Three -Pion Correlations

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First of all, we mention the situation of empirical analyses on 3rd order BEC (Bose-Einstein Correlation) at RHIC. Second, we introduce several theoretical formulae / approaches. Third we present our analyses of data in Au+Au at 130 GeV by STAR and preliminary data in Au+Au at 200 GeV by PHENIX Collaborations. Our results also contain analyses by means of core-halo model. Finally, we estimate that the volume of interaction in Au + Au collisions at 130 GeV is 500 fm$^3$, which is compared with $V = R_{long}R_{out}R_{side} \sim 300$ fm$^3$ in Pb + Pb collision at 2.76 TeV by ALICE Collaboration. Moreover, usefulness of empirical analyses on $(2\pi^+)\pi^−$ and $(2\pi^−)\pi^+$ combinations at RHIC and LHC energies is remarked.

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1. Situation of Empirical analyses on 3rd order BEC in Au+Au collision at RHIC

As shown in Table 1, STAR and PHENIX Coll. have reported their analyses at 130 GeV and 200 GeV, respectively. In their analyses, STAR Coll. \( Q_{inv} \) \( \sqrt{q_i^2} \leq Q_3 \) (Inside of a globe in Fig. 1),

\[
Q_{inv,3}^2 = q_{12}^2 + q_{23}^2 + q_{31}^2.
\]

On the other hand, PHENIX Coll. \( \sqrt{q_3} \) (1.1) used data on diagonal line of a cube in Fig. 1,

\[
q_3 = \langle q_{12} \rangle = \langle q_{23} \rangle = \langle q_{31} \rangle \quad \text{i.e.} \quad Q_{inv,3}^2 = 3q_3^2.
\]

Of course this relation holds, as the number of data increases. \( \langle \cdots \rangle \) is an average value.

| \( \sqrt{s_{NN}} \) | STAR | PHENIX |
|-----------------|------|--------|
| 130 GeV         | raw and corrected data by \( Q_{inv} \) | preliminary raw and corrected data by \( q_3 \) |
| 200 GeV         |      |        |

**Figure 1:** Data ensembles of STAR (left) and PHENIX Coll (right).

Here we compare two kinds of data. In Figs. 2, data by PHENIX Coll. are rearranged by \( \sqrt{3} q_3 \). Coincidence among data by STAR and PHENIX Coll is fairly good. Error bars in raw data by PHENIX Coll are smaller than those of STAR Coll.

2. Several theoretical formulae

In many analyses on BEC, the following formulae based on plane wave function are used.

\[
N^{(2\pi)} / N^{BG} = c \left[ 1 + \lambda e^{-(RQ)^2} \right],
\]

\[
N^{(3\pi)} / N^{BG} = c \left[ 1 + \lambda \sum_{i>j} e^{-(RQ_{ij})^2} + 2\lambda^{1.5} e^{-0.5(RQ_3)^2} \right].
\]

In laser optical (LO/GL) approach, the following formulae with a degree of chaoticity \( p \) have been proposed \( \lambda \) and utilized,

\[
N^{(2\pi)} / N^{BG} = 1 + 2p(1-p)E_{2B} + p^2 E_{2B}^2,
\]

\[
N^{(3\pi)} / N^{BG} = 1 + 6p(1-p)E_{3B} + 3p^2(3-2p)E_{3B}^2 + 2p^3 E_{3B}^3,
\]

where \( E_{2B} = \exp(-R^2Q^2) \) (Gaussian form) and/or \( E_{2B} = \exp(-R \sqrt{Q^2}) \) (exponential form), and \( E_{3B} = \exp(-R^2Q^2) \) and so on.
Hereafter, we use the following expression; the numerical factor “2/3” in the exponential function is important \([6, 7]\).

\[
\Psi_{k_1k_2}(x_1, x_2) = \Gamma(1 + i\eta_{ij})e^{x\eta_{ij}/2}e^{ik_1 r_{ij}}F[-i\eta_{ij}, 1; i(k_1 r_{ij} - k_2 r_{ij})],
\]

(2.5)

where, \(r_{ij} = x_i - x_j\), \(k_{ij} = (k_1 - k_2)/2\), \(r_{ij} = |r_{ij}|\), \(k_{ij} = |k_{ij}|\) and \(\eta_{ij} = e_i e_j \mu_{ij}/k_{ij}\). \(\mu_{ij}\) : reduced mass of \(m_i\) and \(m_j\), \(F[a, b; x]\) : confluent hypergeometric function, \(\Gamma(x)\) : Gamma function.

Using of Eq. (2.5), the 2nd order BEC with \(\lambda\) and Gaussian form for \(\rho(x_i)\) is calculated as,

\[
N^{(2\pi)} = \frac{1}{2} \sum_{i=1}^{2} \rho(x_i) d^3 x_i \left[ \frac{1}{2} \left( |\psi_{k_1k_2}(x_1, x_2)|^2 + |\psi_{k_1k_2}(x_2, x_1)|^2 \right) + \lambda \Re \left( \psi_{k_1k_2}(x_1, x_2) \psi^{\ast}_{k_1k_2}(x_2, x_1) \right) \right].
\]

(2.6)

The 3rd order BEC is computed based on the 3-body Coulomb wave function in \([6]\),

\[
\Psi_f = \psi_{k_1k_2}(x_1, x_2) \psi_{k_3k_3}(x_2, x_3) \psi_{k_1k_3}(x_3, x_1),
\]

(2.7)

Hereafter, we use the following expression; \(\psi_{k_1k_2}(x_1, x_2) = e^{i(2/3)k_1 r_{ij}} \phi_{k_1k_2}(r_{ij})\). Notice that the numerical factor “2/3” in the exponential function is important \([6, 8]\).

