Application of the interacting boson model and the interacting boson-fermion model to $\beta$ decays

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Abstract. The $\beta$ decay is studied in the interacting boson model. The application to single-$\beta$ decay is extended to two-neutrino double-$\beta$ decay.

1. Introduction
The interacting boson model (IBM) [1, 2], including the interacting boson-fermion model (IBFM) [3] and the interaction boson-fermion-fermion model (IBFFM) [4], has been successful in describing the energy levels and the electromagnetic properties of various kinds of nuclei. One of the important applications is $\beta$ decay [5, 6, 7, 8, 9, 10, 11, 12, 13, 14] because $\beta$ decay is related to neutrino physics. In this school, I talk about the application of IBM to the $\beta$-decay. The decays include single-$\beta$ decay from odd-A nuclei as well as double $\beta$-decay from even-even nuclei [15].

2. Description of $\beta$-decay in IBM
Single-$\beta$ decay between odd-even nuclei has been studied in wide regions in IBM [5, 6, 8, 9, 10]. Double-$\beta$ in IBM decay was studied by Scholten and Yu [7]. Recently, Barea, Kotila and Iachello studied extensively both neutrino-less double-$\beta$ decay ($0\nu\beta\beta$) and two-neutrino double-$\beta$ decay ($2\nu\beta\beta$) [12]. These works use the closure approximation in treating the intermediate states in odd-odd nuclei. I talk about the work on $2\nu\beta\beta$ with Iachello [15].

For $2\nu\beta\beta$, the Gamow-Teller (GT) $M_{2\nu}^{GT}$ and the Fermi (F) matrix elements $M_{2\nu}^{F}$ are calculated by [16]

$$M_{2\nu}^{GT} = \sum_N \frac{1}{2} \langle 0^+_N | t^+ \sigma | 1^+_N \rangle \langle 1^+_N | t^+ \sigma | 0^+_I \rangle, \quad M_{2\nu}^{F} = \sum_N \frac{1}{2} \langle 0^+_N | t^+ \sigma | 0^+_N \rangle \langle 0^+_N | t^+ | 0^+_I \rangle$$

where $t^\pm$ is the isospin increasing/decreasing operator, $\sigma = 2s$ is the Pauli spin matrix, while $Q_{\beta\beta}$ is the $Q$ value of the double-$\beta$ decay, and $E_I$ and $E_N$ are the energies of the initial and the intermediate states, respectively. The proton-neutron IBM is used in which the even-even core of the nucleus is treated as a system of proton bosons and neutron bosons of angular momentum zero ($s$-bosons) or angular momentum two ($d$-bosons), which represent proton pairs and neutron pairs outside the closed shell. The microscopic theory of IBM gives the images of the Fermi and
Gamow-Teller transition operators as [5]

\[ t^\pm \rightarrow O^F = \sum_j -\sqrt{2j+1} [P^{(j)}_\pi P^{(j)}_{\nu}]^{(0)} ; \quad t^\pm \sigma \rightarrow O^{GT} = \sum_{j,j'} \eta_{jj'} [P^{(j')}_\pi P^{(j)}_{\nu}]^{(1)} \] (2)

where \( P^{(j)}_\pi \) represents the boson-fermion image of the particle-transfer operator expanded in terms of fermion \( a^\dagger_{j,m} \) and boson \( s^\dagger_{\pi,\nu} \) operators:

\[ P^{(j)}_\nu = \sum_{j'} \zeta_{jj'} a^\dagger_{j,m} + \sum_{j'} \zeta_{jj'} s^\dagger_{\pi,\nu} [\tilde{d} \times a^\dagger_{j',m}] \]

with proper coefficients [17]. From these the \( \log ft \) values can be calculated for \( \beta^- \) and \( \beta^+/EC \) transitions. In many calculations, the closure approximation is adopted in which the summation in (1) over the intermediate states \( N \) is replaced by the average. In the work that I am presenting, no closure approximation is made.

