Observational Constrains on Direct Electron Heating in Hot Accretion Flows from Sgr A* and M87*

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ABSTRACT

An important parameter in the theory of hot accretion flows around black holes is δ, which describes the fraction of “viscously” dissipated energy in the accretion flow that directly heats the electrons. The radiative efficiency of a hot accretion flow is determined by its value if other parameters, such as the accretion rate, are determined. Unfortunately, the value of δ is hard to determine from first principles. The recent Event Horizon Telescope Collaboration (EHTC) results on M87* and Sgr A* provide us with a different way of constraining δ. By combining the mass accretion rates in M87* and Sgr A* estimated by the EHTC with the measured bolometric luminosities of the two sources, we derive good constraints on the radiative efficiencies of the respective accretion flows. In parallel, we use a theoretical model of hot magnetically arrested disks (MAD) to calculate the expected radiative efficiency as a function of δ (and accretion rate). By comparing the EHTC-derived radiative efficiencies with the theoretical MAD model, we find that Sgr A* requires δ ≳ 0.3, with the most likely value being δ ≈ 0.5. A similar comparison in the case of M87* gives inconclusive results, because there is still a large uncertainty in the accretion rate in this source.

Keywords: accretion, accretion disks — Astrophysical black holes — low-luminosity active galactic nuclei (individual: Sgr A*, M87)

1. INTRODUCTION

Accretion flows around black holes (BHs) can be divided into two types: hot accretion flows, which occur at accretion rates below about 1% of Eddington (Narayan & Yi 1994, 1995; Abramowicz et al. 1995, see Yuan & Narayan 2014 for a review), and cold accretion disks which are found at accretion rates closer to Eddington (Shakura & Sunyaev 1973; Novikov & Thorne 1973; Pringle 1981). Most supermassive BHs in the nearby universe have low accretion rates (Ho 2008) and are believed to have hot accretion flows.

The plasma in a hot accretion flow is two-temperature (Shapiro, Lightman, & Eardley 1976), where the electrons and ions have different temperatures, and hence need to be treated with separate energy equations (Narayan & Yi 1995; Yuan & Narayan 2014). The energy equation of electrons takes the form

$$\rho V_r \left( \frac{d e_e}{dR} - \frac{p_e}{\rho^2} \frac{dp_e}{dR} \right) = q_{ie} + \delta q_{vis} - q^-.$$  \hspace{1cm} (1)

Here $\rho$, $V_r$ is the gas density, $V_r$ is the radial velocity, $R$ is the radius, $e_e$ and $p_e$ are, respectively, the specific internal energy and pressure of electrons, and $q^-$ is the radiative cooling rate per unit volume. The first two terms in the right hand side of the above equation correspond to two different mechanisms by which electrons are heated. One mechanism is via Coulomb collisions between ions and electrons, denoted by $q_{ie}$, the other is through direct “viscous heating,” which is a catch-all term describing several physical dissipation processes in the plasma as we explain in the next paragraph. We assume that, out of the total viscous heating rate per unit volume $q_{vis}$, a fraction $\delta q_{vis}$ heats the electrons, and the rest $(1 - \delta)q_{vis}$ heats the ions. Since almost all the radiation of the accretion flow is emitted by electrons, therefore when other model parameters such as the mass accretion rate are given, the value of $\delta$ determines the radiative efficiency $\epsilon$ of the accretion flow, which we define as,

$$\epsilon = \frac{L_{bol}}{M_{BH} c^2} = 10\% \frac{L_{bol}/L_{Edd}}{M_{BH}/M_{Edd}}.$$  \hspace{1cm} (2)

Here $L_{bol}$ is the bolometric luminosity of the accretion flow and $M_{BH}$ is the mass accretion rate at the BH horizon. In the second expression, the luminosity and the accretion rate are normalized to, respectively, the Eddington luminosity $L_{Edd} = 4\pi G M_{BH} M_{BH}/\sigma T = 1.26 \times 10^{44} (M_{BH}/10^8 M_\odot) \text{ erg s}^{-1}$, and the Eddington accretion rate $M_{Edd} = L_{Edd}/(0.1 c^2) = 2.21 \times 10^{-2} (M_{BH}/10^8 M_\odot) M_\odot \text{ yr}^{-1}$.

