Modelling of Raman amplification in silicon-on-insulator optical microcavities

Francesco De Leonardis and Vittorio M N Passaro

1 Dipartimento di Ingegneria dell’Ambiente e per lo Sviluppo Sostenibile, Politecnico di Bari, viale del Turismo n. 8, 74100 Taranto, Italy
2 Dipartimento di Elettrotecnica ed Elettronica, Politecnico di Bari, via Edoardo Orabona n. 4, 70125 Bari, Italy
E-mail: f.deleonardis@poliba.it and passaro@deemail.poliba.it

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Abstract. In this paper, the detailed modelling of Raman amplification in silicon-on-insulator (SOI) guided-wave resonant microcavities is developed for the first time. Theoretical results are compared with experiments in literature for either racetrack or microdisk resonators, demonstrating very good agreement. Investigation of resonant microcavity parameters, including pump and Stokes coupling factors and cavity lengths, is presented and their influence on pump enhancement factor is discussed. Design criteria are derived for both CW Raman lasers and pulsed Raman amplifiers.

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3 Author to whom any correspondence should be addressed.
1. Introduction

In the last decade, increasing interest in developing silicon photonics has been observed. In fact, recent studies [1, 2] have demonstrated that silicon is considered as an ideal platform to overcome some actual limitations that the photonics industry introduces if compared with microelectronics. They can be summarized as follows: (i) a variety of different materials are used in photonics instead of one: InP as substrate for source development, silica as material for fibres, lithium niobate for modulators, other materials for dense wavelength-division-multiplexers (DWDM) and fibre amplifiers, and so on; (ii) no single material or single technology leads the market; (iii) the photonics industry is characterized by many different small companies which only specialize in specific devices; (iv) production technology is still quite primitive. Moreover, chip scale integration of optical components enabling low cost and high reproducibility has not yet been achieved. To develop silicon photonics, it is clear that the areas of main investigation must include selectively guiding and transporting light within the silicon, encoding light, detecting light, amplification and generation of light, packaging the devices and, finally, intelligent control of all these photonic functions.

To these aims, while a wide variety of passive devices have been developed since the 1990s, recent activities have focused on achieving active functionality (mostly light amplification and generation) in silicon-on-insulator (SOI) waveguides. Various approaches have been investigated based on silicon-engineered materials, including nanocrystals [3, 4], Er-doped silicon oxides [5], Si/SiGe structures [6] and porous silicon [7]. However, each of the previous approaches is characterized by nontrivial limitations. Recently, a different approach has been demonstrated, based on stimulated Raman scattering (SRS) [8]–[12]. It exploits the very high SRS gain coefficient in silicon guided-wave structures when compared with silica (about four orders of magnitude larger). Additionally, SOI waveguides allow confinement of the optical field to an area that is approximately 100 times smaller than the modal area in a standard single-mode optical fibre, making SRS observable over the millimetre-scale interaction length, as encountered in integrated optical devices. Starting from 2002, several experimental and theoretical studies based on this effect have been proposed in the literature, such as Raman amplification in SOI waveguides [13]–[18], Stokes and anti-Stokes Raman conversion [19]–[21], the cross phase modulation-based interferometer switch [22], two-photon absorption [23, 24] and lossless modulation [25].

Moreover, the utilization of a resonant microcavity to enhance the Raman effect represents a strong stimulus towards the development of micro-scale Raman lasers with low threshold, as demonstrated in some pioneering studies [26, 27]. In fact, the high quality factors allowed by these microcavities are of wide interest in optics for a number of studies and applications, ranging from fundamental physics such as quantum-electrodynamics to applied areas such as low-threshold and narrow-linewidth lasers, nonlinear all optical gates, high sensitivity transducers and so on.

Several studies have already proposed to induce nonlinear effects in microcavities based on different technologies, such as GaAs–AlGaAs [28, 29] and silica [30]–[33]. However, the integrated optical approach to Raman amplification in SOI optical waveguides in the presence of a coupled microcavity resonator, very useful to improve the Raman device performance, has been not yet modelled and designed.

Therefore, different to other Raman architectures [8]–[12] where the gain medium is represented by a simple SOI straight waveguide, in this study, we theoretically analyse the possibility to enhance the SRS effect in integrated microcavities based on SOI technology, where the resonant microcavity is coupled with an external bus straight waveguide.
Our approach is motivated by the need to design active devices in SOI integrated platform with high performance, i.e., the possibility that a microcavity with high enhancement factor offers a reduction of the threshold limit by some detrimental nonlinear optical effects. The mathematical model and the physical features involved in this study are very general, much more than those analysed in [33], particularly in the quasi-continuous wave (CW) regime. This is due mainly to the simultaneous presence of various considered effects, such as free carrier absorption (FCA) and plasma dispersion effects induced by TPA.

The paper is organized as follows. In section 2, we derive the mathematical model to study the nonlinear effects in a resonant microcavity coupled to the external waveguide. The proposed modelling includes all nonlinear effects involved in the integrated structure for the first time without any a priori assumption, i.e., SRS, TPA, FCA where the free carriers are generated mainly by TPA of the pump pulse, plasma dispersion effect, self-phase-modulation (SPM) and cross-phase-modulation (XPM) effects as induced by Kerr nonlinearity. The model takes into account not only the interaction between pump and fundamental Stokes pulses, but also the mismatch between the input beam wavelengths and the microcavity resonance wavelengths, as well as the coupling mechanism between the microcavity and the bus waveguide. In section 3, we show a number of numerical results, including comparisons between our theory and some experiments presented in the literature on CW Raman lasers based on microdisk or racetrack resonators. Moreover, a parametric study to individuate the influence of the pump enhancement factor on the net Raman gain induced in the SOI microcavity is performed. Conditions for Stokes pulsed excitation under either resonance or off-resonance conditions are also discussed to derive the best performance of pulsed Raman amplifiers using the resonant microcavity. Finally, section 4 summarizes the conclusions.

2. Modelling

In this section, we derive the general physical model to analyse the nonlinear effects, in particular SRS, in a microcavity resonator based on SOI technology. As will be shown, the model is based on two partial differential equations for nonlinear coupling between pump and fundamental Stokes waves inside the microcavity, and on one rate equation for the hole–electron pairs generated in the resonator by TPA effects as induced by the pump. The model presented in this study is a generalization (including resonance effects, cavity behaviour and coupling with the external bus waveguide) of the model proposed in [34] for a simple SOI straight waveguide.

In our analysis, we assume the architecture as sketched in figure 1, where the input pump ($S_p$) and the input Stokes probe ($S_s$) are injected into the resonator by means of the evanescent coupling between the resonant microcavity and the external bus rib waveguide in SOI, $G$ being the gap and $L_{coup}$ the coupling length. In most photonic applications and experimental devices, the microcavity shown in figure 1 is represented by either a microring, a microdisk, or a racetrack resonator. To unify the mathematical treatment for any microcavity geometry, we will consider the curvilinear coordinate $\zeta$ as the propagation direction of each optical wave inside the resonator (see figure 1).

The 3D greyscale view of figure 2 shows the refractive index distribution and the main geometrical parameters of the SOI architecture. In particular, the case of a microring resonator is sketched, having an external radius $R$, a rib total height $H$, a slab height $H_s$ and a ring width $W$. Throughout the paper, the same width $W$ has been always assumed for the straight rib waveguide.
Without any lack of generality, we assume that the electrical field inside the microcavity is predominantly a single transverse mode. This condition is satisfied by choosing rib waveguide sizes to meet the single-mode condition. Thus up to two propagating modes can be confined in the resonator, one quasi-TE (dominant horizontal $x$-component of electric field) and one quasi-TM (dominant vertical $y$-component), where $x$–$y$ designates the cavity cross-section, normal to the $z$ coordinate. A coupled-mode approach is used to describe the power transfer among the pump wave ($p$) and fundamental Stokes wave ($s$) that can be enhanced inside the microcavity.

