MANY-BODY EFFECTS IN $^{16}$O($e, e'p$)

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Abstract

Effects of nucleon-nucleon correlations on exclusive ($e, e'p$) reactions on closed-shell nuclei leading to single-hole states are studied using $^{16}$O($e, e'p$)$^{15}$N (6.32 MeV, $3/2^-$) as an example. The quasi-hole wave function, calculated from the overlap of translationally invariant many-body variational wave functions containing realistic spatial, spin and isospin correlations, seems to describe the initial state of the struck proton accurately inside the nucleus, however it is too large at the surface. The effect of short-range correlations on the final state is found to be largely cancelled by the increase in the transparency for the struck proton. It is estimated that the values of the spectroscopic factors obtained with the DWIA may increase by a few percent due to correlation
effects in the final state.
I. INTRODUCTION

In the past decade the NIKHEF group [1] has accurately measured cross sections on closed-shell nuclei for exclusive \((e,e'p)\) reactions leading to low-energy states in the residual nucleus. Some of these states, denoted by \(|\Psi_h\rangle\), can be regarded as having a quasi-hole in a shell model orbital \(h\). The cross sections are in principle written in terms of the transition matrix element of the nuclear charge-current density operator \(\hat{J}^\mu\) between the initial state \(|\Psi_o\rangle\) describing the target nucleus and the final state \(|\Psi_f\rangle\), which asymptotically corresponds to an ejected proton leaving the residual nucleus in the state \(|\Psi_h\rangle\),

\[
J^\mu = \int d\vec{r} e^{i\vec{q} \cdot \vec{r}} \langle \Psi_f | \hat{J}^\mu(\vec{r}) | \Psi_o \rangle,
\]

where \(\vec{q}\) is the momentum transferred by the virtual photon.

Following the procedure of Ref. [2] it is possible to equivalently rewrite the transition matrix in a one-body representation corresponding to the specific channel selected by the experimental conditions. In momentum space one has

\[
J^\mu = \int d\vec{p} \psi^*_{p}(\vec{p} + \vec{q}) \tilde{J}^\mu(\vec{p}, \vec{q}) \psi_{h}(\vec{p})
\]

where the advantage of working in a one-body representation is paid for by the introduction of an effective charge-current density operator \(\tilde{J}^\mu\). The single-particle wave function

\[
\psi_{h}(\vec{p}) = \langle \Psi_{h} | a(\vec{p}) | \Psi_{o} \rangle
\]

is the overlap between the target state \(|\Psi_{o}\rangle\) and the hole state \(|\Psi_{h}\rangle\) produced by removing a particle with momentum \(\vec{p}\). It is characterized by the hole excitation energy \(E_{h}\) and an additional set of quantum numbers, which are here understood for simplicity. In the following it will be called a quasi-hole wave function. The normalization \(S(E_{h})\) of \(\psi_{h}\) is the spectroscopic factor, which measures the probability that the state \(|\Psi_{h}\rangle\) of the residual nucleus can indeed be considered as a pure hole generated in the target nucleus by the knockout process. A similar definition holds for the scattering state \(\psi_{p}\).
In principle, both wave functions are eigenfunctions of a Feshbach-like non-local Hamiltonian referred to the residual nucleus \[3\]. In practice, calculations have been limited to reactions leading to low-lying states which have a large overlap with single-hole states in the target nucleus. The spectroscopic factors \(S(E_h)\) for these states can often be identified with the quasi-hole renormalization constant \(Z\). In such cases a local energy-dependent mean-field complex potential \(V(E,r)\) is adopted for both bound and scattering states \[3\]. It is separately fitted to single-particle bound state properties and to proton-nucleus elastic-scattering data. The non-locality of the Feshbach Hamiltonian is taken into account by means of the Perey factor for both \(\psi_h\) and \(\psi_p\) \[4,5,6\]

\[
f_P(r) = \left(1 - \frac{\partial V(E,r)}{\partial E}\right)^{1/2},
\]

(1.4)

assuming a linear dependence of \(V\) on the energy. The quasi-hole \(\psi_h\) of Eq. (1.3) is thus approximated as

\[
\psi_h \sim \psi_{WS} = \sqrt{Z} f_P(r) \phi_{WS}(\vec{r}),
\]

(1.5)

where \(\phi_{WS}(\vec{r})\) is the wave function of the single particle state \(h\) at energy \(E_h\) in a Woods-Saxon potential \(V(E_h, r)\). The product function \(f_P(r)\phi_{WS}(\vec{r})\) is normalized to unity so that \(Z\) is the normalization of \(\psi_{WS}\).

The outgoing proton wave function is approximated as

\[
\psi_p = f_P \chi
\]

(1.6)

and is expanded in partial waves. A Schrödinger equation including \(V(E_p, r)\) is solved wave by wave for \(\chi\), where approximately \(E_p \sim E_h + \omega\). The observed cross sections are very well reproduced by varying \(Z\) and the parameters of the bound state Woods-Saxon potential \[7\].

