One-dimensional bound states in the continuum in the $\omega$-$k$ space for nonlinear optical applications

Kaili Sun$^{1,\#}$, Hui Jiang$^{1,\#}$, Dmitry A. Bykov$^2$, Vien Van$^3$, Uriel Levy$^4$, Yangjian Cai$^1$ and Zhanghua Han$^{1,*}$

1. Shandong Provincial Key Laboratory of Optics and Photonic Devices, Center of Light Manipulation and Applications, School of Physics and Electronics, Shandong Normal University, Jinan 250358, China
2. Image Processing Systems Institute — Branch of the Federal Scientific Research Centre “Crystallography and Photonics” of Russian Academy of Sciences, 151 Molodogvardeyskaya st., Samara 443001, Russia
3. Department of Electrical and Computer Engineering, University of Alberta, Edmonton T6G 2V4, Canada
4. Department of Applied Physics, and the center for Nanoscience and Nanotechnology, The Hebrew University of Jerusalem, Jerusalem, Israel

Email: zhan@sdnu.edu.cn

# These authors contribute equally to this work.

The phenomenon of bound state in the continuum (BIC) with infinite quality factor and lifetime has emerged in recent years in photonics as a new tool of manipulating light-matter interactions. However, most of the investigated structures only support BIC resonances at very few discrete positions in the $\omega$-$k$ space, restricting their applications in many fields where random or more input frequencies than the number of supported BIC resonances are required. In this work, we demonstrate that a new set of BIC resonances can be supported by making use of a special composite grating consisting of two ridge arrays with the same period and zero-approaching ridge width difference on a slab waveguide. These BIC resonances are distributed continuously over a broad band along a line in the $\omega$-$k$ space and can thus be considered as one-dimensional BICs. With a slight increase of the width difference, these BICs will switch to quasi-BIC modes and it is possible to choose arbitrarily any frequencies on the dispersion line to achieve significantly enhanced light-matter interactions, facilitating many applications where multiple input wavelengths are required, e.g. sum or difference frequency generations in nonlinear optics.
1. Introduction

The concept of bound states in the continuum (BIC) was first proposed in quantum mechanics by von Neumann and Wigner in 1929 [1], predicting the existence of localized eigenstates of the single-particle Schrödinger equation embedded in the continuum of eigenvalue state solutions. This counterintuitive observation is of fundamental importance in quantum mechanics. Over the years, the phenomenon of BIC has also been popularized and extensively studied in various fields, like acoustics [2] [3], electronics [4] [5] [6] and microwaves [7] [8]. In 2008, the concept of BIC was further extended to optical systems for the first time by D. C. Marinica and A. G. Borisov [9], and has ever since become a new approach of enhancing light-matter interactions. The main idea of BIC is the elimination of the coupling between the resonant modes and all radiation channels in the surrounding space. There are two main approaches of achieving BICs. The first is the symmetry-protected BIC, which typically exists at the Γ point in the reciprocal space and it is based on the symmetry incompatibility between the bound states and the continuum. A bound state of one symmetry class can be embedded in a continuum of another orthogonal symmetry class, and their coupling is forbidden when there is a zero overlap integral between the modes of different symmetry properties. The other type is due to the accidental disappearance of the coupling coefficient with the radiated waves by a successive tuning of one or more system parameters resulting in the formation of the so-called accidental BIC, which is typically observed away from the Γ-point and is also known as off-Γ BIC. The accidental BICs can be explained by the destructive interference of two or more leaking waves, where the radiation from the leaking waves is tuned to cancel each other out completely, a mechanism also known as the Friedrich-Wintgen scenario [10]. Two resonances are usually involved in the accidental BIC and due to the strong coupling between the two resonances, an avoided crossing behavior is usually accompanied [11].

