ABSTRACT The present study discusses the consensus control of dual-rate multi-agent systems, where the sampling/communication interval of quantized data is an integer multiple of the control interval. A conventional multi-agent system uses a dynamic quantizer which is designed in a single-rate system where the intervals are equal, i.e., the control interval length is the same as the communication interval length. However, a dynamic quantizer designed in a dual-rate system is expected to have improved control performance. In the present study, an objective function is divided into a quantization term, which is related to the quantization error, and the remaining term. The proposed dual-rate dynamic quantizer is designed such that the quantization term is minimized. Finally, in numerical examples, the proposed dual-rate method is quantitatively evaluated by comparing with the conventional single-rate method, and the effectiveness of the proposed method is demonstrated.

INDEX TERMS Multi-agent system, quantizer, networked control system, dual-rate.
improve the degradation of consensus performance due to the quantization error, frequent communication, i.e., short sampling intervals, is required. In the continuous time domain, a steady-state optimization method [9], the consensus conditions [10], and nonlinear systems [11] have been studied. Furthermore, an adaptive dynamic quantizer has been proposed for continuous-time second-order multi-agent systems with input quantized [12].

The present study investigates a design method for the consensus control of dual-rate multi-agent systems [13], where the sampling interval of the quantized controlled value is an integer multiple of the updating interval of the control input. Following the conventional method for dual-rate multi-agent systems [14], a controller was previously designed for optimizing not only at the sampling instants but also between them, and the implementation was therefore difficult. In the present study, the conventional single-rate design method [7] is extended to the dual-rate system so that the effect of the quantization error at the sampling instant is minimized.

In the present study, \( \mathbb{N} \) and \( \mathbb{R} \) denote the spaces of integer and real numbers, respectively, and \( \otimes \) is the Kronecker product. \( 0_{i,j} \) is an \( i \times j \) matrix of which all the elements are 0 and \( 1_{i,j} \) is an \( i \times j \) matrix of which all the elements are 1.

**II. CONTROLLED SYSTEM**

Consider a multi-agent system which consists of \( N_a (\in \mathbb{N}) \) agents, in which a network of communication is a non-directed graph. A block diagram for agent \( i \) (\( i = 1, \ldots, N_a \)) is illustrated in Fig. 1, where \( P_i \) and \( C_i \) denote the controlled plant and the controller in agent \( i \), respectively. Furthermore, the state of agent \( i \) is quantized by quantizer \( Q_i \), and the quantized value is transferred through the network. The dynamics of \( P_i \) are described by an integral system as follows:

\[
x_i[k + 1] = x_i[k] + u_i[k] \quad (i = 1, \ldots, N_a)
\]

where \( x_i[k] \in \mathbb{R} \) is the state, and \( u_i[k] \in \mathbb{R} \) is the control input to agent \( i \), which is decided by \( C_i \) designed as a distributed controller given as

\[
u_i[k] = -h \sum_{j=1}^{N_a} a_{ij}(x_Q[k] - x_Q^i/k)]
\]

where \( h \in \mathbb{R} \) is the controller gain, and \( x_Q^i/k] \) is the quantized value of the state of agent \( i, x_i[k] \). Here, \( a_{ij} \) is the \( i, j \)th element of the adjacency matrix \( A \), and is defined by Eq. (3) decided by the edge set of graph \( G, \varepsilon \), as follows:

\[
a_{ij} := \begin{cases} 1 & (i, j) \in \varepsilon \land i \neq j \\ 0 & \text{ others} \end{cases}
\]

The data used in the controller are under the following assumptions:

**Assumption 1:**

- The states of agents are quantized to be transferred in the network as digital signals.
- The state of an agent \( i \), which is itself used in controller \( C_i \), is quantized because the sensor performance is assumed to be equal to the quantization level.

All the states used in the controllers are therefore quantized data.

Quantizer \( Q_i \) in Fig. 1 is implemented by the following dynamic quantizer [8]:

\[
\xi_i[k + 1] = A_Q[k] \xi_i[k] + B_Q(x_Q[k] - x_i[k])
\]

\[
x_Q[k] = q[C_Q \xi_i[k] + x_i[k]]
\]

where \( \xi_i[k] \in \mathbb{R} \) is the state of the quantizer, and the initial value is \( \xi_i[0] = 0 \). Here, \( q \) denotes the static quantizer defined as

\[
q[\alpha] = d(\alpha/d + 0.5)
\]

where \( d \in \mathbb{R} \) is the quantization level. Consequently, \( |\alpha| \) is the maximum integer which is less than or equal to \( \alpha \in \mathbb{R} \). Furthermore, \( A_Q[k] \in \mathbb{R}, B_Q \in \mathbb{R} \) and \( C_Q \in \mathbb{R} \) are the design parameters of the dynamic quantizer. In the present study, the parameters are designed to compensate for the control degradation caused by both the quantization and the long communication interval.

