$B_s^* BK$ vertex from QCD sum rules

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The form factors and the coupling constant of the $B_s^* BK$ vertex are calculated using the QCD sum rules method. Three point correlation functions are computed considering both $K$ and $B$ mesons off-shell and, after an extrapolation of the QCDSR results, we obtain the coupling constant of the vertex. We study the uncertainties in our result by calculating a third form factor obtained when the $B_s^*$ is the off-shell meson, considering other acceptable structures and computing the variations of the sum rules’ parameters. The form factors obtained have different behaviors but their simultaneous extrapolations reach to the same value of the coupling constant $g_{B_s^* BK} = 10.6 \pm 1.7$. We compare our result with other theoretical estimates.

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I. INTRODUCTION

Hadronic three-meson vertex functions or form factors are basic inputs to phenomenological theories of nuclear processes. They play an important role in cross section calculation, which may vary in a few orders of magnitude if they are considered. The form factor depends on the momentum \( Q^2 \) and some parameters. As usual, the form factor is parametrized by an \( \text{ad hoc} \) function, which is not unique and can be Gaussian, exponential or monopolar. From the parametrization, we can extract a cutoff parameter \( \Lambda \) which is associated with the sharpness of the form factor. Therefore, knowing its precise functional form and its cutoff parameter are essential.

Some years ago, we started a program to compute form factors with the QCD sum rules (QCDSR) [1]. Our group has developed a method which leads to less ambiguities in the determination of the coupling constant. The method consists of computing two form factors of the same three meson process: one form factor is evaluated when the heavy meson is off-shell and the other form factor is obtained when the light meson is off-shell. By extrapolating both form factors to the momentum \( Q^2 \) equal to \(-m^2\), where \( m^2 \) is the mass of the off-shell particle, we obtain the coupling constant of the vertex. After our pioneer \( DD\rho \) work [2], we concluded that when the form factor tends to be harder as a function of \( Q^2 \), the off-shell meson in the vertex is heavy, which means the cut-off parameter is larger. A persistent study of vertices involving charmed mesons (\( D^*\Delta \pi \) [3, 4], \( DDJ/\psi \) [5], \( D^*D\pi \) [6], \( D^*J/\psi \) [7, 8], \( DsD^*K \), \( D_s^0DK \) [9] and \( D^*\rho \) [10]) motivated by the development of effective theories related to charm particles interaction, showed a similar conclusion in all of them.

However, in recent years, due to the precise measurements of \( B \) decays performed by BELLE, BES and BABAR, the physics of \( B \) meson has gained a new relevance. The observed \( B_{s1}(5830) \) by CDF Collaboration [11] and \( B_{s2}(5840) \) by CDF and D0 Collaboration [12] stimulated our interest in the physics of bottom mesons interactions [13]. In particular, the unobserved \( B_{s0}^* (5725) \) state and the axial \( B_{s0}^* (5778) \) have been interpreted as hadronic bound states (hadronic molecules), because their masses are close to the thresholds of the corresponding hadronic pairs. In reference [13], the authors used a phenomenological Lagrangian approach to calculate the strong and radiative decay of these new mesons, \( B_{s0}^* (5725) \) and \( B_{s0}^* (5778) \), which were considered as bound states of the \( BK \) and \( B^*K \) mesons respectively, to provide more information about some properties. In particular, the coupling constant of \( B_s^*BK \) vertex was an input in this calculation.

Considering that the form factors are introduced in the development of effective theories to study bottom interactions, in this paper, we calculated the form factors and coupling constant of the \( B_s^*BK \) vertex, using the QCDSR method, which is the only method that permits to extract the coupling constant of the three meson processes without any dependence on other empirical coupling constant. Therefore, we evaluate two form factors, one when \( B \) is the off-shell particle and another when \( K \) is the off-shell and extrapolate these results to obtain the coupling constant \( g_{B_s^*BK} \). We also want to estimate the uncertainties of the QCDSR method by analyzing different sources of errors.

