Impact of clipping noise on the sum rate of NOMA with PD-DCO-OFDM and conventional DCO-OFDM

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ABSTRACT
In visible light communication (VLC) systems, a non-orthogonal multiple access (NOMA) scheme is deemed promising technology to offer better spectral efficiency than the Orthogonal multiple access (OMA) scheme. The feasibility of power domain NOMA for VLC system has been studied with different variants of unipolar OFDM schemes. However, few research works have been presented on the impact of clipping noise on the achievable sum rate of NOMA with DC-biased optical OFDM (DCO-OFDM) and polarity divided DCO-OFDM (PD-DCO-OFDM). This paper presents the impact of clipping noise on the achievable sum rate of NOMA-DCO-OFDM and NOMA-PD-DCO-OFDM VLC systems. Moreover, NOMA-DCO-OFDM and NOMA-PD-DCO-OFDM systems are compared based on achievable sum rates for different total power constraints and signal clipping levels. For the two-user scenario, Simulation results have confirmed that NOMA-PD-DCO-OFDM can offer a better sum rate compared to NOMA-DCO-OFDM system.

1. Introduction
Visible light communication (VLC) is thought to be a promising complement for conventional RF wireless communication to tackle the challenge of the inevitable spectrum crunch in the conventional RF band [1, 2]. The recent huge deployments of LEDs for lighting purposes have created significant motivation towards simultaneous functionalities of LEDs for both lighting and communication purposes. However, the issue of low modulation bandwidth of LEDs is a well-known challenge towards the implementation of practical VLC. Therefore, spectrally efficient modulation and multiple access schemes are highly needed to maximize the capacity of LED-based VLC systems [3, 4]. To accommodate high-speed communication and massive connectivity in cases of internet of things (IoT) and overly growing mobile internet, VLC should adopt spectrally efficient multiple access and modulation schemes such as NOMA and DCO-OFDM, respectively. NOMA is mostly implemented with OFDM to utilize the inherent advantages of OFDM such as resistance to inter symbol interference (ISI), high spectral efficiency, and the possibility of flexible bandwidth allocation [5, 6, 7]. Different variants of OFDM schemes such as asymmetrically clipped optical OFDM (ACO-OFDM), DCO-OFDM, and PD-DCO-OFDM have been proposed for optical wireless communications [8, 9, 10, 11].

Different alternatives of NOMA (power domain NOMA and code domain NOMA) [2, 12] have been adopted recently to improve the spectral efficiency of DCO-OFDM based VLC system. NOMA improves the spectral efficiency of VLC by enabling the sharing of the same frequency resource for multiple users at the same time. In NOMA, users' data are superimposed with the aid of superposition coding at the transmitter. Successive interference cancelation (SCI) is also performed at the receiver to separate the user's information signal.

In those [13, 14] previous works, NOMA has been studied for VLC without consideration of double-sided clipping noise. In [15], the effect of clipping on the achievable rate of NOMA-DCO-OFDM system was presented in comparisons to the performance of orthogonal multiple access (OMA) scheme. In [11], polarity division DCO-OFDM (PD-DCO-OFDM) was introduced with code domain NOMA to reduce the effect of clipping noise. However, it lacks the analytical modeling of clipping noise when power domain NOMA is implemented with PD-DCO-OFDM.

The goal of this paper is to analyze the impact of clipping distortion on the achievable sum rate of power domain NOMA VLC systems implemented with DCO-OFDM and PD-DCO-OFDM schemes. Analytical frameworks are presented for clipping distortions of NOMA PD-DCO-OFDEM VLC system. Moreover, performances of NOMA DCO-OFDM...
and NOMA PD-DCO-OFDM VLC systems are compared based on achievable sum rates.

The other parts of this paper are organized in the following format. Section 2 covers the system model and clipping noise analysis of PD-DCO-OFDM NOMA. Section 3 reviews the system model and clipping noise analysis of DCO-OFDM NOMA. Simulation results are discussed in Section 4 and conclusions are drawn at last in Section 5.

