Fragmentation Function and Hadronic Production of the Heavy Supersymmetric Hadrons

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Abstract

The light top-squark $\tilde{t}_1$ may be the lightest squark and its lifetime may be ‘long enough’ in a kind of SUSY models which have not been ruled out yet experimentally, so colorless ‘supersymmetric hadrons (superhadrons)’ $(\tilde{t}_1\bar{q})$ ($q$ is a quark except $t$-quark) may be formed as long as the light top-squark $\tilde{t}_1$ can be produced. Fragmentation function of $\tilde{t}_1$ to heavy ‘supersymmetric hadrons (superhadrons)’ $(\tilde{t}_1\bar{Q})$ $(\bar{Q} = \bar{c} \text{ or } \bar{b})$ and the hadronic production of the superhadrons are investigated quantitatively. The fragmentation function is calculated precisely. Due to the difference in spin of the SUSY component, the asymptotic behavior of the fragmentation function is different from those of the existent ones. The fragmentation function is also applied to compute the production of heavy superhadrons at hadronic colliders Tevatron and LHC under the so-called fragmentation approach. The resultant cross-section for the heavy superhadrons is too small to observe at Tevatron, but great enough at LHC, even when all the relevant parameters in the SUSY models are taken within the favored region for the heavy superhadrons. The production of ‘light superhadrons’ $(\tilde{t}_1\bar{q})$ $(q = u, d, s)$ is also roughly estimated. It is pointed out that the production cross-sections of the light superhadrons $(\tilde{t}_1\bar{q})$ may be much greater than those of the heavy superhadrons, so that even at Tevatron the light superhadrons may be produced in great quantities.

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I. INTRODUCTION

Supersymmetry (SUSY) is one of the most appealing extensions of the Standard Model (SM). Whereas without knowing the realistic SUSY breaking mechanism, even in the minimum supersymmetry extension of the Standard Model (MSSM), there are too many parameters in the SUSY sector, which need to be fixed via experimental measurements. If the MSSM is rooted in the ‘minimum supergravity model’ (mSUGRA), the numbers of the independent parameters can be deducted a lot, but there are still many un-fixed parameters. Therefore, the spectrum of the SUSY sector in SUSY models still is a quite open problem.

For some kinds of SUSY models and by choosing un-fixed parameters from the region which has not been ruled out yet, it is not difficult to realize a general feature of the two mass eigenstates \( \tilde{t}_1 \) and \( \tilde{t}_2 \) for the SUSY partners of top-quark (top-squark \( \tilde{t}_L \) and \( \tilde{t}_R \)) such as that the comparatively lighter one \( \tilde{t}_1 \) is the lightest squark, and the \( \tilde{t}_1 \)'s lifetime is so ‘long’, that its width is less than \( \Lambda_{QCD} \). In this case, \( \tilde{t}_1 \) may form various colorless hadrons i.e. the superhadrons by QCD interaction, which consist of \( \tilde{t}_1 \) and \( \bar{q} \) (here \( q = u, d, c, s, b \)). On the other hand, the direct experimental searching for the SUSY partners may only set a lower bound of \( \tilde{t}_1 \) mass as \( m_{\tilde{t}_1} \geq 100 \text{ GeV} \) (even lower as 75 GeV).

Therefore in the paper, we would like to focus our attention on the consequences for the possible features of \( \tilde{t}_1 \). Namely, we shall assume that \( \tilde{t}_1 \), the SUSY partners of top-quark, is not very heavy e.g. \( m_{\tilde{t}_1} \sim 120 \sim 150 \text{ GeV} \) and has a ‘quite long’ lifetime \( \Gamma_{\tilde{t}_1} \leq \Lambda_{QCD} \), that \( \tilde{t}_1 \) (after producing and before decaying) has chances to form colorless superhadrons \((\tilde{t}_1 \bar{q})^3\). Moreover we think that the squark (anti-squark) in the superhadrons is a scalar, which is different from a quark in ‘common’ hadrons, hence with such a scalar component the study of the superhadrons is also very interesting from the bound state point of view.

There was a remarkable progress in nineties of the last century in perturbative QCD (pQCD) for double heavy meson studies, i.e., it was realized that the fragmentation function and the production of a double heavy meson such as \( B_c \) and \( \eta_c, J/\psi \) can be reliably computed in terms of pQCD and the wavefunction derived from the potential model.

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1 In fact, all of the available indications on the masses of the SUSY partners are abstracted from experimental measurements and/or astro-observations under assumptions (not direct measurements), so one should consider them only as references.

2 For a summary see [16].

3 This kind of superhadrons are bound states of a quark (anti-quark) and an anti-squark (squark), or two gluinos, or two quarks (antiquarks) and a squark (anti-squark), etc. All of them are colorless and bound via strong interaction (QCD).

4 Exactly to say, here we mean color-singlet mechanism only i.e. only the color-singlet component of the con-
and further progress in formulating the problem under the framework of the effective theory: non-relativistic QCD (NRQCD) [29, 30] was made a couple years late. We should note here that before Refs. [22, 23, 24, 25, 26, 27, 28] there were papers [31, 32]. The inclusive production of a meson [31, 32] and the fragmentation functions of a parton into a meson [32] were calculated. But the authors of [32] precisely claimed that their calculations might be extended to the cases for the heavy-light mesons, such as $D, B$ etc.. In fact, the claim is incorrect and misleading in the key point on the theoretical calculability of the production and the fragmentation functions\(^5\). Since heavy-light mesons contain a light quark, and the light quark creation involving in the fragmentation function is non-perturbative, so it cannot further be factorized out a hard factor as that in the case of the double heavy mesons. It is just the reason why, similar to the case for double heavy mesons, we expect that only the ‘heavy superhadrons’ ($\bar{t}_1 Q$) (but not the ‘light superhadrons’ ($\bar{t}_1 \bar{q}$)), where $Q$ ($q$) denotes a heavy quark, $c$ or $b$ (a light quark, $u$ or $d$ or $s$), and their inclusive production and fragmentation functions may be calculated reliably. The fragmentation functions to the ‘heavy superhadrons’ ($\bar{t}_1 \bar{Q}$) can be simply attributed to a wave function of potential model and a pQCD calculable factor as in the case of the double heavy mesons.

