How Students Build Their Mathematical Dispositions towards Solving Contextual and Abstract Mathematics Problems

Mukhtamilus Sa’diyah¹, Cholis Sa’dijah¹*, Siswo¹, UcikFitri Handayani¹

¹ Mathematics Education, Faculty of Mathematics and Science, Universitas Negeri Malang, Jalan Semarang No. 5, Malang 65145, Indonesia

* cholis.sadijah.fmipa@um.ac.id

Abstract. Mathematical dispositions affect students’ performance when studying mathematics. Teachers should have high awareness of students’ mathematical dispositions when solving mathematical problems. This research identifies students’ viewpoints about problems in mathematics. Understanding students’ viewpoints would assist teachers to elevate students’ mathematical dispositions by developing more efficient test items. This is a case study research using qualitative method, of which students who have high motivation in learning mathematics were asked to answer questions related to their mathematical dispositions. Three main focuses observed on this research were students’ perceptions, attitudes, and efforts. The data show that the students build their mathematical dispositions through some process. Although they have mathematical dispositions, their preference in mathematics problems is different. Most of the students prefer solving abstract problems while others prefer solving contextual problems. This difference affected their performance in solving mathematics problems. This means that teachers should pay more attention to the form and level of problems given to the students, so that students would have less effort to understand the problems and emphasize their works by thinking more deeply when solving problems.

1. Introduction

The most critical barrier when students learning mathematics is the difficulties in resolving the given mathematical problems [1]. As a concept, the word “problem” has multiple academic interpretations.

A number of researchers in the mathematical education field outline “problem” as all practice test items given for train students’ ability related to a newly learned materials[2][3]. Others define “problem” as non-routine and complex test item which require application and association of mathematical concept[4][5]. In this research, the word “problem” refers to all mathematics test items aimed at training students’ competence. There are two type of problems, which are abstract and contextual mathematics problems.

Abstract mathematics problems are defined as mathematics problems that use mathematical symbols and operations. Although it is easy to solve the problems using mathematics procedure, students sometimes have difficulty in understanding the context of usage of the problems. Whereas contextual mathematics problems are defined as mathematics problems that are closely related to students’ everyday life situation and most likely to be experienced by the students in person. Students would have better understanding about the context of the given problems[1]. However, some students are more likely to be confused when distinguishing important information of the problems.
Not all of mathematics problems could be solved immediately by applying the given formulas. A considerable number of problems require further analysis and implementation of multiple steps of solving methods to obtain correct answers. The skill to resolve the problems is called the problem-solving ability. Problem solving is essential for mathematics learning, as it help students to develop their cognitive process and openness when solving more complex problems[6].

Researchers had formulated strategies to reduce students’ difficulties when solving mathematics problems by developing learning methods[7][8], learning media[9][10][11], and also evaluation test items[12][13]. However, it is also constructive to shape students’ attitude towards mathematics. A number of researches had supported the importance of students’ attitudes and desires to learn mathematics[10][14][15][16]. These attitude and desire are known as mathematical dispositions.

Students’ mathematical dispositions refer to students’ tendency to perceive mathematics as useful, reasonable, and important material, aside from their confidences of their skills and efforts[17]. A research conducted by Beyers[18] has introduced mathematical disposition division into three assessment domains, of cognitive, affective, and conative. The cognitive domain refers to students’ awareness when learning new knowledge or concepts. The affective domain refers to students’ emotional responses. The conative domain focuses on students’ efforts to develop and utilize their knowledge when solving mathematical problems. These three domains, then became assessment references to learn about students’ mathematical dispositions.

A considerable number of enquiries have been conducted to discover the significance of students’ mathematical dispositions[17][18][19][20]. Some examined the influence of students’ mathematical dispositions towards performance[21], students’ achievements[22], and students’ problem-solving ability[16]. Students with mathematical dispositions tend to have more confidence about their skills, appear to be more challenged to solve mathematical problems, and realize how mathematics contributes to their daily life problem solving. When students’ mathematical dispositions are improved, students’ performance and learning interest would be improved, then students could easily and happily take part in mathematics learning at school[21][22]. However, there has been inadequate number of researches conducted to learn the construction process of mathematical dispositions and its relations to mathematics problems. In fact, the construction process of students’ mathematical dispositions had a significant influence towards students’ perceptions about mathematics in the future[20]. Students would assume that mathematics is important, if they feel attracted and challenged to solve given problems[16].