In optics and photonics, researchers have realized symmetry-protected BICs and accidental BICs in gratings [11] [12] [13], waveguide arrays [14] [15], photonic crystals with near-zero refractive index [16] [17], integrated photonic circuits [18] [19] and metasurfaces [20] [21]. A range of amazing properties and applications have been achieved to date with photonic BICs in applications such as lasing [22], sensing [23] and Raman spectroscopy [24], and the application of BIC in nonlinear optics [25], twisted light [26] and light-matter interaction [27] is being actively investigated. Unfortunately, most of the reported BICs only occur at very few discrete points in ω–k space for a given structure geometry. This is true for both the symmetry-protected BICs and accidental type of BICs. When one introduces some perturbation into the system geometry to switch the ideal BIC into a quasi-BIC (QBIC) resonance, the number of resonances doesn’t change. Since these conventional BIC or QBIC resonances only happen at fixed points in the ω–k space, they can be considered as zero-dimensional (0D) BICs. Unfortunately, the enhanced light-matter interactions can only be achieved at these limited number of points with fixed frequencies for a specific geometry, restricting the applications of BIC in many circumstances where it is required to tune the working frequency or the number of working frequencies is more than that of the
supported BIC resonances. Due to this reason, most of the BIC applications in nonlinear optics reported to date are focused on higher harmonic generations [28] [29], where only a single light beam at the fundamental frequency is involved. The ultra-high local electric field enhancement associated with the QBIC resonances significantly improves the nonlinear conversion efficiency [25]. However, there are many circumstances in the nonlinear optics where the generation of the target signal frequency requires two or more different input frequencies, e.g. in sum frequency generation (SFG) or difference frequency generation (DFG). To obtain the utmost enhancement based on the BIC effect, one needs to have all the input frequencies at the QBIC resonances. Unfortunately, one can see that significant restrictions emerge with the conventional 0D BICs because in those applications, the input frequencies may not match exactly all the QBIC resonances supported by a structure or the number of required input frequencies is more than that of supported BIC resonances. The situation becomes even worse if one needs to realize a spectral tunability of the target signal frequency.

In this work, we demonstrate that a composite grating composed of two ridge arrays with the same period and different ridge width on a slab waveguide structure supports a new set of BIC resonances and can address the above challenges. We note that a similar structure with regular uniform ridge grating supports the ideal BIC at normal incidence and the QBIC at inclined incidence, both at fixed and limited number of frequencies [12]. In contrast, these BIC resonances supported by the composite grating are distributed continuously along a line over a large spectral range in the \( \omega \sim k \) space and thus can be considered as one-dimensional (1D) BICs. Using the 1D QBICs supported by a structure with fixed geometry, it is now possible to choose arbitrarily any frequency within a broad range to achieve enhanced light-matter interactions. This important feature greatly promotes many applications requiring multiple input wavelengths, such as SFG or SFG, and is expected to significantly push forward the use of BIC in applications such as nonlinear optics.

2. Structure and results

Figure 1. Schematic diagram of the structure supporting the 1D QBICs. The inset presents a magnified view of the grating unit cell, which is assumed to extend infinitely along the y-direction. The red beams indicate the incident, reflected and transmitted light.
Figure 1 shows a schematic sketch of the investigated structure that supports the 1D BICs. The red dashed box in the inset shows a magnified side view of the unit cell. To demonstrate the working principle, Si (dark grey area, refractive index 3.45) is first assumed in this section as the constituent material for both the slab waveguide and ridges on the substrate of SiO$_2$ (blue area, refractive index 1.45). To achieve the BIC effect in the telecom band, we deliberately adjusted the period $P$ to 540 nm, and chose $t = 220$ nm as in the standard silicon on insulator (SOI) wafer specification. For the topmost grating layer, we take the width of one ridge as $w=50$nm and define another as $w+\delta$ by introducing the deviation variable $\delta$, both with a height $h$ of 60nm. For the sake of simplicity and the ease of calculation, it is assumed that all materials are dispersionless. The red arrows in Fig. 1 represent the incident, reflected and transmitted light beams. Without losing generality, we consider only TE-polarization with the electric field parallel to the y-direction and the incident beam is within the $xz$-plane onto the structure at an angle of $\theta$ with respect to $z$ axis. Similar results and conclusions can be obtained as well for the TM-polarization. A finite-element method based commercial software of Comsol Multiphysics together with Floquet periodic boundary conditions in the $x$-direction and perfectly matched layers (PML) in the $z$-direction is used for all the calculations.

