PROTON DECAY

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ABSTRACT

The status of proton decay is described, including general motivations for baryon number violation, and the present and future experimental situation. Grand unification with and without supersymmetry is considered, including possible evidence from coupling constant unification and implications for proton decay and neutrino mass.

1 Motivations

Baryon number is almost certainly not absolutely conserved.

- There is no compelling reason to think that baryon number is conserved. The only convincing mechanism we have for ensuring an absolute conservation law is gauge invariance. For example, electromagnetic gauge invariance guarantees that electric charge is conserved and implies the existence of a massless photon. However, there is no analogous baryon – i.e., there is no long range force coupling to \( B \). We know from Eötvos-type experiments \(^1\) that if there were such a gauge boson its coupling would have to be incredibly small, \( g_B^2/4\pi < 6 \times 10^{-48} \). Hence, baryonic gauge invariance cannot be invoked and there is no good reason to suspect absolute conservation.

- Black holes do not remember baryon number. If a proton were to drop into a black hole its quantum numbers would disappear from the universe, violating baryon number.

- It has been known for some time \(^2\) that baryon number is violated in the weak interactions via the weak anomaly, as shown in Figure \( \text{I} \). The idea is that the vacuum state is not unique and there are degenerate vacua characterized by different values for \( B \). There are nonperturbative tunnelling amplitudes to make transitions from one vacuum to another. However, these are incredibly slow, characterized by rates proportional to \( \exp(-4\pi \sin^2 \theta_W/\alpha) \sim 10^{-172} \), and are irrelevant for proton decay. However, it has been realized and emphasized recently \(^3\) that thermal fluctuations at the time of the electroweak phase transition could lead to transitions, and this has significant implications for the baryon asymmetry of the universe.
Figure 1: Schematic diagram of baryon number violation in the weak interactions. There are degenerate vacua of different baryon number and a possibility of tunnelling between them.

- The baryon asymmetry of the universe is the small difference between the number of baryons and antibaryons, \((n_B - n_{\bar{B}})/n_\gamma \sim 10^{-10}\). This most likely implies \(B\) violation at some time in the history of the universe. It is also possible that there was a small initial asymmetry or that there is a large-scale separation between baryons and antibaryons, but these seem to me to be much less likely.

- Finally, it is easy to construct grand unification or other interactions which involve \(\Delta B \neq 0\).

All of these should be viewed as reasonable motivations to consider the possibility that \(B\) is not conserved and that the proton is not stable.

2 Proton Decay Experiments

It was possible by “simple considerations” to determine the limit \(\tau_p > 10^{20-22}\) \(yr\) in the early 1950’s. Dedicated experiments, either looking for the disappearance of a nucleon from the nucleus or searching directly for the decay products, improved the limits to \(\sim 10^{20}\) \(yr\) by the mid 1970’s. A detailed review can be found in [8].

The modern experiments have pushed the limits still further. Recent experiments that have been completed, are still running, or are under construction are listed in Table 1. A recent review is given in [9].

No experiment has observed proton decay. There are some candidate events in some of the experiments, but these could be due to neutrinos interacting in the detector. The present limits on two particularly interesting modes are

\[
e^+\pi^0 : \quad \tau/B > 10^{33}\ yr
\]
\[
\bar{\nu}K^+ : \quad \tau/B > 10^{32}\ yr.
\]

(1)
| exp        | Type       | Status               | Sensitivity (kT yr) |
|------------|------------|----------------------|--------------------|
| Kolar      | Track. Cal.| running              | 0.8                |
| IMB        | Water Cer. | stopped              | 3.8 − 7.2          |
| NUSEX      | Track. Cal.| running              | 0.36               |
| HPW        | Water Cer. | stopped (85)         | 0.14               |
| Kamiokande | Water Cer. | running              | 3.76               |
| FREJUS     | Track. Cal.| stopped (88)         | 1.5 − 2            |
| Soudan 2   | Track. Cal.| - running (0.7 kT)   | 0.94 (→ 6)         |
|            |            | (1 kT completed ’93) |                    |
| Super-Kamiokande | Water Cer. | approved             | (22 kT completed ’96) | (→ 100) |

Table 1: Modern proton decay experiments. The last column shows their sensitivity in kiloton-years. From [9].

These are interesting in ordinary and supersymmetric grand unified theories, respectively. In the future the $e^+\pi^0$ limit will be improved to $\sim 10^{34}$ yr.

