New inflation, preinflation, and leptogenesis

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Abstract

We present a new inflation model in which the inflationary scenario including subsequent reheating is determined in part by a $U(1)_R \times Z_n$ symmetry. A preinflation epoch can be introduced to yield, among other things, a running spectral index indicated by the WMAP analysis. The inflaton decay into right handed neutrinos, whose masses can be hierarchical because of the $Z_n$ symmetry, leads to a reheat temperature $\gtrsim 10^8$ GeV ($\sim 10^4$–$10^6$ GeV in some cases), followed by non-thermal leptogenesis.

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I. INTRODUCTION

Supersymmetric hybrid inflation \cite{1} and its extensions (for reviews see \cite{2}) are scenarios that can reconcile the amplitude of the primordial density perturbations with a GUT scale symmetry breaking, without any dimensionless parameters that are very small. An attractive feature of these models is the relative ease with which the observed baryon asymmetry of the universe can be explained via leptogenesis \cite{3}, generated by the right handed neutrinos which arise from the inflaton decay \cite{4}.

In these models a $U(1)_R$ symmetry plays an essential role in constraining the form of the superpotential that drives inflation \cite{1}. A hybrid inflation model with a $U(1)_R \times Z_2$ symmetry, named smooth hybrid inflation, was considered in \cite{5}, and generalized to arbitrary $Z_n$ in \cite{6}.\textsuperscript{1} In this paper we discuss a related new inflation model in which the symmetry $U(1)_R \times Z_n$ again plays an essential role, but unlike smooth hybrid inflation, the inflaton rolls away rather than towards the origin. While the $U(1)_R$ symmetry, as usual, constrains the superpotential responsible for inflation, the $Z_n$ symmetry also plays an important role in controlling the reheating process and leptogenesis.

One of our goals in the paper is to realize new inflation in as natural a manner as possible. This leads us to consider preinflation which is used to avoid fine tuning the initial conditions. One possible outcome of this double inflation scenario is a spectral index that can vary significantly with scale as suggested by the WMAP analysis \cite{8}. We also briefly discuss the possibility of reconciling early star formation with a running spectral index \cite{6,9}.

The right handed neutrinos typically acquire masses through a dimension five term in the superpotential after symmetry breaking. For a symmetry breaking scale of order $M_{GUT}$, one would expect these masses to be of order $M^2_{GUT}/m_P \sim 10^{14}$ GeV, where $m_P = 2.4 \times 10^{18}$ GeV is the (reduced) Planck scale. However, with the gravitino constraint on the reheat temperature $T_r \lesssim 10^{10}$ GeV \cite{10}, we require the existence of at least one right handed neutrino of mass $\lesssim 10^{12}$ GeV. The $Z_n$ symmetry is used to ensure such right handed neutrino masses without invoking small dimensionless couplings.

The inflaton decays into right handed neutrinos with masses that scale as $\mu^p/M_{GUT}^{p-1}$, where $\mu$ is the energy scale of inflation, and $2 < p \leq 3$. The energy scale is lower in the

\textsuperscript{1} See also \cite{7} for related models.
new inflation scenario compared to smooth hybrid inflation, which further mitigates the gravitino problem. The out of equilibrium decay of the right handed neutrinos arising from the inflaton satisfactorily explains the observed baryon asymmetry.\textsuperscript{2}

The paper is organized as follows. In Sec. II we consider new inflation and the density fluctuations. In Sec. III we consider the initial condition problem where we invoke preinflation as a solution and also as a way to improve the agreement with the WMAP data. We discuss the right handed neutrino masses and leptogenesis in Sec. IV, and present our conclusion in Sec. V.

