A simple toy model for effective restoration of chiral symmetry in excited hadrons.

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A simple solvable toy model exhibiting effective restoration of chiral symmetry in excited hadrons is constructed. A salient feature is that while physics of the low-lying states is crucially determined by the spontaneous breaking of chiral symmetry, in the high-lying states the effects of chiral symmetry breaking represent only a small correction. Asymptotically the states approach the regime where their properties are determined by the underlying unbroken chiral symmetry.

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One striking feature of the hadron spectrum is the presence of nearly degenerate resonances of opposite parity relatively high in the excitation spectrum. It has been argued that this phenomenon might be a manifestation of an “effective restoration of chiral symmetry”\(^1\,\text{[3, 4, 5, 6, 7, 8]}\). The central idea underlying this notion is the possibility that the coupling of these highly excited states to the dynamics driving spontaneous chiral symmetry breaking could get progressively weaker for progressively massive hadrons leading to the excited states being quite insensitive to the effects of spontaneous chiral symmetry breaking and hence act qualitatively very much as if they were in the Wigner-Weyl mode. In such a situation the resonances would be expected to form approximate linearly realized chiral multiplets whose members were nearly degenerate. Such a scenario naturally predicts multiplets of states with opposite parities. That the linear realization of the chiral symmetry naturally predicts multiplets of states with opposite parities.

Thus, it is important to precisely characterize what is implied under effective restoration in \([1, 2, 3, 4, 5, 6, 7, 8]\). The central idea underlying this notion is the possibility that the coupling of these highly excited states to the dynamics driving spontaneous chiral symmetry breaking could get progressively weaker for progressively massive hadrons leading to the excited states being quite insensitive to the effects of spontaneous chiral symmetry breaking and hence act qualitatively very much as if they were in the Wigner-Weyl mode.

It is important to precisely characterize what is implied under effective restoration in \([1, 2, 3, 4, 5, 6, 7, 8]\), because sometimes it is erroneously interpreted in the sense that the highly excited hadrons are in the Wigner-Weyl mode. The mode of symmetry is defined only by the properties of the vacuum. If symmetry is spontaneously broken in the vacuum, then it is the Nambu-Goldstone mode and the whole spectrum of excitations on the top of the vacuum is in the Nambu-Goldstone mode. However, it may happen that the role of the chiral symmetry breaking condensate of the vacuum becomes progressively irrelevant in excited states. This means that the chiral symmetry breaking effects (dynamics) become less and less important in the highly excited hadrons and asymptotically the states approach the regime where their properties are determined by the underlying unbroken chiral symmetry (i.e. by the symmetry in the Wigner-Weyl mode).

Although the basic idea has been discussed in the past, it is useful to construct a simple model which illustrates clearly the basic idea. Such a model has the virtue of demonstrating explicitly that the idea is consistent with the underlying concepts of chiral symmetry and spontaneous chiral symmetry breaking. This may be of use in preventing confusion about the physical content of “effective chiral restoration”. We will then use the behavior in this model to clarify the physical meaning of the given phenomenon.

Consider as an example the following model which, while having no particular physical significance illustrates clearly the physical content of effective chiral restoration of the type discussed in refs. \([1, 2, 3, 4, 5, 6, 7, 8]\). The model contains an infinite number of \(\pi\) and \(\sigma\) mesons. In this respect the model mimics large \(N_c\) QCD. We denote the \(j\)th pion (\(\sigma\) meson) \(\pi_j\) (\(\sigma_j\)). These fields enter the Lagrangian in a chirally invariant way as members of \(L(\frac{1}{2}, \frac{1}{2})\) chiral multiplets:

\[
[V^a, \pi^b_j] = i\epsilon^{abc} \pi^c_j \quad [V^a, \sigma_j] = 0 \\
[A^a, \pi^b_j] = i\delta^{ab} \sigma_j \quad [A^a, \sigma_j] = i\pi^a_j
\]

where \(V^a\) (\(A^a\)) represent the generators of vector (axial) rotations. To simplify the analysis by reducing the number of possible couplings, in addition to chiral symmetry the model has an infinite number of discrete symmetries: it is invariant under \(\sigma_j \rightarrow -\sigma_j\) for all \(j\). The discrete symmetries ensure that each type of field always enters the Lagrangian in even powers. The Lagrangian is given by

\[
\mathcal{L} = \sum_j \frac{1}{2} (\partial^\mu \sigma_j \partial_\mu \sigma_j + \partial^\mu \bar{\pi}_j \cdot \partial_\mu \pi_j) - \frac{m^2}{2} \left( \alpha (\sigma^2_1 + \bar{\pi}_1 \cdot \pi_1) + \frac{g}{2m_\sigma^2} (\sigma^2_1 + \bar{\pi}_1 \cdot \pi_1)^2 \right)
\]
where \(m_o\) has dimensions of mass and \(\alpha\) and \(g\) are dimensionless constants and \(V_4\) is some function of the fields \(\sigma_j, \pi_j; j > 1\) consistent with the symmetries and with the potential whose terms are quartic in the fields. The model is chosen so that \(j = 1\) fields play a special role in the chiral broken phase: \(\sigma_1\) (and no other fields) acquires a vacuum expectation value and the excitation associated with \(\pi_1\) becomes massless. The parameter \(\alpha\) controls spontaneous symmetry breaking; \(\alpha > 0\) yields the Wigner-Weyl mode while \(\alpha < 0\) yields the Nambu-Goldstone mode. As will be seen below the analysis is independent of the particular form picked for \(V_4\).

