Hybrid Carrier Frequency Modulation Based on Rotor Position to Reduce Sideband Vibro-Acoustics in PMSM Used by Electric Vehicles

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Abstract: In the permanent magnet synchronous motor (PMSM) drive system, the unwilling and ear-piercing vibro-acoustics caused by high-frequency sideband harmonics becomes unacceptable in electric vehicle applications. In this paper, a modified space vector pulse width modulation (SVPWM) technique implemented with a hybrid carrier frequency modulation (HCFM) is provided to reduce the sideband current harmonic components and vibro-acoustic responses. The principle and implementation of the proposed HCFM technique are firstly presented in which the fixed carrier frequency is improved with the sawtooth and random signal-based coupling modulation based on the rotor position. The analytical derivations with the power spectral density method are also proposed. For verification, the experiment tests are conducted on a prototype 12/10 PMSM and microcontroller unit. The effectiveness of the HCFM technique can hence be confirmed considering the sideband vibro-acoustics reduction operated more effectively than that in a conventional random PWM. The proposed approach may provide a new route in noise-cancelling and electromagnetic compatibility for the electric drive powertrain.

Keywords: permanent magnet synchronous motor; sideband harmonic component; space vector pulse-width modulation; carrier frequency modulation; vibro-acoustic responses

1. Introduction

In recent years, permanent magnet synchronous motor (PMSM) driven with pulse width modulation (PWM) techniques have been widely used in electric vehicles (EVs) and hybrid electric vehicles (HEVs) [1–3]. The high-frequency vibro-acoustics responses introduced by sideband voltage/current harmonics are mainly located and contribute to the carrier frequency and its multiples that produce more noise, vibration, and harshness (NVH) challenges during the design and optimization stages [4].

Electromagnetic vibro-acoustics have been confirmed as the most significant contributor to the PMSM system due to electromagnetic forces on the stator and rotor caused by the air-gap magnetic fields. The research on the harmonic components and vibro-acoustics has been investigated by several researchers in the following aspects:

1. Regarding the analytical prediction, the sideband voltage and current harmonic spectrum under pulse width modulation (PWM) are studied as the mechanism by using the double Fourier series [5]. The analytical model of the symmetrical regular sampled the expressions of sideband phase voltage and the corresponding harmonic currents are proposed to derive the pattern of the sideband components in each relation [6]. Further investigations into the sideband electromagnetic vibration are then provided by analyzing...
In sum, the high-frequency sideband vibro-acoustics is related to the voltage source inverter (VSI) and space vector PWM (SVPWM). The fixed carrier frequency causes the current harmonic components to contribute around the carrier frequency and its multiples. The ear-piercing vibro-acoustic responses can thus be produced. In order to reduce the sideband components, some effective methods have been presented. The most representative method is called the harmonics spread spectra (HSS) technique, used in modulations such as periodic signal-based dithering modulation \cite{14,15} and random pulse width modulation (RPWM) \cite{16}. The periodic dithering modulation transfers the fixed carrier frequency to the determined periodic ways including the sawtooth, sinusoidal, and square \cite{12,15}. Alike to the RPWM, the carrier frequency would be randomly distributed within a defined range \cite{17}. However, the related single-HSS technique would exhibit a saturating and limiting effect such that the sideband components cannot be further reduced \cite{18}, in addition to other negative influences, increasing the total harmonic distortion (THD) \cite{19} and electromagnetic interference (EMI) \cite{20} on the power converter system.

Consequently, this paper presents a novel hybrid carrier frequency modulation (HCFM) method to modify the conventional space vector pulse width modulation (SVPWM) technique. Based on the single-HSS method, the fixed carrier frequency in the HCFM can be dithered by coupling the sawtooth and random signal-based patterns with different rotor position angles. Moreover, the explanation of sideband harmonics reduction has been also proposed with the power spectral density method. The corresponding experimental tests are presented with great practical significance to verify the reduction effects of the proposed HCFM.

