Optimization of RFID Tags Coil's System Stability under Delayed Electromagnetic Interferences

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Abstract This article discusses the very crucial subject of RFID TAG's stability. RFID equivalent circuits of a label can be represented as Parallel circuits of Capacitance (Cpl), Resistance (Rpl), and Inductance (Lpc). We define V(t) as the voltage that develops on the RFID label therefore making dV(t)/dt the voltage-time derivative. Due to electromagnetic interference, there are different time delays with respect to RFID label voltages and voltage time derivatives. We define V(t) as V(t) and V2(t) as dV(t)/dt. The delayed voltage and voltage derivative are V1(t-τ1) and V1(t-τ2) respectively (τ1,τ2). The RFID equivalent circuit can be represented as a delayed differential equation that depends on variable parameters and delays. The investigation of RFID's differential equation is based on bifurcation theory [1], which is the study of possible changes in the structure of the orbits of a delayed differential equation as a function of variable parameters. This article first illustrates certain observations and analyzes local bifurcations of an appropriate arbitrary scalar delayed differential equation [2]. RFID label stability analysis is done under different time delays with respect to label voltage and voltage derivative. Additional analysis of the bifurcations of RFID's delayed differential equation on the circle. Bifurcation behavior of specific delayed differential equations can be condensed into bifurcation diagrams. This serves to optimize dimensional parameters analysis of RFID TAGs under electromagnetic interferences to get ideal performances.

Keywords RFID, stability, locus

1. Introduction

This article discusses a very critical and useful subject of passive RFID TAGs system stability analysis under electromagnetic interferences. RFID TAG system has two main variables—TAG-voltage and TAG-voltage derivative with respect to time which may be subject to delay as a result of electromagnetic interferences. We define τ1 as time delay respect to TAG's voltage and τ2 as time delay respect to TAG's voltage derivative. RFID Equivalent circuits of a Label can be represented as parallel circuits of Capacitance (Cpl), Resistance (Rpl), and Inductance (Lpc). Our RFID TAG system delay differential and delay differential model can be utilized for analysis of the dynamics of delay differential equations. Incorporation of a time delay is often necessary during some stage. It is often difficult to
analytically study models with delay dependent parameters, even if only a single discrete delay is present. Practical guidelines exist that combine graphical information with analytical work to effectively study the local stability of models involving delay dependent parameters. The stability of a given steady state is determined simply through the use of the graphs of a function of $\tau_1, \tau_2$, which can be expressed distinctly and thus can be easily depicted by Matlab and other popular software—we need only look at one such function and locate the zeros. This function often has only two zeros, providing thresholds for stability switches. As time delay increases, the stability fluctuates. We emphasize the local stability aspects of certain models with delay-dependent parameters. In addition there is a general geometric criterion that, theoretically speaking, may be applied to models that include many delays or even distributed delays. The simplest case is that of a first order characteristic equation which provides more user-friendly geometric and analytic criteria for stability switches. The analytical criteria provided for the first and second order cases can be used to obtain insightful analytical statements and can be helpful for conducting simulations.

2. RFID Equivalent Circuit and Representation of Delay Differential Equations

Figure 1. NXP/PHILIPS ICODE IC Parallel equivalent circuit and simplified complete equivalent circuit of the label (L1 is the antenna inductance) [6].

![Diagram of ICODE RFID TAG](image)

RFID TAG can be represented as a parallel Equivalent Circuit of Capacitor and Resistor in Parallel. For example, see NXP/PHILIPS ICODE IC Parallel equivalent circuit and simplified complete equivalent circuit of the label (L1 is the antenna inductance) [6].

The dynamic equation system:

$$\frac{dV_1}{dt} = \frac{dV_2}{dt}, V_1 = V_2$$

The following Variable settings: $V_2 = \frac{dV_1}{dt} = \frac{dV}{dt}$, $V_1 = V$. The dynamic equation system:

$$\frac{dV_1}{dt} = V_1, \quad \frac{dV_2}{dt} = -\frac{1}{C_1 R_1} V_2 - \frac{1}{C_1 L_1} V_1$$

(1)

![Diagram of RFID coil dimensional parameters](image)

Figure 2. RFID's coil dimensional parameters

$$d = 2 \cdot (t + w) / \Pi$$

$$A_{avg} = a_0 - N_c \cdot (g + w)$$

$$B_{avg} = b_0 - N_c \cdot (g + w)$$

$a_0, b_0$ – Overall dimensions of the coil. $A_{avg}$, $B_{avg}$ – Average dimensions of the coil. $t$ – Track thickness, $w$ – Track width, $g$ – Gap between tracks. $N_c$ – Number of turns, $d$ – Equivalent diameter of the track. Average coil area $A_{cavg} = A_{avg} * B_{avg}$. Integration of all of these parameters gives the equations for inductance calculation:

$$X_1 = A_{avg} \ln \left( \frac{2 * A_{avg} * B_{avg}}{d * (A_{avg} + B_{avg})^2} \right)$$

(3)

$$X_2 = B_{avg} \ln \left( \frac{2 * A_{avg} * B_{avg}}{d * (A_{avg} + B_{avg})^2} \right)$$

(4)

$$X_3 = 2 \sqrt{A_{avg} + B_{avg} - \sqrt{A_{avg}^2 + B_{avg}^2}}$$

(5)

$$X_4 = (A_{avg} + B_{avg}) / 4$$

(6)

The RFID's coil calculation inductance is

$$L_{calc} = \left[ \frac{\mu_0}{\pi} \left[ X_1 + X_2 - X_3 + X_4 \right] \cdot N_c \right], L_1 = L_{calc}$$

(7)

Definition of limits, Estimations: Track thickness $t$, $A_{l}$ and $C_1$ coils ($t > 30\mu m$).
Due to electromagnetic interferences we get RFID TAG’s voltage and voltage derivative with delays $\tau_1$ and $\tau_2$ respectively $V_1(t)\rightarrow V_1(t-\tau_1)$; $V_2(t)\rightarrow V_2(t-\tau_2)$. We consider no delay effect on $dV_1/dt$ and $dV_2/dt$. The RFID TAG’s differential equations under the effects of electromagnetic interferences (we consider electromagnetic interferences (delay terms) influence only RFID TAG voltage $V_i(t)$ and the voltage derivative $V_i(t)$ with respect to time. There is no influence on $dV_i(t)/dt$ and $dV_i(t)/dt$:

$$\frac{dV_1}{dt} = V_2(t - \tau_2) \quad \frac{dV_2}{dt} = -\frac{1}{C_1 R_1} V_1(t - \tau_1)$$

To find the Equilibrium points (fixed points) of the RFID TAG system is by $\lim_{t\to\infty} V_1(t - \tau_1) = V_1(t)$;

$$\lim_{t\to\infty} V_2(t - \tau_2) = V_2(t); \frac{dV_1(t)}{dt} = 0; \frac{dV_2(t)}{dt} = 0 \quad (10)$$

$\forall t \gg \tau_1; t \gg \tau_2: (t - \tau_1) \approx t; (t - \tau_2) \approx t, t \to \infty$

We get two equations and the only fixed point is: $E(0)(V_1(0), V_2(0)) = (0,0)$.

Stability analysis: The standard local stability analysis about any one of the equilibrium points of RFID TAG system consists of adding arbitrarily small increments of exponential form $[v_1, v_2]e^{i\lambda t}$ to coordinates $[V_1, V_2]$, and retaining the first order terms in $v_1, v_2$. The system of two homogeneous equations leads to a polynomial characteristics equation in the eigenvalues $\lambda$. The polynomial characteristics equations accept by set the following voltage and voltage derivative with respect to time into two RFID TAG system equations.

