Analytical approach of late-time evolution in a torsion cosmology

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Abstract

In this letter, we study the late-time evolution of a torsion cosmology only with the spin-0+ mode. We find three kinds of analytical solutions with a constant affine scalar curvature. In the first case, it is not physical because the matter density will be negative. In the second case, it shows that the dark energy can be mimicked in the torsion cosmological model. In the third case, the characteristic of late-time evolution is similar to that of the universe of matter dominant. And we also find a kind of expression with the non-constant curvature that the periodic character of numerical calculation is only the reflection of solution in a specific period of evolution. Using these expressions, we shall be able to predict the evolution over the late-time. From this prediction, we know the fate of universe that the universe would expand forever, slowly asymptotically to a halt.

1 Introduction

The cosmological revolutions of the past quarter century have changed everything about our understanding of the future of universe. The current observations, such as SNeIa (Supernovae type Ia), CMB (Cosmic Microwave Background) and large scale structure, converge on the fact that a spatially homogeneous and gravitationally repulsive energy component, referred to as dark energy, accounts for about 70 % of the energy density of universe. Some heuristic models that roughly describe the observable consequences of dark energy were proposed in recent years [1]. Dark energy can even behave as a phantom which effectively violates the weak energy condition [2]. When the

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parameter of equation of state $w$ is a constant less than $-1$, the universe ends up with a big rip singularity which is characterized by the divergence of curvature of the universe after a finite interval of time \[3\]. For the quintessence \[4\] and phantom \[5\] models, the future course of evolution is shown to critically depend on the potential. Specific phantom field models may be proposed to avoid the cosmic doomsday of big rip \[6\].

On the other hand, it is inspiring that one can replace physical field by a geometry quantity in the dark energy model. The PGT (Poincaré Gauge Theory of gravity) has \textit{a priori} independent of local rotation and translation potentials, which correspond to the metric-compatible connection 1-form $\Gamma^\mu_{\nu\rho} = \Gamma_{a}^{[\mu\nu]} dx^a$ and orthonormal coframe $\vartheta^\mu = e^\mu_a dx^a$, where the metric is $g = -\vartheta^0 \otimes \vartheta^0 + \delta_{ij} \vartheta^i \otimes \vartheta^j$ and $\mu, \nu, \rho \cdots$ are 4$d$ coordinate (holonomic) indices and $i, j, k \cdots$ are 3$d$. The gauge vector potential associates with field strength. In our case, they are curvature and torsion. PGT has been regarded as an interesting alternative of GR (general relativity) because of its gauge structure and geometric properties \[7\]. The bouncing cosmological model with torsion was suggested in Ref. \[8\], but the torsion was imagined as playing role only at high densities in the early universe. Goenner \textit{et al.} made a general survey of the torsion cosmology \[9\], in which the equations for all the PGT cases were discussed although they only solved in detail a few particular cases. Recently some authors have begun to investigate torsion as a possible reason of the accelerating universe \[10\].

There are six possible dynamic connection modes \[11\], carrying certain spins and parity: $2^\pm, 1^\pm, 0^\pm$. Some investigations showed that $0^\pm$ may well be the only acceptable dynamic PGT torsion modes \[12\]. The pseudoscalar mode $0^-$ is naturally driven by the intrinsic spin of elementary fermions, therefore it naturally interacts with such sources. Consequently, it is generally thought that axial torsion must be small and have small effects at the late time of cosmological evolution. This is a major reason why one does not focus on this mode at the late time. On the other hand, the scalar mode $0^+$ does not interact in any direct obvious fashion with any known type of matter \[13\], therefore one can imagine it as having significant magnitude and yet not being conspicuously noticed. Furthermore, there is a critical non-zero value for the affine scalar curvature since $0^+$ mode can interact indirectly through the non-linear equation. Nester and collaborators \[14\] consider an accounting for the accelerated universe in term of the dynamic scalar torsion. In Ref. \[15\], it was shown that the dynamic Riemann-Cartan geometry could contribute an oscillating aspect to the acceleration expansion rate of the Universe. We have appeared analyzing the dynamics of this model \[16\]. Applying the statefinder diagnostic to the torsion cosmology, we find that there are some typical characteristics \[17\]. An extension model with the spin-$0^+$ and spin-$0^-$ modes was also considered, but the acceleration mechanism is still due to the spin-$0^+$ mode \[18\].
In this paper, we study the late-time evolution of a torsion cosmology only with spin-0\(^+\) mode. In a constant affine scalar curvature case, we find three analytical solutions: the first is not physical, conflicting with the assumption of energy positivity; the second shows that the dark energy can be mimicked in the torsion cosmological model; the third shows that the behavior of late-time evolution is analogous to that of the universe of matter dominant. To satisfy the energy positivity requirement \([12]\), there is only a critical point \((0, 0, 0)\) for the nonlinear system which is an asymptotically stable focus in the phase space \((H, \Phi, R)\) \([16]\). In non-constant curvature case, we also find a kind of expression showing that the periodic character of numerical calculation is only the reflection of solution in a specific period of evolution. Using these expressions, we shall be able to predict the evolution over the late-time. From the prediction, we know the fate of universe that the universe would expand forever, slowly asymptotically to a halt.

