Field theory model giving rise to "quintessential inflation" without the cosmological constant and other fine tuning problems

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A field theory is developed based on the idea that the effective action of yet unknown fundamental theory, at energy scale below the Planck mass $M_p$, has the form of expansion in two measures:

$$ S = \int d^4x [\Phi L_1 + \sqrt{-g} L_2] $$

where the new measure $\Phi$ is defined using the antisymmetric tensor field

$$ \Phi d^4x = \partial_\alpha A_{\beta\gamma\delta} dx^\alpha \wedge dx^\beta \wedge dx^\gamma \wedge dx^\delta. $$

A shift $L_1 \rightarrow L_1 + const$ does not affect the equations of motion whereas a similar shift when implementing with $L_2$ causes a change which in the standard GR would be equivalent to that of the cosmological constant (CC) term. The next basic conjecture is that the Lagrangian densities $L_1$ and $L_2$ do not depend of $A_{\mu\nu\lambda}$. The new measure degrees of freedom result in the scalar field $\chi = \Phi/\sqrt{-g}$ alone. The constraint appears that determines $\chi$ in terms of matter fields. After the conformal transformation to the new variables (Einstein frame), all equations of motion take the canonical GR form of the equations for gravity and matter fields and, therefore the models we study are free of the well-known "defects" that distinguish the Brans-Dicke type theories from Einstein’s GR. All novelty is revealed only in an unusual structure of the effective potentials and interactions which turns over our intuitive ideas based on our experience in field theory. For example, the greater $\Lambda$ we admit in $L_2$, the smaller magnitude of the effective inflaton potential $U(\phi)$ will be in the Einstein picture. Field theory models are suggested with explicitly broken global continuous symmetry, which in the Einstein frame has the form $\phi \rightarrow \phi + const$. The symmetry restoration occurs as $\phi \rightarrow \infty$. A few models are presented where the effective potential $U(\phi)$ is produced with the following shape: for $\phi \lesssim -M_p$, $U(\phi)$ has the form typical for inflation model, e. g. $U = \lambda \phi^4$ with $\lambda \sim 10^{-14}$; for $\phi \gtrsim -M_p$, $U(\phi)$ has mainly the exponential form $U \sim e^{-a\phi/M_p}$ with variable $a$: $a = 14$ for $-M_p \lesssim \phi \lesssim M_p$, that gives the possibility for nucleosynthesis and large-scale structure formation; $a = 2$ for $\phi \gtrsim M_p$, that implies the quintessence era. There is no need of any fine tuning to prevent appearance of the CC term or any other terms that could violate the flatness of $U(\phi)$ at $\phi \gg M_p$. $\lambda \sim 10^{-14}$ is obtained without fine tuning as well. Quantized matter fields models, including spontaneously broken gauge theories, can be incorporated without altering the mentioned above results. Direct coupling of fermions to the inflaton resembles the Wetterich’s model but there is a possibility to avoid any observable effect at the late universe. SSB does not raise any problems with the CC in the late universe.

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1. INTRODUCTION

Recent high-redshift and CMB data suggest that a small effective cosmological constant gives a dominant contribution to the energy density of the present universe. Among the attempts to describe this picture, the idea to profit by the properties of a slow-rolling scalar field (quintessence model) seems to be the most attractive and successful. In such approach, the present vacuum energy density \( \rho_{\text{vac}} \sim 10^{-120} (\text{GeV})^4 \) has to be imitated by the energy density of a slowly rolling scalar field down its potential \( U(\phi) \) which presumably approaches zero as \( \phi \to \infty \). However all known quintessence models contain two fundamental problems:

1. The cosmological constant problem remains in the quintessence models as well: particle physics and cosmology must give a distinct mechanism that enforces the effective cosmological constant to decay from an extremely large value in the very early universe to the extremely small present value without fine tuning of parameters and initial conditions.

2. All known quintessence models are based on the choice of some specific form for the potential \( U(\phi) \). The general feature of the potentials needed to realize quintessence is that \( U(\phi) \) must be flat enough as \( \phi \) is large enough in order to provide conditions for the slow-roll approximation. However it is not clear what happens with other possible terms unless an extreme fine tuning is assumed for the mass and self-couplings. For example, the restriction of the flatness feature of the potentials needed to realize quintessence is that

\[
U(\phi) \text{ must be flat enough as } \phi \text{ is large enough.}
\]

In this paper I am going to present a field theory model that resolves the above fine tuning problems and besides that, this model is able to give a broad range of tools for constructive answers to more important questions:

3. In the framework of a model where potential \( U(\phi) \) of the exponential or inverse power low (or there combinations) form plays the role of a quintessential potential as \( \phi \) is large enough, the question arises what is the cosmological role of such \( U(\phi) \) as \( \phi \) is close to zero or negative. If some other scalar field is responsible for an inflation of the early universe, then a field theory has to explain why the potential \( U(\phi) \) of the scalar field \( \phi \) is negligible as \( \phi \) is close to zero or negative. However, if the same quintessence field \( \phi \) plays also the role of the inflaton (in the early universe) then again a field theory has to explain an origin of the relevant effective potential. Of course this is a nontrivial problem. For example, Peebles and Vilenkin have presented an interesting model of a single scalar field that drives the inflation of the early universe and ends up as quintessence. They adopt the monotonic potential

\[
U(\phi) = \lambda m^4 \left[ 1 + (\phi/m)^4 \right] \quad \text{for } \phi < 0,
\]

\[
= \frac{\lambda m^4}{1 + (\phi/m)^{\alpha}} \quad \text{for } \phi \geq 0
\]

where \( \alpha = \text{const} > 0 \) (for example 4 or 6) and the parameters \( \lambda = 10^{-14} \) and \( m = 8 \times 10^5 \text{GeV} \) were adjusted in to achieve a satisfactory agreement with the main observational constraints. It is well known that such extremely small value of \( \lambda \) is dictated in the \( \lambda \phi^4 \) theory of the chaotic inflation scenario by the necessity to obtain a density perturbation \( \delta \rho / \rho \sim 10^{-5} \) in the observable part of the universe. In other words, the potential of this "quintessential inflation" model includes both the fine tuning required by the inflation of the early universe and the fine tuning dictated by the quintessence model of the late universe. As it is pointed out in Ref., it seems also to be an unnatural feature of this model that a small mass \( m = 8 \times 10^5 \text{GeV} \ll M_p \) must appear in the potential of the inflaton field \( \phi \) interacting only with gravity. And finally, one should apparently believe that such sort of "quintessential inflation" potential must be generated by some field theory without fine tuning. These problems are typical for the quintessential inflation type models.

4. It is well known that the "coincidence problem" can be avoided in the framework of the quintessence models that make use of "tracker potentials". The exponential potential with \( a = \text{const} \)

\[
U(\phi) = U_0 e^{-a\phi/M_p}
\]

is a special example of a tracker solution. In spatially flat models with such potential, the ratio of the scalar field \( \phi \) energy density to the total matter energy density rapidly approaches a constant value determined by \( a \) and the matter equation of state (see also Ref. where a similar result was achieved in the context of Kaluza-Klein-Casimir cosmology). However the strong constraint on \( \Omega_\phi \) dictated by cosmological nucleosynthesis \( \Omega_\phi \lesssim 0.2 \) predetermines for the \( \phi \)-fraction to remain subdominant one in the future that apparently contradicts the observable accelerated expansion. A possible resolution of this problem proposed by Wetterich consists in the idea that \( a \) in might be \( \phi \)-dependent. In that case it would be again very attractive to develop a field theory model where the exponential potential with an appropriate \( \phi \)-dependent \( a \) is generated in a natural way.
5. Since the mass of excitations of the $\phi$-field has to be extremely small in the present-day universe ($m_\phi \lesssim H_0 \sim 10^{-33} \text{eV}$), possible direct couplings of $\phi$ to the standard matter fields should give rise to very long-range forces which do not obey the equivalence principle [19]. To prevent such undesirable effects, the very strong upper limits on the coupling constants of the quintessence field to the standard matter fields have to be accepted without any known reason: an attempt to construct a model where an unbroken symmetry could support zero mass of $\phi$-excitations [20] inevitably runs against the necessity to start from a trivial potential [19]; without knowledge of a mechanism for the breaking of this symmetry, such small coupling constants may be introduced into a theory only by hand.

It will be shown in this paper that one can answer all the above questions 1-5 in the framework of the field theory model based on the hypothesis that the effective action of the fundamental theory at the energy scales below the Planck mass can be represented in a general form including \textit{two measures and respectively, two Lagrangian densities}

\[ S = \int \left[ \Phi L_1 + \sqrt{-g} L_2 \right] d^4x \]

Here $\sqrt{-g}$ is the standard measure of integration in the action principle of Einstein’s General Relativity (GR) and other gravitational theories making use the general coordinate invariance. The measure $\Phi$ is defined using the antisymmetric tensor field

\[ \Phi d^4x = \partial_{[\alpha} A_{\beta\gamma\delta]} dx^\alpha \wedge dx^\beta \wedge dx^\gamma \wedge dx^\delta \]

and (3) is also invariant under general coordinate transformations. Notice that the measure $\Phi$ is a total derivative and therefore a shift $L_1 \rightarrow L_1 + \text{const}$ does not affect equations of motion whereas a similar shift when implementing with $L_2$ causes a change which in the standard GR would be equivalent to that of the cosmological constant term.

The next basic conjecture is that the Lagrangian densities $L_1$ and $L_2$ do not depend of $A_{\mu\nu\lambda}$. In this paper I refer to this theory as the “two measures theory” (TMT).

The main features of TMT have been studied in series of papers [21]- [26].

\section*{II. SOME GENERAL FEATURES OF TMT}

Let us consider a simple model with the scalar field $\phi$

\[ S = \int d^4x \left[ \Phi \left( -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V_1(\phi) \right) + \sqrt{-g} V_2(\phi) \right] \]

The case where $V_2(\phi) \equiv \text{const}$ was studied in Ref. [24] and the general case was studied by Guendelman in Ref. [25]. TMT gives desirable results if we proceed in the first order formalism (metric $g_{\mu\nu}$ and connection $\Gamma^\lambda_{\mu\nu}$ are independent variables as well as the antisymmetric tensor field $A_{\mu\nu\lambda}$) and $R(\Gamma, g) = g^{\mu\nu} R_{\mu\nu} (\Gamma)$, $R_{\mu\nu} (\Gamma) = R^\lambda_{\mu\nu\lambda} (\Gamma)$ and

\[ R^\lambda_{\mu\nu\sigma} (\Gamma) \equiv \Gamma^\lambda_{\mu\nu,\sigma} + \Gamma^\lambda_{\alpha\sigma} \Gamma^\alpha_{\mu\nu} - (\nu \leftrightarrow \sigma). \]

At this stage no specific form for $V_1(\phi)$ and $V_2(\phi)$ is assumed.

Variation of the action with respect to $A_{\mu\nu\lambda}$ results in equation $\epsilon^{\mu\nu\lambda\sigma} \partial_\lambda L_1 = 0$ which means that

\[ L_1 = -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V_1(\phi) = s M^4 = \text{const}, \]

where $s M^4$ is an integration constant, $s = \pm 1$ and $M$ is a constant of the dimension of mass. Variation with respect to $g^{\mu\nu}$ leads to

\[ -\frac{1}{\kappa} R_{\mu\nu} (\Gamma) + \frac{1}{2} \phi_{,\mu} \phi_{,\nu} - \frac{1}{2\chi} V_2(\phi) g_{\mu\nu} = 0 \]

where the scalar field $\chi$ is defined by

\[ \chi \equiv \frac{\Phi}{\sqrt{-g}} \]

Consistency condition of equations (7) and (8) takes the form of the constraint
\[ V_1(\phi) + sM^4 - \frac{2V_2(\phi)}{\chi} = 0 \]  

(10)

Solution of equations obtained by variation of the action with respect to \( \Gamma^{\lambda}_{\mu\nu} \) can be represented (see [23 - 24]) as a sum of the Christoffel’s connection coefficients \( \{ \lambda_{\mu\nu} \} \) of the metric \( g_{\mu\nu} \), and a non-Riemannian part which is a linear combination of \( \sigma_{\mu} \) where \( \sigma \equiv \ln \chi \).

