Developing Algebraic Knowledge: Foundation Programme ex-Mathematical Literacy Students’ Perceptions

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Abstract
South African Mathematical Literacy (ML) learners are unlikely to have mastered the mathematical skills essential for undergraduate degree studies in Business Science and are most likely to commence tertiary studies in a foundation programme (FP). One such FP at one private higher education institution included an algebra course and provided an opportunity for such students to articulate to undergraduate degree studies in Business Science. Essentials, an algebra reteaching strategy, supported underprepared students. The perceptions of 11 students from ML were probed in semi-structured interviews on aspects influencing the development of algebraic knowledge. Four themes: introducing the ML student, beneficial aspects, aspects to address for improvement, and personal aspects emerged from deductive coding. Perceptions identified may sensitise future ML students entering higher education studies through similar pathways about previous students’ experiences of FP algebra courses, and inform FP algebra course teachers about ML learners’ needs during their development of academic knowledge.

Keywords: algebraic knowledge, foundation programme, mathematical literacy, students

INTRODUCTION
Following the 1994 elections, South Africa’s (SA’s) education curriculum was redeveloped to take into account the needs of all learners¹. During Grade 9, the ninth year of schooling, when learners make subject choices for their final three years of secondary education, they must choose a mathematics-related subject. The mathematics-related choices are either the national senior certificate (NSC) Mathematics or NSC Mathematical Literacy (ML). Students who choose Mathematics are exposed to a pedagogy that develops mathematical reasoning and skills in preparation for studies at higher education institutions (HEIs) (South African Department of Basic Education [DBE], 2011a), whereas the subject ML was developed to transform the country’s low levels of numeracy skills (South African Department of Education [DoE], 2008), and elementary mathematical concepts were contextualised to prepare learners to become “self-managing individuals” who are “contributing workers” (South African DBE, 2011b, p. 8).

The ML syllabus is numeracy-based (Mhakure & Mokoena, 2011), and learners may choose ML rather than Mathematics for several reasons: Mathematics may not be mandatory for their future career or study direction (South African DBE, 2003), ML is easier than Mathematics (Sofowora, 2014), or mathematics teaching may outpace the knowledge development rate of individuals (Pritchett & Beatty, 2015). Furthermore, learners may select ML because of negative attitudes toward learning mathematics, or the school may have issued a directive for the learner to take ML based on their Mathematics results (Spangenberg, 2012).

Consequently, learners who choose ML do not meet the entry requirements for undergraduate degree studies in Business Science at most HEIs in SA. Even alternative access routes² seldom cater for learners who chose ML and aspire to pursue undergraduate degree studies.

¹ In South Africa, those attending primary or secondary schooling are known as learners; those attending tertiary or higher education are referred to as students.
² Alternative access routes in South Africa include extended or augmented degree programmes and foundation programmes.

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studies in Business Science. At the time of this study, the HEI under investigation offered a foundation programme (FP) that provided access to students who met the entrance requirements, including those from ML backgrounds. In addition to the FP, the HEI offered undergraduate and postgraduate degree studies in numerous fields, including Business Sciences. This international HEI is reputable, and students from South Africa and many African countries apply to study there. Students enrolled into the HEI’s FP may have previously completed a variety of mathematics curricula, including NCS (Mathematics or ML), Cambridge (ordinary or advanced subsidiary levels), or the West African Examination Council. This article considered ML students enrolled into the FP.

Most year-long, non-credit bearing FPs provide academic support to prepare previously disadvantaged or underprepared students for the rigour of higher education (HE) studies (Dietrich, 2017). The SA government has endorsed FPs as a strategy to address the nation’s educational equity goals (Kirby & Dempster, 2018). Support for FPs may yield fruit, as students completing a FP prior to undergraduate enrolment may experience greater academic success than students enrolled in undergraduate studies who narrowly met the entry criteria (Kirby & Dempster, 2018). Students from ML backgrounds wishing to enrol into the FP at the private HEI under study needed to have achieved, inter alia, an admission point score (APS) of at least 23, and a minimum of 50% for ML. In this particular FP, students completed two 12-week semesters with four courses per semester. Three of the FP streams: Business Science, Computer and Information Science, and Health Science required successful completion of an algebra-rich mathematics course (Baumgartner et al., 2018).

The main topics in the algebra course were linear, quadratic, and exponential expressions, equations, and functions, which were briefly revised before each topic was applied to business and economics. Learners choosing ML early in their secondary schooling years have had very little engagement with algebraic content since ML is numerical and context-driven, including real-life problems that are language-laden (Baumgartner, 2016; Mhakure & Mokoena, 2011). ML students were thus presumed to be underprepared for the content of the FP algebra course, and potentially underprepared to study algebra in other ways too. The motivation for this article arose from a desire to comprehend the perceptions of ML students on the development of algebraic knowledge in the FP algebra course in order to sensitise future ML students entering HE studies through similar pathways about previous students’ experiences of FP algebra courses, but also to inform FP algebra course teachers about ML students’ needs in their development of academic knowledge.

The algebra course was taught as a full-time, face-to-face course, and students were expected to attend a one-hour lecture and three one-hour tutorials each week during the first 12-week semester. Teacher-centred lectures were presented to groups of up to 250 students and covered theory and limited examples. Smaller groups of approximately 25 students attended tutorials, where the classroom culture depended on the pedagogy of the tutorial teacher and students engaged with exercises and activities that aligned with the theory of the week.

An additional voluntary reteaching class, Essentials, was timetabled for all students, but targeted mathematically underprepared students. Essentials comprised two additional hours of contact time per week for those students who wished to attend. Essentials was a hybrid of the lecture and tutorial pedagogy, since the week’s theory was revised, and students completed exercises and activities in a setting where collaborative learning was encouraged. Eleven students from ML backgrounds who regularly attended the Essentials were purposively selected to participate in the study, as they were most able to offer a holistic perception on aspects influencing their development of algebraic knowledge in the algebra course, including the reteaching aspect. Participants could thus share their perceptions in relation to lectures, tutorial classes, and Essentials, or a combination of these.

