Impulse distributions in dense granular flows: signatures of large-scale spatial structures

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In this paper we report the results of simulations of a 2D gravity driven, dissipative granular flow through a hopper system. Measurements of impulse distributions $P(I)$ on the simulated system show flow-velocity-invariant behavior of the distribution for impulses larger than the average impulse $\langle I \rangle$. For small impulses, however, $P(I)$ decreases significantly with flow velocity, a phenomenon which can be attributed exclusively to collisions between grains undergoing frequent collisions. Visualizations of the system also show that these frequently colliding particles tend to form increasingly large linear clusters as the flow velocity decreases. A model is proposed for the form of $P(I)$, given distributions of cluster size and velocity, which accurately predicts the observed form of the distribution. Thus the impulse distribution provides some insight into the formation and properties of these “dynamic” force chains.

Introduction

Granular materials exhibit a wide spectrum of behavior ranging from gaseous to liquid to solid. Remarkably, all of these phases of granular matter respond to external stimuli in a manner strikingly different from ordinary fluids and solids [1, 2]. In static granular piles, the spatially inhomogeneous manner in which stress is transmitted from the bulk of the pile to the lar piles, the spatially inhomogeneous manner in which stress is transmitted from the bulk of the pile to the boundary has been well documented [2, 11]. Experiments have shown that the force distribution $P(f)$ at the walls is exponential at large forces and exhibits a plateau at small forces [2]. In addition, the highly stressed grains in static packings are organized into linear structures termed “force chains” [3, 4]. The appearance of similar large scale structures in flowing granular matter would have significant implications for both continuum theories and descriptions of jamming in non-thermal systems. A recent proposal for a unified picture of jamming in thermal and non-thermal systems suggests that jamming occurs due to the formation of force chains whose presence is signalled by the appearance of a plateau in $P(f)$ [6]. A continuum description of steady-state flow on an inclined plane models the system as a collection of transient 1D chains immersed in a viscous fluid [7]. Indeed, transient “clusters” have been identified experimentally in granular surface flows [8] and shear flows [9], and simulations of chute flows have shown evidence for a plateau in $P(f)$ as the system approaches jamming [10].

Recent experiments have been performed in a two dimensional hopper geometry to explore the presence of incipient force chains in purely collisional gravity-driven flow [11]. Measurements of the time trace of the impulse delivered to a transducer placed at the side wall of the hopper have shown that the distribution of impulses, $P(I)$, displays an exponential decay at large $I$, as for the case of static materials. This exponential form of the distribution is maintained for all flow velocities from the largest measured ($v_f = 60.0$ cm/s) to the minimum flow velocity prior to the point at which the system no longer exhibits sustained flow ($v_f = 9.4$ cm/s). However, at small $I$, $P(I)$ develops an upward trend which becomes increasingly more evident as the flow velocity decreases.

In this letter, we report the results of event-driven simulations of a system of inelastic, monodisperse hard disks in the experimental geometry of Ref. [11]. Our simulations provide clear evidence of an increasing proportion of collisions with small impulses as the flow velocity is decreased. This results in the formation of a plateau in $P(I)$ as the minimum flow velocity for sustained flow is approached. In addition, we observe the formation of clusters of disks which collide “frequently” and are reminiscent of the “collapse strings” observed in freely cooling granular matter [12]. We present a model calculation which strongly suggests that the increase of small impulse events is associated with the growth of these clusters.

Simulations

The grain dynamics used in the simulations are as in Ref. [13] (momentum is conserved and at each interparticle collision the energy loss is proportional to $(1 - \mu^2)$ where $\mu$ is the coefficient of restitution). To ensure that the pressure is independent of the height the side walls must absorb some vertical momentum, therefore we impose the condition that collisions with the walls are inelastic in the tangential direction. The flow velocity is controlled similarly to the experiments, by adjusting the width of the hopper opening. However, as we wish to observe the system over many events, particles exiting the system at the bottom must be replaced at the top to create sustained flow. This necessitates the introduction of a probability of reflection $p$ at the bottom of the hopper (this would be equivalent to the presence of a sieve in experiment). In our case, it provides another parameter with which we can tune the flow velocity. Typically, our simulations were done on systems of 500 disks, with $\mu = 0.9$ and $p = 0.4$. The simulation was run for $2 \times 10^6$ events for each flow velocity, with $1.5 \times 10^5$ discarded initially to allow the system time to reach steady state.

Simulation Results

The physical quantity that is most closely related to $P(f)$ in our flowing hard-disk system is the distribution of impulses transferred at each
A peak is seen. In addition, our upturn occurs only at a minimum impulse curve, while in our distribution the experiments. That feature was a smooth continuum, we believe it is not the same upward trend visible in our analysis, we will consider the impulse distribution without the \( I = 0 \) point.

The basic form of the impulse distribution, a peak at a finite value of \( I \) and an exponential tail can be understood on the basis of a flow of uncorrelated hard disks. If the disks were uncorrelated, the impulse distribution would be a convolution of the individual momentum (velocity) distributions. Since there is an average flow velocity, this would give rise to a peak in the distribution. The exponential tail would be observed if the individual velocity distributions also had exponential tails. The velocity distribution observed in our simulations can, to a first approximation, be described by Gaussians with exponential tails. This argument would then suggest that the exponential tail arises from uncorrelated particles and is a consequence of the shape of the velocity distribution. Viewed from this perspective, the lack of change in shape of \( P(I) \) at large \( I \) as the flow approaches jamming is not surprising. We will discuss this further in the context of a simple model to be presented below.

