Noncommutative geometry-inspired dirty black holes

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Abstract

We provide a new exact solution of the Einstein equations which generalize the noncommutative geometry-inspired Schwarzschild metric, we previously obtained. We consider here a more general relation between the energy density and the radial pressure and find new geometries describing a regular ‘dirty black hole’. We discuss strong and weak energy condition violation and various aspects of the regular dirty black hole thermodynamics.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

In a recent series of papers we obtained exact solutions of the Einstein equations describing both neutral and charged black holes free of curvature singularities in the origin \cite{1–8}. We reached these conclusions after a long path starting from an original approach to noncommutative geometry which is based on coordinate coherent states \cite{9–18}, in alternative to the mathematically correct, but physically hard to implement, ‘star-product’ formulation.

However, our results are ‘model independent’ in the sense that our approach improves the short-distance behavior of the Einstein equations by taking into account the presence of a quantum gravity-induced minimal length, whatever it is.

String theory, noncommutative geometry, generalized uncertainty principle, etc, all point out the existence of a lower bound to distance measurements. Thus, the very concept of ‘point-like’ particle becomes physically meaningless and must be replaced with its best
approximation consistent with the tenets of quantum mechanics, i.e. a minimal width Gaussian distribution of mass/energy. Solving the Einstein equations for a static, minimal width, mass–energy distribution centered around the origin, we found a black hole-type solution smoothly interpolating between de Sitter spacetime at a short distance and Schwarzschild geometry at a large distance. The characteristic length scale of this system is given by the matter distribution width $\sqrt{\theta}$.

The geometric and thermodynamics features of the solution can be summarized as follows:

(i) there is no curvature singularity in $r = 0$; the center of the black hole is a regular ‘ball’ of de Sitter vacuum accounting for the short-distance quantum fluctuations of the spacetime manifold, whatever is their physical origin;

(ii) even in the neutral case, there exist an outer and an inner horizon. At the end of the Hawking evaporation the two horizons coalesce into a single, degenerate horizon, corresponding to an extremal black hole;

(iii) the Hawking temperature reaches a finite maximum value and drops down to zero in the extremal configuration.

To achieve this regular behavior the choice of the matter equation of state is instrumental. The stability of the solution is guaranteed by choosing

$$\rho(r) = -p_r(r)$$

(1.1)

where $\rho(r)$, $p_r(r)$ are the Gaussian matter density and radial pressure, respectively. The tangential pressure $p_\perp(r)$ was determined by $\rho(r)$ and $p_r(r)$ through the hydrodynamic equilibrium equation. Condition (1.1) has the same form of the vacuum equation of state. Thus, one can expect that near the origin the metric will be de Sitter with an effective cosmological constant $\Lambda \propto G_N \rho(0)$. On the other hand, if the width of the matter distribution is $\sqrt{\theta}$, at a large distance one sees a small sphere of matter with radius about $\sqrt{\theta}$. Thus, the Birkhoff theorem assures the metric to be Schwarzschild. In the intermediate region the metric is neither de Sitter nor Schwarzschild, and can be analytically written in terms of a lower incomplete gamma function.

In this paper we are going to relax equation (1.1) and look for a new solution describing a dirty black hole. The starting point is the minimal width, Gaussian, mass/energy distribution

$$\rho(r) = \frac{M}{(4\pi \theta)^{3/2}} \exp \left( -\frac{r^2}{4\theta} \right),$$

(1.2)

which we obtained from our coordinate coherent state approach to noncommutative geometry. From this perspective the parameter $\theta$ is a length squared quantity defining the scale where spacetime coordinates become non-commuting (quantum) objects. However, in a more general framework the distribution (1.2) represents the most localized energy density which is compatible with the existence of a minimal length, whatever it is. Thus, the width of the bell-shaped function (1.2) can be consistently related to the string length $\sqrt{\alpha}$, the TeV quantum gravity scale, etc. In a recent paper [19] a Gaussian source has been used to model phantom energy-supported wormholes.

The constant $M$ is the total mass energy given by

$$M \equiv 4\pi \int_0^\infty dr\ r^2 \rho(r).$$

(1.3)

The mass distribution (1.2) is the component $T^0_0$ of the energy–momentum tensor. Before proceeding we have to define the remaining components. We model our source through a fluid-type $T^\mu_\nu$ of the following form:

$$T^\mu_\nu = \text{Diag}(-\rho(r), p_r(r), p_\perp(r), p_\perp(r))$$

(1.4)

where $p_r$ is the radial pressure and $p_\perp$ is the tangential pressure.
In order to be a viable source for the Einstein equations, the condition \( T^\mu_\nu ; \mu = 0 \) must be satisfied by \( \rho, p_r, p_\perp \).

