No Holography for Eternal AdS Black Holes

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Abstract: It is generally believed that the eternal AdS black hole is dual to two conformal field theories with compact spatial sections that are together in a thermofield double state. We argue that this proposal is incorrect, and by extension so are the “entanglement=geometry” proposal of Van Raamsdonk and “ER=EPR” proposal of Maldacena and Susskind. We show that in the bulk there is an interaction needed between the two halves of the Hilbert space for connectivity across the horizon; however, there is no such interaction between the CFTs. This rules out the possibility of the dual to the CFTs being the eternal AdS black hole. We argue the correct dual “geometries” resemble the exterior of the black hole outside the stretched horizon but cap off before the global horizon. This disallows the possibility of a shared future (and past) wedge where Alice falling from one side can meet Bob falling from the other. We expect that in the UV complete theory the aforementioned caps will be fuzzballs.
1 Introduction

A standard piece of the AdS/CFT dictionary, first advocated in [1], is the duality between eternal AdS black holes and the thermofield double formalism for finite temperature conformal field theories.
We argue here that the proposed duality does not exist except in its most limited form. While the two asymptotic CFTs do describe the physics outside the black hole’s stretched horizons, they do not describe any black hole interior degrees of freedom. This is not the first argument against a description of the shared forward wedge of eternal AdS black holes in terms of a thermofield double state of two CFTs (for instance, see [2–5]); however, the arguments presented here are new and stronger. We argue that there is no extension or completion of the two CFTs that describes a shared forward wedge that would allow observers from the two asymptotic regions to meet.

This has implications for two other proposals as well. When studying free or weakly interacting field theories, one finds that the ground state is highly entangled in position space; in fact, it is “maximally entangled”, by which we mean as entangled as is possible given conservation of energy. Based on [1] one might propose the converse: that entanglement implies a connected space. One might take as a slogan for this point of view “entanglement=spacetime”. Certainly, this theme runs through [8–11].

Further, in response to recent information theoretic arguments against smooth black hole horizons [5, 12–14], Maldacena and Susskind [16] proposed a more concrete incarnation of these ideas, which has been called “ER=EPR”. (For recent refinements, see [17, 18]..) The idea is that quantum entanglement (for instance, of the EPR type), can be geometrized as an Einstein–Rosen bridge. Since these proposals are based on the understanding of how the eternal AdS black hole is realized in the AdS/CFT correspondence [1, 19], we expect (at the very least) that the simplest interpretations of entanglement=spacetime and ER=EPR fail to hold.

It is important to note, as emphasized in [16], that entanglement is not a good observable; there is no linear, Hermitian operator that measures the entanglement of a state. The best one can do is check whether the system is in a particular entangled state. This already suggests that we proceed cautiously when trying to relate entanglement to spacetime connectivity.

Without further ado, let us consider the eternal AdS black hole. The full spacetime has two asymptotically AdS regions. The AdS/CFT dictionary suggests that each has its own dual CFT. To put a single CFT into a thermal state one traditionally puts a black hole in the geometry, and so it is natural to suppose that one should think of the two-sided black hole as dual to two copies of the CFT maximally entangled in the so-called thermofield double state [1, 19].

At this point, we would like to emphasize two elementary points:

1. All Hilbert spaces of the same dimension are isomorphic, so a particular “state” is only meaningful if we agree what operators we are going to act with before hand.

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1 The statement is also expected to hold for reasonable, strongly interacting theories. The same is not true in momentum space: the vacuum of free field theory in momentum space is entirely unentangled. See [6, 7] for investigations of entanglement in momentum space for interacting theories.

2 An oversight in the pivotal argument of [12] was addressed in [15].

3 For a two-qubit system, this process is called a Bell measurement. We discuss it and its relevance to the current work in Appendix C.
2. If Alice is going to measure two degrees of freedom, she must certainly interact with them both. In the context of black holes, we can say that moving across the horizon corresponds to interacting with the inside-horizon and the outside-horizon modes and performing a Bell measurement. See Appendix C for details.

What is the relevance of these points? Recall the thermofield double formalism, which as its name suggests involves doubling the degrees of freedom. If we have a thermal state, \( \rho \sim \exp(-\beta H) \), we can imagine purifying the system by adding additional degrees of freedom; the simplest way is to double every degree of freedom and entangle each degree of freedom in the thermal system with its partner:

\[
|\Psi\rangle = N_\beta \sum_E e^{-\frac{1}{2}E} |E\rangle_L \otimes |E\rangle_R.
\] (1.1)

One can think of the second copy as modeling the role of the environment. Our point: this state does not have a particular meaning unless we agree what operators we act on it with. In particular the same state can be evolved with different Hamiltonians. For instance, consider the Hamiltonian

\[
H_{\text{doubled}} = H_L + H_R.
\] (1.2)

This Hamiltonian has no interaction term between the two copies. This is equivalent to the standard approximation of ignoring interactions that bring a system to thermality once it is in equilibrium.

However, one can also consider Hamiltonians of the form

\[
H_{\text{doubled-interacting}} = H_L + H_R + H_{\text{int}}.
\] (1.3)

which involves an interaction term between the two copies. We argue that for any quantum or collection of quanta to pass through the horizon of the eternal AdS black hole and thereby ascertain whether or not there is a firewall necessitates an interaction term between the left and right degrees of freedom. While such an interaction term is present in the eternal AdS black hole, it is missing between the two CFTs. This shows that the proposed duality of [1] does not work.

Below, we outline the major points we make in the paper.

1.1 A Toy Model

In Section 2, we start with a toy problem that demonstrates several of the above points. We study a free 1 + 1 dimensional field theory by arbitrarily breaking it into left and right halves.

The first observation we make is that there is an explicit interaction between the two halves, which ultimately derives from the \( (\partial_x \phi)^2 \) term in the Lagrangian. This is what allows excitations to travel from one side to the other.

Our second observation is that one can evolve \textit{the same state with an alternate Hamiltonian} with a mirror at \( x = 0 \). For instance, one may take the highly entangled global
degree of freedom.
(“interacting”) vacuum at $t = 0$, and evolve it forward with the mirror Hamiltonian. This results in a divergent pulse of radiation traveling outward along null rays. This “free” Hamiltonian does not have the interaction terms coupling the two halves. Thus, in this field theory example, one can explicitly see that it is the interactions that connect the two halves, and not entanglement.

Our third observation is that a Rindler observer accelerating away from the origin is completely ignorant of the potential existence of a fatal light-like pulse. The Rindler coordinates freeze the interactions at the origin. Moreover, since all correlators are identical in the Rindler wedge for the mirror and for the non-mirror case, if one analytically continues correlators in the Rindler wedge to the global Minkowski spacetime, there is no signature of the radiation that will prove fatal for an inertial observer. Analytic continuation gives an incorrect result.

In the context of holography, the evolution without the mirror corresponds to the bulk AdS black hole spacetime and we see it involves an interaction between the left and the right. The outside of the black hole evolves with Schwarzschild time and is the analogue of Rindler wedges. These are oblivious to this interaction. The CFTs living on the boundary have to be non-interacting on account of being causally disjoint. If there is some way to extend or complete the CFTs so that they interact interact, then the dual bulk microstates will also interact, and the AdS black hole being dual to the CFTs would be consistent. Conversely, if there is no such extension of the CFTs, then the bulk AdS black hole cannot be dual to the two CFTs. The dual of the CFTs would in the latter case be analogous to the mirror system, with no connectivity between the left and the right and thus no shared future or past wedges. We give examples of both kinds in following sections.

1.2 The Rindler-AdS/CFT Correspondence

In Section 3, we use the recent Rindler-AdS/hyperbolic CFT correspondence. This serves as an example where two CFTs can be completed into a global CFT describing a connected spacetime (global AdS). In this section and the remainder of the paper, we specialize to $AdS_3$, but we expect our arguments to hold unmodified for higher dimensions.

First, we argue that the two hyperbolic CFTs describe the physics of the two Rindler wedges, but do not capture the physics behind the Rindler horizons.

Second, we note that the Rindler horizons correspond to the boundaries of the hyperbolic CFTs, so that connectivity of the bulk gets related to connectivity of the boundary. The two hyperbolic CFTs are analogous to the Rindler wedges in the toy model; they are ignorant of whether there are sharp insertions at the connecting point that spoil smooth global AdS. Moreover, the Hilbert spaces associated with the two CFTs interact if we map back to the global time, just as the two parts of the global AdS interact when switching away from the Rindler coordinates.

1.3 The Eternal AdS Black Hole

In Section 4, we finally turn to the case of interest, the eternal AdS black hole. In three bulk dimensions, we have the BTZ black hole, which was also the case explicitly studied in [1].

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We recall that the BTZ black hole can be understood as an orbifolding of $\text{AdS}_3$. The orbifolding is particularly simple in Rindler-$\text{AdS}$ coordinates. The orbifolding of the bulk translates into an orbifolding of the hyperbolic CFT, which places the CFT out of contact with the bulk horizon. The CFT becomes a cylinder instead of an open interval.

Before orbifolding, an interaction between the two systems can be made manifest by returning to global $\text{AdS}$ in the bulk and the original CFT on a cylinder in the boundary. In the CFT, the interaction arises just from piecing the two open regions into a complete whole. After orbifolding, in the bulk one can switch to Kruskal-like coordinates to pass through the horizon so the interaction is still present; however, since the two CFTs are already on cylinders there is no way to patch them together and the interaction is turned off.

