Generalization Techniques Empirically Outperform Differential Privacy against Membership Inference

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Abstract—Differentially private training algorithms provide protection against one of the most popular attacks in machine learning: the membership inference attack. However, these privacy algorithms incur a loss of the model’s classification accuracy, therefore creating a privacy-utility trade-off. The amount of noise that differential privacy requires to provide strong theoretical protection guarantees in deep learning typically renders the models unusable, but authors have observed that even lower noise levels provide acceptable empirical protection against existing membership inference attacks.

In this work, we look for alternatives to differential privacy towards empirically protecting against membership inference attacks. We study the protection that simply following good machine learning practices (not designed with privacy in mind) offers against membership inference. We evaluate the performance of state-of-the-art techniques, such as pre-training and sharpness-aware minimization, alone and with differentially private training algorithms, and find that, when using early stopping, the algorithms without differential privacy can provide both higher utility and higher privacy than their differentially private counterparts. These findings challenge the belief that differential privacy is a good defense to protect against existing membership inference attacks.

I. INTRODUCTION

Machine learning technologies are widely popular and we can find them as part of the services provided by major companies such as Amazon or Google. It is well-known that machine learning models memorize information about the data they are trained with, and this makes them vulnerable to different attacks (Song et al., 2017; Carlini et al., 2019). One of the most prominent attacks against machine learning models is the Membership Inference Attack (MIA) (Shokri et al., 2017; Yeom et al., 2018), where an adversary that has access to a target model wants to determine whether or not a particular data sample was in the model’s training set. In many cases, leaking the membership of a sample can be highly sensitive (e.g., when the model has been trained with patients of a particular hospital).

Fortunately, the research community has developed defense mechanisms that protect against MIAs, such as adversarial regularization (Nasr et al., 2018) and output obfuscation (Jia et al., 2019). Among existing defenses, those based on the notion of Differential Privacy (DP) (Dwork et al., 2006) have been getting particular attention (Chaudhuri et al., 2011; Abadi et al., 2016; Yu et al., 2019), where the classification problem is very easy to learn (Tramer and Boneh, 2020). However, researchers have shown that noise levels that keep the model’s accuracy above reasonable levels, while not providing meaningful theoretical guarantees, can still successfully deter existing MIAs, thus achieving empirical privacy (Jayaraman and Evans, 2019). This suggests that DP and, in particular, DP-SGD, is a useful defense against existing MIAs.

One of the main advantages of DP as a privacy notion is that it provides provable theoretical privacy guarantees. However, achieving such guarantees in deep learning requires noise levels that are so high that they render the model unusable for classification (Jayaraman and Evans, 2019), except for cases where the classification problem is very easy to learn (Tramer and Boneh, 2020). However, researchers have shown that noise levels that keep the model’s accuracy above reasonable levels, while not providing meaningful theoretical guarantees, can still successfully deter existing MIAs, thus achieving empirical privacy (Jayaraman and Evans, 2019). This suggests that DP and, in particular, DP-SGD, is a useful defense against existing MIAs.

In this work, we challenge this belief by evaluating DP-SGD along with other techniques that provide higher privacy and/or higher utility than DP-SGD. We do not come up with new defenses, but instead evaluate existing techniques that are considered good practice in ML, such as pre-training, Sharpness-Aware Minimization (SAM), and early stopping. Intuitively, these techniques contribute to generalizing the model, i.e., they make the model perform well on samples it has not been trained on, while avoiding overfitting the model to the training set. These generalization techniques, that were designed to improve the model’s classification accuracy (i.e., utility), also improve the protection against MIAs, since a generalized model will typically react similarly to members and non-members of the training set. By increasing both privacy and utility, these techniques can break from the privacy-utility trade-off inherent to DP defenses.
We select two state-of-the-art MIAs and evaluate them against models trained with different generalization techniques and/or DP-SGD, studying how the privacy-utility trade-off varies after a certain number of training epochs. We consider two datasets: CIFAR-100, an example of a complex classification problem, where previous works struggle to achieve both high privacy and utility (Jayaraman and Evans, 2019); as well as CIFAR-10, a simpler problem that allows high noise levels in DP-SGD without sacrificing a lot of utility. In order to achieve reasonable classification accuracy levels in these datasets, we use models pre-trained on ImageNet[1] which increases utility while not affecting privacy. We find that, in both datasets, DP is underwhelming in terms of the empirical protection it provides for a certain utility performance. Among the different parameters of DP-SGD that we test, we empirically assess the performance of a training algorithm 