II. NEW INFLATION

We consider the superpotential

\[ W_1 = S \left( -\mu^2 + \frac{(\Phi \Phi)^m}{M_s^{2m-2}} \right), \]

where \( \Phi \Phi \) denote a conjugate pair of superfields transforming as nontrivial representations of some gauge group \( G \), and \( S \) is a gauge singlet superfield. Here \( M_s \) is a cut-off scale and \( m \) is an integer \( \geq 2 \). Under the \( U(1)_R \) symmetry, the superfields transform as \( S \rightarrow e^{2i\alpha} S \), \( \Phi \rightarrow \Phi \), \( \Phi \rightarrow \Phi \), and \( W \rightarrow e^{2i\alpha} W \). Under the discrete symmetry \( Z_n \), \( \Phi \) and \( \Phi \) each has unit charge, so that \( m = n \) for odd \( n \) and \( m = n/2 \) for even \( n \).

The vanishing of the F- and D-terms imply that the SUSY vacua lie at \( \langle S \rangle = 0 \), \( \langle \Phi \Phi \rangle^* = \langle \Phi \rangle = \pm M \), where the symmetry breaking scale \( M \) is given by

\[ M = (\mu M_s^{m-1})^{1/m}. \]

With \( |\Phi| = |\Phi| \) along the D-flat direction of the scalar potential, we can write the Kähler potential as\textsuperscript{3}

\[ K_1 = |S|^2 + 2 \left( |\Phi|^2 + \kappa_1 \frac{|\Phi|^4}{4m_p^2} + \kappa_2 \frac{|S|^2 |\Phi|^2}{m_p^2} \right) + \kappa_3 \frac{|S|^4}{4m_p^2} + \ldots. \]

The scalar potential is given by

\[ V = e^K \left[ \left( \frac{\partial^2 K}{\partial z_i \partial \overline{z}_j} \right)^{-1} D_z W D_{\overline{z}} W^* - 3|W|^2 \right] + V_D, \]

\textsuperscript{2} Leptogenesis in the framework of a similar new inflation model was considered in [11]. Unlike our case, there the inflaton is a gauge singlet and \( Z_n \) is not associated with the neutrino masses.

\textsuperscript{3} We employ the same letters for superfields and their scalar components.
with
\[ D_{zi} W = \frac{\partial W}{\partial z_i} + \frac{\partial K}{\partial z_i} W. \] (5)

Using Eqs. (1), (3), and the D-flatness condition, the scalar potential for \(|\Phi|, |S| \ll m_P\) is found to be
\[ V \simeq \mu^4 \left(1 - \kappa_3 \frac{|S|^2}{m_P^2} + 2(1 - \kappa_2) \frac{|\Phi|^2}{m_P^2} - 2 \frac{|\Phi|^{2m}}{M^{2m}} + \frac{|\Phi|^{4m}}{M^{4m}} \right). \] (6)

In contrast to smooth hybrid inflation, where \(\kappa_3\) is taken to be small and positive, for new inflation we take \(\kappa_3 < -1/3\). The \(S\) field acquires a positive mass squared larger than \(H^2\), where the Hubble parameter \(H \simeq \mu^2/\sqrt{3} m_P\) during inflation, and therefore rapidly settles to zero.

Hereafter we set the symmetry breaking scale \(M = M_{\text{GUT}} \simeq 2 \times 10^{16}\) GeV, and use units such that \(2M \equiv 1\). Defining \(\beta \equiv \kappa_2 - 1 \geq 0\) and the canonically normalized real fields \(\phi \equiv 2\text{Re}\Phi, \sigma \equiv \sqrt{2}\text{Re}\,S\), the inflaton potential near the origin is given by
\[ V \simeq \mu^4 \left(1 - \frac{\beta}{2} \frac{\phi^2}{m_P^2} - 2\phi^{2m} \right). \] (7)

This is very similar to the potential of the new inflation model discussed in [12]. The slow roll parameters are given by
\[ \epsilon \simeq \frac{1}{2} \left( \beta \frac{\phi}{m_P} + 4mm_P \phi^{2m-1} \right)^2, \quad \eta \simeq -\left( \beta + 4m(2m-1)m_P^2 \phi^{2m-2} \right). \] (8)

