Review On: The Service Robot Mathematical Model

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Abstract

After nearly 30 years of development, service robot technology has made important achievements in the interdisciplinary aspects of machinery, information, materials, control, medicine, etc. These robot types have different shapes, and mainly in some are shaped based on application. Till today various structure are proposed which for the better analysis’s need to have the mathematical equation that can model the structure and later the behaviour of them after implementing the controlling strategy. The current paper discusses the various shape and applications of all available service robots and briefly summarizes the research progress of key points such as robot dynamics, robot types, and different dynamic models of the differential types of service robots. The current review study can be helpful as an initial node for all researchers in this topic and help them to have the better simulation and analyses. Besides the current research shows some application that can specify the service robot model over the application.

Keywords: Service robot, AGV, Two Wheel Differential, Three Wheel, Four Wheel, Four Wheel Skid Steering, Six Wheel, Ackermann AGV, Crawler Robot, Legged Robot, Snake Robot medical image recognition, character recognition.

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1. Introduction

As an inevitable extension of computer technology and modern comprehensive technology, home service robot technology known as service robots will achieve a breakthrough at an unprecedented speed. Service robots are entering various places from home to industries and are even recently accepted as home appliances. The history of robotics shows that the service robot chapter started with the Automated Guided Vehicle (AGV) [1] and then various applications bring the various shapes furnished with different structures. AGV’s were initially as the load carriers but are now being used for more applications [2-3]. Then, for a better analysis of a service robot, researchers must examine mathematical modeling for a better control algorithm [4] and dynamic simulation as necessary stages in the robotic domain and studies. This equation which called Dynamic modelling involves deriving equations that clearly describe the relationship between forces and motions in a system. The paper's main aim is to describe the different types of service robot models, especially the Automated Guided Vehicle (AGV), showcase work related to the individual types, and state their proposed dynamic models. This paper is set as follows initially, the service robot overview is discussed, then based on the different structures of service robots, review the different kinematic and dynamic models described. Finally, the comparison between the shapes and application are discussed at the end.

2. Service Robot Models

Currently, Service Robots are an umbrella term for various types of robots and consider as any type of robot that can give the service or in place of a human. One of the most famous groups of service robots, which initially started from industry but are now used as a part of home applications, are Automated Guided Vehicle (AGV). Initially, AGV was known as a complex machine representing a complete material handling solution, installed in many industries and widely used to efficiently...
transport materials through a facility [5], AGV can increase efficiency and productivity and reduce product damage and labour costs. The overall view of service robots shows that usually, a mobile robot has two subsystems, Drive Subsystem and Mechanical Subsystem, as shown in figure 1.

![Mobile Robot Diagram](image)

**Figure 1.** An electrical robot’s decomposition

As it is shown in figure 1, the Drive subsystem takes in voltages and controls the motor functions. The mechanical subsystem has dynamics and kinematics, which require various algorithms and has models that help determine steering or tracking of paths, i.e., positioning or orientation of the robot. Understanding the mathematical equation and relation between each part will help for better control and design of the robot. The reported shape and structure of Service robots can be named as Two Wheel Differential, three-wheel, four-wheel AGV, Four Wheel Skid Steering AGVs, Six Wheel AGVs, Ackerman AGV, crawler and legged robot as described in the following:

### 2.1 Two Wheel Differential

The differential drive is a two-wheel drive system with independent actuators for each wheel. The name refers to that the robot's motion vector is the sum of independent wheel motions. The robot's driving wheels are normally mounted on both sides, facing forward. [6]. Pure tracking in this type of robot is at least as good as the others and is easier to implement and understand [7]. Considering the mathematical equation, the different variables of the two-wheel differential robot are shown in Figure (2).

![Two Wheel Differential Robot Schematic](image)

**Figure 2.** Two Wheel Differential Robot Schematic.

As it is shown in Figure 2, The ICC, which stands for instantaneous centre of curvature, and the front of the robot have two freewheels, as shown in figure (2), with L being the difference between the two back motor wheels. ϕ is the angle at which the robot is turned from the centerline to the x-axis. R is the distance from ICC to the midpoint between the two back wheels. Considering the above variables, to follow the robot and find the robot velocity. The inputs required for the motion of a mobile robot are the linear velocity (V) and orientation [8-9]. The rate of change of position of robot along the x-direction is ẋ and that in the y-direction is ẏ are stated by

\[
\dot{x} = V\cos(\theta) \\
\dot{y} = V\sin(\theta)
\]  

and the angular velocity of the robot is given by

\[
\dot{\theta} = \omega = \frac{(v_r - v_l)}{L} 
\]  

Substituting the linear velocity V.

\[
\dot{x} = \frac{(v_r + v_l)}{2}\cos(\theta) \\
\dot{y} = \frac{(v_r + v_l)}{2}\sin(\theta)
\]

The velocity V of the robot in a fixed reference coordinate system is therefore given by

\[
V = \sqrt{\dot{x}^2 + \dot{y}^2}
\]

\[
V = \sqrt{\left(\frac{(v_r + v_l)}{2}\cos(\theta)\right)^2 + \left(\frac{(v_r + v_l)}{2}\sin(\theta)\right)^2} = \frac{v_r + v_l}{2}
\]

The individual velocities, Vr and Vl, can now be calculated using the equations:

\[
V_r = \left(V + \frac{L}{2}\omega\right)
\]

\[
V_l = \left(V - \frac{L}{2}\omega\right)
\]

The outputs, Vr and Vl then used to generate the output ẋ, ẏ and ω using the above equations [10-13].

The following equations (7) are the forward kinematic mathematical equations stated by the researchers, which in them the Angular momentum of the robot presented

\[
\omega(R + 1/2) = v_r
\]

\[
\omega(R - 1/2) = v_l
\]

Vr is the velocity of the right wheel while vL is the velocity of the left wheel, then from (7): the value (R distance from ICC) and (ω Angular momentum) defined as shown in equation (7)

\[
R = 1/2 \times (v_r + v_l)/(v_r - v_l)
\]

\[
\omega = (v_r - v_l)/v_r
\]

The forward kinematics equation (9) (concerning time):
\[
\frac{dx}{dt} = m(t) \cos(\theta(t)) \\
\frac{dy}{dt} = m(t) \sin(\theta(t))
\]

Then the change of direction with respect to time is the same as the angular rate \( \omega \). Therefore,

\[
\frac{d\theta}{dt} = \omega = (v_r - v_l)/1
\]

Integrating the equation(4) gives a function for the robot's orientation with respect to time. The robot initial orientation \( \theta(0) \) is also replaced by \( \theta_0 \):

\[
\theta(t) = (v_r - v_l)t/1 + \theta_0
\]

\[
\frac{dx}{dt} = [(v_r + v_l)/2] \cos(\theta(t))
\]

\[
\frac{dy}{dt} = [(v_r + v_l)/2] \sin(\theta(t))
\]

Taking the initial positions to be \( x(0) = x_0 \), and \( y(0) = y_0 \) to get:

\[
x(t) = x_0 + 1/2(v_r + v_l)/(v_r - v_l)
\]

\[
sin((v_r - v_l)t/1 + \theta_0) - sin(\theta_0)
\]

\[
y(t) = y_0 - 1/2(v_r + v_l)/(v_r - v_l)
\]

\[
[cos((v_r - v_l)t/1 + \theta_0) - cos(\theta_0)]
\]

As \( 1/2(v_r + v_l)/(v_r - v_l) = R \), the robots turn radius, and \( (v_r - v_l)/1 = \omega \).

