Radiative Resistance at The Nano-scale: Thermal Barrier

N. Zolghadr and M. Nikbakht

Department of Physics, University of Zanjan, Zanjan 45371-38791, Iran.

(Dated: June 9, 2020)

In present article the radiative thermal resistance in a system of parallel slabs is investigated. The system is placed in an environment with a constant temperature and subjected to a constant temperature gradient, which causes a radiative energy flux through the system. We have calculated the steady-state temperatures profile of the system, assuming that the material and thickness of the middle slab could be different from the other slabs. Here we propose the exact formulation of thermal current and thermal resistance for radiative heat transfer in many-body systems. According to our results, the middle slab acts as a thermal barrier and depending on the width of this barrier, an extreme thermal isolation is achievable. Simulation results indicate that the thermal resistance of the barrier is an increasing function of the thickness for near-field separation distances but it is virtually insensitive to the barrier width in far field regime. The long range character of the radiative heat transfer, which occurs in system with identical slabs is also discussed.

I. INTRODUCTION

In recent years, many studies have been conducted on radiative heat transfer between objects with separation distances less than the thermal wavelength. Because the radiative flux at this scale violates Stephen Boltzmann’s law, heat flux management by controlling geometrical parameters and system-specific features has attracted much attention. Dependence on parameters is an interesting part of radiative heat transfer in two-body systems. These parameters can be either internal, such as size, shape, orientation, distance and material composition of objects, or an external parameter such as magnetic field in magneto-optical systems, thermal boundary conditions, the properties of surrounding media, or an electric field in metallic material systems. The quantitative form of the radiative heat flux can change as parameters are varied. In particular, the heat flux can be enhanced or decreased, or the net direction can change. As we move up from two-body to three-body systems, the radiative heat transfer can be tuned by changing the parameters, but theoretically and more recently it has been shown experimentally that three-body systems can provide the possibility to enhance radiative heat transfer over two-body counterparts. There has been a large amount of literature seeking to improve thermal rectification, thermal switching and thermal splitting by controlling various material and structural parameters. As the number of objects in the system increases, the many-body effects become very significant and as expected it influences the dynamics of temperature. On the other hand, the dynamics and the steady-state radiative heat flux may exhibit sensitive dependence on parameters, initial conditions and also on the thermal boundary conditions. Recent theoretical work on the thermal bistability has highlighted the importance of an external heat flux on the thermal switching in near-field radiative heat transfer. It has been previously established that the many-body parallel planar systems can provide distinctive properties for significant enhancement of near-field heat transport, and the geometrical properties as well as the initial condition for temperatures can have a remarkable effect on the temperature evolution.

The transfer of large amounts of energy between system components in the near-field regime results in a very strong temperature coupling at these scales. However, finding ways to minimize radiation heat transfer in systems that require thermal insulation is particularly important. In conductive heat transfer this isolation is mainly carried out using multi-layer structures. The temperature profile in such structures does not show a monotonic trend across an interface between different materials. Instead, there is a temperature difference athwart the boundaries. Similarly, a system of planar objects can resist radiation heat flux and it is expected to cause discontinuities in temperature profile. In other words, the components of the system provide a thermal resistance that must be considered in thermal design or analysis.

In this article, we have investigated the radiative thermal current and steady state temperature profile in a parallel planar system which is subjected to an external temperature gradient. Inspired by the idea of Kapitza resistance and interfacial thermal resistance, we have introduced a radiative thermal resistance in parallel planar objects that exchanges heat in the form of radiation and demonstrate the possibility of extreme radiative thermal isolation. We have introduced radiative thermal barrier and the linear and non-linear resistances are calculated for barriers with different materials. It is shown that both thermal current and temperature profile in steady-state regime depend strongly on the width and the composition of the barrier. The numerical results indicate that for a given composite system, due to the existence of a thermal barrier, the radiative thermal resistance depends not only on the width of the barrier but also on the separation distances. While we have used Silicon Carbide (SiC) and hexagonal Boron Nitride (hBN) as typical materials for slabs, the proposed formalism is general and can be applied to any planar system with arbitrary parameters (materials, widths, vacuum gap distances). Moreover, the proposed quantities (thermal resistance and radia-
local thermal equilibrium and there is no time variation of physical variables. The temporal behaviour of each slab is governed by equation:

$$\rho_i c_i \Delta_i \frac{\partial T_i}{\partial t} = \varphi_i(T_i, T_1, \cdots, T_{N-1}, T_R) \quad i = 2, \cdots, N - 1$$

