Nucleon Form Factors in Point-Form Spectator-Model Constructions

T. Melde
Theoretische Physik, Institut für Physik, Karl-Franzens-Universität, Universitätsplatz 5, A-8010 Graz, Austria

We discuss electromagnetic currents in the point-form formulation of relativistic quantum mechanics. The construction is along a spectator model and implies that only one quark is explicitly coupled to the photon, but nevertheless many-body contributions are present in the current operator. Such effects are unavoidable in relativistic constructions and resulting ambiguities are notably reduced by imposing charge normalization and time-reversal invariance. The residual theoretical indetermination introduces small but sizeable changes in the nucleon form-factors, particularly at higher $Q^2$ values, with the data generally centered in the middle of the theoretical band.

Keywords: Electromagnetic nucleon form-factors; Relativistic constituent quark model; Theoretical uncertainty estimation

1. Introduction

Nucleon form factors are of considerable importance, as they can give additional information on the internal structure of the nucleons. Renewed interest in the form factors is due to new polarization transfer measurements\textsuperscript{1–4} for the proton, that produced data at variance with respect to earlier Rosenbluth data.\textsuperscript{5–7} Accurate measurements\textsuperscript{8,9} of new generation revealed the existence of a discrepancy that cannot be explained\textsuperscript{10} with Coulomb distortion effects. It has been suggested that two-photon contributions could be the source of the disagreement,\textsuperscript{11,12} but to date the experimental situation is still under discussion (see also\textsuperscript{13–15}). On the theoretical side, two-photon corrections\textsuperscript{16–20} and additional $\Delta$ contributions\textsuperscript{21} have been advocated to resolve this problem. A detailed discussion on the theoretical and experimental status of nucleon form factors can be found in two recent reviews.\textsuperscript{22,23}

Here, we reinvestigate the description of nucleon form factors within the Goldstone-boson exchange (GBE) constituent-quark model (CQM).\textsuperscript{24} The
approach is complementary to field-theoretic ones, like e.g. Refs.,\textsuperscript{25,26} as it is valid for low and intermediate momentum transfers. The electromagnetic (EM) ground state properties of the nucleon already were reproduced quite well\textsuperscript{27–30} with the point-form spectator model (PFSM) applied to the GBE CQM. With the advent of newer and more precise data for the nucleon form-factor, it becomes necessary to assess more carefully the predictive power of the PFSM construction and possible sources of indetermination.\textsuperscript{31}

In the last Cortona meeting,\textsuperscript{32} we presented arguments that translational invariance of the transition amplitude implies the presence of many-body contributions. In Ref.\textsuperscript{33} it has been shown that in connection to these many-body effects certain ambiguities arise which cannot be fully constrained by Poincaré invariance alone. Here, we further constrain the PFSM construction by requiring charge normalization and time-reversal invariance of the elastic electromagnetic nucleon transition amplitude.

2. The Electromagnetic Current

The reduced matrix element of the PFSM EM current between three constituent quarks with individual momenta $p_i$ and helicities $\sigma_i$ reads\textsuperscript{33}

$$\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{J}^{\mu=3} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle = 3N e_1 \bar{u} (p'_1, \sigma'_1) \gamma^\mu u (p_1, \sigma_1) 2p_20 \delta^3 (p_2 - \bar{p}'_2) 2p_30 \delta^3 (p_3 - \bar{p}'_3) \delta_{\sigma_2 \sigma'_2} \delta_{\sigma_3 \sigma'_3}. \hspace{1cm} (1)$$

For the nucleon electromagnetic form-factors the transition amplitude between the incoming and outgoing baryon eigenstates is given by the follow-
In the Breit frame, the nucleon Sachs form-factors are related to the transition amplitude $F_{\Sigma',\Sigma}^\mu(Q^2)$ according to the expressions

$$F_{\Sigma',\Sigma}^\mu(Q^2) = 2m_N G_E(Q^2) \delta_{\Sigma',\Sigma}$$

$$\hat{F}_{\Sigma',\Sigma}^\mu(Q^2) = iQG_M(Q^2) \chi_{\Sigma',\Sigma}^\dagger (\vec{\sigma} \times \vec{e}_z) \chi_{\Sigma},$$

where $\Sigma', \Sigma = \pm \frac{1}{2}$ are the spin projection along the $z$-axis of the nucleon and $\chi$ the corresponding Pauli spinors.
As has been observed in Ref.\textsuperscript{33} a normalization factor $N$ appears in the PFSM current of (1), which can be parametrized in the following way

$$N(x, y) = \left( \frac{M}{\sum_i \omega_i} \right)^x \left( \frac{M'}{\sum_i \omega'_i} \right)^{x(1-y)}.$$ \hfill (5)

