Modular networks of spiking neurons for applications in time-series information processing

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Received February 5, 2020; Revised May 26, 2020; Published October 1, 2020

Abstract: Spiking neural networks with complex spatiotemporal dynamics support efficient information processing of time-series signals. Here, we investigate the relationship between complexity of network dynamics and modular topology of networks using numerical simulations and discuss their effect on the classification performance of spoken-digit recognition tasks. The results show that modular networks generate spatially complex dynamics in which partially and globally synchronous bursts coexist. The classification rate of the modular reservoir network was approximately 75%, a value of which was comparable to that of a random network. This was caused by the randomly-connection structure between the input-reservoir and reservoir-readout layers, thus appropriate inference methods and asymmetry of connections should be introduced to take advantage of the complex dynamics in modular networks.

Key Words: neural network, reservoir computing, integrate-and-fire model, modular organization, synchronous activity

1. Introduction

Information processing systems that mimic the brain have been shown to be effective in performing specific information processing tasks more efficiently as compared to conventional von Neumann computers. Recently, reservoir computing has gained attention as a method of utilizing the spatiotemporal dynamics of neural networks for information processing [1–3]. In the reservoir computing framework, nonlinear elements, such as neurons, are connected recurrently to form a high-dimensional dynamical system, or a reservoir. The reservoir then maps time-series inputs to higher dimensions, allowing the network to efficiently process temporal information by classifying reservoir states with simple
classifiers such as linear regression [1].

When constructing reservoir networks, neurons are usually connected randomly. In contrast, recent connectomics analysis has revealed several important structural features in brain networks, including the modular organization [4]. Studies on modular-reservoir networks have reported that performance and robustness can be increased compared to randomly-connected reservoirs [5–7]. For example, Klampfl et al. showed that modular structures that code specific inputs self-organize in a reservoir network when STDP is implemented in synapses within the reservoir and that the computational capability of linear readout neurons was enhanced after the STDP-based tuning of the synaptic weights [6]. More recently, Sakaguchi et al. showed that modularity of a recurrent neural network enhances the separability of LSMs since modular reservoirs exhibit unique and input-dependent dynamics [7]. Moreover, Schrauwen et al. reported that multiple reservoir systems support the generation of network dynamics in various time scales [8], which can be beneficial in improving the computational performance of reservoirs [9]. Using in vitro experiments, we have previously shown that modular topology promotes biological neuronal networks to exhibit complex dynamics in which synchronous and asynchronous states are well-balanced in a single network [10]. Such dynamically complex activity pattern is also observed in vivo and is thought to underlie proper information processing in the brain [11]. However, the relationship between the spatiotemporal dynamics in modular networks of spiking neurons and their computational performance is still unclear.

In this study, we computationally study the dynamics that appear in spiking neural networks with random and modular structures, and investigate the effects of the structural differences on the dynamics and computational performance from the viewpoint of the reservoir computing framework. This paper is organized as follows. In the next Section, methods to construct network models with modular topology and to analyze network dynamics are explained. In Section 3, the effect of modularity on network dynamics is analyzed, using statistical measures such as correlation coefficients and functional complexity. The Section also discusses the impact of modularity and dynamical complexity on the computational performance of the networks. Finally, conclusions are given in Section 4.

2. Methods

2.1 Neuron and network models

The reservoir computing system consisted of a reservoir network which was coupled to an input and a readout layers as shown in Fig. 1. The spiking neural network that served as a reservoir was constructed by connecting leaky integrate-and-fire neurons, which are modified to reproduce the behavior of cultured neurons [10, 12–15]. In this simulation, the ratio of excitatory and inhibitory neurons was fixed at 80% and 20%, respectively. This ratio is consistent with the ratio of excitatory and inhibitory neurons in the cerebral cortex [16]. The membrane potentials of each neuron are calculated as

\[
\tau^E \frac{dV^E(t)}{dt} = V^E_L - V^E(t) + R^E(I^E(t) + I^I(t) + I_{K(Ca)}(t) + I_{ref}(t)),
\]

