Trapping atoms in the vacuum field of a cavity

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The aim of this work is to find ways to trap an atom in a cavity. In contrast to other approaches we propose a method where the cavity is basically in the vacuum state and the atom in the ground state. The idea is to induce a spatial dependent AC Stark shift by irradiating the atom with a weak laser field, so that the atom experiences a trapping force. The main feature of our setup is that dissipation can be strongly suppressed. We estimate the lifetime of the atom as well as the trapping potential parameters and compare our estimations with numerical simulations.

INTRODUCTION

Cavity QED constitutes one of the most important experimental set-ups where the basic properties of Quantum Mechanics can be controlled, observed, and tested. During the last years, a significant experimental progress has taken place, allowing to observe quantum phenomena in the interaction of a single atom with a single mode of the electromagnetic field, both in the optical and microwave regime. Some of these experiments are currently limited by the fact that (neutral) atoms typically move almost freely in the cavity and eventually leave it, which restricts the duration of the experiment as well as its controllability. For example, in the optical regime, the coupling between the atoms and the cavity mode strongly depends on the position of the atom, and thus when it moves this can strongly affect the interaction.

In order to overcome these problems, several strategies to trap an atom in a cavity have been put forward. Some of them involve using some external laser fields which exert a confining force to the atom, something that has been successfully realized in recent experiments. In a far-off resonant trap (FORT) this is achieved by employing a far-off resonant trapping beam along the cavity axis. A more intriguing approach consists of using the cavity mode itself to confine the atom. In remarkable experiments it has been possible to keep an atom in a cavity just using the force provided by a single photon. In this work we will show that it is, in principle, possible to trap an atom in its ground state in a cavity which is basically in the vacuum state. Apart from its fundamental interest, our method may have some practical advantages with respect to the previous one in that since the atom and the cavity mode are in their ground state, losses are appreciable reduced.

Atom trapping in cavities has interesting applications in the field of quantum information. All the proposals of quantum computation using atoms interacting via a common cavity mode require that the atoms are trapped in the cavity in a fixed position. Moreover, this is also required to build quantum networks involving cavity QED setups. In those cases the idea is to store the quantum information in two internal ground levels of each atom, $|g\rangle$ and $|g'\rangle$, and to entangle them by using real or virtual photon exchange through the cavity mode. Note that in this context a single spontaneous emission or cavity loss may have dramatic effects for all quantum information tasks (see, however, Ref. [26]). Thus, it is not only important to trap the atoms in the cavity but also to reduce the decoherence time as much as possible. By trapping the atoms in the vacuum state of the cavity our scheme achieves a strong reduction of the decoherence processes.

The plan of this paper is as follows. In Section II we give a qualitative description of our scheme, estimating its operating conditions such as the depth of the trapping potential and the lifetime of the state. In Section III we give a full description of the method including dissipative processes. The analytical results and estimations are checked numerically in Section IV. Finally, Section V contains a summary of our results.

DESCRIPTION

We consider an atom with two internal ground levels $|g\rangle$ and $|g'\rangle$, which are resonantly coupled by two cavity modes to two excited levels $|e\rangle$ and $|e'\rangle$, respectively. Additionally, an external plane-wave laser field detuned by $\Delta$ excites the same transition (see Fig. 1). Note that
this laser field does not exert any force on the atoms. In the following we will consider only the levels $|g\rangle$ and $|e\rangle$, since for the other two levels the same description applies (they are independent).

