Analysis of rain fade duration models for Earth-to-satellite path based on data measured in Malaysia

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Abstract. Statistical analysis of rain fade duration is crucial information for system engineer to design and plan a fade mitigation technique (FMT) for the satellite communication system. An investigation is carried out based on data measured of one year period in Kuala Lumpur, Malaysia from satellite path of MEASAT3. This paper presents statistical analysis of measured fade duration on high elevation angle (77.4˚) in Ku-band compared to three prediction models of fade duration. It is found that none of the models could predict measured fade duration distribution accurately.

1. Introduction
Duration statistical knowledge such as probability of duration for rain fades at a given attenuation, what time and how frequent it occurs and when it may be next happened is important key to get insight into the performance availability and quality of service. Service provider must consider such aspects to implement an appropriate forward error correction (FEC), choice of modulations and the range of adaptive power control (APC) during severe rain fade events in the overall design of their satellite communication systems, especially for the low-margin and Ku-band and higher frequencies systems [1].

The number of events for low attenuations and short durations mainly depends on scintillation and receiver noise amplitude. In literatures, different values are usually proposed to identify and separate fade events into short and long durations between 10 and 30s [2]. The number of events of filtered and unfiltered data of this link was already shown in [3]. In this case, duration above 30s was effective for separating long duration caused by attenuation from short duration due to fast fluctuations of scintillation.

Many works have been studied and published on fade duration statistics both satellite and terrestrial path links whereas it is no model for interfade duration. All models fundamentally developed a cumulative distribution based on power law, lognormal and two lognormal distributions.

This paper is presented statistical analysis of fade duration based on measured data compared to three prediction models of cumulative distribution function (CDF), namely ITU-R, Timothy and Cheffana-Ayama models [4-6]. Testing method for comparing error of fade duration predictions were also calculated and presented in this paper.

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2. Measurement setup and data processing
The measurement was monitored the MEASAT3 signal at 10.982 GHz (Ku-band), vertical polarization with QPSK modulation and was sampled for time interval length of 0.1s and subsequently processed to 1s time interval length. The signal was collected for 12 months during 1st September 2011 to 31st August 2012 located in the International Islamic University Malaysia (IIUM), Gombak, Kuala Lumpur. Tipping bucket rain gauge of 0.2mm was synchronized with time of attenuation logging system to determine raining events. Details of the experimental setup are available in [7-8].

3. Fade duration distribution models
The statistics of fade duration distribution is analysed from unfiltered measured data of 1s time interval length. The data points for calculating the number of events of fade duration \(N(d>D | a>A)\) and probability of occurrence \(P(d>D | a>A)\) are preferably used corresponding ITU-R P 311-12 [9], namely 1, 10, 30, 60, 120, 180, 300, 600, 900, 1200, 1500, 1800, 2400 and 3600 s, respectively. The number of events \(N(d>D | a>A)\) statistics and calculation method of probability of occurrence \(P(d>D | a>A)\) are already presented in [3].

3.1 ITU-R model. ITU-R model is able to calculate fade duration statistics including effect of gases, clouds, rain and scintillation for earth-satellite path links. The model presents long-term of fade duration follows a lognormal distribution and short-term of fade duration follows a power-law distribution [4].
Calculation of \(P(d>D | a>A)\) of duration \(d\) longer than \(D\) given that attenuation \(a\) is greater than \(A\) as (1-2).

\[
P(d > D | a > A) = D^{-\gamma} \quad \text{for } 1 \leq D \leq D_t
\]
\[
P(d > D | a > A) = D_t^{-\gamma} \cdot \frac{Q\left(\frac{\ln(D) - \ln(D_t)}{\sigma}\right)}{Q\left(\frac{\ln(D_t) - \ln(D_2)}{\sigma}\right)} \quad \text{for } D > D_t
\]

Where: exponent \(\gamma\) of the power-law distribution of the fraction of fading time due to fades of short duration and \(D_t\) is boundary between short and long fade durations. Details of the ITU-R model are available in [4].

