Achieving High-Fidelity Single-Qubit Gates in a Strongly Driven Charge Qubit with $1/f$ Charge Noise

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Charge qubits formed in double quantum dots represent quintessential two-level systems that enjoy both ease of control and efficient readout. Unfortunately, charge noise can cause rapid decoherence, with typical single-qubit gate fidelities falling below 90%. Here, we develop analytical methods to study the evolution of strongly driven charge qubits, for general and $1/f$ charge-noise spectra. We show that special pulsed techniques can simultaneously suppress errors due to strong driving and charge noise, yielding single-qubit gates with fidelities above 99.9%. These results demonstrate that quantum dot charge qubits provide a potential route to high-fidelity quantum computation.

Building high-quality qubits is a key objective in quantum information processing. Achieving high-fidelity gates requires both precise control and effective measures to combat decoherence arising from the environment. Semiconductor based quantum dot charge qubits, for example, suffer from strong coupling to charge noise that causes voltage fluctuations on the control electrodes, which has so far limited gate fidelities to below 90%. To be suitable for scalable quantum computation, the fidelity must be increased to at least 99.9%.

One strategy for achieving higher fidelities is to operate the qubits as fast as possible, for example, by driving the qubits with strong microwaves. However this can potentially cause strong-driving effects, including Bloch-Siegert shifts of the resonant frequency and fast oscillations superimposed on top of Rabi oscillations. While Bloch-Siegert shifts can be accommodated by adjusting the driving frequency or gate time, fast oscillations may be difficult to control, resulting in gate errors. There are several known approaches for mitigating control errors, including pulse-shaping methods that suppress oscillations by engineering the pulse envelopes. While such schemes can be applied to charge qubits, we explore a different method here, based on rectangular pulse envelopes engineered to produce nodes in the fast oscillations at the end of a gate. In principle, the scheme explored here can also be combined with pulse-shaping methods, although this lies outside the scope of the present work.

The paper is organized as follows. We first explain how to synchronize the fast oscillations and Rabi oscillations in a strongly driven charge qubit, in the absence of noise. We then introduce charge noise, focusing on a $1/f$ noise spectrum, and characterize the qubit dynamics both numerically and analytically. Finally, we demonstrate that gates with fidelities higher than 99.9% can be achieved, for typical charge noise magnitudes.

Noise-free evolution. The basis states of a double quantum dot charge qubit, $|L\rangle$ and $|R\rangle$, represent the localized positions of an excess charge in the left or right dot, as indicated in Fig. 1(a). We consider ac gating of a single qubit, with the Hamiltonian $H_{\text{sys}}=H_q+H_{\text{ac}}$, where $H_q=-\varepsilon/2\sigma_z-\Delta\sigma_z$, the $\sigma_i$ are Pauli matrices, $\varepsilon$ is the detuning parameter (defined as the energy difference between the two dots), and $\Delta$ is the tunnel coupling between the dots. Here we have expressed $H_{\text{sys}}$ in the eigenbasis $|0\rangle=(|L\rangle-|R\rangle)/\sqrt{2}, |1\rangle=(|L\rangle+|R\rangle)/\sqrt{2}$, corresponding to the charge qubit “sweet spot” $\varepsilon=0$, where it is first-order insensitive to electrical noise. Unless otherwise noted, we assume that the nominal operating point is $\varepsilon=0$ throughout the remainder of this work. When a microwave signal is applied to $\varepsilon$, the driving Hamiltonian is given by $H_{\text{ac}}=(A_c/2)\sigma_x \sin(\omega d+\phi)$, where $A_c$ is the driving amplitude, $\omega_d$ is its angular frequency, and $\phi$ is the phase at time $t=0$, when the drive is initiated.

First, we follow Ref. [8] and obtain exact solutions for strongly-driven qubits in the absence of noise, up to arbitrary order in the strong-driving parameter $\gamma=A_c/(16\Delta)$. Expanding the time-evolution operator order-by-order as $U_0(t)=\sum_{n=0}^{\infty} \frac{\gamma^n}{n!} U_0^{(n)}$, we obtain

$$U_0^{(0)}(t) = \begin{pmatrix} e^{i\tilde{\omega}_{\text{res}} t/2} \cos(\Omega t/2) & -e^{i\phi} \sin(\Omega t/2) \\ -e^{-i\phi} \sin(\Omega t/2) & e^{i\tilde{\omega}_{\text{res}} t/2} \cos(\Omega t/2) \end{pmatrix},$$

with higher-order terms given in [12]. Here, we consider only resonant driving, $\omega_d=\tilde{\omega}_{\text{res}}$, where $\tilde{\omega}_{\text{res}}=2\Delta(1+4\gamma^2)$ is the renormalized resonant angular frequency, including Bloch-Siegert corrections, and $\Omega=\gamma/(1+\gamma^2)/2$ is the renormalized Rabi angular frequency.

In the rotating frame defined by $H_{\text{rot}}=U_{\text{rot}}^\dagger H_{\text{sys}} U_{\text{rot}} - i\hbar \frac{d}{dt} U_{\text{rot}}$, with $U_{\text{rot}}=\text{diag}[e^{i\omega t/2}, e^{-i\omega t/2}]$, the ideal evolution term $U_0^{(0)}$ generates smooth, sinusoidal, Rabi oscillations, corresponding to rotations about the $(\cos \phi, -\sin \phi, 0)$ axis of the Bloch sphere. The $U_0^{(n)}$ term represents the dominant fast oscillations associated with strong driving, with amplitude $\gamma$. Both drive components can be observed in the top panel of Fig. 1(b), where we show that our analytical results (shown here up to $O(\gamma^2)$) agree well with the results of numerical simulations of the full Hamiltonian.

Fast oscillations can cause gate infidelity. For example, if we consider an $R_\theta(\phi)$ rotation of angle $\theta$ about the
FIG. 1. (a) Energy level diagram of a charge qubit. The insets depict a double quantum dot in three regimes of detuning, $\varepsilon$. (A potential charge noise fluctuation is shown as a dashed line.) Here, filled circles indicate the position of the excess electron in the ground state, and the barrier between the dots induces a tunnel coupling, $\Delta$. (b), (c) Time evolutions of the density matrix element $\rho_{00}$ (brackets $\langle \cdot \rangle$ denote a noise average), including numerical simulations (solid lines), analytical calculations (dashed lines), and their differences (dotted lines). The evolution in (b) corresponds to the laboratory frame, while (c) corresponds to the interaction frame. In all cases, we use $\{\varepsilon, \Delta, A_c\}/\hbar = \{0, 5, 4\}$ GHz, $\phi = \pi/4$ and initial state $|0\rangle$. The charge noise follows the $1/f$ spectrum of Eq. (S90), with frequency cutoffs $\omega_c/2\pi = 0.193$ MHz and $\omega_c/2\pi = 80.8$ GHz, and noise amplitudes $c_\varepsilon$ as indicated. The insets of (b) show blow-ups of the evolution near the end of a $\pi$-rotation ($t = t_\pi$), decomposed into their Rabi (dark purple) and fast-oscillation components (red). The oscillations are synchronized at $t_\pi$ when $N = (2\theta\omega_{\text{res}})/\pi\Omega$ is an even integer, resulting in high-fidelity gates. (The main panels also use $N = 10$.) Charge noise causes a slight decay of $\langle \rho_{00} \rangle$ at the end of the simulation period ($2t_\pi \approx 0.5$ ns), which can be observed more clearly at long times in (c). Here, the inset shows a short-time blow-up of the interaction-frame simulations (solid lines), our full analytical calculations obtained from Eq. (518) (dashed white line), and the simple asymptotic expression from Eq. (5) (dashed cyan line) with corrections to $K_\varphi$ up to $O[\gamma^3]$ and corrections to $K_M$ and $K_{\text{ interaction}}$ up to $O[\gamma^2]$, as discussed in [12].

$(\cos \phi, -\sin \phi, 0)$ axis, the fast oscillations may prevent the density matrix element $\rho_{00}$ from reaching 0 at the end of a gate period, $t_\pi$. We see this more clearly by plotting the fast and Rabi oscillation components separately in the top inset of Fig. [1]b. On the other hand, we may adjust the pulse parameters $A_c$ and $\phi$ to synchronize the fast oscillations with the slower Rabi oscillations, as shown in the bottom inset, to obtain an $R_{\theta}(\phi)$ gate with much higher fidelity.

We characterize the infidelity arising from the fast oscillations by computing the process fidelity $F_\theta(\phi)$, defined by comparing the ideal evolution operator $U_0^{(0)}$, to the actual evolution $U_\infty$, for $R_{\theta}(\phi)$ in the rotating frame. In [12], we show that specific $A_c$’s give rotations with perfect fidelity when $\theta = \pi, 2\pi, 3\pi, \ldots$ for any $\phi$. More importantly, when $\phi = \pi/4, 3\pi/4, 5\pi/4, \ldots$, we obtain

$$1 - F_\theta(\phi) = 2\gamma^2[1 - \cos(2\theta\omega_{\text{res}}/\Omega)] + 4\gamma^3 \sin \theta \sin(2\theta\omega_{\text{res}}/\Omega) + O[\gamma^4],$$

where, again, $\gamma = A_c/(16\Delta)$. For these values of $\phi$, the infidelity due to strong-driving errors is bounded above by $\sim 4\gamma^2$. Moreover, the oscillations are synchronized, yielding perfect fidelity (up to $O[\gamma^4]$), when $2\theta\omega_{\text{res}}/\Omega = N\pi$, with $N$ an even integer. Since this condition can be met for a continuous range of $\theta$ by adjusting $\Omega$ (i.e., $A_c$), and since $\phi = \pi/4$ and $3\pi/4$ represent orthogonal rotation axes, the rotations $\{R_{\theta}(\pi/4), R_{\theta}(3\pi/4)\}$ therefore generate a complete set of high-fidelity single-qubit gates. Additional phase control is provided by adjusting the waiting time between ac pulses. Unless otherwise noted, we set $\phi = \pi/4$ for the remainder of our analysis.

