Momentum imbalance observables as a probe of gluon TMDs

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QCD Evolution 2015
Thomas Jefferson National Accelerator Facility
Newport News, VA (USA)
May 26 - 30, 2015
Outline

- Gluon TMDs for a spin-1/2 hadron

- Observables depending on transverse momentum imbalance in:
  - Electroproduction of heavy quark and jet pairs
  - Hadroproduction of Higgs + jet
  - Hadroproduction of \( J/\psi (\Upsilon) + \gamma \)

- Conclusions
Gluon correlator

The gluon correlator describes the hadron $\rightarrow$ gluon transition

Gluon momentum $\quad p^\alpha = x P^\alpha + p^T_\alpha + p^- n^\alpha$, with $n^2=0$ and $n \cdot P=1$

transverse projectors: $\quad g^{\alpha\beta}_T \equiv g^{\alpha\beta} - P^\alpha n^\beta - n^\alpha P^\beta$, $\quad \epsilon^{\alpha\beta}_T \equiv \epsilon^{\alpha\beta\gamma\delta} P^\gamma n^\delta$

Spin vector: $\quad S^\alpha = S_L \left(P^\alpha - M_h^2 n^\alpha\right) + S_T$, with $S_L^2 + S_T^2 = 1$

\[
\Phi_{g}^{\alpha\beta} \equiv \Gamma_{\alpha\beta} = \frac{n_\rho n_\sigma}{(p \cdot n)^2} \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} \ e^{ip \cdot \xi} \langle P, S | \text{Tr} \left[ F^{\alpha\rho}(0) U_{[0,\xi]} F^{\beta\sigma}(\xi) U'_{[\xi,0]} \right] | P, S \rangle \mid_{\text{LF}}
\]

Mulders, Rodrigues, PRD 63 (2001) 094021
**Parametrization of $\Phi^{\alpha\beta}$ (at “Leading Twist” and omitting gauge links)**

\[
\Phi_U^{\alpha\beta}(x, p_T) = \frac{1}{2x} \left\{ -g_T^{\alpha\beta} f_1^g(x, p_T^2) + \left( \frac{p_T^{\alpha} p_T^{\beta}}{M_h^2} + g_T^{\alpha\beta} \frac{p_T^2}{2M_h^2} \right) h_{1L}^\perp g(x, p_T^2) \right\} \quad \text{[unp. hadron]}
\]

\[
\Phi_L^{\alpha\beta}(x, p_T) = \frac{1}{2x} S_L \left\{ i\epsilon_T^{\alpha\beta} g_{1L}^g(x, p_T^2) + \frac{p_T^\rho \epsilon_T^{\rho\{\alpha\beta\}}}{M_h^2} h_{1L}^\perp g(x, p_T^2) \right\} \quad \text{[long. pol. hadron]}
\]

\[
\Phi_T^{\alpha\beta}(x, p_T) = \frac{1}{2x} \left\{ g_T^{\alpha\beta} \frac{\epsilon_T^{\rho\sigma} p_T^\rho S_T^\sigma}{M_h} f_{1T}^g(x, p_T^2) + i\epsilon_T^{\alpha\beta} \frac{p_T \cdot S_T}{M_h} g_{1T}^\perp g(x, p_T^2) \\
+ \frac{p_T^\rho \epsilon_T^{\rho\{\alpha\beta\}}}{4M_h} + S_T^\rho \epsilon_T^{\rho\{\alpha\beta\}} \right\} h_{1T}^\perp g(x, p_T^2) - \frac{p_T^\rho \epsilon_T^{\rho\{\alpha\beta\}}}{2M_h^2} \frac{p_T \cdot S_T}{M_h} h_{1T}^\perp g(x, p_T^2) \right\} \quad \text{[transv. pol. hadron]}
\]

- $f_1^g$: unpolarized TMD gluon distribution
- $h_{1L}^\perp g$: (helicity-flip, rank-2 in $p_T$) distribution of linearly polarized gluons inside an unpolar. hadron. It is $T$-even $\Rightarrow h_{1L}^\perp g \neq 0$ in absence of ISI or FSI
  Mulders, Rodrigues, PRD 63 (2001) 094021
- $f_{1T}^\perp g$: $T$-odd distribution of unp. gluons inside a transversely pol. hadron
  Sivers, PRD 41 (1990) 83
Phenomenology of gluon TMDs

All TMDs receive contributions from ISI/FSI, which can render them process dependent and even lead to factorization breaking effects.

