Abstract—The dynamics of human skeletons have significant information for the task of action recognition. The similarity between trajectories of corresponding joints is an indicating feature of the same action, while this similarity may subject to some distortions that can be modeled as the combination of spatial and temporal affine transformations. In this work, we propose a novel feature called spatio-temporal dual affine differential invariant (STDADI). Furthermore, in order to improve the generalization ability of neural networks, a channel augmentation method is proposed. On the large scale action recognition dataset NTU-RGB+D, and its extended version NTU-RGB+D 120, it achieves remarkable improvements over previous state-of-the-art methods.

I. INTRODUCTION

Skeleton-based action recognition has received great attention in recent years, as the dynamics of human skeletons has significant information for the task of action recognition. Compared to other modalities of human action, for example, video, depth images and optical flows, human skeletons have the advantage of small amount of data and high information density. The dynamics of human skeletons can be seen as time series of human poses, or the combination of human joint trajectories. Among all the human joints, the trajectory of important joints indicating the action class conveys the most significant information. It is also worth noting that when performing the same action under different attempts, trajectories of these joints are subject to some distortions. In this work, we propose a novel invariant feature under these distortions and then utilize them for facilitating skeleton-based action recognition.

When performing the same action, two similar trajectories of corresponding joints should share a basic shape. However, due to individual factors, these two trajectories always appear in diverse kinds of distortions. These distortions are caused by spatial and temporal factors. Spatial factors include the change of viewpoints, different skeleton sizes and action amplitude ([1], [2]), while temporal factors indicate time scaling along the time series ([3], [4]). All the spatial factors can be modeled by the affine transformation in 3D space, whereas the uniform time scaling is commonly discussed case, which can be seen as affine transformation in 1D space. We combine these two kinds of distortions as the spatio-temporal dual affine transformation.

In this paper, we propose a general method for constructing Spatio-Temporal Dual Affine Differential Invariant (STDADI). Specifically, we utilize the rational polynomial of derivatives of joint trajectories to obtain the invariants. By bounding the degree of polynomial and the order of derivatives, we generate 8 independent STDADIs and combine them as an invariant vector at each moment for each human joint.

Recently, researchers tend to explore the potential of date-driven methods for skeleton-based action recognition. When considering to improve the generalization ability of neural networks under different transformations, a common practice is data augmentation. However, additional data preprocessing generates more samples and takes longer time in the training phase. In this paper, we propose an intuitive yet effective method, extending input data with STDADI along the channel dimension for training and evaluation, and call this practice as channel augmentation. Experiments show that channel augmentation based on STDADI not only achieves stronger performance and generalization, but also provides more insights for skeleton-based action recognition.

The main contributions of this work are the following:

1) We propose a novel feature called spatio-temporal dual affine differential invariant (STDADI).
2) In order to improve the generalization ability of neural networks, a channel augmentation method is proposed.
3) We validate the effectiveness of the proposed feature and method on the large scale action recognition dataset NTU-RGB+D [5] and its extended version NTU-RGB+D 120 [6], and get superior performance when compared to previous state-of-the-art methods.

II. RELATED WORK

Skeleton-based action recognition Before the rising of deep learning, some handcrafted-feature-based methods were proposed to solve skeleton-based action recognition. Wang et al. [7] proposed to use relative locations of joints as motion features. Hussein et al. [8] exploited the covariance matrices of joint trajectories. Vemulapalli et al. [9] utilized rotations and translations between joint locations to capture the dynamics of
human skeletons. However, the performance of these methods is limited as designed features do not cover all factors affecting the recognition. Thanks to the success of deep learning, data-driven methods achieve better performance than before. These methods can be further divided as RNN-based, CNN-based and GNN-based approaches. RNN-based approaches take the sequence of human joint coordinate vectors as input and predict the action label in a recursive manner ([2], [5], [10], [11], [12], [13], [14], [15]). CNN-based approaches express the skeleton data as a pseudo-image for conventional CNNs ([16], [17], [18], [19], [20]) or as a sequence of coordinate vectors for temporal CNNs ([21], [22], [23]). Compared to these two kinds of methods, GNN-based approaches are modeled based on the natural connections between human joints, thus better to characterize the dynamics of human skeletons. Recently, Yan et al. [24] proposed the spatial-temporal graph convolutional network (ST-GCN) and achieved evident improvements over previous methods.

