Abstract

We present an improved version of our program package oneloop which – written as a package for MAPLE [1, 2] – solves one-loop Feynman integrals. The package is calculating one-, two- and three-point functions both algebraically and numerically to any tensor rank. In addition to the original version oneloop 2.0 also calculates infrared divergent integrals. Higher powers of propagator terms and the $O(\varepsilon)$ parts relevant for two-loop calculations are now supported.
NEW VERSION SUMMARY

Title of new version: oneloop.ma, version 2.0

Reference to original program: oneloop.ma

Reference in CPC: Comput. Phys. Commun. 85 (1995) 153

Does the new version supersede the old program? Yes

Licensing provisions: None

Computers: Any platform which has MAPLE V, Release 3

No. of processors used: 1

Operating system under which the new version has been tested: (i) Unix (Linux 2.0) (ii) VMS 6.2 (iii) MS-DOS 6.0

Programming language: MAPLE V, Release 3

Subprograms used: cfcn.ma, r.ma, simple.ma, pv.ma, mess.ma

Memory required to execute with typical data: Less than 8 MB RAM

Typical running time: The algebraic or numeric evaluation of a three-point function up to the rank-3-tensor on a DEC Alpha 8400 needs less than 2s

No. of lines in distributed program including subprograms: 4019

No. of bytes in distributed program including test data: 168 189

Distribution format: binary (zipped ASCII files)

Keywords: Electroweak theory, Feynman diagrams, one-loop corrections, renormalization

Nature of physical problem: The theoretical determination of cross sections in particle processes requires the calculation of radiative corrections. The most important contribution comes from the level of one-loop Feynman diagrams which arise from the model under consideration, usually the standard model of elementary particles

Method of solution: This package is designed to evaluate automatically one-loop integrals. It is making use of the properties of $\mathcal{R}$ functions, a class of special functions which simplifies the evaluation of Feynman integrals

LONG WRITE-UP

1 Introduction

In particle physics the calculation of one- and higher loop corrections to particle processes is mandatory to keep track with the increasing accuracy of particle colliders. For this
reason the necessary calculations were automated by virtue of computer programs during recent years. In this context the program oneloop was developed at the one-loop level [3]. It provides the necessary integrals which are needed for Feynman diagrams with at most three external legs.

Several improvements of the original version now merged in a new version 2.0. It contains the following new features:

- Arbitrary powers of the propagator terms in the denominator are allowed (cf. sect. 2). The original version was restricted to propagators of power one.

- Infrared divergences can be calculated in dimensional regularization. They will appear as poles in the dimensional regulator $\varepsilon = (4 - D)/2$ like in the ultraviolet case (cf. sect. 3).

- The contributions of $\mathcal{O}(\varepsilon)$ can be calculated as well. Although these terms are not contributing in pure one-loop calculations they become relevant in two-loop applications (cf. sect. 3).

- Several kinematical points in which the original version ran into numerical instability or terminated in a division by zero are now solved in a different, numerically stable way (cf. sect. 3).

- Additional functions $\text{OneLoopTensnPt}$ accept squared momenta as input and return full tensors (cf. sect. 3).

- The usage of a library of one-loop integrals is generalized (cf. sect. 4).

- Some other minor changes have to be reported (cf. sect. 5):
  - In the case of the three-point function the convention of the output changed slightly. It has now the same structure as for the other functions.
  - Some new functions which abbreviate the output are introduced in the case where the functions are calculated analytically.

This note describes all changes in more detail.

2 Arbitrary powers of propagators

In addition to the original functions

- $\text{OneLoop1Pt}(p, m)$
- $\text{OneLoop2Pt}(p_0, p_1, q, m_1, m_2)$
- $\text{OneLoop3Pt}(p_0, p_1, p_2, q_1, q_20, q_21, m_1, m_2, m_3)$

– which need the tensor degree, the external momenta $q_n$ and the masses $m_n$ as input – there exist now the following generalizations

- $\text{OneLoop1Pt}(p, m, t)$
- $\text{OneLoop2Pt}(p_0, p_1, q, m_1, m_2, t_1, t_2)$
- $\text{OneLoop3Pt}(p_0, p_1, p_2, q_1, q_20, q_21, m_1, m_2, m_3, t_1, t_2, t_3)$. 
These functions additionally expect the powers $t^p$ of the propagator terms in the denominator. In detail they directly correspond to the following integrals:

- **One-point function:**

  \[
  \text{OneLoop1Pt}(p, m, t) = A^{(p)(t)}(m) = \int d^D l \frac{(l^2)^t}{[l^2 - m^2 + i\varrho]^t}
  \]  

  Abbreviations:

  \[
  l_\parallel = \frac{l \cdot q}{\sqrt{q^2}}; \quad l_\perp = \sqrt{l_\parallel^2 - l^2}
  \]  

- **Two-point function:**

  \[
  \text{OneLoop2Pt}(p_0, p_1, q, m_1, m_2, t_1, t_2) = B^{(p_0 p_1)(t_1 t_2)}(q, m_1, m_2)
  = \int d^D l \frac{(l_\parallel)^{p_0} (l_\perp)^{p_1}}{[(l + q)^2 - m_1^2 + i\varrho_1]^t_1 [l^2 - m_2^2 + i\varrho_1]^t_2}
  \]  

  Abbreviations:

  \[
  l_\parallel = \frac{l \cdot q}{\sqrt{q^2}}; \quad l_\perp = \sqrt{l_\parallel^2 - l^2}
  \]  

- **Three-point function:**

  \[
  \text{OneLoop3Pt}(p_0, p_1, p_2, q_1, q_2, q_2', m_1, m_2, m_3, t_1, t_2, t_3)
  = C^{(p_0 p_1 p_2)(t_1 t_2 t_3)}(q_1, q_2, m_1, m_2, m_3)
  = \int d^D l \frac{(l_\parallel)^{p_0} (l_\perp)^{p_1} (l_\parallel)^{p_2}}{[(l + q_1)^2 - m_1^2 + i\varrho_1]^t_1 [(l + q_2)^2 - m_2^2 + i\varrho_1]^t_2 [l^2 - m_3^2 + i\varrho_1]^t_3}
  \]
Abbreviations:

\[
\begin{align*}
    l_0 \parallel &= \frac{l \cdot q_1}{\sqrt{q_1^2}}; \\
    l_1 \parallel &= \frac{l \cdot q'_2}{\sqrt{q_2^2}}; \\
    q'_2 &= q_2 - \frac{q_1 \cdot q_2}{q_1^2} q_1 \\
    l_\perp &= \sqrt{l_0^2 - l_1^2}; \\
    q_{20} &= \frac{q_1 \cdot q_2}{\sqrt{q_1^2}}; \\
    q_{21} &= \sqrt{q_{20}^2 - q_2^2}
\end{align*}
\]

Our notation for the OneLoop\textsubscript{\textit{nPt}} functions distinguishes between parallel and orthogonal space which is reflected by the definitions of (3) and (5) for the momentum components. Please keep in mind the fact that in this notation the indices \(p_0, p_1, \ldots\) represent the powers of the different components of the loop momentum \(l\) which should not be mixed with Lorentz indices. In our notation \(l_\parallel, l_0 \parallel, l_1 \parallel\) describe the components of \(l\) which are parallel to the external momenta \(q, q_1, q_2\), whereas \(l_\perp\) represents the orthogonal complement.

If the powers \(t_n\) of the propagator terms in the denominator are omitted the program assumes the default value 1 for each \(t_n\).

3 Two-loop relevant parts of one-loop diagrams

The results of the OneLoop\textsubscript{\textit{nPt}} functions are Laurent expansions in terms of the ultraviolet regulator \(\varepsilon\). Therefore the output of the OneLoop\textsubscript{\textit{nPt}} procedures consists of a list where the significant coefficients ([\(\mathcal{O}(\varepsilon^{-1}), \mathcal{O}(\varepsilon^0)\)]) of this expansion are given.