The RFID TAG system fixed values with arbitrarily small increments of exponential form $[v_1, v_2]e^{i\lambda t}$ are: $i=0$ (first fixed point), $i=1$ (second fixed point), $i=2$ (third fixed point).

$$V_i(t) = V_i^{(0)} + v_i e^{i\lambda t}; V_i(t = t - \tau_i) = V_i^{(0)} + v_i e^{i\lambda(t - \tau_i)} \quad (12)$$

We choose the above expressions for our $V_1(t), V_2(t)$ as small displacement $[v_1, v_2]$ from the system’s fixed points at time $t=0$.

$$V_1(t=0) = V_1^{(0)} + v_1; V_2(t=0) = V_2^{(0)} + v_2 \quad (13)$$

When $\lambda < 0$, $t > 0$ the selected fixed point is stable, however when $\lambda > 0$, $t > 0$ is unstable. Our system tends towards the selected fixed point exponentially for $\lambda < 0, t > 0$ and otherwise will deviate exponentially from the selected fixed point. $\lambda$ is the eigenvalue parameter which establishes whether the fixed point is stable or unstable. In addition, the absolute value ($|\lambda|$) establishes the speed of flow toward or away from the selected fixed point [1] [2].

$$\begin{array}{c|c}
\lambda < 0 & \lambda > 0 \\
\hline
t = 0 & \\
V_1(t=0) = V_1^{(0)} + v_1 e^{i\lambda t} & V_1(t=0) = V_1^{(0)} + v_1 e^{i\lambda t}
\end{array}$$

The speeds of flow toward or away from the fixed point for RFID TAG system voltage and voltage derivative respect to time are

$$\frac{dV_1(t)}{dt} = \lim_{\Delta t \to 0} \frac{V_1(t + \Delta t) - V_1(t)}{\Delta t}$$

$$\frac{dV_2(t)}{dt} = \lim_{\Delta t \to 0} \frac{V_2(t + \Delta t) - V_2(t)}{\Delta t} \quad (14)$$

$$\frac{dV_1(t)}{dt} = \frac{V_1(t) + v_1 e^{i\lambda t} - V_1(t)}{\Delta t} = \frac{V_1(t) + v_1 e^{i\lambda t} - V_1(t)}{\Delta t} \quad (15)$$

$$\frac{dV_2(t)}{dt} = \frac{V_2(t) + v_2 e^{i\lambda t} - V_2(t)}{\Delta t} = \frac{V_2(t) + v_2 e^{i\lambda t} - V_2(t)}{\Delta t}$$

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and the time derivative of the above equations:

\[
\frac{dV_1(t)}{dt} = v_1 \cdot \lambda \cdot e^{\lambda t} \; ; \quad \frac{dV_2(t)}{dt} = v_2 \cdot \lambda \cdot e^{\lambda t}
\]  

(16)

\[
\frac{dV_1(t-\tau_1)}{dt} = v_1 \cdot \lambda \cdot e^{\lambda (t-\tau_1)} = v_1 \cdot \lambda \cdot e^{\lambda t} \cdot e^{-\tau_1 \lambda} \\
\frac{dV_2(t-\tau_2)}{dt} = v_2 \cdot \lambda \cdot e^{\lambda (t-\tau_2)} = v_2 \cdot \lambda \cdot e^{\lambda t} \cdot e^{-\tau_2 \lambda}
\]  

(17)

First we take the RFID TAG’s voltage \(V_1\) differential equation: \(\frac{dV_1}{dt} = V_2\) adding arbitrarily small increments in exponential form \([v_1, v_2] \cdot e^{\lambda t}\) to the coordinates \([V_1, V_2]\) and retaining the first order terms in \(v_1, v_2\).

\[
\lambda \cdot v_1 \cdot e^{\lambda t} = V_2^{(i)}(t) + v_2 \cdot e^{\lambda t} \; ; \quad V_2^{(i=0)} = 0 \; ; \quad \lambda_1 = \frac{v_2}{v_1} \approx 1 > 0
\]  

(18)

Second we take the RFID TAG’s voltage \(V_2\) differential equation: \(\frac{dV_2}{dt} = V_1\) and adding arbitrarily small increments in exponential form \([v_1, v_2] \cdot e^{\lambda t}\) to the coordinates \([V_1, V_2]\) and retaining the first order terms in \(v_1, v_2\).

\[
\frac{dV_2}{dt} = \left\{ -\frac{1}{C_1 \cdot \frac{\mu_0}{\pi} \left[ X_1 + X_2 - X_3 + X_4 \right] \cdot Nc^p} \right\} V_1(t-\tau_1) - \frac{1}{C_1 \cdot R_1} V_2(t-\tau_2)
\]  

(20)

\[
\lambda \cdot v_2 \cdot e^{\lambda t} = \left\{ -\frac{1}{C_1 \cdot \frac{\mu_0}{\pi} \left[ X_1 + X_2 - X_3 + X_4 \right] \cdot Nc^p} \right\} (V_1^{(i)} + v_1 \cdot e^{\lambda t}) - \frac{1}{C_1 \cdot R_1} (V_2^{(i)} + v_2 \cdot e^{\lambda t})
\]  

(21)

\[
V_1^{(i=0)} = V_2^{(i=0)} = 0 \; \text{then} \; \frac{v_1}{v_2} \approx 1 \; ; \\
\lambda_2 = \left[ -\frac{1}{C_1 \cdot \frac{\mu_0}{\pi} \left[ X_1 + X_2 - X_3 + X_4 \right] \cdot Nc^p} + \frac{1}{R_1} \right]
\]  

If

\[
\frac{1}{C_1 \cdot \frac{\mu_0}{\pi} \left[ X_1 + X_2 - X_3 + X_4 \right] \cdot Nc^p} + \frac{1}{R_1} > 0
\]  

then we have saddle fixed point otherwise it is an unstable node (both eigenvalues are positive). We define

\[
V_1(t-\tau_1) = V_1^{(i)} + v_1 \cdot e^{\lambda(t-\tau_1)} \; ; \\
V_2(t-\tau_2) = V_2^{(i)} + v_2 \cdot e^{\lambda(t-\tau_2)}
\]  

(22)

which gives us two delayed differential equations adding arbitrarily small increments of exponential form \([v_1, v_2] \cdot e^{\lambda t}\) to the coordinates \([V_1, V_2]\).

\[
v_1 \cdot \lambda \cdot e^{\lambda t} = V_2^{(i)} + v_2 \cdot e^{\lambda(t-\tau_2)} \; ; \\
v_2 \cdot \lambda \cdot e^{\lambda t} = V_2^{(i)} + v_2 \cdot e^{\lambda(t-\tau_2)}
\]  

(23)

In the equilibrium fixed point \(V_1^{(i=0)} = V_2^{(i=0)} = 0\) and

\[
\lambda \cdot v_1 \cdot e^{\lambda t} = \left\{ -\frac{1}{C_1 \cdot \frac{\mu_0}{\pi} \left[ X_1 + X_2 - X_3 + X_4 \right] \cdot Nc^p} \right\} V_1^{(i)} + \frac{1}{C_1 \cdot R_1} V_2^{(i)} = 0
\]  

Then we get

\[
\lambda \cdot v_2 \cdot e^{\lambda t} = \left\{ -\frac{1}{C_1 \cdot \frac{\mu_0}{\pi} \left[ X_1 + X_2 - X_3 + X_4 \right] \cdot Nc^p} \right\} V_2^{(i)} + \frac{1}{C_1 \cdot R_1} V_1^{(i)} = 0
\]  

(25)

We define

\[
f_s(X_1, X_2, \text{etc.}) = \left[ \frac{\mu_0}{\pi} \left[ X_1 + X_2 - X_3 + X_4 \right] \cdot Nc^p \right]
\]  

(26)

The small Jacobian increments of our RFID TAG system

\[
\begin{bmatrix}
-\lambda & e^{-\lambda \tau_2} & 0 \\
-\frac{1}{C_1} \cdot f_1 & -\frac{1}{C_1} \cdot e^{-\lambda \tau_1} & -\frac{1}{C_1} \cdot e^{-\lambda \tau_2} & -\lambda
\end{bmatrix}
\]  

(27)
\[ A - \lambda \cdot I = \begin{bmatrix} -\lambda & e^{-\lambda t_1} \\ -\frac{1}{C_1 \cdot f_s} \cdot e^{-\lambda t_1} & -\lambda - \frac{1}{C_1 \cdot R_1} \cdot e^{-\lambda t_1} \end{bmatrix} \]  \tag{28}

\[ \det | A - \lambda \cdot I | = 0 \]  \tag{29}

\[ D(\lambda, \tau_1, \tau_2) = \lambda^2 + \lambda \cdot \frac{1}{C_1 \cdot R_1} \cdot e^{-\lambda (\tau_1 + \tau_2)} \]  \tag{30}

We have three stability analysis cases: \( \tau_1 = \tau ; \tau_2 = 0 \) or \( \tau_2 = \tau ; \tau_1 = 0 \) or \( \tau_1 = \tau_2 = \tau \) otherwise \( \tau_1 \neq \tau_2 \).

