Joint Spectrum Sensing and Resource Scheduling for Cognitive Radio Networks Via Duality Optimization

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1. Introduction

With the increasing growth of wireless spectrum demand and the corresponding weak supply, wireless bandwidth has been reported as a new black gold of the new decade by Time News. To resolve the well known dilemma between wireless bandwidth scarcity and underutilization, cognitive radio (CR) (J. Mitola, 2000) is a promising technology in which secondary user (SU) can be allowed to opportunistically access a spectrum hole (S. Haykin, 2005) without interrupting primary user (PU)`s communication. During the past decade, worldwide researchers have created a mass of interesting results to promote CR technology and a comprehensive survey on the recent advances can be found in (Q. Zhao, et al., 2007). However, many challenges are still ahead.

Spectrum sensing and resource scheduling are two critical components of enabling CR technology. Generally, the former aims at finding the spectrum hole quickly and reliably (P. Cheng, et al., 2008, Y. Chen, et al., 2008), and the latter's objective is to obtain a network performance (e.g., system throughput) gain as much as possible (R. Wang, et al., 2008). Due to receiver uncertainty, wireless channel fading and shadowing effect, sensing capability of single spectrum sensor is limited (Z. Chair & P. K. Varshney, 1986). Previous work (Z. Quan, et al., 2008) has shown that cooperation among SUs can significantly improve the sensing reliability by exploiting multi-user spatial diversity. On the other hand, adaptive scheduling spectrum resource (spectrum band, power, etc.) among SUs with different channel conditions can obtain a higher network performance by exploiting multi-user frequency diversity (J. Ma, et al., 2008).

In the most existing designs of CR systems, spectrum sensing and resource scheduling are always implemented separately or sequentially (P. Cheng, et al., 2008, Y. Chen, et al., 2008). In these related works, two main steps are included as follows,

**Step 1.** A spectrum sensing module makes a one-bit hard decision (i.e., idle or busy) based on the soft sensing information collecting from all local spectrum sensors.

**Step 2.** A resource scheduling module implements spectrum assignment and/or power allocation based on the one-bit hard decision sensing information (HSI).

However, intuitively, the separated/isolated design of spectrum sensing and resource scheduling might not be the best choice to improve wireless spectrum efficiency. For one
thing, any process (e.g., the hard decision-making) of passive information processing can bring information loss according to classical information theory (G.M. Antonio, et al., 2009). For another, a joint design or cross-layer design can often outperform a separated/isolated design by exploiting correlation between different components/layers. Moreover, (Y. Chen, et al., 2008) shows that a joint PHY sensing and MAC access strategy outperforms a separation design by considering a scenario with single SU.

On the shoulder of previous valuable works, our contributions in this chapter will be twofold:

- First of all, we will design a joint PHY layer cooperative spectrum sensing and MAC layer resource scheduling scheme. In the joint design, the soft sensing information (SSI) collecting from all local spectrum sensors will be directly fused at a base station for resource scheduling, without any hard decision-making process, to decrease the information loss and further exploit the spectrum opportunity.
- Secondly, we will formulate the joint design as a non-convex optimization problem and find the asymptotic optimum solution based on recent advances in duality optimization theory.

2. System model

In this paper, we consider a centralized CRN consists of a secondary base station (SBS) and K SUs (Fig.1). In each frame, every SU independently senses the PU’s activity and sends its sensing results to the SBS. Based on the results collecting from all SUs, the SBS performs resource scheduling decision-making, i.e., spectrum assignment and/or power allocation.

![Fig. 1. System model of CRN](image)

2.1 Cooperative spectrum sensing and sensing confidence level

In general, CSS can be divided into two major steps: Local processing and Global processing. For subsequent comparison, we further classify CSS into three classes:

1. Local 1 bit hard decision and Global 1 bit hard decision (LHGH) (Z. Chair & P. K. Varshney, 1986).
2. Local 1 bit hard decision and Global non-hard fusion (LHGN) (R. Wang et al., 2008).
3. Local soft sensing and Global soft fusion (LSGS) (Z. Quan, et al., 2008).
Let $S_m$ denotes the real instantaneous state of channel $m$ in a frame where $S_m = 0$ if the $m$-th channel is idle and $S_m = 1$ otherwise. We denote the occupy probability of the channel as $Q_p$. Note that the aim of spectrum sensing is to determine whether a channel is occupied. Due to capability limit of the spectrum sensor, noise uncertainty and something else, sensing error is inevitable. Therefore, we shall concern that how much confidence we should put on the sensing results, which can be formulated as follows:

$$\phi_m = E[S_m | \hat{S}_m] = \frac{Q_p \Pr[\hat{S}_m | S_m = 1]}{Q_p \Pr[\hat{S}_m | S_m = 1] + (1 - Q_p) \Pr[\hat{S}_m | S_m = 0]}$$

