Analysis of near-field subwavelength focusing of hybrid amplitude–phase Fresnel zone plates under radially polarized illumination

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Abstract
The near-field focusing properties of actual hybrid amplitude–phase binary subwavelength Fresnel zone plates (HBSFZPs) are studied theoretically. The analysis based on the exact vector angular spectrum method is done for a radially polarized beam incident on the HBSFZPs. The results show that the near-field subwavelength focusing with a long depth of focus can be obtained using an HBSFZP, which is very useful for near-field subwavelength photolithography and high-resolution microscopy. The position of the actual focus in the near-field focusing HBSFZPs depends on the evanescent wave rather than the propagating wave. The etch depth has an important influence on focusing properties of HBSFZPs.

Keywords: subwavelength Fresnel zone plate, near-field focusing, angular spectrum theory

1. Introduction
Radially polarized (RP) beams are of particular interest. A highly focused RP beam can generate a strong longitudinal electric field at the focus [1]. This strong longitudinal component forms a tight focal spot that can improve the resolution of microscopy [2, 3], enhance laser cutting ability in material processing [4], decrease the size of the recording spot in optical storage [5], and trap microscopic Rayleigh particles [6–9]. Liu et al analyzed the far-field focusing ability of a binary amplitude-only superoscillatory lens illuminated by an RP beam [10]. Recently, several research works have proposed the use of an RP beam as an incident beam illuminating subwavelength Fresnel zone plates to achieve superresolution focus [11, 12]. Mote et al [11] analyzed the subwavelength focusing behavior of a high-numerical-aperture (NA) phase-only Fresnel zone plate (FZP) using the vector diffraction theory of Richards and Wolf [13]. It is known that the vector diffraction theory of Richards and Wolf is based on the Debye approximation where the diffraction effect due to the aperture edge is neglected [14]. However, the effect of the aperture edge is important for a high-NA FZP in near-field diffraction, which has to be considered. Carretero et al calculated the near-field focusing of high-NA amplitude-only FZPs using the FDTD simulation [12]. However, they did not analyze in detail the contribution of evanescent and propagating waves in the near-field focusing region of $z < \lambda$. On the other hand, an actual FZP is in general a hybrid amplitude–phase zone plate rather than a phase-only or amplitude-only zone plate.

The vector angular spectrum theory is an exact diffraction one [10, 15, 16]. The effects of evanescent and propagating waves on the diffraction field can be separately analyzed using this theory. In this paper, we use the vector angular spectrum theory to exactly compute the focusing properties.
of the actual hybrid amplitude–phase binary subwavelength Fresnel zone plates (HBSFZPs) in the near-field region of $z < \lambda$. The contribution of evanescent and propagating waves to the diffraction field generated by an HBSFZP is analyzed in detail under RP illumination. The influence of the etch depth on focusing properties of an actual HBSFZP is discussed. We find that the considered HBSFZPs under an RP beam of incidence can generate a sharper spot with a longer depth of focus in the near-field region, which will be very useful for near-field subwavelength photolithography and high-resolution microscopy.

2. Angular spectrum representation for diffraction fields of an HBSFZP

Figure 1 depicts the schematic structure of a binary circular FZP that will be considered in our analysis. The FZP is etched in the dielectric film with refractive index $n$ and thickness $h$. The etch depth of the FZP is $d$ and $F$ denotes the designed focus. According to the angular spectrum representation [16], the electric field in the $z > 0$ half-space can be expressed, in the cylindrical-coordinate system, as

$$E_z(\rho, \eta, z) = -\frac{1}{(2\pi)^2} \int_0^\infty \int_0^{2\pi} A_k(\xi, \phi) \times \exp[i\xi \rho \cos(\phi - \eta)] \exp(i\xi_z z) \xi \, d\xi \, d\phi,$$

$$E_x(\rho, \eta, z) = -\frac{1}{(2\pi)^2} \int_0^\infty \int_0^{2\pi} A_k(\xi, \phi) \times \exp[i\xi \rho \cos(\phi - \eta)] \exp(i\xi_x z) \xi \, d\xi \, d\phi,$$

