Development of new muon source for muon g-2 measurement and muon magnetic microscope

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Abstract. In various fields, from particle physics to condensed matter physics, extremely low emittance muon beam is eagerly awaited. We started a new project to realize the extremely low emittance and high intensity muon beam further beyond previous R&D study performed at RIKEN-RAL. To reduce the muon beam emittance, we plan to utilize the room temperature muonium generator, and furthermore, we apply muonium motion-cancellation mechanism using Doppler shift of 1S-2P transition energy. By the cancellation, one can expect the beam emittance better than $\Delta x \Delta \theta_x \approx 100 \mu m \cdot mrad$ even at 100 kV extraction field, which can be focused to the size of $\sim 1 \mu m$ by applying electron microscope technique. We are also developing high intensity and extremely stable VUV laser for the muonium ionization, which can efficiently ionize muonium and produce high muon yield of $\sim 10 kHz$ even at RIKEN-RAL (240kW proton with conventional surface muon channel). This new R&D study will open a totally new application to the muon science, namely a Muon Magnetic Microscope ($M^3$). This muon beam enable us to observe sub-µmeter magnetic structure of the condensed matter with the depth resolution less than 10 nano-meter.

1. Introduction

At RIKEN-RAL muon facility, we started a new facility oriented project to realize transcendent polarized muon beam, whose characteristics could exceeds previously proposed ultra-slow muon R&D works (RIKEN-KEK collaboration) [1, 2] in many aspects, such as intensity, momentum dispersion, emittance (sharp focusing capability), and time resolution. The limits, we have encountered in the previous ultra-slow muon project, are about $3 \times 10^{-5}$ in extraction efficiency normalized by the number of incoming muons, focusing of about 3 mm$\phi$ in diameter, and about 8 nsec in time resolution. Compared to the moderator-based slow muon using DC source at PSI [3], the present limits already have several advantages, though the improvement in many aspects are vitally required for the realistic application of the muon based researches. In the new project, we would like to achieve better muon beam characteristics of $\sim 100$ times in extraction efficiency, $\sim 100 \times 100$ times in focus (horizontal and vertical), and $\sim 10$ times in time resolution at minimum, starting from room temperature muonium ($\sim 300$ K) instead of hot tungsten foil ($\sim 2100$ K). At the same time, we also propose to cool down the muon below 300 K by applying chirped laser and vertical motion compensation field to have better emittance in one direction.

If one can realize $\sim$ micron focusing, it would be feasible to build a muon microscope sensitive to the magnetic properties of condensed matters having spatial resolution of $\sim 1 \mu m$, namely a Muon Magnetic Microscope ($M^3$). Because muon is a self-analyzing particle, the microscopic magnetic property can be studied precisely if one can sharply focus the beam at low momentum.
At about 100 keV re-acceleration, the muon range in matter is few µm (1 µm in copper), so novel three-dimensional magnetic-property mapping become feasible for the first time. Further post-acceleration of the beam up to $\gamma = 3$, one can store the beam in a simple dipole magnet without focusing, which opens a new generation muon $g$-2 measurement beyond the present systematic error.

2. Muonium in Vacuum

A muonium in vacuum from the muon stopping target (muonium generator, such as hot tungsten and silica powder) is quite dilute system so that the muonium should have Maxwell-Boltzmann distribution $P_{Mu} \Delta p$, which can be written as

$$P_{Mu} \Delta p \propto \exp \left( -\frac{p^2}{2m_\mu kT} \right) p^2 \Delta p = \prod_{j=1}^{3} \exp \left( -\frac{p_j^2}{2m_\mu kT} \right) \Delta p_j,$$

where $k$ is the Boltzmann’s constant, $T$ is the temperature of the muonium-generator, $p$ and $p_j$ are the muonium momentum and the projection to each axis, and the $m_\mu$ is the mass of the muonium ($\approx$ that of muon). Thus the muonium momentum distributes in a Gaussian function of

