Local Group mass estimators from cosmological simulations

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ABSTRACT
We identify Local Group (LG) analogs in the IllustrisTNG cosmological simulation, and use these to study two mass estimators for the LG: one based on the timing argument (TA) and one based on the virial theorem (VT). Including updated measurements of the Milky Way-M31 tangential velocity and the cosmological constant, we show that the TA mass estimator slightly overestimates the true median LG-mass, though the ratio of the TA to the true mass is consistent at the approximate 90% c.l. These are in broad agreement with previous results using dark matter-only simulations. We show that the VT estimator better estimates the true LG-mass, though there is a larger scatter in the virial mass to true mass ratio relative to the corresponding ratio for the TA. We attribute the broader scatter in the VT estimator to several factors, including the predominantly radial orbits for LG satellite galaxies, which differs from the VT assumption of isotropic orbits. With the systematic uncertainties we derive, the updated measurements of the LG mass at 90% c.l. are $4.75^{+2.22}_{-2.41} \times 10^{12} \, M_\odot$ from the TA and $2.0^{+2.1}_{-1.5} \times 10^{12} \, M_\odot$ from the VT.

Key words: Local Group – galaxies: kinematics and dynamics – dark matter

1 INTRODUCTION
The Milky Way (MW) and M31 and their associated dark matter (DM) halos are the dominant mass components of the Local Group (LG). The kinematics of stars and satellite galaxies bound to these galaxies provide an estimate of the DM halo masses of the MW (Boylan-Kolchin et al. 2013; Patel et al. 2017; Callingham et al. 2019; Cautun et al. 2020; Fritz et al. 2020) and M31 (van der Marel et al. 2012). These analyses show that M31 is likely more massive than the MW, and that the total mass in these systems is $M_{MW} + M_{M31} \approx [2 - 3] \times 10^{12} \, M_\odot$.

In addition to the MW and M31, there are dozens of dwarf galaxies in the LG (McConnachie 2012; Simon 2019), the most massive of which, the Large Magellanic Cloud (LMC) and M33, being less than approximately 10% of the MW/M31 mass (Peñarrubia et al. 2016; Erkal et al. 2019). Some of these dwarf galaxies are satellites that are bound to either the MW or M31, while others are bound to the LG system. In sum, the dwarf galaxies represent a small fraction of the total LG mass.

The dynamics of the MW/M31 and the associated dwarf galaxies encode important information on the LG. The “timing argument” (TA) is straightforward and often-used dynamical model to estimate the LG mass, $M_{LG}$. The TA assumes that the MW and M31 have broken off from the cosmic expansion and they can be modeled as a system on a two-body Keplerian orbit (Kahn & Woltjer 1959). Applications of the TA, including both the radial and tangential velocities of M31, find $M_{LG} \approx 5 \times 10^{12} \, M_\odot$ (van der Marel & Guhathakurta 2008; van der Marel et al. 2012). TA mass estimates tend to be larger than the sum $M_{MW} + M_{M31}$ as deduced from satellite kinematics, and the origin of this discrepancy has been subject of several recent studies, most of which investigate possible systematic uncertainties in the LG mass as determined from the TA. For example, the TA mass is sensitive to the local circular speed of the Sun, and may be affected by the LMC (Peñarrubia et al. 2016; Partridge et al. 2013) and Peñarrubia et al. (2014) find that including a cosmological constant term in the equations of motion increases the deduced LG mass by $\sim 13\%$. More generally, the TA mass depends on the cosmological model of dark energy (McLeod & Lahav 2020) and possible past encounters which manifest in the form of additional angular momentum in the LG (Benisty et al. 2019).

Cosmological simulations may also be used to estimate the LG mass. Identifying LG-like systems in the dark matter-only Millennium simulation, Li & White (2008) showed that the TA-mass estimator is largely unbiased. These authors provide an estimate of the intrinsic (or “cosmic”) scatter that must be accounted for when deducing $M_{LG}$ from the TA. González et al. (2014) use the dark matter-only Bolshoi cosmological simulation and include the impact of the MW/M31 tangential velocity and environmental constraints and show that the TA overestimates the true mass by a factor $\sim 1.3 - 1.6$. The LG mass may also be extracted from simulations using likelihood analyses. For given cuts on the velocity, separation, and environmental properties, including both the tangential and radial velocities, and depending on the specific method, these estimates span a range $M_{LG} \sim [1 - 5] \times 10^{12} \, M_\odot$ (González et al. 2014; McLeod et al. 2017; Carlesi et al. 2017; Lemos et al. 2021). Zhai et al. (2020) use simulations with semi-analytic models to deduce a LG mass at the higher end of this range. This range of LG masses is also consistent with that obtained by Phelps et al. (2013) using a numerical least action method.

Given the large spread in the LG mass as determined from the...
TA and from simulations, it is important to consider independent estimates of the LG mass. Such an additional LG mass estimate comes from the application of the Virial Theorem (VT) to the MW, M31, and the dwarf galaxies in the LG. Courteau & van den Bergh (1999) first considered this method, starting with the assumption that the kinematics of LG galaxies may be described by the VT, finding that $M_{\text{LG}} \sim 2 \times 10^{12} \, M_\odot$. Application of this method with a larger sample of galaxies by Diaz et al. (2014) corroborate this result. This value for $M_{\text{LG}}$ as determined from the VT is also consistent with the corresponding value deduced from examination of the properties of the Hubble Flow (Karachentsev et al. 2009).

Motivated by understanding the systematics in how the LG mass is deduced from cosmological simulations, in this paper we identify LG-like systems in the large volume Illustris simulation (Vogelsberger et al. 2014). We select systems based on a series of criteria, including MW-M31 separation, relative velocity, absolute magnitudes, and density of local environment. With this sample, we provide an updated calibration of the TA mass estimator, and we determine the first estimate of the cosmic scatter in $M_{\text{LG}}$ from the VT. With Illustris, we are able to provide an estimate of both the TA and the VT-deduced masses for the LG using a hydrodynamical simulation.