Since \(\epsilon\) is negligible, inflation ends when \(|\eta| \simeq 1\), at
\[ \phi \simeq \frac{1 - \beta}{(4m(2m-1)m_P^2)^{1/(2m-2)}} \equiv \phi_f. \] (9)

The number of e-folds after the comoving scale \(l\) has crossed the horizon is given by
\[ N_l = \frac{1}{m_P^2} \int_{\phi_f}^{\phi_l} \frac{V d\phi}{V'} \simeq \frac{1}{(2m - 2)\beta} \left[ \ln \left( \frac{4m(1 + \beta)m_P^2}{1 - (2m - 2)\beta} \frac{\beta + 4mm_P^2 \phi_l^{2m-2}}{\phi_l^{2m-2}} \right) \right], \quad (\beta \neq 0), \] (10)

where \(\phi_l\) is the value of the field at the comoving scale \(l\). From Eqs. (8) and (10), for \(\beta \gg 4mm_P^2 \phi_l^{2m-2}\), the spectral index \(n_s\) is found to be
\[ n_s \simeq 1 - 2\eta \simeq 1 - 2\beta \left[ 1 + \frac{(2m - 1)(1 - (2m - 2)\beta)}{1 + \beta} \cdot e^{-(2m-2)\beta N_l} \right], \] (11)
giving \(n_s \simeq 1 - 2\beta\) for \(\beta \gg 1/[(2m - 2)N_l]\). For \(\beta = 0\) we find \(n_s = 1 - 2(2m-1)/(2m-2)N_l\).

A numerical calculation of the spectral index, for \(m = 2\) to 5, is given in Fig. 1. Here the
values of $n_s$ correspond to the comoving scale $l_0 = 2\pi/k_0$, with $k_0 \equiv 0.002 \text{ Mpc}^{-1}$. The number of e-folds $N_0$ corresponding to this scale is 50–55 depending on the energy scale $\mu$.\footnote{$N_0 \simeq 54 + (1/3) \ln(T_r/10^9 \text{ GeV}) + (2/3) \ln(\mu/10^{14} \text{ GeV})$, where $T_r$ is the reheat temperature.} The running of the spectral index is negligible, with $dn_s/d\ln k \lesssim 10^{-3}$.

Note that the WMAP analysis suggests a running spectral index, with $n_s \lesssim 0.95$ and $dn_s/d\ln k \lesssim 10^{-3}$ disfavored at the 2$\sigma$ level \footnote{8}. Other analyses relax this bound to $n_s \lesssim 0.92$ \footnote{13}, which still requires $\beta \lesssim 0.04$.

The amplitude of the curvature perturbation $\mathcal{R}$ is given by
\begin{equation}
\mathcal{R} = \frac{1}{2\sqrt{3}\pi m_P^3 |V'|} \simeq \frac{\mu^2}{2\sqrt{3}\pi(\beta m_P \phi + 4m m^3_P \phi^{2m-1})}. \tag{12}
\end{equation}

Using the WMAP best fit $\mathcal{R} \simeq 4.7 \times 10^{-5}$ at $k_0 \equiv 0.002 \text{ Mpc}^{-1}$ \footnote{8}, we obtain $\mu \sim 2 \times 10^{13}$ GeV for $m = 2$ and $\mu \sim 2 \times 10^{14}$ GeV for $m = 5$. The cut-off scale from Eq. (2) is $M_* \simeq 2 \times 10^{19}$ GeV ($\simeq 7 \times 10^{16}$ GeV) for $m = 2$ ($m = 5$) (see Fig. 2).

### III. INITIAL CONDITIONS AND PREINFLATION

As shown in Fig. 3 the value of the inflaton field $\phi$ at $k_0$ is found to be $\phi_0 \sim 10^{-4}$ ($10^{-1}$) for $m = 2$ ($m = 5$), in units $2M \equiv 1$. In other words, the initial value of the inflaton is close to the local maximum of Eq. (7) at $\phi = 0$, whereas the true minimum is at $\phi = 1$. Such an initial condition demands an explanation.