## 3 Grand Unification

Many of the problems of the standard model can be solved in a grand unified theory [5, 6]. The basic idea is that the strong, weak, and electromagnetic interactions are unified, i.e., embedded in a simple group $G$. If one could probe the theory at a large momentum scale $Q \geq M_X$ for which symmetry breaking can be ignored one would observe a single coupling constant. In practice $G$ is broken to the standard model group, $G \rightarrow SU_3 \times SU_2 \times U_1$, at some large scale $M_X$ so that at lower energies the three interactions appear different. One of the predictions is that the coupling constants that we observe at low energy, when run theoretically up to high scales, should meet at $M_X$. Grand unification is an elegant idea but it does not incorporate gravity.

In addition to the coupling constants the $q, \bar{q}, l, \bar{l}$ are all unified. That is, they are related in the same multiplets. This typically explains charge quantization, i.e., why the the atom is electrically neutral, and implies new interactions which mediate proton decay.

The simplest grand unified theory is the Georgi-Glashow $SU_5$ model [3]. The fermions are placed in a complicated reducible representation, consisting of a 5-plet,

$$5^*: \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \bar{d}_L, \quad (2)$$

1Or to a subgroup of $G$ which contains the standard model.
Figure 2: Diagram for proton decay in the Georgi-Glashow ($SU_5$) and similar models.

and a 10-plet,

$$10 : \begin{pmatrix} e^+_L \\ u_d \\ \bar{u}_L \end{pmatrix}$$

for each fermion family. One expects that the group will break

$$SU_5 \rightarrow SU_3 \times SU_2 \times U_1 \rightarrow SU_3 \times U_1Q$$

to the standard model. There are new colored gauge bosons $X$ and $Y$ with charges 4/3 and 1/3 which mediate transitions between quarks and leptons or between quarks and antiquarks. These can mediate proton decay, as shown in Figure 2. One expects the lifetime to be of order

$$\tau_{p \rightarrow e^+ \pi^0} \sim \frac{M_X^4}{\alpha^2 m_p^5} \approx 10^{30} \text{ yr [1980]},$$

where the experimental limit is from 1980. Thus, one required

$$M_X \gtrsim 10^{14} \text{ GeV [1980]}$$

to avoid too fast proton decay, with a grand desert between $M_X$ and $M_Z$. This is an enormous mass scale compared to those of the standard model – it was a daring extrapolation. Still, $M_X$ is sufficiently small compared to the Planck scale, $M_P = G_N^{-1/2} \sim 10^{19} \text{ GeV}$, that the partial unification without gravity is consistent.

One can also consider larger grand unified theories, in which more particles are related by the symmetry. For example, there is the $SO_{10}$ model, $SU_5 \subset SO_{10}$, in which each family is placed in a 16-plet, $16 = 5^* + 10 + 1$, which includes a new neutrino $\bar{N}_L$, which may be superheavy. There is also the even larger group $SO_{10} \subset E_6$, in which each family is placed in a $27 = 16 + 10 + 1$, which consists of the 16-plet, a heavy down-type quark ($D_L, \bar{D}_L$) which has no charged current weak interactions (it is an $SU_2$ singlet), a heavy lepton doublet \( \begin{pmatrix} E^+ \\ E^0 \end{pmatrix}_L \) \( \begin{pmatrix} \bar{E}^0 \\ E^- \end{pmatrix}_L \) in which both the left and
right-handed components transform as weak doublets, and an additional (possibly superheavy) neutrino $S_L$.

Grand unified theories have many interesting prediction. In addition to the coupling constant unification, charge quantization, and proton decays, some of the simplest give correct predictions for the ratio $m_b/m_\tau$. Also the baryon number violating interactions could generate a baryon asymmetry of the universe. However, as alluded to earlier, such an asymmetry could be later erased at the time of the electroweak transition unless the GUT asymmetry had a non-zero $B - L$.

4 Supersymmetry

Supersymmetry (SUSY) is a new type of symmetry which relates fermions and bosons. No known particles can be partners, so supersymmetry requires a doubling of the particle spectrum. There are two major motivations for considering such schemes. One is the problem of the Higgs mass renormalization. In the standard model the loop diagrams in Figure 3 lead to

$$m_H^2 = m_{H^0}^2 + \delta m^2,$$  \hfill (7)

where $m_{H^0}$ is the bare mass and

$$\delta m^2 = O(g^2, \lambda, h^2) \Lambda^2$$  \hfill (8)

is the correction. The diagrams are quadratically divergent, so that $\Lambda$ is a cut off. In practice it should be viewed as the scale at which new physics comes in to turn off the standard model and regulate the integrals. For example, if the next scale in nature is the gravity (Planck) scale $M_P = 10^{19}$ GeV one requires an enormous fine-tuning to ensure that the physical mass is of order 1 TeV or less.

