Coalescing binaries as possible standard candles

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(Dated: January 9, 2010)

Gravitational waves detected from well-localized inspiraling binaries would allow to determine, directly and independently, both binary luminosity and redshift. In this case, such systems could behave as “standard candles” providing an excellent probe of cosmic distances up to $z < 0.1$ and thus complementing other indicators of cosmological distance ladder.

**Keywords:** gravitational waves, standard candles, cosmological distances

I. INTRODUCTION

A new type of standard candles, or, more appropriately, standard sirens, could be achieved by studying coalescing binary systems [1,2]. These systems are usually considered strong emitter of gravitational waves (GW), ripples of space-time due to the presence of accelerated masses in analogy with the electromagnetic waves, due to accelerated charged. The coalescence of astrophysical systems containing relativistic objects as neutron stars (NS), white dwarves (WD) and black holes (BH) constitute very standard GW sources which could be extremely useful for cosmological distance ladder if physical features of GW emission are well determined. These binaries systems, as the famous PSR 1913+16 [3–5], have components that are gradually inspiralling one over the other as the result of energy and angular momentum loss due to (also) gravitational radiation. As a consequence the GW frequency is increasing and, if observed, could constitute a “signature” for the whole system dynamics. The coalescence of a compact binary system is usually classified in three stages, which are not very well delimited one from another, namely the *inspiral phase*, the *merger phase* and the *ring-down phase*. The merger phase is the process that proceeds until the collision of the bodies and the formation of a unique object. Its duration depends on the characteristics of the originating stars and emission is characterized by a frequency damping in the time. In merger phase, stars are not modelled as rigid sphere due to the presence of a convulsive exchange of matter [6]. GW emission from merger phase can only be evaluated using the full Einstein equations. Because of the extreme strong field nature of this phase, neither a straightforward application of post-Newtonian theory nor any perturbation theory is very useful. Recent numerical work [7–9] has given some insight into the merger problem, but there are no reliable models for the waveform of the merger phase up to now. Gravitational radiation from the ring-down phase is well known and it can be described by quasi-normal modes [10]. The relevance of ring-down phase is described in [11]. Temporal interval between the inspiral phase and the merger one is called *coalescing time*, interesting for detectors as the American LIGO (Laser Interferometer Gravitational-Wave Observatory) [12] and French/Italian VIRGO [13]. Coalescence is a rare event and therefore to see several events per year, LIGO and/or VIRGO must look far beyond our Galaxy. For example, the expected rate for a NS-NS system (determined from the observed population of NS-NS binaries) is found to be $80^{+210}_{-70} \text{Myr}^{-1}$ per galaxy [14]. From this figure, one finds that the expected rate for the today available sensitivities of today LIGO and VIRGO is of the order $35^{+90}_{-30} \times 10^{-3} \text{yr}^{-1}$, while for the advanced version of such interferometers, the rate is more interesting being $190^{+470}_{-150} \text{yr}^{-1}$, that is the probability ranges from one event per week to two events per day. A remarkable fact about binary coalescence is that it can provide an *absolute measurement* of the source distance: this is an extremely important event in Astronomy. In fact, for these systems, the distance is given by the measure of the GW polarization emitted during the coalescence. One of the problems which affects the utilization of coalescing binaries as standard candles is the measure of the redshift of the source as well as the measure of the GW polarization (at present GWs have not been still experimentally observed). A solution for the redshift determination could be the detection of an electromagnetic counterpart of the coalescing system (e.g. the detection of an associated gamma ray burst) or the redshift measurement of the host galaxy or galaxy cluster at their barycenter [15]. Recent evidence supports the hypothesis that many short-hard gamma-ray bursts could be associated with coalescing binary systems indeed [14,15]. In this paper, we want to show that such systems could be used as reliable standard candles.

