New-Physics Search through $\gamma\gamma \rightarrow t\bar{t} \rightarrow \ell X/bX$ †

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ABSTRACT

We probe optimal beam polarizations for new-physics search in top-quark and Higgs-boson sectors at Photon Linear Colliders (PLC). Expressing possible non-standard effects generated by $SU(2) \times U(1)$ gauge-invariant dimension-6 effective operators as anomalous top and Higgs couplings, we estimate expected statistical sensitivities of these couplings in $\gamma\gamma \rightarrow t\bar{t} \rightarrow \ell X/bX$, using the optimal-observable method.

PACS: 14.65.Fy, 14.65.Ha, 14.70.Bh
Keywords: anomalous top-quark couplings, $\gamma\gamma$ colliders

†Presented by K. Ohkuma at the 7th International Symposium on Radiative Corrections (RADCOR2005), Shonan Village, Japan, October 2-7, 2005.
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1. INTRODUCTION

Recent neutrino experiments strongly indicate that the Standard Model (SM) has to be modified to explain $\nu$-oscillation phenomena [1]. However this does not necessarily require any new particles and we have no information yet on SM behavior in much higher-energy region where even new heavy particles could produce some effects directly or indirectly. Since top-quark/Higgs-boson interactions might receive corrections via such new-physics effects, precise studies of their reactions are expected to be a window on possible beyond-the-SM physics.

Current measurements of top quark at Fermilab Tevatron, which is the unique facility for top-quark production at present, are not sufficient yet to study if top-quark can be fully described in the framework of SM [2], but in the near future Large Hadron Collider (LHC) [3] and International Linear Collider (ILC) [4] will work as top-quark factories and realize its precise studies. Considering that the top-Higgs coupling is proportional to $m_t$, these colliders will also give us an opportunity to study the Higgs sector.

Focusing on ILC, we have been carrying out those studies intensively. Here we would like to show some of our results about drawing possible anomalous top and Higgs couplings from the processes $\gamma\gamma \to t\bar{t} \to \ell/bX$ at Photon Linear Collider (PLC), an interesting option of ILC, and discuss optimal beam polarizations that could lead us to the smallest statistical uncertainties. This report is organized as follows; We briefly review the framework of the analysis in section 2. Our numerical results are presented in section 3, and then a summary and some discussions are contained in section 4.

2. FRAMEWORK

In this section, we briefly explain the effective-Lagrangian approach [5] and optimal-observable method [6] used in our analysis. In addition, polarization parameters of PLC are reviewed.

2.1. Effective Lagrangian

We assumed that all non-standard particles are heavier than the lowest new-physics scale $\Lambda$ and decouple in lower-energy processes. In the effective-Lagrangian ap-
proach with this assumption, effects from such heavy particles can be parameter-
ized as coupling constants of effective operators which are composed of standard
fields alone.

Let us describe this scenario more concretely. The SM Lagrangian $\mathcal{L}_{\text{SM}}$ is
modified by the addition of a series of $SU(3) \times SU(2) \times U(1)$ gauge-invariant oper-
ators with coefficients suppressed by inverse powers of $\Lambda$. Among those operators
the largest contribution comes from dimension-6 operators, denoted as $\mathcal{O}_i$, since
dimension-5 operators violate lepton number \[5\] and are irrelevant for the processes
considered here. Consequently we have

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \sum_i (\alpha_i \mathcal{O}_i + \text{h.c.}) + O(\Lambda^{-3}) \quad (1)$$

as the basis of our analysis.

The operators relevant here lead to the following non-standard top-quark- and
Higgs-boson-couplings: (i) $\text{CP}$-conserving and $\text{CP}$-violating $t\bar{t}\gamma$ vertices, (ii) $\text{CP}$-
conserving and $\text{CP}$-violating $\gamma\gamma H$ vertices, and (iii) the anomalous $tbW$ vertex.
The corresponding coupling constants are denoted respectively as $\alpha_{\gamma 1}$, $\alpha_{\gamma 2}$, $\alpha_{h 1}$,
$\alpha_{h 2}$ and $\alpha_d$. The explicit expressions for these couplings in terms of the coefficients
of dimension-6 operators are to be found in Ref. \[7, 8\].

2.2. Optimal-observable method

The optimal-observable method \[6\] is a useful tool for estimating expected statisti-
tical uncertainties in various coupling measurements, which we summarize below.

