Abstract. It is experimentally demonstrated that a quantum state, destroyed by uncontrollable natural decoherence, can be purified by using results of projective measurement and converted into a desired target pure state. The physical system is a cluster of seven dipolar-coupled nuclear spins of single-labelled $^{13}$C-benzene in liquid crystal. $^{13}$C spin plays the role of a device for measuring the protons’ ‘cat’ state, a superposition of states with six spins up (alive) and six spins down (dead). Information about the state, stored in the $^{13}$C spin, is used to bring the protons’ subsystem into the target alive state, while the excess entropy produced by decoherence is transferred to the ‘measuring device’, the $^{13}$C spin.
of states with six spins up (alive) and six spins down (dead). After decoherence, information about the state, stored in the $^{13}$C spin, is used to bring the protons’ subsystem into the alive state, while the excess entropy produced by decoherence is transferred to the $^{13}$C spin. Therefore, after the quantum state of the seven-spin system is irreversibly destroyed by decoherence, one part of the system can be brought into a desired pure state by using information stored in the other part.

When each of an ensemble of identical systems is characterized by the same quantum state, such an ensemble is said to be in a pure state and can be described by a wavefunction $|\Psi\rangle$ or, equivalently, by the corresponding density matrix $\rho = |\Psi\rangle\langle\Psi|$. When a system is in one of the two states: $|u\rangle$ (alive) or $|d\rangle$ (dead), in a basis where $|u\rangle$ and $|d\rangle$ are eigenstates, the density matrix has only one nonzero matrix element. For the superposition state $1/\sqrt{2}(|u\rangle + |d\rangle)$, the density matrix of this pure state contains four nonzero elements in the same basis. Two diagonal elements are the populations and the two off-diagonal elements describe a coherence between the two states. Interaction with the environment can destroy the coherence. The result of this process, called decoherence, is the mixed state of an ensemble. Individual systems are no longer in the same quantum state but can be found in one of the two states, $|u\rangle$ or $|d\rangle$, with equal probability.

To be more specific, let us consider a system of $N$ coupled two-level systems (qubits). A cluster of spins 1/2 is a natural physical implementation of such system. As the two most distinct states, one can choose the ferromagnetic states $|u\rangle = |\uparrow\uparrow\cdots\uparrow\rangle$ and $|d\rangle = |\downarrow\downarrow\cdots\downarrow\rangle$ with all spins up and down, respectively. These two states have the maximum difference in polarization (magnetization). The off-diagonal part, $N$-quantum (NQ) coherence, of the cat state density matrix $\rho = 1/2(|u\rangle + |d\rangle)(|u\rangle + |d\rangle)$ has the operator form $|u\rangle\langle d| + |d\rangle\langle u| = S_1^+S_2^+\cdots S_N^+ + S_1^-S_2^-\cdots S_N^-$, where $S_i^+$ and $S_i^-$ are the raising and lowering operators for individual spins. When individual spins are rotated by angles $\phi_i$ around their quantization axes, the two product operator terms in the NQ coherence acquire the phases $\pm\sum_i \phi_i$. Therefore, only the sum of the phases is important and, as an example, rotation of all spins by $\phi$ and rotation of a single spin by $N\phi$ produce the same result. This unique feature of the cat state opens a way to interesting applications. Firstly, $N$ times faster rotation of the phase of the cat state, compared to that of an individual spin, can be used in high-precision measurements of phases and frequencies [1]. On the other hand, local rotation of a single spin can produce a global change of the phase for the entire system, which can be then converted into a change of macroscopic observables. This creates a base for amplified quantum detection [3, 4] and state measurement [5].

The negative side of these useful properties is the high sensitivity of the cat state to phase noise. When interaction with the environment produces uncorrelated rotations of individual spins by $\Delta\phi_i$, the change in the phase of the NQ coherence, $\Delta\varphi$, can be estimated as $\langle\Delta\varphi^2\rangle \approx N\langle\Delta\phi_i^2\rangle$. This explains why even weak interaction with the environment may cause fast decoherence in macroscopic systems. Random phases acquired by the NQ coherences of individual systems average out the off-diagonal part of the cat state density matrix and convert it to the diagonal mixed state. Decoherence can be viewed as a loss of information about the phase of the NQ coherence of an individual system. This also means a loss of reversibility because, in order to dynamically convert a state into some target state, one needs to know the exact starting state. A degree of irreversibility is quantified by the entropy, which changes from zero for the pure cat state to $k_B \ln 2$ for the mixed state.

