Lack of evidence for an odderon at small $t$

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Abstract

It is fundamental that the phase of an elastic scattering amplitude is related to its energy variation. We repeat a previous fit to $pp$ and $p\bar{p}$ elastic scattering data from 13 to 13,000 GeV, taking better account of the very high accuracy of the 13 TeV data. The conclusion remains that there is no evidence for the existence of an odderon in the small-$t$ data.

In an analysis of its highly-accurate 13 TeV elastic scattering data at small momentum transfer $t$, the TOTEM collaboration concluded that the ratio $\rho$ of the real to the imaginary part of the forward amplitude was about 0.1. This is somewhat smaller than predictions from extrapolating from its value at lower energies and the collaboration attributed this to the onset of a significant contribution from odderon exchange.

We disagreed with this analysis and concluded that the value of $\rho$ at 13 TeV is rather close to 0.14, and that therefore there is no need for odderon exchange. Our main criticism of the TOTEM analysis was that it ignored information linking the phase of the amplitude to its variation with energy, and that therefore it is not valid to extract the value of $\rho$ from the 13 TeV data alone.

Our approach has in turn been criticised by the collaboration on the grounds that we did not take sufficient account of the very high accuracy of the 13 TeV data. This has led us to repeat our least-\(\chi^2\) fit to all the $|t| < 0.1$ GeV$^2$ $pp$ elastic scattering data from 13.76 GeV to 13 TeV, now including only the statistical errors in the 13 TeV data, where previously we combined them in quadrature with the systematic errors.

The effect on the fits to the lower-energy data is hardly noticeable, but it gives a much more accurate fit to the 13 TeV data. The value for $\rho$ is still close to 0.14.

For the 13 TeV data beyond the Coulomb peak, $|t| > 0.02$ GeV$^2$, we obtain $\chi^2 = 0.75$ per data point. A similar calculation for the 8 TeV data gives $\chi^2 = 0.06$ per data point. See figure 1. We do not show the 7 TeV or 2.76 TeV data, for the reasons we explained before.

The fit is just as before, with the new parameter values

\[
\epsilon_p = 0.1083 \quad X_p = 165.7 \quad X_+ = 202.4 \quad X_- = 120.0 \quad \alpha'_p = 0.323 \text{ GeV}^{-2} \\
A = 0.594 \quad a_1 = 0.32 \text{ GeV}^{-2} \quad a_2 = 8.192 \text{ GeV}^{-2}
\]

with $C$ still fixed at 0.5. Note that if we rounded the parameters to the accuracy we gave in our previous paper the values of $\chi^2$ quoted above would be very significantly worse.

Our fit used the data in the Coulomb peak and so it included in the $pp$ amplitude the term

\[
8\pi\alpha_{\text{EM}}F_1(t)^2/t
\]

where $F_1(t)$ is the Dirac form factor. Figure 2 shows that this works quite well. It is the correct form to use at very small $t$, where the difference from the more correct Rosenbluth form is negligible. We disagree
Figure 1: fit to the 13 and 8 TeV data

Figure 2: fit to the 13 and 8 TeV data in the Coulomb peak
Figure 3: Fits to lower-energy data. In the second plot the lower points are pp elastic scattering, the upper points p\bar{p} multiplied by 2.

Figure 4: The calculated ratio \( \rho(t) \) of the real to the imaginary part of the hadronic amplitude at 13 TeV.

with the claim by West and Yennie\cite{4} that including contributions from hadron exchange together with Coulomb exchange introduces a phase factor, since if the photon is accompanied by any other exchange the result is not\cite{5} singular at \( t = 0 \).

As we have said, the fits to the data at lower energies are almost as before. Figure 3 shows two examples, at 546 and 53 TeV.

It is a matter of fundamental theory that if the variation with energy of the amplitude varies with \( t \), then so does its phase. Figure 4 shows that the phase of the hadronic part of the amplitude at 13 TeV in the range \( 0 < |t| < 0.1 \) GeV\(^2\) is nowhere near constant.

Of course a fit to the 13 TeV data alone would improve the already-good agreement between data and fit in the Coulomb region. With the simple form (2) the value of \( \rho \) reduces to a little over 0.12, while with the West-Yennie modification to (2) it becomes less than 0.1, as TOTEM found\cite{1}. However this destroys the agreement with the lower-energy data and so is not a correct thing to do.

There is no need for an odderon at \( t = 0 \). The data can be fitted perfectly well without it.
References

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