In addition to the determination of the LG mass, we use our sample to study the kinematics of low-mass dwarf satellite galaxies in the LG that are not bound to the MW or M31. We determine the ratio of the radial to the tangential velocity dispersions for this satellite population, and find that for many LG-like systems these galaxies are on preferentially radial orbits. This result has important implications for both mass estimates of the LG and for understanding its formation history.

This paper is organized as follows. In section 2 we give a brief description of the Illustris simulation, and the cuts implemented to the simulation to gather our sample of LG analogs. Section 3 discusses the two methods used to estimate the mass of the mock LGs, the TA and the VT. Section 4 presents the results of our analysis, and in section 5 we discuss the implications of our results.

## 2 SELECTION OF LOCAL GROUPS

In this section we outline our method for identifying LG-like systems in the Illustris simulation. Our selection criteria is based on the properties of the two most massive galaxies in the system. Within this sample, we characterize the population of fainter galaxies bound within the system, focusing specifically on the number of fainter galaxies and their kinematics.

### 2.1 Galaxies and halos in the Illustris Simulation

IllustrisTNG is a suite of cosmological gravo-magnetohydrodynamic simulations which follows the evolution of dark matter, cosmic gas, luminous stars, and supermassive black holes within the centers of galaxies. Within this suite, the TNG300-1 is the simulation with the largest number of particles, $N = 2500^3$, with a dark matter particle mass of $m_{\text{dm}} = 4.0 \times 10^7 \, M_\odot \, h^{-1}$ and a baryonic particle mass of $m_{\text{part,sys}} = 7.6 \times 10^6 \, M_\odot h^{-1}$. The TNG300-1 simulation is evolved in the large comoving volume of $205 \, h^{-1}\, \text{Mpc}$. The initial conditions, in agreement with Planck 2015 results (Aghanim et al. 2016), are determined by the cosmological parameters: $\Omega_{\Lambda,0} = 0.6911$, $\Omega_{(m,0)} = 0.3089$, $\Omega_{(b,0)} = 0.0486$, $\sigma_8 = 0.8159$, $n_s = 0.9667$, and $h = 0.6774$. This yields an age of the Universe of 13.8 Gyr.

The Illustris database provides the properties of the galaxies and dark matter halos identified in 100 snapshots each associated with a unique redshift. Here we use the SubLink algorithm (Nelson et al. 2019) to access the data associated with different merger trees. To identify LG-like systems, we are interested in the following kinematic galaxy and halo properties: 1) The position of the center of mass of each halo, which Illustris reports as the spatial position within the halo volume for the particle with the lowest gravitational potential; 2) the velocity, which is calculated as the sum of all particles velocity weighted by mass; 3) the maximum circular velocity, $v_{\text{max}}$, which is calculated from the spherically-averaged rotation curve; and 4) the mass $M_{\text{halo}}$ for each halo, which is the sum of all particle masses bound to the subhalo. Note, $M_{\text{halo}}$ does not include particle masses bound to subhalos of this halo.

In addition to these kinematic properties, we extract the absolute magnitudes for each galaxy. IllustrisTNG calculates the absolute magnitude as the sum of the luminosity for each stellar particle of the halo. IllustrisTNG reports these magnitudes in eight bands; here we utilize the B-band magnitude and denote this as $M_B$.

### 2.2 The Local Group Sample

In the $z = 0$ snapshot, the subfind algorithm identifies $14.5 \times 10^6$ dark matter halos in TNG300-1, with the vast majority of these halos hosting galaxies. From this sample of halos, we choose systems that best resemble the LG, based on the following criteria. Our cuts and the corresponding sample sizes subsequent to each cut are summarized in Table 1.

We begin by identifying galaxies with B-band magnitude similar to that of the MW and M31. For the MW the estimated B-band magnitude is $M_B = -20.9$, while for M31 it is $M_B = -20.7$ (van den Bergh 1999). To include a sufficiently large number of pairs in our sample, we expand this range and select galaxies with B-band magnitude within $-22.3 < M_B < -19.3$. This cut results in a sample of 97,574 galaxies.

From this sample of galaxies chosen based on absolute magnitude, we add a cut to restrict to pairs of galaxies that have a separation similar to that of MW and M31. The distance between the MW and M31 is $D = 783 \pm 25 \, \text{kpc}$ (McConnachie 2012). Cutting on galaxy pairs within this observed range would result in an unreasonably small sample of LG systems. In order to obtain a sufficiently large sample size of LG pairs, we take as our distance cut pairs of galaxies with a separation 500 kpc $< |r| < 1000$ kpc. The lower limit of 500 kpc is motivated to ensure that the dark matter halos of the two pairs galaxies do not overlap, while the upper limit of 1000 kpc is...
motivated to ensure that the galaxies are bound and are detached from the Hubble Flow.

Even after performing the above distance cut, we must be careful to ensure that there is not a third, massive galaxy in close proximity to the identified pair. To account for this possibility, we keep only those pairs that have two unique halo ids, so that each member of the pair is not also assigned to a third pair. With this additional cut, our combined sample after absolute magnitude and distance cuts contains 4133 pairs.

To compare our magnitude cut to previous cuts that consider stellar masses of galaxies, Zhai et al. (2020) found 12 pairs in Illustris TNG100-1 using a stellar mass cut of $4.0 \times 10^{10} \text{M}_\odot < M_{\text{MW}} < 8.0 \times 10^{10} \text{M}_\odot$ for the Milky Way mass, and $8.0 \times 10^{10} \text{M}_\odot < M_{\text{M31}} < 13.0 \times 10^{10} \text{M}_\odot$ for M31 mass. In this work, they also use a slightly stricter distance cut of 600 kpc $< |\vec{r}| < 1000$ kpc. When we perform these stellar mass cuts and distance cuts to Illustris TNG300-1, we find 427 pairs. Accounting for the difference between the simulation volumes of TNG100-1 and TNG300-1, the sample of LGs that we obtain is consistent with that of Zhai et al. (2020).

