Sliding Mode Variable Structure Control Strategy Applied on Position Servo System of EMA

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Abstract. This paper introduces the basic elements of Electronic-Mechanical Actuator (EMA), and gives the equations. The mathematical model-state equations is established based on the basic equations, and the sliding mode variable structure control strategy, based on the reaching law is applied to control the position servo of EMA. We design a position-speed integrated sliding mode variable structure controller. The models of EMA machinery, electrical motor and controller are established in AMESim and Matlab, and joint simulation is carried out. Finally, this control strategy is compared with the three loop PID control strategy, and the experiment is carried out. The result shows that the sliding mode variable structure control strategy based on reaching law used to control EMA can improve the frequency response and control accuracy of the system, can also improve the dynamic performance of the position servo system.

1. Introduction
As for the small size, light weight and no leakage, EMA will be widely used in aircraft actuating system in the future. However, when EMA is applied on the actuating system of aircraft, the main problems are:

- Surface load condition of the actuation system is not only a function of the rudder angle, but also a complex non-linear function associated with many factors, such as air parameters, aircraft attitude, flight Machr and so on, which led to the force condition of EMA changes constantly[1];
- EMA is a multivariable, strongly coupled and nonlinear system;
- Because of the impact resistance of the rolling screw is poor and easy to reverse, the influence of load disturbance must be overcome effectively to ensure the system stiffness [2].

Although the PID control strategy is simple and easy to apply, there are some disadvantages, such as the poor robustness of the time variant perturbation and parameter perturbation, etc.[2], which makes it difficult to solve the three problems above. Therefore, it is an urgent task to adopt a better control strategy to realize the position servo control with high speed, high accuracy and no overshoot.

The sliding mode variable structure control is discontinuous, and the switching characteristic of the system structure with the change of time, which is the essential difference between the sliding mode control and other control strategies. It forces the system to operate along the plan, and the motion with small amplitude and high frequency. So it is called sliding mode motion[3]. The sliding mode can be
managed artificially, and it is independent of the change of system parameters and the degree of external disturbance, which makes the system to be more robust. The variable sliding mode structure control strategy is applied on EMA position servo control system to improve the system tracking accuracy and dynamic performance and robustness. Through co-simulation analysis of AMESim and MATLAB, the rapidity, frequency response and anti-disturbance ability are compared with tricyclic control strategy. This paper will analyze the merits of the control effect of two control strategies, to verify the correctness and effectiveness of the research results, by experimental verification.

2. Mathematical model of EMA position servo system

EMA is usually composed of servo motor, rolling screw mechanism or gear rotating mechanism. EMA is used to change the rotation motion of the motor into linear motion output. The position servo system of EMA in this paper is a parallel EMA, and a planet roller screw is used as the rolling screw mechanism. The basic structure of the roller screw is shown in “Fig. 1” [4]. It is mainly composed of permanent magnet synchronous motor, planet roller screw rod and other components. Based on the study of the basic components of EMA, the basic equation is established in this paper, which is the basis of the EMA mathematical model.

![Figure 1. Basic structure of EMA.](image)

2.1. Voltage balancing equation of permanent magnet synchronous torque motor

\[
\begin{bmatrix}
    u_d \\
    u_q
\end{bmatrix} =
\begin{bmatrix}
    R_s & -\omega_n L_q \\
    \omega_n L_d & R_s
\end{bmatrix}
\begin{bmatrix}
    i_d \\
    i_q
\end{bmatrix} +
\begin{bmatrix}
    L_d & 0 \\
    0 & L_q
\end{bmatrix}
\begin{bmatrix}
    \frac{di_d}{dt} \\
    \frac{di_q}{dt}
\end{bmatrix}
+ \begin{bmatrix}
    0 \\
    \omega_n \psi_f
\end{bmatrix}
\]

(1)

\[T_m = \frac{3}{2} p \psi_f i_q\]

(2)

Using the coordinate transformation theory, we establish the model in d-q coordinate system[4], controlling \(i_q\) with as \(i_d=0\), make the electromagnetic torque and current PMSM linearization, the PMSM decoupling control, so this method is simple, it is the most popular [5].

