World-Volume Interactions on D-Branes

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ABSTRACT

We examine in detail various string scattering amplitudes in order to extract the world-volume interactions of massless fields on a Dirichlet brane. We find that the leading low-energy interactions are consistent with the Born-Infeld and Chern-Simons actions. In particular, our results confirm that the background closed string fields appearing in these actions must be treated as functionals of the non-abelian scalar fields describing transverse fluctuations of the D-brane.
1 Introduction

Recent years have seen dramatic progress in the understanding of nonperturbative aspects of string theory[6]. With these studies has come the realization that extended objects, other than just strings, play an essential role. An important tool in these investigations has been the Dirichlet brane[2], which within the framework of perturbative string theory provide an exact description of Ramond-Ramond charged solitons.

The world-volume theory of a single D-brane includes a massless U(1) vector $A_a$, a set of massless scalars $X^i$, describing the transverse oscillations of the brane, and their super-partner fermions $\psi^i$. The leading order low-energy action for these fields corresponds to a dimensional reduction of a ten dimensional U(1) super-Yang Mills theory. As usual in string theory, there are higher order $\alpha' = \ell_s^2$ corrections, where $\ell_s$ is the string length scale. As long as derivatives of the field strengths (and second derivatives of the scalars) are small compared to $\ell_s$, then the action takes a Born-Infeld form [4]. This action also has a supersymmetric extension[5]. To take into account the couplings of the open string states with closed strings, the Born-Infeld action can be extended to include background closed string fields, in particular, the metric, dilaton and Kalb-Ramond field. Thus one arrives at the following world-volume action for a Dp-brane:

$$S_{BI} = -T_p \int d^{p+1} \sigma \; \text{Tr} \left( e^{-\Phi} \sqrt{-\det(\tilde{G}_{ab} + \tilde{B}_{ab} + 2\pi\ell_s^2 F_{ab})} \right)$$

(1)

where $T_p = (2\pi\ell_s)^{-p}/g$ is the brane tension with $g$ being the asymptotic closed string coupling[4]. Here, $F_{ab}$ is the field strength of the world-volume gauge field, while the metric and antisymmetric tensors are the pull-backs of the bulk tensors to the D-brane world-volume, e.g.,

$$\tilde{G}_{ab} = G_{ab} + 2G_{i(a} \partial_{b)}X^i + G_{ij}\partial_aX^i\partial_bX^j.$$  

(2)

The D-branes are also carry RR charge and so to account for the coupling of the massless closed string RR states, the above action is supplemented by a Chern-Simons action of the form[5, 4]

$$S_{CS} = \mu_p \int \text{Tr} \left( e^{\tilde{B} + 2\pi\ell_s^2 F} \sum_n \tilde{C}_{(n+1)} \right)$$

(3)

where $\tilde{C}_{(n+1)}$ are the pull-backs of the $(n+1)$-form RR potentials and $\mu_p = \sqrt{2\pi(2\pi\ell_s)^{3-p}}$. Thus a Dp-brane is naturally charged under the $(p+1)$-form RR potential. However in the presence of background Kalb-Ramond fields or world-volume gauge fields, it may also carry a charge of the RR potentials with a lower form degree, as allowed with the couplings induced by the exponential factor[6]. Note that as well as the derivative couplings induced by the pull-back as in eq. (2), in general the background fields would be functionals of the scalars $X^i$ so that even in the leading low energy approximation these fields would be governed by a non-linear sigma model type action.

One of the most remarkable aspects of D-brane physics is that the U(1) gauge symmetry of an individual D-brane is enhanced to a non-abelian U(N) symmetry for N coincident D-branes[8]. When N parallel D-branes approach each other, the ground state modes of strings stretching between the D-branes become massless. These extra massless states carry the appropriate charges to then fill out U(N) representations and the $U(1)^N$ of the individual D-branes is

\[\text{Our convention will be that the dilaton field }\Phi\text{ vanishes asymptotically far from the D-branes.}\]
enhanced to $U(N)$. Hence $A_a$ becomes a nonabelian gauge field and the scalars $X^i$ becomes scalars in the adjoint representation of $U(N)$. Understanding how to accommodate this simple yet remarkable modification in the world-volume actions for general backgrounds is the primary motivation for the present paper, and we report some progress in this direction. In trivial backgrounds, the extension of the Chern-Simons action (3) is apparently straightforward with the addition of the trace over gauge indices of the non-abelian field strength which now appears in the exponential factor. Because of its highly nonlinear nature, the nonabelian extension of the Born-Infeld action is unclear even for a flat space background. Tseytlin has made compelling arguments to suggest that one should supplement the usual Born-Infeld form with a symmetrized trace over gauge indices, i.e., to leading order all commutators of the field strengths should be dropped. Unfortunately this does not seem to capture the full physics of the infrared limit, and it appears nontrivial commutators must be included at sixth-order in the field strength. To go beyond trivial backgrounds is a question which has received only limited attention. Douglas has proposed that the background fields should be functionals of the non-abelian scalars (rather than, e.g., only the $U(1)$ or center-of-mass component of $X^i$). An additional point noted by Hull is that it is natural to assume that the pull-backs in both eqs. (1) and (3) are modified to be defined in terms of covariant derivatives of the full adjoint scalar fields. That is $\partial_a X^i \to D_a X^i$ in eq. (2). As D-branes are objects originally defined within perturbative string theory, these questions can be resolved by studying string scattering amplitudes. This is the approach of our present investigation, and we are able to confirm proposals by Douglas and Hull. Thus the trace appearing in the two pieces of the action above, eqs. (1) and (3), acts implicitly on not only the gauge fields but also on the implicit gauge dependence in the background fields, both in the form of the derivatives of the pull-backs and the internal functional dependence.

