Mechanism for a Bolt and Nut Self Loosening under Repeated Bolt Axial Tensile Load*

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Abstract
The mechanism for a bolt and nut self loosening under repeated bolt axial tensile load has yet to be clarified, despite much investigation into this phenomena. In this paper, the self loosening mechanism is derived from basic strength of materials equations, which results in the following conclusions. (1) When load is applied, a slip occurs on the screw thread surface, and the bolt shank twists clockwise, that is it descends on the lead angle of the screw thread. At the end of this process, counterclockwise restitution torque \( T_{S1} - \) is generated by the twisted angle of the bolt shank. However, there is no rotation of the nut. (2) When the load is released, if \( T_{S1} - \) exceeds the friction torque \( T_{W0} \) on the nut bearing surface generated by the decreased bolt axial tension, a slip occurs on the nut bearing surface, and the bolt and the mating nut rotate counterclockwise as one unit. To satisfy the relationship \( T_{S1} - > T_{W0} \), the maximum bolt axial tension \( F_1 \) must be larger than \( cF_0 \). Here \( F_0 \) is the minimum bolt axial tension and the coefficient \( c \) depends on the bolt shape and the friction coefficient. The rotational behavior of the bolt and the nut derived from this analysis concurs with experimental and FEM calculation results of other researchers. For steel joints, it is believed that rotational loosening rarely occurs when there is no separation between joined parts.

Key words: Fixing, Screw, Bolt, Nut, Loosening Mechanism, Tensile Load, Slip, Contact Surface, Friction

1. Introduction

Much research has been conducted into the self loosening mechanism of bolts and nuts for various loads applied to the bolt axis. The main loads are the shear load, torsional load and tensile load. The self loosening mechanism for a shear load joint has been investigated by Yamamoto et al(1), Sakai(2), Kasei et al(3) and Pai et al(4), and for torsional load joint by Sakai(5), and for tensile load joint by Goodier et al(6), Sato et al(7) and Izumi et al(8). In summary of selected papers on the self loosening mechanism for a tensile load joint, which is investigated here more precisely, the following has been noted. First, Goodier et al(6) pointed out that a radial micro slip is generated both on the screw thread surface and on the bearing surface during the bolt axial tension change as the result of the radial contraction of the bolt and the radial expansion of the nut by the amount of Poisson’s ratio deformation, and derived an equation for loosening torque. But, the self loosening mechanism was not considered. Sato et al(7) derived the equation to calculate the loosening angle of the bolt and nut based on the micro slip locus on the screw thread surface and on the bearing surface assuming that the descending micro slip occurs both when load is applied and released. Izumi et al(8) calculated the detailed loosening behavior of the bolt and nut under axial tension load using the three-dimensional finite element method, which provided very useful data for analysis of the self loosening mechanism, but did not clarify it in classical mechanics. The assumption by Sato that a descending micro slip occurs both as load is applied and released does not concur.
with Izumi’s calculation results.

In this paper, based on the strength of materials, the twisting or rotational behavior of the bolt and nut during axial loading and release are analyzed, and the self loosening mechanism is derived from basic strength of materials equations.

**Nomenclature**

- $F$: Bolt axial tension
- $F_0$: Initial bolt axial tension
- $F_1$: The maximum bolt axial tension
- $\alpha$: Flank angle
- $\beta$: Lead angle of thread
- $\phi$: Load factor
- $P$: Thread pitch
- $d_2$: Pitch diameter
- $d_W$: Equivalent friction diameter of the bearing surface
- $L$: Distance between the bearing surfaces of bolt and nut
- $\mu_S$: Friction coefficient on the screw thread surface
- $\mu_W$: Friction coefficient on the bearing surface
- $T_{S-}$: Descending clockwise torque that makes the bolt screw thread slip down on the screw thread surface at $F = F$ (Friction torque is subtracted.)
- $T_{S1-}$: Descending clockwise torque of the bolt at $F = F_1$ (Friction torque is subtracted.)
- $T_{SO+}$: Residual clockwise torque generated on the bolt by the initial tightening to $F = F_0$ (Friction torque is added.)
- $T_{ST}$: Torque to make the bolt screw thread slip up on the screw thread surface
- $T_W$: Friction torque on the bearing surface
- $\theta_0$: Twist angle of the bolt shank generated by the initial tightening to $F = F_0$
- $\theta_1$: Twist angle of the bolt shank at the end of loading, $F = F_1$
- $\theta_F$: Final residual angle of the bolt shank
- $\Delta \theta_R$: Counterclockwise returning angle of the bolt and the nut during the unloading process
- $\Delta \theta_L$: Loosening angle of the nut per one cycle

2. Theoretical analysis of the mechanism for self loosening under repeated bolt axial tensile load

2.1. Theoretical analysis model

A right-hand screw thread with a nut is shown in Fig.1. Under the condition that the bolt axis is fixed, if the nut rotates clockwise the bolt axial tension increases (the bolt is tightened), and if the nut rotates counterclockwise the tension decreases (the bolt loosens). On the other hand, under the condition that the nut is fixed, if the bolt axis rotates clockwise the bolt screw thread ridge goes down on the lead angle and the bolt loosens, whereas if the bolt rotates clockwise the bolt screw thread ridge goes up on the lead angle and the bolt tightens.