In the formula, 
\(i_d\): q axis is current (A); \(U_q\): q is axis voltage; \(i_d\): d is axis current (A); \(U_d\): d is axis voltage (V); \(R_s\): motor stator winding(\(\Omega\)); \(L_d\): d is axis inductance(\(H\)); \(L_q\): q is axis inductance (\(H\)); \(\omega_m\): motor rotor angular velocity (rad/s); \(\psi_f\): rotor permanent magnet flux psi (Wb).

2.2. Motor end torque balance equation

\[T_m = T_L + B_m \omega_m + J_m \dot{\theta_m}\]

(3)

In the formula, \(T_L\) is the external load that applied on the motor torque (Nm); \(B_m\) is the equivalent viscous friction coefficient when the motor rotates (rotating resistance coefficient) (Nm s/rad); \(J_m\) motor and screw rotational inertia (kg-m^2)); \(\theta_m\) is the mechanical angle of PMSM (rad).

2.3. Torque and force transfer

\[F_n = \eta_{em} k_{em} T_L R\]

(4)
\[ k_{ema} = 2\pi / P_{s} \]  

(5)

In the formula, \( F_{n} \): the screw force after motor transformation (N); \( \eta_{ema} \): transmission efficiency of the roller screw; \( k_{ema} \): roller screw transmission ratio; \( R \): reducer transmission ratio (R=1); \( P_{s} \): roller screw (mm).

2.4. Force analysis of planetary roller screw

\[ F_{n} = m_{ema} \dot{x}_{ema} + F_{ema} \]  

(6)

\[ \theta_{m} = k_{ema} x_{ema} \]  

(7)

In (4), drive efficiency of the roller screw \( \eta_{ema} \) contains viscous friction of the screw, viscous friction of the screw transmission will not be considered [6].

In the model, \( m_{ema} \): roller screw self-mass (Kg); \( x_{ema} \): EMA output linear displacement (m); \( F_{ema} \): external force (N).

2.5. State equation of position servo EMA system

According to (2) - (7), the system input is \( x_{ema}, \omega_{m} \), output is \( i_{qd} \), and system disturbance is \( F_{ema} \).

\[ x = x_{ema}, x_{1} = x, x_{2} = \dot{x} \quad \ddot{x} = -f(x, t) + bu - Nd(t) \]  

(8)

In which,

\[ f(x, t) = \frac{\eta_{ema} k_{ema}^2 B_{no} R}{m_{ema}} x_{2} + \frac{\eta_{ema} k_{ema} R \omega_{m}}{m_{ema}} \dot{\theta}_{m} \]  

(9)

\[ b = \frac{1.5 \eta_{ema} k_{ema} R \omega_{m}}{m_{ema}}; \quad u = i_{q}; \quad N = \frac{1}{m_{ema}}; \quad d(t) = F_{ema} \]

3. Design of Position servo controller

The block diagram of position servo controller is as follows:

![Figure 2. Schematic diagram of position servo integrated sliding mode variable structure controller.](image)

3.1. Design of position and speed integrated sliding mode controller

Using sliding mode variable structure control strategy, an external loop regulator with position loop and speed loop is designed. The input is \( x_{d} \), position feedback signal \( x_{ema} \) and velocity feedback \( \omega \). As is known, \( x = F_{ema} \), obtained by (8).

3.1.1. Error equation[3] Actual output location \( x_{ema} = x \), Set the position control expectation value \( x_{ema} = x_{d} \) then

Definite error:
\[ e = x_d - x \]  

The corresponding error is divided into

\[ \dot{e} = \dot{x}_d - \dot{x} \]  

3.1.2. Design of control law  The design switching function is

\[ s = ce + \dot{e} \]  

The upper differential is obtained

\[ \dot{s} = c\dot{e} + \ddot{e} = c(\dot{x}_d - \dot{x}) + \ddot{x}_d - \ddot{x} = c(\dot{x}_d - \dot{x}) + \ddot{x}_d + f(x, t) - bu + N\dot{d}(t) \]  

Take \( s \) as exponential approaching law

\[ \dot{s} = -ks - \epsilon \text{sgn}(s) - ks - \epsilon \text{sgn}(s) = c(\dot{x}_d - \dot{x}) + x_d + f(x, t) - bu + N\dot{d}(t) \]  

