Astrophysics independent bounds on the annual modulation of dark matter signals

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We show how constraints on the time integrated event rate from a given dark matter (DM) direct detection experiment can be used to set a stringent constraint on the amplitude of the annual modulation signal in another experiment. The method requires only very mild assumptions about the properties of the local DM distribution: that it is temporally stable on the scale of months and spatially homogeneous on the ecliptic. We apply the method to the annual modulation signal in DAMA/LIBRA, which we compare to the bounds derived from the constraints on the time-averaged rates from XENON10, XENON100, CDMS and SIMPLE. Assuming a DM mass of 10 GeV, we show that a DM interpretation of the DAMA/LIBRA signal is excluded at 6.3σ (4.6σ) for isospin conserving (violating) spin-independent interactions, and at 4.9σ for spin-dependent interactions on protons.

Introduction. Dark matter (DM) constitutes a significant fraction of the energy density in the universe, ΩDM = 0.229±0.015 [1]. This conclusion is based entirely on gravitational effects of DM. A fundamental question is whether DM interacts also non-gravitationally. There are a number of experiments searching for signs of such DM interactions. Direct detection experiments, for instance, are looking for a signal of DM particles from the galactic halo that would scatter in underground detectors. A characteristic feature of the resulting signal will be an annual modulation, because the earth rotates around the sun, while at the same time the sun moves relative to the DM halo [2].

At present two experiments are reporting annually modulated signals, DAMA/LIBRA [3] (DAMA for short) and CoGeNT [4], with significances of 8.9σ and 2.8σ, respectively. Are these signals due to DM? The answer is readily obtained by (i) assuming a specific local DM velocity distribution and (ii) postulating the predominant DM–nucleus interaction. Usually a simple Maxwellian DM halo is adopted. If interpreted in terms of elastic spin-independent DM scattering both claims are in tension [5, 6] with bounds on time integrated rates from other direct detection experiments such as XENON10 [7], XENON100 [8], or CDMS [9]. The situation may change other direct detection experiments such as XENON10 [7], XENON100 [8], or CDMS [9]. The situation may change in the case of non-standard DM halos with, e.g., highly anisotropic velocity distributions, DM streams or DM deflimb flows. Recently CDMS provided a direct bound on the anisotropic velocity distributions, DM streams or DM deflimb flows. The resulting bounds present roughly an order of magnitude improvement over [11, 12] and [13].

The bounds are (almost completely) astrophysics independent. Only very mild assumptions about DM halo properties are used: (i) that it does not change on the time-scales of months, (ii) that the density of DM in the halo is constant on the scales of the earth-sun distance, and (iii) that the DM velocity distribution is smooth on the scale of the earth velocity ve = 29.8 km/s. If the modulation signal is due to DM, then the modulation amplitude has to obey the bounds. In the derivation an expansion in ve over the typical DM velocity ∼ 200 km/s is used. The validity of the expansion can be checked experimentally, by searching for the presence of higher harmonics in the time-stamped DM scattering data [13].

Bounds on the annual modulation. We focus on the case of DM χ elastically scattering off a nucleus (A, Z) and depositing the nuclear recoil energy Enr in the detector. The differential rate in events/keV/kg/day is then given by

\[
R_A(E_{nr}, t) = \frac{\rho_\chi \sigma_0^{SI}}{2m_\chi} \frac{\mu_A^2}{\mu_X^2} F_A^2(E_{nr}) \eta(v_m, t),
\]

