Post-inflationary thermalization with hadronization scenario

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We study thermalization of the early Universe when the inflaton can decay into the Standard Model (SM) quarks and gluons, using QCD arguments. We describe the possible formation of the thermal plasma of soft gluons and quarks well before the completion of reheating. Relevant interaction rates of leading order processes and the corresponding thermalization time scale is presented. We discuss hadronization while thermalizing the decay products of the inflaton, when the reheat temperature of the Universe is below the QCD phase transition but above the temperature of the Big Bang nucleosynthesis.

I. INTRODUCTION

After the end of inflation, which is required to solve some of the outstanding issues of hot Big Bang cosmology [1], the homogeneous inflaton starts oscillating around its minimum. During this period the average oscillations of the inflaton mimics the equation of state of a pressureless fluid. The inflaton oscillations persist until the inflaton decays and fragments in order to provide a bath of relativistic species in kinetic and chemical equilibrium. In the literature the temperature of the relativistic thermal bath acquired from the complete decay products of the inflaton is known as the reheat temperature: \( T_{\text{rh}} \). The equilibration of the bath takes finite time and the whole process is known as thermalization of the inflaton decay products [2]. The time scale of thermalization depends sensitively on the inflaton coupling to the decay products, which is usually not known when the inflaton is considered to be a SM and SU(3) gauge singlet, [3].

Note that it is important for Big Bang nucleosynthesis (BBN) that at the time when the electroweak interaction between neutrons and protons freezes out (around the temperature of few MeV) the Universe must be dominated by the SM relativistic degrees of freedom. Therefore it becomes important that the inflaton must couple to the SM fields if not directly but via non-renormalizable interactions *.

Although a thermal bath of temperature \( \sim 1 \text{ MeV} \) is necessary, there is no direct evidence of a thermal history beyond the BBN era. Therefore, the reheat temperature could in principle lie anywhere between \( m_\phi \geq T_{\text{rh}} \geq O(1) \text{ MeV} \), where \( m_\phi \) is the inflaton mass. In supersymmetric (SUSY) theories there is an upper bound on reheat temperature arising from thermal production of the superpartner of the graviton: the gravitino [6,7]. For the gravitino mass \( \sim 100 \text{ GeV} \), \( T_{\text{rh}} \leq 10^9 \text{ GeV} \) [7] (assuming that the Universe expands adiabatically from \( T_{\text{rh}} \) onwards).

The issue of thermalization has been addressed in many papers [8–15] and very recently some concrete ideas have been put forward in [16,17]. In earlier papers [8,9,13] elastic interactions, such as \( 2 \to 2 \) scattering and annihilation have been considered, while in [10,11], it was shown that elastic collisions lead to kinetic equilibrium while redistributing the energy density, and \( 2 \to 3 \) (particle number changing) processes that lead to chemical equilibrium take a longer time. In [14], it was suggested that elastic scattering followed by prompt decay might lead to rapid thermalization. The new idea behind rapid thermalization was proposed in [16], where inelastic scattering such as \( 2 \to 3 \) process have been invoked from the very beginning. It was found that thermalization time scale is roughly given by the inverse of the inelastic scattering rate \( T_{\text{inel}} \). In this paper, we consider thermalization with a hadronization scenario, where \( T_{\text{rh}} \) might be below the QCD scale \( \Lambda_{\text{QCD}} \sim 0.2 \text{ GeV} \). Especially when the inflaton scale is as low as \( m_\phi \sim O(1) \text{ TeV} \) [18,19], and the reheat temperature is below the QCD scale \( \sim 200 \text{ MeV} \), then the importance of thermalization and hadronization of the quark gluon plasma (QGP) becomes an interesting issue. Further note that the inflaton decay products still have energy ranging from \( O(m_\phi) \) down to \( T_{\text{rh}} \). The hard hitting quarks and gluons must lose their initial momentum towards the last stages of reheating. Depending on whether there is a hadronic bath, or a soft bath of quarks and gluons, the inflaton decay products will lose their energy differently. Our main goal in this paper is a qualitative understanding of thermalization and hadronization process in a cosmological context with an order of magnitude estimations.