A. Membership Inference Attacks

Membership Inference Attacks (MIAs) aim at guessing whether or not a particular data sample was part of the training set of a target model. Formally, given a data sample \( z \) and a trained model \( a \in \mathcal{A} \), a MIA is a function \( M(a, z) \) that outputs an estimation of whether or not \( z \) was part of the training set of \( a \). We use \( M(a, z) = 1 \) to denote that the MIA decided that \( z \) was in the training set of \( a \). We assume the attack also knows the training algorithm \( A \) and the training set size \( n \) (Yeom et al., 2018), but we omit them from the arguments of \( M \) for notational simplicity.

We consider only black-box MIAs, i.e., the attacker can query the model \( a \) (any number of times) with a data sample to obtain confidence score vectors for that sample. Black-box MIAs can broadly be classified into those that require to train Neural Networks (NN) to perform the inference, and those that do not. Shokri et al. (2017) proposed the first NN-based attack, also known as the Shadow Model attack. This attack trains so-called shadow models on auxiliary data to mimic the behavior of the target model, and then trains an attacker model with the outputs of the shadow models, where the labels denote the membership attribute. Finally, the attacker model is used on the outputs of the target model to predict the membership of samples. Salem et al. (2018) proposed a refinement of this attack that only requires one shadow model to function, achieving similar performance.

Among the attacks that do not require training neural networks, the attack by Yeom et al. (2018) is perhaps the most popular one. This attack considers that the adversary knows the loss function and the average loss of the training set. The attack evaluates the target sample \( z \) on the target model \( a \), and decides that it is a member if its loss is below a threshold (typically, the average training set loss). Despite its simplicity, this threshold attack achieves similar or even better performance than the NN-based attacks. Song and Mittal (2021) recently showed that using different thresholds per class leads to a better performance.

In summary, our work questions the usefulness of DP techniques in deep learning to protect against MIAs: reasonable theoretical privacy guarantees require noise levels that destroy the model’s accuracy, and empirical privacy performance can be achieved more efficiently by following good ML practices (not designed with privacy goals in mind).

II. Preliminaries

In this section, we introduce our notation, explain membership inference attacks as well as differential privacy, and introduce the metrics that we use to assess the performance of machine learning models.

We use \( z = (x, y) \in \mathcal{Z} \) to denote a data sample, which consists of a vector of features \( x \) and a label \( y \). A training set \( S \) is a set of data samples. A training algorithm \( A \) is a (possibly randomized) algorithm that receives a training set \( S \) and outputs a trained model \( a \in \mathcal{A} \). A model \( a \) evaluated on a vector of features \( x \) returns a confidence score vector, with one confidence value for each class, which can be used to predict its label \( y \).

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\[ P_{\text{err}} = \frac{\text{FPR} + \text{FNR}}{2}. \]  

This definition implicitly assumes a balanced prior, i.e., this is the error when the true membership of the target sample \( z \) is chosen by flipping an unbiased coin. In Section V-D
we also study the protection that training algorithms offer over members and non-members separately. Note that a naive adversary that randomly outputs a bit achieves $P_{err} = 0.5$. Therefore, $P_{err}$ typically ranges from 0 (no privacy) to 0.5 (total privacy). We note that other works (Yeom et al., 2018; Jayaraman and Evans, 2019) use a privacy loss metric known as the membership advantage (ADV), which can be written as $ADV = 1 - 2P_{err}$. Our findings also extend to this metric, since $P_{err}$ and ADV are linearly related.