2 The model

Nester and collaborators \([14]\) consider an accounting for the accelerated universe in terms of a Riemann-Cartan geometry: dynamic scalar torsion. The torsion and curvature 2-forms are defined by

\[
T^\mu \equiv \frac{1}{2} T^{\mu}_{\alpha \beta} \partial^\alpha \wedge \partial^\beta = d\partial^\mu + \Gamma^\mu_{\nu} \wedge \partial^\nu \tag{1}
\]

\[
R^{\mu \nu} \equiv \frac{1}{2} R^{\mu \nu}_{\alpha \beta} \partial^\alpha \wedge \partial^\beta = d\Gamma^{\mu \nu} + \Gamma^\mu_{\rho} \wedge \Gamma^{\rho \nu} \tag{2}
\]

which satisfy the Bianchi identities, respectively,

\[
DT^\mu \equiv R^\mu_{\nu} \wedge \partial^\nu, \quad DR^\mu_{\nu} \equiv 0 \tag{3}
\]

Theoretical analysis of PGT led us to consider tendentiously two spin-0 modes. In this case, the gravitational Lagrangian density is

\[
\mathcal{L}[\partial, \Gamma] = \frac{1}{2\kappa^2} \left[ -a_0 R + \sum_{n=1}^{3} a_n \left( \frac{(n)}{T^2} + \frac{b^+}{12} R^2 + \frac{b^-}{12} E^2 \right) \right] \tag{4}
\]

where \((n)\) is the algebraically irreducible parts of the torsion, \(R\) is the scalar curvature and \(E\) is the pseudoscalar curvature \([18]\). Note that \(a_0\) and \(a_n\) are dimensionless parameters, and \(b^\pm\) have the same dimension with \(R^{-1}\).

Since current observations favor a flat universe, we will work in the spatially flat Robertson-Walker cosmological model. The isotropic orthonormal coframe
has the form:

\[ \vartheta^0 = t, \quad \vartheta^i = a(t)\chi^i \]  \hspace{1cm} (5)

Because of isotropy, the only non-vanishing torsion tensor components are

\[ T^i_{j0} = -\frac{\Phi(t)}{3}\delta^i_j, \quad T^i_{jk} = -2\chi(t)\epsilon^i_{jk} \]  \hspace{1cm} (6)

where \( \epsilon_{ijk} = \epsilon_{0ijk} \) is the usual asymmetric tensor. Using the equation obtained by the variation with respect to the connection, we know the \( 0^- \) part couples to the axial spin vector of spin-\( \frac{1}{2} \) fermions, but \( 0^+ \) mode does not couple to any known source[13]. Therefore, one can consider only spin-\( 0^+ \) mode. In other words, we only discuss the case of \( \chi(t) = 0 \). Furthermore, from the field equation one can finally give the necessary equations for the matter-dominated era to integrate (for a detailed discussion, see Ref.[13] and we have made the replacement \( A_0 \to -a_0 \) and \( A_n \to 2a_n \), which is consistent with an earlier work [16].