In order to understand the mechanism that we propose, it is convenient first to revise the method used in previous experiments [13, 14, 17, 18] to keep an atom in a cavity. The interaction between the atom and cavity mode is characterized by a coupling constant $g(x)$, which depends on the atomic position $x$. In Fig. 2 we show the set-up, as well as the instantaneous energy levels of an atom as a function of its position (in one dimension). The ground state of the composed atom–cavity system is $|g,0\rangle$, where $|n\rangle$ is the cavity state with $n$ photons (in this case $n = 0$). The corresponding energy, $E_0$ is position independent. The first two excited levels are the dressed states of the Jaynes–Cummings Hamiltonian [27]. $|\pm\rangle = \frac{1}{\sqrt{2}}(|g,1\rangle \pm |e,0\rangle)$, with corresponding energies $E_{\pm}(x) = \pm g(x)$ in the interaction picture, where we have taken $\hbar = 1$. As Fig. 2 shows, the position–dependence of $E_{\pm}(x)$ provides the atom with a confining potential at the center of the cavity. Thus, if the atom can be prepared in the state $|-\rangle$ with a kinetic energy smaller than $E_0 = g(0)$, it will remain trapped [11, 12, 13, 14, 15, 16, 17, 18]. As the state $|-\rangle$ contains a linear combination involving one photon, one can state that the atom is trapped by a single photon. On the other hand, the state $|-\rangle$ can be efficiently prepared by starting in the state $|g,0\rangle$ and tuning the external laser field to be resonant with the transition $|g,0\rangle \rightarrow |-\rangle$ near $x = 0$, as indicated in Fig. 2 [28]. Note that in the case studied in many references [12, 13, 17, 18] the trapping force may be velocity dependent since they are also interested in laser cooling, whereas for us this is not the case. In this sense it will be difficult to exactly compare the results of both approaches.

The above discussion has omitted an important element which is present in all experiments, namely the dissipation mechanism. On the one hand, excited atoms may decay very fast (as long as the state $|e\rangle$ does not correspond to some Zeeman level, which is coupled to the cavity mode by some Raman transition [29]). More importantly, cavities have usually losses, so that the photons will leave the cavity after some time $t \approx 1/\kappa$, where $\kappa$ is the cavity damping rate. Any of these mechanisms will induce the spontaneous transition $|-\rangle \rightarrow |g,0\rangle$, and therefore the atom will no longer experience the trapping force. The typical time scale of this processes is of the order of $\Gamma^{-1}$ and $\kappa^{-1}$, where $\Gamma$ and $\kappa$ are the spontaneous emission and the cavity damping rate, respectively. In practice [11, 12, 14, 17] the atom can be promoted several times to the state $|-\rangle$ by the external laser, so that the trapping time inside the cavity can be several hundreds of $\kappa^{-1}$. Note, however, that these spontaneous transitions will break the atomic coherence if we are using more internal levels to store, for example, some quantum information in the atom (see Fig. 1).

Our idea is to detune the external laser slightly below the transition $|g,0\rangle \rightarrow |-\rangle$ at $x = 0$. If the laser intensity is low enough, its only effect will be to produce an AC–Stark shift for the level $|g,0\rangle$, whose energy $E_0(x)$ will now depend on the position, as shown in Fig. 3. Thus, if the atom is in the level $|g,0\rangle$, it will experience a trapping force towards $x = 0$, and therefore, it can be trapped (as long as the corresponding potential supports bound states). In this sense it will be difficult to exactly compare the results of both approaches. In practice [11, 12, 14, 17] the atom can be promoted several times to the state $|-\rangle$ by the external laser, so that the trapping time inside the cavity can be several hundreds of $\kappa^{-1}$. Note, however, that these spontaneous transitions will break the atomic coherence if we are using more internal levels to store, for example, some quantum information in the atom (see Fig. 1).
states). Note also, that since the atom is basically in the ground state and no photon is present, all the dissipative mechanisms may be drastically reduced.

In the following sections we will compute the performance of our scheme. In the rest of this section we will use very simple estimates to characterize the trapping potential and the corresponding time scales.

Denoting by $\Omega$ the Rabi frequency of the external laser, and by $\Delta$ its corresponding detuning with respect to the $|g\rangle \rightarrow |e\rangle$ transition ($\Delta < 0$), we have that the regime of validity of our analysis will be

$$\Omega \ll |\Delta + g_0| \ll g_0.$$  \hspace{1cm} (1)

In this case, the depth of the trapping potential $V_0$ will be approximately equal to the AC–Stark shift of the level $|g, 0\rangle$ due to its coupling to $|\rangle$ at $x = 0$, i.e.

$$V_0 \simeq \frac{\Omega^2}{8|\Delta + g_0|}.$$  \hspace{1cm} (2)

On the other hand, losses will be due to the small contamination of level $|g, 0\rangle$ with level $|\rangle$ given by the off–resonant coupling. The population of this level will be of the order of $10^{-14}$, and therefore the lifetime of the state will be

$$\tau \simeq \frac{4|\Delta + g_0|^2}{\Omega^2} \text{min}(\Gamma^{-1}, \kappa^{-1}).$$  \hspace{1cm} (3)

Equations (2) and (3) indicate that the lifetime can be made arbitrarily big at the expense of reducing the potential depth.

