Role of spinon and spinon singlet pair excitations on phase transitions in $d$–wave superconductors

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We examine the roles of massless Dirac spinon and spin singlet pair excitations on the phase transition in $d$–wave superconductors. Although the massless spinon excitations in the presence of the spin singlet pair excitations do not alter the nature of the phase transition at $T = 0$, that is, the XY universality class, they are seen to induce an additional attractive interaction potential between vortices, further stabilizing vortex-antivortex pairs at low temperature for lightly doped high $T_c$ samples.

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Recent thermal hall conductivity measurements\(^1\) suggest the existence of vortices in the pseudogap (PG) phase. This implies that preformed pairs are present in the PG phase\(^2\). In the PG phase, vortex-antivortex pairs remain broken to cause a state of globally incoherent but locally coherent Cooper pairs. Vortex induced phase transitions in underdoped regions have been an issue of great interest\(^3-8\). In this paper, by applying a duality transformation we extend the $U(1)$ gauge Lagrangian\(^9\) obtained from the slave-boson mean field theory\(^10,11\) in order to examine how vortex induced phase transitions in $d$–wave superconductors at low energy are affected by the presence of both the massless spinon and spinon singlet pair excitations. In other study\(^7,8\), the flavor number of the massless Dirac fermions without the spinon singlet pair excitations is shown to alter the nature of the phase transition. According to this study\(^7\), as the flavor number increases, the type II superconductivity is preferred showing the second order phase transition which deviates from the XY universality class. In the case of small flavor number leading to the type I superconductivity, it becomes the first order transition owing to the strong fluctuations of the massless fermions. Our present study differs from other previous studies\(^5,8\) in that in our case the $U(1)$ effective gauge field of interest becomes massive as a result of the spinon singlet pair excitations. Particle-hole excitations of the massless Dirac spinons lead to the renormalized kinetic energy of the $U(1)$ gauge field\(^12,13\) (the second term in Eq. [3]). However, we find that the XY universality class is not altered despite the presence of the massless Dirac fermions (spinons). Thus it is irrespective of the flavor number as long as the effective gauge particle (Eq. [5] and Eq. [6]) remains sufficiently massive. The $U(1)$ Berry gauge field can emerge from coupling between the massless Dirac fermions and the vortices\(^4-6,8\), which may affect the XY universality class. However, the Berry gauge field becomes massive owing to the charge fluctuations\(^5\) and as a consequence the effective dual Lagrangian stays robust to maintain the XY universality class. It is shown from the present study that the interaction potentials between vortices are modified to bring an additional attractive interaction (Eq. [7] and Eq. [11]) as a consequence of the massive gauge field.

Our primary focus is to examine how low energy excitations affect the vortex induced phase transition. Here low energy excitations refer to the phase fluctuations of both the spinon singlet pair and the single holon order parameter and the massless Dirac spinon excitations near the $d$–wave nodes of the spinon singlet pair. Gauge field fluctuations are introduced to allow the presence of internal flux responsible for energy lowering. Thus considering proper phase fluctuations (involved with $\phi_{sp} = e^{i\theta_{sp}}$, $\phi_h = e^{i\theta_h}$) for the spinon pairing order field and the single holon order field, we rewrite the low energy effective Lagrangian of Lee\(^9\) in compact form,

$$L = \frac{K_{b,\mu}}{2} |\partial_\mu \theta_b + a_\mu - A_\mu|^2 + \frac{K_{sp,\mu}}{2} |\partial_\mu \theta_{sp} + 2a_\mu|^2 + \psi_1^{|[\partial_x + v_F^2 \tau^3 i\partial_y + v_\Delta \tau^1 i\partial_y]} \psi_1 + (1 \to 2, x \to y) + iJ_{f\mu}(\partial_\mu \psi_{sp} + 2a_\mu) + i\bar{\rho}_{sp}(\partial_\mu \theta_{sp} - 2\partial_\tau \theta_b + 2A_0), \quad (1)$$

