A Reliability of Measurement Based Algorithm for Adaptive Estimation in Sensor Networks

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Abstract—In this paper we consider the issue of reliability of measurements in distributed adaptive estimation problem. To this aim, we assume a sensor network with different observation noise variance among the sensors and propose new estimation method based on incremental distributed least mean-square (IDLMS) algorithm. The proposed method contains two phases: I) Estimation of each sensors observation noise variance, and II) Estimation of the desired parameter using the estimated observation variances. To deal with the reliability of measurements, in the second phase of the proposed algorithm, the step-size parameter is adjusted for each sensor according to its observation noise variance. As our simulation results show, the proposed algorithm considerably improves the performance of the IDLMS algorithm in the same condition.

Index Terms—adaptive filter, distributed estimation, sensor network, IDLMS algorithm.

I. INTRODUCTION

C onsider a wireless sensor network composed of distributed sensor nodes as shown in Fig. 1. The purpose is to estimate an unknown vector \( w^o \) from multiple spatially independent but possibly time-correlated measurements collected at \( N \) nodes in a network. Each node \( k \) has access to time-realizations \( \{d_k(i), u_{k,i}\} \) of zero-mean spatial data \( \{d_k, u_k\} \) where each \( d_k \) is a scalar measurement and each \( u_k \) is a \( 1 \times M \) row regression vector. We assume that the unknown vector relates to the as:

\[
d_k(i) = u_{k,i}w^o + v_k(i),
\]

(1)

where \( v_k(i) \) is observation noise with variance \( \sigma_v^2 \) and is independent of \( \{d_k(i), u_{k,i}\} \). A number of studies have considered such a distributed estimation problem \([1]–[3]\). In \([4]–[12]\), distributed adaptive estimation algorithms using incremental optimization techniques are developed and their transient and steady-state performance analysis are also provided. The IDLMS and distributed recursive least mean-square (DRLS) \([5]\) are the examples of such algorithms. These algorithms are distributed, cooperative, and able to respond in real time to changes in the environment. In these algorithms, each node is allowed to communicate with its immediate neighbor in order to exploit the spatial dimension while limiting the communications burden at the same time. In \([13]–[19]\), diffusion implementation of distributed adaptive estimation algorithms are developed. In these algorithms, each node can communicate with all its neighbors as dictated by the network topology. Both LMS-based and RLS-based diffusion algorithms are given in the literature. In addition, for both of these cases the performance analysis can be found in \([6]\) and \([13]\) respectively. In comparison with incremental based algorithms, diffusion based methods need more communication resources while have better estimation performance. Both diffusion LMS and diffusion RLS algorithm are introduced in the literature.

In all of the mentioned distributed adaptive estimation algorithms, either equal observation noise is assumed for all the nodes in the network or same strategy is used for different variance condition. The motivation for a new estimation method stems from the following facts: 1) The equal observation noise variance is not a suitable assumption in practice, and 2) It is clear that if the issue of reliability of observations is considered, better estimation performance can be expected. In this paper, to deal with the mentioned problems and especially the issue of reliability of observations, we propose a new distributed adaptive estimation algorithm. In the proposed method which is based on IDLMS, first each sensor’s observation noise variance is estimated and in the next step, based on the estimated variances, the step-size parameter is adjusted according to estimated observation noise variances.

II. ESTIMATION PROBLEM AND THE ADAPTIVE DISTRIBUTED SOLUTION

A. Notation and Assumptions

A list of the symbols used through the paper, for ease of reference, are shown in Table I.

The subsequent equations rely on the following assumptions

\[
\begin{align*}
&u_{k,i} \text{ independent of } u_{l,i} \text{ for } k \neq l, \text{ (spatial independence)}.
&\text{For every } k, \text{ the sequence } u_{k,i} \text{ is independent over time (time independence).}
&\text{The variances of observation noise for all of the sensors do not vary with time.}
\end{align*}
\]
The optimal solution satisfies the normal equations\(^\dagger\) 
\[
\begin{align*}
\text{have access to the global statistical information} \\
\text{Note that in order to use (5) to compute} \\
\text{where in (6), the symbol } \ast \text{ denotes the Hermitian transform.}
\end{align*}
\]  

where
\[
J_k \triangleq E \left\{ |d_k - U_k w|^2 \right\}.
\]

