Special Relativity as a Physical Theory

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(Dated: October 18, 2004)

A modest aim of this pedagogical presentation is to analyze, critically, certain fundamental physical concepts to illustrate the physical principles behind the special theory of relativity and, hence, to also illustrate the limitations of its applicability.

Expanded version of the talk presented during a One-Day Seminar
100 Years of Special Relativity
organized jointly by
VMV Arts, JMT Commerce and JJP Science College, Wardhaman Nagar, Nagpur
and
Central India Research Institute, Nagpur
on 19th September 2004

It was in 1905 that Einstein’s monumental work titled Zur Elektrodynamik bewegter Körper (On the electrodynamics of moving bodies) appeared in print in Annalen der Physik - a German journal of research in physics [1]. This fundamentally important research work is the reason behind our present endeavors. It would then be also appropriate if, on this occasion, we pause to take due cognizance of the formation and the development of some fundamental ideas in Physics. It is therefore my modest aim to present here a critical analysis of certain fundamental physical concepts in relation to the special theory of relativity and thereby to illustrate the physical principles as well as the limitations of the applicability of this theory.

Our this critical analysis must begin with the formation and sharpening of some fundamental physical concepts that took place during Newton’s times. It is during this Golden Era of Science that many gifted scientists conceived, formulated and defined sharply some of the fundamental physical concepts that we use today.

Probably, most of us use these fundamental conceptions without realizing the great struggles leading ultimately to their accepted meaning. As a consequence, most of us, probably, also accept the generalizations of these conceptions uncritically. As a consequence, an impression may also be left with us that some concepts are unchangeable and must always hold. Perhaps, it is also why we, many times, rigidly adhere to them even beyond their natural applicability.

A primary concept of Physics is that of the inertia of a material body. Ordinarily, a body needs to be “pushed” to produce its motion. It then displays the inertia or lethargy to move.

Here, motion is to be conceived as a change in the position of the material body in relation to the experimenter or the observer. Then, speed is the rate of change of position of a material body with time. It characterizes the motion of that material body in relation to the experimenter - the observer. Therefore, the inertia and the speed of a material body are physical conceptions that we have derived from our ordinary, day to day, experiences involving physical bodies.

We could then postulate that every material body has this inertia for motion and that some “push” is always required to move any material body. This is a generalization that we make about all the conceivable physical bodies.

We could then state that a physical or a material body continues to remain in its state of rest relative to an observer or the experimenter unless and until it is acted upon by a “push” - an external agency or the cause of the motion. This statement could be taken as a physical law - the Law of Inertia of material bodies.

In fact, this above was taken to be the statement of the Law of Inertia at the beginning of the aforementioned golden era of science. The quantitative measure of this inertia was then called the mass, denoted by $m$, of that material body.

From our ordinary experiences, we could then also assert that the mass $m$ of a physical body is always a positive real number since we do not, ordinarily, encounter a physical body that “aids” the “push” producing its motion.

[Then, a body that displays properties contrary to related ordinary experiences could be considered to possess negative mass. However, to this date, no such material body is known to exist.]
On the face value, there does not appear to be anything problematic with this statement of the Law of Inertia of material or physical bodies. However, Galileo Galilei of Pisa in Italy was the one to sharpen this conception on the basis of certain experiments conducted by him.

Galileo’s greatness lies not only in conceiving these experiments but also in developing further the concept of the inertia for motion on their basis. Before him, the emphasis was on philosophical considerations of natural phenomena. Galileo supported his philosophy with appropriate experiments and observations. It is a change of attitude from only “philosophical” analysis to verifiable or scientific analysis.

Galileo observed that when we place a material body on an inclined plane, it has tendency to descend down it. An idea then occurred to Galileo of placing another inclined plane next to the first one in such a manner that the descending body would climb up this second inclined plane.

He observed that the material body climbs the second inclined plane up to the height above the ground from which it was released on the first inclined plane, this when the surfaces (of contact between the plane and material body) were made smooth, that is to say, as frictionless as possible. Galileo, as extraordinarily gifted scientist as he was, then conceived a series of careful experiments to test how far up the second inclined plane the material body climbs when the angle of incline of that second plane is changed.

He then noticed that as the second inclined plane is made more and more horizontal, tangent to the surface of the Earth, the material body travels larger and larger distance along this plane but reaches the same height from which it was released on the first inclined plane.

He also noticed that the material body possesses, relative to the experimenter or the observer in the laboratory, the same speed, say, \( v \), at the bottom of the inclined plane when the height from which it is released on the plane is the same, this irrespective of the angle of incline of the plane. The speed \( v \) was also found to vary with the (square root of the) height from which the material body was released. Moreover, he also noticing that this speed \( v \) is the same for different material bodies (wooden, iron, glass etc.) under the same situations.

[In modern terms, the total energy \( E \) is conserved in the situation. Then, if mass of the material body is \( m \), its height above the ground is \( h \) and \( g \) is to denote the acceleration due to gravity then, the material body has \( E = mgh \) to begin with since it begins from the state of rest at height \( h \) above the ground. At the bottom of the inclined plane, this potential energy is converted into kinetic energy of the body: \( mv^2/2 \). Equating the two, we obtain: \( v = \sqrt{2gh} \), an expression independent of the mass of the material body but varying as the square root of the height \( h \). Of course, this holds only in the absence of friction.]

Galileo, then, logically argued: if the second inclined plane (of infinite spatial extent) were made completely horizontal, ie, exactly tangent to the surface of the Earth, then, the material body descending down the first inclined plane would travel an infinite distance rectilinearly along the second inclined plane with the uniform speed \( v \). By varying the height at which the material body is released on the first inclined plane, we would also obtain different uniform speeds \( v \).

The uniform rectilinear motion of a material body relative to an experimenter therefore has no special status. A material body, as it reaches infinite separation from the Earth, moves with uniform speed even when there is no “push” acting on it relative to the experimenter.

(The existence of an infinite plane is an obvious impossibility. But, a reader of Galileo is compelled to draw the above conclusion.)

We should then state that a physical or a material body continues to remain in its state of rest or of uniform rectilinear motion relative to an observer or the experimenter unless and until acted upon by “push” - an external agency or the cause of the motion. Here, we may add further that the material body under considerations be also far removed from other physical bodies in the universe. This is then the correct Law of Inertia of material or physical bodies.

[The “correctness” of this law of inertia is as far as Galileo’s aforementioned experiments are concerned. We also note here that if some material body were found to move with the same uniform speed relative to all the experimenters or observers then, that body would have zero inertia in the sense described above. Nothing of Galileo’s conceptions prevents the existence of such material bodies. We will return to this issue.]

It is our routine to state this law - Newton’s First Law of Motion - in this form. We mostly learn it as a statement of facts without, perhaps, learning this interesting history.

However, in Galileo’s times, he had to struggle to establish it. Genuinely speaking, it was not any easy to realize to eliminate friction from the experimental setup. In fact, elimination of friction is really the key element of Galileo’s experiments. If we do not eliminate friction then, a constant “push” is evidently needed to make a material body move with “uniform” speed. Importance of this fact must be adequately recognized by any student of physics.
Today, the word *inertia* has associated with it the meaning of the *opposition* of a material body to a change in its state of rest or of uniform motion of rectilinear character.

At this place, let us also note that one of the problems of foremost importance for the present Physics is to "explain" the origin of inertia of material bodies.

Clearly, the next primary conception is that of the "push" or the "external influence" that causes the motion of a physical body. This concept is then related to the concept of change in the state of motion (as defined, to begin with, in the Law of Inertia that we have stated earlier) of a material body. Then, much of what we shall consider next will deal with the conceptions of what this "push" means and also of what we really mean by a "material" body.

Now, let us move on to consider some more ideas. Even before Newton, many others had realized the importance of the concept of inertia as a fundamental property of physical bodies. Then, efforts began to obtain the Laws of Motion using this fundamental conception.

In this connection, Descartes then made the mathematical construction of the Cartesian coordinate system. He realized that, to locate a material body, we require three suitable numbers in relation to another material body. In his mathematical construction, Descartes selected a point as the origin of the coordinate system and chose the three numbers such that the distance of another point from the chosen origin is the Euclidean distance: \( x^2 + y^2 + z^2 \) where \( x \), \( y \), \( z \) denote the coordinates of the point in relation to the origin and the three coordinate axes which are perpendicular to each other and meeting at the origin.

We must realize here that this above is entirely a mathematical construction and that it has no physical implications. That is to say, we can consider a triplet, \((x, y, z)\), of three real numbers and consider that they form the (Euclidean) function: \( x^2 + y^2 + z^2 \) which remains invariant when the triplet is changed to, say, \((x', y', z')\).

The transformations which keep invariant the Euclidean distance \( x^2 + y^2 + z^2 \) are called Galilean Transformations. They form, mathematically, a group - called the Galilean Group. This group consists of translations of coordinates and rotations of the cartesian coordinate system.

Now, as a crucial step, let us represent a material body as a material point in this cartesian system. Then, let \( m \) be its mass or inertia. In order to describe its motion, we must give the values of its coordinates as functions of the time.

But, we must be really careful here in these associations of physical character.

