The $\eta'$ Photoproduction Off the Nucleon In The Quark Model

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Abstract

The reaction $\gamma + p \rightarrow \eta' + p$ is investigated in the quark model approach. The quark model predicts that the off-shell contributions from the resonance $S_{11}(1535)$ dominate the $\eta'$ production in the threshold region. We find that the coupling constant $\alpha_{\eta' NN}$ from the fit to a few total cross section data is small. The results of the total and differential cross sections are presented at different energies, which can be tested in the future experiments. If the future total cross data indeed show a resonance structure around 2 GeV suggested in a recent study, it would be an evidence of a non $qqq$ structure in its wavefunction.

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There had been a vacuum in investigating the $\eta'$ photoproduction

$$\gamma + p \rightarrow \eta' + p$$  \hspace{1cm} (1)

since the early experiments by Aachen-Berlin-Bonn-Hamburg-Heidelberg -München (ABBHHM) collaboration\textsuperscript{[1]} in the sixties and Aachen-Hamburg-Heidelberg -München (AHHM) collaboration\textsuperscript{[2]} in the seventies, and no theoretical attempt was made in studying this reaction. The construction of the continuous electron beam accellerator facility (CEBAF) has revived the interests in this reaction. The new experiment proposed\textsuperscript{[3]} at CEBAF will generate more systematic data with much better energy resolution and statistics.

There is little knowledge on the reaction mechanism of $\eta'$ photoproductions, except several studies\textsuperscript{[4]} suggesting that the $\eta'NN$ coupling constant should be small. Because the threshold energy of this reaction is above the second resonance region, data from this reaction may provide us an important probe into the structure of the resonances around 1.9 $\sim$ 2.0 GeV. This was emphasized in the recent investigation\textsuperscript{[5]} by the RPI group, in which the effective Lagrangian approach was used. However, there is a large truncation of the model space in the effective Lagrangian approach, in which the off-shell contributions from the resonances in 1.5 $\sim$ 1.7 GeV are excluded. It remains to be seen whether the resonances around 2.0 GeV, in particular the resonance $N^*(2080)$, would be as dominant as suggested in Ref.\textsuperscript{[5]}.

In this note, we investigate the $\eta'$ photoproduction in the quark model. This is a natural extension of the framework developed recently\textsuperscript{[6, 7]} for meson photoproductions of the nucleon. It begins with the low energy QCD Lagrangian, in which the $\pi$, $\eta$ and $K$ are regarded as Goldstone bosons so that their interaction with the quarks inside hadrons are invariant under the chiral transformation. Our studies in Kaon and $\eta$ photoproductions have shown that the quark model gives a very good description of the meson photoproductions with far less free parameters. Although the $\eta'$ could not be regarded as a Goldstone boson, it has the same quantum numbers as those of the $\eta$. Thus, the reaction mechanism for both photoproductions should be very similar, and the transition amplitudes for both photoproductions should have the same expressions for a pointlike $\eta'$. Of course, it may be unrealistic to treat the $\eta'$ as a pointlike particle. Nonetheless, we expect that the qualitative features from this investigation will survive the finite size effects.

We shall discuss briefly the quark model approach to the $\eta'$ photoproductions of the nucleon, as the detailed formalism in the quark model has been given in Ref.\textsuperscript{[7]}. Because the $\eta'$ is a charge neutral particle, there are only
two components in the quark model for the $\eta'$ photoproduction; the s- and u-
channel contributions from the baryon resonances. The transition amplitude is
\[
|\mathcal{M}_{if}| = |\mathcal{M}_s| + |\mathcal{M}_u|.
\]
(2)
The t-channel contributions, such as the $\rho$ and $\omega$ exchanges that played an
important role in the effective Lagrangian approach, are excluded here with
the input of the duality hypothesis[8, 9]. Whether this hypothesis is valid
remains to be investigated for the $\eta'$ photoproduction, although the studies
suggest that it may be true for the Kaon photoproduction[3].