**Modeling charge noise.** We introduce charge noise into our analysis through the Hamiltonian term $H_\infty = h_n\delta \varepsilon(t)$, where $\delta \varepsilon(t)$ is a random variable affecting the detuning parameter and $h_n = -\sigma_x/2$ is referred to as the noise matrix. The noise is characterized in terms of its time correlation function $S(t_1 - t_2) = \langle \delta \varepsilon(t_1) \delta \varepsilon(t_2) \rangle$, where the brackets denote an average over noise realizations, and the corresponding noise power spectrum is $S(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t}S(t)$ [15]. Although we obtain analytical solutions for generic noise spectra in [12], below we focus on $1/f$ noise, including in our simulations, due to its relevance for charge noise in semiconducting devices [16] [17]:

$$\tilde{S}(\omega) = \begin{cases} \frac{c_\varepsilon^2}{\omega_\varepsilon^2} & (\omega \leq |\omega_\varepsilon|) \\ 0 & (\text{otherwise}) \end{cases},$$

where $c_\varepsilon$ is related to the standard deviation of the detuning noise, $\sigma_\varepsilon$, via $\sigma_\varepsilon = c_\varepsilon[2\ln(\sqrt{2\pi}c_\varepsilon/\omega_\varepsilon)]^{1/2}$ [18] [19], and $\omega_\varepsilon$ (\omega_\varepsilon) are low (high) cutoff angular frequencies.

We now present numerical simulations of a strongly driven, noisy charge qubit by averaging the density matrix $\langle \rho(t) \rangle$ over noise realizations. Time sequences for
\[ \langle \rho^I(t) \rangle = \exp \left\{ -\frac{1}{\hbar^2} \int_0^t dt_1 \int_0^{t_1} dt_2 \mathcal{L}(t_1)\mathcal{L}(t_2)S(t_1 - t_2) \right\} \rho^I(0), \]

where we have assumed that the noise is stationary, with zero mean \((\langle \delta \varepsilon \rangle = 0)\). We can rewrite Eq. (S18) in the form \(r^I(t) = \exp[K(t)]r^I(0)\) by defining \(\langle \rho^I \rangle = 1/2(I_2 + r^I_x \sigma_x + r^I_y \sigma_y + r^I_z \sigma_z)\). Here, \(I_2\) the \(2 \times 2\) identity matrix, \(r^I = (r^I_x, r^I_y, r^I_z)\) is the Bloch vector, and \(K(t)\) is a \(3 \times 3\) evolution matrix, to be determined. Since \(h^I(\delta \varepsilon)\) can be expanded into Fourier components, Eq. (S18) is solved by evaluating simple integrals of the form \(I(t, \omega_1, \omega_2) \equiv \int_0^t dt_1 \int_0^{t_1} dt_2 e^{i\omega_1 t_1} e^{i\omega_2 t_2} S(t_1 - t_2)\). Details of the calculations are provided in [12], while the main results are discussed below. The accuracy of our cumulant approach is evident in Figs. 1(b) and 1(c), where the theoretical results (dashed white lines) are seen to capture all the fine structure of the simulations. Indeed, the bottom panel of Fig. 1(b) indicates that the analytical and numerical solutions differ by \(<10^{-3}\) over the entire range plotted.

**Results.** The physics of noise-averaged qubit dynamics is encoded in \(K(t)\), which can be decomposed into a sum of Markovian terms \(K_{M}\), and non-Markovian terms. The latter may be further divided into pure-dephasing terms \(K_{\varphi}\) and non-Markovian-non-dephasing terms \(K_{nMn}\). Pure-dephasing terms are conventionally associated with the integral \(I(t, \omega_1 = 0, \omega_2 = 0) \sim t^2 \ln(1/\omega t)\) [23,31,32]. However, since \(K\) is defined in a rotating frame, “pure dephasing” has a different meaning than in the laboratory frame [23,25]: here, the leading order contributions to \(K_{\varphi}\) are proportional to \(\gamma^2\), and are therefore attributed to strong driving. Markovian terms are associated with the integral \(\text{Re}[I(t, \omega, -\omega)]\), corresponding to short correlation times [12], and exponential decay \((\sim e^{-T_1})\). The dominant non-Markovian-non-dephasing terms are associated with the integral \(\text{Im}[I(t, \omega, -\omega)]\), yielding slow oscillations in the rotating frame, as well as the fast oscillations in the inset of Fig. 1(c). Since it is common in the literature to treat the dephasing and depolarizing channels separately [25], it is significant that our method encompasses both phenomena (and other behavior, including \(K_{nMn}\)) within a common framework, allowing us to compare and contrast their effects.

We can compute the asymptotic behavior of \(r^I(t)\) analytically, using the cumulant expansion [22,33], truncated at \(O((\delta \varepsilon/\hbar \Omega)^2)\):

\[ r^I(t) = e^{-\Gamma_{t\epsilon}/\sqrt{2}} \left[ \sin(\Gamma_{nMn} t), \sin(\Gamma_{nMn} t), \sqrt{2} \cos(\Gamma_{nMn} t) \right], \]

where \(\delta \varepsilon(t)\) are obtained by generating a white noise sequence, then scaling its Fourier transform by an appropriate spectral function [12,20], such as Eq. (S90). A typical result is shown in the middle panel of Fig. 1(b), where the suppression of Rabi oscillations is a direct consequence of the charge noise. To differentiate the effects of decoherence from those arising from strong driving, we present the same results in an interaction frame defined by \(U_0\), \(\rho^I = U_0^\dagger \rho U_0\), in which the fast oscillations due to strong driving are not observed. Figure 1(c) shows the resulting long-time decay of the density matrix, while the inset shows the short-time behavior on an expanded scale. Note that the fast oscillations observed here do not arise directly from strong driving, but rather from non-Markovian noise terms, as discussed below.

**Analytical solutions, with charge noise.** Several theoretical techniques have been applied to noisy, driven two-level systems, including master equations [19,21–29], dissipative Lander-Zener-Stückelberg interferometry [27,28], and treatments of spin-Boson systems [29–31]. Here, we solve the dynamical equation \(i\hbar d\rho^I/dt = \delta \varepsilon(t)\mathcal{L}\rho^I\) in the interaction frame, where \(\mathcal{L}\rho^I \equiv [h^I_n, \rho^I]\) and \(h^I_n = U_0^\dagger h_n U_0\). We then average over the noise via a cumulant expansion [22,33], truncated at \(O((\delta \varepsilon/\hbar \Omega)^2)\):
Infidelity, $1-F$

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\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$A_\varepsilon$ (μeV) & 0.0 μeV & 0.5 μeV & 1.0 μeV \\
\hline
\end{tabular}
\end{table}
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FIG. 3. Dependence of the infidelity $1-F_\varepsilon(\pi/4)$ of a strongly driven charge qubit on (a) the driving amplitude $A_\varepsilon$, with $\Delta/h=5$ GHz, and on (b) the tunnel coupling $\Delta$, with $N=(28\omega_{\text{res}})/\pi=10, 12, 14, 16$ held constant. Here, $\varepsilon=0$, and the charge noise is given by Eq. (2), with $\omega_1/2\pi=1$ Hz, $\omega_\varepsilon/2\pi=256$ GHz, and the values of $c_\varepsilon$ are indicated. Note that low frequencies ($<0.3$ GHz) are approximated as quasistatic in these simulations, while the analytical results are obtained from Eq. (5) as $O[\gamma^4]$ (or $O[\gamma^3]$ for $c_\varepsilon=0$ [12]). In (b), the infidelities are computed at the “dips” indicated in (a).

for the initial state $|0\rangle$. Here, the decoherence is dominated by the integral $I(t,\omega, -\omega) \approx \hat{S}(\omega)/2 + \hat{S}_{\text{imag}}(\omega)t$, whose imaginary part is given by $\hat{S}_{\text{imag}}(\omega) = c_\varepsilon^2(2i/\omega)|\omega/\omega_1|$. The real part describes exponential decay, giving the Markovian decoherence rate for driven evolution, $\Gamma_{\text{Markov}} = (1/16\hbar^2) \{4\hat{S}(\omega_{\text{res}})+\hat{S}(\omega_{\text{res}}+\Omega)+\hat{S}(\omega_{\text{res}}-\Omega)\}$ as observed in Fig. 1(c), which can also be derived from Bloch-Redfield theory [25, 26]. The imaginary part corresponds to a non-Markovian-non-dephasing noise-induced rotation with frequency $\Gamma_{\text{non-Markov}} = (-i/8\hbar^2) \{\hat{S}_{\text{imag}}(-\omega_{\text{res}}+\Omega)+\hat{S}_{\text{imag}}(\omega_{\text{res}}+\Omega)\}$, originating from the integrated low-frequency (“quasistatic”) portion of the noise spectrum, $\int_{-\infty}^{\infty} d\omega' S(\omega')/\pi|^{1/2} = c_\varepsilon^2(2\ln(\omega/\omega_1))^{1/2}$. Additional high-order results are presented in [12].

We can define the figure of merit (FOM), $J_{\text{Rabi}}/\Gamma_{\text{Rabi}}$, which corresponds to the number of coherent $R_\pi/\pi$ rotations (not including strong-driving control errors), where the Rabi decay rate $\Gamma_{\text{Rabi}}$ is determined from Eq. (5) as $\langle \rho_{00} \rangle (t=1/\Gamma_{\text{Rabi}}) = (1+e^{-t})/2$. By exploring a range of control parameters in Fig. 2 we first confirm that the FOM is strongly enhanced at the sweet spot $\varepsilon=0$ (lower inset). Increasing the tunnel coupling $\Delta$ and the driving amplitude $A_\varepsilon$ both enhance the FOM, as shown in the main panel. By increasing $\Delta$ and $A_\varepsilon$ simultaneously, as shown in the upper inset, we find that the FOM can exceed 700 for a physically realistic charge noise amplitude of $c_\varepsilon=0.5 \mu eV$ ($\sigma_\varepsilon=3.12 \mu eV$) [11].

For the noise levels considered in Fig. 3(a), which are consistent with recent experiments [11] [10] [11], we obtain $F_{\text{max}}<99\%$, which is insufficient for achieving high-fidelity gates. However, the following procedure can be used to suppress both control errors and decoherence. First, $A_\varepsilon$ is tuned to a dip. Then, $\Delta$ and $A_\varepsilon$ are simultaneously increased while holding $\gamma = A_\varepsilon/10\Delta$ (and thus $N$) fixed. In this way, we remain in a dip, while increasing the gate speed to suppress noise effects. The results are shown in Fig. 3(b). Here, when $c_\varepsilon=1 \mu eV$ ($\sigma_\varepsilon=6.36 \mu eV$), we obtain fidelities $>99\%$ when $\Delta>40 \mu eV$, and $>99.9\%$ when $\Delta>120 \mu eV$. The corresponding qubit frequencies, $2\Delta/h=29.3$ GHz and 58.0 GHz, are comparable to the qubit frequency of the quantum dot spin qubit in Ref. [26], and the Rabi frequencies $\approx 4\Delta/hN$ are generally lower.