The main feature of the structure is the composite grating on the slab waveguide, which is composed of two ridge arrays with the same period and slightly different ridge width. When $\delta$ is not zero, the composite grating has a period of $P$ to accommodate two ridges within one unit cell and the well-known phenomenon of guided-mode resonance (GMR) [30] is achieved, exhibiting a sharp dip in the transmission spectrum when the following phase-matching condition is satisfied:

$$k_0 \sin \theta + m \frac{2\pi}{P} = k_0 n_{\text{eff}}$$  \hspace{1cm} (1)

where $\theta$ is the incidence angle, $k_0$ is the wavenumber in vacuum, $n_{\text{eff}}$ is the effective index of guided mode within the slab waveguide and $m$ is the order of diffraction by the grating. For simplicity, we only consider the $\pm 1$ diffraction orders in this paper for simplicity. The prominent characteristic of the composite grating is that, when $\delta$ decreases to 0, it will become a regular uniform ridge grating and the period will switch from $P$ to $0.5P$, resulting in a leap of the supported GMR from the original frequency to the double of it. The dependence of resonance Q factor as a function of $\delta$ at normal incidence and at a random incident angle of 5° are calculated and presented in Fig. 2(a). It is quite evident that the Q factor increases with the decrease of the ridge asymmetry. A polynomial fitting of the Q values as a function of $\delta$ are shown in the inset of Fig. 2(a) and a quadratic dependence of Q versus $\delta$ is found, consistent with the BIC resonances supported by asymmetric metasurfaces [31]. When $\delta$ approaches 0, an infinite value of Q will be achieved, indicating the occurrence of the ideal BIC. In this case, the structure period effectively decreases to the half of its original value and the phase matching condition in (1) is not fulfilled. Thus the original GMR will not be excited, the same as ideal BIC which cannot be excited externally as well. The blue curves in Fig. 2(a) also show that the quadratic dependence of Q on $\delta$ as well as the...
supporting of ideal BIC resonances also happen at the inclined incidence angle of 5°. Actually, the same dependence works for any incident angle because the switching of the composite grating period from $P$ to $0.5P$ in equation (1) works when $\delta$ decreases to 0, regardless the value of $q$. That is to say, when $\delta$ approaches 0, the composite grating on the slab waveguide supports the ideal BIC resonances at any incident angle over a broad spectral range determined by equation (1). The trend of the BIC resonance as a function of $k_x$ will follow a continuous curve, giving rise to the term of a 1D BIC in the $\omega$–$k$ space. This phenomenon is in large contrast with Friedrich–Wintgen BIC which only happens at the certain positions of avoided crossing between two different resonances [2].

When $\delta$ is non-zero yet small, QBIC resonances with finite yet ultra-high Q factors are excited, and they inherit the 1D distribution behavior of BIC in the $\omega$–$k$ space. One can choose any wavelength within a broad spectral range to have the QBIC resonance, whose Q factor can be further controlled by choosing a proper deviation of $\delta$ from 0. These QBIC resonances are of special importance for real applications, thanks to the benefit of relieved excitation requirement while the local enhancement of electromagnetic fields is weakly affected.

![Fig. 2 (a) The dependence of resonance Q-factor as a function of ridge width difference $\delta$, at two incident angles of 0°(red) and 5°(blue) respectively. (b) The band structure of a QBIC mode supported by composite grating waveguide structure when $\delta$ is 10nm. The dotted line represents the light line in the free space. (c) The transmission spectra of a conventional grating structure GMR (red line) and composite gratings (black line) at normal incidence.](image)

To further verify that the composite grating structure with a non-zero $\delta$ supports the QBIC resonances, we show in Fig. 2(b) the calculated dispersion properties of the
resonances in the ω–k space with a special value of δ as 10nm. The dashed line represents the dispersion of light in air, above which it is the continuum region of radiation. The red and the blue solid lines represent two resonant modes supported by the structure. The position of those resonances above the light line, as well as the dependence of their Q-factors as a function of the system asymmetry confirm that these resonances belong to the QBIC category. One can also observe that there are two branches of QBIC resonances in Fig. 2(b). This is due to the excitation of two counter-propagating guided modes in the waveguide, as will be explained in the subsequent part. We should note that for the QBIC resonances supported by resonating metasurface elements, although a continuous distribution of ω–k dependence can also be found [25] [29], the dispersion curve is much flatter within a much narrower spectral band. The 1D BIC resonance presented in this work is governed by the conditions in equation (1), and is principally completely different.