$$E_y(\rho, \eta, z) = \frac{1}{(2\pi)^2} \int_0^\infty \int_0^{2\pi} \left[ \frac{\xi_x}{\xi_z} A_x(\xi, \phi) + \frac{\xi_y}{\xi_z} A_y(\xi, \phi) \right] \times \exp[i\xi \rho \cos(\phi - \eta)] \exp(i\xi_z z) \xi \, d\xi \, d\phi,$$

where $k = 2\pi / \lambda$ is the wavenumber and $\lambda$ is the illumination wavelength. $\xi_x = \sqrt{k^2 - \xi_z^2}$ for propagating waves and $\xi_z = i\sqrt{\xi_z^2 - k^2}$ for evanescent waves. $A_x$ and $A_y$ are the inverse Fourier transforms of the fields ($E_{0x}$ and $E_{0y}$) in the $z = 0$ plane,

$$A_x(\xi, \phi) = \int_0^\infty \int_0^{2\pi} E_{0x} \exp[-ix \cos(\phi - \eta)] r \, dr \, d\phi,$$

$$A_y(\xi, \phi) = \int_0^\infty \int_0^{2\pi} E_{0y} \exp[-ix \cos(\phi - \eta)] r \, dr \, d\phi.$$

To obtain the electric field transmitted from the FZP at the plane $z = 0$, we first calculate the transmission field $E'$ of a uniform parallel film with thickness $w$. The transmission electric field $E'$ in the interface of the film can be expressed as

$$E' = R^{-1} TRE_i,$$

where

$$R = \begin{bmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T = \begin{bmatrix} t_p & 0 & 0 \\ 0 & t_s & 0 \\ 0 & 0 & t_p \end{bmatrix}.$$  \hspace{1cm} (7)

The matrix $R$ describes the rotation of the coordinate system around the optical axis ($z$-axis), and $T$ represents the amplitude transmission (Fresnel coefficients) through the film. $t_p$ and $t_s$ are the amplitude transmission coefficients of the film for TE and TM waves, respectively, which can be calculated according to the method in the film optics [14]. Assuming the RP beam is normally incident on the FZP, the incident field can be written as

$$E_i = \begin{bmatrix} \cos \varphi \\ \sin \varphi \\ 0 \end{bmatrix}.$$  \hspace{1cm} (8)

Substituting equations (7) and (8) into (6) yields

$$E'(r, \varphi) = \begin{bmatrix} (t_p \cos^2 \varphi + t_s \sin^2 \varphi) \cos \varphi + (t_p - t_s) \sin^2 \varphi \cos \varphi \\ (t_p \sin^2 \varphi + t_s \cos^2 \varphi) \cos \varphi + (t_p - t_s) \sin \varphi \cos^2 \varphi \\ 0 \end{bmatrix}.$$  \hspace{1cm} (9)

Generally, for $t_p \neq t_s$, the polarization direction of the transmission field $E'$ is different from that of the incident field $E_i$, that is, an optical film has the function of depolarization. When lights are normally incident on the FZP, however, we find $t_p = t_s(=t)$ and then equation (9) becomes

$$E'(r, \varphi) = \begin{bmatrix} t \cos \varphi \\ t \sin \varphi \\ 0 \end{bmatrix}.$$  \hspace{1cm} (10)

Thus, the electric field in the plane $z = 0$ is obtained as

$$E_{0x}(r, \varphi) = \cos \varphi [t_A C(r) + t_B D(r) \exp(ikd)],$$  \hspace{1cm} (11)

$$E_{0y}(r, \varphi) = \sin \varphi [t_A C(r) + t_B D(r) \exp(ikd)],$$  \hspace{1cm} (12)

where $t_A$ and $t_B$ are the transmission coefficients from the dielectric films with thicknesses of $h$ and $(h - d)$ in ridge and...
groove zones of the FZP, respectively, given by

\[ t_A = \frac{a \exp(iknh)}{1 + b \exp(2iknh)} \] (13)
\[ t_B = \frac{a \exp(iknh - d)}{1 + b \exp(2iknh - d)} \] (14)

