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Abstract

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Gauge Theories with a Layered Phase

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ABSTRACT

We study abelian gauge theories with anisotropic couplings in $4 + D$ dimensions. A layered phase is present, in the absence as well as in the presence of fermions. A line of second order transitions separates the layered from the Coulomb phase, if $D \leq 3$. 
1 Introduction

The phase structure of gauge theories with isotropic couplings is known since the advent of lattice gauge theories. For $U(1)$ theories, there are two phases: a strong coupling and a weak coupling (Coulomb) phase, above and in four spacetime dimensions, while below this limit there is only one, strong coupling (confining) phase.

It was realized some time ago \cite{1} that lattice gauge theories with anisotropic couplings may have an extra, “layered”, phase. The prototype of such a system is the $4 + 1$-dimensional $U(1)$ theory, in which the plaquettes in four dimensions have the same coupling constant $\beta = 1/g^2$, whereas all plaquettes with links in the fifth dimension have coupling $\beta' = 1/g'^2$.

In the layered phase all four-dimensional hyperplanes, along the fifth direction, decouple from one other and in each hyperplane we have a four-dimensional Coulomb phase. This takes place for $\beta'$ small enough (typically $\leq 1/d$, where $d$ is the dimensionality of the layer, in our case $d = 4$) and for $\beta$ large enough (typically $\beta \geq O(1)$).

The intuition of Fu and Nielsen \cite{1} for the occurrence of the layered phase is worthwhile bringing up at this point.

Let us suppose that we are in the $5d$ Coulomb phase (so $\beta \approx \beta'$ large). Between two test charges we have a Coulomb force in this phase. Imagine making $\beta'$ smaller but keeping $\beta$ fixed. Then the force between the charges, in the fifth direction, will increase and will become, for small enough $\beta'$, confining and create a string—whereas the force will still be Coulomb-like in the four other directions. Thus we expect the Wilson loops to behave like

\begin{align*}
W_{\mu \nu}(N, M) &\approx \exp -NM\sigma \quad \text{(strong coupling)}; \\
W_{\nu \mu}(N, M) &\approx \exp -(N + M)\tau \quad \text{(Coulomb phase)}; \quad 1 \leq \mu, \nu \leq 5 \text{ and} \\
W_{\mu \nu}(N, M) &\approx \exp -(N + M)\tau', \quad 1 \leq \mu, \nu \leq 4; \\
W_{\mu \nu}(N, M) &= \exp -NM\sigma' \quad \text{(layered phase)}.
\end{align*}

There isn’t any layered phase with the roles of $\beta$ and $\beta'$ reversed. At small enough $\beta$ one has either strong coupling or weak coupling and Monte Carlo simulations \cite{3} seem to corroborate this picture, though only single plaquette averages were calculated. This asymmetry is due to the asymmetric role of $\beta$ and $\beta'$: whereas $\beta$ is the coupling restricted to the $4d$ space, $\beta'$ not only acts in the space orthogonal to the $4d$ space, but also couples the links in the fifth direction to those in the others.

The layered phase is due to the simultaneous presence of Coulomb forces in the physical direction and confining forces in the other directions. So we need at least 4 dimensional layers for an Abelian theory (i.e. $4 + D$ spacetime dimensions) and at least 5 dimensions for a non-Abelian theory ($5 + D$ spacetime dimensions).

In this paper we will concentrate on two issues: in section 2 on the order of the various phase transitions and in section 3 on the presence of fermions, both by

\[\ldots\]
mean field techniques and Monte Carlo methods (section 4). In section 5 we present results from mean field and Monte Carlo methods and conclusions and outlook are set forth in section 6.

2 Mean Field Theory for 4+D Abelian gauge theory and order of the transition

Mean field techniques for gauge theories typically give first order transitions—contrast to spin systems. The reason may be found in the structure of the fixed-point equation for the order parameter, as will be made explicit below and may be summarized by the remark that a plaquette has four links (gauge system) while there are two spin variables per link (spin system).