We are going to solve Einstein equations assuming the metric to be spherically symmetric, static, asymptotically flat. Thus, we can write the general form of the line element in terms of two independent functions \( \Phi(r) \) and \( m(r) \) as

\[
ds^2 = -e^{2\Phi(r)} dt^2 + \frac{dr^2}{1 - 2m(r)/r} + r^2(d\psi^2 + \sin^2 \psi d\phi^2) \tag{1.5}
\]

where \( m(r) \) is the shape function and \( \Phi(r) \) is the redshift function [20, 21]. Both unknown functions must be determined by the Einstein equations. To this purpose, we now briefly recall a recent solution one obtains by assuming in place of (1.1) the condition

\[
p_r = -\frac{1}{4\pi r^3} m(r). \tag{1.6}
\]

Thus, the resulting line element reads

\[
ds^2 = -dt^2 + \frac{dr^2}{1 - 4M_\gamma(3/2; r^2/4\theta)/\sqrt{\pi r}} + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \tag{1.8}
\]

which describes a wormhole geometry. Its properties have been recently investigated in [22] and we are not going to repeat them here. We have recalled this wormhole solution since it turns out to be useful to understand the procedure we shall follow in the next section. Indeed to derive the dirty black hole solution, we are going to relax condition (1.6) in favor of a more general one motivated by physical requirements.

2. Dirty black hole

The term ‘dirty black holes’ refers to black hole solutions of the Einstein equations in interaction with various kinds of matter fields, some remarkable examples are

- gravity + electromagnetism + dilaton [23–26];
- gravity + electromagnetism + axion [27–29];
- gravity + electromagnetism + Abelian Higgs field [30];
- gravity + electromagnetism + dilaton + axion [31];
- gravity + non-Abelian gauge fields [32–35];
- gravity +axion + non-Abelian gauge fields [36].

In our model we do not select any specific kind of field theory, but we simply introduce a smeared energy/pressure distribution, which is a classical parameterization for the energy/pressure ‘stored’ in some kind of field. However, the condition which provides the relation between energy and radial pressure is crucial to determine the new solution. Among the huge variety of possible choices of this condition, we require that the pressures \( p_r \) and \( p_\perp \) satisfy the following physical conditions:

(1) \( p_r \) and \( p_\perp \) must be asymptotically vanishing;
(2) \( p_r \) and \( p_\perp \) must be finite at the horizon(s);
(3) \( p_r \) and \( p_\perp \) must be finite at the origin.

This kind of solutions are alternatively referred to as ‘hairy’ black holes.
Condition 1 assures that the solution matches the Minkowski space at infinity while conditions 2 and 3 warrant the regularity at the horizon(s) and at the origin, a fact that is in the spirit of all the previous solutions generated by a smeared source term.

The line element we are looking for is of the form

$$\text{d}s^2 = -e^{2\Phi(r)}(1 - 2m(r)/r) \text{d}t^2 + \frac{\text{d}r^2}{1 - 2m(r)/r} + r^2 (\text{d}\psi^2 + \sin^2 \psi \text{d}\phi^2),$$

which can be obtained from (1.5) by the following local rescaling of the redshift function

$$\Phi(r) \rightarrow \Phi(r) + \frac{1}{2} \ln(1 - 2m(r)/r).$$

Therefore, the Einstein equation reads

$$\frac{dm}{dr} = 4\pi r^2 \rho,$$

$$\frac{1}{2g_{00}} \frac{dg_{00}}{dr} = \frac{m(r) + 4\pi r^3 p_r}{r(r - 2m(r))},$$

$$\frac{dp_r}{dr} - \frac{1}{2g_{00}} \frac{dg_{00}}{dr} (\rho + p_r) + \frac{2}{r} (p_\perp - p_r)$$

where $g_{00} \equiv -e^{2\Phi(r)}(1 - 2m(r)/r)$.

According to recipes 1, 2 and 3, pressures regularity requirements are satisfied by the following condition replacing (1.6):

$$\rho(r) + p_r(r) \equiv -\sqrt{\theta} (1 - 2m/r) \frac{d\rho}{dr} = \frac{1}{2\sqrt{\theta}} \rho (1 - 2m/r),$$

which can be rewritten as

$$p_\perp(r) = -\rho(r) \left[ 1 - \frac{r}{2\sqrt{\theta}} (1 - 2m/r) \right].$$

This is the simplest choice, satisfying the physical conditions above. In principle, one can still consider further conditions, involving additional terms $\sim (r/\sqrt{\theta})^n (1 - 2m/r)^n$ in the square brackets on the rhs. In such a case, one ends up with a more complicated solution, endowed only with subleading contributions to the solution we are going to derive, in the three physically meaningful regions (infinity, horizon(s) and origin).