Just as in all of the above stability analysis cases, we need to identify characteristic equations. We study the occurrence of any possible stability switching resulting from the increase in value of the time delay \( \tau \) for the general characteristic equation \( D(\lambda, \tau) \).

\[ D(\lambda, \tau) = P_a(\lambda, \tau) + Q_a(\lambda, \tau) \cdot e^{-\lambda \tau} \]  \tag{31}

The expression for \( P_a(\lambda, \tau) \) is

\[ P_a(\lambda, \tau) = \sum_{k=0}^{n} P_{k}(\tau) \cdot \lambda^k = P_0(\tau) + P_{1}(\tau) \cdot \lambda + P_{2}(\tau) \cdot \lambda^2 + \ldots \]  \tag{32}

The expression for \( Q_a(\lambda, \tau) \) is

\[ Q_a(\lambda, \tau) = \sum_{k=1}^{m} q_{k}(\tau) \cdot \lambda^k = q_{0}(\tau) + q_{1}(\tau) \cdot \lambda + q_{2}(\tau) \cdot \lambda^2 + \ldots \]  \tag{33}

3. RFID Tag System Second Order

**CHARACTERISTIC EQUATION** \( \tau_1 = \tau ; \tau_2 = 0 \)

The first case we analyze involves a delay in RFID Label voltage with no delay in voltage time derivative [4] [5].

\[ D(\lambda, \tau_1 = \tau, \tau_2 = 0) = \lambda^2 + \lambda \cdot \frac{1}{C_1 \cdot R_1} + \frac{1}{C_1 \cdot f_s} \cdot e^{-\lambda t_1} \]  \tag{34}

\[ D(\lambda, \tau) = P_a(\lambda, \tau) + Q_a(\lambda, \tau) \cdot e^{-\lambda \tau} \]  \tag{35}

The expression for \( P_a(\lambda, \tau) \) is

\[ P_a(\lambda, \tau) = \sum_{k=0}^{n} P_{k}(\tau) \cdot \lambda^k = P_0(\tau) + P_{1}(\tau) \cdot \lambda + P_{2}(\tau) \cdot \lambda^2 + \ldots \]  \tag{36}

\[ Q_a(\lambda, \tau) = \sum_{k=1}^{m} q_{k}(\tau) \cdot \lambda^k = q_{0}(\tau) + q_{1}(\tau) \cdot \lambda + q_{2}(\tau) \cdot \lambda^2 + \ldots \]  \tag{37}

Our RFID system second order characteristic equation is:

\[ D(\lambda, \tau) = \lambda^2 + a(\tau) \cdot \lambda + b(\tau) \cdot \lambda^2 + c(\tau) + d(\tau) \cdot e^{-\lambda \tau} \]  \tag{38}

Then

\[ a(\tau) = \frac{1}{C_1 \cdot f_s} ; b(\tau) = 0 ; c(\tau) = 0 ; d(\tau) = \frac{1}{C_1 \cdot f_s} \]  \tag{39}

\( \tau \in R_{+} \) and \( a(\tau), b(\tau), c(\tau), d(\tau) : R_{+} \rightarrow R \) are differentiable functions of class \( C^1(R_{+}) \) such that \( c(\tau) + d(\tau) = \frac{1}{C_1 \cdot f_s} \neq 0 \) for all \( \tau \in R_{+} \) and for any \( \tau, b(\tau), d(\tau) \) are not simultaneously zero. We have

\[ P(\lambda, \tau) = P_a(\lambda, \tau) = \lambda^2 + a(\tau) \cdot \lambda + c(\lambda) = \lambda^2 + \frac{1}{C_1 \cdot R_i} \cdot \lambda \]  \tag{40}

\[ Q(\lambda, \tau) = Q_a(\lambda, \tau) = b(\tau) \cdot \lambda + d(\tau) = \frac{1}{C_1 \cdot f_s} \]  \tag{41}

We assume that \( P_a(\lambda, \tau) = P_a(\lambda) \) and \( Q_a(\lambda, \tau) = Q_a(\lambda) \) cannot have common imaginary roots. That is, for any real number \( \omega \) :

\[ p_a(\lambda = i \cdot \omega, \tau) + Q_a(\lambda = i \cdot \omega, \tau) \neq 0 \]  \tag{42}

\[ \frac{1}{C_1 \cdot f_s} - \omega^2 + i \cdot \omega \cdot \frac{1}{C_1 \cdot R_i} \neq 0 ; \]  \tag{43}

\[ F(\omega, \tau) = |P(i \cdot \omega, \tau)|^2 - |Q(i \cdot \omega, \tau)|^2 = (c - \omega^2 \cdot \omega^2 - a^2 - (\omega^2 \cdot b^2 + d^2) \]  \tag{44}

\[ F(\omega, \tau) = \omega^4 + \omega^2 \cdot \frac{1}{(C_1 \cdot R_i)^2} - \frac{1}{(C_1 \cdot f_s)^2} \]  \tag{45}

Hence

\[ F(\omega, \tau) = 0 \]  \tag{46}
\[ \omega^4 + \omega^2 \frac{1}{(C_1 \cdot R_1)^2} - \frac{1}{(C_1 \cdot f_s)^2} = 0 \] (45)

And its roots are given by
\[ \omega_i^2 = \frac{1}{2} \left\{ (b^2 + 2 \cdot c - a^2) + \sqrt{\Delta} \right\} = \frac{1}{2} \left\{ \sqrt{\Delta} - \frac{1}{(C_1 \cdot R_1)^2} \right\} \] (46)
\[ \omega_i^2 = \frac{1}{2} \left\{ (b^2 + 2 \cdot c - a^2) - \sqrt{\Delta} \right\} = -\frac{1}{2} \left\{ \sqrt{\Delta} + \frac{1}{(C_1 \cdot R_1)^2} \right\} \] (47)

\[ \Delta = (b^2 + 2 \cdot c - a^2)^2 - 4 \cdot (c^2 - d^2) = \frac{1}{C_1^2} \left\{ \left( \frac{2}{f_s} \right)^2 - \frac{1}{R_1^2} \right\} \] (48)

Therefore the following holds true:
\[ 2 \cdot \omega_{i-}^2 - (b^2 + 2 \cdot c - a^2) = \pm \sqrt{\Delta} \] (49)
\[ 2 \cdot \omega_{i-}^3 + \frac{1}{(C_1 \cdot R_1)^2} = \pm \sqrt{\Delta} \] (50)

Furthermore \( P_i(i \cdot \omega, \tau) = c(\tau) - \omega^2(\tau) = -\omega^2(\tau) \) (51)
\[ P_i(i \cdot \omega, \tau) = \omega(\tau) \cdot a(\tau) = \omega(\tau) \cdot \frac{1}{C_1 \cdot R_1} \] (52)
\[ Q_i(i \cdot \omega, \tau) = d(\tau) = \frac{1}{C_1 \cdot f_s} \] (53)
\[ Q_i(i \cdot \omega, \tau) = \omega(\tau) \cdot b(\tau) = 0 \] hence
\[ \sin(\theta(\tau)) = \frac{P_i(i \cdot \omega, \tau) \cdot Q_i(i \cdot \omega, \tau) + P_i(i \cdot \omega, \tau) \cdot Q_i(i \cdot \omega, \tau)}{|Q(i \cdot \omega, \tau)|^2} \] (54)
\[ \cos(\theta(\tau)) = \frac{P_i(i \cdot \omega, \tau) \cdot Q_i(i \cdot \omega, \tau) + P_i(i \cdot \omega, \tau) \cdot Q_i(i \cdot \omega, \tau)}{|Q(i \cdot \omega, \tau)|^2} \] (55)
\[ \sin(\theta(\tau)) = \frac{(c - \omega^2) \cdot \omega \cdot b + \omega \cdot a \cdot d}{\omega^2 \cdot b^2 + d^2} = \omega \cdot \frac{f_s}{R_1} \] (56)
\[ \cos(\theta(\tau)) = \frac{(c - \omega^2) \cdot d + \omega^2 \cdot a \cdot b}{\omega^2 \cdot b^2 + a^2} = \omega^2 \cdot C_1 \cdot f_s \] (57)

Which jointly with \[ \omega^4 + \omega^2 \cdot \frac{1}{(C_1 \cdot R_1)^2} - \frac{1}{(C_1 \cdot f_s)^2} = 0 \] (58)
defines the maps \( S_n(\tau) = \tau - \tau_n(\tau); \ n \in \mathbb{N}_0 \) (59)

which are continuous and differentiable in \( \tau \) based on Lema 1.1 (see Appendix A). Hence we use theorem 1.2 (see Appendix B). This proves theorem 1.3 (see Appendix C) and theorem 1.4 (see Appendix D).

Remark: \( a, b, c, d \) parameters are independent of delay parameter \( \tau \) even if we use \( a(\tau), b(\tau), c(\tau), d(\tau) \).

