The classification of single traveling wave solutions for the fractional coupled nonlinear Schrödinger equation

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Abstract
The main purpose of this paper is to study the single traveling wave solutions of the fractional coupled nonlinear Schrödinger equation. By using the complete discriminant system method and computer algebra with symbolic computation, a series of new single traveling wave solutions are obtained, which include trigonometric function solutions, Jacobi elliptic function solutions, hyperbolic function solutions, solitary wave solutions and rational function solutions. As you can see, we give all the classification of single traveling wave solutions for the fractional coupled nonlinear Schrödinger equation. The obtained results substantially improve or complement the corresponding conditions in the literature (Esen and Sulaiman in Optik 167:150-156, 2018), (Eslami in Appl. Math. Comput. 258:141-148, 2016), (Han et al. in Phys. Lett. 395:127217, 2021). Finally, in order to further explain the propagation of the fractional coupled nonlinear Schrödinger equation in nonlinear optics, two-dimensional and three-dimensional graphs are drawn.

Keywords Fractional coupled nonlinear Schrödinger equation · Complete discriminant system method · Computer algebra · Traveling wave solutions

1 Introduction
It is well known that nonlinear evolution equations (NLEEs) model various physical phenomena and play an important position in the investigation of numerous fields, such as combustion theory, fluid dynamics, ecological system, signal processing, nonlinear optics, engineering, statistical mechanics, and plasma physics. As a result, it is one of the critical problems to seek the exact solutions of these NLEEs in nonlinear science. However, due to the complexity of NLEEs, giving all the exact solutions of a NLEE with a unified technique seems to be impossible. Over the decades, a lot of efficient methods have been established and developed to fabricate exact solutions through the efforts of many mathematicians, such as the bifurcation theory and planar portraits analysis method (Tang 2021), $G'/G$-expansion method (Tang and Chen 2022), the extended simplest equation method

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(Elsayed et al. 2019), the Riccati sub equation method (Khodadad and Nazari 2017), the Jacobi elliptic function method (Parks et al. 2002; Tchier et al. 2016), Painlevé analysis (Bountis and Vanhaecke 2016; Marinakis and Bountis 2000; Tzirtzilakis et al. 2002a; Tzirtzilakis et al. 2002b; Weiss et al. 1983), exp-function method (Hosseini et al. 2019; Dehghan et al. 2011; Manafian 2015), Lax pairs (Ma and Strampp 1999), Bäcklund transformation (Huang 2013), \( \tan(\phi/2) \)-expansion method, (Manafian et al. 2020; Manafian 2016; Llhan et al. 2020), bilinear transformation (Manafian et al. 2020), the multiple rogue wave solutions method (Lu et al. 2020), the F-expansion method (Li et al. 2020), etc.

In the current study, the fractional coupled nonlinear Schrödinger equation (FCNLSE) is considered in the following form (Zhang et al. 2018; EI-Shiekh and Gaballah 2020; Abdou et al. 2020)

\[
\begin{align*}
\mathcal{D}_t^\alpha \psi_1 + \mathcal{D}_x^{2\beta} \psi_1 + \delta(|\psi_1|^2 + \gamma |\psi_2|^2)\psi_1 &= 0, \\
\mathcal{D}_t^\alpha \psi_2 + \mathcal{D}_x^{2\beta} \psi_2 + \delta(\gamma |\psi_1|^2 + |\psi_2|^2)\psi_2 &= 0,
\end{align*}
\tag{1.1}
\]

where \( i \) is imaginary unit, \( i^2 = -1 \), \( \alpha, \beta \in (0, 1] \), \( \psi_1 = \psi_1(x, t) \) and \( \psi_2 = \psi_2(x, t) \) are complex functions, which represent the wave amplitudes in two polarizations, \( x \) represents the normalized propagation and \( t \) denotes the retard time. \( \delta \) represents self-focusing and \( \gamma \) denotes cross-phase modulation are nonzero constants. When \( \alpha = \beta = 1 \), it is known to all that Eq. (1.1) is the coupled nonlinear Schrödinger equation (Sulaiman and Bulut 2020; Boyd 1992).

