COMPLEX MONOPOLES IN THE PATH INTEGRAL

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Complex monopole configurations dominate in the path integral in the Georgi-Glashow-Chern-Simons model and disorder the Higgs vacuum. No cancellation is expected among Gribov copies of the monopole configurations.

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1 Georgi-Glashow-Chern-Simons model

The Georgi-Glashow model is a $SO(3)$ gauge theory with a triplet Higgs scalar field $\vec{h}$ in which the gauge symmetry is spontaneously broken to $U(1)$ by the Higgs mechanism. The vacuum is ordered with nonvanishing $\langle \vec{h} \rangle \neq 0$.

In three dimensions instantons, or monopoles, disorder the Higgs vacuum; $\langle \vec{h} \rangle = 0$. Electric charges are linearly confined, forming an electric flux string. The model is dual to the Josephson junction system in the superconductivity.

Further the Chern-Simons term can be added to the Lagrangian. This defines the Georgi-Glashow-Chern-Simons model. The $U(1)$ gauge boson acquires a topological mass, and electric charges are screened.

How about the Higgs vacuum? Is the vacuum still disordered such that $\langle \vec{h} \rangle = 0$? In disordering the vacuum, monopole configurations play an important role. It has been argued in the literature, however, that monopole configurations would become irrelevant once the Chern-Simons term is added; monopole solutions would have infinite action, and for configurations of finite action their Gribov copies would lead to cancellation. We are going to show that this is not the case. There are complex monopole solutions of finite action, and Gribov copies do not lead to cancellation.

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2 Monopole ansatz

The most general form of the spherically symmetric monopole ansatz is

\[ h^a(\vec{x}) = \hat{x}^a h(r) \]
\[ A^a_\mu(\vec{x}) = \frac{1}{r} \left[ \epsilon_{a\mu
u} \hat{x}^\nu (1 - \phi_1) + (\delta_{a\mu} - \hat{x}_a \hat{x}_\mu) \phi_2 + r S \hat{x}_a \hat{x}_\mu \right] \]  

(1)

where \( \hat{x}^a = x^a/r \). The regularity of configurations at the origin and the finiteness of the action impose boundary conditions \((h, \phi_1, \phi_2) = (0, 1, 0)\) at \( r = 0 \) and \((h, \phi_1, \phi_2, S) = (v, 0, 0, 0)\) at \( r = \infty \).

Under a gauge transformation \( A \rightarrow \Omega A \Omega^{-1} + \Omega d \Omega^{-1} \) where \( \Omega = \exp \left\{ \frac{1}{2} f(r) \hat{x}^a \sigma^a \right\} \) and \( f(0) = 0 \),

\( \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos f & \sin f \\ -\sin f & \cos f \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \), \( S \rightarrow S - f' \)  

(2)

The Chern-Simons term, \( I_{CS} = -\left( \frac{i \kappa}{g} \right) \int \text{tr} (A \wedge dA + \frac{1}{3} A \wedge A \wedge A) \), is not gauge invariant; \( \delta I_{CS} = \left( \frac{4 \pi i \kappa}{g} \right) f(\infty) \). On \( S^3 \), \( f(\infty) \) is a multiple of \( 2 \pi \) so that the quantized Chern-Simons coefficient guarantees the gauge invariance. On \( R^3 \), however, there is a priori no reason to demand that \( f(\infty) \) be quantized.

3 Path integral and complex monopoles

In the path integral the gauge fixing condition is inserted;

\[ Z = \int \mathcal{D}A^a_\mu \mathcal{D}h \Delta_{FP}[A] \delta[F(A)] e^{-I} . \]

(3)

We look for configurations which extremize the action \( I \) within the subspace specified with \( F(A) = 0 \).

In the radial gauge \( S = 0 \) the extremization of the action leads to

\[ \phi_1'' + \frac{1}{r^2} \left( 1 - \phi_1^2 - \phi_2^2 \right) \phi_1 + i \kappa \phi_1' - \lambda h^2 \phi_1 = 0 \]
\[ \phi_2'' + \frac{1}{r^2} \left( 1 - \phi_1^2 - \phi_2^2 \right) \phi_2 - i \kappa \phi_2' - \lambda h^2 \phi_2 = 0 \]
\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dh}{dr} \right) - \lambda (h^2 - v^2) h - \frac{2}{r^2} (\phi_1^2 + \phi_2^2) h = 0 \]  

(4)

Since eq. (5) contains complex terms, solutions necessarily become complex.

Eq. (5) is solved by an ansatz \( \phi_1 = \zeta(r) \cosh \frac{1}{2} kr \) and \( \phi_2 = i \zeta(r) \sinh \frac{1}{2} kr \).

The solution is depicted in fig. 1. \( \phi_2(r) \) is pure imaginary. The action is real and finite. The \( U(1) \) field strengths are given exactly by those of a real magnetic monopole. Non-Abelian field strengths are complex. There is no Gribov copy in this gauge.

In the original form of the path integral, field configurations are integrated along real axes. We have found that the saddle points of \( I[A, h] \) are located
off the real axes. In the saddle point method for the integration, the integration path is deformed such that a new path pass the saddle points. The complex monopole configurations approximate the integral, and dominate the path integral. They are relevant in disordering the Higgs vacuum. Without monopole-type configurations the perturbative Higgs vacuum cannot be disordered and \( \langle \vec{h} \rangle \) remains nonvanishing. With complex monopoles taken into account \( \langle \vec{h} \rangle = 0 \) but \( \langle \vec{h}^2 \rangle \sim v^2 \).

We remark that if the gauge is not fixed and the action is varied with respect to arbitrary gauge field configurations, then one would obtain one more equation to be solved. This equation is not satisfied by our solution. But in the path integral the configuration space is restricted by the gauge condition as in (3). This subtlety arises due to the gauge non-invariance of the Chern-Simons term.

4 Gribov copies

The radiation gauge does not uniquely fix gauge field configurations. In the monopole ansatz the radiation gauge condition \( \partial_\mu A^a_\mu = 0 \) is maintained if \( f(r) \) in (3) obeys \( f'' + (2/r)f' - (2/r^2)\{\phi_1 \sin f + \phi_2 (1 - \cos f)\} = 0 \). Solutions to this equation define Gribov copies.

These copies have a significant effect in the Chern-Simons theory. The Chern-Simons term is not gauge invariant. Gribov copies carry an extra phase factor, \( \exp \{(4\pi i\kappa/g^2)f(\infty)\} \), which could lead to cancellation in the path integral.

Solutions \( f(r) \) are uniquely determined by \( f(0) = 0 \) and \( f'(0) \). In fig. 2 we have plotted \( f(\infty) \) as a function of \( f'(0) \) for the BPS monopole solution.
Figure 2. $f'(0)$ vs $f(\infty)$ for Gribov copies of the BPS monopole.

The range of the asymptotic value is $-3.98 < f(\infty) < +3.98$. It is quite unlikely that these Gribov copies of the BPS monopole lead to the cancellation $\sum e^{-4\pi i n f(\infty)/g^2} = 0$ in the presence of the Chern-Simons term. Monopole configurations remain important in the path integral.

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