QUARK MIXING AND CP VIOLATION — THE CKM MATRIX

ULRICH NIERSTE

Institut für Theoretische Teilchenphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany
E-mail: nierste@particle.uni-karlsruhe.de

I present the status of the elements and parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and summarise the related theoretical progress since Lepton-Photon 2003. One finds $|V_{ub}| = 0.2227 \pm 0.0017$ from K and $\tau$ decays and $|V_{cb}| = (41.6 \pm 0.5) \cdot 10^{-3}$ from inclusive semileptonic $B$ decays. The unitarity triangle can now be determined from tree-level quantities alone and the result agrees well with the global fit including flavour-changing neutral current (FCNC) processes, which are sensitive to new physics. From the global fit one finds the three CKM angles $\theta_{12} = 12.9^\circ \pm 0.1^\circ$, $\theta_{23} = 2.38^\circ \pm 0.03^\circ$ and $\theta_{13} = 0.223^\circ \pm 0.007^\circ$ in the standard PDG convention. The CP phase equals $\delta_{13} = (58.8 \pm 5.8)^\circ$ at 1$\sigma$ CL and $\gamma = (58.8 \pm 11.2)^\circ$ at 2$\sigma$ CL. A major progress are first results from fully unquenched lattice QCD computations for the hadronic quantities entering the UT fit. I further present the calculation of three-loop QCD corrections to the charm contribution in $K^+ \to \pi^+\nu\bar{\nu}$ decays, which removes the last relevant theoretical uncertainty from the $K \to \pi\nu\bar{\nu}$ system. Finally I discuss mixing-induced CP asymmetries in $b \to s\nu$ penguin decays, whose naive average is below its Standard Model value by $3\sigma$.

1 Flavour in the Standard Model

In the Standard Model transitions between quarks of different generations originate from the Yukawa couplings of the Higgs field to quarks. The non-zero vacuum expectation value $v$ of the Higgs field leads to quark mass matrices $M^u$ and $M^d$ for the up-type and down-type quarks, respectively. The transformation to the physical mass eigenstate basis, in which the mass matrices are diagonal, involves unitary rotations in flavour space. The rotation of the left-handed down-type quarks relative to the left-handed up-type quarks is the physical Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$. It appears in the couplings of the $W$ boson to quarks and is the only source of transitions between quarks of different generations. $V$ contains one physical complex phase, which is the only source of CP violation in flavour-changing transitions.

Flavour physics first aims at the precise determination of CKM elements and quark masses, which are fundamental parameters of the Standard Model. The second target is the search for new physics, pursued by confronting high precision data with the predictions of the Standard Model and its extensions. To this end it is useful to distinguish between charged-current weak decays and flavour-changing neutral current (FCNC) processes. The determination of CKM elements from the tree-level charged-current weak decays, discussed in Sect. 2, is practically unaffected by possible new physics. By contrast, FCNC processes are very sensitive to virtual effects from new particles with masses at and above the electroweak scale, even beyond 100 TeV in certain models of new physics. FCNC processes are discussed in Sect. 3.

$V$ can be parameterised in terms of three mixing angles $\theta_{12}$, $\theta_{23}$, $\theta_{13}$ and one complex phase $\delta_{13}$, which violates CP . Adopting the PDG convention, in which $V_{ud}$, $V_{us}$, $V_{cb}$ and $V_{tb}$ are real and positive, these parameters $\theta_{12}$ are real and positive, these parameters $\theta_{12}$ are real and positive, these parameters

Still new physics can be revealed if the $3 \times 3$ CKM matrix $V$ is found to violate unitarity: One may then infer the existence of new (for example iso-vector) quarks which mix with the known six quarks. Further leptonic decays of charged mesons are tree-level, but sensitive to effects from charged Higgs bosons.
can be determined through
\[
V_{us} = \sin \theta_{12} \cos \theta_{13}, \quad V_{ub} = \sin \theta_{13} e^{-i \delta_{13}}, \\
V_{cb} = \sin \theta_{23} \cos \theta_{13}.
\]
(1)

The Wolfenstein parameterisation\(^2\)
\[
V = \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 (\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}
\]
is an expansion of \(V\) in terms of \(\lambda \simeq 0.22\) to order \(\lambda^3\). It shows both the hierarchy of the CKM elements and their correlations, like \(|V_{us}| \simeq |V_{cd}|\) and \(|V_{cb}| \simeq |V_{ts}|\). The apex of the standard unitarity triangle (UT), which is shown in Fig. 1 is defined by\(^3\)
\[
(\rho, \eta) \equiv -\frac{V_{us}^* V_{ud}}{V_{cd} V_{cb}} = \left| \frac{V_{us}^* V_{ud}}{V_{cd} V_{cb}} \right| e^{i \gamma}
\]
(3)

The numerical results presented in the following have been prepared with the help of the Heavy Flavor Averaging Group (HFAG)\(^4\) and the CKMfitter\(^5\) and UTFit\(^6\) groups. CKMfitter uses a Frequentist treatment of theoretical uncertainties, while UTFit pursues a Bayesian approach, using flat probability distribution functions for theoretical uncertainties.

