CONSTRaining PHYSICAL PROPERTIES OF Type IIn Supernovae THrough Rise Times and Peak luminosities

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ABSTRACT

We investigate the diversity in the wind density, supernova ejecta energy, and ejecta mass in Type IIn supernovae based on their rise times and peak luminosities. We show that the wind density and supernova ejecta properties can be estimated independently if both the rise time and peak luminosity are observed. The peak luminosity is mostly determined by the supernova properties and the rise time can be used to estimate the wind density. We find that the ejecta energies of Type IIn supernovae need to vary by factors of 0.2–5 from the average if their ejecta masses are similar. The diversity in the observed rise times indicates that their wind densities vary by factors of 0.2–2 from the average. We show that Type IIn superluminous supernovae should have not only large wind density but also large ejecta energy and/or small ejecta mass to explain their large luminosities and the rise times at the same time. We also note that shock breakout does not necessarily occur in the wind even if it is optically thick, except for the case of superluminous supernovae, and we analyze the observational data both with and without assuming that the shock breakout occurs in the dense wind of Type IIn supernovae.

Key word: supernovae: general

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1. INTRODUCTION

Type IIn supernovae (SNe IIn) are a class of SNe in which the signatures of the interaction between the SN ejecta and the circumstellar medium are observed (Schlegel 1990; Filippenko 1997). The estimated circumstellar density required to explain the observational properties is much higher than that expected from the standard stellar evolution theory (e.g., Langer 2012). It is generally assumed that the high circumstellar density is due to the high mass-loss rates of the SN IIn progenitors. The estimated mass-loss rates are typically higher than \(10^{-3} M_\odot \text{yr}^{-1}\) (e.g., Kiewe et al. 2012; Taddia et al. 2013; Fransson et al. 2013; Moriya et al. 2014).

SNe IIn are heterogeneous. For example, their peak luminosities spread over more than two orders of magnitudes (e.g., Richardson et al. 2014; Li et al. 2011). This diversity can be caused by many factors, e.g., diversity in circumstellar density, SN ejecta mass, and SN ejecta energy. It is important to understand which observational properties are affected by which physical parameters. By disentangling the origins of the observational diversities, we can constrain the physical properties of the progenitor systems and obtain a better understanding of SNe IIn and their progenitors.

In this Letter, we investigate a method of disentangling the information of the wind density, SN ejecta energy, and SN ejecta mass based on the early light curves (LCs) of SNe IIn. We suggest that the wind and SN properties can be constrained independently if both the rise time and the peak luminosity of a SN IIn are observed. By using the observational rise times and peak luminosities of SNe IIn recently reported by Ofek et al. (2014a), we show how diverse the wind and SN properties should be to explain the observational diversities of SNe IIn.

Ofek et al. (2014a) also performed a similar analysis by using their data to mainly focus on the rise times of their reported observations. However, they did not use the peak luminosities to constrain the SN properties. In addition, they assumed that the shock breakout always occurs in the dense wind in SNe IIn. The shock breakout does not necessarily occur in the wind even if it is optically thick and makes SNe IIn. Here, we also investigate the case in which shock breakout does not occur in the wind.

2. DIFFUSION TIME AND CHARACTERISTIC LUMINOSITY

We analytically estimate the diffusion time \(t_d\) and the characteristic luminosity \(L_p\) of SNe IIn at \(t_d\) using the method presented by Chevalier & Irwin (2011).

2.1. Diffusion Time

The diffusion time \(t_d\) is estimated as \(t_d = \frac{\tau_w \Delta R}{c}\), where \(c\) is the speed of light, \(\tau_w\) is the Thomson scattering optical depth of the wind from the radius \(R_b\) where photons begin to be emitted in the wind, and \(\Delta R\) is the length between \(R_b\) and the radius where the wind optical depth becomes unity (\(r = R_{\tau_w=1}\)). We assume that the steady-wind density structure \(\rho_w = D r^{-2}\). \(D\) can be expressed using the progenitor’s mass-loss rate \(\dot{M}\) and the wind velocity \(v_w\) as \(D = \frac{\dot{M}}{4 \pi v_w} \). Assuming that the wind radius and \(R_{\tau_w=1}\) are much larger than \(R_b\), the diffusion time is expressed as

\[
\tau_w \Delta R = \frac{\kappa^2 D^2}{c R_b},
\]

