Chiral Symmetry Breaking  
in the Dual Ginzburg-Landau Theory

Hiroshi Toki, Shoichi Sasaki, Hiroko Ichie and Hideo Suganuma  
Research Center for Nuclear Physics (RCNP), Osaka University  
Mihogaoka 10-1, Ibaraki, Osaka 567, Japan

Confinement and chiral symmetry breaking are the most fundamental phenomena in Quark Nuclear Physics, where hadrons and nuclei are described in terms of quarks and gluons. The dual Ginzburg-Landau (DGL) theory, which contains monopole fields as the most essential degrees of freedom and their condensation in the vacuum, is modeled to describe quark confinement in strong connection with QCD. We then demonstrate that the DGL theory is able to describe the spontaneous breakdown of the chiral symmetry.

1 Introduction

In Quark Nuclear Physics (QNP), the most essential phenomena are confinement of quarks and gluons and the chiral symmetry breaking. Quarks are not found in free space, while they are seen in deep inelastic scattering. Quarks are present in hadrons and hence the understanding of confinement is essential for the sizes of hadrons.

The chiral symmetry is found in the QCD lagrangian. In the u-d sector, the current mass is considered negligible (∼5MeV) as compared to the hadron mass (∼1GeV), and hence the chiral symmetry is to be realized with high accuracy. We expect then the chiral partners, saying simply parity doublets, to be degenerate in the u-d meson spectrum. The nature, on the other hand, shows that the pion (0−) has 139MeV and no 0+ partner is found. The rho meson (1−) is 500MeV apart from its chiral partner, the a1 meson (1+). The same feature is found also for the baryon spectrum. This property of hadron spectra tells us that the chiral symmetry is broken spontaneously and the pion appears as its Goldstone particle. This chiral symmetry breaking should be understood for full description of QNP. Chiral symmetry breaking would provide the constituent quark masses and even the small mass pions, which are responsible for N-N interaction.

How do these phenomena happen? For this let us see the QCD coupling strength αS, which runs with relevant momentum scale obeying the renormalization group of QCD. At large momenta, αS diminishes and the theory becomes asymptotic free, and hence the momentum dependence of deep inelastic scattering is calculable perturbatively. At small momenta, where confinement and chiral symmetry breaking are expected, αS blows up and...
hence we face highly non-perturbative processes. We ought to find the essential degrees of freedom to get the insight of these phenomena.

2 Dual Ginzburg-Landau theory

It is Nambu, who introduced an interesting view on color confinement in 1974\cite{2}. Suppose we insert a superconductor in a magnetic field. The superconductor does not allow the magnetic field to pass through. If it were to allow the magnetic field, in the superconductor of second kind, the magnetic field should be confined in a vortex-like configuration. This is known as the Meissner effect. Nambu takes its dual version for quark confinement. If the vacuum is normal, the color electric field should look like the one of the Coulomb potential between a positive and a negative color charges. If the vacuum is superconductor-like (dual superconductor), then the vacuum dislikes the electric field to pass through and hence the color electric flux ought to be confined in a vortex-like configuration. This is then named as dual Meissner effect. This picture, however, does not become popular, because it requires color magnetic monopoles. In the superconductor, the charged object, Cooper pair, is to condense, while in the QCD vacuum, the magnetically charged object, color magnetic monopole, to condense.

‘t Hooft was the one, who demonstrated the natural appearance of color magnetic monopoles in QCD\cite{3}. In the non-abelian gauge theory like QCD, he introduced a particular gauge named abelian gauge, to reduce it to the abelian gauge theory like QED. From a topological argument, color magnetic monopoles appear naturally in the abelian space. This work then supports the idea of Nambu for confinement. Hence, QCD naturally reduces to QED with magnetic monopoles, which is the Maxwell equation with magnetic charges and currents, studied by Dirac\cite{4}. This Maxwell equation has the duality symmetry, which naturally arises in the special gauge of QCD.

It took then about 10 years before the above idea was formulated in the form of lagrangian\cite{5}. The dual Ginzburg-Landau (DGL) theory is expressed with the following lagrangian,

$$L_{DGL} = L_{\text{dual}} + \bar{q}(i\gamma_\mu \partial^\mu - m + e\gamma_\mu A^\mu)q + \text{tr}[\hat{D}_\mu, \chi]^{\dagger}[\hat{D}_\mu, \chi] - \lambda \text{tr}(\chi^{\dagger}\chi - v^2)^2 \quad (1)$$

Here, $L_{\text{dual}}$ denotes the dual version of the gauge field tensor. $q$ is the quark field and $\chi$ is the monopole field. $\hat{D}_\mu$ is the dual covariant derivative, $\hat{D}_\mu = \partial_\mu + igB^\mu$. $B^\mu$ is the dual gauge field and $g$ is the dual coupling constant, which satisfies the Dirac condition, $eg = 4\pi$. The last term is the Higgs term to cause monopole condensation, where $\lambda$ and $v$
are the parameters of the DGL lagrangian. It is important to note that this DGL lagrangian is derived from the QCD lagrangian by assuming the existence of the monopole field and the abelian dominance\cite{5}. It is at the same time supported by the recent lattice QCD calculations\cite{6}. This is the dream lagrangian of Dirac, which ought to appear in some non-abelian gauge theory like QCD.

