Squeezing concentration for Gaussian states with unknown parameter

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A continuous-variable analog of the Deutsch’s distillation protocol locally operating with two copies of the same Gaussian state is suggested. Irrespectively of the impossibility of Gaussian state distillation, we reveal that this protocol is able to perform a squeezing concentration of the Gaussian states with unknown displacement. Since this operation cannot be implemented using only single copy of the state, it is a new application of the distillation protocol which utilizes two copies of the same Gaussian state.

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I. INTRODUCTION

The exploitation of entangled states of quantum systems makes new quantum information protocols possible such as quantum teleportation [1], entanglement swapping [2] and quantum cryptography [3]. The common feature of all these protocols is that they require to transmit the entangled state from the common source to the distant partners. In practice, the transmission is always accompanied by losses that contaminate the shared state and thus reduce the degree of entanglement. Since the efficiency of the protocols is considerably dependent on the degree of entanglement shared by the partners, the question that naturally arises is whether the partners can eliminate the influence of losses and to enhance the entanglement employing only local operations and classical communication (LOCC). Such the protocols, commonly called distillation protocols, leading to the concentration of entanglement contained in several copies of partially entangled two-qubit states into a smaller number of singlet states has been proposed [4, 5] and also experimentally realized [6]. Further, efficient distillation procedure converting any two-qubit state with fidelity relative to maximally entangled state $F > 1/2$ has been suggested in [7]. Also entanglement swapping based distillation of partially entangled pure states of two qubits has been proposed in the literature [8]. The higher-dimensional generalizations of the qubit distillation protocols have been given in [9] and [10].

With increase of the interest in quantum information processing with continuous variables [11], [12] the natural need for distillation of continuous-variable (CV) entangled states has arisen. The existing proposals of CV distillation schemes allow either to distill the bipartite Gaussian states at the expense of non-Gaussian operations such as photon number measurement [13] or they are capable to distill the bipartite non-Gaussian states [14]. Unfortunately, any of these protocols is not satisfactory with respect to the current experiment, in which only Gaussian operations with Gaussian states are well managed. Recent results which apply to the "purely" Gaussian distillation protocols have led to the conclusion that no such a protocol exists. Namely, it was shown by means of the Gaussian completely positive (CP) maps technique that any trace-decreasing LOCC Gaussian CP map acting on known Gaussian state can be replaced by trace-preserving LOCC Gaussian map CP [15]. From that it follows particularly, that a single copy of two-mode bipartite entangled Gaussian state cannot be distilled to a more entangled state using only LOCC Gaussian operations. Further it was proved, that the bipartite symmetric two-mode Gaussian state cannot be distilled to more entangled state with the aid of another identical copy of the state using only local Gaussian operations [16]. The general proof that Gaussian states cannot be distilled by local Gaussian operations and classical communication was given in [17]. Having these facts in mind the question that naturally arises is whether there exists some Gaussian operation on bipartite entangled Gaussian state that two distant observers employing only local Gaussian operations wish to carry out that can be performed only at the expense of utilization of more than single copy of the state. This is the subject of the present article. The paper is organized as follows. In Sec. II we investigate the continuous-variable analog of the Deutsch’s qubit distillation protocol. The example of Gaussian operation requiring ‘collective’ local operations on two copies of entangled Gaussian state is given in Sec. III. Finally, Sec. IV contains conclusion.

II. CV ANALOG OF DEUTSCH’S PROTOCOL

The properties of the CV analog of the Deutsch’s protocol [7], which will be described below, are best studied on the example of the displaced two-mode squeezed vac-
CV then consists of (i) local QND measurement of source tension of the Deutsch’s qubit distillation protocol \[7\] to and Bob (\(B\)), as is depicted in Fig. 1. The natural e- x

sition and momentum quadrature operators of modes\(A\) and\(B\), respectively;\(x_0\) is an unknown coherent displacement of quadrature\(X_A\). Note, that as\(\sigma_-\) decreases,\(\sigma_+\) increases, and this state approaches the well-known Einstein-Podolsky-Rosen state. The state \([1]\) can be prepared by mixing of momentum-squeezed vacuum state with squeezing\(r_1\) and position-squeezed vacuum state with squeezing\(r_2\) at a balanced beam-splitter \([12]\), followed by unitary displacement operation \(U = \exp(-ix_0P_A)\) on mode\(A\).

