N=2 Super-Born-Infeld Theory Revisited

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Abstract

I discuss the symmetry structure of the N=2 supersymmetric extension of the Born-Infeld action in four dimensions, and confirm its interpretation as the Goldstone-Maxwell action associated with partial breaking of N=4 extended supersymmetry down to N=2, by revealing a hidden invariance of the action with respect to two non-linearly realized supersymmetries and two spontaneously broken translations. I also argue about the uniqueness of supersymmetric extension of the Born-Infeld action, and its possible relation to noncommutative geometry.

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1 Introduction

In ref. [1] I proposed the N=2 supersymmetric extension of the four-dimensional Born-Infeld (BI) action. I interpreted it as the Goldstone-Maxwell action associated with spontaneous (partial) breaking of (rigid) N=4 supersymmetry down to N=2, and the N=2 (abelian) vector supermultiplet of Goldstone fields. The basic idea behind this interpretation was the anticipated equivalence (modulo a non-linear field redefinition) between the N=2 super-BI action in four dimensions and the gauge-fixed world-volume action of a D3-brane propagating in six dimensions. This equivalence was verified in ref. [1], in the leading and subleading orders only (see ref. [2] too), while no direct argument was presented. In this Letter I report on a progress in obtaining the transformation laws of the hidden non-linearly realized symmetries (including spontaneously broken translations and extra N=2 supersymmetry) which determine the form of the N=2 super-BI action and prove its Goldstone nature. The uniqueness of N=2 superextension of the BI action is also discussed. I give an N=2 superconformal extension of the BI theory, and speculate about its possible relation to noncommutative geometry.

2 Featuring the bosonic BI action

In this introductory section I recall some well-known facts about the bosonic BI action, in order to provide a basis for the subsequent discussion of the N=2 supersymmetric extension in sect. 3.

The bosonic BI action in flat four-dimensional spacetime with Minkowski metric $\eta_{\mu\nu}$, $\mu, \nu = 0, 1, 2, 3$, reads \footnote{The overall normalization of the BI action yields the Maxwell term, $-\frac{1}{2b}F_{\mu\nu}F^{\mu\nu}$, as the leading contribution. The D3-brane action has, in addition, the inverse string coupling constant in front of the action.}

$$S_{\text{BI}} = -\frac{1}{b^2} \int d^4x \sqrt{-\det (\eta_{\mu\nu} + b F_{\mu\nu})},$$ (1)

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $b > 0$ is the dimensionful parameter. For instance, in string theory one has $b = 2\pi\alpha'$, whereas in N=1 supersymmetric QED one has $b = e^2/(2\sqrt{6}\pi m^2)$. In what follows, I choose $b = 1$ for simplicity.

The BI theory (1) can be thought of as the particular covariant deformation of Maxwell electrodynamics by higher order terms depending upon $F$ only. In fact,
the BI theory also shares with the Maxwell theory some other physical properties, such as causal propagation, positive energy density and electric-magnetic duality (see, e.g., refs. [1], [2] and references therein). Unlike the Maxwell theory, its BI generalization gives rise to the celebrated taming of the Coulomb self-energy, i.e. it smears the singularity associated with a point-like charge in classical electrodynamics. Supersymmetry is known to be compatible with causality, positive energy and duality, so that one expects from supersymmetric BI actions the similar (properly generalized) properties. It is indeed the case for the N=1 BI action [3], and it should be the case for the N=2 BI action [4] too.

As is also quite clear from its origin, either in open string theory or in N=1 scalar QED, the BI action is the *effective* action obtained by summing up certain quantum corrections (to all orders in $b$) that are independent upon spacetime derivatives ($\partial F$) of the Maxwell field strength $F$. The effective action is dictated by S-matrix, being defined modulo local field redefinitions. This does not, however, make the BI action to be ambiguous since it depends upon the vector gauge potential $A$ only via its field strength $F$, while any local reparametrization of $A$ merely results in the additive $\partial F$-dependent terms which are to be disregarded by definition of the BI action,

$$\delta S = \int d^4 x \delta \mathcal{L}(F) = \int d^4 x \frac{\partial \mathcal{L}}{\partial F_{\mu \nu}} \partial_{\mu} \delta A_{\nu} = - \int d^4 x \partial_{\mu} \frac{\partial \mathcal{L}}{\partial F_{\mu \nu}} \delta A_{\nu} = O(\partial F) . \quad (2)$$

In other words, the BI action is the effective action of slowly varying (but not necessarily small) abelian gauge fields, which is dependent upon $F$, being independent upon $\partial F$. In supersymmetric BI theories the rôle of $F$ is played by the gauge superfield strength $W$, so that the super-BI actions in superspace are defined modulo spacetime derivatives of $W$.