2 CKM elements from tree-level decays

The standard way to determine the magnitudes of the elements of the first two rows of \(V\) uses semileptonic hadron decays, depicted in Fig. 2. From Eq. (2) one realizes that an accurate determination of \(V_{us}\) or \(V_{ud}\) determines \(V_{cd}\) and \(V_{cs}\) as well. Therefore measurements of semileptonic \(c \to d\) and \(c \to s\) decays are usually viewed as test of the computation of the hadronic form factors entering the decay amplitudes. Charm decays are covered by Iain Stewart.\(^7\)

2.1 \(V_{ud}\)\n
\(V_{ud}\) can be determined from superallowed \((0^+ \to 0^+)\) nuclear \(\beta\) decay and from the \(\beta\) decays \(n \to p \ell \bar{\nu}_\ell (\gamma)\) and \(\pi^- \to \pi^0 \ell \bar{\nu}_\ell (\gamma)\). Since no other decay channels are open, the semileptonic decay rate can be accessed through lifetime measurements. All three methods involve the hadronic form factor of the vector current:
\[
\langle f | \sigma_{\gamma\mu} d | i \rangle,
\]
where \((i,f) = (0^+,0^+),(n,p)\) or \((\pi^\pm,\pi^0)\).

The neutron decay further involves the form factor of the axial vector current:

\[
\langle f | \overline{\tau}_\mu \gamma_5 d | i \rangle
\]

The form factors parameterise the long-distance QCD effects, which bind the quarks into hadrons. The normalisation of the vector current is fixed at the kinematic point of zero momentum transfer \(p_i - p_f\) in the limit \(m_u = m_d\) of exact isospin symmetry.

The Ademollo-Gatto theorem\(^9\) assures that corrections are of second order in the symmetries in the Dalitz plot. Experimentally, the highest precision in the determination of \(V_{ud}\) is achieved in the nuclear \(\beta\) decay, but \(n \to p \ell \overline{\nu}_\ell(\gamma)\) starts to become competitive. However, there is currently a disturbing discrepancy in the measurement of the neutron lifetime among different experiments.\(^8\)

From a theoretical point of view progress in \(n \to p \ell \overline{\nu}_\ell(\gamma)\) and, ultimately, in the pristine \(\pi^- \to \pi^0 \ell^+ \overline{\nu}_\ell(\gamma)\) decay are highly desirable to avoid the nuclear effects of \(0^+ \to 0^+\) transitions. On the theory side QED radiative corrections must be included to match the experimental accuracy, recently even dominant two-loop corrections to \(n \to p \ell \overline{\nu}_\ell(\gamma)\) have been calculated.\(^10\)

The world average for \(V_{ud}\) reads\(^11\):

\[
V_{ud} = 0.9738 \pm 0.0005 \quad (5)
\]

2.2 \(V_{us}\)

\(V_{us}\) can be determined from Kaon and \(\tau\) decays. The most established method uses the so-called \(K\ell3\) decays \(K^0 \to \pi^- \ell^+ \nu_\ell\), \(K^0 \to \pi^0 \mu^+ \nu_\mu\), \(K^+ \to \pi^0 \ell^+ \nu_\ell\) and \(K^+ \to \pi^0 \mu^+ \nu_\mu\). The decay rates schematically read

\[
\Gamma(K \to \pi^+ \ell^+ \nu_\ell) \propto |V_{us}|^2 F^\ell(0)
\]

The horizontal band is the range quoted in Eq. (6). Courtesy of Vincenzo Cirigliano.\(^11\)

\[
V_{us}^2 \left| f_+^{K^0\pi^-}(0) \right|^2 \left[ 1 + 2\Delta_{SU(2)}^K + 2\Delta_{em}^K \right].
\]

The hadronic physics is contained in \(\Delta_{SU(2)}^K\) and \(\Delta_{em}^K\), and QED corrections are contained in \(\Delta_{em}^K\). The Ademollo-Gatto theorem\(^9\) ensures \(f_+^{K^0\pi^-}(0) = 1 + O(m_s - m_d)^2/\Lambda_{had}^2\), \(f_+^{K^0\pi^-}(0) - 1\) can be calculated with the help of Chiral Perturbation Theory (\(\chi PT\))^12, which exploits the fact that the pseudoscalar mesons are Goldstone bosons of a dynamically broken chiral symmetry of QCD. \(\chi PT\) amounts to a systematic expansion in \(p/\Lambda_{had}\), \(M/\Lambda_{had}\), \(m_\ell/\Lambda_{had}\) and the electroweak coupling \(\epsilon\). Here \(p\) and \(M\) denote meson momenta and masses and \(m_\ell\) is the lepton mass. There has been a substantial progress in the calculation of both \(\Delta_{em}^{K\ell3}\) and \(f_+^{K^0\pi^-}(0)\) since \(LP'03\). Significant effects of \(O(\epsilon^2 p^2)\) QED corrections on differential distributions were found; they must be included in Monte Carlo simulations. The value for \(V_{us} f_+^{K^0\pi^-}(0)\) extracted from various experiments is shown in Fig. 3. The world average reads:\(^11\)

\[
f_+^{K^0\pi^-} V_{us} = 0.2175 \pm 0.0008. \quad (6)
\]
Combining the results from $\chi^P T$ at order $p^6$ and quenched lattice gauge theory (new) to\textsuperscript{14} 

\[ f_+^{K^0\pi^-} = 0.972 \pm 0.012 \]

one arrives at

\[ V_{us} = 0.2238 \pm 0.0029 \quad (7) \]

from $K\ell 3$.

