Testing Vector Gravity with Gravitational Wave Interferometers

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Recently, the author proposed an alternative vector theory of gravity. To the best of our knowledge, vector gravity also passes all available tests of gravity, and, in addition, predicts the correct value of the cosmological constant without free parameters. It is important to find a new feasible test which can distinguish between vector gravity and general relativity and determine whether gravity has a vector or a tensor origin. Here we propose such an experiment based on measurement of propagation direction of gravitational waves relative to the perpendicular arms of a laser interferometer. We show that transverse gravitational wave in vector gravity produces no signal when it propagates in the direction perpendicular to the interferometer plane or along one of the arms. In contrast, general relativistic wave yields no signal when it propagates parallel to the interferometer plane at 45\(^\circ\) angle relative to an interferometer arm. The test can be performed in the nearest years in a joint run of the two LIGO and one Virgo interferometers.

I. INTRODUCTION

Recently, the author proposed an alternative vector theory of gravity which assumes that Universe has fixed background geometry - four dimensional Euclidean space, and gravity is a vector field in this space which breaks the Euclidean symmetry [1]. Direction of the vector gravitational field gives the time coordinate, while perpendicular directions are spatial coordinates. Similarly to general relativity, the theory postulates that gravitational field is coupled to matter universally and minimally through a metric tensor which, however, is not an independent variable but rather a functional of the vector gravitational field.

Such assumptions yield a unique theory of gravity. To the best of our knowledge, it passes all available tests, including the recent detection of gravitational waves by LIGO [2,3]. Vector gravity predicts lack of black holes and gravitational waveforms measured by LIGO are interpreted as being produced by orbital inspiral of massive neutron stars that can exist in the theory. The measured waveforms can be fitted in the framework of vector gravity as accurately as in general relativity [4].

For cosmology vector gravity predicts the same evolution of the Universe as general relativity with cosmological constant and zero spatial curvature. However, vector gravity provides explanation of the dark energy as energy of longitudinal gravitational field induced by the Universe expansion is the mysterious dark energy. Contrary to matter, it has negative energy density and accelerates expansion of the Universe. Despite of fundamental differences, vector gravity and general relativity yield for the experimentally tested regimes quantitatively very close predictions which allowed both theories to pass available tests. In particular, vector gravity and general relativity are equivalent in the post-Newtonian limit. At strong field, however, vector gravity substantially deviates from general relativity and yields no black holes. Namely, the end point of a gravitational collapse is not a point singularity but rather a stable star with a reduced mass. One should mention that black holes have never been observed directly and "evidences" of their existence are based on the presumption that general relativity describes gravity for strong field. Until signatures of the event horizons are found the existence of black holes will not be proven.

Prediction of the matter behavior at strong gravity and interpretation of the universe evolution on large scales depends on the theory of gravity we are using. Thus, there is a need for a feasible test which can distinguish between vector gravity and general relativity and rule out one of the two theories. Here we propose such an experiment that can be done in the nearest years using gravitational wave interferometers.

Both in general relativity and vector gravity the polarization of gravitational waves emitted by orbiting binary objects is transverse, that is wave yields motion of test particles in the plane perpendicular to the direction of wave propagation. However, as we show, dependence of the laser interferometer signal on the orientation of the interferometer arms relative to the propagation direction of the gravitational wave is different in the two theories.

Gravitational wave produces motion of the interferometer mirrors and changes phase velocity of light. Both of these effects contribute to the relative phase shift of light traveling in the perpendicular arms of the Michelson interferometer. For certain propagation directions of the gravitational wave relative to the arms the two contributions cancel each other yielding zero net phase shift.
Those are the directions of zero response of the interferometer for which gravitational wave cannot be detected for any transverse polarization. As we show, directions of the zero response are different for transverse gravitational waves in vector gravity and general relativity. Detection of a wave in the direction of the zero response predicted by a theory of gravity will rule out such theory.