4. RFID Tag System Second Order

CHARACTERISTIC EQUATION \( \tau_1 = 0; \tau_2 = \tau \)

The second case we analyze involves no delay in RFID Label voltage but does have a delay in voltage time derivative [9].

\[ D(\lambda, \tau_1 = 0, \tau_2 = \tau) = \lambda^2 + \frac{1}{C_1 \cdot R_1} \cdot e^{+\lambda \tau} + \frac{1}{C_1 \cdot f_s} \cdot e^{-\lambda \tau} \] (60)
\[ D(\lambda, \tau_1 = 0, \tau_2 = \tau) = \lambda^2 + (\lambda^2 + \frac{1}{C_1 \cdot R_1} + \frac{1}{C_1 \cdot f_s}) \cdot e^{-2\lambda \tau} \] (61)
\[ D(\lambda, \tau) = P_n(\lambda, \tau) + Q_n(\lambda, \tau) \cdot e^{-2\lambda \tau} \] (62)

The expression for \( P_n(\lambda, \tau) \) is
\[ P_n(\lambda, \tau) = \sum_{i=1}^{n} P_i(\tau) \cdot \lambda_i = P_n(\tau) \cdot \lambda + P_i(\tau) \cdot \lambda + P_i(\tau) \cdot \lambda = \lambda \] (63)
\[ P_1(\tau) = 1; P_2(\tau) = 0; P_3(\tau) = 0 \]

The expression for \( Q_n(\lambda, \tau) \) is
\[ Q_n(\lambda, \tau) = \sum_{i=1}^{n} Q_i(\tau) \cdot \lambda_i = \lambda \frac{1}{C_1 \cdot R_1} + \frac{1}{C_1 \cdot f_s} \]
\[ q_1(\tau) = 1; q_2(\tau) = 0; q_3(\tau) = 0 \] (64)

Our RFID system second order characteristic equation:

\[ D(\lambda, \tau) = \lambda^2 + a(\tau) \cdot \lambda + b(\tau) \cdot \lambda \cdot e^{-2\lambda \tau} + c(\tau) + d(\tau) \cdot e^{-2\lambda \tau} \] (65)

Then
\[ a(\tau) = 0; b(\tau) = \frac{1}{C_1 \cdot R_1}; c(\tau) = 0; d(\tau) = \frac{1}{C_1 \cdot f_s} \] (66)

And like in our previous case analysis:
\[ P(\lambda, \tau) = P_n(\lambda, \tau) = \lambda^2 + a(\tau) \cdot \lambda + c(\lambda) = \lambda^2 \] (67)
\[ Q(\lambda, \tau) = Q_m(\lambda, \tau) = b(\tau) \cdot \lambda + d(\tau) = \lambda \cdot \frac{1}{C_1 \cdot R_1} + \frac{1}{C_1 \cdot f_\omega} \]  \hspace{1cm} (68)

we assume that \( P_n(\lambda, \tau) = P_1(\lambda) \) and \( Q_n(\lambda, \tau) = Q_1(\lambda) \) cannot have common imaginary roots. That is, for any real number \( \omega \):

\[ p_n(\lambda = i \cdot \omega, \tau) + Q_n(\lambda = i \cdot \omega, \tau) \neq 0 \]  \hspace{1cm} (69)

\[ \frac{1}{C_1 \cdot f_\omega} - \omega^2 + i \cdot \omega \cdot \frac{1}{C_1 \cdot R_1} \neq 0; \]  \hspace{1cm} (70)

\[ F(\omega, \tau) = \left| P(i \cdot \omega, \tau) \right|^2 - \left| Q(i \cdot \omega, \tau) \right|^2 \]
\[ = (c - \omega^2)^2 + \omega^2 \cdot a^2 - (\omega^2 \cdot b^2 + d^2) \]  \hspace{1cm} (71)

\[ F(\omega, \tau) = \omega^4 - \omega^2 \cdot \frac{1}{(C_1 \cdot R_1)^2} - \frac{1}{(C_1 \cdot f_\omega)^2}; \]  \hspace{1cm} (72)

Therefore \( F(\omega, \tau) = 0 \) implies

\[ \omega^4 - \omega^2 \cdot \frac{1}{(C_1 \cdot R_1)^2} - \frac{1}{(C_1 \cdot f_\omega)^2} = 0 \]  \hspace{1cm} (73)

And its roots are given by

\[ \omega_1^2 = \frac{1}{2} \{ (b^2 + 2 \cdot c - a^2) + \sqrt{\Delta} \} = \frac{1}{2} \{ (b^2 + 2 \cdot c - a^2) + \sqrt{\Delta} \} \]  \hspace{1cm} (74)

\[ \omega_2^2 = \frac{1}{2} \{ (b^2 + 2 \cdot c - a^2) - \sqrt{\Delta} \} = \frac{1}{2} \{ (b^2 + 2 \cdot c - a^2) - \sqrt{\Delta} \} \]  \hspace{1cm} (75)

\[ \Delta = (b^2 + 2 \cdot c - a^2) - 4 \cdot (c^2 - d^2) = \frac{1}{C_1^2} \left( \left( \frac{2}{f_\omega} \right)^2 + \frac{1}{R_1^2} \right) \]  \hspace{1cm} (76)

Therefore the following holds:

\[ 2 \cdot \omega_{1,2} = - (b^2 + 2 \cdot c - a^2) = \pm \sqrt{\Delta} \]  \hspace{1cm} (77)

\[ 2 \cdot \omega_{1,2}^2 + \frac{1}{(C_1 \cdot R_1)^2} = \pm \sqrt{\Delta} \]  \hspace{1cm} (78)

Furthermore \( P_R(i \cdot \omega, \tau) = c(\tau) - \omega^2(\tau) = -\omega^2(\tau) \)  \hspace{1cm} (79)

\[ P_L(i \cdot \omega, \tau) = \omega(\tau) \cdot a(\tau) = 0 \]  \hspace{1cm} (80)

\[ Q_R(i \cdot \omega, \tau) = d(\tau) = \frac{1}{C_1 \cdot f_\omega}; \]  \hspace{1cm} (81)

\[ Q_L(i \cdot \omega, \tau) = \omega(\tau) \cdot b(\tau) = \omega(\tau) \cdot \frac{1}{C_1 \cdot f_\omega}, \]  \hspace{1cm} (82)

\[ \sin \theta(\tau) = \frac{-P_L(i \cdot \omega, \tau) \cdot Q_L(i \cdot \omega, \tau) + P_R(i \cdot \omega, \tau) \cdot Q_R(i \cdot \omega, \tau)}{|Q(i \cdot \omega, \tau)|^2} \]  \hspace{1cm} (83)

\[ \cos \theta(\tau) = \frac{-P_L(i \cdot \omega, \tau) \cdot Q_L(i \cdot \omega, \tau) + P_R(i \cdot \omega, \tau) \cdot Q_R(i \cdot \omega, \tau)}{|Q(i \cdot \omega, \tau)|^2} \]  \hspace{1cm} (84)

Which, along with \( \omega^4 - \omega^2 \cdot \frac{1}{(C_1 \cdot R_1)^2} - \frac{1}{(C_1 \cdot f_\omega)^2} = 0 \) \hspace{1cm} (85)

Defines the maps \( S_n(\tau) = \tau - \tau_n(\tau) \); \( \tau \in I \), \( n \in \mathbb{N}_0 \)

Defines the maps \( S_n(\tau) = \tau - \tau_n(\tau) \); \( \tau \in I \), \( n \in \mathbb{N}_0 \)

which are continuous and differentiable in \( \tau \) based on Lema 1.1 (see Appendix A). Hence we use theorem 1.2 (see Appendix B). This proves the theorem 1.3 (see Appendix C) and theorem 1.4 (see Appendix D).

Remark: \( a, b, c, d \) parameters are independent of the delay parameter \( \tau \) even we use \( a(\tau), b(\tau), c(\tau), d(\tau) \) \hspace{1cm} [4][5].

5. RFID Tag System Second Order

CHARACTERISTIC EQUATION \( \tau_1 = \tau ; \ \tau_2 = \tau \)

The third case we analyze is when there is delay in both the RFID Label voltage and in the voltage time derivative \[ \hspace{1cm} [4][5]. \]

\[ D(\lambda, \tau_1 = \tau, \tau_2 = \tau) = \lambda^2 + \frac{1}{C_1 \cdot R_1} \cdot e^{\lambda \cdot \tau} + \frac{1}{C_1 \cdot f_\omega} \cdot e^{\lambda \cdot \tau_2} \]  \hspace{1cm} (86)

\[ D(\lambda, \tau_1 = \tau, \tau_2 = \tau) = \lambda^2 + \frac{1}{C_1 \cdot R_1} \cdot e^{\lambda \cdot \tau} + \frac{1}{C_1 \cdot f_\omega} \cdot e^{\lambda \cdot \tau_2} \]  \hspace{1cm} (87)

\[ D(\lambda, \tau_1 = \tau, \tau_2 = \tau) = \lambda^2 + \frac{1}{C_1 \cdot R_1} \cdot e^{\lambda \cdot \tau} + \frac{1}{C_1 \cdot f_\omega} \cdot e^{\lambda \cdot \tau_2} \]  \hspace{1cm} (88)
\[ D(\lambda, \tau) = P_n(\lambda, \tau) + Q_m(\lambda, \tau) \cdot e^{-\lambda \tau} \]  

(89)

The expression for \( P_n(\lambda, \tau) \) is

\[ P_n(\lambda, \tau) = \sum_{k=0}^{n} P_k(\tau) \cdot \lambda^k = P_0(\tau) + P_1(\tau) \cdot \lambda + P_2(\tau) \cdot \lambda^2 = \lambda^2 \]  

