DELTA-ISOBAR PRODUCTION IN ANTIPROTON
ANNIHILATION ON THE DEUTERON

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Abstract

The annihilation of antiprotons on deuterons at rest via the channels $\bar{p}d \rightarrow \pi\Delta$ and $\bar{p}d \rightarrow \pi N$ is studied. The two-step mechanism is investigated by analysing these processes when either $\pi^0 n$, $\pi^0 \Delta^0$ or $\pi^- p$, $\pi^- \Delta^+$ are produced in the final state. Predictions for the branching ratios of the annihilations $\bar{p}d \rightarrow \pi^- p$ and $\bar{p}d \rightarrow \pi^- \Delta^+$ are presented. From comparison of the presented results with the experimental data new information on the mechanism of the $\Delta$-isobar production in antiproton-deuteron annihilation can be deduced.

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It is known that antiproton annihilation on a deuteron occurs mostly on quasi-free nucleons leaving one nucleon a "spectator" with a typical momentum below 100 MeV/c. But this so-called impulse approximation holds until the "spectator" nucleon is measured. However, in the case of annihilation when the final baryon has a higher momentum, the main mechanism

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envisaged is a two-step process. The antiproton annihilates on one nucleon producing at least two mesons, one of which is absorbed by the second nucleon forming a nucleon or a baryonic resonance, for example, a \( \Delta \)-isobar. Pontecorvo reactions such as \( \bar{p}d \rightarrow \pi N \), where the final nucleon has a momentum of the order of 1 GeV/c, have already been considered in the framework of this mechanism in refs. [1, 2] and [3, 4]. In this paper, annihilation processes at rest on the deuteron of the type of \( \bar{p}d \rightarrow \pi N \) and \( \bar{p}d \rightarrow \pi \Delta \) are analysed. Some predictions for branching ratios and the ratio between reactions \( \bar{p}d \rightarrow \pi^- p \) and \( \bar{p}d \rightarrow \pi^- \Delta^+ \) are presented. Moreover, the branching ratios and the corresponding ratio between the reactions \( \bar{p}d \rightarrow \pi^0 n \) and \( \bar{p}d \rightarrow \pi^0 \Delta^0 \) are calculated and compared with the experimental data of the Crystal Barrel Collaboration [3, 4] and KEK [7]. The analysis of these processes is performed within the framework of the two-step mechanism of \( \bar{p}d \) annihilation. Comparison of these predictions with the experimental data results in a new view on the mechanism of the absorption process of the virtual meson by the virtual nucleon inside the deuteron, especially in the case when the \( \Delta \)-isobar is produced.

The general expression for the cross section of the annihilation \( \bar{p}d \rightarrow \pi B \) in flight where \( B \) is a baryon, either a nucleon or a \( \Delta \)-isobar, can be written in the following form:

\[
\sigma_{\bar{p}d \rightarrow B}(s) = \frac{(2\pi)^4}{F} \int \delta^4(p_p + p_d - p_\pi - p_B) |F_{\bar{p}d \rightarrow \pi B}|^2 \frac{d^3p_\pi d^3p_B}{(2\pi)^32E_\pi(2\pi)^32E_B},
\]

where \( F \) is the so-called flux factor, i.e., \( F = 2\lambda^{1/2}(s, m^2, M_d^2) = 2(s - (m + M_d)^2)^{1/2}(s - (M_d - m)^2)^{1/2}, \) here \( s \) is the square initial energy in the c.m.s. of \( \bar{p}d, m \) and \( M_d \) are the nucleon and deuteron masses respectively; \( F_{\bar{p}d \rightarrow \pi B} \) is the amplitude of the reaction. Eq.(1) holds valid for annihilation in flight but
cannot be calculated at rest, because at rest the flux factor is zero. Therefore, in the case of annihilation at rest it would be better to analyse the branching ratio of the reaction, \( Br_{\bar{p}d \rightarrow \pi B} = \frac{\sigma_{\bar{p}d \rightarrow \pi B}}{\sigma_{\bar{p}d}} \), where \( \sigma_{\bar{p}d} \) is the total cross section of \( \bar{p}d \) annihilation.