The scalar field \( \phi \) equation is

\[ (-g)^{-1/2} \partial_{\mu}(\sqrt{-g}g^{\mu\nu}\partial_{\nu}\phi) + \sigma_{\mu} \phi^{\mu} + \frac{dV_1}{\chi \phi} - \frac{1}{\chi} \frac{dV_2}{d\phi} = 0, \]  

(11)

In the conformal frame defined by the conformal transformation

\[ g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x) = \chi g_{\mu\nu}(x); \quad \phi \rightarrow \phi; \quad A_{\mu\nu\lambda} \rightarrow A_{\mu\nu\lambda} \]  

(12)

the non-Riemannian contribution into the connection disappears: \( \Gamma^{\lambda}_{\mu\nu} \rightarrow \Gamma^{\lambda}_{\mu\nu} = \{ \lambda_{\mu\nu} \} \) (here \( \{ \lambda_{\mu\nu} \} \) are the Christoffel’s connection coefficients of the Riemannian space-time with the metric \( g'_{\mu\nu} \)). Tensors \( R^\lambda_{\mu\nu\sigma}(\Gamma) \) and \( R_\mu(\Gamma) \) transform to the Riemann \( R^\lambda_{\mu\nu\sigma}(g'_{\alpha\beta}) \) and Ricci \( R_{\mu\nu}(g'_{\alpha\beta}) \) tensors respectively in the Riemannian space-time with the metric \( g'_{\mu\nu} \). After making use the solution for \( \chi \) as it follows from the constraint \( [14] \), the gravitational equations \( [8] \) and the scalar field equation \( [13] \) in the new conformal frame obtain the standard form of the Einstein’s GR equations for the selfconsistent system of gravity \( (g'_{\mu\nu}) \) and scalar field \( \phi \) with the TMT effective potential (for details see \([24]\) and \([20]\))

\[ U(\phi) = \frac{1}{\chi^2} V_2(\phi) = \frac{1}{4V_2(\phi)}[sM^4 + V_1(\phi)]^2 \]  

(13)

Notice that just \( U(\phi) \) plays the role of the true potential that governs the dynamics of the scalar field \( \phi \) while \( V_1(\phi) \) and \( V_2(\phi) \) have no sense of the potential energy densities themselves but rather they generate the potential energy density. This is why we will use the term pre-potentials for \( V_1(\phi) \) and \( V_2(\phi) \). Notice that our choice of the sign in front of the pre-potential \( V_2(\phi) \) is opposite to the usual one that would be in the case of the standard GR. This is doing just for convenience in what follows.

In order to provide disappearance of the cosmological constant, one demands usually that the effective potential is equal to zero at the minimum, i.e. it is necessary that the effective potential and its first derivative are equal to zero at the same point. As a matter of fact this is the essence of the cosmological constant problem treated in the "old" sense when there was no need in explanation of a small but non-zero cosmological constant at present. If we want to avoid the necessity to fulfill this fine tuning, TMT gives us such an opportunity and it has been explored in Refs. \([24]\), \([20]\). In fact, independently of the shape of the nontrivial pre-potential \( V_1(\phi) \), infinite number of initial conditions exists for which \( V_1 + sM^4 \neq 0 \) at some value \( \phi = \phi_0 \). If \( V_1(\phi) \) and \( V_2(\phi) \) are regular at \( \phi = \phi_0 \), \( V_1'(\phi_0) \neq 0 \) and \( V_2(\phi) \) is positive definite then \( \phi = \phi_0 \) is the absolute minimum of \( U(\phi) \) with the value \( U(\phi_0) = 0 \). We will refer to such a situation as the first class scenario.

In the present paper we will study the models with such a pre-potential \( V_1 \) that there will be infinite number of initial conditions for which \( V_1 + sM^4 \neq 0 \) at any value of \( \phi \) (we will refer to such a situation as the second class scenario). Then the stable vacuum may for instance be realized asymptotically as \( \phi \rightarrow \infty \), which is actually the idea used in the quintessence models.

The assumption that \( V_2(\phi) \) is positive definite will be our choice in what follows.

**III. EXTREMELY BROAD CLASS OF TMT MODELS DOES NOT REQUIRE FINE TUNING TO PROVIDE Quintessence.**

A. General idea: the inverse power low quintessential potential as a simple example.

In contrast to standard gravitational theories where the quintessential potential must be slow decreasing function as \( \phi \rightarrow \infty \), in TMT we have an absolutely new option: the quintessential behaviour of the TMT effective potential \( U(\phi) \) for \( \phi \) large enough may be achieved with increasing prepotentials \( V_1(\phi) \) and \( V_2(\phi) \). This circumstance enables to avoid both the cosmological constant problem and the problem of the flatness of the quintessential potential.
For illustration of these statements let us notice that starting from the positive power low pre-potentials $V_1$ and $V_2$

\[ V_1 = m_1^{(4-n_1)} \phi^{n_1}; \quad V_2 = \frac{1}{4} m_2^{(4-2n_2)} \phi^{2n_2}. \]  

(14)

with $n_2 > n_1$, we obtain the TMT effective potential which for $\phi$ large enough has the inverse power low form

\[ U \approx \frac{m_1^{2(4-n_1)}}{m_2^{2(2-n_2)}} \frac{1}{\phi^{2(n_2-n_1)}} \]  

(15)

and does not depend on the integration constant. Another interesting case is $V_1 \equiv 0$ (remind that adding a constant to $V_1$ is equivalent just to a redefinition of the integration constant $sM^4$) and, for example, $V_2 \equiv \lambda \phi^4$. Then $U(\phi) = M^8/\lambda \phi^4$.

Although there exists a possibility for generation of a negative power low potential in the models with dynamical supersymmetry breaking (see, for example [29]), such potentials still looks to be exotic one in the context of the standard field theory. As we see, in TMT such quintessential form of the effective potential is obtained very easy and in a natural way.

Besides, adding any subleading (as $\phi \to \infty$) terms to (14) does not alter the above results since their relative contributions to $U(\phi)$ will be suppressed as $\phi$ is large enough. In particular, adding the term $\int \sqrt{-g} d^4x$, $V^{(0)}_2 \equiv \text{const}$, which in GR would have the sense of the cosmological constant term, does not affect $U(\phi)$ as $\phi$ is large enough.

Thus starting from the polynomial form of the pre-potentials $V_1$ and $V_2$ with an appropriate choice of the powers $n_1$ and $n_2$ of the leading terms, one can in fact provide a generation of the inverse power low quintessential potential in such a way that neither the cosmological constant problem nor the the problem of the flatness of the quintessential potential do not appear at all.

**B. The exponential form of the TMT effective potential $U(\phi)$.**

A simple way to realize an exponential asymptotic form of the TMT effective potential $U(\phi)$, Eq.(13), is to define the pre-potentials $V_1$ and $V_2$ as follows:

\[ V_1 = s_1 m_1^4 e^{\alpha \phi/M_p}; \quad V_2 = \frac{1}{4} m_2^4 e^{2\beta \phi/M_p}. \]  

(16)

Here $s_1 = \pm 1$ and we assume that $\alpha$ and $\beta$ are positive constants. The restrictions formulated after Eq. (10) have to be taken into account. The effective TMT potential corresponding to the pre-potentials (16)

\[ U = \frac{1}{m_2^4} \left( s_1 m_1^4 e^{-\beta (\alpha - \beta) \phi/M_p} + s M^4 e^{-\beta \phi/M_p} \right)^2 \]  

(17)

contains two particular cases of special interest.

a. The case $\alpha = \beta$  

This case corresponds to a sort of the scale invariant theory studied by Guendelman [25]. In fact, in this case the theory (13) is invariant under global transformations

\[ g_{\mu \nu} \to e^\theta g_{\mu \nu}; \quad A_{\mu \nu \lambda} \to e^\theta A_{\mu \nu \lambda} \]  

(18)

whereas the scalar field $\phi$ undergoes the shift

\[ \phi \to \phi - \frac{M_p}{\beta} \theta. \]  

(19)

In such a model, the TMT effective potential has the form

\[ U(\phi) = \frac{m_1^8}{m_2^8} \left[ 1 + \frac{s_1}{s} \left( \frac{M}{m_1} \right)^4 e^{-\beta \phi/M_p} \right]^2 \]  

(20)

and the observation that $U(\phi)$ has an infinite flat region as $\phi \to \infty$ and approaches a nonzero constant $\frac{m_1^4}{m_2^4}$, has been used in Ref. [25] for discussion of possible cosmological applications with the choice $\frac{s_1}{s} = -1$. The first possibility
is related to the very early universe: a slow rolling (new inflationary) scenario might be realized assuming that the universe starts at a sufficiently large value of \( \phi \). Another scenario discussed by Guendelman in Ref. [25] is based on a possibility for \( \frac{m^4}{m^2} \) to be very small and this approach has the aim to construct a scenario for the very late universe. In this scenario there could be a long lived stage with almost constant energy density \( \frac{\phi^2}{M^2} \) that will eventually disappear when the universe achieves its true vacuum state with zero cosmological constant. This occurs when the expression in parenthesis in Eq. (20) becomes zero and therefore no fine tuning is needed. It turns out (see Refs. [25], [26]) that in the presence of a matter, which is introduced in a way respecting the global symmetry (18), (19), the change of the constraint (10) leads to a correlation between \( U(\phi) \) (close but not equal to zero) and the matter energy density.

In the case \( \beta = \alpha \), the TMT effective potential (24) is not a constant due to the appearance of a non-zero integration constant \( M \), that is, actually, due to a spontaneous breaking of the global continuous symmetry (18), (19). Guendelman noticed (24) that in terms of the dynamical variables used in the Einstein frame, that is \( g'_{\mu \nu} \) and \( \phi \), the symmetry transformations (18), (19) are reduced to shifts (19) alone (\( g'_{\mu \nu} \) is invariant under transformations (18), (19)). Thus in terms of the dynamical variables of the Einstein frame, the spontaneous symmetry breaking is just that of the global continuous symmetry \( \phi \rightarrow \phi - \frac{\alpha}{\beta} \theta \). It is important that, as it was mentioned in Ref. [25], this global continuous symmetry is restored as \( \phi \rightarrow \infty \).

b. The case \( \beta > \alpha > 0 \). This is the most interesting case from the viewpoint of the quintessence. For \( \beta \phi \gg M_p \), the TMT effective potential (17) behaves as a decaying exponent:

\[
U \simeq \frac{m^4}{m^2} e^{-2(\beta-\alpha)\phi/M_p} \quad \text{as} \quad \beta \phi \gg M_p.
\] (21)

If we want to achieve the quintessential form of the TMT effective potential (21) for not too large values of \( \phi \) and with not too big difference in orders of \( m_1 \) and \( M \) (this point will be explained in the next section) then we need the condition

\[
0 < \beta - \alpha \ll \beta
\] (22)

And of course the most evident argument in favour of this condition consists in the demand to provide the flatness of the \( \phi \)-potential at the late, \( \phi \)-dominated universe where it has to imitate the present "cosmological constant". This is possible only if \( \beta - \alpha \) is less or of order 1 while there are no reasons for \( \beta \) not to be large in general.

Comparing this condition for \( \alpha \) and \( \beta \) with that of the model of Ref. [25] discussed just above, one can observe that the model under consideration can be interpreted as that with a small explicit violation of the global symmetry (18), (19). Notice that the expression for \( U(\phi) \) as \( \beta \phi \gg M_p \) does not include the integration constant \( M \) and the exponent is proportional to \( \beta - \alpha \). This reflects the fact that such asymptotic behavior of \( U(\phi) \) results from the explicit violation of the global continuous symmetry (18), (19).

It is very interesting that although the discussed global continuous symmetry (18), (19) is broken in this model explicitly, the equations of motion show that the symmetry is also restored as \( \phi \rightarrow \infty \), just as in the case \( \alpha = \beta \) with only spontaneous symmetry breaking. Therefore, in terms of the dynamical variables used in the Einstein frame, that is \( g'_{\mu \nu} \) and \( \phi \), in the model where the condition (22) holds, the approximate global symmetry \( \phi \rightarrow \phi - \frac{\alpha}{\beta} \theta \) is restored as \( \phi \rightarrow \infty \).

