Optimal placement design for picking and packing in distribution centers

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Abstract
An efficient distribution center (DC) is one that receives, stores, picks and packs products into new logistics units and then dispatches them to points of sale at the minimal operating cost. The picking and packing processes represent the highest operating cost of a DC, and both require a suitable space for their operation. An effective coordination between these zones prevents bottlenecks and has a direct impact on the DC’s operational results. In the existing literature, there are no studies that optimize the distribution of the picking and packing areas simultaneously while also reducing operating costs. This article proposes an integer nonlinear integer programming model that minimizes order preparation costs. It does so by predicting customer demand based on historical data and defining the ideal area for picking and packing activities. The model is validated through a real case study of seven clients and fifteen products. It achieves a 15% reduction in operating costs when the optimal allocation of the picking and packing areas is made.

Keywords
Picking, packing, distribution center, non-linear optimization

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Introduction
The exchange of globalized goods and services necessitates intricate shipping and relocation patterns that impact the productive process, in particular, the sales point.¹ Distribution centers (DC) are thus utilized to help reduce the time required to transport goods from the production plants to the sales points by maintaining an adequate inventory of products and minimizing stock shortages.²

The DCs are logistic structures that receive and store products according to clients’ requirements and prepare orders (picking). Thus, the products are separated from their original load units to constitute a new loading unit. Then, these new loading units are packed in efficient units (packing) to be transported safely (in pallets) to the sales point. A warehouse management system (WMS) helps minimize handling costs by tracking the location of the products and the status of the orders in the DC.³

According to the description of the DC operations, four sections are identified: reception, warehouse, preparation of orders and dispatch. Picking and packing operations in the order picking area often cause a bottleneck due to the complexity of their automation. The literature indicates that these operations amount to two-thirds of the DC’s operating costs and time. One way to improve the productivity of a DC is to facilitate the flow of materials by optimizing the facility’s design.

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In the 80s, the attempts to reduce the picking time of orders in DCs only considered the ends of the aisles. The computational effort required is linear and in direct relation to the number of corridors. However, Gumma et al. proposed a mixed formulation of non-linear programming to address the problem of assigning picking locations and thus minimize operating costs.

Tarczyński compares strategies for planning a picking route through a simple warehouse by using an algorithm that relates the length of the route with the attributes of the DC.

Peng et al. propose the method of nested partitions to solve problems of global optimization; this method systematically divides the feasible regions and intensifies the search in promising regions. The diversity of the stock unit (SKU) in the warehouse and the product request volume can be factors that affect the storage system.

When evaluating different picking, storage, and routing policies, it is possible to determine which process provides the highest percentage of savings relative to baseline policies. Lagodimos et al. analyze the effects of the size of the order, the form of the depot, the location of the collection/return point and the distribution of the demand on the performance. Wu et al. consider the case where all the items of the order are available for picking in the inventory, and the unfulfilled orders are transferred to the next day with higher priority using a linear programming model.

The picking process is critical in the supply chain of all DCs. Most optimization tools provide efficient solutions but only consider uniform pallet dimensions. Once the picking is finished, the new logistics units are transported for consolidation in the packing area. The system will depend on the number of workers assigned to a single zone; a greater number of zones reduces the picking time (since the travel time is reduced). However, the waiting time to complete packing increases. Derpich and Sepúlveda seek to determine the optimal number of zones so as to minimize the total picking time and thus improve the packing operation.

Venkatadri et al. contribute to optimizing the placement of products in a DC tuning tunnel in the soda industry, thus reducing congestion and improving performance. To minimize the total replenishment effort, the number of collection locations for each product is optimized, which has a positive impact on performance.

At present, DCs are characterized by a high diversity of logistic units that must be repackaged and incorporated in the same pallet. Derpich and Sepúlveda propose a model that integrates the design of the DC, the storage and distribution policies and the selection process for a product, thus minimizing transportation costs.

The reviewed literature sheds light on improvements in the zone selection network as it allows routing flexibility in the order fulfillment process. Ho and Lin analyze two problems: batch dispatch and order selection; they consider the picking time as well as the order size, location, and how these factors affect the performance of the process. Jiang et al. consider routing with heterogeneous equipment, storage locations and inventory. They use metaheuristics such as Genetic Algorithms, Tabu Search, and a hybrid system (2-Opt Exchange y 2-Opt Insertion), which were integrated into the WMS in real time.

