From cradle to grave: how burial history controls the rock-physics properties of quartzose sandstones

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ABSTRACT
Rock-physics properties of sands and sandstones are strongly affected by geological processes of the past, including deposition, compaction and exhumation. By honouring these geological processes, the rock-physics modelling will be more predictive in areas with limited well control. This study performs rock-physics modelling constrained by a given geological history, starting from deposition to mechanical and chemical compaction. Different geological factors, including sorting, grain size and clay coating, will affect the quartz cementation and rock stiffening of quartzose reservoir sandstones. By combining compaction models with rock-physics contact theory, we can model the rock-physics properties of sands/sandstones as a function of geological time. We have demonstrated the approach on well log data from three selected wells on the Norwegian Continental Shelf, where the burial histories of the target reservoir sandstones are significantly different. We conclude that rock-physics modelling constrained by burial history can be used to predict elastic properties quite accurately in these wells. The integrated approach presented in this study allows for more realistic rock-physics depth trends in areas with complex burial history that can be used in AVO studies or to estimate net erosion associated with tectonic uplift.

Key words: Borehole geophysics, Reservoir geophysics, Rock physics.

INTRODUCTION
Present-day rock-physics properties and associated seismic signatures of sedimentary rocks are strongly affected by various geological processes, including deposition, mechanical compaction, chemical diagenesis (dissolution and cementation), tectonic uplift and possibly erosion and re-deposition (e.g. Avseth et al., 2010). To fully understand the physical and seismic properties of rocks, we should account for the geologic processes through time in the rock-physics modelling.

One common approach to investigate geologic trends in porosity–velocity crossplots is the rock-physics diagnostic method presented by Dvorkin and Nur (1996) and further developed by Avseth et al. (2000), where contact theory is combined with modified/heuristic Hashin–Shtrikman models. This hybrid approach can be used to model porosity–velocity values for any combination of sorting (depositional trend) and cementation (diagenetic trend). The method has been used to diagnose rock texture (i.e. quantify sorting and cement volume) from well log data (e.g. Avseth et al., 2009, 2010; Lehocki and Avseth, 2010; Hossain and MacGregor, 2014). Critical and disputed aspects of the contact theory comprise the following:
1. Grain contacts and associated stress distributions are assumed uniform
2. Pressure effects are ignored/eliminated when cementation starts
3. The granular medium assumptions break down at lower porosities
4. Cementation effects are handled before sorting, which is geologically inconsistent

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5. Post-depositional porosity reduction and texture alteration associated with mechanical compaction are often ignored. Bachrach and Avseth (2008) and Sain et al. (2016) addressed point 1, which is the main reason why contact theory overpredicts shear moduli. The shear moduli can be adjusted heuristically (i.e., modification of theoretical model by calibration/estimation of a so-called ‘slip-factor’ from data observations; see equation (17) in Bachrach and Avseth, 2008) to take into account heterogeneous stress and nonuniform grain contacts, yielding improved prediction with contact theory. Point 2 was addressed by Avseth and Skjei (2011) and Avseth et al. (2016), who proposed the hybrid patchy cement model, where some grains are assumed unconsolidated and pressure-sensitive. In contrast, other grains are assumed contact-cemented during the initial cementation. Point 3 was addressed by Dvorkin et al. (1999) and Avseth et al. (2014), where contact theory was combined with inclusion models to better predict seismic velocities over the whole range of porosities in sandstones. Dræge et al. (2006) combined detailed diagenetic modelling with various rock-physics models, including contact theory, to capture complex diagenetic trends and transitions.

However, the shortcomings associated with points 4 and 5 above have not been thoroughly studied yet (Avseth and Lehocki, 2017; Zhou et al., 2017). The main objective of this paper is to link the geological processes to rock-physics models in a geologically consistent way. Then, mechanical compaction should be modelled prior to cementation. This has been done by Vernik and Kachanov (2010) using a completely different approach to model the elastic properties of siliciclastics. They used an inclusion-based Mori–Tanaka scheme, strictly valid for low-to-intermediate-porosity, consolidated sandstones. For high-porosity, unconsolidated to poorly consolidated sands, where mechanical compaction is the dominant process, the Mori–Tanaka scheme breaks down (Berryman and Berge, 1996), and Vernik and Kachanov used some simple semi-empirical relations to capture this regime.

This study shows how we can combine contact theory with diagenetic modelling from deposition to present-day burial, including both mechanical and chemical compaction (Fig. 1). The mechanical compaction is accounted for using empirical porosity–depth trends. The predicted porosities are then inserted into the Hertz–Mindlin contact theory model. For the chemical compaction domain, quartz cement is calculated using the Walderhaug kinetic model. The predicted porosities and cement volumes are inserted into the Dvorkin–Nur (1996) contact theory model for cemented sandstones. The goal is to see if geologically consistent modelling can better explain porosity–velocity data than the traditional rock-physics diagnostics approach. Furthermore, by honouring the burial history and geological processes through time, the rock-physics modelling will be more predictive in an explorational setting. We perform a sensitivity study of depositional porosities (i.e., sorting), average grain size and clay coating. We compare our geology-consistent modelling with the traditional rock-physics diagnostic approach and discuss the results. Finally, we investigate the reservoir sandstone interval in three selected wells from the Norwegian Continental Shelf (two from the Barents Sea and one from the North Sea). We use our integrated approach to model rock-physics depth trends and burial-consistent rock-physics templates and to estimate net uplift from the observed well-log data.

**CONTROLS ON SANDSTONE RESERVOIR QUALITY**

Reservoir quality in siliciclastic sands and sandstones is controlled by a wide range of geologic factors, including depositional and diagenetic processes (Table 1). In this study, the focus is on quartzose sands and sandstones. At deposition, the controlling factors on porosity are the distance and transport mechanisms from the provenance, and the energy level of the depositional environment. Mineralogic composition, sorting, texture, grain size, grain shape, clay content and clay location...
are all depositional variables that will impact the porosity. Typically, the depositional porosity of clean and well-sorted sand is around 40%–45% (Ramm et al., 1997).

Table 1 Overview of geological processes acting on sands and sandstones from deposition to deep burial and uplift, and how these relate to rock-physics. The processes, together with controlling parameters and modelling approaches considered/utilized in this study, are shown in bold. It is important to note that we ignore processes like early calcite cement, feldspar dissolution, overpressure and fracturing.

