Symmetry relating Gravity with Antigravity: A possible resolution of the
Cosmological Constant Problem?

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(Dated: March 24, 2022)

I discuss possible implications a symmetry relating gravity with antigravity might have for smoothing out of the cosmological constant puzzle. For this purpose, a very simple model with spontaneous symmetry breaking is explored, that is based on Einstein-Hilbert gravity with two self-interacting scalar fields. The second (exotic) scalar particle with negative energy density, could be interpreted, alternatively, as an antigravitating particle with positive energy.

PACS numbers: 98.80.Es, 04.20.Cv, 04.20.-q, 11.30.-j, 11.30.Qc

One of the most profound mysteries in fundamental physics is the cosmological constant problem (see references [1] for recent reviews on this subject). Recently there have been revealed two faces of this problem: 1) Why is the cosmological constant so small? and 2) Why is it comparable to the critical density of the Universe precisely at present? In the present letter I will focus on point 1) of the problem, leaving point 2) for further research.

The physical basis for the cosmological constant \( \Lambda \) are the zero-point vacuum fluctuations. The expectation value of the energy-momentum tensor for vacuum can be written in the Lorentz invariant form \( \langle T_{ab} \rangle_{\text{vac}} = (\Lambda/8\pi G)g_{ab} \), where \( G \) is Newton’s constant. It is divergent for both bosons and fermions. Since bosons and fermions (of identical mass) contribute equally but with opposite sign to the vacuum expectation of physical quantities, supersymmetry was expected to account for the (nearly) zero value of the cosmological constant, through an accurate balance between bosons and fermions in Nature. However, among other objections, the resulting scenario is not the one it is expected to occur if a Universe with an early period of inflation (large \( \Lambda \)) and a very small current value of \( \Lambda \) is to be described [2]. Although other mechanisms and principles, among them a running \( \Lambda \) and the anthropic principle [3,2], have been invoked to solve the cosmological constant (vacuum energy density) puzzle, none of them have been able to give a definitive answer to this question and the problem still remains a mystery.

At present, there is no known fundamental symmetry in Nature which will set to zero the value of \( \Lambda \) [1,2]. Hence, the search for such a symmetry remains a challenge. In the present letter I want to put forward the possibility that such a fundamental symmetry could be the one under the interchange of gravity and antigravity. The idea behind this possibility is that, once this symmetry is assumed, to each gravitating standard model partner (antiparticle), it corresponds an antigravitating partner, whose contribution to the vacuum energy exactly cancels gravitating particle’s contribution [6]. This can be visualized as if there were two distinct vacua: one gravitating and other one that antigravitates so that, the resulting “total” (averaged) vacuum does not gravitates at all. The kind of symmetry I am proposing to account for the (nearly) zero value of the cosmological constant, opens up the possibility that such exotic entities like antigravitating objects, might exist in Nature [7]. Hence, why this kind of objects are not being observed in our Universe? A similar question, this time for antimatter, has been raised before. In this last case, a possible mechanism for generating the desired amount of baryon asymmetry [4], relies on three necessary (Sakharov’s) conditions: i) Baryon number non-conservation, ii) C and CP violation and iii) Deviations from thermal equilibrium. In the same fashion, an answer to the problem of gravitating-antigravitating matter asymmetry could be approached. In this sense, one should expect non-conservation of the charge associated with gravity-antigravity symmetry.

In this letter I call gravity-antigravity transformations (G-aG transformations for short) the following set of simultaneous transformations: \( G \rightarrow -G \) and \( g_{ab} \rightarrow -g_{ab} \). This means that, simultaneous with the interchange of gravity and antigravity, interchange of time-like and space-like domains is also required [8]. It is straightforward noting that, the purely gravity part of the Einstein-Hilbert action \( S = \int d^4x \sqrt{|g|} R/(16\pi G) \), is G-aG symmetric. Actually, under \( g_{ab} \rightarrow -g_{ab} \), the Ricci tensor is unchanged \( R_{ab} \rightarrow R_{ab} \), while the curvature scalar \( R \rightarrow -R \). The introduction of a \( \Lambda \) term in the above Einstein-Hilbert action breaks this symmetry. This fact

[6] Gravitating and antigravitating partners differ only in the sign of the mass parameter, otherwise, in the character of their gravitational interactions. All the other quantum numbers are identical.
[7] This would double the number of existing standard model particles (antiparticles).
[8] The interchange of time-like and space-like domains, simultaneous with the transformation \( G \rightarrow -G \), is justified since, the last transformation can be visualized as the interchange of Planck mass squared and negative (tachyonic) Planck mass squared: \( M_P^2 \rightarrow -M_P^2 \).
hints at the possibility that, precisely, this kind of symmetry could account for a zero value of the cosmological constant $\Lambda$.