Two main cases are investigated in this paper. In the former, the Stokes probe at the waveguide input is zero, and we investigate the performance of Raman lasers in CW regime. In the latter, a probe is present and the potential of pulsed Raman amplifiers are analysed. Hereinafter, we assume the input pump ($S_p$) and Stokes probe ($S_s$) as aligned with quasi-TE and quasi-TM polarization, respectively (as usually encountered in experiments). Neglecting the polarization cross-coupling between waveguide and microcavity (condition usually satisfied in SOI directional couplers), pump and Stokes waves inside the resonant cavity can hold the same polarizations as $S_p$ and $S_s$, respectively.
In addition, the assumption of translational invariance along \( \zeta \) (normal to the cavity cross-section) is made. Therefore, we apply the variable separation principle, and the electric field inside the microcavity can be written as

\[
\mathbf{E}(x, y, \zeta, t) = \hat{x} \left[ \sum_m C_{p,m} A_{p,m}(\zeta, t) F_{p,m}(x, y) e^{i(\beta_{p,m} \zeta - \omega_{p,m} t)} + \text{c.c.} \right] \\
+ \hat{y} \left[ \sum_q C_{s,q} A_{s,q}(\zeta, t) F_{s,q}(x, y) e^{i(\beta_{s,q} \zeta - \omega_{s,q} t)} + \text{c.c.} \right],
\]

where c.c. indicates the conjugate complex terms, the subscripts \( m \) and \( q \) designate all possible longitudinal modes in the cavity for pump and Stokes waves, respectively, \( \omega_{p,m} \) (\( \omega_{s,q} \)) is the resonant angular pulsation of the pump (Stokes) mode inside the cavity, and \( \beta_{p,m} \) (\( \beta_{s,q} \)) is the propagation constant of the pump (Stokes) wave under the resonance condition. In equation (1), \( F_{p,m}(x, y) \) and \( F_{s,q}(x, y) \) represent the optical mode distribution in \( x-y \) cross-section (solutions of Helmholtz’s wave equation) for pump and Stokes waves, respectively, while \( C_{p,m} \) (\( C_{s,q} \)) is a normalization constant, as:

\[
C_{p,m} = \left( \int \int_{-\infty}^{\infty} |F_{p,m}(x, y)|^2 \, dx \, dy \right)^{-1/2}, \\
C_{s,q} = \left( \int \int_{-\infty}^{\infty} |F_{s,q}(x, y)|^2 \, dx \, dy \right)^{-1/2}.
\]

Finally, \( A_{p,m}(\zeta, t) \) and \( A_{s,q}(\zeta, t) \) are the slowly-varying amplitudes of pump and Stokes waves travelling in the resonator, respectively.

It is clear that only a few terms are to be considered in the summations of equation (1), if the resonator is excited from an external optical beam travelling in the bus waveguide. In fact, if the input pump (\( S_p \)) and the input Stokes probe (\( S_s \)) are launched in the bus waveguide with angular frequencies \( \omega_p \) and \( \omega_s \), respectively (\( \omega_p - \omega_s = \Omega_R \)), where \( \Omega_R \) is the Raman frequency shift, only those resonant modes whose angular frequencies are close to \( \omega_p \) and \( \omega_s \) will give a contribution to summations in equation (1). These modes are characterized by a longitudinal order, namely \( \bar{m} \) and \( \bar{q} \), given by the resonance condition as:

\[
\bar{m} \sim \omega_p n_{\text{eff},p} R/c \text{ and } \bar{q} \sim \omega_s n_{\text{eff},s} R/c
\]

for a ring cavity, \( n_{\text{eff},p} \) (\( n_{\text{eff},s} \)) being the effective index of the pump (Stokes) wave inside the ring resonator and \( c \) the light velocity in vacuum.

The presence of only two resonant modes in the microcavity holds if the cavity free spectral range (FSR) is larger than the input pulse bandwidth, \( \Delta \omega_{\text{pulse}} \). For example, assuming a Gaussian input pump pulse with full-wave-half-maximum (FWHM) time width \( T_{\text{FWHM}} \), the condition FSR > \( \Delta \omega_{\text{pulse}} \) gives \( L_{\text{cavity}} < (2\pi c T_{\text{FWHM}})/(2\sqrt{2\ln 2} n_{\text{eff},p}) \), \( L_{\text{cavity}} \) being the cavity perimeter. For example, by assuming \( T_{\text{FWHM}} = 100 \) ps, we find \( L_{\text{cavity}} < 2.8 \) cm to consider only two resonant modes. This usually occurs in experiments on integrated structures.

To derive the basic equations that govern the power transfer among pump and fundamental Stokes waves, the starting point is the general wave equation, written in terms of current density \( \mathbf{J} \) as

\[
\nabla \times \nabla \times \mathbf{E} + \mu_0 \frac{\partial \mathbf{J}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2},
\]

where the induced polarization \( \mathbf{P} \) and electric field \( \mathbf{E} \) vectors are related by the phenomenological relation \( \mathbf{P} = \varepsilon_0 (\chi^{(1)} \times \mathbf{E} + \chi^{(3)} \times \mathbf{E} \mathbf{E} \mathbf{E}) \), \( \varepsilon_0 \) being the vacuum permittivity and \( \chi^{(j)} \) (\( j = 1, 3 \)) the \( j \)th order susceptibility. Thus the induced polarization consists of one linear and one nonlinear contribution, \( \mathbf{P}(r, t) = \mathbf{P}^L(r, t) + \mathbf{P}^{NL}(r, t) \), where the linear contribution \( \mathbf{P}^L \) is mainly related to the FCA coefficient.
In general, the components of induced nonlinear polarization $\mathbf{P}^{NL}$ are calculated by means of the susceptibilities of nonlinear processes involved in the interaction among pump and Stokes waves. Because of the crystal symmetry, third order nonlinearities in bulk silicon still exist. The Raman–resonant susceptibility can be written as [19]

$$
\chi^R(\omega_1; \omega_p - \omega_s, \omega_s) = \frac{2\Omega_R\Gamma_R\xi_R}{2j\Gamma_R\Delta \omega + \Omega_R^2 - \Delta \omega^2},
$$

where $\xi_R = 11.2 \times 10^{-18}$ (m$^2$ V$^{-2}$) [19] is the Raman susceptibility when $\Delta \omega = \Omega_R$, $\Gamma_R = 2\pi \times 53$ GHz is the resonance half-width, $\Omega_R = 15.6$ THz (Raman frequency shift) and $\Delta \omega = \omega_p - \omega_s$ is the frequency shift between pump and fundamental Stokes input waves. The other contributions to the third order nonlinearity in silicon are due to both electronic resonant cavity group velocity.

The overall quality factor of the resonant cavity is given by $1$ the decay process for the pump and Stokes photons, as induced by this coupling. Finally, the TPA effect is taken into account by considering the TPA susceptibility as $j\chi^{(TPA)} = \beta^{(TPA)} c n_{eff,g,p}/(\Delta \omega)$, with $\beta^{(TPA)} = 0.5$ cm GW$^{-1}$ the TPA coefficient [15]. In general, since the mutual interactions between pump and Stokes waves depend on Raman gain, pump depletion, walk-off effect, SPM, XPM, TPA and plasma dispersion, all these contributions have to be included in the nonlinear induced polarization, $\mathbf{P}^{NL}$.

Moreover, $\mathbf{J}$ has been included in the wave equation (2) to take into account the other cavity losses. It is expressed in terms of the fictional conductivity $\sigma$ as $\mathbf{J} = \sigma \mathbf{E}$, where $\sigma$ is assumed to be the sum of two contributions, $\sigma_l$ and $\sigma_c$. The former includes all losses induced by the resonant cavity (such as propagation, bending and radiation loss induced by sidewall roughness) on the optical mode. It is related to the loss quality factor $Q_l$ of the cold cavity by means of the relationship $\sigma_l = (\varepsilon \omega p s)/Q_{p,s}^l$, where $\varepsilon$ is the cavity permittivity. This loss quality factor is found as $1/Q_i^l = 1/Q_i^{l,prop} + 1/Q_i^{l,bend} + 1/Q_i^{l,scatt} + \cdots$, where $Q_i^{l,prop}$, $Q_i^{l,bend}$ and $Q_i^{l,scatt}$ designate the contributions related only to propagation, bending or scattering loss, respectively. Further, $Q_i^l$ can be also given as a function of the overall linear loss coefficient ($\sigma_{loss}$) as $Q_i^l = \omega_i \tau_i$, $\tau_i = 1/(\sigma_{loss} v_{g,i})$ being the decay time related to losses and $v_{g,i}$ the wave group velocity.

The contribution $\sigma_c$ to fictional conductivity is included to take into account the power leaving the cavity due to the coupling with the external waveguide. It is defined by $\sigma_c = (\varepsilon \omega p s)/Q_{c,p,s}^c$, where $Q_{c,p,s}^c$ represents the quality factor only related to the waveguide-cavity coupling process. It has been evaluated as $Q_i^c = \omega_{p,s} \tau_{c,(p,s)}$, where $\tau_{c,(p,s)}$ is the time constant of the decay process for the pump and Stokes photons, as induced by this coupling. Finally, the overall quality factor of the resonant cavity is given by $1/Q_i = 1/Q_i^l + 1/Q_i^c$.