| Burial domain | Geological process | Geological parameter | Physical parameter | Rock-physics modelling |
|---------------|--------------------|----------------------|-------------------|------------------------|
| At/near surface | • Weathering/erosion  
• Sediment transport (suspension or saltation)  
• Deposition  
• Leaching of felspar and mica  
• Bioturbation | • Mineralogic composition  
• Grain size  
• Grain shape  
• Sorting  
• Depositional porosity  
• Pore structure  
• Lamination | • Effective mineralogic moduli  
• Clay volume  
• Grain diameter  
• Critical porosity  
• Net-to-gross | • Elastic mixing models (Hill’s average, Hashin-Shtrikman models)  
• Suspension models (lower bound Hashin-Shtrikman or Reuss bounds) |
| Mechanical compaction domain (≤ 70 °C, ca. 0-2 km) | • Grain packing  
• Grain crushing/ripping  
• Creep/dilatation deformation  
• Early carbonate cementation  
• Authigenic clays  
• Porosity reduction  
• Opal-CT → Opal-CT  
• Undercompaction/overpressure  
• Remobilisation/inejcties | • Packing degree  
• Grain alignment  
• Clay coating (authigenic)  
• Calcite cement  
• Primary porosity | • Intergranular volume  
• Framework stability  
• Specific surface area  
• Contact area  
• Coordination number  
• Effective stress  
• Shear weakening (i.e. stress relaxations) | • Contact theory (Hertz-Mindlin, Walton)  
• Modified contact theory (Bachrach and Avseth, 2008)  
• Frangible sand model (Dvorin and Nur, 1996)  
• Combined mechanical compaction and contact theory (this paper) |
| Chemical compaction domain (> 70 °C) | • Stylolite dissolution  
• Quartz cementation  
• Feldspar dissolution  
• Carbonate dissolution  
• Dolomitization  
• Clay diagenesis | • Stylolite distance  
• Cement volume  
• Cement location  
• Secondary porosity  
• Clay volume/location  
• Burial/Temperature history | • Contact cement volume  
• Pore-filling cement volume  
• Temperature  
• Temperature gradient  
• Time | • Contact cement theory (Dvorin and Nur, 1996)  
• Constant cement model (Avseth et al., 2000)  
• Patchy cement model (Avseth et al., 2016)  
• Combined chemical compaction and contact theory (this paper) |
| Deeply buried, then uplifted | • Tectonic uplift  
• Exhumation  
• Unloading  
• Fracturing/Brittle failure | • Continued cementation (pore-filling)  
• Tensile cracks  
• Pore shape  
• Net erosion estimate | • Pore-filling cement volume  
• Aspect ratio  
• Crack density  
• Effective stress  
• Anisotropy | • Inclusion based models (e.g. DEM, Meri-Tanaka)  
• Hybrid models (e.g. Avseth et al., 2014; Bredesen et al.,2019)  
• Combined mechanical/chemical compaction and contact theory (this paper) |

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mechanical compaction when temperatures are lower than 70°C, and with quartz cementation when temperatures are higher than 70°C. Following the Walderhaug model (Walderhaug, 1996), we account for grain size, clay content and potential coating of grain surfaces. It is also known that the presence of oil wetting the grain surfaces can inhibit the quartz cementation process (e.g. Worden et al., 1998; Worden and Morad, 2000). In this study, however, only water-wet grain surfaces with no fluid–rock interactions are assumed. Table 1 summarizes the various geologic controls on reservoir quality in quartzose sands and sandstones and highlights which factors are included (in bold text) and which are ignored (not bolded text) in this study.

TRADITIONAL ROCK-PHYSICS DIAGNOSTICS AND ITS LIMITATIONS

One of the most common methodologies in rock physics is to investigate the relationships between porosities and seismic velocities. The goal is either to interpret observed data or forward model expected seismic responses as a function of geological factors, like clay content, rock texture, pressure or pore fluid saturations. There is a whole range of rock-physics models that can be utilized in rock-physics diagnostics (Mavko et al., 2020). The most common approach for high-porosity sands and sandstones is the Dvorkin-Nur (1996) approach, including the contact-cement model and the friable sands model, further extended with the constant-cement model by Avseth et al. (2000) (all shown in Fig. 2). The diagenetic trend comprises the Dvorkin–Nur (D-N) contact-cement model at high porosities and the modified Hashin–Shtrikman Upper Bound (HSUB) at intermediate-to-low porosities. The friable sand model is a combination of Hertz–Mindlin contact theory at high-porosity end-member and modified lower-bound Hashin–Shtrikman for decreasing porosities. The constant-cement model is a combination of the friable sand model and the contact-cement model. It is a useful model for a cemented sandstone reservoir at a given burial depth, assuming that cement volume is more or less constant. In contrast, porosity will vary as a function of depositional porosity. The formulas for the models are listed and explained in Appendices A–C, and further theoretical details can be found in Mavko et al. (2020). One of the critics of this approach is that it does not take into account mechanical compaction. It is also common to add cement first (as cement volume is needed to constrain the high-porosity end members of the constant-cement models), then do the sorting variability afterwards by using a modified lower-bound Hashin–Shtrikman model. This is geologically incorrect. Furthermore, it is not pressure-sensitive, nor is it applicable to low-porosity sandstones with crack-like porosities. Nevertheless, this modelling approach has proven to be useful for seismic reservoir characterization and rock-physics template analysis of high-porosity sandstones in many places worldwide, as long as good calibration data are available (e.g. Ødegaard and Avseth, 2004). For intermediate-to-low-porosity sands and sandstones, as well as shales, inclusion-based models have been found to be favourable (e.g. Berryman, 1980; Smith et al., 2009; Vernik and Kachanov, 2010; Avseth et al., 2014).

COMBINED COMPACTION/DIAGENESIS AND ROCK-PHYSICS MODELLING

Next, we want to extend the rock-physics diagnostic approach using contact theory by honouring the complete burial history of sandstone (i.e. from deposition to deep burial). More specifically, we want to combine the modelling of burial history (mechanical and chemical compaction) with rock-physics modelling. Similar studies have been performed by Helset et al. (2004), Dræge et al. (2006) and Avseth and Lehocki (2016). In this study, the focus is on improved understanding of the geological trends and pathways in porosity–velocity cross-plots, and to investigate if we can explain the observations in well log data from three selected wells with different burial histories.
Step 1: Compaction modelling of sands becoming sandstones

First, we perform modelling of mechanical compaction and diageneesis of sands as a function of burial history. Mechanical compaction can be modelled via empirical relationships between porosity and burial depth (Avseth and Lehocki, 2016). The porosity reduction rate for sands and shales is normally faster at shallow depths and slows at a greater burial depth. The porosity as a function of burial depth can be expressed with the following exponential decay function (first written by Athy, 1930):

\[
\phi(z) = \phi_0 \exp(-cz),
\]

where \(\phi\) is the porosity at burial depth \(z\), \(\phi_0\) is the depositional (i.e. critical) porosity at the seafloor \((z = 0\, \text{m})\), and \(c\) is the exponential decay constant. Both \(\phi_0\) and \(c\) will vary depending on lithology and clay content. Equation (1) can be modified to include clay content in sandstones (see Ramm, 1992; Avseth et al., 2005). Alternatively, porosity reduction can be expressed in terms of effective stress instead of burial depth (e.g. Lander and Walderhaug, 1999).

We assume hydrostatic pressure and normal compaction for the sands, yet overpressure may also be included in the modelling.

