HOT QUARK-GLUON MATTER WITH DECONFINED HEAVY QUARKS

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Abstract

The phase diagram of the quark-gluon matter evolution is presented for the SU(3)-model with a new phase of heavy deconfined quarks which exists in a rather wide range of temperatures and densities. Fitting the chiral phase transition data to fix the model parameters we establish another (deconfinement) phase transition which separates a new phase from hadronic matter. The parameters and properties of the phase diagram are discussed in comparison with lattice and other results.

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At present for many applications which are based on hot QCD predictions and for future experiments it is very essential to establish a reliable \((\mu, T)\)-phase diagram of the QCD-thermodynamical evolution and to calculate quantitatively the parameters which determine its phase properties. However this is not the case today since the properties of many (at least, two) phases are not elaborated in detail. Only the quark-gluon plasma phase (QGP-phase) which takes place at high temperatures and densities is reliably studied since due to a small coupling constant it can be treated perturbatively. This is the phase of the "light" and deconfined quarks and gluons which are massless in the first approximation. We know as well that there is a phase which presents hadronic matter (H-phase) where quarks and gluons are confined to form the observable nuclear matter. However, there is a question about the intermediate phase (Q-phase) which should present heavy and deconfined quarks. This phase is very natural and has a good physical meaning but till now it is not established firmly. Of course, it has been studied in many papers (in [1] and then in other papers) but no reliable predictions have been given. Today, the interest in this problem was revived again by paper [2] where the intermediate Q-phase is explicitly found and its parameters are determined within the bag model ideology.

The goal of this paper is to derive the QCD-pattern of the Q-phase directly from the Lagrangian approach using the standard calculations within the temperature Green function technique. Fitting the chiral phase transition data to fix the model parameters we establish another (deconfinement) phase transition which separates the Q-phase from hadronic matter. Here we consider that the quark mass sharply arises after crossing the chiral phase transition line and the Q-phase demonstrates itself as a phase of heavy quarks with a rather strong interaction. The deconfinement phase transition occurs when the temperature decreases and within our scenario it does not coincide with the chiral transition. There are two well-separated phase transitions and below we specify their parameters.

The QCD Lagrangian is usual and in covariant gauges it has the form

\[
\mathcal{L} = - \frac{1}{4} G_{\mu \nu}^{a} 2 + N_f \bar{\psi} [\gamma_\mu (\partial_\mu - \frac{1}{2} i g \lambda^a V_\mu^a) + m] \psi \\
- \mu N_f \bar{\psi} \gamma_\lambda \psi + \frac{1}{2 \alpha} (\partial_\mu V_\mu^a)^2 + \bar{C}^a (\partial_\mu \delta^{ab} + gf^{abc} V_\mu^c) \partial_\mu C^b
\]  

(1)

where \(G_{\mu \nu}^a = \partial_\mu V_\nu^a - \partial_\nu V_\mu^a + gf^{abc} V_\mu^b V_\nu^c\) is the Yang-Mills field strength; \(V_\mu\)
is a nonabelian gauge field; $\psi$ (and $\bar{\psi}$) are the quark fields in the SU(N)-fundamental representation ($\frac{1}{2}\lambda^a$ are its generators and $f^{abc}$ are the SU(N)-structure constants) and $C^a$ (and $\bar{C}^a$) are the ghost Fermi fields. In Eq.(1) $\mu$ and $m$ are the quark chemical potential and bare quark mass, respectively, $N_f$ is the number of quarks flavours and $\alpha$ is the gauge fixing parameter ($\alpha = 1$ for the Feynman gauge). The metric is chosen to be Euclidean and $\gamma^2_{\mu} = 1$.