In this method of analysis the \((e,e'p)\) reaction is essentially considered as a one-body process occurring in the average potential produced by the \((A - 1)\) nucleons of the residual nucleus in the state \(|\Psi_h\rangle\). Effects due to two-body currents in \(\hat{J}^\mu(\vec{p}, \vec{q})\) have been analyzed in Refs. \[8,9\]. If the mean-field approximation were to be exactly valid, then the quasi-hole function would have normalization \(Z = S(E_h) = 1\). Typically \(Z \sim 0.6\) is required to
reproduce the NIKHEF data on closed-shell nuclei from $^{16}\text{O}$ to $^{208}\text{Pb}$ [4]. This value of $Z$ is consistent with the observed quenching of single-particle contributions to several other nuclear properties [10].

Correlations between nucleons in the nucleus reduce the value of $Z$, and $1 - Z$ can be regarded as a measure of their strength [10]. The observed value of $Z$ suggests that correlations have a significant effect on the $(e,e'p)$ reaction. Ideally, many-body calculations should then be used to compute the $\psi_h, \psi_p$ and $\tilde{J}$ of Eq. (1.2).

In the past decade, improved models of the nuclear Hamiltonian of the form

$$H = -\frac{\hbar^2}{2m} \sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

(1.7) have become available. The two-nucleon interaction $v_{ij}$ is required to reproduce the nucleon-nucleon scattering data, and the three-nucleon interaction $V_{ijk}$ is chosen to reproduce the binding energies of light nuclei and the density of nuclear matter. In the present work we use the Argonne $v_{14}$ model of $v_{ij}$ [11] and the Urbana model VII of $V_{ijk}$ [12].

The ground state wave functions of $^3\text{H}$ and $^4\text{He}$ [13], and $^{16}\text{O}$ [14] have been obtained with this $H$ using the variational method. In the few-body nuclei, the variational $|\Psi_o\rangle$ appears to be fairly accurate from comparisons with exact Green’s function Monte Carlo calculations [15], and we can hope that they have useful accuracy for $^{16}\text{O}$. However, the $|\Psi_f\rangle$ is much more difficult to calculate from a realistic $H$. In order to study $^4\text{He}(e,e'p)^3\text{H}$, measured at NIKHEF [16] and Saclay [17], Schiavilla [18] used the approximation

$$|\Psi_f\rangle = \mathcal{A}\{\mathcal{S} \prod_{i=1,3} \mathcal{F}_{4i}\} |\Psi_o(^3\text{H}, x_1, x_2, x_3) \chi(x_4)\rangle + \text{orthogonality corrections.}$$

(1.8)

Here $x_i$ denotes $\vec{r}_i, \vec{\sigma}_i$ and $\vec{\tau}_i$ of the $i$-th nucleon, $\mathcal{F}_{4i}$ denote correlations between the outgoing nucleon and residual nucleons, and $\mathcal{A}$ and $\mathcal{S}$ are antisymmetrization and symmetrization operators. The $|\Psi_f\rangle$ is orthogonalized to $|\Psi_o\rangle$ boosted with momentum transfer $\vec{q}$. The $\chi(x_4)$ is determined from an optical potential, the correlation operator $\mathcal{F}_{4i}$ is estimated by using plane waves for particles 4 and $i$, and the transition matrix element is calculated with the Monte Carlo method.
Such a calculation may be possible for $^{16}$O($e,e'p$) reactions leading to the $p_{1/2}$ and $p_{3/2}$ hole states of $^{15}$N, for which cross sections have been measured by the NIKHEF group [19], using the available variational wave functions $|\Psi_h\rangle$ for these $^{15}$N states [20]. However, it is numerically much more difficult, and it will probably need an improved treatment of the correlation operator $F_{Ai}$ describing correlations between the struck, high-energy nucleon $A$ and the bound nucleons $i = 1, \ldots, 15$ in $^{15}$N.

In the present work we consider a less ambitious treatment of this reaction retaining a complete one-body charge-current operator [2] for $\tilde{J}$, using a many-body calculation for $\psi_h$ and considering some approximate many-body effects on $\psi_p$. Our two objectives are to test the variational wave functions for $^{16}$O and $^{15}$N and to estimate the effect of plausible correlations in the final state on the analysis of $(e,e'p)$ reactions.

The quasi-hole wave function, $\psi_h$, is calculated in Section II from the overlap of the variational wave functions of $^{16}$O and $^{15}$N using methods developed earlier for helium liquid drops [21]. In the interior of the nucleus this wave function is found to be very close to the $\psi_h$ obtained by fitting the NIKHEF data [19]. However, it is too large in the surface region, suggesting that the variational wave functions of Refs. [14,20] do not describe the nuclear surfaces very well.