Regarding the quantization error in agent \( i \) as noise \( w_i[k] \), Eq. (5) is arranged as

\[
x_Q[k] = C_Q \xi_i[k] + x_i[k] + w_i[k].
\]

Substituting the control law (Eq. (2)) into the plant (Eq. (1)), the next relational expression is given as

\[
x_i[k + 1] = x_i[k] - h \sum_{j=1}^{N_a} a_{ij}(x_Q[k] - x_Q^i/k]).
\]

Using the above-mentioned expression, all the agents are summarized as follows:

\[
\begin{bmatrix}
x_1[k + 1] \\
x_2[k + 1] \\
\vdots \\
x_{N_a}[k + 1]
\end{bmatrix} =
\begin{bmatrix}
x_1[k] \\
x_2[k] \\
\vdots \\
x_{N_a}[k]
\end{bmatrix} - hL
\begin{bmatrix}
x_Q^1[k] \\
x_Q^2[k] \\
\vdots \\
x_Q^{N_a}[k]
\end{bmatrix}
\]

where \( L \) is the graph Laplacian of \( G \), defined as

\[
L = D - A
\]

FIGURE 1. Block diagram of agent \( i \).
in which degree matrix $D$ is given as follows:

$$D = \text{diag}(d_1^{in}, d_2^{in}, \ldots, d_{Na}^{in})$$

$$d_i^{in} = \sum_{j=1}^{Na} a_{i,j}. \quad (11)$$

Eq. (9) can be rewritten in vector form as

$$X[k + 1] = X[k] - hLX_Q[k]$$

$$X[k] = [x_1[k] \, x_2[k] \, \cdots \, x_{Na}[k]]^T$$

$$X_Q[k] = [x_Q1[k] \, x_Q2[k] \, \cdots \, x_QN_a[k]]^T. \quad (12)$$

Additionally, the dynamic quantizer is also described in vector form as

$$Z[k + 1] = A_Q[k]Z[k] + B_Q(X_Q[k] - X[k])$$

$$X_Q[k] = C_QZ[k] + X[k] + W[k]$$

where

$$Z[k] = [\xi_1[k] \, \xi_2[k] \, \cdots \, \xi_{Na}[k]]^T$$

$$W[k] = [w_1[k] \, w_2[k] \, \cdots \, w_{Na}[k]]^T$$

$$A_Q[k] = \text{diag}(A_Q[k] \, \cdots \, A_Q[k])$$

$$B_Q = \text{diag}(B_Q \, \cdots \, B_Q)$$

$$C_Q = \text{diag}(C_Q \, \cdots \, C_Q).$$

### III. DUAL-RATE MULTI-AGENT SYSTEM

In the present study, because of the network performance constraints, the communication interval, where the state of agents is transferred through networks, is an integer ($N > 1$) multiple of the updating interval of the control input.

Since it is assumed that agents communicate with adjacent agents at every step ($N = 1$), $X[k + i]$ ($i = 1, \cdots, N$) are listed as

$$X[k + 1] = X[k] - hLX_Q[k]$$

$$X[k + 2] = X[k + 1] - hLX_Q[k + 1]$$

$$\vdots$$

$$X[k + N] = X[k + N - 1] - hLX_Q[k + N - 1].$$

The equations are then summarized as follows:

$$X[K + 1] = X'[K] - hL \otimes X'_Q[K]$$

$$X[K] = [X[K - N + 1] \, X[K - N + 2] \cdots X[K]]^T$$

$$X'[K] = [X[K] \, X[K + 1] \cdots X[K + N - 1]]^T$$

$$X'_Q[K] = [X_Q[K] \, X_Q[K + 1] \cdots X_Q[K + N - 1]]^T. \quad (15)$$

where the step length of $K \in \mathbb{N}$ is $N$ times that of $k$, and $K + 1$ denotes $K + N$.

Thus, the control input is updated at every step, whereas the state of agents is measured every $N$ steps. The system is therefore a dual-rate system, where the states of agents are not measured at steps $K + j$ ($j = 1, \cdots, N - 1$). The relationship between the interval of control input $u_i[k]$ and that of agent state $x_i[k]$ is illustrated in Fig. 2.

