Inclusive Electron Scattering from Nuclei at \( x \simeq 1 \)

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The inclusive \( A(e, e') \) cross section for \( x \simeq 1 \) was measured on \(^2\)H, C, Fe, and Au for momentum transfers \( Q^2 \) from 1–7 (GeV/c)\(^2\). The scaling behavior of the data was examined in the region of transition from \( y \)-scaling to \( x \)-scaling. Throughout this transitional region, the data exhibit \( \xi \)-scaling, reminiscent of the Bloom-Gilman duality seen in free nucleon scattering.

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In inclusive electron scattering, scaling functions are important in the study of constituent sub-structure and interactions. Scaling is typically a sign that a simple reaction mechanism dominates the process, allowing one to extract information on structure in a model independent way. The most familiar scaling occurs in the limit of large energy transfer \( \nu \) and momentum transfer \( Q^2 \) where the deep inelastic structure functions \( M W_1(\nu, Q^2) \) and \( \nu W_2(\nu, Q^2) \) become functions only of the Bjorken variable \( x = Q^2/2M\nu \), where \( M \) is the nucleon mass. In this limit, \( x \) can be interpreted as the longitudinal momentum fraction of the struck quark, and \( M W_1 \) and \( \nu W_2 \) are related to the quark longitudinal momentum distribution. Violations of Bjorken scaling in the free nucleon exist at low \( Q^2 \) due to target mass and higher-twist effects. To correct for the effects of target mass at finite \( Q^2 \), the Nachtmann variable \( \xi = 2x/(1 + (1 + 4M^2x^2/Q^2)^{1/2}) \) has been used in place of \( x \). This has been shown to be the correct variable in which to study the logarithmic QCD scaling violations in the nucleon.\(^1\)

Similarly, for quasielastic electron-nucleus scattering at high momentum transfer, the ‘reduced’ cross section was predicted\(^3\), and later observed\(^4\) to exhibit scaling in the variable \( y(q, \nu) \). In the simplest picture of \( y \)-scaling, the electron-nucleus cross section is divided by the elastic electron-nucleus cross section and \( F(y) \) is independent of \( Q^2 \) in the plane wave impulse approximation. In the scaling limit, \( y \) can be interpreted as the nucleon’s initial momentum along the momentum transfer direction, and \( F(y) \) is related to the nucleon’s momentum distribution in the nucleus. Thus, \( y \) plays a similar role for nucleons in a nucleus as \( x \) does for quarks in a nucleon.

In the limit of high \( Q^2 \), the scaling variables \( x, y, \) and \( \xi \) are related. In the parton model, \( \xi \) replaces \( x \) as the scaling variable when the target mass is not neglected. At large \( Q^2 \), \( \xi \) can also be expressed as a function only of \( y \) (with the leading scale-breaking term \( M^2/Q^2 \)\(^3\)). There may also be a relationship between quasielastic and inelastic scattering at more modest momentum transfers.

In the case of the free nucleon, Bloom and Gilman\(^3\) discovered that the resonance peaks in the structure function have the same \( Q^2 \) behavior as the deep inelastic contribution when viewed as a function of \( \omega \), a modified version of the Bjorken scaling variable. It was later shown\(^4\) that this connection between the high \( Q^2 \) structure function and the resonance form factors, called local duality, was expected from perturbative QCD and should be valid for the nucleon elastic peak as well as the resonance peaks if the structure function is analyzed in terms of \( \xi \).
and resonance peaks decrease rapidly with $Q^2$, but move to higher $\xi$, keeping a nearly constant strength with respect to the deep inelastic structure function, which falls with $\xi$. Thus the strong $Q^2$ dependence of the higher twist effects (the elastic and resonance peaks) is removed when the structure function is averaged over a range in $\xi$. In the case of electron scattering from a nucleus, the Fermi motion can perform this ‘averaging’ of the structure function. Thus, when examining $\nu W_2^A$ as a function of $\xi$, scaling may be observed at lower momentum transfers where $x$-scaling is not yet valid due to the quasielastic contribution.