The remainder of the paper is organized as follows: In section 2, we examine amplitudes involving one closed string and one or two open string states. These amplitudes allow us to identify interactions linear and quadratic in the world-volume fields of the D-branes. In section 3, we consider the world-volume interactions which can be inferred from amplitudes for three open strings and those for two closed strings. Note that in both of these sections, we only consider massless bosonic modes of the string. Finally, we present a brief discussion of our results in section 4.

Before continuing with our calculations, let us make a comment on conventions. In the scattering amplitudes below, we will set $\ell_s^2 = \alpha' = 2$. Our index conventions are that lowercase Greek indices take values in the entire ten-dimensional spacetime, e.g., $\mu, \nu = 0, 1, \ldots, 9$; early Latin indices take values in the world-volume, e.g., $a, b, c = 0, 1, \ldots, p$; and middle Latin indices take values in the transverse space, e.g., $i, j = p + 1, \ldots, 9$. Thus, for example, $G_{\mu\nu}$ denotes the entire spacetime metric, while $G_{ab}$ and $G_{ij}$ denote metric components for directions parallel and orthogonal to the D-branes, respectively.

## 2 Scattering Calculations

In this section, we investigate the open-closed string couplings by an analysis of the appropriate scattering amplitudes. We will consider scattering amplitudes with one closed string and one open string in the next subsection, and with one closed string and two open strings in the
subsequent subsection. The techniques for these calculations (and in fact the amplitudes in the second case) are available in refs. [15, 16, 17, 18], and we will refer the reader there for details. One useful observation is that these amplitudes can all be related to purely open string scattering amplitudes with unusual kinematics, and so in fact one can determine the desired amplitudes from much older calculations of open superstrings[19]. In fact, this result can be extended all tree-level open/closed string amplitudes[20].

The results should be compared to those for the various interactions arising in the actions (1) and (3). So we begin by calculating the interactions expected from these world-volume actions. In particular, we expand the actions for fluctuations around flat empty space, i.e., \( G_{\mu\nu} = \eta_{\mu\nu}, \ B_{\mu\nu} = 0 = \Phi = C(n) \). The fluctuations should be normalized as the conventional field theory modes which appear in the string vertex operators. As a first step, we transform from the string frame to Einstein frame metric with \( G_{\mu\nu} = e^{\Phi/2} g_{\mu\nu} \). The latter only affects the Born-Infeld action which becomes

\[
S_{BI} = -T_p \int d^{d+1}\sigma \ Tr \left( e^{\Phi/2} \sqrt{-det(g_{ab} + e^{-\Phi/2}B_{ab} + 2\pi\ell_s^2 e^{-\Phi/2}F_{ab})} \right)
\]

(4)

Now with conventions of [16], the string mode fluctuations take the form

\[
g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu} \]
\[
\Phi = \sqrt{2}\kappa\phi \]
\[
B_{\mu\nu} = -2\kappa b_{\mu\nu} \]
\[
C(n) = 4c(n) \]

where in the present conventions \( \kappa = 2^3\pi^{7/2}\ell_s^4g \). In the following, it is only the relative normalizations of these modes which will be important. Similarly the open string modes are normalized as

\[
A_a = \frac{1}{\sqrt{T_p2\pi\ell_s^2}} a_a \]
\[
X^i = \frac{1}{\sqrt{T_p}} \lambda^i
\]

(5)

Now continuing with the Born-Infeld action, it is straightforward, although somewhat tedious, to expand eq. (1) using

\[
\sqrt{det(\delta^a_b + M^a_b)} = 1 + \frac{1}{2} M^a_a - \frac{1}{4} M^a_b M^b_a + \frac{1}{8}(M^a_a)^2 + \frac{1}{6} M^a_b M^b_c M^c_a - \frac{1}{8} M^a_a M^b_c M^c_b + \frac{1}{48}(M^a_a)^3 + \ldots
\]

to produce a vast array of interactions. We are interested in the interactions linear in the closed string fluctuations, and linear or quadratic in the open string fields.

We begin with the linear couplings of the closed strings to the D-brane source itself (these will be useful in section 3, below)

\[
\mathcal{L}_0 = -T_p\kappa \left( h^a_a + \frac{p-3}{2\sqrt{2}}\phi \right)
\]

(6)

Next there are interactions involving one closed string mode and one open string mode

\[
\mathcal{L}_0' = -\sqrt{T_p}\kappa \ Tr \left( \lambda^i \partial_i h^a_a + \frac{p-3}{2\sqrt{2}} \lambda^i \partial_i \phi \right)
\]
\[
\mathcal{L}_1 = -\sqrt{T_p}\kappa \ Tr \left( 2h_{ia} \partial^a \lambda^i - b_{ab} f^{ab} \right)
\]

(7)
where \( f_{ab} = \partial_a a_b - \partial_b a_a \). We have divided these interactions into two sets: those in \( \mathcal{L}_1 \) appear naturally in the naive expansion where \( f_{ab} \) appears explicitly while \( \partial_a \lambda^i \) appears in the pull-back of the metric (\( \mathcal{L}_0 \)). The interactions in \( \mathcal{L}_0' \) arise essentially from a Taylor expansion in the transverse coordinates of the terms appearing in eq. (3). Thus these terms arise because the transverse fluctuations of the D-branes feel the variations of the background fields. Because only a single field appears under the gauge trace, these interactions only involve the U(1) or center-of-mass fluctuations. Finally we will need to compare to the interactions involving a single closed string mode and two open string modes

\[
\begin{align*}
\mathcal{L}_0'' &= -\frac{\kappa}{2} \text{Tr} \left( \lambda^i \lambda^j \partial_i \partial_j h^a_a + \frac{p-3}{2\sqrt{2}} \lambda^i \lambda^j \partial_i \partial_j \phi \right) \\
\mathcal{L}_1' &= -\kappa \text{Tr} \left( 2\lambda^j \partial_j h_{ia} \partial^a \lambda^i - \lambda^j \partial_j b_{ab} f^{ab} \right) \\
\mathcal{L}_2 &= -\kappa \text{Tr} \left( \frac{1}{2} h^a_a(\partial \lambda) - h^{ab} \partial_a \lambda^i \partial_b \lambda_i + h^{ij} \partial_i \lambda^j \partial^a \lambda^i \right) + \frac{p-3}{4\sqrt{2}} \phi(\partial \lambda)^2 \\
&\quad + \frac{1}{4} h^a_a(f)^2 - h_{ab} f_{ac} f^b_c + \frac{p-7}{8\sqrt{2}} \phi(f)^2 - 2 b_{ab} \partial_b \partial^i \partial^a \lambda_i f^{ab} \right) (8)
\end{align*}
\]