1.4 Conclusions

From the above arguments, we conclude that while two hyperbolic CFTs in a thermofield double state do have shared future and past wedges in the bulk, two cylindrical CFTs in a thermofield double state do not, contrary to the proposal in [1]. Being entangled is necessary, but not sufficient for connectedness of spacetime. Moreover, the eternal AdS black hole has a smoothly connected forward wedge, and relatedly has quasi-normal modes. The CFT on a compact space has no quasi-normal modes.\(^5\) This suggests that the two CFTs in a thermofield double state is dual to the left and right wedges of the eternal black hole cut off at the stretched horizons. There is no interior description, although we expect the UV complete description of the geometry to replace the stretched horizon by fuzzball microstates.

2 Breaking a Field Theory in Twain

As a demonstration of the key ideas, we introduce the following toy problem. We consider breaking 1 + 1-dimensional field theory into (strongly interacting) left and right halves. We explicitly demonstrate, in case there was any doubt, that the connectivity of the space derives from an interaction term. Moreover, we consider evolving the original vacuum state with the noninteracting Hamiltonian, which results in a divergent stress tensor propagating outward from the break point. This is analogous to a firewall along the Rindler horizon of an accelerating observer.

In the context of holography, this example illustrates that for observers in different asymptotic regions of the eternal AdS black hole to meet in the future, there must either be an interaction term directly between the two CFTs or between each CFT and a common third system. Neither of which is the case in the standard thermofield double formalism. At which point, one may consider trying to extend the CFT side of the duality in some way to describe a shared future, or one may consider altering the bulk side of the duality so that the two asymptotic AdS regions are not connected.

\(^5\)Technically quasi-normal modes are defined for the bulk. In the CFT the signature of bulk quasi-normal modes are poles of the retarded 2-point function in the lower half complex plane in momentum space. In this paper we refer to a CFT having such complex poles as having quasi-normal modes.
2.1 Global aspects

Consider a free, massless scalar field with Dirichlet boundary conditions:

\[ L = \frac{1}{2}(\dot{\phi}^2 - \phi'^2) \quad \phi(t, -\frac{L}{2}) \equiv \phi(t, \frac{L}{2}) \equiv 0 \quad \forall t \in \mathbb{R}. \]  

(2.1)

We put the field theory in a box as a convenient IR regulator. The field can be expanded in modes

\[ \phi_m(t, x) = \frac{1}{\sqrt{\omega_m L}} \sin(\omega_m (x + \frac{L}{2})) e^{-i\omega_m t} \quad \omega_m = \frac{m\pi}{L} \quad m \in \mathbb{Z} \]  

(2.2)

in the usual way

\[ \phi(t, x) = \sum_{m=1}^{\infty} \left( a_m \phi_m + a_m^\dagger \phi_m^* \right) \quad [a_m, a_n^\dagger] = \delta_{m,n}. \]  

(2.3)

We are interested in now formally breaking the field into a left and right field \((x\) negative and positive, respectively):

\[ \phi(t, x) = \begin{cases} 
\psi(t, x) & -\frac{L}{2} \leq x < 0 \\
\chi(t, x) & 0 < x \leq \frac{L}{2} 
\end{cases} \]  

(2.4)

Let \(\psi(t, x)\) be the left field with modes \(b_m\) and \(\chi(t, x)\) be the right field with modes \(c_m\). We arbitrarily impose Dirichlet boundary conditions on both fields at \(x = 0\). The expansions are given by

\[ \psi_m = \sqrt{\frac{2}{\kappa_m L}} \sin(\kappa_m x) e^{-i\kappa_m t} \quad \kappa_m = \frac{2m\pi}{L} \]  

(2.5)

\[ \chi_m = \sqrt{\frac{2}{\kappa_m L}} \sin(\kappa_m x) e^{-i\kappa_m t}. \]

These expansions can be thought of as in the interaction picture, where the time-dependence due to the interaction between the two halves has not been included.

We are left with two field expansions that we equate at \(t = 0\):

\[ \sum_{m=1}^{\infty} \left( a_m \phi_m + a_m^\dagger \phi_m^* \right) = \begin{cases} 
\sum_{n=1}^{\infty} \left( b_n \psi_n + b_n^\dagger \psi_n^* \right) & x < 0 \\
\sum_{n=1}^{\infty} \left( c_n \chi_n + c_n^\dagger \chi_n^* \right) & x > 0 
\end{cases} \]  

(2.6)

Using the orthogonality of the mode functions we may then extract the Bogolyubov coefficients relating the different modes:

\[ a_{2k} = \frac{(-1)^k}{\sqrt{2}} (b_k + c_k) \]  

(2.7)

\[ a_m = \frac{(-1)^{m-1}}{L} \sum_{n=1}^{\infty} \sqrt{\frac{2\kappa_n}{\omega_m}} \left( b_n - c_n + \frac{b_n^\dagger - c_n^\dagger}{\omega_m - \kappa_n} + \frac{b_n^\dagger - c_n^\dagger}{\omega_m + \kappa_n} \right) \quad (m \text{ odd}). \]

The mixing between the lowering operator and raising operators implies that the notions of vacua do not agree. It turns out, one can write

\[ |0_a \rangle = \prod_{m,n} \exp \left[ \frac{1}{2} \gamma_{m,n} (b_m^\dagger - c_m^\dagger)(b_n^\dagger - c_n^\dagger) \right] |0_b \rangle |0_c \rangle, \]  

(2.8)
with
\[ \gamma_{m,n} = \frac{2\sqrt{\kappa_m \kappa_n}}{L^2} \left( \sum_{k \in \text{odd}+} \frac{1}{\omega_k (\kappa_m - \omega_k) (\kappa_n + \omega_k)} \right) = \frac{2\sqrt{\kappa_m \kappa_n}}{L^2} s_{m,n}. \] (2.9)

This equation implies, as expected, that the \( a \)-vacuum is highly entangled between the left and right halves.

One may be concerned with this decomposition, since in the \( \psi/\chi \) description we have imposed Dirichlet boundary conditions at \( x = 0 \) where our original theory had no such condition. We expect that generically this is not a problem since the fields \( \psi \) and \( \chi \) can converge to any \( \phi \) in an \( L^2 \)-norm sense, for instance. Some issues may arise if operators are inserted exactly at \( x = 0 \), as became evident in a related set-up explored by the current authors [20].

Let us proceed to rewrite the original Hamiltonian,
\[ H_a = \sum_m \omega_m \left( a_m^\dagger a_m + \frac{1}{2} \right) \] (2.10)
in terms of the left and right modes. The number operator is straightforward to write out. For the even modes
\[ a_{2m}^\dagger a_{2m} = \frac{1}{2} \left( b_m^\dagger b_m + c_m^\dagger c_m + b_m^\dagger c_m + c_m^\dagger b_m \right). \] (2.11)
Note that the latter two terms already have the form of an interaction which “moves” degrees of freedom between the left and right sides. The first two terms are just the “free” Hamiltonian terms for the left and right separately. For odd modes
\[ a_m^\dagger a_m = \frac{2}{\omega_m L^2} \sum_{k,l=1}^{\infty} \sqrt{\kappa_k \kappa_l} \left[ \frac{1}{(\omega_m - \kappa_k)(\omega_m - \kappa_l)} + \frac{1}{(\omega_m + \kappa_k)(\omega_m + \kappa_l)} \right] (b_k^\dagger c_k^\dagger)(b_l - c_l) \]
\[ + \frac{(b_k^\dagger - c_k^\dagger)(b_l^\dagger - c_l^\dagger)}{(\omega_m - \kappa_k)(\omega_m + \kappa_l)} + \frac{(b_k - c_k)(b_l - c_l)}{(\omega_m + \kappa_k)(\omega_m - \kappa_l)} \] + \epsilon_m, \] (2.12)
where
\[ \epsilon_m = \frac{4}{L^2 \omega_m} \sum_{k=1}^{\infty} \frac{\kappa_k}{(\omega_m + \kappa_k)^2} \] (2.13)
is the (divergent) \( \phi_m \) mode contribution to the difference between the \( bc \) vacuum energy and the \( a \) vacuum energy.

Our purpose here is to explicitly exhibit the interaction term between the left and right halves of the theory. While we can think of the \( b^\dagger c^\dagger \) and \( bc \) type terms as being related to the shift in the vacuum, the \( b^\dagger c \) and \( c^\dagger b \) type terms are related to the ability for excitations on the left to travel to the right and vice-versa. It is precisely this kind of interaction term that equilibrates between the left and right halves, and allows for the two otherwise disjoint systems to be entangled in the first place.

In particular, we can compare the Hamiltonian \( H_a \) discussed above, to the natural Hamiltonian for the left and right halves with Dirichlet boundary conditions,
\[ H_{\text{mirror}} = H_b + H_c = \sum_{k=1}^{\infty} \kappa_k (b_k^\dagger b_k + c_k^\dagger c_k), \] (2.14)
which has no interaction terms. This theory corresponds to inserting a two-sided mirror at $x = 0$ of the original system. It is clear that excitations in this system cannot travel between the two halves. This is the “free” Hamiltonian which gives the time dependence of the interaction picture modes $\psi$ and $\chi$ in Equation (2.5).