The amplitude \( F_{\bar{p}d \rightarrow \pi B} \), according to the above, can be calculated within the framework of the two-step mechanism which in the general form is presented in Fig.1. It has been shown in refs.\[3, 4\] that the \( \pi \)-meson exchange in the triangle graph for the Pontecorvo reaction (see Fig.1) \( \bar{p}d \rightarrow \pi N \) yields the dominant contribution, about 90\% as compared with the \( \rho, \omega \) contribution. One can suggest the same for \( \bar{p}d \rightarrow \pi \Delta \) annihilation, too, for two reasons: first, the excitation of \( \Delta \) by \( \rho \) and \( \omega \) absorption has a smaller probability than the simple absorption of the virtual pion by the nucleon because the off-shellness of \( \rho \)- or \( \omega \)-mesons is very large in comparison to the corresponding one of the pion; second, \( \Delta(1232) \) decays mainly into a pion and a nucleon. Therefore, one may take only the \( \pi \)-meson exchange into account and calculate the triangle graph presented in Figs.(1,2).

The general form of the amplitude \( F_{\bar{p}d \rightarrow \pi B} \) corresponding to the graphs of Figs.(1,2) can be written, for example, within the framework of the formalism presented in \[3, 4\], as follows:

\[
F_{\bar{p}d \rightarrow \pi B} = -\frac{1}{(2\pi)^3} \int \Phi_d(k) F_\pi^B \Gamma_{\pi NB} G_\pi^B(k) f_{\bar{p}N \rightarrow \pi \pi} \frac{d^3k}{4E(k)\epsilon_\pi(q_\pi)} , \quad (2)
\]

here \( \Phi_d(k) \) is the deuteron wave function (d.w.f.), \( f_{\bar{p}N \rightarrow \pi \pi} \) is the amplitude of the annihilation process \( \bar{p}N \rightarrow \pi \pi \), \( F_\pi^B \) is the formfactor corresponding to the vertex \( \pi NB \), \( G_\pi^B(k) \) is the propagator of the virtual \( \pi \)-meson which has the following form:

\[
G_\pi^B(k) = \left( E_B(p_B) - E(k) - \epsilon_\pi(q_\pi) + i\epsilon \right)^{-1} , \quad (3)
\]
where $E_B(p_B)$ and $E(k)$ are the energies of the final baryon and of the internal nucleon inside the deuteron; $\epsilon_\pi(q_\pi)$ is the energy of the virtual pion in the intermediate state, $q_\pi = p_B - k$ is its three-momentum; $p_B$ is the three-momentum of the final baryon; $\Gamma_{\pi NB}$ corresponds to the lower vertex of Fig.1 or Fig.2, i.e. to absorption of the virtual pion by the nucleon inside the deuteron, in particular, $\Gamma_{\pi NN} = g_{\pi NN} (\sigma \cdot \tau)$, $\tau = b(p_N E_N(p_N) + m)/2m$; $\sigma$ is the vector Pauli matrix, $g_{\pi NN}$ is the corresponding coupling constant, $g_{\pi NN}^2/4\pi = 14.7$ \cite{10, 11}; $\Gamma_{\pi N\Delta} = g_{\pi N\Delta}(E(k) + m)^{1/2}(E_{\Delta}(p_\Delta) + m_{\Delta})^{1/2}$, $g_{\pi N\Delta}$ is the $\pi N\Delta$ coupling constant and $g_{\pi N\Delta}^2/4\pi = 56 \div 71$ according to \cite{10, 11} and \cite{12, 13}.

Substituting $F_{\bar{p}d \to \pi B}$ into eq.(1) and neglecting the interference between different isotopic triangle graphs we can get the branching ratios of the processes $\bar{p}d \to \pi N$ and $\bar{p}d \to \pi \Delta$. In particular, for the final states containing the neutral particles $\pi_0 n$ and $\pi_0 \Delta_0$ we have:

$$Br_{\bar{p}d \to \pi_0 n} = (2Br_{\bar{p}m \to \pi^+ \pi^0} + Br_{\bar{p}p \to \pi^0 \pi^0})c_N | J_N |^2 g_{\pi NN}^2 , \quad (4)$$

$$Br_{\bar{p}d \to \pi_0 \Delta_0} = (2Br_{\bar{p}m \to \pi^+ \pi^0} + Br_{\bar{p}p \to \pi^0 \pi^0})c_\Delta | J_\Delta |^2 g_{\pi \Delta N}^2 , \quad (5)$$

