Inflation in Wess–Zumino Models

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ABSTRACT

We show that a class of Wess–Zumino models lead to inflation in supersymmetry and supergravity. This is due to the existence of a classically flat direction generic to these models. The pseudomodulus that parametrizes this flat direction is the inflaton and obtains a small mass due to either one–loop or supergravity corrections giving rise to slow–roll inflation. At the end of inflation, the fields roll to a supersymmetric vacuum that arises from explicit R symmetry breaking.

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1. Introduction

A necessary condition for slow–roll inflation[1,2] is the presence of a field, i.e. the inflaton, which corresponds to an almost flat direction of the scalar potential. Supersymmetric models generically have subspaces of field space with nonzero energy and classically flat directions parametrized by pseudomoduli. In such states, supersymmetry is necessarily broken and as a result, the pseudomoduli obtain potentials due to one–loop quantum effects. These potentials are logarithmic, giving rise to small masses for the pseudomoduli and causing them to roll slowly towards supersymmetric vacua. Thus, the pseudomoduli play the role of the inflaton and inflation is naturally realized in supersymmetric models.

By now it is well–known that there are supersymmetric models with metastable nonsupersymmetric vacua[3]. These models, which have been investigated for supersymmetry breaking purposes[4,5], also have pseudomoduli with classically flat potentials which are lifted due to one–loop effects in global supersymmetry or supergravity corrections. Therefore, they are naturally good candidates for models of inflation. An important example of this class are Wess–Zumino models which are supersymmetric models with the most general renormalizable superpotentials subject to (discrete and/or continuous) global symmetries. In fact, recent results on Wess–Zumino models may be used to show that they are ideally suited for inflation in supersymmetry and supergravity. In this paper we show that, in both cases, Wess–Zumino models lead to F–term inflation.

In the context of supersymmetry breaking, the metastable vacua in Wess–Zumino models have to be classically and quantum mechanically stable[3,4,5]. On the other hand, in order to exit inflation with vanishing vacuum energy, the fields have to relax to supersymmetric vacua. Thus, for a successful end to inflation, we have to demand exactly the opposite, that the models be classically unstable along the trajectory of the inflaton.

1 Note that these are not real moduli since the models of inflation have isolated vacua with fixed VEVs.
This paper is organized as follows. In section 2, we briefly review the general results on Wess–Zumino models that are relevant for inflation. We then describe a Wess–Zumino model that realizes inflation in supersymmetry. In section 3, we show that the same model can give rise to inflation in supergravity. Section 4 includes ideas on how to realize Wess–Zumino inflation in string theory, a discussion of our results and our conclusions.

2. Wess–Zumino Inflation in Supersymmetry

Wess–Zumino models have been studied in great detail for supersymmetry breaking purposes[4,5,6]. The new insights gained from these studies show that Wess–Zumino models can lead to inflation in supersymmetry and supergravity. Below, we summarize their properties that are important for inflation. Consider a generic Wess–Zumino model with chiral superfields $\phi_i$ and the (renormalizable) superpotential

$$W = F_i \phi_i + \frac{1}{2} m_{ij} \phi_i \phi_j + \frac{1}{6} \lambda_{ijk} \phi_i \phi_j \phi_k$$

(1)

We assume that all fields have canonical Kahler potentials. In ref. [6,7] it was shown that, in a supersymmetry breaking vacuum, there is a classically flat direction, i.e. a pseudomodulus. In this vacuum all other fields are stabilized and get VEVs. One can perform a unitary transformation and redefine the fields so that the superpotential becomes

$$W = X (F + \frac{1}{2} \lambda_{ab} \phi_a \phi_b) + \frac{1}{2} m_{ab} \phi_a \phi_b + \frac{1}{6} \lambda_{abc} \phi_a \phi_b \phi_c$$