In Sect. [11] we briefly sketch the GW emission from coalescing binary systems in circular orbit at cosmological distance. In Sect. [11] we simulate various coalescing binary systems (WD-WD, NS-NS, BH-BH) at redshift $z < 0.1$ because of the observational limits of ground-based-interferometers as LIGO and VIRGO. The goal of the simulation is the measure of the Hubble constant and consequently the use of these systems as standard candles [20]. This new type of standard candles will be able to increase the confidence level on the other "traditional" standard candles in Astronomy and, moreover, it could constitute an effective tool to measure distances at larger redshifts. Conclusions are drawn in Sect. [15].
II. GRAVITATIONAL RADIATION FROM A COALESCING BINARY SYSTEM

For a detailed exposition of GW theory see, e.g. [21–23]. Let us consider here an isolated, far away and slowly moving source. In this approximation, we can write the GW solution as:

$$\ddot{h}_{\mu\nu}(t, x) = \frac{2G}{r} \frac{d^2 I_{ij}}{dr^2}(t_R),$$

where $I_{ij}$ is the quadrupole momentum tensor of the energy density of source, conventionally defined as:

$$I_{ij}(t) = \int y^i y^j T^{00}(t, y) d^3 y,$$

a tensor defined at any constant time surface, and $t_R$ is the retarded time [21]. The distance between source and observer is denoted as $r$.

The gravitational wave produced by an isolated non-relativistic object is therefore proportional to the second derivative of the quadrupole momentum of the energy density at the point where the past light cone of the observer intersects the source. In contrast, the leading contribution to electromagnetic radiation comes from the changing dipole momentum of the charge density.

The above result can be specified for two point masses $M_1$ and $M_2$ in a circular orbit. In the quadrupole approximation, the two polarization amplitudes of GWs at a distance $r$ from the source are given evaluating Eq. (1) to lowest order in $v/c$, that is:

$$h_+(t) = \frac{4}{r} \left( \frac{GM_C}{c^2} \right)^{5/3} \left( \frac{\pi f(t_R)}{c} \right)^{2/3} \left( \frac{1 + \cos^2 i}{2} \right) \times$$
$$\cos[\Phi(t_R)] ,$$

$$h_\times(t) = \frac{4}{r} \left( \frac{GM_C}{c^2} \right)^{5/3} \left( \frac{\pi f(t_R)}{c} \right)^{2/3} \cos i \sin[\Phi(t_R)] ,$$

where $i$ is the binary inclination angle such that $i = 90^\circ$ corresponds to a system visible edge-on. These are traditionally labeled "plus" and "cross" from the lines of force associated with their tidal stretch and squeeze (see Fig. 1). Here $f$ is the frequency of the emitted GWs (twice the orbital frequency). The rate of the frequency change is [22]:

$$f = \frac{96}{5} \frac{\pi^{8/3}}{\left( \frac{GM_C}{c^2} \right)^{5/3}} f^{11/3} ,$$

where $t_R$ is the so called "retarded time" and the phase $\Phi$ is given by the expression:

$$\Phi(t) = 2\pi \int_{t_0}^{t} dt' f(t') .$$

Note that for a fixed distance $r$ and a given frequency $f$, the GW amplitudes are fully determined by $\mu M^{2/3} = M_C^{5/3}$, where the combination:

$$M_C = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}} ,$$

is called chirp mass of the binary, here $M = M_1 + M_2$ is the total mass of the system and $\mu = \frac{M_1 M_2}{M_1 + M_2}$ is the reduced mass of the system. Introducing the coalescence time $\tau = t_{\text{coal}} - t$ and integrating Eq. (8), we get:

$$f \approx 130 \left( \frac{1.21 M_C}{M} \right)^{5/8} \left( \frac{1}{\tau} \right)^{3/8} \text{ Hz} .$$

Eq. (8) predicts coalescence times of $\tau \sim 17 \text{ min}, 2 \text{ sec}, 1 \text{ msec}$ for $f \sim 10, 100, 1000 \text{ Hz}$.

After averaging over the orbital period and the orientations of the binary orbital plane, one arrives at the average (characteristic) GW amplitude:

$$h(f, M_C, r) = \langle h_+^2 \rangle + \langle h_\times^2 \rangle^{1/2} =$$

$$\left( \frac{32}{5} \right)^{1/2} \frac{G^{5/3} M_C^{5/3}}{c^4} \frac{r}{\pi f^{2/3}} .$$

For a binary at the cosmological distance, i.e. at redshift $z$ where GWs propagate in a Friedmann-Robertson-Walker Universe, these equations are modified in a very straightforward way:

- The frequency that appears in the above formulae is the frequency measured by the observer, $f_{\text{obs}}$, which is red-shifted with respect to the source frequency $f_s$, i.e. $f_{\text{obs}} = f_s/(1 + z)$, and similarly $t$ and $t_R$ are measured with the observer clocks.
• The chirp mass $M_C$ has to be replaced by $M_C = M_C(1 + z)$.