The paper is organized into five sections. Following this introduction, the principle and implementation of the HCFM technique are presented in Section 2. The experimental setup is presented in Section 3. Then, the sideband current harmonics and vibro-acoustic results among the SVPWM, RPWM, and the proposed HCFM are presented in Section 4. Finally, the conclusion is summarized in Section 5.

2. Hybrid Carrier Frequency Modulation

2.1. Principle and Implementation of the HCFM

The implementation of the conventional SVPWM is shown in Figures 1 and 2. Considering Sector I as an example, the output PWM signals, q-axes voltages, and q-axes current are shown in Figure 3.
Figure 1. Implementation of the control system.

Figure 2. Principle of the conventional SVPWM.
With the $d$-$q$ voltage calculated models, the harmonic analysis can be developed with quantitative analysis of the $q$-axes voltage that confirms that the sideband harmonic components are related to the time duration of the zero-vectors $T_0$ and $T_7$. Moreover, the time durations are also related to the rotor position [19]. Hence, the hybrid carrier frequency modulation is based on the rotor position to combine the corresponding periodic and random signal-based dithering methods that are presented in Figure 4.

As can be observed in Figure 4, the carrier frequency in this study is set at 8000 Hz. The periodic signal, associated with a sawtooth form, is employed while the regular random signal is also employed. The dithering range is set as 1000 Hz that indicates the modified carrier frequency is distributed between 7000 Hz and 9000 Hz. Changing with the rotating angles of the rotor, the carrier frequency dithers per 30 degrees to which the triangle waves of the carrier frequency are also presented.
2.2. Explanation of the Sideband Harmonics Reduction

Neglecting the stator resistance, the d-q axes voltages in Sector I can be expressed as

\[
\begin{align*}
U_d &= -\omega_c L_d i_d \\
U_q &= \omega_c (L_d i_d + \psi_f)
\end{align*}
\]  

(1)

where \(\omega_c\) is the rotating electric angle; \(i_d\) and \(i_q\) are the d-q axes current components; \(\psi_f\) is the PM flux; and \(L_d\) and \(L_q\) are the d-q axes synchronous inductance components.

The working angle in the steady-state can be expressed as \(\delta\). Acting with \(U_d\) and \(U_q\) separately, d-q-axes voltages can be expressed, respectively, as

\[
\begin{align*}
U_{d4d} &= \frac{2}{3} U_{dc} \sin(\theta - \delta) \\
U_{d4q} &= \frac{2}{3} U_{dc} \cos(\theta - \delta) \\
U_{q6d} &= -\frac{2}{3} U_{dc} \sin(\delta - \theta + \frac{\pi}{3}) \\
U_{q6q} &= \frac{2}{3} U_{dc} \cos(\delta - \theta + \frac{\pi}{3})
\end{align*}
\]  

(2)

The analysis of the sideband harmonics can be proposed by the quantitative analysis of the q-axis voltages.

\[
U_q = \frac{T_4}{T_s} U_{4q} + \frac{T_6}{T_s} U_{6q}
\]  

(4)

where \(T_4\) and \(T_6\) can be expressed by using the phase voltage amplitude \((U_m)\) and the vector clamping angle \((\theta)\).

\[
\begin{align*}
T_4 &= \sqrt{3} \frac{U_m}{U_{dc}} T_s \sin(60^\circ - \theta) \\
T_6 &= \sqrt{3} \frac{U_m}{U_{dc}} T_s \sin \theta \\
T_0 &= T_s - T_4 - T_6 \\
&= T_s \left[ 1 - \frac{\sqrt{3} U_m}{U_{dc}} \cos(\theta - 30^\circ) \right]
\end{align*}
\]  

(5) \hspace{1cm} (6) \hspace{1cm} (7)