For the ‘light superhadrons’ ($\bar{t}_1 \bar{q}$) where $q$ indicates a light quark: $u$ or $d$ or $s$, due to the non-perturbative nature for producing the involved light quark $q$, the ‘story’ about the calculation of the fragmentation function is very different, i.e., the fragmentation function cannot be attributed to a wave function and a hard factor of pQCD. Due to the non-perturbative QCD effects in the fragmentation function of the ‘light-heavy mesons’ such as $B, D$ and etc, so far practically the way to obtained the fragmentation functions of the ‘light-heavy meson’ is that they are formulated in terms of theoretical considerations and parametrization first, and then the parameters in the formulation are fixed via experimental measurement(s). With the fragmentation functions the production cross-section of a ‘light-heavy meson’, in experiences, generally is greater than that of the respective double heavy meson (the $q$ quark in ‘light-heavy meson’ is replaced by a charm quark $c$) by a factor $10^{3\sim4}$. Since there is no experimental observation on superhadrons, so we cannot play the same way for the ‘light superhadrons’ as concerned double heavy meson, which is the biggest in Fock space expansion, is taken into account in calculating the fragmentation function and production as well. While since the color-octet matrix element appearing in the formulation for color-octet mechanism production (fragmentation function) cannot be calculated theoretically so far, so the color-octet mechanism is not the case.

\(^5\) To present the calculations of the fragmentation functions, we would like also to remind here that there are some substantial contributions from the phase-space integration which might be missed if enough care were not paid. In fact, as pointed out in [29], they were missed in [32] indeed.
‘light-heavy mesons’ at all. Alternatively, as a magnitude order estimate, we expect that the fragmentation function and the production of the light superhadrons ($\tilde{t}_1 \bar{q}$) are also greater than that of the respective heavy superhadrons ($\tilde{t}_1 \bar{Q}$) by a factor $10^3 \sim 4$, no matter how heavy $\tilde{t}_1$ is, that is very similar to the case of a double heavy meson vs a heavy-light meson. Thus, based on the quantitative computation of the fragmentation function and the production of the heavy superhadrons, we simply extend the results of the the production to the light superhadrons at the end of the paper by referring the cases of the ‘double heavy mesons’ vs the ‘light-heavy mesons’ experientially. For convenience, later on we will denote ($\tilde{t}_1 \bar{Q}$) as $\tilde{H}$ throughout the paper.

Since the spectrum and the wave function accordingly of a double heavy-quark binding system i.e. a system of a heavy quark and a heavy anti-quark system, ($Q' \bar{Q}$), can be obtained theoretically in terms of non-relativistic potential model inspired on QCD quite well, so the ‘heavy superhadrons’ $\tilde{H}$, as the double heavy system ($Q' \bar{Q}$), may also be depicted by the non-relativistic potential model as long as the difference in spin is taken into account carefully[20, 21]. Therefore, there is no problem to obtain the wave functions of ‘heavy superhadrons’ $\tilde{H}$ that appear in the fragmentation functions.

In the literature, there are two approaches for estimating the direct production of a double heavy meson in NRQCD framework: the ‘fragmentation approach’ vs the complete ‘lowest-order-calculation’ approach. It is known that, of the two approaches, the former is much simpler than the latter one in computation, but the former is ‘good’ only in the region where the transverse momentum of the produced double heavy meson is large ($p_T \gtrsim 15\text{GeV}$)[33, 34, 35]. The situation for the production of the heavy superhadrons is similar to the cases of the double heavy mesons, so of the two approaches, we adopt the fragmentation approach when estimating the production of the superhadrons for rough estimation.

This paper is organized as follows: In Section II, we show how to derive the fragmentation function of the lightest top-squark $\tilde{t}_1$ to the ‘heavy superhadrons’ $\tilde{H}$, and try to present the obtained results i.e. its general features properly. In Section III, we compute the cross sections for hadronic production of the superhadrons at colliders Tevatron and LHC in terms of the so-called fragmentation approach. The Section IV is devoted to the discussions and conclusions.
II. FRAGMENTATION FUNCTION OF THE LIGHT TOP-SQUARK $\tilde{t}_1$ TO THE HEAVY SUPERHADRONS $\tilde{H}$

In this paper we adopt the fragmentation approach to estimate the $\tilde{H}$ production, and in the present section, we are computing the fragmentation function first, that is one of the key factors of the fragmentation approach.

According to pQCD, with leading logarithmic (LL) terms being summed up, a fragmentation function of a ‘parton’ $i$ to a heavy superhadron $\tilde{H}$ is depicted by DGLAP Equation as below

$$
\frac{dD_{i \rightarrow \tilde{H}}(z, Q^2)}{d\tau} = \sum_j \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{dy}{y} P_{i \rightarrow j}(z/y) D_{j \rightarrow \tilde{H}}(y, Q^2),
$$

where $\tau = \log(Q^2/\Lambda_{QCD}^2)$, $P_{i \rightarrow j}(x)$ is the splitting function. For example, the splitting function for the supersymmetric top-squark [39, 40] reads

$$
P_{\tilde{t}_1 \rightarrow \tilde{t}_1 g}(x) = \frac{4}{3} \left[ \frac{1 + x^2}{(1 - x)_+} - (1 - x) + \delta(1 - x) \right].
$$

Since Eq.(1) is an integro-differential equation, so to have definite solution, a ‘boundary (initial) condition’ for the equation i.e. $D_{j \rightarrow \tilde{H}}(z, Q_0)$, the fragmentation function at the energy scale $Q_0 \approx m_{\tilde{t}_1}$, is needed. Now the task is to obtain the boundary condition. Fortunately, the boundary condition for heavy superhadron $\tilde{H}$ can be derived in terms of pQCD and the relevant wave function precisely as double heavy mesons [22, 23, 24, 25, 26, 27, 28]. Hereafter, to simplify our notation, we shall always use $D_{\tilde{t}_1 \rightarrow \tilde{H}}(z)$ instead of $D_{j \rightarrow \tilde{H}}(z, Q_0)$.

Since the fragmentation functions are universal by definition, i.e. they are independent of the concrete process, so for the ‘boundary (initial) condition’ of the fragmentation function of the light top-squark $\tilde{t}_1$, we would like to choose a relevant simple process to calculate the ‘boundary condition’ $D_{\tilde{t}_1 \rightarrow \tilde{H}}(z)$. In order to simplify the derivation as much as possible, we furthermore assume a fictitious “$Z$”, which, except the mass, has the same properties as that of the physical $Z$ boson. The fictitious “$Z$” has such a great mass that it may decay to a pair of $\tilde{t}_1$ and $\bar{\tilde{t}}_1$.

