A first study of the semi-leptonic decay of the $\Lambda_b$ baryon.  

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We present the preliminary results of the first Lattice study of the baryonic Isgur and Wise function obtained from the matrix element of the weak current between $\Lambda$-baryon external states. Its dependence on the heavy and light quark masses is studied. Some result on the semi-leptonic decay $\Lambda_b \to \Lambda_c + l \nu$ are given.

We present the results of the first non-perturbative study of the semi-leptonic decay $\Lambda_b \to \Lambda_c + l \nu$, carried out using Lattice QCD. This study will provide an independent measurement of the CKM matrix element $V_{cb}$, as experimental data become available.

We evaluate the hadronic matrix element of the weak current $J_\mu = \bar c \gamma_\mu (1 - \gamma_5) b$, computing the correlators
\begin{equation}
C(t_x) = \sum_{\vec x} e^{-i\vec p \cdot \vec x} \langle \mathcal O^Q(x) \mathcal O^Q(0) \rangle
\end{equation}
\begin{equation}
C_\mu(t_x, t_y) = \sum_{\vec x, \vec y} e^{-i(\vec p \cdot \vec x + \vec q \cdot \vec y)} \langle \mathcal O^Q(x) (J_\mu(y)) \mathcal O^Q(0) \rangle
\end{equation}
on 60 $24^3 \times 48$ lattices at $\beta = 6.2$, and using the $O(a)$–improved fermion action [1]. In (1) $\mathcal O^Q$ is the interpolating operator of the $\Lambda_b$. In this study we consider only correlators with initial and final momenta either zero or $p_{\text{min}} = 2\pi / L_a$ and transitions with equal initial and final heavy quark masses $m_Q$. The matrix element can be decomposed into six form factors (FF), which are invariant functions of $\omega = v \cdot v'$
\begin{equation}
\langle \Lambda_b(v') | J_\mu | \Lambda^Q(v) \rangle = \bar u(v') \Gamma_i (F_i^V - \gamma_5 F_i^A) u(v).
\end{equation}

with $\Gamma_i = \gamma_\mu, v_\mu$ and $v'_\mu$. In this basis, one can use the Heavy Quark Effective Theory (HQET) analysis to relate the six FF to the Isgur-Wise (IW) function [2], through the correction coefficients $N_i^{(5)}$

\begin{equation}
N_i(\omega) \hat \xi_{iQ'Q}(\omega) = F_i^V(\omega) \quad N_i^{(5)}(\omega) \hat \xi_{iQ'Q}(\omega) = F_i^A(\omega).
\end{equation}

We will study the quantities
\begin{equation}
\hat \xi_{iQ'Q}(\omega) = \frac{F_i^A(\omega) N_i^{(5)}(\omega)}{F_i^A(1) N_i^{(5)}(1)} = \frac{\sum_i F_i^V(\omega) \sum_i N_i(1)}{\sum_i F_i^V(1) \sum_i N_i(1)}
\end{equation}
which are independent of the current renormalization constants, and exhibit a stable signal. The equality (2) is valid up to $O(1/m_Q)$, and we neglect higher order corrections.

The explicit flavour-dependence of $\hat \xi_{iQ'Q}$ is studied by linearizing it about $\omega = 1$
\begin{equation}
\hat \xi_{iQ'Q}(\omega) = 1 + \rho^2 (1 - \omega)
\end{equation}
and measuring the slope $\rho^2$ as a function of $m_Q$. By keeping the light quark masses fixed to $\kappa = 0.14144$, around that of the strange quark, and varying $m_Q$ around the charm mass, we obtain
\begin{align}
\rho^2 &= 2.4 \pm 0.4 \quad \text{at } \kappa_Q = \kappa'_Q = 0.121 \\
\rho^2 &= 2.4 \pm 0.4 \quad \text{at } \kappa_Q = \kappa'_Q = 0.125 \\
\rho^2 &= 2.4 \pm 0.3 \quad \text{at } \kappa_Q = \kappa'_Q = 0.129 \\
\rho^2 &= 2.4 \pm 0.3 \quad \text{at } \kappa_Q = \kappa'_Q = 0.133,
\end{align}
suggesting that the flavour dependence of $\hat \xi_{iQ'Q}$ can be neglected at our masses or above. A global fit, shown in Figure 1, to the four sets of determinations, yielding $\rho^2 = 2.4 \pm 0.4$, is our best estimate.

