Pulsar motions from neutrino oscillations induced by a violation of the equivalence principle

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We analyze a possible explanation of the pulsar motions in terms of resonant neutrino transitions induced by a violation of the equivalence principle (VEP). Our approach, based on a parametrized post-Newtonian (PPN) expansion, shows that VEP effects give rise to highly directional contributions to the neutrino oscillation length. These terms induce anisotropies in the linear and angular momentum of the emitted neutrinos, which can account for both the observed translational and rotational pulsar motions. The violation needed to produce the actual motions is completely compatible with the existing bounds.

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It is very difficult to obtain precise evidence on the characteristics of the gravitational interaction beyond the range where the Newtonian approximation holds. Only systems with very large densities of mass in rapid motion can provide suitable laboratories for such a phenomenology. One well known example is the orbital behavior of binary pulsars, which gives support to the production of gravitational waves. Type II supernovas are another interesting scenario. In this case the intense neutrino flux produced during the gravitational collapse can be sensible to subtle characteristics of the gravitational interaction. In this letter we analyze some effects on this flux that could test a possible violation of the equivalence principle.

Perhaps one of the most intriguing characteristic of the pulsar dynamics related with the supernova stage is their anomalous proper motions. There is strong observational evidence that translational velocities of pulsars include a significant component from kicks given when they are formed. Several mechanisms have been proposed to explain such kicks, but none of them is completely satisfactory. Recently it has also been pointed out that the observed rotation periods are several orders of magnitude shorter than the predictions for the cores of the protoneutron stars. Thus the spin of the pulsars are probably produced by the same mechanism that gives them their translational velocities during the formation stage. Moreover, there is significant observational evidence that seems to indicate a polarization of the motion of young pulsars along a direction near the plane of the galaxy. This correlation could mean that kicks involve a characteristic length at least of the order of the galaxy radius, which is very difficult to explain on the basis of the proposed mechanisms.

An appealing possibility that could account for the translational kick is a 1% anisotropy in the momentum carried by the neutrinos emitted during pulsar formation. However small, such anisotropy is not easy to obtain. Kusenko and Segre (KS) have proposed a mechanism based on the deformation of the resonance surface when neutrinos undergo matter oscillations in the presence of a magnetic field. Unfortunately the necessary magnetic field is relatively high, $B \gtrsim 10^{15}$ G. Furthermore, the condition that the resonance surface has to lie between the neutrinospheres implies $m_\nu \sim 100$ eV. The existence of such heavy neutrinos is cosmologically ruled out unless they are unstable.

A less orthodox mechanism for neutrino oscillation was proposed several years ago. It requires a flavor dependent coupling of neutrinos to gravity, and no neutrino mass. Consequences of such a violation of the equivalence principle (VEP) in the neutrino sector have been analyzed in a number of papers. In particular, in Ref. it was applied to the problem of the translational motion of pulsars. In this case the desired kick can be achieved with massless neutrinos, but the intensity of the magnetic field is similar to the one required by the KS mechanism.

In this work we propose a purely gravitational explanation for both the translational and rotational motion of pulsars, where the neutrino oscillation and the momentum anisotropy are induced by VEP effects and that do not rely on the magnetic field of the protostar. We work within the framework of a generalized parametrized post-Newtonian (PPN) formalism, previously applied to the solar neutrino problem, that naturally includes the effect of a preferred reference system. Our approach generalizes the usual VEP scheme by including the effect of potentials of the next PPN order to the Newtonian potential $U$, and a tensorial potential of the same order than $U$. In principle all these terms should be present if the equivalence principle is violated. In this context the neutrino oscillations are a manifestation of a VEP effect, and the momentum anisotropy is signature of the preferred reference system. The accuracy of the equivalence principle may be characterized by limits in the differences of the PPN
parameters for different neutrinos. As we show, violations of the equivalence principle consistent with the present bounds generate the necessary kicks to produce the observed pulsar motions.

