On-shell Supersymmetry Anomalies and the Spontaneous Breaking of Gauge Symmetry

J. A. Dixon

Center for Theoretical Physics
Physics Department
Texas A & M University
College Station, Texas 77843

September, 1993

ABSTRACT

A search for supersymmetry anomalies requires an examination of the BRS cohomology of supersymmetric Yang-Mills coupled to chiral matter, and the physically interesting (on-shell) anomalies are those which cannot be eliminated using the equations of motion. An analysis of this cohomology problem shows that the simplest situation where a physically interesting supersymmetry anomaly can arise is when:

1. the anomaly occurs in the renormalization of a composite antichiral spinor superfield operator constructed from the chiral matter in the theory,

2. the anomalous diagrams are one-loop triangle diagrams containing a gauge propagator,

3. the gauge symmetry (but not supersymmetry) is spontaneously broken,

4. the initial operator has a dimension such that the triangle diagram is at least linearly divergent.

1Supported in part by the U.S. Dept of Energy, under grant DE-FG05-91ER40633, Email: dixon@phys.tamu.edu
5. the anomaly contains only massless chiral superfields, although (appar-ently harder to calculate) supersymmetry anomalies can also contain massive chiral superfields,

6. the theory contains vertices which, after gauge symmetry breaking, couple massless matter fields to massive matter fields and massive gauge fields, like the $e^{-\nu_e}W^+$ vertices of the standard model.

The supersymmetry anomalies considered here are all ‘soft’, in the sense that they must vanish when certain masses go to zero. It appears that the order parameter of the resulting supersymmetry breaking may be the vacuum expectation value that breaks the gauge symmetry. Nonzero values for the anomalies, if they exist, appear to generate supersymmetry breaking for observable particles with a cosmological constant that is naturally zero.

A specific example of a possibly anomalous operator which arises in this way is then examined. An analysis is made of the most efficient way to try to calculate supersymmetry anomalies for a simple case. It remains to do some careful analyses and Feynman graph calculations of the relevant coefficients for some specific examples.
1 Introduction and Discussion

Supersymmetry breaking is considered by many physicists to be one of the chief problems in modern elementary particle theory. It is amusing and ironical that this is so, since there is so little phenomenological evidence for supersymmetry. The reason for this strange situation is of course that supersymmetry is theoretically very attractive. Some of the problems of dynamical supersymmetry breaking and some references to the recent literature can be
found in [1]. See also [2] for a recent discussion of the ‘soft terms’ approach to supersymmetry breaking.

Recently it was suggested [3] that there might be a mechanism whereby supersymmetry breaks itself through supersymmetry anomalies. This would be a pretty phenomenon if it works, because it would be calculable in perturbation theory, inevitable rather than contrived, and it also leaves the cosmological constant at a zero value in a natural way after supersymmetry breaking, assuming it was zero before supersymmetry breaking, which is often the case in superstring theory. This mechanism for supersymmetry breaking affects only composite operators which can act as interpolating fields for the bound states (like the hadrons) or the broken phase states (like the electron), because this is where the supersymmetry anomalies can occur, according to the BRS cohomology results.

These superanomalies can occur only in certain composite operators in supersymmetric theories. This explains why they have escaped detection up to now, even if they actually exist with non-zero coefficients. It is a real chore to find them. This chore requires a lot of knowledge of the BRS cohomology space before one even begins a Feynman graph calculation. Then the calculation is very model dependent and also depends on the operator chosen. Also, since the effect is mass dependent, the number of terms and the integrals involved appear to make the problem fairly labour intensive. For the example given below in section [19], the individual diagrams that
need calculation are certainly not zero, but they might nevertheless give a zero anomaly after regularization, addition and variation by $\delta$. I do not at present know a reliable method to try to calculate supersymmetry anomalies or alternatively show that they are absent. It does appear to be impossible to find a regularization procedure that respects both gauge invariance and supersymmetry, so that it seems possible that the anomalies discussed here do appear with non-zero coefficients. It is tantalizing that the superanomalies do seem to require both gauge invariance and supersymmetry for purely cohomological reasons as explained below. However, it may be that the coefficients are always zero. How could the regularization problem give rise to mass dependent anomalies? Is there some sort of choice between gauge anomalies and supersymmetry anomalies going on here? These seem to be hard questions. The first step at any rate is to do the present cohomology analysis in order to even know which diagrams to calculate. The problem of calculating them must come later.

So the purpose of this paper is to analyze the BRS cohomology to the point necessary to have a reasonable chance of finding a superanomaly. Some of the discussion of this section and the bare bones of the example to follow formed part of a recent talk [4]. The present paper contains more complete results.

To motivate the problem analyzed in this paper, I shall try to explain some of the fairly obvious reasons that such anomalies are not present in
some simple cases. Then we will examine a case which appears to require a
detailed calculation to determine whether a supersymmetry anomaly is or is
not present.

It is easy to write down the simplest examples where a supersymmetry
anomaly could conceivably arise. The BRS cohomology of these theories
indicates that there could be anomalies in the renormalization of composit-
ete operators (made from the elementary chiral superfields $S$ of the theory)
which are antichiral spinor superfields. These composite operators satisfy
the antichiral constraint:

$$D_\alpha \Psi_\beta = 0$$

and take forms such as:

$$\Psi_{1\alpha} = D^2[S_1 D_\alpha S_2]$$

$$\Psi_{2\alpha} = S_1 D^2[S_2 D_\alpha S_3]$$

$$\Psi_{3\alpha} = S_1 D^2[D^2 S_1 D_\alpha S_3]$$

$$\Psi_{4\alpha} = D^2[\overline{D}^2 S_1 \overline{D}^2 D_\alpha S_2]$$

One could add more chiral superfields $S$ or more supercovariant derivatives
of course. The main things to keep in mind are:

1. The expression for $\Psi_\alpha$ should not vanish

2. It is frequently necessary to use more than one flavour of superfield

$S$ to prevent the expression from vanishing, because such expressions
may be antisymmetric under interchange of flavour indices
3. It is probably necessary that the resulting integral should be at least linearly divergent to give rise to an anomaly, though it is not entirely clear that this is either necessary or sufficient.

To calculate the anomaly, one would couple such composite operators to the action with an elementary (i.e. not composite) antichiral spinor source superfield $\Phi^\alpha$. This means that one simply adds the following term to the usual action of the theory:

$$S_{\Phi} = \int d^6 \bar{\zeta} \, \Phi^\alpha \Psi_\alpha$$

(6)

where $\Psi_\alpha$ is some composite antichiral spinor superfield, some examples of which are given above in (2-5).

Then the anomaly would appear in the form:

$$\delta \Gamma_{\Phi} = m^k \int d^6 \bar{\zeta} \, \Phi^\alpha c_\alpha \bar{\Phi}^n$$

(7)

where $\Gamma_{\Phi}$ is the one-particle irreducible generating functional with one insertion of the source $\Phi_\alpha$, $\delta$ is the nilpotent BRS operator, $\int d^6 \bar{\zeta}$ is an integral over antichiral superspace, $c_\alpha$ is the constant ghost parameter of rigid supersymmetry, $m^k$ is the mass parameter $m$ to some power $k$ required by simple dimensional analysis, and $\bar{\Phi}^n$ is the $n^{th}$ power of the antichiral superfield (this might include a sum over indices which distinguish different superfields from each other).

To count masses we use the following assignments for the variables and
the superfields:

\[ m = 1; \partial_\mu = 1; \theta_\alpha = -\frac{1}{2}; S = 1; c_\alpha = -\frac{1}{2}; \Phi_\alpha = \frac{1}{2}; \]  

(8)

Now we define the component fields:

\[ S(x, \theta, \bar{\theta}) = A(y) + \theta^\alpha \psi_\alpha(y) + \frac{1}{2} \theta^2 F(y) \]  

(9)

where

\[ y^\mu = s^\mu + \frac{1}{2} \theta^\alpha \sigma_\alpha^\mu \bar{\theta}^\beta \]  

(10)

satisfies

\[ \overline{D}_\alpha y^\mu = D_\alpha \bar{y}^\mu = 0 \]  

(11)

Similarly we have:

\[ \Phi_\alpha(x, \theta, \bar{\theta}) = \phi_\alpha(\bar{y}) + W_{\alpha\beta}(\bar{y}) \bar{\theta}^\beta + \frac{1}{2} \theta^2 \chi_\alpha(\bar{y}) \]  

(12)

The dimensions of these component fields are then:

\[ A = 1; \psi_\alpha = \frac{3}{2}; F = 2; \chi_\alpha = \frac{3}{2}; \phi_\alpha = \frac{1}{2}; W_{\alpha\beta} = 1. \]  

(13)

An examination of examples shows that elementary dimensional counting prevents the powers of \( m \) from working correctly to yield (7) whenever the only vertices of the diagram are chiral vertices involving only chiral fields. It should be possible to show this by a dimensional argument, but this has not yet been done—at any rate it certainly seems to hold for a wealth of examples, one of which can be found in [3].
However when there is at least one gauge propagator in the diagram, the powers of $m$ easily work out correctly to yield (7). But then one has to confront another problem, which is that one has to analyze the cohomology again in the presence of the gauge fields. This unsolved problem has been partially and sufficiently finessed in the present paper.

Another problem that was also unsolved and also necessary for our present purposes is the problem of solving the full BRS cohomology of any supersymmetric theory including the sources that are necessary to formulate the full BRS identity. Essentially, this brings in the complication of ensuring that the BRS cohomology space is orthogonal to the equations of motion of the fields. This is the main subject of the present paper.

Whenever one formulates a BRS identity in the manner pioneered by Zinn-Justin, it is necessary to also include sources $\tilde{f}_i$ for the variation of the fields $f_i$, and in the resulting ‘full’ BRS operator, these give rise to terms that involve the equations of motion of the corresponding fields. This turns out to be more or less equivalent to the Batalin-Vilkovisky quantization method. The essential point is that this eliminates from the cohomology space anything which vanishes by the equation of motion, i.e. anything which vanishes ‘on-shell’. At the simplest level, as explained more fully below, this will eliminate all those objects in the cohomology space which involve superfields $\mathbf{S}$ which have mass terms in the action, as well as a number of higher order terms that are of no concern at present.
So we are now interested only in computing diagrams where the possible supersymmetry anomaly involves massless antichiral fields $\overline{\psi}$ in (7). But this raises another problem. The most promising simple case (see below) seems to involve a triangle diagram with the $\Phi^\alpha$ superfield at one vertex, two chiral (or antichiral) superfields emerging from that vertex and the exchange of a vector superfield between these two lines. Now the mass counting implies that the anomaly (7) generally has a higher power of mass than the composite operator (6) from which it arises. The only way this can happen is if some of the interior lines are massive. Is there any way that interior massive lines can give rise to exterior massless lines while exchanging a vector superfield? The answer to this question is of course well known–this will happen if and only if the gauge symmetry is spontaneously broken, provided the representations are chosen correctly, as is discussed below. We will therefore assume that gauge symmetry is spontaneously broken and that supersymmetry is not spontaneously broken. Since we are looking for supersymmetry breaking through anomalies, it is reasonable to assume that it is not otherwise broken.

This combination is in fact very easy to achieve–as is well known, gauge symmetry breaking is natural and very easy to achieve in rigid supersymmetry, but spontaneous supersymmetry breaking can only be achieved with very contrived models, particularly if the gauge group is semisimple.

So now, if we want to examine the question of supersymmetry anomalies, we are forced to consider a supersymmetric gauge theory with spontaneous
breaking of the gauge symmetry. But there are more conditions, at least for the supersymmetry anomalies that involve massless matter superfields. In order for the relevant diagrams to exist, we must have matter multiplets which break under the gauge breaking into a combination of massive and massless fields, so that a massive vector superfield can have a vertex with a massless and a massive chiral superfield.

This happens of course for the Higgs multiplet itself, but then the massless Goldstone supermultiplets do not contribute to the relevant BRS cohomology space, as will be shown below. We must have additional (non-Higgs) matter multiplets which break under the gauge breaking into a combination of massive and massless fields. There are many ways to do this, and an example is given below. Note that this happens also in the standard model, where the neutrino remains massless after spontaneous breaking of $SU(2) \times U(1)$ to $U(1)_{EM}$ simply because there is no right handed neutrino for it to form a mass with (and because lepton conservation prevents the formation of a Majorana neutrino mass, in the minimal standard model at least). The relevant discussion of the standard model will be the subject of a forthcoming paper [6].

So if we want to find a simple supersymmetry anomaly, we are driven to models with gauged supersymmetry and spontaneous breaking of the gauge symmetry through Higgs multiplets which develop a VEV in their ‘A’ components (but not their ‘F’ components–that would break supersymmetry). In
addition these models must have matter which is massless at tree level, but
which gets split into massive and massless components as a result of gauge
breaking. These are the simplest models that have a chance of developing
supersymmetry anomalies in some of their composite operators at the one
loop level. Such models are of course very reminiscent of a supersymmetric
version of the standard model of strong, weak and electromagnetic interac-
tions. It is just within the realm of possibility that these anomalies could
account for the experimentally observed lack of supersymmetry in the world
with no additional assumptions in the model at all—in which case we could
say that supersymmetry breaks itself. But there is plenty of work to do before
we can determine whether this notion is right. Even if the supersymmetry
anomalies exist, considerable work will be necessary to deduce the form of
the supersymmetry breaking they give rise to.

A rather interesting and new feature is that we can see that the particular
‘soft’ mass-dependent supersymmetry anomalies we are examining here, if
their coefficients are non-zero, would give rise to a kind of supersymmetry
breaking that is a function of the VEV that breaks the gauge symmetries,
and which vanishes in the gauge symmetric limit. This is still consistent with
the conjecture mentioned above that such anomalies might also provide a
natural mechanism whereby ‘supersymmetry breaks itself’, while at the same
time retaining the cosmological constant at the zero value it naturally has in
many unbroken supersymmetric theories. Spontaneous breaking of the gauge
symmetry would not interfere with this feature, because it does not change
the vacuum energy so long as supersymmetry is not spontaneously broken at
the same time.

There is still a possibility of ‘hard’ supersymmetry anomalies too, which
we do not consider here, since they look more difficult to compute.

2 Supersymmetric gauge theory with spontaneous breaking of gauge symmetry

Pursuant to the above discussion, we will now consider a general supersym-
metric gauge theory coupled to chiral matter, where the gauge symmetry is
spontaneously broken. We will consider this action in terms of component
fields in the Wess-Zumino gauge, and we will fix the gauge in the way pio-
neered by ‘t Hooft for spontaneously broken theories. Naturally much of this
is simpler in superspace, but the method we use to find cohomology actu-
ally brings us back to components anyway, and also it is probably better to
use components to compute diagrams when one is looking for something as
obscure and tricky as anomalies.