One way to realize the initial condition dynamically is through an earlier stage of so-called preinflation \footnote{14}. Although any suitable preinflation with a high enough energy scale can suppress the initial value of $\phi$, to be specific we will assume the superpotential to be of the same form as Eq. (1):
\begin{equation}
W_2 = X \left(-v^2 + \frac{(\overline{\Psi}\Psi)^m}{M_{S(2m-1)}^2}\right), \tag{13}
\end{equation}
\begin{equation}
K_2 = |X|^2 + 2 \left(|\overline{\Psi}|^2 + \alpha_1 \frac{|\overline{\Psi}|^4}{4m_P^2} + \alpha_2 \frac{|X|^2|\overline{\Psi}|^2}{m_P^2} + \alpha_3 \frac{|X|^4}{4m_P^2} + \ldots \right). \tag{14}
\end{equation}

However, unlike $\kappa_3$ in Eq. (3), we take $\alpha_3$ to be positive, so that smooth hybrid inflation is realized. This particular preinflation stage has recently been considered in Ref. \cite{6}. Note that while $\alpha_3 \lesssim 10^{-2}$ is required to obtain the required 50–55 e-folds in a single stage hybrid
or smooth hybrid inflation, if the number of e-folds after horizon exit is of order 10, as in the case of a double inflation model, then $\alpha_3 \sim 0.1$ is possible.

The inflaton field $\phi$ acquires an effective mass during preinflation, given by

$$m_{\text{eff}} = c \frac{v^2}{m_p} = \sqrt{3}cH,$$

(15)

where $v$ is the energy scale of preinflation, and $c$ is a parameter which depends on the details of the Kähler potential. For example, if the latter has a term $2g|\Phi|^2|X|^2$, the effective mass is equal to $\sqrt{1-gv^2/m_P}$. The evolution of $\phi$ is given by

$$\ddot{\phi} + 3H\dot{\phi} + m_{\text{eff}}^2\phi = 0,$$

(16)

with the solution

$$\phi \propto \text{Re} \left[ a^{-(3/2)+\sqrt{(9/4)-3c^2}} \right],$$

(17)

where $a$ is the scale factor of the universe. If we define $\zeta \equiv 3/2$ for $c > \sqrt{3}/2$, and $\zeta \equiv (3/2) - \sqrt{(9/4)-3c^2}$ otherwise, at the end of preinflation $\phi$ takes the value

$$\phi \simeq \phi_{\text{min}} + (\phi_b - \phi_{\text{min}})e^{-\zeta N_{\text{pre}}} \equiv \phi_e.$$  

(18)

Here $\phi_b$ is the value of $\phi$ at the beginning of preinflation, and $N_{\text{pre}}$ is the total e-fold number of preinflation. The field $S$ also settles to its minimum the same way. To find these minima we consider the potential during preinflation, obtained from $W = W_1 + W_2$ and $K = K_1 + K_2$:

$$V \simeq v^4 \left( 1 + \frac{|S|^2}{m_P^2} + \frac{2|\Phi|^2}{m_P^2} \right) - \frac{\mu^2 v^2}{m_P^2} (SX^* + S^*X) \left( 1 + \frac{(m-1)|\Phi|^{2m}}{M^{2m}} \right)$$

$$+ \mu^4 \left( -\frac{2|\Phi|^{2m}}{M^{2m}} + \frac{|\Phi|^{4m}}{M^{4m}} + \frac{2m^2 |S|^2 |\Phi|^{4m-2}}{M^{4m}} \right),$$

(19)

where we have omitted other higher order terms, and assumed $v > \mu$. Identifying the inflaton during preinflation with $x = \sqrt{2}\text{Re}X$, Eq. (19) is minimized for $\text{Im}S = 0$. We therefore can express the potential in terms of the normalized real scalar fields. Eq. (19) has no minimum for $\phi \geq 1$, and for $\phi < 1$ it simplifies to

$$V \simeq v^4 \left( 1 + \frac{\sigma^2}{2m_P^2} + \frac{\phi^2}{2m_P^2} \right) - \frac{\mu^2 v^2 \sigma x}{m_P^2}.$$  

(20)

Hence the minima are at $\sigma_{\text{min}} \simeq x\mu^2/v^2$, where $x \sim \langle \Psi \rangle$ at the end of preinflation, and $\phi_{\text{min}} = 0.5$.

