Nematic phase of the two-dimensional electron gas in a magnetic field

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The two dimensional electron gas (2DEG) in moderate magnetic fields in ultra-clean AlAs-GaAs heterojunctions exhibits transport anomalies suggestive of a compressible, anisotropic metallic state. Using scaling arguments and Monte Carlo simulations, we develop an order parameter theory of an electron nematic phase. The observed temperature dependence of the resistivity anisotropy behaves like the orientational order parameter if the transition to the nematic state occurs at a finite temperature, \( T_c \approx 65mK \), and is slightly rounded by a small background microscopic anisotropy. We propose a light scattering experiment to measure the critical susceptibility.

Recently, two of us introduced the concept of liquid crystal phases of the two-dimensional electron gas (2DEG) in large magnetic fields, as an extension of earlier work on high temperature superconductors. Electronic liquid crystal phases are quantum mechanical analogues of classical liquid crystals, and are predicted to be a generic feature of strongly correlated fermionic systems. In the case of the 2DEG the competing effects of repulsive (Coulomb) interactions and the quenching of the kinetic energy of electrons in Landau levels lead naturally to the existence of such phases. Pursuing this analogy, the phases of a 2DEG in order of increasing symmetry breaking, were characterized as: (a) isotropic liquids, (b) nematic liquids, (c) smectic liquid crystals, and (d) insulating crystals. The fluid character of states (a) and (b) is obvious, as they are translationally invariant. The smectic breaks translational symmetry in only one direction, and so is also a fluid. Insulating crystals break the translational symmetry down to a discrete subgroup, such that there are an integer number of electrons per unit cell.

Among the isotropic liquids are the various quantum Hall fluids, while insulating crystals are simply the Wigner crystal and its generalizations. A smectic (or stripe) phase has been found in Hartree-Fock calculations, presumably accurate in high Landau levels, which provides a qualitative picture of a smectic state. Moreover, a recent exact diagonalization study of a system with 12 electrons in the \( N \geq 2 \) Landau levels found results consistent with a smectic (stripe) ground state, up to yet poorly understood finite size effects. Wigner crystals and “bubble phases” (crystalline states with several particles per unit cell) have also been found as Hartree-Fock variational states. In addition, a predicted charge-density-wave instability of the smectic phase at Hartree-Fock level, \( J_{\parallel} \) (with a rectangular unit cell. However, a microscopic theory of the nematic phase does not presently exist.

The low energy physics of a quantum smectic can be understood in terms of a theory of quantum fluctuations of the smectic order parameter, including the phase transition to the stripe crystal phase. The long-distance behavior of a quantum nematic phase is completely determined by symmetry and by the associated Goldstone modes. Both smectic and nematic states break rotational symmetry, and as such transport properties of both states is anisotropic. In Ref. \( \text{[1]} \) we argued that while the smectic and nematic states are both natural candidates to explain the anisotropy observed in recent experiments on 2DEG in high mobility AlAs–GaAs heterostructures, at least at finite temperature, there are strong reasons to favor the nematic. The smectic has an infinite conductivity, at least in one direction, whereas the measured (anisotropic) conductivity has a finite \( T \rightarrow 0 \) limit. In addition, the data shows a pronounced temperature dependence of the resistivity, consistent with the existence of a finite temperature phase transition; since the energy of a dislocation is still finite even for a Coulomb interaction, the smectic always melts at any non-zero temperature. In the present paper we explore the universal properties of the 2D electron nematic.

