\[ \varepsilon' / \varepsilon \text{ in and beyond the Standard Model} \]

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Abstract

Estimates of the CP violating observable \( \varepsilon' / \varepsilon \) have gained some attention in the past few years. Depending on the long-distance treatment used, they exhibit up to 2.9\( \sigma \) deviation from the experimentally measured value. Such a deviation motivates the investigation of New Physics (NP) effects in the process \( K \to \pi \pi \). In my talk I will review the Standard Model (SM) prediction for \( \varepsilon' / \varepsilon \), with a special focus on the Dual QCD approach. On the NP side, I will discuss a recent computation of the hadronic matrix elements of NP operators. Furthermore a master formula for BSM effects in \( \varepsilon' / \varepsilon \) is presented. Finally, a treatment of \( \varepsilon' / \varepsilon \) using the SM effective theory (SMEFT) will be discussed together with possible correlations to other observables.
1 Introduction

CP violation in the Standard Model (SM) has first been measured in the Kaon sector. The CP violating parameter measured in the famous Cronin-Fitch experiment \[1\] is \(\varepsilon_K\), which describes the mixing between CP and mass eigenstates of the neutral Kaon system. The parameter \(\varepsilon_K\) measures the so-called CP violation through mixing. On the other hand, Kaons can also decay through direct CP violation. This CP violating decay is parametrized by the quantity \(\varepsilon'\). The ratio of the two CP violating parameters \(\varepsilon'/\varepsilon\), where we suppress \(K\) in \(\varepsilon_K\), is also accessible experimentally, namely through a confrontation of the \(K_L \rightarrow \pi^+\pi^-\) and \(K_L \rightarrow \pi^0\pi^0\) decay widths. It has been measured by the NA48 \[2\] and KTeV \[3,4\] collaborations and leads to an experimental world average of

\[
(\varepsilon'/\varepsilon)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}
\]

The SM estimates for this observable depend on the long-distance (LD) treatment used to compute the \(K \rightarrow \pi\pi\) hadronic matrix elements. As can be seen from Tab. 1, the SM prediction differs for the three types of LD approaches and consequently there is some controversy over which treatment to use. The results obtained with Lattice QCD (LQCD) inputs as well as the ones in the Dual QCD (DQCD) approach are in good agreement with each other and exhibit about a 2.9\(\sigma\) deviation from the experimental value in eq. (1). The Chiral Perturbation Theory (\(\chi\)PT) approach leads to a value consistent with the SM, however exhibiting large uncertainties. Moreover the lower part of the error is consistent with the values obtained using Lattice or DQCD and therefore the situation is not conclusive.

Taking the discrepancy between the SM prediction and the experimental value for granted, it is interesting to study beyond the SM (BSM) effect that could explain such deviations. In the following section I will review the SM prediction for \(\varepsilon'/\varepsilon\) based on the DQCD approach. In Sec. 3 the computation of the BSM matrix elements relevant for \(\varepsilon'/\varepsilon\) is discussed. In Sec. 4 a master formula for BSM effects in \(\varepsilon'/\varepsilon\) is presented and in Sec. 5 the relation between \(\varepsilon'/\varepsilon\) and the SM effective theory (SMEFT) is discussed, before I summarize in Sec. 6.

2 \(\varepsilon'/\varepsilon\) in the SM

To describe \(\varepsilon'/\varepsilon\) in a model-independent way, we use the effective Hamiltonian of three quark flavours which generates a \(\Delta S = 1\) transition. It consists of local operators multiplied by their corresponding Wilson coefficients and can be written as follows \[5,8\]:

\[
\mathcal{H}_{\Delta S=1}^{(3)} = - \sum_i C_i(\mu) O_i.
\]

