Can there be a Fock state of the radiation field?

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We analyze possible hurdles in generating a Fock state of the radiation field in a micromaser cavity.

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We plan to answer this question with the Munich micromaser in mind [1]. It consists of a superconducting cavity maintained at $T = 0.3K$. Hence, the average thermal photons present in the cavity is $\bar{n}_{th} = 0.033$. The cavity dissipation parameter $\kappa = \nu/2Q$ stands at 3.146 Hz with the cavity $Q = 3.4 \times 10^9$ and $\nu$ being the masing frequency. A clever velocity selector sends $^{85}Rb$ atoms in the upper of its two Rydberg levels into the cavity at such a rate that at most one atom is present there at a time. Also, the velocity selector maintains a constant flight time for each and every atom through the cavity. This is crucial for the Jaynes-Cummings [2] interaction between the single mode of the cavity and the atom present there. The attempt is to generate a Fock state of the cavity radiation field. To start with, the cavity is in thermal equilibrium having the normalized variance

$$v = \sqrt{\langle n^2 \rangle - \langle n \rangle^2}/\langle n \rangle = \sqrt{1 + \bar{n}_{th}}.$$ (1)

The cavity at $T = 0.3K$ has $v = 1.0164$. The evolution of $v$ has to be from this value to zero if one plans to generate a photon Fock state in the cavity.

Our earlier analysis [3] indicated such a possibility if and only if $\bar{n}_{th} = 0$, that is, the cavity temperature has to be at $T = 0K$, a feat unattainable experimentally. However, the theory there followed an iterative procedure. Surely, we have to adopt an exact procedure in order to get a correct answer to the question in the title of this letter. Further, the reservoir effects have to be properly addressed to since the Fock states are very amenable to the dissipative forces. For this reason we find other approaches in the literature [4] unsuitable for the present purpose since the cavity dissipation is completely neglected ($Q = \infty$) there during the atom-field interaction. Hence, we look for a solution of the equation of motion

$$\dot{\rho} = -i[H, \rho] - \kappa(1 + \bar{n}_{th})(a a^\dagger \rho - 2a \rho a^\dagger + \rho a^\dagger a)$$
$$-\kappa\bar{n}_{th}(aa^\dagger \rho - 2a^\dagger \rho a + \rho a a^\dagger)$$ (2)

describing the situation whenever a atom is present in the cavity [5]. $H$ is the Jaynes-Cummings Hamiltonian [2] in interaction picture given by

$$H = g(S^+ a + S^- a^\dagger)$$ (3)

with $g$ representing the strength of the atom-field coupling. $a$ is the photon annihilation operator and $S^+$ and $S^-$ are the Pauli pseudo-spin operators for the two-level system. As mentioned earlier, a atom takes a time $\tau$ to pass through the cavity. These atomic events are seperated by random durations, $t_{cav}$, during which the cavity evolves under its own dynamics. Hence we set $H = 0$ in Eq. (2) during $t_{cav}$. Processes like these atomic events seperated by random intervals are known as Poisson processes in literature encountered in various branches of physics, for example, radioactive materials emitting alpha particles. A sequence of durations of such processes can be obtained from uniform deviates, also called random numbers, $x$ generated using a computer such that $0 < x < 1$, and then by using the relation [6]

$$t_R = -\mu \ln(x)$$ (4)

where $t_R = t_{cav} + \tau$. $\mu = 1/R$ where $R$ is the flux rate of atoms.

We have carried out numerical simulation of the dynamics with the data taken from the experimental arrangements

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[1] in which $g = 39 \, kHz$ and the $\tau = 40 \, \mu s$ was one of the atom-field interaction times. This gives $g\tau = 1.56$, a condition required for generating a Fock state of $n_0$ photons where $n_0$ satisfies $\sin g\tau \sqrt{n_0 + 1} = 0$ in an ideal cavity ($Q = \infty$) [7]. Since the experimental arrangements are close to ideal situation, it was hoped that such Fock states could be attained experimentally. Indeed, such results have been reported in Ref. 1. However, our numerical simulations [8] does not confirm these conclusions. Instead, it gives photon fields with very narrow distribution functions (sub-Poissonian) centred about $n$. Figs. 1 and 2 display distribution function $P(n)$ narrowly centred about $n_0 = 14$.

The reason for these results is simple. The cavity dissipation, although very small, effects the coherent atom-field interaction and moreover the randomness in $t_{cav}$ makes the photon distribution function fluctuate all the time centred about $n_0$ in addition to making it broader.

In this experiment [1], the atoms coming out of the cavity are subjected to measurements from which state of the cavity field is inferred. The atoms enter the cavity in the upper $|a\rangle$ of the two states $|a\rangle$ and $|b\rangle$. The exiting atom is, in general, in a state

$$|\psi\rangle = a|a\rangle + b|b\rangle$$

with $p_a = |a|^2$ and $p_b = |b|^2$ are the probabilities of the atom being in the states $|a\rangle$ and $|b\rangle$ respectively. According to the Copenhagen interpretation of quantum mechanics [9], this wave function collapses (or is projected) to either $|a\rangle$ or $|b\rangle$ the moment a measurement is made on it. Due to this inherent nature of quantum mechanics, a noise is associated with the measurement which is known as quantum projection noise [10]. We define the projection operator $J = |a\rangle\langle a|$. The variance in its measurement is given by

$$(\Delta J)^2 = \langle J^2 \rangle - \langle J \rangle^2 = p_a(1 - p_a)$$

We find that $(\Delta J)^2 = 0$ only when $p_a = 1$ or 0. For the generation of a Fock state, it is necessary that the atom should leave the cavity unchanged in its upper state [3,4,7]. Hence, for such a situation we must have $p_a = 1$ in which case $(\Delta J)^2$ should be 0. We find from our numerical simulations that that $p_a = P(a)$ is mostly about 0.8 [Fig. 3] and, hence, $(\Delta J)^2 \neq 0$ always. This obviously indicates that the cavity field is in a linear superposition
of Fock states giving a photon distribution function with the normalized variance $v > 0$ (For a Fock state $v = 0$). Indeed, we find that the $v$ is about 0.5 in our calculations, presented in Fig. 4, indicating a sub-Poissonian nature of the cavity field. By itself, it carries a signature of quantum mechanics. We further notice in Fig. 4 that there are small fluctuations in $v$ due to the fluctuations in $P(n)$ [Figs 1 and 2]. Also, $v$ is nowhere near 0 in Fig. 4.

The field ionization techniques used in Ref. [1] to detect the outgoing atomic states, obviously, cannot incorporate the above quantum projection noise. The photon statistics inferred from the measured atomic statistics would then be correct only to the extent one could afford to neglect the quantum noise. But, our observations in Figs. 1-4 clearly show that this is crucial for the generation of a photon Fock state. In other words, the situation $\Delta J = 0$ just does not happen due to the non-stop dissipation of the cavity field and also due to the randomness in $t_{cav}$. Further, as mentioned earlier, the small but finite $\bar{n}_t$ in the equation of motion (Eq. 2) has a major influence on this dissipation [5]. Hence, the analysis of the micromaser dynamics in Ref. 11 does not show cavity field dissipation clearly since the effects of the finite cavity temperature has not been properly included there.

We have carried out simulation until about ten thousand atoms passed through the cavity and carrying out the simulations any further would only be a repetition of the fluctuations in Figs. 1-4. In other words, the nagging reservoir dynamics does not allow the cavity field to settle down to a photon number state. A similar conclusion can also be inferred in a recent report [12] where the authors show that Fock states are fragile in thermal baths.

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