(90)

\[ P_0(\tau) = 1; \quad P_1(\tau) = 0; \quad P_2(\tau) = 0 \]

The expression for \( Q_m(\lambda, \tau) \) is

\[ Q_m(\lambda, \tau) = \sum_{i=0}^{m} q_i(\tau) \cdot \lambda^i = \lambda \cdot \frac{1}{C_1 \cdot R_i} + \frac{1}{C_1 \cdot f_s} \cdot e^{-\lambda \tau} \]  

(91)

Taylor expansion: \( e^{-\lambda \tau} \approx 1 - \lambda \cdot \tau + \frac{\lambda^2 \cdot \tau^2}{2} \) since we need \( n > m \) for [BK] analysis we choose \( e^{-\lambda \tau} \approx 1 - \lambda \cdot \tau \) then we get

\[ Q_m(\lambda, \tau) = \sum_{i=0}^{m} q_i(\tau) \cdot \lambda^i = \lambda \cdot \frac{1}{C_1 / R_i} \cdot \frac{1}{C_1 \cdot f_s} - \frac{1}{C_1 \cdot f_s} \cdot q_2(\tau) = 0 \]  

(92)

Our RFID system second order characteristic equation:

\[ D(\lambda, \tau) = \lambda^2 + a(\tau) \cdot \lambda + b(\tau) \cdot \lambda \cdot e^{-\lambda \tau} + c(\tau) + d(\tau) \cdot e^{-\lambda \tau} \]  

(93)

Then

\[ a(\tau) = 0; \quad b(\tau) = \frac{1}{C_1} \left( \frac{1}{R_i} - \frac{\tau}{f_s} \right); \quad c(\tau) = 0; \quad d(\tau) = \frac{1}{C_1 \cdot f_s} \]  

(94)

And much like our previous case analysis:

\[ P(\lambda, \tau) = P_0(\lambda, \tau) = \lambda^2 \]  

(95)

\[ Q(\lambda, \tau) = Q_m(\lambda, \tau) = \lambda \cdot \frac{1}{C_1 / R_i} \cdot \frac{1}{C_1 \cdot f_s} \]  

(96)

we assume that \( P_0(\lambda, \tau) = P_0(\lambda) \) and \( Q_m(\lambda, \tau) \) can’t have common imaginary roots. That is for any real number \( \omega \); \( P_0(\lambda = i \cdot \omega, \tau) + Q_m(\lambda = i \cdot \omega + \tau) \neq 0 \)

\[ \omega^2 + i \cdot \omega \cdot \frac{1}{C_1} \cdot \frac{1}{R_i} + \frac{1}{C_1 \cdot f_s} \neq 0; \]  

(97)

\[ F(\omega, \tau) = |P(i \cdot \omega, \tau)|^2 = |Q(i \cdot \omega, \tau)|^2; \quad P(i \cdot \omega, \tau) = -\omega^2 \]  

(98)

\[ F(\omega, \tau) = P_i \cdot P_0 + Q_i \cdot Q_0 = \left( P_0 \cdot Q_0 + P_r \cdot Q_r \right) \]  

(99)

\[ P_i = 0; \quad P_0 = 0; \quad P_r = 0; \quad P_1 = 0 \]  

(100)

The expressions for \( U \) and \( V \) can be derived easily [BK]:

\[ U = (P_0 \cdot P_i - P_1 \cdot P_0) - (Q_0 \cdot Q_i - Q_1 \cdot Q_0) \]  

(101)

\[ V = (P_0 \cdot P_1 - P_0 \cdot P_1) - (Q_0 \cdot Q_1 - Q_0 \cdot Q_1) \]  

(102)
\[ V = \frac{\omega}{C_i^2 \cdot f_2^2} \quad \text{and} \quad U = \frac{1}{C_i^2 \cdot f_2} \cdot \left[ R - \frac{1}{R_1} \right] \]  

(115)

Then, we get the expression:

\[ \omega = -\frac{F_2}{F_{\omega}} \]  

(116)

Define the maps \( S_n(\tau) = \tau - \tau_n(\tau) \) \( \tau \in I, n \in \mathbb{N}_0 \)

which are continuous and differentiable in \( \tau \) based on Lema 1.1 (see Appendix A). Hence we use theorem 1.2 (see Appendix B). This proves theorem 1.3 (see Appendix C) and theorem 1.4 (see Appendix D).

Remark: Taylor approximation for \( e^{-\lambda \tau} \approx 1 - \lambda \cdot \tau \) gives us good stability analysis approximation for restricted delay time intervals only.

6. RFID Tag System Stability Analysis under Delayed Variables in Time

Our RFID homogeneous system for \( v_1, v_2 \) leads to a characteristic equation for the eigenvalue \( \lambda \) having the form

\[ P(\lambda) + Q(\lambda) \cdot e^{-\lambda \tau} = 0 \]  

where \( \tau_1 = \tau, \tau_2 = 0 \)

\[ D(\lambda, \tau_1 = \tau, \tau_2 = 0) = \lambda^2 + \frac{1}{C_1 \cdot R_1} + \frac{1}{C_1 \cdot f_2} \cdot e^{-\lambda \tau} \]  

(117)

We use different parameter terminology in this case:

\[ k \to j; p_k(\tau) = a_j; q_k(\tau) = c_j; n = 2; m = 0 \]

Additionally \( P(\lambda) \to P(\lambda), Q(\lambda) \to Q(\lambda) \)

then \( P(\lambda) = \sum_{j=0}^{\lambda^2 + \frac{1}{C_1 \cdot R_1}} a_j \cdot \lambda^j \) and \( Q(\lambda) = \sum_{j=0}^{c_j \cdot \lambda^j} \)  

(118)

\[ P(\lambda) = \lambda^2 + \frac{1}{C_1 \cdot R_1}; Q(\lambda) = \frac{1}{C_1 \cdot f_2} \]  

(119)

\( n, m \in \mathbb{N}_0, n \geq m \) and \( a_j, c_j : \mathbb{R}_{>0} \to \mathbb{R} \) are continuous and differentiable functions of \( \tau \) such that \( a_0 + c_0 \neq 0 \). In the following \( \lambda \) denotes complex and conjugate. \( P(\lambda), Q(\lambda) \)

Are analytic functions in \( \lambda \) and differentiable in \( \tau \).

The coefficients:

\[ \{a_n(C_1, R_i), c_j(C_1, \text{antenna parameters})\} \in \mathbb{R} \]

depend on RFID C, R, \( \omega \) values and antenna parameters but not on \( \tau \).

\[ a_0 = 0, a_1 = \frac{1}{C_1 \cdot R_1}, a_2 = 1, a_3 = 0; c_1 = \frac{1}{C_1 \cdot f_2}, c_1 = c_2 = 0 \]

Unless absolutely necessary, the designation of the variation arguments \( (R_1, C_1, \text{antenna parameters}) \) will subsequently be omitted from \( P, Q, a_n, c_j \). The coefficients \( a_n \) are continuous, and differentiable functions of their arguments, and direct substitution shows that

\[ a_0 + c_0 \neq 0; \frac{1}{C_1 \cdot f_2} \neq 0 \]

\( \forall C_1, \text{ antenna parameters} \in \mathbb{R}, \text{i.e. } \lambda = 0 \) is not a root of the characteristic equation. Furthermore \( P(\lambda), Q(\lambda) \) are analytic functions of \( \lambda \) for which the following requirements of the analysis (see Kuang, 1993, section 3.4) can also be verified in the present case [4] [5].

(a) If \( \lambda = i \cdot \omega, \omega \in \mathbb{R} \) then \( P(i \cdot \omega) + Q(i \cdot \omega) \neq 0 \), i.e. \( P \) and \( Q \) have no common imaginary roots. This condition was verified numerically in the entire \( (R_1, C_1, \text{antenna parameters}) \) domain of interest.

(b) \( |Q(\lambda)/P(\lambda)| \) is bounded for \( |\lambda| \to \infty \), \( \text{Re } \lambda \geq 0 \). No roots bifurcation from \( \infty \).

Indeed, in the limit

\[ |Q(\lambda)/P(\lambda)| = 1 \]  

(120)

\[ (c) F(\omega) = |P(i \cdot \omega)|^2 - |Q(i \cdot \omega)|^2 \]  

(121)

\[ F(\omega, \tau) = \omega^4 + \frac{1}{(C_1 \cdot R_1)^2} - \frac{1}{(C_1 \cdot f_2)^2} \]  

(122)

Has at most a finite number of zeros. Indeed, this is a bicubic polynomial in \( \omega \) (second degree in \( \omega^2 \)).

(d) Each positive root \( \omega(C_1, R_i, \text{antenna parameters}) \)
of $F(\omega)=0$ is continuous and differentiable with respect to $C_1, R_i$, antenna parameters. This condition can only be assessed numerically.