**PERSPECTIVES ON ACCESS PROGRAMMES**

Studies of access programmes, of which a FP is one type, have explored many facets relating to student perceptions and experiences at the programme, rather than the course level. Access programmes situated on separate campuses may initially cause a sense of
isolation and exacerbate stigmatisation or perceptions of academic inferiority (Dietrich, 2017; Ogude et al., 2019). Some students mentioned experiencing discrimination or stigmatisation due to attending an access programme (Dietrich, 2017), although Millennials, as a non-homogeneous group, are thought to acknowledge and accept diversity (Shushok & Kidd, 2015). Students may view the access year as a positive experience (Potgieter et al., 2015) and advocate for access programmes (Ogude et al., 2019). In addition to possible negative access programme perceptions, ML students have a history of deficit thinking, having been labelled as “stupid”, unable to do mathematics, and less successful (Machaba & Du Plooy, 2019, p. 366).

Access programmes should provide a nurturing and supportive learning environment (Daniels & Jooste, 2018; Ogude et al., 2019) where students can develop self-efficacy and learn to cope with failure (Ogude et al., 2019) while building self-confidence (Daniels & Jooste, 2018). Many access programmes offer relatively small tutorial classes, which students view as a positive feature (Potgieter et al., 2015) supporting learning development (Daniels & Jooste, 2018). In smaller classes it is possible to reduce the pace of teaching which aids learning when students are less prepared for HE study (Engelbrecht et al., 2010).

Collaborative learning and assistance from more knowledgeable others (MKOs) aid the construction and development of knowledge (Vygotsky, 1978). Individualised learning to internalise and connect new knowledge with previously learned knowledge is also important (Vygotsky, 1978). Peer support and collaboration is crucial as a form of academic support in access programmes (Ogude et al., 2019) and should be incorporated into tutorial learning (Daniels & Jooste, 2018). Good performers have been found to engage in collaboration more easily and effectively than poor performers (Potgieter et al., 2015) and where peer support is encouraged, students feel assisted to succeed in their studies (Daniels & Jooste, 2018). Encouraging peer collaboration outside of scheduled contact time may improve student engagement (Case et al., 2013). Seeking help from, and studying with peers, is effective for academic skills acquisition (Zander et al., 2018), consolidation of learning (Hattie & Donoghue, 2016), and outperformance of those who do not study with others (Pai et al., 2015). Technology may also be considered a MKO of sorts, and Millennials employ technology for many reasons, one of which may be to stay interested in their learning, as they are easily distracted (Shushok & Kidd, 2015).

Aspects Influencing the Development of Algebraic Knowledge: A Comparison of ML and Other Students

The development of algebraic knowledge is related to the thinking style of the student (Wan Muda et al., 2020) and students from ML backgrounds are likely to have different thinking styles from other students (Spangenberg, 2012). If teachers know the thinking styles of their students, they are able to design resources that enable students to learn more effectively (Wan Muda et al., 2020). Spangenberg’s study (2012) on thinking styles elucidated that Mathematics learners choose an internal scope of self-management, preferring to work alone, and they display a liberal learning style, being open to trying new methods. By contrast, ML learners are more likely to favour an external scope of self-management and approach learning from a conservative stance.

Students who are motivated to work hard, display commitment and dedication (Sibanda et al., 2015), and attend classes regularly (Zhu et al., 2019), are more likely to develop knowledge successfully. Differences in levels of interest, motivation, and perseverance have been elucidated between learners of Mathematics and ML (Botha, 2012; Spangenberg, 2012). ML learners are less motivated to study mathematics (Botha, 2012), and may lack the discipline and effort regulation required (Spangenberg, 2012) to develop algebraic knowledge. They are fearful or anxious about, and exhibit negative attitudes toward mathematics (Botha, 2012; Spangenberg, 2012). These states may result in, or be the result of low attainment in mathematics. Students from ML backgrounds self-reported significantly lower levels of intrinsic motivation, task value, and self-efficacy, and significantly higher levels of test anxiety than non-ML students in the early weeks of a FP algebra course (Baumgartner et al., 2018). Such states may endure or escalate in HE studies when ML students begin to develop algebraic knowledge alongside students who have previously studied Mathematics.

Students from ML backgrounds may differ from non-ML students in terms of learning attributes, which include learning strategies, beliefs, management, and support. While ML students have self-reported similar results to other students regarding learning strategies, they self-reported significantly lower levels of effort regulation than other students in a FP mathematics course (Baumgartner et al., 2018). This may be particularly relevant in light of the strong positive correlations between effort regulation and intrinsic motivation, self-efficacy, task value, time and study environment management, and metacognitive self-regulation (Baumgartner, 2016). Bowles et al. (2014) found that variables relating to study, such as study skills, time management, and help-seeking behaviours, as well as those relating to effort, such as motivation and commitment enabled transition. Prior studies (Baumgartner, 2016; Baumgartner et al., 2018) did not elucidate significant differences between ML students and other students in a FP algebra course relating to learning strategies, learning beliefs or learning support. The perceptions of ML students relating to these learning attributes may thus reveal new insights about the development of algebraic knowledge.
Teaching approaches also differ in ML and Mathematics classrooms (Machaba, 2018; Machaba & Du Plooy, 2019) as ML involves reasoning, whereas Mathematics involves following rules (Machaba, 2018). In the ML classroom, learner-centred and problem-solving approaches are often followed, whereas mathematics classrooms are more teacher-centric and mathematics is taught procedurally (Machaba & Du Plooy, 2019). Different teaching approaches could impose a further adjustment to HE learning for ML students.

## RESEARCH QUESTIONS

Based on the introduction and background provided, this article aims to address the following question:

What are the perceptions of students from Mathematical Literacy who are enrolled in a Foundation Programme algebra course on aspects influencing their development of algebraic knowledge?

Research sub-questions probed positive and negative perceptions. The three research sub-questions are as follows:

- What aspects of an algebra course do Mathematical Literacy students perceive to be beneficial to their development of algebraic knowledge?
- What aspects of an algebra course do Mathematical Literacy students perceive could be addressed to improve their development of algebraic knowledge?
- What personal aspects do Mathematical Literacy students perceive to influence their development of algebraic knowledge?

## METHODOLOGY

This exploratory research design utilised semi-structured interviews to gain insights on the academic development of ML students. Eleven purposively selected participants from a population comprising 104 ML students enrolled in a semester-long FP algebra course, aiming to articulate to undergraduate degree studies in Business Science were interviewed until data saturation was achieved. These participants were prior ML learners and were all enrolled in the FP algebra course during the same semester. They attended the Essentials classes regularly and participated voluntarily.