II. INFLATON DECAY TO QUARKS AND GLUONS

After inflation has ceased, the inflaton field \( \phi \) begins to execute coherent oscillations about the minimum of its potential. The perturbative decay occurs over several
such oscillations\(^1\). The initial stage of reheating begins right after \(H_{inf} \approx m_\phi\), where \(H_{inf}\) denotes the Hubble expansion of the Universe towards the end of inflation, and \(m_\phi\) denotes the mass of the inflaton. Note that the inflaton oscillations always dominate the energy density until the inflaton decay is completed.

\[
H^2(a) \approx \frac{\rho_i}{3M_P^2} \left( \frac{a_{in}}{a} \right)^3 \sim \Gamma_\phi^2, \tag{1}
\]

where \(\Gamma_\phi\) is the inflaton decay rate, and \(\rho_i\) is the energy density stored in the inflaton sector, and \(a\) is the scale factor of expansion. The subscript \(in\) denotes the initial time which can be taken to be the onset of inflaton oscillations. The inflaton oscillations dominate until \(\tau \sim \Gamma_\phi^{-1}\). The inflaton energy goes into a thermal bath of relativistic particles whose energy density is determined by the reheating temperature \(T_{reh}\), given by

\[
T_{reh} \sim g_*^{-1/4} (\Gamma_\phi M_P)^{1/2}, \tag{2}
\]

where \(g_*\) is the number of relativistic degrees of freedom. The above estimation is simple and it still stands as the only valid estimate of the reheating temperature.

As \(\phi\) is a gauge singlet with respect to all gauge symmetries under consideration, it follows that for the case of QCD with quarks, it can couple only to color singlets, and to SU(2) gauge singlets constructed from the left-handed quark field which forms a doublet in the standard model.

In this paper we mainly concentrate upon interactions

\[
\mathcal{L}_{\phi gg} = \frac{\phi}{M_P} F^{\mu \nu} F_{\mu \nu}, \tag{3}
\]

which describes the perturbative decay of the inflaton to gluons. Inflatons could also decay into SM quarks through

\[
\mathcal{L}_{\phi qq} = \frac{\phi}{M_P} (H \bar{q}_L q_R), \tag{4}
\]

where \(H\) is the Higgs doublet, \(q_L\) is the SU(2) doublet, and \(q_R\) is singlet. The neutral Higgs scalar eventually decays into the SM quarks and leptons via Yukawa couplings.

We can now estimate the number density of hard hitting quarks and gluons at time: \(\tau > \tau_{in}\), where \(\tau_{in}\) is the reference time when the inflaton starts oscillating. The number density is given by

\[
n_\chi(\tau) = 2n_\phi(a_{in})(1 - e^{-\Gamma_\phi(\tau - \tau_{in})}) \left( \frac{a_{in}}{a} \right)^3, \tag{5}
\]

where \(n_\phi(a_{in}) = \rho_\phi(a_{in})/m_\phi\), and we collectively designate hard hitting quarks and gluons by \(\chi\). Note that we are neglecting here the particle number changing processes in the above estimation.

### III. FORMATION OF THE THERMAL BATH OF SOFT QUARKS AND GLUONS

The inflaton decay rapidly builds up a large number density of quarks and gluons. Note that fragmentation or hadronization cannot occur in the early stages of the inflaton decay, the formation of bound states being prevented by the IR cutoff imposed by the finite expansion rate of the universe. The time scale of expansion during inflaton oscillations is \(\tau_{exp} \sim 1/H(\tau) \geq 1/m_\phi\) which is much smaller than the typical time scale of strong interactions \(\tau_s \sim GeV^{-1}\) (for \(m_\phi \gg O(1)\) GeV). However, these mechanisms will be important towards the end of the inflaton decay, and we will discuss these in section V.

We now ask the question as to how this dense system of hard quarks and gluons will thermalize, what the corresponding temperature would be and estimate the time to reach the final reheating temperature \(T_{reh}\). A rigorous study of thermalization would involve solving numerically the relativistic Boltzmann equation for the time evolution of the quark and gluon densities, with splitting functions incorporated from pQCD, and consequent logarithmic corrections to the quark and gluon densities. A full calculation of the parton cascade evolution needs to be done in the expanding space-time background, which is beyond the scope of the present work. It is worthwhile at this stage to review and recast the arguments for rapid thermalization that appear in references [16,17] in a QCD based approach.

