A cognitively driven weighted-entropy model for embedding semantic categories in hyperbolic geometry

Eugene Yu Ji
<yuj1@uchicago.edu>
Department of Psychology, The University of Chicago

Abstract
In this paper, an unsupervised and cognitively driven weighted-entropy method for embedding semantic categories in hyperbolic geometry is proposed. The model is driven by two fields of research in cognitive linguistics: the first is the statistical learning theory of language acquisition and the proposal of using high-dimensional networks to represent semantic knowledge in cognition, and the second is the domain-specific approach to semantic communication. Weighted conditional entropy of word co-occurrence is proposed as the embedding metric, and the two weighting parameters are collocation diversity and conditional probability ranking in the corresponding statistical distribution. The Boltzmann distribution is then used on the weighted-entropy metric and embedded into a hyperbolic Poincare disk model. Testing has been in particular performed in the domains of basic color and kinship words, which belong to the classes that domain-specificity focused research in cognitive semantics has most intensively investigated. Results show that this new approach can successfully model and map the semantic relationships of popularity and similarity for most of the basic color and kinship words in English and have potential to be generalized to other semantic domains and different languages.

1. Introduction
Recent modeling methods of natural language semantics have been heavily influenced by work in computational linguistics (CL) and natural language processing (NLP). Though some research demonstrates that a number of CL and NLP models can also be applied to modeling and testing language cognition as well (Chersoni et al. 2021, Wu & Ettinger 2021), the design, training, and testing of language models in CL and NLP are still not cognitively driven or tested in general. Unsupervised language models in CL and NLP have achieved tremendous success in recent years (for example, BERT in Devlin et al. 2018, GPT-3 in Brown et al. 2020). These models, however, need to rely on large-size textual data which are usually billions of word tokens, and tasks for testing these models are often designed according to researchers’ folk knowledge of natural language, rather than drawing work from other branches of linguistics, such as formal linguistics, cognitive linguistics, and sociolinguistics.

Arguably, language cognition and processing by the human brain still suggests far more effective and efficient computational strategies for comprehending and processing language phenomena than current language models in CL and NLP. Language learning by humans relies on much smaller sizes of linguistic input and shows much greater flexibility and interpretative capacity. Hence, one crucial direction of further research on developing language models is to draw more lessons from language learning and processing by the human brain, and develop methods and models that are more cognitively driven or inspired.

This paper proposes a cognitively-driven embedding methodology for modeling semantic categories. The model is related to cognitive linguistics in two aspects. First, the design of the model is inspired by both statistical learning theory and semantic network theory of language acquisition and processing in cognitive linguistics (Aslin et al. 1998, Jackson & Bolger 2014, Saffran 2003, Steyvers & Tenenbaum 2005). Second, the testing of the model is based upon a specific research line in cognitive linguistics for building a computational-cognitive model of the basic-level semantic categories (Regier et al. 2015, Regier & Xu 2017, Zaslavsky et al. 2019). In a way, the two research traditions are complementary to each other for modeling human semantic knowledge, and one main intended contribution of the paper is to incorporate the strength of both approaches. Developing a weighted-entropy metric and an embedding method based upon such a metric in hyperbolic space, this paper develops an
unsupervised learning model that maps the “popularity-similarity tradeoff” for the semantic relationships of basic-level categories in English. Further, it only relies on small to medium-size raw textual corpus (typically millions of word tokens), without the need of pre-trained data or pre-determined semantic relationships such as WordNet (Fellbaum 1998).

2. Related work

Statistical learning theory proposes that infants have the capacity of learning certain aspects of phonology, syntax, and semantics of natural language from statistical patterns of sound or word co-occurrence in the language input from the environment (Aslin et al. 1998, Saffran 2003). Evidence from empirical studies that attempt to support this theory, however, faces difficulties in representation and generalization. During the last two decades, cognitive scientists have proposed various approaches for extracting statistical knowledge of natural language, and represent such knowledge in the high-dimensional vector space (Steyvers & Tenenbaum 2005, Jackson & Bolger 2014).