Scaling in inclusive scattering from nuclei (He, C, Fe and Au) was examined in a previous measurement \[7,4\] for $Q^2$ values of 0.3-3.1 (GeV/c)$^2$. For these values of momentum transfer, where the quasielastic contribution dominates the cross section, the data exhibit $y$-scaling for $y < 0$ ($x > 1$). The positive $y$ values represent the high energy-transfer side of the quasielastic peak where the $y$-scaling breaks down due to the increasing inelastic contribution at higher $Q^2$. This same experiment examined $x$- and $\xi$-scaling in the nucleus \[5\]. For low values of $x$, the structure function $\nu W_2$ is a function only of $x$, as predicted. For values of $x$ near or above 1, scaling was not observed due to the contribution of quasielastic scattering. If one examines the structure function vs the Nachtmann variable $\xi$, a scaling behavior is suggested. At lower $\xi$, the data are nearly independent of $Q^2$, while at higher $\xi$, the data approach a universal curve. More recent data at higher $Q^2$ show the same approach to scaling for inclusive scattering from aluminum \[6\]. The beginning of scaling in this region suggests that $\xi$-scaling is not just applicable to deep inelastic scattering, but is also connected to quasielastic scattering and $y$-scaling. Here we examine scaling in the transition region from quasielastic to deep inelastic scattering, to further study the connection between $\xi$- scaling and $y$-scaling.

The data presented here are from the NE18 experiment \[2,4\], a coincidence A(e,e'p) measurement performed in End Station A at the SLAC Nuclear Physics Facility (NPAS). Electron singles were recorded as well as the electron-proton coincidences and this data was analyzed to measure the inclusive cross section. Scattering was measured from cryogenic liquid $^1$H and $^2$H targets and solid C, Fe, and Au targets with beam energies of 2.02, 3.19, 4.21, and 5.12 GeV, at angles of 35.5°, 47.4°, 53.4°, and 56.6° respectively ($Q^2=1, 3, 5, \text{and } 6.8 \text{ (GeV/c)}^2$). The scattered electrons were detected in the SLAC 1.6 GeV/c spectrometer. The pion rate in the spectrometer was up to 500 times the electron rate for runs on Au at the highest $Q^2$. A CO$_2$ gas Čerenkov counter and lead glass shower counter were used to eliminate the pions. Tight cuts were used in the final analysis, resulting in a pion rejection of 15000 to 1, while maintaining an electron efficiency of 90%.

In order to extract the cross section, corrections for spectrometer acceptance, detector efficiencies, data acquisition deadtimes, and radiative corrections were applied to the data. The acceptance was determined using a Monte-Carlo model of the spectrometer, and deadtime corrections were measured on a run-by-run basis. Radiative corrections were applied using an iterative procedure following the formulae of Stein et al. \[11\], which are based on the work of Mo and Tsai \[12\] and Tsai \[13\]. A model cross section was radiated and compared to the data to determine a smooth correction to the model cross section. The ‘corrected’ model was then radiated again, and the procedure repeated until the radiated model was consistent with the data. The model dependence of the radiative correction procedure was tested by varying the initial model cross section. We also compared the radiatively corrected cross sections calculated from runs using targets of different thickness. A final error of 3% was assigned to the radiative correction procedure.

Extracting the structure functions from the measured cross section without performing a Rosenbluth separation requires a knowledge of the ratio of the absorption cross sections for longitudinal and transverse virtual photons, $R = \sigma_L/\sigma_T$. However, the error in extracting $\nu W_2^A$ due to uncertainty in $R$ is small for forward angles and for $R < 1$. We have assumed $R = 0.5/Q^2$ with an uncertainty of 50%, which is consistent with impulse approximation predictions as well as a recent measurement \[8\]. This leads to a worst case contribution to the uncertainty in $\nu W_2^A$ of ±3%. The scaling function $F(y)$ was extracted from the measured cross section using the same method as Day et al. \[7\] and Potter and Feldman \[14\]. The extracted scaling function $F(y)$ for iron is shown in figure 1, along with the previous SLAC NE3 data \[7\]. While the $y < 0$ data exhibited $y$-scaling for the previous data, the scaling clearly breaks down at high $Q^2$ for all $y$ values measured ($y > -80 \text{ MeV/c}$). The breakdown of $y$-scaling is due to the transition from quasielastic scattering to inelastic scattering. To test $y$-scaling in this region, one must calculate and subtract off inelastic contributions to the cross section. This introduces a model dependence and can only be done reliably when the inelastic contributions do not dominate the cross section. It is clear that in the case of inclusive scattering, the applicability of $y$-scaling is limited to lower momentum transfers, where quasielastic scattering dominates the cross section.

