Super-accelerating bouncing cosmology in asymptotically-free non-local gravity

Gianluca Calcagni,\textsuperscript{a} Leonardo Modesto,\textsuperscript{b} Piero Nicolini\textsuperscript{c}

\textsuperscript{a}Instituto de Estructura de la Materia, CSIC, Serrano 121, 28006 Madrid, Spain
\textsuperscript{b}Department of Physics & Center for Field Theory and Particle Physics, Fudan University, 200433 Shanghai, China
\textsuperscript{c}Frankfurt Institute for Advanced Studies (FIAS) & Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität, 60438 Frankfurt am Main, Germany

E-mail: calcagni@iem.cfmac.csic.es, lmodesto@fudan.edu.cn, nicolini@fias.uni-frankfurt.de

\textbf{Abstract.} Recently, evidence has been collected that a class of gravitational theories with certain non-local operators is renormalizable. We consider one such model which, at the linear perturbative level, reproduces the effective non-local action for the light modes of bosonic closed string field theory. Using the property of asymptotic freedom in the ultraviolet and fixing the classical behaviour of the scale factor at late times, an algorithm is proposed to find general homogeneous cosmological solutions valid both at early and late times. Imposing a power-law classical limit, these solutions (including anisotropic ones) display a bounce instead of a big-bang singularity, and super-accelerate near the bounce even in the absence of an inflaton or phantom field.

\textbf{Keywords:} Quantum Gravity Phenomenology, Alternatives to Inflation, Cosmic Singularity
1 Introduction

Asymptotic freedom is an attribute of field theories such that interactions are negligible in the ultraviolet (UV), where the theory possesses a trivial fixed point. This property has been used, directly or implicitly, to construct field theories of gravity where the Laplace–Beltrami operator $\Box$ is replaced by one or more non-local operators $f(\Box)$ in the kinetic terms of the action. On a Minkowski background, if the Fourier transform $\tilde{f}(k^2) \to \infty$ for large momenta $k$, these terms dominate in the UV over the interactions, which can be ignored. Asymptotic freedom then, if realized, ensures that the correct UV behaviour of the theory be encoded in the free propagator. A class of these models is of particular relevance inasmuch as the kinetic operator is of the exponential form $\Box e^\Box$ or $e^\Box$ and is inspired, respectively, by open string field theory (OSFT; see [1–4] for reviews) and the p-adic string [5–10]. In the simplest classical cosmological applications, gravity is local and the only non-local content is a scalar field [11–35], sometimes identified with the tachyon of bosonic OSFT or of super-symmetric OSFT on an unstable brane.

Non-local gravity sectors and their cosmology have been proposed in [36–51], following various criteria including avoidance of ghosts, improved renormalizability, and the possibility to construct non-perturbative solutions. The gathered results (also in theories without non-local operators at the tree level [52]) point towards a resolution of gravitational singularities thanks to asymptotic freedom. These approaches do not stem from closed string field theory, i.e., the SFT sector containing the perturbative graviton mode [53–67]. Yet, many of them are inspired from it in the sense that the kinetic functions are exponential (or somewhat more general) operators as in effective closed SFT. In this context, one bypasses the technical difficulties in getting effective non-local actions directly from SFT [56, 67] and concentrates on phenomenological but more manageable models.

Cosmological dynamical solutions were obtained either directly, by solving the non-local equations of motion, or indirectly, by solving an Ansatz for the Ricci curvature (or via the diffusion equation [40]) respecting the equations of motion. Here we pursue a different but no less economic alternative, using asymptotic freedom as a key ingredient.
The strategy is the following. (i) First, in section 2 we define a model of non-local gravity inspired by closed SFT and falling into the class of actions considered in [41, 42]. (ii) In the UV, all interactions can be ignored and, thus, it is sufficient to find cosmological backgrounds compatible with the Green equation for the propagator. The scale factors $a(t)$ representing such backgrounds do not collapse into a big bang but, rather, display a bounce. At large scales, they reduce to known profiles of ordinary cosmology, the details of which depend on the choice of matter content. In particular, in section 3 we will find, with two different methods, backgrounds following a power law at late times. These profiles are approximate solutions of the full equations of motion valid both at very early times (when, roughly, the cosmological horizon scale is near the UV asymptotically-free fixed point) and at late times. All such solutions have a bounce and accelerate near it without invoking inflaton-like matter content.1 (iii) Next, we write down an effective Friedmann equation of the form

$$H^2 = \frac{\kappa^2}{3} \rho_{\text{eff}} := \frac{\kappa^2}{3} \rho \left[ 1 - \left( \frac{\rho}{\rho_*} \right)^\beta \right],$$

(1.1)

where $H := \dot{a}/a = \partial_t a/a$ is the Hubble parameter, $\kappa^2 = 8\pi G$, $G$ is Newton’s constant, $\rho$ is the energy density of the universe, $\rho_*$ is the critical energy density at which the bounce occurs, and $\beta > 0$ is a real parameter. The exponent $\beta$ is determined by plugging the profile $a(t)$ found under the provision of asymptotic freedom into eq. (1.1) for a given energy density profile $\rho(a)$. Since this fitting is generally rather good, we can conclude that the class of asymptotic solutions found in step (ii) is reasonably valid also at intermediate times, and that the bouncing-accelerating scenario of the theory is well described by the effective Friedmann equation (1.1).

2 The model

2.1 Effective action from closed string field theory

We start by recalling the derivation of the effective SFT Lagrangian for the closed string through a mass-level truncation scheme. In the non-polynomial bosonic closed SFT, the action has the following compact form:

$$S_{\text{SFT}} = \int \frac{1}{\alpha'} \Phi * Q b_0^\alpha \Phi + g \sum_{N=3}^{+\infty} \frac{(g \alpha')^{N-3}}{2^{N-3}N!} \Phi * [\Phi^{N-1}],$$

(2.1)

where $\Phi$ is the closed-string field, $Q$ is the BRST operator, $b_0^\alpha = (b_0 - \bar{b}_0)/2$ is a combination of antighost zero modes, $\alpha'$ is the Regge slope and $g$ is the string-field coupling constant. The integral $\int \Phi_1 * \Phi_2$ represents the string-field scalar product, while the symbol $[\Phi^{N-1}] = [\Phi_1 \ldots \Phi_{N-1}]$ denotes the string field obtained combining $N - 1$ fields $\Phi_1, \ldots, \Phi_{N-1}$ using the $N$-string vertex function.

The idea that light states dominate physical processes justifies the following truncation of the string field in terms of oscillators and particle fields:

$$\Phi = c_0 \left[ \phi + A_{\mu\nu} \alpha_\mu - \alpha_\nu \right] + \frac{(a_+ + a_-) \bar{b}_- - \bar{c}_- - 1}{\sqrt{2}} + \frac{(a_+ - a_-) c_1 - \bar{b}_+}{\sqrt{2}} + ic_0 \left( j_1 \alpha_{\mu_1} \bar{b}_- + j_2 \alpha_{\nu_1} \bar{a}_- \right) |0\rangle,$$

(2.2)

1See [68, 69] for inflation-without-inflaton scenarios descending from other mechanisms.
where $A_{(\mu\nu)} := (A_{\mu\nu} + A_{\nu\mu})/2$ is the graviton field, $A_{[\mu\nu]} := (A_{\mu\nu} - A_{\nu\mu})/2$ is an antisymmetric rank-2 tensor field, $\alpha_{\pm}, j_{1\mu}$ and $j_{2\mu}$ are auxiliary fields, and $b$ and $c$ are first-quantized ghost oscillators. Greek indices run over spacetime directions and are lowered via the Minkowski metric $\eta_{\mu\nu} = \text{diag}(+,-,\ldots,-)$. The state $|0\rangle = \tilde{c}_j|\Omega\rangle \otimes c_j|\bar{\Omega}\rangle$ is the first-quantized string vacuum, with $|\Omega\rangle$ and $|\bar{\Omega}\rangle$ the left and right $SL(2,\mathbb{R})$-invariant vacua. Further details can be found, e.g., in [1].