From equation (2.12) one finds that the angular pressure $p_\perp$ is

$$p_\perp(r) = p_r(r) + \frac{r}{2} \frac{dp_r}{dr} + \frac{1}{4\sqrt{\theta}} \rho(m(r) + 4\pi r^3 p_r).$$

We note that both $p_r$ and $p_\perp$ enjoy the above physical conditions at the origin, horizon(s) and asymptotically. From equations (2.11) and (2.14) one has

$$\frac{d\Phi}{dr} = 4\pi r (\rho + p_r) \frac{\sqrt{\theta}}{1 - 2m/r} = \frac{2\pi r^2}{\sqrt{\theta}} \rho(r)$$

with the boundary condition $\Phi(\infty) = 0$ in order to reproduce Minkowski geometry at infinity. The solution reads

$$\Phi(r) = \frac{MG}{\sqrt{\theta}} \psi((3/2, r^2/4\theta)) - \frac{MG}{2\sqrt{\theta}}$$

where for the sake of clarity we momentarily re-introduced the Newton constant $G$.

As a result the noncommutative Schwarzschild ‘dirty’ black hole is described by the following line element:

$$\text{d}s^2 = -\exp \left[ -\frac{MG}{\sqrt{\theta}} \left( 1 - \frac{2}{\sqrt{\pi}} \psi((3/2, r^2/4\theta)) \right) \right] (1 - 2Gm(r)/r) \text{d}t^2$$

$$+ \frac{\text{d}r^2}{1 - 2Gm(r)/r} + r^2 \text{d}\Omega^2.$$
Before discussing thermodynamical properties, let us check the regularity of the geometry at the origin. From the Einstein equations one finds

\[ R(0) = -8\pi GT_{\mu}^{\nu}(0) = 8\pi G [\rho(0) - p_r(0) - 2p_\perp(0)]. \]  

From equations (2.13), (2.14) and (2.15) we see that all the pressures approach \(-\rho(0)\) near the origin. Thus, we find

\[ R(0) = 32\pi G\rho(0) = \frac{4GM}{\sqrt{\pi}0^{3/2}} \equiv 4\Lambda_{\text{eff}}. \]

which is the Ricci scalar for a de Sitter metric characterized by a positive effective cosmological constant \(\Lambda_{\text{eff}}\). The same conclusion can be obtained by expanding the line element near \(r = 0\):

\[ ds^2 = -e^{-M/\sqrt{\beta}} \left( 1 - \frac{\Lambda_{\text{eff}}}{3} r^2 \right)^{-1} dr^2 + \left( 1 - \frac{\Lambda_{\text{eff}}}{3} r^2 \right)^{-1} dr^2 + r^2 (d\psi^2 + \sin^2 \psi d\phi^2) \]

where the constant \(e^{-M/\sqrt{\beta}}\) can be reabsorbed into a rescaled time coordinate in order to have the ordinary de Sitter line element. Also in the case of the dirty black hole there is no curvature singularity and the central black hole geometry results to be a de Sitter spacetime.

Regarding the horizon(s), we have to study the equation \(M = U(r_H)\) where the ‘effective potential’ \(U(r_H)\) is defined to be

\[ U(r_H) = \frac{4}{\sqrt{\pi}} \frac{r_H}{\gamma(3/2, r_H^2/4\beta)}. \]

For a positive given value of \(M\), equation (2.22) can admit two, one, or no solution. We studied in detail the solutions of this equation in [1] and we are not going to repeat all the discussion here. It may be useful to summarize the main conclusion, i.e. the existence of a lower bound for a black hole mass given by \(M_0 \approx 1.9\sqrt{\beta}/G\). Thus, we find again three possible cases:

- \(M > M_0\) non-extremal dirty black hole with two horizons \(r_+ > r_-\); the outer horizon \(r_+\) is said to be of canonical type, while \(r_-\) is a Cauchy horizon.
- \(M = M_0\) extremal dirty black hole with one degenerate horizon \(r_+ = r_- \equiv r_0 \approx 3\sqrt{\beta}\); the function \(g_{rr}\) does not change sign at the horizon, which is said to be of non-canonical type.
- \(0 < M < M_0\) dirty minigravastar, no horizons.