There are two known ways to avoid this fine-tuning. One is to replace the elementary Higgs fields by some sort of dynamical symmetry breaking. However, this
approach leads to other difficulties and no satisfactory models have been constructed. The other is to introduce supersymmetry. In this case the new particles of the theory enter the corrections with opposite signs, so that

$$m_H^2 = m_{H^0}^2 + \delta m^2 - \delta m_{\text{SUSY}}^2$$

If supersymmetry were exact there would be a complete cancellation. Since supersymmetry has not yet been observed it must be broken, with the new superpartners heavy. If there is a soft breaking of the supersymmetry the quadratic divergences still cancel, but one expects to have finite remaining terms $O(g^2, \lambda, h^2)(m^2 - \tilde{m}^2)$, where $m$ and $\tilde{m}$ represent the masses of an ordinary particle and its superpartner. We must therefore choose

$$|\tilde{m}| \lesssim O(\text{TeV})$$

as the typical mass scale of the new supersymmetric particles to ensure that the Higgs mass is not too much larger than the desired electroweak scale. Supersymmetry predicts a rich structure of new particles. In addition to the superpartners there must be two Higgs doublets (and their partners) instead of one. All of the new particles may be in the hundreds of GeV range where they will be difficult to observe. However, the motivation for supersymmetry requires that they cannot be too heavy. They can be searched for at high energy colliders, and there may be indirect indications from low energy precision experiments.

Another motivation for supersymmetry is gravity. When one has a gauged SUSY there is necessarily a spin-3/2 gauge particle known as the gravitino, which is the superpartner of a spin-2 graviton. Supergravity theories automatically bring gravity into the game. However, they are not by themselves renormalizable.

One can consider supersymmetry either with or without grand unification. Superstring theories assume that the low energy theory is supersymmetric.

### 5 Unification of Coupling Constants

Now let us consider the running of the scale-dependent effective coupling constants, using the two-loop renormalization group equations

$$\frac{d\alpha^{-1}_i}{d\ln \mu} = -\frac{b_i}{2\pi} - \sum_{j=1}^{3} \frac{b_{ij}\alpha_j}{8\pi^2},$$

where $\alpha_i = g_i^2/4\pi$, $i = 1, 2, 3$, is the coupling of the $SU_3$, $SU_2$, or $U_1$ groups, respectively. These equations can be solved to yield

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_X) - \frac{b_i}{2\pi} \ln \left( \frac{\mu}{M_X} \right) + \sum_{j=1}^{3} \frac{b_{ij}}{4\pi b_j} \ln \left[ \frac{\alpha_j^{-1}(\mu)}{\alpha_j^{-1}(M_X)} \right],$$
Figure 4: Running of the inverse coupling constants. They can be measured at low energies and extrapolated theoretically in a grand unified theory. They should meet at the unification scale up to small threshold corrections.

where $\mu$ is an arbitrary momentum and $M_X$ is an arbitrary reference point. To an excellent first approximation one can neglect the last (2-loop) term, in which case the inverse coupling constant varies linearly with $\ln \mu$. The 2-loop terms are small but not entirely negligible.

In a grand unified theory one expects that the three couplings will meet at $M_X$ \cite{12}–\cite{18}, up to threshold corrections \cite{19}, as shown in Figure 4:

$$\alpha_i^{-1}(M_X) = \alpha_G^{-1}(M_X) + \delta_i + \Delta_i.$$ (13)

The $\delta_i$ are small corrections associated with the low energy threshold, from effects such as $m_t > M_Z$ and the non-degeneracy of the new particles, $m_{t^{\text{new}}} \neq M_{\text{SUSY}}$. Similarly, there may be corrections $\Delta_i$ associated with $m_{\text{heavy}} \neq M_X$ at the high scale, or with non-renormalizable operators.

If one knows the values of the couplings at low energy then they can be extrapolated theoretically in terms of calculable coefficients. The 1-loop terms are

$$b_i = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + F \left( \begin{pmatrix} \frac{4}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} \right) + N_H \left( \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} \right),$$ (14)

assuming the standard model. $F$ is the number of fermion families and $N_H$ is the number of Higgs doublets. In the minimal supersymmetric extension of the standard model (MSSM), in which one has the minimal number of new particles,

$$b_i = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + F \left( \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \right) + N_H \left( \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix} \right),$$ (15)

where the difference is due to the fact that additional particles enter the loops. The 2-loop coefficients can be found in \cite{19}.
6 Couplings at $M_Z$