While we have a distinct Hamiltonian with a new vacuum $|0_b\rangle|0_c\rangle$, there is no obstacle to considering the same highly entangled state in Equation (2.8) or small excitations on top of it. Suppose, for example, we suddenly insert a mirror at $x = 0$ and $t = 0$. At $t = 0$, we have the same state.\footnote{By “suddenly”, we mean fast compared to the time-scale set by the UV cutoff, so that there is no time for the system to react.}

This is why we emphasize in the Introduction that we need to be clear what operators we are using on our states. The state (2.8) of itself cannot tell us whether there is a mirror or not. This is determined by which Hamiltonian we use: $H_a$ or $H_{\text{mirror}}$. That is to say, we could move from a Schrodinger picture at $t = 0$, and then evolve with whichever is the physically relevant Hamiltonian.

What happens to the example under discussion for $t > 0$? Inserting the mirror can be thought of as a local quench similar to the kind studied in \cite{21}. Thus, we expect excitations to travel outward causally from the quench site. In the current case of a massless field, this means excitations traveling outward along the light cone. Let us compute the energy density of this state using the mirror Hamiltonian. The energy density is given by

$$T_{tt} = \mathcal{H} = \frac{1}{2}(\dot{\phi}^2 + \phi'^2), \quad (2.15)$$

and we want to evaluate the difference from the $bc$ vacuum:

$$t_{tt} = \langle 0_a | T_{tt} | 0_a \rangle - \langle 0_b,c | T_{tt} | 0_b,c \rangle. \quad (2.16)$$

Let us just focus on $x > 0$. After some manipulation,

$$t_{tt} = 4\pi^3 \sum_{m,n=1}^{\infty} \kappa_m \kappa_n \left[ \tilde{s}_{m,n} \cos \left( (\kappa_m - \kappa_n) x \right) \cos \left( (\kappa_m - \kappa_n) t \right) + s_{m,n} \cos \left( (\kappa_m + \kappa_n) x \right) \cos \left( (\kappa_m + \kappa_n) t \right) \right]. \quad (2.17)$$

Both $s_{m,n}$ and $\tilde{s}_{m,n}$ can be expressed in terms of harmonic numbers:\footnote{We remind the reader that the harmonic numbers for integer $n$ can be expressed as $H_n = \sum_{k=1}^{n} \frac{1}{k}$ or for general $n$ as $H_n = \int_0^1 \frac{1-x^n}{1-x} dx$.}

\begin{align*}
    s_{m,n} &= \frac{m(2H_{2n} - H_n) + n(2H_{2m} - H_m)}{8mn(m+n)} \quad (2.18a) \\
    \tilde{s}_{m,n} &= \frac{m(2H_{2n} - H_n) - n(2H_{2m} - H_m)}{8mn(m-n)} \quad (2.18b)
\end{align*}

For $m = n$, one must take the limit as $m \to n$ in the above expression for $\tilde{s}$; the limit is finite and well-defined. The above expressions are manifestly symmetric in $m$ and $n$.\footnote{By “suddenly”, we mean fast compared to the time-scale set by the UV cutoff, so that there is no time for the system to react.}
and for \( m \neq n \) are rational, positive numbers. The expression for the stress-tensor (2.17) diverges. We can regulate the series by putting in a cutoff, \( m, n \leq \Lambda \). The stress-tensor is plotted for \( \Lambda = 30 \) in Figure 1(a). As \( \Lambda \to \infty \), \( t_{tt} \) tends to zero away from the pulse emitted by the insertion site, whereas the peak diverges like \( \Lambda^2 \log \Lambda \). The scale of the log is presumably given by the insertion timescale.

### 2.2 Rindlerization freezes interactions

We saw in the previous section that we can evolve the vacuum state of the box with Dirichlet boundary conditions at \( x = \pm L/2 \) (i.e. the vacuum of the hamiltonian (2.10)) with the hamiltonian corresponding to having a mirror at \( x = 0 \) (i.e. the hamiltonian (2.14)) starting at \( t = 0 \). This results in a divergent stress tensor, which moves outwards from \( x = 0 \) and \( t = 0 \) towards the two walls of the original box. The pulse of radiation eventually bounces off the walls; however, we introduced the box only as a convenient IR regulator. Therefore we are interested in the physics for \( x, t \ll L \).

Taking the limit of \( L \to \infty \), the original state becomes the Minkowski vacuum. We do not have to worry about how the quench is inserted at \( t = 0 \). Instead, we can imagine the initial state at \( t = -\infty \) was created such that the evolution with the mirror hamiltonian produced the Minkowski vacuum at \( t = 0 \). Or we can simply be indifferent to how the state was created at \( t = 0 \) and evolve it with the mirror hamiltonian. If we introduce massive fields then the divergent stress tensor would also be present in the future and past wedges. The resulting scenario is shown in Figure 1(b).

As is clear from the figure, an accelerated observer forever stays away from the divergent stress tensor. In fact, the local state seen by the accelerated observer is the Minkowski

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**Figure 1.** In (a), \( t_{tt} \) is plotted at time \( t < L/2 \) after the insertion of the mirror. We cut off the sums at \( m, n = 30 \) in Equation (2.17). The left pulse is created at the mirror \( (x = 0) \) and propagates away along the lightcone. Note that box wall is at \( x = L/2 \). The pulse at \( x = L - t \) is actually the left-moving bounced pulse that only enters the physical region once the pulse leaving the quench site at \( x = 0 \) hits the wall. In (b), an accelerated observer escapes the divergent stress-tensor, and does not notice that a mirror was inserted.
vacuum although the global state will be changing with time. More formally, if we Rindlerize the state in the usual way, the Rindler wedges for the mirror hamiltonian will be identical to Rindler wedges for the Minkwoski hamiltonian. In particular correlation functions with all points inside the Rindler wedge will be identical. Correlation functions with points outside will be different in general.\textsuperscript{8}

The Rindler coordinates natural to the accelerating observer slice the spacetime in a peculiar way. As one approaches the coordinate singularity along the horizon, time steps forward by smaller and smaller increments until it stops and the coordinates degenerate. Thus any left–right interactions become frozen. In order for the mirror to leave no imprint on both Rindler wedges, one requires that the mirror be infinitesimally thin and inserted instantaneously, as compared to the theory’s UV cutoff. In what follows, we argue the accelerating observer is analogous to a single CFT’s description of the eternal AdS black hole, with some important differences related to the mirror “thickness”.

We hope the relevance of this toy problem to the firewall discussion is clear: the accelerated observer cannot tell whether the system evolves with $H_a$ from (2.10), allowing “free infall” via its required interaction terms, or with $H_{\text{mirror}}$ from (2.14), corresponding to a “firewall”. Let us emphasize that the entanglement between left and right halves is the same in both situations. Within field theory, when there is no dynamical theory of gravity, entanglement does not imply a smoothly connected spacetime. Using AdS/CFT, we extend the argument into the bulk.

We can also use this example to address the issue of analytic continuation. One can imagine analytically continuing a Green’s function in the right Rindler wedge onto the whole Minkowski space; however, since all correlators are identical on the wedge with and without the mirror, the existence of a smooth analytic continuation does not imply a smooth Rindler horizon.

3 Rindler-AdS

3.1 Review of Rindler-AdS

Global $AdS_{d+1}$ space can be “Rindlerized” pretty much like flat spacetime resulting in bulk acceleration horizons \cite{23–25}. To simplify our discussion, we focus solely on the case of $AdS_3$. The Carter–Penrose diagram of $AdS_3$ with acceleration horizons is shown in Figure 2(a). These “black holes” have horizons, but no singularities as is evident from the figure.

The metric of global $AdS_3$ is is given by

$$ds^2 = \sec^2 u (-d\tau^2 + du^2 + \sin^2 u \, d\phi^2)$$

(3.1)

where the coordinate $u \in [0, \pi/2)$. These coordinates are helpful in visualising global AdS as conformal to a finite radius cylinder. According to AdS/CFT duality, the dual CFT is

\textsuperscript{8}It should be noted that we are talking about Rindlerization with a mirror on the state identical to the Minkowski vacuum at $t = 0$. The case with the vacuum state of the mirror hamiltonian is not relevant for us but was recently studied in \cite{22}.
(a) Bulk AdS with acceleration horizons and Bulk-Alice falling in. $\tau$ increases in the vertical direction and $\phi$ increases anti-clockwise.

(b) Causal diamonds on global boundary with Boundary-Alice escaping. $\tau$ increases in the vertical direction and $\phi$ increases towards the right.

**Figure 2.** Global AdS can be “Rindlerized”. In (a) the associated acceleration horizons and closed string Bulk-Alice falling through one of them is shown. In (b) the associated causal diamonds and open string Boundary-Alice escaping it is shown.

defined on a background geometry given by the conformal boundary of the AdS space; in this case $\mathbb{R} \times S^1$.

The global coordinates, $\tau$, $u$ and $\phi$, can be exchanged for Rindler-AdS coordinates, $t, r$ and $\chi$. The details are given in Appendix A. The metric then becomes

$$ds^2 = -(r^2 - 1)dt^2 + \frac{dr^2}{r^2 - 1} + r^2 d\chi^2$$

where $\chi \in (-\infty, \infty)$. Note that the acceleration horizon is at $r = 1$ and the coordinates cover only one of the Rindler-AdS wedges shown in Figure 2(a). There is a second set of coordinates that covers the other Rindler-AdS wedge. As is obvious from the large $r$ behaviour, the dual CFT is defined on $\mathbb{R}^{1,1}$.