The FCNLSE is a classical nonlinear model which can describe lots of physical nonlinear systems. The equation can be applied to many fields, such as biology, fluid mechanics, nonlinear optics, circulation system of chemical industry, heat pulses in solids and so on. Due to the importance of FCNLSE, the equation has been investigated by many researchers (Younis 2013; Bekir et al. 2015; Wen 2020; Du et al. 2019; Zhang et al. 2011; Guo and Liu 2020). As a result, it is an important work to seek the exact solutions of the fractional differential equation. So far, lots of effective methods have been established about the traveling wave solutions of the FCNLSE (Gadzhimuratov et al. 2020; Xie et al. 2018; Li et al. 2021; EI-Shiekh 2019). In Ref. (Esen and Sulaiman 2018), Esen and his co-workers considered the space-time fractional \( (1 + 1) \)-dimensional coupled nonlinear Schrödinger equation, a series of exact solutions including dark, mix dark-bright and mixed singular optical solitons are obtained via the extended sinh-Gordon equation expansion method. By applying the Kudryashov method, the traveling wave solutions to the time FCNLSE were derived by Eslami (Eslami 2016). The traveling wave solutions of Eq. (1.1) were obtained by Han and his collaborators (Han et al. 2021) by using the bifurcation theory and planar portraits analysis method. Although there are lots of methods to construct the exact solutions of the FCNLSE, the discriminant system method to study the exact solutions of the FCNLSE, it seems as far as we know, not available in the literature. Especially in recent years, with the development of computer algebra theory, by using the mathematical software Maple or Mathematica, a series of traveling wave solutions can be obtained by solving complex algebraic equations. In 1996, with the help of computer algebra, a complete discrimination system of high-order polynomials has been derived by Yang and his co-workers (Yang et al. 1996). As a matter of fact, it is a powerful tool to seek traveling wave solutions of NLEEs. Therefore, a range of solutions of different forms are obtained (Zheng and Lai 2008).

In this paper, the complete discriminant system method is employed to seek exact solutions of the FCNLSE, with the assistance of computer algebra and symbolic computation, according to the root-classifications, a series of new traveling wave
solutions are obtained. As you can see, although there are many references about the traveling wave solutions of FCNLSE, the classification of all single wave solutions of this equation has not been reported in the above literature as far as we know. The obtained results in this paper improve or complement the corresponding conditions in the literature (Esen and Sulaiman 2018; Eslami 2016; Han et al. 2021).

The organization of this paper is as follows. In Sect. 2, we review the definition of conformable fractional derivatives. In Sect. 3, the description of the complete discriminant system method is given. In Sect. 4, by applying the complete discriminant system method, the new traveling wave solutions to Eq. (1.1) are obtained by adapting the inverse transformation. The last section summarizes the results of the current study.

2 An overview of the conformable derivative

It is known to all that the fractional derivative has a long history. Theoretical speaking, it can be traced back to the time on September 30, 1965. There is a story about the fractional derivative. The day on September 30, 1965 is a special day when L’Hospital asked Leibniz the problem about the order of derivative turns into non-integer. On the other words, “Can the definition of integer derivative be extended to non-integer order derivative?” Hence, the time on September 30, 1965 is supposed to be the birth date of fractional derivative. Over 300 years of development, there are many definitions about fractional derivative, such as Caputo derivative (Sabrina et al. 2020), Atangana-Baleanu derivative ( Sarwar 2020), Conformable derivative (Shi and Zhang 2020; Ganaini and Alamr 2019), Riemann-Liouville derivative (Das et al. 2018; Choi et al. 2017) and so on. As is known to all that Riemann-Liouville fractional derivative is the classical fractional derivative which has been widely used. But unfortunately, we can easy find that the Riemann-Liouville fractional derivative which modified by Jumarie does not obey the chain rule (Jumarie 2006). The conformable derivative which is defined in Ref. (Khalil et al. 2014) not only satisfies chain rule but also Leibniz formula. Therefore, we just consider conformable derivative in the current study.