Firstly, we must remember that real material bodies are not point-like but possess spatial extension. Therefore, our this representation of a material body as a point is, in reality, an idealization in which we simply replace that extended physical body by a suitable point, let us call it the center of inertia, to which we attribute the entire mass \( m \) of that material body.

Whether the above idealization "correctly" represents real material bodies is to be checked only by experimentation involving real material bodies. That is, by verifying whether the real material body follows the "path of the center of inertia" in an experimental situation.

Further, the Laws of Motion obtainable on the basis of these considerations must "demonstrate" that this replacement of an extended material body by a single material point is indeed true in that the path of the material point (as the center of inertia) must be obtainable from the paths of motions of other material points (being considered to form the material body). That is, the Laws of Motion must be consistent with the concept of the center of inertia. This signifies the internal consistency of the associated ideas.

Secondly, it is also necessary for us to realize that the "time" here must be measurable by a physical clock. That is to say, the position of a material point is to be checked against the reading of a physical clock. Thus, we have to say that when the material point is at such and such location given by the three cartesian space coordinates, the physical clock is simultaneously showing such and such time. This simultaneity is inherent in these physical associations.

If this simultaneity is not assumed then, there is no genuine physical sense in saying that the material point has the corresponding position. Furthermore, in the absence of any correspondence with a physical clock, the "time" is simply a label or a parameter (taking real values) and any other arbitrary label would equally do. Time is also a parameter that is independent of the cartesian space coordinates. It is, therefore, an implicit assumption in these considerations that such a correspondence with a physical clock exists and can be made, as and when desired.

Then, we have the concept of the speed of the material body: the rate of change of its (Euclidean) distance (in relation to the origin of the coordinate system) with respect to the time.

Now, in Newton’s times, collisions of material bodies were considered to be the simplest interaction between them. Collision changes the (initial) speeds of material bodies. Then, the above considerations could be “applied” to collision of material bodies to test their usefulness.
In other words, what we look here for are some universal laws which hold in a collision of material points and verify these laws in an actual collision of material bodies.

Descartes then conceived the quantity of motion: \(mv\) and stated that this quantity is conserved in a collision of material bodies. Descartes’s assertion could not hold. [Notice that speed is a scalar quantity. It was yet to be realized that we need a vector quantity - velocity - that has amplitude as well as direction. Obviously then, Descartes’s assertion could not have been true.]

Huygens, on the other hand, realized that the quantity \(mv\) gets conserved in a collision only if we assign to it a positive or negative sign in a suitable manner by a convention.

So, consider the collision of two material points, initially located on the X-axis, with motions completely along the X-axis. Let the material point \(m_1\) move away from the origin towards the direction of the positive X-axis with initial speed \(v_1^i\) with positive sign and let the material point \(m_2\) move closer to the origin, i.e., towards the direction of the negative X-axis, with initial speed \(v_2^i\) with negative sign. Let the directions of motion of involved material points be reversed as a result of their collision. Then, let their speeds after the collision be \(v_1^f\) with negative sign and \(v_2^f\) with positive sign. Then, as Huygens showed, in this case, we obtain the result:

\[m_1 v_1^i - m_2 v_2^i = -m_1 v_1^f + m_2 v_2^f.\]

As we realize today, this above is a correct result. It is an application of the Law of Conservation of Linear Momentum.

It is within this scheme of physical conceptions that Newton developed the theoretical foundation for his famous three Laws of Motion.

Now, Galileo did not state the Law of Inertia of Material Bodies in the final form as we have done. It was Newton who stated it, in his famous book, *The Principia*, as the First Law of Mechanics developed by him. That is why we call it Newton’s First Law of Motion.

From the Euclidean geometry and the “association” of the inertia of a material body with a point, material point, of the Euclidean space, it is clear that the motion of a material body is representable as a curve in this geometry.

A material point moving with, for example, uniform rectilinear velocity along the X-axis is representable as the “curve” X-axis.

Alternatively, different types of curves of the Euclidean geometry, straight line, circle etc. represent then possible motions of a material body within this newtonian scheme.

Clearly, the “push” that produces the motion of a material body is related to some appropriate property of the curve in a Euclidean geometry. It was “the mathematical genius” - Newton - who realized what this property really is.

Let us follow Newton further from here.

Firstly, we notice that the displacement of a material point is definable as a tangent to the curve in this Euclidean geometry. But, velocity is the displacement per unit time and, hence, it also can be considered to be tangential to the same curve. Similarly, Descartes’s quantity of motion, now, \(mv\), the momentum vector \(\vec{p}\), is also tangential to the same curve.

A change in the velocity vector of a material body, the acceleration, is then a vectorial quantity. Then, the “push” must be related, within this newtonian scheme, to the rate of change, with time, of some quantity tangential to the curve. The push, that will, henceforth, be called the force, is then another vectorial quantity.

Almost prophetically, Newton then postulated his Second Law of Motion that the force is equal to the rate of change of the vector of the momentum. In terms of our usual notations:

\[\vec{F} = \frac{d\vec{p}}{dt}\]

This equation of Newton’s Second Law of Motion appears almost prophetic because Newton could easily have chosen the force to be proportional to the rate of change of velocity. But, in that case, we would have

\[\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m \ddot{\vec{x}} = m \frac{d^2\vec{x}}{dt^2}\]

an expression that holds only when the mass or the inertia is a constant.

What is then the source or cause of this force? It is important to recognize that, within this newtonian scheme, only another material point can be the source of this force. A material point “here” acts on a material point “there” with the specified force. The newtonian scheme is then an action at a distance framework.

We can then consider a physical body as many material points and vectorially add the forces exerted by each one on the other.

Now, it remains to check whether these conceptions are applicable to real material bodies in that these conceptions should be self-consistent in the sense described earlier. Then, the center of inertia or mass is to be obtained from a distribution of material points and it ought to be shown that the center of inertia follows a path that is obtainable from the paths of motion of the material points considered to form the material body.

It is then History that Newton analyzed related conceptions for a situation of two material points.
An immediate question then arises of the difference between the force exerted by the first material point on the second material point, let us call this force the action, and that exerted by the second material point on the first material point, let us call this force the reaction.

Surely, the most natural assumption here is the equality of these two forces: action and reaction forces. But, force is a vectorial quantity with amplitude and direction, both.

Newton then realized that for the internal consistency of this theoretical framework it is necessary that these two forces must be equal in amplitude but opposite in direction. Hence, his Third Law of Motion: Forces of Action and Reaction are equal and oppositely directed.

It is the remarkable simplicity of the newtonian scheme that it is based on just these three Laws of Motion. The remaining are just deductions that follow from these three basic laws.

On the basis of only these remarkably simple three laws of motion, it was then possible to calculate the planetary motions. It was also possible to develop the theory of tides, the theory of the equilibrium configurations of rotating bodies, the calculation of the speed of sound etc.

Primarily, one of the simplest forms of a general physical law is to assert the conservation of some physical quantity when material bodies participate in different physical processes. That a given physical quantity is really subject to a conservation principle is then to be decided only by performing experiments with material bodies.

It was then the great triumph of the newtonian scheme that various experiments confirmed different conservation principles of this scheme: for example, those of the mass, energy, linear momentum, angular momentum, etc.

This is the reason as to why the impact of the newtonian scheme completely overshadowed the developments in Physics for the next few centuries. That the mathematical structure of the newtonian scheme developed by many others after Newton required no new experiments or observations is testimony enough to say that the physical foundation laid by Newton was completely sufficient to support these developments.

This led the physicists of later generations to the erroneous belief that the entirety of physics could be reduced to the newtonian mechanics. In other words, they failed to recognize clearly the limitations of the newtonian framework as we have outlined above in brief.

It should now be clear at this stage that the limitations of Newton’s three laws of motion are really embedded within the limitations of this newtonian scheme itself.

Of particular concern are the use in this scheme of the Cartesian conceptions of Euclidean geometry and the associations of properties of material bodies with the points of this space.

In this connection, we note that the coordinate transformations which keep the Euclidean distance invariant are very special transformations of the triplets $(x, y, z)$ of real numbers. Newton’s laws of motion are invariant only under these special (the Galilean) transformations.

Moreover, the association of the inertia of a material body with the points of the Euclidean space does not produce any change in the Euclidean space. Then, the Euclidean space is an inert background for the material bodies. This scheme then explains all phenomena as relations between objects existing in space and time.

A coordinate system, the construction of the coordinate axes and clocks, must also be using the material bodies. But, precisely this construction is left outside the scope of this newtonian scheme. Therefore, we have to treat the construction of the coordinate system as the one using rigid rods and clocks which never get affected by anything happening with the material bodies. This must be recognized as an important and inherent drawback of the newtonian scheme.

[Let us construct a cartesian coordinate system using “sufficiently long” material rods, say, of iron. Ever imagined a heavy-duty bulldozer or road-roller crossing, say, the rod representing the X-axis, but “not doing anything” to that rod?]

The cartesian coordinate system / space is then an absolute, meaning unchanging, coordinate system / space in this scheme.

Within Newton’s scheme, the acceleration of a material point is, at a fundamental level, then to be referred to only this absolute background space or the unchanging coordinate system. Such coordinate systems are then fundamental, special, to this description of physical systems.

