Linking axionlike dark matter to neutrino masses

C. D. R. Carvajal
Instituto de Física, Universidad de Antioquia,
Calle 70 No. 52-21, Medellín, Colombia and
INFN, Laboratori Nazionali di Frascati,
C.P. 13, 100044 Frascati, Italy

B. L. Sánchez-Vega
Universidade Estadual Paulista (Unesp), Instituto de Física Teórica (IFT),
São Paulo, R. Dr. Bento Teobaldo Ferraz 271,
Barra Funda, São Paulo - SP, 01140-070, Brasil

O. Zapata
Instituto de Física, Universidad de Antioquia,
Calle 70 No. 52-21, Medellín, Colombia
(Dated: December 25, 2017)

We present a framework linking axionlike particles (ALPs) to neutrino masses through the minimal inverse seesaw (ISS) mechanism in order to explain the dark matter (DM) puzzle. Specifically, we explore three minimal ISS cases where mass scales are generated through gravity-induced operators involving a scalar field hosting ALPs. In all of these cases, we find gravity-stable models providing the observed DM relic density and, simultaneously, consistent with the phenomenology of neutrinos and ALPs. Remarkably, in one of the ISS cases, the DM can be made of ALPs and sterile neutrinos. Furthermore, other considered ISS cases have ALPs with parameters inside regions to be explored by proposed ALPs experiments.

I. INTRODUCTION

The discovery of neutrino oscillations [1, 2] and the fact that baryonic matter only yields a few percent contribution to the energy density of the Universe [3] are two experimental evidences calling for physics beyond the standard model (SM). On the theoretical side, the apparent absence of CP violation in the QCD sector is also a strong motivation for going beyond the SM since it can be dynamically explained by the Peccei-Quinn mechanism [4], which requires to extend the SM gauge group with a global symmetry and the existence of a pseudo-Nambu-Goldstone boson, the axion [5, 6]. Besides elegantly solving the strong CP problem [7–10], the Peccei-Quinn mechanism may be also related to the solution of DM and neutrino puzzles by offering a candidate for cold DM, the axion itself, [11–13] and a connection to the neutrino mass generation [14–20].

In the context of ALPs models, the approximate continuous symmetry is typically assumed to be remnant of an exact discrete gauge symmetry as gravity presumably breaks the global symmetries through Planck-scale suppressed operators. In other words, since the global symmetry is highly unstable it is usually stabilized by imposing a discrete gauge symmetry [36–39] such as a $Z_N$ symmetry [23, 40] (see Refs. [42–49] for $Z_N$ realizations in QCD axion models).

This discrete gauge symmetry protects the ALPs mass against large gravity-induced corrections and it can also be used to stabilize other mass scales present in the theory. In particular, with the aim of generating neutrino masses the authors in Refs. [23, 40] used these types of discrete gauge symmetries in order to protect the associated lepton-number-breaking scale. In this work, we go further by building a self-consistent framework of ALPs DM [50] and neutrino masses via the ISS mechanism [51, 52]. For this purpose we make use of appropriate $Z_N$ discrete gauge symmetries to protect the suitable ALPs mass reproducing the correct DM relic abundance as well as to stabilize the mass scales present in the ISS mechanism. It turns out that the ISS mass terms are determined up to some factor by $v_σ^n/M_{Pl}^{n-1}$, where $v_σ$ is the vacuum expectation value (VEV) of the scalar field $σ$ that spontaneously breaks the global symmetry $U(1)_A$ and hosts the ALPs, $a$. $n$ is an integer that is determined
by the invariance of such terms under the symmetries of the model and some phenomenological constraints.

In order to implement the ISS mechanism we extend the SM matter content by introducing \( n_{N_R} \times n_{S_R} \) generations of SM-singlet fermions \( N_R(S_R) \) as is usual. In this work, we consider the minimal number of singlet fermionic fields that allows to fit all the experimental neutrino physics: the \( (n_{N_R}, n_{S_R}) = (2,2), (2,3) \) and \( (3,3) \) cases [53]. In each case, the ALPs plays the role of the DM candidate. Moreover, for the \( (2,3) \) ISS case there is a possibility of having a second DM candidate: the sterile neutrino (the unpaired singlet fermion) [54,56]. Motivated by that, we also build a multicomponent DM framework where the DM of the Universe is composed by ALPs and sterile neutrinos, with the latter generated through the active-sterile neutrino mixing [57] and accounting for a fraction of the the DM relic density.

As far as phenomenological issues are concerned, since in each framework the approximate continuous symmetry is anomalous respect to the electromagnetic gauge group (through an exotic vectorlike fermion) instead of being anomalous respect to QCD, it is possible to build an effective interaction term involving the axion field and the electromagnetic field strength and its dual [58,59]. This in turn implies that the ALPs may be detected in current-and/or proposed-experiments that use the ALPs-photon coupling as their main interaction channel to search for ALPs [21,22,55,60]. Moreover, considering this particle as the dark matter candidate it can be part of the Milky Way DM halo and could resonantly convert into a monochromatic microwave signal in a microwave cavity permeated by a strong magnetic field [61,62]. On the other hand, since a large portion of the parameter space of ALPs (e.g. low masses and couplings) is relatively unconstrained by experiment since the conventional experiments, Helioscopes, Haloscopes and others [60], are only sensitive to axion particles whose Compton wavelength is comparable to the size of the resonant density in order to cover other regions of the parameter space. To reach smaller values for the ALPs mass and ALPs-photon coupling is necessary a different experimental approach like the one associated to the ABRAZADABRA proposal [62,65], where it is suggested a new set of experiments based on either broadband or resonant detection of an oscillating magnetic flux, designed for the axion detection in the range \( m_a \in [10^{-14}, 10^{-6}] \) eV. And it is precisely these kinds of searches that can be used to probe the benchmark regions that we study in the (2,2) and (3,3) ISS cases.

The rest of the paper is organized as follows: in Sec. II we discuss phenomenological and theoretical conditions that lead to a successful protection of the ALPs mass and the ISS texture against gravity effects. In Sec. III we search for viable models simultaneously compatible with DM phenomenology, neutrino oscillation observables and lepton-flavor-violating processes. Finally, we present our discussion and conclusions in Sec IV.

II. FRAMEWORK

The goal of this section is to present the main ingredients of a SM extension in order to link ALPs to neutrino mass generation, and at the same time, to offer an explanation for the current DM relic density reported by Planck Collaboration [3]. In order to achieve that, the SM matter content must to be extended with some extra fields. Besides the scalar \( \sigma \) and fermionic \( S_R \) and \( N_R \) fields, an extra electrically charged fermion \( E \) is also added to the SM to make possible the coupling of ALPs to photons, \( g_{\sigma \gamma} \). That is necessary because \( g_{\sigma \gamma} \) is anomaly induced and there is no any \( U(1)_A \) symmetry anomalous in the electromagnetic group just with the SM charged fermions. The main role of the anomalous \( U(1)_A \) symmetry is to induce an ALPs coupling to two photons. This brings as a consequence that the ALPs can be found, in principle, in current and/or proposed experiments that make use of the ALPs-photon coupling. We will show target regions of some experiments searching for ALPs in Fig 1. Also, it will be found that ALPs in some ISS cases discussed in this paper are inside the regions of planned experiments [64,65].