The truncation (2.2) allows one to derive the cubic effective Lagrangian $\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$. Working in the Siegel–Feynman gauge $b_0^\mu \Phi = 0$ (which sets $j_{1\mu} = j_{2\mu} = 0$), the kinetic and mass terms read

$$\mathcal{L}_{\text{free}} = \frac{1}{2} \partial_\lambda A_{\mu\nu} \partial^\lambda A^{\mu\nu} + \frac{1}{2} \partial_\lambda \phi \partial^\lambda \phi + \frac{2}{\alpha'\phi^2} + \frac{1}{2} \partial_\lambda \alpha_+ \partial^\lambda \alpha_+ - \frac{1}{2} \partial_\lambda \alpha_- \partial^\lambda \alpha_- ,$$

while $\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{int}}(\partial, \tilde{\phi}, \tilde{A}, \tilde{\alpha}_{\pm})$ has a number of interaction terms with derivatives. For a given field $\varphi(x)$ in the kinetic term, we have the corresponding “dressed” field

$$\varphi(x) = e^{-\Box/(2\Lambda^2)} \varphi(x), \quad \Box = \partial_\mu \partial^\mu$$

in the interaction part, where $1/\Lambda^2 = \alpha' \ln(3\sqrt{3}/4) \approx 0.2616 \alpha'$.

The above Lagrangian $\mathcal{L}$ can be recast in an equivalent form by shifting the smearing functions from the interaction term to the kinetic and mass ones by the field redefinition $\varphi(x) \to e^{\Box/(2\Lambda^2)} \varphi(x)$. Ignoring the auxiliary fields, the free part of the Lagrangian reads

$$\mathcal{L}_{\text{free}} = -\frac{1}{2} A_{\mu\nu} \Box e^{\Box/(2\Lambda^2)} A^{\mu\nu} - \frac{1}{2} \phi \left( \Box - \frac{2}{\alpha'} \right) e^{\Box/(2\Lambda^2)} \phi ,$$

which leads to the following stringy modifications of the Laplace–Beltrami operator in the zero-level truncation scheme:

$$\Box \to \Box e^{\Box/(2\Lambda^2)} .$$

The associated propagator generally leads to a ghost-free spectrum, the intuitive reason being that entire functions $f(\Box)$ do not introduce extra poles [14, 25, 38, 70]. Quantum field theories with exponential propagators have been argued long since to be super-renormalizable [71]. Operators of the form (2.6) also appear in the context of non-commutative geometries where a minimal length is effectively induced [72–75].

### 2.2 Non-local gravity model

From the tree-level effective action for the graviton and matter, we can argue about the form of its non-linear covariant extension. On the ground of a recently introduced candidate model of super-renormalizable gravity [41–43, 45, 48, 49] based on earlier results [36, 37] (see also [76] for considerations in a local higher-derivative theory), we propose the action\(^2\)

$$S = \frac{1}{2\kappa^2_D} \int d^Dx \sqrt{|g|} \left[ R - G_{\mu\nu} \gamma(\Box) R^{\mu\nu} \right] + S_{\text{matter}} ,$$

$$S_{\text{matter}} = \int d^Dx \sqrt{|g|} \left[ \frac{1}{2} \nabla_\mu \phi V^{-1}(\Box) \nabla^\mu \phi + \frac{1}{2\pi n!} e^{\epsilon \phi} F_{[n]} V^{-1}(\Box) F_{[n]} \right] ,$$

---

\(^2\)This action differs from the theory of [40], which is of scalar-tensor type. There, the curvature invariants appear only in the exponential non-local operator in order to allow for non-perturbative solutions via a method based on the diffusion equation.
where $G_{\mu\nu}$ and $R_{\mu\nu}$ are, respectively, the Einstein tensor and the Ricci tensor associated with the $D$-dimensional target spacetime metric $g_{\mu\nu}$, and, adopting the terminology used in string theory, $\phi$ is the dilaton field coming from the trace of the field $A_{\mu\nu}$, $n = p + 2$, and $F[n]$ are the $p$-field strengths corresponding to the gauge potentials. The value of the parameter $c$ controls the interaction of the scalar field $\phi$ with the field strength $F[n]$. The key ingredients of the above action are the operators

$$\gamma(\Box) := \frac{V^{-1}(\Box) - 1}{\Box}, \quad V(\Box) := e^{-\Box/\Lambda^2},$$

(2.8)

where $\Lambda$ (proportional to $\alpha'^{-1/2}$ in SFT) is the invariant energy scale above which quantum gravity effects become non-negligible. This form of the kinetic terms correctly reproduces the non-local operator (2.6) (and its inverse, the propagator $V(\Box)/\Box$) when linearizing the fields [36, 37]. It also leads to the improved renormalizability of the model. However, in a spacetime of even dimension the effective action is not generally finite but only super-renormalizable because one-loop diagrams are still superficially divergent [36, 37, 42, 43, 45, 48, 49, 76, 77]. Things go differently in a spacetime of odd dimension, because at the one-loop level there are no local operators which can serve as counter-terms for pure gravity and the theory results to be finite [78, 79]. When, in the case of super-symmetry, matter is added to fill up the super-gravity multiplet, the theory remains finite [80].

In this paper, we study asymptotic profiles which are approximate cosmological solutions of the gravitational system (2.7a). The classical action (2.7a) is a “non-polynomial” or “semi-polynomial” extension of quadratic Stelle theory [76, 81]. All the non-polynomiality is incorporated in the form factor $\gamma(\Box)$. The entire function $V$ has no poles in the whole complex plane, which preserves unitarity, and it has at least logarithmic behaviour in the UV regime to give super-renormalizability at the quantum level.

Here we only consider corrections to the classical solutions coming from the bare two-point function of the graviton field. The reason is that this class of theories is asymptotically free and the leading asymptotic behaviour of the dressed propagator is dominated by its bare part. In fact, according to power counting arguments [37, 42], the self-energy insertions, which are constant or at most logarithmic, do not contribute to it.

3 Cosmology

3.1 General solution in asymptotically-free gravity

To find general homogeneous and isotropic cosmological solutions in the UV, we adapt a procedure used in ordinary perturbative quantum gravity [82–84]. Later on, we shall comment on the main differences of our setting with respect to earlier applications. We fix the number of dimensions to $D = 4$. Consider a homogeneous and isotropic Friedmann–Robertson–Walker (FRW) metric $g_{\mu\nu}$ with zero curvature; the anisotropic case is straightforward and will be discussed later. We split the metric into a flat Minkowski background plus a homogeneous fluctuation $h_{\mu\nu}$,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a(t)^2 \delta_{ij}dx^i dx^j,$$

(3.1)

where $i = 1, 2, 3$. By definition, the fluctuation is small around a certain time $t_i$, where $g_{\mu\nu}(t_i) = \eta_{\mu\nu}$. Thus, the scale factor $a$ and the fluctuation $h_{\mu\nu}$ are

$$a^2(t) = 1 - \kappa h(t), \quad h(t) = 0, \quad g_{\mu\nu}(t = t_i) = \eta_{\mu\nu},$$

(3.2a)

$$h_{\mu\nu}(t) = h(t) \text{diag}(0, \delta_{ij}) =: h(t) \mathbb{I}_{\mu\nu} \text{.}$$

(3.2b)
The tensor $h_{\mu\nu}$ can be rewritten in harmonic gauge by the transformation

$$h_{\mu\nu}(x) \rightarrow h'_{\mu\nu}(x) := h_{\mu\nu}(x) + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad \xi_\mu(t) = -\frac{3\kappa}{2} \text{diag} \left[ \int_{t_i}^t dt' h(t'), 0, 0, 0 \right]. \quad (3.3)$$