2) Utility: We measure the utility of a model $a$ as the probability that the model correctly classifies a data sample that was not in the training set, also known as the testing accuracy. This utility metric measures how useful the model is towards the classification task for which it has been trained, and ranges from 0 (no utility) to 1 (perfect utility).

3) Algorithm Comparison: Given two training algorithms $A$ and $A'$, we say that $A$ outperforms $A'$ if it achieves higher privacy and no less utility than $A'$, or higher utility and no less privacy than $A'$.

C. Differential Privacy in Machine Learning

We define DP in the context of membership inference attacks. Let $S \in Z^n$ be a dataset of $n$ samples. We say $S$ and $S'$ are neighbouring datasets if $S'$ is a copy of $S$ with one sample added or removed. Differentially private training algorithms $A$ ensure that neighbouring datasets have similar probabilities of producing a model $a \in A$. Formally,

**Definition II.1 ($\epsilon, \delta$)-DP.** A training algorithm $A$ is ($\epsilon, \delta$)-DP if, for every pair of neighbouring datasets $S$ and $S'$, and any subset of trained models $R \subseteq A$, the following condition holds:

$$\Pr(A(S) \in R) \leq \Pr(A(S') \in R) \cdot e^{\epsilon} + \delta. \quad (2)$$

The parameter $\epsilon$ characterizes the privacy level, and $\delta$ is a slack variable that relaxes the condition and should be chosen to be smaller than $1/n$ (Dwork and Roth, 2014). Small values of $\epsilon$ force the probabilities in (2) to be closer, and therefore denote high privacy regimes, while large values of $\epsilon$ denote low privacy regimes.

DP is a strong worst-case protection in the sense that, even if the adversary somehow knows $S$ and $S'$, deciding which of the two is the actual training set after observing the model $a$ is non-trivial.

One of the most efficient approaches to provide DP during the training process (Chaudhuri et al., 2011; Al-Rubaie and Chang, 2019) is the DP version of Stochastic Gradient Descent (SGD) by Abadi et al. (2016). This algorithm, known as DP-SGD, follows the standard SGD algorithm, working in steps or batches. In each batch, the algorithm selects a subset of the training samples and computes the gradient of the loss function evaluated on each. Then, instead of averaging these gradients to update the model’s weights (like standard SGD), DP-SGD first clips the norm of the gradients using a threshold $C$, averages the clipped gradients, applies Gaussian noise with standard deviation $\sigma$ to them, and then updates the weights. Here, $\sigma$ is known as the noise multiplier and tunes the amount of noise that the algorithm adds. The clipping plus noising step ensures DP guarantees. Following the moments accountant method (Abadi et al., 2016), one can compute the actual $(\epsilon, \delta)$ guarantee of DP-SGD, which depends on $\sigma$ as well as other model parameters like the training set size and number of epochs.

III. Problem Statement

In this section, we explain the limitations of DP as a theoretical protection against MIA, and argue that existing generalization techniques might empirically perform better than DP. This motivates our experiments below.

A. Limitations of Differential Privacy

The main strength of DP, that sets it apart from other privacy notions, is that it provides *provable theoretical guarantees* against attacks. Kairouz et al. (2017) show that an $(\epsilon, \delta)$-DP algorithm ensures the following bounds involving the FNR and FPR of an adversary trying to distinguish between the neighbouring datasets, which by extension applies to MIAs:

$$\text{FPR} + e^\epsilon \cdot \text{FNR} \geq 1 - \delta,$$
$$\text{FNR} + e^\epsilon \cdot \text{FPR} \geq 1 - \delta. \quad (3)$$