Conclusions. Based on numerical and analytical calculations with $1/f$ detuning noise, we demonstrated that high-fidelity gate operations can be achieved in charge qubits employing strong-driving methods. We identified rotations that synchronize Rabi and fast oscillations, yielding a complete set of single-qubit gates that suppress control errors. We also outlined a protocol for suppressing decoherence caused by charge noise. Our
protocol, and our analytical formalism, are both applicable to other solid-state systems, including superconducting flux qubits \cite{37,38} and quantum-dot singlet-triplet qubits \cite{10,39}, and can be extended to systems with multiple levels, including quantum-dot hybrid qubits \cite{3,8,40–44} and charge-quadrupole qubits \cite{45,46}.

One challenge for our proposal is the requirement of large tunnel couplings, or fast qubits. However, by employing high-order synchronized oscillations ($N \sim 10$), we can reduce gate speeds to be compatible with current experiments. Improvements in ac control technology and materials with lower charge noise can also mitigate the technical challenges. On the other hand, large tunnel couplings determine the phonon-induced relaxation rate \cite{47–49}, which sets an upper bound on qubit coherence. Moving forward, we note that the phase, $\phi$, represents an important control knob in our proposal, and can be viewed as a simple pulse-shaping tool. In future work, we therefore expect more sophisticated pulse-shaping techniques to improve the gate fidelities \cite{8,50}.

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SUPPLEMENTARY INFORMATION

In these Supplemental Materials, in Sec. , we provide the definition of the process fidelity, and in Sec. we present a brief derivation of the fidelity formula, Eq. (2) in the main text. In Sec. , we sketch the derivation of the analytic formula for the qubit time evolution for general noise spectra, and derive the asymptotic formulas for Markovian noise, quasi-static noise, and $1/f$ noise. Sec. provides the details of the simulations shown in the main text, including the method used to generate the noise realizations and a method used to speed up the simulations. In Sec. , we show that high-fidelity rotations about an axis tilted away from $X$-axis by $\phi = \pi/4$ by an arbitrary angle $\theta$, $R_\theta(\phi = \pi/4)$, can be achieved using a method similar to the one discussed in the main text.

PROCESS FIDELITY

Following Ref. \cite{51}, a generic quantum process $\mathcal{E}$ on a two dimensional Hilbert space can be expressed as

$$\mathcal{E}(\rho_0) = \sum_{m,n} E_m \rho_0 E_n^\dagger \chi_{mn},$$

(S1)

where $\rho_0$ is the initial-state density matrix, \{ $E_m$ \} = \{ $I, \sigma_z, -i\sigma_y, \sigma_z$ \} is a basis for the vector space of $2 \times 2$ matrices, and $\chi_{mn}$ is a $4 \times 4$ process matrix, commonly referred to as the chi matrix.

The process fidelity is then defined as

$$F = \text{Tr}[\chi_{\text{sys}} \chi_{\text{ideal}}],$$

(S2)

where $\chi_{\text{sys}}$ is the process matrix for the actual physical evolution, including strong-driving effects, and $\chi_{\text{ideal}}$ is the process matrix for the ideal rotation.

DERIVATION OF FIDELITY FORMULA IN THE ABSENCE OF CHARGE NOISE

We now evaluate Eq. (S2) in the absence of charge noise, including only strong-driving effects. Additional effects due to charge noise are presented in Sec. .

For a resonantly driven charge qubit operated at the charge anti-crossing ($\varepsilon = 0$) with only detuning drive ($A_\Delta = 0$), the Hamiltonian in the eigenbasis of the qubit is given by

$$\mathcal{H}_{\text{sys}} = -\Delta \sigma_z + (A_\varepsilon/2) \sigma_x \cos(\tilde{\omega}_{\text{res}} t + \phi),$$

(S3)

where $\tilde{\omega}_{\text{res}}$ is the resonant angular frequency. The evolution operation of the system in the absence of noise, $\mathcal{E}_{\text{sys}}(\rho_0) = U_0 \rho_0 U_0^\dagger$, can be obtained analytically up to arbitrary order in the strong-driving parameter $\gamma \equiv A_\varepsilon/(16\Delta)$.
as \( U_{0}(t) = \sum_{n=0}^{\infty} \gamma^{n} U_{0}^{(n)} \). The ideal unitary evolution of an \( R_{0}(\phi) \) rotation corresponds to the \( O[\gamma^{0}] \) terms in this expansion, which can also be obtained from the rotating wave approximation, giving

\[
U_{\text{ideal}} = U_{0}^{(0)} = \begin{pmatrix}
\cos(\omega_{\text{res}} t_0) + 2\phi \sin(\theta/2) & e^{-i\phi} \sin(\omega_{\text{res}} t_0) \cos(\theta/2) \\
e^{i(\omega_{\text{res}} t_0 + \phi)} \sin(\omega_{\text{res}} t_0) \cos(\theta/2) & -e^{-i\phi} \cos(\omega_{\text{res}} t_0) + 2\phi \sin(\theta/2)
\end{pmatrix},
\]

where \( t_0 = \theta/\Omega \) is the duration of the rotation. The leading order contribution to the fast oscillations which we treat here as infidelity, occurred at \( O[\gamma^{1}] \), and are given by

\[
U_{0}^{(1)} = 2i \begin{pmatrix}
\cos(\omega_{\text{res}} t_0) + 2\phi \sin(\theta/2) & e^{-i\phi} \sin(\omega_{\text{res}} t_0) \cos(\theta/2) \\
e^{i(\omega_{\text{res}} t_0 + \phi)} \sin(\omega_{\text{res}} t_0) \cos(\theta/2) & -e^{-i\phi} \cos(\omega_{\text{res}} t_0) + 2\phi \sin(\theta/2)
\end{pmatrix}.
\]

We now calculate the \( \chi \) matrix of \( \mathcal{E}_{\text{sys}} \) order by order, \( \chi_{\text{sys}} = \chi_{\text{ideal}} + \sum_{n=1}^{\infty} \gamma^{n} \chi_{\text{sys}}^{(n)} \). Since \( \mathcal{E}_{\text{sys}} \) describes the unitary evolution (without decoherence), we must have \( 1 = \text{Tr}(\chi_{\text{sys}} \chi_{\text{sys}}) = 1 + 2\gamma \text{Tr}(\chi_{\text{ideal}} \chi_{\text{sys}}) + \gamma^{2}(\text{Tr}(\chi_{\text{ideal}} \chi_{\text{sys}}) + 2 \text{Tr}(\chi_{\text{ideal}} \chi_{\text{sys}})) + \cdots \). Equating terms order by order in \( \gamma \), we arrive at

\[
\text{Tr}(\chi_{\text{ideal}} \chi_{\text{sys}}^{(1)}) = 0,
\]

\[
\text{Tr}(\chi_{\text{ideal}} \chi_{\text{sys}}^{(2)}) = -1/2 \text{Tr}(\chi_{\text{sys}} \chi_{\text{sys}}),
\]

\[
\text{Tr}(\chi_{\text{ideal}} \chi_{\text{sys}}^{(3)}) = - \text{Tr}(\chi_{\text{sys}} \chi_{\text{sys}}),
\]

\[
\text{Tr}(\chi_{\text{ideal}} \chi_{\text{sys}}^{(4)}) = - \text{Tr}(\chi_{\text{sys}} \chi_{\text{sys}}) - 1/2 \text{Tr}(\chi_{\text{sys}}^{(2)}).
\]

Using Eq. (S2) and Eq. (S6)-(S9) and applying our analytical results for \( U_{0} \), i.e. Eqs. (S4) and (S5) and high-order contributions, the fidelity \( F \) up to \( O[\gamma^{3}] \) is given by

\[
F_{\theta}(\phi) = 1 - 2\gamma^{2}[1 - \cos^{2}(\theta/2) \cos(2\theta \omega_{\text{res}}/\Omega) + \sin^{2}(\theta/2) \cos(2\theta \omega_{\text{res}}/\Omega + 4\phi)] - 4\gamma^{3} \sin(\theta) \sin(2\theta \omega_{\text{res}}/\Omega).
\]

If \( \phi = (2m + 1)\pi/4 \) for any integer \( m \), this expression can be simplified to yield Eq. (2) in the main text:

\[
F_{\theta}(\phi) = 1 - 2\gamma^{2}[1 - \cos(N\pi)] - 4\gamma^{3} \sin(\theta) \sin(N\pi),
\]

where \( N \equiv (2\theta \omega_{\text{res}})/\pi(\Omega) \). Note here that \( F_{\theta}(\phi) \) vanishes up to \( O[\gamma^{3}] \) whenever \( N \) is an even integer. The \( O[\gamma^{4}] \) contribution for \( \phi = (2m + 1)\pi/4 \) for any integer \( m \) is

\[
F_{\theta}^{(4)}(\pi/4) = 16[1 - \cos(2\theta \omega_{\text{res}}/\Omega)] - 16 \sin^{2}(\theta/2) \cos(4\theta \omega_{\text{res}}/\Omega) + 2(-1)^{m} \cos^{2}(\theta/2) \sin(4\theta \omega_{\text{res}}/\Omega)
\]

\[
-(5/2)[1 - \cos(8\theta \omega_{\text{res}}/\Omega)] - (-1)^{m} \sin(8\theta \omega_{\text{res}}/\Omega),
\]

which can be simplified as \( F_{\theta}^{(4)}(\phi) = -16 \sin^{2}(\theta/2) \) when \( N \) is an even integer. This represents an upper bound on the gate fidelity in any dip, as discussed in Sec. below, and in the main text.

We also note that, for arbitrary \( \phi \) and any integer \( k \), the fidelity can be written as

\[
F_{\theta}(\phi) = 1 - \begin{cases}
2\gamma^{2}[1 - \cos(2\theta \omega_{\text{res}}/\Omega)] + O[\gamma^{4}] & \text{if } \theta = 2k\pi \\
2\gamma^{2}[1 - \cos(2\theta \omega_{\text{res}}/\Omega + 4\phi)] + O[\gamma^{4}] & \text{if } \theta = (2k + 1)\pi
\end{cases},
\]

which equals unity, corresponding to perfect fidelity, whenever \( (\theta \omega_{\text{res}}/\Omega) \) is an integer multiple of \( \pi \) for \( \theta = 2k\pi \), or whenever \( (\theta \omega_{\text{res}}/\Omega + 4\phi) \) is an integer multiple of \( \pi \) for \( \theta = (2k + 1)\pi \).

**Analytic Formula for Dynamics in the Presence of Detuning Noise with Different Spectra**

In this section we analytically solve the dynamics of a strongly driven two-level system in the presence of classical detuning noise with different spectra. We first outline the derivation, and then present the results for noise with a generic spectrum. We then discuss in detail the asymptotic formulas that describe the large-time behavior for three different spectra: Markovian noise, quasi-static noise, and 1/f noise.

