A Strain Rate Dependent Progressive Damage Model for Carbon Fiber Woven Composites under Low Velocity Impact

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Abstract. A progressive damage model was presented for carbon fiber woven composites under low velocity impact, considering the strain rate sensitivity of both mechanical properties and failure mechanisms. In this model, strain rate dependency of elastic modulus and nominal strength along in-plane direction are considered. Based on the Weibull distribution, stiffness progressive degradation is conducted by introducing strain rate dependent damage variables for distinct damage modes. With the model implemented in ABAQUS/Explicit via user-defined material subroutine (VUMAT), the mechanical behavior and possible damage modes of composites along in-plane direction can be determined. Furthermore, a bilinear traction separation model and a quadratic stress criterion are applied to predict the initiation and evolution of interlaminar delamination. Comparisons are made between the experimental results and numerical simulations. It is shown that the mechanical response and damage characteristics under low velocity impact, such as contact force history and delamination, are more consistent with the experimental results when taken the strain rate effect into consideration.

Keywords. Strain rate, progressive damage, woven composites, low velocity impact.

1. Introduction
Carbon fiber reinforced composites are applied widely in many fields, such as aviation, aerospace and automobile. However, composites are vulnerable to impact loading due to the poor interlaminar properties. Even low velocity impact issues may cause serious matrix cracking and delamination, which significantly threatens the safety of composite structures.

In recent decades, various researchers have numerically investigated the impact resistance of composite laminates [1-3]. As we known, the key problem in simulation is to reasonably selecting the constitutive model which can accurately predict the mechanical response and damage mechanism of composites. The continuum damage mechanics (CDM) model, in which the mechanical property parameters decrease gradually with the increase of damage, is the most commonly used constitutive model [4-6].

However, most of the previous studies tried to deal with the impact issues based on the quasi-static method, neglecting the strain rate dependence of composites. In fact, due to the viscoplastic characteristics of epoxy, the strain rate sensitivity of mechanical response should be taken into consideration when subjected to impact loadings [7, 8]. Kara [9] showed that with the increase of strain rate, the modulus and peak stress of the E-glass/polyester composites exhibit increasing trend, especially along the in-plane direction. Kushvaha [10, 11] studied the dynamic fracture behavior for different composite materials, the results showed that loading rate is the most significant variable in predicting the stress intensity factor. Even for the low velocity impact issues, a better fit can be
obtained when considering the effects of strain rate in the numerical model[12]. Thus, in order to accurately describe the mechanical behavior of composites subjected to low velocity impact, a strain rate dependent constitutive model should be established.

The main damage modes of carbon fiber reinforced composites are fiber breakage, matrix cracking and delamination, et al. when subjected to impact loadings. Generally, the mechanical behavior of carbon fiber is considered to be rate independent. However, for the woven composites, the elastic modulus, nominal strength, even the damage mechanism along the fiber direction show obvious strain rate dependency [13]. This is due to the fiber bundles of woven composites are interlaced by specific pattern. The mechanical response of curved fiber bundle is different from the unidirectional fiber bundle especially under compression. Additionally, the viscoplastic behavior cannot be neglect because of the resin-rich zones located at the interlacing points. Therefore, in order to accurately describe the damage evolution for woven composites under low-velocity impact, the strain rate dependence of both mechanical properties and damage evolution along in-plane directions should be included.

The aim of the present work is to develop a three-dimensional progressive damage constitutive model with strain rate effect for a carbon fiber woven composite. The mechanical performance of composites under in-plane loading, such as the elastic modulus, nominal strength and damage evolution in different modes, has been expressed as functions of strain rate. Combined with the cohesive model to depict the interlaminar delamination, the rate dependent progressive damage model is incorporated in ABAQUS/Explicit. The numerical and experimental results are compared, and the necessity of considering the strain rate effect in predicting the damage evolution under low velocity impact is analyzed.

