Interfacing Superconducting Qubits and Telecom Photons via a Rare-Earth Doped Crystal

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We propose a scheme to couple short single photon pulses to superconducting qubits. An optical photon is first absorbed into an inhomogeneously broadened rare-earth doped crystal using controlled reversible inhomogeneous broadening. The optical excitation is then mapped into a spin state using a series of \( \pi \)-pulses and subsequently transferred to a superconducting qubit via a microwave cavity. To overcome the intrinsic and engineered inhomogeneous broadening of the optical and spin transitions in rare-earth doped crystals, we make use of a special transfer protocol using staggered \( \pi \)-pulses. We predict total transfer efficiencies on the order of 90%.

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Superconducting quantum circuits are a promising candidate for scalable quantum computation. Impressive improvements have been made in the last few years: coherence times were increased by new qubit designs [1, 2], single qubit operations were performed [3] and by coupling two qubits to a microwave cavity [4, 5], the realization of two qubit gates was shown [6, 7, 8, 9, 10]. One major limitation of superconducting qubits (SCQ) is that they are essentially stationary, with both qubits and microwave cavities fixed on a single chip. Telecom-wavelength photons on the other hand, are the best candidates for transporting quantum information, due to the availability of low loss optical fibers. By interfacing short-pulse photons and SCQs, a fast quantum network could be realized where one uses the stationary SCQ for quantum information processing and photons for communication between different nodes.

There are several proposals for such hybrid devices [11], all of which have in common the coupling of a spin ensemble, e.g. cold gas [12], NV-centers in diamond [13], or rare-earth doped crystals [14, 15] to a microwave cavity. Coupling spin states of NV-centers to a SCQ via a microwave cavity has been realized recently [16], also direct coupling of the spin ensemble to a superconducting flux qubit was proposed and implemented in [17, 18]. However, NV-centers have the disadvantage of being incompatible with the telecom bandwidth and typically have smaller optical depth [19]. In principle wavelength incompatibility could be addressed by single photon frequency conversion [20], which is however accompanied by large noise [21]. Both issues can be overcome by using rare-earth doped crystals. Inspired by the recent experimental demonstrations of coupling a spin ensemble in a rare-earth doped crystals (REDC) to a microwave cavity we here present and analyze a scheme for coupling optical photons to superconducting qubits mediated by a

![FIG. 1. Sketch of the transfer scheme from telecom-wavelength photons to a superconducting qubit employing rare-earth doped crystals coupled to a tunable superconducting cavity. Left: scheme for absorbing a single photon into a collective spin excitation in the REDC. Right: Scheme to move the crystal spin into an excitation of the SCQ.](image-url)
fast population transfer from the symmetric collective state into non-symmetric ones, which do not couple to the microwave resonator. Here we propose and theoretically analyze a specific protocol using a staggered series of \( \pi \)-pulses which compensates the effect of the induced broadening and facilitates the transfer to a SCQ with above 90% efficiency.

The total efficiency for transfer between the telecom photon and a SCQ can be expressed as: \( \eta = \eta_S \eta_T \), where \( \eta_S \) is the efficiency of storing a photon as a collective spin state that can be directly coupled to the microwave cavity, and \( \eta_T \) is the efficiency of moving the collective spin excitation to an excitation of the qubit. We will see that \( \eta_S \) is essentially determined by the spectral shape of the input pulse and \( \eta_T \) is dominated by the incoherent loss of population during the transfer process to non-coupled spin states due to intrinsic inhomogeneous broadening of the spin state. Whereas the dephasing due to \textit{induced} broadening can be reversed, population losses due to \textit{intrinsic} broadening constitute the main bottleneck of any transfer protocol to microwave photons.

