Phase-dependent heat transport through magnetic Josephson tunnel junctions

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We present an exhaustive study of the coherent heat transport through superconductor-ferromagnet(S-F) Josephson junctions including a spin-filter(I\textsubscript{s f}) tunneling barrier. By using the quasiclassical Keldysh Green’s function technique we derive a general expression for the heat current flowing through a S/F/I\textsubscript{s f}/F/S junction and analyze the dependence of the thermal conductance on the spin-filter efficiency, the phase difference between the superconductors and the magnetization direction of the ferromagnetic layers. In the case of non-collinear magnetizations we show explicitly the contributions to the heat current stemming from the singlet and triplet components of the superconducting condensate. We also demonstrate that the magnetothermal resistance ratio of a S/F/I\textsubscript{s f}/F/S heat valve can be increased by the spin-filter effect under suitable conditions.

I. INTRODUCTION

Two fields have been attracting increasing attention among several research groups in the recent years: Spintronics with superconductors\textsuperscript{1–4} and coherent caloritronics\textsuperscript{5–21}. Both fields exploit phase-dependent phenomena which are key characteristics of superconducting circuits. On the one hand, superconducting spintronics is emerging as a possible technology from the discovery of spin-polarized supercurrents\textsuperscript{3} in superconductor-ferromagnet (S/F) hybrid nanostructures. Such super currents are due to existence of triplet superconducting correlations created by magnetic inhomogeneities\textsuperscript{22}. Once generated, triplet correlations can penetrate over long distances into ferromagnets as observed in experiments on S/F/S Josephson junctions\textsuperscript{23–26}. These experiments suggest the possibility of using S/F hybrids in spintronic circuits with the aim of lowering the dissipation\textsuperscript{4}. On the other hand, the study of heat transport in nanoscale devices, \textit{i.e.} caloritronics, also attracts the attention of researchers working on nanodevices\textsuperscript{27–29} containing for example normal metal, ferromagnets\textsuperscript{30,31} and superconductors\textsuperscript{32,33}. Of particular interest is the recent experimental control of the heat current flowing through a Josephson junction by tuning the macroscopic phase-difference between two superconducting reservoirs\textsuperscript{5,34,35}, as predicted in several theoretical works\textsuperscript{36–40}.

The interplay between superconductivity and ferromagnetism in the context of heat transport has been recently used to describe a phase-tunable heat-valve in a recent theoretical work\textsuperscript{41}. The valve is a F/S/I/F Josephson junction (I denotes a non-magnetic tunneling barrier) and its operating principle is based on both phase-coherence and spin-dependent transport. Moreover, it is well known that in junctions containing S/F elements both singlet and triplet pair correlations are generated and contribute to the Josephson (charge) current and, as we will prove below, to the phase-dependent part of the heat current. If a spin-filter with a large efficiency is used as a tunneling barrier, the singlet contribution to the Josephson current is suppressed and a highly spin-polarized supercurrent can be achieved in a S/F/I\textsubscript{s f}/F/S junction provided that the magnetizations are non-collinear\textsuperscript{40} (I\textsubscript{s f} denotes the spin-filter tunneling barrier). As we shall show in the present work, this also applies for the phase-coherent part of the heat current flowing through a S/F/I\textsubscript{s f}/F/S junction.

The spin-filter effect has been intensively studied in europium chalcogenides tunneling barriers\textsuperscript{42,43,45}. This type of barriers possess very large spin-filter efficiencies (typically larger than 95\%) and, therefore, they are ideal candidates for the creation of spin-polarized currents. In tunnel junctions made of superconducting electrodes and spin-filter barriers, measurements of the tunneling conductance have revealed that the interaction between conducting electrons in the leads and the localized magnetic moments of the barrier lead to a Zeeman-splitting in the density of states of the supercon-
ducting electrodes, as theoretical expected. An experiment performed on NbN/GdN/NbN junctions has shown that the temperature dependence of the Josephson current flowing through a GdN barrier (with a spin-filter efficiency of \( \sim 75\% \)) clearly deviates from that expected in conventional S/I/S junctions, thus suggesting an interplay between magnetism of the barrier and superconducting condensate of the electrodes, as described recently in a theoretical work.