The above equations can be reduced to:

\[
x(t) = x_0 + R[sin(n(\omega t + \theta_0)) - sin(\theta_0)]
\]

\[
y(t) = y_0 - R[cos(\omega t + \theta_0) - cos(\theta_0)]
\]

Replacing the terms \( v_r \) and \( v_l \) with \( v_r \) and \( v_l \), which indicates displacements instead of speeds, allows dropping the time variable \( t \). Here \( s_r \) and \( s_l \) do the left and right wheels travel the distances respectively. Finally, the equation becomes:

\[
\frac{dx}{dt} = [x_0 + s_r + s_l]/(s_r - s_l)[sin((s_r - s_l)/1 + \theta_0) - sin(\theta_0)]\]

\[
y(t) = y_0 - [s_r + s_l]/(s_r - s_l)[cos((s_r - s_l)/1 + \theta_0) - cos(\theta_0)]
\]

Another mathematical model with the aim of showing in the kinematic equations relating to differential drive robots are discussed and implemented by [14]. For autonomous navigation of the robot, it must always know its position, i.e., the translation matrix and the rotation matrix. When the robot speed of the wheel change, the robot has to rotate around a point on the common axis of the two driving wheels, called the Instantaneous Centre of Curvature (ICC). Suppose the robot is at some location \( (x, y) \), making an angle of \( \theta \) with the \( X \) axis. After some time, the robot's position will shift, and a new one will be \( (x', y') \) and the new angle is \( \theta' \).

\[\text{ICC} = [x - R \sin(\theta), y + R \cos(\theta)]\]

and at time \( t + \delta t \) the new positions would be:

\[
\begin{bmatrix}
x'(t) \\
y'(t)
\end{bmatrix} = \begin{bmatrix}
\cos(\omega \delta t) & -\sin(\omega \delta t) & 0 \\
\sin(\omega \delta t) & \cos(\omega \delta t) & 0
\end{bmatrix} \begin{bmatrix}
x - \text{ICC}_x \\
y - \text{ICC}_y
\end{bmatrix} + \begin{bmatrix}
\text{ICC}_x \\
\text{ICC}_y
\end{bmatrix}
\]

by using equation (16), the user can find the robot's position at any instant. The above equations can be described as the position of the robot moving in a particular direction \( \theta \) at a given velocity \( V \) (where \( V \) is the average of the left wheel and the right wheel velocity) by:

\[
\begin{align}
P_x &= x_t + \int V \cdot \sin \theta \cdot d\theta \\
P_y &= y_t + \int V \cdot \cos \theta \cdot d\theta
\end{align}
\]

\[
\theta = \int \omega \cdot dt
\]

The dynamic model of Two Wheel Differential robot (Khepera IV) [15] is derived from the kinematic and dynamic relations as follows:

\[
F_L + F_R = m \cdot a
\]

\[
\frac{(F_R - F_L)\delta}{2} = J \cdot \varepsilon
\]

Where the dynamic parameters of the robot are, \( F_L \) and \( F_R \) are Applied forces to the left and right wheels, respectively, \( m \) as Mass of the robot, \( a \) as tangential acceleration, \( J \) as moment of inertia and \( \varepsilon \) as angular acceleration. The following are the model's inputs, outputs, and state variables:

\[
\begin{align}
x(t) &= [x_1(t), x_2(t), x_3(t), x_4(t)] \\
u(t) &= [u_1(t), u_2(t)] \\
y(t) &= [y_1(t), y_2(t)]
\end{align}
\]

Where \( U_L \) and \( U_R \) are the voltages applied to the DC motors that drive the robot's wheels.

\[
x_1(t) = v(t)
\]

We have after deriving from the sides of the equation \( \dot{x}_1(t) = a(t) \) and using equation (29) the following relation will be obtained:

\[
\dot{x}_1(t) = \frac{F_L + F_R}{m} = \frac{1}{m} F_L + \frac{1}{m} F_R
\]

From equation (29), \( u_1(t) = F_L \) and \( u_2(t) = F_R \). As a result, equation (13) can be written as follows:

\[
\dot{x}_1(t) = \frac{1}{m} u_1(t) + \frac{1}{m} u_2(t)
\]
With this equation $x_2(t) = \omega(t)$ we can get a deduction from the equation $\dot{x}_2(t) = \epsilon(t)$. The second state equation is given by applying equation (9):

$$
\dot{x}_2(t) = \frac{-D}{2J} u_1(t) + \frac{D}{2J} u_2(t) \tag{23}
$$

The angular velocities $U_L$ and $U_R$ were generated using $\omega_L$ and $\omega_R$, respectively. The following equations describe the voltage values of the motors:

$$
J\dot{\omega}_L(t) + F\omega_L(t) + F_R = U_L
$$

$$
J\dot{\omega}_R(t) + F\omega_R(t) + F_L = U_R \tag{35}
$$

The angular accelerations of the wheels are $\epsilon_L$ and $\epsilon_R$, and the friction force is $F$. By opting for the third state, variable $x_3(t) = \omega_L$ and fourth state variable $x_4(t) = \omega_R$. The following state equations can be constructed by using $U_L$ and $U_R$ as third and fourth input variables, respectively:

$$
\dot{x}_3(t) = -\frac{F}{J} x_3(t) - \frac{r}{J} u_1(t) + \frac{1}{J} u_3(t)
$$

$$
\dot{x}_4(t) = -\frac{F}{J} x_4(t) - \frac{r}{J} u_2(t) + \frac{1}{J} u_4(t) \tag{36}
$$

As a result, there are four state equations that can be expressed as a state-space matrix, as shown below:

$$
\dot{x}(t) = 
\begin{bmatrix}
0 & 0 & 0 & 0 & \frac{1}{m} & \frac{1}{m} & 0 & 0 \\
0 & 0 & 0 & 0 & -D & \frac{D}{2J} & 0 & 0 \\
0 & 0 & 0 & -\frac{F}{J} & 0 & -\frac{r}{J} & 0 & \frac{1}{J} \\
0 & 0 & 0 & -\frac{F}{J} & 0 & -\frac{r}{J} & 0 & \frac{1}{J}
\end{bmatrix}

\begin{bmatrix}
x(t) \\
u(t)
\end{bmatrix}
$$

$$
y(t) = 
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}

\begin{bmatrix}
x(t)
\end{bmatrix} \tag{37}
$$

2.2 Three Wheel

The three-wheeled robots are divided into two parts, Differential steering (2 driving wheels with additional free rotating wheels to maintain body balance) and Two wheels powered by a single power source and one for the third cycle power steering. The robot's steering can be changed by changing the relative rate of rotation of the two separate drive wheels for differential steering wheels. The robot will go straight ahead if both wheels are propelled in the same direction and at the same speed. Otherwise, the centre of rotation can fall anywhere on the straight line joining the two wheels, depending on the speed and direction of rotation. [16]. Figure 3 shows the schematics of a three-wheeled robot. The back wheels represent the motor wheels, and the front is the steering wheel.

As shown in [8] [9], the robot velocity can be defined as

$$
v_x = v_u \sin \psi + v_w \cos \psi
$$

$$
v_y = v_u \cos \psi - v_w \sin \psi \tag{38}
$$

In (39) we observe $v_x$ and $v_y$ which are the lateral parts of the velocity of the AGV mass centre in the UCW frame. $\psi$ is the position of the vehicle's steering wheel at any given time. Both UCW and XOY have an angular velocity of $w = \dot{\psi}'. \delta$ is the steering angle, which is defined as the angle between the steering wheel and the vehicle's longitudinal axis. In the two coordinate frames, $a_u$, $a_w$, $a_x$, and $a_y$ are employed for acceleration components. The resultant of the forces produced by the tires on the vehicle has longitudinal ($u$) and lateral ($w$) components, as indicated. The subscripts R and L stand for Right and Left (for rear tires), respectively, while the subscripts r and f stand for rear and front, respectively. Thus, the lateral ($w$) components of the force ($F$) from the rear ($r$) left ($L$) tire are denoted as $(F_{wr})_L$. The longitudinal components of the forces generated by the two rear tires, on the other hand, are the same and are represented by (40).