This Hamiltonian is invariant under the unbroken gauge-group \(SU(3)_c \times U(1)_{\text{em}}\) and contains all the fields lighter than the charm quark as dynamical degrees of freedom. The minus sign is chosen to be in accord with the SMEFT conventions.
In the SM, the sum in eq. (2) contains seven four-quark operators consisting of \((V \pm A)\) currents as well as the chromomagnetic operator. The four-quark operators are generated through tree-level and box diagrams containing a \(W\) boson and a gluon, as well as from QCD and Electroweak (EW) penguin diagrams. The seven effective operators can be written as linear combinations of the following vector-vector operators:

\[
O^q_{VAB} = (\bar{s}_i \gamma_\mu P_A d_j)(\bar{q}_j \gamma_\mu P_B q_i), \quad \tilde{O}^q_{VAB} = (\bar{s}_i \gamma_\mu P_A d_j)(\bar{q}_j \gamma_\mu P_B q_i),
\]

where \(P_A, B\) \((A, B = L, R)\) denote the chirality projection operators, \(i, j\) are colour indices and \(q = u, d, s\). The chromomagnetic operator reads:

\[
O_{8g} = m_s (\sigma^{\mu \nu} T^A P_L d) G^A_{\mu \nu},
\]

with \(\sigma^{\mu \nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\), \(T^A\) being the \(SU(3)_c\) generators and \(G^A_{\mu \nu}\) the gluonic field-strength tensor.

Having the Hamiltonian of eq. (2) at hand allows to compute the \(\varepsilon'/\varepsilon\) observable, which is given by:

\[
\frac{\varepsilon'}{\varepsilon} = -\frac{\omega}{\sqrt{2}|\varepsilon_K|} \left[ \frac{\text{Im} A_0}{\text{Re} A_0} - \frac{\text{Im} A_2}{\text{Re} A_2} \right].
\]

Here \(\omega = \text{Re} A_2/\text{Re} A_0 \approx 1/22\), reflecting the \(\Delta I = 1/2\) rule, and \(\varepsilon_K\) is the Kaon mixing parameter mentioned before. The expression is therefore determined by the isospin amplitudes \(A_{0,2}\) defined by

\[
A_{0,2} = \left( \langle \pi\pi |_{I=0,2} \left| \mathcal{H}^{(3)}_{\Delta S=1}(\mu) \right| K \right).
\]

After having fixed the Wilson coefficients of \(\mathcal{H}^{(3)}_{\Delta S=1}\) by performing a matching procedure, the only remaining task is to compute the hadronic matrix elements of the local operators in eq. (2). In the following subsection, we will look into this computation by employing the DQCD approach.

### Table 1: SM estimates for \(\varepsilon'/\varepsilon\), using different treatments of the long-distance effects.

| Long-distance | SM prediction | Group | Ref. |
|---------------|---------------|-------|------|
| Lattice       | \((1.4 \pm 6.9) \times 10^{-4}\) | RBC-UKQCD | \[9, 10\] |
|               | \((1.9 \pm 4.5) \times 10^{-4}\) | Buras/Gorbahn/Jamin/Jäger | \[11\] |
|               | \((1.1 \pm 5.1) \times 10^{-4}\) | Kitahara/Nierste/Tremper | \[12\] |
| DQCD          | \(< (6.0 \pm 2.4) \times 10^{-4}\) | Buras/Gérard | \[13\] |
|               | \(B_6 < B_8 = B_8\) (LQCD) | | |
| \(\chi PT\)   | \((15 \pm 7) \times 10^{-4}\) | Gisbert/Pich | \[14\] |
2.1 Long-distance effects in the DQCD approach

The DQCD is based on the large $N_c$ limit, first studied by t’Hooft \[15,16\] and Witten \[17,18\] for strong interactions. To study hadronic weak decays, the following truncated Chiral Lagrangian is used \[19–21\]:

\[
L_{tr} = \frac{F^2}{8} \left[ \text{Tr}(D^\mu UD_\mu U^\dagger) + r \text{Tr}(mU^\dagger + \text{h.c.}) - \frac{r}{\Lambda^2} \text{Tr}(mD^2U^\dagger + \text{h.c.}) \right],
\]

with the unitary chiral matrix and the octet of lowest-lying pseudoscalars

\[
U = \exp(i\sqrt{2}\Pi/F), \quad \Pi = \sum_{\alpha=1}^{8} \lambda_\alpha \pi^\alpha.
\]

