Fourier decomposition of segmented magnets with radial magnetization in surface-mounted PM machines

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This paper presents a generic field model of radial magnetization (RM) pattern produced by multiple segmented magnets per rotor pole in surface-mounted permanent magnet (PM) machines. The magnetization vectors from either odd- or even-number of magnet blocks per pole are described. Fourier decomposition is first employed to derive the field model, and later integrated with the exact 2D analytical subdomain method to predict the magnetic field distributions and other motor global quantities. For the assessment purpose, a 12-slot/8-pole surface-mounted PM motor with two segmented magnets per pole is investigated by using the proposed field model. The electromagnetic performances of the PM machines are intensively predicted by the proposed magnet field model which include the magnetic field distributions, airgap flux density, phase back-EMF, cogging torque, and output torque during either open-circuit or on-load operating conditions. The analytical results are evaluated and compared with those obtained from both 2D and 3D finite element analyses (FEA) where an excellent agreement has been achieved.

Key words: Fourier decomposition, magnet segmentation, permanent magnet synchronous machine, radial magnetization

1 Introduction

The rare-earth magnet materials such as Neodymium Iron Boron (NdFeB) and Samarium Cobalt (SmCo) have been widely utilized in the production of surface-mounted permanent magnet synchronous machines (PMSMs). However, these high energy magnets tend to make the PMSMs suffer from excessive cogging torque and distorted phase back-EMF, resulting in undesired torque ripples and vibration which are detrimental to the overall performance and capability of the motor [1, 2]. One of the methods to suppress the torque vibration in PM machines is by segmenting the surface-mounted magnet into several magnet blocks per rotor pole [1, 3, 4]. Since 1992, many researchers have employed the Fourier series space decompositions to emulate different magnetization patterns in PMSMs, for example, prediction of motor performance due to the magnet fields with radial magnetization (RM) [5] and parallel magnetization (PaM) [6]. However, they only considered single magnet segment per rotor pole [5, 6]. Conversely, the analytical models of multiple segmented magnets per magnetic pole were reported in [7] and [8], but, the models were derived for various Halbach configurations. Additionally, the magnet field models were developed to emulate RM produced by several magnet blocks per rotor pole [1, 3, 4]. Nevertheless, the model reported by Lateb is derived based on an approximated Fourier coefficient [1], therefore, the magnetic field prediction is inaccurate. Also, the formulations of magnet field models that proposed by Ashabani [3] and Tiang [4] are complex since they superimpose a high number of virtual PM blocks in order to obtain the equivalent field distributions produced by multi-segmented magnets.

Therefore, this motivates us to investigate further on this topic. A field model based on Fourier series decomposition is considered here to represent the net effect of the magnetic field distributions produced by multi-segmented magnets mounted on the rotor surface in polyphase PMSMs. This is important because the analytical method can provide valuable information and physical insights of the machine performance during initial design stage.

2 Fourier series space decomposition of magnet segmentations for radial magnetization

The radial magnetization (RM) pattern produced by single magnet per pole is first considered, and its radial and tangential components, $M_\phi$ and $M_\alpha$, respectively, are illustrated in Fig. 1. In this case, the rotor positions, $\alpha_{p1}$ to $\alpha_{p4}$, are indicated, $\alpha_{pl}$ is the magnet pole-pitch ratio, and $p$ is the number of pole-pairs. Noted that the magnet pitch ratio of single magnet segment is solely determined by $\alpha_{pl}$.

Table 1 describes the radial component of magnetization function with its rotor positions for RM pattern produced by single magnet segment per magnetic pole.

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Table 1. Radial component of magnetization function with its rotor positions for RM pattern produced by single magnet segment per pole

| $M_{r1}$ | Rotor angular position |
|---------|-----------------------|
| $B_r/\mu_0$ | $-\alpha_{p1}\pi/(2p) \leq \alpha_r \leq \alpha_{p1}\pi/(2p)$ |
| $-B_r/\mu_0$ | $(2 - \alpha_{p1})\pi/(2p) \leq \alpha_r \leq (2 + \alpha_{p1})\pi/(2p)$ |