The linearized Dirac equation for massless neutrinos in a static gravitational field leads to the dispersion relation \([15]\):

\[
E = p \left[ 1 + h_{oo} \hat{p}_i - \frac{1}{2} h_{ij} \hat{p}_i \hat{p}_j - \frac{1}{2} h_{oo} U \right],
\]

where the \(h^{\mu \nu}\) fields are defined by \(g^{\mu \nu} = \eta^{\mu \nu} + h^{\mu \nu}\), referred to the Minkowskian metric. In deriving this relation we have neglected the spatial derivatives of the gravitational potentials, which is justified for neutrinos in astrophysical systems. Up to third order in the velocity of the source \(w\) we can write \((G = \hbar = c = 1)\):

\[
h_{oo} = 2 \gamma U + \mathcal{O}(w^4),
\]

\[
h_{oi} = -7 \frac{2}{\Delta_1} V_i - \frac{1}{2} \Delta_2 W_i + (\alpha_2 - \frac{1}{2} \alpha_1) v_i U - \alpha_2 v_j U_{ij} + \mathcal{O}(w^4),
\]

\[
h_{ij} = 2 \gamma U \delta_{ij} + \Gamma U_{ij} + \mathcal{O}(w^4).
\]

The adimensional parameters of the PPN expansion are \(\gamma, \gamma', \Delta_1, \Delta_2, \Gamma, \nu, \alpha_1, \) and \(\alpha_2\). The parameters \(\alpha_1\) and \(\alpha_2\) vanish in Lorentz covariant theories, but if there exists a preferred reference frame, characterized by a velocity \(\nu\), they should be non-null.

The general expressions for the potentials \(U, V_i, W_i\) and \(U_{ij}\) can be found in Ref. \([14]\). In the present case, the source of the gravitational field is the protoneutron star. Considering a spherical configuration and a rigid rotation, the PPN potentials become

\[
U = 4 \pi \int_0^R dr' r^2 \left[ \frac{1}{r} \theta (r - r') + \frac{1}{r'} \theta (r' - r) \right] \rho (r'),
\]

\[
U_{ij} = \hat{r}_i \hat{r}_j I (r) + \delta_{ij} J (r),
\]

\[
V_i = W_i = w_i J (r).
\]

where \(\rho (r)\) is the mass distribution of the star and \(w_i = \epsilon_{ijk} \Omega_j r_k\). Here \(\Omega\) is the angular velocity and

\[
I = \frac{4 \pi}{3} \int_0^R dr' r'^3 \left[ \frac{\rho (r - r')}{r'^3} + \frac{\rho (r')}{r^3} \right] \rho (r') + \gamma (r - r') \rho (r'),
\]

\[
J = \frac{4 \pi}{3} \int_0^R dr' \rho (r - r') \rho (r') + \gamma (r - r') \rho (r') + \gamma (r - r') \rho (r').
\]

In presence of VEP all the PPN parameters can depend on the flavor numbers. We assume that deviations from a metric theory are small, so that in a very good approximation there is a common coordinate frame for all flavors. Since the parameters are flavor dependent, distinct neutrinos will undergo different phase shifts when passing through the same sector of space. In the presence of neutrino mixing phase shift differences become observable as neutrino oscillations. For simplicity, in what follows we consider two neutrino flavors, \(\nu_e\) and \(\nu_{\mu}\) or \(\nu_{\tau}\). They are supposed to be linear superpositions of the gravitational eigenstates \(\nu^0\) and \(\nu^2\), with a mixing angle \(\theta_g\). Along the neutrino path flavor evolution is governed by

\[
\frac{d}{dr} \begin{pmatrix} \nu_e \\ \nu_{\mu} \end{pmatrix} = \frac{\Delta_0}{2} \begin{pmatrix} \cos \theta_g & \sin \theta_g \\ -\sin \theta_g & \cos \theta_g \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_{\mu} \end{pmatrix},
\]