2. System model of NOMA PD-DCO-OFDM

In this paper, downlink transmission NOMA for a total of two users is considered. Hence, a maximum of two users is sharing the same time-frequency resources to reduce the receiver complexity and error propagation due to imperfect successive interference cancellation (SCi). As shown in Figure 1, the information bits of both users are given as input to the serial and parallel converters and later mapped to QAM symbols and allocated to the available subcarriers.

Assuming K subcarriers are available in the system, the QAM symbol of the user at kth subcarrier is represented as $X_k$, $k = 0, 1, 2, ..., K - 1$, $j = 1, 2$. Hermitian symmetry is imposed to obtain real bipolar time-domain signal at the output of the IFFT modules. The QAM symbols which are mapped to subcarriers satisfy the conditions $(X_k = X_{K-k}$ and $X_0 = X_{K-2} = 0)$ [8]. According to the central limit theorem (for enough number of available subcarriers, $K \geq 64$), The time-domain signal $x_n$ at the output of the IFFT module follows a zero-mean Gaussian distribution with variance $\sigma^2 = E[|x_n|^2]$.

As illustrated in Figure 1, superposition of the bipolar signal of the two users is done in power domain. Let $P_T$ is the available electrical power for transmission of both users information signals, the superimposed time-domain signal $x_n$ can be written in the following form [15, 16]:

$$x_n = \sqrt{\beta_1} P_T x_{n1} + \sqrt{\beta_2} P_T x_{n2}$$

where $\beta_1$ and $\beta_2$ are power allocation factors for user-1 and user-2 respectively. Assuming normalized variances for both users ($\sigma^2_{x1} = \sigma^2_{x2} = 1$), $x_n$ becomes a zero-mean Gaussian random signal with variance $\sigma^2 = P_T$. Let $h_1$ and $h_2$ are the optical channel gains of user-1 and user-2 respectively. Assuming user-1 is the nearest user to the transmitter, the power allocation factors can be obtained from normalized gain power allocation (NGDPA) method as [16]:

$$\beta_1 = \frac{\beta_1 (h_1 - h_2)}{h_1^2}, \quad \beta_2 = \frac{h_1}{h_1 + h_1 - h_2}$$

For large number of users in the case of indoor multi-users VLC, user grouping strategies based on their channel gains along with different power allocation algorithms can be used to reduce both interference and receiver complexity [1, 17, 18].

In PD-DCO-OFDM NOMA system, positive unipolar $x_n^+$ and negative $x_n^-$ unipolar signals are constructed by the polarity divider as:

$$x_n^+ = \frac{1}{2} (x_n + |x_n|)$$

$$x_n^- = \frac{1}{2} (x_n - |x_n|)$$

A DC-offset $B_d$ is added only on $x_n^-$ to obtain a positive unipolar signal to fulfill the requirement of IM/DD system. Let $\lambda_i$ and $\lambda_b$, respectively, are the upper and lower dynamic ranges of the LED. Assuming $\lambda_i > B_d$, both $x_n^+$ and $x_n^-$ will experience a single-sided clipping, I.e. upper sided clipping on $x_n^+$ at the point $\lambda_i$ and lower sided clipping on $x_n^-$ at point $\lambda_b - B_d$.

A. Clipping distortion of PD-DCO-OFDM

The exact distribution of the time domain clipping noise for practical clipping bounds is yet to be investigated. However, different pieces of literature [19, 20, 21] have modeled the time domain clipping noise in OFDM based OWC as a Gaussian random variable with the aid of Bussgang theorem. The main assumption of this approach is that the time domain signal remains Gaussian after clipping is performed. On the contrary, the assumption of clipping noise as Gaussian distribution is claimed to be valid only for particular conditions [22]. In [22], Kurtosis function is used to measure the normality of the clipped signal distribution and the authors have claimed that the assumption of the time domain clipping noise as Gaussian distribution is accurate only if the range of truncation is wide enough to assure the better part of the signal untouched.