In section II, we describe the QCDSR technique; in II.A, the QCD side for this vertex and in II.B, the phenomenological side. In section III, we show the results; in section IV, we estimate the uncertainties and, finally, conclude.

II. THE SUM RULE FOR THE \( B_s^*BK \) VERTEX

Following our previous works, specifically in Ref. [10], the three-point function associated with the \( B_s^*BK \) vertex is given by

\[
\Gamma^{(B)}_{\mu}(p, p') = \int d^4x d^4y \ e^{i p'^\cdot x} e^{-i (p' - p) \cdot y} \langle 0 | T \{ j_\mu^K(x) j_5(B^1(y)) j_\mu^{B_s^*}(0) \} | 0 \rangle
\]

(1)

for an off-shell \( B \) meson, and:

\[
\Gamma^{(K)}_{\mu}(p, p') = \int d^4x d^4y \ e^{i p'^\cdot x} e^{-i (p' - p) \cdot y} \langle 0 | T \{ j_\mu^{B_s^*}(x) j_5^K(y) j_\mu^{B_s^*}(0) \} | 0 \rangle,
\]

(2)

for an off-shell \( K \) meson. The expressions for the vertices [11] and [2] contain different numbers of Lorentz structures, and each structure can be a different Sum Rule (SR) to be considered. Equations (1) and (2) can be calculated in two different ways: using quark degrees of freedom—the theoretical or the QCD side—using hadronic degrees of freedom—the phenomenological side. In the QCD side, the correlators are evaluated using the Wilson operator product expansion (OPE), which incorporates the effects of the QCD vacuum through an infinite series of condensates of increasing dimension. The phenomenological side is written in terms of hadronic degrees of freedom, which is the responsible for the introduction of the form factors, decay constants and masses. After performing a double Borel transformation, both representations are matched involving the quark-hadron global duality.
A. The OPE side

In the OPE or theoretical side, each meson interpolating field appearing in Eqs. (1) and (2) can be written in terms of the quark field operators. For the $B$ off-shell case, Eq. (1), we use the following meson currents:

$$
\begin{align*}
  j^{B*}_\mu(0) &= \bar{b}\gamma_\mu s \\
  j^{B}_5(y) &= i\bar{q}\gamma_5 b \\
  j^K_\mu(x) &= i\bar{q}\gamma_\mu\gamma_5 s 
\end{align*}
$$

(3)

and when the $K$ is off-shell, we have chosen the pseudoscalar current for it, following the same procedure as in previous works as $D^*D\pi$ [4] and $D^*_sDK$ [10]:

$$
  j^K_5(x) = i\bar{q}\gamma_5 s, 
$$

(4)

this choice of current is motivated by the coupling constant that is obtained by extrapolating the $g^{(K)}_{B_sB_K}(Q^2)$ to momentum $Q^2$ equal to the square mass of the particle, that in this case will be $m_K^2$. Otherwise, if we use the axial vector current when $K$ meson is off-shell, the sum rule obtained is also good and the coupling constant result is very similar to the pseudoscalar case.

The $u$, $d$, $s$ and $b$ are the up, down, strange and bottom quark fields, respectively. Each one of these currents has the same quantum numbers of the associated mesons.

In order to obtain the QCD side, we use the Cutkosky rule and we obtain an expression where all the possible structures appear. We can obtain a sum rule for each structure that have an invariant amplitude, which can be written in a double dispersion relation over the virtualities $p^2$ and $p'^2$ holding $Q^2 = -q^2$ fixed:

$$
\Gamma^T(p^2, p'^2, Q^2) = -\frac{1}{4\pi^2} \int_{s_{\text{min}}}^{\min} ds \int_{u_{\text{min}}}^{\min} du \frac{\rho^T(s, u, Q^2)}{(s - p^2)(u - p'^2)}, 
$$