For our next cut, we ensure that each pair is sufficiently isolated from another massive system that would gravitationally perturb the LG pair. For this cut, we remove pairs if a galaxy with $M_B < -19.3$ is found within a distance of 3 Mpc from the barycenter of the pair, or if a dark matter halo with $V_{\text{max}} > 150$ km s$^{-1}$ is found within 1 Mpc of the barycenter of the pair. This isolation criteria is similar to that of Fattahi et al. (2016), who use an isolation cut of 2.5 Mpc. These cuts are implemented to identify pairs with a similar local environment to the observed LG. Our magnitude cut is motivated by demanding that nearby galaxies are fainter than the most luminous nearby galaxy outside of the LG, Centaurus A, which is $\sim 3$ Mpc from the LG. The $V_{\text{max}}$ cut ensures the pair dominates the gravitational potential within its local volume, and thus dominates the dynamics of the LG-like system. After this isolation cut we are left with 798 isolated pairs.

The MW and M31 are observed to approach one another with a velocity (in the Galactocentric frame) of $V_r = -109.3 \pm 4.4$ km s$^{-1}$. The bound on the tangential velocity from HST observations is $V_t < 34$ km/s (van der Marel et al. 2012), while recent measurements from Gaia DR2 (van der Marel et al. 2019) and EDR3 (Fardal et al. 2021) indicate larger mean values of $V_t \approx 79$ km/s. All these measurements are still consistent with the observed MW-M31 system being on a nearly radial orbit. To match this kinematic criteria with our sample of pairs, we calculate the total relative radial velocity, $V_r$, for the two main galaxies of each pair. Since the simulation provides peculiar velocities, we add in the expansion velocity to obtain the total relative radial velocity for the pair. If we cut our above sample to simply ensure that the galaxies are approaching with $V_r < 0$ km s$^{-1}$, on top of the above magnitude, distance, and isolation cuts we are left with a sample of 658 pairs. In the analysis below we examine the impact of restricting the range of allowed radial velocities even further.

Note that in choosing our sample based on absolute magnitude, we have neglected the impact of the dark matter halo mass, or similarly the maximum circular velocity, in our cuts. To better understand the range of maximum circular velocities that our absolute magnitude cuts correspond to, in Figure 1 we show the relation between absolute magnitude and maximum circular velocity. As indicated, there is a significant scatter in $V_{\text{max}}$ over the absolute magnitude range that we consider. To remove galaxies with $V_{\text{max}}$ much larger than that of the MW or M31 galaxies, we add the constraint $V_{\text{max}} < 300$ km s$^{-1}$ on top of our above cuts, which results in a sample 613 galaxy pairs.

Our cut in $V_{\text{max}}$ is similar to that implemented in Li & White (2008), who used the Millenium simulation to identify LG-like systems based on the range $150$ km s$^{-1} \leq V_{\text{max}} < 300$ km s$^{-1}$ as their primary sample. Examining Figure 1 suggests that our initial magnitude cut above corresponds to a range in $V_{\text{max}}$ that is larger than that considered in Li & White (2008), and does indeed introduce unreasonably massive halos in our sample.

For our final cut, we require that the system be gravitationally-bound, with total energy $E < 0$. We calculate the energy of the pair as

\[ E = \frac{1}{2} \mu |\vec{V}|^2 - \frac{GM}{|\vec{r}|} - \frac{1}{2} \mu H_0^2 \Omega(\Lambda,0) |\vec{r}|^2, \]

where $H_0$ is the Hubble constant, $\mu$ is the reduced mass defined as $\mu = M_1 M_2 / M$, $M = M_1 + M_2$ is the total mass, $|\vec{V}|$ is the total relative velocity, and $|\vec{r}|$ is the separation. Enforcing this cut yields a final sample of 580 isolated bound pairs, which we take as our main LG sample from TNG300-1.

### 2.3 Outer Local Group Members

In addition to the two massive galaxies, we will be interested in the fainter galaxies that are gravitationally-bound to the LG systems. In particular, we are interested in the subsample of galaxies that are bound to the LG system, but are not bound to either of the two massive galaxies. We refer to these galaxies as outer LG members (OLGM).

From our LG sample identified above, we consider two cuts to extract OLGMs: first a cut on the absolute magnitude, and second a cut on the dark matter mass. For an absolute magnitude limit, we take the limit on the B-band magnitude to be $M_B < -10$, which approximately corresponds to the faintest satellite in the observed LG.
that is not bound to either the MW or to M31 (McConnachie 2012; Simon 2019). For our dark matter halo cut, we take the lower bounds on the dark matter halo mass of $M_{dm} > 10^{10} M_\odot$ and $M_{dm} > 10^9 M_\odot$. This corresponds to a cut on systems with dark matter mass approximately larger than the most massive luminous satellite galaxies of the MW and M31.

On top of these mass and luminosity cuts, we must be careful to identify OLGMs that are bound to either of the two massive galaxies or are too far out into the Hubble flow. For the former, we tag the specific OLGMs that are within < 350 kpc from either of the two most massive galaxies. This cut tags galaxies that are within the approximate virial radius of the MW and M31. For the latter, we exclude OLGMs that are > 1.5 Mpc from either of the two massive galaxies, thereby excluding systems that are strongly affected by the Hubble flow. Though we only utilize the > 1.5 Mpc cut for our fiducial analysis below, we do compare to the results when the < 350 kpc cut is also implemented and find a negligible difference. Note that the > 350 kpc and < 1.5 Mpc distance cut is similar to that used in the observational sample in Diaz et al. (2014).