II. RESPONSE OF INTERFEROMETER ON GRAVITATIONAL WAVE IN VECTOR GRAVITY AND GENERAL RELATIVITY

In vector gravity for a weak transverse plane gravitational wave propagating along the \( x \)-axis the equivalent metric reads \[ g_{ik} = \eta_{ik} + \begin{pmatrix} 0 & 0 & h_{0y}(t, x) & h_{0z}(t, x) \\ 0 & 0 & 0 & 0 \\ h_{0y}(t, x) & 0 & 0 & 0 \\ h_{0z}(t, x) & 0 & 0 & 0 \end{pmatrix}, \] (1)

where \( \eta_{ik} \) is Minkowski metric and \( h_{0y} \), \( h_{0z} \) are small perturbations obeying the wave equation. A rest particle (or mirrors of an interferometer) will move under the influence of the gravitational wave \[ \eta^{\alpha\beta} \] with a time-dependent velocity \( V^\alpha = h_{0\alpha}c \) \( (\alpha = x, y, z) \) perpendicular to the direction of the wave propagation. By making a coordinate transformation into the co-moving frame of the test particle

\[ x'^\alpha = x^\alpha - \int^t V^\alpha dt, \]

the metric (1) reduces to

\[ g_{ik} = \eta_{ik} + \begin{pmatrix} 0 & 0 & h_{xy}(t, x) & h_{xz}(t, x) \\ 0 & 0 & 0 & 0 \\ h_{xy}(t, x) & 0 & 0 & 0 \\ h_{xz}(t, x) & 0 & 0 & 0 \end{pmatrix}, \] (2)

where \( h_{xy} = h_{0y}, h_{xz} = h_{0z} \). Metric (2) is written in the coordinate system in which test particles do not move under the influence of the gravitational wave.

On the other hand, in general relativity for a weak gravitational wave propagating along the \( x \)-axis the metric in the co-moving frame evolves as \[ g_{ik} = \eta_{ik} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & h_{yy}(t, x) & h_{yz}(t, x) & 0 \\ 0 & h_{yz}(t, x) & -h_{yy}(t, x) & 0 \end{pmatrix}. \] (3)

Here we investigate a response of a laser interferometer with perpendicular arms on a gravitational wave in the two theories of gravity. In gravitational field with a metric \( g_{ik} \) the Maxwell’s equations for the electromagnetic field vector \( A^k \) in the absence of charges read

\[ \frac{\partial}{\partial x^k} \left[ \sqrt{-g} g^{kl} g^{im} \left( \frac{\partial A_m}{\partial x^l} - \frac{\partial A_l}{\partial x^m} \right) \right] = 0, \] (4)

while the Lorenz gauge equation is

\[ \frac{\partial}{\partial x^k} \left( \sqrt{-g} A^k \right) = 0, \] (5)

where \( A^k = g^{km} A_m \) and \( g = \text{det}(g_{ik}) \). Gravitational wave causes oscillation of \( g_{ik} \) in space and time. However, since frequency of the gravitational waves is much smaller then frequency of the electromagnetic waves traveling in the interferometer one can disregard derivatives of the metric in Eqs. (4) and (5). Then Eqs. (4) and (5) reduce to

\[ g^{kl} \frac{\partial^2 A^l}{\partial x^k \partial x^l} - g^{im} \frac{\partial^2 A^k}{\partial x^m \partial x^k} = 0, \] (6)

\[ \frac{\partial A^k}{\partial x^k} = 0. \] (7)

Combining them together we obtain

\[ g^{kl} \frac{\partial^2 A^l}{\partial x^k \partial x^l} = 0. \] (8)

In the co-moving frame there is no motion of the interferometer mirrors and the phase shift of light traveling along the two arms appears due to difference in the light phase velocity. Substitute \( g^{kl} = \eta^{kl} + h^{kl} \) in Eq. (8), where \( h^{kl} \) is a small perturbation that has only spatial components \( h^{\alpha\beta} \) \( (\alpha, \beta = x, y, z) \), yields the following propagation equation for the electromagnetic wave

\[ \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + h^{\alpha\beta} \frac{\partial^2}{\partial x^\alpha \partial x^\beta} \right) A^i = 0. \] (9)