Several processes have been suggested to access $f_{1T}^\perp g$.

$h_{1}^\perp g$ is still unknown experimentally. It can be probed by looking at the transverse momentum imbalance of two particles or jets:

- **In $pp$ collisions**, i.e. $pp \rightarrow \gamma\gamma X$ or $J/\psi\gamma X$ (RHIC, LHC)
  
  Qiu, Schlegel, Vogelsang, PRL 107 (2011) 062001
  
  den Dunnen, Lansberg, CP, Schlegel, PRL 112 (2014) 212001

- **In $ep$ collisions**, i.e. simpler measurements of azimuthal asymmetries in heavy quark or jet pair production (EIC, LHeC)

  $A_{2\phi} \sim \cos 2\phi \ h_{1}^\perp g$  
  [Only one TMD involved]

  Boer, Brodsky, Mulders, CP, PRL 106 (2011) 132001
Heavy quark pair production in DIS
Outline of the calculation

**Electroproduction of heavy quarks**

\[ e(\ell) + h(P) \rightarrow e(\ell') + Q(K_1) + \bar{Q}(K_2) + X \]

the \( Q\bar{Q} \) pair is almost back to back in the plane \( \perp \) to \( q \) and \( P \)

\( q \equiv \ell - \ell' \): four-momentum of the exchanged virtual photon \( \gamma^* \)

\[ q_T \equiv K_{1\perp} + K_{2\perp} \]

\[ K_{\perp} \equiv (K_{1\perp} - K_{2\perp})/2 \]

\[ \phi_\ell = \phi_{\ell'} = 0 \]

\[ \Rightarrow \text{Correlation limit:} \quad |q_T| \ll |K_{\perp}|, \quad |K_{\perp}| \approx |K_{1\perp}| \approx |K_{2\perp}| \]
Heavy quark pair production in DIS
Outline of the calculation

Calculation of the cross section

**TMD Master Formula**

\[
\frac{d\sigma}{ds} = \frac{1}{2s} \frac{d^3\ell'}{(2\pi)^3} \frac{d^3K_1}{2E'_e} \frac{d^3K_2}{2E_1} \left(2\pi\right)^4 \delta^4(q+p-K_1-K_2) \int dx \, d^2p_T \frac{d^3\ell}{(2\pi)^3} \sum_{a,b,c} \frac{1}{Q^4} L(\ell, q) \otimes \Phi_a(x, p_T) \otimes \left| H_{\gamma^* a \rightarrow b c}(q, p, K_1, K_2) \right|^2
\]

**Leptonic tensor:**

\[
L^{\mu\nu}(\ell, q) = -g^{\mu\nu} Q^2 + 2(\ell^{\mu} \ell'^{\nu} + \ell^{\nu} \ell'^{\mu}), \quad Q^2 = -q^2
\]

At LO in pQCD:

\[
\left| H_{\gamma^* a \rightarrow b c} \right|^2 = \left| H_{\gamma^* g \rightarrow Q \bar{Q}} \right|^2 \text{ from the diagrams}
\]

**\(\gamma^* g \rightarrow Q \bar{Q}:\)**

- Diagram 1: \(q\) \(\rightarrow\) \(K_1\) \(\rightarrow\) \(p\) \(\rightarrow\) \(K_2\)
- Diagram 2: \(q\) \(\rightarrow\) \(K_1\) \(\rightarrow\) \(K_2\)
Angular structure of the cross section

In the photon-hadron cms: \( y_1 \) (\( y_2 \)) rapidity of \( Q \) (\( \bar{Q} \))