Here again the interactions in \( \mathcal{L}_0'' \) and \( \mathcal{L}_1' \) arise from the Taylor expansion of the previous interactions. At the level of these quadratic interactions, we begin to include the non-Abelian parts of the fluctuations.

The expansion to determine the interactions from the Chern-Simons action (3) is somewhat simpler. First we begin with the linear couplings of the closed strings to the D-brane

\[
\tilde{\mathcal{L}}_0 = 4\mu_p \frac{1}{(p+1)!} (c_{(p+1)})^{a_0 \cdots a_p} \epsilon^{v}_{a_0 \cdots a_p} \tag{9}
\]

where \( \epsilon^{v}_{a_0 \cdots a_p} \) is the volume-form on the D-brane. Next we have interactions linear in both the RR fields and open string modes

\[
\begin{align*}
\tilde{\mathcal{L}}_0' &= \frac{4\mu_p}{\sqrt{T_p}} \frac{1}{(p+1)!} \text{Tr} \left( \lambda^i \lambda^j \partial_i (c_{(p+1)})^{a_0 \cdots a_p} \right) \epsilon^{v}_{a_0 \cdots a_p} \\
\tilde{\mathcal{L}}_1 &= \frac{4\mu_p}{\sqrt{T_p}} \text{Tr} \left( \frac{1}{p!} \partial^a \lambda^i (c_{(p+1)})^{a_1 \cdots a_p} + \frac{1}{2(p-1)!} f^{a_0 a_1} (c_{(p-1)})^{a_2 \cdots a_p} \right) \epsilon^{v}_{a_0 \cdots a_p} \tag{10}
\end{align*}
\]

Finally we will also compare to the quadratic couplings

\[
\begin{align*}
\tilde{\mathcal{L}}_0'' &= \frac{4\mu_p}{T_p} \frac{1}{2(p+1)!} \text{Tr} \left( \lambda^i \lambda^j \partial_i (c_{(p+1)})^{a_0 \cdots a_p} \right) \epsilon^{v}_{a_0 \cdots a_p} \\
\tilde{\mathcal{L}}_1' &= \frac{4\mu_p}{T_p} \text{Tr} \left( \frac{1}{p!} \lambda^i \partial^a \lambda^j \partial_i (c_{(p+1)})^{a_1 \cdots a_p} + \frac{1}{2(p-1)!} f^{a_0 a_1} \lambda^i \partial_i (c_{(p-1)})^{a_2 \cdots a_p} \right) \epsilon^{v}_{a_0 \cdots a_p} \\
\tilde{\mathcal{L}}_2 &= \frac{4\mu_p}{T_p} \text{Tr} \left( \frac{1}{2(p-1)!} \partial^a \lambda^i \partial^1 \lambda^j (c_{(p+1)})^{a_2 \cdots a_p} + \frac{1}{2(p-2)!} f^{a_0 a_1} \partial^2 \lambda^i (c_{(p-1)})^{a_3 \cdots a_p} \\
&\quad + \frac{1}{8(p-3)!} f^{a_0 a_1} f^{a_2 a_3} (c_{(p-3)})^{a_4 \cdots a_p} \right) \epsilon^{v}_{a_0 \cdots a_p} \tag{11}
\end{align*}
\]

We have again divided the interactions between those arising in the Taylor expansion of the background (closed string) fields of previous lower order terms and new terms coming from additional factors of \( f_{ab} \) or the pull-backs.
Note that certain interactions do not contribute at leading order. For example, interaction $\text{Tr}[B_{ab}F_{bc}F_{ca}]$ does not contribute in (8), because the quadratic term vanishes under the gauge trace. However, this interaction will make contributions at higher orders as the background field $B$ is Taylor expanded.

2.1 Linear couplings

Here, we wish to compare the linear couplings in eqs. (7) and (10) to the results of the appropriate scattering amplitudes. As mentioned above, since these terms involve a single open string state the gauge trace will select out only the U(1) component of the open string fields – below we will keep the Chan-Paton factors explicit in any event. To compare to the Born-Infeld interactions (7), we consider the scattering amplitude for one open NS and one closed NS-NS state. The amplitude is given by

$$A_{\text{NS,NS-NS}} \sim \int dx_1 d^2z_2 \text{Tr} < V_{\text{NS}}(k_1, \zeta_1, x_1) V_{\text{NS-NS}}(p_2, \varepsilon_2, z_2, \bar{z}_2) >$$

