Kinematics of the H$_2$O masers at the centre of the planetary nebula K3–35

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ABSTRACT
We have studied the kinematics traced by the water masers located at the centre of the planetary nebula (PN) K3–35, using data from previous Very Large Array (VLA) observations. An analysis of the spatial distribution and line-of-sight velocities of the maser spots allows us to identify typical patterns of a rotating and expanding ring in the position–velocity diagrams, according to our kinematical model. We find that the distribution of the masers is compatible with tracing a circular ring with a $\sim$0.021-arcsec ($\sim$100-au) radius, observed with an inclination angle of 55$^\circ$ with respect to the line of sight. We derive expansion and rotation velocities of 1.4 and 3.1 km s$^{-1}$, respectively. The orientation of the ring, projected on the plane of the sky, at position angle (PA) $\cong$ 158$^\circ$, is almost orthogonal to the direction of the innermost region of the jet observed in K3–35, suggesting the presence of a disc or torus that may be related to the collimation of the outflow.

Key words: masers – planetary nebulae: general – planetary nebulae: individual: K3–35.

1 INTRODUCTION

High angular resolution observations of molecular gas have revealed the presence of dense equatorial discs and tori towards several late asymptotic giant branch (AGB) stars and young planetary nebulae (PNe); see for instance, Bieging & Nguyen-Quang-Rieu (1988), Forveille et al. (1998), Sahai et al. (1998) and Bujarrabal et al. (2003). The interaction between the post-AGB wind and such equatorial structures has been proposed as one of the possible physical mechanisms in shaping the bipolar and multipolar morphologies seen in PNe and proto-PNe (PPNe) (Balick, Preston & Icke 1987; Mellema 1995; Soker & Rappaport 2000; Icke 2003).

The origin of molecular discs and tori around late AGB stars is not completely clear, but a possible explanation is the presence of a binary system (Morris 1987; Livio & Soker 1988; Taam & Bodenheimer 1989; Soker 2006). In this case, when one of the stars enters the AGB phase, some of the ejected material can be retained, generating an extended torus. However, in the case of the high-velocity jets observed in some late AGB stars, post-AGB stars and young PNe (Feibelman 1985; Sahai & Trauger 1998; Imai et al. 2002; Riera et al. 2003), it is believed that a more effective collimation mechanism(s) should be present in the innermost region of the object, such as an accretion disc (Morris 1987; Soker & Livio 1994; Soker & Rappaport 2000), or a stellar magnetic field produced by a rotating star (Pascoli 1985; Chevalier & Luo 1994; García-Segura 1997). Furthermore, recent observations reveal compelling evidence for magnetic fields, in post-AGB stars and PNe, associated with equatorial discs and/or jets (e.g. Vlemmings, Diamond & Imai 2006; Sabin, Zijlstra & Greaves 2007).

The process for the formation of accretion discs in PPNe has been investigated in detail by Reyes-Ruiz & López (1999). They find that a disc forms when a close binary system (with a substellar companion) undergoes common envelope evolution. For the case in which a low-mass secondary is disrupted during a dynamically unstable mass transfer process, an accretion disc, with a radius of $\sim$10 au and a mass of $\sim$2 $\times$ 10$^{-5}$ $\solar$ forms within $\sim$100 yr. Recently, Rijkhorst, Mellema & Icke (2005) from their three-dimensional (3D) simulations, based on a two-wind model with a warped disc, suggested that, to explain the observed multipolar and point-symmetric shape of PNe, the required discs are quite small ($\sim$10–100 au). Furthermore, these disc-like structures should be dense (10$^3$–10$^5$ cm$^{-3}$) and in Keplerian rotation.

Interferometric CO observations show larger toroidal molecular structures with sizes in the range of $\sim$1000–6000 au in PPNe and young PNe. These structures seem to be systematically in expansion with a mean velocity of $\sim$7 km s$^{-1}$, such as in M1–92 (Bujarrabal, Alcolea & Neri 1998), M2–9 (Zweigle et al. 1997), M2–56 (Castro-Carrizo et al. 2002), or KjPn8 (Forveille et al. 1998). These velocities are comparable to or below those found in

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expanding AGB envelopes (Huggins 2007). Importantly, rotation has been observed in the Red Rectangle, as well as slower expansion (∼0.8 km s⁻¹), superimposed on rotation, according to Bujarrabal et al. (2005). Note that the sizes of the tori are about one or two orders of magnitude more than the sizes of the disc-like structures proposed by Reyes-Ruíz & López (1999) and Rijkhorst et al. (2005).

