The analysis of work of presses continuous action when processing wood

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Abstract. In case of application of smooth hold-back rolls from wood laminated plastics and a compregnated wood which elastic modules are close to an elastic modulus of the wood given by rollers, deformations of the roller and wood can be determined by the given formulas. Deformations are arcwise connected with loadings and gradually increase with their increase.

In recent years interest in research of deformations of wood in processing has increased [1-6].

In case of wood supply by smooth metal rollers, the question of deformation of the roller at its contact with wood does not arise as the module of elasticity of metal is much higher than the module of elasticity of wood. When using as the giving bodies of rollers from wood-fiber plastic and improved laminated plastic and the pressed wood which elasticity modules slightly differ from the module of elasticity of wood interaction of the roller and wood at their contact will be characterized by mutual deformation.

The purpose of this work is determination of sizes of deformations as the roller manufactured of improved laminated plastic or the pressed wood, and the given wood at their contact.

We will imagine the cylinder lying on the plane (figure 1,a). If the cylinder is affected by no forces, then it will adjoin to the plane on forming, passing through a point A.

The running force applied vertically to the cylinder P will cause deformation both the cylinder, and the plane, and not the straight line, but a strip, width 2a (figure 1, b) will be their contact any more.

Figure 1. The cylinder on the plane.
As it is known from the theory of elasticity [7], a half of width of a strip of contact at compression of the elastic cylinder with the elastic plane can be determined by a formula:

\[
a = \sqrt{\frac{4pR}{\pi} \left( \frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2} \right)}
\]  

(1)

where \( p \) – the loading falling on unit of the cylinder, N/mm; \( R \) – radius of the cylinder, mm; \( \mu_1 \) and \( \mu_2 \) – Poisson's coefficients of the plane and cylinder; \( E_1 \) and \( E_2 \) – modules of elasticity of the plane and cylinder.

For determination of size of deformation of a \( \Delta l_1 \) and cylinder \( \Delta l_2 \) (figure 2) we will use the solution of a task of Flaman [2] which gives the chance to determine tension and movements for any point of elastic half-plane from the concentrated vertical running force (figure 3) applied to it.

Under the influence of force \( P \) point of elastic half-plane with coordinates of \( \theta \) and \( r \) will receive movements which can be determined by the following formulas:

\[
u = -\frac{2(1-\mu^2)}{\pi \cdot E} \cdot \left[ \ln \frac{r}{h} \cos \theta + \frac{1-2\mu}{2(1-\mu)} \cdot \theta \cdot \sin \theta \right];
\]

(2)

\[v = -\frac{2(1-\mu^2)}{\pi \cdot E} \cdot \left[ \ln \frac{r}{h} + \frac{1}{2(1-\mu)} \cdot \sin \theta - \frac{1-2\mu}{2(1-\mu)} \cdot \theta \cdot \cos \theta \right];
\]

(3)

where \( u \) – movement of a point in the direction \( r \), mm; \( v \) – movement of a point perpendicular to \( r \), mm; \( \mu \) – Poisson's constant; \( E \) – elastic modulus; \( P \) – force having per unit length, H/mm; \( r \) and \( \theta \) – coordinates of the considered point; \( h \) – distance from limit of half-plane to the point lying on the line of action of force \( P \), for which under a condition \( \theta=0, u=0, \) mm.

Figure 2. Determination of size of deformation of the plane and cylinder.

Vertical movement of any point which is on limit of half-plane is defined from a formula (3) if to accept a condition \( \theta = \frac{\pi}{2} \).
\[ V_{\theta=\frac{\pi}{2}} = \Delta l = \frac{2(1-\mu^2)}{\pi E} P \ln \frac{r}{h} + \frac{(1+\mu)P}{\pi E}; \]  

(4)

At contact of the cylinder with the plane deformation of the last is caused by not concentrated force, but the contact pad distributed on width 2a.

As it is known on Hertz, specific pressure \( q \) have to be proportional to ordinates of the semi-circle constructed on diameter 2a (figure 4), mm.

\[ q = \frac{q_{\text{max}}}{a} \sqrt{a^2 - x^2} \]  

(5.1)

For definition of vertical movement of a point \( A \) from a distributed load, we will allocate on a site of contact a strip with width \( dx \) at distance \( x \) from the vertical axis passing through point \( A \).

\[ dp = qdx = \frac{q_{\text{max}}}{a} \sqrt{a^2 - x^2} \cdot dx \]  

(5.2)
Integrating, we receive:

\[
P = 2 \int_0^a qdx = 2 \frac{q_{\text{max}}}{a} \sqrt{a^2 - x^2} \cdot dx = \frac{\pi \cdot a \cdot q_{\text{max}}}{2}, \quad \text{from where } q_{\text{max}} = \frac{2P}{\pi \cdot a} \tag{6}
\]

Under the influence of elementary loading \(dP\) the point \(A\) of half-planes will receive vertical movement which according to a formula (4) is equal:

\[
dV = \frac{2(1 - \mu^2)}{\pi \cdot E} \cdot dP \cdot \ln \frac{x}{h} + \frac{1 + \mu}{\pi \cdot E} \cdot dP = \frac{2(1 - \mu^2)}{\pi \cdot E} \left[ \ln xdP + \left( \frac{1}{2(1 - \mu)} - \ln h \right) dP \right]; \tag{7}
\]