with \(\Delta_0 = E^2 - E^1\). For a rotating protoneutron star we have

\[
\Delta_0 = \left\{ -(\delta \gamma' + \delta \gamma) U - \delta \Gamma J - \delta \Gamma I (\hat{r} \cdot \hat{p})^2 \\
+ \left[ (\delta \alpha_2 - \frac{1}{2} \delta \alpha_1) U - \delta \alpha_2 J \right] \nu \cdot \hat{p} - \delta \alpha_2 I (\hat{r} \cdot \nu) \hat{r} \cdot \hat{p} \\
- \frac{1}{2} (7 \delta \Delta_1 + \delta \Delta_2) J \Omega \times r \cdot \hat{p} \right\} E,
\]

where \(E = p\) is the neutrino energy, \(\delta \gamma = \gamma^2 - \gamma^1\), and the same for the difference between the other PPN parameters. Here \(\Delta_0\) plays the same role as the quantity \((m^2_2 - m^2_1)/2E\) in the mass mechanism for neutrino oscillations. Note
that in our case the potentials depend on \( r \) and hence \( \Delta_0 = \Delta_0(r) \). Terms with \( \mathbf{v} \) appear whenever a preferred frame exists. In principle \( \mathbf{v} \) could also depend on the gravitational flavor, but the observed position offset for pulsar-supernova remnant pairs \( \mathbf{p} \) can be interpreted as the existence of a translational effect associated to a preferred direction. For this reason we take \( \mathbf{v} \) as a flavor independent parameter. Its action is analogous to the one produced by a magnetic field in the KS mechanism.

As is well known, neutrino oscillations in matter differ from the oscillations in vacuum. The interaction of neutrinos with the background modifies their dispersion relations, and under favorable conditions leads to the MSW phenomena of resonant flavor transformation. If electrons are the only leptons present in the medium, the term \( \frac{G_F N_e(r)}{\sigma_3} \) has to be added to the matrix in Eq. (10), where \( \sigma_3 \) is the Pauli matrix and \( N_e(r) \) denotes the electron number density. The resulting Hamiltonian can be diagonalized at every point by a local rotation, with the mixing angle in matter \( \theta_m(r) \) given by

\[
\sin 2\theta_m(r) = \frac{\Delta_0(r)}{2G_F N_e(r)} \tan 2\theta_g \cos 2\theta_g .
\]

(12)

There is a resonance when the diagonal elements of the Hamiltonian vanish, i.e. when \( \sqrt{2G_F N_e(r_R)} = \Delta_0(r_R) \cos 2\theta_g \). The efficiency of the flavor transformation depends on the adiabaticity of the process, which is characterized by the parameter

\[
\kappa = \left| \frac{d\theta_m}{dr} \right|_{r=r_R} = \left| \frac{\Delta_0 \sin 2\theta_g \tan 2\theta_g}{h_{N_e}^{-1} - h_{\Delta_0}^{-1}} \right|_{r=r_R} .
\]

(13)

where the scale heights are \( h_{N_e}^{-1} = \frac{d}{dr} \ln N_e \) and \( h_{\Delta_0}^{-1} = \frac{d}{dr} \ln \Delta_0 \). The transition will be adiabatic whenever \( \kappa \gg 1 \).

The translational kick comes from the anisotropy in the radial momentum carried by neutrinos emerging from the resonance surface. The resulting effect on the motion of the pulsar is obtained integrating over all the surface. To this integration will only contribute the radial component of \( \mathbf{p} \). Therefore, to estimate the translational kick we use a simplified situation with a purely radial neutrino flux. In this case,

\[
\Delta_0 = [A(r) + B(r)v \cos \chi] E ,
\]

(14)

where \( \chi \) is the angle between \( \mathbf{r} \) and \( \mathbf{v} \). The functions \( A(r) \) and \( B(r) \) are given by

\[
A = - (\delta \gamma' + \delta \gamma) U - \delta \Gamma (I + J) ,
\]

(15)

\[
B = (\delta \alpha_2 - \frac{1}{2} \delta \alpha_1) U - \delta \alpha_2 (I + J) .
\]

