Additivity and non-additivity of multipartite entanglement measures
Huangjun Zhu, Lin Chen and Masahito Hayashi 2010 New J. Phys. 12 083002

New Journal of Physics 13 (2011) 019501
Received 19 September 2010
Published 28 January 2011
Online at http://www.njp.org/
doi:10.1088/1367-2630/13/1/019501

In the appendix, equations (A.2), (A.3) and (A.4) are incorrect. In (A.2) and (A.3), $\rho_N'$ on the right hand side of the equal sign should be replaced by its complex conjugate $\rho_N'^*$. In (A.4), $\rho_N'$ in the last three lines should be replaced by $\rho_N'^*$. There are three consequences of this error in the main text. Firstly, on the right hand side of the last equal sign in equation (60) and equation (61), $\rho_N'$ should be replaced by $\rho_N'^*$. Fortunately, this replacement does not affect any of the results in section 4.

Secondly, proposition 23 should be replaced by the following.

Proposition 23. Suppose $\rho_N$ and $\rho_N'$ are two $N$-partite states on the Hilbert space $\bigotimes_{j=1}^N \mathcal{H}_j$ with $\text{Dim} \mathcal{H}_j = d_j$; define $d_T = \prod_{j=1}^N d_j$; then $G(\rho_N \otimes \rho_N') \leq \log d_T - \log \text{tr}(\rho_N \rho_N'^* N)$. In particular, $G(\rho_N \otimes \rho_N'^*) \leq \log d_T - \log \text{tr}(\rho_N^2 N)$; $G^\infty(\rho_N) \leq \frac{1}{2} G(\rho_N^{\otimes 2}) \leq \frac{1}{2} \log d_T - \frac{1}{2} \log \text{tr}(\rho_N^2 N)$.

The implications are also changed as follows. If the GM of $\rho_N$ is strong additive and thus $G(\rho_N \otimes \rho_N'^*) = G(\rho_N) + G(\rho_N'^*) = 2G(\rho_N)$, proposition 23 implies that $G(\rho_N) \leq \frac{1}{2} \log d_T - \frac{1}{2} \log \text{tr}(\rho_N^2 N)$. In other words, GM cannot be strong additive (as opposed to ‘additive’ in the published version) if the states are too entangled with respect to GM. For states with real entries in the computational basis, proposition 23 sets a universal upper bound for $G(\rho_N^{\otimes 2})$ and $G^\infty(\rho_N)$; that is, $G^\infty(\rho_N) \leq \frac{1}{2} G(\rho_N^{\otimes 2}) \leq \frac{1}{2} \log d_T - \frac{1}{2} \log \text{tr}(\rho_N^2 N)$. As a result, GM cannot be additive if the states are too entangled with respect to GM.

Thirdly, as a result of the revision in proposition 23, theorem 24 must be weakened as follows.

Theorem 24. Suppose pure states are drawn according to the Haar measure from the Hilbert space $\bigotimes_{j=1}^N \mathcal{H}_j$ with $N \geq 3$ and $\text{Dim} \mathcal{H}_j = d_j$ ($d_j \geq 2, \forall j$); define $d_T = \prod_{j=1}^N d_j$ and $d_S = \sum_{j=1}^N d_j$. The fraction of pure states whose GM is strong additive is smaller than $\exp\left(-\frac{1}{2} \sqrt{d_T} + d_S \ln(59Nd_T)\right)$; the fraction of pure states $|\psi\rangle$ such that $[\log d_T - \log(d_S \ln d_T) - \log \frac{d}{2}] \leq G(|\psi\rangle) \leq G(|\psi\rangle \otimes |\psi^*\rangle)$ is larger than $1 - d_T^{-d_S}$. For pure states with real entries in the computational basis, the fraction of pure states whose GM is additive is smaller than $\exp\left(-\frac{1}{2} \sqrt{d_T} + d_S \ln(59Nd_T)\right)$; the fraction of pure states $|\psi\rangle$ such that
\[
[\log d_T - \log(d_S \ln d_T) - \log 9] \leq G(|\psi\rangle) \leq G(|\psi\rangle^\otimes 2) \leq \log d_T \text{ is larger than } 1 - d_T^{-ds}.
\]

The second part of the new version of the theorem, which is concerned with pure states with real entries in the computational basis, is derived in the same way as the first part; more details can be found in the latest version of [1]. Now, theorem 24 only implies that GM is not strong additive (as opposed to ‘non-additive’ in the published version) for almost all pure multipartite states, provided that the number of parties is sufficiently large and the dimensions of the local Hilbert spaces are comparable. However, concerning pure states with real entries in the computational basis, theorem 24 does imply that GM is non-additive for almost all pure multipartite states, and the GM of one copy and two copies of identical states, respectively, are almost equal. Finally, the implications of additivity property for one-way quantum computation and for asymptotic state transformation remain the same as in the published version.

References

[1] Zhu H, Chen L and Hayashi M 2010 arXiv:1002.2511 [quant-ph]