This observation opens unexpected chance to solve the problem discussed by Carroll [19] (problem 5 in the list of problems in Introduction) which consists of the following. There are no reasons to ignore a possibility that the scalar field \( \phi \) interacts directly to usual matter fields. Suppose that such interactions have the form of the coupling \( \int \frac{m}{p} \phi L \) where \( L \) is any gauge invariant dimension-four operator, \( m \) is a mass scale and \( f_i \) is a dimensionless coupling constant. The flatness of the quintessential potential of the field \( \phi \) means that excitations of \( \phi \) are almost massless. Therefore in the presence of direct interactions of the \( \phi \)-field to the usual matter fields, one has to expect the appearance of the very long-range forces which do not obey the equivalence principle. Observational restrictions on such "fifth force" impose small upper limits on the coupling constants \( f_i \).

To explain smallness of \( f_i \) ’s, Carroll proposed to postulate that the theory possesses an approximate global continuous symmetry of the form \( \phi \rightarrow \phi + \text{const} \) (the idea similar to what is used in pseudo-Goldstone boson models of quintessence [19] where however, an explicit breaking of the continuous chiral symmetry reduces it to a discrete symmetry). In the framework of Einstein’s GR, such exact continuous symmetry is incompatible with a nontrivial potential of the scalar field \( \phi \). This means that if we were working in Einstein’s GR, then started from the model with the exact symmetry \( \phi \rightarrow \phi + \text{const} \) and therefore with a constant potential, we would want to achieve a nontrivial, quintessential potential (passed also across a fine tuning purgatory) as a result of some mechanism for a symmetry breaking. Such a picture looks even more problematic one than the fine tuning problem itself. In addition, in the
framework of such general idea about a breaking of the symmetry \( \phi \rightarrow \phi + \text{const.} \), it is impossible to point out the parameters of the theory which could produce, after a symmetry breaking, the small coupling constants \( f_i \).

In contrast to GR, in TMT one can suppose that in yet unknown more fundamental theory the global continuous symmetry (18), (19) is an exact one and \( \alpha = \beta \). At energies below the Plank mass the symmetry is breaking and it is assumed that the effective action describing the relevant physics, has the form of TMT, Eq. (9) (inclusion of the usual matter will be studied in Sec. VI), with the nontrivial pre-potentials (10). The only thing we need from a mechanism for a symmetry breaking consists of a small relative shift of the magnitudes of \( \alpha \) and \( \beta \) satisfying the condition (22). If the symmetry breaking generates couplings of the scalar field \( \phi \) to the usual matter fields, then the corresponding dimensionless coupling constants \( f_i \) must be proportional\(^{\text{I}} \) to some positive power of \((\beta - \alpha)/\beta\).

Notice that an unbounded increase of the pre-potentials as \( \phi \rightarrow \infty \) does not produce problems, at least on the classical level, since as was already mentioned in Sec. II, the pre-potentials have no sense of a potential energy density. The real potential is the TMT effective potential (13) that in the model under consideration approaches zero according to Eq. (21) as \( \phi \rightarrow \infty \).

An evident generalization of the pre-potentials (14) that maintains the behavior of \( U(\phi) \) as \( \beta \phi \gg M_p \), Eq. (21), consists of adding to them the terms with lower degree of growth. They may be for example, polynomials in \( \phi \) as it was in the case discussed in previous subsection. Relative contributions of all adding terms into the TMT effective potential \( U(\phi) \) will be exponentially suppressed for large \( \phi \). If these additional terms appear as a result of breaking of the symmetry (18), (19) (reminded that \( \alpha = \beta \) in the case of the exact symmetry), then coefficients in front of them have to be proportional to some positive power of the small parameter \((\beta - \alpha)/\beta\). The latter will be used in the next section. For the same reasons as it was before, the symmetry (18), (19) is restored as \( \phi \rightarrow \infty \).

Simple reasoning adduced here as well as in the previous subsection, does not look like trivial one if we recall that in GR adding any constant and/or increasing (as \( \phi \rightarrow \infty \)) term to the potential destined to be a quintessential one, causes a drastic violation of its desirable features: an arbitrary cosmological constant appears and/or the flatness conditions are destroyed if no extreme fine tunings are made. The basis for the resolution of these problems in TMT consists in a possibility to achieve a quintessence form of the effective potential as \( \phi \) is large enough, starting from pre-potentials increasing as \( \phi \rightarrow \infty \). As a matter of fact, this is the main advantage of the studied TMT models over the quintessence models formulated in the framework of the standard GR.

In the conclusion it is worthwhile to notice for the following discussion that in all cases considered in this section, \( \chi^{-1} \) as the solution of the constraint (10), asymptotically approaches zero as \( \phi \rightarrow \infty \).

IV. PROBE MODELS: TOWARDS EFFECTIVE TMT POTENTIAL OF THE "QUINTESSENTIAL INFLATION" TYPE

A. Some clarifications to the rest part of the paper

The previous sections served a preparatory role in the formulation and solution of the main problems of this paper. In Sec. III our attention was concentrated on the possibilities of TMT to generate without fine tuning the scalar field \( \phi \) potential which for \( \phi \) large enough provides a quintessence. It turns out however that some of such TMT effective potentials can appear to be also well defined to drive the early universe evolution. In this paper I have no aim to look for a precise values of all parameters of the potential that could be able to provide an adequate description of the cosmological evolution from slightly after Planck time up to now and answer all demands of the realistic cosmology. But I do want to exhibit that the field theory models based on TMT provide the existence of a broad spectrum of tools giving us the firm belief that such a potential can be generated without fine tuning. More precisely, in this section I am going to demonstrate that exploring the results of the previous section one can make sure that TMT is able to generate \((\text{without any sort of fine tuning})\) the effective potential of such a form that in the main could answer basic demands of the realistic cosmology.

\(^{\text{I}}\)Notice that the exponents in the pre-potentials (16) contain actually dimensional factors \( \alpha M_p^{-1} \) and \( \beta M_p^{-1} \). Therefore the dimensionless parameter that could characterize the symmetry breaking has to be of the form \((\beta M_p^{-1} - \alpha M_p^{-1})/\beta M_p^{-1} = (\beta - \alpha)/\beta\).
Such qualitative examination is enough for the purposes of this paper which consist mainly in studying of some basic field-theoretic problems of TMT that turn out to be in the very close interrelation with some fundamental features of the cosmological scenario. The essence of the matter is that generally speaking the price for the success of TMT in the resolution of the cosmological constant problem is serious enough. In fact, in order to incorporate the matter fields into the simplifying picture reviewed in Sec. II in such a way that the TMT effective equations of motion of all fields in the Einstein frame would have the form of the equations of motion of the standard field theory based on GR, in Ref. [24] we were forced to start from the very nonlinear (in the matter fields) original TMT action. This circumstance makes the quantization of TMT a very hard problem even on a semiclassical level (i.e. the matter fields quantization in the background curved space-time). We will see below that for the second class cosmological scenarios (see the end of Sec. II) with the appropriate choice of the pre-potentials, it is enough to start from the original TMT action with exactly the same degree of nonlinearity in matter fields as in the standard theory in order to achieve the standard matter field theory in a background (pseudo-Riemannian) space-time. Then the matter fields quantization reduces to the standard procedure of the matter fields quantization in curved space-time [27]. Fortunately, it turns out that the choice of the initial cosmological conditions and the pre-potentials needed to provide such successful construction of the matter field theory in the context of TMT, corresponds to the class of models where the TMT effective potential allows to solve all five problems mentioned in Introduction.

B. Models based on the hypothesis that the theory possesses the explicitly broken global symmetry [18], [19]

The pre-potentials of the form [18] with additional (subleading as \( \phi \to \infty \)) terms provide the possibility to generate the TMT effective potential \( U(\phi) \) with an asymptotic quintessence behavior that mimics the current effective cosmological constant. For this to be done there is no need of any sort of fine tuning and the enough condition for this is \( 0 < \beta - \alpha < \beta \) in [18]. If however, one wants to extend the range of applicability of the TMT effective potential of the same single scalar field \( \phi \) to satisfy constraints of the realistic cosmology from inflation of the early universe up to the present-day universe, then we have too big arbitrariness in the choice of the additional terms to (16). I restrict myself by models based on the idea that the action [6] is the effective one of a more fundamental theory at the energy scales below the Planck mass. It seems then to be natural to suppose that transition from the fundamental theory to the effective one is accompanied by breaking of some fundamental symmetries. I will assume that one of such symmetries is the global one [18], [19]. Such approach to the choice of prepotentials enables to narrow the amount of the suitable versions. In particular, for models leading to the asymptotic (as \( \phi \to \infty \)) inverse power low TMT effective potentials (discussed in Sec. III.A), one can not point out a range where the symmetry (18), (19) is restored. This is why I am obliged to restrict myself by studying models of the type discussed in subsection III.B and more precisely, by models where the condition [22] holds.

Below we will formulate three models where the modifications of the pre-potentials [16] will be realized by adding the simplest terms explicitly breaking the symmetry [18], [19]. The Planck mass \( M_p \) is chosen as the typical scale for parameters of the dimension of mass corresponding to the limit where the global symmetry (18), (19) is unbroken. Then the appearance of the mass parameters smaller than \( M_p \) is a manifestation of a symmetry breaking by the appropriate terms since those parameters can be represented as \( \left( \frac{2-\alpha}{s} \right)^n M_p, \quad n > 0 \). In the framework of such an approach one can maintain that the model is free of a fine tuning if orders of all such mass parameters are not too much differ from \( M_p \) (in this connection see also discussions after Eqs. [21], [22] and footnote 1)

\[
I. Model 1
\]

\[ V_1(\phi) = m_1^2 e^{\alpha \phi/M_p}; \quad V_2(\phi) = \frac{1}{4} \left( 4V_2^{(0)} + m_2^4 e^{2\beta \phi/M_p} \right) \]  

(23)

With the choice of the parameters \( m_2 = M_p \), \( 4V_2^{(0)} = (10^{-3}M_p)^4 \), \( m_1 = 10^{-2}M_p \), \( \beta = 7 \), \( \alpha = 6 \), and with the integration constant \( M^4 = (3q \times 10^{-2}M_p)^4 \), \( 0 < q \leq 1 \) (s = +1), the TMT effective potential \( U(\phi) \), Eq. [13], is a monotonically decreasing function with the shape that is convenient to describe in a piecewise form with the following four typical regions:

\[ ^2 \text{Of course, without knowledge of the fundamental theory one can not discuss a mechanism for the symmetry breaking.} \]
$U(\phi) \approx q^8 M_p^4$ for $\phi < -2.2 M_p$, 
\[ \approx \frac{q^8 M_p^4}{1 + 10^{12} e^{14 \phi / M_p}} \text{ for } -2.2 M_p < \phi < -1.8 M_p \]
\[ \approx 10^{-12} M_p^4 e^{14 \phi / M_p} \text{ for } -1.8 M_p < \phi < 0.6 M_p \]
\[ \approx 10^{-16} M_p^4 e^{-2 \phi / M_p} \text{ for } \phi > 1.2 M_p \] (24)

2. Model 2

$V_1(\phi) = \frac{1}{2} \mu_1^2 \phi^2 + m_4 e^{\alpha \phi / M_p}$; $V_2(\phi) = \frac{1}{4} \left( 4V_2^{(0)} + m_2^2 e^{2 \beta \phi / M_p} \right)$ (25)

With the choice of the parameters $m_2 = M_p$, $4V_2^{(0)} = (\sqrt{2} M_p)^4$, $\mu_1 = 10^{-4} M_p$, $m_1 = 10^{-3} M_p$, $\beta = 7$, $\alpha = 6$ and with the integration constant $M^4 = (\frac{1}{\sqrt{2}} 10^{-2} M_p)^4$, $(s = +1)$, the TMT effective potential $U(\phi)$, Eq. (13), is a monotonically decreasing function with the shape that one can describe in a piecewise form with the following three typical regions:

\[ U \approx \frac{1}{4} \lambda \phi^4, \quad \lambda = 10^{-14} \text{ for } \phi < -\frac{1}{3} M_p \]
\[ \approx 10^{-16} M_p^4 \left[ 10^{-1} + \frac{1}{2} \left( \frac{\phi}{M_p} \right)^2 \right]^2 e^{-14 \phi / M_p} \text{ for } 0 < \phi < 1.1 M_p \]
\[ \approx 6 \times 10^{-28} M_p^4 e^{-2 \phi / M_p} \text{ for } \phi > 1.4 M_p \] (26)

where in the interval $0 < \phi < 1.1 M_p$ the factor in front of the exponential function varies very slowly.

