On the formulation and solution of the boundary value problem of drilling well by differently mounted drilling bits

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Abstract. The tensile stress of the drilled rock under the influence of two circular and annular drilling bits rotating in opposite directions is of profound interest. The aforesaid method solves the problem of spin moment removal from the drill string. Assessment of the tensile stress determines the formation stimulation during drilling. Inasmuch as the drill bits are circular in the axial plane of the aperture, the elastic strain range is not simply-connected. To solve this problem, it is proposed to consider the equations of the elasticity theory in a toroidal coordinate. This statement corresponds to the axisymmetric torsion problem articulated by A.I. Lurie [1]. The task is formulated and solved in the displacements indicating the subsequent transition to tensions. An outstanding feature in comparison with the Laplace equation is the presence of an additional summand. The boundary conditions are displacements' equality to zero on the axis and on the walls of the aperture. Then an additional solution of the problem is carried out in a cylindrical coordinate, shifted to the bottom of the slaughter. Two solutions are compared.

1. Introduction
The timeliness of this task is related to those areas where a minimum stress is required on the drilled layers. First of all, it is associated with flexodrilling, where it is necessary to remove the reaction torque that occurs during the operation of drilling out from a hose. This approach can be useful in drilling out of small-mass space bodies — asteroids, comets, inasmuch as it provide minimal requirements for fixing a drilling device.

The basis is the method of drilling and the device for its implementation, proposed in an article [2]. It proposed to divide the drilling bit into the annular and circular parts and rotate them in different directions, simultaneously changing the drilled areas, by changing the angle of rotation of the drilling bits in the axial plane of the pore, and equalizing due to this the countertorques, with increasing lading of one of the drilling bits. A differential device with the axis of the satellites of which is connected with the system of leads turning the drill bits in the axial plane of the well is used for automatic redistribution of drilled areas and equalization of reactive torques.

2. Formulation of the problem
The boundary value problem is posed as follows - figure 1. The bottom of the well is described by a torus with a radius equal to one-fourth the diameter of the well. A distributed axial load $P$, technologically ensuring the well deepening process, and a distributed tangential load from bit rotation and rock fracture, which in the first approximation can be connected with the axial load through a friction
coefficient $k$ act on this bottom. The equality of the reactive torques on the annular and circular drill bits leads to the equality of two curvilinear integrals (figure 1).

$$\int_{0}^{\varphi} kP\sin (R - R\cos \varphi) Rd\varphi = \int_{\varphi}^{\varphi^*} kP\sin (R - R\cos \varphi) Rd\varphi$$

where $\varphi$ - the angle measured from the center of the well inside the drilled rock.

After integration, we obtain the trigonometric equation

$$\cos^2 \varphi^* - 2\cos \varphi^* - 1 = 0$$

Solving first the quadratic equation we get

$$\cos^2 \varphi^* - 2\cos \varphi^* - 1 = 0$$

Finally, we determine the inclination angle of the drill bits

$$\varphi^* \approx 114.5^\circ$$

However, it is necessary to solve the application task in generalized analytic functions [3], since the range of proportionality is not simply connected.

3. Results and discussion

In the book [1], it was shown that the axisymmetric problem can be divided into axial and torsion problems. The solution of the torsion problem is reduced to solving an equation in displacements [4,5].

$$\left( \nabla^2 - \frac{1}{r^2} \right) u = \frac{1}{H_1H_2r} \left[ \frac{\partial}{\partial q_1} \left( \frac{rH_2}{H_1} \frac{\partial u}{\partial q_1} \right) + \frac{\partial}{\partial q_2} \left( \frac{rH_1}{H_2} \frac{\partial u}{\partial q_2} \right) \right] - \frac{u}{r^2} = 0.$$
The solution of this equation in accordance with figure 1 must be found in toroidal coordinates [6], the connection with which is carried out as follows

\[ r = \frac{\cosh \xi}{\cosh \xi - \cos \eta}; z = \frac{\sin \eta}{\cosh \xi - \cos \eta}. \]  

Consequently, the Lamé coefficients in (5) are equal

\[ H_1 = H_2 = \frac{c}{(\cosh \xi - \cos \eta)}. \]  