In the present work we combine ideas from S/F hybrid structures and caloritronics studies in order to analyze the phase-dependent heat transport through such structures. We extend the model proposed in Ref. for the heat transport through S/I/S and F/S/I/S/F junctions and derive compact expressions for the thermal conductance. With the help of our model we are able to study in detail the dependence of the heat conductance on the spin-filter efficiency, the superconducting phase and the relative angles between the magnetization of the ferromagnetic layers. In analogy to the charge supercurrent we shall demonstrate that the phase-dependent part of the heat current consists of two contributions stemming from singlet and triplet pair correlations, respectively. Moreover, as for the charge transport studied in Ref., the spin-filter effect suppresses the singlet contribution to thermal transport leading to spin-polarized heat currents. Finally, we show how the spin-filter barriers can be used for the enhancement of the magnetothermal resistance of Josephson heat valves as those recently proposed in Ref.

The paper is organized as follows: In Sec. II we derive a general expression describing the heat current flowing through a generic spin-filter junction. With the help of this expression, in section III we first analyze the heat conductance through a S/I/S junction as the one used in the experiments. We demonstrate that while for a zero-phase difference between the superconductors the thermal conductance increases by increasing the spin-filter efficiency, the opposite regime is achieved if the phase difference \( \phi \) equals to \( \pi \). This behavior holds in the presence of a Zeeman splitting in the superconductors and also if we neglect this field. We also show that for a large spin-filter efficiency of the barrier the maximum value of the thermal conductance depends non-monotonically on the amplitude of the Zeeman splitting. In section IIIB we consider a triplet Josephson junction consisting of a F/S/I/S/F structure, for which the magnetization direction of the outer F layers can point in arbitrary direction with respect to the spin quantization axis determined by the magnetization of the I barrier. We explicitly show the contributions of the singlet and triplet part of the condensate to the heat conductance. In section IIIC we discuss the ferromagnetic Josephson thermal valve and show that the magnetothermal resistance ratio in the structure can reach values as large as \( 10^6 - 10^8 \) at low temperature depending on the macroscopic phase and on the spin-filter efficiency of the barrier. Finally, we summarize our results in Sec. IV.

II. THE MODEL

We consider the generic Josephson junction sketched in Fig. 1(a). It consists of two S/F electrodes tunnel-coupled by a spin-filter barrier \( I_f \). The thin F layers may model the effective exchange field induced in the S electrodes due to the presence of the magnetic barrier. This model is accurate if one assume that the F and S layers are in good electric contact and their thicknesses are small enough. The junction is phase- and temperature-biased. The phase difference between the left (L) and right (R) electrode is denoted by \( \phi \), while their temperatures are kept constant, at \( T_L \) and \( T_R \), respectively. In order to describe the electronic transport in the junction we introduce the quasiclassical Green’s functions (GFs) in the L and R electrodes which are \( 8 \times 8 \) matrices in the Nambu-spin-Keldysh space:

\[
G_{R/L} = \begin{pmatrix} \hat{G}^R_{R(L)} & \hat{G}^K_{R(L)} \\ \hat{G}^K_{R(L)} & \hat{G}^A_{R(L)} \end{pmatrix},
\]

where \( \hat{G}^{RAK} \) are the retarded, advanced and Keldysh components, respectively, which are \( 8 \times 8 \) matrices in the Nambu-spin space.

