Firefly Algorithm for optimization problems with non-continuous variables: A Review and Analysis

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Abstract

Firefly algorithm is a swarm based metaheuristic algorithm inspired by the flashing behavior of fireflies. It is an effective and an easy to implement algorithm. It has been tested on different problems from different disciplines and found to be effective. Even though the algorithm is proposed for optimization problems with continuous variables, it has been modified and used for problems with non-continuous variables, including binary and integer valued problems. In this paper a detailed review of this modifications of firefly algorithm for problems with non-continuous variables will be discussed. The strength and weakness of the modifications along with possible future works will be presented.

Keywords: Firefly algorithm, optimization, bio-inspired algorithm, discrete variables, discrete firefly algorithm

1. Introduction

Optimization problems are problems of finding values for the variables which will give an optimum functional value of the objective function. This kind of problems exists beyond our daily activity. They are common problems, in engineering, decision science, agriculture, computer science, economics and many
other disciplines [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Based on the decision variables a solution can be classified into three categories as continuous variable, non-continuous variables and mixed variables. Continuous variables are when the variable can have any value in the given interval and if that is not the case then we have a problem with non-continuous variables which includes integer and binary variables. When some of the decision variables can be assigned with continuous and the rest with non-continuous values then it is called a mixed problem. Most of real optimization problems involves non-continuous variables like number of products and human resource.

For an optimization problem there are different solution methods. One class of solution methods is metaheuristic algorithms. These algorithms are a non-deterministic solution method which search the solution space based on an educated guess and 'trail and error' approach based on a given randomness term. Swarm based algorithms are a class of metaheuristic optimization algorithms which are inspired by the social behavior of animals. Firefly algorithm is one of swarm based metaheuristic algorithm inspired by the flashing behavior of flashing bugs also called fireflies [11]. Firefly algorithm is an easy to implement algorithm which can easily be implemented and can also easily be parallelized. It is also tested to be effected on problems from different problem domain. Even though it has a number of strength it also prone to parameter setting and also on controlling the exploration and exploitation of the search space. Hence, different modified versions are proposed to improve its performance as well as to make it useful for problems with non-continuous variables.

Hence in this paper, the modifications proposed to make firefly algorithm suitable for optimization problems with non-continuous variables will be discussed. A discussion on the strength and weakness of the modifications will be presented along with possible future work. In the next section a general discussion on optimization problems will be given followed by a discussion on firefly algorithm. In section 3, a discussion on modified versions of firefly algorithms will be given followed by a general discussion on the modifications in section 4. In section 5
conclusion will be presented.

2. Preliminary

2.1. Optimization problems

A given optimization problem has decision variables $x = (x_1, x_2, ..., x_n)$, for which we are search a value for, an objective function, $f(x)$, which is a function of the decision variable and also used to measure the performance of the values assigned to the decision variables and a feasible region, $S$, from which the decision variable can take values. A minimization problem can then be given as in equation (1):

$$\min_{x} \{ f(x) | x \in S \subseteq \mathbb{R}^n \}$$

The search space or the feasible region $S$ can be continuous, non-continuous or mixed i.e. continuous for some of the variables and non-continuous for the others possibly binary or integer.

A solution $x^*$ is said to be global (local) optimal solution for the minimization problem given in equation (1) if and only if $x \in S$ and $f(x^*) \leq f(x)$ for all $x \in S$ (for all $x$ in the neighborhood of $x^*$).

2.2. Firefly algorithm

Nature has been a motivation to different metaheuristic algorithms\cite{11, 12, 13}. The interaction between a predator and its pray, the attraction between bodies and the swarm behavior of fishes or birds can be mentioned as an example\cite{14, 15, 16, 17}. Firefly algorithm is inspired by the flashing behavior of flashing bugs also called fireflies. It is proposed for optimization problems with continuous variables \cite{11}. A randomly generated feasible solutions will be considered as fireflies where their brightness is determined by their performance on the
objective function. The algorithm is guided by three rules. The first rule is that fireflies are unisex, that means any firefly can be attracted to any other firefly. The second rule is the brightness of a firefly depends on its performance in the objective function. The attraction of a firefly depends on its brightness and decreases with distance. This means, since we are considering a minimization problem, a solution with smaller functional value is brighter. The light intensity has an inverse square law as given in equation (2).

\[ I \propto \frac{1}{r^2} \]  

(2)

where \( I \) is the intensity and \( r \) is the distance. The brightness follows similar rule as the light intensity with respect to the distance. Furthermore, suppose the light is passing through a medium with a light absorption coefficient of \( \gamma \). The the brightness of a firefly at a distance \( r \) can be summarized using the equation given in equation (3).

\[ \beta = \beta_0 e^{-\gamma r^2} \]  

(3)

where \( \beta \) is the brightness of the firefly at a distance \( r \) and \( \beta_0 \) is the brightness at the source, i.e. \( r = 0 \).

