Microscopic description of shape evolution in medium-mass nuclei

P Sarriguren\textsuperscript{1}, R R Rodríguez-Guzmán\textsuperscript{2}, L M Robledo\textsuperscript{3}

\textsuperscript{1} Instituto de Estructura de la Materia, CSIC, Serrano 123, E-28006 Madrid, Spain
\textsuperscript{2} Departamento de Física Aplicada, Universidad de Huelva, 21071 Huelva, Spain
\textsuperscript{3} Departamento de Física Teórica C-XI, Universidad Autónoma de Madrid, 28049-Madrid, Spain

E-mail: sarriguren@iem.cfmac.csic.es

Abstract. The evolution of the ground-state nuclear shapes with the number of nucleons is studied within a self-consistent Hartree-Fock-Bogoliubov formalism based on Gogny and Skyrme density-dependent interactions. Potential energy surfaces including triaxial degrees of freedom are studied in various isotopic chains. Signatures for the E(5) critical point symmetry are analyzed in Pd, Xe, and Ba isotopes, while those for the X(5) are studied in Nd, Sm, Gd, and Dy isotopes. Oblate to prolate shape transitions are also studied in Yb, Hf, W, Os, and Pt isotopic chains. The transitions are discussed in terms of the deformation dependence of the single-particle energies.

1. Introduction
The evolution of the atomic nuclei ground-state shapes as the number of their constituents changes is an active research field \cite{1} that has been recently addressed both experimentally \cite{2} and theoretically \cite{3, 4, 5, 6, 7, 8, 9}. From the experimental side, low-lying spectroscopy is one of the most powerful sources of information about nuclear shapes and shape transitions, since one can establish signatures correlating the excitation energies with the deformation properties. From the theoretical side, the problem has been addressed from many different points of view. Geometrical and algebraic models, as well as microscopic models based on shell model or mean field approaches have been considered. The shape evolution is described differently depending on the approach adopted. Thus, in algebraic models like the Interacting Boson Model (IBM) \cite{10}, shape changes correspond to the breaking of the dynamic symmetries of the model and are considered as phase transitions. In the shell model, nuclear deformation and its evolution occur due to configuration mixing. From a mean field point of view, shape changes arise when the deformed single-particle levels are energetically favored to a different degree in open shell nuclei.

Iachello introduced some years ago, the E(5) and X(5) critical point symmetries, which provide parameter free (up to scale factors) predictions of excitation spectra and strengths for nuclei at the critical point of a phase shape transition \cite{11}. These symmetries were obtained within the framework of the collective Bohr Hamiltonian under some simplifying approximations. In particular, the potential in the $\beta$ degree of freedom was approximated by a simple square well potential, which is decoupled from the potential in the $\gamma$-variable. In the case of E(5), which corresponds to the transition from spherical vibrational U(5) to deformed $\gamma$-unstable O(6), the potential is constant in the $\gamma$-direction. In the case of X(5), related to the transition from U(5)
to axially symmetric prolate SU(3), a harmonic oscillator potential is used in the $\gamma$-direction. Empirical evidence of these transitional symmetries at the critical points has been observed in several isotopes of Ba, Pd, and Xe for E(5), and in $^{152}$Sm, $^{150}$Nd, $^{154}$Gd, and $^{156}$Dy for X(5).

In this work we investigate shape transitions within non-relativistic microscopic models, based on effective interactions between nucleons that provide a unified description of nuclear properties along the nuclear chart. In particular, we study whether the assumptions made on the $\beta$-$\gamma$ potentials to construct the point symmetries are justified microscopically. Thus, we study various isotopic chains in which the occurrence of shape transitions has been predicted and we show results for the potential energy surfaces (PES) corresponding to constrained mean field calculations [4, 5, 6, 7]. We also focus our attention to the oblate-prolate shape transition in neutron-rich Yb, Hf, W, Os, and Pt isotopes. The theoretical framework is based on both the self-consistent Hartree-Fock-Bogoliubov approximation with the finite-range and density-dependent Gogny interaction [12] with the parametrization D1S [13], and the self-consistent Hartree-Fock + BCS with short-range density-dependent Skyrme interactions (SLy4) [14] and a zero-range density-dependent interaction in the pairing channel. In the latter case we use the code EV8 [15] that solves the HF equations in a coordinate space mesh. The role of triaxiality is also considered and discussed in those nuclei where this degree of freedom could be relevant.