We also calculate the energy of each OLGM with respect to the associated mock-LG center of mass. Imposing the requirement that the OLGM is bound reduces the number of OLGMs identified per LG analog. To summarize our cuts above, in Table 2 we show the number of pairs with $N > 0$, whose OLGMs are also energetically bound to the mock-LG center of mass.

| Selection criteria | $D < 1.5$ Mpc | $350$ kpc $< D < 1.5$ Mpc |
|--------------------|---------------|---------------------------|
| $M_{dm} > 10^{10} M_\odot$ | 580 (579)     | 580 (576)                 |
| $M_{dm} > 10^9 M_\odot$ | 580 (580)     | 580 (580)                 |
| $MB < -10$          | 579 (577)     | 567 (546)                 |

In addition to their number, we are also interested in characterizing the kinematics of the OLGM sample. To examine the kinematics, we first identify the position of the barycenter, where the barycenter is calculated as the center of mass of the two massive galaxies in each system. We then determine the radial velocity, $V_r$, of each OLGM relative to the LG barycenter, excluding the two massive galaxies.

The radial velocity dispersion of the OLGMs is then calculated as $\sigma^2_r = \frac{1}{N-1} \sum_{i=1}^{N} (V_{r,i} - \bar{V}_r)^2$, where $\bar{V}_r$ is the mean radial velocity of the sample. This assumes that the radial velocity dispersion is constant throughout the LG, which is an appropriate assumption given the small sample sizes that we are considering.

Figure 3 (left) plots the distribution of the radial velocity dispersion for the sample with the distance cut of $D < 1.5$ Mpc and the additional cuts $N \geq 10$ and $0.3 \leq m \leq 3.3$ described above. Implementing these cuts yields 169 pairs, 234 pairs, 425 pairs for the absolute magnitude, $M_{dm} > 10^{10} M_\odot$, and $M_{dm} > 10^9 M_\odot$ cuts respectively. The histogram distribution of the radial velocity dispersion for the three samples are similar. We see the sample of mock-LG’s with OLGMs identified with the luminosity cut has the largest mean radial velocity dispersion of 97.3 km s$^{-1}$. The mean radial velocity dispersions are 79.2 km s$^{-1}$ and 82.0 km s$^{-1}$ for $M_{dm} > 10^{10} M_\odot$, and $M_{dm} > 10^9 M_\odot$ respectively.

The velocity dispersions we obtain for the three samples are similar, and further are consistent with the radial velocity dispersion measured from the observed LG (Diaz et al. 2014). Note that the observed velocities are heliocentric radial velocities, which must then be converted to a barycentric frame by measuring the location of the MW-M31 barycenter. While this conversion adds uncertainty to the velocity dispersion as deduced from observations, it does not add an uncertainty to our analysis given that we identify the barycenter directly from the simulation.

Figure 3 (right) shows the corresponding ratio of the radial to the tangential velocity dispersion, $\sigma^2_r / \sigma^2_t$, for each LG in the sample. The tangential velocity is defined so that for an isotropic distribution, $\sigma^2_r / \sigma^2_t = 0.5$, where this value is denoted by the vertical line. The peak at $\sigma^2_r / \sigma^2_t = 2$ implies that OLGMs are on dominantly radial orbits, likely due to infall into the LG. Below we discuss the implications of $\sigma^2_r / \sigma^2_t$ on the equilibrium nature of the LG.

### 3 MASS ESTIMATES FOR LOCAL GROUP
In this section we discuss our mass estimation methods for the LG, starting with the timing argument and then moving on to the virial mass estimator.
Figure 2. Distribution of $N$, the number of outer Local Group members per Local Group, found within $D < 1.5$ Mpc. The left plot shows this distribution for the luminosity cut sample in shaded red and the dark matter halo mass sample $> 10^{10} M_\odot$ in shaded blue. The region where these distributions overlap appears in purple. The right plot shows the distribution of $N$ for the relaxed dark matter halo mass cut sample of $> 10^9 M_\odot$.

Figure 3. The left plot shows the distribution of the barycentric radial velocity dispersion for the $M_B < -10$ cut sample in shaded red, dark matter halo mass cut sample $> 10^{10} M_\odot$ in shaded blue, and dark matter halo mass cut sample $> 10^9 M_\odot$ in black. The right plot shows the distribution of the ratio of the radial velocity dispersion to the tangential velocity dispersion squared for these same samples. For both plots, we keep only those mock-Local Group’s with $N \geq 10$, and $M_31$ to $M_{MW}$ mass ratios $0.3 \leq m \leq 3.3$. The vertical dashed-black line on the right at 0.5 indicates the isotropic value.

3.1 Timing Argument

The TA has been long studied as a LG mass estimator (Kahn & Woltjer 1959). The TA assumes that the MW and M31 can be described by a Keplerian orbit, with $z = 0$ boundary conditions given by present-day separation, relative velocity, and the age of the Universe. From these assumptions and observations, the total LG mass $M_{L,G} = M_{MW} + M_{M31}$ may be determined.

Implementing the TA involves solving for the evolution of the MW-M31 separation. In $\Lambda$CDM, this magnitude of the relative separation, $|\vec{r}|$, is given by

$$\frac{d^2|\vec{r}|}{dt^2} = -\frac{GM_{L,G}}{|\vec{r}|^2} + H_0^2\Omega(\Lambda,0)|\vec{r}|.$$  \hspace{1cm} (2)

Equation 2 is solved for $|\vec{r}(t)|$, given the above boundary conditions and the condition that $|\vec{r}| \to 0$ as $t \to 0$.

In some prior implementations of the TA, two simplifications have been invoked. The first involves neglecting the vacuum energy
term in Equation 2. A second simplification involves neglecting the tangential component of the relative motion between the MW and M31. This is a reasonable approximation, given that the magnitude of the tangential velocity is small relative to the radial velocity. In order to obtain the most accurate solution, we numerically solve Equation 2 and account for both the vacuum energy term and the non-zero tangential velocity.

For each mock LG in our sample, we determine the separation $|\vec{r}|$, as well as the relative velocity components $V_r$ and $V_t$. Then for a chosen $M_{LG}$, we take these as initial conditions to numerically solve Equation 2 by integrating backwards in time. The best solution for $M_{LG}$ is the one for which the separation between the two massive galaxies approaches zero as $t \to 0$ (Gyr). Practically, the solution for $M_{LG}$ is obtained by extending the integration to times $t < 0$, and generating a library of $M_{LG}$ and times at which the separation goes to zero. The best value for $M_{LG}$ is obtained by interpolating between the smallest positive $r$ and the smallest negative $r$ obtained. Then given this solution for the “timing argument mass,” $M_{TA}$, we can compare to the true LG mass from the simulation to estimate the bias in the TA mass estimator.