The acceleration horizons intersect the boundary cylinder at the edges of causal diamonds. The inside of these diamonds describe the causal development of the open intervals $\phi \in (-\pi/2, \pi/2)$ and $\phi \in (\pi/2, 3\pi/2)$ at $t = 0$ (see Figure 2(b)).

It was shown in [26] that the conformal transformation,

$$\tan(\tau) = \frac{\sinh(t)}{\cosh(\chi)}, \quad \tan(\phi) = \frac{\sinh(\chi)}{\cosh(t)},$$

maps the causal development of the interval $\phi \in (-\pi/2, \pi/2)$ with metric

$$ds^2 = -dt^2 + d\phi^2$$
into a space conformally related to $\mathbb{R}^{1,1}$:

$$ds^2 = \frac{2}{\cosh(2\chi) + \cosh(2t)}(-dt^2 + d\chi^2).$$  \hspace{1cm} (3.5)

Under this map, the vacuum of the full CFT gets mapped into a thermal state on the space (3.5). The coordinates $t, \chi \in \mathbb{R}$, thus making the causal diamond a complete theory by itself. This is a purely field theory result, but the map can be seen as a projection of the bulk coordinate transformation (A.4) and (A.5). The space and time dependent conformal factor comes from (A.3) and captures the effect of mapping the CFT on constant $r$ surfaces to that on constant $u$ surfaces. In higher dimensions the CFT is defined on hyperbolic spacetime [23–25], so we will refer to the two dimensional CFT on (3.5) as the hyperbolic CFT.

### 3.2 How much does a hyperbolic CFT capture?

So far, we have only discussed the global vacuum state. What we are really interested in is tracking a bulk excitation passing through the acceleration horizon and interpreting this motion in the CFT. To this end, we ask what is the region of bulk spacetime which is uniquely fixed by the state on a Cauchy slice which covers only a part of the boundary cylinder? For small enough perturbations initially localised in the causal diamond of the Cauchy slice we can then say that the dual bulk excitations are completely determined by the associated hyperbolic CFT as long as they are in the corresponding bulk region. Recall that since the CFT is local, one can write the complete Hilbert space in a product form

$$H = H_A \otimes H_{\bar{A}},$$ \hspace{1cm} (3.6)

where $H_A$ is associated with the relevant Cauchy slice and $H_{\bar{A}}$ with its complement. So the question is given a state on $H_A$ how much of the bulk is fixed.

If the map between boundary degrees of freedom and bulk spacetime is sufficiently non-local it could happen that data from the entire boundary is needed to construct any subregion of the bulk. However, there are reasons to believe that the situation is not so bad.

One reason to expect a subregion duality is the following. If we do not take the decoupling limit, then the core region of a stack of D-branes is AdS-like and a closed string tunnelling through the graybody factors and falling in has an alternate description as the closed string hitting the stack of branes and becoming open strings [27–34]. The essential idea of AdS/CFT is that in the decoupling (low-energy) limit, the core geometry becomes asymptotically AdS and the D-brane theory flows to a CFT. When the closed string is close to the boundary, the open strings are close to each other and the action of “plucking” the D-branes is very localised. As the closed string goes deeper, the open strings spread out [32]. Thus, it seems likely that when the bulk excitation is close to the boundary, it is captured by local operators in the boundary.

Another reason to expect a subregion duality is that the asymptotic behavior of the fields in the bulk is directly related to the expectation values of local operators in the boundary field theory (together with the field theory action). One expects that using the
boundary behavior of the bulk fields in some region and the bulk field equations, one may integrate the field equations into some neighbourhood of the boundary subregion [9].

Intuition from AdS/CFT and causality would suggest that the boundary data should in the very least be enough to construct the Rindler-AdS wedge corresponding to the boundary causal diamond. Such a picture has indeed been advocated in [9, 35, 36]. We work under that assumption.

Now imagine introducing an excitation called Alice into the system. We imagine that bulk-Alice, made of closed strings, is introduced very close to the boundary in the center of some causal diamond as shown in Figure 2(a). From the above discussion, the boundary-Alice will be made of left- and right-moving open strings that are entangled in general. Further, from the above discussion we expect the motion of bulk-Alice while she is in the Rindler wedge to be captured by the hyperbolic CFT living in the causal diamond as shown in Figure 2(b).

When bulk-Alice crosses the acceleration horizon, boundary-Alice can no longer be described solely by the hyperbolic CFT she was created in since that hyperbolic CFT can only uniquely determine the bulk up to the acceleration horizon. When bulk-Alice crosses her future horizon, she might be hit by any excitation created in the other Rindler wedge—say bulk-Bob—in the region marked “Future” in Figure 2(a). Similarly, boundary-Alice after coming out of the causal diamond may get hit by the dual to bulk-Bob—boundary-Bob who was created in the antipodal hyperbolic CFT—in the region marked “Future” in Figure 2(b).

More formally, even without the details of the construction, it is clear that a bulk two-point function with both points inside the Rindler-AdS wedge, corresponding to the propagator that keeps bulk-Alice inside the Rindler-AdS wedge, can be written in terms of \( n \)-point functions of the corresponding hyperbolic CFT. However, the bulk two-point function with one point inside the Rindler-AdS wedge and the other in the forward wedge, corresponding to the propagator that moves bulk-Alice across the acceleration horizon, has to be written in terms of \( n \)-point functions with points outside the hyperbolic CFT. In this sense, the global CFT is the theory of the inside of the Rindler-AdS “black hole”.

Note that the hyperbolic CFTs cannot describe boundary-Alice and boundary-Bob leaking out since the boundary time tends to infinity at the edges. In fact the hyperbolic CFTs are defined on open intervals and do not include the boundaries. Recall that the boundaries of the diamond are projections of the bulk acceleration horizons. The Rindler-AdS bulk spacetimes also cannot describe bulk-Alice and bulk-Bob crossing the horizons since Rindler time tends to infinity at the horizons. Switching to global coordinates in the bulk and the boundary we see that Alice and Bob have no trouble crossing the respective horizons and meeting in the bulk and similarly the boundary versions come out of the causal diamond and meet.

### 3.3 What is dual to the horizon?

Let us discuss if the two hyperbolic CFTs together are equivalent to the global CFT. At \( t = 0 \) they together span the open interval \((-\pi/2, \pi/2) \cup (\pi/2, 3\pi/2)\) and thus together the two CFTs miss the points \( \phi = \pm \pi/2 \); however, since these are just points one may wonder
how much Cauchy data is missing. The correct way to handle this is to introduce UV
cutoffs in the global CFT so that the point has some finite thickness. For the map (3.3) it
can be shown that the interval \((\pi/2 - \delta\phi, \pi/2)\) maps to \((\chi_{\text{max}}, \infty)\) where

\[e^{-\chi_{\text{max}}} \approx \frac{\delta\phi}{2 \cosh t}.\] (3.7)

In ref. [26] this was used to argue that a UV cutoff in the global CFT is related to an IR
cutoff in the hyperbolic CFT. Thus, if we remove the IR cutoff on the two hyperbolic CFTs
they indeed are together equivalent to the global CFT because the two missing points are of
zero measure. There is no room to insert a “reasonable” operator at \(\phi = \pm\pi/2\). However,
one can imagine introducing a very sharp operator at the point where the two causal
diamonds meet and this will propagate along the forward edges of the causal diamonds
in the boundary. In the bulk the corresponding signal will move along a null path and
depending on the precise operator may collide with Alice crossing the acceleration horizon.
So, the acceleration horizons in the bulk are only captured by the hyperbolic CFTs in the
absence of any IR cutoff. This will be crucial when we discuss getting the BTZ black hole
by orbifolding \(AdS_3\).

3.4 Signatures of connectivity in the CFT

We saw that connectivity in the bulk is associated with connectivity in the boundary:
when the two Rindler-AdS wedges are secretly part of global AdS, the hyperbolic CFTs
are part of the global CFT. In the general relativistic description of the bulk, there are clear
diagnostic signals indicating that one can extend the solution past the bulk horizon: the
geometric invariants are all smooth and finite at the horizon, and timelike geodesics reach
it in a finite amount of proper time. Are there corresponding indications of the “complete-
ability” of the hyperbolic CFT? Here we are interested in isolating the conditions in the
CFT that led to connectivity/complete-ability, or in other words allowed Alice, a creature
created inside one of the causal wedges, to escape unharmed and possibly meet Bob who
has been created in the antipodal causal wedge. Below we list these conditions.

- **Interaction:** As discussed earlier, the two hyperbolic CFTs are complete theories
  by themselves and are in particular non-interacting. It would seem counterintu-
tive that excitations in two non-interacting theories can meet. Let us begin by
discussing how this comes about. Recall from Section 2 that Rindlerization makes
the theory in the Rindler wedge indifferent to the nature of interaction, or lack
thereof, between two halves of the system. This is because their inner boundaries—
the acceleration horizons—move away at speed of light to escape any effect of an
interaction between the two halves. This is also true of Rindlerization of AdS.
Of course, the two halves of global AdS do interact. Similarly, though the two
halves of the cylinder \(\phi \in (-\pi/2, \pi/2)\) and \(\phi \in (\pi/2, 3\pi/2)\) do interact for all \(\tau\),
the way the hyperbolic CFTs are defined—only inside the causal diamond—they

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\(^9\)Points in the horizon are contained within many different Rindler-AdS wedges so for any given point
asking which subregion of CFT is dual to it does not make sense. However, once we have chosen specific
Rindler-AdS wedges we can ask the question of what is dual to the horizon.
are ignorant of this interaction. Thus, while the two hyperbolic CFTs are non-interacting, it is via a carefully tuned evolution of initial Cauchy data with “ever slowing time” that this is arranged. Connectivity actually comes from the two sub-Hilbert spaces, corresponding to the two halves of the cylinder, interacting. What allows this interaction is of course the fact that the global CFT lives on a connected manifold. It is precisely this interaction which causes the entanglement of the two sub-Hilbert spaces in the first place. So, interaction, entanglement and connectedness go hand in hand in this example and we cannot really say one causes the other.

