Wilsonian Proof for Renormalizability of $\mathcal{N} = \frac{1}{2}$ Supersymmetric Field Theories

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Abstract: We provide Wilsonian proof for renormalizability of four-dimensional quantum field theories with $\mathcal{N} = 1/2$ supersymmetry. We argue that the non-hermiticity inherent to these theories permits assigning noncanonical scaling dimension both for the Grassman coordinates and superfields. This reassignment can be done in such a way that the non(anti)commutativity parameter is dimensionless, and then the rest of the proof amounts to power counting. The renormalizability is also stable against adding standard four-dimensional soft-breaking terms to the theory. However, with the new scaling dimension assignments, some of these terms are not just relevant deformations of the theory but become marginal.

Keywords: string theory, noncommutative geometry, supersymmetry.

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1. Introduction

Recently, deformations of superspace geometry have attracted renewed attention, partly motivated by the quest to better understand Ramond-Ramond background in string theory [1, 2, 3, 4]. For instance, in type IIB superstring compactified on a Calabi-Yau threefold $X$ one can consider the dynamics of space-filling branes in the transverse direction to the Calabi-Yau geometry in the presence of a self-dual graviphoton flux on $\mathbb{R}^4$. Under these conditions the $\mathcal{N} = 1$ superspace is deformed to $\mathcal{N} = \frac{1}{2}$ superspace, whose chiral and antichiral Grassman-odd coordinates obey the Clifford and the Grassman algebras, respectively,

\[
\{\theta^\alpha, \theta^\beta\} = C^\alpha_\beta, \quad \{\bar{\theta}^\dot{\alpha}, \bar{\theta}^\dot{\beta}\} = 0, \quad [y^m, y^n] = 0, \quad (1.1)
\]

where $C^\alpha_\beta$ refers to the self-dual graviphoton flux measured in units of string scale and $(y^m, \theta^\alpha, \bar{\theta}^\dot{\alpha})$ denotes the $\mathcal{N} = 1$ superspace coordinates in the chiral basis. Accordingly, once the graviphoton background is turned on, the Euclidean worldvolume dynamics of the D3-branes gets modified, and is governed by a non(anti)commutative gauge theory with $\mathcal{N} = \frac{1}{2}$ supersymmetry.
With such motivations, various aspects of field theories defined on the deformed superspace have been studied extensively [3, 5, 6, 7, 8, 9, 10, 11]. It was pointed out [3] that the deformation (1.1) induces local operators multiplied by the non(anti)commutativity parameter $C^{\alpha\beta}$. These induced operators are typically of higher scaling dimension, and they might well render the deformed theories nonrenormalizable. Surprisingly, it turned out the deformed theories are renormalizable to all orders in perturbation theory. The first important observation was that the deformed Wess-Zumino model is renormalizable up to two loops [8]. The renormalizability was then extended to all orders in perturbation theory for the deformed Wess-Zumino model in [9, 10], and for deformed gauge theories with(out) matter in [11]. In particular, these works show that, though they carry scaling dimensions larger than four, the deformation-induced operators are radiatively corrected at most logarithmically.

In this work, we offer an intuitive proof for renormalizability of deformed quantum field theories with $N = \frac{1}{2}$ supersymmetry. The key observation is that the deformation (1.1) is chirally asymmetric and renders operators induced by the deformation non-hermitian. This implies that a field theory defined on deformed superspace is non-unitary, so we might as well relax or drop out other assumptions normally required for unitary field theories from the very beginning. The main idea is then to use a different scaling dimension for the superspace coordinates and superfields that appear in the supersymmetric Lagrangian so that, with the new scaling dimension assignment, perturbations around a Gaussian fixed point are marginal or relevant by power-counting. Furthermore, we show that there are only finitely many relevant and marginal operators, so this constitutes a proof of perturbative renormalizability. Any notion and meaning that should be assigned to the new dimensional analysis at nonperturbative level will not be addressed.

This work is organized as follows. In section 2, we study general operator analysis in deformed non(anti)commutative field theories, and provide Wilsonian proof for the renormalizability. In section 3, we discuss various points worthy of mention. we show that the renormalizability is stable against adding soft-breaking operators and variant choices of the superpotentials. We also draw analogy and comparison to known nonunitary conformal field theories.