The fluctuation now reads

$$h'_{\mu\nu}(t) = h(t) \text{diag}(-3, \delta_{ij}), \quad h'_{\mu}^{\mu}(t) = -6h(t). \quad (3.4)$$

The standard gravitational field $\tilde{h}_{\mu\nu}$ is then

$$\tilde{h}_{\mu\nu} := h'_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h'_{\chi}^{\chi} = h(t) \text{diag}(0, -2\delta_{ij}) = -2h(t)I_{\mu\nu}, \quad \partial^\mu \tilde{h}_{\mu\nu} = 0. \quad (3.5)$$

The Fourier transform of the above field is given by

$$\tilde{h}_{\mu\nu}(E, \vec{k}) = -2\tilde{h}(E)(2\pi)^3\delta(\vec{k}) I_{\mu\nu}. \quad (3.6)$$

At this point, we exploit asymptotic freedom to avoid solving the full equations of motions of (2.7) on a flat FRW background, and to recognize the profile (3.2) as an actual asymptotic solution of our model. We express the classical propagator for the excitation $\tilde{h}_{\mu\nu}$ via a dimensionless source $\varrho$, representing the 00 component of an effective “energy-momentum” tensor $T^{\mu\nu}$. Denote its Fourier transform with a tilde. The gauge-independent part of the graviton propagator [42, 88] for the theory (2.7a) is then

$$\mathcal{O}^{-1}(k) = \frac{V(k^2)}{k^2} \left( P^{(2)} - \frac{P^{(0)}}{2} \right) \quad \Rightarrow \quad \tilde{h}_{\mu\nu}(x) = \kappa \int \frac{d^4k}{(2\pi)^4} \mathcal{O}^{-1}_{\mu\nu,\rho\sigma}(k) \tilde{T}^{\rho\sigma}(k) e^{-ik\cdot x}, \quad (3.7)$$

where $P^{(0)}$ and $P^{(2)}$ are Van Nieuwenhuizen projectors in four dimensions [89]. The standard “classical” case is obtained for $V(k^2) \rightarrow 1$, but for the theory (2.7a) we find, using the graviton propagator after Wick rotation,

$$h(t) = -\frac{\kappa}{2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2V^{-1}(k^2)} \tilde{g}(E, \vec{k}) e^{-ik\cdot x} = \int dE \frac{2\pi}{\Lambda} \tilde{h}(E) e^{Eit}. \quad (3.8)$$

Assuming that the kernel $\tilde{g}$ does not depend on the cut-off $\Lambda$, the quantum-corrected profile $a(t)$ can be found from its “classical” limit $a_{cl}(t)$. The procedure is the following: (i) fix an $a_{cl}(t)$ and, via eq. (3.2), a profile $h_{cl}(t)$; (ii) set temporarily $V = 1$ in eq. (3.8) and obtain $\tilde{h}(E)$ and, from that, the cut-off-independent distribution $\tilde{g}(E, \vec{k})$; (iii) plug $\tilde{g}$ back into eq. (3.8) and perform the four-momentum integral with the full $V(k^2) \neq 1$, to obtain $h(t)$; (iv) use (3.2) to get $a(t)$. The profile $a_{cl}(t)$ is thus recovered perturbatively at times $t \gg t_i$.

This procedure is similar to the one employed in [82–84], but with an important difference. There, in order to go beyond the classical theory, one introduces one-loop quantum corrections to the graviton propagator. In our case, however, we already have modifications at the classical level and, therefore, we use only the bare propagator. This can be justified by noting, as mentioned above, that one-loop corrections to the propagator in this class of non-local theories are UV sub-dominant with respect to the tree-level contribution. Thus, a general conclusion is that any asymptotically-free theory of gravity with a two-point function of the form (3.7), with sufficiently strong damping factor $V$, will admit an asymptotic UV solution of the form (3.2) solely found at the tree level in perturbation theory.
3.2 General solution with power-law regime

We start by considering a profile compatible, at late times, with a power law. We recall that we are not in vacuum and the matter source shapes the expansion of the universe. The classical profile is very simple, namely,

\[ a_{cl}(t) = \left| \frac{t}{t_i} \right|^p \text{ and } h_{cl}(t) = \frac{1}{\kappa} \left[ 1 - \left| \frac{t}{t_i} \right|^{2p} \right], \tag{3.9} \]

where \( t_i \) is the pivot time around which one centers the perturbative expansion of the metric, \( t = 0 \) is the big-bang singularity time, and \( p > 0 \) is a constant. Equation (3.9) is solution to the ordinary Einstein equations, recovered at low curvature also in our model when higher-order Riemann terms are negligible. \( a_{cl}(t) \) is an even function of time due to time-reversal symmetry in the standard classical Friedmann equations and, in turn, it will determine an even quantum-corrected profile \( a(t) = a(|t|) \). This will also prevent a possible issue with the procedure (i)–(iv) detailed above. The propagator is integrated after Wick rotation, when the form factor \( V \) makes it convergent. For the purpose of field-theory calculations, going to imaginary time poses no particular problem, provided the Osterwalder–Schrader conditions are satisfied [37, section 5]. However, the solution \( a(t) \) is found with the Wick-rotated propagator, and its analytic continuation back in Lorentzian time \( t \rightarrow -it \) may no longer be sensible for the Lorentzian system. In the present case, however, the solution depends on \( t^2 = |t|^2 \) (\( t \) is a real parameter) and it correctly reaches the classical power-law solution of the Lorentzian theory at late times. Therefore, the profile \( a(t) \) is unaffected by the analytic continuation. In section 3.3, we will recover the same solution with an independent method.

Given the condition (3.9), we can calculate the Fourier transform \( \tilde{h}(E) \) defined in eq. (3.6),

\[ \tilde{h}(E) = \frac{1}{\kappa E^2 V^{-1}(E^2)} \left[ \pi E^2 \delta(E) + \frac{\sin(\pi p)\Gamma(2p+1)}{t_i^{2p}|E|^{2p-1}} \right], \tag{3.10} \]

and, from eq. (3.8), the source \( \tilde{\rho} \) in momentum space:

\[ \tilde{\rho}(E, \vec{k}) = \frac{4}{\kappa^2} \left[ \pi E^2 \delta(E) + \frac{\sin(\pi p)\Gamma(2p+1)}{t_i^{2p}|E|^{2p-1}} \right] (2\pi)^3 \delta(\vec{k}). \tag{3.11} \]

For the form factor in eq. (2.8), the fluctuation resulting from the integral (3.8) is

\[ \kappa h(t) = 1 - \left( \frac{2}{\Lambda t_i} \right)^{2p} \sqrt{\frac{\Gamma\left(\frac{1}{2} + p\right)}{\pi}} \int_{ \Gamma(b) }^{ \Gamma(c) } \Gamma(b + l) \ z^l \frac{z^l}{l!} \ 1 F_1 \left( -p; \frac{1}{2}; -\frac{1}{4} t_i^2 \Lambda^2 \right), \tag{3.12} \]

where \( 1 F_1 \) is confluent hypergeometric function of the first kind (Kummer’s function):

\[ 1 F_1 (b; c; z) := \frac{\Gamma(c)}{\Gamma(b)} \sum_{l=0}^{+\infty} \frac{\Gamma(b + l) \ z^l}{\Gamma(c + l) \ l!}. \tag{3.13} \]

The transcendental equation (3.12) determines the value of \( \Lambda t_i \) such that \( h(t_i) = 0 \). This only affects unimportant time rescalings and the normalization of the scale factor

\[ a(t) = \left( \frac{2}{\Lambda t_i} \right)^p \sqrt{\frac{\Gamma\left(\frac{1}{2} + p\right)}{\pi}} \int_{ \Gamma(b) }^{ \Gamma(c) } \Gamma(b + l) \ z^l \frac{z^l}{l!} \ 1 F_1 \left( -p; \frac{1}{2}; -\frac{1}{4} t_i^2 \Lambda^2 \right). \tag{3.14} \]
Figure 1. Typical bouncing profile of the scale factor for $p < 1$ (left plot, $p = 1/2$, radiation) and $p > 1$ (right plot, $p = 3$, accelerating universe), for $\Lambda = 1 = t_i$. The convexity of the curve changes sign at $p = 1$. Dashed curves show the corresponding power-law profiles $a \sim t^p$ with big bang singularity.