**DIS variables:**
\[
\begin{align*}
    x_B &= \frac{Q^2}{2P \cdot q}, \\
    y &= \frac{P \cdot q}{P \cdot \ell} \\
    q_T &= |q_T|(\cos \phi_T, \sin \phi_T) \\
    K_\perp &= |K_\perp|(\cos \phi_\perp, \sin \phi_\perp)
\end{align*}
\]

\[
\frac{d\sigma}{dy_1 dy_2 dy dx_B d^2 q_T d^2 K_\perp} \propto \left\{ A_0 + A_1 \cos \phi_\perp + A_2 \cos 2\phi_\perp \right\} f_1^g
\]
\[
+ \frac{q_T^2}{M_h^2} h_1^{\perp g} \left\{ B_0 \cos 2(\phi_\perp - \phi_T) + B_1 \cos(\phi_\perp - 2\phi_T) \\
+ B_1' \cos(3\phi_\perp - 2\phi_T) + B_2 \cos 2\phi_T + B_2' \cos 2(2\phi_\perp - \phi_T) \right\}
\]

Integrating over \( \phi_T, \phi_\perp \implies A_0 f_1^g \)
Heavy quark pair production in DIS
Outline of the calculation

$q_T$-imbalance observables

Example of diagram contributing to $B_i^{(i)}$:
gluon helicities flip

$A_i$: gluon helicities do not flip

The different contributions can be isolated by defining

$$\langle W(\phi_\perp, \phi_T) \rangle = \frac{\int d\phi_\perp d\phi_T W(\phi_\perp, \phi_T) d\sigma}{\int d\phi_\perp d\phi_T d\sigma} , \quad W = \cos 2(\phi_\perp - \phi_T) , \ldots$$

Positivity bound:

$$|h_{1\perp}^g (x, p_T^2)| \leq \frac{2M_h^2}{p_T^2} f_1^g (x, p_T^2) \quad p_T^2 = q_T^2$$
Maximum asymmetries in $ep \rightarrow e' Q \bar{Q} X$

$R$: upper bound on $|\langle \cos 2(\phi_\perp - \phi_T) \rangle|$
Maximum asymmetries in $ep \rightarrow e'Q\bar{Q}X$

$R$: upper bound on $|\langle \cos 2\phi_T \rangle|$
Dijet production in \( ep \) and \( pp \) collisions

Results for \( eh \rightarrow e' \text{jet jet} X \) can be obtained by taking \( M_Q = 0 \) in the expressions for the asymmetries in \( ep \rightarrow e'Q\bar{Q}X \).

The denominator receives a contribution also from \( \gamma^* q \rightarrow gq \)

\( h_1 g \) contributes to the dijet imbalance in hadronic collisions, commonly used to extract the average partonic \( p_T \).

Boer, Mulders, CP, PRD 80 (2009) 094017

Azimuthal asymmetries in \( pp \rightarrow Q\bar{Q}X \) and \( pp \rightarrow \text{jet jet} X \) suffer from factorization breaking contributions and would allow us to quantify the importance of ISI/FSI.

Rogers, Mulders, PRD 81 (2010) 094006
Higgs production

\[ h_1 \perp g \quad \text{in} \quad pp \rightarrow H X \]

Talks by D. Boer and M. Echevarria

Higgs boson production happens mainly via \( gg \rightarrow H \)

Pol. gluons affect the Higgs transverse spectrum at NNLO pQCD

Catani, Grazzini, NPB 845 (2011) 297

The nonperturbative distribution can be present at tree level and would contribute to Higgs production at low \( q_T \)

Boer, den Dunnen, CP, Schlegel, Vogelsang, PRL 108 (2012) 032002

Echevarria, Kasemets, Mulders, CP, arXiv:1502.05354
Higgs production

Transverse spectrum of the Higgs boson

\[ \frac{1}{\sigma} \frac{d\sigma}{dq_T^2} \propto 1 + R_0(q_T^2) \]

\[ R_0 = \frac{h_{1g}^T \otimes h_{1g}^T}{f_1^g \otimes f_1^g} \]

\[ |h_{1g}^T(x, p_T^2)| \leq \frac{2M_p^2}{p_T^2} f_1^g(x, p_T^2) \]