Maps of water maser emission allow the identification of a disc-like structure with a radius of 0.12 arcsec in IRAS 17347–3139 (de Gregorio-Monsalvo et al. 2004), which corresponds to ∼100–750 au at the source distance (Gómez et al. 2005a). In this case, the gas kinematics suggests the presence of both rotation and expansion in the disc traced by the water masers. In this context, it is very important to study in detail the kinematics of much smaller disc-like structures that might be related to the collimation of the observed bipolar outflows in some PNe.

K3–35 is a young PN that shows a bipolar outflow in optical images (Miranda et al. 2000). At radio wavelengths, K3–35 exhibits a bright core and two bipolar lobes with a S-shape (Miranda et al. 2001). The distance to this object has been estimated to be ∼5 kpc (Zhang 1995), using statistical method. However, we note large uncertainty in this type of estimate, since the application of different statistical methods could give distances varying by factors of ∼3 (Phillips 2004). The characteristic S-shape morphology of the radio lobes can be successfully reproduced by a precessing jet, evolving in a dense circumstellar medium (Velázquez et al. 2007).

Water maser emission has been found in three regions: two regions located at the tips of the bipolar radio jet about ∼1 arcsec from the centre (regions N and S) and another region towards the core of the nebula within ∼0.02 arcsec (region C), suggesting the presence of a torus (Miranda et al. 2001). In addition, OH maser emission has been detected towards the centre of K3–35 (within ∼0.04 arcsec), showing circular polarization that suggests the presence of a magnetic field (Miranda et al. 2001; Gómez et al. 2005b).

We decided to study the spatiokinematical distribution of the water masers, reported by Miranda et al. (2001), towards the centre of K3–35 to identify possible expansion and/or rotation motions. The paper is organized as follows. In Section 2, we present a simple kinematical model of a ring, including both expansion and rotation, and we calculate the pattern delineated in the position–velocity diagrams. We also describe the least-squares fit procedure that we used. In Section 3, we present the current observational data of H₂O masers in the PN K3–35. We then apply our model to the H₂O masers located towards the centre of the PN K3–35, making a comparison between the results and the observations. Finally, in Section 4, we discuss the implications of our results.

2 ROTATING AND EXPANDING RING MODEL

2.1 Model

We assume a narrow, uniform, rotating and expanding ring of radius \( R \), arbitrarily oriented with respect to the line of sight. Its projection on the plane of the sky is an ellipse with semimajor and semiminor axes \( a \) and \( b \), respectively. We define the two frames of reference shown in Fig. 1. Both coincide with the plane of the sky; one of them has its origin at the centre of the ellipse and is oriented such that the \( \chi' \)-axis is along the major axis of the projected ellipse and the other has the axes parallel to the right ascension (RA) and declination (Dec.) axes. The semimajor and semiminor axes are related to the ring radius by \( a = R \) and \( b = R \cos i \), where \( i \) is the inclination angle between the line of sight and the normal to the ring plane, as shown in Fig. 2.

The equation of the ellipse is given by

\[
\frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1, \tag{1}
\]

and the transformation equations between the coordinate systems are

\[
x' = (x - x_0) \cos \theta + (y - y_0) \sin \theta, \tag{2}
\]

\[
y' = -(x - x_0) \sin \theta + (y - y_0) \cos \theta, \tag{3}
\]

where \((x_0, y_0)\) is the position of the centre of the ellipse and \( \theta \) is the angle between the \( x \)-axes of the two frames of reference and is defined as positive clockwise. The angle \( \theta \) is related to the position angle (PA) of the major axis of the ellipse by \( \text{PA} = 90^\circ - \theta \).

Let \( v_s, v_{\text{rot}}, \) and \( v_{\text{exp}} \) be the local standard of rest (LSR) systemic velocity, the rotation velocity and the expansion velocity of the ring, respectively. Then the observed LSR velocity of a point in the ring can be expressed as

\[
V_{\text{LSR}} = v_s + \frac{x'}{a} v_{\text{rot}} \sin i + \frac{y'}{a} v_{\text{exp}} \tan i. \tag{4}
\]