The early- and late-time behaviour can be easily found from the asymptotics of Kummer’s function (e.g., [23]):

$$1F_1(b; c; -z^2) \overset{z \to \pm\infty}{\sim} \frac{\Gamma(c)}{\Gamma(c - b)} (z^2)^{-b} , \quad 1F_1(b; c; -z^2) \overset{z \to 0}{\sim} 1 .$$

We obtain

$$a(t) \overset{t \to \pm\infty}{\sim} \left| \frac{t}{t_i} \right|^p , \quad a(t) \overset{t \to 0}{\sim} a_* := \sqrt{\frac{\Gamma \left( \frac{1}{2} + p \right)}{\sqrt{\pi}}} \left( \frac{2}{\Lambda t_i} \right)^p .$$

The transition between these two regimes is set by the critical time $1/\Lambda$. The general picture is that, at late times, the universe expands as a power law determined by its matter content. Common cases are radiation ($p = 1/2$), dust matter ($p = 2/3$), and acceleration-inducing components such as inflaton and quintessence ($p > 1$). In all these scenarios, there is a finite bounce at $t = 0$, whose value increases with $p$ if $\Lambda t_i \sim 1$. The most direct agent responsible for the bounce is asymptotic freedom: when the energy scale $\Lambda$ is sent to infinity, $a_* \to 0$.

Figures 1 and 2 illustrate the bouncing behaviour of, respectively, the scale factor and the Ricci scalar $R = 6(\ddot{a}/a + \dot{a}^2/a^2)$ for radiation and for a large-$p$ example. In the first case, we used the special form of Kummer’s function $1F_1(-1/2; 1/2; -z^2) = e^{-z^2} + \sqrt{\pi} z \text{erf}(z)$, where erf is the error function:

$$a(t) = \frac{2e^{-\frac{1}{4}\Lambda t_i^2}}{\sqrt{\pi} \Lambda t_i} + \frac{t}{t_i} \text{erf} \left( \frac{\Lambda t_i}{2} \right) .$$

An important consequence of eq. (3.14) is that one always has super-acceleration near the bounce independently of the value of $p$. This mechanism of super-inflation, which does not need any slow-rolling or exotic (e.g., phantom) scalar field, may be viewed as due to the vacuum energy associated with the graviton fluctuation, which is present for any type of matter content. Figure 3 shows positivity of the acceleration $\ddot{a}$, and a graceful exit from inflation when the matter content obeys the classical dominant energy condition (i.e., non-inflationary matter).

The Hubble parameter associated with the solution (3.14) is

$$H(t) = \frac{\dot{a}}{a} = \frac{p \Lambda^2 t}{2} 1F_1 \left( 1 - p; \frac{3}{2}; -\frac{1}{4} t_i^2 \Lambda^2 \right) 1F_1 \left( -p; \frac{1}{2}; -\frac{1}{4} t_i^2 \Lambda^2 \right) .$$
Figure 2. Ricci scalar $R$ for $p = 1/2$ (left) and $p = 3$ (right). The curvature increases towards the bounce, where it acquires a non-singular value.

Figure 3. $\ddot{a}(t)$ for $p = 1/2$ (left) and $p = 3$ (right). In the presence of non-accelerating matter content (in this case, late-time radiation), when $\ddot{a} = 0$ the (super-)inflationary era near the bounce naturally ends.

At the bounce, $H = 0$. Asymptotically,

$$H(t) \underset{t \to \pm\infty}{\sim} \frac{p}{t}, \quad H(t) \underset{t \to 0}{\sim} \frac{p\Lambda^2}{2t},$$

(3.19)

and, integrating at small times, we have

$$a(t) \sim \exp\left(\frac{p}{4}\Lambda^2 t^2\right) \quad \text{as} \quad t \to 0,$$

(3.20)

thus getting the asymptotic behaviour of the bounce in a neighborhood of $t = 0$. The scale factor $a(t)$ for small times has a typical super-acceleration profile. This is remindful of an early contribution on cosmology in higher-derivative gravity [90], even if, as just stated, in our case we do not need to invoke the inflaton.

We conclude this section with two comments. First, Kasner solutions are also straightforward. In ordinary Einstein–Hilbert gravity, the solution for a flat homogeneous anisotropic universe is $a_i(t) = t/t_i|^{p_i}$, where $i = 1, 2, 3$ and the exponents $p_i$ obey the two conditions $\sum_i p_i = 1 = \sum_i p_i^2$. Following the above procedure (see [84] for Kasner solutions in a local quantum-gravity model), from the propagator we get three copies of eq. (3.14). This formula is automatically well-defined for all the values of $p_i$ allowed by the Kasner conditions. Each
Figure 4. The three scale factors $a_i(t)$ of a Kasner solution with $p_1 = -1/4$ (dashed curve), $p_2 = (5 - \sqrt{5})/8 \approx 0.35$ (solid curve), and $p_3 = (5 + \sqrt{5})/8 \approx 0.90$ (dotted curve), with $\Lambda = 1 = t_i$.

scale factor reaches its local extremum (a minimum for two $p_i > 0$ and a maximum for $p_i < 0$) at different values dictated by eq. (3.16). Going forwards in time towards $t = 0$ and beyond, two directions contract, undergo a bounce, and begin to expand. In the meanwhile, the third direction expands from a minimum value $a_i^\ast$, reaches a maximal extension at the bounce, and then contracts back to $a_i^\ast$. Figure 4 shows one such solution.

Finally, we also checked that, in models where Wick rotation is not necessary, the bounce picture persists. Let us recall that Wick rotation was required because, in momentum space, the form factor (2.8) is not convergent when integrating in $k^0$, $V(k^2) = \exp(\Lambda^2 k^2) = \exp[\Lambda^2(k_0^2 - |\vec{k}|^2)]$. On the other hand, using even powers of the Laplace–Beltrami operator renders the form factor convergent without transforming to imaginary time, and the bouncing-accelerating scenario still holds. For instance, in Krasnikov’s model with the operator $V(\Box) = e^{-\Box^2/\Lambda^4}$, the solution with power-law asymptotic limit is a superposition of generalized hypergeometric functions $_qF_s$, bounded from below (respectively, above) for $p > 0$ ($< 0$) by a symmetric bounce at some $a_s \neq 0$.

3.3 Alternative derivation of the solution

The solution (3.14) can be also found via the diffusion-equation method [85], which has proven to be a powerful tool both to address the Cauchy problem in exponential-type non-local systems [86] and to find non-perturbative tachyon solutions in string theory at the level of target actions [25, 85, 87].

We linearize the action up to second order in the fluctuations and end up with eq. (2.5). Here, we do not throw away self- and matter interactions completely, and we encode them into an effective mass $m_{\text{eff}}$ for the linearized gravitational field. The latter obeys the equation of motion $\Box e^{\Box^2/\Lambda^4} h_{\mu\nu} + m_{\text{eff}}^2 h_{\mu\nu} = 0$. To solve this equation (or, approximately, its non-linear extensions), we promote $h_{\mu\nu}(x) \rightarrow h_{\mu\nu}(x, r)$ to a field living in $D + 1$ dimensions, where $r$ is an artificial extra direction (dimensionally, a squared length), and assume that $h$ obeys the diffusion equation

$$(\partial_r - \Box)h_{\mu\nu}(x, r) = 0, \quad h_{\mu\nu}(x, 0) = h_{\mu\nu}^{\text{cl}}(x),$$

with a given set of initial conditions $h_{\mu\nu}^{\text{cl}}(x)$. Here $\Box$ is the Laplace–Beltrami operator in the background metric, i.e., $\Box = \eta^{\mu\nu} \partial_\mu \partial_\nu$ in our case. Once the solution is found, the parameter
$r$ is fixed at some constant value $r = r_*$ such that the equation of motion $\Box h_{\mu\nu}(x, r_* + 1/\Lambda^2) + m_{eff}^2 h_{\mu\nu}(x, r_*) = 0$ is solved.

