TRANSLATION-FINITE SETS

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ABSTRACT. The families of right (left) translation finite subsets of a discrete infinite group \( \Gamma \) are defined and shown to be ideals. Their kernels \( Z_R \) and \( Z_L \) are identified as the closure of the set of products \( pq (p \cdot q) \) in the \( \check{\text{C}} \)ech-Stone compactification \( \beta \Gamma \). Consequently it is shown that the map \( \pi : \beta \Gamma \rightarrow \Gamma^{WAP} \), the canonical semigroup homomorphism from \( \beta \Gamma \) onto \( \Gamma^{WAP} \), the universal semitopological semigroup compactification of \( \Gamma \), is a homeomorphism on the complement of \( Z_R \cup Z_L \).

INTRODUCTION

This note is an elaboration on the beautiful work of Ruppert \[3\] from 1985. Given a discrete infinite group \( \Gamma \) we define right and left versions of the combinatorial property (of subsets of \( \Gamma \)) of being translation finite. Then, using the ultrafilter representation of the \( \check{\text{C}} \)ech-Stone compactification \( \beta \Gamma \), we show that the collections of sets with these properties form ideals (Theorem 2.3). This yields a new proof of Ruppert’s theorem which asserts that the collection of translation finite sets forms an ideal. We then use these results to obtain some unexpected information about the map \( \pi : \beta \Gamma \rightarrow \Gamma^{WAP} \), the canonical semigroup homomorphism from \( \beta \Gamma \) onto \( \Gamma^{WAP} \), the universal semitopological semigroup compactification of \( \Gamma \) (Theorem 2.4).

1. THE \( C^* \)-ALGEBRAS \( \ell_\infty(\Gamma) \) AND \( WAP(\Gamma) \)

Let \( \Gamma \) be a countable discrete infinite group with unit element \( e \). We briefly review some basic properties of the \( C^* \)-algebras \( \ell_\infty(\Gamma) \), of bounded complex-valued functions on \( \Gamma \), and \( WAP(\Gamma) \), the closed subalgebra comprising the weakly almost periodic functions on \( \Gamma \). Recall that \( f \in \ell_\infty(\Gamma) \) is weakly almost periodic if its orbit under translations \( \{ f \circ \gamma : \gamma \in \Gamma \} \) is a weakly precompact subset of the Banach space \( \ell_\infty(\Gamma) \). We are mostly interested in their Gelfand (or maximal ideal) spaces: \( \beta \Gamma \), the \( \check{\text{C}} \)ech-Stone compactification of \( \Gamma \), and \( \Gamma^{WAP} \), the universal WAP-compactification of \( \Gamma \), respectively.
The compactification $\beta \Gamma$ can be viewed as the collection of ultrafilters on $\Gamma$, where an element $\gamma \in \Gamma$ is presented as the principal ultrafilter $e_\gamma = \{ A \subseteq \Gamma : \gamma \in A \}$. Then the left translation of an ultrafilter $q \in \beta \Gamma$ by $\gamma$ is the ultrafilter $\gamma q = \{ A \subseteq \Gamma : \gamma^{-1}A \in q \}$ (note that this extends the product on $\Gamma$ as $\gamma e_\delta = e_{\gamma \delta}$). These translations define a left action of $\Gamma$ on $\beta \Gamma$ and the resulting pointed dynamical system $(\beta \Gamma, e, \Gamma)$ is the universal ambit (or point transitive pointed system). That is, for any point transitive pointed $\Gamma$ dynamical system $(Y, y_0, \Gamma)$ there is a unique homomorphism of pointed dynamical systems $\pi : (\beta \Gamma, e, \Gamma) \to (Y, y_0, \Gamma)$.

This $\Gamma$ action on $\beta \Gamma$ can be extended to a multiplication on $\beta \Gamma$ as follows: for $p, q \in \beta \Gamma$

$$m_R(p, q) = pq = \{ A \subseteq \Gamma : \{ \alpha \in \Gamma : \alpha^{-1}A \in q \} \in p \}.$$  

This multiplication has the property that for each fixed $q \in \beta \Gamma$ the map $R_q : \beta \Gamma \to \beta \Gamma$, defined by $p \mapsto pq = m_R(p, p)$ is continuous. Thus this product makes $\beta \Gamma$ a right topological semigroup. It can be shown that this right topological semigroup can be identified with the enveloping semigroup $E(\beta \Gamma, \Gamma)$ of the dynamical system $(\beta \Gamma, \Gamma)$.