First of all, the conformable derivatives can be defined as follows (Chen et al. 2020; Tang 2020; Hammad and Khalil 2014; Khalil et al. 2014; Ghanbari et al. 2019; Rezazadeh et al. 2020).

**Definition 2.1** Let \( u : [0, +\infty) \rightarrow \mathbb{R} \), \( \alpha \in (0, 1] \). The conformable derivative of \( u \) of order \( \alpha \) is defined as

\[
T_\alpha(u)(t) = \lim_{\epsilon \to 0} \frac{u(t + \epsilon t^{1-\alpha}) - u(t)}{\epsilon}, \quad \forall \ t \geq 0, \tag{2.1}
\]

the function \( u \) is \( \alpha \)-conformable differentiable at a point \( t \) if the limit in Eq. (2.1) exists.

**Theorem 2.2** Assume that \( u, v : (0, \infty) \rightarrow \mathbb{R} \) be differentiable and also \( \alpha \) differentiable functions, then chain rule holds

\[
T_\alpha(uv)(t) = t^{1-\alpha}v(t)\alpha-1v'(t)T_\alpha(u(t))|_{t=v(t)}. \tag{2.2}
\]
3 Analysis of the method

Considering the nonlinear partial differential equation in the form:

\[ G(u, D^\alpha_t u, D^\beta_x u, D^\gamma_t D^\delta_x u, D^\delta_t D^\beta_x u, \ldots) = 0, \quad 0 < \alpha, \beta < 1. \]  

(3.1)

First of all, by traveling wave transformations and some other suitable transformations, the Eq. (3.1) can be transferred into a nonlinear ordinary differential equation as follows:

\[ u_\xi^2 = F(u), \]  

(3.2)

where \( u_\xi := \frac{d}{d\xi} u \) and \( F(u) = a_2 u^2 + a_1 u + a_0 \) is the double degree polynomial with the parameters \( a_2, a_1, a_0 \). Then integrating the above (3.2), we can obtain

\[ \xi - \xi_0 = \int \frac{du}{\sqrt{F(u)}}, \]  

(3.3)

where \( \xi_0 \) is the integration constant. Therefore, we need to solve Eq. (3.3). However, it is a challenging work to decide the range of the parameters, which can be accomplished by complete discrimination system functions. We can derive its complete discrimination system

\[ \Delta = a_1^2 - 4a_2a_0. \]  

(3.4)

Finally, according to the root-classifications, the parameters mentioned above can be obtained from the integral Eq. (3.3). And then by an inverse transformation, we can obtain the exact solutions of the original partial differential equation.

4 Traveling wave solutions for FCNLSE

In this section, we consider the traveling wave solutions for the Eq. (1.1), we assume that Eq. (1.1) has the following traveling wave transformation

\[ \psi_1(x, t) = Z_1(\xi)e^{\eta}, \quad \psi_2(x, t) = Z_2(\xi)e^{\eta}, \quad \xi = m \left( \frac{x^\beta}{\beta} - c \frac{t^\alpha}{\alpha} \right), \quad \eta = -\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0, \]  

(4.1)

where \( m, c, \lambda \) and \( \mu \) are undetermined real constants, \( \eta_0 \) is an arbitrary constant.

Substituting (4.1) into Eq. (1.1), decomposes real parts and imaginary parts of Eq. (1.1), the FCNLSE can be reduced into

\[
\begin{aligned}
m^2Z_1'' + \delta Z_1^3 + \delta \gamma Z_2^2Z_1 - (\lambda^2 + \mu)Z_1 &= 0, \\
m^2Z_2'' + \delta Z_2^3 + \delta \gamma Z_1^2Z_2 - (\lambda^2 + \mu)Z_2 &= 0, \\
c &= -2\lambda.
\end{aligned}
\]  

(4.2)

Suppose that there is a linear relationship between \( Z_1 \) and \( Z_2 \), namely \( Z_2 = kZ_1 (k \neq 0) \), substituting \( Z_2 = kZ_1 (k \neq 0) \) into the first Equation of (4.2), we can obtain the following form:

\[ m^2Z_1'' + (\delta + \delta \gamma k^2)Z_1^3 - (\lambda^2 + \mu)Z_1 = 0. \]  