Furthermore, improving mathematical dispositions should be supported by selecting proper problems. Identifying preferred and suitable problems could also help students increasing their performance in learning mathematics. Investigating students’ mathematical dispositions when solving problems would help teachers depicting students’ affective, cognitive, and conative viewpoints[18]. Researches related to mathematical dispositions mostly use systems of multiple-choice questionnaire[17][23]. The use of questionnaire is limited to statements written or chosen, and unable to disclose in-depth meaning. It also holds back students’ flexibility to state their standpoints. By selecting open-ended questions, the answers given by students would be more diverse and receptive[14].

This research discloses variations of test items that appear more attractive to the students. In addition, teachers would have further references to elevate their students’ mathematical dispositions by observing the process of solving mathematical problems and the type of test items preferred by the students. The results would also be implemented by teachers to develop teaching methods aiming at improving students’ mathematical dispositions.

2. Method
This is a qualitative descriptive research[24]. Research subjects were selected from 8th graders in Malang who took extra mathematics lesson independently. The researcher gave questionnaire inquiring their mathematics learning motivation. Through the questionnaire, the researcher collected five students who had high motivation in learning mathematics as subjects. This research identifies
how they built their mathematical dispositions towards solving mathematics problems and showing their preferred problem. Instrument utilized was essay questions including rationale behind students’ motivation to learn mathematics, students’ knowledge about activities related to mathematics, forms of mathematical problems, and their difficulties when solving problems. There were five students, namely S1, S2, S3, S4, S5 who had been asked to answer the questionnaires. The problem given was being connected with general activities which is more likely to be experienced by students. Figure 1. and Figure 2. exhibit test items given to the students. As the possibilities for short and lack of information answers provided in the questionnaires, the researchers had prepared interview sessions to obtain in-depth responses and further confirmations.

Test Item A
Enter data on the graph based on the given table

| a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|
| 3 | 2a | 3b | a + 10 | b − 5 | b | c − 10 |

Of which \( a − g \) is value is the most? What is the value? Which one has the smallest value? What is the value? Explain how you got the answer.

Figure 1. A mathematics problem using an approach of abstract representation

Test Item B
From the diagram, it appears that Steve has 5 marbles. If it it known that
- Rosalie has twice as many as what Steve has
- Mark has 4 times marbles more than Rosalie
- Peter has 4 times marbles more than Steve
- Alice has as many as what Steve has
- Jane has 5 times marbles less than Rosalie
- Helen lost all marbles she had
- Monica has as many marbles as Rosalie.

Finish the bar chart above based on the given information!

Who has the most marbles? How many marbles does she/he have? Who has the less marbles? How many marbles does she/he have? Explain how you got the answer.

Figure 2. A mathematics problem using an approach of contextual representation

At the end of data collection phase, students’ worksheets and interview transcripts were identified based on Beyer’s indicators [18] of students’ mathematical dispositions, of: perception, interest, and effort when solving contextual mathematical problems. The data were analysed thematically, based on students’ dominant answers and other unique findings. Afterwards, conclusion was drawn by considering variations of students’ answers.

3. Result

3.1. Students’ self-perception towards mathematics

The first question in the essay was related to students’ early opinion about mathematics. Of all 5 students, there were two students who claimed that they have pleasant experiences about mathematics, which were S3 and S4. S3 stated that the mathematics teacher had been teaching with clarity making them more interested in studying mathematics. Meanwhile, S4 wrote that the high mark on mathematics at the previous education level was the main factor for loving mathematics. This made the S4 keeping at studying mathematics. The answers imply the important role of the teachers to build students’ confidence towards mathematics, either from test items formulation, or the teaching methods. These two side would further assist students to build their motivations to learn mathematics.