One can also define a left product on $\beta \Gamma$ by

$$m_L(p, q) = p \cdot q = \{ A \subseteq \Gamma : \{ \alpha \in \Gamma : A\alpha^{-1} \in p \} \in q \}.$$  

This extension of the product on $\Gamma$ to a product on $\beta \Gamma$ makes $\beta \Gamma$ a left topological semigroup, i.e. one in which the maps $L_q : \beta \Gamma \to \beta \Gamma$, defined by $p \mapsto q \cdot p = m_L(p, p)$, are continuous.

The remainder space of $\Gamma$ is the compact space $X := \beta \Gamma^* = \beta \Gamma \setminus \Gamma$. Clearly $X$ is a subsemigroup of $\beta \Gamma$ with respect to both right and left multiplications. We let $Z_R := \text{cls} X^2 = \text{cls} \{ pq : p, q \in X \}$ and $Z_L := \text{cls} X^{-2} = \text{cls} \{ p \cdot q : p, q \in X \}$. We also set $Z = Z_R \cup Z_L$.

As the algebra $C_0(\Gamma)$, comprising the functions on $\Gamma$ which vanish at infinity, is contained in the algebra $WAP(\Gamma)$ we deduce that $WAP(\Gamma)$ distinguishes points in $\Gamma$ and that consequently the natural compactification map of $\Gamma$ into $\Gamma^WAP$ is an isomorphism. We will therefore consider $\Gamma$ as a dense discrete subset of both $\beta \Gamma$ and $\Gamma^WAP$.

A dynamical system $(X, \Gamma)$ is called weakly almost periodic (WAP) if for every $F \in C(X)$, its orbit $\{ F \circ \gamma : \gamma \in \Gamma \}$ forms a weakly precompact subset of the Banach space $C(X)$. A theorem of Ellis and Nerurkar which is based on well known results of Grothendieck asserts that a system $(X, \Gamma)$ is WAP iff its enveloping semigroup $E(X)$ consists of continuous
maps, iff \( E(X) \) is a *semitopological semigroup* (that is, one in which both right and left multiplications are continuous). It then follows that the dynamical system \( \Gamma^{WAP} \) is the universal WAP point transitive dynamical system. Moreover, \( \Gamma^{WAP} \) is isomorphic to its own enveloping semigroup and is therefore also the maximal semitopological semigroup compactification of \( \Gamma \).

Let \( \pi : \beta \Gamma \to \Gamma^{WAP} \) denote the canonical homomorphism of the corresponding dynamical systems. With our identifications of \( \Gamma \) as a subset of both \( \beta \Gamma \) and \( \Gamma^{WAP} \) we have \( \pi(\gamma) = \gamma \) for every \( \gamma \in \Gamma \). We set \( Y := \Gamma^{WAP} \setminus \Gamma \). As a direct consequence of the discussion above we see that for every \( p, q \in \beta \Gamma \) we have \( \pi(pq) = \pi(p)\pi(q) \) and \( \pi(p \cdot q) = \pi(p)\pi(q) \). Consequently \( \pi(Z_R \cup Z_L) = \text{cls } Y^2 \). A result of our analysis shows that the restricted map

\[
\pi : \beta \Gamma \setminus Z \to \Gamma^{WAP} \setminus \text{cls } Y^2
\]

is a homeomorphism (Theorem 2.4 below). This extends results of Ruppert and Hindman and Strauss (see [2, Theorem 21.22]).

2. **Translation-finite sets**

2.1. **Definitions.**

1. Let \( Z_R = \text{cls } X^2 \subset X \). Set

\[
J_R = \{ A \subset \Gamma : \text{cls } A \cap Z_R = \emptyset \}.
\]

Set \( \mathcal{F}_R = \{ B \subset \Gamma : B^c \in J \} = \{ B \subset \Gamma : \text{cls } B \supset Z_R \} \). Clearly \( J_R \) is an ideal and \( \mathcal{F}_R \) is a filter.