![Fig. 1 Nut with a right-hand thread](image)

Rotational direction is the view from the nut side.

Here, a triple-start thread with a large lead angle is shown to assist understanding.
counterclockwise, the bolt tightens. Here, the rotational direction is defined by the view from the nut side. When a fluctuating tensile load is applied to parts joined by a bolt and nut, the bolt axial tension varies with the applied load. Under these circumstances, the rotational loosening of a bolt and nut, that is, the descending slip of the bolt screw thread on the nut screw thread is being investigated based upon basic strength of materials equations. For this investigation, the period during which the bolt axial tension is increased by increasing the applied tensile load, is called the “loading process”, and the period during which the bolt axial tension is decreased, is called the “unloading process”.

2.2. Behavior during the loading process

When bolt axial tension $F$ is applied, a tangential force $W = F \tan \beta$ is generated between the bolt screw thread and the nut screw thread because the screw thread surface is inclined toward the plane perpendicular to the bolt axis by the lead angle $\beta$, and the bolt screw thread and the nut screw thread push (jostle) each other [Fig.2]. For a right-hand thread, the bolt is twisted clockwise, while the nut is pushed back counterclockwise. Namely, this tangential force tends to twist both the bolt and nut in their loosening directions.

During the loading process, it is important to first determine the location where the slip occurs, on the screw thread surface or on the bearing surface.

2.2.1. Theoretical study of slip surface

(a) Condition for slippage to occur only on the screw thread surface and not on the bearing surface

In order for the slip to occur on the screw thread surface, the tangential force $F \sin \beta$ on the screw thread surface must be larger than the friction force $\mu_S F \cos \beta / \cos \alpha$ on the screw thread surface. Here, $\mu_S$ is the friction coefficient on the screw thread surface, $\alpha$ is a flank angle.

$$F \sin \beta > \frac{F \cos \beta}{\cos \alpha} \mu_S$$

$$\therefore \mu_S < \frac{\tan \beta}{\cos \alpha}$$

Using the relationship $\tan \beta = \frac{P}{\pi d_2}$, Eq.(2) can be also expressed as follows ($P$ is the thread pitch, and $d_2$ is the pitch diameter).

$$\frac{P}{\pi} = \frac{d_2}{\cos \alpha} \mu_S > 0$$

According to Table 1 which shows the critical values $\mu_{S,c}$ of $\mu_S$ to satisfy Eq.(2), $\mu_S$ must be less than at least 0.05 to satisfy Eq.(2) (Here, thread sizes range from M6 to M20. This is applied to the following.). Hereafter calculation results are shown only when Eq.(2) is satisfied, that is $\mu_S \leq 0.05$.

Next, for the bolt screw thread to slip down on the screw surface, the torque $T_S$ must be smaller than the frictional resistant torque $T_W$ on the bearing surface (Otherwise, the slip would occur on the bearing surface).
Table 1  Critical friction coefficient $\mu_{Scr}$ and $\mu_{cr}$ [Eq.(2) and Eq.(6)]

| Dimensions of bolt and nut | Critical friction coefficient |
|----------------------------|------------------------------|
| Nominal thread designation | $d_w^*$                      |
|                            | For slippage to occur on screw thread surface | To satisfy self-supporting condition of screw thread |
| M6                         | 7.72                         | 0.0515 | 0.0229 |
| M8                         | 10.28                        | 0.0479 | 0.0214 |
| M10                        | 12.81                        | 0.0458 | 0.0206 |
| M12                        | 15.03                        | 0.0444 | 0.0202 |
| M14                        | 17.39                        | 0.0434 | 0.0199 |
| M16                        | 19.84                        | 0.0375 | 0.0173 |
| M18                        | 22.51                        | 0.0421 | 0.0192 |
| M20                        | 24.96                        | 0.0375 | 0.0172 |

*a*d_w : Equivalent friction diameter of bearing surface for a standard hexagon head with washer face. (This applies to the following tables.)