where \(\kappa\) is opacity and is assumed to be 0.34 cm\(^2\) g\(^{-1}\). \(R_b\) differs depending on whether or not the shock breakout occurs in the wind. We derive \(t_d\) in the two cases separately.
If the shock breakout occurs in the wind, then photons in the shock are released when the following condition is satisfied:

$$\tau_w \simeq \frac{c}{v_s}$$

(2)

where $v_s$ is the shock velocity. We assume that the SN ejecta density structure has two density components, $\rho_{ej} \propto r^{-n}$ outside and $\rho_{ej} \propto r^{-3}$ inside (see, e.g., Chevalier & Irwin 2011), and the SN ejecta expands homologously. Then, the radius and velocity of the shock evolve following a power-law analytic formula with time $t$ (e.g., Chevalier 1982; Moriya et al. 2013b). The shock breakout occurs at

$$t_{br} \simeq \frac{n - 3}{n - 2} \frac{\kappa D}{c}.$$ 

(3)

Using the power-law formula presented in Moriya et al. (2013b),

$$r_s(t) = C_1 D^{\frac{n-5}{n-2}} M_{ej}^{\frac{n-5}{n-2}} E_{ej}^{\frac{1}{n-2}} t^{\frac{n-5}{n-2}},$$

(4)

to estimate the shock radius at $t = t_{br}$ which is $R_b$, we obtain the diffusion time $t_d$ for the case of the shock breakout in the dense wind:

$$t_d \simeq C_2 \kappa^{\frac{1}{n-2}} D^{\frac{n-5}{n-2}} M_{ej}^{\frac{n-5}{n-2}} E_{ej}^{\frac{1}{n-2}},$$

(5)

where $E_{ej}$ is the kinetic energy of the SN ejecta and $M_{ej}$ is the mass of the SN ejecta (see Table 1). The constants $C_1$ and $C_2$ are shown in the Appendix.

### 2.1.2. No Shock Breakout Model

If the total optical depth of the wind is smaller than $c/v_s$ when the shock reaches the inner radius $R_i$ of the dense wind, then the shock breakout does not occur in the dense wind. In this case, $R_b = R_i$ and the diffusion time in the wind is

$$t_d \simeq \frac{\kappa^2 D^2}{c R_i}.$$ 

(6)

For example, when $M = 10^{-3} M_\odot yr^{-1}$ and $\nu_w = 100 \text{ km s}^{-1}$, the total wind optical depth does not exceed $c/v_s \simeq 30$ if $R_i \gtrsim 6 \times 10^{12} \text{ cm}$ with a standard $v_s \simeq 10,000 \text{ km s}^{-1}$. It is possible that the progenitor radius is larger than $\simeq 6 \times 10^{12} \text{ cm}$ and that the wind does not become optically thick enough to cause the shock breakout inside it. This dividing radius is less than those of red supergiants (RSGs) and luminous blue variables (LBVs) ($\sim 100 R_\odot$). Even if the progenitor radius is smaller than $\simeq 6 \times 10^{12} \text{ cm}$, it is possible that the dense wind does not start just above the progenitor and that there exist a “void” between the progenitor and the dense part of the wind.

### Table 1

| Model                  | $D$  | $R_i$ | $M_{ej}$ | $E_{ej}$ |
|------------------------|------|-------|----------|----------|
| Breakout (general)     | $\frac{n-5}{n-2}$ | 0     | $\frac{n-5}{n-2}$ | $-\frac{n-5}{n-2}$ |
| Breakout ($n = 10$)   | 1.25 | 0     | 0.313    | -0.438   |
| Breakout ($n = 7$)    | 1.4  | 0     | 0.2      | -0.4     |
| No breakout            | 2    | -1    | 0        | 0        |

Note. Based on Equations (5) and (6).

### Table 2

| Model                  | $R_i$ | $t_d$ | $M_{ej}$ | $E_{ej}$ |
|------------------------|------|-------|----------|----------|
| Breakout (general)     | 0    | $\frac{n-5}{n-2}$ | $\frac{(n-5)M_{ej}}{n-2}$ | $\frac{(n-5)E_{ej}}{n-2}$ |
| Breakout ($n = 10$)   | 0.125| -1.09 | 1.53     |
| Breakout ($n = 7$)    | -0.314| -0.657| 1.31     |
| No breakout (general) | $\frac{n-5}{n-2}$ | 0     | $\frac{(n-5)M_{ej}}{n-2}$ | $\frac{(n-5)E_{ej}}{n-2}$ |
| No breakout ($n = 10$)| 0.313| -0.0625| -0.938 | 1.31     |
| No breakout ($n = 7$) | 0.2  | -0.4  | -0.6     | 1.2      |

Note. Based on Equations (9) and (10).