Different states of the same system have different sensitivity to noise. Some of them are resistant to relaxation and decoherence [6]–[9]. The cat state is the most fragile in terms of the decoherence rate, but the damage produced to this state by decoherence is relatively small.
In quantum mechanics, the density matrix gives the most complete description of a system. This means that no physical measurement can distinguish between the mixed state \( \rho = 1/2(|u\rangle\langle u| + |d\rangle\langle d|) \), resulting from averaging out the NQ coherences, and the mixture of systems in one of the two states, \( |u\rangle \) or \( |d\rangle \). Therefore, the only missing information is which of the two states has been chosen by a system. This information can be stored in only one additional qubit.

Let us consider a combined system of \( N + 1 \) qubits. The symbols \( |↑\rangle \) and \( |↓\rangle \) will denote the two states of the additional qubit. It will be called a control qubit because its state will be used to control a change of the state of the \( N \)-qubit system. We will start with the system in some superposition of the states \( |u\rangle \) and \( |d\rangle \) and the control qubit in its ground state \( |↑\rangle \):

\[
|\Psi\rangle_{\text{in}} = |↑\rangle(a|u\rangle + b|d\rangle), \quad |a|^2 + |b|^2 = 1.
\] (1)

By using interaction between the control qubit and the system, one can design a reversible unitary evolution \([5]\), which converts the state (1) into the state

\[
|\Psi\rangle_{\text{out}} = a|↑\rangle|u\rangle + b|↓\rangle|d\rangle.
\] (2)

Decoherence eliminates the \((N+1)Q\) coherence in this state and produces the mixed state with the density matrix

\[
\rho_{\text{mix}} = aa^*|↑\rangle\langle ↑| \otimes |u\rangle\langle u| + bb^*|↓\rangle\langle ↓| \otimes |d\rangle\langle d|.
\] (3)

In this mixture of two states, information about the state of the \( N \)-qubit system (is it \( |u\rangle \) or \( |d\rangle \)?) is stored in the state of the control qubit. It is possible to use this information for producing a dynamic evolution, conditioned by the state of the control qubit, which will bring the \( N \)-qubit system to a desired pure state, for example, the \textit{alive} state \( |u\rangle \). Of course, most important here is purification of the state of the \( N \)-qubit system. Other states can be obtained with suitable unitary transformations. In our experiment, we have chosen the target \textit{alive} state \( |u\rangle \) as the most optimistic outcome with a simple spectrum.

Alternatively, the density matrix (3) can be viewed as resulting from projective measurement, where the control qubit is used as a ‘measuring device’. The initial superposition state of the \( N \)-qubit system, \( a|u\rangle + b|d\rangle \), is collapsed into one of the two definite states \( |u\rangle \) or \( |d\rangle \), while the remaining uncertainty is compensated by the measurement result, stored in the state of the control qubit.

For experimental implementation of this scheme we have chosen a cluster of seven dipolar-coupled nuclear spins \( (N = 6) \) in \(^{13}\text{C}\)-labelled benzene molecule oriented by a liquid-crystalline matrix. The sample is 5% of single-labelled \(^{13}\text{C}\)-benzene (Aldrich) dissolved in liquid crystal MLC-6815 (EMD Chemical). Each benzene molecule contains seven nuclear spins, one \(^{13}\text{C}\) and six protons, coupled by residual dipole–dipole interactions. All intermolecular spin–spin interactions are averaged out by fast molecular motions. Therefore, the system is an ensemble of non-interacting spin clusters, where each benzene molecule contains seven dipolar-coupled nuclear spins. The experiment has been performed with a Varian Unity/Inova 500 MHz nuclear magnetic resonance (NMR) spectrometer at room temperature (25°C). At present, clusters of nuclear spins explored with NMR provide the largest and the most complex systems with individually addressable quantum states. With liquid-state NMR, coherent manipulations have been demonstrated for systems of up to seven spins \([10, 11]\).
At thermal equilibrium, nuclear spins are in a highly mixed state, which means that an individual system can be, with some probability, in any of the possible quantum states. The most convenient way to address individual states is by using so-called pseudopure states [12]–[14]. In a pseudopure state, populations of all but one state are made equal. As a result, the spin density matrix is the sum of a maximally mixed background, which is proportional to the unity matrix, and a deviation part, which is proportional to a density matrix of a pure state. Since the unity matrix does not contribute to observables and is not changed by unitary transformations, the behaviour of a system in a pseudopure state is exactly the same as it would be at zero spin temperature. For dipolar-coupled spins, we have recently created pseudopure states [15], including the cat state [16], for a system of twelve nuclear spins of fully $^{13}$C-labelled benzene in liquid crystal. More recently, a 12-qubit pseudopure state has been reported for a weakly coupled NMR system [17].

