Cluster structures of excited states in $^{11}$B

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Abstract. The cluster structure of excited states in $^{11}$B is investigated with a method of $\beta-\gamma$ constrained antisymmetrized molecular dynamics in combination with the generator coordinate method. For negative-parity states, we identify a band with a $2\alpha+t$ cluster structure. This band starts from the $3/2^-_t$ state and may correspond to the experimental band observed recently. We find that the composition of the $3/2^-_s$ state is quite similar to that of the $0^+_t$ state in $^{12}$C.

1. Introduction

Recently, the cluster structure of $^{11}$B has attracted much interest. Some theoretical and experimental studies have suggested that cluster structures develop well in the negative-parity states above or near $2\alpha+t$ the threshold energy [1, 2, 3]. In the recent experiment of $\alpha$ resonant scattering on $^7$Li [4], Yamaguchi et al suggested a new negative-parity band consisting of 8.56 MeV ($3/2^-$), 10.34 MeV ($5/2^-$), 11.59 MeV ($7/2^-$) and 13.03 MeV ($9/2^-$) states, which have large $\alpha$ decay widths. It is interesting to know what kind of the cluster structure this band has.

Moreover, the analogy between the cluster features of $^{11}$B and the three-$\alpha$-cluster structure of $^{12}$C is a fascinating problem to be clarified. In previous works [2, 5], the $3/2^-_t$ state was suggested to have a dilute cluster structure with a $2\alpha+t$ configuration, and to be an analogue of the $0^+_t$ state in $^{12}$C, which has a dilute $3\alpha$ structure [6, 7, 8]. However, in a recent work [3], it was argued that $^{11}$B($3/2^-_s$) cannot correspond to $^{12}$C($0^+_t$). The relation between the states $^{11}$B($3/2^-_s$) and $^{12}$C($0^+_t$) is controversial and further studies are required.

In this article, we investigate the structures of some excited states in $^{11}$B with a method of $\beta-\gamma$ constrained antisymmetrized molecular dynamics (AMD) in combination with the generator coordinate method (GCM). To clarify the correspondence between the $3/2^-_s$ state of $^{11}$B and the $0^+_t$ state of $^{12}$C, we compare the squared overlaps of their resulting wave functions with the basis functions on the $\beta-\gamma$ plane.

2. Formulation

In this section, we briefly explain the $\beta-\gamma$ constrained AMD + GCM [9]. In the method of AMD, a basis wave function $|\Phi\rangle$ is a Slater determinant of single-particle wave functions $|\phi_i\rangle$. The $i$th single-particle wave function consists of the spatial part $|\phi_i\rangle$, spin part $|\chi_i\rangle$, and isospin part $|\tau_i\rangle$ of the following form

$$|\phi_i\rangle = |\phi_i\rangle |\chi_i\rangle |\tau_i\rangle,$$  \hspace{1cm} (1)

$$\langle \bf{r}|\phi_i\rangle = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \exp \left[-\nu \left(\bf{r}-\bf{Z}_i\right)/\sqrt{\nu}\right] + \frac{1}{2} Z_i^2,$$  \hspace{1cm} (2)

$$|\chi_i\rangle = \xi_{i\uparrow} |\uparrow\rangle + \xi_{i\downarrow} |\downarrow\rangle.$$  \hspace{1cm} (3)
Figure 1. The black crosses are the calculated negative-parity states in $^{11}$B. $5.0 \ e^2 \text{fm}^4 < B(E2) \leq 10.0 \ e^2 \text{fm}^4$, $10.0 \ e^2 \text{fm}^4 < B(E2) \leq 20.0 \ e^2 \text{fm}^4$, and $20.0 \ e^2 \text{fm}^4 < B(E2) \leq 40.0 \ e^2 \text{fm}^4$ transitions are denoted by broken, black solid and red bold solid arrows, respectively. The blue circles belong to the experimental band found in Ref. [4]. Dotted line: $2\alpha + t$ threshold.

The isospin part $|\tau_i\rangle$ is fixed to be up (proton) or down (neutron). In a basis wave function $|\Phi\rangle$, $\{Z_1, \xi_1, Z_2, \xi_2, \ldots, Z_A, \xi_A\}$ are complex variational parameters and they are determined by the energy optimization with constraints on the quadrupole deformation parameters $\beta$ and $\gamma$.