2. Set \( Z_L = \text{cls } X^2 \subset X \), where \( X^2 = \{ p \cdot q : p, q \in X \} \). The ideal \( J_L \) and the filter \( \mathcal{F}_L \) are then defined as above with \( Z_R \) replacing \( Z_L \).

3. Let \( Z = Z_R \cup Z_L \subset X \). Set

\[
J = \{ A \subset \Gamma : \text{cls } A \cap Z = \emptyset \} = J_R \cap J_L.
\]

Set \( \mathcal{F} = \{ B \subset \Gamma : B^c \in J \} = \{ B \subset \Gamma : \text{cls } B \supset Z \} \). Clearly then \( J \) is an ideal and \( \mathcal{F} = \mathcal{F}_R \cap \mathcal{F}_L \) is a filter.

4. A subset \( A \subset \Gamma \) is called *right translation-finite* (RTF for short) if for every infinite \( D \subset \Gamma \) there is a finite \( F \subset D \) such that \( \cap_{\delta \in F} A\delta^{-1} \) is finite. We denote by \( J_{RTF} \) be the collection of RTF subsets of \( \Gamma \). We say that a subset \( B \subset \Gamma \) is *co-right-translation-finite* (CRTF) if \( B^c = \Gamma \setminus B \) is RTF and denote the collection of CRTF
sets by $\mathcal{F}_{RTF}$. Thus a subset $B \subset \Gamma$ is CRTF if for every infinite subset $D \subset \Gamma$ there is a finite subset $F \subset D$ such that $\cup_{\delta \in F} B\delta^{-1}$ is co-finite in $\Gamma$. These notions have obvious left analogues, LTF subsets of $\Gamma$, $\mathcal{J}_{LTF}$, etc. Following Ruppert we say that elements of $\mathcal{I}_{TF} := \mathcal{J}_{LTF} \cap \mathcal{J}_{RTF}$ are translation-finite sets (TF).

5. We let $\mathcal{I}_W$ be the collection of sets $A \subset \Gamma$ such that $\text{cls } A$ is an open subset of $\Gamma$ with $\text{cls } A \cap \text{cls } Y^2 = \emptyset$. Then $\mathcal{F}_W = \{ A^c : A \in \mathcal{I}_W \}$.

6. We say that $A \subset \Gamma$ is a $W$-interpolation set if $\text{cls } A \subset \Gamma$ is an open subset of $\Gamma$ which is homeomorphic to $\beta A$. We let $\mathcal{I}_{IW}$ denote the collection of $W$-interpolation sets, and let $\mathcal{F}_{IW} = \{ A^c : A \in \mathcal{I}_{IW} \}$.

Recall the following theorems of Ruppert (Theorem 7 and Proposition 13 in [3]).

2.2. Theorem. 1. $\mathcal{I}_{TF}$ is an ideal and $\mathcal{I}_{TF} = \mathcal{I}_W = \mathcal{I}_{IW}$.

2. Every infinite subset of $\Gamma$ contains an infinite TF subset.

Ruppert’s main tools in analyzing the TF property were the universal WAP compactification of $\Gamma$ and Grothendieck’s double limit characterization of WAP functions. Our approach is through the Čech-Stone compactification of $\Gamma$ and the combinatorial definition of the product of ultrafilters.

2.3. Theorem. 1. $\mathcal{F}_R = \mathcal{F}_{RTF}$, in particular $\mathcal{F}_{RTF}$ is a filter.

2. $\mathcal{F}_L = \mathcal{F}_{LTF}$, in particular $\mathcal{F}_{LTF}$ is a filter.

3. $\mathcal{F} = \mathcal{F}_{TF} = \mathcal{F}_{RTF} \cap \mathcal{F}_{LTF}$, hence

$$\mathcal{I}_{TF} = \mathcal{I}_{LTF} \cap \mathcal{J}_{RTF} = \mathcal{I}_L \cap \mathcal{J}_R = \mathcal{J} = \mathcal{J}_W = \mathcal{I}_{IW}$$

Proof. We prove the two inclusions of claim (1) below. The claim (2) then holds by symmetry and claim (3) is obtained by taking the appropriate intersections and applying Ruppert’s theorem.