Many cognitive scientists criticize the statistical learning and semantic space/network approaches because they do not appear to be sufficient for modeling language acquisition and cognition. Typical critiques include that statistical learning mechanism is too generic to learn the whole human language repertoire, that the high-dimensional representation of language knowledge is too thin to represent language (Steyvers & Tenenbaum 2005), and that both approaches ignore the fundamental interactional and cultural characteristics of human language.

Alternatively, one important line of cognitive-driven semantic models starts with the assumptions that semantic knowledge is related to sensation and perception, hence specific to sensual and perceptual domains, and that semantic knowledge is formed and represented in communication. Modeling studies based on these assumptions develop a domain-specific and interaction-based “informativeness” hypothesis of semantic cognition and have achieved tremendous success in testing with basic-level semantic categories in the domains of color, kinship, and shape across multiple languages (Regier & Xu 2017, Kemp & Regier 2012).

The domain-specific and informativeness approach, nonetheless, faces the difficulties of obtaining empirical data across domains and languages. Testing the domain-specific informativeness hypothesis has to rely on collecting large empirical data from semantic categorization tasks in real human communication, hence it is difficult for researchers to replicate studies that focus on one specific sensory domain in a language to another domain in the same or another language.

The general motivation of the investigation here is to develop a model that incorporates and develops the principles and methods of statistical learning and high-dimensional representation of semantic knowledge, which at the same time is capable of learning both domain-specific and domain-general types of semantic knowledge from corpus or language input data. The goal is to develop a model that, on the one hand, is able to extract semantic knowledge as rich as the domain-specific approach suggests and, on the other hand, can maintain the computational strength of the statistical learning and semantic space/network approaches that is able to model semantic knowledge directly from textual or language input data. Current testing based on English corpus suggests that the model can extract a tradeoff relationship between popularity and similarity for modeling basic-level semantic categories in the domains of color and kinship, the results of which can be fruitfully compared with and tested by the cognitive research based on the domain-specificity and informativeness hypothesis of natural language semantics.

3. Building the model
3.1 The weighted-entropy metric: conditional entropy weighted by two network-related parameters

The core component of the model is a new basic metric for measuring the correlative relationship between word co-occurrence statistics extracted from textual corpus or language input data. The metric is constituted by the target word’s conditional entropy upon occurrences of its context words. The innovation of the model is that the word conditional entropy
is weighted by two parameters inspired by concepts from statistical and network theories of word semantics. The first parameter \( d \) is the co-occurrence diversity of the target word. Given the target word \( w_a \), its co-occurrence diversity \( d \) is equal to \( f(w_a) / \text{count}(w) \), where \( f(w_a) \) is the frequency of the target word \( w_a \) in a given corpus or language input data, and \( \text{count}(w) \) is the count of the type number of each context word \( w \) that co-occurs in a given length of the context window adjacent to the target word \( w_a \). The second parameter \( r \) is equal to the rank of conditional probability \( f(w_i \mid w_j) \) among all \( f(w_i \mid w \in W) \), where \( w_i \) is the target word, \( w_j \) is the context word of \( w_a \), and \( W \) is the whole vocabulary. It is convenient to express \( d \) and \( r \) together as a single weighting parameter \( q \).

Proposing \( d \) and \( r \) as weighing parameters for conditional entropy of single words are driven by the concept of semantic network, in which word semantics is understood as determined by the network properties of semantics. Collocation diversity is intrinsically a network property which has been used in previous work on semantic modeling (Vincze 2015). Ranking of probability values in statistical distributions is demonstrated in network theory to be relevant to hierarchical and geometrical characteristics of networks (Krioukov et al, 2010). Nonetheless, to the author’s knowledge, no work has yet explicitly applied this specific parameter to language modeling.