The Fourier field model of RM pattern produced by single magnet segment per magnetic pole can be obtained in [2] and rewritten as below:

$$M_{r1} = \sum_{k=1,3,5,...} M_{rck1} \cos(k\alpha) + M_{rsk1} \sin(k\alpha),$$

$$M_{\alpha1} = \sum_{k=1,3,5,...} M_{\alpha ck1} \cos(k\alpha) + M_{\alpha sk1} \sin(k\alpha),$$

where $\alpha_0$ is the rotor position at initial location, $\omega_r$ is the angular speed of the rotor, $M_{r1}$ and $M_{\alpha1}$ are the normal and circumferential components of magnetization vector with the $k$th harmonic number for RM pattern produced by single magnet segment per pole. As can be seen in Fig. 1(c), the tangential component of magnetization vector is always zero, therefore, $M_{\alpha k1}$ is zero; meanwhile, $M_{r k1}$ can be expanded into Fourier series as

$$M_{rk1} = \frac{pB_r}{\pi \mu_0} \{ \int_{\alpha_{-2}}^{\alpha_1} \cos(k\alpha_r) d\alpha_r - \int_{\alpha_1}^{\alpha_{-2}} \cos(k\alpha_r) d\alpha_r \} = \sum_{k/p=1,3,5,...} 4pB_r \sin(k\pi \alpha_{p1}/2p)/k\pi \mu_0,$$

Next, the RM pattern produced by two segmented magnets per magnetic pole is considered and its radial and tangential components, $M_{r2}$ and $M_{\alpha2}$, respectively, are illustrated in Fig. 2. In this case, the rotor positions, $\alpha_{r1}$ to $\alpha_{r8}$, are indicated, while $\alpha_{p1}$ and $\alpha_{p2}$ are the pitch ratios. Noted that the magnet spans are determined by the differences between $\alpha_{p1}$ and $\alpha_{p2}$. The radial component of magnetization function with its rotor positions for RM pattern produced by two segmented magnets per magnetic pole is fully outlined in Tab. 2

Table 2. Radial component of magnetization function with its rotor positions for RM pattern produced by two segmented magnets per pole

| $M_{r2}$ | Rotor angular position |
|---------|-----------------------|
| $B_r/\mu_0$ | $-\alpha_{p1}\pi/(2p) \leq \alpha_r \leq -\alpha_{p2}\pi/(2p)$ |
| $B_r/\mu_0$ | $\alpha_{p2}\pi/(2p) \leq \alpha_r \leq \alpha_{p1}\pi/(2p)$ |
| $-B_r/\mu_0$ | $(2 - \alpha_{p1})\pi/(2p) \leq \alpha_r \leq (2 - \alpha_{p2})\pi/(2p)$ |
| $-B_r/\mu_0$ | $(2 + \alpha_{p2})\pi/(2p) \leq \alpha_r \leq (2 + \alpha_{p1})\pi/(2p)$ |

Therefore, the field model of RM pattern produced by two segmented magnets per magnetic pole can be obtained as

$$M_{r2} = \sum_{k=1,3,5,...} M_{rck2} \cos(k\alpha) + M_{rsk2} \sin(k\alpha),$$

$$M_{\alpha2} = \sum_{k=1,3,5,...} M_{\alpha ck2} \cos(k\alpha) + M_{\alpha sk2} \sin(k\alpha),$$

$$M_{rck2} = M_{rck1} \cos(k\omega_r t + k\alpha_0),$$

$$M_{\alpha ck2} = M_{\alpha ck1} \sin(k\omega_r t + k\alpha_0),$$

$$M_{rsk2} = M_{rsk1},$$

$$M_{\alpha sk2} = M_{\alpha sk1}.$$
As can be seen in Fig. 3(c), the tangential component of magnetization vector is always zero, therefore, \( \alpha_{sk2} \) is zero; meanwhile, \( \alpha_{sk2} \) can be expanded into Fourier series as

\[
M_{\text{sk2}} = 0.
\]