Only the symmetric collective state can couple through the microwave cavity to the SCQ therefore, it is important to store the telecom photon as a collective spin state with spatially homogeneous probabilities. So we will first consider how to create this state. To achieve this we suggest following one of the possible implementations of the CRIIB protocol where the transition frequency is a function of distance into the medium in the direction of photon propagation \[26, 27\]. The preparation protocol is as follows: We start with the inhomogeneously broadened optical line and perform hole burning by optically pumping a large frequency window into a long-lived shelving state. Then using a narrow band laser, some of the atoms are pumped back into the excited state such that there is now a narrow frequency peak. The remaining atoms in the shelving state do not participate in the optical interaction. Finally this peak is frequency broadened by applying a magnetic field (Zeeman shift) linearly varying in space along the crystal length \( L \), leading to a frequency profile with slope \( \omega_0 - \alpha L / 2 \) to the maximum frequency \( \omega_0 + \alpha L / 2 \), chosen to match the frequency width of the incoming photon.

As will be seen later it is important that this magnetic field gradient also induces inhomogeneous broadening of the spin energy levels.

The maximal amount of the coherent excitation which can be mapped into the cavity is given by the spatial overlap of the wave function with the symmetric Dicke state:

\[
\mathcal{F} = \langle \Psi_S | \Psi(t) \rangle = \sum_k \langle g, 0 | \hat{\sigma}_{ge}^k(t) | g, 1 \rangle.
\]  

where the sum is over the individual spins, so we need the optical coherence which can be found in the continuum limit. To calculate the optical coherence, resulting from the interaction of an input probe pulse with the prepared medium, we start with the linearized equations of motion for the slowly varying operators for the probe field \( \hat{E} \) and the optical coherence \( \hat{\sigma}_{ge} \) with spatial dependent detuning in the co-moving frame:

\[
\begin{align*}
\partial_t \hat{\sigma}_{ge}(z,t) &= -i \alpha z \hat{\sigma}_{ge}(z,t) + i g_{ge} \hat{E}(z,t), \\
\partial_z \hat{E}(z,t) &= i \frac{g_{ge} n}{\epsilon} \hat{\sigma}_{ge}(z,t),
\end{align*}
\]

where \( g_{ge} = d_{ge} \sqrt{\frac{\omega_p}{2 \epsilon_0 \hbar}} \) is the coupling constant with optical resonance frequency \( \omega_s \), beam cross-section \( A \), and atomic density \( n \). We neglect spontaneous emission from the excited state, since for a REDC the decay time is much longer than the usual pulse times.

To ensure that the incoming photon is fully absorbed we require the photons spectral width to fit within the induced inhomogeneous broadening such that \( \Delta \omega_p < \alpha L \). In this limit we can follow the analysis of \[28\] and approximate the solution for the coherence by:

\[
\hat{\sigma}_{ge}(z,t) = -g_{ge} e^{-\frac{\pi d}{d^2}} \int_0^t dt' e^{-i \frac{d^2}{d^2} \log((t-t')^2 \alpha L/2)} \cdot e^{-i \alpha z (t-t')} \hat{E}(-L/2, t'),
\]

where \( \Gamma(ix) \) is the complex gamma function (with \( x \) real) and \( d = \frac{d}{\alpha L} \) is the optical depth of the medium. We can further simplify this expression for \( t \) much larger than the pulse time, \( t \gg T_p \) to get a compact expression for the coherence:

\[
\hat{\sigma}_{ge}(z,t) = -g_{ge} e^{-\frac{\pi d}{d^2} \frac{e^{-i \frac{d^2}{d^2} \alpha L/2}}{\Gamma(-i d)}} e^{-i \alpha z t} \sqrt{2 \pi} \hat{E}(-L/2, \alpha z)
\]

where \( \hat{E}(-L/2, \omega) = \frac{1}{\sqrt{2 \pi}} \int dt e^{i \omega t} \hat{E}(-L/2, t) \) is the Fourier transform of the incoming field at the beginning of the medium. The phase term \( \exp(-i \alpha z t) \) is due to the induced inhomogeneous broadening, which is essential to prevent reemission of the excitation as a photon echo.