• The distance $r$ to the source has to be replaced by the luminosity distance $d_L(z)$.

Inserting the following quantity:

$$h_c(t) = \frac{4}{d_L(z)} \left( \frac{G M_C(z)}{c^2} \right)^{5/3} \left( \frac{\pi f(t)}{c} \right)^{2/3}, \quad (10)$$

we can rewrite the expressions for the polarization “+” and “×” as:

$$h_+(t) = h_c(t_R) \frac{1 + \cos^2 i}{2} \cos[\Phi (t_R)] , \quad (11)$$

and

$$h_\times(t) = h_c(t_R) \cos i \sin[\Phi (t_R)] . \quad (12)$$

Explicating the dependence on the chirp mass redshift, we can obtain the luminosity distance $d_L$ using the equation (10) linked directly to the GW polarization:

$$d_L(z) = \frac{4}{h_c(t)} \left[ \frac{G M_C(1 + z)}{c^2} \right]^{5/3} \left[ \pi f(t) \right]^{2/3} = \frac{4(1 + z)^{5/3}}{h_c(t)} \left[ \frac{G M_C}{c^2} \right]^{5/3} \left[ \pi f(t) \right]^{2/3} . \quad (13)$$

Let us recall that the luminosity distance $d_L$ of a source is defined by

$$F = \frac{L}{4\pi d^2_L}, \quad (14)$$

where $F$ is the flux (energy per unit time per unit area) measured by the observer, and $L$ is the absolute luminosity of the source, i.e. the power that it radiates in its rest frame. For small redshifts, $d_L$ is related to the present value of the Hubble parameter $H_0$ and to the deceleration parameter $q_0$ by

$$\frac{H_0 d_L}{c} = z + \frac{1}{2} (1 - q_0) z^2 + \ldots . \quad (15)$$

The first term of this expansion gives the Hubble law $z \simeq (H_0/c)d_L$, which states that redshift is proportional to the distance. The term $O(z^2)$ is the correction to the linear law for moderate redshifts. For large redshifts, the Taylor series is no longer appropriate, and the whole expansion history of the Universe is encoded in a function $d_L(z)$. For a spatially flat Universe, one finds

$$d_L(z) = c (1 + z) \int_0^z \frac{dz'}{H(z')} . \quad (16)$$

where $H(z)$ is the value of the Hubble parameter at redshift $z$. Knowing $d_L(z)$, we can therefore obtain $H(z)$. This shows that the luminosity distance function $d_L(z)$ is an extremely important quantity, which encodes the whole expansion history of the Universe. Coalescing binaries could be standard candles (or precisely standard sirens) in the following sense. Suppose that we can measure the amplitudes of both polarizations $h_+$, $h_\times$, as well as $f_{\text{obs}}$. From the ratio of $h_+$ and $h_\times$, we can obtain the value of the inclination of the orbit, besides, evaluating $f_{\text{obs}}$ at a given frequency, we can obtain $M_C$. If we are capable of measuring the redshift $z$ of the source, we have found a gravitational standard candle since we can obtain the luminosity distance from Eq. (13) and then evaluate the Hubble constant $H_0$. The difference between gravitational standard candles and the "traditional" standard candles is that the luminosity distance is directly linked to the GW polarization and there is no theoretical uncertainty on its determination a part the redshift evaluation. Various possibilities have been proposed. Among these there is the possibility to see an optical counterpart. In fact, it can be shown that observations of the GWs emitted by inspiralling binary compact systems can be a powerful probe at cosmological scales. In particular, short GRBs appear related to such systems and quite promising as potential GW standard sirens [44]. On the other hand, the redshift of the binary system can be associated to the barycenter of the host galaxy or the galaxy cluster as we are going to do here.

