Universal scaling and pairing symmetry for high-temperature cuprate superconductors

Huan-Qiang Zhou\textsuperscript{1,2}, Zu-Jian Ying\textsuperscript{1,2,3}, Mario Cuoco\textsuperscript{2,3}, Canio Noce\textsuperscript{2,3}
\textsuperscript{1}Centre for Modern Physics, Chongqing University, Chongqing 400044, People’s Republic of China
\textsuperscript{2}Dipartimento di Fisica “E. R. Caianiello”, Università di Salerno, I-84084 Fisciano, Salerno, Italy
\textsuperscript{3}CNR-SPIN, I-84084 Fisciano, Salerno, Italy

(Dated: February 23, 2010)

Since the discovery of high-temperature superconductivity in the copper oxides (cuprates), intense research has been devoted to identify universal trends among various physical properties, aimed at providing essential information to understand the mechanisms of electron pairing and condensation.

A first step in this direction was made by Uemura and coworkers\textsuperscript{1}, who discovered a linear relation between the zero-temperature superfluid density, $\rho_s(0)$, and the transition temperature, $T_c$. This relation works successfully in the under-doped regime but fails in the optimally-doped and overdoped materials. A similar trend has been uncovered for a scaling relation between $\rho_s(0)$ and the d.c. conductivity, $\sigma_{DC}$, measured at $T_c$\textsuperscript{2}. Recently, a remarkable relation $\rho_s(0) \propto \sigma_{DC}$ at $T_c$ was proposed by Homes and coworkers\textsuperscript{3}; it is claimed to work for all the cuprate superconductors, thus supporting the $d$-wave pairing symmetry. However, it has been subsequently observed that the Homes relation breaks down at least in the over-doped regime\textsuperscript{3}, if the data for $\sigma_{DC}$ are exploited from the electrical resistivity measurements. This strongly suggests that the pairing symmetry in the cuprates is not pure $d$-wave. Here, we report a scaling relation between $\rho_s(0)$ and the product of the pseudogap temperature $T^*\sigma_{DC}$ and $T_c$, with $T^*$ rescaled by the maximum transition temperature $T_{c_{\text{max}}}$

This scaling relation holds universally in the entire doping range for the hole-doped cuprates.

One of its ramifications is that the ratio $T^*/T_{c_{\text{max}}}$ reflects the extent to which the pairing symmetry deviates from pure $d$-wave pairing. It also supports the idea that the pseudogap coexists with the superconducting gap in the superconducting phase.

PACS numbers:

The scaling relation, which holds universally for a variety of the hole-doped cuprates, takes the form:

$$\rho_s(0) \equiv \lambda_{ab}^{-2} = C \frac{T^*}{T_{c_{\text{max}}}} \sigma_{DC} T_c,$$ \hfill (1)

where $\rho_s(0)$ is the superfluid density at absolute zero temperature and $\lambda_{ab}$ represents the in-plane London penetration depth, with $C \sim 0.2 \Omega (K \mu m)^{-1}$ being a universal constant. The transition from a normal state to a superconducting state occurs at the transition temperature $T_c$, usually with a narrow transition width. $T_{c_{\text{max}}}$ is the maximum transition temperature at the optimal doping level. The d.c. conductivity $\sigma_{DC}$ is extracted from the in-plane resistivity $\rho_{ab}$ measured just above the normal to superconducting transition temperature $T_c$. The pseudogap temperature $T^*$ is defined to be the temperature, below which the resistivity drops below its linear temperature dependence at high temperature\textsuperscript{2}. The nuclear magnetic resonance (NMR) relaxation rate divided by temperature $T$, $1/T_1T$, also takes a maximum value at $T^*$\textsuperscript{0}.