Here \( a = 4n/(n + 1)^2 \) and \( b = (n - 1)^2/(n + 1)^2 \). It is noted that the amplitude and phase of \( t_A \) and \( t_B \) vary with the etch depth. Thus, an actual FZP is a hybrid amplitude–phase one. \( C(r) \) and \( D(r) \) in equations (11) and (12) are related to the structure of the FZP. For a binary FZP as shown in figure 1, \( C(r) \) and \( D(r) \) can be written as

\[ C(r) = \sum_{n=0}^{N} [\text{circ}(r/r_{2n+1}) - \text{circ}(r/r_{2n})] \] (15)
\[ D(r) = \sum_{n=1}^{N} [\text{circ}(r/r_{2n}) - \text{circ}(r/r_{2n-1})] \] (16)

where the circular function \( \text{circ}(a) = 1 \) for \( r < a \) and 0 for \( r \geq a \). \( r_j \) is the radius of the \( j \)th zone, which is written as

\[ r_j = \sqrt{[(\lambda/2)^2 - d^2]}. \] (17)

Here \( f_0 \) is the designed focal length and \( 2N \) is the number of the FZP’s zones. Using the inverse Fourier transform on equations (11) and (12), it is easy to find

\[ A_\alpha(\xi, \phi) = 2i\pi \cos \phi T(\xi), \] (18)
\[ A_\nu(\xi, \phi) = 2i\pi \sin \phi T(\xi), \] (19)

with

\[ T(\xi) = t_A \sum_{n=0}^{N} \int_{r_{2n+1}}^{r_{2n}} J_1(\rho \xi) r dr \\
+ t_B \exp(ikd) \sum_{n=1}^{N} \int_{r_{2n-1}}^{r_{2n}} J_1(\rho \xi) r dr. \] (20)

Substituting equations (18) and (19) into (1)–(3) and transforming to the cylindrical coordinates, we find that the azimuthal component of the electric field is zero in the whole diffraction space. Nonzero radial and axial components are obtained as

\[ E_\rho(\rho, \xi) = \int_0^\infty T(\xi) J_1(\rho \xi) \exp(iz\xi) d\xi, \] (21)
\[ E_\xi(\rho, \xi) = i \int_0^\infty \frac{\xi}{\xi z} T(\xi) J_0(\rho \xi) \exp(iz\xi) d\xi. \] (22)

In equations (20)–(22), \( J_n \) is the \( n \)th-order Bessel function of the first kind.

For subwavelength focusing HBSFZPs in the near-field region, evanescent waves are important. To investigate the contribution of evanescent and propagating waves, we should separate the integration performed over \( \xi \) as \( \int_0^\infty = \int_0^k + \int_{x_\xi}^\infty \), where the first and second integrations correspond, respectively, to the propagating and evanescent waves. To avoid the problem that occurs at \( \xi = k \) (which implies \( \xi_\xi = 0 \)) for \( E_\xi \), we perform the integration over the normal component \( (\xi_\xi) \) of the wavevector. Hence, one can write

\[ E_\rho(\rho, \xi) = \left( \int_0^k + \int_{x_\xi}^\infty \right) T(\xi) J_1(\rho \xi) \exp(iz\xi) d\xi, \] (23)
\[ E_\xi(\rho, \xi) = i \left( \int_0^k + \int_{x_\xi}^\infty \right) T(\xi) J_0(\rho \xi) \exp(iz\xi) d\xi. \] (24)

In a more detailed form equations (23) and (24) become

\[ E_\rho,\text{pro}(\rho, \xi) = \int_0^k T(\sqrt{k^2 - \xi_\xi^2}) J_1(\rho \sqrt{k^2 - \xi_\xi^2}) \exp(iz\xi), \] (25)
\[ E_\rho,\text{eva}(\rho, \xi) = \int_0^\infty T(\sqrt{k^2 + \alpha^2}) J_1(\rho \sqrt{k^2 + \alpha^2}) \exp(-\alpha \xi_\xi) d\alpha, \] (26)
\[ E_\xi,\text{pro}(\rho, \xi) = i \int_0^k T(\sqrt{k^2 - \xi_\xi^2}) J_0(\rho \sqrt{k^2 - \xi_\xi^2}) \exp(iz\xi), \] (27)
\[ E_\xi,\text{eva}(\rho, \xi) = i \int_0^\infty T(\sqrt{k^2 + \alpha^2}) J_0(\rho \sqrt{k^2 + \alpha^2}) \exp(-\alpha \xi_\xi) d\alpha, \] (28)

where the indices ‘eva’ and ‘pro’ mean the integration corresponds to the evanescent and propagating waves, respectively. It is immediately found from equations (25)–(28) that the diffraction field is rotationally symmetrical about the optical \( z \) axis for the RP beam vertically incident on the FZP. The radial component is zero on the optical axis but the axial component is nonzero.

