Quenched QCD with domain wall fermions

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We report on simulations of quenched QCD using domain wall fermions, where we focus on basic questions about the formalism and its ability to produce expected low energy hadronic physics for light quarks. The work reported here is on quenched $8^3 \times 32$ lattices at $\beta = 5.7$ and 5.85, using values for the length of the fifth dimension between 10 and 48. We report results for parameter choices which lead to the desired number of flavors, a study of undamped modes in the extra dimension and hadron masses.

1. INTRODUCTION

Domain wall fermions [1] were originally proposed as a technique for putting chiral fermions on the lattice. For fermions coupled to a vector gauge theory (like QCD), this approach has given a way to treat massless (or at least controllably small mass) fermions, while preserving the full continuum symmetry group. Subsequent improvements to the original formulation [2] made simulations more practical and studies of the Schwinger model [3], including dynamical fermion effects, and quenched QCD [4] have been very encouraging. Here we report a reasonably systematic study of low energy, non-anomalous QCD physics using domain wall fermions. For a review and further references, see [5].

2. BASIC QUESTIONS

For quenched domain wall simulations, there are three input parameters for the fermions: $m_f$, the explicit four-dimensional bare quark mass; $L_s$, the extent of the lattice in the fifth dimension and $m_0$, the five-dimensional bare quark mass. Except for different symbols for the three parameters listed above, our conventions follow [2].

A first basic question involves choosing parameters so that the desired number of flavors of light fermions is being studied. For free fermions, one easily sees that for $m_0 < 0$ there are no light surface states at the ends of the fifth dimension, for $0 < m_0 < 2$ surface states for a single light quark flavor appear, for $2 < m_0 < 4$ surface states for four light flavors appear and for $4 < m_0 < 6$ there are surface states for six light quark flavors. There is a symmetry under $m_0 \rightarrow 10 - m_0$, which remains when coupling to gauge fields is introduced, although then these values of $m_0$ shift.

Figure 1 shows one way to probe the number of light fermions. The graph is for quenched simulations at $\beta = 5.85$ on an $8^3 \times 32$ lattice. For a given value of $m_0$, we have measured the chiral condensate for $m_f \rightarrow 0$.02, 0.04, 0.06, 0.08 and 0.10, and calculated $\langle \bar{\psi} \psi(m_f \rightarrow 0) \rangle$ (simple linear fits work well). This extrapolation was done for three different values of $L_s$, to make sure we were close to the large $L_s$ limit. The figure shows this extrapolated chiral condensate as a function of $m_0$.

For $m_0 < 0.9$ the extrapolated chiral condensate is zero, indicating no spontaneous chiral symmetry breaking, due to the absence of surface states. For larger values of $m_0$ the chiral condensate is non-zero and increasing, until leveling off for $1.65 < m_0 < 2.15$. This gives a region where there appears to be a single surface state. (For free fermions, the normalization of the fields depends on $m_0$; this region of insensitivity to $m_0$ does not appear to have been anticipated.) For

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Figure 1. The four-dimensional chiral condensate extrapolated to zero quark mass as a function of the five-dimensional domain wall mass.

larger $m_0$ the chiral condensate increases as the change from a one flavor to a four flavor theory occurs. Above about 3.4, a region of relative insensitivity to $m_0$ occurs, where there are four flavors of light fermions. The chiral condensate in this region is numerically very close to four times its value for $1.65 < m_0 < 2.15$.

A second basic question is whether the mixing between the chiral modes at $s = 0$ and $s = L_s - 1$ can be controlled by choosing $L_s$ large. It is known that a gauge field with a zero eigenvalue for the four-dimensional Wilson Dirac operator (with mass $m_0$), has no damping for modes propagating in the fifth dimension. We wanted to explicitly measure the effect of this undamped mode on non-anomalous low energy QCD physics. To this end, we have studied its effect on the chiral condensate (for related work see [6]).

The four-dimensional chiral condensate is defined through the inverse propagator for domain wall fermions, where $s = 0(L_s - 1)$ is a source for right (left) handed quarks and $s = L_s - 1(0)$ is a sink for left (right) handed quarks. We have studied a generalized condensate, where the distance from source to sink is variable. When this distance is $L_s - 1$, we have the conventional condensate.

For a few thermalized configurations, we have measured the values of $m_0$ where there should be undamped modes in the fifth dimension. Figure 2 shows a measurement of the generalized chiral condensate as a function of the separation between the source and sink in the fifth dimension for $m_0$ very close to the value where a zero of the four-dimensional Wilson operator occurs.

Figure 2. A generalized version of the four-dimensional chiral condensate, where the separation in the fifth dimension between the source and sink is $s$. $s = L_s - 1$ is the usual condensate.

One can clearly see the presence of the expected translationally invariant mode in the fifth dimension. However, this mode contributes about 1% to the value of the chiral condensate, for this one lattice where its effect should be largest. For an ensemble of lattices, at any given $m_0$ only some will have largely undamped modes. (This number could be a set of measure zero in the full average, although this needs to be studied as a function of lattice volume.) Even if undamped modes do survive the ensemble average, our example shows
that they can make a very small contribution to an observable.

3. HADRON MASSES

To test the formulation in a more practical way, we have measured hadron masses for quenched domain wall fermions on $8^3 \times 32$ lattices with $L_s = 10, 16, 24$ and 48 for $\beta = 5.7$ and $m_0 = 1.65$. (Since the conference, we have also simulated at $L_s = 32$.) We have chosen $m_f$ in the range 0.02 to 0.22. For $m_f = 0.02$, we have not gotten good plateaus with 70-90 configurations and consequently only report results using $m_f > 0.02$.

The rho mass shows no dependence on $L_s$ between 10 and 48, while the nucleon mass decreases somewhat for larger $L_s$. For $L_s = 48$, we find $m_\rho = 0.787(12) + 2.28(5)m_f$ and $m_N = 1.19(3) + 3.71(15)m_f$. For $L_s = 48$, we find $m_\rho = 0.792(28) + 2.15(12)m_f$ and $m_N = 1.12(4) + 3.79(19)m_f$. These are correlated fits to five or more valence quark masses and all have $\chi^2/dof < 1$. Figure 3 shows the nucleon and rho masses for $L_s = 48$. The $m_f = 0$ value for $m_N/m_\rho$ is 1.42 ($L_s = 48$), compared to the staggered fermion values of $\approx 1.50$.

4. CONCLUSIONS

Our studies of domain wall fermions for quenched QCD have shown the technique capable of reproducing expected physics. The non-zero value for $m_\pi^2(0)$, even for large $L_s$, deserves more attention. Since other formulations have shown similar effects for $m_\pi^2(0)$, we must check again at weaker coupling and larger volumes before concluding that this is a new phenomena revealed by domain wall fermions in the quenched approximation.

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