Territorial Technosphere as Managed Dynamic System

O Bezborodova1, O Bodin2, V Polosin2

1Department "Technosphere safety", Penza State University, 40, Krasnaya Str., Penza, 440026 Russia
2Department "Information and Measuring Equipment", Penza state University, 40, Krasnaya Str., Penza 440026, Russia

E-mail: oxana243@yandex.ru

Abstract. The territorial technosphere is a dynamic system described by a system of differential equations. The initial data of these equations take into account the conditions of normal functioning of the territorial technosphere and emergency situations. Changing the parameters of the impact of the technosphere allows you to influence the quality of the environment and to maintain it in a stable state. An effective mechanism for controlling the territorial technosphere as a dynamic system should be the elimination of inoperable States using: the formation of a set of informative parameters; control and registration of values of informative parameters; creation of a database of normative and actual values of informative parameters, formation of control actions.

1. Introduction

The forecast of the behavior of the territorial technosphere (TT) in the normal and emergency modes of its operation is necessary to reduce the severity of the consequences of emergency situations. To make a forecast, models that take into account the interaction of all objects that make up the TT are required. The purpose of developing such models is to establish the possibility of forming a control action aimed at improving the quality of the environment.

TT is a managed dynamic system that includes subsystems of natural and artificial origin. The composition of the TT is determined by the specifics of the development of the region in which it is located, and the totality of all objects located in the territory (settlements, industrial or agricultural enterprises, links in the transport and social structure, etc.) with established links between them. The parameters that characterize the activities of each of these objects change over time and affect the entire TT. Therefore, it is more correct to consider the TT as a complex and self-developing dynamic system, changing in time under the influence of internal and external factors. The influence of the technosphere on the environment is an internal factor and reduces the efficiency of the functioning and reproduction of its individual components.

2. Relevance, scientific significance of the issue with a brief review of literature

When studying the TT, mathematical modeling is used where the object of research is not available and experiments with it are expensive or can lead to serious environmental consequences, as well as when it is necessary to study not a single concrete phenomenon but a wide class of related phenomena.

Mathematical modeling of processes in the environment began to be considered in the first third of the XX century. The most suitable models the Lotka-Volterra [1], Kolmogorov A [2], Khlebopros P...
They modeled biological processes by introducing a large number of boundary conditions. But the models were so successful that they were used in other fields of science to describe economic, social, military, and medical processes at various levels.

The method of constructing the Lotka-Volterra model "predator-prey" is given in [1]. The system of the Lotka-Volterra differential equations describes the population dynamics in their interaction. Nevertheless, this model has found wide application for the description of closed systems in other branches of science. The analysis of the application of the "predator-prey" model in economics and technology is given in Table 1.

Table 1. Analysis of the application of the "predator-prey" model in economics and technology.

| System        | Economic                  | Technical                  |
|---------------|---------------------------|----------------------------|
| energy carrier| capital                   | information                |
| territory     | city [4]                  | enterprise [7]             |
| the participant| relative population and per capita income |                          |
| model         | buyers - sellers of shares | transfer of capital from the Central Bank to ordinary banks and further to individuals and companies |

\[
\dot{x}_i = x_i \left( -\alpha_i - \alpha \bar{y}_i + \alpha x_i \right)
\]

\[
\dot{y}_i = y_i \left( \beta_i - \beta x_i \right)
\]

where \( i \) is the viewing area; \( x \) is the population; \( y \) - average income; \( \alpha \) and \( \beta \) are dynamic parameters.

\[
\frac{dX}{dt} = a_i X - b_i X^2 + c_{xy} X Y
\]

\[
\frac{dY}{dt} = a_i Y - b_i Y^2 + c_{xy} X Y \times \left( \left( a_i - b_i x_i \right) U + c_{xy} x_i \right) v
\]

where \( X \) - sellers of shares; \( Y \) - buyers of shares; \( a \) - volume of demand; \( b \) - the volume of the offer; \( c_{xy} \) and \( c_{yx} \) are the interaction constants between the two competitors.

\[
\dot{x}_i = x_i \left( -a_i - b_i x_i + c_{xy} \right)
\]

\[
\dot{y}_i = y_i \left( a_i - b_i x_i \right) \left( a_i - b_i x_i \right) v
\]

where \( x_i, x, y_i \) - capital belonging to the Central Bank, banks, individuals and companies; \( a_i, b_i, u, v \) - factors influencing the redistribution of capital.

\[
\frac{dM}{dt} = M \left( a_u - b_u M - c_{xy} B - c_{xy} \right)
\]

\[
\frac{dB}{dt} = B \left( a_u - b_u B - c_{xy} M - c_{xy} \right)
\]

\[
\frac{dE}{dt} = E \left( a_E - b_E E - c_{xy} M - c_{xy} \right)
\]

where \( a_M, b_M \) and \( a_E \) are the production volumes of technologies \( M \), \( B \) and \( E \), respectively; \( c_{xy}, c_{xy} \) and \( c_{xy} \) are the interaction coefficients for each technology; \( c_{xy}, c_{xy} \) and \( c_{xy} \) are the interaction coefficients.

