CP Violation, Sneutrino Oscillation and Neutrino Masses in R-parity Violating Supersymmetric Standard Model

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Abstract

In supersymmetric theories, sneutrino–anti-sneutrino mixing can occur with the oscillation time \( \sim 0.01 \text{ ps} \) corresponding the atmospheric neutrino mass scale \( \sim 0.05 \text{ eV} \). We explore the possibility of observing sneutrino oscillation phenomena and CP violation when R-parity violation explains the observed neutrino masses and mixing. It is shown for some parameter region in the bilinear model of R-parity violation that the tiny sneutrino mass splitting and time-dependent CP violating asymmetries could be measured in the future experiments if the tau sneutrino is the lightest supersymmetric particle.
The atmospheric neutrino data from the Super-Kamiokande experiment strongly suggest the presence of sub-eV neutrino masses. As is commonly accepted, such tiny masses are most likely to be of the Majorana type breaking lepton number by two units ($\Delta L = 2$). In supersymmetric theories where each neutrino species $\nu$ is accompanied by a complex scalar $\tilde{\nu}$, similar lepton number violation should occur in the scalar (sneutrino) sector. The sneutrinos $\tilde{\nu}$ and $\tilde{\nu}^\ast$ generally have the mass terms,

$$- \mathcal{L} = m_\tilde{\nu}^2 \tilde{\nu} \tilde{\nu}^\ast + \frac{1}{2} \left( m_\tilde{\nu}^2 \tilde{\nu} \tilde{\nu} + h.c. \right),$$

where $m_\tilde{\nu}$ is the usual slepton mass of the order 100 GeV and $m_\tilde{\nu}^2$ carries the same lepton number as the Majorana neutrino mass $m_\nu$. As we will see, the soft $B$-terms are the origin of the mass $m_\tilde{\nu}^2$ in our case. This ‘Majorana’ sneutrino mass term generates mass splitting and mixing in the sneutrino-anti-sneutrino system. One can easily find from Eq. (1) that the sneutrino mass-squared eigenvalues are $m_\tilde{\nu}^2 \pm |m_\tilde{\nu}^2|$. In general supersymmetric models, one finds $m_\tilde{\nu}^2 \sim m_\nu m_\tilde{\nu}$ and thus the sneutrino mass splitting of the order of the neutrino mass; $\Delta m_\tilde{\nu} \sim |m_\tilde{\nu}^2|/m_\tilde{\nu} \sim m_\nu$. Similarly to the case of neutrinos, such a small mass difference could only be measured through the observation of sneutrino oscillation, providing a novel opportunity to probe $\Delta L = 2$ lepton number violating phenomena in the future collider experiments.

Mixing phenomena in the sneutrino–anti-sneutrino system are analogous to those in the $B-\bar{B}$ system for which we refer the readers to the reviews in Ref. [4]. Extending the original investigation in Ref. [3], we wish to formulate CP violating effects in time-dependent sneutrino oscillation and apply them to a specific model of neutrino masses, namely, supersymmetric standard model with R-parity violation. For this, we assume the neutrino masses explaining the Atmospheric neutrino data with the largest neutrino mass $m_{\nu_3} \sim 0.05$ eV and the large $\nu_\mu-\nu_\tau$ mixing [4]. It is amusing to notice that the neutrino mass $m_{\nu_3}$ corresponds to the proper sneutrino oscillation time, $t_{osc} = \Delta m_\tilde{\nu}^{-1} = \frac{m_\tilde{\nu}}{m_p} L \sim 0.013$ ps where $L$ measures the distance between the sneutrino production and decay vertex and $\frac{m_\tilde{\nu}}{m_p}$ is the boost factor of a few. Therefore, considering a spatial resolution of a few hundredths of ps [4], the atmospheric neutrino mass scale could be directly probed by observing time-dependent sneutrino oscillation in the future experiments.
For sneutrino oscillation to occur, it is required that the sneutrino life-time is longer than its oscillation time;

\[ \Gamma_{\tilde{\nu}} < \Delta m_{\tilde{\nu}}. \]

In the supersymmetric see-saw model considered in Ref. [3], the sneutrino mass splitting is given by \( \Delta m_{\tilde{\nu}}/m_{\nu} \approx 2(A + \mu \cot \beta)/m_{\tilde{\nu}} \) where \( A \) is the trilinear soft mass and \( \tan \beta \) is the ratio of the vacuum expectation values of two Higgs bosons. With \( m_{\nu} \sim 0.05 \text{ eV} \), one generically expects \( \Delta m_{\tilde{\nu}} < 1 \text{ eV} \). Then, the requirement \( \Gamma_{\tilde{\nu}} < 1 \text{ eV} \) can be arranged in a limited region of parameter space under the assumption that the sneutrino has only three-body decay channels and the stau \( \tilde{\tau}_R \) is the ordinary lightest supersymmetric particle (LSP) [3]. In this scheme, CP violating effects could arise if a nontrivial phase exists in the ‘Majorana’ sneutrino mass.