2. Strain Rate Dependent Progressive Model

The compressive and tensile stress-strain curves of carbon fiber woven composites under quasi-static and dynamic loading are shown in figure 1. For different loading conditions, the stress-strain plots exhibit linear elasticity at small strain state and nonlinear behavior presents at larger strain state. The variation trend of stress-strain curves of composites under compression is similar to the one under tension as the loading rate are same. Nevertheless, the mechanical behavior is sensitive to the loading rate and a more brittle failure is observed under quasi-static loading. Therefore, for the carbon fiber woven composites studied in this work, it can be considered that the stress-strain behavior is only dependent on the strain rate.

\[
E = kE_{r1} + (1 - k)E_{m0} \left[ 1 + A_m \log \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right]
\]  

Figure 1. Tensile and compressive stress-strain curves at different strain rates.

As been previously described in detail [13], the strain rate dependent modulus of woven composites along in-plane direction, \( E \), can be expressed as:

\[
E = kE_{r1} + (1 - k)E_{m0} \left[ 1 + A_m \log \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right]
\]  

where $E_{f0}$ and $E_{m0}$ are the modulus of carbon fiber and epoxy at reference loading rate $\dot{e}_0$ (0.001/s), parameter $k$ indicates the proportion of carbon fiber in modulus of composite, $A_m$ represents the strain rate dependency of epoxy matrix.

The degradation of modulus is a macroscopic reflection of the micro damage accumulation. In order to predict nonlinear behavior caused by damage accumulation, a damage variable should be adopted. Thus, the continuous damage mechanical constitutive model of composites can be given by:

$$\sigma = (1 - d) E \varepsilon$$

(2)

where $d$ is the damage parameter depicting the stiffness deterioration. The value of $d$ ranges from 0 to 1, where 0 represents undamaged state and 1 represents fully damaged state.

A statistical mechanism based on Weibull distribution is used to describe the strain rate dependent damage variable. As noted above, the stress-strain behaviors of woven composites are similar under tensile and compressive loading. Thus, the in-plane damage variable for different failure modes can be given as equations (3)-(5), and the detailed derivation process can be found in our previous work [13].

$$d_i = 1 - \exp \left[ -\frac{1}{n_i} \left( \frac{E_i \varepsilon_{eq}}{\sigma_{0,i}} \right)^n \right] ; \quad i = x_t, x_c, y_t, y_c$$

(3)

where $\sigma_{0,i}$ is the nominal strength; $n_i$ is the shape parameter that determines the nonlinear part of stress-strain plot; $\varepsilon_{i,eq}$, the equivalence strain and the expression for different failure modes, are given in table 1; subscript $i$ represents a failure mode respectively. With the increase of $n_i$, the stress-strain behavior transforms from gradual degradation to brittle failure. Further, the parameters $\sigma_{0,i}$ and $n_i$, which dominantly influence the damage parameter, are also sensitive to the loading rate:

$$\sigma_{0,i} = \sigma_{0,i}^{ref} \left\{ 1 + A_{i,0} \left[ \log(\dot{e} / \dot{e}_0) \right]^{p_i} \right\}$$

(4)

$$n_i = n_{0,i} \left[ 1 + A_{i} \log \left( \frac{\dot{e}}{\dot{e}_0} \right) \right]$$

(5)

where $\sigma_{0,i}^{ref}$ and $n_{0,i}$ are nominal strength and shape parameter at referenced strain rate; $A_{i,0}^{ref}$, $p_i$ and $A_i$ are strain rate sensitivity parameters. Considering the orthogonal woven structure of composites, it is considered that the mechanical properties in warp and weft direction are consistent. Therefore, the elastic modulus and damage variable of composites along warp and weft direction can be given in the same expressions.

| Failure mode                        | Equivalence strain                          |
|------------------------------------|---------------------------------------------|
| Warp fiber tensile failure         | $\varepsilon_{eq} = \left\{ (\varepsilon_{11})^2 + \alpha \left( \varepsilon_{12}^2 + \varepsilon_{13}^2 \right) \right\}^{1/2}$ |
| Warp fiber compressive failure     | $\varepsilon_{eq} = \left\{ (\varepsilon_{11})^2 \right\}$ |
| Weft fiber tensile failure         | $\varepsilon_{eq} = \left\{ (\varepsilon_{22})^2 + \alpha \left( \varepsilon_{23}^2 + \varepsilon_{23}^2 \right) \right\}^{1/2}$ |
| Weft fiber compressive failure     | $\varepsilon_{eq} = \left\{ (\varepsilon_{22})^2 \right\}$ |