In three dimensions, one can easily estimate the condition for a potential to possess a bound state. It is given by $2mV_0L^2/\hbar^2 \gtrsim 1$, where $L$ is the cavity length, $m$ is the atomic mass, and we have included $\hbar$ to make the dimensions more explicit. We can rewrite this as $V_0(L/\lambda)^2 \gtrsim E_R$, where $\lambda$ is an optical transition wavelength and $E_R$ the corresponding energy of one photon recoil. Since $L \gtrsim \lambda$ in all cases we see that by taking $V_0 > E_R$ we will always have an atomic bound state. Note that in a one dimensional set–up there is always a bound state for any value of $V_0$. \[3\]

So far, we have shown that it is possible to have atoms trapped in the cavity with basically zero photons and in the atomic ground state. However, the trapping potential may become very weak. Thus, in order to trap atoms it will be required that they move very slowly in the cavity in the state $|g, 0\rangle$ and then, when they are close to $x = 0$, the external field is turned on. Let us estimate what will be, in this case, the lifetime of the trapped state. We will assume that we have Rb atoms and the kinetic energy is of the order of one optical recoil ($E_R = \hbar^2 k^2 / 2m$, where $k$ is the optical wavevector). Thus, we take $E_R = 4\text{kHz} < V_0 = 10\text{kHz}$. Let us analyze separately the optical and microwave regimes.

For the optical regime we take the parameters from \[17\]. There the $5^2S_{1/2}/F = 3 \rightarrow 5^2P_{3/2}/F = 4$ transition of $^{85}\text{Rb}$ with a frequency of $3.8 \times 10^{13}\text{Hz}$ was used. The maximal coupling between cavity and atom is $g_0 \approx 16 \times 2\pi \text{MHz}$, the cavity loss rate is $\kappa \approx 1.4 \times 2\pi \text{MHz}$ and the spontaneous decay rate is $\Gamma \approx 3 \times 2\pi \text{MHz}$. We estimate a decay time of $2.1 \times 10^{-9}\text{s}$. For the microwave regime we consider circular Rydberg states, so we have $g_0 \approx 67 \times 2\pi \text{kHz}$, $\kappa \approx 1.6 \times 2\pi \text{Hz}$ and $\Gamma \approx 6 \times 2\pi \text{Hz}$, where $\Gamma$ is the spontaneous decay rate of the Rydberg transition. We reach a life time of $40\text{s}$ \[6\].

These estimates look very promising. They will be optimized and compared with numerical calculations in the following sections. On the other hand, let us stress that we have calculated here the time for a single loss event, since it will destroy the coherence present in the atomic state. For the reference \[12, 13, 14, 15, 16, 17, 18\], in the optical experiments in which the atom is trapped in a cavity both the effective decay rate and the potential depth seen by the atom scale proportionally to the population of the excited level. However, in our case the potential depth \[2\] scales in a different way. Thus we expect that our scheme will be useful under appropriate conditions (small initial velocities). In the following sections we will also analyze the trapping time if several loss events are allowed.

**MODEL**

In this section we will introduce in detail the model that describes the situation we have in mind. In the first subsection we will start with the full Hamiltonian characterizing the atom–cavity interaction and perform some approximations in order to derive the estimates given in the previous section. Then we will introduce the decay mechanisms in this picture.

**Hamiltonian dynamics**

The Hamiltonian describing the dynamics of the atom and the cavity mode can be written as follows

$$\hat{H} = \frac{\hat{p}^2}{2m} + \omega_0 (a^\dagger a + \frac{1}{2}\sigma_z) + g(x) (\sigma^+ a + a^\dagger \sigma^-) + \frac{\Omega}{2} (\sigma^+ e^{-i\omega_L t} + \sigma^- e^{i\omega_L t}).$$  \hspace{1cm} (4)

