Dilepton production spectrum above $T_c$ with a lattice quark propagator

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The dilepton production rate from the deconfined medium is analyzed with the photon self-energies constructed from quark propagators obtained by lattice numerical simulation for two values of temperature $T = 1.5T_c$ and $3T_c$ above the critical temperature $T_c$. The photon self-energy is calculated by the Schwinger-Dyson equation with the lattice quark propagator and a vertex function determined so as to satisfy the Ward-Takahashi identity. The obtained dilepton production rate at zero momentum exhibits divergences reflecting van Hove singularity, and is significantly enhanced around $\omega \approx T$ compared with the rate obtained by the perturbative analysis.

I. INTRODUCTION

Ultra-relativistic heavy ion collisions are the unique method to produce the deconfined medium experimentally on the Earth [1,2]. Various observables are measured in the experiments [1] to reveal properties of the deconfined medium and a variety of phenomena which come into play during the time evolution of the hot medium. Among these observables, the dilepton production yield has a characteristic feature that the yield provides us direct signals from the primordial deconfined medium [3], because dileptons once produced in the hot medium do not interact with and pass through the medium owing to their colorless nature.

The dilepton yield observed experimentally consists of the sum of the dilepton production in each stage of the time evolution of the hot medium. The dilepton production in heavy ion collisions is roughly classified into three processes except for final state hadronic decays. The first one is the hard process, in which dileptons are produced by scatterings of hard partons in the colliding nuclei. The second and third ones are thermal radiations from the deconfined medium and confined medium, respectively. The dileptons in low invariant mass region are usually expected to be dominated by these thermal radiations, while dileptons from the hard process have relatively high transverse momenta and large invariant masses. Experimental results on the dilepton production yield are usually compared with the baseline called cocktail [4,5], although the discrepancy on the magnitude of the enhancement between the two experimental groups has not been settled down yet. The result of PHENIX Collaboration shows that the yield at $m \approx 500$ MeV is about one order larger than the cocktail [4].

When one performs an estimate of the dilepton production yield, one first calculates the dilepton production rate per unit time and unit volume from a static medium. The dilepton production yield is then given by the space-time integral of the rate from each volume element of the medium. The dilepton production rate of a static medium is proportional to the imaginary part of virtual photon self-energy [7,9]. When the temperature ($T$) is asymptotically high, the photon self-energy can be calculated perturbatively. Using the hard thermal loop (HTL) resummed perturbation theory [10,11], the dilepton production rate was calculated in Ref. [12] for lepton pairs with zero total three-momentum, and the result was extended in Ref. [13] to nonzero momentum. It is, however, nontrivial whether or not such perturbative analyses well describe the production rate from the deconfined medium near the critical temperature $T_c$, which has turned out to be a strongly-coupled system [1]. Moreover, it is known that the perturbative analyses in Refs. [12,13] are modified by proper inclusion of higher-order terms [14]. The analysis of higher order terms, however, is complicated and it is still under debate whether the scheme is valid for the whole kinetic region [14,15]. For the description of the dilepton production rate in the deconfined phase near $T_c$, therefore, it is desirable to evaluate the rate without resort to perturbative methods. In particular, the large enhancement observed at PHENIX [4] suggests a possibility that the dilepton production from the strongly-coupled medium above $T_c$ has a large enhancement compared with the perturbative results used in the previous analyses [10].

There are several attempts of non-perturbative analyses for the dilepton production rate with lattice QCD [17,18]. When one investigates the rate on the lattice one must take an analytic continuation from the imaginary-time correlator computable on the lattice to the real-time photon self-energy. This procedure, however, is an ill-posed problem, since the information on the imaginary-time correlators for discrete imaginary-time points ob-
tained on the lattice is insufficient to reconstruct the continuous real-time function by itself [19]. In Ref. [18], an ansatz for the spectral function is introduced to avoid this problem. An alternative way is to use the Bayesian analysis such as the maximum entropy method [19]. The lattice correlator, however, is insensitive to the structure of the spectrum in the low energy region [20]. The estimate of the low-energy spectrum on the lattice, therefore, is a difficult problem with the presently existing methodologies.

The exact non-perturbative photon self-energy can be calculated by the Schwinger-Dyson equation (SDE) if we have the full quark propagator and the photon-quark vertex function. Recently, an analysis of the non-perturbative quark propagator above \( T_c \) was performed on the lattice in the quenched approximation in Landau gauge [21–23]. In this series of analyses, the quark propagator was analyzed with the two-pole ansatz for the real-time propagator in order to carry out the analytic continuation. This pole ansatz was employed motivated by the study of fermion propagator at nonzero temperature [24–25]. It has been shown [21–23] that this simple ansatz can reproduce the quark correlator obtained on the lattice over a rather wide range of bare quark mass and momentum. It is therefore expected that the obtained quark propagator well grasps the gist of the non-perturbative nature of the quark propagator.

The purpose of the present study is to analyze the dilepton production rate using this quark propagator. We construct the SDE with the quark propagators obtained on the lattice in Ref. [23] and with the vertex function constructed so as to satisfy the Ward-Takahashi identity (WTI). Our formalism, therefore, fulfills the conservation law of electric current. In this analysis, we show that the obtained dilepton production rate exhibits an enhancement of one order or more compared with the one from the free quark gas. Compared with the perturbative result in Ref. [12], our result has a qualitatively similar behavior at low \( m \) region, while it exhibits an enhancement around \( m \sim T \) owing to van Hove singularity. The effect of the vertex correction is also discussed in detail.

The outline of this paper is as follows. In the next section we introduce the SDE for the photon self-energy and its components, lattice quark propagator and a vertex function satisfying the WTI. In Sec. III, we then solve the SDE and present the form of the dilepton production rate in our formalism. The rate without the vertex correction is also calculated in this section. We then present the numerical result in Sec. IV. The final section is devoted to a short summary.

