Modelling the effect of laser focal spot size on sheath-accelerated protons in intense laser–foil interactions

C M Brenner1,2, P McKenna2 and D Neely1

1 Central Laser Facility, STFC, Rutherford Appleton Laboratory, Didcot, Oxon, OX11 0QX, UK
2 Department of Physics, SUPA, University of Strathclyde, Glasgow G4 0NG, UK

Received 29 November 2013, revised 2 May 2014
Accepted for publication 6 May 2014
Published 22 July 2014

Abstract
We present an approach to modelling the effect of the laser focal spot size on the acceleration of protons from ultra-thin foil targets irradiated by ultra-short laser pulses of intensity $10^{16}$–$10^{18}$ W cm$^{-2}$. An expression is introduced for the proton acceleration time, which takes account of the time taken for the laser-accelerated electrons, which expand laterally at the rear surface of the target, to escape the region of the sheath. When incorporated into an analytical model of plasma expansion, this approach is found to provide a good fit to measured scaling of the maximum proton energy as a function of intensity at large focal spot sizes.

Keywords: laser proton beams, sheath acceleration, plasma

1. Introduction
High power laser-plasma driven ion acceleration is currently studied by many in the laser-plasma community in light of the compactness of the source and unique properties of the ion beam that could be useful for a variety of applications (see [1, 2] for recent reviews of the field). It is clear that a detailed understanding of how the spectrum and maximum energy of these bright and directional proton beams vary with laser properties is required in order to establish mechanisms for control and optimisation and therefore fully harness their application potential.

Under the laser interaction conditions described in this paper, proton acceleration occurs as the result of electrons accelerated by the laser pulse at the target front surface penetrating through the foil and establishing an evolving electric field, driven by charge separation [3], which acts to ionize and accelerate ions at the rear surface. The presence of contaminant layers, containing hydrogen, on the target rear surface results in protons being preferentially accelerated due to their high charge-to-mass ratio. The spectral properties of the resultant proton beam are therefore closely related to the dynamics of the energetic (hot) laser-accelerated population of electrons driving the plasma-sheath acceleration.

There have been many attempts at quantitatively modelling the maximum proton energy in terms of the laser intensity, based on the properties of the sheath field. These include contributions from, among others, Mora [4, 5], Schreiber et al [6], Wilks et al [3] and more recently Zani et al [7], Robson et al [8], Lefebvre et al [9] and Coury et al [10]. Macchi et al [2] provide a comprehensive review of the literature and introduction to the main approaches to modelling sheath-accelerated proton beams alongside comparison with experimental scaling studies. Many of the descriptions derive the maximum proton energy, $E_{\text{pmax}}$, as a function of the hot electron temperature, $T_{\text{hot}}$, whereas Shreiber et al derive $E_{\text{pmax}}$ in terms of the laser power and the radius of the charge distribution at the target rear surface. A collection of papers can also be found that make explicit reference to the effect of changing the laser focal spot size on $E_{\text{pmax}}$, with experimental work [11–13] highlighting the benefit of employing a larger focal spot size and theoretical work [7, 14] exploring the scaling predictions of codes that simulate varying laser energy only compared to varying laser spot size only. However, the majority of models of the sheath acceleration of protons calculate the scaling of $E_{\text{pmax}}$ with respect to changes...
in laser energy or laser pulse duration, at tight focus (<10 \mu m) only.

In this paper, the effect of increasing the drive laser pulse focal spot size on proton acceleration is considered and then included into an established model with a comparison made with experimental observation. Attention is spent on modelling the acceleration time to incorporate the increase in this parameter associated with the use of a large laser spot diameter, particularly when ultra-thin targets are used in combination with ultra-short pulses. Using ultra-thin targets, for which electron beam divergence during transport within the target is negligible, the initial lateral extent of the hot electron population on the rear surface will be of the order of the laser spot size. Therefore, defocusing the laser to larger spot sizes will give rise to a proportional increase in the initial lateral size of the hot electron source at the rear surface. However, this approximation applies only to the case in which target thickness is much smaller than the laser spot size. Under these conditions, there is no significant lateral spreading of the hot electron population as it is transported from the front to the rear side of the target. By contrast, Coury et al [10] have demonstrated that in the case of the irradiation of 100 \mu m-thick foils with a defocused laser spot, for which there is significant divergence of the transported hot electrons, the resultant electron sheath distribution at the target rear surface is strongly peaked on axis.