Table 1. Equivalence strain for different failure modes.
Once the damage variables for different failure mode are determined, the stiffness coefficients of the damaged element can be degraded accordingly. The damaged stiffness matrix can be given by:

\[
C_d = \frac{1}{\Delta} \begin{bmatrix}
    d_{\mu}E_{11}(1-d_{\mu}\nu_{12}\nu_{21}) & d_{\mu}d_{\nu}E_{11}(\nu_{12}+\nu_{21}\nu_{31}) & d_{\mu}E_{11}(\nu_{12}+d_{\mu}\nu_{21}\nu_{31}) \\
    d_{\mu}E_{12}(1-d_{\mu}\nu_{12}\nu_{21}) & d_{\mu}d_{\nu}E_{12}(\nu_{12}+d_{\mu}\nu_{12}\nu_{31}) & d_{\mu}E_{12}(\nu_{12}+d_{\mu}\nu_{21}\nu_{31}) \\
    E_{11}(1-d_{\mu}d_{\nu}\nu_{12}\nu_{21}) & \Delta d_{\mu}d_{\nu}G_{12} & \Delta d_{\mu}G_{13} \\
    \Delta d_{\mu}d_{\nu}G_{12} & \Delta d_{\mu}G_{13} & \Delta d_{\mu}G_{13}
\end{bmatrix}
\]

(6)

\[
\Delta = 1 - d_{\mu}d_{\nu}\nu_{12}\nu_{21} - d_{\mu}\nu_{12}\nu_{32} - d_{\mu}\nu_{13}\nu_{31} - 2d_{\mu}d_{\nu}\nu_{12}\nu_{32}\nu_{13}
\]

(7)

\[
d_{\mu} = 1 - \max(d_{\mu}, d_{\nu})
\]

(8)

\[
d_{\nu} = 1 - \max(d_{\mu}, d_{\nu})
\]

(9)

where \(d_{\mu}\), \(d_{\nu}\) are the damage variables for fiber bundle along warp and weft directions; \(d_{\mu}\), \(d_{\nu}\), \(d_{\mu}\), and \(d_{\nu}\) represent the damage variables corresponding to the four different failure modes: warp fiber tension, warp fiber compression, weft fiber tension, weft fiber compression.

In order to ensure a smooth stiffness degradation process, the viscosity correction is applied in the numerical simulation. By introducing the viscous damage variable, the calculation efficiency of the model can be effectively improved without significantly affecting the calculation results. Based on the regularization model proposed by Duvaut and Lions [14], a simple form of strain rate dependent viscous damage coefficient is given by:

\[
\dot{d}_i^{vis} = \frac{(d_i - d_i^{vis})\dot{\varepsilon}}{\eta}
\]

(10)

Delamination has been identified as a key damage mode of composites under impact loading. In this paper, a bilinear traction separation model based on the critical energy release rate is applied to describe the delamination process. A quadratic stress criterion and the B-K criterion are applied to determine the damage initiation and evolution of interlaminar delamination.

3. Numerical Modelling of Impact Behavior

3.1. The Numerical Model

To investigate the damage characteristics of composites, separate layers, which simplified as orthotropic material are established. The progressive damage constitutive model considering the strain rate sensitivity of composites derived previously is incorporated in ABAQUS as a VUMAT. Additionally, zero thickness cohesive elements are introduced in adjacent layers to simulate the delamination.

Figure 2 shows the numerical model of woven composites when subjected to low velocity impact. The hemispherical head cylindrical impactor and the metal base are modelled as rigid bodies. Reduced integral elements C3D8R are applied for all parts in the numerical model. All degrees of freedoms of the rubber clamps and metal base are fixed.
Figure 2. Numerical model of composites under low velocity impact.

In order to determine the material variables required in the constitutive model, compressive and tensile experiments under different loading rates are conducted. The elastic modulus and nominal strength of carbon fiber woven composites along the in-plane direction are obtained. Further, the strain rate sensitivity coefficient of mechanical properties and damage variables are fitted and validated based on the experimental results. The shear strength and modulus are obtained by quasi-static test, while the elastic modulus along normal direction is cited from the Ref [15]. All the material parameters in the progressive damage constitutive model for woven composites are listed in Table 2.

