Trajectory Plotting Algorithm for a Self-Driving Road Grader

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Abstract. The deployment of self-driving technologies occurs in various industries and economic sectors. Self-driving taxi cabs can be found in city streets, in Russia as well as in other countries. The introduction of self-driving technologies in construction, namely, construction machinery, is a promising area that will develop rapidly in the nearest few years. A road grader is the construction machine whose control algorithms resemble those of self-driving cars the most. One of the first development stages for the self-driving road grader is trajectory plotting.

This article presents the developed trajectory plotting algorithm for the road grader taking into account its design features (minimum turn radius).

1. Introduction

All of the road grader navigation systems can be divided into global and local ones. The global one stands for trajectory plotting before the work begins using the data from the 3D project and the digital map. The local one stands for navigation during movement and changing the trajectory in case of unforeseen obstructions.

There are several global projects on the automation of construction machines: SafeAI; Built robotics; Autonomous Solutions; Robo Industries; Autonomous Haulage System, Carnegie Mellon University; HEAP, ETHZ [3, 5].

In Russia, similar projects are developed at the Innopolis University, Cognitive Technologies, and Cifra [3, 5].

The majority of works on robot navigation use two-wheeled turning robots are the basic platform [1, 4, 9]. This turning method provides almost unlimited maneuverability because the platform can turn around in one place. Machines using classing turning systems have a much lower maneuverability, which is a significant complication for the trajectory plotting algorithm [2, 6, 7].

2. Working arrangements

Working arrangements for the road grader depend on the type of work and the type of earth structure. The average road grader bay length is 200-400 m. Road grader operations consist of ground relocation and spreading [8, 10].

To construct earth banks out of lateral reserves, road graders make circular movements along the bank from the edges to the middle (Figure 1). The work is generally arranged as follows: ground cutting, ground relocation, ground spreading, trimming [8, 10].
Figure 1. Working arrangements [8]:

a) for the road grader moving clockwise;
b) for the road grader moving counter-clockwise

Road grader trajectory on the site can be divided into working passes (straight passes limited by the bay entry and exit points) and turns (lines connecting bay exit points to bay entry points). The road grader must enter the bay after it aligns its course.

Thus, any trajectory can be described mathematically using the trajectory points that the grader must pass in a strict sequence. Apart from the coordinates, the trajectory point must contain the information on the azimuth angle, i.e. the heading for the further movement that the grader should take after it reaches the point in question (Figure 2). The points alternate: bay entry point (the beginning of the working pass) and the bay exit point (the beginning of the idle pass).

3. Calculation model
The location of trajectory points depends on the shape and size of the working site, as well as the specifications of the road grader (implement width, nip angle).

Figure 2. Trajectory points for grader movement.
The working pass trajectory is, as a rule, a straight line or a very large radius curve. In other words, the headings of entry and exit are either the same or have slight variations.

The idle pass trajectory generally requires the machine to turn around (about 180°) and go in the opposite direction.

**Figure 3. Calculation model**

4. Algorithm

Initial data for the trajectory plotting algorithm:

a) current point coordinates – point $A_i$ coordinates $(x_i, z_i, \phi_i)$;
b) destination point coordinates – point $A_{i+1}$ coordinates $(x_{i+1}, z_{i+1}, \phi_{i+1})$;
c) minimum turning radius $R_{\text{min}}$.

The algorithm implementation results in the trajectory plotting from point $A_i$ to point $A_{i+1}$.

Review the calculation model for the algorithm (Figure 3). We must plot the trajectory from point $A_i$ with heading $\phi_i$ to point $A_{i+1}$ with heading $\phi_{i+1}$.

The shortest trajectory, in this case, can be divided into 3 simple motion sections: circular movement (turning) with the radius $R_{\text{min}}$ (minimum machine turning radius), direct motion, and circular movement (turning) with the radius $R_{\text{min}}$. Intermediary trajectory points should be located where one movement type turns into another.

The algorithm should provide the answer to the first question: are we turning left or right in the first section? This depends on the location of point $A_{i+1}$ that can be to the right or the left of heading $\phi_i$.