The u-channel $\mathcal{M}_u$ in Eq. 2 includes the contributions from the nucleon
and the excited baryons with isospin 1/2, of which the formulae have been
given in Ref. [7]. The excited baryons are treated as degenerate so that their
total contributions can be written in a compact form in the quark model. This
is a good approximation since the contributions from the u-channels resonances
are not sensitive to their precise mass positions. The transition amplitude $\mathcal{M}_s$
in Eq. 1 is[7]
\[
|\mathcal{M}_s| = \sum_R \frac{2M_R}{s - M_R(M_R - i\Gamma_R(q))} e^{-\frac{k^2 + q^2}{a^2}} O_R,
\]
(3)
where the resonance $R$ has the mass $M_R$ and total width $\Gamma_R$, $k$ and $q$ are the
momenta of incoming photons and outgoing mesons, and $\sqrt{s}$ is the total energy
of the system. The operator $O_R$ in Eq. 3 depends on the structure of resonances. It is devided into two groups; the s-channel resonances below 2 GeV
and those above 2 GeV that could be regarded as continuum contributions.
The electromagnetic transitions of s-channel baryon resonances and their meson
decays have been investigated extensively in the quark model[10, 11, 12, 13]
in terms of the helicity and the meson decay amplitudes. These transition am-
plitudes for s-channel resonances below 2 GeV have been translated into the
standard CGLN[14] amplitudes in Refs. [6, 7] for the proton target and [15]
for the neutron target in the harmonic oscillator basis. The advantage of
the standard CGLN variables is that the kinematics of the meson photoproduc-
tions has been thoroughly investigated[16], the various observables of the
meson photoproductions could be easily evaluated in terms of these amplitudes.
Those resonances above 2 GeV are treated as degenerate, since few
experimental information is available on those resonances. Qualitatively, we
find that the resonances with higher partial waves have the largest contributions
as the energy increases. Thus, we write the total contributions from the
resonances belonging to the same harmonic oscillator shell in a compact form,
and the mass and total width of the high spin states, such as $G_{17}(2190)$ for
\[ n = 3 \text{ harmonic oscillator shell, are used. The contributions from the resonances above 2 GeV are particularly important for the } \eta' \text{ photoproduction, since the threshold energy is higher than those in the Kaon and } \eta \text{ photoproductions. We have included the resonances belonging } n = 4 \text{ and } n = 5 \text{ with the resonance mass up to 2.6 GeV. Thus, our calculation becomes less reliable for } E_{\text{lab}} \geq 3.0 \text{ GeV, in which the continuum contributions in our calculation are no longer adequate.} \]

We assume that the relative strength and phases of each term in s- and u-channels are determined by the quark model wavefunction with exact \( SU(6) \otimes O(3) \) limit. The masses and decay widths of the s-channel baryon resonances are obtained from the recent particle data group[17]. Thus, there are three parameters in this calculation; the coupling constant \( \alpha_{\eta'NN} \), the constituent quark masses \( m_q \) for up or down quarks, and the parameter \( \alpha^2 \) from the harmonic oscillator wavefunctions in the quark model. The quark masses \( m_q \) and the parameter \( \alpha^2 \) are well determined in the quark model, they are
\[ m_u = m_d = 0.34 \text{ GeV} \]
\[ \alpha^2 = 0.16 \text{ GeV}^2. \] (4)

The finite size effects of the \( \eta' \) could be partially taken into account by adjusting the parameter \( \alpha^2 \), which we leave it to the future investigation. This leaves only one free parameter, the coupling constant \( \alpha_{\eta'NN} \), to be determined in our calculation.

In Fig. 1, we show the result of our calculation for the total cross sections of the \( \eta' \) photoproduction off the proton target. The coupling constant \( \alpha_{\eta'NN} \) is 0.35 from the fit to a few total cross section data, which is indeed consistent with the recent studies[4] suggesting it to be small. Of course, there is a large uncertainty due to the poor quality of the data. Our result does not exhibit the dominance of any particular resonance around 2 GeV region. This can be understood by the relative strength of the CGLN amplitudes \( O_R \) in Eq. 3 between the resonance \( S_{11}(1535) \) and the resonances around 2.0 GeV in the quark model. The operator \( O_R \) for the resonance \( S_{11}(1535) \) is
\[ O_{S_{11}(1535)} = \frac{k}{3} \left( \frac{\omega_{\eta'}}{m_q} + \frac{A \cdot q}{3\alpha^2} \right) \left( 1 + \frac{k}{2m_q} \right) \sigma \cdot \epsilon \] (5)

where
\[ A = -\left( \frac{\omega_{\eta'}}{E_f + M_N} + 1 \right) q, \] (6)
and \( \sigma \) and \( \epsilon \) are the total spin operator and the polarization vector of the incoming photons. According to the quark model classification, the \( S \) or \( D \) wave resonances around 2 GeV belong to \( n = 3 \) in the harmonic oscillator basis. The operator \( \mathcal{O}_R \) for the \( n = 3 \) resonances is

\[
\mathcal{O}_{n=3} = -\frac{1}{12m_q^3} i \sigma \cdot A \sigma \cdot (\epsilon \times k) \left( \frac{k \cdot q}{3\alpha^2} \right)^3 \\
+ \frac{1}{6} \left[ \frac{\omega q^2 k}{m_q} \left( 1 + \frac{k}{2m_q} \right) \sigma \cdot \epsilon + \frac{k}{\alpha^2} \sigma \cdot A \epsilon \cdot q \right] \left( \frac{k \cdot q}{3\alpha^2} \right)^2 \\
+ \frac{\omega q^2 k}{9\alpha^2 m_q} \sigma \cdot k \epsilon \cdot q \left( \frac{k \cdot q}{3\alpha^2} \right). \tag{7}
\]