On the other side, three other students asserted that bad experiences when learning had caused them dislike mathematics. For S1, the difficulties were in understanding the given problems and memorizing related formulas. S2 had a similar experience by stating that hardships when memorizing necessary formulas had withheld the subject from learning mathematics, but the subject express the willingness to learn mathematics further due to the consciousness about the importance of mathematics. Whereas, S5 experienced mood swings when learning mathematics, depending on the learned topics.
All the answers imply the complexity of mathematics as the main set back from becoming interested in learning mathematics.

In the next question, students were asked about activities and games related to mathematics. All the five students wrote activities related to numbers and basic calculations, whereas the mathematics materials that they had learned in the junior high school have not been exemplified in the daily life. On talking about games, three students, which were S1, S2, and S4 mentioned number-related games, while the two others admitted that they did not have references about games that are closely related with mathematics. This signifies that the students could not implement the use and advantages of mathematics in their daily life situations. In other words, mathematics is perceived as a learning subject in formal education settings of all levels. The games that the subjects familiar with were popular online games such as Hago, Line Gets Rich, PUBG, and CS:60. Skills learned from mathematics should be applicable for these games. Further, teachers also should be aware that these games could be utilized as a means of attracting students’ interest in learning mathematics. Students’ lack of comments on the relations between their popular games and mathematics exhibits the fact that mathematics problem solving given had failed depicting students’ everyday life situations. As a result, students were less aware about the importance of mathematics in their life.

3.2. Students’ interest in solving mathematical problems

In order to understand students’ interest in solving mathematics problems, two test items were given to the research subjects with different representations. One test item was an abstract problem with numbers of mathematical operations and symbols. While the other test item was more contextual.

When students were given a choice of questions, four out of five students chose the abstract mathematical problems compared to the contextual one. But interestingly, the reasons the had given were anonymous, which was: it is easy. Students who preferred abstract problems stated that they were easier as they were short and not confusing. Students claimed that they had only needed to apply learned formulas or resolution procedures. These students stated that, to solve problems, they need to memorize and study the formulas they had been taught before. This had also become the reason why they dislike contextual problems. According to their statements, contextual problems were more complicated as it has too many words and given information. Similar justifications were given by students who preferred solving contextual problems. It was easier to solve as the students had a better frame of reference and it was relatable. This helped the students to map out problem solving procedures based on information obtained from their daily life experiences.

Their answers to the interviews were also noteworthy. The word “easy” refers also to the procedure of problem solving. When solving abstract problems, they just had to utilize general and standard procedure to solve the problem. Contextual problems appeared to be more difficult to be solved as narrative test items contain too many words, which confused them to choose the proper information. On the contrary, students who preferred solving contextual problems admitted that they could be more flexible when solving the problems as they did not need to be fixated on any problem-solving procedures. When students happened to forget the formula needed, their experience would become practical for seeking resolution and problem-solving strategies for given test items. Furthermore, the problems had become more familiar and relevant for the students.

3.3. Students’ effort to solve mathematical problems

To discover students’ effort, firstly, the students were asked about their difficulties when solving mathematics problems. Figure 1. and Figure 2. have relatively simple problems, but it appeared that students had been still encountering obstacles to finish the test item. When solving the abstract problem, they were frequently asking about required formulas and resolving procedures. Conversely, when solving the contextual problems, they were frequently asking about relevant information for answering the given questions. The type of difficulties faced by the students was a difficulty in basic work. As a matter of fact, the problems given on this research instrument was a simple one, which did
not require multiple formulas to finish. There are more complex and require more advanced level of thinking to be solved.

Regarding their efforts, students were asked about their attempts to solve the problems. Their answers were divided into two. S1 and S2 had been relying to other people’s assistance when solving problems. They confessed that they consulted the teacher or friends about the necessary procedures to solve the problems, aside from peer-comparing their answers. On the contrary, S3, S4, and S5 had preferred to re-read the problems and tried to better understand the given information. The different efforts demonstrate contrast in the learning methods. A number of students tend to be depended to other people to help them, while the other half prefer to be independent in figuring out the answers for the given problems.