$$T_S = \frac{d_z}{2} \left( F \sin \beta - \frac{F \cos \beta}{\cos \alpha} \mu_S \right) \frac{1}{\cos \beta} < T_w = \frac{d_w}{2} F \mu_W$$  \hspace{1cm} (4)

$$\therefore \mu_W > \frac{1}{d_w} \left( \frac{P}{\pi} - \frac{d_z}{\cos \alpha} \mu_S \right)$$ \hspace{1cm} (5)

Here, $d_w$ is the equivalent friction diameter of the bearing surface, $\mu_W$ is the friction coefficient on the bearing surface. Eq.(5) is equal to Eq.(6), that is, the self-supporting condition of screw thread.

$$\frac{d_z}{\cos \alpha} \mu_S - \frac{P}{\pi} + d_w \mu_W > 0$$ \hspace{1cm} (6)

Therefore, if the self-supporting condition of screw thread is satisfied and the joined parts exist as one piece, Eq.(5) is always satisfied and no slip will occur on the bearing surface before a slip occurs on the screw thread surface. Assuming $\mu_S = \mu_W$, the critical friction coefficient $\mu_{cr}$ that satisfies the self-supporting condition is calculated and shown in Table 1 (The bearing surface is assumed to be a standard washer faced hexagon which is easily loosened.).

(b) Condition for slippage to occur on the bearing surface  Under this condition, the inequality sign of Eq.(5) is reversed and the self-supporting condition is not satisfied. Therefore, no slippage occurs on the bearing surface during the loading process in a normal bolted joint.

From the above, it can be concluded that slippage occurs only on the screw thread surface during the loading process in a normal joint.

2.2.2. Deformation of bolt and nut during the loading process  During the loading process, a nut is compressed by the force applied to the screw thread ridges causing it to contract axially and expand radially due to Poisson’s ratio effect. Similarly, Poisson’s ratio effect causes a bolt stretched by the increasing axial tension to elongate axially, and then contract radially. As a result, relative micro slippage occurs radially both on the screw thread surface and on the bearing surface\(^{(6)(9)(10)}\). During the unloading process, as both deformation directions are inverted, this radial micro slippage also occurs.

Therefore, if the bolt axial tension changes, the radial micro slippage is generated both on the screw thread surface and on the bearing surface.

2.2.3. Friction coefficient between surfaces which slide relative to each other  
(Concept of the friction circle)  If a small force $W'$ is applied to a body being pushed by another large force $W$ and is sliding to the same direction as $W$, the friction coefficient $\mu_{W'}$
in the direction of $W'$ becomes very small and approaches 0. The reason is as follows. $W$ is larger than the friction force when another small force $W'$ is applied (Here, for simplicity, $W'$ is perpendicular to $W$). If the resultant force of $W$ and $W'$ is greater than the friction force (a resultant force outside the friction circle. Fig.3(a)), slippage occurs in the direction of the resultant force. That means the small force $W'$ causes slippage in the direction of $W'$. As the apparent friction coefficient in the direction of $W'$ is calculated by $W'/F$ ($F$ is the vertical force to the sliding surface), if $W'$ is very small, the apparent friction coefficient in the direction of $W'$ becomes very small. This is called the friction circle concept.

Fig. 3 Friction circle

$W$ and $W'$ on the screw thread surface are shown in Fig.3(b). $W$ is the radial force generated by the radial micro slippage between the bolt and nut. $W'$ is the circumferential force generated by the descending clockwise torque $T_{S-}$. If the radial micro slippage between the bolt and nut occurs, the friction coefficient $\mu_S$ in the circumferential direction on the screw thread surface becomes very small.

2.2.4. Summary of behaviors during the loading process

From the above, when the bolt axial tension changes, radial micro slippage is generated on the screw thread surface, and the friction coefficient $\mu_S$ in the circumferential direction on the screw thread surface becomes very small. As a result, Eq.(2) holds, and descending slippage occurs on the screw thread surface during the loading process.

2.2.5. Condition to produce clockwise torsion on the bolt

When the bolt axial tension increases to $F_1$, after loading, descending clockwise torque $T_{S-1}$ is generated on the bolt. When torque $T_{S-1}$ is greater than the residual clockwise torque $T_{S0+}$, generated by the initial tightening to $F_0$, the bolt further twists clockwise.

$$T_{S1-} = \frac{F_1}{2} \left( \frac{P}{\pi} - \frac{d_2}{\cos \alpha} \mu_S \right) > T_{S0+} = \frac{F_0}{2} \left( \frac{P}{\pi} + \frac{d_2}{\cos \alpha} \mu_S \right)$$

(7)

$$\therefore \frac{F_1}{F_0} > \frac{P}{\pi} + \frac{d_2}{\cos \alpha} \mu_S$$

(8)

Therefore, if $F_1$ is larger than $aF_0 = F_{Tcr}$, a clockwise twisted deformation is added the bolt during the loading process.