The standard way to do mean field theory in a 4 + $D$ anisotropic system with gauge action

$$S_G = \beta \sum_{1 \leq \mu < \nu \leq 4,x} (1 - \Re U_{\mu \nu}(x)) + \beta' \sum_{x, 1 \leq \mu \leq 4, 5 \leq \nu \leq 4+D} (1 - \Re U_{\mu \nu}(x)) + \beta' \sum_{5 \leq \mu < \nu \leq 4+D,x} (1 - \Re U_{\mu \nu}(x))$$

(1)

goes as follows (we use standard notation—the link variables $U_{\mu}(x) \equiv e^{i\phi_{\mu}(x)}$ in the $x \rightarrow x + \mu$ direction, multiplied around a plaquette give the product $U_{\mu \nu}(x)$).

The partition function $Z$ is

$$Z = \int \prod_{l} U(l) \exp -S_G$$

(2)

and may be trivially rewritten, by inserting the identity operator

$$1 = \int \prod_{l} Dv_{r}(l)Dv_{i}(l) \delta(v_{r}(l) - \Re U(l)) \delta(v_{i}(l) - \Im U(l))$$

(3)

and using the integral representation of the delta function,$\delta(v-U) = \int d\alpha \exp -(i\alpha(v-U))$. The result is

$$Z = \int \prod_{l} DU(l)D\alpha_{r}(l)D\alpha_{i}(l)Dv_{r}(l)Dv_{i}(l) \times \exp(i\alpha_{r}(l)\Re U(l) + \alpha_{i}(l)\Im U(l)) \times \exp -(S(v) + i\alpha_{r}(l)v_{r}(l) + i\alpha_{i}(l)v_{i}(l))$$

(4)
$S(v)$ is obtained by substituting $v_r(l)$ for $\Re U(l)$ and $v_i(l)$ for $\Im U(l)$ in eq. (1).

The $U(l)$ integrations now decouple, at the expense of introducing the new link variables $\alpha(l)$, and give a contribution $\propto \sum_l \log \left( I_0 \left( \sqrt{\alpha_r^2(l) + \alpha_i^2(l)} \right) \right)$ to the exponential in eq. (4), leading to an effective action

$$S_{\text{eff}} = S(v) + \sum_l \log \left( I_0 \left( \sqrt{\alpha_r^2(l) + \alpha_i^2(l)} \right) \right)$$

Traditionally one introduces axial gauge in the 4 direction\(^4\), $U_4(x) \equiv 1$ for all points.

The equations of motion for the fields $v(l)$ and $\alpha(l)$ may now be explicitly written down. The correct solution is the configuration that minimizes the free energy, $-\log Z$. Fu and Nielsen \(^1\) guessed that $\alpha_i(l) = v_i(l) = 0$ for all links $l$ and, furthermore, $v_\mu(x) \equiv v$ for $\mu = 1, \ldots, 3$, while $v_\mu(x) \equiv v'$ for $\mu = 5, \ldots, 4+D$, where $v, v'$ are independent of $x$. With this simplifying Ansatz the equations of motions become

$$4\beta v^3 + 2D\beta' v'^2 v + 2\beta v = \alpha$$

$$v = I_1(\alpha) / I_0(\alpha)$$

$$6\beta' v^2 v' + 2\beta'(D-1) v^3 + 2\beta' v' = \alpha'$$

$$v' = I_1(\alpha') / I_0(\alpha')$$

and the free energy/site (for the $D=1$ case, on which we focus in the remainder of this section) may be written as

$$f = 3(-\beta v^2(v^2 + 1) + \alpha v - \log I_0(\alpha)) + \beta'(-3v^2v'^2 - v'^2) + \alpha'v' - \log I_0(\alpha')$$

and delivers plaquette expectation values through

$$\frac{\partial}{\partial \beta} f = \sum_{\mu, \nu=1}^4 \langle U_{\mu\nu} \rangle$$

$$\frac{\partial}{\partial \beta'} f = \sum_{\mu=1}^4 \langle U_{\mu5} \rangle$$

In ref. \(^1\) these equations were analyzed and three régimes identified:

- $v = 0, \ v' = 0$ (strong coupling, I);
- $v \neq 0, \ v' = 0$ (layered phase, II);

\[^4\text{This is a gauge choice that allows one to compute the free energy } - \log Z \text{ easily. Formally, axial gauge would be } U_{4+D}(x) \equiv 1 \text{ for all points } x \text{—but the mean field equations become less transparent.}\)
• \( v \neq 0, \ v' \neq 0 \) (weak coupling,III).

The transition on the axis \( \beta = 0 \) is an artifact of mean field theory—along this line the theory splits into a sum of one-dimensional spin theories. Mean field theory is known to break down in low dimensions and corrections shift the “transition point” significantly to the right [1].

Let us now argue the order of the transition(s) from mean field theory.

The transitions separating I and II, I and III are first order. This is the expected result from mean field theory, since the \( v^3 \) term in eq. (3) leads to a jump from \( v = 0 \) (I) to \( v \neq 0 \) (II,III). For the strong to layered transition, our result is in agreement with the latest Monte Carlo data, obtained for \( \beta' = 0 \), i.e. four-dimensional compact \( U(1) \) theory [4]. Along the (II,III) line, however, \( v \) stays non-zero; the transition is marked by \( v' \) becoming non-zero. The mean field equation (7) is now relevant—and for \( D = 1 \) \( v' \) enters linearly, due to the fact that \( U_5 \) comes in only quadratically in the 4+1 dimensional anisotropic model! The transition is second order—and Monte Carlo results [3, 5] do indeed confirm this. For \( D > 1 \) this argument is no longer valid, since \( v' \) enters cubically—however, the coefficient of this term is not large enough to drive the transition first order—at least as long as \( D \leq 3 \). Monte Carlo results are not yet available for the higher dimensional case \( (D > 1) \), so we cannot check the validity of this mean field prediction.

### 3 Coupling fermions

The presence of fermions renders the order parameter inefficient—it will always indicate perimeter behavior. What one should look at instead is the fermion spectrum, which is different in the three phases:

• bound states with strong coupling (I);

• fermions move freely in the four-dimensional subspace as in a four-dimensional Coulomb phase, but feel a confining force in the extra directions (II);

• free fermions, positronium states in \( 4+D \) dimensions.

We have checked, by mean field methods, that the phase diagram described above is not qualitatively altered by the presence of the fermions. In this section we will explain these methods.

Our starting point is the fermionic action, with a Wilson term

\[
S_F = -\frac{1}{2} \sum_{x,\mu} \overline{\Psi}(x) \gamma_{\mu} [U_\mu(x)\Psi(x + \hat{\mu}) - U_\mu^\dagger(x - \hat{\mu})\Psi(x - \hat{\mu})] \\
+ \frac{\gamma}{2} \sum_{x,\mu} \overline{\Psi} \left( U_\mu(x)\Psi(x + \hat{\mu}) - 2\Psi(x) + U_\mu^\dagger(x - \hat{\mu})\Psi(x - \hat{\mu}) \right)
\]
\[ M \sum_x \overline{\Psi}(x) \Psi(x) \equiv \sum_{x,x'} \overline{\Psi}(x) \mathcal{M}(x,x') \Psi(x') \quad (10) \]

The total action is the sum of the above and the gauge action, equation (11)

\[ S = S_G + S_F \]

The partition function now reads

\[ Z = \int \prod_x D\overline{\Psi}(x) D\Psi(x) \prod_l DU(l) \exp -S \quad (12) \]

