Coupling the Superstring to a D-Brane
Ramond-Ramond Background

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ABSTRACT: We propose a new approach for coupling the type II superstring to the Ramond-Ramond background of D-branes in the RNS formalism, alternative to introducing RR vertex operators. It is based on the mixing between Ramond-Ramond p-form excitations in the closed string spectrum and transversally polarized excitations in the open string spectrum.

KEYWORDS: Superstrings and Heterotic Strings, Ramond-Ramond Background, RNS Formalism, D-branes.
1. Introduction

D-branes have played a crucial role in the recent development of string theory. For example, they have been important for realizing the duality relations between the five consistent string theories and have given rise to phenomenologically interesting models. For some of them and for the AdS/CFT correspondence, it is desirable to improve the understanding of string theory in the presence of the background fields of Dp-branes. The major difficulty is the handling of the Ramond-Ramond (RR) background. In the Ramond-Neveu-Schwarz (RNS) formalism, the usual method for coupling the string to a background field fails as superconformal invariance is spoiled if the RR vertex operators are used to deform the action.

There have been many approaches to this problem in recent years, for example, using different formalisms. In the RNS formalism, the interaction Lagrangian with RR vertex operators has been investigated in \[\text{[5]}\]. The approaches \[\text{[6, 7, 8, 9, 10]}\] are either based on the Green-Schwarz (GS) formalism or on a mixture between the RNS and the GS formalism, such as the hybrid formalism. In \[\text{[6]}\], the coset supermanifold \(\text{SU}'(2,2|4)/(\text{SO}(4,1) \otimes \text{SO}(5))\) is taken as target space for the GS superstring in the \(\text{AdS}_5 \otimes S^5\) RR background. The coupling to the RR background found this way agrees to some extent with the one constructed by performing double dimensional reduction and a T-duality transformation of the 11-dimensional supermembrane action coupled to the supergravity background \[\text{[11]}\]. The approach \[\text{[7]}\] in the hybrid formalism is based on the topological string theory in RNS variables and partly introduces GS variables. The RR vertex operators can then be used to deform the action of the free superstring to achieve the coupling to the RR background.

In this letter we propose a new way for coupling the superstring to the RR background of a flat D-brane in the RNS formalism, alternative to introducing RR vertex operators.

2. Coupling of 1-Forms to the RNS Superstring

In the bosonic string theory, the coupling of a 1-form background field is given by integrating the pullback of the 1-form \(A^{(1)}\) over the string boundary \(\partial \Sigma\). With the embedding \(X\) and the pullback \(X^{*}\), it can be written as

\[
\int_{\partial \Sigma} X^{*} A^{(1)} = \int_{\partial \Sigma} A^{(1)}(X(z, \overline{\sigma})) = \int_{\partial \Sigma} A^{(1)}_{\mu}(X(z, \overline{\sigma})) dX^\mu(z, \overline{\sigma})
\]

\[
= \int_{\partial \Sigma} A^{(1)}_{\mu}(X(z, \overline{\sigma})) \{\partial X^\mu d\sigma + \overline{\partial} X^\mu d\overline{\sigma}\}.
\]

This has to be modified if Dp-branes are present. In Cartesian coordinates and with \(i \in \{0, \ldots, p\}\) denoting indices parallel and \(t \in \{p+1, \ldots, 9\}\) indices transverse to the
\[ \int_{\partial \Sigma} A_i^{(1)}(X) \{ \partial X^i dz + \bar{\partial} X^i d\bar{z} \} + \int_{\partial \Sigma} \Phi_t(X) \{ \partial X^t dz - \bar{\partial} X^t d\bar{z} \}, \]  

