A new insight into the negative-mass paradox of gravity and the accelerating universe

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The discovery of acceleration of the universe expansion in recent astrophysics research prompts the author to propose that Newton’s gravitation law can be generalized to accommodate the antimatter: While the force between matters (antimatters) is attractive, the force between matter and antimatter is a repulsive one. A paradox of negative-mass in gravity versus a basic symmetry \((m \rightarrow -m)\) based on quantum mechanics is discussed in sufficient detail so that the new postulate could be established quite naturally. Corresponding modification of the theory of general relativity is also suggested. If we believe in the symmetry of particle and antiparticle as well as the antigravity between them, it might be possible to consider a new scenario of the expansion of universe which might provide some new insight into the interpretation of cosmological phenomena including the accelerating universe observed.

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I. INTRODUCTION

Let us begin with a “paradox of negative-mass” in the theory of gravity. As is well known, the Newton’s gravitation law reads:

\[ F(r) = -Gm_1m_2/r^2, \tag{1} \]

where \(m_1\) or \(m_2\) is the gravitational mass of a particle or a macroscopic body with spherical symmetry, \(r\) is the distance between them and \(G\) is the gravitational constant. The minus sign in \((1)\) means that the force \(F\) between them is always an attractive one, which in turn implies that the gravitational potential energy of them is negative:

\[ U(r) = -Gm_1m_2/r. \tag{2} \]

On the other hand, all experiments and the theory of special relativity (SR) have been verifying the equivalence of mass \(m\) and energy \(E\), i.e., the Einstein’s equation:

\[ E = mc^2, \tag{3} \]

where \(m\) is the inertial mass. It is defined by another Newton’s law of dynamics:

\[ F = ma, \tag{4} \]

where \(a\) is the acceleration of the particle (body).

Consider a system (body) being composed of many particles, the gravitational binding energy shown by \((2)\) would render the total mass of the body decreasing. Then an acute problem arises: can this body have a negative mass? If so, a very bizarre phenomenon would occur as discussed by Bondi, Schiff and Will respectively \([1]\): Suppose that such a body (with mass \(m_1 < 0\)) is brought close to a normal body (with mass \(m_2 > 0\)). According to Eqs. \((1)\) and \((4)\), the positive-mass body \((m_2)\) would attract the negative-mass body \((m_1)\) whereas the negative-mass body \((m_1)\) would repel the positive-mass body \((m_2)\). The pair (a “gravitational dipole”) would accelerate itself off, without any outside help or use of propulsion! The conservation law of momentum and that of energy would all be violated.

Incredible! No one can believe in that. Hence we may name the above problem a “negative-mass paradox” in gravity. There are a number of paradoxes in physics, aiming at pushing the contradiction hidden in the theory to a situation that enables one to clarify what was wrong or missing in the basic concept.

To get rid of this “negative-mass paradox”, a “positive-energy theorem” was posed in the middle of 1960s, saying that the total asymptotically determined mass of any isolated body in general relativity (GR) must be nonnegative. This theorem had been proved since 1979 in a variety of ways and is in total conformity with Einstein’s equation \((3)\) because the observed inertial mass and energy are always positive definite.

However, being a paradox as acute as that hidden in Eq. \((1)\), it does provide a much stronger hint than a negation of negative mass. Remembering that the appearance of negative energy is inevitable in relativistic quantum mechanics (RQM) and is intimately related to the existence of antiparticles, I will manage to claim that the whole solution to the “negative-mass paradox” must be a generalization of Newton’s gravitation law into the following form:

\[ F(r) = \pm Gm_1m_2/r^2, \tag{5} \]

where the minus sign holds for \(m_1\) and \(m_2\) (both positive) being both matters or both antimatters whereas the plus sign holds for one of them being antimatter, which means that matter and antimatter repel each other.
II. SYMMETRY OF SPACE-TIME INVERSION

Starting from RQM, we consider the wavefunction (WF) of a freely moving (along $x$ axis) particle:

$$\psi \sim \exp \left[ i(p \cdot x - E t)/\hbar \right], \quad (6)$$

where $p$ is the momentum and $E(>0)$ the total energy.