(1)

\[
\tau^I \frac{dV^I(t)}{dt} = V^I_L - V^I(t) + R^I(I^E(t) + I^I(t)),
\]

(2)

where superscripts of E and I represent excitatory and inhibitory neurons, respectively. \(V^E_L = -74\) mV and \(V^I_L = -70\) mV are resting potentials, \(R^E = 40\) M\(\Omega\) and \(R^I = 50\) M\(\Omega\) are the input resistance. \(I^E(t)\) and \(I^I(t)\) represent the total excitatory and inhibitory synaptic currents, respectively. \(I_{K(Ca)}(t)\) is an inhibitory K current depending on the intracellular Ca\(^{2+}\) concentration. \(I_{ref}(t)\) is the relative refractory current. When membrane potential \(V(t)\) above threshold values \(V^E_T = -54\) mV and \(V^I_T = -50\) mV, spike is generated and \(V(t)\) reset to \(V^E_R = V^I_R = -60\) mV. The full description of the neuron model is explained in our previous paper [13, 17].

A modular structure was realized by connecting \(M\) equally-sized random networks, with the total number of neurons \(N\) and connections \(T_1\). The degree of modularization of the network was controlled by tuning the total number of connections between modules \(T_O\) [17]. The inter-modular connections were formed between neurons which were randomly selected from modules. This approach was adopted
from our previous work, which showed that the experiment on cultured neuronal networks was best modeled by assuming a constant intra-modular connection density (defined by $T_I/N$) and tuning modularity by increasing the number of inter-modular connections (defined by $T_O$) \[10\]. The modular networks using as reservoir was generated with $N = 100, M = 4, T_I = 1500$ and $T_O = 40, 120, 400$. Randomly connected reservoirs also generated as a reference ($N = 100, M = 1, T_I = 1500$ and $T_O = 0$). Figure 2 illustrates the network structures with different $T_O$. The boundary between separate modules becomes ambiguous with increasing $T_O$.

The degree of modularization of network is quantified using the Newman modularity \[18–20\].

$$Q = \frac{1}{T} \sum_{i,j} \left[ a_{ij} - \frac{k_{in}^i k_{out}^j}{T} \right] \delta_{m_i, m_j} \tag{3}$$

where $T = T_I + T_O$ is the total number of connections in the reservoir, $a_{ij}$ is an element of the connectivity matrix in the reservoir, $k_{in}^i$ the in-degree of the neuron $i$, $k_{out}^j$ the out-degree of the neuron $j$, $\delta$ the Kronecker delta symbol, and $m_i$ the label of a community to which neuron $i$ is assigned.

The input layer was composed of $N_I = 78$ excitatory neurons which followed the same model as the reservoir neurons. The number of connections between input neurons and the reservoir was set to 780, which corresponded to 10% of all possible connections between the input and the reservoir layers ($0.1NN_I$). The connections were formed randomly to both excitatory and inhibitory neurons. Finally, the readout layer consisted of $N_y = 10$ rate neuron units, whose connection to the reservoir was expressed by an $N \times N_y$ weight matrix $W$. 

Fig. 1. The configuration of speech classification task using reservoir computing.
2.2 Analysis of network dynamics

Network dynamics were analyzed based on the raster plot, correlation coefficients, and functional complexity. The correlation coefficient was calculated from a binned binary spike train \( F(t) = \{ f_i(t) \} \), where \( f_i(t) = 1 \) when neuron \( i \) fired more than once within a time bin (10 ms) and \( f_i(t) = 0 \) otherwise. The width of time bin was set to 10 ms in order to adequately capture the coherent firing of presynaptic and postsynaptic neurons. Note that a synaptic delay was set to 2.8 ms and that a single time bin could contain more than one spike. The correlation coefficient between neurons \( i-j \), \( r_{ij} \) was calculated as follows:

\[
    r_{ij} = \frac{\sum_i (f_i(t) - \bar{f}_i)(f_j(t) - \bar{f}_j)}{\sqrt{\sum_i (f_i(t) - \bar{f}_i)^2 \sum_j (f_j(t) - \bar{f}_j)^2}},
\]

where \( f_i(t) \) is a binned binary spike train of neuron \( i \) and \( \bar{f}_i \) is the temporal average of \( f_i(t) \).

