Effect of Austenitization Temperature and Holding Time on Transformation Plasticity in a Three-Point Bending System†

by

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The influences of austenitization temperature and holding time on transformation plasticity behaviors were experimentally investigated in a three-point bending test, wherein the test specimen is simply supported from one end and fixed at the opposite end. The specimens were fabricated from S45C steel and heated to different austenitization temperatures (850, 900, and 950 °C). The temperatures of the specimens in the austenite phase were kept constant for 5 or 20 min, and then naturally cooled to room temperature. During the cooling process, the specimens were subjected to the maximum bending stress (less than the yield stress) before the transformation start point (around 800 °C). Under the bending stress, the transformation plasticity due to austenite to pearlite occurred. From the measured maximum transformation plasticity deflections, the transformation plasticity coefficients were determined. These coefficients were found to strongly depend on the austenitization temperature and holding time.

Key words:

Transformation plasticity, Austenite, Pearlite, Heat treatment, Three-point bending system, Bending deformation, Transformation plasticity coefficient

1 Introduction

The properties of materials can be modified by changing the microstructure and composition of the material. The microstructure is the most important property of metallic materials, and can be optimized by understanding the transformation plasticity behaviors. Phase transformation plasticity can occur under below-yield external stresses imposed by heat treatment and welding processes. The relationships between temperature, stress/strain, microstructure and the coupling effects among the properties are very complex and must be investigated by reliable simulation. For this purpose, some factors such as the transformation plasticity coefficient must be experimentally determined.

Transformation plasticity has been attributed to two different phase transformation mechanisms.

(1) Magee mechanism3): According to Magee, transformation plasticity occurs when the newly formed phase is oriented by the applied stress.

(2) Greenwood–Johnson mechanism5): According to Greenwood and Johnson, transformation plasticity is due to the compactness difference between the parent and product phases.

Transformation plasticity has been studied by various test methods under the same austenitization conditions at specific temperatures and holding times. These methods include the axial loading system6,7,8, four-point bending system9,10, three-point bending system with simple supports11,12, and three-point bending system with a simple support at one end and a fixed support at the other13. In addition, other researchers have investigated the influence of tensile–torsional systems on martensite transformation14 at different austenitization temperatures. The tensile–torsional system results15 show that under uniaxial torsional loading, transformation plasticity is independent of the prior austenite grain size (AGS), but in the uniaxial tensile loading case, it is increasing function of the AGS. Furthermore, under tensile loading tests16 at different austenitization temperatures and holding times, martensite transformation generated different transformation plastic deformations. At higher heating rates, the materials with small AGS generated high transformation plasticity; at lower heating rates, the transformation plasticity is a slightly increasing function of AGS17. Moreover, dilatometric tests18 show that the dilatation of the martensite transformation decreases with increasing austenitization temperature. The pearlite volume fraction of pearlite transformation19 of hypo–eutectoid steel also increases with increasing AGS.

In this study, the macroscopic transformation plasticity phenomena are represented by constitutive equations with an internal state variable, which may not be macroscopically observable. The changing microstructure during the phase transformation is modeled by the average volume fraction of new phase as an internal state variable.
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Literature on transformation plasticity kinetics is too scarce to evaluate the effect of austenitic conditions at different temperatures and holding times. Therefore, in this study, the pearlite transformation plasticity behavior of S45C steel under different austenitic conditions (different temperatures and holding times) is investigated using a very simple test method (a three-point bending system, wherein the sample is fixed at one end and simply supported at the other). The effects of the austenitic conditions on the transformation plasticity coefficient are also examined.

2 Theory

2.1 Kinetics of Transformation Plasticity

The constitutive equations assume uniaxial stress, in which only bending stress is applied. In general, the total strain rate \( \dot{\varepsilon} \) is the sum of the elastic, plastic, thermal expansion, transformation expansion and transformation plastic strain rates, represented by the superscripts \( e, p, T, ph, \) and \( tp \), respectively.

\[
\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p + \dot{\varepsilon}^T + \dot{\varepsilon}^{ph} + \dot{\varepsilon}^{tp}
\]

In this study, since the applied stress is smaller than the yield stress, it will not cause plastic deformation; therefore, the plastic strain rate \( \dot{\varepsilon}^p \) is 0. The specimen is assumed sufficiently small so that the temperature is uniformly distributed across its cross-sectional area during cooling. In this case, the total strain rate is unaffected by the transformation and thermal expansions. The elastic strain rate is

\[
\dot{\varepsilon}^e = \frac{\sigma}{E},
\]

where \( \sigma \) and \( E \) are the stress rate and Young’s modulus, respectively.

