Self-tuning of the P-vortices

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We observe that on the currently available lattices the non-Abelian action associated with the P-vortices is ultraviolet divergent. On the other hand, the total area of the vortices scales in physical units. Since both the ultraviolet and infrared scales are manifested and there is no parameter to tune, the observed phenomenon can be called self tuning.

1. INTRODUCTION

The Abelian monopoles and central vortices seem to be most promising effective degrees of freedom to explain the confinement, for review and references see, e.g., [1]. Straightforward interpretation of the lattice data is made difficult, however, by the fact that monopoles and vortices are defined in terms of projected fields. One still can, upon detection of the vortices or of monopoles by means of projections, measure their entropy and non-Abelian action. In case of the monopoles, it turns out that there is a non-trivial behavior of the action in the ultraviolet, see [2] and references therein. In other words, the monopoles can be observed only as a result of cancellation between the mass and entropy, for discussion see [3]. The point-like facet of the monopoles is manifested also in properties of the monopole clusters [4].

In this note we summarize the first steps made to realize a similar program of studying the anatomy of the P-vortices [5]. Our main result is that the non-Abelian action of the P-vortices is comparable to the action of the zero-point fluctuations and is ultraviolet divergent in the limit \( a \to 0 \):

\[
S_{vortex} \approx 0.54 \cdot A_{vort} \cdot a^{-2},
\]

where \( A_{vort} \) is the area of the vortex, the measurements refer to the \( SU(2) \) case and the P-vortices defined in the so called indirect central gauge, for details see [5]. Note that naively one would assume that for non-perturbative fluctuations \( S_{n-pert} \sim A \cdot L_{QCD}^2 \).

In Sect. 2 we present the results of the measurements while in Sect. 3 we discuss in more detail structure of the P-vortices and some implications of our measurements.

2. RESULTS

We have performed our calculations in pure \( SU(2) \) lattice gauge theory for \( 2.35 \leq \beta \leq 2.6 \). The lattice spacing \( a \) is fixed using \( \sqrt{\sigma} = 440 \text{MeV} \) where \( \sigma \) is the string tension. At each value of \( \beta \) we have considered 20 statistically independent configurations generated on symmetric \( L^4 \) lattices. The lattice size was \( L=16 \) for \( \beta=2.35 \), \( L=24 \) for \( \beta=2.4.2.45.2.5 \) and \( L=28 \) at \( \beta=2.55.2.6 \). The indirect maximal center gauge [6] was employed to define the P-vortices. For further details see [5].

In Fig. 1 we summarize the results of the measurements of the P-vortex density, \( \rho_{vortex} \equiv \langle N_{PV} / (6L^4a^2) \rangle \), where \( N_{PV} \) is the number of plaquettes occupied by the P-vortex. Alternatively,
Figure 1. The density of P-vortices vs. lattice spacing

\[ A_{\text{vort}} = 6 \rho_{\text{vort}} \cdot V_4 \] where \( V_4 \) is the volume of the lattice. We see that the vortex density is approximately a constant in physical units:

\[ \rho_{\text{vort}} \approx 4 \, (fm)^{-2}. \] (2)

Note that the scaling of the vortex area was observed earlier \[7\]. We confirm this result on better statistics and for smaller values of \( a \).

In Fig. 2 we summarize measurements of the non-Abelian action associated with P-vortices. In more detail, we measure the average action density, \( S_{PV} \), on the plaquettes dual to those forming P-vortices. We subtract from it the average plaquette action, \( S_{\text{vac}} = \beta (1 - \langle Tr U_P \rangle / 2) \). Both \( S_{PV} \) and \( S_{\text{vac}} \) are measured in the lattice units. The difference \( (S_{PV} - S_{\text{vac}}) = S_{\text{vortex}} \) is shown on the Fig.2 by circles and is approximately a constant in the lattice units. Note that for particular value of \( \beta = 2.4 \) this difference was measured first in Ref. \[8\] and we agree with the results obtained therein.

To further probe the structure of the vortices we have measured also the average action density next to the P-vortex world sheet. Geometrically, there are in fact two various types of plaquettes neighboring the plaquettes belonging to the P-vortex, for details see \[9\]. We have measured the average action for these two types of plaquettes separately (up and down triangles on the Fig.

Figure 2. The excess of the non-Abelian action on and next to the P-vortices

2). In the both cases, the average action is very close to that of vacuum. Thus, we conclude that the vortex thickness is smaller than the resolution available:

\[ R_{\text{vort}} \leq 0.06 \, fm , \] (3)

where 0.06 \( fm \) is the smallest lattice spacing used in our simulations.

3. SELF TUNING

Thus, the vortices represent quite a unique type of the vacuum fluctuations which exhibits both the ultraviolet and infrared scales. Let us remind the reader that the most common examples demonstrate sensitivity to one of the two scales. Thus, zero-point fluctuations are controlled by \( \Lambda_{\text{QCD}} \). Much less common are examples of fine tuned fluctuations when one can tune a parameter to ensure that, say, inverse size is much larger than the mass. For example, for the monopole mass in case of the compact \( U(1) \) one has

\[ m_{\text{mon}}^2 \approx \text{const} (e^{-2} - e_{\text{crit}}^{-2}) a^{-2} , \] (4)

where \( e \) is the electric charge and \( e_{\text{crit}}^2 \) is fixed (for a review and references see, e.g., \[3\]). Tuning \( e^2 \) to \( e_{\text{crit}}^2 \) one ensures that \( m_{\text{mon}}^2 \ll a^{-2} \).
In case of a non-Abelian theory there is no parameter to tune since the coupling is running. Nevertheless, the radiative mass of the monopole is largely cancelled by the entropy, for discussion see [4,3]. Thus, the underlying gauge theory ensures a kind of self tuning for the monopoles.

Now we observe a similar phenomenon for the P-vortices: their area scales in the physical units while the tension is a constant in the lattice units. Suppression of the vortices due to their action is approximately

\[ \exp(-S_{\text{vortex}}) \approx \exp(-0.54 \cdot A/a^2) \]  

For the area of the vortices not to shrink this factor is to be cancelled by the entropy. P-vortices represent closed surfaces and their entropy can be estimated knowing the position of the phase transition to the vortex percolation in case of the lattice $Z(2)$:

\[ (\text{Entropy})_{\text{vortex}} \approx \exp(+0.88 \cdot A/a^2) \]  

(Entropy)_{vort}

(5)

(6)

(for discussion of the $Z(2)$ theories and further references see, e.g., [9]).

Apparently, the action and entropy factors do not cancel each other to the accuracy needed. In other words, the self tuned vortices cannot be described in terms of a local action density alone but are to possess further structure. And, indeed, it was observed, for a particular $\beta$ that the vortices are populated by the monopoles [8]. We confirm this picture for all the values of $\beta$ tested. Moreover, the plaquettes shared by the monopoles and vortices have even higher action than the P-vortices on average, see Fig. 2. Accounting for the monopoles might bridge the gap between [3] and [4] although no explicit calculation is available.

Our final remark is that we defined the vortex size in terms of the distribution of the non-Abelian action. Another manifestation of the small physical size of the vortices is the linearity of the heavy quark potential associated with vortices [10]. However, one could define thickness of the vortices in terms of their flux, for discussion see [11]. Then the vortex seems to be not localized to the ultraviolet cut off. Indeed, the action which we observe is still considerably smaller than that of the negative plaquettes.

In conclusion let us mention that further observations on the interplay between the monopoles and vortices were presented in another talk at this conference [11].

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