To do this, we need to calculate the azimuth angle for section $A_iA_{i+1}$ and compare it to angle $\phi_i$, (since the inverse tangent function returns angle values from -90° to 90°, we add a summand to the formula to get the values from -180° to 180°).

$$\phi_{AA} = 90 \cdot \left( \frac{X_{i+1} - X_i}{|X_{i+1} - X_i|} \right) \left( 1 - \frac{Z_{i+1} - Z_i}{|Z_{i+1} - Z_i|} \right) + \arctan \left( \frac{Z_{i+1} - Z_i}{X_{i+1} - X_i} \right)$$

If angle $\phi_{AA}$ is greater than angle $\phi_i$, we turn left. If it is smaller, we turn right.

The second question is whether we reach point $A_{i+1}$ from the left or the right.

To do this, we need to compare azimuth angle $\phi_{AA}$ and azimuth angle $\phi_{i+1}$. If angle $\phi_{AA}$ is greater than angle $\phi_{i+1}$, we reach the destination from the right. If it is smaller, we reach the destination from the left.

As a result, we get 4 options for the possible locations of the points (Figure 4).
Figure 4. Calculation models for various angle proportions:

a) angle $\varphi_{AA}$ is smaller than angle $\varphi_i$ and greater than angle $\varphi_{i+1}$ – right and right;

b) angle $\varphi_{AA}$ is smaller than angle $\varphi_i$ and smaller than angle $\varphi_{i+1}$ – right and left;

c) angle $\varphi_{AA}$ is greater than angle $\varphi_i$ and smaller than angle $\varphi_{i+1}$ – left and left;

d) angle $\varphi_{AA}$ is greater than angle $\varphi_i$ and greater than angle $\varphi_{i+1}$ – left and left;

Obtain the coordinates for turning centers $A_i', A_{i+1}'$:

$$
\begin{align*}
Z_i' &= \begin{cases} 
Z_l - R \sin \varphi_i @ AA_i \min \\
Z_l + R \sin \varphi_i @ AA_i \min 
\end{cases} \\
X_i' &= \begin{cases} 
X_l - R \cos \varphi_i @ AA_i \min \\
X_l + R \cos \varphi_i @ AA_i \min 
\end{cases} \\
Z_{i+1}' &= \begin{cases} 
Z_{i+1} - R \sin \varphi_i @ AA_{i+1} \min \\
Z_{i+1} + R \sin \varphi_i @ AA_{i+1} \min 
\end{cases} \\
X_{i+1}' &= \begin{cases} 
X_{i+1} + R \cos \varphi_i @ AA_{i+1} \min \\
X_{i+1} - R \cos \varphi_i @ AA_{i+1} \min 
\end{cases}
\end{align*}
$$

Obtain the coordinates of intermediary points $B_i$ and $C_i$. To do this, we need to find the angle of the straight line formed by points $A_i'$ and $A_{i+1}'$:

$$
\varphi_{AA}' = 90 \cdot \frac{X_{i+1}' - X_i'}{X_{i+1}' - X_i'} \cdot \left( 1 - \frac{Z_{i+1}' - Z_i'}{Z_{i+1}' - Z_i'} \right) + \arctan \left( \frac{Z_{i+1}' - Z_i'}{X_{i+1}' - X_i'} \right)
$$

For a) and c) in Figure 4.3.4, we find the coordinates using the following formulae:

$$
\begin{align*}
Z_B &= \begin{cases} 
Z_l + R \sin \varphi_{AA}' @ AA_{i} \min \\
Z_l - R \sin \varphi_{AA}' @ AA_{i} \min 
\end{cases} \\
X_B &= \begin{cases} 
X_l + R \cos \varphi_{AA}' @ AA_{i} \min \\
X_l - R \cos \varphi_{AA}' @ AA_{i} \min 
\end{cases}
\end{align*}
$$
For b) and d), we need to find angle $\theta$ (Figure 5)

$$
\theta = \arccos \frac{2R_{\text{min}}}{A_i'A_{i+1}'}
$$

where

$$
A_i'A_{i+1}' = \frac{Z_{i+1}' - Z_i'}{\cos \phi_{AA}'} .
$$

Figure 5. Calculation model for internal tangent construction.