The mechanism therein involves creating a soft thermal bath of gauge bosons from scattering of hard particles accompanied by the emission of a soft gluon from one of the legs of the \(2 \rightarrow 2\) diagram. This is the gluon bremsstrahlung. The emitted soft gluon can now be involved in a scattering process with another quark (or gluon), thereby leading to an exponential growth in the number density of soft particles. Due to pair annihilation processes, the thermal bath in QCD will be composed of soft quarks as well as soft gluons. The soft quarks and gluons can form a thermal plasma with an instantaneous temperature [2]

\[
T_{inst} \sim \left( g_*^{-1/2} H(\tau_{inst}) \Gamma_\phi M_P^2 \right)^{1/4}, \tag{6}
\]

where \(H(\tau_{inst}) \geq \Gamma_\phi\) is the Hubble parameter at the time when the soft thermal bath is created. This instantaneous temperature reaches its maximum \(T_{max} \leq m_\phi\) soon after the inflaton field starts to oscillate around the

\(^1\)The initial stages of the inflaton oscillations could give rise to non-thermal production of bosons and fermions, usually known as preheating [20,21]. Alternatively the inflaton condensate could fragment due to running mass [22,23]. It is important to highlight that preheating does not give rise to thermalization. Preheating produces multi particle non-thermal spectrum at initial stages which still requires to be thermalized with chemical and kinetic equilibrium.
minimum of its potential. Also an important point to note here is that since the inflaton energy density is still dominating \( \rho_\phi \approx H^2 M_\phi^2 \sim a^{-3/2} \), the instantaneous temperature falls as

\[
T_{\text{inst}} \sim a^{-3/8} \left( \frac{g_*^{-1/2} \Gamma_\phi}{H} \right)^{1/4},
\]

instead of \( T_{\text{inst}} \sim a^{-1} \). Numerical simulations also support this argument [24]. In the next section we estimate \( T_{\text{max}} \).

For elastic processes, one cannot reduce the average energy of the system quickly since no new particles are created (we are ignoring the red-shift of the particle momenta for the moment). The Bremsstrahlung process redistributes the energy among softer constituents and these soft particles act as a very efficient sink of energy once the bath starts to form. An analogous mechanism for heavy-ion collisions was suggested by Shuryak [25] that a thermally equilibrated quark-gluon plasma (QGP) would be created in a two-step process ("hot-glue scenario"), wherein the gluonic bath would be created in a much shorter time than it took the quarks to thermalize.

We now proceed to make some quantitative estimates for the cross-sections and scattering rates. Note that these are order of magnitude estimates, and more accurate methods such as using Boltzmann equations with collision terms taken from perturbative QCD may be developed as has been done for the finite size system in heavy-ion collisions [26]. However, in the cosmological context, it is adequate to first obtain an order of magnitude estimate for a proposed physical scenario.

The \( 2 \rightarrow 3 \) processes and higher order \( (2 \rightarrow n) \) gluon branching will lead to the formation of a plasma of soft quarks and gluons. When this is eventually formed, exchanged gluons will be screened over a distance scale \( \mu^{-1} \sim (1/gT) \) corresponding to the momentum scale \( gT \) [27]. In the case of quarks being exchanged in a plasma, we can use the cutoff given by the kinetic theory which gives the quarks an effective mass of \( \mathcal{O}(gT) \). A more consistent perturbative approach in a thermal plasma would be to the Braaten-Pisarski re-summation scheme [28], which incorporates effective propagators and vertices for the quark and gluons, and amounts to a re-ordering of perturbation theory. Since we are interested only in order of magnitude estimations here, we choose the bare propagators and minimal expressions for the cutoff.

For a sufficiently dilute system of quarks and gluons, the concept of a thermal plasma does not make sense, in which case, we choose to regulate in the infra-red by using, at any instant, the inverse of the inter-particle separation of the decay products of the inflaton (hard quarks and gluons which we denote by \( \chi \)). This is determined by Eq. (5) at any instant of time. For \( \Gamma_\phi \leq H(\tau) \), we expand the right hand side of the expression Eq. (5) in terms of \( \Gamma_\phi/H \), and with the help of Eq. (6), we immediately obtain the number density \( n_\chi \) of hard quarks and gluons, and from it, the infra-red cutoff

\[
n_\chi^{-1/3} \sim \left( \frac{T_{\text{inst}}^4}{m_\phi} \right)^{-1/3}.
\]

It should be noted that the expressions for energy transfer will be less divergent than those for the absolute cross-section since the energy transfer also tends to zero for very soft scattering. This reduces the efficacy of soft processes for thermalization.

As emerged in studies of thermal and chemical equilibration in heavy-ion collisions, inelastic processes \( (2 \rightarrow n) \), where \( n > 2 \) are likely to be the dominant mechanism for rapid thermalization and entropy generation. Within perturbative QCD itself, it has been shown that processes that are higher order in \( g_\alpha \) can be more important for equilibration. Thus, gluon branching from a quark or gluon line must be taken into account.

