Thermal Evolution of a Dual Scale Cosmology

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Abstract

Previous work developed a space-time metric with two cosmological scales; one that conveniently describes the classical evolution of the dynamics, and the other describing a scale associated with macroscopic quantum aspects like vacuum energy. The present work expands upon the dynamics of these scales to demonstrate the usefulness of these coordinates for describing early and late time behaviors of our universe. A convenient parameter, the fraction of classical energy density, is introduced as a means to parameterize the various early time models for the microscopic input.

1 Introduction

The appearance of multiple scales of cosmological relevance from observational evidence such as supernovae accelerations\(^1\) and CMB fluctuations\(^2\) motivates the development of models with scale dependence in the early and late times, but spatial scale invariance during intermediate times. A metric with coordinates that conveniently express such dependencies was developed in a previous paper\(^3\). The present paper will further develop the usefulness of this description of space-time for exploring the dynamics of our cosmology.

It will be assumed that the dynamics can be accurately described using the Einstein equation without a cosmological constant during the period under consideration:

\[
G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G_N}{c^4} T_{\mu\nu},
\]

where the energy-momentum tensor takes the form of an ideal fluid

\[
T_{\mu\nu} = P g_{\mu\nu} + (\rho + P) u_\mu u_\nu.
\]

This fluid form considerably constrains the possible forms of the geometric dynamics of the cosmology.

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1.1 A hybrid metric

Previous work\cite{3} has motivated the use of the following hybrid metric well suited for the exploration of the early and late time behaviors of the cosmology:

$$g_{\mu\nu} = -c^2 dt^2 + R^2(t) \left( dr - \frac{r}{R(t)} c dt \right)^2 + R^2(t) \left( r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right). \tag{1.3}$$

This metric is convenient in its explicit inclusion of a dynamical parameters that can generate a deSitter geometry in late times. The parameter $R_v$ provides an intuitive scale that describes the evolution of the cosmological (quantum) vacuum modes, while the parameter $R$ describes the evolution of the (classical) thermal state of the cosmology. The hydrodynamic parameters can be immediately calculated using Eq. (1.1):

$$\rho = \frac{3e^4}{8\pi G_N} \left( \frac{1}{R_v} + \frac{\dot{R}}{R} \right)^2 \equiv \frac{3e^4}{8\pi G_N} \left( \frac{\dot{R}}{R} \right)^2, \tag{1.4}$$

$$P + \rho = -\frac{e^4}{4\pi G_N \dot{c} t} \left( \frac{1}{R_v} + \frac{\dot{R}}{R} \right) = -\frac{e^4}{4\pi G_N \dot{c} t} \left( \frac{\dot{R}}{R} \right).$$

The dynamics in Eq. (1.4) can be expressed solely in terms of the energy content:

$$\frac{d}{d\dot{c}t} \rho = -\sqrt{\frac{24\pi G_N}{c^4}} \rho (P + \rho), \tag{1.5}$$

which, if the energy content is partitioned in terms of the gravitational vacuum energy and thermal energy $\rho = \rho_v + \rho_{\text{thermal}}$, and the equation of state takes the usual form $P = w\rho$, gives an equation for the evolution of the energy density of the form

$$\frac{d}{d\dot{c}t} \rho = -(1 + w)\rho_{\text{thermal}} \sqrt{\frac{24\pi G_N}{c^4}} \rho. \tag{1.6}$$

1.2 Temporal and spatial scales

The early time microscopic dynamics will define a temporal scale associated with the thermalization of any quantum coherent initial state of the cosmology. This time scale is expected to be associated with the small spatial scale cutoff of the macroscopic gravitational physics, and will be referred to as $\tau_{UV}$. Early temporal evolution is defined by times less than
or comparable to this scale. Similarly, late time behavior is characterized by the domination of dark energy, and hence will be determined in terms of the temporal scale $\tau_\Lambda$. For the cosmology of interest, $\tau_\Lambda \gg \tau_{UV}$. The intermediate, thermal evolution occurs for periods between these extremes, when the energy content is dominated by radiation and/or matter.

