Controllable Josephson-Like Tunneling in Two-Component Bose-Einstein Condensates Coupled with Microwave via Feshbach Resonance and Trapping Potential

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We put forward a scheme for controlling Josephson-like tunneling in two-component Bose-Einstein condensates coupled with microwave field via Feshbach resonance and tuning aspect ratio of trapping potential. We prove how to realize a perfect periodic oscillation from a fast damped and irregular oscillation on relative number of atoms in future experiment. In particular, intensity of Josephson-like tunneling can be successfully controlled through controlling speed of recovering the initial value of intra-atomic interaction and aspect ratio of trapping potential. Interestingly, we find that relative number of atoms represents two different types of oscillation in respond to periodic modulation of attractive intra-atomic interaction.

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The existence of a Josephson current is a direct manifestation of macroscopic quantum phase coherence and has numerous important applications in condensed matter physics, quantum optics and cold atom physics, for example, precision measurement, quantum computation. The physical origin of the Josephson current is the temporal interference of the two systems which must both have a well defined quantum phase and a different average energy per particle, respectively. Recently, the experimental realization of multi-component Bose-Einstein condensates (BECs) of weakly interacting alkali atoms has provided a route to study Josephson effect in a controlled and tunable way by means of Feshbach resonance in far unattainable in charged systems. The physical origin of the low-energy Feshbach resonances is the low-energy binary collisions described by the difference of the initial and intermediate state energies which can be effectively altered through variations of the strength of an external magnetic field.

In this Letter, we put forward a scheme for controlling Josephson-like tunneling in two-component BECs coupled with microwave field via Feshbach resonance and tuning aspect ratio of trapping potential. We find that, through time-dependent tuning attractive intra-atomic interaction, a perfectly periodic oscillation can be obtained successfully. Especially, by controlling speed of recovering the initial value of attractive intra-atomic interaction and aspect ratio of trapping potential, we can successfully control intensity of Josephson-like tunneling. Furthermore, relative number of atoms represents two different types of oscillation in respond to periodic modulation of attractive intra-atomic interaction.

We consider a system of longitudinal elongated two-component BECs coupled by the microwave field with the effective Rabi frequency \( \Omega \) and finite detune \( \delta \), where the \( \Omega \) and \( \delta \) are independent upon time and space coordinate as experimental case. At zero temperature, this system can be described by macroscopic wave function \( \psi_j \) (normalized to unity, i.e., \( \int |\psi_j|^2 dz = 1 \)) which obey the dimensionless one-dimensional Gross-Pitaevskii equations (GPEs) as

\[
\imath \frac{\partial}{\partial t} \psi_j(z,t) = \left[ -\frac{\partial^2}{\partial z^2} + V(z) \right] \psi_j(z,t) + [G_{jj}(t)|\psi_j|^2 + G_{jk}(t)|\psi_k|^2] \psi_j(z,t) + (-1)^j \frac{\omega_\perp}{\omega} \psi_j(z,t) + \frac{\Omega}{\omega} \psi_k(z,t),
\]

where \( j, k = 1, 2 \), \( j \neq k \), and \( \omega_\perp \) is radial harmonic trapping frequency, time and coordinate unit are \( 2/\omega_\perp \) and \( a_\perp = \sqrt{\hbar/m\omega_\perp} \), respectively, and \( V(z) \) is the longitudinal confining potential. 