In this form $x$ and $y$ are free parameters which we consider in the range $0 \leq x, 0 \leq y \leq 1$, while $M$ and $M'$ are the mass eigenvalues of the baryon states (here, we restrict our investigation to the elastic case with $M = M'$). Conversely, $\sum_i \omega_i = \sum_i \sqrt{m_i^2 + k_i^2}$ is the invariant mass of the free incoming three-quark system, where $k_i$ are internal momenta, and similarly for $\sum_i \omega'_i$ for the outgoing. Obviously, the normalization factors so defined are all Lorentz invariant. Here, we further constrain the construction by adhering to additional global requirements for the EM amplitudes.

Charge normalization requires $x = 3$ (for any $y$ values), because in Fig. 1(a) only for this value the proton charge is recovered at zero momentum transfer. This result substantiates the findings in Ref.\textsuperscript{33} where one can see that only when the exponent takes the $x = 3$ value, one can recover a genuine one-body operator at zero momentum transfer. If we impose that the EM-current amplitude is time-reversal invariant, then it has been demonstrated (see Refs.\textsuperscript{47,48}) that the third component of $F_{\Sigma, \Sigma'}$ has to vanish identically in the Breit frame. In Fig. 1(b) we present the corresponding transition amplitude exhibiting an antisymmetric structure around the value $y = \frac{1}{2}$. To obtain a vanishing amplitude the obvious choice clearly is the zero-crossing at $y = \frac{1}{2}$, leading to the symmetric (geometric) factor

$$N_S = \left( \frac{M}{\sum_i \omega_i} \right)^{\frac{3}{2}} \left( \frac{M'}{\sum_i \omega'_i} \right)^{\frac{3}{2}}.$$ \hfill (6)

However, the antisymmetric structure of the curves in Fig. 1(b) allows to consider also other, equally valid, PFSM normalizations. For example, physically valid constructions are

$$N(z) = \frac{1}{2} \left[ \left( \frac{M}{\sum_i \omega_i} \right)^{3z} \left( \frac{M'}{\sum_i \omega'_i} \right)^{3(1-z)} + \left( \frac{M'}{\sum_i \omega'_i} \right)^{3z} \left( \frac{M}{\sum_i \omega_i} \right)^{3(1-z)} \right].$$ \hfill (7)

In Eq. (7), one can sort out two special cases, namely $N_S$ for $z = 0.5$ and $N_{\text{ari}}$ for $z = 0$ (or $z = 1$). Here, $N_S$ is given by Eq. (6), while $N_{\text{ari}}$ takes on
the form of an arithmetic combination

$$N_{\text{ari}} = \frac{1}{2} \left[ \left( \frac{M}{\sum_i \omega_i} \right)^3 + \left( \frac{M'}{\sum_i \omega_i'} \right)^3 \right]. \quad (8)$$

All forms implied by Eq. (7) lead to the correct charge normalization and fulfill time-reversal invariance\( ^a \). In the \( Q^2 \to 0 \) limit all different expressions implied by Eq. (7) lead to the same transition amplitudes.

In Fig. 2 we show the ratio of magnetic nucleon to standard dipole form factors in comparison to experiment. As one moves to higher \( Q^2 \) momenta, one observes a broadening of the band between the curves \( N_S \) and \( N_{\text{ari}} \), indicating that the normalization factor effects mainly the higher momentum results. The predictions with \( N_S \) (full line) exhibit a strong fall off above \( Q^2 = 1 \text{GeV}^2 \) following approximately the lower bounds of the experimental neutron data, but considerably underestimating the proton data. The dashed line, representing the results with \( N_{\text{ari}} \), exhibit an overprediction of the experimental data. We also have performed a one-parameter ”best fit” (denoted by the dash-dotted line) to the experimental data of the dipole ratios \( G_M^p/G_D \) and \( G_M^n/G_D \) at \( 3 \text{GeV}^2 \) momentum transfer. The value \( z = 1/6 \) in Eq. (7) produces a theoretical curve that lies still below the experimental data for the proton, while it is already above for the neutron. These results are denoted as \( N_{\text{fit}} \) and represented by the dash-dotted lines in Figs. 2,3. The two curves with \( N_{\text{geo}} \) and \( N_{\text{ari}} \) envelop the experimental data in most cases, while the curve with \( N_{\text{fit}} \) lies in-between.

