Formation of spatial responses of the optical field in the region of location of the optical antenna array

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Abstract. This article describes a model of an optical antenna array operating in the mode of receiving electromagnetic waves in the frequency range 30-300 THz (in this case, the wavelength varies from 10 µm to 1 µm). In the course of numerical experiments (to solve the system of Maxwell equations in the integral formulation, the Weiland method [31] was used) the accuracy of approximation of the spatial structure of the optical field at the points of the “virtual” antenna array elements using the auxiliary field sources method was studied. A study was also carried out on the possibility of using a virtual antenna array formed on the plane of arrangement of elements of a real array to increase the resolution of a multichannel optical system. At the upper frequency of the analyzed range of 300 THz, the wavelength is 1 µm. The lattice period was chosen equal to half the minimum wavelength of d=0.5 µm, so that it was possible to approximate the spatial distribution of the field in the space between the elements of the antenna array, in accordance with the requirement of the Kotelnikov-Shannon theorem.

1. Introduction
During the simulation, the following parameters of the optical antenna array (figure 1) were considered: \(w=0.25\) µm crystal width; crystal height \(h=0.25\) µm [1, 2, 3]; the lattice period in coordinates \(x, y\) is \(d=0.5\) µm; the wave with vertical polarization falls in the plane \(yOz\), the angle of the point of incidence of the wave is measured from the \(z\) axis. The material of the crystals is silicon dioxide. The substrate is the perfect metal. It was believed that the intensity \(E_x\) of the field components was measured at points corresponding to the midpoints of the upper edges of the optical lattice crystals (\(z=0.25\) µm).

Figure 1. Geometry of the optical antenna array, plane incident wave and field probes (shown by arrows)
In the course of numerical experiments, we study an accuracy of approximating the spatial structure of the optical field at locations of the “virtual” antenna array elements using the method of auxiliary field sources. Also, we study a method to increase the resolution of a multi-channel optical system by use of virtual antenna array, formed at the same plane as real array elements were located.

At the upper frequency of the analyzed range (300 THz), the wavelength is \(1 \text{ m} \). The array period was equal to half the minimum wavelength (0.5 \( \text{ m} \)), so we can approximate the spatial distribution of the field in space between the antenna array elements in accordance with the requirement of the Nyquist–Shannon theorem.

The upper arrows in figure 1 correspond to field probes at the points of formation of the “virtual” antenna array (in fig. 1, the “virtual” antenna array is elevated along the \(z\) coordinate by 0.5 m from the upper edges of the optical grating (\(z=0.75 \text{ m}\)). The coordinates \(x, y\) the «real» antenna array and the “virtual” antenna array, as well as at the auxiliary point sources of the field (not shown in fig. 1) in this case coincide (later we will also consider other cases of the arrangement of the “virtual” antenna array and auxiliary field sources) [4].

Auxiliary sources of the field (figure 1) are located above the “virtual” antenna array: \(z_{\text{auxill.array}} > z_{\text{VAA}}\).

In addition, when calculating the approximated field, the model takes into account the specular reflection of auxiliary sources in the ideal metal substrate.

It was believed that the amplitudes of specular reflections of auxiliary sources are equal to the amplitudes of auxiliary sources, and the phases of specular reflections of auxiliary sources are opposite to the phases of auxiliary sources [5].

To find the complex amplitudes of the auxiliary sources of the field, find the solution of the following system of linear algebraic equations:

\[
\begin{align*}
U_n^{\text{auxill}} &= \sum_{n=1}^{N} \frac{\exp\left( -ik_0 \left( \left( x_n^{\text{auxill}} - x_k^{\text{RAA}} \right)^2 + \left( y_n^{\text{auxill}} - y_k^{\text{RAA}} \right)^2 + \left( z_n^{\text{auxill}} - z_k^{\text{RAA}} \right)^2 \right) \right)}{\left( x_n^{\text{auxill}} - x_k^{\text{RAA}} \right)^2 + \left( y_n^{\text{auxill}} - y_k^{\text{RAA}} \right)^2 + \left( z_n^{\text{auxill}} - z_k^{\text{RAA}} \right)^2} U_k^{\text{RAA}},
\end{align*}
\]

Where \(N\) - the number of analyzed elements of the "real" antenna array (RAA); \(U_n^{\text{auxill}}\) - complex amplitude of \(n\)-th auxiliary source located at the point with coordinates \((x_n^{\text{auxill}}, y_n^{\text{auxill}}, z_n^{\text{auxill}})\); \(k_0\) - free space wave number; \(U_k^{\text{RAA}}\) - complex amplitude of the signal at the output of \(k\)-th element of the RAA, located at a point with coordinates \((x_k^{\text{RAA}}, y_k^{\text{RAA}}, z_k^{\text{RAA}})\).

