DISK ACCRETION ONTO MAGNETIZED NEUTRON STARS: THE INNER DISK RADIUS AND FASTNESS PARAMETER

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ABSTRACT

It is well known that the accretion disk around a magnetized compact star can penetrate inside the magnetospheric boundary, so that the magnetospheric radius \( R_0 \) does not represent the true inner edge \( R_{in} \) of the disk, but controversies exist in the literature concerning the relation between \( R_0 \) and \( R_{in} \). In the model of Ghosh & Lamb, the width of the boundary layer is given by \( \delta = R_0 - R_{in} \ll R_0 \), or \( R_{in} \approx R_0 \), while Li & Wickramasinghe recently argued that \( R_{in} \) could be significantly smaller than \( R_0 \) in the case of a slow rotator. Here we show that if the star is able to absorb the angular momentum of the disk plasma at \( R_0 \) appropriate for binary X-ray pulsars, the inner disk radius can be constrained by 0.8 \( \leq R_{in}/R_0 \leq 1 \), and the star reaches spin equilibrium with a relatively large value of the fastness parameter (\( \sim 0.7 - 0.95 \)). For accreting neutron stars in low-mass X-ray binaries (LMXBs), \( R_0 \) is generally close to the stellar radius, \( R_\star \), so that the toroidal field cannot transfer the spin-up torque efficiently to the star. In this case, the critical fastness parameter becomes smaller, but \( R_{in} \) is still near \( R_0 \).

Subject headings: accretion, accretion disks — stars: magnetic fields — stars: neutron — stars: rotation

1. INTRODUCTION

The interaction between a magnetized, rotating star and a surrounding accretion disk has been of continued interest since the discovery of X-ray pulsars in binary systems (e.g., Giacconi et al. 1971). Its solution may hold the key to understanding fundamental problems such as the structure and dynamics of magnetized accretion disks, angular momentum exchange between the disk and the star, etc., in a variety of astrophysical systems, including cataclysmic variables, X-ray binaries, T Tauri stars, and active galactic nuclei. A detailed model for the disk-star interaction was first developed by Ghosh & Lamb (1979a, 1979b), who argued that the stellar magnetic field penetrates the disk via Kelvin-Helmholtz instability, turbulent diffusion and reconnection, producing a broad transition zone joining the unperturbed disk flow far from the star to the magnetospheric flow near the star. The transition zone is composed of two qualitatively different regions: a broad outer zone, where the angular velocity is nearly Keplerian, and a narrow inner zone or boundary layer, where it departs significantly from the Keplerian value. The total torque \( N \) exerted on a star with mass \( M \) is then divided into two separate components:

\[
N = N_{in} + N_{out}, \tag{1}
\]

where the subscript represents the appropriate zone in the disk. The “material” torque of the (inner) accretion flow is given by

\[
N_{in} \approx N_0 = M R_0^3 \Omega_{\phi 0}, \tag{2}
\]

where \( \Omega_{\phi 0} \equiv (GM/R_0^3) \) is the angular velocity of the disk plasma at the magnetospheric radius \( R_0 \) (where \( G \) is the gravitational constant), and \( M \) is the accretion rate. Here we have adopted cylindrical coordinates \( (R, \phi, z) \) centered on the star, and the disk is assumed to be located on the \( z = 0 \) plane, which is perpendicular to the star’s spin and magnetic axes. The torque in the outer zone, supplied by the Maxwell stress, is given by

\[
N_{out} = - \int_{R_0}^{\infty} B_\phi B_z R^2 \, dR, \tag{3}
\]

where \( B_\phi \) is the azimuthal field component generated by shear motion between the disk and the vertical field component \( B_z \). The torque \( N_{out} \) can be positive or negative, depending on the value of the fastness parameter \( \omega_A \equiv \Omega/\Omega_{\phi 0} = (R_0/R_A)^{3/2} \), where \( \Omega \) is the angular velocity of the star and \( R_A \equiv (GM/\Omega^2)^{1/3} \) is the corotation radius.