### 3. Numerical results

In the following investigation, it is assumed that the circular HBSFZP is formed in a SiO\(_2\) film with refractive index \( n = 1.5426 \) and thickness \( h = 1 \) nm and the number of zones is \( 2N = 16 \). The HBSFZP is vertically illuminated by an RP beam with wavelength \( \lambda = 633 \) nm.

Figure 2 shows the total intensity distribution of radiation in a meridian plane of lights. The parameters of the HBSFZP are \( d = 0.6 \) \( \mu \)m and \( f_0 = 0.5 \) \( \mu \)m (< \( \lambda \)). It is clear that the intensity reaches its largest value at \( r = 0 \) for each \( z \) and the actual focus shifts toward the FZP for this FZP. To analyze the contribution of evanescent and propagating waves to the total field, in figure 3, we plot the intensity distributions of evanescent and propagating waves along the \( z \) axis for three different values of \( f_0 = 0.3, 0.5, \) and 0.8 \( \mu \)m. It is seen that, although the propagating wave dominates the total intensity distribution, the evanescent wave has an important influence. The propagating wave can be focused and its focusing point is far away from the designed focus for the three FZPs in figure 3. The focal shift is due to the effect of the aperture, which is a common phenomenon in far-field
Figure 2. Intensity distribution of radiation in a meridian plane. The parameters of the HBSFZP are \( N = 8, \ d = 0.6 \ \mu m, \) and \( f_d = 0.5 \ \mu m. \) The incident RP’s wavelength is \( \lambda = 633 \ \mu m. \) The vertical dash–dot line denotes the designed focal plane.

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optical systems [17, 18]. When the designed zone boundary of an FZP is \( r_j = \sqrt{j\lambda f_d}, \) the focal shift of the propagating wave is negative, i.e., the actual focus shifts toward the FZP [19]. For the considered FZP whose designed radius is given in equation (17), however, we find that the focal shift of the propagating wave is positive, which is due to the factor of \((j\lambda/2)^2\) in equation (17). When the designed focal length is smaller than the illumination wavelength, the position of the actual focus is dominated by the evanescent wave and the focal shift is irrelevant to the propagating wave. We find that the focus of the evanescent wave is relevant to the zone number of the zones rather than to the designed focal length, as shown in figures 3(a) and (b).

The evanescent wave is gradually attenuating and cannot be focused when \( f_d > \lambda \) (see figure 3(c)). However, when \( f_d < \lambda, \) the evanescent wave, like the propagating wave, can be focused and the actual focusing point of the total intensity depends on the focusing point of the evanescent wave rather than on that of the propagating wave. We find that the focal shift in figure 3(a) is negative (i.e., the actual focal length of \( f_a = 0.387 \ \mu m\) is smaller than the designed focal length of \( f_d = 0.5 \ \mu m\)), which is similar to the result for a linearly polarized incident beam [20], but the focal shift in figure 3(b) is positive (\( f_d = 0.387 \ \mu m > f_a = 0.3 \ \mu m\)). If the depth of focus (DoF) is defined as the distance where the intensity becomes half of the maximum axial intensity, DoF = 0.947 \( \mu m \) (1.496\( \lambda \)) for the two HBSFZPs of \( f_d = 0.5 \) and 0.3 \( \mu m. \)

Figure 4(a) shows the intensity distributions in the actual focal plane, while figure 4(b) shows the full-width at half-maximum (FWHM) and peak intensity of the spot as a function of the observation plane. From figure 4(a) it is seen that the axial component \( (I_z) \) dominates the total intensity \( (I) \) and the contribution of the radial component \( (I_r) \) can be ignored. The FWHM of the spot at the actual focus of \( f_d = 0.387 \ \mu m \) is 0.374\( \lambda \), in figure 4(a), which is a small difference from the 0.378\( \lambda \) FWHM at the designed focus of \( f_d = 0.5 \ \mu m. \) In fact, the variation of FWHM is quite small in the whole DoF range of 1.496\( \lambda \), where the maximum FWHM is 0.379\( \lambda \), and the minimum FWHM is 0.363\( \lambda \) in figure 4(b). The long DoF and high resolution are useful for near-field photolithography and microscopy systems [21, 22]. By contrast, for an aplanatic lens with the same illumination and numerical aperture, our calculations