The details of the vertex operators and the techniques in calculating may be found in refs. [15, 16, 17, 18]. The final result is

$$A_{\text{NS,NS-NS}} \sim i [2k_{1a}(\varepsilon_2 \cdot D)^{\mu a} \zeta_{1\mu} - 2k_{1a}(\varepsilon_2 \cdot D)^{\alpha \mu} \zeta_{1\mu} - p_{2\mu} D^{\mu \nu} \zeta_{1\nu} \text{Tr}(\varepsilon_2 \cdot D)] \text{Tr}(T_1)$$

where $D^\mu$ is a diagonal matrix with values +1 on the world-volume and -1 in the transverse space [16]. Substituting the appropriate polarizations, one finds the following scattering amplitudes for different states:

$$A(\lambda, h) \sim 2i(2k_{1a}(\varepsilon_2 \cdot D)^{\mu a} \zeta_{1\mu} + p_2^i \zeta_{1i} \varepsilon_2^{a}) \text{Tr}(T_1)$$

$$A(\lambda, \phi) \sim i \frac{p - 3}{\sqrt{2}} \zeta_{1i} p_2^i \text{Tr}(T_1)$$

$$A(a, b) \sim -2i(k_{1a} \zeta_{1b} + k_{1b} \zeta_{1a}) \varepsilon_2^{ab} \text{Tr}(T_1)$$

while $A(\lambda, b), A(a, h), A(a, \phi) = 0$. Comparing these results to eq. (7), it is clear that these string amplitudes are precisely reproduced by a field theory calculation with those interactions. The terms in $L_1$ reproduce $A(a, b)$ and the first term in $A(\lambda, h)$, while those in $L_0'$ yield $A(\lambda, \phi)$ and the second term in $A(\lambda, h)$. Meanwhile the vanishing of the remaining amplitudes is consistent with the fact that there are no corresponding interactions at this order.

To compare with the Chern-Simons interactions (10) at this order, we examine the scattering amplitude between one NS open and one RR closed string state, which is given by

$$A_{\text{NS,RR}} \sim \text{Tr} \int dx_1 d^2z_2 < V_{\text{NS}}(k_1, \zeta_1, x_1)V_{\text{RR}}(p_2, \varepsilon_2, z_2, \bar{z}_2) >$$

Again, we refer the reader to refs. [16, 17] for the details of the calculations, and simply state the final result

$$A_{\text{NS,RR}} \sim \text{Tr}(T_1) \zeta_{1\mu_0} (F_{(3)})_{\mu_1 \ldots \mu_n} \epsilon_{\alpha_0 \ldots \alpha_p} \text{Tr}[\gamma^{\mu_0 \gamma^{\mu_1} \ldots \gamma^{\mu_n} \gamma^{a_0} \ldots \gamma^{a_p}(1 + \gamma_{11})]$$

(13)
where \( F_{(n)} \) is the (Fourier transform of) the RR field strength for the \((n - 1)\)-form potential \( c_{(n-1)} \), i.e., \( (F_{(n)})_{\mu_1 \ldots \mu_n} = i n p_{2 \mu_1} \varepsilon_{2 \mu_2 \ldots \mu_n} \) — see \([14]\). Now choosing various explicit polarizations and performing the trace over gamma matrices, one finds two nonvanishing amplitudes:

\[
A(\lambda, c_{(p+1)}) \sim \zeta_i (F_{(p+2)})_{\lambda} e^{a_0 \ldots a_p} \epsilon_{a_0 \ldots a_p} \text{Tr}(T_1) \\
A(a, c_{(p-1)}) \sim \zeta_1^{a_0} (F_{(p)})^{a_1 \ldots a_p} \epsilon_{a_0 \ldots a_p} \text{Tr}(T_1) \\
\sim ip k_1 a_0 \zeta_1^{a_1} \epsilon_{a_2 \ldots a_p} e_{a_0 \ldots a_p} \text{Tr}(T_1)
\]

where we have used momentum conservation in the world-volume to rewrite the amplitudes in both cases, i.e., \( k_1 + p_\| = 0 \). In this second form, it is clear that these amplitudes are reproduced by the interactions given in eq. \([10]\). \( \tilde{L}_1 \) yields \( A(a, c_{(p-1)}) \) and the second term in \( A(\lambda, c_{(p+1)}) \), while \( \tilde{L}_0 \) corresponds to the first term in \( A(\lambda, c_{(p+1)}) \).

Hence in accord with expectations, it is clear that the background fields in the world-volume actions \([1] \) and \([3]\) are functionals of at least the U(1) or center-of-mass components of the scalar fields \( X^i \). This is necessary as the interactions arising from the Taylor expansion of the closed string fields were necessary to properly reproduce the string scattering amplitudes. A comment on these contributions to the amplitudes is called for as for a strictly physical configuration they will vanish. This arises because of the simple two particle kinematics. Both particles are massless, i.e., \( k_1^2 = 0 = p_2^2 \), and combined with world-volume momentum conservation i.e., \( k_1 + p_\| = 0 \), this implies that \( p_\perp^2 = 0 \) as there are no null directions in the transverse space. Hence one only really sees these interactions, in an analytic continuation of the momentum in which one allows for \( (p_\perp^2)^2 = 0 \) without having \( p_\perp^2 = 0 \).

A curious feature of the RR scattering amplitudes is that initially they appear in a form involving the RR field strength, so that the gauge invariance of these fields is manifest. However to match to the expansion of the Chern-Simons action \([3]\), an integration by parts is necessary and this gauge invariance is no longer manifest.

### 2.2 Quadratic couplings

Next, we investigate the couplings of closed strings quadratic in the open string fields. In fact, the appropriate scattering amplitudes with two open strings and one closed string have already been calculated in ref. \([17]\), and we need only interpret their results. Again, the reader may find the details of the calculations there. One begins with an amplitude of the form

\[
A \sim \int dx_1 dx_2 d^2 z_3 \text{Tr} < V^{\text{NS}}(k_1, \zeta_1, x_1) V^{\text{NS}}(k_2, \zeta_2, x_2) V^{\text{closed}}(p_3, \varepsilon_3, z_3, \bar{z}_3)>
\]

where the last vertex will either be a NS-NS or RR closed string vertex. The calculation here may be related to that of a four point amplitude of open superstrings, and the final amplitude takes the form \([17]\)

\[
A \sim \frac{\Gamma[-2t]}{\Gamma[1 - t]^2} K(1, 2, 3)
\]

where \( t = -2k_1 \cdot k_2 \) and the kinematic factor may be obtained from type I calculations \([19]\) with an appropriate substitution of momenta and polarizations. This amplitude has \( t \)-channel poles
at $t = 1, 2, 3, ...$ which correspond to propagation of on-shell open string states. Since we are only interested in the leading low energy contact terms, we take the limit $t \to 0$ for which eq. (15) simplifies to

$$A \sim -\frac{1}{2t} K(1, 2, 3).$$

There is actually no pole at $t = 0$ in this expression as the kinematic factor will provide a compensating factor of $t$.