(2)

where $X$ is the pseudomodulus that parametrizes the flat direction and $\phi_a$ are the transformed $\phi_i$ fields shifted by their VEVs. In this basis, the supersymmetry breaking vacuum is at $\phi_a = 0$ and arbitrary $X$. Since supersymmetry is broken, the bosonic masses of $\phi_a$ are shifted relative to those of their fermionic superpartners. As a result, there is a one–loop effective potential for $X$ generated by loops of $\phi_a$ and given by[8]
$$V_1 = \frac{1}{64\pi^2} STr\mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2}$$

where $STr$ is the supertrace operator, $\mathcal{M}$ is the (bosonic or fermionic) mass matrix and $\Lambda$ is a cutoff. We see that eq. (3) gives rise to a logarithmic potential and therefore a small mass for $X$. Thus, we conclude that the pseudomodulus $X$ is a good inflaton candidate. It has a classically flat potential which is lifted by one-loop effects that lead to a slow-roll towards the origin of field space (which is the metastable non-supersymmetric vacuum).

In order to have a successful model of inflation, at the end of inflation the fields have to relax to a supersymmetric vacuum. However, not all Wess–Zumino models preserve supersymmetry. It is well-known that, a sufficient condition for a generic model to have a supersymmetric vacuum is a superpotential that breaks R symmetry explicitly[9]. Therefore, in order to have a supersymmetric vacuum, the superpotential given by eq. (2) has to break R symmetry. When both types of vacua are present, depending on the parameters of the model, the non-supersymmetric vacuum may be classically or quantum mechanically stable. Fortunately, it is quite easy to build Wess–Zumino models with superpotentials that break R symmetry explicitly.

In models of metastable supersymmetry breaking, one demands that there are no tachyonic (unstable) directions over the whole pseudomodulus space i.e. the complex line $X$. In ref. [6] it was shown that this is equivalent to demanding that the matrix $m\lambda^{-1}$ be nilpotent. Moreover, one needs to make sure the metastable vacuum is also quantum mechanically stable, i.e. stable against tunneling. These demands for stability constrain the parameters such as $m_{ab}$ and $\lambda_{ab}$. On the other hand, for purposes of inflation, there is no need to demand the absence of unstable directions after inflation ends. All that is required is a long enough slow-roll era that generates 60 e–folds and large enough scalar perturbations. In fact, since at the end inflation the fields have to reach a supersymmetric vacuum with vanishing energy, the requirement for a model of inflation is exactly the opposite; there
has to be an unstable direction in field space either the trajectory of \( X \), which leads to the supersymmetric vacuum. This will guarantee a successful exit from inflation. (Throughout the paper we sometimes refer to the origin of field space as the metastable vacuum even though it has to be unstable due to the above reasons.)

If there are no classical instabilities, an alternative way to exit inflation may be for the metastable vacuum to tunnel quickly to the supersymmetric one. This would lead to the old inflationary scenario[10] with all the problems associated with it[11]. Nevertheless, we will briefly consider this possibility in the following.

Consider the Wess–Zumino model with three chiral superfields \( X, \phi_1, \phi_2 \) and the superpotential[6] (We assume that all fields have canonical Kahler potentials throughout the paper.)

\[
W = -FX + \mu\phi_1\phi_2 + \frac{1}{2}\lambda X\phi_1^2 + \frac{1}{6}g\phi_1\phi_2^2
\]  

(4)

The scalar potential is given by

\[
V = | -F + \frac{1}{2}\lambda\phi_1^2 |^2 + | \mu\phi_2 + \lambda X\phi_1 + \frac{1}{6}g\phi_2^2 |^2 + | \mu\phi_1 + \frac{1}{3}g\phi_1\phi_2 |^2
\]  

(5)

For \( \mu^2 > \lambda F \) this model has a nonsupersymmetric vacuum at \( \phi_1 = \phi_2 = 0 \) and arbitrary \( X \). Thus, \( X \) is a pseudomodulus that parametrizes the flat direction and plays the role of the inflaton. As mentioned above, in order to exit inflation, the origin of field space has to be classically unstable (i.e. it has to be a saddle point of \( V \)) which is the case for \( \mu^2 < \lambda F \). (In this range of parameters, the scalar potential in eq. (5) has another saddle point at \( \phi_1 = 0, \phi_2 = [-3\mu \pm \sqrt{\mu^2 - (2/3)g\lambda X\phi_1}]/g \) and arbitrary \( X \), which we do not consider in the following.)