In the calculations of the energy levels, we superpose parity- and total-angular-momentum-projected AMD wave functions $\hat{P}^J_{MK} \hat{P}^\pm |\Phi(\beta, \gamma)\rangle$. Thus, the final wave function for the $J^\pm_n$ state is given by a linear combination of the basis wave functions, $|\Phi_{J^\pm_n}\rangle = \sum_K \sum_i f_n(\beta_i, \gamma_i, K) \hat{P}^J_{MK} \hat{P}^\pm |\Phi(\beta_i, \gamma_i)\rangle$. The coefficients $f_n(\beta_i, \gamma_i, K)$ are determined by solving the Hill-Wheeler equation.

3. Results

In this section, we briefly show the results for negative-parity states. For more details, including the results for positive-parity states, the reader is referred to Ref. [10].

In Fig. 1, we show the calculated negative-parity energy levels as functions of the angular momentum $J(J + 1)$ grouped according to the $E2$ transition strengths. We also show the experimental band [4]. At first, we explain the structures of ground and excited states. The calculated low-lying states have large overlaps with the basis wave functions that are in between the shell-model structure and the cluster structure. In the high-lying states near and above the $2\alpha + t$ threshold energy, we obtain various well-developed cluster states having significant overlaps with the basis wave functions which represent $2\alpha + t$ cluster structures. In particular, the $3/2^-_3$ state, which is found to have a dilute cluster structure with a $2\alpha + t$ configuration, is described by the linear combination of various $2\alpha + t$ spatial configurations.

For the $3/2^-_3$, $5/2^-_3$, $7/2^-_3$, and $9/2^-_3$ states, the $E2$ transition strengths are as large as 20–30 $e^2 \text{fm}^4$, and therefore, we consider these states as members of a band. Although the calculated excitation energies are higher than the experimental ones of the band identified in Ref. [4] by 2.5–4.5 MeV, the systematics of the level structure corresponds well with the experimental ones. In both results, the higher angular momentum states satisfy the $J(J + 1)$ rule. Only the $3/2^-_3$ state deviates from the straight line. Moreover, the present result suggests that clustering is well-developed in this band, and its members have large $\alpha$ decay widths. A confirmation of this
conjecture would support the correspondence between the theoretical and experimental bands.

We discuss the relation between the $3/2^-_3$ state in $^{11}$B and the $0^+_2$ state in $^{12}$C by comparing the squared overlaps between the GCM wave functions and the basis wave functions on the $\beta$-$\gamma$ plane, $|\langle \Phi(\beta, \gamma) | \Phi_{J^\pm n} \rangle|^2$, in Fig. 2. The results for the $0^+_2$ states in $^{12}$C are calculations with the $\beta$-$\gamma$ constrained AMD + GCM taken from Ref. [9]. The distributions of the squared overlaps between the GCM wave functions and the basis wave functions for these states are very similar to each other. Their comparison indicates that the $3/2^-_3$ state in $^{11}$B is described by a linear combination of various $2\alpha + t$ configurations similar to the linear combination of the $3\alpha$ configurations in the $0^+_2$ state of $^{12}$C. That is, when an $\alpha$ cluster is replaced by a $t$ cluster in the $0^+_2$ state of $^{12}$C, one obtains the $3/2^-_3$ state of $^{11}$B.

4. Summary
We investigated the cluster structures of excited states in $^{11}$B with the method of $\beta$-$\gamma$ constrained AMD + GCM. By analysing the $E2$ transition strengths as well as the squared overlaps between the GCM wave functions and the basis wave functions, we discussed the negative-parity band starting from $3/2^-_3$, in which a $2\alpha + t$ structure is well developed. This band is a good theoretical candidate for a band which was identified in a recent experiment of alpha resonant scattering on $^7$Li. To make the correspondence between the calculated and experimentally observed states more clear-cut, the partial decay widths of these excited states remain to be determined in the future. We found that the squared overlaps between the GCM wave functions and the basis wave functions of the $3/2^-_3$ state in $^{11}$B are quite similar to those of the $0^+_2$ state in $^{12}$C.

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