We present initial results from our ongoing program to image the Sunyaev–Zel’dovich (SZ) effect in galaxy clusters at 143 GHz using Bolocam; five clusters and one blank field are described in this manuscript. The images have a resolution of 58 arcsec and a radius of ~6–7 arcmin, which is approximately $r_{500}$–$2r_{500}$ for these clusters. We effectively high-pass filter our data in order to subtract noise sourced by atmospheric fluctuations, but we are able to obtain unbiased images of the clusters by deconvolving the effects of this filter. The beam-smoothed rms is $\sim 10 \mu K_{\text{CMB}}$ in these images; with this sensitivity, we are able to detect the SZ signal to beyond $r_{500}$ in binned radial profiles. We have fit our images to beta and Nagai models, fixing spherical symmetry or allowing for ellipticity in the plane of the sky, and we find that the best-fit parameter values are in general consistent with those obtained from other X-ray and SZ data. Our data show no clear preference for the Nagai model or the beta model due to the limited spatial dynamic range of our images. However, our data show a definitive preference for elliptical models over spherical models, quantified by an $F$ ratio of $\sim 20$ for the two models. The weighted mean ellipticity of the five clusters is $\epsilon = 0.27 \pm 0.03$, consistent with results from X-ray data. Additionally, we obtain model-independent estimates of $Y_{500}$, the integrated SZ $\gamma$-parameter over the cluster face to a radius of $r_{500}$, with systematics-dominated uncertainties of $\sim 10\%$. Our $Y_{500}$ values, which are free from the biases associated with model-derived $Y_{500}$ values, scale with cluster mass in a way that is consistent with both self-similar predictions and expectations of a $\sim 10\%$ intrinsic scatter.

Key words: cosmology; observations – galaxies: clusters: individual (A697, A1835, MS 0015.9+1609, MS 0451.6–0305, MS 1054.4) – methods: data analysis

Online-only material: color figures

1. INTRODUCTION

Galaxy clusters are the largest collapsed objects in the universe, making them excellent tools for studying cosmology and the astrophysics of gravitational collapse. They are rare excursions in the matter density field and the formation history of clusters is closely tied to the composition and evolution of the universe. As a consequence, clusters have been used extensively to constrain cosmology. For example, they provided the first evidence that the matter density, $\Omega_m$, was insufficient to close the universe (Bahcall et al. 1997). Additionally, clusters provided the most reliable information about the amplitude of scalar perturbations, $\sigma_8$, prior to the release of Wilkinson Microwave Anisotropy Probe (WMAP) results (Viana & Liddle 1999; Pierpaoli et al. 2001, 2003; Borgani et al. 2001; Allen et al. 2003b; Spergel et al. 2003).

Moreover, the cluster-derived measurements of $\sigma_8$ can be combined with measurements of the normalization of the cosmic microwave background (CMB) power spectrum to constrain the total neutrino mass via the growth of density perturbations (Pierpaoli 2004; Allen et al. 2003a; Vikhlinin et al. 2009). In addition, galaxy clusters have been forming during the same epoch that dark energy has evolved to dominate the energy density of the universe. As a result, the number of galaxy clusters as a function of redshift is sensitive to the dark energy density, $\Omega_\Lambda$, and its equation of state, $w$ (Hallman et al. 2001; Holder et al. 2000, 2001). Recently, large surveys of galaxy clusters have been completed, or have released initial results, that constrain the total neutrino mass and/or the properties of dark energy (Allen et al. 2008; Mantz et al. 2010b; Vikhlinin et al. 2009; Vanderlinde et al. 2010). Additionally, clusters are being used to test gravity on large scales via studies of their internal structure and distribution in space (Schmidt et al. 2009; Diaferio & Ostorero 2009; Martino et al. 2009; Wu et al. 2010; Moffat & Toth 2010; Rapetti et al. 2010).

Galaxy clusters also provide excellent laboratories for studying the astrophysics of structure formation. In general, galaxy clusters are well-behaved objects, and their properties can be predicted to fairly good precision using simple gravitational collapse models and self-similar scaling relations (Kaiser 1986). However, non-gravitational effects, such as radiative cooling, star formation, turbulence, magnetic field support, and cosmic-ray pressure, produce deviations from the simple gravitational models, and the data clearly favor models that include non-gravitational processes (Kravtsov et al. 2006; Nagai et al. 2007).

A wide range of observational techniques are used to study galaxy clusters. Optical/infrared and radio measurements can be used to study the individual galaxies within the cluster (e.g., Pipino et al. 2010; Lin et al. 2009; Gralla et al. 2010); parameters such as velocity dispersion and richness can then be used to understand the global properties of the cluster (e.g., Menanteau et al. 2010b; Rines & Diaferio 2010; Serra et al. 2010; Szabo et al. 2010; Hao et al. 2010). Additionally, X-ray observations are sensitive to the bremsstrahlung emission from the hot gas in the intra-cluster medium (ICM), which contains $\sim 90\%$ of the baryonic mass of the cluster (e.g., Vikhlinin et al. 2006; Gonzalez et al. 2007; Vikhlinin et al. 2009; Arnaud et al. 2010; Zhang et al. 2010). Galaxy clusters are also efficient gravitational lenses, and detailed observations of the background galaxies can be used to determine the matter distribution within...
the cluster, ≈90% of which is in the form of dark matter (Clowe et al. 2006; Allen et al. 2008).

The ICM can also be studied using the Sunyaev–Zel’dovich (SZ) effect (Sunyaev & Zel’dovich 1972), where the background CMB photons inverse Compton scatter off of the electrons in the ICM. Studies of galaxy clusters using the SZ effect are quickly maturing. The South Pole Telescope (SPT) and the Atacama Cosmology Telescope are conducting large untargeted surveys and have already published catalogs with dozens of clusters (Vanderlinde et al. 2010; Menanteau et al. 2010a); they expect to detect hundreds of clusters when the surveys are complete. Due to the redshift independence of the SZ surface brightness, the clusters discovered in these surveys are, on average, at significantly higher redshifts than those discovered with X-ray or optical surveys. Consequently, SZ-selected cluster catalogs are expected to play a leading role in further constraining cosmological parameters such as $\Omega_\Lambda$ and $w$ (e.g., Carlstrom et al. 2002).

These large-scale SZ surveys are operating based on the prediction that the total integrated SZ signal over the cluster, $Y$, which is proportional to the total thermal energy of the ICM, scales in a robust way with total cluster mass (e.g., Kravtsov et al. 2006). In practice, the integral does not generally extend to the edge of the cluster, which is effectively at $\gtrsim 5r_{500}$, and can be performed in one of the two ways: over a spherical volume using a deprojected SZ profile or over a cylindrical volume using a projected SZ profile. Most groups have calculated the integrated SZ signal by performing the cylindrical integration over the cluster face within a well-defined aperture (generally $r_{2500}$ or $r_{500}$) (Morandi et al. 2007; Bonamente et al. 2008; Mroczkowski et al. 2009; Marrone et al. 2009; Plagge et al. 2010; Huang et al. 2010; Culverhouse et al. 2010; Andersson et al. 2010), although a couple groups have also performed the spherical integral (Mroczkowski et al. 2009; Andersson et al. 2010). With the exception of Plagge et al. (2010, hereafter P10), these groups have exclusively used parametric profiles to describe the SZ signal in order to determine $Y$. The initial scaling results from these $Y$ measurements are roughly consistent with the expectation that it is a low-scatter ($\approx 10\%$) proxy for the total cluster mass.

