Curved-space classical solutions of a massive supermatrix model

Takehiro AZUMA† and Maxime BAGNOUD‡

†Department of Physics, Kyoto University, Kyoto 606-8502, Japan
‡Institut de Physique, Université de Neuchâtel, CH-2000 Neuchâtel, Switzerland

Abstract

We investigate here a supermatrix model with a mass term and a cubic interaction. It is based on the Lie superalgebra \( \mathfrak{osp}(1|32, \mathbb{R}) \), which could play a rôle in the construction of the eleven-dimensional M-theory. This model contains a massive version of the IIB matrix model, where some fields have a tachyonic mass term. Therefore, the trivial vacuum of this theory is unstable. However, this model possesses several classical solutions where these fields build noncommutative curved spaces and these solutions are shown to be energetically more favorable than the trivial vacuum. In particular, we describe in details two cases, the \( SO(3) \times SO(3) \times SO(3) \) (three fuzzy 2-spheres) and the \( SO(9) \) (fuzzy 8-sphere) classical backgrounds.

---

1 e-mail address : azuma@gauge.scphys.kyoto-u.ac.jp
2 e-mail address : Maxime.Bagnoud@unine.ch
1 Introduction

Despite the fact that perturbative superstring theory provides us with a consistent unified theory of fundamental interactions, we still lack a completely satisfactory nonperturbative formulation of superstring theory. As a consequence, although this theory has a plethora of possible vacua (the dynamics in some of these vacua has already been studied in details), there is no way to select the true vacuum of the theory and compare the physical implications of superstring theory with known phenomenological data. It is thus instrumental to find a constructive definition of superstring theory in order to predict the real world or/and falsify the theory.

One of the successful proposals for a constructive definition of superstring theory[1, 2, 3, 5] is a formulation through a large $N$ reduced model. A candidate model of this kind is the so-called IIB matrix model[2, 4], which is defined by the following action:

$$S = \frac{1}{g^2} Tr \left( \frac{1}{4} [A_\mu, A_\nu] [A_\mu, A_\nu] + \frac{1}{2} \bar{\psi} \Gamma^\mu [A_\mu, \psi] \right), \quad (1.1)$$

where the indices $\mu, \nu, \cdots$ run over the 10-dimensional Minkowskian spacetime. It is a large $N$ reduced model[32, 33, 34] of 10-dimensional $\mathcal{N} = 1$ super Yang-Mills theory with $U(N)$ gauge symmetry. Here, $\psi$ is a 10-dimensional Majorana-Weyl spinor field, and $A_\mu$ and $\psi$ are $N \times N$ Hermitian matrices. The IIB matrix model has $\mathcal{N} = 2$ supersymmetry, which exhibits a particular structure that allows us to interpret the eigenvalues of the large $N$ matrices describing the bosonic fields as space-time coordinates[7, 8] (IIB matrix model is extensively reviewed in [9]).

Another intriguing attempt for a constructive definition of superstring theory is a background-independent matrix model based on the Lie superalgebra $\mathfrak{osp}(1|32, \mathbb{R})[11, 13, 15, 16, 21]$. It is a natural generalization of the IIB matrix model, in which both bosons and fermions are unified into a single supermultiplet. $\mathfrak{osp}(1|32, \mathbb{R})$ has been known as the unique maximal simple Lie superalgebra with 32 fermionic generators[12]. In a 10-dimensional representation, the smallest irreducible spinors are the 16-component chiral spinors, so that the 32 fermionic generators can be decomposed in two chiral spinors of equal or opposite chiralities. The former and the latter respectively correspond to the type IIA and IIB superstring theories. In this sense, we speculate that the IIB matrix model would be extracted from this supermatrix model by integrating out some degrees of freedom. In the papers [13, 16], it has been attempted to clarify the relation between a purely cubic $\mathfrak{osp}(1|32, \mathbb{R})$ supermatrix model and the IIB matrix model by paying particular attention to the structure of the supersymmetry algebra. In the paper

\[ Z = \int dA d\bar{\psi} e^{+ S_E}, \]

where $S_E$ is defined in the 10-dimensional Euclidean space; i.e. $S_E$ is defined by Wick-rotating $A_0$ as $A_0 \rightarrow i A_0$ and replacing the gamma matrices for the $SO(9,1)$ Clifford algebra with those for $SO(10)$ in the action $S$. 

\[ Z = \int dA d\bar{\psi} e^{+ S_E}, \]
such a cubic supermatrix model supplemented by a mass term has been investigated to elucidate its relation with the BFSS matrix model[1].

Since large \( N \) reduced models are expected to be eligible frameworks to describe gravitational interactions, it is essential to have the possibility of describing curved spaces manifestly in their framework. As the IIB matrix model only possesses flat noncommutative spacetime as a classical solution (the same holds true of the non-gauged \( \mathfrak{osp}(1|32, \mathbb{R}) \) supermatrix model[15]), it is impossible to study perturbations around curved backgrounds.

Some generalization is thus necessary in order to overcome this difficulty. A possible approach is to identify large \( N \) matrices with differential operators[15, 20, 23, 24]. Large \( N \) matrices have both aspects of differential operators and spacetime coordinates. The former appears clearly in the twisted Eguchi-Kawai model[33, 34] while the latter is the essential feature of the IIB matrix model. These two aspects can be related by expanding the IIB matrix model around its flat noncommutative background[10]. This bilateral character is interpreted as the T-duality of string theory. The advantage of identifying matrices with differential operators lies in the fact that differential operators act on fields on a curved spacetime in a natural way.

Another approach is to consider a matrix model which has some curved space as a classical solution, so that it becomes possible to perform perturbations around this curved background. To achieve this, some modification of the IIB matrix model[14, 17, 25, 30] is needed. In [14], a Chern-Simons term has been added to the IIB matrix action to construct a noncommutative gauge theory on the \( SO(3) \) fuzzy sphere[35]. Another possible alteration is the addition of a tachyonic mass term to the bosonic part of the IIB matrix model[17]:

\[
S = \frac{1}{g^2} Tr \left( \frac{1}{4} [A_\alpha, A_\beta][A_\alpha, A_\beta] + \lambda^2 A_\alpha A_\alpha \right),
\]

(1.2)

where the indices \( \alpha, \beta \) run over \( \alpha, \beta = 1, 2, 3, 4 \). The indices are contracted with respect to the four-dimensional Euclidean space metric, and the model has \( SO(4) \) global symmetry. Its equations of motion

\[
[A_\beta, [A_\alpha, A_\beta]] + 2\lambda^2 A_\alpha = 0
\]

(1.3)

have classical solutions given by a set of fields satisfying some Lie algebra. Thus, such a massive IIB matrix model can be expanded around various curved spaces. In [17], expansions around the two-dimensional fuzzy sphere and the two-dimensional fuzzy torus have been studied.