The action consists of the following parts, each of which is separately
supersymmetric and gauge invariant (before spontaneous breaking):

\[
S_{\text{Total}} = S_{\text{YM}} + S_{\text{Matter}} + S_{\text{Chiral}}
\]

\[
+ \overline{S}_{\text{Chiral}} + S_{\text{Ghost}} + S_{\text{Sources}} + S_{\Phi} + \overline{S}_{\Phi}
\] (14)

We will discuss each of these terms in the next section.
3 Component Form of Action

The Yang-Mills supersymmetric action in the Wess-Zumino gauge takes the form:

\[ S_{YM} = \int d^4x \left\{ -\frac{1}{4} G_{\mu \nu}^a G^{a \mu \nu} - \frac{1}{2} \lambda^{\alpha \sigma \mu} D_{\mu} \bar{\lambda}^{\beta} + \frac{1}{2} D^a D^a \right\} \]  \hspace{1cm} (15)

where we will assume that the gauge group is semisimple, and

\[ G_{\mu \nu}^a = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + f^{abc} V^b_\mu V^c_\nu \] \hspace{1cm} (16)

\[ D_{\mu}^{ab} \bar{\lambda}^{a b} = \partial_\mu \bar{\lambda}^{a b} + f^{abc} V^b_\mu \bar{\lambda}^{a c} \] \hspace{1cm} (17)

We will assume that the matter is in some (generally reducible) representation of the compact semi-simple gauge group. The matrices \( T \) satisfy:

\[ [T^a, T^b] = i f^{abc} T^c \] \hspace{1cm} (18)

and have indices of the form

\[ T^{a i}_j \] \hspace{1cm} (19)

In general these matrices are Hermitian complex matrices, which means that:

\[ [T^{a i}_j]^* = T^{b i}_j = T^{a j}_i \] \hspace{1cm} (20)

We will assume that the VEV of the ‘A’ component takes the form

\[ < A^i >= m u^i \] \hspace{1cm} (21)

where \( u^i \) is a dimensionless quantity that describes the direction and magnitude in group space of the breaking, and \( m \) is the mass parameter of the
theory. Then the ‘kinetic’ part of the matter action is, after a shift:

\[
S_\text{Matter} = -\int d^4x \left\{ D^i_{\mu j}(mu^j + A^j)D^k_{\mu k}(mu^k + \overline{A}_k) + \psi^{\alpha i} \sigma^{\mu}_{\alpha \beta} (D_\mu \overline{\psi}^\beta) - F^i F_i + \psi^{\alpha j} T^\alpha_j \lambda^\alpha_i (mu_i + \overline{A}_i) \right. \\
+ \left. \overline{\psi}_j T^\alpha_j \lambda^\alpha_i (mu^i + A^i) \right. \\
+ D^\alpha T^\alpha_i (mu_i + \overline{A}_i) \right\} 
\]

(22)

Next we must discuss the chiral part of the action. We assume that it is such that there is a minimum at some non-zero VEV as discussed above. We will consider non-renormalizable terms up to fourth order here just to show how they would work, though a renormalizable theory would probably be sufficient for all purposes.

After a shift, this takes the form

\[
S_\text{Chiral} = \int d^4x \left\{ mg_{ij}(u)[2A^i F^j + \psi^{\alpha i} \psi^j] + 3 g_{ijk}(u)[A^i A^j F^k + A^i \psi^{\alpha j} \psi^{\alpha k}] + \frac{1}{m} g_{ijkl}[4A^i A^j A^k F^l + 6A^i A^j \psi^{\alpha k} \psi^l] \right\} 
\]

(23)

where

\[
g_{ij}(u) = g_{ij} + 3g_{ijk}u^k + 6g_{ijkl}u^k u^l \\
g_{ijk}(u) = g_{ijk} + 4g_{ijkl}u^l
\]

(24)

(25)

Here \( g_{ij} \), \( g_{ijk} \) and \( g_{ijkl} \) are dimensionless group invariant tensors under the action of the matrices \( T \). The parameter \( u^i \) in the above is a solution of the equations:

\[
< F_i > = 2g_{ij}u^j + 3g_{ijk}u^j u^k + 4g_{ijkl}u^j u^k u^l = 0 
\]

(26)
These equations state that the VEV of the auxiliary fields $F$ and $D$ are zero. This in turn insures that supersymmetry is preserved even though there is a non-zero VEV $<A^i> = m u^i$ of the unshifted scalar field $A^i$. We assume here for simplicity that there are no linear terms in the chiral action.

Now let us consider the ghost and gauge-fixing action:

$$S_{\text{Ghost}} = -\delta \int d^4x \left\{ \xi^a [k^2 Z^a + \partial^\mu V^a_\mu + k m T^{aj}_i u^i A^j + k m T^{aj}_i \pi_j A^i] \right\}$$

The field $\omega^a$ (used below) is the (real scalar) Fadeev-Popov ghost and $\xi^a$ is the (real scalar) Fadeev-Popov antighost. $Z^a$ is a (real scalar) auxiliary field used for convenience in gauge fixing. The $u^i$ dependent terms are included to eliminate mixing between gauge bosons and Goldstone bosons in the gauge fixed theory \cite{footnote1}. $k$ is a gauge parameter, and any physically meaningful quantity (such as the coefficient of an on-shell supersymmetry anomaly) should be independent of it. Here $\delta$ is the BRS variation of the relevant terms, given below. When this is expanded out using the formulae below in section \cite{footnote2}, we find:

$$S_{\text{Ghost}} = \int d^4x \left\{ (Z^a + e^\mu \partial_\mu \xi^a) \left[ k^2 Z^a + \partial^\mu V^a_\mu + k m T^{aj}_i u^i A^j + k m T^{aj}_i \pi_j A^i \right] + \xi^a \frac{k}{2} (c^\alpha \sigma_\alpha \sigma_\beta \partial_\mu \xi^a + e^\mu \partial_\mu Z^a) \\
+ \partial^\mu \xi^a D^a_\mu \omega^b + \frac{1}{2} c^\alpha \sigma_\alpha \sigma_\beta \bar{\psi}_j \chi_j + \frac{1}{2} \lambda^a \sigma_\alpha \sigma_\beta \bar{\psi}_j \chi_j + e^\nu \partial_\nu V^a_\mu \right\}$$

$$+ k m \xi T^{aj}_i u^i \left[ \bar{\psi}_j \psi_j + T^{aj}_i \omega^a (m \pi_k + \bar{A}_k) + e^\mu \partial_\mu \bar{A}_j \right]$$
\[ + k \, m \, \xi^a \mathcal{T}^{aj}_{i} \pi_j \left[ c^a \psi^i_{\alpha} + T^a_{j} \omega^a(m \, u^j + A^j) + \epsilon^\mu \partial_\mu A^i \right] \]  \quad (28)

All the \( \epsilon^\mu \) terms in \( S_{\text{Ghost}} \) cancel. So finally this takes the form:

\[
S_{\text{Ghost}} = \int d^4 x \left\{ Z^a_{i} \frac{k}{2} Z^a + \partial^\mu V^a_{\mu} + k \, m \, T^{aj}_{i} u^i \overline{A}_j + k \, m \, \mathcal{T}^{aj}_{i} \pi_j A^i \right. \\
+ \left. \xi^a \frac{k}{2} \left( c^a \sigma_{\alpha \beta} \overline{c}^\beta \partial_\mu \xi^a \right) + \partial^\mu \xi^a \left[ D^a_{\mu \beta} \omega^a + \frac{1}{2} c^a \sigma^\mu_{\alpha \beta} \overline{\lambda}^\beta + \frac{1}{2} \lambda_{\alpha \alpha} \sigma^\mu_{\alpha \beta} \overline{c}^\beta \right] \\
+ k \, m \, \xi^a T^{aj}_{i} u^i \left[ \tau^j \overline{\psi}_j \alpha + T^{ak}_{j} \omega^a \left( m \, \pi_k + \overline{A}_k \right) \right] \\
+ k \, m \, \xi^a \mathcal{T}^{aj}_{i} \pi_j \left[ c^a \psi^i_{\alpha} + T^a_{j} \omega^a \left( m \, u^j + A^j \right) \right] \right\} \quad (29)
\]

Now we come to the ‘Source’ part of the action. This is necessary in order to formulate the Ward identity for the theory in the form first advocated by Zinn-Justin \[8\]. We introduce sources \( \tilde{f} \) for the BRS variation \( \delta f \) of each field \( f \) in the action. The (Bose-Fermi) statistics of the source is opposite to that of the field, so that the action has even statistics (since \( \delta \) is odd).

\[
S_{\text{Sources}} = \int d^4 x \left\{ \tilde{F}^i \delta A^i + \tilde{\psi}_i^{\alpha} \delta \psi^i_{\alpha} + \tilde{A}_i \delta F^i \\
+ \tilde{\overline{F}}^j \delta \overline{A}_j + \tilde{\psi}_i^{\alpha} \delta \overline{\psi}_i^{\alpha} + \tilde{\overline{A}}_i \delta \tilde{F}^i \\
+ \tilde{V}^{ai} \delta V^a_{\mu} + \tilde{\lambda}_a \delta \lambda^a_{\alpha} + \tilde{\overline{\lambda}}^{a\dot{\alpha}} \delta \overline{\lambda}_{\alpha} + \tilde{\overline{D}}^a \delta D^a \\
+ \tilde{\omega}^{a} \delta \omega^a + \tilde{\xi}^{a} \delta \xi^a + \tilde{Z}^{a} \delta Z^{a} \right\} \quad (30)
\]

When expanded using the form of \( \delta \) below, this takes the form:

\[
S_{\text{Sources}} = \int d^4 x \left\{ \tilde{F}^i [c^a \psi^i_{\alpha} + T^{aj}_{j} \omega^a \left( m \, u^j + A^j \right) + \epsilon^\mu \partial_\mu A^i] \right\}
\]

15
Finally we come to the most important part of the action for present purposes. This is the part $S_\Phi$ that introduces the operator that may have a supersymmetry anomaly. It is not possible to write this in a general way.

We refer the reader to section (19) for a specific example of such an operator.
4 Ward Identity and Construction of BRS operator $\delta$

Once we have the action in the above form, the Ward identity takes the Zinn-Justin [8] ‘Master Equation’ form (a complete derivation of this for D=4 super Yang-Mills can be found in [2]):

$$\int d^4x \left\{ \frac{\delta \Gamma}{\delta X_i} \frac{\delta \Gamma}{\delta X_i} \right\} = 0$$

(32)

Here $X_i$ represents all the fields and sources in the action. For one loop amplitudes this reduces to the form

$$\delta \Gamma = 0$$

(33)

where the BRS operator is found from the action as follows:

$$\delta = \int d^4x \left\{ \frac{\delta S_{\text{Total}}}{\delta X_i} \frac{\delta}{\delta X_i} + \frac{\delta S_{\text{Total}}}{\delta X_i} \frac{\delta}{\delta X_i} \right\}$$

(34)

Now the invariance of the action, and hence the Master Equation, result from the invariance of the YM, chiral and Matter actions, and from the nilpotence of the transformations of the fields. We can recover these transformations as follows from the action:

$$\delta X_i(x) = \frac{\delta S_{\text{Total}}}{\delta X_i(x)}$$

(35)

and the transformations of the sources in $\delta$ are given by:

$$\delta \bar{X}_i(x) = \frac{\delta S_{\text{Total}}}{\delta X_i(x)}$$

(36)
5 The Full BRS Operator

The following transformations are the gauge and supersymmetry transformations for the theory with spontaneous breaking of the gauge symmetry. They were used in $S_{\text{Ghost}}$ and $S_{\text{Sources}}$ in the above to derive the form of the action itself.

\[ \delta A^i = c^i \psi^i + T^{ai}_j \omega^a (m w^j + A^j) + \epsilon^\mu \partial_\mu A^i \]

\[ \delta \psi^i_\alpha = (D_\mu (mu + A))^{i \alpha} \sigma^{\mu}_{\alpha \beta} \bar{c}^\beta + F^i c_\alpha + T_{j}^{ai} \omega^a \psi^j_\alpha + \epsilon^\mu \partial_\mu \psi^i_\alpha \]

\[ \delta F^i = (D_\mu \psi)^{ia} \sigma^{\mu}_{\alpha \beta} \bar{c}^\beta + T_{j}^{ai} \omega^a F^j + \epsilon^\mu \partial_\mu F^i \]

\[ \delta V^a_\mu = D_{\mu \nu} \omega^a - \frac{1}{2} c^{\alpha} \sigma^{\nu}_{\alpha \beta} \bar{c}^\beta + \epsilon^\nu \partial_\nu V^a_\mu \]

\[ \delta \lambda^a_\alpha = \frac{1}{2} G^{a \mu \nu} \sigma^{\mu \nu}_{\alpha \beta} \bar{c}^\beta - f^{abc} \lambda^b_\alpha \omega^c + \epsilon^\nu \partial_\nu \lambda^a_\alpha \]

\[ \delta D^a = -\frac{i}{2} c^{\alpha} \sigma^{\mu}_{\alpha \beta} D^{ab}_\mu \bar{c}^b \bar{c}^\beta + \frac{i}{2} D^{ab}_\mu \lambda^a_\alpha \sigma^{\mu}_{\alpha \beta} \bar{c}^\beta + f^{abc} D^{b} \omega^c + \epsilon^\nu \partial_\nu D^a \]

\[ \delta \omega^a = -\frac{1}{2} f^{abc} \omega^b \omega^c + c^{\alpha} \sigma^{\mu}_{\alpha \beta} \bar{c}^\beta V^a_\mu + \epsilon^\nu \partial_\nu \omega^a \]

\[ \delta \xi^a = Z^a + \epsilon^\mu \partial_\mu \xi^a \]

\[ \delta Z^a = c^{\alpha} \sigma^{\mu}_{\alpha \beta} \bar{c}^\beta \partial_\mu \xi^a + \epsilon^\mu \partial_\mu Z^a \]

\[ \delta \epsilon_\mu = -c^{\alpha} \sigma^{\mu}_{\alpha \beta} \bar{c}^\beta \]

\[ \delta c^\alpha = 0 \]

\[ \delta \bar{c}^\dot{a} = 0 \]
The above transformations on the fields are nilpotent—this is all that is needed to derive the Master Equation, which then implies that $\delta$ is also nilpotent on the sources.

Next we find the variations of the sources using (36) and the foregoing expressions (14), (15), (22), (23), (29) and (31) for the parts of the action:

$$
\delta \tilde{F}_i = D^j_{\mu} D^{\mu k}_{j} (m \tilde{u}_k + \tilde{A}_k) \\
+ \bar{\psi}_j T^{ao}_i \chi_{\bar{a}}^o + D^a T^{ao}_i (m \tilde{u}_j + \tilde{A}_j) \\
+ 2mg_{ij}(u) F^j + 3g_{ijk}(u)(2A^j F^k + \psi^{\alpha j} \psi^{\beta k}) + \frac{12}{m} g_{ijkl}(A^j A^k F^l + \psi^{\alpha j} \psi^{\beta k} A^l) \\
+ Z^a k \ m T^{ao}_i \pi_j + \xi^a k \ m T^{ao}_i \pi_j T^{ao}_i \omega^a \\
+ T^{ao}_i \tilde{F}_j \omega^a + (D_{\mu} \tilde{\psi})^o_{\alpha} \sigma^o_{\alpha \beta} \bar{\chi}^\beta + \epsilon^\mu \partial_{\mu} \tilde{F}_i \tag{49}
$$

$$
\delta \tilde{\psi}_{i \alpha} = \sigma^o_{\alpha \beta} (D_{\mu} \tilde{\psi})^o_{i \beta} + \bar{\psi}_j T^{ao}_i \lambda_{\bar{a}} + \frac{12}{m} g_{ijkl} A^j A^k \psi_{\alpha l} + k \ m \xi^a T^{ao}_i \pi_j c_{\alpha} \\
+ \tilde{F}_i c_{\alpha} + T^{ao}_i \omega^a \tilde{\psi}_j + (D_{\mu} \tilde{\psi})^o_{i \beta} \sigma^o_{\alpha \beta} \bar{\chi}^\beta + \epsilon^\mu \partial_{\mu} \tilde{\psi}_{i \alpha} \tag{50}
$$

$$
\delta \tilde{A}_i = - \tilde{F}_i + 2mg_{ij}(u) A^j + 3g_{ijk}(u) A^j A^k + \frac{4}{m} g_{ijkl} A^j A^k A^l \\
+ c^\alpha \tilde{\psi}_{i \alpha} + T^{ao}_i \omega^a \tilde{A}_j + \epsilon^\mu \partial_{\mu} \tilde{A}_i \tag{51}
$$