The experiments of Refs. \( \text{[1]} \) and \( \text{[2]} \) have revealed the existence of regimes of magnetic fields in which the 2DEG exhibits characteristics of a compressible fluid with an unexpectedly large and temperature-dependent anisotropy in its transport properties. This occurs when the Landau level index \( N \) lies in the range \( 2 \leq N \leq 6 \). The large anisotropy is seen only in high mobility samples. In the same samples, similar behavior has also been seen in the first \( N = 1 \) Landau level when the magnetic field is tilted \( \text{[3]} \), while in the lowest \( N = 0 \) Landau level, a large number of fractional quantum Hall (FQH) states are observed, including a subtle FQH state at \( \nu = 5/2 \). To summarize the experimental facts: (i) Measurements on square samples show that, as the temperature is lowered below 100 mK, the longitudinal resistivity \( R_{xx} \) grows very rapidly while \( R_{yy} \) becomes smaller; as \( T \rightarrow 0 \), their ratio approaches a constant in the range \( 100 < R_{xx}/R_{yy} < 2,350 \) where \( x \) and \( y \) are orthogonal lattice directions. (ii) In Hall bars, \( R_{xy} \) is essentially temperature independent while \( R_{xx} \) increases by a factor of \( 5-10 \) as the temperature is lowered below 100 mK. (iii) The compressible (dissipative) regime occupies a finite
range of magnetic fields $\Delta B$, centered around the middle of the partially filled Landau level, and unlike the conventional transition between plateaus, $\Delta B$ does not shrink as the temperature is lowered. (iv) In the same range of magnetic fields the Hall resistance varies continuously with the magnetic field. (v) At the peak, the conductivity $\sigma_{yy}$ is typically of the order of $e^2/h$ (see below). (vi) Applying an in-plane magnetic field tunes the anisotropy through zero, and reverses the roles of the $x$ and $y$ directions. (viii) In the region of the resistivity peak, the $I$-$V$ curves are highly non-linear but do not show any threshold (depinning) behavior. (ix) There are reentrant integer quantum Hall plateaus symmetrically located for magnetic fields outside the compressible regime. (x) The anisotropy has not been reported in lower mobility samples which show instead the (expected) phase transition between quantum Hall states.

A natural interpretation of the experiments is that, in regimes in which interactions dominate over the effects of disorder, instead of the expected transition between plateaus in the middle of the partially filled Landau level, the 2DEG forms a compressible anisotropic fluid. Because a continuous (rotational) symmetry cannot be spontaneously broken in D=2, for such an anisotropy to be observable the sample must have a small background microscopic anisotropy whose effect is greatly amplified at low temperatures by the collective properties of the state. Specifically, we will show that the experimentally observed temperature dependence of the anisotropy can be understood as evidence for a finite temperature Kosterlitz-Thouless (KT) transition from a two-dimensional nematic to an isotropic fluid, which is rounded by a symmetry breaking field representing the effects of the background anisotropy. We present an analysis (Fig. 1) of the experimental data of Lilly et al., and a fit with the results of a Monte Carlo simulation of a model of a classical nematic in a symmetry breaking field (Fig. 2). The results strongly support our earlier claim that the anisotropic transport occurs where the 2DEG is in a nematic phase (at least at finite temperature). They also give some indirect support to the further conjecture made in Ref. [4] that there is a direct transition as a function of $B$ from the nematic state to an insulating stripe crystal phase, which was identified with the innermost of the reentrant quantum Hall liquids. (See, also Ref. [5]).

In 2D, the nematic has only quasi-long-range order; its finite temperature transition to a disordered liquid can be described by the two-dimensional classical XY-model with a director order parameter. Such a description should fail at (very) low temperatures where quantum fluctuations (and/or quenched disorder) become important. Since the order parameter of the nematic state is a director field, $\vec{m}(\vec{r})$, it is periodic under rotations by $\pi$, and has the form $m_x(\vec{r}) + im_y(\vec{r}) = \exp(2i\theta(\vec{r}))$. The classical Hamiltonian of this system is thus

$$H = -J \sum_{\vec{r},\mu=x,y} \cos(2\Delta_p \theta(\vec{r})) + h \sum_{\vec{r}} \cos(2\theta(\vec{r}))$$  (1)

where, for simplicity we have used a square lattice of unit spacing whose sites are labelled by the lattice vectors $\vec{r}$. In Eq. (1) we have use the notation $\Delta_p \theta(\vec{r}) = \theta_{x+\hat{e}_x} - \theta_{y+\hat{e}_y}$, where $e_{\mu}$ is a unit vector along the direction $\mu = x,y$, and $J$ is the stiffness, the energy required to rotate two nearby regions by a small angle. The quantity $h$ breaks rotational symmetry explicitly. It represents the effects of a background symmetry breaking field, such as the anisotropy and/or the effects of a parallel magnetic field.