The Lagrangian depends on the quark mass matrix and the chiral enhancement factor

\[
m = \text{diag}(m_u, m_d, m_s), \quad r = \frac{2m_K^2}{m_s^2 + m_d^2}.
\]

It contains a hadronic mass scale $\Lambda_\chi$ corresponding to higher resonances. Employing now the large $N_c$ limit, the Lagrangian of eq. (7) can be matched onto the regular QCD Lagrangian containing quark and gluon fields only. In the chiral limit and at order $O(p^2)$ the quark currents are then given by:

\[
(\gamma^\mu P_L)^{ba} = i\frac{F^2}{4} (\partial^\mu U U^\dagger)^{ab}, \quad (P_L)^{ba} = -\frac{F^2}{8} r(U)^{ab}, \quad (\sigma^{\mu\nu} P_L)^{ab} = 0,
\]

for the flavour indices $a,b$. The chirality flipped versions are obtained by the replacement $U \leftrightarrow U^\dagger$. These relations allow to express the local operators in terms of the lowest-lying mesons and therefore to compute their corresponding matrix elements. Furthermore, this framework allows to study the renormalization group (RG) evolution of the matrix elements up to a scale of $O(1\text{GeV})$ until where the theory is valid. This RG evolution is dubbed meson evolution.

The DQCD approach was first employed in the context of $K \to \pi\pi$ matrix elements in \[19,21,22\]. Its validity is confirmed by results obtained within LQCD. Among them is the correctly predicted hierarchy of the bag factors for the SM operators $Q_6$ and $Q_8$ \[13\]

\[
B_6^{(1/2)} \leq B_8^{(3/2)} < 1.
\]

Also the explicit calculations for $B_6^{(1/2)}(m_c)$, $B_8^{(3/2)}(m_c)$ are in good agreement with the Lattice results \[9,10\]. Not only for the SM four-quark operators but also for the matrix element of the chromomagnetic operator of eq. (4), DQCD \[23\] agrees well with LQCD \[24\]. Furthermore, the impact of final state interactions has been analysed within the DQCD approach in \[25\] and has been shown to be less important for $\varepsilon'/\varepsilon$ than for the $\Delta I = 1/2$ rule, and less important than meson evolution which is responsible for (11).

Finally DQCD also allows, with the help of meson evolution, to understand the pattern of the BSM $K^0 - \bar{K}^0$ mixing matrix elements \[26\] obtained by LQCD \[27–29\]. More information on DQCD can be found in the original papers and in the reviews in \[22,30\].
3 BSM matrix elements for $\varepsilon'/\varepsilon$

Generalizing the SM Hamiltonian by allowing for all possible Lorentz- and gauge invariant operators, one finds that there are 13 additional four-quark operators to be added to $\mathcal{H}^{(3)}_{\Delta S=1}$. Three of them are vector-vector operators which are independent of the seven operators generated within the SM. They can also be written as linear combinations of the operators in eq. (3). The other BSM operators consist of scalar or tensor bilinears and can be written as linear combinations of the following operators:

\begin{align}
O^q_{SAB} = (\bar{s}^i P_A d^j)(\bar{q}^j P_B q^i), & \quad \tilde{O}^q_{SAB} = (\bar{s}^i P_A d^j)(\bar{q}^j P_B q^i), \\
O^q_{TA} = (\bar{s}^i \sigma^{\mu\nu} P_A d^j)(\bar{q}^j \sigma^{\mu\nu} P_A q^i), & \quad \tilde{O}^q_{TA} = (\bar{s}^i \sigma^{\mu\nu} P_A d^j)(\bar{q}^j \sigma^{\mu\nu} P_A q^i),
\end{align}

for $q = u, d, s$. Two equivalent bases for the 13 BSM operators can be found in [31]. The $K \to \pi\pi$ matrix elements of these BSM operators have been calculated for the first time in [31], using the DQCD approach. They were computed first at the factorization scale $\mu_F$ at which the meson representation of eq. (10) holds. The factorization scale corresponds to very low momenta of $O(p^2 \approx 0)$. Since the observable $\varepsilon'/\varepsilon$ is usually computed at the charm scale $\mu_c = O(m_c)$, the running of the matrix elements has to be performed from the factorization scale up to the scale $\mu_c$ via the meson evolution for scales below 1 GeV followed by the usual QCD evolution.