$$
\delta \tilde{V}_{\mu a} = (D^\nu \bar{G}_\nu)^a_{\mu} - \frac{1}{2} f^{abc} \chi^a_{\alpha \beta} \sigma^o_{\alpha \beta} \bar{\chi}^\beta \\
+ T^{ai}_j (m u^j + A^j) \bar{D}^{\mu k}_{i} (m \tilde{u}_k + \tilde{A}_k) + D^{\mu k}_{i} (m \tilde{u}_j + \tilde{A}_j) T^{ao}_j (m \tilde{u}_j + \tilde{A}_j) \\
+ T^{ao}_j \psi_{o} \sigma^o_{\alpha \beta} \tilde{\psi}^\beta_i - \partial^\nu Z^a + T^{ai}_j (m u^j + A^j) \tilde{\psi}^a_{i} \sigma^o_{\alpha \beta} \bar{\chi}^\beta + \epsilon^\mu \partial_{\mu} \tilde{A}_i \\
+ T^{ai}_j \psi^{\alpha j} \bar{D}^{\mu k}_{i} (m \tilde{u}_k + \tilde{A}_k) + D^{\mu k}_{i} (m \tilde{u}_j + \tilde{A}_j) T^{ao}_j (m \tilde{u}_j + \tilde{A}_j) \tag{19}
$$
\begin{align}
&+ T_j^{ai} \tilde{A}_i \psi^j a \sigma_{\alpha \beta}^{\mu} \sigma_{\alpha \beta}^{\nu} c^\gamma + T_j^{aj} \tilde{A}_j \psi^i \sigma_{\alpha \beta}^{\mu} c^\beta \\
&+ (D_\nu \tilde{\lambda})^{\alpha \alpha} \sigma_{\alpha \beta}^{\mu} c^\beta + (D_\nu \tilde{\lambda})^{\alpha \beta} \sigma_{\alpha \beta}^{\mu} c^\beta \\
&+ f^{abc} \tilde{D}^b [\frac{-i}{2} e^a \sigma_{\alpha \beta}^{\mu} \tilde{\lambda} c^\beta + \frac{i}{2} \lambda c a \sigma_{\alpha \beta}^{\mu} c^\beta] \\
&+ f^{abc} (\tilde{V}^{\mu b} - \partial^\nu \xi^b) \omega^c + \tilde{\omega} a c \sigma_{\alpha \beta}^{\mu} c^\beta + \epsilon^\nu \partial_\nu \tilde{V}^{\mu a} \\
\delta \tilde{\lambda}_a^\alpha &= - \frac{1}{2} \sigma_{\alpha \beta}^{\mu} (D_\mu \tilde{\lambda}) a \beta + T_j^{ai} (m \overline{u}_i + A_i) \psi a j - \frac{1}{2} (\tilde{V}_a^\alpha - \partial_\mu \xi^a) \sigma_{\alpha \beta}^{\mu} c^\beta \\
&+ f^{abc} \tilde{\lambda}_a^b \omega^c + \frac{i}{2} (D_\mu \tilde{D}^a) \sigma_{\alpha \beta}^{\mu} c^\beta + \epsilon^\nu \partial_\nu \tilde{\lambda}_a^a \\
\delta \tilde{D}^a &= D^a + T_i^{aj} (m \overline{u}_j + \overline{A}_j) (m u^i + A^i) \\
&+ i \tilde{\lambda}_a^c c_a - i \tilde{\lambda}_a^a c^\alpha + f^{abc} \tilde{D}^b \omega^c + \epsilon^\nu \partial_\nu \tilde{D}^a \\
\delta \tilde{\omega}^a &= D^{\mu a b} (\tilde{V}_a^\mu - \partial_\mu \xi^a) + \epsilon^\nu \partial_\nu \tilde{\omega}^a \\
&+ k m \xi^b T_i^{bj} u^i T_j^{ai} (m \overline{u}_k + \overline{A}_k) + k m \xi^b T_j^{aj} \overline{u}_j T_i^{ai} (m u^i + A^i) \\
&+ [\tilde{F}_i T_j^{ai} (m u^i + A^i) + \tilde{\psi}_a^j T_j^{ai} \psi^j + \tilde{A}_i T_j^{ai} F^j + \text{comp. conj.}] \\
&- f^{abc} \tilde{\lambda}_a^b \lambda^c + f^{abc} \tilde{\lambda}_a^b \tilde{\lambda}_a^c + f^{abc} \tilde{D}^b \omega^c + f^{abc} \tilde{\omega}^b \omega^c \\
\delta \tilde{\xi}^a &= \frac{k}{2} c^a \sigma_{\alpha \beta}^{\mu} c^\beta \partial_\mu \xi^a - \partial_\mu [D_\mu \omega^b + \frac{1}{2} c^a \sigma_{\alpha \beta}^{\mu} \tilde{\lambda} c^\beta + \frac{1}{2} \lambda a c \sigma_{\alpha \beta}^{\mu} c^\beta] \\
&+ k m T_j^{ai} \omega^i \tilde{\psi}_a^j + \tilde{T}_j^{ak} \omega^a (m \overline{u}_k + \overline{A}_k) \\
&+ k m \tilde{T}_j^{ai} \overline{u}_i [c^a \psi_a^j + T_k^{aj} \omega^a (m u^k + A^k)] \\
&+ c^a \sigma_{\alpha \beta}^{\mu} c^\beta \partial_\mu \tilde{Z}^a + \epsilon^\mu \partial_\mu \tilde{\xi}^a \\
\delta \tilde{Z}^a &= k Z^a + \partial^\mu V_\mu^a + k m T_j^{ai} \omega^i \overline{A}_i \\
&+ k m \tilde{T}_j^{ai} \overline{u}_i A^i + \tilde{\xi}^a + \epsilon^\mu \partial_\mu \tilde{Z}^a
\end{align}
6 The ‘linear’ part of $\delta$

We will find it useful to collect certain terms in $\delta$ when they have a common heritage. The entire operator can be written in the form:

$$\delta = c^\alpha \Lambda_\alpha + \bar{c}^\dot{\alpha} \bar{\Lambda}_{\dot{\alpha}} + \bar{c}^\dot{\alpha} \bar{\nabla}_{\dot{\alpha}} + \bar{\nabla}^\alpha \Sigma_\alpha + mc^\alpha \tilde{\Sigma}_\alpha$$

$$+ \delta_{\text{Kinetic}} + m\delta_{\text{Mass}} + \delta_{\text{Nonlinear}} + \epsilon^\mu \partial_\mu - c^\alpha \sigma^\mu_{\alpha\beta} \tilde{c}^\beta (\epsilon^\mu)^\dagger$$  

where we take terms which are homogeneous in the fields or homogeneous with one ghost $c$ or $\bar{c}$ for our 'linear' terms. These decompose further into:

$$\Lambda_\alpha = \Lambda_\alpha \text{ Matter} + \Lambda_\alpha \text{ Gauge} + \tilde{\Lambda}_\alpha \text{ Matter} + \tilde{\Lambda}_\alpha \text{ Gauge}$$  

$$\nabla_{\dot{\alpha}} = \nabla_{\dot{\alpha}} \text{ Matter} + \nabla_{\dot{\alpha}} \text{ Gauge} + \tilde{\nabla}_{\dot{\alpha}} \text{ Matter} + \tilde{\nabla}_{\dot{\alpha}} \text{ Gauge} + \nabla_{\dot{\alpha}} \text{ Ghost}$$

$$\delta_{\text{Kinetic}} = \delta_{\text{Kinetic Matter}}$$

$$+ \tilde{\delta}_{\text{Kinetic Matter}} + \tilde{\delta}_{\text{Kinetic Gauge}} + \tilde{\delta}_{\text{Kinetic Ghost}}$$

$$\delta_{\text{Mass}} = \delta_{\text{Mass Matter}} + \tilde{\delta}_{\text{Mass Matter}} + \delta_{\text{Mass Gauge}}$$

$$+ \tilde{\delta}_{\text{Mass Matter} \rightarrow \text{Gauge}} + \tilde{\delta}_{\text{Mass Gauge} \rightarrow \text{Matter}}$$

$$+ \tilde{\delta}_{\text{Mass Ghost}} + \tilde{\delta}_{\text{Mass Matter} \rightarrow \text{Ghost}} + \tilde{\delta}_{\text{Mass Ghost} \rightarrow \text{Matter}}$$

It is convenient to introduce the following abbreviations:

$$T^{ai} = T_j^{ai} u^j$$

$$\mathbf{T}_i^a = T_i^{aj} \mathbf{u}_j = T_i^{aj} \mathbf{u}_j$$
\[ G^{ab} = T_{ai} T_{bi} - T_{ai} T_{bi} \]  

Then the above operators can be written in the form:

\[ \Lambda_\alpha \text{ Matter} = \int d^4x \left\{ \psi_i^\alpha \frac{\delta}{\delta A_i} + F_i^\alpha \frac{\delta}{\delta \psi_i^\alpha} \right\} \]

\[ \tilde{\Lambda}_\alpha \text{ Matter} = \int d^4x \left\{ \tilde{\psi}_i^\alpha \frac{\delta}{\delta \tilde{A}_i} + \tilde{F}_i^\alpha \frac{\delta}{\delta \tilde{\psi}_i^\alpha} \right\} \]

\[ \nabla_\alpha \text{ Matter} = \int d^4x \left\{ \partial_\mu \psi_i^\alpha \sigma_{\alpha \beta} \delta F_i^\beta + \partial_\mu \tilde{A}_i \sigma_{\alpha \beta} \delta \tilde{\psi}_i^\beta \right\} \]

\[ \tilde{nabla}_\alpha \text{ Matter} = \int d^4x \left\{ \partial_\mu \tilde{\psi}_i^\alpha \sigma_{\alpha \beta} \delta F_i^\beta + \partial_\mu \tilde{A}_i \sigma_{\alpha \beta} \delta \tilde{\psi}_i^\beta \right\} \]

\[ \Lambda_\alpha \text{ Gauge} = \int d^4x \left\{ \frac{1}{2} \sigma_{\alpha \beta} \lambda_{\beta}^\alpha \frac{\delta}{\delta V^\mu a} + i D^a \frac{\delta}{\delta \lambda_{\beta}^\alpha} \right\} \]

\[ \tilde{\Lambda}_\alpha \text{ Gauge} = \int d^4x \left\{ -\frac{1}{2} \sigma_{\alpha \beta} \tilde{\lambda}_{\beta}^\alpha \frac{\delta}{\delta \tilde{V}^\mu a} + i \tilde{\lambda}_{\alpha}^\alpha \frac{\delta}{\delta D^a} \right\} \]

\[ \nabla_\alpha \text{ Gauge} = \int d^4x \left\{ \frac{1}{2} \partial_\mu V^\alpha_\nu \sigma^{\mu \nu}_{\alpha \beta} \frac{\delta}{\delta \lambda_{\beta}^\alpha} - \frac{i}{2} \sigma_{\alpha \beta} \partial_\mu \tilde{\lambda}_{\beta}^\alpha \frac{\delta}{\delta D^a} \right\} \]

\[ \tilde{nabla}_\alpha \text{ Gauge} = \int d^4x \left\{ \sigma^{\mu \nu}_{\alpha \beta} \partial_\nu \lambda_{\beta}^\alpha \frac{\delta}{\delta V^{\mu a}} + \sigma_{\alpha \beta} \partial_\mu \tilde{D}^a \frac{\delta}{\delta \tilde{\lambda}_{\beta}^\alpha} \right\} \]

\[ \delta \text{ Kinetic Matter} = \int d^4x \left\{ \Box A_i \frac{\delta}{\delta F_i} + \sigma^\mu_{\alpha \beta} \partial_\mu \psi_i^\beta \frac{\delta}{\delta \psi_i^\alpha} - F_i^\alpha \frac{\delta}{\delta A_i} \right\} \]

\[ \delta \text{ Kinetic Gauge} = \int d^4x \left\{ \left( \Box V^\mu a - \partial^\mu \partial^\nu V^\nu_{\mu a} \right) \frac{\delta}{\delta V^\mu a} + \sigma^\mu_{\alpha \beta} \partial_\mu \lambda_{\beta}^\alpha \frac{\delta}{\delta \lambda_{\alpha}^\beta} + D^a \frac{\delta}{\delta D^a} \right\} \]

\[ \delta \text{ Mass Matter} = \int d^4x \left\{ F^i_j \frac{\delta}{\delta F_i} + \psi_j^\alpha \frac{\delta}{\delta \psi_i^\alpha} + A_i^\beta \frac{\delta}{\delta A_i} \right\} \]

\[ \delta \text{ Mass Gauge} = m \int d^4x G^{ab} V^{bj \mu} \frac{\delta}{\delta V^{\mu a}} \]
\[ \delta \text{Mass Matter}\to\text{Gauge} = \]
\[ \int d^4x \left\{ (\partial_\mu V^{\mu a} + D^a)T^i_a \right\} \frac{\delta}{\delta F_i} + \delta \frac{\partial_\mu \lambda^a_\alpha}{\delta \psi^{\mu a}_{i\alpha}} \]  
\[ \delta \text{Mass Gauge}\to\text{Matter} = \int d^4x \left\{ [T^{ai} \partial^\mu \bar{A}_i + T^i_a \partial^\mu A^i] \frac{\delta}{\delta V^{\mu a}} + \right. \]
\[ + T^b_i \psi^j_\alpha \frac{\delta}{\delta \lambda^a_\alpha} + T^a_i \psi^j_\alpha \frac{\delta}{\delta \lambda^a_\alpha} + \left( T^a_i A^i + T^{ai} \bar{A}_i \right) \frac{\delta}{\delta D^a} \]  
(79)

Here are some ghost and gauge-fixing dependent supersymmetry transformations:

\[ \Sigma_\alpha = \int d^4x \left\{ kT^b_i \psi^j_\alpha \frac{\delta}{\delta \xi^a_\alpha} + k \xi^a_\alpha T^b_i \frac{\delta}{\delta \psi^{\mu a}_{i\alpha}} \right\} \]  
(80)

\[ \nabla_\alpha \text{Ghost} = \int d^4x \left\{ \frac{1}{2} \sigma^\mu_{\alpha \beta} \partial_\mu \bar{\chi}^{\alpha \beta} \frac{\delta}{\delta \xi^a_\alpha} + \frac{1}{2} \sigma^\mu_{\alpha \beta} \partial_\mu \bar{\xi}^{\alpha \beta} \frac{\delta}{\delta \chi^a_\beta} \right\} \]  
(81)

Here is a rather amorphous collection of ghost and gauge-fixing dependent linear terms:

\[ \delta \text{Kinetic Ghost} = \int d^4x \left\{ -\partial_\mu Z^a \frac{\delta}{\delta V^{\mu a}} - \Box \omega^a \frac{\delta}{\delta \omega^a} \right. \]
\[ + (kZ^a + \partial^\mu V^{\mu a} + \bar{\xi}^a) \frac{\delta}{\partial Z^a} + T^a_i \omega^a \frac{\delta}{\partial A^i} + \right. \]
\[ + \partial_\mu \omega^a \frac{\delta}{\partial V^{\mu a}} + \partial_\mu (\bar{V}^{\mu a} - \partial^\mu \xi^a) \frac{\delta}{\partial \omega^a} + \bar{Z}^a \frac{\delta}{\delta \xi^a} \]  
(82)

\[ \delta \text{Mass Ghost} = \int d^4x G^{ab} \left\{ k \omega^b \frac{\delta}{\delta \xi^a} + k \xi^b \frac{\delta}{\delta \omega^a} \right\} \]  
(83)

\[ \delta \text{Mass Matter}\to\text{Ghost} = k Z^a T^i_a \frac{\delta}{\delta F_i} \]  
(84)

\[ \delta \text{Mass; Ghost}\to\text{Matter} = \]
\[ \int d^4x \left\{ k(T^{ai} \bar{A}_i + T^a_i A^i) \frac{\delta}{\delta Z^a} + (T^{ai} \bar{F}_i + T^a_i \bar{F}) \frac{\delta}{\delta \bar{\omega}^a} \right\} \]  
(85)
Finally there are a large number of terms that are non-linear which can all be deduced given the total form of $\delta$ above. Unfortunately it is necessary to write them all down and include them in the grading of $\delta$ even though most of them will play no role in the spectral sequence that we will use to find the cohomology. The problem is that only by going through all the steps can one be sure that nothing important has been missed. In fact some of the non-linear terms do play an important role, even though most of them do not.

