General consensus with circular opinion under attractive and repulsive mechanisms

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In this work, we study a nonlocal opinion dynamics in a ring of agents with circular opinion in the presence of both attractive and repulsive interactions. We identified three types of consensus in this model, including global consensus, local consensus, and chimera consensus. In global consensus, both local agreement among adjacent agents and global agreement among all agents are achieved. In local consensus, local agreement is satisfied but global agreement fails. There are two domains in chimera consensus, one preserves local agreement and the other breaks the local agreement. The relation between the opinion difference between adjacent agents and the interaction radius is investigated and a scaling law is found. The transitions between local consensus and chimera consensus are exemplified.

I. INTRODUCTION

Everyday, we encounter a variety of situations in which we have to make decisions following our opinions, typical examples including political campaigns, human society could be free of conflicts if all agents share the same opinion on issues they encounter. On the other hand, human society may be more energetic if different opinions coexist with each other. Opinion dynamics models the opinion formation by focusing on the interaction and the communication among individuals. There exist different types of models on opinion dynamics. Opinion model may be classified into discrete opinion models and continuous opinion models by whether opinion is represented as discrete numbers or continuous ones. The voter model, the Galam majority-rule model, and the Sznajd model are examples for discrete opinion models. Deffuant-Weisbuch (DW) model, and Hegselmann-Krause model are typical continuous opinion models.

The DW model is the one of the most famous continuous opinion models. In the DW model, there exists a bounded confidence. Two agents interact with each other by reducing their opinion difference with a convergence rate if their opinion difference is less than the bounded confidence. Depending on the bounded confidence, the model displays different asymptotic states, fragmentation where several opinion clusters coexist, polarization where there only exist two opinion clusters, and consensus where all agents share a same opinion. The concept of consensus is widely applied in different disciplines, such as sociophysics, management, and automatical control, which has drawn much attention on it. Concerning with the consensus, many mechanisms have been proposed, including the introduction of inflexible minorities, heterogeneous bounded confidence, heterogeneous convergence rate, effects of social power, effects of leadership, evolutionary games based model, and the effects of communication burstiness.

The DW model is actually defined on an all-to-all network in which each agent may interact with the rest of population provided that the opinion difference between them is within the bounded confidence. In reality, interaction among agents always forms sparse complex networks. Recently, opinion dynamics on sparse complex networks has been investigated where agents can interact with their nearest neighbors. Besides all-to-all networks accounting for global interaction among agents and sparse networks for local interaction, nonlocal interaction where every agent interacts with a finite fraction of population also draws interests from scientists. Nonlocal interaction may induce interesting dynamics. It has been shown that nonlocally coupled oscillators may produce a intriguing dynamical state, chimera state, in which coherent domains coexist with incoherent ones. It is interesting to investigate whether nonlocal interaction can induce interesting dynamics in opinion models. Then, the DW model considers the attractive interaction in which agents tend to reduce their opinion differences. However, repulsive interaction is also common in reality, which has been modeled in many social systems. In opinion models, the role of repulsive interaction has been studied by assigning links among agents to be either attractive ones or repulsive ones. However, consider two agents. It is more likely for them to be enemy if their opinions are far away from each other and to be friend otherwise. In modelling opinion dynamics, friends tend to adopt attractive interaction while enemies tend to adopt repulsive interaction. In most of continuous opinion models,
opinions are represented as real numbers in the range from 0 to 1. The description of opinion like this allows for the existence of extreme opinions, for example the opinion 0 and the opinion 1, and its simplification goes to binary opinion. There are some works considering circular opinion where opinions are represented as real numbers on a ring with unit length and some interesting phenomena have been found [31]. In the description of circular opinion, the extreme opinions are absent. The circular opinion could be justified by social phenomenon that people convert their faith from one religion to another [32].

In this work, we investigate a nonlocal DW model with circular opinion under attractive and repulsive interactions on a ring-like network. We identify three types of consensus, global consensuses, local consensus, and chimera consensus. All agents share a same opinion in the global consensus. In local consensus, the opinions of adjacent agents are close to each other while the opinion difference between distant agents are sufficiently large. In other words, local agreement among agents exists while global agreement is absent in local consensus. Chimera consensus is a new type of state involving consensus in which local agreement is violated within a domain of agents and is preserved in rest of population.

The rest of this paper is organized as follows. In section II, we introduce the model. The numerical results and discussions are presented in section III. The properties of three types of consensus and their dependence on the interaction range of agents are investigated. Finally, the conclusion is drawn in section IV.