Creation of the pseudopure state $|↑⟩|u⟩$ for our seven-spin system is the first part (step A) of the experimental scheme in figure 1. It starts with a sequence of carbon pulses and gradients to saturate the $^{13}$C magnetization. The multi-pulse sequence [18] excites the protons’ multiple-quantum coherences. Phase cycling is used to filter the highest-order coherence, and the evolution caused by dipole–dipole interaction between the carbon and the protons’ spins during alternating

![Diagram of NMR pulse sequence](image)

**Figure 1.** NMR pulse sequence: (step A) six proton spins and a carbon spin are initialized in the pseudopure state; (step B) the proton’s cat state is prepared; (step C) the carbon spin is entangled with the proton spins; (step D) decoherence brings the whole system into a mixed state; (step E) the ‘controlled-not’ operation, conditioned on the carbon spin state, creates the target state of the proton subsystem.
Figure 2. Linear-response $^1$H NMR spectra at thermal equilibrium (a, a’) and for four pseudopure states (b–e and b’–e’). The left column shows $^{13}$C-decoupled spectra.

intervals $m\tau (m=0, 1$ and $\tau = 201 \mu s$) selects only the protons’ highest-order coherence with the carbon spin up. The second multi-pulse sequence returns the coherence back to the diagonal states, and partial saturation performed with selective sinc-pulses creates the pseudopure state $|\uparrow\rangle|u\rangle$. The details of the technique are described elsewhere [3, 5, 15]. $^1$H linear-response spectra of the four pseudopure states are shown in figure 2. The states $|\uparrow\rangle|d\rangle$, $|\downarrow\rangle|u\rangle$, and $|\downarrow\rangle|d\rangle$ are
obtained from the state |\uparrow\rangle_u by applying ‘hard’ 180° carbon and proton pulses. The left column in figure 2 displays the spectra with decoupled 13C spin. The system is too complex to allow a complete reconstruction of the density matrix. At the same time, all the spectra, both 13C-coupled and 13C-decoupled, coincide precisely with the theoretical spectra calculated for these states [5]. Each of the decoupled spectra (left column) consists of a single peak. The 13C-coupled spectra for the states |\uparrow\rangle_u (figure 2(b′)) and |\downarrow\rangle_d (figure 2(c′)) have two peaks, and there are one large and two smaller peaks in the spectra for the states |\uparrow\rangle_d (figure 2(c′)) and |\downarrow\rangle_u (figure 2(d′)). For comparison, the thermal equilibrium spectra are shown in figures 2(a) and (a′).

Step B of the experiment (figure 1) creates the six-spin cat state for the proton subsystem and, therefore, the total state becomes that of equation (1) (at $a = b = 1/\sqrt{2}$), which is the starting point of our ‘resurrection’ scheme.

The next step C (figure 1) is designed to convert the protons’ six-spin cat state of equation (1) into the seven-spin cat state of the entire system, described by equation (2). This step includes 90° carbon pulse, carbon–proton evolution delay, and protons multiple-quantum evolution period. The 13C-decoupled and 13C-coupled spectra of this seven-spin cat state are shown in figures 4(a) and (a′), respectively.

The variable-length step D is a delay when decoherence takes place and converts the pure state of equation (2) into the mixed state of equation (3). Before proceeding to the next step, we measured the decay times of the off-diagonal, i.e. the seven quantum (7Q) coherence, and the diagonal elements of the seven-spin cat state density matrix to see how decoherence transforms a superposition state into a mixture. The results are shown in figure 3. Since the 7Q coherence is not directly observable, we followed step D by an additional step, which is a time-reversal of step C (not shown in figure 1). After that, the reading pulse produced the spectra with the intensities of the peaks proportional to the intensity of the 7Q coherence. The decay of the diagonal elements of the cat state has been measured by simply applying a reading pulse after step D. Different symbols in figures 3(a) and (b) show the results obtained by using intensities of different peaks in the spectra. Decays of both the off-diagonal (figure 3(a)) and the diagonal (figure 3(b)) elements of the cat state are well described by single-exponential curves with the average lifetimes of 29 and 490 ms, respectively. Note that the timescales in figures 3(a) and (b) are different. The lifetime of the 7Q coherence, or decoherence time of the cat state, is much shorter than the relaxation time of its diagonal elements. This difference between decoherence and the relaxation times is supposed to increase with the system size. For our seven-spin cluster, the difference is already significant and allows clean implementation of the proposed scheme.