To test the unification one must know the values of the couplings at low energy. We define the couplings $g_s = g_3$, $g = g_2$, and $g' = \sqrt{3/5}g_1$ of the standard model $SU_3 \times SU_2 \times U_1$ group, and the fine-structure constants $\alpha_i = g_i^2/4\pi$. The extra factor in the definition of $g_1$ is a normalization condition [12]. The couplings are expected to meet only if the corresponding group generators are normalized in the same way. However, the standard model generators are conventionally normalized as

$$\text{Tr}(Q_T^2) = \text{Tr}(Q_L^2) = \frac{5}{3}\text{Tr}(Y/2)^2,$$

so the factor $\sqrt{3/5}$ is needed to compensate. The couplings are related to $e$, the electric charge of the positron, by

$$e = g \sin \theta_W,$$

where the weak angle is

$$\sin^2 \theta_W = \frac{g'^2}{g^2 + g'^2} = \frac{g_2^2}{\frac{5}{3}g_2^2 + g_1^2} \rightarrow \frac{3}{8}. \hspace{1cm} (16)$$

One expects $\sin^2 \theta_W = 3/8$ at the unification scale [12] for which $g_1 = g_2$.

Precision experiments determine the weak angle very well. For comparing with grand unification it is convenient to use the modified minimal subtraction ($\overline{MS}$) renormalization scheme. Experimentally,

$$\sin^2 \hat{\theta}_W(M_Z) = \frac{\alpha_1(M_Z)}{\frac{5}{3}\alpha_2(M_Z) + \alpha_1(M_Z)} = 0.2325 \pm 0.0007, \hspace{1cm} (17)$$

where most of the uncertainty is from the top quark mass. Similarly,

$$\alpha^{-1}(M_Z) = 127.9 \pm 0.2 = \frac{\alpha_2^{-1}(M_Z)}{\sin^2 \hat{\theta}_W(M_Z)} \hspace{1cm} (18)$$

is obtained by extrapolating the observed electromagnetic fine structure constant from low energies, where it is measured precisely, to $M_Z$. Much of the uncertainty is associated with the low energy hadronic contribution to the photon vacuum polarization and some is from $m_t$.

For the strong coupling I will use

$$\alpha_s(M_Z) = 0.12 \pm 0.01. \hspace{1cm} (19)$$

This value is somewhat higher than has been used in the past. It is motivated by a new theoretical analysis of event shapes at LEP which yields $0.123 \pm 0.005$, as shown in Table 2. Previous analysis of event shapes using the same data gave the

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2 This value is obtained assuming the standard model. In the MSSM, one has $\sin^2 \hat{\theta}_W(M_Z) = 0.2324 \pm 0.0006$. The difference is due to the lower value (50-150 GeV) expected for the mass of the lightest Higgs scalar in the MSSM. (The second Higgs doublet and the superpartners do not significantly affect the precision observables.)

3 These same hadronic uncertainties lead to the major theoretical uncertainty in $g_\mu - 2$ and in the relationship between $\sin^2 \hat{\theta}_W - M_Z$.  

7
Table 2: Values of $\alpha_s(M_Z)$, adapted from [20]. $R_\tau$ refers to the ratio of hadronic to leptonic $\tau$ decays; $DIS$ to deep-inelastic scattering; $\Upsilon, J/\Psi$ to onium decays; and LEP($R$) to the ratio of hadronic to leptonic $Z$ decays. LEP($\text{events}$) refers to the event topology in $Z \rightarrow \text{jets}$.

somewhat lower value 0.119±0.006. However, resummed QCD [21], in which one uses both $O(\alpha_s^2)$ and next to leading logarithm corrections in the theoretical expressions, yields the higher value and better agreement between different determinations. The uncertainty is essentially all theoretical, from scale ambiguities. This value is in good agreement with that obtained from the hadronic $Z$ width. It is somewhat higher than the values obtained by low energy experiments such as deep inelastic scattering, $\tau$ hadronic decays, $\Upsilon$ decays, $F_2'\gamma$, and jet production. It is not clear whether the uncertainties stated in Table 2 for low energy determinations are realistic. There may be larger theoretical uncertainties. The value is (19) is a reasonable average of the various measurements with a conservative uncertainty.

One can now extrapolate the couplings to higher energy to see whether they meet at a point. This is shown for the standard model (SM) and for the minimal supersymmetric extension (MSSM) in Figure 5. For the standard model the couplings do not meet. This means that the old-fashioned simple grand unified theories such as minimal $SU_5$ are excluded. These have also been ruled out for some time by the nonobservation of proton decay, but the additional evidence is welcome. However, the couplings do meet in the supersymmetric extension. In Figure 5 it is assumed that all of the new particles, (i.e., the superpartners and extra Higgs particles) have a common mass $M_{\text{SUSY}} = M_Z$. They still meet within uncertainties for $M_{\text{SUSY}} = 1 \text{ TeV}$ or anywhere between. The unification scale $M_X$ is $O(10^{16} \text{ GeV})$, which is sufficiently high to suppress proton decay via heavy gauge boson exchange. However, as we will see there are new sources of proton decay that may be dangerous.