3.2 Virial Theorem

Next we consider a LG mass estimator based on the VT. This estimator was first employed in Courteau & van den Bergh (1999) given the sample of OLGs known at the time, and has been developed more recently with updated data on OLGs in Diaz et al. (2014).

For a system in equilibrium, the VT relates the kinetic energy ($K$) and the potential energy ($U$) via $2K + U = 0$. Assuming that the system is isotropic, so that the random motions are the same in each of the coordinate directions, the kinetic energy is $K = 3/2N M_0 \sigma_t^2$, where $N$ is the number of galaxies in the sample, $M_0$ is the mass of each galaxy, and $\sigma_t$ is the one-dimensional radial velocity dispersion of the system. The factor of 3 arises from the assumption of isotropy.

We take two contributions to the potential energy $U$: the gravitational contribution from the two massive galaxies, and the contribution from the cosmological constant. On the $t^{th}$ galaxy, the potential is then

$$\phi = -GM_{MW} p_t - GM_{M31} q_t - \frac{4\pi}{3} G \rho_\Lambda |\vec{r}|^2,$$

where $\rho_\Lambda$ is the present value of the vacuum energy density, and $p_t = \left( |\vec{r}| - R_{MW} \right) \left( |\vec{r}| - R_{M31} \right) - \left( a_{MW} \right)^{-1}$

$$q_t = \left( |\vec{r}| - R_{M31} \right) \left( |\vec{r}| - R_{MW} \right) - \left( a_{M31} \right)^{-1}.$$

Here $a_{MW}$ is the scalelength of the MW’s dark matter halo, and $a_{M31}$ is the corresponding scalelength of the M31’s dark matter halo. To model the DM density profiles of the MW and M31, we have followed Diaz et al. (2014) and adopted a Hernquist profile, and for the scale radii we take $a_{MW} = a_{M31} = 40$ kpc. We find that reasonable changes in the scale radius for each halo do not affect our results.

Taking the approximation that all the satellite galaxies have equal mass, the LG mass can be estimated as

$$M_{LG} = \frac{-3N(1+m)\sigma_t^2 - (8\pi/3)G\rho_\Lambda(1+m) \sum N |\vec{r}|^2}{G\sum N |\vec{r}| \cdot \vec{V} (p_t + m q_t)},$$

where $\rho_\Lambda = 0.69 \times 1.5 \times 10^7 M_0$ Mpc$^{-3}$, and as defined above $m = M_{M31}/M_{MW}$ is the mass ratio between the MW and M31. Note that this formula is similar to that derived in Diaz et al. (2014), the only difference being the vacuum energy term which is added to the kinetic energy term.

With our LG sample, we implement Equation 6 by determining $\sigma_t$ and the mass ratio $m$ for each system, and from these determine $M_{LG}$. We refer to this solution as the virial theorem mass, $M_{VT}$. Note that since $m$ is not a priori known for the MW-M31 system, we must take realistic bounds on this quantity when comparing to observations. We discuss in detail below how the assumption for $m$ affects our results.

4 RESULTS

In this section we present the results of our comparison of the TA and the VT mass estimators to the LGs identified in the Ilustris simulation. We estimate the systematics that arise from the “cosmic scatter” in each of these measurements, and use these to provide robust estimates of the LG mass using each method.

4.1 Calibration of the Timing Argument mass for the Local Group

We begin with an analysis of the TA mass estimator. Our analysis follows a similar approach to that of Li & White (2008), who utilize the dark matter-only Milennium simulation. These authors determine the bias in the TA estimate, $M_{TA}$, as compared to the true LG mass as obtained from the simulation, where the true mass of the LG from the simulation, $M_{TA,halo}$, is defined as the sum of the MW/M31 halo analogs, $M_{TA,halo} = M_{halo,1} + M_{halo,2}$. These authors define $A \equiv M_{TA,halo}/M_{TA}$, and study the distribution of this quantity for their mock LG sample from Milenium. We use our sample of LGs to obtain a distribution of $A$, which provides an estimate of the bias in the TA mass estimator for the Ilustris simulation.

To get a sense of the relative importance of the different velocity cuts, we consider three subsamples of our 580 LG pairs identified above. In the first subsample, we restrict the tangential velocity to be less than the absolute value of the radial velocity, i.e., $V_t < |V_r|$. This is our loosest cut, motivated to simply ensure that the dominant component of the velocity is in the radial direction.

In the second subsample, we restrict the radial velocity range to $-195 \text{ km s}^{-1} \leq V_r \leq -65 \text{ km s}^{-1}$, reducing our sample to 251 pairs. This cut is motivated to span the uncertainty in the observed radial velocity between the MW and M31, and was implemented with similar motivation by Li & White (2008). For our third subsample, we consider the union of the first and second cuts. This is our smallest subsample, resulting in a total of 177 pairs.

To compare our magnitude cut to previous cuts that consider $V_{max}$, Li & White (2008) found 11388 pairs in Millennium simulation using $150 \text{ km s}^{-1} < V_{max} < 300 \text{ km s}^{-1}$ as their initial cut. For this sample of 11388 pairs, they use the same distance cut of 500 kpc $< |\vec{r}| < 1000$ kpc, and radial velocity cut of $-195 \text{ km s}^{-1} \leq V_r \leq -65 \text{ km s}^{-1}$. When we perform the $V_{max}$, distance and radial velocity cuts on Ilustris TNG300-1, we find 1051 pairs. Accounting for the difference in the simulation volumes, 500 $h^{-1}$Mpc and 205 $h^{-1}$Mpc for the Millennium and IlustrisTNG simulations respectively, our final number of mock-LG pairs is consistent with the number of pairs found by Li & White (2008).

