Radiative Corrections in Vector-Tensor Models

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Abstract

We consider a two-form antisymmetric tensor field $\phi$ minimally coupled to a non-abelian vector field with a field strength $F$. Canonical analysis suggests that a pseudoscalar mass term $\mu^2 \frac{1}{2} \text{Tr}(\phi \wedge \phi)$ for the tensor field eliminates degrees of freedom associated with this field. Explicit one loop calculations show that an additional coupling $m \text{Tr}(\phi \wedge F)$ (which can be eliminated classically by a tensor field shift) reintroduces tensor field degrees of freedom. We attribute this to the lack of the renormalizability in our vector-tensor model. We also explore a vector-tensor model with a tensor field scalar mass term $\mu^2 \frac{1}{2} \text{Tr}(\phi \wedge *\phi)$ and coupling $m \text{Tr}(\phi \wedge \ast F)$. We comment on the Stueckelberg mechanism for mass generation in the Abelian version of the latter model.

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1 Introduction

We consider a model in which an antisymmetric tensor field $\phi^{a}_{\mu\nu}$ and a vector field $W_{a}^{\mu}$ interact, both through covariant derivatives and through direct coupling of $\phi^{a}_{\mu\nu}$ with the field strength $F_{\mu\nu}^{a}(W_{a}^{\lambda})$. A pseudoscalar mass term for the tensor field has been shown to eliminate degrees of freedom associated with this field [1, 2] and the consequences of this has been explored in the calculation of the vector and the tensor self-energy in [3, 4]. In section 2 we compute the the one-loop corrections to the mixed vector-tensor propagator and show that the direct coupling between $\phi^{a}_{\mu\nu}$ and $F_{\mu\nu}^{a}$ results in the breakdown of an identity derived in [4]. Thus, although tensor field degrees of freedom can be eliminated classically, they must reappear at one-loop level. We attribute this to the lack of renormalizability in our model.

In section 3 we discuss a slight variant of a vector-tensor model [3, 4]. Specifically, we replace the tensor field pseudoscalar coupling and the mass term with the scalar ones. For a generic tensor field mass $\kappa$ and a coupling $m$ the theory is non-renormalizable. When $\kappa^2 + 2m^2 = 0$ the $U(1)$ version of the model is renormalizable. It generalizes the Stueckelberg model for a massive vector boson to that of a massive rank-two tensor field. We point out that the generalized Stueckelberg invariance leads to non-local transformations on the coupled matter fields.

2 Anomalies in vector-tensor models with pseudoscalar tensor field mass term and $\text{Tr}(\phi \wedge F)$ coupling

The Lagrange density\footnote{We use mostly negative signature convention.}

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu,a}F^{\mu\nu,a} + \frac{1}{12}G_{\mu\nu\lambda,a}G^{\mu\nu\lambda,a} + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}\phi_{a}^{\mu\nu}F^{a\lambda\sigma} + \frac{\mu^2}{8}\epsilon_{\mu\nu\lambda\sigma}\phi_{a}^{\mu\nu}\phi^{a\lambda\sigma}$$

with

$$F_{\mu\nu}^{a} = \partial_{\mu}W_{a}^{\nu} - \partial_{\nu}W_{a}^{\mu} + gf^{abc}W_{a}^{\mu,b}W_{a}^{\nu,c}$$

$$G_{\mu\nu\lambda}^{a} = D_{\mu}^{ab}\phi_{a\lambda,b} + D_{\nu}^{ab}\phi_{a\mu,b} + D_{\lambda}^{ab}\phi_{a\mu,\nu}$$

$$D_{\mu}^{ab} = \delta^{ab}\partial_{\mu} + gf^{abc}W_{a}^{\mu,c}$$

for the vector field $W_{a}^{\mu}$ and the adjoint antisymmetric tensor field $\phi_{\mu\nu}^{a}$ was shown in [1] to have only two dynamical degrees of freedom (those of the transverse polarization of
the vector $W^a_\mu$) provided $\mu^2 \neq 0$. This is consistent with the results of [2] where the $m \to 0$, $g \to 0$, $U(1)$ limit of this Lagrange density was considered.