In the traditional analysis of $(e,e'p)$, correlation effects on the initial state of the struck proton are included through the $Z$ and by varying the $\phi_{WS}$ to fit the data, but those on the final state are neglected. In Section III the effects of correlations in the final state are studied. We consider the following three modifications of the final state: (1) When the quasi-hole function $\psi_h$ is calculated from the overlap of $|\Psi_o\rangle$ and $|\Psi_h\rangle$, the Perey factor $f_p(r)$ cannot be calculated from Eq. (1.4). It is identified with the effective mass correction $\sqrt{m^*(r)/m}$ [22]. (2) The $\chi$ is multiplied by a factor $\sqrt{Z(\bar{r})}$, estimated with the local density approximation [21], to take into account the effect of short-range correlations between the struck proton and the residual nucleons. (3) The struck proton is a part of the $^{16}$O ground state. Hence, its final state interactions on the way out are driven by the ground state density weighted by the pair distribution function. This “correlation-hole” effect is observed
in inclusive \((e,e'p)\) \cite{23} and \((e,e')\) \cite{24} reactions, and it is estimated by multiplying the \(\chi(x)\) by a factor \(\lambda(\vec{r}')\) calculated from the pair distribution function.

II. THE QUASI-HOLE WAVE FUNCTION

The variational wave function used in Ref. \cite{14} to describe the ground state of \(^{16}\text{O}\) has the form

\[
|\Psi_o\rangle = \prod_{IT}(1 + U_{ijk})\{S \prod_{i<j}(1 + U_{ij})\}|\Psi_{J,o}\rangle, \quad (2.1)
\]

\[
|\Psi_{J,o}\rangle = \prod_{i<j}f_c(r_{ij})A|\Phi_o\rangle. \quad (2.2)
\]

Here \(|\Phi_o\rangle\) is an independent particle wave function, \(f_c(r_{ij})\) represents spatial pair correlations, the operators \(U_{ij}\) and \(U_{ijk}\) describe two- and three-particle spin, isospin, tensor and spin-orbit correlations, \(IT\) denotes a product of \((1 + U_{ijk})\) containing only independent triplets, and \(S\) and \(A\) denote symmetrization and antisymmetrization operators. The \(\Phi_o\) is a simple product of single-particle wave functions \(\phi_n(\vec{r}_i - \vec{R}_A)\) in the \(A\)-nucleon center of mass frame:

\[
\vec{R}_A = \frac{1}{A} \sum_{i=1}^{A} \vec{r}_i. \quad (2.3)
\]

It thus contains many-body correlations required to make \(|\Phi_o\rangle\), and therefore \(|\Psi_o\rangle\), translationally invariant.

Wave functions for single-hole states of \(^{15}\text{N}\) are approximately obtained by removing a nucleon from \(\Phi_o\) in the state \(h\). The single-particle wave functions of \(|\Phi_h\rangle\) are \(\phi_n(\vec{r}_i - \vec{R}_{A-1})\), where

\[
\vec{R}_{A-1} = \frac{1}{(A-1)} \sum_{i=1}^{A-1} \vec{r}_i, \quad (2.4)
\]

so that \(|\Phi_h\rangle\) is also translationally invariant. The two- and three-body correlations in \(^{15}\text{N}\) hole states are assumed to be the same as in \(^{16}\text{O}\) in this approximation. The \(|\Psi_{J,h}\rangle\) and \(|\Psi_h\rangle\) are obtained from \(|\Phi_h\rangle\) with equations like (2.1) and (2.2).
If the state that is removed from $\Phi_o$ has $\ell = 1$, $m_\ell = -1$ and $m_s = -1/2$, then the $|\Phi_h\rangle$ and $|\Psi_h\rangle$ have $J, M = 3/2, 3/2$. It was argued in [20] that this state should be associated with the centroid at $\sim 6.87$ MeV of the discrete $3/2^-$ strength in $^{15}\text{N}$. It was shown that it reproduces reasonably the $^{15}\text{N}(3/2^-) - ^{16}\text{O}$ energy difference and, with the corresponding $1/2^-$ state, the spin-orbit splitting in $^{15}\text{N}$. Here we use it to calculate the quasi-hole orbital.

The $\psi_h$ is function of $\vec{r} = \vec{r}_A - \vec{R}_A - 1$ and can be written as

$$
\psi_h(x) = \psi(r) Y_{ljs}(\hat{r}, \vec{\sigma}) = \langle \Psi_h|a(x)|\Psi_o \rangle
$$

(2.5)

$$
\psi_h(r) = \frac{\sqrt{A} (\Psi_h(x_1\ldots x_{A-1}) Y_{ljs}(x_A) \delta(r - |\vec{r}_A - \vec{R}_{A-1}|) |\Psi_o(x_1\ldots x_A) \rangle}{\langle \Psi_h|\psi_h \rangle^{1/2} \langle \Psi_o|\psi_o \rangle^{1/2}},
$$

(2.6)

where $Y_{ljs}$ are standard spin-angle functions, for example

$$
Y_{3/2 1/2 3/2}^{3/2} = Y_{11}(\hat{r}) |m_s = 1/2\rangle.
$$

(2.7)

The factor $\sqrt{A}$ in Eq. (2.6) takes into account the possibility that the removed particle can be any of the $A$ nucleons in $|\Psi_o\rangle$. The quasi-hole normalization $Z_h$ is given by

$$
Z_h = \int r^2 dr \psi_h^2(r).
$$

(2.8)

In momentum space the $\psi_h(p)$ is defined as

$$
\psi_h(p) = \sqrt{A} (\Psi_h(x_1\ldots x_{A-1}) \eta(p, x_A) |\Psi_o(x_1\ldots x_A) \rangle / \langle \Psi_o|\psi_o \rangle / \langle \Psi_h|\psi_h \rangle)
$$

(2.9)

where

$$
\eta(p, x_A) = j_\ell(p |\vec{r}_A - \vec{R}_{A-1}|) Y_{ljs}^{m}.
$$

(2.10)

The normalization of $\psi_h(p)$ is $Z_h \pi/2$.

