The problem of exotic states: view from complex angular momenta

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Abstract

Having in mind present uncertainty of the experimental situation in respect to exotic hadrons, it is important to discuss any possible theoretical arguments, pro and contra. Up to now, there are no theoretical ideas which could forbid existence of the exotic states. Theoretical proofs for their existence are also absent. However, there are some indirect arguments for the latter case. It will be shown here, by using the complex angular momenta approach, that the standard assumptions of analyticity and unitarity for hadronic amplitudes lead to a non-trivial conclusion: the S-matrix has infinitely many poles in the energy plane (accounting for all its Riemann sheets). This is true for any arbitrary quantum numbers of the poles, exotic or non-exotic. Whether some of the poles may provide physical (stable or resonance) states, should be determined by some more detailed dynamics.

1 Exotic hadrons: brief overview

The problem of exotic states of hadrons has long history, nearly as long as quark themselves. After several unsuccessful experimental searches, it was formulated most clearly by Lipkin[1], as a problem for theorists: “Why are there no strongly bound exotic states ..., like those of two quarks and two antiquarks or four quarks and one antiquark?” This question could be eliminated by experimental observations of the first exotic baryon $\Theta^+$, just consisting of at least four quarks (two $u$’s and two $d$’s) and one antiquark ($\bar{s}$). However, other experimental publications, with null results, cast doubts on its existence (recent history and present experimental status see in the talk [2]). Therefore, Lipkin’s question may be still burning, having no answer. It seems reasonable in such a situation to analyze in detail all arguments, pro and contra exotic hadrons. Here we briefly discuss some of them.
The current experimental situation is rather uncertain and not decisive yet. In the plenary talk of Burkert at NSTAR2005, it was summarized by the words: “The narrow $\Theta^+$ pentaquark is not in good health, but it is too early to pronounce it dead”.

What about theoretical situation, the general postulates of Quantum Field Theory (QFT) do not provide any way to discriminate between exotic and non-exotic particles. Its particular case, Quantum Chromodynamics (QCD), believed to underlie the strong interactions and hadron spectroscopy, also cannot forbid exotic hadrons vs. non-exotic ones (at least, at the present level of understanding the structure and properties of QCD).

Moreover, in the framework of QCD, any hadron should be seen in some conditions (e.g., for short time intervals) as a multi-quark system. Experiments on hard processes confirm this. But it is then difficult to understand why such multi-quark systems must be bound in all cases to have quantum numbers of a 3-quark (or quark-antiquark) system.

Attempts to calculate hadronic spectra in various approaches, which are assumed to be based on QCD (bag model, soliton-like models, sum rules, lattice calculations, and so on), as a rule, also demonstrate exotic states (though with their properties strongly dependent on the model used).

Rather unexpectedly, the method of complex angular momenta (CAM), usually related only to high-energy asymptotics of hadronic amplitudes, has also something to say in the problem of exotic states. It can suggest a new (indirect) argument for existence of exotic hadrons. This argument has been recently published [3]. Here it will be explained in more detail.

2 CAM and exotics

Let us begin with some necessary preliminaries, partly forgotten now. For simplicity, at first we neglect particle spins.

Consider a process 2-hadrons-into-2-hadrons. Its amplitude $A$ is a function of two independent variables, for which one may choose the c.m. energy $W$ and c.m. scattering angle $\theta$. Another possibility is to use invariant variables. There are three of them (Mandelstam variables): one is the c.m. energy squared $s = W^2$; two others are squares of two c.m. momentum transfers $t$ and $u$, between a particular initial hadron and one or the other final hadron. Usually, $t$ is taken as proportional to $z = \cos \theta$, with $u$ proportional to $-z$. Evidently, they are not independent. Moreover, the sum $s + t + u$ equals just to the sum of the squared masses for two initial and two final particles (being thus independent of both $W$ and $z$).