Consequently, in selecting any coordinate frame accelerated with respect to the background coordinate system, we will have to introduce fictitious forces, the pseudo-forces, to account for the phenomena involving material bodies. For example, in selecting a rotating frame for describing the motion of a material body, we have to introduce the notion of the Coriolis Force to account for the “observed” phenomena.

Coordinate frames in which fictitious forces do not occur are defined to be inertial frames of reference. Then, unambiguous physical construction of inertial frames is a problem in Newton’s theory since pseudo-forces are to be defined in relation to only these frames in this scheme. This is obviously unsatisfactory vicious circle.
Furthermore, this scheme has four independent coordinates: three space coordinates and one time coordinate. This, the newtonian, scheme is therefore four-dimensional of character. However, it is, not any four-dimensional distance, but, the three-dimensional Euclidean distance that is invariant under the galilean transformations.

A further difficulty of the newtonian scheme is then the following. Clearly, for the path of a material body, we use the space coordinates as functions of the time coordinate. Then, mathematically, the path of a material point is a “curve” in the four-dimensional space of \((x, y, z, t)\).

But, it is the peculiarity or the oddity of this newtonian scheme that no transformations of the time axis are, in any way, involved in it. In other words, the time coordinate is the same for all the observers. Why should this be so? Newton’s theory offers no explanation here.

Moreover, it should also be clear now that, when \(m = 0\), the newtonian scheme offers no laws for the motion of a material body. Clearly, \(\ddot{a} = \frac{\vec{F}}{m}\) and the acceleration has no meaning for \(m = 0\). That is to say, the newtonian scheme cannot describe the motion of a material body that moves with the same uniform speed in relation to all (the inertial) observers. This must be recognized as a limitation of the newtonian scheme if such inertia-less material bodies existed in reality.

Notice, now, that the newtonian scheme is based on two independent considerations: first, those of the law of motion and second, those of the law of force. Then, unless and until we specify the force acting on a material point, Newton’s Second Law of Motion will be unable to provide us the path followed by that material point.

It is precisely for this reason that Newton had to postulate a separate Law of Gravitation - Newton’s Law of Gravitation.

Here, Newton introduced a new notion of the source properties of material bodies. Precisely, if \(M\) is the source or the gravitational mass of one material point, \(m\) is the gravitational mass of another material point situated at distance \(d\) from the first body then, the gravitational force of attraction (produced by \(M\) and acting on \(m\)) is given by the famous expression:

\[
\vec{F}_g = -G \frac{mM}{d^2} \hat{d}
\]

where \(G\) is Newton’s constant of gravitation and \(\hat{d}\) is the outwardly directed unit vector along the line joining the two material points with origin of the coordinates being at the location of the material point of gravitational mass \(M\).

Then, in this Law of Gravitation, the force varies inversely with the square of the distance separating the material points. This must be recognized as an important assumption.

[Why not any other power of \(d\)? Why should the expression for the force not contain any derivatives of the space coordinates?]

Furthermore, it is essential to distinguish between the inertial mass and the gravitational mass of a material point. These two are conceptions of very different physical origins.

Now, consider that various bodies of differing inertia fall freely under the action of Earth’s gravity after being released from the same distance above the ground. In terms of Newton’s Laws of Motion: let \(m_i\) be the inertial mass and \(m_g\) be the gravitational mass of a material body. Then, from Newton’s Second Law of Motion and Newton’s Law of Gravitation, we have:

\[
\vec{F} = m_i \ddot{a} = -G \frac{m_g M}{r^2} \hat{r} \equiv m_g \ddot{g}
\]

where \(M\) denotes the gravitational mass of the Earth, \(r\) is the distance to the material body from the center of the Earth and \(\ddot{g}\) denotes the acceleration due to Earth’s gravity.

Thus, the (linear) acceleration is related to the acceleration due to gravity as:

\[
\ddot{a} = \frac{m_g}{m_i} \ddot{g}
\]

Now, if the ratio of \(m_g\) to \(m_i\) were different for different material bodies then, they would fall with different accelerations even when released from the same distance above the surface of the Earth. Galileo’s experiments at the leaning tower of Pisa showed that this is not the case.

Hence, the inertial and the gravitational mass are equal to a high degree of accuracy for known material bodies. This is an experimental result. But, the fact that these two quantities are equal must be recognized as another additional assumption of the newtonian scheme.

Furthermore, it also follows that the origin of the source properties of material bodies like the gravitational mass is unexplainable within the newtonian scheme. This is so because the newtonian scheme treats the cause of the motion of a material body, the force, as an external agency to be postulated or specified by hand. Therefore, although Newton’s Law of Gravitation specifies the inverse-square behavior for the force of gravity, this law is an assumption that is not any “explanation” of the phenomenon of gravitation.

Now, if this scheme is to be applicable to every material body, as Newton’s First Law of Motion asserts, then, every physical phenomenon must be explainable as an interaction of material points. In
other words, every material body must be treatable as a material point.

Therefore, Light must also be treatable as a material point within this newtonian scheme. This is, precisely, the reason behind Newton proposing the Corpuscular Theory of Light.

Then, the observation that Light propagates in a straight line is consistent with this picture of Light as a material point: a material point of Light moves in a straight line unless acted upon by a force changing its direction of motion. The reflection of Light from the surface of a mirror is also explainable on the basis of collisions of material points of Light with the mirror.

However, it was known in those times that the shadow of an object illuminated by Light has two distinguishable regions: first, the dark one, called the Umbra and second, relatively less darker one, called the Penumbra. Light also penetrates the geometrical shadow region near the edge of the object and diffracts. In Newton's scheme, this must be because of some force acting on the material points of Light. This force is then different for different material points of Light since the penetration by Light in the geometric shadow occurs at various depths behind the object.

Also, in Newton's own experiments with Light, Newton observed the phenomenon of concentric (Newton's) rings. A bright ring is a ring-shaped region of Light. A dark ring is another ring-shaped region of no Light. There also are more than one such concentric bright and dark rings. Once again, within the newtonian scheme, this must be because of some force acting on the material points of Light. This force is then evidently different for different material points of Light.

It is then thinkable that an explanation of these phenomena on the basis of some hypothetical force acting on the material points of Light is obtainable within the newtonian scheme. Any such explanation is, however, unsatisfactory.

The pivotal reason for this is that any explanation must be universal of character. It is only in such a situation that the involved explanation is also the simple one. This principle of simplicity of an explanation has been the driving impetus behind scientific theories.

In relation to the (hypothetical) force acting on the material points of Light postulated within the newtonian scheme, we could then ask: What causes the required behavior of this force acting on the material points of Light? Is there some universal, rational explanation for this?

Evidently, no such universal, rational explanation is permissible in Newton's scheme as any force is an assumption for it. (Newton, perhaps, recognized this fact.) Furthermore, the phenomenon of polarization of Light has no conceivable explanation in Newton's scheme.

We have gone to great lengths in describing this evolution of newtonian ideas here because the conception of an inertia of a material body is one of the basic concepts of even the modern physical theories. Newton's theory deals with the conception of an inertia of a material body only in one respect: by ascribing it to a point of the space. But, the newtonian framework does not explain the origin of inertia of a material body. Clearly, this is also an additional limitation of Newton's theoretical framework.

For us, of much later generations, these limitations of the newtonian scheme may appear obvious. But, it must be kept in mind that many of these limitations were pointed out during that Golden Era of Science itself! For example, Descartes had pointed out the rigid nature of the coordinate system; Newton himself was uncomfortable with the absolute nature of the space.

It was of course recognized that properties of Light are not explainable on the basis of the Corpuscular Theory of Light. In particular, Huygens developed the Wave Theory of Light on the basis of the hypothesis that Light is, for example, like a wave propagating in a medium - ether. It was then possible to explain the phenomena of Light. In particular, the polarization of Light received an explanation with the wave theory.

However, all these limitations of the newtonian scheme do not lessen the stature of either Newton as a scientist or that of Newton's theory. The concepts created by him, by others, as well as by those who erected the mathematical framework for these concepts are still important, except that we now know their limitations.

Having presented an in-depth critique of the newtonian theoretical framework, Einstein once wrote [4]: Newton, forgive me; you found the only way which, in your age, was just about possible for a man of highest thought and creative power. The concepts, which you created, are even today still guiding our thinking in physics, although we now know that they will have to be replaced by others farther removed from the sphere of immediate experience, if we aim at a profounder understanding of relationships. Indeed, true this.

Now, let us then turn to modifications of the newtonian scheme that are necessary to explain the phenomena displayed by Light.

Then, at the present stage, we know that Light does not follow the newtonian laws of motion since the explanations based on these laws are not satisfactory ones. Importantly, the property of polarization of Light is not even explainable within the newtonian framework.
But, we only have two types of material bodies left out of the newtonian scheme - those with negative inertia and those with vanishing inertia. Then, it is thinkable that Light is one of these two types of material bodies. This, notwithstanding Huygens’s Wave Theory for Light, is an option open for further exploration of ideas.

But, no property of Light indicates that it has negative inertia. [For example, speed of Light does not increase when it collides with the mirror, say. Here is a “Push” acting on Light, but Light does not “help” it.] Then, in Newton’s scheme, the only option is of treating Light as a material body with vanishing inertia.