The dynamics of a strongly driven two-level system coupled to classical noise affecting the detuning \( \varepsilon \) can be described using the Hamiltonian

\[
\mathcal{H} = \mathcal{H}_{\text{sys}} + \mathcal{H}_{n} = -(\hbar \omega_{n}/2) \sigma_{z} + [A_{i} \sigma_{x} + A_{i} \sigma_{z}] \cos(\omega_{d} t + \phi) + [h_{n,x} \sigma_{x} + h_{n,z} \sigma_{z}] \delta \varepsilon(t),
\]

(S14)
where \( \omega_q \) is the qubit angular frequency, \( \omega_d \) is the angular frequency of the drive, \( A_t \) is the transverse (longitudinal) amplitude of the drive, and \( h_{n,x} \) (\( h_{n,z} \)) is the transverse (longitudinal) coupling to the detuning noise \( \delta \varepsilon \). Here we start with a more general situation to include, taking a charge qubit as an example, the case of driving tunnel coupling at the sweet spot \( (A_t \neq 0 \text{ when } \varepsilon = 0) \), and also the case of working away from the sweet spot \( (\varepsilon, A_t, h_{n,x}, h_{n,z} \neq 0) \).

In the main text, we only consider driving detuning while working at the sweet spot, the special case where \( \hbar \omega_q = 2 \Delta, A_t = 0, h_{n,x} = -1/2, \) and \( h_{n,z} = 0 \).

In the absence of noise, the evolution operator \( U_0 \) satisfies the equation

\[
\frac{ih}{\hbar} \frac{d}{dt} U_0 = H_\text{sys} U_0, \tag{S15}
\]

where \( U_0(t = 0) = I \). In Ref. [8], we showed how to obtain \( U_0 \) order-by-order in the perturbation parameter \( \gamma \sim A/\hbar \omega_d \), where \( A = A_1, A_\ell \), using a dressed-state formalism. Transforming into the interaction picture, the equation of motion in the presence of the detuning noise can be written as

\[
\frac{ih}{\hbar} \frac{d}{dt} \rho^I = \delta \varepsilon(t) \mathcal{L} \rho^I, \tag{S16}
\]

where \( \rho^I = U_0^\dagger \rho U_0 \) is the density matrix in the interaction picture, and \( \mathcal{L} \rho^I \equiv [h_n^I, \rho^I] \) with \( h_n^I = U_0^\dagger h_n U_0 \) and \( h_n = h_{n,x} \sigma_x + h_{n,z} \sigma_z \). The evolution can be expressed as a cumulant expansion [32, 33]

\[
\langle \rho^I(t) \rangle = \exp \left\{ -\frac{1}{\hbar^2} \int_0^t dt' \int_0^{t_1} dt_2 \mathcal{L}(t_1) \mathcal{L}(t_2) S(t_1 - t_2) \right\} \rho^I(0), \tag{S17}
\]

where \( \langle \cdots \rangle \) is the ensemble average over \( \delta \varepsilon(t) \) and \( \langle \cdots \rangle_c \) is the cumulant average. To \( O((\delta \varepsilon/\hbar \Omega)^2) \), this can be written as

\[
\langle \rho^I(t) \rangle = \exp \left\{ -\frac{1}{\hbar^2} \int_0^t dt' \int_0^{t_1} dt_2 \mathcal{L}(t_1) \mathcal{L}(t_2) \right\} \rho^I(0), \tag{S18}
\]

where \( \langle \cdots \rangle \) describes the ensemble average over \( \delta \varepsilon(t) \), and \( S(t_1 - t_2) \equiv \langle \delta \varepsilon(t_1) \delta \varepsilon(t_2) \rangle \) is the time autocorrelation function of \( \delta \varepsilon(t) \). We note that the power spectrum density of the noise, \( \tilde{S}(\omega) \), is related to \( S(t) \) by [15]

\[
\tilde{S}(\omega) = \int_{-\infty}^{\infty} dt \ e^{i \omega t} S(t). \tag{S19}
\]

To simplify the calculation, we express the qubit state as a Bloch vector \( r^I \) in the interaction frame, \( [\rho^I = 1/2(I_z + r^I_x \sigma_x + r^I_y \sigma_y + r^I_z \sigma_z)] \), and the super-operator \( \mathcal{L} \) as a matrix by using \( \{ \sigma_x, \sigma_y, \sigma_z \} \) as the basis of its domain (two-by-two Hermitian operator with vanishing trace). Eq. (S18) can then be written as

\[
r^I(t) = \exp[K(t)] r^I(0). \tag{S20}
\]

By expressing \( h_n^I(t) \) in terms of the Pauli matrices, \( h_n^I(t) = h_{n,x}^I(t) \sigma_x + h_{n,y}^I(t) \sigma_y + h_{n,z}^I(t) \sigma_z \), and further expanding the coefficients in Fourier series, \( h_n^I(t) = \sum_{i,\omega} \alpha_i \ e^{i \omega t} \), the matrix \( K(t) \) in Eq. (S20) can be written as

\[
[K(t)]_{ij} = -\frac{4}{\hbar^2} \int_0^t dt' \int_0^{t_1} dt_2 [h_{n,y}^I(t_1) h_{n,z}^I(t_2) - h_{n,z}^I(t_1) h_{n,y}^I(t_2) ] \delta_{ij} S(t_1 - t_2)
\]

\[
= -\frac{4}{\hbar^2} \sum_{\omega_1, \omega_2} \alpha_{j,\omega_1} \alpha_{i,\omega_2} - \delta_{ij} \sum_{k=1}^3 \alpha_{k,\omega_1} \alpha_{k,\omega_2} I(t, \omega_1, \omega_2), \tag{S21}
\]

where

\[
I(t, \omega_1, \omega_2) = \int_0^t dt' \int_0^{t_1} dt_2 e^{i \omega_1 t_1} e^{i \omega_2 t_2} S(t_1 - t_2), \tag{S22}
\]

\[
h_n^I = [h_{n,x}^I, h_{n,y}^I, h_{n,z}^I]. \tag{S23}
\]
In general, $K(t) = K_M + K_\varphi + K_{nMn}$ where $K_M$ is a Markovian term and $K_\varphi$ is a pure-dephasing term \[25\] \[34\] \[35\]. We denote the remaining term, $K_{nMn}$, as the non-Markovian-non-dephasing term; it is the correction to the Markovian approximation. The Markovian term is defined as the decoherence within the Markovian approximation (i.e., when the correlation time is much smaller than the characteristic time scale of the system dynamics). If only the Markovian term is present, the decoherence yields an exponential decay and $K(t)$ is linear in time $t$. For further discussion, please see Sec. and the main text. The pure-dephasing term describes pure dephasing in the rotating frame defined by $U_{rot} = \text{diag}[e^{i\omega_1 t/2}, e^{-i\omega_1 t/2}]$, and is associated with $I(t, \omega_1 = 0, \omega_2 = 0) \ [25\] \ [34\] \ [35\]. The non-Markovian-non-dephasing term is the correction to the Markovian approximation, describing, for example, the decoherence due to low-frequency noise, and oscillatory terms typically ignored in the asymptotic limit $t \gg 2\pi/\omega$, where $\omega$ represents any relevant angular frequency in the system.

In the following subsections, we evaluate the three terms of $K(t)$, \{$K_M, K_\varphi, K_{nMn}$\}, for general $S(t_1 - t_2)$, and then describe the asymptotic form of $K(t)$ for three cases: Markovian noise, quasistatic noise, and $1/f$ noise. To be concise, we assume the qubit is driven resonantly ($\omega_4 = \bar{\omega}_{\text{res}}$).

We note that this technique is easy to use and is applicable to many other kinds of noise that are present in condensed matter devices such as Lorentzian noise and power-law noise, and can also be generalized to multi-level systems such as quantum-dot hybrid qubits \[3\] \ [8\] \ [40\] \ [41\].

### Analytic formula for general noise spectra

For a generic noise spectrum $\tilde{S}(\omega)$, we define the symmetric and antisymmetric versions of Eq. \[S22\] as follows:

$$I_S(t, \omega_1, \omega_2) = \frac{1}{2} \int_0^t dt_1 \int_0^{t_1} dt_2 \left[ e^{i\omega_1 t_1} e^{i\omega_2 t_2} + e^{i\omega_2 t_1} e^{i\omega_1 t_2} \right] S(t_1 - t_2),$$  \tag{S24}$$

$$I_A(t, \omega_1, \omega_2) = \frac{1}{2} \int_0^t dt_1 \int_0^{t_1} dt_2 \left[ e^{i\omega_1 t_1} e^{i\omega_2 t_2} - e^{i\omega_2 t_1} e^{i\omega_1 t_2} \right] S(t_1 - t_2),$$  \tag{S25}$$

such that $I(t, \omega_1, \omega_2) = I_S(t, \omega_1, \omega_2) + I_A(t, \omega_1, \omega_2)$. The partially evaluated integrals for different cases of $\{\omega_1, \omega_2\}$ are given by

$$I_S(t, \omega_1, \omega_2) = \begin{cases} -[f_1(t) - t f_0(t)] & (\omega_1 = \omega_2 = 0), \\ -[f_{1\cos}(t, \omega_1) - t f_{1\cos}(t, \omega_1)] & (\omega_1 = -\omega_2 \neq 0), \\ -\frac{e^{i\omega_1 t}}{\omega_1} \left[ \cos(\frac{\omega_1^2}{2} t) f_{1\sin}(t, \omega_1) - \sin(\frac{\omega_1^2}{2} t) (f_{1\cos}(t, \omega_1) + f_t(t)) \right] & (\omega_1 \neq 0, \omega_2 = 0), \\ -\frac{e^{i\omega_2 t}}{\omega_2} \left[ \cos(\frac{\omega_2^2}{2} t) f_{1\sin}(t, \omega_2) - \sin(\frac{\omega_2^2}{2} t) (f_{1\cos}(t, \omega_2)) \right] & (\omega_1 = 0, \omega_2 \neq 0), \\ -\frac{e^{i\omega_1 t}}{\omega_1 + \omega_2} \left[ \cos(\frac{\omega_1 + \omega_2}{2} t) (f_{1\sin}(t, \omega_1) + f_{1\sin}(t, \omega_2)) - \sin(\frac{\omega_1 + \omega_2}{2} t) (f_{1\cos}(t, \omega_1) + f_{1\cos}(t, \omega_2)) \right] & (\text{Otherwise}), \\ \end{cases}$$  \tag{S26}$$

