Appendix for Seasonality, density dependence and spatial population synchrony
A Winter Weather Patterns

Figure 1: Winter mean temperature and winter total precipitation for the 19 stations, color-coded according to their region.
B Model coefficients

Model II

Figure 2: Autoregressive coefficient estimates for model II. Associated 95% credible intervals are represented by the bars.

Model III

Figure 3: Autoregressive coefficient estimates for model III. Associated 95% credible intervals correspond to the bars, with the coefficients associated with each region represented with the respective color (blue for $R_1$, green for $R_2$ and yellow for $R_3$).
Model IV

Figure 4: Autoregressive coefficient estimates for model IV, according to the different seasons. Associated 95% credible intervals correspond to the bars, with the coefficients associated with each region represented with the respective color (blue for $R_1$, green for $R_2$ and yellow for $R_3$).
C Simulations

Figure 5: Correlations between 1000 time-series of length 200 simulated according to the seasonal model IV, for spring (left) and fall (right). From left to right on each panel: 1) Correlations estimated using the raw densities (Model I), 2) Correlations estimated using the residuals of an annual AR2 model (Model II), 3) Correlations estimated using the residuals of the seasonal model (Model IV), and 4) Correlations of the noise term in the simulated time-series (“true observed correlation”).

In the empirical case study, we observed season-specific changes in synchrony when comparing annual to spring and fall spatial correlations, with a stronger decrease in synchrony in fall than in spring. To assess the robustness of these changes and potential sources of bias, we used simulations with known values of population processes, including process variances, and synchrony (correlation). We used these simulations to understand how estimated correlations were dependent on the models used (I to IV).

We simulated 1,000 correlated time-series of length 200, of two populations corresponding to a true process described by model IV (see Methods), with regression parameters approximating those of the real data set \( \{\beta_1 = 0.5, \beta_2 = 0.1, \beta_3 = -0.4, \beta_4 = 0.1, \gamma_1 = 0.8, \gamma_2 = 0.1, \gamma_3 = -0.1, \gamma_4 = -0.1\} \). We then compared (a) the true correlation in the noise terms for each pair (which corresponds to a parameter in the simulation; we used \( r = 0.5 \) for the spring correlation and \( r = 0.3 \) for the fall correlation), against: (b) the correlations when using year-to-year raw abundances for either spring and fall (model I); and (c) correlations in the residuals of annual AR(2) for each season separately (model II). Figure 5 displays the correlation estimates for each of the approaches. As expected, using the (correct) seasonal model attained accurate estimates of the correlation between the two populations, whereas using either the raw densities or annual AR2 model led to biases in correlation estimates. The bias was stronger for the fall correlations than for the spring correlations, and the direction of the bias was season-dependent. This confirms that the observed changes may come from considering the seasonal model (IV) instead of the annual models (I–III), rather than artifacts related to the data. This shows that fluctuating (i.e., seasonal) environments can enhance or decrease estimates of synchrony, but the exact pattern obtained using annual time series will depend on the noise structure and which season is monitored Vasseur (2007).

D Bayesian R-squared

For Bayesian models, the computation of the \( R^2 \) measure is not straightforward as the classical definition (variance explained by the predicted values divided by the total variance in the data) cannot be directly applied. First, there is the risk of the explained variance to surpass the total variance, yielding an \( R^2 > 1 \), and second, it is important to reflect the posterior uncertainty in the coefficients. To overcome both problems, Gelman et al. (2019) proposed an alternative formula and framework, extensively described in their manuscript and briefly
summarized here. The general formula is given by

$$\text{Alternative } R^2 = \frac{\text{Explained Variance}}{\text{Explained Variance} + \text{Residual Variance}} = \frac{\text{Var}_{\text{fit}}}{\text{Var}_{\text{fit}} + \text{Var}_{\text{res}}},$$

where $\text{Var}_{\text{fit}}$ corresponds to the variance in the fitted values, while $\text{Var}_{\text{res}}$ is the variance in the modeled residuals ($\sigma^2$ for a simple linear regression). Considering the Bayesian framework, rather than a point estimate for the model coefficients, we obtain a distribution for the model coefficients which the fitted values (and the residuals) are a function of. We can sample from the approximated posterior distribution to obtain the

$$\text{Bayesian } R^2_{(s)} = \frac{\text{Var}^{(s)}_{\text{fit}}}{\text{Var}^{(s)}_{\text{fit}} + \text{Var}^{(s)}_{\text{res}}},$$

for each sample $(s)$, where the number of samples is given by $s = 1, \ldots, S$. For this study, we generated $S = 1000$ samples from the posterior distribution using the `inla.posterior.sample` function, available in the R-INLA package [Martins et al. (2013)]. The precise code for the computation of the Bayesian R-squared is given in the scripts published with the manuscript.

References

Gelman, A., Goodrich, B., Gabry, J., and Vehtari, A. (2019). R-squared for bayesian regression models. *Am. Stat.*, 73(3):307–309.

Martins, T. G., Simpson, D., Lindgren, F., and Rue, H. (2013). Bayesian computing with INLA: new features. *Comput. Stat. Data Anal.*, 67:68–83.

Vasseur, D. A. (2007). Environmental colour intensifies the Moran effect when population dynamics are spatially heterogeneous. *Oikos*, 116(10):1726–1736.