(1)

where $\Delta_i$ is the thickness of the slab with mass density $\rho_i$ and heat capacity $c_i$. For a given thermal boundary condition and initial condition for temperatures, the net radiative heat flux per unit surface received by the i-th slab is given by:

$$\varphi_i = \sum_{j=0}^{N+1} \left[ \int_0^\infty \frac{d\omega}{2\pi} \int_0^\infty \frac{dk}{2\pi} \sum_{p=(S,P)} \Theta(\omega, T_j) T^{j,i}(\omega, k, p) \right],$$

(2)

for $i = 2, \cdots, N - 1$. In this relation, the second summation runs over two physical polarization states of the radiating field, i.e. S and P polarizations respectively. Moreover, $\Theta(\omega, T_j) = \hbar \omega / (e^{\hbar \omega/k_B T_j} - 1)$ denotes the mean energy of Planck oscillator at temperature $T_j$. The contribution of each slab in the heat flux is given by the transmission coefficient $T^{j,i} = T^{j,i}(\omega, k, p)$, which depends on the geometrical and intrinsic features of the system (see the appendix for more detail). The solution of Eq. (1) can be visualized as a trajectory in a (N-2)-Dimensional phase space. However, we are only interested in the long-time behavior of the system, i.e. the steady-state temperature profile that the system is able to reach as $t \to \infty$. Since the right-hand-side of Eq. (1) does not depend on $t$ explicitly, the system is autonomous and we only need to find fixed point of the system. The steady state temperature profile of Eq. (1) is defined by the fact that the net energy flux on each slab vanishes, i.e. $\varphi_i(T^*) = 0$ for $i = 2, \cdots, N - 1$, where $T^* = (T_L, T_1, T_2^{eq}, \cdots, T_{N-1}^{eq}, T_N, T_R)$ is the fixed point of the system in phase space. For further investigation of the steady-state of the system, we summarized and extend Eq. (2) to cover all system components, including slabs, environments and reservoirs. Hence

$$\varphi_i = F_{i,i} + \sum_{j \neq i} F_{j,i} + F_{i}^{ext},$$

(3)

for $i = 0, 1, \cdots, N, N + 1$. In second term the value of index $j$ runs from 0(L) to $N+1$ (R) (including the reservoirs and the external environment). Here $F_{i,i} \leq 0$ is a radiative cooling of the i-th component which could be a slab, a reservoir or an environment. Moreover, $F_{j,i} \geq 0$ is the power transferred from the j-th component to the i-th one. Finally, an external amount of heat ($F_{i}^{ext}$) should be transfer from, or to, heat reservoirs and environments in order to keep them in constant temperatures. It should be emphasized that $F_{i}^{ext} = 0$ for slabs with varying temperature, i.e. $i = 2, \cdots, N - 1$ to match Eq (2).

Suppose the left environment absorbs the rate of heat $F_{L}^{ext}$ and first slab absorbs the rate of heat $F_{1}^{ext}$. According to the conservation of energy $\sum_{i=0}^{N+1} \varphi_i = 0$, it is easy
show that \( F^\text{ext}_L(t \to \infty) + F^\text{ext}_R(t \to \infty) = -[F^\text{ext}_R(t \to \infty) + F^\text{ext}_N(t \to \infty)] \), which can be referred to as the left-right symmetry at steady state. Specifically, we can say now that the same amount of heat which is given to the left environment and reservoir at a time is equal to the one that taken from the right environment and reservoir, or visa versa. By setting \( \varphi_i = 0 \) in Eq. (3) for \( i = 0, 1 \) and using Eq. (2), it is straightforward to show that

