Modelling and simulation of gyrotrons for ITER

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Abstract. Powerful gyrotrons of the megawatt class will be used for electron cyclotron resonance heating (ECRH) and current drive (ECCD) of magnetically confined plasma in the thermonuclear reactor ITER. For computer-aided design (CAD), analysis and optimization of their performance numerical experiments based on adequate physical models are used. In this paper, we outline and illustrate the current status of both the available software tools for numerical simulation of such gyrotrons, as well as the novel computer codes of the problem oriented GYREOSS software package which is under development now.

1. Introduction

Gyrotrons are the most powerful sources of coherent radiation in the sub-terahertz frequency range of the electromagnetic spectrum and are being considered as essential and indispensable components of the electron cyclotron wave system for electron cyclotron resonance heating (ECRH) and current drive (ECCD) of fusion plasmas in various thermonuclear reactors (tokamaks, stellarators etc.) including the ITER, as well as for stability control and diagnostics. The success of the first generation of megawatt class tubes operating in long-pulse and CW regime has motivated a European endeavor for the development of multi-megawatt class gyrotrons [1], most notably the 2 MW, 170 GHz gyrotron with a coaxial cavity. It is quite clear that such an increase of the output power would result in a reduction of the number of required units, power supplies, and transmission lines and, eventually, would decrease the installation cost of the whole system and at the same time would increase its performance. In this short paper we outline briefly the physical models and the computer codes (both available and under development) used for computer-aided design (CAD) and optimization of such powerful radiation sources.

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2. Physical models for simulation of gyrotrons
The overall operational performance of the gyrotron depends on the synergy between several subsystems (e.g., electron-optical system (EOS), electro-dynamical system (resonant cavity), quasi-optical output coupler and so on). Below, however, we describe the simulation tools (models and software) pertinent only to the EOS and the resonator. Moreover, the limited space forces us to outline the generic form of the used physical models rather than their detailed computational form.

The longitudinal profile \( f_n(z,t) \) of the field intensity in the cavity is given by the following equation

\[
\frac{\partial^2 f_n(z,t)}{\partial z^2} + \omega_n^2 - \frac{\omega_{\perp,n}^2}{c^2} f_n(z,t) - \frac{2\omega_n}{c^2} \frac{\partial}{\partial t} f_n(z,t) = R_n, \tag{1}
\]

where \( \omega_n \) is the frequency of the \( n \)th mode, \( k_{\perp,n}^2 = \omega_{\perp,n}^2 / c^2 \) is the squared transverse wave number, \( z \) is the axial coordinate and \( t \) is time. The excitation factor \( R_n(t, I_b, U_b, \phi(\alpha, \nu)) \) depends on the beam parameters (such as the current \( I_b \), the voltage \( U_b \) as well as the distribution \( \phi \) of electron velocities \( \nu \) and the pitch factor \( \alpha \) ) and is calculated taking into account the energy balance of the interaction between the beam electrons and the RF electromagnetic field. The solution to equation (1) must satisfy the following radiation boundary conditions at the entrance \( (z = z_{\text{in}}) \) and at the exit \( (z = z_{\text{out}}) \) of the resonator

\[
\frac{\partial f_n^{\text{in}}(z_{\text{in}})}{\partial z} = +ik_{\|n} f_n^{\text{in}}(z_{\text{in}}), \quad \frac{\partial f_n^{\text{out}}(z_{\text{out}})}{\partial z} = -ik_{\|n} f_n^{\text{out}}(z_{\text{out}}), \tag{2}
\]

where \( k_{\|n} = [(\omega_n^2 - \omega_{\perp,n}^2)/c^2]^{1/2} \) is the axial wave number. The motion of the electrons with the rest mass \( m_0 \), velocity \( \nu(r) \) and momentum \( p(r) \) is governed by the equation

\[
\frac{dp(r)}{dt} = F[f_n(z), E_{\text{HF}}(r), B_{\text{HF}}(r), B_0(r)], \quad p(r) = \gamma m_0 \nu(r). \tag{3}
\]

Here \( F(r) \) is the Lorentz force acting on the electrons in the high-frequency electric and magnetic fields \( E_{\text{HF}}(r), B_{\text{HF}}(r) \) and an external magnetic field \( B_0(r) \), and \( \gamma \) is the relativistic Lorentz factor. Both components of the high-frequency electromagnetic field \( (E_{\text{HF}}(r) \text{ and } B_{\text{HF}}(r)) \) are superposition of the corresponding modal fields in the cavity. In fact, an averaged equation for the “slowly varying” quantities is used in the codes instead of equation (3). The more notable simplifications of this self-consistent model are the static (time-independent) model when \( \partial f_n/\partial t = 0 \) and the so-called cold cavity model \( (R_n = 0) \), when the influence of the electron beam on the field in the cavity is neglected.