(1)

where $\phi_m \in [0, 1]$ is defined as sensing confidence level that the SBS believes the $m$-th channel is busy. $\hat{S}_m$ denotes the sensing results (vector) of the $m$-th subcarrier collected by the SBS. For different CSS classes mentioned above, sensing results (vector) shall have different forms and the corresponding sensing confidence levels are given by:

$$\phi_{m\_LHGH} = \frac{Q_{p,m} \Pr[\hat{S}_m | S_m = 1]}{Q_{p,m} \Pr[\hat{S}_m | S_m = 1] + (1 - Q_{p,m}) \Pr[\hat{S}_m | S_m = 0]}$$

(2)

$$= \begin{cases} 
\frac{Q_{p,m}(1 - Q_{cd}^m)}{Q_{p,m}(1 - Q_{cd}^m) + (1 - Q_{p,m})(1 - Q_{cf}^m)}, & \hat{S}_m = 0 \\
\frac{Q_{p,m}Q_{cd}^m}{Q_{p,m}Q_{cd}^m + (1 - Q_{p,m})Q_{cf}^m}, & \hat{S}_m = 1 
\end{cases}$$

where $\hat{S}_m$ denotes the global 1 bit HSI. $Q_{cf}^m$ and $Q_{cd}^m$ represent the probability of false alarm and detection after LHGH, respectively.

$$\phi_{m\_LHGS} = \frac{Q_{p,m} \Pr[\hat{S}_m | S_m = 1]}{Q_{p,m} \Pr[\hat{S}_m | S_m = 1] + (1 - Q_{p,m}) \Pr[\hat{S}_m | S_m = 0]}$$

(3)

$$= \frac{Q_{p,m} \left( \prod_{i=1}^{K} \prod_{j=1}^{N} Q_{i,m}^{d} (1 - Q_{f,j,m}) \right)}{Q_{p,m} \left( \prod_{i=1}^{K} \prod_{j=1}^{N} Q_{i,m}^{d} (1 - Q_{f,j,m}) + (1 - Q_{p,m}) \prod_{i=1}^{K} \prod_{j=1}^{N} Q_{f,i,m}^{d} (1 - Q_{f,j,m}) \right)}$$

where $Q_{cf,i,m}$ and $Q_{cd,i,m}$ represent the probability of false alarm and detection of the $i$-th SU after local 1 bit hard decision. $N$ is the number of users whose local 1 bit HSI is 0.
where $f(\hat{S}_{k,m} | S_m = 0)$ and $f(\hat{S}_{k,m} | S_m = 1)$ represent the probability density of the instantaneous SSI of the $k$-th SU under given hypothesis, respectively. For detail expressions of them, one can refer to (J. Ma, et al., 2008).

2.2 System throughput and interference constraints

In this paper, we consider the downlink of the centralized CRN with an OFDMA air interface. We assume that the SBS could transmit only when PUs are inactive and it has perfect channel state information (CSI) knowledge.

With the imperfect sensing information and perfect CSI, the SBS make decision of resource scheduling based on the following definitions:

**Definition 1 (Exclusive subcarrier assignment policy $A$):** For any feasible realization of CSI $H$ and sensing information $\hat{S}$, we characterize an exclusive subcarrier assignment policy by a subcarrier assignment indicator matrix $A$, whose element $a_{k,m} = 1$ means that the $m$-th subcarrier is assigned to the $k$-th SU and $a_{k,m} = 0$ otherwise.

**Definition 2 (Power allocation policy $P$):** For any feasible realization of CSI $H$ and sensing information $\hat{S}$, we characterize the power allocation policy with a matrix $P$, whose element $p_{k,m}$ denotes the power that the SBS allocates to the $k$-th SU over the $m$-th subcarrier. As the peak power budget of the SBS should not be exceeded, we have $\sum_{k,m} p_{k,m} \leq P_{\text{max}}$.