### Participant Profiles

Table 1 introduces participants through pseudonyms and summary information. Three of the four participants who chose ML from the start of Grade 10 failed the algebra course; all other participants passed the course. From Table 1, it is clear that the participants have diverse academic backgrounds. In South Africa, public schools are managed by the state and supply education to a diverse learner population, whereas private schools are independently managed and usually target the wealthier income groups. Townships are racially segregated residential areas, where, under the apartheid era, black people were relocated, and are areas considered synonymous with “poverty, crime and violence” (Mampane & Bouwer, 2011, p. 114). Inner city schools, by contrast, are located in the central business district and service learners who are often from a lower socio-economic status and living in the city centre or a township adjoining that city (McKay et al., 2018).

Colleagues interrogated the data collection instruments to ensure that the questions would inform the research question and sub-questions. Interviews commenced with casual conversation about prior mathematical background.

In order to acquaint the reader with students from ML, participants were encouraged to elaborate on the following statement:

1. Tell me a little about your secondary schooling experiences and the decision to choose ML.

Thereafter, interview questions probed participants’ perceptions about aspects that benefitted the development of algebraic knowledge, or aspects that could be improved in order to develop algebraic knowledge were asked. The second interview question aimed to address the first research sub-question:

### Table 1. Introducing the participants

| Gender | Type of school          | Started ML              |
|--------|-------------------------|-------------------------|
| Lerato | Female                  | Public                  | Start of Grade 12 |
| Lindive| Female                  | Public                  | Second term, Grade 10 |
| Alile  | Female                  | Private, boarding       | Start of Grade 12 |
| Sergio | Male                    | Public                  | Second term, Grade 11 |
| Xola   | Male                    | Private, township       | Second term, Grade 12 |
| Bianca | Female                  | Inner-city              | Start of Grade 10  |
| Thembu | Male                    | Public                  | Start of Grade 10  |
| Kagiso | Male                    | Private                 | Start of Grade 10  |
| Ravi   | Male                    | Private                 | Start of Grade 12  |
| Thandi | Female                  | Public school           | Third term, Grade 11 |
| Neo    | Female                  | Private                 | Start of Grade 11  |
2. What aspects of the various algebra classes (lectures, tutorials and/or Essentials) do you perceive would equip students to feel confident about understanding algebra? The third interview question focused on addressing improvement; the subject of the second research sub-question:

3. What aspects of the various algebra classes (lectures, tutorials and/or Essentials lessons) do you suppose could be improved to better equip students to feel confident about understanding algebra?

4. What advice would you give to new students from ML enrolled in the algebra course? You can provide advice relating to both the do’s and the don’ts.

The final interview question explored participants’ perceptions that related to the third research sub-question:

5. If you could go back in time and change anything that you did during the algebra course to improve your own algebraic understanding or academic success, what changes would you make?

A pilot interview was held with a volunteer participant from the prior semester’s cohort who had regularly attended the Essentials in that semester, and had previously completed ML. Interviews with participants were employed at different stages of the students’ studies, aiming to elucidate similarities and differences that might be perceived while sustaining credibility. Participants chose an appropriate time to meet, based on their schedule. Transcribed conversations were checked against the recordings and participant audits of the transcriptions to ensure correctness and accuracy were encouraged. Findings were compared against the analysed data and these were compared with the raw data to ensure dependability and confirmability of the data (Koonin, 2014).

The HEI overseeing the study granted Ethics clearance (2013-066), and ethical measures were upheld throughout the research process. These measures included informed consent, voluntary participation, participant anonymity, withdrawal for the study without penalty, and confidentiality.

**DATA ANALYSIS**

During the interviews, the first author became immersed in the data to appreciate the perceptions of each participant on the development of algebraic knowledge (Burnard, 1991). Although deductive and provisional codes had emerged from an examination of the literature, descriptive and emotional coding (Miles et al., 2013) were used during first cycle coding, to assign labels. The computer-assisted qualitative data analysis software (CAQDAS) package, NVivo 12, aided analysis and organisation of unstructured texts, and theme identification. Pattern coding was utilised during second round coding to group summaries into themes. Open coding methods were chosen and themes, categories, and codes that transpired are presented in Table 2 (Saldaña, 2016).

**FINDINGS**

Students from ML are seldom envisioned to enrol in HE mathematics studies, as they have not developed the mathematical knowledge required for such studies and are considered underprepared for mathematics studies

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3 The institution at which the study took place also granted ethics clearance, but no ethics clearance number was attached.
at this level. This section thus begins by introducing these students to the reader. Thereafter, aspects of the algebra course that ML students perceived to be beneficial to their development of algebraic knowledge are discussed. Aspects of the algebra course that ML students perceived could be addressed to improve their development of algebraic knowledge are then probed. Finally, personal aspects that are perceived to influence the development of algebraic development are presented.

Introducing ML Students to the Reader

In response to the first interview question, participants shared their secondary schooling experiences and their decision to choose ML. As the literature predicted, reasons for choosing ML were varied. For Lindiwe, “the course I was wanting to study didn’t actually need pure maths,” while Sergio changed to ML to “boost [his] APS”. Thandi “had difficulties with pure maths” and Alile worried that if her friend was “struggling then I’m not gonna cope”. Kagiso prioritised rugby, so “Maths Lit was … a first choice”. The Principal of Ravi’s school insisted he should “drop down to Maths Lit.”

Regardless of why ML was chosen, the result is a hiatus in the development of algebraic knowledge. ML students may not have studied algebra for as long as the past three years. In the FP they must, within a 12-week period, develop the algebraic knowledge others have previously acquired alongside the content and applications of the algebra course. Students who “did ML, like baby maths really, … take a lot longer than someone who had done maths, to do a question” [Sergio] or to develop the algebraic knowledge required of an FP algebra course. Learning in an algebra course is easier “for the people who did pure maths” [Lindiwe], but “it’s something new to us people who didn’t do pure maths”. The teacher “assumes that you know, because you did maths in high school” [Lerato], but ML students have not. “They expect you to be on the same maths level” [Sergio], but ML students are not.