Using the phenomenological relation between induced polarization and electric field, substituting equation (1) into equation (2) and after some algebraic manipulations, we obtain the following set of partial differential equations for the pump and Stokes pulses inside the resonant cavity

$$
v_{g/p} \frac{\partial A_p}{\partial \xi} + \frac{\partial A_p}{\partial t} = j(\omega_{p,\tilde{m}} - \omega_p) A_p - \frac{1}{2} \varepsilon_p A_p - \frac{1}{2} v_{g,p}^{(FCA)} A_p - 0.5 v_{g,p}^{(TPA)} f_{p,p} |A_p|^2 A_p
$$

$$
+ j v_{g,p} f_{p,p} |A_p|^2 A_p + j 2 v_{g,p} f_{s,s} |A_s|^2 A_p + j v_{g,p} \frac{2\pi}{A_p} \Delta n_p A_p - \frac{1}{2} v_{g,p}^{(GR)} f_{p,s} \frac{\omega_p}{\omega_s} |A_s|^2 A_p + \xi_p S_p,
$$

(3)
In equations (3) and (4), the group velocity dispersion (GVD) is not included as second order time derivative, since this effect is negligible in SOI micro-scale structures [34]. Moreover, the power transfer to higher order Stokes waves is also negligible for short interaction lengths and relatively large values of $T_{\text{FWHM}}$, as used in this study. However, our model can easily take into account the power coupling to higher order Stokes waves, as a generalization of the model in [34]. The coefficients in the system (3) and (4) are defined as

$$ \gamma_{i,i} = \frac{\omega_i}{2cn_{\text{eff},i}} f_{i,i} (\chi^\text{NR}_{xxx} + \chi^R (0)) ; \quad \gamma_{i,j} = \frac{\omega_i}{2cn_{\text{eff},i}} f_{i,j} (\chi^\text{NR}_{xxyy}) \quad i \neq j, $$

where $i, j = p, s$. The overlap integrals $f_{i,j}$ are given by

$$ f_{i,j} = \frac{\iint |F_i(x, y)|^2 |F_j(x, y)|^2 dx dy}{\iint |F_i(x, y)|^2 dx dy \iint |F_j(x, y)|^2 dx dy}, \quad i, j = p, s. $$

In particular, $f_{i,i}^{-1} = A_{\text{eff},i}$ represents the effective core area of the optical mode relevant to the $i$th pulse, $i = p, s$, determining the efficiency of any nonlinear device in SOI technology. Moreover, $n_{\text{eff},i}$ is the mode effective index for either pump or Stokes wave, $i = p, s$. In equations (3) and (4), $g_R = 4\omega_s \chi^R (\Omega_R)/(cn_{\text{eff},s})$ is the Raman gain. In particular, $+g_R$ designates the SRS effect, while $-g_R$ determines the pump depletion occurring together with the TPA effect (see term in $\beta^{(\text{TPA})},g_{s,p}$). Finally, the coefficients $\gamma_{i,i}$ and $\gamma_{i,j} (i \neq j)$ take into account the SPM and XPM effects as induced by Kerr nonlinearity, respectively.

The partial differential equation system (3) and (4) clearly represents a generalization of the model for a simple SOI straight waveguide [34]. This aspect is mainly evident observing the first, second and last term on the right hand of equations (3) and (4). In fact, $(\omega_{p,m} - \omega_p)$ and $(\omega_{s,q} - \omega_s)$ indicate the mismatch from the resonance condition of input pump and Stokes probe frequency, respectively. The term $\tau_p$ ($\tau_s$) represents the overall photon decay time of the pump (Stokes) pulse inside the cavity. It is related to the overall resonator quality factor by means of the relationship $Q_m = \omega_p \tau_p$, where $\tau_p$ is given by

$$ \frac{1}{\tau_p} = \frac{1}{\tau_{l,p}} + \frac{1}{\tau_{c,p}}, $$

being the two contributions related to loss ($\tau_{l,p}$) and coupling ($\tau_{c,p}$) time constants. A similar definition holds for $\tau_s$. Moreover, the coefficients $\xi_p$ and $\xi_s$ are related to the power fraction transferred into the resonator from the input pump ($S_p$) and input Stokes probe ($S_s$) as: $\xi_p = \sqrt{\frac{\kappa_{p,s}^2}{\kappa_{p,s}^2 \tau_{c,p}}} \frac{1}{\ell_p}$ and $\xi_s = \sqrt{\frac{\kappa_{p,s}^2}{\kappa_{p,s}^2 \tau_{c,s}}} \frac{1}{\ell_p}$, $v_{g,p}$ and $v_{g,s}$ being the pump and Stokes group velocities, respectively. For convenience, it is useful to write these coefficients in terms of the coupling factor $\kappa_p^2 (\kappa_s^2)$, defined as the power fraction of the input pump (Stokes) pulse injected into the resonator as coming from the bus waveguide. It is possible to demonstrate [33] that the coupling factor

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\( \kappa_p^2 (\kappa_s^2) \) is related to the coupling time constant \( \tau_{c,p} (\tau_{c,s}) \) by means of the following:

\[
\kappa_p^2 = \frac{1}{\tau_{c,p}} \frac{L_{\text{cavity}}}{v_{g,p}}, \quad (6a)
\]

\[
\kappa_s^2 = \frac{1}{\tau_{c,s}} \frac{L_{\text{cavity}}}{v_{g,s}}, \quad (6b)
\]

Moreover, in equations (3) and (4) \( \alpha_{i}^{(\text{FCA})} \), \( i = p, s \), is the contribution to the total losses due to FCA, as induced by the change of free carrier density generated mainly by TPA of pump pulse. We have evaluated \( \alpha_{i}^{(\text{FCA})} \) according to Soref’s relationship [35] as

\[
\alpha_{i}^{(\text{FCA})} = 8.5 \times 10^{-18} \left( \frac{\lambda_i}{1.55} \right)^2 \Delta N_e + 6.0 \times 10^{-18} \left( \frac{\lambda_i}{1.55} \right)^2 \Delta N_h = \sigma_i \times N_c = \sigma_0 \left( \frac{\lambda_i}{1.55} \right)^2 N_c,
\]

where \( N_c = \Delta N_e = \Delta N_h \) is the density of electron–hole pairs generated by TPA process. The coefficient \( \sigma_0 = 1.45 \times 10^{-17} \text{ cm}^{-2} \) [19] is the FCA cross-section measured at \( \lambda = 1.55 \mu \text{m} \), and \( \lambda_i \) is the relevant mode (pump or Stokes wave) wavelength.

In the system (3) and (4), \( \Delta n = -8.8 \times 10^{-22} (\lambda_i/1.55)^2 \Delta N_e - 8.5 \times 10^{-18} (\lambda_i/1.55)^2 (\Delta N_h)^{0.8} \approx -1.66 \times \delta_i \times N_c \) [35] is the change of effective index due to plasma dispersion effect as induced by free carriers, being \( \delta_i = 8.8 \times 10^{-22} (\lambda_i/1.55)^2 \) [19].

Finally, in order to obtain a consistent mathematical model, the system (3) and (4) has been coupled to the rate equation governing the free carrier dynamics into the waveguide core, given by [15]

\[
\frac{dN_c}{dt} = -\frac{N_c}{\tau_{\text{eff}}} + \frac{\beta^{(\text{TPA})}}{2\hbar \omega_p} (|A_{p,p}(\zeta, t)|^2 f_{p,p})^2,
\]

where \( \tau_{\text{eff}} \) is the relevant effective recombination lifetime for free carriers, and \( \hbar \) is the reduced Planck constant.

Some physical comments can be derived from equations (3) and (4). Through the general formalism used, the walk-off effect (first-order time derivative) between pump and Stokes waves is considered inside the resonator. As demonstrated in [34], this effect is dominant in SOI waveguides for pulses with ultrashort FWHM time widths \( T_{\text{FWHM}} \approx 1 \text{ ps} \), where the walk-off length \( L_w = T_0 / (|v_{g,p}^{-1} - v_{g,s}^{-1}|) \) is typically shorter than the waveguide length \( (T_0 = T_{\text{FWHM}}/1.665 \text{ for a Gaussian pulse}) \). Then, SRS is limited by the group velocity mismatch and occurs only over distances \( z \sim L_w \), even when the propagation length is much larger than \( L_w \). On the contrary, in the case of SRS effect induced in the resonant microcavity, the limiting factor for the process should not be the group velocity mismatch, but the increase of pump energy inside the resonator, i.e., the enhancement factor of the pump pulse. It can be defined by

\[
\Gamma = \frac{|A_p|}{S_p},
\]

where \( A_p \) is the maximum of the pump amplitude inside the cavity and \( S_p \) is the input pump peak, outside the cavity.
Therefore, the design of the architecture of figure 1 requires the condition $\Gamma \gg 1$, in order to achieve better performance with respect to a simple straight waveguide, in terms of lower threshold and smaller sizes. As a first step, the enhancement factor of the pump under pulsed excitation can be estimated by considering a simple linear coupling mechanism, i.e., by neglecting all nonlinear effects in equation (3). Thus in a first approximation, we determine the influence of the pulse width, coupling factor and resonator sizes on the pump enhancement factor by solving the simplified equation derived from equation (3)

$$\frac{dA_p}{dt} = j(\omega_p - \bar{\omega}_m)A_p - \frac{1}{2} \frac{1}{\tau_p} A_p + \xi_p S_p,$$

whereas an observer moving with the optical wave is considered. Results are presented in the next section.

3. Numerical results and discussion

3.1. CW laser operation

In several applications, it is important to use the SRS for CW laser operation. From the system of equations (3), (4)–(7), we can observe that a quasi-CW regime occurs if the input pump time width $T_{\text{FWHM}}$ in the bus waveguide satisfies the condition: $T_{\text{FWHM}} \gg \max(\tau_p, \tau_{\text{eff}})$. Since $\tau_p$ is of the order of a few tens of picoseconds in most cases, it must be that $T_{\text{FWHM}} \geq 100$ ps and thus $\tau_{\text{eff}}$ becomes the dominant factor for the quasi-CW regime. Moreover, a large $T_{\text{FWHM}}$ means that the group-velocity mismatch does not influence the SRS process for microcavities. Then, we can assume that the pump and the Stokes waves travel with the same velocity inside the resonator, and thus the partial differential equations (3) and (4) can be transformed into ordinary differential equations by writing [36]

$$\frac{dA_{p,s}}{dt} = \frac{\partial A_{p,s}}{\partial t} + v_g \frac{\partial A_{p,s}}{\partial \varsigma},$$

where $v_{g,p} \cong v_{g,s} = v_g$.