Complexity of network dynamics was quantified using functional complexity \( \chi \) [21]:

\[
    \chi = 1 - \frac{1}{C_m} \sum_{\mu=1}^{m} \left| p_\mu(r_{ij}) - \frac{1}{m} \right|,
\]

where \( p \) designates the probability, \( m \) is the number of bins, and \( C_m \) is a normalization factor. This index was originally introduced to capture the balance of integrated (synchronous) and segregated (asynchronous) states in neural networks. Functional complexity \( \chi \) approaches one when the distribution of \( r_{ij} \) is uniform, a state in which synchronous and asynchronous node pairs coexist in the system. It has been reported that \( \chi \) tends to increase when a network possesses a non-random structural feature such as the modular organization [21].

2.3 Spoken-digit recognition task

In order to evaluate the relationship between functional complexity and their computational performance, spoken-digit recognition tasks were performed on reservoir networks with different degree of modularization. TI-46 dataset was used as input, which is speech sounds of females saying numbers from zero to nine [22–24].

TI-46 audio dataset was first converted to a spectrogram by the Lyon cochlear filter (Fig. 3), which is an audio filtering algorithm modeling the cochlea, with a sampling rate of 12.5 kHz and a decimation factor of 5 [25]. The signal amplitude of the spectrogram was multiplied by a factor, \( A \), and then the signal was fed into the input neurons as current. In our simulations, \( A \) was set to 8 \( \mu \)A. Reservoir neurons received synaptic currents from input neurons and other reservoir neurons, and then generated a spike when the membrane potential reached the threshold value.

The state of the readout layer \( Y \) was calculated by taking the product-sum of the reservoir state \( x \) and weight matrix \( W \), namely, \( Y = WX \cdot x \). Here, \( x = [x_i] \) was defined as the exponentially filtered firing rate of the reservoir neurons and was calculated as:

\[
    \frac{dx_j(t)}{dt} = \sum_{k} \delta(t - t_{j,k}) - \frac{x_j(t)}{\tau_x},
\]

where \( j \) is the neuron index, \( \tau_x = 200 \) ms time constant of \( x \), \( t_{j,k} \) the time of the \( k \)th firing of neuron \( j \), and \( \delta(t) \) a function which is equal to one when \( t = 0 \) and equal to zero otherwise. We adopted the winner-take-all method in which the largest element in \( Y \) was regarded as the estimated answer of the network.

During the training phase, the weight \( W \) was updated using ridge regression:

\[
    W = \hat{Y}X^T(XX^T + \lambda I)^{-1},
\]

where \( \hat{Y} \) is the desired output matrix, \( \lambda = 1 \) the scale factor, and \( I \) an \( N \times N \) identity matrix. The training data consisted of 10 words spoken by five speakers 10 times each, resulting in a total of 500 inputs. The classification rate \( CR \) was evaluated by taking a percentage of successful classifications for 100 words in the test phase.
3. Results and discussion