(4.3)
Thus,

\[
Z''_1 = -\frac{\delta + \delta \beta k^2}{m^2} Z'_1 + \frac{\lambda^2 + \mu}{m^2} Z_1. \tag{4.4}
\]

By multiplying (4.4) with \(Z'\), we derive

\[
Z'_1 Z''_1 = -\frac{\delta + \delta \beta k^2}{m^2} Z'_1 Z'_1 + \frac{\lambda^2 + \mu}{m^2} Z_1 Z'_1. \tag{4.5}
\]

Integrating Eq. (4.4) once, we obtain

\[
(Z'_1)^2 = a_4 Z'_4 + a_2 Z'_2 + a_0, \tag{4.6}
\]

where \(a_4 = -\frac{\delta + \delta \beta k^2}{2m^2}, \quad a_2 = \frac{\lambda^2 + \mu}{m^2}, \quad a_0 \) is a integral constant.

By a suitable transformation as follows

\[
\begin{cases}
Z_1 = \pm \sqrt{(4a_4)^{-\frac{1}{3}} W}, \\
b_1 = 4a_2 (4a_4)^{-\frac{2}{3}}, \\
b_0 = 4a_0 (4a_4)^{-\frac{1}{3}}, \\
\xi_1 = (4a_4)^{\frac{1}{3}} \xi,
\end{cases}
\tag{4.7}
\]

then the Eq. (4.6) can be rewritten as

\[
(W_{\xi_1})^2 = W (W^2 + b_1 W + b_0). \tag{4.8}
\]

Integrating Eq. (4.8) once, we derive

\[
\pm(\xi_1 - \xi_0) = \int \frac{dW}{\sqrt{W (W^2 + b_1 W + b_0)}}, \tag{4.9}
\]

where \(\xi_0\) is the integration constant. Denoting \(F(W) = W^2 + b_1 W + b_0\), thus we can establish the second order complete discrimination system as

\[
\Delta = b_1^2 - 4b_0. \tag{4.10}
\]

According to the root-classifications of (4.10), there are four cases to be discussed.

**Case 1** Suppose that \(\Delta = 0\). As for \(W > 0\), we have

\[
\pm(\xi_1 - \xi_0) = \int \frac{dW}{(W + \frac{b_1}{2}) \sqrt{W}}. \tag{4.11}
\]

If \(b_1 > 0\), it follows from Eq. (4.11), we can obtain

\[
W = \frac{b_1}{2} \tan^2 \left[ \frac{1}{2} \sqrt{\frac{b_1}{2}} (\xi_1 - \xi_0) \right]. \tag{4.12}
\]

According to the Eqs. (4.7), (4.12) and \(Z_2 = kZ_1 (k \neq 0)\), the solution of Eq. (1.1) can be obtained as follows (see Fig. 1)
If $b_1 < 0$, it follows from Eq. (4.11), we can obtain

$$
W = -\frac{b_1}{2} \tanh^2 \left[ \sqrt{-\frac{b_1}{2}} \times \frac{1}{2} (\xi_1 - \xi_0) \right]
$$

and

$$
W = -\frac{b_1}{2} \coth^2 \left[ \sqrt{-\frac{b_1}{2}} \times \frac{1}{2} (\xi_1 - \xi_0) \right].
$$

According to the Eqs. (4.7), (4.14) and $Z_2 = kZ_1 (k \neq 0)$, the solution of Eq. (1.1) can be obtained as follows (see Figs. 2 and 3)
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Fig. 2 The graphics of $\psi_{1,2}(x,t)$ in Eq. (4.15) at $\lambda = \gamma = k = m = 1, \mu = -2, \delta = -1, c = 2, \xi_0 = 0, a = \beta = \frac{1}{2}$

Fig. 3 The graphics of $\psi_{1,3}(x,t)$ in Eq. (4.16) at $\lambda = \gamma = k = m = 1, \mu = -2, \delta = -1, c = 2, \xi_0 = 0, a = \beta = \frac{1}{2}$
If \( b_1 = 0 \), it follows from Eq. (4.11), we can obtain

\[
W = \frac{4}{(\xi_1 - \xi_0)^2}.
\]