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5 If $K \supset |\Phi|^2|X|^2 + \ldots$, the mass terms are multiplied with $c$, but this does not affect the minimum of $\phi$ as long as $c$ is positive.
After preinflation, $W_2$ vanishes and hence the potential has minima at the origin for both $\phi$ and $\sigma$. During the matter dominated era between the two stages of inflation, these fields acquire effective masses $m_{\text{eff}}^2 = (3/2)H^2$. Consequently, from Eq. (16) the fields scale as $a^{-3/4}$ until $H \simeq \mu^2/m_P$. Since the energy density scales as $a^{-3}$, the value of $\phi$ at the onset of new inflation is

$$\phi \simeq \frac{\mu}{v} \phi_e \equiv \phi_i.$$  

(21)

From Eq. (18), we see that for $\zeta = 3/2$ and $N_{\text{pre}} \gtrsim 10$, $\phi_e \approx 0$ so that $\phi_i \ll \phi_0$ and new inflation takes over before the cosmological scales exit the horizon.

Hybrid and smooth hybrid inflation in supergravity provide a natural way to obtain a blue spectrum at large scales [17]. Hence, in the light of the WMAP results, it is of interest to consider the cases where the onset of new inflation corresponds to cosmological [15] or galactic scales, with preinflation responsible for the density fluctuations on larger scales. In particular, the latter case can accommodate early star formation as well as the running spectral index favored by WMAP [6, 9]. This can be realized in our setup if we take $c < \sqrt{3}/2$. As an example, for $c = 1/2$ we obtain $\zeta = (3 - \sqrt{6})/2$. Assuming $\phi_P \simeq m_P$, $N_{\text{pre}} \simeq 20$ for the central WMAP values $n_s \simeq 1.13$ and $dn_s/d\ln k \simeq -0.055$ at $k_0$ [8], and Eq. (18) gives $\phi_e \simeq 0.25$. Taking $\mu/v = 1/50$ in Eq. (21), we find $\phi_i \simeq 0.005$ (see Fig. 3). Higher values of $\phi_i$ are obtained with slightly lower values of $c$ (e.g. $c = 1/3$ yields $\phi_i \simeq 0.12$).

As shown in Ref. [15], for $c > \sqrt{3}/2$, the quantum fluctuations of $\phi$ during preinflation reenter the horizon at the beginning of new inflation with an amplitude

$$\delta \phi \simeq \frac{H_{\text{pre}}}{3^{1/4}2\pi} \left(\frac{\mu}{v}\right)^2,$$  

(22)

which is a factor $3^{1/4}$ smaller than the fluctuations induced during new inflation. Moreover, the fluctuations produced during preinflation are suppressed for smaller wavelengths. Thus, their contribution to the curvature perturbation can be neglected.

For $c < \sqrt{3}/2$, Eq. (22) is replaced by

$$\delta \phi \simeq \frac{H_{\text{pre}}}{2\pi} \left(\frac{\mu}{v}\right)^{1+2\zeta/3},$$  

(23)

and the corresponding ratio of fluctuations (preinflation/new inflation) is

$$r \equiv \left(\frac{v}{\mu}\right)^{1-2\zeta/3}.$$  

(24)
As an example, \( c = 1/2 \) and \( v/\mu = 50 \) yields \( r \approx 24 \). Such a crest in the curvature perturbation at scales \( \lesssim 100 \) kpc could help resolve the apparent discrepancy between the WMAP predictions of the running spectral index and early star formation \[18\].