2. Intuitive Proof of Renormalizability

2.1 Standard and non-standard dimensional analysis

Consider a $N = 1$ supersymmetric quantum field theory, described by the Lagrangian $L_0$, so it consists of D-, F-, and $\bar{F}$-terms. After the deformation (1.1), the deformed theory has $N = \frac{1}{2}$
supersymmetry only, and is described by the Lagrangian of the form:

$$L = L_0(\mathcal{O}, \mathcal{O}^\dagger) + \sum_i \tilde{O}_i(C).$$

The first part, $L_0$, is the Lagrangian of the undeformed theory consisting of local operators $\mathcal{O}, \mathcal{O}^\dagger$. The remainder consists of local operators $\tilde{O}_i$ induced by the deformation (1.1). We include the explicit dependence on the non(anti)commutativity parameter $C^{\alpha\beta}$ into the definition of these operators. Typically, the deformation-induced operators are operators of dimension four or higher, and the operators $\tilde{O}_i$ are suppressed in the (anti)commutative limit $C^{\alpha\beta} \to 0$. Other than this, for the foregoing discussions, explicit form of the operators is not needed.

Quite surprisingly, though involving higher-dimensional operators, such deformed theories exhibits perturbative renormalizability. It was shown, by explicit computation or by operator analysis and power-counting, that both Wess-Zumino model [8, 9, 10] and gauge theories [11] defined on the deformed superspace are renormalizable. Certainly, this is not a feature inherent to ordinary quantum field theory, and calls for a better way of understand this issue.

An important clue is provided by the observation that the deformation parameter $C^{\alpha\beta}$ in (1.1) acts as the R-symmetry-breaking parameter. If we start with the undeformed $\mathcal{N} = 1$ theory with a U(1) R-symmetry $^1$. That is, under R-symmetry

$$\theta^\alpha \to e^{+i\delta} \theta^\alpha, \quad \bar{\theta}^\dagger \to e^{-i\delta} \bar{\theta}^\dagger,$$

the non(anti)commutativity parameter carries R-charge $-2$: $C^{\alpha\beta} \to e^{-2i\delta} C^{\alpha\beta}$. We thus interpret $C^{\alpha\beta}$ as a spurion of R-charge $-2$. We also recall that, in the ordinary $\mathcal{N} = 1$ superspace, from the analytic continuation from Lorentzian spacetime $\mathbb{R}^{3,1}$, $\theta^a$ and $\bar{\theta}^\dagger$ carry equal scaling dimensions, so

$$[\theta^a] = -\frac{1}{2}, \quad [\bar{\theta}^\dagger] = -\frac{1}{2}, \quad [y^m] = -1.$$  \hspace{1cm} \text{(2.2)}$$

In Lorentzian spacetime $\mathbb{R}^{3,1}$, the first two follow from the unitarity requirement of a given theory, viz. $\bar{\theta}^\dagger = (\theta^a)^\dagger$. In Euclidean spacetime $\mathbb{R}^4$, we ordinarily continue working with the same scaling dimensions of the Grassman-odd coordinates. With such R-symmetry and scaling dimension assignments, the D-term in the Lagrangian is always hermitian, while the F-term is hermitian conjugate to the F-term. Likewise, real superfield $V$ continues to be real, and chiral superfield $\Phi$ and antichiral superfield $\bar{\Phi}$ are hermitian conjugate each other. Moreover, local operators $\mathcal{O}$ and their hermitian-conjugate operators $\mathcal{O}^\dagger$ carry the same scaling dimensions:

$$[\mathcal{O}] = \Delta \iff [\mathcal{O}^\dagger] = \Delta.$$  \hspace{1cm} \text{(2.3)}$$

$^1$Here, the R-symmetry may not be an exact symmetry but is broken by various couplings. In this case, by R-symmetry, we refer to (pseudo) R-symmetry in which R-symmetry breaking parameters are promoted to appropriate superfields.
and all of these results are true from unitarity considerations.