The viscous heating rate of electrons $\delta q_{vis}$ includes several microphysical processes. Many attempts have been made to estimate $\delta$ from first-principle investigations of these processes, including magnetic reconnection (Bisnovatyi-Kogan
& Lovelace 1997; Quataert & Gruzinov 1999; Ding et al. 2010; Hoshino 2012, 2013; Numata & Loureiro 2015; Sironi & Narayan 2015; Rowan et al. 2017, 2019; Ball et al. 2018), MHD turbulence (Quataert 1998; Quataert & Gruzinov 1999; Blackman 1999; Medvedev 2000; Lehe et al. 2009; Howes 2010; Ressler et al. 2015; Ryan et al. 2017), dissipation of pressure anisotropy in a collisionless plasma (Sharma et al. 2007), or low Mach number shocks (Guo et al. 2017, 2018). Unfortunately, there is no consensus, and the value of $\delta$ remains poorly determined.

Yuan et al. (2003) considered an alternative approach to constrain the value of $\delta$. By calculating the dynamics and radiation of the accretion flow using a height-integrated model, and comparing the results with observations of the supermassive black hole Sagittarius A* (Sgr A*) at the center of our Galaxy, they estimated $\delta \sim 0.5$.

In this work, we follow the approach of Yuan et al. (2003) and try to estimate the value of $\delta$ using new and updated observational data and a more modern theoretical model. Specifically, we make use of results from the Event Horizon Telescope (EHT), an Earth-size sub-millimeter radio interferometer, which has recently captured, at 230 GHz, the image of the innermost BH horizon scale region of the low-luminosity supermassive BHs, Sgr A* (EHTC 2022a) and M87* (EHTC 2019, 2021). Key results from the work of the EHT Collaboration (EHTC) are estimates of the mass accretion rates $\dot{M}_{\text{BH}}$ for the two sources to within a factor of several, see Sec. 2 below. By combining these estimates with a reliable measurement of the bolometric luminosities of the sources from their broad-band spectra, we are able to evaluate the radiative efficiencies $\epsilon$ (cf. Equation 2) of the two sources. We compare these measurements with the predictions of the model described in Xie & Zdziarski (2019) and thus evaluate the value of $\delta$.

Before going into details, we briefly introduce additional background information on hot accretion flows. In the last decade or two, it has become clear that the degree of magnetization of the accretion flow plays an important role in the dynamics and observational characteristics of hot accretion flows. Broadly, one distinguishes between two kinds of models. One has relatively weak magnetic fields and is referred to as the standard and normal evolution (SANE) model, while the other has the maximum saturated level of magnetic field, and is called the magnetically arrested disk (MAD) model (Narayan et al. 2003; Igumenshchev et al. 2003; Bisnovatyi-Kogan & Ruzmaikin 1974, 1976; Tchekhovskoy et al. 2011; McKinney et al. 2012; Liska et al. 2020). The dynamical differences between SANE and MAD are investigated in detail in Narayan et al. (2012).

The accumulation of a considerable quantity of poloidal magnetic flux around the BH in the MAD state has the attractive feature that it provides the ideal magnetic field configuration to launch a powerful relativistic jet (Tchekhovskoy et al. 2011; Liska et al. 2020; Narayan et al. 2022). Indeed, there is accumulating evidence that systems with strong relativistic jets are generally MAD, e.g., Zamaninasab et al. (2014); Ghisellini et al. (2014); Zdziarski et al. (2015) for a sample of blazars. Meanwhile, the EHT has provided evidence that M87* and Sgr A* may also both be MAD (EHTC 2021, 2022a). The MAD nature of M87* is confirmed later by an analysis to the observed rotation measure (Yuan et al. 2022). These results lead one to speculate that perhaps the MAD configuration is the inevitable final state of most hot accretion flows in nature (e.g., Narayan et al. 2022).