Since models with spontaneous symmetry breaking are relevant to the cosmological constant problem [2], I will explore a very simple model, that is based on general relativity plus self-interacting scalar fields with symmetry breaking potentials, as sources of gravity. The starting point will be the following action which includes a single scalar field $\phi$:

$$S = \int \frac{d^4x \sqrt{|g|}}{16\pi G} \left\{ R - (\nabla \phi)^2 - 2V(\phi) \right\},$$

(1)

where $V(\phi)$ is the self-interaction (symmetry breaking) potential. This action respects reflection symmetry $\phi \to -\phi$ if $V$ is an even function. However, it is invariant under G-aG transformations, with the inclusion of reflection, only if, simultaneously, $V(\phi) \to -V(\phi)$. The last transformation, under reflection, restricts the self-interaction potential to be an odd function of $\phi$. Therefore, a very wide class of symmetry breaking potentials (including the typical "Mexican hat" potential) is ruled out by G-aG symmetry. In order to extend this symmetry to any kind of potential, one can introduce a second self-interacting scalar field $\bar{\phi}$, with the wrong sign of both kinetic and potential energy terms, in the action [11] and, at the same time, to introduce the innocuous factor $\epsilon \equiv G/|G|$ ($\epsilon = +1$ for gravity and $\epsilon = -1$ for antigravity) in both kinetic terms for $\phi$ and $\bar{\phi}$. The improved action is:

$$S = \int \frac{d^4x \sqrt{|g|}}{16\pi \epsilon |G|} \left\{ R - \epsilon (\nabla \phi)^2 - 2V(\phi) \right\}
+ \epsilon (\nabla \bar{\phi})^2 + 2V(\bar{\phi}).$$

(2)

Notice I kept the same symbol $V$ for the self-interacting potential, meaning that the functional form of both $V(\phi)$ and $V(\bar{\phi})$ is the same. The fact that both kinetic and potential energies of $\phi$ enter with the wrong sign, means that the energy density of the second scalar field $\bar{\phi} = \epsilon (\nabla \bar{\phi})^2/2 - V(\bar{\phi})$ is negative if the potential $V$ is a positive definite function. A second interpretation could be that, $\phi$ has positive energy ($\rho_\phi = \rho_+ = -\rho_\phi > 0$), but it antigravitates (\epsilon \to -\epsilon). This second interpretation is apparent if one realizes that, in the right-hand side (RHS) of Einstein’s equations, that are derivable from [2], one has the combination: $8\pi\epsilon |G|(T^\phi_{ab} - T^{\bar{\phi}}_{ab})$, where the stress-energy tensor for scalar field degrees of freedom is defined in the usual way (except for the innocuous factor $\epsilon$): $T^\chi_{ab} = \epsilon (\nabla_a \chi) (\nabla_b \chi - \frac{1}{2} g_{ab} (\nabla \chi)^2) - g_{ab} V(\chi)$ ($\chi$ is the collective name for $\phi$ and $\bar{\phi}$). Hence, one could hold the view that, the minus sign in the second term of the RHS of Einstein’s field equations, could be absorbed into the innocuous factor $\epsilon$: $8\pi |G| [\epsilon T^\phi_{ab} + (-\epsilon) T^{\bar{\phi}}_{ab})$. Any way, $\phi$ represents a kind of exotic particle, whose existence could be justified only in quantum systems like the quantum vacuum [10].

The action [2] is explicitly invariant under the (enhanced) set of G-aG transformations [11]:

$$\epsilon \to -\epsilon, \ g_{ab} \to -g_{ab}, \ \phi \leftrightarrow \bar{\phi}. \quad (3)$$

A remarkable property of this model is that the Klein-Gordon equations for both $\phi$ and $\bar{\phi}$

$$\square \phi = \frac{dV(\phi)}{d\phi}, \ \square \bar{\phi} = \frac{dV(\bar{\phi})}{d\bar{\phi}}, \quad (4)$$

coincide. The consequence is that both fields will tend to run down the potentials towards smaller energies. Therefore, if $V$ has global minima, both $\phi$ and $\bar{\phi}$ will tend to approach one of these minima. This is, precisely, the key ingredient in the present model to explain the small value of the vacuum energy density, through weak violation of G-aG symmetry. To illustrate this point, let us consider the "Mexican hat" potential:

$$V(\chi) = V_0 - \frac{\mu^2}{2} \chi^2 + \frac{\lambda_\chi}{4} \chi^4, \ \lambda_\chi > 0. \quad (5)$$