The first application is the $q\bar{q}$ static potential by putting $q\bar{q}$ pair with distance $r$\cite{7}. The potential comes out to have a Yukawa term and a linear confining term. We can fix the parameters of the DGL lagrangian by fitting to the phenomenological potential. As for the glueball mass, which appears in the DGL theory and has a strong connection with the QCD vacuum and confinement, it turns out $M(0^+) \sim 1.5$ GeV. Its appearance of the linear potential is not surprising, since it is modeled in the DGL theory. It is worthwhile to stress, however, that there are no other models, which are able to realize confinement of colors and at the same time have a strong link with QCD. The real challenge is now the chiral symmetry breaking, which is discussed next.

3 Chiral symmetry breaking

Chiral symmetry breaking is directly related with the quark mass generation in the QCD vacuum. How quarks then behave in monopole condensed vacuum? It corresponds to solving the Schwinger-Dyson equation, where quarks get the self-energy corrections due to the non-perturbative interaction with gluons\cite{8}. This seems, however, unphysical, because quarks are confined. It means that whenever a quark is present, there should be an anti-quark or a di-quark to make the system color-singlet. In principle, therefore, we ought to solve multi-body system to talk about a single quark. Suppose we have the Schwinger-Dyson equation as schematically written as,

$$S^{-1}(p) = S_{0}^{-1}(p) + \int_{0}^{\infty} S(p - q) D(q) dq.$$ \hspace{1cm} (2)

The quark, which is confined, should be seen from the position of anti-quark. Then, the probability to find the quark is finite only within the distance of the hadronic scale; $\sim$1fm. Therefore, gluons cannot travel freely any distance. Rather, it is confined also within the hadronic distance. This indicates that there should be the infrared cut-off, which is of order of the inverse of the confining distance $R$ as $q > q_c = \frac{1}{R}$. Hence, the SD equation is modified simply to

$$S^{-1}(p) = S_{0}^{-1}(p) + \int_{q_c}^{\infty} S(p - q) D(q) dq.$$ \hspace{1cm} (3)

3
We show in Fig.1 the result of chiral symmetry breaking, expressed in terms of the quark mass $M(p)$. It becomes finite with increasing the strength of monopole condensation. We find also the pion decay constant and the quark condensate to have the numbers close to the semi-experimental values. This calculation demonstrates that monopole condensation is the source of both the confinement and the chiral symmetry breaking\[8\].

We discuss also the recovery of the chiral symmetry at finite temperature\[9\]. We can formulate this in the imaginary time formalism. We in fact find the recovery of the chiral symmetry as indicated in Fig.2 by the ratio of quark condensate at finite and zero temperatures $\langle \bar{q}q \rangle_T$. $\langle \bar{q}q \rangle_T$ decreases with temperature and eventually drops to zero, indicating the recovery of the chiral symmetry. We note that the temperature of the phase transition; $T_c \sim 0.11\text{GeV}$ seems smaller than the results of lattice QCD. This difference would be caused by the use of temperature independent parameters in the Higgs term. Since this term is introduced at zero temperature, it is likely that they depend on temperature as the case of the superconductor. In addition, the hadronic scale should also depend on temperature. Here, the point of showing this result is merely to demonstrate that the DGL theory provides phase transition to the normal phase at finite temperature.

We can even talk about the confinement-deconfinement phase transition at finite temperature\[10\]. In the quenched approximation, we can write the DGL lagrangian in terms of the dual gauge fields by integrating out the gauge fields; $A_\mu$. It amounts to calculate the partition functional and we can derive the effective potential; the therodynamical potential. The result is shown in Fig.3, where the effective potential is plotted as a function of the monopole condensate, $\chi$. The absolute minimum appears at finite value of $\chi$ in the lower temperature side and jumps to $\chi=0$. This indicates deconfinement phase transition of first order. We can calculate the string tension as a function of temperature, the result of which is shown in Fig. 4. Adjusting $\lambda$ as a temperature dependent parameter so as to reproduce the critical temperature at 0.2GeV, we find the string tension to reproduce the lattice QCD results\[11\].