The scaling relation is plotted in Fig.\textsuperscript{1} for different families of the hole-doped cuprate superconductors: Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (Bi-2212), La$_{2-x}$Sr$_x$CuO$_4$ (La-214), Tl$_2$Ba$_2$CuO$_{6+\delta}$ (Tl-2201), YBa$_2$Cu$_3$O$_{6+\delta}$ (Y-123), Y$_{1-x}$Ca$_x$Ba$_2$Cu$_3$O$_{7-\delta}$ (Ca-Y123), YBa$_2$Cu$_3$O$_{6+\delta}$ (Y-124), Bi$_2$Sr$_{2-x}$La$_x$CuO$_{6+\delta}$ (La-Bi2001), Y$_{1-x}$Pr$_x$Ba$_2$Cu$_3$O$_{7-\delta}$ (Pr-Y123), and HgBa$_2$CaCu$_2$O$_{6+\delta}$ (Hg-1212). Inside the brackets are the corresponding abbreviations for these families. The families cover a wide hole doping range under the superconducting dome: $p \sim [0.05, 0.27]$, with the under-doped (over-doped) regime below (above) the optimal doping around $p \sim 0.16$. Specifically, $p \sim [0.01, 0.26]$ for Bi-2212, $p \sim [0.06, 0.24]$ for La-214, $p \sim [0.075, 0.25]$ for YCa-123, $p \sim [0.13, 0.26]$ for Tl-2201, $p \sim [0.13, 0.185]$ for La-Bi2201, and up to optimal doping for other families. Moreover, the number of the copper oxide layers per unit cell varies from one (La-214, La-Bi2201, Tl-2201) to two (Bi-2212, Hg-1212, Y-123, Ca-Y123, Pr-Y123, Y-124). As one sees in Fig.\textsuperscript{1} all the data, for compounds with a wide doping range and varied layer structures, collapse onto one single straight line. That is, the universal scaling relation (1) is valid. We remark that the d.c. conductivity is determined from the CuO$_2$ plane electrical resistivity just above $T_c$. The London penetration depth $\lambda_{ab}$, $\sigma_{DC}$ and $T_c$ for La-214, Bi-2212, YCa-123 and Tl-2201 are taken from Refs.\textsuperscript{4,7}, while the data for other compounds are taken from Refs.\textsuperscript{8–10} for Y-123, Refs.\textsuperscript{11,12} for Y-124, Refs.\textsuperscript{13,14} for Pr-Y123, Refs.\textsuperscript{15,16} for La-Bi2201, and Refs.\textsuperscript{17,19} for Hg1212. Here, the $\lambda_{ab}$ data are measured by the ac susceptibility for La-214, the field-dependent specific heat for Bi-2212, the equilibrium magnetization via a superconducting quantum interference device (SQUID) for Hg1212 and the muon spin rotation ($\mu$SR) for all the other compounds. We take the $a$ axis $\sigma_{DC}$ values versus the oxygen excess per CuO$_2$ plane for Y-123 and Y-124 to minimize the chain influence along the $b$ axis. The
inset illustrates the evolution of $T^*$ and $T_c$ with respect to the hole doping $p$, with Bi-2212 as a typical example. The data are taken from the resistivity measurements [5], with $T^*$ being the same as measured in the NMR experiments [6]. Actually, the data for $T^*/T_{c\text{max}}^*$, as shown in the inset, are also used for other compounds, since they collapse between Bi-2212 and La-214, Bi-2212 and Ti-2201 [21], La-214 and La-Bi2201 [16], as measured by means of different probes. The $T^*/T_{c\text{max}}^*$ data for Hg-1212 are similar, as shown in the NMR experiments [22], if the doping $p$ is rescaled by the optimal doping value. The evolution of $T^*$ with respect to the oxygen excess in the CuO$_2$ plane for Y-123 [23] and for Y-124 [24] are taken from the NMR measurements. In addition, $T^*$ versus Pr doping for Pr-Y123 is from the resistivity measurements [14].

We stress that, the deviation, as observed in Fig. 1 for some samples for YCa-123 (Ca doping $x = 0.2$), may be attributable to the fact that the d.c. conductivity data are extracted from thin film samples [4]. Indeed, it is expected that the d.c. conductivity, if available, would be larger for single crystal samples. The YCa-123 data will be driven closer to our scaling line, as one may judge from an estimation in terms of the available d.c. conductivity data for single crystal samples with Ca doping $x = 0.12, 0.14$ [25] by assuming that the d.c. conductivity for Ca doping $x = 0.2$ exhibits a similar behavior.