But what is the WF $\psi_c$ of an antiparticle? Before 1956, it was assumed to be a consequence of the operation of a so-called charge-conjugate transformation $C$ which can bring a charged particle (say an electron with charge $-e$) to its antiparticle (say the positron with charge $e$) [2]:

$$\psi_c = C \psi \sim \psi^* \sim \exp \left[ i(-p \cdot x + E t)/\hbar \right]. \quad (7)$$

We see that the negative-energy $-E < 0$ emerges immediately due to the basic operators in quantum mechanics (QM):

$$\hat{p} = -i \hbar \frac{\partial}{\partial x}, \quad \hat{E} = i \hbar \frac{\partial}{\partial t}. \quad (8)$$

The negative-energy difficulty at the level of QM was remedied to some extent by the so-called “hole theory for positron” (which is obviously impossible for the Klein-Gordon particle) and solved formally at the level of quantum field theory (QFT) by the redefinition of creation (annihilation) operators.

Since the discovery of parity violation, i.e., the violation of space-inversion $P(x \rightarrow -x)$ symmetry in 1956-1957, physicists realize that not only $P$ but also $C$ transformations are not conserved in the weak-interaction processes. So Eq. (7) is not applicable in general and the WF of antiparticle should be redefined as:

$$\psi_c = CPT \psi \sim \exp \left[ -i(p \cdot x - E t)/\hbar \right], \quad (9)$$

where the so-called “time-reversal transformation” $T$ is defined as:

$$\psi \rightarrow T \psi = \psi^*(x,-t) \sim \exp \left[ i(-p \cdot x - E t)/\hbar \right]. \quad (10)$$

Note that: First, the name “time-reversal” is actually a misnomer [3, 4]. What the transformation (10) means is merely a reversal of motion ($p \rightarrow -p$). Second, the correctness of definition of antiparticle WF (9) depends on the validity of the CPT theorem which in turn is ensured by basic principles of SR and QFT. Third, as the complex-conjugate operations in $C$ and $T$ cancel each other, what a combined CPT transformation in (9) means is merely a sign change of coordinates $(x,t)$ in comparison with Eq. (6) [2]. But the original meaning of $C$, $P$ and $T$ implies that Eq. (9) should describe an antiparticle with the same $p$ and $E(>0)$ as that of the particle described by (6). Hence for antiparticles, we should forget the “hole theory” and use the following operators instead of (8):

$$\hat{p}_c = i \hbar \frac{\partial}{\partial x}, \quad \hat{E}_c = - i \hbar \frac{\partial}{\partial t}. \quad (11)$$

In fact, Eq. (11) had been proven to be the direct and unique outcome of the full solutions to the EPR paradox and Klein Paradox [5,4,6].

Fourth, once we accept Eqs.(9) and (11), the CPT theorem becomes a new fundamental postulate, i.e., a basic symmetry which can be stated in the following form:

Under the (newly defined) space-time inversion denoted by $\mathcal{PT}$, meaning merely $x \rightarrow -x$, $t \rightarrow -t$, the theory of RQM remains invariant with its concrete solution, e.g., a particle WF transforming to its antiparticle WF (denoted by $C$) automatically. It means that our postulate reads:

$$\mathcal{PT} = C. \quad (12)$$

For example, the Schrödinger equation is nonrelativistic. But the following coupled Schrödinger-like equation

$$\begin{cases} 
    i \hbar \frac{\partial}{\partial t} \varphi = m c^2 \varphi - \frac{\hbar^2}{2m} \nabla^2 (\varphi + \chi), \\
    i \hbar \frac{\partial}{\partial t} \chi = -m c^2 \chi + \frac{\hbar^2}{2m} \nabla^2 (\chi + \varphi),
\end{cases} \quad (13)$$

is just the relativistic Klein-Gordon equation

$$(i \hbar \frac{\partial}{\partial t})^2 \psi = -\frac{c^2 \hbar^2}{m^2} \nabla^2 \psi + m^2 c^4 \psi, \quad (14)$$

with relation first pointed out by Feshbach and Villars in 1958:

$$\begin{cases} 
    \varphi = (\psi + i \hbar \psi/ m c^2)/2, \\
    \chi = (\psi - i \hbar \psi/ m c^2)/2.
\end{cases} \quad (15)$$

Now we see that under the space-time inversion ($x \rightarrow -x, t \rightarrow -t$) and the transformation:

$$\varphi(-x,-t) \rightarrow \chi(x,t), \quad \chi(-x,-t) \rightarrow \psi(x,t), \quad (16)$$

the Eq. (13) does remain invariant while a particle WF (6) with $|\varphi| > |\chi|$ (due to $E > 0$, see (15)) turning to its antiparticle WF (9) with $|\chi_c| > |\varphi_c|$ (due to $E < 0, E_c = -E > 0$, see (16)).