$$
2F_{wr} + F_{uf} \cos \delta - F_{wf} \sin \delta = ma_u
$$

$$
(F_{wr})_L + (F_{wr})_R + F_{uw} \cos \delta + F_{uf} \sin \delta = ma_w \tag{40}
$$

$$
-(F_{wr})_R + (F_{wr})_L \hat{\theta} + (F_{uw} \cos \delta + F_{uf} \sin \delta)a = I \dot{\omega} \tag{41}
$$

where $a$ and $b$ are the two distances from two of the wheels (one in the front centre and one from the back out of two) toward the perpendicular from the centroid of the vehicle. $I$ is the moment of inertia about a vertical axis passing through the centre of mass, $a_u$ and $a$ is the $u$ and $w$ components of the acceleration of point $c$, the mass centre, and $\omega$ is the angular acceleration. It can be shown that $a$ and $b$ can be written in the following form.
\[ a_u = \dot{v}_u - v_y \omega \]
\[ a_w = \dot{v}_w + v_y \omega \]  
(42)

where \( v_u \) and \( v_w \) are the components of the velocity in the vehicles coordinate system. Then the dynamic equations of the plane motion for the vehicle are

\[
m \dot{v}_u = m v_w \omega + 2 F_w + F_f \cos \delta - F_w \sin \delta \]
\[
m \dot{v}_w = -m v_u \omega + (F_w)_1 + (F_w)_2 + F_r \cos \delta + F_f \sin \delta \]
\[
l \dot{\omega} = -[(F_w)_2 + (F_w)_1] b + (F_r \cos \delta + F_f \sin \delta) a \]  
(43)

The above equations have been used for the simulations carried out in this study. As it was mentioned earlier, it is assumed that the side slip angles are small, and thus, the corresponding side forces are determined from (44) and (45).

\[
F_{wr} = C_f \beta_f \]  
(44)

\[
(F_{wr})_{LR} = C_r (\beta_r)_{LR} \]
(45)

where \( \beta_f, \beta_r \) and \( \beta_r \) are the slip angles for the front, rear left and rear right tires, respectively, and \( C_f \) and \( C_r \) are the cornering stiffness of the front and rear wheels, respectively. The slip angles for the front and rear tires can be determined as follows:

\[
\beta_f = \delta - \tan^{-1} \frac{v_w + d \omega}{v_u} \]  
(46)

\[
(\beta_r)_R = \tan^{-1} \frac{b u - v_u}{v_u + d \omega} \]  
(47)

\[
(\beta_r)_L = \tan^{-1} \frac{b u - v_u}{v_u + d \omega} \]

Where \( 2d \) is the distance between the rear tires.

### 2.3. Four Wheel AGVs

The four-wheeled robot is the most balanced robot among other wheeled robots. Although three wheels are enough to maintain static stability, the three-wheeled robot can lose its balance while moving. Four-wheeled robots rarely lose their stability when moving. The four-wheeled robot can be controlled using differential steering and a steering method similar to a car. This model was introduced in 2 Degree of Freedom (DOF) and 7 DOF, as shown in the following.

\[
F_{xx} + F_{xt} \cos \delta - F_{xy} \sin \delta + F_{xfl} + F_{xrr} \]
\[
(F_{xx} + F_{xt}) \sin \delta + F_{yrl} + F_{yyr} \]
\[
(F_{xx} + F_{xt}) \cos \delta + \]
\[
(F_{xx} + F_{xfl}) \sin \delta - F_{yrl} + F_{yyr} \]
\[
(F_{xx} + F_{xfl}) \sin \delta + F_{yrl} + F_{yyr} \]
(50)

then the Yaw movement as shown in equation (62)

\[
l \dot{\omega} = a (F_{yrl} + F_{yyr}) \cos \delta + \frac{b}{2} (F_{xfl} - F_{xrr}) \sin \delta - a (F_{xx} + a)
\]
\[
F_{xfl} \sin \delta + \frac{b}{2} (F_{xrr} - F_{xrl}) \]
(52)

and finally, the rotation of the wheel equation (53) described the wheel rolling inertia wheel rational rate relation to wheel brake torque.

\[
J_\omega \dot{\omega} = T_{dij} - T_{bij} - F_{xij} R_\omega \]
(54)
In (63), $F_{xf}, F_{xfr}, F_{xrb}, F_{yfr}, F_{yfb}, F_{yrr}$ force components for front left, front right, rear left and rear right tire along $x, y$ coordinates respectively; $d$ distance between leaf and right wheels; $J_ω$ wheel rolling inertia; $\omega_{ij}$ wheel rational rate; $T_{bij}$ wheel brake torque ($i = f, r$ front, rear-wheel, $j = l, r$ left, right wheel); $T_{bij}$ rear wheel driving torque; $F_{sl}i$ wheel longitudinal force; $R_w$ wheel rolling.

Another model introduced [18] is that the vehicle follows the desired velocity profile, enabling a comfortable and safe ride. Catered the lateral dynamic model with two approaches: pure pursuit and model predictive control. Their model is as follows:

$$\ddot{x} = \psi \dot{y} + \alpha_x,$$
$$\ddot{y} = -\dot{\psi} \dot{x} + \frac{2}{m} (F_c \cos (\delta) + F_c),$$
$$\ddot{\psi} = \frac{1}{I_\theta} (I_F \dot{F}_x - I_c \dot{F}_y),$$

(55)

where, the longitudinal and lateral speed is represented by $x$ and $y$, respectively, with respect to the body frame. The lateral forces on the front and back wheel are denoted by $F_{cf}$ and $F_{cr}$, respectively. The yaw rate, vehicle's mass, and yaw inertia are denoted by $\psi$, $m$, and $I_\theta$, respectively. For the linear tire model, the $F_{ci}$ is defined by the equation

$$F_{ci} = -C_{ai} \alpha_i,$$

(56)

where, $i \in f, r$ and $\alpha_i$ is the slip angle given by equation (12)-(13) and $C_{ai}$ is the tire cornering stiffness.

$$\alpha_f = \arctan \left( \frac{\nu_y + \frac{I_\theta}{I_c} \dot{\nu}_y}{\nu_x} \right) - \delta$$

(57)

$$\alpha_r = \arctan \left( \frac{\nu_y - \frac{I_\theta}{I_c} \dot{\nu}_y}{\nu_x} \right).$$

(58)

In this model the speed of the differential module is synthesized by the speeds of left and right driving wheels, and the speeds of the two differential modules can be written as [19]:

$$\nu_f = \frac{\nu_{fr} + \nu_{rl}}{2},$$
$$\nu_r = \frac{\nu_{rr} + \nu_{rl}}{2}.$$

(59)

The prerequisite for the steering of the differential module is that it has a certain angular velocity, which is realized by the speed difference between the two driving wheels of the differential module. Set the speed difference between the two driving wheels of the front differential module to be $\Delta \nu_f$, and the rear differential module to be $\Delta \nu_r$; then the speed of the four driving wheels can be written as:

$$\nu_{fr} = \nu_f + \frac{\Delta \nu_f}{2},$$
$$\nu_{fl} = \nu_f - \frac{\Delta \nu_f}{2},$$
$$\nu_{rr} = \nu_r + \frac{\Delta \nu_r}{2},$$
$$\nu_{rl} = \nu_r - \frac{\Delta \nu_r}{2}.$$

(70)

The angular velocity of the two differential modules $\alpha_f, \alpha_r$, can be written as:

$$\omega_f = \frac{|\nu_{fr} - \nu_{fl}|}{D} = \frac{\Delta \nu_f}{D},$$
$$\omega_r = \frac{|\nu_{rr} - \nu_{rl}|}{D} = \frac{\Delta \nu_r}{D}.$$  

(71)

The velocity components $\nu_{xf}, \nu_{yf}$ of $\nu_f$ on the x and y axes are:

$$\nu_{xf} = \nu_f \cdot \sin |\theta_f|,$$
$$\nu_{yf} = \nu_f \cdot \cos |\theta_f|.$$  

(72)