### 7 Non-linear terms

The non-linear terms in the above BRS transformations are:

\[
\delta_{\text{Nonlinear}} A^i = T^a_i \omega^a A^i
\]  
(86)

\[
\delta_{\text{Nonlinear}} F_i = V^j_{\mu} \omega^\mu \bar{A}_j + \partial_{\mu} [V^j_{\mu} \bar{A}_k] + V^j_{\mu} \bar{V}^k_{\mu} (m \bar{\pi}_k + \bar{A}_k)
\]
\[
+ \psi^j \sigma^a T^a_i \bar{\lambda}^\alpha + D^a T^a_i \bar{A}_j
\]
\[
+ 3g_{ijk}(u)[2A^i F^k + \psi^{\alpha j} \psi^k_{\alpha}] + \frac{12}{m} g_{ijkl}[A^i A^k F^l + A^i \psi^{\alpha k} \psi^l_{\alpha}]
\]
\[
+ \xi^a k m \bar{T}^a_i \bar{\pi}_j T^a_k \omega^\alpha
\]
\[
+ T^a_k \bar{F}_j \omega^\alpha + (V_{\mu} \bar{\psi})^\alpha_i \sigma^\mu_{\alpha \beta} \bar{\pi}^\beta
\]
(87)

\[
\delta_{\text{Nonlinear}} \psi^i_{\alpha} = (V_{\mu} A)^i \sigma^\mu_{\alpha \beta} \bar{\pi}^\beta + T^a_i \omega^a \psi^i_{\alpha}
\]  
(88)

\[
\delta_{\text{Nonlinear}} \bar{\psi}^i_{\alpha} = \sigma^\mu_{\alpha \beta} (V_{\mu} \bar{\psi})^i_{\beta} + T^a_i \bar{\lambda}^a_{\alpha} \bar{A}_i
\]
\[
+ 6g_{ijk}(u) A^j \psi^k_{\alpha} + \frac{12}{m} g_{ijkl} A^j A^k \psi^l_{\alpha}
\]
\[ + T_i^{aj} \omega^a \bar{\psi}_j + (V_\mu \tilde{A})_i \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta \]

\[ \delta_{\text{Nonlinear}} F^i = (V_\mu \bar{\psi})^{i \alpha} \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta + T_j^{ai} \omega^a F^j \]

\[ \delta_{\text{Nonlinear}} \tilde{A}_i = 3g_{ijk}(u) A^j A^k + \frac{4}{m} g_{ijkl} A^j A^k A^l \]

\[ + T_i^{aj} \omega^a \tilde{A}_j \]

\[ \delta_{\text{Nonlinear}} V^a_\mu = V^{ab}_\mu \omega^b \]

\[ \delta_{\text{Nonlinear}} \bar{V}^{\mu a} = f^{abc} V^{\nu b} [\partial^\mu V^c_\nu - \partial_\nu V^c_\mu] + f^{abc} \bar{\psi}^{i \alpha} V^c_\nu + f^{abc} V^{\nu b} f^{cde} V^{\mu d} V^e_\nu \]

\[ - \frac{1}{2} f^{abc} \lambda^{\alpha \beta} \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta \]

\[ + T_j^{ai} A^j \partial^\mu \tilde{A}_i + T_j^{ai} \tilde{A}_i \partial^\mu A^j + T_j^{ai} m \bar{\psi}^i \bar{V}^{\mu k}_i \tilde{A}_k + V^{\mu k}_i m u^i \bar{T}^{aij}_k \tilde{A}_j \]

\[ + T_j^{ai} A^j \bar{V}^{\mu k}_i m \bar{u}_k + V^{\mu k}_i \bar{A}_k \bar{T}^{aij}_k m \bar{u}_j + T_j^{ai} A^j \bar{V}^{\mu k}_i \tilde{A}_k + V^{\mu k}_i A^j \bar{T}^{aij}_k \tilde{A}_j \]

\[ + T_j^{ai} \psi^{i \alpha} \sigma^\mu_{\alpha \beta} \bar{\psi}^j + T_j^{ai} A^j \psi^{i \alpha} \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta + T_j^{ai} \bar{A}_i \lambda \bar{\psi}^{i \alpha} \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta \]

\[ + T_j^{ai} \tilde{A}_i \lambda \psi^{i \alpha} \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta + T_j^{ai} A^j \psi^{i \alpha} \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta \]

\[ + (V_\mu \tilde{\lambda})^{\alpha a} \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta + (V_\mu \bar{\lambda})^{ai} \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta \]

\[ + f^{abc} \tilde{D}^b \left[ \frac{-i}{2} e^a \sigma^\mu_{\alpha \beta} \bar{\lambda}^\beta + \frac{i}{2} \lambda^{\alpha \beta} \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta \right] \]

\[ + f^{abc} (\bar{V}^{\mu b} - \partial^\mu \xi^b) \omega^c + \bar{\omega}^a \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta \]

\[ \delta_{\text{Nonlinear}} \lambda^a_\alpha = \frac{1}{2} f^{abc} V^{\nu b}_\nu \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta - f^{abc} \lambda^b_\alpha \omega^c \]

\[ \delta_{\text{Nonlinear}} \bar{\lambda}^a_\alpha = \frac{1}{2} \sigma^\mu_{\alpha \beta} (V_\mu \bar{\lambda})^{\alpha \beta} + T_j^{ai} A_i \psi^{i \alpha j} \]

\[ + f^{abc} \bar{\lambda}^b_\alpha \omega^c + \frac{i}{2} (V_\mu \bar{D})^a \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta \]

\[ \delta_{\text{Nonlinear}} D^a = \frac{-i}{2} e^a \sigma^\mu_{\alpha \beta} V^{\mu b} \bar{\lambda}^\beta + \frac{i}{2} V^{\nu b}_\nu \lambda^{\alpha \beta} \sigma^\mu_{\alpha \beta} \bar{\epsilon}^\beta + f^{abc} D^b \omega^c \]

25
\[ \delta_{\text{Nonlinear}} \tilde{D}^a = T_i^a \overline{\theta}_j A^i \]
\[ + f^{abc} \tilde{D}^b \omega^c \]  \hspace{1cm} (97)
\[ \delta_{\text{Nonlinear}} \omega^a = -\frac{1}{2} f^{abc} \omega^b \omega^c + c^\alpha \sigma^\mu_{\alpha \beta} \tilde{\tau}^\beta V^a \]  \hspace{1cm} (98)
\[ \delta_{\text{Nonlinear}} \tilde{\omega}^a = V^{\mu ab}(\tilde{V}^b - \partial_{\mu} \xi^a) \]
\[ + k m \xi^b \tilde{T}_j^{b j} u^i T_j^{a k} \omega^k A_k + k m \xi^b \tilde{T}_i^{b j} u_j^{a i} A^j \]
\[ + [\tilde{F}_i T_j^{a i} A^j + \tilde{\psi}_i T_j^{a i} \psi_j^a + \tilde{A}_i T_j^{a i} F^j + \text{comp. conj.}] \]
\[ - f^{abc} \tilde{\lambda}^b \lambda^c \tilde{\lambda} + f^{abc} \tilde{\lambda}^b \lambda^c + f^{abc} \tilde{D}^b D^c + f^{abc} \tilde{\omega}^b \omega^c \]  \hspace{1cm} (99)
\[ \delta_{\text{Nonlinear}} \xi^a = 0 \]  \hspace{1cm} (100)
\[ \delta_{\text{Nonlinear}} \tilde{\xi}^a = -\partial_\mu [V^{ab}_{\mu} \omega^b] \]
\[ + k m T_j^{a i} u^i T_j^{a k} \omega^k A_k \]
\[ + k m \tilde{T}_j^{a i} u_j^{a i} T_k^{a j} \omega^k A^k \]
\[ + c^\alpha \sigma^\mu_{\alpha \beta} \tilde{\xi}^\beta \partial_\mu \tilde{\xi}^a \]  \hspace{1cm} (101)
\[ \delta_{\text{Nonlinear}} \tilde{Z}^a = c^\alpha \sigma^\mu_{\alpha \beta} \tilde{\tau}^\beta \partial_\mu \xi^a \]  \hspace{1cm} (102)
\[ \delta_{\text{Nonlinear}} \tilde{Z}^a = 0 \]
\[ \delta_{\text{Nonlinear}} \xi_\mu = -c^\alpha \sigma^\mu_{\alpha \beta} \tilde{\xi}^\beta \]  \hspace{1cm} (103)
8 Gradings

We want to examine some aspects of the BRS cohomology of the foregoing formidable operator \( \delta \). To do this we shall use a spectral sequence, which in turn is generated by a grading. Familiarity with the methods and results of [10] [11] [12] [13] [14] and [15] will be assumed here. See also [16] [17] [18] for a different approach. The choice of the grading is far from unique. Each grading has some advantages and some disadvantages. To find the present one, all the terms in \( \delta \) were put on a microcomputer spreadsheet and a number of possibilities were tried. For this purpose, \( \delta \) was divided into 97 different terms. In general, for an arbitrary (integral) grading, \( \delta \) will split up into a sum of the form

\[
\delta = \sum_{i=-N_-}^{N_+} \delta_i
\]

However a spectral sequence arises in the manner contemplated in [10] only when the lower limit satisfies \( N_- = 0 \). We will call this a positive grading. By experimenting on the spreadsheet one quickly finds a number of gradings that grade \( \delta \) positively, and many more that give \( N_- < 0 \). It is necessary to try to choose one of the positive gradings that yields a sufficiently simple form for the low \( \delta_i \) and their Laplacians \( \Delta_i \). There is often an ‘equivalence class of gradings’ which all give rise to the same \( E_\infty \), and, in such cases, it does not make much difference which grading in the class one chooses.
A grading that seems very useful for the present problem is:

\[ N_{\text{Grading}} = 3[N'_{\text{Matter}} + \tilde{N}'_{\text{Matter}} + \overline{N}_{\text{Matter}} + \overline{N}'_{\text{Matter}}] \]

\[ + 2[N'_{\text{Gauge}} + \tilde{N}'_{\text{Gauge}}] + 7[N_{\text{Matter}} + \overline{N}_{\text{Matter}}] + 11N_{\text{Gauge}} \]

\[ + 3[N(\epsilon) + N(\bar{\epsilon})] + N_{\text{GaugeFixing}} + 4\tilde{N}_{\text{GaugeFixing}} \]

\[ + N(\epsilon) + 17N(\omega) + 2N(\tilde{\omega}) \]  

(105)

where

\[ N_{\text{Matter}} = N[A] + N[\psi] + N[F] \]  

(106)

\[ \tilde{N}_{\text{Matter}} = N[\tilde{A}] + N[\tilde{\psi}] + N[\tilde{F}] \]  

(107)

\[ N'_{\text{Matter}} = 3N[A] + 2N[\psi] + N[F] \]  

(108)

\[ \tilde{N}'_{\text{Matter}} = 3N[\tilde{A}] + 2N[\tilde{\psi}] + N[\tilde{F}] \]  

(109)

\[ N_{\text{Gauge}} = N[V] + N[\lambda] + N[\overline{\lambda}] + N[D] \]  

(110)

\[ \tilde{N}_{\text{Gauge}} = N[\tilde{V}] + N[\tilde{\lambda}] + N[\overline{\tilde{\lambda}}] + N[\tilde{D}] \]  

(111)

\[ N'_{\text{Gauge}} = 3N[V] + 2N[\lambda] + 2N[\overline{\lambda}] + N[D] \]  

(112)

\[ \tilde{N}'_{\text{Gauge}} = N[\tilde{V}] + 2N[\tilde{\lambda}] + 2N[\overline{\tilde{\lambda}}] + 3N[\tilde{D}] \]  

(113)

\[ N_{\text{GaugeFixing}} = N[Z] + N[\xi] \]  

(114)

\[ \tilde{N}_{\text{GaugeFixing}} = N[\tilde{Z}] + N[\tilde{\xi}] \]  

(115)

These definitions are motivated by the simple relations:

\[ [N'_{\text{Matter}}, \Lambda_{\alpha\text{Matter}}] = -\Lambda_{\alpha\text{Matter}} \]  

(116)
\[ [N'_\text{Matter}, \nabla \dot{\alpha}_{\text{Matter}}] = \nabla \dot{\alpha}_{\text{Matter}} \] (117)

with similar relations for all the other cases. This grading was found by trying to duplicate the success of the grading used for the pure chiral case without gauge fields or sources for BRS variations. It is also adapted for separating the gauge fields from the matter fields, since mixing them causes difficulties. It would be nice to treat the chiral fields and sources in exactly the same way, but this is incompatible with a positive grading and the other requirements. It seems a good idea to keep the degrees of \( V \) and \( \omega \) identical in view of the BRS cohomology of pure Yang-Mills, so that the operator \( \partial_\mu \omega^a \delta \frac{\delta}{\delta V^a_\mu} \) occurs in \( \delta_0 \) and so eliminates all derived \( \omega \) fields. Next one wants to ensure that the operator \( \epsilon^\mu \partial_\mu \) occurs after the operator \( \partial_\mu \omega^a \delta \frac{\delta}{\delta V^a_\mu} \) so that the results of [10] can eventually be used. The operator \( c^\alpha \sigma^{\mu\nu} \pi^i (\epsilon^\mu)^I \) must be of higher order than \( \epsilon^\mu \partial_\mu \) or else there are very difficult mixings.

Once these criteria are met, there is very little freedom in choosing the grading left in the problem. The other terms are determined so that \( \delta_0 \) generates some strong and simple restrictions.

Actually, it is clear that a great deal of information is obtained from the existence of a grading like the present one. It is susceptible of much more exploitation. I believe that the full problem can also be solved using this or a similar grading, but it requires lots more work, and there are still some tricky problems.
We note that the ghost number is given by the grading operator

\[ N_{\text{Ghosts}} = N[\omega] + N[\epsilon] + N(c) + N[\overline{\epsilon}] - N[\xi] \]

\[-N[\tilde{V}] - N[\tilde{\psi}] - N[\overline{\psi}] - 2N[\tilde{\omega}] - N[\overline{\lambda}] - N[\overline{\lambda}] \]

\[-N[\tilde{F}] - N[\overline{\tilde{F}}] - N[\tilde{\Lambda}] - N[\overline{\tilde{\Lambda}}] - N[\tilde{D}] - N[\overline{\tilde{D}}] \] (118)

It satisfies the simple relations

\[ [N_{\text{Ghosts}}, \delta] = \delta \] (119)

\[ [N_{\text{Ghosts}}, S_{\text{Total}}] = 0 \] (120)

\section{Space \( E_1 \)}

Using the grading above, we find

\[ \delta = \sum_{i=0}^{i=50} \delta_i \] (121)

and all these \( \delta_i \) (some of which are zero) will now be presented and discussed in the context of the limited result that we want to establish in this paper.

The first operator in the series is:

\[ \delta_0 = c^a \Lambda_{\alpha, \text{Matter}} + \bar{c}^\dot{\alpha} \bar{\Lambda}_{\dot{\alpha}, \text{Matter}} + c^a \tilde{\Lambda}_{\alpha, \text{Matter}} + \bar{c}^{\dot{\alpha}} \tilde{\bar{\Lambda}}_{\dot{\alpha}, \text{Matter}} \]

\[ + \int d^4x \{ Z^a \frac{\partial}{\partial \xi^a} + \tilde{\xi}^a \frac{\partial}{\partial \tilde{Z}^a} + \partial_\mu \omega^a \frac{\partial}{\partial V_\mu^a} + \partial_\mu \tilde{\tilde{\omega}}^a \frac{\partial}{\partial \tilde{V}_\mu^a} \} \] (122)

The cohomology of the \( c \)-dependent parts of this operator are already known and have been analyzed at length in \[ \mathbb{I} \]. The next two terms simply
eliminate all dependence on the four fields $\xi, Z, \tilde{\xi}$ and $\tilde{Z}$ from the cohomology space $E_1$. Their presence here means that we can now also ignore all the terms in the higher operators $\delta$ that depend on these fields, since these will all give zero when sandwiched between projection operators onto the space $E_1$. The next two terms are also simple to analyze—see [10]. In summary, the equations determining the space $E_1$ are:

\begin{align*}
(\omega^a_{\mu_1\ldots\mu_k})^\dagger E_1 &= 0; \ (k \geq 1) \\
(V^a_{(\mu_1\ldots\mu_k)})^\dagger E_1 &= 0; \ (k \geq 1) \\
(\tilde{\omega}^a_{\mu_1\ldots\mu_k})^\dagger E_1 &= 0; \ (k \geq 0) \\
(\tilde{V}^a_{\mu\mu_1\ldots\mu_k})^\dagger E_1 &= 0; \ (k \geq 0) \\
(\tilde{\xi}^a_{\mu_1\ldots\mu_k})^\dagger E_1 &= 0; \ (k \geq 0) \\
(\tilde{Z}^a_{\mu_1\ldots\mu_k})^\dagger E_1 &= 0; \ (k \geq 0) \\
(\xi^a_{\mu_1\ldots\mu_k})^\dagger E_1 &= 0; \ (k \geq 0) \\
(Z^a_{\mu_1\ldots\mu_k})^\dagger E_1 &= 0; \ (k \geq 0) \\
[\Lambda_\alpha \text{ Matter} + \tilde{\Lambda}_\alpha \text{ Matter}] E_1 &= 0 \\
[\tilde{\Lambda}_\alpha \text{ Matter} + \Lambda_\alpha \text{ Matter}] E_1 &= 0 \\
N(c)[N_{\text{Matter}} + \tilde{N}_{\text{Matter}}] E_1 &= 0 \\
N(\bar{c})[\bar{N}_{\text{Matter}} + \tilde{\bar{N}}_{\text{Matter}}] E_1 &= 0
\end{align*}
We must now refer to [15] for a discussion of the solution of the equations involving $\Lambda$. The current problem is no different except that there are additional variables $\tilde{A}$ etc.