### IV. NEUTRINO MASSES AND LEPTOGENESIS

The mass matrix \( M_R \) of right handed neutrinos is generated by the superpotential coupling

\[
\gamma_{ij} \left( \Phi \Phi \right) s \frac{\Phi \Phi}{M_*} N_i N_j. \tag{25}
\]

Here \( i, j \) denote the family indices, \( \gamma_{ij} \) are dimensionless coupling constants assumed to be of order unity, and the vevs of \( \Phi, \Phi \) along their right handed neutrino components \( \nu_H^c, \nu_H \) break the gauge symmetry. Eq. (25) yields \( M_R = \gamma_{ij} M_0 \epsilon^{2s} \); with \( M_0 \equiv 2M^2/M_* \) and \( \epsilon \equiv M/M_* \). We assume that \( \Phi, \Phi \) each has unit charge under the discrete symmetry \( Z_n \).

Denoting the \( Z_n \) charge of \( N_i \) by \( q_i \), \( s \) is given by \( q_i + q_j + 2s + 2 = 2m \). Assigning \( q_3 = m-1, q_1 = q_2 = 0 \), the neutrino mass matrices are of the form

\[
M_R \sim M_0 \begin{pmatrix}
\epsilon^{2(m-1)} & \epsilon^{2(m-1)} & 0 \\
\epsilon^{2(m-1)} & \epsilon^{2(m-1)} & 0 \\
0 & 0 & 1
\end{pmatrix}, \tag{26}
\]

for even \( m \), and

\[
M_R \sim M_0 \begin{pmatrix}
\epsilon^{2(m-1)} & \epsilon^{2(m-1)} & \epsilon^{(m-1)} \\
\epsilon^{2(m-1)} & \epsilon^{2(m-1)} & \epsilon^{(m-1)} \\
\epsilon^{(m-1)} & \epsilon^{(m-1)} & 1
\end{pmatrix}, \tag{27}
\]

for odd \( m \), with coefficients of order unity. Denoting the mass eigenvalues as \( \nu_e^c \) with masses \( M_i \), we have \( M_1(2) = \gamma_{1(2)} \epsilon^{2m-2} M_0 \) and \( M_3 = \gamma_3 M_0 \). Here \( \gamma_i \) are coefficients of order unity and we take \( \gamma_1 < \gamma_2 = \gamma_3 = 1 \). Using Eq. (2), we find \( M_2 = 2(M^{-m} \mu^{2m-1})^{1/(m-1)} \) and \( M_3 = 2(M^{m-2} \mu)^{1/(m-1)} \). \( M_2 \) and \( M_3 \) are in the range \( 10^7 \text{–} 10^{12} \) GeV and \( 10^{13} \text{–} 10^{16} \) GeV respectively, for \( 2 \leq m \leq 5 \) (see Fig. 2).

The terms responsible for \( M_R \) also cause the decay of the inflaton. After inflation ends, the system performs damped oscillations about the SUSY vacuum. The oscillating inflaton field is \( \theta = (\delta \nu_H^c + \delta \nu_H^c)/\sqrt{2} \) (where \( \delta \nu_H^c, \delta \nu_H^c \) are the deviations of \( \nu_H^c, \nu_H^c \) from \( M \), with
mass $M_\theta = \sqrt{2m_\mu^2/M_1}$. Since $M_{1,2} \leq M_\theta/2 < M_3$, it decays into $\nu_{1,2}$ via Eq. (25), and the decay width is

$$\Gamma = \frac{1 + y^2}{8\pi} \left( \frac{M_2}{M} \right)^2 M_\theta,$$

where $y \equiv M_1/M_2$. The decay width of $\theta$ to sneutrinos is $(M_2/M_\theta)^2 \Gamma \ll \Gamma$, since $M_2 \ll M_\theta$. (Note that $S$ rapidly settles to its minimum during inflation and so does not oscillate afterward.) For the MSSM spectrum the reheat temperature $T_r$, given by