In [8-10], a detailed review of the most significant economic situations was carried out using the Lotka-Volterra models, with a concise conclusion on each of them.

3. Formulation of the problem

The mathematical model of the TT must take into account the conditions of normal functioning (for example, the efficiency of the environmental protection equipment, the soundness of the process control and control systems, the possibility of comparing the actual environmental parameters with the maximum permissible values) and critical situations, when the effectiveness of the decisions taken depends on the state of the emergency object and the values parameters that characterize this state.

An important place in the mathematical modeling of processes in the technosphere is occupied by nonlinear mathematical models that most fully and accurately describe existing processes. When studying the stability of the TT, the Lotka-Volterra models are most adequate, for the description of which nonlinear systems of differential equations are needed.

With this in mind, it is proposed to use the model based on the Lotka-Volterra equations to assess the effectiveness of the TT. It is necessary to determine its components, their characteristics and to use the statistical data to predict its behavior.
4. Theoretical part

At the meetings of victims and predators, whose frequency is directly proportional to the product \((X_1 \cdot X_2)\), the victims are consumed (lost) with the coefficient \(\beta\) and new predators are born with the coefficient \(\varepsilon\). Taking this into account, we obtain a system of equations characterizing the dynamics of ecosystem parameters over time:

\[
\begin{align*}
\frac{dX_1}{dt} &= \alpha \cdot X_1 - \beta \cdot X_1 \cdot X_2 \\
\frac{dX_2}{dt} &= -\delta \cdot X_2 + \varepsilon \cdot X_1 \cdot X_2 
\end{align*}
\]  

(1)

where \(X_1\) is the magnitude of the victim population; \(X_2\) - the size of the predator population; \(\alpha\) is the birth rate of the victims; \(\beta\) - loss rate of victims; \(\varepsilon\) - birth rate of predators; \(\delta\) - coefficient of predator mortality; \(t\) is time.

The interaction between pollution and the environment can be interpreted as a special case of the predator-prey model, when the environment acts as a victim and the pollution is a predator. The main assumption underlying the model is that the environment actively assimilates pollution to a certain limit and is considered as a closed system ("bubble principle").

It is theoretically determined that the assimilative capacity of the environment is the ability of the biota to compensate for the external impact through the work of negative feedbacks generated by these influences. It is established that the growth of the assimilative capacity of biota is possible only up to a certain level of pollution effects. If this level is exceeded, the biota loses its ability to stabilize the environment. As a result, local and global changes may occur. Biota in this altered state itself begins to distort the environment. And then the process of environmental loss of stability is accelerating.

There are three scenarios for interaction of participants in the analyzed system "pollution - environment" [11].

The mathematical model described by the system of equations (1) is structurally unstable, that is, a small change in the right-hand parts in it can lead to a qualitative change in the behavior of the solution. This is its main drawback. When one adds to the right-hand sides of equations (1), taking into account, for example, the mutual influence of the contaminants, the model becomes stable and, in general, equation (1), applied to the TT, can be rewritten as follows:

\[
\begin{align*}
\frac{dX_1}{dt} &= \alpha \cdot X_1 - \beta \cdot X_1 \cdot X_2 - f(X_1, X_2) \\
\frac{dX_2}{dt} &= -\delta \cdot X_2 + \varepsilon \cdot X_1 \cdot X_2 - h(X_1, X_2) 
\end{align*}
\]  

(2)

Considering the quasistable situation in the example of the atmosphere, when the atmosphere partly assimilates pollutants and partially accumulates non-assimilated pollution with an increase in the emission of pollutants depending on external conditions and random factors, we obtain an expression for estimating the dynamics of the state of the atmosphere:

\[
\begin{align*}
\frac{dX_1}{dt} &= \alpha \cdot X_1 - \beta \cdot X_1 \cdot X_2 + \beta \left(1 - e^{-\alpha \cdot X_1}\right) \\
\frac{dX_2}{dt} &= -\delta \cdot X_2 + \varepsilon \cdot X_1 \cdot X_2 - X_2 \left(1 - e\right) 
\end{align*}
\]  