Because the spatial wavefunctions for the \( S \) and \( D \) wave resonances are orthogonal to that of the \( S_{11}(1535) \), the dependence on \( k \) and \( q \) of the CLGN amplitudes for the \( S \) and \( D \) wave resonances around 2 GeV should be very different from those for \( S_{11}(1535) \) and \( D_{13}(1520) \). Indeed, Eq. 7 shows that the amplitude for the \( S \) and \( D \) wave resonances with \( n = 3 \) is at least proportional to \( q^2 \) comparing to the \( q \) dependence of the amplitude of the \( S_{11}(1535) \) in Eq. 5. Thus, the \( S \) and \( D \) wave resonances around 2 GeV give little contribution to the \( \eta' \) productions in the threshold region. Moreover, Eq. 7 represents the sum of every resonance with \( n = 3 \), and the \( G \) wave resonance has the largest amplitude among these resonances. The magnitude of the CGLN amplitudes for \( S \) and \( D \) wave resonances is even smaller that that for the \( S_{11}(1535) \), since the magnitude of Eq. 7 is about 10 times smaller than that of Eq. 5. Thus, the dominance from a particular resonance around 2 GeV in the threshold region, similar to the dominance of the resonance \( S_{11}(1535) \) in the threshold region of the \( \eta \) photoproduction, is not expected, unless a resonance has a non \( qqq \) structure in its wavefunction. This suggests that the s-wave resonances in 1.5 \~{} 1.7 GeV region, in particular \( S_{11}(1535) \), play quite important role in the threshold region of the \( \eta' \) photoproduction. To show the importance of the off-shell contributions from the resonance \( S_{11}(1535) \), we show the result of our calculation without the contribution from the resonance \( S_{11}(1535) \); the resulting total cross section is only about 20\% of that with the contribution from the \( S_{11}(1535) \). The differential cross sections in Fig. 2 also show the S-wave dominance in the threshold region, although it is slightly forward peaked. Thus, the off-shell contributions from the \( S_{11}(1535) \) is dominant in the threshold region of the \( \eta' \) production. This result represents an important prediction of the quark model, because the relative strength and phases of the CGLN amplitudes for each resonance are determined by the quark model wavefunctions.
Fig. 1 also shows a small bump around $E_{lab} = 2.1$ GeV region, and it comes from the resonances $G_{17}(2190)$ and $H_{19}(2220)$. This suggests that if there is indeed a dominance from the resonance $N^*(2080)$ with the magnitude shown in Ref. [4], its magnitude of the CGLN amplitude should be much larger than that expected in the quark model, and it is only possible for a non $qqq$ state. Thus, the search of the missing resonances requires more elaborate studies of the polarization data proposed in Ref. [18], as the quark model predicts that $S$ and $D$ wave resonances around 2 GeV do not make significant contributions to total and differential cross sections of the $\eta'$ photoproduction.

It should be also pointed out that the resonance structure around $W = 2$ GeV is far from being established in the $\eta'$ photoproduction data. There are total four points of data in $E_{lab} \leq 2.4$ GeV from ABBHHM collaboration (triangle) and from AHHM collaboration (square). On the surface, the data seem to show a resonance structure with very large uncertainties. However, if one separates the two sets of the data by ABBHHM and AHHM collaborations, the resonance structure disappears. Consequently, the total cross section decreases monotonously as the energy increases, consistent with the results of the quark model calculations. Moreover, the dominance of $D_{13}(2080)$ shown in Ref. [5] requires further support from the differential cross section data, which do not exist at present.

Thus, our investigation here provides an interesting challenge to the future experiments. A strong peak centered around 1.9 GeV shown in Ref. [5] in the total cross section data suggests the existence of a resonance that should not be a normal $qqq$ state. On the other hand, the contribution from the resonance $S_{11}(1535)$ is determined by the off-shell behaviour of its CGLN amplitudes. Whether the CGLN amplitudes from the quark model give a good description of the off-shell behaviour of baryon resonances remains to be seen, which are usually evaluated on-shell. The differential and total cross section data in the threshold region would provide an important test to the off-shell behaviour of the CGLN amplitudes from the quark model.

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Figure Caption

1. The total cross sections for $\gamma + p \rightarrow \eta' + p$. The difference between the solid and dashed lines represents the importance of the contribution from the resonance $S_{11}(1535)$. The experimental data are from Refs. [1] (triangle) and [2] (square). See text.

2. The differential cross sections at $E_{lab} = 1.6 \text{ GeV}$ (solid), $1.8 \text{ GeV}$ (dashed) and $2.0 \text{ GeV}$ (dot-dashed).