Hence the observed \( V_{\text{LSR}} \) will be a linear function of either \( x' \) or \( y' \), if only one type of motion (rotation or expansion, respectively) is present in the ring (Uscanga et al. 2005). Using equation (1), equation (4) can be written in terms of either the \( x' \)- or the \( y' \)-coordinate as

\[
\left[ V_{\text{LSR}} - v_s - (x'/a) v_{\text{rot}} \sin i \right]^2 \frac{x'^2}{(v_{\text{exp}} \sin i)^2} + \frac{x'^2}{a^2} = 1, \tag{5}
\]

\[
\left[ V_{\text{LSR}} - v_s - (y'/a) v_{\text{exp}} \tan i \right]^2 \frac{y'^2}{(v_{\text{rot}} \sin i)^2} + \frac{y'^2}{(a \cos i)^2} = 1. \tag{6}
\]

Therefore, equations (5) and (6) indicate that the observed \( V_{\text{LSR}} \) has a quadratic form (ellipse) expressed in terms of \( x' \) or \( y' \), when both motions are present.

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Figure 1. Reference systems. Both the \( x'y' \)- and the \( xy \)-coordinate systems are in the plane of the sky. The \( x'y' \)-system has its origin at the centre of the ellipse \((x_0, y_0)\) and \( \theta \) is the angle between the \( x \)-axis and the \( x' \)-axis. The \( x \)- and \( y \)-axes are parallel to the RA and Dec. axes.
In this model, we do not solve the radiative transfer through the ring, but we assume that the emission at a given \( V_{\text{LSR}} \) comes from the point of the ring, having this line-of-sight velocity component. We then use this information to construct position–velocity diagrams.

The orientation of the major axis of the ellipse in the position–velocity \((x'-V_{\text{LSR}})\) or \((y'-V_{\text{LSR}})\) diagrams changes depending on the value of the inclination angle and on whether the sense of rotation is clockwise or counter-clockwise, as seen from the observer’s point of view, as shown in Fig. 2. Accordingly, we are able to distinguish between a positive or a negative value of the inclination angle and the sense of the rotation by making a comparison between the position–velocity diagrams delineated by the maser emission and the position–velocity diagrams expected for a rotating and expanding ring. It is important to note that similar position–velocity diagrams can be obtained considering contraction instead of expanding the ring, but changing the sign of the inclination angle and the sense of rotation. This ambiguity can be solved by constraining the value of the inclination angle (positive or negative) with additional information (see Section 3.2).

2.2 Least-squares fit

We carried out a least-squares fit of an ellipse to the observed emission to estimate its spatial distribution on the sky. To do this, we considered the curve on the \(xy\)-plane given by equations (1)–(3) for a given set of values of the parameters \((x_0, y_0)\) (the centre of the ellipse on the plane of the sky), \(a\) and \(b\) (the semimajor and semiminor axes, respectively) and \(\theta\) (the angle between the \(x\)-axes of the two frames of reference). We then compute the minimum distances \(d\) between the position \((x_j, y_j)\) of each of the masers and the ellipse (these distances are measured along straight lines that pass through the maser and intersect the ellipse at right angles to the curve). With these distances, we define the chi-square as

\[
\chi^2 = \frac{1}{N - 5} \sum_{j=1}^{N} \left( \frac{d_j}{\sigma_j} \right)^2,
\]

where \(N\) is the number of data points (i.e. the factor \(N - 5\) is the number of degrees of freedom) and \(\sigma_j\) is the error associated with the position of the maser spots. We then find the set of parameters \((x_0, y_0, a, b, \theta)\) (see above), which give the minimum of chi-square.

As a first approximation, we began by setting the values of \(\sigma_j\) equal to the observational uncertainties \(\Delta_j\) for the measured positions of the masers and then finding the ellipse that gives the minimum value of chi-square (see equation 7). The actual structure of the maser-emitting region could be a ring of finite width for which a broad ellipse is a rough approximation of its projection on the plane of the sky. Therefore, we do not expect maser spots to exactly trace an ellipse. We can characterize the width of the ring, assuming that the fitted ellipse traces the mean projected angular distance of the maser emission to the central star, and the actual emission will be distributed around this ellipse, with a dispersion \(\Delta_e\). We treat \(\Delta_e\) as a source of error for the ellipse fit, additional to the measured error, so that \(\sigma_j^2 = \Delta_j^2 + \Delta_e^2\). We then try different values of the width parameter \((\Delta_e)\), until we obtain a fit with a minimum \(\chi^2(\Delta_e) = 1\). We consider \(2\Delta_e\) as the characteristic width of the maser ring.