In this paper, the time domain clipping noise has been assumed to be Gaussian and the truncation ranges are kept large enough to satisfy the claims on [22]. After clipping has been performed on lower side of $x_n$ and upper side of $x_n^+$, the clipped signal $\tilde{x}_n^+$ and $\tilde{x}_n^-$ can be written by using Bussgang theorem as [8, 20]:

$$\tilde{x}_n^+ = \alpha_1 x_n + d_n^+$$

$$\tilde{x}_n^- = \alpha_2 x_n + d_n^-$$

Where $\alpha_1$ and $\alpha_2$ are attenuation coefficients and $d_n^+$ and $d_n^-$ are clipping noise components uncorrelated with $x_n$. Since the input to both processes represented in Eqs. (5) and (6) is the zero-mean Gaussian process $x_n$, the clipping operator $C^+$ and $C^-$ for $\tilde{x}_n^+$ and $\tilde{x}_n^-$ respectively, can be given as [23]:

![Figure 1. NOMA PD-DCO-OFDM transmitter block diagram.](image-url)
Cov\(E(x)\) \begin{align*}
C^+(x) &= \begin{cases}
\lambda_x, & x > \lambda_x \\
\lambda_x - B_k, & 0 \leq x \leq \lambda_x \\
0, & x < 0
\end{cases} \\
C^-(x) &= \begin{cases}
0, & x > \lambda_k - B_k \\
\lambda_k - B_k - x, & \lambda_k - B_k \leq x \leq 0 \\
\lambda_k - B_k, & x < \lambda_k - B_k
\end{cases}
\end{align*}
(7)

Where \(\alpha\) is the overall attenuation coefficient and \(d_k\) is the total clipping noise component which is uncorrelated with \(x_n\). The clipping operator \(C(x)\) of \(x_n\) can be obtained from Eqs. (7) and (8) by combining \(C^+(x)\) and \(C^-(x)\): \(C(x)\) as:
\begin{equation}
C(x) = \begin{cases}
\lambda_x, & x > \lambda_x \\
\lambda_x - B_k, & 0 \leq x \leq \lambda_x \\
\lambda_k - B_k, & \lambda_k - B_k \leq x \leq 0 \\
0, & x < \lambda_k - B_k
\end{cases}
(10)
\end{equation}

As presented in [20, 23], the attenuation factor \(\alpha\) can be expressed as:
\begin{equation}
\alpha = \frac{\text{Cov} [x_n, \tilde{x}_n]}{\sigma^2} = \frac{E[x_n \tilde{x}_n]}{\sigma^2} = \frac{1}{\sigma^2} \int_{-\infty}^{\infty} xC(x)\exp\left(-\frac{x^2}{2\sigma^2}\right)dx
\end{equation}
(11)

Where \(\alpha = \lambda_x/\sigma_x, b = (\lambda_k - B_k)/\sigma_x, \text{Cov}[\cdot]\) is covariance operator, and \(\text{erf}(\cdot)\) denotes error function.

The variance of the clipping noise can be calculated as:
\begin{equation}
\sigma^2_{\text{clip}} = \text{Var}[d_k] = E[(d_k - \mu_k)^2] = E[d_k^2] - E[d_k]^2
\end{equation}
(12)

Where \(\mu_k = E[d_k]\) is the mean of the clipping noise.

The electrical power of the clipped signal is also calculated from Eq. (12) as:
\begin{equation}
E[\tilde{x}_n^2] = \alpha^2 E[x_n^2] + E[d_k^2]
\end{equation}
(13)

From Eqs. (12) and (13), the variance of the clipping noise is given by:
\begin{equation}
\sigma^2_{\text{clip}} = E[\tilde{x}_n^2] - \alpha^2 E[x_n^2] - E[d_k]^2
\end{equation}
(14)