Another key point of the framework is the existence of a \( Z_N \) discrete gauge symmetry. In order to understand its role, firstly, note that to impose an anomalous \( U(1)_A \) symmetry to the Lagrangian does not seem sensible in the sense that in the absence of further constraints on very high energy physics we should expect all relevant and marginally relevant operators that are forbidden only by this symmetry to appear in the effective Lagrangian with coefficient of order one. However, this symmetry follows from some other free anomaly symmetry, in our case from the a \( Z_N \) discrete gauge symmetry, all terms which violate it are then irrelevant in the renormalization group sense. Secondly, the \( Z_N \) symmetry also protects both the ALPs mass and the ISS texture against gravity effects as we will explain in more detail later on. For these reasons, the effective Lagrangian will be invariant under a \( Z_N \) discrete gauge symmetry. Due to the ALPs mass is very low and only protected by the \( U(1)_A \) symmetry which is explicitly broken by gravity effects, the \( Z_N \) symmetry will have a high order. This fact also happens in models with QCD axions and it is shared by all models with this type of stabilization mechanism [23,40,43,44,49,66].

A. Lagrangian

The effective Lagrangian that we consider to relate the ISS mechanism to ALPs DM reads

\[
\mathcal{L} \supset \mathcal{L}_{\text{Yuk}}^{\text{SM}} + \mathcal{L}_\sigma + \mathcal{L}_{\text{ISS}} + \mathcal{L}_E,
\]

where \( \mathcal{L}_{\text{Yuk}}^{\text{SM}} \) is nothing more than the Yukawa Lagrangian of the SM

\[
\mathcal{L}_{\text{Yuk}}^{\text{SM}} = Y_{ij}^{(u)} \overline{Q_L}^i H u_{Rj} + Y_{ij}^{(d)} \overline{Q_L}^i H d_{Rj} + Y_{ij}^{(l)} \overline{L_i} H l_{Rj} + \text{H.c.},
\]
with the usual $Q_L$, $u_R$, $d_R$ and $L_i$, $t_R_i$ fields denoting the quarks and leptons of the SM, respectively. $H$ is the Higgs SU(2)$_L$ doublet with $\bar{H} = i\tau_2 H^*$ ($\tau_2$ is the second Pauli matrix).

The term in $\mathcal{L}_\sigma$ (Lagrangian involving the $\sigma$ field) which is relevant in our discussion is the following non-renormalizable operators

$$\mathcal{L}_\sigma \supset g^2 \frac{\sigma^D}{M_{\text{Pl}}^4} + \text{H.c.},$$

with $g = e^{i\delta} |g|$ and $D$ being an integer. The $\sigma$ field is parametrized as $\sigma(x) = \frac{1}{\sqrt{2}} [v_\sigma + \rho(x)] e^{i\frac{\phi}{\sqrt{2}}} x^2$, with $a(x)$ being the ALPs field and $\rho(x)$ the radial part that will gain a mass of order of the vacuum expectation value

$$10^9 \lesssim \sqrt{2} \langle \sigma \rangle \equiv v_\sigma \lesssim 10^{14} \text{ GeV}. \tag{4}$$

With the operators in Eq. (3) and the $\sigma(x)$ parametrization, the ALPs mass term is written as follows

$$m_a = |g| \frac{1}{2} DM_{\text{Pl}} \lambda^\frac{2}{3} \gamma^3 - 1, \tag{5}$$

where $10^{-10} \lesssim \lambda \lesssim \frac{v_\sigma}{\sqrt{2} M_{\text{Pl}}} \lesssim 10^{-5}$ and $M_{\text{Pl}} = 2.44 \times 10^{18}$ GeV is the reduced Planck scale.

Now, we turn our attention to the coupling of ALPs to photons which is determined by the interaction term

$$\frac{1}{4} a F_{\mu\nu} F^{\mu\nu},$$

where $F_{\mu\nu}$ and $F^{\mu\nu}$ are the electromagnetic field strength and its dual, respectively. This term is anomaly induced and given by

$$g_{a\gamma} = \frac{\alpha}{2\pi} \frac{C_{a\gamma}}{v_\sigma}, \tag{6}$$

where $C_{a\gamma}$ is the electric charge of the fermion $\psi$, and $X_{\psi_L R}$ is its charge under the U(1)$_A$ symmetry. This anomaly coefficient is of order of one (or 2 more specifically) in our models and it directly determines the width of the red band in Figure 1 where ALPs are DM candidates. Also, it is important to note that the existence of a non-null anomaly coefficient guarantees that $g_{a\gamma} \neq 0$. This is the reason for the total Lagrangian in Eq. (1) is invariant under an anomalous U(1)$_A$ global symmetry. Nevertheless, only with SM model fermions and the neutral $S_{R\alpha}$ and $N_{R\beta}$ fermions is not possible to have an anomalous U(1)$_A$ symmetry in the electromagnetic group. Therefore, we need include the SU(2)$_L$ singlet fermion, $E$, with an unit of electric charge.

On the other hand, the dimension $D$ of the gravity-induced mass operator in Eq. (3) must be, in general, larger than 4 because the astrophysical and cosmological constraints on the properties of ALPs. To be more specific, we show, in Figure 1, some regions of the ALPs space of parameters $g_{a\gamma}$ vs $m_a$ where ALPs give an explanation for some astrophysical anomalies and others forbidden regions.

Regarding the neutrino mass generation, we have that, once introduced the $N_{R\beta}$ and $S_{R\alpha}$ fields, the $\mathcal{L}_{\text{ISS Lagrangian}}$ reads as

$$\mathcal{L}_{\text{ISS}} = y_{i\beta} \bar{L}_i \bar{H} N_{R\beta} + \zeta_{\alpha\beta} \frac{\sigma^\rho}{M_{\text{Pl}}} \bar{S}_{R\alpha} (N_{R\beta})^C + \eta_{\alpha\alpha'} \frac{\sigma^q}{2 M_{\text{Pl}}} \bar{S}_{R\alpha} (S_{R\alpha'})^C + \theta_{\beta\beta'} \frac{\sigma^r}{2 M_{\text{Pl}}} \bar{N}_{R\beta} (N_{R\beta'})^C + \text{H.c.}, \tag{8}$$

where the $y_{i\beta}$, $\zeta_{\alpha\beta}$, $\eta_{\alpha\alpha'}$, $\theta_{\beta\beta'}$, coupling constants, with $i, j = 1, 2, 3$, $\alpha, \alpha' = 1, 2, (or 3)$ and $\beta, \beta' = 1, 2, (or 3)$, are generically assumed of order one. The exponents $p, q, r$ are integer numbers chosen for satisfying some phenomenological constraints discussed below. Negative values for these exponents will mean that the term is $\sim \sigma^* n$ instead of $\sim \sigma^n$. Note that, without loss of generality, the exponent $p$ can be assumed to be positive. We will only consider the minimal number of neutral fermionic fields, $S_{R\alpha}$ and $N_{R\beta}$, that allow to fit all the experimental neutrino physics. Specifically, we study the (2, 2), (2, 3) and (3, 3) cases.