Gaussian model for both \( f_1^g \) and \( h_{1g}^T \):

\[ f_1^g(x, p_T^2) = \frac{f_1^g(x)}{\pi \langle p_T^2 \rangle} \exp \left( -\frac{p_T^2}{\langle p_T^2 \rangle} \right) \]

\[ h_{1g}^T(x, p_T^2) = \frac{M^2 f_1^g(x) 2(1 - r)}{\pi \langle p_T^2 \rangle^2} \frac{1}{r} \exp \left( 1 - \frac{1}{r} \frac{p_T^2}{\langle p_T^2 \rangle} \right) \quad 0 < r < 1 \]

\[ \langle p_T^2 \rangle = 7 \text{ GeV}^2 \]
Higgs plus jet production

Motivations: azimuthal asymmetries can be defined [≠ pp → HX]
study of the TMD evolution by tuning the hard scale
Nonuniversality and factorization breaking effects
Boer, CP, PRD 91 (2015) 074024

TMD Master Formula

\[
d\sigma = \frac{1}{2s} \frac{d^3 K_H}{(2\pi)^3 2K_H^0} \frac{d^3 K_j}{(2\pi)^3 2K_j^0} \sum_{a,b,c} \int dx_a dx_b d^2 p_aT d^2 p_bT (2\pi)^4 \\
\times \delta^4(p_a + p_b - q) \text{Tr} \left\{ \Phi_a(x_a, p_{aT}) \Phi_b(x_b, p_{bT}) \right| \mathcal{M}^{ab \rightarrow Hc} \left|^{2} \right. \}
\]

Higgs and jet almost back to back in the \( \perp \) plane: \( |q_T| \ll |K_\perp| \)

\[
q_T = K_{HT} + K_{jT}, \quad K_\perp = (K_{HT} - K_{jT})/2
\]
Feynman diagrams

At LO in pQCD the partonic subprocesses that contribute are
\[ g g \rightarrow H g \quad g q \rightarrow H q \quad q \bar{q} \rightarrow H g \]

Quark masses taken to be zero, except for \( M_t \rightarrow \infty \)
Kauffmann, Desai, Risal, PRD 55 (1997) 4005

No indications that TMD factorization can be broken due to color entanglement
Rogers, Mulders, PRD 81 (2010) 094006
Angular structure of the cross section

Focus on $gg \rightarrow Hg$ (dominant at the LHC). In the hadronic c.m.s.:

$$q_T = |q_T|(\cos \phi_T, \sin \phi_T) \quad K_\perp = |K_\perp|(\cos \phi_\perp, \sin \phi_\perp) \quad \phi \equiv \phi_T - \phi_\perp$$

$$d\sigma \equiv \frac{d\sigma}{dy_H \ dy_j \ d^2 K_\perp \ d^2 q_T} \quad \frac{d\sigma}{\sigma} \equiv \frac{d\sigma}{\int_0^{q_T^{\text{max}}} dq_T^2 \int_0^{2\pi} d\phi \ d\sigma}$$

Normalized cross section for $pp \rightarrow H \text{jet} \ X$

$$\frac{d\sigma}{\sigma} = \frac{1}{2\pi} \sigma_0(q_T^2) \left[1 + R_0'(q_T^2) + R_2(q_T^2) \cos 2\phi + R_4(q_T^2) \cos 4\phi\right]$$

$$\sigma_0(q_T^2) \equiv \frac{f_1^g \otimes f_1^g}{\int_0^{q_T^{\text{max}}} dq_T^2 f_1^g \otimes f_1^g}$$
TMD observables

The three contributions can be isolated by defining the observables

\[ \langle \cos n\phi \rangle_{q_T} \equiv \int_0^{2\pi} d\phi \cos n\phi \frac{d\sigma}{d\sigma} \quad (n = 0, 2, 4) \]

such that

\[ \langle \cos n\phi \rangle = \int_0^{q_T^2_{\text{max}}} dq_T^2 \langle \cos n\phi \rangle_{q_T} \]

\[ \frac{1}{\sigma} \frac{d\sigma}{d^2 q_T} \equiv \langle 1 \rangle_{q_T} \quad \Rightarrow \quad 1 + R_0' \propto f_1^g \otimes f_1^g + h_{1\perp g} \otimes h_{1\perp g} \]

\[ \langle \cos 2\phi \rangle_{q_T} \quad \Rightarrow \quad R_2 \propto f_1^g \otimes h_{1\perp g} \]

\[ \langle \cos 4\phi \rangle_{q_T} \quad \Rightarrow \quad R_4 \propto h_{1\perp g} \otimes h_{1\perp g} \]
Models for the TMD gluon distributions