Equation (5) takes the form

\[ \frac{(\cosh \xi - \cos \eta)^2}{c^2 \sinh \xi} \left( \frac{\partial}{\partial \xi} \left( \frac{\sinh \xi}{\cosh \xi - \cos \eta} \frac{\partial u}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\sinh \xi}{\cosh \xi - \cos \eta} \frac{\partial u}{\partial \eta} \right) \right) - \frac{(\cosh \xi - \cos \eta)^2}{c^2 \sinh \xi} u = 0 \]  

We are looking for a solution in the form

\[ u = v \sqrt{2 \cosh \xi - 2 \cos \eta}. \]  

Substituting it into equation (8), we have

\[ \frac{\partial^2 v}{\partial \xi^2} + \frac{\partial v}{\partial \eta} + \frac{\partial v}{\partial \xi} \cosh \xi + \frac{v}{4} - \frac{v}{\sinh^2 \xi} = 0 \]  

Now you can separate the variables

\[ v = A(\xi)B(\eta) \]  

\[ \frac{\partial^2 A}{\partial \xi^2} + \frac{\partial A}{\partial \xi} \cosh \xi + \left( \frac{1}{4} - K^2 - \frac{1}{\sinh^2 \xi} \right) A = 0 \]

\[ \frac{\partial^2 B}{\partial \eta^2} + K^2 B = 0 \]  

Solution of the second equation

\[ B = \frac{C_2}{K^2} \cos(K\eta - C_1) \]  

In the first equation, apply the substitution [7]

\[ A = \exp \left( -\frac{1}{2} \int \csc \xi d\xi \right) \]  

Then the first equation is converted to

\[ \frac{\partial^2 z}{\partial \xi^2} + \left( \frac{1}{4} - K^2 - \frac{1}{2 \sinh^2 \xi} \right) z = 0 \]  

Transfer it into the Riccati equation [8] by substitution \( \frac{\partial z}{\partial \xi} = zU \)

\[ \frac{\partial U}{\partial \xi} + U^2 = K^2 - \frac{1}{4} + \frac{1}{2 \sinh^2 \xi} \]  

We will solve this equation numerically, for example, by Picard's method [9] using the iteration formula
\[ U_n = U_0 + \int_0^{\xi} \left( K^2 - \frac{1}{4} U_{n-1}^2 \right) \, d\xi - U_{n-1}^2 (\xi_n - 0.885) \]  \hspace{1cm} (17)

or

\[ U_n = U_0 + \left( K^2 - \frac{1}{4} U_{n-1}^2 \right) (\xi_n - 0.885) - \frac{1}{2} \cosh \xi \Big|_{0.885} \]  \hspace{1cm} (18)

Substituting this expression into (14), we get

\[ A = \frac{1}{\sqrt{\sinh \xi}} \exp \left( U_0 + \left( K^2 - \frac{1}{4} U_{n-1}^2 \right) (\xi_n - 0.885) - \frac{1}{2} \cosh \xi \Big|_{0.885} \right) \]  \hspace{1cm} (19)

Returning to the displacement and taking \( U_0 = 0 \), we have the following dependence on its toroidal coordinates

\[ u(\xi, \eta) = \frac{\sqrt{2} C_2}{K^2} \cos (K \eta - C_1) \exp \left( \int_0^{\xi} Ud\xi \right) \sqrt{\frac{\sinh \xi - \cos \eta}{\sinh \xi}} \]  \hspace{1cm} (20)

Analyzing the resulting formula, we come to the conclusion that it is not applicable on the axis \( z(\xi = 0) \) in virtue of the appearance of uncertainty \( \eta = 0 \).