\( R \) symmetry is explicitly broken by the superpotential. With the charge assignments \( R[X] = R[\phi_2] = 2 \) and \( R[\phi_1] = 0 \) the last term in eq. (4) breaks the \( R \)
symmetry. As a result, there are supersymmetric vacua\cite{9} given by

\[ X = \frac{3}{2} \frac{\mu^2}{g\sqrt{2}\lambda F} \quad \phi_1 = \pm \sqrt{ \frac{2F}{\lambda} } \quad \phi_2 = -\frac{3\mu}{g} \] (6)

As expected, when \( g \to 0 \), the supersymmetric vacuum in eq. (6) escapes to infinity and disappears.

We initially start with large values for all fields. More precisely, we assume that initially \( X \) and \( \phi_2 \) have large values whereas \( \lambda\phi_1 < H \) so that \( X \) does not have a large tree level mass. Even though this initial state in somewhat unnatural, it is similar to the ones assumed in models of hybrid inflation\cite{12}. In those models, initially the inflaton has a large value whereas the value of the trigger field is relatively small for the same reason. With this assumption, the massive fields \( \phi_1 \) and \( \phi_2 \) roll quickly to one of their extrema. Whether this is the origin of field space or the supersymmetric vacuum (or the second saddle point) obviously depends on the details of these initial values as well as properties of the field space. Therefore, answering this question requires a detailed analysis of the six dimensional field space and is quite complicated. (Note that when \( \phi_1, \phi_2 \) are in the supersymmetric vacuum given by the eq. (6), \( X \) has a large mass, \( m_X \sim H \), and inflation cannot take place. However, in the second saddle point \( m_X \) vanishes at tree level since \( \phi_1 = 0 \) and this may lead to inflation.) Here we will simply assume that \( \phi_1 \) and \( \phi_2 \) roll towards the nonsupersymmetric metastable vacuum. Thus, the fields roll to \( \phi_1 = \phi_2 = 0 \) and we can drop the last two terms in the superpotential.

The remaining superpotential contains only the pseudomodulus \( X \) and leads to a constant scalar potential. \( X \) is the inflaton with a flat potential at tree level.

In this vacuum, the bosonic masses squared are

\[ m_B^2 = \frac{1}{2} [ 2\mu^2 + \lambda^2 |X|^2 - \epsilon \lambda F \pm \sqrt{ (\lambda^2 |X|^2 - \epsilon \lambda F)^2 + 4\mu^2 \lambda^2 |X|^2 } ] \] (7)

whereas the fermionic masses squared are
\[ m_{F}^2 = \frac{1}{2}[2\mu^2 + \lambda^2|X|^2 \pm \sqrt{\lambda^2|X|^4 + 4\mu^2\lambda^2|X|^2}] \]  

(8)

where \( \epsilon = \pm 1 \). Since supersymmetry is broken \( (F_X = -F) \) loops of \( \phi_1 \) and \( \phi_2 \) generate a one–loop potential for \( X \) given by eq. (3). With the masses in eqs. (7) and (8) and for \( X >> \mu, F \) we get the total potential for \( X \) [8]

\[ V = V_0 + V_1 = F^2 \left(1 + \frac{\lambda^2}{16\pi^2}\log\left(\frac{X^2}{\Lambda^2}\right)\right) \]  

(9)

where \( \Lambda \) is a cutoff. The inflaton mass is \( m_X = \lambda F/2\sqrt{2\pi}X \) which is smaller than the Hubble constant \( H = F/\sqrt{3}M_P \) for \( X > \lambda M_P/2\pi \). Inflation occurs when the inflaton \( X \) slowly rolls down this potential and ends when one of the slow–roll conditions \(|\epsilon|, |\eta| << 1 \) is violated.