### 3 N=2 BI action and its symmetries

The N=1 BI action is well-known to be the Goldstone-Maxwell action associated with spontaneous partial supersymmetry breaking $N=2 \rightarrow N=1$ and the N=1 vector supermultiplet of Goldstone fields [3], [2]. Both N=1 and N=2 gauge field theories are most naturally formulated in superspace, with manifest off-shell N=1 or N=2 supersymmetry, respectively, which makes a study of partial breaking $N=2 \rightarrow N=1$ rather straightforward, by starting from a linear off-shell realization of N=2 supersymmetry and imposing a non-linear constraint. Partial breaking $N=4 \rightarrow N=2$ in N=2 superspace is more complicated since a natural (off-shell and N=4 supersymmetric) formulation of N=4 gauge theories does not exist.
The N=2 supersymmetric BI action can be formulated in the standard N=2 superspace parametrized by $Z = (x^a, \theta_i^\alpha, \bar{\theta}_i^\alpha)$, where $\alpha = 1, 2$ and $i = 1, 2$. The N=2 flat superspace covariant derivatives $(\partial_{\alpha}, D^i_{\alpha}, \bar{D}^i_{\alpha})$ satisfy the algebra

$$\{D^i_{\alpha}, \bar{D}^j_{\alpha'}\} = -2i\delta^i_j \partial_{\alpha'} \quad \text{and} \quad \{D^i_{\alpha}, D^j_{\beta}\} = \{D^i_{\alpha}, \bar{D}^j_{\alpha'}\} = 0.$$  

The standard realization is given by

$$D^i_{\alpha} = \frac{\partial}{\partial \theta_i^\alpha} + i\bar{\theta}_{ai}^\alpha \partial_{\alpha}, \quad \bar{D}^i_{\alpha} = -\frac{\partial}{\partial \bar{\theta}^i_{\alpha}^\alpha} - i\theta_{ai}^\alpha \partial_{\alpha}.$$  

The SL(2, C) and SU(2) indices are raised and lowered by the use of Levi-Civita ($\varepsilon$) symbols, as usual. I use the notation

$$D^i_{\alpha} = \frac{1}{2} D^i_{\alpha} D^i_{\alpha}, \quad D_{\alpha\beta} = \frac{1}{2} D_{\alpha} D_{\beta}, \quad (D^3)^{\alpha}_{i} = \frac{\partial}{\partial D^i_{\alpha}}, \quad D^4 = \prod_{\alpha,i} D^i_{\alpha}.$$  

The abelian N=2 superfield strength is described by an N=2 restricted chiral superfield $W$ satisfying the off-shell N=2 superspace constraints

$$\bar{D}^i_{\alpha} W = 0 \quad \text{and} \quad D^4 W = \square W.$$  

The second constraint (7) is just the N=2 Bianchi identity that implies $\square (D_{ij} W - \bar{D}_{ij} \bar{W}) = 0$ and, hence, $D_{ij} W = \bar{D}_{ij} \bar{W}$. A solution to eq. (7) in components reads (in N=2 chiral superspace parametrized by $y^\mu = x^\mu - \frac{1}{2} \theta^\alpha_{ai} \sigma^\mu_{\alpha} \bar{\theta}^{ai} \theta_{ai}^\alpha$)

$$W(y, \theta) = a(y) + \theta^\alpha_{ai} \psi^i_{ai}(y) - \frac{1}{2} \theta^\alpha_{ai} (\bar{\tau})^i_{j} \theta_{ai}^\alpha \quad \text{and} \quad \bar{D}(y)$$

$$- i (\theta^3)^{ia}_{\alpha} \partial_{\alpha\beta} \psi^i_{ai}(y) + \theta^4 \partial \bar{a}(y),$$

where I have introduced the complex (physical) scalar $a$, the chiral (physical) spinor isodoublet $\psi^i_{ai}$, the real (auxiliary) isotriplet $\bar{D} = \frac{1}{2} (\bar{\tau})^i_{j} D^j_{i}$, and the Maxwell field strength $F_{\mu\nu}$ subject to the Bianchi identity

$$\varepsilon^{\mu\nu\lambda\rho} \partial_\lambda F_{\mu\rho} = 0,$$

whose solution is just $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$.  

