A New Interpretation of Compensate Effect

M.X Shao *, Z. Zhao †

Department of Physics, Beijing Normal University, Beijing 100875, P.R.China.

Abstract

A new interpretation of compensate effect is presented. The Hawking effect in general space-time can be taken as a compensate effect of the scale transformation of coordinate time on the horizon in generalized tortoise coordinates transformation. It is proved that the Hawking temperature is the pure gauge of compensate field in tortoise coordinates. This interpretation does not refer to a zero-temperature space-time.

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1 Introduction

Recently Zhao et al. developed an interpretation of Hawking effect as a compensate effect of the coordinate scale transformation [1] [2] [3] [4] [5] [6] [7]. In this understanding the temperature appears to be the \( x^1 \) component of a pure gauge [8]. It is of importance that the zero-temperature space-time must be singled out. Otherwise an uncertainty in this scheme will arises. It is confident that all is determined whenever the metric of space-time is given. Therefore we look a scheme that the compensate effect is determined by the space-time itself. In section two a brief review of Zhao’s idea is given via a simple example. In section three the temperature is presented to be \( x^1 \) component of a pure gauge on the horizon in generalized tortoise coordinates. It is proved for a general case and unnecessary to point out a zero-temperature space-time.

2 Zhao’s Compensate Effect

It is known that via the technique of conformal flat [9] [10] [11] [12] [13] [14] the line element in \( x^0, x^1 \) subspace is conformal to Minkowski space near the event horizon. Zhao et al. analyze the conformal factor and present

*E-mail: shaomingxue@hotmail.com
†E-mail: blackhole@ihw.com.cn
the compensate effect. We now use Schwarzschild metric in advanced Eddington-Finkelstein coordinate to go over the skeleton.

\[ ds^2 = -(1 - \frac{2m}{r})dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2 \theta d\varphi^2). \]  

(1)

In tortoise coordinates

\[ r_* = r + \frac{1}{2\kappa} \ln \left( \frac{r}{2m} - 1 \right), \]  

(2)

the line element can be rewritten as

\[ ds^2 = \Omega_2^2 ds_2^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2), \]  

(3)

in which

\[ \Omega_2^2 = 1 - \frac{2m}{r} = \frac{e^{2\kappa(r_* - r)}}{2\kappa r}, \]  

(4)

\[ ds_2^2 = -dv^2 + 2dvdr_*. \]  

(5)

Introducing the generalized null Kruskal coordinates

\[ U = -\frac{1}{\kappa} e^{-\kappa u}, \quad V = \frac{1}{\kappa} e^{\kappa v}, \]  

(6)

where the retarded Eddington-Finkelstein coordinate is defined as \( u = v - 2r_* \). Its relation to the kruskal coordinates is

\[ T = \frac{1}{2}(V + U), \quad R = \frac{1}{2}(V - U). \]  

(7)

The line element Eq.(3) can be written as

\[ ds^2 = \Omega_1^2 ds_1^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2), \]  

(8)

in which

\[ \Omega_1^2 = \frac{e^{-2\kappa r}}{2\kappa r}, \]  

(9)

\[ ds_2^2 = -dV^2 + 2dVdR. \]  

(10)

Since the line element keeps invariant, from Eqs.(3)(8), it is clear

\[ ds_2^2 = \frac{ds_1^2}{\Omega^2}, \quad \Omega^2 = \frac{\Omega_2^2}{\Omega_1^2}. \]  

(11)

Eq.(11) can be regarded as a conformal isometry.

Now consider a infinitesimal 2-dimensional vector \( B_\mu \) whose proper length is

\[ L = |ds| = \sqrt{B_\mu B^\mu} = \sqrt{g_{\mu\nu} B^\mu B^\nu}, \quad \mu, \nu = 0, 1 \]  

(12)

whose coordinate lengths are defined respectively

\[ l_1 = |ds_1|, \quad l_2 = |ds_2|. \]  

(13)
Obviously
\[ L = \Omega_1 l_1 = \Omega_2 l_2. \] (14)

Since \( L \) is invariant under parallel transfer in two dimensional subspace,
\[ \delta \ln l_1 = -d \ln \Omega_1, \quad \delta \ln l_2 = -d \ln \Omega_2 \] (15)
is obtained. On the other hand, the contractions of the affine connection of 2-dimensional subspace have the relations
\[ \Gamma^\alpha_{\alpha \mu} dx^\mu = d(\ln \sqrt{-g_1}) = 2 d(\ln \Omega_1) \] (16)
\[ \tilde{\Gamma}^\alpha_{\alpha \mu} dx^\mu = d(\ln \sqrt{-g_2}) = 2 d(\ln \Omega_2) \] (17)
where \( g_1 \) and \( g_2 \) are the metric determinants. Define 1-form \( A_1 \) and \( A_2 \)
\[ - A_1 = \delta \ln l_1, \quad - A_2 = \delta \ln l_2, \] (18)
which rapidly results in
\[ A_1 = \frac{1}{2} \Gamma^\alpha_{\alpha \mu}, \quad A_2 = \frac{1}{2} \tilde{\Gamma}^\alpha_{\alpha \mu}. \] (19)