(4.17)

Thus, the solution of Eq. (1.1) can be obtained as follows (see Fig. 4)

\[
\psi_{1,2}(x,t) = \pm \sqrt{-\frac{\lambda^2+\mu}{\delta-\delta y k^2}} \tanh \left\{ \frac{2^{-\frac{7}{6}} \sqrt{-\frac{\lambda^2+\mu}{\delta-\delta y k^2}}}{\frac{2 m^2}{m^2}} \left( \frac{-2\delta-2\delta y k^2}{m^2} \right) \right\} \exp \left( i \left( \frac{-\lambda^2+\mu}{\delta y} + \frac{\mu}{\alpha} + \eta_0 \right) \right),
\]

(4.15)

\[
\psi_{2,2}(x,t) = \pm k \sqrt{-\frac{\lambda^2+\mu}{\delta-\delta y k^2}} \tanh \left\{ \frac{2^{-\frac{7}{6}} \sqrt{-\frac{\lambda^2+\mu}{\delta-\delta y k^2}}}{\frac{2 m^2}{m^2}} \left( \frac{-2\delta-2\delta y k^2}{m^2} \right) \right\} \exp \left( i \left( \frac{-\lambda^2+\mu}{\delta y} + \frac{\mu}{\alpha} + \eta_0 \right) \right).
\]

(4.16)

\[
\psi_{1,3}(x,t) = \pm \sqrt{-\frac{\lambda^2+\mu}{\delta-\delta y k^2}} \coth \left\{ \frac{2^{-\frac{7}{6}} \sqrt{-\frac{\lambda^2+\mu}{\delta-\delta y k^2}}}{\frac{2 m^2}{m^2}} \left( \frac{-2\delta-2\delta y k^2}{m^2} \right) \right\} \exp \left( i \left( \frac{-\lambda^2+\mu}{\delta y} + \frac{\mu}{\alpha} + \eta_0 \right) \right),
\]

(4.18)

\[
\psi_{2,3}(x,t) = \pm k \sqrt{-\frac{\lambda^2+\mu}{\delta-\delta y k^2}} \coth \left\{ \frac{2^{-\frac{7}{6}} \sqrt{-\frac{\lambda^2+\mu}{\delta-\delta y k^2}}}{\frac{2 m^2}{m^2}} \left( \frac{-2\delta-2\delta y k^2}{m^2} \right) \right\} \exp \left( i \left( \frac{-\lambda^2+\mu}{\delta y} + \frac{\mu}{\alpha} + \eta_0 \right) \right).
\]

Fig. 4 The graphics of \( \psi_{1,4}(x,t) \) in Eq. (4.18) at \( \gamma = k = m = 1, \delta = -1, c = 2, \xi_0 = 0, \alpha = \beta = \frac{1}{2} \).
\[
\psi_{1,4}(t, x) = \pm 2^\frac{1}{6} \left( \frac{-\Delta - \delta y k^2}{2\omega^2} \right)^{-1/6} \left[ \left( \frac{-2\Delta - 2\delta y k^2}{m^2} \right) \xi - \xi_0 \right]^{-1} \times \exp \left( i \left( -\frac{\lambda y}{\beta} + \frac{\mu}{\alpha} + \eta_0 \right) \right),
\]
\[
\psi_{2,4}(t, x) = \pm 2^\frac{1}{6} k \left( \frac{-\Delta - \delta y k^2}{2\omega^2} \right)^{-1/6} \left[ \left( \frac{-2\Delta - 2\delta y k^2}{m^2} \right) \xi - \xi_0 \right]^{-1} \times \text{times exp} \left( i \left( -\frac{\lambda y}{\beta} + \frac{\mu}{\alpha} + \eta_0 \right) \right). \tag{4.18}
\]