In the bibliographic review, no literature was found that solves both of the aforementioned problems. Therefore, integrating the design of the picking and packing sections for multiple products is a novel contribution to hard engineering. This strategy defines an operational flow that achieves an agile and timely dispatch by minimizing the number and severity of bottlenecks in the DC. The efficient design of the picking and packing areas facilitates automation in warehouse management, which is part of Industry 4.0, since it is an activity that consumes significant resources and is applicable to all engineering fields.

This article presents a non-linear programming model that optimizes the design of the picking and packing area. The costs associated with picking, turnover rate, inventory management, and ABC classification of products are thus minimized. Additionally, it considers the speed of picking and packing, and solves the aforementioned problem.

The article is structured as follows: in Section “Problem Formulation,” we present the integer non-linear integer optimization model. In Section “Industrial Case,” we present the results of the implementation, applied to a case study based on real industrial data. In Sections “Conclusions,” the main conclusions and areas for future study are presented.

### Problem formulation

The following is an integer non-linear programming model called Design of the Picking and Packing Area (DPPA). The DPPA model is described in this section; its aim is to minimize the operating costs of picking and packing by optimizing the design of their respective areas. The parameters are presented with their respective units and the model variables. The operational requests are also described.

#### Sets

- **Products set:** \{1, \ldots, i, \ldots, J\}.
- **Customer set:** \{1, \ldots, j, \ldots, J\}.

#### Parameters

- **a:** Number of orders in period (order).
- **d:** Number or orders in a batch (order).
• $o_i$: Average number of orders of product $i$ in the period ($order/time$).
• $u_i$: Average number of items of product $i$ in an order ($units/order$).
• $m_i$: Amount of product $i$ in the period, which can be stored at each picking location ($units/position$).
• $w$: Width of the picking locations, for the product ($length/position$).
• $l_{si}$: Length of the picking position ($length$).
• $r_j$: Average number of customer orders $j$ in the period ($order/time$).
• $q_j$: Average number of orders per pallet for customer $j$ in the period ($order/pallet$).
• $cf$: Average fixed cost of replenishing a position in the picking area in the period ($\$/time$), where $\$\$ represents Chilean Peso.
• $cv_i$: Average variable unit cost of moving product $i$ to a position in the picking area from its storage location ($\$/time/quantity/position$).
• $Tr_i$: Rate of rotation of product $i$, according to sales in the period ($Cost_{of\_merchandise\_sold} / Average_{value\_inventory}$).
• $ABC_i$: An ABC inventory analysis is used to determine the demand for product $i$ in the period. A two-year history is used, and its allocation by category is:

\[
ABC_i = \begin{cases} 
0.80, & \text{if } i \text{ belongs to the category A} \\
0.15, & \text{if } i \text{ belongs to the category B} \\
0.05, & \text{if } i \text{ belongs to the category C} 
\end{cases}
\]

• $cri$: Cost to pick up in the picking area ($\$/time$).
• $cit$: Cost of infrastructure of the picking area in the period ($\$/area$).
• $vci$: Average picking speed in the picking area ($length/time$).
• $vpi$: Average speed to supply a picking position in the period ($units/time/position$).
• $vpa$: Average speed of assembling pallets in the picking area in the period ($pallet/time/position$).
• $caa$: Cost of assembling the pallets in the picking area in the period ($\$/pallet)$.
• $cia$: Cost of infrastructure by position in the picking area for the period ($\$/area$).
• $Aapi$: Maximum area available for picking (area).
• $Aipi$: Minimum area available for picking (area).
• $Aapa$: Maximum area available for packing (area).
• $Aipa$: Minimum area available for packing (area).

**Decision variables**

• $s_i$: Quantity of positions in the picking area assigned to product $i$ in the period ($position$).
• $t_j$: Quantity of floor positions (pallets) of the customer $j$ within the packing area in the period ($position$).