\[
F^\text{ext}_1 = -\sum_{j=0}^{N+1} \int_0^\infty \frac{d\omega}{2\pi} \int_0^\infty \frac{dk}{2\pi} k \sum_p \Theta(\omega, T_j) T^{j+1},
\]

and

\[
F^\text{ext}_L = -\sum_{j=0}^N \int_0^\infty \frac{d\omega}{2\pi} \int_0^\infty \frac{dk}{2\pi} k \sum_p \Theta_{j,R} T^{j,L},
\]

with \( \Theta_{j,R} = \Theta(\omega, T_j) - \Theta(\omega, T_R) \). It is clear that these external powers are time-dependent during the initial stage of the dynamics of temperatures in the system. However, they eventually approach the steady-state values as the system reaches local thermal equilibrium. In addition, we know that due to the temperature difference caused by the boundary conditions, the radiative thermal current flows along the z direction either to the left or to the right. Hence, we define the net current flow along the system as:

\[
J^\text{eq} = F^\text{ext}_L + F^\text{ext}_1 \equiv -(F^\text{ext}_R + F^\text{ext}_N),
\]

which remains constant in the steady-state regime. Here, \( J^\text{eq} \) represents the radiant energy passing through the system in local thermal equilibrium. It is important to emphasize that for the system under consideration, the transmission probabilities do not depend on temperature as in phase changed materials, which implies that the power dissipated in each slab (\( \varphi_i \)) is a continuous function of the right and diagonal elements in the Jacobian matrix of the system \( \left( \frac{\partial \varphi_i}{\partial \varphi_j}, i \neq j \right) \) have constant sign, independent of the system’s state. As a result, the system of equations \( \varphi_i = 0 \) have only one fixed point and since the system is non-conservative, the fixed point is stable. On the other hand, the steady-state and so the equilibrium thermal current do not depend on the choice of initial condition of the system, i.e. \( [T_2(0), \cdots, T_{N-1}(0)] \). Using Eqs. (4) to (6), the steady-state thermal current passing through the system can be expressed as

\[
J^\text{eq} = \int_0^\infty \frac{d\omega}{2\pi} \int_0^\infty \frac{dk}{2\pi} k S^\text{eq}(\omega, k),
\]

with

\[
S^\text{eq}(\omega, k) = \sum_{j=0}^N \sum_p \Theta^\text{eq}_{R,j}[T^{j,L}(\omega, k, p) + T^{j,1}(\omega, k, p)].
\]

Here, \( \Theta^\text{eq}_{R,j} = \Theta(\omega, T^\text{eq}_R) - \Theta(\omega, T^\text{eq}_j) \) and \( S^\text{eq}(\omega, k) \) can be interpreted as the dispersion relation of the thermal current passing through the system. It should be emphasize that, \( S \) depends not only on the choice of materials and system geometrical properties, but also on the steady-state temperature profile. For very large separation distances, where the far-field interaction dominates, the thermal current and so the resistances could mainly decided by the coupling of the middle slabs with the thermal baths (depending on the boundary conditions and materials) rather than the reservoirs temperature gradient. For the sake of simplicity we take \( T_L = T_R = 300 \text{ K} \) in our calculations. It is also easy to show that, for the special case of \( N = 2 \) and in the absence of thermal baths \( T_L = T_R = 0 \text{ K} \), Eq. (7) reduces to the well known formula for the heat flux exchanged between two parallel slabs.

There is an electrical analogy with radiative heat transfer that can be used to exploited in, see Fig. 1 From this perspective the radiative heat flux is equivalent to the electric current and each slab is a pure resistance to radiative heat flux. Hence, we can define slab resistance as

\[
R_i = \frac{\Delta T_i}{J_i},
\]

where by definition, \( \Delta T_i = (T_{i-1} - T_{i+1})/2 \) is the temperature difference at the position of \( i \)-th slab and \( J_i \) is the net radiative thermal current passing through it. It is plausible that the thermal current and so the temperatures are function of times. However, when the system reaches steady-state, they depend only on the temperature of the two reservoirs and environments, the slab properties and their separation distances. Since the thermal current has only one path to take in the system under consideration, it is the same through all slab at the steady-state regime, i.e. \( J_i \to J^\text{eq} \). It also seems reasonable to postulate that thermal resistances are independent of environment and reservoirs temperatures in linear regime where \( T_L \sim T_1 \sim T_N \sim T_R \). To this end, a more useful quantity to work with is the linear resistance of slabs

\[
R^\text{eq}_i = \lim_{J \to 0} \frac{\partial(\Delta T_i)}{\partial J}_{\text{eq}}.
\]