In this equation, \( h^{\alpha\beta} \) can be approximately treated as constants since they vary slowly as compared to the fast
variation of $A_t$. Looking for solution of Eq. (9) in the form $A_t \propto e^{-i\omega t + ik \tau}$, where $k$ is the wave vector of the electromagnetic wave, we obtain the following dispersion relation for light

$$\frac{\omega^2}{c^2} = k^2 - h^{\alpha\beta} k_{\alpha} k_{\beta},$$

and, hence, the phase velocity of the electromagnetic wave is (see also [6])

$$V_{ph} = \frac{\omega}{k} \approx c \left( 1 - \frac{1}{2} h^{\alpha\beta} \hat{k}_{\alpha} \hat{k}_{\beta} \right),$$

where $\hat{k} = k/k$.

Equation (10) shows that presence of the gravitational wave leads to the change of the phase velocity of light which depends on the direction of the light propagation $k$. If arms of the interferometer are oriented along unit vectors $\hat{a}$ and $\hat{b}$ then difference in the phase velocities of the laser light propagating along the two arms is

$$\Delta V_{ph} = \frac{c}{2} h^{\alpha\beta} \left( \hat{a}_{\alpha} \hat{a}_{\beta} - \hat{b}_{\alpha} \hat{b}_{\beta} \right).$$

Signal of the LIGO-like (Michelson) interferometer with arms of length $L$ oriented along the directions $\hat{a}$ and $\hat{b}$ is proportional to the relative phase shift $\Delta \varphi$ of electromagnetic waves traveling a roundtrip distance $2L$ along the two arms [3]

$$\Delta \varphi = \frac{2L}{c} \Delta V_{ph} = \frac{\omega L}{c} h^{\alpha\beta} \left( \hat{a}_{\alpha} \hat{a}_{\beta} - \hat{b}_{\alpha} \hat{b}_{\beta} \right),$$

where $\omega$ is the frequency of electromagnetic wave and $h^{\alpha\beta}$ is the spatial perturbation of the metric in the reference frame in which interferometer mirrors do not move (frame of Eqs. (2) and (3)).

For the gravitational wave propagating along the $x-$axis, Eq. (11) yields for the gravitational wave in general relativity

$$\Delta \varphi = \frac{\omega L}{c} \left[ h^{yy} \left( \hat{a}_y \hat{a}_y - \hat{b}_y \hat{b}_y + \hat{b}_z \hat{b}_z + \hat{a}_z \hat{a}_z \right) + 2h^{yz} \left( \hat{a}_y \hat{a}_z - \hat{b}_y \hat{b}_z \right) \right],$$

while for the transverse wave (2) in vector gravity we obtain

$$\Delta \varphi = \frac{2\omega L}{c} \left[ h^{xy} \left( \hat{a}_x \hat{a}_y - \hat{b}_x \hat{b}_y + \hat{b}_z \hat{b}_z \right) + h^{xz} \left( \hat{a}_x \hat{a}_z - \hat{b}_x \hat{b}_z \right) \right].$$

Equations (12) and (13) show that vector gravity and general relativity predict qualitatively different effect of the gravitational wave on the interferometer signal. Namely, general relativistic gravitational wave of any polarization (arbitrary $h^{yy}$ and $h^{yz}$) produces no signal when gravitational wave propagates parallel to the interferometer plane at $45^\circ$ angle relative to one of its perpendicular arms (see Fig. [11]). E.g., this is the case for $\hat{a} = (1, 1, 0) / \sqrt{2}$ and $\hat{b} = \pm (1, -1, 0) / \sqrt{2}$. For these orientations the gravitational wave in vector gravity will produce signal, namely,

$$\Delta \varphi = \frac{2\omega L}{c} h^{xy}.$$

On the other hand, gravitational wave in vector gravity (for arbitrary $h^{yy}$ and $h^{xz}$) yields no signal if gravitational wave propagates in the direction perpendicular to the interferometer plane, e.g. $\hat{a} = (0, 1, 0)$ and $\hat{b} = (0, 0, 1)$ (see Fig. [11]), or along one of the interferometer arms, e.g. $\hat{a} = (1, 0, 0)$, $\hat{b} = (0, 0, 1)$ or $\hat{a} = (0, 1, 0)$, $\hat{b} = (1, 0, 0)$. For these orientations the gravitational wave in general relativity can produce signal.