For b) and d), we find the coordinates using the following formulae

$$
Z_B = \begin{cases} 
Z_i' + R \cos (\phi_{AA} + \theta) @ AA_{i \text{min}} \\
Z_i' + R \cos (\phi_{AA}' - \theta) @ AA_{i \text{min}} 
\end{cases}
$$

$$
X_B = \begin{cases} 
X_i' + R \sin (\phi_{AA} + \theta) @ AA_{i \text{min}} \\
X_i' + R \sin (\phi_{AA}' - \theta) @ AA_{i \text{min}} 
\end{cases}
$$

$$
Z_C = \begin{cases} 
Z_{i+1}' - R \cos (\phi_{AA}' - \theta) @ AA_{i+1 \text{min}} \\
Z_{i+1}' - R \cos (\phi_{AA} + \theta) @ AA_{i+1 \text{min}} 
\end{cases}
$$

$$
X_C = \begin{cases} 
X_{i+1}' - R \sin (\phi_{AA}' - \theta) @ AA_{i+1 \text{min}} \\
X_{i+1}' - R \sin (\phi_{AA} + \theta) @ AA_{i+1 \text{min}} 
\end{cases}
$$

The first section of the trajectory is a turnaround point $A_i'$ from the angle

$$
\phi_i' = \begin{cases} 
\phi_i + 90, @ \phi_{AA} \geq \phi_i; \\
\phi_i - 90, @ \phi_{AA} < \phi_i;
\end{cases}
$$

to angle

$$
\phi_i'' = \begin{cases} 
\phi_{BC} + 90, @ \phi_{AA} \geq \phi_i; \\
\phi_{BC} - 90, @ \phi_{AA} < \phi_i;
\end{cases}
$$

where $\phi_{BC}$ is the angle of the section formed by points $B_i$ and $C_i$. 
\[ \phi_{BC} = 90 \left( \frac{X_C - X_B}{|X_C - X_B|} \right) \left( 1 - \frac{Z_C - Z_B}{|Z_C - Z_B|} \right) + \arctan \left( \frac{Z_C - Z_B}{X_C - X_B} \right). \]

The curve begins in point \( A_i \) and ends in point \( B_i \). We can find the rectangular coordinates of the trajectory points using the following formulae

\[ Z_{1j} = R_{\min} \cos \varphi_j + Z'_i, \]
\[ X_{1j} = R_{\min} \sin \varphi_j + X'_i, \]

Where \( \varphi'_i \geq \varphi_j \geq \varphi''_i \) for right turns and \( \varphi'_i \leq \varphi_j \leq \varphi''_i \) left turns.

The second trajectory section is a straight line from point \( B_i \) to point \( C_i \)

\[ X_{2j} = k \cdot Z_{2j} + b \]

We can find the coefficients using the following formulae

\[ k = \frac{X_C - X_B}{Z_C - Z_B}, \]
\[ b = \frac{Z_C \cdot X_B - X_C \cdot Z_B}{Z_C - Z_B}, \]

The third section of the trajectory is a turnaround point \( A_i' \) from the angle \( \varphi_{i+1} \) to angle \( \varphi'_{i+1} \)

\[ \phi'_{i+1} = \begin{cases} \phi_i' + 180, & @ \phi_{AA} < \phi_i \text{ and } \phi_{AA} \geq \phi_{i+1} \\ \phi_i' - 180, & @ \phi_{AA} \geq \phi_i \text{ and } \phi_{AA} < \phi_{i+1} \end{cases} \]

The curve begins in point \( C_i \) and ends in point \( A_{i+1} \). We can find the rectangular coordinates of the trajectory points using the following formulae

\[ Z_{3j} = R_{\min} \cos \varphi_j + Z'_{i+1} \]
\[ X_{3j} = R_{\min} \sin \varphi_j + X'_{i+1} \]

Where \( \varphi'_{i+1} \geq \varphi_j \geq \varphi''_{i+1} \) for right turns and \( \varphi'_{i+1} \leq \varphi_j \leq \varphi''_{i+1} \) left turns.

Thus, the implementation of the described algorithm produces an array of points making up the trajectory from point \( A_i \) to point \( A_{i+1} \). The trajectory consists of three simple movement sections: turning, direct movement, and turning. Figure 6 shows the calculations example by the suggested algorithm.
Figure 6. Trajectory plotting algorithm results.

The algorithm will not work for cases b) and d) if turning radiuses cross, i.e. if section $A_i A_{i+1}$ length is smaller than $4R_{\text{min}}$.

5. Conclusion

The suggested algorithm allows for trajectory plotting between two lines. If the headings in the points used for the plotting of the working pass trajectory are the same, the result will be a straight line. Therefore, the algorithm can be used to plot the entire movement trajectory. Further research will focus on the development of a trajectory point location algorithm for the working bay.

6. References

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