![Diagramatic representation of the Schwinger-Dyson equation for the photon self-energy, Eq. (2). The shaded circles represent the full quark propagator and the full vertex function.](image)

**II. SCHWINGER-DYSON EQUATION FOR PHOTON SELF-ENERGY**

A. Schwinger-Dyson equation

As dileptons are emitted from decays of virtual photons, the dilepton production rate from a medium per unit time and unit volume is related to the retarded self-energy \( \Pi^R_{\mu\nu}(\omega, q) \) of the virtual photon as [7–9]

\[
\frac{d\Gamma}{d\omega dq} = \frac{\alpha}{12\pi^4} \frac{1}{Q^2} \frac{1}{e^{\beta\omega} - 1} \text{Im}\Pi^R_{\mu\nu}(\omega, q) \tag{1}
\]

at the leading order of fine structure constant \( \alpha \) with \( Q^2 = \omega^2 - q^2 \) and the inverse temperature \( \beta = 1/T \). With the SDE in Matsubara formalism, the exact photon self-energy is given by the full quark propagator \( S(P) \) and the full photon-quark vertex \( \Gamma_\mu(P + Q, P) \) as

\[
\Pi_{\mu\nu}(i\omega_m, q) = -\sum_f e_f^2 T \sum_n \int \frac{d^3p}{(2\pi)^3} \text{Tr}_C \text{Tr}_D \left[ S(P) \gamma_\mu S(P + Q) \Gamma_\nu(P + Q, P) \right], \tag{2}
\]

where \( \omega_m = 2\pi T m \) and \( \nu_n = (2n + 1)\pi T \) with integers \( m \) and \( n \) are the Matsubara frequencies for bosons and fermions, respectively, \( P_\mu = (i\nu_n, p) \) is the four-momentum of quarks, and \( e_f \) is the electric charge of a quark with an index \( f \) representing the quark flavor. The color, flavor, and Dirac indices of \( S(P) \) are suppressed for notational simplicity. \( \text{Tr}_C \) and \( \text{Tr}_D \) denote the trace over color and Dirac indices, respectively. We note that since we take Landau gauge in this calculation, off-diagonal elements in color space disappear. As a result, the trace over the color indices gives a factor 3 in Eq. (2). Equation (2) is graphically shown in Fig. 1 in which the shaded circles represent the full propagator and vertex function. The retarded photon self-energy is obtained by the analytic continuation,

\[
\Pi^R_{\mu\nu}(\omega, q) = \Pi_{\mu\nu}(i\omega_m, q)\mid_{i\omega_m \rightarrow \omega + i\eta}. \tag{3}
\]

In the following, we consider the two-flavor system with degenerate \( u \) and \( d \) quarks, in which \( \sum_f e_f^2 = 5e^2/9 \). In this study we also limit our attention to the \( q = 0 \) case.
B. Lattice Quark Propagator and Spectral Function

In the present study, we use the quark propagator obtained on the quenched lattice in Ref. [23] as the full quark propagator in Eq. (2). In this subsection, after a brief review on the general property of the quark propagator we describe how to implement the results in Ref. [23] in our analysis.

On the lattice with a gauge fixing, one can measure the imaginary-time quark propagator,

\[ S_{\mu\nu}(\tau, \mathbf{p}) = \int d^3x e^{-i\mathbf{p}\cdot\mathbf{x}} \langle \bar{\psi}(\tau, \mathbf{x}) \psi(0, 0) \rangle, \]

where \( \bar{\psi}(\tau, \mathbf{x}) \) is the quark field with the Dirac index \( \mu \). Here, \( \tau \) is the imaginary time restricted to the interval \( 0 \leq \tau < \beta \). For the moment, the Dirac indices \( \mu \) and \( \nu \) of the quark propagator are explicitly shown. The Fourier transform of the quark correlator,

\[ S_{\mu\nu}(i\nu_n, \mathbf{p}) = \int_0^\beta d\tau e^{i\nu_n\tau} S_{\mu\nu}(\tau, \mathbf{p}), \]

is written in the spectral representation as

\[ S_{\mu\nu}(i\nu_n, \mathbf{p}) = -\int_{-\infty}^{\infty} d\nu' \rho_{\mu\nu}(\nu', \mathbf{p}) (\nu' - i\nu_n), \]

with the quark spectral function \( \rho_{\mu\nu}(\nu', \mathbf{p}) \). The spectral function is related to the imaginary-time correlator Eq. (4) as

\[ S_{\mu\nu}(\tau, \mathbf{p}) = \int_{-\infty}^{\infty} d\nu \frac{e^{(1/2 - \tau/\beta)\nu}}{e^{\beta\nu/2} + e^{-\beta\nu/2}} \rho_{\mu\nu}(\nu, \mathbf{p}). \]

In the deconfined phase in which the chiral symmetry is restored, the quark propagator anticommutes with \( \gamma_5 \). In this case, the spectral function can be decomposed with the projection operators \( \Lambda_{\pm}(\mathbf{p}) = (1 \pm \gamma_0 \mathbf{p} \cdot \mathbf{\gamma})/2 \) as

\[ \rho_{\mu\nu}(\nu, \mathbf{p}) = \rho_+(\nu, \mathbf{p}) \Lambda_+(\mathbf{p}) \gamma_0 + \rho_-(\nu, \mathbf{p}) \Lambda_-(\mathbf{p}) \gamma_0, \]

with \( \mathbf{p} = |\mathbf{p}|, \mathbf{\hat{p}} = \mathbf{p}/|\mathbf{p}| \), and

\[ \rho_{\pm}(\nu, \mathbf{p}) = \frac{1}{2} \text{Tr}_{\text{QCD}} (\rho(\nu, \mathbf{p}) \gamma_0 \Lambda_{\pm}(\mathbf{p})). \]

It is shown from the anticommutation relations of the Dirac fields that the decomposed spectral functions satisfy the sum rules,

\[ \int d\nu \rho_{\pm}(\nu, \mathbf{p}) = 1. \]

Using charge conjugation symmetry, one can show that \( \rho_{\pm}(\nu, \mathbf{p}) \) satisfy

\[ \rho_{\pm}(\nu, \mathbf{p}) = \rho_{\mp}(-\nu, \mathbf{p}). \]

On the lattice, one can measure the imaginary-time correlator Eq. (4) for discrete imaginary times. To obtain the quark propagator one has to deduce the spectral function from this information. In Refs. [21, 23], the quark correlator in Landau gauge is analyzed on the lattice with the quenched approximation, and the quark spectral function is analyzed with the two-pole ansatz,

\[ \rho_+(\nu, p) = Z_+(p) \delta(\nu - \nu_+(p)) + Z_-(p) \delta(\nu + \nu_-(p)), \]

where \( Z_{\pm}(p) \) and \( \nu_{\pm}(p) \) are the residues and positions of the poles, respectively. The four parameters, \( Z_{\pm}(p) \) and \( \nu_{\pm}(p) \), are determined by fitting the correlators obtained on the lattice for each \( p \). The two poles in Eq. (12) at \( \nu_+(p) \) and \( \nu_-(p) \), respectively, correspond to the normal and plasmino modes in the HTL approximation. In fact, the study of the momentum and bare quark mass, \( m_0 \), dependences of the fitting parameters [21, 23] shows that the behavior of these parameters is consistent with this observation [14, 20]; for example, for large \( m_0 \) or \( p \) the residue of the plasmino mode \( Z_{-}(p) \) becomes small and the propagator approaches that of the free quark. The restoration of the chiral symmetry for massless quarks above \( T_c \) is also checked explicitly on the lattice by measuring of the scalar term in the massless quark propagator [23].

