Realizing controllable noise in photonic quantum information channels

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Controlling the depolarization of light is a long-standing open problem. In recent years, many demonstrations have used the polarization of single photons to encode quantum information. The depolarization of these photons is equivalent to the decoherence of the quantum information they encode. We present schemes for building various depolarizing channels with controlled properties using birefringent crystals. Three such schemes are demonstrated and their effects on single photons are shown by quantum process tomography to be in good agreement with a theoretical model.

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Light depolarization is a fundamental optical phenomenon. It was studied as early as the nineteenth century, when measurement and characterization methods of the polarization state of light were introduced. Methods for complete depolarization of light, such as the Cornu and Lyot depolarizers, have been known for many decades. The Cornu and wedge depolarizers require the light beam to be wide, as the loss of coherence between the two polarizations is achieved via averaging over the spatial degrees of freedom. In the case of the Lyot depolarizer, short coherence length is required and the averaging is over the temporal degrees of freedom. Nevertheless, there are currently no methods that enable control of all of the aspects of the depolarization process.

Quantum information is commonly encoded in the polarization of single photons. Depolarization of such photons acts as quantum noise on the stored information, i.e. the interaction between the information encoding units and the environment results in decoherence. In order to study quantum decoherence in general and its effect on quantum information protocols in particular, it is desirable to create quantum channels with controlled noise. Such channels will be useful for testing quantum error correction and quantum key distribution protocols. Other uses for these channels are to test the existence of decoherence-free subspaces and for generating partially-mixed entangled states.

In recent years, depolarizing channels were studied by several methods. When a single birefringent crystal was used, only dephasing channels were demonstrated, as we will show later. Optical scatterers such as emulsions, multi-mode fibers and ground glass, give variable depolarization, but are also accompanied by a spread in k-space, resulting in considerable loss when collected for further processing. Another approach is to use polarization scramblers of various kinds. These realizations are equivalent to fast polarization rotations and averaging measurements over times longer than the typical rotation periods. Nevertheless, when used with single photons, each photon by itself is completely polarized. Controllable depolarizers were demonstrated by using two wedge depolarizers with variable beam diameter, or with a tunable relative angle. These channels are hard to model and their anisotropy level is uncontrolled as they couple the polarization with many spatial degrees of freedom.

In this letter we present a theoretical and experimental study of various controllable depolarizing channels. We study channels that are composed of a sequence of birefringent crystals and wave plates. The depolarization and its anisotropy depends on the order and relative angles between the channel components. The generated channels are mostly anisotropic and can be tuned continuously between no depolarization and complete dephasing. These channels were characterized by the transmission of polarized single photons, generated by spontaneous parametric down-conversion. Quantum process tomography (QPT) was used to compare the experimental results with theory.

The information in a classical channel can be degraded only by bit-flip errors. Thus, such a channel is completely described by a single parameter - the bit-flip error probability. In comparison, quantum channels can have a constant unitary rotation and three types of errors, represented by the Pauli operators: a bit-flip that swaps the logical |0⟩ and |1⟩ amplitudes, a phase flip between the amplitudes and the combination of the two, which is a third orthogonal operation. Isotropic decoherence is the case when the three error probabilities are equal.

A polarization qubit can be described either by a density matrix operator or equivalently, by a point in the Poincaré sphere. The Cartesian coordinates of this point are the Stokes parameters \( S = (S_1, S_2, S_3) \) which describe the linear \(|h⟩, |v⟩\), diagonal \(|p⟩ = (|h⟩ + |v⟩)/\sqrt{2}\), \(|m⟩ = (-|h⟩ + |v⟩)/\sqrt{2}\) and circular \(|r⟩ = (|h⟩ + i|v⟩)/\sqrt{2}\) polarization components, respectively. The Degree Of Polarization (DOP) is defined as \( D \), the length of the Stokes vector:

\[
D = \sqrt{S_1^2 + S_2^2 + S_3^2} \equiv \sqrt{1 - 4|\det(\hat{\rho})|}.
\]  

The perfectly polarized states are described by the surface of the sphere (\( D = 1 \)) and its center designates the completely unpolarized state (\( D = 0 \)). The inside of the
sphere includes all partially polarized states (0 < \( D < 1 \)). The physical meaning of the DOP is the ratio between the polarized light intensity and the total light intensity.