$$\sigma_{p_x} = \sigma_{p_y} = \sigma_{p_z} = \frac{1}{\sqrt{3}} \sigma_p = \sqrt{m_\mu kT},$$

which is 1.66 keV/c for each axis at 300K ($kT = 0.026$ eV). It is clear that the lower temperature target will give us lower momentum dispersion, and the smaller spatial spread resulting higher efficiency for muonium laser re-ionization. On the other hand, muonium cannot come out and be isolated from the target, if the temperature is too low. It takes already about 100 nsec for 1 mm diffusion even at 300 K. Thus the room temperature target would be an ideal target for our purpose.

The resonant laser wave-length depends on the motion proportional to the momentum of the laser direction. Let us define the laser injection direction to be $y$-axis. The photon wave length of muonium 1S-2P transition $\lambda_0$ at rest (Lyman-α) is 122 nm. The Doppler broadening of this photon caused by the muonium thermal motion along $y$-axis is then given as

$$\sigma_{\nu_y} = \frac{c}{\lambda_0} \sqrt{\frac{kT}{m_\mu}},$$

which is 38 GHz at 300 K. This means we need to have full band width at least $\sim 115$ GHz ($\pm 1.5\sigma$) to ionize the muonium. On the other hand, this also means that one can ionize specific muonium momentum $p_y$ selectively if the laser band width $\Delta \nu$ is much narrower than that caused by the muonium motion. This fact can be utilized for further muon cooling below the target temperature, as described in later section.

The production of muonium in vacuum from room temperature materials such as silica powder was actively studied in the 1980s [5, 6, 7]. Muonium is formed with about 60% probability inside silica powder particles [8] and is emitted from the particles with 97% probability [9]. For laser re-ionization, silica aerogel would be much more suitable for its self-supporting feature. Actually, silica aerogel is essentially same compound having same micro-structure as silica powder, although less muonium production yield per stopped muon at 0.16 g/cm$^3$ target is reported [10]. Very lately, hydrophobic aerogel, whose density can be well controlled up to 1.2 g/cm$^3$, is newly synthesized so that one can search the optimum density in the case of silica aerogel.
To study the best material for the muonium yield at the room temperature, we submitted a proposal S1249 to TRIUMF and the proposal was accepted. In the experiment, we are planning to track a high energy positron from muon decay and detect a left electron at the muonium decay, \((\mu^+e^-)_{\text{atom}} \rightarrow (e^+ + \nu_e + \nu_\mu) + e^-\), by a MCP at the same time, so that we can substantially reduce the background of decay positrons from the target. We are also aiming to study the muonium chemical potential in the silica grain from the minimum muonium velocity in vacuum, which can be obtained from the time evolution of the muonium spatial distribution.

### 3. Extraction field

The longitudinal momentum dispersion is defined by the acceleration field and the initial condition of the ionization laser. In our previous work, we applied a static electric field for the ionization muon extraction. This configuration is required to simplify the target design, to operate the hot tungsten target stably. In the case of a room temperature target, we are free from this difficulty.

Let us compare the longitudinal momentum dispersion given by static and pulsed extraction fields. Fig. 1 shows the schematic space (a) and time (b) relation relative to the ionization laser injection, respectively to the static and pulsed extraction fields. We assume that the field gradients are homogeneous in space and flat in time, both given as \(E_0\), in the region of the acceleration. For simplicity, we ignore the initial kinetic motions of muoniums at the laser ionization, because the effect is negligibly small, as described in the previous section.