2.2.6. Twist angle of bolt

When a bolt is tightened to axial tension $F_0$, the twist angle $\theta_{0+}$ of the bolt shank is expressed as follows.

$$\theta_{0+} = \frac{32L T_{S0+}}{\pi d^4 G_B} = \frac{16L F_0}{\pi d^4 G_B} \left( \frac{P}{\pi} + \frac{d_2}{\cos \alpha} \mu_S \right)$$

(9)

Here, $L$ is the distance between the bearing surfaces of the bolt and nut, $d$ is the diameter of the bolt shank, and $G_B$ is the modulus of rigidity of the bolt material. When the bolt axial tension is increased to $F_1$ during the loading process, the clockwise torque of the bolt is $T_{S1-}$. The twist angle $\theta_1$ of the bolt at $F_1$ can be expressed as follows.
For $F_1 \leq aF_0 = F_{Tcr}$ $(a > 1)$

$$\theta_1 = \theta_0,$$ \hspace{1cm} (10)

For $F_1 \geq aF_0 = F_{Tcr}$ $(a > 1)$

$$\theta_1 = \frac{32LT_{S1-}}{\pi d^4 G_B} = \frac{16L F_1}{\pi d^4 G_B} \left( \frac{p}{\pi} - \frac{d^2}{\cos \alpha} \mu_S \right)$$ \hspace{1cm} (11)

Therefore, the clockwise twist angle of the bolt shank increases by $\Delta \theta$ when the bolt axial tension increases from $F_0$ to $F_1$.

$$\Delta \theta = \theta_1 - \theta_0 = \frac{16L}{\pi d^4 G_B} \left( \frac{p}{\pi} (F_1 - F_0) - \frac{d^2}{\cos \alpha} \mu_S (F_1 + F_0) \right)$$ \hspace{1cm} (12)

2.3. Behavior during the unloading process

When the applied tensile force is decreased, the bolt axial tension also decreases to $F$. This leads to the occurrence of two possibilities. (a) When the bolt shank is twisted clockwise by $\theta_1$, at the point where axial force $F_1$ is applied, counterclockwise restitution torque $T_{S1-}$ is generated. On the other hand, $T_{ST}$ is required for the counterclockwise ascending slippage on the screw thread surface to occur when the bolt axial tension decreases to $F$. If $T_{S1-}$ overcomes the torque $T_{ST}$, the bolt slips against the screw thread surface and rotates counterclockwise (tightening rotation). (b) If $T_{S1-}$ overcomes the friction torque $T_W$ on the nut bearing surface at $F$, slippage occurs on the nut bearing surface and both the bolt and the nut rotate as a unit counterclockwise (In this case, there is no slippage against the screw thread surface and therefore no relative rotation between the bolt and nut occurs.). It is unknown which of these two cases, (a) or (b), will occur. Therefore, the mechanism is further investigated and discussed.

2.3.1. Investigation of slippage surface  
When the bolt shank is twisted clockwise by $\theta_1$, through application of axial force $F_1$, the counterclockwise restitution torque $T_{S1-}$ is as follows.

$$T_{S1-} = \frac{F_1}{2} \left( \frac{p}{\pi} - \frac{d^2}{\cos \alpha} \mu_S \right)$$ \hspace{1cm} (13)

Torque $T_{ST}$ and Torque $T_W$ are as follows.

$$T_{ST} = \frac{F}{2} \left( \frac{d^2}{\cos \alpha} \mu_S + \frac{p}{\pi} \right)$$ \hspace{1cm} (14)

$$T_W = \frac{F \mu_W d_W}{2}$$ \hspace{1cm} (15)

(a) Condition for ascending slippage to occur on the screw thread surface  
For bolt axial tension $F$, when the resistance torques $T_{ST}$ and $T_W$ under counterclockwise rotation on the screw thread surface and on the nut bearing surface are compared, $T_{ST}$ must be smaller than $T_W$ (Otherwise slippage occurs on the bearing surface.).

$$\therefore \mu_W > \frac{1}{d_W} \left( \frac{d^2}{\cos \alpha} \mu_S + \frac{p}{\pi} \right)$$ \hspace{1cm} (16)