Since a successful end to inflation requires the origin to be classically unstable, we examine the bosonic masses in eq. (7) for \( X >> \sqrt{F} >> \mu \). We find that out of the four masses squared, two are very large, \( O(X^2) \), and one is very small and positive, \( \sim 2\mu^2F/\lambda X^2 \). Finally, there is a tachyon with a very small negative mass squared \( \sim -2\mu^2F/\lambda X^2 \) which is the sign of the classical instability. For \( \sqrt{F} >> \mu \) this tachyon mass is much smaller than the inflaton mass \( \sim \lambda F/2\pi X \). This means that as the inflaton slowly rolls down its potential during inflation, the tachyon sits at the top of the saddle point (the origin). In fact, the ratio of the tachyon mass to the inflaton mass is very small and independent of \( X \), \( m_{tach}/m_X \sim 4\pi\mu/\sqrt{\lambda^3 F} << 1 \). After inflation ends and \( X \) rolls to smaller values, both the tachyon and inflaton masses increase since they are both inversely proportional to \( X \). Eventually, the tachyon mass becomes large as \( X \) rolls towards the origin and the fields roll to the supersymmetric vacuum in eq. (6). Inflation fails to occur for larger values of \( \mu \), e.g. for \( X >> \mu > \sqrt{F} \). In this case, the tachyon mass squared is \( -F^2/X^2 \) which is larger than the inflaton mass squared. This means that the fields roll to the supersymmetric vacuum before the inflaton has a chance to roll long enough to inflate the universe by 60 e–folds.
This scenario is basically F–term inflation previously considered in ref. [13]. However, here we show that inflation occurs in a large class of models, i.e. Wess–Zumino models with R symmetry breaking superpotentials. The slow–roll conditions are satisfied if

$$\varepsilon = \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2 = \frac{2\lambda^4 M_P^2}{(16\pi^2)^2} \frac{1}{X^2}$$  \hspace{1cm} (10)

and

$$\eta = M_P^2 \left( \frac{V''}{V} \right) = -\frac{\lambda^2 M_P^2}{8\pi^2} \frac{1}{X^2}$$  \hspace{1cm} (11)

We see that $|\varepsilon| = \lambda^2 |\eta|/16\pi^2$ is very small. Thus, $R = 16\varepsilon$ is also very small and tensor perturbations are negligible. In this model, inflation ends when $|\eta| \sim 1$. Inflating the universe by 60 e–folds requires

$$N = \frac{1}{M_P^2} \int_{X_f}^{X_i} \left( \frac{V}{V'} \right) dX = \frac{4\pi^2}{\lambda^2 M_P^2} (X_i^2 - X_f^2) \sim 60$$  \hspace{1cm} (12)

The magnitude of scalar density perturbations is given by

$$A_s = \frac{H^2}{8\pi^2 M_P^2 \varepsilon} = \frac{16\pi^2}{3\lambda^4} \left( \frac{F^2}{M_P^4} \right) \left( \frac{X^2}{M_P^2} \right) \sim 2 \times 10^{-9}$$  \hspace{1cm} (13)

Using $X_i >> X_f$ and eq. (13) we can obtain enough scalar density fluctuations with $F \sim 10^{-5} \lambda M_P^2$. Then, in order to get at least 60 e–folds we need $X_i > 1.3\lambda M_P$. The condition on the spectral index of the scalar perturbations is $n_s = 1 - 4\varepsilon + 2\eta \sim 0.96$ at $N \sim 60$. Since $|\varepsilon| << |\eta|$ we find from eqs. (11) and (12) that this condition can be satisfied with $\lambda \sim 0.1$.

Inflation ends when $|\eta| \sim 1$ which occurs around $X_f \sim (\lambda/2\sqrt{2}\pi)M_P$. For $X < X_f$, the inflaton rolls down its potential quickly. When $X$ is small enough, as described above, the tachyon mass becomes large and the fields roll to the supersymmetric vacuum.
Note that, even if $X$ gets close to the origin, there is no instability along the $X$ direction. Around $X \sim 0$, the one–loop potential becomes\[4,6\]

\[
V_1 = F^2 + \frac{\lambda^2 \mu^2 |X|^2}{32 \pi^2} F(x) + O(|X|^4) \tag{14}
\]

where

\[
F(x) = \frac{1 + x^2}{x} \log \left| \frac{1 + x}{1 - x} \right| + 2 \log |1 - x^2| - 2 \tag{15}
\]

and $x = \lambda F/\mu^2 > 1$. Since $F(x) > 0$, the point $X = 0$ which is the endpoint of inflaton’s trajectory is stable in the $X$ direction.