Transformations and invariant features for skeleton-based action recognition For skeleton-based action recognition, commonly discussed transformations are geometric, based on action recognition, which are usually caused by the change of viewpoint and the nition, commonly discussed transformations are geometric, based action recognition network (ST-GCN) and achieved evident improvements over previous methods.

The dual affine transformation can be defined as

$$g(u) = Af(t) + T, u = \frac{1}{c}(t - d)$$  (3)

where the matrix A and vector T express the spatial affine transformation, and the scalar c and d are used to denote the temporal affine transformation. This can be detailed as follows:

Spatial affine transformation The matrix A controls the rotation and scaling while the vector T means the translation. Spatial affine transformations are caused by multiple factors, including coordinate system conversion, pose orientation, different skeleton sizes and action amplitude.

Temporal affine transformation The linear transformation of time domain can be considered as the 1D affine transformation. The parameter c means time scaling, indicating different speeds, and d means phase shift, indicating different beginning time. We discuss uniform time scaling here and it assumes a uniform change of the time scale according to the same proportion [4]. We follow this assumption and express it as the temporal affine transformation.

B. Spatio-Temporal Dual Affine Differential Invariant

We utilize the rational polynomial of derivatives of joint trajectories to construct STDADI. Specifically, based on equation [3] we can derive the relationship between 1st derivatives of joint trajectories before and after the transformation:

$$\frac{dg}{du} = \frac{dg}{df} \cdot \frac{df}{dt} \cdot \frac{dt}{du}$$  (4)

$$= A \cdot \frac{df}{dt} \cdot c = cA \cdot \frac{df}{dt}$$

Similarly, we can obtain the relationship between any order derivatives by chain rule:

$$g^{(i)} = c^{i}A\hat{f}^{(i)}$$  (5)

where the superscript \((i)\) denotes the order of derivation. It is worth noting that when \(i\) is equal to 0, formula [5] is equivalent to formula [3] without translation vector \(T\). We can eliminate the effect of translation vector \(T\) by subtracting the mean value. That is,

$$\hat{g} = g - g_{mean} = A \cdot (f - f_{mean}) = c^{0}A\hat{f}$$  (6)

Thus, in Equation [5] we can set \(i\) as a non-negative integer.

Based on the relationship in equation [5] we construct a 3x3 matrix using 3 derivatives of different orders as column and derive their relationship:

$$M_{g}^{ijk} = \begin{pmatrix} g^{(i)} & g^{(j)} & g^{(k)} \\ (c^{i}A\hat{f})^{(j)} & (c^{j}A\hat{f})^{(j)} & (c^{k}A\hat{f})^{(k)} \\ A \cdot (c^{i}f^{(j)}) & A \cdot (c^{j}f^{(j)}) & A \cdot (c^{k}f^{(k)}) \end{pmatrix}$$  (7)

where \(i, j, k\) are all non-negative integers. To ensure the determinant of \(M\) is not equal to 0, \(i, j, k\) are different from each other. We find that the determinant of \(M\) is a relative
invariant which is related to the transformation parameters of $c$ and $A$:

$$\|M_{ij}^g\| = c^{i+j+k} \|A\| \|f(i), f(j), f(k)\|$$

$$= c^{i+j+k} \|A\| \|M_{ij}^f\|$$

(8)

We eliminate the parameters of $c$ and $A$ by constructing the rational formula:

$$\prod_{\sigma=1}^N \|M_{i\sigma}^{j\sigma-k\sigma}\| = \prod_{\sigma=1}^N \|M_{i\sigma}^{j\sigma-m\sigma}\|$$

(9)

This means

$$\prod_{\sigma=1}^N \|M_{i\sigma}^{j\sigma-k\sigma}\| = \prod_{\sigma=1}^N \|M_{i\sigma}^{j\sigma-m\sigma}\| + \epsilon$$