(5)

where $\rho^T(s, u, Q^2)$ equals the double discontinuity of the amplitude $\Gamma^T(p^2, p'^2, Q^2)$, $T$ index means $B$ off-shell or $K$ off-shell, $q = p' - p$ and $s_{\text{min}}$ and $u_{\text{min}}$ are integration limits equal to $s_{\text{min}} = (m_b + m_s)^2$ and $u_{\text{min}} = t - m_b^2 + m_s^2$ for $B$ off-shell and $s_{\text{min}} = m_K^2$ and $u_{\text{min}} = t + m_K^2$ for $K$ off-shell.

The invariant amplitudes receive contributions from all terms in the OPE. The first of those contributions comes from the perturbative term, which is represented in Fig. 1.

![FIG. 1. Perturbative diagrams for the $B$ off-shell (left) and $K$ off-shell (right) correlators.](image)

In order to obtain the form factor, we choose one of the structures in Eqs. (1) and (2), the one that has less ambiguities in the QCD sum rule approach, which means less influence from the higher dimension condensates, better stability as a function of the Borel mass and has larger pole than continuum contribution. The other structures that are “good” sum rules can also be considered to improve the uncertainties of the method. For $B$ off-shell meson, which has two different structures, we choose the $p'_\mu p'_\nu$ because it is the best structure with less ambiguities. For $K$ off-shell meson, we have $p_\mu$ and $p'_\mu$ structures, both being excellent sum rules. The chosen is $p'_\mu$ and the other will be used in the calculation of the uncertainties.

The corresponding perturbative spectral densities, which enter in Eq. (3), are:

$$
\rho^{(B)}(s, u, Q^2) = \frac{3}{2\pi\sqrt{\lambda}} \left[ (2m_b(A - B)) \right] 
$$

(6)

for the $p'_\mu p'_\nu$ structure of the $B$ off-shell case, and

$$
\rho^{(K)}(s, u, Q^2) = -\frac{3}{2\pi\sqrt{\lambda}} \left[ \frac{(u - s - t)}{2} - m_s^2 + m_b^2 + m_s m_s + \frac{s + m_s^2 - 2m_b^2}{2} \right] 
$$

(7)
for the $p_i'$ structure of the $K$ off-shell case, where $\lambda = \lambda(s,u,t) = s^2 + t^2 + u^2 - 2st - 2su - 2tu$, $s = p^2$, $u = p'^2$, $t = -Q^2$ and $A$ and $B$ are functions of $(s,u,t)$, given by the following expressions:

$$A = -\pi \frac{|k|^2}{|p'|} (3 \cos \theta - 1); \quad B = 2\pi \frac{|k|}{|p'|} \cos \theta;$$

where

$$\overrightarrow{|k|} = k_0^2 - m_i^2; \quad \cos \theta = -\frac{2p_0'k_0 - u - m_i^2 - \eta m_i^2}{2|p'||k|};$$

$$p_0' = \frac{s + u - t}{2s}; \quad |p'|^2 = \frac{\lambda}{4s}; \quad k_0 = \frac{s + m_i^2 - \epsilon m_i^2}{2\sqrt{s}};$$

with $i = s, \eta = 0$ and $\epsilon = 1$ for $B$ off-shell and $i = b, \eta = -1$ and $\epsilon = 0$ for $K$ off-shell.

The main OPE contribution is the perturbative. The next terms in the OPE expansion are the quark condensate, the gluon condensate and the mixed quark-gluon condensate. In this work, the quark condensate was calculated and has the following expression obtained for the $K$ off-shell case:

$$\Gamma_{<bK>} = m_s < b > e^{m_s^2/M_s^2} p_{\nu}$$

this contribution is proportional to the bottom condensate, consequently, the contribution is null [1]. The other two quark condensates contributions for the $K$ off-shell case vanish after the application of the double Borel transformation. For $B$ off-shell case, the quark condensates do not contribute for the structure considered. In this calculation, we do not include gluon condensate and mixed quark gluon condensate, because as seen in previous calculation of the form factors of $D^* D \pi$ and $B^* B \pi$ vertices [1], we found out that the gluon contribution is negligible as compared with the perturbative one. Moreover, the gluon condensate decreases with the Borel mass and the most important role of the gluon condensate is to guarantee the stability as a function of the Borel mass. In this vertex, this stability is guaranteed together with a good contribution of the pole.