Generally, the final state scale $R_v$ need not be a fixed constant. However, for the dual scale cosmology discussed here the scale parameters will be assumed to behave as follows:

$$R_I \leq R_v(ct) \Rightarrow R_\Lambda \quad \text{for} \quad 0 \leftarrow t/\tau_{UV} \to \infty$$

$$R_I \leq R(ct) \Rightarrow R_\Lambda \quad \text{for} \quad 0 \leftarrow t/\tau_\Lambda \to \infty \quad (1.7)$$

$$\rho_I \leq \rho_v(ct) = \rho_v + \rho_{\text{thermal}} \Rightarrow \rho_\Lambda \quad \text{for} \quad 0 \leftarrow t/\tau_{UV,\Lambda} \to \infty,$$

where $1/R_I^2 \equiv \frac{8\pi G_N \rho_I}{3c^4}$ and $1/R_\Lambda^2 \equiv \frac{8\pi G_N \rho_\Lambda}{3c^4}$. During the early thermalization of any initial state quantum condensate, the cosmological quantum scale $R_v$ is expected to rapidly vary from the initial vacuum scale $R_I$ to the final deSitter scale $R_\Lambda$. The general dynamics of the cosmology will be explored in reversed temporal order.

## 2 Present to Late Time Cosmology

For the period from matter domination to dark energy domination, the parameters are chosen to satisfy $\rho_v \cong \rho_\Lambda$, $R_v \cong R_\Lambda$, and $w = 0$. Eq. 1.6 takes the form

$$\rho_v\left(\frac{ct}{\sqrt{\rho_{\text{thermal}} + \rho_\Lambda}}\right) = \frac{1}{\sqrt{\rho_{\text{thermal}} + \rho_\Lambda + \rho_\Lambda}} e^{-\frac{\sqrt{2\rho_{\text{thermal}} + \rho_\Lambda}}{\sqrt{2\rho_{\text{thermal}} + \rho_\Lambda}}} = e^{-\frac{\sqrt{2\rho_{\text{thermal}} + \rho_\Lambda}}{\sqrt{2\rho_{\text{thermal}} + \rho_\Lambda}}} e^{-\frac{\sqrt{2\rho_{\text{thermal}} + \rho_\Lambda}}{\sqrt{2\rho_{\text{thermal}} + \rho_\Lambda}}} (ct - ct_*) \quad (2.1)$$

For brevity, define the ratio at time $t_*$ as $A_* \equiv \frac{\sqrt{\rho_v + \rho_\Lambda} - \sqrt{\rho_\Lambda}}{\sqrt{\rho_v + \rho_\Lambda} + \sqrt{\rho_\Lambda}}$. Using Eq. 1.4, the reduced scale factor can be calculated as

$$\frac{R(ct)}{R_*} = \left[ \frac{e^{2ct/2R_\Lambda} - A_* e^{-2ct/2R_\Lambda}}{e^{2ct_*/2R_\Lambda} - A_* e^{-2ct_*/2R_\Lambda}} \right]^{2/3} \quad (2.2)$$

whereas the thermal scale factor satisfies

$$\frac{R(ct)}{R_\infty} = \left[ 1 - A_* e^{-3ct/R_\Lambda} \right]^{2/3} \quad (2.3)$$
For consistency, the thermal scale corresponds to the horizon scale at late times $R_{\infty} = R_{\Lambda}$, which gives the thermal scale at $t_s$ as $\frac{R_{*}}{R_{\Lambda}} = \left(1 - A_s e^{-3c t_{*}/R_{\Lambda}}\right)^{2/3}$. Writing the Hubble rate for the reduced scale parameter at a time $t_s$ during the matter dominated epoch as $\frac{\dot{R}_{*}}{R_{*}} \approx \frac{\dot{R}_{*}}{R_{*}} = \frac{H_s}{c}$, the ratio $A_s$ can be re-expressed using Eq. 1.4 as $A_s \approx 1/(1 + 2H_{s}c/R_{\Lambda})$. Therefore, the thermal scale at matter-radiation equality can be estimated as

$$R_{eq} \approx \left(3c t_{eq} + \frac{2c}{H_{eq}}\right)^{2/3} R_{\Lambda}^{1/3}. \quad (2.4)$$