III. SYMMETRY OF MASS INVERSION

Alternatively, we can restate the above basic symmetry in the following way: Under the mass inversion:

$$m \rightarrow -m, \quad \varphi(x,t) \rightarrow \varphi(x,t), \quad \chi(x,t) \rightarrow \varphi(x,t), \quad (17)$$

the theory, e.g., Eq. (13), remains invariant. Although transformation (16) is equivalent to transformation (17), they share different advantages. The former is relevant to unobservable coordinates $(x,t)$ and so is more essential in RQM and equivalent to even more abstract symmetry of $i$ versus $-i$ (see Eq.(6) versus (9)), while the latter is relevant to observable mass $m$ and so is easily to be generalized to the case of classical theory.
For example, as is well known, the motion equation for an electron is given by the Lorentz-force law:

\[ \mathbf{F} = m \mathbf{a} = -e (\mathbf{E} + \mathbf{v} \times \mathbf{B} / c). \] (18)

Before 1956, its counterpart for positron is obtained via a C transformation \((-e \rightarrow e)\), yielding:

\[ \mathbf{F}_e = m \mathbf{a}_e = e (\mathbf{E} + \mathbf{v} \times \mathbf{B} / c), \] (19)

which is still allowed since C is conserved in the electromagnetic interaction. However, we’d better use the following working rule: to deal with particle (matter) and antiparticle ( antimatter) on an equal footing, a classical theory must be invariant under the mass-inversion transformation \(m \rightarrow -m\). Using this rule to Eq. (18), we get (19) immediately. Note that: First, \(m\) is always positive. Second, being the external field, the electric (magnetic) field strength \(E(B)\) undergoes no change in the transformation. Third, in RQM like (13) or (14), the motion equation for antiparticle is the same one as that for particle. This is because each particle state like (6) contains its hidden antiparticle field under the condition \(|\phi| > |\chi|\) whereas an antiparticle state like (9) contains its hidden particle field under the condition \(|\chi_c| > |\phi_c|\). By contrast, in classical theory, the equation for particle is separated from that for antiparticle as shown by (18) versus (19). We just complement (18) with (19) so that the theory becomes invariant under the mass inversion transformation. In other words, a new equation must be added before it can be complete in the sense of treating the particle and antiparticle equally.

Fourth, to clarify further why we prefer the new postulate (12) instead of C transformation and CPT theorem, we wish to emphasize an important difference between a postulate (law) and a theorem. All quantities in a theorem must be defined in advance separately and unambiguously and the outcome of the theorem is actually contained in its premise implicitly. For example, the definitions of C, P and T are all clear in mathematics and the validity of CPT theorem is ensured by the basic principles of SR and QFT. Once C, P and T are not conserved in experiments, they cease to be meaningful as physical transformations. In this situation, the CPT theorem immediately exhibits itself as a new postulate (12) in which the definition of transformation of particle to antiparticle is just contained in the same equation. In general, a postulate or law can often (not always) accommodate a definition of physical quantity, and the validity of the postulate (law) together with the definition must be verified by experiments. Hence the establishment of a law is a process “from particular to general” or an outcome of “analysis and induction method”. By contrast, to prove a theorem from well-established theories is a process “from general to particular” or a consequence of “deduction method”.

For example, the definition of gravitational mass \(m\) is contained in the gravitational law, Eq. (1). The definition of inertial mass \(m\) is contained in Newton’s law, Eq. (4). The definition of electric-magnetic field strength, \(\mathbf{E}\) and \(\mathbf{B}\), is contained in the Lorentz force law, Eq. (18). What we have done is a similar thing—the definition of particle-antiparticle transformation \(C\) is contained naturally in a new postulate (12) — not one that comes from elsewhere.

IV. GENERALIZATION OF NEWTON’S GRAVITATION LAW TO ANTIMATTER AND ANTAGRAVITY

We are now in a position to establish Eq. (5), performing the rule of mass-inversion on Eq. (1) together with Eq. (4). Note that, however, both \(m_1\) and \(m_2\) belong to a closed system under consideration. When one of them, say \(m_1\), is an antimatter, we just perform a transformation \(m_1 \rightarrow -m_1\) once, regardless of whether it is a source (\(m_1\) appears only at one side of equation) or a detected body (\(m_1\) appears at both sides of equation). This simple procedure brings Eq. (1) to Eq. (5) with a plus sign, implying that \(m_1\) and \(m_2\) repel each other. Furthermore, if \(m_2\) is also an antimatter, one more transformation \(m_2 \rightarrow -m_2\) recovers the minus sign, implying that two antimatters attract each other. Now the whole Eq. (5) is invariant with respect to transformation \(m_i \rightarrow -m_i (i = 1, 2)\).