In two-loop calculations one-loop diagrams get multiplied with divergent \(Z\) factors. Two-loop integrals can factorize into a product of one-loop integrals. This is the reason why one-loop contributions of \(\mathcal{O}(\varepsilon^1)\) are also of interest \(\text{(3)}, \text{(5)}\). For that purpose this coefficient is calculated if the qualifier \textit{more} is used in the function call:

| input | output |
|-------|--------|
| OneLoop1Pt\((p, m, t)\) | \[\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}\] |
| OneLoop1Pt\((p, m, t, \text{more})\) | \[\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}, \varepsilon^1\text{-term}\] |
| OneLoop2Pt\((p_0, p_1, q, m_1, m_2, t_1, t_2)\) | \[\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}\] |
| OneLoop2Pt\((p_0, p_1, q, m_1, m_2, t_1, t_2, \text{more})\) | \[\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}, \varepsilon^1\text{-term}\] |
| OneLoop3Pt\((p_0, p_1, p_2, q_1, q_20, q_21, m_1, m_2, m_3, t_1, t_2, t_3)\) | \[\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}\] |
| OneLoop3Pt\((p_0, p_1, p_2, q_1, q_20, q_21, m_1, m_2, m_3, t_1, t_2, t_3, \text{more})\) | \[\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}, \varepsilon^1\text{-term}\] |

In the output “\(\varepsilon^{-1}\text{-term}\)” is meant to represent the divergent part (the coefficient of \(1/\varepsilon\)) whereas “\(\varepsilon^0\text{-term}\)” is describing the finite part and “\(\varepsilon^1\text{-term}\)” the \(\mathcal{O}(\varepsilon)\) contribution.

As long as the arguments of the OneLoop functions are symbols the output is given algebraically, whereas numbers inserted for the arguments imply a numerical result.

4 Infrared divergences

Infrared divergences are now regulated as well. This kind of divergence may occur for instance for the three-point function if the momenta are on-shell and one mass is set to 0.\footnote{The output of OneLoop3Pt changed slightly compared to the original version (cf. sect. 4).}
This effect also appears for a two-point function where a propagator term with vanishing mass is squared. Since the collinear divergent case is still excluded the divergence will always occur as a pole term of $O(\varepsilon^{-1})$. It means that the output involves a non-vanishing $\varepsilon^{-1}$ term in the notation of sect. \[8]\ There is no explicit distinction made between UV and IR divergences, there is no infrared dimension parameter $\varepsilon_{\text{IR}}$. Both kinds of divergences are described by $\varepsilon = (4 - D)/2$. The result for the collinear case is well-known \[8]. It is the only case which has a pole of $O(\varepsilon^{-2})$. We excluded this simple case, but the user can add it easily himself.

5 Numerical stability

The accuracy and stability of numerical results may suffer from cancellations of large, approximately equal dilogarithms. Since MAPLE supports calculations of arbitrary precision, it is possible to increase the number of digits to improve accuracy. Usually we calculated with 20 or 40 digits.

Nevertheless, at some kinematical points the general procedure breaks down since divisions by 0 or similar errors are encountered. It turns out that these points always belong to a kinematical arrangement – for instance equal or vanishing masses or momenta – which is simpler than the general case. For this reason, even if there were no numerical problems – in the sense of optimizing the code – it is advisable to solve the simpler kinematical situation in a faster than the general way.

This is done completely automatically for the following kinematical points:

**OneLoop2Pt:**

$q = 0$
$q_1 = 0$
$q_20 = 0$
$q_{21} = 0$

$q_{20} = q_{21}$
$q_{20} = -q_{21}$
$q_1 = 0, m_1 = m_3$
$q_20 = 0, q_21 = 0$
$q_{21} = 0, q_1 = q_{20}$
$q_{21} = 0, (q_{20}^2 - m_2^2)q_1 - (q_1^2 - m_1^2)q_{20}$
$q_{20} = 0, q_{21} = 0, m_2 = m_3$
$q_{21} = 0, q_1 = q_{20}, m_1 = m_2$

$q_1 = 0, m_1 = m_3, \sqrt{q_{20}^2 - q_{21}^2} + m_1 + m_2 = 0$
$q_1 = 0, m_1 = m_3, \sqrt{q_{20}^2 - q_{21}^2} - m_1 + m_2 = 0$
$q_1 = 0, m_1 = m_3, \sqrt{q_{20}^2 - q_{21}^2} + m_1 - m_2 = 0$
$q_1 = 0, m_1 = m_3, \sqrt{q_{20}^2 - q_{21}^2} - m_1 - m_2 = 0$