Because two is the lower critical dimension for continuous symmetry breaking, for $0 < T < T_c$ the system is controlled by a line of critical points. In this range of $T$, where classical Goldstone excitations of the nematic (“spin waves”) dominate, the correlation function $G(\vec{r}) = \langle \exp(2i\theta(\vec{r}) - \theta(\vec{0})) \rangle$ of the order parameter has power law behavior, $G(\vec{r}) \sim 1/|\vec{r}|^\eta(T)$, with $\eta = 2T/\pi\kappa(T)$ and a divergent susceptibility. Here $\eta \to 1/4$ as $T \to 0$ and $\kappa(T_c) = 4T_c/\pi$. In the presence of a symmetry breaking field $h$, the order parameter behaves like $m = (\exp(2i\theta(\vec{r})) \sim |h|^{1/\delta}$, where $\delta = 4/\eta - 1$ and $\delta(T_c) = 15$. For $T > T_c$, the correlation length is finite, and diverges at $T_c$ like $\xi \sim \exp(A/\sqrt{T - T_c})$, where $A$ is a (non-universal) constant. At finite $h$ the correlation length is always finite, and the singularities of the KT transition get rounded. In this regime, even a very small symmetry breaking field induces a very large expectation value of the order parameter. For $T > T_c$, $m \sim \chi(T)h$, where $\chi(T) \sim \xi^{7/4}$ is the susceptibility.

How is this thermodynamic transition, which describes the breaking of rotational invariance, related to the transport properties? On general grounds one expects that near a phase transition, quantities which transform in the same way under the symmetry should be related, even if one is a transport coefficient and the other a thermodynamic property. In particular the combination of resistivities $\zeta = (\rho_{xx} - \rho_{yy})/(\rho_{xx} + \rho_{yy})$ transforms like the order parameter $m = (\exp(2i\theta(\vec{r}))$. It should be related to the order parameter through an odd analytic function $\zeta = f(m)$. Therefore, near $T_c$, if the symmetry breaking is small, the linear approximation $f(m) \propto m + O(m^3)$ should be reasonably accurate.

We can determine if the 2DEG is in a nematic phase by analyzing the temperature dependence of the resistivity in terms of the temperature dependence of the order parameter of the nematic in the presence of a symmetry breaking field. What is needed is the function $m = \Phi(T,h)$, the equation of state, which we computed by a Monte Carlo simulation of the classical XY model of Eq. (1). Notice that we relate $m$ to a local (intensive) property such as the resistivity instead of to the resistance, which is extensive and sensitive to significant finite size effects. However, the experimental data gives the resistances $R_{xx}$ and $R_{yy}$ as functions of temperature, not the resistivities. Thus, in order to fit the data, we extracted the resistivities from the measured resistances, using a method discussed below, with the result shown in Fig. 4.
We have done this by means of a Monte Carlo Metropolis simulation of the 2D XY model of Eq. 1 on square lattices of sizes 40 \times 40 through 120 \times 120, for the range of symmetry breaking fields 0.01J < h < 0.5J, and for a wide range of temperatures (see below). In Fig. 2 we show our Monte Carlo data for the order parameter as a function of temperature for h = 0.05J. For this range of symmetry breaking fields, we find that for L = 100 the finite size effects on the order parameter are very small. We have fitted the data by assuming that ζ = (ρ_{xx} - ρ_{yy})/(ρ_{xx} + ρ_{yy}) is actually equal to the order parameter m. Having done so, we fitted the data by finding the best value of J that fits the data for a given value of h, and then changed h to get the best fit.