For the theories under consideration, we are in a different and interesting situation. Since the non(anti)commutative deformation treats $\theta^\alpha$ differently and independently from $\bar{\theta}^\dot{\alpha}$, the deformed theories are defined only in Euclidean spacetime $\mathbb{R}^4$. Wick rotation to Lorentzian spacetime $\mathbb{R}^{3,1}$ is not permitted: first, it violates the Jacobi identities of the non(anti)commutative deformations, and, second, it is inconsistent with the self-duality of the graviphoton background in the context of Type II string theory. As chiral and antichiral coordinates are a priori independent, various properties implicit to ordinary superspace no longer need to hold. For example, a vector superfield $V$ becomes complex-valued because of the $C^{\alpha\beta}$-dependence:

$$V(y, \theta, \bar{\theta}) = V_{\text{ordinary}}(y, \theta, \bar{\theta}) - \frac{i}{4} \theta^\alpha \bar{\theta}^{\dot{\alpha}} C_{\alpha}^{\gamma} \sigma^m_{\gamma \dot{\gamma}} \{ \bar{\lambda}^{\dot{\gamma}}, A_m \}.$$  

A chiral superfield $\Phi(y, \theta)$ and antichiral superfield $\bar{\Phi}(y, \bar{\theta})$ are independent each other (except for the fact that they are paired in the kinetic term), and so are the superpotentials $W(\Phi)$ and $\bar{W}(\bar{\Phi})$. Thus, in the absence of hermiticity property, we can think of the fields $\Phi$ and $\bar{\Phi}$ as chiral and antichiral superfields, independent each other, and assign non-standard but perfectly sensible scaling dimension to the chiral coordinates $\theta^\alpha$ different from that for antichiral coordinates $\bar{\theta}^\dot{\alpha}$. Explicitly, with a continuous parameter $\delta$, we can assign

$$[\theta^\alpha] = -\frac{1}{2} + \delta, \quad [\bar{\theta}^\dot{\alpha}] = -\frac{1}{2} - \delta, \quad [y^m] = -1,$$  

and, for the non(anti)commutativity parameter, $[C^{\alpha\beta}] = -1 + 2\delta$. Notice that the new scaling dimension assignment is compatible with the anticommutation relations involving $\mathcal{N} = \frac{1}{2}$ supercharges $Q_\alpha$:

$$\{ Q_\alpha, Q_\beta \} = 0$$
$$\{ Q_\alpha, \bar{Q}_{\dot{\alpha}} \} = \sigma^{m}_{\alpha \dot{\alpha}} P_m$$
$$\{ \bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}} \} = \sigma^{m}_{\dot{\alpha} \dot{\beta}} \sigma^{n}_{\dot{\alpha} \dot{\beta}} P_m P_n,$$  

where $[Q_\alpha] = 0, [\bar{Q}_{\dot{\alpha}}] = +1$ and $[P_m] = +1$. We can also assign different scaling dimension to chiral superfields differently from their corresponding antichiral superfields, and scaling dimension of a chiral operator differently from that to its hermitian-conjugate antichiral operator. Possibility of the nonstandard scaling dimension assignment (2.4) constitutes the crux for proving renormalizability of $\mathcal{N} = \frac{1}{2}$ supersymmetric field theories defined on deformed superspace (1.1).

\textsuperscript{2}The associated graviphiton background is either complex or it carries energy-momentum and there is back-reaction to the background geometry.

\textsuperscript{3}Notice that because $\bar{Q}_{\dot{\alpha}}$ is not a conserved charge and the third relation is not a derivation operation, these equations do not form a Lie algebra.
2.2 Renormalization group flow

With our nonstandard scaling dimension assignment, we can explain intuitively renormalizability of the non(anti)commutative field theories. Recall that, in the Wilsonian approach [12], renormalizability is ensured if all operators involved in the renormalization group flow retain scaling dimensions equal to four or less and if there are only a finite number of such operators. More precisely, renormalizability follows if flows of renormalized couplings form a finite-dimensional subspace in the infrared in the total space of all possible operators.