Using (9) and (10) the standard gradient-descent implementation of (8) can be rewritten as [3-6]:
\[
w_i = w_{i-1} - \mu \sum_{k=1}^{N} \nabla J_k \left(w_{i-1}\right)
\]

By defining the as the local estimate of the \(\psi_{k}^{(i)}\) at node \(k\) and time \(i\), then \(w_i\) can be evaluated as
\[
\psi_{k}^{(i)} = \psi_{k-1}^{(i)} - \mu \left[ \nabla J_k \left(\psi_{k-1}^{(i)}\right)\right]^{\ast}, \quad k = 1, 2, \ldots, N
\]

This scheme still requires all node to share global information \(w_{i-1}\). The fully distributed solution can be achieved by using the local estimate \(\psi_{k}^{(i)}\) at each node \(k\) instead of \(w_{i-1}\),
\[
\psi_{k}^{(i)} = \psi_{k-1}^{(i)} - \mu \left[ \nabla J_k \left(\psi_{k-1}^{(i)}\right)\right]^{\ast}, \quad k = 1, 2, \ldots, N
\]

Now, we need to determine the gradient of \(J\) and replace it in (13). To do this, the following approximations are used
\[
R_{du,k} \approx d_k(i) u_{k,i}
\]

The resulting IDLMS algorithm is as follows
\[
\begin{align*}
\psi_{0}^{(i)} & \leftarrow w_{i-1} \\
\psi_{k}^{(i)} & = \psi_{k-1}^{(i)} - \mu u_{k,i}^{2} \left[ d_k(i) - u_{k,i} \psi_{k-1}^{(i)} \right]_{k=1,2,\ldots,N} \\
w_i & \leftarrow \psi_{N}^{(i)}
\end{align*}
\]

### III. PROPOSED ALGORITHM

#### A. Motivation

As mentioned in the introduction section, equal observation noise assumption for all nodes could not comply with situations in physical problems. On the other hand, although considering some noisy sensors in the network (as in [20]) is a better assumption for sensor network, but it is still far away from real scenario. Nevertheless, the results obtained in [20] reveal that considering the sensors with high observation noise will cause severe decrease in performance of the distributed adaptive estimation algorithms such as IDLMS. To address this problem and to deal with the issue of reliability measurements, a new adaptive distributed estimation algorithm where each sensor participates in the algorithm according to its observation noise variance is proposed.

#### B. Method

To deal with the mentioned conditions, it is necessary to obtain an estimate of each sensor’s observation noise. To do this, we consider the equation (1) again. If the IDLMS algorithm (i.e. (16)) is done for \(L_s\) times (where \(L_s\) is a suitably chosen integer), it is possible to have a primary estimate of \(w^o\). Now this primary estimate of \(w^o\) is used to obtain each sensors observation noise. It must be noted that this estimate of \(w^o\) is used just to obtain a primary estimate of observation noise at each sensor, and it is not the final

| Symbol | Description |
|--------|-------------|
| \(w_i\) | Weight vector estimate at iteration \(i\) |
| \(u_i\) | Regressor vector at iteration \(i\) |
| \(e(i)\) | Output estimation at iteration \(i\) |
| \(d(i)\) | Value of a scalar variable \(d\) at iteration \(i\) |
| \(u_i\) | Value of a vector variable \(u\) at iteration \(i\) |

**LIST OF THE MAIN USED SYMBOLS**

**Table I**
estimate of $w^o$. Denoting by $\psi_k^{(Ls)}$ as the estimate of $w^o$ in the $i$th iteration in $N$th node we will have:

$$\psi_N^{(Ls)} = \psi_k^{(i)} \bigg|_{k=N,i=Ls}$$

Using (1) and (17), the observation noise at each sensor can be estimated as

$$n_k(i) = d_k(i) - u_k,i \psi_N^{(Ls)}, \quad i = 1, 2, ..., Ls$$

In each node $k$, first the $n_k(i)$ is computed and then the variance of observation noise of the $k$th sensor is estimated by

$$g_k = \left( \frac{1}{Ls} \right) \sum_{i=1}^{Ls} n_k(i).$$

$$\sigma_k = \sum_{i=1}^{Ls} (n_k(i) - g_k)^2$$

As $\sigma_k$ increases, the reliability of $d_k$ decreases, so there is inverse relation between $\sigma_k$ and sensor’s reliability. Motivated by this fact we define the step-size of the our incremental distributed LMS algorithm as