(2.2)

where we have defined \( \Phi_t := A_i^{(1)} \). Supersymmetrizing equation (2.2), i.e. substituting the components \( X^\mu \) of the embedding by the superfields \( X^\mu = X^\mu + \theta \psi^\mu + \bar{\theta} \bar{\psi}^\mu + \theta \theta F^\mu \), the Dolbeaut operators \( \partial \) and \( \bar{\partial} \) by \( D = \partial_\theta + \theta \partial \) and \( \bar{D} = \partial_{\bar{\theta}} + \bar{\theta} \bar{\partial} \), and adding the integration over the complex fermionic coordinates \( \theta \) and \( \bar{\theta} \) yields the coupling

\[ \int_{\partial \Sigma} A_i^{(1)}(X) \{ D X^i dz d\theta + \bar{D} X^i d\bar{z} d\bar{\theta} \} \]

+ \[ \int_{\partial \Sigma} \Phi_t(X) \{ D X^t dz d\theta - \bar{D} X^t d\bar{z} d\bar{\theta} \} \]  

(2.3)

of a 1-form background field to the superstring in the RNS formalism.

3. Construction of the Interaction Lagrangian

The RR background of a Dp-brane is given by a \( p + 1 \) form potential \( C^{(p+1)} \). The translation symmetries which are unbroken by the flat Dp-brane are generated by the \( p + 1 \) vector fields \( V_0 := \frac{\partial}{\partial X^0}, \ldots, V_p := \frac{\partial}{\partial X^p} \). With the interior products \( i V_0, \ldots, i V_p \) and the exterior derivative \( d \) as tools, from \( C^{(p+1)} \) an interesting 1-form,

\[ i V_0 \cdots i V_p dC^{(p+1)} = i V_0 \cdots i V_p F^{(p+2)}, \]  

(3.1)

can be constructed. \( F^{(p+2)} \) is the \( p + 2 \) form field strength. The pullback of this 1-form can now be geometrically coupled to the string boundary as in section 2 or equivalently the closed 2-form \( d i V_0 \cdots i V_p dC^{(p+1)} \) can be coupled to the string itself. Supersymmetrizing and T-dualizing, as in section 2, yields

\[ \mathcal{L}_{\text{int}} = \int_{\partial \Sigma} F_{0\ldots pt}(X) \{ D X^t dz d\theta - \bar{D} X^t d\bar{z} d\bar{\theta} \}. \]  

(3.2)

We propose that this is a viable interaction Lagrangian for the superstring in the RR background of a flat Dp-brane.

In general, interaction Lagrangians can be constructed using the vertex operators corresponding to the desired background field. In the RNS formalism, RR vertex operators contain spin fields and this method leads to the loss of superconformal invariance. On the other hand, the interaction Lagrangian proposed in equation (3.2) can be viewed as being constructed from a vertex operator for an open string state polarized transversally to the Dp-brane. This can be seen more explicitly by writing (3.2) in component fields,

\[ \mathcal{L}_{\text{int}} = \int_{\partial \Sigma} dz F_{0\ldots pt}(X(z, \bar{z})) \{ \partial X^t(z) + \bar{\partial} \psi^t(z) \}, \]  

(3.3)
using the notation $F_0...pt(X)\partial_i := \partial_i F_0...pt(X)$. Thus, for the proposed interaction Lagrangian of equation (3.2) to be appropriate, the vertex operator for the transversally polarized open string should be a proper substitute for the RR vertex operator.

4. Mixing between States of the Open and the Closed String

It is known that a mixing between excitations in the open and closed string spectrum can occur. In order to investigate the relation between closed strings in RR states and open strings polarized transversally to a D-brane, we therefore consider the disk amplitude\(^1\) with the vertex operators

$$V_{RR}(q) = e^{-\frac{i}{2}\phi S^\alpha(z)} f_{\mu_1...\mu_{p+1}} \Gamma_{\alpha\beta}^{\mu_1...\mu_{p+1}} e^{-\frac{i}{2} \overline{S}^\beta(z)} e^{iqX(z,\overline{z})}, \quad (4.1)$$

$$V_{open}(k) = \xi_t e^{-\phi \psi^t} e^{i2k||X(w)}, \quad (4.2)$$

for the RR state in the $(-\frac{1}{2}, -\frac{3}{2})$ ghost picture ($S^\alpha$ are the spin fields) and the transversally polarized open string state in the $(-1)$ ghost picture, respectively. Using the doubling trick, this yields

$$\mathcal{A}(V_{RR}(q)V_{open}(k))^D = \left< \xi(z)c(z)\overline{z}V_{RR}^{(q)}(z,\overline{z})c(w)\overline{c}(w)V_{open}^{(k)}(w) \right>_D \sim f_0...pt \xi \delta(2q|| + 2k||). \quad (4.3)$$

