Towards a complete $\Delta(27) \times SO(10)$ GUT of flavour

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Abstract. We propose a renormalisable model based on $\Delta(27)$ family symmetry with an $SO(10)$ grand unified theory (GUT) leading to a novel form of spontaneous geometrical $CP$ violation. The symmetries are broken close to the GUT breaking scale to yield the minimal supersymmetric standard model with standard $R$-parity. Low-scale Yukawa structure is dictated by the coupling of matter to $\Delta(27)$ antitriplets $\phi$ whose vacuum expectation values are aligned in the CSD3 directions by the superpotential. Light physical Majorana neutrinos masses emerge from the seesaw mechanism within $SO(10)$. The model predicts a normal neutrino mass hierarchy with the best-fit lightest neutrino mass $m_1 \approx 0.3 \text{meV}$, $CP$-violating oscillation phase $\delta_l \approx 280^\circ$ and the remaining neutrino parameters all within 1$\sigma$ of their best-fit experimental values.

1. Introduction

It is well established that the Standard Model (SM) remains incomplete while it fails to explain why neutrinos have mass. Small Dirac masses may be added by hand, but this gives no insight into the Yukawa couplings of fermions to Higgs (where a majority of free parameters in the SM originate), or the extreme hierarchies in the fermion mass spectrum, ranging from neutrino masses of $O$(meV) to a top mass of $O$(100) GeV. Understanding this, and flavour mixing among quarks and leptons, constitutes the flavour puzzle. Other open problems unanswered by the SM include the sources of $CP$ violation ($CPV$) and the origin of three distinct gauge forces.

An approach to solving these puzzles is to combine a Grand Unified Theory (GUT) with a family symmetry which controls the structure of the Yukawa couplings. In the highly attractive class of models based on $SO(10)$, three right-handed neutrinos are predicted and neutrino mass is therefore inevitable via the seesaw mechanism.

In this paper we summarise a recently proposed model [1], renormalisable at the GUT scale, capable of addressing all the above problems, based on $\Delta(27) \times SO(10)$. All SM fermions (and their superpartners) are gathered in a single superfield $\Psi$, an $SO(10)$ spinor that is a triplet under $\Delta(27)$, which couples to (gauge singlet) antitriplet flavons $\phi$. The vacuum expectation values (VEVs) of these flavons are aligned in particular directions in flavour space (known as the CSD3 alignment), which dictate the Yukawa structure at the low scale.

Interactions between the flavons and additional non-trivial singlets of $\Delta(27)$ give rise to $CPV$ phases, which fixes all phases in the lepton mass matrices, and leads to a novel form of spontaneous geometrical $CPV$ (for a partial list of available literature on geometrical $CPV$, see Ref. 6 in [1]).

The model has many attractive features, including the use of only the lower dimensional “named” representations of $SO(10)$, i.e. the singlet, fundamental, spinor or adjoint
representations. $SO(10)$ is broken via $SU(5)$ with doublet-triplet (DT) splitting achieved by a version of the Dimopoulos-Wilczek (DW) mechanism [2]. We find that the MSSM $\mu$ term can naturally be $O$(TeV), and confirm that proton decay is highly suppressed.

The family symmetries are broken close to the GUT breaking scale to yield the minimal supersymmetric standard model (MSSM). The model is realistic in the sense that it provides a successful (and natural) description of the quark and lepton (including neutrino) mass and mixing spectra.

Large lepton mixing is accounted for by the seesaw mechanism with constrained sequential dominance (CSD) [3]. The basic goal of the flavour sector in these models is to couple matter to flavons $\tilde{\phi}_{\text{atm}}, \tilde{\phi}_{\text{sol}}$ and $\tilde{\phi}_{\text{dec}}$, whose VEVs are aligned in the CSD3 direction [4], i.e. where

$$
\tilde{\phi}_{\text{atm}} = v_{\text{atm}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad 
\tilde{\phi}_{\text{sol}} = v_{\text{sol}} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad 
\tilde{\phi}_{\text{dec}} = v_{\text{dec}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},
$$

which has been previously shown to be a very promising and predictive setup for understanding lepton masses [5].

Since $SO(10)$ constrains the Dirac couplings of leptons and quarks to be equal (or nearly so), it is non-trivial that the successful scheme in the lepton sector will translate to success in the quark sector. Remarkably we find that we can attain good fits to data for quark and lepton masses, mixings and phases.