II. MODEL

We consider a population of $N$ agents sitting on a ring. Each agent is represented by a node index $i$, which is taken modulo $N$. We assume nonlocal interaction among agents in which every agent may interact with $k$ neighbors on each side. That is, agents $i$ and $j$ may interact with each other if $\min(|i-j|,N-|i-j|) \leq k$. The interaction among agents reduces to a local one for $k = 1$, while it becomes a global one when $2k = N - 1$. For convenience, we define the interaction radius $p = k/N$, which is in the range $(0, 0.5)$. The opinion of agent $i$ is represented as $x_i(t) \in [0, 1]$ at the time step $t$. We consider circular opinion $x_i \in [0, 1]$ such that $x_i(t) = (x_i(t) \mod 1)$. For circular opinion, the opinion difference between agents $i$ and $j$ is defined as $|\delta_{i,j}| = \min\{|x_i - x_j|, 1 - |x_i - x_j|\}$. Therefore, the upper bound of opinion difference between agents is 0.5. The updating rule of agents’ opinions follows a modified Deffuant-Weisbuch rule incorporating both attractive and repulsive interactions [31], which takes into considerations a pairwise interaction one. There are two parameters, the bounded confidence $\sigma \in (0, 0.5]$ and the convergence rate $\mu \in (0, 0.5]$. Initially, each agent is assigned an opinion randomly chosen from the interval $[0, 1)$ or specified. At each time step, two neighboring agents, agent $i$ and agent $j$, are chosen at random. If the opinion difference between them is less than the bounded confidence ($|\delta_{i,j}| < \sigma$), they adopt attractive interaction and update their opinions to get closer. If their opinion difference is larger than the bounded confidence ($|\delta_{i,j}| > \sigma$), they adopt the repulsive interaction to drive their opinions further away from each other. Combining the attractive and repulsive interactions together, the opinion updating rule is written as

$$\begin{align*}
x_i(t + 1) &= x_i(t) + \mu \Theta(\sigma - |\delta_{i,j}(t)|) \delta_{i,j}(t), \\
x_j(t + 1) &= x_j(t) + \mu \Theta(\sigma - |\delta_{i,j}(t)|) \delta_{i,j}(t).
\end{align*}$$

where $\Theta(y)$ is defined as $\Theta(y) = 1$ for $y > 0$ and $\Theta(y) = -1$ otherwise. One Monte Carlo time step consists $N$ such events. Throughout this work, the opinions of agents are updated asynchronously.

We characterize the asymptotic states in the model using two methods. Firstly, we observe the opinion profile displaying the agents’ opinions $x_i(t)$ against their locations. Secondly, we monitor the average opinion difference between adjacent agents, which is defined as

$$\Delta = \langle \sum_{i=1}^{N} |\delta_{i+1,i}|/N \rangle_t$$

where $\langle \cdot \rangle_t$ means the time average over 100 Monte Carlo time steps after $10^6$ transient time steps.

III. RESULTS AND DISCUSSION

We focus on consensus. The bounded confidence determines the asymptotic state in opinion model and $\delta > 0.5$ assures the realization of consensus in typical DW models. The convergence rate $\mu$ is always set to be 0.5 in most of works. However, it has been shown that $\mu$ plays important role in opinion dynamics by controlling the time scale in the evolution of opinion. Considering the existence of both the attractive and repulsive interactions among agents, we set $\mu = 0.2$ to foster the competition between these two types of interaction. Furthermore, we set $\sigma = 0.3$ for the purpose to realize consensus. The
the situation with local interaction. Clearly, different realization of \( N \) size large opinion difference between distant agents breaks the full turns around the opinion space when it traverses \( n \). Typical DW models. Its opinion profile is a straight horizontality but in the sense of an overall trend. In comparison with the situation with local interaction, the exact order of \( x_i \) with \( i \) is lost since agents with a little large distance may still be neighboring ones and they may interact with each other. As a result, the opinion difference between adjacent agents fluctuates around zero and the fluctuation increases with \( p \). These features reflect the characteristics of \( LCs \). In \( LCs \), neighboring agents tend to reach local agreement among them. For large \( p \), every agent is required to be in local agreement with his neighbors on both sides. This trades off the loss of exact order of \( x_i \) with \( i \). On the other hand, the attractive interaction among the neighboring agents assures the opinion convergence locally even though the opinion differences among them may be large. Thus, the opinion profiles of \( LCs \) for large \( p \) display fluctuations and the monotonic variation with \( i \) in an overall way. To be mentioned, since the local agreement exists in \( LCs \), attractive interaction among agents still plays dominant role.