- **Subregion CFTs are open systems**: Alice can escape the causal diamond and possibly meet Bob only if the propagators on the global boundary are vacuum propagators. Consider the two-point function of an operator of dimension (1, 1) on the vacuum state on the cylinder,

\[
\langle O(\tau, \phi)O(0, 0) \rangle = \frac{1}{16} \frac{1}{\sin^2\left(\frac{\tau-\phi}{2}\right)\sin^2\left(\frac{\tau+\phi}{2}\right)},
\]

(3.8)

where we have taken the spatial extent of the CFT to be \(2\pi\). Using the conformal map (3.3) the two-point function for the hyperbolic CFT becomes

\[
\langle O(t, \chi)O(0, 0) \rangle = \frac{1}{16} \frac{1}{\sinh^2\left(\frac{t-\chi}{2}\right)\sinh^2\left(\frac{t+\chi}{2}\right)}
\]

(3.9)

which is the two-point function on the infinite line with temperature \(\frac{1}{2\pi}\). Reversing the logic, given a thermal two-point function for a CFT on an infinite spatial line, we can conformally compactify the associated causal diamond and embed it in a bigger space in various ways—we can have more than two causal diamonds or we can have one big and one small one—and analytically continue the two-point function. The maps in the other cases are of course different from that in 3.3. Analytic continuation only gives the correct answer if the state near the edges of the causal diamond is the vacuum state with respect to the global time of the particular map in question. From the discussion earlier about the relation between UV cutoffs of the global CFT and IR cutoffs of the hyperbolic CFT, we see that for connectivity, hyperbolic CFTs have to be in thermal states to arbitrarily low energy scales. Said differently, this means that there is no IR cutoff and in particular there is no mass gap. This is related to the conformally compactified intervals in the global CFT being open. A possible answer to the question of “complete-ability” of the CFT could simply be that only when the CFT is hyperbolic and thermal to arbitrarily low energies can it be completed since that implies it is an open interval in the vacuum state. This answer seems trivial and contradicts the proposal in [1]; however, we argue against parts of the duality for eternal AdS black holes below. This answer may be somewhat disappointing because it simply says the bulk is completable and therefore connected when the boundary is too.
Quasi-normal modes: Thermal CFTs on infinite volume have quasi-normal modes. In fact, in the case of 1 + 1 dimensional CFT at finite temperature and infinite spatial extent, the quasi-normal modes are fixed by conformal invariance [37]. We saw that the vacuum on open intervals is mapped to a thermal state on infinite intervals. Thus if the previous conjecture about “complete-ability” is true, quasi-normal modes in the CFT are a necessary (but not sufficient) condition for smooth horizons in the bulk.

Trans-horizon excitations: Another signature of connectivity, related to the preceding discussion, is the possibility of excitations which do not fit into any of the individual intervals. From the point of view of the hyperbolic CFTs these are analogous to “trans-horizon excitations”. In fact, even in the bulk they correspond to transhorizon excitations.

Inhomogeneity: The previous point is also related to an inhomogeneity coming from the fact that when we look at the conformally compact causal diamond it has an edge through which excitations can leak out. In other words there are “edges” to the domain of dependence which break homogeneity.

Perhaps the most important of these points is that the two sub-Hilbert spaces interact. The associated CFTs are non-interacting only because they are defined inside causal diamonds. AdS/CFT tells us that the dual states in the bulk should also be interacting. This is indeed the case for the bulk $AdS_3$ geometry when viewed as two halves of the solid cylinder, even though the Rindler wedges are oblivious to the interaction. This tells us that this particular piece of the AdS/CFT dictionary, a couple of hyperbolic CFTs in the thermofield double state being dual to global $AdS_3$ is consistent.

4 Eternal AdS

Let us now move on to the case that really interests us: the eternal AdS black hole. The eternal $AdS_d$ black hole has two asymptotic boundaries with cylindrical geometry $\mathbb{R} \times S^{d-2}$. The state in the bulk is the Hartle–Hawking state [38]. As this geometry has two boundaries one would expect that if there were a dual description, it would involve two decoupled CFTs living on the two boundaries. Indeed in [1], Maldacena proposed that an eternal AdS black hole is dual to two CFTs living on cylinders that are together in a thermofield double state. The motivation for the choice of state seems to be that the bulk Hartle–Hawking state is a thermofield double state with respect to the two outside regions [19].

This proposal has quite remarkable implications. Imagine introducing uncorrelated localised perturbations on the two spheres $S^{d-2}$ on which the CFTs live. As before, we will call them boundary-Alice and boundary-Bob. The duality tells us there are bulk versions of these perturbations called bulk-Alice and bulk-Bob close to the respective boundaries. The proposed duality further tells us that since bulk-Alice and bulk-Bob can meet behind the horizons and can, say, exchange qubits, there must be a dual process in the two CFT-on-spheres system. Now recall that the two CFTs are decoupled and so a meeting of such perturbations is quite counterintuitive. This proposal has recently led to other
proposals which in short can be summarised as “entanglement=geometry” [10, 11] and “ER=EPR” [16]. The idea is that if two systems are entangled they also share a non-traversable wormhole.

Our claim is that the original proposal is incorrect and so are the ones based on it. We will go about demonstrating this by looking at the system originally considered in [1], the BTZ black hole. The advantage of this system is that while it is an eternal AdS black hole with two asymptotic boundaries in its own right, it also comes from global AdS by orbifolding. In fact, this orbifolding is most easily seen in the Rindler-AdS coordinates. After briefly reviewing the construction we will show that the effect of orbifolding on the boundary is to change the causal structure such that the process that allowed excitations to leave the causal diamond does not work anymore. The boundary, which was two causal diamonds touching at their ends, becomes two cylinders. It is manifest that excitations created on the two cylinders cannot meet since they cannot leak out of cylinders unlike casual diamonds. However, the bulk does contain horizons and excitations in the two asymptotic regions that can meet behind the horizon. We take this as evidence that the proposal in [1] does not work, i.e. the eternal AdS black hole is not dual to the two CFTs on cylinders.

4.1 BTZ as orbifold of AdS$_3$

Following the work of [39], it is well known that the BTZ black hole can be viewed as an orbifold of AdS$_3$. What is perhaps less well known is that said orbifolding is very simply related to AdS$_3$ expressed in terms of Rindler-AdS coordinates. We give the details of this in appendix B (see also [40, 41] for good reviews), however, for our purposes the bottom line is that the orbifolding is the identification

\[ \chi \rightarrow \chi + 2\pi \]  

(4.1)

on (3.2) defined in the Left region and its analytic continuation to the Future, Past and Right regions in Figure 2(a). The orbifolding turns the horizon into a cylinder with the periodic direction parameterised by $\chi$. The rest of the bulk outside the horizon becomes a solid annulus. Importantly, connectivity is maintained across the horizon. This is shown in Figure 3(a).

The acceleration horizon intersects the boundary cylinder at $\chi = \pm \infty$. However, under the identification (4.1) the CFT causal diamond “becomes” a cylinder (something we elaborate on shortly) and the bulk horizon gets “disconnected” from the edge of the diamonds in the boundary. This is shown in Figure 3(b). This means that while we could disturb the state at the horizon by inserting an operator in the global boundary at $\chi = \infty$ which would send a shock wave along the relevant horizon as discussed in Section 3 for Rindler-AdS, we cannot do so after the orbifolding.

4.2 Orbifolding breaks connectivity in the boundary

It is useful to investigate the effect of the orbifolding (4.1) on the CFT in detail as this is a potentially confusing issue.
Figure 3. In (a) the effect of orbifolding in the bulk is shown. Connectivity across the horizon is maintained and thus so is the interaction. In (b) the effect of orbifolding on the boundary is shown. Hyperbolic CFTs are replaced by cylindrical ones which are not connected and do not interact.

- **No interaction:** Before orbifolding the two halves of the boundary cylinder were interacting. While this interaction was not visible in the hyperbolic CFTs, it was nevertheless present in the full theory and was responsible for excitations in the two CFTs meeting. Upon orbifolding, the two hyperbolic CFTs get replaced by two CFTs on cylinders. While non-interacting hyperbolic CFTs are defined on open intervals may be a part of a bigger system that may allow interaction, cylinders are periodic and therefore complete systems. This is clear from the Carter–Penrose diagrams of the two systems in Figure 3(b). It seems obvious that cylinders cannot be embedded in a bigger theory that allows interaction and therefore connectivity. We will nevertheless give some more evidence for this.