In addition, if $v_s$ is smaller, then the shock breakout radius can be smaller. For instance, if $v_s \simeq 5000 \text{ km s}^{-1}$, which is indicated in some SNe IIn in Ofek et al. (2014a), then $R_b$ needs to be smaller than $3 \times 10^{12} \text{ cm}$ to cause the shock breakout. Thus, we do not assume that the shock breakout always occurs in the dense wind and investigate the case in which the shock breakout does not occur.

### 2.2. Characteristic Luminosity

We estimate the characteristic luminosity at $t_d$ by assuming that a fraction of the kinetic energy in the SN ejecta shocked in $t_d$ is radiated in $t_d$. The total available kinetic energy $E_p$ is

$$E_p = \int_{r(t_d)}^\infty 4\pi r^2 \frac{1}{2} \nu v_{ej}^2 dr,$$

(7)

where $v_{ej}$ is the SN ejecta velocity. Equation (4) is used to estimate $r(t_d)$ when deriving Equation (7).

Assuming that a fraction $\epsilon$ of the kinetic energy is emitted in the diffusion time, we obtain the characteristic luminosity $L_p$:

$$L_p \simeq \frac{\epsilon E_p}{t_d}.$$ 

(8)

By using the $t_d$ obtained in the previous section, $L_p$ can be expressed as a function of $t_d$, $M_{ej}$, and $E_{ej}$ (see Table 2). When the shock breakout occurs in the wind, we get the characteristic luminosity:

$$L_p \simeq C_3 \epsilon \kappa^{\frac{n-5}{n-2}} t_d^{\frac{n-5}{n-2}} M_{ej}^{\frac{(n-5)M_{ej}}{n-2}} E_{ej}^{\frac{(n-5)E_{ej}}{n-2}}.$$ 

(9)

For the no shock breakout case, we get

$$L_p \simeq C_4 \epsilon \kappa^{\frac{n-5}{n-2}} R_i^{\frac{n-5}{n-2}} t_d^{\frac{n-5}{n-2}} M_{ej}^{\frac{n-5}{n-2}} E_{ej}^{\frac{n-5}{n-2}}.$$ 

(10)

The constants $C_3$ and $C_4$ are shown in the Appendix.

### 2.3. Summary

We summarize the dependence of $t_d$ and $L_p$ on the wind and SN properties in Tables 1 and 2 using specific examples for typical $n$. $t_d$ is strongly dependent on the wind properties, while $L_p$ is strongly dependent on the SN properties.

Using the relations obtained in this section, the physical properties of the SN ejecta and the wind can be constrained separately as

$$M_{ej}^{\frac{(n-5)M_{ej}}{n-2}} E_{ej}^{\frac{(n-5)E_{ej}}{n-2}} = C_3^{-1} \epsilon^{-1} \kappa^{\frac{n-5}{n-2}} L_p^{\frac{2(n-5)M_{ej}}{n-2}} t_d^{\frac{2(n-5)E_{ej}}{n-2}},$$

(11)

$$D = C_2 \frac{n-5}{n-2} \epsilon^{\frac{n-5}{n-2}} \kappa^{\frac{n-5}{n-2}} L_p^{\frac{n-5}{n-2}} t_d^{\frac{n-5}{n-2}}.$$ 

(12)
3. DIVERSITY IN TYPE IIn SUPERNOVAE

In this section, we investigate the origin of the observational diversity in the rise times and the peak luminosities of SNe IIn and relate this observational diversity to the diversity in the wind density and SN ejecta properties in SNe IIn. We use the analytic estimates for $t_d$ and $L_p$ obtained in the previous section for this purpose. Ofek et al. (2014a) recently summarized the rise times and peak luminosities of SNe IIn. Based on their Table 1, we plot the rise times and peak luminosities of SNe IIn in Figure 1. The peak luminosities are based on the R-band data and we do not adopt any bolometric corrections in the figure. The bolometric corrections for SNe IIn change with time, but are typically within 0.5 mag and small (Ofek et al. 2014b). We assume that the rise time corresponds to $t_d$ and the peak luminosity corresponds to $L_p$.