**Case 2** Suppose that \( \Delta > 0 \) and \( b_0 = 0 \). As for \( W > -b_1 \), we have
\[
\pm (\xi_1 - \xi_0) = \int \frac{dW}{W \sqrt{W + b_1}}. \tag{4.19}
\]
If \( b_1 > 0 \), it follows from Eq. (4.19), we can obtain
\[
\begin{aligned}
W &= \frac{b_1}{2} \tanh^2 \left[ \sqrt{\frac{b_1}{2}} \times \frac{1}{2} (\xi_1 - \xi_0) \right] - b_1, \\
W &= \frac{b_1}{2} \coth^2 \left[ \sqrt{\frac{b_1}{2}} \times \frac{1}{2} (\xi_1 - \xi_0) \right] - b_1. \tag{4.20}
\end{aligned}
\]
According to the Eqs. (4.7), (4.20) and \( Z_2 = kW_1 (k \neq 0) \), the solution of Eq. (1.1) can be obtained as follows:
\[
\begin{aligned}
\psi_{1,5}(x, t) &= \pm \sqrt{\frac{\lambda^2 + \mu}{-\delta - \delta y k^2}} \times \exp \left( i \left( -\frac{\lambda y}{\beta} + \frac{\mu}{\alpha} + \eta_0 \right) \right) \\
&\times \left\{ \tanh^2 \left[ 2^{-\frac{1}{6}} \sqrt{\frac{\lambda^2 + \mu}{m^2}} \left( \frac{2m^2}{\delta - \delta y k^2} \right)^{1/3} \left( \frac{-2\Delta - 2\delta y k^2}{m^2} \right)^{1/3} (\xi - \xi_0) \right] - 2 \right\}^{\frac{1}{2}}, \tag{4.21}
\end{aligned}
\]
\[
\begin{aligned}
\psi_{2,5}(x, t) &= \pm k \sqrt{\frac{\lambda^2 + \mu}{-\delta - \delta y k^2}} \times \exp \left( i \left( -\frac{\lambda y}{\beta} + \frac{\mu}{\alpha} + \eta_0 \right) \right) \\
&\times \left\{ \coth^2 \left[ 2^{-\frac{1}{6}} \sqrt{\frac{\lambda^2 + \mu}{m^2}} \left( \frac{2m^2}{\delta - \delta y k^2} \right)^{1/3} \left( \frac{-2\Delta - 2\delta y k^2}{m^2} \right)^{1/3} (\xi - \xi_0) \right] - 2 \right\}^{\frac{1}{2}}, \tag{4.22}
\end{aligned}
\]
If \( b_1 < 0 \), it follows from Eq. (4.19), we can obtain
\[
W = -\frac{b_1}{2} \tan^2 \left[ -\frac{b_1}{2} \times \frac{1}{2} (\xi_1 - \xi_0) \right] - b_1. \tag{4.23}
\]
According to the Eqs. (4.7), (4.23) and \( Z_2 = kZ_1 (k \neq 0) \), the solution of Eq. (1.1) can be obtained as follows:

\[
\psi_{1,7}(x, t) = \pm \sqrt{-\frac{\lambda^2 + \mu}{-\delta - \delta y k^2}} \times \exp \left( i \left( -\lambda \frac{v}{\beta} + \mu \frac{n}{a} + \eta_0 \right) \right) \\
\times \left\{ \tan^2 \left[ \frac{2 \frac{7}{2}}{\frac{7}{2}} \sqrt{-\frac{\lambda^2 + \mu}{m^2} \left( -\frac{2 \delta - 2 \delta y k^2}{m^2} \right) \left( -\frac{2 \delta - 2 \delta y k^2}{m^2} \right) \frac{1}{3} \left( -\frac{2 \delta - 2 \delta y k^2}{m^2} \right) \frac{1}{3} \xi - \xi_0 \right] \right] + 2 \right\}^{\frac{1}{2}},
\]

\[
\psi_{2,7}(x, t) = \pm k \sqrt{-\frac{\lambda^2 + \mu}{-\delta - \delta y k^2}} \times \exp \left( i \left( -\lambda \frac{v}{\beta} + \mu \frac{n}{a} + \eta_0 \right) \right) \\
\times \left\{ \tan^2 \left[ \frac{2 \frac{7}{2}}{\frac{7}{2}} \sqrt{-\frac{\lambda^2 + \mu}{m^2} \left( -\frac{2 \delta - 2 \delta y k^2}{m^2} \right) \left( -\frac{2 \delta - 2 \delta y k^2}{m^2} \right) \frac{1}{3} \left( -\frac{2 \delta - 2 \delta y k^2}{m^2} \right) \frac{1}{3} \xi - \xi_0 \right] \right] + 2 \right\}^{\frac{1}{2}}. \tag{4.24}
\]