In Fig. 3 we show the ratio of the electric and magnetic proton form-factor (left) and the ratio of the electric proton form-factor to standard dipole parametrization (right). Again, the theoretical spread enlarges with increased momentum transfer. The dash-dotted line follows the experimental values and is congruent to latest data with the asymmetric beam-target experiment by Jones et al.\(^{14} \) Note that this line was obtained with the same \( N_{\text{fit}} \), which was determined previously. To further decrease the theoretical spread, one needs to find additional conditions that further constrain the construction of PFSM operators. However, for the time being one could assume the band between the two curves as an estimate for the theoretical uncertainty of the PFSM construction of the nucleon EM form-factors.

\(^a\) a linear combination of the type \( a N_S + (1-a) N_{\text{ari}} \) also is an allowed construction.
3. Summary and Conclusion

In this contribution we have reconsidered the construction of EM operators in the PFSM formalism. The spectator-like nature of the construction implies that only one quark is explicitly coupled to the current operator, but nevertheless the operator contains many-body effects. Such many-body effects appear in the kinematics and in the occurrence of a normalization factor that is explicitly required by the theory. The specific expression for the factor, however, is not completely determined since Poincaré invariance and proton-charge normalization alone are not sufficient to uniquely determine the current-operator. Time-reversal invariance produces an important additional constraint that further restricts the possible choices, but still allows for a residual indetermination. This spread is zero at zero-momentum transfer and increases at higher $Q^2$-values. We have estimated the corresponding theoretical uncertainty and found that it remains reasonably small, at least for the EM form-factors of the nucleon. We have finally determined an optimal choice utilizing one single free parameter, obtaining results overall in good agreement with the experimental data.

Acknowledgements

The results reported in this contribution originated in a collaborative effort with L. Canton, W. Plessas and R. F. Wagenbrunn. The author is particularly thankful to L. Canton for his help in completing this manuscript and to R. F. Wagenbrunn for his expertise in the numerics. The author is grateful to the INFN Sezione di Padova and the University of Padova for their hospitality. This work was supported by the Austrian Science Fund (Projects P16945 and P19035). Finally, the author would like to thank the organizers for accepting this contribution to the Cortona meeting.

