Thermodynamic Studies of Transcritical CO$_2$ and N$_2$O Combined Power Refrigeration Cycles

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Abstract

Objective: Researchers have started proposing the use of natural refrigerants like CO$_2$ and N$_2$O in place of CFCs and HCFCs. The cycles using these refrigerants operate in transcritical region. Thus, there is a scope of combined power generation with cooling. This has been analyzed in this work. Methods: The energetic and exergetic analysis are carried out on transcritical CO$_2$ and N$_2$O combined power and refrigeration cycles. The equations thus generated for COP and exergetic efficiency are studied for their behavioral dependence on various influencing parameters like evaporator temperature, gas cooler outlet temperature, gas heater pressure and turbine inlet temperature. In the entire analysis the standard property code is used in the program for evaluation of properties. Findings: For a comparison, the Grassmann diagrams for both CO$_2$ based and N$_2$O based combined power and refrigeration cycles have been presented at a given operating condition. It is noticed that the performances of transcritical N$_2$O combined power cycle is better than transcritical CO$_2$ combined power cycle as it has higher COP and exergetic efficiency under all operating conditions. It is also found that there exists an optimum evaporator temperature at which the exergetic efficiency attains maximum value. Specific exergy loss is found more for throttle valve and compressor. Application: The natural gas based combined power and refrigeration cycle can be used in large applications like cooling and heating in ships or large ice plants etc.

1. Introduction

Use of Natural refrigerants, hydrocarbon, water, carbon dioxide, ammonia etc. is on upfront since the two menaces; global warming and ozone depletion were identified due to use of halogenated refrigerants CFCs and HCFCs. In recent years CO$_2$ which is present in the atmosphere has been attracted the attention of researchers and is already in the path of revival as a formidable natural refrigerant.

The natural refrigerant CO$_2$ was revived and successfully demonstrated to use as a refrigerant in transport air conditioning with an altogether new approach to use it, transcritical cycle. Further, excellent thermo physical and heat transfer properties coupled with low cost, easy availability, non-toxicity, non-flammability characteristics make the CO$_2$ as an excellent alternate refrigerant. In$^4$ described the cycle modifications and system design issues with the use of CO$_2$as refrigerant and emphasized the advantageous side of CO$_2$, low-pressure ratio and high volumetric capacity that enables the CO$_2$ as a wiser choice as an appropriate alternative.

In transcritical vapour compression cycle, there exists an optimum as cooler pressure for a given gas cooler exit temperature at which the COP of the cycle is maximum$^{5,6}$. In$^7$ in their work established a correlation to determine this optimum gas cooler pressure for CO$_2$ based combined heating and cooling application cycle. Later$^8$ have successfully demonstrated the existence of
such a pressure in their work of optimization of a CO$_2$-C$_3$H$_8$ based binary refrigeration cycle. In$^2$ studied three different two-stage transcritical CO$_2$ heat pump cycles. They carried out simultaneous optimization of the intermediate pressure in intercooler and the pressure of heat rejection in gas cooler and suggested an empirical correlation to predict these pressures.

A novel combined power - refrigeration cycle is studied by$^{10}$ and optimized its thermal performance using ammonia-water binary mixture as a working fluid. Later$^{11}$ optimized a combined power refrigeration transcritical CO$_2$ cycle for maximum COP based on the steady state thermodynamic analysis. An exergy loss analysis is carried out by$^{12}$ using different mixtures such as (R-404 A, R 410-A, R 410-B and R 507). In$^{13}$ undertook a study on first law and second law analysis of a simple refrigeration transcritical carbon dioxide cycle using throttle valve and expander.

In$^{14}$ have presented a comparative study of CO$_2$ based power cycle and R-123 based Rankine cycle in which they have established that CO$_2$ based cycle with low grade waste heat as the source has higher efficiency as compared to the rankine cycle charged with R-123 as the working fluid.

The exergy analysis is an effective way to study of refrigeration and heat pump system to get the true insight of the system$^{15-18}$. In$^{19}$ have undertaken a theoretical research study to compare between two-stage transcritical N$_2$O based and CO$_2$ based cycles. Results are presented based on the cycle simulation simultaneous optimizing intercooler and gas cooler pressures.

The work of$^{20}$ is important to mention here. In his work he has exhaustively studied and compared CO$_2$ and N$_2$O based transcritical cycle with heat exchanger and optimized the compressor discharge pressure. He conclusively established that, as compared to CO$_2$ based transcritical cycle, consideration of internal heat exchanger yields much less improvement of COP and reduction in optimum gas cooler pressure in N$_2$O based cycle.

In$^{21}$ conducted a study of thermodynamic analysis of combined CO$_2$ power and cooling cycle. Heat recovery from exhaust gas has been considered in the power source of the combined cycle. They have reported an improvement of COP by 40% and very little change in the optimal gas cooler pressure.