**Assumptions**

• The products processed in picking can be handled manually (i.e. they are small and lightweight).
• The DC works with standard American pallets ($1.2m^2$) in the packing area.
• The replenishment of a location in the picking area is done using pallets containing only one type of product.
• The fixed costs in the picking area are related to the operating costs of the equipment used in the operation and the daily salary of the employees.
• The variable costs in the picking area are derived from the boxes, film, special labeling and transport due to the location of the pallet in the storage area.
• In the packing area, the full pallets can have an arrangement with several SKU.
• In the packing area, a position is occupied by a pallet plus a percentage of area (85%) destined to corridors. Consequently, the floor area per packing position working with standard pallets is $2.22m^2$.
• Neither the cost of moving the products to the dispatch area nor the dispatch to the final client are included.
• The model minimizes costs for one period.

**Formulation of the DPPA model**

\[
\begin{aligned}
\min_{s,t} \quad & z = \sum_{i=1}^{I} \frac{1}{Tr_i \cdot ABC_i} \left[ Cost1(s_i) + Cost2(s_i) + Cost3(s_i) \right] + \sum_{j=1}^{J} \left[ Cost4(t_j) + Cost5(t_j) \right] \\
\text{Subject to:} \\
& vpi \cdot s_i \geq o_i \cdot u_i, \quad \forall i \\
& vpa \cdot t_j \geq \left( \frac{r_j}{q_j} \right), \quad \forall j \\
& s_{\min} \leq s_i \leq s_{\max}, \quad \forall i
\end{aligned}
\]
The objective function (1) has five costs. The first three correspond to (1) the picking area multiplied by the (2) inverse ratio between the rotation rate, and (3) the ABC classification of products. Thus, the higher a product’s rate of rotation and ABC classification, the greater priority it will receive when the number of positions in the picking area is assigned. The last two costs are associated with the packing area.

The cost of collecting items in the picking area in the period is given by (10), where \( (a/d) \) represents the average number of times a resource passes through a location. The path traveled to collect the product \( i \) is given by \( (w \cdot s_i) \). Then, \( (a/d) \cdot w \cdot s_i \) represents the total distance traveled during the period in the collection operation. The distance traveled divided by the collection speed \( (vci) \) expresses the time dedicated to the operation in the period. Finally, the cost of collection is attained by multiplying the resulting expression by the average unit cost of collection \( cri \).

\[
Cost1_s(s_i) = cri \cdot \frac{(a/d) \cdot w \cdot s_i}{vci} \tag{10}
\]

The replacement cost for product \( i \) in the picking area in the period is defined by (11) and corresponds to the sum of the fixed costs and the variable costs. The first term expresses how many times per day the picking positions of a product must be replenished to satisfy the demand \( \left( \frac{u_i \cdot o_i}{m_i \cdot s_i} \right) \), and it is multiplied by the fixed cost of replacing said product \( (cf) \). The second term is the variable cost \( (cv_i \cdot s_i) \) of moving a product from its storage place to its position in the picking area, and it is multiplied by the units required in the period \( (u_i \cdot o_i) \).

\[
Cost2_s(s_i) = cf \cdot \frac{u_i \cdot o_i}{m_i \cdot s_i} + cv_i \cdot s_i \cdot u_i \cdot o_i \tag{11}
\]

The cost per product space \( i \) in the picking area for the period is defined by (12), in which the area occupied by the product in the warehouse \( (w \cdot s_i \cdot lsi) \) is multiplied by the cost of infrastructure \( (cii) \) in that area.

\[
Cost3_s(s_i) = cii \cdot w \cdot s_i \cdot lsi \tag{12}
\]

The cost per customer \( j \), which is associated with the space utilized in the packing area during the period, is defined by (13). In this equation, the area occupied per client in packing \( (2.22 \cdot t_j) \) is multiplied by the cost of the infrastructure \( cia \).

\[
Cost4_{t_j} = 2.22 \cdot cia \cdot t_j \tag{13}
\]

Equation (14) gives the average cost per customer \( j \). Because the products are grouped into new pallets, the average cost per customer is determined by multiplying the number of pallets that can be formed in the period \( (vpa \cdot t_j) \) by the cost of assembling a pallet \( (caa) \).

\[
Cost5_{t_j} = caa \cdot vpa \cdot t_j \tag{14}
\]

Additionally, Restriction (2) establishes that the available units of each product in the picking area \( (vpi \cdot s_i) \) must be at least equal to the number of units required in the period \( (u_i \cdot o_i) \). Restriction (3) establishes that the number of pallets assembled in the packing area during the period \( (vpa \cdot t_j) \) must at least satisfy the demand of the period \( (r_j/q_j) \). Restriction (4) defines the lower and upper bounds of the number of positions per type of product in the picking area, and Restriction (5) defines the maximum and minimum levels for the number of floor positions per customer in the packing area. The limits on restrictions (4) and (5) are defined according to the historical requirements. The restrictions (6) and (7) define the area for picking and packing operations, respectively, according to availability. Finally, restrictions (8) and (9) express the nature of the variables.