![Figure 1. Illustration of the extraction / acceleration field for the laser ionized muon. In the case of a static field (left), the final kinetic momenta are simply defined by the ionization position, and it is difficult to minimize the resulting momentum dispersion, keeping muon yield. In the case of a pulsed field (right), the momentum dispersion is defined by the laser pulse width, which can be as short as a few 100 psec.](image)

In the case of the static field, if the spatial uniformity of the extraction field near the laser ionization position is good enough, then the longitudinal momentum dispersion \(\Delta p/l I_{\text{static}}\) after the acceleration due to \(\pm \Delta l_{lp}\) is given by the spatial distribution of the ionization position as

\[
\left(\frac{\Delta p}{p I}\right)_{\text{static}} = \frac{1}{2} \frac{\Delta T_{\text{kin}}}{T_{\text{kin}}^{\text{static}}} = \frac{1}{2} \int_0^\infty eE(l)dl = \frac{1}{2} eE_0 \Delta l_{lp} = \frac{1}{2} \Delta l_{acc},
\]

where \(\Delta l_{lp}\) is the laser pulse width, \(T_{\text{kin}}^{\text{static}}\) is the kinetic energy, and \(\Delta l_{acc}\) is the effective
acceleration distance, given as
\[ \Delta l_{\text{acc}} = \frac{1}{E_0} \int_{0}^{\infty} E(l)dl. \] (5)

Here we denote the longitudinal axis as \( l \), because the acceleration direction is undecided. It is clear that the dispersion is independent of the ionization timing within a laser pulse.

On the other hand, in the case of the pulsed field, the longitudinal momentum dispersion after the acceleration due to \( \pm \Delta t_{lp} \) is given by the acceleration period, independent of the position. The dispersion is given as
\[ \left( \frac{\Delta p_l}{p_l} \right)_{\text{pulse}} = \frac{\Delta \int_{0}^{\infty} eE(t)dt}{\int_{0}^{\infty} eE(t)dt} = \frac{eE_0 \Delta t_{lp}}{eE_0 \Delta t_{acc}} = \mp \Delta t_{lp}, \] (6)

where \( \Delta t_{lp} \) is the time width of the laser pulse and \( \Delta t_{acc} \) is that of the effective acceleration, given as
\[ \Delta t_{acc} = \frac{1}{E_0} \int_{0}^{\infty} E(t)dt, \] (7)

if the field gradient is unchanged during the laser irradiation. Note that these dispersions are insensitive to the field inhomogeneity in space close to \( \Delta l_{acc} \) and to the time dependence around \( \Delta t_{acc} \).

A pulsed field extraction method is preferable for further muon cooling, and the static method is preferable for a shorter muon bunch width, hence time width, after the re-buncher. The preferred acceleration configuration may be different depending on the application and its purpose. Further design study is needed between initial acceleration and the LINAC for the choice of the extraction method.

4. Additional cooling by angular dispersion compensation
The angular dispersion caused by 300 K muonium thermal motion is already not far from the requirements. In the case of the \( M^3 \) source, smaller dispersion is preferable for better focusing in general.

In the case of the \( g \)-2 source, the situation is a bit more complicated. The horizontal motion (vertical to the magnetic field) is naturally confined in the ring by the magnetic field, when one put the muon in a storage ring of magnetic dipole field. On the other hand, vertical motion is not confined by the magnetic field. Thus one should minimize the motion parallel to the field, not to let the muon escape from the observation region of the ring.

Even if one can accelerate the beam without heating the muon beam bunch, one cannot be free from the transverse thermal motion. After injecting a parallel muon beam bunch to the storage ring, the beam bunch expands due to the thermal motion, which can be written as
\[ \Delta y(t) = \sqrt{\Delta y_{inj}^2 + \left( \frac{\Delta p_y}{m_\mu} \right)^2 \gamma^2 t^2} \quad \text{and} \quad \Delta p_y = \sqrt{m_\mu kT}, \] (8)

where \( \Delta y_{inj} \) is the beam size at the injection and \( t \) is time in Lab. frame. For the \( g \)-2 measurement, one need to re-accelerate the muon up to \( \gamma = 3 \) or more [4]. It means that the muon bunch have minimum expansion of about 15 cm in the standard deviation, during the observation time (five times of the muon life (\( \tau_\mu \)) at \( \gamma = 3 \) and \( T = 300 \) K). Obviously, this expansion is still not small enough for the precise \( g \)-2 measurement, because it is difficult to design the effective region of the decay positron observation much larer than 10 cm\( \phi \). Therefore, further cooling by factor 4 ~ 5 is required in this direction.
Fig. 2 shows the proposed muon extracting configuration, to make it possible to cool down the muon beam in the vertical direction. Parallel pulsed-extraction field along the muon injection direction and applying chirped laser, whose central frequency is shifted by $\pm \Delta \nu$ in a laser pulse, which is described by