The velocity components $\nu_{rx}, \nu_{ry}$ of $\nu_r$ on the x and y axes are:

$$\nu_{rx} = \nu_r \cdot \sin |\theta_r|,$$
$$\nu_{ry} = \nu_r \cdot \cos |\theta_r|.$$  

(73)

The speeds of AGV centre on the x and y axes are:

$$\nu_x = \nu_{fx} - \nu_{rx},$$
$$\nu_y = \nu_{fy} - \nu_{ry}.$$  

(74)

The steering of the AGV is caused by the sub-speed of $\nu_f$ and $\nu_r$ on the x-axis (two sub-speeds are reversed), and the angular velocity of AGV center can be written as:

$$\omega_c = \frac{|\nu_{rx} + |\nu_{xx}|}{L} = \frac{|\nu_{rx} \sin |\theta_r| + |\nu_{xx} \cos |\theta_r|}{l}.$$  

(75)

Another mathematical model [20] developed for the wheeled robot by Lagrange or Newton Euler equation. In the dynamics model, $F$ and $M$ are the fore and torque generated by wheel system, the output variable is body coordinates $(x_1, x_2) \in R^2$ and azimuth angle $\alpha$, the input variables are longitudinal force acted on wheels and control signal $u_\phi$ of wheels. For simplicity, it was assumed that: the body of the robot is rigid body; the contact type between wheel system and land is point contact. The development of the wheeled mobile robot: the wheel system is installed on the body of robot. The position of the robot in Cartesian coordinate system $X \in R^2$ is represented by vector $x = (x_1, x_2)$. The point $C$ is the center of the robot, angle $\alpha$ is azimuth angle. In the Y axis, the movement of the wheeled mobile robot can be described as

$$\ddot{x} = V, mV = F, \ddot{\alpha} = \omega, J \ddot{\omega} = M.$$  

(76)

Where $V \in R^2$, absolute linear velocity, $F = (F_f, F_r)$ : external force, $\omega$ : angular velocity, $M$ : torque, $m$ : mass, $J$ : moment of inertia. In order to analysis the velocity and
force in relative coordinate system, the following matrix was introduced:

\[ T(\alpha) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \]  

(77)

The linear velocity of \( C_i \) can be calculated by

\[ V^i = \begin{bmatrix} V^i_x \\ V^i_y \end{bmatrix} = T(\theta^i)(V_p + \omega E^T z^i) \]  

(78)

Where \( V_p \)-velocity of robot body, \( V^i_j \)-vertical velocity of wheel \( i \), \( V^i_j \)-tangential velocity of wheel \( i \).

So the kinematics model of the mobile robot is

\[ \left[ \begin{array}{c} \dot{\xi}_{id} \\ \dot{\eta}_{id} \end{array} \right] = \left[ \begin{array}{c} V^i_y \end{array} \right] - R(\alpha) \left[ \begin{array}{c} F_p^i \\ M^i \end{array} \right] \gamma = \left[ \begin{array}{c} |M| \end{array} \right]; u = |V^i_j F_p^i + \omega M|; |M| = T_i(\theta)u + T_2(\theta)f \]

\[ f = -k_F r_i = -k_F T_i(\theta) \right]_{\alpha \theta} \quad \gamma = \eta \alpha \theta \]

(79)

Where, \( R(\alpha) = \begin{bmatrix} T(\alpha) & 0 \\ 0 & 1 \end{bmatrix} \cdot J \)

\[ = \begin{bmatrix} m \alpha & 0 \\ 0 & f \end{bmatrix}; u_\theta \text{-control signal, } u_\gamma \]

\[ = (u_\theta, u_\gamma, \ldots u_n); K_\theta \text{- matrix of transmission coefficient, } K_\theta = \text{diag}(k_P, k_D, \ldots, k_D) \]

\[ u_d = \eta \dot{\alpha} + k_D \ddot{\alpha} + \eta \dot{\alpha} \]

\[ u_\gamma = \eta \dot{\gamma} - k_\gamma \dot{\gamma} - k_\gamma \gamma \]

2.4 Four Wheel Skid Steering AGVs

The Skid Steering is a movement mechanism widely used in mobile robots. For the skid steered robot, as shown in figure 5, there is no steering mechanism, and the direction of Movement is changed by turning the wheels left and right at different speeds. The design of the mechanism makes the robot mechanically robust and easy to navigate outdoors. Due to the complex wheel/ground interaction and kinematic constraints, it remains difficult to obtain precise kinematic and dynamic models for sliding guided mobile robots [21].

\[ F_i = \frac{r_i}{r}\]  

(83)

Assuming that the vertical force \( N_i \) acts from the surface to the wheel. Considering the four wheels of the vehicle and neglecting additional dynamic properties, obtained are the following equations of equilibrium:

\[ N_1 = N_2 = \frac{b}{a+b} m g \]

\[ \sum_{i=1}^{4} N_i = m g, \]

\[ N_1 = N_2 = \frac{b}{a+b} m g \]

Assume that the vector \( F_{sl} \) results from the rolling resistant moment \( r_{ri} \) and the vector \( F_{pi} \) denotes the lateral reactive force. These reactive forces can be regarded as friction ones. However, it is important to note that friction modeling is quite complicated since it is highly nonlinear.
and depends on many variables. Therefore, in most cases, only a simplified approximation describing the friction \( F_f \) as a superposition of Coulomb and viscous friction is considered. It can be written as

\[
F_f(\sigma) = \mu_c N \text{sgn}(\sigma) + \mu_v \sigma, \tag{86}
\]

where \( \sigma \) denotes the linear velocity, \( N \) is the force perpendicular to the surface, while \( \mu_c \) and \( \mu_v \) denote the coefficients of Coulomb and viscous friction, respectively. Since for the SSMR the velocity \( \sigma \) is relatively low, especially during lateral slippage, the relation \( \mu_c N [\mu_c, \sigma] \) is valid, which allows to neglect the term \( \mu_v \sigma \) to simplify the model. It is critical to emphasize that the function \( f(\sigma) \) is not differentiable at \( \sigma = 0 \). Since a continuous and time differentiable model of the SSMR should be obtained, the following approximation of this function is proposed:

\[
\text{sgn}(\sigma) = \frac{2}{\pi} \arctan(k_c \sigma) \tag{87}
\]

where \( k_c \rightarrow 1 \) is a constant which determines the approximation accuracy according to the relation

\[
\lim_{k_c \to 1} \frac{2}{\pi} \arctan(k_c \sigma) = \text{sgn}(\sigma). \tag{88}
\]

Based on the previous deliberations, the friction forces for one wheel can be written as

\[
F_{li} = \mu_{cli} m g \text{sgn}(v_{yi}) \]

\[
F_{si} = \mu_{sci} m g \text{sgn}(v_{xi}) \tag{89}
\]

where \( \mu_{cli} \) and \( \mu_{sci} \) denote the coefficients of the lateral and longitudinal forces, respectively.