Next, we model the quartz cementation for the sandstone layers that are buried at temperatures \((T)\) high enough for cementation to occur (i.e. \(T \geq 70^\circ\text{C}\)). We need to know the temperature history during the geological time in order to model quartz cementation. Also, parameters like grain size and amount of clay coating will affect the degree of cementation. The volume of quartz cement precipitated in a specific time interval is given by the following analytical formula (Walderhaug, 1996):

\[
V_{cem} = \phi_{0cc} \left\{ 1 - \exp \left( \frac{-MaA_0}{\rho\phi_{0cc}bC\ln(10)} \right) \times \left( 10^{b(T_2 - T_1)} - 10^{b(T_1)} \right) \right\},
\]

where the following symbols (and units) are used:
- \(V_{cem}\) (cm\(^3\)): the amount (volume) of quartz cement precipitated within a given time interval (from \(t_1\) to \(t_2\))
- \(\phi_{0cc}\) (–): porosity at the start of cementation (at time \(t_1\))
- \(T_1, T_2\) (°C): temperatures at times \(t_1\) and \(t_2\), respectively
- \(M\) (g/mol): molar mass of quartz
- \(\rho\) (g/cm\(^3\)): density of quartz
- \(A_0\) (cm\(^2\)): initial quartz surface area (at time \(t_1\))
- \(a\) (mol/cm\(^2\)): constant
- \(b\) (1/°C): constant
- \(c\) (°C/s): heating rate (estimated from temperature gradients and burial history curves for different stratigraphic intervals)

We use the estimates by Walderhaug (1996), where \(a = 1.98 \times 10^{-22}\) mol/cm\(^2\) and \(b = 0.022\) 1/°C. Moreover, \(M = 60.09\) g/mol and \(\rho = 2.65\) g/cm\(^3\). In the modelling, we assume that mechanical compaction continues after the start of cementation. In reality, however, we may expect cementation to retard the mechanical compaction. For the derivation of equation (2), please refer to Appendix D.

Finally, the fraction of quartz cement \((f_{cem})\) in a unit volume \((V)\) of the sandstone can be defined as

\[
f_{cem} = \frac{V_{cem}}{V},
\]

where \(f_{cem}\) is a unitless quantity.

Step 2: Rock-physics modelling

Based on the compaction modelling (porosity and cement volume with burial depth), we can model the corresponding rock-physics and seismic properties of sandstones with different pore fluid types. We use the Hertz–Mindlin contact theory for unconsolidated sands and the Dvorkin-Nur contact-cement model for cemented sandstones (Avseth et al., 2005) combined with Gassmann fluid substitution (Gassmann, 1951). In the modelling we assume clean sands/sandstones with predominantly quartz (90%–100% of the solid fraction) and occasionally some clay minerals (0%–10% smectite and/or illite). A shear weakening factor is applied to account for nonuniform grain contacts and associated stress relaxations not governed by the contact theory models (Bachrach and Avseth, 2008). Sorting can be varied at the depositional porosity before the mechanical compaction, which is consistent with geology. One of the advantages of combined compaction and rock-physics modelling is that we can model realistic rock-physics depth trends that honour the true burial history. These models can be used in amplitude versus offset (AVO) studies (e.g. Avseth and Lehocki, 2016) or to build low-frequency models or facies depth trends in seismic inversions (e.g. Ristad et al., 2012). They can also be used to estimate net erosion and uplift (e.g. Johansen, 2016; Gatemann and Avseth, 2017). More deeply buried rocks that are uplifted tend to be fractured and cracked, and this should also be honoured in the rock-physics modelling if relevant. This is further investigated by Avseth et al. (2014) and Bredesen et al. (2019).
Figure 3 Combined burial (mechanical and chemical compaction) and rock-physics modelling using contact theory. The endpoints in blue represent the present-day properties, whereas the light blue curves show the rock-physics properties as a function of geological time. Note the good match between the endpoints in subplot (d) and the constant-cement model (2% cement), showing the validity of the lower-bound Hashin–Shtrikman for modelling of sorting.

SENSITIVITY ANALYSIS OF KEY GEOLOGICAL INPUT PARAMETERS

Figure 3 shows an example of combined burial and rock-physics modelling of P-wave velocity versus porosity, where we couple the mechanical compaction (grain packing and crushing) with the chemical compaction (cementation). We test the sensitivity of various key geologic input parameters on the joint diagenesis – rock-physics modelling, including varying clay coating (Fig. 3b), grain size (Fig. 3c) and depositional sorting (Fig. 3d). Both clay coating and grain size will affect the degree of cementation, in addition to temperature and time, in accordance with the Walderhaug model. This shows that the cement volume can be different at a given reservoir depth, and the velocities can vary along a diagenetic trend if there are significant variations in grain size and clay coating. If we assume constant grain size ($d = 1$ mm), no clay coating and only vary the sorting, we obtain three different paths in the porosity–velocity crossplot (Fig. 3d). The endpoints of these three paths (blue dots) show a trend very similar to the constant-cement model. This confirms the validity of the constant-cement model if we assume constant grain size and no clay coating.

Figure 4 shows the sensitivity analysis of combined burial and rock-physics modelling on the evolution of elastic properties and AVO signatures with burial depth, where we assume that the maximum burial is known, but the timing of the burial is unknown (Fig. 4a). This is often the case in areas of significant uplift, like the Barents Sea, where geochemical studies (e.g. vitrinite reflectance) or shale compaction trends have been used to estimate net erosion or uplift (e.g. Japsen, 1999; Baig et al., 2016), but where the timing of the uplift is still uncertain. Some studies have favoured maximum burial of Jurassic reservoir sandstones in Late Cretaceous. In contrast, others have indicated most likely maximum burial in the Cenozoic age (see Zattin et al., 2016). The sensitivity study shows that for a sand burial curve constrained by geologic age and present depth, and a known constant
maximum burial, the present-day sandstone will have quite similar rock-physics properties, regardless of the timing of the maximum burial (Fig. 4b–e). Moreover, the seismic response (visualized in intercept–gradient crossplot in Fig. 4f) varies only slightly for the given elastic properties of the overburden. This point is even better illustrated in Fig. 4(f'), which shows intercept and gradient with broader (and equal) limits. Note that in all three cases tested, the calculated intercept–gradient pairs/points plot in a very concentrated part of the class I domain. A more complex burial curve would further complicate the picture (e.g. repeated uplift and burial episodes). Nevertheless, to first-order, the uncertainty in the timing of the uplift does not have a large impact on the rock-physics properties.

Figure 5 shows what happens if we vary the maximum burial, but keep the timing of the burial and uplift the same. Then, we see that the rock-physics properties will change drastically (Fig. 5b–e), where velocities increase with increasing maximum burial. The observations in Figures 4 and 5 demonstrate the power of the combined burial and rock-physics modelling approach. First, it can be used to invert for uplift from observed velocities in reservoir sandstones. Second, it can be used to forward model velocities if uplift maps are available, but velocities are unknown, even if the timing of the maximum burial is not well constrained. The bottom line is that the velocities will be a function of cement volume, which again will be an integral over time and temperature history. In practice, the area of the curve under the dashed line (i.e. the onset of chemical compaction) in Figures 4 and 5 will correlate strongly with the increase in seismic velocity and reduction in porosity. This correlation is observed both during burial and uplift, as long as we are within the chemical compaction domain. Finally, we also show the effect of grain size variation.
Figure 5  Sensitivity study of maximum burial for combined burial and rock-physics modelling. The same subplots are plotted as in Figure 4. The variation of maximum burial significantly impacts cement volume fraction and porosity, consequently the elastic properties. This translates into a wide separation of AVO responses for three cases in intercept–gradient crossplot (as seen in f and f’). Thus, it is crucial to know the maximum burial depth the studied formation has reached.