Then, obviously with an hindsight now, we can say that Light needs to be treated as a material body with vanishing inertia, this if we are to follow, faithfully, the overall nature of the (scientific) development of the physical ideas beginning with the Golden Era of Science.

But, any material body with vanishing inertia moves with the same speed for all the inertial observers. That is, speed of a material body of vanishing inertia is a universal constant as far as inertial observers are concerned. Then, speed of Light (in vacuum) must be a universal constant for the inertial observers.

Now, the Galilean transformations of coordinates are clearly insufficient to accommodate the above universality of the speed of Light for the inertial observers. It therefore follows that we will have to “extend” these transformations to some suitable others.

However, a lesson from Newton’s theory, namely that, the laws of motion of material objects retain their form in all inertial frames, need not be discarded. (Or, equivalently, a lesson from Galileo’s experiments that uniform rectilinear motion of inertial observers has no special status.) Hence, we should look here for those transformations of the space and the time coordinates that keep the laws of motion invariant.

We therefore arrive at the starting principles used by Einstein for the formulation of his Special Theory of Relativity:

- **The Principle of (Special) Relativity:** The laws of Physics (eg, of motion of material bodies) retain the same form in all inertial frames of reference.

- **The Principle of the Constancy of the Speed of Light:** The speed of Light (in vacuum) is a universal constant (with same value) for all the inertial observers.

On the basis of our considerations so far, it should then be expected that these two principles (of the Special Theory of Relativity) would be sufficient to provide us a logically consistent framework for the theory.

In this connection, we note that the first of these two principles is, in fact, the basis of Newton’s theory and, at the present stage of our theoretical considerations, there do not exist any reasons to give up this characteristic of the newtonian framework. The second of these two postulates is a direct consequence of our assumption that Light is a material body with vanishing inertia.

The theoretical framework based on these two principles may then be expected to provide us logically consistent description of the motion of material bodies with vanishing and non-vanishing (positive) inertia, both. In other words, we can then expect that this theory would describe the motion of material bodies moving not only with speed less than but moving also with the speed of Light relative to inertial observers.

Now, by definition, we have

\[
\text{Speed of Light} = \frac{\text{Light path}}{\text{Time interval}}
\]

Then, the required transformations of coordinates will also involve suitable transformations of the time coordinate when we demand the constancy of the speed of Light for all the inertial observers. This issue then brings us to Einstein’s analysis of the simultaneity of events, an event being a physical happening in space at some instant of time. Below, we follow Einstein’s original presentation of this analysis from his 1905 paper.

At point \( A \) of space, let there be a clock using which an observer at \( A \) determines the time values of events in the immediate vicinity of \( A \) by associating the positions of the hands of the clock with these events. Similarly, at another point \( B \) of the space, let there be a clock, identical in all respects to the clock at the point \( A \), using which an observer at \( B \) determines the time values of events in the immediate proximity of \( B \).

Now, it is important to recognize that we are yet to establish the existence of a common time for the separated locations \( A \) and \( B \). Evidently, this is to be done by sending (Light) signal from location \( A \) to location \( B \) and reflecting it back to \( A \) so that an observer at \( A \) can compare readings of clocks at these separate locations.

If a ray of Light starts from \( A \) at time \( t_A \), reaches and is reflected in the direction of \( A \) at \( B \) at time \( t_B \), and arrives again at \( A \) at time \( t'_A \), then, the clocks at \( A \) and \( B \) synchronize, show same time, if

\[
t_B - t_A = t'_A - t_B
\]

Having this procedure for synchronism of clocks at
different space locations, we can then extend it to all of the space.

Again, it is important to note that this above procedure for comparing clocks at spatially separated locations is a common, day-to-day, experience. It is by detecting a ray of Light emitted by an object or reflected from an object that we “see” that object. The above procedure is an appropriate adaptation of this common experience. Then, the adopted procedure of synchronization of clocks is a logical abstraction derived from our this day-to-day experience.

We therefore assume that the above definition of synchronism of clocks is free of any contradictions and that the following are universally valid:

- **Reflexivity of Synchronism Relation:** If the clock at B synchronizes with the clock at A then, the clock at A synchronizes with the clock at B

- **Associativity of Synchronism Relation:** If the clock at A synchronizes with the clock at B and also with the clock at C then, the clocks at B and C also synchronize with each other in the above procedure

Clearly, these are assumptions and we will have to assume their consistency.

Then, the hypothesis of the constancy of the speed of Light, when expressed in terms of these quantities, is

\[ c = \frac{2d(A, B)}{t'_A - t_A} \]

where \( c \) is a universal constant and \( d(A, B) \) is the “distance” separating \( A \) and \( B \).

Here, we can clearly recognize that the newtonian scheme assumes an infinite speed of propagation for (Light) signals. Recall that the distance separating points \( A \) and \( B \) is the same in all the inertial frames of reference, ie, it is an invariant of the galilean transformations. Furthermore, the absolute nature of time in galilean transformations implies that \( t'_A = t_A \) in all the inertial frames if it holds in one inertial frame. Therefore, an infinite speed of propagation for signals is, in principle, allowed in Newton’s theory.

Now, chose an inertial frame, to be called a stationary frame with all its paraphernalia of coordinate axes, measuring rods and clocks. (Remember that the unambiguous definition of an inertial frame is a problem in Newton’s theory. We rely on the approximate validity of Newton’s laws of motion for this above purpose.)

Moreover, consider a stationary rigid rod lying lengthwise along the \( X \)-axis of the stationary frame; and let its length be \( \ell \) as measured by a measuring-rod which is also stationary in the same frame of reference.

Next, imagine that this rigid rod is imparted a uniform speed \( v \) along the \( X \)-axis of the stationary frame. Then, the length of the moving rod can be established in the following two obvious ways:

(a) The observer moves together with a measuring rod and the rod to be measured. The length of the rod is then obtained directly by superposing the measuring rod, in just the same way as if all three were at rest. Clearly, this length must be equal to \( \ell \).

(b) The observer establishes the system of stationary clocks and synchronizes them as per the adopted procedure. Then, the observer ascertains the locations of the two ends of the rod, whose length is to be measured, in the stationary frame at a definite time. The distance between these two points, measured by the measuring-rod already employed, is the length of the moving rod in the stationary frame of reference.

To ascertain the length of the moving rod in the stationary frame of reference, we adopt the procedure (b) as follows.

Then, consider that at the ends \( A \) and \( B \) of the rod, clocks are placed which synchronize with those of the stationary frame. Imagine also that there is a moving observer passing each of these clocks, and that these observers apply the method of synchronism to both the clocks.

Let a ray of Light be emitted from \( A \) at the stationary frame time \( t_A \), let it be reflected at \( B \) at time \( t'_B \), and let it reach \( A \) again at time \( t'_A \). If \( R_{AB} \) is the length of the moving rod measured in the stationary frame then, from the principle of the constancy of the speed of Light, we have

\[ t_B - t_A = \frac{R_{AB}}{c - v} \quad \text{and} \quad t'_B - t_B = \frac{R_{AB}}{c + v} \]

Clearly, \( t_B - t_A \neq t'_B - t_B \). The observers moving with the rod then find that the two clocks were not synchronized even when the stationary frame had them synchronized!

Consequently, it follows that there is no absolute significance to the concept of the simultaneity of events under the postulate of the constancy of the speed of Light together with the use of Light to “measure” length and time.

But, the length, \( R_{AB} \neq \ell \), of a moving rod is then different in different inertial frames in motion relative to each other!
This, of course, is only a kinematical effect without any absolute significance. That is, no “physical” change in the length of the rod is implied here in the sense that the molecules of the body get pressed together. Rather, it is only that the observer for whom the rod is in motion must, for consistency of the ideas, consider the length of the moving rod to be \( R_{a \beta} \neq \ell \) in physical statements about that rod.

Here, we already see that the Laws of Motion for material bodies with vanishing inertia, when used as signals, have direct implications also for the Laws of Motion for material bodies of non-vanishing inertia. Of specific importance are the kinematic effects as above.

In Newton’s theory however, the lengths of a rod determined from the two methods (a) and (b) are precisely equal. As we have seen earlier, this is a consequence of the galilean transformations used by Newton’s theory. Therefore, the kinematic effects of Einstein’s Special Theory of Relativity do not have any newtonian analogues.

Now, let us turn to our task of finding the required transformations of coordinates consistent with the hypothesis of the constancy of the speed of Light. It will of course be simplified by choosing the reference frames in some convenient manner. This can be done as follows.

In the stationary space, consider two systems of coordinates, each with three rigid material lines, perpendicular to one another and issuing from a point. [Here, we want to emphasize the physical nature of the construction of the coordinate system.] Let each system be provided with rigid measuring rods, alike in all respects, and clocks, which too are alike in all respects. Let the clocks of each of these coordinate systems be synchronized as per the adopted procedure.