After the above definitions, system throughput is given by:

$$R(A,P) = \mathbb{E}_S \left[ \sum_{m=1}^{M} (1 - S_m) \sum_{k=1}^{K} a_{k,m} \log_2 (1 + p_{k,m} r_{k,m}) \right]$$

where $r_{k,m}$ denotes the squared channel gain between the SBS and the $k$-th SU over the $m$-th subcarrier, which is assumed normalized by the receiver noise variance.

Note that imperfect spectrum sensing implies aggressive access to busy subcarriers and thus potential interference to the PU. The interference constraint over the $m$-th subcarrier is:
\[ T_m = \text{El} \sum_{k=1}^{K} a_{k,m} p_{k,m} \sigma^2_{p,m} s_m | \hat{S}_m | \]
\[ = \sum_{k=1}^{K} a_{k,m} p_{k,m} \sigma^2_{p,m} \phi_{m} \leq T_{th,m} \]  

(6)

where \( \sigma^2_{p,m} \) denotes the squared channel gain from the SBS to the \( p \)-th primary receiver over the \( m \)-th subcarrier and the interference threshold is \( T_{th,m} \).

3. Joint spectrum sensing and resource scheduling

3.1 Problem formulation

In this section, the proposed joint spectrum sensing and resource scheduling scheme is formulated as follows:

Given a feasible realization of CSI \( H \) and sensing information \( \hat{S} \), find the optimal exclusive subcarrier assignment policy \( A \) and Power allocation policy \( P \) such that the system throughput is maximized, while satisfying the peak power budget of the SBS and the interference constraint of the PU on each subcarrier simultaneously. The mixed integer nonlinear programming problem can be modelled by

\[
\begin{align*}
g^* = & \max_{P,A} \sum_{m=1}^{M} \left( 1 - \phi_m \right) \sum_{k=1}^{K} a_{k,m} \log_2(1 + p_{k,m} r_{k,m}) \\
\text{s.t.} & \sum_{m=1}^{M} \sum_{k=1}^{K} a_{k,m} p_{k,m} \leq P_{\text{max}} \\
& \sum_{k=1}^{K} a_{k,m} p_{k,m} \sigma^2_{m} \phi_{m} \leq T_{th,m} \quad \forall m \in \{1, \ldots, M\}
\end{align*}
\]

(7)

3.2 Duality optimization

Note that two optimal variables \( P \) and \( A \) are coupled in the problem above, so global optimal solution is often difficult to obtain. However, we will propose an approximately global optimal solution based on duality theory (Y. Wei & R. Lui, 2006).

Firstly, by introducing nonnegative dual variables \( \lambda \) and \( \mu = [\mu_1, \mu_2, \ldots, \mu_m] \), the Lagrange function is:

\[
L(A,P,\lambda,\mu) = \sum_{m=1}^{M} \left( 1 - \phi_m \right) \sum_{k=1}^{K} a_{k,m} \log_2(1 + p_{k,m} r_{k,m}) \\
+ \lambda \left( P_{\text{max}} - \sum_{m=1}^{M} \sum_{k=1}^{K} a_{k,m} p_{k,m} \right) + \sum_{m=1}^{M} \mu_m \left( T_{th,m} - \sum_{k=1}^{K} a_{k,m} p_{k,m} \sigma^2_{m} \phi_{m} \right)
\]

(8)

\[ = \sum_{m=1}^{M} \sum_{k=1}^{K} l(a_{k,m}, p_{k,m}, \lambda, \mu_m) + \lambda P_{\text{max}} + \sum_{m=1}^{M} \mu_m T_{th,m} \]
where

\[ l(a_{k,m}, P_{k,m}, \hat{\lambda}, \mu_m) = a_{k,m}[(1 - \phi_m)\log_2(1 + P_{k,m}n_{k,m}) - \hat{\lambda}P_{k,m} - \mu_mP_{k,m}\sigma_m^2]\]  \hspace{1cm} (9)

Secondly, Lagrange dual function can be obtained by

\[ D(\hat{\lambda}, \mu) = \max_{P,A} L(A,P,\hat{\lambda}, \mu) \]  \hspace{1cm} (10)