Figure 4 plots the cumulative (left) and histogram (right) distributions of $A$ for all three velocity samples. In Figure 4 (left) also plots the best fit gaussian curve (dashed-black) for the preferred velocity sample with $-195 \text{ km s}^{-1} \leq V_r \leq -65 \text{ km s}^{-1}$. From this we see that the distribution is very close to gaussian. In particular, comparing the mean of 0.75 to 0.77 and standard deviation of 0.22 to
In Table 3 we summarize our statistical results, reporting the in-
distribution of $A$ under three differing velocity restrictions denoted
in the velocity cut column.

| Velocity Cut                     | 5%  | 25% | 50% | 75% | 95% | # of pairs |
|----------------------------------|-----|-----|-----|-----|-----|-----------|
| $V_t < |V_r|$                     | 0.42| 0.67| 0.83| 0.98| 1.22| 284       |
| $-195 \text{ km s}^{-1} < V_r < -65 \text{ km s}^{-1}$ | 0.39| 0.61| 0.74| 0.88| 1.16| 251       |
| $-195 \text{ km s}^{-1} < V_r < -65 \text{ km s}^{-1}, V_t < |V_r|$ | 0.39| 0.64| 0.79| 0.90| 1.16| 177       |

It is interesting to compare our median values of the distribution of $A$ to that of Li & White (2008). In addition to selecting LG systems based on maximum circular velocity, these authors do not include the effect of the cosmological constant term, nor do they include the tangential velocity. Assuming $V_t = 0$ yields a lower bound to the TA mass estimate. In addition, excluding the cosmological term has the effect described above. Thus the combination of $V_t = 0$ and $\Lambda = 0$ results in their slightly calculated bias in the median value of $A \approx 0.99$. This is consistent with González et al. (2014), who find that including the tangential velocity overestimates the true mass by a factor $\sim 1.3 - 1.6$.

The different velocity cuts that we consider impact the tail of the $A$ distributions. In particular, constraining the range of the radial velocity to $-195 \text{ km s}^{-1} \leq V_r \leq -65 \text{ km s}^{-1}$ reduces the width of the distribution. For different velocity cuts, the distribution widens because the sample includes systems with either very large or very small radial velocities. In addition requiring the tangential velocity to be less than the radial velocity, $V_t < |V_r|$, the width of the distribution is unchanged, though the median values shift up slightly. This upward shift in the median values reflects the smaller TA mass estimate for systems with small tangential velocity.

Figure 4. Cumulative probability distributions (left) and histogram distributions (right) for $A$, the ratio of the true mock-LG mass to the Timing Argument mass. We plot for three different velocity restrictions on the main sample. First, restricting the tangential velocity to be less than the radial velocity between the main pair, $V_t < |V_r|$ (red). Second, the radial velocity restriction of $-195 \text{ km s}^{-1} < V_r < -65 \text{ km s}^{-1}$ (blue). Then the union of these two velocity cuts, $-195 \text{ km s}^{-1} < V_r < -65 \text{ km s}^{-1}, V_t < |V_r|$ (black). The dashed-black curve is the best Gaussian fit for the preferred velocity sample, $-195 \text{ km s}^{-1} \leq V_r \leq -65 \text{ km s}^{-1}, V_t < |V_r|$. 0.27 for the curve fit and Gaussian fit respectively, we see our sample has a relatively normal distribution.

In Table 3 we summarize our statistical results, reporting the interquartile and the 90% range of the distributions for all three velocity cuts. The median value of $A$ and the interquartile ranges are less than unity, with $A \approx 0.74 - 0.83$, depending on the specific velocity cut implemented. This implies that there is a slight bias in the TA mass estimator to predict a mass higher than the true mass obtained from the simulation. However, when considering the 90% range about the mean, this bias is removed, with the upper bounds on the distribution in the range $A \approx 1.16 - 1.22$.

To understand this bias, we also calculate the timing argument mass assuming no cosmological constant, i.e. $\Lambda = 0$. This calculation yields a median value of $A$ in the range $A \approx 0.82 - 0.93$. Thus, including the cosmological term in calculating the TA mass estimator results in a shift to a median value of $A \approx 0.74 - 0.83$, indicating a larger timing argument mass estimate. This particular shift in the TA mass estimator can be understood by considering the current distance and approach velocity between the pair. When we include the cosmological term in the calculation, we account for a repulsive term in the potential. Thus, a more massive system is estimated in order to account for the current distance and approach velocity. Recall these distributions record $M_{tr, halo}/M_{TA}$, thus a higher estimate in the timing argument mass lowers the distribution.

0.27 for the curve fit and Gaussian fit respectively, we see our sample has a relatively normal distribution.
4.2 Timing Argument Application to Local Group

We are now in position to calculate the TA mass of the LG, and to apply a correction to this estimate based on our analysis above. We assume \( \Omega_{\Lambda,0} = 0.69 \) and we utilize the observed Local Group kinematics of \( V_r = 79 \) km s\(^{-1} \), \( V_t = -109.3 \) km s\(^{-1} \) and \( r = 785 \) kpc. For these assumptions, we calculate a TA mass of 

\[
M_{TA, LG} = 6.01 \times 10^{12} \, M_\odot. \tag{7}
\]

The observational uncertainty on the TA mass is primarily due to the measured uncertainties of the radial and tangential velocities. van der Marel et al. (2012) quote a statistical uncertainty of \( 0.45 \times 10^{12} \, M_\odot \). Noting 5% and 95% containment points of the distribution for our preferred sample are separated by a factor of 2.97, we see that the systematic uncertainty of the TA mass is much larger than the measured uncertainty.

From Table 3, the median value for the distribution of \( A \) for the preferred sample of mock-LGs is 0.79. Thus, the corrected value to the timing argument mass is

\[
M'_{TA, LG} = 4.75 \times 10^{12} \, M_\odot. \tag{8}
\]

The 5% and 95% c.l. on the \( A \) distribution are 0.39 and 1.16 respectively. This yields a range of \( 2.34 \times 10^{12} \, M_\odot \leq M'_{TA, LG} \leq 6.97 \times 10^{12} \, M_\odot \) with 90% confidence.