$$T_r = \left( \frac{45}{2\pi^2 g^*} \right)^{1/4} \sqrt{\Gamma m_P} \simeq \frac{\sqrt{1 + y^2}}{16} \sqrt{\frac{m_P M_\theta}{M}} M_2,$$

ranges from $10^4$ GeV for $m = 2$, to $10^8$ GeV for $m = 3$ and $10^{10}$ GeV for large $m$. Note that $T_r \sim 10^{10}$ GeV may lead to overproduction of gravitinos, although the gravitino constraint on $T_r$ varies, depending on the SUSY breaking mechanism and the gravitino mass $m_3/2$.\footnote{For gravity mediated SUSY breaking models with unstable gravitinos of mass $m_3/2 \simeq 0.1–1$ TeV, $T_r \lesssim 10^7–10^9$ GeV\cite{20}, while $T_r \lesssim 10^{10}$ for stable gravitinos\cite{21}. In gauge mediated models $T_r$ is generally constrained more severely, although $T_r \sim 10^9–10^{10}$ GeV is possible for $m_3/2 \simeq 5–100$ GeV\cite{22}.}

The inflaton decays into the right handed neutrinos $\nu_1^c$ and $\nu_2^c$ with branching ratios $\text{Br}_1 = y^2/(1+y^2)$ and $\text{Br}_2 = 1/(1+y^2)$ respectively. The decay of these neutrinos into lepton and electroweak Higgs superfields creates a lepton asymmetry, which is partially converted into baryon asymmetry via the electroweak sphaleron effects\cite{3}. Using the experimental value of the baryon to photon ratio $\eta_B \simeq 6.1 \times 10^{-10}$\cite{23}, the required lepton asymmetry is found to be $|n_L/s| \simeq 2.5 \times 10^{-10}$\cite{24}.

The lepton asymmetry is given by

$$\frac{n_L}{s} \simeq \sum_{i,j} \frac{3}{2 M_\theta} \text{Br}_i \epsilon_i^{(j)},$$

where $\epsilon_i^{(j)}$ is the contribution from the decay of each $\nu_i^c$ with $\nu_j^c$ in the loop. We assume that $M_1 \gg T_r$ so that washout effects are negligible. We also neglect the small contribution from the decay of the sneutrinos. The $\epsilon_i^{(j)}$ are given by\cite{25}

$$\epsilon_i^{(j)} = \frac{1}{8\pi (h^\dagger h)_{ii}} \text{Im} \left[ \{(h^\dagger h)_{ij}\}^2 \right] f \left( \frac{M_i^2}{M_\theta^2} \right),$$

with

$$f(x) \simeq \frac{\sqrt{x}}{2} \left[ \ln \left( 1 + \frac{1}{x} \right) + \frac{2}{x - 1} \right].$$

\footnote{For gravity mediated SUSY breaking models with unstable gravitinos of mass $m_3/2 \simeq 0.1–1$ TeV, $T_r \lesssim 10^7–10^9$ GeV\cite{20}, while $T_r \lesssim 10^{10}$ for stable gravitinos\cite{21}. In gauge mediated models $T_r$ is generally constrained more severely, although $T_r \sim 10^9–10^{10}$ GeV is possible for $m_3/2 \simeq 5–100$ GeV\cite{22}.}
Assuming hierarchical Dirac masses $M^D_i$, the leptonic mixings in the right handed sector are small, and we obtain \[ \langle h^\dagger h \rangle_{ij} \sim \frac{M^D_i M^D_j}{v^2}, \] giving \[ \epsilon^{(j)}_i \sim \frac{1}{8\pi} f \left( \frac{M^2_j}{M^2_i} \right) \frac{M^D_j}{v^2}, \] where $v = 174$ GeV is the electroweak vev. To estimate the lepton asymmetry we take the Dirac masses to be $M^D_1 \sim 0.1$ GeV, $M^D_2 \sim 1$ GeV, and $M^D_3 \sim 100$ GeV. Here the assumption is that the Dirac masses coincide with the up-type quark masses at tree level, with large radiative corrections to the first two family masses.\footnote{As discussed in Ref. \cite{26}, setting the Dirac masses strictly equal to the up-type quark masses and fitting to the neutrino oscillation parameters generally yields strongly hierarchical $\nu^c$ masses, with $M_1 \sim 10^5$ GeV. The lepton asymmetry in this case is too small by several orders of magnitude.}