(Fig. 6), one of the input parameters in the Walderhaug model. Smaller grains will allow for more quartz cement precipitating within a unit volume for the same temperature history, due to larger specific surface area when grains are smaller. Hence, fine-grained sandstones will tend to be more cemented than more coarse-grained sandstones, for the same burial history. This shows that it is also important to understand the depositional environment of a reservoir sandstone during combined burial and rock-physics modelling. Other depositional effects that can affect the cementation process include sorting, clay content and grain coating (the latter can also be a diagenetic effect; cf. Bjørlykke and Jahren, 2015).

REAL DATA DEMONSTRATIONS FROM THE NORWEGIAN CONTINENTAL SHELF

We demonstrate the workflow presented above on elastic log data (P-wave velocity, S-wave velocity and density) from three wells located in different locations on the Norwegian Continental Shelf (Fig. 7a). Total porosity is estimated from the density logs (corrected for the pore fluid), whereas quartz cement volume can be calculated from elastic moduli combined with porosity (cf. Lehocki and Avseth, 2010). The target zones of these three wells have different degrees of diagenesis. Two wells used in this study are located in the Barents Sea, where uplift of the prolific Stø Fm (quartzose, shallow marine sandstones of Mid Jurassic age) varies from around 1000 m to more than 2000 m; see uplift maps in Figure 7. The uplift map to the upper right (Fig. 7b) is adapted from Baig et al. (2016). It is an arithmetic average of uplift estimates from three different approaches, including shale compaction at well locations, refraction velocities from seismic shot gathers and vitrinite reflectance. The uplift map to the lower right is based on shale compaction analysis from interval velocities and is modified from Johansen et al. (2017) (see also Johansen, 2016). One well is from the Alvheim field, North Sea, where the reservoir zone is composed of Palaeocene-age deep-water sandstones. In this area, only a minor Miocene tilting and Quaternary glacial-related erosion and isostatic rebound have caused a couple of hundred metres uplift after maximum burial during Cenozoic
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Figure 6  Sensitivity study of grain size for combined burial and rock-physics modelling. Case 1: \( d = 0.2 \) mm; case 2: \( d = 0.3 \) mm; case 3: \( d = 0.6 \) mm. The same subplots are plotted as in Figures 4 and 5. It can be seen in (b) that the smallest grain size (case 1) produces the largest cement volume fraction (thus filling more pore space, i.e. reducing porosity), and the largest P-wave velocity. Therefore, for this case, the AVO response will be the stiffer, as seen in (f). The grain size (in an idealized sense, diameter) is an important geologic parameter that can strongly impact the observed seismic response.

(see Avseth and Lehocki, 2016). The geologically consistent rock-physics diagnostics was used to model seismic properties as a function of the burial history for the target reservoir sandstones at the different well locations. The resulting modelled rock-physics properties of the target sandstone intervals were compared with observed well log data. The maximum burial and sandstone grain size were then adjusted until an optimal fit was obtained between the modelled and observed velocities and porosities. The grain size is often well constrained from facies information and petrographic inputs (i.e. thin sections are available at two of the wells in this study). When an optimal fit was obtained, the maximum burial and hence the net uplift for each well were adjusted, if needed. Note that the initial assumptions of uplift from regional studies/maps are quite uncertain (± several hundred metres) and could vary significantly from the local estimates at well positions. The principal reasons for the substantial discrepancy are:

- the limited spatial resolution in interpolated uplift maps derived from vitrinite reflectance analysis in sparse wells
- shale compaction studies using coarse-gridded seismic interval velocities.

Case 1: Hoop area, Barents Sea

Figure 8 shows the combined burial and rock-physics analysis of well 7324/7-2 (Hanssen discovery) in the Barents Sea. Most of the overburden is characterized by siliciclastic shales and sands. Varying shale volume will affect both porosity values and estimated cement volume fractions displayed as logs in Figure 8. However, the rock-physics modelling focuses on the sandy unit of the Stø Fm highlighted with yellowish colour in Figure 8. According to the modelling, the maximum burial depth of the top of the hydrocarbon-bearing zone is nearly 2000 m under the seafloor, just beneath the cementation onset (dashed brown lines in the upper subplots). From the present-day burial depth (around 200 m), it is easy to calculate the uplift magnitude, which is ca. 1850 m. The grey lines in Figure 8(a–d, top) are well logs ‘despiked’ by a simple

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moving median filter with 5-m window size. The corresponding grey circles in the porosity–$V_P$ crossplot are the well log data, brown filled circles are the overlying shale points, and the yellow points come from the reservoir zone. The porosity–$V_P$ crossplot clearly shows reservoir points slightly above the unconsolidated sand line, fitting with the 1% constant-cement model. Combined burial history and rock-physics modelling explain the observed sandstone data in the porosity–$V_P$ crossplot quite nicely. Yet, part of the reservoir sandstones seems to be more poorly sorted than what is assumed in the modelling. Finally, in the intercept–gradient crossplot (Fig. 8f), we create AVO probability ellipses as described by Avseth and Lehocki (2016). The presence of a tiny amount of cement to the rock frame will indeed affect the fluid sensitivities and AVO signatures. However, for this well, the modelling shows that we expect a class II–III AVO signature when the reservoir is hydrocarbon-filled (yet significant overlap between gas case ($S_g = 0.8$) and light oil case ($S_o = 0.8$, gravity = 30 API). In contrast, the brine response is a class IIp and nicely separated from the hydrocarbon case, as indicated by the simulated uncertainty ellipses in Figure 8 (68% iso-probability). In the same figure, we also include the modelling of a ‘what-if’ scenario where the reservoir sandstones have not yet reached the cementation window. In this case, the sands will still be unconsolidated after burial and uplift. Then, we expect a drastically different AVO signature, with strong AVO class III for both gas- and oil-filled sands, whereas brine-filled sands will have AVO class II. This ‘what-if’ scenario illustrates how important it is to take into account the burial history, as local softening of AVO signatures away from well control can be associated with varying diagenesis instead of pore fluid variations. The modelling results for this well align with the AVO analysis constrained by burial history published by Lehocki et al. (2020).