According to pQCD factorization theorem, the differential width for the fictitious “$Z$” decaying to $\tilde{H}$ may be factorized as

$$
d\Gamma(“Z” \rightarrow \tilde{H} + X) = \int_0^1 dz d\tilde{\Gamma}(“Z” \rightarrow \tilde{t}_1 + \bar{\tilde{t}}_1, \mu_f)D_{\tilde{t}_1 \rightarrow \tilde{H}}(z, \mu_f),
$$

where $z = \frac{2E}{\sqrt{S_{eff}}}$ and $\mu_f$ is the energy scale for factorization. By definition, $D_{\tilde{t}_1 \rightarrow \tilde{H}}(z, \mu_f)$ is the fragmentation function, which represents the probability of $\tilde{t}_1$ fragmenting into the superhadron with energy fraction $z$. 
order $pQCD$ calculation, the Feynman diagrams for the inclusive decay of the fictitious particle $Z$ with the hard virtual-gluon attached to the light top-squark $\tilde{t}_1$ and its anti-particle $\bar{\tilde{t}}_1$ in different ways.

To calculate $D_{\tilde{t}_1 \rightarrow \bar{H}}(z)$, let us calculate the process $``Z'' \rightarrow \bar{H} + X$ precisely. Of the lowest-order $pQCD$ calculation, the Feynman diagrams for the inclusive decay of the fictitious particle $``Z''$ into a $\left(\frac{1}{2}\right)^-$ superhadron $\tilde{H}$ (with mass $M$), $``Z'' \rightarrow \bar{H} + X$, are described by the three diagrams (A), (B) and (C) as shown in FIG. 1. The intermediate gluon in each of the Feynman diagrams should ensure the production of a heavy quark-antiquark pair, so its momentum squared should be bigger than $(p_2 + q_2)^2 \geq 4m_Q^2 \gg \Lambda_{QCD}^2$, thus $pQCD$ calculation and the factorization theorem are reliable. The corresponding amplitudes are

$$ M = \frac{4g_2^2}{3\sqrt{3}} \cos \theta_W \int d^2 q \text{ Tr} \left\{ \chi^{(1/2)-}(p,q)(h_A^\mu + h_B^\mu + h_C^\mu) \frac{G_{\mu\nu}}{(p_2 + q_2)^2} \bar{\pi}(q_2)\gamma^\nu \right\} $$

$$ \approx \frac{4g_2^2}{3\sqrt{3}} \cos \theta_W g_B(h_A^\mu + h_B^\mu + h_C^\mu) \frac{G_{\mu\nu}}{(p_2 + q_2)^2} L^\nu, $$

with

$$ h_A^\mu = \frac{(p_1 + k - q_1)^\mu}{(k - q_1)^2 - m_{\tilde{t}_1}^2}(k - 2q_1) \cdot \epsilon,$$

$$ h_B^\mu = -2\epsilon^\mu,$$

$$ h_C^\mu = \frac{(p_1 - k - q_1)^\mu}{(k - p_1)^2 - m_{\tilde{t}_1}^2}(2p_1 - k) \cdot \epsilon$$

and

$$ L^\nu = \bar{\pi}(q_2)\gamma^\nu v(p), \quad G_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu(q_2 + p_2)_\nu + q_\nu(q_2 + p_2)_\mu}{q_1 \cdot (q_2 + p_2)}, \quad g_B = \frac{\phi(0)}{2\sqrt{m_{\tilde{t}_1}}},$$

where $k, p_1, p_2, q_1$ and $q_2$ are the four momenta of $``Z''$, top-squark $\tilde{t}_1$, antiquark $\bar{Q}$ ($\bar{b}$ or $\bar{c}$), top anti-squark $\bar{\tilde{t}}_1$ and quark $Q$ ($b$ or $c$) respectively. $p$ is the four momentum of $\tilde{H}$ and $q$ is the relative momentum between the two constituents inside $\tilde{H}$, so we have $p = p_1 + p_2$, $q = \alpha_2 p_1 - \alpha_1 p_2$ with $\alpha_1 = -\frac{m_1}{m_1 + m_2}$, $\alpha_2 = \frac{m_2}{m_1 + m_2}$, where $m_1$ and $m_2$ are masses of $\tilde{t}_1, \bar{Q}$ respectively. $\phi(0)$ is the wave function at origin for the superhadron $\tilde{H}$. If neglecting the $q$-dependence of the integrand of Eq. (3), which can be considered as the lowest term in the expansion of $q$ for the integrand, all the non-perturbative effects can be attributed into the wave function at origin.
TABLE I: The wavefunction at origin $\phi(0)$ for $(\bar{t}_1\bar{b})$ or $(\bar{t}_1\bar{c})$. Here the needed parameters are set: $m_b = 5.18$ GeV and $m_c = 1.84$ GeV.

| $m_{\bar{t}_1}$ | 120 GeV | 150 GeV |
|-----------------|---------|---------|
| $\phi(0)_{(\bar{t}_1\bar{b})}$ $(\text{GeV}^{3/2})$ | 2.502 | 2.530 |
| $\phi(0)_{(\bar{t}_1\bar{c})}$ $(\text{GeV}^{3/2})$ | 0.693 | 0.695 |

after doing the integration over $q$. $\epsilon$ is the polarization vector for the fictitious particle “Z”. $c_{11} = I_3 L \cos^2 \theta_q - e_q \sin^2 \theta_W$ \[41\]. For convenience, the variables: $S_{\text{eff}} = k^2$, $x = \frac{2p_k}{S_{\text{eff}}}$, $y = \frac{2q_{1\mu}}{S_{\text{eff}}}$, $z = \frac{2q_{2\mu}}{S_{\text{eff}}}$, $M = m_{\bar{t}_1} + m_Q$ and $d = \frac{M}{\sqrt{S_{\text{eff}}}}$ are introduced. Keeping the leading term for $d^2$, the maximum and minimum values of $y$ are $y_{\text{max}} = 1 - \frac{d^2(1 - \alpha_1 x)^2}{x(1-x)}$, $y_{\text{min}} = 1 - x + \frac{d^2(1 - x + \alpha_1 x)^2}{x(1-x)}$. In the calculation the axial gauge $n_\mu = q_{1\mu}$ is adopted. Under the axial gauge, it can be found that only the two amplitudes $M_A$ and $M_B$, which correspond to the first two Feynman diagrams in FIG 1, have contributions to the fragmentation function.