Formally, the following formulation of a weighted entropy-based metric \( H \) of word embedding is proposed:

\[
H(w_i \mid w_j) = -q_{ab} f(w_i, w_j) \log(q_{ab} \frac{f(w_i, w_j)}{f(w_j)})
\]

\[
= -q_{ab} f(w_i, w_j) \log(q_{ab} f(w_i, w_j)) \quad (1)
\]

where \( q_{ab} = \frac{d}{r_{max} + 1} \)

\[
= \frac{f(w_i \mid w_j)}{\text{count}(w_j) - 1} \log(1 + \text{rank}[f(w_i \mid w_j)] \text{count}(w_j) \text{count}(w \in W)) \quad (2)
\]

Equation (1) is the formula of the weighted conditional-entropy metric \( H \) for measuring co-occurrence relationship between the target word \( w_a \) and any context word \( w_b \), given the data of textual input. Equation (2) is the formula of the weighting parameter \( q_{ab} \), which is a linear function of \( d \) and \( r \). Both \( w_a \) and \( w_b \) are extracted from the vocabulary of the whole corpus of language input \( W \).

The model further develops the distributional and statistical hypothesis by defining the target word \( w_i \) as the Boltzmann distribution \( P \) according to its corresponding values of the weighted conditional-entropy metric \( H \), as expressed in equation (3):

\[
P(w_i \mid w_j) = \frac{1}{Z_{ij}} \exp(H(w_i \mid w_j)) \quad (3)
\]

where \( Z_{ij} = \sum_j \exp(H(w_i \mid w_j)) \)

The weighted word co-occurrence matrix for the vocabulary of the whole textual or input corpus is thus defined by Boltzmann distributions as in \( P \) in terms of \( H \) as in equation (3), where each element \( M_{ij} \) of the matrix \( M \) is \( P(w_i \mid w_j) \).

### 3.2 Similarity measure through weighted set intersection

The model then contains the following two modules for extracting semantic characters of basic-level categories. Based on matrix \( M \), the first module named module (a) uses the sets of common context words to extract target words that share the highest degree of common contexts. Two specific extraction strategies are performed for the extraction of words that maximally share common context words, and the intersection of these words is generated from the two algorithms to produce the final result of words that are semantically closest to the target word.

Specifically, let \( l(w) \) be the number of non-zero values of \( P(w \mid w_j) \) for \( w_i \) in \( M \). For any chosen target word \( w_a \), let \( l^{[a]}(w) \) be the number of non-zero values of \( P(w_i \mid w_j) \) and \( P(w_a \mid w_j) \) conditioned by every context word \( w_j \) commonly shared by the chosen word \( w_a \) and any
where \( w \) represents the mean value \( P \), to polar geometry. The mapping for the selected word on a Poincare disk model in hyperbolic similarity in a hyperbolic Poincare disk model

3.3 Mapping semantic popularity and similarity in a hyperbolic Poincare disk model

The second module named module (b) maps the words that are selected by module (a) on a Poincare disk model in hyperbolic geometry. The mapping for the selected word \( w_i \) is first through projecting two measures, \( P(w_j|W_i) \) and \( P(W_j|w_i) \), to polar coordinates, where \( P(w_j|W_i) \) represents the mean value of the Boltzmann distributions for \( w_j \) given all its context words \( w_s \), and \( P(W_j|w_i) \) represents the mean value of the Boltzmann distributions for every \( w_j \) being treated as a target word and conditioned upon \( w_i \) as the context word. These two measures are then transformed to the open unit Poincare disk. Specifically, letting \( x = P(w_i|W_j) \) and \( y = P(W_j|w_i) \), \( x \) and \( y \) are first transformed into polar coordinates:

\[
\rho = x^2 + y^2
\]

\[
\theta = \arctan \left( \frac{y}{x} \right);
\]

and then to the open unit Poincare disk model \((\rho', \theta')\):

\[
\rho' = \frac{1}{2} \ln \left( \frac{1 + \theta}{1 - \theta} \right)
\]

\[
\theta' = \theta.
\]

Results in the testing (section 4.2) show that semantic popularity and typicality can be mapped in the hyperbolic dimension \( \rho' \), and semantic similarity can be mapped in the hyperbolic dimension \( \theta' \).

4. Experiments and results

Testing based on English wikipedia corpus in the domains of color and kinship words show that the model can extract coarse-grained word similarity relationships from small-sized corpus data at least as effectively as the classic word embedding method does. Importantly, testing shows that the model is capable of mapping fine-grained popularity and similarity relationships for the semantics of basic-level categories in the domains of color and kinship in English, according to the baseline characterized by extensive research in cognitive linguistics on these domains.