The radiative properties of MAD flows are found to be fairly similar to those of SANE flows (Xie & Zdziarski 2019), except that, for a given accretion rate, MAD is brighter by about a factor of $\sim 3$. This motivates us to use a model appropriate to MAD systems when attempting to estimate the electron heating parameter $\delta$.

This paper is organized as follows. In Sec. 2, we provide the observational and modeling results for Sgr A* and M87*, with an emphasis on the accretion rate $\dot{M}_{\text{BH}}$ and radiative efficiency $\epsilon$. In Sec. 3, we present theoretical calculations of the radiative efficiency corresponding to different values of $\delta$ for MAD flows. We then compare the theoretical results with the values of $\epsilon$ inferred by the EHTC for Sgr A* and M87*, and we thereby constrain the value of $\delta$. The final section is devoted to a brief summary.

2. OBSERVATIONAL CONSTRAINTS ON THE RADIATIVE EFFICIENCY

Both Sgr A* and M87* are low-luminosity systems with hot accretion flows. They are the main targets of the EHT project. Below we provide basic properties of the two sources and derive their mass accretion rates and radiative efficiencies.

2.1. M87*

M87* is located at a distance of $d = 16.9$ Mpc, and the BH mass is measured to be $M_{\text{BH}} = 6.2 \times 10^9 M_{\odot}$ (Gebhardt et al. 2011; EHTC 2019). By comparing the values of four physical quantities obtained from theoretical predictions and those derived from the reconstructed EHT image of M87* and ALMA-only (Atacama Large Millimeter/submillimeter Array) measurements, the EHTC concluded that the accretion flow in M87* is very likely in a MAD state (EHTC 2021). This result has been confirmed recently by Yuan et al. (2022) by comparing the predicted rotation measure in M87* with that measured along the jet. The advantage of the latter work is that it does not suffer from uncertainties in the electron temperature and the contribution of nonthermal electrons in the accretion flow, which are a challenge for the EHTC analysis. The spin of the BH in M87* is hard to determine (EHTC 2021); but some recent works have begun to constrain its value by comparing observed images of the M87* jet with theoretical predictions (Cruz-Osorio et al. 2022; Yang et al. 2022).

Broad-band quasi-simultaneous observations of M87* indicate that the turnover frequency of the sub-mm bump in the $\nu L_\nu$ plot is located at $\sim 230$ GHz (Algha et al. 2021, see also Hada et al. 2011), and the bolometric luminosity of M87* is approximately $L_{\text{bol}} \approx 8.5 \times 10^{41}$ erg s$^{-1} \approx 1.1 \times 10^{-6} L_{\text{Edd}}$. In estimating the bolometric luminosity,
we only include nuclear emission, cf. Fig. 16 and Model 1a of Algaba et al. (2021)\(^1\). We adopt a 20\% uncertainty in \(L_{\text{bol}}\).

Using the EHT polarimetric data, together with additional observational results and theoretical modeling, the EHTC came up with the following constraint on the mass accretion rate near the BH horizon in M87*: \(M_{\text{BH}} \approx (3 - 20) \times 10^{-4} M_\odot \text{yr}^{-1}\) (EHTC 2021), or equivalently, \(M_{\text{BH}} / M_{\text{Edd}} \approx (2.2 - 15) \times 10^{-6}\). Combining this with the bolometric luminosity estimated in the previous paragraph, the derived radiative efficiency (cf. Equation 2) of M87* is

\[
7.5 \times 10^{-3} \lesssim \epsilon \lesssim 5.0 \times 10^{-2},
\]

where the range is driven almost entirely by the uncertainty in \(M_{\text{BH}}\). The two extreme values of the radiative efficiency and their geometric mean are shown in Figure 2 as green filled circles. The gray solid curve connecting the three points represents the allowed combinations of \(\epsilon\) and \(M_{\text{BH}}\) in M87*.