The goal of this subsection is to determine the threshold condition for the SRS effect in the resonator under quasi-CW regime. In this hypothesis, a number of considerations can be made on the equation system (3)–(4)–(7) under assumption (9). In fact, quasi-CW regime means a steady-state analysis, where the threshold condition for SRS effect leads to neglect of the terms depending on Kerr and plasma dispersion effects. In particular, SPM and XPM effects can be considered under threshold when the Raman effect is assumed at the threshold. In fact, Raman dominates over Kerr effect in SOI technology. Finally, in order to minimize the input pump power at the threshold, we assume that the angular frequencies of the pump and the Stokes waves are both very close to resonance $(\omega_{p,s} \cong \omega_{\bar{m},\bar{q}})$. Therefore, the equation system (3)–(4)–(7) is simplified as follows

$$-\frac{1}{2} \frac{1}{\tau_p} A_p - v_g \frac{g_p}{2} |A_s|^2 A_p - \frac{1}{2} v_g \beta^{\text{TPA}} |A_p|^2 A_p - \frac{1}{2} v_g \alpha_{\text{FCA}}^p A_p + \xi_p S_p = 0,$$

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Therefore, equation (15) imposes a limitation on the total cavity decay time decreases by decreasing factor for the input pump of SRS threshold depend on both TPA and FCA effects. In addition, for a given coupling resonator. In fact, by substituting equation (13) into (10) and considering with \( \kappa \) relationship is verified in equation (11). Using the definition for terms \( \alpha_{\text{p}}^{\text{FCA}} \) and \( \alpha_{\text{s}}^{\text{FCA}} \), where the free carrier density is evaluated by equation (12), the double solution of equation (11) is given by

\[
A_{\text{p}} = \begin{cases} 
A_{\text{p},1} = \sqrt{\frac{v_{g,s} \left( g_s / 2 - \frac{\beta_{\text{TPA}}}{A_{\text{eff},p}} \right) + \sqrt{v_{g,s}^2 \left( g_s / 2 - \frac{\beta_{\text{TPA}}}{A_{\text{eff},p}} \right)^2 - \frac{v_{g,s} \sigma_s \eta}{\tau_s}}}{v_{g,s} \sigma_s \eta}} \\
A_{\text{p},2} = \sqrt{\frac{v_{g,s} \left( g_s / 2 - \frac{\beta_{\text{TPA}}}{A_{\text{eff},p}} \right) - \sqrt{v_{g,s}^2 \left( g_s / 2 - \frac{\beta_{\text{TPA}}}{A_{\text{eff},p}} \right)^2 - \frac{v_{g,s} \sigma_s \eta}{\tau_s}}}{v_{g,s} \sigma_s \eta}}
\end{cases}
\]  

with \( \eta = \frac{\tau_{\text{eff}} \beta_{\text{TPA}}}{2h \omega_{\text{p}}} \). In turn, the solution for the pump amplitude \( A_{\text{p}} \) includes two possible threshold levels for the input pump \( (S_{\text{p}}) \), in order to excite the Stokes wave by SRS in the microcavity resonator. In fact, by substituting equation (13) into (10) and considering \( A_{\text{p}} = 0 \) (threshold condition), we obtain

\[
P_{\text{th}} = \left| S_{\text{p}}^{\text{th}} \right|^2 = \begin{cases} 
\left| \frac{1}{\xi_{\text{p}}} \left( \frac{2A_{\text{p},1}}{\tau_{\text{p}}} + \frac{v_{g,p} \beta_{\text{TPA}}}{2A_{\text{eff},p}} |A_{\text{p},1}|^2 A_{\text{p},1} + \frac{v_{g,p} \sigma_p \eta}{2} |A_{\text{p},1}|^4 \right) \right|^2 = P_{\text{th},1}, \\
\left| \frac{1}{\xi_{\text{p}}} \left( \frac{2A_{\text{p},2}}{\tau_{\text{p}}} + \frac{v_{g,p} \beta_{\text{TPA}}}{2A_{\text{eff},p}} |A_{\text{p},2}|^2 A_{\text{p},2} + \frac{v_{g,p} \sigma_p \eta}{2} |A_{\text{p},2}|^4 \right) \right|^2 = P_{\text{th},2}.
\end{cases}
\]

Several considerations can be derived from equations (13) and (14). First, the two values of SRS threshold depend on both TPA and FCA effects. In addition, for a given coupling factor for the input pump \( \kappa_{\text{p}}^2 \), the upper threshold \( (P_{\text{th},1}) \) increases while the smaller level \( (P_{\text{th},2}) \) decreases by decreasing \( \kappa_{\text{p}}^2 \). In contrast, for a given value of \( \kappa_{\text{p}}^2 \), both threshold levels increase by decreasing \( \kappa_{\text{p}}^2 \).

Finally, considering a real field amplitude of the input pump \( S_{\text{p}} \), i.e., both real solutions \( A_{\text{p},1} \) and \( A_{\text{p},2} \), the SRS effect in the resonator can have a finite threshold only if the following relationship is verified

\[
v_{g,s}^2 \left( g_s / 2 - \frac{\beta_{\text{TPA}}}{A_{\text{eff},p}} \right)^2 - \frac{v_{g,s} \sigma_s \eta}{\tau_s} \geq 0.
\]  

Therefore, equation (15) imposes a limitation on the total cavity decay time \( \tau_s \) for a Stokes wave. In particular, it leads to individuating the range for \( \kappa_{\text{p}}^2 \) needed to find the SRS effect threshold.
into the microcavity resonator. In fact, using equations (6b), (15) can be re-written as

$$\kappa_s^2 < \frac{L_{\text{cavity}}}{v_{g,s}} \left( \frac{\tau_{l,s} - \tilde{\tau}}{\tau_{l,s} \tilde{\tau}} \right) \quad \text{if} \quad \tau_{l,s} > \tilde{\tau}, \quad (16)$$

where $\tilde{\tau} = \eta/\tau_{l,s}(v_g / 2 - \beta_{\text{TPA}} (A_{p,l})^2)$. The term $\tilde{\tau}$ represents the time constant related to the nonlinear effects, including SRS, TPA and FCA. Thus equation (15) or similarly equation (16), states that the Stokes photon decay rate related to coupling has to assume an appropriate value to guarantee that the photon generation rate induced by SRS effect can compensate the overall photon decay rate (due to linear losses, FCA and coupling processes).

Then, the Raman emission power can be evaluated by equations (10) and (13) inside a microcavity having $L_{\text{cavity}} < L_{\text{NL}}$, where $L_{\text{NL}} = 1/(|A_p|^2 \gamma_{p,l})$ represents the (nonlinear) length over which SPM and XPM effects become important. After straightforward algebra, we obtain

$$P_s = |A_s|^2 = \xi_s \left( \sqrt{P_{\text{inc}}} - \sqrt{P_{\text{th},i}} \right) \frac{2}{v_{g,p} g_{p} A_{p,l}^2}, \quad i = 1, 2, \quad (17)$$

where $P_{\text{inc}} = |S_p|^2$ is the pump power injected into the bus waveguide. It is evident that we can calculate two levels of emitted Raman power, depending on the considered threshold.

According to a worst case scenario (higher pump power), assuming $|A_p|^2$ of the order of Watts, equation (17) holds since $L_{\text{NL}} \sim 9 \text{ m}$, i.e., it is applicable to a very large number of experimental cases. Moreover, the laser Stokes power calculated at the bus waveguide end is given by

$$P_{\text{out}} = \kappa_s^2 P_s e^{-\alpha_s L_{\text{out}}}, \quad (18)$$

where $\alpha_s$ is the linear propagation loss inside the bus waveguide for Stokes wave, and $L_{\text{out}}$ represents the waveguide length from the coupling region to the end (see figure 1). Finally, the laser external efficiency $\eta_{\text{ex}}$ is evaluated by linearizing the previous expression for the input pump power $P_{\text{inc}} = |S_p|^2$ close to the threshold condition, as

$$\eta_{\text{ex}} = \frac{d P_{\text{out}}}{d (P_{\text{inc}})} = e^{-\alpha_s L_{\text{out}}} \kappa_s^2 \xi_p \frac{2}{g_{p} v_{g,p} A_{p,l}^2} \frac{1}{2 \sqrt{P_{\text{th},i}}}, \quad i = 1, 2. \quad (19)$$

Differently from Raman lasers based on a Fabry–Perot cavity, equation (17) demonstrates that the output Stokes power presents a square root dependence from the input pump power, as a consequence of the pump-to-Raman conversion influenced by the coupling between resonator and waveguide (see $\xi_p$ in equation (10)). In addition, the previous equations show that, by increasing $\kappa_s^2$, both output power and external efficiency increase at the expense of a larger threshold level, i.e., $P_{\text{th},2}$. Thus equations (14)–(18)–(19) are very useful relationships to find the best trade-off among different physical parameters, with the aim to design efficient CW Raman lasers based on SOI microcavity resonators.

