Testing the Sprint Curve Model using the 150m Bailey–Johnson Showdown

J. R. Mureika*

Department of Computer Science
University of Southern California
Los Angeles, California 90089 USA

PACS No. : Primary 01.80; Secondary: 02.60L

March 31, 2022

Abstract

Recently, a simple model was derived to account for a sprinter’s energy loss around a curve, based on previous sprint models for linear races. This paper offers a quick test of the model’s precision by comparing split times from Donovan Bailey’s 150m “Challenge of Champions” race at Skydome on June 1st, 1997. The discrepancy in the track configuration which almost prompted Bailey to drop from the race is also addressed.

In a highly controversial showdown at Skydome in Toronto on June 1st, Canadian sprinter Donovan Bailey proved that he is the “World’s Fastest Man” by defeating opponent Michael Johnson of the United States over an unconventional distance of 150m. Bailey clocked a time of 14.99s, a mere 0.02s off the “official” World Record of 14.97s, held by Britain’s Linford Christie.

The showdown was set in motion by each athlete’s remarkable performance at the 1996 Olympic Games in Atlanta, Georgia. Bailey captured gold in the 100m with a World Record time of 9.84s, while Johnson obliterated the existing 200m World Record in a breathtaking 19.32s. Following the latter of these sprints, public opinion and media influence (not to mention American nationalism) split the vote on who actually should be the bearer of the title “World’s Fastest Man”, a designation traditionally reserved for

*newt@sumatra.usc.edu
the 100m champion.

A number of predictions were made in the buildup to this race as to the value of the winning time. Even contests were held, with prizes going to the individual who could correctly guess the victor and his corresponding victorious time. The majority of these predictions were well under the official 14.99s finish (see e.g. [1, 2, 3]), and most fell in the range of 14.70 ∼ 14.80s. So, to add to the disappointment of Johnson’s dropping out mid-way due to injury was Bailey’s curiously “slow” victory.

In this brief report, Bailey’s performance will be discussed in light of a simple model to calculate short sprint times for races which are partially run off a curve [5]. To quickly review the model and associated history, one must recall the underlying assumptions of the original model. In the early 1970s, J. Keller [4] proposed that the distance covered by a sprinter over a given time could be expressed as the solution to the differential equation

\[ \dot{v}(t) = f(t) - \tau^{-1}v(t) , \]  

where \( v(t) \) is the velocity at time \( t \), \( f(t) \) is a measure of force/unit mass exerted by the athlete, and \( \tau^{-1} \) is a decay term (which crudely models fatigue–related factors). From his analysis (an optimization problem), Keller determined that \( f(t) = f \equiv \text{constant} \) for short sprints [4]. The solution to (1) is found subject to the constraint \( v(0) = 0, f(t) \leq f \), and the resulting distance \( d \) traveled in time \( T \) is simply

\[ d = \int_0^T dt \, v(t) . \]  

There is an additional equation which couples with (1) if the race is longer than 291m, but this will not be addressed here. The interested reader is directed to the citations in [4] for further reading.

Prompted by the 150m showdown between Bailey and Johnson, R. Tibshirani revised (1) in a statistical analysis based on the Atlanta races which predicted Bailey would win by a margin of (0.02, 0.19) seconds at a 95% confidence level [2]. His adjustment to (1) consisted of the modification of \( f(t) \), reasoning that it is erroneous to set \( f(t) = f = \text{const} \) \( \forall t \). Rather, since a sprinter must experience fatigue at some rate, it would make more sense to assume that \( f(t) = f - ct \), for some \( c > 0 \).

Tibshirani notes [2], however, that neither Keller’s nor his model take into account the effects of the curve, i.e. their power of predictability drastically diminishes for races longer than 100m. In [4], a simple term propor-
tional to the centrifugal force felt by the runner is introduced to Tibshirani’s modification of (1). Since ideally the two forces are normal to each other, the terms are added vectorally. The resulting equation of motion is

\[ \dot{v}(t) = -\tau^{-1}v(t) + \sqrt{(f - ct)^2 - \lambda^2 \frac{v(t)^4}{R^2}}, \]

for a track with curve of radius \( R \).

The term \( \lambda \) is introduced to account for the fact that a sprinter does not feel the full value of the centrifugal force exerted on him/her (by means of leaning into the turn, banked curves, different use of leg muscles, etc...). It is a simplified attempt to model a seemingly non-trivial mechanism.