Adding these two expressions together and writing the bound in terms of the probability of error $P_{err}$ in (1), we get:

$$P_{err} \geq \frac{1 - \delta}{1 + e^\epsilon}. \quad (4)$$

Recall that the probability of error of an adversary that runs an attack no worse than random guessing is $0 \leq P_{err} \leq 0.5$. Assuming a very small $\delta < 1$, the bound in (4) means that a small $\epsilon = 0.1$ ensures that the adversary’s error will be very close to random guessing $P_{err} \geq 0.47$, while an $\epsilon = 1$ provides $P_{err} \geq 0.26$. A higher value like $\epsilon = 5$ can only ensure $P_{err} \geq 0.0067$, which is certainly a very weak guarantee, close to the trivial bound $P_{err} \geq 0$. We can reasonably conclude that the theoretical protection that DP offers for values of $\epsilon > 1$ is not strong, and stops being useful altogether when $\epsilon > 5$. Values of $\epsilon < 5$ can only be achieved while providing reasonable test accuracy levels in simple classification tasks, such as MNIST and CIFAR-10. More complex tasks like CIFAR-100 require higher $\epsilon$ values (Jayaraman and Evans, 2019), which cannot provide useful theoretical protection guarantees.

The actual *empirical* privacy that DP algorithms provide against existing MIAs, however, is significantly above the bound in (4). This is not because the bound is loose (Nast et al., 2021) proved the bounds in (3), and by extension (4), are indeed tight), but because the bound holds against a rather unrealistic adversary that knows all the entries in the dataset but one (i.e., $S$ and $S'$ in our notation). In their evaluation, Jayaraman and Evans (2019) show that even values of $\epsilon = 100$ or even $\epsilon = 1000$ provide a significant amount of protection against existing (realistic) MIAs, while keeping utility high. This suggests that DP is a reasonable empirical protection guarantee, if the data owner can afford its privacy-utility trade-off.

B. Generalization Techniques as an Alternative to Differential Privacy in Deep Learning

Previous work shows that a model’s vulnerability against MIAs is connected to overfitting (Yeom et al., 2018), and it is well-known that generalization techniques can help against
MIAs (Choquette-Choo et al., 2021; Li and Zhang, 2020; Nasr et al., 2018; Salem et al., 2018; Song and Mittal, 2021) recently showed that early stopping alone can outperform ad hoc defenses like adversarial regularization (Nasr et al., 2018).

Motivated by this, in this work we evaluate the privacy and utility guarantees of existing off-the-shelf generalization techniques, and compare them with differentially private training. There is a plethora of ML techniques and model configurations that provide high generalization and utility performance. We pick a small subset of these techniques only to illustrate why there are alternatives better than DP when aiming for empirical privacy, and leave a more detailed evaluation as subject for future work. A key aspect of our evaluation is that we assess the privacy-utility trade-off of our models after every training epoch so that we better understand the effect that early stopping has in the model’s performance.

IV. EXPERIMENTAL SETUP

Our code runs in Python and we use PyTorch ML library. We use the DP-SGD implementation from Opacus DP library version 0.14, which follows the algorithm by Abadi et al. (2016). We ran our experiments in a private computer server and Google Colab, both of which use Tesla P100 GPUs. (We note that running times are not relevant for this paper.) Our code will be available upon request.

a) Datasets: We consider two datasets for our evaluation: CIFAR-100 and CIFAR-10. CIFAR-100 contains 60 000 32 × 32 RGB images distributed into 100 different classes. The dataset is already divided into a (default) training set of 50 000 images, and a (default) testing set of 10 000 images; each set has even representation of each class. CIFAR-10 is similar to CIFAR-100, but with 10 classes instead.