Let these coordinate systems be \((x, y, z, t)\) and \((x', y', z', t')\). Let the primed \((K')\) and unprimed \((K)\) systems coincide at \(t' = t = 0\). Then, let the system \(K'\) (along with its entire construction of the three rigid material lines, measuring rods and clocks) be moving with the uniform speed \(v\) in the direction of the positive \(x\)-axis relative to the system \(K\). Let the \(y\) and \(y'\) axes be parallel to each other and so also be the case with the \(z\) and \(z'\) axes, when \(K'\) is in motion.

Therefore, any event in the stationary space can be unambiguously defined in place and in time by the two coordinate systems, in \(K\) by \((x, y, z, t)\) and in \(K'\) by \((x', y', z', t')\). Our required coordinate transformations are then the relations connecting these coordinate quantities.

Now, since the stationary space is homogeneous by construction, meaning no particular coordinate location of an event is preferable over any other location in it, it should be clear that the required equations must be linear.

Furthermore, if we take \(\zeta = x - vt\), a point at rest in \(K'\) must have coordinate values \(\zeta, y, z\), independent of time \(t'\).

From the origin of the system \(K'\), let a ray of light be emitted at time \(t'_1\) along the \(x\)-axis to \(\zeta\). Let this ray of light be reflected at \(\zeta\) at time \(t'_2\), and arrive back at the origin at time \(t'_3\). Then, by construction, we must have

\[
\frac{1}{2} \left( t'_0 + t'_2 \right) = t'_1
\]

Now, by inserting the arguments of the function \(t'\) and using the principle of the constancy of the speed of Light:

\[
\frac{1}{2} \left[ t'(0,0,0,t) + t' \left( 0,0,0,t + \frac{\zeta}{c-v} + \frac{\zeta}{c+v} \right) \right] = t' \left( \zeta,0,0,t + \frac{\zeta}{c-v} \right)
\]

Then, for infinitesimal \(\zeta\), we obtain:

\[
\frac{1}{2} \left( \frac{1}{c-v} + \frac{1}{c+v} \right) \frac{\partial t'}{\partial t} = \frac{\partial t'}{\partial \zeta} + \frac{1}{c-v} \frac{\partial t'}{\partial t}
\]

Or, finally, as

\[
\frac{\partial t'}{\partial \zeta} + \frac{v}{c^2-v^2} \frac{\partial t'}{\partial t} = 0
\]

It should be noted that we could have chosen here any other point to be the origin of the ray of Light to obtain the same equation.

Analogous considerations of a ray of Light for \(y\) and \(z\) axes give us

\[
\frac{\partial t'}{\partial y} = 0 \quad \frac{\partial t'}{\partial z} = 0
\]

Since \(t'\) is a linear function, we therefore obtain

\[
t' = \phi(v) \left( t - \frac{v}{c^2-v^2} \zeta \right)
\]

where, for brevity, it is assumed that at the origin of system \(K'\), \(t' = 0\), when \(t = 0\) and \(\phi(v)\) is an unknown function.

Now, a ray of Light is also propagated with velocity \(c\) in the moving frame \(K'\). Expressing this in equations will then provide us the required quantities \(x', y', z'\). For example, for a ray of Light emitted in the direction of increasing \(x'\) at time \(t' = 0\) at the origin of \(K'\), we have

\[
x' = ct' = c \phi(v) \left( t - \frac{v}{c^2-v^2} \zeta \right)
\]
But, the same ray of Light moves with the velocity \( c - v \) when measured in the system \( K' \) so that

\[
\frac{\zeta}{c - v} = t
\]

Whence, we obtain

\[
x' = \phi(v) \frac{c^2}{c^2 - v^2} \zeta
\]

In summary, we therefore obtain

\[
\begin{align*}
t' &= \phi(v) \gamma (t - vx/c^2) \\
x' &= \phi(v) \gamma (x - vt) \\
y' &= \phi(v) y \\
z' &= \phi(v) z
\end{align*}
\]

(1)

where

\[
\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}
\]

(2)

Note that an additive constant will have to be placed on the right hand side of each of these equations if no assumptions are made as to the initial position of the moving frame \( K' \).

Note also that these transformations must form a (mathematical) group. Then, using some of the group properties, in particular, the existence of an inverse transformation, it can be easily shown that \( \phi(v) = 1 \), very generally.

Now, we have still not proved that the hypothesis of the constancy of the speed of Light is compatible with the hypothesis of the equality of all the inertial frames. Then, to check the internal consistency of our formulation, we need to show it explicitly that any ray of Light also has the same, universally constant, speed \( c \) when measured from the moving frame.

To this end, at \( t' = t = 0 \), let a spherical wave of Light be emitted from the common origin of these two frames. Then, if \( c \) were to denote the (constant) speed of Light, the equation of the spherical wavefront of Light wave in the unprimed frame would be

\[
x^2 + y^2 + z^2 = c^2 t^2
\]

at any later (unprimed) time \( t \).

By transforming this equation with the help of the relations (1), the equation of the same spherical wavefront of Light in the primed frame is obtainable as

\[
(x')^2 + (y')^2 + (z')^2 = c^2 (t')^2
\]

at any later (primed) time \( t' \). Thus, the wave under consideration is also a spherical wave propagating with speed \( c \) in the moving frame.

The relations (1), now with \( \phi(v) = 1 \), are the famous Lorentz transformations with \( \gamma \) being called the Lorentz factor.

These transformations were already obtained by Lorentz and Fitzgerald, and were available in the literature before Einstein propounded his special theory of relativity in 1905. It should also be noted here that Einstein was not aware of these transformations before 1905.

However, Lorentz and Fitzgerald, both, had in mind the considerations related to the “fictitious” ether. Consequently, the genuine credit of showing that these transformations are kinematical of character is entirely that of Einstein’s who also derived these transformations on the basis of only the two postulates mentioned earlier.

Furthermore, a special mention must also be made of Henri Poincaré who had realized that Newton’s laws need modifications. Therefore, in a sense, “The solution anticipated by Poincaré was given by Einstein in his memoir of 1905 on special relativity. He (Einstein) accomplished the revolution which Poincaré had foreseen and stated at a moment when the development of physics seemed to lead to an impasse.”

Let us now turn to physical implications of the Lorentz transformations. To this end, let us consider a rigid sphere of radius \( R \) at rest in the moving frame \( K' \), a material body possessing spherical shape when examined at rest, with the center of the sphere being coincident with the origin of the coordinates of \( K' \).

Then, the equation of the surface of this spherical body moving relative to the system \( K' \) with velocity \( v \) is

\[
(x')^2 + (y')^2 + (z')^2 = R^2
\]

The equation of this surface when expressed in \((x, y, z)\) coordinates at time \( t = 0 \) is

\[
x^2 \gamma^2 + y^2 + z^2 = R^2
\]

It therefore has the shape of an ellipsoid of revolution with the axes

\[
\frac{R}{\gamma}, \ R, \ R
\]

Therefore, to an observer in the stationary frame, the \( x \)-dimension of the body appears to be shortened in the ratio \( 1 : \sqrt{1 - v^2/c^2} \). For \( v = c \), all moving objects, when viewed from the “stationary” system, shrink into plain figures.

Clearly, when viewed from the moving frame, the same result holds good for bodies at rest in the stationary frame.

This is then the Lorentz-Fitzgerald contraction of a material body in motion. It is of course only a
kinematical effect. In other words, to consistently describe the physics of these bodies an observer in the stationary frame has to consider their shortened dimensions while there being no objectively real contraction of these bodies.

Furthermore, we can now consider a clock that marks the time \( t \) when at rest in the stationary frame \( K \), and the time \( t' \) when at rest relative to the moving frame \( K' \). What is then the rate of this clock in the latter situation, relative to the stationary frame?

Considering that the clock is at rest at the origin of the frame \( K' \), we have \( x = vt \) and \( t' \) as in (1). Hence,

\[
t' = t\sqrt{1 - \frac{v^2}{c^2}} = t - \left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)t
\]

Consequently, the time marked by the clock is slow by \( 1\sqrt{1 - \frac{v^2}{c^2}} \) seconds per second, when viewed from the stationary frame.

Clearly, the clock in the stationary frame \( K \) would also run slow when viewed from the moving frame \( K' \). Therefore, this is again only a kinematical effect with no objectively real slowing of the clock taking place.

Now, in Newton’s theory, two given velocity vectors could be vectorially added (as 3-vectors) to obtain the resultant velocity: \( \vec{V} = \vec{v} + \vec{w} \). This Law of Composition of Velocities also changes under the Lorentz transformations.

In the system \( K' \), let a point move as per the equations:

\[
x' = \alpha t', \quad y' = \beta t', \quad z' = 0
\]

where \( \alpha, \beta \) are constants. Then, using the Lorentz transformations, the motion of the point relative to the system \( K \) is described by the equations:

\[
x = \frac{\alpha + v}{1 + \alpha v/c^2} t
\]

\[
y = \frac{\sqrt{1 - v^2/c^2}}{1 + \alpha v/c^2} \beta t
\]

\[
z' = 0
\]

If we now set

\[
V^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2
\]

\[
w^2 = \alpha^2 + \beta^2
\]

\[
\theta = \tan^{-1}(\beta/\alpha)
\]

then, \( \theta \) is the angle between the two velocity 3-vectors \( \vec{v} \) and \( \vec{w} \).