Additionally, several groups have made detailed SZ observations of previously known galaxy clusters, and these data have been used to constrain the properties of the ICM beyond $r_{500}$ (Halverson et al. 2009; Nord et al. 2009; Basu et al. 2010; Mroczkowski et al. 2009; Plagge et al. 2010). For example, the SPT has measured radial profiles in 15 clusters to a significant fraction of the virial radius, and they find best-fit model parameters that are consistent with previous studies using X-ray data. APEX-SZ has published detailed studies of three clusters, using a joint SZ/X-ray analysis to constrain mass-weighted temperature profiles beyond $r_{500}$ in two of these clusters (Halverson et al. 2009; Nord et al. 2009; Basu et al. 2010). Note that, in addition to providing complementary information about the ICM in these previously studied clusters, these SZ data will help constrain and improve systematic uncertainties in blind SZ surveys (e.g., constraining the typical SZ profiles and resulting detection biases due to the cooling properties of the central gas; Pipino & Pierpaoli 2010).

High resolution ($\approx 10$ arcsec) SZ measurements are also now being made using MUSTANG (Mason et al. 2010; Korngut et al. 2010) and SZA/CARMA (Carlstrom et al. 2002). These observations will provide insights into the internal structure of clusters that will be complementary to the information obtained via X-ray observations. Among other applications, these data can be used, in combination with optical observations, to infer the merging history and evolution of the cluster (Croston et al. 2008).

In this work, we present Bolocam SZ observations of five massive galaxy clusters. We are able to measure the SZ signal in our two-dimensional images to $\approx r_{500}$ and to well beyond $r_{500}$ in azimuthally averaged radial profiles. Using these data, we are able to constrain the broad morphologies of these clusters and compute observables such as $Y_{SZ}$ in a model-independent way. A companion paper to this manuscript, S. Ameglio et al. (2011, in preparation), presents joint X-ray/SZ deprojections of density and temperature profiles for these same five clusters using Bolocam and Chandra data (Ameglio et al. 2007).

2. CLUSTER OBSERVATIONS WITH BOLOCAM

Bolocam is an millimeter-wave imaging camera that operates from the Caltech Submillimeter Observatory (CSO) with 144 bolometric detectors covering a circular 8 arcmin field of view (FOV; Glenn et al. 1998; Haig et al. 2004). For the observations described in this manuscript, Bolocam was configured to observe at 143 GHz. To image the clusters, we scanned the telescope in a Lissajous pattern (Kovacs et al. 2006), where the telescope is driven in two orthogonal directions using sine waves with incommensurate periods. We used an amplitude of 4 arcmin for the sinusoids, which were oriented along the R.A. and decl. directions with periods of 6.28 and 8.89 s. These parameters were chosen to keep the FOV on the cluster center 100% of the time while scanning as fast as possible at the CSO ($\approx 4$ arcmin s$^{-1}$) to modulate the cluster signal above the low-frequency atmospheric noise. An example of the resulting integration time per pixel is given in Figure 1. The data were collected via 10 minute long observations, with the periods of the two orthogonal sinusoids exchanged between observations. Typically, we complete $\approx 100$ observations per cluster, which corresponds to approximately 50 ks and yields a beam-smoothed rms of $\approx 10 \mu K_{CMB}$ (see Table 1).
In this paper, we present the results from five clusters and one blank field, which are described below. Due to the size of our resulting maps ($r \sim 6$–7 arcmin), we have chosen to focus primarily on high-redshift clusters, which have virial radii within the extent of our maps. All of the clusters presented in this manuscript are beyond $z = 0.25$, and three of the five are beyond $z = 0.50$. For reference, $r_{500}$ lies within the extent of our map for a typical $10^{15} M_\odot$ cluster at $z = 0.25$, and at $z = 0.50$, the virial radius lies within our map. Note that, throughout this work, we assume a $\Lambda$CDM cosmology with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

### Table 1

| Target      | R.A.   | Decl.   | Redshift | Bolocam Time (ks) | $\text{rms} (\mu\text{K}_{\text{CMB}})$ | $r_{500}$ (Mpc) | $M_{\text{gas,}500}$ (M$_\odot$) |
|-------------|--------|---------|----------|-------------------|-----------------------------------------|----------------|----------------------------------|
| A697        | 08:42:58 | +36:21:56 | 0.28     | 52                | 8.9                                      | 1.65±0.09       | $(19.6\pm2.7)\times10^{13}$ |
| A1835       | 14:01:02 | +02:52:42 | 0.25     | 50                | 8.7                                      | 1.49±0.06       | $(14.1\pm1.2)\times10^{13}$ |
| MS 0015.9+1609 | 00:18:34 | +16:26:13 | 0.54     | 38                | 10.2                                    | 1.28±0.08       | $(17.5\pm1.9)\times10^{13}$ |
| MS 0451.6–0305 | 04:54:11 | −03:00:53 | 0.55     | 53                | 7.7                                      | 1.45±0.12       | $(15.6\pm2.2)\times10^{13}$ |
| MS 1054.4–0321 | 10:56:59 | −03:37:34 | 0.83     | 66                | 6.7                                      | 1.07±0.13       | $(11.5\pm2.4)\times10^{13}$ |
| SDS1        | 02:18:00 | −05:00:00 |          | 37                | 9.1                                      | ...            | ...                             |

**Notes.** A list of the clusters presented in this manuscript. From left to right, the columns give the R.A. and decl. of the cluster in J2000 coordinates, the redshift of the cluster, the amount of Bolocam integration time, the median rms per beam-smoothed pixel in the Bolocam map, the radius of the cluster, and the mass of the cluster. The values for $r_{500}$ and $M_{\text{gas,}500}$ were taken from Mantz et al. (2010a; A697 and A1835) and Ettori et al. (2009; MS 0015.9+1609, MS 0451.6–0305, and MS 1054.4–0321).

### 3. DATA REDUCTION

In general, our data reduction followed the procedure described in Sayers et al. (2009, hereafter S09), with some minor modifications. We briefly describe the techniques below; along with the changes relative to S09.

#### 3.1. Calibration

Bright quasars located near the clusters were observed for 10 minutes once every $\sim$90 minutes in order to determine the offset of our focal plane relative to the telescope pointing coordinates. These observations were used to construct a model of the pointing offset as a function of local coordinates (az, el), with a single model for each cluster. The uncertainty in the pointing models is $\lesssim 5$ arcsec. This pointing uncertainty is quasi-negligible for Bolocam’s 58 arcsec FWHM beams, especially for extended objects such as clusters. We made two 20 minute long observations each night of Uranus, Neptune, or a source in Sandell (1994) for flux calibration. Using the quiescent detector resistance as a proxy for detector responsivity and atmospheric transmission, we then fit a single flux-calibration curve to the entire data set. We estimate the uncertainty in our flux calibration to be 4.3%, with the following breakdown: 1.7% from the Rudy temperature model of Mars scaled to measured WMAP values (Halverson et al. 2009; Wright 1976; Griffin et al. 1986; Rudy et al. 1987; Muhleman & Berge 1991; Hill et al. 2009), 1.5% in the Uranus/Neptune model referenced to Mars (Griffin & Orton 1993), 1.4% due to variations in atmospheric opacity (S09), 3.1% due to uncertainties in the solid angle of our point-spread function (PSF) (S09), and 1.5% due to measurement uncertainties (S09).

#### 3.2. Atmospheric Noise Subtraction

The raw Bolocam timestreams are dominated by noise sourced by fluctuations in the water vapor in the atmosphere, which have a power spectrum that rises sharply at low frequencies. In order to optimally subtract the atmospheric noise, we have used a slightly modified version of the average subtraction algorithm described in Sayers et al. (2010, hereafter S10). We have modified the S10 algorithm because these cluster data contain additional atmospheric noise caused by the Lissajous scan pattern. Since we are scanning the telescope parallel to R.A. and decl., the airmass we are looking through is constantly changing. As a result, our data contain a large amount of atmospheric noise in narrow bands centered on the two fundamental scan frequencies.