with ρ_χ the local DM density, σ_0^{SI} the total DM–nucleus scattering cross section at zero momentum transfer, m_χ the DM mass, and \mu_X/\mu_A the reduced mass of the DM–nucleus system. F_A(E_{nr}) is a nuclear form factor. For SI interactions with a nucleus (A, Z), σ_0^{SI} can be written as σ_0^{SI} = \sigma_p[Z + (A - Z)(f_n/f_p)]^2 \mu_X^2/\mu_A^2, where \sigma_p is the DM–proton cross-section and f_n/p are coupling strengths to neutron and proton, respectively. Apart from a common overall factor \rho_\chi the astrophysics enters the pre-
predicted rate in Eq. (1) through the halo integral
\[ \eta(v_m, t) = \int_{v>v_m} d^3v f_{\text{det}}(v, t) \frac{1}{v}, \quad v_m = \sqrt{\frac{m_A E_{nr}}{2\mu^2_{\chi A}}}, \]  
(2)

where \( v_m \) is the minimal velocity required to have at least \( E_{nr} \) energy deposited in the detector. The function \( f_{\text{det}}(v, t) \) describes the distribution of DM particle velocities in the detector rest frame with \( f_{\text{det}}(v, t) \geq 0 \) and \( \int d^3v f_{\text{det}}(v, t) = 1 \). It is related to the velocity distribution in the rest frame of the sun by \( f_{\text{det}}(v, t) = f_{\text{sun}}(v + v_e(t)) \), where \( v_e(t) \) is the velocity vector of the earth. The rotation of the earth around the sun introduces a time dependence in the DM-nucleus scattering rate through \( \eta(v_m, t) = \eta(v_m) + \delta\eta(v_m, t) \), where
\[ \delta\eta(v_m, t) = A_{\eta}(v_m) \cos 2\pi[t − t_0(E_{nr})], \]  
(3)

when expanding to first order in \( v_e = 29.8 \text{ km/s} \ll v_{\text{sun}} \simeq 230 \text{ km/s} \). Here, \( A_{\eta}(v_m) \) is defined to be positive.

Let us now assume that \( f_{\text{sun}}(v) \) is smooth on the scale of \( v_e \), and the only time dependence comes from the rotation of the earth around the sun and \( f_{\text{sun}}(v) \) itself is constant in time and space. Then the modulation amplitude \( A_{\eta}(v_m) \) can be bounded in terms of the unmodulated halo integral \( \eta \) in the following way [13]:
\[ A_{\eta}(v_m) \leq v_e \left[ -\frac{d\eta}{dv_m} + \frac{\eta(v_m)}{v_m} - \int_{v_m} dv \frac{\eta(v)}{v^2} \right]. \]  
(4)

The first term in Eq. (4) is positive since \( \eta(v_m) \) is a monotonously decreasing function of \( v_m \). If we further assume that the DM halo is symmetric, so that there is only one single direction related to the DM flow (see [13] for details), then one obtains a more stringent constraint:
\[ \int_{v_1}^{v_2} dv_m A_{\eta}(v_m) \leq \sin \alpha v_e \left[ \eta(v_1) - v_1 \int_{v_1}^{v_2} \frac{\eta(v)}{v^2} \right]. \]  
(5)

Here \( \alpha \) is the angle between the DM flow and the direction orthogonal to the ecliptic. The most conservative bound is obtained for \( \sin \alpha = 1 \) (which would correspond to a DM stream parallel to the ecliptic). However, in many cases the DM flow will be aligned with the motion of the sun within the galaxy. This holds for any isotropic velocity distribution and, up to a small correction due to the peculiar velocity of the sun, also for tri-axial halos or a significant contribution from a possible dark-disc. In this case we have \( \sin \alpha \simeq 0.5 \).

In the following we will use time averaged rates from various experiments to derive an upper bound on \( \eta(v_m) \). In order to be able to apply this information we integrate Eq. (4) over \( v_m \) and drop the negative terms in Eqs. (4) and [15]. This gives the bounds
\[ \int_{v_1}^{v_2} dv_m A_{\eta}(v_m) \leq v_e \left[ \eta(v_1) + \int_{v_1}^{v_2} \frac{\eta(v)}{v} \right], \]  
(6)
\[ \int_{v_1}^{v_2} dv_m A_{\eta}(v_m) \leq \sin \alpha v_e \eta(v_1), \]  
(7)

In practice the integrals on the l.h.s. are replaced by a sum over bins. Below we will refer to the relations (6) and [7] as the bounds from “general halo” and “symmetric halo” (where we will take \( \alpha = 0.5 \)), respectively.