$V_{us}$ can also be determined from the $K\mu 2$ decay $K^+ \rightarrow \mu^+ \nu_\mu(\gamma)$.\textsuperscript{15} The hadronic quantity entering this decay is the Kaon decay constant $F_K$. Uncertainties can be better controlled in the ratio $F_K/F_\pi$ and one considers

\[ \frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu(\gamma))}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu(\gamma))} = \frac{V_{us}^2}{V_{ud}^2} \frac{M_K^2 - m_\mu^2}{M_\pi^2 - m_\mu^2} \left[ 1 - \frac{\alpha}{\pi} (C_\pi - C_K) \right] \]

with QED corrections $C_\pi - C_K = 3.0 \pm 1.5$. Using the result\textsuperscript{16} $F_K/F_\pi = 1.210 \pm 0.004 \pm 0.013$ computed by the MILC collaboration with 2+1 dynamical quarks, one finds (with $V_{ud}$ from Eq. (5)):

\[ V_{us} = 0.2223 \pm 0.0026 \quad (8) \]

from the $K\mu 2$ decay. This is astonishingly precise and $K^+ \rightarrow \mu^+ \nu_\mu(\gamma)$ may constrain mass and couplings of a charged Higgs boson, which can mediate this decay as well.\textsuperscript{15}

The third possibility to measure $V_{us}$ used hadronic $\tau$ decays to the inclusive final state with strangeness $|S| = 1$. The experimental inputs are the ratios

\[ R_{\tau s} = \frac{\Gamma_{\Delta S=1}(\tau \rightarrow \text{hadrons } \nu_\tau(\gamma))}{\Gamma(\tau \rightarrow e^+\nu_\tau(\gamma))} \propto V_{us}^2 \]

\[ R_{\tau d} = \frac{\Gamma_{\Delta S=0}(\tau \rightarrow \text{hadrons } \nu_\tau(\gamma))}{\Gamma(\tau \rightarrow e^+\nu_\tau(\gamma))} \propto V_{ud}^2 \]

Here $S$ is the strangeness. The optical theorem allows to relate $R_{\tau s,d}$ to the QCD current-current correlators $\Pi_{s,d}^T$ and $\Pi_{s,d}^L$:

\[ R_{\tau s,d} = 12\pi \int_0^1 dz (1-z)^2 \times \left[ (1+2z) \text{Im } \Pi_{s,d}^T(z) + \text{Im } \Pi_{s,d}^L(z) \right] \]

with $z = s/M_\pi^2 = (p_\tau - p_\nu)^2/M_\pi^2$. This relationship is depicted in Fig. 4. $\Pi_{s,d}^T$ can be computed through an operator product expansion (OPE). The leading term is massless perturbative QCD, subleading operators entering $\Pi_{s,d}^T$ are $m_s^2$ and $m_s(qq)$. The OPE amounts to an expansion in $\Lambda_{QCD}/M_\tau$, $m_s/M_\tau$ and $\alpha_s(m_\tau)$. In the limit $m_s = 0$ of exact SU(3)$_F$ symmetry the ratio $R_{\tau s}/R_{\tau d}$ would directly determine $V_{us}^2/V_{ud}^2$. Hence it suffices to compute the (small) SU(3)$_F$ breaking quantity\textsuperscript{17}

\[ \delta R_{\tau} \equiv \frac{R_{\tau d}}{V_{ud}^2} - \frac{R_{\tau s}}{V_{us}^2} \]

With $\delta R_{\tau} = 0.218 \pm 0.026$\textsuperscript{18} and experimental data from OPAL\textsuperscript{19} one finds $R_{\tau d} = 3.469 \pm 0.014$, $R_{\tau s} = 0.1694 \pm 0.0049$ and finally\textsuperscript{18}:

\[ V_{us} = \sqrt{\frac{R_{\tau s}}{R_{\tau d}/V_{ud}^2 - \delta R_{\tau}}} \]

\[ = 0.2219 \pm 0.0033_{\text{exp}} \pm 0.0009_{\text{th}} \]

\[ = 0.2219 \pm 0.0034. \quad (9) \]

The dominant source of uncertainty in $\delta R_{\tau}$, which enters Eq. (9) as a small correction, is from $m_s$. In the near future it should be possible to improve on $V_{us}$ with data from BaBar and BELLE.

In summary one finds an excellent consistency of the three numbers for $V_{us}$ from $K\ell 3$, $K\mu 2$ and $\tau$ decays. This is remarkable, since

\[ s \rightarrow \tau \nu_\tau \]

\[ W \]
the three methods use very different theoretical tools to address the strong interaction: Chiral perturbation theory, lattice gauge theory and the operator product expansion. The result nicely reflects the tremendous progress of our understanding of QCD at low energies. Averaging the results of Eqs. (7), (8) and (9) one finds:

\[ V_{us} = 0.2227 \pm 0.0017 \quad (10) \]

With \( V_{ud} \) in Eq. (5) one can perform the first-row unitarity check

\[ V_{us}^2 + V_{ud}^2 + |V_{ub}|^2 - 1 \simeq V_{us}^2 + V_{ud}^2 - 1 = -0.0021 \pm 0.0012 \]

The Cabibbo matrix is unitary at the 1.8σ level, just as at \( LP'03 \):^{20} \n
\[ V_{us}^2 + V_{ud}^2 - 1 = -0.0031 \pm 0.0017 \]

### 2.3 \( V_{cb} \)

\( V_{cb} \) can be determined from inclusive or exclusive \( b \to c\ell\nu_\ell \) decays. Exclusive decays are not discussed here. The analysis of the inclusive decay employs an OPE,\(^{21} \) similarly to the determination of \( V_{us} \) from \( \tau \) decay discussed in Sect. 2.2. The optical theorem relates the inclusive decay rate \( \mathcal{B} \to X_c\ell\nu_\ell \) to the imaginary part of the \( B \) meson self energy, depicted on the LHS of Fig. 5. The OPE matches the self energy diagram to matrix element of effective operators, whose coefficients contain the short-distance information associated with the scale \( m_b \) and can be calculated perturbatively. Increasing dimensions of the operators on the RHS of Fig. 5 correspond to decreasing powers of \( m_b \) in the coefficient functions, so that the OPE amounts to a simultaneous expansion in \( \Lambda_{QCD}^3/m_b^3 \) and \( \alpha_s(m_b) \). Since \( \langle B |\bar{b}b|B \rangle = 1 + O(\Lambda_{QCD}^2/m_b^2) \) and there are no dimension-4 operators, non-perturbative parameters first occur at order \( \Lambda_{QCD}^3/m_b^3 \). They are

\[ \mu_\pi^2 \propto -\langle B |\bar{b}D_\pi^2 b|B \rangle \]

\[ \mu_G^2 \propto \langle B |\bar{b}i\sigma_{\mu\nu}G^{\mu\nu} b|B \rangle \]