The interaction terms in the model of Eq. (2) are controlled by the parameter \(g\). For \(g \ll 1\), the theory is weakly coupled and hence can be treated classically. The Lagrangian is parameterized in such a way that the dependence of the mass spectrum on \(g\) only arises through loop contributions which can be neglected in the weak-coupling limit. Note, however, that even in the weak coupled limit the interaction terms play an essential role when \(\alpha < 0\) since it determines the amount of chiral symmetry breaking. We consider here the weak coupling limit of the theory which is analytically tractable—indeed trivial—but is a perfectly legitimate chiral theory. We will study the theory in the weakly coupled regime where it is tractable.

If one imposes isospin invariance then it is easy to see that the minimum of the potential is given by:

\[
\langle \sigma_j \rangle = 0 \quad \text{for} \quad \alpha > 0
\]

\[
\langle \sigma_j \rangle = \pm \delta_{j1} m_o \sqrt{-\alpha \over g} \quad \text{for} \quad \alpha \leq 0 .
\]  

By expanding quadratically around the minimum of the potential one can find the mass spectrum.

\[
- {m_o^2 \over 2} \sum_{j=2}^{\infty} \left( j^2 (\sigma_j^2 + \pi_j \cdot \pi_j) + {g \over j m_o^2} \left( (\sigma_j \sigma_{j+1} + \pi_j \cdot \pi_{j+1})^2 + (\sigma_{j+1}^2 + \pi_{j+1} \cdot \pi_{j+1}) (\sigma_j^2 + \pi_j \cdot \pi_j) \right) \right) + g V_4 \]  

This procedure is legitimate since we are working in the weak coupling limit and quantum (loop) corrections are suppressed. The spectrum is shown in Fig. 1. It is apparent from Fig. 1 that this model in the spontaneously broken phase exhibits the phenomenon of effective chiral restoration. Consider, for example, the region near \(\alpha = -1\). While the lowest-lying states have no hint of a chiral multiplet structure, as one goes higher in the spectrum the states fall into nearly degenerate multiplets which to increasingly good approximation look like pions and \(\sigma\)-meson in linearly realized \((1, 1 \over 2, 1 \over 2)\) representations.

Suppose that there is a “no-go theorem” forbidding effective chiral restoration in excited hadrons and the

\[
\begin{align*}
   m_{\pi_1}^2 &= 0 \\
   m_{\pi_1}^2 &= -2\alpha m_o^2 \\
   m_{\pi_1}^2 &= j^2 m_o^2 + {2g |\sigma_1|^2 \over j} \\
   &= (j^2 + 2 \alpha) m_o^2 \quad (j \geq 2) \\
   m_{\pi_1}^2 &= j^2 m_o^2 + {4g |\sigma_1|^2 \over j} \\
   &= (j^2 + 4 \alpha) m_o^2 \quad (j \geq 2)
\end{align*}
\]  

for \(\alpha \leq 0\)

\[
\begin{align*}
   m_{\pi_1}^2 &= \alpha m_o^2 \\
   m_{\pi_1}^2 &= \alpha m_o^2 \\
   m_{\pi_j}^2 &= j^2 m_o^2 \quad (j \geq 2) \\
   m_{\pi_j}^2 &= j^2 m_o^2 \quad (j \geq 2)
\end{align*}
\]  

for \(\alpha > 0\)
existence of approximate parity doublets in the Nambu-Goldstone phase cannot be a manifestation of chiral symmetry. If this were correct then the appearance of approximate parity doublets in the Nambu-Goldstone phase of this model must be unrelated to chiral symmetry. However, this is obviously not the case: the near degeneracy of the parity doublets reflects the underlying chiral symmetry of the model. The key point is that high-lying states are very insensitive to the chiral order parameter and the spectrum approximates a Wigner-Weyl mode spectrum increasingly accurately.

The scenario in which high mass resonances become increasingly insensitive to the chiral order parameter and hence form into approximate chiral multiplets in the spectrum is precisely what was meant by effective chiral restoration in refs. [1, 2, 3, 4, 5, 6, 7, 8]. Thus, since there is at least one chirally invariant model in which this happens, there cannot be any “no-go theorem” for effective chiral restoration.

One might legitimately argue that the model in Eq. (2) is highly artificial. The model was designed—with malice aforesight—to ensure effective chiral restoration. This was done in part by fixing the term in the Lagrangian which couples to the chiral order parameter to scale like \( \frac{1}{j} \) and hence become weak for large \( j \). This is, of course, true. However, the point of the exercise was simply to demonstrate that there is a solvable model that exhibits effective chiral restoration and this model helps to clarify a physical meaning of this phenomenon. However artificial the model, it is adequate for this purpose.