Let us begin by considering the Born-Infeld couplings in detail, and so we choose a NS-NS vertex in eq. (14). Making the appropriate explicit choices of polarizations, we find

$$A(\lambda, \lambda, h) = (2k_1 \cdot k_2 \zeta_1 \cdot \epsilon_3 \cdot \zeta_2 + k_1 \cdot k_2 \zeta_1 \cdot \zeta_2 \cdot \epsilon_{3a} + + \zeta_1 \cdot p_3 \zeta_2 \cdot p_3 \cdot \epsilon_{3a} + -2k_1 \cdot \epsilon_3 \cdot k_2 \zeta_1 \cdot \zeta_2 + 4\zeta_1 \cdot \epsilon_3 \cdot k_1 \zeta_2 \cdot p_3 + (1 \leftarrow 2)) \text{Tr}[T_1T_2]$$

$$A(\lambda, \lambda, \phi) = \frac{p - 3}{2\sqrt{2}} (k_1 \cdot k_2 \zeta_1 \cdot \zeta_2 + \zeta_1 \cdot p_3 \zeta_2 \cdot p_3 + (1 \leftarrow 2)) \text{Tr}[T_1T_2]$$

$$A(\lambda, a, b) = -2i(2k_1^a \zeta_1 f_{2ab} \epsilon^b_3 - \zeta_1 \cdot p_3 f_{2ab} \epsilon^a_3) \text{Tr}[T_1T_2]$$

$$A(a, a, h) = 2 \left( \epsilon_{3ab} f_1^ac f_2^b c - \frac{1}{4} f_{1ab} f_2^a b \epsilon^a_3 + (1 \leftarrow 2) \right) \text{Tr}[T_1T_2]$$

$$A(a, a, \phi) = -\frac{p - 7}{4\sqrt{2}} (f_{1ab} f_2^a b + (1 \leftarrow 2)) \text{Tr}[T_1T_2]$$

where in the last three amplitudes we have suggestively introduced $f_{iab} = i(k_{ia}\zeta_{ib} - k_{ib}\zeta_{ia})$. One also finds that $A(\lambda, \lambda, b) = A(\lambda, a, h) = A(\lambda, \lambda, \phi) = A(a, a, b) = 0$. It is not hard to verify that these amplitudes are those precisely accounted for by the interactions given in eqs. (8).

Next we consider scattering amplitude of two NS open and one RR closed string states to compare with the quadratic interactions in the Chern-Simons action. With a RR vertex, one finds that the amplitude (14) reduces to

$$A_{NS,NS,RR} \sim \left(f_{(a)}^{3}\right)_{\mu_1...\mu_n} \epsilon^v_{a_0...a_p} \text{Tr}[(\gamma^{\mu_1}...\gamma^{\mu_n} \gamma^a_0...\gamma^a_p (1 + \gamma_{11}) \gamma \cdot \zeta_2 \gamma \cdot (k_1 + p_3/2) \gamma \cdot \zeta_1] + \text{Tr}[(\gamma^{\mu_1}...\gamma^{\mu_n} \gamma^a_0...\gamma^a_p (1 + \gamma_{11}) \gamma \cdot \zeta_1] k_1 \cdot \zeta_2$$

$$\text{Tr}[(\gamma^{\mu_1}...\gamma^{\mu_n} \gamma^a_0...\gamma^a_p (1 + \gamma_{11}) \gamma \cdot \zeta_2] k_2 \cdot \zeta_1$$

$$\text{Tr}[(\gamma^{\mu_1}...\gamma^{\mu_n} \gamma^a_0...\gamma^a_p (1 + \gamma_{11}) \gamma \cdot k_1] \zeta_2 \cdot \zeta_1) \text{Tr}(T_1T_2)$$

Now it is straightforward to perform the trace over the Dirac matrices and find $A_{NS,NS,RR}$ in terms of only polarization and momenta. In the end, one finds only three nonvanishing amplitudes for a Dp-brane, which may be expressed as

$$A(\lambda, \lambda, c_{(p+1)}) \sim (\zeta_1 \cdot p_3 \zeta_2 \cdot p_3 \cdot \epsilon_{3a} \cdot a_0...a_p + 2(p + 1) \zeta_1^a k_{10}^a \cdot p_3 \cdot \epsilon_{3a} \cdot a_1...a_p$$

$$+p(p + 1) \zeta_1^a k_1^a k_2^a \cdot \epsilon_{3a} \cdot a_2...a_p) \epsilon^v_{a_0...a_p} \text{Tr}(T_1T_2) + (1 \leftarrow 2)$$

$$A(\lambda, a, c_{(p-1)}) \sim i(\zeta_1 \cdot p_3 f_{2a}^a \epsilon_{3a} \cdot a_2...a_p + (p - 1) \zeta_1^i f_{2a}^a k_{12}^a \cdot \epsilon_{3a} \cdot a_3...a_p) \epsilon^v_{a_0...a_p} \text{Tr}(T_1T_2)$$

$$A(a, a, c_{(p-3)}) \sim -f_{1a}^a \cdot f_{2a}^a \epsilon_{3a} \cdot a_4...a_p \epsilon^v_{a_0...a_p} \text{Tr}(T_1T_2)$$

It is again straightforward to verify that these are the amplitudes precisely accounted for by the interactions in eq. (14).
3 Additional Amplitudes

In this section, we consider two classes of amplitudes which do not address directly the question of interactions between open and closed string states. However, they do add to the coherent picture of consistency between the string scattering amplitudes and the effective actions discussed in the introduction. The amplitudes considered here are for (i) three open strings and (ii) two closed strings.