Suppose we have a cross section

$$\frac{d\sigma}{d\phi}(\equiv \Sigma(\phi)) = \sum_i c_i f_i(\phi), \quad (2)$$

where $f_i(\phi)$ are known functions of the final-state phase-space variables $\phi$ and $c_i$’s
are model-dependent coefficients. The goal is to determine the $c_i$’s. This can be
done by using appropriate weighting functions $w_i(\phi)$ such that $\int w_i(\phi) \Sigma(\phi)d\phi = c_i$.
In general, different choices for $w_i(\phi)$ are possible, but there is a unique choice for
which the resultant statistical error is minimized. Such functions are given by

$$w_i(\phi) = \sum_j X_{ij} f_j(\phi)/\Sigma(\phi), \quad (3)$$
where $X_{ij}$ is the inverse matrix of $M_{ij}$ which is defined as

$$
M_{ij} \equiv \int \frac{f_i(\phi)f_j(\phi)}{\Sigma(\phi)} \, d\phi.
$$

When we use these weighting functions, the statistical uncertainty of $c_i$ becomes

$$
\Delta c_i = \sqrt{X_{ii} \sigma_T/N},
$$

where $\sigma_T \equiv \int (d\sigma/d\phi) \, d\phi$ and $N$ is the total number of events.

In order to adapt this method to our processes, we decompose cross sections into a sum of several functions which are proportional to each non-standard couplings\(^\#1\). In this way, the angular and energy distributions of the secondary fermions $\ell/b$ in the $e\bar{e}$ CM frame are expressed as

$$
\frac{d\sigma}{dE_{\ell/b} \, d\cos \theta_{\ell/b}} = f_{SM}(E_{\ell/b}, \cos \theta_{\ell/b}) + \sum_i \alpha_i f_i(E_{\ell/b}, \cos \theta_{\ell/b}),
$$

where $f_{SM}$ and $f_i$ are calculable functions: $f_{SM}$ denotes the SM contribution, $f_{\gamma_1,\gamma_2}$ describe the anomalous $CP$-conserving and $CP$-violating $t\bar{t}\gamma$-vertices contributions respectively, $f_{h1,h2}$ those generated by the anomalous $CP$-conserving and $CP$-violating $\gamma\gamma H$-vertices, and $f_d$ that by the anomalous $tbW$-vertex (see, e.g., Ref.[7] for detailed calculations).

### 2.3. Polarization parameters of PLC

At PLC, a colliding photon beam originates as a laser beam back-scattered off an electron ($e$) or positron ($\bar{e}$) beam. The polarizations of the initial state are characterized by the electron and positron longitudinal polarizations $P_e$ and $P_{\bar{e}}$, the maximum average linear polarizations $P_t$ and $P_{\bar{t}}$ of the laser photons with the azimuthal angles $\varphi$ and $\bar{\varphi}$ (defined in the same way as in [9][10]), and their average helicities $P_\gamma$ and $P_{\bar{\gamma}}$.

The photon polarizations $P_{t,\gamma}$ and $P_{\bar{t},\bar{\gamma}}$ satisfy

$$
0 \leq P_t^2 + P_\gamma^2 \leq 1, \quad 0 \leq P_{\bar{t}}^2 + P_{\bar{\gamma}}^2 \leq 1.
$$

For the linear polarization, we fixed the relative azimuthal angle $\chi \equiv \varphi - \bar{\varphi}$ to be $\pi/4$, because we found it the optimal value to study $\alpha_{\gamma 2}$ and $\alpha_{h 2}$ through checking $\chi$ dependence of $\sigma(\gamma\gamma \to t\bar{t})$.

\(^\#1\)We kept only linear terms in non-standard couplings.
3. NUMERICAL RESULTS

We searched for the polarizations that could make the statistical uncertainties $\Delta \alpha_i$ minimum for $\sqrt{s_{\text{ee}}} = 500$ GeV and $\Lambda = 1$ TeV, varying their parameters as $P_{e,e} = 0, \pm 1$, $P_{t,\bar{t}} = 0, 1/\sqrt{2}, 1$, and $P_{\gamma,\bar{\gamma}} = 0, \pm 1/\sqrt{2}, \pm 1$. We also changed the Higgs mass as $m_H = 100, 300$ and $500$ GeV, which lead to the width $\Gamma_H = 1.08 \times 10^{-2}, 8.38$ and $73.4$ GeV respectively according to the standard-model formula.