For a homogeneous setting, the problem is drastically simplified. The only non-vanishing components of the metric are the diagonal spatial ones, and we need only to consider one diffusion equation for $a^2(t, r)$, with Laplace–Beltrami operator $\Box = \partial_r^2$ (spatial derivatives are immaterial). The initial condition at $r = 0$ is nothing but the asymptotic classical profile at $t \to \pm\infty$ because the solution only depends on the ratio $-t^2/(4r)$ (this can be checked either a posteriori or beforehand by a simple scaling argument). Given a power-law initial condition, the solution of the diffusion equation

$$(\partial_r - \partial_r^2) a^2(t, r) = 0, \quad a^2(t, 0) = a^2_0(t) = \left| \frac{t}{t_i} \right|^{2p}, \quad (3.22)$$

is, when evaluated at $r = r_*$, eq. (3.14) with $\Lambda^2 = 1/r_*$. The proof of this statement is essentially the calculation in [23] for a scalar-field profile in non-local cosmologies with exponential operators. Summarizing the procedure in a nutshell, one expands the initial condition $a^2(t, 0)$ as an integral superposition of the eigenfunctions $e^{\pm iEt}$ of the exponential operator $V^{-1}(\Box)$. Applying $V^{-1}(\Box)$ to the initial condition and performing the integration, one obtains a linear combination $C_1 a^2_1(t, r) + C_2 a^2_2(t, r)$ of the two solutions to the second-order equation (3.22), where

$$a^2_1(t, r) = \frac{2\Gamma(1 + p)}{\sqrt{\pi}} \left( \frac{4r}{l_i^2} \right)^p \sqrt{\frac{l_i^2}{4r}} i \Gamma_1 \left( \frac{1}{2} - p; \frac{3}{2} - \frac{t^2}{4r} \right), \quad (3.23)$$

$$a^2_2(t, r) = \left( -\frac{4r}{l_i^2} \right)^p \Psi \left( -p; \frac{1}{2} - \frac{t^2}{4r} \right), \quad (3.24)$$

and $\Psi$ is confluent hypergeometric function of the second kind. The solution $a^2_1$ has a big-bang singularity and is not well-defined (positive definite, finite, and so on) for all values of $p > 0$, while $a^2_2$ is complex-valued for general $p$. The choice of coefficients $C_1 = -i \tan(\pi p)$ and $C_2 = e^{-i\pi p} / \cos(\pi p)$ gives (the square of) our solution (3.14). This linear combination is real-valued, big-bang free, super-accelerating near the bounce, and valid for all positive $p$.

### 3.4 Effective dynamics

The solution (3.14) is approximate. Thanks to asymptotic freedom, it is valid at early times $t \lesssim t_i$ where interactions are negligible. It is valid also at late times, where the theory reduces to ordinary Einstein–Hilbert gravity plus sub-leading quadratic terms (Stelle model). To check its viability outside these regimes, it is interesting to fit the Hubble parameter (3.18) with the effective energy density $\rho_{eff}$ defining the modified Friedmann equation (1.1). In the right-hand side of that equation, we take the energy density $\rho$ to be the one of a standard power-law cosmology, but with $a = |t/t_i|^p$ replaced by eq. (3.14),

$$\rho(t) = \frac{3p^2}{k^2 l_i^2} \frac{1}{a^{2/p}(t)}, \quad (3.25)$$

---

3The reader can track down the steps in [86] from eqs. (45) to (81), with the following mapping from the symbols used there to those adopted here: $\psi = \psi_3 \to a^2$, $\phi_0 \to t^{-2q}$, $\theta/4 \to p$, $\nu \to 1/2$, $H_0 \to 0$, $\alpha \to -1$. In [86], the opposite convention $\eta = \text{diag}(-, +, \cdots, +)$ is used for the spacetime signature.
Figure 5. Plot of $H^2$ (eq. (3.18), solid curve) and the effective energy density (3.27) (dashed line), for $p = 1/2, \beta = 0.83$ (left plot) and $p = 3, \beta = 1.3$ (right plot), with $\Lambda = 1$.

While the critical energy density $\rho_\star \neq \rho_{\text{eff}}(0)$ is fixed by the bounce scale factor $a_\star = a(0)$ in (3.16),

$$
\rho_\star = \frac{3p^2}{\kappa^2 t_1^2 a_\star^{2/p}} = \frac{3p^2 \Lambda^2}{4\kappa^2} \left[ \frac{\sqrt{\pi}}{\Gamma \left( \frac{1}{2} + p \right)} \right]^\frac{1}{p} = \frac{3p^2}{32\pi} \left[ \frac{\sqrt{\pi}}{\Gamma \left( \frac{1}{2} + p \right)} \right]^\frac{1}{p} \frac{\Lambda^2}{m_{\text{Pl}}^2} \rho_{\text{Pl}},
$$

where $m_{\text{Pl}} = \sqrt{\hbar c/G} \approx 1.2209 \times 10^{19}$ GeV is the Planck mass and $\rho_{\text{Pl}} = m_{\text{Pl}}^4 \approx 2.2 \times 10^{76}$ GeV$^4$ is the Planck energy density. Overall,

$$
\frac{\kappa^2}{3} \rho_{\text{eff}}(t) = \frac{p^2}{t_1^2 a_\star^{2/p}(t)} \left\{ 1 - \left[ \frac{a_\star}{a(t)} \right]^{\frac{2\beta}{p}} \right\},
$$

which is the quantity plotted in figure 5. For $p = 1/2$ (radiation case), we achieved a good qualitative fit with $\beta \approx 0.83$, while for $p = 3$ we plot $\beta \approx 1.3$.

Notice that there may be no compelling reason to impose an all-scale fitting as in the figure. Using the asymptotic limits (3.15), we have eq. (3.19) and

$$
\sqrt{\frac{\kappa^2}{3} \rho_{\text{eff}}(t)} \overset{t \to \pm \infty}{\sim} \frac{p}{t}, \quad \sqrt{\frac{\kappa^2}{3} \rho_{\text{eff}}(t)} \overset{t \to 0}{\sim} \sqrt{\frac{2\beta}{t_1^2 a_\star^{2/p} \Lambda^2}} \frac{p\Lambda^2}{2}. \quad (3.28)
$$

The relative error at the origin is minimized for

$$
\beta = \frac{a_\star^{\frac{2}{p}} \Lambda^2 t_1^2}{2} = 2 \left[ \left( \frac{\frac{1}{2} + p}{\sqrt{\pi}} \right) \right]^{\frac{1}{p}}, \quad (3.29)
$$
equal to $\beta \approx 0.64$ for $p = 1/2$ and $\beta \approx 2.47$ for $p = 3$. For these values (which neither correspond to the fit of the figure nor to a least maximal error in the fit), there is a significant difference in the height of the local symmetric maxima at intermediate times. Conversely, in the plots the maximal relative error occurs at the origin. We believe that eq. (3.29) better represents the asymptotic solutions, since higher-order curvature and quantum corrections are expected anyway to modify the evolution at mesoscopic scales, where the two lumps are located.
The effective pressure $P(t) = -\rho_{\text{eff}} - \dot{\rho}_{\text{eff}}/(3H)$ for $p = 1/2$ (left) and $p = 3$ (right). At the bounce, it tends to a non-zero finite value $P(0) = -\hat{R}(0)/(3\kappa^2)$. Here we set $\kappa^2 = 1 = \Lambda$.