where $K_{b,\mu} \equiv (1/u_b, K_b, K_b)$ with $1/u_b$ ($\sim 1/t$), the compressibility and $K_b$ ($\sim 2\chi_0\delta$), the phase stiffness of the single holon field and $K_{sp,\mu} \equiv (1/u_{sp}, K_{sp}, K_{sp})$ with $1/u_{sp}$ ($\sim 1/J$), the compressibility and $K_{sp}$ ($\sim J\Delta^3_0$), the phase stiffness of the spinon pair order field. $\psi_{n\sigma} = \left( e^{-i\theta_{sp}/2} f_{n\sigma}, e^{-i\theta_{sp}/2} f_{n\sigma} \right)$ is the renormalized Nambu spinor associated with the $d$–wave gap nodes $n$. $v_F$ ($\sim J\chi_0$) and $v_\Delta$ ($\sim J\Delta^3_0$) are the fermi and gap velocities of the Dirac spinons respectively. $J_{f\mu} = \frac{1}{2}(\sum_{\sigma} \psi_1^{|[\tau^3 i\sigma, i\sigma, \tau^3 i\sigma, i\sigma, \tau^3 i\sigma, i\sigma]} \psi_{1\sigma})$ is the three current of the spinon quasiparticles and $\bar{\rho}_{sp}$, the average density of spinon pairs.

By introducing the unitary gauge transformation $\tilde{a}_\mu = 2a_\mu + \partial_\mu \theta_{sp}$, we rewrite Eq. [1]

$$L = \frac{K_{b,\mu}}{8} |\partial_\mu \theta_b - \tilde{a}_\mu + 2A_\mu|^2 + \frac{K_{sp,\mu}}{2} \tilde{a}_\mu^2 + iJ_{f\mu} \tilde{a}_\mu + \psi_1^{|[\partial_x + v_F^2 \tau^3 i\partial_y + v_\Delta \tau^1 i\partial_y]} \psi_1 + (1 \to 2, x \to y) + i\bar{\rho}_{sp}(\partial_\mu \theta_{sp} - 2A_0), \quad (2)$$
where $\theta_p = \theta_{sp} - 2\theta_h$, $\theta_p$ is the phase of the Cooper pair order parameter $\Delta_{\text{cooper}}(k) = |\Delta_{\text{cooper}}(k)| \phi_p$ where $\Delta_{\text{cooper}}(k) = <c_{k \uparrow} c_{-k \downarrow} > + <b_{k \uparrow} b_{-k \downarrow} >$ of the effective gauge field $\tilde{\mu}$. The mass of the effective gauge field $\tilde{\mu}$ is defined by the phase stiffness $K_{sp}$ of the spinon pairing order parameter associated with the PG phase of the doped Mott insulator. In the above equation, the fluctuating fields of $\theta_p$, $\tilde{\mu}$, and $\psi_n$, are U(1) gauge invariant, thus satisfying the Elitzur’s theorem. Integrating over the Nambu spinor fields and expanding the resulting logarithmic term up to second order in $\tilde{\mu}$, we get an effective U(1) Lagrangian associated with the massive gauge field $\tilde{\mu}$, 

$$Z = \int D\psi D\tilde{\mu} e^{-\int d^2x L},$$

$$\mathcal{L} = \frac{K_{b,\mu}}{2} |(\psi \partial \psi - \tilde{\mu} + 2A_\mu)|^2 + \frac{N}{16} (\partial \times \tilde{\mu}) \cdot (\tilde{\psi} \tilde{\psi} + \tilde{\mu}^2 + 2A_\mu) + i\tilde{\nu}_{sp} (\partial \psi + 2A_0),$$

where $K_{b,\mu} = K_{b,\mu}/4$ and $m_{\mu}^2 = K_{sp,\mu}$. $N$ is the number of flavors (i.e., the number of nodal points) of the Dirac fermions. The kinetic energy term (the second term) of the effective gauge field $\tilde{\mu}$ arises as a result of the massless excitations of the spinon quasiparticles (Dirac fermions). The Berry phase contribution $i\tilde{\nu}_{sp} (\partial \psi + 2A_0)$ is related to the Cooper pair boson density. If we ignore the spinon quasiparticles and thus consider only the $N = 0$ limit (which corresponds to the isotropic $s$-wave superconductivity), it is obvious that the kinetic energy term of the gauge field disappears. Integrating over the effective gauge field $\tilde{\mu}$ in Eq. [3], we obtain