### B. The phenomenological side

The three-point functions from Eqs. (11) and (12), when written in terms of hadron masses, decay constants and form factors, are the phenomenological side of the sum rule, which is based on the interactions at the hadronic level. These interactions are described here by the following effective Lagrangian [10]:

$$\mathcal{L}_{B^*B K} = ig_{B^*B K}\left[ B^*_{\mu}\bar{B}\partial_\mu K - \partial_\mu \bar{B} K - B^*_{\mu}\bar{K}\partial_\mu B - \partial_\mu B K \right],$$

from where we can extract the matrix element associated with the $B^* B K$ vertex.

The meson decay constants $f_K$, $f_B$ and $f_{B^*}$ are defined by the following matrix elements:

$$\langle 0 | j_{\nu}^K | K(p) \rangle = if_K p_\nu,$$

$$\langle 0 | j_{\mu}^{B^*} | B^*(p) \rangle = m_{B^*} f_{B^*} \epsilon_{\mu}^*(p),$$

and

$$\langle 0 | j_{\mu}^B | B(p) \rangle = \frac{m_B^2}{m_b} f_B,$$

where $\epsilon_{\mu}^*$ is the polarization vector for the $B^*$ meson. Saturating Eqs. (11) and (12) with $K$ and $B^*$ states and using Eqs. (12) - (13), we arrive at

$$\Gamma_{\mu\nu}^{(B)}(p,p',q) = g_{B^*B K}(q^2) \left[ \frac{f_{B^*} f_B m_{B^*} m_B^2}{m_B^2} \left( p'^2 - m_{B^*}^2 \right)^2 \left( q^2 - m_{B^*}^2 \right)^2 \left( p'^2 - m_{K}^2 \right) \right] \times \left[ 2p'_\mu p'_\nu - p'_\mu p_\nu \left( 1 + \frac{(m_{K}^2 - q^2)^2}{m_{B^*}^4} \right) \right] + \text{"continuum"},$$

(15)
when \( B \) is off-shell. Using the matrix element of \( K \) meson equal to
\[
\langle 0 | j^K_\mu | K(p) \rangle = \frac{m_K^2}{m_s} f_K,
\]
we arrive at an expression for \( K \) off-shell:
\[
\Gamma^{(K)}(p, p', q) = -g^{(K)}_{B^*BK}(q^2) \frac{f_B f_K f_{B^*} m_B^2 m_{B^*}^2 m_K^2}{m_B m_s (p^2 - m_B^2) (p'^2 - m_{B^*}^2) (q^2 - m_K^2)} \times \left[ 2 p_\mu + p'_\mu \left( 1 + \frac{(m_B^2 - q^2)}{m_{B^*}^2} \right) \right] + \text{“continuum”}.
\]
(17)

In the Eqs. (15) and (17), we can see all the structures that appear in the QCDSR and the “continuum” contribution, which may be subtracted assuming that its contribution is equal to the contribution of the QCD side, by introducing the continuum thresholds \( s_0 \) and \( u_0 \) as upper integration limits in Eq. (5).

In order to improve the matching between the two sides of the sum rule, we perform a double Borel transformation in the variables \( P^2 = -p^2 \rightarrow M^2 \) and \( P'^2 = -p'^2 \rightarrow M'^2 \), on both invariant amplitudes \( \Gamma^{(T)} \) (QCD side) and \( \Gamma^{(T)}_{ph} \) (phenomenological side). Equating the results, we obtain the form factors \( g^{(K)}_{B^*BK}(Q^2) \) that appear in Eqs. (15) and (17).