With Equations (2), (3), and (7) in sum, the phase wave in one period can be obtained as depicted in Figure 3. Then, the first and second orders of current harmonics, \(\Delta i_{q1}\) and \(\Delta i_{q2}\), can be presented as

\[
\begin{align*}
\Delta i_{q1} &\approx \frac{U_m T_s}{4 \sqrt{3} L_q} \cdot \left[ \sin(60^\circ + \theta) - \sin \frac{\theta}{3} \right] \\
\Delta i_{q2} &\approx \frac{U_q T_s}{4 L_q} \cdot \left[ 1 - \frac{\sqrt{3} U_m}{2 U_{dc}} \right]
\end{align*}
\]  

(8) \hspace{1cm} (9)

In reference to Equations (8) and (9), the second order of current harmonics mainly affects the q-axis current harmonic components. Moreover, the magnitude of \(\Delta i_q\) is determined by \(T_0, L_q,\) and \(U_q\). In this study, \(T_0\) can thus be optimized by the rotor position to reduce the sideband harmonics. The optimized configuration is shown in Figure 4 and expressed as

\[
f_c = \begin{cases}
  f_{i1} + R_i \Delta f & \frac{k_2 \pi}{6} < \theta < \frac{(k+1) \pi}{6} \\
  f_c + f_{i2} \Delta f & \frac{(k+1) \pi}{6} < \theta < \frac{(k+2) \pi}{6}
\end{cases}
\]  

(10)

In Equation (9), \(f_c\) is the carrier frequency and \(\Delta f\) is the dithering index defining the harmonics spread range that sets 1000 Hz in this study. In addition, \(f_{i1}\) is the frequency of the sinusoidal wave, \(f_{i2}\) is the frequency of the sawtooth wave meeting as \([-1, 1]\), and \(R_i\) is the random modulation index. According to the above analysis, the novel HCFM based
on rotor position not only reduces harmonic currents but also achieves a wider spectrum extension.

2.3. Power Spectral Density Explanation of HCFM

The proposed HCFM can be considered as the sawtooth and random signal-based coupling modulation based on the rotor position. Single-HSS modulation based on the sawtooth signal method causes the sideband current harmonics from the concentrated distribution to become a wider spectrum. The sideband power can thus be reduced and the power spectral density \( S_H \) can be satisfied with the following Formula (12).

\[
S_H(f, \beta) = \sum_{n=1}^{+\infty} C_n \left\{ j_0(\frac{4n\beta}{\pi}) \delta(f - nf_c) + \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{(2k-1)^2} \left[ j_1(\frac{4n\beta}{\pi}) \delta(f - nf_c - k(2k-1)f_m) + (-1)^k \delta(f - nf_c - k(2k-1)f_m) \right] \right\}
\]

where \( f_c \) is the carrier frequency, \( f_m \) is the frequency of the sawtooth signal, \( C_n \) is the magnitude of the sideband current harmonics, \( n = 1, 2, 3 \ldots \) \( f(\cdot) \) is the Bessel function, and \( k = 1, 2, 3 \ldots \) \( \beta \) is the modulation factor related to the frequency and amplitude of the sawtooth signal.

Another single-HSS modulation based on the random signal can be expressed as (12) in which the magnitude and spreading width are determined by the mathematical expectation factor \( E(\cdot) \) and random degree coefficient \( R_e \).

\[
S_R(f, T) = \frac{1}{E[T]} \left\{ E[|S(f)|^2] + 2R_e \left[ E[S(f)e^{2\pi j f T}] E[S^*(f)] \right] \right\}
\]

where \( T \) is the switching period of the random carrier frequency, \( S(f) \) is the power spectral density of the original SVPWM, and \( S^*(f) \) is the complex conjugate function.