A solution \( x_i \) will be attracted by a brighter firefly \( x_j \), this means \( x_i \) moves towards \( x_j \) using (4).

\[ x_i := x_i + \beta_0 e^{-\gamma r_{ij}^2} (x_j - x_i) \]  

(4)

In addition it will explore using a random movement given by (5).

\[ x_i := x_i + \alpha (rand() - 0.5) \]  

(5)
for a step length for the random movement $\alpha$ and $rand()$ is an $n$ vector whose entries are generated randomly from a uniform distribution between zero and one.

Therefore, for the updating equation of a firefly $x_i$, by putting equations (4) and (5), is given as in equation (6).

$$x_i := x_i + \beta_0 e^{-\gamma r^2_{ij}} (x_j - x_i) + \alpha (rand() - 0.5)$$ (6)

for a brighter firefly $x_j$. If there is no brighter firefly than $x_i$ it will perform a random move only as given in equation (5).

The algorithm is summarized in Table 1.

| Table 1: The standard firefly algorithm |
|----------------------------------------|
| Set algorithm parameters ($\gamma, \alpha$) |
| Set simulation set-up (Maximum number of iteration ($MaxGen$), Number of initial solutions ($N$)) |
| Randomly generate $N$ feasible solutions ($x_1, x_2, ..., x_N$) |
| for iteration = 1 : $MaxGen$ |
| Compute the brightness |
| Sort the solutions in such a way that $I_i \geq I_{i-1}$, $\forall i$ |
| for $i = 1 : n - 1$ |
| for $j = i + 1 : n$ |
| if ($I_i > I_j$) |
| move firefly $i$ towards firefly $j$ |
| end if |
| end for |
| end for |
| move firefly $N$ randomly |
| end for |
3. Modified firefly algorithms for discrete optimization problems

Firefly algorithms has been modified and used for discrete optimization problems. Based on the space where the updating is performed, these modifications can generally be categorized into two categories. The first category is when the updating is done in the continuous space and a discretization mechanism is used to change the values to discrete numbers whereas the second is when the updating is done on the discrete space.

3.1. Updating in the continuous space

In this category the updating procedure of the standard firefly algorithm will be used and the result will be converted to discrete values.

Perhaps the first modification of firefly algorithm for discrete problems, specifically binary problems, is done for a job scheduling problem in [18]. After updating a solution $x_i$ using the updating equations of the standard firefly algorithm, each component $k$ of the solution vector $x_i$ will be converted to 0’s and 1’s based on the sigmoid function,

$$S(x_i(k)) = \frac{1}{1 + e^{-x_i(k)}}$$

as the probability that $x_i(k)$ will be 1. A similar approach in which

$$x_i(k) = \begin{cases} 1, & \text{rand} < S(x_i(k)) \\ 0, & \text{otherwise} \end{cases}$$

is used in [19, 20, 21, 22, 23]. In addition using the sigmoid function the tan hyperbolic function has also been used in some studies [24, 25, 26]. In [26], rather than using a random number to determine the value of $x_i(k)$, a given threshold, $\tau$, is used, i.e.

$$x_i(k) = \begin{cases} 1, & \tau < \text{tan}|x_i(k)| \\ 0, & \text{otherwise} \end{cases}$$

Furthermore, in [27] both the sigmoid as well as the tan hyperbolic functions are used to limit the value of $x_i(k)$ in between 0 and 1. The two functions are shown on Fig. 1 where the sigmoid function falls as an S-shaped function and the tan hyperbolic function as a V-shaped function.