**Case 3** Suppose that \( \Delta > 0 \) and \( b_0 \neq 0 \) and \( \omega_1 < \omega_2 < \omega_3 \). Then, we adopt the assumption that one of \( \omega_1, \omega_2, \omega_3 \) is zero and the rest of them are two different real roots of \( F(W) = 0 \). Taking the transformation \( W = \omega_1 + (\omega_2 - \omega_1) \sin^2 \theta \), it is clear that

\[
\pm (\xi_1 - \xi_0) = \frac{2}{\sqrt{\omega_3 - \omega_1}} \int \frac{d\theta}{\sqrt{1 - n_1^2 \sin^2 \theta}}, \tag{4.25}
\]

where \( n_1^2 = \frac{\omega_3 - \omega_0}{\omega_3 - \omega_1} \). It follows from Equation (4.25), we obtain

\[
W = \omega_1 + (\omega_2 - \omega_1) \sin^2 \left( \sqrt{\omega_3 - \omega_1} \left( \frac{1}{2} (\xi_1 - \xi_0) \right), n_1 \right). \tag{4.26}
\]

According to the Eqs. (4.7), (4.26) and \( Z_2 = kZ_1 (k \neq 0) \), the solution of Eq. (1.1) can be obtained as follows

\[
\psi_{1,8}(x, t) = \pm \left( \frac{-2 \delta - 2 \delta y k^2}{m^2} \right) \frac{1}{2} \left\{ \omega_1 + (\omega_3 - \omega_1) \sin^2 \left[ \sqrt{\omega_3 - \omega_1} \left( \frac{1}{2} (\xi_1 - \xi_0) \right), n_1 \right] \right\}^{\frac{1}{2}} \\
\times \exp \left( i \left( -\lambda \frac{v}{\beta} + \mu \frac{n}{a} + \eta_0 \right) \right),
\]

\[
\psi_{2,8}(x, t) = \pm k \left( \frac{-2 \delta - 2 \delta y k^2}{m^2} \right) \frac{1}{2} \left\{ \omega_1 + (\omega_3 - \omega_1) \sin^2 \left[ \sqrt{\omega_3 - \omega_1} \left( \frac{1}{2} (\xi_1 - \xi_0) \right), n_1 \right] \right\}^{\frac{1}{2}} \\
\times \exp \left( i \left( -\lambda \frac{v}{\beta} + \mu \frac{n}{a} + \eta_0 \right) \right). \tag{4.27}
\]

For another transformation \( W = -\frac{\omega_1 \sin^2 \theta + \omega_3}{\cos^2 \theta} \), it follows from Eq. (4.25), we obtain

\[
W = \frac{-\omega_2 \sin \left( \sqrt{\omega_3 - \omega_1} \left( \frac{1}{2} (\xi_1 - \xi_0) \right), n_1 \right) + \omega_3}{\sqrt{\omega_3 - \omega_1} \left( \frac{1}{2} (\xi_1 - \xi_0) \right), n_1}. \tag{4.28}
\]