$$\mathcal{L} = \frac{K_{p,\mu}}{2} |(\partial \psi + 2A_\mu)|^2 + i\tilde{\nu}_{sp} (\partial \psi + 2A_0),$$

where $K_{p,\mu} = \frac{K_{b,\mu} K_{p,sp}}{K_{b,\mu} + K_{p,sp}}$. This expression shows that the phase stiffness $K_{p,\mu}$ of the Cooper pair field is in a reduced “mass” form and the usual logarithmic type of interaction between vortices arises.

The duality transformation\textsuperscript{[5,15–17]} of Eq. [3] with the introduction of vortex mass and self interaction terms leads to an effective Lagrangian for the vortex field, 

$$Z = \int D\psi D\tilde{\mu} e^{-\int d^2x L},$$

$$\mathcal{L} = \frac{1}{2} |(\partial \psi + 2A_\mu)|^2 + i\tilde{\nu}_{sp} (\partial \psi + 2A_0) + \frac{N}{16} (\partial \times \tilde{\mu}) \cdot (\tilde{\psi} \tilde{\psi} + \tilde{\mu}^2 + 2A_\mu),$$

The fifth term represents an additional kinetic energy of the dual gauge field resulting from coupling of the massless Dirac spinon field to the Cooper pair field via the massive effective gauge field $\tilde{\mu}$. Not considering the external magnetic field $\mu$, despite the contribution of the massless Dirac fermions (as shown in the fifth term) the nature of the XY universality class will not be affected as long as the mass of the gauge field, that is, the phase stiffness $K_{sp}$ of the spinon singlet pair order parameter and the original $U(1)$ gauge field $\tilde{\mu}$, is substantially large; the lower temperature, the larger $K_{sp}$ in the PG phase.

By considering a static case of the vortex field we calculate the dual gauge field propagator to obtain the interaction potential between vortices. The dual field propagator associated with the fourth and fifth terms in Eq. [6] is obtained,

$$P_{ij}(q) = P(q) \left( \delta_{ij} - \frac{q_i q_j}{q^2} \right),$$

$$P(q) = \frac{K_b + \frac{8}{N} m_a^2}{q^2 + \frac{8}{N} m_a^2 + \frac{4}{N} K_b},$$

where $K_b = \frac{K_{sp} + 4K_p}{K_{sp} + 4K_p}$, $M = \sqrt{\frac{8}{N} (K_{sp} + K_b/4)}$ and $z_j = \frac{K_b}{K_{sp} + 4K_p}$, $i, j = 1, 2$ denotes the space component index $(x, y)$. In the real space, the above equations lead to the total interaction potential between a vortex and an antivortex,
where $Y_0(x)$ is the zeroth order Bessel function of the second kind and Struve $H_0(x)$ is the zeroth order Struve function. The additional interaction between vortices shows a power law decay at large separations between vortices while it is logarithmic in nature at short separations. In Fig. 1, we plot this interaction potential as a function of distance in the underdoped region. The correction term yields a large modification to the logarithmic behavior of the Cooper pair field. $K_p$ is linearly dependent on $\delta$ particularly in the lightly doped region. On the other hand, the strength of the additional interaction shows a nonlinear (quadratic) dependence of $\delta$ owing to the linear doping dependence of $K_b(\sim \delta)$ and $z_J(\sim \delta)$ in the lightly doped region.