Side 1: We first show that $\mathcal{F}_{RTF} \subset \mathcal{F}_R$. Consider $B \in \mathcal{F}_{RTF}$ and suppose $A \subset \Gamma$ has the property that there are $p, q \in X$ with $A \in pq$; i.e. $Ap^\leftarrow := \{ \gamma \in \Gamma : A\gamma^{-1} \in p \} \in q$. Then $|Ap^\leftarrow| = \infty$ and by assumption there is a finite subset $F \subset Ap^\leftarrow$ such that $\cup_{\delta \in F} B\delta^{-1}$ is cofinite in $\Gamma$. As $p$ is an ultrafilter this implies that for some $\delta \in F$ we have $B\delta^{-1} \in p$. Now, as both $B\delta^{-1}$ and $A\delta^{-1}$ are in $p$ so is $(A \cap B)\delta^{-1}$. In particular we conclude that
A \cap B \neq \emptyset. This discussion shows that for any two ultrafilters p, q in X their product pq is in cls B; hence cls B ⊃ Z_R, i.e. B ∈ \mathcal{F}_R.

**Side 2:** Next we show that \mathcal{F}_R \subset \mathcal{F}_{RTF}. Suppose then that A ⊂ \Gamma is not in \mathcal{F}_{RTF}; i.e. there is an infinite D ⊂ \Gamma such that for every finite F ⊂ D we have |(AF^{-1})^c| = |\Gamma \setminus \cup_{\delta \in F} A\delta^{-1}| = \infty. Clearly then the collection of sets of the form (AF^{-1})^c, with F ⊂ D finite, is a filter, say \mathcal{L}, on \Gamma. Choose some ultrafilter p ⊃ \mathcal{L}. Now choose an ultrafilter q with D ∈ q. We will show that A \not\in pq, whence A \not\in \mathcal{F}_R, as required.

Assuming A \in pq we have Ap^c = \{γ \in \Gamma : Aγ^{-1} \in p\} ∈ q. However if δ \in D then (Aδ^{-1})^c ∈ \mathcal{L}, hence (Aδ^{-1})^c ∈ p, hence Aδ^{-1} \not\in p, hence D^c ⊃ Ap^c ∈ q, hence D^c ∈ q. This is a contradiction and we conclude that indeed A \not\in pq. □

2.4. **Theorem.**

1. We have \pi^{-1}(\text{cls} Y^2) = Z = Z_R \cup Z_L, hence \pi^{-1}(Y \setminus \text{cls} Y^2) = X \setminus Z.

2. The restriction of \pi to the open dense subset X \setminus Z of X is a homeomorphism from X \setminus Z onto Y \setminus \text{cls} Y^2.

**Proof.**

**Step 1:** Given y \in U \subset (\Gamma^{WAP} \setminus \text{cls} Y^2), where y \in Y and U is an open subset of \Gamma^{WAP}, let V be an open subset of \Gamma^{WAP} such that y \in V \subset \text{cls} V \subset U. The set \tilde{V} = \pi^{-1}(V) is an open subset of \beta\Gamma such that cls \tilde{V} ∩ Z = \emptyset (since \pi is a homomorphism of semigroups we have \pi(X^2) = \pi(X^2) = Y^2, for both right and left semigroup structures on \beta\Gamma). Let A = \Gamma \cap \tilde{V}, then cls_{\beta\Gamma} A = cls \tilde{V} and therefore A ∈ J. By Theorem 2.3 we have A ∈ J_{TF} and then, by Theorem 2.2 A ∈ J_W. We conclude that cls_{\Gamma^{WAP}} A is a clopen neighborhood of y which is contained in U. Thus we have shown that the collection of sets of the form cls A with A ∈ J_{TF}, is a basis for the topology on \Gamma^{WAP} \setminus \text{cls} Y^2.

**Step 2:** If A is any set in J_{TF} then again by Theorem 2.2 A ∈ J_W = J_{IW} and we conclude that cls A is a clopen subset of \Gamma^{WAP} which is homeomorphic to \beta A. By the universality of \beta A it follows that \pi : cls_{\beta\Gamma} A → cls_{\Gamma^{WAP}} A is a homeomorphism.