The total distance of races run off the curve can be expressed as \( d = d_c + d_s \), where

\[ d_c = \int_0^{t_1} dt \, v_c(t), \]
\[ d_s = \int_{t_1}^{T} dt \, v_s(t), \]

with \( v_c(t) \) the solution to Equation (3), and \( v_s(t) \) the velocity as expressed in the modified (1) (with \( f(t) = f - ct \)), subject to the boundary condition \( v_c(t_1) = v_s(t_1) \). Here, \( t_1 \) is the time required to run the curved portion of the race (distance \( d_c \)).

Split times (e.g. the time \( t_1 \) to run the curve) have never been recorded accurately until recently, so there has been up till now an unfortunate lack of empirical data which could be used to make or break such a model. Luckily, the 150m race held at Skydome was well–documented, and splits were obtained for the 50m and 100m marks; the former is on the curve, the latter not. The official splits for Donovan Bailey are given in Table 1. If the model is an accurate representation of the physical process, a value of \( \lambda^2 \) can be found to reproduce these times subject to the equations of motion.

The track used in Skydome was an unconventional configuration of \( d_c = d_s = 75m \) (herafter denoted as 75m+75m), and was to have had a radius of curvature corresponding to lanes 8 and 9 of a standard outdoor track. The corresponding value of \( R \) can be calculated as

\[ R = \left( \frac{100}{\pi} + 1.25(p - 1) \right) \text{metres}, \]
for $p$ the lane number. The form of (3) roughly results from the IAAF regulations governing curvature of outdoor tracks. The total curve length in lane $p = 1$ (the smallest radius of curvature) must be exactly 100m (the complete set of IAAF rules and regulations can be obtained from [8]).

For the equations of motion in (4), the parameters obtained for Bailey from a least square fit of his official Atlanta splits (see [5] for specific details) are used. These are $(f, \tau, c) = (7.96, 1.72, 0.156)$.

The night prior to competition, Bailey threatened to drop out of the race, citing that the track did not conform to the specifications agreed upon in a signed contract. In particular, he claimed that the curvature of his lane corresponded more to lane 3 of an outdoor track rather than lane 8. Additionally, he submitted that the curve was 10m longer than anticipated, giving a 85m+65m configuration instead of 75m+75m [9].

Thus, (3) is solved for the split distances 50m, 100m, as well as the final 150m mark. The model in [5] does not account for the sprinter’s reaction time, which must be added on to the resulting calculated times. For Bailey, this was $t_{\text{react}} = +0.171s$ [6].

Tables 2, 3 give the splits for a 75m+75m configuration total as run by Bailey in lanes 3 and 8, respectively. Tables 4, 5 present the same information for an 85m+65m configuration. Note that in this case, the splits would be equal up to 75m, since (for equal lane assignments) the curve is of the same radius despite the fact that it is longer. So, splits are only given for 85m and beyond. The sixth column lists the sum of difference of squares (a loose measure of relative error from a small sample space) $\Sigma^2 = \sum_{i=50,100,150} \Delta_i^2/T_i^2$, where $\Delta_i = t_i - T_i$, for $T_i$ the official splits of Table 1 and $t_i$ the associated model predictions.

The findings in [5] suggest that $\lambda^2$ could realistically assume a value between 0.50 and 0.80, so similar values are used here. For interest’s sake, possible 200m times for Bailey are extrapolated, to gauge whether or not he could be a viable contender to Michael Johnson in the 200m, as stated in his post-race interview.

Upon first inspection, the model [5] reproduces the official splits and race time surprisingly well. The smallest value of $\Sigma^2$ for each configuration is taken to be the closest fit to the official race splits. These are:

- 75m+75m, lane 3: $\lambda^2 = 0.50; \Sigma^2 = 8.48 \times 10^{-5}$
- 75m+75m, lane 8: $\lambda^2 = 0.80; \Sigma^2 = 7.90 \times 10^{-5}$
- 85m+65m, lane 3: $\lambda^2 = 0.50; \Sigma^2 = 2.71 \times 10^{-5}$
• 85m+65m, lane 8: \( \lambda^2 = 0.70; \Sigma^2 = 4.22 \times 10^{-5} \)

Interestingly enough, the closest match above comes from the 85m+65m lane 3 configuration, which is the configuration allegedly used contrary to the signed contracts. However, it is obvious that a more precise choice of \( \lambda^2 \) could easily yield a closer match for any of the configurations. The different solutions arise from the readjustment of the ratio \( \lambda/R \); hence, to narrow down an “exact” solution (if indeed one exists), one needs to analyze other race splits. An interesting test of the model will come at the end of June 1997, when Donovan Bailey will run a 150m race at a Grand Prix meet in Sheffield, England (most likely on a 50m+100m configuration; see [1] for Bailey’s possible 150m times on such a track).