The partition function is determined by the functional integral in the standard manner [3]

$$Z = N \int D(\psi, C, V) \exp \left[ \int d^3 x \int_0^\beta dx_4 \mathcal{L}(\psi, C, V) \right]$$

(2)

where $\beta$ is the inverse temperature and we calculate the thermodynamical potential for hot QCD following the formula

$$\Omega = - T \ln Z.$$  

(3)

The two-loop approximation for $\Omega$ is considered at first but then we go beyond the perturbative theory. In the two-loop approximation the four diagrams (or three ones if the axial gauge is used) should be calculated. The calculations are performed with the aid of the bare Green and vertex functions in the Matsubara technique. The Feynman gauge is more preferable although the final result does not depend on the gauge choice. The renormalization prescription is well-known [4,5] and due to the quark loop (where is the mass renormalization) it is not trivial and more complicated then for hot gluodynamics. Such calculations were made many years ago in [4] for hot QED where the thermodynamical potential with nonperturbative corrections was found and then in [5,6,7] for hot QCD. In a more general case, when the quark mass and chemical potential $m, \mu \neq 0$, the two-loop thermodynamical potential was calculated in [7] and this expression is given by

$$\frac{\Omega^{(2)}}{V} = - \frac{\pi^2 (N^2 - 1)}{45} T^4 + \frac{g^2 (N^2 - 1) N}{144} T^4 - \frac{NN_f}{3\pi^2} \int_0^\infty \frac{dp}{E_p} p^4 n_p$$

$$+ \frac{N_f (N^2 - 1) g^2 T^2}{24\pi^2} \int_0^\infty \frac{dp}{E_p} p^2 n_p + \frac{N_f (N^2 - 1) g^2}{32\pi^4} \int_0^\infty \frac{dp dq}{E_p E_q} p^2 q^2$$
\[ \times \left[ \left( 2 + \frac{m^2}{\text{pq}} \ln \frac{E_p E_q - m^2 - \text{pq}}{E_p E_q - m^2 + \text{pq}} \right) \left( n_p^- n_q^- + n_p^+ n_q^+ \right) + \left( n_p^- n_q^+ + n_p^+ n_q^- \right) \left( 2 + \frac{m^2}{\text{pq}} \ln \frac{E_p E_q + m^2 + \text{pq}}{E_p E_q + m^2 - \text{pq}} \right) \right] \tag{4} \]

where \( n_p = n_p^+ + n_p^- \) and \( n_p^\pm = [\exp(\beta E_p \pm \mu) + 1]^{-1} \) are the usual quark occupation numbers. Here \( E_p = (p^2 + m^2)^{1/2} \) is the quark energy.

Our goal is to go beyond the perturbative expansion. There are several ways of doing it but phenomenologically (as a convenient fit) it is possible to replace \( g^2 \) by the running constant \( g^2(Q^2) \) which further can be used for any temperatures and densities. Here we put our fit into agreement with the chiral phase transition data and then the Q-phase with the massive quarks will be investigated to find the deconfinement phase transition. The proposed fit has a rather simple form

\[ g^2(Q^2) = \frac{\pi^2 b_1}{b_0 \ln \left[ 1 + \frac{Q}{16b_0} \right]} \tag{5} \]

where \( b_0 = (11N - 2N_f)/3 \) and \( b_1 = (34N^2 - 13NN_f + 3N_f/N)/3 \) are the standard renormalization group coefficients (for other more complicated fit see [8]). The ansatz (5) correctly reproduces the high momentum (temperature or density) behaviour and demonstrates the interaction strengthening when a small momentum region is considered. In this region where the infrared divergencies are dominant the canonical dimensions are removed from the theory (like in QED see e.g. in [3]) and anomalous ones arise.

At first, we study the simplest case within the SU(3)-model when \( \mu, m = 0 \) to check the fit (5) and to fix the parameter \( \Lambda \). When the temperature decreases the chiral phase transition should occur and namely this case is more convenient for lattice simulations and for analytical calculations as well: here all integrals in Eq.(4) are calculated exactly. The two-loop thermodynamical potential is given by

\[ \frac{\Omega^{(2)}}{V} = -\frac{\pi^2 T^4}{45} \left[ (N^2 - 1) + \frac{7}{4} NN_f \right] + g^2 T^4 \left( \frac{N^2 - 1}{144} \right) (N + \frac{5}{4} N_f) \tag{6} \]

however Eq.(6) being a pure perturbative expansion does not demonstrate any phase transition [9]. The useful model arises only after replacing \( g^2 \) with
the aid of Eq.(5) and we study its properties below. The phase transition is determined by the condition \((-p = \Omega(Q_c) = 0)\) which here results in the equation

\[
\frac{b_1}{2(\frac{b_1}{16b_0})} = \frac{144[(N^2 - 1) + \frac{7}{4}NN_f]}{45(N^2 - 1)(N + \frac{3}{4}N_f)}
\]