III. RESULTS AND DISCUSSION

Table I shows the parameters used in the present calculation. We have used the experimental value for \( f_K \) of Ref. [18], for \( f_{B^*} \) and \( f_B \) from Ref. [19]; for \( m_s \) from Ref. [20] and for \( m_B \) we used recent results of Ref. [21, 22]. The continuum thresholds are given by \( s_0 = (m_i + \Delta_s)^2 \) and \( u_0 = (m_o + \Delta_u)^2 \), where \( m_i \) is the mass of the incoming meson, \( m_o \) is the mass of the out coming meson and \( \Delta_u \) and \( \Delta_s \) are usually around 0.5 GeV, but this value can be varied to obtain a good pole-continuum contribution and stability of the sum rule with the Borel mass parameter.

| \( q \) (GeV) | \( s \) | \( b \) | \( K \) | \( B \) | \( B^* \) |
|---------------|-------|-----|------|------|------|
| 0.007         | 0.13  | 1.20 | 0.49 | 5.27 | 5.41 |
| \( J \) (MeV) | -     | -   | 160  | 208  | 250  |

TABLE I. Parameters used.

A. B off-shell form factor

In order to obtain the \( B \) off-shell form factor, we choose the structure \( p'_\mu p_\mu \) which has an excellent stability and its pole contribution is larger than the continuum contribution in a Borel region between \( 10 < M^2 < 24 \, \text{GeV}^2 \). The first order in the OPE, the quark condensate does not contribute in this structure. In this paper, the next order in the OPE, the gluon condensate was not calculated, because in our previous work of \( D^*D\pi \) form factor [4] and also in \( J/\Psi D^*D \) [22], we found out that the gluon condensate contribution is less than 10\% for a Borel mass of \( 5 \, \text{GeV}^2 \). In this case, we are dealing with heavy mesons and so we are working with a Borel mass that is \( 21 \, \text{GeV}^2 \) with which we are expecting the gluon condensate to be negligible and it does not contributing to modify the result of the perturbative term.

Performing the match between the QCD and the phenomenological side, in Fig. (2b), we show the Borel window stability for different values’ combinations of thresholds and in the same Fig. (2a), we can see the pole and continuum contributions observing that the pole contribution is always bigger than the continuum contribution in the same Borel window of stability, therefore we have good credibility for this sum rule. Joining this criteria with the thresholds \( \Delta_s = 0.6 \, \text{GeV} \) and \( \Delta_u = 0.5 \, \text{GeV} \), we obtain the form factor for \( B \) off-shell: \( g^{(B)}_{B^*BK}(Q^2) \), where we have used \( M^2 = \frac{m_B^2}{m_{B^*}^2 - m_q^2} \) as a Borel mass relation, which is usual when light and heavy mesons are involved.

The triangles in Fig. (3) correspond to the sum rule result, which can be fitted by the monopolar function represented by the dashed line in Fig. (3) and given by
The coupling constant is obtained when the form factor is extrapolated to \( Q^2 = -m_T^2 \), where \( m_T \) is the mass of the off-shell meson \([2, 10, 11]\). Using \( Q^2 = -m_B^2 \) in Eq. (18), the resulting coupling constant is equal to:

\[
g_{B^* BK} = 10.6
\]  

For the \( B \) meson off-shell case, we have other structure, \( p'_\nu p_\mu \), to work with. This sum rule has quark condensate contribution but it has not good stability and its continuum contribution is larger than the pole contribution, therefore this sum rule is not considered a good sum rule.