Lastly, the students were given questions about their motivations to learn mathematics. S1, S2, and S3 had an inclination for good marks to be easier to embark upon higher level of education at a reputable school. These students were grade-oriented, so the type of mathematics problems that could motivate them to a greater extent is the solvable and easy problems. S4 dan S5 had given different answers. S4 explained that mathematics is needed in all fields, so that the subject tried to learn mathematics in discrete ways. Although S4 showed preference for solving abstract problems, the subject was aware about the importance of studying how to solve contextual problems, as it requires higher level of comprehension. S5 also answered in accordance to S4. The subject was willing to learn further the problem that are challenging, by mastering the materials. This indicates that the student has a high motivation for learning and becoming an expert about mathematical concepts.

3.4. Students’ insight about contextual problems
It emerges that most of the students tend to avoid solving contextual problems. It has become a common assumption that contextual problems should be easier for students, because the problem is adapted from students’ daily lives, as stated by S2. However, the other four students, although admitting the closeness between the test item and everyday life situations, had argued that the narration was too long and complicated, thus become hard to be understood. S1 asserted that the main trouble was interpreting abundant words and given information. The subject felt lazy to read the problem. In addition, S2 had been avoiding contextual problems for its complexity, as S4 and S5 also disclosed in the interview. Meanwhile, S3 articulated that contextual problems have not made him more interested in studying mathematics as the problems given by the teacher were trivial and unattractive. This made students become reluctant in solving contextual problems.

4. Discussion
4.1. Students’ perception
The students’ answers had revealed that they are fond of mathematics they had built a favourable history with mathematic since they were in the elementary school. Especially when they got good grades on tests or examinations. Their passions towards mathematics then has been continued until their junior high schools, and make them like activities related to mathematics, one of which was their choice of games. It also has become apparent that students’ good grades at the elementary school has affected their positive preferences for mathematics. The preceding impression of mathematics affects students’ attitudes at the next level of education[20]. The high standard achievements foster students’ strong confidence when learning mathematics. On the contrary, students with bad grade would have a propensity to feel reluctant to study mathematics at the next level of education.

Students’ achievements in Indonesia are measured at the final examination, especially at the end of a level of education. Students’ statements confirmed that the final examination namely national exam had been an indication of students’ confidence level. If their national exam grades are good, they would be highly motivated to learn mathematics further at the next level of education. This should be a significant consideration for teachers to further build students’ confidence both in the learning and in the evaluation processes. Teacher should be able to adjust test items given to their students accordingly to their level of competence to avoid scepticism due to hardships when solving
mathematics problems. The level of difficulties should be tailored fitting to students’ level of competences. The same treatment should be given on the evaluation phase. A good mathematical problem is characterized with a complexity, but still solvable if students spare more efforts. The accommodation of ideal problems would help students improving their learning motivations. It is obligatory for teacher to build their students’ level of confidence by providing comprehensive test items that also able to stimulate students’ deep thinking. Not only that students would be more interested in solving the problems given, but also would have opportunities to practice their abilities gradually. This idea is supported by Lee[25], who states that an ideal problem should have an easy entry method, but requires higher mathematical abilities to find the solutions. Besides, in the problem learning process, teachers should provide varied forms of problem-solving representations, as proposed by Sa’dijah[26], that introducing multiple forms of representations would support students to adjust their interpretations with their learning styles.

4.2. Students’ Motivation

Students’ priorities towards mathematical problems are discrete, depending on their perceptions about the problems. Either students who preferred solving abstract problems or the ones who preferred to solve contextual problems, had provide the same reason with different angles of “easy.” This signifies that there are no differences in the level of difficulties between abstract and contextual problems[4][27]. But it is recognized that the purpose of the two distinctive type of problems are unalike. Abstract problems are given to introduce students towards mathematical concepts and mathematical forms that simplified their efforts to learn mathematical formulas and creating a network of mathematical concepts. Whereas, contextual problems aim at introducing the implementation of mathematics in students’ everyday life situations. This differences also refer to the concepts that are being taught and the approaches that are being applied. In addition, it is more likely that students’ age and their prior achievements in mathematics might affect the role of this abstract and contextual problems[27].