The justification of our scaling relation, unveiled in this Letter, resides in the pairing symmetry: the Homes scaling relation, if it were valid for high-temperature cuprate superconductors, would be convincing evidence for pure $d$-wave pairing symmetry for the cuprates [26]. However, as observed subsequently by Tallon and coworkers [4], there is a significant deviation from the Homes relation, with a salient feature that the deviation is increasing with increasing doping. This strongly suggests that the pairing symmetry realized in the (hole-doped) cuprates is not pure $d$-wave. In fact, growing experimental evidence [27–37] points to an admixture of an $s$-wave component to the predominant $d$-wave superconductivity in the hole-doped cuprate superconductors. In addition, an unbiased numerical simulation of the two-dimensional $t-J$ model has demonstrated that the pairing symmetry is of $d+s$-wave nature, with the $s$-wave component increasing with increasing doping [38, 39], consistent with the electronic Raman scattering experiments [29, 31]. Here, we emphasize that such an admixture arises from the fact that the tetragonal lattice symmetry is broken locally, as evidenced by the simulation of the $t-J$ model [38, 39]. This in turn explains why a $d+s$-wave superconductivity model works so well in the hole-doped cuprates [29]. Therefore, it is necessary to remove an extra contribution from the $s$-wave superconducting component to the superfluid density. Equivalently, a proper weight that accounts for the contribution to the $d$-wave component should be included in the normal state d.c. conductivity $\sigma_{\text{DC}}$. Remarkably, this weight tracks the ratio of the pseudogap temperature $T^*$ and the maximum transition temperature $T_{c\text{max}}^*$. That is, the ratio, $T^*/T_{c\text{max}}^*$, reflects the extent to which the pairing symmetry deviates from pure $d$-wave pairing. It is worth mentioning that the pseudogap temperature $T^*$ plays a similar role in a two-fluid description of the under-doped cuprate superconductors [40]: $T^*$ tracks the weight of the spin liquid component versus a non-Landau Fermi liquid.

Last but not least, our results lend further support to the picture that the pseudogap coexists with the superconducting gap in the superconducting phase [21], due to the fact that the scaling relation simultaneously involves both $T^*$ and $T_c$, two important temperature scales that dominate the phase diagram of the high-temperature cuprate superconductors.

Helpful discussions with Carmine Attanasio are acknowledged. Huan-Qiang Zhou would like to thank the Dipartimento di Fisica, Università di Salerno for hospitality during his stay.

[1] Uemura, Y. J. et al. Universal correlations between $T_c$ and $n_s/m^*$ (carrier density over effective mass) in high-Tc cuprate superconductors. Phys. Rev. Lett. 62, 2317-2320 (1989).
[2] Pimenov, A. et al. Universal relationship between the penetration depth and the normal-state conductivity in YBCO. Europhys. Lett. 48, 73-78 (1999).
[3] Homes, C. C. et al. A universal scaling relation in high-temperature superconductors. Nature 430, 539 (2004).
[4] Tallon, J. L., Cooper, J. R., Naqib, S. H. & Loram, J. W. Scaling relation for the superfluid density of cuprate superconductors: origins and limits. Phys. Rev. B 73, 180504(R)(1-4) (2006).
[5] Oda, M. et al. Strong pairing interactions in the under-doped region of Bi$_2$Sr$_2$CaCu$_2$O$_8 +\sigma$. Physica C 281, 135-142 (1997).
[6] Ishida, K. et al. Pseudogap behavior in single-crystal Bi$_2$Sr$_2$CaCu$_2$O$_8 +\sigma$ probed by Cu NMR. Phys. Rev. B 58, R5960-R5963 (1998).
[7] Tallon, J. L. et al. Superfluid density in cuprate high-$T_c$ superconductors: a new paradigm. Phys. Rev. B 68, 180501(R)(1-4) (2003).
[8] Takenaka, K., Mizuhashi, K., Takagi, H. & Uchida, S. Interplane charge transport in YBa$_2$Cu$_3$O$_7-\gamma$: spin-gap effect on in-plane and out-of-plane resistivity. Phys. Rev. B 50, 6534-6537 (1994).
[9] Lee, Y. S. et al. Electrodynamics of the nodal metal state in weakly doped high-$T_c$ cuprates. Phys. Rev. B 72, 054529-(1-13) (2005).
[10] Zimmermann, P. et al. Muon-spin-rotation studies of the temperature dependence of the magnetic penetration depth in the YBa$_2$Cu$_3$O$_y$ family and related compounds. Phys. Rev. B 52, 541-552 (1995).
[11] Shengelaya, A. et al. Muon-spin-rotation measurements...
of the penetration depth in the YBa$_2$Cu$_4$O$_8$ family of superconductors. *Phys. Rev. B* **58**, 3457-3461 (1998).

[12] Bucher, B., Steiner, P., Karpinski, J., Kaldis, E. & Wachter, P. Influence of the spin gap on the normal state transport in YBa$_2$Cu$_4$O$_8$. *Phys. Rev. Lett.* **70**, 2012-2015 (1993).

