the parameters of the plasma flow, \( \kappa \) and \( \gamma \). Besides that, \( r_p \) strongly depends on the angle \( \theta \), which introduces a significant uncertainty in the estimate (3). The value of \( \theta \) may differ within an order of magnitude (\( \theta \sim 0.1 - 1 \)) for different pulsars and for the rays observed at different longitudinal ranges in a given pulsar. On the whole, one can conclude that \( r_p \) lies inside the light cylinder in the majority of the pulsars for all the rays forming the pulse profile.

It should be noted that the wave mode coupling holds only if the wave vector goes outside the initial plane of magnetic lines. In the pulsar case, this condition is certainly satisfied, e.g. because of the magnetosphere rotation. Let us choose the Cartesian coordinate system with the \( z \)-axis along the wavevector and the \( x \)-axis in the initial plane of magnetic lines (see Fig. 1). The ray is emitted along a field line of the dipolar magnetic field at an angle \( \psi \) to the magnetic axis. In the non-rotating magnetosphere, the ray would propagate in the plane of magnetic lines and at distances much larger than the emission altitude would make the angle \( \sim \psi \) with the ambient magnetic field because of the divergence of the magnetic lines, so that \( b_z \sim \psi, b_x = 0 \). Because of the magnetosphere rotation, \( b_z \) increases with distance \( \propto z/r_L \) (the exact formulas for \( b_x \) and \( b_y \) which allow for the rotational aberration, are given by equation 3 in Petrova 2003). The quantity \( \mu \equiv (b_y/b_z)_{\|} \sim r_p/(r_L \psi) \) is the

\[
\Delta n = \frac{2\omega_p^2}{\gamma^2 \omega^2 \delta^2}.
\]
The process of the wavemode coupling holds as long as \( L_0 \sim \mu \cdot r_p \). As the plasma number density decreases further, this process ceases and the waves propagate as in a vacuum, preserving their polarization state. For this reason, the wavemode coupling is usually called the ‘polarization-limiting effect’. However, it is actually the wave ellipticity arises and changes purely on account of the wavemode coupling and cyclotron absorption. An opposite approach has recently been developed by Luo & Melrose (2004) and Melrose & Luo (2004a), who investigated the characteristics of the elliptically polarized natural waves at \( r \sim r_p \) ignoring the wavemode coupling in this region. The effect of cyclotron absorption on the elliptical natural waves has been considered in Melrose & Luo (2004b).

An exact treatment of polarization transfer in pulsar plasma taking account of the wavemode coupling and cyclotron absorption can be found in Petrova (2006). The evolution of the Stokes parameters of the natural waves is described by the following set of equations:

\[
\frac{dI}{dw} = -\eta^2 s F_1 \left( \frac{1 - \mu^2}{\eta^2 w^2} \right) - 2\mu \eta F_2, \\
\frac{dQ}{dw} = -\eta^2 s F_1 \left( \frac{1 - \mu^2}{\eta^2 w^2} \right) - 2\mu F_1, \\
\frac{dU}{dw} = \eta^2 w F_1 \left( \frac{1 - \mu^2}{\eta^2 w^2} \right) - 2\mu F_2, \\
\frac{dV}{dw} = -\eta^2 w F_1 \left( \frac{1 - \mu^2}{\eta^2 w^2} \right) - 2\mu F_1 q.
\]

Here \( w = r_c/z, s = 1 + \mu^2 \left([1 + \mu^2/(\eta^2 w^2)]^2\right) \),

\[
F_1 = v_p \left[ \int f(\gamma) d\gamma \left( \frac{\gamma}{\gamma_0} \right)^3 \left[ 1 - \left( \frac{\gamma}{\gamma_0} \right)^3 w^{-6} \right] \right],
\]

\[
F_2 = \frac{\pi}{2} w^{-6} \gamma_0 f \left[ w^3 \gamma_0 \right],
\]

v.p. means that the integral is taken in the principal value sense, \( f(\gamma) \) is the particle distribution function with the normalization \( \int f(\gamma) d\gamma = 1 \), \( \gamma_0 \) is the characteristic Lorentz-factor of the plasma particles, \( r_c \), the characteristic radius of cyclotron resonance defined as \( \omega_0(r_c) = \omega_{\gamma 0} \beta^2 / 2 \). In equation (5) we have omitted the factors common to all the Stokes parameters, because they are irrelevant to the problem of polarization evolution that is considered.² The set of equations (5) is a generalization of equations (17) in Petrova (2006) for the case of arbitrary \( \mu \). At the same time, we still do not include the terms responsible for rotational aberration explicitly and assume that they enter \( b_i \) and \( b_0 \) as factors of the order of unity (for more detail see Petrova 2006). The terms containing \( F_1 \) describe polarization evolution as a result of the wavemode coupling, while those containing \( F_2 \) correspond to cyclotron absorption. The initial conditions for the natural waves read: \( (I, Q, U, V) = (1, 0, 0, 0) \), where

1 Note, however, that the cyclotron absorption can markedly affect the total intensity of the pulsar radiation. In the short-period pulsars, \( P \sim 0.1 \) s, the absorption depth can exceed unity (Lyubarskii & Petrova 1998). As is shown in Petrova (2002), the effect of resonant absorption can account for the observed statistical features in the energetic characteristics of the short-period pulsars.