Three massless open strings: Here there are various distinct amplitudes depending on the choices for the polarizations of the particles. For three vectors (i.e., all polarizations parallel to the D-brane world-volume), the amplitude takes the form

\[ (\zeta_1 \cdot \zeta_2 \cdot k_1 + \zeta_2 \cdot \zeta_3 \cdot k_2 + \zeta_1 \cdot \zeta_3 \cdot k_3) \text{Tr}(T_1[T_2,T_3]) \]

(16)

where the commutator of the U(N) Chan-Paton factors arises from including the two distinct cyclic orderings of the vertex operators. This result is of course identical to that in ten-dimensional open string theory (D9-branes backgrounds) which was calculated long ago, and was known to reproduce the three-point interaction for three non-Abelian gauge particles arising in the minimal action \( \text{Tr}(F_{ab}F^{ab}) \) \( [21, 22] \). This is consistent then with an expansion to leading order of the Born-Infeld action (1). Also of interest is the observation that the superstring scattering amplitude \( [22] \) does not contain any terms of order \( k^3 \), and so the low energy action does not contain a cubic term of the form \( \text{Tr}(F_{ab}F^{bc}F^{ca}) \). The absence of this term is also consistent with the expansion around flat space of the Born-Infeld action \( [1] \) accompanied with a symmetrized trace prescription for the gauge indices. The latter observation was also recently made from an analysis of the beta functions \( [23] \). Such terms do arise for the bosonic string, and can be accommodated by including a Born-Infeld action with an antisymmetric trace prescription \( [24] \). In fact for the superstring, one could extend this analysis to consider four-point functions which would reveal an \( F^4 \) contact term \( [25] \), which is in fact again consistent with the expansion of Born-Infeld with a symmetrized trace. It is known that certain inconsistencies arise, however, between string theory and this low energy action at order \( F^6 \) \( [10, 11] \).

The other nonvanishing amplitude involves two scalars (i.e., polarizations orthogonal to the D-brane world-volume) and one vector. Although the details of the scalar vertex operators differ, the resulting amplitude is essentially the same as eq. (16) above, with the restriction that \( \zeta_{1,2} \) are orthogonal to all momentum vectors (which lie in the D-brane world-volume). Hence, one finds

\[ \zeta_1 \cdot \zeta_2 \cdot k_1 \text{Tr}(T_1[T_2,T_3]) \]

(17)

This result reproduces the appropriate non-Abelian gauge interaction occurring in the scalar action \( \text{Tr}(D_a X^i D^a X^i) \), which of course arises from a dimensional reduction of \( F^2 \) in ten dimensions. This again consistent with the expansion of the Born-Infeld action around flat space, as long as the pull-backs are extended to involve gauge covariant derivatives of the non-abelian scalar fields, e.g., eq. \( [2] \) is replaced by

\[ \tilde{G}_{ab} = G_{ab} + 2G_{i(a} D_b)X^i + G_{ij} D_a X^i D_b X^j \]

(18)

Hull \( [14] \) noted that this modification is a natural choice which accommodates the non-Abelian gauge symmetry.
Finally one may observe that the vanishing of the scattering amplitudes with three scalars or two vectors and a scalar is also consistent with the leading order expansion of the Born-Infeld action (1). One can also check the consistency of the four-point amplitudes involving scalars with the expansion of the Born-Infeld action [26], and of course consistency arises from the dimensional reduction of the $F^4$ terms in ten dimensions which are known to agree [9].

Two closed strings: The scattering amplitudes for scattering two massless closed strings from a D-brane were presented in refs. [15, 16]. In ref. [16], it was verified that the scattering was consistent with the propagation of the closed string in the long range fields of a supergravity solution corresponding to a D-brane. Essentially, this analysis showed that the linear coupling of the closed string fields agreed with that appearing in the low-energy world-volume action (14). For example, the Born-Infeld term gave in eq. (6)

\[
L_0 = -T_p \kappa \left( h^a_a + \frac{p-3}{2\sqrt{2}} \phi \right)
\]  

(19)

Thus the linear coupling of antisymmetric Kalb-Ramond field $B^{\mu \nu}$ is zero. Similarly the linear source term for the RR potentials agrees with that from the Chern-Simons action (3), as given above in eq. (9)

\[
\tilde{L}_0 = 4\mu_p \frac{1}{(p+1)!} \epsilon_{a_0 \ldots a_p} c_{(p+1)}^{a_0 \ldots a_p} e_v^{a_0 \ldots a_p}
\]

(20)

This analysis of scattering amplitudes can be extended to the case of D-branes with a constant background gauge field or equivalently $B$ potential [27]. These backgrounds modify the world-sheet boundary conditions and the above source terms. For example, a linear coupling to the antisymmetric $B$-field appears.

A more careful analysis of these amplitudes can also determine the quadratic contact terms for the closed strings themselves on the D-brane world-volume. In the small momentum limit, i.e., $\ell_s \to 0$, one finds massless poles in both the $s$- and $t$-channels. The latter, which were considered in the previous analysis, result from the absorption of a massless closed string by the D-brane. In the $s$-channel, these poles arise because the closed strings can couple to a single massless open string through the interactions in eqs. (4) and (8), which then can propagate on-shell (or nearly on-shell) along the world-volume. Subtracting the field theory amplitudes in both of these channels from the string amplitudes leaves a set of quadratic contact terms for the closed strings on the world-volume. To leading order in the derivative expansion, there are non-derivative couplings, which we will extract below.