ML students realise their knowledge of algebra is different from other students: “they work with x and y which brings confusion to people, whereas Maths Lit, you worked with words and numbers” [Lindiwe]. They realise too that their ML background may hinder their development of algebraic knowledge “in literacy, I think the level is a bit low” [Bianca], but hope the course will “bridge the gap between maths core and maths lit” [Thandi]. Psychological aspects surrounding the current consequences of previously choosing ML may thus aggravate what students already perceive to be an insurmountable task. This rationale, along with the lack of mathematical knowledge, could be why so few HEIs in South Africa have provided opportunities for ML students to enrol in undergraduate degree studies in Business Science.

ML students are likely to be particularly underprepared for HE studies of a mathematical nature. ML students previously chose ML for different reasons at various stages of their secondary schooling, thereby developing disparate levels of algebraic knowledge from each other and their FP peers.

Aspects of the Algebra Course Perceived to Benefit the Development of Algebraic Knowledge

The findings reported in this section were largely elucidated from participants’ responses to the second interview question. The dominant theme extracted from responses to this question was support. Lindiwe recalled, “because I didn’t do maths, it’s just difficult, you need somebody” and Alile agreed saying, “you have to be open to the fact that you’re struggling and you need help”. Participants mentioned support from peers and teachers.

Learning with peers, who may be MKOs, inside and outside of the classroom, was perceived to benefit algebraic knowledge development. Sergio’s classmate was “very helpful, a lot of the time he even, he offers, ‘if you don’t understand this, please ask.’ … He really doesn’t mind helping me out at all”, which was appreciated, because “just to get to the first step, I need to ask someone: ‘Listen, how did you get here?’” [Sergio]. Thandi and her friends “would go through every question to help each other understand” and she advised ML students to have “friends who did maths core … to help them” [Thandi]. Ravi agreed, remembering: “if I don’t understand, she’ll explain and if I understand, I can explain it to her”.

Often the distinct aim of befriending others was to progress algebraic knowledge development: “university friends are just semester friends” [Ravi]; “they were just there to help you get through the course all together” [Alile], “I don’t really know them anymore” [Ravi]. Lerato knew she’d “do a little better” if she “were to join a study group,” supporting Xola’s strategy that ML students should “find partners, people who know maths core, who can teach them.” Students who do not learn collaboratively may learn less effectively, as Lerato realised: “I haven’t been able to make proper friends, so the fact that I’m thinking about it would probably make me concentrate a little less on my school work.” The willingness of students from ML to work with others is promising; as such strategies consolidate learning [Hattie & Donoghue, 2016].

Although participants described learning with peers as beneficial to the development of algebraic knowledge, Kagiso cautioned against possible negative consequences of peer learning: “you can talk about partying … for at least three hours. Now you’ve wasted three hours where you could have done maths.” Ravi
also cautioned that when working with MKO peers, “they will understand something, but you won’t understand that thing, and because of them understanding, you’ll say, ‘OK, if you understand, I understand as well’”.

ML students seem to be social learners who enjoy learning with and appreciate support from peers. These perceptions of peer learning align with both the social nature of Millennials, who like group learning (Shushok & Kidd, 2015), and the social adaptability of ML students (Spenenberg, 2012). Individual learning could, however, be a less straightforward skill for ML students, as Lindiwe lamented, “if only I could get help at home”. Opportunities for students to work together should be provided and encouraged, while guidance on how to approach learning alone at home may assist ML students to enhance the quality of their individualised learning.

The active support of the teachers of the algebra course was perceived to be beneficial to the algebraic knowledge development of ML students. Sergio’s teacher “always walks around, … he doesn’t just sit there doing nothing”, and Xola found the same: “she walks around … she assists”. Participants appreciated tutorial teaching where, due to “small number of students, the [tutorial] teacher can attend to you individually” [Bianca], and “you’ve actually got the opportunity to ask questions” [Sergio]. Xola’s tutorial teacher encouraged peer collaboration, asking students who understood the content to “help those who don’t understand”. Alile’s teacher “tries and helps us. It’s not like he just tells us and then leaves us ... he actually explains”. Xola liked that the teacher “always finds space to make a joke, so that the students don’t get bored or distracted”, while in Lerato’s class, “because of her [the teacher] energy, you are more like into the whole tutorial and you wanna know more”.

ML students believe a progressive pedagogy such as a light-hearted, student-centred teaching approach supports active engagement and facilitates student learning (Case et al., 2013). Personalised learning opportunities where students may clarify aspects taught further benefits their algebraic knowledge development (Potgieter et al., 2015). While the benefits of knowledge gains from individualised learning are perceived, ML students may still struggle to harness these.

Mathematical Literacy students further perceived that the Essentials reteaching strategy supported their learning and understanding of algebra. The Essentials classes “are a big help to Math Lit students. People understand, and they enjoy and constantly want to come to Essentials” [Xola]. In Essentials, “we do a lot more examples [and] have a lot more time to do the examples” [Sergio]. Content is “explain[ed] slower” [Lindiwe, Sergio]; we “really break down the topic on how to do things in a simpler method” [Thandi] and “take it step-by-step” [Aliile]. “I understand when I’m at Essentials” [Lindiwe]; they “actually help” by “supporting tutorials” [Aliile]. This reteaching strategy, like that of Daniels and Jooste (2018), may build students’ self-confidence, play a supportive role in the development of knowledge, and link social and individualised learning. Aliile mentioned that at Essentials “we’re able to understand” and “then when we have time in our own private lives to actually go back and say, ‘we can do this’”.

The course’s structure and the teaching approach were also perceived to benefit ML students’ learning. Ravi felt the algebra course was “really well structured ... you go to your lecture, then you go to your tutorial, then you go to Essentials if you still don’t understand”. The different lesson types were perceived to support learning and each other. Bianca explained: if “you didn’t understand” some explanations in the lecture, then “you get a little bit of knowledge” during the tutorial classes, “by breaking the thing down into bits, ... students begin to understand” at Essentials. Lectures delivered “teaching in a broad way” [Xola] and in tutorials “you’ve got the opportunity to ask questions and do even more examples in different ways” [Sergio]. Essentials are “slower than the lectures” with “more examples” and “more time to do the examples” so “you get a lot more things that you missed in the lecture” [Sergio]. Participants generally felt sufficient teaching support was provided: “they’re doing enough” [Lerato]; “there is enough support for everyone” [Lindiwe]; “they can’t do any more” [Ravi]. Lindiwe echoed Daniels and Jooste (2018) on the value of the supportive role of teachers: “I don’t think you guys really need to push us to, like, do work and all that. All that you need to do is just support us”. Participants thus acknowledge the teacher’s role as prominent during knowledge acquisition and they also recognise that the responsibility to consolidate learning is, as Hattie and Donoghue (2016) indicated, the students’. Underprepared students, such as ML students may, however, require support and coaching that enables them to take charge of their learning so they are able to undertake effective independent revision and study.