Table 2. Model parameters of woven composites.

| Material       | $E_f$ (GPa) | $E_m$ (GPa) | $E_{33}$ (GPa) | $v_{12}$ | $v_{13}$, $v_{23}$ | $G_{12}$ (GPa) | $G_{13}$ (GPa) | $G_{23}$ (GPa) | $X_{0t}$, $Y_{0t}$ (MPa) |
|----------------|-------------|-------------|---------------|----------|-------------------|----------------|----------------|----------------|-------------------------|
| Woven composites | 230         | 3.5         | 8.7           | 0.050.1  | 4.45              | 3.2            | 3.2            | 602            |
| $X_{0t}$, $Y_{0t}$ (MPa) | 525         | 124         | 83            | 1.6      | 0.220.35          | 8.3, 8.8       | 0.013, 0.0180.055, 0.0470.68, 0.81 |

It should be noted that although the same expression of damage variable is adopted for both tensile and compressive condition, two different viscous damage variables are applied. By simulating tensile and compressive tests, the viscous damage variables are determined to be 0.05 and 0.06, respectively. Figure 3 compares the stress-strain curves obtained by experiments and numerical predictions. The numerical predictions agreed well with the experimental results with a maximum error less than 7%.

Figure 3. Comparisons of experimental and numerical results under: (a) tensile and (b) compressive loading.
3.2. Model Validation

The carbon fiber woven composites with a stacking sequence of \([\left[\pm 45^\circ\right]/\left[0^\circ/90^\circ\right]/\left[\pm 45^\circ\right]\_6\]_6\) were tested. Low velocity impacts were conducted at 40J with the velocity of 3.5m/s. To evaluate the effectiveness of introducing the strain rate dependency, both rate dependent (RD) and rate independent (RI) numerical model are performed to describe the impact response and damage characteristics.

Figure 4 shows the contact force-time curves obtained by experiments and simulations (RD model and RI model). The experimental result presents an obvious drop at the initial period, this may be caused by the contact vibration and localized damage at the contact area. Then, attribute to the combination of impact vibration and damage propagation in different modes, the contact force increases quickly and fluctuates violently [12]. After that, the rebound procedure began and the contact force smoothly reduced with time. When ignoring the strain rate effects, the contact force-time curve exhibits smaller initial slope, and the time at maximum contact force is 1.58ms, which increases 17% as compared to the experimental result. However, the initial slope, time at maximum contact force and the decreasing rate at rebounding state are found to be more consistent with the experimental result for the RD model. The maximum contact force achieved at 1.21ms, which has been advanced by 10.4%. Figure 5 shows the delamination area obtained by experiments and simulations. The delamination of woven composites exhibits elliptic shape when subjected to low velocity impact. With the strain rate considered, the maximum relative error of delamination dimension decreased from 11.3% to 5.6%.

Above all, the RD model can better predict the mechanical response of composites, including the contact force history curve at different stage and the delamination area. Further, with the impact energy or impact velocity increasing, the damage degree and strain rate effect will be more significant. Therefore, it is necessary to consider the strain rate sensitivity of both mechanical property and damage evolution in the progressive damage model for carbon fiber woven composites under low velocity impact.

![Figure 4](image_url)

**Figure 4.** Comparisons of contact force-time curves obtained from simulation and experiment

![Figure 5](image_url)

**Figure 5.** Comparisons of delamination of composites under low velocity impact.

4. Conclusions

A progressive damage model combining the strain rate dependent damage model and cohesive model has been established to depict the mechanical response and damage evolution of composites. The
strain rate sensitivity of modulus, nominal strength, and failure parameters of composites along in-plane directions are taken into account. The rate dependent parameters in the progressive damage model are determined and validated by the tensile/compressive experiments at different loading rates. The proposed progressive damage model is implemented by using a VUMAT subroutine, and various damage mechanisms, including warp/weft fiber breakage under tensile/compressive loading and interlaminar delamination are considered. With the strain rate dependence of both mechanical properties and damage variables taken into consideration, the numerical predictions show better agreement with the experimental observations. Therefore, the strain rate effect should be introduced in order to precisely predict the progressive damage of carbon fiber woven composites under low velocity impact loading.

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