Digital model to optimize the rotor assembly for aircraft engine compressor models

M A Bolotov¹, V A Pechenin¹, E Yu Pechenina¹, N D Pronichev¹

¹Samara National Research University, Institute of Engines and Power Plants, Moskovskoye Shosse 34, Samara, Russia, 443086

Abstract. The article focuses on the problems of knowledge-intensive products assembly quality in aircraft engine industry. A mathematical model has been proposed to determine optimal conditions for mating assembly units of a low-pressure compressor (LPC) rotor of an aircraft engine. Parameters optimized include relative angular positions of parts in the rotor. Three optimization criteria have been considered: assembly geometry; imbalance value and integral criterion combining both above criteria. The model has been implemented in the MATLAB system. The calculations have been made using actual part models created based on measurements of the LPC parts geometry. The best optimization criterion has been selected.

1. Introduction
Fulfilling the requirements for reducing the products market launch and ensuring their high quality is a complex contradictory problem which can be solved using advanced digital technologies. Production and repair of complex knowledge-intensive products have a high production cost, long cycles and considerable investments of floating assets. Complex knowledge-intensive products which quality is a critical indicator include aircraft engines, power and propulsion plants, and rocket engines. The quality of the above products depends on a variety of factors, mainly, the geometrical accuracy of assembly units and products.

The ways to achieve dimensional accuracy are determined at the stage of design-engineering process preparation [1, 2, 3]. Solving the above tasks can be simplified at each stage with the use of available mathematical models [4] and tools that allow forecasting and determining the actual value of geometrical parameters [5] characterizing the achievable accuracy.

The work considers the creation of a digital model of the LP compressor rotor assembly used to find optimal assembly positions included in the part assembly before actual assembly works.

2. Subject of the research
As it was said before, the object of the research is the assembly of the LP compressor rotor. The sketch of the assembly is shown in Fig. 1.

The assembly includes the following parts (figure 1): rear shaft (9), and front shaft (1); discs 1, 2, and 3 (2, 5, and 8, figure 1); first and second spacer rings (3 and 6) and rings 1 and 2 (4 and 7 in Fig. 1). Rings 1 and 2 of the assembly were not used in the experiments. The LPC rotor is installed into the engine on the cylindrical surfaces B and C resting upon the faces A and D. The assembly is performed
in three stages: 1) assembly of the rear shaft, disc 3, and spacer ring 2; 2) insertion of disc 2 and spacer ring 1; 3) insertion of disc 1 and front shaft. The parts are connected with the help of fit bolts.

![Figure 1. Sketch of assembling seven LPC parts.](image)

The surfaces $A$ and $B$ are the basic elements (used to control the assembly parameters) in the assembly. The face $\delta_{rt}$ and radial $\delta_{rr}$ runouts are controlled at each stage of the assembly. The surfaces $E$ and $F$ of the spacer ring (6) are controlled at the first stage; the surfaces $G$ and $H$ of the spacer ring (3) are controlled at the second stage and the surfaces $C$ and $D$ of the front shaft are controlled at the third stage.

The goal of the assembly is to achieve the allowable assembly parameters (the error for all the runouts under consideration is 0.1 mm) and ensure the allowable imbalance of parts and assembly in general [6] (the error for the residual imbalance for the rotor under consideration is 600 g•mm). All the parts are symmetrical; hence different combinations of part assemblies are possible. The goal of the research is to reduce the labor intensity at the assembly stage by selecting the best positions of the assembled parts by means of a virtual calculation. Below is the summary of the mathematical model of the assembly under consideration.

### 3. Rotor assembly model

The assembly modelling includes three stages: modelling parts that include the actual geometry containing production deviations; calculating mated parts [7, 8] and calculating geometrical parameters and imbalance.

#### 3.1. Creating models of parts with the actual geometry

Information about the actual geometry in the form of data on part surface measurements is required to model it. The part surfaces were measured on a coordinate measuring machine (CMM) of DEA GlobalPerformance. The parts were fixed on the CMM plate with the help of special tools.