To achieve thermalization at a temperature well below \( m_\phi \), one needs to generate many soft particles so that while the energy density remains constant, the number density rapidly increases. Then the average energy per particle also decreases. The creation of a soft gluonic bath can proceed via Bremsstrahlung emission of a gluon from a quark or gluon participating in a scattering process \( (2 \rightarrow 3 \) process). The energy loss due to Bremsstrahlung can be mitigated by the Landau-Pomeranchuk-Migdal (LPM) effect [29] in a dense medium. This effect has been studied in a QED [30] as well as in a QCD plasma, with significant modifications in the energy loss profile of a parton jet traversing a QCD plasma [31]. If the mean free path of the incident parton is small compared to the formation time of the emitted gluon (which can become large for small angle scattering by uncertainty principle arguments), interference effects between multiple scattering has to be taken into account. This issue has been addressed in calculations of photon and gluon production rates from an equilibrated QGP, as well as in the phenomenon of jet quenching. The importance of this effect can be gauged as follows.

Suppose the incident quark of energy \( E_q \) emits at a small angle \( \theta \), a gluon \( (k^\mu = (\omega_g, k_z, k_l)) \) with energy \( \omega_g \sim T_{\text{inst}} \). The formation time of the emitted gluon would be given by the uncertainty principle as [31]

\[
\tau(k) \sim \frac{2\omega_g}{k^2} \sim \frac{2}{\omega_g \theta^2},
\]

where \( \theta = k_l/\omega_g \). The ratio of the mean free path to the formation time is given by

\[
\frac{\lambda}{\tau(k)} \sim \frac{1}{\omega_g \theta^2}.
\]
\[ \frac{L}{\tau} \sim \frac{1}{n_\chi \sigma_{\text{inel}}} \frac{\omega_\rho g^2}{2}, \quad (10) \]

where \( \sigma_{\text{inel}} = \sigma_{\chi \chi g} \) is the inelastic cross-section. It has a logarithmic dependence on \( t \) from the Bremsstrahlung emission, and a \( 1/t \) dependence from the exchanged momentum. The extra emitted gluon gives one extra power of \( \alpha_s \) compared to the elastic case leading to

\[ \sigma_{\text{inel}} \sim \frac{\alpha_s^3}{t_{\text{min}}} \log \left( \frac{m_\phi^2}{t_{\text{min}}} \right), \quad (11) \]

Substituting the above equation into Eq. (10), and using the fact that \( t_{\text{min}} \sim g^2 T_{\text{inst}}^2 \) and \( n_\chi \sim T_{\text{inst}}^3/m_\phi \), we find

\[ \frac{L}{\tau} \sim \frac{m_\phi}{g^4 T_{\text{inst}}^4 \log \left( \frac{m_\phi^2}{g^2 T_{\text{inst}}^2} \right)} \quad \text{(12)} \]

If this ratio is much less than one, the LPM effect causes suppression of energy loss, and consequently, the hard quarks (or gluons) lose their energy at a slower rate. This can weaken the functional dependence of \( \frac{dE}{d\tau} \) on the incident energy [31], leading to a suppression of energy loss, and an increase in thermalization time. From Eq. (12), it is evident that for a given \( 4\pi \alpha_s \ll 1 \), the assumption of additive energy loss from successive scattering applies, since the fraction \( T_{\text{inst}}/m_\phi \) is much less than unity implying that \( L/\tau \gg 1 \) (the logarithm cannot counteract the dependence from the prefactor).

Continuing with the inelastic process in the limit \( L/\tau \gg 1 \) (additive energy loss), the time scale for an inflaton decay product to lose an energy \( \sim m_\phi \) by such processes turns out to be much smaller than in the elastic case, as was shown in [16]. The reason is that the soft gluon population (neglecting its annihilation to fermions) grows exponentially since each soft gluon produced radiatively also acts as a subsequent scattering center for the production of further soft gluons. The bath will be actually composed of a certain number of fermions as well, due to the forward and backward annihilation processes between quarks and gluons. However, for our estimation purposes, it only matters that hard quarks/gluons are scattering off soft particles, whose total number is important though not its species content. Due to the (almost) exponential increase in the number of soft particles, we estimate that the rate of energy loss is given by

\[ \frac{dE}{d\tau} \approx n_g \sigma_{\text{inel}} T_{\text{inst}}, \quad (13) \]

where \( n_g \) is the number density of the soft quarks (and gluons) \( n_g \sim g_s T_{\text{inst}}^3 \) \( ^\S \)

\[ ^\S \text{We are assuming, as in [16], that the average energy loss is} \]