- **CFTs are closed systems without quasi-normal modes:** The two-point function on the hyperbolic CFT (3.9) is basically the two-point function of a 1 + 1 dimensional CFT with infinite spatial extent and temperature $\frac{1}{2\pi}$ and is thus fixed by conformal invariance. Naively, one would think that the two-point function after orbifolding can be obtained by the method of images (see [42] for example) but this is incorrect for reasons we explain in Appendix D. One would like to know what the correct correlation function is to understand the properties of the
bulk microstates. Unfortunately, the answer is not universal and will be hard to evaluate for a strongly coupled theory.\textsuperscript{10} We do know it will be spatially periodic with period $2\pi$ and will have a temporal periodicity also with a very long period coming from discreteness of spectra and the resulting Poincare recurrence. This rules out quasi-normal modes \cite{1, 44}. Recall, from Section 3.4 that a lack of mass gap and presence of quasi-normal modes in the CFT were necessary conditions for connectivity in the Rindler-AdS case. This is extra evidence that connectivity is broken when we orbifold and the two cylindrical CFTs cannot be embedded in a bigger theory, unlike the hyperbolic CFTs.

- **No analytic continuation:** Upon orbifolding, the CFTs’ hyperbolic spacetime is replaced by cylinders. The analogues of the Future and Past regions of Figure 2(b) do not exist after orbifolding. Thus, on physical grounds it seems that boundary-Alice and boundary-Bob cannot meet. However, one may still wonder if it is possible to analytically continue the correct non-decaying two-point function from within the fundamental region of orbifolding inside the diamond to outside the diamond under the map (3.3) (see Figure 4). If it were indeed possible, this propagator could allow boundary-Alice and boundary-Bob to meet. This is of course seems nonsensical from a physical point of view but if the answer came

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fundamental_domains}
\caption{The fundamental domains of orbifolding are obtained by identifying the dotted lines. This results in the boundary of BTZ becoming cylinders. There are also the other fundamental domains obtained by identifying the dashed lines. These produce closed timelike curves and are truncated to get the BTZ geometry. These are not relevant for our purposes. If one tries to analytically continue periodic two-point functions defined in the fundamental domain, one encounters essential singularities at the edges of the diamond. This shows that boundary-Alice and boundary Bob cannot meet in the Future region anymore. This is of course consistent with the physical intuition that cylinders, unlike causal diamonds, are not open and therefore not “leaky” and orbifolding thus effectively removes the Future region.}
\end{figure}

\textsuperscript{10}For correlation functions at the “orbifold-point” in the D1-D5 system for “long strings” see for instance \cite{1, 43}.
out in the affirmative we would be forced to think that two CFTs on cylinders could somehow be embedded in a bigger theory despite the arguments given above. It turns out that reasonable periodic functions oscillate increasingly fast as one approaches the boundary of the diamond and there is an essential singularity at the boundary. Formally, one can still step into the complex plane and analytically continue the two point function into the inside of the next diamond but this needs to be interpreted carefully. The edge of the causal diamond was associated with the horizon in the bulk. We already saw that the orbifolding disconnected the horizon from the boundary cylinder. The essential singularity at the old connecting surface further shows that a perturbation cannot “reach” the edge of the diamond; however, one can still insert operators inside two different diamonds and get finite two-point functions. This is possibly related to matching of thermofield double correlators with geodesics passing through the forward wedge [45, 46]. That aside, let us emphasize that our main point is that there is no process in the CFT that captures the crossing of the horizons in the bulk.

4.3 What does the CFT tell us about the bulk?

The orbifolding that gives BTZ black holes form global $AdS_3$ replaces the non-interacting boundary hyperbolic CFTs by non-interacting cylindrical CFTs. On physical grounds two non-interacting cylindrical CFTs cannot be a part of a bigger system where the associated Hilbert spaces interact (unlike two non-interacting hyperbolic CFTs). Nevertheless, we gave a number of evidences for this fact.

Now let us understand the implication of this for the bulk. The result of Strominger–Vafa for BPS D1-D5-P system [47], and follow-up results for near-extremal D1-D5-P system [48] tell us that the dimensionality of the Hilbert space of two CFT system is the same as that of the bulk BTZ black hole. However, as emphasised in the introduction, all Hilbert spaces of the same dimension are isomorphic, so a particular state is only meaningful if we agree what operators we are going to act with beforehand. In particular we have to specify which Hamiltonian to evolve it with. In Section 2 we demonstrated that the same Minkowski vacuum state evolves differently for the system with a mirror and one without. In particular the system with a mirror has non-interacting left and right parts. Since, the two cylindrical CFTs do not interact in any way, it stands to reason that their bulk duals do not interact either. Without an interaction, they cannot have shared Future and Past wedges. Thus, the bulk dual is manifestly not the BTZ black hole which has an interaction between the left and right side and thus has the Future and Past wedges.

5 Conclusions

5.1 Summary

If we divide $1 + 1$ dimensional Minkowski spacetime into two parts $x > 0$ and $x < 0$ then it seems intuitive that moving across the shared boundary involves an interaction between the two subsystems. Nevertheless, we demonstrated this explicitly. We also showed that
the effect of removing this interaction from the Minkowski vacuum, i.e. putting a two sided mirror at \( x = 0 \), is to have divergent stress tensor in the future wedge. Furthermore, after putting in the mirror, excitations on either side cannot meet each other and hit a “firewall” while crossing the Rindler horizon. Within the left and right Rindler wedges, however, the physics is indistinguishable between the two systems. Thus Rindlerization is oblivious to the potential existence of a mirror inserted at \( x = 0 \).

The point of discussing this toy model is to show that connectivity across a horizon and the possibility of Alice from the left Rindler wedge and Bob from the right Rindler wedge meeting in the future region exists only if the Hilbert spaces corresponding to the two Rindler systems interact, even though this interaction is invisible with the Rindler evolution. Within holography, since connectivity in the bulk comes with interactions between the two subsystems, the holographic dual field theories also have to be interacting for the bulk to be connected.

We then discussed an example with interactions in the bulk and also in the boundary. Global AdS can be Rindlerized and bulk-Alice and bulk-Bob introduced in Left and Right Rindler wedges can meet in the Future region as shown in Figure 2(a). The CFTs dual to the Rindler wedges are hyperbolic CFTs. These are defined on open intervals and are part of the global CFT. Boundary-Alice and boundary-Bob can meet because of the interaction between the Hilbert spaces corresponding to the two hyperbolic CFTs (although the CFTs themselves are oblivious to such interactions) as shown in Figure 2(b). Thus the bulk duals to the hyperbolic CFT should be connected and this is consistent with the bulk dual being global AdS.

Finally, we turned to the eponymous subject of the paper, the eternal AdS black hole and its purported dual non-interacting thermofield double CFT system on cylinders. We specifically discussed the BTZ black hole, which has the advantage of being obtainable by orbifolding Rindler-AdS; this allows easy comparison between Rindler-AdS and the BTZ black hole. The effect of orbifolding in the bulk is to replace the hyperbolic horizon by a cylindrical one. This maintains the smoothness across the horizon and shows that the two Hilbert spaces in the bulk remained interacting after the orbifolding. On the other hand, the effect of orbifolding on the boundary is radically different. It replaces hyperbolic CFTs by cylindrical CFTs. While it is obvious the two CFTs cannot be embedded in a bigger system to make the associated Hilbert spaces interacting, we nevertheless gave further arguments to show this. This lack of interaction in the boundary implies that the bulk dual systems also do not interact. This rules out the possibility of the BTZ being the bulk dual.

5.2 Discussion

Having established that the thermofield double state of two CFTs on cylinders is not dual to the eternal AdS black hole, what is the dual bulk description? No connectivity between the two systems in the dual picture is like the situation with the mirror in the toy example of Section 2. If the analogy were exact then the bulk states would be identical to the black hole all the way to the horizon and abruptly change. However, one does not expect to be able to make a mirror of infinitesimal thickness in quantum gravity and a reasonable guess
Figure 5. The idea advocated in [24], based on the proposal of [1], (a) is that generic geometric-microstates dual to decoupled CFTs resemble a black hole outside the stretched horizon but then differ sharply. However, the thermofield double state would involve a topology change in the bulk which creates an eternal AdS black hole. Alice and Bob falling from different sides can then meet in the future wedge. In (b) we summarise our results which is that for all kinds of generic states including the thermofield double state there is no forward wedge. The wedges end in fuzzballs, which maybe thought of as a microscopic realisation of the stretched horizon. Alice and Bob falling on the two sides thermalise on the fuzz but do not meet each other.

would be that the mirror width is Planck scale or string scale. Thus, it seems that the dual geometry would resemble a black hole outside the stretched horizon on both sides, but due to an absence of interactions between the two stretched horizons there would be no future and past wedges.

This picture is deceptively close to the one advocated in [24] where the bulk duals to generic states of decoupled CFTs are claimed to be similar to black holes up to the stretched horizon, but then differ sharply and have no future and past regions. However, the claim there is that if the bulk duals are in the thermofield double state then there is a topology change in the bulk leading to future and past wedges. Our claim is that even if the two sides are in a thermofield double state, there are no future or past wedges. These two viewpoints are summarised in Figure 5. While at this level of analysis, we cannot say much about the nature of the “non-completable” stretched horizons in Figure 5(b) except that there is no interior description of a common shared future wedge, we expect the UV complete description of the geometry to replace the stretched horizon by fuzzball microstates. Alice and Bob falling into either side will thermalise after hitting the respective “fuzz” and in time be re-emitted in a unitary fashion as shown in Figure 5(b).

Our analysis has been for a particular eternal AdS black hole, the BTZ black hole,
which is specific to three spacetime dimensions. This made some arguments clearer since we could then contrast it with the case of Rindler-AdS as the two are related by orbifolding. However, the lessons are quite general. The two decoupled CFTs on $S^d \times \mathbb{R}$ do not exhibit decaying correlators. Thus, we do not expect them to be embeddable in a bigger system and have their Hilbert spaces interact. If they cannot be made interacting then their bulk duals would also be disconnected, and we expect a picture like that shown in Figure 5(b).