2. Results
In Figure 1 we can see the contour plot of the PES for $^{130}$Xe, obtained from Gogny D1S, as a function of the quadrupole moment $Q_0$ (b) and $\gamma$ (deg) angle. This nucleus has been proposed as a candidate for the E(5) symmetry. Along with the triaxial plot, there is a cut corresponding to the axially symmetric shape ($\gamma = 0^\circ$ and $\gamma = 60^\circ$). We can see that the potential is rather flat in $\beta$ ($Q_0$) around sphericity and that it is practically constant in the $\gamma$-direction at the $\beta$ deformation corresponding to the minimum of the energy. It is also interesting to notice that the oblate axial minimum becomes a saddle point when triaxiality is considered. All of this

![Figure 1. Contour plot of the PES for $^{130}$Xe with the Gogny D1S force.](image)
Figure 2. (Left panel) Axial PES for Sm isotopes with SLy4. (Right panel) Contour plot of the PES for $^{152}\text{Sm}$ with the Gogny D1S force (notice that $Q$ is defined as $2Q_0$).

makes this nucleus a reasonably good candidate to show E(5)-like behavior. Similar conclusions are obtained for the other candidates $^{108,110}\text{Pd}$, $^{128,132}\text{Xe}$, and $^{130-134}\text{Ba}$ [4, 5].

In Figure 2 we study the X(5)-like behavior on the example of Sm isotopes, and in particular on $^{152}\text{Sm}$. Constrained HF+BCS calculations for Sm isotopes with SLy4 and zero-range pairing force are shown in the left panel of Figure 2. The axial plots show a clear shape transition from spherical $^{144}\text{Sm}$ ($N=82$) to well-developed prolate shapes at $^{156-158}\text{Sm}$. The isotope $^{152}\text{Sm}$ ($N=90$) shows a transitional behavior with a shallow minimum on the prolate side and an additional minimum on the oblate side with an energy barrier rather high between these minima. On the right panel, the triaxial contour plot for the X(5) candidate $^{152}\text{Sm}$ shows that for $Q_0$ values in the vicinity of the prolate minimum, the energy presents a parabolic behavior as a function of $\gamma$. This is particularly true for low $\gamma$ angles but tends to become flat as $\gamma$ approaches 60°. The oblate axial minimum becomes also a saddle point when looking at the $Q_0-\gamma$ plane. Thus, the assumption of an infinite square well made for the $\beta$-potential to arrive to the X(5) symmetry, is not realized within non-relativistic microscopic calculations that produce systematically potential energy barriers at $\beta=0$, excluding the spherical configuration from the coexisting shapes. Similar results are obtained for the other X(5) candidates $^{150}\text{Nd}$, $^{154}\text{Gd}$, and $^{156}\text{Dy}$ [4, 5]. The results also agree with those obtained from relativistic calculations [8, 9].

We next consider the chain of W isotopes as examples of oblate-prolate shape transitions occurring in this mass region. Figure 3 shows the evolution of the axial shapes with increasing neutron number, calculated from various forces, Skyrme SLy4 and Gogny D1S. The transition takes place at $N=116-118$, where the transitional nuclei show oblate and prolate minima coexisting at close energies. In Figure 4 we can see the contour plots of the PES with SLy4 in the $^{186-196}\text{W}$ isotopes. We can see in $^{186}\text{W}$ a real prolate minimum and an oblate saddle point. $^{190}\text{W}$ shows a triaxial ground state with the axial prolate and oblate minima transformed into saddle points. There is a very soft behavior of the PES along the $\gamma$ degree of freedom, developing a very shallow triaxial minimum. Finally, in $^{196}\text{W}$ the oblate minimum has been
developed, while the prolate one is now a saddle point. Similar results are found in Yb, Hf, Os, and Pt isotopes [6, 7]. In general, for all of these isotopes, the transition from prolate to oblate shapes takes place at $N = 116 - 118$, where the energies of oblate and prolate shapes are nearly degenerate. The triaxial analysis of these nuclei shows that the axial oblate and prolate minima, which are separated by high potential barriers in the $\beta$ degree of freedom, are linked very softly in the $\gamma$ degree of freedom, making these minima saddle points in the extended $\beta - \gamma$ plane.