According to HQET, the IW function only depends on the quantum numbers of light quarks. Previous studies on the lattice [3] demonstrated that such a dependence is not negligible in the case of mesons, where the “brown muck” contains only one light quark. Thus we expect to measure an even stronger dependence of the baryonic IW function.
Table 1
Estimates of $\hat{\xi}_{QQ}(\omega)$ form both axial and vector form factors, at $\kappa_Q = \kappa_Q' = 0.129$ and at three values of the light quark masses and at the chiral limit. Errors on $\omega$ are on the digit beyond the last shown.

| $\kappa_{12}/\kappa_{13}$ | $J_{\mu}$ | $\omega$ | $\hat{\xi}_{QQ}$ | $\omega$ | $\hat{\xi}_{QQ}$ | $\omega$ | $\hat{\xi}_{QQ}$ | $\omega$ | $\hat{\xi}_{QQ}$ |
|--------------------------|-----------|----------|-----------------|----------|-----------------|----------|-----------------|----------|-----------------|
| 0.14144/                 | A         | 1.037    | 0.94 $^{+3}_{-3}$ | 1.08     | 0.88 $^{+9}_{-9}$ | 1.0      | 1.02 $^{+11}_{-11}$ | 1.15     | 0.68 $^{+7}_{-6}$ |
|                          | V         | 1.037    | 0.96 $^{+3}_{-3}$ | 1.08     | 0.90 $^{+8}_{-8}$ | 1.0      | 0.98 $^{+12}_{-12}$ | 1.15     | 0.62 $^{+8}_{-7}$ |
| 0.14144/                 | A         | 1.040    | 0.93 $^{+4}_{-4}$ | 1.08     | 0.86 $^{+12}_{-11}$ | 1.0      | 0.97 $^{+15}_{-16}$ | 1.16     | 0.71 $^{+9}_{-9}$  |
|                          | V         | 1.040    | 0.97 $^{+4}_{-4}$ | 1.08     | 0.90 $^{+10}_{-10}$ | 1.0      | 0.86 $^{+16}_{-17}$ | 1.16     | 0.65 $^{+8}_{-8}$  |
| 0.14226/                 | A         | 1.043    | 0.93 $^{+5}_{-5}$ | 1.09     | 0.82 $^{+18}_{-15}$ | 1.0      | 0.85 $^{+24}_{-25}$ | 1.18     | 0.75 $^{+13}_{-12}$ |
|                          | V         | 1.043    | 0.95 $^{+5}_{-5}$ | 1.09     | 0.80 $^{+13}_{-14}$ | 1.0      | 0.63 $^{+27}_{-27}$ | 1.18     | 0.68 $^{+13}_{-11}$ |
| Chiral/                  | A         | 1.048    | 0.92 $^{+7}_{-7}$ | 1.10     | 0.90 $^{+22}_{-20}$ | 1.0      | 1.02 $^{+23}_{-24}$ | 1.19     | 0.78 $^{+17}_{-17}$ |
|                          | V         | 1.048    | 0.96 $^{+8}_{-8}$ | 1.10     | 0.95 $^{+19}_{-19}$ | 1.0      | 0.79 $^{+26}_{-32}$ | 1.19     | 0.74 $^{+16}_{-15}$ |

The slope $\rho^2$ at the chiral limit,

$$\rho^2 = 1.2 \pm 0.8 \pm 11,$$

is obtained by extrapolating the three estimates of both $\omega$ and $\hat{\xi}_{QQ}(\omega)$ linearly in the sum of the two light quark masses, Our results are presented in Table 1.

Our estimate of the IW function can, in turn, be used to quantify the $1/m_Q$ corrections, affecting $F_1$ and $F_{2,3}$, in terms of the baryonic binding energy $\bar{\Lambda}$, as detailed in [4]. There, it was found that $\bar{\Lambda} = 0.37 \pm 0.11$ GeV. Its value is necessary to reconstruct the FF at the physical limit, i.e. for the decay $A_b \rightarrow A_c + e^+\nu$.