The expression for the charge current \( I_q \) taking into account the spin-filter effect was derived in Refs. and reads

\[
I_q = \frac{1}{16eN_N(\mathcal{T}^2 + \mathcal{U}^2)} \int d\epsilon \text{Tr} \left\{ \hat{\Phi}_1 \left[ \Gamma \hat{G}_R(\epsilon)\Gamma^\dagger, \hat{G}_L(\epsilon) \right]^K \right\},
\]

where \( \mathcal{T} \) and \( \mathcal{U} \) are the tunneling spin-independent and spin-dependent matrix elements (for simplicity we neglect their momentum dependence), \( \Gamma = \mathcal{T} + \mathcal{U} \tau_3 \otimes \sigma_3 \), \( N_N = \left[ 4\pi e^2 N_L(0)N_R(0)(\mathcal{T}^2 + \mathcal{U}^2) \right]^{-1} \) is the junction resistance in the normal state, \( N_{R(L)} \) are the density of the states at the Fermi level in the left or right electrode, respectively, and \( e \) is the electron charge. In analogy and following the derivation carried out in Ref., one can demonstrate that the heat current \( \dot{Q} \) is given by

\[
\dot{Q} = \frac{1}{16e^2N_N(\mathcal{T}^2 + \mathcal{U}^2)} \int d\epsilon d\epsilon' \text{Tr} \left\{ \left[ \Gamma \hat{G}_R(\epsilon)\Gamma^\dagger, \hat{G}_L(\epsilon') \right]^K \right\}.
\]

The GF’s in Eqs. (2-3) have the general structure

\[
\hat{G}^{R(A)}_{R(L)} = \hat{g}^{R(A)} \tau_3 + \hat{f}^{R(A)}(\cos(\phi/2)i/\sin(\phi/2)i)\tau_2,
\]

\[
\hat{G}^K_{R(L)} = (\hat{G}^R - \hat{G}^A) \tanh(\frac{\epsilon}{2T_{R(L)}}),
\]

where \( \tau_{1,2,3} \) are the Pauli matrices in Nambu space, \( \hat{g}^{R(A)} \) is the normal and \( \hat{f}^{R(A)} \) the anomalous component of the retarded (advanced) GFs. The latter are \( 2 \times 2 \) matrices in the spin-space and are determined by solving the quasiclassical equations in the F/S electrodes. Thus, both \( I_q \) and \( \dot{Q} \) are given by Eqs. (2,3) after substituting the values of the GFs at the interface. For simplicity we assume that the thickness of the S and F layers \( \langle t_S, t_F \rangle \) is smaller than the characteristic length over which the GFs vary. In such a case one can average the quasiclassical equations over the thickness of the F/S bilayer that is now described by an effective exchange field \( \hat{h} \) and superconducting order parameter.
have assumed that \( \delta \) defined by \( h/\hbar_0 = N_F(0)\tau_F(N_S(0)I_S + N_F(0)\tau_F)^{-1} \) and \( \Delta_0 = N_S(0)I_S(N_S(0)I_S + N_F(0)\tau_F)^{-1} \), respectively. In the expressions above, \( h_0 \) is the bare exchange field existing in each ferromagnetic layer, \( \Delta_0 \) the bulk superconducting energy gap, and \( N_{F_S}(0) \) is the density of states at the Fermi level in the F or S layer, respectively. The normal and anomalous functions in Eqs. (4,5) are given by \( (\Delta) \) defined by \( h/\hbar_0 = N_F(0)\tau_F(N_S(0)I_S + N_F(0)\tau_F)^{-1} \) and \( \Delta_0 = N_S(0)I_S(N_S(0)I_S + N_F(0)\tau_F)^{-1} \), respectively. In the expressions above, \( h_0 \) is the bare exchange field existing in each ferromagnetic layer, \( \Delta_0 \) the bulk superconducting energy gap, and \( N_{F_S}(0) \) is the density of states at the Fermi level in the F or S layer, respectively. The normal and anomalous functions in Eqs. (4,5) are given by \( (\Delta) \)

\[
\hat{g} = \frac{g_+ + g_-}{2} + \frac{g_+ - g_-}{2} \sigma_3
\]

\[
\hat{f} = f_+ + f_- \sigma_3
\]

where \( f_+ = (f_+ + f_-)/2 \) is the singlet, and \( f_+ = (f_+ - f_-)/2 \) is the triplet (with vanishing total spin projection) components of the condensate, and

\[
\tilde{g}^F = \frac{(\epsilon + h)}{\sqrt{(\epsilon + h + i\eta)^2 - \Delta^2}}
\]

\[
\tilde{f}^F = \frac{\Delta}{\sqrt{(\epsilon + h + i\eta)^2 - \Delta^2}}
\]