We should mention that we used subregion duality crucially in the above discussion. Ref. [49] has an alternate proposal where the outside of the horizon is captured by a coarse-grained subsector of the CFT and the inside by a fine-grained one. First, let us emphasise that this does not affect our main result that two CFTs in the thermofield double state do not have a bulk description with a shared future (and past) wedge. Second, in the context of obtaining BTZ from orbifolding Rindler-AdS we find the proposal incorrect. If the CFT were indeed to split into two subsectors then in the case of global AdS the two Rindler wedges are entangled, so the two coarse-grained subsectors have to be entangled. The fine-grained ones may or may not be depending on the proposal. However, upon orbifolding if each side is to become a one-sided black hole (instead of fuzzballs ending on the stretched horizon), the orbifolding has to make each side’s coarse-grained subsector maximally entangled with the fine-grained one. We do not see how an operation like orbifolding can accomplish the entanglement swapping.

Finally, we would like to comment on the implications of our work for states dual to just one CFT. Our main result is that when two cylindrical CFTs are in the thermofield double state, or any other state for that matter, the bulk description does not involve shared future and past wedges. Further, assuming a subregion duality of the kind advocated in [9, 35, 36], the CFT duals match the black hole outside the stretched horizon and then start differing and capping off outside the semiclassically anticipated horizon. In effect, the fact that the two CFTs were entangled made no difference. Then it is easy to see that typical states of one CFT would have the same behaviour as the ensemble and the geometric dual will end in a fuzz cap at the stretched horizon with there being no region behind the future and past horizons. This is to be contrasted with the traditional picture where an excitation in a single CFT settles down to a pure state which is described by a black hole formed from collapse.

It would of course be interesting to see how this comes about and the detailed dynamics that prevent a black hole from forming in the bulk, despite being predicted by equations of motion. One possibility has been advocated by Mathur [50] where the exponential number of bulk fuzzball microstates invalidates the saddle point approximation, allowing a thin shell to tunnel into (a superposition of) fuzzball states. This still leaves open the question of what the general mechanism responsible for the existence of so many states with horizon-scale structure is.

If, as we have argued from AdS/CFT, there are unanticipated quantum gravity effects well below the Planck scale (under special circumstances), this opens up the possibility of other new effects. Obviously, we do not expect new quantum gravitational physics on solar system or intergalactic scales, but there could be important effects in early cosmology.
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A Global vs. Rindler-AdS coordinates

Often global AdS is written in terms of coordinate $\rho, \tau$ and the angular coordinates as
\[
d s^2 = -(1 + \rho^2) d \tau^2 + \frac{d \rho^2}{1 + \rho^2} + \rho^2 d \Omega_{d-1}^2
\]  
(A.1)
however the boundary is then at $\rho = \infty$. To see the physics of the boundary better, it is useful to use a different radial coordinate given by the relation $\tan u = \rho$. Then the metric is
\[
d s^2 = \sec^2 u (-d \tau^2 + du^2 + \sin^2 u \, d \Omega_{d-1}^2)
\]  
(A.2)
where the radial coordinate has the range given by $u \in [0, \pi/2)$. The Rindler AdS-coordinates treat the polar angle of $S^{d-1}$, $\phi$, separately and are related to global coordinates by
\[
\tan^2 u = (r^2 - 1) \frac{\cosh(2\chi) + \cosh(2t)}{2} + \sinh^2(\chi),
\]  
(A.3)
\[
\tan \phi = \frac{r}{\sqrt{r^2 - 1}} \frac{\sinh(\chi)}{\cos(\t)},
\]  
(A.4)
\[
\tan \tau = \frac{\sqrt{r^2 - 1}}{r} \frac{\sin(\t)}{\cosh(\chi)}.
\]  
(A.5)
We note that $\phi \in [0, \pi]$ for $d > 1$ and $\phi \in [0, 2\pi)$ with $\phi \sim \phi + 2\pi$ for $d = 1$. The inverse relations are given by
\[
r^2 - 1 = -\sin^2 \tau \sec^2 u + \cos^2 \phi \tan^2 u,
\]  
(A.6)
\[
\coth \chi = \frac{\cos \tau}{\sin \phi \sin u},
\]  
(A.7)
\[
\tanh \t = \frac{\sin \tau}{\cos \phi \sin u}
\]  
(A.8)
where the radial coordinate is written shifted so that the RHS gives the Rindler horizon. Note that $\chi \in (-\infty, \infty)$. In terms of these coordinates the metric is
\[
d s^2 = -(r^2 - 1) d\t^2 + \frac{dr^2}{r^2 - 1} + r^2 \left[ d\chi^2 + \sinh^2(\chi) d\Omega_{d-2}^2 \right].
\]  
(A.9)
From (A.7) we see that the intersection of the Rindler horizon $r = 1$ and the boundary cylinder $u = \pi/2$ is given by
\[ \sin \tau = \pm \cos \phi. \] (A.10)
From this it is easy to see that at the edges of the causal diamond $t, \chi = \pm \infty$. We will see that orbifolding $AdS_3$ to get BTZ requires $\chi \sim \chi + 2\pi$ and therefore we see that the horizon gets disconnected from the boundary cylinder.

\section*{B BTZ as an orbifold of $AdS_3$}

It is well known that BTZ can be viewed as an orbifold of $AdS_3$ \cite{btz}. We follow the analysis in \cite{btz}. $AdS_3$ is the group manifold of $SL(2, \mathbb{R})$ and one can quotient it
\[ g \sim hgh \]
where $h$ is a hyperbolic element of $SL(2, \mathbb{R})$ to obtain the BTZ geometry. Let us parameterise $g$ as
\[ g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \cos(\tau) \sec(u) + \sin(\phi) \tan(u) & \sin(\tau) \sec(u) + \cos(\phi) \tan(u) \\ -\sin(\tau) \sec(u) + \cos(\phi) \tan(u) & \cos(\tau) \sec(u) - \sin(\phi) \tan(u) \end{pmatrix} \] (B.2)

From (A.3), (A.4) and (A.5) we see that the Rindler-AdS coordinates are related to $g$ by
\[ a = r e^\chi, \quad b = \sqrt{r^2 - 1} e^t \] (B.3)
\[ c = \sqrt{r^2 - 1} e^{-t}, \quad d = r e^{-\chi}. \] (B.4)
The metric is then given by
\[ ds^2 = -(r^2 - 1)dt^2 + \frac{dr^2}{r^2 - 1} + r^2 d\chi^2. \] (B.5)
These coordinates only cover one of the exterior regions but one can analytically continue to the other exterior and the future and past regions by the Kruskal extension
\[ u = -\sqrt{\frac{r - 1}{r + 1}} e^{-t}, \quad v = \sqrt{\frac{r - 1}{r + 1}} e^t. \] (B.6)
They are related to $g$ as
\[ a = \frac{1 - uv}{1 + uv} e^\chi, \quad b = -\frac{2u}{1 + uv} e^t \] (B.7)
\[ c = \frac{2v}{1 + uv} e^{-t}, \quad d = \frac{1 - uv}{1 + uv} e^{-\chi}. \] (B.8)
The metric is then given by
\[ ds^2 = \frac{1}{(1 + uv)^2} (-4dudv + (1 - uv)^2 d\chi^2). \] (B.9)
If we quotient $AdS_3$ by $h = e^{\pi \sigma_3}$ we get

$$
\begin{pmatrix}
\cos(\tau) \sec(u) + \sin(\phi) \tan(u) & \sin(\tau) \sec(u) + \cos(\phi) \tan(u) \\
-\sin(\tau) \sec(u) + \cos(\phi) \tan(u) & \cos(\tau) \sec(u) - \sin(\phi) \tan(u)
\end{pmatrix}
= \begin{pmatrix}
\sin(\tau) \sec(u) + \cos(\phi) \tan(u) & \cos(\tau) \sec(u) - \sin(\phi) \tan(u) \\
\cos(\tau) \sec(u) + \sin(\phi) \tan(u) & e^{2\pi}(\cos(\tau) \sec(u) - \sin(\phi) \tan(u))
\end{pmatrix}
$$

(B.10)

In particular this implies

$$\chi \sim \chi + 2\pi$$

(B.11)

which converts the Rindler-AdS geometry which has the same metric as the BTZ black hole, into a true BTZ black hole with a cylindrical boundary. The BTZ horizons are at $uv = 0$, and the boundary at $uv = -1$. There is also a orbifold singularity at $uv = 1$ beyond which the geometry is cutoff because of closed timelike curves.

C Infall as Bell measurement

Immediately after AMPS’s firewall paper [13], one of us proposed that the correct way to analyse the situation would be think of the observer as part of the complete system and measurements as coming from decoherence between the observer (or her apparatus) and the rest of the system [51]. This appendix is based on the same theme and can be seen as a motivation for the rest of the paper. We will consider bulk-Alice, a closed string object moving around in the eternal AdS black hole, and ask what does it take for her to verify that the horizon is smooth. We will then consider boundary-Alice, an open string excitation moving on one of the boundary cylinders, and ask the same question. We will demonstrate an inconsistency and claim that the thermofield double CFTs are not dual to the eternal AdS black hole.