**B. \( K \) off-shell form factor**

In the case \( K \) off-shell, we have two structures in Eq. (17) that can be used: \( p'_\nu \) and \( p_\nu \). Both structures give good sum rule results, that means a good pole-continuum contribution and good stability. The quark condensate of the bottom is considered, but the contribution is null. At first, we have chosen the \( p'_\nu \) structure to obtain the form factor. The stability for different thresholds are showed in Fig. (3b). For \( \Delta s = 0.7 \text{GeV}, \Delta u = 0.7 \text{GeV} \) and for a Borel mass of \( 7.2 \text{GeV}^2 \), we obtain a pole contribution of about 70% of the total contribution (Fig. (3) part b)).

![FIG. 2. a) \( g_{B^* BK}^{(B)}(Q^2 = 1 \text{GeV}^2) \) as a function of the Borel mass \( M^2 \) and different thresholds and b) pole-continuum contributions.](image)

![FIG. 3. a) \( g_{K^* BK}^{(K)}(Q^2 = 2 \text{GeV}^2) \) stability as a function of the Borel mass for different threshold values, structure \( p'_\nu \) and b) pole and continuum contributions.](image)
The white circles in Fig. (6) represent the sum rule result fitted by a Gaussian function that is the short dashed line in the same figure and is given by

$$g^{(K)}_{B^*_sBK}(Q^2) = 10.71 e^{-(Q^2/7.49)^2},$$

(20)

by extrapolating Eq. (20) to $Q^2 = -m^2_K$, we obtain the coupling constant:

$$g_{B^*_sBK} = 10.7$$

(21)

We can see an excellent agreement between the results obtained in Eqs. (19) and (21), as expected.

We noted the same vertex behavior concerning the shape of the form factor that we obtained in previous works, when the heavy meson is the off-shell particle, the form factor is “harder” and has a big cutoff parameter $\Lambda$, which is defined from Eqs. (18) and (20), which have a general Gaussian form:

$$g_{B^*_sBK}(Q^2) = A e^{-(Q^2/\Lambda)^2}$$

(22)

and a monopolar form

$$g_{B^*_sBK}(Q^2) = \frac{A}{\Lambda + Q^2}$$

(23)

where $\Lambda$ parameter is equal to $34.99 \ GeV^2$ for $B$ off-shell meson and is $7.49 \ GeV^2$ for $K$ off-shell meson.

IV. UNCERTAINTIES OF THE QCDSR METHOD

The uncertainties in the QCDSR calculation come from different sources such as decay constant errors, mass uncertainties, other “good” structures which also can be considered and the QCDSR variation parameters themselves. In the next section, we discuss each case separately in sub-sections and use the results to find an error of the method in our result.

A. $B^*_s$ off-shell

At first, we calculate a third sum rule with the other heavy meson of the vertex off-shell: the $B^*_s$. Since this meson has a small mass difference when compared with the $B$ meson mass, we expect to obtain a similar result to the one obtained for $B$ off-shell. We have two structures, $p'_\mu p'_\nu$ and $p_\mu p'_\nu$, to work with in the method. The $p_\mu p'_\nu$ structure does not present good stability in the Borel mass, therefore it is not considered. The other structure, $p'_\mu p'_\nu$, has good stability and the pole is larger than the continuum contribution, for $\Delta_s = 0.7 \ GeV$ and $\Delta_u = 0.4 \ GeV$. Fig. 4 a) shows the Borel mass stability and in b) the pole-continuum contribution.

![FIG. 4. (a) Stability of $g^{(B^*_s)}_{B^*_sBK}(Q^2 = 1 \ GeV^2)$ as Borel mass and for different thresholds and (b) pole-continuum contributions.](image)
We extrapolate the QCDSR results, the dots in Fig. 6, by using a monopolar function until reaching \( Q^2 = -m_B^2 \), and we obtain
\[
\begin{align*}
g^{(B^*_s)}_{B^*_s B K}(Q^2) &= \frac{93.03}{37.80 + Q^2} \\
g_{B^*_s B K} &= 10.25
\end{align*}
\] (24)
for the form factor, and
\[
g_{B^*_s B K} = 10.25
\] (25)
for the coupling constant. Both results, Eq. (24) and Eq. (18) show the same extrapolation function with similar parameters.

