\( \Delta F=2 \) processes in the MSSM in large \( \tan \beta \) limit

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Abstract

We discuss corrections to \( \Delta M_{B_d}, \Delta M_{B_s} \) and to the CP violation parameter \( \varepsilon \) in two examples of (generalized) minimal flavour violation models: 2HDM and MSSM in the large \( \tan \beta \) limit. We show that for \( H^+ \) not too heavy, \( \Delta M_{B_s} \) in the MSSM with heavy sparticles can be substantially smaller than in the SM due to the charged Higgs box contributions and in particular due to the growing like \( \tan^4 \beta \) contribution of the double penguin diagrams involving neutral Higgs boson exchanges.

The determination of the CKM parameters is the hot topic in particle physics. In view of forthcoming precise results from B-factories, it is particularly important to discuss possible effects of new physics contributions to \( \Delta F = 2 \) processes: \( \Delta M_{B_d} \) and \( \Delta M_{B_s} \) mass differences and to the \( \bar{K}^0-K^0 \) CP violation parameter \( \varepsilon \). In general models of new physics fall into the two following broad classes:

- Models in which the CKM matrix remains the unique source of flavour and CP violation - so-called minimal flavour violation (MFV) models [1, 2] and their generalizations (GMFV models). In the GMFV models the non SM-like operators contribute significantly to the effective low energy Hamiltonian [3].
- Models with entirely new sources of flavour and/or CP violation.

On the basis of the analysis [3] we discuss here two examples of the GMFV models: the large \( \tan \beta \) limit of the 2HDM(II) and of the MSSM, in which the CKM matrix is the only source of flavour and CP violation (see e.g. [3]).

The effective weak Hamiltonian for \( \Delta F = 2 \) transitions in the GMVF models can be written as follows

\[
H_{\Delta F=2}^{\text{eff}} = \frac{G_F^2 M_W^2}{16\pi^2} \sum_i V_{i}^{\text{CKM}} C_i(\mu) Q_i .
\]  

(1)

where \( Q_i \) are the set of 8 dimension six \( \Delta F = 2 \) operators [3] and \( V_{i}^{\text{CKM}} \) are the appropriate CKM factors. \( \Delta M_{B_d}, \Delta M_{B_s} \) and \( \varepsilon \) can be expressed in terms of, respectively, three real functions [3]

\[
F_{tt}^d = S_0(x_t)[1 + f_d] , \quad F_{tt}^s = S_0(x_t)[1 + f_s] , \quad F_{tt}^\varepsilon = S_0(x_t)[1 + f_\varepsilon] .
\]  

(2)

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\[ S_0(x_t \equiv \bar{m}_t^2/M_W^2) \approx 2.38 \pm 0.11 \text{ for } \bar{m}_t(m_t) = (166 \pm 5) \text{ GeV.} \] We have then
\[ \Delta M_q = \frac{G_F^2 M_W^2}{6\pi^2} M_{B_q} \eta_B \hat{B}_{B_q} F^q_{B_q} |V_{tq}|^2 F^q_{tq}, \quad q = d, s \] (3)

The functions \( F^{d}_{tt}, F^{s}_{tt} \) and \( F^{c}_{tt} \) can be expressed in terms of \( C_i \) as
\[ F_{tt} = \left[ S_0(x_t) + \frac{1}{4r} C^{\text{VLL}}_{\text{new}}(\mu_t) \right] + \frac{1}{4r} C^{\text{VRR}}_{1}(\mu_t) + \tilde{P}^{\text{LR}}_1 C^{\text{LR}}_{1}(\mu_t) + \tilde{P}^{\text{LR}}_2 C^{\text{LR}}_{2}(\mu_t) \]
\[ + \tilde{P}^{\text{SLL}}_1 \left[ C^{\text{SLL}}_{1}(\mu_t) + C^{\text{SRR}}_{1}(\mu_t) \right] + \tilde{P}^{\text{SLL}}_2 \left[ C^{\text{SLL}}_{2}(\mu_t) + C^{\text{SRR}}_{2}(\mu_t) \right] \] (4)

where \( r = 0.985 \) and the factors \( \tilde{P}^a_i \) are given in [3, 5].