2.2. Sgr A*

The second object is Sgr A*. It has a distance of \(d = 8.13\) kpc and a BH mass of \(M_{\text{BH}} = 4.14 \times 10^6 M_\odot\) (both are average values, as adopted in EHTC 2022a). According to the EHT work, the accretion flow in Sgr A* is probably in a MAD state; specifically, EHTC (2022a) showed that MAD models are more consistent with observational constraints in Sgr A* compared to SANE models, although the models are generally too variable compared to observations (a presently unsolved problem).

The spectrum of Sgr A* peaks at the sub-millimeter waveband (von Fellenberg et al. 2018; Bower et al. 2019; EHTC 2022b), and the luminosity of this sub-mm bump is \(\approx 5.0 \times 10^{35} \text{erg s}^{-1}\) (cf. Bower et al. 2019; EHTC 2022b). The near-infrared emission (Witzel et al. 2018), which also originates from the inner accretion flow, should be included when we estimate the bolometric luminosity. Adopting a spectrum that takes the form of a power-law with exponential cutoff near \(10^{13}\) Hz (i.e. \(F_\nu \propto \nu^a \exp(-\nu/10^{13}\text{Hz})\), Bower et al. 2019), we empirically derive a luminosity of \(\approx 1.9 \times 10^{35} \text{erg s}^{-1}\) for the infrared emission (we take the 50th percentile of Witzel et al. (2018) as representative, see their Figure 19). Combining the sub-millimeter and infrared contributions, the bolometric luminosity of Sgr A* can be estimated to be \(L_{\text{bol}} \approx 6.9 \times 10^{35} \text{erg s}^{-1} = 1.5 \times 10^{-9} L_{\text{Edd}}\). This model-independent measurement of \(L_{\text{bol}}\) agrees with values derived from MAD models (EHTC 2022a), which are in the range \(L_{\text{bol}} = (6.8 - 9.2) \times 10^{35} \text{erg s}^{-1}\). The X-ray emission of Sgr A* during the quiescent state mainly originates from the region \(R > 10^3 R_g\) (Here \(R_g = GM_{\text{BH}}/c^2\) is the gravitational radius of BH). Detailed analysis of the high-resolution Chandra observations indicates that nuclear (\(R < 10^3 R_g\)) X-ray emission has only \(\nu L_{\nu} \approx 1.0 \times 10^{32} \text{erg s}^{-1}\) at 5 keV (Roberts et al. 2017). It is three orders of magnitude fainter than the sub-millimeter bump, thus we ignore this contribution. For the uncertainty in our \(L_{\text{bol}}\) measurement of Sgr A*, we again assume that it is 20\% of \(L_{\text{bol}}\).

The EHT data on Sgr A*, combined with detailed theoretical modeling, tightly constrain the mass accretion rate at the BH horizon (EHTC 2022a and references therein for earlier theoretical/observational works) in the range \(M_{\text{BH}} = (5.2 - 9.5) \times 10^{-3} M_\odot \text{yr}^{-1}\), or equivalently \(M_{\text{BH}} / M_{\text{Edd}} = (5.7 - 10.4) \times 10^{-8}\). The radiative efficiency of Sgr A* is thus constrained to be

\[
1.3 \times 10^{-3} \lesssim \epsilon \lesssim 2.3 \times 10^{-3}.
\]

The above range of values is shown in Figure 2 as cyan diamonds, connected by a gray solid curve that shows the allowed combinations of \(\epsilon\) and \(M_{\text{BH}}\) in Sgr A*. Thanks to the factor of \(< 2\) uncertainty in \(M_{\text{BH}}\), the constraint on \(\epsilon\) is much tighter in the case of Sgr A* compared to M87*.

2.3. *Comparing the Two Objects*

The radiative efficiencies of M87* and Sgr A*, together with the large difference in their Eddington-scaled accretion rates, confirm an important theoretical prediction of hot accretion flow theory. All models, however much they may differ in physics details or parameter choices, agree that the radiative efficiency of hot accretion flows should decrease with decreasing Eddington-scaled mass accretion rate \(M_{\text{BH}} / M_{\text{Edd}}\) (e.g., Narayan & Yi 1995; Narayan et al. 1998; Xie & Yuan 2012; Xie & Zdziarski 2019, and Figure 2 below). This is clearly borne out by the EHT-derived results on M87* and Sgr A*. However, due to the large uncertainty in \(M_{\text{BH}} / M_{\text{Edd}}\) (especially the factor of \(\sim 7\) uncertainty in the case of M87), it is not possible to reliably estimate the slope of the \(\epsilon - M_{\text{BH}} / M_{\text{Edd}}\) relation based on current observational data.