In Monte Carlo calculations both $\psi_h(r)$ and $\psi_h(p)$ are simultaneously calculated in order to avoid Fourier transforms of data with sampling errors. For brevity we discuss the calculation of only $\psi_h(p)$. It is convenient to write Eq. (2.9) as

$$
\psi_h(p) = \sqrt{A} Q(p) / \sqrt{M}
$$

(2.11)
and evaluate $Q$ and $M$ using cluster expansions [14]. The overlaps $\langle O \rangle$ are defined as

$$\langle O \rangle = \frac{\langle \Psi_{J,h}(x_1...x_{A-1}) \eta(p,x_A) | O | \Psi_{J,o} \rangle}{\langle \Psi_{J,o} | \Psi_{J,o} \rangle}$$  \hspace{1cm} (2.12)$$

and calculated with the Monte Carlo method. The cluster expansion of $Q$ is then given by

$$Q = q_o + \sum_{i<j} q_{ij} + \sum_{i<j<k} q_{ijk} + ....$$  \hspace{1cm} (2.13)$$

We obtain

$$q_o = \langle 1 \rangle,$$  \hspace{1cm} (2.14)$$

$$q_{ij} = \frac{\langle (1 + U_{ij}^\dagger)(1 + U_{ij}) \rangle - q_o}{1 + d_{ij}^A} \quad \text{for } j \neq A,$$

$$= \frac{\langle U_{iA} \rangle}{1 + d_{iA}^A} \quad \text{for } j = A.$$  \hspace{1cm} (2.15)$$

The $d_{ij}^A$ are given by

$$d_{ij}^A = \langle (1 + U_{ij}^\dagger)(1 + U_{ij}) \rangle - 1$$  \hspace{1cm} (2.16)$$

for all $j$ and $i$. For the sake of brevity we do not give detailed expressions for $q_{ijk}$, $d_{ijk}^A$, etc.; they can be easily obtained from methods given in Ref. [14].

The ratio $M$ is written as

$$M = \frac{\langle \Psi_h | \Psi_h \rangle}{\langle \Psi_o | \Psi_o \rangle} = M_u M_J,$$  \hspace{1cm} (2.17)$$

and

$$M_u = \frac{\langle \Psi_h | \Psi_h \rangle}{\langle \Psi_{J,h} | \Psi_{J,h} \rangle} \frac{\langle \Psi_{J,o} | \Psi_{J,o} \rangle}{\langle \Psi_o | \Psi_o \rangle}$$  \hspace{1cm} (2.18)$$

is evaluated using a cluster expansion. Defining

$$d_{ij}^{A-1} = \frac{\langle \Psi_{J,h}(1 + U_{ij}^\dagger)(1 + U_{ij}) | \Psi_{J,h} \rangle}{\langle \Psi_{J,h} | \Psi_{J,h} \rangle} - 1,$$  \hspace{1cm} (2.19)$$

and $d_{ijk}^{A-1}$, etc., analogously we get
In practice we express $M_u$ as a sum of irreducible cluster terms as in Eq. (2.13). The ratio

$$M_J = \frac{\langle \Psi_{J,h} | \Psi_{J,h} \rangle}{\langle \Psi_{J,o} | \Psi_{J,o} \rangle}$$  

(2.21)

can be calculated exactly without cluster expansions. It is written as

$$M_J = \frac{\langle \Psi_{J,h} | \phi_h (\vec{r}_A - \vec{R}_{A-1}) \phi_h (\vec{r}_A - \vec{R}_{A-1}) | \Psi_{J,h} \rangle}{\langle \Psi_{J,o} | \Psi_{J,o} \rangle}$$  

(2.22)

for convenient Monte Carlo evaluation, using a normalized $\phi_h$:

$$\int |\phi_h(\vec{r})|^2 d\vec{r} = 1.$$  

(2.23)

The complete calculation of $\psi_h$ thus requires two separate Monte Carlo walks: (i) an $A$-body walk in which the $q$’s of Eq. (2.13), the $d^A$’s of Eq. (2.19) and the $M_J$ of Eq. (2.22) are calculated; and (ii) an $(A-1)$-body walk in which the $d^{A-1}$’s of Eq. (2.19) are calculated. Attempts to evaluate the $d^{A-1}$’s from the $A$-body walk led to larger sampling errors.