It is well-known that the GMR effect themselves can generate a sharp resonance effect with large Q-factors [30]. However, the QBIC resonances exhibit a much narrower bandwidth because it is a perturbation of the ideal BIC case. To have a straightforward comparison between the GMR and the QBIC effects supported by a conventional ridge grating and the composite grating respectively, we present in Fig. 2(c) the transmission spectrum at normal incidence for both the GMR (red line) and the QBIC (black line) with the inset showing the structure schematics of the two cases. Here δ is also chosen as 10 nm for the composite grating. It is clearly shown that a much sharper transmission dip is present for the QBIC resonance than the GMR. Detailed calculations show that the QBIC resonance has a Q-factor higher than 10⁴ and is two orders of magnitude higher than that of the GMR shown in the same figure.

By studying the transmission spectra of the composite grating at different incidence angles, the underlying physics of the QBIC resonance can be further revealed. For the composite grating structure with δ=10nm, in Fig.3 (a) we present the calculated resonance wavelength versus incident angle. There are two sets of resonances, the long-wavelength branch displayed in red and the short-wavelength branch in blue. When the incident angle increases, the long-wavelength branch undergoes a red shift from 1545nm at normal incidence to around 1750nm at an incident angle of 30°, while the short-wavelength branch experiences a blue shift to the opposite direction. These trends suggest that the long-wavelength branch results from the grating excitation with m=−1 while the other branch is with m=1 in equation (1). We choose four spectral positions in Fig. 3(a) marked a, b, c and d to demonstrate the formation of BIC/QBIC modes for different cases. a and c correspond to the resonances at normal incidence, while b and d correspond to the incident angle of 5°. We present both the transmission spectrum and the field distributions around these four positions in Figs. 3(b)–(e) respectively. The transmission spectrum in Fig.3(b) presents the results for position a, and is an enlargement of that in Fig. 2(c). The inset shows the electric field distribution as well as the Poynting vectors by the white arrow. One can see that at normal incidence, guided modes propagating along two different directions are excited simultaneously. The electric field is mainly concentrated within the waveguide layer directly below the ridges. Note that due to the asymmetry in the ridge width, the electric field is not strictly
anti-symmetric. So there is still a weak coupling into this mode when a plane wave is normally incident onto the composite grating. When $\delta$ approaches 0, the electric field profile at position $a$ is closer to being an anti-symmetric, leading to an infinitesimal overlap with the normally incident plane wave. In other words, the resonance at position $a$ is a quasi-BIC resonance of the symmetry-protected type.

For a larger incident angle, the wavelength of the QBIC has a red-shift. For example, when the incident angle is $5^\circ$, the QBIC wavelength increases from 1545.3nm to 1573.2nm, as marked by position (b) in Fig. 3(a). The corresponding Q-factor increases slightly, but the electric field distribution is no longer symmetrical. As is shown in Fig. 3(c), the $E_y$ field has a larger distribution on the right side of one unit cell and the power flow is propagating to the left, contradictory with the incident beam direction. These field distributions indicate that the QBIC resonance does not belong to symmetry-protected type. Since the composite grating is composed of two ridge arrays with the same period and slightly-different width, two independent GMRs will be excited at the same resonance wavelength. However, the coupling efficiencies back to free space which is the origin for the high reflectance at GMR resonance will be different for the two GMRs, especially in the phases, leading to destructive interferences in the far field for certain wavelengths. As a result, a significant narrowing of the GMR resonance is achieved, leading to the formation of the QBIC resonance. It’s worth noting that the destructive interferences occur at any incident angle, and the QBIC wavelength can extend continuously over a large bandwidth, as shown in Fig. 3(a). Similar QBIC resonances for the short-wavelength branch at inclined incidence happen resulting from the destructive interferences between the two coupling processes from the GMR to free space, which is the case for position $d$ whose mode distributions are presented in Fig. 3(e). The main difference between positions $d$ and $b$ is that guided mode propagating along the same direction as the incident beam is excited, which corresponds to the grating diffraction order $m$ to be 1.