\footnote{The fields are assumed to vanish at infinity.}
The N=2 supersymmetric extension of the BI action, proposed in ref. [1], reads
\[ S = \frac{1}{2} \int d^4x d^4\theta W^2 + \frac{1}{8} \int d^4x d^8\theta \mathcal{Y}(K, \bar{K}) W^2 \bar{W}^2, \]  
(10)
where \( K = D^4W^2 \) and \( \bar{K} = D^4\bar{W}^2 \), and
\[ \mathcal{Y}(K, \bar{K}) = 1 - \frac{1}{4}(K + \bar{K}) - \sqrt{(1 - \frac{1}{4}K - \frac{1}{4}\bar{K})^2 - \frac{1}{4}KK} \]  
(11)
\[ = 1 + \frac{1}{4}(K + \bar{K}) + O(K^2). \]

The action (10) can be rewritten to the form
\[ S = \frac{1}{4} \int d^4x d^4\theta X + \frac{1}{4} \int d^4x d^4\bar{\theta} \bar{X} + O(\partial W), \]  
(12)
where the N=2 chiral lagrangian \( X \) is the iterative solution to the N=2 non-linear constraint \( \mathcal{Y}(K, \bar{K}) = 1 + \frac{1}{4}(K + \bar{K}) + O(K^2). \)

The uniqueness of the N=2 BI action was questioned in ref. [5] by presenting a calculation of some terms in the iterative solution to eq. (13), which are absent in the perturbative expansion of the action (10), for example,
\[ \int d^4x d^8\theta W^2 \bar{W}^2 \left[ (D^4W^2) \bar{D}^4(\bar{W}^2 D^4W^2) + (\bar{D}^4\bar{W}^2) D^4(W^2 \bar{D}^4\bar{W}^2) \right] \]  
(14a)
versus
\[ \int d^4x d^8\theta W^2 \bar{W}^2 \left[ (D^4W^2)^2 \bar{D}^4\bar{W}^2 + (\bar{D}^4\bar{W}^2)^2 D^4W^2 \right]. \]  
(14b)
However, it is not difficult to verify, by the use of eqs. (3) and (7), that the difference between eqs. (14a) and (14b) amounts to the \( \partial W \)-dependent terms which do not belong to the N=2 BI action because they are ambiguous (cf. sect. 2). It was also explicitly demonstrated in ref. [5] that the N=2 BI action (12) is self-dual with respect to an N=2 supersymmetric electric-magnetic duality (claimed in ref. [1] too), by keeping all terms in the solution to eq. (13), including the \( \partial W \)-dependent ones. This means that taking into account some \( \partial W \)-dependent terms is apparently needed to demonstrate the N=2 supersymmetric electric-magnetic duality of the N=2 BI action. In general, however, it does not make sense to keep some \( \partial W \) (or \( \partial F \)) dependent terms in the effective BI action originating either from a quantized open superstring theory or from a quantized supersymmetric gauge theory, while ignoring other possible \( \partial W \) (or \( \partial F \)) dependent quantum corrections. Perhaps, the N=2 electric-magnetic self-duality may, nevertheless, be useful for a study of derivative corrections to the N=2 BI action in a more fundamental framework than just N=2 supersymmetry.
The Goldstone interpretation of the N=2 BI action implies that the complex scalar \( W = a = P + iQ \) is the Goldstone field associated with two spontaneously broken translations (in the directions orthogonal to a D3-brane world-volume in six dimensions). Hence, the action (10) or (12) should possess hidden invariance with respect to spontaneously broken (non-linearly realized) translations, \( \delta a = \lambda + \ldots \), where \( \lambda \) is the complex (rigid) parameter. This symmetry is obvious from the viewpoint of a (1,0) supersymmetric BI action in six dimensions [1], which is related to the four-dimensional N=2 BI action via dimensional reduction. Indeed, the six-dimensional action depends upon its gauge fields via their field strength only, while one can identify \( A_4 + iA_5 = a \). Hence, the dimensionally reduced action actually depends upon the derivatives of \( a \), and not upon \( a \) itself, though it is not manifest in eq. (10). Similarly, the spinor components \( \psi^i_{\alpha} \) of \( W \) in eq. (8) are supposed to be the Goldstone fermions associated with two spontaneously broken (non-linearly realized) supersymmetries in four dimensions, \( \delta \psi^i_{\alpha} = \lambda^i_{\alpha} + \ldots \), where \( \lambda^i_{\alpha} \) are the (rigid) spinor parameters.