Aspects of the Algebra Course That Could be Addressed to Improve the Development of Algebraic Knowledge

The findings reported in this section were largely elucidated from participants’ responses to the third and fourth interview questions. Mathematical Literacy students believe that increased contact time and a slower pace of teaching could improve their algebraic knowledge development. All participants mentioned the pace of the course, which was often too brisk, occasionally just right, but never too slow for ML students. Lerato felt her teacher “goes a little too fast sometimes, but sometimes, he goes at an adequate speed” while Xola suspected other students may “get bored at times because they...
already know what is happening”. Lindiwe felt overwhelmed by the pace of the classes, belabouring Engelbrecht et al.’s (2010) point that the pace of teaching in first year classes should decrease. Lindiwe felt her teacher explained

... tutorials, I think they’re just moving too fast, slower, … yeah, ‘cos he moves too fast … the thing is there’s less time. … give us an extra hour, two hours, yeah, and maybe at least have three Essentials for maths per week … because time really, it’s just time … and the time, it’s the time, the problem is time, there’s less time.

Reducing the teaching pace aids understanding: Essentials “don’t go as fast as … the tutorials [so] it helps to understand a lot more” [Lerato]. If learning is to be undertaken with understanding, students must be permitted to learn at their own individual paces. Students must, however, understand that the onus of preparing for lessons and the need for post lesson effort is theirs: “you should attempt the homework before coming to class so you can ask questions” [Ravi].

Students from ML do not learn effectively by watching their teacher “just solve questions quick-quick” in a way that is “just too fast for us to comprehend” [Thandi]. Lindiwe explained that ML students need time to develop algebraic knowledge because “it’s something new to us people who didn’t do pure maths. We really need time to learn.” ML students would rather take “five minutes of time to solve the question and then we can solve it together” [Ravi].

Supportive learning approaches (Daniels & Jooste, 2018) and pace management strategies must be sought, so that cumulative learning is not compromised (Pritchett & Beatty, 2015). Where future learning is built on current learning, any compromise in understanding may be detrimental to knowledge development. It may be that this very circumstance has occurred in the learning history of ML students, leading them to currently recognise the importance of pace.

Creative solutions may be required to maintain a balance in the pace of learning in the classroom; whereas ML students may require more time on task, other students “get bored at times because now they already know what is happening” [Xola]. The development of pertinent video material to explain new concepts slowly and clearly in a step-by-step manner may be beneficial, as ML students can choose to watch these as often as they need to. They may pause such material and make notes where necessary and in so doing, grasp important concepts, which may be lost when learning at a brisker pace in the classroom.

A second aspect that ML students perceived could be addressed to improve their learning was assistance to feel confident in their understanding of the content. When “different methods” [Lindiwe, Ravi] of teaching are employed, “it gets to confuse some of us” [Lindiwe], and ML students may lose confidence in their ability to understand the algebraic content. ML students want to feel confident that those teaching them understand the content. Neo described how a tutor4 “wasn’t understanding the question, so he was busy confusing us more than we already were.” Paradoxically, however, Neo later advised future ML students to “go to tutors more often”, which Bianca and Alile supported, with Alile’s proviso that students “find tutors that can help them.”

Some participants described feeling comfortable when HE teaching mirrored that of school: “I feel like when you’re in a tutorial, it’s … a normal class, … like at high school where they [the teachers] get to explain everything step by step” [Lindiwe]. It may be that learning is improved when the teaching of new topics is simplified. “Take it step-by-step” [Alile], “break down the topic” [Thandi], ML students are possibly still building their foundational algebraic knowledge, and they may want to understand the why and how of the algebraic concepts that they are learning so that they are able to develop a firm fundamental understanding on which further knowledge can be built. By explaining basic concepts and equipping students to determine why one process follows from another, students could be more likely to develop algebraic knowledge.

Thandi hoped, “you can just be revising towards the exams … maybe you have clues about the test”, while Bianca suggested maybe “a test every Friday of that week that actually encourages students to work?” These suggestions are nostalgic of high school teaching practices that have equipped ML students to feel confident about their learning and understanding at school.

Within the learning of new knowledge, curiosity could evoke the need for clarification questioning. Students who are learning new concepts that other students have already assimilated are, however, likely to feel reticent about voicing clarifying questions or uncertainties in large public groups. In lectures, “I’d like to ask questions, but … there’s so many people it makes you nervous … with the tutorials you’ve actually got the opportunity to ask questions” [Sergio]. Strategies that enable students to seek and gain clarity should be implemented in the classroom. Students must also be aware that if they wish to receive immediate clarification, they need to be bold enough to ask questions, otherwise they should write their question down so they can receive clarification later. Dedicated

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4 Tutors are students (currently pursuing undergraduate degree studies) who previously completed the algebra course. The FP contracts tutors to support the current cohort.
consultation times can be scheduled where underprepared students can seek clarification if there is not enough time in the teaching timetable.

Personal Aspects Perceived to Influence the Development of Algebraic Knowledge

The findings reported in this section were largely elucidated from participants’ responses to the fourth and fifth interview questions. Students may recognise their personal responsibility in the development of their algebraic knowledge in HE studies: “in school the teachers are always after you, but in lectures, ... it’s really up to you. If you want to pass, it’s up to you to do the work” [Sergio]. “Students should be determined in what they are doing, they should know the reason why they’re at school and they should work according to the pace” [Thandi]. While ML students may acknowledge their role, they may not know what is required: “I wanted to know, in order for me to get better, to get there, to know maths, what do I need to do?” [Lindiwe].