Point $c$ has a completely different property as compared to other positions. Eigen frequency analysis as well as the transmission spectrum calculations demonstrate that a resonance with an infinite Q factor can be found, and correspondingly this resonance cannot be excited by a plane wave at normal incidence (see Fig. 3(d)). These results suggest that point $c$ corresponds to the occurrence of an ideal BIC resonance. Although there is a slight level of structural asymmetry between the ridges($\delta=10$nm), the whole structure is still symmetric if one uses a vertical plane cross the center of either ridge. So the resonance at point $c$ is an ideal BIC of the symmetry-protected type, and thus cannot be excited by a plane wave at normal incidence. In striking difference to the field distribution for position $a$, the electric field distribution is located between the grating ridge intervals, and it is distributed with perfect anti-symmetry in the waveguide layer. Due to the existence of the true BIC resonance for the short-wavelength branch at normal incidence, any other points along the blue line away from point $c$ can also be interpreted as QBIC resonances and can be excited simply by the tuning of incident angle. As a result, the QBIC resonance in the short-wavelength branch typically has a Q factor which deceases significantly with the increase in incident angle.
Figure 3. (a) The relationship between the resonant wavelength of BIC/QBIC modes and the incident angle in the composite grating structure. The red solid line represents the long-wave branch of the QBIC mode and the blue solid line is for the short-wave branch. The four circles of a, b, c and d represent the BIC/QBIC modes at different positions at different angles (0° or 5°). (b), (c), (d) and (e) present the local transmission spectrum close to the four positions marked in Fig.3 (a), with the inset showing the field distribution of the real part of $E_y$ within a single periodic unit. The white arrows represent the vectorial distributions of the Poynting vector. The three black arrows in each figure represent incidence, reflection and transmission respectively, where (b) and (d) are obtained at normal incidence, (c) and (e) are calculated at 5° tilt angle.

The above behavior in Fig. 3 can be actually theoretically explained within the framework of coupled-mode theory. As we described above, the quality factor of the QBIC depends on both the incidence angle $\theta$ and ridge width difference $\delta$. Here we adopt the spatiotemporal formulation of this theory for gratings presented in paper [32], and write the homogeneous coupled-mode equations:
Here \( u \) and \( v \) define the amplitudes of the two counter-propagating modes of the slab layer; \( v_g \) is the group velocity of these modes, and \( c_1 \) and \( c_2 \) are the coupling coefficients. Making use of the energy conservation law [33] we can show that 
\[ i(c_1 - c_2) \] is real for lossless structures.

Taking the Fourier transform of Eq. (2) we can arrive at the system of linear equations having non-trivial solutions when 
\[
\frac{\partial u}{\partial t} = -v_g \frac{\partial u}{\partial x} + c_1 u + c_2 v, \\
\frac{\partial v}{\partial t} = v_g \frac{\partial v}{\partial x} + c_1 v + c_2 u.
\]  
(2)

where \( k_x = k_0 \sin \theta \) and \( \omega = 2\pi c / \lambda \) are the wave number and angular frequency of the incident light; \( \omega_{p1} = i(c_1 + c_2) \) is the complex frequency of the symmetric mode marked as (a) in Fig. 3(a), and \( \omega_{p2} = i(c_1 - c_2) \) is the real frequency of the antisymmetric mode marked by (c). Equation (2) is the dispersion equation describing the hyperbola-like dispersion law seen in Fig. 3(a).

Note that all the parameters used in Eqs. (2) and (3) depend on the ridge width difference \( \delta \). For the considered structure it is the dependence of \( \text{Im} \omega_{p1} \) on \( \delta \) is the most important. As we demonstrated previously, the eigenmodes of the structure are not exited at \( \delta = 0 \). Therefore, the frequency \( \omega_{p1} \) of the symmetric mode is real at \( \delta = 0 \) and \( \text{Im} \omega_{p1} \approx \delta \cdot \alpha \) where \( \alpha \) is a real parameter. When \( \delta \) is zero all coefficients in Eq. (3) become real and we obtain the dispersion law for the 1D BIC:

\[
\frac{v_g^2 k_x^2}{\epsilon} = (\omega - \omega_{p1})(\omega - \omega_{p2}).
\]  
(3)

When the ridge width difference is non-zero we can solve Eq. (3) with respect to the complex frequency \( \omega \) and obtain the quality factor \( Q = \text{Re} \omega / (-2 \text{Im} \omega) \) of the QBIC.