The explicit expressions and numerical values of all the matrix elements at the charm scale as well as further details of the computation can be found in [31]. Here, we summarize only quantitatively the results of the analysis. For the different types of BSM operators, one finds for their respective matrix elements at the factorization scale $\mu_F$ and at the charm scale $\mu_c$:

- Vector operators: small at $\mu_F$ and at $\mu_c$.
- Scalar operators: large at $\mu_F$, moderate at $\mu_c$.
- Tensor operators: zero at $\mu_F$, large at $\mu_c$.
- Scalar/Tensor operators containing three $s$ quarks: zero at $\mu_F$ and at $\mu_c$.

4 Master formula for BSM effects in $\varepsilon'/\varepsilon$

Knowing the matrix elements for the complete set of local effective operators relevant for $\varepsilon'/\varepsilon$ allows for a model-independent analysis of the BSM effects. In this section we provide the means for such an analysis in the form of a master formula for $\varepsilon'/\varepsilon$ [32]. For this purpose, we split the observable in the following way:

$$\frac{\varepsilon'}{\varepsilon} = \left( \frac{\varepsilon'}{\varepsilon} \right)_{\text{SM}} + \left( \frac{\varepsilon'}{\varepsilon} \right)_{\text{BSM}},$$

and focus on the BSM part. Since many NP scenarios contain heavy degrees of freedom with a mass scale above the EW scale, it is reasonable to provide a master formula evaluated at the EW scale $\mu_W$. Consequently, a NP analysis of a particular model only
requires a simple tree-level matching at $\mu_W$. To evaluate eq. (5) at the EW scale, the RG evolution of the matrix elements from $\mu_c$ up to $\mu_W$ has to be taken into account [33,34]. In the running up to the EW scale new operators containing $c$ and $b$ quarks will be generated through QCD and QED mixing, leading to the more general Hamiltonian of five flavours $H^{(5)}_{\Delta S=1}$. The master formula will therefore depend on the Wilson coefficients of all such effective operators. Setting the parameter $\varepsilon_K$ as well as Re($A_0$) and Re($A_2$) appearing in eq. (5) to their experimental values [35] one finds the following master formula:

$$
\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{BSM}} = \sum_i P_i(\mu_W) \text{Im} \left[C_i(\mu_W) - C'_i(\mu_W)\right] \times (1 \text{ TeV})^2,
$$

(15)

with

$$
P_i(\mu_W) = \sum_j \sum_{I=0,2} P_{ij}^{(I)}(\mu_W, \mu_c) \left[\frac{\langle O_j(\mu_c)\rangle_I}{\text{GeV}^3}\right].
$$

(16)

Here, the $P_{ij}^{(I)}$ contain the evolution from $\mu_c$ to $\mu_W$. The matrix elements $\langle O_j(\mu_c)\rangle_I$ are taken from LQCD [9,10] for the SM operators and from DQCD [31] for the BSM operators. The crucial objects determining the impact of each Wilson coefficient on $\varepsilon'/\varepsilon$ are the $P_i$ values. These were obtained using the public codes wcxf [36] for the basis change, wilson [37] for the RG running and flavio [38] to compute $\varepsilon'/\varepsilon$ at the EW scale. The $P_i$ values of the full set of operators contained in $H^{(5)}_{\Delta S=1}$ can be grouped into five classes ($A$ – $E$), which are listed in Tab. 2. The operators either give a direct BSM contribution to $\varepsilon'/\varepsilon$ through their matrix element (ME) or contribute to the observable indirectly through RG mixing. For further details and the explicit values of the $P_i$’s as well as their respective uncertainties we refer to [32].