The \((x_0, y_0, a, b, \theta)\) parameters obtained from the minimization of chi-square give the best elliptical fit to the observed positions of the masers. With these spatial parameters, we carried out a kinematical fit, using the LSR velocities of the maser components to define a \(\chi^v\) of the form

\[
\chi^v = \frac{1}{N - 3} \sum_{j=1}^{N} \frac{1}{\sigma_v^j} \left( V_{\text{LSR},j} - v - \frac{y'_j}{a} v_{\text{rot}} \sin i - \frac{y'_j}{a} v_{\text{exp}} \tan i \right)^2,
\]

where \(N - 3\) is the number of degrees of freedom and \((x'_j, y'_j)\) are given by equations (2) and (3). Here \(\sigma_v\) is the uncertainty in the observed LSR velocity that we adopt as the spectral resolution of the observations. The minimization of \(\chi^v\) yielded the best values for the systemic \((v_s)\), rotation \((v_{\text{rot}})\) and expansion \((v_{\text{exp}})\) velocities.
There are only two sets of Very Large Array (VLA) water maser observations towards the PN K3–35: the VLA 1999.7 epoch observations reported by Miranda et al. (2001) and the VLA 2002.3 epoch observations reported by de Gregorio-Monsalvo et al. (2004). In the latter observations, only a group of four maser spots was detected towards the central region of this source. No maser emission was detected at the tips of the bipolar lobes of the PN.

Note that in the de Gregorio-Monsalvo et al. (2004) paper, the position of the continuum peak, used to align the positions of the masers at the two epochs, was not used with enough precision, resulting in a spurious shift of the maser spots from one epoch to another (see their fig. 4). Using the position with the adequate precision, we find the positions of the masers at the two epochs to be consistent within the uncertainties, 0.01 arcsec (2σ).

### 3.2 Model application and results

In our analysis, we have used the water maser data from the VLA 1999.7 epoch observations towards the central region of K3–35, reported by Miranda et al. (2001). At this epoch, the number of maser spots detected was larger than during the other epoch, allowing us a better identification of possible expansion and/or rotation motions at the centre of K3–35. The velocity resolution of the VLA observations was 1.2 km s\(^{-1}\) and the accuracy in the relative positions of the water maser spots was of order of milliarcseconds. The positions of the observed water maser spots towards the core (region C) are listed in Table 1. We adopt the position of the 1.3-cm continuum-emission peak as the origin of the xy-coordinate system.

Based on a least-squares fit to the positions of the maser spots (see Section 2.2) and using the observational uncertainties given in Table 1, we found that the H\(_2\)O masers located towards the core of K3–35 can be fitted by a circular ring of radius \(R \approx 0.021\) arcsec (\(\approx 100\) au at the estimated distance of \(\sim 5\) kpc) with an angular width of \(2\Delta \alpha = 0.003\) arcsec, observed at an inclination angle of \(\pm i \approx 55^\circ\) (see Table 2 and Fig. 3).

Spectroscopic observations show that the north-eastern lobe of the outflow is blueshifted and the south-western one is redshifted (Miranda et al. 2000). If we assume that the ring traced by the water masers is perpendicular to the bipolar lobes, then the inclination angle should be positive (\(i \approx +55^\circ\)). This means that the western half of the ring is closer to the observer.

### 3 H\(_2\)O MASERS IN K3–35 (REGION C)

### 3.1 Observational data

The calculated position of the centre of the ellipse, relative to the position of the 1.3-cm continuum-emission peak \((x_0, y_0)\) is given in Table 2. Both positions are in agreement within the uncertainties (see the overlap of these positions in Fig. 3).

The kinematical trend is shown in Fig. 4. Since we have determined that the inclination angle of the ring is positive, the ambiguity between expansion and contraction can be solved when we compare the position–velocity diagrams delineated by the water masers (see the top panel of Fig. 4) and the position–velocity diagrams of the model (see Fig. 2). We have found that the ring traced by the masers rotates clockwise, as seen from the observer at a velocity \(v_{\text{rot}} \approx 3.1\) km s\(^{-1}\) and expands at a velocity \(v_{\text{exp}} \approx 1.4\) km s\(^{-1}\) (see Table 2). The rotation and expansion velocities are estimated from a purely kinematical fit to the LSR velocities of the maser spots (see equation 8). The kinematical fit yields a \(X^2\) value of 1.94, which means that the fit is good, assuming a conservative confidence level of 90 per cent. Although all maser spots may not be completely independent, given the limited angular and spectral resolution, the observed velocity trend and the good kinematical fit suggest that these motions are real and systematic. We note that we have considered a single value of \(v_{\text{rot}}\) instead of a Keplerian rotation law. This is reasonable, since the velocity gradient over the width of the ring would be only \(\sim 7\) per cent. Tracing subtler velocity variations would require more data points than the ones available.