In Figure 5 we can see the single-particle energies (SPE) for the Gogny force corresponding
Figure 5. Single-particle energies for protons (left) and neutrons (right) for $^{190}$W with the D1S Gogny force.

to $^{190}$W. In the plot for protons we observe the presence of the $3s_{1/2}$ level just above the Fermi level and the levels $1h_{11/2}, 2d_{3/2}$ and $2d_{5/2}$ below. For neutrons we have the $3p_{1/2}$ level above the Fermi level and almost degenerate $3p_{3/2}, 2f_{5/2}$ and $1i_{13/2}$ orbitals just below the Fermi level. A couple of MeV below we find degenerate $1h_{9/2}$ and $2f_{7/2}$ orbitals. Those levels evolve with deformation and at $Q_{20}$ around 6.6 b a gap in the SPE spectrum signaling a region of low level density appears both in the proton and neutron spectra that is responsible for the prolate minimum observed in the axially symmetric potential energy curve. In the oblate side, at $Q_{20} = -6.6$ b the neutron’s Fermi level approaches another gap that is responsible for the oblate minimum observed in the axially symmetric case. Both minima are in fact saddle points as long as the $\gamma$ degree of freedom is considered. In the case of $^{184}$W, the Fermi level for neutrons is lowered and the SPE spectrum shows a gap near the Fermi level for $Q_{20} = 8$ b. This fact together with the proton’s gap also observed in that region of $Q_{20}$ favors the development of the prolate minimum observed. In the oblate side a gap is developed in the neutron’s SPE spectrum at $Q_{20} = -8$ b that is responsible for the observed oblate minimum. For the nucleus $^{196}$W, the neutron’s SPE spectrum shows a gap around the Fermi level for oblate deformations with $Q_{20}$ in the range between -1 b and -5 b that is responsible for the oblate minimum observed in this case. Therefore the prolate-oblate transition seen at $N = 116$ is a consequence of the two gaps in the neutron’s SPE, one in the prolate and the other in the oblate side as the Fermi level crosses them. On the other hand, the proton’s SPE spectrum seems to favor the appearance of coexisting oblate and prolate configurations as $Z$ increases that are the precursors of the triaxial instability observed in that case.

We can also look at the onset of deformation in this region by using the ideas developed by Federman and Pittel (FP) [16] in trying to unify the description of deformation both for light nuclei and heavy ones. The argument of Refs. [16] is that deformation is driven by the $T = 0$ neutron-proton interaction and this is particularly intense between spin orbit partners. Next in the range of relevance of the n-p interaction strength we find interactions between orbitals with
the same radial quantum number and large orbital angular momenta differing by one unit (i.e., $n_p = n_n$ and $l_p = l_n \pm 1$). By looking at the SPE plots in Figure 5 we find the relevant role of the $1h_{11/2}$ orbital for protons which is very close to the Fermi level for all the nuclei considered in the region. According to FP’s argument this orbital could interact with its neutron spin orbit partner, namely the $1h_{9/2}$ orbital but this one is well below the Fermi level and can be considered as inert. Near the neutron’s Fermi level we have a $1i_{13/2}$, $2f_{5/2}$ and $3p_{3/2}$. Obviously, it is the first one that fulfills the above criteria of $n_p = n_n$ and $l_p = l_n \pm 1$ and therefore is the strongly interacting one with the $1h_{11/2}$ orbital. For values of $N$ around 110 the $1i_{13/2}$ is in the middle of the Fermi level favoring the observed prolate deformation with well established and deep prolate wells. As $N$ increases the $1i_{13/2}$ gets more and more occupied and at some point it ceases to play a role that is transferred to the $2f_{5/2}$ and $3p_{3/2}$ orbitals. Among them, only the $2f_{5/2}$ can interact with the $2d_{3/2}$ of protons but as the $l$ values are low we do not expect a strong interaction. This explains why as $N$ increases the depth of the deformation wells decreases favoring triaxial deformations.

3. Summary and Conclusions

We have carried out a systematic mean-field exploration of the triaxial PESs in various isotopic chains where nuclear shape transitions occur and where the realization of the critical point symmetries E(5) and X(5) have been proposed. We use the HFB framework based on the finite-range density-dependent Gogny interaction (D1S), as well as the HF+BCS formalism based on the short-range density-dependent Skyrme interaction (SLy4) in connection with a zero-range density-dependent interaction in the pairing channel.

We have found that the assumptions of flat potentials in the quadrupole deformation $\beta$ and constant potentials in the $\gamma$ direction, which are characteristics of the E(5) critical point symmetry, are supported by our results in $^{108,110}$Pd, $^{128−132}$Xe, and $^{130,134}$Ba. In the case of the rare-earth isotopes with $N = 88,90$, it is shown that they present a transitional behavior between spherical and axially deformed shapes. However, we do not find the characteristic flat potentials of the X(5) symmetry but rather a potential barrier at zero deformation. In these cases we find that, at equilibrium, the potentials in $\gamma$ exhibit a quadratic behavior centered at $\gamma = 0$ that becomes flat as $\gamma$ increases. These results, together with those obtained within the relativistic mean-field approach, allow us to conclude that all the available state-of-the-art mean-field approximations, support the assumption of flat potentials compatible with the ones postulated in the E(5) critical point symmetry for some selected isotopes. However, we have not found satisfactory examples for PES showing patterns similar to those of the X(5) symmetry.

