On the Mass Transfer Rate in SS Cyg

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ABSTRACT

The mass transfer rate in SS Cyg at quiescence, estimated from the observed luminosity of the hot spot, is \( \log \dot{M}_{\text{tr}} = 16.8 \pm 0.3 \). This is safely below the critical mass transfer rates of \( \log \dot{M}_{\text{crit}} = 18.1 \) (corresponding to \( \log T_{\text{crit}} = 3.88 \)) or \( \log \dot{M}_{\text{crit}} = 17.2 \) (corresponding to the "revised" value of \( \log T_{\text{crit}} = 3.65 \)). The mass transfer rate during outbursts is strongly enhanced.

Key words: accretion, accretion disks – binaries: cataclysmic variables, stars: dwarf novae, stars: individual: SS Cyg

1. Introduction

SS Cyg is one of the best studied dwarf novae. Its system parameters have recently been reliably determined by Bitner et al. (2007). Earlier Harrison et al. (1999) determined the new, highly accurate trigonometric parallax \( \pi = (6.02 \pm 0.46) \) mas, corresponding to a distance of \( d = 166 \pm 12 \) pc, i.e. 1.5-2.0 times larger than estimated previously.

Schreiber and Gänsicke (2002) discussed the consequences of this new distance and found that the mass transfer rate in SS Cyg at quiescence is significantly larger than the critical mass transfer rate. Their analysis was later repeated with new system parameters (from Bitner et al. 2007) by Schreiber and Lasota (2007) who found \( \dot{M}_{\text{tr}} = (8.8 - 9.2) \times 10^{18} \) g/s, compared to \( \dot{M}_{\text{crit}} = (9.2 - 9.1) \times 10^{17} \) g/s. This discrepancy led them to the conclusion that "Either our current picture of disc accretion in these systems must be revised or the distance to SS Cyg is \( \sim 100 \) pc".

Common to both these investigations was the assumption that the mass transfer rate at quiescence can be calculated as the amount of mass accreted during outburst (estimated from the absolute visual magnitude at outburst maximum and its duration) divided by the length of the cycle. This assumption would be correct only in the case of a constant mass transfer rate throughout the entire dwarf nova
A. A.
cycle. However, as already pointed out by Schreiber and Lasota, there is growing
evidence that during outbursts and superoutbursts of dwarf novae the mass transfer
rate is strongly enhanced.

The mass transfer rate during quiescence can be determined directly from the
luminosity of the hot spot. This will be done in the present paper.

2. System Parameters and the Critical Mass Transfer Rate

2.1. System Parameters

We adopt system parameters of SS Cyg determined by Bitner et al. (2007). These are:

\[ M_1 = 0.81 \pm 0.19 \, M_\odot, \quad M_2 = 0.55 \pm 0.13 \, M_\odot, \quad \text{and} \quad i = 49^{+6}_{-4} \]  
(note that \( i = 49 \) corresponds to \( M_1 = 0.81 \) and \( M_2 = 0.55 \)).

The mean magnitude and color of SS Cyg at quiescence compiled by Bruch
and Engel (1994) are: \( V = 11.9, \quad (B - V) = 0.53 \). After being corrected for
\( E_{B-V} = 0.07 \) they become: \( V_0 = 11.7, \quad (B - V)_0 = 0.46 \). Using distance modulus
of \( (m - M) = 6.10 \pm 0.15 \), corresponding to the trigonometric parallax measured
by Harrison et al. (1999), we get \( M_V = 5.6 \pm 0.15 \). In addition, however, we must
note that – due to variations of the V magnitude at quiescence – the uncertainty of
this value is likely to be bigger. In what follows we will adopt: \( M_V = 5.6 \pm 0.5 \).

2.2. The Hot Spot in SS Cyg at Quiescence

Vološina and Khruzina (2000a; English translation: 2000b) published mean
UBV light curves of SS Cyg at quiescence showing double-humped modulation
due – obviously – to the non-spherical shape of the secondary. The two maxima are
of nonequal heights, the one at phase \( \phi \sim 0.75 \) being higher. This was interpreted
by them and also by Bitner et al. (2007), as being due to the contribution from the
hot spot.