References

1. M. K. Jones et al., Phys. Rev. Lett. 84, 1398 (2000).
2. O. Gayou et al., Phys. Rev. Lett. 88, p. 092301 (2002).
3. V. Punjabi et al., Phys. Rev. C 71, p. 055202 (2005).
4. G. MacLachlan et al., Nucl. Phys. A764, 261 (2006).
5. L. Andivahis et al., Phys. Rev. D 50, 5491 (1994).
6. R. C. Walker et al., Phys. Lett. B224, 353 (1989).
7. G. Höhler, E. Pietarinen, I. S. Stefanescu, F. Borkowski, G. G. Simon, V. H. Walther and R. D. Wendling, Nucl. Phys. B114, p. 505 (1976).
8. M. E. Christy et al., Phys. Rev. C 70, p. 015206 (2004).
9. I. A. Qattan et al., Phys. Rev. Lett. 94, p. 142301 (2005).
10. J. Arrington and I. Sick, *Phys. Rev. C* **70**, p. 028203 (2004).
11. J. Arrington, *Phys. Rev. C* **69**, p. 022201 (2004).
12. J. Arrington, *Phys. Rev. C* **71**, p. 015202 (2005).
13. V. Tvaskis *et al.*, *Phys. Rev. C* **73**, p. 025206 (2006).
14. M. K. Jones *et al.*, *Phys. Rev. C* **74**, p. 035201 (2006).
15. E. Tomasi-Gustafsson, *hep-ph/0610108* (2006).
16. P. A. M. Guichon and M. Vanderhaeghen, *Phys. Rev. Lett.* **91**, p. 142303 (2003).
17. P. G. Blunden, W. Melnitchouk and J. A. Tjon, *Phys. Rev. Lett.* **91**, p. 142304 (2003).
18. P. G. Blunden, W. Melnitchouk and J. A. Tjon, *Phys. Rev. C* **72**, p. 034612 (2005).
19. Y. C. Chen, A. Afanasev, S. J. Brodsky, C. E. Carlson and M. Vanderhaeghen, *Phys. Rev. Lett.* **93**, p. 122301 (2004).
20. A. V. Afanasev, S. J. Brodsky, C. E. Carlson, Y.-C. Chen and M. Vanderhaeghen, *Phys. Rev. D* **72**, p. 013008 (2005).
21. S. Kondratyuk, P. G. Blunden, W. Melnitchouk and J. A. Tjon, *Phys. Rev. Lett.* **95**, p. 172503 (2005).
22. J. Arrington, C. D. Roberts and J. M. Zanotti, *nucl-th/0611050* (2006).
23. C. F. Perdrisat, V. Punjabi and M. Vanderhaeghen, *hep-ph/0612014* (2006).
24. L. Y. Glozman, W. Plessas, K. Varga and R. F. Wagenbrunn, *Phys. Rev. D* **58**, p. 094030 (1998).
25. M. Oettel, R. Alkofer and L. von Smekal, *Eur. Phys. J. A* **8**, 553 (2000).
26. R. Alkofer, A. Holl, M. Kloker, A. Krassnigg and C. D. Roberts, *Few Body Syst.* **37**, 1 (2005).
27. R. F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas and M. Radici, *Phys. Lett. B* **511**, 33 (2001).
28. L. Y. Glozman, M. Radici, R. Wagenbrunn, S. Boffi, W. Klink and W. Plessas, *Phys. Lett. B* **516**, 183 (2001).
29. S. Boffi, L. Glozman, W. Klink, W. Plessas, M. Radici and R. Wagenbrunn, *Eur. Phys. J. A* **14**, 17 (2002).
30. K. Berger, R. F. Wagenbrunn and W. Plessas, *Phys. Rev. D* **70**, p. 094027 (2004).
31. T. Melde, L. Canton, W. Plessas and R. F. Wagenbrunn, *nucl-th/0612013* (2006).
32. T. Melde, W. Plessas, R. F. Wagenbrunn and L. Canton, Proceedings of the 10th Conference on Theoretical Nuclear Physics in Italy, Cortona, Italy, 6-9 Oct. 2004 (World Scientific, Singapore, 2004), p. 213.
33. T. Melde, L. Canton, W. Plessas and R. F. Wagenbrunn, *Eur. Phys. J. A* **25**, 97 (2005).
34. A. Lung *et al.*, *Phys. Rev. Lett.* **70**, 718 (1993).
35. P. Markowitz *et al.*, *Phys. Rev. C* **48**, 5 (1993).
36. S. Rock *et al.*, *Phys. Rev. Lett.* **49**, p. 1139 (1982).
37. E. E. W. Bruins *et al.*, *Phys. Rev. Lett.* **75**, 21 (1995).
38. H. Gao *et al.*, *Phys. Rev. C* **50**, 546 (1994).
39. H. Anklin *et al.*, *Phys. Lett. B* **336**, 313 (1994).
40. H. Anklin et al., *Phys. Lett.* **B428**, 248 (1998).
41. W. Xu et al., *Phys. Rev. Lett.* **85**, 2900 (2000).
42. G. Kubon et al., *Phys. Lett.* **B524**, 26 (2002).
43. W. Xu et al., *Phys. Rev.* **C67**, p. 012201 (2003).
44. A. F. Sill et al., *Phys. Rev. D* **48**, 29 (1993).
45. W. Bartel et al., *Nucl. Phys.* **B58**, 429 (1973).
46. B. D. Milbrath et al., *Phys. Rev. Lett.* **80**, 452 (1998).
47. L. I. Durand, P. C. DeCelles and R. B. Marr, *Phys. Rev* **126**, p. 1882 (1962).
48. F. J. Ernst, R. G. Sachs and K. C. Wali, *Phys. Rev* **119**, p. 1105 (1960).