As the $\sigma$ field gets a VEV the gravity-induced terms in Eq. (8) give the mass matrix for light (active) and heavy neutrinos. Specifically, we can write the mass matrix

$$\begin{bmatrix} M_\nu & 0 \\ M_D & M_R \end{bmatrix}, \text{ with } \begin{bmatrix} M_D & M_R \end{bmatrix} = \begin{bmatrix} M_{D}^\nu & M_{R}^\nu \end{bmatrix}, \tag{9}$$

where $m_D$, $M$, $\mu_N$, $\mu_S$ are matrices with dimension equal to $n_{N_{R\beta}} \times 3$, $n_{N_{R\beta}} \times n_{S_{R\alpha}}$, $n_{N_{R\beta}} \times n_{N_{R\beta}}$, $n_{N_{R\beta}} \times n_{S_{R\alpha}}$, respectively. The energy scales of the entries in these matrices are determined essentially by $\sqrt{2} \langle H \rangle \equiv v_{SM} \approx 246$ GeV, $\lambda$ (or $v_\sigma$) and $M_{\text{Pl}}$ GeV as follows

$$m_{D \beta} = y_{i\beta} \frac{v_{SM}}{\sqrt{2}}, \quad M_{a\beta} = \zeta_{a\beta} M_{\text{Pl}} \lambda^p, \quad \mu_{S \alpha \alpha'} = \eta_{\alpha \alpha'} M_{\text{Pl}} \lambda^{[p]}, \quad \mu_{N \beta \beta'} = \theta_{\beta \beta'} M_{\text{Pl}} \lambda^{[r]}. \tag{11}$$
The mass matrix in Eq. 9 allows light active neutrino masses at order of sub-eV without resorting very large energy scales in contrast to the type I seesaw mechanism 12 – 17. In more detail, assuming the hierarchy \( \mu_N \lesssim \mu_S \ll M_D < M \) (note that making \( \mu_S \) and \( \mu_N \) small is technically natural) and taking a matrix expansion in powers of \( M^{-1} \), the light active neutrino masses, at leading order, are approximately given by the eigenvalues of the matrix 78 – 79

\[
m_{\nu_{\text{light}}} \approx m_D^T M^{-1} \mu_S (M^T)^{-1} m_D.
\] (13)

On the other hand, the heavy neutrino masses are given by the eigenvalues of \( m_{\nu_{\text{heavy}}} \approx M_R \). Note from Eq. 13 that \( \mu_N \) does not contribute to the light active neutrino masses at the leading order 78 – 79. Actually, the presence of \( \mu_N \) term gives a subleading contribution to \( m_{\nu_{\text{light}}} \). This is a factor \( \mu_S \mu_N / M^2 \) smaller than the leading contribution 40.

Very motivated scales for \( M \) and \( \mu_S \), \( \mu_N \) are TeV and keV scales, respectively. These scales allow getting active neutrino masses in the sub-eV scale without considering smaller Yukawas and, in some scenarios, such as the (2, 3) ISS case, the existence of a keV sterile neutrino as a warm dark matter (WDM) candidate 80. In addition, \( M \) has to satisfy

\[
M \gtrsim \sqrt{10^{\frac{\mu_S}{\text{keV}}} \text{ TeV}},
\] (14)

because light active neutrino masses are in sub-eV scale and \( m_P \) is of order of \( O(\mu_{\text{SM}}) \).

Another constraint on the \( M \) scale comes from the fact that the mixing matrix that relates the three left-handed neutrinos with the three lightest mass-eigenstate neutrinos is not longer unitary. This implies that deviations of some SM observables may be expected, such as additional contributions to the \( b \nu W \) vertex and to lepton-flavor and CP-violating processes, and non-standard effects in neutrino propagation 81 – 82. For example, in the inverse seesaw model, the violation of unitary is of order of \( \epsilon^2 \), with \( \epsilon \equiv m_D M^{-1} \) being approximately the mixing between light active and heavy neutrinos 79. Roughly speaking, \( \epsilon^2 \) at the percent level is not excluded experimentally 81 – 83 – 85.

Taking into account the previous considerations, the ranges chosen for \( M \) and \( \mu_S \) are

\[
1 \leq M \leq 25 \text{ TeV}, \quad 0.1 \leq \mu_S \leq 50 \text{ keV}.
\] (15)

Once established that scales of the mass matrices and using Eqs. 11 and 12 (and following a similar procedure as in Ref. 14), the integers \( p \) and \( q \) in Eq. 5 can only take the values

\[
(p, |q|) = (2, 3) \quad \text{for} \ 6 \times 10^{10} \lesssim v_\sigma \lesssim 1 \times 10^{11} \text{ GeV},
\] (16)

\[
(p, |q|) = (3, 5) \quad \text{for} \ 2 \times 10^{13} \lesssim v_\sigma \lesssim 8 \times 10^{13} \text{ GeV}.
\] (17)

That happens because the same VEV simultaneously provides \( M \) and \( \mu_S \) scales. Note that for both possibilities in Eqs. 16 and 17 the light active neutrino mass matrix in Eq. 13 is simplified to

\[
m_{\nu_{\text{light}}} = \left[ y^T \zeta^{-1} \eta (\zeta^T)^{-1} y \right] \frac{v_{SM}^2}{\sqrt{2} v_\sigma}.
\] (18)

Moreover, the exponent \( r \) of the term that generates \( \mu_N \) in Eq. 8 is also constrained to be \( r \geq |q| \), because \( \mu_N \) must be \( \lesssim \mu_S \).

Finally, we have that \( \mathcal{L}_E \), the Lagrangian involving the E charged fermion, is written as

\[
\mathcal{L}_E \supset \partial_i \frac{\sigma_s}{M_{Pl}} \frac{t_i H E_R}{\sin \theta} + \frac{\kappa}{M_{Pl}} \frac{\sigma_L}{M_{Pl}^2} t_i H E_R + \text{H.c.},
\] (19)

where \( \partial_i \) and \( \kappa \) are Yukawas, in principle, assumed of order one. These two terms are also subjected to phenomenological and theoretical constraints as follow. Because the term \( \sim \sigma \sigma_L H E_R \) must give a mass large enough for the E fermion to satisfy its experimental constraints. For stable charged heavy lepton, \( m_E > 102.6 \text{ GeV} \) at 95\% C.L. 86, or for charged long-lived heavy lepton, \( m_E > 574 \text{ GeV} \) at 95\% C.L. assuming mean life above \( 7 \times 10^{-10} - 3 \times 10^{-8} \text{ s} \) 87 – 88, \( t \) must be less or equal than 3. It must be different from zero because the electromagnetic anomaly must be present. On the other hand, \( s \) can take the values 1 or 2 because \( \sim \sigma \sigma_L H E_R \) determines the interaction of the E fermion with the SM leptons, and whether \( s \) is larger than 2, the charged E fermion becomes stable enough to bring cosmological problems, unless its mass is \( \gtrsim \text{TeV} \). Another constraint comes from searches for long-lived particles in pp collisions 87 – 88.

Now an important discussion about the stability of both the ISS mechanism and the ALPs mass is in order. In general, the gravitational effects must be controlled to give a suitable ALPs mass. With this aim, we introduced a gauge discrete \( Z_N \) symmetry assumed as a remnant of a gauge symmetry valid at very high energies 89. Thus, to truly protect the ALPs mass against those effects, \( Z_N \) must at least be free anomaly 90, i.e.,

\[
A_2(Z_N) = A_3(Z_N) = A_{\text{grav}}(Z_N) = 0 \quad \text{Mod} \frac{N}{2},
\] (20)

where \( A_2, A_3 \) and \( A_{\text{grav}} \) are the [SU(2)\(_L\)]\(^2 \times Z_N\), [SU(3)\(_C\)]\(^2 \times Z_N\) and [gravitational]\(^2 \times Z_N\) anomalies, respectively. Other anomalies, such as \( Z_3 \), do not give useful low-energy constraints because they depend on some arbitrary choices concerning to the full theory.