One can also predict the value of $\sin^2\hat{\theta}_W(M_Z)$ from $\alpha^{-1}(M_Z) = 127.9 \pm 0.2$ and $\alpha_s(M_Z) = 0.12 \pm 0.01$. Recall that this should be $3/8$ if there were no symmetry breaking. The predictions are compared with the experimental data in Figure 6. Again, it works beautifully for the MSSM for reasonable values of $M_{\text{SUSY}}$, but not for ordinary grand unified theories. The predictions are also shown for the standard model and the MSSM for two values of $M_{\text{SUSY}}$ in Table 3.

We see that the $\sin^2\theta$ predictions are in excellent agreement with grand unification
Figure 5: Extrapolation of the gauge coupling constants in the standard model and its minimal supersymmetric extension, assuming $\alpha^{-1}(M_Z) = 127.9 \pm 0.2$, $\sin^2 \theta_W(M_Z) = 0.2325 \pm 0.0007$, and $\alpha_s(M_Z) = 0.12 \pm 0.01$, updated from [15].

| Model                        | $\sin^2 \theta_W(M_Z)$       | $M_X(GeV)$               |
|------------------------------|-------------------------------|--------------------------|
| Standard Model               | $0.2100 \pm 0.0026$           | $4.6^{+1.7}_{-1.8} \times 10^{14}$ |
| MSSM ($M_{SUSY} = M_Z$)     | $0.2334 \pm 0.0026$           | $2.5^{+1.3}_{-0.9} \times 10^{16}$ |
| MSSM ($M_{SUSY} = 1 TeV$)   | $0.2315 \pm 0.0026$           | $2.0^{+1.6}_{-0.7} \times 10^{16}$ |
| Experiment                  | $0.2325 \pm 0.0007$           | --                       |

Table 3: Predictions for $\sin^2 \theta_W(M_Z)$ and $M_X$ in the standard model and the MSSM compared with the experimental value.
Figure 6: Predictions of $\sin^2 \hat{\theta}_W(M_Z)$ in the ordinary and supersymmetric grand unified theories compared with the experimental data. Updated from [13].
Figure 7: Predictions for $\sin^2 \theta_W$ compared with with the experimental data (solid circles), for the standard model (open circles) and for the supersymmetric extension with $M_{\text{SUSY}} = M_Z$ (boxes) and 1 TeV (triangles).

of the supersymmetric standard model but not with non-SUSY unification. This has actually been known since 1980 [6, 13, 14], as shown in Figure 7. However, the new high precision determination of the low energy couplings from LEP make the agreement especially striking.

A number of comments are in order.

• Is supersymmetry proved? Absolutely not! The agreement could very well be an accident, or one can modify ordinary grand unified theories in some ad hoc fashion.

• The predictions are independent of the actual GUT group, $SU_5$, $SO_{10}$, $E_6$, etc., to an excellent approximation, if the charge normalization is preserved.

• At the tree level, $\sin^2 \hat{\theta}_W$ and $M_X$ and the meeting of the couplings are independent of the number of fermion families. (This is not the case for $\alpha_i^{-1}(M_Z).$) The reason is that a fermion family forms a full multiplet of the $SU_5$ group and thus changes the slope of each $\alpha_i^{-1}$ by the same amount.

• On the other hand, there is a strong dependence on the number of Higgs doublets, $N_H$. That is because a Higgs doublet is part of a split multiplet associated with heavy partners. It therefore affects some couplings differently from other. In particular it does not affect the strong coupling.
• One could improve the $SU_5$ prediction for $\sin^2 \theta_W$ by increasing the number of Higgs doublets, but that would also decrease the unification scale, aggravating the proton lifetime problem.

• Supersymmetry was originally predicted and motivated assuming that the breaking scale was not too large, e.g., $M_Z < M_{\text{SUSY}} \lesssim 1 \text{ TeV}$. As we have seen, the coupling constant predictions are successful for $M_{\text{SUSY}}$ anywhere in this range. One should not, in my opinion, view $M_{\text{SUSY}}$ as a parameter to be fit to the data. There is no motivation for taking it outside of this range.

• The new higher values of $\alpha_s$ favor the smaller values of $M_{\text{SUSY}}$. One can turn around the logic and use $\alpha + \sin^2 \hat{\theta}_W$ to predict

$$\alpha_s(M_Z) \simeq \left\{ \begin{array}{ll}
0.072 \pm 0.001 & \text{standard model} \\
0.125 \pm 0.002 & \text{MSSM}(M_Z) \\
0.118 \pm 0.002 & \text{MSSM}(1 \text{ TeV})
\end{array} \right. \quad (20)$$

Comparing with the results of Table 3 one sees again the strong motivation for the MSSM, especially with the smaller values of $M_{\text{SUSY}}$.