The fermions may then be integrated out, leading to the effective action

\[ S_{\text{eff}} \equiv S - \text{Tr} \log \mathcal{M}(U) \quad (13) \]

The mean field approximation goes through exactly the same steps as in the pure gauge case, leading to

\[ Z = \int_{-i\infty}^{i\infty} \prod_l D\alpha(l) \int D\nu(l) \exp - (S_{\text{eff}} + \alpha \nu - \log I_0(\alpha)) \quad (14) \]

The difference with the pure gauge case is that the fermionic determinant leads to a non-local action for the \( \nu \)-variables. The equations of motions now become

\[ 4 \beta v^3 + 2D \beta' v'^2 v + 2 \beta v + j(v,v') = \alpha \]

\[ v = I_1(\alpha)/I_0(\alpha) \]

and

\[ 6 \beta' v^2 v' + 2 \beta'(D-1)v'^3 + 2 \beta v' + j'(v,v') = \alpha' \]

\[ v' = I_1(\alpha')/I_0(\alpha') \]

The currents \( j(v,v') \) and \( j'(v,v') \) are short-hand for

\[ j(v,v') = \frac{\delta}{\delta v_{\mu}(x)} \text{Tr} \log \mathcal{M}|_{v_{\mu}(x) = v, v_{\nu}(x) = v'} \quad (15) \]

and

\[ j'(v,v') = \frac{\delta}{\delta v'_{\nu}(x)} \text{Tr} \log \mathcal{M}|_{v_{\mu}(x) = v, v'_{\mu}(x) = v'} \quad (16) \]
So $j(v, v')$, $j'(v, v')$ are the expectation values of the components of the fermionic current in the background field $(v, v')$. Straightforward manipulations lead to the expressions

$$j_\mu(v, v') = \text{Tr} \left( \frac{\gamma_\mu + r}{2} G_{x,x+\hat{\mu}}(v, v') \right)$$

$$+ \text{Tr} \left( -\frac{\gamma_\mu + r}{2} G_{x+\hat{\mu},x}(v, v') \right), \quad \mu = 1, 2, 3, 5, \ldots, 4 + D \quad (19)$$

where $G(v, v')$ is the fermion propagator in the background $(v, v')$ and the trace is only over Dirac indices.

Since the background is constant in space, the currents $j, j'$ can be computed implicitly as functions of $v, v'$ through a $4 + D$ dimensional momentum integral.

Let us now introduce the variables

$$W \equiv M - r \left( \sum_{\lambda=1}^{3} (1 - v \cos p_\lambda) + 1 - \cos p_4 + \sum_{\lambda=5}^{4+D} (1 - v' \cos p_\lambda) \right) \quad (20)$$

and

$$P \equiv \sum_{\lambda=1}^{3} v^2 \sin^2 p_\lambda + \sin^2 p_4 + \sum_{\lambda=5}^{4+D} v'^2 \sin^2 p_\lambda + W^2 \quad (21)$$

in terms of which the currents may be written as

$$j_\mu = 4 \int_{-\pi}^{\pi} \frac{d^{4+D} p}{(2\pi)^{4+D}} \left[ v \sin^2 p_\mu + r \cos p_\mu W \right] \frac{1}{P} \quad \mu = 1, 2, 3 \quad (22)$$

and

$$j_\nu = 4 \int_{-\pi}^{\pi} \frac{d^{4+D} p}{(2\pi)^{4+D}} \left[ v' \sin^2 p_\nu + r \cos p_\nu W \right] \frac{1}{P} \quad \nu = 5, \ldots, 4 + D \quad (23)$$

Obviously, if $v' = 0$ then $j' = 0$ (c.f. equation (16)–or, more explicitly, by combining (20) and (23)) and the equations of motion are identical with or without fermions.