\( \mu_G^2 \), which parameterises the matrix element of the chromomagnetic operator, can be determined from spectroscopy. Hence to order \( \Lambda_{QCD}^3/m_b^3 \) one only has to deal with the three quantities \( m_b \), \( m_c \) and \( \mu_\pi^2 \), which quantifies the Fermi motion of the \( b \) quark inside the \( B \) meson.

The OPE can further be applied to certain spectral moments of the \( B \to X_c\ell\nu_\ell \) decay, the distributions of the hadron invariant mass \( M_X \) and of the lepton energy. Further the same parameters govern different inclusive decays, for instance also \( B \to X_s \gamma \). Therefore there is a lot of redundancy in the determination of \( V_{cb} \), providing powerful checks of the theoretical framework. The state of the art are fits to order \( \Lambda_{QCD}^3/m_b^3 \), which involve 7 parameters.\(^{22} \) The result of a global fit to hadron and lepton moments in \( B \to X\ell\nu_\ell \) and photon energy moments in \( B \to X_s \gamma \) from BaBar, BELLE, CDF, CLEO, DELPHI\(^{23} \) can be seen in Fig. 6. It gives

\[ V_{cb} = 41.6 \pm 0.3_{\text{exp}} \pm 0.3_{\text{OPE moments}} \]

\[ \pm 0.3_{\text{OPEcalc}} \]

\[ = (41.6 \pm 0.5) \cdot 10^{-3} \quad (11) \]

from inclusive \( B \to X\ell\nu_\ell \).

### 2.4 \( |V_{ub}| \)

I discuss the determination of \( |V_{ub}| \) from inclusive \( B \to X_u\ell\nu_\ell \) decays. Exclusive decays are discussed in.\(^7 \) In principle one could determine \( |V_{ub}| \) in the same way as \( V_{cb} \), if there were no background from \( B \to X_u\ell\nu_\ell \) decays. Its suppression forces us to impose cuts on the lepton energy \( E_\ell \), the hadronic energy \( E_X \), the hadron invariant mass \( M_X \)

\[ \text{Figure 5. OPE for } \mathcal{B} \to X_c\ell\nu_\ell \text{. The leading operator } \mathcal{B} \text{ has dimension 3.} \]
and the shape function $S$ with scales of order $m_{\Lambda_{QCD}}$. Here is a schematic form:

$$d\Gamma \propto H \int_{0}^{P_+} d\omega \ J(m_b(P_+ - \omega)) \ S(\omega)$$

Here $H$ contains the hard QCD, associated with scales of order $m_b$. The jet function $J$ and the shape function $S$ contain the physics from scales of orders $M_X \sim m_b\Lambda_{QCD}$ and $\Lambda_{QCD}$, respectively. $P_+$ and $P_-$ are defined as $P_N = E_X \mp |\vec{P}_X|$. From $\Lambda_{QCD} \ll P_+ \sim M_X \sim \sqrt{m_b\Lambda_{QCD}} \ll P_- \leq m_b$ one realizes that one has to deal with a multi-scale problem, which is more complicated than $B \rightarrow X_c\ell\nu_\ell$. The second frontier of research in $B \rightarrow X_u\ell\nu_\ell$ deals with subleading shape functions $s_\ell$, which occur at order $1/m_b$. They are different in $B \rightarrow X_c\ell\nu_\ell$ and $B \rightarrow X_s\gamma$, but their moments can be related to OPE parameters like $\mu_2^2$, which gives some guidance to model these functions. Meanwhile an event generator for $B \rightarrow X_u\ell\nu_\ell$ decays is available, with formulae which contain all available theoretical information and smoothly interpolate between the shape function and OPE regions. It is pointed out that a cut on the variable $P_+$, which is directly related to the photon energy in $B \rightarrow X_s\gamma$, makes the most efficient use of the $S(\omega)$ extracted from the radiative decay. Alternatively one can eliminate $S(\omega)$ altogether by forming proper weighted ratios of the endpoint photon and lepton spectra in $B \rightarrow X_s\gamma$ and $B \rightarrow X_u\ell\nu_\ell$, respectively. Using also the information from $B \rightarrow X_c\ell\nu_\ell$ on $m_b$ and $\mu_2^2$ the data from CLEO, BELLE and BaBar combine to the world average:

$$V_{ub} = (4.39 \pm 0.20_{exp} \pm 0.27_{th,m_b,\mu_2^2}) \cdot 10^{-3}$$
$$= (4.39 \pm 0.34) \cdot 10^{-3} \quad (12)$$

from inclusive $B \rightarrow X_u\ell\nu_\ell$. Eq. (3) implies that $|V_{ub}/V_{cb}|$ defines a circle in the $(\rho,\eta)$ plane which is centered around $(0,0)$. With Eqs. (11) and (12) its radius is constrained to

$$R_u \equiv \sqrt{\rho^2 + \eta^2} = 0.45 \pm 0.04. \quad (13)$$

2.5 arg $V_{ub}$

$\gamma = \arg V_{ub}^* \gamma$ can be determined from exclusive $B \rightarrow D^{\pm}X$ decays, where $X$ denotes one or several charmless mesons. This method exploits the interference of the tree-level $b \rightarrow c\bar{q}q$ and $b \rightarrow u\bar{q}q$ amplitudes, where $q = d$ or

![Graphical representation](image-url)
The probability to solve for \( \gamma \) instead of averaging the inferred values of \( \gamma \) is involved, the Tevatron experiments are submitted to prochep: Wyler (GLW) method shown in Fig. 7. The Gronau-London-Wyler method combines the rates of \( B^\pm \to D^0 \to f_i \phi \) for different final states \( f_i \).