![Figure 2. Dual-rate communication.](image)

### IV. OPTIMAL DESIGN OF QUANTIZED CONSENSUS CONTROL

#### A. TRANSFORMATION OF THE DUAL-RATE SYSTEM

In the dual-rate system, $X'[K]$ and $X'_Q[K]$ are unavailable, and Eq. (15) is replaced with Eq. (16):

$$X[K + 1] = A_P X[K] - hLX'_Q[K]$$

$$A_P = [0_{Na,Na(N-1)} \, I_{Na,Na}]$$

$$L = \begin{bmatrix}
D_{eg} & 0_{Na,Na} & \cdots & 0_{Na,Na} \\
\vdots & \vdots & \ddots & \vdots \\
0_{Na,Na} & \cdots & 0_{Na,Na} \\
NA_{adj} & 0_{Na,Na} & \cdots & 0_{Na,Na}
\end{bmatrix}$$

$$\quad (16)$$

where $D_{eg}$ and $A_{adj}$ are the degree matrix and the adjacent matrix of the dual-rate network system.

Next, the dynamics of the quantizers between the sampling instants are summarized as follows:

$$Z[K + 1] = \tilde{A}_Q[K]Z[K] + \tilde{B}_Q[K](X'_Q[K] - X'[K])$$

$$Z[K] = [Z[K - N + 1] \, Z[K - N + 2] \cdots Z[K]]^T$$

$$\tilde{A}_Q[K] = \begin{bmatrix}
0_{Na,Na(N-1)} & \cdots & 0_{Na,Na} \\
I_{Na} & 0_{Na,Na} & \cdots & 0_{Na,Na} \\
\vdots & \vdots & \ddots & \vdots \\
\prod_{i=1}^{N-1} A_{Q}[K + i] & \cdots & 0_{Na,Na} \\
\prod_{i=1}^{N-1} A_{Q}[K + i] & \cdots & 0_{Na,Na} \\
0_{Na,Na} & \cdots & 0_{Na,Na} \\
\prod_{i=1}^{N-1} A_{Q}[K + i] & \cdots & 0_{Na,Na}
\end{bmatrix} \otimes B_Q$$

$$\quad (17)$$

Since the future variables at steps $K + j$ ($j = 1, \cdots, N - 1$) are included in $X'[K]$ and $X'_Q[K]$ used in Eq. (16) and
Using Eq. (16), \( \mathbf{X}'[\mathbf{K}] \) is written as follows:

\[
\mathbf{X}'[\mathbf{K}] = \mathbf{X}[\mathbf{K} + N - 1] = A_p \mathbf{X}[\mathbf{K}] - h \mathbf{L} \mathbf{X}'[\mathbf{K}]
\]

\[
\mathbf{L} = \begin{bmatrix}
\mathbf{D}_{\mathbf{X}} & \cdots & \mathbf{D}_{\mathbf{X}}
\end{bmatrix} - \begin{bmatrix}
\mathbf{N}_d & \cdots & \mathbf{N}_d
\end{bmatrix}
\]

\[
\mathbf{L}' = \mathbf{L} - \begin{bmatrix}
\mathbf{N}_d & \cdots & \mathbf{N}_d
\end{bmatrix}
\]

Expanding the elements which consist of \( \mathbf{X}'[\mathbf{K}] \), \( \mathbf{X}'[\mathbf{K}] \) can then be rewritten as follows:

\[
\mathbf{X}'[\mathbf{K}] = \mathbf{X}[\mathbf{K}] + \mathbf{W}'[\mathbf{K}]
\]

In the same way, Eq. (17) can also be rewritten as

\[
\mathbf{Z}[\mathbf{K} + 1] = (\bar{\mathbf{A}}[\mathbf{K}] + \bar{\mathbf{B}}[\mathbf{K}] \bar{\mathbf{R}}[\mathbf{K}] \bar{\mathbf{C}} \bar{\mathbf{A}}[\mathbf{K}]) \mathbf{Z}[\mathbf{K}]
\]

\[
+ \bar{\mathbf{B}}[\mathbf{K}] \bar{\mathbf{R}}[\mathbf{K}] \bar{\mathbf{W}}'[\mathbf{K}]
\]

Note that \( \mathbf{W}'[\mathbf{K}] \) included in Eq. (21) and Eq. (22) represents unknown future noise. In the present study, the design parameters of the dynamic quantizer are designed so that the effect of \( \mathbf{W}'[\mathbf{K}] \) is minimized.