5  $\varepsilon'/\varepsilon$ meets SMEFT

Assuming that NP manifests itself at scales much higher than the EW scale, the SMEFT [39,40] consists of a valid low-energy effective theory of such a NP scenario. Therefore it is reasonable to adopt the SMEFT as an intermediate theory between any NP model and the SM. This procedure allows to describe NP effects in a model independent way. The complete tree-level matching of the SMEFT onto the weak effective theory is done in [41,42] and in [43] all the SMEFT operators relevant for $\varepsilon'/\varepsilon$ have been identified. There are:

- vector four-quark operators: $O^{(1,3)}_{qq}, O^{(1,8)}_{qu}, O^{(1,8)}_{qd}, O^{(1,8)}_{ud}, O_{dd}$,
- scalar four-quark operators: $O^{(1,8)}_{qud}$,
- modified W and Z couplings: $O^{(1,3)}_{Hq}, O_{Hd}, O_{Hud}$,
- chromomagnetic dipole operator: $O_{dG}$.

An effect in $\varepsilon'/\varepsilon$ stemming from SMEFT operators can result in correlations with other observables. This occurs for operators containing a quark doublet after changing from the
### Table 2: $P_i$ values of the effective operators relevant for $\varepsilon'/\varepsilon$ at the EW scale, grouped into five classes (A-E). The operators either contribute via their matrix element (ME) or through mixing effects to the observable.

| Class | Type          | Operators | $P_i$  | Impact |
|-------|---------------|-----------|--------|--------|
| A     | SM            | $O_{VAB}^{u,d}, \tilde{O}_{VAB}^{u,d}, O_{SLR}^d$ | can be large | ME     |
|       |               | $O_{VAB}^{s,c,b}, \tilde{O}_{VAB}^{s,c,b}, O_{SLR}^{s,c,b}$ | small       | Mixing |
| B     | Chromomagnetic| $O_{8g}$  | small   | Mixing |
|       | Scalar: $s,c,b$ | $O_{SLL}^{s,c,b}, \tilde{O}_{SLL}^{s,c,b}$ | small       | Mixing |
|       | Tensor: $s,c,b$ | $O_{TLL}^{s,c,b}$ | small       | Mixing |
| C     | Scalar: $u$   | $O_{SLL}^u, \tilde{O}_{SLL}^u$ | small       | ME     |
|       | Tensor: $u$   | $O_{TLL}^u, \tilde{O}_{TLL}^u$ | large       | ME     |
| D     | Scalar: $d$   | $O_{SLL}^d$ | small       | ME     |
|       | Tensor: $d$   | $O_{TLL}^d$ | large       | ME     |
| E     | Scalar LR: $u$ | $O_{SLR}^u, \tilde{O}_{SLR}^u$ | can be large | ME     |

flavour to the interaction basis, or through flavour dependent RG mixing effects. In [43] correlations of $\varepsilon'/\varepsilon$ to $\Delta S = 2$ and $\Delta C = 1, 2$ processes, semileptonic Kaon decays, the electroweak $T$ parameter, collider constraints as well as the neutron electric dipole moment (EDM) have been analysed. Furthermore, several tree-level mediator scenarios have been studied, which are summarised in Tab. 3. Further details on correlations of $\varepsilon'/\varepsilon$ and the observables mentioned here can be found in [43].

### 6 Summary

The hadronic matrix elements for the BSM operators relevant for $\varepsilon'/\varepsilon$ have been presented for the first time in [31]. The newly acquired matrix elements allowed for the first time to derive a master formula for $\varepsilon'/\varepsilon$, depending on SM and BSM operators. This master formula is presented in [32] and is already included in several public codes, such as flavio [38] and smelli [44]. Based on this master formula, different correlations of $\varepsilon'/\varepsilon$ to other observables have been analysed in the context of the SMEFT in [43].

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Table 3: Tree-level models, which can have a sizable effect in $\varepsilon'/\varepsilon$ and their correlations to other observables.

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