As discussed in Ref. [14], the optimal variational $|\Psi_o\rangle$ that minimizes the ground state energy does not give a good representation of the experimental $^{16}$O charge form factor. It is possible to reproduce the observed $^{16}$O charge form factor by changing only the one-body part $|\Phi_o\rangle$ of the optimum $|\Psi_o\rangle$. In this paper we are interested in the study of spatial wave functions with the $(e,e'p)$ reaction. It therefore seems reasonable to start with the wave function $|\Psi_o\rangle$ that gives an accurate description of the charge form factor and contains optimal correlation operators $f_c(r_{ij}), U_{ij}$ and $U_{ijk}$. The results presented in this paper are obtained with such $|\Psi_o\rangle$ and $|\Psi_h\rangle$. However, none of our conclusions change significantly when the optimal variational wave functions are used instead. The $|\Psi_o\rangle$ constrained to reproduce the observed charge form factor of $^{16}$O gives $\sim 5\%$ less binding energy than the optimal wave function [14].

The results for $\psi_h(p)$ are shown in Fig. 1. The dotted line shows $\phi_h^2(p)$, where $\phi_h$ is the single-particle wave function in $\Phi_o$. The norm of this wave function is, of course, unity. The
dash-dot curve shows the $\psi^2_{h,cm}(p)$ obtained when the center-of-mass effects are included, but no other correlations. In this approximation the $q_{ij}$, $d_{ij}^A$, $d_{ij}^{A-1}$, etc. are all zero, so that $M_u = M_J = 1$ and

$$Q(p) = q_{o,cm}(p),$$
$$\psi_{h,cm}(p) = \sqrt{A} q_{o,cm}(p).$$

(2.24)

A full $A$-body integration is necessary to calculate the $\psi_{h,cm}$. The $\psi^2_{h,cm}(p) < \phi^2_h(p)$ and its normalization is $Z_h = 0.88$. The dashed curve shows the further effect of including central correlations $f_c(r_{ij})$ only. In this approximation

$$\psi_{h,J}(p) = \sqrt{A} q_o(p) / \sqrt{M_J},$$

(2.25)

with $\sqrt{M_J} = 1.013(3)$ and $Z_h = 0.87$. Fig. 1 shows that the function $\psi_{h,J}(p)$ is rather close to $\psi_{h,cm}$. Note that the present calculation of $\psi_{h,cm}$ and $\psi_{h,J}$ from the chosen many-body wave function is exact.

At present it is necessary to use cluster expansions to calculate overlaps of wave functions with spin-isospin correlations. Two- and three-body cluster contributions are retained in the calculation of the $\psi_h(p)$ from the complete variational wave functions $\Psi_o$ and $\Psi_h$. The two-body terms reduce $Z_h$ by 0.09 to 0.78, and the three-body terms increase $Z_h$ by 0.04 to its present value of 0.82. We expect that a complete calculation will give $Z_h = 0.81 \pm 0.02$ with present wave functions. The $\psi^2_h(p)$ is shown by the full line in Fig. 1.

Müther and Dickhoff \[25\] have recently studied the quasi-hole orbitals in $^{16}$O using Brueckner theory. They neglect the effect of center-of-mass motion and obtain $Z_h = 0.91$ for the $p_{3/2}$ state. We obtain a similar value ($Z_h = 0.90$) by including $f_c(r_{ij})$, $U_{ij}$ and $U_{ijk}$ correlations, but no center-of-mass corrections. It is interesting that in a light nucleus like $^{16}$O the center-of-mass effects seem to account for almost half of the reduction of $Z_h$ from its unit value.

Most of the low-energy $p_{3/2}$ strength observed in $^{16}$O($e,e'p$) reactions goes to the $3/2^-$ states at 6.32, 9.93 and 10.7 MeV in $^{15}$N \[13\]. The state at 6.32 MeV has 86% of the total
strength in these three states. We assume that the $p_{3/2}$ quasi-hole state fragments due to mixing with other more complex states. The quasi-hole wave function

$$\psi_{6.32} \equiv \langle \Psi(6.32 \text{ MeV in } ^{15}\text{N})|a(x)|\Psi_o \rangle$$

is then given by

$$\psi_{6.32} = \sqrt{0.86} \psi_h,$$

assuming that the more complex states have negligible overlap with the state $a(x)|\Psi_o \rangle$. The empirical wave function $\psi_{WS}$, obtained by fitting the $(e,e'p)$ cross-section to the 6.32 MeV state with the parameterization (1.5), is therefore compared with $\sqrt{0.86} \psi_h$ in Fig. 2. We note that at small $r$ the calculated and fitted wave functions are very similar, however, at large $r$ the $\psi_h$ is too large. The $Z$ of $\psi_{WS}$ is 0.53, whereas the above many-body calculation gives

$$Z_{6.32} = 0.86Z_h = 0.70.$$  

These differences between the calculated and fitted wave functions are indicative of the limitations of the variational wave functions (2.1). They may not be able to describe the clustering of nucleons in the surface of the nucleus. Such a clustering can also lead to fluctuations in the shape of the nucleus.