Adopting this interpretation we determine the absolute visual magnitude of the
spot at its maximum. Using the observed difference between the amplitudes of
two maxima \( \Delta f_V / < f_V > \approx 0.047 \pm 0.005 \) (see Fig. 2 in Vološina and Khruzina
2000ab, or Fig. 4 in Bitner et al. 2007) and \( M_V = 5.6 \pm 0.5 \) (as adopted above) we
obtain \( M_{V,\text{sp}}^{\text{max}} = 8.9 \pm 0.5 \).

The B and V light curves (Fig. 2 in Vološina and Khruzina 2000ab) show
nearly identical amplitudes. This implies that the color of the hot spot is: \( (B - V)_{\text{sp}} \approx (B - V)_0 = 0.46 \). Using calibration based on Kurucz (1993) model atmospheres
we get for the effective temperature of the spot: \( \log T_{\text{sp}} \approx 3.82 \).

2.3. The Critical Mass Transfer Rate

The critical mass transfer rate, below which the thermal instability sets in and
results in the dwarf nova behavior, is defined by the critical temperature corre-
sponding to the upper bend in the \( \Sigma - T_c \) relation (see Lasota 2001 or Smak 2002
and references therein). The value of this critical temperature depends on relevant parameters as (Smak 2002, Eq.13)

$$\log T_{\text{crit}} = \log T_{\text{crit}}^0 - 0.085 \log R_{d,10} + \frac{0.085}{3} \log M_1 + 0.010 \log (\alpha/0.1) , \quad (1)$$

where $M_1$ is in solar units, $R_{d,10} = R_d/10^{10}$ and $T_{\text{crit}}^0$ corresponds to $M_1 = 1$, $R_d = 1 \times 10^{10}$ and $\alpha = 0.1$.

The critical mass transfer rate is then related to the critical temperature by

$$\sigma T_{\text{crit}}^4 = \frac{3}{8\pi} M_{\text{crit}} \frac{G M_1}{R_d^3} \left[ 1 - (R_1/R_d)^{1/2} \right] . \quad (2)$$

The commonly adopted value of the critical temperature $\log T_{\text{crit}}^0 \approx 3.88$ is based on the shape of the $\Sigma - T_e$ relations resulting from numerical integrations of the vertical structure equations. However, there is evidence (Smak 2002), based on the analysis of CV’s with stationary accretion and, particularly, of Z Cam stars at their standstills, which strongly suggest that its value is much lower, namely: $\log T_{\text{crit}}^0 \approx 3.65$.

Using system parameters of SS Cyg and $r_d = R_d/A = r_{\text{tid}} = 0.9 \, r_{\text{Roche}}$ we get for those two values of $T_{\text{crit}}^0$

$$\log \dot{M}_{\text{crit}}^{3.88} = 18.1 , \quad \text{and} \quad \log \dot{M}_{\text{crit}}^{3.65} = 17.2 . \quad (3)$$

3. Mass Transfer Rate at Quiescence from the Hot Spot

3.1. Luminosity of the Hot Spot

The mean bolometric luminosity of the hot spot can be written as

$$< L_{\text{bol,sp}} > = \frac{1}{2} \Delta V^2 \eta M_{\text{fr}} = \frac{1}{2} \Delta v^2 \left( \frac{2\pi P}{A} \right)^2 M_{\text{fr}} , \quad (4)$$

where $1/2\Delta V^2$ is the energy dissipation per 1 gram of the stream material, $M_{\text{fr}}$ is the mass transfer rate, and $\Delta v^2$ is the dimensionless equivalent of $\Delta V^2$ which depends on the mass ratio and on the distance from the disk center, i.e. on the radius of the disk $r_d = R_d/A$ or $r_d/r_{\text{Roche}}$.