\( q = s \). The prototype is the Gronau-London-Wyler (GLW) method shown in Fig. 7. The decays \( B \to D^0 X \) and \( B \to \bar{D}^0 X \) interfere, if both subsequent decays \( D^0 \to f \) and \( \bar{D}^0 \to f \) are allowed. One needs four measurements to solve for the magnitudes of the branching fractions and their relative weak phase, which is the desired UT angle \( \gamma \). For example one can combine the information of the branching fractions of \( B^+ \to \bar{D}^0 \to K^\pm \pi^\mp K^\pm \) and \( B^- \to D^0 \to K^\mp \pi^\pm K^\pm \). This works with untagged non-flavour-specific decays as well. E.g. the final state \( \bar{D}^0 \phi \) does not reveal whether the decaying meson was a \( B_s \) or \( B_s \). Still, when at least three pairs of \( \bar{B}^0_s \to D^0 \to f_i \phi \) and \( B_s \to \bar{D}^0 \to f_i \phi \) branching fractions are measured, where \( f_i = \text{CP} f_i \) (and the \( f_i \)'s are not CP eigenstates), one has enough information to solve for \( \gamma \). Since no flavour tagging is involved, the Tevatron experiments may contribute to these class of \( \gamma \) determinations. The described determination of \( \gamma \) from tree-tree interference is modular, that is measurements in different decay modes can be combined, as they partly involve the same hadronic parameters. One should further first average the branching ratios from different experiments and then determine \( \gamma \) instead of averaging the inferred values of \( \gamma \) obtained from different experiments. Combining (almost) all \( B^+ \to D^0 K^{+(\pm)} \) data gives (preliminary)

\[
\gamma = (70^{+12}_{-14})^\circ \tag{14}
\]

and the second solution \( \gamma - 180^\circ \sim -110^\circ \). This is \( \gamma = \arg V_{ub}^* \) determined from the tree-level \( b \to u\pi s \) amplitude.

Within the Standard Model \( b \to u\pi d \) decays of tagged \( B^0 \) mesons are used to determine the UT angle \( \alpha \). \( b \to u\pi d \) decays involve both a tree and a penguin amplitude. The penguin component can be eliminated, if several decay modes related by isospin are combined, as in the Gronau-London method which uses \( B^+ \to \pi^+ \pi^0, B^0 \to \pi^+ \pi^- \) and \( B^0 \to \pi^0 \pi^0 \). The \( B \to \rho \pi \) and \( B \to \rho \rho \) decay modes are better suited for the determination of \( \alpha \), because the penguin amplitude is smaller. A combined analysis of the \( \pi \pi, \rho \pi \) and \( \rho \rho \) systems gives

\[
\alpha_{\exp} = (99^{+12}_{-9})^\circ \tag{15}
\]

and the second solution \( \alpha_{\exp} - 180^\circ \sim -81^\circ \). The experimental result \( \alpha_{\exp} \) could differ from the true \( \alpha = \arg(-V_{tb}^* V_{td}^\prime/(V_{ub}^* V_{ud})) \), if new physics alters the \( B_q - \bar{B}_q \) mixing amplitude. However, the influence from new physics is fully correlated in \( \alpha_{\exp} \) and the CP asymmetry measured in \( b \to u\pi s \) decays. From the latter (see Eq. (21) below) we infer the \( B_d - \bar{B}_d \) mixing phase \( 2\beta_{\exp} = (43.7 \pm 2.4)^\circ \). The \( B_d - \bar{B}_d \) mixing phase cancels from the combination \( 2\gamma = 360^\circ - 2\alpha_{\exp} - 2\beta_{\exp} \), so that one obtains

\[
\gamma = (59^{+9}_{-12})^\circ \tag{16}
\]

and the second solution \( \gamma - 180^\circ \sim -121^\circ \). Since the isospin analysis eliminates the penguin component, this is \( \gamma = \arg V_{ub}^* \) determined from the tree-level \( b \to u\pi s \) amplitude.

The results in Eqs. (14) and (16) are in good agreement. Their naive average is

\[
\gamma = (63^{+7}_{-9})^\circ \tag{17}
\]

The successful determination of a CP phase from a tree-level amplitude is a true novel result compared to LP’03. For the first time we
can determine the UT from tree-level quantities alone, the result is shown in Fig. 8. This is important, because the tree-level UT can only be mildly affected by new physics and therefore likely determines the true values of \( \rho \) and \( \eta \).

### 3 CKM elements from FCNC processes

In the Standard Model FCNC processes are suppressed by several effects: First they only proceed through electroweak loops. Second they come with small CKM factors like \( |V_{ts}| \sim 0.04 \) and \( |V_{td}| \sim 0.01 \). Loops with an internal charm quark are further suppressed by a factor of \( m_{c}^{2}/M_{W}^{2} \) from the GIM mechanism. Radiative and leptonic decays further suffer from an additional helicity suppression, because only left-handed quarks couple to W bosons and undergo FCNC transitions. All these suppression mechanism are accidental, resulting from the particle content of the Standard Model and the unexplained smallness of most Yukawa couplings. They are absent in generic extensions of the Standard Model (like its supersymmetric generalisations) making FCNC highly sensitive to new physics, probing scales in the range of 200 GeV to 100 TeV, depending on the model considered. This feature is a major motivation for the currently performed high-statistics experiments in flavour physics. Comparing different constraints on the UT from FCNC processes and the tree-level constraints discussed in Sect. 2 therefore provides a very powerful test of the Standard Model.