After some simple calculations, we then obtain:

\[
V = \frac{\sqrt{v^2 + w^2 + 2vw \cos \theta} - (vw \sin \theta/c^2)^2}{1 + vw \cos \theta/c^2}
\]

an expression for \( V \) in which \( v \) and \( w \) obviously enter symmetrically.

Clearly, if \( \vec{w} \) also has a direction of the axis of \( x \) then, we have

\[
V = \frac{v + w}{1 + vw/c^2}
\]

an expression which is, most often, to be found in standard text books [3].

In this last expression, let \( v = c - \kappa \) and \( w = c - \lambda \), where \( \kappa \) and \( \lambda \) are positive so that the \( v \) and \( w \) are less than \( c \). Then,

\[
V = c \frac{2c - \kappa - \lambda}{2c - \kappa - \lambda + \kappa \lambda/c} < c
\]

Clearly, in a composition of two velocities which are less than \( c \), there then always results a velocity less than \( c \).

Furthermore, it also follows that the velocity of Light cannot be altered by composition with a velocity less than that of light:

\[
V = \frac{c + w}{1 + w/c} = c
\]

It is also clear that such “parallel transformations” (all the involved velocities being in the same direction as is the case here) form a sub-group of the full group of Lorentz transformations.

[One may now be tempted to do something like \( v = c \) and \( w = c \) in the expression (1) and also obtain \( V = c \). However, one must remember that no observer or the experimenter can move with the speed of Light. It therefore does not make any sense to calculate the relative velocity between two particles of Light as being equal to \( c \). A student of special relativity must be careful against such obvious traps.]

Now, it is clear that Maxwell’s electromagnetism is consistent with the two postulates of the special theory of relativity. Still, to begin with, Einstein explicitly showed the consistency of the postulates of special relativity and Maxwell’s theory of electromagnetism.

Consider then a ray of Light (an electromagnetic wave) having energy \( E \) and making an angle \( \phi \) with the \( x \)-axis of the system \( K \). Let us introduce the system \( K' \) that is moving in uniform parallel translation along the \( x \)-axis of the system \( K \) with speed \( v \). Then, when measured by the system \( K' \), that ray of Light can be shown to possess energy \( E' \) given by:

\[
E' = E \frac{1 - \frac{v}{c} \cos \phi}{\sqrt{1 - v^2/c^2}}
\]

We will use this result in the following.
Imagine now a stationary body in the frame $K$ and let its energy be $E_o$ as referred to that frame. Let the energy of the same body be $H_o$ relative to the system $K'$ moving in parallel translation along the $x$-axis of $K$ with speed $v$.

Let this body now send out a ray of Light (of energy $L/2$ as referred to the frame $K'$) in a direction making an angle $\phi$ with the $x$-axis and, simultaneously, an equal quantity of Light in the opposite direction so that the body, in the meanwhile, remains at rest in the frame $K$. Let its energy now be $E_1$ relative to the system $K$ and $H_1$ relative to the system $K'$.

Clearly, the principle of conservation of energy must apply to this process and, by the principle of (special) relativity, with respect to both the inertial systems $K$ and $K'$.

Then, we have the relations

\[
E_0 = E_1 + \frac{1}{2}L + \frac{1}{2}L
\]

\[
H_o = H_1 + \frac{L}{2} \left( \frac{1 - c \cos \phi}{\sqrt{1 - v^2/c^2}} \right) + \frac{L}{2} \left( \frac{1 - c \cos \phi}{\sqrt{1 - v^2/c^2}} \right)
\]

\[
= H_1 + \frac{L}{\sqrt{1 - v^2/c^2}}
\]

And, on subtraction, we then obtain

\[
(H_o - E_o) - (H_1 - E_1) = L \left( \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right)
\]

Now, the quantity $H$ is the energy relative to the system $K'$ and the quantity $E$ is the energy relative to the system $K$. Hence, $H - E$ is the difference in energy as referred to two inertial frames with the body being at rest in one of them. Consequently, $H - E$ can differ from the kinetic energy $J$ of the body with respect to $K'$ only by an additive constant $C$. That is, we have $H_o - E_o = J_o + C$ and $H_1 - E_1 = J_1 + C$ since $C$ does not change during the emission of Light.

Therefore, we have the result

\[
J_o - J_1 = L \left( \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) \quad (5)
\]

That is to say, the kinetic energy of the body referred to the system $K'$ decreases as a result of the emission of Light and the amount of decrease is independent of the properties of that body. It however depends on the velocity $v$.

To second order of magnitude, we then obtain:

\[
J_o - J_1 \approx \frac{1}{2} \left( \frac{L}{c^2} \right) v^2
\]

and we can now conclude that if a body gives off energy $E$ in the form of electromagnetic radiation, its mass decreases by $E/c^2$.

This is another of celebrated results of the special theory of relativity: the equivalence of mass and energy. Einstein obtained this result in an independent publication again in 1905.

It also follows now that the inertia or the mass of a material body depends upon its velocity relative to the stationary frame as

\[
m = \frac{m_o}{\sqrt{1 - v^2/c^2}} \quad (6)
\]

where $m_o$ is the mass as observed by an inertial frame in which that material body is at rest. This is an important result in many respects.

The reason for our adapting here Einstein’s 1905 paper is the following one. His methods are based precisely on the way we make measurements - the way we use Light to communicate. A critical analysis of fundamental concepts must begin with only such considerations and then can it lead to theoretical advancement. (History has also shown this before.) It will also be important at a later stage of our present considerations.

In the early days of special relativity, this theory was misunderstood to a large extent because kinematical effects predicted by it were misconceived to be objectively real. This misunderstanding then gave rise to many paradoxes. One of the famous early paradoxes of special relativity is known as the Paradox of Twins.

Imagine a pair of twins borne at the same earth-time. One of them stays on the earth and the other travels in space for some earth-years and returns to meet his counterpart on the earth. Will the age of the traveller be less than that of the one who stayed on the Earth?

It is important to recognize that this question is unanswerable within the special theory of relativity since this situation necessarily involves “non-inertial” frames of reference. For example, to return to the Earth, the traveller twin has to decelerate and accelerate in the path. But, special theory of relativity stays silent on as to how to deal with such frames of reference.

Now, it should be evident as to what the limitations of Einstein’s this special theory of relativity really are. Once again, noteworthy for us are the use here of the Cartesian conceptions of Euclidean geometry for the space and the associations of properties of material bodies with the points of that Euclidean space.

These above are essentially the same conceptions as that of Newton’s theory. Only the coordinate transformations used by the special theory of relativity are then different.
It is here that we note the contributions of H. Minkowski. Minkowski showed that different cumbersome formulas of special relativity can be elegantly recast into concise mathematical forms if we treat the 3-space and the time as forming a 4-dimensional spacetime.

Consider different quadruplets \((x, y, z, t)\) of real numbers forming a 4-dimensional (mathematical) space endowed with a metric function:

\[ (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2 (\Delta t)^2 = ds^2 \tag{7} \]

where \(\Delta x, \Delta y, \Delta z, \Delta t\) are the infinitesimal changes in the values of \(x, y, z, \) and \(t\). Clearly, “Lorentz transformations” of quadruplets keep this metric function form-invariant.

Notice that the above metric function is not a positive-definite one. In mathematical vocabulary, such a metric function is called a pseudo-metric and the corresponding space is called a pseudo-metric space. Then, we are considering here a 4-dimensional pseudo-metric space.

(The 4-dimensional spacetime of special relativity is therefore a pseudo-metric space.)

Let a specific point of this spacetime be called an event. Notice that different events can now be classified on the basis of whether the 4-distance \(ds\) to that event from the origin of the coordinates is positive, zero, or negative.

When \(ds = 0\), we call the trajectory a null trajectory. When \(ds > 0\), we call the trajectory to be spacelike and when \(ds < 0\), we call the trajectory to be timelike.

[We have chosen the signature of the metric function to be \((+, +, +, -)\). We could instead have chosen it to be \((-,-,-,+).\) In that case, these above definitions of “timelike” and “spacelike” will have to be interchanged.]

We call this 4-dimensional (pseudo-metric) space the Minkowski spacetime. It is only a mathematical construction and no physical implications are incorporated into it unless, of course, we make the required physical associations.

Again, all these physical associations are exactly as they were in Newton’s theory. Recollect that we have, after all, only extended the newtonian formalism to material bodies with vanishing inertia and have not discarded any of the physical associations of Newton’s theory.

Of course, in extending Newton’s formalism to material bodies with vanishing inertia, we also use the fact that we “observe” material bodies using the electromagnetic radiation, Light, as a material body of vanishing inertia.

Clearly, any modifications can then occur for only those newtonian concepts which rely on the (implicit) assumption that signals can, in principle, propagate with infinite speed. Evidently, simultaneity of events and the absolute nature of time are two such conceptions.

Now, when we carry out the required physical associations, it follows that the null trajectory of the Minkowski spacetime is the path followed by Light (as a material body with vanishing inertia), a timelike trajectory is the path followed by a material body moving with speed less than the speed of Light, while the spacelike trajectory would be followed by a (hypothetical) material body moving with speed faster than the speed of Light relative to inertial observers.