(10)

is an invariant feature with respect to the spatio-temporal dual affine transformation, namely, STDADI. In this expression, $N$ is a positive integer named as the degree of polynomials, and the degree of numerator and denominator should be equal to guarantee the elimination of the matrix $A$. The max value of derivatives is named as the order of STDADI. To ensure the elimination of the parameter $c$, the following needs to be met,

$$\sum_{\sigma=1}^N (i_{\sigma} + j_{\sigma} + k_{\sigma}) = \sum_{\sigma=1}^N (l_{\sigma} + m_{\sigma} + n_{\sigma})$$

(11)

To ensure that every determinant is not equal to 0, it is also needed that $i_{\sigma} \neq j_{\sigma} \neq k_{\sigma}, l_{\sigma} \neq m_{\sigma} \neq n_{\sigma}, \forall \sigma$. The parameter $\epsilon$ is a small value for computational stability.

For computation simplicity, we set the upper limit of the degree and order to be 2 and 4, respectively, and we obtain 55 invariants in total. We select 8 of them which are function-independent [31] from each other, which means weaker correlation and better description ability. The 8 invariants are listed as follows ($\epsilon$ are ignored here for compact expression):

$$\|M^{123}\|, \|M^{123}\|, \|M^{134}\|$$

$$\|M^{012}\|, \|M^{024}\|, \|M^{124}\|$$

$$\|M^{102}\|, \|M^{213}\|, \|M^{123}\|, \|M^{023}\|, \|M^{124}\|$$

$$\|M^{013}\|, \|M^{014}\|, \|M^{214}\|$$

$$\|M^{102}\|, \|M^{123}\|, \|M^{134}\|$$

$$\|M^{012}\|, \|M^{013}\|, \|M^{014}\|, \|M^{023}\|, \|M^{124}\|$$

$$\|M^{012}\|, \|M^{013}\|, \|M^{014}\|, \|M^{023}\|, \|M^{124}\|$$

(12)

In practice, we approximate the derivatives of joint trajectories using a 5th order B-spline curve. Then we calculate STDADIs following formula [10] and [12]. Finally we arrange the obtained invariants as an 8-dimension invariant feature vector at each moment for each human joint.

C. Channel Augmentation

Compared to other handcrafted features, our STDADI focuses on describing joint trajectories under the spatio-temporal dual affine transformation. As not all factors are covered, STDADI itself is not efficient enough for the recognition task. However, as the feature is beneficial for recognizing actions under different transformations, it can help improve the generalization of data-driven methods. In this case, we propose an intuitive yet effective method named channel augmentation.

IV. RESULTS

In this section we validate the effectiveness of the proposed feature and method on the large scale action recognition dataset NTU-RGB+D [5] and its extended version NTU-RGB-D 120 [6]. In addition to the original ST-GCN, we adopted a data augmentation technique as the baseline method. As illustrated in [2], the data augmentation technique involves rotation, scaling and shear transformations of 3D skeletons during training. For all the experimental methods, we used the same training strategy and hyperparameters as suggested in [24].

A. Datasets & Evaluation Metrics

NTU-RGB+D and its extended version, NTU-RGB-D 120 are currently the largest action recognition datasets with 3D joint annotations captured in a constrained indoor environment using Microsoft Kinect V2 cameras. Both of them provide 3D skeleton data containing 3D locations of 25 major body joints in the camera coordinate system. NTU-RGB-D contains 56880 samples in 60 action classes performed by 40 subjects, and NTU RGB+D 120 extends the original by adding 57600 more samples, expanding the number of action classes and subjects to 120 and 106, respectively. Both datasets have
Comparisons of the validation accuracy with state-of-the-art methods on NTU-RGB+D and NTU-RGB+D 120. "*" indicates that for [20], we report here the results on NTU-RGB+D using only skeleton data. Best results are labeled in bold.