The effective barotropic index $w(t) = -P(t)/\rho_{\text{eff}}(t)$ for $p = 1/2$ (left) and $p = 3$ (right), with $\Lambda = 1$. For $w < -1/3$, one has acceleration and for $w < -1$ (small times) super-acceleration, in accordance with eq. (3.20). The lines $w = -1 + 2/(3p)$ (dot-dashed), $w = -1/3$ (dotted) and $w = -1$ (dashed) are shown for reference.

Assuming a standard Raychaudhuri equation $\dot{\rho}_{\text{eff}} + 3H(P + \rho_{\text{eff}}) = 0$, one can also obtain an effective pressure $P$ and barotropic index $w$,

$$P(t) := -\rho_{\text{eff}} - \frac{\dot{\rho}_{\text{eff}}}{3H} = -\frac{1}{\kappa^2} \left( H^2 + \frac{\ddot{a}}{a} \right), \quad w(t) := \frac{P}{\rho_{\text{eff}}},$$

which are depicted in figures 6 and 7, respectively. At the bounce, one has a non-vanishing finite effective pressure $P(0) = -\hat{R}(0)/(3\kappa^2)$, where $\hat{R}(0) = 3p\Lambda^2$ is the Ricci scalar. At late times, $w \to -1 + 2/(3p)$, while at early times

$$w(t) \sim -1 - \frac{4\Lambda^2}{3p^2 t^2},$$

where one has super-acceleration ($w < -1$).

4 Conclusions

In this paper, we have proposed a non-local model of gravity with improved UV behaviour. We have focussed only on its cosmology, and used the property of asymptotic freedom to
find approximate solutions valid both at early and late times. In general, these solutions possess a bounce and avoid the big bang singularity, and have an early era of acceleration with natural exit in the absence of inflaton fields. Specifically, the universe is characterized by a super-acceleration regime at the bounce, where we found an effective barotropic index $w < -1$.

Since we have not solved the full equations of motion exactly, we have bypassed the problem of getting the dynamics of a theory with all sectors (both gravity and matter) being non-local. Instead, we have matched the solution (3.14) with an effective dynamics encoded in the modified Friedmann equation (1.1). The good agreement between this solution and eq. (1.1) at all scales suggests that the problem is not unsolvable. The diffusion equation method, applied in section 3.3 to find an alternative derivation of the profile (3.14), might be a useful tool in this respect. Intriguingly, equation (3.22) acts as a “beta function,” determining the running of the metric with the cut-off length scale $1/\Lambda$, which plays the role of diffusion time $\sim \sqrt{r}$. The possibility to study the dynamics of this class of non-local theories via the diffusion method is a direct consequence of their renormalization properties [85].

It is remarkable that the bouncing dynamics of the present model can be reproduced semi-quantitatively by an effective equation with only one free parameter. The value of the critical energy density $\rho_\ast$ is also plausible. From eq. (3.26) and setting the cut-off to its natural value $\Lambda = m_{\text{Pl}}$, we get $\rho_\ast \approx 0.02\rho_{\text{Pl}}$ for $p = 1/2$ and $\rho_\ast \approx 0.22\rho_{\text{Pl}}$ for $p = 3$. Models such that $\rho \leq \rho_\ast \leq \rho_{\text{Pl}}$ must have $p \leq p_{\text{max}} \approx 12.67$. For $p > p_{\text{max}}$, the critical energy density exceeds $\rho_{\text{Pl}}$, and the energy density of the universe can become trans-Planckian near and at the bounce. Therefore, scenarios with too-large $p$ are not well represented by eq. (1.1). This is not an issue in our model, since we have early-universe acceleration by default also for small values of $p$.

The relative error between the approximate solution $H^2$ and the effective energy density $\kappa^2 \rho_{\text{eff}}/3$ could be reduced by a more refined Ansatz for the effective Friedmann equation. Further study of this method may turn out to shed some light on the exact dynamics, which should be developed in parallel starting from the actual equations of motion. For instance, once derived the actual equations of motion of the theory one could plug the solution $a(t)$ found here, and check whether a reasonable matter sector is recovered. In particular, from our modified Kasner solution it should be possible to check whether some form of BKL chaos survives in the full anisotropic dynamics.

This should not only clarify whether the bounce picture is robust in our theory, but also to which class of singularity-free cosmologies the model belongs to. As an exact dynamical equation, expression (1.1) with $\beta = 1$ appears also in braneworlds with a timelike extra direction [91] and in purely homogeneous loop quantum cosmology (for reviews consult, e.g., [92, 93]) only in the parameter choice for the so-called “improved” mini-superspace dynamics [94, 95] (for other parametrizations, eq. (1.1) with $\beta = 1$ no longer holds [96]). Although there is no relation between our framework and these high-energy cosmological models, they all share the same type of bounce where the right-hand side of the Friedmann equation receives a negative higher-order correction in the energy density. On the other hand, there is a different class of models where the correction is of the “dark radiation” form $-1/a^4$, which is responsible for the bounce at $H = 0$. Such is the case for the Randall–Sundrum braneworld with a spacelike extra dimension [97], Hořava–Lifshitz gravity without detailed balance [98, 99], and cosmologies with fermionic condensates [100, 101]. As far as we pushed the analysis in this paper, the present model apparently lies in the first category, with the added bonus that we have an alternative mechanism of inflation of purely geometric origin.
Acknowledgments

The authors thank G. Mena Marugán for comments on the manuscript and are grateful to J. Moffat for early discussions on related topics. G.C. and L.M. acknowledge the i-Link cooperation programme of CSIC (project ID i-Link0484) for partial sponsorship. The work of G.C. is under a Ramón y Cajal tenure-track contract; he also thanks Fudan University for the kind hospitality during the completion of this article. The work of P.N. has been supported by the German Research Foundation (DFG) grant NI 1282/2-1, and partially by the Helmholtz International Center for FAIR within the framework of the LOEWE program (Landesoffensive zur Entwicklung Wissenschaftlich-Ökonomischer Exzellenz) launched by the State of Hesse and by the European COST action MP0905 “Black Holes in a Violent Universe”.

References

[1] K. Ohmori, A review on tachyon condensation in open string field theories, hep-th/0102085.
[2] A. Sen, Tachyon dynamics in open string theory, Int. J. Mod. Phys. A 20 (2005) 5513 [hep-th/0410103].
[3] E. Fuchs and M. Kroyter, Analytical solutions of open string field theory, Phys. Rep. 502 (2011) 89 [arXiv:0807.4722].
[4] Y. Okawa, Analytic methods in open string field theory, Prog. Theor. Phys. 128 (2012) 1001.
[5] P.G.O. Freund and M. Olson, Nonarchimedean strings, Phys. Lett. B 199 (1987) 186.
[6] P.G.O. Freund and E. Witten, Adelic string amplitudes, Phys. Lett. B 199 (1987) 191.
[7] L. Brekke, P.G.O. Freund, M. Olson and E. Witten, Nonarchimedean string dynamics, Nucl. Phys. B 302 (1988) 365.
[8] V.S. Vladimirov and Ya.I. Volovich, On the nonlinear dynamical equation in the p-adic string theory, Theor. Math. Phys. 138 (2004) 297 [math-ph/0306018].
[9] V. Vladimirov, Nonlinear equations for p-adic open, closed, and open-closed strings, Theor. Math. Phys. 149 (2006) 1604 [arXiv:0705.4600].
[10] T. Biswas, J.A.R. Cembranos and J.I. Kapusta, Thermal duality and Hagedorn transition from p-adic strings, Phys. Rev. Lett. 104 (2010) 021601 [arXiv:0910.2274].
[11] I.Ya. Aref’eva, Nonlocal string tachyon as a model for cosmological dark energy, AIP Conf. Proc. 826 (2006) 301 [astro-ph/0410443].
[12] I.Ya. Aref’eva and L.V. Joukovskaya, Time lumps in nonlocal stringy models and cosmological applications, JHEP 0510 (2005) 087 [hep-th/0504200].
[13] I.Ya. Aref’eva, A.S. Koshelev and S.Yu. Vernov, Stringy dark energy model with cold dark matter, Phys. Lett. B 628 (2005) 1 [astro-ph/0505605].
[14] G. Calcagni, Cosmological tachyon from cubic string field theory, JHEP 0605 (2006) 012 [hep-th/0512259].
[15] I.Ya. Aref’eva and A.S. Koshelev, Cosmic acceleration and crossing of $w = -1$ barrier from cubic superstring field theory, JHEP 0702 (2007) 041 [hep-th/0605085].
[16] I.Ya. Aref’eva and L.V. Volovich, On the null energy condition and cosmology, Theor. Math. Phys. 155 (2008) 503 [hep-th/0612098].
[17] N. Barnaby, T. Biswas and J.M. Cline, p-adic inflation, JHEP 0704 (2007) 056 [hep-th/0612230].
[18] A.S. Koshelev, *Non-local SFT tachyon and cosmology*, JHEP **0704** (2007) 029 [hep-th/0701103].