Same expressions hold for the advanced GFs if we substitute \( i\eta \) by \( -i\eta \). The latter parameter describes the inelastic scattering rate within the relaxation time approximation and it is set \( \eta = 10^{-3}\Delta_0 \) throughout the article. The density of the states of the electrodes is given by the real part of \( \tilde{g}^F + \tilde{g}^S \). Notice that the order parameter \( \Delta \) in Eqs. (8,9) has to be calculated self-consistently from the gap equation \( \ln(\Delta_0/\Delta) = \frac{\hbar}{\omega_D} d\epsilon (\epsilon^2 + \Delta^2)^{-1/2} f_+ (\epsilon) + f_- (\epsilon)) \), where \( f_\pm (\epsilon) = \left[ 1 + \exp \left( \frac{\epsilon}{\sqrt{\epsilon^2 + \Delta^2}} \right) \right]^{-1} \) and \( \omega_D \) is the Debye frequency of the superconductor. Eqs. (3-9) are used in the next sections in order to analyze the heat transport through a variety of tunneling junctions based on the prototypical example of Fig. 1(a).

III. RESULTS

We now use the above derived equations to determine the heat transport through Josephson junctions with spin filters. While the charge current (quasiparticle and Josephson components) in such structures has been analyzed both experimentally (in Al/EuS/Al,42 and NbN/GdN/NbN49 junctions) and theoretically discussed48,50, heat transport in S/I/S has not been studied so far. In what follows we present the results for the thermal conductance, \( \kappa = Q/\delta T \), in different structures. \( \kappa \) can be obtained from Eq. (3), and in the case of identical electrodes is given by

\[
(\Delta) \text{ defined by } h/\hbar_0 = N_F(0)\tau_F(N_S(0)I_S + N_F(0)\tau_F)^{-1} \text{ and } \Delta_0 = N_S(0)I_S(N_S(0)I_S + N_F(0)\tau_F)^{-1} \text{, respectively. In the expressions above, } h_0 \text{ is the bare exchange field existing in each ferromagnetic layer, } \Delta_0 \text{ the bulk superconducting energy gap, and } N_{F_S}(0) \text{ is the density of states at the Fermi level in the F or S layer, respectively. The normal and anomalous functions in Eqs. (4,5) are given by } (\Delta) \text{ defined by } h/\hbar_0 = N_F(0)\tau_F(N_S(0)I_S + N_F(0)\tau_F)^{-1} \text{ and } \Delta_0 = N_S(0)I_S(N_S(0)I_S + N_F(0)\tau_F)^{-1} \text{, respectively. In the expressions above, } h_0 \text{ is the bare exchange field existing in each ferromagnetic layer, } \Delta_0 \text{ the bulk superconducting energy gap, and } N_{F_S}(0) \text{ is the density of states at the Fermi level in the F or S layer, respectively. The normal and anomalous functions in Eqs. (4,5) are given by } (\Delta) \text{ defined by } h/\hbar_0 = N_F(0)\tau_F(N_S(0)I_S + N_F(0)\tau_F)^{-1} \text{ and } \Delta_0 = N_S(0)I_S(N_S(0)I_S + N_F(0)\tau_F)^{-1} \text{, respectively. In the expressions above, } h_0 \text{ is the bare exchange field existing in each ferromagnetic layer, } \Delta_0 \text{ the bulk superconducting energy gap, and } N_{F_S}(0) \text{ is the density of states at the Fermi level in the F or S layer, respectively. The normal and anomalous functions in Eqs. (4,5) are given by } (\Delta) \)