Fig. 1. Geometry of the vectors \( \mathbf{k} \) and \( \mathbf{b} \) in the coordinate system described in the text. \( B_0 \) is the initial orientation of the magnetic axis.
waves as they propagate in the pulsar magnetosphere. One can see that the polarization evolution can indeed be significant and differs markedly for the two modes.

3 STATISTICS OF THE FINAL POLARIZATION PARAMETERS

3.1 Histograms of the PA and ellipticity

As is discussed above, polarization evolution is determined by the parameters $\mu$ and $\eta$ and is affected by the particle distribution function. For the rays observed at a fixed pulse longitude, the locations of the regions of the wavemode coupling and cyclotron resonance may vary from pulse to pulse because of fluctuations in the number density and the characteristic Lorentz factor of the plasma particles. Besides that, the fluctuations in the plasma distribution may affect the refraction of the waves, so that the rays of a certain orientation may follow somewhat different trajectories in the open field line tube. All this is believed to influence the final polarization states of the original natural waves. Note that although the modern theories of the pulsar pair creation cascade say nothing about the fluctuations in the resultant distribution of the secondary plasma, an idea of such fluctuations is supported by the variability of the observed individual pulse profiles.

In the upper panel of Fig. 3, the left-hand plot shows the histogram of the final PA of the original natural waves in case of randomly varying parameters $\mu$ and $\eta$. One can see that the peaks are separated by not exactly $90^\circ$, the scatter of the PA values around the peaks is about $10^\circ$ and the ordinary mode (that with positive PA) shows somewhat less scatter than the extraordinary one. The histogram of mode efficiencies for the same $\mu$ and $\eta$ is shown in the right-hand plot of the upper panel in Fig. 3. Note that the peaks are not symmetrical with respect to zero and the extraordinary mode exhibits a more pronounced scatter.

The middle and bottom panels of Fig. 3 show the histograms of the PA and ellipticity for the sum of the two polarization modes given random variations in $\mu$, $\eta$ and the mode intensity ratio. In the present consideration, we assume that initially only the ordinary mode is emitted, whereas the extraordinary mode arises as a result of the partial conversion of the ordinary mode deep inside the magnetosphere (see Petrova 2001). This is supported by the recently discovered anticorrelation of the mode intensities (Edwards & Stappers 2004). Because the mode conversion is a propagation effect, its efficiency is also determined by the instantaneous distribution of the plasma and is believed to fluctuate. Thus, the coefficient of conversion, $\tau$, is considered as a random quantity. Generally speaking, it may be correlated with $\mu$ and $\eta$, but as the process of mode conversion takes place far from $r_p$ and $r_c$, the correlation is thought to be weak and is neglected throughout the paper. In the middle panel of Fig. 3, $\tau$ is taken to be distributed over the whole interval from 0 to 1, whereas in the bottom panel $\tau \in [0.6, 1]$.

Although in the former case the probability to dominate is the same for the two types of original natural waves, the humps at the PA histograms are substantially distinct: the peak corresponding to the ordinary waves is much more pronounced. This is a consequence of cyclotron absorption, which suppresses the extraordinary constituent of the wave polarization more strongly. The extraordinary-mode hump looks more smeared and is connected to the ordinary-mode one with a bridge. In the bottom panel, the humps change in dominance, but the ordinary mode is still present, despite the complete dominance of the extraordinary waves just after conversion. This is again because of the differential action of cyclotron absorption.
Figure 3. Numerically simulated histograms of the final PA and ellipticity after polarization evolution in the fluctuating plasma. The parameters $\mu$ and $\eta$ are uniformly distributed over the intervals $[0.2, 0.4]$ and $[0.9, 1.1]$, respectively. The upper panel shows the histograms for the original ordinary and extraordinary modes. The middle and lower panels correspond to the sum of modes with random intensity ratio. The coefficient of conversion is uniformly distributed in the intervals $[0, 1]$ and $[0.6, 1]$, respectively.
absorption on the two types of natural waves. The bridge between the humps looks more pronounced, and the histogram on the whole is similar to the observed ones (e.g. McKinnon 2003).