3. SFG as an example

To demonstrate that it is possible to select randomly any incident wavelength within a specific range and generate the light in the desired spectral range with a larger freedom of choice by using this 1D quasi-BIC resonances, we choose the process of SFG as a simple example. Here an x-cut (the optical axis is along y direction) LiNbO\(_3\)
film structure is employed to make use of its relatively high second-order nonlinear susceptibility along the TE polarization. The structure is schematically shown as the inset of Fig. 4(a) and its geometrical parameters are adjusted due to a smaller refractive index \((n_o=2.22, n_e=2.14)\) of LiNbO3 compared to Si. Fig. 4(a) presents the calculated transmission spectra for TE polarization at three different incident angles. It is apparent that a sharp resonance is associated with each incident angle, and the resonance has a blue shift and slightly increasing bandwidth at a larger incident angle. We note that these resonances belong to the lower-wavelength branch as shown in Fig. 3(a).

![Figure 4(a) Transmission spectra through the LiNbO3 thin film composite grating structure at three different incident angles of 2°, 3° and 5°. The geometrical parameters are as follows: \(P=835\)nm, \(t=350\)nm, \(h=80\)nm, \(W=80\)nm, \(\delta=5\)nm. (b) and (c) SFG spectra when one input wavelength is 1490.665nm at a fixed incident angle of 2° while the other input beam is fixed at 3° (b) and 5°(c) respectively while its wavelength is tuned. The red dashed lines correspond to the SFG through the bare LiNbO3 thin film of the same thickness.

The SFG enhancement is most significant when both incident wavelengths match a certain quasi-BIC resonance respectively. We first choose one incident plane wave with a fixed wavelength of \(\lambda_1=1490.665\)nm at the incident angle of 2°. The second incident beam has an incident angle of 3° while its wavelength \(\lambda_2\) is scanned continuously. Both the incident plane waves are assumed to have an electric field amplitude of \(1 \times 10^6\)V/m, corresponding to an intensity of 0.133MW/cm² in vacuum. The SFG is calculated using the FEM method by only considering the \(d_{33}\) value of LiNbO3 as \(-41.7\)pm/V [34]. This simplification is valid because \(d_{33}\) is one order of magnitude higher than other components and is the dominant factor in the second-order nonlinear process. As shown in Fig. 4(b), the SFG is most significant when \(\lambda_2\) is tuned as 1478.818nm which is exactly the quasi-BIC resonance for the incident angle of 3°, leading to a SFG wavelength of \(\lambda_1 \lambda_2 / (\lambda_2 + \lambda_2) = 742.359\)nm. In the SFG calculations, we assume a grating length of 1cm is used in the y direction to have a valid 2D grating structure, and the power at SFG frequency is calculated by an integral of its pointing
vector only at the lower output port. From the results one can see that the SFG efficiency when both input wavelengths match the quasi-BIC wavelength is enhanced by a factor of $10^8$, compared to the SFG effect through a bare LiNbO$_3$ thin film of the same thickness. If one aspires to have the enhanced SFG at a different target wavelength, one can keep the wavelength of $\lambda_1$ at 2° and simply tune the incident angle of $\lambda_2$, which will tune the quasi-BIC resonance to a different wavelength value. For example, when the incident angle of $\lambda_2$ is increased to 5°, the quais-BIC resonance will switch to 1454.755nm and the enhanced SFG will be around 736.246nm now. The SFG results for this case are presented in Fig. 4(c).

4. Discussions and Conclusion

The most significant feature of the BIC resonances with the composite grating is that the BIC can be supported continuously following the relation between the resonance wavelength and the incident angle governed by equation (1). As a result, one can achieve the BIC resonance over a large spectrum by tuning the incident angle, and switch the BIC resonance to be QBIC with controlled Q-factor by introducing some structural asymmetry. The continuous distribution of the QBIC resonances over a broad spectral range is a significant advantage over traditional QBIC resonances which can only occur around very few discrete positions. As a result, one can manipulate the light-matter interactions at any wavelength within the range, by simply choosing the proper incident angle. As an example, we have demonstrated in section 3 the enhancement of the SFG process with some spectral tunability by simply changing the incident angle of one input beam.

In summary, we have demonstrated in this work that a composite grating structures composed of two ridge arrays with the same period and slightly different ridge width located on a waveguide slab can be employed to support the QBIC resonances continuously over a large spectral range, forming a phenomenon of a 1D QBIC along a continuous curve in the $\omega$–$k$ space. The occurrence of the BIC/QBIC resonances at any wavelength over a broad spectral range for a structure with fixed geometry makes it possible to achieve the enhanced light-matter interactions with more freedom compared to the traditional BICs. We believe that these 1D QBICs can greatly promote many applications requiring multiple input wavelengths, and has great applications in general nonlinear optics. Furthermore, although we use the simple 1D grating structures to demonstrate the formation of the 1D BIC resonances, the same methodology can be easily extended to more sophisticated 2D composite periodic elements or metasurface structures and to other spectral range of the electromagnetic spectrum to have enhanced interactions.