In general, the luminosity of the hot spot represents only part of the energy dissipated during stream collision. To account for this effect Eq.(4) contains factor $\eta < 1$. In what follows we will adopt $\eta = 0.5$. This is probably a lower limit to the true value of this parameter. If so, the resulting value of $M_{\text{fr}}$ will form an upper limit to the true mass transfer rate, appropriate in the context of our considerations.

The luminosity of the spot can also be expressed as

$$< L_{\text{bol,sp}} > = \pi s^2 A^2 \sigma T_{sp}^4 , \quad (5)$$
where $s$ is the dimensionless radius of the spot (assumed to be circular) and $T_{sp}$ – its effective temperature. By comparing Eqs.(4) and (5) we obtain $s$ as a function of other parameters

$$s = \left[ \frac{\frac{1}{2} \Delta v^2 \eta (2\pi/P)^2 M_{tr}}{\pi \sigma T_{sp}^4} \right]^{1/2}.$$  

(6)

The mean visual luminosity of the spot can now be calculated as

$$< L_{V,sp} > = \pi s^2 A^2 f_V(T_{sp}),$$  

(7)

where $f_V(T_{sp})$ is the visual flux, to be obtained from Kurucz (1993) model atmospheres.

Turning to the maximum luminosity of the spot observed at orbital inclination $i$ we have (Paczyński and Schwarzenberg-Czerny 1980, see also Smak 2002)

$$L_{V,sp}^{\max} = \frac{12}{3-u} \left( 1 - u + u \sin i \right) \sin i < L_{V,sp} > ,$$  

(8)

where $u$ is the limb darkening coefficient. Converting $L_{V,sp}^{\max}$ to magnitudes we finally obtain

$$M_{V,sp}^{\max} = f(M_{tr}, r_d/r_{Roche}, T_{sp}, u).$$  

(9)

Calculations show that – with parameters applicable to SS Cyg – the largest uncertainty in the resulting $M_{V,sp}^{\max} - M_{tr}$ (Eq.9) and $s - M_{tr}$ (Eq.6) relations comes from $r_d$. Using results obtained from spot eclipse analysis in U Gem (Smak 2001) and IP Peg (Smak 1996) we adopt: $r_d = (0.7 \pm 0.1) r_{Roche}$. For the limb darkening coefficient we adopt $u = 0.6$; additional calculations with $u = 0.2$ show that $M_{V,sp}^{\max}$ is rather insensitive to this parameter. The effects of $T_{sp}$ will be discussed below.

The $M_{V,sp}^{\max} - M_{tr}$ relation depends of course also on system parameters. Fortunately, it turns out that in this case the effects of higher masses at lower inclination (and vice versa) largely compensate each other.

The resulting $M_{V,sp}^{\max} - M_{tr}$ and $s - M_{tr}$ relations are shown in Fig.1. In addition to relations calculated with log $T_{sp} = 3.82$ (Section 2.2) shown are also relations corresponding to log $T_{sp} = 4.00$; note that such a temperature would already require the spot to have $B-V \approx 0.0$, i.e. be bluer than observed by nearly 0.5 mag. Even in such a case the $M_{V,sp}^{\max} - M_{tr}$ relation differs only slightly from the relation calculated with log $T_{sp} = 3.82$.

3.2. Comments on $M_{tr} = (1.1 - 3.8) \times 10^{18} \text{ g/s}$

Such a value was obtained by Schreiber and Lasota (2007) from their estimates involving the amount of mass accreted during outburst. Apart from our direct determination of the much lower value of $M_{tr}$ (Section 3.3) there are several other arguments which imply that such a high mass transfer rate is simply impossible.
Fig. 1. The $M_{V,sp}^\text{max}$ (bottom) and $s-M_{tr}$ (top) relations calculated from Eqs.(5)-(9) with $\eta = 0.5$. The solid and dotted lines correspond to $r_d = (0.7 \pm 0.1) r_{\text{Roche}}$ and $\log T_{sp} = 3.82$. The broken lines – to $r_d = 0.7 r_{\text{Roche}}$ and $\log T_{sp} = 4.00$. The horizontal lines in the lower plot represent the absolute visual magnitude of the spot at maximum (Section 2.2): $M_{V,sp}^\text{max} = 8.9 \pm 0.5$.