**B. \( K \) off-shell, structure \( p_\nu \)**

For the light meson off-shell, \( K \), we also have the \( p_\nu \) structure, which gives a good sum rule and is used here to calculate the uncertainties of the method. For this structure, we analyze the Borel mass stability for different threshold parameters, Fig. 5 a), and the pole-continuum contribution, showed in Fig. 5 b). We obtain a Gaussian function by extrapolating the QCDSR result given by:
\[
g^{(K)}_{B^*_s B K}(Q^2) = 10.65e^{-(Q^2/4.99)^2},
\] (26)
and making \( Q^2 = -m_K^2 \), we obtain the coupling constant of the vertex which is equal to:
\[
g_{B^*_s B K} = 10.7
\] (27)
In Fig. 6 we summarize the results obtained with both structures showing an excellent agreement.

**C. Uncertainties of parameter variations.**

In order to know, how much does the QCDSR depend on the parameters’ variations, we analyze, in this work, the experimental data uncertainties for the decay constant \( f_K \) [18], the theoretical uncertainties for \( m_s \) [20], \( m_b \) [21, 22], \( f_B \) and \( f_{B^*_s} \) [19] and the variations over the QCDSR parameters that are the Borel mass, \( M^2 \), and the thresholds \( \Delta s \) and \( \Delta u \).

To compute the error, we proceed in the following way: after calculating the coupling constant, we compute a new coupling constant, but now with all parameters kept fixed, except one, which is changed according with its intrinsic error, given in Table II. Then, we move to the next parameter to be varied, keeping all others fixed. At the end of the procedure, we know how sensitive is this vertex respect to each parameter. In table II that contain the percentual
deviation for the three cases ($B$, $K$ and $B_s^*$ off-shell mesons), we observe that the biggest variation on the coupling constant is produced when $f_B$, $f_{B^*}$ and $m_b$ were varied. Finally, the uncertainties for each coupling constant, which is obtained with $B$, $K$ and $B_s^*$ off-shell are:

$$g_{B_s^* B K}^{(B)} = 10.6 \pm 1.5$$

for $B$ off-shell meson,

$$g_{B_s^* B K}^{(K)} = 10.3 \pm 1.7$$

for $K$ off-shell, where were used both structures that give good SR, and

$$g_{B_s^* B K}^{(B_s^*)} = 11.0 \pm 1.5$$

for $B_s^*$ off-shell. Fig. (6) summarize these results, where the error bar represents the uncertainties for each calculation.

Considering the three off-shell cases, for all possible and "good" sum rules structures, we obtain the coupling constant of the $B_s^* B K$ vertex. After computing all the uncertainties, we obtain the mean value:

$$g_{B_s^* B K} = 10.6 \pm 1.7$$

| Parameters                          | Deviation % |
|-------------------------------------|-------------|
|                                     | $B$ off-shell | $K$ off-shell | $B_s^*$ off-shell |
| $f_K = 159.8 \pm 1.4 \pm 0.44$ (MeV) | 1.04        | 1.04          | 1.02             |
| $f_B = 208 \pm 10 \pm 29$ (MeV)     | 15.92       | 15.97         | 15.94            |
| $f_{B^*} = 250 \pm 10 \pm 35$ (MeV) | 15.22       | 15.25         | 15.29            |
| $M^2 \pm 10\%$ (GeV)               | 6.05        | 5.9           | 2.41             |
| $m_b = 4.20 + 0.17 - 0.07$ (GeV)    | 13.69       | 28.80         | 14.18            |
| $m_s = 104 + 26 - 34$ (MeV)        | 8.98        | 14.67         | 5.59             |
| $\Delta s \pm 0.1 \ e \Delta u \pm 0.1$(GeV) | 13.51 | 2.79 | 14.30 |

TABLE II. Percentage deviation related with each parameter.