In the \( 2\nu\beta\beta \) decay from \(^{128}\)Te to \(^{128}\)Xe, the states in the intermediate nucleus \(^{128}\)I are accounted for in the proton-neutron IBFFM. Some of the low-lying states are shown in Fig. 1. In the summation for the matrix elements, the states \( 1^+ \) up to \( 3 \) MeV in excitation energy are included.

From \(^{128}\)I, some of single-\( \beta \) decay (\( \beta^- \), \( \beta^+/EC \)) are experimentally observed. Table 1 shows the \( \log ft \) values of electron capture (EC) from \(^{128}\)I, while those to \(^{128}\)Xe are shown in Table...

**Table 1.** The \( \log_{10} ft \) values of EC from \(^{128}\)I to \(^{128}\)Te. The data are from [18].

| transition | exp | cal | quenched |
|------------|-----|-----|----------|
| \( 1^+_1 \rightarrow 0^+_1 \) | 5.049 (7) | 3.836 | 5.15 (9) |

2. Introducing a common hindrance factor: \( h \approx 4.5 \), which is equivalent to a quenched axial vector coupling constant: \( g_{A,\text{eff}} = 1.269/h = 0.28 \), we obtain a reasonable agreement, as shown as “quenched” in the Tables. The \( B(GT) \) values from \(^{128}\)I have been also extracted from \((^3\text{He}, t)\) reaction [19]. The related values are: \(^{128}\)I, \( B(GT)_{[^3\text{He}, t]} = 0.079 \) (8); \( \sum = 0.829 \) (50). The corresponding IBM values are: \(^{128}\)I= 1.676, \( \sum = 15.09 \). If we use the same quenched value of...
Table 2. The log_{10} ft values of $\beta^-$ decay from $^{128}$I to $^{128}$Xe. The data are from [18].

| transition | exp  | cal  | quenched |
|------------|------|------|----------|
| $1^+_1 \to 0^+_1$ | 6.061 (5) | 4.665 | 5.98 (9) |
| $1^+_1 \to 0^+_2$ | 7.748 (24) | 5.262 | 6.57 (9) |
| $1^+_1 \to 0^+_3$ | 7.84 (6) | 5.712 | 7.02 (9) |
| $1^+_1 \to 2^+_1$ | 6.495 (7) | 5.212 | 6.52 (9) |
| $1^+_1 \to 2^+_2$ | 6.754 (9) | 6.446 | 7.76 (9) |

$g_{A,\text{eff},\beta} = 0.28$, then we have $B(\text{GT})[\text{IBM-quenched}] = 0.082$, $\sum[\text{IBM-quenched}] = 0.735$. These values are in good agreement with the ($^{3}{\text{He}}, t$) values.

Figure 2 shows the contributions from the intermediate states in $^{128}$I to the GT matrix element in Eq. (1). The single-state dominance (SSD) discussed in Refs. [13, 14], namely, the dominance of 1$_1^+$ in the summation, is seen in the figures. Similar analysis has been made for the Fermi decay, as well as those from $^{130}$Te. Table 3 shows the nuclear matrix elements calculated by (1) and a similar formula for the decay to a state 2$_2^+$. The inverse half-life of $0^+_1 \to 0^+_F$ can be calculated from

$$|M_{2\nu}^\text{calc}| = g_A^2 |M_{2\nu}^{\text{GT}} - \left(g_V/g_A\right)^2 M_{2\nu}^F|$$

(3)

by multiplying the lepton phase-space integral. Table 4 shows the thus obtained nuclear matrix element as “calu”. By introducing the same quenched $g_{A,\text{eff},\beta\beta} = g_{A,\text{eff},\beta} = 0.28$ in

$$|M_{2\nu}^\text{quenched}| = g_{A,\text{eff},\beta\beta}^2 |M_{2\nu}^\text{calc}|,$$