3. Model 3

$V_1(\phi) = \frac{1}{2} \mu_1^2 \phi^2 + m_4 e^{\alpha \phi / M_p}$; $V_2(\phi) = \frac{1}{4} \left( 4V_2^{(0)} + \frac{1}{2} \mu_2^2 \phi^2 + m_2^2 e^{2 \beta \phi / M_p} \right)$ (27)

With the choice of the parameters $m_2 = M_p$, $4V_2^{(0)} = (10^{-1} M_p)^4$, $\mu_1 = 10^{-4} M_p$, $m_1 = 10^{-3} M_p$, $\mu_2 = 10^{-2} M_p$, $\beta = 7$, $\alpha = 6$ and with the integration constant $M^4 = (\frac{1}{\sqrt{3}} 10^{-2} M_p)^4$, $(s = +1)$, the TMT effective potential, Eq. (13), is a monotonically decreasing function with the shape that one can describe in a piecewise form with the following three typical regions:

\[ U \approx \frac{1}{2} m^2 \phi^2, \quad m = 10^{-6} M_p \text{ for } \phi < -0.7 M_p \]
\[ \approx 10^{-16} M_p^4 \left[ 10^{-1} + \frac{1}{2} \left( \frac{\phi}{M_p} \right)^2 \right]^2 e^{-14 \phi / M_p} \text{ for } -0.6 < \phi < 1.5 M_p \]
\[ \approx 10^{-24} M_p^4 e^{-2 \phi / M_p} \text{ for } \phi > 1.7 M_p \] (28)

where in the interval $-0.6 M_p < \phi < 1.5 M_p$ the factor in front of exponential function varies very slowly.

C. Some general features of the models 1 - 3.

As it was already noted, the exact fitting of all parameters to satisfy the requirements of the realistic cosmology is over and above the plan of this paper. Our aim here is rather a demonstration of extremely broad spectrum of tools giving by TMT to solve some fundamental problems of the realistic cosmology.

9
1) In each of the models 1-3 with the action (8), the global continuous symmetry (18), (19) is violated by all terms of \( V_1 \) and \( V_2 \) except for the last term of \( V_2 \). The symmetry is restored at the limit \( \phi \to \infty \). All mass parameters (including mass parameters corresponding to "Λ-terms" in each of the models) have orders equal or slightly less than the Planck mass (but not less than the GUT scale).

2) One can see that the TMT effective potential \( U(\phi) \) of each of the models 1 - 3 has a region that can be responsible for an inflation of the early universe. Let us refer to this region of \( U(\phi) \) as the "inflationary region" of \( U \).

In model 1 the inflationary region of \( U \) is the infinite interval \(-\infty < \phi < -1.8M_p \) with practically constant value \( U(\phi) \approx q^2 M_p^4 \) that smoothly passes on a slowly decreasing region. Such inflationary region of \( U \) might be responsible for an initial stage of a new inflationary scenario [30].

In models 2 and 3, the inflationary regions of \( U \) have the form of the power low potentials \((\frac{1}{4} \lambda \phi^4 \) and \( \frac{1}{2} m^2 \phi^2 \) respectively) driving the chaotic inflation [14]. Parameters of the pre-potentials are chosen in such a way that the inflationary region of \( U \) satisfies the requirements of the realistic cosmology. It is very important to stress that this can be done without strong tuning of the parameters, in contrast with the GR approach to the chaotic inflation models where the strong enough tuning is needed. The choice of \( \beta \) and \( \alpha \) does not affect practically the inflationary region of \( U(\phi) \).

3) The TMT effective potential \( U(\phi) \) of each of the models 1 - 3 behaves as

\[
U(\phi) \approx \frac{m_1^8}{M_p} e^{-2(\beta-\alpha)\phi/M_p} \quad \text{as} \quad \phi > \phi_b = \tilde{\omega} M_p, \tag{29}
\]

where the constant factor \( \tilde{\omega} \) of order 1 is very sensitive to the choice of parameters. Let us refer to this region of \( U(\phi) \) as the "quintessential region" since it can serve for the quintessential model of the present universe. One should make here an important remark. The quintessential region of \( U \) has the form [23] where the value of \((\beta-\alpha)/\beta \ll \beta \) determines a strength of the symmetry breaking. The choice of \( \beta - \alpha = 1 \) and \( \beta = 7 \) in the models 1-3 has just an illustrative aim and it is not a problem to adjust the value of \( \beta - \alpha \) to satisfy the observable value of \( \Omega_\phi \) at present.

4) Between the inflationary and quintessential regions, there exists an intermediate region of great interest. The TMT effective potential \( U(\phi) \) in the intermediate region can be represented in the general form

\[
U(\phi) = f(\phi) M_p^4 e^{-2\beta \phi/M_p} \tag{30}
\]

where \( f(\phi) \) is a very slowly varying function compared to the exponential factor. There is a remarkable property of the intermediate region of \( U \) that provides possibilities for resolution of some fundamental problems of the realistic cosmology: by an appropriate choice of \( \beta \) one can achieve a very rapid decreasing of \( U(\phi) \) after inflationary epoch that provides conditions for transition to the radiation and matter dominated era. For instance, in model 3 the TMT effective potential \( U(\phi) \) at the end of the intermediate region \( (\phi \approx 1.5M_p) \) is roughly \( 10^{28} \) times less than at the beginning of the intermediate region \( (\phi \approx -0.6M_p) \). This property of the intermediate region of \( U \) may be very useful for resolution of problems of cosmological nucleosynthesis constraints and large-scale structure formation [3], [8], [18], [19]. The exact shape of the intermediate region of \( U \) (steepness and the range of definition) dictated by the realistic cosmology can be adjusted by the choice of the magnitudes of \( \alpha, \beta \) and dimensional parameters (like \( M, \mu_1 \) etc.).

5) Combining the intermediate and the quintessential regions one can see that the post-inflation region of the TMT effective potential can be represented approximately in the exponential form described by Eq. (8) with \( \phi \)-dependent parameter \( a \) (see Ref. [26])

\[
a = a(\phi) = 2\beta \quad \text{as} \quad \phi < \phi_b
\]

\[
= 2(\beta - \alpha) \quad \text{as} \quad \phi > \phi_b \tag{31}
\]

where \( \phi_b \) (see Eq. (24)) is a boundary value of \( \phi \) between the intermediate and quintessential regions of \( U(\phi) \). It seems to be very attractive that this result is obtained in a natural way in the framework of the field theory model without any assumptions specially intended for this. The only thing have been assumed is that the model possesses the approximate global continuous symmetry (18), (19) and the value of \( \beta - \alpha \ll \beta \) depending on a strength of the symmetry breaking should not be large in order to provide the flatness of the TMT effective potential in the quintessential region.

6) It turns out that in the models 2 and 3 the shape of \( U(\phi) \) in the buffer range between the inflationary and intermediate regions can be very sensitive to variations of the parameters entering into \( V_1 \) and \( V_2 \). By means of a suitable change of the parameters one can achieve (without altering the qualitative properties of the discussed above regions), for instance an almost flat shape of \( U \) in this buffer range or even successive local minimum and maximum immediately after the inflationary region. This feature of the models may be very important if for example one wants to realize a scenario where the instant preheating [31] occurs before entry into the intermediate region.
The final remarks concerns the terminology. Since the scalar field $\phi$, in context of models 1-3, dominates both at the very early and at the late universe, acting in such a way that the universe expands with acceleration, let us call it the inflaton field following the terminology by Peebles and Vilenkin [13].

V. INCLUSION OF USUAL MATTER FIELDS

A. Outline of the approach to the problem

Inclusion of the ordinary matter fields (like vector bosons, fermions, etc.) in TMT is a very nontrivial problem. In the framework of the first class cosmological scenarios, there was shown in Ref. [24] that the field theory model exists where in the conformal Einstein frame, the classical equations of motion of the gauge unified theories as well as the GR equations are exactly reproduced. The merit of this model is that the spontaneous symmetry breaking (SSB) does not generate the cosmological constant term. However a serious defect of this model consists of the necessity to use the artificial enough form of how the gauge fields kinetic terms and the fermions selfinteractions enter into the original action. This creates a situation where it is unclear how one can approach the matter fields quantization.

The origin of the problem is practically reduced to the role of the constraint (10) which is modified in the presence of usual matter fields. In fact, matter fields in general contribute to the constraint and then the $\chi$-field becomes depending of matter fields. Therefore when starting with Lagrangians $L_1$ and $L_2$ including the matter fields in a form similar to the canonical one, the resulting matter fields equations of motion in the Einstein picture (obtained with the use of the conformal transformations [13] or their generalization in the presence of fermions) can appear in general to be very nonlinear.

Inclusion of the usual matter fields in the context of the models of Sec. IV.B permits to avoid this problem. In fact, following the idea that the only mass scale typical for the inflaton physics in the limit where the symmetry (18), (19) is exact, is the Planck mass, and terms that explicitly breaks this symmetry, contain mass parameters only a few orders of magnitude less than $M_p$, we provide a situation where the usual matter fields contributions to the constraint appear to be negligible in comparison with the inflaton contributions throughout the history of the universe. At the late universe, the unbounded increase of the pre-potentials (as $\phi \to \infty$) reinforces this effect. As a result of this, the scalar field $\chi$ with high accuracy is determined by the same constraint (14) as it was in the absence of the usual matter fields. This allows, starting from the Lagrangians similar to usual ones, one to keep after transition to the Einstein frame the desirable basic features of the usual matter fields sector. Together with the basic idea about the broken continuous global symmetry (18), (19) modified to the case of the presence of fermions, this approach provides possibilities for constructing gauge models in the context of TMT and, at the same time, to solve problems 1-5 of Introduction.

B. Action of a gauge abelian model and continuous global symmetry

In the framework of the formulated above general ideas let us consider a toy model that possesses gauge abelian symmetry and contains the following matter fields: a complex scalar field $\xi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2)$, an abelian gauge vector field $A_\mu$ and a fermion $\Psi$. Generalization to non-abelian gauge theories can be performed straightforward.

In the presence of fermions, the vierbein-spin-connection formalism [22], [33] has to be used instead of the first order formalism of Sec. II. The action of the model has the general form as in Eq. (3) with

$$\begin{align*}
L_1 &= -\frac{1}{\kappa}R(\omega, V) + \frac{1}{2} g^{\mu\nu}\phi_{\mu\nu} + g^{\mu
u}(\partial_{\mu} - ieA_{\mu})\xi(\partial_{\nu} + ieA_{\nu})\xi^* - V_1(\phi, |\xi|) + \\
&\quad + \frac{1}{2} \gamma^{a\mu}_{\nu} \left( \bar{\gamma} \frac{1}{2} \omega_{\mu}^{\sigma \tau} \sigma_{\tau \sigma} - i e A_{\mu} \right) - \left( \bar{\gamma} \frac{1}{2} \omega_{\mu}^{\sigma \tau} \sigma_{\tau \sigma} + i e A_{\mu} \right) \gamma^{a\mu}_{\nu} \right) \Psi
\end{align*}$$

$$L_2 = V_2(\phi) - \frac{1}{4} g^{\alpha\beta} g^{\mu\nu} F_{\alpha\mu} F_{\beta\nu} - h \Psi \xi |\phi/\xi|^{M_p}$$

Here the following definitions are used [32]:

$$R(\omega, V) = V^{\mu\nu} V^{\lambda\rho} R_{\mu\nu\rho\lambda}(\omega); \quad R_{\mu\nu\rho\lambda}(\omega) = \partial_{\mu} \omega_{\nu\rho\lambda} + \omega_{\mu\rho} \omega_{\nu\lambda} - (\mu \leftrightarrow \nu).$$

$$V = \frac{1}{2} \sqrt{g} \left( \frac{1}{\kappa} \bar{\gamma} \gamma^{\mu\nu} \right) R_{\mu\nu}$$
where $V^{ab} = g^{ab}V_b^\mu$, $g^{ab}$ is the diagonal 4 $\times$ 4 matrix with elements (1, $-1$, $-1$, $-1$) on the diagonal, $V_\mu^a$ are the vierbeins and $\omega^{ab}_\mu = -\omega^{ba}_\mu$ ($a,b = 0,1,2,3$) is the spin connection.