The constitutive equation of the transformation plastic strain rate is given by Greenwood and Johnson

\[
\dot{\varepsilon}^{tp} = 2K(1 - \xi)\sigma_t
\]

where \( K \) and \( \xi \) are the transformation plasticity coefficient and volume fraction of new phase, respectively.

This study adopts the Greenwood–Johnson mechanism because the sample was naturally cooled.

2.2 Basic equation of bending

The bending of a one-dimensional beam is investigated by the following relationship. According to Euler–Bernoulli bending theory, the strain \( \varepsilon \) and its rate \( \dot{\varepsilon} \) is given by

\[
\varepsilon = \frac{\eta}{R}
\]

\[
\dot{\varepsilon} = \frac{\dot{\eta}}{R^2t^2}
\]

where \( R \) and \( \eta \) are the radius of curvature and the distance from a neutral plane, respectively. Assuming very small slope of the beam \( |dy/dx| \ll 1 \) and defining the beam deflection profile by \( y = y(x) \) at a given time, the radius of curvature is approximated as

\[
\frac{1}{R} = \frac{d^2y}{dx^2}
\]

Moreover, considering the equilibrium state of the moment at a given cross-section, the bending moment \( M \) becomes

\[
M = \int_A \sigma\eta dA,
\]

where \( A \) is the cross-sectional area of the beam. Furthermore, under a constant applied load, the bending moment rate is given by

\[
M = \int_A \sigma\eta dA = 0.
\]

2.3 Deflection Profile

The deflection \( y \) is assumed as a function of position \( x \) and time \( t \) from the instant of loading.

\[
y(x, t) = \delta(t)g(x)
\]

\[
0 \leq g(x) \leq 1
\]

\[
\delta^e \leq \delta(t) \leq \delta_{\text{max}}
\]

Where \( \delta^e \) and \( \delta_{\text{max}} \) are the maximum elastic deflection and final maximum deflection, respectively and \( g(x) \) can be considered a function of the dimensionless deflection. Then,

\[
\frac{d^2y}{dx^2} = \frac{\delta(t)g(x)}{\sigma}
\]

By the Euler–Bernoulli bending theories of strain rate and beam curvature, the second integral in Eq. (9) equals the bending moment \( M \); therefore, the moment equilibrium equation can be written as

\[
I \frac{d\delta}{dt} \frac{d^2g(x)}{dx^2} + 2K(1 - \xi)\dot{\varepsilon}M = 0,
\]

where \( I \) is the geometrical moment of inertia.

The total strain in bending deformation is contributed solely by the transformation plastic strain. Therefore, the deflection under bending strain is due to the transformation plasticity alone.

It is considered that a single type of transformation occurs and that the Young’s modulus is identical in each phase. Finally, by integrating Eq. (14) from the loading start time \( t = 0 \) to the end of transformation \( t = t_0 \), and setting \( \delta = \delta^e \) and \( \xi = 0 \) at \( t = 0 \), and \( \delta = \delta_{\text{max}} \) and \( \xi = 1 \) at \( t = t_0 \), the differential equation for dimensionless deflection function is obtained as

\[
\frac{d^2g(x)}{dx^2} = \frac{KM}{(\delta_{\text{max}} - \delta^e)} - \frac{KM}{(\delta_{\text{max}} - \delta^e)I}
\]

where \( \delta^p = \delta_{\text{max}} - \delta^e \) is the transformation plastic deflection imposed by the bending stress. The Young’s modulus and other parameters are assumed independent of...
temperature. Introducing
\[ E' = \frac{\delta_{\text{max}} - \delta^e}{K} = \frac{\delta_{\text{tp}}}{K}, \]  
Eq. (15) can be rewritten as
\[ d^2g(x) = -\frac{M}{E'I} \]  
2.4 Transformation Plasticity Coefficient