$q_{20} = 0, q_{21} = 0, m_2 = m_3, q_1 + m_1 + m_3 = 0$
$q_{20} = 0, q_{21} = 0, m_2 = m_3, q_1 - m_1 + m_3 = 0$
$q_{20} = 0, q_{21} = 0, m_2 = m_3, q_1 + m_1 - m_3 = 0$
$q_{20} = 0, q_{21} = 0, m_2 = m_3, q_1 - m_1 - m_3 = 0$
$q_{20} = 0, q_{21} = 0, m_2 = m_3, q_1 + m_1 + m_3 = 0$
$q_{20} = 0, q_{21} = 0, m_2 = m_3, q_1 - m_1 + m_3 = 0$
$q_{20} = 0, q_{21} = 0, m_2 = m_3, q_1 - m_1 - m_3 = 0$
$q_{20} = 0, q_{21} = 0, m_2 = m_3, q_1 + m_1 - m_3 = 0$
$q_{20} = 0, q_{21} = 0, m_2 = m_3, q_1 + m_1 + m_3 = 0$
$q_{20} = 0, q_{21} = 0, m_2 = m_3, q_1 - m_1 + m_3 = 0$
$q_{20} = 0, q_{21} = 0, m_2 = m_3, q_1 - m_1 - m_3 = 0$

$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
$q_20 = m_1^2 - m_2^2 = 0$
\[ q_{20}^2 - q_{21}^2 + m_1^2 - m_2^2 = 0 \]
\[ q_{20}^2 - q_{21}^2 = m_2^2, q_1^2 = m_1^2, m_3 = 0 \]
\[ (q_{20} - q_1)^2 - q_{21}^2 = m_3^2, q_{20} - q_{21}^2 = m_3^2, m_2 = 0 \]
\[ (q_{20} - q_1)^2 - q_{21}^2 = m_3^2, q_1^2 = m_3^2, m_1 = 0 \]

Several of these points correspond to different thresholds of the integrals. There are only two remaining sources of difficulties:

- if an integral is first solved in a general manner and then – in the result – special values are substituted which correspond to one of the aforementioned kinematical points. Solution: the values have to be assigned from the beginning of the calculation.
- if the kinematical arrangement is not in, but very close to (less than roughly \(10^{-10}\) compared to the relevant scale) one of the above listed kinematical points. Solution: the value of \texttt{Digits} has to be increased.

6 Tensors

The functions \texttt{OneLoopTens} expect squared momenta \(q^2\) for two-point functions, \(q_1^2, q_2^2\) and \((q_2 - q_1)^2\) for three-point functions – as arguments and return a full tensor as output – which is completely equivalent to the notation of (3) and (5) but perhaps the more familiar representation [3]. They replace the similar \texttt{PassVelt} functions of the original version which now are no longer needed.

The following types of arguments – here only demonstrated for the two-point case – are allowed for all \texttt{OneLoopTens} functions:

- \texttt{OneLoopTens}\(2\)\(Pt(i)\):
  The rank \(i\) tensor decomposition of the two-point function is given. The procedure returns a list consisting in different coefficients, which are expressed in terms of the \texttt{OneLoop} tensor integrals, and the defining equation for the coefficients, for instance in the case \(i = 2\):
  \[
  \begin{bmatrix}
  C_{21} = -\frac{\text{OneLoop2Pt}(0, 2)}{3 - 2\varepsilon} - \frac{\text{OneLoop2Pt}(0, 2)}{q^2 (-3 + 2\varepsilon)} \\
  C_{20} = -\frac{\text{OneLoop2Pt}(2, 0)}{q^2}, C_{20}q_{\mu_1\mu_2} + C_{21}g_{\mu_1\mu_2}
  \end{bmatrix}
  \]

- \texttt{OneLoopTens}\(2\)\(Pt(i, full)\):
  The function inserts the results of the \texttt{OneLoop} tensor integrals explicitly. Here again the case \(i = 2\):
  \[
  \begin{bmatrix}
  C_{21} = [\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}], C_{20} = [\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}], C_{20}q_{\mu_1\mu_2} + C_{21}g_{\mu_1\mu_2}
  \end{bmatrix}
  \]

- \texttt{OneLoopTens}\(2\)\(Pt(i, q^2, m_1, m_2, t_1, t_2)\):
  The function returns the same as \texttt{OneLoopTens}\(2\)\(Pt(i, full)\), but expressed in the user defined terms for \(q^2, m_1\) and \(m_2\). Of course numerical values are also allowed. \(t_1\) and \(t_2\) are optional.