4.1 Word similarity

The first set of tests compares the performance of module (a) of the model (section 3.2) with the word2vec method in word embedding (Mikolov et al. 2013). Testing is performed on the domains of color and kinship words on two English Wikipedia data of the
same size (8M word tokens for each corpus). A number of words not in the domain of color or kinship are also used in the testing for the sake of comparison (data not shown).

For the color and kinship domains, 10 basic color words and 12 kinship words are modeled by both methods (“red”, “yellow”, “green”, “blue”, “white”, “black”, “orange”, “pink”, “purple”, “gray” for the domain of color; “father”, “mother”, “son”, “daughter”, “brother”, “sister”, “husband”, “wife”, “uncle”, “cousin”, “grandfather”, “grandmother” for the domain of kinship). These words are chosen based on the relevant research on domain-specific semantic categories in cognitive linguistics (Kemp & Regier 2012, Zaslavsky et al. 2019). Both the word2vec and weighted-entropy models extract words that are semantically close to each chosen target word in color and kinship domains (Table 1).

Since this model takes a cognitive position for modeling semantics, the testing does not consider comparisons with most of the NLP models that have to be trained on corpus usually in the size of billions of word tokens.

Testing is run on two 8M word Wikipedia corpuses and Table 1 selects results of four target words from testing on one corpus. The results suggest the following observations:

(i). Word2vec and the weighted entropy methods perform at similar levels for extracting basic color words that are semantically close to the most typical color words (for example, “red” and “blue” in the 1st and 2nd columns, tables 1.1 and 1.2);

(ii). The weighted entropy method performs much better on less typical basic color words (such as “purple” in the 3rd column, tables 1.1 and 1.2). It also performs well on modeling three generic terms of color in English (“color”, “colour”, “colors”) for both typical and less typical basic color words, which the word2vec method does not model well.
(iii). Performance on kinship terms is varied. For example, for the target word “brother”, the weighted entropy method successfully extract three basic kinship words as the semantically closest ones (“son”, “daughter”, “sister”; 4th column, table 1.2) better than the word2vec method does; yet overall, the word2vec method extracts four more basic kinship terms (“father”, “husband”, “mother”, “sons”; 4th column, table 1.1) than the weighted-entropy method does.

Overall, for the Wikipedia English corpus of 8M word size, the word2vec and weighted entropy methods are both relatively robust in extracting semantic neighborhoods for the selected words and semantic domains, with slight variations.

4.2 Fine-grained mapping of within-domain semantic popularity and similarity relationships in a Poincare disk model for basic color and kinship words

Built upon the same weighted entropy metric, the second set of tests on module (b) (section 3.3) shows that the model is capable of mapping the fine-grained popularity and similarity relationships of word semantics in hyperbolic geometry. As previously mentioned, semantic relationships like popularity and similarity have been computationally and empirically studied under the informativeness hypothesis in cognitive science. Yet the latter approach is not able to extract semantic relationships directly from corpus data or language input. On the other hand, researchers in CL and NLP have begun to develop methods of embedding categorical and hierarchical semantic relationships in hyperbolic geometry. Although theoretically, hyperbolic space is demonstrated to be optimal for embedding tree-like hierarchical relationships, empirical semantic embedding in hyperbolic space often needs to rely on predetermined semantic hierarchy such as WordNet, or only tests narrowly defined semantic hierarchy like hypernym-hyponym relationships that are driven by WordNet.

Results based on the testing of module (b) shows that the model has potential in developing a hyperbolic embedding method which is both unsupervised and cognitively driven. Cognitive research on basic color terms ranks the typicality of the 10 basic color words in English as follows: dark and light colors\(^2\) > “red” > “yellow and green” > “blue” > “purple”, “pink”, “orange”, “gray” (Berlin & Kay 1969, Regier et al. 2015). Figure 1 shows the results of mapping 10 basic color words in English based on the weighted-entropy metric in the Poincare disk model (\(\rho’, \theta’\)). In the results:

(i). All 10 basic color words are mapped closely to one another in the similarity axis, with “red”, “yellow”, “green”, “blue”, “white”, “black” being mapped as more similar to one

\(^2\) which can but not necessarily be “black” or “white”.