Notice that the Wilsonian renormalization group flow, as summarized above, does not rely explicitly on how one assigns scaling dimension to each elementary field and each Grassman-odd coordinate of the superspace. In particular, if we assume we have a Gaussian fixed point in the ultraviolet, the perturbative expansion will be given by the same Feynmann diagrams with any other assignment of the scaling dimensions. It is thus advantageous to assign scaling dimensions to be the most convenient of all possible choices. Such a freedom is not available for theories with hermiticity. In the present context, the full theory is given by (2.1) and is non-hermitian. Since the deformation-induced operators \( \tilde{O} \), carry dimensions higher than four in the standard scaling dimension assignment, we would look for a new assignment wherein these induced operators (along with the Gaussian terms) carry scaling dimension four. Such assignment is motivated in part by the observation [8, 9, 10, 11] that possible radiative corrections to these operators are logarithmic only. It is not hard to see that the most suitable choice is \( \delta = 1/2 \) in (2.4), viz.

\[
\delta = \frac{1}{2} : \quad [\theta^\alpha] = 0, \quad [\bar{\theta}^\alpha] = -1, \quad [y^m] = -1. \tag{2.6}
\]

It immediately follows that the non(anti)commutativity \( C^{\alpha\beta} \) parameter can be promoted to a dimensionless coupling parameter. This means that for the free field theory the non(anti)commutative deformation does not introduce any scaling violations, and can still be considered as an ultraviolet fixed point. This should be sharply contrasted against the noncommutative field theories, where the deformation parameter \( \Theta^{mn} \) carries always dimensions and spoils the idea that, in the ultraviolet, there is a Wilsonian renormalization-group fixed point, from which one is perturbing away to obtain the infrared dynamics.

The new scaling dimension assignment leads to the following virtue when a theory is deformed by the non(anti)commutativity (1.1). As \( [\theta^\alpha] \) and \( [C^{\alpha\beta}] \) are assigned dimensionless, the star product,

\[
\star = \exp \left( -\frac{1}{2} C^{\alpha\beta} \overleftarrow{Q}_\alpha \overrightarrow{Q}_\beta \right) \quad \text{where} \quad Q_\alpha := \frac{\partial}{\partial \theta^\alpha},
\]

used for the non(anti)commutative deformation (1.1) would not generate operators with higher scaling dimensions. Therefore, once the undeformed theory is renormalizable (with the new counting...
of dimension), the new theory deformed with non(anti)commutative is manifestly renormalizable as well.

2.3 Wess-Zumino model

This much said, we now examine explicitly the lagrangian for the deformed Wess-Zumino model and test our argument further.

Consider the deformed Wess-Zumino model with

\[ K_\ast(\Phi, \bar{\Phi}) = \bar{\Phi} \Phi, \quad W_\ast(\Phi) = \left(\frac{m}{2} \Phi^2 + \frac{g}{3} \Phi^3\right)_\ast, \quad \bar{W}_\ast(\bar{\Phi}) = \left(\frac{\bar{m}}{2} \bar{\Phi}^2 + \frac{\bar{g}}{3} \bar{\Phi}^3\right)_\ast. \]

(2.7)

The undeformed theory is unitary and renormalizable. With the deformation (1.1), the theory is a sum of ordinary Wess-Zumino model with \( \mathcal{N} = 1 \) supersymmetry and a deformation-induced operator \[ \tilde{\mathcal{O}} = -\frac{g}{3} |C| F^3, \]

(2.8)

where \( |C| \equiv \det C^{\alpha\beta} \). Evidently, \( \tilde{\mathcal{O}} \) is nonhermitian, and the theory is nonunitary.