And the dual problem is

\[ d^* = \min_{\lambda \geq 0, \mu \geq 0} D(\hat{\lambda}, \mu) . \]  \hspace{1cm} (11)

which can be decomposed into three sub problems (P. Cheng, et al., 2008, G.M. Antonio, et al., 2009, R. Wang, et al., 2009):

**Sub problem 1 (Power allocation):** Given the dual variables \( \hat{\lambda} \) and \( \mu \), for any \( k \in \{1, \ldots, K\} \) and any \( m \in \{1, \ldots, M\} \), maximizing (10) will bring the optimized variable as follows:

\[ P_{k,m}^* = \arg\max_{P_{k,m}} l(a_{k,m}, P_{k,m}, \hat{\lambda}, \mu_m) = \left( \frac{1 - \phi_m}{[\hat{\lambda} + \mu_m\sigma_m^2]\ln 2 - \frac{1}{n_{k,m}}} \right)^+ \]  \hspace{1cm} (12)

where \([x]^+ = \max\{x, 0\}\).

Sub problem 2 (Subcarrier assignment): Substituting (12) into (9) will bring \( l^*(a_{k,m}, P_{k,m}^*, \hat{\lambda}, \mu_m) \), and maximizing which will bring the best SU of each subcarrier,

\[ a_{k,m}^* = \begin{cases} 1, & k = \arg\max_{k} l^*(a_{k,m}, P_{k,m}^*, \hat{\lambda}, \mu_m) \\ 0, & \text{otherwise} \end{cases} \]  \hspace{1cm} (13)

and we have \( l^0(a_{k,m}, P_{k,m}^*, \hat{\lambda}, \mu_m) = \max_{a_{k,m}} l^*(a_{k,m}, P_{k,m}^*, \hat{\lambda}, \mu_m) \).

Sub problem 3 (Dual variables update): The optimal dual variables can be obtained by solving its dual problem:

\[ (\lambda^*, \mu^*) = \min_{\lambda \geq 0, \mu \geq 0} \sum_{m=1}^{M} \sum_{k=1}^{K} l^0(a_{k,m}^*, P_{k,m}^*, \hat{\lambda}, \mu_m) \]  \hspace{1cm} (14)

\[ + \lambda P_{\max} + \sum_{m=1}^{M} \mu_m T_{th,m} \]

Because dual function is always convex (S. Boyd & L. Vandenberghe, 2004), a gradient-type search is guaranteed to converge to the global optimum. However, dual function is not necessarily differentiable and thus it does not always have a gradient. Here we use a subgradient (a generalization of gradient) update method,
\[ \lambda^{n+1} = [\lambda^n - s^n (P_{\text{max}} - \sum_{m=1}^{M} \sum_{k=1}^{K} a_{k,m}^* p_{k,m}^*)] \]

\[ \mu_{m}^{n+1} = [\mu_{m}^n - s^n (\bar{\mu}_{\text{ih},m} - \sum_{k=1}^{K} a_{k,m}^* p_{k,m}^* \sigma_{m}^2 \phi_{m})] \quad \forall m \]

where \( n \) is the iteration number. The above sub-gradient update is guaranteed to converge to the optimal dual variables as long as the sequence of scalar step \( s^n \) is chosen appropriately (Y. Wei & R. Lui, 2006). The duality gap \( g^* - d^* \) can be zero as long as the number of subcarrier is sufficiently large (Y. Wei & R. Lui, 2006).

### 3.3 Distributive implementation

Without loss of optimality, we proposed a distributive iterative algorithm to alleviate the computing overhead of the SBS.

**Distributive Algorithm:**

**Step 1.** Each SU performs local spectrum sensing on each subcarrier and sends the sensing information to the SBS.

**Step 2.** The SBS computes the sensing confidence level according to (2)/(3)/(4) and broadcasts \( \phi_{m} \) and initial dual variables \( \lambda^1 \) and \( \mu_{m}^{1} \) to all SUs.

**Step 3.** In each iteration, first each SU solves subproblem 1 and sends \( P_{k,m}^* \) and \( \lambda^* \) to the SBS. Secondly the SBS solves subproblem 2 and assigns each subcarrier to the corresponding best SU. Then the SBS update the dual variables according to (16) and (17).