In addition, since the coefficients in $P$ and $Q$ are real, we have

$$P(-i \cdot \omega) = P(i \cdot \omega), \quad \text{and} \quad Q(-i \cdot \omega) = Q(i \cdot \omega)$$

thus $\lambda = i \cdot \omega, \; \omega > 0$ may be on eigenvalue of characteristic equation. The analysis consists in identifying the roots of characteristic equation situated on the imaginary axis of the complex $\lambda$ – plane, where by increasing the parameters

$C_1, R_i$, antenna parameters and delay $\tau$, the root $\lambda$ may, at the crossing $\pm$, change its sign from (-) to (+), i.e. from a stable focus

$E^{(0)}(V_1^{(0)}, V_2^{(0)}) = (0, 0)$ to an unstable one, or vice versa.

This feature may be further assessed by examining the sign of the partial derivatives with respect to $C_1, R_i$ and antenna parameters.

$$\lambda^{-1}(C_1) = \left( \frac{\partial \text{Re} \lambda}{\partial C_1} \right)_{\lambda=i \cdot \omega}, \; R_1, \text{antenna parameters} = \text{const}$$

$$\lambda^{-1}(R_i) = \left( \frac{\partial \text{Re} \lambda}{\partial R_i} \right)_{\lambda=i \cdot \omega}, \; C_1, \text{antenna parameters} = \text{const}$$

$$\lambda^{-1}(f_c) = \left( \frac{\partial \text{Re} \lambda}{\partial f_c} \right)_{\lambda=i \cdot \omega}, \; C_i, R_i = \text{const};$$

$$\lambda^{-1}(\tau) = \left( \frac{\partial \text{Re} \lambda}{\partial \tau} \right)_{\lambda=i \cdot \omega}, \; C_i, R_i, \text{antenna parameters} = \text{const};$$

For the first case $\tau_1 = \tau; \; \tau_2 = 0$ we get the following results

$$P_k(i \cdot \omega) = -a_2 \cdot \omega^2 + a_0 = -\omega^2$$

$$P_l(i \cdot \omega) = -a_3 \cdot \omega^3 + a_1 \cdot \omega = \omega \cdot \frac{1}{C_i \cdot R_i}$$

$$Q_k(i \cdot \omega) = -c_2 \cdot \omega^2 + c_0 = \frac{1}{C_i \cdot f_c}$$

$$Q_l(i \cdot \omega) = c_1 \cdot \omega = 0; \; F(\omega) = 0$$

yield to

$$\omega = \pm \sqrt{\frac{1}{2(1 - (C_i \cdot R_i)^2)} \pm \frac{1}{2} \sqrt{\frac{1}{(C_i \cdot R_i)^4} + 4 \cdot \frac{1}{[C_i \cdot f_c]^2}}}$$

$$\frac{1}{(C_i \cdot R_i)^4} + 4 \cdot \frac{1}{[C_i \cdot f_c]^2} > 0 \text{ always and additional for }$$

$$\omega \in R: \frac{1}{2(1 - (C_i \cdot R_i)^2)} \pm \frac{1}{2} \sqrt{\frac{1}{(C_i \cdot R_i)^4} + 4 \cdot \frac{1}{[C_i \cdot f_c]^2}} > 0$$

There are two options: first, it is always true that

$$\frac{1}{2(1 - (C_i \cdot R_i)^2)} + \frac{1}{2} \sqrt{\frac{1}{(C_i \cdot R_i)^4} + 4 \cdot \frac{1}{[C_i \cdot f_c]^2}} > 0$$

Not exist and always negative for any RFID TAG overall parameters values.

When writing $P(\lambda) = P_k(\lambda) + i \cdot P_l(\lambda)$ and $Q(\lambda) = Q_k(\lambda) + i \cdot Q_l(\lambda)$, and inserting $\lambda = i \cdot \omega$ into RFID characteristic equation, $\omega$ must satisfy the following:

$$\sin \omega \cdot \tau = g(\omega) = \frac{-P_k(i \cdot \omega) \cdot Q_l(i \cdot \omega) + P_l(i \cdot \omega) \cdot Q_k(i \cdot \omega)}{|Q(i \cdot \omega)|^2}$$

$$\cos \omega \cdot \tau = h(\omega) = \frac{-P_k(i \cdot \omega) \cdot Q_l(i \cdot \omega) + P_l(i \cdot \omega) \cdot Q_k(i \cdot \omega)}{|Q(i \cdot \omega)|^2}$$

Where $|Q(i \cdot \omega)|^2 \neq 0$ in light of requirement (a) above, and $(g, h) \in R$. Furthermore, it follows that in the $\sin \omega \cdot \tau$ and $\cos \omega \cdot \tau$ equations, by squaring and adding the sides, $\omega$ must be a positive root of

$$F(\omega) = |P(i \cdot \omega)|^2 - |Q(i \cdot \omega)|^2 = 0.$$ 

Note that $F(\omega)$ is independent of $\tau$. It is important to notice that if $\tau \notin I$ (assume that $I \subseteq R_0$) is the set where $\omega(\tau)$ is a positive root of $F(\omega)$ and for $\tau \notin I$, $\omega(\tau)$ is not define. Then for all $\tau$ in $I$ $\omega(\tau)$ it satisfies that $F(\omega, \tau) = 0$.

Then there are no positive $\omega(\tau)$ solutions for $F(\omega, \tau) = 0$, and we cannot have stability switches. For any $\tau \in I$ where $\omega(\tau)$ is a positive solution of
\[ F(\omega, \tau) = 0, \] we can define the angle \( \theta(\tau) \in [0, 2 \cdot \pi] \) as the solution of

\[ \sin \theta(\tau) = \frac{-P_2(i \cdot \omega) \cdot Q_2(i \cdot \omega) + P_1(i \cdot \omega) \cdot Q_1(i \cdot \omega)}{|Q(i \cdot \omega)|^2} \] (133)

\[ \cos \theta(\tau) = \frac{-P_1(i \cdot \omega) \cdot Q_1(i \cdot \omega) + P_2(i \cdot \omega) \cdot Q_2(i \cdot \omega)}{|Q(i \cdot \omega)|^2} \] (134)

And the relation between the angle \( \theta(\tau) \) and \( \omega(\tau) \) for \( \tau \in I \) must be

\[ \dot{\theta}(\tau) \cdot \tau = \theta(\tau) + n \cdot 2 \cdot \pi \quad \forall \quad n \in \mathbb{N}_0. \]

Hence we can define the angle \( \theta(\tau) \in [0, 2 \cdot \pi] \) as the solution of

\[ 2^n \sin \theta(\tau) - \sin \theta(\tau) \cdot \theta(\tau) = 0 \] (135)

Upon separating this into real and imaginary parts, with

\[ P = P_2 + i \cdot P_1; \quad Q = Q_2 + i \cdot Q_1; \quad P_0 = P_{0x} + i \cdot P_{0x}, \] (137)

\[ Q_w = Q_{0w} + i \cdot Q_{1w}; \quad P_{Rx} = P_{Rx} + i \cdot P_{Rx} \] (138)

\[ P^2 = P_2^2 + P_1^2. \]

When \( \omega(\tau) \) can be any RFID TAG parameters \( R_i, C_i \), and time delay \( \tau \) etc., for convenience, we have dropped the arguments \( (i \cdot \omega, \tau) \), and

\[ F(\omega, \tau) = 2 \cdot ((P_{Rw} \cdot P_{Rx} + P_{0w} \cdot P_{0x}) - (Q_{Rw} \cdot Q_{Rx} + Q_{0w} \cdot Q_{0x})), \] (139)

\[ F_x = 2 \cdot ((P_{Rx} \cdot P_{Rx} + P_{Rx}) - (Q_{Rx} \cdot Q_{Rx} + Q_{Rx} \cdot Q_{Rx})). \] (140)

\[ \omega_x = \frac{F_x}{F_0}. \]

We define \( U \) and \( V \):

\[ U = (P_{Rx} \cdot P_{Rx} - P_{Rw} \cdot P_{Rx}) - (Q_{Rx} \cdot Q_{Rx} - Q_{Rw} \cdot Q_{Rx}). \] (141)

\[ V = (P_{Rx} \cdot P_{Rx} - P_{Rw} \cdot P_{Rx}) - (Q_{Rx} \cdot Q_{Rx} - Q_{Rw} \cdot Q_{Rx}). \] (142)

We choose our specific parameter as time delay \( x = \tau \).

\[ \frac{\partial U}{\partial \omega} = \frac{\omega^2}{C_1 \cdot R_1}; \quad P^2 = \omega^4 + \omega^2 \cdot \frac{1}{(C_1 \cdot R_i^2)}; \quad F_x = 0 \] (143)

\[ P_{Rx} = 0; \quad Q_{Rx} = 0; \quad Q_{Rx} = 0 \quad \Rightarrow \quad V = 0 \]

\[ \frac{\partial F}{\partial \omega} = F_0 = 2 \cdot [2 \cdot \omega^2 + \omega^2 \cdot \frac{1}{(C_1 \cdot R_i^2)}] \quad F(\omega, \tau) = 0 \] (144)