Figure 2 shows the measured structure function for iron and carbon as a function of $x$. Clearly the data do not scale in this range but the $Q^2$ dependence is decreasing as $Q^2$ increases. The structure function is nearly identical for all of the targets except deuterium, where the smaller Fermi momentum causes a peak in the structure function near $x = 1$. The larger Fermi momentum in the heavier nuclei washes out the quasielastic peak, leading to a lower structure function near $x = 1$. The difference between carbon and iron decreases as $x$ gets...
further from 1 and at higher $Q^2$. Table 1 gives the ratio of the structure functions for different targets at each $Q^2$, for $0.95 < x < 1.05$, and the ratio of iron to deuterium for $x = 1$. The $Q^2$ behavior of the ratio to deuterium is consistent with the behavior found for aluminum [4]. In figure 3, $\nu W_2$ is plotted vs $\xi$ and an approach to scaling is observed. The new data are all centered at $x = 1$, but move to higher $\xi$ as $Q^2$ increases, lying on the universal curve.

To better understand the transition, we calculated the contributions due to the different scattering processes using the convolution model of Ji and Filippone [6] with a Woods-Saxon spectral function, dipole electric form factor, and the magnetic form factor of Gari and Krümpelmann [15]. Figure 4 shows the approach to scaling for fixed $\xi$ along with calculations showing the quasielastic and deep inelastic contributions to the structure function. Also included is the NE11 data for aluminum [4], which are in good agreement when the structure function is scaled by the number of nucleons. As a function of $Q^2$, we see an increase in the structure function on the low $Q^2$ side of the quasielastic peak, and then a decrease to the high-$Q^2$ value, where inelastic scattering dominates. While the structure function is not independent of $Q^2$ for a fixed $\xi$, it shows less $Q^2$ dependence than when viewed at a constant $x$. More importantly, the measured structure function has relatively little $Q^2$ dependence in the region of transition from quasielastic to inelastic scattering, even though the quasielastic contribution is falling rapidly with $Q^2$. This is true for all $\xi$ values measured, indicating a connection between the quasielastic and inelastic cross sections, reminiscent of local duality in the nucleon.

To summarize, we have extracted the scaling function $F(y)$ and the structure function $\nu W_2$ near $x = 1$ for nuclei with $A$ ranging from 2–197 at $Q^2$ values from 1–7 (GeV/c)$^2$. At the higher $Q^2$ values, $y$-scaling breaks down for all measured values of $y$ as deep inelastic scattering begins to dominate. When examining $\nu W_2$, we do not yet see scaling in $x$, but we do begin to see scaling in the Nachtmann scaling variable $\xi$. This suggests a connection between quasielastic and inelastic scattering, similar to the case of local duality in the nucleon. $\xi$-scaling may prove to be a useful tool in understanding nuclear structure functions, but better coverage in $\xi$ at the present $Q^2$ values, as well as higher $Q^2$ measurements (e.g. at CEBAF [4]) are needed to fully understand the scaling of the structure function, and the relation between $\xi$-scaling and $y$-scaling.

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FIG. 1. $F(y)$ vs $y$ for iron for the present experiment and the previous NE3 measurement. Errors in the new data (solid points) are dominated by a 4% systematic error, but are smaller than the points shown.

FIG. 2. $\nu W_2/A$ vs $x$ for iron (solid points) and carbon (hollow points).

FIG. 3. $\nu W_{2e}^F$ vs $\xi$ for the present experiment and the NE3 measurement.

FIG. 4. $\nu W_{2e}^F$ is plotted vs $Q^2$ at $\xi = 0.85$. The lines are calculations of the total (solid), quasielastic (dashed), and deep inelastic (dotted) contributions to the structure function.

TABLE I. Ratio of Structure Functions for different targets near $x = 1$

| $Q^2$ (GeV/c$^2$) | C/Fe   | Fe/Au  | Fe/D   |
|-------------------|--------|--------|--------|
| 1.0               | 1.14±0.05 | 1.11±0.02 |        |
| 3.0               | 1.14±0.05 | 1.19±0.05 | 0.48±0.03 |
| 5.0               | 1.07±0.05 | 1.16±0.05 | 0.69±0.04 |
| 6.8               | 1.05±0.05 | 0.96±0.05 | 0.95±0.06 |