As mentioned above, if $\mu^2 > \lambda F$ and there are no classical instabilities, the metastable vacuum may still decay to the supersymmetric one by tunneling. The decay rate of the metastable vacuum is given by $\Gamma \sim e^{-S_I}$ where the action of the bounce is\[14\]

\[
S_I = \frac{27 \pi^2}{2} \frac{\langle \Delta X \rangle^4}{\Delta V} \tag{16}
\]

Taking $\Delta X \sim \mu^2/\sqrt{F}$ and $\Delta V \sim F^2$ we find that $S_I \sim (27 \pi^2/2)(\mu^2/F)^4$. If the origin is classically stable then $\mu^2 > \lambda F$ and we find the decay rate is greater than $\Gamma > e^{130 \lambda^4}$. For $\lambda < 0.3$ the decay rate is very large and the false vacuum tunnels quickly to the supersymmetric vacuum. However, this leads to the old inflationary scenario\[10\] with all its well–known problems\[11\].

Using the above prescription, we can easily build many models of Wess–Zumino inflation. As another example, consider the model with the fields $X, \phi_1, \phi_{-1}, \phi_3$ (where the subscripts on the fields denote their R charges) and the superpotential

\[
W = FX + \lambda X \phi_1 \phi_{-1} - m_1 \phi_1^2 + m_2 \phi_{-1} \phi_3 + m_3 \phi_3^2 \tag{17}
\]

which was previously proposed (without the last term) as a model of supersymmetry breaking in ref. \[15\]. We see that with the choice $R[X] = 2$, last term in
eq. (17) breaks R symmetry explicitly. For \( m_3 = 0 \) the model has a nonsupersymmetric vacuum\[15\]. When \( m_3 \neq 0 \) a supersymmetric vacuum appears (in addition to the nonsupersymmetric one which becomes metastable). The scalar potential is given by

\[
V = |F + \lambda \phi_1 \phi_1| + |\lambda X \phi_1 - 2m_1 \phi_1|^2 + |\lambda X \phi_1 + m_2 \phi_3|^2 + |m_2 \phi_1 + 2m_3 \phi_3|^2 \tag{18}
\]

The nonsupersymmetric metastable vacuum is given by \( \phi_1 = \phi_{-1} = \phi_3 = 0 \) with arbitrary \( X \) at tree level. Thus, as before, \( X \) is a pseudomodulus which parametrizes this flat direction and plays the role of the inflaton. Supersymmetry is broken in this vacuum since \( F_X = -F \) and, as a result, \( X \) gets a potential (and mass) from one–loop effects. This model also has a supersymmetric vacuum given by

\[
\phi_1^2 = -\frac{F}{2m_1} X \quad \phi_{-1} = \frac{F}{\lambda \phi_1} \quad \phi_3 = -\frac{m_2}{\lambda m_3} \frac{F}{\phi_1} \tag{19}
\]

where

\[
X = \pm \frac{m_2}{\lambda} \sqrt{\frac{m_1}{m_3}} \tag{20}
\]

As expected the VEVs of \( X, \phi_1, \phi_3 \) are inversely proportional to \( m_3 \) which means that when \( m_3 \to 0 \) the supersymmetric vacuum escapes to infinity and disappears. If all the masses are of the same order of magnitude we find \( \phi_1 \sim \phi_{-1} \sim \phi_3 \sim \sqrt{F/\lambda} \) and \( X \sim m/\lambda \). We will not go into the details of inflation in this model since it leads to an inflationary scenario which is very similar to the one considered above.