where $Br_{\bar{p}m \to \pi^+ \pi^0}$ and $Br_{\bar{p}p \to \pi^0 \pi^0}$ are the branching ratios of the annihilation processes $\bar{p}m \to \pi^+ \pi^0$ and $\bar{p}p \to \pi^0 \pi^0$ at rest, respectively, $c_N = 1/E_N(p_N)$ and $c_\Delta = (E_{\Delta}(p_\Delta) + m)/E_{\Delta}(p_\Delta)$. Eqs. (4-5) were derived under the assumption that d.w.f. decreases with the internal momentum $k$ more rapidly than the amplitude $f_{\bar{p}N \to \pi \pi}$. The expressions for the integrals $J_N$ and $J_\Delta$ are the following:

$$J_N = \frac{1}{4\sqrt{2\pi}} \int_{-1}^{1} d\cos(k)_{\bar{p}} \int_{0}^{k_{\text{max}}} \Phi_d(k)\tau_z F_{\pi N}^D(q^2)G_N(k)k^2dk/E(k) \quad (6)$$
and

\[ J_\Delta = \frac{1}{4\sqrt{2}\pi} \int_{-1}^{1} d\cos(k) \Delta \int_0^{k_{\text{max}}} \Phi_d(k)(E(k) + m)^{1/2} F^\Delta (q^2)(G_\Delta(k) \frac{k^2 dk}{E(k)}) , \]

(7)

where \( k_{\text{max}} \simeq 0.8 - 1.0 \text{ GeV/c} \) is the maximum value of the internal momentum of the nucleon inside of the deuteron, \( F^N_\pi \) and \( F_\Delta^\pi \) are the pion formfactors for the processes \( \bar{p}d \to \pi N \) and \( \bar{p}d \to \pi \Delta \), respectively, \( \tau = b(p_N/(E_N(p_N) + m) - k/(E(k) + m))p_N/ | p_N |; \theta(k)_N \) and \( \theta(k)_\Delta \) are the angles between the momenta of the final baryon, \( N \) or \( \Delta \), and the internal nucleon inside of the deuteron.

The ratio \( R_1 \) between (4) and (5) has the following form:

\[ R_1 = \frac{Br_{\bar{p}d \to \pi^0 n}}{Br_{\bar{p}d \to \pi^0 \Delta^0}} \approx \frac{|J_N|^2 g_{\pi NN}^2 c_N}{|J_\Delta|^2 g_{\pi \Delta N}^2 c_\Delta} . \]

(8)

Under isospin invariance for the \( |\pi^0 n >, |\pi^0 \Delta^0 > \) and \( |p^- p >, |p^- \Delta^+ > \) final states the corresponding relations can be deduced for \( \bar{p}d \to \pi^- p \) and \( \bar{p}d \to \pi^- \Delta^+ \) annihilation processes:

\[ Br_{\bar{p}d \to \pi^- p} = 2Br_{\bar{p}d \to \pi^0 n} \]

(9)

\[ Br_{\bar{p}d \to \pi^- \Delta^+} = \frac{1}{2} Br_{\bar{p}d \to \pi^0 \Delta^0} \]

(10)

The corresponding expected ratio of the branchings is:

\[ R_2 = \frac{Br_{\bar{p}d \to \pi^- p}}{Br_{\bar{p}d \to \pi^- \Delta^+}} = 4R_1 . \]

(11)
• **Kinematics**

The conservation law at the vertex of the absorption of the virtual π-meson by the nucleon inside the deuteron is expressed by

\[(q + k)^2 = m_B^2 \tag{12}\]

where \(q\) and \(k\) are the four-momenta of the π-meson and the above mentioned nucleon, respectively; \(m_B\) is the mass of either the nucleon or the \(\Delta\)-isobar.

Solution of eq.(12) yields the following form for the square four-momentum of the virtual pion:

\[q^2 = m_B^2 + m^2 + 2(\mathbf{p}_B \cdot \mathbf{k} - EE_B) \tag{13}\]

where \(\mathbf{p}_B\) is the three-momentum of the outgoing baryon \(B\), i.e. of either the nucleon or the \(\Delta\)-isobar.