The induced inhomogeneous broadening, can be controlled in order to rephase the excitation at a specific time. To that end, we use a series of three \( \pi \)-pulses between the spin and excited levels. After the photon is absorbed into the optical transition \( |e\rangle - |g\rangle \) the corresponding coherence starts to dephase. At a time \( \tau_1 \) a \( \pi \)-pulse on the \( |e\rangle - |s\rangle \) transition is applied to move the population into the spin state, where it continues to dephase due to the inhomogeneous broadening of the spin state, but now with a different rate. Then after some time \( \tau_2 \) we simultaneously flip the magnetic field gradient to begin rephasing while using a \( \pi \)-pulse to bring the population back into \( |e\rangle \). Once the atoms have spent the same time in the excited state as during storage, the dephasing due to inhomogeneous broadening of the excited state is compensated and the \( \exp(-i \alpha z t) \) term vanishes. Note however, that due to the accumulated phase caused by the inhomogeneous broadening of the spin state there will be no photon echo. Then applying the third \( \pi \)-pulse
we again move the excitation to the spin state, where it can now complete rephasing. For faithful transfer the pulses must have a Rabi frequency larger than the inhomogeneous width of the spin transition, and be well timed to get the proper pulse area.

In the continuum limit, we can use Eq. (6) in Eq. (4), and after normalizing the variables for space $z = \xi L$ and time $\tau = aL/\hbar$ and introducing dimensionless field operator $\tilde{\mathcal{E}} = \sqrt{Lac}$ we get the following expression for the absolute square of the overlap:

$$\eta_S = |\mathcal{F}|^2 = (1 - e^{-2\pi d}) \left| \int_{-1/2}^{1/2} d\xi \tilde{\mathcal{E}}(\xi) \right|^2. \quad (6)$$

Since the first factor on the right is almost unity for optical depths $d > 1$ we see that the overlap is given by the square of the integral of the input field’s Fourier transform which is maximal for constant $\tilde{\mathcal{E}}$, whereas pulses with standard shapes, like for example a photon with a Gaussian frequency distribution could reach an overlap above 90%.

Now that we have shown how to create a symmetric collective spin state, let us discuss the dynamics of the transfer of this spin excitation to a SCQ. The combined spin-cavity-qubit system, see Fig. 1(right), is described by the Hamiltonian:

$$H = \sum_j \hbar \omega_j \hat{S}_j^z + \sum_j \hbar \kappa \left( \hat{a} \hat{\tilde{S}}_j + \hat{\tilde{S}}_j^\dagger \hat{a}^\dagger \right) + \hbar \omega_c \hat{a} \hat{a}^\dagger + \hbar G \left( \hat{\sigma} \hat{a} \hat{a}^\dagger + \hat{a}^\dagger \hat{\sigma} \hat{a} + \hbar \omega_c \hat{\sigma}^2, \right. \quad (7)$$

where $G$ is the coupling between the cavity and the qubit. $\hat{\sigma}$ is the qubit spin flip operator, and $\hat{\sigma}^2$ describes spin population operator for the SCQ. $\kappa$ denotes the coupling between an individual spin in the REDC and the cavity and is assumed to be the same for every interacting spin. $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators of the microwave cavity mode. $\hat{S}_j$ is the collective spin flip operator in the REDC, $\hat{S}_j^2$ corresponds spin population operator, $\omega_j$ is the frequency of each spin, $\omega_c$ is the resonance frequency for the cavity, and $\omega_q$ is the frequency of the SCQ.

To discuss the transfer protocol let us consider a quantum state with a single excitation:

$$|\Psi(t)\rangle = \sum_j \xi_j(t) |1_j\rangle_s |0\rangle_c |\downarrow_q\rangle + c(t) \langle 0\rangle_s |1\rangle_c |\downarrow_q\rangle + q(t) \langle 0\rangle_s |0\rangle_c |\uparrow_q\rangle, \quad (8)$$

where $|1_j\rangle_s = |g\rangle_1 |g\rangle_2 \ldots |s\rangle_j \ldots |g\rangle_N$ represents the state where the $j$-th atom is in the spin state and the rest are in the ground state, $\langle 0\rangle_s$ describes the total ground state, $|0\rangle_c$ denotes the empty cavity state, $|1\rangle_c$ implies a single photon is in the cavity, $|\downarrow_q\rangle$ is the ground state of the qubit, and $|\uparrow_q\rangle$ is the excited state of the qubit. The sum runs over the $N$ crystal spins that are in the interaction region of the cavity.