3. Wess–Zumino Inflation in Supergravity

The above scenario of Wess–Zumino inflation in global supersymmetry can be extended to supergravity. This is possible due to the special form of the \( X \) dependent terms in the superpotential given by eq. (4). It is well–known that it
is difficult to realize F–term inflation in supergravity due to the inflaton mass or η problem (which lead to the advent of D–term inflation[16] as an alternative). As shown in ref. [13], this problem does not arise, if the inflaton superpotential has a special form (as in eq. (4) above). The supergravity scalar potential is given by

\[ V = e^K (|W_i + K_i W|^2 - 3|W|^2) \] (21)

where the subscript \( i \) denotes differentiation with respect to a field and \( K \) is the Kahler potential which we take to be canonical for all fields. In eq. (21) we omitted all factors of \( M_P \) which are easy to replace when necessary. Using eq. (4), the supergravity F–terms, \( F_i = W_i + K_i W \) are given by

\[ F_X = -F + \lambda \phi_1^2 + X (-FX + \frac{1}{2} \lambda X \phi_1^2) \] (22)

\[ F_{\phi_1} = \mu \phi_2 + \lambda X \phi_1 + \frac{g}{6} \phi_2^2 + \bar{\phi}_1 (-FX + \frac{1}{2} \lambda X \phi_1^2) \] (23)

and

\[ F_{\phi_2} = \mu \phi_1 + \frac{g}{3} \phi_1 \phi_2 + \bar{\phi}_2 (-FX + \frac{1}{2} \lambda X \phi_1^2) \] (24)

The scalar potential can be obtained by plugging eqs. (22)-(24) and (4) into eq. (21). First, we need to make sure that, as in the globally supersymmetric case, there are initial conditions that lead \( \phi_1 \) and \( \phi_2 \) to roll to the origin quickly. From the scalar potential it is easy to see that \( m_X^2 \) only gets corrections of order \( O(\phi^2 F/M_P^2) \) or \( O(F^2/M_P^2) \) where \( \phi = \phi_1, \phi_2 \). For \( \phi^2, F << M_P^2 \) these are about the same order of magnitude as the one–loop inflaton mass \( \sim \lambda^2 F^2/8\pi^2 X^2 \). Thus, after taking supergravity corrections into account, \( m_X \) is still small enough to lead to slow–roll inflation.\(^2\) Moreover, it is easy to see that \( \phi_1 \) and \( \phi_2 \) have large masses as in the globally supersymmetric case.

\(^2\) We show that self–interactions of \( X \) do not lead to a large mass term below.
In addition, we need to make sure that the origin of the field space remains a saddle point in supergravity. When $\phi_1 = \phi_2 = 0$, $W_i = K_i = 0$ and $W = -FX$, $\phi_1, \phi_2$ masses squared get a negative supergravity contribution of $\sim -F^2X^2/M_P^4$. For the values of the parameters we used in the supersymmetric case, $F \sim 10^{-5}M_P^2$, $\lambda \sim 0.1$ and $X \sim M_P/10$, we find that these supergravity contributions are much smaller than even the (small) tree level masses $\sim 2\mu^2F/\lambda X^2$. Therefore, there is still one tachyon with a very small mass at the origin. This tachyon will sit at the top of the potential at the origin for a long time allowing slow–roll inflation in supergravity.

The potential for $X$ is obtained from eqs. (4) and (21)

$$V_X = e^{X\bar{X}}(|F + F\bar{X}X|^2 - 3|FX|^2)$$
$$= (1 + X\bar{X} + \frac{1}{2}(X\bar{X})^2 + \ldots)(|F + F\bar{X}X|^2 - 3|FX|^2)$$
$$= |F|^2(1 + \frac{1}{2}(X\bar{X})^2 + \ldots)$$

where the dots indicate higher order terms that we neglect. We see that $X$ does not have a tree level mass but a quartic (and higher order) coupling. When $X$ has a large VEV this term induces a mass of $m_X^2 \sim X\bar{X}F^2/M_P^4$ (in addition to the one that arises from one–loop effects). The slow–roll of $X$ requires $m_X < H = F/\sqrt{3}M_P$ which means we need $X < \sqrt{3}M_P$ which is easily satisfied. Another possible contribution to $m_X$ arises from nonrenormalizable terms in the Kahler potential which we assumed to be canonical at tree level. The Kahler potential may include nonrenormalizable terms like

$$V_K = \frac{c}{M_P^2} \int d^4\theta(X\bar{X})^2 = c \left(\frac{F_X}{M_P}\right)^2 X\bar{X}$$