In the following discussion, we consider two-hyperfine spin states of \(^7\)Li Bose atom which the magnitude and sign of the atomic interaction can be tuned to any value via a magnetic-field dependent Feshbach resonance. And we choose dimensionless \( V(z) = \gamma^2 z^2 \), where \( \gamma = \omega_z/\omega_\perp \) is aspect ratio of trapping potential, \( \omega_z \) is longitudinal harmonic trapping frequency and \( \omega_\perp = 2\pi \times 625 \) Hz (so, the time and coordinate units are 0.51 ms and \( a_\perp = 1.51 \mu m \) respectively). Moreover, considering the condensates will be unstable and collapse when the number of particles is large enough, so we choose \( N = 6000 \), which provides a safe range of parameters for avoiding instability and collapse occurring. And the effective Rabi frequency and finite detune are chosen as \( \Omega = 1.5\omega_\perp \) and \( \delta = 0.5\omega_\perp \) considering experimental feasibility.
First of all, we consider the simplest case, i.e., intra-and inter-species effective atomic interaction potentials are time-independent attractive or repulsive case, which can be easily realized using Feshbach resonance experimentally. Here, we have chosen $a_{11} = a_{22}$ is just considering the experimental feasibility and assumed that tuning atomic interaction wouldn’t destroy the condensates in this system. We denote relative number of atoms between two species as $N_d$, i.e., $N_d = N_1 - N_2$, where $N_1 = \int |\psi_1|^2 dz$ and $N_2 = \int |\psi_2|^2 dz$, respectively. We study Josephson-like tunneling in this case and the results are shown in Fig. 1, which is based on numerical simulation of Eqs. (1) for the initial Gaussian wave function under the various aspect ratio of trapping potential. As shown in Fig. 1a corresponding to attractive atomic interactions, during the evolution process, exchange of atoms is always damped very fast and almost suppressed completely irrespective of aspect ratio of trapping potential. But, on the contrary, in repulsive atomic interactions corresponding to Fig. 1b, exchange of atoms vary significantly in response to various aspect ratio of trapping potential. Detailed analyses of above phenomena will be given in the following.

From Eqs. (1) and taking boundary condition into account, it is not difficult to derive following equations,

$$
\frac{dN_d}{dt} = \frac{4\Omega}{\omega_1} \int (\sqrt{\rho_1 \rho_2} \sin \phi_d) dz,
$$

where wave functions have been chosen as $\psi_1 = \sqrt{\rho_1} \exp(i\phi_1)$, $\psi_2 = \sqrt{\rho_2} \exp(i\phi_2)$, respectively, and $\phi_d = \phi_1 - \phi_2$. From Fig. 1a, we can see that maximum $\rho_1$ will be much smaller than maximum $\rho_2$ very quickly for all aspect ratio of trapping potential, so maximum $\sqrt{\rho_1 \rho_2}$ will decrease rapidly. Moreover, if considering well localized density profiles of two species as shown in Fig. 1a, from Eqs. (2), we can immediately conclude that amplitude of oscillation of $N_d$ will be damped very fast and suppressed completely in this case. But from Fig. 1b corresponding to repulsive atomic interactions, we can see that the atomic densities of two species vary significantly in response to various aspect ratio of trapping potential and represent nonlocalized profiles, so time evolution of amplitudes of $N_d$ represents very different behavior from attractive case.

As pointed out above, $N_d$ will represent a very irregular and fast damped oscillation in time-independent attractive atomic interactions and represent a more irregular oscillation in time-independent repulsive case. But, from the experimental point of view, valuable and applicable systems usually have regular and stable characteristics rather than irregular and unstable one. Fortunately, we...
find out that, through time-dependent tuning atomic interaction via Feshbach resonance, regular and stable oscillation of $N_d$ can be successfully realized in the attractive case, but not be realized in the repulsive case. Now, let us to understand physics from both theoretical analysis and numerical simulation aspects.

In order to study how to control Josephson-like tunneling in this system, we diagonalize linear parts of Eqs. (1), by means of applying an unitary transformation $U$ on both sides of Eqs. (1), then it is transformed into following form,

$$\frac{\partial}{\partial t} \varphi_{\pm}(z,t) = \left(-\frac{\partial^2}{\partial z^2} + V(z)\right)\varphi_{\pm}(z,t) + \varsigma^\pm_1 |\varphi_{\pm}|^2 \varphi_{\pm}(z,t) + \varsigma^\pm_2 |\varphi_{\pm}|^2 \varphi_{\pm}(z,t) + \Sigma(\eta),$$

where

$$U \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \varphi_+ e^{i\eta t} \\ \varphi_- e^{-i\eta t} \end{pmatrix} \equiv \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}, U = \begin{pmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{pmatrix},$$

$$\Sigma(\eta) = \varsigma^\pm_1 \varphi^*_+ \varphi^+_e^{\pm i2\eta t} + \varsigma^\pm_2 |\varphi_{\pm}|^2 \varphi_{\pm} e^{\mp 2\eta t} + \varsigma^\pm_3 \varphi^*_e^{\pm 4\eta t} + \varsigma^\pm_4 |\varphi_{\pm}|^2 \varphi_{\pm} e^{\pm 2i\eta t},$$