Pre-potential $V_2(\phi)$ is the same as in the models of Sec. IV.B. Pre-potential $V_1(\phi, |\xi|)$ is chosen in the form

$$V_1(\phi, |\xi|) = V_1(\phi) + P(|\xi|)e^{\alpha\phi/M_P}$$  \hspace{1cm} (35)$$

where $V_1(\phi)$ is the same as in the models of Sec. IV.B, that is $e^{\alpha\phi/M_P}$ is the common factor in front of $m_1^2 + P(|\xi|)$ in (33).

The transformations of the continuous global symmetry $[4]$, $[5]$ are generalized now to the form $[6]$

$$V_a^\mu \rightarrow e^{-\theta/2}V_a^\mu; \quad g_{\mu\nu} \rightarrow e^{\theta}g_{\mu\nu}; \quad A_{\mu\nu\lambda} \rightarrow e^{\theta}A_{\mu\nu\lambda}; \quad \phi \rightarrow \phi - \frac{M_P}{\beta}\theta$$
$$\xi \rightarrow \xi; \quad A_\mu \rightarrow A_\mu; \quad \Psi \rightarrow e^{-\theta/4}\Psi; \quad \overline{\Psi} \rightarrow e^{-\theta/4}\overline{\Psi}$$

(36)

The term $\int P(|\xi|)e^{\alpha\phi/M_P}\Phi d^4x$ breaks the symmetry $[36]$ by the same manner as the pre-potential $V_1(\phi)$. For the "Yukawa coupling type" term

$$S_{Yuk} = -h\int \overline{\Psi}\Psi|\xi|e^{\gamma\phi/M_P}\sqrt{-g}d^4x$$

(37)

to be invariant under transformations (36), the parameter $\gamma$ must be $\gamma = \frac{4}{3}\beta$. The value of $\gamma$ preferable from the dynamical point of view will be discussed later and we will see that $\gamma < 2\beta$. All other terms describing the usual matter fields are invariant under transformations (36). If $\gamma \neq \frac{4}{3}\beta$ then the symmetry is explicitly broken only by the Yukawa coupling type term and by pre-potentials $V_1(\phi, |\xi|)$ and $V_2(\phi)$. Thus, similar to the models of Sec. IV.B, in the model with the Lagrangian densities (32) and (33), the global continuous symmetry $[4]$ is restored as $\phi \rightarrow \infty$.

It is interesting that the form of the $\phi$-dependence of the "Yukawa type" term dictated by the symmetry (36) is very similar to a motivated by string theories "nucleon-scalar coupling" discussed by Wetterich $[3]$ in the context of a quintessence type model with exponential potential.

Note finally that for "pedagogical" reason we have started from the simplified model where the Yukawa type term appears only with the measure $\sqrt{-g}$. We will see later (see Sec. VI.H) that an additional Yukawa type term in (32), that is with the measure $\Phi$, is needed to provide a possibility to avoid the long-range force problem.

C. Connection, equations of motion and constraint

Variation of the action with respect to $\omega^{ab}_\mu$ leads to the equation solution of which is represented in the form $[8]$

$$\omega^{ab}_\mu = \omega^{ab}_\mu(V) + K^{ab}_\mu(\sigma) + K^{ab}_\mu(V, \overline{\Psi}, \Psi)$$

(38)

where $\omega^{ab}_\mu(V)$ is the Riemannian part of the connection $[32], [33]$ and

$$K^{\alpha\beta}_\mu(\sigma) = \frac{1}{2}\sigma,\alpha(V^a_\mu V^{\alpha b} - V^b_\mu V^{\alpha a}), \quad \sigma \equiv \ln \chi,$$

(39)

$$K^{ab}_\mu(V, \overline{\Psi}, \Psi) = \frac{\kappa}{8}\eta_{\alpha\beta\gamma}V_{\mu\nu\lambda}e^{\alpha\beta\gamma\delta\epsilon}/\gamma^\gamma\Psi.$$

(40)

For short we omit here equations obtained by variations of vierbeins, $A_{\mu\nu\lambda}$ as well as of the matter fields $\phi$, $\xi$, $A_\mu$, $\Psi$ and $\overline{\Psi}$. Combining equations obtained by variation of vierbeins and $A_{\mu\nu\lambda}$ and using equations of motion for $\Psi$ and $\overline{\Psi}$, one can eliminate $R(\omega, V)$ and the result is the constraint

$$sM^4 + V_1(\phi) + P(\phi)e^{\alpha\phi/M_P} = \frac{2}{\chi}\left[V_2(\phi) - \frac{3}{4\sqrt{2}}h\overline{\Psi}\Psi e^{\gamma\phi/M_P}\right]$$

(41)

which is a direct generalization of the constraint (11) to the model we study here.

One of the aims of this toy model consists in a demonstration of a possibility to construct realistic gauge unified theories (like electro-weak and GUT) in the context of cosmological scenarios dictated by models of Sec. IV.B. Introducing the scalar field $\xi$ is intended for realization of the Higgs phenomenon. Since $P(\phi)$ and $m_1^2$ appear in the
combination $m_1^4 + P(\varphi)$, the constant part of $P(\varphi)$ can be always absorbed by $m_1^4$. Then it is natural to assume\(^3\) that $|P(\varphi)| \ll m_1^4$. Later, turning to quantum effective potential, we will discuss a concrete model where $P(\varphi) = \frac{1}{3!} \varphi^4$ and then the idea explained in Sec. VI.A becomes more clear: the choice of the mass parameters in the models of Sec. IV.B allows to provide a situation where the contribution of the Higgs field $\varphi$ to the constraint (11) is negligible with respect to the inflaton field $\phi$-contribution and hence it can give only extremely small corrections to the main picture. If fluctuations of fermionic fields are not anomaly large, it is natural to expect that the same conclusion is true for fermionic contribution to the constraint (11) as well. So, the $\chi$-field determined by the constraint (11), in practically interesting cases coincides with the $\chi$-field determined by the constraint (11) which holds in the model free of the usual matter at all. For short, in what follows, when neglecting the usual matter fields contribution to the constraint, we will use the term "A-approximation". This notion will be very useful in the next subsection where we are going to represent equations of motion in the Einstein frame.

**D. Equations of motion for the selfconsistent problem in the Einstein frame**

In the presence of fermions, the transition to the "Einstein frame" (more suitable term for this case would be the Einstein-Cartan frame) is carried out by the transformations to the new variables \(^4\)

\[
V_{\mu a}(x) \rightarrow V'_{\mu a}(x) = \chi^{1/2}(x)V_{\mu a}(x); \quad g_{\mu \nu}(x) \rightarrow g'_{\mu \nu}(x) = \chi(x)g_{\mu \nu}(x); \\
\Psi(x) \rightarrow \Psi'(x) = \chi^{-1/4}(x)\Psi(x); \quad \overline{\Psi}(x) \rightarrow \overline{\Psi}'(x) = \chi^{-1/4}(x)\overline{\Psi}(x); \\
\phi \rightarrow \phi'; \quad A_{\mu \nu \chi} \rightarrow A'_{\mu \nu \chi}; \quad \varphi \rightarrow \varphi'; \quad A_{\mu} \rightarrow A_{\mu},
\]

(42)

where $\chi$ is determined by the constraint (11).

In fact, after transition to the new variables defined by the transformations (42), the $\sigma$-contribution \(^5\) to the spin connection is canceled and the transformed spin connection takes the form

\[
\omega'^{\mu c d} = \omega_{\mu c d}(\gamma') + \frac{\kappa}{8}\eta_{c d}V'_{\mu e a b c d}^\gamma \gamma^5 \gamma^4 \Psi'.
\]

(43)

that coincides with the well-known solution for the spin connection in the context of the first order formalism approach to the Einstein-Cartan theory \(^6\) where a Dirac spinor field is the only source of a non-riemannian part of the connection. Hence the curvature tensor \(^7\) expressed in terms of the new connection \(^8\) becomes the curvature tensor of such an Einstein-Cartan theory \(^9\).

At the same time, in the fermionic field equation, all terms containing $\sigma_{\mu}$ also disappear \(^4\) in the Einstein-Cartan frame and the result is

\[
\left\{ i \left[ V'_{\mu a} \gamma^a (\partial_\mu - i e A_\mu) + \gamma^a C_{ab} \right] + \frac{\kappa}{4} \epsilon^{c d e f} \xi_{c d e f} \gamma^5 \gamma^b V'^{b a}_{\mu} - \frac{\hbar}{\sqrt{2}} e^{\gamma\phi/M_p} \frac{e^{\gamma\phi/M_p}}{\chi^{3/2}} \right\} \Psi' = 0
\]

(44)

where $C_{ab}^{\mu}$ is the trace of the Ricci rotation coefficients \(^8\) in the new variables and the unitary gauge is used: after a shift we define

\[
\xi = \frac{1}{\sqrt{2}} \varphi \equiv \frac{1}{\sqrt{2}} (\nu + \tilde{\nu}(x)); \quad \nu = \text{const}
\]

(45)

Equation for $\overline{\Psi}$ has similar structure. The only difference of these fermionic equations from the standard Dirac equations in the Einstein-Cartan theory \(^8\) in the Einstein-Cartan frame is related to an unusual Yukawa type term and it will be discussed later. Notice that for purposes of realistic particle physics one can neglect the second term in Eq. (43) that leads to a "spin-spin contact interaction" \(^8\) with coupling constant $M_p^{-2}$. For short, in what follows, when neglecting this interaction, we will use the term "B-approximation".

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\(^3\)Recall that $m_1$ appears in the definition of the pre-potential $V_1(\phi)$ in models of Sec. IV.B, and the values of $m_1$ are chosen such that $m_1^4 = (10^{-2} M_p)^4$ in the model 1 and $m_1^4 = (10^{-3} M_p)^4$ in models 2 and 3.

\(^4\)Notice that in the original frame, the terms including $\sigma_{\mu}$ (recall that $\sigma \equiv \ln \chi$) originate a non-metricity and therefore TMT in the original variables has no form of an Einstein-Cartan theory.
Other equations of motion in the Einstein-Cartan frame have the following form:

\[
\frac{1}{\sqrt{-g}} \partial_{\mu} \left( \sqrt{-g} g^{\mu \nu} \partial_{\nu} \phi \right) + \frac{1}{\chi} \left[ \frac{dV_1}{d\phi} - 1 \frac{dV_2}{d\phi} + \frac{\alpha}{M_p} P(\phi) e^{\chi \phi/M_p} \right] - \frac{\hbar \gamma}{\sqrt{2} M_p} \sqrt{\Psi} \varphi e^{\chi \phi/M_p} = \frac{\chi}{\sqrt{2}} \frac{\hbar \gamma}{M_p} \sqrt{\Psi} \varphi e^{\chi \phi/M_p} \chi^{3/2};
\]

\[
\frac{1}{\sqrt{-g}} \partial_{\mu} \left( \sqrt{-g} g^{\mu \nu} \partial_{\nu} \tilde{\varphi} \right) + \frac{e^{\chi \phi/M_p}}{\chi} \frac{dP(\phi)}{d\varphi} - e^2 \varphi^2 g^{\alpha \beta} A_\alpha A_\beta = \frac{\hbar \gamma}{\sqrt{2}} \frac{\hbar \gamma}{M_p} \sqrt{\Psi} \varphi e^{\chi \phi/M_p} \chi^{3/2};
\]

\[
\frac{1}{\sqrt{-g}} \partial_{\mu} \left( \sqrt{-g} g^{\mu \alpha} g^{\nu \beta} F_{\alpha \beta} \right) + \frac{e^2}{2} \varphi^2 g^{\mu \nu} A_\mu = -e \sqrt{\Psi} \chi^{\alpha} V_a^{\mu \alpha} \Psi'.
\]

It is very important to stress that in the A and B-approximations, all matter fields equations, have the canonical structure of the corresponding matter fields equations in a Riemannian space-time. The only specific features of these equations are concentrated in unusual forms of the effective potentials and some of the interactions.