The momentum of particle \(B\) can be deduced from the conservation law for the Pontecorvo reaction at rest:

\[(p_\pi + p_B)^2 = (m_N + m_d)^2 \tag{14}\]

here \(p_\pi\) and \(p_B\) are the four-momenta of the final π-meson and nucleon or \(\Delta\), respectively; \(m_N\) and \(m_d\) are the nucleon and deuteron masses, respectively. Finally, the square three-momenta of the final nucleon or \(\Delta\)-isobar are respectively:

\[|\mathbf{p}_N|^2 = \frac{a_N^2 - \mu^2 m_N^2}{9m_N^2} \tag{15}\]

and

\[|\mathbf{p}_\Delta|^2 = \frac{a_\Delta^2 - \mu^2 m_\Delta^2}{9m_N^2} \tag{16}\]

where for the final nucleon \(a_N \simeq 4m_N^2 - \mu^2/2\) and for the \(\Delta\)-isobar \(a_\Delta \simeq \frac{1}{2}(9m_N^2 - m_\Delta^2 - \mu^2)\), here the binding energy of the deuteron (\(\simeq 2.2\) MeV) is neglected and \((m_N + m_d)^2 \simeq 9m_N^2\) is assumed.
Formfactors $F^N_\pi(q^2)$ and $F^\Delta_\pi(q^2)$

The choice of the formfactors (FF) $F^N_\pi(q^2)$ and $F^\Delta_\pi(q^2)$ in eqs. (6) and (7) is very important. For example, for the annihilation $\bar{p}d \rightarrow \pi B$ the form factor can be chosen in the following form $[11, 12]$

$$F^B_\pi(q^2) = \left( \frac{\Lambda^2_B - \mu^2}{\Lambda^2_B + |q^2|} \right)^n$$

(17)

where either $n = 1$ (so called monopole FF) or $n = 2$ (dipole FF).

In principle, the values of the cut-off parameter $\Lambda_B$ and of $n$ could be different for $\bar{p}d \rightarrow \pi N$ and $\bar{p}d \rightarrow \pi \Delta$ annihilations. For example, according to refs. $[1, 2]$ and $[3]$, $\Lambda_N = 1.2 - 1.4$ GeV/c and $n = 1$ for the first process. As to the second reaction, the form of the FF can be chosen like in the description of the processes $\Delta N \rightarrow \Delta N, \Delta N \rightarrow \Delta \Delta$ etc. For example, following $[13]$ we can use

$$F^\Delta_\pi(q^2) = \left( \frac{\Lambda^2_\Delta}{\Lambda^2_\Delta + |q^2|} \right)^n$$

(18)

or, according to $[14]$, the exponential form:

$$F^\Delta_\pi(q^2) = \exp\left(-\frac{q^2}{2\Lambda^2_\Delta}\right)$$

(19)

We calculated the branchings using these FF $[18]$ and $[19]$. However the exponential form of FF $[13]$ didn't reproduce the experimental data by applying the values of cut-off parameter $\Lambda_\Delta = (2 - 3)\mu$ presented by $[14]$. Therefore we limited ourselves to the application of the more conventional FF of type $[18]$.

Results and Discussions.

We now present the resultant calculated branching ratios $BR$ for the reactions $\bar{p}d \rightarrow \pi N$ and $\bar{p}d \rightarrow \pi \Delta$. They were calculated using different forms for the FF and different values of the cut-off parameters $\Lambda_N$ and $\Lambda_\Delta$. 
The results obtained for the annihilation $\bar{p}d \rightarrow n\pi^0$ using Paris d.w.f. \[17\] and FF (17) with the $n = 1$ standard choice are presented in Table 1 for different values of the cut-off parameter $\Lambda_N$. As a consequence of the monopole choice, the range of values of the cut-off parameter to be explored should be close to the nucleon mass (see, for example, [16] and references quoted therein), as confirmed also by the results presented in [1], [2] and [3]. Using FF (18) with $n = 2$ for the annihilation $\bar{p}d \rightarrow \pi^0\Delta^0$ and the coupling constant $g_{\pi N\Delta}/4\pi = 71$ according to [12, 17] we obtain the plot of the branching ratio versus $\Lambda_\Delta$ presented in Fig.3. When FF (18) is adopted - or, equivalently, FF (17) - with the monopole choice it is impossible to reproduce the experimental data by keeping the cut-off parameter reasonably close to the nucleon mass.

