Zero temperature phase diagram of finite connectivity spin glasses

Florent Krzakala

Dipartimento di Fisica, INFM and SMC, Università di Roma “La Sapienza”,
P. A. Moro 2, 00185 Roma, Italy

The zero temperature phase diagram of spin glasses on finite connectivity graphs is investigated, with or without magnetic field and/or ferromagnetic bias, for mean field (using the cavity method) and Edwards-Anderson (using numerical ground states computations) models. In the mean field case we show that the phase diagram is complex and compute the equivalent of the zero temperature de Almeida-Thouless line. In the 3d model however, we found a trivial phase diagram. This paper presents new analytical results as well as a rapid review of numerical works on ground states.

Since its proposal in 1975[1] the Edwards-Anderson (EA) model of spins glasses (SG) — an Ising model with ferromagnetic and anti-ferromagnetic interactions — has been the subject of many studies and controversies[2]. A simple question such as the qualitative shape of the phase diagram is still matter of debates[3]. Here, we investigate the generic phase diagram of finite connectivity spin glasses at zero temperature, with magnetic field and/or ferromagnetic bias. It is motivated by the recent progresses in both numerical studies of $T = 0$ spin glasses (mainly obtained by borrowing tools from computer science such as combinatorial optimization[4]) as well as in analytical studies of finite connectivity mean field systems[5] that last years have witnessed, thus allowing for a direct comparison. In the following, we first consider spin glasses on random graphs where new analytical results for the phase diagram and the zero-temperature equivalent of the de Almeida-Thouless (dAT) line[6] are presented. We then consider 3d spin glasses, using numerical computation of ground states, finding in that case a trivial phase diagram, without any dAT line.

Mean field results- Most of mean field predictions were derived within the fully connected Sherrington-Kirkpatrick model using the replica trick solution of G. Parisi[3] characterized by a so-called Replica Symmetry Breaking (RSB), or many-valleys picture, in which the energy landscape is divided into many different non ergodic phases with a complex ultrametric structure[3]. The phase diagram in presence of a field or with a ferromagnetic bias is quite rich too: there is a SG phase at low enough field and, when the concentration of ferromagnetic bonds is enhanced, a ‘mixed’ phase with both RSB and spontaneous magnetization appears. However, real spin glasses do have a finite connectivity and therefore finite connectivity mean field models should be used to study them; the simplest way of defining them is to consider fixed connectivity random graphs[7] that we will refer to as Bethe lattices. At $T = 0$, this problem allows many simplifications and computing the phase diagram using the cavity[8] method turns out to be somehow easy[9]. We present our

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[1] Our $T = 0$ computation of the dAT line was made using a RSB stability analysis equivalent to the bug proliferation method[10] (we send the reader to[11] for a similar computation in the absence of a magnetic field) and independently confirmed by a numerical 2-replicas cavity analysis a la[12].
new results for the phase diagram of mean field SG with discrete interactions in fig.1 where we considered as parameters the applied magnetic field and the excess concentration of ferromagnetic bonds. The most important features of this phase diagram are (a) the existence of an equivalent to the dAT line at $T = 0$, that separates the paramagnetic phase at high field from a SG phase at lower field *) and (b) like in the fully connected model, the presence of a mixed phase for high enough concentration of ferromagnetic bonds, where both a SG ordering and a spontaneous magnetization do set in. The ferromagnetic ordering was studied here at the RS level, but RSB corrections can be computed: we have shown recently that in fact that they even increase the size of the mixed phase.9)

So far we have derived prediction for mean field models; however, and this is the whole point of the controversy, it is well known that in the scaling/droplet approach11), 12) the phase diagram is trivial (see for instance the Imry-Ma arguments in11),13) or the Migdal-Kadanoff approach of14)): there is no dAT line nor there is a mixed phase and the phase diagram is the same as for usual ferromagnet. The two theories (mean field vs scaling) are therefore in conflict and it is still a very debated question to determine which theory is correct.4) Now that we have a clear idea of what are their predictions at zero temperature, let us see what we can say for the 3d EA model. To do so, we will now resort to ground state computations.6)

3d results- We focus on two questions: (1) Is there a SG phase when a magnetic field is applied? (2) Is there a mixed phase when the concentration of ferromagnetic bond is increased? To detect such a SG phase in ground state simulations, one has to find a relevant order parameter. From what we know about spin glasses, a very