(16)

The radius of a point on the distorted resonance surface can be written as \( r_R = r_o + \delta \cos \chi (\delta \ll r_o) \). The radius of the unperturbed resonance sphere \( r_o \) is determined by

\[
A(r_o) = \frac{\sqrt{2G_F N_e(r_o)}}{\cos 2\theta_g} E ,
\]

(17)

and

\[
\delta = \frac{B}{A h_{N_e}^{-1} - h_{A}^{-1}} \frac{v}{r_o} ,
\]

(18)

where we keep only the terms linear in \( \delta \), and \( h_{A}^{-1} = \frac{d}{dr} \ln A(r) \).

At the moment there is no agreement about the details of the production of a kick by a distorted neutrinosphere. To explore the possibilities of the VEP mechanism we will now consider this effect in the context of the main neutrinosphere models proposed.

For a hard neutrinosphere model in thermal equilibrium as considered in Refs. [3,4], the momentum asymmetry in the \( \mathbf{v} \) direction is generated by the emission at points with different temperatures on the resonance surface: \( \Delta p / p \approx \frac{2}{3} h_T^{-1} \delta \), where \( h_T^{-1} = \frac{d}{dr} \ln T \). In the case of a quasi-degenerate gas of relativistic electrons with a constant chemical potential \( \mu_e \approx (3\pi^2 N_e)^{1/3} \) and \( \frac{dN_e}{dT} = \frac{3}{2} T \mu_e \). Then
\[ \frac{\Delta p}{p} \approx Q \frac{B v}{A}, \]  

(19)

with \( Q = \frac{v^2 A}{2} \), where \( \eta = \mu_e / T \) is the degeneracy parameter for the electrons and \( \Lambda = h_A / (h_A - h_{N_e}) \). Another possibility is to assume that the electron fraction \( Y_e \) remains constant and \( \rho \sim T^3 \). In this case \( h_{N_e} \sim h_T / 3 \), and \( Q = \frac{2}{3} \Lambda \).

A different kick model in the literature uses a soft neutrinosphere \[1, 11\]. In such a case there is an important reduction in the anisotropy given by the ratio \( \delta \rho / \rho \) of the density at the resonance and the density at the core. The momentum asymmetry can also be written as in Eq. (13), with \( Q = \rho_0 h_{N_e} / 18 m_e \), where \( m_e = f_{r_e} \rho \, dr \) is the integral of the mass density between the central core and the surface of the star. In all the cases considered above the adimensional parameter \( Q \) depends only on the specific model and the remaining factors contain the PPN parameters.

For a quantitative estimation of the effects of VEP on the neutrinosphere, we use the density profile \( \rho(r) = \rho_c \) for \( r < r_c \) and \( \rho(r) = \rho_c (r_c / r)^n \) for \( r > r_c \). We take \( \rho_c = 8 \times 10^{14} \) g/cm\(^3\), \( r_c = 10 \) km, and \( 5 \leq n \leq 7 \) that give a good description of the supernova SN1987A \[14\]. The resonance surface has to lie below the \( \nu_e \) neutrinosphere and above the \( \nu_\mu \) neutrinosphere. If we take \( \rho_0 \sim 10^{-11} \) g/cm\(^3\) and \( Y_e \sim 0.1 \), then we obtain \( \left( \delta \gamma + \delta \gamma' + 0.95 \delta \Gamma \right) \cos 2\theta_g \approx -6 \times 10^{-10} \). For \( \delta \Gamma = 0 \) our result agrees with the one obtained in Ref. \[3\]. As pointed out in this work the adiabaticity condition is achieved provided that \( \theta_g > 10^{-4} \), \( h_{N_e}^{-1} \lesssim h_\Lambda^{-1} \), and hence \( \Lambda \approx 1 \) for every value of \( n \). The value of the momentum asymmetry is

\[ \frac{\Delta p}{p} \approx -Q (\delta \alpha_1 - 0.1 \delta \alpha_2) v \cos 2\theta_g \times 10^9. \]  

(20)

For \( T = 3 \) MeV, \( Q \sim 0.1 \) in the hard neutrinosphere models, and \( Q \approx 4 \times 10^{-5} \) for a soft neutrinosphere. Taking \( \delta \alpha_1 \approx \delta \alpha_2 \approx \delta \alpha \), and requiring \( \Delta p / p \sim 0.01 \), we obtain \( \delta \rho / \rho \sim 10^{-10} \) and \( \delta v \alpha \sim 10^{-7} \), respectively.