The students who showed preferences towards abstract problems expressed that formulas and procedures are significant for them. Listening procedures for solving problems have become habitual for these students. It might be caused by the tendency for their teachers to provide descriptive explanations. This is supported by a research conducted by Murtafiah et al.[28] towards pre-service teachers. By memorizing problem solving procedures, students assumed that they would be able to find the answer for the test items. However, there are setbacks that occurred once they were given more complex problems. In addition to learn about procedures, students should practice their abilities to connect their learned concepts and procedures to systematically solve problems. This could be trained to the students when they solve contextual mathematical problems.

Contextual mathematical test items introduce closely related problems of students’ life. This situational approach assists students to identify which mathematical concepts relevant for solving given problems. Further, students would be able to use their knowledge of everyday life situations to determine applicable problem-solving procedures[29]. But when solving contextual problems, it is suggested that the students are equipped with capabilities to analyze relevant information to solve the problems. These skills were less mastered by the students, so that they felt only a little interested in solving contextual problems.

4.3. Students’ Efforts

It is emerged from this research that the first thing came up to these students’ mind when solving mathematical problems is formulas. This has also become the main reason why students prefer abstract problems to contextual ones, as abstract problems are more likely to be directly solved procedurally by following formulas that were previously introduced to the students. Whereas, it is assured that to solve contextual problems, students should understand the framework before finding resolutions for the problems[2]. In fact, there are mathematical problems that require simple solutions without applying any formulas.
The contrasting standpoints stated by the students were strongly influenced by the way they manage information. Students who showed preferences towards solving abstract problems tend to experience difficulties in sorting and managing information obtained. They were confused in determining which information is relevant for solving the problems. The amount of information given in contextual problems made them avoiding the problems. On the contrary students who had preference in solving contextual problems take the advantage of their everyday situations to solve problems.

4.4. Contextual mathematical problems
Contextual mathematical problems define as problems that require less procedural problem solving and have tendency to be solved by informal strategies[30]. Students who preferred solving contextual mathematical problems expressed little interest towards standard formulas and problem-solving procedures. They claimed that they only need basic concepts to solve problems. Majority of these students rarely used formulas given by the teachers in class. They tend to trust their cognitive comprehension about the problems. Mostly, procedures that were implemented by the students are different with ones that are taught by the teacher.

Students’ habit when experiencing difficulties in solving problems also need to be highlighted. The strategy chosen by the students in finding resolutions would affect their performance when solving problems[31]. A number of students have been independently seeking resolutions by learning the situations or looking for examples of problem-solving strategies from books or their teaching handouts. Others would prefer to ask completion strategies to their teacher or their fellow students. The variance of information management and problem-solving strategies amongst students are influenced by their cognitive styles[32]. The difference of cognitive styles affects students’ ability to solve problems[31]. There are students who prefer to work individually, and others who prefer to work in groups. There are students who are capable to directly understand the given problems, while others are dependently in need of assistance. These differences also cause varied time of completion amongst the students, and also the gap of thinking. It is strongly suggested to the teacher to be conscious about these gaps and provide adjustable treatments without exhibiting their preferences. It is obliged for teacher to accommodate students’ cognitive style differences to further elevate students’ mathematical dispositions.

5. Conclusions and Suggestions
The construction of these students’ mathematical dispositions was started in their perceptions of problems given to them. They choose problems which they think are easier to solve. According to the research, some students were fixated towards mathematical formulas and representations, so that they were displaying a positive preference towards solving abstract problems. Other students who prefer contextual problems enjoy solving problems related to their daily life. Thus, in promoting the establishment of students’ mathematical dispositions, it is critical for teacher to introduce varied forms of problems with gradual level of difficulties. Once students’ perceptions are formed, students’ motivation could be further improved by introducing various mathematical problems that are closely related to their lives. By cultivating students’ interest in solving mathematical problems, students would display more efforts in finding solutions for the problems.

The tendency towards these students’ preference of either abstract of contextual problems are caused by students’ comprehension abilities and their information management towards given problems. This is also related to the students’ cognitive styles. Further research on the connection between this research and students’ cognitive styles is recommended to find out students’ attitude when solving problems. This would further help teacher to determine appropriate problems that are also interesting for the students.

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