Using the Lagrange-Euler formula with Lagrange multipliers to include the nonholonomic constraint, the dynamic equation of the robot can be obtained. Next, it is assumed that the potential energy of the robot \( PE(q) = 0 \) because of the planar motion. Therefore, the Lagrangian \( L \) of the system equals the kinetic energy:

\[
L(q, \dot{q}) = T(q, \dot{q}). \tag{90}
\]

Considering the kinetic energy of the vehicle and neglecting the energy of rotating wheels, the following equation can be developed:

\[
T = \frac{1}{2} m v^T v + \frac{1}{2} I \omega^2 \tag{91}
\]

where \( m \) denotes the mass of the robot and \( I \) is the moment of inertia of the robot about the COM. For simplicity, it is assumed that the mass distribution is homogeneous. Since \( v^T v = v_x^2 + v_y^2 = x^2 + y^2 \), Equation (90) can be rewritten as follows:

\[
T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} I \dot{\theta}^2 \tag{92}
\]

After calculating the partial derivative of kinetic energy and its time-derivative, the inertial forces can be obtained as

\[
d \frac{d}{dt} \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix} = M \ddot{q} \tag{93}
\]

where

\[
M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

Consequently, the forces which cause the dissipation of energy are considered. The following resultant forces expressed in the inertial frame can be calculated:

\[
F_{rx}(q) = \cos \theta \sum_{i=1}^{4} F_{xi}(v_{xi}) - \sin \theta \sum_{i=1}^{4} F_{yi}(v_{yi})
\]

\[
F_{ry}(q) = \sin \theta \sum_{i=1}^{4} F_{xi}(v_{xi}) + \cos \theta \sum_{i=1}^{4} F_{yi}(v_{yi}) \tag{94}
\]

The resistive moment around the mass centre \( M_r \) can be calculated as

\[
M_r(q) = -a \sum_{i=1}^{4} F_{xi}(v_{xi}) + b \sum_{i=2}^{3} F_{yi}(v_{yi}) + c \left[ -\sum_{i=1}^{2} F_{xi}(v_{xi}) + \sum_{i=2}^{4} F_{yi}(v_{yi}) \right].
\]

To define generalized resistive forces, the vector

\[
R(q) = [F_{rx}(q) \quad F_{ry}(q) \quad M_r(q)]^T \tag{95}
\]

is introduced. The active forces generated by the actuators that make the robot move can be presented in the inertial frame as follows:

\[
F_x = \cos \theta \sum_{i=1}^{4} \tau_i,
\]

\[
F_y = \sin \theta \sum_{i=1}^{4} \tau_i \tag{96}
\]

The active torque around the COM is calculated as

\[
M = c(-F_1 - F_2 + F_3 + F_4). \tag{97}
\]

In consequence, the vector \( F \) of active forces has the following form:

\[
F = [F_x \quad F_y \quad M]^T \tag{98}
\]

Assume that each wheel’s radius is the same then,

\[
F = \frac{1}{r} \begin{bmatrix} \cos \theta \sum_{i=1}^{4} \tau_i \\ \sin \theta \sum_{i=1}^{4} \tau_i \\ c(-\tau_1 - \tau_2 + \tau_3 + \tau_4) \end{bmatrix} \tag{99}
\]
To simplify the notation, a new torque control input $\tau$ is defined as

$$
\tau = \begin{bmatrix} \tau_l \\ \tau_R \end{bmatrix} = \begin{bmatrix} \tau_1 + \tau_2 \\ \tau_3 + \tau_4 \end{bmatrix}
$$

where $\tau_l$ and $\tau_R$ signify the torques generated by the wheels on the vehicle's left and right sides, respectively. Combining (99) and (100), we get

$$
F = B(q)\tau,
$$

where $B$ is the input transformation matrix defined as

$$
B(q) = \begin{bmatrix} \cos \theta & \cos \theta \\ \sin \theta & \sin \theta \end{bmatrix}
$$

Next, using (92), (95) and (101), the following dynamic model is obtained:

$$
M(q)\ddot{q} + R(q) = B(q)\tau.
$$

It should be noted that (103) describes the dynamics of a free body only and does not include the nonholonomic constraint. Therefore, a constraint has to be imposed on (103). To this end, a vector of Lagrange multipliers, $\lambda$, is introduced as follows:

$$
\dot{M}(q)\ddot{q} + R(q) = B(q)\tau + \dot{A}^T(q)\lambda.
$$

For control purposes, it would be more suitable to express (104) in terms of the internal velocity vector $\eta$. Therefore, (104) is multiplied from the left by $S^T(q)$, which results in

$$
S^T(q)M(q)\ddot{q} + S^T(q)R(q) = S(q)^Tb(q)\tau + S^T(q)\dot{A}^T(q)\lambda.
$$

After taking the time derivative,

$$
\ddot{q} = \dot{S}(q)\eta + S(q)\ddot{q}.
$$

Hence, combining (106) and (104), the dynamic equations become

$$
\ddot{M}\dot{\eta} + \dot{C}\eta + \ddot{R} = \ddot{B}\tau
$$

Where,

$$
\dot{C} = S^TM\dot{S} = m_{ICR} \begin{bmatrix} 0 & \dot{\theta} \\ -\dot{\theta} & \ddot{x}_{ICR} \end{bmatrix}
$$

$$
\ddot{M} = S^TMS = \begin{bmatrix} m & 0 \\ 0 & m_{ICR} + l \end{bmatrix}
$$

$$
\ddot{R} = S^TR = \begin{bmatrix} F_{xx}(\dot{q}) \\ x_{ICR}F_{yy}(\dot{q}) + M_r \end{bmatrix}
$$

To have a better model [23] propose a kinematic approach for tracked mobile robots in order to improve motion control and pose estimation. $(V_x, V_y)$ is the linear velocity of the robot's left and right tracks in relation to the robot frame. Then, on the plane, direct kinematics can be described as follows:

$$
(V_x, V_y, \omega_z) = f_{\theta}(V_x, V_y)
$$

where $\mathbf{v} = (v_x, v_y)$ is the translational velocity of the vehicle in relation to its local frame, and $\omega_z$ is its angular velocity. Conversely, finding control actions that result in the desired motion can be expressed as the inverse kinematics problem:

$$
(V_x, V_y) = f_{\theta}(v_x, v_y, \omega_z)
$$

Local coordinates for the vehicle and track ICRs can be obtained geometrically as a function of the vehicle's angular and translational velocities as

$$
\begin{align*}
\dot{x}_{ICR} &= \frac{v_y}{\omega_z} \\
\dot{y}_{ICR} &= \frac{v_x - v_{ICR}}{\omega_z}
\end{align*}
$$

Instantaneous translational and rotational speeds with respect to the local frame can be obtained as

$$
\begin{align*}
\dot{x}_{ICR} &= \frac{v_x}{\omega_z} \\
\dot{y}_{ICR} &= \frac{v_y}{\omega_z}
\end{align*}
$$

These equations represent the vehicle's direct kinematics assuming that the ICRs of the left and right treads could be properly estimated. If the ICRs lie precisely on the track plane (i.e., $y_{ICR} = 0$), then the vehicle's velocity $\mathbf{v}$ has no transversal component (i.e., $v_x = 0$). On the other hand, inverse kinematic relations can be expressed by

$$
\begin{align*}
V_l &= v_y + x_{ICR}\omega_z \\
V_r &= v_y - x_{ICR}\omega_z
\end{align*}
$$

which includes a non-holonomic restriction, since $v_x$ references cannot be directly imposed. Besides, according to these equations, the same control input values are computed regardless of $y_{ICR}$ in the model. The inverse of the normalized distance between the wheels can be used to calculate the vehicle's steering efficiency index $\chi$, track ICRs.
\[ \chi = \frac{L}{x_{ICR_r} - x_{ICR_l}}; \ (0 < \chi < 1), \]  

(119)

\[ e = \frac{x_{ICR_r} + x_{ICR_l}}{x_{ICR_r} - x_{ICR_l}} \]  

(120)

Index \( e \) is zero when track ICRs are symmetrical with respect to the local Y-axis. For friction, in the case of hard-surface soils, the resulting force for a point in the track is a function of its slip velocity \( v_s \) as expressed by Coulomb’s law

\[ \tau = -[\mu] p \frac{v_s}{|v_s|} \]  

(121)

where \( p \) is the pressure under the track, and \([\mu]\) is a coefficient matrix, which, in the general case of anisotropic friction on the \( XY \) plane, has the following form:

\[ [\mu] = \begin{bmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & 0 \end{bmatrix}. \]

(122)

Moreover, if the inertia frame coincides with the vehicle frame, \([I]\) is a diagonal matrix expressed as

\[ [I] = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}. \]

(123)

2.5 Six Wheel AGVs

Six-wheel AGVs have three wheels on each side, totalling six wheels to guide it on its path automatically. All the wheels run at different speeds and power to control the vehicle’s steering. [24]