The inelastic scattering rate is then given by

\[ \Gamma_{\text{inel}} = \frac{1}{E} \frac{dE}{d\tau} \sim \frac{g_s \alpha_s^2 T_{\text{inst}}^2}{4\pi m_\phi} \log \left( \frac{m_\phi^2}{g^2 T_{\text{inst}}^2} \right). \quad (14) \]

Note that if we were to consider higher order gluon emission (2 or more emitted gluons), \( \Gamma_{\text{inel}} \) would be suppressed by further powers of \( \alpha_s \) and will not compete with the \( 2 \to 3 \) process. Furthermore, emission of gluons softer than \( T_{\text{inst}} \), which would ultimately require handling collinear divergences are not considered since those ultra soft gluons would have to be re-scattered back up to a momentum of \( T_{\text{inst}} \). As we are interested only in the time scale to create a thermalized plasma at roughly \( T_{\text{inst}} \), this aspect will not concern us here [16]. We will return to the point about the \( 2 \to n \) processes at the end of this section. Continuing with our estimate, the time to lose an energy \( m_\phi \) is then

\[ \tau_{\text{inel}} = \Gamma_{\text{inel}}^{-1} \sim \frac{m_\phi}{g_\phi \alpha_s^2 T_{\text{inst}}^2} \sim \frac{4\pi m_\phi}{g_\phi \alpha_s^2 T_{\text{inst}}^2} \log \left( \frac{m_\phi^2}{g^2 T_{\text{inst}}^2} \right). \quad (15) \]

Note that this result is a factor \( 4\pi \alpha_s/g_\phi \) different than previous estimates [16], because the lower cutoff is taken as the screening mass rather than the typical temperature (which would be the energy of the individual constituents themselves, and would be ignorant of many-body effects).

Now we are able to estimate the largest temperature of the instantaneous thermal bath before reheating. By using \( H(t_{\text{inst}}) \simeq \Gamma_{\text{inel}}, \) Eq. (6), and \( \Gamma_{\text{inc}} = g_s^{1/4} T_{\text{inst}}^2/M_P \), we obtain the maximum instantaneous temperature

\[ \frac{T_{\text{max}}}{T_{\text{rh}}} \approx \left( g_s^{3/4} \alpha_s^2 \frac{M_P}{4\pi m_\phi} \right)^{1/2}. \quad (16) \]

Note that \( T_{\text{max}} \) is couple of magnitudes larger than the reheat temperature, but still smaller compared to the inflaton mass: \( T_{\text{max}} \leq m_\phi \).

It is sufficient for us to estimate the thermalization time by studying \( 2 \to 3 \) processes, since further gluon branching \( (2 \to n, n > 3) \) will come with additional factors of \( \alpha_s \ll 1 \). The lower scattering cross-section for such processes renders them higher-order corrections to the inelastic scattering rate and the thermalization time scale. This conclusion can change if the mean free path of the hard particles becomes comparable to the formation time of the emitted gluon, i.e., if the LPM suppression is severe.

\[ \sim T_{\text{inst}}. \] This is not strictly true if one considers the collinear behavior of the Bremsstrahlung process, but we can ignore \( \log(m_\phi/T) \) corrections which only speed up the thermalization rate by a numerical factor of order one.
IV. TOWARDS HADRONIZATION

Thus far, we have described a generic scenario towards reheating the Universe without bothering too much on the reheating temperature itself. An implicit assumption was made that the reheating temperature was sufficiently above the QCD scale. However, a priori there is no reason that the Universe could not be reheated at a temperature lower than the QCD scale and above the BBN temperature. Such a scenario is plausible if the inflaton scale is sufficiently low. Keeping the reheating temperature $T_{rh} < \Lambda_{QCD} \approx 200$ MeV, we will concentrate upon the hadronization issue.

Naturally if the thermal bath of soft quarks and gluons is formed near $T_{inst} \sim T_c$, where $T_c$ is the critical temperature ($\sim 200$ MeV), the quarks and gluons will rapidly hadronize. However, if the thermal bath is initially formed at a much higher temperature than a GeV, it will hadronize at a later time when the expansion of the universe has red-shifted the energy down to the confinement scale. This is particularly important if thermalization occurs within a Hubble time $H^{-1}$.