\[
(\Delta) \text{ defined by } h/\hbar_0 = N_F(0)\tau_F(N_S(0)I_S + N_F(0)\tau_F)^{-1} \text{ and } \Delta_0 = N_S(0)I_S(N_S(0)I_S + N_F(0)\tau_F)^{-1} \text{, respectively. In the expressions above, } h_0 \text{ is the bare exchange field existing in each ferromagnetic layer, } \Delta_0 \text{ the bulk superconducting energy gap, and } N_{F_S}(0) \text{ is the density of states at the Fermi level in the F or S layer, respectively. The normal and anomalous functions in Eqs. (4,5) are given by } (\Delta) \text{ defined by } h/\hbar_0 = N_F(0)\tau_F(N_S(0)I_S + N_F(0)\tau_F)^{-1} \text{ and } \Delta_0 = N_S(0)I_S(N_S(0)I_S + N_F(0)\tau_F)^{-1} \text{, respectively. In the expressions above, } h_0 \text{ is the bare exchange field existing in each ferromagnetic layer, } \Delta_0 \text{ the bulk superconducting energy gap, and } N_{F_S}(0) \text{ is the density of states at the Fermi level in the F or S layer, respectively. The normal and anomalous functions in Eqs. (4,5) are given by } (\Delta) \text{ defined by } h/\hbar_0 = N_F(0)\tau_F(N_S(0)I_S + N_F(0)\tau_F)^{-1} \text{ and } \Delta_0 = N_S(0)I_S(N_S(0)I_S + N_F(0)\tau_F)^{-1} \text{, respectively. In the expressions above, } h_0 \text{ is the bare exchange field existing in each ferromagnetic layer, } \Delta_0 \text{ the bulk superconducting energy gap, and } N_{F_S}(0) \text{ is the density of states at the Fermi level in the F or S layer, respectively. The normal and anomalous functions in Eqs. (4,5) are given by } (\Delta) \)
for instance, due to the presence of a non-magnetic oxide between the $\text{I}_f$ and $\text{S}$ layers. In such a case, one can set in Eq. (10) $N_0 = N$ and $M_0 = M$. Figures 1(b) and (c) show the temperature dependence of $\kappa$ for two values of $\phi$ and different spin-filter efficiencies. Throughout the paper the thermal conductance is shown normalized to that in the normal state, $\kappa^{\text{th}} = L_0 T / R_N$, where $L_0 = \pi^2 k_B^2 / 3 e^2$ is the Lorenz number and $k_B$ is the Boltzmann constant. If $\phi = 0$ the contribution to $\kappa$ from the phase-dependent part is negative, and therefore by decreasing $r$, (i.e., by increasing the efficiency of the spin-filter) the thermal conductance increases [see Fig. 1(b)]. On the contrary, for $\phi = \pi$ the anomalous contribution to $\kappa$ is positive, and the thermal conductance decreases with $r$. With the exception of $r = 1$ and $\phi = 0$ case, $\kappa$ always shows a maximum at a certain finite temperature ($T \approx 0.55 T_c$).

If we now assume a good $\text{I}_f/S$ contact and thin $\text{S}$ layers the density of states of the latter shows a Zeeman splitting which acts as an effective exchange field $h$ inside the superconductor in accordance with Eq. (8). This is induced by the magnetic proximity effect of the $\text{I}_f$ barrier. We note that our model can also describe SF/ $\text{I}_f$ /S structures with two thin ferromagnetic films [see Fig. 1(a)]. In Figs. 2(a) and 2(b) we have chosen $h = 0.4 A_0$ and calculated the temperature dependence of $\kappa$ for $\phi = 0$ and $\phi = \pi$, respectively. Due to the presence of the exchange field the superconducting critical temperature of the SF electrodes is reduced by a factor $\sim 0.875$ with respect to the bulk $T_c$. The black curves in Figs. 2(a) and 2(b) correspond to a perfect spin-filter with $\mathcal{P} = 1$ ($r = 0$). According to Eq. (10), in this case, the only contribution to $\kappa$ comes from the quasiparticle channel. As in the zero exchange field case, if $r \neq 0$ the corrections to $\kappa$ from the phase-dependent anomalous term in Eq. (10) are negative for $\phi = 0$ and positive for $\phi = \pi$. This explains why for $\phi = 0$ the amplitude of the thermal conductance decreases by increasing $r$ [see Fig. 2(a)], whereas for $\phi = \pi$ the thermal conductance increases with $r$ [see Fig. 2(b)].