$$I_A(t, \omega_1, \omega_2) = \begin{cases} 0 & (\omega_1 = \omega_2 = 0), \\ -i \left[ f_{1\sin}(t, \omega_1) - t f_{1\sin}(t, \omega_1) \right] & (\omega_1 = -\omega_2 \neq 0), \\ -i \frac{e^{i\omega_1 t}}{\omega_1} \left[ -\cos(\frac{\omega_1^2}{2} t) (f_{1\cos}(t, \omega_1) - f_0(t)) - \sin(\frac{\omega_1^2}{2} t) f_{1\sin}(t, \omega_1) \right] & (\omega_1 \neq 0, \omega_2 = 0), \\ -i \frac{e^{i\omega_2 t}}{\omega_2} \left[ -\cos(\frac{\omega_2^2}{2} t) (f_0(t) - f_{1\cos}(t, \omega_2)) + \sin(\frac{\omega_2^2}{2} t) f_{1\sin}(t, \omega_2) \right] & (\omega_1 = 0, \omega_2 \neq 0), \\ i \frac{e^{i\omega_1 t}}{\omega_1 + \omega_2} \left[ \cos(\frac{\omega_1 + \omega_2}{2} t) (f_{1\cos}(t, \omega_1) - f_{1\cos}(t, \omega_2)) + \sin(\frac{\omega_1 + \omega_2}{2} t) (f_{1\sin}(t, \omega_1) - f_{1\sin}(t, \omega_2)) \right] & (\text{Otherwise}), \\ \end{cases}$$  \tag{S27}$$
We note that in some cases, \(I\) provide the results for the functions defined in Eqs. (S28)-(S33), the corresponding integrals in, \(K\) asymptotic formulas for these quantities. We express where \(\omega\) resonant angular frequency contribution to \(K\) plotted on a log scale. \(\omega\) the oscillatory terms. The asymptotic forms of \(K\) we note that the deviations are quite small, on the order of \(10^{-3}\) over the time shown in the figure. \(\gamma\) and simulation of the dynamics of a strongly driven charge qubit coupled to \(1/f\) charge noise. The analytic formula captures the main features of the simulation; the deviation of the two is smaller than \(10^{-3}\) over the time shown in the figure. Figs. 3(a) and 3(b) in the main text compare the analytical results and the simulation results of the gate fidelity \(F_x(\phi = \pi/4)\). The analytical results match the simulation results in the regimes of greatest interest, for example when the driving amplitude is large. In Fig. 3(a), deviations between theory and simulations begin to appear when the driving amplitude is small because the higher order effects of the noise become visible when the gate time is long. In Fig. 3(b), deviations also arise from the higher order terms in the noise \(\delta\varepsilon\) when the tunnel coupling is small. When the tunnel coupling is large, deviation appears again because of the higher order terms in \(\gamma\). However, we note that the deviations are quite small, on the order of \(10^{-4}\), and are mainly visible here because the data are plotted on a log scale.

In order to describe the asymptotic behavior of the dynamics at long times, we calculate the asymptotic form of \(K(t)\) by taking the limit \(t \gg 2\pi/\omega\), where \(\omega\) represents any relevant angular frequency in the system, and neglecting the oscillatory terms. The asymptotic forms of \(K_M(t), K_\varphi(t),\) and \(K_{nMn}\) typically can be expressed as

\[
[K_M]_{ij} = -\frac{4}{\hbar^2} \sum_{\omega_1} [\alpha_{j,\omega_1}\alpha_{i,-\omega_1} - \delta_{ij} \sum_{k=1}^3 \alpha_{k,\omega_1}\alpha_{k,-\omega_1}] I_S(t, \omega_1, -\omega_1),
\]

\[
[K_\varphi]_{ij} = -\frac{4}{\hbar^2} \sum_{\omega_1} [\alpha_{j,0}\alpha_{i,0} - \delta_{ij} \sum_{k=1}^3 \alpha_{k,0}\alpha_{k,0}] I_S(t, 0, 0),
\]

\[
[K_{nMn}]_{ij} = -\frac{4}{\hbar^2} \sum_{\omega_1} [\alpha_{j,\omega_1}\alpha_{i,-\omega_1} - \delta_{ij} \sum_{k=1}^3 \alpha_{k,\omega_1}\alpha_{k,-\omega_1}] I_A(t, \omega_1, -\omega_1).
\]

We note that in some cases, \(I(t, \omega_1 = 0, \omega_2 \neq 0)\) also contributes to the asymptotic form of \(K_{nMn}\).

In the following, we focus on three typical types of noise: Markovian noise, quasi-static noise, and \(1/f\) noise. We provide the results for the functions defined in Eqs. (S28)-(S33), the corresponding integrals \(I(t, \omega_1, \omega_2)\), and the asymptotic formulas for these quantities. We express \(K(t)\) as a perturbation series in \(\gamma\), using \(K^{(i)}_x\) to label the \(O[\gamma^i]\) contribution to \(K_x\), where \(x\) can be \(M, \varphi,\) or \(nMn\).

Markovian Noise

In the Markovian limit, we assume that the correlation time \(\tau_c\) is much smaller than the time scale we are interested in, \(t\), and \(1/\omega\), where \(\omega\) is the characteristic frequency of the system (i.e., for a resonantly driven charge qubit, the resonant angular frequency \(\omega_{res}\), the Rabi angular frequency \(\Omega\) or their linear combinations), such that \(t/(2\pi) \gg\)
1/ω ≫ τr/(2π). By setting S(t′) ≈ 0 when t′ > τc and neglecting any term that scales with τc, Eqs. (S28)–(S33) can be evaluated to yield

\begin{align*}
    f_{t,\cos}(\omega) &\approx 0, \\
    f_{t,\sin}(\omega) &\approx 0, \\
    f_{1}(\omega) &\approx 0, \\
    f_{\cos}(\omega) &\approx \frac{1}{2} \tilde{S}(\omega), \\
    f_{\sin}(\omega) &\approx 0, \\
    f_{0}(\omega) &\approx \frac{1}{2} \tilde{S}(0).
\end{align*}

\hspace{1cm} (S37) \hspace{1cm} (S38) \hspace{1cm} (S39) \hspace{1cm} (S40) \hspace{1cm} (S41) \hspace{1cm} (S42)

Inserting Eq. (S37) to Eq. (S42) in Eq. (S26) and Eq. (S27) and neglecting O[1/ω] terms, we obtain I = I_S + I_A, with:

\begin{align*}
    I_S(t, \omega_1, \omega_2) &\approx \begin{cases} 
        \tilde{S}(\omega_1)t/2 & (\omega_1 = -\omega_2), \\
        0 & \text{Otherwise},
    \end{cases} \\
    I_A(t, \omega_1, \omega_2) &\approx 0. \\
\end{align*}

\hspace{1cm} (S43) \hspace{1cm} (S44)

Applying this formula to Eq. (S14) and only keeping the terms up to O[γ^0] (consistent with the rotating wave approximation) yields

\[ K^{(0)}(t) = K_M^{(0)}(t) = -t \begin{pmatrix} \Gamma_x & \Gamma_{xy} & 0 \\ \Gamma_{xy} & \Gamma_y & 0 \\ 0 & 0 & \Gamma_z \end{pmatrix}, \]

\hspace{1cm} (S45)

where

\begin{align*}
    \Gamma_x &= \frac{\hbar^2}{\hbar^2} \left[ \frac{3 + \cos(2\phi)}{2} \tilde{S}(\Omega) + \frac{h_{n,x}^3}{\hbar^2} \sin^2(\phi) \tilde{S}(\tilde{\omega}_{\text{res}}) + \frac{1 + \cos^2(\phi)}{4} \tilde{S}(\tilde{\omega}_{\text{res}} + \Omega) + \frac{1 + \cos^2(\phi)}{4} \tilde{S}(\tilde{\omega}_{\text{res}} - \Omega) \right], \\
    \Gamma_y &= \frac{\hbar^2}{\hbar^2} \left[ \frac{3 - \cos(2\phi)}{2} \tilde{S}(\Omega) + \frac{h_{n,x}^3}{\hbar^2} \cos^2(\phi) \tilde{S}(\tilde{\omega}_{\text{res}}) + \frac{2 - \cos^2(\phi)}{4} \tilde{S}(\tilde{\omega}_{\text{res}} + \Omega) + \frac{2 - \cos^2(\phi)}{4} \tilde{S}(\tilde{\omega}_{\text{res}} - \Omega) \right], \\
    \Gamma_z &= \frac{\hbar^2}{\hbar^2} \tilde{S}(\Omega) + \frac{h_{n,x}^3}{\hbar^2} \tilde{S}(\tilde{\omega}_{\text{res}}) + \frac{1}{4} \tilde{S}(\tilde{\omega}_{\text{res}} + \Omega) + \frac{1}{4} \tilde{S}(\tilde{\omega}_{\text{res}} - \Omega), \\
    \Gamma_{xy} &= -\frac{h_{n,x}^3}{\hbar^2} \sin(2\phi) \tilde{S}(\Omega) + \frac{h_{n,x}^3}{\hbar^2} \sin(2\phi) \tilde{S}(\tilde{\omega}_{\text{res}}) - \frac{1}{4} \tilde{S}(\tilde{\omega}_{\text{res}} + \Omega) - \frac{1}{4} \tilde{S}(\tilde{\omega}_{\text{res}} - \Omega),
\end{align*}

\hspace{1cm} (S46) \hspace{1cm} (S47) \hspace{1cm} (S48) \hspace{1cm} (S49)

and \( K_\varphi = K_{\text{MNP}} = 0 \). Here, \( \Omega \) is the Rabi angular frequency. If we apply this result on the detuning-driven charge qubit at the sweet spot (\( \varepsilon = 0 \)), as discussed in the main text, and focus on the small drive regime, \( \tilde{\omega}_{\text{res}} \gg \Omega \), the results can be further simplified as

\[ \Gamma_x = \frac{h_{n,x}^3}{\hbar^2} \left[ 1 + \frac{1}{2} \sin^2(\phi) \right] \tilde{S}(\tilde{\omega}_{\text{res}}), \quad \Gamma_y = \frac{h_{n,x}^3}{\hbar^2} \left[ 1 + \frac{1}{2} \cos^2(\phi) \right] \tilde{S}(\tilde{\omega}_{\text{res}}), \quad \Gamma_z = \frac{3h_{n,x}^3}{2\hbar^2} \tilde{S}(\tilde{\omega}_{\text{res}}), \quad \Gamma_{xy} = \frac{h_{n,x}^3}{4\hbar^2} \sin(2\phi) \tilde{S}(\tilde{\omega}_{\text{res}}). \]