For both datasets, we split the default training and testing set in half. We follow a stratified split, i.e., we run the split per class, ensuring every class has the same representation in our halves. We use one half of the default training set as our actual training set (i.e., the members), and one half of the default testing set as our testing set (which are also the non-members). We use the remaining halves of the built-in training and testing sets to train the neural-network MIA by Salem et al. (2018).

b) Membership Inference Attacks: We consider two state-of-the-art MIAs: the Neural Network (NN) attack by Salem et al. (2018) and the threshold attack by Yeom et al. (2018). In order to measure privacy, we measure the FPR of the attack by evaluating it on the non-members, and the FNR by evaluating it on the members. Then, we compute the probability of error P_{err} from the FPR and FNR following [1].

c) Model configuration: Our model configuration choices are based on state-of-the-art results for CIFAR-100 by Foret et al. (2021). Even though Foret et al. achieve the best results using an EffNet model, this is not directly compatible with Opacus’ DP-SGD implementation. Instead, we use a ResNet50 architecture pre-trained on ImageNet.

Our baseline experiments use the SGD optimizer, and we also test DP-SGD from the Opacus library and the Sharpness-Aware Minimization (SAM) optimizer (Foret et al., 2021). SAM modifies SGD by adding an extra regularization term to the loss function that makes the algorithm look for a minimum whose neighbourhood is also at a low loss value (i.e., a “wide” minimum). We use a PyTorch implementation of SAM and set its hyperparameter (neighbourhood size) to ρ = 0.05, following Foret et al. (2021). We also use an ℓ2 regularization coefficient of η = 0.0005 by default in all of our experiments (unless mentioned otherwise). We train each model for 15 epochs, using the step decay for the learning rate from the SAM implementation: we start with a rate of λ that we keep for the first five epochs, the next four epochs use 0.2 · λ, the next three use 0.22 · λ, and the last three 0.23 · λ. We use a batch size of 32.

Besides SAM, we consider two generalization techniques also evaluated in previous work: ℓ2 regularization (Krogh and Hertz, 1992) and dropout (Srivastava et al., 2014).

We save the model after each epoch in order to understand how the privacy and utility guarantees of the model evolve with training. We repeat all of our experiments 10 times to account for the randomness in training, and plot average results as well as 95% confidence intervals.

V. EVALUATION

We first consider the CIFAR-100 dataset, which is a more realistic and harder classification problem than CIFAR-10. We begin our evaluation with a simple experiment to explain our plotting methodology. Figure 1 shows the performance of our baseline experiment (SGD optimizer) with an initial learning rate of λ = 0.01 against the threshold attack. Each point in the plot represents the performance (privacy vs. utility) of the model after one epoch (a total of 15 points). We represent the results of the first epoch with a triangle (△) and the last epoch with a star (*). Intermediate epochs (●) are joined by a thin dotted line. The crosses behind the points show the 95% confidence intervals for the privacy (vertical line) and the utility (horizontal line). Note that when these intervals are not visible, it means they are smaller than the marker size.

As the number of epochs increases, the model memorizes more about the training set. This causes an increase in utility and a decrease in privacy. The large gap between the fifth and sixth epochs is due to the change in learning rate that happens at that point; we see this effect in our other experiments as well. We see that, after 15 epochs, the model reaches an accuracy of 81%, which is a high value for this dataset (note that our training set is only of size 25 000, since we do not use all 50 000 samples for training).

We ran all of our experiments for the threshold and NN attacks, but found only small differences between them. Due to space constraints, we show only the performance of the threshold attack in this paper, with the exception of Figure 5.
A. Effect of Differential Privacy

We perform an initial experiment to look for reasonable values for the learning rate $\lambda$ and the clipping threshold $C$ for DP-SGD in CIFAR-100 (Figure 2). We fix the noise multiplier $\sigma = 0.01$ and $\delta = 10^{-5}$. This $\sigma$ is low enough so that the test accuracy does not decrease significantly, and we empirically see it provides high privacy levels $P_{\text{err}} > 0.4$. The $\delta$ ensures that $\delta < 1/n \approx 4 \cdot 10^{-5}$, which is recommended (Dwork and Roth, 2014). For reference, this configuration yields an $\epsilon \approx 186,000$ after 15 epochs, which does not provide a meaningful theoretical protection. Figure 2 shows that, out of the $\lambda$ and $C$ values we tried (in the legend), $C = 1$ and $\lambda = 0.01$ (green markers) achieves the best results: for every non-green point in the plot, there is a green point that achieves (roughly) higher privacy and utility. We verified the same is true when setting $\sigma = 0.02$. Therefore, we fix $\lambda = 0.01$ and $C = 1$ for our remaining experiments in CIFAR-100.

