From neutrino oscillations to baryogenesis *

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The evidence for neutrino oscillations leads to small neutrino masses, which can be realized by means of the seesaw mechanism. In this framework, baryogenesis may be achieved from leptogenesis.

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I. Brief history of neutrino oscillations

The concept of neutrino oscillations was introduced by B. Pontecorvo in 1957 \[1\]. He considered $\nu - \overline{\nu}$ oscillations in vacuum, in analogy to $K - \overline{K}$ oscillations \[2\].

Flavor mixing was proposed by Maki, Nakagawa and Sakata in 1962 \[3\]. According to this idea, two weak (flavor) eigenstates $\nu_\alpha$ are related to two mass eigenstates $\nu_i$ by a rotation $U$, that is $\nu_\alpha = U_{\alpha i} \nu_i$. More generally, $U$ is a unitary matrix.

Flavor oscillations of Majorana neutrino were introduced by B. Pontecorvo in 1967 \[4\]. In this paper he also anticipated the solar neutrino problem, since he pointed out that, due to neutrino oscillations, the observed flux of solar neutrinos should be half of the expected flux. In fact, in 1968, a deficit of solar neutrinos ($\nu_e$) was found \[5\], with respect to the theoretical calculation on the basis of the solar model \[6\]. Then, in 1969, Gribov and Pontecorvo proposed the solution of the solar neutrino problem by means of neutrino oscillations in vacuum \[7\]. The pattern of oscillations is modified in matter \[8\]. Several years later, the atmospheric neutrino anomaly was discovered \[9\], that is a deficit in atmospheric neutrinos ($\nu_\mu$).

Evidence for neutrino oscillations in atmospheric neutrinos was indeed found in 1998 by the SuperKamiokande experiment \[10\]. Then, in 2002, evidence for neutrino oscillations in solar neutrinos has been also found \[11\]. Finally, in 2003, terrestrial evidence for $\overline{\nu}_e$ oscillations from reactor neutrinos \[12\] and terrestrial evidence for $\nu_\mu$ oscillations from accelerator neutrinos \[13\] are achieved.

Hence, the solar neutrino problem is solved by $\nu_e - \nu_{\mu,\tau}$ matter oscillations in the sun, while the atmospheric neutrino anomaly is explained by $\nu_\mu - \nu_\tau$ vacuum oscillations.

II. Neutrino masses and mixings

The oscillation formula is given by the expression

$$P \simeq \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E},$$

where $P$ is the probability of transition at distance $L$ from the source, $E$ is the energy of neutrinos, $\theta$ is the mixing angle, and $\Delta m^2$ is the square mass difference between the two mass eigenstates involved in the process. Therefore, neutrino oscillations imply neutrino
masses and mixings. From oscillation data, the following values are inferred:

\[ \Delta m_{32}^2 = m_3^2 - m_2^2 \simeq 2.7 \cdot 10^{-3} \text{eV}^2, \]  
\[ \Delta m_{21}^2 = m_2^2 - m_1^2 \simeq 7.1 \cdot 10^{-5} \text{eV}^2, \]  

where the three masses \( m_1, m_2, m_3 \) are the effective neutrino masses. The lepton mixing matrix, is given by the approximate form

\[ U \simeq \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \]  

This expression yields large \( U_{e2} \), near maximal \( U_{\mu 3} \), and small \( U_{e3} \). Hence, lepton mixings can be large and even maximal, while quark mixings are all small.

Neutrino masses are very small with respect to charged fermion masses. In fact, from beta decay experiments we get

\[ m_{\nu_e} = (U^2_{ei} m_i^2)^{1/2} < 2.2 \text{eV}, \]  

which gives \( m_i \lesssim 1 \text{eV} \). Also from cosmology we get \( \sum m_i \lesssim 1 \text{eV} \). Moreover, for Majorana neutrinos, a upper limit comes from neutrinoless double beta decay,

\[ M_{ee} = (U^2_{ei} m_i) < 0.86 \text{eV}. \]  

In contrast, charged fermion masses span the range going from \( m_e \sim 1 \text{MeV} \) to \( m_t \sim 100 \text{GeV} \). We should find a mechanism for generating very small neutrino masses.

### III. Dirac and Majorana masses

A Dirac mass term can be written in the form

\[ m_D \bar{\psi}_R \psi_L + h.c.. \]  

It conserves electric charge and lepton number.

Majorana mass terms for left-handed and right-handed particles can be written as

\[ m_L (\bar{\psi}_L)^c \psi_L + h.c., \]
respectively. They violate electric charge and lepton number, thus are allowed only for neutral particles, in particular neutrinos.

A Dirac neutrino is expressed by the field \((\nu_L, \nu_R)\), which under charge conjugation goes into \((\nu_R^c, \nu_L^c)\). Instead, Majorana neutrino fields are given by \((\nu_L, (\nu_L)^c)\) and \((\nu_R^c, \nu_R)\), which are both self-conjugate. Majorana fields contain only one type of Weyl spinor, left or right, while Dirac fields contain both types of spinors.

IV. Seesaw mechanism

If both Dirac and Majorana masses are present, the full mass matrix of neutrinos is

\[
M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix},
\]

Let us assume that \(m_L = 0\), and \(m_R \gg m_D\). Then \(M\) has mass eigenvalues nearly equal to \(m_R\) and \(m_D^2/m_R\). The latter can be written as \((m_D/m_R)m_D\), so that it is much smaller than \(m_D\). This is the seesaw mechanism \[14\]. The corresponding two eigenstates are of the Majorana kind, see for example \[15\]. For three generations of fermions, we have the seesaw formula for mass matrices,

\[
M_L \simeq M_\nu M_R^{-1} M_\nu^T,
\]

where \(M_L\) is the effective mass matrix of (light) left-handed neutrinos, \(M_\nu\) is the Dirac neutrino mass matrix, and \(M_R\) is the mass matrix of (heavy) right-handed neutrinos.