3.1. Diversity in Supernova Ejecta

The analytic estimates for the peak luminosity (Equations (9) and (10)) show that if the diffusion time is known, then the peak luminosity is mostly determined by the SN properties ($M_{ej}$ and $E_{ej}$). In Figure 1, $L_p$ is plotted for two sets of SN ejecta properties, ($E_{ej}/1$ B, $M_{ej}/M_\odot$, $n$) = (1.5, 5, 10) and (1, 10, 7),4 for the cases with and without the shock breakout. The conversion efficiency $\epsilon$ is set as 0.3.

The suggested conversion efficiency range is $\epsilon \simeq 0.1$–0.5 in the literature and we choose an average value (see, e.g., Fransson et al. 2013).

We set $\delta = 0$ and choose two $n$ based on previous studies (e.g., Chevalier & Irwin 2011; Fransson et al. 2013; Matzner & McKee 1999). The characteristic luminosities from the two parameter sets roughly correspond to the average peak luminosity of SNe IIn in Figure 1. The exact values of $M_{ej}$ and $E_{ej}$ which give the average $L_p$ depend on the model assumptions as in $\epsilon$, but the diversity does not. As we can see in Figure 1, $L_p$ does not strongly depend on $t_d$ and is in fact mostly determined by the SN ejecta properties. We can see in Figure 1 that the differences in the observational peak luminosities are roughly within the factors of 0.1–10 to the analytical average lines. If the shock breakout occurs in the wind, then this means that the diversity in the SN properties is roughly within the following range:

$$0.1 < \eta \equiv \left( \frac{M_{ej}}{M_{ej,s}} \right) \left( \frac{E_{ej}}{E_{ej,s}} \right) < 10,$$

where $E_{ej,s}$ and $M_{ej,s}$ are the standard values. In our case, $E_{ej,s} = 1.5$ B and $M_{ej,s} = 5 M_\odot$ for $n = 10$, and $E_{ej,s} = 1$ B and $M_{ej,s} = 10 M_\odot$ for $n = 7$. In Figure 2, we show the

4 1 B = $10^{51}$ erg.
estimated diversity in the SN properties of SNe IIn. For example, if the exploding stars in SNe IIn have similar ejecta masses \((M_{ej} \approx M_{ej,s})\), then the ejecta energy needs to be diversified roughly by factors of 0.2–5 from the standard value. If the ejecta energy is roughly the same in SNe IIn \((E_{ej} \approx E_{ej,s})\), then the ejecta mass should be diversified by about factors of 0.1–8 \((n = 10)\) or 0.03–40 \((n = 7)\) from the standard ejecta mass. Most SNe IIn are found in the observed luminosity range in Figure 1 (Richardson et al. 2014; Li et al. 2011) and the diversities in SN ejecta properties estimated here are presumed to exist generally in SNe IIn.

So far, we have used the shock breakout model to discuss diversity. The total wind optical depth estimated from Equation (6) exceeds \(\approx 30\) if \(\tau_{wd} \gtrsim 3.5\) days with \(R_i = 10^{13} \text{ cm} = 140 \text{ pc}\) and the shock breakout may not occur in some SNe IIn shown in Figure 1, assuming the typical SN shock velocity of \(10000 \text{ km s}^{-1}\). The progenitor radius can be larger than \(140 \text{ pc}\) if the progenitor is an RSG or an LBV. Alternatively, if the dense wind is detached and \(R_i = 10^{14} \text{ cm}\), for example, the shock breakout only occurs in SNe IIn with \(\tau_{wd} \gtrsim 35\) days. The detachment can occur in SN IIn progenitors if they have variable mass-loss rates. We can see from Figure 1 that most SNe IIn have \(\tau_{wd}\) of less than 35 days. Assuming \(R_i = 10^{14} \text{ cm}\), the observational diversity indicates

\[
0.1 < \xi \equiv \left( \frac{M_{ej}}{M_{ej,s}} \right)^{-3} \left( \frac{E_{ej}}{E_{ej,s}} \right)^{3} < 10. \tag{16}
\]

The expected diversity does not differ much from that expected from the shock breakout model (Figure 2). However, \(L_p\) also depends on \(R_i\) in this case. The difference in \(R_i\) by a factor of 10 can make the difference in the luminosity by a factor of about two (Equation (10)).