2. The model

The most important superfields in the model [1] are given in table 1. It includes the MSSM matter superfield $\Psi$, various Higgs fields that break $SO(10)$, $SU(5)$ and electroweak gauge symmetries, and flavons $\phi_i$ that produce the phenomenologically successful form of the quark and lepton mass matrices. It also includes the singlet $\xi$, which acquires a VEV below the GUT scale, $\langle \xi \rangle \lesssim 0.1 M_{\text{GUT}}$ and ultimately controls the hierarchies present in the model. $CP$ is conserved at the high scale and spontaneously broken by flavons, while GUT and flavour singlets $Z,Z''$ break $Z_4^R$ $R$-symmetry to regular $R$-parity [6].

| Field   | $\Delta(27)$ | $SO(10)$ | $Z_4^R$ | Role                          |
|---------|--------------|----------|---------|-------------------------------|
| $\Psi$  | 3            | 16       | 1       | Contains SM fermions          |
| $H^u,d_{i0}$ | 1         | 10       | 0       | Break electroweak symmetry    |
| $H_{i6,\bar{16}}$ | 1     | 16,16    | 0       | Break $SO(10)$                |
| $H_{45}$ | 1            | 45       | 0       | Breaks $SU(5)$                |
| $H_{DW}$ | 1            | 45       | 2       | Gives DT splitting via DW mechanism |
| $\phi_i$ | 3            | 1        | 0       | Produces CSD3 mass matrices   |
| $\xi$   | 1            | 1        | 0       | Gives mass hierarchies, $\mu$ term |
| $Z,Z''$ | 1            | 1        | 2       | Break $Z_4^R \rightarrow Z_2^R$ ($R$-parity) |
| $A_i$   | 3            | 1        | 2       | Aligns triplet flavons $\tilde{\phi}_i$ |
| $O_{ij}$ | 1           | $ij$     | 2       | Aligns triplet flavons $\tilde{\phi}_i$ |

Table 1. Core superfields of the model and their representations under the symmetries.

The flavour structure for quark and lepton matrices is determined by the CSD3 framework, where the fermions couple to flavons that acquire VEVs like in Eq. 1. More precisely, its implementation can be understood as follows: at the renormalisable level, $\Psi, \phi_i$ and $H$ couple to
SO(10) spinor superfields $\chi_i$, which act as messengers. Integrating these out produces Yukawa-like terms
\[
\lambda \psi \psi H_0 \bar{\phi}_i \phi_j \frac{\xi^n}{M_X^{n+2}},
\]
where $\lambda$ are $O(1)$ couplings, $n$ are integer powers, and $M_X \gtrsim M_{\text{GUT}}$. The presence of different such terms with different powers of $\xi$ establishes a hierarchy between elements of the fermion mass matrices, and consequently a natural explanation for the large range of fermion masses. $\xi$ is also responsible for generating a $\mu$ term that can be $O(\text{TeV})$. After $SO(10)$ is broken, all but two of the Higgs doublets and all triplets contained in $H_{10}^{u,d}$ and $H_{16,\overline{10}}$ must be very heavy, while exactly two doublets, $H_{u,d}$, are light. Constructing the effective mass matrix for Higgs doublets, it has one light eigenvalue with $\mu \sim \langle \xi \rangle^8 / M_{\text{GUT}}^2 \ll M_{\text{GUT}}$.

Guided by $SO(10)$ unification, ultimately all mass matrices take the same generic form:
\[
m' = m_1^f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_2^f \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix} + m_3^f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]
where $m_i^f$ are complex numbers. An exception is the term $\Psi \Psi H_{10}^{u} \xi \bar{\phi}_{\text{sol}} \phi_{\text{dec}}$, which is allowed by the symmetries, giving an additional contribution to $m^u$. It does not affect neutrino physics.

This matrix structure can successfully accommodate all quark and lepton masses and mixing angles, and is predictive in the lepton sector. It predicts a Normal Hierarchy with best fit parameters
\[
\theta_{12} = 33.1^\circ, \quad \theta_{13} = 8.55^\circ, \quad \theta_{23} = 40.8^\circ, \quad m_1 = 0.32 \text{ meV},
\]
\[
\delta_{\text{CP}} = 280^\circ, \quad m_2 = 8.64 \text{ meV}, \quad m_3 = 49.7 \text{ meV}.
\]
Once lepton masses are fitted, the PMNS matrix is completely fixed with no freedom, including the Dirac phase $\delta_{\text{CP}}$, which is a prediction of the model and not fitted. It agrees well with the current experimental hints.

3. Conclusion
This is one of few flavour $SO(10)$ models with realistic fits to both quark and lepton data, and the most complete, as it is renormalisable and accounts for gauge coupling unification, the $\mu$ term, DT splitting, and (lack of) proton decay. It is predictive in the lepton sector, and may be probed by precise measurements of PMNS parameters, particularly $\delta_{\text{CP}}$, and ruled out by observation of an Inverse Hierarchy, or non-observation of SUSY.

References
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