In this paper, we take this latter approach in order to describe a curved background spacetime by considering an \( \mathfrak{osp}(1|32, \mathbb{R}) \) supermatrix model with a mass term. We analyze how the massive supermatrix model incorporates the non-commutative curved-space classical solutions.

This paper is organized as follows: In Section 2, we give a brief review of the Lie superalgebra \( \mathfrak{osp}(1|32, \mathbb{R}) \) and the associated supermatrix model. In Section 3, we suggest an ansatz that allows us to solve the equations of motion of the massive supermatrix model and leads to solutions of the fuzzy-sphere type. We describe in detail two of these solutions, one exhibiting \( SO(3) \times SO(3) \times SO(3) \) symmetry and the other exhibiting...
SO(9) symmetry and compare their stability properties. This leads us to a more general discussion of a possible brane nucleation process in such totally reduced matrix models. Then, we make a few remarks on the structure of the supersymmetry transformations in our model. Finally, we summarize the results presented in this work in section 4 and indicate there a few directions for future research on this topic.

2 \ osp(1|32, \mathbb{R}) supermatrix model with a mass term

L. Smolin proposed a cubic matrix model\cite{11, 13} based on the Lie superalgebra \osp(1|32, \mathbb{R}). The action is constructed from a matrix \( M \) belonging to \osp(1|32, \mathbb{R}) whose entries are promoted to large \( N \) Hermitian matrices. In this paper, we follow the notation of \cite{14}, in which details about \osp(1|32, \mathbb{R}) are given. Its relation with the 11-dimensional super-Poincaré algebra is described in \cite{12, 21}.

The (even part) of the Lie superalgebra \osp(1|32, \mathbb{R}) is constituted from matrices of the form:

\[
M = \begin{pmatrix}
m & \psi \\
-i\psi & 0
\end{pmatrix},
\]

where \( \psi \) is a 32-component Majorana spinor and \( m \) belongs to the Lie algebra \sp(32, \mathbb{R}). We take the metric of the 10-dimensional Minkowskian spacetime as

\[
\eta^{\mu\nu} = \text{diag}(-1, +1, \cdots, +1).
\]

2.1 Action

Here, we consider an \osp(1|32, \mathbb{R}) supermatrix model with a mass term included, expecting similarities with the massive IIB matrix model studied in \cite{17}. We consider the following action, with a mass term added to the pure cubic \osp(1|32, \mathbb{R}) supermatrix model:

\[
S = Tr_{\text{osp}(1|32)} \left[ \text{str}_{\text{osp}(1|32)} \left( -3\mu M^2 + iM [M, M] \right) \right]
= Tr_{\text{osp}(1|32)} \left[ 3\mu (-m_p^q m_q^r + 2i\bar{\psi}\psi) + i \left( m_p^q [m_q^r, m_r^p] - 3i\bar{\psi}\psi [m_p^q, \psi^q] \right) \right].
\]

(2.3)

where \( p, q, r, \cdots = 1, \ldots, 32 \). In this model, each element of the \osp(1|32, \mathbb{R}) supermatrices is promoted to an \( N \times N \) Hermitian matrix. This action is invariant under \( U(N) \) gauge transformations and \( OSP(1|32, \mathbb{R}) \) orthosymplectic transformations, and these two symmetries are decoupled, since they do not act on the same indices. Since we want to consider this model in a 10-dimensional spacetime context, we decompose the bosonic part \( m \) as follows:

\[
m = WT^2 + A_\mu \Gamma^\mu + B_\mu \Gamma^{\mu 2} + \frac{1}{2!} C_{\mu_2 \mu_2} \Gamma^{\mu_1 \mu_2} + \frac{1}{4!} H_{\mu_1 \cdots \mu_4} \Gamma^{\mu_1 \cdots \mu_4 2} + \frac{1}{5!} Z_{\mu_1 \cdots \mu_5} \Gamma^{\mu_1 \cdots \mu_5},
\]

(2.4)

where \( \mu_i = 0, \ldots, 9 \) and \( \Gamma^2 \) is the chirality operator. Then, the relevant part of the action (2.3) is expressed as (writing simply \( Tr \) instead of \( Tr_{\text{osp}(1|32)} \) from now on):

\[
S = 96\mu Tr \left( -W^2 - A_\mu A^\mu + B_\mu B^\mu + \frac{1}{2} C_{\mu_1 \mu_2} C^{\mu_1 \mu_2} - \frac{1}{4!} H_{\mu_1 \cdots \mu_4} H^{\mu_1 \cdots \mu_4} - \frac{1}{5!} Z_{\mu_1 \cdots \mu_5} Z^{\mu_1 \cdots \mu_5} + \frac{i}{16} \bar{\psi}\psi \right) + 32i Tr \left( 3C_{\mu_1 \mu_2} [B^{\mu_1}, B^{\mu_2}] + C_{\mu_1 \mu_2} [C^{\mu_2}_{\mu_3}, C^{\mu_3}_{\mu_1}] \right) + \text{cubic interactions involving } (W, A, H, Z, \psi),
\]

(2.5)
while the full result can be found in the first appendix and the detailed computation in [16]. In the purely cubic supermatrix model (without mass term, which has been studied in [11, 13, 15, 16]), the rank-2 field $C_{\mu_1\mu_2}$ possesses a cubic interaction term but has no quadratic term. This has been a severe obstacle to the appearance of a Yang-Mills-like structure in the supermatrix model, because it has been impossible to identify $C_{\mu_1\mu_2}$ with the commutators of the rank-1 fields $[B_{\mu_1}, B_{\mu_2}]$ (or $[A_{\mu_1}, A_{\mu_2}]$). In the 11-dimensional case, this difficulty has been overcome in [21] through the addition of a mass term, and we thus expect this model to contain the massive IIB matrix model, the bosonic part of which has been studied in [17] to investigate perturbation theory around noncommutative curved-space backgrounds.