A previous study of rapidly cooled SCM440 steel was conducted on a three-point bending system with simple supports\(^{12}\). Since the deflection results were moderately scattered, we here propose a new test method, in which one end is simply supported and the other is fixed. The new three-point bending system is illustrated in Fig. 1. The distance \( l \) between the two supports is 120.0 mm. The reactions of the simple and fixed supports are denoted \( R_1 \) and \( R_2 \), respectively, the moment at the fixed support is \( M_2 \), and the load \( W \) is applied at distance \( l/2 \) from each support. The bending moment along the beam is calculated from the moment equilibrium as

\[
M = \begin{cases} 
R_1x & \text{if } 0 \leq x \leq \frac{l}{2}, \\
R_1x - W(x - \frac{l}{2}) & \text{if } \frac{l}{2} \leq x \leq l.
\end{cases}
\]

Solving the differential equation (17), the deflection function \( g(x) \) is obtained as
\[ g(x) = \frac{Wl^3}{16EI} \left( \frac{5}{36}x^3 - \frac{1}{6}x^2 - \frac{l^2}{32}x \right). \]  
Here, the symbol in Macaulay’s brackets \( (x - \frac{l}{2})_+ \) is given by
\[ (x - \frac{l}{2})_+ = \begin{cases} 
0 & \text{if } x - \frac{l}{2} < 0, \\
x - \frac{l}{2} & \text{if } x - \frac{l}{2} \geq 0.
\end{cases} \]

The maximum deflection occurs at \( x = \frac{l}{2} \), i.e., \( g(\frac{l}{2}) = 1 \) in Eq. (19), the transformation plasticity coefficient \( K \) is then determined as
\[ K = \frac{48\sqrt{\pi}l}{Wl^3}(\delta_{\text{max}} - \delta^e) = \frac{48\sqrt{\pi}l}{Wl^3}\delta_{\text{tp}}. \]  
According to Eq. (10), the deflection \( y \) is a function of position \( x \) and time \( t \) from the loading start. Denoting the time at the end of transformation by \( t' \), the final maximum transformation plastic deflection at \( x = \frac{l}{\sqrt{\pi}} \) is written as
\[ \delta_{\text{tp}}(t') = \delta_{\text{tp}}(t)g \left( \frac{l}{\sqrt{\pi}} \right). \]

And the final (measured) transformation plastic deflection at the loading point (\( x = \frac{l}{2} \)) is written as
\[ \delta_{\text{tp}} = \delta_{\text{tp}}(t')g \left( \frac{l}{2} \right). \]  

The deflection function \( g(x) \) at \( x = \frac{l}{\sqrt{\pi}} \) and \( x = \frac{l}{2} \) is given by Eq. (19) as follows,
\[ g \left( \frac{l}{\sqrt{\pi}} \right) = \frac{Wl^3}{48\sqrt{\pi}EI} \]  
\[ g \left( \frac{l}{2} \right) = \frac{Wl^3}{7Wl^3}. \]

Solving Eqs. (21)–(25), the transformation plasticity coefficient at the loading point is obtained as
\[ K = \frac{768l}{7Wl^3}\delta_{\text{tp}}. \]  

Therefore, given the beam shape and the applied load, the transformation plasticity coefficient \( K \) can be calculated by measuring the maximum deflection at the loading point.

3 Experimental Apparatus and Material

The experiment is performed in a three-point bending system. The specimen is fixed at one end and simply supported at the other, and subjected to a bending stress between the two supports. The specimens are fabricated from S45C steel rods of volume 1.0 \( \times \) 8.0 \( \times \) 133.0 mm\(^3\). The chemical composition of the steel (S45C) is shown in Table 1.

Table 1  Chemical composition of S45C (wt.%).
| Element | C  | Si  | Mn  | P   | S   | Cu  | Ni  | Cr  |
|---------|----|-----|-----|-----|-----|-----|-----|-----|
|         | 0.46| 0.16| 0.83| 0.14| 0.17| 0.04| 0.03| 0.15|