Where \(E[\tilde{x}_n^2]\) is the 2\(^{nd}\) moment of the truncated Gaussian random signal \(x_n\) with lower truncation point at \(\lambda_k - B_k\) and upper truncation point at \(\lambda_x\). Therefore, the \(E[\tilde{x}_n^2]\) term in Eq. (14) is obtained as [24, 25]:
\begin{equation}
E[\tilde{x}_n^2] = \sigma^2 - \sigma^2_{\text{clip}}\left(\frac{\phi(u) - b\phi(b)}{\Phi(u) - \Phi(b)}\right) = \sigma^2_{\text{clip}}\left[\frac{u^2 + b^2 - (u^2 - 1) \text{erf} \left(\frac{u}{\sqrt{2}}\right)}{\sqrt{2}}\right] + (b^2 - 1) \text{erf} \left(\frac{b}{\sqrt{2}}\right) + 2b\phi(b) - 2\phi(u)
\end{equation}
(15)

Where \(\phi(\cdot)\) and \(\Phi(\cdot)\) are probability distribution function (PDF) and communicative distribution function (CDF) of standard normal distribution respectively. The term \(E[d_k]^2\) in Eq. (14) can also be calculated as the square of the mean of the truncated Gaussian random signal \(\tilde{x}_n\):
\begin{equation}
E[d_k]^2 = E[\tilde{x}_n^2] = \left[-\sigma^2_{\text{clip}}\left(\frac{\phi(u) - b\phi(b)}{\Phi(u) - \Phi(b)}\right)\right]^2
\end{equation}
(16)

\section{3. The system model of NOMA DCO-OFDM}

The system model of downlink NOMA DCO-OFDM is shown in Figure 2 for the two-user scenario. After the information bits of both users mapped to QAM symbols, each QAM symbol is mapped to subcarriers. Let \(X_k\) be the QAM symbol of \(k\)th user at \(k\)th subcarrier, Hermitian symmetry is imposed on half of the subcarriers to obtain real bipolar signal at the output of the IFFT module, i.e. \(X_k = X_{k,k}\) and \(X_0 = X_{0,2} = 0\) for \(K\) available subcarriers in the system. Power domain superposition coding is performed on the bipolar time-domain signals \(x_{1,n}\) and \(x_{2,n}\) of user-1 and user-2 respectively. For total available power \(P_T\) for transmission and power allocation factors \(P_1\) and \(P_2\) for user-1 and user-2 respectively, the superimposed signal \(x_n\) can be written as:
\begin{equation}
x_n = \sqrt{P_1}x_{1,n} + \sqrt{P_2}x_{2,n}
\end{equation}
(22)

Since \(x_n\) is a bipolar signal, a DC-offset \(B_0\) should be added on \(x_n\) to obtain a positive unipolar signal. Due to the limited dynamic range of LED, the bipolar signal \(x_n\) will experience a double-sided clipping at \(\lambda_k - B_k\) from the top and at \(\lambda_k - B_k\) from the bottom.

\subsection{A. Clipping noise and achievable rate of NOMA DCO-OFDM}

According to Bussgang theorem, the clipped signal \(\tilde{x}_n\) can be modeled as:
\begin{equation}
\tilde{x}_n = \alpha x_n + d_n
\end{equation}
(23)

Where \(\alpha\) is the attenuation coefficient and \(d_n\) is the clipping noise component which is uncorrelated with \(x_n\). The clipping distortion operator \(C(x)\) can also be obtained based on Bussgang theorem as [15]:
\begin{equation}
C(x) = \begin{cases}
\lambda_x - B_k, & x > \lambda_x - B_k \\
\lambda_x - B_k, & \lambda_x - B_k \leq x \leq \lambda_x - B_k \\
\lambda_k - B_k, & \lambda_k - B_k \leq x \leq 0 \\
\lambda_k - B_k, & x < \lambda_k - B_k
\end{cases}
\end{equation}
(24)

The variance of the clipping noise can be obtained as:
\begin{equation}
\sigma^2_{\text{clip}} = E[\tilde{x}_n^2] - \alpha^2 E[x_n^2] - E[d_n]^2
\end{equation}
(25)
Figure 2. NOMA DCO-OFDM transmitter block diagram.