3.2 Timothy model. The model [1, 5] developed by normalizing a lognormal distribution that depends on the average fade duration \(\overline{D}\). If fractions of fade events exceeding the normalised fade duration, the fade duration distribution approximation is given by (3).

\[
P(D_t \geq x) = \frac{1}{2} \text{erfc} \left(\frac{\ln x - \ln \overline{x}}{\sigma \sqrt{2}}\right) \quad \text{(3.1)}
\]
\[
= \frac{1}{x \sigma \sqrt{2 \pi}} \int_x^\infty e^{-\frac{1}{2} \left(\frac{\ln x - \ln \overline{x}}{\sigma}\right)^2} dx \quad \text{(3.2)}
\]

Where \(x = D/\overline{D}\) is the normalised fade duration, \(\overline{D}\) is the average fade duration for a particular threshold, \(\sigma\) is the standard deviation of \(\ln(D/\overline{D})\), \(\text{erfc}\) is the complementary of error function, and \(\ln x\) is the mean of \(\ln x\).
3.3 Cheffena-Ayama model. The model [6] was modelled and tested the database created covering Ku, Ka, and Q/V frequency bands, elevation angles between 14°-89° in various climatic conditions such as tropical, subtropical and temperate. The model was developed the prediction model of a cumulative distribution function (CDF) of probability of occurrence of fade duration, by the sum of two lognormal functions. The conditional fade duration distribution is given by (4).

\[
P(d > D | a > A) = \alpha \left( \frac{\ln(D/m_r)}{\sigma_s} \right) + (1 - \alpha) \left( \frac{\ln(l/m_r)}{\sigma_r} \right)
\]

Where: \( D \geq 1 \) (s) and \( Q \) is the standard cumulative distribution function for a normal distributed variable as same as equation in [4].

\[
\alpha = 0.8881 - 0.3168\beta + 0.1636\exp\left( -\frac{A}{2.61} \right)
\]

\( A \) is the attenuation threshold (dB), \( \beta \) is the rain convectivity parameter obtained from recommendation ITU-R P.837 [10], \( f \) is frequency (10-50 GHz) and \( \epsilon \) is elevation angle (5°-90°).

The mean \( m_s \) and standard deviation \( \sigma_s \) of the first lognormal function are defined as

\[
m_s = 0.3636 - 2.0411 \cdot 10^{-6} f^3 + 11117 \exp(-\epsilon)
\]

\[
\sigma_s = 1.6462 + 29.8038 \exp\left( -\frac{f}{3.5} \right) - 1.3671 \cdot 10^{-6} A^3
\]

Similarly to the second lognormal function, mean \( m_r \) and standard deviation \( \sigma_r \) are expressed as

\[
m_r = 686.59 - 173.51 \log(f)
\]

\[
\sigma_r = 0.621 + 4.3516 \cdot 10^{-3} f^{1.5} + 3.3637 A^{-2}
\]

4. Comparison of fade duration distribution models and measurement

Probability of occurrence \( P(d > D | a > A) \) of measured data compared to three prediction models is presented in figure 1-3. The ITU-R model predicts the measured statistics quite well for short duration (<30s) except 1dB attenuation as shown in figure 1. However, the model underestimated the measured data for long duration (>30s) that mainly caused by rain attenuation.
Figure 1. Probability of occurrences $P(d>D \mid a>A)$ comparison between ITU-R model (ITU) and measured data (MEA) for given attenuation thresholds.

Figure 2 depicts probability of occurrence $P(d>D \mid a>A)$ comparison between Timothy model and measured data for given attenuation thresholds. The Figure clearly shows that the model overestimates both short and long durations for all attenuation thresholds.

Figure 2. Probability of occurrence $P(d>D \mid a>A)$ comparison between Timothy model (TMY) and measured data (MEA) for given attenuation thresholds.