\[ \dot{H} = \frac{\mu}{6a_2}R - \frac{\kappa \rho_m}{6a_2} - 2H^2 \]  \hspace{1cm} (7)

\[ \dot{\Phi} = \frac{a_0}{2a_2}R - \frac{\kappa \rho_m}{2a_2} - 3H\Phi + \frac{1}{3}\Phi^2 \]  \hspace{1cm} (8)

\[ \dot{R} = -\frac{2}{3}\left( R + \frac{6\mu}{b} \right)\Phi \]  \hspace{1cm} (9)

where \( b \equiv b^+ \), \( \mu = a_2 - a_0 \), \( H = \dot{a}/a \) is Hubble parameter, and the energy density of matter component is

\[ \kappa\rho_m = \frac{b}{18}(R + \frac{6\mu}{b})(3H - \Phi)^2 - \frac{b}{24}R^2 - 3a_2H^2 \]  \hspace{1cm} (10)

The Newtonian limit requires \( a_0 = -1 \).

3 The solutions of constant scalar curvature

From Eq. (9), it is easy to find the scalar affine curvature remains a constant \( R = -6\mu/b \) forever as long as its initial data has this special value [14]. In this case, Eq. (7) can be rewritten as

\[ \dot{H} = -\frac{3}{4}a_2b - \frac{3}{2}H^2 \]  \hspace{1cm} (11)
The positivity of the kinetic energy requires \(a_2 > 0\) and \(b > 0\) [14], so we have the solution

\[
H(t) = \zeta \tan \left[\frac{3\zeta}{2}(t_0 - t) + \arctan \left(\frac{H_0}{\zeta}\right)\right]
\] (12)

where \(\zeta = \mu / \sqrt{2a_2b}\) and \(H_0 = H(t_0)\). However, such a choice conflicts with the assumption of energy positivity in the \(R = -6\mu/b\) case.

If we audaciously relax the parameter requirement for positive kinetic energy, i.e., \(a_2 < -1\) and \(\mu < 0\), this phantom scenario will turn out to be interesting. Now we have the solution

\[
H(t) = \frac{\xi(\xi + H_0)(\xi - H_0)^{-1} \exp \left[3\xi(t - t_0)\right] - 1}{(\xi + H_0)(\xi - H_0)^{-1} \exp \left[3\xi(t - t_0)\right] + 1}
\] (13)

where \(\xi = \mu / \sqrt{-2a_2b}\). When \(t\) tends to infinity, \(H(t) \to \xi\), so that the dark energy can be mimicked in the torsion cosmological model with a constant affine scalar curvature. Using the dynamical analysis, we have pointed out that there is a late-time de Sitter attractor [16]. Note that the solution (13) is just corresponding to the de Sitter attractor.

Especially, as \(a_2 = -1\), we have a solution

\[
H = \frac{2H_0}{2 + 3H_0t}
\] (14)

\[
\Phi = \frac{3H_0[2 + (2 + 2H_0t)^{\frac{2}{3}}\eta]}{(2 + 3H_0t)[1 + (2 + 3H_0t)^{\frac{2}{3}}\eta]}
\] (15)

\[
R = 0
\] (16)

where

\[
\eta = \frac{2^{2/3}(-3H_0 + \Phi_0)}{3H_0 - 2\Phi_0}
\] (17)

From Eq. (14), we have

\[
a = a_0(\frac{2 + 3H_0t}{2 + 3H_0t_0})^{\frac{2}{3}}
\] (18)

Note that the behavior of the late-time evolution is analogous to that of the universe of matter dominant. In Fig. 1, we plot the trajectories in the phase space \(H-\Phi\) when we take \(a_2 \leq -1, b > 0\) and \(R = -6\mu/b\). Obviously, if \(a_2 = 0\)
or \(-1\), the slightest change in \(a_2\) leads to a radical change in the behavior of solutions. Therefore, we have a bifurcation at \(a_2 = 0\) or \(-1\) in Eqs. (7)-(8) and \(R = -6\mu/b\).

