A solution to the double crack’s J-integral in the plain plate

H Li¹ and X Y Zhang¹²
¹College of Aerospace and Civil Engineering, Harbin Engineering University, Harbin, China
E-mail: zhangxueyi@hrbeu.edu.cn

Abstract. Based on the relevant criteria and theory of fracture mechanics and the description and definition of corresponding fracture parameters, the J integral factor of double crack propagation in plain plate is analyzed. In order to obtain the J integral factor of double crack propagation, the method of defining seam is used to preset the path of crack’s propagation. With the variation of load, the crack would be acquired a propagation forward to the presetting seam, furthermore, and simulated the singularity of crack tip, the stress field, displacement field and strain field at the crack tip are analyzed for the causes and regular changes. The model under the same load is also calculated using the model of the viscoelastic plate, and the corresponding regular changes and discussions are also given. The result is showed that the distribution of stress about crack tip is more larger than other district, concentration of stress is so obvious, the value of stress is more decreasingly less than other distribution; the strain of crack tip would acquire an increasing tendency to the direction of crack propagation, the displacement’s magnitude also presents a significantly growth across the central point toward both sides, and the value of displacement is largest of the both sides of plate. Compared to the classical elastic plate, the J integral factor is smaller, and the strain energy according to the crack tip is less.

1. Introduction
The occurrence of a large number of fracture problems in actual engineering, especially for the fracture of the plate and shell structures, is an important measure for the calculation and determination of the corresponding fracture parameters. And the corresponding degree of damage and the form of failure are also reflected. The reference and judgment, especially the stress field, the displacement field and strain field at the crack tip have a certain description and mechanism [1].

2. Basic theory

2.1. The elastic plain plate

2.1.1. The process of the model. According to the size requirements, the geometric model of the corresponding flat plate is established [2-5], as shown in the figure 1. The geometric model is shown in figure 2, the element meshing is shown in figure 3.
Figure 1. Load diagram of the entire plate.

Figure 2. The geometric model of the plain plate.

Figure 3. Corresponding finite element meshing.
2.1.2. Mesh. Divide the finite element mesh and preset the crack model to refine the mesh at the crack.

2.1.3. Result. As a result of the deformation of the plain plate after biaxial stretching, the crack growth has become into a symmetrical shape distribution. As shown in figure 4.

![Figure 4. The result of crack propagation.](image)

2.1.4. Crack tip stress field distribution. The simulation results show that stress distribution at the crack tip is extraordinary large, the stress concentration occurs at the same time, the stress value decreases along the crack propagation direction, and at the stress concentration, the value is the largest, it may be a serious concentration at crack tip, in the figure 5, which is about 13.60 MPa.

![Figure 5. Stress field distribution of corresponding crack.](image)

2.1.5. Crack tip displacement field distribution. The strain value at the tip increases in the crack propagation direction [6-8], and the total displacement gradually increases along the center point toward both sides. From the figure 6, the displacement at the boundary between the two ends of the plate is maximal, and the maximum value is $2.82 \times 10^{-6}$ m.

On the one hand, the method of defining the seam is used to preset the crack propagation path. As the load is applied, the crack will expand along the seam. It’s different from debonding method, the latter is limited in practical application range, the former is convenient to calculate the fracture parameters such as stress intensity factor and J-integral, and in the former method, the approach of defining the crack front is to select the crack tip (2D) or the crack tip line (3D) directly. Using this method to define the crack front does not need to define the next crack tip/line, which is relatively simple.
2.1.6. J integral factor. Solution of corresponding parameters after crack propagation (J integral factor).

Elastic plain plate:

| parameters |.parameters 1 | parameters 2 | parameters 3 | parameters 4 | parameters 5 | parameters 6 | parameters 7 | parameters 8 |
|------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Left crack | 1.3042E-02   | 1.3019E-02   | 1.3027E-02   | 1.3028E-02   | 1.3028E-02   | 1.3028E-02   | 1.3028E-02   | 1.3029E-02   |
| Right crack| 1.3043E-02   | 1.3019E-02   | 1.3027E-02   | 1.3028E-02   | 1.3028E-02   | 1.3028E-02   | 1.3028E-02   | 1.3029E-02   |

The crack at the left end expands to the right, and the crack at the right end expands to the left, as shown in the table 1, the J integral factor according to the crack propagation at the left and right ends is stable at 1.30e-02.

2.2. The viscoelastic plain plate

2.2.1. Crack tip stress field distribution (static step). The crack tip stress distribution in the visoelastic plain plate law is basically the same as the elastic plain plate, as shown in the figure 7, the largest value is 14.89 Mpa.

2.2.2. Crack tip displacement field distribution. The displacement field at the tip is the same as that of the elastic plain plate. The total displacement increases gradually from the center point to the sides, and the displacement at the two ends of the plate is the largest. From the figure 8, the maximum value is
4.67×10^{-6} \text{ m.}

Figure 8. Displacement field distribution of corresponding crack.

2.2.3. \textit{J integral factor}. The \textit{J} integral factor according to the crack propagation at the left and right ends are stable at 7.94e-03, as shown in the table 2, the value is less than in the elastic plain plate.

| parameters | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Left crack | 7.9459E-03 | 7.9318E-03 | 7.9365E-03 | 7.9372E-03 | 7.9374E-03 | 7.9372E-03 | 7.9372E-03 | 7.9376E-03 |
| Right crack| 7.9462E-03 | 7.9320E-03 | 7.9366E-03 | 7.9372E-03 | 7.9374E-03 | 7.9372E-03 | 7.9372E-03 | 7.9375E-03 |

2.2.4. \textit{Crack tip stress field distribution (visco step)}. The crack tip stress distribution in the viscoelastic plain plate law is basically the same as the plane elastic plate, from the figure 9, the largest value is 14.87 Mpa.

Figure 9. Corresponding crack stress field distribution.

2.2.5. \textit{Crack tip displacement field distribution}. The displacement field at the tip is the same as that of the elastic plain plate. The total displacement increases gradually from the center point to the sides, and the displacement at the two ends of the plate is the largest. From the figure 10, the maximum value is 2.84×10^{-6} \text{ m.}
2.2.6. J integral factor. The J integral factor according to the crack propagation at the left and right ends are stable at 8.00e-03, as shown in the table 3, the value is less than in elastic plain plate and larger than in the static step of viscoelastic plain plate, there is creep during the stretching process of the viscoelastic plate, the plastic deformation, resulting in a small J integral value than before.

Due to the existence of the creep, as shown in the table 4, C-integral is less than the J integral factor in the viscoelastic plain plate.

3. Conclusion
The simulation of the biaxially stretched plate, whether elastic or viscoelastic, it shows that the stress distribution at the crack tip is relatively large, the stress concentration is obvious, and the stress value decreases along the crack propagation direction [9]; the strain value at the tip decreases first, then increasing along the direction of crack propagation, the total displacement gradually increases along the center point toward both sides, and the displacement at the boundary between the two ends of the plate is the largest [10]. And in the viscoelastic plain plate, the J integral factor is less than in the elastic plain plate, and deformation during load application has appeared and creep affects crack propagation and energy release.

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