### III. NUMERICAL SIMULATION

We have simulated various coalescing binary systems at redshifts $z < 0.1$ In this analysis, we do not consider systematic errors and errors on redshifts to obviate the absence of a complete catalogue of such systems. The choice of low redshifts is due to the observational limits of ground-based interferometers like VIRGO or LIGO. Some improvements are achieved, if we take into account the future generation of these interferometers as Advanced VIRGO [33] and Advanced LIGO [36]. Advanced VIRGO is a major upgrade, with the goal of increasing the sensitivity by about one order of magnitude with respect to VIRGO in the whole detection band. Such a detector, with Advanced LIGO, is expected to see many events every year (from 10s to 100s events/year). For example, a NS-NS coalescence will be detectable as far as 300 Mpc. In the simulation presented here, sources are slightly out of LIGO-VIRGO band but observable, in principle, with future interferometers. Here, we have used the redshifts taken by NASA/IPAC EXTRAGALACTIC DATABASE [27], and we have fixed the redshift using $z$ at the barycenter of the host galaxy/cluster), and the binary chirp mass $M_C$, typically measured, from the Newtonian part of the signal at upward frequency sweep, to $\sim 0.04\%$ for a NS/NS binary and $\sim 0.3\%$ for a system containing at least one BH.
The distance to the binary $d_L$ ("luminosity distance" at cosmological distances) can be inferred, from the observed waveforms, to a precision $\sim 3/\rho \lesssim 30\%$, where $\rho = S/N$ is the amplitude signal-to-noise ratio in the total LIGO network (which must exceed about 8 in order that the false alarm rate be less than the threshold for detection). In this way, we have fixed the characteristic amplitude of GWs, and frequencies are tuned in a range compatible with such a fixed amplitude, then the error on distance luminosity is calculated by the error on the chirp mass with standard error propagation.

The systems considered are NS-NS, BH-BH and WD-WD. For each of them, a particular frequency range and a characteristic amplitude (beside the chirp mass) are fixed. We start with the analysis of NS-NS systems ($M_C = 1.22M_\odot$) with characteristic amplitude fixed to the value $10^{-22}$. In Table I we report the redshift, the value of $h_C$ and the frequency range of systems analyzed.

| Object            | $z$     | $h_C$  | Freq. (Hz) |
|-------------------|---------|--------|------------|
| NGC 5128          | 0.0011  | $10^{-22}$ | 0 ÷ 10     |
| NGC 1023 Group    | 0.0015  | $10^{-22}$ | 0 ÷ 10     |
| NGC 2997          | 0.0018  | $10^{-22}$ | 5 ÷ 15     |
| NGC 5457          | 0.0019  | $10^{-22}$ | 10 ÷ 20    |
| NGC 5033          | 0.0037  | $10^{-22}$ | 25 ÷ 35    |
| Virgo Cluster     | 0.0042  | $10^{-22}$ | 30 ÷ 40    |
| Fornax Cluster    | 0.0044  | $10^{-22}$ | 35 ÷ 45    |
| NGC 7582          | 0.0050  | $10^{-22}$ | 45 ÷ 55    |
| Ursa Major Groups | 0.0057  | $10^{-22}$ | 50 ÷ 60    |
| Eridanus Cluster  | 0.0066  | $10^{-22}$ | 55 ÷ 65    |

Table I: Redshifts, characteristic amplitudes, frequency range for NS-NS systems.

In Fig. 2 the derived Hubble relation is reported, and in Fig. 3 residuals are reported.

The Hubble constant value is $72 \pm 1$ km/s/Mpc in agreement with the recent WMAP estimation (Wilkinson Microwave Anisotropy Probe [25]). The same procedure is adopted for WD-WD systems ($M_C = 0.69M_\odot$, $h_C = 10^{-23}$) and BH-BH systems ($M_C = 8.67M_\odot$, $h_C = 10^{-21}$). In Tables II and III, we report the redshift, the value of $h_C$ and the frequency range for BH-BH and WD-WD systems respectively. These simulations are reported in Fig. 4 and in Fig. 5; the corresponding residuals have been checked and their goodness is the same of previous case so we omit them. For these simulations, the Hubble constant value is $69 \pm 2$ km/s/Mpc and $70 \pm 1$ km/s/Mpc for BH-BH and WD-WD systems respectively, also in these cases in agreement with WMAP estimation.

IV. CONCLUSIONS

In this paper, we have considered simulated binary systems whose redshifts can be estimated considering the barycenter of the host astrophysical system as galaxy, group of galaxies or cluster of galaxies. In such a way, the standard methods adopted to evaluate the cosmic distances (e.g. Tully-Fisher or Faber-Jackson relations) can be considered as "priors" to fit the Hubble relation. We have simulated various situations assuming NS-NS, BH-BH, and WD-WD binary systems. Clearly, the leading parameter is the chirp mass $M_C$, or its redshifted counter-part $M_C$, which is directly related to the GW amplitude. The adopted redshifts are in a well-tested range of scales and the Hubble constant value is in good agreement with WMAP estimation. The Hubble-luminosity-distance diagrams of the above simulations show the possibility to use the coalescing binary systems.
as distance indicators and, possibly, as standard candles. The limits of the method are, essentially, the measure of GW polarizations and redshifts. Besides, in order to improve the approach, a suitable catalogue of observed coalescing binary-systems is needed. This is the main difficulty of the method since, being the coalescence a transient phenomenon, it is very hard to detect and analyze the luminosity curves of these systems. Furthermore, a few simulated sources are out of the LIGO-VIRGO band.