Differentiating with respect to \( \tau \), we get

\[ F_{\tau x} \cdot \frac{\partial \omega}{\partial \tau} + F_x = 0; \quad \tau \in I \quad \Rightarrow \quad \frac{\partial \omega}{\partial \tau} = \frac{F_x}{F_0} \] (145)

\[ \lambda^{-1}(\tau) = \left( \frac{\partial \Re \lambda}{\partial \tau} \right)_{\omega = 0} \] (146)

\[ \lambda^{-1}(\tau) = \Re \left\{ \frac{2 \cdot (U + \tau \cdot |P|^2) + i \cdot F_x}{F_x + i \cdot 2 \cdot (V + \omega \cdot |P|^2)} \right\} \quad \frac{2 \cdot \omega^2 + \frac{1}{(C_1 \cdot R_i^2)}}{\omega^4 + \omega^2 \cdot \frac{1}{(C_1 \cdot R_i^2)}}, \] (147)
\[ \text{sign} \{ \wedge^{-1} (\tau) \} = \text{sign} \{ \left( \frac{\partial \text{Re} \lambda}{\partial \tau} \right)_{\tilde{\omega} = \omega} \} \]  
(148)

\[ \text{sign} \{ \wedge^{-1} (\tau) \} = \text{sign} \{ F_{\omega} \} \cdot \text{sign} \{ \tau \} \cdot \left( \frac{\partial \omega}{\partial \tau} + \omega + \frac{U}{|P|^2} \right) \]  
(149)

\[ \frac{\partial \omega}{\partial \tau} = \omega - \frac{F_{\omega}}{F_{\omega}} ; \quad F_{\omega} = 0 \Rightarrow \frac{\partial \omega}{\partial \tau} = 0 \text{ then we get} \]  

\[ \text{sign} \{ \wedge^{-1} (\tau) \} = \text{sign} \{ 2 \cdot \omega^2 \left[ 2 \cdot \omega^2 + \frac{1}{(C_1 \cdot R_1)} \right] \} \cdot \text{sign} \{ \omega \} \]  
(150)

Result: \( \wedge^{-1} (\tau) > 0 \) for all \( \omega, R_1, C_1 \) values. The sign of \( \wedge^{-1} (\tau) \) is independent of \( \tau \) values so in the first case \( \tau_1 = \tau ; \quad \tau_2 = 0 \) there is no stability switch for different values of \( \tau \).

We now inspect the third interesting case wherein \( \tau_1 = \tau ; \quad \tau_2 = \tau \) when there are delays both in RFID Label voltage and voltage time derivative [4][5].

\[ D(\lambda, \tau_1 = \tau, \tau_2 = \tau) = \lambda^2 + \lambda \cdot \frac{1}{C_1 \cdot R_1} \cdot e^{-\lambda \cdot \tau} + \frac{1}{C_1 \cdot f_s} \cdot e^{-\lambda \cdot \tau^2} \]  
(151)

Taylor expansion: \( e^{-\lambda \cdot \tau} \approx 1 - \lambda \cdot \tau + \frac{\lambda^2 \cdot \tau^2}{2} \) since we need \( n > m \) [BK] analysis we choose \( e^{-\lambda \cdot \tau} \approx 1 - \lambda \cdot \tau \) and then we get our RFID system second order characteristic equation:

\[ D(\lambda, \tau) = \lambda^2 + a(\tau) \cdot \lambda + b(\tau) \cdot \lambda \cdot e^{-\lambda \cdot \tau} + c(\tau) + d(\tau) \cdot e^{-\lambda \cdot \tau} \]  
(152)

\[ a(\tau) = 0 ; \quad b(\tau) = \frac{1}{C_1} \left( \frac{1}{R_1} - \frac{\tau}{f_s} \right) ; \quad c(\tau) = 0 ; \quad d(\tau) = \frac{1}{C_1 \cdot f_s} \]

\[ F(\omega, \tau) = \left| P(i \cdot \omega, \tau) \right|^2 - \left| Q(i \cdot \omega, \tau) \right|^2 = (c - \omega^2)^2 + \omega^2 \cdot a^2 - (\omega^2 \cdot b^2 + d^2) \]  
(153)

\[ F(\omega, \tau) = \omega^4 - \omega^2 \cdot \left[ \frac{1}{(C_1 \cdot R_1)^2} - \frac{1}{(C_1 \cdot f_s)^2} \right] = 0 \]  
(154)

Hence \( F(\omega, \tau) = 0 \) implies

\[ \omega^4 - \omega^2 \cdot \left[ \frac{1}{(C_1 \cdot R_1)^2} - \frac{1}{(C_1 \cdot f_s)^2} \right] = 0 \]  
(155)

And its roots are given by

\[ \omega_1^2 = \frac{1}{2} \left[ (b^2 + 2 \cdot c - a^2) + \sqrt{\Delta} \right] \]  
(156)

\[ = \frac{1}{2} \left[ \sqrt{\Delta} + \frac{1}{C_1^2} \left( \frac{1}{R_i} - \frac{\tau}{f_s} \right)^2 \right] \]

\[ \omega_2^2 = \frac{1}{2} \left[ (b^2 + 2 \cdot c - a^2) - \sqrt{\Delta} \right] \]  
(157)

\[ = \frac{1}{2} \left[ \sqrt{\Delta} + \frac{1}{C_1^2} \left( \frac{1}{R_i} - \frac{\tau}{f_s} \right)^2 \right] \]

\[ \Delta = (b^2 + 2 \cdot c - a^2) - 4 \cdot (c^2 - d^2) \]  
(158)

\[ = \frac{1}{C_1^2} \left( \frac{1}{R_i} - \frac{\tau}{f_s} \right)^2 + \frac{4}{(C_1 \cdot f_s)^2} \]

Therefore the following holds:

\[ 2 \cdot \omega^2_3 - (b^2 + 2 \cdot c - a^2) = \pm \sqrt{\Delta} \]  
(159)

\[ \sin \theta(\tau) = \frac{-P_a(i \cdot \omega, \tau) \cdot Q_b(i \cdot \omega, \tau) + P_b(i \cdot \omega, \tau) \cdot Q_a(i \cdot \omega, \tau)}{|Q(i \cdot \omega, \tau)|} \]  
(160)

\[ \cos \theta(\tau) = \frac{-P_a(i \cdot \omega, \tau) \cdot Q_b(i \cdot \omega, \tau) + P_b(i \cdot \omega, \tau) \cdot Q_a(i \cdot \omega, \tau)}{|Q(i \cdot \omega, \tau)|} \]  
(161)

\[ \sin \theta(\tau) = \frac{(c - \omega^2) \cdot \omega \cdot b + \omega \cdot a \cdot d}{\omega^2 \cdot b^2 + d^2} \]  
(162)

\[ = \omega^2 \cdot \left[ \frac{1}{(C_1 \cdot R_i)^2} - \frac{1}{(C_1 \cdot f_s)^2} \right] \]

\[ \cos \theta(\tau) = \frac{(c - \omega^2) \cdot d + \omega^2 \cdot a \cdot b}{\omega^2 \cdot b^2 + d^2} \]  
(163)

For our stability switching analysis we choose typical RFID parameter values:

\[ C_i = 23 \text{pF} ; \quad R_i = 100k \Omega = 10^3 \text{ } \Omega ; \quad L_{\text{calc}} = f_s = 2.65 mH \]
Then \( \frac{1}{C_2} = 1.89 \times 10^{21} \); \( \frac{1}{C_2^2 \cdot f_s^2} = 2.69 \times 10^{26} \)

We find those \( \omega, \tau \) values which fulfill \( F(\omega, \tau) = 0 \). We ignore negative, complex, and imaginary values of \( \omega \) for specific \( \tau \) values. The following table gives the list. \( \tau \in [0.001..10] \) and can be expressed using a straight line (\( \omega = \tau \cdot 1.64 \times 10^{13} \))

| \( \tau \) | \( \omega \) |
|---|---|
| 0.001 | \( 1.64 \times 10^{10} \) |
| 0.01 | \( 1.64 \times 10^{11} \) |
| 0.05 | \( 8.2 \times 10^{10} \) |
| 0.1 | \( 1.64 \times 10^{12} \) |
| 0.2 | \( 3.28 \times 10^{12} \) |
| 1 | \( 1.64 \times 10^{13} \) |
| 5 | \( 8.2 \times 10^{13} \) |
| 10 | \( 1.64 \times 10^{14} \) |

Figure 3. RFID TAG \( F(\omega, \tau) \) function for \( \tau = \tau = \tau \)

Remark: In the above figure \( \omega \) variable is \( 10^{10} \) units.