Consider an arbitrarily polarized wave packet that is passing through a birefringent crystal\[19\]. The temporal walk-off \( \tau = L \Delta n / c \) between two wave packets, each polarized along one of the symmetry axes of the crystal, depends on the crystal length \( L \), its refractive index difference \( \Delta n \) and the speed of light \( c \). We assume that the coherence time of the wave packets \( t_c \) is shorter than the walk-off \( \tau \). If the light wave packet is not polarized linearly along one of the crystal symmetry axes, its two components acquire temporal distinguishability. Thus, the polarization and temporal degrees of freedom become entangled. The role of the environment in general decoherence models is fulfilled here by the temporal degrees of freedom. As the detectors are insensitive to the short temporal walk-offs, they cannot distinguish between the wave packets, effectively tracing out the temporal degrees of freedom. The result is an effective depolarization since no coherence can be observed between the two orthogonal polarizations. The depolarization operation is described in the Poincaré sphere picture by a projection of the initial Stokes vector on the direction that represents the crystal principal axes. For example, a birefringent crystal aligned along the \( h-v \) directions will project any initial state onto the \( S_1 \) direction. This kind of operation is referred to as a 'dephasing channel'\[3\].

A single crystal configuration can apply any level of depolarization to any initial linear polarization. On the other hand, for such a configuration there is always another polarization direction that experiences no depolarization whatsoever. Thus, we consider a second crystal, that is placed after the first one\[4\]. For historical reasons, let us first assume that the second crystal is twice as long as the first one. The two crystals are coupling orthogonal temporal degrees of freedom, as the first crystal couples \( t = 0 \) with \( \tau \), while the second couples \( t = 0 \) with \( 2\tau \) and \( t = \tau \) with \( 3\tau \). Thus, this configuration can be described as two consecutive projections of the initial polarization state onto the Stokes directions defined by the crystals' axes. In the case where the orientation of the crystals differ by \( 45^\circ \), the two projections are perpendicular, resulting in a final state at the sphere center (\( D = 0 \), the completely unpolarized state) for any initial state. This configuration in known as the 'Lyot depolarizer'\[2, 4\].

The relevant error rates for practical tests of quantum information protocols are below 20%\[20\]. For this reason, it is desirable to have a depolarizing scheme that can be tuned to small values of depolarization or even to zero depolarization. Thus, we investigate a configuration of two identical crystals. If the second crystal is oriented at \( 90^\circ \) with respect to the first one, the polarization time delay created by the first crystal is exactly compensated for by the second. For any other relative angle, there can be up to three different temporal modes: \( t = 0 \) and \( \tau \) are coupled by the first crystal, while the second crystal couples between them and an additional third delay \( t = 2\tau \). Changing the relative angle between the two crystals affects the occupation of each of the three modes, which results in a different depolarization. Hence, by tuning this angle we control the channel depolarization level. The modal occupation can be easily calculated for a given initial polarization and tuning angle. The polarization two-dimensional density matrix \( \hat{\rho}_f \) of the final polarization state is the normalized sum of the polarization states of all the occupied temporal modes \( |\psi_t\rangle \):

\[
\hat{\rho}_f = \sum_{t=1}^{T} \alpha_t |\psi_t\rangle \langle \psi_t |,
\]

where \( T \), \( t \) and \( \alpha_t \) are the number of relevant temporal modes, their index and weights, respectively. This calculation is equivalent to partially tracing over the temporal degrees of freedom of the \( 2T \)-dimensional polarization-time density matrix. In addition, the effect of wave-plates is simple to calculate as they rotate each of the temporal modes separately. With this method, it is possible to calculate the final polarization state for every initial state that passes through a sequence of birefringent crystals (a "depolarizer"). From the resulting density matrix it is possible to calculate the DOP by Eq. 1.