The symmetric state $(\phi, \bar{\phi}) = (0, 0)$ is unstable and the system settles in one of the following ground states $(\phi, \bar{\phi})$: $(\sqrt{\mu^2/\lambda}, \sqrt{\mu^2/\lambda})$, $(-\sqrt{\mu^2/\lambda}, -\sqrt{\mu^2/\lambda})$, $(-\sqrt{\mu^2/\lambda}, \sqrt{\mu^2/\lambda})$, $(-\sqrt{\mu^2/\lambda}, -\sqrt{\mu^2/\lambda})$. This means that the reflection symmetry $\phi \to -\phi, \bar{\phi} \to -\bar{\phi}$, inherent in theory [2] with potential [5], is spontaneously broken. The immediate consequence is that the stress-energy tensor of the vacuum of the theory takes the Lorentz invariant form

$$T_{ab} = \frac{\Lambda}{8\pi |G|} g_{ab}, \quad (6)$$

where the cosmological constant $\Lambda = ((\mu^4/\lambda) - (\bar{\mu}^4/\lambda))/4\epsilon \ (\mu \equiv \mu_\phi, \bar{\mu} \equiv \mu_{\bar{\phi}}, \text{etc.})$. Now it is apparent that the resulting theory (with broken reflection symmetry), that is given by the action $S = \int d^4x \sqrt{|g|} \left\{ R - \frac{\Lambda}{16\pi |G|} \right\}$, is invariant under G-aG transformations [4] only if $(\mu, \bar{\mu}) = (\bar{\mu}, \mu) \Rightarrow \Lambda = 0$. In consequence, the small observed value $\Lambda \sim 10^{-47} \text{GeV}^4$, denotes a weak

[9] I chose the following signature for the metric: $(- + + +)$.

[10] To learn about the recurrence of negative energy densities in physics, see reference [3].

[11] If the potential $V$ is an even function, then the reflection $\phi \to -\phi, \ \bar{\phi} \to -\bar{\phi}$, is also a symmetry of [2].
violation of G-aG symmetry, that is due to a degeneracy of reflection symmetry energy scales \( \alpha \equiv \mu^4 / 8\pi |G|\lambda \) and 
\( \bar{\alpha} \equiv \bar{\mu}^4 / 8\pi |G|\bar{\lambda} \).

Which physical mechanism is responsible for a small violation of G-aG symmetry, is a question that could be clarified only once exotic (in principle, antigravitating) fields like \( \bar{\phi} \), are built into a fundamental theory of the physical interactions, including gravity.[12] In the absence of such fundamental theory of the unified interactions, one might only conjecture on the physical origin of the degeneracy in reflection symmetry energy scales. In this regard, a possible origin of the aforementioned degeneracy could be associated with the following reasoning. It is an observational fact that, almost all real (observable) standard model particles in Nature gravitate. Therefore, their interactions with gravitating and antigravitating vacuum particles are of different nature. The difference is accentuated in the early stages of cosmic evolution, due to strongest character of gravitational interactions for high-energy scales, while, at present, the difference is very tiny, due to the very weak intensity of gravitating effects at the low-energies prevailing in the Universe. This will fit well a scenario, in which a large value of \( \Lambda \) is required to produce the due amount of inflation in the early Universe, while a small current value of \( \Lambda \) will reproduce the present stage in the cosmic evolution.[2] A less physically inspired possibility is that, during the course of the cosmic evolution, the relative difference between reflection symmetry breaking scales is nearly constant: \( (\alpha - \bar{\alpha}) / \alpha \sim \text{const} \). Therefore, for larger (reflection) symmetry breaking scales, the cosmological constant \( \Lambda = 2\pi |G| (\alpha - \bar{\alpha}) / \varepsilon \) is larger.

Although the model explored in this letter is far from giving a final answer to the cosmological constant puzzle and, besides, it is incomplete in that it gives no explanation about a realistic mechanism for present small violation of G-aG symmetry, nevertheless, it hints at a possible connection between this (would be) fundamental symmetry and the vacuum energy density. The study of this symmetry could be relevant, also, to the understanding of the role orbifold symmetry plays in Randall-Sundrum (RS) brane models[5]. Actually, in RS scenario \( M_{P,1}^2 \propto \int dy M_{P,1}^3 \) (\( y \) accounts for the extra coordinate) so, symmetry under \( y \to -y \Leftrightarrow M_{P,1}^2 \to -M_{P,1}^2 (G \to -G) \). Treatment of the second face of the cosmological constant puzzle (see the introductory part of this letter), within the present approach, requires of further research

I am grateful to the MES of Cuba for financial support of this research.

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