Let us assume that two identical copies, conventionally called source (S) and target (T), of the state \([1]\) where\(x_0 = 0\) are distributed between two observers, Alice (\(A\)) and Bob (\(B\)), as is depicted in Fig. 1. The natural extension of the Deutsch’s qubit distillation protocol \([4]\) to CV then consists of (i) local QND measurement of source position quadrature (CV analog of XOR-gate)

\[|x\rangle_S|x'\rangle_T \rightarrow |x\rangle_S|x' - x\rangle_T,\] (2)

performed both by Alice and by Bob, followed by (ii) local homodyne measurement of the target quadratures\(X_{Ta}\) and\(X_{Tb}\). Alice and Bob then can communicate their measurement outcomes via classical channel and, as in the original protocol, discard both the pairs if the outcomes do not coincide. A straightforward calculation reveals, however, that irrespectively of the measurement outcomes all the output states of the protocol can be brought into the single state that is independent on the measurement outcomes, by applying two local unitary displacement transformations

\[x \rightarrow x + \chi/2, \quad y \rightarrow y + \chi'\] (3)
on Alice’s and Bob’s side, if the measurement outcomes obtained by them are\(\chi\) and\(\chi'\), respectively. Thus the selection of the subensemble of the output states based on the results of target measurements is not useful and the protocol becomes deterministic. This property of the CV analog of the Deutsch’s scheme is in sharp contrast with the original protocol for qubits. It has been shown recently \([13]\), that this behaviour is characteristic to all Gaussian distillation protocols for apriori known input Gaussian states, i.e. that any such ‘purely’ Gaussian distillation protocol would be deterministic.

After application of this CV protocol and displacements \([6]\), one finds the output state of the form \([6]\) with the variances changed according to the rule \(\bar{\sigma}_\pm = \pm \frac{\sigma_\pm}{\sqrt{2}}\). Irrespectively of the increase of the position correlations, the marginal purity \(P = 2\sqrt{\sigma_+\sigma_-}/(\sigma_+ + \sigma_-)\) of outgoing pure state remains unchanged and the entanglement in the state \([1]\) is not enhanced. In order to decrease the marginal purity \(P\) and consequently to obtain more entangled state, we would need different behaviour in these variances, for example, \(\sigma_-\) decreases and \(\sigma_+\) remains constant at a time.

Surprisingly, our CV procedure can be substituted by the unconditional scheme employing only single copy of the source state. It consists of two single-mode squeezers, performing the following squeezing transformations \(X_A = \frac{1}{\sqrt{2}}X_A\) and \(X_B = \frac{1}{\sqrt{2}}X_B\) on Alice’s and Bob’s side. This procedure is capable of transformation of nonsymmetrically entangled states with \(\sigma_+\sigma_- > 1\) to symmetrical ones, having \(\sigma_+\sigma_- = 1\), and vice versa, as has been previously discussed in \([13]\). Thus, due to the equivalence between aforementioned two-copy and single-copy schemes, one can have doubt about the usefulness of our CV procedure based on operations on two copies of the input state. The question that naturally arises in this context is, whether the utilization of two copies of a two-mode Gaussian state enables us to perform operation, which cannot be equivalently carried out only on single copy of the state. In the following Section, we demonstrate that our CV analog of the Deutsch’s protocol can be useful in this respect, particularly for squeezing concentration of the Gaussian states \([1]\) with an unknown displacement\(x_0\).

### III. SQUEEZING CONCENTRATION

We start from a simple example illustrating the usefulness of the protocol discussed in the previous Section in manipulating with an unknown Gaussian state. First, we analyse a local part of the protocol, for example, on the Alice’s side. Let us consider the following single-mode
pure Gaussian state

\[ |x_0, \sigma_X \rangle = \left(2\pi \sigma_X \right)^{-1/4} \int_{-\infty}^{\infty} \exp\left[ -\frac{(x-x_0)^2}{4\sigma_X^2} \right] |x\rangle dx \]  

where \( x_0 = \langle X \rangle \) is an unknown parameter representing a coherent signal and \( \sigma_X = \langle (\Delta X)^2 \rangle = e^{-2r}/2 \), \( r \) is the squeezing parameter. This state can be prepared by displacing the squeezed vacuum state by the value \( x_0 \). The displacement is often used to encode information into the quantum state, for example, in the CV dense coding [13]. Since the transmission channel introduces extra noise into the state [13], it is desirable to reduce the noise affecting the transmitted information before subsequent processing. It can be achieved by means of appropriate squeezing in the position quadrature. Employing only single copy of the state [4], we can simply squeeze the position fluctuations in the single-mode squeezer, however, in this case also the value of unknown parameter \( x_0 \) is changed. The reason is that single-mode squeezer reduces both the mean value \( \langle X \rangle \) and the variance \( \sigma = \langle (\Delta X)^2 \rangle \) of the position quadrature. If we do not know the displacement \( x_0 \), then we cannot restore the mean value to the original one without errors. Because the information is encoded in the mean value \( \langle X \rangle \), its preservation during squeezing is important. On the other hand, knowing the parameter \( x_0 \), we can restore the initial mean value by a suitable displacement.