Following the algorithm in S10, we first create a template of the atmosphere by averaging the signal from all of our detectors at each time sample (i.e., the average signal over the FOV). In S10, this template is subtracted from each detector’s timestream after weighting it by the relative gain of that detector, which is determined from the correlation coefficient between the timestream and the template. We use a single correlation coefficient for each detector for each 10 minute long observation. However, a significant fraction of the atmospheric noise at the fundamental scan frequencies remains in the data after application of the S10 algorithm, indicating that we have slightly misestimated the correlation coefficients. Therefore, we modified the S10 algorithm to compute the correlation coefficients for the template based only on the data within a narrow band centered on the two fundamental scan frequencies.
The atmospheric noise power in these narrow frequency bands is roughly an order of magnitude above the broadband atmospheric noise at nearby frequencies; consequently, the data in these narrow bands provide a high signal-to-noise estimate of each detector’s response to atmospheric signal. The narrowband atmospheric noise features are completely removed using this modified S10 algorithm, and the amount of residual broadband atmospheric noise is slightly reduced compared to the results from the original S10 algorithm.

After applying this average subtraction algorithm to the timestream data, we then high-pass filter the data according to

$$ f = 1 - \frac{1}{1 + (10^{-f_0 - 1})^\kappa} $$

with $f_0 = 250$ mHz and $\kappa = 8$. The value of $f_0$ was chosen to maximize the spatially extended signal-to-noise ratio (S/N) for the typical cluster in our sample based on tests with $f_0$ varying from 0 to 400 mHz, and the value of $\kappa$ was chosen to produce a sharp cutoff with minimal ringing. Figure 2 shows a typical pre- and post-subtraction timestream noise power spectral density (PSD).

3.3. Transfer Function of the Atmospheric Noise Filtering

In addition to subtracting atmospheric noise, the FOV-average subtraction and timestream high-pass filter also remove some cluster signal. Since we use the data timestreams to both determine the atmospheric fluctuation template and the correlation coefficient of each detector’s data timestream with the template, the FOV-average subtraction acts on the data in a nonlinear way. Consequently, its impact on the data depends on the cluster signal. As described below, we quantify the effects of the FOV-average subtraction and the timestream high-pass filter via simulation by processing a known cluster image through our data-reduction pipeline. These simulations are computationally intensive, and we find, in practice (see Section 6.1), that the filtering is only mildly dependent on the cluster signal. Thus, in the end, we determine the effects of the filtering for a particular cluster’s data set using the cluster model that best fits those data. We use the term transfer function to describe the effect of the filtering, although this terminology is not rigorously correct because the filtering depends on the cluster signal.

To compute the transfer function, we first insert a simulated, beam-smoothed cluster profile into our data timestreams by reverse mapping it using our pointing information. These data are then processed in an identical way to the original data, and an output image, or map, is produced. When processing the data-plus-simulated-cluster timestreams we use the FOV-average subtraction correlation coefficients that were determined for the original data. This ensures that the simulated cluster is processed in an identical way to the real cluster in our data. In the limit that the best-fit cluster model is an accurate description of the data, this process is rigorously correct. The original data map is then subtracted from this data-plus-simulated-cluster map to produce a noise-free image of the processed cluster. In Figure 3, we show an example cluster image, along with the noise-free processed image of the same cluster. The Fourier transform of this processed cluster image is divided by the Fourier transform of the input cluster to determine how the cluster is filtered as a function of two-dimensional Fourier mode (i.e., what we term the transfer function, see Figure 4).

At small angular scales, there is very little signal in the beam-smoothed input cluster, and numerical noise prevents us from accurately characterizing the transfer function at these scales. The transfer function is expected to be unity at small angular scales, and is asymptoting to this value at the larger scales where we can accurately characterize it. Therefore, we set the transfer function to a value of 1 for $u > 0.75$ arcmin$^{-1}$. Deconvolving

![Figure 2. Timestream noise power spectral density (PSD) for a typical Bolocam detector. The black curve shows the raw PSD recorded by the detector; spectral lines at the fundamental scan frequencies are clearly seen above the broadband atmospheric noise. The red curve shows the noise PSD after subtracting the atmospheric noise using the average signal over the FOV. This timestep is then high-pass filtered at 250 mHz to produce the green PSD. Note that there is very little cluster signal above 1 Hz, where there are some spectral lines due to the readout electronics. The dashed horizontal line provides an estimate of the photon, or BLIP, noise.](A color version of this figure is available in the online journal.)

![Figure 3. Images of the best-fit spherical Nagai model for MS 0451.6–0305. The left image is the model and the right image is the model after being processed through our data reduction pipeline, which high-pass filters the image in a complex way. This filtering significantly reduces the peak decrement of the cluster and creates a ring of positive flux at $r > 2$ arcmin. Note that the processed image is not quite azimuthally symmetric.](A color version of this figure is available in the online journal.)
the processed cluster image using the transfer function (see Section 5), which has been approximated as 1 at small angular scales, produces an image that is slightly biased compared to the input cluster. The residuals between these two images are approximately white, with an rms of $\lesssim 0.1 \mu K_{\text{CMB}}$. This transfer-function-induced bias is negligible compared to our noise, which has an rms of $\pm 10 \mu K_{\text{CMB}}$.

As noted above, the transfer function (weakly) depends on the profile of the cluster; larger clusters are more heavily filtered than smaller clusters. Therefore, we determine a unique transfer function for each cluster using the best-fit elliptical Nagai model for that cluster (Nagai et al. 2007, hereafter N07). The details of this fit are given in Section 4. Since the transfer function depends on the best-fit model, and vice versa, we determine the best-fit model and transfer function in an iterative way. Starting with a generic cluster profile, we first determine a transfer function, and then fit an elliptical Nagai model using this transfer function (i.e., the Nagai model parameters are varied while the transfer function is held fixed). This process is repeated, using the best-fit model from the previous iteration to calculate the transfer function, until the best-fit model parameters stabilize. This process converges fairly quickly, usually after a single iteration for the clusters in our sample. The model dependence of the resulting transfer function is quantified in Section 6.1.

3.4. Noise Estimation

In order to accurately characterize the sensitivity of our images, we compute our map-space noise directly from the data via 1000 jackknife realizations of our cluster images. In each realization, random subsets of half of the $\pm 100$ observations are multiplied by $-1$ prior to adding them into the map. Each jackknife preserves the noise properties of the map while removing all of the astronomical signal, along with any possible fixed-pattern or scan-synchronous noise due to the telescope scanning motion. Since these jackknife realizations remove all astronomical signal, we estimate the amount of astronomical noise in our images separately, as described below. After normalizing the noise estimate of each map pixel in each jackknife by the square root of the integration time in that pixel, we construct a sensitivity histogram, in $\mu K_{\text{CMB}} s^{1/2}$, from the ensemble of map pixels in all 1000 jackknives. The width of this histogram provides an accurate estimate of our map-space sensitivity (see Figure 5). We then assume that the noise covariance matrix is diagonal and divide by the square root of the integration time in each map pixel to determine the noise r.m.s in that pixel. This method is analogous to the one used in S09, where it is described in more detail.