Let us imagine that there already exists a plasma of soft quarks and gluons. Since the temperature of the plasma is inversely related to the scale factor of the FRW metric as $T \propto a^{-3/8}$, it follows that during the inflaton oscillation dominated phase

$$\frac{T(\tau)}{T(\tau_0)} = \left(\frac{\tau_0}{\tau}\right)^{1/4}$$

and

$$\frac{s(\tau)}{s(\tau_0)} = \frac{T^3(\tau)a^3(\tau)}{T^3(\tau_0)a^3(\tau_0)} = \left(\frac{\tau}{\tau_0}\right)^{5/4}.$$

where $\tau_0$ denotes any reference time. It is interesting to mention here that similar relations are derived for the evolution of the relativistic plasma of quarks and gluons formed in heavy-ion collisions **.

At time $\tau = \tau_c$ when $T = T_c \sim 200$ MeV which is the critical temperature for hadronization, the fluid is dilute enough that strong non-perturbative forces recombine the quark-gluon soup into colorless hadrons. This constitutes a phase transition whose nature depends on the number of light quark flavors (from universality arguments [33]) and the quark masses (which act as external fields). Assuming the strange quark mass to be light on the $\Lambda_{QCD}$ scale, we can say that the hadronization proceeds by a first order phase transition with bubbles of hadron matter appearing within the quark-gluon plasma, which grows in number till the end of the phase transition [34]. The time for this transition to occur is denoted by $\tau_h$; the hadronization time.

It is possible to estimate $\tau_h$ as follows [35]. Throughout the phase transition, the temperature remains constant at $T_c$, while entropy is decreased in going from the quark-gluon to the hadronic phase. This means that Eq. (17) cannot be applicable in describing the phase transition. It is helpful to imagine the quark gluon plasma as a subsystem where the temperature remains $T(\tau_0) = T(\tau) = T_c \sim 200$ MeV for the duration of the phase transition. Then, the ratio of the entropy densities is simply the ratio of $\epsilon + p$ of the plasma, which scales as the fourth power of temperature. From Eq. (17), it follows that

$$\frac{s(\tau)}{s(\tau_c)} = \frac{\epsilon(\tau) + p(\tau)}{\epsilon(\tau_c) + p(\tau_c)} \cdot \frac{T_c}{\tau_c}.$$

where

$$\frac{s(\tau)}{s(\tau_c)} = \left(\frac{T(\tau)}{T(\tau_0)}\right)^{5/4}.$$  \hspace{1cm} (18)

In the case of heavy-ion collisions, a rerun of the above steps yields the relation $s(\tau)/s(\tau_c) = (\tau_c/\tau)^{4/3}$.

This means that the phase transition in the early universe occurs at a slower rate than in heavy-ion collisions, since the entropy/particle is some fixed quantity in the two different phases. We will see that this fact is born out by our final expression for the hadronization time.

During the transition, the system can be described by a mixed phase, with a fraction $f(\tau)$ in the quark-gluon phase ($1 - f(\tau)$ in the hadronic phase). Since entropy is additive, we find using Eq. (18),

$$f(\tau) = \frac{1}{s_{ng}(T_c) - s_h(T_c)} \left(\frac{\left[f(\tau_c)s_{ng}(T_c) + [1 - f(\tau_c)]\right]}{s_h(T_c)} \times \frac{\tau_c}{\tau} - s_h(T_c)\right).$$

The entropy densities in the two phases can be traded for the respective degeneracies ($g_{ng} \sim 37$ and $g_h \sim 3$) to give

$$f(\tau) = \frac{1}{g_{ng} - g_h} \left(\frac{f(\tau_c)g_{ng} + [1 - f(\tau_c)]g_h}{\tau_c - g_h}\right).$$

At $\tau = \tau_h$, the quark-gluon phase is completely hadronized. Thus, it follows from the above equation that

$$\tau_h = \left[\frac{g_{ng}}{g_h} f(\tau_c) + 1 - f(\tau_c)\right] \tau_c.$$  \hspace{1cm} (21)

Starting with quark-gluon matter at $f(\tau_c) = 1$, we find that $\tau_h \sim 10\tau_c$. In heavy-ion collisions, this relation gives $\tau_h \sim 6\tau_c$. **

** There, it is appropriate to adopt Bjorken’s idealized hydrodynamical scenario of the quark-gluon plasma as an expanding relativistic fluid [32]. Approximate longitudinal boost invariance and the ideal equation of state for a relativistic fluid imply that the temperature and entropy density depend on the proper time $\tau$ as $T(\tau)/T(\tau_0) = (\tau_0/\tau)^{1/3}$, and $s(\tau)/s(\tau_0) = (\tau_0/\tau)$. **