By coupling the sawtooth and random signals, the power spectral density of the HCFM in one rotor rotating period can hence be defined as (13). Compared with the signal-HSS method, the proposed HCFM method can further improve the reduction effect of sideband current harmonics while the uneven distribution of the RPWM random performance can be fixed as well.

\[
S_N(f, \beta) = \frac{1}{E[1/f_m]} \left\{ E[|S_H(f, \beta)|^2] + 2R_e \left[ E[S_H(f, \beta)e^{\frac{2\pi j f}{f_m}}] E[S_H^*(f, \beta)] \right] \right\}
\]

3. Experimental Test Setup

As shown in Figures 5 and 6, the test bench as set up by the prototype PMSM and the power-driven system and measurements are presented to verify the effectiveness of the proposed HCFM. The prototype 12/10 PMSM adopts the rear axle driven unit in an EV application whose parameters can be found in Table 1. The DC power is fed by 540 V–4.5 kw. Three full-bridge power modules are employed by circuit boards with the Infineon-BSM75GB120DN2. Based on the dSPACE-MicroLabBox 1103, several PWM strategies can be established with the MATLAB/Simulink and RTI Electric Motor Control Blockset. The corresponding pulse signals can be generated and controlled online.

| Table 1. Parameters of the prototype PMSM. |
|--------------------------------------------|
| **Item**                  | **Value** | **Item**                  | **Value** |
|---------------------------|-----------|---------------------------|-----------|
| Number of slots           | 12        | Type of controller        | VSI       |
| Number of poles           | 10        | Carrier frequency         | 8000 Hz   |
| Rated speed               | 2000 r/min| DC link voltage           | 540 V     |
| Rated torque              | 8 Nm      | Rated power               | 1.8 kW    |
The vibro-acoustic tests are measured by the ICP triaxial accelerometer with a sensitivity of 50 mV/g and a sound-pressure microphone (GRAS-46AE) with the sensitivity of 47.23 mV/Pa. The phase current signals are recorded by the Télétroïnix A622 current probe and processed by the Télétroïnix TDS2024c.

The operational condition of all experimental tests is set at 1000 r/min and 4 Nm in order to obtain the sideband current harmonics and vibro-acoustic responses clearly. Based on the setting rotating speed of 1000 r/min, the rotation frequency and fundamental frequency are 16.67 Hz and 83.34 Hz, respectively.
4. Results and Discussion

4.1. Sideband Current Harmonics

The experimental results of sideband current harmonics are shown in Figure 7 in which the comparison between the conventional SVPWM, RPWM, and HCFM techniques is presented by the power spectral density (PSD). The sideband components in the conventional SVPWM obtain a specific pattern characterized by two symmetric sidebands appearing around the first carrier frequency. The sidebands at $f_c \pm 2f_0$ and $f_c \pm 4f_0$, where $f_0$ is the fundamental frequency of the electrical supply, can be calculated by the rotation frequency $f_r$ (i.e., $f_r = n/60$ and $f_0 = p \cdot f_r$) Considering that $f_0$ is 83.34 Hz under 1000 r/min, the main sidebands can be obtained with four main orders where the peak magnitude is $-27.56$ dB/Hz.

![Figure 7. The sideband current harmonics with the conventional SVPWM.](image)

In terms of the RPWM and HCFM, the current harmonic reduction is presented in Figures 8 and 9. In contrast to the conventional SVPWM, the obvious orders disappear. The concentrated harmonics are extended to the specified frequency domain with a 1000 Hz dithering range as the setting. The peak magnitude in the RPWM is $-39.56$ dB/Hz, while in the HCFM it is $-48.78$ dB/Hz or even lower. The reduction effort in the proposed HCFM is more significant than that in the RPWM.

![Figure 8. The sideband current harmonics with random PWM.](image)
4.2. Sideband Vibro-Acoustic Responses

The experimental vibro-acoustics results are conducted to the frequency domain. The comparison of the sideband vibro-acoustic responses is shown in this chapter. As presented in the following figures, the vibro-acoustic responses obtain a significant correlation with the current harmonics. In Figure 10, the sideband orders of the conventional SVPWM in vibro-acoustic responses are located with \( f_c \pm f_0, f_c \pm 3f_0, \) and \( f_c \pm 5f_0. \)

As a result of the reduction techniques in Figures 11 and 12, the relatively concentrated sideband components are spread to a wider frequency band that has been cut off at the lower and upper frequencies of 7000 Hz and 9000 Hz. The original narrow-band acoustics are reduced below 50 dBA and even 40 dBA by using the HCFM. All results are presented in Table 2. As demonstrated, the proposed HCFM obtains the best sideband current harmonics reduction and sideband vibro-acoustics.
Figure 11. The sideband vibro-acoustic responses with random PWM.