After some algebraic manipulations with equations resulting from variation of the vierbeins, transition to the new variables by means of (42) and making use the fermionic equation (44) and similar equation for \( \Psi \), we obtain canonical gravitational equations of the Einstein-Cartan theory. If finally one to write down these equations in the B-approximation, we come to the canonical GR gravitational equations

\[
G_{\mu \nu} = \frac{\kappa}{2} T_{\mu \nu},
\]

where \( G_{\mu \nu} \) is the Einstein tensor of the Riemannian space-time with metric \( g'_{\mu \nu} \) and the energy-momentum tensor has a canonical GR structure [27].

\[
T_{\mu \nu} = \phi_{,\mu} \phi_{,\nu} - \frac{1}{2} g'_{\mu \nu} \phi_{,\alpha} \phi_{,\beta} g'^{\alpha \beta} + \frac{1}{\chi^2} V_2(\phi) g'_{\mu \nu} + \tilde{\varphi}_{,\mu} \tilde{\varphi}_{,\nu} - \frac{1}{2} g'_{\mu \nu} \tilde{\varphi}_{,\alpha} \tilde{\varphi}_{,\beta} g'^{\alpha \beta}
\]

\[
+ \frac{1}{4} g'_{\mu \nu} F_{\alpha \beta} F^{\alpha \beta} - F_{\mu \rho} F^{\rho \nu} = -2 g'_{\mu \nu} \varphi_{,\alpha} \beta g'^{\alpha \beta}
\]

\[
+ \frac{1}{2} \left[ \sqrt{\Psi} \gamma^a V'_a_{(\mu} \sqrt{\Psi} \gamma^b V'_{\nu) a} \Psi \right] - (\nabla_{(\mu} \sqrt{\Psi}) \gamma^a V'_{(\nu) a} \Psi \right] - \frac{1}{\chi} \left[ \frac{M^4 + V_1(\phi)}{2 V_2(\phi)}
\]

In what follows, all discussion will be performed in the framework of A- and B-approximation.

E. Effective classical action for usual matter fields in the background

To study the matter fields sector of the system of equations (44)- (51) one has to define an appropriate background. In the models where the usual matter was absent and the inflaton field \( \phi \) was the only field of the non-gravitational sector, the gravitational background in the variables of the Einstein frame is described by external field \( g'_{\mu \nu} \). However, we want to construct quantum theory of the usual matter fields then it seems to be natural to start from the approximation where in the addition to the gravitational background, the inflaton field \( \phi \) is also regarded as the background one. This can be done since in a course of its evolution, the classical inflaton field \( \phi \) remains practically constant during a typical time of quantum fluctuations of the matter fields.

So, let us study some features of the particle physics model in the background which, in terms of variables of the Einstein picture, consists of two external fields: \( g'_{\mu \nu} \) and \( \phi \). For short I will refer to this issue as the "particle physics model in the cosmological background".
The effective classical action for the particle physics model corresponding to the system of equations (44), (45) and (48), in the cosmological background, can be written down in the following form (in the unitary gauge)

\[ S_{\text{class}}^{\text{background}} = \int \sqrt{-g'} \left[ \frac{1}{2} g'^{\mu \nu} \partial_\mu \varphi \partial_\nu \varphi - V_{cl}(\varphi; \phi) + \frac{\epsilon^2}{2} \partial_\mu A_\mu g'^{\mu \nu} - \frac{1}{4} g'^{\alpha \beta} g'^{\mu \nu} F_{\alpha \mu} F_{\beta \nu} + L_{\text{kin}}(\Psi', \varphi', A_\mu) + L_{Yuk}(\Psi' \varphi'; \phi) \right] \]

(52)

where \( V_{cl}(\varphi; \phi) \) is the classical TMT effective potential for the matter (Higgs) scalar field \( \varphi \) in the presence of the background inflaton field \( \phi \):

\[ V_{cl}(\phi; \varphi) = P(\varphi) \frac{M^4 + V_1(\phi)}{2V_2(\phi)} e^{\alpha \phi/M_p}; \]

(53)

\( L_{\text{kin}}(\Psi', \varphi', A_\mu) \) is the standard kinetic term for the fermion field in a Riemannian space-time with metric \( g'_{\mu \nu} \), including also the gauge coupling to the vector field \( A_\mu \). And finally, the TMT effective "Yukawa coupling type" term \( L_{Yuk}(\Psi' \varphi'; \phi) \) is

\[ L_{Yuk}(\Psi' \varphi'; \phi) = - \frac{h}{\sqrt{2}} \Psi' \varphi' \frac{e^{\gamma \phi/M_p}}{\chi^{3/2}} = - \frac{h}{4} \Psi' \varphi \left[ \frac{M^4 + V_1(\phi)}{V_2(\phi)} \right]^{3/2} e^{\gamma \phi/M_p}. \]

(54)

F. Massless scalar electrodynamics model in the cosmological background and SSB

Up to now the function \( P(\varphi) \) was unspecified. Ignoring here technical questions (in particular, the question of renormalizability that requires a non-minimal coupling \( \eta R |\varphi|^2 \), let us attract attention to a quantum effective potential when choosing \( P(\varphi) = \frac{\alpha}{4} \varphi^4 \), \( \Lambda_0 = \text{const} \). This means that (ignoring the fermion field), we are dealing with massless scalar electrodynamics in curved space-time where the classical potential (the tree approximation) is given by

\[ V_{cl}(\varphi; \phi) = \lambda_0(\phi) \frac{\varphi^4}{4!}; \]

(55)

and \( \lambda_0(\phi) \) depends on the background field \( \phi \):

\[ \lambda_0(\phi) = \frac{\Lambda_0(\phi)}{\Lambda_0}; \quad k(\phi) \equiv M^4 + V_1(\phi) \frac{2V_2(\phi)}{V_2(\phi)} e^{\alpha \phi/M_p}. \]

(56)

Numerical estimations of \( k(\phi) \) in the models of Sec. IV.B give the following results: \( 0 < k(\phi) < 3.5 \) for model 1; \( 0 < k(\phi) < 1.2 \cdot 10^{-8} \) for model 2; \( 0 < k(\phi) < 3 \cdot 10^{-7} \) for model 3. In all models \( k(\phi) \) asymptotically approaches zero as \( \phi \to \pm \infty \). Thus, in all cases \( \lambda_0(\phi) \) is of the same order or less than \( \Lambda_0 \).

The computation technics of the effective potential for the "massless" scalar electrodynamics in the one-loop approximation is well-known issue [34]. However, the problem we study here is not quite usual: the quartic coupling "constant" depends actually on the cosmic time via the inflaton field \( \phi \). Taking into account that in a course of its evolution, the classical field \( \phi \) remains practically constant during a typical time of quantum matter fields fluctuations, it is natural to consider the problem in the adiabatic approximation. Therefore computing the effective potential we can regard \( \lambda_0(\phi) \) as a constant. Then the computation becomes quite standard. The only additional issue we have to clear up is a possible physical effect that the adiabatically changing \( \lambda_0(\phi) \) might be on the \( \varphi \)-effective potential.

One can check that the first point where we encounter necessity to decide this problem, is the renormalization procedure. In fact, performing calculations with the bare coupling constant \( \lambda_0 \) we have no need to think about its adiabatic \( \phi \) dependence. But when we turn to the use of the renormalized (finite) parameter \( \lambda \) defined by \( \lambda_0 = \lambda + \delta \lambda \) where \( \delta \lambda \) is the counter term (which, as one knows, is divergent in perturbation theory), we have to take into account a possible \( \phi \)-dependence of the effective \( \lambda \).

The vector boson loops contribution to the effective potential in the one-loop approximation has the order of \( e^4 \) and does not depend on \( \phi \) (see Eq. (52)). Therefore, just as in the standard scalar electrodynamics, one can assert that in spite of possibility for \( \lambda_0(\phi) \) to be very small, the effective \( \lambda(\phi) \) can not be too small. On the other hand it is
important also that \( \lambda(\phi) \) can not be large: since \( \phi \)-dependence of \( \lambda_0 \) acts in the direction of decrease in comparison with \( \lambda_0 \), there are no reasons for a possible \( \phi \)-dependence of \( \lambda \) to act in the opposite direction.

The scalar loops contribution has the order of \( \lambda^2 \). Therefore, in the same way as in the standard scalar electrodynamics, in the one-loop approximation, one can neglect the scalar loops contribution with respect to the vector boson loops contribution.

The one-loop effective potential for the scalar field \( \phi \) evaluated at the fixed value of the background inflaton field \( \phi = \phi_1 \) can be written in the form

\[
V_{\text{eff}}(\phi; \phi_1) = \frac{\lambda(\phi_1)}{4!} \phi^4 + \frac{3e^4}{(8\pi^2)^2} \phi^4 \left( \ln \frac{\phi^2}{\mu^2} - \frac{25}{6} \right),
\]

where

\[
\lambda(\phi_1) = \frac{d^4V_{\text{eff}}}{d\phi^4}\bigg|_{\phi=\mu}.
\]

Let us assume that \( \phi_1 \) is the value of the background inflaton field where \( \lambda(\phi) \) has a maximal possible magnitude (but it is still small!). Suppose also that the renormalization mass \( \mu \) is chosen such that \( \lambda(\phi_1) \sim e^4 \). This can be always done as is well known from the renormalization group analysis [34]. The final form of the effective potential

\[
V_{\text{eff}}(\phi; \phi_1) = \frac{3e^4}{(8\pi^2)^2} \phi^4 \left( \ln \frac{\phi^2}{\mu^2} - \frac{1}{2} \right),
\]

is determined in terms of two free parameters: renormalized gauge coupling constant \( e \) and VEV \( \phi > < \nu = 0 \).

To verify whether the change of the value of the background inflaton field \( \phi \) has some physical consequences, let us suppose that we want to repeat the same computation of the one-loop effective potential at an another fixed value of the background inflaton field \( \phi = \phi_2 \) where the order of magnitude of \( \lambda(\phi_2) \) is less than \( e^4 \), if we take the same renormalization mass \( \mu \). According to results of the renormalization group analysis [34], one can move \( \lambda(\phi_2) \) to the magnitude of the order of \( e^4 \) by a change in the renormalization mass that does not change the order of magnitude of \( e \). This can be always done if \( \lambda(\phi_2) \) is small. Then the computation of the one-loop effective potential at \( \phi = \phi_2 \) in the same approximation leads to the same effective potential as it was at \( \phi = \phi_1 \), Eq. (58), with the same order of magnitude of the free parameter \( e \). One can conclude therefore that in the used approximation, the \( \phi \)-dependence of \( \lambda \) has no physical effect.

I will ignore on this stage of investigation the fermion loops contribution into the \( \phi \) effective potential. The nonminimal coupling of the Higgs field \( \varphi \) to curvature, which appears in the quantum effective action in curved space-time [35], might have some interesting but, most likely, weak enough effect, and this question exceeds the limits of the present paper.

So, for the usual enough form of the function \( P(\varphi) \), we obtain, in a cosmological background, the effective quantum potential for the scalar (Higgs) field \( \varphi \) typical for gauge theories with dynamical symmetry breaking. Notice again that the term \( \frac{3e^4}{(8\pi^2)^2} A_\mu A_\nu g^{\mu\nu} \) in Eq. (58) does not depend on the inflaton field \( \varphi \). Thus, SSB and Higgs phenomenon occur in a standard way.