In the system under consideration, we have used \( T_L = T_R = 300 \text{ K} \), since the contribution of environments in the thermal current is small compared to that of reservoirs (i.e. \( |F^\text{ext}_L| \ll |F^\text{ext}_R| \) especially for small separation distances), the derivative evaluated at thermal equilibrium in the limits of \( T_1 \sim T_N \). The total thermal resistance of the system could be calculated by simply adding up the resistance values of the individual resistors, i.e.

\[
R^\text{eq}_\text{total} = \sum_{i=1}^N R^\text{eq}_i = \lim_{\Delta T \to 0} \frac{\Delta T}{J^\text{eq}},
\]

with \( \Delta T = T_1 - T_N \). We could also define a thermal boundary resistance, similar to the Kapitza resistance, as the ratio of the temperature variation at the interface to the heat current across it. As a result, like a series circuit, the resistance of each slab would be the addition
of its interface resistances

\[ R_{eq}^i = R_{di}^i + R_{ir}^i, \]  

(12)

where \( R_{eq}^i = (T_{eq,i}^i - T_{eq,i+1}^i)/2J_{eq}^i \) is the resistance of the left interface of the slab and \( R_{ir}^i = (T_{i}^i - T_{i+1}^i)/2J_{eq}^i \) is the right part.

III. RESULTS AND DISCUSSION

Based on the framework built above, we can calculate the steady-state temperature profile of a typical radiative thermal barrier. The system that we consider here consists of 15 slabs, which is a hBN or a SiC slab with thickness \( \Delta_8 = \Delta_{\text{barrier}} \) sandwiched between 14 identical SiC slabs. The separation between slabs are equal, i.e. \( \delta \), and the system is positioned in an environment with constant temperature \( T_L = T_R = 300 \text{ K} \). The thermal gradient is applied by maintaining the two reservoir slabs at constant but different temperatures. For the non-linear regime, the temperature of hot reservoir and cold reservoir are fixed at \( T_1 = 400 \text{ K} \) and \( T_{15} = 300 \text{ K} \), respectively. However, we have used \( T_1 \sim T_{15} = 300 \text{ K} \) to calculate resistances in linear regime. For the complex dielectric function of SiC and hBN, we used the Lorentz-Drude model:

\[ \varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_L^2 - \omega^2 - i\Gamma\omega}{\omega_L^2 - \omega^2 - i\omega\Gamma}, \]  

(13)

where the parameters for silicon carbide (SiC) are as follow: \( \varepsilon_{\infty} = 6.7 \) is the high frequency dielectric constant, \( \omega_L = 1.83 \times 10^{14} \text{ rad/s} \) is the longitudinal optical frequency, \( \omega_T = 1.49 \times 10^{14} \text{ rad/s} \) is the transverse optical frequency, and \( \Gamma = 1.0 \times 10^{12} \text{ rad/s} \) is damping coefficient. While for hexagonal Boron Nitride (hBN) these constants are, \( \varepsilon_{\infty} = 4.9 \), \( \omega_L = 3.03 \times 10^{14} \text{ rad/s} \), \( \omega_T = 2.57 \times 10^{14} \text{ rad/s} \), and \( \Gamma = 1.0 \times 10^{12} \text{ rad/s} \).

A. TEMPERATURE PROFILE

Figure 2 shows the stationary state temperature profile of a 15-body parallel planar system as a function of normalized position of slabs \( Z_i \). The temperature of the reservoirs and environments are \( (T_L, T_1, T_{15}, T_R) = (300, 400, 300, 300) \text{ K} \). The slab separation distances are \( \delta = 100 \text{ nm} \) and the thickness of the barrier is chosen as \( \Delta_8 = \Delta_{\text{barrier}} = 200 \text{ nm} \) equal to the rest of the slabs.

