Programming wave propagation problem in cable of wirerope barrier systems with mathematica

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Abstract. The article discusses the issues of modeling of longitudinal and shear waves propagation in the cables of a road cable barrier. The problem was solved using explicit methods with finite deformation. In the first part of the article, the procedure considered, that allows to solve the problem. In the second part, the procedures for P and S waves verified, using analytical solutions and the commercial LS-DYNA code.

1. Introduction

Despite the frequent occurrence of the wire rope safety barriers, there is still a number of understudied aspects, in particular, the relation between the structural specifics (of wire ropes, legs) and the operating properties (deflections, accelerations at car points).

The finite-element complexes [1] and simplified physical models [2] are mainly used to describe the behavior of the wire rope safety barrier when hit by a car that is an impact. The latter do not include the dynamics of the wire rope itself. Each approach does not allow conducting detailed parametric studies of the structure parameters. For instance, it is difficult to set the geometry parameters in the software complexes, while the analytical model without the wire rope dynamics does not allow studying the wire rope vibrations and their behavior under the load. This work represents the first stage to unify these two approaches. A simplified model of the structure behavior is planned to be developed as part of this work for the parametric study of the safety barrier with consideration of longitudinal and transversal wire rope vibrations.

2. Setting and Solving the Task

The interaction process between the car and the wire rope safety barrier, when the latter is hit, represents one of the frequent diagonal impact cases (impact at an angle to the wire rope and wire rope safety barrier axial line) resulting in longitudinal and transversal wire rope vibrations. In the present work the wire rope was considered to be in the simplest position as a rod. The main equation used to describe the dynamics was the virtual power equation [2]. After discretizing the equation by space and choosing the form functions, the equation takes the form of the equation of dynamics:

\[ [M][\ddot{v}] + \{f^{int}\} - \{f^{ext}\} = 0 \]  

\( \{v\}, \{f^{int}\}, \{f^{ext}\} \) – vectors of acceleration, external and internal forces, respectively. \([M]\) – lumped matrix.
Equation (1) does not require the calculation of the stiffness matrix or the damping matrix compared with implicit methods. Damping and stiffness is included in the value of the internal force, which is obtained by integrating the stresses. Stresses can be found by strain-displacements relations and constitutive equations.

According to J. Hallquist, it is the lumped mass matrix that shall be used in the calculation, as non-physical vibration modes occur if a distributed mass matrix is applied. Another advantage of the concentrated mass is its small width and the inverse matrix is determined faster.

The lumped mass matrix of elements is calculated:

$$ M_{ij} = \delta_{ij} \int_{\Omega} \rho N_i N_j d\Omega $$

$\delta$ is the Kronecker delta, $\rho$ is the density, and $N$ is the element shape functions.

To solve equations (1), they must be integrated over time. At the first stage of solution equation (3) should be solved:

$$ \{\dot{v}\} = [M]^{-1}\{f^{int} - f^{ext}\} = [M]^{-1}\{f^n\} $$

$f^n$ – forces vector at time n.

The initial conditions in the form of forces and velocities are imposed on equation (3) at the time t = 0. To find the displacements, a central-difference time integration scheme is used, so the velocities are found for the time n + 0.5 from the velocity values at the time n-0.5. If the velocity at time n are known, then the velocity at time n + 0.5 are determined by the following equation:

$$ \{v^{n+0.5}\} = \alpha \Delta t [M]^{-1}\{f^n\} + \{v^{n+0.5-\alpha}\} $$

$\Delta t$ – time step. $\alpha$ – parameter, for t=0 is $\alpha$ =0.5 and for other time steps $\alpha = 1$.

After finding the velocities, essential boundary conditions are imposed on the system and displacements are determined in the system for the next time step:

$$ \{u^{n+1}\} = \{u^n\} + \Delta t\{v^{n+0.5}\} $$

$u$ - nodes displacements.

Then nodal forces effective in the system were calculated. Forces can be found either through stress integration or by the use of the resulting elements directly connecting the displacements and forces. The resulting definitions were used in the said algorithm.

3. Internal Force Calculation

Elongation of for each element (figure 1 (a)) were calculated using equation (6). The finite length of the element (L) was determined based on the known displacements in the longitudinal ($u_1, u_2$) and lateral ($w_1, w_2$) directions, as well as the initial node coordinates ($X_1, Y_1$) and ($X_2, Y_2$):

$$ L = (((X_2 + u_2) - (X_1 + u_1))^2 + ((Y_2 - w_2) - (Y_1 - w_1))^2)^{0.5} $$

When solving the ask of free wave propagation along the wire rope, a numerical instability was identified in the course of the calculation under formula 2 (figure 1 (b)) in case of large wire rope displacements and the use of single precision numbers.
Figure 1. (a) Initial and final element positions in space, (b) point displacement under the impulse: blue under formula 2, brown under formula 3.

