Getting to the top with extra dimensions

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ABSTRACT

The prospect of large extra dimensions and an effective theory of gravity at around a TeV has interesting experimental consequences. In these models, the Kaluza-Klein modes interact with Standard Model particles and these interactions lead to testable predictions at present and planned colliders. We investigate the effect of virtual exchanges of the spin-2 Kaluza-Klein modes in the production cross-section of $t\bar{t}$ pairs at the Tevatron and the LHC and find that the $t\bar{t}$ cross-section can be an effective probe of the large extra dimensions. This enables us to put bounds on the effective low-energy scale.

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While traditionally it has been assumed that quantum gravity effects are large only near the Planck scale, $M_P = 1.2 \times 10^{19}$ GeV, recently it is becoming evident that the effects of gravity propagating in dimensions higher than four can completely change this simple picture, if these higher dimensions are compactified to have relatively large sizes. The proposal finds its most natural setting in a string-theoretic framework: the feature of large extra dimensions is quite generic and can be realised in the context of several string models. One starts with a theory in $D$ dimensions and the extra dimensions are then compactified to obtain the effective low-energy theory in 3+1 dimensions, and it is assumed that $n$ of these extra dimensions are compactified to a common scale $R$ which is relatively large, while the remaining dimensions are compactified to much smaller length scales which are of the order of the Planck scale.

One immediate advantage of the string construction is that matter is localised on a 3-brane because the open strings, to which correspond the SM particles, end on the brane and are therefore confined to the 3 + 1-dimensional spacetime, while gravitons (corresponding to closed strings) propagate in the 4 + $n$-dimensional bulk. The relation between the scales in 4 + $n$ dimensions and in 4 dimensions is given by

$$M^2_{Pl} = M^2_S + 2 R^n,$$

where $M_S$ is the low-energy effective string scale. Then it follows that if we need $M_S$ to be of the order of a TeV then $R = 10^{32/n-19}$ m. One immediate consequence of this relation is that for this value of $R$ (for any given $n$) we will expect to see deviations from Newton’s law of gravitation. The case $n = 1$ is obviously excluded, but for $n = 2$ or larger it is possible to have the scale $M_S$ to be of the order of 1 TeV and still be consistent with experiment. For $n = 2$ the compactified dimensions are of the order of 1 mm and just below the experimentally tested region for the validity of Newton’s law of gravitation, and within the possible reach of ongoing experiments.

For particle physics, one very important consequence of the lowering of the string scale is the nullification of the hierarchy problem. This is because the string scale is now of the order of a TeV and of the same order as the electroweak scale. In fact, it has been shown that it is possible to construct a phenomenologically viable scenario with large extra dimensions, which can survive the existing astrophysical and cosmological constraints. Recently, the effect of the Kaluza-Klein states on the running of the gauge couplings i.e. the effect of these states on the beta functions of the theory have been studied and it has been shown that the GUT scale can be also lowered down to scales close to the electroweak scale.

Let us describe the effective theory below the scale $M_S$ that emerges in such a scenario. The particle content of the theory below the scale $M_S$ is as follows: there are the usual Standard Model (SM) particles and the graviton and also other light modes related to the brane dynamics. In particular, there are the $Y$ modes which are related to the deformation of the brane and these are the massless Nambu-Goldstone bosons (in the higher-dimensional theory) due to the spontaneous breaking
of translational invariance in the transverse directions. In the effective theory, however, some of these modes could acquire mass \[6\]. In any case, the $Y$ modes couple to the SM matter only in pairs so their effects, compared to the graviton, are subleading. The graviton corresponds to a tower of Kaluza-Klein modes which contain spin-2, spin-1 and spin-0 excitations. The spin-1 modes do not couple to the energy-momentum tensor and their couplings to the SM particles in the low-energy effective theory is not important. The scalar modes couple to the trace of the energy-momentum tensor, so they do not couple to massless particles.

The interesting aspect of the large extra dimensions is that it is possible to have experimentally observable consequences of the effects of the infinite tower of Kaluza-Klein states in present and planned high-energy collider experiments. As explained above, the effects of the spin-1 Kaluza-Klein states can be neglected. Further for collider experiments, it is usual to consider the particles in the initial state as massless. In such a case, the scalar contributions can also be neglected. The only states that contribute are the spin-2 Kaluza-Klein states. These correspond to a massless graviton in the $4 + n$ dimensional theory, but manifest as an infinite tower of massive gravitons in the low-energy effective theory. For graviton momenta smaller than the scale $M_S$, the effective description reduces to one where the gravitons in the bulk propagate in the flat background and couple to the SM fields which live on the brane via a (four-dimensional) induced metric $g_{\mu\nu}$. Moreover, it turns out that the couplings of the modes in the Kaluza-Klein expansion of the energy-momentum tensor, $T_{\mu\nu}$, to the SM fields are universal. This model-independence of the couplings is crucial because it allows us to make definite predictions for the experimental consequences of these interactions.