Case 2: Loppa High/Hammerfest Basin, Barents Sea

The combined burial and rock-physics modelling for the Sto Formation in well 7120/1-2 (Myrsildre) further south in the Barents Sea is shown in Figure 9. Again, we display data from a broader interval of mostly siliciclastic shales and sands, but our modelling is confined to the highlighted target zone of Sto Formation. The uplift map of Baig et al. (2016) shows an uplift of ca. 1250 m for this well. Using our modelling approach, with temperature gradients reported from the area (38°C/km; cf. Johansen, 2016), we obtain 1000 m for the Sto Fm sandstone interval. The Myrsildre well is located on a terrace down-flank from the Loppa High, towards the Hammerfest Basin in the South West Barents Sea (Lehocki and Avseth, 2015). The Loppa High has experienced repeated
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Figure 8 Burial-constrained rock-physics and AVO modelling for the Barents Sea Hanssen discovery (well 7324/7-2). The same subplots are plotted as in Figures 4, 5 and 6. Blue curves are contact theory models combined with the Hashin–Shtrikman model. The (oil-to-brine fluid substituted) well log data (plotted as grey curves in b, c and d and grey circles in e) have been smoothed by utilizing a running average filter with a window size of 5 m. The cement volume fraction (b) was quantified from porosity–$V_P$ space (e), as introduced by Lehocki and Avseth (2010). Moreover, within the same subplot, a zoomed subplot (b’) is shown, in which the reservoir zone (Stø Formation) is magnified and shows a cement volume fraction equal to 0.01 (i.e. 1% of the unit rock volume). Note that it nicely matches the cement volume fraction obtained with rock-physics modelling (blue curve). As the burial history curve suggests (a), the reservoir sands have been just barely buried into the chemical compaction domain (i.e. below the dashed horizontal brown line) before they were uplifted to present-day depth, only a few hundred metres beneath the seafloor. Finally, brown and yellow filled circles in (e) correspond to the filtered Fuglen Fm overburden and Stø Fm reservoir sands, respectively.

tectonic episodes (Zattin et al., 2016), both of local and regional character. Hence, net uplifts may vary significantly over short distances, which will not necessarily be picked up by relatively smooth uplift maps like the ones included in Figure 7. Still, the match between our estimate and the uplift map of Baig et al. (2016) is reasonable and within expected error bars (ca. ±200 m). By fitting the models to observed well log data (green and yellow zones are the key shale and sandstone layers, respectively), we estimated the maximum burial to be ca. 2900 m. This is about 1200 m below the cementation onset line (found at around 1700 m burial depth). Therefore, the reservoir zone is well cemented (which is nicely seen in porosity–$V_P$ crossplot). Consequently, the fluid sensitivity becomes very weak (virtually, the fluid change in the reservoir can go completely unnoticed if the herein presented approach is used), as seen in the intercept–gradient crossplot where the brine, oil and gas ellipses almost wholly overlap.

We expect a stiffer AVO response of Top Stø Fm in this well than what we observed in the Hanssen well further north because the reservoir sandstones are more cemented. From the Walderhaug modelling, we estimate around 10% quartz cement in the Stø Fm for the Myrsildre well, as opposed to only 1% in the Hanssen well. Furthermore, we expect AVO class I regardless of pore fluid, and it will be challenging to discriminate pore fluids using AVO for this type of reservoir sandstone.

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Figure 9 Combined burial and rock-physics modelling for well 7120/1-2 in the Barents Sea. The same annotation has been used, as in Figure 8.

Case 3: Alvheim Field, North Sea

The final well log example demonstrated in this study is shown in Figure 10. This well, 24/6-2 (Kameleon), is the discovery well on the Alvheim Field, with turbidite reservoir sandstones of Palaeocene age (marked by the yellow colour in Fig. 10). In this well, we have thin-section estimates of cement volume fraction available (see also Lehocki and Avseth, 2010). Small amounts of quartz cement have been confirmed in the reservoir zone, indicating that the chemical compaction has just started. The western flank of Viking Graben in the North Sea has mainly experienced continuous subsidence. Still, due to tectonic movements of the East Shetland Platform (west of Alvheim) throughout Cenozoic, there has been some tilting and associated small uplift of the reservoir in Miocene/Oligocene times (Rimstad et al., 2012). Also, deglaciation and associated isostatic rebound could have had some impact. Avseth and Lehocki (2016) estimated an uplift of around 200 m for this well. Even this small uplift is important to consider during combined burial and rock-physics modelling, since the reservoir sandstones are currently located very close to the transition from mechanical to chemical compaction domain. The temperature gradient in this well is somewhat lower than what we see for the Barents Sea wells, as we are located closer to a basin axis, and temperature gradient is found to be around 34°C/km for this well (Avseth and Lehocki, 2016; Avseth et al., 2020). The cement volume fraction from the rock-physics diagnostics (Lehocki and Avseth, 2010) shown as a curve in Figure 10(b) fits nicely with the thin-section cement volumes. The quartz cement volume fraction from the Walderhaug modelling is also matching the thin-section estimates when the uplift is assumed to be 200 m. Moreover, the combined burial and rock-physics modelling nicely explain porosity–velocity relationships. In this case, we expect class I–IIp for the brine case, and class II for the gas case, which is in good agreement with AVO observations at this well location (see Avseth et al., 2008, 2009; Avseth and Lehocki, 2016).

Evaluation and comparison of modelling results for different cases

Finally, in Figure 11, we show the burial history curves (Fig. 11a, c, e) together with the expected brine (light blue) and gas (dark red) AVO ellipses (Fig. 11b, d, f) for all three
studied interfaces. With increasing burial depth/degree of diagenesis, the rock frame undergoes large stiffening, which drastically decreases fluid sensitivity. If the cementation has just begun (case 1; see also Fig. 7), the separation between the centres of the brine and gas ellipses decreases (compared with the ‘what-if’ unconsolidated scenario, case 1’). However, the gas response can still easily be discerned from the brine response. If the cement volume percentage fraction is 2%–3%, as in case 2 (for well 24/6-2), the centres of the ellipses come much closer together, and the 68% iso-probability contours of the AVO ellipses start to overlap. When the formation has travelled way below the cementation line, i.e. for moderately to heavily cemented reservoirs, the fluid sensitivity is virtually non-existent. Therefore, it is impossible to say which fluid saturates the pore space. Note that the fluid substitution for the given well has a more considerable impact on the change of intercept. In contrast, the increasing burial depth for the given fluid type and studied interfaces will mostly affect the AVO gradient.

**DISCUSSION**

It is crucial to bear in mind that in the modelling presented in this study, we assume a quartzose sandstone regime, where reservoir quality and rock properties can be affected by a wide range of geologic parameters (cf. Table 1).

Both the Stø Fm in the Barents Sea and the Heimdal Fm in the North Sea are known to be texturally and compositionally mature (i.e. quartz-rich, well rounded, well-sorted, fine-to-medium grained) sandstones with extensive distances from the original provenance. This yields breakdown of relatively unstable feldspar grains and lithic fragments that are more typical when the source of erosion is more proximal (Bjørlykke and Jahren, 2015). The Stø Fm has been washed and transported by waves and currents over a shallow epicontinental shelf far away from the Ural Mountains and Norway hinterlands, and clay particles tend to be sorted out during the sedimentary processes. The Heimdal Fm sandstones represent deep-water turbidite sands deposited far away from the source.
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Figure 11 Burial history curves (a, c, e) and the statistical brine (abbreviated with B) and gas (G) AVO responses (b, d, f) for the studied interfaces at three well locations (see Fig. 7 for map view). The positions of the brine and gas AVO ellipses on the right subplots clearly show that the sensitivity to fluid change decreases as the degree of chemical compaction (which is a function of burial depth) increases. This is manifested in a more extensive overlap of brine and gas ellipses (d and f). Note in (b) that two models are plotted: one where the formation has just ‘touched’ the cementation window (dashed ellipses; $f_{cem} = 0\%$) and another one where the formation has been slightly cemented (solid ellipses; $f_{cem} = 1\%$). The separation between centre points of ellipses decreases significantly from case 1’ to case 1 because of the stiffer pore space caused by the initial cementation. If this initial cementation is not accounted for, gas-filled pore space could easily be misinterpreted as oil-filled pore space (which would plot between brine and gas ellipses).

in elevated East Shetland platform. Still, within these sandstones, the shaliness can vary over short distances due to rapid facies changes in deep-water sedimentary systems, often with clay-laminated sands. Some depositional parameters are assumed (spherical grain shape, depositional porosity), whereas others are given by direct thin-section observations (grain size, mineralogic composition). What is observed in selected thin sections can vary significantly even at a well log scale, representing an uncertainty.