For comparison, assuming that \( A = 0 \), we obtain

\[
M_{TA, LG} = 5.43 \times 10^{12} \, M_\odot. \tag{9}
\]

For \( A = 0 \), the median of the \( A \) distribution is 0.87, and the 5% and 95% range yield values of 0.45 and 1.29, respectively. Thus, using the median value to correct the TA mass yields

\[
M'_{TA, LG} = 4.72 \times 10^{12} \, M_\odot. \tag{10}
\]

in a range of \( 2.44 \times 10^{12} \, M_\odot \leq M'_{TA, LG} \leq 7.01 \times 10^{12} \, M_\odot \) with 90% confidence. Note that the 90% confidence ranges for the \( M_{true, LG} \) with and without the cosmological constant term are very similar.

4.3 Calibration of the Virial Method mass for the Local Group

We now move on to an analysis of the VT mass estimator, and compare this to the LG mass derived from the simulations. Here we define the VT mass estimate as \( M_{VT} \), and the ratio of the VT mass estimate to the true mass as \( B \equiv M_{VT,halo}/M_{VT} \). We consider the same velocity cuts as above, and again define \( M_{VT,halo} \) as the sum of the masses of the two most massive galaxies. For each sample, we use Eq. 6 to calculate the mass of each mock-LG, \( M_{VT} \). To begin, we calculate the mass ratio \( m \) for each mock-LG by setting the MW and M31 masses to their true values as extracted from the simulation.

Figure 5 shows the distributions of \( \log_{10} B \). We plot the sample with the OLGM distance criteria of \( D < 1.5 \) Mpc and the additional cuts, \( N \geq 10 \) and \( 0.3 \leq m \leq 3.3 \). The left plot of Figure 5 shows this distribution for all three samples in cumulative form. The dashed curves indicate the best fit gaussian for the three samples. The right plot of Figure 5 shows histogram distribution of \( B \) for all three samples.

In Table 4 we summarize our statistical results, reporting the interquartile and the 90% points of the distributions for all three OLGM identification cuts. The median value of \( B \) and the interquartile ranges are less than unity, with \( B \approx 0.80 - 1.07 \), depending on the initial OLGM cut implemented. This implies that in general there is a slight bias in the VT mass estimator depending on the specific choice of OLGM sample. In particular in the case of our main sample, \( M_{dm} > 10^9 \, M_\odot \), we find a slight bias, \( B = 0.8 \), with a 95% containment interval of [0.20, 1.63].

As noted above, current estimates of \( m = M_{M31}/M_{MW} \) indicates that the mass of M31 is about a factor of two larger than that of the MW. This motivated an additional cut to our Local Group sample, keeping only those with \( 0.3 \leq m \leq 3.3 \). If we instead relax this restriction on \( m \), but still use the true values of their masses in the VT, there is minimal change in the \( B \) distribution, yielding median values of \( B = 0.79 - 1.01 \).

When we instead consider the sample with the initial OLGM distance criteria of \( 350 \) kpc < \( D < 1.5 \) Mpc and the additional cuts, \( N \geq 10 \) and \( 0.3 \leq m \leq 3.3 \), we find a larger range for the median value between cuts of \( B \approx 0.82 - 1.18 \). This larger range may be in part because the stricter initial OLGM distance cut yields a smaller number of pairs in our sample.

For all of the above analysis, we chose \( m \) to be the true value by extracting the true M31 and MW masses from the simulation. To check the results are not sensitive to the choice of \( m \), we simply set \( m = 1 \) in the VT, implying that the masses of the MW and M31 are equal. Note setting \( m = 1 \) implies the barycenter is located at the midpoint of the pair, and this assumption propagates in calculating the velocity dispersion. Referring to our main sample in which we use the distance cut of \( D < 1.5 \) Mpc and restrict the true value of \( m \) to be within \( 0.3 \leq m \leq 3.3 \), we find the median value to be \( B \approx 0.80 - 1.09 \). Comparing this to \( B \approx 0.80 - 1.07 \) when we use the true value of \( m \), we find the VT mass estimate is not sensitive to the value of \( m \) assumed in the VT.

4.4 Virial Theorem Application to Local Group

Diaz et al. (2014) have estimated the LG mass from OLGMs using the VT. These authors derive this result using radial velocity data on OLGMs within a distance of \( 350 \) kpc < \( D < 1.5 \) Mpc. The result of their analysis is an LG mass estimate of

\[
M_{VT, LG} = 2.5 \pm 0.4 \times 10^{12} \, M_\odot. \tag{11}
\]

Referring to our preferred sample with a median of \( B = 0.8 \) and a 90% containment interval of 0.20-1.63, the corrected value for the LG mass estimate from the VT is then

\[
M'_{VT, LG} = 2.0 \times 10^{12} \, M_\odot. \tag{12}
\]

This correction was performed with our preferred sample of all OLGMs < 1.5 Mpc, though as noted above the \( B \) distribution changes only slightly if OLGMs < 350 kpc are excluded. The uncertainties in Equation 12 are just larger than the measured uncertainty of \( 0.4 \times 10^{12} \, M_\odot \) quoted in Diaz et al. (2014). The upper and lower limits of this estimate imply a range of \( 0.5 \times 10^{12} \, M_\odot \leq M'_{VT, LG} \leq 4.1 \times 10^{12} \, M_\odot \) with 90% confidence. This indicates, when estimating the mass using the VT from observations, the derived value is an overestimate of the true mass, though the correction is small.

The fact that the VT estimator is largely unbiased, i.e. \( B \approx 1 \), may come as a surprise, given the simplicity of this estimator and the assumptions made in deriving it. The two key assumptions in deriving the VT are: 1) that the LG is in dynamical equilibrium, and 2) that the orbits of the OLGMs are statistically isotropic. The former assumption is most likely violated simply because the MW and M31 are falling towards one another for the first time. The application of the VT is further complicated given the fact that a large fraction of LG satellites are bound to either the MW or M31. Regarding the second assumption, this is also manifestly violated in our LG sample since
the OLGMs are on largely radial orbits (Fig. 3 right). If the system is assumed to be isotropic, but the underlying orbits are anisotropic, the result would be that the VT overestimates the true mass. This is indeed the trend that we observe in our distribution of $B$, albeit with a substantial scatter.