\( f_1^g: \) Gaussian + tail

\[
f_1^g(x, p_T^2) = f_1^g(x) \frac{R^2}{2\pi} \frac{1}{1 + p_T^2 R^2} \quad R = 2 \text{ GeV}^{-1}
\]

\( h_1^\perp g: \) Maximal polarization and Gaussian + tail

\[
\begin{align*}
  h_1^\perp g(x, p_T^2) &= \frac{2M_p^2}{p_T^2} f_1^g(x, p_T^2) \quad [\text{max pol.}] \\
  h_1^\perp g(x, p_T^2) &= 2 f_1^g(x) \frac{M_p^2 R_h^4}{2\pi} \frac{1}{(1 + p_T^2 R_h^2)^2} \\
  R_h &= \frac{3}{2} R
\end{align*}
\]

Boer, den Dunnen, NPB 886 (2014) 421
$q_T$-distribution

Configuration in which the Higgs and the jet have same rapidities

- $K_\perp = 10$ GeV
- $K_\perp = 100$ GeV
- Lin. pol. = 0

Effects largest at small $q_T$ (hard to measure), but model dependent!
Azimuthal $\cos 2\phi$ asymmetries

Sensitive to the sign of $h_1 g$:

$$\langle \cos 2\phi \rangle_{q_T} < 0 \implies h_1 g > 0$$

$q_{T\text{max}} = \frac{M_H}{2}$

$$\langle \cos 2\phi \rangle \approx 12\% \text{ at } K_\perp = 100 \text{ GeV}$$
Azimuthal \( \cos 4\phi \) asymmetries

\[ \langle \cos 4\phi \rangle_{q_T} [\text{GeV}^{-2}] \]

\[ q_T^{\text{max}} = \frac{M_H}{2} \]

\[ \langle \cos 4\phi \rangle \approx 0.1 - 0.2\% \text{ at } K_\perp = 100 \text{ GeV} \]
Gaussian model for the unpolarized TMDs

\[ q_T^{\text{max}} = K_\perp / 2 \, , \, \langle \cos 2\phi \rangle \approx 9\% \, , \, \langle \cos 4\phi \rangle \approx 0.4\% \, \text{at} \, K_\perp = 100 \, \text{GeV} \]
The unpolarized gluon TMD distribution

\[ p p \rightarrow J/\psi + \gamma X \]

**J/\psi + \gamma** production in hadronic collisions

First determination of \( h_{1g}^\perp \) and \( f_{1g}^g \) could be possible now at the LHC

Color Singlet mechanism dominates: TMD factorization might hold

\[
\frac{1}{\sigma} \frac{d\sigma}{d^2q_T} \equiv S_{q_T}^{(0)} \equiv \langle 1 \rangle_{q_T} \quad \rightarrow \quad f_{1g}^g \otimes f_{1g}^g
\]

\[
S_{q_T}^{(2)} \equiv \langle \cos 2\phi \rangle_{q_T} \quad \rightarrow \quad f_{1g}^g \otimes h_{1g}^\perp
\]

\[
S_{q_T}^{(4)} \equiv \langle \cos 4\phi \rangle_{q_T} \quad \rightarrow \quad h_{1g}^\perp \otimes h_{1g}^\perp
\]

Polarized proton with \( S_T = |S_T| (\cos \phi_S, \sin \phi_S) \quad \rightarrow \quad f^g_{1T} \) Possible at the future AFTER@LHC and, in principle, at RHIC

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den Dunnen, Lansberg, CP, Schlegel, PRL 112 (2014) 212001
The gluon Sivers function

\[ p^\uparrow p \rightarrow J/\psi + \gamma X \]

Angular structure of the cross section for \( p^\uparrow p \rightarrow J/\psi + \gamma X \)

\[
\frac{d\sigma_{UT}}{dy_\psi \, dy_\gamma \, d^2K_\perp \, d^2q_T} \propto \sin \phi_S \ f_1^g \otimes f_{1T}^g + B \left\{ \sin(\phi_S - 2\phi) \ f_1^g \otimes h_{1T}^g \right. \\
+ \sin \phi_S \cos 2\phi \ [f_1^g \otimes h_{1T}^g + h_{1T}^g \otimes f_1^g] \\
+ \sin \phi_S \cos 4\phi \ [h_{1T}^g \otimes h_{1T}^g + h_{1T}^g \otimes h_{1T}^g] \\
+ \cos \phi_S \sin 2\phi \ [f_1^g \otimes h_{1T}^g + h_{1T}^g \otimes f_1^g] \\
+ \cos \phi_S \sin 4\phi \ [h_{1T}^g \otimes h_{1T}^g + h_{1T}^g \otimes h_{1T}^g] \left\} \quad \phi \equiv \phi_T - \phi_\perp
\]