According to the factorization Eq. (2), the fragmentation function versus $z$ at energy scale $Q_0$ can be derived by dividing the differential decay width with $\Gamma_0$:

$$D_{\bar{t}_1\rightarrow \bar{H}}(z) = \frac{1}{\Gamma_0} \frac{d\Gamma}{dz}, \quad (4)$$

where $\Gamma_0$ is the decay width for the fictitious particle “Z” decaying into top-squark $\bar{t}_1$ and top anti-squark $\bar{t}_1$. Thus the result for the fragmentation function may be presented as follows:

$$D_{\bar{t}_1\rightarrow \bar{H}}(z) = F_{\bar{t}_1} \cdot f_{\bar{t}_1}(z), \quad (5)$$

where

$$F_{\bar{t}_1} = \frac{16\alpha_s(4m_{\bar{t}_1}^2)|\phi(0)|^2}{27\pi m_{\bar{t}_1}^2},$$

$$f_{\bar{t}_1}(z) = \frac{1}{6} \left( \frac{(1-z)^2 z^2}{(1-\alpha_1 z)^2} \right) \left[ 2\alpha_1^2 (z - 4) z + \alpha_1^3 (3\alpha_1 z - 2z + 2) z + 3\alpha_1^2 - 6\alpha_1 + 6 \right]. \quad (6)$$

At preset, there is no experiment data for the superhadron at all, so we adopt potential model with the Cornell potential $(-\frac{z}{r} + \frac{r}{\alpha^2})$ to estimate the wave function at origin, $\phi(0)$. For definiteness, we assume that the potential of the heavy scalar-antiquark binding system is the same as that of double heavy quark-antiquark systems. The relevant parameters are taken as:

\[^6\] $c_{11}$ is a factor in the effective coupling “Z” $-\bar{t}_1 - \bar{t}_1$, which will not appear in the final result of the fragmentation function because of cancelation between the numerator and denominator in the computation formula Eq. (4).
\( \kappa = 0.52, a = 2.34 \text{ GeV}^{-1} \), \( m_{t_1} = 120 \) or 150 GeV, \( m_b = 5.18 \) GeV and \( m_c = 1.84 \) GeV.

The corresponding wave functions at origin \( \phi(0) \) for the systems \( (t_1 \bar{b}) \) and \( (t_1 \bar{c}) \) are listed in TAB.I.

The fragmentation function \( D_{t_1 \rightarrow \bar{H}}(z) \) obtained, see Eq.(5), is just a boundary condition for the DALAP evolution equation Eq.(1). Solving the DGLAP equation, one may obtain the fragmentation function with the energy-scale evolution to \( Q^2 \). The relevant Feynman diagrams for the boundary condition \( D_{h \rightarrow \bar{H}}(z) \) with \( (j = q, \bar{q}, g) \) are of higher order in \( \alpha_s \) than the Feynman diagrams for \( D_{t_1 \rightarrow \bar{H}}(z) \), see FIG.II, therefore only the case with \( i, j = \bar{t}_1 \) shall be taken into account in solving Eq.(1), so as to meet the LL approximation criterion.

We solve Eq.(1) with the method developed by Field [43]. When \( Q^2 \gg m_{t_1}^2 \), according to the Field’s method, we have the following solution:

\[
D_{t_1 \rightarrow \bar{H}}(z, Q^2) = \tilde{D}_{t_1 \rightarrow \bar{H}}(\frac{8}{3} z, Q^2) + \kappa \int_z^1 \frac{dy}{y} \tilde{D}_{\bar{t}_1 \rightarrow \bar{H}}(\frac{8}{3} z/y, Q^2) P_\Delta(y) + O(\kappa^2),
\]

\[
D_{g \rightarrow \bar{H}}(z, Q^2) = \tilde{D}_{g \rightarrow \bar{H}}(6 z, Q^2) + \kappa \int_z^1 \frac{dy}{y} \tilde{D}_{g \rightarrow \bar{H}}(6 z/y, Q^2) P_{\Delta g}(y) + O(\kappa^2),
\]

with

\[
P_\Delta(x) = 4 \left[ \frac{1 + x^2}{1 - x} + \frac{2}{\log(x)} + \frac{3}{2} - 2 \gamma_E \delta(1 - x) - (1 - x) \right],
\]

\[
P_{\Delta g}(x) = 6 \left[ \frac{x}{1 - x} + \frac{1 - x}{\log(x)} + \frac{1}{x} + x(1 - x) \right] + \frac{11}{12} - \frac{1}{18} n_f \gamma_E \delta(1 - x)
\]

and

\[
\tilde{D}_{t_1 \rightarrow \bar{H}}(a, z, Q^2) = \int_z^1 \frac{dy}{y} D_{t_1 \rightarrow \bar{H}}(z/y, Q^2) \frac{(-\log(y))^{(ak-1)}}{\Gamma(ak)} ,
\]

\[
\tilde{D}_{g \rightarrow \bar{H}}(b, z, Q^2) = \int_z^1 \frac{dy}{y} D_{g \rightarrow \bar{H}}(z/y, Q^2) \frac{(-\log(y))^{(bk-1)}}{\Gamma(bk)} ,
\]

\[
\tilde{D}_{\bar{t}_1 \rightarrow \bar{H}}(z, Q^2) = \frac{1}{6} \int_z^6 \frac{da}{a} D_{\bar{t}_1 \rightarrow \bar{H}}(a, z, Q^2) ,
\]

\[
\tilde{D}_{g \rightarrow \bar{H}}(b, z, Q^2) = \frac{1}{3} \int_z^6 \frac{db}{b} D_{g \rightarrow \bar{H}}(b, z, Q^2) .
\]

\footnote{For \( D_{g \rightarrow \bar{H}}(z) \), the relevant part of the Feynman diagrams must have one more strong coupling vertex \( g \rightarrow t_1 \bar{t}_1 \) in \( \alpha_s \) than the relevant part \( \bar{t}_1 \rightarrow \bar{H} \) in FIG.I and for \( D_{q \bar{q} \rightarrow \bar{H}}(z) \) the relevant Feynman diagrams must have two more strong coupling vertex \( q \rightarrow g g \) and \( g \rightarrow t_1 \bar{t}_1 \) in \( \alpha_s \) than those \( \bar{t}_1 \rightarrow \bar{H} \).}
FIG. 2: Behavior of the fragmentation function (amplified by a scale factor $10^3$ for convenience) for the light top-squark $\tilde{t}_1$ to $\tilde{H}$ (here $\tilde{H}$ precisely is the $S$-wave $(\tilde{t}_1\tilde{b})$ superhadron). The dotted and dashed lines are those stand for Eq.(5), the ‘initial’ fragmentation function, with $m_{\tilde{t}_1} = 150$ GeV and $m_{\tilde{t}_1} = 120$ GeV respectively. The solid and dash-dot lines are those stand for the fragmentation function evolving to the energy scale $Q = 2$ TeV (a typical energy scale) with $m_{\tilde{t}_1} = 150$ GeV and $m_{\tilde{t}_1} = 120$ GeV respectively.