Students who grasp the value of learning algebra and realise “you’re going to use it in first year” [Neo] are less likely to engage in obstructive behaviours, some of which were identified by ML students. Students “don’t listen, or even though we listen, we are not actually listening” [Xola], being “on their phones ... not concentrating” [Alile] or “just copying and pasting” quiz answers. Participants acknowledge that the outcome of such behaviour is that students “don’t actually learn” [Alile].

Self-belief may influence the development of algebraic knowledge, as Neo explained:

Everything starts in your mind, so once you hear everyone say that: ‘Oh my gosh! It’s hard!’ it will always stay in your subconscious mind that it’s hard, and every time you actually want to practice or everything, it will just prevent you from answering the question. So if you tell yourself: ‘Ok, it is possible and I can do it’, then firstly it will relieve stress and secondly whenever you practice, it will actually empower you and motivate you ... so it starts in your mind.

Lerato described how this explanation applied to her: “I was scared before the test, like the preparation wasn’t as hard. But now ... I’m going to study a little bit more and I’m going to try to be more attentive when it comes to the tests and the tutorials.” It may be that Millennials in general, and ML students in particular, need to understand why they should learn algebra, and they want to see results before choosing to redouble their efforts. Students may know when others are working or not, in the same way they are likely to admit when they are industrious in their learning or not. Perhaps more than other students, ML students are encouraged when achieving success in algebra assessments, as they are unlikely to have experienced this often in the past. Opportunities that enable ML students to experience success may improve the desire to attend classes and learn algebra.

Kagiso believed attendance to be an important expeditor of algebraic development “for students like me who did Maths Lit and obviously need that help.” Lerato would advise new students, “firstly, you need to attend”, and Ravi shared that part of his success in algebra was that “I went for all my tutorials and I went for all my Essentials and I went for all my lectures for maths only.” Themba stressed the importance of active attendance and explained “it will not be helpful if the students attend the Essentials while she or he doesn’t make an effort for listening.” ML students noticed that not all students persisted in attending classes: “in the first few weeks, everyone was there [Essentials] and then during the mid-semester no one was there” [Alile]. Lectures “the same, in the beginning you see it’s packed and then ... it falls away slightly, slightly” [Ravi]. Kagiso believed all classes “should be compulsory in a way that you take register.” Participants expressed astonishment that others found it acceptable to miss algebra classes, and wondered if students missed classes because they were “overly confident or because they don’t care” [Alile]. It is encouraging that participants recognise the importance of attendance, as attendance may be linked to motivation (Sibanda et al., 2015) and academic performance (Zhu et al., 2019). Active class attendance is likely to be important for ML students given that they are learning a new subject and have previously provided evidence of the struggle to learn algebra independently. A positive, welcoming learning environment may encourage regular attendance.

ML students may employ learning strategies similar to those of other students when learning mathematics (Baumgartner et al., 2018). Themba believed “maths will be simple if I do work hard” and Sergio explained: “I’ve needed to work so much harder, because I need to know.” Working hard, when clarified, equated to practice. Lerato’s studying strategy was to “practice. Like, I go over what we did in class and, you know, practice over and over again. It’s just, I practice a lot.” Ravi advised future ML students to “practice, practice, practice. You have to practice. Practice it ... I practiced.” Sergio also emphasised rehearsal, saying “I’ll redo it and redo it and redo it,” clarifying that “the more we practiced it, the more I got it.” Sergio elaborated that he “put more time into” work he thought “would be more difficult” and tried to “really know what I’m doing with one thing so I can apply it somewhere else.”

Participants realised that time management was their responsibility: “at the age and place we are ... we should know how to do that for ourselves” [Sergio] although some ML students struggled to manage time effectively. Xola confided “in the first few weeks, I wasn’t on point, I wasn’t up to date ... If I could go back in time, I would
do that assignment in time and submit it in time, um so that I can have enough time to study” algebra. Neo agreed that “the worst thing to do is leaving anything to the last minute,” because “it puts you under pressure and pressure is not good and then you’re gonna make mistakes.” Lerato emphasised the timeliness of seeking help, suggesting, “ask them to help you immediately instead of waiting till the last minute.”

While rehearsal strategies consolidate learning (Hattie & Donoghue, 2016), students who are new to algebra should be mentored in learning strategies other than rehearsal. Ravi recalled organising his learning when his study-group “made one whole sheet, it was A4 just formulas … those formulas really helped us”. Ravi was the only participant to mention a learning strategy other than rehearsal. A range of learning strategies can advance the development of algebraic knowledge. In addition to encouraging students to attempt new learning strategies, a range of suitable resources could be made available to support this learning.

Anxiety relating to learning mathematics or writing mathematics assessments may hamper ML students’ ability to learn or demonstrate their algebraic knowledge development. Lerato was “scared of doing maths because I had done maths before and I didn’t do as well”, which resulted in her being “very nervous when I got into the test”. Consequences of anxiety include that “when you’re really afraid of your first paper, you tend to forget things you should know” [Alile] resulting in “very careless mistakes” [Sergio], leading one to start believing “I’m going to lose marks anyway” [Sergio]. Longer-term consequences may be that students, like Lerato, begin “procrastinating my studying for maths” because she “didn’t understand half the stuff that was happening” and start thinking, “I’m not good at it anyway, so what’s the point?” Alile advised anxious students to “have that calm sense of feeling, not like stressing ‘cos it’s maths again,” because “stressing will cause mistakes” [Kagiso]. Teachers and students should explore and implement strategies that reduce anxiety relating to learning mathematics and writing mathematics assessments so that ML students are able to learn and demonstrate accurate evidence of that learning.

DISCUSSION

As the literature predicted, learners chose ML for different reasons and at different times in their schooling (Sofowora, 2014; Spangenberg, 2012). The prior content knowledge of ML students is thus disparate, and dichotomous from that of other students (Baumgartner, 2016) and determining where to begin new learning may be one aspect to address in order to improve the development of algebraic knowledge.