In Table 2, the minimum values of $\mu_W/\mu_S$ for Eq.(16) to hold are shown. From Table 2, the relationship $\mu_W > 1.6\mu_S$ is required for the equation to hold. Assuming that $\mu_S$ and $\mu_W$ are independent and follow a normal distribution, and the mean values for both friction coefficients are equal, even when $\mu_S$ and $\mu_W$ scatter very widely (the variation coefficients of $\mu_S$ and $\mu_W$ are 0.10), the probability of Eq.(16) to hold resulting in an ascending slippage is 0.13% or less. This means that the ascending slippage on the screw thread surface, that is, tightening rotation of the bolt, rarely occurs during the unloading process for normal cases in which $\mu_S$ and $\mu_W$ are nearly equal.
Table 2 Critical friction coefficient ratio $\mu_w/\mu_s$ for slippage to occur on screw thread surface during the unloading process [Eq.(16)]

| Nominal thread designation | $d_w$ | $\mu_w/\mu_s$ when $\mu_s = 0.05$ | $\mu_s = 0.04$ | $\mu_s = 0.03$ | $\mu_s = 0.02$ |
|---------------------------|-------|-------------------------------|----------------|----------------|----------------|
| M6                        | 7.72  | 1.63                          | 1.83           | 2.18           | 2.86           |
| M8                        | 10.28 | 1.58                          | 1.78           | 2.10           | 2.74           |
| M10                       | 12.81 | 1.56                          | 1.75           | 2.06           | 2.68           |
| M12                       | 15.03 | 1.58                          | 1.76           | 2.07           | 2.69           |
| M14                       | 17.39 | 1.58                          | 1.76           | 2.06           | 2.67           |
| M16                       | 19.84 | 1.50                          | 1.66           | 1.93           | 2.46           |
| M18                       | 22.51 | 1.55                          | 1.72           | 2.02           | 2.61           |
| M20                       | 24.96 | 1.49                          | 1.65           | 1.91           | 2.45           |

In the shaded areas, either Eq.(2) or Eq.(6) is not satisfied.
(This applies to the following tables as well.)

When the mean value of $\mu_w$ is much greater than that of $\mu_s$ and Eq.(16) holds, to permit the tightening rotation of bolt, the next equation must hold.

$$T_{S1-} = \frac{F_1}{2} \left( \frac{P}{\pi} - \frac{d_2}{\cos \alpha} \mu_s \right) > T_{ST} = \frac{F}{2} \left( \frac{P}{\pi} + \frac{d_2}{\cos \alpha} \mu_s \right)$$

$$\therefore \frac{F}{F_1} < \frac{P/\pi - d_2 \mu_s / \cos \alpha}{P/\pi + d_2 \mu_s / \cos \alpha}$$

(17)

(18)

Table 3 shows the maximum values of $F/F_1$ to satisfy Eq.(18). When $\mu_s$ is rather large, in order to cause a tightening rotation of the bolt, the bolt axial tension must be decreased to a very small value $F$. In this case, though there is slippage on the screw thread surface, since there is no slippage on the bearing surface, the nut does not rotate and rotational loosening does not occur.

Table 3 Critical bolt axial tension ratio $F/F_1$ to rotate the bolt counterclockwise (ascending rotation) during the unloading process [Eq.(18)]

| Nominal thread designation | $\mu_s = 0.05$ | $\mu_s = 0.04$ | $\mu_s = 0.03$ | $\mu_s = 0.02$ |
|---------------------------|----------------|----------------|----------------|----------------|
| M6                        | 0.02           | 0.13           | 0.26           | 0.44           |
| M8                        | -0.02          | 0.09           | 0.23           | 0.41           |
| M10                       | -0.04          | 0.07           | 0.21           | 0.39           |
| M12                       | -0.06          | 0.05           | 0.19           | 0.38           |
| M14                       | -0.07          | 0.04           | 0.18           | 0.37           |
| M16                       | -0.14          | -0.03          | 0.11           | 0.30           |
| M18                       | -0.09          | 0.03           | 0.17           | 0.36           |
| M20                       | -0.14          | -0.03          | 0.11           | 0.30           |
(b) Condition for slippage to occur on the nut bearing surface

In this condition, the inequality sign of Eq.(16) is reversed.

\[ \mu_W < \frac{1}{d_W} \left( \frac{d_2}{\cos \alpha} \mu_S + \frac{P}{\pi} \right) \]  

(19)

Under the above-mentioned assumption, there is a 99.87% or greater probability that Eq.(19) is valid. And, the following equation must hold to slip and rotate the nut counterclockwise on the bearing surface, while the nut is supported by the bolt.

\[ T_{S1} - \frac{F_1}{2} \left( \frac{P}{\pi} - \frac{d_2}{\cos \alpha} \mu_S \right) > T_W = \frac{d_W \mu_W}{2} F \]  

(20)

\[ \therefore \frac{F}{F_1} < \frac{P/\pi - d_2 \mu_S / \cos \alpha}{\mu_W d_W} = b \]  

(21)

Assuming \( \mu_S = \mu_W \), the maximum values of \( F/F_1 = b \) to satisfy Eq.(21) are calculated and shown in Table 4. The critical bolt axial tension to cause the counterclockwise loosening rotation angle \( \Delta \theta_R \) of the nut is referred as \( bF_1 = F_{Kcr} \).