To test the analytical formulae and the physical assumptions for the Raman effect induced in microcavities under CW operation, we have compared our numerical results with some experiments proposed in the literature. First, we have examined the Raman effect in toroidal and spherical resonators based on silica technology [33] for the case of a toroidal microcavity.
with a radius \( R = 30 \, \mu \text{m} \) and microcavity loss quality factor measured as \( Q^1 = 10^8 \). The quasi-TE input pump wave is launched at the wavelength 1.550 \( \mu \text{m} \). Of course, to compare our results with the measurements in [33], equations (14)–(17) have been applied to the silica case by setting: \( \beta^{(\text{TPA})} = 0 \), \( g_R = 10^{-13} \, \text{m W}^{-1} \) and \( \Delta \omega = 13.2 \, \text{THz} \). It is evident that, if the TPA effect vanishes, FCA also tends to zero and, thus the two thresholds degenerate in only one level. In addition, the chromatic dispersion of the silica refractive index \( n(\omega) \) has been approximated by the Sellmeier equation [37]. The Raman emissions depending on input pump power as measured [33] or calculated by our model at the end of the input waveguide have been compared, showing very good agreement. For instance, the threshold is evaluated as 277.8 \( \mu \text{W} \) in comparison with measured value of 234.5 \( \mu \text{W} \), while the external efficiency is calculated as 49\% (instead of 45\%). It is important to note that the same coupling conditions as used in [33] have been assumed for this comparison, so no arbitrary choice or any model fitting parameter was needed.

A very interesting set of comparisons with experimental results involves the CW Raman laser based on an SOI resonator. The architecture used in the experimental setup proposed in [38] is equal to that in figure 1, where the microcavity consists of an SOI racetrack resonator. The device proposed has a rib width \( W = 1.5 \, \mu \text{m} \), height \( H = 1.55 \, \mu \text{m} \), and rib etch depth equal to 0.7 \( \mu \text{m} \). The total length of the racetrack cavity is 3 cm and the bend radius is 400 \( \mu \text{m} \). Finally, the measured optical parameters include \( \alpha_{\text{loss,p}} = \alpha_{\text{loss,s}} = \alpha_{\text{loss}} = 0.6 \pm 0.1 \, \text{dB cm}^{-1} \), \( \tau_{\text{eff}} = 1 \, \text{ns} \), \( g_R = 9.5 \, \text{cm GW}^{-1} \) and \( \beta_{\text{TPA}} = 0.5 \, \text{cm GW}^{-1} \).

In the experiments [38], Raman laser devices with different coupling factors for pump and Stokes waves were considered. Thus figure 3 shows the laser output power \( P_{\text{out}} \) versus \( P_{\text{inc}} \) (pump input power) for three devices characterized by different \( \kappa_p^2 \) and \( \kappa_s^2 \). The markers represent the experimental data, the solid lines designate our numerical results as evaluated by means of equation (18). The numbers in the parenthesis indicate the experimental coupling factors for pump and Stokes waves at 1550 and 1686 nm, respectively. We have to focus on two fundamental aspects. Firstly, the upper threshold value \( (P_{\text{th,1}}) \) assumes a level too high to be practical because of the large racetrack length and, secondly, equation (16) does not impose any limitation on the coupling factor \( \kappa_p^2 \) to be used. The same coupling conditions as in [38] have been assumed for these comparisons, so no arbitrary fitting parameters were used.

The plots show very good agreement with the experimental data in terms of threshold values, output powers above threshold, and external efficiencies. This comparison has been summarized in table 1 in terms of measured and simulated threshold values. As shown in table 1, our formula leads to evaluation of the threshold power \( (P_{\text{th,2}}) \) with high accuracy, the percentage error being significantly smaller than in the model proposed in [38]. In addition, different to the mathematical approach followed in [38], our model allows estimation of the output power and the external efficiency by means of equations (18) and (19), in the case of CW Raman lasers based on a microcavity with \( L_{\text{cavity}} < L_{\text{NL}} \). This is possible essentially because equation (9) allows the nonlinear interaction between pump and Stokes waves inside the microcavity to be investigated in the time domain, instead of the space domain.

It is important to outline that these experimental devices cannot show double threshold levels for the input pump. In fact, the toroidal cavity based on silica technology does not experience any TPA effect. On the other hand, the SOI racetrack resonators in experiments [38] have too large cavity perimeters to induce a double solution in equation (14).

Therefore, we have investigated the CW Raman threshold for micro-scale SOI cavity resonators. Hereinafter, microring resonators will be often considered as laser resonant microcavities. In particular, three ring resonators have been considered with ring radius \( R = 10 \),
Figure 3. Comparison between experimental data and modelling of this paper in terms of CW Raman laser output power versus input pump power for three racetrack resonators.

Table 1. Comparison between measured [38] and calculated Raman laser thresholds.

| Device #   | Measured $P_{th}$ [38] (mW) | Calculated $P_{th}$ (this paper) (mW) | $\Delta$ (%) | Calculated $P_{th}$ [38] (mW) | $\Delta$ (%) |
|------------|-----------------------------|-------------------------------------|--------------|--------------------------------|--------------|
| Device #1  | 231                         | 245.45                              | 6.25         | 256                            | 10.82        |
| Device #2  | 208                         | 210.17                              | 1.04         | 211                            | 1.44         |
| Device #3  | 170                         | 174.52                              | 2.66         | 162                            | 4.70         |

15 and 20 $\mu$m, respectively. In order to apply equation (14), it is essential to estimate both pump and Stokes coupling factors. To this aim, a number of simulations have been carried out by means of the finite difference time domain (FDTD) method [39], and results are summarized in table 2. The pump and Stokes wavelengths have been assumed to be equal to 1433.2 and 1586.1 nm, respectively. As is evident in table 2, for each value of ring radius $R$ and gap $G$, $\kappa_s^2 > \kappa_p^2$ always results.

The physical reason for this behaviour is related to the different refractive index contrast between cover and guiding structure (larger for the pump than for the Stokes wave). Moreover, coupling to micro-size ring resonators is not negligible only in the region of smallest separation. This means that, for such small structures, the coupling length cannot be a design parameter as it usually is in large-scale racetrack resonators [38].

Using the simulation results in table 2, the threshold conditions have been found. In figure 4 the input threshold pump power is sketched as a function of Stokes coupling factor $\kappa_s^2$, for different ring radii, assuming $\beta^{(\text{TPA})} = 0.5 \text{ cm GW}^{-1}$, $g_R = 10.5 \text{ cm GW}^{-1}$, $\alpha_{\text{loss, p}} = \alpha_{\text{loss, s}} = \alpha_{\text{loss}} = 0.6 \text{ dB cm}^{-1}$, $\tau_{\text{eff}} = 1 \text{ ns}$. Hereinafter, calculations of field distributions, effective indices and modal areas were carried out by the full-vectorial finite element method (FEM) [40].
Table 2. FDTD results for pump (1433.2 nm) and Stokes (1586.1 nm) coupling factors.

| G (µm) | \( R = 10 \mu m \) | \( R = 15 \mu m \) | \( R = 20 \mu m \) |
|--------|-----------------|-----------------|-----------------|
| 0.1    | \( \kappa_p^2 \) (%) | \( \kappa_s^2 \) (%) | \( \kappa_p^2 \) (%) | \( \kappa_s^2 \) (%) | \( \kappa_p^2 \) (%) | \( \kappa_s^2 \) (%) |
| 0.2    | 19.32           | 29.54           | 28.05           | 41.97           | 46.57           | 62.67           |
| 0.25   | 3.41            | 4.55            | 3.66            | 7.41            | 6.94            | 10.67           |
| 0.3    | 1.43            | 1.78            | 1.32            | 3.11            | 2.67            | 4.40            |
| 0.35   | 0.6             | 0.7             | 0.47            | 1.30            | 1.03            | 1.81            |
|        | 0.25            | 0.27            | 0.17            | 0.55            | 0.4             | 0.75            |

Figure 4. Input threshold pump power versus Stokes coupling factor for various ring radii.

The plots clearly demonstrate that two values of laser threshold still exist for each ring radius, namely one upper and one lower. This is true only if \( \kappa_s^2 \) satisfies equation (16), i.e., when both TPA and FCA effects induce two possible solutions \( (A_{p1}, A_{p2}) \) for the field amplitude \( A_p \) (see equation (13)). The two arms of the curves in figure 4 merge at one limit point, around 0.01 W. Thus the SRS effect can be excited in CW regime into the microring cavity only if the coupling factor \( \kappa_s^2 \) is not larger than a critical value, depending on the ring radius. Finally, the figure indicates that the upper threshold value decreases a little with decreasing \( R \), as a result of the improved pump enhancement factor.