- \texttt{OneLoopTens}\(2\)\(Pt(i, q^2, m_1, m_2, t_1, t_2, more)\):
  The function returns also the \(O(\varepsilon)\) contribution. Again, \(t_1\) and \(t_2\) are optional:
  \[
  \begin{bmatrix}
  C_{21} = [\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}, \varepsilon^1\text{-term}], C_{20} = [\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}, \varepsilon^1\text{-term}], \\
  C_{20}q_{\mu_1\mu_2} + C_{21}g_{\mu_1\mu_2}
  \end{bmatrix}
  \]
7 Library

The usage of a library of integrals was restricted to the PassVelt procedures in the original version. It is now applicable for all OneLoop and OneLoopTens procedures, so that in any case the procedures speed up. The package now includes the functions

- OneLoopLib2Pt(i, j)
- OneLoopLib3Pt(i, j)

which generate a library of all OneLoop2Pt and OneLoop3Pt functions respectively up to the tensor rank i. All powers of the denominators from 1 to j are considered. The second parameter j is optional. If it is omitted the procedures assume the value 1 for j, so that no powers of denominators higher than one are calculated.

If this library shall be written in any other than the current directory one has to assign the variable LibPath which substitutes the variable tensorpath in the original version

LibPath:=⟨path⟩;

Now the library will be written to ⟨path⟩. The procedures will only look for the library if LibPath is assigned. If the procedures search for an integral which is not contained in the library, an error message is returned.

In practice it is necessary to store not only the general mass case of each integral. Several special cases mentioned in sect. 5 are also needed. The necessary integrals are automatically created by the OneLoopLib routines. Each integral corresponds to one file.

8 Minor changes

We changed the convention for the three-point function compared with the original version: The vanishing $\varepsilon^{-2}$-term which was kept for testing reasons is dropped now. The list of abbreviations which came with the three-point function also disappeared, because it is no longer needed.

Instead the output now is written in the algebraic case by using several new abbreviations:

$$R2ex1(x, y) = \sqrt{1 - \frac{x}{y}} \left[ \ln \left( 1 - \sqrt{1 - \frac{x}{y}} \right) - \ln \left( 1 + \sqrt{1 - \frac{x}{y}} \right) + i\pi \right] - \ln(-x) - i\pi$$

$$R2ex2(x, y) = \left( 1 + \sqrt{1 - \frac{x}{y}} \right) \text{Li}_2 \left( 1 - \frac{1 - \sqrt{1 - \frac{x}{y}}}{1 + \sqrt{1 - \frac{x}{y}}} \right) + \left( 1 + \sqrt{1 - \frac{x}{y}} \right) \left[ \ln \left( 1 - \sqrt{1 - \frac{x}{y}} \right) \right]^2 + \left( 1 - \sqrt{1 - \frac{x}{y}} \right) \text{Li}_2 \left( 1 - \frac{1 + \sqrt{1 - \frac{x}{y}}}{1 - \sqrt{1 - \frac{x}{y}}} \right) + \left( 1 - \sqrt{1 - \frac{x}{y}} \right) \left[ \ln \left( 1 + \sqrt{1 - \frac{x}{y}} \right) \right]^2$$

(6) (7)
\[ + \frac{1}{2} (\ln y)^2 + 2 (\ln x)^2 - 2 \ln x \ln y \]
\[ + (\ln y - 2 \ln x) \left[ \left( 1 + \sqrt{1 - \frac{x}{y}} \right) \ln \left( 1 - \sqrt{1 - \frac{x}{y}} \right) \right. \]
\[ + \left. \left( 1 - \sqrt{1 - \frac{x}{y}} \right) \ln \left( 1 + \sqrt{1 - \frac{x}{y}} \right) \right] \]
\[ - i \pi \frac{\sqrt{x - y}}{\sqrt{-y}} [2 \ln 2 + \ln(x - y)] \]