#### 3.1 Meson-antimeson mixing

\( K^{+}K^{-} \) mixing, \( B_{d}^{0} \rightarrow \bar{B}_{d}^{0} \) mixing and \( B_{s}^{0} \rightarrow \bar{B}_{s}^{0} \) mixing are all induced by box diagrams, depicted in Fig. 9. Each meson-antimeson system involves two mass eigenstates, their mass difference \( \Delta m \) measures the magnitude of the box diagram and therefore constrains magnitudes of CKM elements. The phase of box diagram and thereby the phases of the CKM elements involved are constrained through CP-violating quantities. Tab. 1 shows the relationship of the measurements to the CKM phenomenology. The quantities in the first two columns of Tab. 1 are well-measured and there is a lower bound on \( \Delta m_{B_{s}} \).
Table 1. Relationship of meson-antimeson mixing to CKM and UT parameters. $\beta = \arg V_{ts}^*$ is one of the UT angles in Fig. 1 and $\beta_s = \arg (V_{ts}) \approx \lambda^3 \pi$.

\[
\begin{array}{|c|c|c|c|}
\hline
& K - \bar{K} mixing & B_d - \bar{B}_d mixing & B_s - \bar{B}_s mixing \\
\hline
\text{CP-conserving quantity:} & \Delta m_K & \Delta m_{B_d} & \Delta m_{B_s} \\
\text{CKM information:} & |V_{cb} V_{cd}|^2 & |V_{tb} V_{td}|^2 & |V_{tb} V_{ts}|^2 \\
\text{UT constraint:} & \text{none} & R_\text{lt} = \sqrt{(1 - \eta^2)} & \text{none} \\
\hline
\text{CP-violating quantity:} & \epsilon_K & a_{\text{mix}}^{\text{CP}}(B_d \rightarrow J/\psi K_S) & a_{\text{mix}}^{\text{CP}}(B_s \rightarrow J/\psi \phi) \\
\text{CKM information:} & \text{Im}(V_{ts} V_{td}^*)^2 & \sin(2\beta) & \sin(2\beta_s) \\
\text{UT constraint:} & \bar{\eta}(1 - \eta) + \text{const.} & \bar{\eta} & \bar{\eta} \\
\hline
\end{array}
\]

3.2 $\epsilon_K$

While $\epsilon_K$, which quantifies indirect CP violation in $K \rightarrow \pi \pi$ decays, is measured at the percent level, its relationship to $\text{Im} V_{td}^2 \propto \bar{\eta}(1 - \eta)$ is clouded by hadronic uncertainties in the matrix element

\[
\langle K^0 | \bar{s}_{V - A} \bar{d}_{V - A} | \bar{K}^0 \rangle = \frac{8}{3} f_K^2 M_K^2 B_K.
\]

This defines the hadronic parameter $B_K$, which must be computed by non-perturbative methods like lattice QCD. $M_K$ and $f_K$ are the well-known mass and decay constant of the Kaon. This field has experienced a major breakthrough since LP03, since meanwhile fully unquenched computations with 2+1 dynamical staggered quarks are available. Using MILC configurations the HPQCD collaboration reports a new result\textsuperscript{35}

\[
B_K(\mu = 2 \text{ GeV}) = 0.618 \pm 0.018_{\text{stat}} \pm 0.019_{\text{chiral extrapolation}} \\
+ 0.030_{\text{discret}} \pm 0.130_{\text{pert. matching}} \\
= 0.618 \pm 0.136
\]  \hspace{1cm} (18)

in the $\overline{\text{MS}}$--NDR scheme. The conventionally used renormalisation scale and scheme independent parameter reads

\[
\bar{B}_K = 0.83 \pm 0.18 \hspace{1cm} (19)
\]

The uncertainty from the perturbative lattice–continuum matching dominates over the statistical error and the errors from chiral extrapolation and discretisation in Eq. (18). This matching calculation was performed in\textsuperscript{36}. The error in Eq. (18) is a conservative estimate of the unknown two-loop contributions to this matching. If one instead takes twice the square of the one-loop result of\textsuperscript{36} as an estimate of the uncertainty, one finds 0.036 instead of 0.130 in Eq. (18) and

\[
B_K(\mu = 2 \text{ GeV}) = 0.618 \pm 0.054 \\
\bar{B}_K = 0.83 \pm 0.07 \hspace{1cm} (20)
\]

$\epsilon_K$ fixes $\bar{\eta}(1 - \eta)$, so that it defines a hyperbola in the $(\bar{\eta}, \eta)$ plane.