**Bounds on the unmodulated halo integral.** Let us first consider SI scattering with \( f_n = f_p \). Generalization to isospin violating scattering with \( f_n \neq f_p \) and to SD scattering is straightforward. The predicted number of events in an interval of observed energies \( [E_1, E_2] \) is given by
\[ N_{[E_1, E_2]}^{\text{pred}} = MTA^2 \int_{0}^{\infty} dE_{nr} F^2_{\chi}(E_{nr}) G_{[E_1, E_2]}(E_{nr}) \eta(v_m). \]  
(8)

Here \( G_{[E_1, E_2]}(E_{nr}) \) is the detector response function, which describes the contribution of events with true nuclear-recoil energy \( E_{nr} \) to the observed energy interval \( [E_1, E_2] \). It may be non-zero outside the \( E_{nr} \in [E_1, E_2] \) interval due to the finite energy resolution and includes also (possibly energy dependent) efficiencies. \( M \) and \( T \) are the detector mass and exposure time, respectively, and we defined
\[ \tilde{\eta} = \frac{\sigma_p p_{\chi}}{2m_{\chi} \mu^2_{\chi p}} \eta, \]  
(9)

where \( \tilde{\eta} \) has units of events/kg/day/keV.

Now we can use the fact that \( \tilde{\eta} \) is a falling function [11] (see also [14] [15]). Among all possible forms for \( \tilde{\eta} \) such that they pass through \( \tilde{\eta}(v_m) \) at \( v_m \), the minimal number of events is obtained for \( \tilde{\eta} \) constant and equal to \( \tilde{\eta}(v_m) \) until \( v_m \) and zero afterwards. Therefore, for a given \( v_m \) we have a lower bound \( N_{[E_1, E_2]}^{\text{obs}}(v_m) \geq \mu(v_m) \) with
\[ \mu(v_m) = MTA^2 \int_{0}^{E_{\text{pred}}} dE_{nr} F^2_{\chi}(E_{nr}) G_{[E_1, E_2]}(E_{nr}), \]  
(10)

where \( E(v_m) \) is given in (2). Suppose an experiment observes \( N_{[E_1, E_2]}^{\text{obs}} \) events in the interval \( [E_1, E_2] \). Then we can obtain an upper bound on \( \tilde{\eta} \) for a fixed \( v_m \) at a confidence level CL by requiring that the probability of obtaining \( N_{[E_1, E_2]}^{\text{obs}} \) events or less for a Poisson mean of \( \mu(v_m) \) is equal to 1−CL. Note that this is actually a lower bound on the CL, since Eq. (10) provides only a lower bound on the true Poisson mean. For the same reason we cannot use the commonly applied maximum-gap method to derive a bound on \( \tilde{\eta} \). If several different nuclei are present, there will be a corresponding sum in Eqs. (6) and (10).

The limit on \( \tilde{\eta} \) can then be used in the r.h.s. of Eq. (6) or (7) to constrain the modulation amplitude. For concreteness we first focus on the annual modulation in DAMA. If \( m_{\chi} \) is around 10 GeV, then DM particles do not have enough energy to produce iodine recoils above the DAMA threshold. We can thus assume that the DAMA signal is entirely due to the scattering on sodium. We define \( A_{\eta} = \sigma_p p_{\chi}/(2m_{\chi} \mu^2_{\chi p}) A_{\eta} \), which is related to the
observed modulation amplitude $A_{\eta}^{obs}$ by

$$A_{\eta}^{obs}(v_m^i) = \frac{A_{\eta}^{obs} q_{Na}}{A_{Na}^{i}(F_{Na}^i/v_{Na})}.$$  \hspace{1cm} (11)

Here $q_{Na} = dE_{ee}/dE_{nr}$ is the sodium quenching factor translating keVee into keVnr, for which we take $q_{Na} = 0.3$. The index $i$ labels energy bins, with $v_m^i$ given by the corresponding energy bin center using Eq. \hspace{1cm} (2).