[19] I.Ya. Aref’eva, L.V. Joukovskaya and S.Yu. Vernov, *Bouncing and accelerating solutions in nonlocal stringy models*, JHEP **0707** (2007) 087 [hep-th/0701184].

[20] I.Ya. Aref’eva and I.V. Volovich, *Quantization of the Riemann zeta-function and cosmology*, Int. J. Geom. Methods Mod. Phys. **4** (2007) 881 [hep-th/0701284].

[21] J.E. Lidsey, *Stretching the inflaton potential with kinetic energy*, Phys. Rev. D **76** (2007) 043511 [hep-th/0703007].

[22] N. Barnaby and J.M. Cline, *Large non-gaussianity from nonlocal inflation*, JCAP **0707** (2007) 017 [arXiv:0704.3426].

[23] G. Calcagni, M. Montobbio and G. Nardelli, *Route to nonlocal cosmology*, Phys. Rev. D **76** (2007) 126001 [arXiv:0705.3043].

[24] L.V. Joukovskaya, *Dynamics in nonlocal cosmological models derived from string field theory*, Phys. Rev. D **76** (2007) 105007 [arXiv:0707.1545].

[25] G. Calcagni and G. Nardelli, *Nonlocal instantons and solitons in string models*, Phys. Lett. B **669** (2008) 102 [arXiv:0802.4395].

[26] L. Joukovskaya, *Rolling solution for tachyon condensation in open string field theory*, arXiv:0803.3484.

[27] I.Ya. Aref’eva and A.S. Koshelev, *Cosmological signature of tachyon condensation*, JHEP **0809** (2008) 068 [arXiv:0804.3570].

[28] L. Joukovskaya, *Dynamics with infinitely many time derivatives in Friedmann–Robertson–Walker background and rolling tachyons*, JHEP **0902** (2009) 045 [arXiv:0807.2065].

[29] N. Barnaby and N. Kamran, *Dynamics with infinitely many derivatives: variable coefficient equations*, JHEP **0812** (2008) 022 [arXiv:0809.4513].

[30] N.J. Nunes and D.J. Mulryne, *Non-linear non-local cosmology*, AIP Conf. Proc. **1115** (2009) 329 [arXiv:0810.5471].

[31] A.S. Koshelev and S.Yu. Vernov, *Cosmological perturbations in SFT inspired non-local scalar field models*, Eur. Phys. J. C **72** (2012) 2198 [arXiv:0903.5176].

[32] G. Calcagni and G. Nardelli, *Cosmological rolling solutions of nonlocal theories*, Int. J. Mod. Phys. D **19** (2010) 329 [arXiv:0904.4245].

[33] S.Yu. Vernov, *Localization of non-local cosmological models with quadratic potentials in the case of double roots*, Class. Quantum Grav. **27** (2010) 035006 [arXiv:0907.0468].

[34] S.Yu. Vernov, *Localization of the SFT inspired nonlocal linear models and exact solutions*, Phys. Part. Nucl. Lett. **8** (2011) 310 [arXiv:1005.0372].

[35] A.S. Koshelev and S.Yu. Vernov, *Analysis of scalar perturbations in cosmological models with a non-local scalar field*, Class. Quantum Grav. **28** (2011) 085019 [arXiv:1009.0746].

[36] N.V. Krasnikov, *Nonlocal gauge theories*, Theor. Math. Phys. **73** (1987) 1184 [Teor. Mat. Fiz. **73** (1987) 235].

[37] E.T. Tomboulis, *Superrenormalizable gauge and gravitational theories*, hep-th/9702146.

[38] T. Biswas, A. Mazumdar and W. Siegel, *Bouncing universes in string-inspired gravity*, JCAP **0603** (2006) 009 [hep-th/0508194].

[39] J. Khoury, *Fading gravity and self-inflation*, Phys. Rev. D **76** (2007) 123513 [hep-th/0612052].
[40] G. Calcagni and G. Nardelli, Nonlocal gravity and the diffusion equation, Phys. Rev. D 82 (2010) 123518 [arXiv:1004.5144].

[41] T. Biswas, T. Koivisto and A. Mazumdar, Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity, JCAP 1011 (2010) 008 [arXiv:1005.0590].

[42] L. Modesto, Super-renormalizable quantum gravity, Phys. Rev. D 86 (2012) 044005 [arXiv:1107.2403].

[43] T. Biswas, T. Koivisto and A. Mazumdar, Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity, JCAP 1011 (2010) 008 [arXiv:1005.0590].

[44] L. Modesto, Super-renormalizable higher-derivative quantum gravity, arXiv:1202.0008.

[45] T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar, Towards singularity and ghost free theories of gravity, Phys. Rev. Lett. 108 (2012) 031101 [arXiv:1110.5249].

[46] A.S. Koshelev, Modified non-local gravity, Rom. J. Phys. 57 (2012) 894 [arXiv:1112.6410].

[47] T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar, Towards singularity and ghost free theories of gravity, Phys. Rev. Lett. 108 (2012) 031101 [arXiv:1110.5249].

[48] A.S. Koshelev and S.Yu. Vernov, On bouncing solutions in non-local gravity, Phys. Part. Nucl. 43 (2012) 666 [arXiv:1202.1289].

[49] L. Modesto, Super-renormalizable multidimensional quantum gravity: theory and applications, Astron. Rev. in press [arXiv:1202.3151].

[50] L. Modesto, Towards a finite quantum supergravity, arXiv:1206.2648.

[51] T. Biswas, A.S. Koshelev, A. Mazumdar and S.Yu. Vernov, Stable bounce and inflation in non-local higher derivative cosmology, JCAP 1208 (2012) 024 [arXiv:1206.6374].

[52] F. Briscese, A. Marcianò, L. Modesto and E.N. Saridakis, Inflation in (super-)renormalizable gravity, Phys. Rev. D 87 (2013) 083507 [arXiv:1212.3611].

[53] B. Hasslacher and E. Mottola, Asymptotically free quantum gravity and black holes, Phys. Lett. B 99 (1981) 221.

[54] M. Saadi and B. Zwiebach, Closed string field theory from polyhedra, Ann. Phys. (N.Y.) 192 (1989) 213.

[55] T. Kugo, H. Kunitomo and K. Suehiro, Nonpolynomial closed string field theory, Phys. Lett. B 226 (1989) 48.

[56] T. Kugo and K. Suehiro, Nonpolynomial closed string field theory: action and its gauge invariance, Nucl. Phys. B 337 (1990) 434.

[57] V.A. Kostelecký and S. Samuel, Collective physics in the closed bosonic string, Phys. Rev. D 42 (1990) 1289.