Another interesting scheme for generating neutrino masses and mixing is to allow R-parity violation in the supersymmetric standard model [7]. In this case, the LSP is unstable and decays through R-parity violating couplings which generate both neutrino masses and sneutrino mass splittings. Therefore, it is expected that R-parity violation can naturally lead to the suppressed decay rate and observable sneutrino oscillation phenomena if a sneutrino is the LSP. Whether or not a sneutrino can be the LSP depends on the models of supersymmetry breaking. In the constrained framework of minimal supergravity [8] or gauge-mediated supersymmetry breaking [9], the sneutrino LSP may be obtained in a limited parameter space where the D-terms can make \( m_{\tilde{\nu}}^2 \) smaller than \( m_{\tilde{\tau}_R}^2 \). But, such a possibility appears to be ruled out [10] by current experiments on the invisible Z width, providing the limit \( m_{\tilde{\nu}} > 44.7 \text{ GeV} \) [3]. Therefore, we assume less constrained (non-universal) soft masses so that slepton doublets are lighter than slepton singlets and thus sneutrinos lighter than charged sleptons. Most favorable framework for this is SU(5) grand unification theory where the common soft mass of 5 multiplets is smaller than that of 10 multiplets or gauginos. Let us remark that our discussion can be complimentary to the case of a neutralino LSP in testing the R-parity violation model of neutrino masses and mixing [11].

We begin our main discussion by considering the general sneutrino-fermion-fermion
couplings as follows:

\[ \mathcal{L} = -\frac{g}{\sqrt{2}} \bar{\nu}_i \gamma_{\mu} \left[ P_L K_{L,i}^{ff} + P_R K_{R,i}^{ff} \right] f' + h.c. \]  

(2)

where \( P_{L,R} = (1 \mp \gamma_5)/2 \) and the index \( i \) refers to the lepton flavor. Neglecting mixing between different sneutrino flavors, each sneutrino species \( \tilde{\nu}_i \) has the mass term \( [\text{I}] \) with complex \( m_i^2 \). The mass eigenstates denoted by two real fields, \( \tilde{\nu}_{i1} \) and \( \tilde{\nu}_{i2} \), have the following decay widths:

\[ \Gamma_{\tilde{\nu}_i} \equiv \Gamma_{\tilde{\nu}_{i1}} + \Gamma_{\tilde{\nu}_{i2}} = \frac{g^2}{16\pi} \sum_{f,f'} \left[ |K_{L,i}^{ff'}|^2 + |K_{R,i}^{ff'}|^2 \right] \]  

(3)

\[ \Delta \Gamma_{\tilde{\nu}_i} \equiv \Gamma_{\tilde{\nu}_{i1}} - \Gamma_{\tilde{\nu}_{i2}} = \frac{g^2}{16\pi} \sum_{f,f'} \left[ K_{L,i}^{ff'} K_{R,i}^{ff'} e^{i\phi_M} + c.c. \right] \]  

(4)

where \( \phi_M \equiv \text{Arg}(m_i^2) \). The time-evolution of the state identified as the sneutrino \( \tilde{\nu}_i \) or the anti-sneutrino \( \tilde{\nu}_i^* \) at an initial time \( t = 0 \) is given respectively by

\[ |\tilde{\nu}_i(t)\rangle = g_+(t)|\tilde{\nu}_i\rangle + g_-(t)e^{-i\phi_M}|\tilde{\nu}_i^*\rangle \]

\[ |\tilde{\nu}_i^*(t)\rangle = g_+(t)|\tilde{\nu}_i^*\rangle + g_-(t)e^{+i\phi_M}|\tilde{\nu}_i\rangle \]  