3. RADIATIVE EFFICIENCY OF THE MAD MODEL

3.1. The MAD model

In our numerical model of hot accretion flows in the MAD state, we assume for simplicity a non-spinning BH and adopt a pseudo-Newtonian Paczyński-Wiita gravitational potential (Paczynski & Wiita 1980) to mimic the potential of the BH. Since relativity is not explicitly taken into account in the dynamics of our model, the radial velocity can unphysically exceed the speed of light \(c\) near the BH horizon. To correct for this, we follow Xie et al. (2010) and interpret the velocity \(V_{\nu,\text{PW}}\) in our Paczyński-Wiita potential model as \(\gamma V_{\nu,\text{real}}\), where \(\gamma\) is the bulk Lorentz factor of the accreting gas and \(V_{\nu,\text{real}}\) is the corrected radial velocity.

The neglect of the BH spin parameter \(a\) in the model is reasonable. Although the jet power depends sensitively on spin (e.g., Tchekhovskoy et al. 2010), the emission from the main body of the accretion flow has only a weak dependence. This is because, unlike the cold accretion disk model (Shakura & Sunyaev 1973) where the location of the disk inner edge varies substantially with BH spin, a hot
The estimation of the $\alpha$ parameter for the turbulent stress. The black solid and blue dotted curves show the density-weighted radial velocity of GRMHD simulations of MAD and SANE respectively, around a non-spinning BH (i.e. $a = 0$; Narayan et al. 2022). The purple dashed curve is from our height-integrated MAD model with $\alpha = 0.08$. The shadowed region ($3R_g < R < 6R_g$) shows where most of the observed radiation comes from.

accretion flow extends down to the BH horizon $R_{\text{horizon}}$ which is much less sensitive to $a$. Dynamical properties of the flow outside $(3 - 5)R_{\text{horizon}}$ are mostly unaffected by BH spin, while emission from regions inside of $\sim 2R_{\text{horizon}}$ (e.g., radius of marginally bound orbit) is mostly beamed toward the BH and lost through the horizon.

The details of our MAD model and its radiation calculation can be found in Xie & Zdziarski (2019). Below we highlight the main points. Guided by numerical simulations, Xie & Zdziarski (2019) solve the height-integrated equations of a steady MAD system, paying attention to the effects of non-axisymmetric gas streams/spirals, which are a dominant feature of hot accretion flows in the MAD regime. We follow the GRMHD simulations of McKinney et al. (2012) to set a $R^{-1.1}$ profile of the global vertical component of the magnetic field. Other components are calculated numerically according to the gas dynamics. Most importantly, unlike most MHD simulations where a single-fluid equation is adopted, we use separate energy equations for the electrons and ions (Narayan & Yi 1995; Yuan & Narayan 2014; Xie & Zdziarski 2019). As usual, in our model we mimic the turbulent stress term, which is automatically present in MHD simulations, via a radius-independent $\alpha$ parameter (see Sec. 3.1 for the determination of its value).