Note that, if \( \phi = 0 \), the expressions for \( \Gamma_x \) and \( \Gamma_z = \Gamma_y \) are equivalent to those obtained using Bloch-Redfield theory [15][25][26]. For this special case, the elements of the Bloch vector exhibit an exponential decay,

\[ \mathbf{r}(t) = \begin{pmatrix} e^{-\Gamma_x t} & 0 & 0 \\ 0 & e^{-\Gamma_y t} & 0 \\ 0 & 0 & e^{-\Gamma_z t} \end{pmatrix} \mathbf{r}(0). \]

\hspace{1cm} (S50)

If one now restore the O(1/Ω) terms, previously neglected while deriving Eqs. (S43) and (S44), one obtains an equation of motion for the Bloch vector within the rotating wave approximation,

\[ \frac{d}{dt} \mathbf{r}(t) = \begin{pmatrix} -\Gamma_x & 0 & 0 \\ \eta \sin(\Omega t) & -\Gamma_y - \lambda \cos(2\Omega t) & \text{sgn}(A_t) \lambda \sin(2\Omega t) \\ \text{sgn}(A_t) \eta \cos(\Omega t) & \text{sgn}(A_t) \lambda \sin(2\Omega t) & -\Gamma_z - \lambda \cos(2\Omega t) \end{pmatrix} \mathbf{r}(t), \]

\hspace{1cm} (S51)

where \( \eta = \frac{1}{2} h_{n,x}^3 (\tilde{S}(\tilde{\omega}_{\text{res}} + \Omega) - \tilde{S}(\tilde{\omega}_{\text{res}} - \Omega)), \lambda = -\frac{1}{2} h_{n,x}^3 (\tilde{S}(\tilde{\omega}_{\text{res}} + \Omega) + \tilde{S}(\tilde{\omega}_{\text{res}} - \Omega)) + h_{n,x}^3 \tilde{S}(\Omega), \) and \( \text{sgn}(x) = +1 (-1) \) when \( x > 0 (x < 0) \). We note that Equation (S51) is equivalent to the master equation results derived in [21]. However, the current method also provides a systematic approach for including higher-order terms in γ, beyond the rotating-wave approximation.
For quasistatic noise with standard deviation $\sigma_\varepsilon$, the time correlation function is

$$S(t) = \sigma_\varepsilon^2,$$  \hspace{1cm} (S52)

and the corresponding noise spectral density is

$$\tilde{S}(\omega) = 2\pi\sigma_\varepsilon^2\delta(\omega).$$  \hspace{1cm} (S53)

Using these forms in Eqs. (S28)-(S33) yields

$$f_{t\cos}(t,\omega) = \sigma_\varepsilon^2 - \frac{1}{\omega^2} + \cos(\omega t) + \frac{\omega t}{\omega^2} \sin(\omega t),$$  \hspace{1cm} (S54)

$$f_{t\sin}(t,\omega) = \sigma_\varepsilon^2 - \frac{\omega t \cos(\omega t)}{\omega^2} + \frac{\sin(\omega t)}{\omega},$$  \hspace{1cm} (S55)

$$f_t(t) = \sigma_\varepsilon^2 \frac{t^2}{2},$$  \hspace{1cm} (S56)

$$f_{\cos}(t,\omega) = \sigma_\varepsilon^2 \frac{\sin(\omega t)}{\omega},$$  \hspace{1cm} (S57)

$$f_{\sin}(t,\omega) = \sigma_\varepsilon^2 \frac{1 - \cos(\omega t)}{\omega},$$  \hspace{1cm} (S58)

$$f_0(t) = \sigma_\varepsilon^2 t.$$  \hspace{1cm} (S59)

Taking the limit $t/(2\pi) \gg 1/\omega$ for any relevant angular frequencies of the system $\omega$, and keeping only the non-oscillatory terms to $O[1/\omega]$, the integrals can be approximated as

$$I(t,\omega_1,\omega_2) \approx \begin{cases} \sigma_\varepsilon^2 t^2/2 & (\omega_1 = \omega_2 = 0), \\ i\sigma_\varepsilon^2 t/\omega_1 & (\omega_1 = -\omega_2 \neq 0), \\ i\sigma_\varepsilon^2 t/\omega_2 & (\omega_1 = 0 \text{ and } \omega_2 \neq 0), \\ 0 & \text{(Otherwise)}. \end{cases}$$  \hspace{1cm} (S60)

For quasistatic noise, $K(t) = K_\varphi + K_{nMn_\varphi}(t)$, and $K_M = 0$. The leading order term in $K_{nMn_\varphi}(t)$ is $O[\gamma^0]$:

$$K_{nMn_\varphi}^{(0)}(t) = -\Gamma_{nMn_\varphi} t \begin{pmatrix} 0 & 0 & \sin(\phi) \\ 0 & 0 & \cos(\phi) \\ -\sin(\phi) & -\cos(\phi) & 0 \end{pmatrix},$$  \hspace{1cm} (S61)

where

$$\Gamma_{nMn_\varphi} = -\frac{i}{2\hbar^2} \text{sgn}(A_1)\{4\hbar^2 n_z \tilde{S}_{\text{imag}}(\Omega) + \hbar^2 n_x (\tilde{S}_{\text{imag}}(-\omega_{\text{res}} + \Omega) + \tilde{S}_{\text{imag}}(\omega_{\text{res}} + \Omega))\},$$  \hspace{1cm} (S62)

$$\tilde{S}_{\text{imag}}(\omega) \equiv i\sigma_\varepsilon^2 \frac{\omega}{\omega^2}.$$  \hspace{1cm} (S63)

The leading order contribution to $K_\varphi(t)$ is $O[\gamma^2]$:

$$K_\varphi^{(2)}(t) = -\Gamma_\varphi t^2 \begin{pmatrix} \sin^2(\phi) & \cos(\phi) \sin(\phi) & 0 \\ \cos(\phi) \sin(\phi) & \cos^2(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$  \hspace{1cm} (S64)

where

$$\Gamma_\varphi = 2\sigma_\varepsilon^2 n_x A_1^2 \frac{\hbar^2}{K_{\omega_{\text{res}}}}.$$  \hspace{1cm} (S65)

We note that, for the charge qubit operated at the sweet spot as discussed in the main text, this term is non-vanishing only if the tunnel coupling driving amplitude is not zero. This indicates that one should only drive detuning in order to suppress this decoherence effect, as in the main text.
To provide a physical picture, we note that $\hbar d\Omega/\omega = 2\hbar \omega_n A_{\Omega}/\hbar \omega_n$. By writing $\Gamma_\omega = \sigma_e^2 (\hbar d\Omega/\omega)^2/2$, we interpret $K_\omega(t)$ as the dephasing in the rotating frame, where the eigenenergies of the dressed states are $\pm \hbar \Omega$.

These two terms cause the elements of the Bloch vector to decay as a Gaussian with a sinusoidal oscillation:

$$ r^I(t) = \begin{pmatrix} 1 + |e^{-\Gamma_\omega t^2} \cos(\Gamma_{nMn} t) - 1| \sin^2(\phi) & |e^{-\Gamma_\omega t^2} \cos(\Gamma_{nMn} t) - 1| \sin(\phi) \cos(\phi) & -e^{-\Gamma_\omega t^2} \sin(\Gamma_{nMn} t) \sin(\phi) \\ |e^{-\Gamma_\omega t^2} \cos(\Gamma_{nMn} t) - 1| \cos(\phi) \sin(\phi) & 1 + |e^{-\Gamma_\omega t^2} \cos(\Gamma_{nMn} t) - 1| \cos(\phi) \sin(\phi) & -e^{-\Gamma_\omega t^2} \sin(\Gamma_{nMn} t) \cos(\phi) \\ e^{-\Gamma_\omega t^2} \sin(\Gamma_{nMn} t) \sin(\phi) & e^{-\Gamma_\omega t^2} \sin(\Gamma_{nMn} t) \cos(\phi) & e^{-\Gamma_\omega t^2} \cos(\Gamma_{nMn} t) \end{pmatrix} r^I(0). $$ (S66)

1/f noise

The 1/f power spectrum was presented in Eq. (3) of the main text

$$ S(\omega) = \begin{cases} c/|\omega| : |\omega| \leq \omega_l \\ 0 : \text{otherwise} \end{cases}, $$ (S67)

where $\omega_l$ ($\omega_h$) is the low (high) angular frequency cut-off. The corresponding time correlation function is

$$ S(t) = 2c/|\omega| [C_i(\omega_h |t|) - C_i(\omega_l |t|)], $$ (S68)

where $C_i(x) \equiv \int_x^\infty dx' \cos(x'/x')$ is the cosine integral function, and $S_i(x) \equiv \int_0^x dx' \sin(x'/x')$ is the sine integral function. We further define functions

$$ C_i(\omega) \equiv 2 [C_i((\omega + \omega_h)t) + C_i((\omega - \omega_h)t)] - C_i((\omega + \omega_l)t) - C_i((\omega - \omega_l)t), $$ (S69)

$$ S_i(\omega) \equiv 2 [S_i((\omega + \omega_h)t) + S_i((\omega - \omega_h)t)] - S_i((\omega + \omega_l)t) - S_i((\omega - \omega_l)t), $$ (S70)

and obtain

$$ f_{\text{cos}}(t, \omega) = \frac{t}{\omega} \sin(\omega t) S(t) + \frac{1}{\omega} \left\{ \frac{1}{\omega} \cos(\omega t) S(t) - \frac{c^2}{2\omega} \right\} \left[ C_i(\omega) - 2 \log \left( \frac{\omega_h^2 - \omega^2}{\omega_h^2 - \omega^2} \right) \right], $$

$$ f_{\text{sin}}(t, \omega) = \frac{1}{\omega} t \cos(\omega t) S(t) + \frac{1}{\omega} \left\{ \frac{1}{\omega} \sin(\omega t) S(t) - \frac{c^2}{2\omega} S_i(\omega) \right\} $$

$$ + \frac{c^2}{\omega} \left[ \frac{\sin((\omega + \omega_l)t)}{\omega + \omega_l} + \frac{\sin((\omega - \omega_l)t)}{\omega - \omega_l} - \frac{\sin((\omega + \omega_l)t)}{\omega + \omega_l} - \frac{\sin((\omega - \omega_l)t)}{\omega - \omega_l} \right], $$ (S72)

$$ f_1(t) = \frac{1}{2} t^2 S(t) - c^2 t \left[ \frac{\sin(\omega t)}{\omega t} - \frac{\sin(\omega t)}{\omega t} \right] + c^2 \left[ \frac{1 - \cos(\omega t)}{\omega t^2} - \frac{1 - \cos(\omega t)}{\omega t^2} \right], $$ (S73)

$$ f_0(t) = t S(t) - \frac{c^2}{2\omega} S_i(\omega), $$ (S74)

We can obtain an asymptotic formula for 1/f noise by taking the long time limit, $1/\omega_l \gg t/(2\pi) \gg 1/\omega \gg 1/\omega_h$, where $t$ is the qubit evolution time we are interested in and $\omega$ is any angular frequency relevant to the system, and dropping the oscillatory terms in the full formula. The integrals $I(t, \omega_1, \omega_2)$ can be approximated as

$$ I(t, \omega_1, \omega_2) \approx \begin{cases} c^2 t^2 \log(1/\omega t) - \gamma_E + 3/2 & (\omega_1 = \omega_2 = 0), \\ c^2 t^2 (\pi + 2\cos(1/\omega t)\log(\omega/\omega_1)) / \omega t_1 & (\omega_1 = \omega_2 \neq 0), \\ 2 c^2 t^2 (1/\omega t) - \gamma_E + 1 / \omega_2 & (\omega_1 = 0 \text{ and } \omega_2 \neq 0), \\ 0 & (\text{Otherwise}), \end{cases} $$

where $\gamma_E \approx 0.577$ is the Euler’s constant.
For a resonantly driven two-level system, only $K_M(t)$ and $K_{nM_n}(t)$ have contributions of $O[(A/\hbar \omega)^0]$, corresponding to the rotating-wave approximation. The lowest-order contributions to $K_\phi(t)$ is $O[(A/\hbar \omega)^2]$. Below, we report the leading order contributions of $K_M(t)$, $K_{nM_n}(t)$, and $K_\phi(t)$, respectively. The explicit expressions of $O[\gamma]$ contributions of $K_M(t)$, $K_{nM_n}(t)$ are, however, omitted for brevity.