Gravitational effects can also generate terms such as \( \frac{\sigma_n}{M_{Pl}} \frac{N R}{M_{Pl}^2} \), \( \frac{\sigma_n}{M_{Pl}} \frac{N R}{M_{Pl}^2} \), \( \frac{\sigma_n}{M_{Pl}} \frac{N R}{M_{Pl}^2} \) or \( \frac{\sigma_n}{M_{Pl}} \frac{N R}{M_{Pl}^2} \) - and the scales of the ISS mechanism. Thus, \( Z_N \) will be chosen such that it also prevents those undesirable terms from appear.

In general, the \( Z_N \) symmetry can be written as a linear combination of the continuous symmetries in the model:
the hypercharge $Y$, the baryon number $B$ and the generalized lepton number $L$. The charge assignments for $B$ and $L$ symmetries are shown in Table I whereas the assignment for $Y$ symmetry is the canonical one. Nevertheless, since the hypercharge is free anomaly by construction, the $Z_N$ charges $(Z)$ of the fields can be written as $Z = c_1 B + c_2 L$, where $c_1, c_2$ are rational numbers in order to make the $Z_N$ charges integers [90]. Now, substituting the charges in Table I into the general form of the $Z_N$ symmetry (see Refs. [93,99]) we can obtain the anomaly coefficients. Doing so, we find that $A_3(Z_N) = 0$ and

$$A_2(Z_N) = \frac{3}{2} [c_1 + c_2],$$

$$A_{\text{grav}}(Z_N) = c_2 \left[ 3 - n_R - n_S \times \left(\frac{qd}{2}\right) + sd \right].$$

Note that $A_2(Z_N)$ and $A_{\text{grav}}(Z_N)$ are not, in general, 0 Mod $N/2$ which implies strong constraints on the choice of the $Z_N$ discrete symmetry.

**B. ALPs and sterile neutrino dark matter**

Since the ALPs are very weakly interacting light particles and cosmologically stable, they can be considered as DM candidates [25]. In fact, ALPs may be nonthermally produced via the misalignment mechanism in the early Universe and survive as a cold dark matter population until today. Specifically, its relic density is determined from the following equation [25,63,68,90–94].

$$\Omega_{\nu_S,\text{DM}} h^2 = 1.1 \times 10^7 \sum_{\alpha} C_{\alpha}(m_S)|U_{\alpha S}|^2 \left[ \frac{m_{\nu_S}}{\text{keV}} \right]^2;$$

$$\alpha = e, \mu, \tau,$$

(24)

where $C_{\alpha}(m_S)$ are active flavor-dependent coefficients which are calculated solving numerically the Boltzmann equations (an appropriate value in this case is $C_{\alpha}(m_S) \approx 0.8$ [95]). We also have that the sum of $U_{\alpha S}$, the elements of the leptonic mixing matrix, is the active-sterile mixing, i.e., $\sum_{\alpha} |U_{\alpha S}|^2 \sim \sin^2(2\theta)$. For the case $m_{\nu_S} < 0.1 \text{ keV}$ there is a simpler expression written as follows [80,96]

$$\Omega_{\nu_S,\text{DM}} h^2 = 0.3 \left[ \frac{\sin^22\theta}{10^{-10}} \right] \left[ \frac{m_{\nu_S}}{100 \text{ keV}} \right]^2. $$

(25)

After imposing bounds coming from stability, structure formation and indirect detection, in addition to the constraints arising from the neutrinos oscillation experiments, it was found that the sterile neutrino as WDM in the $(2,3)$ ISS provides a sizable contribution to the DM relic density for $2 \lesssim m_{\nu_S} \lesssim 5 \text{ keV}$ and active-sterile mixing angles $10^{-8} \lesssim \sin^2(2\theta) \lesssim 10^{-11}$ [95], where the maximal fraction of DM made of $\nu_S$ is achieved when $m_{\nu_S} \approx 7 \text{ keV}$ [97,100].

Once established the DM candidates and the parameters that determine the relic density in each case, we are going to search for models satisfying all mentioned conditions in Section III as well as

$$\Omega_{\text{DM}}^{\text{Planck}} h^2 = \Omega_{\nu_S,\text{DM}} h^2 + \Omega_{\alpha,\text{DM}} h^2,$$

(26)

where $\Omega_{\text{DM}}^{\text{Planck}} h^2 = 0.1197 \pm 0.0066 \text{ (at 3}\sigma)$ is the current relic density as reported by Planck Collaboration [8].

**III. MODELS**

In the previous section, we have introduced the general and minimal constraints that models have to satisfy. Now, we proceed to find specific models that give an explanation to the dark matter observed in the Universe. In particular, the $(2,2)$, $(3,3)$ and $(2,3)$ cases of the ISS mechanism are studied in detail. For each model we check the compatibility (at $3\sigma$) with the experimental neutrino physics [82] for the normal mass ordering and vanishing CP phases by varying the free Yukawa couplings $y_{ij\beta}, \zeta_{ij\beta}, \eta_{ij\alpha}, \theta_{ij\beta}$ in the range $\sim (0.1, 3.5)$. Additionally, we also analyze the lepton flavor violating processes such as $\ell_\beta \rightarrow \ell_\alpha + \gamma$, which are induced at one loop by the W boson and the heavy neutrinos.
Among the minimal configuration of the ISS mechanism consistent with the experimental neutrino physics and lepton-flavor-violating (LFV) processes \cite{53,80,115} (for a recent review see Ref. \cite{116}), we, firstly, study the (2, 2) ISS case because this is the minimal configuration that satisfy all the constraints coming from experimental neutrino physics. For this case, in the neutrino mass spectrum there are two heavy pseudo-Dirac neutrinos with masses \( \sim M \) and three light active neutrinos with masses of order of sub-eV coming from the mass matrix \( \text{Eq. (13)} \) \cite{53}. We verify that each ISS model is compatible with the current experimental limits \( \text{Br}(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13} \) \cite{113}, \( \text{Br}(\tau \rightarrow e\gamma) < 3.3 \times 10^{-8} \) and \( \text{Br}(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8} \) \cite{114}.

A. (2, 2) ISS case

In order to find the main features of the model, we find useful to rewrite \( \Omega_{a,\text{DM}}h^2 \) in terms of \( D \) - the exponent of the mass operator for \( \sigma \), \text{Eq. (3)} - and \( m_a \). Thus,

\[ \Omega_{a,\text{DM}}h^2 = \Omega_{a,\text{DM}}h^2. \]
The gravitational anomaly. In fact, the ISS mechanism shown in Eqs. (9) and (10). We have considered a fine tuning in couplings. Nevertheless, we have as a consequence that: the ALPs mass, the mass operators with those dimensions \( v, \) and \( v_\sigma \) in Eq. (23), we find that

\[
\Omega_{\text{a,DM}} h^2 \simeq 0.49 |g|^4 \sqrt{D} \exp \left[ -\frac{D}{4} \ln \frac{\sqrt{2M_{\text{Pl}}}}{1 \text{ GeV}} \right] \times \left[ \frac{v_\sigma}{1 \text{ GeV}} \right]^{D_{\text{a,DM}}},
\]

(28)

where \( g \) is assumed to be \( 10^{-3} \leq |g| \leq 2 \). Thus, we can see that \( \Omega_{\text{a,DM}} h^2 \) only depends on \(|g|, D, v_\sigma\). In Table I, we show \((D, v_\sigma)\) values for the cases where \( \Omega_{\text{a,DM}} h^2 = \Omega_{\text{Planck}}^0 h^2 \) and \( \Omega_{\text{a,DM}} h^2 = 0.57 \times \Omega_{\text{Planck}}^0 h^2 \). The last case applies only for the (2, 3) ISS case and will be discussed in Section III. C.