In the current work, the degree of modular organization, or modularity $Q$, in neural networks was varied by adding a small number of connections between densely interconnected modules, i.e. <5% of the total number of connections. Networks with different values of $Q$ were found to exhibit different spatiotemporal dynamics even though they were constructed with identical numbers of neurons and a similar number of connections. The value of functional complexity $\chi$ of the networks with various modularity is compared in Table I. A network with the highest modularity was generated with intermodular connections $T_O = 40$, in which the highest value of $\chi$ was observed. To further investigate this result in detail, the raster plots and the correlation matrices of networks with different modularity were compared (Fig. 4). In highly modularity networks ($T_O = 40, Q = 0.72$), partially and globally synchronous states coexisted in the activity patterns due to an intermediate density of inter-modular connections that balanced the two states [Fig. 4(a)]. On the other hand, only the globally synchronous state was observed in weakly modular networks with $T_O = 400$ and $Q = 0.48$ [Fig. 4(b)]. Weakly synchronous activities were observed in intermediately modular networks ($T_O = 120, Q = 0.65$). As a general trend, increase in the number of inter-modular connections facilitated the propagation of burst firing from one module to another, which increased the frequency of globally synchronized bursts.

Figure 5 shows the distribution of correlation coefficients in networks with different modularity. As a control, the distribution for a random network is also depicted. In the random (green) and weakly modular (blue) networks, correlation coefficients are clustered around $\sim 0$ or $\sim 1$. This indicates that most of the neural pairs were asynchronously and synchronously activated in the random and weakly modular networks, respectively. Contrarily, the correlation coefficients of the highly modular

| Type               | $T_O$ | $Q$  | $\chi$ | $CR$   |
|--------------------|-------|------|--------|--------|
| Highly modular     | 40    | 0.72 | 0.62   | 72.1%  |
| Intermediately modular | 120  | 0.65 | 0.48   | 76.6%  |
| Weakly modular     | 400   | 0.48 | 0.34   | 74.3%  |
| Random             | -     | 0.15 | 0.19   | 75.6%  |

Table I. Functional complexity and classification performance in network with different modularity.
Fig. 4. Raster plots and correlation matrices of networks with different modularity. (a) highly modular network, $T_O = 40$, (b) intermediately modular network, $T_O = 120$, (c) weakly modular network, $T_O = 400$.

(red) and intermediately modular (yellow) network were broadly distributed between 0 and 1. This indicates that the network is comprised of a mixture of strongly and weakly coherent neuron pairs, which underlies the increase in the value of functional complexity.

De la Rocha et al. reported that spike correlation depends on the firing rate of the neurons [26]. However, in the current model, modularity of the network, which depends on the number of inter-modular connections ($T_O$), is the primary factor behind the different distribution of correlation coefficients. For instance, comparison of the distribution of correlation coefficients for two networks with different modularity ($Q = 0.72$ and 0.48) but with identical firing rate ($f = 31.0$ and 30.9 Hz) show that the correlation coefficient for the high modularity network is broadly distributed between zero and one, whereas that for the low modularity network is clustered around one (data not shown). Such variation in the distribution of correlation coefficients is caused by the different activity patterns in the networks—the former generating synchronized bursting confined within each module, whereas the latter dominated by globally synchronized bursts.

Table I also summarizes the rate of obtaining correct classification, $CR$, when a spoken-digit classification task was performed using the spiking neural networks as reservoirs. The relationship between $CR$ and the inter-modular connections in more detail was also shown in Fig. 6. Contrary to our expec-
tation, the difference in network modularity and the resulting difference in the functional complexity had a subtle influence on CR. To analyze the mechanism behind this observation, we compared the state matrices of the networks $X$ used for the training of output weights, $W$. Here, state matrices $X$ refer to exponentially-filtered spike trains, whose product with $W$ is the output of the readout layer. Figure 7 show examples of state matrices in network with $T_O = 40, 120,$ and $400$. Clearly, the state matrix reflected the differences in network structure. However, the classification performance was not influenced by the network structure. Our current interpretation is that this result due to the learning method used to train $W$. In the current simulation, since the nodes in the readout layer were connected with all the nodes in the reservoir, the activity within all the reservoir nodes was used for classification. In addition, neural activity was pre-sampled in both the spatial and temporal dimensions and was used for training. Therefore, the system was able to learn the correct answer from a slight difference in the firing rate for different inputs. Thus, networks that exhibited only the global synchronization was also capable of achieving comparable scores of CR.