However, if we consider two copies of the state [4], we are able to squeeze the position fluctuations without changing unknown mean value \( \langle X \rangle = x_0 \), as is depicted in Fig. 2. It can be achieved by above discussed protocol, employing, for instance, only Alice’s part of the scheme outlined in Fig. 1. The procedure produces single copy of the state [4] with new reduced variance \( \bar{\sigma} = \frac{\sigma}{N} \) and without changing the mean value \( \langle X \rangle \). Since it cannot be achieved employing only single copy of the state [4], this example illustrates a new application of the CV analog of the distillation protocol in manipulation with unknown Gaussian states.

In general, this protocol can be looked at as a state transformation of the source Wigner function \( W_S(x, p) \) with the help of target Wigner function \( W_T(x, p) \). After QND measurement, position measurement on the target mode and subsequent displacement, we arrive at the output source Wigner function

\[ \tilde{W}_S(x, p) = \int_{-\infty}^{\infty} W_S(x-\chi/2, p') \times W_T(x+\chi/2, p-p') dp' d\chi. \]  

If we are interested in the position and momentum distributions of the outgoing source mode separately, we can integrate the Wigner function [3] over momentum \( p \) and position \( x \), respectively and obtain the following marginal distributions

\[ \tilde{p}_S(x) = \int_{-\infty}^{\infty} p_S(x-\chi/2)p_T(x+\chi/2) d\chi, \]
\[ \tilde{p}_S(p) = \int_{-\infty}^{\infty} p_S(p')p_T(p-p') dp', \]

where \( p_S \) and \( p_T \) are input source and target marginal distributions, respectively. Assuming both the source and target state to be in the same Gaussian state with marginal distributions

\[ p_S(x) = p_T(x) = \frac{1}{\sqrt{2\pi(\Delta X_S)^2}} \exp\left[ -\frac{(x-x_0)^2}{2(\Delta X_S)^2} \right], \]
\[ p_S(p) = p_T(p) = \frac{1}{\sqrt{2\pi(\Delta P_S)^2}} \exp\left[ -\frac{p^2}{2(\Delta P_S)^2} \right], \]

substituting them into the formulas [3] and performing the integrations, we obtain that the unknown mean value \( \langle \bar{X}_S \rangle = \langle X_S \rangle = x_0 \) is preserved, whereas the position fluctuations are squeezed to half of initial value

\[ \langle (\Delta \bar{X}_S)^2 \rangle = \frac{\langle (\Delta X_S)^2 \rangle}{2}, \quad \langle (\Delta \bar{P}_S)^2 \rangle = 2(\langle \Delta P_S \rangle^2) \quad (8) \]

around this mean value. Consequently, due to principle of complementarity, the momentum fluctuations are enhanced. Employing \( N \) identical copies of the same Gaussian state, we are able to squeeze the position fluctuations to \( 1/N \) of initial value, without changing the mean value of the position quadrature. Naturally, similar procedure can be constructed for the squeezing of momentum fluctuations. Note, that a discrete-variable analogue of this procedure has been discussed as the qubit purification protocol [20].

Let us apply now the idea of this procedure to the two-mode case, considering two copies of the state [4], for simplicity. If the parameter \( x_0 \) is known, then we can utilize the Bowen’s “concentrating” procedure and manipulate with two-mode correlations on single copy [13]. If, however, the value \( x_0 \) is a priori not known, then we are not able to coherently manipulate with the variance \( \langle (\Delta (X_A-X_B))^2 \rangle \), without changing the unknown parameter \( x_0 \). On the other hand, employing entire two-mode setup depicted in Fig. 1, we are able to squeeze coherently the fluctuations \( \bar{\sigma}_+ = \sigma_+/2, \bar{\sigma}_- = \sigma_-/2 \) again without changing the unknown parameter \( x_0 \). In terms of Wigner functions, our concentrating procedure can be expressed, in analogy with the single-mode case [3], by the formula

\[ \bar{W}_S(x, p) = \int_{-\infty}^{\infty} W_S(x-\chi/2, p') \times W_T(x+\chi/2, p-p') dp' d\chi. \]
If we are interested separately in position correlations and momentum correlations between Alice and Bob, we can integrate the function over momentum variables $p_A, p_B$ to obtain joint position distribution

$$\tilde{p}_S(x_A, x_B) = \int \int p_S(x_A - x/2, x_B - y/2) \times
$$

$$p_T(x_A + x/2, x_B + y/2) d\lambda dy \quad (10)$$

or we can integrate over position variables $x_A, x_B$ to obtain joint momentum distribution

$$\tilde{p}_S(p_A, p_B) = \int \int p_S(p_A', p_B') \times
$$

$$p_T(p_A - p_A', p_B - p_B') dp_A' dp_B'. \quad (11)$$

Let us assume, that the source and target have the same joint probability distributions of the form