The classical nematic does indeed explain the temperature dependence; the data is consistent with a thermodynamic Kosterlitz-Thouless transition at T_c(h = 0) = 0.88J \sim 65mK slightly rounded by a background anisotropy field of magnitude h \sim 0.05J \sim 3.5mK, which is a very small energy scale. Notice that both the stiffness J \sim 73mK and h are much smaller than the Coulomb energy, although they are comparable with the gap in the ν = 5/2 (presumably paired) state, which hints a possible common origin. Below 55mK the fit is not as good. In this temperature range the XY order parameter is big (larger than 1/2) so there is no reason to expect ζ \sim m. However, ζ strikingly saturates (unlike m which shows the characteristic linear temperature behavior of classical spin wave theory) so the discrepancy may be indicating that quantum mechanical effects (or disorder) are important at low T.

FIG. 1. Resistivities ρ_{xx} and ρ_{yy} determined from the resistance data of Lilly et al. [11] [16] at ν = 9/2, after deconvoluting the effects of the geometry; ρ_{yy} is essentially constant for the entire range of temperatures, as in Hall bars. Inset: the function x(T).

One result of this analysis is that the resistivities for the square sample behave exactly in the same way as the resistances of the Hall bars. Given the low T values of ρ_{xx} and ρ_{yy} in Fig. 1, and the (large) measured value of the Hall conductance, one finds that the peak value of the conductivity is σ_{yy} = 1.12e^2/h and σ_{xx} = 0.11e^2/h. Notice that ρ_{xx} saturates rather sharply below 55mK and that both ρ_{xx} and ρ_{yy} approach non-zero (and different) values as T → 0. Thus, the 2DEG remains in an anisotropic compressible (metallic) state, down to the lowest accessible temperatures.

Having determined the temperature dependence of the resistivities we can now see if it is consistent with a (rounded) phase transition from a high temperature isotropic fluid phase to a low temperature nematic phase. We have done this by means of a Monte Carlo Metropolis analysis of the experiments strongly indicates that the 2DEG in large magnetic fields in clean samples has regimes where it behaves like a nematic fluid, an anisotropic metal. Such a metallic state should have a strong signature in polarized light scattering experiments. In particular, a nematic has long range fluctuations in the orientational order, which will cause the
polarization tensor correlation function (and the corresponding longitudinal and transverse susceptibilities $\chi_L$ and $\chi_T$) to have a singularity at $T_c$ (cutoff by the anisotropy). This effect is similar to critical opalescence but for orientational order instead of density fluctuations. For non-zero and small background anisotropy $h$, for $T < T_c$, $\chi_L$ is $\chi_L \sim h^{-\alpha}$, with $\alpha = 1 - 1/8$, where $\alpha = 14/15$ at $T_c$. For $T > T_c$, $\chi_L$ can be written in a scaling form as $\chi(h,T) \sim \xi^2 \phi_0(\xi^{15/8})$ where $\phi_0(0) = 1$ and $\phi_0(x) \sim x^{-\frac{14}{15}}$ as $x \rightarrow \infty$; as discussed above, $\xi(T) \sim \exp(-A/\sqrt{T})$, where $t = T/T_c - 1$. Thus, at fixed but small $h$, as the temperature is lowered, $\chi_L$ increases very rapidly to a maximum above $T_c$, with a crossover to a critical behavior $\sim h^{-\alpha(T)}$, where $\alpha(T) = 2(7 + t)/(15 + t)$ for $|h| \ll T$, and $\chi_L \sim (\pi T/2h)(1/(4\pi^2 J + h))$, for $T \ll |h|$. On the other hand, by Goldstone’s Theorem, $\chi_T = m/h$, where $m$ is the order parameter (Fig. 3). $\chi_L$ and $\chi_T$ are shown in Fig. 3. Although the specific heat of the 2DEG is very sensitive to the temperature, the resistivities from the measured resistances. It was observed recently that the 2DEG has an anisotropic but homogeneous redistribution in square samples. If one assumes the data from Hall bars and square samples can be understood to be a consequence of a large distortion of the nematic phase, while at $T_c$ there is a very weak essential singularity (rounded by the anisotropy).