The model above was formulated in terms of the fields that transform linearly under the chiral group. Then a question arises whether this model illustrates some generic behavior or it is only specific to the linear realization of the chiral symmetry? Indeed, in the Nambu-Goldstone mode one can always make a field redefinition to the standard nonlinear realization in which fields of opposite parity are decoupled under, i.e. do not transform into each other under chiral transformation. In this case the act of making an axial rotation does not transform a field into its chiral partner but instead creates a massless Goldstone boson (pion) from the vacuum. However, such a redefinition is unphysical and the Lagrangian of Eq. (2) rewritten in terms of these new fields cannot alter the spectrum. This reflects a general situation that the field redefinitions themselves cannot modify any physical content of the theory. In this context it is useful to recall that the physics is not in the fields, but in the states, which appear once one applies fields on the vacuum. The physics of these states is controlled only by the microscopical theory. Clearly the chiral symmetry can impose no constraints on these decoupled fields in the spontaneously broken phase. However this does not rule out effective chiral restoration. Generically, in the Nambu-Goldstone mode, the properties of states are sensitive to the dynamics of chiral symmetry breaking—this is why the states generically do not form \textit{exact} chiral multiplets.

If however, there exist states in the spectrum which for some reason are insensitive or weakly sensitive to the dynamics of chiral symmetry breaking, there will be approximate relations between the properties of the levels dictated by the underlying chiral symmetry (in the limit of complete insensitivity the states must look identical to exact linear chiral multiplets). The reason for this is simple: one can reverse the transformation from the standard nonlinear realization and rewrite things in terms of a set of linear realized fields; theory in terms of these fields is equally physical as the standard nonlinear realization. These fields correspond to states which are related to each other under chiral symmetry except due to the interactions with the chiral order parameter and to the Goldstone bosons. As these interactions become small approximate chiral multiplets emerge. This is explicitly seen in our toy model since the high-lying states do decouple from the vacuum condensate and from the Goldstone bosons. Indeed, it was argued in ref. [6, 20] that the high-lying hadrons do decouple from the Goldstone bosons once they decouple from the quark condensate.

Thus, the issue is a quantitative one and not merely a qualitative one. In the Nambu-Goldstone mode the states should be sensitive to the dynamics of chiral symmetry breaking. The question is how much. As demonstrated above, it is certainly possible in some models for high lying states to have very small coupling to the dynamics of symmetry breaking and, hence, to very good approximation to fall into chiral multiplets. The conjecture of effective chiral restoration in hadronic physics is that QCD behaves in the same manner as this simple model.

As noted above the model in Eq. (2) is highly artificial. However, it does illustrate many of the salient points relevant to the issue. Firstly, it illustrates the most important point: while the physics of the low-lying states is crucially determined by the spontaneous breaking of chiral symmetry, in the high-lying states the effects of chiral symmetry breaking represent only a small correction. Secondly, it shows the gradual nature of the conjectured effect. The effect is never absolute but always approximate; for any given strength of the coupling \( \alpha \) it becomes increasingly accurate as one goes up in the
spectrum. Thirdly, it makes very clear that the key issue is the coupling of the state to the dynamics responsible for spontaneous chiral symmetry breaking—in this case the coupling to $\sqrt{g}\langle\sigma_1\rangle$ which plays the role of the chiral order parameter.

Although we have constructed a simple tractable toy model explicitly exhibiting the effective restoration of chiral symmetry in excited hadrons, the essential question of whether the parity partners seen in excited hadrons are due to effective chiral restoration remains open. As discussed in [2,7,8], there are general arguments leading to the expectation that as one goes to asymptotically high mass states the sensitivity to the chiral condensate decreases. This in turn leads to the expectation of effective chiral restoration in the hadron spectrum, provided that discernable hadrons still exist in the regime where the sensitivity to the chiral condensate becomes negligible. However, at present there is no reliable theoretical tool which allows one to answer the question of whether discernable hadronic resonances persist high enough in the spectrum to reach the regime of effective chiral restoration or whether the spectrum essentially melts into the QCD continuum before this point. There is an interesting theoretical limit—the large $N_c$ limit—in which the meson spectrum is infinite and remains discrete and the phenomenon of effective chiral restoration ought to occur. Some discussions of the rate of symmetry restoration can be found in [17,18]. The results of the solvable model of the ’t Hooft type in 3+1 dimensions are presented in [19]. While the large $N_c$ argument is interesting theoretically and shows how the phenomenon may come about, it does not provide a compelling theoretical argument for the $N_c = 3$ world. Similarly, the present empirical evidence is not compelling but we take it to be suggestive.

There is an empirical way to verify the idea, however. If the parity doubling observed at present is indeed due to chiral restoration, then some of the missing states in the chiral multiplets with approximately known masses should be experimentally found. This point is a legitimate subject for discussion.

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