Lansberg, CP, Schlegel, in preparation

Feynman diagrams at LO pQCD

(Color Singlet Model)
Upper bounds of the Sivers asymmetry

\[ A_N^{\sin \phi_S} \equiv \frac{\int d\phi_S \sin \phi_S \left[ d\sigma(\phi_S) - d\sigma(\phi_S + \pi) \right]}{\int d\phi_S \left[ d\sigma(\phi_S) + d\sigma(\phi_S + \pi) \right]} = \frac{\int d\phi_S \sin \phi_S d\sigma_{UT}}{\int d\phi_S d\sigma_{UU}} \]

Positivity bound

\[ |p_T| f_T^g(x, p_T^2) \leq f_T^g(x, p_T^2) \]

Gaussian model for \( f_T^g \)

\[ f_T^g(x, p_T^2) = \frac{M f_T^g(x)}{\pi \langle p_T^2 \rangle^{3/2}} \sqrt{\frac{2e(1-r)}{r}} \exp \left( -\frac{1}{r} \frac{p_T^2}{\langle p_T^2 \rangle} \right) \quad 0 < r < 1 \]
Transverse spectra of $\eta_Q$ and $\chi_{QJ}$ ($Q = c, b$)

\[ \frac{1}{\sigma(\eta_Q)} \frac{d\sigma(\eta_Q)}{dq_T^2} \propto 1 - R_0(q_T^2) \quad \text{[pseudoscalar]} \]

\[ \frac{1}{\sigma(\chi_Q)} \frac{d\sigma(\chi_{Q0})}{dq_T^2} \propto 1 + R_0(q_T^2) \quad \text{[scalar]} \]

Effects of $h_1^{g \perp}$ on higher angular momentum states are suppressed

Boer, CP, PRD 86 (2012) 094007

Proof of TMD factorization at NLO only for $\eta_Q$ production

Ma, Wang, Zhao, PRD 88 (2013), 014027; PLB 737 (2014) 103
The gluon Sivers function
C-even quarkonium production

$A_N^{\sin \phi_S}$ for $p^\uparrow p \rightarrow \eta Q X$ and $p^\uparrow p \rightarrow \chi_{QJ} X$

$A_N^{\sin \phi_S} (\eta Q) = \frac{|S_T|}{2(1 - R_0)} f_1^g \otimes f_1^g \left\{ f_1^g \otimes f_{1T}^g + h_{1T}^g \otimes h_1^g + h_1^g \otimes h_{1T}^g \right\}$

$A_N^{\sin \phi_S} (\chi_{Q0}) = \frac{|S_T|}{2(1 + R_0)} f_1^g \otimes f_1^g \left\{ f_1^g \otimes f_{1T}^g - h_{1T}^g \otimes h_1^g - h_1^g \otimes h_{1T}^g \right\}$

$A_N^{\sin \phi_S} (\chi_{Q2}) = \frac{|S_T|}{2 f_1^g \otimes f_1^g} f_1^g \otimes f_{1T}^g$

Upper bounds (Gaussian models for TMDs)
Conclusions

▶ The cleanest way to probe gluon TMDs would be to look at $q_T$-distributions and azimuthal asymmetries in $e p^{(\uparrow)} \rightarrow e' Q \bar{Q} X$ and/or $e p^{(\uparrow)} \rightarrow e' \text{jet jet} X$

▶ $h_1^g$ produces a modulation of the transverse spectrum of $H + \text{jet}$ and leads to azimuthal asymmetries in $pp \rightarrow H \text{jet} X$

▶ First determination of $h_1^g$ and $f_1^g$ could come from $J/\psi(\Upsilon) + \gamma$ production at the running experiments at the LHC. In experiments with polarized protons, $f_{1T}^g$ could also be accessed