A frequency correction method based on axisymmetric function

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Abstract. In this paper, a frequency correction method based on axisymmetric function is presented first and the analytical solutions of three forms of axisymmetric equations with three points are developed to correct the frequency. A complex exponent signal with exact solution is used to demonstrate the accuracy and theory of the proposed method by comparing it to the traditional methods with fast Fourier transform (FFT). Moreover, the proposed method is applied in solving for a frequency identification of free attenuation vibration signal based on finite element model of actual building structure and the results show a good agreement with experimental and structural modal simulation results, which demonstrates the validity of the proposed method for solving frequency problems of building structure and others.

1. Introduction
The application of discrete fourier transform (DFT) spectrum analysis method has been widely used in scientific research and engineering signal analysis. In particular, frequency identification played an important role in the analysis of building structure data. However, in the DFT analysis, the error caused by non-integral period sampling, and the error caused by energy leakage, which due to window truncation, could not be avoided. Theoretically, the methods to reduce the error could only be realized by increasing the data length N or reducing the sampling frequency. Unfortunately, as the results of building structures were mostly fixed time signals, neither of these methods could be easy to implement. Therefore, in order to obtain the frequency more accurately, different spectrum correction methods were proposed based on mathematical methods.

At present, there were three main analysis methods: 1. Based on spectrum zoom method. 2. Based on phase difference method. 3. Based on Energy centrobaric correction method. Among these methods, firstly, the amount of calculation of the spectrum zoom method was generally large, and its data length should not be too small. Secondly, the phase difference method generally needed additional data for correction, but, in the analysis of building structure, the signal length was limited. Finally, energy centrobaric correction method, one or two spectral lines next to the main spectral lines were used to correct the spectrum by the law of the center of gravity between spectral lines, was affected by window truncation.

This paper presented a kind of frequency correction method based on three-point coordinates. Its principle was similar to that of energy centrobaric correction method, and can correct multi-frequency
signals. Moreover, numerical examples were given to verify its performance in structural analysis of buildings.

2. Theoretical Analysis

In this section, it could be mainly introduced that a frequency correction method based on three-point coordinates. In this method, the hypothetical axis symmetry function was used to simulate the attenuation process of the main spectral line. Moreover, it was given that the theoretical analysis of three common axisymmetric functions as hypothetical functions.

2.1. Frequency correction method based on axisymmetric function

In this paper, the correction method could assume that the attenuation spectral line function \( f(x) \) was a axisymmetric function with no inflection point within a frequency resolution (\( \Delta f \)) pre and post the main spectral line. Then, on the DFT spectrum of the signal, the main frequency spectrum point, the previous secondary frequency spectrum point and the post secondary frequency spectrum point should be on the axisymmetric function. Therefore, by specifying the form of the function and considering the known three-point coordinates, the specific function expression can be determined. Then, its abscissa of symmetrical axis was the corrected frequency value.

![Figure 1. Schematic of frequency correction method based on axisymmetric function.](image)

The schematic diagram of its principle can be referred to Figure 1. In the spectrogram, the coordinates of the main spectrum and the previous and back post points were \( K (x_K,y_K) \), \( M (x_M,y_M) \) and \( N (x_N,y_N) \), respectively. And, it must satisfy the inequality \( y_K \leq y_M \leq y_N \). From the above, the correction frequency could be obtained.

Defining the frequency resolution of the DFT spectrum \( \Delta f \), coordinates of M and N points could be expressed as

\[
\begin{align*}
    y_M &= my_K \\
    y_N &= my_K \\
    x_M &= x_K \pm \Delta f \\
    x_N &= x_K \mp \Delta f
\end{align*}
\]

(1)

Where \( m \in [0,1] \), \( n \in [0, m] \) and \( n < 1 \).
Because of the inequality \( y_K \geq y_M \geq y_N \), the distance from point M to point K must be greater than or equal to the distance from point N to point K, then the correction frequency \( x_O \) should be between \( x_K \) and \( x_M \). Besides, \( f(x) \) was an axisymmetric function without inflection point, so it could be shown \( x_O \in [x_K - \Delta f / 2, x_K + \Delta f / 2] \). Therefore, the \( f(x) \) boundary conditions must be shown as:

\[
x_O = x_K \pm 0.5 \Delta f, \quad m = 1
\]

\[
x_O = x_K, \quad m = n
\]

2.2. Analytic solutions of different axisymmetric functions

In this part, through using the method of this paper, it was given that the analytical solutions of three common axisymmetric functions.