Finally, the input threshold pump power is sketched in figure 5 as a function of Stokes coupling factor \( \kappa_s^2 \), for different propagation loss coefficients, assuming \( \beta^{(TPA)} = 0.5 \text{ cm GW}^{-1} \) \( g_R = 10.5 \text{ cm GW} \), \( R = 20 \mu m \), \( \tau_{\text{eff}} = 1 \text{ ns} \). It is possible to observe that both arms of the curves shift towards higher power values by increasing \( \alpha_{\text{loss}} \), while the limit point shifts towards lower values of \( \kappa_s^2 \). This latter effect indicates that too large propagation loss coefficients do not allow an SRS threshold to be induced for each value of \( \kappa_s^2 \). Thus our calculations show that the SRS effect cannot be obtained when \( \alpha_{\text{loss}} \geq 5.8 \text{ dB cm}^{-1} \).
3.2. Cavity enhancement factor

As a first step, we have investigated the cavity enhancement factor in the linear regime by solving equation (8). As is known, the enhancement factor $\Gamma$ assumes a monotonically decreasing shape versus $R$ for each value of $T_{\text{FWHM}}$ and for a given value of $\kappa_p^2$. In particular, assuming $\kappa_p^2 = 3\%$, $T_{\text{FWHM}} = 1$ ps and $R = 5 \mu$m, the power enhancement factor falls to 0.66, while it is $\Gamma \geq 10$ for $T_{\text{FWHM}} = 100$ ps and $R < 12 \mu$m. Further improvements can be obtained by changing $\kappa_p^2$. In fact, our calculations show that the shape of $\Gamma$ versus $\kappa_p^2$ presents an absolute maximum, and this peak shifts towards a smaller value of $\kappa_p^2$ with increasing $T_{\text{FWHM}}$. Thus the best operation region for micro-scale ring resonators in the linear regime requires pump pulses with $T_{\text{FWHM}} \geq 100$ ps and coupling factors $\kappa_p^2$ in the range 1\%–5\%.

As will be clear in the following, the calculations in the linear regime can be considered a good approximation of the pump enhancement factor even in the presence of the nonlinear interaction between pump and Stokes pulses, if FCA is negligible and $T_{\text{FWHM}} \ll \tau_{\text{eff}}$. However, the condition $T_{\text{FWHM}} \geq 100$ ps gives an increase of $\Gamma$ even in the presence of the FCA effect. Moreover, using $T_{\text{FWHM}} \geq 100$ ps and $\lambda_p$ around 1.55 $\mu$m, the walk-off length is of the order of $L_w \sim 3.3$ m. This means that the group velocity mismatch does not influence again the SRS process for microcavity resonators. Then, we can still assume that pump and Stokes waves travel together with the same velocity inside the cavity, thus confirming the applicability of equation (9).

To point out the positive influence of a large pump enhancement factor, we compare the cavity resonator with respect to the simple straight waveguide. To this aim, our results (solution of the equation system (3)–(4)–(7) under condition (9)) have been compared with some numerical and experimental results proposed in the literature for SOI waveguides with large cross-section [15]. Of course, the model given by equation (3) and (4) can be still used to describe the behaviour of a Raman straight waveguide when $R \rightarrow \infty$, $\xi_p = \xi_s = 0$, $\omega_{p,m} = \omega_p$ and $\omega_{s,q} = \omega_s$, in the presence of different boundary conditions (see also [34]).

**Figure 5.** Input threshold pump power versus Stokes coupling factor for various propagation loss coefficients.
Figure 6. Comparison between waveguide and racetrack resonator in terms of net Raman gain versus pump peak intensity (experimental points and numerical results).

The SOI waveguide proposed in [15] has rib total height \( H = 1.45 \, \mu m \), rib width \( W = 1.52 \, \mu m \) and slab height \( H_s = 0.82 \, \mu m \). The pump pulse is launched at the wavelength 1.545 \( \mu m \) with \( T_{\text{FWHM}} = 17 \, \text{ns} \) and the Stokes probe signal has a peak power of 2 mW at 1.680 \( \mu m \) (according to Raman shift). The pump and Stokes signals interact for a length \( L = 4.8 \, \text{cm} \) and are aligned with quasi-TE and quasi-TM polarized fundamental modes, respectively.

In order to achieve a consistent comparison using the architecture shown in figure 1, we have assumed the same values as in [15] for \( T_{\text{FWHM}} \), input pump and input probe wavelengths, peak power of input probe and, in particular, for the effective recombination lifetime of free carriers, \( \tau_{\text{eff}} \). Since it is directly proportional to the optical mode width [23], the estimated value \( \tau_{\text{eff}} \sim 25 \, \text{ns} \) is obtained using a cavity resonator with the same cross-section as in the straight waveguide [15]. This induces consideration of a racetrack resonator in the architecture of figure 1 as the best choice to realize our comparison. In addition, we have assumed the same cross-section for the external rib waveguide to achieve technological compatibility and improve the coupling with the resonator. Thus the structure used in the simulation consists of a racetrack resonator with a total length of \( L_{\text{cavity}} = 0.3770 \, \text{cm} \) and bend radius of 50.28 \( \mu m \), gap \( G = 0.7 \mu m \), and coupling length \( L_{\text{coup}} = 191.5 \, \mu m \).

Simulations performed by means of the beam propagation method (BPM) [41] have given the following coupling factors: \( \kappa_s^2 = 17.55\% \) and \( \kappa_p^2 = 10\% \).

Figure 6 shows the net Raman gain as a function of the input pump peak intensity. In particular, asterisks represents the experimental data measured in [15] for the straight waveguide, the black line designates the numerical results obtained by our rigorous mathematical model proposed in [34] for SOI waveguides, and the blue line represents the numerical solution given by equations (3)–(4)–(7). For each case, we have assumed \( \beta_{\text{TPA}} = 0.5 \, \text{cm GW}^{-1} \), \( g_R = 10.5 \, \text{cm GW}^{-1} \) and \( \alpha_{\text{loss}} = 0.22 \, \text{dB cm}^{-1} \) [15].

Before any comment on the physical features derived from figure 6, some considerations must be emphasized about the definition of net Raman gain used in the case of SRS effect inside the microcavity. By using the definition for straight waveguides [15]–[34], the net Raman gain
in the presence of the microcavity should become

\[ G_{\text{net}} = 10 \log |A_s/A_s^L|^2, \quad (20) \]

where \( A_s \) represents the Stokes field amplitude inside the microresonator (solution of equation system (3)–(4)–(7) with assumption (9)), while \( A_s^L \) represents the solution of equation (4) when all nonlinear and loss contributions are neglected. Thus equation (20) leads to calculation of the Raman amplification as separated from the enhancement effect experienced by the input probe signal \( (S_s) \).

Thus definition (20) allows one to compare in a consistent way the SRS effect as induced in microresonators and straight waveguides, as shown in figure 6. The plot shows that the racetrack resonator with \( L_{\text{cavity}} = 0.3770 \text{ cm} \) and peak pump intensity larger than 45 MW cm\(^{-2} \), induces a saturation net gain similar to the waveguide case, but with an interaction length approximately 13 times smaller. In addition, figure 6 emphasizes that the SRS effect in microcavity induces a linear net gain of about 3.2 dB, three times larger than the value measured in the SOI waveguide proposed in [15]. In turn, this gives a threshold for the input pump of 0.46 MW cm\(^{-2} \) in the racetrack resonator, considerably lower than 6.74 MW cm\(^{-2} \) as measured in [15]. Thus the advantages obtainable with the microresonator in terms of reduced threshold and sizes are evident, as due to a pump enhancement factor \( \Gamma \gg 1 \). However, our calculations show that the \( \Gamma \) value is smaller than that estimated by solving equation (8). In fact, in this case the pulse width and free carrier recombination lifetime are both of the same order of magnitude, so inducing the FCA effect induced by TPA to become dominant, and increasing the energy depletion inside the resonator. Our simulations have been carried out on a very large number of points, although all figures show only a few calculated points for clarity.

The influence of the interaction length and FCA on \( \Gamma \) can be better understood by considering the simulations presented in figures 7 and 8. Figure 7 shows the net Raman gain as a function of \( T_{\text{FWHM}} \) for various resonator sizes. In particular, we have considered three racetrack cavities with \( L_{\text{cavity}} = 0.0628, 0.1885 \text{ and } 0.3770 \text{ cm} \) and bend radii 8.4, 25.14 and 50.28 \( \mu \text{m} \), respectively.

For comparison, figure 7 also shows the net Raman gain for three straight waveguides with length \( L_{\text{WG}} \) equal to the racetrack perimeters, \( L_{\text{cavity}} \). The simulations for straight waveguides have been performed using the rigorous mathematical model proposed in [34]. For both racetrack resonators and straight waveguides we have assumed \( H = 1.45 \mu \text{m}, W = 1.52 \mu \text{m}, H_s = 0.82 \mu \text{m}, \lambda_p = 1.545 \mu \text{m} \) and \( \lambda_s = 1.680 \mu \text{m} \). We have set the Stokes probe signal and pump pulse with a peak power of 2 mW and 0.4 W, respectively, and \( \tau_{\text{eff}} = 25 \text{ ns} \). Coupling factors, coupling length and gap hold the same values as before, i.e., \( \kappa_p^2 = 17.55\%, \kappa_s^2 = 10\% \), \( L_{\text{coup}} = 191.5 \mu \text{m} \) and \( G = 0.7 \mu \text{m} \). Finally, it is important to outline that the relatively large cavity perimeter means having a very small FSR, i.e., \( \lambda_p \) and \( \lambda_s \) matched to the resonant wavelengths of the racetrack cavities.