Consider massless fields in $1 + 1$ dimensions. The equations of motion split the fields into left and right movers. We consider only the left movers and the right movers behave the same. It can shown (see [52] for example) that the Minkowski vacuum can be expressed in terms of Rindler modes as

$$|0_M\rangle = \frac{1}{\sqrt{\prod \lambda}} \prod e^{\tanh \theta_\lambda b^{\dagger}_\lambda R} b^{\dagger}_\lambda L |0_R\rangle |0_L\rangle. \tag{C.1}$$

where $Z_\lambda = Tr[e^{-2\pi \lambda/a}]$ and $\tanh \theta_\lambda = e^{-\pi \lambda/a}$ where $a$ is the acceleration of the Rindler observer. Different modes given by different $\lambda$ decouple and we can focus on the vacuum for a particular $\lambda$

$$|0_{M,\lambda}\rangle = \frac{1}{\sqrt{Z_\lambda}} \sum \tanh^n \theta_\lambda |n_{\lambda, R}\rangle |n_{\lambda, L}\rangle. \tag{C.2}$$

Note that if we consider the high temperature limit and restrict to fermionic modes then the above truncates to

$$|0_{M,\lambda}\rangle = \frac{1}{\sqrt{2}} (|0_{\lambda, R}\rangle |0_{\lambda, L}\rangle + |1_{\lambda, R}\rangle |1_{\lambda, L}\rangle) \tag{C.3}$$
and we can simplify our analysis by just considering qubits. The right moving observer will encounter left moving modes localised inside and outside the horizon and will find the state as the vacuum only if together they are in the state (C.3).

There is a simple generalization of the Minkwoski vacuum state (C.3) which is a maximally entangled state between the two subsystems. One can write down four such orthogonal states

\[|\varphi_1\rangle := \frac{1}{\sqrt{2}}(|\hat{0}\rangle|0\rangle + |\hat{1}\rangle|1\rangle),\]
\[|\varphi_2\rangle := \frac{1}{\sqrt{2}}(|\hat{0}\rangle|0\rangle - |\hat{1}\rangle|1\rangle),\]
\[|\varphi_3\rangle := \frac{1}{\sqrt{2}}(|\hat{0}\rangle|1\rangle + |\hat{1}\rangle|0\rangle),\]
\[|\varphi_4\rangle := \frac{1}{\sqrt{2}}(|\hat{0}\rangle|1\rangle - |\hat{1}\rangle|0\rangle),\] (C.4)

and these are referred to as Bell states. The $|\hat{0}\rangle$ and $|\hat{1}\rangle$ are eigenstates of $\hat{\sigma}_z$ and similarly $|0\rangle$ and $|1\rangle$ are eigenstates of $\sigma_z$. Observe that in a simplified qubit model the Minkwoski state corresponds to the first Bell state.

The reduced density matrix of the hatted and the unhatted systems for all four states are

\[\hat{\rho} = \frac{1}{2}(|\hat{0}\rangle\langle\hat{0}| + |\hat{1}\rangle\langle\hat{1}|), \quad \rho = \frac{1}{2}(|0\rangle\langle0| + |1\rangle\langle1|)\] (C.5)

which means that Charlie with access to only one of the systems (i.e. with access to operators $\hat{I} \otimes \sigma_x$, $\hat{I} \otimes \sigma_y$ and $\hat{I} \otimes \sigma_z$) will get identical response from all four states and will be unable to distinguish between them. This does not, however, mean that the four states are indistinguishable. These states are eigenstates of the operators $\hat{\sigma}_x \otimes \sigma_x$, $\hat{\sigma}_y \otimes \sigma_y$ and $\hat{\sigma}_z \otimes \sigma_z$. The eigenvalues are shown in the table below.

| state | $\hat{\sigma}_x \otimes \sigma_x$ | $\hat{\sigma}_y \otimes \sigma_y$ | $\hat{\sigma}_z \otimes \sigma_z$ |
|-------|-------------------------------|-------------------------------|-------------------------------|
| $|\varphi_1\rangle$ | +1 | -1 | +1 |
| $|\varphi_2\rangle$ | -1 | +1 | +1 |
| $|\varphi_3\rangle$ | +1 | +1 | -1 |
| $|\varphi_4\rangle$ | -1 | -1 | -1 |

Thus, an observer, Alice, can distinguish between the four states by measuring the expectation value of any of the two operators, say $\hat{\sigma}_x \otimes \sigma_x$ and $\hat{\sigma}_z \otimes \sigma_z$. This is called a Bell measurement.

In light of this, our previous comment about a right moving observer finding the left movers in the vacuum only if they are in the state (C.3) can be restated in the following way. Accelerating observers who stay inside the Rindler wedge have access to only half the system can only perform non-Bell measurements and cannot tell of the full state is the Minkwoski vacuum or any other state that leaves the right wedge density matrix the same (see Figure 6(a)). However, inertial observers can measure the full state of the system and in fact do so while crossing the horizon. They can thus tell if the full state is the Minkwoski
vacuum or some other state. Thus inertial observers perform Bell measurements. This is shown in Figure 6(b).

![Diagram](image)

**Figure 6.** An accelerating observer only intersects the modes of the right wedge so can only do non-Bell measurements. These do not measure the actual state of the system but instead collapse the system into a different state. An inertial observer on the other hand intersects both modes and thus can perform a Bell measurement to verify that the full state is the Minkowski vacuum.

This has immediate consequences for Maldacena’s proposal that the eternal AdS black hole is dual to two decoupled CFTs on cylinders in a thermofield double state [1]. Bulk-Alice, a creature created in the left exterior of the eternal AdS black hole (see Figure 5(a)) performs a Bell measurement on the modes inside and outside while falling into the black hole. According to the AdS/CFT proposal, when bulk-Alice is close to the corresponding boundary, the dual boundary-Alice is an excitation in the CFT living on that boundary. However, since the two CFTs are defined on cylinders and there is no interaction term between them, there is no way boundary-Alice can perform a Bell measurement to ascertain that the two CFTs are indeed in the thermofield double state. Thus, there is a contradiction. Since, there is nothing wrong with the bulk or the boundary systems by themselves, the conclusion is that they are not dual to each other.

### D Why the method of images gives the wrong correlation function

Naively one would think that the correlator after orbifolding the hyperbolic CFT can be obtained by the method of images from (3.9)

\[
\langle O(t, \chi) O(0, 0) \rangle_{\text{orb}}^{\text{naive}} = \sum_{n=-\infty}^{\infty} \frac{1}{16} \frac{1}{\sinh^2 \left( \frac{t-\chi-2n\pi}{2} \right) \sinh^2 \left( \frac{t+\chi+2n\pi}{2} \right)}.
\]

(D.1)

However, this is incorrect. The point is that (3.9) decays with time and thus when Fourier transformed has quasi-normal modes i.e. it has poles away from the real line [37, 42]. This comes from the CFT being defined on an infinite line and physically corresponds to a wave
function exploring the infinite phase space. However, on closed intervals the phase space is finite and while there is quasi-periodicity, there are no quasi-normal modes \[1, 44\]. The problem with the correlation function obtained from method of images (D.1) is that it still has quasi-normal modes so it cannot be the correct correlation function for a CFT of finite extent and temperature.

The problem is easily illustrated even at the level of free fields. Consider a circular string whose transverse oscillations are governed by a CFT (it could be the closed string of string theory). Then it has left movers and right movers and we have

\[
\partial X = \sum_{n \in \mathbb{Z}^+} \sqrt{\frac{\omega_n}{2L}} (a_{L,\omega_n} e^{-i\omega_n x_-} + a_{L,\omega_n}^\dagger e^{i\omega_n x_-})
\]

\[
\bar{\partial} X = \sum_{n \in \mathbb{Z}^+} \sqrt{\frac{\omega_n}{2L}} (a_{R,\omega_n} e^{-i\omega_n x_+} + a_{R,\omega_n}^\dagger e^{i\omega_n x_+})
\]

where \(\omega_n = \frac{2\pi n}{L}\) and \(x_{\pm} = t \pm y\). For a thermal state with inverse temperatures \(\beta_L, \beta_R\) we get the correlator

\[
\langle (\partial X \bar{\partial} X)(t,y)(\partial X \bar{\partial} X)(0) \rangle_{\beta_L, \beta_R} = \sum_{n \in \mathbb{Z}} \frac{\omega_n}{2L} e^{\beta_L \omega_n - 1} e^{i\omega_n x_-} \times \sum_{n \in \mathbb{Z}} \frac{\omega_n}{2L} e^{\beta_R \omega_n - 1} e^{i\omega_n x_+}.
\]

If we approximate the summation by an integral we get

\[
\sum_{n \in \mathbb{Z}} \frac{\omega_n}{2L} e^{\beta_L \omega_n - 1} e^{i\omega_n x_-} \rightarrow \frac{\pi}{4\beta^2} \frac{1}{\sinh(\frac{\pi x_-}{\beta})^2}
\]

Which is the same as (3.9) up to a conventional normalisation factor. This obviously had to work because the two-point function is fixed by conformal invariance when we take \(L \rightarrow \infty\), which is basically what we do when we replace the sum by an integral. However, this example clearly shows what happens. The two-point function with the sum did not have any quasi-normal modes and was the correct two-point function on the cylinder. If we first take an infinite cylinder limit and then use the method of images we obviously do not recover the correct answer.

For a CFT at finite temperature and with compact spatial extent (thus on a torus upon Euclideanization) the two-point function is not universal but the method of images gives the \textit{wrong} universal answer.

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