3 Resolution of the equations of motion

We proceed to search for possible curved-space classical configurations solving the equations of motion that follow from the action (2.5). To get a clearer picture of the problem, we now set the fermions and the positive squared-mass bosonic fields to zero:

$$\psi = W = A_\mu = H_{\mu_1...\mu_4} = Z_{\mu_1...\mu_5} = 0.$$ (3.1)

Since their masses are positive (at least in the spatial directions, while the time-like direction of quantum fields is generally unphysical), (3.1) is a stable classical solution. Furthermore, we choose to identify the tachyonic 10-dimensional vector field $B_\mu$, rather than the well-defined $A_\mu$ with the bosonic fields of the massive IIB matrix model, in order to obtain a possibly stable curved-space classical solution. The classical equations of motion for the remaining tachyonic fields $B_\mu$ and $C_{\mu\nu}$ following from (2.5) are

$$B_\mu = -i\mu^{-1}[B_\nu, C_{\mu\nu}],$$ (3.2)
$$C_{\mu\nu} = -i\mu^{-1}([B_\mu, B_\nu] + [C_\rho, C_{\mu\rho}]).$$ (3.3)

Although it is difficult to solve these equations in full generality, the equation of motion for $C_{\mu\nu}$ suggests to take $C_{\mu\nu} \propto [B_\mu, B_\nu]$ for $B_\mu$’s satisfying a fairly simple commutator algebra. If we look for objects having a clear geometrical interpretation, it is tempting to look for solutions building fuzzy spheres.

3.1 $SO(3) \times SO(3) \times SO(3)$ classical solution

The simplest tentative solution is the product of three fuzzy 2-spheres with the symmetry $SO(3) \times SO(3) \times SO(3)$. Such a system is described by $N \times N$ hermitian matrices building a representation of the $so(3)$ Lie algebra in the following way:

$$[B_i, B_j] = i\mu_\epsilon_{ijk}B_k, \quad B_1^2 + B_2^2 + B_3^2 = \mu^2\epsilon^{2N^2}_{ijk} = 1_{N \times N} \quad \text{for } (i,j,k = 1,2,3)$$ (3.4)

with similar relations for $i, j, k = 4, 5, 6$ and $i, j, k = 7, 8, 9$, trivial commutators for indices that do not belong to the same group of 3, and $B_0 = 0$ ($\epsilon_{ijk}$ is defined as usually).

This set of fields (3.4) describes a space formed by the Cartesian product of three fuzzy spheres located in the directions $(x_1, x_2, x_3)$, $(x_4, x_5, x_6)$ and $(x_7, x_8, x_9)$, whose radii are
all $\mu r \sqrt{N^2 - 1}/2$. $(N^2 - 1)/4$ is the quadratic Casimir operator of the $\mathfrak{so}(3)$ Lie algebra. Note that any positive-integer value of $N$ is possible here, since $N$ indexes the dimensions of irreducible representations. For $SO(3)$, the irreps have dimensions $N = 2j + 1$, for all integer values of the spin $j$. However, we can also use spinorial representations with half-integer spins in this case. We have to consider this classical solution instead of the single $SO(3)$ fuzzy sphere

$$[B_i, B_j] = i\mu r \epsilon_{ijk} B_k \quad (\text{for } i, j, k = 1, 2, 3), \quad B_\mu = 0 \quad (\text{for } \mu = 0, 4, 5, \cdots, 9),$$

(3.5)

because the solution $B_4 = \cdots = B_9 = 0$ is unstable in the directions 4 to 9 due to the negative squared mass\(^4\) of the rank-1 fields $B_\mu$. Without restricting the generality, we can focus on the first sphere located in the direction $(x_1, x_2, x_3)$, since the three fuzzy spheres all share the same equations of motion.

In the framework of fuzzy 2-spheres, we can solve the equations of motion (3.2) and (3.3) with the following ansatz for the rank-2 field $C_{ij}$:

$$C_{ij} = f(r) \epsilon_{ijk} B_k,$$

(3.6)

where $f(r)$ is a function depending on the radius parameter $r$. Indeed, the equation of motion (3.3) reduces then to:

$$\epsilon_{ijk} B_k (-f(r) + r + rf^2(r)) = 0.$$  

(3.7)

(3.7) has two solutions: $f_\pm(r) = \frac{1 \pm \sqrt{1 - 4r^2}}{2r}$. When we plug this result in the equation of motion for $B$ (3.2), this leads to

$$B_i(1 - 2rf_\pm(r)) = 0.$$  

(3.8)

This gives the same condition on the radius parameter $r$ for both $f_+(r)$ and $f_-(r)$, namely:

$$\sqrt{1 - 4r^2} = 0.$$  

(3.9)

Therefore, when we assume the ansatz (3.6), we obtain the classical solution (3.4) with the radius parameter set to $r = \frac{1}{2}$, which is fortunately real. Indeed, $r^2 \leq 0$ would indicate that the fuzzy sphere solution is unstable. For example, in the IIB massive matrix model described by (1.2), the sign of the squared radius of the fuzzy 2-sphere is linked to the sign of the mass term in the action and it would become negative for a correct-sign mass term, which is to be expected, since in that case, the trivial commutative solution becomes the stable vacuum of the theory.

We next want to discuss the stability of the $SO(3) \times SO(3) \times SO(3)$ classical solution in more qualitative terms\(^5\). To this end, we compare the energy of the trivial commutative solution $B_\mu = 0$ with that of the fuzzy-sphere solution. The classical energy for $B_\mu = 0$ is obviously $E_{B_\mu=0} = -S_{B_\mu=0} = 0$. \footnote{The classical solution with $B_0 = 0$ has no problem, because it has a positive mass unlike the other directions of the field $B$.}

\footnote{Recall that in our notation, the action is minus the potential. Since we now consider a classical solution with $B_0 = 0$ (thus no need of Wick rotation), the energy is simply minus the classical action in which we substitute the solution.}
In the $SO(3) \times SO(3) \times SO(3)$ fuzzy-sphere background, the 2-form field $C_{ij}$ is

$$C_{ij} = \epsilon_{ijk} B_k.$$  \hfill (3.10)

Therefore, the total energy is

$$E_{SO(3)^3} = -S_{SO(3)^3} = -64\mu \sum_{\mu=1}^{9} Tr(B_\mu B^\mu) = -3 \times 64\mu \sum_{i=1}^{3} Tr(B_i B_i)$$

$$= -12\mu^3 N(N-1)(N+1).$$  \hfill (3.11)

This result shows that the $SO(3) \times SO(3) \times SO(3)$ fuzzy-sphere classical solution has a lower energy compared to the trivial commutative solution and hence a higher probability.