We attempted to study the possibility of surface vibrations reducing the $Z_h$ by including a factor

$$\left\{ 1 + \sum_\ell \alpha_\ell \sum_{i<j} r_i^\ell r_j^\ell \sum_{m,m'} [Y_{\ell m}(\hat{r}_i) \times Y_{\ell m'}(\hat{r}_j)]_{J=0} \right\}$$

in the $^{16}\text{O}$ wave function. However, values of $\alpha$ that had a significant (few percent) effect on $Z_h$ resulted in less binding energy for $^{16}\text{O}$ and hence are variationally excluded.

The $(e,e'p)$ cross sections are very sensitive to the $Z$ and the radius of the quasi-hole wave function. In Fig. 3 for example the experimental momentum distribution for the $^{16}\text{O}(e,e'p)^{15}\text{N}$ leading to the $(3/2)^-$ state of $^{15}\text{N}$ at 6.32 MeV is shown. The proton is
ejected quasi-elastically with 90 MeV of kinetic energy in the center-of-mass system and with its momentum lying in the \( \vec{q} \) direction, i.e. in the so-called parallel kinematics [19]. By varying \( q \) itself, it is possible to extract the distribution of the missing momentum \( p_m \) (which is related to the momentum of the bound nucleon) keeping the ejectile energy fixed, i.e. keeping the final state interactions fixed.

The theoretical reduced cross sections obtained with \( \sqrt{0.86} \psi_h \) and \( \psi_{WS} \) are shown with the dashed and solid lines, respectively. The \( \psi_{WS} \) gives a much better description of data while \( \sqrt{0.86} \psi_h \) overestimates them at \( p_m < 200 \text{ MeV}/c \). For both cases the ejected proton wave function \( f_P \chi \) is given by the optical potential of Ref. [26] and the Coulomb distortion of electron waves has been taken into account through the effective momentum approximation [27].

### III. WAVE FUNCTION OF THE EJECTED PROTON

In this section we study the effect of three possible improvements in \( \psi_p(x = \vec{r}, \vec{\sigma}, \vec{\tau}) \), the wave function of the ejected proton. In the one-body representation it can be calculated, like the \( \psi_h(x) \) of Eq. (1.3), from the overlap

\[
\psi_p(x) = \langle \Psi_h | a(x) | \Psi_f \rangle, \tag{3.1}
\]

where \( |\Psi_f\rangle \) is the \( A \)-nucleon final state asymptotically corresponding to an outgoing proton of energy \( E_p \) leaving the nucleus in the \((A - 1)\)-nucleon state \( |\Psi_h\rangle \). In the analysis of NIKHEF data the \( \psi_p(x) \) is approximated by \( f_P(r)\chi(x) \) with a Perey factor consistent with the adopted phenomenological optical potential.

In mean-field (MF) approximation the outgoing distorted wave may be calculated from either a local energy-dependent, or a non-local momentum-dependent optical potential. The waves obtained from these two, respectively denoted by \( \chi(x) \) and \( \chi'(x) \), are approximately related by

\[
\chi'(x) = \sqrt{\frac{m^*(\vec{r})}{m}} \chi(x), \tag{3.2}
\]
where \( m^*(\vec{r}) \) is the effective mass of the outgoing proton at \( \vec{r} \). Outside the nucleus \( m^* = m \), \( \chi' = \chi \), and hence both give the same nucleon-nucleus scattering observables. However, inside the nucleus, where \( m^*(\vec{r}) \neq m \), only the \( \chi'(x) \) conserves the proton flux. Therefore \( \chi'(x) \) should be used to calculate the transition matrix element.

The problem with using \( \chi \), and the underlying physics of Eq. (3.2), is most easily seen by considering a beam of nucleons impinging on nuclear matter occupying the \( z > 0 \) half of space. As usual we have incident, reflected and transmitted waves denoted by \( A e^{ikz} \), \( B e^{-ikz} \) and \( C e^{ik'z} \) respectively, where \( k \) and \( k' \) are the momenta of the incident nucleons outside and inside nuclear matter. The flux conservation is given by

\[
|C|^2 = (|A|^2 - |B|^2) \frac{k}{k'} \frac{m^*(k')}{m},
\]

where \( m^*(k') \) is the effective mass in nuclear matter. When an energy-dependent local potential is used to describe the nucleon beam one obtains the incorrect flux conservation corresponding to \( m^*(k') = m \) inside matter.

The effective mass \( m^*(r) \), at energy \( E_p \), is given by the well known equation \[22\]

\[
\frac{m^*(r)}{m} = 1 - \frac{\partial V(E, r)}{\partial E} \bigg|_{E_p} \equiv f^2_P(r),
\]

which shows that the Perey factor is responsible for the flux conservation. When the \( V(E, r) \) is linear in \( E \) over the entire range \( E_h \leq E \leq E_p \) one recovers the approximation for \( \chi'(x) \) adopted in the analysis of NIKHEF data.