(7)

where \(Q_c = T_c/\Lambda\) is a scaled variable and \(N = 3\). Solving this equation one finds the critical \(Q_c\)-points for each \(N_f\) as follows (see Table 1).

| \(SU(3)\) | \(N_f = 2\) | \(N_f = 3\) | \(N_f = 4\) |
|---|---|---|---|
| \(Q_c^2\) | 0.704 | 0.529 | 0.346 |
| \(T_c(MeV)\) | 186 | 160 | 130 |

Table 1: The data summary for the chiral phase transition within the SU(3)-model. Here \(\Lambda = 222\,\text{MeV}\) to reproduce \(T_c = 260\,\text{MeV}\) since \(Q_c^2 = 1.375\) for \(N_f = 0\). For comparison, see lattice data in [10] and about \(\Lambda\) in [11].

Of course, the data in Table 1 can be improved but the order of their values and the tendency (the monotonic decreasing \(T_c\) with \(N_f\)) are correct. The proposed model reliably reproduces the chiral transition temperature for any \(N_f\).

Now the model is fixed and we study the Q-phase with massive quarks to establish the deconfinement phase transition when the temperature becomes lower. These calculations are more complicated since all integrals in Eq.(4) are not treated exactly when the quark mass \(m\) is non-zero. However for \(\mu = 0\) Eq.(4) keeps a rather convenient form

\[
\frac{\Omega(2)}{V} = -\frac{\pi^2(N^2 - 1)T^4}{45} - \frac{2NN_fT^4\omega^4}{3\pi^2}I_1(\omega) + \frac{g^2(N^2 - 1)NT^4}{144} + \frac{N_f(N^2 - 1)g^2T^4}{12}\left[\frac{\omega^2}{\pi^2}I_2(\omega) + \frac{3}{4}I_3(\omega)\right]
\]

(8)

since all integrals in this case depend on the scaled parameter \(\omega = m/T\).
These integrals are

\[ I_1(\omega) = \int_0^\infty \frac{x^4 dx}{\sqrt{x^2 + 1} \exp(\omega \sqrt{x^2 + 1}) + 1} \]

\[ I_2(\omega) = \int_0^\infty \frac{x^2 dx}{\sqrt{x^2 + 1} \exp(\omega \sqrt{x^2 + 1}) + 1} \]

\[ I_3(\omega) = \int_0^\infty \frac{x dx}{\sqrt{x^2 + 1}} \frac{y dy}{\sqrt{y^2 + 1}} \ln\left(\frac{(x-y)^2}{(x+y)^2}\right) \exp(\omega \sqrt{x^2 + 1}) + 1 \exp(\omega \sqrt{y^2 + 1}) + 1 \]  

and they have to be calculated numerically. But, there is a problem with the \( I_3 \)-integral which is not prepared properly for numerical calculations. However this integral being rather small for \( \omega \geq 1 \) does not affect essentially the final result and due to this fact its logarithm can be slightly redefined

\[ \ln\left(\frac{(x-y)^2}{(x+y)^2}\right) \rightarrow \ln\left(\frac{(x-y)^2 + 0.01}{(x+y)^2 + 0.01}\right) \]  

(10)

to make this integral convenient for the numerical calculations as well.

Let us find the critical point \( T_d \) which determines the deconfinement phase transition in our scenario. We consider the SU(3)-model with \( N_f = (2 \text{ and } 3) \): the experimental data seem to be between these values. It is also assumed that all constituent quarks have the same masses which are generated due to the chiral phase transition and have the magnitude of \( 180 \text{MeV} \leq m \leq 350 \text{MeV} \). This quantity being a free parameter is fitted with the aid of the \( \omega \)-variable to define another (deconfinement) phase transition. The equation which determines the critical temperature of this transition is established through the "standard" requirement \( (-p = \Omega(Q_d) = 0) \) and using Eq.(8) its explicit form is given by

\[ \frac{8}{45} + \frac{2N_f \omega_d^4}{\pi^4} I_1(\omega_d) = \frac{g^2(Q_d^2)}{\pi^2} \times \left\{ \frac{1}{6} \left[ \frac{\omega_d^2}{\pi^2} I_2(\omega_d) + \frac{\omega_d^4}{\pi^4} \left[ 3 I_2^2(\omega_d) + \frac{3}{4} I_3(\omega_d) \right] \right] \right\} \]  