Next, we evaluate the effect of different noise multipliers $\sigma$ in DP-SGD. We set $C = 1$, $\lambda = 0.01$, and $\delta = 10^{-5}$, and vary $\sigma \in \{0, 0.01, 0.02, \ldots, 0.05\}$. We show the results in Figure 3. As expected, increasing $\sigma$ improves the privacy protection and decreases the utility of the model. We can see this, for example, by looking at the final trained model ($\star$). However, the privacy-utility trade-off caused by increasing $\sigma$ is worse than the cause by early stopping. Roughly speaking, for every point in the graph, there is a $\sigma = 0$ point that achieves no less privacy and utility than it. This means that, paradoxically, the best DP-SGD configuration is the one that does not provide differential privacy at all ($\sigma = 0$ implies $\epsilon = \infty$).

This experiment also confirms that it is not possible to achieve theoretical protection ($\epsilon < 5$) in CIFAR-100 while keeping a reasonable utility loss: even the largest $\sigma$ we test ($\sigma = 0.05$) yields a huge $\epsilon \approx 7,000$.

B. Comparison of Regularization Techniques

Next, we compare three regularization techniques:

- $\ell_2$-regularization: for this experiment, we use SGD with an $\ell_2$ regularization coefficient of $\eta = 0.025$.
- Dropouts: we add a dropout layer with dropout rate of 0.6 before the last layer of the pre-trained ResNet50 model, and train using the SGD optimizer.
- SAM: we use the Sharpness-Aware Minimization optimizer with hyperparameter $\rho = 0.05$.

Figure 4 shows these results, as well as the baseline SGD. If we were to judge by the last epoch only ($\star$), we see that SAM leaks the most, followed by the baseline and $\ell_2$-regularization, and then the dropouts. However, using the last epoch as comparison is misleading: by looking at the evolution of the privacy-utility trade-off with the training epochs, we see that SAM provides the best performance.

C. Differential Privacy vs. Regularization

Next, we compare the best-performing DP-SGD model (i.e., $\sigma = 0$) with the best-performing regularization technique that we have tried (i.e., SAM). We show the results in Figure 5 for both the threshold and NN attacks. We cannot compare DP-SGD and SAM directly, since the models produced with SAM are in a lower privacy and higher utility region than the models produced with DP-SGD. This is because, with our parameters, SAM learns too fast. In order to compare them, we found that clipping the gradients of SAM provides utility-privacy points in the same region as DP-SGD. When comparing SAM with $C = 1$ and DP-SGD, we see that SAM clearly outperforms the latter.

To further confirm that adding noise to the gradients is not a good idea, we ran a differentially private version of SAM that we call DP-SAM. We achieve this in our code by simply plugging Opacus’s Privacy Engine into the SAM optimizer (for completeness, we include the pseudo-code of this algorithm in Appendix B). We use a noise multiplier of $\sigma = 0.1$, and we see that the effect of adding noise is the same that we saw in Figure 3 given a single epoch. DP-SAM achieves more privacy and less utility than SAM with clipping, but when considering early stopping, SAM with clipping $C = 1$ clearly outperforms DP-SAM.

D. Outliers Protection

Previous works show that, even when the model generalizes well on average, there are certain samples, also called outliers, that can still be consistently unprotected (Long et al., 2018; Choquette-Choo et al., 2021). Measuring MIA performance with average metrics such as $P_{\text{err}}$ would not show such vulnerable samples. In this section we identify outliers in our previous experiments, and study whether DP can protect such samples. We consider that a sample is an outlier if it was correctly identified by our attack across all 10 runs.