which gives $m_X = \sqrt{c}F_X/M_P = \sqrt{c}F/M_P$. We can assume that $\sqrt{c} \sim 1/4\pi$ since $V_K$ arises from one–loop effects and then $m_X < H$. Thus, $X$ obtains a small mass due to one–loop effects and/or supergravity corrections and therefore is a good inflaton candidate.
Inflation occurs as $X$ slowly rolls towards the origin as a result of either the one–loop or supergravity potentials in eqs. (9) and (25) respectively. The dynamics of $X$ is determined by the competition between the one–loop superpotential in eq. (9) and the supergravity quartic term in eq. (25). By comparing these, we find that the former dominates at late times, i.e. for $X < \sqrt{\lambda} M_P/3$. Before that era, for larger values of $X$, i.e. for $\sqrt{\lambda} M_P/3 < X < \sqrt{3} M_P$, we need to take supergravity contributions into account. Following ref.[13], the number of e–folds obtained at later times, the supersymmetric era dominated by the one–loop potential is $N_{\text{susy}} \sim 4/\lambda$. At earlier times when $X$ is larger, the supergravity potential contributes $N_{\text{sugra}} \sim 9/\lambda$ to give the total number of e–folds $N = N_{\text{susy}} + N_{\text{sugra}} = 13/\lambda$. If $\lambda \sim 0.1$ as in the globally supersymmetric case, we find that inflation lasts for $N \sim 130$ and the last 40 e–folds are due to the one–loop potential. Inflation ends when the slow–roll condition is violated, i.e. $|\eta| \sim 1$. The discussion of the scalar density perturbations and the scalar spectral index is exactly as in ref. [13] and will not be repeated here.

At the end of inflation, $X$ reaches a critical value for which the mass of the tachyon at the origin is large. Then, the fields quickly roll to the supersymmetric vacuum. In a supersymmetric vacuum the supergravity F–terms in eqs. (22)-(24) have to vanish. Again, these are given by the supersymmetric F–terms plus corrections that are suppressed by $\phi^2/M_P^2$. Even though it is hard to solve these constraints analytically, clearly there must be a supersymmetric vacuum close to the globally supersymmetric one. Unfortunately, this supersymmetric vacuum described by the vanishing F–terms in eqs. (22)-(24) does not have vanishing energy. In supergravity, a supersymmetric vacuum has vanishing energy only if the superpotential vanishes in the vacuum. From eq. (21) and the VEVs in eq. (6) we see that this is not the case. We can remedy this by adding a fine–tuned constant to the superpotential in eq. (4). This does not affect the results of the previous section, i.e. Wess–Zumino inflation global supersymmetry, but leads to a vanishing vacuum energy in supergravity.
4. Conclusions and Discussion

In this paper we showed that a large class of Wess–Zumino models lead to inflation in both supersymmetry and supergravity. The main property of Wess–Zumino models that make this possible is the existence of a pseudomodulus with a classically flat potential, (i.e. the inflaton), which obtains a small mass either at one–loop order or in supergravity. Inflation occurs as the inflaton slowly rolls toward the origin of field space. During this era with a large $X$, the tachyon at the origin is stable due to its very small mass (which is much smaller than the inflaton mass). After the slow–roll era ends, the inflaton reaches a value for which the tachyon mass becomes large and the fields roll to the supersymmetric vacuum.

Thus, Wess–Zumino models realize F–term inflation in both supersymmetry and supergravity. An important difference between the above models and that of ref. [13] is the nature of the instability that causes the fields to roll to the supersymmetric vacuum. In F–term inflation, which is a type of hybrid inflation, initially the origin is classically stable and becomes unstable only after the inflaton rolls below a critical value. Then, the trigger field becomes tachyonic and the fields roll to the supersymmetric vacuum. In Wess–Zumino models of the type we discussed above, on the other hand, there is a classical instability from the beginning signaled by the tachyonic direction at the origin of field space. However, this tachyon has a very small ($X$ dependent) mass until inflation ends and the inflaton rolls to a small value. Eventually, when $X$ becomes small enough, the tachyon mass becomes large and the fields settle at the supersymmetric vacuum.