The amplitude $A(s, z)$, as function of $z$, may be decomposed into partial waves. Every partial-wave amplitude $f_l(s)$ corresponds to a definite value $l$ of the orbital momentum. For the physical amplitudes, $l$ may only be equal to a non-negative integer number. In the case of purely elastic scattering, the physical amplitudes $f_l(s)$ satisfy the elastic unitarity condition

$$f_l(s) - f^*_l(s) = 2ik f_l(s) f^*_l(s),$$

where $k$ is the c.m. momentum.
Now we make two assumptions:

1) Amplitudes \( f_l(s) \) admit unambiguous analytical continuation to non-integer, and even complex, values of \( l \).

2) There are no massless hadrons and no massless hadron exchanges.

The first assumption is not trivial. Every function, defined on a set of discreet points, can be analytically continued, but the continuation is, generally, very ambiguous. Only in some cases there exists a preferred continuation, which may be clearly separated from all others. This is fulfilled, e.g., if the amplitude \( A(s, z) \) satisfies dispersion relations (DR) in momentum transfers \( t, u \); the arising continuation is described by the integral Gribov-Froissart (GF) formula[4] (for more details see the monograph [5]). Such DR have never been formally proved, neither in general QFT, nor in QCD. Nevertheless, they (and their analogs) are widely and actively used in phenomenology of strong interactions. Up to now, they have not encountered any inconsistency. Note that the DR provide a sufficient condition for the unambiguous continuation; necessary conditions are essentially weaker.

The second assumption ensures a finite range of interactions, and also the threshold behavior \( \sim k^{2l} \) for the elastic scattering amplitudes \( f_l(s) \) with physical values of \( l \), when \( k \to 0 \). The GF formula, where it is applicable, provides the same threshold behavior for the continued amplitudes \( f_l(s) \). But the elastic unitarity condition for the continued amplitudes has somewhat modified form

\[
 f_l(s) - f_l^*(s) = 2ik f_l(s) f_l^*(s),
\]

which coincides with eq.(1) only at real \( l \).

It is easy to see now that the continued unitarity relation (2) is not always consistent with the threshold behavior \( f_l(s) \sim k^{2l} \). Indeed, near threshold, each of the left-hand side terms is \( \sim k^{2 \Re l} \), while the right-hand side is \( \sim k^{4 \Re l + 1} \). Since the left-hand side terms may subtract each other, but not enhance, the two sides may be consistent with each other only at \( \Re l > -1/2 \). This problem was first discovered by Gribov and Pomeranchuk[6]. To solve it, they studied in more detail the small-\( k^2 \) region and showed that there are Regge poles, which condense near threshold to the point \( l = -1/2 \) and, thus, invalidate the \( k^{2l} \)-behavior of \( f_l(s) \) for \( \Re l < -1/2 \). Near the threshold, these poles have trajectories

\[
 l_n(s) \approx -\frac{1}{2} + \frac{i \pi n}{\ln(R\sqrt{|k^2|})},
\]

with \( R \) being the effective interaction radius. The number \( n \) takes any positive and/or negative integer values, \( n = \pm 1, \pm 2, ... \). Hence, there are infinitely many reggeons condensing to \( l = -1/2 \) at \( k^2 \to 0 \).

Till now, we have neglected particle spins. Taking them into account changes the orbital momentum \( l \) by the total angular momentum \( j \). The reggeon condensation still exists at a two-particle threshold, though with the shifted limiting point [7]. For the spins \( \sigma_1 \) and \( \sigma_2 \) it is

\[
 j = -1/2 + \sigma_1 + \sigma_2.
\]
instead of \( j = -1/2 \), without spins. The reason is simple: the condensation point, as before, corresponds to \( l = -1/2 \), and the highest value of \( j \) at fixed \( l \) equals \( l + \sigma_1 + \sigma_2 \). The movement of the condensing reggeons in the \( j \)-plane, near a threshold energy for two spinning particles, is also described by trajectories \( \mathbf{3} \), but with the shifted limiting point \( \mathbf{4} \).

The main conclusion of the above consideration is that the unitarity and the possibility of unambiguous analytical continuation in \( j \) (standard analytical properties, in particular), taken together, imply existence of the Gribov-Pomeranchuk (GP) threshold condensations of reggeons. They collect infinite number of reggeons and, therefore, imply that the total number of reggeons is always infinite.