Table 4 Critical bolt axial tension ratio \( F/F_1 = b \) for counterclockwise rotation of bolt and nut to occur during the unloading process [In the case of \( \mu_S = \mu_W \). Eq.(21)]

| Nominal thread designation | \( d_W \) | \( \mu_S = 0.05 \) | \( \mu_S = 0.04 \) | \( \mu_S = 0.03 \) | \( \mu_S = 0.02 \) |
|-----------------------------|----------|-----------------|-----------------|-----------------|-----------------|
| M6                          | 7.72     | 0.02            | 0.23            | 0.57            | 1.26            |
| M8                          | 10.28    | -0.03           | 0.16            | 0.48            | 1.13            |
| M10                         | 12.81    | -0.07           | 0.12            | 0.43            | 1.05            |
| M12                         | 15.03    | -0.09           | 0.09            | 0.40            | 1.02            |
| M14                         | 17.39    | -0.11           | 0.07            | 0.38            | 0.99            |
| M16                         | 19.84    | -0.21           | -0.05           | 0.21            | 0.75            |
| M18                         | 22.51    | -0.13           | 0.04            | 0.34            | 0.93            |
| M20                         | 24.96    | -0.21           | -0.05           | 0.21            | 0.74            |

From the above mentioned fact, Eq.(19) holds in the usual case when \( \mu_S \) and \( \mu_W \) are nearly equal, and if Eq.(21) holds, the nut slips counterclockwise on the bearing surface supported by the bolt (There is no relative rotation between bolt and nut.).

2.3.2. Counterclockwise returning angle \( \Delta \theta_R \) of the bolt and the nut

When the counterclockwise restitution torque \( T_{S1} \) of the bolt is reduced to the friction torque \( T_W \) on the nut bearing surface, \( \Delta \theta_R \) is equal to the twisted angle caused by the counterclockwise torque \( (T_{S1} - T_W) \), and is derived as follows.

\[ \Delta \theta_R = \frac{32L}{\pi d^2 G_B} (T_{S1} - T_W) = \frac{16L}{\pi d^2 G_B} \left\{ F_1 \left( \frac{P}{\pi} - \frac{d_2}{\cos \alpha} \mu_S \right) - F_{\mu W} d_W \right\} \]  

(22)

2.3.3. Final residual angle \( \theta_F \) when bolt axial tension decreases to \( F \)

The final residual angle \( \theta_F \) is obtained by subtracting \( \Delta \theta_R \) from \( \theta_1 \) [Eq.(11)].

\[ \theta_F = \frac{32L}{\pi d^2 G_B} (T_{S1} - (T_{S1} - T_W)) \]

\[ = \frac{32L}{\pi d^2 G_B} (T_W) = \frac{16L}{\pi d^2 G_B} (F_{\mu W} d_W) \]  

(23)
\( \theta_F \) is equal to the clockwise angle which is maintained by the friction torque \( T_W \) on the bearing surface.

If \( F = F_0 \), \( \theta_F < \theta_{0+} \) holds because \( T_{ST} > T_W \) [Eq.(19)] holds when a slip occurs on the bearing surface. That is, the counterclockwise returning angle \( \Delta \theta_R \) during the unloading process is larger than the clockwise twist angle \( \Delta \theta \) during the loading process, and the final residual angle \( \theta_F \) is smaller than the initial twist angle \( \theta_{0+} \) at \( F = F_0 \).

\[
\theta_F(F = F_0) = \frac{16L}{\pi d^4 G_B} F_0 \mu W d_W = \theta(T_{WF+F_0})
\]

2.3.4. Condition for \( \theta_F \) to be 0  
From Eq.(23), the bolt axial tension \( F \) must be reduced to 0, so that \( \theta_F \) is completely eliminated.

2.4. Behavior after repeated loading and unloading

2.4.1. First cycle  
The relationship between bolt axial tension \( F \) and rotation angle \( \theta \) of the bolt and nut is shown in Fig.4 (the thick solid line indicates \( \theta \) for the bolt, and the broken line indicates \( \theta \) for the nut). During the loading process, when \( F \) is less than \( F_{Tcr} \), the twist angle \( \theta_B \) of bolt is the constant value \( \theta_0 + \) and there is no increase in rotation angle [line AB in Fig.4]. If \( F \) is greater than \( F_{Tcr} \), \( \theta_B \) increases according to Eq.(11) [line BC in Fig.4]. The nut does not rotate.