A porosity decay factor for sands is selected from ‘global’ empirical relations for the mechanical compaction. This parameter can vary from one area to another as a function of shaliness and grain size. Even within the same well, different types of sands can experience different degrees of mechanical compaction depending on texture, sorting, grain size and composition. The porosity evolution presented in Figures 8–10 is representative for the clean, well-sorted end members of the sands/sandstones within the studied target intervals (Stø Fm in the Barents Sea and Heimdal Fm in the North Sea). Hence, the scatter we observe in the data, not captured by the ‘end-member’ modelling, is likely caused by reservoir heterolithics and deteriorating sorting. These effects can also be included in the modelling (e.g. Avseth et al., 2010), but this is beyond this paper’s scope. One future modification to the modelling performed in this study is to account for grain size/shape effect on the mechanical compaction and associated porosity reduction. Grain size and grain shape will affect both the depositional porosity (Fawad et al., 2011; angular grains cause
larger depositional porosity) and the mechanical compaction rate (smaller grains will resist mechanical compaction more than larger grains).

During the chemical compaction, we have assumed that initial cementation is occurring equally around all grains. Hence, we have ignored any pressure sensitivity of sandstones during the chemical compaction. This is an effect that will be included in the future (cf. Avseth et al., 2016). We also ignore the possible impact of stylolite distance on cement volume. Walderhaug and Bjørkum (2003) showed that for Stø Fm in the Barents Sea, this was secondary to the effect of temperature and time on quartz cement volume. We also ignore potential secondary porosity effects, which is a fair assumption for quartzose sandstones. Clay coating can be included in the modelling, but is not abundant in Stø nor Heimdal Fm-s. The fluvial/deltaic Snadd Fm sandstones in the Barents Sea (Triassic age) are known to be commonly affected by coating, and coating should be accounted for in these types of sandstones, as it inhibits the quartz cementation.

Moreover, there are occasionally calcite stringers in both Stø Fm and the Heimdal Fm sandstones. These can give rock-physics signatures similar to that of quartz cement, as sand grains are consolidated together. However, calcite cement tends to occur in thin stringers, often with close to zero porosity, causing huge velocity spikes in velocity logs. These are often filtered away before further well log analysis of reservoir quality from well log data (cf. Avseth et al., 2020). Finally, we have ignored brittle failure and cracking that can be caused during massive uplifts. Reservoirs that are tight but fractured are better modelled with inclusion-based models than using contact theory, which is known to work best for high-to-intermediate-porosity sandstones. Hence, the modelling strategy presented here is useful for shallow to intermediate burial where sands and sandstones have porosities in the range from around 0.15 to 0.4. For low-porosity sandstones, the reader is recommended to utilize alternative modelling approaches (e.g. Vernik and Kachanov, 2010; Avseth et al., 2014; Bredesen et al., 2019).

The key takeaway from this study is that it is possible to link rock-physics modelling to geologic processes and use this link in a predictive and realistic way in areas with limited well control, given certain assumptions and constraints. The quartzose sands and sandstones offshore Norway, and many other locations worldwide, are suitable for applying a combined Walderhaug–Dvorkin–Nur modelling of cemented sandstones. No model scheme can handle all the possible outcomes. The key is to make the modelling as simple as possible, but still useful for predictive purposes. Limitations and uncertainties should then be addressed concurrently, and for that purpose, Table 1 can be used as a guide.

CONCLUSIONS

We have investigated porosity–velocity trends of sandstones associated with geological processes starting from deposition to mechanical compaction and chemical compaction. We have combined compaction models with rock-physics contact theory and shown how different geological factors, including sorting, grain size and clay coating, will affect the observed trends. This approach validates the constant cement model. Still, porosities will typically be reduced due to mechanical compaction before cementation starts. This should be taken into account in rock-physics modelling of cemented sandstones. The geologically consistent modification of rock-physics diagnostics, honouring mechanical and chemical compaction, makes the contact theory models more consistent with real data observations.

We have applied the combined burial and rock-physics modelling on reservoir data (with varying degrees of diagenesis) from three wells from both the North Sea and the Barents Sea. Good fits are obtained between observed and modelled porosity–velocity relations for all the cases when the local burial histories are accounted for. Furthermore, it is shown that as the degree of diagenesis increases, the fluid sensitivity decreases. More specifically, it is challenging to discriminate between different pore fluids from the seismic in moderately to heavily cemented sandstones.

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DATA AVAILABILITY STATEMENT

All well log data used in this study are currently released by the Norwegian Petroleum Directorate and available for members of Diskos (The Norwegian National Data Repository for Petroleum data: www.npd.no/en/diskos).

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APPENDIX A: FRIABLE (UNCONSOLIDATED) SAND MODEL

The friable sand model (Figure A) describes porosity–velocity behaviour versus sorting at a specific effective pressure (Avseth et al., 2005). It is a combination of Hertz–Mindlin theory (Mindlin, 1949) and the modified lower Hashin–Shtrikman bound. Below, the formulas for calculation of this model are given. For a detailed description of this and other models described in the appendices, the reader is referred to Avseth et al. (2005) and Mavko et al. (2020).

The symbols (and units) used in the formulas are defined as follows:

- $\phi$ (-): total porosity and critical/depositional porosity
- $\phi_c$ (-): critical/depositional porosity
- $\nu_{\text{min}}$ (-): Poisson ratio of the grain material
- $C$ (-): coordination number, i.e. the average number of contacts per grain
- $f$ (-): slip-factor: an ad hoc coefficient between 0 (perfect adhesion) and 1 (perfect frictionless grains)
- $B$ (-): Skempton’s coefficient
- $\mu_{\text{eff}}$ (Pa): effective pressure
- $K_{\text{min}}, \mu_{\text{min}}$ (Pa): average bulk and shear modulus of the mineral, respectively
- $K_{\text{HM}}, \mu_{\text{HM}}$ (Pa): effective bulk and shear modulus of a dry, random, identical-sphere packing for the right-end member (Hertz–Mindlin – HM)
- $K_{\text{eff}}, \mu_{\text{eff}}$ (Pa): effective bulk and shear modulus of a dry, random, identical-sphere packing for all interpolated porosities
- $K_{\text{sat}}, \mu_{\text{sat}}$ (Pa): effective bulk and shear modulus of a saturated, random, identical-sphere packing
- $K_{\text{fi}}$ (Pa): bulk modulus of the fluid
- $\rho_{\text{sat}}$ (kg/m$^3$): bulk density of a saturated rock
- $V_P \text{ sat}$, $V_S \text{ sat}$: P-wave and S-wave velocity of the saturated rock, respectively