Whether or not the shock breakout occurs in the wind also strongly depends on the shock velocity. If the shock velocity is \(5000 \text{ km s}^{-1}\), then the shock breakout occurs in SNe IIn with \(\tau_{wd} > 14\) days even in wind \(R_i = 10^{13} \text{ cm} = 140 \text{ pc}\), which is compatible with RSG and LBV radii. Therefore, SNe IIn both with and without the shock breakout may commonly exist (Figure 1). Ofek et al. (2014a) assumed that the shock breakout always occurs in the dense wind of SNe IIn and they tried to constrain \(u_w\) by using the relation \(r_w \simeq c / u_w\). However, this does not necessarily occur in every SN IIn. Photon diffusion in the wind without the shock breakout may commonly occur in SNe IIn.

Figure 2 indicates that there may exist two separate populations in the ejecta properties, since SNe IIn do not exist at \(n \simeq 1\) or \(\xi \simeq 1\). However, the number of observations is still small and this remains to be investigated.

### 3.2. Diversity in Wind

If the shock breakout occurs in the dense wind in SNe IIn, then the diffusion time \(\tau_{wd}\) depends both on the wind properties and the SN ejecta properties (Equation (5)), but it is more sensitive to the wind density. The wind density can be estimated with Equation (12) using \(\tau_{wd}\) and \(L_p\). Figure 2 shows the mass-loss rates of SN IIn progenitors obtained by the estimated wind density \((M = 4\pi r_w \nu_w D)\). We find that the wind density in SNe IIn differs by factors of roughly 0.2–2 from the average when the shock breakout occurs for the standard sets of \(M_{ej}\) and \(E_{ej}\). The estimated mass-loss rate in Figure 2 is over the range \(\sim 10^{-3} - 10^{-2} M_\odot \text{ yr}^{-1}\) and is consistent with those estimated in the previous SN IIn studies (e.g., Fox et al. 2011; Kiewe et al. 2012; Taddia et al. 2013; Moriya et al. 2014).

If the shock breakout does not occur in the wind, then the wind density can be estimated solely from \(L_p\) with Equation (14). Since \(\tau_{wd}\) in Figure 1 are roughly between 1 and 30 days, the corresponding diversity in the wind density is by factors of 0.3–1.7, assuming a constant \(R_i\) and an average \(\tau_{wd}\) of 15 days. If \(R_i = 10^{14} \text{ cm}\), then we obtain \(D = 1.5 \times 10^{15} \text{ g cm}^{-1}\) for \(\tau_{wd} = 1 \text{ day}\) and \(D = 8.2 \times 10^{15} \text{ g cm}^{-1}\) for \(\tau_{wd} = 30 \text{ days}\). Figure 1 shows that most \(\tau_{wd}\) are between 1 and 30 days in SNe IIn and the wind density is presumed to differ by factors of 0.3–1.7 if \(R_i\) is constant. If the wind velocity \(u_w\) is 100 \text{ km s}^{-1}, then the corresponding mass-loss rates of the progenitors are \(2.9 \times 10^{-3} M_\odot \text{ yr}^{-1}\) and \(1.6 \times 10^{-2} M_\odot \text{ yr}^{-1}\), respectively (Figure 2).

### 4. SUPERLUMINOUS SUPERNOVAE

The peak magnitudes of superluminous supernovae (SLSNe) are brighter than \(-21\) mag or roughly \(10^{44}\) erg \text{s}^{-1} (Gal-Yam 2012). Quimby et al. (2013) constructed the pseudo-bolometric LCs of SLSNe. The rise times of SLSNe IIn are typically larger than 40 days. This means that SLSNe IIn have both large \(\tau_{wd}\) and \(L_p\). The large diffusion time indicates that the wind is generally dense enough to cause the shock breakout in SLSNe, as is suggested by previous works (e.g., Chevalier & Irwin 2011).

The peak luminosities are typically more than about one order of magnitude larger than our standard \(L_p\) in Figure 1. We have shown that the peak luminosity does not strongly depend on the wind properties in the shock breakout model and is mostly determined by the SN ejecta properties. This means that if the SN ejecta mass of SLSNe is similar to other SNe IIn, then their SN kinetic energy needs to be higher by more than a factor of five to explain the huge luminosities (Equation (15)). Alternatively, the SN ejecta mass can be smaller by a factor of less than 0.1 \((n = 10)\) or 0.03 \((n = 7)\) if their SN ejecta energy is similar to the standard SNe IIn. The total energy emitted just by radiation in SLSNe IIn is typically more than \(10^{51}\) erg and it is likely that the SN energy is higher than for usual SNe.