(3)

where \(\frac{dX_1}{dt}\) - change in the concentration of pollutants in the atmosphere for a time \(t\); \(\frac{dX_2}{dt}\) - change in the concentration of pollutants in the release for a time \(t\); \(X_1\) - background concentration of pollutants in the atmosphere at the initial time \(t\); \(X_2\) - the concentration of pollutants in the emission (emission) at
time \( t \); \( \alpha \) - coefficient of accumulation of pollutants in the atmosphere; \( \beta \) - coefficient of assimilation of pollutants in the atmosphere; \( \varepsilon \) - efficiency of the environmental protection equipment (if any), in fractions of a unit; \( \delta \) - the amount of pollutants passed through the environmental protection equipment, in fractions of a unit; \( \beta(1-e^{-\alpha t}) \) - trophic predator function, characterizing, as applied to the situation analyzed, the change in the assimilative capacity of the atmosphere; \( X_2(1-\varepsilon) \) - the trophic function of the predator, characterizing, as applied to the situation analyzed, the intensity of the incoming pollutant into the atmosphere from the source; \( t \) is time.

Taking into account, in accordance with [13], the migration of pollutants from one medium to another, we show the physical meaning of the coefficients entering into formula (3). Graphically, the migration of pollutants is shown in Figure 1.

By changing pollutant emission parameters, it is possible to control the quality of the environment. For this, it is necessary to carry out a comprehensive monitoring of the environment of the TT, excluding its transition to an unstable state leading to collapse.

5. **Practical significance, suggestions and results of implementation, the results of experimental studies**

To maintain the TT in a certain stable state, it is necessary to form an effective management mechanism. This mechanism should exclude the entry of the TT into permanently inoperable conditions. To prevent these processes, it is necessary to form a set of informative parameters, to monitor and register their values, forming databases of normative (reference) and actual values of the parameters [14]. The object of management, in this case, is the activity of objects of the technosphere (industrial or agricultural enterprises, transport and social infrastructure, etc.). The control effect can be realized in the form of a legal act prescribing the object of the technosphere to change the parameters of functioning due to, for example, the intensification of the work of the environmental protection equipment. Such control and management can be realized using the method proposed in [15]. The algorithm for implementing this method is shown in Figure 2.

The concept of control and management, applied to the TT, consists in the fact that the database of standard parameters of the distribution law of the output parameter is first formed and the real parameters of the distribution of the output parameter are determined from the expressions [16,17]:

![Figure 1. The scheme of migration transfer of pollution in the atmosphere.](image-url)
\[
\begin{align*}
K_e &= \frac{A}{\sigma} \\
k &= \frac{\mu_4 - 3 \cdot \sigma^4}{\mu_4 - 3 \cdot \sigma^4} \\
\end{align*}
\]

where \(K_e\) and \(k\) are the entropy and counterexcess coefficients, respectively.

For these calculations, the entropy potential \((A_e)\) is calculated by the formula:

\[
A_e = 0.5 \cdot \Delta y \cdot N \cdot \exp \left( -\frac{1}{N} \sum_{j=1}^{m} n_j \cdot \ln(n_j) \right)
\]

where \(\Delta y\) is the width of the intervals of the grouping of the output parameter; \(n_j\) - number of values of the output parameter in the \(j\)-th grouping interval; \(N\) is the number of sample values of the output parameter; \(m\) is the number of grouping intervals.

The central moment of the fourth order of the distribution of the output parameter \((\mu_4)\) is determined by the formula:

\[
\mu_4 = \frac{1}{N} \sum_{j=1}^{m} (y_j - M)^4 \cdot n_j
\]

where \(y_j\) is the average value of the output parameter in the \(j\)-th grouping interval.