(11)
where \( \omega_d = m/T_d \) and \( Q_c = T_d/\Lambda \). In Eq.(11) all integrals are calculated numerically for the chosen \( \omega_d \)-values and then the equation obtained is solved to find the \( Q_d^2 \)-parameter. The results are summarized in Table 2.

| \( N_f = 2 \) | \( N_f = 3 \) |
|-----|-----|
| \( \omega_d \) | \( Q_d^2 \) | \( T_d (\text{MeV}) \) | \( m (\text{MeV}) \) | \( Q_d^2 \) | \( T_d (\text{MeV}) \) | \( m (\text{MeV}) \) |
| 0.7 | 0.577 | 169 | 118 | 0.400 | 140 | 98 |
| 1.0 | 0.540 | 163 | 163 | 0.361 | 135 | 133 |
| 1.3 | 0.523 | 160 | 208 | 0.339 | 129 | 168 |
| 1.9 | 0.543 | 164 | 312 | 0.343 | 130 | 247 |
| 2.5 | 0.616 | 174 | 435 | 0.395 | 140 | 350 |

Table 2: The data summary for the deconfinement phase transition within the SU(3)-model for \( \mu = 0 \). We keep \( \Lambda = 222\text{MeV} \) to estimate \( T_d \). For comparison see [2] where the similar results are discussed.

From Table 2 we see that indeed \( T_d = 166\text{MeV} \) (for \( N_f = 2 \)) and \( T_d = 140\text{MeV} \) (for \( N_f = 3 \)) seems to be the upper limit for this value since the quark mass is of the order of 350Mev or less. But in any case \( T_d \neq T_c \) (for \( T_c \) see Table 1) and this fact is more important. These results completely confirm the scenario where the Q-phase separates QGP from the usual hadronic mater: the two well-separated phase transitions are established with \( \Delta T_{\text{max}} = T_c - T_d \sim 30\text{MeV} \).

**Below the case \( T = 0 \) is investigated.** Treating this case we start again from the two-loop approximation for \( \Omega \) which for \( T = 0 \) is simpler since all integrals can be calculated analytically. Here one finds that the two-loop thermodynamical potential for cold quark-gluon matter has the form [7]

\[
\frac{\Omega^{(2)}}{V} = - \frac{NN_f \mu^4}{12\pi^2} \left[ \sqrt{1 - \theta^2} (1 - \frac{5}{2} \theta^2) + \frac{3}{2} \theta^4 \ln \frac{1 + \sqrt{1 - \theta^2}}{\theta} \right] \\
+ \frac{g^2 N_f (N_f^2 - 1) \mu^4}{64\pi^4} \left\{ 3 \left[ \sqrt{1 - \theta^2} - \theta^2 \ln \frac{1 + \sqrt{1 - \theta^2}}{\theta} \right]^2 \\
- 2(1 - \theta^2)^2 \right\}
\]

(12)

where \( \theta = m/\mu \) is a new parameter and the link between \( \mu \) and the quark
density $\rho$ is given by

$$\rho = \frac{N_f \mu^3}{3\pi^2} (1 - \theta^2)^{3/2}.$$  

(13)

At first we consider the chiral phase transition which occurs in the quark-gluon plasma with the light (here massless) quarks. In this limit Eq.(12) becomes very simple and, for $N = 3$, it is reduced to [6]

$$p = -\frac{\Omega^{(2)}}{V} = \frac{N_f \mu^4}{4\pi^2} \left(1 - \frac{g^2(Q^2)}{2\pi^2}\right)$$

(14)

where $Q = \mu/\mu_0$. The phase transition occurs when $p = 0$ and the equation which results from this condition has the form

$$1 = \frac{b_1}{2 \, b_0^2 \ln\left[1 + Q_c\left(\frac{b_1}{16b_0}\right)\right]}$$