In order to obtain the Lagrangian in this scenario, we search for discrete symmetries for the two possibilities showed in Eqs. (16-17) and different values of \( r, s, t \) according its respective constraints as follow: considering Eqs. (16-17) and the Table I we can see that, for the range of values of \( v_\sigma \) established in the Section II.A only the values \( D = 9, 10, 16, 19 \) to reproduce the correct relic density to ALPs. Thus we searched for discrete symmetries \( Z_N \) that allows the mass operators with those dimensions \( D \), with the following results:

- The \( Z_{9,10} \) symmetries allow terms such as \( \tilde{H} \tilde{R} S_R \), \( \tilde{H} \tilde{S}_R \), and since \( H \) and \( \sigma \) get VEVs, these terms do not give the appropriate zero texture of the ISS mechanism shown in Eqs. (9) and (10). We have searched for all the possible combinations of \( r, s, t \) values in the Lagrangian of Eq. (5) without any success. On the other hand, the \( Z_{16,18} \) symmetries are not free of the gravitational anomaly. In fact, the \( Z_{N \leq 20} \) discrete symmetries that satisfy all the anomaly constraints and stabilize the ISS mechanism are \( Z_{17,19} \).

In the case of \( Z_{17} \) symmetry, the Lagrangian, \( \mathcal{L}_{Z_{17}} \), is given by Eq. (1) with the parameters \( D = 17 \) and \((p, q, r, s, t) = (3, −5, −6, 2, 2) \) in Eqs. (3), (8) and (19), respectively. An assignment of the \( Z_{17} \) (with \( Z_{17} = 6B + 11U(1)_L \)) charges and the anomalous \( U(1)_A \) symmetry for this case is shown in Table II. Note that, for this model the term \( \sigma^6 N_{R3} \) in Eq. (8) gives a negligible contribution for the light active neutrino masses.

The corresponding \( g_{a\gamma} \) and \( m_a \) for this model is given by

\[
g_{a\gamma} \simeq 7.54 \times 10^{-17} \left[ \frac{3.08 \times 10^{13} \text{ GeV}}{v_\sigma} \right] \text{ GeV}^{-1},
\]

\[
m_a \simeq 5.59 \times 10^{-10} \left| g \right|^2 \left[ \frac{v_\sigma}{3.08 \times 10^{13} \text{ GeV}} \right]^{15/2} \text{ eV}.
\]

(29)

The benchmark region for this case is denoted as M22a in the Figure 1 where we have considered \( 10^{-3} \leq |g| \leq 2 \) and \( 2.9 \times 10^{13} \leq v_\sigma \leq 4.2 \times 10^{13} \) GeV. These values for \( g_{a\gamma} \) and \( m_a \) allow that the ALPs explain 100% of the DM relic density.

Sharp predictions for neutrinos masses are not possible with just the knowledge of the \( p, q, r, s, t \) values and \( v_\sigma \). However, the order of magnitude of the mass matrices can be estimated from Eqs. (11) and (12) to be (using \( v_\sigma \simeq 3.08 \times 10^{13} \) GeV)

\[
M \simeq \zeta \times 1.73 \text{ TeV}, \quad m_S \simeq \eta \times 0.13 \text{ keV},
\]

\[
m_{\text{light}} \simeq \left[ y \zeta^{-1} \eta (\zeta^{-1} y) \right] \times 1.38 \text{ eV},
\]

(30)

which is appropriate to satisfy the constraints coming from experimental neutrino physics and unitarity without resorting a fine tuning in couplings. Nevertheless, we have to admit that some care must be taken in order to generate the benchmark region M22a in agreement with bounds coming from LFV processes such as \( \mu \to e + \gamma \). Specifically, due to \( m_{\nu_i} \sim M \gg m_W \) the loop function tends to \( G(x) \to 1/2 \) and the mixing terms are generically given by \( U \sim m_D/M \). This leads to the decay rate for \( \mu \to e + \gamma \) of the order of

\[
Br(\mu \to e\gamma) \sim 1.1 \times 10^{-13} \left( \frac{m_D}{10 \text{ GeV}} \right)^4 \left( \frac{3 \text{ TeV}}{M} \right)^4,
\]

(31)

which implies that small \( y \sim 0.1 \) couplings must be required.

For the case with \( Z_{19} \), the effective Lagrangian is characterized by \( (p, q, r, s, t) = (3, −5, −8, −2, 1) \), and the results, roughly speaking, are quite similar to the model with \( Z_{17} \), in the sense that as the \( p, q, |s| \) values are equals for both models, the neutrino spectrum is similar in both cases. Nevertheless, since \( D, t \) values are not equals, we have as a consequence that: the ALPs mass, the mass term for the exotic fermion \( E \) and the ALPs-photon coupling, \( g_{a\gamma} \), are different. Specifically, from the Table III.
and Eq. (7), in the $Z_{19}$ model the ALPs parameters are

$$
g_{a\gamma} \cong 1.12 \times 10^{-17} \left[ \frac{1.0 \times 10^{14}}{v_\sigma} \right] \text{GeV}^{-1},$$

$$m_a \cong 1.87 \times 10^{-10} |g|^2 \left[ \frac{v_\sigma}{1.04 \times 10^{14}} \text{GeV} \right]^{17/2} \text{eV}.
$$

(32)

The benchmark region in this model corresponding to this case in Figure 4 is denoted as M22b. We also show the values for the neutrino mass spectrum in Table IV. Concerning to the upper bound on $\mu$, the values for the neutrino mass spectrum in Table IV are not free of gravitational anomalies, therefore these are not suitable symmetries. However, the $Z_{17,19}$ symmetries forbid the dangerous terms and allow an effective Lagrangian.

In the case of $Z_{17}$ symmetry, the Lagrangian in Eq. (1) is characterized by the parameters $(p, q, r, s, t) = (3, -5, 7, 2, 1)$ and $D = 17$. Note then that this model has a Lagrangian very similar to the (2,2) LS Lagrangian. However, in this case, the mass term for the exotic fermion $E$ has the exponent equal to one and the term associated with $\mu_N$ is not allowed with dimension less than seven. Because the parameters $(p, |q|) = 3$ and $7$ are equal in both cases, the neutrino spectrum is the same as in the M22a model (see Eqs. (3)). Moreover, note that in this case, the term $\sim \sum_n \lambda_n (N_R^\dagger N_R)_{CC}$ gives a negligible contribution for the light active neutrino masses. On the other hand, the fact that the mass term for the exotic fermion differs from (2,2) ISS model implies that the anomaly coefficient $C_{a\gamma}$ be different (see charge in the Table III and Eq. (7)), such that the ALPs-photon coupling has also a different value. Possible assignments for the $Z_{17}$ and $U(1)_A$ symmetries are shown in Table III with $Z_{17} = 9B + 11U(1)_L$.

$$g_{a\gamma} \cong 3.77 \times 10^{-17} \left[ \frac{3.08 \times 10^{13}}{v_\sigma} \text{GeV} \right] \text{GeV}^{-1},
$$