Restriction (7) defines the total area of the packing zone; the shape of the area is left to the decision maker’s discretion. However, if the pallets are left side by side and have a width of 1m and a depth of 1.2m (American pallet measure), the length and width are defined by equations (15) and (16), respectively, where 1.22 is the width of the corridor in meters.

\[
\text{Width} = 1.2 + 1.22 \tag{16}
\]

\[
\text{Large} = 1.0 \cdot \sum_{j=1}^{J} t_j \tag{15}
\]

**Industrial case**

The integer nonlinear programming model is validated in a company with seven clients and fifteen products. The results show an increase in the efficiency of the process in the picking and packing area as compared to a standard layout. The model is written in GAMS and executed with the Knitro solver, which converges to an overall solution in 0.047s.

**Input data**

The DC works 8h a day with a daily volume of orders \( (a) \) equal to 1500 and a lot size \( (d) \) of 20. The five
average fixed operating costs, the operating speed, the size of the picking position and the available area are presented in Table 1.

The daily requirements, variable replacement costs per product and the storage capacity of a picking position are presented in Table 2. Meanwhile, the daily requirements per customer in the packing area, along with the maximum and minimum number of positions per product, are presented in Table 3.

Finally, Table 4 presents the number of positions available in packing, the ABC inventory classification and the rotation rate per product. The demand for products was estimated using historical data, and the turnover rate for the period was calculated by dividing the cost of the required product by the value of the inventory for each product.

### Results

The minimum value of the objective function is $769444.01 when considering the ABC classification of inventories and the turnover rate. The costs derived from each of the activities are presented in Table 5.

The cost of infrastructure is not very high compared to the other costs because the DC is on the outskirts of the city. It should be noted, however, that the cost of infrastructure in picking is higher than that of packing; this is due not only to the area required by each respective operation, but also to their relative infrastructure requirements.
In the picking area, the cost of replacement is less than the cost of collection because replacement is done at the pallet level with the help of forklifts while collection is done manually at the product level. Finally, the optimal position assignments per product and the number of floor positions per customers are presented in Table 6.

In Table 6, the product that requires the most picking positions is product 11 with 13 units, so the DC must have a total of 90 pick and aisle positions in an area of 216.0 m². The demand of the period is thus satisfied, since there is an available capacity of 13 500 units, which is higher than the 12 276 units required for picking. The customer with the most packing positions is customer 1, with 22 positions. In all, the DC must have a total of 64 packing positions and corridors in an area of 142.08 m². With this provision, the demand of the period is satisfied since there is an available capacity of 768.0 pallets, which is significantly more than the required quantity of 734.5.

Idle capacity is available in both picking and packing due to the discrete nature of the positions (pallet positions cannot be split). Nevertheless, when the optimal allocation is compared with the results of the method that is normally employed, a 15% reduction in operating costs is observed. Regarding the case study, this reduction in the objective function was 15.02%. Based on these results, we present our conclusions in the following section.

## Conclusions

An integer non-linear optimization model called DAPP was presented; it minimizes the operating costs and identifies the optimal number of positions per product and per customer in picking and packing operations for small products. By using this model, it is possible to define the area of these operations that best meets the DC’s daily operational requirements.

The proposed model allows a free flow between picking and packing operations to reduce costs and increase the volume of these operations. When comparing the proposed model with others found in the literature, a significant reduction in bottlenecks and the delivery of the wrong products to customers was found.

The proposed model can be applied to the operation of all types of DCs, regardless of the size of the operation. It should be noted that the Knitro solver, independent of the user interface (GAMS, AMPL, Front Line Solver, etc.) can smoothly solve large, non-linear optimization problems, such as the one presented in this research.

For future work, we propose extending the proposed method to multiple periods and incorporating seasonal changes in demand into the model. Another interesting
extension would be to address the uncertainty in demand by using robust distributional optimization to find solutions that are immune to disturbances caused by this type of fluctuation.

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