In the middle histogram of the ellipticity, the hump corresponding to the extraordinary mode is barely resolved, whereas the ordinary mode peaks at $\chi \approx 0$. On the whole, this distribution looks like a unimodal one with a long tail. As can be seen in the bottom histogram of $\chi$, only positive values are met and the two humps are barely resolved. This distribution can also be regarded as a unimodal one, in contrast to the corresponding histogram of the PA. It is not our aim here to fit the concrete distributions observed, but Fig. 3 demonstrates the principal possibility of such fits within the framework of the propagation model of the pulsar polarization.

### 3.2 Two-dimensional scatter plots

Propagation origin of the pulsar polarization implies a certain correlation between the mode ellipticity and PA: both these quantities are determined by the instantaneous state of the plasma and vary from modulation between the mode ellipticity and PA: both these quantities are.

If one considers $\rho$ and $\lambda$ as the polar coordinates (the radius and azimuth, respectively), we come to the Lambert azimuthal equal area projection of the Poincaré sphere. This projection is interrupted at the equator, and for the Stokes vectors lying in the southern hemisphere (i.e. for the second mode) the projection of the sphere with the polar axis along $s_m = -s_p$ is considered (for more detail see Edwards & Stappers 2004).

Fig. 4 shows the two-dimensional plots for the two original natural modes after their evolution in the plasma in case of negligible cyclotron absorption, $\eta \sim 0.01$, and moderate mode coupling, $\mu \sim 0.1$. Here $s_p$ corresponds to the original ordinary mode. One can see that the process of mode coupling in the fluctuating plasma results in the unambiguous relation of the polarization characteristics, which has already been proposed as a basis for the diagnostics of the pulsar plasma (Petrova 2003). At these plots, the points prefer certain azimuths, which is in qualitative agreement with the observational result of Edwards (2004) (see fig. 5 there), though the scatter of the observational points is enormously large. Note that this scatter cannot be reproduced by considering the sum of the two modes with random intensities. Introducing a slight non-orthogonality in the modes because of cyclotron absorption ($\eta \sim 0.1$) makes the plot more realistic (see Fig. 5 for the case of completely dominating original ordinary mode, $\tau \in [0, 0.3]$), though the scatter of the points is still insufficient.

Fig. 6 shows the two-dimensional scatter plots in the case of strongly non-orthogonal modes ($\eta \sim 1$). In the upper panel, the fiducial Stokes vector, $s_p$, corresponds to the state with the dominant ordinary mode, whereas the coefficient of conversion is uniformly

![Figure 4](https://example.com/f4.png)

**Figure 4.** Two-dimensional scatter plots of the final Stokes parameters of the original ordinary (left-hand panel) and extraordinary (right-hand panel) modes in the Lambert azimuthal equal area projection (for more detail see text); $\mu \in [0.1, 0.5]$ and $\eta \in [0.01, 0.05]$. The one-to-one correspondence of the polarization parameters is evident.
The role of cyclotron absorption is not negligible, \( \eta \in [0.1, 0.4] \); \( \mu \in [0.1, 0.5] \).

Figure 5. The Lambert azimuthal equal area projection of the final Stokes parameters of the sum of modes with the dominant ordinary mode; \( \tau \in [0, 0.3] \); the role of cyclotron absorption is not negligible, \( \eta \in [0.1, 0.4] \); \( \mu \in [0.1, 0.5] \).

Figure 5. The Lambert azimuthal equal area projection of the final Stokes parameters of the sum of modes with the dominant ordinary mode; \( \tau \in [0, 0.3] \); the role of cyclotron absorption is not negligible, \( \eta \in [0.1, 0.4] \); \( \mu \in [0.1, 0.5] \).

4 DISCUSSION AND CONCLUSIONS

The hot, magnetized, weakly inhomogeneous plasma of the pulsars may substantially affect the radio wave propagation. The polarization states of the original natural waves may change markedly because of the wavemode coupling and cyclotron absorption. The former process turns the original linearly polarized waves into the elliptical ones, which are still purely orthogonal at the Poincaré sphere. The mode coupling efficiency is determined by the location of the coupling region, \( r_p \), in the tube of open magnetic lines. The role of cyclotron absorption in the evolution of wave polarization depends on how close the regions of mode coupling and cyclotron resonance are, being most prominent in the case of their approximate coincidence, \( \eta = r_p/r_c \approx 1 \). Typically \( r_c \) lies beyond \( r_p \), in which case cyclotron absorption contributes to the non-orthogonality of the waves. Note that the original natural waves remain completely polarized in all the magnetosphere.