The reason we can get rid of the “negative-energy paradox” is as follows. If one thought that the detected body \(m_1\) is really a negative mass \((m_1 < 0)\) while \(m_2 > 0\) seriously, the paradox was inevitable. But here every mass is positive regardless of whether it is matter or antimatter. Even if we make a careless mistake of performing transformation \(m_1 \rightarrow -m_1\) twice, it is still not a serious mistake—we merely do a trivial thing to return back to the minus sign. Yet by another transformation on either \(m_1\) or \(m_2\) will the other equation with plus sign be supplemented. Eventually, the correct interpretation of whole Eq. (5) is independent of the procedures used to reach it. The bizarre phenomenon which occurred in the paradox will never happen again.

In some sense, the paradox stemmed from the incorrect conception that the difference between positive and negative masses is absolute, whereas now we regard the difference between \(m\) and \(-m\) merely relative. We merely perform a symmetry transformation \(m \rightarrow -m\) to show the equal existence of matter versus antimatter, but eventually there is no negative mass at all. This reminds us of the experience (i.e., lesson) of Einstein. He struggled for eight years before inventing the theory of GR. What Einstein eventually realized is that space and time have no absolute meaning but systems of relations (as stressed by Smolin [7]).
V. GENERALIZATION OF EINSTEIN FIELD EQUATION IN GENERAL RELATIVITY

Consider a positronium and an atom of matter. If Eq. (5) is correct, there will be no gravitational force between them. This means that the gravitational mass \( m \) (grav.) of positronium is zero! However, the energy or the relevant inertial mass \( m \) (inert.) (see Eq. (3)) of positronium is definitely nonzero. Hence we see that in the case of coexistence of particles and antiparticles, the equivalence principle (EP) in the (weak) sense that [8]

\[ m \text{ (grav.)} = m \text{ (inert.)} \] (20)
cannot be valid.

As is well known, the EP served as a starting point in establishing the theory of general relativity (GR). The possible invalidity of EP in the presence of antimatter implies that GR is dealing with the gravitation of pure matter without the coexistence of antimatter. Indeed, let us look at the Einstein field equation [9]:

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi GT_{\mu\nu} \] (21)

On the left side, the Ricci tensor \( R_{\mu\nu} \), curvature scalar \( R \) and the metric tensor \( g_{\mu\nu} \) are all functions of coordinates \( x_{\mu} \). While on the right side, the energy-momentum tensor \( T_{\mu\nu} \) is introduced to describe the existence of matter in the vicinity (a macroscopic small volume) of \( x_{\mu} \). Then under a transformation of mass inversion \( m \to -m \) to reflect that of matter to antimatter, \( T_{\mu\nu} \) should change its sign due to its proportionality to the mass \( m \). Hence Eq. (21) changes sign on the right side whereas not on the left side. This reflects the fact that GR is a classical field theory and so cannot treat the matter and antimatter on an equal footing.

To keep Eq. (21) invariant under the mass inversion, we need to modify its right side by a generalization as:

\[ T_{\mu\nu} \to T_{\mu\nu}^{\text{eff}} = T_{\mu\nu} - T_{\mu\nu}^{\text{c}} \] (22)

(where the superscript \( c \) means antimatter) since under the mass inversion, \( T_{\mu\nu} \to -T_{\mu\nu}^{\text{c}} \) and \( T_{\mu\nu}^{\text{c}} \to -T_{\mu\nu} \). Notice that the form of the energy-momentum tensor is the same for both matter and antimatter. We stress once again that the distinction between \( m \) and \( -m \) is merely relative, not absolute.

VI. ANTIGRAVITY AND COSMOLOGY

Evidently, Eq. (5) cannot be tested by experiments on earth. We can only await and see new developments in astrophysical research. Surprisingly, in recent years, the careful observation of some distant Type Ia supernovae with redshift \( z = 0.39 - 0.9 \) reveals that the expansion of the universe is accelerating rather than decelerating as physicists pondered several years ago (see the excellent papers [10,11,12]). Many physicists tend to explain the acceleration by resorting to the cosmological constant (with dimension (length)^{-2}) originally introduced by Einstein, unsuccessfully trying to interpret it as a type of “vacuum zero-point energy” in QFT. It is often called “dark energy”, which may account for up to 70% of the entire matter and energy in the universe. Actually, nobody understands what the dark energy really is.