where $\kappa = \frac{6}{33-2n_f} \log(\alpha_s(Q^2_0)/\alpha_s(Q^2))$, $\gamma_E$ is Euler constant. Furthermore, at LL level we have the boundary condition $D_{g\rightarrow \tilde{H}}(z/y, Q^2_0) = 0$, thus the solution Eq.(7) becomes

$$D_{\tilde{t}_1\rightarrow \tilde{H}}(z, Q^2) = \frac{8}{3} \rightD_{\tilde{t}_1\rightarrow \tilde{H}}(z, Q^2) + \kappa \int_z^1 \frac{dy}{y} D_{\tilde{t}_1\rightarrow \tilde{H}}(\frac{8}{3}, z/y, Q^2) P_\Delta(y) + O(\kappa^2),$$

$$D_{g\rightarrow \tilde{H}}(z, Q^2) = \kappa \int_z^1 \frac{dy}{y} D_{g\rightarrow \tilde{H}}(z/y, Q^2) P_{g\rightarrow \tilde{t}_1}(y) + O(\kappa^2).$$

(9)

Numerically, it can be found that the first term for $D_{\tilde{t}_1\rightarrow \tilde{H}}(z, Q^2)$ is much greater than the other terms in the right hand side of the first equation of Eq.(9), and then it is quite accurate to consider the first term only. Moreover, due to the fact that the splitting function $P_{g\rightarrow \tilde{t}_1\tilde{t}_1}$ must be suppressed greatly $\mathcal{O}(m_Q^2/m_{\tilde{t}_1}^2)$ as $\tilde{t}_1$ is heavy ($m_{\tilde{t}_1} \geq 120$ GeV), we may safely conclude that $D_{g\rightarrow \tilde{H}}(z, Q^2) \sim 0$ when $Q^2$ is not very great.

To precisely see the general behavior of the fragmentation function obtained, we draw its curves in FIG.2. It can be found that when the mass of the top-squark becomes heavier, the peak of the curve for the fragmentation function increase higher accordingly.

Furthermore, to see the characters of the obtained fragmentation function, let us compare it with those for quarks ($Q$). The fragmentation function for an anti-quark $\bar{Q}$ into a double heavy meson ($\bar{Q}Q'$), e.g., a bottom anti-quark $\bar{b}$ into $B_c$, which can be found in Refs.[22, 23].
FIG. 3: The comparison of the two types of fragmentation functions. The upper curve (the solid one) is $D'_b(z)$, and the lower curve (the dashed one) is $D'_s(z)$. The relevant parameters are taken as $m_{t_1} = m_b = 5.18$ GeV, $m_c = 1.84$ GeV. The definitions and the artificial assumption are taken as those in text.

24, 25, 26, 27, 28:

$$D_b(z) = F_b \cdot f_b(z),$$ (10)

where $F_b = \frac{8\alpha_s^2|\psi_0(0)|^2}{27Mm_c^2}$ and

$$f_b(z) = \frac{z(1-z)^2}{(1-\lambda_1 z)^6} \left\{ [12\lambda_2 z - 3(\lambda_1 - \lambda_2)(1-\lambda_1 z)(2-z)](1-\lambda_1 z)z + 6(1+\lambda_2 z)^2(1-\lambda_1 z)^2 - 8\lambda_1 \lambda_2 z^2(1-z) \right\},$$ (11)

where $\psi_0(0)$ is wavefunction at origin of $B_c$, $\lambda_1 = \frac{m_b}{M}$, $\lambda_2 = \frac{m_c}{M}$ and $M = m_b + m_c$.

To highlight the difference between the two types of fragmentation functions, we remove the irrelevant factors $F_b$ and $F_{t_1}$ away and introduce the functions $D'_b(z)$ and $D'_s(z)$:

$$D'_b(z) = f_b(z) \quad \text{and} \quad D'_s(z) = f_{t_1}(z).$$

One may see the differences clearly in asymptotic behaviors of the two kinds of fragmentation functions that for $D'_b(z)$ and $D'_s(z)$ they are $z$ and $z^2$ as $z \rightarrow 0$ respectively; and the same $(1-z)^2$ as $z \rightarrow 1$. FIG. 3 depicts the two kinds of fragmentation functions quantitatively. In the figure, the function $D'_b(z)$ is taken precisely as the fragmentation function for the $\bar{b}$ quark fragmenting into $S$-wave pseudoscalar state of a double heavy meson $B_c$ or $B^*_c$, while the function $D'_{t_1}(z)$ is the fragmentation function for the light top-squark $\tilde{t}_1$ fragmenting into $S$-wave superhadron $S_0^c$. 

\[\]
\[ \tilde{H} = (\tilde{t}_1 \bar{c}) \]. To contrast with the difference between these two kinds of fragmentation functions, in FIG. 3 we have assumed \( m_{\tilde{t}_1} = m_b \simeq 5.18 \text{ GeV} \) artificially.

III. PRODUCTION OF THE SUPERHADRON AT HADRONIC COLLIDERS

Here we are adopting the fragmentation approach to estimate the production of the superhadrons at hadronic colliders. According to the NRQCD factorization theorem, the cross section of \( \tilde{H} \)-production by collision of hadrons \( H_1 \) and \( H_2 \), \( d\sigma_{H_1 H_2 \rightarrow \tilde{H}X} \), can be factorized into three factors as below:

\[
\sum_{ijk} \int dx_1 \int dx_2 \int dz f_{ij/H_1}(x_1, \mu_f) f_{j/H_2}(x_2, \mu_f) \cdot d\hat{\sigma}_{ij \rightarrow kX}(x_1, x_2, z; \mu_f, \mu_R) \cdot D_{k \rightarrow \tilde{H}}(z, \mu_f),
\]

where \( i, j \) and \( k \) are parton species; \( \mu_f \) corresponds to the energy scale where the factorization is made; \( \mu_R \) is the renormalization energy scale for the hard subprocess; \( d\hat{\sigma}_{ij \rightarrow kX}(x_1, x_2, z; \mu_f, \mu_R) \) is the cross section for the ‘hard subprocess’ \( ij \rightarrow kX \); \( D_{k \rightarrow \tilde{H}}(z, \mu_f) \) is the fragmentation function of ‘parton’ \( k \) to \( \tilde{H} \); \( f_{ij/H_1}(x_1, \mu_f) \) and \( f_{j/H_2}(x_2, \mu_f) \) are the parton distribution functions (PDFs) in collision hadrons \( H_1 \) and \( H_2 \) respectively. In this paper, as in most pQCD calculations, we choose \( \mu_f = \mu_R \simeq \sqrt{m_{\tilde{H}}^2 + p_T^2} \), and, as a consequence of the choice, we may further set \( D_{g \rightarrow \tilde{H}}(z, Q^2) = 0 \) quite safely as argued above. Therefore, \( k \) in Eq. (12) ’runs over’ \( \tilde{t}_1 \) only, i.e. \( k = \tilde{t}_1 \).