The Euclidean space propagators for these fields appeared in [3],

$$\langle W^a_\mu, W^b_\nu \rangle = \frac{\delta_{\mu\nu} \delta^{ab}}{k^2} \quad (2.3)$$

$$\langle \phi^a_{\alpha\beta}, \phi^b_{\gamma\delta} \rangle = \frac{\delta^{ab}}{\mu^4} \left( 1 + \frac{m^2}{k^2} \right) \left( \delta_{\alpha\gamma} k_\beta k_\delta - \delta_{\beta\gamma} k_\alpha k_\delta + \delta_{\beta\delta} k_\alpha k_\gamma - \delta_{\alpha\delta} k_\beta k_\gamma \right) - \frac{\delta^{ab} \epsilon_{\alpha\beta\gamma\delta}}{\mu^2} \quad (2.4)$$

$$\langle W^a_\mu, \phi^b_{\alpha\beta} \rangle = - \frac{i m \delta^{ab}}{\mu^2 k^2} (\delta_{\alpha\mu} k_\beta - \delta_{\beta\mu} k_\alpha) \quad (2.5)$$

where the Feynman-'t Hooft gauge (with the gauge fixing term $L_{gf} = -\frac{1}{2} (\partial_\alpha W^a_\alpha)^2$) was used.

The classical analysis showed in [1, 2] that the tensor field has no dynamical degrees of freedom. Explicit computation [3] of the one loop radiative corrections to the two point function $\langle W^a_\mu, W^b_\nu \rangle$ leads to a cancellation of all contributions from the diagrams containing the tensor field (i.e., the pure gauge theory result is recovered).

One can also see from (2.4) that if $m = 0$, then the tensor propagator has no poles. This is consistent with the tensor having no physical degrees of freedom when $\mu \neq 0$. It also implies that all radiative corrections vanish in the limit that there is no vector field even if some self interactions for the tensor field such as $(\phi^a_{\mu\nu} \phi^a_{\mu\nu})^2$ or $(\phi^a_{\mu\nu} \phi^a_{\alpha\beta} \phi^b_{\lambda\sigma} \phi^b_{\mu\sigma})$ were present. This is most easily seen when one uses dimensional regularization, as in this case we only encounter integrals of the form $\int d^n k f(k)$ where $f$ is a polynomial function; such tadpoles are regulated to zero.

However, if $m \neq 0$ a pole does appear in the propagator of (2.4). This would appear to render the theory non-renormalizable as the asymptotic behavior of this propagator for large value of $k^2$ is that it grows as $k^2$. However, as pointed out in [4], the shift

$$\phi^a_{\mu\nu} = \chi^a_{\mu\nu} - \frac{m}{\mu^2} F^a_{\mu\nu} \quad (2.6)$$

in (2.1) leads to (using $\epsilon^{\mu\nu\lambda\sigma} D^{ab}_{\nu\lambda} F^{b}_{\lambda\sigma} = 0$)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu,a} F^{\mu\nu,a} + \frac{1}{12} H_{\mu\nu\lambda,a} H^{\mu\nu\lambda,a} + \frac{\mu^2}{8} \epsilon^{\mu\nu\lambda\sigma} \chi^a_{\mu\nu} \chi^a_{\lambda\sigma} + \frac{m^2}{8\mu^2} \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu,a} F^{\lambda\sigma,a} \quad (2.7)$$

where $H^a_{\mu\nu\lambda}$ is defined in terms of $\chi^a_{\mu\nu}$ and $W^a_\lambda$ in the same way that $G^a_{\mu\nu\lambda}$ is defined in terms of $\phi^a_{\mu\nu}$ and $W^a_\lambda$. Since the last term in (2.7) is topological, we see that it cannot
contribute to a perturbative calculation; the coupling $m$ in (2.1) has been removed and hence there is no pole in the propagator $\langle \chi_{\mu\nu}^a \chi_{\lambda\sigma}^b \rangle$.