G. Yukawa coupling type term and fermion mass

As a result of SSB, the Yukawa coupling type term (54) (see also Eq. (43)) produces the TMT effective fermion mass \( m_f \) depending on the inflaton field \( \varphi \):

\[
m_f = m_f(\varphi) = \frac{h}{4} \left[ \frac{M^4 + V_1(\varphi)}{V_2(\varphi)} \right]^{3/2} e^{\gamma(\varphi/M_p)}.
\]

For \( \varphi > M_p \) (the region corresponding to the late universe), the fermion mass becomes

\[
m_f^{(\text{late})} \approx m_f^{(0)} e^{-[3(\beta - 4\alpha) - \gamma] \varphi/M_p}, \quad m_f^{(0)} = 2h [m_1/m_2]^6, \quad \text{as} \quad \varphi > M_p
\]

We see that in the late universe the fermion mass approaches the nonzero constant \( m_f^{(0)} \) if
\[ \gamma = 3(\beta - \frac{\alpha}{2}). \] (62)

Notice that if \( \gamma \) indeed satisfies the relation \( (62) \) then with the choice as in Sec. IV (i.e. \( \alpha = 6 \) and \( \beta = 7 \)), we obtain \( \gamma = 12 \) that is close to the value \( \frac{3}{2}\beta = 10.5 \) dictated by the symmetry \( (36) \) (see discussion after Eq. \( (37) \)). So in the framework of our working hypothesis about approximate symmetry \( (39) \) one can ensure a successful mass generation for fermions in the present cosmological epoch in a way typical for the standard model and, at the same time, one to keep the direct coupling \( , \) Eq. \( (53) \), of fermionic matter to inflaton field (compare this with Wetterich’s model \( \{ \) ). It is very interesting that only if the relation \( (62) \) holds, the equations of motion in the Einstein frame (see subsection VID) display the asymptotic restoration of the symmetry \( \phi \to \phi + \text{const} \) as \( \phi \to \infty \).

One has to notice that a formal generalization of the toy (abelian) model we study here, to a non-abelian one (like \( SU(2) \times U(1) \) or \( SU(5) \)) can be performed straightforward. Then we have to worry about scales of the particles mass generated as a result of SSB. In this connection it would be interesting to estimate the order of magnitude of the fermion mass in the present universe that one could expect on the basis of Eqs. \((61) \) and \((62) \). With the mass parameters \( m_1 \) and \( m_2 \) of the models of Sec. IV.B (that implies \( m_1/m_2 = 10^{-2} \) for model 1 and \( m_1/m_2 = 10^{-3} \) for models 2 and 3) and with \( v \sim 10^2 \text{GeV} \), estimations give too small values for fermion mass at the late universe: \( m_f^{(0)} \sim h \cdot 10^{-1} \text{eV} \) in model 1 and \( m_f^{(0)} \sim h \cdot 10^{-7} \text{eV} \) in models 2 and 3. This is because of the presence in \( (61) \) of the very small factor \( (\frac{m_1}{M_2})^6 \).

Masses of the vector bosons, as it was explained at the end of the previous subsection, do not depend on the inflaton field \( \phi \) and their values are defined as in the standard gauge unified models. For the mass generation of fermions we have more freedom than in the standard models. According to the basic ideas of the model developed in the present paper, the general structure of Eq. \((60) \) for masses of fermions is the same for field theory models with different symmetry groups. The only free parameters, besides the inflaton field \( \phi \), are the VEV \( v \) of the appropriate scalar boson and \( \gamma \). If the values of \( \gamma \)'s are determined by Eq. \((62) \) then masses of all fermions in the present universe are constants. If however, the parameter \( \gamma \) corresponding to some of the fermions is such that \( 3(\beta - \frac{1}{2}\alpha) - \gamma \) is very small but non-zero, then \( m_f \) becomes \( \phi \)-dependent according to Eq. \((61) \) even at the late universe. Namely, since in the quintessence model with exponential potential \( (35) \), the inflaton field \( \phi \) changes \( (\frac{3}{2})^\beta \) in cosmic time as \( \phi \propto \frac{M_p}{\beta - \alpha} \ln t \), we obtain that \( m_f^0(\phi(t)) \) will change in such a case as \( t^{-3[3(\beta - \frac{1}{2}\alpha) - \gamma]/(\beta - \alpha)} \). If \( \gamma < |3(\beta - \frac{1}{2}\alpha)| \ll |\beta - \alpha| \) (in models of Sec.IV.B this means \( |\beta - \frac{1}{2}\alpha| \ll 1 \)), the rate of change of \( m_f \) might be very small in the present universe. Depending on the sign of \( 3(\beta - \frac{1}{2}\alpha) - \gamma \), that should not be the same for all fermions, \( m_f \) could be either increasing or decreasing. Notice that the case \( 3(\beta - \frac{1}{2}\alpha) - \gamma \leq 0 \) corresponds in some sense to the model studied by Wetterich \( \{ \) .

Concerning the very early universe, that is for \( \phi < -M_p \), one can see that the model predicts for the TMT effective fermion mass, Eq. \((58) \), to be extremely small: \( m_f \to 0 \) as \( \phi \to -\infty \). For example, in model 3 of Sec. IV.B, \( m_f \approx \hbar v 10^{-4} \text{eV}/M_p \) as \( \phi < -M_p \). At the same time, the gauge coupling of \( \Psi ' \) to \( A_\mu \) (see Eq. \((58) \)) is the standard one and, in particular, it does not depend on the inflaton field \( \phi \). \( \{ \)

H. The long-range force problem

The r.h.s. of Eq. \((60) \) describes a model with the direct coupling of the inflaton to fermionic matter. For all models of Sec. IV.B, at the present universe, i.e. in the quintessential region (see Sec. IVC, item 3), the effective Lagrangian of this coupling takes the form

\[ L_{eff, present}^{(Yuk)} = -\gamma \frac{m_f^{(late)}}{M_p} \bar{\Psi} \Psi \phi. \] (63)

Assuming the condition \( (62) \) for constancy of \( m_f^{(late)} \) and with the choice \( \beta = 7, \alpha = 6 \), we get that the coupling constant of the present day effective Yukawa coupling of the inflaton to fermion is \( 12\frac{m_f^{(0)}}{M_p} \). Existing of such coupling

5 The value \( \gamma = 12 \) is as close to \( \frac{3}{2}\beta = 10.5 \) as \( \beta = 7 \) is close to \( \alpha = 6 \).

6 This can be an interesting example of the model \( \{ \) of massless spinor electrodynamics realized as the limit of a massive theory as \( \phi \to -\infty \).
would produce a too strong scalar long-range force. Fortunately, TMT gives us additional tools that allow to solve this problem.

In the model \[26\] with the only spontaneous breaking of the global continuous symmetry \[36\], Guendelman studied the case where the direct fermion-inflaton couplings similar to \[37\] present in the original TMT action both with the measure \(\Phi\) and with the measure \(\sqrt{-g}\). In such a model the constant fermion mass is also achieved \[26\]. Having this idea in mind, let us modify our model \[32\], \[33\] with the explicit breaking of the symmetry \[36\], by including an additional Yukawa coupling type term which enters into the action with the measure \(\Phi\):

\[
\tilde{S}_{\text{Yuk}} = -\tilde{h} \int \overline{\Psi} \psi e^{\tilde{\gamma}\phi/M_p} \Phi d^4 x. \tag{64}
\]

For this term to be invariant under transformations \[36\], the parameter \(\tilde{\gamma}\) must be \(\tilde{\gamma} = \frac{1}{2}\beta < \alpha\). The magnitude of \(\tilde{\gamma}\) preferable from the dynamical point of view will be discussed below.

One can check that in this modified model, the fermion mass at the late universe becomes

\[
m_{f,\text{modified}}^{(\text{late})} \approx \frac{\beta - 1}{2} \alpha \left( \frac{m_1}{m_2} \right)^2 \left( \frac{\beta - 1}{2} \alpha \right) \left[ \frac{6}{2} h \left( \frac{m_1}{m_2} \right)^4 + \tilde{h} e^{-\left( \frac{1}{2} \alpha - \tilde{\gamma} \right) \phi/M_p} \right] \] as \(\phi > M_p\). \tag{65}

The constancy of \(m_{f,\text{modified}}^{(\text{late})}\) is achieved now if the condition

\[
\tilde{\gamma} = \beta - \frac{1}{2} \alpha \tag{66}
\]

holds together with \[36\]. For \(\beta = 7\) and \(\alpha = 6\), the constancy of the fermion mass at the late universe implies that \(\tilde{\gamma} = 4\) which is as close to \(\tilde{\gamma} = \frac{1}{2}\beta = 3.5\) as \(\alpha\) is close to \(\beta\).

With the conditions for constancy of the fermion mass at the late universe, Eqs. \[32\] and \[34\], the modified effective Yukawa coupling of the inflaton to fermionic matter takes now the form

\[
L_{\text{Yuk,modified}}^{\text{eff,present}} = -\frac{\nu}{M_p} \left( \frac{m_1}{m_2} \right)^2 \left( \beta - \frac{1}{2} \alpha \right) \left[ 6h \left( \frac{m_1}{m_2} \right)^4 + \tilde{h} \right] \overline{\Psi} \psi \phi. \tag{67}
\]

We see that in the modified model there exists a possibility to prevent the appearance of such danger interaction. To realize this opportunity we have to require

\[
\tilde{h} = -6 \left( \frac{m_1}{m_2} \right)^4. \tag{68}
\]

This is actually strong enough tuning since for instance, in the context of models 2 and 3 of Sec. IV.B, it implies \(|\tilde{h}/h| \sim 10^{-12}\). If we recall that \(h\) and \(\tilde{h}\) are the Yukawa type coupling constants of the Higgs scalar to fermion, it appears to be surprisingly that their ratio has to be of the order of magnitude that shows the degree of the hierarchy problem in GUT: \(m_W/m_X \sim 10^{-12}\).

With conditions \[32\], \[36\] and \[38\], the fermion mass at the late universe becomes

\[
m_{f,\text{modified}}^{(\text{late})} = \frac{2}{3} \frac{\tilde{h} \nu}{M_p} \left( \frac{m_1}{m_2} \right)^2 \] as \(\phi > M_p\). \tag{69}

A possible relation of the discussed question to the hierarchy problem in GUT, as well as other problems that appear in the attempts to generate a realistic unified gauge theories in the context of TMT, will be studied elsewhere.

VI. DISCUSSION AND CONCLUSION

Before summarizing and discussing main results of this paper I would like to stress again that the first impression that the studied models belong to a sort of a scalar-tensor theories, is wrong. The ratio of two measures, that is the scalar field \(\chi\), Eq. \[3\], is the only object entering into equations of motion and carrying information about the measure \(\Phi\) degrees of freedom. If we restrict ourselves by models where \(L_1\) is linear in the scalar curvature (see Eqs. \[3\], \[5\] and \[12\]) and \(L_2\) does not contain curvature, then in the first order formalism, the constraint appears that
determines \( \chi \) in terms of matter fields (see Eqs. (10) or (11)). This means that in such models the scalar field \( \chi \) does not carry an independent degree of freedom. All deviations from the Einstein or Einstein-Cartan theory existing in the original variables are caused by derivatives of \( \sigma \equiv \ln \chi \) and they disappear in the new variables obtained by the conformal transformations (12) or (42). By an appropriate choice of \( L_1 \) and \( L_2 \) one can provide that all equations of motion in the new variables have canonical GR form of equations for gravity and matter fields. All novelty is revealed only in an unusual structure of the effective potentials and interactions. And just this novelty enables to solve a number of problems (questions 1-5 of Introduction) most of which in the framework of GR require fine tuning.

a. Towards a resolution of the cosmological constant problem. Let us return for the moment to the simple model of Sec. II. If one takes \( V_2(\phi) \equiv -\Lambda = \text{const} \) that in GR would correspond \( \Lambda \) to a model with a cosmological constant \( \Lambda \), then we see that the greater \( |\Lambda| \) we admit, the smaller TMT effective potential, Eq. (13), we obtain in the Einstein picture. This is a direct result of existence of two measures and two Lagrangians in the original TMT action, Eq. (1). We see that TMT turns over our intuitive ideas based on our experience in field theory.

The resolution of the cosmological constant problem in models studied in Refs. 25, 26 was based on the assumption that a cosmological scenario belongs to the first class (see Sec. II). In the context of such types of scenarios, those TMT models predict that if \( V_2(\phi) \) is positive definite, the stable vacuum with zero energy density is realized without any sort of fine tuning at a finite value of \( \phi = \phi_0 \) where \( V_1(\phi_0) + sM^4 = 0 \).