4 The solution of non-constant scalar curvature

The numerical analyses show that \(H\), \(\Phi\) and \(R\) have a periodic character at late-time of the evolution for \(a_2 > 0\) and \(b > 0\), approximatively [15]. Using the dynamical analysis [16] and statefinder diagnostic [17], we find that this character is corresponding to an asymptotically stable focus. In Fig. 2, we plot evolving trajectory in \(H-\Phi-R\) space, where we have chosen \(a_2 = 2\) and \(b = 2/t_0^2\). We find easily that the evolving trajectories tend to the focus \((0,0,0)\). If we consider the linearized equations, then Eqs.(7)-(9) can be reduced to

\[
\begin{align*}
\dot{H} &= \frac{\mu}{6a_2} R, \\
\dot{\Phi} &= \frac{1}{2a_2} R, \\
\dot{R} &= -\frac{4\mu}{b} \Phi
\end{align*}
\]

The linearized system (19) has an exact periodic solution
Fig. 2. We illustrate evolution trajectory in the phase space, where have chosen \( a_2 = 2, b = 2/t_0^2 \). The surfaces are the nullclines of nonlinear dynamical system, and the curve is the evolution trajectory. It shows clearly that at late time the phase line has a oscillation feature.

\[
H = -\alpha R_0 \sin \omega t + \frac{\mu}{3} \Phi_0 \cos \omega t + H_0 - \frac{\mu}{3} \Phi_0 \\
\Phi = -\beta^{-1} R_0 \sin \omega t + \Phi_0 \cos \omega t \\
R = R_0 \cos \omega t + \beta \Phi_0 \sin \omega t \tag{20}
\]

where \( \omega = \sqrt{\frac{2 \mu}{a_2 b}}, \alpha = \sqrt{\frac{\mu b}{72 a_2}}, \beta = \sqrt{\frac{8 \mu a_2}{b}} \) and \( H_0 = H(0), \Phi_0 = \Phi(0) \) and \( R_0 = R(0) \) are initial values. Obviously, \((H, 0, 0)\) is a critical line of center for the linearized solution. In other words, there are only exact periodic solutions for the linearized system, but there are quasi-periodic solutions near the focus for the coupled nonlinear equations. This property of quasi-periodic also appears in the statfinder diagnostic with the case of \( a_2 \geq 0 \) [17]. According to nonlinear equations (7)-(9), we can obtain the critical points and study the stability of these points. To study the stability of the critical point \((0, 0, 0)\), we write the variables near \((0, 0, 0)\) in the form \( H = \Delta H, \Phi = \Delta \Phi \) and \( R = \Delta R, \) where \( \Delta H, \Delta \Phi \) and \( \Delta R \) are the perturbations of the variables near the critical point \((0, 0, 0)\). Substituting the expressions into Eqs. (7)-(9), we obtain the corresponding eigenvalues \((0, -\sqrt{-\frac{2 \mu}{a_2 b}}, \sqrt{\frac{2 \mu}{a_2 b}})\). Therefore, the critical point \((0, 0, 0)\) is a stable focus when \( a_2 > 0 \) and \( b > 0 \). We illustrate the evolution trajectory in the phase space \((H, \Phi, R)\) in Fig.2, where have chosen \( a_2 = 2, b = 2/t_0^2 \). Nullcline surfaces in Fig. 2 divided the phase-space \((H, \Phi, R)\) into some domains, in each of which there is definite behavior of the evolution trajectory. It is obvious that the evolution trajectory helically tends to the critical point \((0, 0, 0)\) after crossing the nullcline surface.

For the late-time behavior of non-linear system Eqs.(7)-(9), we should determine the behavior at infinity. We substitute \( \tau = \frac{1}{t} \) in Eqs. (7)-(9), and show the Laurent expansions around the point \( \tau = 0 \). Clearly, as \( t \to \infty, \tau \to 0 \). Thus, we are interested in the behavior of equations for \( \tau = 0 \). The non-linear system clearly shows that the solutions have regular behavior at
\( \tau = 0 \), and have the Taylor expansions: \( H(t) = \sum_{n=1}^{\infty} h_n \tau^n \), \( \Phi(t) = \sum_{n=3}^{\infty} \phi_n \tau^n \) and \( R(t) = \sum_{n=2}^{\infty} r_n \tau^n \). So we find an approximate formula up to \( t^{-N} \) order as follows