In panels (c) and (d) of Fig. 2 we compare the $\kappa(T)$ dependence in the presence and in the absence, respectively, of a spin-filter barrier. Here we set a zero phase difference, $\phi = 0$. If the tunneling barrier is non-magnetic, $r = 1$, the transition to the superconducting state leads to a decrease of the thermal conductance as shown in Fig. 2(d). Notably, in this case for any temperature $\kappa$ increases monotonically by enhancing the amplitude of the effective exchange field $h$. By contrast, if the tunneling barrier has a finite spin-filter efficiency ($r = 0.5$ which corresponds to $\mathcal{P} \approx 0.88$), below the superconducting transition temperature, $T \lesssim T_c$, the thermal conductance increases by decreasing the exchange field. By further decreasing the temperature, $\kappa$ shows a maximum, and then decays to zero [see Fig. 2(c)]. The maximum value of $\kappa$ ($\kappa_{\text{max}}$) depends non-monotonically on $h$: For small enough values of $h$, $\kappa_{\text{max}}$ decreases by increasing $h$, however for $0.4 A_0 < h < 0.5 A_0$ it turns out to increase.

From Eq. (10) it clearly appears that for a spin-filter with 100% efficiency ($r = 0$), the anomalous contribution to $\kappa$ vanishes [i.e., the last term in Eq. (10) is zero] and therefore the heat transport will not depend on the phase difference $\phi$.

**B. Triple junctions with spin-filter**

In order to detect the spin triplet supercurrents, long-range Josephson effect has been measured in a variety of multilayered ferromagnetic structures with inhomogeneous magnetic configurations. According to the theoretical prediction\textsuperscript{3}, such inhomogeneity induces the triplet pair correlations with equal spin-projection in the ferromagnetic bridge. Here we aim to understand the heat transport through SF hybrid structures containing tunneling barriers. For that sake we consider the structure shown in Fig. 3(a). It consists of two FS bilayers tunnel-coupled by a spin-filter barrier. We set the $z$-axis (spin quantization axis) parallel to the magnetization of the $\text{I}_f$ layer, and define the angles, $\alpha$ and $\beta$, which describe the direction of magnetization of the left and right ferromagnets, respectively [see Fig. 3(b)]. For a good contact between the $\text{S}$ and $\text{F}$ layers and small enough thicknesses this structure is equivalent to the one shown in Fig. 1(a).

The generalized expression for the thermal conductance in this case can be derived from Eq. (3) with the help of the technique used in Refs.\textsuperscript{48,50}. We obtain $\kappa = \kappa_{\text{qp}} + \kappa_{\phi}$, where $\kappa_{\text{qp}}$ is the contribution from the quasiparticles to thermal transport given by

$$\kappa_{\text{qp}} = \frac{1}{4 e^2 R_N} \int d\varepsilon \left[ \left( \frac{\partial F}{\partial T} \right) \left\{ \left[ N_+ + N_- \right]^2 + \left[ N_+ + N_- \right] \cos \alpha \cos \beta + r \sin \alpha \sin \beta \right\} \right] \left[ M_+^2 \cos \alpha \cos \beta + M_-^2 \sin \alpha \sin \beta \right],$$