Here, $\omega_L$ and $\omega_0$ are the laser and atomic transition frequency, respectively, $\Omega$ is the Rabi frequency and $g(x)$ the position dependent coupling constant between the cavity mode and the atom. Note that we have not included the position dependence of the laser plane wave to make more explicit the fact that the laser exerts no force on the atom (in any case, since this laser only gives rise to AC-Stark shift, its position dependence will cancel out).
In order to make the analysis simpler, we will project our system in the subspace spanned by the states \(|g,0\rangle, |e,0\rangle, |g,1\rangle\). In any case, the reader can easily verify that the population of all other levels will be much smaller than the last two, which will be scarcely populated. The Hamiltonian in this subspace can be rewritten as \(H = p^2/2m + H'(x)\), and this last can be diagonalized exactly in the rotating frame. Instead of doing that, we calculate the eigenstates and eigenvalues of \(H'(x)\) in lowest order perturbation theory with respect to \(\Omega\), which is assumed to be small with respect to \(\Delta\) for all values of \(x\). Then, by perturbation theory, we can find an effective non-Hermitian Hamiltonian \(H_{\text{eff}}\) and its corresponding eigenvalues that we can adiabatically eliminate the levels \(|\Delta + g(x)|\) for all values of \(x\) (see Fig. 3), where \(\Delta = \omega_L - \omega_0\). We obtain

\[
|\Psi_0\rangle = |g,0\rangle + \frac{\Omega/2}{\Delta^2 - g(x)^2} (g(x)|g,1\rangle + \Delta|e,0\rangle) \\
|\Psi_1\rangle = \frac{1}{\sqrt{2}} (|g,1\rangle - |e,0\rangle - \frac{\Omega/2}{\Delta + g(x)}|g,0\rangle) \\
|\Psi_2\rangle = \frac{1}{\sqrt{2}} (|g,1\rangle + |e,0\rangle - \frac{\Omega/2}{\Delta - g(x)}|g,0\rangle)
\]

and the corresponding eigenvalues

\[
\lambda_0(x) = \frac{\Delta}{2} + \frac{\Omega^2}{8} \left(\frac{1}{\Delta + g(x)} + \frac{1}{\Delta - g(x)}\right) \\
\lambda_1(x) = -\frac{\Delta}{2} - g(x) - \frac{\Omega^2}{8(\Delta + g(x))} \\
\lambda_2(x) = -\frac{\Delta}{2} + g(x) - \frac{\Omega^2}{8(\Delta + g(x))}
\]

We introduce a cavity decay rate \(\kappa\) and a spontaneous decay rate \(\Gamma\) for the atom. To take both into account we use the master equation that describes the time evolution of this open quantum systems. The state of the system, which is now in general a mixed one, is given by a density matrix \(\rho\). For this system we obtain

\[
\dot{\rho} = -i[H, \rho] + (L_{\text{cav}} + L_{\text{at}})\rho.
\]

Here,

\[
L_{\text{cav}}\rho = \kappa (2a \rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a)
\]

describes cavity damping, whereas

\[
L_{\text{at}}\rho = \Gamma \left(2 \int_{-1}^1 N(u) \sigma^- e^{-iu x} \rho e^{iu x} \sigma^+ du - \sigma^+ \sigma^- \rho - \rho \sigma^+ \sigma^-\right)
\]

describes spontaneous emission. The first term in this expression accounts for the photon recoil experienced by the atom after photon emission. We have included here a one dimensional version, since in our numerical calculations we have investigated this case.

To simulate a single trajectory we use the Quantum Jump Approach [31, 32, 33, 34]. Therefore one defines an effective non-Hermitian Hamiltonian \(H_{\text{eff}}\) which describes the time evolution of the system under the condition that no emission takes place. The master equation can than be written in the form

\[
\dot{\rho} = -i \left[(H_{\text{eff}} + \frac{p^2}{2m}), \rho\right] + 2\kappa a \rho a^\dagger + 2\Gamma \int_{-1}^1 N(u) \sigma^- e^{-iu x} \rho e^{iu x} \sigma^+ du.
\]