In Fig. 2, we show the fitting result of each parameter in Eq. (12) for massless quarks as a function of \( p \) for \( T = 1.5T_c \) and \( 3T_c \) obtained in Ref. [23]. These analyses are performed on the lattice with the volume \( 128^3 \times 16 \), where both the lattice spacing and finite volume effects
are found to be small \[23\]. In the upper panel, \( p \) dependences of \( \nu_\pm(p) \), i.e. the dispersion relations of the normal and plasmino modes, are shown by the open symbols. The vertical and horizontal axes are normalized by the thermal mass \( m_T \) defined by the value of \( \nu_\pm(p) \) at \( p = 0 \). The value of \( m_T \) obtained on the lattice after the extrapolation to the infinite volume limit is \( m_T/T = 0.768(11) \) and \( 0.725(14) \) at \( T = 1.3T_c \) and \( 3T_c \), respectively \[23\].

The lower panel shows the relative weight of the plasmino residue, \( Z_-(Z_+ + Z_-) \). This figure shows that the weight becomes smaller as \( p \) increases, which indicates that the quark propagator for large \( p/T \) is dominated by the normal mode. A result similar to that shown in Fig. 2 is obtained by the Schwinger-Dyson approach for the quark propagator \[27\].

Although the lattice data are available only for discrete values of \( p \), we must have the quark propagator as a continuous function of \( p \) to solve the SDE. For this purpose, we take the interpolation and extrapolation of the lattice data by the cubic spline method. From the charge conjugation symmetry one can show that \( d\nu_+(p)/dp = -d\nu_-(p)/dp \), \( d^2\nu_+(p)/dp^2 = d^2\nu_-(p)/dp^2 \), and \( Z_+(p) = Z_-(p) \) for \( p = 0 \) \[21]\ [23]. These properties are taken into account in our cubic spline interpolation. The lattice data are available only in the momentum range \( p/T \lesssim 4.7 \). To take extrapolations to higher momenta, we extrapolate the parameters using an exponentially damping form for \( Z_-/(Z_+ + Z_-) \),

\[
Z_-/(Z_+ + Z_-) = \text{Re}^{-\alpha p},
\]

and \( \nu_\pm(p) \) are extrapolated by functions,

\[
\nu_\pm(p) = p + \beta_1^\pm e^{-\beta_2^\pm p},
\]

which exponentially approach the light cone for large \( p \). The parameters \( R, \alpha \), and \( \beta_2^\pm \) are determined in the cubic spline analysis. The \( p \) dependence of each parameter determined in this way is shown by the solid lines in Fig. 2. We tested another extrapolation form by a polynomial, \( \nu_\pm(p) = p + \beta_1^\pm + \beta_2^\pm p^2 + \cdots \), but found that it hardly changes the dispersion relation. Finally, we fix

\[
Z_+ + Z_- = 1
\]

throughout this paper to satisfy the sum rule Eq. (10).

With the two-pole form of the spectral function Eq. (12), the quark propagator reads

\[
S(i\nu_n, p) = S_+(i\nu_n, p)\Lambda_+(p)\gamma_0 + S_-(i\nu_n, p)\Lambda_-(p)\gamma_0,
\]

where

\[
S_s(i\nu_n, p) = \frac{Z_+(p)}{i\nu_n - s\nu_+(p)} + \frac{Z_-(p)}{i\nu_n + s\nu_-(p)},
\]

and \( s = \pm 1 \) on the right-hand side are understood as the numbers \( \pm 1 \). Correspondingly, the inverse propagator is given by

\[
S^{-1}(i\nu_n, p) = \sum_{s=\pm} S_s^{-1}(i\nu_n, p)\gamma_0\Lambda_s(p),
\]

with

\[
S_s^{-1}(i\nu_n, p) = \frac{(i\nu_n - s\nu_+(p))(i\nu_n + s\nu_-(p))}{i\nu_n - sE(p)},
\]

and

\[
E(p) = -Z_+(p)\nu_- + Z_-(p)\nu_+.
\]

Note that the inverse propagator has poles at \( \nu_n = \pm E(p) \). These poles inevitably appear in the multipole ansatz, because the propagator Eq. (17) has one zero point in the range of \( \omega \) surrounded by two poles. The form of the inverse propagator Eq. (19) will be used in the construction of the vertex function. We will see that the poles at \( \nu_n = \pm E(p) \) give rise to additional terms in the dilepton production rate.

C. Vertex Function

The SDE, Eq. (2), requires the full photon-quark vertex \( \Gamma_\mu(P + Q, P) \) besides the full quark propagator. So far, the evaluation of \( \Gamma_\mu(P + Q, P) \) on the lattice at nonzero temperature has not been performed to the best of the authors’ knowledge. In the present study, we construct the vertex function from the lattice quark propagator respecting the Ward-Takahashi identity (WTI) as follows.