The following number of measured points, which was optimal in terms of accuracy, was obtained during the measurement [9, 10]: 200–300 points on the mating planes and cylindrical surfaces; 100 points on the control face D. The part ends were measured in cross-sections. In case of cylindrical surfaces, cross-sections represent intersection lines of the surface and planes which are perpendicular to the rotation axes. The cross-sections for face surfaces represent intersection lines of the surface and cylindrical surfaces which axis and center coincide with the normal plane vector. So the cross-sections represent circles. The coordinates of the measured points were saved as *.txt files for further analysis made in the MATLAB system.

After downloading the point coordinates on the surfaces, they are processed and brought to the structure for further creation of actual surfaces. Processing of the point coordinates lies in smoothing outliers and
calculating point coordinates missing to build the data structure. Smoothing was performed with the moving average method. Calculation of the point coordinates lies in creating cross-sections of the part surfaces by approximating or interpolating the measured sets of surface point coordinates using spline functions in the form of profiles or surfaces [11].

In general, complex part surfaces are represented in a portion way, like a patchwork quilt. Complex curves and surfaces in CAD systems and metrology software of measuring equipment are described using spline equations. A 3rd degree normalized cubic spline, namely the Hermite curve, was used for mathematical representation of spatial curves [12]. To describe the part surfaces that have geometrical deviations of the form, the surfaces made of bicubic portions (Kuns’ portions [12]) were used.

So the actual part models represent a set of related part surfaces used in the assembly and control.

3.2. Virtual calculation of the part assembly, result saving
To solve the contact task using the surface models, an iterative algorithm has been developed; it allows calculating the parts mating without taking into account deformation of the parts in the process of assembly detailed in [7]. The algorithm for determining the mated state assumes iterative movement of one mating surface in relation to the other one, with the stress application vector of the surface assembly $\Delta$. To ensure the best adjustment, the iterative algorithm of nearest points (ICP) is used [13, 14]. According to this algorithm, the rotation and movement angles along the coordinate axes are calculated at each iteration with the non-linear optimization search methods. To eliminate intersections of two surfaces, the system of inequalities limiting the gap function $G(\vec{v})$, which is described in the work [15], is used. The use of the algorithm results in calculating a rotation matrix and moving part movement vector that determines the conversion of its initial coordinate system into the coordinate system in the assembled state.

3.3. Calculating the assembly parameters and imbalance
The radial runout between the cylindrical surfaces and the base $B$ is calculated in the following order:
- The system of element coordinates is adjusted over the surface $B$ by mating the normal vector of the surface $B$ to the axis $X$.
- Distances from the measured points of the second cylindrical end to the axis of the surface $B$ are calculated.
- The radial runout $\delta_r$ is calculated as the difference between the maximum $d_{\text{max}}$ and minimum $d_{\text{min}}$ values from the measured points to the axis $B$.

Figure 2 explains the determination of the “radial runout” geometrical parameter.

The face runout, for example, the one used for the face $D$ of the “front shaft” part is calculated as the difference between the maximum and minimum distances from the measured points of the face $D$ to the plane perpendicular to the axis formed by the base $B$.

The total assembly imbalance is calculated by the following equation:

$$D \sum = |\vec{r}_\Sigma| M \sum,$$

where $|\vec{r}_\Sigma|$ is a module of the sum vector of the assembly mass centre displacement;

$M \sum$ is the total mass of the assembly, which is composed of the sum of masses of the parts included in it.

Components of the sum vector $\vec{r}_\Sigma$ along the coordinate axes $OX$, $OY$ and $OZ$ are calculated as follows:

$$r_j = \left(\sum_{i=1}^{N} l_{ij} \cdot M_i \right) / M \sum,$$
where \( r_j \) is a component of the vector along the \( j^{th} \) axis (\( OX, OY \) or \( OZ \)); \( N \) is the number of parts in the assembly; \( t_{ij} \) is a component of the movement vector of the mass center of the \( i^{th} \) part along the axis \( j \); \( M_i \) is the mass of the \( i^{th} \) part.

\[
\text{Figure 2. Graphical explanation of the radial runout calculation, 1 – surface for which the radial runout is calculated.}
\]