The efficiency of the interaction \( \eta \) is calculated from \( \eta = \langle (\gamma_0 - \gamma_1)/(\gamma_0 - 1) \rangle \), where \( \gamma_0 \) and \( \gamma_1 \) are respectively the initial and the final values of the relativistic factor and \( \langle \ldots \rangle \) denotes an averaging over the whole ensemble of particles representing the electron beam.

The generic physical model, which is implemented in practically all computer codes for simulation (ray tracing) of the electron-optical system (EOS) of gyrotrons, consists of a self-consistent set of equations:

\[
\nabla^2 \phi(r) = -\frac{\rho(r)}{\varepsilon_0}, \quad \forall r \in \Omega, \quad E(r) = -\nabla \phi(r), \tag{4}
\]

\[
\phi_{\Gamma_i} = \phi_i, \quad \frac{\partial \phi}{\partial t} |_{\Gamma_i} = 0, \quad \Gamma = \sum_k \Gamma_k = \partial \Omega, \tag{5}
\]

\[
\frac{dp(r)}{dt} = F[E(r), B(r)], \quad p(r) = m_0 \nu(r), \tag{6}
\]

\[
p_0(r) = p(r) |_{\Omega_{\Sigma_i}} = f_i (\nu_0, r_0). \tag{7}
\]
Here (4) is the Poisson equation with boundary conditions (5) for the electrostatic potential distribution \( \phi(r) \) and electric field \( \mathbf{E}(r) \) in the domain \( \Omega \) with a closed boundary \( \Gamma \) composed of the segments \( \Gamma_k \), \( \rho(r) \) is the space charge density and \( \varepsilon_0 \) is the permittivity of vacuum. Equation (6) is the relativistic equation of motion for electrons with the initial conditions (7) specified on \( N_S \) different regions \( \Omega_{Se_i} \) \( \Omega_{Se_i} = \bigcup \Omega_{Se_i}, \ \Omega_{Se_i} \cap \Omega_{Se_j} = 0, \ \text{diam}(\Omega_{Se_i}) << 1, \ \ i = 1, 2, \ldots, N_S \) on which the emitting surface \( \Omega_{Se} \) of the cathode is decomposed. The space charge density \( \rho(r) \) is calculated and allocated to the nodes of the computational grid taking into account the conservation of the beam current. The magnetic field \( \mathbf{B}(r) \) is a superposition of the external magnetic field and the magnetic field of the beam current.

3. Computer codes for simulation and CAD of gyrotrons

The physical models outlined above are realized in two groups of software packages for simulation of: (i) the beam–wave interaction in the cavity, and (ii) the formation of the helical electron beam in the EOS, respectively. The first one is represented by the problem-oriented CAVITY software package developed at KIT-IHM by S Kern and co-workers [2]. Its structure and content are illustrated in figure 1. This hierarchy of codes reflects the usual steps in the process of CAD, which habitually starts with an analysis of the geometry and the mode spectrum of the cavity, followed by simplified considerations using for example static, cold cavity and/or single mode physical models and finally utilizes the more adequate (but also the most sophisticated) self-consistent, time-dependent multimode physical model. The latter is implemented in the SELFT code. All components of the CAVITY package are united by a shell (GUI) that allows one to input and edit the data for simulations, to execute a sequence of codes, as well as to visualize the results.

![Figure 1. Structure and content of the CAVITY package (KIT-IHM) [2].](image)

Similar functionality have the cavity codes belonging to the GYROSIM package [3] (developed in FIR-FU, Japan), which is also available to our team. In its current version, however, it is restricted only to the conventional resonators, while CAVITY treats coaxial structures (including such with a corrugation insert) as well.

Figure 2 shows a collage of typical screenshots that illustrate both the graphical user interface (GUI) and the visualization capabilities of the CAVITY package.