The gauge invariance requires that the vertex function must fulfill the WTI

\[
Q_\mu \Gamma_\nu(P + Q, P) = S^{-1}(P + Q) - S^{-1}(P),
\]

with the inverse quark propagator \( S^{-1}(P) \). For \( q = 0 \), the temporal component \( \Gamma_0 \) is completely determined only by this constraint as follows. First, in this case \( q \cdot \Gamma \) should vanish provided that \( \Gamma_i \) \( (i = 1, 2, 3) \) are not singular at \( q = 0 \). Then, by substituting \( q \cdot \Gamma = 0 \) in Eq. (21) one obtains

\[
\Gamma_0(i\omega_m + i\nu_n, P + i\nu_n, P) = \frac{1}{\omega_m} [S^{-1}(i\omega_m + i\nu_n, P) - S^{-1}(i\nu_n, P)].
\]

On the other hand, the spatial components \( \Gamma_i \) cannot be determined only with Eq. (21) [28]. In the present study, we employ an approximation to neglect the \( q \) dependence of \( \Gamma_0(i\omega_m + i\nu_n, P + q, i\nu_n, P) \) at \( q = 0 \). In other words we assume that

\[
\partial \Gamma_0(i\omega_m + i\nu_n, P + q, i\nu_n, P)/\partial q|_{q=0} = 0.
\]

With this approximation and Eq. (21), one obtains

\[
q^i \Gamma_i(i\omega_m + i\nu_n, P + q, i\nu_n, P) = S^{-1}(i\omega_m + i\nu_n, P + q) - S^{-1}(i\omega_m + i\nu_n, P).
\]


By taking the leading-order terms in $\mathbf{q}$ on the both sides, one has

\begin{align}
\Gamma_i(i\omega_m + i\nu_n, \mathbf{p}; i\nu_n, \mathbf{p}) & = \frac{\partial S^{-1}}{\partial \mathbf{p}^i}(i\omega_m + i\nu_n, \mathbf{p}) \\
& = \sum_{s=\pm} \frac{\partial S^{-1}_s(i\omega_m + i\nu_n, p)}{\partial p^i} \gamma_0 \Lambda_s(p) \\
& + \sum_{s=\pm} S^{-1}_s(i\omega_m + i\nu_n, p) \gamma_0 \frac{\partial \Lambda_s(p)}{\partial p^i},
\end{align}

(25)

where in the second equality, we used Eq. (18).

We note that there is no a priori justification of Eq. (23). By expanding $\Gamma_0$ with respect to $\mathbf{q}$ at $\mathbf{q} = 0$, one obtains

\begin{align}
i\omega_m \Gamma_0(i\omega_m + i\nu_n, \mathbf{p} + \mathbf{q}; i\nu_n, \mathbf{p}) & = S^{-1}(i\omega_m + i\nu_n, \mathbf{p}) - S^{-1}(i\nu_n, \mathbf{p}) \\
& + \mathbf{q} \cdot \mathbf{p} \gamma_0 A(i\omega_m + i\nu_n, i\nu_n, \mathbf{p}^2) \\
& + \mathbf{q} \cdot \gamma C(i\omega_m + i\nu_n, i\nu_n, \mathbf{p}^2) + O(q^2),
\end{align}

(26)

where $A$, $B$, and $C$ are unknown functions. Our approximation corresponds to neglecting these functions. Although these functions do not affect Eq. (22) at $\mathbf{q} = 0$, the corresponding terms appear in Eq. (25) when these functions are nonzero. The determination of the non-perturbative form of the photon-quark and gluon-quark vertices is generally difficult, and various approximations have been employed in studies of the SDE [28–31]. It should be emphasized that the vertex functions, Eqs. (22) and (25), satisfy the WTI and thus are advantageous in light of the gauge invariance among various ansätze on the vertex function. $\Gamma_0$, Eq. (23), is the same as that obtained in Ref. [28] since it is uniquely determined only from the WTI. On the other hand, $\Gamma_i$ differ from the ones in Ref. [28], even when only the longitudinal part in Ref. [28] is concerned. Introduction of the functions given in Eq. (26) fills this difference.

III. DILEPTON PRODUCTION RATE

The goal of the present study is to obtain the dilepton production rate with the lattice quark propagators and the vertex function discussed in the previous section. In this section, however, before the analysis of the full manipulation we first see the dilepton production rates in simpler cases: (1) free quark gas in Sec. IIIA and (2) case with the lattice quark propagators but with the bare vertex function in Sec. IIIB. The full analysis is then presented in Sec. IIIC.

A. Free quark gas

The photon self-energy for the massless free quark gas is obtained by substituting the free quark propagator $S(i\nu_n, \mathbf{p}) = 1/(i\nu_n - \mathbf{p} \cdot \gamma)$ and bare vertex function $\Gamma_\mu = \gamma_\mu$ into Eq. (2). The result of the dilepton production rate for the massless two-flavor case is given by

\begin{equation}
\frac{d\Gamma}{d\omega d^2 \mathbf{q}} |_{\omega = 0} = \frac{5\alpha^2}{36\pi^4} \left( f(\frac{\omega}{2}) \right)^2,
\end{equation}

(27)

where $f(\omega) = 1/(e^{\beta \omega} + 1)$ is the Fermi distribution function.

B. Dilepton production rate without vertex correction

Next, we calculate the photon self-energy and the dilepton production rate with the lattice quark propagators Eq. (16) but with $\Gamma_\mu = \gamma_\mu$. The photon self-energy obtained in this way, of course, does not fulfill the gauge invariance. The result obtained here, however, is helpful in understanding the effect of the vertex correction, i.e. the role of the WTI.

The photon self-energy with the bare vertex is given by

\begin{equation}
\Pi_{\mu\nu}(i\omega_m, \mathbf{q}) = -\frac{5\epsilon^2}{3} T \sum_n \int \frac{d^3 p}{(2\pi)^3} \text{Tr}_D[S(i\nu_n, \mathbf{p}) \gamma_\mu S(i\omega_m + i\nu_n, \mathbf{p} + \mathbf{q}) \gamma_\nu].
\end{equation}

(28)

By substituting Eq. (16) into this formula, we obtain

\begin{equation}
\Pi_{\mu\nu}(i\omega_m, 0) = -\frac{5\epsilon^2}{3} T \sum_n \int \frac{d^3 p}{(2\pi)^3} \sum_{s,t=\pm} S_s(\nu_n, p) S_t(i\omega_m + i\nu_n, p) \\
\times \text{Tr}_D[\Lambda_s \gamma_\mu \Lambda_t \gamma_\nu].
\end{equation}

(29)

The trace in Eq. (29) is calculated with

\begin{equation}
\text{Tr}_D[\Lambda_s(\mathbf{p}) \Lambda_t(\mathbf{p})] = 2\delta_{st},
\end{equation}

(30)

\begin{equation}
\text{Tr}_D[\Lambda_s(\mathbf{p}) \gamma_\mu \Lambda_t(\mathbf{p}) \gamma_\nu] = 2\delta_{s,-t} + 2st\tilde{p}_t^2,
\end{equation}

(31)

where it is understood that

\begin{equation}
\delta_{++} = \delta_{--} = 1, \quad \delta_{+-} = \delta_{-+} = 0,
\end{equation}

(32)

\begin{equation}
\delta_{s,-t} = \delta_{s,t} \quad (\text{for } t = \pm).
\end{equation}