$$\nu_0 + \Delta \nu \frac{t - t_0}{\Delta t_{lp}}, \text{ where } |t - t_0| < \Delta t_{lp}. \quad (9)$$

The $\Delta \nu$ is defined by the Doppler shift caused by the thermal motion in the $y$ direction. In this way, one can selectively ionize the muonium at $p_y$ by selecting the laser central energy and its band width. Using a narrow band width and sweeping the laser energy during a laser pulse (chirped laser), then the angular dispersion of the extracted muon beam after the acceleration will have a linear correlation with the extracted time. If one wishes to ionize in the range of $\pm n \sigma_{p_y} (= \sqrt{kT/m_\mu})$ by a laser pulse, then the formula can be written as

$$= \nu_0 + \left( n \frac{c}{\lambda_0} \frac{kT}{m_\mu} \right) \frac{t - t_0}{\Delta t_{lp}}. \quad (10)$$

With this laser, one can select upward moving muonium (positive in $y$ direction) in the earlier timing of the laser, and downward one in the later timing. If one makes the band width narrow enough, then one can linearly map the $y$ motion into the momentum dispersion in the acceleration direction as described in section 3.

In this condition, the bunch size in space expands to

$$\Delta z_{bunch} = \sqrt{\Delta z_{lp}^2 + \left( \nu_0 \frac{\Delta t_{lp}}{\Delta t_{acc}} \frac{L}{v_0} \right)^2} = \sqrt{\Delta z_{lp}^2 + \left( \frac{\Delta t_{lp}}{\Delta t_{acc}} \frac{L}{\Delta z_{lp}} \right)^2}, \quad (11)$$

after the free space motion of distance $L$, depending on the ionization timing. The ratio of the spatial expansion $M$, compared to the original spatial size $\Delta z_{lp}$, is given as

$$M = \sqrt{1 + \left( \frac{\Delta t_{lp}}{\Delta t_{acc}} \frac{L}{\Delta z_{lp}} \right)^2}. \quad (12)$$

One can utilize its dispersion to compensate the angular dispersion, and the improvement factor one can expect by this compensation can be naturally defined by $M$. 
Fig. 3 shows a typical motion of the muon bunch by a rather weak acceleration field pulse of $E_0 = 500$ V/cm and for $\Delta t_{acc} = 30$ nsec, assuming the ionization laser size of $\pm \Delta z_{lp} = \pm 0.5$ mm and the laser pulse width of $\pm \Delta t_{lp} = \pm 0.5$ nsec. The ionization timings are plotted in different colors in a step of 0.25 nsec. As shown in the figures, the muon bunch moves about 2 cm during the effective acceleration field pulse, and each ionization timing region is separated at about 70 nsec after the laser irradiation around 7 cm away from the muonium target.

One can compensate the angular dispersion in the $y$-direction, as shown in Fig. 4. For the compensation, one should magnify the initial $z$ distribution $\Delta z_{lp}$ to apply compensation field according to the central $p_y$ for given timing inside a muon pulse. It would be feasible to switch off the electric field in the order of 10 nsec, well before all the muons move out from the acceleration region, because the initial muon motion is very slow.

5. Cylindrical acceleration stage

The simplest and ideal muon $\sim 100$ kV acceleration scheme, to realize 1$\mu$m resolution $M^3$ and to inject LINACs for new muon $g$-2 measurement, can be achieved by parallel and static electric field, namely Cockcroft-Walton type, because there exist absolutely no thermal perturbation source during the acceleration. However, simple parallel and static field is not ideal, because the laser-ionized muon is rather large in size. Before the injection to the static acceleration field, one need to focus the laser-ionized muon to utilize only homogeneous region of the field, otherwise the system becomes unreasonably huge.