With the above assumptions and starting from a linearized gravity Lagrangian in $D$ dimensions, the four-dimensional interactions can be derived after a Kaluza-Klein reduction (on a $n$-dimensional torus) has been performed. The interaction of the SM particles with the graviton can be derived from the following Lagrangian:

$$\mathcal{L} = -\frac{1}{\bar{M}_P} G^{(j)}_{\mu\nu} T^{\mu\nu}, \quad (2)$$

where $j$ labels the Kaluza-Klein mode and $\bar{M}_P = M_P/\sqrt{8\pi}$. Using the above interaction Lagrangian the couplings of the graviton modes to the SM particles can be calculated \[8, 9\].

Very recently, there have been papers which have studied the consequences at colliders of this TeV scale effective theory of gravity. In particular, direct searches for graviton production at $e^+ e^-$ and $p\bar{p}$ and $pp$ colliders have been suggested \[8, 10, 9\]. These lead to spectacular single photon + missing energy or monojet + missing energy signatures. Constraints coming from indirect effects (i.e. the study of the effects of virtual gravitons in various experimental observables) can also be made. The virtual effects in $e^+ e^- \rightarrow f\bar{f}$ and in high-mass dilepton production at Tevatron and LHC have
been studied [11]. In view of the fact that the effective Lagrangian given in Eq. 2 is suppressed by \(1/M_p\), it may seem that the effects at colliders will be hopelessly suppressed. However, in the case of real graviton production, the phase space for the Kaluza-Klein modes cancels the dependence on \(M_p\) and, instead, provides a suppression of the order of \(M_S\). For the case of virtual production, we have to sum over the whole tower of Kaluza-Klein states and this sum when properly evaluated [8, 9] again substitutes the scale \(M_S\) for \(M_p\).

In the present work, we study the effect of the virtual graviton exchange on the production cross-section of the \(t\bar{t}\) at hadron colliders. In addition to the SM cross-section, we have new \(s\)-channel production mechanisms for \(t\bar{t}\) production where the graviton modes couple to the \(q\bar{q}\) or \(gg\) initial state. Given the vertices for the \(q\bar{q}G^{(j)}\) coupling and the \(ggG^{(j)}\) coupling and summing over all the graviton modes, as discussed explicitly in Refs. [9, 8], we find the following expressions for the cross-sections involving the virtual graviton exchange:

\[
\frac{d\hat{\sigma}}{dt}(q\bar{q} \rightarrow t\bar{t}) = \frac{d\sigma}{dt}_{SM}(q\bar{q} \rightarrow t\bar{t}) + \frac{\pi\lambda^2}{64\hat{s}^2M_S^8}
\left[5\hat{s}^2(\hat{t} - \hat{u})^2 + 4(\hat{t} - \hat{u})^4 + 8\hat{s}(\hat{t} + \hat{u})^2(\hat{t} + \hat{u}) - 2\hat{s}^3(\hat{t} + \hat{u}) - \hat{s}^4\right],
\tag{3}
\]

and

\[
\frac{d\hat{\sigma}}{dt}(gg \rightarrow t\bar{t}) = \frac{d\sigma}{dt}_{SM}(gg \rightarrow t\bar{t}) - \frac{\pi}{16\hat{s}^2} \left[\frac{3\lambda^2}{M_S^8} - \frac{2\alpha_s}{M_S^4(\hat{M}_t^2 - \hat{t})(\hat{M}_t^2 - \hat{u})}\right]
\times \left[6\hat{M}_t^8 - 4\hat{M}_t^6(\hat{t} + \hat{u}) + 4\hat{M}_t^2\hat{t}\hat{u}(\hat{t} + \hat{u}) - \hat{t}\hat{u}(\hat{t}^2 + \hat{u}^2) + \hat{M}_t^4(\hat{t}^2 - 6\hat{t}\hat{u} + \hat{u}^2)\right].
\tag{4}
\]

In the above \(M_t\) refers to the mass of the top quark and the coupling \(\lambda\) is the effective coupling at the scale \(M_S\). \(\lambda\) is expected to be of \(O(1)\), but its sign is not known \textit{a priori}. In our work we will explore the sensitivity of our results to the choice of the sign of \(\lambda\). There is no interference between the SM diagram and the graviton exchange diagram in the \(q\bar{q}\)-initiated cross-section, but there is a non-vanishing interference in the \(gg\)-initiated cross-section. With the above expressions for the subprocess cross-sections at hand, it is straightforward to compute the integrated top-quark cross-section, using the formula

\[
\sigma(AB \rightarrow t\bar{t}) = \sum \int dx_1dx_2d\hat{t} \left[f_{a/A}(x_1)f_{b/B}(x_2) + x_1 \leftrightarrow x_2\right] \frac{d\hat{\sigma}}{dt},
\tag{5}
\]

where \(A, B\) are the initial hadrons (either \(p\bar{p}\) or \(pp\)), and \(f_{k/h}\) denotes the probability of finding a parton \(k\) in the hadron \(h\). The sum in Eq. 5 runs over the contributing subprocesses.