It is unsurprising that $1/f$ noise causes both Markovian and non-Markovian types of decoherence. At $O[\gamma^0]$,

$$K^{(0)}_M(t) = -t \begin{pmatrix} \Gamma_x & \Gamma_{xy} & 0 \\ \Gamma_{xy} & \Gamma_y & 0 \\ 0 & 0 & \Gamma_z \end{pmatrix},$$

$$K^{(0)}_{nM_n}(t) = -\Gamma_{nM_n}t \begin{pmatrix} 0 & 0 & \sin(\phi) \\ 0 & 0 & \cos(\phi) \\ -\sin(\phi) & -\cos(\phi) & 0 \end{pmatrix},$$

where

$$\Gamma_x = \frac{\hbar_n}{\hbar^2} \left[ 3 + \cos(2\phi) \sin^2(\phi) \tilde{S}(\tilde{\omega}_{res}) + \frac{1 + \cos^2(\phi)}{4} \tilde{S}(\tilde{\omega}_{res} + \Omega) + \frac{1 + \cos^2(\phi)}{4} \tilde{S}(\tilde{\omega}_{res} - \Omega) \right],$$

$$\Gamma_y = \frac{\hbar_n}{\hbar^2} \left[ 3 - \cos(2\phi) \cos^2(\phi) \tilde{S}(\tilde{\omega}_{res}) + \frac{2 - \cos^2(\phi)}{4} \tilde{S}(\tilde{\omega}_{res} + \Omega) + \frac{2 - \cos^2(\phi)}{4} \tilde{S}(\tilde{\omega}_{res} - \Omega) \right],$$

$$\Gamma_z = \frac{\hbar_n}{\hbar^2} \tilde{S}(\tilde{\omega}_{res}) + \frac{\hbar_n}{\hbar^2} \tilde{S}(\tilde{\omega}_{res} + \Omega) + \frac{1}{4} \tilde{S}(\tilde{\omega}_{res} + \Omega) + \frac{1}{4} \tilde{S}(\tilde{\omega}_{res} - \Omega),$$

$$\Gamma_{xy} = \frac{\hbar_n}{2\hbar^2} \sin(2\phi) \tilde{S}(\tilde{\omega}_{res}) - \frac{\hbar_n}{2\hbar^2} \cos(2\phi) \tilde{S}(\tilde{\omega}_{res}) - \frac{1}{4} \tilde{S}(\tilde{\omega}_{res} + \Omega) - \frac{1}{4} \tilde{S}(\tilde{\omega}_{res} - \Omega),$$

$$\Gamma_{nM_n} = -\frac{i}{2\hbar^2} \text{sgn}(A_t) [4\hbar_n^2 \tilde{S}_{imag}(\tilde{\omega}_{res}) + \hbar_n^2 \tilde{S}_{imag}(\tilde{\omega}_{res} + \Omega) + \tilde{S}_{imag}(\tilde{\omega}_{res} + \Omega)],$$

$$\tilde{S}_{imag}(\omega) \equiv (2i c^2/\omega) \ln |\omega/\omega_i|.$$  

Note that $\Gamma_{nM_n}$ is the rotating frequency induced by the low-frequency part of the $1/f$ noise. For a driven charge qubit operated at a sweet spot $\varepsilon = 0$, as considered in the main text, $h_{n,x} = 0$ so that terms proportional to $\tilde{S}(\Omega)$ or $\tilde{S}_{imag}(\Omega)$ vanish. Since the remaining terms in $\Gamma_{nM_n}$ now have opposite signs and similar magnitudes, we see that $K_{nM_n}$ is small compared to $K_M$. If we further take $\phi = \pi/4$ as in the main text, the expressions are simplified as $\Gamma_x = \Gamma_y$ and $\Gamma_x + \Gamma_{xy} = \Gamma_z$, yielding Eq. (5) of the main text when the initial state is $|0\rangle$.

The leading order contribution to $K_\phi(t)$ is $O[\gamma^2]$:

$$K_\phi(t) = -4t^2 (\ln(1/\omega_t) - \gamma_E + 3/2) h_{n,x} \frac{A_t^2}{\hbar^2 \omega_{res}^2} \begin{pmatrix} \sin^2(\phi) & \cos(\phi) \sin(\phi) & 0 \\ \cos(\phi) \sin(\phi) & \cos^2(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  

As $\hbar d\omega/d\varepsilon = 2h_{n,x} A_t/\hbar \omega_{res}$, we again interpret $K_\phi(t)$ as the dephasing in the rotating frame, in which the eigenenergies of the dressed states are $\pm \hbar \Omega$.

For the data of the asymptotic formula shown in the main text [Fig. 1(c) and Fig. 2], we also include contributions to $K_\phi$ up to $O[\gamma^5]$, and contributions to $K_M$ and $K_{nM_n}$ up to $O[\gamma]$. To capture the essence of the dynamics in a simple and illustrative way, we did not include the $O[\gamma^5]$ contributions to $K_M$ and $K_{nM_n}$. This is justified by the fact that the lower-order contributions already capture the features of the simulation as shown in the Fig. 1(c) in the main text. In fact, the $O[\gamma^5]$ contributions to $K_M$ and $K_{nM_n}$ are expected to have small effects. This is contrary to $K_\phi$ where the leading-order terms are $O[\gamma^2]$ and the dephasing function $I(t, \omega_1 = 0, \omega_2 = 0) \sim t^2 \log(t) \gg t$ when $t$ is large, suggesting appreciable effects on the dynamics, as shown in the lower inset of Fig. 2.

**DETAILS OF NUMERICAL SIMULATIONS**

We now describe the method used for the numerical simulations. The objective is to simulate the evolution of the ensemble average of the density matrix, $\langle \rho \rangle$, in the presence of detuning noise $\delta \varepsilon(t)$, where $\langle \cdots \rangle$ denotes the ensemble average over $\delta \varepsilon(t)$.

The dynamics is governed by the Schrödinger equation

$$i\hbar \frac{d}{dt} \langle \psi(t) \rangle = \left[ H_{sys} + H_n \right] \langle \psi(t) \rangle = \left[ H_{sys} + \delta \varepsilon(t) h_n \right] \langle \psi(t) \rangle.$$  

(S86)
Here, $\mathcal{H}_{\text{sys}} = \mathcal{H}_q + \mathcal{H}_{\text{ac}}$ is the Hamiltonian of the system, where $\mathcal{H}_q$ describes the qubit and $\mathcal{H}_{\text{ac}}$ describes the ac driving, and $\mathcal{H}_n = h_n\delta(t)$ describes the effect of charge noise on the system, where $h_n$ is the noise matrix. Using the states $\{|0\rangle = (|L\rangle - |R\rangle)/\sqrt{2}, |1\rangle = (|L\rangle + |R\rangle)/\sqrt{2}\}$ as the basis, we can express $\mathcal{H}_q = -(\epsilon/2)\sigma_x - \Delta\sigma_z$, $\mathcal{H}_{\text{ac}} = (A_\epsilon/2)\sigma_x \cos(\omega t)$, and $h_n = -\sigma_x/2$, where the $\sigma_i$ are Pauli matrices.

The most obvious way to proceed is to first generate a set of noise realizations, $\{\delta\varepsilon^a(t)\}_{\alpha=1,...,\alpha_{\text{max}}}$, having the desired spectrum, $\tilde{S}(\omega)$, then solve the Schrödinger equation for a given noise realization $\delta\varepsilon^a(t)$ to obtained the final state $|\psi^a\rangle$ and the corresponding density matrix $\rho^a = |\psi^a\rangle\langle \psi^a|$. The ensemble average of the density matrix is then given by

$$\langle \rho \rangle \approx \frac{1}{\alpha_{\text{max}}} \sum_{\alpha=1}^{\alpha_{\text{max}}} \rho^a.$$  \hfill (S87)

Performing numerical simulation in this manner, however, is very time-consuming, especially when $\alpha_{\text{max}}$ is large.

We therefore develop a more efficient method to simulate the Schrödinger equation over a large number of noise realizations. We now first describe the generation of the noise realizations, and then the simulation method.