††In the heavy-ion case, there is a power of 3/4 for the term
Coming back to Eq. (18), we observe that $\tau_c$ denotes the time for the plasma to cool down to the critical temperature for hadronization $T_c$. The origin of time is taken to be the moment when the inflaton begins its oscillations. Taking $\tau_{\text{infl}} \sim \frac{1}{m_\phi}$, we can estimate an upper bound on $\tau_c$ as

$$\tau_c \leq \Gamma_\phi^{-1} \sim \frac{1}{m_\phi} \alpha_\phi^{-1}, \quad (22)$$

if the universe is to be reheated below the QCD phase transition. Here, $\alpha_\phi$ is the non-renormalizable coupling of the inflaton to the SM quarks and leptons. For a low reheating temperature, a small inflaton coupling as set by the above equation is therefore necessary. It is known that this time in the lab frame must be on the $\mu$s time scale.

Unlike for the case of heavy-ion collisions, where hadrons, once formed, decouple chemically and thermally from each other after a certain time and stream freely to the particle detectors, that may not be the case here. Since the hadronization time is short compared to the inflaton decay lifetime, it follows that the (hard) inflaton decay products will continue impinging on the newly-formed hadrons and cause break-up due to the large energy transfer. How efficient this process is depends on the number density of these hard particles. One also needs some estimate of the rate of energy lost by these hard particles traveling now in a hadronic phase rather than a soft gluonic bath. It is interesting to compare this mode of energy loss for the hard particles to the previously considered inelastic scattering process. We also expect fragmentation to be important for lowering the average energy, since energetic partons can now fragment into a large number of less energetic hadrons, which can re-scatter inelastically among themselves, or resonance decay into lighter hadrons. We comment on the relative importance of these processes below.

V. FINAL STAGES OF HADRONIZATION

A. Deep-inelastic scattering (DIS)

The break-up of hadrons by hard-hitting quarks can be assessed through DIS. The structure functions at some $x, Q^2$ can be obtained from the parton distributions that appear in the DGLAP equation [36]

$$\frac{d^2 f_i(x, Q^2)}{dx} = \frac{\alpha_s}{4\pi} \int_{x}^{1} \frac{dy}{y} P_{ij}(z) f_j(y, Q^2) \quad (23)$$

within square brackets in Eq. (21), which follows from the different scaling law of entropy density with time. The fractional power implies a reduced hadronization time in heavy ion collisions.

where the momentum fraction of the daughter parton $x = zy$, with $y$ the momentum fraction of the parent quark. $P_{ij}(z)$ are splitting kernels that could be written down in lowest order pQCD, for example. Rather than perform a detailed DGLAP evolution, we wish to estimate the time scale for the hard quarks to lose the bulk of their energy. From Eq. (5), we can conclude that the number density of hard particles at temperature of order the hadronization scale ($\sim 200$ MeV) is very low ($n_q \leq T^4/m_\phi \sim 10^{-6}$ fm$^{-3}$), for an inflaton mass of $m_\phi = 10^5$ GeV (relatively small inflaton mass which gives rise to reheat temperature smaller than $\Lambda_{\text{QCD}}$) and the temperature being around the QCD phase transition. At the same temperature, the number of hadrons is $n_h \sim (m_h T_h)^{3/2} e^{-m_h/T}$. The Boltzmann suppression is least for the lightest hadron, namely pions, whose mass $m_\pi = 140$ MeV is very close to $T_h$, where the hadronization temperature $T_h \approx 150$ MeV. Therefore $n_h \sim T_h^{3/2} e^{-1} \sim 10^{-1}$ fm$^{-3}$. This implies that while not many hadrons break up, the hard quarks can lose energy quite efficiently due to the large number of participants. The average fractional energy loss of a hard quark per hard collision is not much less than 1/2 or 1/3. The mean free path of the hard quark is approximately $l \sim (n_h \sigma_{\text{DIS}})^{-1}$. Assuming a typical cross-section of micro-barns for the integrated deep-inelastic cross-section at a momentum transfer $Q \sim 1$ GeV [37]††, we find $l \sim 10^4$ fm. Since these quarks are relativistic, this implies that the hard quarks would lose most of their energy within a time $\sim l/c \approx 10^{-19}$s.