The first type of consensus is the global consensus (GC) which refers to a state where all agents hold a same opinion (the global agreement) and is the one found in typical DW models. Its opinion profile is a straight horizontal line, as shown in Fig. 1. The opinion difference between adjacent agents is 0 in the time limit \( t \to \infty \). However, different from typical DW models \cite{33, 34}, the average opinion of the initial opinion may not be the final opinion of all agents, due to the circular opinion mechanism. In GC, only the attractive interaction takes effect. To be noted, GC is always stable in the model, which is independent of the system size \( N \) and the interaction radius \( p \). Perturbation to GC dies off quickly due to the attractive interaction.

The second type of consensus is the local consensus (LC). Figure 2 shows the opinion profiles for \( LC \) at different parameters. In a typical \( LC \), the opinion makes \( n \) full turns around the opinion space when it traverses from agent 1 to agent \( N \) and, at the same time, \( x_i \) changes with \( i \) monotonically. We denote \( LC \) with \( n \) full turns of opinion as \( LC_n \). The local agreement refers to the situation in which the opinion differences between any adjacent agents are sufficiently small. In an \( LC \), the local agreement among adjacent agents is achieved but the large opinion difference between distant agents breaks the global agreement. Figures 2(a-c) show \( LCs \) for \( p = 1/N \) (the situation with local interaction). Clearly, \( LC_n \) with different \( n \) may be realized depending on initial condition and the realization of \( LC_n \) is independent of the system size \( N \). Then, we present \( LCs \) in Figs. 2(d-f) for nonlocal interaction. We find that \( x_i \) still changes with \( i \) monotonically but in the sense of an overall trend. In comparison with the situation with local interaction, the exact order of \( x_i \) with \( i \) is lost since agents with a little large distance may still be neighboring ones and they may interact with each other. As a result, the opinion difference between adjacent agents fluctuates around zero and the fluctuation increases with \( p \). These features reflect the characteristics of \( LCs \). In \( LCs \), neighboring agents tend to reach local agreement among them. For large \( p \), every agent is required to be in local agreement with his neighbors on both sides. This trades off the loss of exact order of \( x_i \) with \( i \). On the other hand, the attractive interaction among the neighboring agents assures the opinion convergence locally even though the opinion differences among them may be large. Thus, the opinion profiles of \( LCs \) for large \( p \) display fluctuations and the monotonic variation with \( i \) in an overall way. To be mentioned, since the local agreement exists in \( LCs \), attractive interaction among agents still plays dominant role.

GCs result from attractive interaction while circular opinion, together with attractive interaction, are responsible for \( LC \). When the effect of repulsive interaction is involved, we find the third type of consensus, chimera consensus (CC). Typical CCs are shown in Fig. 3. Figures 3(a-c) show the results for local interaction with \( p = 1/N \). The opinion \( x_i \) makes a half turn when agent goes from 1 to \( N \) denoted as \( CC_1 \) in Fig. 3(a) while one and a half turns (denoted as \( CC_2 \) in Figs. 3(b,c)). The local agreement is violated only for one pair of agents whose opinion difference becomes \( \sigma \), which implies the repulsive interaction between them. Figures 3(d-f) for nonlocal interaction show that \( x_i \) also performs an extra half turn after several full turns when \( i \) goes through 1 to
N and the local agreement fails in a finite domain. In the domain breaking the local agreement, agents’ opinions have a gap around 0.5, which is allowed by the repulsive interaction, and they fluctuate greatly from one agent to another, which is due to the nonlocal interaction. The size of the domain breaking the local agreement is determined by the interaction radius \( p \) and displays a positive correlation with \( p \). The coexistence of domains supporting the local agreement and breaking the local agreements in CCs resembles the dynamical chimera states in nonlocally coupled systems [35] if we treat the domain supporting (or breaking) the local agreement as coherent (or incoherent) domain. In addition, in a CC, agents’ opinions in the domain holding the local agreement are frozen while they fluctuate in time in the domain breaking the local agreement. Briefly, CCs are much similar to LCs except that the local agreement is broken in the incoherent domain.