The typical temperature profile of Fig. 2(a) which corresponds to SiC barrier (SiC–SiC–SiC system) is clearly continuous. This profile has a part that varies roughly linearly across a large portion of the system with large gradients at the two ends due to boundary effect. Since the slabs are very close to each other, near field radiative heat transfer is the dominant mechanism that determines the steady-state temperatures. As a result, the coupling with the left and right reservoirs is strong and the temperature gradient in the profile is large. Figure 2(b) displays a similar thermal profile when the material used for the barrier (slab no. 8) is hBN (i.e. SiC–hBN–SiC system). This profile is dramatically different from the system with SiC barrier (Fig. 2(a)) in that a sharp discontinuity in temperature appears across the barrier position. In addition, there are pronounced boundary effects as for the SiC case. It can be seen that the left slabs \( (Z_i < 0.5) \) coupled to the left reservoir \( (T_1 = 400 \text{ K}) \) and the right slabs \( (Z_i > 0.5) \) are isolated from the left reservoir and well coupled to the right reservoir \( (T_{15} = 300 \text{ K}) \).

B. THERMAL RESISTANCE

In Fig. 2(a) the temperature drop at the barrier \( \Delta T_{eq}^8 \) is plotted against steady state thermal current \( J_{eq}^8 \) that
corresponds to different boundary conditions. As it can be seen, the temperature drop increased linearly with thermal current for both SiC and hBN barriers, but for very large thermal currents, their behaviour becomes non-linear, to some extent. The linear regression is used to determine the slope that is the resistance of the barrier in both linear and non-linear regimes. As an example, the solid black dots on the diagram correspond to a linear regime caused by boundary condition \((T_L, T_1, T_{15}, T_R) = (300, 310, 300, 300) \text{ K}\). On the other side, the solid green dots correspond to a non-linear regime for boundary condition \((T_L, T_1, T_{15}, T_R) = (300, 400, 300, 300) \text{ K}\). The temperature profiles of the latter case are those shown in Fig. 2. Compared with the linear thermal resistance of 0.018 \(\text{Km}^2\text{W}^{-1}\) for SiC barrier, the thermal resistance is 16 times higher for hBN barrier. It is clear from Fig. 3(a) that larger temperature gradients \(\Delta T\), create a larger temperature difference across the barrier \(\Delta T_{\text{eq}}\) and results in higher thermal current \(J_{\text{eq}}\). We observe that thermal resistances decrease with temperature in a power law form, and the decrease is larger for interface with weaker coupling (here the hBN barrier).

Using Eq. (10), we have calculated linear thermal resistance \(R_{\text{eq}}\) of all slabs and results are shown for both SiC–SiC–SiC and SiC–hBN–SiC systems in Fig. 3(b) as a function of normalized position of slabs. We observe that both systems show similar trends on the sides, however, the resistance of the hBN slab and its neighbours are (\(~20\) times) greater in SiC–hBN–SiC system compared to the SiC barrier in SiC–SiC–SiC system.

**C. BARRIER WIDTH EFFECT**

To analyze the effect of barrier width on the radiative thermal transport properties, in Fig. 4(a) we present the
the temperature profiles of the SiC–hBN–SiC system for different hBN thicknesses. As can be seen, the increase in the thickness of the hBN barrier is associated with the increase in temperature discontinuity on both sides of the barrier. The corresponding linear resistance profiles which are shown in Fig. 4(b) confirm that as the barrier thickness increases, its thermal resistance increases.

In Figure 5(a), the steady state thermal current dispersion relation $\mathcal{S}_8^{eq}$ in $(\omega, k)$ space fixing $\delta = 100$ nm and $\Delta_{SiC} = 200$ nm for boundary condition $(T_L, T_1, T_{15}, T_R) = (300, 400, 300, 300)$ K. The horizontal lines represent $\omega_T$ and $\omega_L$ of SiC and hBN. (a) SiC barrier, $\Delta_8 = 200$ nm. (b) hBN barrier, $\Delta_8 = 200$ nm. (c) hBN barrier, $\Delta_8 = 500$ nm. (d) hBN barrier, $\Delta_8 = 1000$ nm.