In [5], some possible 200m times were obtained which Bailey might be able to run under peak conditions. It was found that \( \lambda^2 \in [0.5, 0.7] \) yielded 200m times between 20.29s–20.59s, as run in lane 4 of an outdoor track, which are reproduced in Table 6. These closely approximated Bailey’s 1994 personal bests of 20.76s / 20.39s wind-assisted [5]).

This season (1997), Bailey has clocked a 20.65s 200m [10], which agrees with the cited range of \( \lambda^2 \). Assuming that the same value of \( \lambda^2 \) held for the 150m race in Skydome (between 0.7–0.8 [5]), then according to Table 3, then a possible configuration of the track would have been 75m+75m with a curve radius equivalent to lane 8 (5.78s, 10.27s, 14.92s). Accounting for the fact that Bailey was undoubtedly much more mentally and physically prepared for the Skydome match, then it is likely that his equivalent outdoor 200m time would drop. The values \( \lambda^2 = 0.6 \) give 200m times between 20.29s – 20.40s, which is a reasonable range for Bailey to hit if he runs all-out. It is not inconceivable that he could run a low 20s race, despite his higher PB of 20.65s. Both Frank Fredricks of Namibia and Ato Boldon of Trinidad are comparable 100m runners, and each has clocked a sub–20s 200m time (19.66s and 19.80s, respectively [8]). Undoubtedly, it is high muscular endurance which allows them to do this.

For each case listed above, Bailey’s extrapolated 200m times of Tables 2–5 are generally less than for a standard outdoor track. This is simply due to the fact that the longer curve outdoors (100m v.s. 75m) creates a larger drain on \( f(t) \). Whether or not such seemingly large time discrepancies are physically realizable, or are just a manifestation of the model, are unknown. The proof is left to the sprinter.

Acknowledgements Thanks to R. Mureika (U. of New Brunswick Department of Mathematics and Statistics) for some useful suggestions, and to C.
Georgevski (Director, U. of Toronto Canadian High Performance Track and Field Centre) for making it possible for me to observe first-hand the cited experiment in progress.

References

[1] J. R. Mureika, “And the Winner Is... Predicting the Outcome of the 150m Showdown”, Athletics: The National Canadian Track and Field / Running Magazine (June 1997).

[2] R. Tibshirani, “Who is the fastest man in the world?”, Amer. Stat. (May 1997)

[3] The Globe and Mail, p. C-2, "One–to–One" special insert, 29 May 1997.

[4] J. B. Keller, “A theory of competitive running”, Physics Today, 43, Sept. 1973; J. B. Keller, “Optimal velocity in a race”, Amer. Math. Monthly 81, 474 (1974)

[5] J. R. Mureika, “A Simple Model for Predicting Sprint Race Times Accounting for Energy Loss on the Curve”, submitted to Can. J. Phys.

[6] “How the race was won”, The Toronto Star, p. D-5 (June 2, 1997).

[7] Toronto Slam! Sports web site, http://www.canoe.ca/BaileyJohnson/jun1_fax.html the contents of a fax sent to Ray Flynn, Donovan Bailey’s agent, concerning the track configurations (01 June, 1997)

[8] International Amateur Athletic Federation web site, http://www.iaaf.org/

[9] Toronto Slam! Sports web site, http://www.canoe.ca/BaileyJohnson/jun1_protest.html a public statement of Bailey’s protests (01 June 1997)

[10] Official 1997 Outdoor Canadian Rankings, Ontario Track and Field Association web site, http://www.lglobal.com/~ontrack/.
Distance (m) | Split (s)
---|---
0 | 0.171
50 | 5.74
100 | 10.24
150 | 14.99

Table 1: Donovan Bailey’s official splits for the Challenge of Champions 150m race at Skydome, Toronto, 01 June 1997 [6].