The present work is aimed to carry out the theoretical analysis of transcritical N$_2$O combined refrigeration power cycle including the comparison with transcritical CO$_2$ combined refrigeration power cycle.

### 2. Mathematical Modeling

#### 2.1 Process Analysis of Combined Cooling and Power Cycle

Figure 1 presents the schematic sketch of a system which generates power along with cooling effect. The system effectively consists of components like two internal heat exchangers, turbine, gas heater and other conventional refrigeration cycle components. The thermodynamic cycle of such a system is presented in Figure 2.
The following assumptions have been made in this work:

- Heat exchange with ambient is negligible in the internal heat exchangers.
- The process of compression is irreversible.
- The expansion process in turbine is irreversible.
- Expansion process in throttle valve is constant isenthalpic.
- Evaporation, gas cooling, gas heating and heat exchanger processes are isobaric.

2.2 Specific Energy Analysis of Combined Power and Refrigeration Cycle

Refrigerating effect of evaporator:
\[
q_{\text{evap}} = h_6 - h_5
\]  
(1)

Work input to compressor:
\[
\dot{w}_{\text{comp}} = h_2 - h_1
\]  
(2)

Heat rejected in gas cooler:
\[
q_{\text{gc}} = h_{10} - h_3
\]  
(3)

Heat supplied in gas heater:
\[
q_{\text{gh}} = h_8 - h_7
\]  
(4)

Work output in turbine:
\[
\dot{w}_{\text{tur}} = h_8 - h_9
\]  
(5)

The compressor isentropic efficiency is expressed as:
\[
\eta_{\text{is.comp}} = \frac{h_{2s} - h_1}{h_2 - h_1}
\]  
(6)

The turbine isentropic efficiency is expressed as:
\[
\eta_{\text{is.tur}} = \frac{h_8 - h_9}{h_8 - h_{9s}}
\]  
(7)

The internal heat exchanger II effectiveness is expressed as:
\[
\varepsilon_{\text{hex II}} = \frac{h_7 - h_2}{h_9 - h_2}
\]  
(8)

2.3 Loss of Specific Exergy in Combined Power and Cooling Cycle

Loss of exergy in the compression process:
\[
\dot{i}_{\text{comp}} = T_o (s_2 - s_1)
\]  
(11)

Loss of exergy in the evaporator:
\[
\dot{i}_{\text{evap}} = T_o (s_6 - s_5) - \frac{T_o}{T_{\text{evap}}} (h_6 - h_5)
\]  
(12)

Loss of exergy in the throttle valve:
\[
\dot{i}_{\text{tv}} = T_o (s_5 - s_4)
\]  
(13)

Loss of exergy in the gas cooler:
\[
\dot{i}_{\text{gc}} = (h_{10} - h_3) - T_o (s_{10} - s_3)
\]  
(14)

Loss of exergy in the gas heater:
\[
\dot{i}_{\text{gh}} = T_o (s_8 - s_7) - \frac{T_o}{T_{\text{gh}}} (h_8 - h_7)
\]  
(15)

Loss of exergy in turbine:
\[
\dot{i}_{\text{tur}} = T_o (s_9 - s_8)
\]  
(16)

Loss of exergy in internal heat exchanger I:
\[
\dot{i}_{\text{hex I}} = T_o (s_1 + s_4) - (s_1 + s_4)
\]  
(17)

Loss of exergy in internal heat exchanger II:
\[
\dot{i}_{\text{hex II}} = T_o (s_7 + s_{10}) - (s_2 + s_9)
\]  
(18)

2.4 Specific Exergy Change in Combined Power and Refrigeration Cycle

Change of exergy in gas cooler:
\[
e_{\text{gc}} = (h_{10} - h_3) - T_o (s_{10} - s_3)
\]  
(19)

Change of exergy in the evaporator:
\[
e_{\text{evap}} = (h_6 - h_5) - T_o (s_6 - s_5)
\]  
(20)

Change of exergy in the compressor:
\[
e_{\text{comp}} = (h_2 - h_1) - T_o (s_2 - s_1)
\]  
(21)

Change of exergy in turbine:
\[
e_{\text{tur}} = (h_8 - h_9) - T_o (s_8 - s_9)
\]  
(22)
Change of exergy in the gas heater:

\[ e_{gh} = (h_8 - h_7) - T_0(s_8 - s_7) \]  \hspace{1cm} (23)

Actual energy supplied to the compressor:

\[ w_{act, comp.} = \frac{h_2 - h_1}{\eta_{mech}} \]  \hspace{1cm} (24)

\[ II \text{ law efficiency } (\eta_{II}) \text{ in } \%: \]

\[ \eta_{II} = \frac{\text{Exergy output}}{\text{Exergy input to the system}} \times 100 \% = \]

\[ \frac{e_{\text{evap}} + e_{\text{tur}}}{(w_{act, comp.} - w_{\text{tur}}) + e_{gh}} \times 100 \% \]  \hspace{1cm} (25)

The cycle working condition of combined power refrigeration cycles are listed in Table 1.