B. Fragmentation and hadron-hadron scattering

The mechanism of energy loss by 2→ 3 inelastic scattering can still occur, though it is not correct to use one gluon exchange as the only interaction when quarks are so dilute. If we continue to use the perturbative estimate, at the temperature $T_h$, from Eq. (15), we obtain $\tau_{\text{inel}} \sim 10^{-19}$s. However, we believe that this would not be an accurate estimate because of the strong non-perturbative forces active at this separation scale of the quarks. In light of these strong non-perturbative forces, we may consider the fragmentation process whereby the energy of the quark is degraded by conversion to hadrons. Late thermalization and hadronization can happen also by fragmentation of the hard quarks followed by hadron-hadron scattering. For purposes of estimation, we neglect the effect of the hadronic medium and using a typical proper time of formation $\tau_T \approx 1 - 2$ fm/c, the time in

†† Although, the possibility of larger momentum transfers also implies a much shorter time scale, since the exchange of high energy gluons (at $Q^2 \gg 1$ GeV$^2$) would include an enhancement factor of $\alpha_s^2/\alpha_{\text{em}}^2$ [38].
the lab frame (time dilated) is given by [39]
\[ t_{\text{frag}} \sim \langle \tau \rangle \frac{E_q}{m_h} \]  
(24)

Since the energy of the hard quark \( E_q \sim m_\phi \sim 10^5 \text{ GeV} \) and the hadronic mass is about 0.1 GeV, we find \( t_{\text{frag}} \sim 10^{-18}\text{s} \). Compared to the time scale for the DIS process, this is only slightly slower, and the two processes might be comparable within a more detailed treatment. The main difference between them is the dependence of the time scale on the inflaton mass (logarithmic for DIS, and linear for fragmentation). For \( m_\phi \sim 10^5 \text{ GeV} \), they are almost the same. Since the late hadronization happens in a time that is very short compared to the inflaton decay lifetime, complete hadronization is achieved only when all inflatons have decayed. It should be noted however that fragmentation alone is not sufficient since one must also take into account that one produces hadrons with a large multiplicity and with energies considerably above \( T_h \sim 0.1 \text{ GeV} \). These hadrons will inelastically scatter among themselves, and lose energy. Since these integrated cross-sections (\( \sigma_{hh} \)) are of order millibarns, using the formula \( l = 1/(n_s \sigma_{hh}) \), it is found that the time to reach the equilibrium temperature of the hadron soup is of order \( 10^{-22}\text{s} \). This is much smaller than the time for the initial fragmentation, so the overall conclusion is that one must compare the fragmentation process against the deep inelastic process. For an inflaton mass \( m_\phi \sim 10^5 \text{ GeV} \), this leads to the picture that the late inflaton decay products thermalize via an initial parton shower, followed by DIS, then fragmentation followed by hadron-hadron scattering, within a time much shorter than the inflaton decay lifetime.

VI. CONCLUSION

We have studied the perturbative decay of the inflaton to quarks and gluons, focusing particularly on the aspect of thermalization and energy loss of these hard particles in an expanding universe. For a given inflaton mass \( m_\phi \), the coupling has to be small enough in order that thermalization is achieved before inflaton decay is completed. The thermalization time is short, driven principally by \( 2 \rightarrow 3 \) inelastic processes. After the thermal plasma is formed, it continues to cool in the background of an expanding universe, until a critical temperature for hadronization is reached. The laws of expansion are different than those of a relativistic fluid expanding on account of it's own pressure, as is the case with heavy-ion collisions.

While we have not examined in detail the hadronization process, which occurs in a mixed phase at roughly a constant temperature and is accompanied by a reduction in entropy, we have estimated that the hadronization time is likely to be on the order of micro-seconds as well (\( \sim 10\mu s \)) based on the relationship between the maximum temperature of the plasma and the reheat temperature, which should be below \( T_{QCD} \).

This state of hadrons may not be immediately stable, however, on account of the small cool-down and thermalization time in comparison to the inflaton decay lifetime. Hard inflaton decay products will impinge on this bath of hadrons, and cause some break up into quarks and gluons once again. One can also safely conclude, on account of the small hadronization time, that complete formation of a hadronic bath will be stable only when all inflatons have decayed and the number density of hard particles is negligible. The temperature characterizing that state of the thermal hadronic bath composed of non-relativistic particles can be considered to be the reheat temperature \( T_{rh} \).

In this paper, we have borrowed ideas and expressions from relativistic heavy-ion physics to discuss cosmological issues such as the thermalization of the inflaton decay products. This intermingling is natural especially near the hadronization scale where current experiments are active, thus the issue of low reheat temperature for a QCD plasma could be successively improved by a more rigorous treatment of the early non-equilibrium stages prior to thermalization.

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