Temporal fluctuations in the plasma flow are believed to underlie the fluctuations of the individual pulse polarization. The plasma distribution in the open field line tube can be strongly variable because of the random character of the pair creation process. Hence, for the rays observed at a fixed pulse longitude, both \( r_p \) and \( r_c \) can change from pulse to pulse due to variations of the number density and characteristic Lorentz-factor of the plasma. Besides that, the variations of the plasma distribution may affect refraction of waves, so that the rays observed at a given pulse longitude may follow somewhat different trajectories in the magnetosphere and have different tilts to the ambient magnetic field while passing through the regions of mode coupling and cyclotron resonance.

The intensity ratio of the modes fluctuates as well, and it is also thought to result from the plasma fluctuations in the magnetosphere. The point is that the extraordinary natural waves have vacuum dispersion and can hardly be generated directly by any conceivable emission process in the plasma. Therefore they are likely to originate as a result of the partial conversion of the ordinary waves, which can take place in the regions of quasi-longitudinal propagation deep inside the magnetosphere (Petrova 2001). The idea of mode conversion is supported by the recently discovered anticorrelation of the mode intensities (Edwards & Stappers 2004). In the framework of this view, the mode intensity ratio is determined by the coefficient of conversion and changes from pulse to pulse because of fluctuations in the plasma flow.

Thus, the propagation model of the pulsar polarization incorporates two superposed, completely polarized modes with randomly varying polarization states and intensities. It is important to note that, because of cyclotron absorption, the superposed modes can become non-orthogonal, whereas the original natural waves are purely orthogonal by definition.

In the present paper, the statistics of the individual pulse polarization have been simulated under the assumption that the parameters \( \mu \) and \( \eta \) as well as the coefficient of conversion, \( \tau \), are the random quantities with uniform distributions over some intervals. The resultant histograms of the PA exhibit two humps which are markedly smeared and connected by a bridge. The peak separation can markedly differ from \( \sim 90^\circ \). The histograms of the resultant ellipticity look like the unimodal ones because of a very small separation between the peaks of the two observational modes. Furthermore, it may happen that in the whole sample the ellipticity is purely of one sign, whereas the corresponding histogram of the PA is bimodal. Although direct fits to the observational data are beyond the scope of the present paper, our results confirm the ability of the propagation model to account for the main features of the observed histograms of the PA and ellipticity.

The propagation origin of the pulsar polarization implies a certain correlation between the mode ellipticity and PA. Given that the contribution of the cyclotron absorption is negligible and the final polarization is determined solely by the mode coupling, there is a one-to-one correspondence between the mode ellipticity and PA. However, it is not proved by the observational data available. Although evidence for the expected relation is indeed present in the polarization of PSR B0818-13 (Edwards 2004), the scatter of the observational points appears dramatic, and it cannot be reproduced by taking into account the fluctuations of the mode intensity ratio. In the case of moderately weak cyclotron absorption, a slight non-orthogonality of the fluctuating modes causes an additional scatter of the polarization parameters and allows one to reproduce the observational plots on a qualitative level, though the scatter is still insufficient.
Statistics of the pulsar polarization

Figure 6. Two-dimensional scatter plots of the final Stokes parameters in case of strong non-orthogonality of the modes, \( \eta \in [0.5, 1.5] \). In the upper and lower panels, \( \tau \in [0.5, 1] \) and \( \tau \in [0, 1] \), respectively, and \( s_p \) is centred on the ordinary- and extraordinary-wave positions, correspondingly.

quantitatively. In the case of strong non-orthogonality of the modes, our model qualitatively reproduces the characteristic arc-like features present in the observational plots for PSR B0329+54 (Edwards & Stappers 2004), though the magnitude of non-orthogonality and the total scatter of the simulated points are again less than in observations.

Thus, on a qualitative level, the expected correlations of polarization parameters are compatible with the observational data. This is a strong argument in favour of the propagation model of the pulsar polarization. Apparently, to achieve better quantitative agreement with the observational results it is necessary to improve and further develop the model suggested. In the present consideration, we have ignored the net charge density in the magnetosphere and treated the process of cyclotron absorption in the limit of small pitch-angles of the particles. In reality, both these assumptions can be violated, so that our results on the individual-pulse polarization and its statistics can be somewhat modified. Besides that, pulsar emission can be considered as a sum of contributions from multiple subsources (e.g. Gil & Lyne 1995; Melrose et al. 2006), in which case the possibilities of modelling the resultant polarization are much wider.

In conclusion, it should be pointed out that the model suggested allows a unique possibility of diagnostics of the pulsar plasma by means of the individual-pulse polarization.

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