Before we discuss the results of our computation, a few remarks about the significance of higher-order corrections are in order. The cross-sections presented above are
at the lowest order in perturbation theory. In the QCD case, significant progress has been made in computing higher-order corrections to heavy quark production. Not only have the next-to-leading order corrections been calculated a long time ago [12], but the resummation of soft gluons and its effect on the total cross-section have been recently computed [13, 14]. In principle, a reliable estimate of the cross-section for the case under consideration can also be made only when we have (at least) the corrections to these processes at next-to-leading order. But for want of such a calculation, the best we can do is to use the leading order QCD and the resummed QCD cross-sections [13] to extract a 'K-factor'. We note that the resummed cross-sections of Ref. [14] would yield a different $K$-factor, but our bounds on the new physics scale are not affected seriously by this change. We work with the approximation that the new physics will also be affected by QCD corrections in a similar fashion so that we can fold in our cross-sections for this case by the same $K$-factor. Clearly, more work is needed in this direction but we do not expect that our results will be qualitatively changed by higher order QCD corrections.

Figure 1: Illustrating the variation of the $t\bar{t}$ cross-section with variation in the scale $M_S$ at (a) the Tevatron and (b) the LHC. For the Tevatron, dashed lines show the experimental data from Run I, when the CDF and D0 results are combined. For the LHC, the dashed lines correspond to errors on the SM cross-section of 0.3, 0.5 and 1 pb respectively.

We present the results of our numerical computations in Fig. 1. In Fig 1 (a), we have plotted the cross-section as a function of the scale $M_S$ for $p\bar{p}$ collisions at the
Tevatron energy of $\sqrt{s} = 1.8$ TeV. We have used the CTEQ4M densities \[13\] and the parton distributions are taken from PDFLIB \[14\]. As explained earlier, we have set the magnitude of the coupling $\lambda = 1$, but we still have the freedom to choose its sign. We have plotted the curves for both choices of sign. We find that the cross-section for the case $\lambda = -1$ is larger than the case $\lambda = 1$. This is due to the fact that in the former case the interference contribution in the $gg$-initiated channel makes a positive contribution. For the experimental value of the cross-section we have used the CDF-D0 average given in Ref. \[17\]. The $1\sigma$, $2\sigma$, $3\sigma$ bands of this average cross-section is shown in Fig 1 (a). We see that for the case $\lambda = -1$, a bound of about 665 GeV results at the 95% confidence level. In the $\lambda = 1$ case, this turns out to be about 660 GeV. These bounds are expected to get much better at Tevatron Run II. We expect (assuming an integrated luminosity of 2 fb) at the Tevatron the $2\sigma$ bound to be about 800 GeV for $\lambda = \pm 1$.

Going from Tevatron to LHC ($pp$ collisions at $\sqrt{s} = 14$ TeV), does affect the results quite significantly. Because of the dominant $gg$ channel at the LHC energy we find that the effect of the interference term is seen much more dramatically in the results, when $\lambda$ changes from $+1$ to $-1$ (see Fig. 1 (b)). The value of the cross-section at the LHC energy is large (about 868 pb) and with an expected luminosity of 10 fb$^{-1}$, we expect of the order of $8.6 \times 10^6$ events at LHC. The statistical error is consequently negligibly small and the error is expected to be dominated by the systematics. We assume errors of the order of 0.3, 0.5 and 1.0 pb (where 0.3 pb is the $1\sigma$ statistical error) and with these assumptions, we find that at LHC we can probe the effective string scale to mass values of the order of 2.5–5 TeV.

In summary, recent proposals about strong effects of gravity at TeV scale can be tested at colliders. We have probed the effects of the interactions of the spin-2 Kaluza-Klein modes with SM matter in the production of $t\bar{t}$ pairs at the Tevatron and LHC. We find that this process can be a useful channel to put bounds on the effective theory scale $M_S$ by using the experimentally measured $t\bar{t}$ cross-sections. The bounds from Tevatron are around 665 GeV at the 95% confidence level for $\lambda = -1$ and about 660 GeV for $\lambda = 1$. More accurate measurements of the top production cross-section at Tevatron Run II will improve these bounds by about 25 GeV. At LHC, it is expected that $M_S$ values of about 2.5–5 TeV can be probed in this channel.
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