Table 1. Basic CO₂ and N₂O combined power cycles operating parameters

| Operating Parameter                          | Operating Value | Unit |
|---------------------------------------------|-----------------|------|
| Ambient Temperature (T₀)                   | 303             | K    |
| Evaporator Temperature for Simple cycle (T_{evap}) | 270             | K    |
| Super heat after evaporator for Combined cycle | 5 (Fixed Value) | K    |
| Gas cooler exit Temperature (Tₐ)            | 308             | K    |
| Gas Heater Pressure (P_{gh})                | 140             | Bar  |
| Gas Heater temperature (T_{gh})             | 460             | K    |
| Compression Isentropic efficiency for Simple and Combined cycles (η_{c, comp}) | 0.75             | ---  |
| Turbine isentropic efficiency for combined cycle (η_{t, tur}) | 0.8             | ---  |
| Mechanical efficiency of compressor for Simple and Combined cycles (η_{c, comp}) | 0.8             | ---- |

A computer code has been developed for the energetic and exergetic study of combined power and cooling cycles at different operating parameters. Employing property codes CO2PROP [7] and N2OPROP the subcritical and supercritical thermodynamic and transport properties of CO₂ and N₂O are estimated.

3. Results and discussion

The thermo dynamic performances comparison of the CO₂ and N₂O combined power and cooling cycle is presented based on the different operating parameters like evaporator temperature, outlet temperature of the gas cooer, pressure in gas heater and inlet temperature of turbine. The isentropic efficiency of compressor and turbine, the effectiveness of IHEX II and mechanical efficiency of the compressor are considered as 75%, 80%, 0.7 and as 80% respectively. In IHEX-I the refrigerant gas is superheated by 5° C. The gas cooler pressure is calculated as the geometric mean of inlet and outlet pressure of compressor. Component exergy percentage loss is defined as the exergy loss in the component to the total exergy loss in the cycle expressed in percentage.

3.1 Performances of CO₂ and N₂O Cycles Based on Evaporator Temperature

Figure 3 shows the effect of evaporator temperature on COP considering P = 170 bar, Tₘ = 500 K and Tₐ = 308 K. As evaporator temperature increases COP increases for both CO₂ and N₂O combined refrigeration and power cycles. However, the increase in COP found marginally better in CO₂ cycle in comparison with N₂O cycle at higher evaporator temperature.

Figure 3. Variation of COP with evaporator temperature for Tₐ = 308 K, Tₘ = 500 K and P = 170 Bar.

Figures 4 and 5 present the percentage exergy loss of components with evaporator temperature for both CO₂ and N₂O combined cycles. As evaporator temperature
increases the exergy loss percentage increases for both gas cooler and evaporator, whereas the exergy loss percentage decreases for throttle valve. The exergy loss in other components is marginal in both the cycles.

![Component exergy loss % for CO₂ cycle with evaporator temperature for T₃ = 308 K, T₈ = 500 K and P = 170 Bar.](image)

Figure 4. Component exergy loss % for CO₂ cycle with evaporator temperature for T₃ = 308 K, T₈ = 500 K and P = 170 Bar.

![Component exergy loss % for N₂O cycle with evaporator temperature for T₃ = 308 K, T₈ = 500 K and P = 170 Bar.](image)

Figure 5. Component exergy loss % for N₂O cycle with evaporator temperature for T₃ = 308 K, T₈ = 500 K and P = 170 Bar.

![Percentage Exergetic efficiency with evaporator temperature for T₃ = 308 K, T₈ = 500 K and P = 170 Bar.](image)

Figure 6. Percentage Exergetic efficiency with evaporator temperature for T₃ = 308 K, T₈ = 500 K and P = 170 Bar.

Figure 6 shows the effect of evaporator temperature on second law efficiency for both CO₂ and N₂O combined power cycles. It can be noticed that the exergetic efficiency initially increases and reaches maximum value of 35.31% and 36.48% for CO₂ and N₂O cycles respectively. Then it starts decreasing for both the cycles with further increase in evaporator temperature.

### 3.2 Effect of Gas Cooler Exit Temperature

Effect of gas cooler outlet temperature on COP is shown in Figure 7 for both the cycles. The COP decreases for both the cycles with gas cooler outlet temperature. However, the decrease in COP for CO₂ cycle is relatively less in comparison to N₂O cycle.

![Variation of COP versus outlet temperature of gas cooler for Tₑvap = 275 K, T₈ = 500 K and P = 170 Bar.](image)

Figure 7. Variation of COP versus outlet temperature of gas cooler for Tₑvap = 275 K, T₈ = 500 K and P = 170 Bar.