(5)

where \( g_{\pm}(t) = \frac{1}{2}\exp(-\frac{1}{2}\Gamma_{\tilde{\nu}_i} t - im_{\tilde{\nu}_i} t)[1 \pm \exp(\frac{1}{2}\Delta \Gamma_{\tilde{\nu}_i} t + i\Delta m_{\tilde{\nu}_i} t)] \). Then, the time-dependent CP asymmetry for the sneutrino decay to the final state \( ff' \) is

\[ A_{CP}^i(ff';t) \equiv \frac{\Gamma(\tilde{\nu}_i(t) \rightarrow ff') - \Gamma(\tilde{\nu}_i^*(t) \rightarrow ff')}{\Gamma(\tilde{\nu}_i(t) \rightarrow ff') + \Gamma(\tilde{\nu}_i^*(t) \rightarrow ff')} = \frac{\cos(\Delta m_{\tilde{\nu}_i} t)X_i^{ff'} - 2\sin(\Delta m_{\tilde{\nu}_i} t) \text{Im}(Y_i^{ff'})}{c(t) - s(t) \text{Re}(Y_i^{ff'})} \]  

(6)

where \( c(t) = (e^{\Delta \Gamma_{\tilde{\nu}_i} t/2} + e^{-\Delta \Gamma_{\tilde{\nu}_i} t/2})/2, s(t) = (e^{\Delta \Gamma_{\tilde{\nu}_i} t/2} - e^{-\Delta \Gamma_{\tilde{\nu}_i} t/2})/2 \) and

\[ X_i^{ff'} \equiv \frac{|K_{L,i}^{ff'}|^2 + |K_{R,i}^{ff'}|^2 - (f \leftrightarrow f')}{|K_{L,i}^{ff'}|^2 + |K_{R,i}^{ff'}|^2 + (f \leftrightarrow f')}, \]

\[ Y_i^{ff'} \equiv \frac{e^{i\phi_M} K_{L,i}^{ff'} K_{R,i}^{ff'} + (f \leftrightarrow f')}{|K_{L,i}^{ff'}|^2 + |K_{R,i}^{ff'}|^2 + (f \leftrightarrow f')} \]  

(7)

For the ‘flavor-specific’ final state \( ff' \) arising from the decay of \( \tilde{\nu}_i \), that is, \( \tilde{\nu}_i \rightarrow ff' \Leftrightarrow \tilde{\nu}_i^* \), we have \( X_i^{ff'} = 1 \) and \( Y_i^{ff'} = 0 \). Thus, the CP asymmetry in Eq. (6) becomes

\[ A_{CP}^i(ff';t) = \cos(\Delta m_{\tilde{\nu}_i} t) \quad \text{for} \quad f \neq f' \]  

(8)
when $\Delta \Gamma_\nu \approx 0$. This time-dependence could be measured to determine the sneutrino mass splitting as far as $x_i$ is not too small. On the other hand, the time-integrated mixing probability \[3\],
\[ \chi_i \equiv \frac{x_i^2 + y_i^2}{2(x_i^2 + 1)} \] (9)
where $x_i \equiv 2\Delta m_{\tilde{\nu}_i}/\Gamma_{\tilde{\nu}_i}$ and $y_i \equiv \Delta \Gamma_{\tilde{\nu}_i}/\Gamma_{\tilde{\nu}_i}$, can be used to determine the sneutrino mass splitting when $x_i \sim 1$. Recall that $\chi_i$ can be measured by counting the ‘same-sign’ and ‘opposite-sign’ lepton events, $\tilde{\nu}_i \tilde{\nu}_i \to ll\bar{l}\bar{l}$ and $\tilde{\nu}_i \tilde{\nu}_i^* \to ll'\bar{l}'\bar{l}'$, analogous to the $B$ system.

As we will see, our model generically has $y_i \ll 1$. For the final state $f\bar{f}$ which is shared by the decays of $\tilde{\nu}_i$ and $\tilde{\nu}_i^*$, we get $X_{ff}^i = 0$ and thus the time-dependent CP asymmetry (for $y_i \approx 0$) becomes
\[ A_{CP}^i(f\bar{f};t) = \frac{2|\rho_i|}{1 + |\rho_i|^2} \sin(\phi_D - \phi_M) \sin \Delta m_{\tilde{\nu}_i} t \] (10)
where $\rho_i = |\rho_i|e^{i\phi_D} \equiv K_R^{f^*f}/K_L^{f^*f}$. This is analogous to the CP asymmetry from $B^0$ and $\bar{B}^0$ decays to CP eigenstates [3].