### 3.2 Other curved-space solutions and the fuzzy 8-sphere

So far, we have analyzed the simplest curved-space solution, the $SO(3) \times SO(3) \times SO(3)$ triple fuzzy spheres. Here, we consider other curved-space classical solutions. The fuzzy $2k$-spheres\[6, 18, 19, 29], which exhibit a $SO(2k+1)$ symmetry, are constructed by the following $n$-fold symmetric tensor product of $(2k+1)$-dimensional gamma matrices:

$$B_p^{SO(2k+1)} = \frac{\mu r}{2} (\Gamma_p^{(2k)} \otimes 1 \otimes \cdots \otimes 1) + \cdots + (1 \otimes \cdots \otimes 1 \otimes \Gamma_p^{(2k)})_{\text{sym}},$$  \hfill (3.12)

where $p$ runs over $1, 2, \cdots, 2k+1$. $\Gamma_p^{(2k)}$ are $2^k \times 2^k$ gamma matrices, and build a representation of the $SO(2k+1)$ Clifford algebra; i.e. $\{\Gamma_p^{(2k)}, \Gamma_q^{(2k)}\} = 2\delta_{pq} 1_{2^k \times 2^k}$. These matrices satisfy the following algebraic relations:

$$B_p^{SO(2k+1)} B_p^{SO(2k+1)} = \frac{\mu^2 r^2}{4} n(n+2k) 1_{N_k \times N_k},$$  \hfill (3.13)

$$B_p^{SO(2k+1)} B_q^{SO(2k+1)} = -\left(\frac{\mu r}{2}\right)^2 8kn(n+2k) 1_{N_k \times N_k},$$  \hfill (3.14)

$$[B_p^{SO(2k+1)}, B_s^{SO(2k+1)}] = \mu^2 r^2 (-\delta_{ps} B_q^{SO(2k+1)} + \delta_{qs} B_p^{SO(2k+1)}),$$  \hfill (3.15)

$$[B_p^{SO(2k+1)}, \Gamma_p^{SO(2k)}] = \mu^2 r^2 (\delta_{qs} B_{pt}^{SO(2k+1)} + \delta_{pt} B_{qs}^{SO(2k+1)} - \delta_{ps} B_{qt}^{SO(2k+1)} - \delta_{qt} B_{ps}^{SO(2k+1)}),$$  \hfill (3.16)

where $B_p^{SO(2k+1)} = [B_p^{SO(2k+1)}, B_q^{SO(2k+1)}]$ furnishes (up to a normalization factor) a representation of the $so(2k+1)$ Lie algebra and $N_k$ is the dimension of the fully symmetrized irreducible representation for the $SO(2k+1)$ fuzzy sphere. The commutation relations (3.13) and (3.16) are inherited from those of the gamma matrices. Thanks to these relations, we expect that the equations of motion can be solved for all even-dimensional fuzzy spheres in a similar fashion to the fuzzy 2-spheres. In other words, this means that the $SO(2k+1)$ fuzzy spheres will provide us with a whole set of curved classical solutions for some precise values of the parameter $r$. In addition, the $B_p^{SO(2k+1)}$'s satisfy the following self-duality relation:

$$\epsilon_{p_1 \cdots p_{2k+1}} B_{p_1}^{SO(2k+1)} B_{p_2}^{SO(2k+1)} \cdots B_{p_{2k}}^{SO(2k+1)} = \left(\frac{\mu r}{2}\right)^{2k-1} m_k B_{p_{2k+1}}^{SO(2k+1)},$$  \hfill (3.17)
where

\[ m_1 = 2i, \quad m_2 = 8(n + 2), \quad m_3 = -48i(n + 2)(n + 4), \quad m_4 = -384(n + 2)(n + 4)(n + 6), \]

which is a generalization of the \( \mathfrak{so}(3) \) Lie algebra\(^6\) and a consequence of the duality relation for odd-dimensional Gamma matrices. We give the computation of these coefficients in the second appendix.

Another possible classical solution of our massive supermatrix model is the single \( \text{SO}(9) \) fuzzy sphere. The analysis goes in the same way as in the \( \text{SO}(3) \times \text{SO}(3) \times \text{SO}(3) \) fuzzy spheres. Here, the indices \( p, q, \ldots \) run over 1, 2, \ldots, 9. For the \( \text{SO}(9) \) fuzzy-sphere classical solution, we likewise assume the following ansatz for the rank-2 fields \( C^{\text{SO}(9)}_{pq} \):

\[ C^{\text{SO}(9)}_{pq} = -i\mu^{-1}g(r)B^{\text{SO}(9)}_{pq}. \]  (3.19)

Then, the equation of motion (3.3) implies

\[ \frac{-i}{\mu} B^{\text{SO}(9)}_{pq}(-g(r) + 1 + 7r^2g^2(r)) = 0. \]  (3.20)

We again have two choices for the function \( f(r) \):

\[ g_{\pm}(r) = \frac{1 \pm \sqrt{1 - 28r^2}}{14r^2}. \]  (3.21)

The equation of motion (3.2) for the rank-1 field \( B^{\text{SO}(9)}_p \) gives

\[ B^{\text{SO}(9)}_p(1 - 8r^2g_{\pm}(r)) = 0. \]  (3.22)

Now, unlike the case of the \( \text{SO}(3) \times \text{SO}(3) \times \text{SO}(3) \) fuzzy spheres, \( 1 - 8r^2g_{-}(r) = 0 \) does not have any real positive solution for \( r \). However, there is exactly one such solution for \( 1 - 8r^2g_{+}(r) = 0 \), which is \( r = \frac{1}{8} \).

More generally, for an \( \text{SO}(2k + 1) \) fuzzy sphere, the same ansatz would give

\[ g_{\pm}(r) = \frac{1 \pm \sqrt{1 - 4(2k - 1)r^2}}{2(2k - 1)r^2}, \]

\[ 1 - 2kr^2g_{\pm}(r) = 0, \text{ solvable only for } g_{+}(r) \text{ at } r = \frac{1}{2k}. \]  (3.23)

\(^6\) It generalizes the fuzzy 2-sphere case where \( B_i^{\text{SO}(3)}B_i^{\text{SO}(3)} \) is proportional to the identity on the totally symmetric space of dimension \( n + 1 \). For \( N = n + 1 \), the radius of the \( \text{SO}(3) \) fuzzy sphere is indeed

\[ \frac{\mu^2r^2}{4}n(n + 2) = (\nu r)^2N^2 - \frac{1}{4}. \]

The relation (3.13) actually corresponds to the Casimir of the \( \mathfrak{so}(3) \) Lie algebra. And (3.17) is trivially equivalent to the commutation relation \([B_i^{\text{SO}(3)}, B_j^{\text{SO}(3)}] = i\nu r\epsilon_{ijk}B_k^{\text{SO}(3)}\).
We discuss the stability of the $SO(9)$ fuzzy-sphere classical solution by computing its classical energy. At the classical level, we obtain

$$E_{SO(9)} = -\frac{5}{8} \mu^3 n(n+8)N_4.$$  

(3.24)