Cheffena-Ayama model and measured data statistics for given attenuation thresholds is presented in figure 3. In general, the model predicted similar trend to measured data statistics for given attenuation thresholds. However, the model relatively overestimated the measured data for all attenuation thresholds. The overestimation was much lower than Timothy model as shown in figure 2.
In order to compare with prediction models, measured data are tabulated for fixed individual duration $D$ in the range of 1 to 3600 s and for fixed attenuation threshold $A$, namely 1, 5, 10 and 13 dB. The test variable is defined by the logarithm ratio of the predicted probability to the measured probability as proposed by ITU-R P.311 [9]. For testing methods of the probability of occurrence $P(d>D | a>A)$, the test variable is defined as the natural logarithm of the ratio of predicted probability $P_p(d>D | a>A)$ to measured probability $P_m(d>D | a>A)$, for each $A$ and $D$.

The mean, standard deviation and root mean square (RMS) for each attenuation threshold $A$ and each duration $D$ are then calculated to evaluate the error of different models.

### Table 1: Prediction test variables of $P(d>D | a>A)$ with different models for given attenuations

| Models            | A (dB) | 1     | 5     | 10    | 13    |
|-------------------|--------|-------|-------|-------|-------|
|                   | Mean   | Std   | RMS   | Mean  | Std   | RMS   | Mean  | Std   | RMS   | Mean  | Std   | RMS   |
| ITU-R             | 0.369  | 0.096 | 0.161 | 1.472 | 0.085 | 0.763 | 0.284 | 0.0001| 0.069 |
|                   | -0.089 | 0.064 | 0.021 | 1.124 | 0.052 | 0.656 | 0.113 | -0.009 | 0.033 |
|                   | 0.030  | 0.048 | 0.041 | 0.892 | -0.303 | 0.512 | 0.278 | -0.120 | 0.084 |
|                   | 0.110  | 0.017 | 0.063 | 0.747 | -0.466 | 0.414 | 0.301 | -0.172 | 0.104 |
| Timothy           |        |       |       |       |       |       |       |       |       |
|                   | Mean   | Std   | RMS   | Mean  | Std   | RMS   | Mean  | Std   | RMS   | Mean  | Std   | RMS   |
|                   | 1.472  | 0.085 | 0.763 | 0.284 | 0.0001| 0.069 |
|                   | 1.124  | 0.052 | 0.656 | 0.113 | -0.009 | 0.033 |
|                   | 0.892  | -0.303 | 0.512 | 0.278 | -0.120 | 0.084 |
|                   | 0.747  | -0.466 | 0.414 | 0.301 | -0.172 | 0.104 |
| Cheffena-Ayama    |        |       |       |       |       |       |       |       |       |
|                   | Mean   | Std   | RMS   | Mean  | Std   | RMS   | Mean  | Std   | RMS   | Mean  | Std   | RMS   |
|                   | 0.284  | 0.0001| 0.069 |
|                   | 0.113  | -0.009 | 0.033 |
|                   | 0.278  | -0.120 | 0.084 |
|                   | 0.301  | -0.172 | 0.104 |
|                   | 0.414  |       |       |
|                   | 0.301  |       |       |
|                   | 0.104  |       |       |

Three distribution prediction models of fade duration were presented and compared with measured data. Table 1 shows calculation results of test variables of probability of occurrence $P(d>D | a>A)$ by mean, standard deviation (Std) and root mean square (RMS) for three models with different attenuations. Calculation details are proposed by ITU-R P.311 [9]. In general, the ITU-R model predicts better than other models followed by Cheffena-Ayama and Timothy, respectively.
5. Conclusion
Rain fade duration data were measured for one year in Kuala Lumpur, Malaysia from satellite path of MEASAT3. The probability distributions of fade duration were found almost similar in short and long durations. Three distribution prediction models of fade duration were presented and compared with measured data. It was found that none of the models predicted measured fade duration distribution accurately. ITU-R predicted the measured data statistics well for short duration except 1dB attenuation whereas long duration, the model underestimated the measured data. Prediction by Timothy was much overestimated for both short and long duration. Cheffana-Ayama model predicted similar trend to the measured data statistics and overestimated for both short and long durations. However, the overestimation by Cheffana-Ayama was much lower than Timothy model.

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