The above figure 6 shows the schematics of an example six-wheel AGV. This model is represented by [25] as shown in equation (124)

\[ \dot{\xi} = R(\theta) \cdot \dot{\xi} \]

(124)

\[ \dot{\xi} = [x \ y \ \theta]^T \]

(125)

\[ \dot{\xi}_r = \{c\theta \ s\theta \ 0\}^T \]

(126)

\[ \dot{\xi}_l = \{c\theta \ -s\theta \ 0\}^T \]

(127)

The dynamic model concerning robot coordinate system:

\[ \frac{d}{dt}(m\ddot{v}) = \sum F_{Ext} \]

(128)

\[ m \begin{bmatrix} a_x \\ a_y \end{bmatrix} + \begin{bmatrix} w_x \\ w_y \end{bmatrix} \times m \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \sum F_x \]

\[ m \begin{bmatrix} a_x + v_x w_y - v_y w_x \\ a_y + v_y w_x - v_x w_y \end{bmatrix} = \sum F_y \]

In this case, considering non-slip condition and movement at the plane \((x,y)\) the equation system is:

\[ m \begin{bmatrix} x_r \dot{x}_r \\ y_r \dot{y}_r \end{bmatrix} = \begin{bmatrix} F_{fx} \\ F_{fy} \end{bmatrix} \]

(129)

\[ m \begin{bmatrix} r_1 \phi_1 \phi_1 + r_2 \phi_2 \phi_2 \\ r_1^2 \phi_1^2 + r_2^2 \phi_2^2 \end{bmatrix} = \begin{bmatrix} F_{fx} \\ F_{fy} \end{bmatrix} \]

(130)
The moment equations to the robot, are described at equations where \( I \) is the inertia tensor of robot:

\[
\frac{d}{dt}(\dot{I}) = \Sigma M_{\text{Ext}}
\]

\[
\frac{d}{dt}(Y\dot{w}) + \dot{w} \times (I\dot{w}) = \Sigma M_{\text{Ext}}
\]

\[
(\alpha_x, \alpha_y, \alpha_z) + (w_x, w_y, w_z) \times (v_x, v_y, v_z) = \sum M_x
\]

\[
(\alpha_x, \alpha_y, \alpha_z) + (w_x, w_y, w_z) = \sum M_x
\]

\[
\left[ \begin{array}{c}
I_{xx} & I_{xy} & I_{xz} \\
I_{xy} & I_{yy} & I_{yz} \\
I_{xz} & I_{yz} & I_{zz}
\end{array} \right] \left[ \begin{array}{c}
\alpha_x \\
\alpha_y \\
\alpha_z
\end{array} \right] + \left[ \begin{array}{c}
w_x \\
w_y \\
w_z
\end{array} \right] \times \left[ \begin{array}{c}
v_x \\
v_y \\
v_z
\end{array} \right] = \sum M_x
\]

\[
\left[ \begin{array}{c}
I_{xx} & I_{xy} & I_{xz} \\
I_{xy} & I_{yy} & I_{yz} \\
I_{xz} & I_{yz} & I_{zz}
\end{array} \right] \left[ \begin{array}{c}
\alpha_x \\
\alpha_y \\
\alpha_z
\end{array} \right] = \sum M_x
\]

The robot will be symmetric at plane (\( zr, xr \)) and (\( zr, yr \)) thus the inertia tensor is a diagonal matrix \( I_{xx} \).

\[
\left[ \begin{array}{ccc}
I_{xx} & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & I_{zz}
\end{array} \right]
\]

\[
\sum M_x = \sum \{ m_x \}
\]

\[
\sum \{ m_y \}
\]

\[
\sum \{ m_z \}
\]

\[
\left( \begin{array}{c}
I_{xx} \\
I_{yy} \\
I_{zz}
\end{array} \right) = \sum \begin{bmatrix} m_x & a_y \end{bmatrix} = \sum \begin{bmatrix} m_y & a_y \end{bmatrix} = \sum \begin{bmatrix} m_z & a_y \end{bmatrix}
\]

With non-slip conditions and movement only at the plane (\( x, y \)) the equations system is:

\[
\left\{ \begin{array}{c}
0 \\
0 \\
l_{xx} \alpha_x
\end{array} \right\} + \left( \begin{array}{c}
w_x l_{xx} v_x \\
w_y l_{yy} v_y \\
l_{zz} a_y
\end{array} \right) = \left( \begin{array}{c}
0 \\
0 \\
0
\end{array} \right) + \left( \begin{array}{c}
I_{xx} \dot{\alpha}_x \\
I_{yy} \dot{\alpha}_y \\
I_{zz} \dot{\alpha}_z
\end{array} \right)
\]

\[
\left( \begin{array}{c}
l_{xx} \alpha_x \\
l_{yy} \alpha_y \\
l_{zz} a_y
\end{array} \right) = \left( \begin{array}{c}
0 \\
0 \\
0
\end{array} \right)
\]

\[
\left( \begin{array}{c}
l_{xx} \alpha_x \\
l_{yy} \alpha_y \\
l_{zz} a_y
\end{array} \right) = \sum \begin{bmatrix} m_x & a_y \end{bmatrix}
\]

\[
\sum \begin{bmatrix} m_y & a_y \end{bmatrix} = \sum \begin{bmatrix} m_z & a_y \end{bmatrix}
\]

In function of wheel variables and dimensions of robot, is presented the equations:

\[
\left( \begin{array}{c}
l_{xx} \dot{\alpha}_x \\
l_{yy} \dot{\alpha}_y \\
l_{zz} \dot{a}_y
\end{array} \right) = \left( \begin{array}{c}
I_{xx} \dot{\alpha}_x \\
I_{yy} \dot{\alpha}_y \\
I_{zz} \dot{a}_z
\end{array} \right)
\]

\[
\left( \begin{array}{c}
l_{xx} \dot{\alpha}_x \\
l_{yy} \dot{\alpha}_y \\
l_{zz} \dot{a}_z
\end{array} \right) = \left( \begin{array}{c}
N_{\text{fright}} - N_{\text{front}} \\
0 \\
0
\end{array} \right)
\]

\[
\left( \begin{array}{c}
l_{xx} \dot{\alpha}_x \\
l_{yy} \dot{\alpha}_y \\
l_{zz} \dot{a}_z
\end{array} \right) = \left( \begin{array}{c}
N_{\text{fright}} - N_{\text{front}} \\
N_{\text{back}} - N_{\text{front}} \\
(F_{\text{fright}} - F_{\text{fleft}}) - (F_y + F_y) \\
(F_{\text{fright}} - F_{\text{fleft}}) - (F_y + F_y)
\end{array} \right)
\]

\[
\left( \begin{array}{c}
l_{xx} \dot{\alpha}_x \\
l_{yy} \dot{\alpha}_y \\
l_{zz} \dot{a}_z
\end{array} \right) = \left( \begin{array}{c}
N_{\text{fright}} - N_{\text{front}} \\
N_{\text{back}} - N_{\text{front}} \\
(F_{\text{fright}} - F_{\text{fleft}}) - (F_y + F_y)
\end{array} \right)
\]

For this robot type (six-wheel mobile robot), the dynamic equation of the high-speed motion [14] is shown in equation (134).
Among them, \( u \) represents the x-axis velocity component of the unmanned platform in the local coordinate system, \( v \) represents the y-axis velocity component, and \( r \) represents the angular velocity. Analyse and calculate the rotational movement of the six wheels of the mobile robot, and obtain the equation of motion as:

\[
\begin{aligned}
\{ T_i \dot{\omega}_i &= T_{el} - F_{xi} R - T_{fi} \\
T_{fi} &= F_{fi} R
\end{aligned}
\]  

(141)