2.2.1. Primary absolute value function

First of all, it was assumed that the function was primary absolute value function (PA), and its form could be shown as:

\[
y = a|x - b| + c
\]

The expression of the abscissa \( x_O \), which was also the correction frequency, of its symmetrical axis was

\[
x_O = x_K + k \Delta f
\]

Where \( k \) was defined as a correction scale factor, which could represent the extent to which the correction frequency \( x_O \) deviates from \( x_K \). Utilizing equation (1)- equation (5), \( k \) should be defined

\[
k = \begin{cases} 
\frac{n - m}{2(n - 1)}, & x_M > x_N \\
\frac{m - n}{2(n - 1)}, & x_M < x_N
\end{cases}
\]

2.2.2. Quadratic function

The quadratic function (Quad) was a natural axisymmetric function, and its expression could be given

\[
y = a(x - b)^2 + c
\]

The coordinate of points M, N and K were substituted into the equation (7), and \( k \) should be defined

\[
k = \begin{cases} 
\frac{n - m}{2(m + n - 2)}, & x_M > x_N \\
\frac{m - n}{2(m + n - 2)}, & x_M < x_N
\end{cases}
\]

2.2.3. Absolute value of inverse proportional function

Usually inverse function was centrosymmetric function, but after taking the absolute value of its independent variable, the function was axisymmetric
in the first and second quadrant. And, the absolute value of inverse proportional function (AIP) could be shown as

\[ y = a |x - b|^{-1} + c \]  \hfill (9)

The coordinate of points M N and K were also substituted into the equation (9), and \( k \) should be defined

\[
k = \begin{cases} 
\frac{m - n - [(8 + m - 9n)(m - n)]^{1/2}}{4(n-1)}, & x_M > x_N \\
\frac{-m + n + [(8 + m - 9n)(m - n)]^{1/2}}{4(n-1)}, & x_M < x_N
\end{cases}
\hfill (10)

2.3. Theoretical analysis of results of different axisymmetric functions

Through the definition of analytical equation (6), equation (8) and equation (10), the relationship among \( m, n \) and \( k \) was the focus of the research. The diagrams of relationship among \( k, m \) and \( n \) in PA, Quad and AIP methods could be shown Figure 2, Figure 3 and Figure 4, respectively. (Only the case of \( x_m \geq x_n \) was considered in this section, on the contrary, the result was similar.)

![Figure 2](image2.png)

**Figure 2.** Relationships among \( k, m \) and \( n \) in PA method

![Figure 3](image3.png)

**Figure 3.** Relationships among \( k, m \) and \( n \) in Quad method.
Figure 4. Relationships among $k$, $m$ and $n$ in AIP method.

Firstly, the basic form of equation (4), equation (7) and equation (9) was identical, except that the exponents were different. So, it could be analyzed together. In Figure 2, when $m$ was constant, $k$ would increase with the decrease of $n$, and when $n$ was constant, $k$ would increase as $m$ increased. In other word, the greater the difference between $m$ and $n$, the greater the deviation of the correction frequency $x_d$ from $x_K$. Secondly, it can be found out in Figure 3, the variation law of Quad method was roughly the same as that in AP method. Moreover, except at the boundary, the result of Quad method was smaller than that of PA method. Furthermore, at the same conditions, the deviation degree of correction frequency of Quad method was lower than that of PA method. Lastly, the result of AIP method was also smaller than that of PA method. But, at the same conditions, the deviation degree of correction frequency of AIP method was higher than that of PA method. Therefore, the correction frequency deviation of AIP correction method was greater than that of the other methods at the same conditions. In addition, it was shown in Figure 4 that the surface protruded obviously near the $m = n$ line. Thus, through in AIP method, when the difference between $m$ and $n$ was small, it would have a great influence on $k$, but the change of $k$ value tended to be smooth with the larger the difference between $m$ and $n$.

3. Numerical tests

In this section, numerical simulation data and finite element analysis data would be used to verify the principles of the three equations and analyze their accuracy. And, the shaking table data of the real structure could not be selected, because of considering that the method in this paper needed the target frequency to verify the accuracy. However, the actual structural model could not determine the exact frequency value, so it was not suitable to be used as an example of numerical test.

3.1. Dual-frequency complex exponent signal without noise

For single frequency signal without noise, there were not spectrum interference problem and multi-frequency non-integer sampling problem. Therefore, a dual-frequency signal was constructed for numerical verification. Set the signal sampling frequency $f_s=1$Hz, data length $N=64$, the rest of the information should be shown in table 1.

| Frequency(Hz) | Amplitude | phase (°) |
|--------------|-----------|-----------|
| 1st frequency | 0.12 | 4 | 100 |
| 2nd frequency | 0.15 | 1 | 30 |

Table 1. The information of Dual frequency model
It can be known by definition, the frequency resolution of the signal was $\Delta f = 0.015325\text{Hz}$, and the difference between the first-order frequency and the second-order frequency was 0.03Hz. Therefore, there could be interference between the two frequency lines. And, using less data as far as possible, to meet architectural structural analysis requirements. Through the above analysis, the expression of the signal could be assumed that

$$x(n) = e^{(2\pi n(0.12+100/360))} + e^{(2\pi n(0.15+30/360))}, n = 0, 1, ..., N-1$$  \hspace{1cm} (11)$$

The frequency values corrected by three different equations were shown in Figure 5 and Figure 6.

