Abstract

We propose to use the Kolmogorov-Smirnov test to uncover non-statistical differences between events created in heavy ion collisions within the same centrality class. The advantage of the method over other approaches which are currently in use, is that it is sensitive to any difference between the events and is not restricted to simple moments of the distribution of hadrons. The particular application examined here is the identification of the fireball decay due to spinodal fragmentation and/or sudden rise of the bulk viscosity.

The hot matter created early in ultrarelativistic heavy ion collisions expands very quickly and cools down. At RHIC, data from jet quenching suggest that the system spends a substantial amount of time in the deconfined phase [1]. The onset of deconfinement is suspected at collision energies around $\sqrt{s_{NN}} = 7 - 8$ AGeV [2]. Qualitatively the phase diagram of QCD matter shows a smooth though rapid crossover at small baryon densities, which are created at RHIC, while a first order phase transition line appears at some non-vanishing baryonic chemical potential.

Generally, the initial conditions for the fireball and the inner pressure lead to a very fast expansion of the fireball. Thus, the matter may pass the phase transition/crossover not as slowly, as required for a description in terms of equilibrium thermodynamics. Such an explosive expansion can lead to non-equilibrium phenomena, like supercooling, or even spinodal decomposition. The latter appears in case of a very fast expansion through a first-order phase transition when the matter reaches an inflection point of the dependence of entropy on some extensive variable. Spinodal decomposition is known to happen in nuclear collisions at lower energies where the liquid-gas phase transition is probed [3]. Hence, the fireball decays into smaller fragments which recede from each other and subsequently decay into final state hadrons.

Such a scenario might seem irrelevant at RHIC as it requires a first order phase transition. However, it has been suggested recently that also in the low baryon density region of the phase diagramme, fragmentation of the fireball might appear [4]. Here it is due to the sudden increase of the bulk viscosity, which has a sharp peak at the critical temperature $T_c$. i.e., the expansion starts early in the partonic phase and is already very strong when the critical temperature is reached. In this moment the peak in the bulk viscosity suddenly tries to stop the expansion. As a result of the competition between inertia and the bulk viscosity the fireball can fragment.

Thus, the fragmentation phenomenon might be present in nuclear collisions studied currently at RHIC. It is therefore relevant to explore methods which can identify the source break-up mechanism. We realise that fragmentation leads to hadronic distributions which will be different in
each event. In fact, hadrons are produced with velocities close to those of the emitting fragments. Hence, in the distribution of hadron momenta from a single event one expects clusters centered around values given by the fragment velocities. In each event, these clusters will be at different positions, so there will be non-statistical differences between the events even if a sample from a very narrow centrality interval is selected.

Up to now, many techniques have been proposed to investigate the presence of clusters in momentum distributions. Among them are rapidity correlations [5, 6], correlations in azimuthal angle and pseudorapidity [7], multiplicity fluctuations [8] and mean $p_t$ fluctuations [8, 9]. However, these methods always focus on certain moments of the momentum distribution. In contrast to this, here we propose a method which compares the complete event shapes.

We use the Kolmogorov-Smirnov (two-sample two-sided) test (KS test), which can be used to measure the similarity between two empirical sets of data [10, 11]. It answers the question, to what extent two sets of data are generated by the same mechanism with the same underlying probability density. To compare two events, one first constructs the empirical cumulative distribution function for each event. We use here the measured rapidities of hadrons. On the abscissa we put all the measured rapidities in one event. Then we draw a “staircase” by putting at each of the positions of rapidities a step of the height $1/n$, where $n$ is the multiplicity of the event. This is done for two events and the maximum vertical distance between the staircases (denoted $D$) is taken as the measure of the difference between the events. For large multiplicities, the cumulative distribution function of the quantity $\sqrt{n}D$ is known, provided that the events are generated from the same distribution $P(\sqrt{n}D) = 1 - Q(\sqrt{n}D) = \sum_{k=-\infty}^{\infty} (-1)^k \exp(-2k^2nD^2) + O(n^{-1/2})$, $n = \frac{n_i n_j}{n_i + n_j}$. (1)

The variable $Q$ acquires values between 0 and 1.

Let us now explain the method in more detail. We start from a large sample of events. For each pair of events we measure $D$ and determine $Q$ from equation (1). (Details of the precise calculation are shown in [12].) If all events correspond to the same underlying probability distribution, then the differences between them are only statistical and we obtain $Q$’s which are distributed uniformly. Any departure from the uniform distribution indicates that some of the following assumptions are not fulfilled: i) within one event all particles are produced independently from each other; ii) all events are generated from the same probability distribution.