Assuming a second order transition between the layered and the Coulomb phases we find the shift of the transition point induced by the presence of the fermions (for $v = 1$) as

$$\beta'_c = \frac{1}{4} - \frac{\delta}{8} \quad (24)$$

where

$$\delta = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{d^4 p}{(2\pi)^4} \left[ \frac{(1 + r^2)S^2 + (1 - r^2)W^2}{P^2} \right] \quad (25)$$

where $S = \sum_{\mu=1}^{4} \sin^2 p_\mu$, $W$ and $P$ as above, but for $v' = 0, v = 1$. Note that the shift is negative and very small (except for special values of $M$ where $P$ has poles),
i.e. the layered phase, in the presence of fermions, loses ground against the Coulomb phase. Physically the sign may be understood from the fact that the presence of fermions weakens the confining forces, responsible for the layered phase. Thus there is a critical number of fermions, beyond which there isn’t any layered phase. The fermionic currents $j$ are independent of $\beta'$ in the layered phase (since $v$ is). The current $j'$ is identically zero in that phase. For fixed $\beta$ it grows in the Coulomb phase, cf. figs. [1], [2].

4 Monte Carlo Methods

The size of the five dimensional system was taken to be $4^5$ and $8^5$. For the pure gauge simulations we used a 5-hit Metropolis algorithm with dynamically adjustable stepsizes $\epsilon_{sp}$ and $\epsilon_5$ in order to maintain an acceptance rate of roughly 50% in each direction.

As we are interested in the phase structure, we performed three thermal runs on each lattice size. Such a run consisted of 5000 updates at the starting values of the couplings $\beta$ and $\beta'$ for thermalization, followed by 1000 updates and measurements, after which either $\beta$ or $\beta'$ was increased by 0.01, and another 1000 updates and measurements were made without rethermalization. After having reached the final values of $\beta$ and $\beta'$, the process is reversed until the starting values have been reached.

\[\text{\[1\]}\]

\[\text{\[2\]}\]

In our original paper [2] there is an unfortunate error in the drawing of the phase diagram.
again. There were no thermalization sweeps done at the start of the second leg of the run.

The number of 1000 measurements was chosen with the following reasoning in mind. In case of a first order phase transition, there are, loosely speaking, two autocorrelation times at play: the first being the tunnelling time $\tau_T$, and the second being the relaxation times $\tau^i_s$ within the states $i$. The number of measurements, $N_{\text{meas}}$, should then satisfy

$$\tau^i_s < N_{\text{meas}} \ll \tau_T.$$ 

For second and higher order phase transitions, $\tau_T$ is absent, and we are left with the condition

$$\tau^i_s < N_{\text{meas}}.$$ 

Although changing the couplings will bring the system out of equilibrium, we might hope that this disturbance is relatively small, and the system will relax to equilibrium within $\tau^s$ sweeps.

As observables to measure we choose the average plaquettes $U_{sp}$ and $U_5$,

$$U_{sp} = \left\langle \frac{1}{6V} \sum_{x,\mu,\nu} \mathbb{R}U_{\mu\nu}(x) \right\rangle$$ (26)
\[ U_5 = \left\langle \frac{1}{4V} \sum_{x, \mu} \Re U_{\mu 5}(x) \right\rangle \]  

(27)

and the Polyakov line correlators \( P_{sp} \) and \( P_5 \)

\[ P_{sp} = \left\langle \frac{1}{6V} \sum_{x, \mu} \Re \left( p_{\mu}^\dagger(x) p_{\mu}(x + \frac{N}{2} \hat{\nu}) \right) \right\rangle \]  

(28)

\[ P_5 = \left\langle \frac{1}{4V} \sum_{x, \mu} \Re \left( p_{\mu}^\dagger(x) p_5(x + \frac{N}{2} \hat{\nu}) \right) \right\rangle \]  

(29)

where \( p_{\mu}(x) \) is the Polyakov line starting at point \( x \), running in the \( \hat{\nu} \) direction and \( N \) is the lattice size in the corresponding direction. As the phase transitions in this model are conjectured to be confining-deconfining transitions in one or more dimensions, these observables are indeed order parameters for these phase transitions. For a confining phase, \( P \) will be zero as the system will be disordered over long distances. In a Coulombic phase, however, \( P \) will be non-zero, as now such long correlations are allowed.