(33)

Substituting Eqs. (30), (31), and $\sum_t\tilde{p}_t^2 = 1$ into Eq. (29), one obtains

\begin{equation}
\Pi_{\mu}(i\omega_m, 0) = \frac{20\epsilon^2}{3} T \sum_n \int \frac{d^3 p}{(2\pi)^3} \sum_{s=\pm} S_s(i\nu_n, p) S_{-s}(i\omega_m + i\nu_n, p),
\end{equation}

(34)
where

$$S_{-s}(i\nu_n + i\omega_m, p) = S_{\pi}(i\nu_n + i\omega_m, p) \quad (\text{for } s = \pm).$$  

(35)

Using Eq. [17] and taking the Matsubara sum and the analytic continuation $i\omega_m \to \omega + i\eta$, we obtain

$$\Pi^{R,\mu}_\mu(\omega, 0) = \frac{-4\alpha}{3\pi} \int_0^\infty dp \, p^2 \left\{ \frac{Z_+(p)^2 (1 - 2f(\nu_+(p)))}{\omega - 2\nu_+(p) + i\eta} + \frac{Z_-(p)^2 (1 - 2f(\nu_-(p)))}{\omega - 2\nu_-(p) + i\eta} + \frac{2Z_+(p)Z_-(p)(f(\nu_-(p)) - f_+(\nu_+(p)))}{\omega - \nu_+(p) + \nu_-(p) + i\eta} \right\}. \quad (36)$$

Taking the imaginary part of this result, we have

$$\text{Im}\Pi^{R,\mu}_\mu(\omega, 0) = \frac{4\alpha}{3} \int_0^\infty dp \, p^2 \left\{ \frac{Z_+(p)^2 \delta(\omega - 2\nu_+(p)) (1 - 2f(\nu_+(p)))}{\omega - 2\nu_+(p) + i\eta} + \frac{Z_-(p)^2 \delta(\omega - 2\nu_-(p)) (1 - 2f(\nu_-(p)))}{\omega - 2\nu_-(p) + i\eta} + \frac{2Z_+(p)Z_-(p)\delta(\omega - \nu_+(p) + \nu_-(p)) (f(\nu_-(p)) - f_+(\nu_+(p)))}{\omega - \nu_+(p) + \nu_-(p) + i\eta} \right\}. \quad (37)$$

The imaginary part of the photon self-energy is the difference between the annihilation and production rates of virtual photons in medium. The three terms in Eq. (37) represent different annihilation and production processes of a virtual photon. The first and second terms in Eq. (37) represent the production of a virtual photon through the pair annihilation of two normal modes and two plasmino modes, respectively, which are dia-

FIG. 3. Photon production processes

gramatically shown in Fig. 3 (a), and their inverse processes. This can be checked from the arguments of the $\delta$-functions and thermal factors in Eq. (37). The $\delta$-function in these terms represents the energy conservation during these processes, and the thermal factor which is rewritten as

$$1 - 2f(\omega) = (1 - f(\omega))^2 - f(\omega)^2,$$  

(38)

is the difference between the products of Pauli blocking effects and thermal distributions. The existence of the residues $Z_\pm(p)$ in these terms is similarly understood. The last term in Eq. (37) represents the Landau damping between normal and plasmino modes, which is diagrammatically shown in Fig. 3 (b). Correspondingly, the thermal factor in this term can be rewritten as

$$f(\omega_1) - f(\omega_2) = f(\omega_1)(1 - f(\omega_2)) - f(\omega_2)(1 - f(\omega_1)).$$  

(39)

The $\delta$-functions in Eq. (37) can be integrated out. The form of the dilepton production rate after the integration over $p$ reads

$$\text{Im}\Pi^{R,\mu}_\mu(\omega, 0) = \frac{4\alpha}{3} \left\{ \frac{p^2 Z_+(p)^2}{2[dv_+(p)/dp]} (1 - 2f(\nu_+(p))) \bigg|_{\omega=2\nu_+(p)} + \frac{p^2 Z_-(p)^2}{2[dv_-(p)/dp]} (1 - 2f(\nu_-(p))) \bigg|_{\omega=2\nu_-(p)} \right\} + \sum_l \left\{ \frac{2p_l^2 Z_+(p_l)Z_-(p_l)}{|dv_+(p_l) - \nu_-(p_l)|/dp} (f(\nu_-(p_l)) - f(\nu_+(p_l))) \bigg|_{\omega=\nu_+(p_l) - \nu_-(p_l)} \right\} \quad (40)$$

where the momentum $p$ in each term is given by the condition arising from the $\delta$-functions in Eq. (37). Each term can take nonzero values only when there exist momenta satisfying this condition for a given $\omega$. This gives
the condition for \( \omega \) at which each term takes a nonzero value. For example, the first term takes nonzero values for \( \omega > 2m_T \). Because the second and third terms can have multiple solutions of \( p \) for a fixed \( \omega \), we represent this possibility by the sum over \( \ell \). It is also notable that each term in Eq. (40) is inversely proportional to the derivative of \( \nu \) with respect to the density of states of each mode. Accordingly, the dilepton production rate diverges when the derivatives vanish. Such divergence is known as van Hove singularity. In Sec. IV, we will see the appearance of such singularities in the dilepton spectrum.

### C. Dilepton production rate with vertex correction

Now, let us calculate the dilepton production rate with the lattice quark propagator Eq. (11) and the full vertex functions Eqs. (22) and (25).