The specimens were heated from room temperature to austenitization temperatures of 850, 900, and 950 °C in an electric infrared radiation furnace. Homogeneous austenitization heating of the specimens was performed in two steps. First, the specimen was heated at 15 °C/s from room temperature to 750 °C. Second, it was heated at 1 °C/s from 750 °C to the maximum austenitization temperature, where it was held for 5 or 20 min. After the holding process, the specimens were naturally cooled to room temperature. When its temperature had reduced to 800 °C (before the transformation plasticity was induced by the transformation), experiments were quadruplicated at each austenitization temperature and holding time. Moreover, transformation plastic deflection and cooling temperature for each specimen were measured. The specimen was cooled uniformly from 750 °C to the maximum austenitization temperature; the Alumel and Chromel terminals were welded at the loading point \((x = \frac{l}{\sqrt{\pi}})\) and at points \( \pm 40.0 \) mm from the loading point. Fig. 2 shows the cooling path of S45C at the loading point on a continuous cooling transformation (CCT) diagram\(^{18}\). Since the specimens were naturally cooled, their phases altered from austenite to pearlite in all experiments. Fig. 2 represents the temperature and transformation plastic deflection at the loading point, respectively. When the austenitization heating of the specimens was performed in two steps, first, the specimen was heated at 15 °C/s from room temperature to 750 °C. Second, it was heated at 1 °C/s from 750 °C to the maximum austenitization temperature, where it was held for 5 or 20 min. After the holding process, the specimens were naturally cooled to room temperature. When its temperature had reduced to 800 °C (before the transformation start point), each specimen was subjected to its maximum bending stress (25.5 MPa; approximately half the yield stress). The same bending stress was applied under all experimental conditions (all austenitization temperatures and both holding times). The deflection at the loading point (\( l/2 \) from the supports) was measured with a laser sensor and the temperature was measured by type K thermocouples spot-welded to the lower surface of the specimens. In the temperature measurements, one thermocouple was fixed at the loading point of every specimen; two more
thermocouples were placed ±40 mm from the loading point on one of the specimens at each austenitization temperature and holding time. For precisely determining the specimen temperature, the Alumel and Chromel terminals were welded 3 mm apart. To ensure accurate bending deflections, the experiments were quadruplicated at each austenitization temperature and holding time.

4 Results and Discussion

4.1 Transformation Plastic Deflection

The experiments were performed on S45C steel under six austenitic conditions; austenitization temperatures of 850, 900, and 950 °C; and holding times of 5 and 20 min at each temperature. The maximum deflections at the loading point induced by the transformation plasticity were measured. Since the specimens were naturally cooled, their phases altered from austenite to pearlite in all experiments. Fig. 2 shows the cooling path of S45C at the loading point on a continuous cooling transformation (CCT) diagram. According to this figure, the cooling curves of 5 and 20 min holding times coincide at each temperature. However, as the austenitization temperature increases, the time of cooling to the start of the transformation accordingly increases.

![Temperature distribution during cooling of a steel specimen.](image)

Fig. 3 shows the temperature distribution during the cooling process along the length of one specimen held at 950 °C for 20 min. The horizontal and vertical axes show the temperature of the loading point and the temperature difference between the loading point and at points ±40.0 mm distant from the loading point, respectively. Because both supports require holding structures, it is difficult to uniformly cool the specimen along its entire length; consequently, the temperature at the measured points differed by approximately 10 °C at maximum. However, since the specimen is very thin, the temperature distribution can be assumed uniform throughout its cross-section. Fig. 3 is representative of the study results, since the distributions at all austenitic conditions (850, 900, 950 °C) and both holding times (5 and 20 min) were investigated by the same experimental procedure.

![Temperature distribution](image)

Figs. 4, 5 and 6 plot the relationships between the transformation plastic deflection and cooling temperature for steel bars heated to 850, 900 and 950 °C, respectively. The holding times are 5 and 20 min. The abscissa and ordinate represent the temperature and transformation plastic deflection at the loading point, respectively. When the cooling specimen experiences a bending stress in the austenite phase, it undergoes elastic deflection. Once the specimen begins phase transformation (at approximately 700 °C), its deflection rapidly increases to the final deflection. Therefore, the plastic deflection at transformation is obtained by subtracting the elastic deflection from the final deflection. As previously mentioned, experiments were performed four times under each austenitic condition (Tests 01–04). Figs. 4–6 show that the transformation plastic deflections of the quadruplicated experiments almost coincide and decrease with increase in the austenitization temperature and holding time. Moreover, transformation under the highest austenitization temperature and holding time (950 °C for 20 min) begins sooner than the other conditions (730 °C versus 700 °C under the other conditions). This reduction is caused by the increased holding time.