3.3 $V_{td}$ from $B_d - \bar{B}_d$ mixing

The $B_d - \bar{B}_d$ mixing mixing amplitude involves the hadronic matrix element

\[
\langle B^0 | \bar{t}_{dV - A} \bar{d}_{V - A} | \bar{B}^0 \rangle = \frac{8}{3} M_{B_d}^2 f_{B_d}^2 B_{B_d}.
\]

Since the decay constant $f_{B_d}$ is not measured, the whole combination $f_{B_d}^2 B_{B_d}$ must be obtained from lattice QCD. The hadronic matrix element, however, cancels from the “gold-plated” mixing induced CP asymmetry $a_{\text{mix}}^{\text{CP}}(B_d \rightarrow J/\psi K_S)$, which determines $\beta = \arg V_{td}$ essentially without hadronic uncertainties. Combining all data from $b \rightarrow c \bar{s} s$ modes results in\textsuperscript{37,4}

\[
\sin(2\beta) = 0.69 \pm 0.03, \hspace{1cm} \cos(2\beta) > 0 \\
\Rightarrow \hspace{0.5cm} \arg (\pm V_{td}^*) = \beta = (21.8 \pm 1.2)\degree \hspace{1cm} (21)
\]

The precisely measured $\Delta m_{B_d} = 0.509 \pm 0.004 \text{ ps}^{-1}$ is proportional to $|V_{td}|^2 f_{B_d}^2 B_{B_d}$.
The HPQCD collaboration has computed $f_{B_d} = 216 \pm 22$ MeV with 2+1 dynamical staggered quarks. This measurement is discussed in detail in Ref. 7. Combining this with $B_{B_d}$ from older quenched calculations results in $f_{B_d}\sqrt{\hat{B}_{B_d}} = (246 \pm 27)$ MeV, where $\hat{B}_{B_d} = 1.52B_{B_d}(\mu = m_b)$ is the conventionally used scale and scheme independent variant of $B_{B_d}$. Then from $\Delta m_{B_d}$ alone we find

$$|V_{td}| = 0.0072 \pm 0.0008,$$

where the error is reduced by a factor of 2/3 compared to the old determination from quenched lattice QCD.

### 3.4 $|V_{td}|/|V_{ts}|$ from $B \to \bar{B}$ mixing

A measurement of the ratio $\Delta m_{B_d}/\Delta m_{B_s}$ will determine $|V_{td}|/|V_{ts}|$ via

$$\frac{|V_{td}|}{|V_{ts}|} = \sqrt{\frac{\Delta m_{B_d}}{\Delta m_{B_s}}} \sqrt{\frac{M_{B_s}}{M_{B_d}}} \xi$$

with the hadronic quantity $\xi = f_{B_s}\sqrt{\hat{B}_{B_s}}/(f_{B_d}\sqrt{\hat{B}_{B_d}})$ which equals $\xi = 1$ in the limit of exact SU(3)$_F$. A new unquenched HPQCD result for $f_{B_d}/f_{B_s}$ presented in Ref. 7 can be used to refine the prediction for $\xi$. The lower bound $\Delta m_{B_s} \geq 14.5$ ps$^{-1}$ implies $|V_{td}/V_{ts}| \leq 0.235$ which constrains one side of the unitarity triangle:

$$R_t \equiv \sqrt{(1 - \bar{\eta})^2 + \eta^2} = \left| \frac{V_{td}}{V_{ts}} \right| \leq 1.06$$

### 3.5 Global fit to the unitarity triangle

The result of a global fit of $(\bar{\eta}, \eta)$ to state-of-the-art summer-2005 data is shown in Fig. 10. It uses $\hat{B}_K = 0.85 \pm 0.02 \pm 0.07$ where the first error is Gaussian and the second is scanned over according to the standard CKMFitter method. For the remaining input see Ref. 8. The fit output is summarised in this table:

| quantity | central ± CL ≡ 1σ | ± CL ≡ 2σ |
|----------|---------------------|----------|
| $\rho$   | $0.204^{+0.035}_{-0.033}$ | $0.095$  |
| $\eta$   | $0.336^{+0.021}_{-0.021}$ | $0.045$  |
| $\alpha$ (deg) | $98.4^{+6.1}_{-5.6}$ | $16.8$ |
| $\beta$ (deg)  | $22.77^{+0.87}_{-0.83}$ | $1.92$ |
| $\gamma$ (deg) | $58.8^{+5.3}_{-5.8}$ | $11.2$ |
| $|V_{ub}|$ $[10^{-3}]$ | $3.90^{+0.12}_{-0.12}$ | $0.29$ |
| $|V_{td}|$ $[10^{-3}]$ | $8.38^{+0.32}_{-0.44}$ | $0.56$ |

The output of the global fit agrees well with the pure tree-level determinations in Eqs. (12) and (17) and Fig. 8.

We can use Eq. (1) to determine $\theta_{13} = 2.23^\circ \pm 0.007^\circ$ from the fitted $|V_{ub}|$ in the table. With Eq. (1) one finds $\theta_{12} = 12.9^\circ \pm 0.1^\circ$ from Eq. (10) and $\theta_{23} = 2.38^\circ \pm 0.03^\circ$ from Eq. (11). Since $1 - \cos\theta_{13}$ is negligibly small, the Wolfenstein parameters $\lambda$ and $A\lambda^2$ defined in Eq. (4) are simply given by $V_{us}$ in Eq. (10) and $V_{cb}$ in Eq. (11), respectively.

### 3.6 $K \to \pi \nu \bar{\nu}$

The rare decays $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ provide an excellent opportunity to determine the unitarity triangle from $s \to d$ transitions. With planned dedicated experiments $(\bar{\eta}, \eta)$ can be determined with a similar precision as today from $b \to d$ and $b \to u$ transitions at the B factories. This is a unique and very powerful probe of the CKM picture of FCNCs. $Br(K_L \to \pi^0 \nu \bar{\nu})$ is proportional to $\bar{\eta}^2$ and dominated by the top contribution. The theoretical uncertainty of the next-to-leading order (NLO) prediction is below 2%.$Br(K^+ \to \pi^+ \nu \bar{\nu})$ defines an ellipse in the $(\bar{\eta}, \eta)$ plane and has a sizeable charm contribution, which inflicts a larger theoretical uncertainty on the next-to-leading order (NLO) prediction, leading to $O(5 - 10\%)$ uncertainties in extracted CKM parameters. Parametric uncertainties from $V_{cb}$ and $m_t$ largely drop out, if $\sin(2\beta)$ is calculated from $Br(K_L \to \pi^0 \nu \bar{\nu})$ and $Br(K^+ \to \pi^0 \nu \bar{\nu})$. For Publisher’s use
\(\pi^+\nu\bar{\nu}\). Therefore the comparison of \(\sin(2\beta)\) determined in Eq. (21) from the B system with \(\sin(2\beta)\) inferred from \(K \to \pi\nu\bar{\nu}\) constitutes a pristine test of the Standard Model.\(^{42}\) The impact of a future 10% measurements of these rare decay modes on the UT is shown in Fig. 11.