As before, let us begin by determining the interactions which an expansion of the actions (1) (or rather (4)) and (3) predict. First from the Born-Infeld action, one finds:

\[
L_{0,2} = -T_p \kappa^2 \left( \frac{1}{2} (h_a^a)^2 - h_{ab} h^{ab} + b_{ab} b^{ab} + \left( \frac{p-3}{4} \right)^2 \phi^2 + \frac{p-3}{2\sqrt{2}} \phi h_a^a \right)
\]

(21)

while the Chern-Simons action yields

\[
\tilde{L}_{0,2} = -8\kappa \mu_p \frac{1}{2(p-1)!} \epsilon_{a_0 a_1} c_{(p-1)}^{a_2 \ldots a_p} e_v^{a_0 \ldots a_p}
\]

(22)

\footnote{Throughout the following, we ignore the non-Abelian trace, which would have the trivial effect of introducing an overall factor of $N$ in the amplitudes and interactions.}
Next consider the amplitudes for two closed string states scattering off of a D-brane, which were calculated in \([16]\). Expanding the amplitudes around small \(t = -k^2\) and \(s = -q^2\), one finds\(^3\)

\[
A = -i\kappa^2 T_p \left( \frac{a_1}{k^2} - \frac{a_2}{4q^2} + \ldots \right) \tag{23}
\]

where \(a_1\) and \(a_2\) are expressions involving the closed string polarizations and are quadratic in momenta — see \([13]\). An important point in the following is that all of the higher order terms which are not explicitly shown above are at least quadratic in the momenta.

To extract the desired contact terms, one now needs to calculate the corresponding field theory scattering amplitudes, in \(s\)-channel using the action (1), and \(t\)-channel using bulk supergravity action. We will present some of the details for the calculation for scattering with a graviton and a dilaton. For this case, the field theory \(t\)-channel amplitude appears in \([16]\):

\[
A'_t(h, \phi) = i\kappa^2 T_p \frac{p - 3}{\sqrt{2}} p_2 \cdot \varepsilon_2 \cdot k \tag{24}
\]

Now the \(s\)-channel amplitude can be calculated as:

\[
A'_s(h, \phi) = \bar{V}^i_{h\lambda} \tilde{G}^{ij}_{\lambda} \bar{V}^j_{\phi\lambda}
\]

where \(\tilde{G}^{ij}_{\lambda} = -iN^{ij}/q^2\) is the scalar propagator on the D-brane world-volume, and the vertex functions are derived for the linear interactions in eq. (6)

\[
\bar{V}^i_{h\lambda} = \kappa \sqrt{T_p} (\varepsilon_{1a} \bar{p}_1^i - 2p_1^a \varepsilon_{1a}^i)
\]

\[
\bar{V}^i_{\phi\lambda} = \kappa \sqrt{T_p} p - 3 \frac{1}{2\sqrt{2}} p_2^i
\]

Substituting these into field theory scattering amplitude above, one will find

\[
A'_s(h, \phi) = -i\kappa^2 T_p \frac{p - 3}{2\sqrt{2}} q^2 \left( \varepsilon_{1a} \bar{p}_1 \cdot N \cdot p_2 + 2p_1 \cdot N \cdot \varepsilon_{1a} \cdot N \cdot p_2 \right) \tag{25}
\]

Now subtracting these field theory amplitudes (24,25) from the corresponding string amplitude (23), one finds

\[
A(h, \phi) - A'_t(h, \phi) - A'_s(h, \phi) = -i\kappa^2 T_p \frac{p - 3}{2\sqrt{2}} \varepsilon_{1a} + \ldots \tag{26}
\]

Again, the important point to note is that the terms implicitly denoted by the ellipsis are all at least quadratic in momenta. Hence the world-volume theory must include a non-derivative interaction between the graviton and dilaton field. In fact, it is not hard to see that the above amplitude (26) is precisely reproduced by the final term in eq. (21).

Similar calculations for the other massless closed string modes yield

\[
A(h, h) - A'_t(h, h) - A'_s(h, h) = -i\kappa^2 T_p \left( \varepsilon_{1a} \varepsilon_{2b} - 2Tr(\varepsilon_1 \cdot V \cdot \varepsilon_2 \cdot V) \right) + \ldots
\]

\[
A(b, b) - A'_t(b, b) - A'_s(b, b) = 2i\kappa^2 T_p Tr(\varepsilon_1 \cdot V \cdot \varepsilon_2 \cdot V) + \ldots
\]

\[
A(\phi, \phi) - A'_t(\phi, \phi) - A'_s(\phi, \phi) = -2i\kappa^2 T_p \left( \frac{p - 3}{4} \right)^2 + \ldots
\]

\(^3\)Note that there is a change in conventions between [16] and the present paper: \((T_p)_{\text{there}} = (\kappa T_p)_{\text{here}}.\)
These amplitudes are precisely reproduced by the remaining terms in eq. (21). One also finds that the string amplitudes $A(b, \phi)$ and $A(h, b)$ vanish, as well as the corresponding field theory amplitudes. Hence no quadratic contact terms appear in these cases, which is again consistent with the absence of such interactions in eq. (21).

Similar calculations for scattering amplitudes involving the massless RR states yield quadratic contact terms consistent with the interactions in eq. (22). That is the only non-derivative contact terms appear for the combination of $b$ and $c_{(p-1)}$.

4 Discussion

By directly examining various string scattering amplitudes, we have extracted various world-volume interactions for the effective field theory on D-branes. Our results are entirely consistent with the Born-Infeld and Chern-Simons actions (1,3), as long as these expressions are properly interpreted. Our analysis suggests that the pull-backs of the background field tensors must be constructed using gauge covariant derivatives of the non-abelian scalar fields, as in (13) for example. Also the background closed string fields in both expressions are functionals of these non-abelian scalars. The latter conclusion is evident from the analysis of the quadratic couplings considered in section 2.2. Thus the gauge trace appearing in eqs. (1) and (3) applies not only on the gauge fields, but also on the non-abelian scalars in the background fields, both in the form of the covariant derivatives of the pull-backs and the internal functional dependence. Our scattering amplitude analysis can be extended to the case of mixed boundary conditions, as in [27]. Such boundary conditions represent D-branes with a constant U(1) gauge field or equivalently $B$ potential, and so extra interactions are revealed in the scattering amplitudes. Although we have not pursued these calculations in the same detail as those presented above, the results again seem to be entirely consistent with the structure of the Born-Infeld and Chern-Simons actions.