The advantage of the method is that it is sensitive to any effects breaking the two assumptions. To demonstrate the power of the method, we provide an example of data where two-particle correlations show nothing but the KS test gives a non-trivial result. We generate toy events with a mean multiplicity of 1000. Particles have the same mass and are distributed into 100 groups. Hence, on average there are ten particles per group. Their momenta are generated from a uniform and spherical distribution. The momentum of the last particle in each group is calculated so that the total momentum within the group vanishes. In this construction, there is no two-particle correlation between the particles, as presented in Figure 1. (Note that a ten-particle correlation would show a signal.) The KS test, however, shows very clear deviations from a uniform distribution (Fig. 1). To quantify the deviation, we introduce the parameter

$$R = \frac{N_0 - \frac{N_{\text{tot}}}{B}}{\frac{N_{\text{tot}}}{B}}$$

(2)
where \( N_0 \) is the number of pairs in the first bin, \( N_{\text{tot}} \) is the total number of pairs, and \( B \) is the number of bins of the \( Q \)-histogram. Momentum conservation leads to negative \( R \). Later we show that fireball fragmentation causes large positive \( R \).

To test the method on realistic data, we generate event samples with the Monte Carlo event generator DRAGON [13]. DRAGON generates the momenta of hadrons as if they were produced from a fragmented fireball. The pattern of expansion is that of the blast-wave model. Here, the blast wave model is used to generate the positions and velocities of the fragments which subsequently radiate hadrons. Some hadrons can be produced also from the space between the fragments. The chemical composition is determined according to the grand-canonical ensemble and resonance decays are taken into account. Choosing different values for the parameters of the model (temperature, fireball sizes, droplet sizes, etc.) allows then to simulate different physical situations.

With DRAGON, sets of \( 10^4 \) events are generated out of which we randomly choose \( 10^5 \) pairs. On these pairs we evaluate the variable \( Q \) and fill histograms. As mentioned above, a departure from a flat distribution indicates non-statistical differences between the events or that the particles within one event are not emitted independently from each other.

Figure 1: (Left) Two-particle correlation function in rapidity difference between particles generated by the toy generator with simple momentum conservation. (Right) The \( Q \)-histogram of \( 10^5 \) pairs of events from the same data.

Figure 2: \( Q \)-histograms from a simulation for central Au+Au reactions at \( \sqrt{s_{\text{NN}}} = 200 \) GeV (RHIC) with assumed fragmentation into fragments with mean volume of 5 fm\(^3\) (solid red lines). For comparison, results from a non-fragmented fireball at RHIC (dashed blue lines) and FAIR (\( E_{\text{beam}} = 30 \) AGeV) (dotted brown lines) are shown. Left: rapidities of all charged hadrons are used for the KS test. Right: only rapidities of charged pions are used.
In Figure 2, we show how the fragmentation of the fireball is reflected in $Q$-histograms. Results from a simulation with fragments with an average volume of 5 fm$^3$ are compared to two cases with direct emission of hadrons. Parameters in the simulation with fragments are chosen to correspond to Au+Au collisions at RHIC. The KS test is performed with the rapidities of the hadrons. In the histogram obtained for all charged hadrons one clearly observes that fragmentation leads to a pronounced peak at small $Q$. For comparison, simulations without fragments (i.e., the hadrons are emitted directly from the source) are made and studied. Naively, one would expect flat $Q$-histograms in these cases. However, one observes that it is not the case (Fig. 2 left). For charged hadrons, there are correlations between the produced hadrons due to resonance decays, which actually act as small clusters. This (unwanted) correlation can be minimized by using only pions of one charge, as shown in the right panel. Indeed, the peak at small $Q$ disappears (note the different scales). However, we have decreased the multiplicity of hadrons entering the procedure and this leads to a peak close to $Q = 0$ due to applicability limits of the approximative formula (1) (even though we actually use an improved formula).

We have also checked that bigger droplets and a larger abundance of hadrons emitted from the droplets leads to a more pronounced low-$Q$ peak.

In summary, analyzing data with the Kolmogorov-Smirnov test can provide novel insights and identify interesting effects not seen before with other methods. Here, the proposed particular use was for the identification of a possible fragmentation of the fireball via spinodal decomposition or due to sudden appearance of the bulk viscosity. We showed that such a fragmentation can be clearly identified with the help of the KS technique. While momentum and charge conservation yield only minor modifications, the influence of other processes needs yet to be studied. It is often assumed that in narrow centrality interval all events develop according to the same physics scenario. We propose to test this assumption with the method described here.

Acknowledgments

BT and IM thank for support by VEGA 1/4012/07 (Slovakia). BT also acknowledges support from MSM 6840770039 and LC 07048 (Czech Republic). The work of GT, SV, and MB was supported by the Helmholtz International Center for FAIR within the framework of the LOEWE program (Landesoffensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz) launched by the State of Hesse. The collaboration between the Slovak and the German group was facilitated by funding from DAAD.

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