![Fig. 4 Relationship between bolt axial tension and rotation angle of the bolt and nut](image)

During the unloading process, when \( F \) is greater than \( F_{Rcr} \), neither the bolt nor the nut rotates [line CD in Fig.4]. If \( F \) is less than \( F_{Rcr} \), both the bolt and nut rotate counterclockwise as a unit according to Eq.(22) [line DE and D’E’ in Fig.4]. If \( F \) is reduced to \( F_0 \), the bolt rotates counterclockwise over \( \theta_{0+} \) and reaches \( \theta_F(F = F_0) \) [point E in Fig.4].

2.4.2. Behavior from the second cycle on under constant load change in axial tension between \( F_0 \) and \( F_1 \)  
At the end of the first cycle, the clockwise twist angle is maintained by the bearing surface friction torque \( T_W \) at \( F_0 \) and remains in the bolt. In this state, when \( F \) fluctuates between \( F_0 \) and \( F_1 \) repeatedly, the behavior of the bolt [line segments ABCDE shown in Fig.5(a)] is repeated in further cycles. At the same time, the counterclockwise rotation angle of the nut is accumulated cycle by cycle by the amount of \( \Delta \theta_R \) [The angle between D’E’ in Fig.5(a)]. These behaviors are shown in Fig.5(b).

If the clockwise torque \( T_{S1} \) at \( F = F_1 \) is less than the nut bearing surface friction torque \( T_{WF+F_0} \) at \( F = F_0 \), no relative rotation between the bolt and the nut is generated during the loading process. Under such circumstances, even if the applied load is repeated, rotational loosening does not occur. Accordingly, when the following equation for the relationship \( F_1 = cF_0 \) is satisfied, it indicates that no rotational loosening of the bolt and nut will occur.

\[
\frac{F_1}{2} \left( \frac{P}{\pi} - \frac{d_2}{\cos \alpha} \mu \right) \leq \frac{F_0}{2} \mu W d_W \\
\therefore \frac{F_1}{F_0} = \frac{\mu W d_W}{\left( \frac{P}{\pi} - \frac{d_2}{\cos \alpha} \mu \right)} = c = \frac{1}{b}
\]

(25)
The coefficient $c$ is the critical ratio of $F_1$ to $F_0$ for which there is no rotational loosening, and it is equal to the reciprocal of $b$ shown in Table 4.

Conversely, the condition for rotational loosening to occur under constant load change requires that the inequality sign of Eq.(25) be reversed, which results in the same equation as Eq.(21).

$$\frac{F_1}{F_0} > \mu W dW \left( \frac{F_1}{\pi} - \frac{d^2}{\cos \alpha} \mu_S \right) = c = \frac{1}{b} \quad (26)$$

### 2.4.3. Loosening angle $\Delta \theta_L$ per one cycle under constant load change

When the behavior shown in Fig.5(a) is repeated, the loosening angle $\Delta \theta_L$ per one cycle is derived by subtracting the residual angle $\theta_F(F = F_0)$ from the angle $\theta_1$ at $F = F_1$.

$$\Delta \theta_L = \theta_1 - \theta_F(F = F_0) = \frac{16L}{\pi d^2 G_B} \left\{ F_1 \left( \frac{P}{\pi} - \frac{d^2}{\cos \alpha} \mu_S \right) - F_0 \mu W dW \right\} \quad (27)$$

### 3. Comparison of test and FEM results

#### 3.1. Behavior during the loosening process

The behavior that the relative loosening rotation between the bolt and nut is generated during the loading process, and that the relative loosening rotation does not occur during the unloading process as shown in Fig.5 (b) concurs with Tsumura’s experimental results(11). Furthermore, the rotational behavior of the bolt and nut shown in Fig.5 (a) concurs with the FEM calculation results of the bolt and nut during the loading and unloading process presented by Izumi et al(8).

#### 3.2. Loosening angle

This analysis cannot determine the friction coefficient of fasteners during the actual loosening process, though it is assumed that this will be a very small value. Therefore, the friction coefficient resulting in the same loosening angle as that in Sato’s experiment(7) is calculated backward by Eq.(27), and the validity of this analysis is checked against the calculated friction coefficient value.