Different to straight waveguides, figure 7 indicates that the curves do not have a monotonically decreasing shape for each value of \( L_{\text{cavity}} \), but they present an absolute maximum for low values of \( T_{\text{FWHM}} \), depending on peaks in the enhancement factor shape (see figure 8). Moreover, these peaks shift toward higher values of \( T_{\text{FWHM}} \) with increasing \( L_{\text{cavity}} \). In fact, larger values of \( L_{\text{cavity}} \) extend the range of \( T_{\text{FWHM}} \) where the detrimental effect induced by FCA can be compensated, although partially. The net Raman gain monotonically decreases with increasing \( T_{\text{FWHM}} \), with an approximately exponential shape as a result of the strong FCA influence (see figure 7). Moreover, we have found that for \( L_{\text{cavity}} = 0.0628 \) and 0.1885 cm a value of \( T_{\text{FWHM}} \)
exists (not represented in the plot), where the net Raman gain becomes negative (SRS effect under threshold). This occurs because the energy depletion inside the cavity from FCA induces a pump enhancement factor which is too low to enforce the pulse along such a short interaction length. Anyway, figure 7 clearly emphasizes that it is always possible to find a range of $T_{\text{FWHM}}$ where the enhancement factor is improved and, thus the Raman net gain is considerably larger than in straight waveguides, even for short interaction lengths.

Figure 8 shows the relevant $\Gamma$ versus $T_{\text{FWHM}}$ for different racetrack lengths, obtained by solving the system (3)–(4)–(7) with assumption (9). For comparison, the figure also shows the
approximated solution for $\Gamma$ as obtained using equation (8). Figure 8 shows that this approximated value is significantly larger than the rigorous solution in any case and, further, no peaks can be observed in the approximated curve shapes. The approximated solutions are in good agreement with the rigorous calculations only in the short range 100–150 ps for $L_{\text{cavity}} = 0.0628$ cm, 100–200 ps for $L_{\text{cavity}} = 0.1885$ cm and 100–299 ps for $L_{\text{cavity}} = 0.3770$ cm. This means that FCA induced by the TPA effect is negligible in these ranges, i.e., no energy depletion is induced inside the microcavity. In fact, the influence of FCA becomes dominant only when $T_{\text{FWHM}}$ is of the same order of magnitude as $\tau_{\text{eff}}$, in this case equal to 25 ns (see figure 8). Thus the rigorous values present an absolute maximum and a slope change around 1.34 ns for $L_{\text{cavity}} = 0.0628$ cm, 1.97 ns for $L_{\text{cavity}} = 0.1885$ cm and 5 ns for $L_{\text{cavity}} = 0.3770$ cm. Both the position of the peak and its value essentially depend on the cavity length, pump depletion induced by TPA and SRS effects, and FCA. In contrast, the slope change of the curve is exclusively due to TPA.

3.3. Pulsed Raman amplification in high $Q$ microring cavities

The previous discussion outlines that the potential of the architecture proposed is enhanced if microcavity resonators with high quality factor $Q$ and large $\Gamma$ are used. Higher $Q$ factors can be obtained by reducing the total resonator losses (propagation, bending, scattering and so on), decreasing the coupling factor and increasing the ring radius. It is evident that the reduction of bending losses requires larger ring radii, which is a conflicting requirement with monolithic integration needs. Fortunately, the SOI technology leads to a solution for this problem. In fact, the high index contrast facilitates light guiding in micro-scale structures, thus making the bending loss in SOI microcavities either negligible or very small. In that case, microcavity losses are mainly limited by sidewall roughness scattering. At the same time, low values of the coupling factor and ring radius induce large pump enhancement factors, allowing low values of $\tau_{\text{eff}}$ to be realized and large pulse widths, $T_{\text{FWHM}} > 10$ ps, to be used. In fact, the last two conditions lead to a minimization of the FCA effect inside the resonator. The reduction of $\tau_{\text{eff}}$ can be carried out by an appropriate design of the resonator cross-section. A good estimation of $\tau_{\text{eff}}$ requires free carrier diffusion to be considered, in addition to the recombination lifetime. If diffusion carriers move out from the modal area, an effective lifetime shorter than the recombination lifetime in SOI structures, $\tau_e$, can occur. If $\tau_e$ is the transit time, then $1/\tau_{\text{eff}} = 1/\tau_t + 1/\tau_e$. It has been already demonstrated [23] that $\tau_t \sim 100$ ns depends on the surface-recombination velocity ($S = 10^3$ cm s$^{-1}$) at the interface between the top silicon and buried oxide. Transit time $\tau_t$ also depends on the optical mode size as $\tau_t = 0.5w\sqrt{H/(SD)}$, where $w$ is the optical mode width and $D$ is the ambipolar diffusion coefficient [23]. Thus a small cross-section is needed to obtain a low value for $\tau_{\text{eff}}$. We have considered a resonator with small cross-section, a few hundreds of squared nanometres. We have chosen the rib total height $H = 500$ nm ($\sim\lambda/3$), $r = H_o/H = 0.3$ and $W = 300$ nm, in order to achieve very high confinement in the rib and negligible bending losses. For these values, $\tau_{\text{eff}}$ is estimated as about 1 ns.

Figure 9 shows the net Raman gain versus pump coupling factor for different microcavities. In particular, three ring resonators with ring radius $R = 10$, 15 and 20 $\mu$m, respectively, are considered. In the simulations, we assume $T_{\text{FWHM}} = 100$ ps, $\beta^{\text{(TPA)}} = 0.5$ cm GW$^{-1}$, $g_R = 10.5$ cm GW$^{-1}$, $\alpha_{\text{loss}} = 0.6$ dB cm$^{-1}$, $|S_p|^2 = 0.4$ W and $|S_s|^2 = 2$ mW. The input pump wavelength is tuned at the same resonance wavelength for the ring microcavities ($\lambda_{p,m} = 1432.5358$ nm). The input Stokes probe is launched at the wavelength corresponding to the Raman shift with respect to the input pump. Under these conditions, the shift between input
Figure 9. Raman net gain versus pump coupling factor for various ring radii, under resonance or off-resonance conditions for Stokes pulse.

Probe and resonance wavelengths is estimated as 1.99 nm for \( R = 10 \) or 20 \( \mu m \) and 2.49 nm for \( R = 15 \mu m \), i.e., the Stokes pulse is not at resonance for the cavity (off-resonance conditions). Thus figure 9 shows the realistic case for off-resonance conditions, and the ‘ideal’ case where the Stokes wave is also assumed to be at resonance for the ring resonator, as the pump. The coupling factors \( \kappa^2_s \) and \( \kappa^2_p \) have been obtained by extracting the interpolating functions on a number of FDTD simulation points, a few of which are reported in table 2. The plots in figure 9 show clearly the detrimental effect induced by the mismatch between Stokes and resonant wavelengths in terms of reduced Raman gain for each \( \kappa^2_p \). This is due to the partial destructive interference that the Stokes wave experiences when travelling inside the microring. In addition, off-resonance conditions induce a decreasing shape of the net gain, with a weak ripple for values of \( \kappa^2_p \), where the ideal case assumes a flat shape.

Anyway, some significant physical aspects can be noted by examining the ideal case. The curve shape shows an absolute maximum whose coupling factor corresponds to critical coupling. As is well known [36], the coupling condition can be described in terms of the ratio \( K = \tau_0 / \tau_{c.p} \) between total loss decay time \( \tau_0 = [v_{gp}(\alpha_{loss,p} + \alpha_{FCA,p})]^{-1} \) and pump coupling decay time, \( \tau_{c.p} \). Following the standard conventions, under-coupling is denoted by \( K < 1 \), over-coupling by \( K > 1 \), and critical coupling by \( K = 1 \). This last condition corresponds to a vanishing transmitted pump at the bus waveguide end, and thus it represents the optimal conditions to maximize the Raman net gain and minimize the Raman threshold. In any case, it is clear that in our general approach \( \tau_0 \) is related to both linear propagation loss and FCA, different to the definition used in [33]. Thus an estimation of the maximum position from the relationship \( \tau_{1.p} = \tau_{c.p} \) is a first approximation, while only the rigorous numerical solution of the system (3)–(4)–(7) can determine the exact position. However, as will be clear in the following simulations, the critical coupling for the input pump pulse could not be the best way to excite the Stokes wave inside the resonator. In fact, this can be observed from figure 10, where the time evolutions of pump and Stokes waves and free carriers generated by TPA are sketched by assuming \( R = 20 \mu m \) and
\( \kappa_p^2 = 0.066\% \) (critical coupling). The value of \( \kappa_p^2 \) used in this simulation has been obtained by considering a gap between ring resonator and bus waveguide \( G = 0.444 \mu m \), corresponding to \( \kappa_s^2 = 0.14\% \) (found by FDTD simulations).

In figure 10, we have assumed that the input pump pulse \( |S_p|^2 \) and the input Stokes probe \( |S_s|^2 \) are applied with a delay of \( 2T_0 \approx 120 \text{ ps} \). From the second subplot, the Stokes pulse peak inside the microring occurs at 233.5 ps. This further delay depends on the time constant due to the coupling mechanism. Then, a low value such as \( \kappa_p^2 = 0.066\% \) allows a large net gain but also a large value of \( \tau_{c,p} \) and \( \tau_{c,s} \), with relevant Stokes pulse delay and distortion (right tail). Thus an optimal choice for \( \kappa_p^2 \) requires a trade-off between conflicting requirements, i.e., high Raman net gain and undistorted Gaussian profile.