**B. PARAMETER DESIGN OF DYNAMIC QUANTIZER**

The objective function \( \delta[\mathbf{K}] \) is defined as

\[
\delta[\mathbf{K}] = \left\| C_d (\mathbf{X}[\mathbf{K}] - \mathbf{X}_{\text{Ideal}}[\mathbf{K}]) \right\|
\]

\[
C_d = \begin{bmatrix}
\mathbf{0}_{\mathbf{N}_d, \mathbf{N}_d (\mathbf{N} - 1)} & \mathbf{I}_{\mathbf{N}_d}
\end{bmatrix}
\]

Here, \( \mathbf{X}_{\text{Ideal}}[\mathbf{K}] \) denotes the ideal state of \( \mathbf{X}[\mathbf{K}] \), in which non-quantized states of agents are transferred with the shortest communication interval \( \mathbf{N} = 1 \), and is set as

\[
\mathbf{X}_{\text{Ideal}}[\mathbf{K}] = \mathbf{H}_{\text{Ideal}}^\mathbf{K} \mathbf{X}[\mathbf{0}]
\]

\[
\mathbf{H}_{\text{Ideal}} = \begin{bmatrix}
\mathbf{0}_{\mathbf{N}_d, \mathbf{N}_d (\mathbf{N} - 1)} & \mathbf{I}_{\mathbf{N}_d}
\end{bmatrix}
\]

The objective function used in the conventional method [14] evaluates the error between the actual state with the ideal state at non-communication steps from \( \mathbf{K} = \mathbf{N} + 1 \) to \( \mathbf{K} = 1 \) as well as at communication step \( \mathbf{K} \). Consequently, extremely high control performance is demanded, and the solution cannot be easily obtained. In contrast, in the present study, a realistic objective function is used where the error at the communication step is evaluated, and the optimal design is therefore achieved.

From Eq. (21), when the initial states are obtained, \( \mathbf{X}[\mathbf{K}] \) is calculated as follows:

\[
\mathbf{X}[\mathbf{K}] = \left\{ \begin{array}{c}
\mathbf{X}[\mathbf{0}] \\
\mathbf{Z}[\mathbf{0}]
\end{array} \right\}
\]

\[
= \left\{ \begin{array}{c}
\mathbf{X}[\mathbf{0}] \\
\mathbf{Z}[\mathbf{0}]
\end{array} \right\}
\]

Finally, \( \mathbf{X}'[\mathbf{K}] \) is obtained as follows:

\[
\mathbf{X}'[\mathbf{K}] = (\mathbf{I}_{\mathbf{N}_d} - h \mathbf{L})^{-1} (\mathbf{A}_p \mathbf{X}[\mathbf{K}]
\]

\[
+ \mathbf{R}_\mathbf{Q}[\mathbf{K}] (\bar{\mathbf{C}} \bar{\mathbf{A}}[\mathbf{K}] \mathbf{Z}[\mathbf{K}] + \mathbf{W}'[\mathbf{K}])
\]

\[
\mathbf{R}_\mathbf{Q}[\mathbf{K}] = (\mathbf{I}_{\mathbf{N}_d} - \bar{\mathbf{C}} \bar{\mathbf{B}}[\mathbf{K}])^{-1}
\]

Using Eq. (18) and Eq. (20), Eq. (16) can be rewritten as

\[
\mathbf{X}[\mathbf{K} + 1] = (\mathbf{I}_{\mathbf{N}_d} - \mathbf{H} \mathbf{L}) \mathbf{X}[\mathbf{K}]
\]

\[
- \mathbf{H} \mathbf{R}_\mathbf{Q}[\mathbf{K}] \bar{\mathbf{C}} \bar{\mathbf{A}}[\mathbf{K}] \mathbf{Z}[\mathbf{K}] + \mathbf{W}'[\mathbf{K}]
\]

\[
\mathbf{H} = h \mathbf{L} (\mathbf{I}_{\mathbf{N}_d} + h \mathbf{L})^{-1}
\]

where

\[
\mathbf{A}_p[\mathbf{K}] = \begin{bmatrix}
\mathbf{A}_{11} & \mathbf{A}_{12}[\mathbf{K}]
\end{bmatrix}
\]

\[
\mathbf{A}_{11} = (\mathbf{I}_{\mathbf{N}_d} - \mathbf{H}) \mathbf{A}_p
\]

\[
\mathbf{A}_{12}[\mathbf{K}] = -\mathbf{H} \mathbf{R}_\mathbf{Q}[\mathbf{K}] \bar{\mathbf{C}} \bar{\mathbf{A}}[\mathbf{K}]
\]

\[
\mathbf{A}_p[\mathbf{K}] = \begin{bmatrix}
\mathbf{A}_{21} & \mathbf{A}_{22}[\mathbf{K}]
\end{bmatrix}
\]

\[
\mathbf{A}_{21} = 0_{\mathbf{N}_d, \mathbf{N}_d}
\]