The change in \( P(I) \) at small \( I \) is reminiscent of the universal jamming scenario although the direction of the change (filling up at small impulses) is different from the one observed in Lennard Jones systems and foams where the probability of finding small forces decreases as the system nears the jamming transition. Nevertheless, the idea that changes in the distribution at small impulses or forces could be related to the appearance of structures akin to force chains is intriguing.

To explore a possible connection between the changes in \( P(I) \) and the appearance of spatial inhomogeneities, we considered a question first asked in studies of inelastic collapse in freely cooling granular gases. “How many collisions does a given grain undergo in a fixed number of events?” We define a minimum frequency of collision for a given particle as \( \omega_0 = \frac{1}{\tau} \) (where \( \tau \) is the average time between events for a given flow velocity and is then identified as the time scale on which the system approaches jamming). We then construct an image of our system at every 1000 events, and color all disks satisfying the criteria (see Fig. 2a). As we decrease the flow velocity, the frequently colliding particles form increasingly larger linear clusters (compare Fig. 2a, where \( \nu_f = 2.03 \) in our units or 35.6 cm/s and Fig. 2b, where \( \nu_f = 0.96 \) or 16.9 cm/s). These 1D structures observed in our simulation are reminiscent of the transient solid chains postulated by the hydrodynamic model of Ref. 7. Comparisons of the impulse distribution of the frequently colliding particles and the impulse distribution of the remaining “rarely colliding” particles.
Impulse Distribution Model

A possible connection between the shape of the impulse distribution and the development of linear clusters of frequently colliding particles can be drawn by investigating the following model. Consider a one-dimensional cluster of particles which are all moving with the same velocity. Now if another particle, travelling at some speed \( v \) relative to the cluster (see Fig. 3a) collides inelastically with one end of the chain, the impulse associated with that collision will be \( (1 - \epsilon)v \) where \( \epsilon \) is related to the coefficient of restitution by \( \epsilon = \frac{(1-\mu)}{2} \). If the chain was only comprised of one grain, the impulse distribution \( P(I, v, 1) \) (given an initial incoming speed \( v \)) would be a single spike of height 1 located at \( (1 - \epsilon)v \). For two particles, \( P(I, v, 2) \) would be a bar of height 1 with limits at \( (1 - \epsilon)^2v \) and \( (1 - \epsilon)v \). Continuing this argument for clusters containing \( S \) particles, then as \( S \) becomes very large, the leftmost limit of \( P(I, v, S) \) approaches zero (Fig. 3b).

Given a distribution of speeds \( P(v) \) for the incident particle, and a distribution of cluster sizes \( P(S) \), the total impulse distribution \( P(I) \) is:

\[
P(I) = \int dSdvP(v)P(S)P(I, v, S)
\] (1)

If the cluster size distribution falls off sharply, then the shape of \( P(I) \) will essentially reflect the shape of \( P(v) \). If, however, the cluster size distribution becomes broad, then the small impulse end of \( P(I) \) will flatten out reflecting the nature of \( P(I, v, S) \) for large \( S \).

As detailed previously, the criterion that we use to identify the clusters in our simulations is that of “frequent” collisions. Due to the inelastic nature of the collisions, the velocities of these particles become highly correlated and they can be idealized as clusters of disks moving with the same velocity. This velocity correlation within spatial clusters of particles has been directly observed in granular surface flows. Since we observe these clusters to be linear and growing in size with decreasing flow rates, it is plausible that the changes in \( P(I) \) observed in our simulations is related to a change in distribution of the size of the clusters of frequently-colliding disks. In order to put this conjecture on a firmer footing, we calculated the impulse distribution from our model using forms of \( P(v) \) and \( P(S) \) which provide good fits to our simulation data. The form of \( P(v) \) that we use is a Gaussian with an exponential tail.
Fig. 4b. The control parameter in the model is remarkably similar to the measured velocity. Since the clusters are essentially the same as the ones identified in freely cooling granular matter, their origin lies in the dissipative nature of the medium. The heterogeneities reflect strong velocity correlations of grains and leave a distinctive signature in the impulse distribution. Whether the clusters that we have identified are indeed incipient force chains remains to be verified. If this connection can be established, for example through the calculation of stress correlations in the flowing medium, then our analysis will provide a natural connection between $P(f)$ and force chains. Moreover, it would indicate that jamming occurs through the formation of system spanning clusters of grains whose velocities are strongly correlated.

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Page 4

FIG. 4: (a) Simulation results for $SP(S)$ for varying flow rates. (b) Results of the proposed model for $P(I)$ with $S_{\text{max}}$ increasing from the bottom curve to the top curve. The inset shows the ratio of $P(I_{\text{peak}})$ to $P(I_{\text{min}})$ as a function of $1/S_{\text{max}}$. The picture which seems to be emerging from our simulations is that of increasingly larger scale spatial heterogeneities developing as the system approaches jamming. Since the clusters are essentially the same as the ones identified in freely cooling granular matter, their origin lies in the dissipative nature of the medium. The heterogeneities reflect strong velocity correlations of grains and leave a distinctive signature in the impulse distribution. Whether the clusters that we have identified are indeed incipient force chains remains to be verified. If this connection can be established, for example through the calculation of stress correlations in the flowing medium, then our analysis will provide a natural connection between $P(f)$ and force chains. Moreover, it would indicate that jamming occurs through the formation of system spanning clusters of grains whose velocities are strongly correlated.

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