There is a non-negligible amount of noise in our maps from two main types of astronomical sources: anisotropies in the CMB and unresolved point sources. The SPT has recently published power spectra for both of these sources at 150 GHz over the range of angular scales probed by our maps, so we use their measurements to estimate the astronomical noise in our maps (Lueker et al. 2010; Hall et al. 2010). Note that these measurements inherently include all astrophysical effects, such as lensing and clustering of point sources. Using the SPT power spectra, we generate simulated maps of our cluster fields assuming Gaussian fluctuations. Note that this is a poor assumption for both the SZ-sourced CMB fluctuations and the signal sourced by background galaxies that are lensed by the cluster. However, since the noise power from both of these sources is quasi-negligible compared to the total noise in our images, any failure of our Gaussian assumption will have a minimal impact on our results. The good match of the SDS1 data to our noise model validates our Gaussian assumption (see Figure 5). These simulated images of the CMB plus point sources are then reverse mapped into our timestream data and processed to estimate how they will appear in our cluster maps. We then add these processed astronomical realizations to our jackknife realizations to provide a complete estimate of the noise in our images. Sensitivity estimates from our signal-free map of SDS1 agree well with the sensitivity estimates from our noise maps, indicating that there are no additional noise sources that have not been included in our estimate (see Figure 5). The properties of this noise are described in more detail in Section 6.2. In general, the noise from astronomical sources is quasi-negligible compared to the other noise in our images. However, the large-scale correlations from CMB

3. We show that there is no measurable fixed-pattern or scan-synchronous noise in our data later in this section and in Section 6.2.
fluctuations are non-negligible in our deconvolved images (see Table 5).

Note that we have chosen to model the signal from point sources as an additional noise term in our maps rather than attempting to subtract any individual point sources. Part of the motivation for this approach is the fact that we do not detect any point sources in our cluster or blank-field images. For reference, P10 mapped 15 clusters to a similar depth using the SPT and only detected two point sources; the combined area of the SPT images is approximately 20 times the combined area of our six images. Although there are several known submillimeter galaxies and active galactic nuclei in our images (Zemcov et al. 2007; Ivison et al. 2000; Cooray et al. 1998), even the brightest sources will have a flux of \(\lesssim 10 \mu K_{\text{CMB}}\) in our observing band, rendering them undetectable given our noise rms of \(\lesssim 10 \mu K_{\text{CMB}}\). Additionally, it is not possible to reliably estimate the flux of these sources in our observing band since most have only been detected at one or two wavelengths, generally separated from our observing band by more than a factor of two in wavelength.

4. CLUSTER MODEL FITTING

4.1. The SZ Effect

As mentioned above, the SZ effect involves CMB photons inverse Compton scattering off of hot electrons in the ICM. Since the electrons are many orders of magnitude hotter than the CMB photons, there is, on average, a net increase in the energy of the photons. The classical distortion relative to the blackbody spectrum of the CMB is given by

\[
f(x) = \frac{e^x + 1}{e^x - 1} - 4,
\]

where \(x = h\nu/k_BT_{\text{CMB}}\). The magnitude of the distortion is proportional to the product of the density and temperature of the electrons in the ICM projected along the line of sight. The frequency-dependent temperature change of the CMB is given by

\[
T_{\text{SZ}} = f(x)yT_{\text{CMB}},
\]

where

\[
y = \int n_e\sigma_T\frac{k_BT_e}{m_ec^2},
\]

and \(n_e\) and \(T_e\) are the density and temperature of the ICM electrons. Relativistic corrections can be included by multiplying \(f(x)\) by \((1 + \delta(x, T))\) (Itoh et al. 1998). Note that the X-ray brightness of the ICM is proportional to \(n_e^2T_e^{1/2}\); the differing density and temperature dependence of the SZ and X-ray signals makes them highly complementary probes of the ICM (e.g., Bonamente et al. 2006, hereafter B06).

4.2. Models

We have fit our data to two types of models: an isothermal beta model (Cavaliere & Fusco-Femiano 1976, 1978) and the pressure profile proposed by N07, hereafter the Nagai profile. The isothermal beta model has been used extensively to describe X-ray and SZ measurements of the ICM, and our beta model fit parameters can be directly compared to these previous results. However, the beta model provides a poor description of deep X-ray data, which generally have a cuspy core and steeper outer profile (e.g., Vikhlinin et al. 2006, N07, and Arnaud et al. 2010). In contrast, the Nagai pressure profile, which is a generalization of the Navarro–Frenk–White dark matter profile, is able to describe deep X-ray observations over a wide range of scales (N07; Arnaud et al. 2010). Therefore, we have also fit our data to the Nagai model. In practice, the beta and Nagai models are highly degenerate over the range of angular scales to which our data are sensitive (1–12 arcmin).

Specifically, the isothermal beta model is described by

\[
p = \frac{p_0}{(1 + r^2/r_c^2)^{\beta/2}},
\]

where \(p\) is the pressure profile, \(p_0\) is the pressure normalization, \(r_c\) is the core radius, and \(\beta\) is the power-law slope. The beta model can be analytically integrated along the line of sight to give the observed SZ signal, with

\[
T_{\text{SZ}} = \frac{f(x)y_0T_{\text{CMB}}}{(1 + r^2/r_c^2)^{(3\beta-1)/2}} + \delta T,
\]

where \(T_{\text{SZ}}\) is the SZ signal in our map (in \(\mu K_{\text{CMB}}\)) and \(y_0\) is the central Comptonization. We fix the value of \(\beta\) at 0.86 for all of our fits; this is the best-fit value found in P10 for SZ data.\(^5\) Since our data timestreams are high-pass filtered, we are not sensitive to the DC signal level in our images. Note that because the timestream data, rather than the map, are high-pass filtered, the DC signal level of the map is in general not equal to 0. However, the DC signal of the map is not physically meaningful due to the filtering and must therefore be included as a free parameter, \(\delta T\), in the model fits. We have also generalized the beta model to be elliptical in the plane of the sky with

\[
T_{\text{SZ}} = \frac{f(x)y_0T_{\text{CMB}}}{(1 + r_1^2/r_c^2 + r_2^2/(1-\epsilon^2r_2^2))^{(3\beta-1)/2}} + \delta T,
\]

where \(r_1\) is oriented along the major axis, described by a position angle of \(\theta\) (in degrees east of north), \(r_2\) is orthogonal to the major axis, and \(\epsilon\) is the ellipticity.

The Nagai model is described by

\[
p = \frac{p_0}{(r/r_s)^\delta[1 + (r/r_s)^\gamma][\delta - \gamma]/\delta},
\]

where \(p\) is the pressure, \(p_0\) is the pressure normalization, \(r_s\) is the scale radius, \(\delta\) is a scaling factor, and \(\gamma\) is a damping factor. The Nagai model is not analytically integrable, so we numerically integrate \(p\) along the line of sight to determine \(T_{\text{SZ}}\). We have also generalized the Nagai model to be elliptical in the plane of the sky, using the same notation as our elliptical generalization of the beta model. Note that, in all of our fits, we have corrected for relativistic effects via the approximations given in Itoh et al. (1998) using gas temperature estimates from X-ray observations of these clusters (Cavagnolo et al. 2008; Jeltema et al. 2001). The relativistic corrections are \(\lesssim 5\%\) for the typical temperatures in these massive clusters (\(\gtrsim 10\) keV).