In GCs, all agents share a same opinion, which means that the average opinion difference \( \Delta \) in the population is zero. However, the global agreement is absent in LCs and CCs, which allows for nonzero average opinion difference \( \Delta \). Actually, \( \Delta \) measures the fluctuation of the opinion differences between adjacent agents. The above results have suggested that the fluctuation of the opinion differences between adjacent agents strongly depends on the interaction radius \( p \) for LCs and CCs. Here, we consider the dependence of \( \Delta \) on \( p \). Since there are plenty of LCs and CCs with opinion \( x_i \) making different full turns, we just consider \( LC_1 \) with one full turn and \( CC_1 \) with a half turn for convenience. Including the opinion gap in the domain breaking the local agreement in CC, opinion in the population transverses a full circle in the opinion space for both \( LC_1 \) and \( CC_1 \). For each consensus state, we consider two situations, one with fixed population size \( N \) and the other with fixed number of neighbors of every agent \( 2k \). As shown in Fig. 4(a) the average opinion difference \( \Delta \) always scales with \( p \) at a same exponent around 1, which is independent of the type of consensus, the population size, and the number of neighbors of an agent. The deviation of the dependence of \( \Delta \) on \( p \) from the scaling law in Figs. 1(a,c) for \( N = 1000 \) is induced by the fact that \( \Delta \) is restricted by its minimum \( 1/N \).

It is of interest to have some discussions on the stabilities of these three types of consistencs. First of all, our model allows for multistability and these three consistenices may coexist in a large range of the interaction radius \( p \). Especially, GC is always locally stable to weak perturbation. On the other hand, CCs are more stable than LCs at large \( p \) while LCs are more stable than CC at small \( p \), which can be hinted in Fig. 4 for example, there is no data for CCs at small \( p \) and \( N = 1000 \) in Fig. 1(c). When \( p \) is sufficiently high, both LCs and CCs yield to GC. We take two examples with the population size \( N = 1000 \) to show the transitions between LCs and CCs. At \( p = 0.005 \), we prepare a \( CC_1 \) as initial conditions in which the opinion gap occurs between agents \( i = 500 \) and \( j = 501 \) and run the model. Figure 5(a) shows five snapshots of the opinion profile at different times and Fig. 5(b) shows the spatiotemporal evolution of agents’ opinions over \( 10^4 \) Monte Carlo time steps. To illustrate how such a transition occurs, we consider two agents lo-
cating on the two sides of the opinion gap, agent $i_1$ (e.g., $i_1 = 498$) and agent $i_2$ (e.g., $i_2 = 502$). As time evolves, $x_{i_1}$ (or $x_{i_2}$) could be pushed up (or pulled down) by his neighbors on the right (or left) side of the gap through the repulsive interaction. When $x_{i_1} - x_{i_2}$ becomes less than $\sigma$, the attractive interaction between agents $i_1$ and $i_2$ steps in and draw them closer. Similar behaviors may occur to other agents near the opinion gap. Resultantly, a $CC_1$ transforms to a $LC_1$. To be noted, when $p = 0.001$ (the local interaction), $CC$ is stable since the absence of nonlocal interaction. At $p = 0.18$, Figs. 3(c,d) show the transition from a $PC_1$ to $CC_1$. In this example, starting from a $PC_1$, the local agreement is quickly broken in a certain area and the opinion profile evolves into a $CC_1$

In closing, we make some discussions. The three consensuses, $GC$, $LC$, and $CC$, are differentiated by the breaking or not of the local agreement and the global agreement. However, the local agreement always exists if we ignore the opinion gap in the incoherent domain in $CCs$. Following this line, the three types of consensuses may be termed under the same rule according to how many turns opinion $x_i$ from agent 1 to agent $N$ has made in the opinion space. For example, $GC$ is actually a special case of $LC$ and can be denoted as $LC_0$. $CC$ can be denoted as $LC_{2n}$ with $n$ an odd integer. Furthermore, we assume that positive $n$ suggests an upward trend in opinion profiles while negative $n$ a downward trend. In this way, all of these consensuses can be represented as $LC_{2n}$ with integer $n$.

IV. CONCLUSION

To conclude, we have studied the opinion dynamics in a ring of agents with circular opinion under nonlocal attractive and repulsive interaction. Three types of consensuses, including the global consensus, the local consensus, and the chimera consensus, are identified by the opinion profiles. The global consensus with the global agreement has been well studied in previous works. The local consensus which only requires the local agreement was found only very recently [31]. The chimera consensus divides the population into two domains, one preserves the local agreement and the other breaks the local agreement. We studied the dependence of the average opinion difference between adjacent agents on the interaction radius and found a scaling law between them. We found that chimera consensus is more stable than local consensus at large interaction radius while local consensus is more stable than chimera state at small interaction radius.

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