---

**Figure 2.** Algorithm for the implementation of the concept of control and management of the territorial technosphere.
To determine the mathematical expectation of the output parameter \( M \), use the expression [18]:

\[
M = \frac{1}{N} \sum_{j=1}^{n} y_j \cdot n_j
\]  

(7)

The entropy-parametric criterion of the domain of optimal control \( \gamma \) is determined by the formula:

\[
\gamma = \left[ \left( \frac{K - K_{00}}{K_{00}} \right)^2 + \left( \frac{k - k_{00}}{k_0} \right)^2 \right]^{1/2} \leq \gamma_{\text{max}}
\]  

(8)

where \( \gamma_{\text{max}} \) is the maximum value of the entropy-parametric criterion of the domain of optimal control. Then, the state of the object belonging to the region of the optimal state is checked and the entropy-parametric potential \( \Delta_{en} \) is determined from the expression:

\[
\Delta_{en} = \left[ \left( \Delta \right)^2 + \left( k_{p,n \rightarrow \infty} \cdot \sigma \right)^2 \right]^{1/2}
\]  

(9)

where \( k_{p,n \rightarrow \infty} \) is the coefficient of the normal standard distribution.

And after that, the entropy-parametric potential \( \Delta_{en} \) of the dynamic system is minimized and the real parameters of the distribution of the output parameter are corrected [19, 20].

5. Conclusions

The analysis of the properties and parameters of the TT makes it possible to characterize it as a dynamic system consisting of a set of elements for which a functional relationship is established between the time and state of each element of the system. Such mathematical dependencies make it possible to study and describe the change of the TT in time, taking into account external and internal influences, and to form a controlling influence aimed at improving the quality of the environment. The proposed method of control and management makes it possible to make the process of forming the control effect more efficient.

References

[1] Volterra V 1976 Mathematical theory of the struggle for existence (Moscow: Nauka) p 286
[2] Kolmogorov A N 1972 Problems of Cybernetics Qualitative Study of Mathematical Models of Population Dynamics 25 pp 101–6
[3] Khlebopros R G and Fet A I 1999 Decision-making in economics, ecology, politics. Models of catastrophes (Novosibirsk) p 345
[4] Kamann D-J F and Nijkamp P 1988 Technogenesis: incubation and diffusion (Amsterdam: Researchmemorandum) p 44
[5] Modis Th 1999 Technological forecasting at the stock market Technological Forecasting and Social Change (New York) 62 pp 173–202
[6] Comes C-A 2012 Banking system: three level lotka-volterra model Procedia Economics and Finance 3 pp 251–5
[7] Ahmadian A 2008 System dynamics and technological innovation system Chalmers University Of Technology, Department of Energy and Environment, Division of Systems Analyses (Göteborg, Sweden) 15 p 77
[8] Bratus A S, Novozhilov A S, and Platonov A P 2010 Dynamic systems and models of biology (Moscow: Fizmatlit) p 400
[9] Romanov V P and Akhmadeev B A 2015 Modeling of the innovation ecosystem based on the predator-prey model Business Informatics 1 (31) pp 7–17
[10] Titov V A and Weinberg R R 2016 Analysis of existing dynamic models based on the lotka-volterra system of equations "predator-prey" Fundamental research 8-2 p 409–13
[11] Holling C S 1965 The functional response of predator to prey density and its role in mimicry and population regulation *Memoirs of the Entomological Society of Canada* 45 pp 1–60
[12] Monod J 1954 Recherches sur la croissance des cultures bacteriennes (Paris, Hermann) p 210
[13] Afanasyev Y A et al 2001 *Monitoring and methods of environmental control* (Moscow: MNEPU Textbook, Publishing House) p 337
[14] Turichin A N et al 1975 Electrical measurements of non-electrical quantities (Leningra: Energy) p 576
[15] *A way of monitoring and managing a dynamic system*: Pat. 2565367 Rus. Federation: IPC G05B 13/00 (2006.01) / Bodin ON, Polosin VG; patent owner and applicant Limited liability company (LLC) Cardiodide - No: 2014111833/08; claimed. 03/27/2014; publ. 10/20/2015 Byul. 29
[16] Lucas V A 2005 *The theory of management of technical systems* (Ekaterinburg: Publishing house of the Ural State Mining University) p 676
[17] *A way of monitoring and controlling a dynamic system*: Pat. 2296356 Russian Federation / Lazarev V.L. - №2005124236 / 09 application; claimed. 29.07.2005; publ. 27.03.2007; Byul. 9
[18] Novitsky P V and Zograf I A 1985 Evaluation of measurement error (Leningrad: Energoatomizdat) p 248
[19] Polosin V G and Pershenkov P P 2014 Information – theoretic method for hypothesis testing with nonsymmetrical distributions *Measurement Techniques* 56 (12) pp 1318–22
[20] Polosin V G 2008 A technique for separating statistical data for a mixture of two distributions using the example of the results of changes in volumetric activity of radon *News of higher educational institutions. Volga region. Physical and mathematical Sciences* 4 (8) pp 79–88