Motivated by the need to better model the effect of laser focal spot size, and therefore initial lateral extent of the electron population, on the sheath acceleration process when ultra-thin targets are employed we present here a modification to the ion acceleration time. This modified definition takes account of the influence of the hot electron population expanding laterally at the rear side of the target and when incorporated into an analytical model we find that this approach gives a better fit to experimentally established scaling laws for proton acceleration as a function of peak laser intensity.

2. Modelling

2.1. Plasma-sheath acceleration

The effect of large focal spot size irradiation of an ultra-thin foil target can be accounted for by exploring the properties of the sheath acceleration process that are directly relevant to foil target can be accounted for by exploring the properties. The total number of hot electrons, \( N_{\text{hot}} \), and hot electron density, \( n_{\text{hot}} \), are given by

\[
N_{\text{hot}} = \frac{\eta E_L}{T_{\text{hot}}} \quad \text{(2)}
\]

and

\[
n_{\text{hot}} = \frac{\lambda_{D}}{\pi \left( \frac{D_i}{2} \right)^2 (2\lambda_{D} + d_i)} \quad \text{(3)}
\]

where

\[
\lambda_{D} = \sqrt{\frac{\varepsilon_{\text{e}}T_{\text{hot}}}{n_{\text{hot}}e^2}} \quad \text{(4)}
\]

\( \eta \) is the conversion efficiency of laser energy \( E_L \), over a laser spot diameter, \( D_i \), into hot electrons and \( \lambda_{D} \) is the Debye length of the electron sheath plasma. The initial hot electron population length used to calculate the hot electron density in equation (4) is determined here using the solid target thickness, \( d_i \), with the addition of the initial longitudinal extension of the hot electron population (approximated as twice the Debye length) from both surfaces of the ultra-thin target.

For this investigation the hot electron temperatures were calculated using the scaling relation obtained by Lefebvre et al [9]:

\[
T_{\text{hot}} (\text{keV}) = 126 \left( \frac{I_L \lambda_{L}^2}{1.37 \times 10^{18}} \right)^{0.6} \quad \text{(5)}
\]

where \( I_L \) is the laser intensity in units of W cm\(^{-2}\) and \( \lambda_{L} \) is the laser wavelength in microns. This scaling law is concurrent in both experimental [17] and numerical [9] studies that were conducted with ultra-short laser pulses under high laser intensity contrast conditions and therefore provides a good match to the laser interaction conditions of interest in this paper.
2.2. Ion acceleration time

For the length of time that hot electrons remain in the vicinity of the sheath, the electric field is maintained and proton acceleration occurs. The laser focal spot size becomes relevant to the proton acceleration time when the focal spot diameter, \(D_z\), is of sufficient size that the time taken for hot electrons travelling from the centre of the sheath with average velocity, \(u_e\), to escape the edge of the initial surface charge area is larger than the laser pulse duration, \(\tau_L\), \(\frac{D_z}{2u_e} > \tau_L\). Using the scaling law given in equation (5), the hot electron temperature can be estimated in order to deduce the average hot electron velocity. For laser intensity \(\sim 10^{18} \text{W cm}^{-2}\) and laser pulse duration of 40 fs, the minimum laser spot diameter that satisfies the condition for the spot size to become relevant is \(\sim 14 \mu\text{m}\). At laser spot diameters in excess of this, the electron escape time is a significant contributor to the acceleration time and therefore to the maximum proton energy, leading to a scaling with intensity that is different from that obtained for a small laser spot size and variable laser pulse energy.

The acceleration time can essentially be described as being made up of the laser pulse duration, \(\tau_L\), with the addition of the time taken for significant expansion of the ion front population beyond the electron sheath region to occur and for transfer of energy from the electrons to the protons to cease, \(\tau_{\text{expansion}}\). Buffechoux et al [18] combined experimental results and simulation data over a wide range of parameters and found a simple relation to describe the latter as \(\tau_{\text{expansion}} \sim 60\nu^{-1}\).