The performances of PD-DCO-OFDM NOMA and DCO-OFDM NOMA systems are presented in Figure 3 for $PT = 26, 30, 36 W, \lambda_b = 0$, and $\lambda_r = 28$. The simulation results show that PD-DCO-OFDM NOMA offers superior performance compared to DCO-OFDM NOMA. It is noted that a DC-bias is not added on the positive samples of PD-DCO-OFDM NOMA. As a result, the upper side clipping noise is reduced and better performance is achieved with PD-DCO-OFDM. For lower DC bias (3–5 dB), the increment in SINR is insignificant when $PT$ is increased since the magnitude of the clipping noise increases for larger $PT$. As a consequence, the performance improvements of NOMA DCO-OFDM is insignificant when $PT$ is increased in the lower DC-bias region. When $PT$ is increased in the DC-bias region beyond 5dB for NOMA DCO-OFDM, the positive and negative samples of the time domain signal will have large peaks and more clipping will be experienced especially on positive valued samples. As a result, the increment in $PT$ is dominated by the increment in the clipping noise which in turn reduces the SNIR, and consequently, the performance of NOMA DCO-OFDM shows a reverse relation with $PT$ for DC-biases beyond 5 dB. Contrarily, the performance of PD-DCO-OFDM NOMA is improved when $PT$ is increased for DC-biases beyond 5 dB. This is because $\lambda_r$ is large enough to avoid upper sided clipping since DC bias is avoided on samples having a positive polarity. Therefore, when the DC bias is increased, the lower side clipping will be negligible and larger SNIR will be achieved for larger $PT$.

From Figure 3, it is also shown that NOMA DCO-OFDM has reached its peak performances at optimum DC bias of 6 dB, 7 dB, and 8 dB for $PT$.
values of 26 W, 30 W, and 36 W respectively. When the DC bias is increased beyond those optimum values, the performance has declined due to the larger upper side clipping noise.

As illustrated in Figures 4 and 5, for lower DC bias (5–7 dB) and large λ values (λ = 32 & 35), NOMA DCO-OFDM with larger PT has shown a slightly better sum rate compared to the NOMA DCO-OFDM system with lower PT. On the contrary, when the DC bias is large (beyond 7 dB), the larger the PT, the lower the sum rate of NOMA DCO-OFDM system since the upper side clipping noise increases because of the positive valued samples having large peaks.

Besides, From Figures 3, 4, and 5, it is shown that for fixed DC bias, for example at 9 dB, the sum rate performance gap between PD-DCO-OFDM NOMA and DCO-OFDM NOMA at the same PT values has been reduced as λ increases. This is because both PD-DCO-OFDM and DCO-OFDM experience equal lower side clipping noise for the same transmitted power PT and the performance gap is mostly due to the unequal upper side clipping noise in the two schemes. However, the upper side clipping noise of NOMA DCO-OFDM can also be minimized when λ is large enough. When λ is large, the simulation results show that the gap of the performance between the PD-DCO-OFDM NOMA and DCO-OFDM is reduced for the fixed PT and DC bias.

5. Conclusions

In this paper, the analytical framework of double-sided asymmetrical clipping noise has been presented for PD DCO-OFDM NOMA with a two-user scenario. Closed-form formulas for both attenuation factor and clipping noise variance have been given for the case of double-sided clipping in PD-DCO-OFDM NOMA. The impact of clipping noise on the achievable sum rate of NOMA PD-DCO-OFDM has also been presented with the aid of simulation results. Moreover, performance comparisons of DCO-OFDM NOMA and PD-DCO-OFDM NOMA have been presented based on the achievable sum rate for the case of equal transmitted power and DC bias. The obtained simulation result confirmed that PD-DCO-OFDM NOMA has superior performance compared to DCO-OFDM NOMA.

Declarations

Author contribution statement

Zelalem Hailu Gebeyehu: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Additional information

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