When including fermions, we used a Hybrid Monte Carlo algorithm with a first order discretization of Hamilton’s equations. A standard Conjugate Gradient algorithm was used for matrix inversion. One sweep consisted of momentum refreshment, followed by 10 integrations with stepsize \( \tau = 0.05 \). An accept/reject step was done in order to maintain balance. The runs were performed as in the pure gauge case, but now with 1500 thermalization sweeps and 250 sweeps at each \((\beta, \beta')\) pair.

The actual simulations were done on the Wuppertal CM-5 in its various incarnations. We are, therefore, unable to make comments about the performance of the two programs on this machine.

5 Results

This section will be subdivided into two parts: one for the pure gauge case and one including the fermions. For each case we shall confront results (namely plaquette values) from mean field theory with Monte Carlo simulations so we work in five spacetime dimensions in what follows and will comment on what happens for \( D = 2, 3 \) at the end.

5.1 Pure gauge results

In this paragraph we report on mean field calculations and Monte Carlo simulations for the pure gauge case. The quantities of interest are the average plaquettes.
Using the equations of motion, (6) and (7), one obtains the following expressions, within mean field theory

\[
\langle U_{sp} \rangle = -\frac{1}{6} \frac{\partial}{\partial \beta} f = \frac{v^4 + v^2}{2} \\
\langle U_{\mu 5} \rangle = -\frac{1}{4} \frac{\partial}{\partial \beta'} f = \frac{v'^2(3v^2 + 1)}{4}
\]

These reflect the fact that the equations of motion cancel all implicit dependence of the free energy on \( \beta \) and \( \beta' \). So the rhs of eq. (30) represents only the explicit dependence and is, therefore, equally valid when fermions are included. In fig. 3 we display typical plaquette values vs. \( \beta \) at fixed \( \beta' \), relevant for the strong ↔ layered transition. Mean field theory predicts a transition at \( \beta = 0.8 \), while the simulations indicate that the transition is at \( \beta = 1.02 \). It should be noted that corrections to mean field theory also indicate an upward shift in the transition point \[1\]. In the layered phase, the mean field prediction is that the space-space plaquette, for fixed \( \beta \), doesn’t depend on \( \beta' \). The reason is that in the layered phase \( v' = 0 \) and \( \beta' \) enters the mean field equations only in conjunction with it; furthermore, from eq. (30), the plaquette in the fifth direction is zero. There is, however, a strong coupling correction \( \delta U = \beta'/2 + O(\beta'^2) \) that must be added-this is a well-known result that may also serve as a check on the simulations.

Finally, we present evidence that the layered↔Coulomb is second order, namely hysteresis loops, in fig. 4. As a comparison, we display hysteresis loops for the strong
↔ Coulomb transition in fig. 3.

For the strong↔layered transition we also display spatial Polyakov line data in Figs. 3. These data are the first Monte Carlo data using the order parameter, not just plaquette expectation values, that show the occurrence of the layered phase. Our results may be summarized by the phase diagram for the pure gauge theory, displayed in fig. 4. The main novelty is the evidence that the layered ↔ Coulomb transition is second order, in agreement with the intuitive argument of the previous section. For higher dimensions of spacetime, relevant for other theories, the situation is the following: mean field theory predicts that the layered ↔ Coulomb phase transition stays second order for $4 + D \leq 7$, while it becomes first order for $4 + D > 7$; this is based on a calculation of the hysteresis loops. Furthermore, as may be seen from fig. 8, the strong coupling phase, for $4 + D = 6$ is significantly reduced, in favor of the Coulomb phase. Unfortunately, Monte Carlo data are not available to check whether this is an artifact of the mean field approximation.