In [28], four variants of the sigmoid functions, S-shaped functions, and four variants of the tan hyperbolic functions, V-shaped functions are given. The four
S-shaped functions are given by $S_1(x_i(k)) = \frac{1}{1+e^{-x_i(k)}}$, $S_2(x_i(k)) = \frac{1}{1+e^{-\frac{x_i(k)}{2}}}$, $S_3(x_i(k)) = \frac{1}{1+e^{-x_i(k)}}$ and $S_4(x_i(k)) = \frac{1}{1+e^{-\frac{x_i(k)}{2}}}$ and the four V-shaped functions are $V_1(x_i(k)) = |\text{erf}(\frac{\sqrt{2}}{\pi}x_i(k))|$, $V_2(x_i(k)) = \tanh(|x_i(k)|)$, $V_3(x_i(k)) = \frac{x}{\sqrt{1+x^2}}$ and $V_4(x_i(k)) = \left|\frac{\pi}{2}\arctan(\frac{\pi}{2}x_i(k))\right|$. The transformation functions are plotted and given in Fig. 1. After the transformation of the values to the interval $[0, 1]$, three methods are used to convert the values to a binary number. These methods are $x_i(k) = \{1, \text{rand} < T(x_i(k))\}$, $x_i(k) = \{x^*(k), \text{rand} < T(x_i(k))\}$, $x_i(k) = \{0, \text{otherwise}\}$ and $x_i(k) = \{(x_i(k))^{-1}, \text{rand} < T(x_i(k))\}$ where $x_i(k)$ is the $k^{th}$ component of $x_i$ from the previous iteration, $x^*$ the best solution so far from the memory and $(x_i(k))^{-1}$ gives the complement of $x_i(k)$, that is if $x_i(k) = 1$ its inverse will be 0 and if it is 0 its inverse will be 1.

Figure 1: S-shape and V-shape transfer functions.
In addition to using sigmoid function for the conversion of the variables to binary numbers, the updating formula was modified in [29, 30]. The distance is modified using sigmoid function using \( S(r) = |tanh(\lambda r)| \) for a parameter \( \lambda \) with value near 1 and based on this the updating formula is done using

\[
  x_i = \begin{cases} 
    x_i + \beta(x_j - x_i) + \alpha(rand() - 0.5), & rand < S(r) \\
    x_i, & otherwise
  \end{cases}
\]

If firefly \( x_i \) is closer \( x_j \) then it has less probability of moving. That may affect the quality of the solution as rather than improving or exploring the solution will stay in its position. In [31], another version of S-shaped sigmoid function given by \( S(x_i(k)) = 0.5(1 + erf(x_i)) \) is proposed.

In addition to modification for binary problems, firefly algorithm has also been used for optimization problems with integer valued variables. In [32, 33], the same updating formulas as the standard firefly algorithm used and the result is rounded to the nearest integer value. A similar approach is used for the mixed integer problem in [34]. Another modification for binary mixed problem is presented in [35]. The modification is done using a new updating formula given by \( x_i(k) := round\left(\frac{1}{1 + e^{-x_i(k) + rand(x_i - x_j)}} - 0.06\right) \).

Another modification in this category is done in [36, 37]. The modification is based on a concept of random key, which is proposed in [38]. The method uses a mapping of a random number space, \([0, 1]\), to the problem space. In other words, it encodes and decodes a solution with real numbers and these numbers, obtained randomly in a (0,1) uniform probabilistic distribution, are keys for sorting other numbers in order to form feasible solutions in an optimization problem and also in the updating process of firefly algorithm. Hence, after updating a solution using the standard firefly algorithm updating mechanism then it will be converted to integers using random key approach.

In addition to modifying the algorithm to suit the problem, different additional modifications are proposed to increase the effectiveness of the algorithm. In [32, 33, 34], the random movement step length was made adaptive as a function of the iteration number, \( Itr \), using \( \alpha = \alpha_0 \theta^{Itr} \), for a new algorithm parameter
where $0 < \theta < 1$. Fig. 2 shows the adaptive step length proposed for different values of $\theta$. Another modification on the random movement step length is given in [35], by $\alpha := \alpha \left( \frac{10}{9} \right) \frac{1}{\text{MaxItr}}$. Additionally, a scaling parameter, $x_{\text{max}} - x_{\text{min}}$, multiplies the random movement [32] [33]. The randomness step length $\alpha$ is modified using $\alpha = \alpha_0 - \frac{1}{1+e^{-\left( Itr - \frac{\text{MaxItr}}{2} \right)}}$, in [27]. Furthermore, in [27], $\alpha$ and $\gamma$ is modified based on the problem property. In [31], both $\alpha$ and $\gamma$ are made to change with iteration using $\alpha = \alpha_{\text{max}} - \frac{Itr}{\text{MaxItr}} (\alpha_{\text{max}} - \alpha_{\text{min}})$ and $\gamma = \gamma_{\text{max}} e^{\frac{Itr}{\text{MaxItr}} \ln \left( \frac{\gamma_{\text{min}}}{\gamma_{\text{max}}} \right)}$. In this modification $\alpha$ decreases linearly and $\gamma$ increases quicker than a linear function. This implies that as the iteration increases the random movement decreases so does the attraction step length. In addition Levy distribution is used to generate a random direction with a scaling parameter which is the difference between the maximum and minimum values of the feasible region. However, it should be noted that Levy distribution will generate a random direction with a step length and also $\alpha$ is used to control the step length hence adding additional scaling parameter may not be effective. That is because it is possible to control the step length based on $\alpha$ and the random vector generated by Levy distribution. Additionally, since $\alpha$ is made to decrease through iteration, its effect may not be seen due to this scaling parameter.
Figure 2: Different modifications proposed for $\alpha$, $\alpha''$ is the modification proposed in [127], $\alpha_L$ from [112], $\alpha_l$ from [108] and $\alpha'$ is from [125, 123, 126] with different values of $\theta$, for all the cases $\alpha_0 = \alpha_{\text{max}} = 2.5$ and for the $\alpha_L$ $\alpha_{\text{min}} = 0.1$.