### 10 The Operator $d_1$

The next operator in the sequence is:

$$
\delta_1 = c^\alpha \Lambda_\alpha \text{Gauge} + \bar{c}^\dot{\alpha} \bar{\Lambda}_{\dot{\alpha}} \text{Gauge} + c^\alpha \bar{\Lambda}_\alpha \text{Gauge} + \bar{c}^\dot{\alpha} \bar{\Lambda}_{\dot{\alpha}} \text{Gauge} + \epsilon^\mu \partial_\mu \\
+ \int d^4x \left\{ \frac{\delta}{\delta A_i} F_i^a \right. + F_i^a \frac{\delta}{\delta \bar{A}^a} + m T^{ai} Z^a \frac{\delta}{\delta \bar{F}_i} + m \bar{T}_i^a Z^a \frac{\delta}{\delta \bar{F}_i} \\
+ m T^{ai} c^\alpha \xi^a \frac{\delta}{\delta \psi_i^\alpha} + m T^{ai} \bar{c}^\dot{\alpha} \bar{\xi}^a \frac{\delta}{\delta \bar{\psi}_i^{\dot{\alpha}}} \\
+ m \bar{T}_i^a \bar{F}_i \frac{\delta}{\delta \bar{\omega}^a} + m T^{ai} \bar{F}_i \frac{\delta}{\delta \bar{\omega}^a} \\
+ m T^{ai} \omega^a \frac{\delta}{\delta A_i} + m T^{ai} \omega^a \frac{\delta}{\delta A_i} \right\} \Pi_1
$$

(135)

Using the properties of $\Pi_1$ above, we immediately deduce that $d_1$ has the considerably simpler form

$$
d_1 = \Pi_1 \delta_1 \Pi_1 = \Pi_1 \left\{ c^\alpha \Lambda_\alpha \text{Gauge} + \bar{c}^\dot{\alpha} \bar{\Lambda}_{\dot{\alpha}} \text{Gauge} + c^\alpha \bar{\Lambda}_\alpha \text{Gauge} + \bar{c}^\dot{\alpha} \bar{\Lambda}_{\dot{\alpha}} \text{Gauge} + \epsilon^\mu \partial_\mu \\
+ \int d^4x \left\{ \frac{\delta}{\delta A_i} F_i^a \right. + F_i^a \frac{\delta}{\delta \bar{A}^a} + m T^{ai} \omega^a \frac{\delta}{\delta A_i} + m T^{ai} \omega^a \frac{\delta}{\delta A_i} \right\} \Pi_1
$$

(136)

Here we shall not take up the large task of analyzing the cohomology of these $\Lambda_\alpha \text{Gauge}$ operators. It looks likely that this may be done in a fairly straightforward way along the lines of [15] and it also appears that the answer may again be highly nontrivial.
11 Masses and the Equation of Motion

In this paper we often have to deal with the dimensional parameter $m$. All other parameters can be chosen to be dimensionless multiples of this parameter to various powers.

Let us examine a very simple example. Suppose that we start with the simplest example of (6)

$S_{\Phi} = \int d^6z \frac{1}{m} \Phi^\alpha D^2 (S_1 D_\alpha S_2) = \int d^4x \frac{1}{m} \{ \chi^\alpha F_1 \psi_2 + \cdots \}$

(137)

and somehow generate the corresponding general form of (7) (We will ignore the index on $S_i$):

$\delta \Gamma_{\Phi} = \int d^6z \frac{1}{m} \Phi^\alpha c_\alpha [g_1 m^3 S + g_2 m^2 \overline{S}^2 + g_3 m \overline{S}^3 + g_4 \overline{S}^4]$}

$= \int d^4x \frac{1}{m} \{ \chi^\alpha c_\alpha [g_1 m^3 \overline{A} + g_2 m^2 \overline{A}^2 + g_3 m \overline{A}^3 + g_4 \overline{A}^4] + \cdots \}$

(138)

Here the $\frac{1}{m}$ is inserted so that $\chi^\alpha$ will have its canonical dimension of $\frac{3}{2}$, and the dimensions of the anomalous part are fixed by simple dimensional counting. All the coefficients $g_i$ are dimensionless. Then clearly some of the terms (138) are related by the pure chiral part of the equation of motion. How do we pick out the physically important part that remains in the cohomology space when the equation of motion part is included?

The first question is whether $m$ should be treated as a spacetime independent field like the supersymmetry ghost $c_\alpha$ or merely as a constant, for the purposes of finding the BRS cohomology of the operator $\delta$. In fact, it is
clearly necessary to treat it as a field for dimensional consistency. We will now see how this works.

Suppose that the $A$ field bit of the pure chiral part of the equation of motion in $\delta$ for this case has the very simple form:

$$\delta = [mA + gA^2] \tilde{A}^\dagger$$

Then its adjoint is:

$$\delta = \tilde{A} [mA + gA^2]^\dagger = \tilde{A} [A^\dagger m^\dagger + gA^\dagger A^\dagger]$$

and we use the relation

$$[m^\dagger, m] = 1$$

thus treating $m$ like a constant field, rather than as a constant.

How does the cohomology work for this case? Let us use the spectral sequence method for this operator, using the grading

$$N_{\text{Grading}} = AA^\dagger + \tilde{A} \tilde{A}^\dagger$$

Then

$$\delta_0 = m\Omega$$

where

$$\Omega = AA^\dagger$$

and

$$\delta_1 = gA^2 \tilde{A}^\dagger$$
We easily see that
\[ \Delta_0 = [N(A) + N(\tilde{A})]N(m) + \Omega^\dagger \Omega \]  
so that
\[ E_1 = X(A) + Y(m) \]  
where \( X \) and \( Y \) are arbitrary functions of the indicated variables and no others. Now clearly
\[ \delta_1 E_1 = 0 \]  
because \( E_1 \) is independent of \( \tilde{A} \) and we have
\[ E_\infty = E_1 \]  
for the same reason: all the operators \( d_r \) are zero for \( r \geq 1 \) because they all need \( \delta_1 \Pi_r = 0 \) in their construction.

But how do we find the correspondence
\[ E_\infty \to H \]  
for use in extracting the physically meaningful part of (138) that is in the cohomology space of the full operator \( \delta \) including the equation of motion term?

One way to see it is to write the above expression as an expression which vanishes by the equation of motion plus a remainder. This is
\[ \int d^4x \frac{1}{m} \{ \chi^\alpha c_\alpha [g_1 m^3 \tilde{A} + g_2 m^2 \tilde{A}^2 + + g_3 m \tilde{A}^3 + g_4 \tilde{A}^4] + \cdots \} \]
\[
= \int d^4 x \frac{1}{m} \left\{ \chi^\alpha c_{\alpha} \left[ [m \bar{A} + g \bar{A}^2] [g'_1 m^2 + g'_2 m \bar{A} + g'_3 \bar{A}^2] + g'_4 \bar{A}^4 + \cdots \right] \right. \\
= \int d^4 x \frac{1}{m} \left\{ \chi^\alpha c_{\alpha} \left[ [\delta \bar{A} [g'_1 m^2 + g'_2 m \bar{A} + g'_3 \bar{A}^2] + g'_4 \bar{A}^4 + \cdots \right] \right. \\
\] (151)

and the coefficient of the last term, which is the anomaly, is easily seen to be

\[
g'_4 = g_4 - gg_3 + g^2 g_2 - g^3 g_1 \] (152)

This should be gauge-invariant and physically meaningful. Clearly the correspondence (150) here is simply an identity—we take the term that has no masses to be our cohomology space and all the others are in the image of \( \delta \), but some care is needed to get the coefficient right as shown above. Let us check that this coefficient is indeed singled out by the cohomology by writing the expression in a different way:

\[
\int d^4 x \frac{1}{m} \left\{ \chi^\alpha c_{\alpha} \left[ [m \bar{A} + g \bar{A}^2] [g'_1 m^2 + g'_2 m \bar{A} + g'_3 \bar{A}^2] + g''_1 m^3 \bar{A} \right] \right. \\
\] (153)

Now the physically meaningfull quantity should be \( g''_1 \). We find that it is given by:

\[
g''_1 = \frac{1}{g_3} [g_4 - gg_3 + g^2 g_2 - g^3 g_1] \] (154)

Note the close relation between (152) and (154). In particular, since \( g \) is a physical coupling constant, it is clear that gauge invariance of one implies gauge invariance of the other. In this formulation the isomorphism (150) is realized in a less obvious way.
It would be possible to introduce more factors of \( \frac{1}{m} \) in this context and generate an infinite series, and in this way ‘eliminate the cohomology space’. Clearly once this process starts it must be continued to all powers of \( \frac{1}{m^k} \) to completely ‘eliminate all anomalies’. However this would not be a natural procedure in the present context, because no more factors of \( \frac{1}{m} \) should be introduced than were present in the starting operator (137). Since the theory is renormalizable so long as we do not propagate the \( \Phi_\alpha \) field and restrict ourselves to treating it to first order, there is no justification for introducing such an infinite series of renormalizations of arbitrary non-renormalizable order.

Incidentally it seems unlikely that an operator as simple as (137) will be likely to develop an anomaly, because the diagrams for it are not linearly (or more) divergent, which is probably necessary to develop an anomaly, if we can judge from the known cases.

12 The Space \( E_\infty \text{Special} \)

It is not at all obvious, looking at the highly complicated operator \( \delta \) above, whether one can find a subspace of the cohomology space \( H = \ker[\delta + \delta^\dagger] \approx E_\infty \) without finding the solution to the whole problem. But, fortunately, this can in fact be done here, as we now explain.

We shall concentrate on the following kind of polynomial and shall find
a set of restrictions on it which imply that it will belong to $E_\infty$.

$$E_0 \text{ Special} = P[A, \psi, F, \bar{A}, \bar{\psi}, \bar{F}, N(\bar{\tau}) \geq 1, N(\epsilon) = 4]$$

(155)

and we will find equations for the spaces

$$E_r \text{ Special} = E_0 \text{ Special} \cap E_r$$

(156)

What we mean here is that this polynomial depends only on the field variables shown and no others, that it contains no derivatives $\partial_\mu$, that it has $N(\bar{\tau}) \geq 1$ and $N(\epsilon) = 4$. Of course the complex conjugate of the above works the same way and we shall not treat it separately.

The problem we must confront is that these polynomials do get mixed with others by the operators $d_r$ in general, and we have to demonstrate that, with suitable restrictions, all the operators $d_r$ and $(d_r)^\dagger$ do in fact give zero on the subspace $E_\infty \text{ Special}$. We will now find a further set of constraints which we can impose to ensure that this is indeed a subspace of $E_\infty$.

13 The Space $E_2 \text{ Special} = E_2 \cap E_0 \text{ Special}$

Our next concern is with the last five terms of (136) since all the operators $c^\alpha \Lambda_\alpha \text{ Gauge} + \bar{c}^\dot{\alpha} \bar{\Lambda}_{\dot{\alpha}} \text{ Gauge} + c^\alpha \bar{\Lambda}_\alpha \text{ Gauge} + \bar{c}^\dot{\alpha} \Lambda_{\dot{\alpha}} \text{ Gauge} \text{ (and their adjoints)}$ certainly give zero on our subspace $E_\infty \text{ Special}$ because it contains no gauge fields. First we have the operator

$$\Pi_1 \epsilon^\mu \partial_\mu \Pi_1$$

(157)
This is zero on our subspace because it has $N(\epsilon) = 4$ and the adjoint is zero because our subspace contains no derivatives. The terms

\[ \int d^4x \left[ F_i \frac{\delta}{\delta A_i} + F^i \frac{\delta}{\delta A^i} \right] \]  

are zero in the antichiral subspace that has $N(c) \geq 1$ and in the chiral subspace that has $N(\bar{c}) \geq 1$, since it takes chiral to antichiral or vice versa (same for adjoints). Finally we have the terms:

\[ \Pi_1 \int d^4x \{ mT^a_i \omega^a \frac{\delta}{\delta A^i} + m\bar{T}^a_i \omega^a \frac{\delta}{\delta A^i} \} \Pi_1 \]  

These are the well known homogeneous terms that prevent the appearance of mass terms for Goldstone bosons in a spontaneously broken theory. The situation here is more complicated because we are already in $E_1$ so that the projection operators have a non-trivial effect in this operator. We will simply impose the equations

\[ [T^a_i A^i] \dagger E_{2\text{Special}} = [T^a_i \psi^i] \dagger E_{2\text{Special}} = [T^a_i F^i] \dagger E_{2\text{Special}} \]

\[ = [T^a_i \bar{A}^i] \dagger E_{2\text{Special}} = [T^a_i \bar{\psi}^i \alpha] \dagger E_{2\text{Special}} = [T^a_i \bar{F}^i] \dagger E_{2\text{Special}} = 0 \]  

which are stronger than we need. These equations eliminate from the cohomology space all those chiral multiplets (and their sources) which contain the Goldstone bosons of the spontaneous breaking of the gauge symmetry. So these equations eliminate any dependence on the Goldstone type multiplets, but still allow dependence of $E_{2\text{Special}}$ on all the other chiral multiplets. A complete treatment of these equations here would lead us into a treatment
of the gauge fields too, since ω plays an important role in the operator above in (153).

14 The Space $E_{3\text{Special}} = E_3 \cap E_{0 \text{Special}}$

The next term in the expansion of δ is:

$$\delta_2 = \int d^4x \left\{ \partial_\mu Z^a \frac{\delta}{\delta V^a_\mu} + \Box \xi^a \frac{\delta}{\delta \bar{\omega}^a} \right\}$$

(161)

It is easy to see that

$$\delta_2 E_{2\text{Special}} = \delta_2^\dagger E_{2\text{Special}} = 0 \quad (162)$$

because this space contains no gauge fields or ghosts. But the complete $d_2$ operator is actually of the form

$$d_2 = \Pi_2 \left\{ \delta_1 \frac{\delta^\dagger}{\Delta_0} \delta_1 - \delta_2 \right\} \Pi_2$$

(163)

Therefore, to check that

$$d_2 E_{2\text{Special}} = d_2^\dagger E_{2\text{Special}} = 0 \quad (164)$$

requires a bit more work. These do in fact give zero because the other terms in $d_2$ all mix chiral with antichiral fields.

15 The Spaces $E_{r\text{Special}} = E_r \cap E_{0\text{Special}}$

For $r \geq 3$, the formula for $d_r$ becomes increasingly complicated in terms of the operators $\delta_r$ and the Laplacians $\Box$. We will imagine for present
purposes that we can simply take

\[ d_r \approx \Pi_r \delta_r \Pi_r \]  \hspace{1cm} (165)  

and we will not need (at present) to return to the problem of analyzing the
correct expression for \( d_r \)–the point is that we will find a subspace of \( H \) in the
following process, and we will therefore be able to verify our result without
the spectral sequence once we have found it. But the spectral sequence helps
us to organize the task and enables us to use results that have already been
derived.