The lepton asymmetry calculated with $y = 0.5$ is shown in Fig. 4 where the solid and dashed curves correspond to lower and upper bounds depending on the relative signs of $\epsilon^{(j)}_i$. For $m = 2$, the contribution from $\epsilon^{(2)}_1$ dominates and the lepton asymmetry is too small unless $y \gtrsim 0.998$. For $m = 3$, the contributions from $\epsilon^{(2)}_1$ and $\epsilon^{(3)}_2$ are of the same order, and sufficient lepton asymmetry can be generated with $y \gtrsim 0.5$. Finally, for $m > 3$ the contribution from $\epsilon^{(3)}_2$ dominates, and sufficient lepton asymmetry can be generated as long as $y \gtrsim T_r/M_2 \gtrsim 10^{-2}$. The contribution from $\epsilon^{(3)}_2$ also dominates for $m = 2$, provided we do not assign the $Z_n$ charges as in Eq. \cite{26} but instead assume $M_i = \gamma_i M_0$ where $\gamma_3 \sim 1$ and $\gamma_2 \sim 10^{-4}$. Sufficient lepton asymmetry can then be generated with $T_r \sim 10^6$ GeV and $y \gtrsim T_r/M_2 \gtrsim 10^{-3}$.

\section{V. CONCLUSION}

We have presented a new inflation model based on a $U(1)_R \times Z_n$ symmetry. The inflaton is dynamically localized near the origin prior to new inflation by a stage of preinflation. With double inflation a running spectral index as well as early star formation indicated by the WMAP analysis can be accommodated.

The discrete symmetry $Z_n$ enables one to realize hierarchical right handed neutrino masses without assuming small dimensionless couplings. We show that sufficient lepton asymmetry
to account for the observed baryon asymmetry can be generated for \( m \geq 3 \), where \( m = n \) for odd \( n \) and \( m = n/2 \) for even \( n \). The reheat temperature is found to be \( \sim 10^8 \) GeV for \( m = 3 \) and \( \sim 10^{10} \) GeV for \( m \geq 5 \). For \( m = 2 \), successful leptogenesis can be realized with a lower reheat temperature \( (10^4-10^6 \) GeV), albeit with some small \( (10^{-3}-10^{-4}) \) dimensionless couplings.

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FIG. 1: The spectral index $n_s$ vs. $\beta$, for $m = 2$ (green), $m = 3$ (blue), $m = 4$ (orange) and $m = 5$ (purple).

FIG. 2: Top to bottom: $M_\ast$ (solid), $M_3$ (long-dashed), $M_\theta$ (dashed), $M_2$ (dot-dashed), and $T_r$ (dotted); for $m = 2$ (green), $m = 3$ (blue), $m = 4$ (orange) and $m = 5$ (purple).
FIG. 3: The value of the inflaton field at $k_0 = 0.002 \text{ Mpc}^{-1}$ in units $2M \equiv 1$ (solid curves), and the ratio of the energy scales $\mu/v$ of new inflation and preinflation, where $v \approx 7 \times 10^{15} \text{ GeV}$ [6] (dashed curves); for $m = 2$ (green), $m = 3$ (blue), $m = 4$ (orange) and $m = 5$ (purple).

FIG. 4: The lepton asymmetry $|n_L/s|$ vs. $\beta$, calculated with $y = 0.5$. The horizontal line corresponds to the observed baryon asymmetry. Solid and dashed curves correspond to lower and upper bounds depending on the relative signs of $\epsilon_i^{(j)}$; for $m = 2$ (green), $m = 3$ (blue), $m = 4$ (orange) and $m = 5$ (purple).