To grasp the origin of the additional attractive interaction between vortices in a different angle, we now take a different procedure. Integrating over the effective gauge field $\tilde{a}_\mu$ first in Eq. [2], we obtain the effective Lagrangian of the U(1) gauge invariant particles containing the Doppler energy shift term, $E = \int J_{\mu} a_\mu + J_{\mu} a_\mu - 2A_\mu$.

$$Z = \int D\psi D\theta e^{-\int \theta_{0} d\tau \int dx^2 \mathcal{L}},$$

$$\mathcal{L} = \frac{K_p}{2} |\partial_\mu \theta_{0} - 2A_\mu|^2 - iz_{J} J_{\mu} (\partial_\mu \theta_{0} - 2A_\mu) + \psi^\dagger \left( i \frac{\partial_\tau}{2} + \frac{J_{\mu}}{2} \right) \psi,$$

$$+ \frac{J^2_{\mu}}{2K_{b,\mu} + K_{\text{spin},\mu}},$$

with $K_{p,\mu} = \frac{K_{b,\mu} K_{p,\mu}}{K_{b,\mu} + K_{\text{spin},\mu}} \equiv (1/u_{\mu}, K_{p}, K_{b})$, the phase stiffness of the Cooper pair order parameters and $z_{J} = \frac{K_{b,\mu}}{K_{b,\mu} + K_{\text{spin},\mu}} \equiv (z_{b, J}, J_{J}, J_{J})$, the effective charge of the spinon quasiparticles. The above Lagrangian is the low energy Lagrangian of the $d$-wave BCS theory with the doping dependent phase stiffness, $K_{p} (\sim \delta)$ and the effective charge, $z_{J} (\sim \delta)$.

To see the effects of the Dirac fermions on the phase fluctuations of the Cooper pair fields, we integrate over the Dirac fermion fields ignoring the local interactions of the Dirac fermions and the temporal fluctuations in Eq. [10] to find

$$\mathcal{L} = \frac{K_p}{2} |\partial_\mu \theta_{0} - 2A_\mu|^2 + \frac{N}{16} \frac{z_{J}^2}{\xi_{J}^2} (\nabla \times \nabla \theta_{0} - 2\nabla \times A) \times \frac{1}{\sqrt{-\xi_{J}^2}} (\nabla \times \nabla \theta_{0} - 2\nabla \times A).$$

The additional attractive interaction between vortices leads to the second term in Eq. [9] as a result of the supercurrent affected by the massless Dirac fermions owing to the Doppler shift term $\partial_{\mu} \theta_{0} - 2A_\mu$. For a brief guidance we list differences in interaction potentials between vortices in the $s$-wave and the $d$-wave superconductors in Table 1 and a comparison of the $d$-wave BCS theory and the present theory in Table 2. We note that in the limit of $q << M^2$ in Eq. [7] the interaction energy between vortices obtained by the $d$-wave BCS formalism is the same as that obtained by our theory. This limit corresponds to the case in which the local interactions of the massless Dirac fermions are ignored in Eq. [10]. The additional interactions may affect the vortex lattice structure. We believe that because our vortex interaction terms reveal the hole doping dependence, it will be of great interest in the future to examine how the vortex dynamics and lattice structure vary with hole concentration.

We noted that the phase transition in underdoped cuprates falls into the XY universality class owing to the presence of the massive gauge field $\tilde{a}_\mu$ which results from the spinon singlet pair excitations. In addition, we showed that the massless Dirac fermions coupled to the phase fluctuations of the spinon singlet pairs (leading to the Cooper pairs) result in the additional attractive interaction of a logarithmic behavior, $\frac{1}{2} K_{b, J} |x - x'|$ at short distances and of a power law behavior, $\frac{1}{2} K_{b, J} |x - x'|^{-1}$ at large distances. The attractive interaction enhances binding of vortex-antivortex pairs at low temperature, thus causing enhanced stability of the superconducting phase. The present study has been made based on the U(1) slave-boson theory concerned with the single holon order parameter. Thus it will be of great interest to apply our recent SU(2) theory involved with holon-pair boson order parameter.