$N_4$ is given in [18][19] by

$$N_4 = \frac{(n+1)(n+2)(n+3)(n+4)^2(n+5)^2(n+6)(n+7)}{302400}. \quad (3.25)$$

In contrast with the $SO(3) \times SO(3) \times SO(3)$ case, $N$ can take here only certain precise values. For example, the smallest non-trivial representation ($n = 1$) has dimension 16, the following one ($n = 2$) 126, then 672, etc... The classical energy for the $SO(3) \times SO(3) \times SO(3)$ triple fuzzy-sphere solution is of the order $O(-\mu^3 n^3) = O(-\mu^3 N^6)$ while that of the $SO(9)$ fuzzy-sphere solution is of the order $O(-\mu^3 n^{12}) = O(-\mu^3 N^{15})$. Therefore, at large $N$, the $SO(3) \times SO(3) \times SO(3)$ triple fuzzy-sphere solution is energetically favored compared to the $SO(9)$ solution at the equal size $N$ of the matrices. The presence of a spherical solution for all $N$ in the $SO(3) \times SO(3) \times SO(3)$ case may indeed be a stabilizing factor. On the other hand, at equal value of $n$, whose physical meaning is less clear, the fuzzy 8-sphere solution has lower energy.

The single $SO(q)$ fuzzy spheres for $q \leq 8$ do not constitute a stable classical solution of our model. When the $SO(q)$ sphere occupies the direction $x_1, x_2, \ldots, x_q$, the solution $B_{q+1}^{SO(q)} = B_{q+2}^{SO(q)} = \cdots = B_9^{SO(q)} = 0$ is trivially unstable because of the negative mass squared. Whereas, the Cartesian product of several fuzzy spheres, such as $SO(3) \times SO(6)$, is a possible candidate for a stable classical solution.

### 3.3 Nucleation process of spherical branes

Starting from a vacuous spacetime, it is interesting to try to guess how spherical brane configurations could be successively produced through a sequence of decays into energetically more favorable meta-stable brane systems. The reader may have noticed that we have so far limited ourselves to the study of curved branes building irreducible representations of their symmetry groups. This could seem at first to be an unjustified prejudice, but it turns out that such configurations are energetically favored at equal values of $N$. For example, for $SO(3)$, an irreducible representation $R_N$ of dimension $N$ contributes as

$$E_{R_N} = -4\mu^3(N^3 - N) \quad (3.26)$$

per fuzzy 2-sphere, while a reducible representation $R_{N_1} \oplus \ldots \oplus R_{N_m}$ of equal dimension $N_1 + \ldots + N_m = N$ would contribute as

$$E_{R_{N_1} \oplus \ldots \oplus R_{N_m}} = -4\mu^3 \sum_{i=1}^{m} (N_i^3 - N_i). \quad (3.27)$$

Generally, $N_k$ is known to be of the order $O(n^{\frac{k(k+1)}{2}})$, and more explicitly,

$$N_1 = (n + 1), \quad N_2 = \frac{(n+1)(n+2)(n+3)}{6}, \quad N_3 = \frac{(n+1)(n+2)(n+3)^2(n+4)(n+5)}{360}.$$
This is obviously a less negative number, especially for big values of $N$. A similar conclusion was reached in [14] for the case of a Euclidean 3-dimensional IIB matrix model with a Chern-Simons term and it seems to be a fairly general feature of matrix models admitting non-trivial classical solutions. This property is particularly clear for low-dimensional branes, since the classical energy is of order $\mathcal{O}(-\mu^{3}N^{3})$ for $SO(3)$, but it remains true for any $SO(2k+1)$ fuzzy-sphere solution, whose energy is of order $\mathcal{O}(-\mu^{3}N^{1+4/(k(k+1))})$, which also shows that low-dimensional configurations are favored. As hinted for in the preceding subsection, this latter fact can be physically understood by remarking that there are more irreps available for low-dimensional fuzzy spheres, which makes it easier for them to grow in radius through energetically favorable configurations. A third obvious fact is that configurations described by representations of high dimensionality are preferred. 

Put together, these comparisons give us a possible picture for the branes nucleation process in this and similar matrix models. As they appear, configurations of all spacetime dimensions described by small representations will be progressively absorbed by bigger representations to form irreducible ones, that will slowly grow in this way to bigger values of $N$. Parallel to that, branes of higher dimensionalities will tend to decay into a bunch of branes of smaller dimensionalities, finally leaving only 2-spheres and noncommutative tori of growing radii. If the size of the Hermitian matrices is left open, as is usually the case in completely reduced models, where the path integration contains a sum on that size, no configuration will be truly stable, since the size of the irreps will grow continuously. 

Of course, this is a relatively qualitative study, which could only be proven correct by a full quantum statistical study of the model. However, it seems to be an interesting proposal for the possible physics of such theories.

### 3.4 Supersymmetry

We next comment on the structure of the supersymmetry. The biggest difference with the purely cubic supermatrix model, due to the addition of the mass term, is that this model is not invariant under the inhomogeneous supersymmetry

$$
\delta_{\text{inhomogeneous}} m = 0, \quad \delta_{\text{inhomogeneous}} \psi = \xi,
$$

which is a translation of the fermionic field. However, this model has 2 homogeneous supersymmetries in 10 dimensions, which are part of the $\mathfrak{osp}(1|32, \mathbb{R})$ symmetry:

$$
\delta_{\epsilon} M = \left[ \begin{pmatrix} 0 & \epsilon \\ i\bar{\epsilon} & 0 \end{pmatrix}, \begin{pmatrix} m & \psi \\ i\bar{\psi} & 0 \end{pmatrix} \right] = \begin{pmatrix} i(\epsilon\bar{\psi} - \psi\bar{\epsilon}) & -m\epsilon \\ i\bar{\epsilon}m & 0 \end{pmatrix},
$$

which transforms the bosonic and fermionic fields as

$$
\delta_{\epsilon} m = i(\epsilon\bar{\psi} - \psi\bar{\epsilon}), \quad \delta_{\epsilon} \psi = -m\epsilon.
$$

In the IIB matrix model, the supersymmetry has to balance between a quartic term $Tr([A_{\mu}, A_{\nu}])^{2}$ and a trilinear contribution $Tr\bar{\psi}\Gamma^{\nu}[A_{\mu}, \psi]$ in the action [13], which implies that the SUSY transformation of the fermionic field has to be bilinear in the bosonic field. On the other hand, the homogeneous supersymmetries are all linear in the fields in the purely cubic supermatrix model [14, 16]. By incorporating the mass term, we are
allowed to integrate out the rank-2 field $C_{\mu_1\mu_2}$ by solving the classical equation of motion iteratively as in [21].