It is interesting to note that, since the its mass is so small, the dynamics of the tachyon may also lead to hilltop inflation[17]. Since the tachyon mass depends on $X$, this requires an era in which $X$ has a large value and is slowly rolling. This is precisely the scenario we described above, i.e. slow–roll inflation for $X$. Thus, it seems that a model similar to the one in section 2 may lead to hilltop inflation[18].

The main open question in the above scenario is our assumption that, at the beginning of inflation, the fields $\phi_1$ and $\phi_2$ quickly roll to the origin rather than to
the supersymmetric vacuum (or the second saddle point). In order to justify this, one should analyze in detail the evolution of the fields in the full six dimensional field space which is hard to do analytically. However, it seems that there must be a nonnegligible region of the initial phase space from which $\phi_1$ and $\phi_2$ roll to the origin. Nevertheless, since the whole scenario depends on this assumption, it is important to justify our assumption by investigating the evolution of the model numerically.

We obtained Wess–Zumino inflation in supersymmetry and supergravity. The natural extension seems to be realization of this scenario in string theory. It is not clear how to obtain generic Wess–Zumino models in string theory. On the other hand, it is very easy to obtain the purely inflaton part of the model, i.e. a model with only the $X$ dependent terms in eq. (2). For example, consider a compactification with an $A_2$ type singularity fibered over the complex plane $C(x)$ and defined by

$$uv = z(z - mx)(z - m(x - a))$$

We wrap one D5 brane on each one of the two nodes ($S^2$s) of the singularity. On the D5 brane world–volume, this gives rise to a field theory with the gauge group $U(1)_1 \times U(1)_2$ and a matter sector with two singlets $\Phi_1, \Phi_2$ and a pair of bifundamentals $Q_{12}, Q_{21}$ with charges $(1, -1)$ and $(-1, 1)$ respectively[19]. The diagonal group $U(1)_d = [U(1)_1 + U(1)_2]/2$ decouples leaving the gauge group $U(1) = [U(1)_1 - U(1)_2]/2$. The superpotential is given by[19]

$$W = Q_{12}Q_{21}(\Phi_2 - \Phi_1) + \frac{1}{2}m\Phi_1^2 - ma\Phi_2$$

At energies below $m \sim M_s$, $\Phi_1$ decouples and we are left with

$$W = \Phi_2(Q_{12}Q_{21} - ma)$$

With the identification $\Phi_2 = X$ and $F = ma$ we obtain the $X$ dependent terms in eq. (2) which are exactly those that give rise to F–term inflation in supersymmetry.
and supergravity as in ref. [13]. (The difference is that $X$ couples to $Q_{12}Q_{21}$ instead of $\phi_1^2$ in eq. (4) but this is not consequential.) This is very similar to inflation models that were obtained on other singular spaces[20] some of which were D–term inflation models. In fact, since these models have $\mathcal{N} = 2$ supersymmetry, F ad D–terms are equivalent (by an $SU(2)$ rotation of the theory) and F and D–term inflation scenarios describe the same physics[21]. We see that it is quite easy to obtain the minimal F–term inflation on D5 branes wrapped on singularities. It would be interesting to generalize this result and obtain generic Wess–Zumino models in string theory that lead to inflation.

An important difference between the Wess–Zumino models in eq. (2) and those that can be obtained on D5 branes is the existence of gauge symmetries in the latter. Usually fields that appear in Wess–Zumino models carry only global charges and the superpotential is the most general one subject to the global symmetries. However, in brane models such as the one above, there is generically an Abelian group for each node. (These can be decoupled by making their gauge couplings very small but this does not affect the superpotential.) It is clear that if the models have Abelian gauge symmetries these will be spontaneously broken at the end of inflation and cosmic strings will be produced[22]. In the brane description, these are D3 branes wrapped on the same singularities and correspond to the strings that arise in F–term inflation[23]. It would be interesting to examine such a brane inflation scenario and find out its implications for cosmic string production.

Acknowledgements

I would like to thank the Stanford Institute for Theoretical Physics for hospitality.
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