When the full vertex function satisfying the WTI is used in Eq. (2), the temporal component \( \Pi_{00} \) for \( q = 0 \) vanishes. One can easily check this explicitly by substituting Eq. (22) into Eq. (2). For \( \sum_{i=1}^{3} \Pi_{ii} \), by substituting Eqs. (16) and (25) into Eq. (2), one has

\[
\sum_{i=1}^{3} \Pi_{ii}(i\omega_m, 0) = \frac{5e^2}{3} T \sum_{n} \int \frac{d^3p}{(2\pi)^3} \sum_{s, \ell, u = \pm} S_s(i\nu_n, p) S_s(i\nu_n + i\omega_m, p) \\
\times \sum_{i=1}^{3} \left( \delta \Pi_{u}^{-1}(i\nu_n + i\omega_m, p), T_{\text{R}}[\Lambda_s\gamma_0\gamma_i\Lambda_s] + \frac{uS_s^{-1}(i\nu_n + i\omega_m, p)}{2p} T_{\text{R}}[\Lambda_s\gamma_0\gamma_i\Lambda_s(\gamma_i - (p \cdot \gamma)\hat{p}_i)] \right).
\]

(41)

We substitute the following relations for the Dirac traces,

\[
\begin{align*}
\text{Tr}_{\text{D}}[\Lambda_s\gamma_0\gamma_i\Lambda_s] &= 2s\delta_{i0}\delta_{\nu}0, \\
\text{Tr}_{\text{D}}[\Lambda_s\gamma_0\gamma_i\Lambda_s\gamma_i] &= 2s_{i-}\delta_{i0} + 2s\delta_{i0}^2, \\
\text{Tr}_{\text{D}}[\Lambda_s\gamma_0\gamma_i\Lambda_s(\hat{p} \cdot \gamma)] &= 2\hat{p}_i\delta_{i0},
\end{align*}
\]

(42) (43) (44)

and obtain

\[
\sum_{i} \Pi_{ii}(i\omega_m, 0) = \frac{10e^2}{3} T \sum_{n} \int \frac{d^3p}{(2\pi)^3} \sum_{s = \pm} sS_s(i\nu_n, p) \left\{ \frac{\partial \ln S_s^{-1}(i\nu_n + i\omega_m, p)}{\partial p} - \frac{1}{p} \left( 1 - \frac{S_s^{-1}(i\nu_n + i\omega_m, p)}{S_s(i\nu_n + i\omega_m, p)} \right) \right\}.
\]

(45)

where to obtain the first term we used

\[
S_s(i\nu_n, p) \frac{\partial S_s^{-1}(i\nu_n, p)}{\partial p} = \frac{\partial \ln S_s^{-1}(i\nu_n, p)}{\partial p}.
\]

(46)

Up to now, the calculation relies only on the decomposition Eq. (16), which is valid for the chiral symmetric quark propagator, and the form for the vertex Eq. (25). The result Eq. (45) thus is valid for any form of the quark propagator \( S_\pm(i\nu_n, p) \). Now, we use the two-pole form of the quark propagator, Eqs. (17) and (19). The term including \( \partial \ln S_s^{-1}/\partial p \) in Eq. (45) is then calculated to be

\[
\begin{align*}
T \sum_{n} sS_s(i\nu_n, p) \frac{\partial \ln S_s^{-1}(i\nu_n + i\omega_m, p)}{\partial p} & = - T \sum_{n} \left( \frac{Z_+}{i\nu_n - sv_+} + \frac{Z_-}{i\nu_n + sv_-} \right) \left( \frac{dv_+}{dp} / (iv_n + i\omega_m + sv_+) - \frac{dv_-}{dp} / (iv_n + i\omega_m + sv_-) - \frac{dE}{dp} / (iv_n + i\omega_m - sE) \right) \\
& = - \left( \frac{Z_+ dv_+}{dp} / (i\omega_m + sv_+ - sv_-) + \frac{Z_- dv_-}{dp} / (i\omega_m - sv_+ - sv_-) \right) (f(sv_+) + f(sv_-) - 1) \\
& - \frac{Z_+ dE}{dp} / (i\omega_m + sv_+ - sE) (f(sv_+ - f(sE)) - \frac{Z_- dE}{dp} / (i\omega_m - sv_- - sE) (f(sv_- - f(sE))),
\end{align*}
\]

(47)
where the Matsubara sum over \( n \) is taken in the last equality. The remaining part of Eq. (45) is calculated as follows:

\[
T \sum_n s S_s(i\nu_n, p) \left( 1 - \frac{S_- s(i\nu_n + i\omega_m, p)}{S_s(i\nu_n + i\omega_m, p)} \right) \frac{1}{p} \]

\[
= T \sum_n \left( \frac{Z_+}{i\nu_n - s\nu_+} + \frac{Z_-}{i\nu_n + s\nu_-} \right) \left( \frac{F_1}{i\nu_n + i\omega_m + sE} + \frac{F_2}{i\nu_n + i\omega_m + s\nu_+} + \frac{F_3}{i\nu_n + i\omega_m + s\nu_-} \right)
\]

\[
= \frac{Z_+ F_1}{i\omega_m + s\nu_+ - sE} (f(sE) - f(s\nu_+)) + \frac{Z_- F_1}{i\omega_m - s\nu_+ - sE} (f(sE) - f(-s\nu_-))
\]

\[
+ \frac{Z_+ F_2}{i\omega_m + 2s\nu_+} (f(-s\nu_+) - f(s\nu_+)) + \frac{Z_- F_2}{i\omega_m + s\nu_+ - s\nu_-} (f(-s\nu_-) - f(-s\nu_-))
\]

\[
+ \frac{Z_+ F_3}{i\omega_m + s\nu_+ - s\nu_-} (f(s\nu_-) - f(s\nu_+)) + \frac{Z_- F_3}{i\omega_m - 2s\nu_-} (f(s\nu_-) - f(-s\nu_-)),
\]

where

\[
F_1 = -\frac{2E(\nu_+ + E)(\nu_+ - E)}{p(\nu_+ - E)(\nu_+ + E)};
\]

\[
F_2 = \frac{2\nu_+(\nu_+ - E)(\nu_+ - \nu_-)}{p(\nu_+ + E)(\nu_+ + \nu_-)};
\]

\[
F_3 = \frac{2\nu_- (\nu_+ - \nu_-)(\nu_- + E)}{p(\nu_+ + \nu_-)(\nu_- - E)}.
\]

Here, each combination of \( \nu_\pm \) and \( E \) in the parentheses is set to become positive; this can be checked by the relation \( \nu_+ > \nu_- > -E > 0 \).