Although I have no research experience in the field of GR and astrophysics, I do share the opinion of Davies [13] that "if we want to keep inflation and account for today's accelerating expansion, we need a theory that explains why antigravity was once tense, then dropped and hovered at just above zero..... One possibility is that the force fades with time. Another is that it varies in space, so that far beyond the limit of our telescopes it may be much bigger..... What we need is a theory that derives the strength of the antigravity force as part of a unified description of all the forces of nature.” I am trying to pose some conjecture as follows.

(a) During the inflation era right after the big bang, once particles and antiparticles were created, they began to repel each other according to Eq. (5). I don’t believe the interpretation that the present baryon asymmetry stemmed from the small CP violation (which is equivalent to \( T \) violation, as discussed above, it has nothing to do with the basic symmetry in time or that in particle and antiparticle). I prefer to think that the total number of particles is equal to that of antiparticles in the entire expanding universe. But antiparticles had a head start during the inflationary expansion and flew away faster than particles did. The latter lagged behind to some extent and our galaxies gradually evolved out of them. On the other hand, some distant stellar objects (quasars?) might be evolved from antiparticles and some of them may be just those observed Type Ia supernovae undergoing acceleration caused by the repulsive force exerted by inner galaxies composed mainly of matters.

(b) Based on the above picture and solving Eq. (21) with modification (22) in the Friedmann model as discussed in [9], we can see that in a large part of universe where the densities of matter and antimatter are nearly equal, the acceleration of the expansion tends to zero. In other words, the flatness of our universe as shown by the recent careful measurement of the fluctuation of cosmic microwave background radiation (see [14]) is critically depending on two factors: first, the inflationary expansion triggered by the repulsive force between matter and antimatter; second, the nearly equal densities of matter and antimatter in a large part of universe after the inflationary expansion. So a careful measurement of the variation of the Hubble constant [11] with distance may reveal the subtle difference in different regions.

(c) Among all particles, neutrinos are of particular interest [15]. After the big bang, superluminal neutrinos and antineutrinos are equally created in three flavors and distributed everywhere isotropically. Their number den-
sity may be $10^8$ times that of protons. However, because the forces between neutrinos and matter just cancel that between antineutrinos and matter, all neutrinos and antineutrinos (even they all have positive tachyon mass, see VII.(d)) escape from the analysis of so-called dark matter.

(d). As the next conjecture, I guess that in the intermediate region where matter and antimatter are overlapping, there may be some probability of collisions between matter and antimatter. Because of the long range repulsive force between them, these collisions may be grazing and so the annihilation process occurs at short distance would have some special character. Is it possible to relate the above conjecture with the mechanism of gamma-ray bursts (GRBs)? It has been noticed for years that GRBs are distributed at a remote distance on the cosmological scale and they tend to have a roughly constant explosive energy (see [16] and references therein). So the careful study of the GRB-Hubble diagram as conducted by Schaefer [16] is very important because the spatial distribution of GRB may provide further evidence of nonuniform distribution of matter and antimatter in the universe.

(e). Furthermore, I think that the properties of cosmic matter composed of pure matter and that of a mixture of matter with antimatter are certainly different. The latter bears some resemblance to plasma (not quite the same) with a “screening length”. The fluctuations in plasma with length scale larger than the screening length should be strongly suppressed. In fact, the star formation rate is rising steadily from remote distance with redshift $z \sim 6$ [16] which seems to me an reflection that the domination of antimatter is increasing steadily there.

VII. SUMMARY AND DISCUSSION

(a). I was pleasantly surprised to read the wonderful paper by Schödel et al [17]. Their delicate observation over ten years and convincing analysis verified the existence of a huge black hole at the center of the Milky way. This should be viewed as a triumph of experimental science, also a triumph of the theory of GR, which was first verified by its successful explanation on the precession of Mercury’s perihelion. However, despite all of these, GR is still a classical theory dealing with the gravitation of pure matter as shown by its noninvariance under the transformation of mass inversion.

(b). Based on the rule of mass-inversion invariance, the Newton’s gravitation law is generalized to Eq. (5) and the Einstein field equation in GR, Eq. (21), is also modified by (22) such that they can treat matter and antimatter equally and explicitly. The antigravity between matter and antimatter may be important to account for many cosmological phenomena including the acceleration of expansion of the universe.