Further, $(F_{Na}^{i})$ is the sodium form factor averaged over the bin width and $f_{Na} = m_{Na}/(m_{Na} + m_i)$ is the sodium mass fraction of the NaI crystal. For the modulation amplitude in CoGeNT we proceed analogously. Note that the conversion factor from $\eta$ to $\tilde{\eta}$ is the same as for $A_{\eta}$ to $\tilde{A}_{\eta}$, and does not depend on the nucleus. Therefore, the bounds (6) and (7) apply to $\tilde{\eta}$, $\tilde{A}_{\eta}$ without change, even if the l.h.s. and r.h.s. refer to different experiments.

Let us briefly describe the data we use to derive the upper bounds on $\tilde{\eta}$. We consider results from XENON10 \hspace{1cm} (XE10) and XENON100 \hspace{1cm} (XE100). In both cases we take into account the energy resolution due to Poisson fluctuations of single electrons. XE10 is sensitive to the interesting region of $v_m$ only because of upward fluctuations from below the threshold. We adopt the best-fit light-yield efficiency $L_{\text{eff}}$ from \hspace{1cm} [5]. The XE10 analysis is based on the so-called S2 ionization signal which allows to go to a rather low threshold. We follow \hspace{1cm} [7] and impose a sharp cut-off of the efficiency below the threshold. From CDMS we use results from a dedicated low-threshold (LT) analysis \hspace{1cm} [4] of Ge data, as well as data on Si \hspace{1cm} [10]. In the case of SD scattering on protons particularly strong bounds are obtained from experiments with a fluorine target. We consider the results from SIMPLE \hspace{1cm} [17], which uses F$_3$C$_2$Cl. We use the observed number of events and expected background events to calculate the combined Poisson probability for Stage 1 and 2. For the prediction we include energy dependent threshold efficiencies from \hspace{1cm} [17].

For all experiments we use the lower bound on the expected events, Eq. \hspace{1cm} (10), to calculate the probability of obtaining less or equal events than observed. For XE100, CDMS Si, and SIMPLE we just use the total number of events in the entire reported energy range. For XE10 and CDMS LT the limit can be improved if data are binned and the corresponding probabilities for each bin are multiplied. This assumes that the bins are statistically independent, which requires to make bins larger than the energy resolution. For XE10 we only use two bins. For CDMS LT we combine the 36 bins from Fig. 1 of [9] into 9 bins of 2 keV where the energy resolution is 0.2 keV.

Results. In Fig. \hspace{1cm} 1 we show the 3$\sigma$ limits (CL = 99.73\%) on $\tilde{\eta}$ compared to the modulation amplitudes $\tilde{A}_{\eta}$ from DAMA and CoGeNT for a DM mass of 10 GeV. Similar results have been presented in \hspace{1cm} [14, 15]. The CoGeNT amplitude depends on whether the phase is floated during data taking at June 2nd \hspace{1cm} [6], which applies to the “general” and “symmetric” halos, respectively. Already at this level XE100 is in tension with the modulation from DAMA (and to some extent also CoGeNT).

We now apply our method. As shown in Fig. \hspace{1cm} 2 the null search results become significantly more constraining after applying the bounds on the integrated annual modulation $f_{v_1}^{v_2} dv A_q$. From Eq. \hspace{1cm} (6), DAMA and CoGeNT are strongly excluded by the bounds from XE100, XE10, CDMS LT even for the general halo. If one were to assume in addition that the halo is symmetric, the bounds would get even stronger. Then also CDMS Si excludes DAMA, and there is some tension with SIMPLE (not shown).