\(^5\) As P10 point out, this value for \(\beta\) is larger than those generally found from X-ray data, in agreement with simulations (Hallman et al. 2007). Additionally, it suggests that the ICM temperature is falling with increasing radius, in agreement with simulations and data (e.g., N07).
ellipticity of the five clusters is \( \nu \) if the clusters were spherical. If we neglect A697, which is not obtain data with equal or greater preference for elliptical models. Elliptical beta model

| Cluster      | \( p_0 \) (10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1} \) | \( r_e \) (arcmin) | \( c_{500} \) | \( \epsilon \) | \( \theta \) (deg) | \( \chi^2/\text{dof} \) | PTE\( \chi^2 \) | PTE\text{Norm} |
|--------------|-----------------|-----------------|------|-------|--------------|---------------|----------------|----------------|
| A697         | 9.3 ± 1.3       | 6.9 ± 1.0       | 0.93 ± 0.14 | 0.37 ± 0.05 | -24 ± 4      | 1289/1117     | 0.00           | 0.00           |
| A1835        | 8.1 ± 1.7       | 6.7 ± 1.5       | 0.94 ± 0.21 | 0.27 ± 0.07 | -16 ± 10     | 966/945       | 0.31           | 0.22           |
| MS 0015.9+1609 | 6.7 ± 1.4       | 5.5 ± 1.1       | 0.62 ± 0.13 | 0.24 ± 0.08 | 68 ± 12      | 1079/1117     | 0.79           | 0.73           |
| MS 0451.6–0305 | 8.7 ± 1.6       | 4.7 ± 0.7       | 0.80 ± 0.14 | 0.26 ± 0.06 | 85 ± 7       | 1188/1117     | 0.07           | 0.08           |
| MS 1054.4–0321 | 6.5 ± 1.9       | 3.6 ± 1.0       | 0.64 ± 0.19 | 0.09 ± 0.07 | -1 ± 32      | 1084/1117     | 0.75           | 0.72           |

| Spherical Nagai model                  |                      |                 |      |       |              |               |                |                |
| A697         | 11.0 ± 1.8       | 4.6 ± 0.8       | 1.40 ± 0.22 | ...   | ...        | 1399/1119     | 0.00           | 0.00           |
| A1835        | 8.9 ± 2.1       | 5.1 ± 1.2       | 1.24 ± 0.27 | ...   | ...        | 1007/947      | 0.08           | 0.07           |
| MS 0015.9+1609 | 6.0 ± 1.1       | 5.4 ± 1.1       | 0.63 ± 0.12 | ...   | ...        | 1100/1119     | 0.66           | 0.61           |
| MS 0451.6–0305 | 10.6 ± 2.1      | 3.5 ± 0.5       | 1.08 ± 0.15 | ...   | ...        | 1220/1119     | 0.02           | 0.02           |
| MS 1054.4–0321 | 6.0 ± 1.8       | 3.6 ± 1.0       | 0.64 ± 0.16 | ...   | ...        | 1087/1119     | 0.75           | 0.75           |

Notes: Best-fit parameters and 1\( \sigma \) uncertainties for our Nagai model fits with the power-law exponents fixed to the best-fit values found in Arnaud et al. (2010; 105, 549, 031). From left to right the columns give the normalization of the Nagai profile, \( p_0 \), the scale radius of the major axis, \( r_e \), the concentration parameter, \( c_{500} \), the ellipticity, \( \epsilon \), the position angle of the major axis in degrees east of north, \( \theta \), the \( \chi^2 \) and dof for the fit, and the goodness of fit quantified by the probability to exceed the given \( \chi^2/\text{dof} \) based on the standard \( \chi^2 \) probability distribution function and also empirically by the fraction of our 1000 noise realizations that produce a larger \( \chi^2 \) value when a model cluster is added to them (see Section 6.2). The values of \( c_{500} \) for the spherical fits can be compared to the nominal value of 1.17 that is given in Arnaud et al. (2010) based on spherical fits of X-ray data. See Section 6.2 for a description of how the parameter uncertainties are calculated. Note that we have included the 4.3% uncertainty in our flux calibration in the error estimates for \( p_0 \).
using the cluster model, computed directly from the isothermal beta model and via a line-of-sight integration of the Nagai pressure model. Next, the model image is convolved with the Bolocam PSF and the measured transfer function. In practice, the transfer function convolution is performed via multiplication in Fourier space. This filtered model is then compared to our data map, using all of the map pixels contained within a radius where the minimum coverage is greater than 25% of the peak coverage. This radius is typically between 6 and 7 arcmin. We use an iterative least-squares technique to determine the best-fit parameters for each model; we estimate the uncertainty on each parameter via the standard deviation of the best-fit parameter values estimated from noise realizations with model clusters added to them (see Section 6.2).

For each model, we fit both a scale radius \( r_c \) for the beta model and \( r_s \) for the Nagai model) and a normalization \( y_0 \) for the beta model and \( p_0 \) for the Nagai model). Additionally, as described above, we fit for the observationally unconstrained DC signal level of the map, \( \delta T \). We also fit for a centroid offset relative to the X-ray pointing center. The offsets for the five clusters range from \( \mp (0\pm20) \pm 5 \) arcsec, indicating there are no major differences in the X-ray and SZ centroids. Finally, when we allow the model to be elliptical in the image plane, we fit for the ellipticity \( \epsilon \) and position angle \( \theta \). A complete list of the best-fit parameters for spherical and elliptical versions of both models is given in Tables 2 and 3.

### 4.4. Discussion and Comparison to Previous Results

All of these clusters have been studied extensively at a wide range of wavelengths, providing us with a large number of published results to compare our model fit parameters. In general, our results agree well with those found from previous studies. In particular, when we fit a spherical beta model to our data using the values of \( \beta \) found by B06, which were determined using Chandra X-ray data and 30 GHz interferometric SZ data, our best-fit values for \( y_0 \) agree quite well, with the exception of A697 (see Table 4). The best-fit values we find for \( r_c \) are also consistent with those determined in B06, with the exception of A1835 and MS 0451.6–0305. Additionally, the ellipticity and orientation of our elliptical fits are in general consistent with previously published results (Maughan et al. 2008; Donahue et al. 2003; De Filippis et al. 2005; Piffaretti et al. 2003; Girardi et al. 2006; McNamara et al. 2006; Schmidt et al. 2001; Neumann & Arnaud 2000). Finally, the best-fit concentration parameters from our Nagai model fits to three of the five clusters are consistent with those found from X-ray data (Arnaud et al. 2010; MS 0015.9+1609 and MS 1054.4–0321 have significantly lower values of \( c_{500} \). We describe each cluster in detail below.

**A697.** We find that A697 has a significant ellipticity, and it is not described well by either a beta model or a Nagai model. The poor model fits result from the cluster appearing significantly extended in the SW direction, and extremely compact in the NE direction. A697 is the only cluster in our sample that is not adequately described by the models. The ellipticity we find for A697, \( \epsilon = 0.37 \pm 0.05 \), is roughly consistent with the ellipticity of \( \pm 0.25 \) found from X-ray data (De Filippis et al. 2005; Maughan et al. 2008; Girardi et al. 2006). We find a position angle of \( -24 \) deg for A697, which is similar to the value of \( -16 \) deg found by Girardi et al. (2006), but somewhat misaligned to the position angle of \( 16 \) deg found by De Filippis et al. (2005).

**A1835.** We detect an ellipticity in our image of A1835, and we find that it is described well by either an elliptical beta or Nagai model. Our best-fit spherical Nagai model parameters with \( A = 0.9 \), \( B = 5.0 \), and \( C = 0.4 \) \((p_0 = 11.1 \pm 2.6 \times 10^{-11} \text{erg cm}^{-3} \), \( r_c = 4.6 \pm 1.1 \text{arcmin} \)) are consistent with the values found in Mruczewska et al. (2009); \( p_0 = 13.6 \times 10^{-11} \text{erg cm}^{-3} \), \( r_c = 4.3 \text{arcmin} \) using a combination of SZA SZ data and Chandra X-ray data. X-ray measurements find an ellipticity of 0.1–0.2 for A1835 (De Filippis et al. 2005; McNamara et al. 2006; Schmidt et al. 2001), consistent with the value of \( \epsilon = 0.27 \pm 0.07 \) we find with Bolocam. We find a position angle of \( -16 \) deg for A697, which is similar to the values of \( 7 \) and \( -20 \) deg found by De Filippis et al. (2005), McNamara et al. (2006), and Schmidt et al. (2001).