Finally, we used the following procedure to extract the resistivities from the measured resistances. It was observed recently that much of the discrepancy between the data from Hall bars and square samples can be understood to be a consequence of a large distortion of the current distribution in square samples. If one assumes that the 2DEG has an anisotropic but homogeneous resistivity tensor, one can calculate the distribution of currents using the method of conformal mapping. One finds that, for an anisotropic homogeneous sample, with its principal axes aligned with the edges,

\[ R_{xx}/R_{yy} = g(x)/g(1/x) \] (2)

where $x = [L_y/L_x]/\sqrt{\rho_{yy}/\rho_{xx}} \equiv x(T)$ measures the aspect ratio $L_y/L_x$ and the ratio of resistivities, and $g(x)$ is given by

\[ g(x) = \ln(\theta_3(i\pi x/2) + k^2 \theta_2(i\pi x/2)/k^2 \theta_2(i\pi x/2)) \] (3)

$\theta_2(z)$ and $\theta_3(z)$ are theta-functions with modulus $k$

\[ k = 4q \prod_{n=1}^{\infty} (1 + q^{2n-1})^4 \] (4)

where $q = \exp(-2\pi x)$ is the period. Given $R_{xx}/R_{yy}$ at different temperatures, and using Eq. (2), we calculated the function $x(T)$ (shown in the inset of Fig. 2). At high $T$, $x(T)$ approaches a value somewhat larger than 1, but it is smaller than 1 at lower temperatures, and both the resistances and the resistivities show a crossing at some high temperature. This effect indicates that the sample is not homogeneous at large scales. A macroscopic inhomogeneity is equivalent to an effective aspect ratio $L_y/L_x \neq 1$, and by choosing a value for $L_y/L_x = 1.12$ we can make the ratio $\rho_{xx}/\rho_{yy}$ approach unity at high temperature.

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1. E. Fradkin and S. A. Kivelson, Phys. Rev. B 59, 8065 (1999).
2. S. A. Kivelson and V. J. Emery, Synthetic Metals, 80, 151-158 (1996), and references therein.
3. S. A. Kivelson, E. Fradkin and V. J. Emery, Nature 393, 550 (1998).
4. H. Fukuyama, P. M. Platzman and P. W. Anderson, Phys. Rev. B 19, 5211 (1979).
5. A. A. Koulakov, M. M. Fogler and B. I. Shklovskii, Phys. Rev. Lett. 76, 499 (1996).
6. R. Moessner and J. T. Chalker, Phys. Rev. B 54, 5006 (1996).
7. T. Stanescu, I. Martin and P. Phillips, cond-mat/9905116.
8. E. H. Rezayi, F. D. M. Haldane and K. Yang, cond-mat/9903258.
9. V. Oganesyan et. al., in preparation.
10. T. Jungwirth, A. H. MacDonald, L. Smereka and S. M. Girvin, cond-mat/9905355.
11. M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer and K. W. West, Phys. Rev. Lett. 82, 394 (1999).
12. R. R. Du, H. Stockman, D. C. Tsui, L. N. Pfeiffer and K. W. West, Solid State Comm. 109, 389 (1999).
13. D. R. Nelson and J. Toner, Phys. Rev. B 24, 363 (1981).
14. W. Pan et. al., cond-mat/9903166.
15. M. P. Lilly et. al., cond-mat/9903193.
16. M. P. Lilly, et. al., unpublished.
17. J. M. Kosterlitz and D. J. Thouless, J. Phys. C 6, 1181 (1973).
18. H. Fertig, Phys. Rev. Lett. 82, 3693 (1999).
19. N. Schultka and E. Manousakis, Phys. Rev. B 49, 12071 (1994).
20. The actual calculation of this function requires a theory of transport in the nematic state.
21. S. H. Simon, cond-mat/9903086.
22. C. Wexler, unpublished.
23. Z. Nehari, Conformal Mapping, Dover, New York (1952).
24. Even for small $m$, there is generally a proportionality constant relating $m$ to $\xi$, which we set equal to unity so as to reduce the number of free parameters.
25. This effect is absent in the data for higher Landau levels.