\[
\mathbf{A}_{22}[\mathbf{K}] = \bar{\mathbf{A}}[\mathbf{K}] + \bar{\mathbf{B}}[\mathbf{K}] \bar{\mathbf{R}}[\mathbf{K}] \bar{\mathbf{C}} \bar{\mathbf{A}}[\mathbf{K}]
\]

\[
\mathbf{B}_w[\mathbf{K}] = \begin{bmatrix}
\bar{\mathbf{B}}[\mathbf{K}] \bar{\mathbf{R}}[\mathbf{K}]
\end{bmatrix}
\]
Using Eq. (25), the objective function is written as

\[
\delta[K] = \left\| C_b \left[ [I_{N_a N} \ 0_{N_a N, N_a N}] \left( \prod_{i=1}^{K-1} A_i[i] \right) \begin{bmatrix} X[0] \\ Z[0] \end{bmatrix} \right] + \sum_{j=1}^{K} \left( \prod_{i=1}^{K} A_i[j-1-1] B_w[K-j-1] \right) \times Wf[K-j-1] - H^K_{\text{ideal}} X[0] \right\|.
\]

The equation is divided into two terms, \( \delta_{QE}[K] \), which is related to the quantization error, and the remaining \( \delta_{ETC}[K] \), as follows:

\[
\delta[K] = \delta_{QE}[K] + \delta_{ETC}[K]
\]

\[
\delta_{QE}[K] = \left\| C_b \left[ [I_{N_a N} \ 0_{N_a N, N_a N}] \times \sum_{i=1}^{K} \left( \prod_{i=1}^{K} A_i[i] \right) \begin{bmatrix} X[0] \\ Z[0] \end{bmatrix} \right] - A_1 H + A_2[K-i-1] \right\|.
\]

To minimize the coefficient of the quantization error, Eq. (30) is rewritten as follows:

\[
\delta_{QE}[K] \leq \left\| C_b \left[ [I_{N_a N} \ 0_{N_a N, N_a N}] \right] \cdot \sum_{j=1}^{K} \left( \prod_{i=1}^{K} A_i[j] \right) \begin{bmatrix} -A_1 H + A_2[K-i-1] \end{bmatrix} \right\|.
\]

The simulation conditions are set as follows: the control interval \( T_c \) is 0.1 s, the communication interval is 0.4 \( (N = 4) \), the number of agents \( N_a \) is 10, the quantization level \( d \) is 5, the controller gain \( h \) is 0.0075, and the network structure is as shown in Fig. 3. Also, the initial values of the states, \( x_i[0] \) \( (i = 1, \cdots, 10) \) are \(-2, -5, 7, 9, -3, -9, -7, 2, -7, \) and \(-4\). Additionally, \( A_Q[k] \) when \( k = jN - 1 \) in Eq. (28) is obtained by using the fminsearch function in MathWorks MATLAB software and is given as

\[
A_Q[k] = 0.8823 \quad (k = jN - 1, \ j \in \mathbb{N}).
\]
The simulation results are shown in Fig. 4. For comparison, the single-rate control results with a long control interval are also shown in Fig. 5, where both the control and communication intervals are 0.4 s and the time-invariant $A_Q$ is 1 [7].

To evaluate the performance quantitatively, 500 trials are simulated under the same conditions except for the initial states $x_i[0]$ which are randomly chosen from a uniform distribution of integers in $[-10, 10]$. The root-mean-square of the variances of each agent in the last 100 steps ($290 \leq t \leq 300$) is shown in Table 1. Furthermore, Table 2 shows the probability that the norm of the difference between the consensus value and the state of the agents is less than or equal to $1/10$ of the quantization level $d$. These results demonstrate that the dual-rate system has superior consensus control performance compared to the single-rate system.

### VI. CONCLUSION

In the present study, we propose an optimal method for a quantizer used in the consensus control of dual-rate multi-agent systems where the quantized states of agents are transferred through a network for which the sampling/communication interval is an integer multiple of the control interval. To minimize the effect of the quantization error, which deteriorates the consensus control performance of multi-agent systems, a dual-rate multi-agent system is designed using a dynamic quantizer. The design method of the dynamic quantizer in this dual-rate system is proposed such that the state of agent is close to the ideal state. Additionally, the effectiveness of the proposed method is demonstrated through numerical examples.

Since the proposed method is restricted to the dual-rate system, where all communication and control intervals are equal, in future studies we plan to extend the method to non-uniform interval systems. Furthermore, communication delay and packet loss, which occur almost universally in network communication systems, are not taken into account in the present study, and hence, must also be coped with or addressed accordingly.

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