• There are several additional uncertainties and corrections [19, 22, 23], including low scale uncertainties associated with the splitting of the masses of the new particles and with $m_t$, high scale uncertainties from the splitting of the heavy particles, and from possible non-renormalizable operators [24]. The are also models with intermediate scales, which break in more than one step to the standard model [25], and epicycle models [26], involving ad hoc representations split into superheavy and light pieces, which can significantly affect the predictions.

7 Complications

There are several complications which introduce theoretical uncertainties and limit the precision of the predictions. At the low scale we have made the simplifying assumptions that all standard model particles, including the $t$ and the Higgs scalar, have $m \leq M_Z$, and that the second Higgs doublet and all of the new SUSY partners have a common mass scale $M_{\text{SUSY}}$. This is clearly unrealistic. Ross and Roberts [27] have studied threshold effects associated with a realistic SUSY spectrum including splittings. They found that the splittings can have a significant effect, mainly because the colored superpartners tend to be heavier than the uncolored ones. A simple parameterization of this effect is given in [23], where it is shown that an effective $M_{\text{SUSY}}$ can always be defined, but it may be very different from the actual superpartner masses.

The high scale is also dangerous [22, 23, 24]. We have implicitly assumed so far that all of the superheavy fermions, scalars, and vectors have a common mass $M_X$. In
fact, they are likely to have splittings, leading to high-scale threshold corrections \[19\]. Including these, we have

\[\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_X) - \frac{b_i}{2\pi} \ln \left( \frac{\mu}{M_X} \right) + 2 \text{ loop} \]
\[+ \frac{1}{3\pi} \text{Tr} \left[ Q_i^2 \ln \frac{M_X}{M_{F,S}} \right] + \frac{1}{24\pi} \text{Tr} \left[ Q_i^2 \ln \frac{M_X}{M_S} \right] \]

(21)

where \(Q_{iF,S}\) are respectively the charges of the heavy fermions and scalars, and \(M_{F,S}\) are their mass matrices. If, for example, one allows the particles to vary by two orders of magnitude around the unification scale, \(M_X/M_{F,S} \sim 10^{\pm 2}\), the prediction for \(\alpha_s\) may vary by \(\Delta \alpha_s(M_Z) = \pm 0.01\). Thus,

\[\alpha_s(M_Z)|_{\text{MSSM}} \sim 0.12 \pm 0.01. \]

(22)

All values in this range are compatible with the experimental value, but due to the theory uncertainties a more precise measurement is useless in this context. Similarly, \(M_{\text{SUSY}}\) cannot be determined in this way. On the other hand, in the nonsupersymmetric case one predicts

\[\alpha_s(M_Z)|_{\text{SM}} \sim 0.072, \]

(23)

which is incompatible with the data for any reasonable uncertainties. There are other high-scale corrections associated with possible non-renormalizable operators \[24\] such as

\[\mathcal{L} \sim \frac{1}{M_P} \text{Tr}(F_{\mu\nu}F^{\mu\nu}\phi). \]

(24)

All of these corrections are studied in detail in reference \[23\].

The above high and low scale corrections are to be viewed as uncertainties in the simple two-scale grand unified theories in which there is indeed a desert between the low and high scales. One can consider more drastic modifications, such as intermediate-scale models \[25\], in which the grand unified theory breaks in two or more steps to the standard model, or models involving ad hoc new split multiplets of fermions and scalars \[26\]. These have sufficient freedom that one can bring the nonsupersymmetric case into agreement with unification. Such models are logical possibilities but have little predictive power.

8 Implications of SUSY Unification

8.1 Proton Decay

In the ordinary nonsupersymmetric grand unified theories, such as those based on \(SU_5\), \(SO_{10}\), and \(E_6\) one predicts a unification scale

\[M_X \sim (2 - 7) \times 10^{14} \text{ GeV} \]