![Figure 1](image)
another, and “pink”, “orange”, “gray”, “purple” being mapped as more similar to one another, respectively.

(ii). “white” is ranked as the most typical color word.

(iii). “white”, “black”, “red”, “blue”, “yellow”, “green” form a cluster of the more popular and typical color words.

(iv). “gray”, “purple”, “orange”, and “pink” are mapped as the less typical or popular colors word group, with the order of “gray” > “purple” > “orange” > “pink”.

Overall, the results show a clear differentiation between the more and less typical color words and within the four least typical color words, yet it does not show an obvious difference in terms of typicality/popularity within the six most typical basic color words.

Figure 2. Mapping of 12 basic kinship words in English: from bottom left (more popular) to upper right (less popular): “father” & “mother”, “son”, “wife”, “brother”, “daughter”, “sister”, “husband” “uncle”, “cousin”, “grandfather” & “grandmother”.

Figures 2 to 4 show the results in the domain of kinship:

(i). The model predicts that among the 12 kinship terms in English, “mother”, “father”, “son”, “daughter”, “brother”, “sister”, “husband”, and “wife” form the cluster of the most typical kinship terms, “uncles” and “cousin” form the less typical, and “grandfather” and “grandmother” form the least typical (Figure 2).

(ii). In particular, the model ranks the words “father” and “mother” as equally popular, and “grandfather” and “grandmother” as equally atypical and much less typical than the pair of “father” and “mother”. (Figure 3). This observation follows well the hypothesis and data on kinship terms in English and many other languages in the relevant research in cognitive linguistics and cognitive anthropology (Hirschfeld 1986, Kemp & Regier 2012).

(iii). Interestingly, the model maps the respective relationships between “man” and “woman”, “son” and “daughter”, and “brother” and “sister” as almost parallel to one another in the Poincare disk. On the other hand, unlike what the most common hypothesis on the regularity of kinship terms in cognitive science and anthropology assumes, the model predicts that “man”, “son”, and “brother” are respectively more typical than “woman”, “daughter”, and “sister” to a similar extent (Figure 4). Unlike hypernym-hyponym relationships formulated and annotated in simplified cognitive or computational settings (such as WordNet), this observation may be relevant to how hierarchical relationships between semantic categories are actually formed and cognized in specific social and cultural
contexts, which is represented in the corresponding corpus data.

Figure 4. The respective semantic relationships between “man” and “woman”, “son” and “daughter”, and “brother” and “sister” are mapped as almost parallel to one another in the hyperbolic space. On the other hand, the mapping predicts that “man”, “son”, and “brother” are respectively more typical than “woman”, “daughter”, and “sister” to a similar extent. This result does not assume or follow the hyponym-hypernym relationships formulated and annotated in the simplified cognitive and computational settings such as WordNet.

4.3 Comparisons with other dimensionality reduction methods: PCA, Isomap, spectral clustering

Besides the hyperbolic Poincare disk model, dimensionality reduction of the matrix $M^p$ is also performed through linear and other nonlinear algorithms such as principal component analysis (PCA), Isomap, and spectral clustering. None of them show results that are obviously meaningful or interpretable (data not shown). Theoretically, it is plausible that beyond hyperbolic geometry, dimensionality reduction in other types of geometry can also be applied to $M^p$ and yield meaningful results. Current results, however, do not show evidence to support that.

5. Discussion

Here I focus on four aspects of this model, the first of which is on the relationship between cognition and computation in the model, the second is on the relationship between this model and the current literature on hyperbolic embedding in CL and NLP, the third is on the questions of domains, and the fourth is on testing.

5.1 Computation and cognition

As mentioned in section (2), the modeling methods that this paper develops are related to two types of computational models of language acquisition and cognition. The first is the statistical learning theory of language and the theory of high-dimensional semantic space and network, and the second is the domain-specific informativeness hypothesis of language communication. The current model intends to develop a methodology based on the assumptions of statistical learning and high-dimensional semantic networks, the theories of which are highly computationally tractable and data-driven. At the same time, it intends to develop a methodology for modeling and testing, using fine-grained baselines from the research on domain-specific semantic categories in cognitive linguistics, which have provided a rich modeling framework and empirical cognitive data for two important aspects of semantic knowledge in multiple semantic domains: the fine-grained tradeoff between semantic similarity and popularity.