Combining these results, Eq. (45) is calculated to be

\[
\sum_{i=1}^3 \Pi_{ii}(i\omega_m, 0) = -\frac{10 \alpha^2}{3} \int \frac{d^3p}{(2\pi)^3} \sum_{s=\pm} s \left\{ \frac{2Z_+^2 \nu_+ \bar{\Omega}}{p(\nu_+ + E)} \left( \frac{1 - 2f(\nu_+)}{\nu_+ + E} \bar{\Omega}E - 2\omega_+ \right) \frac{f(\nu_-) - f(\nu_+)}{i\omega_m - s\nu_+ + s\nu_-} \right.
\]

\[
+ 2Z_+ Z_- \bar{\Omega} \frac{E - 2\nu_+ \nu_-}{p(\nu_+ + E)(\nu_- - E)} \frac{f(\nu_-) - f(\nu_+)}{i\omega_m + s\nu_+ + s\nu_-}
\]

\[
- \left( Z_+ \frac{d\nu_+}{dp} - Z_- \frac{d\nu_-}{dp} \right) \frac{1 - f(\nu_+)}{i\omega_m + s\nu_+ + s\nu_-}
\]

\[
+ \left( -2Z_+ Z_- E(\nu_+ + \nu_-)^2 \right) \frac{dE}{dp} \left( Z_+ \frac{f(\nu_+)}{i\omega_m + s\nu_+ + sE} + Z_- \frac{f(-E) - f(\nu_-)}{i\omega_m + s\nu_- + sE} \right)
\}

\]

with \( \bar{\Omega} = \nu_+ - \nu_- \).

By taking the analytic continuation \( i\omega_m \to \omega + i\eta \), and taking the imaginary part, we obtain

\[
\text{Im} \Pi_{++}(\omega, 0) = -\frac{20 \alpha^2}{3} \int dp \, p^2 \left\{ \frac{Z_+^2 \nu_+}{\nu_+ + E} (1 - 2f(\nu_+)) \delta(\omega - 2\nu_+) + \frac{Z_-^2 \nu_-}{\nu_- - E} (1 - 2f(\nu_-)) \delta(\omega - 2\nu_-) 
\]

\[
- Z_+ Z_- \frac{\bar{\Omega}E - 2\nu_+ \nu_-}{(\nu_+ + E)(\nu_- - E)} \delta(\omega - \nu_+ + \nu_-) \delta(\omega - 2\nu_+) \right\}
\]

\[
+ \left( Z_+ \frac{d\nu_+}{dp} - Z_- \frac{d\nu_-}{dp} \right) (1 - f(\nu_+)) \delta(\omega - \nu_+ - \nu_-)
\]

\[
+ \frac{2Z_+ Z_- E(\nu_+ + \nu_-)^2}{p(\nu_+ + E)(\nu_- - E)} \frac{dE}{dp}
\]

\[
\times \{ Z_+ (1 - f(-E) - f(\nu_-)) \delta(\omega - \nu_+ + E) + Z_- (f(-E) - f(\nu_-)) \delta(\omega - \nu_- - E) \}
\]

\[
+ (\omega \to -\omega).
\]

Now let us inspect the physical meaning of each term in Eq. (51). From the \( \delta \)-functions and thermal factors, one
finds that the two terms in the first line represent the pair creation and annihilation processes of normal and plasmino modes, respectively. The second line corresponds to the Landau damping. These terms have corresponding counterparts in Eq. (37), although the coefficients of these terms are modified by the vertex correction. The term in the third line in Eq. (51) can be interpreted as the pair annihilation and creation of a normal mode and a plasmino one. This process does not appear in Eq. (37), and can manifest itself as a consequence of the vertex correction. We note that the similar process exists in the formula obtained in Ref. [12]. In this way, the terms in the first three lines in Eq. (51) can be understood as the annihilation, creation, and scattering processes of quark quasi-particles. We also note that the Landau damping of two normal or two plasmino modes do not exist in Eq. (51), because such a process can exist only for \( \omega = 0 \) at \( q = 0 \).

On the other hand, one cannot give such interpretations to the terms in the fourth and fifth lines in Eq. (51). From the \( \delta \)-functions and the thermal factors, these terms seem to represent the decay and creation rates with a quasi-particle mode with energy \( \pm E \), which, however, does not exist in the quark propagator Eq. (17). Mathematically, these terms come from the poles in the vertex function Eq. (25). The poles appear in the vertex function via the WTI, Eq. (21) and the fact that the analytic continuation of the propagator \( S_\omega (\nu_n, p) \) gives zero points at energies \( \pm E \). As discussed in Sec. II B the zero in \( S_\omega (\omega, p) \) inevitably appears between the two poles in the two-pole form of the quark propagator Eq. (17).

Another remark on Eq. (51) is the sign of each term in Eq. (51). In Eq. (51), all terms are separately positive definite for \( \omega > 0 \) except for the one in the third line, which becomes negative for sufficiently large \( \omega \). The negative contribution of this term is, however, canceled out by the last term; we have checked that the sum of these terms is always positive. The total dilepton production rate for \( \omega > 0 \) therefore takes a positive value as it should be.

In Fig. 4, we show the dispersion relation of \( E(p) \) for \( T = 1.5 T_c \). For \( p = 0 \), \( E(p) \) vanishes because of chiral symmetry, while \( -E(p) \) approaches \( p \) for large \( p \). The figure shows that \( -E(p) \) is a monotonically increasing function of \( p \). The result for \( T = 3 T_c \) qualitatively the same. In Fig. 3 we also show the combinations of the dispersion relations appearing in the \( \delta \)-functions in Eq. (51),

\[
\nu_+ + \nu_-, \quad \nu_+ - \nu_-, \quad \nu_+ - E, \quad \nu_- + E. \tag{52}
\]

From the figure, one sees that \( \nu_+ - E \) and \( \nu_- + E \) are monotonically increasing and decreasing functions of \( p \), respectively, starting from \( m_T \) at \( p = 0 \). Also, \( \nu_+ - E > m_T \) and \( 0 < \nu_- + E < m_T \) are satisfied. These behaviors become transparent by rewriting these combinations as

\[
\nu_+ - E = Z_+ (\nu_+ + \nu_-), \tag{53}
\]

\[
\nu_- + E = Z_- (\nu_+ + \nu_-). \tag{54}
\]

where we used Eq. (20).

We finally comment on the limiting behaviors of Eq. (51). First, in our two-pole ansatz the quark propagator for massless free quarks is obtained by setting

\[
Z_+ (p) = 1, \quad Z_- (p) = 0, \quad \nu_+ (p) = p. \tag{55}
\]

Equation (51) thus should reproduce the photon self-energy of the free quark gas, when Eq. (55) is substituted. This can be explicitly checked as follows. By substituting \( Z_- = 0 \), all terms including \( Z_- \) vanishes. Since \( E = -\nu_- \) for \( Z_- = 0 \), the third and fourth lines in Eq. (51) cancel out with each other without constraints on \( \nu_- (p) \). Only the first term in Eq. (51) thus survives, which gives the free quark result. Second, our result on the dilepton production rate approaches the free quark one in the large \( \omega \) limit, because the lattice quark propagator used in this study reproduces Eq. (55) at large momentum. This behavior will be explicitly checked in the next section.