Thus, open strings with a transverse polarization mix with closed strings in RR states with the polarization $f_0...pt \neq 0$. Therefore, the open string vertex operators can indeed act as a substitute for the RR vertex operators, which is an argument in favor of the validity of the interaction Lagrangian (3.2).

5. Conclusions

We have proposed an interaction Lagrangian for the superstring in the RR background of a flat Dp-brane, alternative to introducing the RR vertex operator. It has been constructed using the translation symmetries which are unbroken by the Dp-brane and can be viewed as effecting the coupling to the RR background by means of transversally polarized open strings. These states of the open string mix with the RR states of the closed string corresponding to the background of the Dp-brane, as shown in equation (4.3), providing the main argument for the proposed interaction Lagrangian.

The starting point for further investigations, the proposed complete action for the type II superstring in the RR background of a D-brane including the usual

\(^1\)A recent detailed calculation of more general disk amplitudes with RR vertex operators in the $(-\frac{1}{2}, -\frac{3}{2})$ ghost picture can be found in [12].
gravitational background $G_{\mu\nu}$ and the dilaton $\phi$, is thus given by

$$
S = \frac{1}{2} \int_\Sigma d^2z \ d^2\theta \ G_{\mu\nu}(X) \overline{D} X^\mu D X^\nu + \int_\Sigma d^2z \ \phi(X) R^{(2)}
$$
$$
+ \int_{\partial \Sigma} F_{0...pt}(X) \{ D X^t \ dz \ d\theta - \overline{D} X^t \ d\overline{\sigma} \ d\overline{\theta} \}. \tag{5.1}
$$

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References

[1] For an introduction see for example: J. Polchinski, *TASI lectures on D-branes*, hep-th/9611050.

[2] J. Maldacena, *The large N limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. 2 (1998) 231, Int. J. Theor. Phys. 38 (1998) 1113, (hep-th/9711200).

[3] S. S. Gubser, I. R. Klebanov, A. M. Polyakov, *Gauge theory correlators from non-critical string theory*, Phys. Lett. B 428 (1998) 105, (hep-th/9802109).

[4] E. Witten, *Anti-de Sitter space and holography*, Adv. Theor. Math. Phys. 2 (1998) 253, (hep-th/9802150).

[5] D. Berenstein, R. G. Leigh, *Quantization of superstrings in Ramond-Ramond backgrounds*, Phys. Rev. D 63 (2001), (hep-th/9910145).

[6] R. R. Metsaev, A. A. Tseytlin, *Type IIB superstring action in AdS(5) x S(5) background*, Nucl. Phys. B 533 (1998), 109 (hep-th/9805028).

[7] N. Berkovits, C. Vafa, E. Witten, *Conformal field theory of AdS background with Ramond-Ramond flux*, JHEP 9903 (1999), 018 (hep-th/9902098).

[8] D. Polyakov, *On the NSR formulation of string theory on AdS(5) x S**5**, Phys. Lett. B 470 (1999), 103 (hep-th/9812044).

[9] S. Naik, *The sigma model formulation of the type II string theory in the AdS(5) x S(5) backgrounds with Ramond-Ramond flux*, (2001), hep-th/0106116.

[10] N. Berkovits, *Covariant quantization of the superstring*, Int. J. Mod. Phys. A 16 (2001) 801, (hep-th/0008145).

[11] M. Cvetic, H. Lu, C. N. Pope, K. S. Stelle, *T-duality in the Green-Schwarz formalism, and the massless/massive IIA duality map*, Nucl. Phys. B 573 (2000), 149 (hep-th/0007202).

[12] H. Liu, J. Michelson, *-.Trek III: The search for Ramond-Ramond couplings*, (2001), hep-th/0107172.