The next operators are:

\[ \delta_3 = \int d^4x \left\{ k Z^a \frac{\delta}{\delta Z^a} + \sigma^\mu_{\alpha\beta} e^{\alpha} \frac{\delta}{\delta \lambda_\beta} \right\} \]  \hspace{1cm} (166)  

\[ \delta_5 = \epsilon^{\alpha}_{\alpha} \nabla_{\alpha} \text{Gauge} + \epsilon^{\hat{\alpha}} \nabla_{\hat{\alpha}} \text{Gauge} + \epsilon^{\alpha}_{\alpha} \nabla_{\alpha} \text{Matter} + \epsilon^{\hat{\alpha}} \nabla_{\hat{\alpha}} \text{Matter} \]

\[ + c^{\alpha}_{\alpha} \epsilon^\mu \epsilon^\mu \]  \hspace{1cm} (167)  

Since \( E_r \text{Special} \) does not depend on gauge or ghost fields, we easily see that
\( d_r \approx \Pi_r \delta_r \Pi_r \); \( r = 2, 3, 4, 5 \) and their adjoints are all zero on our subspace
\( E_r \text{Special} \).

Next we have

\[ \delta_6 = \epsilon^{\alpha}_{\alpha} \nabla_{\alpha} \text{Matter} + \epsilon^{\hat{\alpha}} \nabla_{\hat{\alpha}} \text{Matter} + \epsilon^{\alpha}_{\alpha} \nabla_{\alpha} \text{Matter} + \epsilon^{\hat{\alpha}} \nabla_{\hat{\alpha}} \text{Matter} \]

\[ + \int d^4x \left\{ c^{\alpha}_{\alpha} \sigma^\mu_{\alpha\beta} \epsilon^\beta \omega^a \frac{\delta}{\delta V_{\mu a}} \right\} \]

\[ + c^{\alpha}_{\alpha} \sigma^\mu_{\alpha\beta} \epsilon^\beta V^a_{\mu} \frac{\delta}{\delta \omega^a} + c^{\alpha}_{\alpha} \sigma^\mu_{\alpha\beta} \epsilon^\beta \partial_{\mu} \tilde{Z}^a \frac{\delta}{\delta \tilde{\xi}^a} \]
\[ + c^\alpha \sigma^\mu_{\alpha\beta} \bar{c}^\beta \partial_\mu \xi^\alpha \frac{\delta}{\delta Z^a} \} \]  

The first part of this operator was discussed in [15]. It is

\[ d_{6;\nabla} = \Pi_6 \delta_{6;\nabla} \Pi_6 = \Pi_6 \{ c^\alpha \bar{\nabla}_\alpha \text{Matter} + \bar{c}^\alpha \nabla_\alpha \text{Matter} \]

\[ + c^\alpha \bar{\nabla}_\alpha \text{Matter} + \bar{c}^\alpha \nabla_\alpha \text{Matter} \} \Pi_6 \]  

The Laplacian for this part is, when operating on chiral polynomials that contain no derivatives:

\[ \Delta_{6;\nabla} = \Pi_6 \{(\nabla_\alpha)\dagger \Pi_6 \nabla_\beta + \pi[M - 4] \]

\[ + (\nabla_\alpha)\dagger \Pi_6 \nabla_\alpha + \pi[M - 4] \} \Pi_6. \]  

where we use the following abbreviations

\[ M = 4N(F) + 2N(\psi) \]  

\[ \overline{M} = 4N(\overline{F}) + 2N(\overline{\psi}) \]

As is discussed at length in [15], this operator restricts us to functions that have at most one \( F \) or two \( \psi \) in them (but still any number of \( A \) fields).

Also there is a nontrivial condition from the equation

\[ \Pi_6 (\nabla_\alpha)\dagger \Pi_6 E_{7 \text{ Special}} = 0 \]  

This equation and all the others so far can be solved by taking a product of those chiral superfields which are not Goldstone superfields of the spontaneous gauge symmetry breaking, with no derivative operators, and then
integrating the result over chiral superspace. The result will be in $E_{7\text{Special}}$.

The rest of $d_6$ and its adjoint clearly give zero on our special subspace.

The next operator yields a number of important restrictions. It is:

$$
\delta_7 = \int d^4 x \left\{ c^\alpha D^a \frac{\delta}{\delta D^a} + \sigma^\mu_{\alpha\beta} \partial_\mu \bar{\psi}_i^\beta \frac{\delta}{\delta \psi_i} 
+ m g^{ij} F_j^i \frac{\delta}{\delta F_i^j} + m g^{ij} \bar{A}_j^i \frac{\delta}{\delta \bar{A}_i^j} + m g^{ij} \bar{\psi}_j^i \frac{\delta}{\delta \bar{\psi}_i^j}
+ m g^{ij} A_j^i \frac{\delta}{\delta A_i^j} + m g^{ij} \bar{\psi}_j^i \frac{\delta}{\delta \bar{\psi}_i^j} + m g^{ij} \bar{\psi}_j^i \frac{\delta}{\delta \bar{\psi}_i^j}
+ c^\alpha T^{ai}_{\alpha\beta} \bar{\psi}_i^\beta \frac{\delta}{\delta V_{\alpha\mu}} \right\} (174)
$$

This is very similar to the problem that we analyzed above in section (11) except that there are more fields and the mass matrices must be diagonalized.

For simplicity of notation, let us assume that it is diagonal. Let us define the operator $\Omega$ by:

$$
m\Omega = \int d^4 x \left\{ m g^{ij} F_j^i \frac{\delta}{\delta F_i^j} + m g^{ij} A_j^i \frac{\delta}{\delta A_i^j} + m g^{ij} \bar{\psi}_j^i \frac{\delta}{\delta \bar{\psi}_i^j} \right\}
\approx m \sum_j e_j \left[ F_j^i \bar{F}_j^i + A_j^i \bar{A}_j^i + \psi_j^i \bar{\psi}_j^i \right] (175)
$$

Note that this operator commutes with supersymmetry

$$\{ \Lambda_\alpha, \Omega \} = 0 (176)$$

so that it is easy to find the solution at this point. The relevant form is

$$E_{8\text{Special}} = P_{\text{Massless}}[m, A, \psi, \bar{A}, \bar{\psi}, \bar{F}, N(\bar{\tau}) \geq 1, N(\epsilon) = 4]$$
\[ \Omega P_{\text{Massive}}[\bar{A}, \bar{\psi}, \bar{F}, N(\tau) \geq 1, N(\epsilon) = 4] \] (177)

where by \( P_{\text{Massless}} \) is we mean that this arbitrary polynomial can depend only on massless chiral superfields but also on the parameter \( m \), whereas by \( P_{\text{Massive}} \) we mean that if any monomial in this arbitrary polynomial depends on a massive chiral superfield, then it cannot also depend on the parameter \( m \). This follows from the following form of the Laplacian:

\[ \Delta_7 = \Pi_7 \{ [N_{\text{Massive}}(A) + N_{\text{Massive}}(\bar{A})] N(m) + \Omega^\dagger \Omega \} \Pi_7 \] (178)

where

\[ N_{\text{Massive}}(A) + N_{\text{Massive}}(\bar{A}) = [\Omega + \Omega^\dagger]^2 \] (179)

So at this stage we have a cohomology space which includes both massive fields to be treated in the rather complicated way indicated in section (11) and also massless fields which can occur accompanied by explicit powers of mass and consequently may be easier to separate from other terms. The equation of motion terms in \( \delta_7 \) again are automatically zero in the subspace \( E_7\text{Special} \) because they mix chiral with antichiral fields or sources. Next we have

\[ \delta_9 = \int d^4x \ m \left\{ T^a_i \chi^a_{\alpha} \frac{\delta}{\delta \psi^1_{i\alpha}} + T^{ai} \chi^i_{\alpha} \frac{\delta}{\delta \psi^1_{i\alpha}} \right\} \]

\[ + T^i \psi^i_{\alpha} \frac{\delta}{\delta \chi^i_{\alpha}} + T^{ai} \bar{\psi}^i_{i\alpha} \frac{\delta}{\delta \chi^i_{\alpha}} \right\} \] (180)

\[ \delta_{10} = \int d^4x \ m \left\{ D^a T^i_\alpha \frac{\delta}{\delta F^i} + \text{c.c.} + (T^a_i A^i + T^{ai} \bar{A}^i) \frac{\delta}{\delta D^a} \right\} \] (181)
These are eliminated because of (160).

\[ \delta_{11} = \int d^4x \left\{ \sigma^\mu_{\alpha\beta} \partial_\mu \lambda^{\alpha\beta} \frac{\delta}{\delta \lambda^\alpha_a} + \sigma^\mu_{\dot{\alpha}\beta} \partial_\mu \lambda^{\alpha\beta} \frac{\delta}{\delta \dot{\lambda}^\alpha_a} \right\} \]  \quad (182)

This is eliminated because our special subspace contains no gauge fields.

\[ \delta_{13} = \int d^4x \left\{ \Box A_i \frac{\delta}{\delta F_i} + \Box A^i \frac{\delta}{\delta F^i} \right\} \]  \quad (183)

These equation of motion terms are zero in the subspace because they mix chiral with antichiral fields.

\[ \delta_{14} = \int d^4x \left\{ \partial_\mu V^{\mu a} T^a_i \frac{\delta}{\delta F_i} + \text{c.c.} + m[T^{ai} \partial^\mu \overline{A}_i + T^a_i \partial_\mu A^i] \frac{\delta}{\delta V^{\mu a}} \right\} \]  \quad (184)

Again, this is zero because of (160) or because it contains derivatives.

\[ \delta_{15} = \int d^4x \left\{ (\Box V^{a\mu} - \partial^\mu \partial^\nu V^a) \frac{\delta}{\delta V^{a\mu}} + kmT^a_i \xi^{\alpha} \frac{\delta}{\delta \xi^\alpha} + \text{c.c.} + km(T^{ai} \overline{A}_i + T^a_i A^i) \frac{\delta}{\delta Z^a} \right\} \]  \quad (185)

Again, this is zero because of (160).

\[ \delta_{16} = \int d^4x \left\{ \partial^\mu V^a \frac{\delta}{\delta Z^a} + \Box \omega^a \frac{\delta}{\delta \xi^a} \right\} \]  \quad (186)

This is zero because it contains none of the fields in our special subspace.

Now we come to an important operator:

\[ \delta_{17} = \int d^4x \left\{ -f^{abc} \bar{\lambda}_b^\alpha \lambda^{\alpha c} + f^{abc} \bar{\lambda}^{bcd} \lambda_{d}^\alpha \frac{\delta}{\delta \omega^a} \right\} \\
- \frac{1}{2} f^{abc} \omega^a \omega^c \frac{\delta}{\delta \omega^a} + \omega^a J^a + c^\alpha \sigma^{\alpha \beta} \partial_\mu \lambda^{a\beta} \frac{\delta}{\delta \xi^\alpha} + \text{c.c.} + c^\alpha \sigma^{\alpha \beta} T^{a} \bar{V}_\mu \bar{\psi}_i \frac{\delta}{\delta F_j} \right\} \]
\[ + km T_i^b \psi_{i\alpha} \frac{\delta}{\delta \xi^a} + \text{c.c.} \]
\[ + \left[ f^{abc} \bar{D}^b D^c + V^{\mu ab} \bar{V}^b + \bar{\psi}_i^a T_j^i \psi_j^a + \bar{\psi}_i^a T_j^i \psi_j^a \right] \frac{\delta}{\delta \bar{\psi}_i^a} \]
\[ + \bar{A}_i T_j^a F^j + \bar{A}_i \bar{T}_j^a \bar{F}^j + A^i T_j^a F^j + \bar{A}_i \bar{T}_j^a \bar{F}^j \] \frac{\delta}{\delta \bar{\psi}_i^a} \right) \quad (187) \]

where

\[ J^a(x) = \sum_{\text{All Fields (except } \xi, Z, \bar{\xi}, \bar{Z})} (\text{Field})^j(x) T_j^a \omega^c \frac{\delta}{\delta (\text{Field})^i(x)} \quad (188) \]

Two new restrictions arise from this operator. They are:

\[ J^a E_{18\text{Special}} = 0 \quad (a = \text{non-goldstone directions only}) \quad (189) \]

At this point it is necessary to point out that only those \( \omega^a \) which are still gauge symmetries survive to this stage. The others were eliminated when the Goldstone modes were eliminated, when the equations (160) were imposed.

Next we have:

\[ \delta_{18} = \int d^4 x \left\{ k m \left[ \xi^b T_i^b u^i T_j^a \bar{A}_k + \xi^b T_i^b u^i T_j^a \bar{A}_i \right] \frac{\delta}{\delta \bar{\omega}^a} \right. \]
\[ + k m \xi^a T_j^a \bar{\omega}^b \frac{\delta}{\delta F_i} + \frac{1}{2} f^{abc} V^{\mu b} V^{\nu c} \sigma^{\mu \nu} \frac{\delta}{\delta \lambda^a} \left. \right\} \quad (190) \]
\[ \delta_{19} = \int d^4 x V^{\mu b} \partial_\mu \xi^b \frac{\delta}{\delta \bar{\omega}^a} \quad (191) \]

These both yield zero.

\[ \delta_{22} = \int d^4 x \left\{ f^{abc} \bar{D}^b \left[ -\frac{i}{2} c^a \sigma^{\mu \alpha} \bar{\psi}^\beta + \frac{i}{2} \lambda^a \sigma^{\mu \alpha} \bar{\psi}^\beta \right] \frac{\delta}{\delta \bar{V}_\mu^a} \right. \]
\[ + (V_\mu D^a)^a \sigma^a \bar{\psi}^\beta \frac{\delta}{\delta \bar{\lambda}_a^a} - \left[ -\frac{i}{2} c^a \sigma^{\mu \alpha} V^{ab} \bar{\chi}^\beta + \frac{i}{2} \lambda^{ab} \sigma^{\mu \alpha} \bar{\psi}^\beta \right] \frac{\delta}{\delta \bar{D}^a} \right\} + \text{c.c.} \quad (192) \]
This yields no restriction. Next

\[
\delta_{23} = \int d^4x \left\{ \left[ V^j_{\mu j} \nabla^\mu_k m \pi_k + g_{ijk} \left[ 2A^j F^k + \psi^{\alpha j} \psi^k_{\alpha} \right] \right] \frac{\delta}{\delta \tilde{F}_i} + (V_\mu \tilde{A})_i \sigma^{\mu}_{\alpha \beta} \tilde{\sigma}^j_{\alpha} \frac{\delta}{\delta \tilde{\psi}_i} + g_{ijk} A^j A^k \frac{\delta}{\delta A_i} + c.c. \right\} \Pi_{23} \tag{193}
\]

This may generate a non-trivial constraint since the argument made in section (11) will not work in general. We shall not attempt a general treatment of it here, except to mention that again this operator commutes with supersymmetry.

\[
\{\delta'_{23}, \Lambda_\alpha + \tilde{\Lambda}_\alpha \} = 0 \tag{194}
\]

where we define

\[
\delta'_{23} = \int d^4x \left\{ g_{ijk} \left[ A^j \psi^k_{\alpha} \frac{\delta}{\delta \tilde{\psi}_i} \right] \frac{\delta}{\delta \tilde{\psi}_i} + \left[ 2A^j F^k + \psi^{\alpha j} \psi^k_{\alpha} \right] \frac{\delta}{\delta \tilde{F}_i} + A^j A^k \frac{\delta}{\delta A_i} + c.c. \right\} \Pi_{23} \tag{195}
\]

Next

\[
\delta_{24} = \int d^4x \left\{ (V_\mu A)^i \sigma^{\mu}_{\alpha \beta} \tilde{\sigma}^j_{\alpha} \frac{\delta}{\delta \tilde{\psi}_i} + \sigma^{\mu}_{\alpha \beta} (V_\mu \tilde{A})_i \frac{\delta}{\delta \tilde{\psi}_i} + c.c. \right\} \tag{196}
\]

The first term needs some thought. It is of course required to make gauge invariant the transformation of the \( \psi \) field when derivatives are present. For our special subspace with no derivatives, it can be ignored. The reason is that for \( k = 1 \), (124) eliminates all \( V_\mu \) with no derivatives from \( E_1 \) (See [10] for details). The second term mixes chiral and antichiral and so it can be
ignored as usual. Now, for reasons already given, all terms vanish up to $\delta_{39}$.