By naive considerations, of all the possible hard subprocesses for the production \( (ij \rightarrow kX, \ k = \tilde{t}_1) \), the gluon-gluon fusion \( g + g \rightarrow \tilde{t}_1 + \bar{\tilde{t}}_1 \) and the quark-antiquark annihilation \( q + \bar{q} \rightarrow \tilde{t}_1 + \bar{\tilde{t}}_1 \) (here \( q \) and \( \bar{q} \) are light quarks) are in the same order of strong coupling \( \alpha_s \), so they may be the most important ones for the production at the colliders: Tevatron and LHC. The gluon component of PDFs in small \( x \) region is the greatest, so the gluon-gluon fusion should be the most important one at LHC; whereas, at Tevatron, due to a comparatively low CM energy, the energy-momentum fraction \( x \) of the gluon parton must be big enough to produce the \( \tilde{t}_1 \bar{t}_1 \) pair, so as the case of top-quark pair production at Tevatron probably the components of valance quarks, instead of the gluon, play more important role (a review of this point can be found in Ref.[44]). Therefore, first of all we highlight these two subprocesses for the production. Note that according to Ref.[45] the production of the top-squark pair \( \tilde{t}_1 \bar{t}_2 \) or \( \tilde{t}_2 \bar{t}_1 \) at the hadronic colliders is small, so we do not take them into account.

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\( ^8 \) One will see later on that the cross-section of the production decrease with \( p_T \) rapidly. Thus at Tevatron and LHC \( \mu_R = \mu_R \simeq \sqrt{m_{\tilde{t}_1}^2 + p_T^2} \) is not high enough that \( m_{\tilde{t}_1} \) can be considered as zero.
FIG. 4: The lowest order Feynman diagrams for the gluon-gluon fusion subprocess $g + g \rightarrow \tilde{t}_1 + \tilde{t}_1$.

Now let us calculate the gluon-gluon fusion subprocesses first. To the lowest order (tree level), there are four Feynman diagrams as shown in FIG. 4. The corresponding amplitudes read

\begin{align}
M_A &= g_s^2 T^{ab} 4p_1^\mu p_2^\nu \epsilon_\mu(k_1) \epsilon_\nu(k_2) (p_1 - k_1)^2 - m_{\tilde{t}_1}^2, \\
M_B &= g_s^2 T^{ba} 4p_1^\mu p_2^\nu \epsilon_\mu(k_1) \epsilon_\nu(k_2) (p_1 - k_2)^2 - m_{\tilde{t}_1}^2, \\
M_C &= g_s^2 (T^{ab} - T^{ba}) (k_2 - k_1)\epsilon_\mu(k_1) \epsilon_\nu(k_2) (p_1 - p_2)_\mu \epsilon_\nu(k_1) \epsilon_\nu(k_2), \\
M_D &= g_s^2 (T^{ab} + T^{ba}) \epsilon_\mu(k_1) \epsilon_\nu(k_2) g_{\mu\nu}.
\end{align}

where $\epsilon$ is the polarization vector of gluon. When taking the axial gauge with a fixed four-vector $n$, the summation of polarization vector reads

$$\sum_\lambda \epsilon_\mu^*(k, \lambda) \epsilon_\nu(k, \lambda) = -g_{\mu\nu} - \frac{k_\mu k_\nu n^2}{(k \cdot n)^2} + \frac{k_\mu n_\nu + k_\nu n_\mu}{k \cdot n}.$$  

The differential cross section is

$$d\sigma(gg \rightarrow \tilde{t}_1 \tilde{t}_1) = \frac{3\pi^2 \alpha_s^2}{16\pi s^2} \left[ 1 - 2A - \frac{1}{9} \right] \left[ 1 - 2\frac{m_{\tilde{t}_1}^2}{A s} \right] \frac{d\hat{t}}{\hat{t}},$$  

where $A = (\hat{t} - m_{\tilde{t}_1}^2)(\hat{u} - m_{\tilde{t}_1}^2)/\hat{s}^2$. $\hat{s}$, $\hat{t}$ and $\hat{u}$ are Mandelstam variables of the subprocess,

$$\hat{s} = (k_1 + k_2)^2,$$  
$$\hat{t} = (p_1 - k_1)^2,$$  
$$\hat{u} = (p_1 - k_2)^2,$$

which satisfy $\hat{s} + \hat{u} + \hat{t} = 2m_{\tilde{t}_1}^2$.

For the quark-antiquark annihilation subprocess, to the lowest order there is only one Feynman diagram and its Feynman amplitude reads:

$$M_{qq \rightarrow \tilde{t}_1 \tilde{t}_1} = g_s^2 T^{aa} (p_1^\mu - p_2^\mu) \bar{v}(k_2) \gamma_\mu u(k_1).$$
TABLE II: Hadronic cross sections (in unit: fb) for the super hadrons \((\tilde{t}_1\bar{c})\) and \((\tilde{t}_1\bar{b})\) with \(J^P = (\frac{1}{2})^-\).

The parameters appearing in the estimate are taken as those stated in text.

| Constituents | LHC (\(\sqrt{S}=14.\) TeV) | TEVATRON (\(\sqrt{S} = 1.96\) TeV) |
|--------------|-----------------|-------------------------------|
| \(m_{\tilde{t}_1} = 120\) GeV | \((\tilde{t}_1\bar{c})\) 114.51 0.36469 0.26975 1.2E-3 | \((\tilde{t}_1\bar{b})\) 30.489 0.10374 0.0696 3.E-4 |
| \(m_{\tilde{t}_1} = 150\) GeV | \((\tilde{t}_1\bar{c})\) 42.176 0.14591 0.0537 2.E-4 | \((\tilde{t}_1\bar{b})\) 11.812 0.0431 0.0142 7.E-5 |

where \(k_1\) and \(k_2\) are the four momenta for the quark and antiquark respectively. The differential cross section is obtained as below

\[
d\hat{\sigma}(q\bar{q} \to \tilde{t}_1\bar{\tilde{t}}_1) = \frac{\pi \alpha_s^2 (s^2 - 4\hat{s}m_{\tilde{t}_1}^2 - (\hat{t} - \hat{u})^2)}{9s^4} d\hat{t}, \tag{22}
\]

where the Mandelstam variables are defined as Eqs.(18,19,20). To calculate the production via the subprocess of quark-antiquark annihilation, as stated above, we are interested in seeing the valance quarks’ contributions, especially, the production at Tevatron, so here we are considering the contributions only from the light quarks in PDFs to the production. In nucleons only the light quarks \(u\) and \(d\) may be their valance quarks.