(25)

\[4\]There are also small mass-independent discontinuities which depend on the renormalization scheme \[19, 23\].
from the observed values of $\alpha$ and $\alpha_s$, as well as the incorrect prediction for $\sin^2 \theta_W$. From the diagrams in Figure 2 one can predict the proton lifetime into $e^+\pi^0$. There are large theoretical uncertainties, from the value of $M_X$ and also from the hadronic matrix element. One finds [6]

$$
\tau_{p \rightarrow e^+e^0} \sim \frac{M_X^4}{\alpha_s^2 m_p^5}
$$

$$
\sim 10^{31\pm0.7} \left( \frac{M_X}{4.6 \times 10^{14}} \right)^4 \text{yr}
$$

$$
\sim 10^{31\pm1} \text{yr}, \tag{26}
$$

which is low compared to the experimental limit [9]

$$
\tau_{p \rightarrow e^+e^0} > 10^{33} \text{yr}. \tag{27}
$$

Thus, these models seem to be excluded on the basis of proton decay. It is interesting that a few years ago the theoretical prediction $\tau_{p \rightarrow e^+\pi^0} \sim 10^{29\pm3} \text{yr}$ was shorter but with a larger uncertainty. The change is due to the exponential dependence of the lifetime prediction on $\alpha_s(M_Z)$. The new higher values of $\alpha_s(M_Z)$ predict a longer lifetime. Therefore, while the new precision measurements tighten the disagreement with the $\sin^2 \theta_W$ predictions, they have weakened the discrepancy based on proton decay.

### 8.2 SUSY-GUT

In the supersymmetric grand unified theories, on the other hand, one has a much higher prediction

$$
M_X \sim 2.5 \times 10^{16} \text{ GeV} \tag{28}
$$

for the unification scale. This strongly suppresses the decay rate via the gauge boson exchange to a comfortably safe

$$
\tau_{p \rightarrow e^+\pi^0} \sim 3 \times 10^{38\pm1} \text{ yr}. \tag{29}
$$

However, SUSY-GUTs have new dimension $d = 5$ operators [28] mediated by the exchange of a heavy Higgsino, as in Figure 8. The basic baryon number-violating process

$$
\tilde{q}\tilde{q} \rightarrow \tilde{q}\tilde{l} \tag{30}
$$

must be dressed with the exchange of a light gaugino, $\tilde{w}, \tilde{z}, \tilde{g}$, in order to generate a diagram for $qq \rightarrow \tilde{q}\tilde{l}$. Since the process is mediated by the exchange of a heavy fermion the lifetime goes like $\tau_p \sim M_X^2$ rather than $M_X^4$, so it is extremely dangerous. It tends to produce decays which change generations, such as into $\nu K^+$. One predicts [28]

$$
\tau_{p \rightarrow \nu K^+} \sim 10^{29\pm4}, \tag{31}
$$

$$
\sim 10^{29\pm4} \text{ yr}.
$$
Figure 8: Diagram for a proton decay in supersymmetric grand unified theories. The basic operator is due to the heavy Higgsino exchange on the right-hand side of the diagram.

which is only marginally compatible with the current experimental limit \[1\]

\[\tau_{p \rightarrow \bar{p}K^+} > 10^{32} \text{yr.}\] (32)

Nath and Arnowitt \[28\] have done a detailed study of the \(d = 5\) constraints in supersymmetric grand unified theories. They find that the so-called “no-scale” models are excluded by proton decay. More general supersymmetric models are still viable, but only if they satisfy constraints on the low energy spectrum, such as \(m_t < 175\) GeV; \(m_\tilde{q} < m_\tilde{g}\); \(M_H < M_Z\); and a chargino and two neutralinos < 100 GeV. It should be commented that the \(d = 5\) operators may be absent in certain theories which are not true grand unified theories, such as some string theories and flipped \(SU_5 \times U_1\).

9 Neutrino Mass

Grand unified theories often have interesting implications for neutrino mass. Typically, they lead to a seesaw prediction \[29\], in which the light neutrino masses are of order

\[m_{\nu_i} \sim c_i \frac{m^2_u}{M_{N_i}},\] (33)

where \(u_i = u, c, t\) are the light charged 2/3 quarks and \(c_i \sim 0.05 - 0.4\) are coefficients which depend on the radiative corrections or the running of the masses from the high scale down to the low scale. The \(N_i\) are heavy majorana neutrinos. Furthermore, simple grand unified theories often predict the relation

\[V_{\text{lept}} \simeq V_{\text{CKM}}\] (34)

between the lepton and quark mixing matrices.
Figure 9: Regions for the neutrino mass and mixing parameters favored by solar neutrino data and other experiments, compared with the typical predictions of various grand unified theories. The two small regions marked “combined” are allowed by the combination of the Kamiokande, Homestake, and GALLEX results [25].

In the “old” supersymmetric grand unified theories which were prevalent before the days of superstring theories one often introduced very large Higgs representations, such as a 126-plet of $SO_{10}$. In such models these Higgs can generate large majorana masses, typically

$$M_{N_i} \sim (10^{-2} - 1)M_X,$$

which implies the very small neutrino masses

$$m_{\nu_e} \approx 10^{-11} \text{ eV}$$
$$m_{\nu_\mu} \sim (10^{-8} - 10^{-6}) \text{ eV}$$
$$m_{\nu_\tau} \sim (10^{-4} - 1) \text{ eV}.$$  \(36\)

These masses are sufficiently small that if one wishes to satisfy the MSW solution to the solar neutrino model one must postulate \[25\] $\nu_e \rightarrow \nu_\tau$. If the equation (34) is satisfied then one expects that this will be in a disfavored small-angle region, as shown in Figure 9.