Following similar steps, the RM pattern produced by three segmented magnets per magnetic pole is now considered and its radial and tangential components, \( \alpha_{r3} \) and \( \alpha_{o3} \), respectively, are illustrated in Fig. 3. In this case, the rotor positions, \( \alpha_{r1} \) to \( \alpha_{r12} \), are indicated, while \( \alpha_{o1} \), \( \alpha_{o2} \), and \( \alpha_{o3} \) are the pitch ratios. Noted that the magnet span of mid-magnet is determined by \( \alpha_{o3} \). Meanwhile, the magnet spans of side-magnets are determined by the differences between \( \alpha_{o1} \) and \( \alpha_{o2} \). The radial component of magnetization function with its rotor positions for RM pattern produced by three segmented magnets per magnetic pole is given in Tab. 3, the Fourier field model of RM pattern produced by three segmented magnets per magnetic pole can be obtained as

\[
M_{r3} = \sum_{k=1,3,5,...} M_{rck3} \cos(k\alpha) + M_{sk3} \sin(k\alpha),
\]

\[
M_{o3} = \sum_{k=1,3,5,...} M_{ock3} \cos(k\alpha) + M_{sk3} \sin(k\alpha),
\]

where \( M_{rk3} \) and \( M_{sk3} \) are the normal and circumferential components of magnetization vector with the \( k \)th harmonic number for RM pattern produced by three segmented magnets per pole.

Table 3. Radial component of magnetization function with its rotor positions for RM pattern produced by three segmented magnets per pole

| \( B_r/\mu_0 \) | \( -\alpha_{o1}/(2p) \leq \alpha_r \leq -\alpha_{o2}/(2p) \) | \( -\alpha_{o3}/(2p) \leq \alpha_r \leq -\alpha_{o5}/(2p) \) | \( -\alpha_{o6}/(2p) \leq \alpha_r \leq -\alpha_{o7}/(2p) \) | \( -\alpha_{o8}/(2p) \leq \alpha_r \leq -\alpha_{o9}/(2p) \) | \( -\alpha_{o10}/(2p) \leq \alpha_r \leq -\alpha_{o11}/(2p) \) | \( -\alpha_{o12}/(2p) \leq \alpha_r \leq -\alpha_{o13}/(2p) \) |
|----------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| \( M_{r3} \) | Rotor angular position |

As can be seen in Fig. 3(c), the tangential component of magnetization vector is always zero, therefore, \( \alpha_{sk3} \) is zero; meanwhile, \( \alpha_{sk3} \) can be expanded into Fourier series as

\[
M_{\text{sk3}} = 0.
\]

Based on the Fourier series space decomposition to emulate the RM pattern produced by single magnet segment, two segmented magnets, and three segmented magnets per magnetic pole, the Fourier coefficients of \( M_{r13} \), \( M_{r23} \), and \( M_{r33} \) are always zero. On the other hand, the Fourier coefficients of \( M_{r11} \), \( M_{r22} \), and \( M_{r33} \) demonstrate a common and generic expression, which can be
Table 4. Main Parameters of 12s/8p Surface-Mounted PM Motor with 2SM per Pole

| Parameters                  | Value | Parameters                  | Value |
|-----------------------------|-------|-----------------------------|-------|
| Slot number, $N_s$          | 12    | Pole number, $2p$           | 8     |
| Remanence, $B_r$ (T)        | 1.12  | Tooth-tips edge (mm)        | 3     |
| Airgap length, $l_g$ (mm)   | 1.0   | Active length, $l_a$ (mm)   | 50    |
| Winding turns/coil, $N_c$   | 30    | Stator yoke height (mm)     | 7.5   |
| Bottom slots radius, $R_{sb}$ (mm) | 42.5 | Magnet thickness, $h_m$ (mm) | 3     |
| PM surface radius, $R_m$ (mm) | 26   | Stator inner radius, $R_s$ (mm) | 50   |
| Stator outer radius (mm)    | 50    | Magnetization pattern       | RM    |
| Winding slot angle, $b_{wa}$ (mech. deg.) | 15    | Relative recoil permeability, $\sigma_{r2}$ (elect. deg.) | 11.2  |
| Slot-opening angle, $b_{wb}$ (mech. deg.) | 5.5   |                              |       |
| $\sigma_{pi}$ (elect. deg.) | 147.6 |                              |       |