The total hadronic cross section is calculated via the two subprocesses in terms of the factorization formulation Eq.(12) and with the help of the fragmentation function Eq.(5). The differential cross-section for the gluon-gluon fusion is in terms of Eq.(17) and the differential cross-section for the quark-antiquark annihilation is in terms of Eq.(22). Since the present calculations are at the lowest order only, so the version CTEQ6L [46] for the parton distribution functions (PDFs) is taken, and for definiteness, we take \(m_b = 5.18\) GeV, \(m_c = 1.84\) GeV and assume two possible values for \(m_{\tilde{t}_1}\): \(m_{\tilde{t}_1} = 120\) or 150 GeV. In addition, the parameter \(\Lambda_{QCD}\) in the running coupling constant \(\alpha_s\) is taken as 0.216 GeV. The energy scale for the QCD factorization formulas is chosen as the ‘transverse mass’ of the produced superhadron: \(\sqrt{m_H^2 + p_T^2}\).

The total hadronic cross sections obtained at Tevatron and LHC are in TABLE II. From the table one may see that the cross section for hadronic production of the superhadron \(\tilde{H}\) at Tevatron is much smaller than that at LHC (almost by three order of magnitude), so when \(\tilde{t}_1\) possesses the behaviors as assumed here and considering the final possible integrated luminosity, it is hopeless to observe \(\tilde{H}\) at Tevatron, but it is hopeful at LHC. TABLE II also shows that the hadronic cross sections of the superhadron \(\tilde{H}\) at Tevatron and at LHC decrease as the mass
FIG. 5: The distributions of the transverse momentum $P_T$ (left figure) and rapidity $y$ (right figure) for the superhadron $\tilde{H}$ produced via gluon-gluon fusion at LHC. For $P_T$ distribution, a rapidity cut $|y| < 1.5$ is made. The upper one of the two dash lines corresponds to the distribution for the superhadron ($\tilde{t}_1\bar{c}$) production with $m_{\tilde{t}_1} = 120$ GeV being assumed, the lower one to that for the superhadron ($\tilde{t}_1\bar{b}$) production with $m_{\tilde{t}_1} = 120$ GeV; The upper one of the two solid lines to the distribution for the superhadron ($\tilde{t}_1\bar{c}$) production with $m_{\tilde{t}_1} = 150$ GeV, the lower one to the distribution for the superhadron ($\tilde{t}_1\bar{b}$) production with $m_{\tilde{t}_1} = 150$ GeV.

FIG. 6: The distributions of the transverse momentum $P_T$ (left figure) and rapidity $y$ (right figure) for the superhadron $\tilde{H}$ produced via quark-antiquark annihilation at LHC. For $P_T$ distribution, a rapidity cut $|y| < 1.5$ is made. The upper one of the two dash lines corresponds to the distribution for the superhadron ($\tilde{t}_1\bar{c}$) production with $m_{\tilde{t}_1} = 120$ GeV being assumed, the lower one to that for the superhadron ($\tilde{t}_1\bar{b}$) production with $m_{\tilde{t}_1} = 120$ GeV; The upper one of the two solid lines to the distribution for the superhadron ($\tilde{t}_1\bar{c}$) production with $m_{\tilde{t}_1} = 150$ GeV, the lower one to the distribution for the superhadron ($\tilde{t}_1\bar{b}$) production with $m_{\tilde{t}_1} = 150$ GeV.
FIG. 7: The $p_T$ distributions of the superhadrons $\tilde{H}$ produced at Tevatron via the gluon-gluon fusion (left figure) and the quark-antiquark annihilation (right figure) respectively. For the distributions, a rapidity cut $|y| < 0.6$ is made. The upper one of the two dash lines corresponds to the distribution for the superhadron ($\tilde{t}_1\tilde{c}$) production with $m_{\tilde{t}_1} = 120$ GeV, the lower one to the distribution for the superhadron ($\tilde{t}_1\tilde{b}$) production with $m_{\tilde{t}_1} = 120$ GeV; The upper one of the solid lines to the distribution for the superhadron ($\tilde{t}_1\tilde{c}$) production with $m_{\tilde{t}_1} = 150$ GeV, the lower one to the distribution for the superhadron ($\tilde{t}_1\tilde{b}$) production with $m_{\tilde{t}_1} = 150$ GeV.

of the light scalar top quark $m_{\tilde{t}_1}$ increasing. Moreover the cross sections for the superhadron production via gluon-gluon fusion are much larger than those via the annihilation at Tevatron and at LHC both. Hence, the contribution from quark-antiquark annihilation can be ignored in comparison to the dominant contribution from the gluon-gluon fusion.

To present more features of the production, we also draw curves to show the distributions of the produced superhadron. The differential cross-sections vs the transverse momentum $p_T$ and the rapidity $y$ of the produced superhadron via gluon-gluon fusion at LHC are drawn in FIG.5 while those via the quark-antiquark annihilation are drawn in FIG.6. The distributions of transverse momentum $p_T$ for the superhadron production at Tevatron via gluon-gluon fusion and quark-antiquark annihilation are drawn in FIG.7 respectively.

IV. DISCUSSION AND CONCLUSIONS

Here the fragmentation function of the light top-squark $\tilde{t}_1$ to heavy superhadrons ($\tilde{t}_1\tilde{c}$) and ($\tilde{t}_1\tilde{b}$) is reliably computed as that in the case of a heavy quark to a double heavy meson. To see the characteristics of the fragmentation function, comparisons of the obtained fragmentation function for the light top-squark with those for heavy quarks are made by drawing curves with
suitable parameters in FIG.3. When \( z \) approaches to zero, the fragmentation functions for the top-squark (a scalar particle) approach to zero as \( z^2 \), instead of those for a heavy quark (a fermion particle) which behave as \( z \); and both of them have a similar asymptotic behavior when \( z \) approaches to 1.

