Bayesian Analysis of Value-at-Risk with Product Partition Models

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Extended Abstract

Value-at-Risk (VaR) measures the maximum potential loss of single assets or portfolio of assets, once a given confidence level and a time horizon have been fixed. VaR has become the standard measure for financial analysts to quantify market risk and it is also important for regulatory purposes. In particular Basel accords impose that all financial institutions have to meet capital requirements based on VaR estimates. Different methodologies have been proposed for an accurate estimation of VaR both in classical and Bayesian framework. If the assets returns follow a normal distribution, a closed-form and easy to implement expression for VaR can be used. Unfortunately this assumption fails to be effective for low liquidity markets and short time horizons and has to be relaxed. One possible solution is to resort to heavy tailed distributions or to abandon the hypothesis of identically distributed returns. In this paper we follow the latter approach and we use a Bayesian methodology based on parametric Product Partition Models (PPM). The main idea is to identify clusters of identically distributed returns. We assume that returns follow a normal distribution with a partition structure on the parameters. We assign a prior distribution on the
space of all possible partitions and consequently we identify cluster of returns sharing the same mean and variance values. We present and compare different scenarios. Firstly we consider a partition structure on the mean vector. This approach turns out to be quite effective for VaR and as a byproduct it provides a useful tool for outliers identification. However, it is very sensitive to the values of some prior parameters. Even a hierarchical prior can not reduce this sensitivity, as we will show. The only solution is to fix these values according to analysts’ experience about market behaviour. This drawback effect is strongly reduced by imposing a partition structure on the vector of variances. We test our methodologies to a Mib30 data set of daily returns from April 2004 to March 2008. We select the four components with the highest excess of kurtosis where the normal model usually fails. We compare our results with those obtained via standard classical approaches based on ML estimators. We find a good agreement with results based on Student-t distributions but, by means of Bayesian PPM, we also provide information about the clustering structure of the data and, eventually, about the presence of outliers.

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