We introduce another factor influencing the plasma sheath dynamics, and hence the ion acceleration time, which is not typically considered in ion acceleration modelling, in terms of the lateral transport of hot electrons at the target rear surface [19, 20]. In applying the Mora model to sheath acceleration, we enable the influence of laser spot size to be incorporated into the calculation of the maximum proton energy with the introduction of a new temporal parameter, \(\tau_{\text{escape}}\), and by incorporating this into an expression for the \(\tau_{\text{acc}}\) outlined below. The maximum contribution to the acceleration time from lateral electron transport can be defined as the time taken for axial hot electrons to escape the initial lateral extent of the plasma sheath, \(\tau_{\text{escape}}\), and is a function of the initial sheath diameter, \(D_z\), and the average velocity of the hot electrons, \(u_e\). The acceleration time is then calculated as the magnitude of all these contributions, to account for the fact that these times are not sequential:

\[
\tau_{\text{acc}} \sim \sqrt{\tau_L^2 + \tau_{\text{expansion}}^2 + \left(\frac{D_z}{2u_e}\right)^2}. \tag{6}
\]

An increase in either \(T_{\text{hot}}, n_{\text{hot}}\) or \(\tau_{\text{acc}}\) in isolation will lead to an increase in the maximum proton energy. Of the three hot electron sheath properties, \(T_{\text{hot}}\) is the dominant contributor in determining the maximum proton energy. However, in exploring the scaling of maximum proton energy as a function of laser intensity one needs to also model the significant contribution made by changes in \(n_{\text{hot}}\) and \(\tau_{\text{acc}}\), by considering explicitly how these parameters vary with varying laser energy, pulse duration and spot size.

3. Results and discussion

The predictive capability of our approach was explored by comparing the maximum proton energy obtained when 25 nm-thick targets are irradiated with 40 fs pulses of intensity range \(10^{16}–10^{18} \text{W cm}^{-2}\) at focal spot sizes of 20 and 60 \(\mu\text{m}\), to experimental observations previously reported [13]. In figure 1 the maximum proton energy has been calculated using equation (1) and the expressions for \(n_{\text{hot}}, T_{\text{hot}}\) and \(\tau_{\text{acc}}\), given here in equations (3), (5) and (6), respectively, with the input parameters being the values of laser intensity and laser energy used to obtain the experimental results in [13]. The laser pulse duration is held constant at \(\tau_L = 40\) fs with the absorption fraction also constant at \(\eta = 0.3\), as measured over a wide range of \(I_L\) when high laser intensity contrast conditions are in use [21].

When the maximum proton energy was determined for both focal spot sizes without the use of \(\tau_{\text{escape}}\) term, the model predicts a single scaling curve for \(E_{\text{pmax}}\) as a function of laser intensity and severely underestimates the maximum proton energy achievable at the larger (60 \(\mu\text{m}\)) focal spot size. This outcome is expected and can be exemplified by considering the following scenario, in which the \(\tau_{\text{escape}}\) term is not included. When the laser energy is held constant and the focal spot size is increased from 20 to 60 \(\mu\text{m}\) there will be a decrease in both the effective rear surface \(n_{\text{hot}}\) as well as \(T_{\text{hot}}\). On closer inspection, an increase of \(\sim\)factor 3 in \(\tau_{\text{acc}}\) is calculated due to the increase in \(\tau_{\text{expansion}}\) that results from a decrease in \(n_{\text{hot}}\). However, the effect on \(E_{\text{pmax}}\) of this increase in \(\tau_{\text{acc}}\) is rendered negligible due to the logarithmic dependence of \(E_{\text{pmax}}\) with \(\tau_{\text{acc}}\), compared to the direct correlation with \(T_{\text{hot}}\) which decreases at larger laser spot size. Whereas for the same scenario but with the \(\tau_{\text{escape}}\) term included a substantial increase in \(\tau_{\text{acc}}\) is calculated and offsets the decrease in the dominant \(T_{\text{hot}}\) parameter, resulting in a much higher estimation of \(E_{\text{pmax}}\) compared to when the

![Figure 1. Measured values of \(E_{\text{pmax}}\) plotted as a function of peak laser intensity (see [13] for details) compared to model calculations, with and without inclusion of the \(\tau_{\text{escape}}\) term in equation (6). At each focal spot size, the laser intensity was varied by varying the laser energy only, while keeping the laser pulse duration constant throughout.](image-url)
\(\tau_{\text{escape}}\) is not considered. As figure 1 clearly demonstrates, by including the escape time in the equation for the acceleration time the model results in two scaling curves, which match well to the measured values.