\[
N^{(3\pi)} = \frac{1}{6} \prod_{i=1}^{3} \rho(x_i) d^3 x_i \left[ \sum_{j=1}^{3} A(j) \right]^2,
\]

(2.8)

where

\[
A(1) = A_1 = \psi_{k_1k_2}(x_1, x_2) \psi_{k_3k_3}(x_2, x_3) \psi_{k_1k_3}(x_3, x_1),
\]

\[
A(2) = A_{23} = \psi_{k_1k_2}(x_1, x_3) \psi_{k_2k_3}(x_3, x_2) \psi_{k_1k_3}(x_2, x_1).
\]

(2.9)
analyses of data by STAR Coll and PHENIX Coll

3-1) For Coulomb corrected data, we employ the conventional formulae, Eqs. (2.1) and (2.2). Ours are given in Table 2 and Fig. 4. It is interesting that $R_{2\pi} \sim R_{3\pi} \sim 8.5$ fm for data by STAR Coll.

| $q_3$ | STAR | $\lambda$ | $\chi^2$/n.d.f. | PHENIX | $\lambda$ | $\chi^2$/N_dof |
|-------|-------|-----------|-----------------|--------|-----------|-----------------|
| 2$\pi^-$ | 8.75±0.31 | 0.58±0.02 | 23.0/25 | 4.77±0.04 | 0.39±0.01 | 178/40 |
| 3$\pi$ | 8.26±0.39 | 0.50±0.02 | 1.88/35 | 6.92±0.82 | 0.34±0.10 | 6.5/14 |

3-2) Corrected data with $q_3 > 0.02$ GeV by PHENIX Coll are analyzed by conventional formula / LO approach. Our results are shown in Figs. 4 and 5, and Tables 2 and 3. Our results are also shown in Fig. 3 and lower-part of Table 3.

3-3) Raw data with $q_3 > 0.02$ GeV are analyzed by the formulae of Coulomb wave function (Eqs. (2.6) and (2.12)). Our results are also shown in Fig. 3 and lower-part of Table 3.
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![Graphs showing N/(N BG) vs q3](image)

**Figure 4:** Analyses of corrected data by STAR Coll. and PHENIX Coll. Eqs. (2.1) and (2.2) are used.

![Graphs showing N/(N BG) vs q3](image)

**Figure 5:** Corrected data are analyzed by Eq. (2.3) and (2.4).

![Graphs showing N/(N BG) vs q3](image)

**Figure 6:** Analyses of raw data by PHENIX Coll. Eqs. (2.6) and (2.12) are used.

3-4) Using two formulae in the core-halo approach (with Gaussian source function, the fraction of core part $f_c$ and the degree of coherent $p_c$ in [5, 6, 7]) we obtain Fig. 7. In raw data, there is no overlapping region. On the other hand, in corrected data, we observe very narrow overlapping region. The reason of the wide region of 3π BEC is due to large error bars of corrected data [2].

4. Summary

4-1) From raw data as well as Coulomb corrected data in Au+Au at 130 GeV by STAR Coll., we get the following interaction ranges $R_{2\pi} = 8.7$ fm and $R_{3\pi} = 8.3$ fm, and can estimate

$$V = R_{3\pi}^3 \sim 500 \text{ fm}^3. \quad (4.1)$$

This value is compared with that of ALICE Coll [8], $V = R_{long}R_{out}R_{side} \sim 300 \text{ fm}^3$ at $dN_{ch}/d\eta = 1500$ and $k_T \sim 0.3$ GeV in Pb+Pb at 2.76 TeV.

4-2) On the contrary, from corrected data at 200 GeV by PHENIX Coll., we obtain the ranges, by utilizing Eqs. (2.1) and (2.2), $R_{2\pi} = 4.8$ fm ($\lambda = 0.39$) and $R_{3\pi} = 6.9$ fm ($\lambda = 0.34$). From raw
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Table 3: Analyses of data by PHENIX Coll. Eqs. (2.3), (2.4), (2.6) and (2.12) are used.

| $E_{2\pi}$ | $R$ [fm] | $\rho$ | $c$ | $\chi^2/N_{dof}$ |
|------------|----------|--------|-----|------------------|
| $2\pi$ Gaussian | 6.58±0.05 | 0.23±0.00 | 0.98±0.00 | 156/40 |
| Exponential | 9.54±0.16 | 0.99±0.01 | 0.99±0.00 | 56/40 |
| $3\pi$ Gaussian | 9.76±1.11 | 0.24±0.08 | 0.99±0.00 | 7.2/14 |
| Exponential | 14.36±2.10 | 1.00±0.07 | 0.99±0.02 | 6.3/14 |

| raw data | $R$ [fm] | $\lambda$ | $c$ | $\chi^2/N_{dof}$ |
|----------|----------|------------|-----|------------------|
| $2\pi$ Eq. (2.6) | 3.77±0.03 | 0.253±0.004 | 1.00±0.00 | 129/40 |
| $3\pi$ Eq. (2.12) | 5.77±0.32 | 0.19±0.02 | 1.00±0.00 | 84/14 |

Figure 7: Analyses of data by core-halo model.

data, we have smaller interaction ranges $R_{2\pi} = 3.8$ fm and $R_{3\pi} = 5.8$ fm. The interaction ranges of data at 200 GeV by PHENIX Coll. are smaller than those of STAR Coll. At present, it is difficult to draw concrete physical picture for Au+Au collision at 200 GeV. Then we are waiting for final empirical analyses by PHENIX Coll.

4-3) Moreover, we also eager for empirical analyses of $(2\pi^+)\pi^-$ and $(2\pi^-)\pi^+$ combinations at RHIC and LHC energies. See [10, 11].

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