Since it is difficult to change the coupling constants $G$ and $\kappa$ as a function of time, this precludes using a STIRAP like procedure to move the population between states. What we can control is the resonance frequency of the spin states, by adjusting the applied fields, as well as the resonance frequency of the cavity. One possible way to move the population is to use adiabatic transfer where the frequency of the cavity is swept through resonance first with the spin state, and then through resonance with the qubit. To simulate this process we numerically solve the Schrödinger equation for the coefficients in Eq. (8) with time dependent detunings. We assume that the initial state of the system is given by the symmetric Dicke state $|\Psi_S\rangle = \frac{1}{\sqrt{N}} \sum_j |1_j\rangle_s |0\rangle_c |\downarrow_q\rangle$, and disregard the cavity decay since the stripline cavities have high quality factors. We found that the adiabatic scheme is too slow to have faithful transfer in available REDC. This is due to the large inhomogeneous broadening of the spin state. Adiabaticity requires that the population spends too long in the initial spin state where there is high dephasing.

Nevertheless, faithful transfer can be realized with a series of staggered $\pi$-pulse interactions. Starting with the qubit and spin far detuned from each other, bring the cavity into resonance with the spin, for an amount of time corresponding to a $\pi$-pulse, $T_S = \pi/(2\kappa\sqrt{N})$, which moves the population into the cavity. Then change the cavity frequency to be in resonance with the SC-qubit, let it interact until it undergoes another $\pi$-pulse transfer $T_C = \pi/(2G)$. Finally, we detune the cavity once again, leaving the excitation in the qubit. On first glance one might think that the induced inhomogeneous broadening $\delta_{inh}$ must be kept much smaller than the REDC-cavity coupling $\kappa\sqrt{N}$ to avoid dephasing during $T_S$, which would substantially limit the bandwidth of the transfer. The results of our simulation show however that by timing the transfer procedure to match up with the rephasing of the spin excitation the transfer protocol can be quite efficient, even with high inhomogeneous broadening. This is demonstrated in Fig 2 where we plot the transfer efficiency as a function of the intrinsic inhomogeneous broadening and the rephasing time $\tau_R$ when we start the transfer.

With proper pulse timings the loss of efficiency is only due to the intrinsic broadening $\delta_{IB}$ of the spin state causing population loss to the uncoupled spin modes. To estimate these losses we can make following considerations: If all of the spins are in resonance, then only the symmetric Dicke state $|\Psi_S\rangle = \frac{1}{\sqrt{N}} \sum_j |1_j\rangle_s |0\rangle_c |\downarrow_q\rangle$ interacts with the cavity. This is the state that would be created by our storage process, if there is no intrinsic inhomogeneous broadening. The presence of this intrinsic broadening couples the Dicke state to the other $N - 1$ non-symmetric eigenmodes, so our actual state can be written as:

$$|\Psi_{inh}\rangle = \frac{1}{\sqrt{N}} \sum_j e^{i\delta_j t} |1_j\rangle_s |0\rangle_c |\downarrow_q\rangle, \quad (9)$$

with the sum over intrinsic spin frequency detunings $\delta_j$. 