### 3 Intermediate (Classical) Evolution

During the intermediate (thermal) period $\tau_{UV} << t << \tau_{\Lambda}$, one assumes that $\rho_v << \rho_{thermal}$ and $\frac{\dot{R}_{*}}{R_{*}} \gg \frac{1}{c} \Rightarrow \frac{\dot{R}}{R} \approx \frac{\dot{R}_{*}}{R_{*}} \approx H_{c}$. The thermal density satisfies

$$\frac{d}{dt}\rho_{thermal} = -\sqrt{\frac{24\pi G N}{c^3}}(1 + w_{\Lambda})\rho_{thermal}^{3/2}$$

$$\left(\frac{\rho_s}{\rho_{thermal}}\right)^{1/2} \approx 1 + \sqrt{\frac{6\pi G N \rho_{*} c^3}{c^3}} \left(1 + w_{\Lambda}\right) (ct - ct_{*}), \quad (3.1)$$

which are the same as the behaviors predicted by the Friedman-Lemaitre equations during the thermal period\[4\]. Using Eq. 1.4 the forms of the reduced scale parameter can be determined:

$$\frac{\mathcal{R}}{\mathcal{R}_{*}} = \left[1 + \frac{3}{2} (1 + w_{\Lambda}) \frac{\dot{\mathcal{R}}_{*}}{\mathcal{R}_{*}} (ct - ct_{*})\right]^{\frac{2}{1 + 3 w_{\Lambda}}}, \quad (3.2)$$

where again $\frac{\dot{R}}{R} \approx \frac{\dot{R}_{*}}{R_{*}} \approx H_{c}$. During these epochs, the reduced and thermal scale parameters are related by

$$\frac{R}{R_{*}} = \frac{\mathcal{R}}{\mathcal{R}_{*}} e^{-c t_{eq}/R_{\Lambda}}. \quad (3.3)$$

The thermal scale ratio is seen to differ significantly from the reduced scale ratio only when the time is comparable to that required for light to traverse the deSitter horizon scale for the dark energy $R_{\Lambda}$.
4 Early Time Evolution

An initial quasi-stationary quantum state is likely in an energy ground state. One can argue that the temporal dynamics begins when the energy content of the cosmology has a non-vanishing thermal fraction. Therefore, the initial state will be taken to have a vanishing thermal fraction. Since the cosmology thermalizes into a radiation dominated epoch, the dynamics of density evolution Eq. 1.6 with \( w = \frac{1}{3} \) gives

\[
\frac{d}{dc} \rho = -\frac{4}{3} \rho_{\text{thermal}} \sqrt{\frac{24 \pi G_N}{c^4}} \rho, \tag{4.1}
\]

It is convenient to define the thermal fraction

\[
f(ct) = \frac{\rho_{\text{thermal}}}{\rho}. \tag{4.2}
\]

The dynamics of the thermal fraction \( f(ct) \) gives microscopic physical input for the early cosmology, and sets the early scales of the system. Its temporal behavior is expected to satisfy \( 0 \Leftrightarrow f(ct) \Rightarrow 1 - \frac{\rho_{\text{eq}}}{\rho_{\text{UV}}} \) for \( 0 \Leftrightarrow ct \to ct_{\text{UV}} \), where the density \( \rho_{\text{UV}} \) represents the thermal energy density of the radiation dominated universe immediately following thermalization. This allows the density in Eq. 4.1 to be determined in the form

\[
\left( \frac{\rho_I}{\rho} \right)^{1/2} = 1 + 2 \sqrt{\frac{8 \pi G_N \rho_I}{3 c^4}} \int_0^{ct} f(ct') dc't = 1 + \frac{2}{R_I} \int_0^{ct} f(ct') dc't, \tag{4.3}
\]

which smoothly connects to the behavior expected during the thermal epoch Eq. 3.1 for \( t > t_{\text{UV}} \) in the form

\[
\left( \frac{\rho_I}{\rho} \right)^{1/2} \approx \frac{R}{R_I} \left[ \int_0^{ct_{\text{UV}}} f(ct') dc't + (ct - ct_{\text{UV}}) \right]. \tag{4.4}
\]