Given the above results, consider the case, for example, of a (test) D-brane propagating in a curved (fixed) background geometry with metric $G_{\mu\nu}^0(x^\rho)$. To implement the necessary functional dependence in (1), we can adapt the spacetime coordinates for a static gauge choice, at least in a neighborhood of the D-brane world-volume, i.e., we make a split of world-volume coordinates, $x^a = \sigma^a$, and transverse coordinates, $x^i = \frac{1}{N} \text{Tr} X^i$. Then the metric functional appearing in the D-brane action would be given a non-abelian Taylor expansion

$$G_{\mu\nu} = \exp \left[ X^i \frac{\partial}{\partial x^i} \right] G_{\mu\nu}^0(\sigma^a, x^i) \big|_{x^i=0} \hspace{1cm} (27)$$

Similar expansions would also be applied for other nontrivial background fields. Such a Taylor expansion naturally produces an expression symmetric in the $X^i$ since the partial derivatives commute. Our analysis, however, leaves open many questions of ordering the various fields under the gauge trace. For example, how are the $X^i$’s in eq. (27) ordered between various components of the background fields appearing in higher order interactions, or with field strength components of the world-volume gauge field.

The latter might be included in the question of finding the complete definition for the gauge trace for the Born-Infeld action (1). However, new ambiguities are also appearing here for
the Chern-Simons action (3) as well. Essentially, our analysis does not resolve these questions because we cannot see any nontrivial commutator terms with the present amplitudes. In principle, our method can be extended to higher point amplitudes to begin to address these ambiguities, however, in practice, extracting the higher order contact interactions would be extremely tedious. We have made some progress in examining scattering amplitudes involving one closed and three open strings [28]. It is evident from other investigations[12, 11] that commutator terms play an important role at higher orders.

Recall that in the amplitude $A^{NS,RR}$ in eq. (12), one starts with a vertex operator written in terms of the RR field strength. Hence the resulting interactions are naturally derived in terms of this field strength, and as a result are invariant under the RR gauge transformations. However, one must integrate by parts and thus the gauge invariance is no longer manifest. Consider the linear interactions involving $c_{(p+1)}$ in eq. (10). Invariance under gauge transformations depending only on world-volume directions is clear, i.e., a total derivative on the world-volume is produced for the term in $\tilde{\mathcal{L}}_1$ which originates in the pull-back. However for transformations depending on transverse directions this term alone is not invariant, but invariance results from a cancellation with a contribution from the term in $\tilde{\mathcal{L}}'_0$ which originates in the Taylor expansion of the background. So there is an important interplay of these two terms, although they have completely different origin in the Chern-Simons action. Similar comments apply for higher order interactions, as well.

A similar but more complicated discussion would apply for the interactions in the Born-Infeld action (1). For example, while the non-abelian Taylor expansion presented in eq. (27) is completely non-covariant, the relevant string scattering amplitudes will be covariant. It may be that gauge invariance can provide a tool to resolve some of the ordering questions raised above. It remains a problem, however, that in general the static gauge choice made to construct the Taylor expansion (27) will only apply in a local neighborhood of the D-brane world-volume. It seems that the description of the world-volume dynamics of D-branes is still lacking at some basic level. The disparity between the role of world-volume and transverse coordinates seems a fundamental problem. Resolving this disparity may provide further insight into the non-abelian nature of spacetime or general backgrounds as seen by string theory.

As a final note, we will comment on the physical significance of some of the interactions considered in this paper. To focus the discussion, consider the interaction $\epsilon^{abc} \text{Tr}[f_{ab}\lambda^i]F_{(2)ci}$ which appears in the Chern-Simons action of a D2-brane. Here, $F_{(2)}$ is the field strength of RR one-form potential. As discussed above, an integration by parts has been performed to combine two terms from $\tilde{\mathcal{L}}_1$ and $\tilde{\mathcal{L}}_2$ in eq. (14) to produce this covariant expression. To understand the physical role played by this interaction, consider two D2-branes which coincide to produce a $U(2) = U(1) \times SU(2)$ gauge symmetry on the world-volume. Now a constant background field strength $f_{ab}$ in the $U(1)$ will induce a coupling to the one-form RR potential through the higher order interactions in the Chern-Simons action [3]. The resulting configuration can be thought of as a bound state of D2- and D0-branes [2, 29]. Giving an expectation value to a $SU(2)$ field strength in, e.g., the $\sigma_3$ part of the gauge group, yields no such coupling since $\text{Tr}f_{ab} = 0$. This indicates there is no net D0-brane charge in the system. One can think of the individual eigenvalues of the gauge matrix as indicating that the two fundamental D2-branes carry equal and opposite densities of D0-branes, and hence the net charge is zero. If now, however, one

\footnote{Note that here we would consider the gauge parameters as functionals of the non-abelian scalars, just as above for the background fields.}
gives an expectation value to some $\lambda^i$ in the $\sigma_3$ part of the gauge group, one is separating one D2-brane from the other. While there is still no net charge, this produces a small separation of positive and negative charges, i.e., a dipole, which should be detectable in scattering closed strings. The coupling above, $\epsilon^{abc}\text{Tr}[f_{ab}\lambda^i]F_{(2)cir}$, provides precisely the desired dipole coupling for the RR one-form. A similar discussion extends to the analogous couplings on other $Dp$-branes. As well, higher order interactions may play the role of higher multi-pole couplings for the appropriate configurations of world-volume fields. These results may be of interest in examining various nontrivial nonabelian field configurations which have been discussed recently [10].

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