3.2. Updating in the discrete space

In [39, 40], the standard firefly algorithm is modified for loading pattern enhancement. The generation of random solutions are using random permutation and the distance between fireflies are measured using hamming distance. Hamming distance of two vectors $x_i$ and $x_j$ is given by $d = |H|$ where $H$ is number of entries, $k$, for which $x_i(k) \neq x_j(k)$. The updating process is separated and made sequentially; first the $\beta$-step, a move due to the attraction and $\alpha$-step, a move due to the random movement. In the $\beta$-step, first same entries with same index for both fireflies, $x_i$ and $x_j$ will be preserved and for the other components an entry from $x_j$ will be copied if $\text{rand} < \beta$, where $\beta = \frac{1}{1+\gamma d^2}$. If $\text{rand} \geq \beta$, then the entry from $x_i$ will be used. After moving $x_i$ using the $\beta$-step the random movement or the $\alpha$-step will be used to update $x_i$ using $x_i := \text{round}(x_i + \alpha(\text{rand}() - 0.5))$ with a swapping mechanism to preserve feasibility. A similar approach, but different way of computing $\beta$ is proposed in [41]. It is computed based on the familiarity degree $P$, which is a random $N$ by $N$ vector initially and updates
by $P_{ij} = P_{ij} + \frac{1}{|rank_i - rank_j|}$ and the $\beta = e^{-\frac{(\max_k(P_{ik}) - P_{ij})^2}{\max_k(P_{ik})}}$. In addition to making the algorithm suitable for non-continuous variables, in [41], the randomness parameter $\alpha$ is made adaptive using $\alpha = \lfloor n - \frac{Itr}{\text{MaxItr}} \rfloor$, which is a function of the iteration number and the dimension of the problem. As can be seen from Fig.3, it decreases with iteration.

The modification proposed in [42] is similar with the modification proposed in [39, 40]. The only difference is in the updating mechanism, which is done based on a given probability parameter $\rho$, $\rho = 0.5 + \frac{0.5Itr}{\text{MaxItr}}$. It is the probability for a firefly $x_i$ to follow another brighter firefly $x_j$. However, if $\text{rand} > \rho$ firefly $x_i$ will move towards the brightest firefly of the fireflies.

In [43], after a randomly integer coded initial solutions are generated, hamming distance is used to compute the distance, $r$, between two solutions. Then a random number $R$ will be generated between 1 and $r$, and $R$ swaps will be done from the brighter firefly. A similar modification is proposed in [44]. In [44], after computing the humming distance, $r$, of two fireflies a random number

![Figure 3: The step length $\alpha$ for dimensions of $n = 2, 3, 4, 5, 6, 7, 8, 9$ and for 100 iterations](image-url)
between 2 and $r\gamma^{Itr}$ will be used to in stead of $r$ in [43]. Only improving solutions will be accepted.

For travel salesman problem, firefly algorithm has been modified in [45]. The distance between two solution is computed using $r = 10\frac{A}{n}$, where $A$ is number of different arcs and $n$ is the number of cities. A firefly $i$ moves towards brighter firefly $j$, $m$ times where $m$ is a new parameter to determine the number of moves, using inversion mutation to preserve feasibility. That is an initial chromosome is selected randomly and other entries will be filled using inversion mutation $m$ times to generate $m$ new solutions. Once all the $N$ solutions are moved $m$ times then the best $N$ solutions will be selected to pass to the next iteration.