Let us now examine how the quantities discussed above arise in the R-parity violating supersymmetric standard model. For the sake of simplicity, we introduce only bilinear R-parity breaking terms in the superpotential $W$ and the soft scalar potential $V_{soft}$ of the supersymmetry standard model;

\[ W \ni -\mu_i L_i H_2, \quad V_{soft} \ni m_{1H}^2 L_i H_1^\dagger + B_i L_i H_2 + h.c. \] (11)

where the same notations $L_i, H_{1,2}$ are used for the lepton and Higgs superfields and their scalar components. This type of model is known to generate viable neutrino mass matrices explaining both the solar and the atmospheric neutrino data [12]. As is well-known, the bilinear terms in Eq. (11) give rise to nonzero sneutrino vacuum expectation values [7];
\[ a_i \equiv \langle \tilde{\nu}_i^* \rangle/\langle H_1^0 \rangle = (m_{1H}^2 + \mu \mu_i + B_i t_\beta)/m_{\tilde{\nu}_i}^2 \]
where $t_\beta = \tan \beta \equiv \langle H_2^0 \rangle/\langle H_1^0 \rangle$ and $\mu$ is the supersymmetric Higgs mass parameter. Then the quantities $\mu_i - \mu a_i$ determine the tree-level neutrino mass matrix;
\[ m_{\nu_i}^2 = -M_Z^2 \xi_i \xi J_{\beta}^2 / F_N \] (12)
where $\xi_i \equiv a_i - \mu_i / \mu$ and $F_N \equiv M_1 M_2 / (M_1 c^2_W + M_2 s^2_W) - M_Z^2 \sin 2\beta / \mu$. Here $M_1$ and $M_2$ denote the $U(1)$ and $SU(2)$ gaugino masses, respectively. Remember that only one neutrino, $\nu_3$, becomes massive from Eq. (12), while the other two will get smaller masses from one-loop corrections. The mass matrix (12) fixes two neutrino mixing angles, $\theta_{23}$ and $\theta_{13}$, corresponding to the atmospheric neutrino and the reactor neutrino mixing angles, respectively, as follows [11]:

$$
\sin^2 2\theta_{23} = 4|\hat{\xi}_\mu|^2|\hat{\xi}_\tau|^2, \quad \sin^2 2\theta_{13} = 4|\hat{\xi}_e|^2(1 - |\hat{\xi}_e|^2)
$$

where $\hat{\xi}_i \equiv \xi_i / |\xi|$ and $|\xi|^2 \equiv \sum_i |\xi_i|^2$. Current experiments require $|\xi_\mu| \approx |\xi_\tau|$ for the large atmospheric neutrino mixing [1] and $|\xi_e| / |\xi_\tau| < 0.3$ for the suppressed reactor neutrino oscillation $\nu_e \rightarrow \nu_{\mu,\tau}$ [13].

R-parity violation induces also nontrivial mixing between sneutrinos and neutral Higgs bosons. Furthermore, their CP-even and CP-odd parts mix together if general complex couplings are allowed. Therefore, we have to deal with a mass matrix of 10 neutral boson fields including the Goldstone mode. To do this, it is convenient to define the ‘proper’ sneutrino fields getting rid of the Goldstone mode by performing the see-saw rotation with the small angle $a_i$. In this basis, one finds that the sneutrino–Higgs mixing term for each sneutrino generation is proportional to $B_i - B \eta_i$ [14]. The next step is to rotate away these mixing terms to find the sneutrino mass splitting:

$$
\Delta m_{\tilde{\nu}_i} = 2m_{\tilde{\nu}_i} M_Z^2 m_A^4 |\eta_i|^2 c^2 s^2 \beta / F_S
$$

where $\eta_i \equiv a_i - B_i / B$ and $F_S \equiv (m_{\tilde{\nu}_i}^2 - m_h^2)(m_{\tilde{\nu}_i}^2 - m_H^2)(m_{\tilde{\nu}_i}^2 - m_A^2)$. Here $m_h$ and $m_H$ denote the light and heavy CP-even Higgs boson masses, respectively and $m_A$ is the CP-odd Higgs boson mass which are defined in the R-parity conserving limit. Eq. (14) is consistent with the result of Ref. [3] for the CP-conserving case. In deriving the above result, we neglected the sneutrino flavor mixing induced by the neutrino flavor mixing. This is a good approximation as far as the mass differences between two sneutrino flavors are much larger than $m_{\nu}, |m_{\tilde{\nu}_i} - m_{\tilde{\nu}_j}| \gg m_{\nu}$, which is usually the case. Remark that the universality in $B$ parameters, $B_i / B = \mu_i / \mu$, implies $\xi_i = \eta_i$ which is not assumed in this
paper. Combining Eqs. (12) and (14), one finds

$$\Delta m_{\tilde{\nu}_i}/m_{\nu_3} \sim (2F_N/m_{\tilde{\nu}_i})(|\eta_i|^2/|\xi|^2)$$