According to the Eqs. (4.7), (4.28) and \( Z_2 = kZ_1 (k \neq 0) \), the solution of Eq. (1.1) can be obtained as follows:
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\[ \psi_{1,9}(x, t) = \pm \left( -\frac{2\delta - \delta \gamma k^2}{m^2} \right)^{-\frac{1}{2}} \left\{ \frac{-\omega_z \sin^2 \left[ \frac{\sqrt{m^2 - n_0^2}}{2} \left( \frac{\Delta}{m^2} \right)^{1/3} \xi - n_0 \right] m_1 + \omega_z}{\cos^2 \left[ \frac{\sqrt{m^2 - n_0^2}}{2} \left( \frac{\Delta}{m^2} \right)^{1/3} \xi - n_0 \right] m_1} \right\}^{1/2} \times \exp \left( i \left( -\frac{x}{\beta} + \frac{\mu}{a} + \eta_0 \right) \right). \]

\[ \psi_{2,9}(x, t) = \pm k \left( -\frac{2\delta - \delta \gamma k^2}{m^2} \right)^{-\frac{1}{2}} \left\{ \frac{-\omega_z \sin^2 \left[ \frac{\sqrt{m^2 - n_0^2}}{2} \left( \frac{\Delta}{m^2} \right)^{1/3} \xi - n_0 \right] m_1 + \omega_z}{\cos^2 \left[ \frac{\sqrt{m^2 - n_0^2}}{2} \left( \frac{\Delta}{m^2} \right)^{1/3} \xi - n_0 \right] m_1} \right\}^{1/2} \times \exp \left( i \left( -\frac{x}{\beta} + \frac{\mu}{a} + \eta_0 \right) \right). \]

**Case 4** Suppose that \( \Delta < 0 \). Taking the transformation \( W = \sqrt{b_0} \tan \frac{\theta}{2} \), it is clear that

\[ \pm (\xi_1 - \xi_0) = (b_0)^{-\frac{1}{2}} \int \frac{d\theta}{\sqrt{1 - n_2^2 \sin^2 \theta}}. \]  

(4.30)

where \( n_2^2 = \frac{2\sqrt{b_0^2 - b_1^2}}{4b_0} \), it follows from Eq. (4.30), we obtain

\[ W = \frac{2\sqrt{b_0}}{1 + cn \left( \frac{b_0 \left( \xi_1 - \xi_0 \right)}{n_2} \right)^{1/2}} - \sqrt{b_0}. \]  

(4.31)

According to the Eqs. (4.7), (4.31) and \( Z_2 = kZ_1(k \neq 0) \), the solution of Eq. (1.1) can be obtained as follows

\[ \psi_{1,10}(x, t) = \pm \left( \frac{2m^2 a_0}{-\delta - \delta \gamma k^2} \right)^{\frac{1}{2}} \times \exp \left( i \left( -\frac{x}{\beta} + \frac{\mu}{a} + \eta_0 \right) \right) \]

\[ \times \left[ \frac{2}{1 + cn \left( \frac{32m^2 a_0^2}{-\delta - \delta \gamma k^2} \right)^{1/2} \left( \frac{\Delta}{m^2} \right)^{1/3} \xi - n_0 \right] m_2 \right]^{1/2}, \]

\[ \psi_{2,10}(x, t) = \pm k \left( \frac{2m^2 a_0}{-\delta - \delta \gamma k^2} \right)^{\frac{1}{2}} \times \exp \left( i \left( -\frac{x}{\beta} + \frac{\mu}{a} + \eta_0 \right) \right) \]

\[ \times \left[ \frac{2}{1 + cn \left( \frac{32m^2 a_0^2}{-\delta - \delta \gamma k^2} \right)^{1/2} \left( \frac{\Delta}{m^2} \right)^{1/3} \xi - n_0 \right] m_2 \right]^{1/2}. \]

(4.32)
5 Concluding remarks

It is known to all that the nonlinear Schrödinger equation is an example of a universal nonlinear model that describes many physical nonlinear systems. In this paper, the complete discriminant system method is employed to seek exact solutions of the FCNLSE, by using the mathematical software Maple, combining computer algebra with symbolic computation, we obtain a series of new traveling wave solutions, including trigonometric function solutions, Jacobi elliptic function solutions, hyperbolic function solutions, solitary wave solutions, rational function solutions. The complete discriminant system method is employed to seek traveling wave solutions of FCNLSE, it seems as far as we know, not available in the literature. Therefore, the research in this paper has an important application and scientific research value.

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Declarations

Conflict of interest The authors declare that there is no conflict of interest regarding the publication of this paper. The authors declare that this paper is original.

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