The decay rates contribute to the effective time evolution as damping terms. Therefore the norm of the state decreases. This means that the probability to find no photon in the time interval \((0, t)\) decreases with time “\(t\)”. Dissipation occurs in our model due to the small contamination of level \(|\Psi_0\rangle\) with the states \(|g,1\rangle\) and \(|e,0\rangle\), which in turn decay due to cavity damping and spontaneous emission, respectively. In order to determine the effective decay rate (or jump time) we take the sum over the probabilities for the excited states \(|g,1\rangle\) and \(|e,0\rangle\) in \(|\Psi_0\rangle\) weighted with the cavity decay rate \(\kappa\) and the spontaneous emission rate \(\Gamma\). For the coupling constant we assume that the atom is in the center of the cavity \(|g(x) = g_0\rangle\). We obtain

\[
\Gamma_{\text{eff}} = \kappa \frac{g_0\Omega/2}{\Delta^2 - g_0^2} + \Gamma \frac{\Delta\Omega/2}{\Delta^2 - g_0^2} = \frac{\Omega^2(\kappa g_0^2 + \Gamma \Delta^2)}{4(\Delta^2 - g_0^2)^2}.
\]
FIG. 4: Effective decay time $\tau_{\text{eff}}$ (A) and potential depth $V_0$ (B) versus laser detuning $\Delta = \omega_L - \omega_0$. For the Rabi frequency of the laser we took $\Omega = 0.70 \times 2\pi \text{MHz}$. The coupling strength between cavity and atom is $g_0 = 16 \times 2\pi \text{MHz}$, the cavity loss rate $\kappa = 1.4 \times 2\pi \text{MHz}$ and the spontaneous decay rate $\Gamma = 3 \times 2\pi \text{MHz}$.

This gives an effective decay time of

$$\tau_{\text{eff}} = \frac{1}{\Gamma_{\text{eff}}} = \frac{4 (\Delta^2 - g_0^2)^2}{\Omega^2 (\kappa g_0^2 + \Gamma \Delta^2)}.$$  \hspace{1cm} (14)

For the estimation of the life time in Eq. (14) we neglected the contribution of the upper dressed level $|+\rangle$. If we consider this and the approximation $\Delta \approx -g_0$ and plug it with $\kappa = \Gamma = \max(\kappa, \Gamma)$ into (14) we end up with the expression (3).

Discussion

In Fig. 4(A) and Fig. 5(A) we have plotted the potential depths $V_0$ and the effective life time $\tau_{\text{eff}}$ versus $\Delta$ and $\Omega$. From Fig. 4(A) we see that in order to get a long decay time it would be desirable to have $|\Delta| \gg |g_0|$. In Fig. 4(B) the region $-g_0 < \Delta < 0$ is not of interest since there one obtains no attractive potential ($V_0 < 0$). One has to find a compromise between $\Delta$ close to $-g_0$ in order to get a deep potential and $|\Delta| \gg |g_0|$ in order to obtain a long decay time. This behavior is not surprising because if the detuning is close to $-g_0$ the population in $|\rangle$ increases. This leads to a short decay time and a deep potential. The same reasoning explains the plots in Fig. 5 since the Rabi frequency $\Omega$ is a measure for the coupling strength between the atomic transition and the laser.

For the parameters from (17) and a potential depth of $V_0 = 10 \text{kHz}$ the longest effective life time we can achieve in the optical regime is $\tau_{\text{eff}} = 0.18 \text{ ms}$. The corresponding values for the laser parameters are

$$\Omega = 0.70 \times 2\pi \text{MHz},$$

$$|\Delta| = 1.90 g_0 = 30 \times 2\pi \text{MHz}.$$  \hspace{1cm} (15)

In the microwave regime (11, 35) we get $\tau_{\text{eff}} = 1.26 \text{ sec}$ with

$$\Omega = 54 \times 2\pi \text{kHz},$$

$$|\Delta| = 2.06 g_0 = 0.14 \times 2\pi \text{MHz}.$$  \hspace{1cm} (16)

It is important to mention that since we used the expressions from (7) and (14) we are not in the limit $\Delta \approx -g_0$ (1). This leads to a significantly longer life time in the optical regime.

NUMERICAL RESULTS

Here we investigate the behavior of the system numerically. For the analytic results we made certain approximations. The comparison with the numerical results will show if these assumption are justified for realistic parameters. Furthermore we will include spontaneous emission and photon recoil.
We denote the state of the system by |Φ⟩. For the simulation we write it as |Φ⟩ = |ϕg0⟩ + |ϕg1⟩ + |ϕe0⟩, where |ϕi⟩ = |i⟩⟨i|Φ⟩. We consider only the contributions of |g, 0⟩, |g, 1⟩ and |e, 0⟩ since the population of the levels with two and more excitations is negligible. As for the analytic estimations we restrict the investigations to one dimension. The probability amplitudes for the system being in the states |g, 0⟩, |g, 1⟩ and |e, 0⟩ at position “x” are given by ϕg0(x) = ⟨x|ϕg0⟩, ϕg1(x) = ⟨x|ϕg1⟩ and ϕe0(x) = ⟨x|ϕe0⟩.