It is interesting to mention that lattice QCD is also doing a very interesting development. The $q\bar{q}$ potential calculated with full lattice QCD agrees with the result of only the abelian part. This agreement indicates that the confinement physics is describable in terms of only the abelian gluons. Other quantities in this direction are being done by several groups\[12, 13\]. All these results indicate that the low momentum phenomena could be described by the abelian gauge gluons (abelian dominance), when the abelian gauge is suitably chosen. At RCNP, we have started numerical experimental program with the use of lattice QCD\[14\].
The largest task is to predict the properties of the glueballs associated with confinement, particularly the decay properties for experimental identification.

4 Conclusion

Quark Nuclear Physics (QNP) is the field to describe nucleons, mesons and nuclei in terms of quarks and gluons. The most essential phenomena in QNP are confinement of quarks and gluons and the chiral symmetry breaking. The confinement is modeled as due to the dual Meissner effect and is expressed in terms of the dual Ginzburg-Landau theory, where the QCD monopoles and their condensation in the QCD vacuum are the essential ingredients. We have demonstrated in this paper that the DGL theory is able also to describe the chiral symmetry breaking. We have then discussed the recovery of these symmetries at finite temperature. Now the DGL theory is ready for exciting experimental phenomena of QNP.

The author is grateful to the members of the theory group of RCNP for exciting collaborations of the theoretical accounts of the Quark Nuclear Physics. We mention also the strong support of the director of RCNP, Prof. H. Ejiri, for continuous encouragement of the theoretical works. H.T appreciates the fruitful organization of this Workshop and thanks A. Thomas and A. Williams for the nice organization and their hospitalities to all the Japanese participants.

This is an invited talk presented by H. Toki in the Joint Japan-Australia Workshop, held in Nov. 15-24, 1995 organized by Adelaide Institute of Theoretical Physics.

References

[1] T. Chen and L. Li, "Gauge Theory of Elementary Particle Theory", Oxford University Press (1984).

[2] Y. Nambu, Phys. Rev. D10 (1974) 4262.

[3] G. ’t Hooft, Nucl. Phys. B190 (1981) 455.

[4] P.A.M. Dirac, Proc. R. Soc. A133 (1931) 60.

[5] T. Suzuki, Prog. Theo. Phys. 80 (1988) 929; 81 (1989) 752.
   S. Maedan and T. Suzuki, Prog. Theo. Phys. 81 (1989) 229.
[6] A.S. Kronfeld et al., Phys. Lett. B198 (1987) 516.
   T. Suzuki and I. Yotsuyanagi, Phys. Rev. D42 (1990) 4257.
   S. Hioki et al., Phys. Lett. B272 (1991) 326.

[7] H. Suganuma, S. Sasaki and H. Toki, Nucl. Phys. B435 (1995) 207.

[8] S. Sasaki, H. Suganuma and H. Toki, Prog. Theor. Phys. 94 (1995) 373.

[9] S. Sasaki, H. Suganuma and H. Toki, Proc. of Int. Conference "Baryons '95" in press.

[10] H. Ichie, H. Suganuma and H. Toki, Phys. Rev. D52 (1995) 2944.

[11] M. Gao, Nucl. Phys. B9 (Proc. Suppl.) (1989) 368.

[12] O. Miyamura and S. Origuchi, Proc. of International RCNP Workshop on "Color Confinement and Hadrons", ed. by H. Toki et al., World Scientific (1995) 235.

[13] H. Shiba and T. Suzuki, Phys. Lett. B343 (1995) 315.

[14] H. Suganuma, A. Tanaka, S. Sasaki and O. Miyamura, Proc. of Lattice'95, Melbourne, July 1995, Nucl. Phys. B (Proc. Suppl.) in press.

[15] H. Ichie, S. Shiba and T. Suzuki, Proc. of INSAM Symposium '95, Hiroshima Univ., Oct. 1995, in press.
**Figure Captions**

Fig.1 The constituent quark mass calculated within the DGL theory with various values of the dual gauge mass, which indicates the strength of monopole condensation, as a function of the Euclidean momentum square. The unit $\Lambda_{\text{QCD}}$ is 200 MeV.

Fig.2 The ratio of quark condensate at finite and zero temperatures within the DGL theory as a function of temperature. The critical temperature is about 0.11GeV.

Fig.3 The effective potential (thermodynamical potential) at various temperatures within the DGL theory as a function of the monopole condensate. The absolute minimum is indicated by $\times$ for each curve, indicating the jump (phase transition of first order) around $T = 0.5$GeV.

Fig.4 The string tension between a quark and antiquark pair for constant and variable lambda’s within the DGL theory as a function of temperature. The dots are the results of the pure-gauge lattice QCD; [10].
Fig. 1
Fig. 2

\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0}

\frac{T}{T_c}
$V_{\text{eff}}(\chi; T) \text{ [fm}^{-4}]$

Fig. 3
$k(T)$ [GeV/fm]

$\lambda = \text{const.}$

$\lambda(T)$

$T$ [GeV]

Fig. 4