A number of simulations performed for different values of \( \kappa_p^2 \) demonstrates both a delay time lowering and a spreading effect reduction in the optical pulse right tail by increasing \( \kappa_p^2 \). Thus a good trade-off can be obtained considering \( \kappa_p^2 = 40\% \), although the net Raman gain drops to 2.92 dB (being maximum at 15.64 dB for \( \kappa_p^2 = 0.066\% \)). In general, the distortion of the Stokes pulse is mainly determined by the off-resonance condition, as demonstrated in figure 11. The plot shows the time evolution of the normalized Stokes wave inside the ring for the optimum value \( \kappa_p^2 = 0.066\% \), for coupling at Stokes resonance (‘ideal’) or off-resonance.

As is evident, the Stokes pulse at off-resonance is characterized by both a different delay time and secondary lobes on the left tail. In contrast, the spreading on the right tail has similar influence as in the ideal case. Differently from CW regime, the small sizes of resonator give some disadvantages in the pulsed regime, due to the Stokes wave mismatch from the resonance, inducing significant distortion of the Gaussian shape. Thus it is evident that an accurate design of micro-scale resonator is required to avoid as much as possible the detrimental effects due to off-resonance on both net Raman gain and Stokes pulse shape.

To this aim, a very efficient method to design the microcavity resonator consists of an appropriate distribution of the resonant angular frequencies, i.e., \( \omega_{p,m} - \omega_{s,q} = \Omega_R \). It is evident...
that this condition gives a discrete set of cavity lengths

\[ L_{\text{cavity}} = \frac{2\pi l c}{n_{\text{eff},p}\omega_{p,\bar{m}} - n_{\text{eff},s}\omega_{p,\bar{m}} + n_{\text{eff},s}\Omega_R}, \]  

where \( l \) is an integer number counting the difference between the pump and Stokes wave longitudinal orders in the cavity. Then, each value of \( l \) determines the cavity length needed to minimize the off-resonance effects. However, the evaluation of \( \omega_{p,\bar{m}} \) in equation (21) also requires knowledge of \( L_{\text{cavity}} \). Thus solutions must be found using a recursive procedure. For example, by setting \( l = 26 \), we obtain a radius \( R = 19.78 \mu \text{m} \) in the case of a microring resonator. This value determines a quasi ideal resonance condition (the difference being 0.015 nm), considerably smaller than the mismatch of 1.99 nm previously obtained in the case of \( R = 20 \mu \text{m} \). Therefore, under this quasi ideal resonance, the detrimental effects induced by off-resonance are found to be negligible.

4. Conclusions

The generalized model presented in this paper allows the evolution of pulses (pump, Stokes) propagating in an SOI microcavity resonator to be accurately predicted, by taking into account the SRS and all other linear and non-linear physical effects. Analytical formulas to design a low threshold CW Raman laser based on a resonant microcavity have been derived. For example, for microcavity perimeters around \( L_{\text{cavity}} = 126 \mu \text{m} \), the Stokes wave coupling factor must be less than 1.43% to find a Raman threshold. Further, large cavity perimeters, \( L_{\text{cavity}} > 8.8 \text{ mm} \), do not imply any limitation on this factor. The influence of the pump enhancement factor on the power exchange between pump and Stokes pulses inside the microcavity, as well as the optimal operation conditions, have been rigorously described and evaluated. A detailed description
of Raman amplification under pulsed excitation has been also presented. Our investigations have demonstrated that the optimal condition to maximize the Raman gain in micro-scale resonators requires nano-scale cross-sections to reduce $\tau_{\text{eff}}$ to about 1 ns, pump pulse widths around $T_{\text{FWHM}} = 100$ ps, critical coupling and under coupling for pump and Stokes probe pulses, respectively. However, critical coupling induces both a very small pump coupling factor ($\kappa_p^2 = 0.066\%$ for $R = 20 \mu m$), and relatively large distortion of the output Stokes pulse. Thus a good trade-off in terms of high net Raman gain and moderate pulse distortion could be obtained using larger coupling factors, such as $\kappa_p^2 = 40\%$, and a ring radius $R = 19.78 \mu m$ for minimizing the Stokes resonance mismatch. Design criteria derived to realize efficient pulsed Raman amplification in microcavities have demonstrated larger net Raman gains and smaller sizes than in straight waveguides.

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References

[1] Reed G T 2004 The optical age of silicon Nature 427 595–6
[2] Pavesi L 2003 Will silicon be the photonic material of the third millennium? J. Phys.: Condens. Matter 15 1169–96
[3] Pavesi L, Negro L D, Mazzoleni G,Franzo G and Priolo S 2000 Optical gain in silicon nanocrystals Nature 408 440–4
[4] Ruan J, Fauchet P M, Negro L D, Cazzanelli M and Pavesi L 2003 Stimulated emission in nanocrystalline silicon superlattices Appl. Phys. Lett. 83 5479–81
[5] Polman A and Van Veggel F C J M 2004 Broadband sensitizers for erbium-doped planar optical amplifiers: review J. Opt. Soc. Am. B 21 871–92
[6] Claps R, Raghunathan V, Boyraz O, Koonath P,Dimitropoulos D and Jalali B 2005 Raman amplification and lasing in SiGe waveguides Opt. Express 13 2459–66
[7] Sirleto L, Raghunathan V, Rossi A and Jalali B 2004 Raman emission in porous silicon at 1.54 $\mu m$ Electron. Lett. 40 1221–2
[8] Boyraz O and Jalali B 2004 Demonstration of a silicon Raman laser Opt. Express 12 5269–73
[9] Krause M, Renner H and Brinkmeyer E 2004 Analysis of Raman lasing characteristics in silicon-on-insulator waveguides Opt. Express 12 5703–10
[10] Rong H, Jones R, Liu A, Cohen O, Hak D, Fang A and Paniccia M 2005 A continuous-wave Raman silicon laser Nature 433 725–8
[11] Rong H, Liu A, Jones R, Cohen O, Hak D, Nicolaescu R, Fang A and Paniccia M 2005 An all-silicon Raman laser Nature 433 292–4
[12] Boyraz O and Jalali B 2005 Demonstration of directly modulated silicon Raman laser Opt. Express 13 796–800
[13] Claps R, Dimitropoulos D, Han Y and Jalali B 2002 Observation of Raman emission in silicon waveguides at 1.54 $\mu m$ Opt. Express 10 1305–13
[14] Claps R, Dimitropoulos D, Raghunathan V, Han Y and Jalali B 2003 Observation of stimulated Raman amplification in silicon waveguides Opt. Express 11 1731–9
[15] Liu A, Rong H, Paniccia M, Cohen O and Hak D 2004 Net optical gain in a low loss silicon-on-insulator waveguide by stimulated Raman scattering Opt. Express 12 4261–8
[16] Xu Q, Almeida V R and Lipson M 2004 Time-resolved study of Raman gain in highly confined silicon-on-insulator waveguides Opt. Express 12 4437–42

New Journal of Physics 9 (2007) 25 (http://www.njp.org/)
[17] Xu Q, Almeida V R and Lipson M 2005 Demonstration of high Raman gain in a submicrometer-size silicon-on-insulator waveguide Opt. Lett. 30 35–7
[18] Espinola R L, Dadap J I, Osgood Jr. R M, McNab S J and Vlasov Y A 2004 Raman amplification in ultrasmall silicon-on-insulator wire waveguides Opt. Express 12 3713–18
[19] Dimitropoulos D, Raghunathan V, Claps R and Jalali B 2003 Phase-matching and nonlinear optical processes in silicon waveguide Opt. Express 12 149–60
[20] Claps R, Raghunathan V, Dimitropoulos D and Jalali B 2004 Anti-Stokes Raman conversion in silicon waveguides Opt. Express 11 2862–72
[21] Raghunathan V, Claps R, Dimitropoulos D and Jalali B 2005 Parametric Raman wavelength conversion in scaled silicon waveguides J. Lightwave Technol. 23 2094–102
[22] Boyraz O, Koonath P, Raghunathan V and Jalali B 2004 All optical switching and continuum generation in silicon waveguides Opt. Express 12 4094–102
[23] Van V, Ibrahim T A, Absil P P, Johnson F G, Grover R and Ho P T 2002 Optical signal processing using nonlinear semiconductor microring resonators IEEE J. Sel. Top. Quantum Electron. 8 705–13
[24] Arik B R and Bennett B R 1987 Electrooptical effects in silicon IEEE J. Quantum Electron. QE-23 123–9
[25] Marcuse D 1983 Computer model of an injection laser amplifier IEEE J. Quantum Electron. QE-19 63–8
[26] Agrawal G P 2001 Nonlinear Fiber Optics 3rd edn (London: Academic) pp 31–58
[27] Rong H, Kuo Y-H, Xu S, Liu A, Jones R, Paniccia M, Cohen O and Raday O 2006 Monolithic integrated Raman laser Opt. Express 14 6705–12