But, it should be evident that the association of the inertia of a material body with the points of the Minkowski spacetime does not then produce any change in the Minkowski 4-space. Then, the Minkowski spacetime is an inert background for the material bodies. Clearly, even the special theory of relativity explains all phenomena as relations between objects existing in space and time. But, as shown by Minkowski, space and time in special relativity are now mathematically treatable as a 4-dimensional spacetime.

Now, the construction of the coordinate axes and clocks must be using the material bodies. But, this construction is left outside the scope also of the special theory of relativity. Hence, the construction of the coordinate system in special relativity is using rigid rods and clocks which never get affected by anything happening with other material bodies. This is surely an important drawback of the special theory of relativity just exactly as it was of Newton’s theory.

The Minkowski spacetime is then an absolute, unchanging, 4-space in physical considerations of the special theory of relativity.

Hence, the acceleration of a material point is, at a fundamental level, then to be referred to only the 4-dimensional background Minkowski spacetime or the corresponding unchanging coordinate system. Such coordinate systems, the inertial frames of reference, are then fundamental or special to this description of physical systems.

Consequently, even in the special theory of relativity, if we select any coordinate frame which is accelerated with respect to the background coordinate system, we will have to introduce fictitious forces, the pseudo-forces, to account for various physical phenomena.

The unambiguous physical construction of the inertial frames of reference is then a problem even in special theory of relativity.

Moreover, it should also be clear now that, when \(v > c\), the special theory of relativity offers no laws for the motion of a material body. Clearly, Lorentz transformations have no (obvious) mean-
ing for \( v > c \). This must be recognized as a limitation of the special theory of relativity if such material bodies with speeds larger than the speed of Light existed in reality. (Several physicists have speculated about the properties of (hypothetical) super-luminal material body, Tachyon, particle so named by G Feinberg.)

Notice now that even the special theory of relativity is based on two independent considerations: first, those of the Law of Motion and second, those of the Law of Force. The Law of Motion is then empty of content without a Law of Force even in special theory of relativity. That is to say, unless and until we specify the “force” acting on a material point, the Special Relativistic Law of Motion will not be able to provide us the path followed by that material point. But, this force is, likewise in Newton’s theory, an action at a distance due to source property of material bodies.

Then, whatsoever is the Law of Force, it must be recognized as an assumption. Any such law is a statement that we postulate regarding the “force” acting between material points. This means that, essentially, we could then always raise the question: Why not any other form for the force acting between material points? Clearly, as long as the law of motion is independent of the law of force, this state of affairs persists.

As an example, consider Coulomb’s law from electrostatics. If \( Q \) and \( q \) are to denote the quantities of electric charge possessed by two material bodies separated by distance \( d \) then, the electrostatic force between them is given by the well known expression:

\[
\vec{F}_e = k \frac{qQ}{d^2} \hat{d}
\]

where \( k \) is a constant and \( \hat{d} \) is a unit vector along the line joining the two charges. This force is attractive or repulsive depending on the relative sign of the two electric charges.

It must be recognized that this law is also an assumption. Furthermore, the quantity of charge is the source property of material body by which one charged body produces the force on another charged body by acting on it at a distance. Then, we could always raise the above question: Why not any other form for this force?

Then, Newton’s Law of Gravitation is also an assumption vis-à-vis the special theory of relativity. But, by imagining an inertial frame in which two separated material bodies are at rest (so that the magnitude of the gravitational force between them is given by Newton’s law), by imagining another inertial frame moving with uniform velocity \( v \) relative to the first one and by the equality of the inertial and the gravitational mass, it is easy to see that Newton’s law of gravitation is not a Lorentz-invariant statement.

Evidently, special theory of relativity requires, therefore, an appropriate modification of Newton’s Law of Gravitation so that the Law of Gravitation be compatible with the postulate of (special) relativity stated earlier.

However, it must be borne in mind that, even with this modification, we will not be able to circumvent the problem of the Law of Force being an assumption in special relativity. No matter what modification of Newton’s Law of Gravitation we may propose, the basic framework would continue to be unsatisfactory. Therefore, special relativity offers no explanation whatsoever for the phenomenon of gravitation. This is exactly as was the case with Newton’s theory.

Since the law of motion (requiring the concept of the inertia for motion of a material body) is independent of the law of force (requiring the concept of the source attribute for a material body), the equality of the inertial and the gravitational mass of a material body is also an explicit assumption of special relativity. Again, this is exactly as it was with Newton’s theory.

Then, it immediately follows that the origin of the source characteristics of material bodies, like the gravitational mass of a material body, will not be explainable in special relativity.

Clearly, this is so because special relativity, similar to Newton’s theory, treats the cause of the motion of a material body, the force, as an agency which is essentially independent of the origin of the inertia of that body.

Surely, the equality of the inertial and the gravitational mass of a material body is to be interpreted to mean that the same quality of a material body manifests itself, according to circumstances, as its inertia or as its weight (heaviness). It should therefore be evident now that whenever the cause of motion is treated as being essentially independent of the origin of the inertia of a material point, we will not be able to explain the origin for the cause of motion.

Clearly, due only to this reason, the origin of “force” - the cause of motion - then remains unexplained in Newton’s theory as well as in special theory of relativity.

Clearly, conclusions of either Newton’s theory or of the special theory relativity based on the law of gravitation, eg. regarding the final or the end-state of the collapse of physical objects such as a star, cannot be “reliable” in their entirety since any “Law of Gravitation” is an assumption of either of these two theories. It is important that this issue be adequately recognized.
Still, it is true that certain experimental justification exists for the laws of force (for example, Newton’s law of gravitation, Coulomb’s law etc.) as assumed by Newton’s theory. This is in the form of the verification of the laws of motion, explanation of the tides, results of many day-to-day laboratory experiments etc.

Similarly, an experimental justification also exists for special relativity in the form of the verification of time dilation effects, for example, for elementary particles moving close to the speed of Light. It also explains results of Michelson-Morley and other experiments.

Consequently, we can trust the predictions of these theories to certain extent. Up to what precise extent can we trust the predictions of these theories? Evidently, this question can be completely answered only on the basis of the theory that explains the origin of inertia. It should also be clear that “correct” answer to the question of the end-state of gravitational collapse of a star, for example, can only be obtained from a theory that explains the origin of inertia.

However, we may also verify the predictions of these theories by laboratory experimentations and astronomical observations. It is such verifications of the “assumed” law of gravitation that are, most often, responsible for our falling in the trap of assuming that the law of gravitation “has the experimental proof” and, therefore, that this law is unchangeable.

Nothing can be really far from the truth than this! A critical analysis of fundamental concepts helps us avoid such misunderstandings by showing us the limitations of such concepts. Another extremely important aspect is that of the quantum considerations. These considerations acknowledge a fundamental limitation in the classical ideas by essentially recognizing that in making a measurement of a physical quantity, an observer will inevitably cause an “uncontrollable” change to the system being observed. This aspect is particularly important for phenomena involving microscopic bodies.

As an example, consider the measurement of the location of an object. From our ordinary day-to-day experiences, we know that to locate an object, we must send a signal, a ray of Light, in the direction of that object and must receive the signal reflected from that object.

But, a ray of Light imparts momentum to the object in the process of reflection, thereby changing the earlier (measured) value of the (linear) momentum of that object. Consequently, there is always to be an interaction of the object and the observing agency in the process of any measurement of the physical quantity.

Now, the issue is whether we can control this “change” in any conceivable manner to whatsoever extent that we desire.

If we could then, we would always be able to simultaneously measure, both, the location and the momentum of the object to any desired accuracy. If, fundamentally speaking, we cannot then, newtonian and special relativistic concepts require, obviously, modifications.

This is of course not the issue of simply being unable to achieve the above task in a particular experimental setup due to experimental limitations of the equipments of measurement. Rather, this is the issue of the theoretical possibility or the impossibility of the measurement of a physical quantity to any desired accuracy.

Here, it is vital to recognize that this issue is related to the issue of the physical construction of the coordinate system, first raised by Descartes during the Golden Era of Science.

In the experiment of the measurement of the location of an object, Light, as a material body, acts as a (tiny) road-roller trying to cross the location of the given object, say, as a part of the coordinate axis. This road-roller must have some action on the object, then. This was the issue raised by Descartes. Then, the earlier issue is whether this action of the road-roller is “controllable” by the experimenter so that the location can be measured to any desired accuracy.

Now, it is the tacit assumption of Newton’s theory as well as of Einstein’s special theory of relativity that this aforementioned action is “precisely controllable” in the experimental setup. That is why the location of a material point is precisely determinable simultaneously with its (linear) momentum in these theories.

But, is it really so? To be sure, we must investigate this issue further. This was what was done by Heisenberg at the beginning of the 20th century – another Golden Era of Science. Apart from those of Galileo, Newton, Descartes and Einstein, Heisenberg’s investigation of this issue is another example of the critical analysis of related fundamental concepts.

Then, from Heisenberg’s analysis, it follows that simultaneously measured values of the location and the momentum of that object satisfy (Heisenberg’s) indeterminacy relation - the basis of the theory of the quantum.