**Step 4.** If \[ |\mu_{m}^{n+1} (\bar{\mu}_{\text{ih},m} - \sum_{k=1}^{K} a_{k,m}^* p_{k,m}^* \sigma_{m}^2 \phi_{m})| \leq \varepsilon, \quad \forall m \] and
\[ |\lambda^{n+1} (P_{\text{max}} - \sum_{m=1}^{M} \sum_{k=1}^{K} a_{k,m}^* p_{k,m}^*)| \leq \varepsilon \] are satisfied simultaneously, then terminate the algorithm. Otherwise, jump to Step 3.

### 4. Numerical results

#### 4.1 Simulation setup

In this section, we compare our proposed scheme LSGS with several previous designs, i.e., LHGH(Z. Chair & P. K. Varshney, 1986), LHGN (R. Wang et al., 2008).

- LHGH here refers to the design that the SBS fuses the 1-bit HSI from all SUs (using an OR rule) to make a 1-bit hard decision on the availability of each subcarrier.
- LHGN refers to the design that the SBS fuses the 1-bit HSI from all SUs for resource scheduling without centralized hard decision.
- The proposed scheme LSGS means that the SBS directly uses the SSI collecting from all SUs for resource scheduling without any hard decision.
In the following simulation, we use an OFDMA system with $K = 64$ subcarriers and the interference constraint of each subcarrier is 0dB. All sensing links from primary transmitters to SUs and all scheduling links from the SBS to SUs are assumed to be i.i.d. Rayleigh fading, respectively.

### 4.2 Sensing performances versus number of cooperative users

Fig. 2 presents the comparison of different schemes in terms of sensing error, which equals to the percentage of the number of incorrect decisions among all decisions. The error can be caused by false alarms (i.e., an idle subcarrier is mistaken as a busy one) or miss detection (i.e., a busy subcarrier is mistaken as an idle one). It is shown in Fig. 2 that the sensing errors of all schemes are highly related to the number of cooperative users. Specifically, the sensing errors of both LHGN and LSGS decrease with the number of cooperative users while LHGH performs a much higher sensing error when the number of cooperative users is large. Note that the proposed LSGS always outperforms the other schemes due to less information loss.

![Fig. 2. Sensing performances versus number of cooperative users](image)

### 4.3 System throughput versus number of cooperative users

In Fig. 3, we compare the performance of different schemes in terms of system throughput. It is shown that the system throughput of all schemes increase with the number of cooperative users $K$. This increment mainly profits from multiuser diversity of CSS. Note that the proposed scheme LSGS always outperforms the other two schemes because of
less sensing information loss, which is consistent with the results in Fig. 2. The separated design, LHGH performs the worst and becomes saturated much earlier than the joint designs, LHGN and LSGS. This fact demonstrates that joint design could exploit spectrum opportunity more effectively, and less sensing information loss means better system performance.

Fig. 3. System throughput versus number of cooperative users at $P_0 = 10\text{dB}, Q_p = 0.5$.

4.4 System throughput versus signal-to-noise ratio (SNR)

Signal-to-Noise Ratio (SNR) is an important parameter that affects the system performance. Fig. 4 indicates that the higher the SNR is, the better the system throughput of all schemes are. System throughput of all schemes increases with SNR. The reason behind is that: given the average noise power, a higher SNR means a bigger transmission power the BS can employ, which in turn brings a higher bit rates.

4.5 System throughput versus peak power budget of the SBS

Fig. 5 shows that the system throughput of all schemes increase with the peak power budget of the SBS and the proposed LSGS performs the best. Especially, in the scenario of heavy load of the primary user activity $Q_p = 0.8$ (i.e., available spectrum for CRN is rare), the proposed scheme LSGS obtains much higher throughput than LHGN and LHGH, which is valuable for CRN design.
Fig. 4. System throughput versus Signal-to-Noise Ratio (SNR) at K=6

Fig. 5. System throughput versus peak power budget at K=6
5. Conclusion and future work

We have developed a joint cooperative spectrum sensing and resource scheduling scheme for cognitive radio networks in this paper. Numerical results tell that joint design exploits spectrum opportunity more effectively than isolated design; especially much higher system throughput performance can be obtained with soft sensing information feedback to the SBS. The next work will focus on quantizing the SSI or/and clustering the spectrum sensors to achieve a better trade-off between performance and feedback overhead.

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