With the result that $\hat{\xi}_{QQ}(\omega)$ is flavour-independent the FF depend on the quark masses only through the factors $N_i^{(5)}$. Given our limited knowledge of the functional form of $\hat{\xi}_{QQ}(\omega)$, we can only model the FF as linear functions of $\omega$.

$$F_i^{A,V} (\omega, m_b, m_c) = \eta_i^{A,V} - \hat{\rho}_i^{A,V} (\omega - 1)$$

where the normalizations $\eta_i^{V,A}$ and the new slopes $\hat{\rho}_i^{A,V}$ are related to the coefficients $N_i^{(5)} (\omega, m_b, m_c)$ and to the slope of the IW function by

$$\eta_i^{V,A} = N_i^{(5)} (1), \quad \hat{\rho}_i^{A,V} = \left. \frac{d N_i^{(5)} (\omega)}{d \omega} \right|_{\omega=1}$$

Our results for $\hat{\rho}_i^{V,A}$ and $\eta_i^{V,A}$ are shown in Table 1.
The decay rates can be written in terms of the FF, in the velocity basis, through the helicity amplitudes \( H \) [3]:

\[
\frac{d\Gamma}{d\omega} = \frac{G_F^2}{(2\pi)^3} 2 |V_{cb}|^2 q^2 M_{\Lambda_b}^2 \sqrt{(\omega^2 - 1)} \times \left( |H_{1/2,1}|^2 + |H_{1/2,-1}|^2 + |H_{0,0}|^2 + |H_{-1/2,0}|^2 \right),
\]

where the subscripts in the helicity amplitudes refer to the polarization of the \( W \) boson and of the daughter baryon \( \Lambda_b \), respectively. The upper integration limit on the decay rates extends to \( \omega \simeq 1.43 \), which is beyond the range of velocity transfer accessible to us (\( \omega \in [1.0, 1.2] \)). We thus define the partially-integrated decay rate,

\[
\Gamma_{\text{part}}(\omega_{\text{max}}) = \int_1^{\omega_{\text{max}}} d\omega \frac{d\Gamma}{d\omega}
\]

as a function of the upper limit of integration. In Table 2 we present our results for the quantities

\[
\Gamma_{\text{part}}(\omega_{\text{max}}) / |V_{cb}|^2 [10^{13} \text{s}^{-1}]
\]

for several values of \( \omega_{\text{max}} \). The masses are taken from the experiments.

At present, a direct comparison of our results with experiments is not possible. In fact, even if the semi-leptonic decay of \( \Lambda_b \) was observed by various experiments [8], a measurement of the decay rate is not yet available. The problem of determining the rate of the \( \Lambda_b \) semi-leptonic decays has been addressed making use of different models (Infinite Momentum Frame, Quark Model, Dipole form factors) [5], [7]. Their predictions, for the total rate, integrated up to the end-point, are reported in Figure 2 and compared with the function \( \Gamma_{\text{part}}(\omega_{\text{max}}) \). To evaluate this function we have assumed \( |V_{cb}| = 0.044 \).

Finally, we note that many other interesting quantities, such as asymmetry parameters (see for example [8]), and the ratio of the longitudinal to transverse rates, could be computed and confronted with the upcoming experiments. However, all these quantities are non-trivial only at \( O(1/m_Q^2) \) or \( O(\omega^2) \); in both cases beyond the precision reached in the present study.

### Table 2

|       | \( F_1^\rho \) | \( F_2^\rho \) | \( F_1^\eta \) | \( F_2^\eta \) | \( F_1^\omega \) | \( F_2^\omega \) |
|-------|---------------|---------------|---------------|---------------|---------------|---------------|
| \( \rho \) | 1.8 \( +9_{-15} \) | -0.4 \( +2_{-1} \) | -0.10 \( +4_{-4} \) | 1.3 \( +8_{-12} \) | -0.4 \( +5_{-2} \) | 0.16 \( +6_{-10} \) |
| \( \eta \) | 1.28 \( +6_{-6} \) | -0.19 \( +4_{-4} \) | -0.06 \( +4_{-1} \) | 0.99 \( +5_{-2} \) | -0.24 \( +6_{-4} \) | 0.09 \( +2_{-2} \) |

**Table 2**

*Normalization and slope of the physical Form Factors, relevant for the \( \Lambda_b \) decay.*

**Figure 2.** Total decay rate with error bands, for the process \( \Lambda_b \rightarrow \Lambda_c + l\nu \), as a function of the limit of integration \( \omega_{\text{max}} \). A comparison with several model estimates is shown at the end-point.

**Table 3**

| \( \omega_{\text{max}} \) | 1.1 | 1.15 | 1.20 | 1.25 |
|-------------------------|-----|-----|-----|-----|
| \( \Gamma_{\text{part}} \) | \( +9_{-7} \) | \( 22_{-20} \) | \( +5_{-4} \) | \( 9_{-7} \) |

**Table 3**

*Partial decay rates for the \( \Lambda_b \).*
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7. M.A. Ivanov and V.E. Lyubovitskij; hep-ph/9502202.
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