Heisenberg’s indeterminacy relation shows that the location and the (linear) momentum of a material body, represented as a material point, cannot be simultaneously determined to any better accuracy than is permitted by this relation. Surely, this indicates limitations of the concepts of Newton’s theory and those of the special relativity.
In spite of this above being the case, special relativity is surely an *advance or a step* in the direction of a theory that can “explain” the origin of inertia and, hence, gravitation.

In this connection, it must be borne in mind that the inertia, the opposition of a material body to a change in its state of motion, can be expected to depend on the state of motion, speed, of that body. This expectation is based on the analogous situation of the opposition to motion experienced by a person located in a crowd. This opposition to motion of a person depends on the state of motion, speed, of that person.

That the expectation based on the above analogy holds is then expressed by the “variation of (inertial) mass with velocity” in special relativity as we have seen earlier in (6).

In a sense, it is only this single result that is conceptually the most important one of the achievements of special relativity. It is of course not that other results such as the “relativization of time” are any less important than this.

But, only the mass-variation with velocity indicates that special relativity is a (right) step in the direction of a theory that could explain the origin of inertia. Indeed, Einstein was fond of referring to special relativity as a step [5].

The origin of inertia of material bodies may then be “explainable” by developing the aforementioned analogy (to a person placed in a crowd) into a suitable new theoretical framework.

Now, concepts of Euclidean geometry will, evidently, not be useful to this new theoretical framework because the physical construction of its coordinate system (coordinate axes, measuring rods and clocks) is required to be unchangeable irrespective of the motion of material bodies in this geometry. This is, as we have now recognized, evidently, unsatisfactory.

Then, we must use that geometry whose (physical) construction of space coordinates changes with the motions of material bodies. Then, as a judicious guess based on our related considerations, it therefore follows that we need to use, for the new theory, geometric considerations more general than those of the Euclidean geometry.

What kind of *non-Euclidean geometry* is this new one required to be?

Here, we must first realize that a “material body” and “geometry of space” ought to be *indistinguishable*. Moving a material body from its given “location” should cause changes to the construction of the coordinate system. And, that will change the “geometry” because the construction of the coordinate system is the basis of the “metric function” of the geometry.

Hence, this new geometry is determined by physical or material bodies. In turn, material bodies are also determined by this new geometry in that “given the (pseudo) metric function of the new geometry” we would know how the material bodies are “located” relative to each other.

Evidently, these ideas are then fundamentally different from those of Newton’s theory as well as from those of special relativity. In these new considerations, the concept of “force” is abandoned and is replaced by suitable properties of the geometry - its transformations.

Even when the basic conceptions of the new theory are required to be quite different, it must be borne in mind that the successes of Newton’s theory as well as those of special relativity will have to be obtainable in the new theory. Mathematically, various formulas of the new theory must “reduce” to those of special relativity and to those of Newton’s theory in approximations.

It is in this above mathematical sense that the new theory will have to incorporate special relativity and Newton’s theory, both.

Now, with due respects to Minkowski’s works, there is nothing fundamentally important about the 4-dimensional formalism in special relativity. In other words, all the physical results, obtained using explicitly the 4-dimensional considerations, can also be obtained using the 3-space and the time, essentially separately.

[As a matter of fact, Einstein’s original paper(s) on the special theory of relativity did not use any of the 4-dimensional methods which were discovered by Minkowski only later.]

However, the *continuum* of the quadruplets of four coordinates, \((x, y, z, t)\) with all the four coordinates varying in continuous manner, may be expected to form the basis of the theory of the future as well. After all, we have abstracted these *four* quantities - three for the space and one for the time - from our ordinary, day-to-day, experiences with material bodies.

Also, to this day, there are no indications, of any kind whatsoever, from our ordinary experiences that, to describe the “motions” of material bodies and, hence, to also describe the origin of their inertia, we need to add any other quantities to this list of the four coordinates.

From the critical analysis of related concepts that we have accomplished above, it has also not emerged that the extra dimensions are any necessary to this description.

Consequently, even if there existed extra dimensions, these must somehow be “suppressed” so as to be not observable at the present level of observations. Why would this always be so? Presently, we simply do not know why.
Then, with due respects to them, it must be said that attempts in attempts to add extra dimensions, over and above those of the four of space and time, have not led to any “genuine advance” in our understanding of the physical world around us.

In the absence of any clear indications about the possible extra-dimensions, it is, conservatively speaking, advisable to restrict ourselves to only the quadruplets of four coordinates: \((x, y, z, t)\). This is what we have followed here.

Then, on the basis of the mathematical “beauty” of Minkowski’s 4-dimensional formulation, we may be tempted to think that any future theory of the physical world should be based only on the relevant 4-dimensional considerations.

This last issue is then that of being able to treat the set of quadruplets as a 4-dimensional “pseudo-metric” manifold when general transformations of quadruplets are invoked. But, transformations of quadruplets need not be continuous, let alone differentiable. Then, usefulness of the differentiable manifold structure or the pseudo-metric structure becomes questionable.

What can then substitute the differentiability (partial differential equations) being used here? This is the issue of the most basic mathematical formalism to describe physics.

Currently, theories of measures and dynamical systems appear basic.\(^8\)

However, any further considerations of the new theory are obviously beyond the scope of our present endeavors.

In conclusion, therefore, I hope to have made it clear to you in this modest presentation as to why the special theory of relativity is only a step in the right direction. Then, the concepts used by special relativity are not to be treated as unchangeable. In fact, we ought to change some of the concepts here if we aim at a profounder understanding of the physical phenomena.

I hope also to have made it clear to you that to advance further from the conceptions in Newton’s theory and in special relativity, we need to “abandon” the concept of “force” as an external cause of motion and “replace” it with some other satisfactory concept.

Einstein had hoped to achieve this with his general theory of relativity. However, he had realized that his dream is not materialized in the formulation that he had proposed for the general relativity. He then searched for a new Unified Field Theory without success.

But, towards the end of his life, he had begun seeing beyond the vision of others, as had always been the case with him in the past.

Acknowledgments

I am grateful to Drs. Dilip A Deshpande, Kishor B Ghormare, Pradeep S Mukti-bodh and other organizers of the Seminar on 100 years of Special Relativity for giving me an opportunity of presenting my thoughts on this occasion.

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[1] See, for example, articles in (1952) The Principle of Relativity: A collection of original papers on the special and general theory of relativity. Notes by A Sommerfeld (Dover, New York)

See, also, Einstein Albert (1968) Relativity: The Special and the General Theory (Methuen & Co. Ltd, London)

[2] See, for example, Galileo Galilei (1955) Dialogue on the Great World Systems (Abridged text edition In the translation of T Salasbury, Edited, corrected, annotated and provided with a historical introduction by Giorgio De Santillana, The University of Chicago Press, Chicago)

[3] See, for example, Goldstein H (1950) Classical Mechanics (John Wiley & Sons, New York)

Kibble T W B (1970) Classical Mechanics (ELBS-McGraw-Hill, London)

Sudarshan E C G and Mukunda N (1974) Classical Dynamics - A modern perspective, (Wiley International, New York)

[4] Einstein Albert (1970) in Albert Einstein: Philosopher Scientist (Ed. P A Schlip, Open Court Publishing Company - The Library of Living Philosophers, Vol VII; La Salle)

[5] Pais Abraham (1982) Subtle is the Lord ..., The science and the life of Albert Einstein (Clarendon Press, Oxford)

[6] Einstein Albert (1905) Does the inertia of a body depend upon its energy content?, (1905) Annalen der Physik, 17

[7] Heisenberg W (1949) The Physical principles of the quantum theory (Dover, New York)

See, also, Bohr N (1928) Nature (Supplement Series), April 14, 1928, p. 580

[8] Wagh S M (2004) Heuristic approach to a natural unification of the quantum theory and the general theory of relativity Database: physics/0409057 and references therein.

See, also, Wagh S M (2004) Einsteinian field theory as a program in fundamental physics Database: physics/0404028 and references therein.

[9] For his purely scientific views, heresies in the eyes of the Roman Catholic Church, Galileo had to face the Cardinal Judges of the Inquisition Board of the Church for this heretic crime.

Galileo was of course not any (religious or other-
wise) fanatic person seeking martyrdom for only the sake of the beliefs he held, particularly when he was already of an advanced age. He felt it advisable to bend before his persecutors.

Galileo then had to recite, kneeling before the Cardinal Judges of the Inquisition Board, some formula of abjuration, this notwithstanding his advanced age and ill health. For details, see Giorgio de Santillana (1955) *The Crime of Galileo* (The University of Chicago Press, Chicago).

But, even after this Inquisition and subsequent abjuration, Galileo was under permanent house-arrest for the remaining eight years of his life to also face total blindness. His scientific works still found the way to reach out to the whole of the Europe and, ultimately, the World.

However, a note was to be found in Galileo’s handwriting, surely written before he turned totally blind, on the margin of his own copy of his famous book *Dialogue on the Great World Systems*, for which he faced the Inquisition, as: *In the matter of introducing novelties. And who can doubt that it will lead to the worst disorders when minds created free by God are compelled to submit slavishly to an outside will? When we are told to deny our senses and subject them to the whim of others? When people devoid of whatsoever competence are made judges over experts and are granted authority to treat them as they please? These are the novelties which are apt to bring about the ruin of commonwealths and the subversion of the state.*