The friction coefficient that provides the same loosening angle as Sato’s experimental results, 0.0035/°cycle, is 0.033 as derived from Eq.(27). This analysis is judged to be appropriate, because this value of 0.033 satisfies both requirements from Eq.(2) ($\mu_S < 0.038$) and Eq.(6) ($\mu_S = \mu_W > 0.017$).
3.3. Value of $F_1/F_0$ for rotational loosening to occur

In both Sato's experiment(7) and Izumi's calculation(8), it is observed that rotational loosening occurs under the conditions: $M_20$, $F_1 = 29.4$ kN and $F_0 < 12.4$ kN. This means the value of $c = F_1/F_0$ is greater than 2.37. The friction coefficient which satisfies $F_1/F_0 > 2.37$ is more than 0.025 as understood from Eq.(26).

The above-mentioned friction coefficient of 0.033 satisfies this condition $\mu_S = \mu_W > 0.025$, and falls into the range that fulfills the following three conditions: $\mu_S < 0.038$, $\mu_S = \mu_W > 0.017$ and $\mu_S = \mu_W > 0.025$.

From the above, it can be concluded that there are no inconsistencies in this analysis.

4. Reason why rotational loosening rarely occurs in actual use

In actual use, rotational loosening rarely occurs(12)(13). The reason for this is considered in the following.

When bolt axial tensile load is applied to joints, if there is any separation between the joined elements, the bolt load sharing increases and fatigue failure of the bolt easily occurs. Therefore, it is a basic design feature to prevent any separation between joined elements. The condition that joined elements are not separated is expressed as follows. Here, $\phi$ is the load factor, $W$ is a tensile load applied to the joint and $c = F_1/F_0$.

$$F_1 - F_0 = \phi W, \quad F_0 > (1 - \phi)W$$

$$\therefore c < \frac{1}{1 - \phi} \quad (28)$$

For joined steel parts, as $\phi$ is generally less than 0.5, the condition that there is no separation of the joined parts is when $c < 2.0$.

According to Sato’s experimental results for M20(7) and M10(14), it is considered that the condition for rotational loosening to occur is when $c$ is larger than approximately 2. From the above mentioned fact, under the supposition that the joints are designed not to separate to avoid fatigue fracturing of the bolt, and the joined parts do not become separated in actual use, the conditions for rotational loosening are not satisfied, which explains why rotational loosening rarely occurs in actual use.

According to experiments by Sauer et al(15), rotational loosening occurs when the amplitude ratio (the ratio of fluctuating bolt axial tension to the mean bolt axial tension) is greater than 0.6. This condition corresponds to $c > 4.0$. Therefore, based on this thesis, it can be concluded that rotational loosening does not occur if the joined parts do not become separated, including the aluminum alloy structures, for which $\phi$ is generally less than 0.7.

5. Conclusions

The mechanism for self loosening of a bolt and nut under repeated axial tensile load was analyzed based on strength of materials equations, and the following conclusions were arrived at.

(1) During the loading process (bolt axial tension $F_0 \rightarrow F_1$)

As a result of radial micro slippage both on the screw thread surface and on the bearing surface, $\mu_S$ and $\mu_W$ become very small values in accordance with the friction circle concept. The descending force component of $F$ on the screw thread surface then exceeds the friction force and the bolt is twisted clockwise resulting in the twist angle of the bolt reaching to $\theta_1$ at $F = F_1$. Under the self-supporting condition, slippage does not occur on the nut bearing surface nor does the nut rotate.

(2) During the unloading process ($F_1 \rightarrow F_0$)

When the counterclockwise restitution torque $T_{31-}$, which is generated in the bolt axis by the clockwise twist angle $\theta_1$ during the loading process, exceeds the bearing surface friction torque $T_W$, as $F$ is decreased below $F_1/c$, slippage occurs on the nut bearing surface and both
the bolt and the nut rotate as a unit counterclockwise by $\Delta \theta_R$ [Eq.(22)]. In this circumstance, there is no relative rotation between the bolt and nut.

(3) Under constant change in axial tension ($F_0 \leftrightarrow F_1$)

If $F_1$ is larger than $c F_0$, the counterclockwise loosening angle $\Delta \theta_L$ [Eq.(27)] is generated on the nut cycle by cycle which is then accumulated. The rotational loosening condition is such that the maximum bolt axial tension $F_1$ must be greater than $c F_0$, here $F_0$ is the minimum bolt axial tension and $c$ is given by Eq.(26).

(4) If $\mu_S$ and $\mu_W$ are approximately 0.03, the results of this analysis concur with Sato’s experiments. And the rotational behaviors of the bolt and nut during the loosening process concur with Izumi’s FEM calculation results. From the above mentioned fact, it is concluded that the rotational loosening mechanism under repeated bolt axial tensile load has been clarified by this analysis.

(5) For steel joints, it is believed that rotational loosening rarely occurs if there is no separation between the joined parts.

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