Figure 12. The sideband vibro-acoustic responses with the HCFM.

Table 2. Sideband current harmonics and vibro-acoustics with the proposed PWM schemes.

| Schemes | Sideband Current Harmonics | Vibration | Acoustics |
|---------|---------------------------|-----------|-----------|
|         | Peak Frequency | Amplitude | Peak Frequency | Amplitude | Peak Frequency | Amplitude |
| SVPWM   | 8167 Hz          | −27.56 dB/Hz | 8250 Hz | 0.154 m/s² | 7750 Hz | 52.7 dBA |
| RPWM    | 8042 Hz          | −39.56 dB/Hz | 7833 Hz | 0.063 m/s² | 7833 Hz | 47.2 dBA |
| HCFM    | 7889 Hz          | −48.78 dB/Hz | 8000 Hz | 0.026 m/s² | 7750 Hz | 41.9 dBA |

5. Conclusions

In this paper, a novel PWM technique applied with hybrid carrier frequency modulation is proposed to reduce the sideband harmonics and vibro-acoustics. The proposed HCFM technique is implemented by the rotor position in which the sawtooth and random signal-based dithering methods are combined to achieve the hybrid carrier waves modulation. As demonstrated by the experimental tests, the magnitude of sideband current harmonics can be reduced by about 20 dB/Hz, resulting in a 0.12 m/s² vibration reduction.
and 10 dBA acoustic noise reduction. The reduction efforts in the HCFM are more significant than those in the RPWM. The results can provide a reference for multi-strategy control to achieve low vibro-acoustics and EMI reduction in EV and HEV application in addition to other PWM-fed motor applications. Further work including efficiency evaluation, torque ripple, and controller loss can be studied based on the proposed investigation.