This difference can be used to test theories of gravity in future polarization experiments with several LIGO-like interferometers. The experiment can be conducted with three interferometers. Simultaneous detection of the gravitational wave by all three instruments allows us to determine the direction of the wave propagation by measuring the wave arrival times at the interferometer locations. By detecting many of such events one can collect statistics and find a distribution of the direction of the detected gravitational waves relative to the interferometer arms. Namely, one can measure the distribution function $N(\theta, \phi)$ defined as $dN = N(\theta, \phi) d\Omega$, where $dN$ is the number of events for which detected gravitational waves propagate inside the solid angle $d\Omega = \sin(\theta) d\theta d\phi$. Here $\theta$ and $\phi$ are the polar and azimuth angles in the spherical coordinate system in the frame of the interferometer arms (see Fig. 2).

Vector gravity predicts that such distribution will have dips in the directions perpendicular to the interferometer plane - $\theta = 0, \pi$ (Fig. [11]) and along the interferometer arms $\theta = \pi/2, \phi = 0, \pi/2, 3\pi/2$. The dips appear because for these propagation directions the interferometer can not detect the transverse gravitational wave. In the
case of general relativity the dips will appear in the directions for which wave propagates in the interferometer plane at 45° angle relative to one of the interferometer arms ($\theta = \pi/2$ and $\phi = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$). Thus, such experiment is able to distinguish between the two theories.

One should mention that vector gravity also predicts existence of longitudinal gravitational waves which, however, are not emitted by orbiting binary starts. For such a wave propagating along the $x$-axis the equivalent metric in the co-moving frame reads

$$g_{ik} = \eta_{ik} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -2h(t, x) & 0 & 0 \\ 0 & 0 & h(t, x) & 0 \\ 0 & 0 & 0 & h(t, x) \end{pmatrix}. \quad (15)$$

Longitudinal gravitational waves produce no interferometer signal if they propagate at equal angles relative to the interferometer arms (more exactly when $\hat{k} \cdot \hat{a} = \pm \hat{k} \cdot \hat{b}$). Thus, propagation direction perpendicular to the interferometer plane shown in Fig. 1 is the direction of zero response for all kind of gravitational waves in vector gravity. However, waves propagating along the arms can produce signal if they are longitudinal.

III. CONCLUSION

To conclude, even though in both theories the gravitational waves generated by orbiting stars are transverse, vector gravity and general relativity predict qualitatively different dependence of the gravitational wave signal on the orientation of the laser interferometer arms. Gravitational wave produces motion of the interferometer mirrors and changes phase velocity of light. Both of these effects contribute to the interferometer signal. For certain directions of the gravitational wave propagation the two contributions cancel each other yielding zero net phase shift for any wave polarization. These directions of the zero response are different in the two theories.

We find that general relativistic gravitational wave produces no signal when it propagates parallel to the interferometer plane at 45° angle relative to one of the arms (see Fig. 1b). On the other hand, transverse gravitational wave in vector gravity yields no signal when it propagates in the direction perpendicular to the interferometer plane (Fig. 1b) or along one of the interferometer arms. The former propagation direction yields no signal even if the wave is longitudinal - it is the direction of the zero response for all type of waves in vector gravity. This difference can be used to distinguish between general relativity and vector gravity in polarization experiments with gravitational wave interferometers.

Such experiment is crucial for our understanding of the nature of gravity and can test whether gravity has a tensor or a vector origin. Simultaneous detection of gravitational waves in at least three instruments is necessary for the experiment. A joint scientific run of the two LIGO interferometers in the US and the Virgo interferometer in Italy is capable of distinguishing between tensor and vector origin of gravity.

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