Next generation of interferometer (as LISA, Advanced-VIRGO or Advanced-LIGO) could play a decisive role to detect GWs from these systems. At the advanced level, one expects to detect at least tens NS-NS coalescing events per year, up to distances of order 2 Gpc, measuring the chirp mass with a precision better than 0.1%. The masses of NSs are typically of order $1.4 M_\odot$. Stellar-mass BHs, as observed in X-ray binaries, are in general more massive, typically with masses of order $10 M_\odot$, and therefore are expected to emit even more powerful GW signals during their inspiralling and coalescing phases. The coalescence of two BHs, each one with $10 M_\odot$, could be seen by Advanced-VIRGO and Advanced-LIGO up to redshifts $z \sim 2 - 3$. Furthermore, the LISA space interferometer, which is expected to fly in about 10 years, will be sensitive to GWs in the mHz region, which corresponds to the wave emitted by supermassive BHs with masses up to $10^6 M_\odot$. Nowadays, supermassive BHs with masses between $10^6$ and $10^9 M_\odot$ are known to exist at the center of most (and probably all) galaxies, including our Galaxy. The coalescence of two supermassive BHs, which could take place, for instance during the collision and merging of two galaxies or in pre-galactic structure at high redshifts, would be among the most luminous events in the Universe. Even if the merger rate is poorly understood, observations from the Hubble Space Telescope and from X-ray satellites such as Chandra have revealed that these merging events could be detectable at cosmological distances. LISA could detect them up

| Object                    | $z$   | $h_\epsilon$ | Freq. (Hz) |
|--------------------------|-------|--------------|------------|
| Eridanus Cluster         | 0.0066| $10^{-23}$   | 5 $\pm$ 10|
| Hydra Cluster            | 0.010 | $10^{-23}$   | 15 $\pm$ 20|
| Pavo-Indus Sup.Cluster   | 0.015 | $10^{-23}$   | 35 $\pm$ 40|
| Perseus-Pisces Sup.Cluster | 0.017 | $10^{-23}$   | 40 $\pm$ 45|
| Abell 569 Cluster        | 0.019 | $10^{-23}$   | 45 $\pm$ 50|
| Centaurus Cluster        | 0.020 | $10^{-23}$   | 45 $\pm$ 50|
| Coma Cluster             | 0.023 | $10^{-23}$   | 55 $\pm$ 60|
| Abell 634 Cluster        | 0.025 | $10^{-23}$   | 60 $\pm$ 65|
| Leo Sup.Cluster          | 0.032 | $10^{-23}$   | 85 $\pm$ 90|
| Hercules Sup.Cluster     | 0.037 | $10^{-23}$   | 100 $\div$ 105|

Table III: Redshifts, characteristic amplitudes, frequency range for WD-WD systems.
to \( z \sim 10 \), and it is expected to measure several events of this kind. The most important issue that can be addressed with a measure of \( d_L(z) \) is to understand “dark energy”, the quite mysterious component of the energy budget of the Universe that manifests itself through an acceleration of the expansion of the Universe at high redshift. This has been observed, at \( z < 1.7 \), using Type Ia supernovae as standard candles \([31, 32]\). A possible concern in these determinations is the absence of a solid theoretical understanding of the source. After all, supernovae are complicated phenomena. In particular, one can be concerned about the possibility of an evolution of the supernovae brightness with redshift, and of interstellar extinction in the host galaxy leading to unknown systematics. GW standard candles could lead to completely independent determinations, and complement and increase the confidence of other standard candles, \([33]\), as well as extending the result to higher redshifts. In the future, the problem of the redshift could be obviate finding an electromagnetic counterpart to the coalescence and short GRBs could play this role.

In summary, this new type of cosmic distance indicators could be considered complementary to the traditional standard candles opening the doors to a self-consistent gravitational astronomy.

Acknowledgments

We wish to thank G. Covone and L. Milano for fruitful discussions and suggestions on the topics of this paper.