**MS 0015.9+1609.** MS 0015.9+1609 appears to be elliptical in our image, and it is described well by either a spherical or elliptical model. As with A697 and A1835, X-ray data favor slightly lower ellipticities, \( \epsilon \lesssim 0.20 \), compared to what we find with Bolocam, \( \epsilon = 0.24 \pm 0.08 \) (De Filippis et al. 2005; Maughan et al. 2008; Piffaretti et al. 2003). We find a position angle of \( 68 \) deg for MS 0015.9+1609, fairly close to the value of \( 47 \) deg found by Piffaretti et al. (2003), but almost orthogonal to the value of \( -49 \) deg found by De Filippis et al. (2005).

**MS 0451.6–0305.** MS 0451.6–0305 also appears to be elliptical in our image and is adequately described by either
an elliptical beta or Nagai model. We find an ellipticity of \( \epsilon = 0.26 \pm 0.06 \) for MS 0451.6–0305, in excellent agreement with ellipticities determined using X-ray data (De Filippis et al. 2005; Donahue et al. 2003). Additionally, our best-fit position angle of 85 deg agrees well with the value of 84 deg found by De Filippis et al. (2005) and the value of −75 deg found by Donahue et al. (2003).

**MS 1054.4–0321.** Our image of MS 1054.4–0321 shows no evidence for ellipticity, and it is described well by either an elliptical or spherical model. Although MS 1054.4–321 does not appear to be elliptical in our data, X-ray data show a clear ellipticity oriented along the east–west direction (Neumann & Arnaud 2000; Jeltema et al. 2001). However, our non-detection of an ellipticity, \( \epsilon = 0.09 \pm 0.07 \), is only marginally inconsistent with the X-ray value determined by Neumann & Arnaud (2000), \( \epsilon = 0.29 \).

**SDS1.** We have attempted to fit cluster models to the SDS1 map, but we find best-fit amplitudes that are consistent with 0 and best-fit scale radii that are large compared to the size of our images (i.e., scale radii that are large enough to produce profiles that are approximately constant over the entire image).

### 5. MODEL-INDEPENDENT IMAGES AND \( Y_{500} \) ESTIMATES

Rather than using models, which at best provide an adequate description of what clusters look like on average, we have chosen to derive observable quantities from our images in a quasi-model-independent way. In Section 3.3, we described how we calculate a unique transfer function for each cluster to quantify the effects of our noise filtering. In the Fourier space of our images, these transfer functions are a set of two-dimensional complex numbers that describe how an input cluster image is filtered as a function of two-dimensional Fourier mode. We obtain an unfiltered, or deconvolved, image of the cluster by Fourier transforming our image, dividing by the two-dimensional complex transfer function, and then Fourier transforming the result back to image space.6 At the largest scales in our map, the transfer function has a magnitude of ≃0.2 (see Figure 3), resulting in a numerically stable, but significant, amplification of the large scale noise. In particular, the residual atmospheric noise and primary CMB fluctuations, which had been filtered to be approximately white, produce a significant low-frequency noise component in our deconvolved images. We estimate the noise in our deconvolved images by deconvolving each of the 1000 noise realizations for each cluster. Since the off-diagonal elements of the noise covariance matrix are significant due to the low-frequency noise, we estimate all of our measurement uncertainties from the standard deviation of measuring the same quantity in each of our 1000 deconvolved noise realizations (see below and Section 6.2). Note that most of the measurement uncertainties in our processed (i.e., filtered) images are computed in the same way, even though their noise covariance matrix is approximately diagonal.

By deconvolving the two-dimensional transfer function of our data processing we obtain unbiased cluster images, modulo smoothing with our PSF and the unconstrained DC signal level. Although the transfer function was computed using the best-fit elliptical Nagai model, and is therefore somewhat model dependent, the dependence is negligible other than the determination of the DC signal offset of the image (see Section 6.1). We do not attempt to recover the information lost due to smoothing by our PSF, but we use the value of \( \delta f \) found from our elliptical Nagai model fits to restore the correct DC signal level to our images. Due to uncertainties in the model itself, especially at large radii where there is little or no observational data, along with cluster-to-cluster deviations from the model, this does introduce a non-negligible model-based bias in our images (the typical model uncertainty in the DC signal offset is 5–10 \( \mu K_{\text{CMB}} \), see Section 6.1). However, we emphasize that this bias only affects the DC signal level of the images; the shapes of the cluster profiles are essentially model-independent.

We have computed a model-independent value for \( Y_{500} \) from these deconvolved images, which is the integrated \( Y \) within \( r_{500} \) (i.e., the cylindrical \( Y_{500} \) rather than the spherical \( Y_{500} \), see Table 5). As mentioned above, since our assumption that the noise covariance matrix is diagonal fails for the deconvolved images, we estimate the uncertainty in our estimate of \( Y_{500} \) via the scatter among the \( Y_{500} \) values determined from each of our noise realizations. As an example of the amount of low-frequency noise present in our deconvolved images on the scale of \( r_{500} \), note that the measurement uncertainty on \( Y_{500} \) is approximately 10 times larger than it would be if the noise was white. For the five clusters in our sample, we are able to determine \( Y_{500} \).

### Table 5

| Cluster             | \( Y_{500} \)       | JK Noise | CMB/PS | Map DC signal | Flux Cal. |
|---------------------|---------------------|----------|--------|---------------|-----------|
| A697                | 59.2 ± 6.3          | 1.2      | 0.5    | 5.6           | 2.5       |
| A1835               | 47.5 ± 4.9          | 1.3      | 0.5    | 4.3           | 2.0       |
| MS 0015.9+1609      | 23.3 ± 2.2          | 1.6      | 0.6    | 0.9           | 1.0       |
| MS 0451.6–0305      | 24.7 ± 2.0          | 1.2      | 0.6    | 0.9           | 1.1       |
| MS 1054.4–0321      | 8.8 ± 1.0           | 0.8      | 0.4    | 0.2           | 0.4       |

**Notes.** Model-independent estimates of \( Y_{500} \), in units of \( 10^{-11} \) ster, along with the associated uncertainties. From left to right the columns give the value and total uncertainty of our estimate of \( Y_{500} \), the uncertainty due to our jackknife noise model (i.e., excluding CMB and point sources), the uncertainty due to CMB and point sources, the uncertainty in the DC signal level of the map (see Section 6.1), and the uncertainty in our flux calibration. Note that the jackknife noise estimate is dominated by large angular-scale residual atmospheric fluctuations; the noise on beam-size scales is negligible.

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6 This method can be compared to the deconvolution method employed by APEX-SZ for their analysis of A2163 and A2204 (Nord et al. 2009; Basu et al. 2010). They first determine what a point-like object looks like in their image after being filtered. Next, they fit this filtered point-source image to the map pixel with the largest S/N and subtract it from the image. The process is repeated until the map is consistent with noise; the sum of all the unfiltered point-like images removed from the map gives the deconvolved cluster image.
where $D_A$ is the angular diameter distance, $E(z) = \sqrt{\Omega_m(1 + z)^3 + \Omega_k}$, and we have assumed $M_{\text{gas}}$ is a good proxy for the total cluster mass (Allen et al. 2008; Mantz et al. 2010b). The scatter in this relation is expected to be $\lesssim 10\%$ (Kravtsov et al. 2006), and current measurements are roughly consistent with this prediction (Morandi et al. 2007; Bonamente et al. 2008; Marrone et al. 2009; Plagge et al. 2010; Huang et al. 2010; Andersson et al. 2010). Given our small sample, we do not attempt to constrain the intrinsic scatter in the $Y - M$ relation, but we follow the formalism of Marrone et al. (2009) and P10 to fit a logarithmic scaling of the form $Y = a + b M$ with the intrinsic scatter set to $10\%$. For the $Y_{500}-M_{\text{gas},500}$ relation we find best-fit values of $a = -5.46 \pm 0.84$ and $b = 1.63 \pm 0.71$, consistent with the self-similar prediction of $b = 5/3$. The overall scatter of our data about the fit is $13\%$, consistent with an intrinsic scatter of $\lesssim 10\%$ given our $\pm 10\%$ uncertainty on $Y_{500}$.