| Method                        | NTU-RGB+D   | NTU-RGB+D 120 |
|-------------------------------|-------------|---------------|
|                               | Cross-subject | Cross-view | Cross-subject | Cross-view |
| Part-Aware LSTM               | 62.9%       | 70.3%      | 25.5%       | 26.3%     |
| Spatio-Temporal LSTM          | 69.2%       | 77.7%      | 55.7%       | 57.9%     |
| GCA-LSTM                      | 74.4%       | 82.8%      | 58.3%       | 59.2%     |
| Two-Stream Attention LSTM     | 76.1%       | 84.0%      | 61.2%       | 63.3%     |
| Skeleton Visualization        | 80.0%       | 87.2%      | 60.3%       | 63.2%     |
| Body Pose Evolution Map(*)    | 82.4%       | 86.7%      | 64.6%       | 66.9%     |
| Multi-Task Learning Network   | 79.6%       | 84.8%      | 58.4%       | 57.9%     |
| Multi-Task CNN with RotClips  | 81.1%       | 87.4%      | 62.2%       | 61.8%     |
| ST-GCN                        | 81.5%       | 88.3%      | 71.7%       | 74.3%     |
| ST-GCN + data augmentation    | 80.6%       | 90.5%      | 72.2%       | 79.0%     |
| ST-GCN + channel augmentation | **83.4%**   | **91.3%**  | **77.3%**   | **78.8%** |

Table II: The validation accuracy of different input settings for channel augmentation on NTU-RGB+D.

| Method          | Cross-subject | Cross-view |
|-----------------|---------------|------------|
| ST-GCN          | 81.5%         | 88.3%      |
| + derivatives   | 80.4%         | 87.6%      |
| + STDADI        | **83.4%**     | **91.3%**  |

the cross-subject evaluation criteria, while NTU RGB+D 120 makes an improvement on the cross-view benchmark by introducing more factors that affect the angle of view, including the height and distance of cameras to the subjects, and renames this benchmark as "cross-setup". We report top-1 recognition accuracy on both datasets with corresponding evaluation metrics.

### B. Comparison with the State-of-the-art

As shown in Table I, our method, ST-GCN + channel augmentation, outperforms most of the previous state-of-the-art methods. Compare to two baseline approaches, ST-GCN and ST-GCN + data augmentation, our method achieves obvious improvements on both benchmarks. For data augmentation, as it is mainly consisted of 3D geometric transformations, it helps much to improve accuracy in cross-view recognition, but contributes little to the cross-subject setting. This also verifies that our spatio-temporal dual affine transformation assumption is valid on both evaluation criteria.

### C. Detailed Analysis

To validate the effectiveness of STDADI, we tried a different input setting using trajectory derivatives as the extended vector for channel augmentation. This vector contains the 1st, 2nd and 3rd derivatives of the joint trajectory and thus is 9-dimensional. Seen from Table II, while the ST-GCN+STDADI has an improvement, the ST-GCN+derivatives has a decrease of accuracy on both the benchmarks. This shows that the improvement of accuracy comes from the invariance expressed by STDADI.

We also compare the improvements of ST-GCN + channel augmentation over ST-GCN of different action classes. As shown in Fig. 2, actions such as pointing to something and salute achieve the greatest performance gain, while actions like brushing hairs suffer performance loss. We find that those action classes with improving accuracy have specific joint trajectory motion patterns. When performing actions like pointing to something and salute, the trajectories of wrist joint of performers are geometrically similar. This indicates that the geometric similarity of important joint trajectories helps to recognize the action class, and our STDADI provides an invariant representation for the similarity under various distortions.

### V. Conclusion

In this paper, we propose a general method for constructing spatio-temporal dual affine differential invariant (STDADI). We prove the effectiveness of this invariant feature using a channel augmentation technique on the large-scale action recognition dataset NTU-RGB+D and NTU-RGB+D 120. The combination of handcrafted features and data-driven methods not only improves the accuracy but also provides more insights. In the future, as the temporal affine transformation may not be efficient to model complex transformations along the time dimension, we are going to explore the invariance under unilinear dynamic time scaling.
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