FIG. 6. The three cases considered here: off-shell $K$, $B$, $B_s^*$ and for all possible sum rules.
The uncertainties of the QCDSR coming from different sources (other structures with the same mesons of shell, other heavy meson off-shell, the extrapolation fit of the sum rule result and the errors in masses, decay constants, condensates, the choice of the Borel mass and the continuum threshold parameters) is near 16%. The usual errors in the QCDSR are around 20% [24].

Also, from Eqs. (18), (20) and (24), we can extract the cut-off parameters, Λ, associated with the form factors. In table III the cut-off parameters are showed and we can observe that they follow the same trend as we had been observed in Refs. [2, 6]: the value of the cut-off is directly associated with the mass of the off-shell meson probing the vertex. A harder form factor means bigger cut-off parameter.

| Meson off-shell | Λ parameter (GeV²) |
|-----------------|--------------------|
| B               | 34.99              |
| B*              | 37.80              |
| K               | 7.49               |

TABLE III. Values of the cut-off parameter Λ.

Recently, our extrapolation procedure [2]-[25] was used to calculate the same coupling constant, \( g_{B^*B_K} \), by the authors of Ref. [26]. Even if the procedure to obtain the coupling constant was the same, the form factors obtained were different. This calculation do not include the analysis of the pole-continuum contributions, exponential extrapolations functions with three parameters were used to fit the form factors and different values for the parameters were used in the SR, resulting in a different value of the coupling constant.

There are some possibilities that could be considerer to compare our result with other theoretical predictions. If we use arguments of heavy hadron chiral perturbation theory (HHChPT), the couplings for the bottom-light vertex \( g_{B^*B_K} \) are related to the charm-light vertex \( g_{D^*D_K} \) through the relation [27]-[28]:

\[
g_{B^*B_K} = g_{D^*D_K} \frac{m_B}{m_D},
\]

where if it is used \( m_B = 5.279, m_D = 1.8693 \) and \( g_{D^*D_K} = 2.84 \) (which is our QCDSR result [10]), we obtain \( g_{B^*B_K} = 8.02 \), that is in complete agreement with the result of this paper.

Also, \( g_{B^*B_K} \) is related to \( g_{B^*B\pi} \) by \( SU(3) \) symmetry and the coupling constants became equal \( g_{B^*B_K} = g_{B^*B\pi} \). In table IV we present other theoretical estimatives from QCDSR, Light Cone SR and other methods of the \( g_{B^*B\pi} \).

| Approach                          | \( g_{B^*B_K} \) violation |
|-----------------------------------|----------------------------|
| QCDSR [30]                       | 20 ± 4 50%                 |
| QCDSR [30]                       | 15 44%                     |
| LCSR [31]                        | 28 ± 6 65%                 |
| QCDSR [32]                       | 14 ± 4 25%                 |
| LCSR [33]                        | 22 ± 9 55%                 |
| QCDSR [34]                       | 42.5 ± 2.6 76%             |
| QCDSR plus meson loops [35]      | 44.7 ± 1.0 73%             |
| dispersive quark model [36]      | 32 ± 5 69%                 |
| Dyson-Schwinger equations [37]    | 30.0^{+1.2}_{-1.4} 73%     |

TABLE IV. Summary of theoretical estimates for \( g_{B^*B\pi} \).

Concluding, we calculate the form factors of the vertex showing that they have two different forms, one when the heavy meson is the off-shell particle and another when the light meson is the off-shell one, with their extrapolations to the on-shell point leading to the coupling constant.

In this paper, we made all the calculations taking control of the SR stability and the pole contribution is always contributing more than the continuum. Besides, we computed all the “good” sum rules and we obtained the same value of the coupling constant. Furthermore, we computed the sum rules taking into account the errors in masses, decay constants, condensates, choice of the Borel mass, continuum threshold parameters, obtaining a final result with errors that are near 16%.
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