We did not probe the Higgs-resonance region, which had been extensively studied previously (see, for example, [11]). This means that the non-standard corrections are expected to be moderate and we may compute the number of secondary fermions, $N_{\ell/b}$, from the SM total cross section multiplied by the lepton/$b$-quark detection efficiency $\epsilon_{\ell/b}$ and the integrated $e\bar{e}$ luminosity $L_{e\bar{e}}$; this leads to $N_{\ell/b}$ independent of $m_H$.

Before showing the corresponding results, we have to explain one serious problem we encountered in Ref.[7]. We have noticed that the numerical results for $X_{ij}$ are often unstable when inverting the matrix $\mathcal{M}_{ij}$: even a tiny variation of $\mathcal{M}_{ij}$ changes $X_{ij}$ significantly. This indicates that some of $f_i$ have similar shapes and therefore their coefficients cannot be disentangled easily. The presence of such instability has forced us to refrain from determining all the couplings at once through this process alone. That is, we have assumed that some of $\alpha_i$’s had been measured in other processes (e.g., in $e\bar{e} \to t\bar{t} \to \ell\pm X$), and we performed an analysis with smaller number of independent parameters.

When estimating the statistical uncertainty in simultaneous measurements, e.g., of $\alpha_{\gamma_1}$ and $\alpha_{h_1}$ (assuming all other coefficients are known), we need only the components with indices 1, 2 and 4. In such a “reduced analysis”, we still encountered the instability problem, and we selected “stable solutions” according to the following criterion: Let us express the resultant uncertainties as $\Delta \alpha_{\gamma_1}^{[3]}$ and $\Delta \alpha_{h_1}^{[3]}$, where “3” shows that we use the input $\mathcal{M}_{ij}$, keeping three decimal places. In addition, we also compute $\Delta \alpha_{\gamma_1}^{[2]}$ and $\Delta \alpha_{h_1}^{[2]}$ by rounding $\mathcal{M}_{ij}$ off to two decimal places. Then, we accept the result as a stable solution if both of the deviations $|\Delta \alpha_{\gamma_1,h_1}^{[3]} - \Delta \alpha_{\gamma_1,h_1}^{[2]}|/\Delta \alpha_{\gamma_1,h_1}^{[3]}$ are less than 10 %.

This is what we faced in [7], and we found that we are not free from this problem
even for the wider range of polarization parameters given at the beginning of this section. That is, we did not find again any stable solution in the four- and five-parameter analysis. Fortunately, however, we did find some solutions not only in the two- but also in the three-parameter analysis [8]. In this report, we show the results of two parameter analysis as examples below. We did not fix the detection efficiencies $\epsilon_{\ell/b}$ since they depend on detector parameters and will get better with development of detection technology. On the other hand, the $N_{\ell/b}$ were estimated for $L_{ee} = 500$ fb$^{-1}$.

Final charged-lepton detection

Independent of $m_H$

- $P_e = P_{\bar{e}} = -1$, $P_t = P_{\bar{t}} = 1$, $P_\gamma = P_{\bar{\gamma}} = 0$, $N_\ell \simeq 1.0 \times 10^4 \epsilon_\ell$
  \[ \Delta \alpha_{h1} = 0.051/\sqrt{\epsilon_\ell}, \quad \Delta \alpha_d = 0.022/\sqrt{\epsilon_\ell}. \]  
  This result is free from $m_H$ dependence since the Higgs-exchange diagrams do not contribute to $\alpha_{h1}$ and $\alpha_d$ determination within our approximation.