$$\mu_k = \mu_{max} e^{-a \sigma_k}$$

where $\mu_{max}$ is the global step-size parameter (which is constant for all sensors) and $a$ is a positive constant. It is obvious from the definition of (21) that larger observation noise variance (i.e. $\sigma_k$) yields smaller step-size parameter. Finally, for $i \geq Ls + 1$ the IDLMS algorithm is modified as follows:

$$\psi_k^{(i)} = \psi_k^{(i-1)} - \mu_k [R_{du,k} - R_{d,k} \psi_{k-1}]^*$$

(22)

After $i \to \infty$, all of the sensors will contain the appropriate estimate of $w^o$, that is

$$\lim_{i \to \infty} \psi_k^{(i)} \to w^o, \quad k \in \{1, 2, \cdots, N\}$$

IV. SIMULATION RESULTS

In this section we present the simulation results of the proposed algorithm and compare it with the IDLMS algorithm of [6]. To this aim, we consider a network with $N = 30$ nodes and Gaussian regressors with $R_{du,k} = I$. We further assume that $\sigma_{\epsilon,k}^2 \in (10^{-3}, 10^{-1})$. The curves are obtained by averaging over 100 experiments with $\mu_{max} = 0.01$ and $M = 4$. In Fig. 2, the performance of proposed algorithm for $Ls = 20$ and $a = 10$ in comparison with the IDLMS algorithm is depicted. To compare the performance of the mentioned algorithms we use the mean-square deviation (MSD) criteria which is defined as follows

$$\text{MSD} = E \left\| w^o - \psi_k^{(Ls)} \right\|^2.$$ (24)

As it is clear from Fig. 2, the proposed algorithm has better performance in a sense of estimation performance. In Fig. 3, the $\sigma_k^2$ and the corresponding step-size parameter for each sensor in plotted.

The performance of the proposed algorithm depends on the value of $Ls$, since it determines how $\psi_k^{(Ls)}$ is close to $w^o$. In Fig. 4 the performance of the proposed algorithm for different values of $Ls$ in comparison with the IDLMS algorithm is shown. As it is clear from Fig. 4, as $Ls$ increases, better primary estimate of $w^o$ is obtained and as a result, a better final estimate of $w^o$ can be expected. It must be noticed that when the algorithm is in its steady-state, increasing the $Ls$ can not provide more better primary estimate of $w^o$. On the other hand, by choosing the $Ls$ such that the algorithm is not in its steady-state, the resulted $\psi_k^{(Ls)}$ is not close enough to $w^o$ which in turn makes a dramatically decrease in the performance of the proposed algorithm. These cases can be easily concluded from the Fig. 5 where the MSD performance of proposed algorithm for different values of $Ls$ is plotted.

The performance of the proposed algorithm also depends on the $a$ parameter, (see (21)). By increasing $a$, the assigned step-size parameters become more smaller and as a result, the proposed algorithms provides better estimation performance (lower MSD) while, on the other hand, the convergence rate
of proposed algorithm decreases. In Fig. 5 the performance of the proposed algorithm for different values of $a$ in comparison with the IDLMS algorithm is shown. In the proposed algorithm by increasing the number of sensors in the network, the convergence rate of the algorithm decreases without change in the steady-state error which is the case for IDLMS algorithm. In Fig. 6 the performance of the proposed algorithm for different number of sensors, $K$, and $a = 10$ and $L_s = 20$ in comparison with the IDLMS algorithm is plotted.

V. CONCLUSION

In this paper we considered the issue of reliability of measurements in distributed adaptive estimation algorithms. To deal with this issue we proposed a distributed adaptive estimation method based on IDLMS algorithm. The proposed algorithm contains two different phases: I) Estimating each sensor’s observation noise and II) Estimating unknown parameter using the estimated observation noise variances. Also In this paper the step-size parameter is assigned to each sensor according to its observation noise variance. As the simulation results show, the proposed method outperforms the IDLMS algorithm in the sense of estimation error under the same conditions. It also must be noticed that although in this paper we consider the IDLMS algorithm as the base for our estimation method, the proposed method can be used in other adaptive estimation algorithm like diffusion least-mean square algorithm and DRLS as we did respectively in [21].

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