First, the formulas for calculating the elastic properties of the right-end member (seen as a black filled circle in Fig. A) are introduced. These (Hertz–Mindlin) formulas are deduced from cradle to grave 645

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under the idealized assumption that the grains are an elastic sphere pack subject to confining pressure.

\[ v_{\text{min}} = \frac{3K_{\text{min}} - 2\mu_{\text{min}}}{2(3K_{\text{min}} + \mu_{\text{min}})}. \]

\[ C = 20 - 34\phi_c + 14\phi_c^2. \]

\[ K_{\text{HM}} = \sqrt{\frac{P_{\text{eff}}}{18}} \left[ \frac{C(1 - \phi_c)\mu_{\text{min}}}{\pi(1 - v_{\text{min}})} \right]^2, \]

\[ \mu_{\text{HM}} = \frac{2 + 3f - v_{\text{min}}(1 + 3f)}{5(2 - v_{\text{min}})} \sqrt{\frac{3P_{\text{eff}}}{2}} \left[ \frac{C(1 - \phi_c)\mu_{\text{min}}}{\pi(1 - v_{\text{min}})} \right]^2. \]

At this point, the modified lower Hashin–Shtrikman formulas can be utilized to calculate the elastic moduli at any porosity along the red line in Figure A:

\[ K_{\text{eff}} = \frac{1}{\frac{2}{K_{\text{HM}} + \frac{1}{2}\mu_{\text{HM}}} + \frac{1 - \frac{2}{\nu_{\text{HM}}}}{\nu_{\text{HM}}}} - 4 \frac{3}{\nu_{\text{HM}}}, \]

\[ \mu_{\text{eff}} = \frac{1}{\frac{2}{\nu_{\text{HM}}}} + \frac{1 - \frac{2}{\nu_{\text{HM}}}}{\nu_{\text{HM}}} - z, \]

where

\[ z = \frac{\mu_{\text{HM}}}{6} \left( \frac{9K_{\text{HM}} + 8\mu_{\text{HM}}}{K_{\text{HM}} + 2\mu_{\text{HM}}} \right). \]

Finally, the pore space of these effective (dry) moduli can be ‘filled’ with brine, oil or gas, via Gassmann’s equations (Gassmann, 1951) to obtain the elastic properties of the wet rock (sand or sandstone):

\[ B = \frac{K_{\text{eff}}}{K_{\text{min}} - K_{\text{eff}}} + \frac{K_{\text{fl}}}{\phi(K_{\text{min}} - K_{\text{fl}})}. \]

\[ K_{\text{sat}} = K_{\text{min}} \frac{B}{1 + B}, \]

\[ \mu_{\text{sat}} = \mu_{\text{eff}}, \]

\[ \rho_{\text{sat}} = \rho_{\text{min}}(1 - \phi) + \rho_f\phi, \]

\[ V_{P_{\text{sat}}} = \sqrt{\frac{K_{\text{sat}} + \frac{2}{3}\mu_{\text{sat}}}{\rho_{\text{sat}}}}, \]

\[ V_{S_{\text{sat}}} = \sqrt{\frac{\mu_{\text{sat}}}{\rho_{\text{sat}}}}. \]

APPENDIX B: DVORKIN-NUR CEMENTED-SAND MODEL (CONTACT-CEMENT MODEL)

The contact-cement model (Fig. B) quantifies the porosity–velocity behaviour versus cement volume (fraction) at high porosities. It is assumed that the starting framework of cemented sand is a dense random pack of identical spherical grains.

The symbols (and units) used in the formulas and not introduced in Appendix A are defined as follows:

- \( \nu_{\text{cem}} \): Poisson ratio of the cement material
- \( K_{\text{cem}}, \mu_{\text{cem}} \): effective bulk and shear modulus of the cement material, respectively
- \( \rho_{\text{cem}} \): density of the cement material
- \( V_{P_{\text{cem}}} \): P-wave velocity of the cement material
- \( M_{\text{cem}} \): P-wave modulus of the cement material
- \( \tilde{S}_n, \tilde{S}_\tau \): parameters proportional to the normal and shear stiffnesses, respectively
- \( \alpha_\cdot \): the amount of contact cement calculated by using Scheme 2 (in which the assumption is that cement is evenly deposited on the grain surface)

\[ C = 20 - 34\phi + 14\phi^2. \]

\[ v_{\text{min}} = \frac{3K_{\text{min}} - 2\mu_{\text{min}}}{2(3K_{\text{min}} + \mu_{\text{min}})}, \]

\[ v_{\text{cem}} = \frac{3K_{\text{cem}} - 2\mu_{\text{cem}}}{2(3K_{\text{cem}} + \mu_{\text{cem}})}, \]
\[ \Lambda_\tau = \frac{\mu_{\text{cem}}}{\pi \mu_{\text{min}}}, \]
\[ \Lambda_n = 2 \Lambda_\tau \frac{(1 - \nu_{\text{min}})(1 - \nu_{\text{cem}})}{1 - 2\nu_{\text{cem}}}, \]
\[ A_\tau = -10^{-2} \left( 2.26v_{\text{min}}^2 + 2.07v_{\text{min}} + 2.3 \right) \Lambda_\tau \left( 0.079v_{\text{min}}^2 + 0.175v_{\text{min}} + 1.342 \right), \]
\[ B_\tau = \left( 0.0573v_{\text{min}}^2 + 0.0937v_{\text{min}} + 0.202 \right) \Lambda_\tau \left( 0.0274v_{\text{min}}^2 + 0.0529v_{\text{min}} + 0.8765 \right), \]
\[ C_\tau = 10^{-4} \left( 9.654v_{\text{min}}^2 + 4.945v_{\text{min}} + 3.1 \right) \Lambda_\tau \left( 0.0186v_{\text{min}}^2 + 0.4011v_{\text{min}} + 1.8186 \right), \]
\[ A_n = -0.024153\Lambda_n^{-1.3646}, \]
\[ B_n = 0.20405\Lambda_n^{-0.89008}, \]
\[ C_n = 0.00024649\Lambda_n^{-1.9846}, \]
\[ \alpha = \sqrt{2(\phi_{\text{cem}} - \phi) / (3(1 - \phi_{\text{cem}}))}, \]
\[ \hat{S}_\tau = f \left( A_\tau \alpha^2 + B_\tau \alpha + C_\tau \right), \]
\[ \hat{S}_n = A_\tau \alpha^2 + B_\tau \alpha + C_n, \]
\[ M_{\text{cem}} = \rho_{\text{cem}}V_{\text{cem}}^2, \]
\[ K_{\text{eff}} = \frac{1}{6} C(1 - \phi_{\text{cem}}) M_{\text{cem}} \hat{S}_n, \]
\[ \mu_{\text{eff}} = \frac{3}{5} K_{\text{eff}} + \frac{3}{20} C(1 - \phi_{\text{cem}}) M_{\text{cem}} \hat{S}_n, \]
\[ B = \frac{K_{\text{eff}}}{K_{\text{min}} - K_{\text{eff}}} + \frac{K_{fl}}{\phi(K_{\text{min}} - K_{fl})}, \]
\[ K_{\text{sat}} = \frac{K_{\text{min}} B}{1 + B}, \]
\[ \mu_{\text{sat}} = \mu_{\text{eff}}, \]
\[ \rho_{\text{sat}} = \rho_{\text{min}}(1 - \phi) + \rho_{fl}\phi. \]

**APPENDIX C: THE CONSTANT-CEMENT MODEL**

The constant-cement model describes the porosity–velocity behaviour versus sorting at a specific cement volume (fraction), which usually corresponds to a constant depth. It is a combination of the Dvorkin-Nur contact-cement theory (see Appendix B) and the modified lower Hashin–Shtrikman bound.