FIG. 5. (Color online) The steady-state thermal current dispersion relation $\mathcal{S}_8^{eq}$ in $(\omega, k)$ space fixing $\delta = 100$ nm and $\Delta_{SiC} = 200$ nm for boundary condition $(T_L, T_1, T_{15}, T_R) = (300, 400, 300, 300)$ K. The horizontal lines represent $\omega_T$ and $\omega_L$ of SiC and hBN. (a) SiC barrier, $\Delta_8 = 200$ nm. (b) hBN barrier, $\Delta_8 = 200$ nm. (c) hBN barrier, $\Delta_8 = 500$ nm. (d) hBN barrier, $\Delta_8 = 1000$ nm.

It is interesting to know that according to the values selected for the thermal boundary conditions $(T_L, T_1, T_{15}, T_R) = (300, 400, 300, 300)$ K, the direction of thermal current $J^{eq}$ is from left to right (i.e. positive). However, the sign of $\mathcal{S}_8^{eq}(\omega, k)$ is not necessarily positive. Since the slabs are very close to each other $\delta = 100$ nm, we observe in Fig. 5(a) that the contribution of evanescent waves in thermal current is positive in $(\omega, k)$ space, while the contribution of propagating waves is mostly negative. Similar argument holds in the case where the SiC thermal barrier is replaced with hBN material of different thicknesses. From the shown plots in Fig. 5(b)-(d), it is also apparent that the coupled surface mode resonances of SiC slabs which are coupled via the vacuum gap in SiC-SiC-SiC configuration persist when replacing the barrier with hBN material. Moreover, by increasing the width of the hBN layer, the contribution of the TM modes in the spectral frequency window between $\omega_T$ and $\omega_L$ of the SiC layer has decreased significantly, and instead new channels for thermal current (with the opposite direction of propagation) are emerged in the diagram. As a result, in addition to the significant change in the dispersion curves of photons, the increase in the opposing thermal currents in $\mathcal{S}_8^{eq}(\omega, k)$ is responsible for the increase in the thermal resistance for large barrier thicknesses.

To compare the thermal resistance of hBN with SiC barrier, we have calculated the linear resistance of barriers with different thicknesses. Shown in Fig. 6(a) are our results for the resistance of SiC barrier as a function of the barrier thickness, for different vacuum gap separation distances. The thickness of the other slabs in the system are $\Delta = 200$ nm, as in the previous figures. It is striking that, for a given width for vacuum gaps, the calculated resistance depends not only on the thickness of the barrier but on the thickness of the other slabs too. It is interesting that this dependence follows a certain rule for SiC barrier. For large separation distances $\delta \geq \Delta$, the thermal resistance of the barrier of width $\Delta_8 \simeq \Delta$ is always minimal. However, as the separation distance decreases, this minimum occurs for smaller barrier thicknesses, i.e. $\Delta_8 \ll \Delta$. The conditions of the hBN barrier are quite different from those of SiC, see Fig. 6(b). In this case, increasing the barrier thickness is accompanied by an increase in thermal resistance. Although this increase is negligible and slightly oscillatory for high separation distances, but similar to the SiC barrier, the resistance is saturated at large barrier thicknesses, as expected.

D. VACUUM GAP EFFECT

Finally, the influence of the width of the vacuum gaps on the linear resistance of the hBN barrier is presented in Fig. 7. The steady-state linear resistance is shown for different barrier thickness values, as a function of vacuum gap widths from near-field to far-field regime. It is
seen that the thermal resistance of the hBN barrier and consequently the thermal resistance of the whole structure in the near field regime is much lower than the far field regime. On the other hand, the rapid decrease in the transmission probabilities between slabs by increasing the vacuum gap widths is responsible for the power law increment of the thermal resistance in the near-field regime which is modulated by logarithmic periodic oscillations and saturates in far-field limit. In agreement with previous results, the barrier resistance increased for larger barrier thicknesses in near-field regime.