(A color version of this figure is available in the online journal.)

with an uncertainty of $\pm 10\%$, limited mainly by systematics in determining the DC signal offset and flux calibration.

We examine the self-similar scaling that is expected between $Y_{\text{SZ}}$ and the total cluster mass using the relation given in Bonamente et al. (2008),

$$Y_{\text{SZ}} D_A^2 E(z)^{-2/3} \propto f_{\text{gas}}^{-2/3} M_{\text{gas}}^{5/3},$$

Figure 6. Scaling relation between $Y_{\text{SZ}}$ and $M_{\text{gas}}$ for the five clusters in our sample. The black squares give the Bolocam model-independent cylindrical $Y_{500}$ estimates and Chandra $M_{\text{gas},500}$ estimates (see Table 1), and the black dashed line represents the best fit to these data. We also show fits of a similar type obtained by various authors—red dashed: SPT beta-model-derived $Y_{500}$ vs. $M_{\text{gas},500}$, primarily obtained from XMM-Newton data (P10); solid blue: same, but within $r_{2500}$; dot-dashed green: OVRO/BIMA beta-model-derived $Y_{500}$ vs. $M_{\text{gas},2500}$ obtained from Chandra data (Bonamente et al. 2008). The difference between the red-dashed line and our data is likely caused by systematic differences between model-derived and model-independent estimates of $Y_{500}$. The scatter of our data relative to our best fit is $13\%$, consistent with the expected intrinsic scatter of $\lesssim 10\%$ given our $\pm 10\%$ uncertainty on $Y_{500}$.

Figure 7. Histograms of the $\chi^2$ value for 1000 separate noise realizations for MS 0451.6–0305, overlaid in green with the predicted distribution assuming the noise covariance matrix is diagonal (i.e., there is no correlated noise). For each noise realization the best-fit elliptical Nagai model profile for MS 0451.6–0305 is added to the noise realization, an elliptical Nagai model fit is to this model-cluster-plus-noise realization, and the value of $\chi^2$ is computed based on the assumption that the noise covariance matrix is diagonal. The vertical red line shows the value of $\chi^2$ for the actual data for MS 0451.6–0305. The left histogram shows the processed data and the right histogram shows the data for the deconvolved image. The predicted and actual $\chi^2$ distributions for the processed data overlap, indicating that there are minimal correlations between map pixels. However, the actual $\chi^2$ distribution for the deconvolved image data is much broader than the predicted distribution, indicating that there are significant noise correlations between map pixels in the deconvolved images.

(A color version of this figure is available in the online journal.)
6.1. Model Dependence of Results

Since we compute the transfer function for each cluster using the best-fit elliptical Nagai model for that cluster, our deconvolved images necessarily have some model dependence. In order to quantify the amount of model dependence, we have computed transfer functions for a range of elliptical Nagai models for one of the clusters in our sample, MS 0451.6−0305. Relative to the best-fit model, we have varied the scale radius, $r_s$, the ellipticity, $\epsilon$, the position angle, $\theta$, and the centroid location, $\delta$R.A. and $\delta$decl., by increasing and decreasing each parameter individually by its $1\sigma$ uncertainty.\(^8\) We then deconvolved our processed map of MS 0451.6−0305 using the transfer function computed from each model and subtracted the resulting map from the one produced using the transfer function for the best-fit model. In each case, the residual map was approximately white, with an rms of 1.5, 0.6, and 0.5 $\mu K_{CMB}$ for variations in $r_s$ and $\epsilon$, $\theta$, and $\delta$R.A. and $\delta$decl., respectively. Since the typical noise rms of our deconvolved maps is $\simeq 10$ $\mu K_{CMB}$, the additional rms introduced by our uncertainty in determining the model used for calculating a transfer function is quasi-negligible. Note that the best-fit elliptical Nagai model will not provide an exact description of a real cluster. However, the elliptical Nagai model does provide an adequate description of four of the five clusters we have observed, indicating that the difference between the true cluster profile and the model profile is in general less than our noise. Therefore, the artifacts in our deconvolved map produced by using a model to describe the cluster will be smaller than the artifacts produced by our measurement uncertainty on the best-fit model.

Additionally, we created a deconvolved map of MS 0451.6−0305 using the transfer function for a point-like source. The resulting profile is significantly different from the profile obtained using the transfer function for the best-fit Nagai model, indicating that the naive calculation of a transfer function using a

\(^8\) The power-law slopes ($A$, $B$, and $C$) were held fixed for all of our model fits. Due to the large degeneracy between these values and $r_s$, we have effectively included variations in the power-law slopes by varying the value of $r_s$. 

Figure A1. A697; from left to right and top to bottom we show the deconvolved image of the cluster, the processed image of the cluster, the residual map between the processed image of the cluster and the best-fit elliptical Nagai model, one of the 1000 noise realizations for the processed data, and a binned radial profile. The contour lines represent an S/N of 2, 4, ...
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Figure A2. A1835; from left to right and top to bottom we show the deconvolved image of the cluster, the processed image of the cluster, the residual map between the processed image of the cluster and the best-fit elliptical Nagai model, one of the 1000 noise realizations for the processed data, and a binned radial profile. The contour lines represent an S/N of 2, 4, . . .

(A color version of this figure is available in the online journal.)

point source is inadequate. Compared to using the transfer function calculated from the best-fit elliptical Nagai model, the peak decrement is reduced by ≃50 μK_{CMB}, while the magnitude of the SZ signal at the edge of the map is increased by ≃50 μK_{CMB} (i.e., the deconvolved cluster image from a point-source transfer function is systematically broader).

As described in Section 4, we use the best-fit elliptical Nagai model to determine the DC signal level in our maps since its value is unconstrained by our data. The measurement uncertainty in the value of the DC signal is small, <1 μK_{CMB}, but there is a large amount of uncertainty in the model at large radii. Based on the results from Borgani et al. (2004), N07, and Piffaretti & Valdarnini (2008) presented in Arnaud et al. (2010), the rms scatter in the pressure profiles from cluster to cluster in simulations is ≲25%. Therefore, we include an additional 25% systematic uncertainty on the DC signal that we add to the deconvolved map. Specifically, we estimate that the model uncertainty in the DC signal level of our map is 25% of the signal level at the edge of the map.