This equation relates the density at radiation-matter density equality to the time \( t_{\text{eq}} \gg t_{\text{UV}} \):

\[
\frac{8 \pi G_N}{3 c^4} \rho_{\text{eq}} \approx \left( \frac{1}{2 t_{\text{eq}}} \right)^2. \tag{4.5}
\]

The general solution for the reduced scale \( R \) using Eq. 4.3 is given by

\[
\frac{R}{R_I} = \exp \left[ \int_0^{ct} \frac{cdt'}{R_I + 2 \int_0^{ct'} f(ct') dc't} \right]. \tag{4.6}
\]
Assuming $f(ct > ct_{UV}) \cong 1$, the solution takes the form

$$\frac{\mathcal{R}}{\mathcal{R}_{UV}} \cong \left[ 1 + \frac{2(ct - ct_{UV})}{R_I + 2 \int_0^{ct_{UV}} f(ct') \, dt'} \right]^{1/2},$$

(4.7)

which is the expected form for the time evolution of the cosmological scale during radiation domination.

Now that correspondence with the observed behavior of the early thermal universe has been established, the useful early time behavior of the model will be explored. The condition of a unique initial cosmological scale $R_I$ with vanishing initial expansion rate for the thermal scale $\dot{R}(0) = 0$ implies that $\dot{\mathcal{R}}(0) = 1$ corresponds to the onset of thermalization. The “vacuum” scale $\mathcal{R}_v$ is determined by the “vacuum energy density” using

$$\frac{1}{R_v^2} = \frac{8\pi G_N}{3c^4} \rho_v = \frac{8\pi G_N}{3c^4} (1 - f) \rho.$$

(4.8)

From Eq. 1.4, the temporal behaviors of the scales during thermalization can be related:

$$\frac{1}{R_v} \equiv \frac{8\pi G_N}{3c^4} \rho_v = \frac{8\pi G_N}{3c^4} (1 - f) \rho.$$

(4.9)

During thermalization, the thermal expansion rate $\frac{\dot{R}}{R}$ increases from 0 to its maximum value $\sqrt{\frac{8\pi G_N \rho_{UV}}{3c^4}}$, and the reduced expansion rate $\frac{\dot{\mathcal{R}}}{\mathcal{R}}$ is seen to decrease from its maximum value $\frac{1}{R_I}$ at time $t = 0$ to essentially the same as the thermal scale expansion rate. The relative contributions of the scales $R_v$ and $R$ to the expansion rate of the reduced scale $\mathcal{R}$ become equal when the thermal fraction takes the value $f = \frac{3}{4}$.

Using Eq. 4.9 and 4.6, the dynamics of the thermal scale during thermalization can be determined:

$$\log \left( \frac{R(ct)}{R_I} \right) = \int_0^{ct} \frac{dct''}{R_I + 2 \int_0^{ct''} f(ct') \, dt'} \left( \frac{f(ct'')}{1 + \sqrt{1 - f(ct'')}} \right).$$

(4.10)

This form directly relates the later time cosmological parameters Eq. 2.4 with the early time behavior

$$\left( \frac{R(ct_{eq})}{R_I} \right) \cong \frac{1}{R_I} \left( 3ct_{eq} + \frac{2c}{H_{eq}} \right)^{2/3} R_{\Lambda}^{1/3}.$$

(4.11)
Further exploration requires a microscopic model that describes the energy density fractions of thermal and vacuum energies during the period of significant macroscopically coherent content.

5 Conclusions

A consistent model of a universe described by a perfect fluid with no cosmological constant has been developed. The model allows for the dynamical evolution of a vacuum energy scale along with a more classically behaved thermal energy scale.

During early times, the classical dynamics is generated by a non-vanishing fraction of thermal energy content, eventually dominating the cosmological expansion during the intermediate period of the expansion of the universe. Particular microscopic physical models specify the detailed temporal dependence of this thermal fraction.

Correspondence of the behavior of the thermal scale with that in the Friedman-Lemaitre equations in standard cosmology has been demonstrated during the intermediate evolution of the model. In addition, the behavior in late times is consistent with dark energy domination as would occur in a cosmology with an additional cosmological constant term added to Einstein’s equation.

A transformation that compares the coordinates and scales of the metric Eq. 1.3 with those of a Friedman-Robertson-Walker space-time will be presented in future work.

References

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