(33)

The corresponding $v_\sigma$ value is the same that in the (2,2) ISS case shown in the Table III for $D = 17$, implying also that the $m_a$ is equal to it given in Eq. (3). Nevertheless, the $g_{a\gamma}$ turn to be because the anomaly coefficient now has a different value. A benchmark region for this case is denoted as M23a in Figure 4, where these values for $g_{a\gamma}$ and $m_a$ allow that the ALP explains 100% of the DM relic density.

| Model | Symmetry | $Q_{Li}$ | $d_{Ri}$ | $u_{Ri}$ | $H$ | $L_i$ | $I_{Li}$ | $N_{R\beta}$ | $S_{R\alpha}$ | $E_L$ | $E_R$ | $\sigma$ |
|-------|----------|----------|----------|----------|-----|------|--------|-------------|------------|-------|-------|-------|
| $Z_{17}$ | $U(1)_A$ | 1 | 2 | 0 | 16 | 14 | 15 | 13 | 8 | 15 | 12 | 7 |
| (2,2) | | 0 | 0 | 0 | 0 | 11/2 | 11/2 | 11/2 | -5/2 | 11/2 | 15/2 | 1 |
| $U(1)_A$ | 0 | 0 | 0 | 0 | 11/2 | 11/2 | 11/2 | -5/2 | 5/2 | 7/2 | 1 |
| $Z_{19}$ | $U(1)_A$ | 1 | 10 | 9 | 8 | 14 | 6 | 5 | 7 | 2 | 15 | 4 |
| (3,3) | | 0 | 0 | 0 | 0 | 11/2 | 11/2 | 11/2 | -5/2 | 9/2 | 7/2 | 1 |
| $Z_{19}$ | $U(1)_A$ | 1 | 10 | 9 | 8 | 14 | 6 | 5 | 7 | 2 | 15 | 4 |
| (3,3) | | 0 | 0 | 0 | 0 | 11/2 | 11/2 | 11/2 | -5/2 | 9/2 | 7/2 | 1 |
| $Z_{19}$ | $U(1)_A$ | 1 | 0 | 2 | 0 | 9 | 7 | 8 | 6 | 6 | 8 | 6 |
| (2,3) | | 0 | 0 | 0 | 0 | 7/2 | 7/2 | 7/2 | -3/2 | 7/2 | 3/2 | 1 |
| $Z_4$ | $U(1)_A$ | 0 | 1 | 3 | 3 | 0 | 1 | 3 | 1 | 1 | 1 | 2 |

TABLE III. Discrete and continuous charge assignments of the fields in the different models.
cient $C_{a\gamma}$. Specifically, the coupling

$$g_{a\gamma} \cong 2.24 \times 10^{-17} \left[ \frac{1.0 \times 10^{14} \text{ GeV}}{v_\sigma} \right] \text{ GeV}^{-1}. \quad (34)$$

The other parameters associated to neutrino spectrum and $m_a$ are similar than in the M22b case, and are shown in Table IV. The benchmark region for this case is denoted as M33b in Figure I.

On the other hand, the constraints and prospects regarding lepton flavor violating processes are similar to the ones in case (2.2) since the mass scale $M$ of the benchmark regions M33a and M33b are the same of the benchmark regions M22a and M22b, respectively.

We remark that a similar effective Lagrangian for the $\nu$-s, $\nu$-t, and $\nu$-m system, the neutrino mass spectrum contains two heavy pseudo-Dirac neutrinos with masses $\sim M$, three light active neutrinos with masses of order of sub-eV. In addition, there is a sterile neutrino, $\nu_S$, with mass of order $\sim \mu_S$. Then, for this model, the presence of both the $\nu_S$ and the ALPs, $a$, brings the possibility of having two DM candidates in the (2,3) scenario [80, 117].

First, let’s consider the case $\Omega_{DM}^{Planck}h^2 = \Omega_{a,DM}h^2$, i.e., when the DM abundance is totally constituted by ALPs. Now, from Eqs. (16) and (17) and Table II, we can see that $(D, v_\sigma) = (9, 0.6 - 1.1) \times 10^{11} \text{ GeV}$ and $(D, v_\sigma) = (10, 1.9 - 3.2) \times 10^{11} \text{ GeV}$ corresponds to the $(p, |q|) = (2,3)$ solution in Eq. (18) (note that $v_\sigma$ corresponding to $D = 10$ is slightly out of allowed range in Eq. (16)). Moreover, the values $D = 9, 10$ restrict the symmetry to $Z_{9,10}$. For these discrete symmetries we find solutions for anomaly free $Z_9$ and $10$ charges, i.e., solutions to Eqs. (21) and (22) with $(p, |q|) = (2,3)$. Nevertheless, all the solutions for the $Z_{9,10}$ and $10$ charges allow terms such as $\sim \sigma N_{R\beta}(N_{R\beta})^C$, $\sim \frac{\sigma^2}{M_{Pl}} N_{R\beta}(N_{R\beta})^C$, $\sim \frac{\sigma^2}{M_{Pl}} \bar{H} S_{R\sigma}$ and other terms in the Lagrangian that do not give the correct texture to the mass matrix in the ISS mechanism. We also have searched for all the possible combinations of $r,s,t$ values in the Lagrangian with $(p, |q|) = (2,3)$ without any success. Therefore, the $(p, |q|) = (2,3)$ case cannot offer a realization for an effective model providing all the observed DM abundance via ALPs when all the constraints in Section II are considered. However, from Table IV we see that for $D = 15, \ldots, 19$ with a larger value of $v_\sigma$ the second solution $(p, |q|) = (3,5)$, cf. Eq. (17) can, in principle, offer a model (note that, strictly speaking, the $v_\sigma$ value corresponding to $D = 15$ is slightly out of allowed range in Eq. (17)). Moreover, the cases of Z15 and Z19 are excluded because the condition for the gravitational anomaly is never satisfied, while in the Z16,18 cases terms as $\sim \frac{\sigma}{M_{Pl}} \bar{H} S_{R\sigma}$ and $\sim \sigma N_{R\beta}(N_{R\beta})^C$ give an incorrect texture for the ISS mass matrix. In fact, after imposing all the constraints, we find that the only symmetry that provides a solution is Z15. In more detail, we find that the discrete symmetry can be written as $Z_{15} = 15 + 11U(1)_L$ (other combinations for $Z_{15}$ are possible). This model has the effective Lagrangian, $\mathcal{L}_{Z_{15}}$, given by Eqs. (11) and (19) with $(p, q, r, s, t) = (3, -5, -4, 2, 2)$. Note that the term $\sim \sigma^4 N_{R\beta}(N_{R\beta})^C$ gives a negligible contribution for the light active neutrino masses.