In Fig. \hspace{1cm} 3 we consider two variations of DM–nucleus interaction. The upper panel is for the case when the DM particle couples to the spin of the proton. The null search result of Xe and Ge experiments are then irrelevant. However, the bound from SIMPLE is in strong disagreement with the modulation signal in DAMA, due to the presence of fluorine in their target. (A comparable limit from fluorine has been published recently by PICASSO \hspace{1cm} [13]). In the lower panel of Fig. \hspace{1cm} 3 we
show the case of SI isospin violating interactions with $f_n/f_p = -0.7$. This choice evades bounds from Xe, but now the DAMA modulation is excluded by the bounds from CDMS Si for the general halo and CDMS Si, LT, and SIMPLE for the symmetric halo.

Let us now quantify the disagreement between the observed DAMA modulation and the rate from another null-result experiment using our bounds. We first fix $v_m$. To each value of $\tilde{\eta}(v_m)$ Eq. (10) provides a Poisson mean $\mu(v_m)$. We can then calculate the probability $p_\eta$ to obtain equal or less events than measured by the null-result experiment. Then we construct the bound on the modulation using the same value $\tilde{\eta}(v_m)$ on the r.h.s. of Eq. (6) or (7) (the integrand $\tilde{\eta}(v)$ in Eq. (6) is calculated using the same $p_\eta$ but with $v > v_m$ in Eq. (10)). We calculate the probability $p_A$ that the bound is not violated by assuming on the l.h.s. of Eq. (6) or (7) a Gaussian distribution for the DAMA modulation signal with the measured standard deviations in each bin. Then $p_{\text{point}}(\tilde{\eta}) = p_\eta p_A$ is the combined probability of obtaining the experimental result for the chosen value of $\tilde{\eta}$. Then we maximize $p_{\text{point}}(\tilde{\eta})$ with respect to $\tilde{\eta}$ to obtain the highest possible joint probability.

The results of such an analysis are shown in Fig. 4. The analysis is performed at the fixed $v_m$ corresponding to the 3rd modulation data point in DAMA, depending on the DM mass $m_\chi$. We find that for all considered interaction types and $m_\chi \lesssim 15$ GeV at least one experiment disfavors a DM interpretation of the DAMA modulation at more than 4σ even under the very modest assumptions of the “general halo”. In the case of SI interactions the tension with XE100 is at more than 6σ for $m_\chi \gtrsim 8$ GeV and saturates at the significance of the modulation data point itself at about 6.4σ for $m_\chi \gtrsim 13$ GeV. The exclusion from XE10 is nearly independent of the DM mass slightly below 6σ. We show also a few examples of the joint probability in case of a “symmetric halo” (dashed curves).

While astrophysics uncertainties are avoided, the obtained bounds are still subject to nuclear, particle physics and experimental uncertainties. For instance, the tension between the DAMA signal and the bounds depends on the value of the Na quenching factor $q_{Na}$, light yield or ionization yield efficiencies in Xe, upward fluctuations from below threshold, and so on. For example, if a value of $q_{Na} = 0.45$ is adopted instead of the fiducial value of 0.3 consistency for SD and isospin violating interactions can be achieved in the case of the general halo at around 3σ, while for SI interactions the XE10 bound still implies tension at more than 5σ for $m_\chi \gtrsim 10$ GeV. Hence, the precise CL of exclusion may depend on systematic uncertainties.

In conclusion, we have presented a powerful method to check the consistency of an annual modulation signal in a DM direct detection experiment with bounds on the total DM scattering rate from other experiments, almost completely independent of astrophysics, for a given type of DM–nucleus interaction. While our bounds strongly disfavor a DM interpretation of present annually modulated signals in the case of SI and SD elastic scattering, the method will be an important test that any future modulated signal will have to pass before a DM interpr-
tation can be accepted.

Acknowledgements: J.H.-G. is supported by the

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