4. Optimization criteria

The assembly parameters of the rotor parts and imbalance value are the criteria of optimal conditions search for the rotor assembly. The parameters optimized in the process of the optimization search are relative angular positions of the parts in the rotor. A set of changed parameters is specified as \( AP = \{ \alpha_{p_1}, \alpha_{p_2}, \alpha_{p_3}, \ldots, \alpha_{p_l} \} \), \( AP \subset R, l = e N \), where \( l \) is the number of changed parameters. The set of changed parameters is related to the set of their allowable values \([AP]\) and the corresponding formal condition \( AP \leq [AP] \subset R \). Allowable values determined by design features of discs including the number of equally spaced mounting holes are specified for the parameters that characterize the relative angular position of the parts. An optimal option of the assembly has been found on the basis of three options differing in target functions.

The first option assumes the best match to the set geometrical assembly parameters. The following criterion is minimized to determine it:

\[
K_{AP} = \sum_{j=1}^{N} w_j \frac{\Delta \alpha_{AP_j}}{T_{AP_j}} \rightarrow \min ,
\]

where \( D \alpha_{AP_j} = A_{SP_j} - [A_{SP_j}] \) is the difference between the \( j^{th} \) geometrical assembly parameter of the rotor \( A_{SP_j} \) and its allowable value \([A_{SP_j}]\); \( T_{AP_j} \) and \( w_j \) are the error and importance weight on the \( j^{th} \) geometrical assembly parameter of the rotor, respectively. Based on the requirements of the design documentation, the geometrical assembly parameters of the rotor shall not exceed their allowable values \( A_{SP} \leq [A_{SP}] \subset R \).

The second option is aimed at achieving the best compliance with the set requirements for the rotor imbalance. Below is the criterion considering the achievement of the best rotor imbalance distribution:
where \( F(DP, AP) \) is the function to record the parameter characterizing the rotor vibration during the research, when the total imbalance is \( D \sum ; [D] \) and \( T_{b} \) are the allowable value and tolerance to achieve the parameter characterizing the rotor vibration, respectively, in accordance with the process or design documentation.

The third option is mixed and provides for a compromise to achieve the most significant geometrical assembly parameters and requirements for the rotor imbalance. The following integral criterion is used:

\[
K = w_{A} K_{A} + w_{D} K_{D} \rightarrow \text{min},
\]

where \( w_{A} \) and \( w_{D} \) are the importance weights of the geometrical assembly parameters and parameters characterizing the rotor vibration, respectively.

Significant geometrical assembly parameters include radial runouts of discs that have a significant effect on the radial gap difference between the blade faces and the stator. A high value of corresponding importance weights should be set for significant geometrical parameters.

Forecasting errors should be estimated to assess the results of the assembly parameter forecast and update the structure of the selected neural network model. The parameter forecasting errors are estimated by two criteria.

5. Results of using the assembly model for the process optimization

Seven parts of the LPC rotor (Fig. 1) were measured, and the part models with the measured geometry were created for the experiments. Each part had four angular positions: 0°, 90°, 180°, and 270°. So, to consider all the assembly combinations of the parts, 4,096 assembly cases were modelled in the MATLAB system. They were calculated in a computer with a AMDRyzen 7 2700 Eight-Core processor, clock rate of 3.2 HZ, RAM 32 Gb, for 30.5 hours. After saving all the results, the three above optimization criteria were used and the best options were selected in the context of the criteria. The allowable value during the optimization \( \sum [A_{s} P_{j}] \) is 0. The tolerance for the assembly parameters \( T_{A_{s} P_{j}} \) is 0.1 mm; \( [D] \) is 0; \( T_{b} \) is 600 g•mm and the importance weights are 1 in the (9-11) criteria.

To compare the obtained options, relative deviations from the parameters in the zero position were calculated, i.e. when the angular position of all the assembled parts is set to 0°. Below is the formula of relative deviation for each parameter:

\[
\Delta_{rel} = \frac{(P_{opt} - P_{o})}{P_{o}} \cdot 100\% ,
\]

where \( P_{opt} \) is the parameter calculated as a result of the optimization; \( P_{o} \) the parameter measured during zero assembly.

Table 1 contains the values of the geometrical parameters \( \delta_{f-r} \) and \( \delta_{t-r} \) of the assembly part surfaces (figure 1), discs 1, 2, and 3 included into the assembly (d. 1, d. 2, and d. 3 in Table 1), total imbalances in zero position, as well as the imbalances after the optimization by the criteria listed in Section 4.