Inside the magnetospheric boundary, the magnetic stress dominates the rotation of disk plasma and forces it to corotate with the magnetosphere at the radius \( R_{in} \). To understand how far the disk can extend into the stellar magnetosphere and how the torque is transferred to the star, it is necessary to obtain solutions for the structure of the magnetosphere. Ghosh & Lamb (1979a) suggested that \( R_0 \) and \( \delta \) be constrained by the relation

\[
(R_B, B_\phi/4\pi \rho_0) 2\pi R_0 \, 2\delta \approx M(\Delta v\rho_0) \tag{4}
\]

(where the subscript “0” denotes quantities evaluated at \( R = R_0 \)), which follows the equation of angular momentum conservation, although the contribution of gas funnel flow to the angular momentum transfer has not been included. Ghosh & Lamb (1979a) also assumed a priori that \( \delta \) corresponds to the electromagnetic screening length, and is much smaller than \( R_0 \). To demonstrate the self-consistency of this assumption, from equation (4) we derive \( \delta \approx (R_0/R_A)^{1/2} R_0 \) when \( \omega_A \ll 1 \), where \( R_A \) is the conventional Alfvén radius in spherical accretion. If \( R_0 \approx 0.5 R_A \), as obtained by Ghosh & Lamb (1979a), then indeed \( \delta \ll R_0 \). However, if \( R_0 \approx R_\star \), as suggested by some recent investigations (e.g., Arons 1993; Ostriker & Shu 1995; Wang 1996; Li 1997), \( \delta \) will be as large as \( R_0 \).

\footnote{According to Spruit & Taam (1990), the accreting gas, nearly corotating with the star, could drift farther inward across the field through an interchange instability.}
Li, Wickramasinghe, & Rüdiger (1996) have presented a class of funnel flow solutions in which the angular momentum is carried by the matter. These authors argue that for an ideally conducting star, the stellar boundary condition requires that the toroidal field be very small, and most of the angular momentum of the matter in the funnel be propagated back to the disk rather to the star. Li & Wickramasinghe (1997) show that one consequence of this result is that the true inner disk radius $R_0$ could be much smaller than $R_0$, with $R_{in}/R_0$ as low as ~0.1–0.2 in slow-rotator case, and the values of the critical fastness parameter at which the net torque vanishes would also change. This “torquess” suggestion was criticized by Wang (1997), who points out that a small toroidal field is enough to produce a significant torque on the star, and that the viscous stress is unable to remove angular momentum back outward from the inner edge of the accretion disk.

This paper is organized as follows. In § 2, based on Wang’s (1997) arguments, we calculate the lower limit of $R_{in}$ that is required for efficient angular momentum transfer, and find it to be at least ~0.8R_0, implying that $R_0$ is a good indicator of the inner edge of the disk. In § 3 we discuss the possibility of inefficient angular momentum transfer, which is likely to occur in LMXBs, and the implications for the resulting variation in the fastness parameter. We conclude in § 4.

2. THE INNER DISK RADIUS

An estimate of $R_0$ can be obtained by setting the magnetic stress to be equal to the rate at which angular momentum is removed in the disk (Wang 1987):

$$B_\phi^2 B_z^2 \frac{M}{(GM R_0)^{1/2}} = \frac{2}{R_0}, \tag{5}$$

where $\Omega = \Omega(R)$ is the angular velocity of disk plasma. Following Li & Wang (1996), we assume that $\Omega(R)$ reaches its maximum at $R = R_0$, i.e., $(d\Omega/dR)_0 = 0$, with $\Omega(R_0) \approx \Omega_{in}$, and we rewrite equation (5) as

$$\frac{B_\phi}{B_z} \frac{R_0}{R_0} = \frac{2}{R_0}. \tag{6}$$

The torque $N_{in}$ in equation (1) in fact contains two components: the real material torque $N_f$ in the funnel flow, and the magnetohydrostatic torque $N_{mag}$ arising from the shearing motion between the corotating magnetosphere and the non-Keplerian disk boundary layer, i.e.,

$$N_{in} = N_f + N_{mag} = \int_{R_0}^{R_{in}} \frac{dM}{dR} R^2 \Omega dR - \int_{R_0}^{R_{in}} B_\phi B_z R^2 dR, \tag{7}$$

where $k = R_0/R_0$.

We first consider the second term on the right-hand side of equation (7). In estimating the toroidal magnetic filed $B_\phi$, we assume as usual that $B_\phi$ is generated by the shearing between the disk and the stellar magnetosphere, and that the growth of $B_\phi$ is limited by diffusive decay produced by turbulent mixing within the disk (e.g., Campbell 1992; Yi 1995; Wang 1995),

$$\frac{B_\phi}{B_z} \propto \frac{\Omega - \Omega}{\Omega}. \tag{8}$$

Several different phenomenological descriptions of disk-magnetosphere interaction (Wang 1995) make little practical difference in our conclusions. Assuming that $B_z$ follows a dipolar field and using equations (6) and (8), we can write the magnetospheric spin-up torque in dimensionless form as

$$\xi_{mag} = \frac{N_{mag}}{N_0} = \frac{4\Omega \Omega}{3(1 - \Omega)} \left[ 1 - \left( \Omega/R_0 \right) \right]^{3/2} dy, \tag{9}$$

where $y = (R/R_0)^{3/2}$.