The expansion velocity we found for the ring is close to that of thermal motions or subsonic turbulence. However, we do not expect that thermal or turbulent motions could produce the

| Parameter Value |
|------------------|
| \(a\) | \(0.021 \pm 0.003\) arcsec |
| \(b\) | \(0.012 \pm 0.002\) arcsec |
| \(x_0\) | \(0.001 \pm 0.001\) arcsec |
| \(y_0\) | \(0.004 \pm 0.004\) arcsec |
| PA | \(158^\circ \pm 10^\circ\) |
| \(i\) | \(55^\circ \pm 7^\circ\) |
| \(v_r\) | \(22.8 \pm 0.5\) km s\(^{-1}\) |
| \(v_{\text{exp}}\) | \(1.4 \pm 0.9\) km s\(^{-1}\) |
| \(v_{\text{rot}}\) | \(3.1 \pm 0.8\) km s\(^{-1}\) |

Note. Uncertainties are 2σ.

\(a\)Coordinates of the centre of the ellipse relative to the position of the 1.3-cm continuum-emission peak.

### Table 1. H\(_2\)O masers in K3–35 (region C).

| \(V_{\text{LSR}}\) (km s\(^{-1}\)) | Flux density (mJy) | \(\alpha(J2000)\) | \(\delta(J2000)\) | Position uncertainty (arcsec) |
|--------------------------|------------------|----------------|----------------|-----------------------------|
| 24.6 | 23 | 19 27 44.0243 | 21 30 03.438 | 0.010 |
| 24.0 | 61 | 19 27 44.0242 | 21 30 03.428 | 0.004 |
| 23.3 | 218 | 19 27 44.0246 | 21 30 03.4460 | 0.0012 |
| 22.6 | 1010 | 19 27 44.02254 | 21 30 03.45 146 | 0.00024 |
| 22.0 | 1572 | 19 27 44.022364 | 21 30 03.45 335 | 0.00014 |
| 21.3 | 945 | 19 27 44.0247 | 21 30 03.4564 | 0.0003 |
| 20.7 | 201 | 19 27 44.0257 | 21 30 03.4625 | 0.0011 |

\(a\)Units of RA are hours, minutes and seconds, and of Dec. are degrees, arcminutes and arcseconds. Data from Miranda et al. (2001) and Gómez et al. (2003).

\(b\)Relative position uncertainties (2σ) between maser spots. The position of the 1.3-cm continuum-emission peak is \(\alpha(J2000) = 19^\text{h} 27^\text{m} 44^\text{s} 023^\text{s}, \delta(J2000) = 21^\circ 30' 03'' 441\). The relative position uncertainty between the continuum and the H\(_2\)O masers is 0.002 arcsec. The accuracy of the absolute positions is 0.05 arcsec.
1131
∼0.8 km s\(^{-1}\)/\(\Delta_1\) 390, 5–8 km s\(^{-1}\) (see Section 2.2). The positional error bars
∼2008 The Authors. Journal compilation
1127–1132 \(\simeq\) 65 \(x\chi/\Delta_1\) \(\simeq\) \(\simeq\) Position–velocity diagrams. Top panel: the ordinate axis corre-
0.6 km s\(^{-1}\) 158 10
2008 RAS, MNRAS \(\simeq\) 1.4 km s\(^{-1}\) 100 au
Positions of the K3–35 water maser spots in offsets relative to the
innermost region of a disc or torus, probably formed at the
end of the AGB phase. Since masers trace regions with very
stringent physical conditions, it is not possible to know whether
the ring is tracing part of a toroidal or a disc-like structure. The
kinematics of the ring in K3–35 suggests the presence of both rotating
and expanding motions, as was also proposed in the young PN
IRAS 17347–3139 (de Gregorio-Monsalvo et al. 2004). The estimated expansion and rotation-velocity values for K3–35 are similar
(a few km s\(^{-1}\)) to those obtained from water maser observations of
IRAS 17347–3139.

The calculated expansion velocity of the ring (\(\simeq\)1.4 km s\(^{-1}\)) in
K3–35 is comparable to, but below, the expansion velocities of the
tori inferred from interferometric CO observations in some PPNe
and young PNe. For instance, the expansion velocities in M1–92,
M2–9, M2–56 and KJPn 8 have values in the range of \(\simeq\)5–8 km s\(^{-1}\)

systematic motions described in the position–velocity diagrams.
The water masers seem to be tracing a spatiokinematical structure,
with organized motions (at macroscopic scales).