An interesting feature of the above line element regards the gravitational redshift \(z \equiv \Delta\lambda/\lambda\), where \(\lambda\) is the wavelength of the electromagnetic radiation at the source and \(\Delta\lambda\) is the difference between the observed and emitted wavelengths. This quantity now depends both on the redshift function \(\Phi(r)\) and on the shape function \(m(r)\). The redshift measured by an asymptotic observer turns out to be

\[ z = e^{\frac{2\beta}{3}(1 - \frac{1}{2} \gamma(3/2, r^2/4\beta))(1 - 2Gm(r)/r)^{-1/2} - 1}, \]

which is a clearly divergent quantity approaching the horizon, while asymptotically vanishes (see figure 1). On the other hand, in the absence of horizons, i.e. the minigravastar case, the redshift \(z\) is finite even at the origin (see figure 2).

In the framework of ‘wormhole engineering’ exotic matter is advocated in order to violate energy conditions and avoid the tunnel to collapse. Thus, weak (WEC) and/or strong energy (SEC) conditions violation are an important issue to discuss in the black hole case, as well. Indeed, an eventual violation would mark a substantial departure from the behavior of any kind of classical matter. This is what we expect as the Gaussian distribution, which is the cornerstone of our approach and was originally derived in [12] in a quantum framework. Figure 3 shows the new features of the dirty black hole.
Figure 1. The asymptotic redshift $z = \Delta \lambda / \Delta \lambda_0$ versus the radius $r / \sqrt{\theta}$. On the right part of the figure, the curves are for the mass $M = 2.5 \sqrt{\theta}/G$, while on the left are for $M = M_0$. The dashed curves correspond to the Schwarzschild case, while the solid ones are for the noncommutative Schwarzschild (thin) and the noncommutative dirty case (thick). We can observe that, for $M = 2.5 \sqrt{\theta}/G$, all the curves coincide and we can conclude that noncommutative effects are not yet important, while for smaller masses in the vicinity of the extremal configuration the curves become distinct. We can conclude that the shape function $m(r)$ lowers the redshift, as in the case of a smaller gravitational field, while the redshift function $\Phi_1(r)$ slightly increases the values of $z$.

Let us start from the dashed curve representing the plot of the function $\rho + p_r + 2p_\perp$. Where it is negative, the SEC is violated. This occurs well for $r < 2.8 \sqrt{\theta}$, which is well inside the event horizon located at $r_s \approx 5 \sqrt{\theta}$, as in the case of the ‘clean’ noncommutative geometry-inspired Schwarzschild black hole. Both in the case of dirty and clean solutions, the SEC is a short-distance effect caused by the underlying non-commutative geometry. Spacetime fluctuations provide an effective gravitational repulsion smearing out the classical curvature singularity. On the other hand, outside the black hole matter behaves in a ‘classical’ manner. However, the memory of the short-distance effects is still present in the black hole temperature as it will be shown below. The new distinctive feature of the dirty black hole is that WEC are violated as well, while in the clean case they are preserved. Violation occurs inside the horizon and more specifically where the two solid curves in figure 3 lower below the $r$-axis. The presence of some ‘dirt’ on the event horizon causes a suppression of the Hawking temperature with respect to the corresponding clean case, in agreement with the general result shown in [21].

The Hawking temperature, mentioned above, is given by ($G = 1$, $\kappa_B = 1$)

$$T_H = \frac{1}{4 \pi r_s} e^{\Phi(r_s)} \left[ 1 - \frac{r_s^3}{4 \theta^{3/2}} \frac{e^{-r_s^2/4 \theta}}{\sqrt{3/2} \left( r_s^2 / 4 \theta \right)} \right], \quad (2.24)$$
Figure 2. The asymptotic redshift $z = \Delta \lambda / \lambda$ versus the radius $r/\sqrt{\theta}$ for the minigravastar case, i.e. $M < M_0$. The dashed curve corresponds to the Schwarzschild line element and exhibit a divergent behavior due to the presence of horizons. The thin solid curve corresponds to the noncommutative Schwarzschild case, while the thick solid curve is for the noncommutative dirty case. As a general prescription, the shape function $m(r)$ provides a regular peak at $r = r_0$, while the redshift function $\Phi(r)$ slightly increases the values of $z$ with respect to the noncommutative Schwarzschild case for all $r$.

\[
\Phi(r_+) \equiv \frac{\sqrt{\pi} r_+}{8 \sqrt{\theta} \gamma(3/2, r_+^2/4\theta)} \left( \frac{2}{\sqrt{\pi}} \gamma(3/2, r_+^2/4\theta) - 1 \right).
\] (2.25)

$T_H$ is defined for any $r_+ \geq r_0$. The profile of $T_H(r_+)$ resembles that found in [15] and implies a ‘SCRAM phase’ at the terminal phase of the evaporation, namely a cooling down to an asymptotic absolute zero configuration. The correction due to the prefactor $e^{\Phi}$ plays a non-trivial role only near the peak of the temperature by lowering of the maximum value, as can be seen in figure 4. This is in agreement with the choice of equation (2.13) and the consequent violations of WEC in a region inside the event horizon $r_+$ [21].