In the first term on the right-hand side of equation (7), $dM/dR$ denotes the vertical mass loss rate from the boundary layer. This can be described by the equation (e.g., Ghosh & Lamb 1979a)

$$\frac{dM}{dR} = 4\pi R \rho c_g g(R), \tag{10}$$

scaling the vertical flow velocity from the boundary layer in terms of the local sound speed $c_g$, and introducing a “gate” function $g(R)$ that describes the radial profile of mass loss out of the boundary layer (where $\rho$ is the mass density). To keep things simple, we assume $g(R) = 0$ outside $R_0$, and $g(R) = 1$ when $R_0 \leq R \leq R_0$. Assuming that the disk plasma around $R_0$ is thermally supported and optically thick to Thomson scattering (appropriate for a binary X-ray pulsar), from the $x$-disk model of Shakura & Sunyaev (1973) we obtain $dM/dR \propto R^{-1}$, or $dM/dR = M_{in} \left( R/R_0 \right)$, which leads to

$$\xi_f = \frac{N_f}{N_0} = \left( \log k \right) \frac{R_{in}}{R_0} \int_{R_0}^{R_{in}} \frac{\Omega}{\Omega_{in}} \frac{R}{R_{in}} dR. \tag{11}$$

For generality, we have also assumed $dM/dR = const.$, and we find that the results do not change significantly.

The value of $k$ can be obtained by setting $\xi + \xi_{mag} = 1$. From equations (9) and (11), it can be seen that the magnitude of $k$ depends on the detailed profile of $\Omega(R)$ between $R_{in}$ and $R_0$, which is unknown before solving the magnetohydrodynamic (MHD) equations. A lower limit of $k$, however, can be estimated in the following way. Because of the steep radial dependence of the stellar magnetic field ($B_z \propto R^{-1}$), the slope of the angular velocity $d\Omega/dR$ should increase from 0 at $R_0$ to a large value at $R_{in}$. This requires that in the $\Omega(R)$ versus $R$ diagram (Fig. 1a), $\Omega(R)$ should always lie beyond the dashed line that represents a linear relation between $\Omega$ and $R$. As we know from equations (9) and (11), the larger the value of $\Omega$, the larger the $\xi$, and the smaller the $\delta$ required. Therefore, the smallest $k$ corresponds to the extreme case in which $\Omega(R)$ decreases linearly from $\Omega_{in}$ to $\Omega$ between $R_0$ and $R_{in}$, i.e.,

$$\Omega(R) = aR + b, \tag{12}$$

where

$$a = \frac{\Omega_{in} (1 - \omega_x)}{R_0} \left( \frac{1}{1 - k} \right), \tag{13}$$

and

$$b = \Omega_{in} \left( \frac{\omega_x - k}{1 - k} \right) \tag{14}$$

In Figure 1b, we plot the calculated values of $k$ as a function of the fastness parameter $\omega_x$ in the dashed curve. It can be seen that $k$ naturally reaches 1 when $\omega_x = 1$, but remains ~0.8 within a large range of $\omega_x$. The solid curves in
Figures 1a and 1b represent an example with more practical boundary conditions, which are always located above the dashed curves.

3. THE FASTNESS PARAMETER

Our calculations in the previous section are based on the assumption that a torque of the order of $MR_s^2\Omega_{\kappa_0}$ can be efficiently transmitted to the star by both the gas funnel flow and the magnetic stress. It has been suggested (Shu et al. 1994; Ostriker & Shu 1995; Li, Wickramasinghe, & Rüdiger 1996) that a star could accrete matter from a magnetically truncated Keplerian disk without experiencing any spin-up torque, because this would require the surface magnetic field to have a large azimuthal twist, which would lead to dynamical instabilities. Wang (1997) estimated the azimuthal pitch $\gamma_*$ at the stellar surface required to transmit this torque to be

$$\gamma_* = \frac{B_{0\theta}}{B_{2\theta}} \approx \frac{\xi}{2^{3/2}} \left( \frac{R_s}{R_0} \right)^{3/2} \frac{R_0}{\delta}, \quad (15)$$

where the subscript asterisk denotes quantities at the stellar surface. Since MHD instabilities are expected to occur only when $|\gamma_*| \gtrsim 1$, one sees that, with $\delta \lesssim 0.2R_0$ and $\xi \approx 1$, $\gamma_*$ could be much less than unity if $R_s \ll R_0$. This condition is satisfied by binary X-ray pulsars, which generally possess surface magnetic fields as strong as $10^{12}$–$10^{13}$ G, and hence $R_0 \sim 10^9$–$10^{10}$ cm, with mass accretion rates of $10^{16}$–$10^{18}$ g s$^{-1}$, much larger than $R_s \sim 10^6$ cm. Thus, the standard accretion disk models based on Ghosh & Lamb (1979a, 1979b) are suitable for these systems, and the value of the critical fastness parameter $\alpha_s$, according to Wang (1995) and Li & Wang (1996), may lie between $0.7$ and $0.95$.