The last step E of the experiment (figure 1) implements the ‘controlled-not’ operation: when the state of the control qubit is |\uparrow\rangle, it does not change the state of the proton subsystem; when the state of the qubit is |\downarrow\rangle, it flips the protons state |d\rangle into the state |u\rangle and vice versa. As a result, a mixture of the two states |\uparrow\rangle_u and |\downarrow\rangle_d after step D is converted into a new mixture of the states |\uparrow\rangle_u and |\downarrow\rangle_d. In both of these states, the proton subsystem is in the target alive state |u\rangle. This is supported by the 13C-decoupled spectra in figures 4(c) and (d) for 100 and 200 ms delays in step D. The spectra are the same as the spectrum of the pseudopure state |u\rangle in figures 2(b) and (d). One can see that even after 200 ms delay time, which is much longer than the seven-spin cat state decoherence time (29 ms) or the protons six-spin cat state decoherence time (42 ms), the target state |u\rangle is well recovered. At the same time, the carbon spin is found in the mixture of the two states |\uparrow\rangle and |\downarrow\rangle, which can be clearly seen in the 13C-coupled spectra in figures 4(c′) and (d′). These spectra are the sums of the spectra for the two pseudopure states |\uparrow\rangle_u and |\downarrow\rangle_u in figures 2(b′) and (d′). Therefore, the excess entropy $k_B \ln 2$ produced by decoherence is transferred to
Figure 3. Lifetime of the seven-spin cat state: (a) decay of the 7Q coherence, the solid line is the function \( \exp(-t/\tau) \) with \( \tau = 0.029 \text{ s} \); (b) decay of the diagonal states, the solid line is the function \( \exp(-t/\tau) \) with \( \tau = 0.49 \text{ s} \).

The \(^{13}\text{C}\) spin. As was mentioned above, this step resembles a projective measurement, where the \(^{13}\text{C}\) spin plays the role of ‘measuring device’. It is interesting that, although the NMR detection is a weak ensemble measurement, the experiment can be designed to provide information about possible outcomes of projective measurement and probabilities of such outcomes [19].

One can notice that quality of the spectra in figure 4 is somewhat lower than that for the pseudopure states in figure 2. It is a consequence of complex dynamics and very long total sequence of 982 radio frequency and gradient pulses. However, even with unavoidable imperfections, the experimental results present a convincing proof of the principle. They show that after a time much longer than the cat state decoherence time, the information recorded in the ‘measuring device’, the \(^{13}\text{C}\) spin, is sufficient to bring the protons’ subsystem into a desired target state. The entropy of the total seven-spin system is increased, and the algorithm cannot be performed one more time. However, if it is possible to reset the control qubit or supply a new one, the whole process starting from building the cat state can be repeated. At the same time, the scheme does not protect the system from relaxation, an energy-exchange process, which decreases intensities of the spectra at increasing delay time (figures 4(c) and (d)).
Figure 4. Linear-response $^1$H NMR spectra for the seven-spin cat state (a, a’), and the restored six-spin alive state $|u\rangle$. The delay times are (b, b’) 0 ms, (c, c’) 100 ms, and (d, d’) 200 ms. The left column shows $^{13}$C-decoupled spectra.

Our purification scheme resembles an active quantum error-correcting algorithm [20]. In both cases, additional resources are used to bring the system back to the target pure state after decoherence or errors. It would be interesting to look for an extension of the current scheme to error correction applications. Simple quantum error-correcting schemes have been experimentally realized for small quantum systems [21]–[23]. Compared to these experiments, demonstrating protection from artificial errors and engineered decoherence effects, our experiment implements a state purification after uncontrollable natural decoherence.

Decoherence is the major obstacle in designing practical devices which will utilize the advantages of quantum dynamics in detection, measurement, or information processing. The experiment performed in this work demonstrates that destructive effect of decoherence can be minimized by using a scheme where information stored in one part of a system is used to purify the quantum state of another part.
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