The better model is electron microscope instead of the simple Cockcroft-Walton. If all the acceleration components have cylindrical symmetry without magnetic field, the equation of the particle tracks close to the beam axis can be represented as

$$\frac{d^2 x}{dz^2} + \frac{1}{2\Phi} \frac{d\Phi}{dz} \frac{dx}{dz} + \frac{1}{4\Phi} \frac{d^2\Phi}{dz^2} x = 0$$  \hspace{1cm} (13)

$$\frac{d^2 y}{dz^2} + \frac{1}{2\Phi} \frac{d\Phi}{dz} \frac{dy}{dz} + \frac{1}{4\Phi} \frac{d^2\Phi}{dz^2} y = 0,$$  \hspace{1cm} (14)

where $\Phi$ is the electric potential on axis, $\Phi = \phi(0,0,z)$. From the equations, it is clear that the tracks are independent of the charge mass ratio $e/m$, thus the trajectories should be same
Figure 4. The compensation field to cancel the muonium motion using Doppler effect.

for electrons and muons. If there is higher order aberration in the acceleration stage, the muon bunch can easily heat-up because uncontrollable relative motions are introduced due to the aberration. The equations have cylindrical symmetry, so the second-order aberration term cannot exist, which simplifies the aberration control drastically. Because there exists an MV class electron microscope, it is feasible to apply that technology for the muon acceleration.

6. Primary design and the goal of the R&D

Fig. 5 shows the schematic setup for the acceleration scheme. The surface muon from the production target is transferred to the muonium generation target by a large solid angle solenoid channel, and accumulated to the central axis by a capillary method [11] using capillary shaped heavy material. The ionization laser will be injected just behind the muonium target to have maximum yield.

The extraction field will be generated by applying slightly higher voltage to the muonium target holder than the bias of the initial focusing element. Double einzel lenses will be used to form a weakly-focused parallel beam to confine the muon beam diameter. After the double einzel lenses, one can place an electrical element to cancel the vertical dispersion in the case where a
pulsed extraction field is applied. The weakly-focused parallel beam is further accelerated by the static parallel field, and the higher order aberration is canceled by an aberration compensator composed by four einzel lenses and two sextupole lenses located behind the static parallel field.

In the present design, the biggest contribution of higher order aberration is $\theta^3$-term, which come from the spherical aberration of the electro-static lenses.

The present conceptual design is suitable for the the case of muon magnetic microscope application. At Cockcroft-Walton type acceleration of about 100 kV, $\sim$ mm-mrad emittance can be feasible without further cooling method using Doppler shift cancellation mechanism. The cylindrical optics for this initial acceleration ensures good focus better than $\sim 10 \mu m$ in diameter, when one compensate the higher order aberration. If Doppler shift cancellation mechanism works as expected, one can improve the focusing size one order of magnitude in one direction.

In the case of g-2 application, one needs extremely short-pulse beam to fulfill injection requirement to the post-acceleration LINACs, so it requires a re-buncher to rotate the phase of the muon bunch to compress the muon beam bunch in time, and then apply final focusing elements before injection into the LINAC for the further acceleration up to $\gamma \sim 3$ or more. One needs more detailed design study to finalize the configuration of the final focusing elements to fulfill the matching condition with LINAC injection.

We are presently developing new muon source both for muon g-2 measurement and for muon magnetic microscope application, aiming at the beam emittance better than $\Delta x \Delta \theta_x \approx 100 \mu m \cdot mrad$ even at 100kV extraction field. With the new high intensity and extremely stable VUV laser, we are expecting the muon yield of $\sim 10$ kHz even at RIKEN-RAL (240kW proton with conventional surface muon channel). We wish to accomplish this goal earlier than five years.

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