where $M_+ = M_+ + M_- = \int f^R d\xi$, and $M_- = M_+ - M_- = f^R - f^L$. Notice that even in the case of a perfect spin-filter efficiency ($r = 0$) there is a phase-dependent contribution to $\kappa$ provided that the magnetization of the $\text{F}$ layers are non-collinear with the one of the barrier (i.e., $\alpha, \beta \neq 0, \pi$). In such a case, the measured $\kappa(\phi)$ dependence is a direct manifestation of the triplet component of the condensate in analogy with the two $\text{FS}$ bilayer S/F/S structures with two thin ferromagnetic films [see Fig. 1(a)]. In Figs. 2(a) and 2(b) we have chosen $h = 0.4 A_0$ and calculated the temperature dependence of $\kappa$ for $\phi = 0$ and $\phi = \pi$, respectively. Due to the presence of the exchange field the superconducting critical temperature of the SF electrodes is reduced by a factor $\sim 0.875$ with respect to the bulk $T_c$. The black curves in Figs. 2(a) and 2(b) correspond to a perfect spin-filter with $\mathcal{P} = 1$ ($r = 0$). According to Eq. (10), in this case, the only contribution to $\kappa$ comes from the quasiparticle channel. As in the zero exchange field case, if $r \neq 0$ the corrections to $\kappa$ from the phase-dependent anomalous term in Eq. (10) are negative for $\phi = 0$ and positive for $\phi = \pi$. This explains why for $\phi = 0$ the amplitude of the thermal conductance decreases by increasing $r$ [see Fig. 2(a)], whereas for $\phi = \pi$ the thermal conductance increases with $r$ [see Fig. 2(b)].
to the finite charge supercurrent flowing through a fully efficient spin-filter, as recently predicted in Ref. 48. Again, the phase dependent contribution $\kappa_\phi$ is proportional to $\cos \phi$ [cf. Eq. (10)] and therefore we expect for $\kappa(T)$ a similar behavior as for the $S/I_s/S$ structure. This is confirmed in panels (a) and (b) of Fig. 4 where we show the temperature dependence of $\kappa$ for the F layers having a magnetization parallel to each other but perpendicular to the magnetization of the barrier, i.e. $\alpha = \beta = \pi/2$. In particular, the thermal conductance can increase considerably with respect to the normal value if $\phi = \pi$. In panels (c) and (d) of Fig. 4 we show the temperature dependence of $\kappa$ for different angles $\beta$ by setting $\alpha = \pi/2$. In the case $\phi = 0$ case maximum values for $\kappa$ are achieved for $\beta = 0$, whereas if $\phi = \pi$ the maximum $\kappa$ is observed for $\beta = \pi/2$.

In the case of a perfect spin-filter ($r \to 0$), one can see from Eq. (12) that only the triplet term $M_r$ contributes to $\kappa$. This term describes the spin-polarized heat current.

In principle one can analyze the contributions from the singlet and triplet pairs density separately by considering the junction of Fig. 3(a) with a non-magnetic tunneling barrier (i.e., $r = 1$). We set the magnetization of one of the F layer fixed ($\alpha = 0$) and then we switch the other F layer magnetization between a parallel ($\beta = 0$) or antiparallel ($\beta = \pi$) configuration. If we now perform a phase-biased experiment and measure $S_\beta$, i.e., the difference between the heat conductance $\kappa(\phi, \beta)$ for $\phi = 0$ and $\phi = \pi$,

$$S_\beta = \kappa(0, \beta) - \kappa(\pi, \beta),$$

in the parallel and antiparallel configuration it is clear from Eqs. (11-12) that $S_0 + S_\pi$ represents the contribution from singlet pairs

$$S_0 + S_\pi = -\frac{1}{e^2R_N} \int d\varepsilon \, (\frac{\partial F}{\partial T}) \, M_s^0 M_s^R,$$

whereas the difference $S_0 - S_\pi$ represents the one from triplet pairs

$$S_0 - S_\pi = -\frac{1}{e^2R_N} \int d\varepsilon \, (\frac{\partial F}{\partial T}) \, M_s^0 M_s^R.$$  

These two contributions are plotted in Fig. 5 as a function of the temperature for different values of the exchange field. In particular, the maximum contribution from the singlet component is achieved for the lowest values of the exchange field around $T \sim 0.5T_c$, whereas the triplet contribution is maximized by increasing the exchange field value (i.e., in the present case $h = 0.5 A_0$) around $T \sim 0.25T_c$. At large enough exchange fields both contributions tend to be similar. We note that at low temperature the amplitude of the singlet component decreases not monotonically by increasing $h$ whereas that of the triplet contribution turns out to monotonically increase by increasing the exchange field.