Thanks to this procedure, the homogeneous SUSY transformation for the fermionic field becomes

$$\delta_\epsilon \psi = \frac{i}{2} [B_{\mu_1}, B_{\mu_2}] \Gamma^{\mu_1\mu_2} \epsilon + \cdots,$$

while the transformation of the field $B_\mu$ is

$$\delta_\epsilon B_\mu = -\frac{1}{32} \text{tr}_{32\times32}(i(\epsilon \bar{\psi} - \psi \bar{\epsilon}) \Gamma_\mu) = -\frac{i}{16} \epsilon \bar{\Gamma}_\mu \psi.$$

In that sense, the mass term is essential to realize the Yang-Mills-like structure for the homogeneous supersymmetries. On the other hand, if we want to preserve the homogeneous supersymmetries, we cannot just put a mass term for $C_{\mu_1\mu_2}$ by hand, the $\mathfrak{osp}(1|32, \mathbb{R})$ symmetry forces all fields to share the same mass, since they all lie in the same multiplet. In particular, we are forced to introduce a mass term for the fermions as well, which breaks the inhomogeneous supersymmetries. In other words, it seems difficult to have Super Yang-Mills-type structure for both homogeneous and inhomogeneous supersymmetries in the context of supermatrix models.

Indeed, in contrast with the purely cubic supermatrix model [13, 10], which has twice as many SUSY parameters, the massive supermatrix model has only $\mathcal{N} = 2$ SUSY in 10 dimensions, because it lacks the inhomogeneous supersymmetries. In consequence, we cannot realize the translation of the vector field $A_\mu$ as a commutator of two linear combinations of the homogeneous and inhomogeneous supersymmetries (3.30) as in the IIB matrix model, where it leads to the interpretation of the eigenvalues of $A_\mu$ as spacetime coordinates. On the contrary,

$$[\delta_\epsilon, \delta_\chi]m = i[\epsilon \bar{\chi} - \chi \bar{\epsilon}], \quad [\delta_\epsilon, \delta_\chi] \psi = i(\epsilon \bar{\psi} - \psi \bar{\epsilon})\psi$$

vanishes up to an $\mathfrak{sp}(32, \mathbb{R})$ rotation. This problem is a serious obstacle for the identification of the supersymmetry of this model with that of the IIB matrix model. More analysis will be reported elsewhere.

4 Conclusion and outlook

In this paper, we have investigated a supermatrix model based on $\mathfrak{osp}(1|32, \mathbb{R})$ with a mass term and a cubic interaction. To be able to describe the gravitational interaction in terms of large $N$ reduced models, we must understand how the reduced models can describe physics in curved spacetimes. Although the IIB matrix model only possesses flat
noncommutative spacetime as a classical solution, by adding a tachyonic mass term as in [17], we can obtain new classical solutions building curved space backgrounds. Following this idea, we have expected that massive supermatrix models could also exhibit similar properties leading to non-trivial classical solutions. In particular, we have investigated fuzzy-sphere solutions with symmetries \(SO(3) \times SO(3) \times SO(3)\) and \(SO(9)\), and calculated the parameter determining the quantization step separating the radii of configurations described by different representations of the Lie algebra. We have then discussed their respective likelihood by comparing their energy at the classical level, which gave us a way to understand a possible dynamical evolution of the solutions through successively more favorable brane configurations.

It is an intriguing issue to search for other stable curved-space classical solutions. For example, an \(SO(3) \times SO(6)\) fuzzy sphere could be a promising candidate. Indeed, the expansion around this classical solution may be related in some way to the BMN matrix model [22, 26, 28, 31], which appears as the discrete light-cone quantization of D0-brane in the M-theory pp-wave background [24]. However, to study this case explicitly, we first have to analyze how the equations of motion (3.2) and (3.3) can be treated in the case of odd fuzzy spheres. Indeed, as is outlined in [29], the commutator \([B_i, B_j]\) does not correspond to a representation of \(SO(2k)\) for odd fuzzy spheres, which makes the analysis much more involved. However, the construction of an \(SO(3) \times SO(6)\) classical solution would show how transverse 5-branes can appear in this model. The \(SO(4) \times SO(5)\) case should proceed along similar lines. Another case that can be investigated is a solution of the type \(SO(2) \times SO(2) \times SO(5)\), in which the two first circles build a noncommutative torus as in [17].

Another difficult problem that could be tackled in the future is a detailed analysis of the stability under perturbations of such noncommutative spaces in this and other similar models, to see if these solutions are really local minima of the potential energy.

We next mention the relation between our massive \(osp(1|32, \mathbb{R})\) supermatrix model and the IIB matrix model. In the quantum field theory, some different models which possess the same symmetry are equivalent in the continuum limit. This property is known as universality. We expect that some similar mechanism may hold true of the large \(N\) reduced models, and hence that various matrix models may have the same large \(N\) limit. If we believe in the matrix-model version of the universality conjecture, it is possible that our massive supermatrix model could be equivalent to the IIB matrix model. In this sense, it is interesting to investigate further whether our model shares the maximal 10-dimensional \(\mathcal{N} = 2\) SUSY with the IIB matrix model and how.

**Acknowledgment**

The authors would like to express their gratitude to Luca Carlevaro, Jean-Pierre Derendinger, Frank Ferrari, Hikaru Kawai, Yusuke Kimura and Christian Römelsberger for valuable discussion. The works of T.A. were supported in part by Grant-in-Aid for Scientific Research from Ministry of Education, Culture, Sports, Science and Technology of Japan (#01282). M.B. acknowledges the financial support provided through the European Community’s Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime and the Swiss Office for Education and Science as well as the Swiss National Science Foundation.
5 Massive supermatrix model action