$$\eta_S = |\mathcal{F}|^2 = (1 - e^{-2\pi d}) \left| \int_{-1/2}^{1/2} d\xi \tilde{\mathcal{E}}(\xi) \right|^2. \quad (6)$$

$$H = \sum_j \hbar \omega_j \hat{S}_j^z + \sum_j \hbar \kappa \left( \hat{a} \hat{\tilde{S}}_j + \hat{\tilde{S}}_j^\dagger \hat{a}^\dagger \right) + \hbar \omega_c \hat{a} \hat{a}^\dagger + \hbar G \left( \hat{\sigma} \hat{a} \hat{a}^\dagger + \hat{a}^\dagger \hat{\sigma} \hat{a} + \hbar \omega_c \hat{\sigma}^2, \right. \quad (7)$$

$$|\Psi(t)\rangle = \sum_j \xi_j(t) |1_j\rangle_s |0\rangle_c |\downarrow_q\rangle + c(t) \langle 0\rangle_s |1\rangle_c |\downarrow_q\rangle + q(t) \langle 0\rangle_s |0\rangle_c |\uparrow_q\rangle, \quad (8)$$

$$|\Psi_{inh}\rangle = \frac{1}{\sqrt{N}} \sum_j e^{i\delta_j t} |1_j\rangle_s |0\rangle_c |\downarrow_q\rangle, \quad (9)$$
Using this expression we can now estimate the influence of the intrinsic broadening in our protocol assuming a homogeneous excitation profile:

$$\eta_T = |\langle S||\Psi_{inh}\rangle|^2 = \frac{\sin(\delta IB T_S)^2}{(\delta IB T_S)^2} \approx e^{-\frac{1}{2}(\delta IB T_S)^2}. \quad (10)$$

This dephasing is thus a loss mechanism in the transfer process, which limits the maximum time $T_S$ that can be spent in the spin state.

For the experimental realization of our scheme we propose to use Er$^{3+}$ YSO for which CRIB has been demonstrated 24. It has been shown 17, that a collective spin-cavity coupling of $\kappa\sqrt{N} = 34$MHz can be achieved with the HWHM of the intrinsic inhomogeneous broadening being reducible to $\delta IB = 12$ MHz. Although, the SCQ is acting as a single spin, it has a very strong dipole moment 4, and thus strong coupling between the qubit and cavity is possible with coupling strength $G$ of several MHz. Making an estimate for an implementation in Er:YSO with the given numbers, the best transfer efficiency can exceed $\eta \geq 90\%$, see Fig 3.

A drawback of the above protocol is the necessity to be able to tune the cavity over a large range. To get perfect transfer it may be required to tune the 6 – 10 GHz cavity up to a 1 GHz in frequency. Some of the reported experimental systems allow tuning 30, but the average seems to be < 500 MHz 31. We can use less sweeping at the cost of the transfer efficiency, but there is also another possibility. We can take advantage of $G$ being weak in comparison to the collective spin coupling, by starting at a smaller fixed cavity-qubit detuning $\delta$, and simultaneously detuning the spin state far away from resonance when we move the cavity to be in resonance with the qubit. In this case there is a factor of 6 improvement in minimizing the cavity sweeping at the cost of needing to strongly detune the spin which should be experimentally easier, however. This would bring the required change in the cavity resonance to < 200 MHz, which is currently achievable.

In summary we have analyzed a scheme to faithfully transfer a quantum state back and forth between short optical photons and a superconducting qubit, which relies on coupling rare-earth doped crystals with engineered inhomogeneous broadening to a microwave cavity that is directly coupled to a superconducting qubit. The incoming photon needs to be stored in the spin coherence as a symmetric Dicke state which can be achieved using the controlled reversible inhomogeneous broadening quantum memory protocol, with a spatially varying frequency. We also show that the main loss mechanism for the transfer procedure is the intrinsic inhomogeneous broadening of the spin state. To overcome this it is necessary to limit the time that the pulse remains as a coherent spin excitation. The best way to accomplish this is to forgo adiabatic transfer schemes, and to use a staggered $\pi$-pulse transfer scheme with proper timing adjusted such that the engineered broadening is compensated. Increasing the effective spin-cavity coupling will give another improvement by reducing the transfer time. Combining proposed storage and transfer schemes, could lead to successful transfer of a short single photon pulse to a SCQ using an Er:YSO crystal with transfer efficiencies on the order of 90%.

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