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**References**

1. Momen, F.; Rahman, K.; Son, Y. Electrical propulsion system design of Chevrolet Bolt battery electric vehicle. In Proceedings of the 2016 IEEE Energy Conversion Congress and Exposition (ECCE), Milwaukee, WI, USA, 18–22 September 2017; IEEE: Piscataway, NJ, USA, 2017.
2. Yang, Z.; Shang, F.; Brown, I.P.; Krishnamurthy, M. Comparative Study of Interior Permanent Magnet, Induction, and Switched Reluctance Motor Drives for EV and HEV Applications. *IEEE Trans. Transp. Electrific.* 2015, 1, 245–254. [CrossRef]
3. A Hannan, M.; Azidin, F.; Mohamed, A. Hybrid electric vehicles and their challenges: A review. *Renew. Sustain. Energy Rev.* 2014, 29, 135–150. [CrossRef]
4. Fang, Y.; Zhang, T. Sound Quality of the Acoustic Noise Radiated by PWM-Fed Electric Powertrain. *IEEE Trans. Ind. Electron.* 2017, 65, 4534–4541. [CrossRef]
5. Liang, W.; Luk, P.; Fei, W.-Z. Analytical Investigation of Sideband Electromagnetic Vibration in Integral-Slot PMSM Drive with SVPWM Technique. *IEEE Trans. Power Electron.* 2016, 32, 4785–4795. [CrossRef]
6. Liang, W.; Fei, W.; Luk, P.-C.-K. An Improved Sideband Current Harmonic Model of Interior PMSM Drive by Considering Magnetic Saturation and Cross-Coupling Effects. *IEEE Trans. Ind. Electron.* 2016, 63, 4097–4104. [CrossRef]
7. Qiu, Z.; Chen, Y.; Liu, X.; Kang, Y.; Liu, H. Analysis of the sideband current harmonics and vibro-acoustics in the PMSM with SVPWM. *IET Power Electron.* 2020, 13, 1033–1040. [CrossRef]
8. Wu, L.J.; Zhu, Z.Q.; Staton, D.A.; Popescu, M.; Hawkins, D. Comparison of Analytical Models of Cogging Torque in Surface-Mounted PM Machines. In Proceedings of the XIX International Conference on Electrical Machines, Rome, Italy, 6–8 September 2010; IEEE: Piscataway, NJ, USA, 2010; pp. 1–6.
9. Fei, W.; Zhu, Z.Q. Comparison of Cogging Torque Reduction in Permanent Magnet Brushless Machines by Conventional and Herringbone Skewing Techniques. *IEEE Trans. Energy Convers.* 2013, 28, 664–674. [CrossRef]
10. Grasso, E.; Palmieri, M.; Corti, F.; Nienhaus, M.; Cupertino, F.; Grasso, F. Detection of stator turns short-circuit during sensorless operation by means of the Direct Flux Control technique. In Proceedings of the 2020 AEIT International Annual Conference (AEIT), Catania, Italy, 23–25 September 2020; pp. 1–6.
11. Huang, Y.; Xu, Y.; Zhang, W.; Kou, J. Hybrid periodic carrier frequency modulation technique based on modified SVPWM to reduce the PWM noise. *IET Power Electron.* 2019, 12, 515–520. [CrossRef]
12. Qiu, Z.; Kang, Y.; Chen, Y.; Liu, X.; Gu, F. Investigation into Periodic Signal-based Dithering Modulations for Suppression Sideband Vibro-acoustics in PMSM Used by Electric Vehicles. *IEEE Trans. Energy Convers.* 2021, 1. [CrossRef]
13. Pindoriya, R.M.; Rajpurohit, B.S.; Kumar, R. A Novel Application of Harmonics Spread Spectrum Technique for Acoustic Noise and Vibration Reduction of PMSM Drive. *IEEE Access* 2020, 8, 103273–103284. [CrossRef]
14. Huang, Y.; Xu, Y.; Li, Y.; Yang, G.; Kou, J. PWM Frequency Voltage Noise Cancelation in Three-Phase VSI Using the Novel SVPWM Strategy. *IEEE Trans. Power Electron.* 2017, 33, 8596–8606. [CrossRef]
15. Xu, Y.; Yao, Y.; Yuan, Q.; Kou, J.; Shang, J. Reduction of the acoustic noise in PMSM drives by the periodic frequency mod-ulation. In Proceedings of the 2011 International Conference on Electrical Machines and Systems, Beijing, China, 20–23 August 2011; pp. 1–5.
16. Ruiz-Gonzalez, A.; Meco-Gutierrez, M.; Heredia-Larrubia, J.; Perez-Hidalgo, F.; Vargas-Merino, F. Pulse width modulation technique with harmonic injection in the modulating wave and discontinuous frequency modulation for the carrier wave to reduce vibrations in asynchronous machines. *IET Power Electron.* 2019, 12, 2865–2872. [CrossRef]

17. Zhang, W.; Xu, Y.; Ren, J.; Su, J.; Zou, J. Synchronous random switching frequency modulation technique based on the carrier phase shift to reduce the PWM noise. *IET Power Electron.* 2020, 13, 892–897. [CrossRef]

18. Wang, G.; Zheng, B.; Xu, Y. A Novel SVPWM Strategy for High-Frequency Noise Suppression of Dual Three-phase PMSM. In Proceedings of the PCIM Asia 2018, International Exhibition and Conference for Power Electronics, Intelligent Motion, Renewable Energy and Energy Management, Shanghai, China, 26–28 June 2018; pp. 1–4.

19. Chen, K.; Xie, Y. Multiphase optimal injection PWM with dual carrier frequency to reduce current THD. *IET Power Electron.* 2017, 10, 1061–1076. [CrossRef]

20. Lee, K.; Shen, G.; Yao, W.; Lu, Z. Performance Characterization of Random Pulse Width Modulation Algorithms in Industrial and Commercial Adjustable-Speed Drives. *IEEE Trans. Ind. Appl.* 2017, 53, 1078–1087. [CrossRef]