$m_H = 100$ GeV

- $P_e = P_{\bar{e}} = -1$, $P_t = P_{\bar{t}} = 1$, $P_\gamma = P_{\bar{\gamma}} = 1/\sqrt{2}$, $N_\ell \simeq 1.9 \times 10^4 \epsilon_\ell$
  \[ \Delta \alpha_{h1} = 0.034/\sqrt{\epsilon_\ell}, \quad \Delta \alpha_d = 0.017/\sqrt{\epsilon_\ell}. \]  

$m_H = 300$ GeV

- $P_e = P_{\bar{e}} = -1$, $P_t = P_{\bar{t}} = 0$, $P_\gamma = P_{\bar{\gamma}} = 1$, $N_\ell \simeq 2.4 \times 10^4 \epsilon_\ell$
  \[ \Delta \alpha_{h1} = 0.013/\sqrt{\epsilon_\ell}, \quad \Delta \alpha_d = 0.015/\sqrt{\epsilon_\ell}. \]  

$m_H = 500$ GeV

- $P_e = P_{\bar{e}} = -1$, $P_t = P_{\bar{t}} = 0$, $P_\gamma = P_{\bar{\gamma}} = 1$, $N_\ell \simeq 2.4 \times 10^4 \epsilon_\ell$
  \[ \Delta \alpha_{h1} = 0.023/\sqrt{\epsilon_\ell}, \quad \Delta \alpha_d = 0.015/\sqrt{\epsilon_\ell}. \]  

- $P_e = P_{\bar{e}} = -1$, $P_t = P_{\bar{t}} = 0$, $P_\gamma = P_{\bar{\gamma}} = 1$, $N_\ell \simeq 2.4 \times 10^4 \epsilon_\ell$
  \[ \Delta \alpha_{h2} = 0.030/\sqrt{\epsilon_\ell}, \quad \Delta \alpha_d = 0.015/\sqrt{\epsilon_\ell}. \]
Final bottom-quark detection

\( m_H = 100 \, \text{GeV} \)

- \( P_e = P_{\bar{e}} = -1, \ P_t = P_{\bar{t}} = -P_\gamma = P_{\bar{\gamma}} = 1/\sqrt{2}, \ N_b \simeq 4.2 \times 10^4 \epsilon_b \)
  \[ \Delta \alpha_{h1} = 0.058/\sqrt{\epsilon_b}, \ \Delta \alpha_d = 0.026/\sqrt{\epsilon_b}. \] (13)

\( m_H = 300 \, \text{GeV} \)

- \( P_e = P_{\bar{e}} = -1, \ P_t = P_{\bar{t}} = -P_\gamma = P_{\bar{\gamma}} = 1/\sqrt{2}, \ N_b \simeq 4.2 \times 10^4 \epsilon_b \)
  \[ \Delta \alpha_{h1} = 0.009/\sqrt{\epsilon_b}, \ \Delta \alpha_{h2} = 0.074/\sqrt{\epsilon_b}. \] (14)

- \( P_e = P_{\bar{e}} = 1, \ P_t = P_{\bar{t}} = -P_\gamma = P_{\bar{\gamma}} = 1/\sqrt{2}, \ N_b \simeq 4.2 \times 10^4 \epsilon_b \)
  \[ \Delta \alpha_{h1} = 0.025/\sqrt{\epsilon_b}, \ \Delta \alpha_d = 0.019/\sqrt{\epsilon_b}. \] (15)

- \( P_e = P_{\bar{e}} = 1, \ P_t = P_{\bar{t}} = -P_\gamma = P_{\bar{\gamma}} = 1/\sqrt{2}, \ N_b \simeq 4.2 \times 10^4 \epsilon_b \)
  \[ \Delta \alpha_{h2} = 0.065/\sqrt{\epsilon_b}, \ \Delta \alpha_d = 0.010/\sqrt{\epsilon_b}. \] (16)

\( m_H = 500 \, \text{GeV} \)

- \( P_e = P_{\bar{e}} = -1, \ P_t = P_{\bar{t}} = 1, \ P_\gamma = P_{\bar{\gamma}} = 0, \ N_b \simeq 4.6 \times 10^4 \epsilon_b \)
  \[ \Delta \alpha_{h1} = 0.030/\sqrt{\epsilon_b}, \ \Delta \alpha_d = 0.018/\sqrt{\epsilon_b}. \] (17)

- \( P_e = P_{\bar{e}} = -1, \ P_t = P_{\bar{t}} = 1, \ P_\gamma = P_{\bar{\gamma}} = 0, \ N_b \simeq 4.6 \times 10^4 \epsilon_b \)
  \[ \Delta \alpha_{h2} = 0.028/\sqrt{\epsilon_b}, \ \Delta \alpha_d = 0.014/\sqrt{\epsilon_b}. \] (18)

Using these results one can find (for known \( m_H \)) the most suitable polarization for a determination of a given pair of coefficients.