The reggeon positions depend on energy and are determined by a relation of the form \( F(s, j) = 0 \). Each of its solution corresponds to an amplitude pole, which may be considered either as a pole in \( j \), with position depending on the energy (on \( s \)), or as a pole in the energy (in \( s \)), with position depending on the total angular momentum \( j \). This provides one-to-one correspondence between reggeons and spin-dependent poles in the energy plane. Therefore, the infinite number of reggeons corresponds to the infinite number of poles in the energy plane.

Note that no assumptions about quantum numbers of the poles have been used. Hence, the hadronic \( S \)-matrix should have infinite number of poles with any quantum numbers, exotic in particular.

The structure and properties of the reggeon condensations may be studied explicitly in the non-relativistic quantum mechanics with a final-range potential. It also generates the threshold behavior \( \sim k^{2l} \) and satisfies elastic unitarity. Detailed investigation \( \mathbf{3} \) for the Yukawa potential \( V(r) = g \exp(-\mu r)/r \) confirms the general character of the GP condensations. It also shows, that poles related to bound states (or resonances) are “initially” members of the set of GP condensing poles, and “evaporate” from it, when attraction increases. The energy plane for the Yukawa potential has many Riemann sheets, and most of the energy-plane poles are “hidden” on remote sheets, while the poles related to bound or resonance states approach to the physical region. The infinite number of the Yukawa energy-plane poles can be “visualized” by taking the limit \( \mu \to 0 \), which transforms the Yukawa potential into Coulomb one and demonstrates the well-known accumulation of the infinite number of Coulomb bound states near the threshold (note that the double limiting transition \( \mu \to 0, k \to 0 \) is not equivalent here to the similar, but reversed limit \( k \to 0, \mu \to 0 \)).

Existence of energy-plane poles with exotic quantum numbers is the necessary condition for existence of exotic hadrons. It appears satisfied under the familiar assumptions of unitarity and analyticity. But only more detailed dynamics may determine whether some of the poles emerge near the physical region, to provide indeed the stable or resonance exotic states.

Two more “technical” notes may be interesting:
1) Reggeons have been used above in a non-standard manner. Usually, to apply the CAM approach for obtaining results in the \( s \)-channel (the channel where the invariant \( s \) has the meaning of the squared c.m. energy), one begins from the crossed channel, where \( t \) and \( s \) are, respectively,
the squared energy and squared momentum transfer (see Ref. [3]). Analytical continuation of
partial-wave amplitudes in this $t$-channel allows, after returning into the $s$-channel, to study
behavior of the invariant amplitude (and cross section) at the high energy $s$ and at a fixed value
of the momentum transfer $t$. To obtain the conclusion about energy-plane poles in the $s$-channel,
we have used complex angular momenta in the same $s$-channel.

2) Gribov and Pomeranchuk suggested one more situation where infinitely many reggeons
might exist near a fixed point in the $l$-plane, this time $l = -1$ [9]. However, accounting for
existence of the moving branch points prevents this reggeon accumulation from emerging. The
threshold GP condensations of reggeons are not influenced by such branch points.

3 Summary and Conclusion

The presented results may be summarized as follows:

- Under the familiar assumptions of unitarity and analyticity, hadronic amplitudes have inﬁ-
  nite number of energy-plane poles with any quantum numbers, both exotic and non-exotic.
  Thus, there is an infinite “reservoir” of poles, which satisﬁes the necessary condition for
  existence of exotics. Most of the poles are “hidden” on remote Riemann sheets of the energy
  plane.

- Real existence (or absence) of exotic hadrons, i.e., of the $S$-matrix poles sufﬁciently near
  the physical region, may be guaranteed only by more detailed dynamics.

Meanwhile, the old wisdom[10], that “...either these states will be found by experimentalists
or our conﬁned, quark-gluon theory of hadrons is as yet lacking in some fundamental, dynamical
ingredient...”, is still alive.

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