Thus, based on the recommendations of T. Belytschko [4], the element elongations were calculated under formula 3:

\[ u_l = \frac{L^2 - L_0^2}{L - L_0} \] (7)

L is the finite element length, \( L_0 \) is the initial element length, and \( u_l \) is the change in the element length.

In case of small displacements (the task of lateral vibrations of the string fixed at both edges with the amplitude of 4.5 mm at the cable length of 1000 mm), as well as the applied machine accuracy of the calculations, no difference was found between the solutions under the formulas (2) and (3).

Then, based on the known displacements, inner element forces in the local coordinate system were calculated and displayed in the global coordinate system:

\[ f = \frac{EAu_l}{L} \{-\cos \theta, -\sin \theta, \cos \theta, \sin \theta\}^T \] (8)

Equations (7) and (8) simplify the calculation of the forces \( f \) for equation (3), since forces are found directly from displacements.

4. Model Verification

Two calculations were made according to the said technique to determine the longitudinal and transversal wire rope vibrations under the above-stated conditions. In case of longitudinal vibrations, an impulse load of 10 kN was applied at one end, while the other end was recorded by displacements. In case of transversal waves, the load was applied to the cable center. The technique stated above was written in Wolfram Mathematica programming language for the wave process modelling in the wire rope. The operation of this code was compared to that of the commercial code for the wave process study (LS-DYNA). The results for the longitudinal impact are shown on figure 2. Local element equations were united into global ones under the congruent transformation method. A model of 50 mm long 10 rod elements with the time integration step of \( \Delta t=10^{-5} \) s, \( E=1.55\cdot10^5 \) MPa and the density of \( \rho_0=7.850\cdot10^{-9} \) t \cdot mm\(^{-3} \) was used to verify the solution.
Figure 2. Modelling results for the longitudinal impact: (a) author code, (b) LS-DYNA.

The comparison gave close results for transversal wire rope vibrations with the author code and LS-DYNA code. A remarkable thing was noted when analyzing the code operation. If the physical wave velocity is $v = 5000 \text{ m/s}$, the impulse at the element length of 100 mm shall be transmitted to the next solution region only in $t = 0.1/5000 = 20 \text{ ms}$. At that, the information in the code is distributed in the element in 1 time step, i.e., with the velocity of $v_m = l_e / \Delta t = 0.1 / 10^4 = 10^4 \text{ m/s}$. It turns out that the information distribution velocity in the model is always higher than in the physical environment and is equal to the latter only if the time increment and the element size have the relation of $v_{ph} = \sqrt{\frac{E}{\rho}} = l_e / \Delta t$ (the lower velocity will result in the solution instability as the Courant-Friedrichs-Levi condition is not met [4]). This circumstance is planned to be studied in further works. In addition, the impulse shape loss shall be noted, which reduces with the increase in the number of elements.

Comparison of the transversal wire rope vibrations is shown on figure 3. Both models demonstrated close solution results.

Figure 3. Modelling results for the transversal impact: (a) author code, (b) LS-DYNA.

When analyzing the results of transversal vibrations, it can be noted that the results obtained when using both models are close by their amplitude and frequency properties.

The wire ropes have a number of specific features complicating their physical descriptions: the wire rope elasticity modulus is a generalized structural indicator and is not related with the real wave propagation velocity in the material, the flexure-torsion and transversal rigidities, as well as the deformation properties are non-linear functions of the respective geometric parameters. It was also found [5, 6] that the deformation velocities influence the wire ropes to a great extent, especially the elasticity modulus. As a result of the experiments it has been determined that the modulus increases by
20-30\% even at the velocities $\dot{e} = 100 \text{ mm} \cdot (\text{s} \cdot \text{mm})^{-1}$ being the operating local velocities for wire ropes when the car hits the safety barrier. Thus, a separate non-resulting definition of the finite elements is planned to be developed through the deformation velocities.

5. Conclusions
The article considers the procedure to solve the task of the longitudinal and transversal vibrations for the wire rope represented as a rod. The program was developed in Wolfram Mathematica to obtain an analytical solution. The model was verified through comparison with the solution obtained when using the LS-DYNA commercial code. Both solutions had close results. Compared to LS-DYNA, the author's program is easier to parameterize, which is supposed to be used to study the influence of the elastic modulus, total length of the barrier, initial tension, stiffness and type of posts on its characteristics. The program is the first step in the work of the authors to obtain a more accurate model of the barrier, which makes it possible to take into account the change in the characteristics of the cables depending on the strain rate.

References
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