Now, we can make the dimensional analysis with the new scaling dimension assignments for \( \theta^{\alpha}, \bar{\theta}^{\dot{\beta}} \). The D-term measure \( \int d^2\theta d^2\bar{\theta} \) has the scaling dimension 2, viz. its dimension does not change. The \( \mathbb{R}^4 \) measure \( d^4y \) has scaling dimension \(-4\). The kinetic term should be marginal in the ultraviolet, so \([\Phi] = 2 - [\bar{\Phi}]\). The F-term measure \( \int d^2\theta \) is dimensionless, while the \( \bar{\Phi} \)-term measure \( \int d^2\bar{\theta} \) has scaling dimension 2. In order for the cubic term in the undeformed antiholomorphic superpotential \( \bar{W}(\bar{\Phi}) = \bar{\Phi}^3 \) to be at most marginal, we need to assign scaling dimensions at the fixed point such that \([\bar{\Phi}] \leq 2/3\). From this, \([\Phi] \geq 4/3\), so the undeformed holomorphic superpotential \( W(\Phi) = \Phi^3 \) is marginal only if \([\Phi] \leq 4/3\). It follows that we need to assign the scaling dimensions such that \([\Phi] = 4/3\) and \([\bar{\Phi}] = 2/3\). It also follows that all of their component fields have positive scaling dimensions.

With these dimensions all of the possible terms in the lagrangian have dimension less than or equal to four. Moreover, the mass parameters \( m, \bar{m} \) have scaling dimensions \( 4/3, 2/3 \), respectively, so one still can promote them as the lowest components of (anti)chiral superfields. Since both \( \Phi, \bar{\Phi} \) and all of their superfield components have positive dimensions, any local operator involving polynomial of them and superspace derivatives carries always a positive scaling dimension, and hence there are only finitely many such operators with scaling dimension less than or equal to four.

2.4 Supersymmetric gauge theories

Consider next the deformed gauge theories. The undeformed part of the theory is the ordinary \( \mathcal{N} = 1 \) supersymmetric gauge theory with massless matter. The Lagrangian has no dimensionful
coupling parameter and has U(1) R-symmetry. The deformation-induced operators consist of
\[\tilde{O}_1 = C^{\alpha\beta} \text{Tr} \{ F_{(\alpha\beta)} \overline{\lambda} \lambda \}, \quad \tilde{O}_2 = |C|^2 \text{Tr} (\overline{\lambda} \lambda)^2 \]
\[\tilde{O}_3 = C^{\alpha\beta} (\nabla_m \overline{\varphi}) \sigma^m_{\alpha\alpha} \overline{\lambda} \psi_\beta, \quad \tilde{O}_4 = C^{mn} \varphi F_{mn}, \quad \tilde{O}_5 = |C|^2 \overline{\varphi} \lambda \lambda F. \quad (2.9)\]
Again, with \[\theta^\alpha = 0\] and \[\overline{\theta}^\lambda = -1\], the superfield analysis yields that \[W_\alpha = 2\] and \[\overline{W}_\dot{\alpha} = 1\]. It assigns scaling dimension unchanged for the vector field, \[V_m = 1\], but changed for the gauginos as \[|\lambda| = 2\], \[|\overline{\lambda}| = 1\]. The deformation of field strength superfields \[\text{is automatically compatible with the new scaling dimension assignment and} \quad C^{\alpha\beta} = 0\]. The new scaling dimensions for matter superfields are the same as the Wess-Zumino model case.

With the nonstandard scaling dimension assignment, the undeformed part of the Lagrangian contains again operators of dimension equal to four only. The gauge coupling parameter remains dimensionless. The deformation-induced operators are \[(2.9)\], and they all have new scaling dimensions equal to four. Hence, the entire deformed Lagrangian contains operators of dimension four only and the theory is manifestly renormalizable.

3. Further Discussions

3.1 Soft-breaking terms

We can also utilize the idea developed in the previous section, and show that the standard soft-breaking terms in four-dimensional supersymmetric field theories are relevant or at most marginal. This is a simple exercise in dimensional analysis with the nonstandard scaling dimension assignment.

The soft-breaking operators consist of the followings. Mass terms for scalar operators are of scaling dimension \(4/3\), \(2\), and \(8/3\), respectively, while cubic terms can be of up to dimension 4 when we take the field of scaling dimension \(4/3\) to the cubic power. Also a gaugino mass term yields terms of dimension 4 and 2 for chiral and antichiral components, respectively.

A surprising fact is that, with the new scaling dimension assignment, some of the standard soft-breaking terms turn into marginal ones. This is because these terms also induce breaking of the R-symmetry, and then they are not protected by the nonrenormalization theorems. Presumably, this would spoil various relations among the renormalized couplings in the full theory and one should instead consider the most general Lagrangian with fields given as above, with all possible operators
of dimension less than or equal to four that can be made out of polynomials of the fields and their superspace derivatives.