The $\mathcal{L}_{Z_{15}}$ is also invariant under a $U(1)_A$ symmetry which is anomalous in the electromagnetic group as must to be a non-null coupling between photons and ALPs, $g_{a\gamma}$ (see Sec. II A). Specifically, for this case we have that the ALPs parameters are given by

$$g_{a\gamma} \cong 2.25 \times 10^{-16} \left[ \frac{1.03 \times 10^{13} \text{ GeV}}{v_\sigma} \right] \text{ GeV}^{-1},$$

$$m_a \cong 4.47 \times 10^{-8} |\xi|^2 \left[ \frac{v_\sigma}{1.03 \times 10^{13} \text{ GeV}} \right]^{13/2} \text{ eV}. \quad (35)$$

We also check that the neutrino mass spectrum for this model is

$$M \cong \xi^2 \times 6.5 \times 10^{-2} \text{ TeV}; \quad \mu_S \cong \eta \times 5.8 \times 10^{-4} \text{ keV};$$

$$m_{\nu_{light}} \cong \left[ \frac{\eta \xi^{-1} \bar{\beta} \left( \bar{\xi} \right)^{-1} \bar{y}}{4.15} \right] \times 4.15 \text{ eV}, \quad (36)$$

where we have used the particular value $v_\sigma \cong 1.03 \times 10^{13} \text{ GeV}$, which is one of the suitable values given in Table I for $D = 15$ giving the 100% of the current DM abundance. For this case, the sterile neutrino as DM candidate has a negligible contribution because the small scales in Eq. (36) imply that the mixing angle between the active and sterile neutrinos has a great suppression. Moreover the mass scale of the sterile neutrino, $\mu_S$, is very small to bring a considerable contribution to DM.

Now, from values of $M, \mu_S, m_{\nu_{light}}$ in Eq. (36) we note that in this scenario there is a some tension to satisfy the unitarity constraint. In more detail, $\left| \frac{\eta \xi^{-1} \bar{\beta} \left( \bar{\xi} \right)^{-1} \bar{y}}{4.15} \right| < \frac{M_{Planck}}{m_{DM}} \cong 10^{-1} = 2.6 \times 10^{-2}$ where we have been conservative choosing a $\epsilon^2$ value of $1\%$ (recall $\epsilon \equiv m_D M^{-1}$). However, this upper bound on $\left| \frac{\eta \xi^{-1} \bar{\beta} \left( \bar{\xi} \right)^{-1} \bar{y}}{4.15} \right|$ implies a lower bound on $\eta > \left| \frac{\eta \xi^{-1} \bar{\beta} \left( \bar{\xi} \right)^{-1} \bar{y}}{4.15} \right| \cong \left| \frac{\eta \xi^{-1} \bar{\beta} \left( \bar{\xi} \right)^{-1} \bar{y}}{4.15} \right|^2 \Delta m^2_{atm} \cong 17.17 \left( \frac{\Delta m^2_{atm}}{4.15} \right) \cong 2.3 \times 10^{-3} \text{ eV}^2$ being the atmospheric squared-mass difference which is not a perturbative value for $\eta$. This happens because the values for $v_\sigma$ corresponding for $D = 15$ is smaller than the values allowed in the range in Eq. (17). Similar conclusions are found if we consider the case when $\Omega_{a,DM}h^2 < \Omega_{Planck}h^2$. Therefore, the effective Lagrangian $\mathcal{L}_{Z_{15}}$ can not provide a natural framework for DM and the neutrino masses in (2,3) ISS case. For this reason we do not show the benchmark region for this model in Figure I.
However, models explaining the DM relic density via ALPs and/or sterile neutrinos for the (2,3) ISS case can be found provided we slightly relax some constraints mentioned in Section II. Actually, if an extra $Z_N$ symmetry is allowed, we found that, for example, the solution $(p, q) = (2, 3)$ in Eq. (10) makes possible a model with $D = 10$ and $(p, q, r, s, t) = (2, -3, -3, 2, 2)$ in Eqs. (1), (8) and (10), where the discrete gauge symmetry $Z_{10} \times Z_4$, with the corresponding charges given in Table II, must be considered with the aim of get the correct DM relic density using $D = 10$ to calculate the ALPs mass. It is straightforward to check that for this model, ALPs provide 100% of the DM abundance provided $v_{\nu} \approx 2.03 \times 10^{11}$ GeV with $g$ of order one. In more details, for this benchmark point, we have that

\[
g_{\text{a} \gamma} \approx 1.14 \times 10^{-14} \frac{2.03 \times 10^{11} \text{GeV}^2}{v_{\sigma}} \text{GeV}^{-1},
\]

\[
m_a \approx 0.29|g|^2 \frac{v_{\sigma}}{2.03 \times 10^{11} \text{GeV}} \text{eV}, \quad (37)
\]

with the neutrino mass spectrum given by

\[
M \simeq \zeta \times 8.4 \text{ TeV}; \quad \mu_S \simeq \left[ \frac{\eta}{10^{-2}} \right] \times 4.96 \text{ keV} ;
\]

\[
m_{\text{light}} \simeq \left[ y^T \zeta^{-1} \left( \frac{\eta}{10^{-2}} \right) (\zeta^{-1})^T y \right] \times 2.11 \text{ eV}. \quad (38)
\]

We note that for $\eta \leq 10^{-2}$ and the other coupling constants of order one, a suitable neutrino mass spectrum is achieved.

In this case, we have also check that the sterile neutrino has a negligible contribution to DM relic density because the mixing angle between the active and sterile neutrinos is smaller than the limits established to consider $\nu_S$ as a DM candidate ($10^{-8} \lesssim \sin^2(2\theta) \lesssim 10^{-11}$, see ref. [80] for more details). For this model, we have shown in Figure I a benchmark region denoted as M23a where ALPs provide 100% of DM abundance.

For the case that the DM abundance is made of ALPs and sterile neutrinos, the scenario slightly changes. We have chosen the case when the DM is made of $\approx 43\%$ of sterile neutrinos and $\approx 57\%$ of ALPs as an illustrative example. However, these can take other values provided the DM abundance make of sterile neutrinos is $\lesssim 50\%$, consistently with the constraints over its parameter space [30, 31, 115]. Doing a similar procedure as in the previous cases, we can obtain

\[
g_{\text{a} \gamma} \approx 1.02 \times 10^{-14} \frac{2.28 \times 10^{11} \text{GeV}^2}{v_{\sigma}} \text{GeV}^{-1},
\]

\[
m_a \approx 0.46|g|^2 \frac{v_{\sigma}}{2.28 \times 10^{11} \text{GeV}} \text{eV}. \quad (39)
\]

and

\[
M \simeq \zeta \times 10.6 \text{ TeV}; \quad \mu_S \simeq \left[ \frac{\eta}{10^{-2}} \right] \times 7.1 \text{ keV} ;
\]

\[
m_{\text{light}} \simeq \left[ y^T \zeta^{-1} \left( \frac{\eta}{10^{-2}} \right) (\zeta^{-1})^T y \right] \times 1.9 \text{ eV}. \quad (40)
\]

In this case for $\eta \approx 10^{-2}$ the sterile neutrino has $m_{\nu_S} \approx 7.1$ keV. In particular, this mass for the sterile neutrino may explain the recently indicated emission lines at 3.5 keV from galaxy clusters and the Andromeda galaxy [33, 34]. The benchmark region for this model is denoted as M23b in Figure I.

It is worth to mention that for both benchmark regions in those models, the constraints and prospects regarding lepton-flavor-violating processes are also similar to the ones in case (2,2). This happens because the contribution of the sterile neutrino to $\text{Br}(\ell_3 \to \ell_3 \gamma)$ is negligible since $G(m_{\nu_S}^2/m_W^2) \to 0$ for $m_{\nu_S} \ll m_W$.

Finally, for clearness, we show in Table IV an overview of the main results of all considered models. Specifically, we show energy scales for the neutrino masses and the ALPs parameter space for each ISS case.