Another modification is proposed in [46]. The updating for a solution $x_i$ towards a brighter firefly $x_j$ is given by $x_i(k) := \begin{cases} S_i(k), \alpha|rand - 0.5| < \beta_0e^{-\gamma r^2} \\ x_i(k), otherwise \end{cases}$, where $S_i(k) = \begin{cases} x_j(k), x_j(k) \neq x_i(k) \\ 0, otherwise \end{cases}$, for each dimension $k$. In addition to this, a firefly $x_i$ will be affected by fireflies in its visual range. That means for a firefly $x_i$ to move towards another firefly $x_j$, firefly $x_j$ should be brighter and also should be in $x_i$’s visual range. The visual range, $dv$, is computed using $dv = \begin{cases} \frac{3(dv_{max} - dv_{min})Itr}{2(MaxItr - 1)}, Itr < \frac{2MaxItr}{3} \\ dv_{max}, otherwise \end{cases}$. As can be seen from Fig. 4, the visual range increases until it reaches to the maximum value. This means that initially a brighter firefly affects solution near its location where in latter stages it can affect any firefly in the maximum allowed visual range.
A modification based on the problem property for supplier selection problem is also proposed in [47]. In addition in [48], firefly algorithm has been modified and used for knapsack problems. The discretization is done based on the problem property. In addition, a firefly $i$ will move towards a brighter firefly $j$ if $\text{rand} < \text{rank}_i^{-\frac{\text{mod}(\text{itr} - 1, \text{MaxIt})}{\text{MaxIt}}}$. The additional condition slowly vanishes as the iteration increases. Furthermore, $\beta$ is modified for computational reason using $\beta = \frac{\beta_0}{\omega + \text{itr}}$, for a very small number $\omega$ to omit the singularity case. A similar modification is used in [49]. In addition to the discretization done in [48], the authors in [49] proposed two additional moves after the updates. The first one is a random flight by 10% of top fireflies with 0.45 probability. The move will be accepted only if it is improving. The second is a local search of the brighter firefly. After 10% of the iterations it will do a local search and the update will be accepted if it is improving.
4. Discussion

Binary problems are one of the main class of problems with non-continuous variables for which firefly algorithm has been modified for. Most of these modifications deals with updating the continuous space using the updating formula of the standard firefly algorithm and changing the resulting solution to a binary number for each dimension. The conversion usually uses sigmoid functions which will convert the result to be in between zero and one. The decimal number then converted to a binary number. Updating the solutions using the updating formula of the standard firefly algorithm and converting the result is done not only for binary problems but also for other problems with non-continuous variables. However, it should be noted that updating on the continuous space push the solutions to the continuous solution which can be far away from the solution for the discrete variables. For example if we consider the function given in Fig. 5, the solution for the continuous case is around \( x = 5.5 \) and the nearest integer is \( x = 5 \) and \( x = 6 \). However, for the integer valued problem the solution is at \( x = 1 \). Hence, updating on the discrete space has an advantage.

![Figure 5: Discrete versus continuous space search where the black circles representing the integer functional value and the red graph is the corresponding continuous function](image)

In updating in the discrete space usually done based on the coding used which
is based on the properties of the problems. The distance between two fireflies are measured based on the difference in the entries or based on the difference sequence of the entries. Swapping using different approaches are used. A research on which approach is better has not be studied and can be studied in the future. Modifying an algorithm to suit a given problem has also been done and it is often effective. Another possible future work is generalizing the approach so that a given modification in one problem domain can be used in another.

Apart from making the algorithm suitable for problems with non-continuous variables the algorithm parameters are modified. Making the randomness step length $\alpha$ decreasing with iteration is a good idea in order to archive quality solutions. In addition it has also been modified based on the problem dimension however appropriate scaling parameter needs to accompany the modification. Perhaps it is also an ideal issue to explore the adaptive step length which varies based on the performance and also which can possibly be increasing and also decreasing based on its current status.

Another possible future work is to merge the search mechanism both on the discrete as well as the continuous space. Perhaps the advantage of the search on continuous space and also discrete space can be combined to give a superior performance.

5. Conclusion

Firefly algorithm has been proposed for continuous problems. However, due to the application of optimization problems with non-continuous problems, it has been modified and used in different studies. The modification basically can be categorized into two. The first category is where the updating mechanism is done on the continuous space and the result is converted to the discrete values. For this purpose different sigmoid and tan hyperbolic functions are used most of the time. The second category is when the update is performed on the discrete space. This approach has an advantage over the first as it will may lead the
solutions in a local minimum or wrong direction. In this paper, a review on these modifications along with possible future works is discussed.

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