In infinite nuclear matter the effective mass depends upon the density \( \rho \) of matter and the energy of the proton, or equivalently the momentum \( k(E, \rho) \). The function \( m^*(k[E, \rho], \rho) \) has been recently studied using realistic nuclear forces \[28,23\]. Using the local-density approximation (LDA)

\[
m^*(\vec{r}) = m^*(k(E_p, \rho(\vec{r})), \rho(\vec{r})),
\]

where \( \rho(\vec{r}) \) is the density distribution of \(^{16}\text{O} \), we can replace the Perey factor by \( \sqrt{m^*(\vec{r})/m} \) in the \( \psi_p(x) \). This change has very little effect on the calculated cross-section in the NIKHEF kinematics as shown in Fig. 4.
In the studies of the quasi-hole orbitals in helium liquid drops [21], it was found that they could be related to MF orbitals with the LDA

$$\psi_h^{LDA}(x) = \sqrt{Z[\rho(\vec{r})]} \phi_h^{MF}(x) \approx \psi_h(x).$$  \hspace{1cm} (3.6)

The $\phi^{MF}(x)$ have a unit norm and are chosen so that

$$\sum_{\text{occupied } i} |\phi_i^{MF}(x)|^2 = \rho(\vec{r}).$$  \hspace{1cm} (3.7)

Interestingly the $f_P(r)\phi_{WS}(x)$ used to fit the NIKHEF data are very similar to the $p$-wave $\phi^{MF}(x)$ required to reproduce the $\rho$ of $^{16}\text{O}$. The $Z(\rho)$ in Eq. (3.0) is interpreted as the renormalization constant in matter at density $\rho$. The $\psi_h^{LDA}$ obtained with the linear expression

$$\sqrt{Z(\rho)} = 1 - \left(1 - \sqrt{Z_o}\right) \frac{\rho}{\rho_o},$$  \hspace{1cm} (3.8)

where $\rho_o$ is the equilibrium density (0.16 fm$^{-3}$) of matter and $Z_o = Z(\rho_o)$, is very close to the $\psi_h(x)$ for $Z_o = 0.64$, as shown in Fig. 2. This value of $Z_o$ is smaller than the $Z_o = 0.71$ estimated from detailed calculations [29] with the Urbana model of $v_{NN}$. However, most of the difference could be because the Argonne model of $v_{NN}$ used in the present work has a stronger tensor force. The $\sqrt{Z(\rho)}$ takes into account the reduction of the overlap wave function (Eqs. 2.5, 3.1) from its MF value due to short-range correlations.

Assuming that $\chi(x)$ is the MF wave function of the ejected proton, we obtain an improved approximation

$$\psi_p(x) = \sqrt{(m^*(\vec{r})/m)} Z[\rho(\vec{r})] \chi(x)$$  \hspace{1cm} (3.9)

with the LDA of Eq. (3.8). The $(e,e'p)$ cross sections obtained with this $\psi_p(x)$ are smaller by $\sim 10\%$ than those obtained with $f_P(r)\chi(x)$, as shown in Fig. 4. Naturally, if this $\psi_p$ is used to analyze the NIKHEF data the extracted values of the spectroscopic factors for the states in $^{15}\text{N}$ will be larger by $\sim 10\%$ than those given in Ref. [19].
The third improvement in $\psi_p$ is meant to take into account the difference between the imaginary potential $W(\vec{r})$ seen in nucleon-nucleus scattering and in the $(e,e'p)$ reaction. The $W(\vec{r})$ in nucleon-nucleus scattering can be regarded as

$$W(\vec{r}) = \frac{1}{2} \sigma_{\text{eff}}(\vec{r}) v(\vec{r}) \rho(\vec{r}), \quad (3.10)$$

where $\sigma_{\text{eff}}(\vec{r})$ is the effective $NN$ cross section, which can have spatial dependence due to density-dependent effects such as Pauli blocking,

$$v(\vec{r}) = \frac{\hbar k(\rho)}{m^*(k(\rho), \rho(\vec{r}))}, \quad (3.11)$$

is the local velocity, and the local momentum $k$ is given by

$$\frac{\hbar^2 k^2}{2m} = E_p - V(E_p, r). \quad (3.12)$$

In an $(e,e'p)$ reaction, let the ejected proton be struck at position $\vec{r}_o$ and time $t_o$. The distribution of nucleons of the residual nucleus at time $t_o$ is then given by the two-nucleon density $\rho_2(\vec{r}_o, \vec{r})$. Assuming that it does not change significantly in the time taken by the ejected nucleon to come out of the nucleus, the imaginary potential $W'(\vec{r})$ seen by the ejected proton is given by

$$W'(\vec{r}) = \frac{1}{2} \sigma_{\text{eff}}(\vec{r}) v(\vec{r}) \rho_2(\vec{r}_o, \vec{r}) \equiv \frac{1}{2} \sigma_{\text{eff}}(\vec{r}) v(\vec{r}) \rho(\vec{r}) g(\vec{r}_o, \vec{r}), \quad (3.13)$$

where $g(\vec{r}_o, \vec{r})$ is the pair distribution function. In practice we must differentiate between the $pp$ and $pn$ distribution functions as discussed in [23]. However, this difference is suppressed here for brevity. Due to the repulsive core in the $NN$ interaction and the Pauli exclusion, $g(\vec{r}_o, \vec{r}) < 1$ at small $|\vec{r}_o - \vec{r}|$. Therefore, transparencies calculated from $W'(\vec{r})$ are larger than those from $W(\vec{r})$ [23] in agreement with the data [30].