![Exergy loss percentage for CO₂ combined cycle components with gas cooler outlet temperature for Tₑvap = 275 K, T₈ = 500 K and P = 170 Bar.](image)

Figure 8. Exergy loss percentage for CO₂ combined cycle components with gas cooler outlet temperature for Tₑvap = 275 K, T₈ = 500 K and P = 170 Bar.

Component wise percentage exergy loss for CO₂ and N₂O combined refrigeration and power cycle is shown in
Figure 8 and 9, respectively. Throttle valve is the most sensitive for the exergy loss with increase in outlet temperature of gas cooler. The effect of gas cooler outlet temperature on exergy loss percentage found for other components are relatively less for CO₂ combined cycle. However, significant variation is observed for N₂O combined cycle.

3.3 Effect of Inlet Temperature of Turbine

Figure 11 shows the effect of turbine inlet temperature on COP for both the cycles. Varying turbine inlet temperature 460 K to 600 K, the turbine output increases in turn COP increases monotonically for both the cycles. It is observed that COP of N₂O cycle is significantly higher in comparison with CO₂ cycle.

Figure 12 presents the influence of inlet temperature of turbine on second law efficiency of the combined power-refrigeration cycles. The second law efficiency increases with inlet temperature of turbine for both the cycles. The variation of second law efficiency for N₂O cycle is more as compared to CO₂ cycle at all the chosen turbine inlet temperature.
3.4 Effect of the Pressure of Gas Heater

Gas heater pressure is a very important operating parameter, which is the exit pressure of the compressor. Figure 13 shows the effect of gas heater pressure on COP for both the cycles. COP increases with gas heater pressure for both the cycles. However, the rise of COP with N₂O combined cycle is faster in comparison to rise in COP with CO₂. The similar trend is observed for exergetic efficiency with gas heater pressure as shown in Figure 14.

![Figure 13. Cycles COP versus gas heater pressure for \(T_{\text{evap}} = 275\, \text{K}, T_{3} = 308\, \text{K} \) and \(T_{8} = 500\, \text{K}\)](image)

![Figure 14. Cycles exergetic efficiency versus gas heater pressure for \(T_{\text{evap}} = 275\, \text{K}, T_{3} = 308\, \text{K} \) and \(T_{8} = 500\, \text{K}\)](image)

3.5 Grassmann Diagram for CO₂ and N₂O Cycles

Figures 15 and 16 show of percentage exergy loss in components for CO₂ and N₂O combined cycles, respectively. It is defined the exergy loss in percentage in components with respect to exergy input (100 %) to the compressor. The operating conditions are chosen as \(T_{\text{evap}} = 275\, \text{K}, P = 175\, \text{bar}, T_{3} = 308\, \text{K} \) for both the cycles. It is found out 51.85 % exergy is recovered in case of CO₂ combined power cycle in turbine and evaporator while 53.69 % exergy recovered for N₂O combined power refrigeration cycle at turbine and evaporator.

![Figure 15. Grassmann diagram for CO₂ combined power cycle](image)

![Figure 16. Grassmann diagram for N₂O combined power cycle](image)

4. Conclusions

Transcritical CO₂ combined power cycle is compared with N₂O combined power cycle based on steady state thermodynamics analysis. Some of the conclusions are outlined below:

1. The thermodynamic performances like COP and second law efficiency of N₂O combined power cycle is better than CO₂ combined power cycle.
2. The variation of percentage exergy loss with evaporator temperature is more prominent in throttle valve for both, CO₂ and N₂O combined power refrigeration cycle.
3. Percentage exergy loss is observed relatively higher in throttle valve and compressor with respect to gas cooler exit temperature for both the cycles.
4. COP and second law efficiency are strongly influenced by gas heater, which increases with gas heater pressure.
for both cycles. It is noticed that the variation of COP and exergetic efficiency is more prominent in $N_2O$ cycle.

Nomenclature:

- $h$: specific enthalpy (KJ/kg)
- $q$: specific heat transfer (KJ/kg)
- $i$: specific exergy loss (KJ/kg)
- $T$: temperature (K)
- $e$: specific exergy (KJ/kg)
- $w$: specific work transfer (KJ/kg)
- $s$: specific entropy (KJ/kg.K)
- $P$: pressure (bar)
- IHEX: internal heat exchanger
- COP: coefficient of performance

Greek symbols:

- $\varepsilon$: effectiveness
- $\eta$: efficiency

Subscripts:

- 1 – 10: state points of refrigerant
- comp: compressor
- evap: evaporator
- tur: turbine
- gc: gas cooler
- comb: combined cycle
- cool: cooling cycle
- gh: gas heater
- tv: throttle valve
- is: isentropic
- mech: mechanical efficiency
- o: ambient
- II: second law
- ht: high temperature

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