Therefore, the design control rate is

\[ u = \frac{1}{\tau}[c(\dot{x}_d - \dot{x}) + x_d + f(x, t) + N\dot{d}(t) + ks + \epsilon \text{sgn}(s)] \]  

3.1.3. Proof of stability

Lyapunov function

\[ V = \frac{1}{2}s^2 \]  

For the upper differential

\[ \dot{V} = ss\dot{e} = s(c\dot{e} + \ddot{e}) = s[c(\dot{x}_d - \dot{x}) + \ddot{x}_d - \ddot{x}] = s[-ks - \epsilon \text{sgn}(s)] = -ks^2 - \epsilon |s| \leq 0 \]  

The control law is proved to be asymptotically stable in Lyapunov sense, namely global convergence.

3.2. Design of current loop PID controller

The current ring is a very important part of the AC motor position servo control system. The current loop can improve the fast response of the system and suppress the internal interference. In this paper, the current loop adopts the \( i_d=0 \) control method and uses the PI control to improve the system stability while taking into account the system precision.

4. Construction of EMA joint simulation platform

EMA is consisted of two parts, mechanical part and control part. In AMESim, EMA is conducive to the establishment of a more accurate mechanical model[7], MATLAB has a big advantage in the control system modeling[8], so the mechanical model established in AMESim as shown in “Fig.3”, the control model set up in the MATLAB/Simulink is as shown in “Fig.4”.

In fact that, the speed, current, voltage and other signals cannot be infinitely large, so certain restrictions need to make for them.

Where the maximum amplitude of the phase current allowed by the motor is expressed in \( I_{max} \), the current limiting condition is:

\[ I_a^2 + I_q^2 \leq I_{max}^2 \]  

When the maximum phase voltage is indicated by \( V_{max} \), the voltage limiting condition is [9]:

\[ \sqrt{U_a^2 + U_q^2} \leq V_{uc} / \sqrt{3} \]  

Finally, the nonlinear factors such as system current limiting and voltage limiting is added into the model of the system control section,
The interface is provided by AMESim, and the mechanical part outputs $F_{ema}$, $T_L$ and $x_{ema}$, which are received by S-Function in MATLAB/Simulink and input into the control system. In this control system, the sliding mode variable structure control strategy is written as the m file, control current $i_{qd}$ is obtained by the signal processing, $i_{dq}$ controls the motor by PI control, output speed signal. Output to AMESim through S-Function, controls the mechanical system, so that the output position is equal to the command position signal.

![Figure 3. AMESim model of EMA mechanical part based on sliding mode variable structure.](image)

![Figure 4. MATLAB model of EMA control part based on sliding mode variable structure.](image)

5. Simulation results and Analysis
Simulation parameters are shown in Table 1.

| Index | Value       | Index | Value       |
|-------|-------------|-------|-------------|
| $L_s$ | 0.0227H     | $k_{ema}$ | 1256.6      |
| $R_s$ | 3.85Ω       | $m_{ema}$ | 15kg        |
| $B_{im}$ | 0.00473N·m·s/rad | $p_n$ | 4          |
| $\psi_f$ | 0.2381Wb  | $m_l$     | 15kg        |
| $P_{rs}$ | 5mm   | $\eta_{ema}$ | 90%         |
| $J_m$ | 0.0013kg·m² |          |             |

Tricyclic PID control strategy is compared with sliding mode control strategy, based on step response, sinusoidal response and anti-jamming capability.

5.1. Compare the indexes of the two different control strategies

| Index | Sliding mode control strategy | Tricyclic PID control strategy |
|-------|-------------------------------|--------------------------------|
| risetime/s | 0.6397                     | 0.6405                        |
As shown in Table 2, the control effect is similar, from the comparison of rise time, regulation time, overshoot and steady state error of the two control strategies. Sliding mode variable structure control strategy is not obvious, even through overshoot is slightly higher than three ring PID.

The step response of sliding mode control strategy and tricyclic PID control strategy is shown in “Fig. 5”.