Using the obtained fragmentation function of the superhadron \( \tilde{H} \) (either \( \tilde{t}_1\tilde{c} \) or \( \tilde{t}_1\tilde{b} \)) and under the fragmentation approach up-to leading logarithm, the cross sections and \( P_T(y) \) distributions for \( \tilde{H} \) have been computed at the energies of Tevatron and LHC. In the computation, the gluon-gluon fusion and light quark-antiquark annihilation as the hard subprocess for the hadronic production of the superhadron \( \tilde{H} \) are taken into account precisely. When calculating \( P_T \) distributions, different rapidity cuts are taken, i.e. \( |y| < 1.5 \) at LHC and \( |y| < 0.6 \) at Tevatron. From the cross sections and \( P_T(y) \) distributions, one may conclude that one cannot collect enough events for observing the superhadron at the hadronic collider Tevatron, even if the parameters of the supersymmetric model are in a very favored region. In contrary, enough events for experimental observation of the superhadrons can be produced (collected) without difficulty at the forthcoming collider LHC. Namely if the coming ‘new physics’ is supersymmetric and the parameters are in the favored region of the concerned superhadrons and allowed by all kinds of the existent experimental observations, Tevatron is not a good ‘laboratory’ to observe the possible superhadron(s), while LHC may be a good one. Moreover, it can be found from TABLE III that the production cross-section via \( q + \bar{q} \to \tilde{t}_1 + \bar{\tilde{t}}_1 \) is much smaller than that via gluon-gluon fusion subprocess at Tevatron and LHC. To compare with the top-quark production, let us note here that the quark-antiquark annihilation for the top-quark production is comparable to the gluon-gluon fusion at LHC [47, 48, 49, 50, 51], while for the so heavy top-quark the quark-antiquark annihilation mechanism is dominant over the gluon-gluon fusion at Tevatron [52, 53, 54, 55]. So the cross-sections for single top production via \( q + q' \to t + \bar{b} \) [56] and \( q + b \to q' + t \) [57], which are comparable to the quark-antiquark annihilation, are also important for the top-quark production.

On the production of the superhadrons at Tevatron and at LHC, for the reason precisely pointed out in the above section we have highlighted the two mechanisms via the hard subprocesses: gluon-gluon fusion and light quark-antiquark annihilation so far. In fact, there may be some other mechanisms for producing the superhadrons which may contribute greater than that via light quark-antiquark annihilation and even so sizable to be comparable with that via gluon-gluon fusion. For instance, when a comparatively light chargino (\( m_{\tilde{\chi}^\pm} \leq O(\text{TeV}) \)) is allowed in the same SUSY models, the ‘single top-squark production’ such as that via \( g + b \to \tilde{t}_1 + \tilde{\chi}_1^{-1/2} \) can be the case: its contribution may be greater than that via light quark-antiquark annihilation.
and even so sizable to be comparable with that via gluon-gluon fusion. It is very similar to the top production\cite{17, 48, 49, 50, 51}: at the LHC the single top production via the process $g + b \rightarrow W + t$ has a cross section of about 60 pb while the cross section via gluon-gluon fusion $g + g \rightarrow t + \bar{t}$ is roughly about 760 pb and the cross section via quark-antiquark annihilation $q + \bar{q} \rightarrow t + \bar{t}$ is roughly about 40 pb. Moreover if the bottom-squark $\tilde{b}_1$ is also comparatively light ($m_{\tilde{b}_1} \leq \mathcal{O}(\text{TeV})$) in the same SUSY models, then the production via $q + \bar{q}' \rightarrow \tilde{t}_1 + \bar{\tilde{t}}_1$ can be quite great too. However, all of the possibilities depends on the parameters of the relevant SUSY models, thus we would not calculate them precisely in the paper. As for the production via the subprocesses such as the annihilation of top-quark and anti-top-quark through gluino $\tilde{g}$ or photino $\tilde{\gamma}$ exchanging $t + \bar{t} \rightarrow \tilde{t}_1 + \bar{\tilde{t}}_1$, and a top-quark ‘scattering’ on a gluon: $g + t \rightarrow \tilde{t}_1 + \tilde{g}(\tilde{\gamma})$ etc, we are sure that their contributions to the superhadron production are very tiny due to the smallness of the PDF of top-quark in the colliding hadrons. Anyway, for the accuracy of the present estimate and such a ‘light’ top-squark $m_{\tilde{t}_1} = 120 \sim 150\text{GeV}$, the contribution via the quark-antiquark annihilation hard subprocess to the production can be negligible in comparison to the dominant gluon-gluon fusion mechanism at Tevatron and LHC both, that is quite different from the top-quark case.

Since the decay of the heavy superhadrons $\tilde{H}$ i.e. $(\tilde{t}_1 \bar{Q})$ with $Q = c, b$ is via the light top-squark $\tilde{t}_1$ or via the involved heavy quark $Q$ with proper relative decay possibility, so there are two typical decay channels for $\tilde{H}$: one is the decay of the light top-squark $\tilde{t}_1$ with the heavy quark $\bar{Q}$ acting as a ‘spectator’ and the other is the decay of the heavy quark $\bar{Q}$ with the light top-squark $\tilde{t}_1$ acting as a ‘spectator’. The second decay channel may be quite different from that of the decay for a light top-squark $\tilde{t}_1$ itself, and then it shall present certain characteristics. Therefore, we think that in order to observe and identify (discover) the light top squark $\tilde{t}_1$ experimentally, one may try to gain some advantages via observing the characteristics of the heavy superhadrons $\tilde{H}$ decay.

If the heavy superhadrons are really observed experimentally, it will be a good news not only for the relevant SUSY model(s) but also for the QCD-inspired potential model, because it will open a fresh field, i.e. the potential model will need to be extended to treat systems with binding of a fermion and a scalar boson.

As it is known, the fragmentation of a heavy quark $b$ or $c$ to a double heavy meson $\eta_b$ or $\eta_c$ is quite smaller than that of the heavy quark to a heavy meson $B$ or $D$ i.e. with a relative possibility about $10^{-4} \sim 10^{-3} \approx 10^{-4}$\cite{22, 23, 24, 25, 26, 27, 28}, thus with the same reason one may be quite sure that the fragmentation function of top-squark $\tilde{t}_1$ to light superhadrons $(\tilde{t}_1 \tilde{q})$, $q = u, d, s$ are much greater than that of top-squark $\tilde{t}_1$ to heavy superhadrons $(\tilde{t}_1 \bar{Q}) Q = c, b$. 
Namely, by conjecture, the fragmentation function of top-squark $\tilde{t}_1$ to light superhadrons ($\tilde{t}_1 \bar{q}$) can be about $10^3 \sim 10^4$ of the top-squark $\tilde{t}_1$ to the heavy superhadron ($\tilde{t}_1 \bar{Q}$) one. With such an enhancement this large, the light superhadrons may be produced numerously, and then one may collect enough events for experimental observation even at Tevatron. However without the additional characteristics due to the decay heavy quark $b$ or $c$ of the heavy superhadrons, the light superhadrons may be comparatively difficult to be identified experimentally.

Finally, we should note here that the computation and discussion in the paper are explicitly based on the assumption that the light color-triplet top-squark does exist in certain SUSY models. As a matter of fact, the results in the present paper are true for a variety of the SUSY models, in which even the light top-squark $\tilde{t}_1$ is not the lightest SUSY object in nontrivial color. As long as in the SUSY models concerned, the lightest SUSY partner is a scalar in color triplet and has a lifetime long enough to form hadrons before decaying, our results as presented here remain meaningful by simply replacing the light top-squark $\tilde{t}_1$ with the corresponding lightest SUSY partner.

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