The Feynman rules for the model defined by (2.7) are thus the same as those of the model of (2.1) in the limit $m \to 0$. It is immediately apparent then that the one loop corrections to the two point functions $\langle \chi_{\mu\nu}^a \chi_{\lambda\sigma}^b \rangle$, $\langle W_{\mu}^a \chi_{\lambda\sigma}^b \rangle$, $\langle \chi_{\mu\nu}^a W_{\lambda}^b \rangle$ all vanish.

By (2.6), we see then that

$$\langle \phi_{\mu\nu}^a \phi_{\lambda\sigma}^b \rangle = \langle \chi_{\mu\nu}^a \chi_{\lambda\sigma}^b \rangle - \frac{m^2}{\mu^2} \left( \langle F_{\mu\nu}^a \chi_{\lambda\sigma}^b \rangle + \langle \chi_{\mu\nu}^a F_{\lambda\sigma}^b \rangle \right)$$

(2.8)

or

$$\langle \chi_{\mu\nu}^a \chi_{\lambda\sigma}^b \rangle = \langle \phi_{\mu\nu}^a \phi_{\lambda\sigma}^b \rangle + \frac{m^2}{\mu^2} \left( \langle F_{\mu\nu}^a \phi_{\lambda\sigma}^b \rangle + \langle \phi_{\mu\nu}^a F_{\lambda\sigma}^b \rangle \right)$$

(2.9)

In (2.8), the only non-zero term on the right side of the equation is the last one; this leads to

$$\langle \phi_{\mu\nu}^a \phi_{\lambda\sigma}^b \rangle = \frac{m^2}{\mu^4} \langle F_{\mu\nu}^a F_{\lambda\sigma}^b \rangle$$

(2.10)

Eq. (2.9) then would result in

$$\langle F_{\mu\nu}^a \phi_{\lambda\sigma}^b \rangle + \langle \phi_{\mu\nu}^a F_{\lambda\sigma}^b \rangle = -\frac{2m}{\mu^2} \langle F_{\mu\nu}^a F_{\lambda\sigma}^b \rangle$$

(2.11)

However, the explicit computation of the 30 one loop diagrams contributing to $\langle \phi_{\mu\nu}^a \phi_{\lambda\sigma}^b \rangle$ in (2.10) leads to a pole contribution at $\epsilon = 2 - \frac{n}{2} = 0$ [4]

$$\frac{m^2}{48p^2 \mu^4} \left( \frac{g^2 C_2 \delta_{ab}}{\pi^2 \epsilon} \right) \left[ (12p^4 + 5p^2 m^2 + 14 \mu^4) (p_{\mu} p_{\lambda} \delta_{\nu\sigma} - p_{\nu} p_{\lambda} \delta_{\mu\sigma} + p_{\nu} p_{\sigma} \delta_{\mu\lambda} \\
- p_{\nu} p_{\sigma} \delta_{\mu\lambda}) - 6 \mu^2 p^4 \epsilon_{\mu\nu\lambda\sigma} + 3 \mu^2 \mu^4 (\delta_{\mu\lambda} \delta_{\nu\sigma} - \delta_{\mu\sigma} \delta_{\nu\lambda}) \right]$$

(2.12)

Not only is this result inconsistent with renormalizability, but also it is clear that with $\langle F_{\mu\nu}^a F_{\lambda\sigma}^b \rangle$ being just the usual (transverse) Yang-Mills result, (2.10) and (2.12) are incompatible.

This has motivated the computation of the mixed propagator $\langle F_{\mu\nu}^a \phi_{\lambda\sigma}^b \rangle + \langle \phi_{\mu\nu}^a F_{\lambda\sigma}^b \rangle$ at one loop order. The two point contribution to these diagrams comes from $\langle \phi_{\mu\nu}^a W_{\lambda}^b \rangle$ which has 30 diagrams contribution at one loop order. The sum contains the pole term

$$\frac{19}{96 \pi^2} \left( \frac{iC_2 mg^2}{p^2 \mu^2 \epsilon} \right) \left[ p_\beta \delta_{\alpha\lambda} - p_\alpha \delta_{\beta\lambda} \right] \delta^{ab}$$