Acknowledgment:

Z.H. acknowledges the support by the National Science Foundation of China (11974221).

References

[1] J. von Neumann and E. Wigner, *Uber Merkwürdige Diskrete Eigenwerte*, Phys. Z 30, 465 (1929).
C. W. Hsu, B. Zhen, A. D. Stone, J. D. Joannopoulos, and M. Soljacic, *Bound States in the Continuum*, Nat. Rev. Mater. **1**, 16048 (2016).

Y. X. Xiao, G. Ma, Z. Q. Zhang, and C. T. Chan, *Topological Subspace-Induced Bound State in the Continuum*, Phys. Rev. Lett. **118**, 166803 (2017).

F. Capasso, C. Sirtori, J. Faist, D. L. Sivco, and A. Y. Cho, *Observation of an Electronic Bound State above a Potential Well*, Nature **358**, 565 (1992).

A. Albo, D. Fekete, and G. Bahir, *Electronic Bound States in the Continuum above (Ga,In)(As,N)/(Al,Ga)As Quantum Wells*, Phys. Rev. B **85**, 115307 (2012).

C. Álvarez, F. Domínguez-Adame, P. A. Orellana, and E. Díaz, *Impact of Electron-Vibron Interaction on the Bound States in the Continuum*, Phys. Lett. A **379**, 1062 (2015).

T. Lepetit, E. Akmansoy, J. P. Ganne, and J. M. Lourtioz, *Resonance Continuum Coupling in High-Permittivity Dielectric Metamaterials*, Phys. Rev. B **82**, 195307 (2010).

T. Lepetit and B. Kanté, *Controlling Multipolar Radiation with Symmetries for Electromagnetic Bound States in the Continuum*, Phys. Rev. B - Condens. Matter Mater. Phys. **90**, (2014).

D. C. Marinica, A. G. Borisov, and S. V. Shabanov, *Bound States in the Continuum in Photonics*, Phys. Rev. Lett. **100**, (2008).

H. Friedrich and D. Wintgen, *Interfering Resonances and Bound States in the Continuum*, Phys. Rev. A **32**, 3231 (1985).

S. I. Azzam, V. M. Shalaev, A. Boltasseva, and A. V. Kildishev, *Formation of Bound States in the Continuum in Hybrid Plasmonic-Photonic Systems*, Phys. Rev. Lett. **121**, 253901 (2018).

D. A. Bykov, E. A. Bezus, and L. L. Doskolovich, *Coupled-Wave Formalism for Bound States in the Continuum in Guided-Mode Resonant Gratings*, Phys. Rev. A **99**, 1 (2019).

K. Sun, Y. Cai, and Z. Han, *A Novel Mid-Infrared Thermal Emitter with Ultra-Narrow Bandwidth and Large Spectral Tunability Based on the Bound State in the Continuum*, J. Phys. D. Appl. Phys. **55**, 025104 (2021).

Y. Plotnik, O. Peleg, F. Dreisow, M. Heinrich, S. Nolte, A. Szameit, and M. Segev, *Experimental Observation of Optical Bound States in the Continuum*, Phys. Rev. Lett. **107**, 28 (2011).

E. N. Bulgakov and A. F. Sadreev, *Robust Bound State in the Continuum in a Nonlinear Microcavity Embedded in a Photonic Crystal Waveguide*, Opt. Lett. **39**, 5212 (2014).

M. Minkov, I. A. D. Williamson, M. Xiao, and S. Fan, *Zero-Index Bound States in the Continuum*, Phys. Rev. Lett. **121**, 263901 (2018).

L. Vertchenko, C. DeVault, R. Malureanu, E. Mazur, and A. Lavrinenko, *Near-Zero Index Photonic Crystals with Directive Bound States in the Continuum*, Laser Photonics Rev. **15**, 1 (2021).