To approximate the field at the point \((x_k^{\text{VAA}}, y_k^{\text{VAA}}, z_k^{\text{VAA}})\) of the location of the “element” of the “virtual” antenna array (VAA) (spatial reference of the field), the following expression is used:
\[ N \sum_{n=1}^{\text{auxill}} \exp \left( -\frac{\text{auxill VAA}_n}{\text{auxill VAA}_k} \right) \left( \frac{\text{auxill VAA}_n}{\text{auxill VAA}_k} \right)^2 \left( \frac{\text{auxill VAA}_n}{\text{auxill VAA}_k} \right)^2 \nonumber \]

\[ = \frac{\exp \left( -\frac{\text{auxill VAA}_n}{\text{auxill VAA}_k} \right) \left( \frac{\text{auxill VAA}_n}{\text{auxill VAA}_k} \right)^2 \left( \frac{\text{auxill VAA}_n}{\text{auxill VAA}_k} \right)^2 }{\sum_{n=1}^{\text{auxill}} \left( \frac{\text{auxill VAA}_n}{\text{auxill VAA}_k} \right)^2 \left( \frac{\text{auxill VAA}_n}{\text{auxill VAA}_k} \right)^2 } \left( \frac{\text{auxill VAA}_n}{\text{auxill VAA}_k} \right)^2 \text{auxill VAA}_k. \]

where \( K \) - the number of formed "elements" of VAA (it can be any, including - substantially larger than the number of elements of RAA \( N \)).

Figure 2 shows the frequency relationships of the real and imaginary parts of the relative dielectric constant of the material of optical lattice (silicon dioxide) crystals.

Below are the relationships that characterize the accuracy of the approximation of the optical field in the central element of the «virtual» antenna array in the frequency range from 30 to 300 THz (\( \lambda_0 \in [10; 1] \) m).

Solid lines indicate the frequency relationships of the amplitudes and phases of the true value \( E_y \) — the field components at the analyzed point, corresponding to a rigorous numerical analysis of the electrodynamic structure using the Weyland method [6]; the dashed lines show the approximated relationships obtained by forming a “virtual” antenna array using the auxiliary field sources method.

2. The study of the accuracy of the approximation of the optical field in relation to the number of elements of the antenna array

In this subsection, the relationship of the accuracy of the approximation of the phases and field amplitudes, carried out using the auxiliary field sources method, in relation to the number of elements of the antenna array (in accordance with figure 2, for an array with \( N \times N \) elements, \( 2 \times N \times N \) auxiliary sources is formed, including specular reflections fields of auxiliary sources) [7].

Figure 3 shows the frequency relationships of the phases and field amplitudes at the point with the coordinates (0;0;0.75) m (this point is located above the central element of the antenna array and is raised to a height of 0.5 m relative to its upper edge). Grids with dimensions 3x3, 5x5 and 7x7 elements were considered.
Figure 3. The frequency relationships of the phases and amplitudes of the field at the point with (0;0;0.75) m coordinates: a) a 3x3-element grid; b) a grid of 5x5 elements; c) a grid of 7x7 elements

In figure 4, 5, it can be seen that, with an increase in the number of elements of the antenna array (and a proportional increase in the number of auxiliary sources of the field), the approximation accuracy rapidly increases.
The studies allow us to establish a very obvious fact - with an increase in the deviation of the incidence angle from the normal, the approximation accuracy decreases, because the phase shift increases from element to element of the array in the plane of wave incidence. The frequency dependences of the field amplitude in case of array with minimum number of elements (3×3) can be approximated with an acceptable error only at small values of the elevation angle (no more than 30°).

The frequency dependences of the phase can be predicted with acceptable accuracy in the long-wave region of the studied frequency band, up to an almost grazing incidence of the wave along the array - at elevation angles up to 70°. Note that in optical location and in measurement of the near-field structure of an optical field, the phase dependences are, as a rule, more informative. Therefore, significant errors in measuring the amplitudes during approximation of the observed field can be acceptable in some practical applications.

Study has shown that with an increase in the distance between the "real" and "virtual" arrays from 0 to 0.4 microns, the approximation accuracy monotonically increases, and with further distance increase between the generated spatial samples and the antenna array, accuracy starts to decrease.
This tendency is easy to explain: at short distance between the "real" and "virtual" arrays, the maximum contribution to the approximated field is made by the field of the nearest auxiliary source. As the distance increases further, two opposing factors act: on the one hand, the contribution of all auxiliary field sources becomes more uniform; on the other hand, with increasing distance from the point at which the value of the function is known, the approximation error is also increase.

3. Conclusion
Thus, in the present work, on the basis of the obtained data of electrodynamic modeling carried out by numerical solution of the system of Maxwell equations with given boundary conditions, it can be concluded that the use of methods and approaches for the formation of VAA developed for the radio frequency range can be used in the optical wavelength range features of the dispersion properties of materials. To convert a holographic image obtained in one layer into a volume hologram (analogue of Denisyuk hologram) we can approximate an optical field measured by the optical antenna array at points lying above the plane of the optical array elements. Study has shown the possibility of increasing the resolution of optical systems with additional "virtual" channels for receiving signals. We studied an accuracy of the field approximation on the plane of the optical antenna array elements, as well as above the plane of the optical antenna array.

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