(15)

where all notations are the same as above. This equation is solved numerically to find $Q_c$ and the results are summarized in Table 3.

| $SU(3)$ | $N_f = 2$ | $N_f = 3$ |
|---------|-----------|-----------|
| $Q_c^2$ | 0.232     | 0.195     |
| $\mu_c (MeV)$ | 700 | 618 |

Table 3: The data summary for the chiral phase transition within the SU(3)-model when $T = 0$. Here we choose $\mu_0 = 1.4GeV$ to estimate the quark chemical potential $\mu_c$ in accordance with [2,12].

The deconfinement phase transition occurs when the quark density decreases and quarks become massive: their mass is generated on the chiral phase transition line. The critical density is determined in the same manner through equation ($-p = \Omega(Q_d^2) = 0$) whose explicit form is found by using Eq.(12). It is given by

$$\frac{b_1}{2 \, b_0^2 \ln\left[1 + Q_d\right]} = \frac{2 \sqrt{1 - \theta_d^2} (1 - \frac{5}{2} \theta_d^2) + 3 \theta_d^4 \ln\frac{1 + \sqrt{1 - \theta_d^2}}{\theta_d}}{3 \left[\sqrt{1 - \theta_d^2 - \theta_d^2 \ln\frac{1 + \sqrt{1 - \theta_d^2}}{\theta_d}}\right] - 2(1 - \theta_d^2)^2}$$

(16)
where $\theta_d = m/\mu_d$. When solving Eq.(16) one finds, at once, that there is an upper bound for $\theta_d$-parameter: only $\theta_d < 0.4$ is acceptable to keep the right side of Eq.(16) positive. This bound means that the quark mass generation is suppressed when $T = 0$ and we find $m_q \sim 60 - 80$ MeV only. Of course, this estimate depends on the scale $\mu_0$ which we fixed above but in any case the chiral phase transition demonstrates itself in the $(T = 0, \mu)$-region very smoothly. Two values for the $\theta_d$-parameter are considered below and for both cases we find approximately the same quark mass but completely different values of $\Delta \mu = \mu_c - \mu_d$ (see the results in Table 4).

| SU(3) | $N_f = 2$ | $N_f = 3$ |
|-------|-----------|-----------|
| $\theta_d$ | $Q^2_\alpha$ | $\mu_d$(MeV) | $m_q$(MeV) | $Q^2_\alpha$ | $\mu_d$(MeV) | $m_q$(MeV) |
| 1/8   | 0.148     | 538       | 67        | 0.106     | 456       | 57        |
| 1/6   | 0.098     | 438       | 73        | 0.068     | 365       | 61        |

Table 4: The data summary for the deconfinement phase transition within the SU(3)-model when $T = 0$. Here we keep $\mu_0 = 1.4$ GeV to estimate $\mu_d$. For comparison see [2] where the similar results are discussed.

From Table 4 one can see that the proposed scenario again presents two well-separated phase transitions with a rather broad Q-phase but, unfortunately, the mass generation in the case $T = 0$ is not expressed sharply.

To conclude two well-separated phase transitions have been established within our scenario. We find that $\Delta T \sim 20$ MeV on the axis $\mu = 0$ and the larger range $\Delta \mu \sim (160-260)$ MeV on the axis $T = 0$. The intermediate $(\mu, T)$-region is not investigated but it is obvious that the scenario should be close. Of course, the present scenario is only a model which should be improved in a selfconsistent manner but it being in agreement with the chiral phase transition data is a solid basis for the theory to come. For removing any arbitrariness it is necessary to solve nonperturbatively the Schwinger-Dyson equation for $m_q$ (e.g. see [13]) and to combine this result with the present scenario. On this way one determines both the dynamical quark mass $m_q(T, \mu)$ and the deconfinement phase transition more exactly. The fit (5) which we used for the running coupling constant is also should be improved. But this question (the same as one about the higher order corrections) being very important and complicated requires additional investigations. Here we choose the simplest fit for $g^2(Q^2)$ to perform the lowest order calculations.
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