In this paper, we studied cosmological scenarios of the second class (see Sec. II) where the true vacuum state is realized asymptotically as \( \phi \to \infty \). This naturally leads us to a need to apply to a quintessence model of the late universe. However, in contrast to quintessence models studied in the framework of GR or Brans-Dicke type models, in TMT we have a new option: one can choose the prepotentials \( V_1 \) and \( V_2 \) increasing at the late universe (that is as \( \phi > M_\text{pl} \)). If \( V_2^0/V_2 \) approaches zero as \( \phi \to \infty \), then the TMT effective potential \( \langle 13 \rangle \) asymptotically approaches zero at the late universe. One can adjust degrees of growth of \( V_1 \) and \( V_2 \) in such a way that the TMT effective potential \( U(\phi) \) will have a desirable flat shape as \( \phi \to \infty \). Unbounded growth of \( V_2 \) as \( \phi \to \infty \) allows adding to \( V_2 \) any constant \( V_2^0 \) without altering \( U(\phi) \) for \( \phi \) large enough (remind that appearance of an additive constant in \( V_1 \) does not affect equations of motion at all). This is actually what we have seen in Sec. III. If appearance of the appropriate term \( \int V_2^0 \sqrt{-g} d^4x \) in the action is a result of quantum vacuum fluctuations then we can conclude that in the framework of the described approach to constructing a quintessence model of the late universe, TMT solves the cosmological constant problem.

However, the impression that the described technical details of the approach to the resolution of the cosmological constant problem in TMT settles a question, is premature. One should remind that the last statement about resolution of the cosmological constant problem implies validity of one more basic conjecture formulated in Introduction (after Eq. (1)) and used in all models of the present paper: Lagrangians \( L_1 \) and \( L_2 \) in the original action (6) do not depend on the measure \( \Phi \) degrees of freedom. In cases when this conjecture is invalid, the cosmological constant problem in TMT can turn into a very nontrivial issue. In fact, till the fundamental theory remains unknown, one can not be sure that the postulated general structure of TMT survives after quantum corrections are taken into account. If it will turn out that the quantum effective action corresponding to the original theory (6), contains the term \( -\int \Phi \chi \Lambda_{\text{eff}} d^4x \), then in the Einstein frame the latter will generate the real cosmological constant \( \Lambda_{\text{eff}} \). This possibility was studied in Ref. 24 (see Sec. VI therein) where a way to prevent the appearance of such a danger term was also discussed. The idea, briefly, is the following. If instead of the antisymmetric tensor field \( A_{\mu\nu\lambda} \) the measure \( \Phi \) is defined by means of four scalar measure fields \( \varphi_a \), \( (a = 1, 2, 3, 4) \),

\[
\Phi \equiv \varepsilon_{a_1a_2a_3a_4} \varepsilon^{\mu\nu\lambda\sigma} \left( \partial_\mu \varphi_{a_1} \right) \left( \partial_\nu \varphi_{a_2} \right) \left( \partial_\lambda \varphi_{a_3} \right) \left( \partial_\sigma \varphi_{a_4} \right),
\]

then the action (6) with \( \varphi_a \) - independent \( L_1 \) and \( L_2 \), is invariant, up to an integral of a total divergence, under transformations \( \varphi_a \to \varphi_a + f_a(L_1) \) where \( f_a(L_1) \) are arbitrary differentiable functions of \( L_1 \). An appearance of the danger term \( -\int \Phi \chi \Lambda_{\text{eff}} d^4x \) in the action would break this local symmetry. Thus, this additional, local symmetry can prevent a generation of the real cosmological constant by quantum corrections to TMT if no anomaly appears.

b. Resolution of the flatness problem of the quintessential potential. The mechanism for the resolution of the flatness problem of the quintessence potential in TMT (question number 2 of Introduction) is actually the same as the one used for the resolution of the cosmological constant problem. Since the TMT effective potential \( U(\phi) \) takes
a quintessence form as $\phi \to \infty$ due to the unbounded growth of the leading terms of the pre-potentials $V_1$ and $V_2$, appearance of any subleading terms (including terms generated by quantum corrections) in $V_1$ and $V_2$ can not alter the shape of $U(\phi)$ as $\phi$ is large enough. There is no any need for coupling constants and mass parameters of the subleading terms to be very small. This is in fact the TMT answer the question raised by Kolda and Lyth [11].

c. "Quintessential inflation" type potential (satisfying also the cosmological nucleosynthesis constraint) obtained without fine tuning. Two basic ideas has been used in this paper to demonstrate that TMT enables to answer questions 3 and 4 of Introduction. The fundamental role belongs to the first idea that in the limit $\phi \to \infty$, the effective theory has to become invariant under shifts $\phi \to \phi + \text{const}$. A basis for this idea is the observation that if we want that effective theory would describe a quintessence as $\phi \to \infty$, the effective potential has to become flat as $\phi \to \infty$.

As it was shown by Gnedinel ̆ and [25], the role of the global continuous symmetries $\phi \to \phi + \text{const}$ in TMT belongs to transformations [13], [14] in the absence of fermions or [36] in the presence of fermions: in terms of the dynamical variables used in the Einstein frame, these transformations are reduced just to shifts of $\phi$.

In models of Ref. [25], where the exponential form for the pre-potentials [14] with $\alpha = \beta$ has been used, the global symmetry [13], [19] is spontaneously broken. And although this symmetry is restored as $\phi \to \infty$, it is impossible in the framework of such a model to realize a quintessence scenario at $\phi > M_p$.

We have seen in the present paper that if a small explicit violation of the global continuous symmetries [13], [19] is present in the TMT original action [1] with the exponential form of the pre-potentials [14], then the TMT effective potential $U(\phi)$, Eq. (17), can be a suitable candidate for a quintessence model as $\beta \phi \gg M_p$. The smallness of the explicit symmetry breaking is formulated as a smallness of the dimensionless parameter $\beta \alpha^2/\beta$, see Eq. (22).

In the absence of a knowledge about the structure of the fundamental theory and without any information about a mechanism leading to an explicit violation of the global continuous symmetry [13], [19], the quantity $\beta \alpha^2/\beta$ is the only small parameter that can be used in attempts to modify the action with simple exponential form of the pre-potentials [14], with the aim to give rise to quintessential inflation type models. This can be done by adding terms that disappear as $(\beta - \alpha)/\beta$ tends to zero. This means that coupling constants in such additional terms have to be proportional to some positive power of this small parameter.

The second basic idea is that in the limit $(\beta - \alpha)/\beta \to 0$ (which leads us to the fundamental theory) the only mass parameter of the theory is the Planck mass $M_p$. This means that the dimensional coupling constants of the symmetry breaking terms have to be powers of the mass parameters $m$ of the form $m = [(\beta - \alpha)/\beta]^n M_p$, $n > 0$.

In the probe models studied in Sec. IV, we have chosen just for illustration $\beta = 7, \alpha = 6$ and hence $(\beta - \alpha)/\beta = 1/7$. Proceeding in the described above way, we reveal a remarkable feature of TMT: it is possible to achieve quite satisfactory quintessential inflation type models (see models 1-3 of Sec. IV) where for adjustment of the parameters it is enough to use only mass parameters of few orders less than $M_p$. We interpret this fact as the absence of a need of a fine tuning.

Besides of the generation of the well-defined inflationary and quintessential regions of the TMT effective potential $U(\phi)$, one more remarkable result consists in the fact that the post-inflationary region of $U(\phi)$ has the exponential form $\propto e^{\gamma \phi/M_p}$ with variable a, Eq. (31). This allows to single out a region of $U(\phi)$, where a familiar approach [2] to a resolution of the problem with the cosmological nucleosynthesis constraint is realized without any additional assumption.

d. Resolution of the problems related to a possible direct coupling of the inflaton field to usual matter. As to the question number 5 of Introduction, the answer is quite clear: if the terms of the form $f_i \omega \mathcal{L}_i$, describing direct couplings of the inflaton field to the usual matter (see Ref. [13]), break the global continuous symmetry [36] they could appear in the original TMT action with small coefficients $f_i \propto [(\beta - \alpha)/\beta]^n$, $n > 0$.

A direct coupling of the inflaton to a fermionic matter is of a special interest. In the modified model studied in Sec. VI.H, such a coupling enters to the original action in the form of two Yukawa coupling type terms, Eq. (17) and (64). The unbounded encrease of $V_1$ and $V_2$ at the late universe works again in the desirable direction: the contributions of the Yukawa coupling type terms to the constraint (41) are negligible compared to $V_1$ and $V_2$. As we have seen, by adjustment of the parameters of the Yukawa coupling type interactions one can provide the presence of the direct coupling of fermionic matter to inflaton without observable effects at the late universe: the fermion mass approaches constant and the correspondent long-range force disappears as $\phi \to \infty$.

It is worthwhile to notice here that the form of the Yukawa coupling type interactions (17) and (64) might be generalized without altering the results obtained for the late universe. In fact, if for example one modifies the interactions of the form $\propto \bar{\Psi} \Phi \phi \sqrt{-g}$ and $\bar{\Psi} \Psi e^{\gamma \phi/M_p} \Phi$ considered in Sec. VI.H, by adding the direct couplings of the form $\propto \bar{\Psi} \Phi \phi \sqrt{-g}$ and $\bar{\Psi} \Psi \Phi$ respectively (which does not respect the global continuous symmetry [13] ) this has no effect on the late universe since the relative contributions of the adding terms are exponentially suppressed as $\phi > M_p$. At the early universe, for instance as $\phi < 0$, modifications like this could lead to observable effects. Such possibilities are additional tools given by TMT for adjustment the field theory parameters to cosmological constraints.
of the early universe. One should stress that this is a merit of TMT that adjustment of the parameters determining the early universe evolution can be performed without any direct influence on the field theory parameters important for the late universe.

e. Matter fields quantization. Quantization of usual matter fields in TMT has some specific problem. In particular, fermionic field $\Psi$ in the model of Sec. VI contributes to the constraint, Eq. (11), and hence $1/\chi$ obtained by solving (11), will depend on $\Psi$. In such a case, equations of motion in the Einstein frame, (44), (46) and (47) would become very nonlinear. In Ref. [24], we have tried to avoid this sort of problems by starting from the original action that was very non-linear in $\Psi$.

In the present paper, where the inclusion of the usual matter is studied in the context of the models of Sec. IV.B, intended to describe quintessential-inflation scenario without fine tuning, the problem of a non-linearity in matter fields does not appear. The reason is just due to a way that we solve the cosmological constant and other fine tuning problems: the parameters of prepotentials $V_1(\phi)$, $V_2(\phi)$ and the integration constant $M^4$ are chosen such that the matter fields contributions to the constraint (41) are negligible compared to $V_1(\phi)$, $V_2(\phi)$ and $M^4$. Then for $1/\chi$ we obtain the expression described by Eq. (51), the same as in the absence of the usual matter. As a result of this, in the Einstein frame the usual matter fields equations in the background have canonical form and their quantization becomes a standard procedure.

f. SSB without generation of the cosmological constant. Reverting to the cosmological constant problem, it is worthwhile to notice in the conclusion that if the scalar (Higgs) field $\phi$ obtains a non-zero VEV, Eq. (45), the appearance of a constant part in $P(\phi)$ leads just to a redefinition of $m_1^4$ (see Eq. (41)). It is very important that in models 1 - 3 of Sec. IV.B, $m_1^4$ has the order of $(10^{-2}M_p)^4$ or $(10^{-3}M_p)^4$. The correction we neglect in the l.h.s. of (11) when replace it by (25), becomes of the order of $Q(\tilde{\phi})/m_1^4$ where $Q$ is a polynomial in $\tilde{\phi}$ ($|\tilde{\phi}| \ll \nu < \sim m_1$) that satisfies the condition $Q(0) = 0$. Thus if $|P(\nu)| < m_1^4$, then spontaneous braking of a gauge symmetry does not affect the magnitude of the effective cosmological constant (at the late universe) imitated by the quintessential potential (29).

Another possibility appears if the whole term $m_1^4e^{\alpha\phi/M_p}$ in the pre-potential $V_1(\phi)$ is generated by SSB. In such a case the quintessential potential becomes

$$U(\phi) \approx \frac{|P(\nu)|^8}{M_p^8}e^{-2(\beta-\alpha)\phi/M_p}$$

(71)

This is the TMT mechanism which together with the shape of $U(\phi)$ in the inflationary region predicted by each of the models 1 - 3 of Sec. IV.B, provides a resolution of one of the most serious aspect of the cosmological constant problem [10]: the need of an enormous fine tuning of initial conditions in models with SSB in order to satisfy the dual requirement of 'large $\Lambda$ in the past + small $\Lambda$ at present'.

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