\[
\delta_{25} = \int d^4x \left\{ \frac{\delta}{\delta F_i} \left[ \psi_j \cdot T_i \cdot \overline{\lambda}_a \cdot \frac{\delta}{\delta \psi_j} + T_i \cdot \overline{\phi}_j \cdot \overline{\sigma}_{\mu \alpha \beta \gamma} \cdot \frac{\delta}{\delta \phi_j} + T_i \cdot \overline{A}_i \cdot \psi^a \cdot \frac{\delta}{\delta \psi^a} \right] \right\} + \text{c.c.} \tag{197}
\]

\[
\delta_{26} = \int d^4x \left\{ D^a \cdot T_i \cdot \overline{A}_i \cdot \frac{\delta}{\delta F_i} + T_i \cdot \psi^a \cdot \overline{\sigma}_{\mu \alpha \beta \gamma} \cdot \frac{\delta}{\delta \phi_j} + T_i \cdot \overline{A}_i \cdot A^i \cdot \frac{\delta}{\delta D^a} \right\} + \text{c.c.} \tag{198}
\]

\[
\delta_{28} = \int d^4x \left\{ \frac{1}{2} f^{abc} \overline{\lambda}_a \cdot \frac{\delta}{\delta \psi_i} + \sigma_{\alpha \beta \gamma} \cdot \frac{\delta}{\delta \phi_j} \right\} \tag{199}
\]

\[
\delta_{30} = \int d^4x \left\{ [V^a \cdot \overline{\lambda}_a \cdot \frac{\delta}{\delta F_i} + T_i \cdot \psi^a \cdot \overline{\sigma}_{\mu \alpha \beta \gamma} \cdot \frac{\delta}{\delta \phi_j} + T_i \cdot \overline{A}_i \cdot A^i \cdot \frac{\delta}{\delta D^a} \right\} \tag{200}
\]

\[
\delta_{32} = \int d^4x \left\{ D^a \cdot T_i \cdot \overline{A}_i \cdot \frac{\delta}{\delta F_i} + T_i \cdot \psi^a \cdot \overline{\sigma}_{\mu \alpha \beta \gamma} \cdot \frac{\delta}{\delta \phi_j} + T_i \cdot \overline{A}_i \cdot A^i \cdot \frac{\delta}{\delta D^a} \right\} \tag{201}
\]

\[
\delta_{33} = \int d^4x \cdot \partial_{\mu} \cdot V^a \cdot \omega^b \cdot \frac{\delta}{\delta \xi^b} \tag{202}
\]

\[
\delta_{35} = \int d^4x \left\{ [V^a \cdot \overline{\lambda}_a \cdot \frac{\delta}{\delta F_i} + T_i \cdot \psi^a \cdot \overline{\sigma}_{\mu \alpha \beta \gamma} \cdot \frac{\delta}{\delta \phi_j} + T_i \cdot \overline{A}_i \cdot A^i \cdot \frac{\delta}{\delta D^a} \right\} \tag{203}
\]

All the above give no new constraints. However $\delta_{39}$ does yield some new equations.

\[
\delta_{39} = \int d^4x \frac{12}{m} f_{ijkl} \left\{ (A^j \cdot A^k \cdot F^l + \psi^a \cdot \psi^b \cdot A^l) \cdot \frac{\delta}{\delta F_i} + A^j \cdot A^k \cdot \psi^a \cdot \frac{\delta}{\delta A^l} + 1 \cdot A^j \cdot A^k \cdot A^l \cdot \frac{\delta}{\delta A_i} \right\} \tag{204}
\]

and once again this operator commutes with supersymmetry. Finally we have

\[
\delta_{49} = \int d^4x \cdot f^{abc} \cdot V^{\mu \nu} \cdot f^{cde} \cdot V^a_{\mu} \cdot V^b_{\nu} \cdot \frac{\delta}{\delta V^a_{\mu}} \tag{205}
\]
\[ \delta_{50} = \int d^4 x \left[ T^a_\cdot A_j^i V^\mu_k \cdot A_k + V^\mu_k A^i T^a_\cdot A_j \right] \frac{\delta}{\delta V^a_\mu} \]  \hspace{1cm} (206)

and these give nothing new.

16 Equations for \( E_{\infty \text{Special}} \)

Collected together we get:

\[ \Lambda_{\alpha \text{Matter}} E_{\infty \text{Special}} = 0 \]  \hspace{1cm} (207)

\[ \Pi_6 \nabla_\beta \text{Matter} E_{\infty \text{Special}} = 0 \]  \hspace{1cm} (208)

\[ \Pi_6 [M - 4] E_{\infty \text{Special}} = 0 \]  \hspace{1cm} (209)

where we use the following abbreviation

\[ M = 4N(F) + 2N(\psi) \]  \hspace{1cm} (210)

So this restricts us to functions that have at most one \( F \) or two \( \psi \) in them. However they can still have any number of \( A \) fields. Next, we also have the equations that result from the requirement that this subspace be independent of the Goldstone modes and not vanish by the equations of motion. The relevant subspace was \([187]\) as restricted by equations \([160]\) and \([189]\), but I have not tried to further explicitly restrict this with the operators \([195]\) and \([204]\). That is best left for specific cases where the tensors are known explicitly.
17 The Space $H_{\text{Special}}$

Now we give rules to construct the space $H_{\text{Special}} \approx E_{\infty\text{Special}}$. The simplest case to analyze is the case when there is no chiral action at all and consequently no spontaneous gauge symmetry breaking or VEVs. In this case the answer is already known from previous work, and the answer is that $H_{\text{Special}}$ contains all possible expressions of the form

$$\int d^4 x d^2 \Phi^\alpha \mathcal{S}_{i_1} \cdots \mathcal{S}_{i_k} t^{i_1, i_2, \ldots, i_k} c_\alpha$$

(211)

where the tensors $t^{i_1, i_2, \ldots, i_k}$ are invariant tensors under the group:

$$\sum_{q=1}^k T^{a i_q}_{a j_q} t^{i_1, i_2, \ldots, j_q, \ldots, i_k} = 0$$

(212)

What we have discovered by the foregoing analysis is very simple. When $S_{\text{Chiral}}$ is non-zero, the set of constraints on the cohomology space $H_{\text{Special}}$ changes in the following ways:

1. When $S_{\text{Chiral}}$ is non-zero, one gets the additional constraints (we assume the renormalizable case again here):

$$[m g_{i j}(u) F^j + g_{i j k}(u) (A^j F^k + \psi^{i j}_{\alpha} \psi^{k \alpha})] H_{\text{Special}} = 0$$

(213)

$$[m g_{i l}(u) A^l + g_{i j k}(u) A^j A^k] H_{\text{Special}} = 0$$

(214)

$$[g_{i l}(u) \psi^i_{\alpha} + g_{i j k}(u) A^j \psi^k_{\alpha}] H_{\text{Special}} = 0$$

(215)

In general I would expect that there are plenty of solutions—equivalently there are usually plenty of supersymmetric polynomials with the
required gauge invariance that are not related by the equations of motion. The simplest such objects will be those which commence with a zero mass superfield.

2. When the VEV is not zero, one gets the additional constraints

\[ [T^a_i A^i]^\dagger H_{\text{Special}} = 0 \]  
\[ [T^a_i \psi^{i\alpha}]^\dagger H_{\text{Special}} = 0 \]  
\[ [T^a_i F^i]^\dagger H_{\text{Special}} = 0 \]  

3. Once the equations for (2) are satisfied, the invariance of the tensor in (212) reduces to invariance under the remaining gauge invariance after gauge symmetry breaking as required by equation (189).

4. It is clear that we do not generate the whole cohomology space in this way, but only a part of it. The rest must wait for a more complete solution of the cohomology problem.

18 Discussion of the Superpotential

Now that we have the equations for \( H_{\text{Special}} \), and we have discussed the solutions, let us discuss the superpotential from this point of view. In most cases of interest, the reducible representation \( T^{ai}_j \) is fully reducible and reduces to a set of irreducible representations of the group:

\[ S^i = \sum_I S^i_I \]  

51
These irreducible chiral superfields then can have the following properties:

1. The first question for each $S_i^I$ is whether any of its $A_i^I$ components develop a VEV or not. If

$$< A_i^I >= \mu^i \neq 0 \quad (220)$$

then we shall call it a ‘Higgs’ multiplet.

2. The second question for each $S_i^I$ is whether any of its components $A_i^I$ contributes to a Goldstone Boson or not. If one or more of them do, we shall call the multiplet a ‘Goldstone’ multiplet. Goldstone bosons always occur in Higgs multiplets because that is the way that the Yang-Mills vector scalar mixing terms arise in the shifted action before ‘t Hooft style gauge fixing.

3. The third question for each $S_i^I$ is whether any of its $A_i^I$ components do not appear in mass terms after spontaneous gauge symmetry breaking even though they are not Goldstone boson–these massless but non-Goldstone bosons can be distinguished by the fact that they do not have mixing terms with the Yang-Mills vector fields, or equivalently, that the multiplet possesses massless scalars but has zero VEV:

$$< A_i^I >= 0 \quad (221)$$

Multiplets of this kind we shall call massless matter chiral superfields. Our example below illustrates this phenomenon, which is well known
in the standard model where there is no right handed partner for the neutrino, so that it must be massless even after spontaneous gauge symmetry breaking.

4. Finally there are multiplets which are none of the above. These have zero VEV, no Goldstone bosons and no other massless bosons. These are the massive matter multiplets. It is currently believed that the quarks are in massive matter multiplets.

In all the cases above, each boson has supersymmetry partners of course, because supersymmetry is not broken.

Now return to our discussion of $H_{\text{Special}}$. Clearly the simplest solutions of these equations involve massless matter chiral superfields, but there are also solutions that involve massive matter multiplets in the way indicated in section (11). We shall not try to discuss the higher constraints in any more detailed way–one simply has to verify that the equations are solved for any particular case.

19 A Simple Example

This example was also presented in [4]. The present discussion will be more complete since the earlier discussion was necessarily rather short by reason of space restrictions.

We consider a supersymmetric gauge theory based on the gauge group $SU(2)$ with matter in two vector multiplets and a singlet: $L^a : I = 1; H^a :$
\( I = 1; R : I = 0 \). These ‘a’ indices transform with \( i\epsilon^{abc} \) and take the values \( a=1,2,3 \). Since the ‘a’ indices are real and since \( \delta_{ab} \) is an invariant tensor of \( SU(2) \), there is no difference when we raise and lower these indices.

Without any good reason, we shall assume that the superpotential does not contain a mass term for the \( L \) field. It is easy to find more natural examples where no mass term is possible for the relevant fields \([3]\).

Since the ‘Higgs field’ \( H^a \) is in a real representation of the gauge group, it can have a mass term in the superpotential.

Now we assume the following form for the superpotential:

\[
W = g_1 L^a H^a R + \frac{g_2 m}{2} H^a H^a + \frac{g_3}{4m} [H^a H^a]^2
\] (222)

Note that renormalizability is not a property of this superpotential.

If \( g_2 g_3 < 0 \), the Higgs field will develop a VEV in its ‘A’ term that breaks the gauge symmetry down to \( U(1) \) while leaving supersymmetry unbroken. The \( L \) and \( R \) fields develop no VEV. Let us denote components as follows:

\[
L^a = A^a + \theta^\alpha \psi_\alpha^a + \frac{1}{2} \theta^2 F^a
\] (223)

\[
H^a = B^a + m\mu^a + \theta^\alpha \phi_\alpha^a + \frac{1}{2} \theta^2 G^a
\] (224)

\[
R = A + \theta^\alpha \psi_\alpha + \frac{1}{2} \theta^2 F
\] (225)

We distinguish \( a = i,3 \) where \( i = 1,2 \). Here we have included a shift by the VEV:

\[
<B^a>_{\text{before shift}} = \delta^{a3} m \sqrt{-\frac{g_2}{g_3}} \equiv \delta^{a3} m h \equiv m\mu^a
\] (226)
Then the ‘F’ term of the superpotential becomes:

\[ W_F = \left[ g_1 L^a (H^a + \delta^a \alpha m h) R + \frac{g_2 m}{2} (H^a + \delta^a \alpha m h)(H^a + \delta^a \alpha m h) \right. \]

\[ + \frac{g_3}{4m} [(H^a + \delta^a \alpha m h)(H^a + \delta^a \alpha m h)]^2 \]  

(227)

In terms of components, this makes the following contribution to the action:

\[
S_{\text{Chiral}} = \int d^4 x W_F = \int d^4 x \left\{ g_1 (m h A^3 F + m h \psi^3 \alpha \psi_\alpha + m h F^3 A) \right. \\
+ g_1 (A^a B^a F + \psi^a \phi^a A + F^a B^a A + A^a \phi^a \psi + A^a G^a A + \psi^a B^a \psi) \\
+ \text{terms involving H superfield only} \right\} 
\]

(228)

The essential point to note here is that there is no mass term like

\[ m(A^i F + \psi^i \alpha \psi_\alpha + F^i A) \]

(229)

for \( i = 1, 2 \) which would give a mass to the \( L^1 \) and \( L^2 \) superfields. They are massless after spontaneous breaking of the gauge symmetry, but they are not Goldstone fields. The Goldstone bosons are contained in the fields \( H^i \).

Equally important for the example below is the fact that \( L \) does not appear quadratically in the superpotential so that there is no term like \( A^i A^i \) in the equations of motion either.

The simplest composite operator (together with a source \( \Phi_\alpha \) that could develop a supersymmetry anomaly seems to be:

\[ S_{\text{Composite}} = \frac{1}{m^3} \int d^4 x d^4 \theta \left\{ \Phi^\alpha D^3 [\bar{L}^\alpha H^\alpha D^3 D_\alpha R] \right\} \]

(230)
The fraction $\frac{1}{m^3}$ is included so that $\Phi_\alpha$ will have its canonical dimension of $\frac{1}{2}$. After translation of the Higgs field, we find the terms

$$S_{\text{Composite}} = \frac{1}{m^2} h \int d^4x \left\{ \chi^\alpha \left[ (\sigma^\mu)^\gamma_\beta \partial_\mu A^3_{\alpha\beta} \sigma^\nu_{\alpha\beta} \partial_\nu \psi_\gamma \right. \\
+ \left. \overline{\psi}^{3\beta} (\sigma^\mu)_{\alpha\beta} \partial_\mu F + \cdots \right] + \cdots \right\}$$

(231)

The form of the supersymmetry anomaly that we would like to calculate is

$$\delta \Gamma_\Phi = m e \int d^4x d^2\theta \Phi^\alpha c_\alpha \sum_{i=1,2} T_i T_i' = m e \int d^4x \chi^\alpha c_\alpha \sum_{i=1,2} A_i A_i' + \cdots$$

(232)

This is in the cohomology space for the reasons given above—it is linearly independent of the polynomials that vanish by the equations of motion of this theory, and because these fields $A^i$ are massless, there is no complicated mixing problem to separate their coefficients like we need to do for massive fields in section (11).

The relevant triangle diagrams here are linearly divergent and it appears dimensionally possible for the anomaly to appear. Do these diagrams add together to preserve supersymmetry or not? Should one introduce a regularization somehow into the calculation? At present I am not sure how to calculate the coefficient $e$ in this or any model. The first question clearly is ‘Which diagrams should one calculate?’ This question will be the topic of the next sections.
20 Tactics for Computing Supersymmetry Anomalies

Now let us consider how one would choose the diagrams to actually compute a supersymmetry anomaly. For this purpose, it is useful to simplify our operator down to a more tractable size, to see how things work.