[58] B. Zwiebach, Closed string field theory: quantum action and the B-V master equation, Nucl. Phys. B 390 (1993) 33 [hep-th/9206084].

[59] A. Sen and B. Zwiebach, A proof of local background independence of classical closed string field theory, Nucl. Phys. B 414 (1994) 649 [hep-th/9307088].

[60] A. Sen and B. Zwiebach, Quantum background independence of closed string field theory, Nucl. Phys. B 423 (1994) 580 [hep-th/9311009].

[61] A. Sen and B. Zwiebach, A note on gauge transformations in Batalin-Vilkovisky theory, Phys. Lett. B 320 (1994) 29 [hep-th/9309027].

[62] Y. Okawa and B. Zwiebach, Twisted tachyon condensation in closed string field theory, JHEP 0403 (2004) 056 [hep-th/0403051].

[63] H. Yang and B. Zwiebach, Dilaton deformations in closed string field theory, JHEP 0505 (2005) 032 [hep-th/0502161].
[63] H. Yang and B. Zwiebach, A closed string tachyon vacuum?, *JHEP* **0509** (2005) 054 [hep-th/0506077].

[64] Y. Michishita, Field redefinitions, T-duality and solutions in closed string field theories, *JHEP* **0609** (2006) 001 [hep-th/0602251].

[65] N. Moeller, Closed bosonic string field theory at quintic order: five-tachyon contact term and dilaton theorem, *JHEP* **0703** (2007) 043 [hep-th/0609209].

[66] N. Moeller, Closed bosonic string field theory at quintic order. II: marginal deformations and effective potential, *JHEP* **0709** (2007) 118 [arXiv:0705.2102].

[67] N. Moeller, A tachyon lump in closed string field theory, *JHEP* **0809** (2008) 056 [arXiv:0804.0697].

[68] S. Alexander, R. Brandenberger and J. Magueijo, Noncommutative inflation, *Phys. Rev. D* **67** (2003) 081301 [hep-th/0108190].

[69] M. Rinaldi, A new approach to non-commutative inflation, *Class. Quantum Grav.* **28** (2011) 105022 [arXiv:0908.1949].

[70] N. Barnaby and N. Kamran, Dynamics with infinitely many derivatives: the initial value problem, *JHEP* **0802** (2008) 008 [arXiv:0709.3968].

[71] G.V. Efimov, Nonlocal Interactions of Quantized Fields [in Russian], Nauka, Moscow, Russia (1977).

[72] A. Smailagic and E. Spallucci, Lorentz invariance, unitarity in UV-finite of QFT on noncommutative spacetime, *J. Phys. A* **37** (2004) 7169 [hep-th/0406174].

[73] E. Spallucci, A. Smailagic and P. Nicolini, Trace anomaly in quantum spacetime manifold, *Phys. Rev. D* **73** (2006) 084004 [hep-th/0604094].

[74] P. Nicolini and M. Rinaldi, A minimal length versus the Unruh effect, *Phys. Lett. B* **695** (2011) 303 [arXiv:0910.2860].

[75] M. Kober and P. Nicolini, Minimal scales from an extended Hilbert space, *Class. Quantum Grav.* **27** (2010) 245024 [arXiv:1005.3293].

[76] M. Asorey, J.L. López and I.L. Shapiro, Some remarks on high derivative quantum gravity, *Int. J. Mod. Phys. A* **12** (1997) 5711 [hep-th/9610006].

[77] L. Modesto, J.W. Moffat and P. Nicolini, Black holes in an ultraviolet complete quantum gravity, *Phys. Lett. B* **695** (2011) 397 [arXiv:1010.0680].

[78] L. Modesto, Finite quantum gravity, arXiv:1305.6741.

[79] G. Calcagni and L. Modesto, Proposal for a quantum M-theory, work in progress.

[80] M.J. Duff and D.J. Toms, Kaluza–Klein–Kounterterms, in *Unification of Fundamental Particle Interactions II*, J. Ellis and S. Ferrara eds., Springer, Amsterdam, The Netherlands (1983).

[81] K.S. Stelle, Renormalization of higher-derivative quantum gravity, *Phys. Rev. D* **16** (1977) 953.

[82] M.J. Duff, Quantum corrections to the schwarzschild solution, *Phys. Rev. D* **9** (1974) 1837.

[83] B. Broda, One-loop quantum gravity repulsion in the early Universe, *Phys. Rev. Lett.* **106** (2011) 101303 [arXiv:1011.6257].

[84] B. Broda, Quantum gravity stability of isotropy in homogeneous cosmology, *Phys. Lett. B* **704** (2011) 655 [arXiv:1107.3468].

[85] G. Calcagni and G. Nardelli, String theory as a diffusing system, *JHEP* **1002** (2010) 093 [arXiv:0910.2160].
[86] G. Calcagni, M. Montobbio and G. Nardelli, *Localization of nonlocal theories*, *Phys. Lett. B* 662 (2008) 285 [arXiv:0712.2237].

[87] G. Calcagni and G. Nardelli, *Kinks of open superstring field theory*, *Nucl. Phys. B* 823 (2009) 234 [arXiv:0904.3744].

[88] A. Accioly, A. Azeredo and H. Mukai, *Propagator, tree-level unitarity and effective nonrelativistic potential for higher-derivative gravity theories in D dimensions*, *J. Math. Phys.* 43 (2002) 473.

[89] P. Van Nieuwenhuizen, *On ghost-free tensor Lagrangians and linearized gravitation*, *Nucl. Phys. B* 60 (1973) 478.

[90] M.D. Pollock, *On super-exponential inflation in a higher-dimensional theory of gravity with higher-derivative terms*, *Nucl. Phys. B* 309 (1988) 513 [Erratum ibid. B 374 (1992) 469].

[91] Y. Shtanov and V. Sahni, *Bouncing brane worlds*, *Phys. Lett. B* 557 (2003) 1 [gr-qc/0208047].

[92] A. Ashtekar and P. Singh, *Loop quantum cosmology: a status report*, *Class. Quantum Grav.* 28 (2011) 213001 [arXiv:1108.0893].

[93] K. Banerjee, G. Calcagni and M. Martín-Benito, *Introduction to loop quantum cosmology*, *SIGMA* 8 (2012) 016 [arXiv:1109.6801].

[94] P. Singh, *Loop cosmological dynamics and dualities with Randall–Sundrum braneworlds*, *Phys. Rev. D* 73 (2006) 063508 [arXiv:gr-qc/0603043].

[95] A. Ashtekar, T. Pawlowski and P. Singh, *Quantum nature of the big bang: improved dynamics*, *Phys. Rev. D* 74 (2006) 084003 [gr-qc/0607039].

[96] G. Calcagni and G.M. Hossain, *Loop quantum cosmology and tensor perturbations in the early universe*, *Adv. Sci. Lett.* 2 (2009) 184 [arXiv:0810.4330].

[97] P. Binétruy, C. Deffayet, U. Ellwanger and D. Langlois, *Brane cosmological evolution in a bulk with cosmological constant*, *Phys. Lett. B* 477 (2000) 285 [hep-th/9910219].

[98] G. Calcagni, *Cosmology of the Lifshitz universe*, *JHEP* 0909 (2009) 112 [arXiv:0904.0829].

[99] E. Kiritsis and G. Kofinas, *Hořava–Lifshitz cosmology*, *Nucl. Phys. B* 821 (2009) 467 [arXiv:0904.1134].

[100] S. Alexander and T. Biswas, *Cosmological BCS mechanism and the big bang singularity*, *Phys. Rev. D* 80 (2009) 023501 [arXiv:0807.4468].

[101] S. Alexander, T. Biswas and G. Calcagni, *Cosmological Bardeen–Cooper–Schrieffer condensate as dark energy*, *Phys. Rev. D* 81 (2010) 043511 [Erratum ibid. 81 (2010) 069902(E)] [arXiv:0906.5161].