Of course there are several caveats that must be kept in mind: (a) corrections to mean field theory may be important—the analysis of ref. [1] indicates that they do not lead to qualitative changes in five dimensions, but are significant in three dimensions; regarding these corrections, it should be noted that they tend to soften the first order transitions usually encountered [7]; (b) regarding the numerical simulations, the analysis of ref. [8] of the order of the transition is certainly relevant here—the results presented are preliminary, certainly indicative of an issue that requires more extensive work [5] and, of course, a very long correlation length cannot be ruled out.
5.2 Fermions

We use the Wilson action, as discussed in the previous section, with one fermion flavor, and set $r = 1$ and $M = 1$. The entire analysis goes through intact—the only calculational complication arising from the fact that every iteration of the mean field equations towards the fixed point requires the numerical evaluation of a four (in the layered phase) or five dimensional integral for which we used the Vegas routine. Computing, once more, the hysteresis loops through the transitions, we find that the presence of the fermions doesn’t change the order of the three transitions. Consistent with this result we find a shift of the layered ↔ Coulomb transition in favor of the Coulomb phase.

To check these predictions we have performed Monte Carlo simulations, measuring the same observables as in the pure gauge case, using the Hybrid Monte Carlo method and our results are summarized in the phase diagram in fig. 5. The caveats stated above apply here as well. High statistics simulations are needed to clarify whether the results obtained for the layered ↔ Coulomb transition do indeed hold up. The claim is that the problem deserves this effort.

5.3 Continuum Limit

The presence of the line of second order transitions in the phase diagrams is intriguing. On both sides of this line we have a phase with zero mass particles and a Coulomb law; the difference is that the force falls off like $1/r^2$ in the four physical
Figure 6: Spatial Polyakov line correlators at $\beta' = 0.2$ from an $8^5$ lattice
Figure 7: Phase Diagram of the five dimensional, $U(1)$ pure gauge theory.

Figure 8: Phase Diagram of the six dimensional, $U(1)$ pure gauge theory.
Figure 9: Phase diagram including fermions of mass $M = 1$ (points). Mean field theory predicts the strong to layered transition at $\beta = 0.8$; Monte Carlo estimates it at 1.02. The pure gauge phase diagram is included for comparison (solid line).

directions in the layered phase and like $1/r^2+D$ in all $4+D$ directions in the weak coupling phase.

In the layered phase the Wilson loops in the extra directions obey an area law. At $\beta' = 0$ the string tension becomes infinite and no correlation in these directions survives—we have a stack of decoupled $4d$ layers, each of unit thickness (for $D = 1$ or “volumes” for $D = 2, 3$). At $\beta' > 0$, but $\beta' < \beta'_c(\beta)$ (so $\nu' = 0$), one has finite, but exponentially damped, correlations between the layers.

Presumably a continuum limit of this system exists at the tricritical point in figs. 7, 8. At $\beta' = 0$ we cannot take any continuum limit.

6 Conclusions and outlook

We have presented evidence, analytical (within the mean field approximation) and numerical (through Monte Carlo simulations) that gauge theories, with anisotropic couplings, naturally support layered phases that admit a non trivial continuum limit.

Our results show unambiguously a line of second order phase transitions, between layered and Coulomb phase in fig. 7. Furthermore, this statement remains true in the presence of fermions, fig. 9. The implications of these findings are quite broad in scope. In the same context one may find a natural setting for investigations of membranes and models with chiral fermions. For this last case the present
calculations are a necessary first step for the location of the chiral layer [2, 9].

In this paper we only looked at abelian theories, in 4 + D dimensions. If we heat up the system, one can include non-abelian systems, as soon as they drop into the deconfined phase. We would expect our results for the Polyakov loops to remain qualitatively the same—with the caveat that only the Polyakov loop in the Euclidean time direction deconfines. Systems of this kind have been studied by other means in ref. [10].

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