C. L. Zou, J. M. Cui, F. W. Sun, X. Xiong, X. B. Zou, Z. F. Han, and G. C. Guo, *Guiding Light through Optical Bound States in the Continuum for*
Ultrahigh-Q Microresonators, Laser Photonics Rev. 9, 114 (2015).

[19] Z. Yu, X. Xi, J. Ma, H. K. Tsang, C.-L. Zou, and X. Sun, Photonic Integrated Circuits with Bound States in the Continuum, Optica 6, 1342 (2019).

[20] K. Koshelev, Y. Tang, K. Li, D. Y. Choi, G. Li, and Y. Kivshar, Nonlinear Metasurfaces Governed by Bound States in the Continuum, ACS Photonics 6, 1639 (2019).

[21] A. S. Kupriianov, Y. Xu, A. Sayanskiy, V. Dmitriev, Y. S. Kivshar, and V. R. Tuz, Metasurface Engineering through Bound States in the Continuum, Phys. Rev. Appl. 12, 1 (2019).

[22] S. T. Ha, Y. H. Fu, N. K. Emani, Z. Pan, R. M. Bakker, R. Paniagua-Dominguez, and A. I. Kuznetsov, Directional Lasing in Resonant Semiconductor Nanoantenna Arrays, Nat. Nanotechnol. 13, 1042 (2018).

[23] Y. Wang, Z. Han, Y. Du, and J. Qin, Ultrasensitive Terahertz Sensing with High-Q Toroidal Dipole Resonance Governed by Bound States in the Continuum in All-Dielectric Metasurface, Nanophotonics 10, 1295 (2021).

[24] S. Romano, G. Zito, S. Managò, G. Calafiore, E. Penzo, S. Cabrini, A. C. De Luca, and V. Mocella, Surface-Enhanced Raman and Fluorescence Spectroscopy with an All-Dielectric Metasurface, J. Phys. Chem. C 122, 19738 (2018).

[25] Z. Han, F. Ding, Y. Cai, and U. Levy, Significantly Enhanced Second-Harmonic Generations with All-Dielectric Antenna Array Working in the Quasi-Bound States in the Continuum and Excited by Linearly Polarized Plane Waves, Nanophotonics 10, 1189 (2021).

[26] E. N. Bulgakov and A. F. Sadreev, Propagating Bloch Bound States with Orbital Angular Momentum above the Light Line in the Array of Dielectric Spheres, J. Opt. Soc. Am. A 34, 949 (2017).

[27] K. L. Koshelev, S. K. Sychev, Z. F. Sadrieva, A. A. Bogdanov, and I. V. Iorsh, Strong Coupling between Excitons in Transition Metal Dichalcogenides and Optical Bound States in the Continuum, Phys. Rev. B 98, 1 (2018).

[28] L. Carletti, K. Koshelev, C. De Angelis, Y. Kivshar, C. De Angelis, Y. Kivshar, C. De Angelis, Y. Kivshar, C. De Angelis, and Y. Kivshar, Giant Nonlinear Response at the Nanoscale Driven by Bound States in the Continuum, Phys. Rev. Lett. 121, 33903 (2018).

[29] Z. Liu, Y. Xu, Y. Lin, J. Xiang, T. Feng, Q. Cao, J. Li, S. Lan, and J. Liu, High-Q Quasibound States in the Continuum for Nonlinear Metasurfaces, Phys. Rev. Lett. 123, (2019).

[30] G. Quaranta, G. Basset, O. J. F. Martin, and B. Gallinet, Recent Advances in Resonant Waveguide Gratings, Laser Photonics Rev. 12, 1 (2018).

[31] K. Koshelev, S. Lepeshov, M. Liu, A. Bogdanov, and Y. Kivshar, Asymmetric Metasurfaces with High-Q Resonances Governed by Bound States in the Continuum, Phys. Rev. Lett. 121, 193903 (2018).

[32] D. A. Bykov and L. L. Doskolovich, Spatiotemporal Coupled-Mode Theory of Guided-Mode Resonant Gratings, Opt. Express 23, 19234 (2015).

[33] S. Fan and W. Suh, Resonance in Optical Resonators, 20, 569 (2003).
[34] L. Kang, H. Bao, and D. H. Werner, *Efficient Second-Harmonic Generation in High Q-Factor Asymmetric Lithium Niobate Metasurfaces*, Opt. Lett. 46, 633 (2021).