IV. NUMERICAL RESULTS

Now let us see the numerical results on the dilepton production rate obtained in the previous section. In Fig. 5 we present the \( \omega \) dependence of the dilepton production rate for \( T = 1.5 T_c \). In the figure, we also plot the result without vertex correction, Eq. (37), together with the rates obtained by the HTL calculation [12] and the free quark gas, Eq. (27). The value of the thermal mass \( m_T \) is taken from the results obtained on the lattice [23].

Figure 5 shows that the production rate with the lattice quark propagators has divergences at two energies, \( \omega/m_T = \omega_1/m_T \simeq 1.1 \) and \( \omega/m_T = \omega_2/m_T \simeq 1.8 \). For \( \omega < \omega_1 \), our result, as a whole, behaves similarly to the HTL one [12], i.e. it increases as \( \omega \) decreases, although our production rate is smaller than the perturbative one.
for small $\omega$. Near $\omega_1$, however, it shows a prominent enhancement and exceeds the latter. The region, where the large production rate is obtained, is located around $m_T$. Therefore, it can be possible that the production yield obtained by integrating this rate has the large enhancement below several hundred MeV, where the enhancement in the experimentally-observed dilepton spectra at RHIC exists. The rate has a discontinuity at $\omega = \omega_1$, and is significantly suppressed compared with Eq. (27) for $\omega_1 < \omega < \omega_2$. The rate has another discontinuity at $\omega = \omega_2$, above which the rate is close to the free quark one. In the dilepton rate without vertex correction, one also finds two divergences at $\omega = \omega_1$ and $\omega_2$, while the rate vanishes for $\omega_1 < \omega < \omega_2$.

In order to understand these results in more detail, we show the contribution of each term in Eq. (51) separately in Fig. 5. In the figure, the rate coming from the pair annihilation of two normal modes (NN), two plasmino modes (PP), and a normal and a plasmino modes (NP) are separately shown, together with those of the Landau damping between quasi-particles (LD) and processes including an E mode with a normal (NE) and a plasmino (PE) modes. From the figure, one finds that the two divergences at $\omega = \omega_1$ and $\omega_2$ come from the LD and PP rates, respectively. As discussed in Sec. III B, these divergences come from van Hove singularity. The photon self-energy Eq. (51) is inversely proportional to derivatives of the dispersion relations, $d\nu_-(p)/dp$ and $d\{\nu_+(p) - \nu_-(p)\}/dp$. As shown in Figs. 2 and 4, each of $\nu_-(p)$ and $\nu_+(p) - \nu_-(p)$ has an extremum at nonzero $p$. Their values at the extrema are $\nu_+(p) - \nu_-(p) = \omega_1$ and $2\nu_-(p) = \omega_2$. At these points the derivatives vanish. This leads to the divergences in the photon self-energy, and accordingly the dilepton production rate.

Figure 5 also shows that each individual process is non-vanishing in a limited range of $\omega$. The range can be read off from the corresponding functions plotted in Fig. 4. The NN and NP rates are nonvanishing for $\omega > 2m_T$. On the other hand, the lower threshold of the PP rate, $\omega = \omega_2$, is slightly lower than $2m_T$, because $\nu_-(p)$ has a minimum smaller than $m_T$ at nonzero momentum. The range of the Landau damping is also kinematically constrained to $\omega < \omega_1$. The NE and PE rates are nonvanishing for $\omega > m_T$ and $\omega < m_T$, respectively. The NE rate gives rise to a nonzero value for $\omega_1 < \omega < \omega_2$. To the dilepton production rate without the vertex correction, only the NN, PP, and LD contribute and the rate vanishes for $\omega_1 < \omega < \omega_2$.

For large $\omega$, the rate is dominated by the NN. This is a consequence of the fact that the quark propagator approaches the free quark one as $p$ becomes larger. A glance at Fig. 6 might give an impression that the NE rate also survives for large $\omega$. Although not shown in Fig. 6, however, the NP rate takes a negative value for $\omega \gtrsim 2.4m_T$, and this term almost cancels out with the NE rate; see the discussion in Sec. III C.

Next, let us address the behavior of the production rate in the $\omega \to 0$ limit. The photon self-energy is identical with the electromagnetic current-current correlation function,

$$ J_{ij}^R(\omega, p) = \int d^4xe^{i\omega t-i\not{p}\cdot\not{x}}[j_i^{\text{EM}}(t, x), j_j^{\text{EM}}(0, 0)]\theta(t),$$

at the leading order in $\alpha$. The low energy behavior of $J_{ij}^R(\omega, p)$ is related to electric conductivity $\sigma$ through the Kubo formula,

$$ \sigma = \frac{1}{6} \lim_{\omega \to 0} \frac{1}{\omega} \sum_{i=1}^{3} J_{ii}^{R}(\omega, 0).$$

Our result shows that $\sum_{i=1}^{3} J_{ii}^{R}(\omega, 0)$ approaches zero faster than $\omega$ in the $\omega \to 0$ limit, and thus the electric...
conductivity vanishes. Incorporation of the width of the quasi particle modes, which is not included in the form of the quark propagator used in this study, may lead to nonzero $\sigma$.

This limiting behavior may depend on the way of the extrapolation of $\nu_{\pm}(p)$ to large momentum.

V. SUMMARY

In this study we have investigated the dilepton production rate using a quark propagator obtained on the lattice with a pole ansatz. The Schwinger-Dyson equation for the photon self-energy is solved with the lattice quark propagator and the photon-quark vertex satisfying the Ward-Takahashi identity. The effect of the vertex correction is discussed by comparing the result with the calculation without vertex correction. Our numerical result shows that the dilepton production rate with the lattice quark propagators is larger by about one order or more compared with that with the free quark ones in the low invariant mass region. Compared with the HTL result, there exists a significant enhancement around $\omega \approx m_T \sim T$ owing to van Hove singularity. This result is interesting since such a large enhancement in the deconfined medium near $T_c$ may explain the excess of the dilepton yield observed at PHENIX [4] in the low invariant mass region. To understand the effect of the enhancement of the dilepton rate to the experimental result quantitatively, the analysis with dynamical models describing the space-time evolution of the hot medium and integration of the dilepton production rate is needed, which would be performed elsewhere. The comparison with other non-perturbative approaches such as Ref. [32] will be an interesting future work.

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