There is the possibility of an accreting star that is unable to absorb the excess angular momentum at $R_0$. This may occur in accreting neutron stars in LMXBs, in which the condition $|\gamma_*| \ll 1$ may not hold unless $\xi \sim 0$, because the surface field strengths in these neutron stars are so low ($10^8$–$10^9$ G or lower) that the accretion disk usually extends close to the star’s surface, i.e., $R_0 \sim R_s$. Since MHD instabilities prevent transmission of the torque of $N_{\text{in}}$, in this case the star only experiences the magnetic torque $N_{\text{out}}$, and the critical fastness parameter decreases to $\sim 0.4$–$0.6$ (Li & Wickramasinghe 1997). Note that the angular momentum of the material at $R_0$ cannot be transported from the boundary layer to the outer parts of the disk by either viscous stress or a stellar magnetic field threading the inner region of the disk (Wang 1997); it is more likely to be removed by a magnetocentrifugal wind. During wind mass loss, some angular momentum of the star could also be lost, equivalent to an extra braking torque exerted on the star, so that the critical fastness parameter would be even smaller.

Observational evidence for the above arguments comes from kilohertz quasi-periodic oscillations (kHz QPOs) recently discovered with the *Rossi X-Ray Timing Explorer* (RXTE) in some 16 neutron star LMXBs (see van der Klis 1997 for a review). These kHz QPOs are characterized by their high levels of coherence (with quality factors $Q \equiv \nu/\Delta \nu$ up to $\sim 200$), large rms amplitudes (up to several 10%), and wide span of frequencies ($\sim 300$–$1200$ Hz), which in most cases is strongly correlated with X-ray fluxes. In many cases, two simultaneous kHz peaks are observed in the power spectra of the X-ray count-rate variations, with the separation frequency being roughly constant (e.g., Strohmayer et al. 1996; Ford et al. 1997; Wijnands et al. 1998). Sometimes a third kHz peak is detected in a few atoll sources during type I X-ray bursts at a frequency equal to the separation frequency of the two peaks (Strohmayer et al. 1996; Ford et al. 1997; Wijnands et al. 1996; Ford et al. 1997) or twice that (Wijnands & van der Klis 1997; Smith, Morgan, & Bradt 1997). This strongly suggests a beat-frequency interpretation, with the third peak at the neutron star spin frequency (or twice that), the upper kilohertz peak at the Keplerian orbital frequency at a preferred radius around the neutron star, and the lower kilohertz peak at the difference frequency between them.\(^2\) If the upper-kilohertz QPOs are associated with the orbital frequency at the magnetospheric radius or the sonic point in the accretion disk (Strohmayer et al. 1996; Miller, Lamb, &
Psaltis 1998; see also Lai 1998), one would expect that the disk could extend toward the marginally stable orbit of a neutron star.

As pointed out by Zhang, Strohmayer, & Swank (1997) and White & Zhang (1997), these neutron stars may have accreted a considerable amount of mass, and so possibly have reached equilibrium spin. The measured spin and QPO frequencies of these neutron stars reveal a critical fastness parameter $\alpha_c$ lying between ~0.2 and 0.7. In addition, considering the spin and orbital evolution in low-mass binary pulsars that are thought to originate from LMXBs, Burderi, King, & Wynn (1996) also found a small value of $\alpha_c \sim 0.1$ from the observed relation between spin period, magnetic field, and orbital period. Although subject to both observational and theoretical uncertainties, these results do indicate a smaller fastness parameter than predicted by current theory for binary X-ray pulsars.

4. CONCLUSIONS

We summarize our results as follows. (1) The inner disk radius $R_{in}$ is always close to the magnetospheric radius $R_\text{m}$. (2) For binary X-ray pulsars, the critical fastness parameter lies in the range $0.71 \sim 0.95$; neutron stars in LMXBs may not absorb angular momentum flux at $R_\text{in}$ as efficiently as X-ray pulsars, resulting in a critical fastness parameter considerably smaller than in X-ray pulsars.

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