The formulas for calculating the elastic properties of the right-end member (seen as an empty black circle in Fig. C) are given in Appendix B. Once the elastic properties of the right-end member are calculated (marked by \( K_b \) and \( \mu_b \) in Fig. C), the modified lower Hashin–Shtrikman formulas can be applied to determine the elastic moduli at any porosity point along the red line in Fig. C:

\[ K_{\text{dry}} = \frac{1}{\phi} \frac{1}{K_b + \frac{4}{3} \mu_b} + \phi + \frac{1 - \phi}{K_b + \frac{4}{3} \mu_b} - \frac{4}{3} \mu_b, \]
\[ \mu_{\text{dry}} = \frac{1}{\phi} \frac{1}{\rho_b + \frac{4}{3} \mu_b} + \phi + \frac{1 - \phi}{\rho_b + \frac{4}{3} \mu_b} - \phi, \]

Figure C Figure 2 with an emphasis on the constant-cement (red thick) line.
APPENDIX D: WALDERHAUG MODEL

Quartz cementation and associated porosity loss is a temperature-related process acting in sandstones buried to temperatures above 60°C (Walderhaug, 1996). In order to model the quartz cementation for the sandstone layers, several parameters must be known, the most important ones being (Wangen, 2010; Avseth and Lehocki, 2016):

- Grain size (i.e., the specific surface area) of the sand particles
- The temperature history of the sedimentary basin during geologic time
- Burial rate of the (sandstone) formation
- Amount of clay coating (i.e., the fraction of quartz grains coated by clay)

Wangen (2010) lists some of the available models proposed to calculate porosity reduction in quartzose sandstones by quartz cementation. In this work, the model proposed by Walderhaug (1996) is implemented. The main formulas for calculation of quartz cement volume in function of time are presented below.

The amount of quartz cement precipitated in a volume of sandstone from time \( t_0 \) to \( t_w \) is calculated as the sum of a series of integrals:

\[
V_{cem} = \frac{M_a}{\rho_d \ln(10)} \left\{ \frac{A_0}{c_1} \left[ 10^{b(c_1 t + d_1)} \right]_{t_0}^{t_1} + \frac{A_1}{c_2} \left[ 10^{b(c_2 t + d_2)} \right]_{t_1}^{t_2} + \ldots + \frac{A_{m-1}}{c_m} \left[ 10^{b(c_m t + d_m)} \right]_{t_{m-1}}^{t_m} \right\}.
\]   (D1)

where the abbreviations used are:

- \( V_{cem} \) (cm³): the amount (volume) of quartz cement precipitated within a given time interval
- \( M \) (g/mol): molar mass of quartz
- \( \rho \) (g/cm³): quartz density
- \( A_0, A_1, \ldots \) (cm³): initial and quartz surface area at \( i \)th time interval, respectively
- \( a \) (mol/cm³): constant
- \( b \) (1°C): constant
- \( c_i \) (°C/s): heating rate (slope) of the \( i \)th (time-temperature) segment
- \( d_i \) (°C): intercept of the \( i \)th time-temperature segment

Each of these integrals gives the volume of quartz precipitated during each time step. By integrating equation (D1), we obtain

\[
V_{cem} = \frac{M_a}{\rho_d \ln(10)} \left\{ \frac{A_0}{c_1} \left[ 10^{b(c_1 t + d_1)} \right]_{t_0}^{t_1} + \frac{A_1}{c_2} \left[ 10^{b(c_2 t + d_2)} \right]_{t_1}^{t_2} + \ldots + \frac{A_{m-1}}{c_m} \left[ 10^{b(c_m t + d_m)} \right]_{t_{m-1}}^{t_m} \right\}.
\]   (D2)

Note that by performing dimension analysis on equation (D1) (or D2), the units of \( V_{cem} \) are cm³’s, instead of the expected cm². This essentially means that, when the sandstone is within the temperature window for cementation, the cement volume is time-dependent, as long as there is pore space available for cement precipitation. In other words, the more time has passed, the larger fraction of the unit volume of sandstone becomes quartz cement.

Note also that equation (D2) must be solved iteratively because the heating rate is a function of time (quartz surface area has to be recalculated at each time interval for the same reason):

\[
V_{cem_i} = V_{cem_{i-1}} + \left( \phi_{0c} - V_{cem_{i-1}} \right) \frac{M_A \alpha a}{b \rho_d \phi_{0c} \ln(10)} \times \left[ 10^{b(c_i t + d_i)} - 10^{b(c_{i-1} t + d_{i-1})} \right],
\]   (D3)

where

- \( \phi_{0c} \) (–): porosity at the start of cementation
- \( V_{cem_i}, V_{cem_{i-1}} \) (cm³): cement volume at \( i \)th \((t_i)\) and \((i-1)\)th \((t_{i-1})\) time steps, respectively

In equation (D3), the assumption is that the time-temperature history takes a linear form.

The initial quartz surface area can be expressed as the cumulative surface area of spheres with a diameter \( D \):

\[
A_0 = \frac{6V}{D} (1 - \text{coat}) \text{.}
\]   (D4)

where

- \( f \) (–): fraction of detrital quartz
- \( V \) (cm³): unit volume of the sandstone
- \( D \) (cm): diameter (size) of the idealized sand sphere (grain)
- \( \text{coat} \) (–): fraction of coated quartz grains

Then, the quartz surface area, \( A \), when \( V_{cem} \) amount of quartz cement has precipitated, is calculated as

\[
A = A_0 \left( 1 - \frac{V_{cem}}{\phi_{0c}} \right) \text{.}
\]   (D5)

Equation (D5) mathematically expresses that the change in quartz surface area caused by precipitation of quartz cement is proportional to the porosity loss caused by quartz precipitation.
Assuming that the temperature changes at a constant rate and that the quartz surface area can be expressed with equation (D5), Walderhaug (1996) offers an analytic solution to equation (D2), which takes the following form:

\[
V_{cem2} = \phi_{0c} - (\phi_{0c} - V_{cem1}) \\
\times \exp \left[ -\frac{MaA_0}{\rho_{\phi_{0c},bc} \ln(10)} \left( 10^{\phi_{T_2}} - 10^{\phi_{T_1}} \right) \right].
\]  

(D6)

where

- \(T_1, T_2\) (°C): temperatures at times \(t_1\) and \(t_2\), respectively.

By setting \(V_{cem1} = 0\) and \(V_{cem2} = V_{cem}\), equation (D6) becomes equation (2).