\[ S = 96 \mu Tr \left( -W^2 - A_\mu A^\mu + B_\mu B^\mu + \frac{1}{2} C_{\mu_1 \mu_2} C^{\mu_1 \mu_2} - \frac{1}{4!} H_{\mu_1 \cdots \mu_4} H^{\mu_1 \cdots \mu_4} \right) 
\]
\[ - \frac{1}{5!} Z_{\mu_1 \cdots \mu_5} Z^{\mu_1 \cdots \mu_5} + \frac{i}{16} \bar{\psi} \psi \right) 
\]
\[ + 32 i Tr \left( - 3 C_{\mu_1 \mu_2} [A^{\mu_1}, A^{\mu_2}] + 3 C_{\mu_1 \mu_2} [B^{\mu_1}, B^{\mu_2}] + 6 W [A_\mu, B^\mu] + C_{\mu_1 \mu_2} [C^{\mu_2}, C^{\mu_3 \mu_4}] \right) 
\]
\[ + \frac{1}{4} B_{\mu_1} [H_{\mu_2 \cdots \mu_5}, Z^{\mu_1 \cdots \mu_5}] - \frac{1}{8} C_{\mu_1 \mu_2} (4[H^{\mu_1 \mu_2 \rho_1 \rho_2 \rho_3}, H^{\mu_2 \rho_1 \rho_2 \rho_3}] + [Z^{\mu_1 \cdots \mu_5}, Z^{\mu_2 \rho_1 \cdots \rho_4}]) \]
\[ + \frac{3}{(5!)^2} \epsilon^{\mu_1 \cdots \mu_{10} \xi} \left( - 2 W [Z_{\mu_1 \cdots \mu_5}, Z_{\mu_6 \cdots \mu_{10}}] + 10 A_{\mu_1} [H_{\mu_2 \cdots \mu_5}, Z_{\mu_6 \cdots \mu_{10}}]) \right) 
\]
\[ + \frac{200}{(5!)^2} \epsilon^{\mu_1 \cdots \mu_{10} \xi} \left( 5 H_{\mu_1 \cdots \mu_4} [Z_{\mu_5 \mu_6 \mu_7}^{\rho \chi}, Z_{\mu_8 \mu_9 \mu_{10} \rho \chi}] + 10 H_{\mu_1 \cdots \mu_4} [H_{\mu_5 \mu_6 \mu_7}^{\rho}, H_{\mu_8 \mu_9 \mu_{10} \rho}] 
\]
\[ + 6 H^{\rho \chi}_{\mu_1 \mu_2} [Z_{\mu_3 \mu_4 \mu_5 \rho \chi}, Z_{\mu_6 \cdots \mu_{10}}]) \right) \]
\[ + 3 Tr \left( \bar{\psi} \Gamma^\mu [W, \psi] + \bar{\psi} \Gamma^\mu [A_\mu, \psi] + \bar{\psi} \Gamma^{\mu \nu} [B_\mu, \psi] + \frac{1}{2!} \bar{\psi} \Gamma^{\mu \nu \sigma} [C_{\mu_1 \mu_2}, \psi] \right) 
\]
\[ + \frac{1}{4!} \bar{\psi} \Gamma^{\mu_1 \cdots \mu_4 \xi} [H_{\mu_1 \cdots \mu_4}, \psi] + \frac{1}{5!} \bar{\psi} \Gamma^{\mu_1 \cdots \mu_5} [Z_{\mu_1 \cdots \mu_5}, \psi] \right) \]

(5.1)

6 Notations and useful formulae

In this appendix, we give hints for the derivations of the coefficients \( m_k \) in the self-duality relation (B.17) for the \( SO(2k+1) \) fuzzy sphere. We first define the \( 2^k \times 2^k \) gamma matrices in the 2\( k \)-dimensional Euclidean space \( \Gamma_p^{(2k)} \) by the following recursive relation:

\[ \Gamma_p^{(2k+2)} = \Gamma_p^{(2k)} \otimes \sigma_2 = \begin{pmatrix} 0 & -i \Gamma_p^{(2k)} \\ i \Gamma_p^{(2k)} & 0 \end{pmatrix}, \quad \Gamma_2^{(2k+2)} = 1_{2^k \times 2^k} \otimes \sigma_1 = \begin{pmatrix} 0 & 1_{2^k \times 2^k} \\ 1_{2^k \times 2^k} & 0 \end{pmatrix}, \]

\[ \Gamma_{2k+3}^{(2k+2)} = 1_{2^k \times 2^k} \otimes \sigma_3 = \begin{pmatrix} 1_{2^k \times 2^k} & 0 \\ 0 & -1_{2^k \times 2^k} \end{pmatrix}, \]

where the index \( p \) runs over \( p = 1, 2, \cdots, 2k+1 \). The 2-dimensional gamma matrices are identical to the Pauli matrices: \( \Gamma_1^{(2)} = \sigma_z \). Under this notation, we obtain

\[ \sigma_1 \sigma_2 = i \sigma_3, \quad \Gamma_1^{(4)} \Gamma_2^{(4)} \Gamma_3^{(4)} \Gamma_4^{(4)} = \Gamma_5^{(4)}, \quad \Gamma_1^{(6)} \Gamma_2^{(6)} \cdots \Gamma_6^{(6)} = -i \Gamma_7^{(6)}, \quad \Gamma_1^{(8)} \Gamma_2^{(8)} \cdots \Gamma_8^{(8)} = -\Gamma_9^{(8)}. \]

(6.2)

It is trivial that \( m_1 = 2i \) as explained in the footnote [3], while the computation of the coefficient \( m_2 \) can be found in [3]. In this appendix, we give formulae that are useful to derive \( m_3 \) and \( m_4 \). We set \( \mu = 1 \) and omit "sym", with the understanding that these formulae are only valid in the fully symmetrized representations.
In general, we have

\[ \sum_{l=1}^{2k+1} (\Gamma^{(2k)}_{l} \otimes \Gamma^{(2k)}_{l}) = (1_{2^k \times 2^k} \otimes 1_{2^k \times 2^k}), \]

\[ \sum_{l=1}^{2k+1} (\Gamma^{(2k)}_{l_1l_2} \otimes \Gamma^{(2k)}_{l_1l_2}) = -2k(1_{2^k \times 2^k} \otimes 1_{2^k \times 2^k}) \] (6.3)

More specifically, to compute \( m_4 \), we also need

\[ (\Gamma^{(6)}_{l_1l_2l_3} \otimes \Gamma^{(6)}_{l_4l_5l_6}) = -18(1_{8 \times 8} \otimes 1_{8 \times 8}), \] (6.4)

\[ (\Gamma^{(6)}_{l_1l_2} \otimes \Gamma^{(6)}_{l_3} \otimes \Gamma^{(6)}_{l_4l_5l_6}) = -6(1_{8 \times 8} \otimes 1_{8 \times 8} \otimes 1_{8 \times 8}), \] (6.5)

\[ (\Gamma^{(6)}_{l_1l_2} \otimes \Gamma^{(6)}_{l_3l_4} \otimes \Gamma^{(6)}_{l_5l_6}) = 24(1_{8 \times 8} \otimes 1_{8 \times 8} \otimes 1_{8 \times 8}), \] (6.6)

\[ (\Gamma^{(6)}_{l_1l_2} \otimes \Gamma^{(6)}_{l_3l_4} \otimes \Gamma^{(6)}_{l_5l_6} \otimes \Gamma^{(6)}_{l_1\ldots l_6}) = -48(1_{8 \times 8} \otimes 1_{8 \times 8} \otimes 1_{8 \times 8} \otimes 1_{8 \times 8}). \] (6.7)

References

[1] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, “M theory as a matrix model: A conjecture,” Phys. Rev. D 55, 5112 (1997) [hep-th/9610043].