Numerical simulations suggest that hot accretion flows, including MADs, have a strong mass outflow (e.g., Narayan et al. 2012; Yuan et al. 2012, 2015; Yang et al. 2021) outside of a certain radius $R_{\text{flat}}$ ($\sim 6 - 10R_g$; refer to Fig. 5 in Yuan et al. 2015 and Fig. 6 in Yang et al. 2021). Inside $R_{\text{flat}}$, the outflow is highly suppressed. Instead of the conventional broken power-law fit, we adopt a smooth function for $\dot{M}(R)$, viz., $\dot{M}(R) = \dot{M}_{\text{BH}} \left[ 1 + (R - R_{\text{BH}})^2/R_{\text{flat}}^2 \right]^{s/2}$, where $s$ measures the strength of the mass outflow. This expression ensures that $\dot{M}(R) \propto R^s$ for $R \gg R_{\text{flat}}$ and $\dot{M}(R) \approx \dot{M}_{\text{BH}}$ for $R \ll R_{\text{flat}}$. In this work we follow the BH spin $a = 0$ MAD case of Yang et al. (2021) and set $R_{\text{flat}} \approx 6R_g$, $s \approx 0.2$. Since the outflow strength we adopt is fairly weak (e.g., $\dot{M}(200R_g) \approx 2\dot{M}_{\text{BH}}$), it has only a minor impact on the radiative efficiency of the model.

### 3.2. Viscous parameter $\alpha$

An important free parameter in our model is the “viscous parameter” $\alpha$ (Shakura & Sunyaev 1973), which has a strong effect on the predicted radiative output of the model. The radiation emitted by a hot accretion flow is largely determined by the gas density $\rho$ which, for a given mass accretion rate, is determined by the efficiency of angular momentum transfer. The latter is proportional to $\alpha$ (note that in our model the stress by global ordered magnetic fields is handled separately, based on numerical simulation results, cf. Sec. 3.1). For a SANE model, we typically have $\rho \propto \dot{M}_{\text{BH}} V_r^{-1} \propto \alpha^{-1}\dot{M}_{\text{BH}}$ (Narayan & Yi 1994). A
similar expression can also be applied for the magnetic field strength.\footnote{We note that for the low-$\dot{M}/\dot{M}_{\text{Edd}}$ regime appropriate to a hot accretion flow, most radiation is in the form of synchrotron, and for optical depth $\tau \lesssim 10^{-6}$ inverse Compton scattering only plays a negligible role. In this case, the radiative luminosity scales as $L_{\text{bol}} \propto \rho B^2 \propto \rho^2 \propto \alpha^{-2} M_{\text{BH}}^2$, (assuming that the electron temperature is un-affected if $\epsilon \ll 1$, cf. Narayan et al. 1998 and Figure 2).}

We use the radial velocity measured in a MAD GRMHD simulation (Narayan et al. 2022) of a non-spinning BH to calibrate the value of $\alpha$ in our model. We focus on the region $R \simeq (3-6) R_q$ from which most of the observed synchrotron radiation in the EHT band (230 GHz) comes from, shown by the shadowed region in Figure 1.

The calculated radial profile of the radial velocity as obtained from our height-integrated MAD model with $\alpha = 0.08$ is shown as the purple dashed line in Figure 1. In comparison, the black solid curve shows the velocity profile seen in the long-duration GRMHD MAD simulation with BH spin $a = 0$ (Narayan et al. 2022). The two profiles agree very well, especially in the shaded region which produces most of the radiation observed by the EHT. Therefore, we use $\alpha = 0.08$ in computing all our model results. For completeness, we also show by the blue dotted curve the GRMHD simulation result for a SANE model with $a = 0$ (Narayan et al. 2022). There is a large difference, suggesting that the global magnetic field in MAD plays an important role in transferring angular momentum and speeding up the radial velocity.

### 3.3. Radiative efficiency

We now calculate the radiative efficiency of our height-integrated MAD model using various values of the electron heating fraction $\delta$. As mentioned earlier, there are two source of heating for electrons in a hot accretion flow: one is through energy transfer from ions via electron-ion Coulomb collisions, and the other is through direct viscous heating at a rate proportional to $\delta$ (see equation 1). The Coulomb heating rate is $\propto \rho^2$, whereas the viscous heating rate is $\propto \rho$ (e.g., Narayan & Yi 1995). Since $\rho \propto M_{\text{BH}}$, we expect the impact of $\delta$ to be most evident at low values of $M_{\text{BH}}/\dot{M}_{\text{Edd}}$. Thus, among the two sources we are considering, Sgr A* is the most likely to give a useful constraint on $\delta$.