C. The Josephson heat valve

A similar junction as the one shown in Fig. 3(a) with a non-magnetic tunneling barrier instead of $I_s/f$ was recently

FIG. 4. (Color online) (a) Thermal conductance vs $T$ calculated for several values of $r$ at $\phi = 0$. (b) The same quantity as in panel (a) calculated at $\phi = \pi$. (c) Thermal conductance vs $T$ calculated for several values of $\beta$ at $\phi = 0$. (d) The same quantity as in panel (c) calculated at $\phi = \pi$. In panels (c) and (d) we set $r = 0$ and $\alpha = \pi/2$. In all the calculations of the figure we assumed $h = 0.2A_0$.

FIG. 5. (Color online) (a) Singlet pairs contribution to thermal conductance vs $T$ calculated for several values of the exchange field in the ferromagnetic layers. (b) Triplet pairs contribution to thermal conductance vs $T$ calculated for the same $h$ values as in panel (a).
In summary, we have presented an exhaustive study of the electronic heat transport in SF/I/F Josephson junctions with magnetic and non-magnetic I/F tunneling barriers. General expressions for the heat current and heat conductance were derived taking into account the spin-filter efficiency \( \kappa \) and in turn the MTR ratio does not depend on \( \phi \), as shown by the black curve in Fig. 6(c). All curves cross at \( \phi = \pi/2 \), which is the phase value separating the two behaviors: If \( 0 \leq \phi < \pi/2 \) the MTR decreases by increasing \( r \) while the opposite behavior is achieved for \( \pi/2 < \phi \leq \pi \). It is worthwhile mentioning that in the parallel configuration the Josephson valve heat conductance is maximized. In contrast, the dc Josephson effect is maximized by the anti-parallel configuration.\(^{51}\) This means that in the P configuration the ferromagnetic Josephson junction behaves as an almost ideal electric insulator whereas in the AP one it behaves as an ideal thermal insulator\(^{41}\).

Panel 6(d) shows the phase dependence of the MTR ratio calculated for a few different values of \( h \) and a moderate spin-filter efficiency \( r = 0.5 \) at \( T = 0.1T_c \). It clearly appears that the larger the splitting field induced in the S layers, the larger is the heat valve effect.

**IV. SUMMARY**

In summary, we have presented an exhaustive study of the electronic heat transport in SF/I/F/SF Josephson junctions with magnetic and non-magnetic I/F tunneling barriers. General expressions for the heat current and heat conductance \( \kappa \) were derived taking into account the spin-filter efficiency \( \mathcal{P} \) of the barrier. It has been shown that \( \kappa \) strongly depends on \( \mathcal{P} \). For a given value of the exchange field two behaviors have been found: In the case of a zero phase difference between the SF electrodes an increasing spin-filter efficiency leads to a decrease of \( \kappa \), whereas the opposite behavior is achieved if \( \phi = \pi \). We have also investigated the heat conductance in the case that the magnetizations of the F layers and the spin-filter are non-collinear. We explicitly computed the contributions to \( \kappa \) stemming from singlet and triplet pair correlations. Finally, we have analyzed a heat valve based on a F/S/F/S/F Josephson junction, and demonstrated that for \( \pi/2 < \phi \leq \pi \) the lowering the spin-filter efficiency of the barrier leads to a sizable enhancement of the magnetothermal resistance ratio.

We finally discuss here some potential applications of the analyzed structures. Ferromagnetic Josephson heat valves can be used whenever a precise tuning and mastering of the temperature is required, for instance, for on-chip heat management as a switchable heat sink. Furthermore, such a valve setup can be useful as well, to tune the operation temperature of radiation sensors.\(^{27,57}\) In the context of quantum...
computation these elements can also be used to influence the behavior and the dynamics of two-level quantum systems through temperature manipulation. Finally, the strong dependence of the Josephson supercurrent on temperature can be exploited for the realization of controllable thermal Josephson junctions of different kinds.

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