On the other hand, in the string-inspired modern versions of supersymmetric
theories it is difficult to introduce large Higgs representations. One therefore expects

\[ M_{N_i} = 0 \]  

in the simplest versions at the lowest order. However, it is quite possible to have

\[ m_{N_i} \sim 10^{-4} M_X \sim 10^{12} \text{ GeV} \]  

generated by non-renormalizable gravity-induced operators, which may well survive the compactification. In this case one would typically expect

\[
\begin{align*}
    m_{\nu_e} &\sim 10^{-7} \text{ eV} \\
    m_{\nu_\mu} &\sim 10^{-3} \text{ eV} \\
    m_{\nu_\tau} &\sim (3 - 21) \text{ eV}.
\end{align*}
\]

One could easily have \( \nu_e \to \nu_\mu \) for the solar neutrinos and observable \( \nu_\mu \to \nu_\tau \) oscillations in the laboratory. Furthermore, \( m_{\nu_e} \) may be in the cosmologically-relevant range of a few eV. Such models have considerable flexibility in the masses and do not make any firm predictions about the mixing angles.

Finally, intermediate scale models (ordinary grand unified theories breaking in two stages to the standard model) give results similar to the “new” SUSY models if the intermediate scale is of order \( 10^{12} \text{ GeV} \). One actually expects a somewhat lower value, \( 10^{10} \text{ GeV} \), in the simplest \( SO_{10} \) models compatible with the coupling constants.

Let me briefly comment on a few other implications of supersymmetric theories.

- **The Supersymmetric Spectrum.** Ross and Roberts \cite{27} have considered realistic SUSY spectra based not only on unification but criteria of naturalness and \( m_b \). Nath and Arnowitt \cite{25} have considered the constraints from proton decay. In both cases they predict rather low scales for the superpartners, such as

\[
\begin{align*}
    m_{\tilde{\gamma}, \tilde{\omega}, \tilde{\zeta}} &\sim 100 \text{ GeV} \\
    m_{\tilde{g}, \tilde{q}} &\sim (300 - 500) \text{ GeV}.
\end{align*}
\]

- **Many simple grand unified theories predict**

\[ m_b = m_\tau \]  

at the GUT scale. Including running effects mainly due to gluon exchange this is renormalized to the prediction

\[ m_b \sim (4 - 5) \text{ GeV} \]  

at the low scale. However, such models also make the bad prediction

\[ \frac{m_d}{m_s} = \frac{m_e}{m_\mu}, \]

which fails by an order of magnitude.
• The connection of these ideas to superstring theories is still rather strained [30, 31]. In the simplest superstring compactifications one would expect a gauge group $G \subset E_6$ to emerge at the compactification scale $M_C \sim 10^{18} \text{ GeV}$. In general one would not expect this to be a simple GUT group. For example, it could be just the standard model itself. Even for schemes for which $G = SU_5, SO_{10}, E_6$ one typically expects $M_X \sim M_C$, while the data are suggesting that $M_X$ is one or two orders of magnitude lower. It is possible [31] that the compactification does result in the standard model group, but that there are large threshold corrections from massive modes which cause the couplings to cross at an effective scale lower than $M_C$. However, there are no compelling or realistic models of this, and other GUT effects, such as the $m_b$ and proton decay predictions, may be lost.

• There are potentially interesting implications of unification for the large scale structure of the universe, but so far this is very speculative.

• What about baryogenesis? The successful prediction of the baryon asymmetry of the universe was a great success of the ordinary GUTs [4]. Now, however, there is the major complication from baryon number violation at the electroweak scale [3]. This may wash out any baryon asymmetry created earlier unless it is either enormous or leads to a non-zero value of $B - L$.

10 Conclusions

• We expect $B$ violation to occur in nature at some level.

• Proton decay has not yet been observed.

• In the future, one can increase the sensitivity of proton decay searches by one or two orders of magnitude.

• There are strong motivations to continue the search for $B$ violation from the coupling constant unification in the MSSM. This could well be an accident, but it may also be a hint that the simple ideas of the grand desert and supersymmetry are on the right track. Such schemes would have many interesting implications, such as for proton decay $p \rightarrow \bar{\nu} K^+$, neutrino mass, $m_b$, the SUSY spectrum, cosmology, etc.

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