### IV. DISCUSSION AND SUMMARY

We have connected two interesting motivations for going beyond the standard model: neutrino masses and ALPs as dark matter. A natural scenario for achieving that is the ISS mechanism. In particular, we have considered the minimal versions of the ISS mechanism in agreement with all the neutrino constraints. Nevertheless, in the considered framework, the mass scales for the ISS mechanism are generated by gravity-induced non-renormalizable operators when the scalar field containing the ALPs gets a vacuum expectation value, $v_{\sigma}$. Naturalness of these scales imposes strong constraints on these operators and, when combining these with the ALPs acceptable range for $v_{\sigma}$, only two solutions are possible: $(p, q) = (2, 3)$ for $6 	imes 10^{10} \lesssim v_{\sigma} \lesssim 1 \times 10^{14}$ GeV and $(p, q) = (3, 5)$ for $2 \times 10^{13} \lesssim v_{\sigma} \lesssim 8 \times 10^{13}$ GeV.

This implies that operators given $M$ and $\mu_S$ scales can only belong to these two categories. Then, a simultaneous application of constraints coming from the texture of ISS mass matrix, the violation of the unitarity, the mass of exotic charged leptons, the stability of the effective Lagrangian against gravitational effects and the suitable ALPs parameter space ($m_a$ and $g_{\text{a} \gamma}$) to provide the total DM density almost set the rest of terms in the Lagrangian, only leaving a few of possibilities for all of ISS cases. These constraints ultimately lead to a concrete prediction for the viable ALPs masses and ALPs-photon couplings and also for the mass scale of the heavy neutrinos necessary to explain the neutrino oscillation data. In other words, both sectors are deeply connected and the observation of a hypothetical signal of the ALP existence within the proper regions will automatically lead to the existence of heavy neutrino states in the TeV and multi TeV scales. In the same way, the nonobservation of an ALP within such regions or the observation of heavy neutrinos below the TeV scale would disfavor the possible linkage between ALP DM and neutrino masses suggested in this work.

Among the minimal ISS mechanisms, the (2,2) and
(3, 3) ISS cases are quite similar. It is due to the fact that in both of them \( n_{N_R} = n_{S_R} \) implying that neutrino mass spectrum is characterized by only two mass scales, \( M \) and \( m_{\text{light}} \). Thus, the results obtained are almost identical. Although, there is a slightly difference in the value of \( g_{a\gamma} \) due to the presence of more fermions in the (3, 3) ISS case. In both cases, we find two effective models denoted as M22a, b and M33a, b in Table LV. Since there is not sterile neutrino in these cases, the total DM density is made of ALPs. We also remark that, although, the ALPs in these models can decay to two photons and, in the (2, 2) ISS mechanism, to two massless active neutrinos, these are cosmologically stable because those decays are strongly suppressed by factors of \( 1/M^2_{\text{Pl}} \) and/or \( 1/v^2_\sigma \).

On the other hand, the (2, 3) ISS case is phenomenologically more interesting due to the presence of a sterile neutrino in the mass spectrum. It implies that the DM density can be made of ALPs and \( v_{S_R} \). We have found a model satisfying all of previously mentioned constraints and, at the same time, offering the total DM. Because sterile neutrinos in the (2, 3) ISS mechanism can give, roughly speaking, at most \( \approx 43\% \) of the DM density, it is necessary that the remaining \( \approx 57\% \) of DM be made of ALPs. It is also possible that ALPs give the total DM density. It occurs when the mixing angle between active and sterile neutrinos is very suppressed in order to make the Dodelson-Widrow mechanism inefficient. Both cases were studied in detail and denoted as M23a and M23b, respectively.

Regarding the search for ALPs, the benchmark regions in Figure IV are out of reach of the current and future experimental searches for axion/ALPs such as ALPS II, IAXO, CAST [60], since these currently have not enough sensitivity to probe the ALPs/axion-photon couplings and masses that are motivated in models with scales \( v_\sigma \lesssim 10^{13} \) GeV. Nevertheless, for the (2, 2) and (3, 3) ISS cases the benchmark regions are remarkably within the target regions in proposed experiments based on LC circuits [61, 65], which are designed to search for QCD axions and ALPs and cover many orders of magnitude in the parameter space of these particles, beyond the current astrophysical and laboratory limits [60, 70, 71]. Specifically, the ABRACADABRA experiment [64] may explore ALPs masses as low as \( \sim 10^{-10} \) eV for a coupling to photons of the order of \( \sim 10^{-18} \) GeV\(^{-1}\), which are well below our benchmark regions (Figure I).

Finally, despite the fact that neutrino mass spectrum is not completely predicted in the models found, the matrix scales in the ISS mechanism are estimated to be in agreement with the neutrino constraints [119]. Moreover, we have numerically checked, in all models, that there are solutions with coupling constants of order one that also satisfy LFV processes and the unitary condition. These processes can easily avoid without fine-tuning in the models discussed in this paper. Specifically, we have found that the \( \text{BR}(\mu \rightarrow e\gamma) \) in all cases are as small as \( \sim 10^{-20} - 10^{-15} \) which are consistent with the current experimental value \( \text{BR}(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13} \) [113] and with future sensitivities around \( 6 \times 10^{-14} \) [120].

**Table IV.** Main features of the models discussed in the text. We have considered the constant \( g \subset [10^{-3}, 2] \) in the mass term of the ALPs, the \( \eta \) Yukawa at order \( 10^{-2} \) in the M23a(b) models, and \( \Omega_{\text{Planck}} m^2 \) at 3\( \sigma \).

| ISS Model | \( m_{a} \times 10^{-11} \) eV | \( g_{a\gamma} \times 10^{-17} \) GeV\(^{-1}\) | \( M \) (TeV), \( \mu_S \) (keV), \( m_{\text{light}} \) (eV) |
|-----------|-----------------|-----------------|-----------------|
| M22a      | (19.0 – 56.0)   | (5.5 – 7.8)     | (1.5 – 4.5), (0.1 – 0.7), (1.0 – 1.4) |
| M22b      | (0.52 – 1.4)    | (1.1 – 1.6)     | (24.3 – 65.6), (11.3 – 58.9), (0.4 – 0.6) |
| M33a      | (19.0 – 56.0)   | (2.7 – 3.9)     | (1.5 – 4.5), (0.1 – 0.7), (1.0 – 1.4) |
| M33b      | (0.52 – 1.4)    | (2.2 – 3.1)     | (24.3 – 65.6), (11.3 – 58.9), (0.4 – 0.6) |
| M23a      | (0.06 – 0.30) \( \times 10^{11} \) | (720 – 1200)   | (7.5 – 21.0), (4.12 – 19.41), (1.33 – 2.24) |
| M23b      | (0.03 – 0.2) \( \times 10^{11} \) | (830 – 1400)   | (5.6 – 15.8), (2.7 – 12.7), (1.4 – 2.5) |

**Acknowledgments**

B. L. S. V. would like to thank Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), Brazil, for financial support. C. D. R. C. acknowledges the financial support given for the Departamento Administrativo de Ciencia, Tecnología e Innovación - COLCIENTÍFICAS (doctoral scholarship 727-2015), Colombia, and the hospitality of Laboratori Nazionali di Frascati, Italy, in the final stage of this work. O. Z. has been partly supported by UdeA/CODI grant IN650CE and by COL-
It is worth mentioning that for all the models the normal spectrum is the preferred neutrino mass spectrum [53] which in turn implies that our scan results are also compatible with the cosmological upper bound on the neutrino mass sum [122, 123].