Note that it is difficult to determine \( \alpha_{\gamma_1} \) and \( \alpha_{\gamma_2} \) together in this analysis. Although we have found some stable solutions that would allow for a determination of \( \alpha_{\gamma_1} \) in the lepton analysis, which we did not find in [7], the expected precision is rather low. Nevertheless this is telling us that the use of purely linear polarization for the laser is crucial for measuring \( \alpha_{\gamma_1} \). Unfortunately, the statistical uncertainty of \( \alpha_{\gamma_2} \) is still large even in this analysis, so we did not list it in the solutions which gave us good statistical precisions. Therefore, we have to look for other suitable processes to determine this parameter, for a review see [12].
On the other hand, it was found that there are many combinations of polarization parameters that make uncertainties of $\alpha_{h1,h2}$ and $\alpha_d$ relatively small. For instance, analyzing the $b$-quark final state with the polarization $P_e = P_{\bar{e}} = -1$, $P_t = P_{\bar{t}} = 1/\sqrt{2}$, $P_\gamma = -P_{\bar{\gamma}} = -1/\sqrt{2}$ enables us to probe the properties of Higgs bosons whose mass is around 300 GeV through the determination of $\alpha_{h1}$ and $\alpha_{h2}$.

As mentioned, the results were obtained for $\Lambda = 1$ TeV. If one assumes the new-physics scale to be $\Lambda = \lambda$ TeV, then all the above results ($\Delta \alpha_i$) are replaced with $\Delta \alpha_i/\lambda^2$, which means that the right-hand sides of eqs.(8)–(18) giving $\Delta \alpha_i$ are all multiplied by $\lambda^2$.

4. SUMMARY

Studying $\gamma\gamma \rightarrow t\bar{t} \rightarrow \ell X/bX$ in detail, we derived optimal beam polarizations that minimize uncertainties in the determination of $t\bar{t}\gamma$, $tbW$ and $\gamma\gamma H$ coupling parameters. To perform this analysis, we have applied the optimal-observable method to the final lepton/$b$-quark angular and energy distributions.

We showed a number of two-parameter solutions, most of which allow for the $\gamma\gamma H$- and $tbW$-couplings determination. The expected precision of the measurement of the Higgs coupling is of the order of $10^{-2}$ (for the scale of new physics $\Lambda = 1$ TeV). This shows that the $\gamma\gamma$ collider is going to be useful for testing the Higgs sector of the SM.

Let us consider the top-quark-coupling determination in an ideal case such that the beam polarizations could be easily tuned and that the energy is sufficient for the on-shell Higgs boson production. Then the best strategy would be to adjust polarizations to construct semi-monochromatic $\gamma\gamma$ beams such that $\sqrt{s_{\gamma\gamma}} \simeq m_H$ and on-shell Higgs bosons are produced. This would allow for precise $\alpha_{h1,h2}$ measurement, so the virtual Higgs effects in $\gamma\gamma \rightarrow t\bar{t}$ would be calculable. Unfortunately, as we have shown earlier, it is difficult to measure $\alpha_{\gamma 2}$ by looking just at $\ell X/bX$ final states from $\gamma\gamma \rightarrow t\bar{t}$. Therefore to fix $\alpha_{\gamma 2}$, one should, e.g., measure the asymmetries adopted in [13] to determine the top-quark electric dipole moment which is proportional to $\alpha_{\gamma 2}$. Then, following the analysis presented here, one can determine $\alpha_{\gamma 1}$ and $\alpha_d$. 
Finally, one must not forget that it is necessary to take into account carefully the Standard Model contribution with radiative corrections when trying to determine the anomalous couplings in a fully realistic analysis. In particular this is significant when we are interested in $CP$-conserving couplings since the SM contributions there are not suppressed unlike the $CP$-violating terms. On this subject, see for instance [14].

ACKNOWLEDGMENTS

This work is supported in part by the State Committee for Scientific Research (Poland) under grant 1 P03B 078 26 in the period 2004–2006, the Grant-in-Aid for Scientific Research No.13135219 and No.16540258 from the Japan Society for the Promotion of Science, and the Grant-in-Aid for Young Scientists No.17740157 from the Ministry of Education, Culture, Sports, Science and Technology of Japan and the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

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