*) We computed $H_c$ in many cases and found (for $c = 6$) $H_c(\rho = 0) \approx 2.45$ for $\pm J$ couplings, $H_c \approx 1.9$ (as10) for Gaussian $J_{ij}$s with uniform field, and $H_c \approx 2.4$ for Gaussian $J_{ij}$s with Gaussian random field of mean 0 and standard deviation $H$. For $c = 8$, we found (resp.) 3.25, 2.7 and 3.7.
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A good candidate is the following: we say that we have a SG phase if, starting from
the ground state, it is possible to find a system-size excitation (that is an excitation
involving a $O(N)$ spins, $N$ being the size of the system) of low energy (typically $O(1)$)
with a probability constant (in mean field) or decreasing with the size of the system
like a power-law (in the droplet/scaling approach)$^\star$. Over the last few years, we have
developed numerical procedures to detect and extract such excitations, based on the
fact that they have a highly non trivial topology, and we named them sponges.$^{15,16}$
In the following the probability that a random excitation $O(1)$ energy is a sponge
will be used as the order parameter. Let us now review the results of our studies.

First, in the absence of any magnetic field and without any ferromagnetic bias,
we found that the probability of finding such excitations increases with the size of
the system and seems to saturates, demonstrating the existence of a SG phase.$^{16}$
In fact that would even suggest the existence of a mean field like SG phase, but
many other mean field ingredients seem to lack: for instance the fractal dimension
of the surface of these excitations is lower than the space dimension —the equality
$d_s = d$ is fundamental for mean field predictions— and we formulated the suggestion
that the whole set of numerical results in 3d spin glasses should be described by
a new scenario, that we named TNT, For Trivial-Non-Trivial.$^{16,17}$

Now, how do these results change when visiting the phase diagram? For the mixed phase, we
shown recently$^{13}$ that, when changing the ferromagnetic bond concentrations, these
spongy excitations disappear at the same ferromagnetic concentration where the
magnetic ordering appears, thus suggesting that there is no mixed phase in 3d or
that it is at least unobservable (see fig.2).

What about the dAT line then? Here finite size effects are more subtle, making
the numerical study more difficult. We argued in$^{13}$ that the putative critical value
is lower than $H_c < 0.65$. To go beyond that, we recently tried$^{20}$ a simple scaling

$^\star$ Note for instance than in a simple disordered ferromagnet this is going exponentially to zero
with $N$. For a more detailed discussion, we refer the reader to$^{15,11,20}$. 

Fig. 2. Results for 3d spin glasses. Left: Absence of mixed phase; the sponge fraction is vanishing
at the same ferromagnetic bond ($\approx 0.385$) concentration where a non zero magnetization sets
in (in inset, the Binder cumulant of the magnetization). Right: finite size scaling analysis of
the sponge fraction under magnetic field, assuming $H_c = 0$; data collapse is very good.
ansatz assuming there is no dAT line: the (very good) result is shown in fig.2, where we used the published data of the fig.2 in ref.12). This strongly suggests an extremely low value for the critical field, if any (similar conclusions have been reached by 13). In fact, simple droplet arguments11 predict that a rescaling by $L^{-\theta}$ in the $y$ axis and by $L^{2/(d-2\theta)}$ in the $x$ axis would collapse the curves; taking the standard value $\theta \approx 0.2$ gives exponents very close to the one used in fig.2. These findings indicate that the mixed phase or the SG phase in field, if any, happens in an extremely small range in the phase diagram of the 3d EA model. However, we saw from mean field computation that no such low values should be expected hence the most direct interpretation is that the 3d phase diagram is trivial. If similar results were found in 4d that would finally answer negatively the question of a mean field phase diagram in finite $d$. This is in accord with experimental results10, as well as with the field theory analysis of 21 where the dAT line disappears at low dimensions (for $d \leq 6$).

**Conclusion**- Working at $T = 0$ allows nice analytical mean field predictions as well as powerful numerical studies in finite dimensions. Although the mean field phase diagram is very rich, we found that the 3d one seems trivial. Finally, we want to remark that, even though many controversies remain,15,19 the TNT picture, where such a trivial diagram is expected15 represents the best resume of the state of art in simulations of 3d spin glasses: we see many low energy excitations that however do not seem to have a mean field structure nor a mean field phase diagram.

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