2.6 Ackermann AGV

Rudolph Ackermann invented the Ackermann in 1816. The steering is based on different steering angles for the left and right wheels of a steering axis. In automotive implementations, this is realized by dedicated mechanics shown on the right side of the picture. Each wheel’s steering angle (also called Ackermann Angle) depends on the vehicle’s dimensions [27]. Land vehicles generally have two types of steering mechanisms. Differential (or non-slip) steering and Ackerman steering. Either way, one of the biggest problems with differential steering is that it wastes energy dragging the wheels across the ground. Ackermann steering is often present in the car, which allows the wheels to turn around at similar turning points. The wheels do not slip sideways when turning; Therefore, energy is not wasted while spinning [28].

\[\cot \varphi_1 = \cot \varphi + d/2l \]
\[\cot \varphi_2 = \cot \varphi - d/2l \]  

(142)

Then we can get equation (142):

\[\cot \varphi_1 - \cot \varphi_2 = \frac{\rho_x + d/2}{l} - \frac{\rho_x - d/2}{l} = \frac{d}{l} \]  

(143)

Define \( \rho_l^1 \) is ICR radius of the left front wheel, \( \rho_l^2 \) is ICR radius of the rear wheel and \( \rho_r^1 \) is ICR radius of the right rear wheel, then we testify \( \rho_l^1 = l/\sin \varphi_1, \rho_l^2 = l/\sin \varphi_2, \rho_r^1 = l/\tan \varphi + d/2, \) and \( \rho_r^2 = l/\tan \varphi - d/2 \). Define inner minus \( \Delta \rho \) as \( \rho_l^1 - \rho_r^1 \) when turning right (when the vehicle turns left \( \Delta \rho \) equals \( \rho_l^2 - \rho_r^2 \)). Different influences on vehicles, such as side slip, brake, and slide, are neglected in the assumptions. But practically, these instances always happen to vehicles, which declare the nonholonomic restriction is destroyed and do not satisfy.

\[\dot{x} \sin \theta - y \cos \theta = 0 \]  

(144)

Assuming \( e_i^p \) as error vector of the rear wheel, it can be defined by \( \sigma \cdot [x_e \ y_e \ \varphi_e] \cdot [x_e,(t), y_e(t) \) and \( \varphi_e(t) \) change by time \( t \). \( \sigma = 1 \) is error of turning angle \( \varphi \).

\[\xi \]  

(145)

\[\xi_i = \begin{bmatrix} v_r + x_e & y_e & \frac{\tan(\psi+\varphi_e)}{l} & v_r \\ v_r \cos \theta & v_r \sin \theta & \frac{v_r \tan \varphi}{l} & \frac{x_e \cos \theta - y_e \sin \theta}{l} & x_e \sin \theta + \frac{y_e \cos \theta \varphi_e}{l} \end{bmatrix} \]  

(146)

In equation \( \phi_e \) is defined:

\[\phi_e = \frac{v_r}{l}, \frac{\tan \psi(1+t^2 \varphi)}{1-\tan \varphi \tan \psi} \]  

(146)

When errors and other factors destroy nonholonomic restriction of vehicle, the range can be declared by \( \kappa_x \) and \( \kappa_y \). \( \kappa_x = x_e \cos \theta - y_e \sin \theta, \kappa_y = x_e \sin \theta + y_e \cos \theta \), which map position error from local coordinate frame to global coordinate frame. Let \( e_i^p = [\kappa_x \ \kappa_y]^T \), and \( e_i^p \) has no relation to the vehicle status but is influenced by different kinds of errors. If not take proper measures to control the accumulation of error \( e_i^p \), it will bring about losing control of vehicles.

2.7 Crawler Robot

Crawler Robots, mainly used for inspecting pipelines [30], large ships or storage tanks, are another type of equipment in the remote visual inspection (RVI) technical kit. The operation of the crawler robot is very similar to that of a remote-control car with a controlled camera when lengthy distances must be travelled to do the check. Robotic lanes...
require a long control distance and long battery life. Most robotic lanes come with tank pedals or miniature construction tires, as foreign debris is a real threat to mobility [31], describes the mathematical model of the suspension of robot traction of crawler type shown in [32], in the mentioned model, to reduce mechanical vibrations caused by the operation of the motors and overcome obstacles in a path.

Forces acting on the right side:

\[ F_D z = F_{Dz} + F_4 z \]

Forces acting on the right side:

\[ F_{Dz} = F_{Kz} + F_{Bz} + F_{K_3 z} + F_{B_3 z} \]  

\[ \sum F_z = m_4 z''(t) \]

\[ -F_I z - F_D z - m_3 g = m_5 z''(t) \]  

For analysis at \( Y \) is followed a similar process than \( Z \).

\[ FK_y = FK_0 \sin \alpha \]

\[ FB_y = FB_0 \sin \alpha \]

\[ FK_0 y = FK_0_0 \sin \alpha \]  

\[ P_{i1-4} y = P_{i1-4}' \sin \alpha \]

Mathematical Analysis in \( Y \) for reference points (1 – 4):

\[ FK_z + FB_z - FK_0 - W = m_0 y_{1-4}' \]

Mathematical analysis regarding the lateral axis \( Y \) (sprung mass). Forces acting on the left side:

\[ F_I y = F_{Iz} y + F_3 y \]  

\[ \sum F_y = m_4 y''(t) \]

\[ -F_I y + F_D y = m_3 y''(t) \]  

\[ Y_{s_1} = Y_s + D_{1} \theta \]

\[ Y_{s_2} = \phi_s + D_{2} \theta \]

\[ Y_{s_3} = \phi_s - D_{2} \theta \]

\[ Y_{s_4} = \phi_s - D_{2} \theta \]

Analysis Roll: \( l_\phi \) is the moment of inertia generated about axis \( X \) (Fig. X).

\[ l_\phi \dot{\phi} = (F_3 z) R_f - (F_2 z) R_f + (F_3 z) R_r - (F_4 z) H_r \]  

\[ 2.8 \text{ Legged Robot} \]

Legged robots are mobile robots that use mechanical limbs to move, like wheeled robots, but move more complexly than wheeled robots, which perform better than wheeled robots on rough terrain and are essential in most applications, such as one-legged robots: two-Legged...
Robot, Three-Legged Robot, Four-Legged Robot, Six-Legged Robot and Multi-Legged Robot [33]. The model presented in [34] a complete kinematical model and dynamic model of a hexapod robot’s leg. In order to obtain a more precise model we divided the mass of each link in two \((M_i -\text{servomotor mass, } m_1-\text{link mass, } M_i > m_i)\).

Using the Lagrange-Euler equations, the following equation is derived by utilizing the kinetic and potential energy of every link in the system as given:

\[
\frac{d}{dt} \frac{\partial K}{\partial \dot{q}_i} - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial \dot{q}_i} = Q
\]

(165)

Where \(K\) and \(P\) represent kinetic and potential energy, respectively and \(Q\) is a vector with non-conservative forces like damping, applied torques and various kinds of friction. Kinetic energy is the amount of energy that a system has. Using the following formulas, we can determine Kinetic and Potential energy for the entire system:

\[
K = \frac{1}{2} q^T (\sum_{i=1}^n m_i J_{wi}^T) q
\]

(166)

We can simplify it as follow:

\[
D = (\sum_{i=1}^n m_i J_{wi}^T) q
\]

(167)

In equation (167) \(D\) is called the inertia matrix. The potential energy of a system with links \(i\) in a gravity field \(g\) with its centre of mass at the position for each link \(O_{ci}\) is:

\[
P = \sum_{i=1}^n m_i g \text{ o}_{ci}
\]

(168)

The equations of motion can be derived using the Lagrange Euler equation, but we can simplify the process by assuming that no external forces are operating on the system other than the applied torques, which can be rewritten as in equation (169):

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \Gamma_i
\]

(169)