We now analyze the effect of the nonradial component of the neutrino momentum. When \( \Omega = 0 \), at a given point of the resonance surface the emitted neutrinos have an azimuthal symmetry respect to the position vector. For nonvanishing angular velocity of the protoneutron star, the last term in Eq. \[14\] brakes this symmetry and produces an angular acceleration of the star. To make a perturbative estimation of this effect we ignore the dependence of \( \Delta \alpha \) on \( v \) and adopt a very simple model of a hard resonance surface at \( r_0 + \delta r \). From the resonant condition we get

\[ \delta r = \frac{C}{A} \Lambda h_{N_e} \Omega \cdot r \times \hat{p} \bigg|_{r_0}, \]  

(21)

where \( C(r) = -\frac{1}{2} (7 \delta \Delta_1 + \delta \Delta_2) J(r) \).

Neutrinos emitted in different directions come from regions at different \( r \) and therefore at different energies. Hence they have different angular momenta. If we adopt the Stefan-Boltzmann law for the neutrino flux at the resonance surface, a neutrino emitted in a direction \( \hat{p} \) has a momentum \( p = E_o (1 + 4 h^{-1} \delta r) \), where \( E_o = E(r_o) \). Therefore it carries an angular momentum

\[ l = r_o E_o (\hat{r} \times \hat{p}) \left[ 1 + 4 h^{-1} \delta r \right]. \]  

(22)

By integrating at each point of the resonance surface over all directions and also over all the points, we compute the angular momentum gained by the star. Because of the symmetry of the system the resulting angular acceleration points along the rotational axis. The time derivative of the total angular momentum can be expressed as

\[ \dot{L} = \frac{C \Lambda h_{N_e} \xi \Omega_{r_o}}{3 \pi A h_T} \left[ \frac{\int_0^\pi \sin \theta \, d\theta \frac{\int_0^\pi \sin \theta \, d\theta' \int_0^{2\pi} \sin \theta' r_o \left( \hat{\Omega} \times \hat{r} \times \hat{p} \right)^2}{\int_0^\pi \sin \theta \, d\theta \frac{\int_0^{2\pi} \sin \theta' \, d\theta'}{\int_0^\pi \sin \theta \, d\theta' \sin \theta}} \right], \]  

(23)

where \( \dot{\xi} \) is the energy carried by the neutrinos per time unit, and a factor \( \frac{1}{6} \) has been included to take into account that although six neutrino and antineutrino species are radiated, only one comes from the distorted neutrinosphere. In the latter expression \( \theta \) is the angle between the radius vector \( \hat{r} \) and the angular velocity \( \Omega \), while \( \theta' \) and \( \varphi' \) are the spherical coordinates for \( \hat{p} \) taking \( \hat{r} \) as the z axis. From Eq. \[23\]

\[ \Omega(t) = \Omega_o \exp \left( \frac{4r_o^2}{27} \int_0^t \frac{C \Lambda h_{N_e} \xi}{A T h_T} \frac{dt}{h_T} \right), \]  