Similar to previous work on statistical learning and semantic space (Jackson & Bolger 2014, Steyers & Tenenbaum 2005), this work does not argue that statistical learning or semantic space or network provide a whole theory of semantic cognition; rather, I make the assumption that word co-occurrence and semantic space and network may form part of the groundwork for a mature semantic theory. For the domain-specific and informativeness approaches, the results from the current testing does not either support or refute their hypotheses, as the model does not investigate any assumption on the process of language communication as the informativeness approach does. It instead models certain aspects of semantic knowledge acquisition in language learning and the consequences of such learning. Developing links between this study and the informativeness approach requires developing a new agent-based modeling method based on the current model, which is beyond the scope of this paper.
5.2 Relations with hyperbolic embedding in computational linguistics and NLP

Embedding hierarchical relationships in hyperbolic geometry has increasingly become a new research paradigm in CL, NLP, and computational neuroscience particularly in recent five years (Nickel & Kiela 2017, Sala et al. 2018, Dhingra et al. 2018, Sharpee 2019). Yet current work on modeling semantic relationships in hyperbolic geometry faces a main issue: training or testing in most of the work heavily relies on artificially and narrowly determined semantic relationships such as WordNet, which are not directly constructed in a data-driven fashion or in the context of real language use. Most of the work on hyperbolic embedding has no clear indications how much their methodology is related to the cognitive aspects of semantic knowledge. The modeling approach proposed in this paper, instead, relies upon established cognitive theories of semantic learning, and tests results based on the baselines generated from a well-developed research field in cognitive linguistics on domain-specific semantic knowledge. Nonetheless, the model also contributes to the hyperbolic embedding literature for the CL and NLP communities. To the author’s knowledge, the model provides the first unsupervised modeling methodology that is capable of learning and testing fine-grained hierarchical semantic knowledge beyond simplistically defined hypernym-hyponym relationships, and it only requires a small to medium sized corpus of unlabeled data.

5.3 The problems of domains

As indicated, the reason that the current testing mainly focuses on the domains of color and kinship words is that research in cognitive linguistics provides rich theoretical and empirical groundwork for semantic cognition in these domains. Empirical baselines for evaluating the semantic relationships of basic color and kinship words are among the most developed in domain-specificity focused cognitive linguistic research. Though tests and applications of the model in principle are not and should not be constrained only by these two domains, much work in cognitive science beyond the color and kinship domains face difficulties in selecting common cognitive baselines as empirically validated as in the domains of color and kinship terms. On the other hand, the model is also able to help generate new testable hypotheses for investigations on the semantic domains that lack cognitive data or face empirical difficulties in cognitive research. It provides a modeling tool for linking inquiries on domain-specific and domain-general semantic categories.

5.4 Future testing

Besides testing in other domains and domain-general cases (as indicated in section 5.3 and footnote 3), further work on testing will mainly focus on four aspects: effects of varying text genres and data sizes, effects of varying context window sizes, modeling polysemy, and modeling semantic categories in different languages.

Figure 5a. In the Poincare disk model, the mapping of each color word starts to converge during the 4th or 5th epochs.

3 In the literature of domain-specific cognitive linguistics, the domains of shape and number also provide good data and baselines for further testing and applying the model, and computationally investigating cross-domain semantic relationships, which empirical cognitive studies also focus on. Testing on these two domains and on the cross-domain cases will be published in upcoming papers. One noticeable domain in current cognitive research that invokes tremendous research interests, but nonetheless faces empirical difficulties as discussed in this section, is the domain of the language of smell (Majid 2021). Applying the model to the domain of smell language is also part of the ongoing work.
after the model being trained on 5 to 6 M words. Each epoch runs on 1M word tokens. The 0th epoch = 1M words. Each color corresponds to the basic color word in English. In this figure, each color-coded number represents the corresponding epoch number in which the color word is trained upon.

Current testing shows that results are relatively robust above 5 million tokens of different Wikipedia data in English (Figures 5a, 5b), yet it is not clear to what extent the resting results would be varied if texts in different genres are used, and what consequences such variations would indicate.