Once the Hawking temperature is known let us look at the area/entropy law. From the first law of black hole thermodynamics, i.e. $dM = T_H dS$, we can write the infinitesimal entropy variation $dS$ in terms of the effective potential $U(r_+)$:

\[
dS = \frac{1}{T_H} \frac{\partial U}{\partial r_+} dr_+.
\] (2.26)

In order to integrate (2.26) in the correct way, we must take into account that the extremal, zero temperature, black hole configuration has zero thermodynamical entropy. Thus, the
integration range starts from the radius \( r_0 \) of the degenerate horizon and runs up to a generic radius \( r_+ > r_0 \). With this choice of the integration range we find

\[
S = \frac{\pi^{3/2}}{2} \left[ \frac{r_+^2}{\gamma(3/2, r_+^2/4\theta)} e^{-\Phi(r_+)} - \frac{r_0^2}{\gamma(3/2, r_0^2/4\theta)} e^{-\Phi(r_0)} \right] - \Delta S \tag{2.27}
\]

where

\[
\Delta S \equiv \frac{\pi^{3/2}}{2} \int_{r_0}^{r_+} dr \ r^2 \left[ \frac{e^{-\Phi(r)}}{\gamma(3/2, r^2/4\theta)} \right]. \tag{2.28}
\]

The first term in (2.27) differs from the corresponding quantity in the clean case only for the presence of the exponential factors. On the other hand, the correction \( \Delta S \) is characteristic of the dirty black hole, and vanishes in the large distance limit. Indeed, for \( r_+, r_0 \gg \sqrt{\theta} \), we recover the classical area/entropy law:

\[
S \to \frac{1}{2}(A_+ - A_0). \tag{2.29}
\]

At first glance, the reader could question why the celebrated Bekenstein–Hawking (2.29) results are valid only asymptotically, and not for any value of \( r_+ \). The key observation is that nowadays everybody is taking for granted that the classical entropy of any black hole ‘is’ one fourth of the area of event horizon, measured in Planck units, up to eventual logarithmic corrections from unspecified quantum gravity effects. Thus, nobody cares anymore to ‘recover’ the area law from basic principles. Rather, the main interest is to match the classical result with the statistical interpretation of entropy in terms of black hole quantum micro-states. We would like to note that the consistency between the geometric and thermodynamical definition of the black hole entropy requires that in any case it must be possible to derive the area/entropy law from the first law (2.26), as we did above, and to assume a priori it is valid. It is a
common feature of regular black holes to display a more complex relation between horizon area and entropy, rather than a simple proportionality law. This effect can be traced back, once again, to the presence of a minimal length or, equivalently, to the granular nature of the ‘quantum’ spacetime. This effect, together with the existence of a finite leftover by the Hawking process, hints to a possible resolution of the information paradox worth of a more in-depth investigation.

3. Conclusions

In this paper we extended our previous investigation of black hole solutions of Einstein equations with a Gaussian source. Gaussian distributions are widely used in physics and it is surprising that nobody considered before the gravitational effects of this kind of sources. From our vantage point, we used this distribution to model the physical effects of short-distance fluctuations of noncommutative coordinates. Generally the covariant divergence-free condition on the energy–momentum tensor in the Einstein equations allows various kinds of physically acceptable conditions between the energy density and radial pressure. In our case the component $T^{00}$ is assigned and the other ones must be determined in a consistent way. After recalling the wormhole spacetime geometry of the type conjectured in [22], we investigated a new dirty black hole-type solution. The novelty of this solution with respect to the corresponding ‘clean’ case is displayed in figure 3. While in the original noncommutative black hole [1] only the strong energy condition is violated, in the dirty case weak energy conditions are broken as well. The former violation removes the curvature singularity in the
origin, the latter decreases the Hawking temperature in agreement with the general result by Visser [21].

If LHC turns to be an effective black hole factory, all the regular objects we have investigated so far are expected to contribute to the cross-section production.

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