(15)

for $m_{\tilde{\nu}_i} \ll m_A$. Therefore, we get $\Delta m_{\tilde{\nu}_i} \sim m_{\nu_3}$ as expected for $F_N \sim m_{\tilde{\nu}_i}$ and $|\eta_i| \sim |\xi|$. Here we note that the sneutrino oscillation time $1/\Delta m_{\tilde{\nu}_i}$ can be made larger than $1/m_{\nu_3} \sim 0.013$ ps for $|\eta_i| < |\xi|$. From the discussion below Eq. (13) implying $|\eta_e| \sim |\xi_e| \ll |\xi|$, one finds that the electron sneutrino $\tilde{\nu}_e$ will generically have the largest oscillation time. But, we will see that the $\tilde{\nu}_e$ decay to charged leptons have too small branching fraction to be observed.

The sneutrino couplings to light fermions $f$ and $f'$ arise from the neutrino-neutralino, the charged lepton–chargino, and the slepton-Higgs mixing. In the bilinear model under consideration, one finds the coupling constants $K_{L,R}$ defined in Eq. (2) as follows:

$$K_{R,i}^{\nu_j \bar{\nu}_k} = \frac{\delta_{ij} M_Z}{c_W F_N} \xi_k c_{\beta},$$

$$K_{L,i}^{\tau_k \bar{\nu}_j} = \frac{m_{\tau} [\delta_{ij} \phi_{k1} c_{\beta} + \delta_{j\tau} \phi_{k1} c_{\beta}]}{c_W M_W},$$

$$K_{L,i}^{\tau_k \bar{\nu}_j} = \frac{m_{\tau} [\delta_{ij} \phi_{k2} c_{\beta} + \delta_{j\tau} \phi_{k2} c_{\beta}]}{c_W M_W},$$

where the coefficients $\omega$ and $\theta$ are given by

$$\omega_{k1} = \frac{-\mu M_2 \xi_k}{\mu M_2 - M_W^2 s_{2\beta}^2} + a_k, \quad \omega_{k2} = \frac{2\mu (\mu + M_2 t_{\beta}) M_W^2 \xi_k c_{\beta}}{(\mu M_2 - M_W^2 s_{2\beta}^2)^2},$$

$$\theta_{i1} = -a_i - s_{2\beta}^2 m_{\tilde{\nu}_i}^2 - m_{\tilde{\nu}_i}^2 m_A^2 - (m_{\tilde{\nu}_i}^2 + m_A^2 c_{2\beta}) M_Z^2 s_{2\beta}^2 m_A^2 \eta_i / F_S$$

$$\theta_{i2} = -c_{2\beta}^2 s_{2\beta}^2 M_Z^2 [m_{\tilde{\nu}_i}^2 - m_A^2 c_{2\beta}] m_A^2 \eta_i / F_S.$$  

(17)

In Eq. (16), we neglected the terms proportional to lighter quark and lepton masses. In the presence of trilinear R-parity violating terms, the couplings (16) will get additional contributions leaving Eqs. (12) and (14) unchanged.

We are ready to discuss how the observable sneutrino mixing phenomena occur in our scheme. From Eqs. (16) and (17), one can see that the dominant decay channels are

$$\tilde{\nu}_i \rightarrow \nu \tilde{\nu} \quad \text{and} \quad \tilde{\nu}_i \rightarrow b \bar{b}, \tau \bar{\tau}$$

for low and large $t_{\beta}$, respectively, if the R-parity violating parameters $\omega_i, \theta_i$, are of the same order. For the latter decay channels, one also finds that
$|K_L|$ is larger than $|K_R|$ for the parameters $M_Z < M_2, \mu$ and large $\tan \beta$. This shows that we generally have $y = \Delta \Gamma / \Gamma \ll 1$ as the decay rate difference $\Delta \Gamma$ gets non-vanishing contributions from $K_{L}^{ff}K_{R}^{ff}$ where $f = \tau, b$. Since we are interested in the region where $\Gamma_{\tilde{\nu}} < \Delta m_{\tilde{\nu}}$, we can put $c(t) = 1$ and $s(t) = 0$ in Eq. (3). Let us now make an estimate of the quantity $x_i$. Taking $\Gamma_{\tilde{\nu}}$ dominated by the couplings $K_{L,\alpha}^{\nu \bar{\nu}}$ or $K_{R,\alpha}^{\tau \bar{\tau}}$, we get

$$x_i \sim \frac{8}{\alpha_2} \cdot \text{Min} \left[ \frac{F_N^2}{m_{\tilde{\nu}_i}^2} \left| \frac{\eta_i}{\xi} \right|^2, \frac{1}{\tan^2 \beta} \left( \frac{m_{\tilde{\nu}_i} m_{\tau_1} t_{\beta}}{m_{\tilde{\nu}_i} m_{\tau_1} t_{\beta}} \right)^2 \right]$$