(Using normalized power spectrum)

Figure 5. First-order correction frequency of different methods.

Figure 6. Second-order correction frequency of different methods.
It could be verified from the diagrams that the correct offset direction of the correction frequency position of the three functions was correct. Thus, under the condition of shorter data length \((N=64)\), all three functions could improve the recognition accuracy of frequency. Moreover, the degree of frequency correction satisfied inequality \(\text{AIP}>\text{PA}>\text{Quad}\), which was consistent with the conclusion of Section 2.3. Further, the error for correcting frequency among three kinds of functions and FFT could be shown in table 2.

|            | FFT | PA | Quad | AIP |
|------------|-----|----|------|-----|
| 1st frequency (%) | 5.817 | 3.125 | 3.708 | 0.650 |
| 2nd frequency (%) | 5.820 | 0.893 | 1.213 | 0.653 |

In table 2, the correction accuracy of the three functions was higher than that of FFT, especially in the case of AIP functions. And, the error of AIP method was less than 1\%, which was close to 10 times that of FFT. Because that, in this case, the real frequency was concentrated in the middle of the two spectral lines, the accuracy of AIP could be higher than that of other functions, considering that it would have a great influence on \(k\) when the difference between \(m\) and \(n\) was small.

3.2. Experimental and verification for structures

In order to test the performance of the three methods in practical building structure engineering, the result of free attenuation vibration was analyzed, based on the finite element modelling of the structure of an industrial factory building. And, the structure of the industrial factory building was complex, and its stiffness and mass distribution were uneven. So, the structural vibration had the participation of higher-order modes, which could affect the accuracy of the first-order frequency. And, the structure was modelled and calculated by SAP2000. The finite element model was modelled in Figure 7, and the free attenuation vibration signal in the Y direction of the vertex was shown in Figure 8.
The acceleration time history sampling frequency $f_s$ was 1000Hz, and its length $N$ was 2048. Through modal analysis, the real frequency of the first-order in Y direction was 8.2515Hz. In order to consider the influence of different length data on the accuracy, the accuracy of frequency correction could be analyzed by taking different data lengths and the result could be shown in Figure 9.

The length of the data in Figure 9 was from 128 to 2048. Firstly, in the case of minimum data ($N=128$), the frequency corrected by PA function was 8.4712Hz, and the error was 2.66%, the frequency corrected by Quad function was 8.4317Hz, and the error was 2.18%, and the frequency corrected by AIP function was 8.5180Hz, and the error was 3.23%. Secondly, in the case of maximum data ($N=2048$), the frequency corrected by PA function was 8.2325Hz, and the error was 0.23%, the frequency corrected by Quad function was 8.2621Hz, and the error was 0.13%, and the frequency corrected by AIP function was 8.1498Hz, and the error was 1.23%.

Therefore, all the three methods can approach the exact solution with the increase of data length, and the precision could meet the needs of the project. Especially, in the case of a small amount of data, it had the advantage of accuracy.

4. Conclusion
In this study, an idea of frequency correction based on axisymmetric function was presented. This idea could easily correct the frequency by making use of the relationship between the main frequency spectral line and the front and back two secondary spectral lines. Specifically, in this method, only one form of axisymmetric function needed to be specified, and the correction frequency could be obtained by determining the symmetry axis of the function through three-point coordinates. Besides, in this
paper, three forms of axisymmetric functions were proposed, of whose the analytical expressions were given, and the internal relations of the correction degrees of each range were analyzed.

Numerical tests had been done to evaluate the proposed method. Firstly, an exact solution example was adopted to demonstrate the accuracy and effectiveness of AP, Quad and AIP methods compared to the method with a classic FFT. Through the free attenuation of the vibration signal of the finite element model of the factory building structure, the results solved by three kinds of method were in good agreement with the modal simulation results of the finite element software. With that, the following conclusions could be obtained:

1. Through the dual-frequency signal analysis, the proposed method used PA, Quad and AIP functions was compared with the classical FFT method, and it could be verified that the correction frequency of these methods had high accuracy compared with the classical FFT method.

2. Through the results of the free attenuation of vibration signals of the finite element model, it was verified that the method of three functions could be used for practical engineering signal analysis, and its accuracy could meet the engineering requirements.

3. Through the results of the dual-frequency signal and the free attenuation vibration signal of the finite element model, it could be verified that the frequency correction method based on the idea of axisymmetric function could achieve higher accuracy in a shorter data length.

Therefore, this method was not only suitable for conventional signal frequency analysis, but also suitable for solving the problem of insufficient frequency accuracy caused by short data. Moreover, it was especially suitable for solving the problem of frequency analysis of building structure.

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