Let the $z$-axis be in the direction of the ejected proton and $\vec{r} = (\vec{r}_\perp, z)$. In the Glauber approximation the damping of the ejected proton wave, emerging from $\vec{r}_o$, is given by

$$D(\vec{r}_o, r \to \infty) = \exp \left\{ -\frac{1}{2} \int_{z_o}^{\infty} dz' \rho(\vec{r}') \sigma_{\text{eff}}(\vec{r}') \right\}$$

$$= \exp \left\{ -\int_{z_o}^{\infty} dz' \frac{W(\vec{r}')}{v(\vec{r})} \right\}, \quad (3.14)$$
when the nucleon-nucleus optical potential is used. However, the correct damping obtained from $W'(\vec{r}')$ is

$$D'(\vec{r}_o, r \to \infty) = \exp \left\{ - \int_{z_o}^{\infty} dz' \frac{W(\vec{r}')}{v(\vec{r}')} g(\vec{r}_o, \vec{r}') \right\}. \quad (3.15)$$

The effect of the increase in the transparency for $(e,e'p)$ reaction can be easily incorporated in the calculation of the cross section by modifying the distorted wave $\chi(x)$:

$$\tilde{\chi}(x) = \lambda(\vec{r}_\perp, z) \chi(x) \quad (3.16)$$

$$\lambda(\vec{r}_\perp, z) = \exp \left\{ + \int_{z_o}^{\infty} dz' \frac{W(\vec{r}')}{v(\vec{r}')}(1 - g(\vec{r}, \vec{r}')) \right\} \quad (3.17)$$

The $\psi_p(x)$ including the effects of increased transparency and the final state correlations, i.e. all three of the above improvements, is given by

$$\psi_p(x) = [Z(\vec{r})m^*(\vec{r})/m]^{1/2} \lambda(\vec{r}) \chi(x). \quad (3.18)$$

The cross sections obtained with it, shown in Fig. 4, are $\sim 3\%$ smaller than those obtained with $f_P(r)\chi(x)$. It thus appears that improvements in the wave function of the struck proton used in the analysis of the $(e,e'p)$ data may increase the extracted values of spectroscopic factors \cite{19} by a few percent.

### IV. CONCLUSIONS

Overlaps of variational nuclear wave functions that include realistic non-central correlations appear to give a reasonable description of the quasi-hole wave function in the interior of the nucleus. However, the resulting $\psi_h$ is too large in the surface. For a light nucleus like $^{16}\text{O}$, about half of the reduction of the $Z$ (the spectroscopic factor) from unity comes from center-of-mass effects. Approximate inclusion of correlation corrections to the optical model wave functions traditionally used to analyze $(e,e'p)$ reactions appears to increase the extracted spectroscopic factors by a few percent. Nevertheless, a calculation using presently available variational wave functions for $^{16}\text{O}$ and $^{15}\text{N}$ and these approximate corrections substantially overpredicts the observed cross sections.
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CAPTIONS

FIG. 1. Various approximations to the $J = 3/2$ quasi-hole $|\psi_h(p)|^2$. The dotted line is the simple $p$-wave single-particle wave function normalized to unity. The dashed-dot line includes also the center-of-mass corrections. The dashed line shows the further effect of central correlations, while the solid line is for the full wave function including the non-central correlations.

FIG. 2. Different models for the single-particle bound state wave function of the $p_{3/2}$ hole in $^{16}$O. The dashed line is for the quasi-hole $\sqrt{0.86} \psi_h$, the solid line is for the effective Woods-Saxon $\psi_{WS}$, the dotted line is for the LDA $\sqrt{0.86} \psi_{h}^{LDA}$ [see text and Eqs. (3.6), (3.8)].

FIG. 3. Theoretical reduced cross sections for the $^{16}$O($e,e'p$)$^{15}$N reaction leading to the $3/2^-$ state at 6.32 MeV. The proton is ejected with 90 MeV of center-of-mass energy in quasi-elastic parallel kinematic conditions. The solid line adopts the effective Woods-Saxon bound state $\psi_{WS}$ while the dashed line uses the quasi-hole $\psi_h$ (see text). The scattering state is from the optical potential of Ref. [26] and is multiplied by the proper Perey factor. The data are from Ref. [19].

FIG. 4. Theoretical reduced cross sections for the same reaction in the same kinematics as in Fig. 3. The solid line is the same as in Fig. 3. The other curves show the result for different modifications of the final scattering state. The dashed line neglects the Perey factor. The dotted line substitutes the equivalent effective mass correction for it. The long dash-dot line further adds the LDA correction [Eq. (3.9)] for short-range correlations. Finally, the short dash-dot line further adds the correction for the correlation-hole on the final state [Eq. (3.18)].
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FIG. 1
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\phi, \psi [\text{fm}^{-3/2}]
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