Spontaneously broken symmetries determine the associated Goldstone action. Since, in our case, the N=2 BI action is fixed by the non-linear constraint (13), there should be a relation between the non-linear transformations in question and the constraint (8). It is now not difficult to find the relevant (without spacetime derivatives) terms in the N=2 superfield transformation laws,

\[
\delta X = 2\Lambda W, \quad \delta W = \Lambda \left( 1 - \frac{i}{4}D^4\bar{X} \right) - \frac{X}{W} \bar{D}^4 \left( \bar{W} \bar{\Lambda} \right) + \ldots ,
\]

(15)

where \( \Lambda \) is the spacetime-independent (rigid) N=2 superfield parameter,

\[
\Lambda = \lambda + \theta^i_{\alpha} \lambda^i_{\alpha} + i \theta^i_{\beta} \sigma^{\mu\nu} \theta^i_{\alpha} \lambda^{\nu}_{\mu} ,
\]

(16)

\( X \) is the iterative (to all orders in \( W \) and \( \bar{W} \), but modulo ambiguous \( \partial W \) and \( \partial \bar{W} \)-dependent terms) solution to the non-linear constraint (13). The dots in eq. (15) stand for the \( \partial W \)-dependent terms needed for the consistency with the second equation (7). Since those terms are ambiguous in the N=2 BI action, I ignore them both in the action and in the transformation laws for simplicity. The invariance of the action (12) under the transformations (15) follows from the fact that

\[
D^4W, \quad (D^3)^i_{\alpha}W \quad \text{and} \quad D_{\alpha\beta}W
\]

(17)

are all total derivatives in \( x \)-space, because of eqs. (8) and (9). The second relation (15) now follows from the first one by varying the constraint (13). Comparing eqs. (8) and (15) shows that \( \lambda \) is the rigid parameter of broken translations, whereas \( \lambda^i_{\alpha} \) are the rigid parameters of two broken supersymmetries. Surprisingly enough, there exists yet
another non-linear symmetry with rigid (real and antisymmetric tensor) parameter \( \lambda_{\mu\nu} \), which is apparently related to the Goldstone nature of the Maxwell field itself.

It is possible to rewrite the action (12) into the ‘free-field’ form (subject to the non-linear constraint)

\[
S = \frac{1}{2} \int d^4x d^4\theta W^2 + \frac{1}{8} \int d^4x d^8\theta \bar{X}X ,
\]

where I have merely substituted the constraint (13) into eq. (12) and used eq. (6). Equation (18) is the \( N=2 \) analogue to the known ‘free-field’ form of the \( N=1 \) BI action, given by the sum of free actions for an \( N=1 \) vector multiplet and an \( N=1 \) chiral multiplet, related by a non-linear constraint [3, 2]. There is, however, an obvious difference between the \( N=1 \) and \( N=2 \) ‘free-field’ actions, because the second term in eq. (18) gives rise to higher derivatives in components. The existence of the field redefinition eliminating the higher derivatives is guaranteed by the existence of the equivalent (gauge-fixed) D3-brane action without higher derivatives but with non-manifest (non-linearly realized or ‘deformed’) unbroken \( N=2 \) supersymmetry [1, 2].

4 Outlook

The \( N=2 \) BI theory can be made (rigidly) \( N=2 \) superconformally invariant by modifying the constraint (13) as (cf. ref. [3])

\[
X = \frac{1}{4} \Phi \bar{D}^4 \left( \frac{X}{\Phi^2} \right) + W^2 ,
\]

where the \( N=2 \) ‘superconformal compensator’ \( \Phi \) is an \( N=2 \) restricted chiral superfield obeying the constraints (7). The original \( N=2 \) BI action is obtained from eqs. (12) and (19) by ‘freezing’ \( \Phi \) to \( \Phi = 1/\sqrt{b} = (2\pi\alpha')^{-1/2} \).

The bosonic BI lagrangian (sect. 2) interpolates between the Maxwell lagrangian, 

\[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \equiv -\frac{1}{4} F^2, \]

for small \( F \) and the total derivative, 

\[\frac{1}{8} \varepsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} \equiv \frac{1}{4} F \bar{F}, \]

for large \( F \) [3]. In the \( b \to 0 \) limit the BI lagrangian reduces to

\[
\frac{F^2}{F \bar{F}} ,
\]

whose \( N=2 \) supersymmetric extension [3]

\[
\int d^8\theta \frac{W^2 \bar{W}^2}{K \bar{K}} \left( \frac{K + \bar{K}}{K - \bar{K}} \right)
\]

follows from eq. (11) in the \( b \to 0 \) limit.
One may, therefore, think of $\Phi$ as a constant non-covariant background containing a constant antisymmetric tensor $B_{\mu\nu}$ on the place of $F_{\mu\nu}$ in eq. (8). The $b \to 0$ limit is then described by sending $\Phi$ to infinity, at large $B_{\mu\nu}$ in particular. The N=2 BI action in this limit is believed to be equivalent to a rank-one (Maxwell) noncommutative N=2 supersymmetric gauge field theory via Seiberg-Witten map \cite{7}, with $B_{\mu\nu}$ being the measure of noncommutativity in $x$-space,

$$[x^\mu, x^\nu] = i(B^{-1})^{\mu\nu}. \quad (22)$$

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