To further examine this point, in Figure 6 we plot the log of the true mock-LG mass vs the log of the mass derived from the VT. We plot the OLGM dark matter mass sample of $>10^9 M_\odot$, where the left plot considers OLGM’s with a distance of $D < 1.5$ Mpc, and the right plot considers the distance of $350 \text{kpc} < D < 1.5$ Mpc. In each plot the dashed-black line indicates the one-to-one line, where the true mass equals the VT mass. By taking bins of roughly 0.1 in log of the true mass from 12.2-12.8, we calculate the median, 16% and 84% points of the VT mass in each of these bins. We show the median with the solid black line, and the 16% and 84% points with the gray lines. The color bar indicates the value of $(\sigma_r/\sigma_t)^2$ where we have set an upper limit of 4 to cover the range on the majority of the sample. There are three points highlighted in these plots. First, in both plots, we generally see the trend of lighter points on the left and darker points on the right of the dashed-black line. This agrees with our conclusion that the anisotropic mock-LGs generally estimate a larger mass using the virial theorem. Second, the lower limit of $(\sigma_r/\sigma_t)^2$ is 0.51 for the left plot and 0.26 for the right plot. This suggests that removing OLGM’s within 350 kpc from one of the main pairs yields systems with larger tangential velocity and thus more circular orbits around the center of mass of the main pair. When we keep OLGM’s with a distance $<350$ kpc of either the mock-MW or mock-M31, we include satellites that follow the bulk motion of the mock-MW or mock-M31 toward the center of mass resulting in a large radial component. Third, in both plots we see the median (solid-black line) of the VT derived mass tracks the one-to-one (dashed-black) line. This suggests the VT gives a reasonable estimate of the true mass for systems with various true masses.

5 DISCUSSION AND CONCLUSIONS

We have identified a sample of Local Group-like systems in the IllustrisTNG simulation, and have used this sample to study two mass estimators for the Local Group: the timing argument and the virial
For the timing argument mass, we update both the slight bias in this estimator and the cosmic scatter, including measurements of the cosmological constant and the Milky Way-M31 tangential velocity. Though our primary Local Group-sample is defined using an absolute magnitude cut in addition to kinematic cuts, compared to previous calculations which identified the Local Group-sample based on dark matter halos, our results are in good agreement (Li & White 2008; Peñarrubia & Fattahi 2017). We find that the timing argument slightly overestimates the Local Group mass, though this estimator is unbiased at the 90% c.l. This result depends weakly on the precise kinematic cuts that we use to define the Local Group-like sample.

Our results are also consistent with previous results in that we find that the cosmic scatter dominates the statistical uncertainty in the determination of the timing argument mass. In particular, for the timing argument we find that the Local Group mass at 90% c.l. is $4.75^{+2.22}_{-2.41} \times 10^{12} M_\odot$, where the errors are due to the cosmic scatter. For comparison, the statistical uncertainty quoted in van der Marel et al. (2012) is 0.45 $\times 10^{12} M_\odot$.

We have provided the first comparison of the Local Group mass as determined from the virial theorem to that obtained in simulations of Local Group-like systems. We find that the virial theorem estimator also overestimates the true Local Group mass, though there is a larger scatter in the virial mass to true mass ratio relative to the corresponding ratio for the timing argument. For the virial theorem, we find the Local Group mass at 90% c.l. is $2.00^{+2.1}_{-1.3} \times 10^{12} M_\odot$, where the errors are due to the cosmic scatter. For comparison, the statistical uncertainty quoted in Diaz et al. (2014) is $0.4 \times 10^{12} M_\odot$. So similar to the timing argument, the cosmic scatter in this mass estimator dominates the uncertainty budget.

We attribute the slight bias and broad scatter in the virial theorem estimator to several factors. First, we find that the orbits are predominantly radial for Local Group galaxies, which differs from the virial theorem assumption of isotropic orbits. In particular, we find that the ratio of radial to tangential orbits for Local Group dwarf galaxies is $\sigma_r^2 / \sigma_t^2 \approx 2$, implying that these galaxies are on largely radial orbits. This small amount of tangential motion is likely to not only impact the virial theorem mass estimator, but also other estimates of the Local Group mass based on pure radial infalling motion (Peñarrubia et al. 2016).

More generally, the radial and tangential motions of the dwarf satellites have important implications for understanding the dynamics of dwarf galaxies in the Local Group. Though the current measurements of the proper motions of isolated dwarfs are not precise enough (McConnachie et al. 2021; Battaglia et al. 2021) to be constrained by our predictions, future Gaia data releases may be able to improve upon the astrometry of the isolated dwarfs.

An additional source of uncertainty in our virial theorem estimator may lie in our choice of the outermost radius that defines the extent of the Local Group. For all Local Group analogs in Illustris, we fix this radius at 1.5 Mpc. The corresponds to the approximate turn-around radius of the Local Group, which is about a factor of two greater than the simple estimate for the Local Group virial radius based upon the dynamical free-fall time (Peñarrubia & Fattahi 2017). We have checked that the true mass to virial mass distribution is unaffected by the precise choice of outer radius for the Local Group. Indeed, defining 0.7 Mpc as the outer radius does not change our statistical distribution of $B$ derived in Section 4, though it does cut down on the sample of galaxies that can be used in the virial analysis. In addition, the ratio $\sigma_r^2 / \sigma_t^2 \approx 2$ remains similar for this sample. This indicates that the deduced mass and the kinematics of the Local Group is independent of where the exact outer radius is set, and that the cosmic scatter may reflect more simplifying assumptions in the underlying nature of the equilibrium of the Local Group.

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