**Step 3:** Again if A is any set in J_{TF} then, by Theorem 2.2 A ∈ J_W and we conclude that cls A is a clopen subset of \Gamma^{WAP}. We claim that \pi^{-1}(cls_{\Gamma^{WAP}} A) = cls_{\beta\Gamma} A. Clearly cls_{\beta\Gamma} A ⊂ \pi^{-1}(cls_{\Gamma^{WAP}} A). Conversely, if p \in \beta\Gamma with \pi(p) = y \in cls_{\Gamma^{WAP}} A, let p = \lim \gamma_\nu for a net \gamma_\nu ∈ \Gamma. Then y = \pi(p) = \lim \pi(\gamma_\nu) = \lim \gamma_\nu u and, as by assumption the set cls_{\Gamma^{WAP}} A is a clopen subset of \Gamma^{WAP}, it follows that eventually \gamma_\nu ∈ A. Thus we have p \in cls_{\beta\Gamma} A as claimed.
Step 4: By Proposition 13 of [3] (Theorem 2.2.2), every infinite subset $B \subset \Gamma$ contains an infinite subset $A \subset B$ with $A \in \mathcal{I}_F$. In view of step 1 above this shows that the set $Y \setminus \text{cls} Y^2$ is a dense open subset of $Y$.

Step 5: Summing up we have shown that (i) the collection of clopen sets $\{\text{cls}_\Gamma W A : A \in \mathcal{I}_F\}$ forms a basis for the topology on $\Gamma^W \setminus \text{cls} Y^2$, (ii) for each $A \in \mathcal{I}_F$, $\pi^{-1}(\text{cls}_\Gamma W A) = \text{cls}_\beta A$ and moreover (iii) $\pi : \text{cls}_\beta A \to \text{cls}_\Gamma W A$ is a homeomorphism. These facts together with the fact that $Y \setminus \text{cls} Y^2$ is a dense open subset of $Y$ prove the assertions of Theorem 2.4. □

3. Divisible properties, IP and D sets

In [1] a collection $\mathcal{P}$ of subsets of $\Gamma$ is called a divisible property if

(i) $\emptyset \notin \mathcal{P}$ and $\Gamma \in \mathcal{P}$,

(ii) $\mathcal{P}$ is hereditary upward (i.e. $A \in \mathcal{P}$ and $B \supset A$ imply $B \in \mathcal{P}$ and

(iii) if $A \in \mathcal{P}$ is a union $A = A_1 \cup A_2$ then at least one of the sets $A_1$ and $A_2$ is in $\mathcal{P}$.

A collection $\mathcal{P}$ is divisible iff the collection $\mathcal{I} = \{A \subset \Gamma : A \notin \mathcal{P}\}$ is an ideal iff the dual collection $\mathcal{F} = \mathcal{P}^* = \{A \subset \Gamma : A \cap B \neq \emptyset, \forall B \in \mathcal{P}\}$ is a filter. When $\mathcal{F}$ is a filter of subsets of $\Gamma$ the compact (nonempty) subset $K = \bigcap\{\text{cls} A : A \in \mathcal{F}\} \subset \beta \Gamma$ is called the kernel of $\mathcal{F}$. Conversely, any compact subset $K \subset \beta \Gamma$ defines a filter

$$\mathcal{F} = \{A \subset \Gamma : \text{cls} A \supset K\}.$$  

The correspondence $\mathcal{F} \leftrightarrow K$ is one to one and we note that

$$\mathcal{I} = \{A \subset \Gamma : \text{cls} A \cap K = \emptyset\} \quad \text{and} \quad \mathcal{P} = \{A \subset \Gamma : \text{cls} A \cap K \neq \emptyset\},$$

are the corresponding ideal and divisible properties respectively.

Expressed explicitly the divisible property which corresponds to the ideal of RTF-sets is the following one: a subset $A \subset \Gamma$ is not right translation finite, an NRTF-set, if there exists an infinite subset $D \subset \Gamma$ such that for every finite subset $F \subset D$ the corresponding intersection $\bigcap_{\delta \in F} A\delta^{-1}$ is infinite. NLTF-sets are defined similarly and a set $A$ is NTF if if there exists an infinite subset $D \subset \Gamma$ such that for every finite subset $F \subset D$ at least one of the two corresponding intersections $\bigcap_{\delta \in F} A\delta^{-1}$ and $\bigcap_{\delta \in F} \delta^{-1} A$ is infinite. In this terminology Theorem 2.3 is stated as follows:
3.1. **Theorem.** The properties NRTF, NLTF and NTF are divisible with corresponding kernels \( Z_R, Z_L \) and \( Z \) respectively.