(2.13)

This is not in accordance with (2.11).
We are thus forced to conclude that the contact term \( \frac{m}{4} \phi_{\mu\nu}^a F^{\mu\nu,a} \) in (2.1) destroys the renormalizability of the theory and thus the identities of (2.10) and (2.11) do not hold beyond the classical level. This is consistent with lack of renormalizability found when \( m \neq 0 \) in the model in which there are the interactions

\[
\gamma \bar{\psi}^\mu \gamma^a W_{\mu,a} \psi + h \bar{\psi} \sigma^{\mu\nu} \gamma^a \phi_{\mu\nu,a} \psi
\]

with a spinor field \( \psi \) [3].

We also note that if \( m = 0 \) and a scalar mass term \( \frac{\kappa^2}{2} \phi_{\mu\nu} \phi^{\mu\nu,a} \) is added to the Lagrangian of (2.1), then the propagator for \( \phi_{\mu\nu}^a \) is inconsistent with renormalizability for all values of \( \mu^2 \).

**3 Vector-tensor models with tensor field scalar mass and \( \text{Tr}(\phi \wedge F) \) coupling**

It is also worth considering a scalar mass for the tensor field and a direct scalar coupling between the tensor and the vector field strength. In this case we have [5]

\[
L = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu} + \frac{1}{12} G_{\mu\nu\lambda} G^{\mu\nu\lambda} + \frac{\kappa^2}{2} \phi_{\mu\nu} \phi^{\mu\nu} + m \phi_{\mu\nu} F^{\mu\nu,a}(3.1)
\]

Again, using the gauge fixing \( L_{gf} = -\frac{1}{2} (\partial_{\alpha} W^\alpha)^2 \), we find that the tensor propagator is

\[
-\frac{4I}{\partial^2 - \kappa^2} + \frac{4(\partial^2 + 4m^2)Q}{\partial^2(\partial^2 - \kappa^2)(\kappa^2 + 2m^2)}(3.2)
\]

where

\[
I_{\alpha\beta\gamma\delta} = \frac{1}{2} (\delta_{\alpha\gamma} \delta_{\beta\delta} - \delta_{\alpha\delta} \delta_{\beta\gamma}),
\]

\[
Q_{\alpha\beta\gamma\delta} = \frac{1}{4} (\delta_{\alpha\gamma} \partial_{\beta}\partial_{\delta} - \delta_{\alpha\delta} \partial_{\beta}\partial_{\gamma} + \delta_{\beta\delta} \partial_{\alpha}\partial_{\gamma} - \delta_{\beta\gamma} \partial_{\alpha}\partial_{\delta}) \quad (3.3)
\]

and the vector propagator is

\[
\frac{2\delta_{\mu\nu}}{\partial^2} + \frac{4m^2(\partial_{\mu}\partial_{\nu} - \partial^2 \delta_{\mu\nu})}{\partial^4(\kappa^2 + 2m^2)} \quad (3.4)
\]

The second term in (3.2) indicates that the model of (3.1) is not renormalizable. However, we also note that if \( \kappa^2 + 2m^2 = 0 \) the propagators of (3.2) and (3.4) are ill defined.
If we set $\kappa^2 = -2m^2$ in (3.1), $L$ reduces to

$$L = \frac{1}{12} G^a_{\mu\nu} G^a_{\mu\nu} - \frac{m^2}{2} \left( \phi^a_{\mu\nu} - \frac{1}{2m} F^a_{\mu\nu} \right)^2$$  \hspace{1cm} (3.5)$$

In the $U(1)$ limit (which serves to pick out those terms in (3.5) that are bilinear in the fields) (3.5) is invariant under the transformation

\begin{align*}
\phi_{\mu\nu} & \rightarrow \phi_{\mu\nu} + \partial_\mu \Theta_\nu - \partial_\nu \Theta_\mu \\
W_\mu & \rightarrow W_\mu + 2m \Theta_\mu 
\end{align*} \hspace{1cm} (3.6)

This gauge invariance is analogous to that of the Stueckelberg model for a massive $U(1)$ vector boson where [6]