A 780 nm Ti:Sapphire pulsed laser of 76 MHz repetition rate was frequency doubled and the 390 nm pulses were focused into and collinearly down-converted in a 1 mm thick type-I BBO crystal (See Fig. 1). The down-converted signal was filtered by a dichroic mirror (DM) and collimated with a lens (L2). One photon of the pair was split by a beam-splitter (BS) and detected, and the second photon was directed to the depolarizer. The polarization state of the depolarized photons was characterized by wave plates and a polarizer (POL). Photons were filtered by 5 nm bandpass filters (IF), corresponding to a coherence time of \( t_c \approx 180 \) fs, and then coupled into single-mode fibers leading to single photon detectors (DET). We characterized the depolarization of three initial states \( |h\rangle, |p\rangle, \) and \( |r\rangle \) which are mutually unbiased\[7\], by quantum state tomography (QST)\[17\]. The \( |v\rangle \) state was also measured as required by QPT.

![FIG. 1. (Color online) The experimental setup and depolarization schemes. See full description in text.](image)
The first depolarizing scheme that we present is composed of two 2 mm long Calcite crystals (C) with two λ/2 wave plates (See Fig. 1). The crystals are fixed perpendicularly, with one wave plate before and the other after the first crystal. Rotation of the wave plates in opposite directions by an angle of θ/2 is equivalent to the rotation of the first crystal by θ. When θ = 0° no depolarization occurs, and when θ = 90° the depolarizer is equivalent to a dephasing channel of a single crystal. Wave plates were used instead of direct rotation of the crystal to eliminate an unwanted angle dependent retardation.

The transformation of purely polarized states through the depolarizer for various angles between the two crystals are presented in Fig. 2. In Figs. 2(a-c), the measured QPT mappings of the initial sphere are shown for three specific cases. These cases are when the polarization of only one initial state is completely lost (θ = 45°), when the channel is isotropic (θ = 54.7°) and when two initial states are depolarized identically (θ = 67.5°). The process fidelities, when compared to the model for all three cases, were higher than 97%. The measured output states for the three mutually unbiased polarizations were obtained using the maximal-likelihood QST algorithm\(^{17}\). They are also presented, up to the respective angles. Figure 2(d) compares these results with theory. The results show a very good agreement with the model. From these results, we have calculated the DOP as a function of θ. The DOP results are presented in Fig. 3 with their theoretical predictions. The three special cases of Figs. 2(a-c) are clearly reproduced in our measurements.

In general, this depolarizer is anisotropic, except for θ = 54.7° = tan\(^{-1}(\sqrt{2})\). Analytical calculation of the DOP for this angle shows that the final value of 1/3 is independent of the initial state, as can be seen in Fig. 3, where all three curves intersect. The isotropy of this configuration is apparent in Fig. 2(b), where the polarization sphere is mapped to another sphere of radius 1/3.

We studied a second depolarizing scheme that was composed of two perpendicularly fixed identical crystals with a quarter-wave plate between them (See Fig. 1). The quarter-wave plate angle θ is set to be zero when the principal axes of the wave plate and the first depolarizing crystal are parallel. For any given initial polarization (|S\(_1\), S\(_2\), S\(_3\)), the final DOP is

\[
D^2 = \frac{1}{4} \left( \frac{19}{8} + \frac{3}{2} \cos (4\theta) + \frac{1}{8} \cos (8\theta) \right) + \frac{S^2}{4} \left( -\frac{7}{8} + \frac{1}{2} \cos (4\theta) + \frac{3}{8} \cos (8\theta) \right). \tag{3}
\]

All states with the same |S\(_1\)| value, result in the same DOP for a certain θ. It is possible to find three mutually unbiased polarization bases that have the same S\(_1\) = ±\(\frac{1}{\sqrt{3}}\) value, and thus, experience the same depolarization. We define this situation as symmetric depolarization. For such bases, the DOP can be tuned between 1 and \(\frac{1}{\sqrt{6}}\) ≈ 0.41 as a function of θ. We generated these states and characterized them after passing through the second depolarizing scheme. The DOP results are shown in Fig. 4(a), and their Poincaré representation in Fig. 4(b). A good agreement with theory is observed.