[2] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, “A large-N reduced model as superstring,” Nucl. Phys. B 498, 467 (1997) [hep-th/9612115].

[3] R. Dijkgraaf, E. Verlinde and H. Verlinde, “Matrix string theory,” Nucl. Phys. B 500, 43 (1997) [hep-th/9703030].

[4] M. Fukuma, H. Kawai, Y. Kitazawa and A. Tsuchiya, “String field theory from IIB matrix model,” Nucl. Phys. B 510, 158 (1998) [hep-th/9705128].

[5] H. Itoyama and A. Tokura, “USp(2k) matrix model: F theory connection,” Prog. Theor. Phys. 99, 129 (1998) [hep-th/9705123].

[6] J. Castelino, S. Lee and W. Taylor, “Longitudinal 5-branes as 4-spheres in matrix theory,” Nucl. Phys. B 526, 334 (1998) [hep-th/9712105].

[7] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa and T. Tada, “Space-time structures from IIB matrix model,” Prog. Theor. Phys. 99, 713 (1998) [hep-th/9802085].

[8] S. Iso and H. Kawai, “Space-time and matter in IIB matrix model: Gauge symmetry and diffeomorphism,” Int. J. Mod. Phys. A 15, 651 (2000) [hep-th/9903217].

[9] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, A. Tsuchiya and T. Tada, “IIB matrix model,” Prog. Theor. Phys. Suppl. 134, 47 (1999) [hep-th/9908038].

[10] H. Aoki, N. Ishibashi, S. Iso, H. Kawai, Y. Kitazawa and T. Tada, “Noncommutative Yang-Mills in IIB matrix model,” Nucl. Phys. B 565, 176 (2000) [hep-th/9908141].
[11] L. Smolin, “M theory as a matrix extension of Chern-Simons theory,” Nucl. Phys. B 591, 227 (2000) [hep-th/0002009].

[12] E. Bergshoeff and A. Van Proeyen, “The many faces of OSp(1|32),” Class. Quant. Grav. 17, 3277 (2000) [hep-th/0003261].

[13] L. Smolin, “The cubic matrix model and a duality between strings and loops,” hep-th/0006137.

[14] S. Iso, Y. Kimura, K. Tanaka and K. Wakatsuki, “Noncommutative gauge theory on fuzzy sphere from matrix model,” Nucl. Phys. B 604, 121 (2001) [hep-th/0101102].

[15] T. Azuma, S. Iso, H. Kawai and Y. Ohwashi, “Supermatrix models,” Nucl. Phys. B 610, 251 (2001) [hep-th/0102168].

[16] T. Azuma, “Investigation of matrix theory via super Lie algebra,” hep-th/0103003.

[17] Y. Kimura, “Noncommutative gauge theories on fuzzy sphere and fuzzy torus from matrix model,” Prog. Theor. Phys. 106, 445 (2001) [hep-th/0103192].

[18] S. Ramgoolam, “On spherical harmonics for fuzzy spheres in diverse dimensions,” Nucl. Phys. B 610, 461 (2001) [hep-th/0105006].

[19] P. M. Ho and S. Ramgoolam, “Higher dimensional geometries from Matrix brane constructions,” Nucl. Phys. B 627, 266 (2002) [hep-th/0111278].

[20] S. Chaudhuri, “Nonperturbative type I-I' string theory,” hep-th/0201129.

[21] M. Bagnoud, L. Carlevaro and A. Bilal, “Supermatrix models for M-theory based on osp(1|32,R),” Nucl. Phys. B 641, 61 (2002) [hep-th/0201183].

[22] D. Berenstein, J. M. Maldacena and H. Nastase, “Strings in flat space and pp waves from N = 4 super Yang Mills,” JHEP 0204, 013 (2002) [hep-th/0202021].

[23] S. Chaudhuri, “Spontaneous breaking of diffeomorphism invariance in Matrix theory,” hep-th/0202138.

[24] T. Azuma and H. Kawai, “Matrix model with manifest general coordinate invariance,” Phys. Lett. B 538, 393 (2002) [hep-th/0204078].

[25] Y. Kimura, “Noncommutative gauge theory on fuzzy four-sphere and matrix model,” Nucl. Phys. B 637, 177 (2002) [hep-th/0204256].

[26] K. Dasgupta, M. M. Sheikh-Jabbari and M. Van Raamsdonk, “Matrix perturbation theory for M-theory on a PP-wave,” JHEP 0205, 056 (2002) [hep-th/0205183].

[27] P. Valtancoli, “Stability of the fuzzy sphere solution from matrix model,” hep-th/0206073.

[28] C. Sochichiu, “Continuum limit(s) of BMN matrix model: Where is the (nonabelian) gauge group?,” hep-th/0206239.
[29] S. Ramgoolam, “Higher dimensional geometries related to fuzzy odd-dimensional spheres,” hep-th/0207111.

[30] Y. Kitazawa, “Matrix Models in Homogeneous Spaces,” Nucl. Phys. B 642, 210 (2002) hep-th/0207115.

[31] K. Sugiyama and K. Yoshida, “Giant graviton and quantum stability in matrix model on PP-wave background,” hep-th/0207190.

[32] T. Eguchi and H. Kawai, “Reduction Of Dynamical Degrees Of Freedom In The Large N Gauge Theory,” Phys. Rev. Lett. 48, 1063 (1982).

[33] A. Gonzalez-Arroyo and M. Okawa, “The Twisted Eguchi-Kawai Model: A Reduced Model For Large N Lattice Gauge Theory,” Phys. Rev. D 27, 2397 (1983).

[34] A. Gonzalez-Arroyo and C. P. Korthals Altes, “Reduced Model For Large N Continuum Field Theories,” Phys. Lett. B 131, 396 (1983).

[35] J. Madore, “The Fuzzy sphere,” Class. Quant. Grav. 9, 69 (1992).