IV. CONCLUSION

In summary, we have used an electrical circuit approach to introduce heat resistance for the components of systems that transmit energy through radiation at the nanoscale. For this purpose we proposed a method for calculating the steady-state radiative current in parallel planar systems. This method can be a useful guide for understanding and optimizing the thermal performance of nanoscale systems. The simulation results indicate that the temperature profile in a parallel planar system, exhibits fantastic changing characteristic around the barriers. We have shown the thermal insulation occurs due to the presence of thermal barrier and the temperature does not show a monotonic trend across the barrier, instead, there is a temperature difference athwart the barrier.

Appendix A: Transmission coefficients in parallel planar systems

The system under consideration consist of $N$ parallel slabs placed at $z_i$ along the z-axis. The separation distance between the consecutive slabs $i$ and $i + 1$ is $\delta_i = z_{i+1} - z_i - \Delta_i/2 - \Delta_{i+1}/2$ where $\Delta_i$ is thickness of the $i$-th slab. The first ($i = 1$) and the last ($i = N$) slabs are connected to reservoirs with fixed temperatures $T_1$ and $T_N$, respectively. The indexes $i = 0 \equiv L$ and $i = N + 1 \equiv R$ are used for the left and the right thermal baths which are kept at fixed temperatures $T_L$ and $T_R$, respectively. The many-body energy transmission coefficients $\mathcal{T}^{i;i}$ take into account the presence of different slabs at the same time and can be fully determined in terms of $\hat{T} = \hat{T}(\omega, k, p)$:

$$\mathcal{T}^{L,i} = \hat{T}_{i-1}^L - \hat{T}_i^L,$$

$$\mathcal{T}^{j,i} = \hat{T}_{i-1}^j - \hat{T}_i^{j-1} - \hat{T}_i^j + \hat{T}_i^{j+1},$$

$$\mathcal{T}^{R,i} = -\hat{T}_{i-1}^N + \hat{T}_i^N,$$

for $j, i = 1, \cdots, N$. The definition of these coefficients are as follow:
These coefficients satisfy the reciprocity relation \( \hat{T}_i^j = \hat{T}_i^j \). The many-body scattering coefficients \( \rho_i^+ \), \( \rho_i^- \), and \( \tau_i \) are given by

\[
\rho_i^+ = \rho_i^+ e^{-ikz_\ell (\Delta_m + 2z_m)}, \\
\rho_i^- = \rho_i^- e^{-ikz_\ell (\Delta_m - 2z_m)}, \\
\tau_i = \tau_i e^{-ikz_\ell \sum_{\ell=\gamma}^m \Delta_\ell},
\]

where

\[
\hat{\rho}_i^+ = \rho_i^+ (e - ikz_\ell \Delta_m)^{-1}, \\
\hat{\rho}_i^- = \rho_i^- (e - ikz_\ell \Delta_m)^{-1}, \\
\hat{\tau}_i = \tau_i (e - ikz_\ell \Delta_m)^{-1}.
\]

Here, \( \rho_j \) and \( \tau_j \) are the scattering coefficient for a single body which are given by

\[
\rho_j = \frac{1 - e^{2ikz_j \Delta_j}}{1 - r_{p,j}^2 e^{2ikz_j \Delta_j}}, \\
\tau_j = \frac{1 - \frac{r_{p,j}^2}{r_{p,j}^2} e^{ikz_j \Delta_j}}{1 - \frac{r_{p,j}^2}{r_{p,j}^2} e^{2ikz_j \Delta_j}}.
\]

In equation (A6), \( r_{p,j} \) is the Fresnel coefficient in which \( p \) indicates the polarization. This coefficient for two possible polarizations including TE and TM, is defined as

\[
r_{TE,j} = \frac{\mu_j k_z - k_{zj}}{\mu_j k_z + k_{zj}}, \\
r_{TM,j} = \frac{\varepsilon_j k_z - k_{zj}}{\varepsilon_j k_z + k_{zj}}.
\]

In the above equations \( \varepsilon_j \) and \( \mu_j \) are electric permittivity and magnetic permeability, representing the optical properties of \( j \)th slab.

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