(c). Eq. (5) should be derived from Eq. (21) with (22) in a weak-field approximation. We follow the standard procedure as discussed in [9] and notice that in the case of one of the bodies being antimatter (the other matter), an extra minus sign will appear in front of the product of their energy-momentum tensors due to Eq. (22). This means antigravity in Eq. (5).

(d). We are dealing with three kinds of symmetries, the particle-antiparticle symmetry, the space-time inversion symmetry, and the mass inversion symmetry. They are essentially equivalent to each other. This can also be seen from the two Casimir invariants in the Poincaré group, which is a group generated by the inhomogeneous Lorentz transformation [18]:

$$\mu^\nu = \Lambda^\mu_\nu x^\nu + a^\mu$$  \hspace{3cm} (23)

Usually, the conditions: $\det |\Lambda| = 1$ and $\Lambda_0 \geq 1$ are imposed. So one is dealing with a proper and orthochronous Poincaré transformation. There are two Casimir invariants (commuting with all ten generators $M_{\mu\nu}$ (the six generators of the Lorentz group) and $P_{\mu\nu}$ (the four generators of the space-time translation group)) being: $P_{\mu\nu} P^{\mu\nu}$ with eigenvalues $m^2$, ($m$ is the particle mass) and $W_\sigma W^\sigma$ ($W_\sigma = -\frac{1}{2} e_{\mu\nu\rho\sigma} M^{\mu\nu} P^\rho$, the Pauli-Lubanski operator) with eigenvalues $-m^2 s (s + 1)$, ($s$ is the particle spin). Now we know that the physical essence of the Lorentz group is that of SR. The latter can be derived from any one of the three discrete symmetries mentioned above based on QM. Hence we can generalize the Poincaré group beyond the proper and orthochronous one to incorporate the antiparticle explicitly, i.e., we may perform the inversion of $P_\mu$ and $M^{\mu\nu}$ so that $P_\mu \rightarrow -P_\mu$, $W_\sigma \rightarrow -W_\sigma$ which means the transformation of a particle to its antiparticle. The invariance of $P_\mu P^\mu$ and $W_\sigma W^\sigma$ implies that their eigenvalues remain invariant under the corresponding inversion of mass: $m \rightarrow -m$. In other words, either the Lorentz group or the Poincaré group is relativistic in essence which is nothing but the equal existence of particle and antiparticle. Moreover, a negative eigenvalue of $P_\mu P^\mu = m^2 < 0$ is also allowed. This implies an analytical continuation of mass: $m \rightarrow im_\sigma$ or $-im_\sigma$ with $m_\sigma$ real (the tachyon mass) to describe a superluminal neutrino or antineutrino [15]. Here the symmetry between $i$ and $-i$ reflects even more deeply the symmetry of mass inversion which is really a relative one rather than an absolute one.

(e). The “photon” with $m^2 = 0$ in the representation of Poincaré group is by no means an ordinary particle. Usually, a photon with linear polarization ($\gamma_x$ or $\gamma_y$) can be viewed as a linear superposition of a photon with left-handed polarization ($\gamma_L$) and that with right-handed polarization ($\gamma_R$). But in our unified theoretical scheme of space-time inversion, $\gamma_L$ and $\gamma_R$ could be viewed as the “particle” and “antiparticle” (or vice versa). So I guess that a photon has no gravitational mass. Alternatively, we may say that a photon exerts no gravitational force on other matter (antimatter). It just moves along the geodesic line in the space-time which is determined by all matter and antimatter in the entire universe. By the way, according to our understanding of so-called “wave-particle duality” [4], a “photon” should be treated theo-
retically as an electromagnetic wave in propagation until detected then it appears as a "particle". To speak of a photon's position before it is detected is meaningless.

(f). The possible violation of the equivalence principle in the case of coexistence of matter and antimatter has far-reaching implication. The energy or inertial mass [linked by Eq. (3)] obeys the conservation law and thus is observable. By contrast, the gravitational mass is not conserved and thus is not observable in the strict sense. It merely obtains its definition from Newton's gravitation law, which, of course, is a classical and approximate one.

(g). I dare not discuss the quantization of GR because I have little knowledge about the theory of quantum gravity (see the excellent introduction by Smolin [7]). However, according to our understanding about SR based on QM [4, 6], a possible "fourth road" to quantize GR might be a localization of the space-time inversion symmetry mentioned above (just like GR being the localization of the Lorentz symmetry in SR) so that both matter and antimatter can be treated explicitly and equally. Regrettably, I have been pondering this problem for many years with nearly no progress. So I hope that more physicists will join this difficult but interesting pursuit.

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