The main aspects that ML students perceived were beneficial their algebraic knowledge development were support offerings, and this finding may be generalised for the learning of underprepared students in any mathematics course. Contrary to Dietrich (2017), and Machaba and Du Plooy (2019), participants did not encounter feelings of academic inferiority in the algebra course or the FP, which may be ascribed to attributes of Millennials or the institution’s culture. Rather, ML students made friends easily (Spangenberg, 2012) and harnessed the support offered through friendships with MKOs to develop algebraic knowledge (Vygotsky, 1978). Students perceive knowledge development to be enhanced when a course is well structured and accompanied by a teaching approach that supports learning and understanding. Institutional support, such as the provision of reteaching strategies, benefits underprepared students who must develop foundational knowledge while simultaneously acquiring current course content knowledge.

One of the key aspects that this study found could be addressed to influence the development of academic knowledge was the pace of teaching. This study reiterates findings of other studies (e.g., Engelbrecht et al., 2010; Pritchett & Beatty, 2015) that the teaching pace informs learning, especially of underprepared students. Algebraic knowledge development may be improved when sufficient learning opportunities are provided at a pace that students can sustain. A slower pace may necessitate more contact time, which students may welcome. Collaborative learning strategies outside the class may be effective where timetabling or budgetary constraints preclude additional class time, and the use of technology may provide solutions in this regard. Moreover, students may require assistance to learn how to learn algebra, or to feel confident that learning has occurred. Strategies that enable students to assess their own knowledge development should be shared so that students may gain autonomy in this aspect.

Students acknowledge that if they hope to develop their knowledge of algebra and other topics, they must take personal responsibility for their learning. Personal responsibility includes attending classes (Sibanda et al., 2015) and utilising resources that enable learning (Botha, 2012). Students may struggle to adapt to aspects of responsibility that have previously been institutionally, rather than personally managed, such as the use of multiple learning strategies, sound time management and anxiety management (Zhu et al., 2019). Interventions embedded within courses to equip students to take ownership of these responsibilities may be required and may be time consuming. Extended rewards when gaining personal responsibility may include the value of learning the subject, and the ability to more easily consolidate learning.

A possible strategy that combines aspects of an algebra course that ML students (1) perceive to benefit their algebraic knowledge, (2) perceive could be improved to benefit their algebraic knowledge, and (3)
includes personal aspects that influence the development of algebraic knowledge may not be unattainable. A course developer could create resources that commence from introductory algebra principles and connects with current content to reduce the gap between prior algebraic knowledge and current expect understanding. Such content should be accessible for effective relearning at a time and pace that is convenient to the student and delivered in manageable portions that allow students to develop and construct knowledge without feeling overwhelmed. Students should be encouraged to work with others in and outside of the classroom. Such a strategy focuses on relearning rather than reteaching, encouraging social and individualised learning to develop a learning practice and culture that can be implemented in all HE studies. The caveat, though, is that students must properly engage with these resources and practise this method of learning.

The findings of this study are important in the light of massification in HE, where students are more often underprepared for tertiary studies in general and mathematical studies in particular. This study focused on the perceptions of ML students, who are South Africa’s most underprepared students for HE studies in mathematics. Their perceptions have not previously been examined in literature, but may create a comparison point for other qualitative studies that have sought to understand the development of knowledge of underprepared students in HE. Until we hear what underprepared students require in order to progress the development of their academic knowledge, we as teachers and researchers cannot be sure that what we provide accommodates those needs.

Limitations and Future Study Implications

While data were managed ethically to limit researcher bias and participant bias, such biases may have arisen. The perceptions on the development of algebraic knowledge of poorly attending ML students may be different, and this could be a topic for future studies. Formal data were collected from a single student cohort at only one private HEI, and students enrolled during a different admission period or attending a different HEI may voice different perceptions. Again, this is an opportunity for further study in this under-researched area.

CONCLUSION

This study has provided an opportunity to learn more about ML students attending a FP algebra course. This pioneering study reveals the voice of this cohort and their perceptions to developing algebraic knowledge in a post-secondary setting. Through understanding the mathematical background of the students, and hearing their perceptions on how the learning and teaching of algebra helps or hinders their development of algebraic knowledge, various insights may be gained. Improved support strategies and resources may be developed. Such narrations may inform the decisions of current learners wondering whether or not to choose ML during their secondary schooling years.

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REFERENCES

Baumgartner, W. L. (2016). The development of a strategy for Mathematical Literacy learners for selected undergraduate studies (Master’s dissertation, University of Johannesburg, Johannesburg, South Africa). https://ujcontent.uj.ac.za/vital/access/manager/Repository/uj:21287

Baumgartner, W. L., Spangenberg, E. D., & Jacobs, G. J. (2018). Contrasting motivation and learning strategies of ex-mathematics and ex-Mathematical Literacy students. South African Journal of Higher Education, 32(2), 8-26. https://doi.org/10.20853/32-2-941

Botha, J. J. (2012). Exploring Mathematical Literacy: The relationship between teachers’ knowledge and beliefs and their instructional practices (Doctoral thesis, University of Pretoria, Pretoria, South Africa). https://repository.up.ac.za/bitstream/handle/2263/28984/Complete.pdf?sequence=8&isAllowed=y

Bowles, A., Fisher, R., McPhail, R., Rosenstreich, D., & Dobson, A. (2014). Staying the distance: Students’ perceptions of enablers of transition to higher education. Higher Education Research and Development, 33(2) 212-225. https://doi.org/10.1080/07294360.2013.832157

Burnard, P. (1991). A method of analysing interview transcripts in qualitative research. Nurse Education Today, 11(6), 461-466. https://doi.org/10.1016/0260-6917(91)90009-Y

Case, J., Marshall, D., & Grayson, D. (2013). Mind the gap: Science and engineering education at the secondary-tertiary interface. South African Journal of Science, 109(7/8), 1-5. https://doi.org/10.1590/sajs.2013/20120113

Daniels, A. D., & Jooste, K. (2018). Support of students by academics in a nursing foundation programme at a university in the Western Cape. Curationis, 41(1), 1-7. https://doi.org/10.4102/curationis.v41i1.1927
Dietrich, B. (2017). Experiences of first year, four year degree programme students; an ethnography of the perceived costs and benefits (Honours dissertation, University of Cape Town, Cape Town, South Africa). http://www.knowledgeeco-op.uct.ac.za/sites/default/files/image_tool/images/155/246_SAEP_cost-benefit%20of%20bridging%20year_Dissertation.pdf

Engelbrecht, J., Harding, A., & Phiri, P. (2010). Are OBE-trained learners ready for university mathematics? Pythagoras, 72, 3-13. https://doi.org/10.4102/Pythagoras.v0i72.16

Hattie, J. A. C., & Donoghue, G. M. (2016). Learning strategies: a synthesis and conceptual model. Science of Learning, 1, 1-13. https://doi.org/10.1038/npjscilearn.2016.13

Kirby, N. F., & Dempster, E. R. (2018). Alternative access to tertiary science study in South Africa: Dealing with ‘disadvantage’, student diversity and discrepancies in graduate success. In C. Agosti & E. Bernat (Eds.), University pathway programs: Local responses within a growing global trend (pp. 85-106). Springer International Publishing AG. https://doi.org/10.1007/978-3-319-72955-5_5

Koonin, M. (2014). Validity and reliability. In F. Du Plooy-Cilliers, C. Davis, & R-M. Bezuidenhout (Eds.), Research Matters (pp. 252-261). Juta.