**Generation Of Noise Realizations for the Numerical Simulations**

In the simulation, a noise realization $\delta\varepsilon(t)$ is discretized into the noise sequences $\{\delta\varepsilon_1, \delta\varepsilon_2, \cdots, \delta\varepsilon_N\}$, such that $\delta\varepsilon(t) = \delta\varepsilon_k$ is constant over the time interval $t_{k-1} \leq t < t_k$ where $t_k = k N/\tau_{\text{tot}}$, $\tau_{\text{tot}}$ is the total time of consideration, and $N$ is the total number of time segments. We follow [20] to generate the noise sequences for the numerical simulation. The method is:

1. Generate a Gaussian white noise sequence $\{u_1, u_2, \cdots, u_N\}$ with zero mean and unit standard deviation.

2. Compute the Fourier transform of the sequence, $\{\tilde{u}_1, \tilde{u}_2, \cdots, \tilde{u}_N\}$, where

$$\tilde{u}_m = \sum_{n=1}^{N} u_n e^{-i2\pi \frac{(m-1)(n-1)}{N}}.$$  \hfill (S88)

3. Let $\tilde{S}(\omega)$ represent the desired noise power spectrum. Scale the white noise in the Fourier space by taking the product $\tilde{u}_m \sqrt{S'_m}$, where $S'_m = \tilde{S}(2\pi(m-1)/\tau_{\text{tot}}) / \Delta t$, and $\Delta t = \tau_{\text{tot}}/N$. Then perform the inverse Fourier transformation to obtain the noise realization in the time domain:

$$\delta\varepsilon_k = \frac{1}{N} \sum_{m=1}^{N} \tilde{u}_m \sqrt{S'_m} e^{i2\pi \frac{(m-1)(k-1)}{N}}.$$  \hfill (S89)

Here, we assume $1/f$ noise such that

$$S'_m = c_2^2 \begin{cases} N/(m-1) & m = m_1, 2, \ldots, N/2 + 1 \\ N/(N-m+1) & m = N/2 + 2, \ldots, N - m_1 + 2 \end{cases},$$

corresponding to the continuous noise spectrum

$$\tilde{S}(\omega) = c_2^2 \begin{cases} \frac{2\pi}{|\omega|} & \omega_l \leq |\omega| \leq \omega_h \\ 0 & \text{otherwise} \end{cases},$$  \hfill (S90)

where the low angular frequency cutoff is $\omega_l = 2\pi(m_1 - 1)/\tau_{\text{tot}}$ and the high angular frequency cutoff is $\omega_h = \pi N/\tau_{\text{tot}}$.

**Method of Simulation**

We wish to solve the Schrödinger equation, Eq. (S86), numerically for a given noise realization $\delta\varepsilon(t)$ defined as in Eq. (S89). Let $U_0(t)$ be the evolution operation satisfying

$$i\hbar \frac{d}{dt} U_0(t) = \mathcal{H}_{\text{sys}} U_0(t),$$  \hfill (S91)
with initial condition $U_0(t = 0) = I$. We transform the Schrödinger equation into the interaction picture, defining $|\psi^I(t)\rangle = U_0(t)^\dagger|\psi(t)\rangle$ and $h_n^I(t) = U_0(t)^\dagger h_n(t) U_0(t)$, so that
\[ ih \frac{d}{dt} |\psi^I(t)\rangle = \delta \varepsilon(t) h_n^I(t)|\psi^I(t)\rangle. \] (S92)

To include the cases where gate times are in between $t_k$ and $t_{k-1}$, we define the evolution operator $U^I(t, t_{k-1})$ as $|\psi^I(t)\rangle = U^I(t, t_{k-1})|\psi^I(t_{k-1})\rangle$ for $t_k \geq t \geq t_{k-1}$, which can be expanded perturbatively as
\[ U^I(t, t_{k-1}) = I + \sum_{r=1}^{\infty} (\delta \varepsilon_k)^r W_r(t, t_{k-1}), \]
where $W_r(t, t_{k-1})$’s are time-ordered integrals,
\[ W_1(t, t_{k-1}) \equiv \left(-i\frac{\hbar}{\alpha}\right) \int_{t_{k-1}}^{t} d \tau_1 h_n^I(\tau_1), \] (S93)
\[ W_r(t, t_{k-1}) \equiv \left(-i\frac{\hbar}{\alpha}\right) \int_{t_{k-1}}^{t} d \tau_1 \cdots \int_{t_{k-1}}^{\tau_{r-1}} d \tau_r h_n^I(\tau_1) \cdots h_n^I(\tau_r), \quad \text{for } r = 2, 3, \cdots. \] (S94)
Truncating the series after $r = r_{\text{max}}$, one can approximate the evolution operator as
\[ U^I(t, t_{k-1}) = I + \delta \varepsilon_k W_1(t, t_{k-1}) + (\delta \varepsilon_k)^2 W_2(t, t_{k-1}) + \cdots + (\delta \varepsilon_k)^{r_{\text{max}}} W_{r_{\text{max}}}(t, t_{k-1}). \] (S95)
where $\{W_r(t, t_{k-1})\}$ can be calculated either analytically or numerically. The evolution operator $U^I(t, 0)$ then can be expressed as
\[ U^I(t, 0) = U^I(t, t_{k-1})U^I(t_{k-1}, t_{k-2}) \cdots U^I(t_1, 0). \] (S96)
The solution of the Schrödinger equation in the lab frame can then be expressed as $|\psi(t)\rangle = U(t)|\psi(0)\rangle$, where $U(t) = U_0(t)U^I(t, 0)$.

The strength of this method is that $W_r(t, t_{k-1})$ does not depend on delta $\varepsilon(t)$, so we only need to perform simulations once to determine $\{W_r(t, t_{k-1})\}_{r=1, \ldots, r_{\text{max}}}$ and $\{W_r(t_l, t_{l-1})\}_{r=1, \ldots, r_{\text{max}}}$ for $l = 1, \ldots, k - 1$, numerically. After computing these matrices, the calculation of $|\psi(t)\rangle$ for all the generated noise sequences directly follows Eqs. (S94) and (S96), with no further simulations required. This allows efficient simulation of the system, even with a large number of noise realizations.

In this work, we take $r_{\text{max}} = 6$, $\alpha_{\text{max}} = 100,000$, and compute $\{W_r\}$ numerically. We have checked the validity of this simplified method by comparing our results to those of full simulations with parameters the same as the simulation shown in Fig. 1(b) and Fig. 1(c) for several different noise sequences, and find the deviations between the resulting density matrices are smaller than $10^{-7}$ for the time period considered. We also study the statistics of the simulated fidelities by dividing the simulation into 10 trials, each with $\alpha_{\text{max}} = 10,000$, and compute the ratio of the standard deviation of these 10 trials, $\sigma_F$, to the average infidelity $\sigma_F/(1 - F)$. For all the parameters considered in the main text, this ratio is around $10^{-2}$, showing that the reported infidelity is accurate to at least two significant figures.

**HIGH-FIDELITY $R_\theta(\phi = \pi/4)$ GATES**

According to Eq. (S10), to $O(\gamma^3)$, the intrinsic infidelity of a resonantly driven $R_\theta(\phi = \pi/4)$ gate performed on a charge qubit in the absence of noise can be expressed as
\[ 1 - F_\theta(\phi = \pi/4) = 2\gamma^2 [1 - \cos(2\theta:\overline{\omega}_{\text{res}}/\Omega)] + 4\gamma^3 \sin(\theta) \sin(2\theta:\overline{\omega}_{\text{res}}/\Omega). \] (S97)
For any rotation angle $\theta$, the intrinsic infidelity vanishes whenever $N \equiv (2\overline{\omega}_{\text{res}}\theta)/(\Omega\pi)$ is an even integer, producing dips like those observed in Fig. 3 of the main text. However, in the presence of the noise, the dips are suppressed and the infidelity increases. This effect can be reduced by increasing the tunnel coupling $\Delta$ and the driving amplitude $A_\Delta$ simultaneously while keeping the ratio $A_\Delta/\Delta$ fixed, so that the system remains in a dip. This is because the strong-driving effects only depend on the ratio of the driving amplitude and the tunnel coupling $A_\Delta/\Delta$; however, by reducing the gate time $(t_g \propto A_\Delta^{-1})$, we can reduce the effects of charge noise, thus improving the fidelity. This method was applied to the special case of $R_\pi(\phi = \pi/4)$ gates in the main text. To demonstrate that the method works for any $\theta$, we now apply it to $R_\theta(\phi = \pi/4)$ gates for $0 < \theta < \pi$. 
FIG. S1. Infidelity of an $R_\theta(\phi = \pi/4)$ gate performed on a strongly driven charge qubit in the presence of detuning noise while operated in the dip with $N = (2\omega_\text{osc})/(\pi\Omega) = 10$. Note that all infidelities plotted here are $< 0.01$. The qubit is operated at the sweet spot where the detuning $\epsilon = 0$, and the different curves show the results for different tunnel couplings from $\Delta/h = 20$ GHz to 50 GHz, as indicated. The noise spectrum is $1/f$, $\tilde{S} = c_\epsilon^2 2\pi/\omega$, with low (high) angular frequency cutoff $\omega_l/2\pi = 1$ Hz ($\omega_h/2\pi = 256$ GHz), where the low frequency part (frequency smaller than 0.3 MHz) is approximated as quasi-static noise in the simulations. The figure shows simulations with noise amplitude $c_\epsilon = 0 \mu eV$ ($\Delta/h = 20$ GHz, black dots), and $c_\epsilon = 1 \mu eV$ ($\sigma_\epsilon = 6.36 \mu eV$) for $\Delta/h = 20$ GHz (orange dots), $\Delta/h = 35$ GHz (triangles) and $\Delta/h = 50$ GHz (× markers), and also analytic results [Eq. (S98)] for $c_\epsilon = 0 \mu eV$ and $\Delta/h = 20$ GHz (black lines). The simulation shows that as $\Delta$ and $A_\epsilon$ increase, the gate time decreases and the infidelities approach the intrinsic infidelity limit, corresponding to $c_\epsilon = 0 \mu eV$. Note that when $\Delta/h \gtrsim 35$ GHz, these gate fidelities are all greater than 99.9%.

In Fig. S1 we compare numerical results for the intrinsic infidelities ($c_\epsilon = 0 \mu eV$) of $R_\theta(\phi = \pi/4)$ gates with infidelities in the presence of $1/f$ detuning charge noise with $c_\epsilon = 1 \mu eV$ ($\sigma_\epsilon = 6.36 \mu eV$), operated in the dip corresponding to $N = 10$. As usual, the system is operated at the sweet spot ($\epsilon = 0$), and we consider the tunnel couplings $\Delta/h = 20, 35, 50$ GHz. Since the intrinsic infidelity vanishes up to $O[\gamma^3]$ when operating in a dip, its leading order contribution, from Eq. (S12), is given by

$$1 - F_\theta(\pi/4) = 16\gamma^4 \sin^2(\theta/2),$$  

(S98)

which increases as $\theta$ increases, and serves as the lower bound of the infidelity in the presence noise. When $c_\epsilon = 1 \mu eV$ ($\sigma_\epsilon = 6.36 \mu eV$), decoherence dominates the infidelity, which is then much larger than the intrinsic infidelity. By increasing $A_\epsilon$ and $\Delta$ while keeping their ratio fixed, as discussed above, the gate time can be reduced, causing the infidelity to approach the intrinsic infidelity limit, as shown in Fig. S1. This demonstrate that our method for improving the fidelity is applicable to a generic $R_\theta(\phi = \pi/4)$ rotation, not just for the case $\theta = \pi$ that is discussed in the main text.

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