(4)

while the ratio $g_V/g_A = 1/1.269$ in (3) is fixed, we obtain the values shown as “quenched” in Table 4, which are consistent with the experimental values.
Table 3. Nuclear matrix elements $M_{2\nu}^{GT}$, $M_{2\nu}^{F}$ of transitions from the ground state of $^{128,130}$Te to some states in $^{128,130}$Xe. The sign of $M_{2\nu}^{GT}$ is chosen to be positive.

| transition       | $^{128}$Te$\rightarrow ^{128}$Xe | $^{130}$Te$\rightarrow ^{130}$Xe |
|------------------|----------------------------------|----------------------------------|
| $0^+_1 \rightarrow 0^+_1$ | 0.297                            | 0.273                            |
| $0^+_1 \rightarrow 2^+_1$ | 0.00718                          | 0.00639                          |
| $0^+_1 \rightarrow 0^+_2$ | 0.668                            |                                  |

| transition       | $^{128}$Te$\rightarrow ^{128}$Xe | $^{130}$Te$\rightarrow ^{130}$Xe |
|------------------|----------------------------------|----------------------------------|
| $0^+_1 \rightarrow 0^+_1$ | -0.0353                          | -0.0309                          |
| $0^+_1 \rightarrow 0^+_2$ | -0.112                            |                                  |

Table 4. Two-neutrino double-$\beta$ decay matrix elements, $|M_{2\nu}|$ in IBFM.

|                | exp      | calc     | quenched |
|----------------|----------|----------|----------|
| $^{128}$Te     | 0.044 (6) | 0.514    | 0.040 (8) |
| $^{130}$Te     | 0.031 (4) | 0.470    | 0.037 (8) |

3. Conclusion

Use of a single value of $g_{A,eff,\beta} = g_{A,eff,\beta}$ appears to describe well both single-$\beta$ and double-$\beta$ decay in a consistent way. The question of the small value of $g_{A,eff}$ is a subject of further study.

References

[1] Arima A and Iachello F 1975 Phys. Rev. Lett. 35 p 1069
[2] Iachello F and Arima A The Interacting Boson Model (Cambridge: Cambridge University Press)
[3] Iachello F and Van Isacker P 1991 The Interacting Boson-Fermion Model (Cambridge: Cambridge University Press)
[4] Brant S and Paar V 1988 Z. Phys. A 329 p 151
[5] Dellagiacoma F 1988 Ph. D. thesis (New Haven: Yale University)
[6] Dellagiacoma F and Iachello F 1989 Phys. Lett. B 218 p 399
[7] Scholten O and Yu Z R 1985 Phys. Lett. B 161 p 131
[8] Yoshida N, Zuffi L and Brant S 2002 Phys. Rev. C 66 014306
[9] Zuffi L, Brant S and Yoshida N 2003 Phys. Rev. C 68 034308
[10] Brant S, Yoshida N and Zuffi L 2004 Phys. Rev. C 70 054301
[11] Brant S Yoshida N and Zuffi L 2006 Phys. Rev. C 74 024303
[12] Barea J and F. Iachello F 2009 Phys. Rev. C 79 044301
[13] Kotila J and Iachello F 2012 Phys. Rev. C 85 034316
[14] Barea J, Kotila J and Iachello F 2013 Phys. Rev. C 87 014315
[15] Yoshida N and Iachello F 2013 Prog. Theor. Exp. Phys. 2013 043D01
[16] Tomoda T 1991 Rep. Prog. Phys. 54 p 53
[17] Scholten O 1980 Ph. D. thesis (The Netherlands: University of Groningen)
[18] Kanbe K and Kitao K 2001 Nucl. Data Sheets 94 p 227
[19] Puppe P, Adachi T, Akimune H, Ejiri H,Frekers D, Fujita H, Fujita Y, Fujiwara M, Ganoğlu E, Grewe E -W, Hatanaka K, Hodak R, Iwamoto C, Khai N T, Lennarz A, Okamoto A, Okamura H, Povinec P P, Susoy G, Suzuki T, Tamii A, Thies J H and Yosoi M 2012 Phys. Rev. C 86 044603