In accordance with the obtained results in Table 1, it may be concluded that the geometrical deviations of the assembly parameters decrease in general during the optimization \( K_{A_{s}P_{j}} \), and the total imbalance values decrease by 62 % as compared to the assembly without optimization. When \( K_{A_{s}P_{j}} \) is used as a criterion, the geometrical deviations increase but the imbalance significantly decreases (by 99 %). When \( K \) is used as a criterion, the geometrical deviations decrease almost by the same values.
as when the criterion is used, and the imbalance decreases by 80%. Considering the fact that arising runouts have an effect on the gas-dynamic performance of the engine in general, it may be concluded that the use of the integral criterion $\Delta$ to optimize the assembly is most reasonable.

| Parameter | Position $P_o$ | Options of target functions | $K_{opt}$ | $K_{loc}$ | $K_{opt}$ | $K_{loc}$ | $K$ |
|-----------|---------------|-----------------------------|-----------|-----------|-----------|-----------|-----|
| $\delta_{r_2r}$ d. 3, mm | 0.031 | 0.031 | 0.041 | 0.031 | 0.00 | 33.97 | 0.00 |
| $\delta_{l_2r}$ d. 3, mm | 0.106 | 0.106 | 0.130 | 0.106 | 0.00 | 22.93 | 0.00 |
| $\delta_{l_2r}$, F, mm | 0.045 | 0.029 | 0.073 | 0.029 | -35.13 | 64.38 | -35.13 |
| $\delta_{l_2r}$, E, mm | 0.015 | 0.023 | 0.029 | 0.023 | 50.54 | 91.99 | 50.54 |
| $\delta_{l_2r}$ d. 2, mm | 0.057 | 0.050 | 0.098 | 0.046 | -12.36 | 71.90 | -19.50 |
| $\delta_{l_2r}$ d. 2, mm | 0.074 | 0.033 | 0.061 | 0.055 | -54.86 | -17.59 | -25.61 |
| $\delta_{l_2r}$, H, mm | 0.050 | 0.011 | 0.096 | 0.010 | -77.49 | 90.71 | -80.56 |
| $\delta_{l_2r}$, G, mm | 0.027 | 0.018 | 0.034 | 0.017 | -34.69 | 23.63 | -36.99 |
| $\delta_{l_2r}$ d. 1, mm | 0.084 | 0.029 | 0.097 | 0.033 | -64.94 | 16.44 | -60.93 |
| $\delta_{l_2r}$ d. 1, mm | 0.039 | 0.028 | 0.045 | 0.025 | -30.06 | 13.90 | -37.16 |
| $\delta_{l_2r}$, C, mm | 0.010 | 0.012 | 0.016 | 0.011 | 12.61 | 49.98 | 2.43 |
| $\delta_{l_2r}$, D, mm | 0.073 | 0.032 | 0.160 | 0.049 | -56.26 | 117.74 | -33.52 |
| $D_{\Sigma}$, g mm | 5,771.42 | 2,193.88 | 59.40 | 1,124.78 | -61.99 | -98.97 | -80.51 |

**6. Conclusion**

The considered digital model of the rotor assembly allows planning assembly woks and completing sets of parts at the stage of acceptance inspection of parts delivered for assembly to achieve the required assembly and balance accuracy. The criteria of search of the optimal part position have been considered, and the comprehensive criterion covering directly the assembly part runouts and imbalance value has been selected. Such an approach will significantly reduce the labor intensity of the assembly as there is no need for several pilot assemblies to find an acceptable part position. This will result in reducing the labor intensity and production cost. The calculated machine time for this assembly was a little more than 24 hours. In case of a greater number of possible assembly options or a greater number of parts included into it, approaches differing from the considered method of simple search should be used to find the optimal state [16, 17]; however, it is the target of further development of the considered model.

**7. Acknowledgements**

The work was supported by the Russian Federation President's grants (project code CII-262.2019.5). Experimental studies were carried out on the equipment of the center for the collective use of CAM technologies of the Samara University (RFMEFI59314X0003).

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