(24)

where \( I \) and \( \Omega_o \) are the momentum of inertia and the initial angular velocity of the protostar. It should be noted that the rotational kick does not require a velocity \( v \) associated to a preferred frame.
As an example, let us consider the density profile introduced above. Assuming that all the quantities in the integrand except $\mathcal{E}$ are constant during the cooling period and taking $\Delta \mathcal{E} \sim 3 \times 10^{55}$ erg, the angular velocity after the angular kick is

$$\Omega_f \simeq \Omega_o \exp \left[ \xi \left( \delta \Delta_1 + \frac{1}{l} \delta \Delta_2 \right) \frac{h_{N_\nu}}{h_T} \cdot 10^8 \right], \quad (25)$$

where $0.1 < \xi < 10$ for $5 < n < 6$. The ratio $h_{N_\nu}/h_T$ is a model dependent quantity of the order of unity. Values usually considered are $h_{N_\nu} \lesssim h_T/3$. The star angular velocity will increase or decrease depending on the sign of $\delta \Delta_1 + \frac{1}{l} \delta \Delta_2$.

If we accept that typical initial angular velocities of the protostar are $\Omega_o \sim 0.01 \Omega_f$, the VEP parameters must be in the range $10^{-6} \lesssim \delta \Delta_1 + \frac{1}{l} \delta \Delta_2 \lesssim 10^{-38}$ to reproduce the observed values for $\Omega_f$.

To estimate the order of magnitude of the translational and rotational accelerations, we have assumed the corresponding kicks decoupled one from the other. In a more realistic situation the rotational motion could produce an average in the translational kick. This effect depends on the relation between the characteristic time of reaccommodation of the neutrinosphere and the period of rotation of the star. The anisotropy axis coincides at every time with $\nu$ and is not affected by the rotation, but the temperature of the resonance surface could change. In a soft neutrinosphere the deformed resonance surface changes the atmosphere opacity over the core and induces a temperature anisotropy in the core-atmosphere interface, which in turn affects the neutrino flux. As the star rotates the resonance surface also rotates with respect to the rest frame of the star inducing a time-changing opacity over the core region.

Therefore we have here to consider the characteristic thermal response time of the system, which is of the order of a few hundred milliseconds [10], in contrast with the pulsar period. Thus, in this case we can expect an averaging of the translational kick. This effect tends to cancel the component orthogonal to the rotational axis and develops a correlation between the translational kick and the axis of rotation. In the case of a hard neutrinosphere the energy flux depends on the temperature at the point from which the neutrinos are radiated from the surface of resonance. The atmosphere here has enough heat capacity to act as a thermal reservoir with a radius dependent temperature. Therefore, there is no effective average and there is no correlation between the translational kick and the rotational axis.

For simplicity, we have assumed that the only mechanism responsible for the pulsar motion is VEP. If this were the only cause for the translational velocity, then all pulsar velocities should show a certain correlation driven by the $\nu$ parameter. This correlation will be more or less accentuated depending on how hard or soft the neutrinospheres are, and also could be blurred by the presence of other kick mechanisms besides the one here considered.

In conclusion, we have shown that resonant VEP neutrino oscillations may be responsible for both the translational and rotational motion of pulsars. Since this mechanism works even for massless neutrinos, it does not clash with cosmological bounds. The strictest boundaries known at present in the neutrino sector are given by accelerator experiments, mainly from CCFR, which correspond to the highest tested energies [17]. These experiments are sensitive to large mixing angles, because they have no access to the MSW effect. The exclusion region for these experiments extends down to $\sin^2 2\theta > 2 \times 10^{-3}$ for $\nu_e \nu_{\mu}$ and $\sin^2 2\theta > 0.2$ for $\nu_{\tau} \nu_{\mu}$, independently of the value of $\Delta_0$. Therefore, the parameter region relevant for the neutrino resonance in neutron stars, taking $10^{-4} < \theta_g < 10^{-3}$ for $\nu_e \nu_{\mu}$ or $10^{-4} < \theta_g < 10^{-1}$ for $\nu_{\tau} \nu_{\mu}$, is well outside the range tested by accelerators [18]. With respect to atmospheric neutrinos, they are not affected by these small mixing angle oscillations, and the MSW effect in the solar neutrinos corresponds to a medium of much lesser density, and thus the involved parameter sector is very different [14]. In this way the kick pulsar physics gives access to a new phenomenological sector of VEP effects.

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