Matlab: \( [w,t]=\text{meshgrid}(1:1:1640,0:0.01:10); \)
\( f=\pi \cdot w \cdot w \cdot w \cdot w \cdot 1.89 \times 10^{10} \cdot 21 \cdot (10^{3} - 5 - (l/(2.65 \times 10^{3} - 3)) \cdot 2 - 2.69 \times 10^{26} ; \) \( \text{meshc}(f); \) \( \% \) \( \omega \rightarrow W, \tau \rightarrow t \)

We plot the stability switch diagram based on different delay values of our RFID TAG system.

\[
\begin{align*}
\text{sign } (F(\omega, \tau)) &= \left( \frac{\partial \text{Re } \lambda}{\partial \tau} \right)_{\lambda=\omega} \\
&= \text{Re } \left\{ -2 \cdot \left[ U + \tau \cdot \left| P_1 \right|^2 \right] + i \cdot F_\omega \left( F_\tau + i \cdot 2 \cdot \left| V + \omega \cdot \left| P_1 \right|^2 \right) \right\} \\
&= 2 \cdot \{ F_\omega \cdot (V + \omega \cdot P^2) - F_\tau \cdot (U + \tau \cdot P^2) \} \left( F_\tau + 4 \cdot (V + \omega \cdot P^2) \right)^2 \\
g(Tau) &= \left( \frac{\partial \text{Re } \lambda}{\partial \tau} \right)_{\lambda=\omega} \\
&= \frac{2 \cdot \{ F_\omega \cdot (V + \omega \cdot P^2) - F_\tau \cdot (U + \tau \cdot P^2) \}}{F_\tau + 4 \cdot (V + \omega \cdot P^2)^2} \\
&= \frac{2 \cdot \{ F_\omega \cdot (V + \omega \cdot P^2) - F_\tau \cdot (U + \tau \cdot P^2) \}}{F_\tau + 4 \cdot (V + \omega \cdot P^2)^2} \\
&= \frac{2 \cdot \{ F_\omega \cdot (V + \omega \cdot P^2) - F_\tau \cdot (U + \tau \cdot P^2) \}}{F_\tau + 4 \cdot (V + \omega \cdot P^2)^2}
\end{align*}
\]

When \( \omega = \omega_+ (\tau) \) if only \( \omega_+ \) is feasible. Additionally when all RFID TAG parameters are known, the stability switch due to various time delay values \( \tau \) is described in the following expression:

\[
\begin{align*}
\text{sign } (F(\omega, \tau)) &= \frac{2 \cdot \{ F_\omega \cdot (V + \omega \cdot P^2) - F_\tau \cdot (U + \tau \cdot P^2) \}}{F_\tau + 4 \cdot (V + \omega \cdot P^2)^2} \\
&= \frac{2 \cdot \{ F_\omega \cdot (V + \omega \cdot P^2) - F_\tau \cdot (U + \tau \cdot P^2) \}}{F_\tau + 4 \cdot (V + \omega \cdot P^2)^2} \\
&= \frac{2 \cdot \{ F_\omega \cdot (V + \omega \cdot P^2) - F_\tau \cdot (U + \tau \cdot P^2) \}}{F_\tau + 4 \cdot (V + \omega \cdot P^2)^2}
\end{align*}
\]
Remark: we know \( F(\omega, \tau) = 0 \) implies it roots \( \omega_j(\tau) \) and finding those delay values \( \tau \) for which \( \omega_j \) is feasible. There are \( \tau \) values for which \( \omega_j \) is complex or imaginary number, leaving us unable to analyze stability \([4][5]\).

7. Conclusion

A RFID TAG environment is characterized by electromagnetic interferences which can influence RFID TAGs stability in time. There are two main RFID TAGs variables which are affected by electromagnetic interferences—voltage developed on the RFID Label and the voltage time derivative. Each RFID Label variable under electromagnetic interference is characterized by respective time delay. The two time delays are not the same but can be categorized to some subcases due to interference behaviors.

The first case is when there is RFID Label voltage time delay but no voltage derivative time delay. The second case is when there is no RFID Label voltage time delay but there is a voltage derivative time delay. The third case is when both RFID Label voltage time delay and voltage derivative time delay exist. For simplicity we consider the two delays in the third case to be the same (the difference exists but it is negligible in our analysis). In each case we derive the related characteristic equation. The characteristic equation is dependent on the RFID Label’s overall parameters and interference time delays. Following mathematical manipulation and \([BK]\) theorems and definitions, we derived the expression which gives us a clear understanding of the RFID Label stability map. The stability map gives all possible options for stability segments where each segment belongs to different time delay values segments. RFID Label stability analysis can be influenced by TAG overall parameter values. We do not discuss this analysis in the current article.

Appendix A: Lemma 1.1

Assume that \( \omega(\tau) \) is a positive and real root of \( F(\omega, \tau) = 0 \) for some \( n \in \mathbb{N}_0 \), then a pair of simple conjugate pure imaginary roots \( \lambda(\tau) = i\omega(\tau), \lambda(-\tau) = -i\omega(\tau) \) of \( D(\lambda, \tau) = 0 \) exist at \( \tau = \tau^* \) which crosses the imaginary axis from left to right if \( \delta(\tau^*) > 0 \) and crosses the imaginary axis from right to left if \( \delta(\tau^*) < 0 \) where

\[
\delta(\tau^*) = \text{sign}\{ \frac{d\text{Re}\lambda}{d\tau} \mid_{\lambda=i\omega(\tau^*)} \} = \text{sign}\{ \frac{dS_n(\tau)}{d\tau} \mid_{\tau=\tau^*} \}.
\]

Appendix C: Theorem 1.3

The characteristic equation: \( \tau_1 = \tau, \tau_2 = 0 \): \( \tau_1 = 0, \tau_2 = \tau \)

\[
D(\lambda, \tau) = \lambda^2 + a(\tau) \cdot \lambda + b(\tau) \cdot e^{\lambda \tau} + c(\tau) + d(\tau) \cdot e^{-\lambda \tau}.
\]

\[
D(\lambda, \tau) = \lambda^2 + \frac{1}{Cl \cdot R_l} e^{\lambda \tau} + \frac{1}{Cl \cdot f_y} e^{-\lambda \tau}.
\]

has a pair of simple and conjugate pure imaginary roots \( \lambda = \pm \omega(\tau^*) \), \( \omega(\tau^*) \) real at \( \tau^* \in I \) if \( S_n(\tau) = \tau - \tau(\tau) = 0 \) for some \( n \in \mathbb{N}_0 \). If \( \omega(\tau^*) = \omega(\tau) \) this pair of simple conjugate pure imaginary roots crosses the imaginary axis from left to right if \( \delta(\tau^*) > 0 \) and crosses the imaginary axis from right to left if \( \delta(\tau^*) < 0 \) where

\[
\delta(\tau^*) = \text{sign}\{ \frac{d\text{Re}\lambda}{d\tau} \mid_{\lambda=i\omega(\tau^*)} \} = \text{sign}\{ \frac{dS_n(\tau)}{d\tau} \mid_{\tau=\tau^*} \}.
\]
If \( \omega_\ast (\tau^\ast) = \omega_\ast (\tau^\ast) = \omega (\tau^\ast) \) then \( \Delta (\tau^\ast) = 0 \) and
\[
\text{sign} \left\{ \frac{d \text{Re} \lambda}{d \tau} \right\}_{\lambda=\omega (\tau^\ast)} = 0,
\]
the same is true when \( S_{\ast} (\tau^\ast) = 0 \).

The following result can be useful in identifying values of \( \tau \) where there have been stability switches.

Appendix D: Theorem 1.4

Assume that for all \( \tau \in I \), \( \omega (\tau) \) is defined as a solution of \( F (\omega, \tau) = 0 \) then
\[
\delta_{\pm} (\tau) = \text{sign} \{ \pm \Delta^{1/2} (\tau) \} \cdot \text{sign} D_{\pm} (\tau)
\]
\[
D_{\pm} (\tau) = \omega_{\pm} \cdot \left\{ [\omega_{\pm}^2 \cdot b^2 + d^2] + a \cdot (c - \omega_{\pm}^2) + b \cdot d - b \cdot d - a \cdot c - \omega_{\pm}^2 \cdot a \right\}
\]
\[
+ \omega_{\pm} \cdot \omega_{\pm} \left\{ [\tau \cdot (\omega_{\pm}^2 \cdot b^2 + d^2) - b \cdot d + a \cdot (c - \omega_{\pm}^2) + 2 \cdot \omega_{\pm}^2 \cdot a \} \right\}
\] (176)

\[
\dot{a} = \frac{da(\tau)}{d \tau} ; \quad \dot{b} = \frac{db(\tau)}{d \tau} ; \quad \dot{c} = \frac{dc(\tau)}{d \tau} ; \quad \dot{d} = \frac{dd(\tau)}{d \tau}
\] (177)

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