For our calculations, we do not focus on detailed microphysics of viscous dissipation (see Introduction). Instead, we take a “model-independent” approach in which we assume a constant electron heating fraction $\delta$ that is independent of radius and explore the observational constraint on its value. We adopt several values: $\delta = 10^{-3}$, 0.1, 0.3, and 0.5. The calculated radiative efficiencies corresponding to these values of $\delta$ are shown as a function of $M_{\text{BH}}/\dot{M}_{\text{Edd}}$ in Figure 2. In our calculations, we limit ourselves to “advection-dominated regime” which corresponds to low accretion rates (Yuan & Narayan 2014) and both M87* and Sgr A* belong to this regime. Its maximal accretion rate at the horizon, above which the radiation is so strong that the flow is no longer advection-dominated but enters into the “luminous hot accretion flow regime” (Yuan 2001; Yuan & Narayan 2014), is determined numerically by having the advection term $f_{\text{adv}} = 1 - q^2/q_{\text{vis}}$ (Narayan & Yi 1994) to be zero at any radius, which is $\lesssim (1 - 3) \times 10^{-5} M_{\text{Edd}}$. Several results can be immediately observed.

First, because the strong poloidal field in a MAD system breaks axisymmetry and causes the accretion to occur in the form of dense gas streams surrounded by dilute magnetic voids (Narayan et al. 2003; Tchekhovskoy et al. 2011; McKinney et al. 2012; White et al. 2019), at a given accretion rate, the density of the gas in the accreting streams in MAD will be higher than that of the more uniform density in SANE. Consequently, the i-e Coulomb heating becomes comparable to the turbulent heating at a lower $M_{\text{BH}}/\dot{M}_{\text{Edd}}$ than for a SANE model (compare with Xie & Yuan 2012).

Second, the maximal accretion rate of ADAF decreases with increasing $\delta$, as indicated in Figure 2. This is naturally expected. At a given accretion rate, a higher $\delta$ means more fraction of dissipated energy goes to electrons. This will makes the electrons hotter, and consequently stronger in outcome radiation.

Third, for a typical ADAF regime with $M_{\text{BH}}/\dot{M}_{\text{Edd}} \lesssim 6 \times 10^{-6}$, we find that the radiative efficiency has a roughly linear dependence on the accretion rate: $\epsilon \propto M_{\text{BH}}^{0.92}$ (for a similar result, see also Xie & Zdziarski 2019). Equivalently, the bolometric luminosity follows $L_{\text{bol}} \propto M_{\text{BH}}^{1.92}$. We note that an estimation of $L_{\text{bol}} \propto M_{\text{BH}}^2$ was derived previously in Narayan et al. (1998).

We now explore the $\delta$-dependence of $\epsilon$ based on our calculations. We find that, at a given accretion rate, the radiative efficiency can differ by a factor of $\sim 5$ for different values of $\delta$. Besides, when $\delta \lesssim 0.05$, the radiative efficiency will become insensitive to the value of $\delta$, because heating by the i-e Coulomb collisions is dominant. For the typical ADAF accretion rate regime with $M_{\text{BH}}/\dot{M}_{\text{Edd}} \lesssim 6 \times 10^{-6}$, we have $\epsilon \propto \max(\delta^{0.7}, 0.1)$, i.e. $\delta = 0.5$ and $\delta = 0.3$ are larger by a factor of $\sim 3$ and $\sim 2.2$ respectively, than the case of $\delta = 0.1$. Combining the dependence on $\delta$ and $M_{\text{BH}}$, our model results can be summarized as,

$$\epsilon \approx 1.9\% \frac{M_{\text{BH}}}{10^{-6} M_{\text{Edd}}}^{0.92} \max(\delta^{0.7}, 0.1),$$

where $\frac{M_{\text{BH}}}{10^{-6} M_{\text{Edd}}} < 6$. (5)

The dependence of $\epsilon$ on $\delta$ becomes weaker when $M_{\text{BH}}/\dot{M}_{\text{Edd}} \gtrsim 6 \times 10^{-6}$, with $\epsilon \approx (2 - 4)\%$. This is because of the increasing importance of i-e Coulomb heating to electrons at high accretion rates.