![Figure 5. Step response (position command =0.1m).](image)

### 5.2. Compare the sinusoidal response’s rapidity precision of the two control strategies

| Frequency/Hz | Tricyclic PID control strategy | Sliding mode control strategy |
|--------------|---------------------------------|--------------------------------|
|              | Amplitude attenuation /%        | Amplitude attenuation /%        | Phase lag /%        | Phase lag /%        |
| 1            | 0.13                            | 0.08                            | 3.3                 |
| 3            | 1.64                            | 21.2                            | 11.2                |
| 5            | 8.5                             | 40.3                            | 34.6                |
| 7            | 29                              | 62.2                            | 58.2                |
| 7.1          | 30                              | 63.4                            | —                   |
| 7.5          | —                               | 29.28                           | 69.2                |

From the response curve of the two control strategies, The limitation of the system's sinusoidal frequency response is not the current limit, but the speed limit. When the system is designed, the system speed limit is slightly increased, and the system frequency response can be enhanced. Two kinds of control strategy, sinusoidal response, amplitude attenuation, phase lag and shock degree were compared as shown in Table 3, sliding mode variable structure control in system response and tracking accuracy (amplitude attenuation and phase lag degree) were slightly better than tricyclic PID control strategy.

### 5.3. Compare the two control strategies against external disturbances

When the two control strategies under the conditions of "0s, 5000N/s ramp force" and "2S, 10kN force", the comparative analysis of the output position is shown in “Fig. 6”.

| regulation time $t_r$/s | 0.7686 | 0.7742 |
|--------------------------|--------|--------|
| overshoot /%             | 0.9    | 0.23   |
| steady state error /%    | 0.05   | 0.15   |
Figure 6. Comparison of the two control strategies of position servo under the condition 1.
When the two control strategies under the conditions of "2s, adding 10kN step force" and "3s, unloading", the comparative analysis of the output position is shown in “Fig. 7”.

Figure 7. Comparison of the two control strategies of position servo under the condition 2.
When the two control strategies under the conditions of "10kN/50mm elastic load", the comparative analysis of the output position is shown in “Fig. 8”.

Figure 8. Comparison of the two control strategies of position servo under the condition 3.
By comparing the output of position servo curves above three conditions, ring PID control strategy to control the output of the system, the external disturbance situation, there will be varying degrees of
jitter, and the sliding mode variable structure control system control strategy, anti disturbance ability was stronger than the tricyclic PID control strategy, robustness.

6. Experimental verification

The experimental verification is carried out. The contrast curve between the instruction position signal, the simulation results and the experimental results are shown in “Fig. 9”.

![Figure 9. Comparison curves between experiment and simulation results.](image)

The experimental results show that the control strategy is correct, effective and feasible.

7. Conclusions

The sliding mode variable structure design is used on EMA position servo system controller, and realize the position control of EMA. Through AMESim and MATLAB/Simulink co-simulation, and compared with ring PID control strategy, we get some experiment results. The results show that the sliding mode variable structure control strategy can improve the system dynamic performance and position accuracy, which ensures the robustness of the system in the external interference. The results show that it is an effective control method.

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References

[1] Wang C 2010 Design and research of electric servo load simulator (Harbin Institute of Technology)
[2] Fu Y Liu H and Pang R 2011 Research on Active Disturbance Rejection Control for Direct Drive Electromechanical Actuator (Journal of Sichuan University Engineering Science Edition) vol 43 pp 121-127
[3] Liu J 2005 Sliding Mode Variable Structure Control MATLAB Simulation (Beijing) (Tsinghua University Press) p 154-167
[4] Wang J 2015 Permanent Magnet Synchronous Motor Intelligent Control Technology (Southwest Jiaotong University Press) p 97-130
[5] Kou B 2008 AC servo system and its control (Mechanical Industry Press) p 214-235
[6] Qiao G Liu G and Ma S 2008 Dynamic Characteristics Analysis of Electromechanical Actuator Based on Planetary Roller Screw Pair vol 35 (Journal of Vibration and Shock) pp 82-101
[7] Fu Y Qi H 2011 LMS imagine. Lab AMESim System Modeling and Simulation Example Tutorial (Beijing) (Beijing Aerospace University Press) p 89
[8] Shi L 2014 MATLAB/Simulink System Simulation Super Learning Manual (Beijing) (People's Posts and Telecommunications Press) p 200-231
[9] Zhu L Wen X and Zhao F 2011 Research on Weak Magnetic Loss Control Mechanism of Permanent Magnet Synchronous Motor and Its Countermeasures vol 31 (Proceedings of the CSEE) pp 67-72