(18)

which shows $x_i \gg 1$ for small $\tan \beta$. With $m_{\tilde{\nu}_i} = M_Z = F_N/2$ and $|\eta_i| = |\xi| = |\theta_{i1}|$, one gets $x_i \sim 1$ with $\tan \beta \sim 20$ for which the time-integrated quantities can be well measured. On the other hand, for low $\tan \beta$, the time-dependent observables may be measured to determine $\Delta m_{\tilde{\nu}}$ or CP asymmetry. If $\tan \beta$ is too large ($x_i \ll 1$), no oscillation effect occurs. The quantity $\rho_i$ determining the magnitude of CP asymmetry is given by

$$\rho_i = \frac{\theta_{i2}}{\theta_{i1}} \text{ or } \left( \frac{\delta_{i\tau_1} \omega_{i2} + \theta_{i2}}{\delta_{i\tau_1} \omega_{i1} + \theta_{i1}} \right)$$

(19)

for the shared final states $b\bar{b}$ or $\tau\bar{\tau}$, respectively. As can be inferred from previous discussions, the magnitude of $\rho_i$ is usually smaller than 1 and drops down with growing $\tan \beta$. This should be compared with the quantity $x_i$ which shows the similar behavior as above. Thus, we typically get $|\rho_i| \ll 1$ in the region where $x_i \sim 1$. For the measurement of the CP asymmetries in the shared final states, one should determine whether the decaying particle is a sneutrino or an anti-sneutrino. Obviously, this can be done by looking at the flavor-specific decays of the other particle, for instance, $\tilde{\nu}_\alpha \rightarrow \alpha \bar{\tau} \neq \bar{\nu}_\alpha^*$ and $\tilde{\nu}_\tau \rightarrow \tau \bar{\tau} \neq \bar{\nu}_\tau^*$ where $\alpha = e, \mu$. Therefore, the branching fractions for the flavor-specific as well as the shared final states should be large enough. To get a reference value, let us consider a future $e^+e^-$ linear collider with an integrated luminosity $1000$ fb$^{-1}$ at $\sqrt{s} = 500$ GeV. For the cross-section of the sneutrino–anti-sneutrino pair production $\sim 50$ fb with $m_{\tilde{\nu}} \sim 100$ GeV \cite{15}, there will be about $5 \times 10^4$ events. Requiring the multiplied branching ratio to be larger than 1 %, one will get about 500 samples to study CP violation. Here, we note that such a value can be hardly obtained for the electron and muon sneutrinos. This is because the decay $\tilde{\nu}_\alpha \rightarrow \alpha \bar{\tau}$ ($\alpha = e, \mu$) comes from the coupling $K_{R,\alpha}^{\nu \bar{\nu}}$ which is suppressed by the factor $m_{\tau_1} / \tan \beta$. Considering the ratio of $K_{R,\alpha}^{\nu \bar{\nu}}$ to $K_{R,\alpha}^{\tau \bar{\tau}}$ and $K_{L,\alpha}^{\nu \bar{\nu}}$, we get
the branching fraction, \( \text{BR}(\alpha \bar{\tau}) = |K_{R,\alpha}^{\alpha \bar{\tau}}/K_{L,\alpha}^{\tau \bar{\tau}}|^2 \approx (m_{\tau} t_\beta M_W/\mu M_Z)^2 \) for low \( t_\beta \), and \( \text{BR}(\alpha \bar{\tau}) = |K_{R,\alpha}^{\alpha \bar{\tau}}/K_{L,\alpha}^{\tau \bar{\tau}}|^2 \approx (M_W^2/t_\beta \mu M_2)^2 \) for large \( t_\beta \), which shows that the branching ratio is usually smaller than 1% for \( M_W < M_2 \sim F_N \sim \mu \).