The third scheme adds the possibility for symmetric depolarization down to complete depolarization. The difference between the second and third schemes is the
doubling of the second crystal’s thickness (See Fig. 1). As before, the final DOP depends only on the initial $S_1$ value and the wave plate angle $\theta$, but now it takes values between 0 and $\frac{1}{\sqrt{3}} \approx 0.58$. This result is due to an effective additional $S_1$ projection to the output of the second scheme. At the $\theta = 45^\circ$ position, the third scheme is exactly a Lyot depolarizer that completely depolarizes any initial polarization by consecutive $S_1$ and $S_3$ projections. Results for an initially polarized state with $S_1 = \frac{1}{\sqrt{3}}$ are shown in Figs. 2(a) and 2(c). As can be seen, when $\theta = 45^\circ$, the state is completely depolarized. The state tomography results (Fig. 2(c)) reveal the difference from the previous scheme as an extra $S_1$ projection.

Although we have demonstrated the effects of the various depolarizing schemes using single photons, these depolarizers would also be effective on any classical light with short enough coherence time. We repeated our measurements with laser pulses, and demonstrated identical results (not presented here). Thus, these results apply not only to polarization encoded qubits, but to any classical scenario where controlled depolarization is required.

In this work we use crystals that are long enough to completely separate the two polarization components. It is possible to deliberately use shorter crystals that will leave a portion of the two wave packets overlapping. The same qualitative results will be achieved this way, but with smaller magnitude. Another scheme that enables control of the channel isotropy level is to combine three setups of the same kind. The crystal lengths should be tripled for the second setup and tripled again for the third, in order for each of them to affect orthogonal temporal modes. If the second setup is positioned at 45° with respect to the first and a quarter-wave plate at 45° is placed between the second and the third setups, then each setup will be oriented at a different Stokes direction. This scheme can generate isotropic channels of any depolarization amount.

In conclusion, we have demonstrated a scheme for the realization of various quantum channels for photon polarization qubits with controllable decoherence. Isotropic and anisotropic depolarization processes are possible. Channels were characterized by QPT using the maximal-likelihood algorithm. All the results are in a good agreement with a simple theoretical model. These depolarizers can be used to evaluate the performance of quantum error correction and quantum key distribution protocols. In addition, they can be utilized in any classical optics setup where controllable depolarization is required. The authors would like to thank the Israeli Science Foundation for supporting this work under grant 366/06.

References:

[1] G.G. Stokes, Trans. Cambridge Phil. Soc. 9, 399 (1852).
[2] N. Hodgson and H. Weber, Laser Resonators and Beam Propagation: Fundamentals, Advanced Concepts and Applications (Second edition), Springer (2005), p. 171-172.
[3] B. Lyot, Annales de l’Observatoire d’astronomie physique de Paris (Meudon) 8, 102 (1929).
[4] A.P. Loeber, J. Opt. Soc. Am. 72, 650 (1982).
[5] M.A. Nielsen and I.L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, UK, 2000).
[6] C.H. Bennett and G. Brassard, Proceedings of the IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India (IEEE, New York, 1984), p. 175.
[7] D. Bruß, Phys. Rev. Lett. 81, 3018 (1998).
[8] P.G. Kwiat et al., Science 290, 498 (2000).
[9] G. Puentes et al., Phys. Rev. A 75, 032319 (2007).
[10] S.P. Morgan, M.P. Khong and M.G. Sornek, Appl. Opt. 36, 1560 (1997).
[11] G. Puentes, D. Voigt, A. Aiello and J.P. Woerdman, Opt. Lett. 30, 3216 (2005).
[12] B.H. Billings, J. Opt. Soc. Am. 41, 966 (1951).
[13] M. Karpiński, C. Radzewicz and K. Banaszek, J. Opt. Soc. Am. B 25, 668 (2008).
[14] Y. Nambu et al., Proc. SPIE Int. Soc. Opt. Eng. 4917, 13 (2002).
[15] G. Puentes et al., Opt. Lett. 31, 2057 (2006).
[16] I.L. Chuang and M.A. Nielsen, J. Mod. Opt. 44, 2455 (1997).
[17] J.B. Altepeter, E.R. Jeffrey and P.G. Kwiat, Adv. At. Mol. Opt. Phys. 52, 105 (2005).
[18] A. Al-Qasimi et al., Opt. Lett. 32, 1015 (2007).
[19] S. Lu and A.P. Loeber, J. Opt. Soc. Am. 65, 248 (1975).
[20] O. Kern and J.M. Renes, Quan. Inf. Comp. 8, 756 (2008).