[13] Seaman, C. L. *et al.* Magnetic penetration depth of Y$_{1-x}$Pr$_x$Ba$_2$Cu$_3$O$_{6.97}$ measured by muon-spin relaxation. *Phys. Rev. B* **42**, 6801-6804 (1990).

[14] Levin, G. A. & Quader, K. F. In-plane resistivity and an explanation for the characteristic $T^*$ in high-$T_c$ cuprates. *Phys. Rev. B* **62**, 11879-11887 (2000).

[15] Russo, P. L. *et al.* Muon spin relaxation study of superconducting Bi$_2$Sr$_2$La$_2$CuO$_{6.4+}$s. *Phys. Rev. B* **75**, 054511-(1-14) (2007).

[16] Ando, Y., Komiyama, S., Segawa, K., Ono, S. & Kurita, Y. Electronic Phase diagram of high-$T_c$ cuprate superconductors from a mapping of the in-plane resistivity curvature. *Phys. Rev. Lett.* **93**, 267001-(1-4) (2004).

[17] Thompson, J. R. *et al.* Equilibrium magnetic studies of Hg-based high-$T_c$ superconductors. *Czech. J. Phys.* **46**, Suppl. S3, 1599-1600 (1996).

[18] Kwon, Y.-T. *et al.* Thermodynamic properties of five-layered HgBa$_2$Ca$_2$Cu$_2$O$_{4+y}$ from equilibrium magnetization. To appear in *Current Applied Physics* (2010).

[19] Shen, L. J. *et al.* The comparison between the pseudo gap opening temperatures in different cuprate superconductors with two CuO$_2$ planes. *Physica C* **341-348**, 941-942 (2000).

[20] Nakano, T. *et al.* Correlation between the doping dependences of superconducting gap magnitude $2\Delta_0$ and pseudogap temperature $T^*$ in High-$T_c$ Cuprates. *J. Phys. Soc. Jpn.* **67**, 2622-2625 (1998).

[21] Hüfner, S., Hossain, M. A., Damascelli, A. & Savatzky, G. A. Two gaps make a high-temperature superconductor? *Rep. Prog. Phys.* **71**, 062501(9pp) (2008).

[22] Itoh, Y. *et al.* study of bilayer HgBa$_2$CaCu$_2$O$_{6.8+}$ — variation of pseudo-spin spectrum. *J. Phys. Soc. Jpn.* **67**, 2212-2214 (1998).

[23] Yashuoka, H. Pseudo spin gap in high-$T_c$’s manifested by the NMR experiments. *Physica C* **282-287**, 119-122 (1997).

[24] Machi, T., Watanabe, N., Itoh, Y. & Koshizuka, N. Pseudo-spin-gap of (Y,Ca)(Ba,La)$_2$CuO$_4$. *Physica C* **412-414**, 342-346 (2004).

[25] Nagasao, K., Masui, T. & Tajima, S. Rapid change of electronic anisotropy in overdoped (Y,Ca)Ba$_2$Cu$_3$O$_{7-\delta}$. arXiv:0802.0061.

[26] Anderson, P.W. Present status of the theory of high Tc cuprates. *J. Low temp. Phys.*, **32**, 282-289 (2006).

[27] Müller, K. A. Two gap behavior observed in YBCO(100) a-axis, (110) c-axis tunnel Junctions. *J. Phys. Soc. Jpn.* **65**, 3090-3091 (1996).

[28] Strohm, T., & Cardona, M. Determination of the s-wave/d-wave gap ratio in YBa$_2$Cu$_3$O$_7$ from electronic Raman scattering and the LMTO band structure. *Solid State Comm.* **104**, 233 (1997).

[29] Nemetschek, *et al.* d + s wave superconductivity: analysis of the electronic Raman data of YBa$_2$Cu$_3$O$_{7-\delta}$ and other cuprates? *Eur. Phys. J. B* **5**, 495-503 (1998).

[30] Lu, D.H. *et al.* Superconducting gap and strong in-plane anisotropy in unwinned YBa$_2$Cu$_3$O$_{7-\delta}$. *Phys. Rev. Lett.* **86**, 4370 (2001).

[31] Masui, T. *et al.* Raman study of carrier-overdoping effects on the gap in high-$T_c$ superconducting cuprates. *Phys. Rev. B* **68**, 060506(R)-(1-4) (2003).