Table 1: BR for Pontecorvo reaction: $\bar{p} + d \rightarrow n + \pi^0$.

| $\Lambda_N$(GeV/c) | BR ($\times 10^{-6}$) |
|--------------------|----------------------|
| 1.1                | 6.278                |
| 1.2                | 7.432                |
| 1.3                | 8.544                |
| 1.4                | 9.600                |

Note that the presented branchings and ratios reproduce the experimental data of Crystal Barrel Collaboration: $BR(\bar{p}d \rightarrow \Delta^0(1232)\pi^0) = (22.1 \pm 2.4)10^{-6}$, $BR(\bar{p}d \rightarrow n\pi^0) = (7.3 \pm 0.72)10^{-6}$ and $R_1 = 0.32 \pm 0.05$ [3, 4].

The corresponding branching ratios for the reactions $\bar{p}d \rightarrow \pi^- p$ and $\bar{p}d \rightarrow \pi^- \Delta^+$ can be obtained from Table 1 and Fig.3 by multiplying the branching ratios for $\bar{p}d \rightarrow n\pi^0$ by a factor 2 and for $\bar{p}d \rightarrow \pi^0\Delta^0$ by a factor 1/2, in
accordance with the isotopic relations (9) and (10) respectively.

- **Conclusions.**

The two-step approach applied to Pontecorvo reactions $\bar{p}d \rightarrow \pi N$ results in branching ratios for $\bar{p}d \rightarrow \pi^0 n$ annihilation which do not contradict existing experimental data [6] within the experimental errors, quite large in the case of the KEK data [7]. A good agreement is also observed with the calculated BR and the most recent measurement on the reaction $\bar{p}d \rightarrow \pi^- p$, performed at the OBELIX Spectrometer [8].

Our approach is close to the one considered in ref. [4] and the results obtained in this paper for $BR(\bar{p}d \rightarrow \pi^0 n)$ approximately coincide with the corresponding ones presented there. This result may serve as a basis for the application of our approach to pion and $\Delta$-isobar production at rest. The triangle graph for $\bar{p}d \rightarrow \pi \Delta$ is distinguished from the one for $\bar{p}d \rightarrow \pi N$ annihilation by the different values of coupling constants $(g_{\pi N\Delta}^2/g_{\pi NN}^2 \simeq 4 - 5)$ and by different spin structures of the corresponding pion-nucleon absorption vertices. Moreover, the pion propagators for the corresponding two graphs (Fig.(1,2)) have different pole positions in the calculation of eqs. (6,7). Such a difference between calculations of the triangle graphs for $\bar{p}d \rightarrow \pi N$ and $\bar{p}d \rightarrow \pi \Delta$ annihilations allows one to assume that the forms of the FF should also be different in order to describe the experimental data on the branching ratio for $\bar{p}d \rightarrow \pi^0 \Delta^0$ [3]. By using a Paris d.w.f. the best fit of CB experimental data for $\bar{p}d \rightarrow \pi^0 n$ annihilation is obtained with monopole FF (17) and cut-off parameter $\Lambda_N = 1.1 - 1.2$ GeV/c. In order to describe the CB measurements on the branching ratio for the reaction $\bar{p}d \rightarrow \pi^0 \Delta^0$, with quite the same value of the previous cut-off parameter ($\Lambda_\Delta \simeq 1$ GeV/c), a FF (17) of a form of higher order than the monopole is required: that is the
dipole formfactor. Using this value of $\Lambda_\Delta$, the results presented in Fig.3 and the isotopic relation (11) one can show the branching ratio for $\bar{p}d \rightarrow \pi^-\Delta^+$ annihilation to be about $(10 - 12) \times 10^{-6}$. This result can be considered as a certain prediction for the OBELIX experiment, that is analysing this channel [13]. The application of other forms of FF doesn’t result in a satisfactory description of the experimental data.

This does not contradict the previous study of Pontecorvo reactions [1, 2, 3, 4] and means that annihilation of the type of $\bar{p}d \rightarrow \pi\Delta$ can be described with the help of the two-step mechanism without introduction of any non-nucleonic six-quark component in the deuteron successfully applied in the description of the prototype Pontecorvo reaction [2] and recently invoked to describe the baryon-baryon content of the deuteron [20]. Nevertheless, in the framework of the two-step model, the investigation of $\Delta$- isobar production in antiproton-deuteron annihilation results in a new view on its production mechanism and, in particular, in new information on the choice of the form-factor of the virtual pion and of the cut-off parameter $\Lambda_\Delta$ which is still quite insufficient [11, 12].

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