We have also studied the oblate-prolate shape transitions in Yb, Hf, W, Os, and Pt isotopes within the same theoretical formalism. We have shown that increasing the proton number in this mass region leads the nuclei to triaxiality. On the other hand, increasing the neutron number, the ground-state shapes in the isotopes studied evolve from axially deformed oblate shapes to axially deformed oblate shapes. The transitional nuclei ($N = 116$) exhibit a $\gamma$ soft behavior with very shallow triaxial minima. The transition occurs with different degrees of stiffness depending on the isotope. It is rather sharp for low $Z$ isotopes (Yb and Hf) and much broader for W, Os, and Pt, where a region of triaxial ground states develops between the regions of prolate and oblate minima. The analysis of the single-particle energies both for axially symmetric and triaxial configurations demonstrates the role of different gaps showing up in the SPE of both protons and neutrons, as well as the role played by the $T = 0$ proton-neutron interaction.

Acknowledgments

This work was partially supported by MICINN (Spain) under contracts Nos. FIS2008-01301 and FPA2007-66069.
References

[1] Wood J L, Heyde K, Nazarewicz W, Huysse M and Van Duppen P 1992 Phys. Rep. 215 101

[2] Wu C Y et al. 1996 Nucl. Phys. A 607 178

Stuchbery A E et al. 1996 Phys. Rev. Lett. 76 2246
Wheldon C et al. 2000 Phys. Rev. C 63 011304(R)
Podolyák Zs et al. 2000 Phys. Lett. B 491 225
Pfützner M et al. 2002 Phys. Rev. C 65 064604
Caamaño M et al. 2005 Eur. Phys. J. A 23 201
Podolyák Zs et al. 2009 Phys. Rev. C 79 031305(R)

[3] Jolie J and Linnemann A 2003 Phys. Rev. C 68 031301(R)
Naik Z et al. 2004 Pramana 62 827
Stevenson P D et al. 2005 Phys. Rev. C 72 047303
Walker P M and Xu F R 2006 Phys. Lett. B 635 286
Rodríguez T R and Egido J L 2008 Phys. Lett. B 663 49
Sun Y, Walker P M, Xu F R and Liu Y X 2008 Phys. Lett. B 659 165
Morales I O, Frank A, Vargas C E and Van Isacker P 2008 Phys. Rev. C 78 024303
García-Ramos J E and Heyde K 2009 Nucl. Phys. A 825 39

[4] Rodríguez-Guzmán R and Sarriguren P 2007 Phys. Rev. C 76 064303
[5] Robledo L M, Rodríguez-Guzmán R and Sarriguren P 2008 Phys. Rev. C 78 034314
[6] Sarriguren P, Rodríguez-Guzmán R and Robledo L M 2008 Phys. Rev. C 77 064322
[7] Robledo L M, Rodríguez-Guzmán R and Sarriguren P 2009 J. Phys. G 36 115104
[8] Fossion R, Bonatsos D and Lalazissis G A 2006 Phys. Rev. C 73 044310
[9] Niksic T, Vretenar D, Lalazissis G A and Ring P 2007 Phys. Rev. Lett. 99 092502
Li Z P, Niksic T, Vretenar D, Meng J, Lalazissis G A and Ring P 2009 Phys. Rev. C 79 054301
[10] Iachello F and Arima A 1987 The Interacting Boson Model (England: Cambridge University Press)
[11] Iachello F 2000 Phys. Rev. Lett. 85 3580
Iachello F 2001 Phys. Rev. Lett. 87 052502
[12] Dehargé J and Gogny D 1980 Phys. Rev. C (21) 1568
[13] Berger J F, Girod M and Gogny D 1984 Nucl. Phys. A 428 23c
[14] Chabanat E, Bonche P, Haensel P, Meyer J, and Schaeffer R 1998 Nucl. Phys. A 635 231
[15] Bonche P, Floreanini M and Longuet P 2005 Comput. Phys. Comm. 171 49
[16] Federman R and Pittel S 1977 Phys. Lett. B 69 385
Federman R and Pittel S 1979 Phys. Rev. C 20 820