Figure 5b. Top: Distance differences on the dimension of similarity $\theta'$ between color words across epochs. The mapping of most words (except “gray”) begins to converge during the 4th epoch after being trained on 5M word tokens. Bottom: Distance differences on the dimension of popularity $\rho'$ between color words across epochs. The mapping of most words (except “pink”) begins to converge around the 5th epoch after being trained on 6M words.

For the size of the context window, current testing only uses 1 and 3 context words as the context window size. Future testing would focus on systematically varying the context window sizes and investigate their consequences. Attention mechanisms would also be added for different window sizes for further enhancing the model.

For modeling polysemy, current literature in CL and NLP has developed rich algorithm repertoires for incorporating polysemy into language embedding (e.g. Niu et al. 2019). Future work will explore whether it is optimal to incorporate some of these algorithms into the model, or to develop an algorithm for modeling polysemy within the framework of the weighted-entropy method itself.

Lastly, certain algorithmic modules in this model need to be modified to accommodate semantic categories in languages that are not linguistically similar to English. For example, words in Chinese and Japanese are formed by thousands of single characters. In CL and NLP, a step of word segmentation is usually required for embedding algorithms to run on Chinese and Japanese texts. Initial evidence demonstrates that the weighted-entropy approach can be also adapted to model such word segmentation in non Indo-European languages, and it is plausible that such an adaptation would only require adding an additional module that is computationally intrinsic to the model itself. Further work needs to be done for verifying the capability of the model on semantics of different natural languages.

6. Conclusion

In this paper I propose a cognitively driven and unsupervised weighted-entropy method for modeling semantic categories in hyperbolic geometry. The model is driven by various theories and hypotheses in two branches of cognitive linguistics. Conditional entropy of word co-occurrence is used as the embedding metric, weighted by collocation diversity and the ranking of conditional probability in the corresponding statistical distribution. The Boltzmann distribution is then used on the
weighted entropy metric and embedded into a hyperbolic Poincare disk model. Testing has been mainly performed in the domains of basic color and kinship, which belong to the classes that the domain-specificity focused research in cognitive semantics has most intensively investigated. Current results show that the new modeling approach can successfully model and map the semantic relationships of popularity and similarity for basic color and kinship words in English. In general, this paper provides new modeling and testing methodologies for investigating semantic categories of natural language. It contributes to computational cognitive semantics as well as the research on network-driven and geometric language embedding in computational linguistics and natural language processing.

Acknowledgements
The author would like to acknowledge the funding support of this work by the Doctoral Fellowship of the Social Sciences Division, Norman H. Anderson Award of the Department of Psychology, and the Center for International Social Sciences Research (CISSR) Dissertation Support Award at the University of Chicago. The author would also like to thank Leslie Kay, Allyson Ettinger, Howard Nusbaum, and John Goldsmith’s suggestions and feedback at various stages of the work.

References
Richard Aslin, Jenny Saffran, Elissa Newport, Elissa. 1998. Computation of Conditional Probability Statistics by 8-Month-Old Infants. Psychological Science. 9 (4): 321–324.

Brent Berlin, Paul Kay. 1969. Basic Color Terms: Their Universality and Evolution. Berkeley: University of California Press.

Tom B. Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, Sandhini Agarwal, Ariel Herbert-Voss, Gretchen Krueger, Tom Henighan, Rewon Child, Aditya Ramesh, Daniel M. Ziegler, Jeffrey Wu, Clemens Winter, Christopher Hesse, Mark Chen, Eric Sigler, Mateusz Litwin, Scott Gray, Benjamin Chess, Jack Clark, Christopher Bener, Sam McCandlish, Alec Radford, Ilya Sutskever, Dario Amodei. 2020. Language Models are Few-Shot Learners. arXiv:2005.14165

Emmanuele Chersoni, Enrico Santus, Chu-Ren Huang, Alessandro Lenci; Decoding Word Embeddings with Brain-Based Semantic Features. Computational Linguistics 2021; 47 (3): 663–698. doi: doi.org/10.1162/coli_a_00412

Bhuwan Dhingra, Christopher J. Shallue, Mohammad Norouzi, Andrew M. Dai and George E. Dahl. 2018. Embedding Text in Hyperbolic Spaces.TextGraphs@NAACL-HLT.

Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. 2018. BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding. arxiv:1810.04805

Christiane Fellbaum. 1998. ed.. WordNet: An Electronic Lexical Database. Cambridge, MA: MIT Press.

Alice Jackson and Donald Bolger. 2014. Using a high-dimensional graph of semantic space to model relationships among words. Front. Psychol. 5:385. doi.org/10.3389/fpsyg.2014.00385

Charles Kemp and Terry Regier. 2012. Kinship categories across languages reflect general communicative principles. Science. vol. 336,6084: 1049-54. doi:10.1126/science.1218811
Lawrence Hirschfeld. 1986. Kinship and Cognition: Genealogy and the Meaning of Kinship Terms. Current Anthropology, 27(3), 217–242.

Dmitri Krioukov, Fragkiskos Papadopoulos, Maksim Kitsak, Amin Vahdat, and Marián Boguñá. 2010. Hyperbolic geometry of complex networks Phys. Rev. E 82, 036106

Ninghao Liu, Qiaoyu Tan, Yuning Li, Hongxia Yang, Jingren Zhou, and Xia Hu. 2019. Is a Single Vector Enough? Exploring Node Polysemy for Network Embedding. In Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining (KDD ’19). Association for Computing Machinery, New York, NY, USA, 932–940. DOI:https://doi.org/10.1145/3292500.3330967

Asifa Majid. 2021. Human olfaction at the intersection of language, culture, and biology. Trends in Cognitive Sciences, 25(2), 111–123. doi.org/10.1016/j.tics.2020.11.005

Tomas Mikolov, Kai Chen, Greg Corrado, and Jeffrey Dean. 2013. Efficient estimation of word representations in vector space. arXiv preprint arXiv:1301.3781.

Maximillian Nickel and Douwe Kiela. 2017. Poincare’ embeddings for learning hierarchical representations. In I. Guyon, U. V. Luxburg, S. Bengio, H. Wallach, R. Fergus, S. Vishwanathan, and R. Garnett, editors, Advances in Neural Information Processing Systems 30, pages 6338–6347. Curran Associates, Inc.

Terry Regier and Yang Xu. 2017. The Sapir-Whorf hypothesis and inference under uncertainty. Wiley Interdisciplinary Reviews: Cognitive Science, 8:e1440.

Terry Regier, Charles Kemp, and Paul Kay. 2015. Word Meanings across Languages Support Efficient Communication. In B. MacWhinney & W. O'Grady (Eds.), The handbook of language emergence. Hoboken, NJ: Wiley-Blackwell, 237-263.

Jenny Saffran. 2003. Statistical language learning: mechanisms and constraints. Current Directions in Psychological Science. 12 (4): 110–114.

Christopher De Sa, Albert Gu, Christopher Re, Frederic Sala. 2018. Representation Tradeoffs for Hyperbolic Embeddings. Proceedings of the 35th International Conference on Machine Learning, 80:4460-4469.

Tatyana Sharpee. 2019. An argument for hyperbolic geometry in neural circuits. Current opinion in neurobiology vol. 58: 101-104. doi:10.1016/j.conb.2019.07.008

Mark Steyvers and Joshua Tenenbaum. 2005. The large-scale structure of semantic networks: statistical analyses and a model of semantic growth. Cogn. Sci. 29, 41–78. doi.org/10.1207/s15516709cog2901_3

Orsolya Vincze. 2015. Learning multiword expressions from corpora and dictionaries. Doctoral Thesis.

Qinxuan Wu and Allyson Ettinger. 2021. Variation and generality in encoding of syntactic anomaly information in sentence embeddings. Proceedings of the Fourth BlackboxNLP Workshop on Analyzing and Interpreting Neural Networks for NLP.

Noga Zaslavsky, Terry Regier, Naftali Tishby, and Charles Kemp. 2019. Semantic categories of artifacts and animals reflect efficient coding. In
Proceedings of the 41st Annual Meeting of the Cognitive Science Society.