We show that the large peak luminosities in SLSNe suggest large \(E_{ej}\) and/or small \(M_{ej}\). However, looking at Table 1, we find that larger \(E_{ej}\) and smaller \(M_{ej}\) both make \(\tau_{wd}\) smaller. However, \(\tau_{wd}\) in SLSNe IIn is much larger than those of SNe IIn. To make large \(\tau_{wd}\) with large \(E_{ej}\) and/or small \(M_{ej}\), the wind density must be very large. This indicates that an extremely large explosion energy (and/or the extremely small ejecta mass) as well as an extremely dense wind are required to explain both the large diffusion times and luminosities of SLSNe. Energetic explosions (and/or explosions with very small mass) need to be somehow accompanied by the formation of the dense wind. Detailed modeling of SLSNe also indicates the necessity of high explosion energy in the extremely dense wind (e.g., Ginzburg & Balberg 2012; Moriya et al. 2013a; Chatzopoulos et al. 2013).

### 5. CONCLUSIONS

We have investigated the diversity in rise times and peak luminosities in SNe IIn and related it to the diversity in the wind and SN properties. We have shown that the peak luminosities are mostly affected by the SN properties. The rise times that we relate to the diffusion time \(\tau_{wd}\) in the wind can be used to estimate the wind properties individually. We also note that the shock breakout does not necessarily occur in the wind, especially if the
progenitors are RSGs or LBVs, and we investigate the models with and without shock breakout.

The expected diversity in the SN ejecta properties estimated from the diversity in the SN IIn peak luminosities is shown in Figure 2. If the SN ejecta mass does not differ much in SNe IIn, then the shock breakout occurs in the wind, and the expected wind density for SNe IIn with large peak luminosities and large rise times (Figure 2). If the shock breakout occurs in the wind or not.

The diversity in the wind density can be estimated using the diversity in the rise times (Figure 2). If the shock breakout occurs in the wind, then the expected wind density for SNe IIn with similar peak luminosities differs if we assume that the shock breakout occurs in the wind or not.

SLSNe IIn show both large peak luminosities and large rise times. We suggest that both the high wind density and the high explosion energy and/or small ejecta mass are required to explain the properties of the SLSNe. The large rise times indicate that the shock breakout occurs in the wind in the SLSNe. The large peak luminosities indicate that the explosion energy is very large and/or the ejecta mass is very small. However, the large explosion energy and/or small ejecta mass make the diffusion time smaller. Thus, the large wind density is required to have large rise times. All together, not only the larger wind density but also the larger SN energy and/or the smaller SN ejecta mass than typical for SNe IIn are required to have large peak luminosities and large rise times at the same time as observed in SLSNe.


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APPENDIX

CONSTANTS

Constants which appear in the main text are

\[
C_1 = \left( \frac{2\pi(n - 4)(n - 3)(n - \delta)}{2\pi(n - 4)(n - 3)(n - \delta)} \right)^{\frac{n+3}{2}} \left[ \frac{2(5 - \delta)(n - 5)}{(3 - \delta)(n - 3)} \right]^{\frac{n+5}{2}} \right]^{\frac{1}{n}},
\]

(A1)

\[
C_2 = \left( \frac{2\pi(n - 4)(n - 3)(n - \delta)}{2\pi(n - 4)(n - 3)(n - \delta)} \right)^{\frac{n+3}{2}} \left[ \frac{2(5 - \delta)(n - 5)}{(3 - \delta)(n - 3)} \right]^{\frac{n+5}{2}} \right]^{\frac{1}{n}},
\]

(A2)

\[
C_3 = \frac{2\pi}{n - 5} \left( \frac{2(5 - \delta)(n - 5)}{(3 - \delta)(n - 3)} \right)^{\frac{n+5}{2}} \left( \frac{n - 3}{n - 2} \right)^{\frac{n+5}{2}} \right]^{\frac{1}{n}} \times \left( \frac{n - 2}{n - 3} \right)^{\frac{n+5}{2}} \right]^{\frac{1}{n}}, \quad (A3)
\]

and

\[
C_4 = \frac{2\pi}{n - 5} \left( \frac{2(5 - \delta)(n - 5)}{(3 - \delta)(n - 3)} \right)^{\frac{n+5}{2}} \left( \frac{n - 3}{n - 2} \right)^{\frac{n+5}{2}} \right]^{\frac{1}{n}} \times \left( \frac{n - 2}{n - 3} \right)^{\frac{n+5}{2}} \right]^{\frac{1}{n}}, \quad (A4)
\]

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