The following transformations are the gauge and supersymmetry transformations for a very simple theory, involving only one chiral superfield. We suppress the Yang-Mills fields and the Goldstone mechanism for present purposes. This operator is the full BRS operator which contains the partial operator analyzed in section (11).

\[ \delta A = c^\alpha \psi_\alpha \]  
\( \delta \psi_\alpha = \partial_\mu A_{\alpha \dot{\beta}} \sigma^\mu_{\alpha \dot{\beta}} \tilde{\psi} + F c_\alpha \)  
\( \delta F = \partial_\mu \psi^\alpha \sigma^\mu_{\alpha \dot{\beta}} \tilde{c}^{\dot{\beta}} \)  
\( \delta \tilde{A} = c^\alpha \tilde{\psi}_\alpha - \tilde{F} + 2mA + 3gAA \)  
\( \delta \tilde{\psi}_\alpha = \partial_\mu \tilde{A}_{\alpha \dot{\beta}} \sigma^\mu_{\alpha \dot{\beta}} \tilde{c}^{\dot{\beta}} + \tilde{F} c_\alpha + \sigma^\mu_{\alpha \dot{\beta}} \partial_\mu \tilde{\psi}^{\dot{\beta}} + 2m\psi^\alpha + 6A\psi_\alpha \)  
\( \delta \tilde{F} = \partial_\mu \tilde{\psi}^\alpha \sigma^\mu_{\alpha \dot{\beta}} \tilde{c}^{\dot{\beta}} + \Box \tilde{A} + 2mF + 6g(AF + \psi^\alpha \psi_\alpha) \)  
\( \delta \phi_\alpha = \tilde{c}^{\dot{\beta}} W_{\alpha \dot{\beta}} \)  
\( \delta W_{\alpha \dot{\beta}} = \partial_\mu \phi_\alpha \sigma^\mu_{\dot{\beta} \gamma} \tilde{c}^\gamma + \chi_\alpha \tilde{c}^{\dot{\beta}} \)  
\( \delta \chi_\alpha = \partial_\mu W_{\alpha \dot{\beta}} \sigma^\mu_{\dot{\beta} \gamma} \tilde{c}^\gamma \)
Note that we have kept a simple non-linear term in the equation of motion, so that we can see its effect.

21 General Form of $P_\Phi(G = 1, D = 2)$

Now let us write down the most general Lorentz invariant integrated local polynomial, linear in the $\Phi$ superfield, of ghost charge 1 with the very low dimension 2. It is:

$$P_\Phi(G = 1, D = 2) = \int d^4 x \left\{ \chi^\alpha c_\alpha [g_1 \overline{A} + g_2 A] ight. \\
+ W^\alpha \overline{\beta} [g_3 c_\alpha \overline{\psi}_\beta + g_4 \overline{\psi}_\alpha \overline{\tau}_\beta] + W^\alpha \overline{\beta} c_\alpha \overline{\tau}_\beta [g_5 \overline{A} + g_6 \overline{A}] \\
+ \phi^\alpha c_\alpha [g_7 \overline{F} + g_8 F + g_9 m A + g_{10} m \overline{A} + g_{11} A^2 + g_{12} A \overline{A} + g_{13} \overline{A}^2] \\
+ \phi^\alpha c_\alpha [g_{14} \overline{\psi}^\beta c_\beta + g_{15} \overline{\psi} \overline{\tau}_\beta] + \phi^\alpha \sigma^\mu_\alpha \overline{\tau}_\beta [g_{16} \partial_\mu A + g_{17} \partial_\mu \overline{A}] \right\}$$

We note that:

1. The dimension of the source superfield $\tilde{S}$ is two whereas the dimension of the superfield $S$ is only one. This limits the possibilities of constructing terms with more than one $c_\alpha$, since such terms have to be accompanied by $\tilde{S}$ fields to compensate the higher ghost charge associated with more than one $c$.

2. There are more terms with $\phi$ than there are with $W$ and more terms with $W$ than there are with $\chi$. This is due to the fact that $\phi, W$ and $\chi$ occur linearly. The number of possibilities is greater for the terms
accompanying $\phi$ than for the terms accompanying $\chi$ because $\phi$ has lower dimension than $\chi$.

3. This general polynomial will not arise in perturbation theory—only the subspace $K_\Phi(G = 1, D = 2) \subset P_\Phi(G = 1, D = 2)$ of such polynomials which satisfy

$$\delta K_\Phi(G = 1, D = 2) = 0 \quad (243)$$

could be expected to arise. One could simply solve this set of equations for the maximal free set of coefficients $g_i$, and one would call the resulting polynomial $K_\Phi(G = 1, D = 2)$ the set of all $\delta$-closed polynomials. But there is another way to construct this space of course, which is the whole point of cohomology theory. We will do that below.

4. The number of terms here will rapidly increase as the dimension $D$ in $P_\Phi(G = 1, D = 2)$ rises. We need to be able to pick out some special minimum number of terms to calculate the physically relevant part while ignoring the physically irrelevant boundary (image of $\delta$) terms.

22 Counterterms and the Anomaly

Continuing with our dimension 2 example, we now write down the most general polynomial linear in $\Phi$ with ghost charge 0 and dimension 2:

$$P_\Phi(G = 0, D = 2) = \int d^4x \{e_1 \phi^\alpha \psi_\alpha + e_2 \phi^\alpha c_\alpha \tilde{A} + e_3 \phi^\alpha c_\alpha \tilde{A}\} \quad (244)$$
Now we can use our cohomology result, which states that the most general solution of the equation (243) takes the form:

\[ K_\Phi(G = 1, D = 2) = \delta P_\Phi(G = 0, D = 2) + e_4 H_\Phi(G = 1, D = 2) \]  

(245)

and we recall that in general we could expect to get the following result in perturbation theory

\[ \delta \Gamma_\Phi = K_\Phi(G = 1, D = 2). \]  

(246)

For the present case, we get from (244) and the above transformations in (233)–(241):

\[
K_\Phi(G = 1, D = 2) = \int d^4x \left\{ e_1 [\bar{c}_\beta W^{\alpha\beta} \dot{\psi}_\alpha + \phi^\alpha (\partial_\mu A^\mu_{\alpha\beta} \bar{c}^\beta + Fc_\alpha) \\
+ e_2 [\bar{c}_\beta W^{\alpha\beta} c_\alpha \tilde{A} + \phi^\alpha c_\alpha (c^\beta \bar{\psi}_\beta - \bar{F} + 2mA + 3gA^2)] \\
+ e_3 [\bar{c}_\beta W^{\alpha\beta} c_\alpha \tilde{A} + \phi^\alpha c_\alpha (\bar{c}^\beta \bar{\psi}_\beta - F + 2mA + 3gA^2)] \\
+ e_4 [\chi^\alpha c_\alpha \tilde{A} + W^{\alpha\beta} c_\alpha \bar{\psi}_\beta + \phi^\alpha c_\alpha \bar{F}] \right\}
\]  

(247)

In this particular case, the term \( \chi^\alpha c_\alpha \tilde{A} \) occurs with the coefficient \( e_4 \), which is the physically meaningful coefficient of the anomaly. What term in \( \Gamma \) could give rise to this term? The only way that this could arise in the expression \( \delta \Gamma_\Phi \) is from the terms

\[
e_4 \int d^4x \chi^\alpha c_\alpha \tilde{A} = \int d^4x [\partial_\mu A^\mu_{\alpha\beta} c_\alpha \frac{\delta}{\delta \bar{\psi}_\beta}] \Gamma(\chi, \bar{\psi}) \\
+ \int d^4x [(\Box \tilde{A} \frac{\delta}{\delta \bar{F}})] \Gamma(\chi, c, \bar{F})
\]  

(248)
The derivative in the functional derivative operator could conceivably convert the non-local functionals \( \Gamma(\chi, \bar{\psi}) \) and \( \Gamma(\chi, c, \tilde{F}) \) into the local cohomologically nontrivial term \( \int d^4x \chi^\alpha c_\alpha A \) with a non-zero coefficient \( e_4 \).

The coefficient \( e_4 \) could also be obtained as the coefficient of the term \( W^{\alpha \beta} c_\alpha \bar{\psi}_\beta \) according to

\[
e_4 \int d^4x W^{\alpha \beta} c_\alpha \bar{\psi}_\beta = \int d^4x [\partial^\mu \bar{\psi}_\beta \sigma^\mu_{\alpha \beta} c_\alpha c_\delta \frac{\delta}{\delta F^\delta}] \Gamma(W, \bar{F})
+ \int d^4x [\partial^\mu W^{\alpha \beta} \sigma^\mu_{\beta \alpha} c_\alpha \frac{\delta}{\delta \chi^\alpha}] \Gamma(\chi, \bar{\psi})
+ \int d^4x [\partial^\mu \bar{\psi}_\alpha \sigma^\mu_{\beta \alpha} \frac{\delta}{\delta \psi_\beta}] \Gamma(W, c, \bar{\psi})
\]

(249)

This computation is harder than the one involving \( \chi \) because it requires knowledge of three parts of \( \Gamma \) rather than two. It is even harder to calculate the coefficient \( e_4 \) using the term \( \phi^\alpha c_\alpha \bar{F} \), because it gets contributions from the boundary term with coefficient \( e_2 \). To obtain \( e_4 \) from \( \phi^\alpha c_\alpha \bar{F} \) one would need to calculate the coefficient \( e_2 \) of the boundary terms also and then subtract. So clearly the \( \chi_\alpha \) method is the easiest. Let us summarize some of the lessons we have found here:

1. One could expect that the coefficient of the anomaly is generated most easily from calculation of the \( \chi_\alpha \) terms in the action, since there are fewer of them and they are more likely to be directly linked to the anomaly coefficient. In higher dimensional cases one would calculate at a minimum the terms \( \Gamma(\chi, \bar{A}, \bar{A}, \cdots, \bar{\psi}) \) and \( \Gamma(\chi, c, \bar{A}, \bar{A}, \cdots, \bar{F}) \) with
one $\chi$ and one $\bar{\psi}$ or $\bar{F}$ and all available numbers of $A$ fields—see section (11).

2. A detailed look at the way the boundary terms contribute to the coefficients will always be needed for specific cases, and this will become increasingly complicated as the dimension increases.

3. The essential equation for finding the coefficient of the anomaly is (248) and its generalization to higher dimensions and more fields.

4. Attempting to find the coefficient of the anomaly using equations like (249) involves more work and is more likely to get mixed up with boundary terms as the dimension increases. Of course, it would be desirable to confirm that this gives the same result if one first finds a non-zero result for the form (248).

5. The present example is useful because it is easy to generate complete expressions which give results of general validity, but it has too low a dimension to have any application to actual Feynman diagrams, since there is no composite antichiral spinor superfield with dimension $\frac{1}{2}$, which is what the present case would require. Indeed the lowest dimension composite antichiral spinor superfield is (5) which has dimension $3\frac{1}{2}$, and even it still looks too low in dimension (from the point of view of divergence of the relevant Feynman diagrams) to have a reasonable chance of developing an anomaly.
6. The discussions in sections (11) and (19) are now seen to deal with the important part of the calculation for practical purposes—namely the $\chi_\alpha$ part.

23 Conclusion

Our result is that when one considers the complete BRS operator for gauge-fixed supersymmetric Yang-Mills theory coupled to chiral matter in four spacetime dimensions with spontaneous breaking of gauge symmetry, but not of supersymmetry, one finds that there is a subspace $H_{\text{Special}}$ of the cohomology space which was described in section (17) and (18), and which can be dealt with along the lines of section (11) and (22).

Supersymmetry anomalies of the simplest kind envisioned here would involve superfields which are massless after spontaneous symmetry breaking but which do not contain Goldstone bosons. A perfect example of such a superfield is the neutrino superfield in the standard model. See [6] for more details of the current analysis in the context of the standard model. For reasons explained above, that kind of anomaly can only occur if the gauge symmetry is spontaneously broken.

If this were the end of the story, it would be interesting but also rather worrying from the phenomenological point of view, because it is hard to imagine that all supersymmetry breaking arises from an anomalous mixing of massive superparticles with the components of the neutrino superfield.
Fortunately, there are a large number of superanomalies involving massive fields also, but these are a little more involved to separate from non-physical parts which vanish by the equations of motion. For these other cases, it is not so obvious that one needs spontaneous gauge symmetry breaking to get the anomalies, but again there is probably a need to ‘mix up’ fields at a gauge-chiral vertex that seems most easily accomplished by a spontaneously broken theory. This needs further investigation.

One might be concerned that the equation of motion of the superfield sources $\Phi_\alpha$ have not been used in the above. Actually these fields have some complicated problems of their own, but it does not appear that including these complications would be likely to make the cohomology space empty or to significantly change the present results. It should also be mentioned that a non-trivial transformation of the $\Phi_\alpha$ sources under $U(1)$ is probably possible, which might be needed to explain the supersymmetry breaking of the charged particles.

In conclusion, it still seems to be possible that these results are the start of a phenomenologically interesting explanation of the breaking of supersymmetry for all observed particles. If this is indeed the explanation, then there must be a wealth of predictions available from pertubation theory with a minimum of uncalculable non-perturbative dynamical assumptions. The first step to test this possibility is to calculate some of the simpler examples to see whether these anomalies do indeed appear with non-zero coefficients.
The main issue there seems to be the question of an appropriate regularization to use for this problem.

Acknowledgments: I would like to thank my collaborators Ruben Minasian and Joachim Rahmfeld for many useful ideas, Ramzi Khuri for some very useful discussions about integrals, Heath Pois for many helpful remarks about phenomenological models of supersymmetry, Chris Pope for helping us to recognize the representations that arose in the solution of the higher spin problems and Mike Duff for his continued insistence that a supersymmetry anomaly be computed.

References

[1] Banks, T., Kaplan, D., Nelson, A.: ‘Cosmological Implications of Dynamical Supersymmetry Breaking’; Preprint UCSD/PTH 93-26, RU-37, hep-th@xxx/9308292.

[2] Brignole, A., Ibanez, L.E. and Munoz, C.: Towards a Theory of Soft Terms for the Supersymmetric Standard Model, Madrid Preprint, hep-ph/@xxx/9308271.

[3] Dixon, J. A.: A natural mechanism for supersymmetry breaking with zero cosmological constant. CTP-TAMU-69/92, to be published in: Duff, M. J., Khuri, R. R., Ferrara, S. (eds.) From superstrings to su-
pergravity. Workshop Proceedings, Erice 1992. Singapore, New Jersey, Hong Kong: World Scientific

[4] Dixon, J. A.: The Search for Supersymmetry Anomalies–Does Supersymmetry Break Itself? (Talk given at the HARC conference on ‘Recent Advances in the Superworld’), Preprint CTP-TAMU-45/93, hep-th/9308088, to be published in the proceedings, Eds. D. Nanopoulos, J. Lopez, A. Zichichi.

[5] O’Raiffeartaigh, L.: Nucl. Phys. B96, 331 (1975).

[6] Dixon, J. A.: Supersymmetry Anomalies and the Standard Model. Preprint CTP-TAMU-47/93 in preparation.

[7] ’t Hooft, G.: Nucl. Phys. B35, 167 (1971).

[8] Zinn-Justin, J.: in Lecture Notes in Physics, Vol 37, Eds J. Ehlers et al., (Springer Verlag, Berlin, 1975).

[9] Dixon, J. A.: Closure of the Algebra and Remarks on BRS Cohomology in D=10 Super Yang-Mills, Preprint UTTG-14-91, Unpublished.

[10] Dixon, J. A.: Calculation of BRS cohomology with Spectral Sequences. Commun. Math. Phys. 139, 495-526 (1991).

[11] Dixon, J. A.: BRS cohomology of the chiral superfield. Commun. Math. Phys. 140, 169-201 (1991)
[12] Dixon, J. A.: Supersymmetry is full of holes. Class. Quant. Grav. 7, 1511-1521 (1990)

[13] Dixon, J. A.: Anomalies, Becchi-Rouet-Stora cohomology, and effective theories. Phys. Rev. Lett. 67, 797-800 (1991)

[14] Dixon, J. A., Minasian, R.: BRS cohomology of the supertranslation in $D = 4$. Preprint CPT-TAMU-13/93 (hep-th@xxx/9304035)

[15] Dixon, J. A., Minasian, R., Rahmfeld, J.: Higher Spin BRS Cohomology of Supersymmetric Chiral Matter in $D = 4$. Preprint CTP-TAMU-20/93

[16] Brandt, F., Dragon, N., Kreuzer, M.: Phys. Lett. B231, 263-270 (1989), Nucl. Phys. B332, 224-249 (1990), Nucl. Phys. B332, 250-260 (1990), Nucl. Phys. B340, 187-224 (1990)

[17] Brandt, F.: Lagrangedichten und Anomalien in vierdimensionalen supersymmetrischen Theorien. PhD-Thesis, Universität Hannover (1991)

[18] Brandt, F.: Lagrangians and anomaly candidates of $D = 1$, $N = 4$ rigid supersymmetry. Nucl. Phys. B392, 928 (1993)