4 DISCUSSION

From our model, we conclude that a ring is a likely explanation for the distribution and kinematics shown by the water masers located towards the centre of the PN K3–35. This ring may be arising from the innermost region of a disc or torus, probably formed at the end of the AGB phase. Since masers trace regions with very stringent physical conditions, it is not possible to know whether the ring is tracing part of a toroidal or a disc-like structure. The kinematics of the ring in K3–35 suggests the presence of both rotating and expanding motions, as was also proposed in the young PN IRAS 17347–3139 (de Gregorio-Monsalvo et al. 2004). The estimated expansion and rotation-velocity values for K3–35 are similar (a few km s\(^{-1}\)) to those obtained from water maser observations of IRAS 17347–3139.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Figure 3. Positions of the K3–35 water maser spots in offsets relative to the position of the 1.3-cm continuum-emission peak (Miranda et al. 2001). Each spot is labelled with its corresponding LSR velocity (in km s\(^{-1}\)). The dashed ellipse corresponds to the least-squares fit to the maser spots’ positions, whose parameters are indicated in Table 2. The open circle indicates the nominal position of the 1.3-cm continuum-emission peak (see Table 1), its size is equal to the uncertainty of this position. The diamond indicates the position of the centre of the ellipse that was obtained from the fit. The straight line shows the direction of the bipolar outflow traced by the innermost region of the jet. The arrows show the sense of rotation of the proposed ring (see Section 2.1). The broad grey ellipse has a width 2\(\Delta_1\), with \(\Delta_1\) fulfilling \(\chi^2(\Delta_1) = 1\) (see Section 2.2). The positional error bars indicate the uncertainties in the relative positions between maser spots given in Table 1.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Figure 4. Position–velocity diagrams. Top panel: the ordinate axis corresponds to the observed LSR velocity and the abscissa axis corresponds to the coordinate \(x’\) or \(y’\). Bottom panel: same as top panel, but the abscissa axis corresponds to RA offset or Dec. offset relative to the position of the 1.3-cm continuum-emission peak. The points correspond to the observed maser spots towards the centre of K3–35 and the dashed ellipses correspond to the kinematical model using the parameters listed in Table 2. The error bars in position are those shown in Fig. 3. The uncertainty in velocity is \(\simeq\)0.6 km s\(^{-1}\).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{Figure 5. Radial velocity versus projected position of the water masers in K3–35. The black circles are the 1.3-cm continuum-emission peaks (e.g. M2–9 and M2–56) and the red squares are the positions of the water masers. The black line represents the velocity of the proposed ring. The ticks indicate the uncertainties in the relative positions between maser spots given in Table 1.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{Figure 6. Position–velocity diagrams. Top panel: the ordinate axis corresponds to the observed LSR velocity and the abscissa axis corresponds to the coordinate \(x’\) or \(y’\). Bottom panel: same as top panel, but the abscissa axis corresponds to RA offset or Dec. offset relative to the position of the 1.3-cm continuum-emission peak. The points correspond to the observed maser spots towards the centre of K3–35 and the dashed ellipses correspond to the kinematical model using the parameters listed in Table 2. The error bars in position are those shown in Fig. 3. The uncertainty in velocity is \(\simeq\)0.6 km s\(^{-1}\).}
\end{figure}
time, similar to the value found by Huggins (2007). However, the dynamical age we derive is not reliable enough as an age estimate. Proper motion studies of the water maser emission (e.g. using e-Multi-Element Radio Linked Interferometer, MERLIN, and Very Long Baseline Array, VLBA) could provide better estimates.

The kinematics of the ring in K3–35 suggests the presence of both expansion and rotation. A rough estimate for the central stellar mass can be obtained by assuming that the total energy (kinetic plus gravitational) of the masing gas is close to zero. In this case, $M \approx \frac{(R/2G)(v^2_{\text{exp}} + v^2_{\text{rot}})}{\frac{D}{5 \text{kpc}}}$, where $R \approx 100(D/5 \text{kpc})$ au is the radius of the ring. Hence, $M \approx [0.7(D/5 \text{kpc})] \pm 0.3 \, M_\odot$, where $D$ is the source distance. The error in the mass includes only the errors in the fitted parameters. This estimate of the central mass is in agreement with the core mass required for a PN, according to evolutionary models (Kwok 2003).

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