However, the situation can be different for the tau sneutrino which allows \( \text{BR}(\tau \bar{\mu}) \sim \text{BR}(\tau \bar{\tau}) \sim \text{BR}(b \bar{b}) \gg \text{BR}(\nu \bar{\nu}) \) for \( |\omega_{\mu 1}| \sim |\omega_{\tau 1}| \sim |\theta_{\tau 1}| \) and relatively large \( t_\beta \), as the relevant coupling \( K_{L,\tau}^{\tau \alpha} \) has no such suppression. To show explicitly the sneutrino oscillation and corresponding CP violation effects, we choose the following typical set of mass parameters as a reference:

\[
M_2 = \mu = 2m_L, \quad m_A = 2.5m_L, \quad m_h = 115 \text{GeV}
\]

(20)

where \( m_L \) denotes the slepton doublet soft mass. Taking \( m_L = 100 \) (200) GeV, the sneutrino masses are, \( m_{\tilde{\nu}} = \sqrt{m_L^2 + M_Z^2 c_2 \beta^2}/2 = 82, 77 \) and 76 (192, 189 and 189) GeV for \( t_\beta = 3, 15 \) and 30, respectively. Concerning the gaugino masses, the unification relation is assumed; \( M_2 = 2M_1 \). In our bilinear model, there are three types of R-parity violating parameters \( \mu_i, B_i \) and \( m_{H}^2 \) without assuming the universality. For our calculation, we will trade these parameters with \( \xi_i, \eta_i \) and \( a_i \). Then, we put \( \xi_\mu = \xi_\tau = 3\xi_\epsilon \), consistently with the experimental data as mentioned before and the value of \( \xi_\tau \) is normalized to yield \( m_{\nu_3} = 0.05 \text{eV} \) in accordance with Eq. (12). In fact, our results are insensitive to the ratio \( \xi_\epsilon/\xi_\tau \). Note that small neutrino masses requires very small R-parity violating numbers, \( \xi_i c_\beta \sim 10^{-6} \), which could be a consequence of some flavor symmetry explaining quark and lepton Yukawa hierarchies [16].

To find the favorable parameter space, we consider the following two regions: (a) \( |\xi_\tau| \sim |\eta_\tau| \sim |a_\tau| \) and (b) \( |\xi_\tau| \gg |\eta_\tau| \sim |a_\tau| \). As discussed before, the latter region is chosen to give larger \( |\xi_\tau| \) and thus larger oscillation time. In the first region, the oscillation time becomes too small for the time-dependence to be observed. In Table A and B, we present the branching ratios and various oscillation parameters for the tau sneutrino LSP taking the following specific numbers for the two regions:

\[
\begin{align*}
(A) & \quad \xi_\tau = -\eta_\tau = -2a_\tau, \\
(B) & \quad \xi_\tau = 6\eta_\tau = -6a_\tau.
\end{align*}
\]

(21)

From Table A, one finds that \( x \) becomes of order one for \( t_\beta \sim 30 \) and the proper
oscillation time $t_{osc}$ is very small. As the branching ratios for $\tilde{\nu}_\tau \rightarrow \tau \bar{\tau}, \tau \bar{\mu}$ become 40–50% for large $\tan \beta$, one could look for the same-sign lepton signal to measure the time-integrated mixing probability $\chi$. In this case, the CP asymmetries $\rho$ are smaller than 1 % and thus can be hardly measured. As alluded before, the set (B) gives observable time-dependence. As can be seen from Table B, $\rho$ becomes smaller and the branching ratios larger as $\tan \beta$ grows. The multiplied branching ratios for the leptonic modes becomes of order 10% for intermediate to large $\tan \beta$ and thus the sneutrino mass splitting can be directly obtained by tracing time-dependent oscillation for the final state $\tau \bar{\mu}$ in this region. Furthermore, the time-dependent CP asymmetry for the final state $\tau \bar{\tau}$ can be a few percent assuming the maximal CP phase $|\sin(\phi_D - \phi_M)| \sim 1$, which could be within the future experimental reach. In fact, the quantity $\rho$ can be made larger for $|\xi| > |\eta_i|$ and lower $\tan \beta$. However, restricting ourselves to the region of $|\xi| < 10|\eta_i|$, we find that $\rho$ can be maximally a few %. Therefore, a better luminosity than mentioned before will be needed to cover more parameter space of our model. We note here that one hardly gets large CP asymmetry for $\tilde{\nu}_\tau \rightarrow b \bar{b}$ as the corresponding $\rho$ is proportional to $t_\beta^{-2}$. In both cases of (A) and (B), one finds that smaller $m_L$ is favored for the observation of various sneutrino oscillation observables.

Let us finally comment on the case of the universal soft parameters giving $\xi_i \approx \eta_i \ll a_i$. From Eq. (13) and (17), one can see that the sneutrino decays dominantly to $b \bar{b}$ and the branching fraction for $\tau \bar{\mu}$ is $m_\tau^2/3m_\theta^2 \sim 5\%$ with much more suppressed rate for $\tau \bar{\tau}$. Thus, almost no oscillation effects can be observed in this case.