\[
H(t) = \sum_{n=1}^{N} \frac{h_n}{t^n} + \frac{1}{t^N} [\beta_N \sin \omega t + \gamma_N \cos \omega t] \quad (21)
\]
\[
\Phi(t) = \sum_{n=3}^{N} \frac{\varphi_n}{t^n} + \frac{1}{t^N} \left[ \frac{3\beta_N}{\mu} \sin \omega t + \frac{3\gamma_N}{\mu} \cos \omega t \right] \quad (22)
\]
\[
R(t) = \sum_{n=2}^{N} \frac{r_n}{t^n} + \frac{1}{t^N} \left[ -12\gamma_N \frac{\sin \omega t}{\omega b} + \frac{12\beta_N}{\omega b} \cos \omega t \right] \quad (23)
\]

where \( \omega = \sqrt{\frac{2\tau}{\omega^2 b}} \) and \( h_n, \varphi_n \) and \( r_n \) are undetermined coefficients. The coefficients \( \beta_N \) and \( \gamma_N \) can be written as the terms of initial values.

From a theoretical point of view, the Eqs. (21)-(23) are equivalent to the Laurent expansions around the point \( \tau = 0 \) when \( N \to \infty \). From a practical point of view, the Eqs. (21)-(23) are more effective than the expansions of minus-power terms because of the latter only determined by the coefficients \( h_1, \cdots, h_{N-1}; \varphi_1, \cdots, \varphi_{N-1}; r_1, \cdots, r_{N-1} \) for a definite \( N \). Therefore, the former is an approximate formula up to \( t^{-N} \) order and the latter is one up to \( t^{-(N-1)} \) order. For example, we take \( N = 3 \) and the approximate solution up to \( t^{-3} \) term is

\[
H(t) = \frac{2}{3t} + \frac{t^2}{t^2} (H_0 - \frac{1}{3t_0} - \frac{\mu \Phi_0}{3} + \frac{\mu}{9t_0}) + \frac{t^3}{t^3} \left( \frac{\mu}{3} \Phi_0 - \frac{\mu}{9t_0} S_0 + \frac{\omega b R_0 C_0 - \omega b}{9t_0^2 C_0} \sin \omega t \right) + \left( -\frac{\omega b}{12} R_0 S_0 + \frac{\omega b}{9t_0} S_0 + \frac{\mu}{3} \Phi_0 C_0 - \frac{\mu}{9t_0} C_0 \right) \cos \omega t \]  
\[
\Phi(t) = \frac{t^3}{t^3} \left[ \frac{b}{\mu t_0^2} + (\Phi_0 S_0 - \frac{1}{3t_0} S_0 + \frac{\omega b R_0 C_0 - \omega b}{4\mu} C_0) \sin \omega t \right] + \left( -\frac{\omega b}{4\mu} R_0 S_0 + \frac{\omega b}{3\mu t_0^2} S_0 + \Phi_0 C_0 - \frac{1}{3t_0} C_0 \right) \cos \omega t \]  
\[
R(t) = \frac{4}{3t^2} + \frac{t^3}{t^3} \left( \frac{AH_0}{t_0} - \frac{4}{3t_0^2} - \frac{4\mu}{3t_0} \Phi_0 + \frac{4\mu}{9t_0} \right) + \left( R_0 S_0 - \frac{4}{3t_0^2} S_0 - \frac{4\mu}{\omega b} \Phi_0 C_0 + \frac{4\mu}{3\omega b t_0} C_0 \right) \sin \omega t + \left( \frac{4\mu}{\omega b} \Phi_0 S_0 - \frac{4\mu}{3\omega b t_0} S_0 + R_0 C_0 - \frac{4}{3t_0^2} C_0 \right) \cos \omega t \]  

where \( H_0 = H(t_0), \Phi_0 = \Phi(t_0) \) and \( R_0 = R(t_0), \) and \( S_0 = \sin \omega t_0, C_0 = \cos \omega t_0 \). Obviously, \( H(t) \to 0, \Phi(t) \to 0 \) and \( R(t) \to 0 \), when \( t \) tends to
Fig. 3. In the non-constant curvature case, we plot the behavior of late-time evolution. In 3(a), we have chosen $a_2 = 1, b = \frac{2}{\pi t_0^2}$ and $t_0 = 20$; In 3(b), we have fixed $a_2 = 1, b = \frac{80000}{\pi t_0^2}$ and $t_0 = 20$.

infinity. Therefore, the fate is that the universe would expand forever, slowly asymptotically to a halt.