It can be find that \( A_1 \) and \( A_2 \) are respectively the relative change rates of the coordinate lengths \( l_1 \) and \( l_2 \). The coordinate length has the same scale as the coordinate time in the two dimensional subspace, so both \( A_1 \) and \( A_2 \) reflect the relative change rates of the coordinate times. Since Eq.(15), the \( A \) transform as a connection in the scale transformation
\[ A_2 = A_1 + d \ln \Omega. \] (20)

The field strength 2-form
\[ F = 0 \] (21)
is immediately obtained since connection \( A \) is an exact-form and \( dd = 0 \). It is easily obtained the property of pure gauge \( A \)
\[ \kappa = \frac{\partial \ln \Omega}{\partial x^1}. \] (22)

It is the \( x^1 \) component of connection that can be regarded as Hawking temperature.

3 Another Interpretation of Conformal Factor

It is wonderful that the Hawking effect can be regarded as the compensate effect of the coordinates. There is also disadvantage: the zero-temperature space-time must be pointed out manually. Otherwise this scheme can not determine the final result uniquely since the method itself does not point out which space has the temperature \( T = \frac{\kappa}{2\pi K_B} \). Here we propose another interpretation. The advantages of the new interpretation
is that the $\kappa$ is determined by the space-time itself and the zero-temperature space-time is unnecessarily to be pointed out. We directly give a general proof in advanced Eddington-Finkelstein coordinates.

Consider the most general case that $\xi = \xi(x^0, x^2, x^3)$ in Eddington coordinates. Tortoise coordinates transformation is

$$x^1_\ast = x^1 + \frac{1}{2\kappa} \ln(x^1 - \xi)$$

with other components invariant.

$$dx^1_\ast = (1 + \frac{1}{\epsilon})dx^1 - \frac{\xi'_\nu}{\epsilon} dx^\nu,$$

in which $\epsilon = 2\kappa(x^1 - \xi)$ and $\xi'_\nu = \frac{\partial \xi}{\partial x^\nu}$. The metric is then obtained in terms of tortoise coordinates

$$ds^2 = \left( \frac{\epsilon g_{11} \xi'_0}{(1 + \epsilon)^2} + \frac{\epsilon g_{10}}{1 + \epsilon} \right) dx^0 dx^0 + \frac{2\epsilon g_{10} \xi'_0 + g_{00}}{1 + \epsilon} dx^0 dx^1 + (\text{others}).$$

The technique of conformally flat require around the horizon the coefficient of $dx^0 dx^0$ in Eq.(25) being $-1$

$$\frac{\epsilon g_{11} \xi'_0 + 2\epsilon g_{10} \xi'_0 + g_{00}}{g_{10} + g_{11} \xi'_0 + \epsilon g_{10}} = -1.$$

So the line element in the two dimensional subspace $x^0, x^1_\ast$ is conformal to Minkowski space

$$ds^2 = \Omega^2(dx^0 dx^0 + 2dx^0 dx^1),$$

where the conformal factor $\Omega$ is

$$\Omega^2 = \frac{\epsilon g_{11} \xi'_0}{(1 + \epsilon)^2} + \frac{\epsilon g_{10}}{1 + \epsilon}.$$

Putting Eqs.(26,28), the technique of conformally flatness at the event horizon obtains

$$ds^2 = \Omega^2|_{x^1 \to \xi}(-dx^0 dx^0 + 2dx^0 dx^1) + (\text{others}),$$

in which when $x^1$ approach the horizon $\xi$ only keeps the first order of $\epsilon$, $\Omega^2$ equals to

$$\Omega^2|_{x^1 \to \xi} = \epsilon(g_{10} + g_{11} \xi'_0).$$

Finally the relation of $\kappa$ and when $x^1 \to \xi$ the $x^1$ component of connection $A$ is obtained as

$$\frac{\partial \ln \Omega}{\partial x^1_\ast} = \frac{1}{2} \frac{\partial \ln \epsilon}{\partial x^1_\ast} + \frac{\partial \ln(g_{10} + g_{11} \xi'_0)}{\partial x^1_\ast} = \frac{1}{2} \frac{\partial \ln \epsilon}{\partial x^1_\ast} = \kappa,$$

where the condition is supposed that the components $g_{10}$ and $g_{11}$ is not divergent on the event horizon $\xi$, i.e. their derivatives with tortoise coordinate $x^1_\ast$ vanish when $x^1 \to \xi$. In fact this hypothesis is not strong. For many models of black holes in advanced Eddington-Finkelstein coordinates, $g_{11} = 0$ and $g_{10}$ is a constant.

It is emphasized that although the Eq.(31) always holds, generally speaking, for the most general horizon $\xi = \xi(x^0, x^2, x^3)$ the $\kappa$ determined by this way is not the one determined by the method of Damour-Ruffini-Zhao[15][16][17][18][19].
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