Also, since rotational invariance of \( \mathbb{R}^4 \) is broken from \( \text{SO}(4) \) to a subgroup which contains only an \( \text{SU}_R(2) \times \text{U}(1) \) isometry subgroup, we should include various operators which are not Lorentz invariant, but they should still respect the above isometry group. These are the terms which can be generated by turning on the non(anti)commutative deformations.

### 3.2 Variants

Extending previous discussion, we can assign \([\Phi] = 1 + \epsilon\) and \([\bar{\Phi}] = 1 - \epsilon\), and so long as both dimension are positive, there are only a finite number of relevant or marginal operators in the field theory.

Recall that, for Euclidean supersymmetries, the superpotentials \( W(\Phi) \) and \( \bar{W}(\bar{\Phi}) \) are independent input data of a given theory. Denote the highest monomial of \( W(\Phi) \) and \( \bar{W}(\bar{\Phi}) \) as \( n, \bar{n} \), respectively. In case \( n = \bar{n} \), by repeating the dimensional analysis with the new scaling dimension assignment, we observe that the theory is renormalizable only if \( n = \bar{n} = 3 \) and \( \epsilon = 1/3 \). This reproduces the standard result. If \( \bar{n} = 2 \), viz. \( \bar{W}(\bar{\Phi}) = \frac{m^2}{2} \bar{\Phi}^2 \), we can take \( n \) up to four and the theory will still be renormalizable. If \( n = 2 \), viz. \( W(\Phi) = \frac{m}{2} \Phi^2 \), then, surprisingly, we find that the dimension of \([\Phi]\) can be made arbitrarily small and positive, so that the antiholomorphic superpotential \( \bar{W}[\bar{\Phi}] \) of any polynomial is permitted while retaining renormalizability. In fact, because of the \( \mathcal{N} = \frac{1}{2} \) (non)renormalization theorem \([5]\), \( \bar{W}[\bar{\Phi}] \) would not be renormalized at all. Evidently, \( \bar{W}[\bar{\Phi}] \) of arbitrary polynomial leads to a richer structure of the \( \mathcal{N} = \frac{1}{2} \) antichiral ring.

Again, in all these variant cases, the non(anti)commutative deformation would not spoil the renormalizability, as the parameter \([C^{\alpha\beta}]\) carries the scaling dimension zero.

### 3.3 Analogy & Comparison

We should think of the pair of (anti)chiral superfields \( \Phi, \bar{\Phi} \) or \( W_\alpha, \bar{W}_{\dot{\alpha}} \) as the four-dimensional equivalent of the \( b,c \)-system in two dimensions. Recall that the \( b,c \)-system is ordinarily a non-unitary free field theory with the Lagrangian \( \int dz d\bar{z} b \partial c \), and with \( b \) and \( c \) having different scaling dimension. The difference is not felt in the correlators of the free fields themselves nor in the Feynman rules if one works in the perturbation theory. However, the energy-momentum tensor of the two-dimensional conformal field theory carries an improvement term: \( T = b \partial c - \alpha c \partial b = T_{\text{ordinary}} + a \partial J \), where \( J \) is the \( \text{U}(1) \) ghost current. Accordingly, scaling dimensions of various operators will be modified depending on the assignment of the \( \text{U}(1) \) ghost charge (the value of \( a \)).

Drawing further analogy and comparison of the \( b,c \)-system with the four-dimensional theories under discussion might uncover surprises.
In this regard, the idea of nonstandard scaling dimension assignment seems to match with the notion of twisting the energy-momentum tensor by R-symmetry current that one does in topological field theories [14]. This may be an indication that there exits a nonperturbative sector in non(anti)commutative field theories which is topological in nature and for which the deformed Lagrangian can be used to compute a certain class of amplitudes or correlators. The antichiral ring (viz. operators annihilated by \( Q_\alpha \) of \( \mathcal{N} = \frac{1}{2} \) supersymmetry) is obviously part of it, and it would be interesting to understand if the sector could be bigger than this.

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