Fig. 4. A typical geometry and its symbols of a surface-mounted PMSM (Region 1, 2, 3, and 4i represent magnet, airgap, the $i^{th}$ winding slot, and the $i^{th}$ slot-opening deduced for the $v^{th}$ segmented magnets per pole as

$$M_{rk} = \sum_{k=1,2,3,...}^{(v+1)/2} \frac{4pB_r}{k\pi\mu_0} \sin \frac{k\pi\alpha_p}{2p}, \quad (25)$$

$$M_{ak} = 0. \quad (26)$$

Also, the normal and tangential components of magnetization functions can be rewritten as

$$M_r = \sum_{k=1,3,5,...} M_{rk} \cos(ka) + M_{rk} \sin(ka), \quad (27)$$

$$M_o = \sum_{k=1,3,5,...} M_{ok} \cos(ka) + M_{ok} \sin(ka), \quad (28)$$

$$M_{rk} = M_r \cos(k\omega t + k\alpha_0), \quad (29)$$

$$M_{ok} = M_o \sin(k\omega t + k\alpha_0), \quad (30)$$

$$M_{ack} = -M_{ak} \sin(k\omega t + k\alpha_0), \quad (31)$$

$$M_{ask} = M_{ak} \cos(k\omega t + k\alpha_0). \quad (32)$$

From here, the magneto-quasi Maxwell equations are solved using separable variables technique to compute the magnetic field in every subdomain [2], of a surface-mounted PMSM, as shown in Fig. 4. Several important assumptions are also considered as reported in [4]. Subsequently, the motor global quantities can be evaluated, e.g. the back-EMFs, cogging torque, and output torque, which can be found in [2].

3 Analysis and validations

In this paper, the proposed field model based on Fourier series space decomposition of RM pattern produced by multiple segmented magnets per pole is integrated with the 2D exact subdomain model [2, 4] to predict the magnetic fields in semi-closed surface-mounted PM machines. The magnetic field distributions and the motor global quantities that include phase back-EMF, cogging torque, and output torque of a three-phase, 12s/8p surface-mounted PMSM are then calculated by the analytical method. Meanwhile, 2D and 3D FEA for the PM motor are evaluated by employing a commercial FE software, i.e., Cobham Vector Field Opera 2D/3D to validate the computed analytical results. The symbols of typical surface-mounted PM motor are displayed in Fig. 4, and the main parameters of 12s/8p, three-phase, surface-mounted PM motor are given in Tab. 4.

Figure 5 shows the magnetic field distributions as predicted by the 2D and 3D FEA for the 12s/8p PM machine during open-circuit and on-load conditions. As can be seen in Figs. 5(a) and 5(c), the airgap spacings between the segmented magnets and magnet interpoles provide alternative flux paths among the PMs, instead of direct interaction between the magnets and stator teeth. The magnetic forces between magnets and slot-openings...
can be minimized by having multiple segmented magnets mounted on the rotor surface. Hence, smaller cogging torque is realized. Whereas, the magnetic field distributions in 3D model are shown in Figs. 5(b) and 5(d).

Furthermore, Fig. 6 shows the magnetic flux density distributions as predicted by the proposed Fourier field model in the mid-airgap ($r = 26.5$ mm) during open-circuit and on-load conditions, which are validated by the...
2D and 3D FEA, for both radial and tangential components. An excellent agreement has been achieved.

Additionally, the predicted phase back-EMF, cogging torque, and output torque by the proposed model using Fourier series space decomposition are presented in Fig. 7. For the output torque computation, the motor has been excited with sinusoidal phase currents of 5A peak. They are also compared and verified with the 2D and 3D FEA respectively. All analytically predicted results have demonstrated a very good agreement with the FE simulated results. Therefore, the 2D and 3D FE simulated results confirm the effectiveness of the proposed field model.

4 Conclusion

This paper has presented a generic field model of radial magnetization (RM) pattern produced by multiple segmented magnets per rotor pole in surface-mounted permanent magnet (PM) machines. The rotor magnets can be formed by odd- or even-number of magnet blocks. Fourier decomposition is used to formulate the field model, and later integrated with the exact 2D analytical subdomain method to predict the magnetic field distributions and other motor quantities such as airgap flux density, phase back-EMF, cogging torque, and output torque during either open-circuit or on-load operating conditions. The analytical results are evaluated and compared with those obtained from both 2D and 3D finite element analyses (FEA) where an excellent agreement has been achieved.

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