Let us consider first the 2HDM(II). At 1-loop only the charged scalar is relevant for the box diagrams contributing to \( K^0-K^0 \) and \( B^0-B^0 \) mixing. For large \( \tan \beta \) and \( M_{H^\pm} \approx m_t \), the leading terms of such contributions to the Wilson coefficients \( C_i \) are of the order of (see [3] for the complete expressions):
\[ \delta^{(+)}C^{\text{VLL}}_1 \sim \frac{4}{3} \cot^2 \beta, \quad \delta^{(+)}C^{\text{LR}}_2 \sim -\frac{8 m_d m_d}{3 m_t^2} \tan^2 \beta \] (5)
for diagrams with \( W^\pm H^\mp \), and
\[ \delta^{(+)}C^{\text{VLL}}_1 \sim \frac{1}{3} \frac{m_t^2}{M_W^2} \cot^2 \beta, \quad \delta^{(+)}C^{\text{VRR}}_1 \sim \frac{1}{3} \frac{m_d m_d}{M_W^2 m_t^2} \tan^4 \beta, \quad \delta^{(+)}C^{\text{LR}}_1 \sim 0 \]
\[ \delta^{(+)}C^{\text{SLL}}_1 \sim 0, \quad \delta^{(+)}C^{\text{SRR}}_1 \sim 0, \quad \delta^{(+)}C^{\text{LR}}_2 \sim -\frac{4}{3} \frac{m_d m_d}{M_W^2} \tan^2 \beta \] (6)
for diagrams with \( H^\pm H^\mp \). For large \( \tan \beta \) the biggest contribution appears in \( \delta^{(+)}C^{\text{LR}}_2 \) and is further enhanced, compared to the SM amplitude, by the QCD renormalization effects [3]. It is of the opposite sign than given by the \( t\bar{W}^\pm \) box diagram and can be significant only for the \( \bar{B}_s^0-B_s^0 \) mixing (so that it can compete with the SM contribution - see fig. [4]), for which it is of the order of
\[ \delta^{(+)}C^{\text{LR}}_2 \approx -\frac{2 \mu s(m_t) m_b(\mu_t)}{M_W^2} \tan^2 \beta \approx -0.14 \times \left( \frac{\tan \beta}{50} \right)^2. \] (7)

Similar contributions for \( \bar{B}_d^0-B_d^0 \) and \( \bar{K}^0-K^0 \) transitions are suppressed by factors \( m_d/m_s \) and \( m_d/m_b \), respectively, and thus very small. Unfortunately, the new CLEO experimental result for the process \( BR(B \rightarrow X_s \gamma) = (3.03 \pm 0.40 \pm 0.26) \times 10^{-4} \) set the bound \( M_{H^\pm} \gtrsim 380 \) GeV [3]. Thus, in the 2HDM(II) for the still allowed range of charged Higgs boson mass the effects in \( \Delta M_{B_s} \) cannot be large.

As a second realistic GMFV model we consider the MSSM with the CKM as the only source of flavour/CP violation. In the limit of heavy sparticles (practically realized already for \( M_{\text{SUSY}} \gtrsim 500 \) GeV) the 1-loop box diagrams involving charginos and stops are negligible. However, for large \( \tan \beta \) even if sparticles are heavy they can still compensate the \( H^\pm \) contribution to the \( b \rightarrow s \gamma \) amplitude allowing for the existence of a light, \( \sim O(150 \text{ GeV}) \), charged Higgs boson [3, 10]. From fig. [4] it follows therefore that, even for \( \tan \beta \lesssim 50 \)
Figure 1: $1 + f_s$ in the 2HDM(II): a) as a function of $\tan \beta$ for $M_{H^+} =$ (from below) 150, 250, 300 and 350 GeV and b) as a function of $M_{H^+}$ for $\tan \beta =$ (from above) 40, 60, 80 and 100.

and already at the 1-loop level the contribution of the MSSM Higgs sector to the $C_{2}^{LR}$ Wilson coefficient can be non-negligible.

At the 2-loop level in the MSSM one has to take into account the dominant 2-loop electroweak corrections, modifying the Yukawa-type interactions. Even for heavy particles these corrections can significantly modify \[ \] the 2HDM(II) relations between the masses $m_{d_i}$ and the Yukawa couplings $Y_{d_i}^I$, as well as induce additional, $\propto \tan \beta$, terms in the charged Higgs-quark couplings \[ \]. For the $\Delta F = 2$ processes the most important (for non-negligible mixing of the top squarks) is however their third effect, that is the generation of the flavour non-diagonal, $\tan \beta$ enhanced couplings of the neutral Higgs bosons to down-type quarks \[ \]. Additional contributions to $C_1^{LL}$, $C_1^{RR}$ and $C_2^{LR}$ are then generated by the double penguin diagrams involving the neutral Higgs bosons exchanges. The dominant terms obtained from such contributions are