[32] Uchiyama, H., Masui, T. & Tajima, S. Gap observation in the photoemission spectra of (Y, Ca)Ba$_2$Cu$_3$O$_{7-\delta}$ with little surface effect. *J. Low Temp. Phys.* **131**, 287 (2003).

[33] Kirtley, J.R. *et al.* Angle-resolved phase-sensitive determination of the in-plane gap symmetry in YBa$_2$Cu$_3$O$_{7-\delta}$. *Nat. Phys.* **2**, 190 (2006).

[34] Khasanov, R. *et al.* Experimental evidence for two gaps in the high-temperature La$_{1.83}$Sr$_{0.17}$CuO$_4$ superconductor. *Phys. Rev. Lett.* **98**, 057007-(1-4) (2007).

[35] Khasanov, R. *et al.* Multiple gap symmetries for the order parameter of cuprate superconductors from penetration depth measurements. *Phys. Rev. Lett.* **99**, 237601-(1-4) (2007).

[36] Furrer, A. Admixture of an s-wave component to the d-wave gap symmetry in high-temperature superconductors. *J. Supercond. Nov. Magn.* **21**, 1-5 (2008).

[37] Bakr, M. *et al.* Electronic and phononic Raman scattering in detwinned YBa$_2$Cu$_3$O$_{6.95}$ and Y$_{0.95}$Ca$_{0.05}$Ba$_2$Cu$_3$O$_{6.95}$: s-wave admixture to the $d_{x^2-y^2}$-wave order parameter. *Phys. Rev. B* **80**, 064505-(1-11) (2009).

[38] Li S.-H., Shi, Q.-Q. & Zhou, H.-Q. Ground-state phase diagram of the two-dimensional $t - J$ model. arXiv:1001.3313.

[39] Zhou, H.-Q. Pairing mechanism for high temperature superconductivity in the cuprates: what can we learn from the two-dimensional $t - J$ model? arXiv:1001.3358.

[40] Barzykin, V. & Pines, D. Universal behavior and the two-component character of magnetically underdoped cuprate superconductors. *Adv. Phys.* **58**, 1-65 (2009).
FIG. 1: The superfluid density $\rho_s(0)$, at absolute zero, versus the product of the pseudogap temperature $T^*$, the d.c. conductivity $\sigma_{DC}$ measured at $T_c$, and the superconducting transition temperature $T_c$, rescaled by the maximum transition temperature $T_c^{\text{max}}$, for a variety of the hole-doped cuprates. The data, with a wide hole doping range, distribute around a universal straight line. The d.c. conductivity is determined from the $a$-$b$ CuO$_2$ plane electrical resistivity just above the normal to superconducting transition temperature $T_c$. The London penetration depth $\lambda_{ab}$, $\sigma_{DC}$ and $T_c$ for La-214, Bi-2212, YCa-123 and Ti-2201 are taken from Refs. [4-7], while the data for other compounds are taken from: Refs. [8-10] for Y-123, Refs. [11, 12] for Y-124, Refs. [13, 14] for Pr-Y123, Refs. [15, 16] for La-Bi2201, and Refs. [17-19] for Hg1212. Here, the $\lambda_{ab}$ data are measured by the ac susceptibility for La-214, the field-dependent specific heat for Bi-2212, the equilibrium magnetization via a superconducting quantum interference device (SQUID) for Hg1212 and the muon spin rotation ($\mu$SR) for all the other compounds. We take the $a$ axis $\sigma_{DC}$ values versus the oxygen excess per CuO$_2$ plane for Y-123 and Y-124 to minimize the chain influence along the $b$ axis. The inset illustrates the evolution of $T^*$ and $T_c$ with respect to the hole doping $p$, with Bi-2212 as a typical example. The data are taken from the resistivity measurements [5], with $T^*$ being the same as that from the nuclear magnetic resonance (NMR) measurements [6]. Actually, the data for $T^*/T_c^{\text{max}}$, as shown in the inset, are also used for other compounds, since they collapse between Bi-2212 and La-214 [20], Bi-2212 and Ti-2201 [21], La-214 and La-Bi2201 [16], as measured by means of different probes. The $T^*/T_c^{\text{max}}$ data for Hg-1212 are similar, as shown in the NMR measurements [22], if the doping $p$ is rescaled by the optimal doping value. The evolution of $T^*$ with respect to the oxygen excess in the CuO$_2$ plane for Y-123 [23] and for Y-124 [24] are taken from the NMR measurements. In addition, $T^*$ versus Pr doping for Pr-Y123 is from the resistivity measurements [14].