| $\lambda$ | $t_{50}$ | $t_{75}$ | $t_{100}$ | $t_{150}$ | $\Sigma^2$ | $t_{200}$ |
|---|---|---|---|---|---|---|
| 0.50 | 5.76 | 7.98 | 10.21 | 14.87 | $8.48 \times 10^{-6}$ | 20.13 |
| 0.60 | 5.79 | 8.03 | 10.28 | 14.94 | $1.02 \times 10^{-4}$ | 20.22 |
| 0.70 | 5.81 | 8.09 | 10.36 | 15.04 | $2.97 \times 10^{-4}$ | 20.33 |
| 0.80 | 5.84 | 8.13 | 10.41 | 15.10 | $6.34 \times 10^{-4}$ | 20.41 |

Table 2: Bailey’s predicted splits as run in lane 3 for a 75m+75m track configuration. All times include reaction time $t_{\text{react}} = +0.17$s.

| $\lambda$ | $t_{50}$ | $t_{85}$ | $t_{100}$ | $t_{150}$ | $\Sigma^2$ | $t_{200}$ |
|---|---|---|---|---|---|---|
| 0.50 | 5.73 | 7.91 | 10.12 | 14.75 | $3.97 \times 10^{-4}$ | 20.00 |
| 0.60 | 5.75 | 7.95 | 10.17 | 14.82 | $1.79 \times 10^{-4}$ | 20.07 |
| 0.70 | 5.77 | 7.99 | 10.22 | 14.88 | $8.49 \times 10^{-5}$ | 20.15 |
| 0.80 | 5.78 | 8.02 | 10.27 | 14.92 | $7.90 \times 10^{-5}$ | 20.20 |

Table 3: Predicted splits as run in lane 8 for a 75m+75m track configuration.

| $\lambda$ | $t_{50}$ | $t_{85}$ | $t_{100}$ | $t_{150}$ | $\Sigma^2$ | $t_{200}$ |
|---|---|---|---|---|---|---|
| 0.50 | 5.76 | 8.89 | 10.26 | 14.94 | $2.71 \times 10^{-5}$ | 20.22 |
| 0.60 | 5.79 | 8.96 | 10.34 | 15.04 | $1.82 \times 10^{-4}$ | 20.33 |
| 0.70 | 5.81 | 9.02 | 10.41 | 15.13 | $5.12 \times 10^{-4}$ | 20.44 |
| 0.80 | 5.84 | 9.08 | 10.48 | 15.22 | $1.09 \times 10^{-3}$ | 20.48 |

Table 4: Predicted splits as run in lane 3 for an 85m+65m track configuration.
Table 5: Predicted splits as run in lane 8 for an 85m+65m track configuration.

| $\lambda^2$ | $t_{50}$ | $t_{85}$ | $t_{100}$ | $t_{150}$ | $\Sigma^2$ | $t_{200}$ |
|-------------|---------|---------|---------|---------|---------|---------|
| 0.50        | 5.73    | 8.80    | 10.15   | 14.80   | $2.41 \times 10^{-4}$ | 20.06   |
| 0.60        | 5.75    | 8.84    | 10.20   | 14.87   | $8.24 \times 10^{-5}$ | 20.13   |
| 0.70        | 5.77    | 8.89    | 10.26   | 14.94   | $4.22 \times 10^{-5}$ | 20.22   |
| 0.80        | 5.78    | 8.93    | 10.30   | 15.00   | $8.34 \times 10^{-9}$ | 20.29   |

Table 6: Bailey’s predicted outdoor 200m times, as run in lane 4.

| $\lambda^2$ | $t_{50}$ | $v_{50}$ | $t_{100}$ | $v_{100}$ | $t_{150}$ | $t_{200}$ | $t_{200} + 0.16$ |
|-------------|---------|---------|---------|---------|---------|---------|------------------|
| 0.36        | 5.55    | 11.60   | 9.98    | 10.85   | 14.69   | 19.96   | 20.12            |
| 0.50        | 5.59    | 11.43   | 10.09   | 10.65   | 14.84   | 20.13   | 20.29            |
| 0.60        | 5.61    | 11.31   | 10.16   | 10.51   | 14.93   | 20.24   | 20.40            |
| 0.70        | 5.63    | 11.20   | 10.24   | 10.39   | 15.09   | 20.43   | 20.59            |