We identify outliers in our CIFAR-100 experiments for the baseline SGD, for DP-SGD with $\sigma = 0$ and $\sigma = 0.05$, and for SAM. We show the percentage of member and non-member outliers vs. the model’s accuracy in Figures 7(a) and
Fig. 2: DP-SGD with different clipping thresholds $C$ and learning rates $\lambda$ (noise magnitude $\sigma = 0.01$).

Fig. 3: DP-SGD with different noise magnitudes $\sigma$ ($C = 1$, $\lambda = 0.01$).

Fig. 4: Privacy-utility trade-off for different regularization techniques.

Fig. 5: Comparison between SAM, SAM with clipping, DP-SGD, and DP-SAM in CIFAR-100.

Fig. 6: Comparison between SAM, SAM with clipping, DP-SGD, and DP-SAM in CIFAR-10.

(b) NN Attack

The fact that DP-SGD protects non-member outliers more than member outliers is a serious privacy issue: typically, correctly identifying the fact that a sample belongs to the training set is more privacy-sensitive than identifying the fact that a non-member was not in the training set. SAM protects members more than non-members (compared to DP-SGD), which is a more desirable situation from a privacy standpoint.

E. Simple Classification Problem

For the following experiments, we consider CIFAR-10 dataset, which represents a simpler classification problem. We perform an initial grid search for DP-SGD using the clipping thresholds $C \in \{0.5, 1, 10\}$ and learning rates $\lambda \in \{0.0005, 0.001, 0.002\}$. We choose a relatively small $\epsilon = 3$, which we achieve by setting the noise multiplier to $\sigma = 0.651$. We find that $C = 1$ and $\lambda = 0.001$ performs best, achieving $P_{err} \approx 0.5$ and test accuracy $\approx 0.86$. This experiment also confirms that lower $\epsilon$ values are feasible in simpler classification problems.

We then compare this configuration of DP-SGD with the baseline SGD, SAM, SAM with $C = 1$, and DP-SAM with $C = 1$ and $\epsilon = 3$ ($\sigma = 0.651$). We show the results in Figure 6. Comparing with our results in CIFAR-100 (Fig. 5), we see that we can achieve higher privacy and utility levels in CIFAR-10, since the classification problem is easier. Qualitatively, our conclusions are the same in both datasets: by looking at the performance across all epochs, we see that SAM (with clipping for lower utility values) outperforms both DP-SGD and DP-SAM.
VI. RELATED WORK

It is well-known that generalization techniques protect against MIAs, which explains why many related works evaluate MIA performance against $\ell_2$-regularization, dropouts, and other techniques (Shokri et al., 2017; Salem et al., 2018; Nasr et al., 2018; Choquette-Choo et al., 2021; Li and Zhang, 2020). Recent works started to consider pre-trained models as well (Choquette-Choo et al., 2021; Song and Mittal, 2021), and, to the best of our knowledge, we are the first to test the SAM optimizer against membership inference. However, these works mostly judge the performance of the model at the last epoch, which we have seen can be misleading. The exception to this, and perhaps the closest work to us, is the work by Song and Mittal (2021), where the authors show that early stopping (without any additional defense) provides no worse protection than adversarial regularization (Nasr et al., 2018), one dedicated defense against MIAs. In this work we studied the privacy-utility performance of DP-SGD after every training epoch and found that the non-DP variant ($\sigma = 0$) performs best than the privacy counterparts, which adds to the findings of Song and Mittal (2021). We additionally found that SAM further improves over DP-SGD and the baseline SGD. Finally, inspired by the work of Long et al. (2018) and Choquette-Choo et al. (2021), we studied the protection that DP-SGD offers against particularly vulnerable samples (outliers), and found that SAM is more efficient than DP-SGD towards protecting member outliers.