\[ R3ex2(x, y, z) = 2 \ln \left( 1 - \frac{x}{z} \right) \eta(x, z) + 2 \ln \left( 1 - \frac{y}{z} \right) \eta(y, z) \]
\[ + 2 \text{Li}_2 \left( 1 - \frac{x}{z} \right) + 2 \text{Li}_2 \left( 1 - \frac{y}{z} \right) + 2 (\ln z)^2 \]  

\[ R3ex3(x, y, z) = 4 S_{12} \left( 1 - \frac{x}{z} \right) - 4 \text{Li}_3 \left( 1 - \frac{x}{z} \right) + 4 S_{12} \left( 1 - \frac{y}{z} \right) - 4 \text{Li}_3 \left( 1 - \frac{y}{z} \right) \]
\[ - 4 \eta \left( x, \frac{1}{z} \right) \left\{ \text{Li}_2 \left( \frac{x}{z} \right) + \ln \left( 1 - \frac{x}{z} \right) \ln \left( - \frac{x}{z} \right) + \frac{1}{2} \left[ \ln \left( 1 - \frac{x}{z} \right) \right]^2 \right\} \]
\[ - 4 \eta \left( y, \frac{1}{z} \right) \left\{ \text{Li}_2 \left( \frac{y}{z} \right) + \ln \left( 1 - \frac{y}{z} \right) \ln \left( - \frac{y}{z} \right) + \frac{1}{2} \left[ \ln \left( 1 - \frac{y}{z} \right) \right]^2 \right\} \]
\[ - 4 \ln z \left[ \text{Li}_2 \left( 1 - \frac{x}{z} \right) + \text{Li}_2 \left( 1 - \frac{y}{z} \right) \right] \]
\[ - 4 \ln z \left[ \eta \left( x, \frac{1}{z} \right) \ln \left( 1 - \frac{x}{z} \right) + \eta \left( y, \frac{1}{z} \right) \ln \left( 1 - \frac{y}{z} \right) \right] \]
\[ + 2 \text{Li}_3 \left( \frac{z - y}{z - x} \right) - 2 \text{Li}_3 \left( \frac{(z - y)x}{(z - x)y} \right) + 2 \text{Li}_3 \left( \frac{x}{y} \right) \]  

\[ + \left[ \ln \left( \frac{z - y}{z - x} \right) \right]^2 \eta \left( \frac{y - x}{z}, \frac{z}{z - x} \right) + 2 \eta \left( y - x, \frac{1}{z} \right) \]
\[ \times \left\{ \frac{1}{2} \left[ \ln \left( \frac{z - y}{z - x} \right) \right]^2 - \ln \left( \frac{z - y}{z - x} \right) \eta \left( \frac{z - y}{z}, \frac{z}{z - x} \right) \right\} \]
\[ + 2 (\ln x - \ln y) \left[ \text{Li}_2 \left( \frac{(z - y)x}{(z - x)y} \right) - \text{Li}_2 \left( \frac{x}{y} \right) \right] \]
\[ - \ln \left( 1 - \frac{x}{z} \right) (\ln x - \ln y)^2 + 2 \zeta(3) - \frac{4}{3} (\ln z)^3 \]
\[ + 2 \left[ \ln x - \ln y - \frac{1}{2} \ln \left( \frac{(z - y)x}{(z - x)y} \right) \right] \ln \left( \frac{(z - y)x}{(z - x)y} \right) \eta \left( \frac{y - x}{y}, \frac{z}{z - x} \right) \]

These functions are related to the corresponding \( R \) functions \([10, 11]\). They represent the coefficients of the Taylor expansion in \( \varepsilon \) making use of (cf. \([12]\))

\[ \zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n} \]

\(^2\)The function \( R3ex3(x, y, z) \) assumes that \( x \) and \( y \) have an imaginary part of different sign which is always the case.
Li_2(z) = - \int_{0}^{z} \frac{\ln(1-s)}{s} ds

Li_3(z) = \int_{0}^{z} \frac{Li_2(s)}{s} ds

S_{12}(z) = \frac{1}{2} \int_{0}^{z} \ln^2(1-s) \, ds

\eta(a,b) = 2\pi i \left[ \theta(-\text{Im } a)\theta(-\text{Im } b)\theta(\text{Im } (ab)) - \theta(\text{Im } a)\theta(\text{Im } b)\theta(-\text{Im } (ab)) \right].