From Fig.1 we find that the luminosity of the hot spot (at maximum) corresponding to $\dot{M}_{tr} = (1.1 - 3.8) \times 10^{18}$ g/s should be $M_{V,sp}^\text{max} = 5.3 \pm 1.3$. If so, the contribution from the spot would not only dominate in the shape of the light curve but would also increase the total luminosity of the system well above the observed $M_V = 5.6 \pm 0.5$.

From Fig.1 we also find that at $\dot{M}_{tr} = (1.1 - 3.8) \times 10^{18}$ g/s the dimensions of the spot should be larger, or even much larger than $s \sim 0.2$, which is simply unrealistic.

In addition, with $\dot{M} \sim 2 \times 10^{18}$ g/s, the amount of mass added to the disk by the quiescent stream would be $\Delta M_D \sim 9 \times 10^{24}$ g. However, the total mass of the disk obtained from dwarf nova model calculations, intended to represent SS Cyg and Z Cam (Hameury et al. (1998, Fig.10; Buat-Ménard 2001, Fig.2) is only $M_D \sim (1.0 - 2.5) \times 10^{24}$ g. The value of $\Delta M_D \sim 9 \times 10^{24}$ g would then be significantly larger than the total mass of the disk. (At this point it may also be worth to mention that $M_{D,\text{max}}$ discussed by Schreiber and Lasota [2007, Eq.5], and used by them to estimate $\dot{M}_{tr}$, is not the mass of the disk at outburst maximum, but an absolute upper limit to disk mass just before the outburst).
3.3. The Mass Transfer Rate at Quiescence

Using the $M_{V,sp}^{\max} - M_{tr}$ relation presented in Fig.1 and $M_{V,sp}^{\max} = 8.9 \pm 0.5$ (Section 2.2) we obtain

$$\log M_{tr} \approx 16.8 \pm 0.3$$

(10)

This is the main result of our analysis. It shows that the mass transfer rate in SS Cyg at quiescence is safely below the critical mass transfer rate (Eqs.3 in Section 2.3). In particular, this is true even in the case of the much lower "revised" value of $\log M_{3.65}^{\max} = 17.2$.

The radius of the spot at $\log M_{tr} \approx 16.8 \pm 0.3$ (see Fig.1–top) is $s \sim 0.05$ which looks reasonable: it is only slightly larger than in other well studied dwarf novae with comparable orbital periods, e.g. U Gem (Smak 1996), or IP Peg (Smak 2001).

From the mass transfer rate and the duration of the cycle we can also calculate the amount of mass added to the disk by the quiescent stream: $\Delta M_D \sim 3 \times 10^{23}$ g. Comparing this with $M_D \sim (1.0 - 2.5) \times 10^{24}$ g (see Section 3.2) we conclude that it represents reasonable fraction – roughly 10-30 percent – of the total mass of the disk.

At this point we recall that our estimate of the luminosity of the spot (Section 2.2), based on the light curves published by Voloshina and Khruzina (2000ab), was rather crude. It is now re-assuring to note that the selfconsistency of results presented above provides an independent argument in favor of the adopted value of $M_{V,sp}^{\max} = 8.9 \pm 0.5$.

3.4. The Mass Transfer Rate during Outbursts

The accretion rate during outburst maxima was estimated by Schreiber and Lasota (2007, Eq.3) as $\dot{M}_{\text{out}} \sim 9 \times 10^{18}$ g/s. Combined with our value of $\dot{M}_{tr} \sim 6 \times 10^{16}$ g/s it implies that the mass transfer rate during outbursts is enhanced by – very roughly – factor of $\sim 100$. Considering all uncertainties involved we note that this is similar to earlier estimates for U Gem (Smak 2005) and for SU UMa type dwarf novae (Smak 2004).

4. Conclusion

Results presented above imply that the answer to the question posed by Schreiber and Lasota (2007) in the title of their paper is quite simple: Nothing is wrong with SS Cyg, nor with the theory of dwarf nova outbursts.

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