The physical model, which describes the formation of a helical electron beam in the EOS (equations (4)–(7)) formulated in two spatial dimensions but taking into account the three components of the electrons’ momentum (i.e., in a 5-dimensional phase space and thus being a 2.5D physical model) is
realized in several packages for trajectory analysis (ray tracing) available to our team, most notably DAPHNE [4] (developed at CRPP-EPFL), ESRAY [5] (developed at KIT-IHM by S Illy) and GUN-MIG/CUSP [3]. Here we outline only the most advanced of them, namely ESRAY. It is characterized by an efficient object-oriented (OO) programming implementation in the C++ language. Both the modular structure of the package (see figure 4) and the OO paradigm of the code make it extensible and allow an easy modification. In ESRAY, the boundary value problem for the electrostatic potential in the physical domain is discretized using a structured boundary-fitted adaptive grid (produced by the GRIDGEN module), which represents the boundary with sufficient accuracy and ensures an appropriate interpolation of all quantities attributed to the nodes. The input to GRIDGEN is a tcl script that describes the geometry of the EOS. The computational grid is created transforming (mapping) the physical domain onto an equidistant logical grid in a rectangle. The magnetic field, produced by a system of solenoids, is calculated by the MAGGEN module. Its off-axis components are computed from the field on the axis using paraxial series expansions. A self-consistent solution to the equations (4)–(7) is obtained in the ESRAY module as a result of an iterative procedure in which the solution of the boundary value problem (Poisson equation) and the integration of electron trajectories follow in a succession until the convergence criterion is satisfied. The results of the numerical experiments are post-processed and can be visualized and exported in a form of informative colour plots. The OVIS GUI is based on an own graphical library (an integral part of the ESRAY package), which is programmed in Tcl/Tk.

Figure 4 illustrates the visualization capabilities of the ESRAY package.

![Figure 4: Visualization capabilities of the ESRAY package.](image)

**Figure 2.** Screenshots, and an example with input and output data of the CAVITY package.

4. **GYREOSS package – a work in progress**

Recently, the current status of the software tools for CAD of the EOS of gyrotrons has been presented
in [4]. A critical examination and benchmarking have revealed not only their advantages and characteristic features but also their limitations. Based on such analysis a concept for further development of the physical models and computer codes have been developed [4]. The realization of this concept is the main aim of the work on the GYREOSS (which stands for GYROtron EOS Simulation). It has been conceived as a problem oriented software package of computer codes united by a common functional assignment to the simulation of the beam formation in the EOS of gyrotrons. A pivotal goal is to include more physics (i.e., to take into account more physical factors and phenomena) extending the currently used 2D models to more adequate self-consistent physical models formulated in three spatial dimensions. A characteristic example in this respect is the work on the development of a 3D emission model, which takes into account the non-uniformity of the extracted beam current and uses more realistic distributions of the initial velocities on the cathode [6]. Such models make the simulations much more computing intensive and require the utilization of efficient numerical methods, algorithms,

**Figure 3.** Main modules of the ESRAY package and their input and output files.

**Figure 4.** Screenshots with typical results from simulations of a coaxial EOS.
programming libraries and integrated development environments. Other two important requirements to the codes under development are their extensibility and portability to different platforms, including high-performance computing (HPC) systems (e.g., multiprocessor workstations, clusters, Grid).

For pre-processing, post-processing and visualization of data from the numerical experiments GYREOSS relies on the 3D finite element mesh generator with a build-in CAD system gmsh [7]. In the first prototype version of the code the boundary value problem for the electrostatic potential was solved using GetDP (General Environment for Treatment of Discrete Problems), which utilizes PETSc or SPARSKIT to solve the corresponding linear systems. It is well known that the two most time consuming procedures in the PIC simulations are the particle pushing and the solution of the field problem. Moreover, the data structure provided by the latter influences strongly the way in which the particles are localized and eventually determines the efficiency of the former procedure. It was found, however, that the algorithms realized using the GetDP are far from optimal. This motivated us to look for an alternative to this solver. Currently we are experimenting with a number of different short-listed libraries. Among them are libmesh, Elmer, GetFEM, FELIB, OpenFOAM, FreeFEM++, just to name a few. Therefore in its current status GYREOSS is mostly a test bed for experimentation with different solvers, programming libraries and algorithms in a pursuit to find the most appropriate building blocks for the novel package modules under development. More information about the current status of the package is available for the reader at the collaborative site of the project (look at http://gyreoss.wikidot.com).

5. Conclusions
Well-validated and efficient simulation packages based on adequate 2D physical models are used for CAD and optimization of high power gyrotrons for ITER. In parallel with their maintenance, constant upgrade and improvement a work on a novel generation of software tools is in progress now. Its goal is the development of computational modules for realization of more informative, adequate and self-consistent physical models formulated in three spatial dimensions.

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