The temperature of specimens decreases (increases) by absorbing (generating) latent heat. In thermomechanical analysis of phase transformation, these changes are attributed to enthalpy changes during heating or cooling processes. In Figs. 4–6, the temperature of specimens subjected to 5 min holding time slowly decreases and increases during phase transformation, whereas the changes are rapid in specimens subjected to 20 min holding time.

4.2 Transformation Plasticity Coefficient $\delta_{TP}$

The transformation plastic deflection $\delta_{TP}$ was experimentally obtained in the three-point bending system.
that is fixed at one end and simply supported at the other. The transformation plasticity coefficients under each austenitic condition were calculated by Eq. (26) and are tabulated in Table 2. According to the experimental results, the transformation plasticity coefficient decreases with decreasing transformation plastic deflection. The relationship between the transformation plasticity coefficient \( K \) and austenitization temperature \( T_{\text{aust}} \) is plotted in Fig. 7.

Specimens with short austenitization time or low austenitization temperature presumably have small AGS. Thus, we suppose that decreasing the AGS increases the boundary area per unit volume of the specimen. If the new phase nucleates at the grain boundary, the number of nucleation sites should increase with decreasing AGS. According to the Greenwood–Johnson model, the progress of the transformation plastic strain with micro deformations of the parent phase arises from the volume difference between the parent and new phases. When the phase transformation is imposed by applying an external stress to the supercooled austenite, the product phase grows at the expense of the parent phase in a certain direction, yielding transformation plasticity strain\(^{1,4,13}\). Therefore, the transformation plastic deflection might be increased by refining the grain size.

At low temperatures, most metallic alloys exhibit the Hall–Petch\(^{19,20}\) effect, in which their strength increases with decreasing grain size. This effect is explained by the resistance of the boundaries to plastic flow. The grain boundaries in polycrystalline materials strongly inhibit dislocation motions at low temperatures (i.e., at \( T < 0.5T_m \), where \( T_m \) is the melting point in °K). Conversely, at high temperatures, the grain boundaries are weak and prone to sliding, potentially leading to plastic flow or void openings along the boundaries\(^21\).

S45C steel melts is approximately 1793 °K. In this study, the austenitization temperatures exceeded 0.5 \( T_m \), and the transformation plastic deflections of the specimens decreased with increasing AGS. In our future study, the AGS effect on martensitic transformation plasticity will be examined in a three-point bending system under the same austenitic conditions. The new results will be compared with those of the present study.

Comparing the transformation plasticity coefficients obtained in this study with those of the tensile tests\(^4,13\), i.e., a four-point bending test\(^10\) and three-point bending tests with simple and fixed supports\(^13\), our results are found to be larger and slightly smaller than the previously reported results at 5 and 20 min holding times, respectively.
deflection and temperature for a steel bar heated to 900 °C and held

deflection and temperature for a steel bar heated to 950 °C and held

Fig. 6  Relationship between measured transformation plastic

TP

m

m

m

Fig. 4  Relationship between measured transformation plastic

Fig. 7  Relationship between transformation plasticity coefficient

and austenitization temperature at holding times of 5 (diamonds)

and 20 min (triangles).

5. Conclusions

In this study, we evaluated the effects of austenitic conditions on the transformation plasticity coefficient of steel. Experiments were performed in a three-point bending system, wherein the test specimens were fixed at one end and simply supported at its other. The specimens were fabricated from S45C steel and heated to different austenitization temperatures, where they were held for 5 or 20 min. Next, the specimens were naturally cooled to room temperature and subjected to identical bending stress. The bending stress induced an austenite–pearlite transformation in all specimens. Finally, the transformation plasticity coefficients of the steel were determined from the transformation plastic deflections at the loading point. According to the experimental results, the transformation plasticity coefficient decreases as the austenitization temperature or holding time increases. This suggests that the transformation plasticity coefficient of the pearlitic transformation of S45C is a decreasing function of the AGS.

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| Holding time | Temperature | 5 min | 20 min |
|--------------|-------------|-------|--------|
| 850 °C       | 11.0        | 9.31  |
| 900 °C       | 10.4        | 8.79  |
| 950 °C       | 10.2        | 7.67  |
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