$$p(x) = \frac{1}{2\pi \sqrt{\det V_x}} \exp \left[ -(x - x_0)^T (2V_x)^{-1} (x - x_0) \right],$$

$$p(p) = \frac{1}{2\pi \sqrt{\det V_p}} \exp \left[ -p^T (2V_p)^{-1} p \right], \quad (12)$$

where $x = (x_A, x_B)$, $x_0 = (x_0, x_0)$, $p = (p_A, p_B)$, $T$ denotes operation of transposition and where

$$<X_i>_i = <\Delta X_i \Delta X_i>, \quad <V_p>_{ij} = <\Delta P_i \Delta P_j> \quad (13)$$

are the elements of position and momentum variance matrices, where $\Delta X_i = X_i - <X_i>$, $\Delta P_i = P_i - <P_i>$, $i, j = A, B$. The simple calculation then yields the output probability distributions of the same form as in Eq. (12), however, with the transformed variance matrices $V_X = V_X/2$ and $V_P = 2V_P$. Thus, the correlations in positions increase as follows

$$<[\Delta(\tilde{X}_A - \tilde{X}_B)^2]> = \frac{<[\Delta(X_A - X_B)^2]>}{2}, \quad (14)$$

whereas the mean values $<X_A>$ and $<X_B>$ are preserved. On the other hand, the momentum anticorrelations decrease

$$<[\Delta(\tilde{P}_A + \tilde{P}_B)^2]> = 2<[\Delta(P_A + P_B)^2]>,$$

and thus the entanglement and total entropy of the source state are preserved. This example illustrates that although the CV analog of the Deutsch’s distillation protocol is not useful for enhancement of entanglement, it can be useful when one wishes to locally concentrate the two-mode squeezing in a partially unknown Gaussian state.

We have analysed the CV analog of specific distillation protocol for two copies of pure two-mode Gaussian state. We have demonstrated that it is only able to manipulate with squeezing, while the entanglement is preserved. Irrespectively to impossibility of entanglement increasing, it can be useful from another point of view. Employing two-copies of a two-mode Gaussian state displaced in the position by an unknown value, we can utilize this local-operation protocol to enhance the particular correlations, without changing the unknown position displacement. Because this procedure cannot be implemented if we have only single copy of this state, we have revealed a new application of the CV analog of the discrete-variable distillation protocol.

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[1] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W.K. Wootters, Phys. Rev. Lett. 70, 1895 (1993).
[2] M. Zukowski, A. Zeilinger, M.A. Horne, and A. K. Ekert, Phys. Rev. Lett. 71, 4287 (1993); J.-W. Pan, D. Bouwmeester, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. 80, 3891 (1998).
[3] A. K. Ekert, Phys. Rev. Lett. 67, 661 (1991).
[4] C.H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Phys. Rev. A 53, 2046 (1996).
[5] C.H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W.K. Wootters, Phys. Rev. Lett. 76, 722 (1996).
[6] J.-W. Pan, C. Simon, C. Bruckner, and A. Zeilinger, Nature 410, 1067 (2001).
[7] D. Deutsch, A. Ekert, R. Jozsa, Ch. Macchiavello, S. Popescu, and A. Sanpera, Phys. Rev. Lett. 76, 722 (1996).
[8] S. Bose, V. Vedral, and P. L. Knight, Phys. Rev. A 60, 194 (1999).
[9] M. Horodecki and P. Horodecki, Phys. Rev. A 59, 4206 (1999).
[10] G. Alber, A. Delgado, N. Gisin, and I. Jex, quant-ph/0102035.
[11] S.L. Braunstein and H.J. Kimble, Phys. Rev. Lett. 80, 869 (1998); R.E.S. Polkinghorne and T.C. Ralph, Phys. Rev. Lett. 86, 4267 (2001).
[12] A. Furusawa, J.L. Sørensen, S.L. Braunstein, C.A. Fuchs, H.J. Kimble, and E.S. Polzik, Science 282, 706 (1998).
[13] L.-M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 84, 4002 (2000); T. Opatrný, G. Kurizki, and D.-G. Welsch, Phys. Rev. A 61, 032302 (2000).
[14] S. Parker, S. Bose, and M. Plenio, Phys. Rev. A 61, 032305 (2000).
[15] J. Fiurasek, quant-ph/0204069.
[16] J. Eisert, S. Scheel, and M. Plenio, quant-ph/0204052.
[17] G. Giedke and J. I. Cirac, quant-ph/0204089.
[18] W.P. Bowen, P.K. Lam and T.C. Ralph, arXiv:quant-ph/0104108 (2001).
[19] S. L. Braunstein and H. J. Kimble, Phys. Rev. A 61, 042302 (2000).
[20] D. Bouwmeester, A. Ekert and A. Zeilinger, The Physics of Quantum Information, Springer-Verlag Berlin Heidelberg 2000.