Next, we give the reason why some numerical calculations look as periodicity as mentioned in Ref. [15]. To put it bluntly, this is a reflection of the late-time solution at specified period of evolution. If we take $t = t_0 + \Delta t$, $\Delta t \in [0, \delta]$ and $\frac{2\pi}{\omega} \ll \delta \ll t_0$, Eqs. (24)-(26) can be reduced to a solution, whose behavior looks like a periodic solution of (20) of the linearized system. Taking the parameter values, we plot Hubble parameter $H(t)$ at the late-times in Fig. 3.

If we take the approximate expression of $N = 2$, when $t > 10, 10^2$ and $2 \times 10^2$, the error $\Delta = \frac{|H_{num} - H_{N=2}|}{H_0} < 0.066, 0.012$ and 0.002, respectively. Therefore, if we want to obtain an effective approximate expression, $N$ is required to be large enough. In Fig. 4, a direct quantitative connection is made between analytical expressions and the numerical solutions presented by Shie et al. [15]. We take initial conditions $H(1) = 1, \Phi(1) = 1.4, R(1) = 1.53, \mu = 1.09$ and $b = 1.4$ which are the same as those in Ref. [15].
Fig. 4. In the non-constant curvature case, we plot the behavior of late-time evolution $H(t)$ for our analytical expression and numerical solution obtained by Shie et al., respectively. Black line represents the numerical solution ($t \leq 3$), and red line represents the analytical solution ($N = 50$).

5 Conclusion

In this paper, we study the evolution of a torsion cosmology only with the "scalar torsion" mode $0^+$. This mode has some distinctive and interesting qualities. It can be considered as a "phantom field", because it does not interact with any known matters directly. It only interacts via the gravitational equations. Therefore, $0^+$ mode can drive the universe to accelerate at present. However, we have to investigate the late-time solution if we want to know the fate of universe. We find a kind of late-time solution which tends to the focus $(0,0,0)$ in the phase space $(H, \Phi, R)$. Under the solution, the feature of evolution is universal with generic choices of the parameters. Distinctive and interesting results are:

i) In the constant affine curvature case, there are a kind of non-physical solution and two kinds of physical solutions at late-time. One of physical solutions corresponds to the de Sitter attractor, and the other is analogous to the universe of matter dominant. In the view-point of dynamical analysis, $a_2 = 0$ and $-1$ are the bifurcations.

ii) In the non-constant affine curvature case, we find a kind of expression corresponding to the universe which would expand forever, slowly asymptotically to a halt. The specific behaviors of late-time evolution differ from those at different periods. If we considered the specified period of evolution $t = t_0 + \Delta t$, $\Delta t \in [0, \delta]$, and satisfied $\frac{2\pi}{\omega} \ll \delta \ll t_0$, the behavior looks like a periodic solution.

Furthermore, if we want to establish a realistic cosmological model, we have to carry out the solutions of $t < t_0$ period, and compare these analytical solutions with observations to determine the model parameters. For example, using SN Ia data, we constrain the parameters of torsion cosmology and get their best
values. Then we can discuss the properties of the realistic cosmological models. These issues will be considered elsewhere.

Recently, the cosmological model with even and odd parity modes in PG has been considered by Baekler, Hehl and Nester \[19\]. They extended the parity violating quadratic Lagrangian $V_-$ to the general gravitational Lagrangian $V_{\pm} = V_- + V_+$, where $V_+$ is the parity conserving quadratic Lagrangian. This model generalized the torsion cosmologies which were presented by Nester et al. \[12,14,15,18\]. Next, we will set about investigations for dynamics of this cosmological model.

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