For an increase in laser energy, at large but constant focal spot size, all of the sheath properties that contribute to determining the maximum proton energy will also increase, apart from \(\tau_{\text{escape}}\) which will decrease as the temperature and thus velocity of the hot electrons increases with laser intensity. By contrast, a change in the laser spot size for either fixed or varying laser energy results in a much more complex interdependency of the main parameters that define the maximum proton energy, because a change in the focal spot size has a significant effect on both \(n_{\text{hot}}\) and \(\tau_{\text{escape}}\). The relative change in \(E_{\text{pmax}}\) can only truly be accounted for by considering all of these parameters and not just what is happening to the dominant \(T_{\text{hot}}\) term. For example increasing the focal spot size (for fixed laser energy) will decrease \(T_{\text{hot}}\) and lead to a decrease in \(n_{\text{hot}}\), despite an increase \(N_{\text{hot}}\), while also increasing \(\tau_{\text{escape}}\). Thus, varying the laser spot size alone results in a weaker intensity dependent scaling of maximum proton energy, compared to varying the laser energy alone. This difference in the scaling of \(E_{\text{pmax}}\) with varying laser energy compared to scaling obtained when the laser spot size is varied is also seen in the theoretical and simulation work presented by Zani et al \[7\]. It is by considering the significant increase in \(\tau_{\text{acc}}\) that occurs at large focal spot size that our approach is also able to reproduce an increase in \(E_{\text{pmax}}\) obtained for data taken with similar laser intensity but increased laser energy and focal spot size, in agreement with numerical results that Passoni et al \[14\] present in figure 3 of their paper on sheath acceleration modelling.

4. Conclusion

The importance of including lateral expansion of hot electrons at the rear surface of a thin foil irradiated by an intense laser pulse in modelling ion acceleration is demonstrated. An additional term in the definition of the ion acceleration time is introduced to account for the time it takes for laterally transported hot electrons to escape the accelerating sheath area on the rear surface, in the case of a large focal spot diameter. An approach that combines the modified acceleration time with a plasma expansion model, along with calculations for the hot electron temperature and density, is used to model the maximum proton energy and is found to be in good agreement with experimental results. Further experimental investigation at large focal spot sizes and various laser and target parameters would be very useful in order to verify that the modelling approach presented in this paper can be expanded beyond the specific irradiation conditions described in this study.

Acknowledgments

This work is financially supported by EPSRC (grant numbers EP/J003832/1 and EP/K022415/1) and by LASERLAB-EUROPE (grant agreement number 284464, EC’s Seventh Framework Programme).

References

[1] Daido H, Nishiuchi M and Pirozhkov A S 2012 Rep. Prog. Phys. 75 056401
[2] Macchi A, Borghesi M and Passoni M 2013 Rev. Mod. Phys. 85 751
[3] Wilks S C et al 2001 Phys. Plasmas 8 542
[4] Mora P 2003 Phys. Rev. Lett. 90 185002
[5] Mora P 2005 Phys. Rev. E 72 056401
[6] Schreiber J et al 2006 Phys. Rev. Lett. 97 045005
[7] Zani A, Sgattoni A and Passoni M 2010 Nucl. Instrum. Methods Phys. Res. A 653 94–7
[8] Robson L et al 2007 Nature Phys. 3 58–62
[9] Lefebvre E et al 2010 New J. Phys. 12 045017
[10] Coury M et al 2012 Appl. Phys. Lett. 100 074105
[11] Green J S et al 2010 New J. Phys. 12 085012
[12] Xu M H et al 2012 Appl. Phys. Lett. 100 084101
[13] Brenner C M et al 2011 Laser Part. Beams 29 345–51
[14] Passoni M, Perego C, Sgattoni A and Batani D 2013 Phys. Plasmas 20 060701
[15] Fuchs J et al 2006 Nature Phys. 2 48–54
[16] Perego C, Zani A, Batani D and Passoni M 2011 Nucl. Instrum. Methods Phys. Res. A 653 89–93
[17] Mordovanakis A G et al 2010 Appl. Phys. Lett. 96 071109
[18] Bufechehoux S et al 2010 Phys. Rev. Lett. 105 15005
[19] McKenna P et al 2007 Phys. Rev. Lett. 98 145001
[20] Tresca O et al 2011 Plasma Phys. Control. Fusion 53 105008
[21] Streeter M J V et al 2011 New J. Phys. 13 023041