Machaba, F. M. (2018). Pedagogical demands in Mathematics and Mathematical Literacy: A case of Mathematics and Mathematical Literacy teachers and facilitators. Eurasia Journal of Mathematics, Science and Technology Education, 14(1), 95-108. https://doi.org/10.12973/ejmste.78243

Machaba, F. M., & Du Plooy, M. (2019). Mathematics and Mathematical Literacy on the career podium – sharing gold? African Journal of Research in Mathematics, Science and Technology Education, 23(3), 363-375. https://doi.org/10.1080/18117295.2019.1694782

Mampane, R., & Bouwer, C. (2011). The influence of township schools on the resilience of their learners. South African Journal of Higher Education, 31(1), 114-126. https://doi.org/10.15700/saje.v31n1a408

McKay, T., Mafanya, M., & Horn, A. C. (2018). Johannesburg’s inner city private schools: The teacher’s perspective. South African Journal of Higher Education, 38(3), 1-11. https://doi.org/10.15700/saje.v38n3a1557

Mhakure, D., & Mokoena, M. A. (2011). A comparative study of the FET phase Mathematical Literacy and Mathematics curriculum. US-China Education Review, 8(3), 309-323. http://hdl.handle.net/11427/3347

Miles, M. B., Huberman, A. M., & Saldaña, J. (2013). Qualitative Data Analysis: A methods sourcebook (3rd ed.). Sage.

Ogude, N. A., Meyer, I. J., Mwambakana, J., & Mthethwa, N. E. (2019). Can extended curriculum programmes be improved through engagement with students using appreciative inquiry? South African Journal of Higher Education, 33(4), 219-236. https://doi.org/10.20853/33-4-2835

Pai, H-H., Sears, D. A., & Maeda, Y. (2015). Effects of small-group learning on transfer: A meta-analysis. Educational Psychology Review, 27(1), 79-102. https://doi.org/10.1007/s10648-014-9260-8

Potgieter, M., Harding, A., Kritzinger, Q., Somo C., & Engelbrecht, J. (2015). Reflections of science students on their experiences of an academic development programme in South Africa. South African Journal of Higher Education, 29(1), 108-131. https://doi.org/10.20853/29-1-446

Pritchett, L., & A. Beatty, A. (2015). Slow down, you’re going too fast: Matching curricula to student skill levels. International Journal of Educational Development, 40, 276-288. https://doi.org/10.1016/j.ijedudev.2014.11.013

Saldana, J. (2016). The coding manual for qualitative researchers (3rd ed.). Sage.

Shushok Jr., F. & Kidd, V. (2015). Millennials in higher education: As students change, much about them remains the same. In J. C Wade, L. I. Marks, & R. D. Hetzel (Eds.), Positive psychology on the college campus (pp. 27-56). Oxford University Press. https://books.google.co.za/books?hl=en&lr=&id=vBqCCQAAQBAJ&oi=fnd&pg=PP1&dq=%22millennials%22+%22higher+education%22+Wade&ots=AnBqzm6_rC&sig=_o3x6GJLdnIPzDMAtqDuQPnBs&redir_esc=y#v=onepage&q=%22millenials%22%20%20higher%20education%22%20Wade&f=false

Sibanda, L., Iwu, C. G., & Benedict, O. H. (2015). Factors influencing academic performance of university students. Demography and Social Economy, 2(24), 103-115. https://doi.org/10.15407/dse2015.02.103

Sofowora, S. O. (2014). Anxiety and lack of motivation as factors affecting success rates in bridging mathematics. (Master’s dissertation, University of South Africa, Pretoria, South Africa). http://uir.unisa.ac.za/handle/10500/19929

South Africa. Department of Basic Education. (2003). National curriculum statement Grades 10-12 (General): Mathematics. https://go-pdf.online/national-curriculum-statement-grades-10-12-general.pdf

South Africa. Department of Education. (2008). National curriculum statement Grades 10-12 (Learning programme guidelines): Mathematical Literacy.
South Africa. Department of Basic Education. (2011a). National curriculum statement, curriculum and assessment policy statement Grades 10 - 12: Mathematics. https://www.education.gov.za/Curriculum/CurriculumAssessmentPolicyStatements(CAPS)/CAPSFET.aspx

South Africa. Department of Basic Education. (2011b). National curriculum statement, curriculum and assessment policy statement Grades 10 - 12: Mathematical Literacy. https://www.education.gov.za/Curriculum/CurriculumAssessmentPolicyStatements(CAPS)/CAPSFET.aspx

Spangenberg, E. D. (2012). Thinking styles of Mathematics and Mathematical Literacy learners: Implications for subject choice. Pythagoras, 33(3), 1-12. https://doi.org/10.4102/pythagoras.v33i3.179

Vygotsky, L. S. (1978). Mind in society. Harvard University Press.

Zander, L., Brouwer, J., Jansen, E., Crayen, C., & Hannover, B. (2018). Academic self-efficacy, growth mindsets, and university students; integration in academic and social support networks. Learning and Individual Differences, 62, 98-107. https://doi.org/10.1016/j.lindif.2018.01.012

Zhu, L., Huang, E., Defazio, J., & Hook, S. A. (2019). Impact of the stringency of attendance policies on class attendance/participation and course grades. Journal of the Scholarship of Teaching and Learning, 19(2), 130-140. https://doi.org/10.14434/josotl.v19i2.23717

http://www.ejmste.com