The charm contribution is expanded in two parameters: \(m_{K}^2/m_{c}^2\) and \(\alpha_s(m_{c})\). The calculations of \(\mathcal{O}(m_{K}^4/m_{c}^2)\) corrections was recently completed, finding a 7% increase of \(Br(K^+ \to \pi^+\nu\bar{\nu})\) with a small residual uncertainty.\(^{43}\) A new result are the next-to-next-to-leading order (NNLO) QCD corrections to the charm contribution.\(^{44}\) This three-loop calculation reduces the theoretical error from unknown higher-order terms well below the parametric uncertainty from \(m_{c}\). The branching ratio is now predicted as

\[
Br(K^+ \to \pi^+\nu\bar{\nu}) = (8.0 \pm 1.1) \times 10^{-11}.
\]

At NNLO one finds the following reduced theoretical uncertainties for parameters extracted from \(Br(K_L \to \pi^0\nu\bar{\nu})\) and \(Br(K^+ \to \pi^+\nu\bar{\nu})\):\(^{44}\)

\[
\delta |V_{td}| = 0.010, \quad \delta \sin(2\beta) = 0.006, \quad \delta \gamma = 1.2^\circ
\]

4 CP violation in \(b \to s\) penguin decays

Within the Standard Model the mixing-induced CP asymmetries in \(b \to s\bar{q}q\) penguin amplitudes are proportional to \(\sin(2\beta)\) which equals \(\sin(2\beta)\) from Eq. (21) up to small corrections from a penguin loop with an up quark. In \(b \to s\bar{q}u\) decays there is also a color-suppressed tree amplitude. In any case the corrections are parametrically suppressed by \(|V_{ub}V_{us}/(V_{cb}V_{cs})| \sim 0.025\). The experimental situation is shown in Fig. 12. A naive average of the measurements of Fig. 12 gives

\[
\sin(2\beta)^{\text{eff}} = 0.51 \pm 0.06,
\]

which is below the value of \(\sin(2\beta)\) from tree-level \(b \to c\bar{s}s\) decays in Eq. (21) by 3σ. Moreover QCD factorisation finds a small and positive correction to \(\sin(2\beta_{\text{eff}})\) from up-
and in a unique way. b decays probes the CKM origin of FCNCs precisely. The current UT from the 2005 global fit is overlaid. The comparison of this UT constructed from s → d FCNCs with the UT found from b decays probes the CKM origin of FCNCs precisely and in a unique way.

Figure 11. Unitarity triangle from a future measurement of \( Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) \) and \( Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \) with 10% accuracy. The current UT from the 2005 global fit is overlaid. The comparison of this UT constructed from s → d FCNCs with the UT found from b decays probes the CKM origin of FCNCs precisely.

\[
\sin(2\beta_{\text{eff}})/\sin(2\phi_{\text{eff}})\]

Figure 12. \( \sin(2\beta_{\text{eff}})/\sin(2\phi_{\text{eff}}) \) from various penguin decays. The small vertical yellow band is \( \sin(2\beta) \) from Eq. (21).

While the significance of the deviation has decreased since the winter 2005 conferences, the mixing-induced CP asymmetries in \( b \rightarrow s \eta' \) decays stay interesting as they permit large effects from new physics. While in \( B_d \) decays the needed interference of a \( B_d \) and \( \bar{B}_d \) decay to the same final state requires a neutral K meson in the final state, \( b \rightarrow s \eta' \) decays of \( B_s \) mesons go to a flavourless \( \pi \eta' \) state, so that the desired CP effects can be studied in any final state. Hence \( B_s \) physics has the potential to become the “El Dorado” of \( b \rightarrow s \eta' \) penguin physics.

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DISCUSSION

Luca Silvestrini (Rome and Munich):
Maybe rather than saying that QCD cannot explain $\sin 2\beta_{\text{eff}}$ in $b \to s$ penguins, one should say that a particular model of power suppressed corrections due to Beneke and Co. cannot do it, but this is not a model-independent statement. If you just want to use data, and you say you do not know anything about power corrections, I do not think that you can infer anything from that plot.

Ulrich Nierste: The parametric suppression of the up-quark pollution by $|V_{ub}V_{us}/(V_{cb}V_{cs})| \sim 0.025$ is undisputed. Further the leading term in the $1/m_b$ expansion of $\sin 2\beta_{\text{eff}} - \sin(2\beta)$ can be reliably computed and results in the finding of Ref.45 that $\sin 2\beta_{\text{eff}} - \sin(2\beta)$ is small and positive for the measured modes. It is true that the size of the modeled power corrections is currently widely debated. Yet I am not aware of any possible dynamical QCD effect in two-body B decays which is formally $\mathcal{O}(1/m_b)$, large in magnitude and further comes with the large strong phase needed to flip the sign of $\sin 2\beta_{\text{eff}} - \sin(2\beta)$. 

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