Substituting its value (5) instead of \(dP\) and integrating (7) on width of a contact pad, we will receive vertical movement of a point \(A\) from pressure upon all strip of contact:

\[
V = \Delta l = \frac{2(1 - \mu^2)}{\pi \cdot E} \int_0^a \ln x \frac{q_{\text{max}}}{a} \sqrt{a^2 - x^2} \cdot dx + 2 \int_0^a \left( \frac{1}{2(1 - \mu)} - \ln h \right) \frac{q_{\text{max}}}{a} \sqrt{a^2 - x^2} \cdot dx = \tag{8}
\]

\[
= \frac{2(1 - \mu^2)}{\pi \cdot E} \left[ 2q_{\text{max}} \int_0^a \ln x \sqrt{a^2 - x^2} \cdot dx \left( \frac{1}{2(1 - \mu)} - \ln h \right) \right];
\]

First composed in square brackets (8) is integral:

\[
\frac{2q_{\text{max}}}{a} \int_0^a \ln x \sqrt{a^2 - x^2} \cdot dx \tag{9}
\]

This integral can be solved only approximately. For this purpose we will spread out expression \(\sqrt{a^2 - x^2}\) to a binomial row:

\[
\sqrt{a^2 - x^2} = a \left[ 1 - \frac{1}{2} \left( \frac{x^2}{a^2} \right) - \frac{1}{8} \left( \frac{x^2}{a^2} \right)^2 - \frac{1}{16} \left( \frac{x^2}{a^2} \right)^3 - \frac{1}{128} \left( \frac{x^2}{a^2} \right)^4 \cdots \right]\]

As \(a \gg x\), and row coefficients quickly decrease, at further calculations it is possible will be limited only to members of the third order of this row, and it is possible with sufficient degree of accuracy to accept

\[
\sqrt{a^2 - x^2} = a \left[ 1 - \frac{1}{2} \left( \frac{x^2}{a^2} \right) - \frac{1}{8} \left( \frac{x^2}{a^2} \right)^2 - \frac{1}{16} \left( \frac{x^2}{a^2} \right)^3 \right] \tag{10}
\]

Having substituted value (10) in (9), we have:

\[
\frac{2q_{\text{max}}}{a} \int_0^a a \left[ 1 - \frac{1}{2} \left( \frac{x^2}{a^2} \right) - \frac{1}{8} \left( \frac{x^2}{a^2} \right)^2 - \frac{1}{16} \left( \frac{x^2}{a^2} \right)^3 \right] \ln x \cdot dx = \tag{11}
\]

\[
= 2q_{\text{max}} \left[ a \int_0^a \ln x \cdot dx - \frac{1}{2a^2} \int_0^a x^2 \cdot \ln x \cdot dx - \frac{1}{8a^4} \int_0^a x^4 \cdot \ln x \cdot dx - \frac{1}{16a^6} \int_0^a x^6 \cdot \ln x \cdot dx \right] =
\]

\[
= 2q_{\text{max}} \cdot a \left[ \ln a \left( 1 - \frac{1}{6} \cdot \frac{1}{40} - \frac{1}{112} \right) - \left( \frac{1}{18} - \frac{1}{200} - \frac{1}{784} \right) \right]
\]
Considering that $\left(1 - \frac{1}{6} - \frac{1}{40} - \frac{1}{112}\right) \approx 0.8$ and $\left(1 - \frac{1}{18} - \frac{1}{200} - \frac{1}{784}\right) \approx 0.96$ we will have

$$\frac{2q_{\text{max}}}{a} \int_{0}^{a} \ln x \sqrt{a^2 - x^2} \cdot dx = 1.6q_{\text{max}} \cdot a(\ln a - 1,2)$$

(12)

Now the formula (8) for determination of size of vertical movement of point A elastic half-plane taking into account (12) takes a form:

$$\Delta l = \frac{2(1-\mu^2)}{\pi \cdot E} \left[1.6q_{\text{max}} \cdot a(\ln a - 1,2) + \left(\frac{1}{2(1-\mu)} - \ln h\right)P\right]$$

(13)

or, taking into account (6), finally we have:

$$\Delta l = \frac{2(1-\mu^2)}{\pi \cdot E_1} \left[\frac{3.2}{\pi} (\ln a - 1,2) + \left(\frac{1}{2(1-\mu)} - \ln h\right)\right]$$

(14)

Thus, at compression of the cylinder and the plane by running force, the point $A_1$ of the plane (figure 2) will receive the vertical movement equal:

$$\Delta l_1 = \frac{2(1-\mu^1)}{\pi \cdot E_1} \left[\frac{3.2}{\pi} (\ln a - 1,2) + \left(\frac{1}{2(1-\mu_1)} - \ln h\right)\right]$$

(15)

With sufficient degree of accuracy vertical movement of the point $A_2$ of the cylinder (figure 2) can be determined by a formula (14) as the strip of contact is small in comparison with cylinder radius, and it can be considered approximately as elastic half-plane.

In this case it is necessary to accept $h = R$, then:

$$\Delta l_2 = \frac{2(1-\mu^2)}{\pi \cdot E_2} \left[\frac{3.2}{\pi} (\ln a - 1,2) + \left(\frac{1}{2(1-\mu_2)} - \ln R\right)\right]$$

(16)

Therefore, in case of use of the smooth giving rollers have made by improved laminated plastics and the pressed wood which modules of elasticity are close to the module of elasticity of the wood given by rollers [1, 8-10], deformations of the roller and wood can be determined by formulas (15) and (16). Sizes of deformations, apparently from the received formulas, depend both on elastic constant $\mu$ and $E$, and on parameters $a$, $h$, and $R$ and in direct ratio to force $P$.

Deformations are linearly connected with loadings and gradually increase with their increase, following Hooke's law. The received expressions of works of elastic deformations allow to determine the power spent for deformation.

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