In the minimal standard model, the neutrino is massless because \(\nu_R\) does not exist. The minimal extension is then adding \(\nu_R\). Gauge extension, such as the left-right model, the Pati-Salam model, and the \(SO(10)\) unified model, do include the right-handed neutrino. It is natural for the Dirac mass to be similar to charged fermion masses, since all are generated from couplings to the same Higgs fields. In particular, we expect \(M_e \sim M_d\) and \(M_\nu \sim M_u\). Instead, the Majorana mass matrix of right-handed neutrinos is generated as bare mass term or by coupling to another Higgs field, so that its value can be very large. These conditions lead to the seesaw mechanism.
Moreover, a seesaw enhancement of lepton mixing can appear [16]. For instance, let us take

\[
M_\nu \simeq \begin{pmatrix}
0 & a & 0 \\
 & a & b & c \\
 & & 0 & 1
\end{pmatrix} m_t,
\]

(12)

with \(a \ll b \sim c \ll 1\), and [17]

\[
M_R \simeq \begin{pmatrix}
c^2 & 0 & 0 \\
0 & c^2 & 0 \\
0 & 0 & 1
\end{pmatrix} m_R.
\]

(13)

Then we obtain

\[
M_L \simeq \begin{pmatrix}
k^2 & k & k \\
k & 1 & 1 \\
k & 1 & 1
\end{pmatrix} \frac{m_t^2}{m_R},
\]

(14)

with \(k = a/c\). Other minimal forms which give large lepton mixing are [17]

\[
M_R \simeq \begin{pmatrix}
ac & 0 & 0 \\
ac & 0 & 0 \\
0 & 0 & 1
\end{pmatrix} m_R.
\]

(15)

\[
M_R \simeq \begin{pmatrix}
\frac{a^2}{c} & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix} m_R.
\]

(16)

In another approach, we may invert the seesaw formula,

\[
M_R \simeq M_\nu^T M_L^{-1} M_\nu,
\]

(17)

in order to determine the heavy neutrino mass matrix. Assuming \(M_\nu \sim M_u\), that is \(a \sim \lambda^6, c \sim \lambda^4\), we get two possible forms [18]

\[
M_R \sim \begin{pmatrix}
\lambda^{12} & \lambda^{10} & \lambda^6 \\
\lambda^{10} & \lambda^8 & \lambda^4 \\
\lambda^6 & \lambda^4 & 1
\end{pmatrix} \frac{m_t^2}{m_k},
\]

(18)
with eigenvalues $M_1/M_2 \sim \lambda^4$, $M_1/M_3 \sim \lambda^{12}$, and

$$M_R \sim \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & \lambda^4 & 1 \\ 0 & 1 & 0 \end{pmatrix} \lambda^6 m_i^2/m_1,$$  \hspace{1cm} (19)$$

with eigenvalues $M_1/M_2 \sim \lambda^6$, $M_1/M_3 \sim \lambda^6$. The first form has small mixings, while the second one has large mixing in the 2-3 sector.

V. Baryogenesis from leptogenesis

It is interesting to see the implications of the previous matrix models for the baryogenesis via leptogenesis mechanism \[19\]. This is based on the out-of-equilibrium decays of the right-handed neutrinos, which produce a lepton asymmetry, partially converted to a baryon asymmetry by electroweak sphalerons. The baryon asymmetry is given by

$$Y_B \simeq \frac{1}{2} \frac{1}{g^*} d \epsilon_1,$$ \hspace{1cm} (20)$$

where $\epsilon_1$ can be written as

$$\epsilon_1 \simeq \frac{3}{16\pi v^2} \left[ \frac{(M_D^2M_D^\dagger)^2_{12} M_1}{(M_D^2M_D^\dagger)^2_{11} M_2} + \frac{(M_D^2M_D^\dagger)^2_{13} M_1}{(M_D^2M_D^\dagger)^2_{11} M_3} \right].$$ \hspace{1cm} (21)$$

Here, $M_D$ is the Dirac neutrino mass matrix in the basis where $M_R$ is diagonal, and $M_i$ are the heavy neutrino masses. The parameter $d < 1$ is a dilution factor, and $g^* \simeq 100$. The allowed value for the baryon asymmetry is $Y_B \simeq 9 \cdot 10^{-11}$.

For matrix models (13) and (15), sufficient baryon asymmetry can be obtained, while matrix model (16) give too small asymmetry.

For the two matrix models (18) and (19), we get a too low baryon asymmetry \[18\]. Then, we proposed another mass matrix model, where the overall mass scale of $M_\nu$ is again $m_t$, but the internal hierarchy is that of $M_d$ and $M_\ell$, namely $a \sim \lambda^3$, $c \sim \lambda^2$. In this case a sufficient amount of baryon asymmetry is achieved \[18\].

For other recent studies of the relation between seesaw mechanism and leptogenesis, see Refs. \[20, 21\]. In particular, baryon asymmetry is enhanced for $M_1 \sim M_2$, as happens for models (13) and (15).
VI. Conclusion

It is quite impressive the chain which takes us from neutrino oscillations to small neutrino masses, to the seesaw mechanism, and to baryogenesis through leptogenesis. If the seesaw mechanism is indeed correct, we should find confirmed evidence for the neutrinoless double beta decay.

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