The function
\texttt{evalRex(}\langle\text{expression}\rangle\texttt{);}

substitutes the \texttt{Rnexprm} functions in \langle\text{expression}\rangle by the corresponding Logarithms and Dilogarithms.

9 Run time

Due to several optimizations in program code the procedures became much faster compared to the original version. Run times typical for the new version are displayed in the following table. We used three different systems: (i) a DEC Alpha 8400 workstation (ii) a VAX 4000/90 workstation (iii) a PC with i486 chip, 16 MB RAM and 66 MHz frequency. All systems are working with MAPLE V. Without using the library we found the following run times:

|          | DEC Alpha 8400 | VAX 4000/90 | PC/DOS |
|----------|----------------|-------------|--------|
|          | tensor degree  | numerical | algebraic | numerical | algebraic | numerical | algebraic |
|          | two-point functions |       |           |           |           |           |           |
| 0        | 0.02 s | 0.04 s | 0.11 s | 0.22 s | 0.50 s | 0.88 s |
| 1        | 0.04 s | 0.21 s | 0.49 s | 1.53 s | 1.64 s | 2.70 s |
| 2        | 0.07 s | 0.28 s | 0.50 s | 1.97 s | 1.79 s | 3.90 s |
| 3        | 0.08 s | 0.41 s | 0.57 s | 2.99 s | 2.14 s | 8.51 s |
|          | three-point functions |       |           |           |           |           |           |
| 0        | 2.54 s | 1.83 s | 19.16 s | 13.82 s | 92.5 s | 25.87 s |
| 1        | 2.57 s | 2.23 s | 20.68 s | 16.66 s | 117.8 s | 26.36 s |
| 2        | 3.01 s | 3.62 s | 20.71 s | 26.31 s | 120.9 s | 38.89 s |
| 3        | 3.78 s | 4.16 s | 28.87 s | 40.30 s | 125.7 s | 117.7 s |

It should be emphasized that these are the times which are necessary to generate the functions once and forever using the \texttt{OneLoopLibp} routines. If they once are stored in the \texttt{LibPath} directory they may be read in quickly. With assigned \texttt{LibPath} we get:
Appendix: Installation and program call

The distribution of the oneloop package is accessible at

http://wwwthep.physik.uni-mainz.de/~xloops/

To install the package, just copy all files with the extension .ma in one directory. After having started the MAPLE session, you can read the oneloop package – just type the following lines:

```
LoopPath:=(path);
read’(path)oneloop.ma’;
```

(path) describes the directory where the .ma files are located. If these files are in the current directory then you can skip the assignment of LoopPath and only type

```
read’oneloop.ma’;
```

Acknowledgements

We would like to thank A. Frink for his check of $\mathcal{O}(\varepsilon)$ contributions.

References

[1] B. W. Char, K. O. Geddes, G. H. Gonnet, B. L. Leong, M. B. Monagan, S. M. Watt. Maple V Language Reference Manual. Springer (1991)

[2] B. W. Char, K. O. Geddes, G. H. Gonnet, B. L. Leong, M. B. Monagan, S. M. Watt. Maple V Library Reference Manual. Springer (1991)

[3] L. Brücher, J. Franzkowski, D. Kreimer. Comp. Phys. Comm. 85 (1995) 153

[4] J. C. Collins. Renormalization. Cambridge University Press (1984)

[5] L. Brücher, J. Franzkowski, A. Frink, D. Kreimer. hep-ph/3611378, Univ. Mainz Preprint MZ-TH/96-39 (1996)
[6] U. Nierste, D. Müller, M. Böhm. *Z. Phys.* **C57** (1993) 605

[7] L. Brücher, J. Franzkowski. *Two-loop Relevant Parts of One-loop Functions.* (in preparation)

[8] R. Gastmans, J. Verwaest, R. Meuldermans. *Nucl. Phys.* **B105** (1976) 454

[9] G. Passarino, M. Veltman. *Nucl. Phys.* **B160** (1979) 151

[10] B. C. Carlson. *Special Functions of Applied Mathematics.* Academic Press (1977)

[11] L. Brücher, J. Franzkowski, D. Kreimer. *Mod. Phys. Lett.* **A9** (1994) 2335

[12] L. Lewin. *Polylogarithms and Associated Functions.* North Holland (1981)