In the first subsection we calculate the ground state of the system with and without the assumptions made above. In the following we include dissipation and compare the decay time with the effective decay time τeff we estimated in the last section. Finally we consider spontaneous emissions and recoil and investigate how long the atom remains in the cavity for different parameters. Apart from one simulation with the parameters from the analytic estimation we will only consider the optical regime in this section.

The ground state

To obtain the ground state we apply the imaginary time evolution to an arbitrary initial state until it remains unchanged. Instead of the time evolution operator e−iHΔt one uses a modified operator e−iHΔt. After a sufficient number of iterations this damps away all states orthogonal to the one with the lowest eigenvalue, which is the ground state of the system.

The Hamiltonian of the system is given in Eq. (4). We denote its ground state by |Φ0⟩. For Ω and Δ we took the values from (13). According to the analytic approximation they should give the maximal decay time which is achievable for a potential depth of 10 kMHz. The numerical simulation of the ground state leads to the probability distribution shown in Fig. 6. The three plots show the population distribution of the three internal states separately. The excited states are only very weakly populated. The probability to find an atom in the center of the cavity with the system being in state |g, 1⟩ or |e, 0⟩ is three to four orders of magnitude smaller than to find it there with the state |g, 0⟩. We also found that the atom is well localized in the center of the cavity. At 0.1 σ, where σ is the width of the cavity, the probability to find the atom is already reduced by more than 1/2.

In order to validate the approximations made for the analytic estimation we calculated the ground state also using the Hamiltonian from Eq. (8). We denote its ground state solution by |ξ0⟩. We find

|Φ0⟩ ≈ |g, 0⟩ ⊗ |ξ0⟩.  

(17)

This means that nearly all the population is in |g, 0⟩. So the approximations in the analytical approach are justified and one can trap an atom in a basically empty cavity.

Dissipation and photon emissions

In this subsection we include the coupling of the system to the environment and as a consequence the spontaneous emission of photons. First we will only consider the time evolution of the system under the condition that no photon is emitted and compare the decay time with the analytic estimation. Then we include also spontaneous emissions and the recoil kick the atom experiences.

In the Quantum Jump Approach one describes the time evolution of the system with an effective Hamiltonian as long as no photon is emitted. The emissions which cause the system to jump in a different state are described by reset operators. We obtain an expression for the effective Hamiltonian by comparing Eqs. (9), (10) and (11) with Eq. (12). This gives

\[ H_{\text{eff}} = \frac{p^2}{2m} - \frac{\Delta}{2} (|g, 1⟩⟨g, 1| + |e, 0⟩⟨e, 0| - |g, 0⟩⟨g, 0|) + g(x) (|e, 0⟩⟨g, 1| + |g, 1⟩⟨e, 0|) + \frac{\Omega}{2} (|g, 0⟩⟨e, 0| + |e, 0⟩⟨g, 0|) - i\kappa |g, 1⟩⟨g, 1| - i\Gamma |e, 0⟩⟨e, 0|, \]

(18)
where we used an interaction picture rotating with the laser frequency $\omega_L$ and assumed that there is at most one excitation in our system.

After preparing the system in the ground state $|\Phi_0\rangle$ using the imaginary time evolution we simulated the time evolution with the effective Hamiltonian $H_{\text{eff}}$. This leads to a damping of the state of the system. So the probability $|\Phi|^2$ that no photon has been emitted also decreases. We compare the time after which this probability has reached $1/e$ with the effective life time $\tau_{\text{eff}}$ we estimated analytically. For the parameters from (15) we obtained $\tau_{\text{eff}} = 0.18$ ms, which agrees with the decay time from the numerical simulation of 0.14 ms.

In the Quantum Jump Approach the jumps are described by reset operators. We obtain them from the master equation (12). For the spontaneous emission of the atom we get

$$e^{-iux} \sqrt{2\Gamma}|g,0\rangle\langle e,0|,$$

where $e^{-iux}$ describes the momentum shift $"u"$ due to the photon recoil. If the cavity emits a photon one has to apply

$$\sqrt{2\kappa} |g,0\rangle\langle g,1|.$$

In both cases the population of the excited level gets shifted to the ground state. After that one has to normalize the wave function.