Considering the division of drilling area when \( \xi = 0.885, \eta = 4.44 \), we have:

\[ u(\xi, \eta) = \frac{\sqrt{2} C_2}{K^2} \cos (K \eta - C_1) \sqrt{\frac{\sinh \xi - \cos \eta}{\sinh \xi}} \]  \hspace{1cm} \text{at } \pi \leq \eta \leq \eta^*

\[ u(\xi, \eta) = \frac{\sqrt{2} C_2}{K^2} \cos (K \eta - C_1) \sqrt{\frac{\sinh \xi - \cos \eta}{\sinh \xi}} \]  \hspace{1cm} \text{at } 2\pi \geq \eta \geq \eta^* \hspace{1cm} (21)

From the equality of zero displacements on the axis and on the perimeter of the well, we obtain two equations for determining the constants:

\[ \begin{align*}
\cos (K \pi - C_1) &= 0 \\
\cos (K 4.44 - C_1) - \cos (K 2\pi - C_1) 0.644 &= 0
\end{align*} \]  \hspace{1cm} (22)

The value of the coefficients is determined by the substitution and solution of the lower transcendental equation: \( K = 0.63, \ C_1 = 0.408 \).

Wherein \( u_{\text{max}} \) is easily calculated using the total torque and the calculation of the torsion deflection of the cantilever-mounted cylinder. Taking it for one, we get \( C_2 = 0.281 \).

We simplify the formulation of the problem by presenting the well in the form of a cylinder. In this case, equation (1) takes a different form

\[ \int_0^{2\pi} Fdr = \int_0^{2\pi} Fdr, \]  \hspace{1cm} (23)

and radius of separation of drill bits \( r^2 = 2R / \sqrt{2} \).

Transferring equation (5) into cylindrical coordinates, we get

\[ \frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} = 0. \]  \hspace{1cm} (24)

Separate the variables
\[
\frac{\partial^2 v}{\partial z^2} - K^2 v = 0
\]  
(25)

\[
\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \left( K^2 - \frac{1}{r^2} \right) w = 0.
\]  
(26)

The solution of these equations is chosen in the form [4], taking into account that there is no movement on the symmetry axis

\[ u = C_1 e^{-Kr} I_1(Kr) \]  
(27)

The displacement increases according to this formula only up to the radius \( r^* \), and then the intentions act in the opposite direction and balance the torque from the previous intentions on the cylindrical surface of the well, hence

\[ u = C_1 e^{-Kr} I_1(Kr), \quad \text{at } 0 < r < r^* \]  
(28)

\[ u = C_1 e^{-Kr} I_1(Kr) - C_2 e^{-Kr} I_1(Kr), \quad \text{at } 2R > r > r^* \]  
(29)

We find \( K \) from the condition of equality to zero of movements at the border of the well

\[ I_1 \left( K \frac{2R}{\sqrt{2}} \right) - I_1(K2R) = 0 \]  
(30)

From the graph of the Bessel function of the first order [10] we find that the equality holds when

\[ K2R = 2.15 \]  
(31)

Constant \( C_1 \) can be defined at the point of maximum displacement \( r^* \), taking that for 1 we get \( C_1 = 1.52 \).

Intentions can be determined by the formulas [2]

\[ \tau_{\psi\rho} = \mu r \frac{\partial}{\partial r} \frac{\partial u}{\partial r} = -K \mu e^{-Kr} I_2(Kr) \]  
(32)

\[ \tau_{\psi\phi} = \mu r \frac{\partial}{\partial r} \frac{\partial u}{\partial r} = -K \mu e^{-Kr} I_1(Kr) \]  
(33)

Inasmuch as the maximum displacement in both cases is the same and equal to one, it is possible to answer the question about the margin of error of the simplified solution of the problem and the exact one. From figure 2 it is clear that starting from the toroidal bottom, the displacement epures are quite close to each other, therefore, further studies related to the dynamics of the interaction of the instrument with the rock can be carried out in cylindrical coordinates.

Displacement graphics also have two regions. It is clear from the formulas that displacements with distance from the bottom of the well, depthward the drilled out rock along its axis, fall off exponentially, that means very quickly.
4. Conclusion

Thus, a boundary task has been set and solved, which allows analyzing the strained state in the drilled out rock under the influence of an original drilling tool, including circular and annular drill bits, rotating in opposite directions. In addition to removing the torque effect from drill string, the equality to zero of the movements on bore hole walls under the investigated method of drilling contributes to maintaining the direction of drilling, the importance of which is noted by many researchers. [11–14].

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