here $\varsigma^\pm_j = \left(\frac{G_{1j} + 2G_{12} + 3G_{22}}{2} \mp 4(G_{1j} - G_{2j}) \cos 2\theta + (G_{1j} + 2G_{12} + G_{22}) \cos 4\theta/8\right)$, $\varsigma^\pm_2 = \left[(G_{11} + 2G_{12} + G_{22}) - (G_{1j} - 2G_{12} + G_{22}) \cos 4\theta/4\right]$, $\varsigma^\pm_3 = \left[2(G_{11} - G_{22}) \sin 2\theta \mp (G_{11} - 2G_{12} + G_{22}) \cos 4\theta/8\right]$, $\varsigma^\pm_4 = \left[2(G_{11} - G_{22}) \sin 2\theta \mp (G_{11} - 2G_{12} + G_{22}) \cos 4\theta/8\right]$, $\varsigma^\pm_5 = \left[\left(G_{11} - 2G_{12} + G_{22}\right) \sin 4\theta/4\right], \varsigma^\pm_6 = \left[2\left(G_{11} - G_{22}\right) \sin 4\theta/8\right].$
perfectly periodic oscillation of $N_d$. In the following, we prove that, only in the attractive case, through time-dependent tuning intra-atomic interaction via Feshbach resonance, we can successfully control the effects of $\Sigma(\eta)$ and obtain a perfectly periodic oscillation on relative number of atoms. Considering the experimental feasibility, in the following case, we choose time-dependent intra-atomic interaction as [18, 19, 20] 

$$G_{jj}(t) = \begin{cases} 
G_{12} \exp(t/\tau), & 0 \leq t \leq t_1; \\
G_{12} \exp[(t_2 - t)/\tau], & t_1 < t \leq t_2; \\
G_{12}, & t > t_2.
\end{cases} \quad (4)$$

where $j = (1, 2)$, $G_{11}(t) = G_{22}(t)$ and $G_{12}$ is time-independent parameter determining inter-atomic interaction, $\tau$ determines speed of tuning intra-atomic interaction depending on the values of $t_1$ and $t_2$ at fixed minimum $G_{jj}(t)$. The results are shown in Fig. 3. It is very interesting that $N_d$ finally represents a perfectly periodic oscillation in the attractive case, but not in the repulsive case. Above phenomena are mainly due to successfully suppress the effects of $\Sigma(\eta)$ on Josephson-like tunneling in the attractive case, but not in the repulsive case, which underlying physics is just because $\varphi_{\pm}(z, t)$ vary significantly over time period $\pi/\eta$ in the repulsive case. So, in the following, we will concentrate on attractive case. It is important to point out that, in the attractive case, the amplitude of $N_d$ is strongly dependent upon aspect ratio of trapping potential as shown in Fig. 3a. Based upon the above analysis, we put forward a scheme for controlling the amplitude of oscillation through controlling speed of recovering the initial value of intra-atomic interaction (i.e., changing value of $t_2$ but fixing other parameters in Eqs. (4)) and tuning aspect ratio of trapping potential.

The results are shown in Fig. 4. It is very interesting to note that oscillatory amplitude of $N_d$ can be controlled successfully not only via Feshbach resonance but tuning aspect ratio of trapping potential. Above phenomena can be well understood as follows: If $\varphi_{\pm}(z, t)$ vary slowly over time period $\pi/\eta$, then the effects of $\Sigma(\eta)$ on Josephson-like tunneling can be effectively suppressed, and $N_d$ will represent a perfectly periodic oscillation which amplitude is determined by atomic density of two species. We also study Josephson-like tunneling in response to periodic modulation of attractive interaction [12] and various aspect ratio of trapping potential. In this case, attractive atomic interactions have been chosen as $G_{11}(t) = G_{22}(t) = G_{12}[1 + 8 \sin^2(0.01t)]$, where $G_{12} = -2.5$. As shown in Fig. 5, it is very interesting to note that $N_d$ represents two different types of oscillation, which one represents small frequency behavior originated from periodic modulation of attractive interaction, but the other represents large frequency behavior originated from coupling microwave. Above effects provide a promising protocol to realize a new type of switch controlled by periodically managed Feshbach resonance in future experiments.

In conclusion, we prove how to successfully control Josephson-like tunneling in attractive two-component BECs coupled by microwave field via Feshbach resonance in respond to various aspect ratio of trapping potential. Recent developments of controlling the scattering length and realization of multi-component BECs in the experiments allow for the experimental investigation of our prediction in future.

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