VII. CONCLUSIONS AND FUTURE WORK

In this work, we have shown the limitations of Differential Privacy (DP) as an empirical privacy defense against Membership Inference Attacks (MIAs). We have evaluated the privacy and utility performance of different techniques, including standard SGD, a state-of-the-art DP-SGD implementation, and the recent Sharpness-Aware Minimization optimizer. By evaluating the performance of the models at each training epoch, we have seen that early stopping, without using any noise, provides a better privacy-utility performance than mechanisms that use DP noise. We have also seen that DP leaves certain (outlier) members particularly vulnerable against MIAs. Out of the generalization techniques we tested, SAM seems particularly promising.

This work illustrates the limitations of DP against MIAs, but we believe we have only scratched the surface of this issue. We have seen that early stopping is critical towards providing a good privacy-utility performance, but more research on how to choose the privacy-utility point needs to be done, perhaps by trying different adaptive learning rate mechanisms or clipping thresholds.

APPENDIX A

ADDITIONAL EXPERIMENT RESULTS

In the main text, we include the results of all our experiments for the threshold attack (Yeom et al., 2018), and we point out that the results for the NN attack (Salem et al., 2018) are similar and do not add anything to our findings. In order to support this claim, we provide a side-by-side comparison of the results for the threshold and the NN attacks in Figures 8-14.

APPENDIX B

PSEUDO-CODE FOR DP-SAM

We implement DP-SAM by plugging Opacus’ Privacy Engine into the SAM implementation. We summarize here the resulting algorithm. Let $w$ be the model’s weights, and let $L_S(w)$ be the training set loss. SAM (Foret et al., 2021) re-defines the loss function as

$$L^{SAM}_S(w) = \max_{\|e\|_{\leq \rho}} L_S(w + e).$$

This means that minimizing $L^{SAM}_S(w)$ requires looking for weights $w$ such that all neighbouring weights (at a certain $\ell_2$-distance around $w$) also yield low loss (i.e., a wide minimum). Foret et al. provide details on how to implement this minimization efficiently, which includes a formula on how to compute $e$ given $w$. We refer to the original paper for details (Foret et al., 2021), and here we just use $g_i(w)$ to denote the gradient approximation of the SAM objective function for the $i$th sample in the batch $z_i \in B$.

We represent DP-SAM in Algorithm 1. DP-SAM first uses the batch samples to estimate the gradient that minimizes the SAM objective function. Then, it clips the gradients evaluated at each sample in the batch, averages them, and adds Gaussian
Algorithm 1 DP-SAM

**Input:** Training set $S \in \mathbb{Z}^n$, batch size $b$, step size $\lambda > 0$, neighborhood size $\rho > 0$, clipping threshold $C > 0$, noise multiplier $\sigma > 0$.

**Output:** Trained model

1: Initialize model’s weights $w_0$, $t = 0$.
2: while not converged do
3: Sample batch $B = \{(x_1, y_1), \ldots, (x_b, y_b)\}$;
4: for each $z_i \in B$ do
5: Compute the SAM gradient: $g_i = \hat{g}_i(w)$;
6: if $\|g_i\|_2 > C$ then
7: Clipping: $g_i = g_i \cdot C/\|g_i\|_2$;
8: end if
9: end for
10: Add Gaussian noise $g = \frac{1}{b} \sum_{i=1}^{b} g_i + \mathcal{N}(0, \sigma^2 C^2 I)$;
11: Update weights $w_{t+1} = w_t - \lambda g$;
12: $t = t + 1$;
13: end while

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Fig. 10: DP-SGD with different noise magnitudes $\sigma$, with $C = 1$ and $\lambda = 0.01$ (CIFAR-100).

Fig. 11: Privacy-utility trade-off for different regularization techniques (CIFAR-100).

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Fig. 12: Comparison between SAM, SAM with clipping, DP-SGD, and DP-SAM in CIFAR-100.

Fig. 13: Comparison between SAM, SAM with clipping, DP-SGD, and DP-SAM in CIFAR-10.

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Fig. 14: Comparison between SAM, SAM with clipping, DP-SGD, and DP-SAM in CIFAR-10.