![Figure 3](image-url) Axial intensity distributions of the HBSFZPs with \( N = 8 \) and \( d = 0.6 \ \mu m. \) The designed focal lengths \( (f_d) \) in (a), (b), and (c) are, respectively, 0.5 \( \mu m, \) 0.3 \( \mu m, \) and 0.8 \( \mu m. \) The solid, dashed, and dotted curves correspond, respectively, to the intensities of the total field, the propagating wave, and the evanescent wave. The vertical dash–dot line denotes the focal plane at the designed focus. Inset plots show the normalized axial intensity distributions.
based on the vector diffraction theory of Richards and Wolf show that the DoF is $1.064\lambda$ and the FWHM in this DoF range varies in the range of $0.553\lambda-0.529\lambda$. This implies that the considered HBSFZP with long DoF and small FWHM may be more useful for near-field subwavelength photolithography and high-resolution microscopy.

Figure 5 shows the effects of the etch depth on the focusing properties of the considered HBSFZP. It is seen from figure 5(a) that the peak intensity and FWHM of the focusing spot oscillate periodically with the etch depth. It is well known that, for a phase-only FZP, the focusing intensity is maximum when the phase shift of the FZP is equal to $\pi$ \cite{10}. However, for the HBSFZP in figure 5, we find that when $d = 0.596 \, \mu m$, which corresponds to the phase shift $0.972\pi$ between the ridge and groove zones, the peak intensity of the spot is maximum. This difference is due to the different amplitude transmission coefficients in the ridge and groove zones in the HBSFZP. If we assume that the minimum acceptable value of the peak intensity is equal to $80\%$ of the maximum intensity at $d = 0.596 \, \mu m$, the HBSFZP in figure 5 presents a usable focusing field only in the interval $d \in [0.41, 0.77] \, \mu m$. In this interval, the FWHM is in the range of $0.371\lambda-0.374\lambda$, the DoF in the range of $1.499\lambda-1.559\lambda$, and the actual focal length in $0.39-0.42 \, \mu m$. In addition, it is of interest to note that, as $d$ increases, the focus shift monotonically increases and the DoF exhibits a stair-like decreasing behavior in figure 5(b). Our explanation for these is as follows. On the one hand, the effect of the evanescent wave becomes weaker for larger $d$ so that the total intensity distribution is closer to that of the propagating wave. Because the DoF of the propagating wave is larger than that of the total wave, the actual DoF increases with the increase of $d$. On the other hand, as $d$ increases, the diffraction distance of the evanescent wave in the groove zones becomes shorter and shorter, and the focus of the evanescent wave shifts to the FZP. We have shown in figure 3 that for the $f_a < \lambda$ HBSFZPs the position of actual focus depends on the focusing point of the evanescent wave and consequently $f_a$ decreases as $d$ increases, as shown in figure 5(b).

4. Conclusion

In conclusion, we give a comprehensive study on the near-field subwavelength focusing properties of actual HBSFZPs illuminated by an RP beam. Our analysis is based on the exact vector angular spectrum theory. By studying the dependence of the focusing properties on the HBSFZP’s parameters including the designed focal length and etch depth, we found that, when the designed focal length is smaller than the illumination wavelength, the evanescent wave can be focused by the HBSFZPs and the actual focal position is dominated by the focusing evanescent wave. When the designed focal length is larger than the illumination wavelength, the evanescent wave is monotonically attenuating with the distance from the HBSFZP. A positive or negative focal shift in the near-field focusing depends on the designed focal length of the FZP. The etch depth has an important influence on the focusing properties, including the FWHM, peak intensity, focal shift, and depth of focus. When a near-field focusing HBSFZP has a suitable etch depth, it can generate the subwavelength focusing spot with a long depth of focus, which is very useful for the near-field subwavelength photolithography and high-resolution microscopy.

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