In the following we will discuss the trapping time $\tau_{\text{trap}}$ of the atom. We define it as the time when the probability to find the atom in the cavity ($|x| < \sigma$) is reduced to 0.5. The atom has an initial kinetic energy and gains a momentum kick when it spontaneously emits a photon. When the motional energy is bigger than the trapping potential the atom leaves the cavity. So it is desirable to achieve a long decay time in order to get a low photon emission rate. On the other hand a deeper potential provides the possibility of a bound state for an atom which experienced more recoil kicks. From our analytic estimations we know that these demands contradict each other. For the simulation we took the parameters from (15) for a potential depth of $V_0 = 10$ kHz and the longest corresponding effective decay time $\tau_{\text{eff}} = 0.18$ ms. We applied the operator $e^{-iux}$ to the ground state which we got from the imaginary time evolution in order to take into account an initial kinetic energy. The momentum shift $"u"$ corresponds to the energy of one photon recoil $E_R = 4$ kHz. After a couple of spontaneous emissions the atom leaves the cavity at $\tau_{\text{trap}} = 0.73$ ms.

In order to achieve a longer trapping time we first varied the detuning $\Delta$ and left the Rabi frequency $\Omega = 0.70 \times 2\pi$ MHz unchanged. The result is shown in Fig. 7. A larger detuning of the laser leads to a smaller potential depth $V_0$ and a longer effective life time $\tau_{\text{eff}}$. From Fig. 7 we see that the longer life time has a bigger influence on the trapping time since $\tau_{\text{trap}}$ increases with growing detuning. It is not surprising that the effective life time has a crucial influence on the trapping time since it determines how fast the motional energy of the atom grows.

A larger Rabi frequency causes a smaller effective life time and a deeper potential. So consistently we expect a decreasing trapping time when we enlarge the Rabi frequency. This is confirmed by Fig. 8 where we plotted $\tau_{\text{trap}}$ versus $\Omega$ for a fixed detuning $|\Delta| = 1.90$ kHz and the spontaneous decay rate $\Gamma = 3 \times 2\pi$ MHz. We compare this plot with the plot in Fig. 7(A) we ascertain a qualitative agreement. This is again what we expect if we assume that the decisive variable for the trapping time $\tau_{\text{trap}}$ is the effective decay time $\tau_{\text{eff}}$.

The longest trapping times we can achieve in the optical regime are of the order of 1 ms. For the microwave regime we took $g_0 = 67 \times 2\pi$ kHz, $\kappa = 1.6 \times 2\pi$ Hz and $\Gamma = 1.6 \times 2\pi$ Hz. The trapping time we obtained for the laser parameters from (15) was $\tau_{\text{trap}} \approx 10$ s. The reason for the good result are the very small decay rates for the Rydberg state and the micro–cavity. Another important
advantage over the optical regime is that the recoil due to spontaneous emissions is practically zero.

**CONCLUSIONS**

We showed that it is possible to trap an atom in the vacuum field of a high Q cavity. To do this we need a weak laser which couples directly to the atom in the cavity. It induces a position dependent AC Stark shift to the ground state of the cavity-atom system. We use this energy shift as a trapping potential and as we showed by an analytic estimation and a numerical simulation it is deep enough to trap an atom with a realistic initial momentum.

The advantage of this approach is the low effective decay rate due to the little amount of excitation in the system. This requires to cool the atom to a lower kinetic energy than the potential depth. In order to obtain a long life time it would be good to have an initial kinetic energy of the order of one photon recoil. This is still difficult to achieve, even though it is possible to cool an atom below one photon recoil with the method of velocity-selective coherent population trapping or Raman-cooling. Another possibility would be a cavity assisted cooling method.

The trapping time we can achieve in the optical regime with our approach is of the same order or even lower as observed already in experiments. The benefit of this method is that the time after which the first jump occurs is longer because there is only very little excitation in the system. As mentioned before this decay time is very important for any kind of quantum information application since the jump destroys the coherence in the atomic state. In the microwave regime the trapping times can be much longer.

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