### 3.4. Constraint on $\delta$ from Sgr A* and M87*

Finally, we apply the above theoretical calculations to the observational constraints from Sgr A* and M87*. From Fig. 2 we find that in the case of Sgr A*, low values of $\delta$ (i.e.
Radiation Efficiency $\epsilon = L_{\text{bol}} / M_{\text{c}}^{\text{bol}}$ at $t_{\text{BH}}$ (refer to Equation 1). The value of $\delta$ describes the fraction of viscous energy that directly heats electrons. Mean-while, by analytically solving the dynamical equations of MAD and calculating the radiative output, we calculate the radiative efficiency as a function of accretion rate and $\delta$. By comparing the theoretical results with those obtained from EHTC, we obtain constraints on the value of $\delta$.

In the case of Sgr A*, we find that $\delta \lesssim 0.3$ is ruled out, and that most likely $\delta \sim 0.5$. This result is in excellent agreement with that obtained in Yuan et al. (2003) in which the constraint on $\delta$ was obtained by modeling the spectral energy distribution of Sgr A*.

In the case of M87*, we are unable to obtain a useful constraint on $\delta$ because EHTC does not provide a sufficiently strict constraint on the mass accretion rate.

\[ \delta \lesssim 0.3 \] are ruled out, and that $\delta \sim 0.5$ is preferred, given the uncertainties in $M_{\text{BH}}$. On the other hand, we cannot put any constraint on $\delta$ based on the current observational data from M87*. A useful constraint at a similar level to Sgr A* will require the mass accretion rate in M87* to be constrained to $M_{\text{BH}} / M_{\text{edd}} \lesssim 8 \times 10^{-6}$ (equivalently, $M_{\text{BH}} \lesssim 10^{-3} M_\odot$ yr$^{-1}$), i.e., the upper limit on $M_{\text{BH}}$ will need to be reduced by a factor $\sim 2$ compared to what the EHT has achieved so far.

4. SUMMARY

In the theory of hot accretion flows around black holes, an important but poorly understood parameter is $\delta$, which describes the fraction of viscous energy that directly heats electrons (refer to Equation 1). The value of $\delta$ determines the energy of electrons and therefore the radiative efficiency of the accretion flow for a given accretion rate. While the underlying microphysics is complicated and the value of $\delta$ is poorly determined from theory, in this paper we try to constrain its value by using the most recent EHTC observational and modeling results on M87* and Sgr A*.

EHTC papers have provided good constraints on the mass accretion rates at the BH horizon for the two sources. These results, combined with the measured bolometric luminosities of the two objects, lead to useful constraints on the radiative efficiencies, as presented in Equations (3) and (4). Mean-while, by analytically solving the dynamical equations of MAD and calculating the radiative output, we calculate the radiative efficiency as a function of accretion rate and $\delta$. By comparing the theoretical results with those obtained from EHTC, we obtain constraints on the value of $\delta$.

In the case of Sgr A*, we find that $\delta \lesssim 0.3$ is ruled out, and that most likely $\delta \sim 0.5$. This result is in excellent agreement with that obtained in Yuan et al. (2003) in which the constraint on $\delta$ was obtained by modeling the spectral energy distribution of Sgr A*.

In the case of M87*, we are unable to obtain a useful constraint on $\delta$ because EHTC does not provide a sufficiently strict constraint on the mass accretion rate.

In this process, we need to determine the value of viscous parameter $\alpha$. We achieve this by asking the analytically obtained radial velocity of the accretion flow at the radius where most of the radiation comes from to be equal to that obtained from GRMHD numerical simulations. The obtained value is $\alpha \sim 0.08$.\footnote{In this process, we need to determine the value of viscous parameter $\alpha$. We achieve this by asking the analytically obtained radial velocity of the accretion flow at the radius where most of the radiation comes from to be equal to that obtained from GRMHD numerical simulations. The obtained value is $\alpha \sim 0.08$.}
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