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{m^2}{2} \left( W_\mu - \frac{1}{m} \partial_\mu \sigma \right)^2$$ \hspace{1cm} (3.7)$$

which is invariant under the transformations

\begin{align*}
W_\mu & \rightarrow W_\mu + \partial_\mu \Theta \\
\sigma & \rightarrow \sigma + m \Theta 
\end{align*} \hspace{1cm} (3.8) \hspace{1cm} (3.9)

The gauge fixing

$$L_{gf} = -\frac{1}{2\alpha} (\partial^\mu \phi_{\mu\nu} - 2\alpha m W_\mu)^2$$ \hspace{1cm} (3.10)$$

when added to $L$ in (3.1) serves to decouple the fields $\phi_{\mu\nu}$ and $W_\mu$. The propagators for both $\phi_{\mu\nu}$ and $W_\mu$ are consistent with renormalizability. However, coupling with matter fields must be consistent with the invariance of (3.6). The vector field $W_\mu$ can be coupled to a matter field $\psi$ (a spinor or scalar) if we use covariant derivative

$$D_\mu \psi = (\partial_\mu + i W_\mu) \psi$$ \hspace{1cm} (3.11)$$

provided $\psi$ undergoes the non-local transformation

$$\psi(x) \rightarrow \exp \left( -2im \int_{x_0}^x dy^\mu \Theta_\mu(y) \right) \psi(x)$$ \hspace{1cm} (3.12)$$

when $W_\mu$ undergoes the transformation of (3.6). Note that if $\Theta_\mu = \frac{1}{2m} \partial_\mu \omega$ (3.12) reduces to the standard gauge transformation of a matter field. The propagator for the field $W_\mu$ is

$$\left[ (\partial^2 + 4\alpha m^2) \delta_{\mu\nu} - \partial_\mu \partial_\nu \right]^{-1} = \frac{\delta_{\mu\nu}}{\partial^2 + 4\alpha m^2} + \frac{\partial_\mu \partial_\nu}{4\alpha m^2 (\partial^2 + 4\alpha m^2)}$$ \hspace{1cm} (3.13)$$
which indicates the presence of a gauge dependent non zero pole in the propagator for \( W_\mu \).

There does not appear to be a way of coupling either the tensor \( \phi_{\mu\nu} \) of (3.5) or the Stueckelberg scalar \( \sigma \) of (3.7) to matter that respect the gauge invariance of (3.6) and (3.9) correspondingly. However, the invariance of (3.6) and (3.8) can be respected when the \( U(1) \) vector field \( W_\mu \) is coupled to matter; this possibility has been incorporated into the Standard Model [7, 8].

The invariance of (3.6) has no non-Abelian extension. However, \( L \) in (3.1) can be replaced by

\[
L = \frac{1}{12} \left[ G_{\mu\nu\lambda}^a + g f^{abc} (F_{\mu\nu,b}^a \sigma_{\lambda,c}^a + F_{\nu\lambda,b}^a \sigma_{\mu,c}^a + F_{\lambda\mu,b}^a \sigma_{\nu,c}^a) \right]^2
- m^2 \left[ \phi_{\mu\nu}^a + (D_{\mu}^{ab} \sigma_{\nu,b}^a - D_{\nu}^{ab} \sigma_{\mu,b}^a) \right]^2
\]

(3.14)

where \( G_{\mu\nu\lambda}^a, F_{\mu\nu}^a \) and \( D_{\mu}^{ab} \) are as in (2.2). Eq. (3.14) is invariant under the usual Yang-Mills gauge transformations as well as the transformations

\[
\begin{align*}
\phi_{\mu\nu}^a &\rightarrow D_{\mu}^{ab} \Theta_{\nu,b}^a - D_{\nu}^{ab} \Theta_{\mu,b}^a \\
\sigma_{\mu}^a &\rightarrow \sigma_{\mu}^a - \Theta_{\mu}^a
\end{align*}
\]

(3.15)

which serves to generalize the transformations of (3.6). The vertices arising in (3.14) are unfortunately not consistent with renormalizability.

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