When reservoir computing is physically implemented in hardware and used in the real environment, the response of the reservoir to an identical input is likely to change due to disturbances, such as noise. Under such conditions, the difference in reservoir dynamics triggered by dissimilar input can, in turn, disappear, which can lead to the degradation of CR [27]. As shown by the above simulations, the activity inside the modules is strongly correlated in modular networks. Such dynamical characteristics of modular networks would be beneficial in increasing the robustness against noise, as partial
Fig. 7. State matrices in networks with different modularity. The horizontal axis represents the time bin, and the vertical axis corresponds to Neuron ID. Color bars represent the value of $x$. (a) Highly modular network, $T_O = 40$, (b) intermediately modular network, $T_O = 120$, (c) weakly modular network, $T_O = 400$.

synchronization would be maintained even when reservoir responses fluctuate by noise [28].

Previous studies on the application of reservoir computing paradigms to spoken-digit recognition tasks have demonstrated the correctness of over 90% [29, 30]. The relatively low performance of our model originates in the restriction that we used a model that faithfully reproduced the behavior of biological experiments. The time step of the input signal which was set to match the temporal scale of neuronal behavior may not have efficiently increased the dimensionality of the input time-series. In such a case, the performance may improve, e.g., by distributing conduction delays in the reservoir layer.

Studies on multi-reservoir networks have reported that the classification performance can be increased by assigning different characteristics of input signals to different reservoirs [9]. Our preliminary experiments also show that the classification performance improves several percent by limiting the reservoir nodes that receive direct inputs from the input layer, which is likely to be caused by
the modular dynamics extracting features in the input signal. Moreover, Paugam-Moisy et al. has previously shown that spatiotemporal spiking patterns can be learned and detected by incorporating axonal conduction delays between reservoir and readout neurons [31]. Therefore, learning rules that optimize not only the synaptic weights but also the conduction delays could also be effective in exploiting the spatiotemporal dynamics and improving the robustness and classification performance of modular reservoirs.

In the current work, the classification was performed based on the firing rate of the read-out neurons. This approach, however, neglects the temporal structure of the reservoir dynamics. Computational studies on spiking neural networks have shown that the information representation capability dramatically increases when the temporal structure of neural dynamics is taken into account [32]. Understanding of how the difference in the number of neurons and modules influences the characteristic dynamics of networks and their recognition performance is also an important topic which we continue to investigate both experimentally and computationally.

4. Conclusions
In this study, we used the reservoir computing framework to elucidate the relationship between the neural dynamics of modular networks and the performance of time-series information processing. Analysis of the network dynamics based on functional complexity and correlation coefficients revealed that the dynamics of high modularity networks are characterized by complex dynamics in which partially and globally synchronous bursts coexist. The correctness of the modular reservoir network was approximately 75%. However, no difference in the performance was observed among the networks with different structures and dynamics. This result is likely to be caused by the symmetrical structure of the input-output connections and the current inference method that does not consider the time-evolution of reservoir dynamics. Introduction of a new inference method that takes into account the temporal structure of the reservoir dynamics, as well as the optimization of input-output couplings, should be considered in order to exploit the functional complexity of modular networks in information processing.

Acknowledgments
The authors thank Prof. Y. Horio (Tohoku University) for fruitful suggestions. This study was supported by the Cooperative Research Project Program of the Research Institute of Electrical Communication of Tohoku University, the Program on Open Innovation Platform with Enterprises, Research Institute and Academia (OPERA) from Japan Science and Technology Agency (JST), JSPS KAKENHI (No. 17K18664, 18J12197, and 18H03325), JST PRESTO (No. JMPJPR18MB) and JST CREST (No. JPMJCR19K3).

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