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1. H. Aubin, K. Behnia, S. Ooi, T. Tamegai, K. Krishana, N. P. Ong, Q. Li, G. Gu, and N. Koshizuka, Science 280, 11
After integrating over the dual gauge field 

\[ V(q) = K_\theta \frac{q}{\sqrt{q^2 + M^2}} \]

we obtain the total interaction energy between vortices,

\[ \frac{1}{2} \left( J_{\mu \nu} - 2 \partial \times A \right)_{ij} P_{\mu \nu} \left( J_{\rho \sigma} - 2 \partial \times A \right)_{kl}, \]

where \( J_{\mu \nu} = -i (\psi_{\mu}^\dagger \partial_{\nu} \psi_{\nu} - \psi_{\nu}^\dagger \partial_{\mu} \psi_{\mu}) \) is the vortex current and \( P_{\mu \nu} \), the renormalized propagator of the dual gauge field (Eq. [7]). The first term \( \frac{K_\theta}{q} \) in Eq. [7] is the same as that obtained by the first term in Eq. [11] and the second term \( \frac{K_\phi}{q} \) is equal to the second term in Eq. [11] in the limit of \( q << M \) (see the text). Since we neglect the local interaction between Dirac fermions in Eq. [11], the second (logarithmic) term in Eq. [8] is not reproduced.

Fig. 1 The total interaction energy (solid line), logarithmic interaction energy (dashed line) and additional interaction energy (dotted line) (in the unit of t) at underdoping \( \delta \sim 0.035 \) as a function of vortex distance \( d \) (in the unit of \( t^{-1} \)).

Table 1 Comparison of the interaction potential between vortices in \( s-wave \) and \( d-wave \) superconductors

Table 2 Comparison of the interaction potential between vortices in the \( d-wave \) BCS formalism and in the present gauge theory formalism

| Table I. |
|---------------------------------|
| \( s-wave \) superconductors |
| \( L = \frac{K_\theta}{2} |\partial \theta| \) |
| \( L_{\text{dual}} = \frac{1}{2kp} [\partial \times c]^2 + ic J_v (J_v \equiv \partial \times \partial \theta) \) |
| \( V(q) = K_\theta q \) |
| \( V(x) = K_\theta \ln |x| \) |

| \( d-wave \) superconductors |
| \( L = \frac{K_\phi}{2} (\partial \theta)^2 - izJ_r \partial \theta + \psi \gamma \partial \psi \) |
| \( L_{\text{dual}} = \frac{1}{2kp} [\partial \times c]^2 + ic J_v + \frac{Nc^2}{M} J_v \sqrt{-\partial^2} \) |
| \( V(q) = K_\phi q^2 + \frac{Nc^2}{M} q^2 \psi \) |
| \( V(x) = K_\phi |x| - \frac{Nc^2}{M} \frac{1}{x} \) |
| $d$ - wave BCS theory | Our theory |
|------------------------|------------|
| $L = \frac{K_p}{2} \left| \partial \theta_p \right|^2 - iz J_f \partial \theta_p + \bar{\psi}_l \gamma \partial \psi_l$ | $L = \frac{\tilde{K}_b}{2} \left| \partial \theta_p - \tilde{a} \right|^2 + \frac{N}{16} (\partial \times \tilde{a}) \frac{1}{\sqrt{-\partial^2}} (\partial \times \tilde{a}) + \frac{1}{2} m_a^2 \tilde{a}^2$ |
| $L_{dual} = \frac{K_p}{2} \left| \partial \times c \right|^2 + ic J_V + \frac{N}{16} z^2 J_V \frac{1}{\sqrt{-\partial^2}} J_V$ ($J_V \equiv \partial \times \partial \theta_p$) | $L_{dual} = \frac{\tilde{K}_b}{2} \left| \partial \times c \right|^2 + ic J_V + \frac{1}{2} (\partial \times c) \frac{1}{\sqrt{-\partial^2 + m_a^2}} (\partial \times c)$ |
| $V(q) = \frac{K_p}{q} + \frac{N z^2}{16}$ | $V(q) = \frac{K_p}{q} + \tilde{K}_b z J \frac{1}{\sqrt{q(q + m_a^2)}}$ |
| $V(x) = K_p \ln |x| - \frac{N z^2}{16} |x|$ for $q << M^2$ | $V(x) = K_p \ln |x| - \frac{\tilde{K}_b z J}{4(q + m_a^2)}$ for $q >> M^2$ |