6.2. Noise Characteristics

As mentioned in Section 3, we make the approximation that the noise covariance matrix is diagonal in our processed maps (i.e., there are no noise correlations between pixels). Although fluctuations in both the atmospheric emission and the CMB are correlated over many pixels, the high-pass filter we apply to our data timestreams eliminates these correlations within our ability to measure them. To test for noise correlations, we added processed cluster images of the best-fit models to the 1000 noise realizations for each cluster (i.e., for a given cluster we separately added each of the four best-fit cluster models from Tables 2 and 3 to each of the 1000 noise realizations). We then fit a model of the same type (e.g., if we added the best-fit elliptical Nagai model to the noise realization, then
we fit an elliptical Nagai model to the cluster-model-plus-noise map to each of these model-cluster-plus-noise maps and examined the distribution of $\chi^2$ values for these fits. For all five clusters, we found that the distribution of $\chi^2$ values matched the predicted $\chi^2$ distribution obtained from the diagonal noise covariance matrix. As an example, the fit quality of our measured $\chi^2$ distribution for the elliptical-Nagai-model-plus-noise realizations to the predicted $\chi^2$ distribution, quantified by a probability to exceed (PTE), is 0.58, 0.02, 0.08, 0.66, and 0.21 for A697, A1835, MS 0015.9+1609, MS 0451.6−0305, and MS 1054.4+321 (see Figure 7). Therefore, we conclude that there is a negligible amount of correlated noise in our processed images and our assumption that the noise covariance matrix is diagonal is valid. Furthermore, we have estimated all of our parameter uncertainties ($p_0$, $r_s$, etc.) directly from the distribution of values calculated from our noise realizations. Consequently, any failure of our assumption that the covariance matrix is diagonal for the processed images will only affect the pixel-weighting and $\chi^2$ values for our model fits.

However, when we perform the same test of fitting a model to our model-cluster-plus-noise maps using noise realizations that have been deconvolved with our transfer function, the result is significantly different. The distribution of $\chi^2$ values calculated from our deconvolved noise realizations is significantly broader than the predicted distribution based on uncorrelated noise (see Figure 7). This result is not surprising since the deconvolution enhances the large-scale signals in the images, including residual atmospheric noise and CMB fluctuations. Since the noise in the deconvolved images is significantly spatially correlated, our assumption that the noise covariance matrix is diagonal fails. Therefore, we estimate the uncertainties for the deconvolved images using the spread in values for the noise realizations rather

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Figure A3. MS 0015.9+1609; from left to right and top to bottom we show the deconvolved image of the cluster, the processed image of the cluster, the residual map between the processed image of the cluster and the best-fit elliptical Nagai model, one of the 1000 noise realizations for the processed data, and a binned radial profile. The contour lines represent an S/N of 2, 4... (A color version of this figure is available in the online journal.)

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9 We have fit for $N_{\text{params}}$ free parameters in our fits to both the actual data and the noise realizations, with $N_{\text{params}} = 5$ or 7 for the spherical or elliptical fits. Therefore, the predicted $\chi^2$ distribution is for $N_{\text{pix}} - N_{\text{params}}$ dof, where $N_{\text{pix}}$ is the number of map pixels.
Figure A4. MS 0451.6−0305; from left to right and top to bottom we show the deconvolved image of the cluster, the processed image of the cluster, the residual map between the processed image of the cluster and the best-fit elliptical Nagai model, one of the 1000 noise realizations for the processed data, and a binned radial profile. The contour lines represent an $S/N$ of 2, 4, ..., (A color version of this figure is available in the online journal.)

than from the diagonal elements of the noise covariance matrix (e.g., the uncertainties in the radial profiles are determined from the rms spread in the radial profiles of the noise realizations). This is the same technique used by Nord et al. (2009) and Basu et al. (2010) to analyze APEX-SZ data.

The model fits to the model-cluster-plus-noise realizations also provide us with estimates of the uncertainties and biases associated with our model-parameter fitting. Specifically, we obtain 1000 best-fit values for each parameter; the standard deviation of these values then gives the uncertainty on our estimate of that parameter in our actual data map. These uncertainties are given in Tables 2 and 3. In addition, if our parameter estimation algorithm is free from biases, then we should, on average, recover the parameters of the input model that was added to the noise realizations. In practice, we find a small, but measurable, bias in our estimates of the pressure normalization and scale radius in our fits; the bias is typically $\lesssim 10\%$ of the uncertainty on each parameter. We find no measurable bias in our estimates of the other fit parameters.

Additionally, we have used our signal-free SDS1 maps to further verify our model-fitting procedure and to search for any components of the noise that have not been included in our noise estimate. First, we inserted model clusters into the SDS1 data timestreams based on the best-fit elliptical Nagai profile for each of the five clusters in our sample. These data were then processed and an elliptical Nagai model was fit to each resulting image. In each fit, there are 6 free parameters ($p_0$, $r_s$, $\epsilon$, $\theta$, $\delta$RA, and $\delta$dec), giving us a total of 30 fit parameters for the 5 model clusters. Of these 30 fit parameters, 17 (57%) are within $1\sigma$ of the input value, 26 (87%) are within $2\sigma$ of the input value, and all 30 are within $3\sigma$ of the input value; these results indicate that our model fitting and parameter error estimation are working properly. Additionally, we obtain a reasonable goodness of fit for the models using these best-fit parameters, quantified by a PTE $\simeq 0.8$, providing further evidence that there is no significant noise in the data that has not been included in our noise estimate. Note that since the five cluster model profiles are fairly similar, we obtain comparable PTEs for all five profiles.
7. SUMMARY

We have presented the first results from our program to image the SZ effect in galaxy clusters with Bolocam. These images have a beam-smoothed rms of $\approx 10 \mu K_{\text{CMB}}$ and a resolution of 58 arcsec. Given this noise level, we are able to measure SZ signal in radial profiles to approximately the edge of our maps, which corresponds to 6–7 arcmin or 1–2 times $r_{500}$. In order to subtract noise from atmospheric fluctuations, we effectively high-pass filter our cluster images. However, we are able to deconvolve the effects of this filter with biases that are negligible compared to our noise level, other than our recovery of the DC signal level. In fitting our images to spherical and elliptical beta and Nagai models, we find no preference between the beta and Nagai models due to the degeneracy between these models over the angular range to which our data are sensitive, but our data do show a definitive preference for elliptical models over spherical models. The weighted mean ellipticity of the five clusters is $\epsilon = 0.27 \pm 0.03$, consistent with results from X-ray data. Additionally, the best-fit model parameters we determine from our data are consistent with those found from previous X-ray and SZ measurements. We have also obtained model-independent estimates of $Y_{\text{SZ}}$, and we find scaling relations between $Y_{\text{SZ}}$ and cluster mass that are consistent with self-similar predictions, with a scatter that is consistent with expectations for a $\approx 10\%$ intrinsic scatter.

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Figure A6. SDS1: from left to right and top to bottom we show the processed image of the field, one of the 1000 noise realizations for the processed data, and a binned radial profile. The contour lines represent an S/N of 2, 4, ... The thin gray lines show the radial profiles for each of the 1000 noise realizations. (A color version of this figure is available in the online journal.)

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Facilities: CSO

APPENDIX

This appendix includes images and radial profiles of the processed and deconvolved maps, an image of the processed residual map after subtracting the best-fit elliptical Nagai model, and an image of one of the 1000 noise estimates generated via jackknife realizations of the data and a model for the astronomical noise (see Figures A1–A6). We have smoothed all of the images using a Gaussian beam with an FWHM of 58 arcsec. A white dot representing the FWHM of the effective PSF for these beam-smoothed images is given in the lower left of each image. The solid white contour lines in the images represent an S/N of −2, −4, ..., and the dashed white contour lines represent an S/N of +2, +4, ... The deconvolved images contain a significant amount of noise that is correlated over large angular scales, along with a model-dependent DC signal offset, and we therefore do not display noise contours on the deconvolved images. The error bars on the radial profiles are estimated from the spread in radial profiles computed from our noise realizations, and therefore do include all of the large-angular-scale noise correlations (although they do not include the uncertainty in the DC signal level of the image). Note that the radial profile bins for the deconvolved images are correlated due to the large-angular-scale noise present in those images.

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