In conclusion, R-parity violation can lead not only to the realistic neutrino masses and mixing, but also to observable sneutrino–anti-sneutrino mixing phenomena. This can occur if the tau sneutrino is the LSP. Among various observable quantities in the sneutrino oscillation, a certain observable will only be within experimental reach given parameter region of the bilinear model of R-parity violation. For the less fine-tuned bilinear R-parity violating parameters, $|\xi| \sim |\eta|$, only time-integrated quantities can be probed, whereas the time-dependence and CP asymmetries in the sneutrino oscillation can be observed for the region where $|\xi| \gg |\eta|$. In the former region, the sneutrino mass splitting can be
determined by measuring the time-integrated mixing probability for large $\tan\beta$ where $x$ becomes order one. In the latter region, the mass splitting can be determined through time-dependent oscillation even for lower $\tan\beta$. Finally, if the sneutrino mass is small ($m_{\tilde{\nu}} < 100$ GeV) and $\tan\beta$ is in the intermediate region, the CP asymmetry in the decay $\tilde{\nu}_r \to \tau\bar{\tau}$ can reach the level of a few percent which could be measured in the future experiments.

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Table A: The branching ratios of the tau sneutrino decay into $\nu \bar{\nu}$, $\tau \bar{\tau}$, $\tau \bar{\mu}$ and $b \bar{b}$ are shown together with the sizes of the oscillation parameters $x, y$ and $\rho$ defined in Eqs. (9) and (10), respectively. The choice of R-parity conserving parameters is described in Eq. (20) and the set (A) in Eq. (21) is used for the R-parity violating parameters.

| $m_L$/GeV | tan $\beta$ | $\nu \bar{\nu}$ | $\tau \bar{\tau}$ | $\tau \bar{\mu}$ | $b \bar{b}$ | $x$ | $y$ | $\rho(\tau \bar{\tau})$ | $\rho(b \bar{b})$ | $t_{\text{osc}}$/ps |
|-----------|-------------|----------------|-----------------|----------------|-----------|-----|-----|-----------------|-----------------|-----------|
| 100       | 3           | 0.68           | 0.03            | 0.07           | 0.22      | 13035| 0.08 | 0.037           | 0.20            | 0.008      |
|           | 15          | 0.007          | 0.38            | 0.47           | 0.09      | 197  | 0.02 | 0.013           | 0.04            | 0.007      |
|           | 30          | $5 \times 10^{-4}$ | 0.40       | 0.48           | 0.08      | 13   | 0.007 | 0.007          | 0.012           | 0.007      |
| 200       | 3           | 0.17           | 0.21            | 0.084          | 0.53      | 12670| 0.062| 0.031           | 0.047           | 0.0048     |
|           | 15          | $4 \times 10^{-4}$ | 0.24       | 0.092          | 0.66      | 35   | 0.005 | 0.003           | 0.003           | 0.0037     |
|           | 30          | $2 \times 10^{-5}$ | 0.24       | 0.092          | 0.66      | 2.2  | 0.002 | 0.001           | 0.0007          | 0.0037     |

Table B: The same as above but with the set (B) in Eq. (21).

| $m_L$/GeV | tan $\beta$ | $\nu \bar{\nu}$ | $\tau \bar{\tau}$ | $\tau \bar{\mu}$ | $b \bar{b}$ | $x$ | $y$ | $\rho(\tau \bar{\tau})$ | $\rho(b \bar{b})$ | $t_{\text{osc}}$/ps |
|-----------|-------------|----------------|-----------------|----------------|-----------|-----|-----|-----------------|-----------------|-----------|
| 100       | 3           | 0.87           | 0.05            | 0.06           | 0.01      | 468  | 0.01 | 0.16             | 0.14            | 0.30       |
|           | 15          | 0.011          | 0.27            | 0.41           | 0.28      | 7.7  | 0.011| 0.026            | 0.005           | 0.25       |
|           | 30          | $7 \times 10^{-4}$ | 0.27       | 0.41           | 0.29      | 0.50 | 0.006 | 0.012           | 0.001           | 0.24       |
| 200       | 3           | 0.48           | 0.06            | 0.14           | 0.30      | 985  | 0.015| 0.034            | 0.017           | 0.17       |
|           | 15          | $2 \times 10^{-3}$ | 0.11       | 0.26           | 0.60      | 4.6  | 0.003 | 0.007           | 0.001           | 0.13       |
|           | 30          | $1 \times 10^{-4}$ | 0.11       | 0.26           | 0.60      | 0.29 | 0.001 | 0.004           | 0.0003          | 0.13       |