Note however that the ideal \( I_W \) is not what we call in [1] the collection of interpolation sets of the algebra \( \text{WAP}(\Gamma) \), as in Definition 2.1.6 we postulate that \( A \in I_W \) when it is a \( \text{WAP}(\Gamma) \) interpolation set which additionally satisfies the requirement that \( 1_D \in \text{WAP}(\Gamma) \).

In [1] (Corollary 5.3.2) we have shown that the collection \( J \) of \( \text{WAP} \)-interpolation sets has the property that if \( \Gamma = \bigcup_{i=1}^{n} A_i \) then at least one of the sets \( A_i \) is not in \( J \). Let \( \Gamma_{\text{dis}}^{\text{WAP}} \) denote the universal totally disconnected semitopological compactification of \( \Gamma \). It is obtained as the quotient \( \Gamma_{\text{dis}}^{\text{WAP}}/\sim \) of \( \Gamma_{\text{dis}}^{\text{WAP}} \) by the equivalence relation: \( x \sim y \iff x \) and \( y \) lie in the same connected component. Let \( \text{WAP}_{\text{dis}}(\Gamma) \) denote the corresponding \( C^* \)-algebra.

3.2. **Problem.** (a) Is the collection of \( \text{WAP}(\Gamma) \)-interpolation sets an ideal?
(b) Is the collection of \( \text{WAP}_{\text{dis}}(\Gamma) \)-interpolation sets an ideal?

For simplicity let us assume next that \( \Gamma \) is abelian. We will denote the group operation by + but keep the notation \((p, q) \mapsto pq\) for the semigroup operation on \( \beta \Gamma \). Recall that a subset \( A \) of \( \Gamma \) is a \( D \)-set if there is an infinite sequence \( \{\gamma_i\}_{i=1}^{\infty} \subset \Gamma \) such that for every \( i \neq j \) at least one of the elements \( \gamma_i - \gamma_j \) or \( \gamma_j - \gamma_i \) is in \( A \). The subset \( A \) is called an \( IP \)-set if there is an infinite sequence \( \{\gamma_i\}_{i=1}^{\infty} \subset \Gamma \) such that for every finite sequence \( i_1 < i_2 < \cdots < i_n \) the element \( \gamma_{i_1} + \gamma_{i_2} + \cdots + \gamma_{i_n} \) is in \( A \). It is well known that Hindman’s theorem is equivalent to the fact that the collection of \( IP \)-sets is a divisible property with the set \( K = \text{cls}\{v \in X : v^2 = v\} \) (the closure of the set of idempotents in \( X \)) as its kernel. Obviously \( K \subset Z \). It is easy to see that every \( IP \)-set is also a \( D \)-set.

The filter which corresponds to the \( IP \)-sets is the collection of \( IP^* \)-sets:
\[
\{A \subset X : \text{cls} A \supset K\} = \{A \subset \Gamma : A \cap B \neq \emptyset, \forall \text{ IP-set } B\}.
\]

Similarly the filter which corresponds to the \( D \)-sets is the collection of \( D^* \)-sets:
\[
\{A \subset X : \text{cls} A \supset K\} = \{A \subset \Gamma : A \cap B \neq \emptyset, \forall \text{ D-set } B\}.
\]

The fact that the collection of \( D \)-sets is a divisible property is equivalent to Ramsey’s theorem and in [1] we have identified the kernel of this divisible property as the following closed subset \( L \subset X \). Define the set \( V \subset X \) as follows: \( p \in X \) is in \( V \) iff there is an element \( q \in X \) and a net \( \gamma_\alpha \) in \( \Gamma \) such that \( \lim \gamma_\alpha = q \) and \( p = \lim \gamma_\alpha^{-1} q \). Now put \( L = \text{cls} V \).
It is easy to see that $V \subset X^2$, whence $L \subset Z$. Thus the identifications of the kernels $K$ and $L$, together with Theorem 2.3 immediately lead to the following corollary.

3.3. Corollary. Every CTF-set (i.e. the complement of a TF-set) is a $D^*$-set and a fortiori an IP$^*$-set.

References

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