$$\delta^{(0)}C_1^{LL} = -\frac{\alpha_{EM}}{4\pi s_W^2} \frac{m_t^4}{M_W^4} \sum_{d_j}^2 X_{d_j}^{2} \tan^4 \beta \mathcal{F}_-$$
$$\delta^{(0)}C_1^{RR} = -\frac{\alpha_{EM}}{4\pi s_W^2} \frac{m_t^4}{M_W^4} \sum_{d_j}^2 X_{d_j}^{2} \tan^4 \beta \mathcal{F}_-$$
$$\delta^{(0)}C_2^{LR} = -\frac{\alpha_{EM}}{2\pi s_W^2} \frac{m_t^4}{M_W^4} \sum_{d_j}^2 X_{d_j}^{2} \tan^4 \beta \mathcal{F}_+ .$$

with $X_{d_j}$ given by (the matrices $Z_+$ and $Z_-$ are defined in \[ \]):

$$X_{d_j} = \sum_{j=1}^2 Z_{d_j}^2 Z_{d_j}^2 \frac{A_t}{m_C} H_{2j} \left( \frac{M_{H_1}^2}{m_{C_j}^2}, \frac{M_{H_2}^2}{m_{C_j}^2} \right) ,$$

(9)
where the function $H_2(x, y)$ is given in [3] and the factor $\mathcal{F}_\mp$ reads as

$$
\mathcal{F}_\mp \equiv \left[ \frac{\cos^2 \alpha}{M_H^2} + \frac{\sin^2 \alpha}{M_h^2} \mp \frac{\sin^2 \beta}{M_A^2} \right]^{(10)}
$$

$\tan^4 \beta$ factor in eq. (8) appears because the dominant part of the effective flavour off-diagonal Yukawa coupling is given [14] by the flavour-changing chargino contribution to the $d$-quark self energy ($\propto \tan \beta$), multiplied by the tree-level Yukawa coupling (also $\propto \tan \beta$). Two powers of the external light quark masses in (8) are canceled by the fermion propagators on internal lines.

For $\tan \beta \gg 1$, $\mathcal{F}_-$ is close to zero [12], but the correction $\delta^{(0)} C_{2\text{LR}}^2$ is proportional to $\mathcal{F}_+$ which is not suppressed in this limit. Approximating the dimensionless factor $X_{\text{IC}}$ by unity, it is easy to see that in the case of the $B_s^0-B_s^0$ mixing this correction can be for $\tan \beta \sim 50$ and $M_{H^+} \sim 200$ GeV as large as $\delta^{(0)} C_{2\text{LR}}^2 \sim 2.5$ i.e. of the same order of magnitude as the SM contribution to $C_{1VLL}$. This is illustrated in fig. 2.

An important features of the double penguin contribution are: i) its fixed negative sign, the same as the sign of the dominant effects of the charged Higgs box diagrams at large $\tan \beta$; ii) its strong dependence on the left-right mixing of the top squarks - from fig. 2 it follows that large values of the stop mixing parameter $A_t$ are excluded already by the present experimental data; iii) it does not vanish when the mass scale of the sparticles is increased uniformly - large effects decreasing $\Delta M_{B_s}$ can be present also for the heavy sparticles provided the Higgs boson masses remain low.

Summarizing, we found that:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{1 + $f_s$ in the MSSM for lighter chargino mass 750 GeV, $M_2/\mu = -0.5$ and stop masses (in GeV) (500,850), (700,1000), (500,850) and (600,1100) (solid, dashed, dotted and dot-dashed lines, respectively) as a function of a) $\tan \beta$ and b) $M_{H^+}$. In panel a) solid and dashed (dotted and dot-dashed) lines correspond to $M_{H^+} = 200$ (600) GeV, and in panel a) solid and dashed (dotted and dot-dashed) lines correspond to $\tan \beta = 50$ (35).}
\end{figure}
- The largest effects of new contributions for large $\tan \beta$ are seen in $\Delta M_{B_s}$. The corresponding contributions to $\Delta M_{B_d}$ and $\varepsilon$ are strongly suppressed by the smallness of $d$-quark mass.
- The dominant contributions to $\Delta M_{B_s}$ for large $\tan \beta$ originate from double penguin diagrams involving neutral Higgs particles and, to a lesser extent, in the box diagrams with charged Higgs exchanges.
- The contribution of double penguins grows like $\tan^4 \beta$ and interferes destructively with the SM contribution, suppressing considerably $\Delta M_{B_s}$. It depends strongly on stop mixing, excluding large values of the mixing parameter $A_t$.

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