Where \(L = K - P\) and \(q_i\) and \(\Gamma_i\) are the link I joint variable and joint torque. Assume that the joint has no damping or friction, and that \(Q\) merely contains joint torques. For each link I the Lagrange-Euler equations are as follows:

\[
\sum_{j=1}^n \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_j} + \sum_{j=1}^n \frac{\partial L}{\partial q_j} \frac{\partial \dot{q}_j}{\partial q_i} + \frac{\partial P}{\partial \dot{q}_i} = \Gamma_i\).
\]

(170)

This in matrix form can be written as:

\[
D \ddot{q} + C \dot{q} + G = \Gamma
\]

(171)

Where \(D\) is inertia matrix, \(C\) is coriolis and centrifugal matrix and \(G\) is gravity vector. The final set point \(\theta_f\) is calculated as:

\[
\theta_f = \begin{bmatrix}
\theta_{1x} \\
\theta_{2x} \\
\theta_{3x} \\
\theta_{4x}
\end{bmatrix} = 0
\]

(172)
The difference between the setpoint and the initial value of the joint angles that is denoted as $\theta_\circ$ is represented as:

\[
e(\theta_1) = \theta_{1s} - \theta_1
\]

(173)

\[
e(\theta_2) = \theta_{2s} - \theta_2
\]

(174)

\[
e(\theta_3) = \theta_{3s} - \theta_3
\]

(175)

\[
e(\theta_4) = \theta_{4s} - \theta_4
\]

(176)

Every system should ideally have an error signal that is set to zero so the initial position is:

\[
\theta_\circ = 0
\]

(177)

In order to implement a PID controller on a system we need to give it the following equation:

\[
\tau = K_p e + K_d \dot{e} + K_i \int e
\]

(178)

We have four different inputs therefore the equation for each is:

\[
\tau_1 = K_p e(\theta_1) + K_d \dot{e}(\theta_1) + K_i \int e(\theta_1)
\]

\[
\tau_2 = K_p e(\theta_2) + K_d \dot{e}(\theta_2) + K_i \int e(\theta_2)
\]

\[
\tau_3 = K_p e(\theta_3) + K_d \dot{e}(\theta_3) + K_i \int e(\theta_3)
\]

\[
\tau_4 = K_p e(\theta_4) + K_d \dot{e}(\theta_4) + K_i \int e(\theta_4)
\]

(179)

In order to solve on Matlab and to understand the programming, the following equations as system equations can be used:

\[
x_1 = \theta_{1s} - \theta_1
\]

\[
x_2 = \theta_{2s} - \theta_2
\]

\[
x_3 = \theta_{3s} - \theta_3
\]

\[
x_4 = \theta_{4s} - \theta_4
\]

(180)

The characteristics of the two controllers were tweaked to have the optimal performance by trial and error. The following were found to be the optimal settings for the parameters:

\[
K_{\text{PID1}} = \begin{bmatrix} K_{P1} \\ K_{D1} \\ K_{I1} \end{bmatrix} = \begin{bmatrix} 15 \\ 3 \\ 2 \end{bmatrix}
\]

\[
K_{\text{PID2}} = \begin{bmatrix} K_{P2} \\ K_{D2} \\ K_{I2} \end{bmatrix} = \begin{bmatrix} 15 \\ 4 \\ 2 \end{bmatrix}
\]

\[
K_{\text{PID3}} = \begin{bmatrix} K_{P3} \\ K_{D3} \\ K_{I3} \end{bmatrix} = \begin{bmatrix} 20 \\ 5 \\ 4 \end{bmatrix}
\]

\[
K_{\text{PID4}} = \begin{bmatrix} K_{P4} \\ K_{D4} \\ K_{I4} \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}
\]

Observe the structure of the preceding ODE with control, we can see that (roughly): $K_p$ is related to direct error and evolution speed. The pace of interaction with changes in states is connected to $K_d$. Overall error cancellation is connected to $K_i$

### 2.9 Snake Robot

Snake Robot is a new type of robot known as the Serpentine Robot. As the name suggests, these robots have multiple drive joints and, therefore, multiple degrees of freedom. This allows them to take advantage of Infinite's vast configuration, adapt, touch, and come close to a large capacity in their workspace [36]. The omni-tread snake robot [37]; Caleb III, presented in [38] designed to locomote on narrow space and rough terrain. Caleb III consists of three parts linked together by 2 DOF joints for pitch and yaw movements. The four sides of each segment have movable orbits that provide propulsion even when the robot is rolling. 2 D.O.F. connectors are actuated by 2 servomotors that produce sufficient torque to lift a front or next segment overcome obstacles.

![Snake Robot Schematic](image)

Figure 7. Snake Robot Schematic

Figure 10 shows the schematic of the snake where Caleb III has articulated steering as shown and the motion of the robot is analyzed by representative points in the front frame, middle frame and rear frame. Then the following equation is obtained:

\[
\theta_f - \theta_m = \phi_1
\]

(181)

\[
\theta_m - \theta_r = \phi_2
\]

Here $\phi_1, \phi_2$ is the steering angle. Eqs. 172 is rewritten as follows by using the angular velocity $\omega_f, \omega_m$ and $\omega_r$ of the front, middle and rear frame:

\[
\omega_f - \omega_m = \frac{d\phi_1}{dt}
\]

\[
\omega_m - \omega_r = \frac{d\phi_2}{dt}
\]

The velocity vector of the front, middle and rear frame $\nu_f, \nu_m$ and $\nu_r$ are expressed as follows based on the assumption of no-slip in the lateral direction:
\[
\begin{align*}
\dot{x}_f(t) &= x_{f0} + \int v_f(t) \cos \theta_f(t) \, dt \\
y_f(t) &= y_{f0} + \int v_f(t) \sin \theta_f(t) \, dt \\
\theta_f(t) &= \theta_{f0} + \int w_f(t) \, dt
\end{align*}
\]  

Here, \(x_{f0}, y_{f0}\) and \(\theta_{f0}\) are initial value. The other shape of this robot type, known as the worm robot prototype, is presented in [39-41] as shown in figure 11.

To assess the torque required in the Sub-Motion, it is necessary to solve the inverse dynamic problem as a function of the given nominal path. Therefore, the dynamic equations which govern each sub-motion must be available. The equations of motion in each sub-motion are established using a set of local coordinates. Therefore, we present the following local coordinate sets, which relate to the global coordinate set shown in Figure 11.

\[
\{\theta_1, \theta_2, \theta_3\}^T \quad \text{in sub-motion } M_1
\]
\[
\{\theta_1, \theta_2, \theta_3, \theta_4\}^T \quad \text{in sub-motion } M_2
\]  

Since the use of equations of motion is aimed at solving inverse dynamic problems rather than controlling, and interpretation is made based on the absolute angle of the connecting rod (measured in a positive direction with respect to the horizontal axis) and is calculated as follows:

\[
\varphi_0 = 0, \quad \varphi_i = \varphi_{i-1} + \theta_i, \quad i = 1, ..., n
\]  

Also, the closed loop constraint in each sub-motion can be formulated as the below:

\[
\sum_{i=1}^{n} \sin \varphi_i = 0
\]  

Thus, the equations of motion governing in sub-motion \(M_1\) has been explained as follows:
This work was supported by Jiangxi University of

Conclusions

Authors thanking

where, \( m \) and \( l \) denote the mass and length of the links, \( g \) is gravity acceleration, and for the sake of abbreviation it's taken \( c_{ij} = \cos(\varphi_i - \varphi_j) \) and \( s_{ij} = \sin(\varphi_i - \varphi_j) \).

Similarly, in sub-motion \( M_2 \) the equations of motion are

\[
\begin{align*}
&\frac{m l^2}{6} \begin{pmatrix}
14 & 9 & c_{12} & 3 & c_{13} & 3 & c_{14} & 3 & c_{24} & 3 & c_{34} & 3 & c_{44} \\
3 & c_{13} & c_{14} & 3 & c_{23} & 3 & c_{24} & 3 & c_{34} & 3 & c_{44} & 3 & c_{44} \\
0 & 9 & 3s_{12} & 3s_{13} & 3s_{14} & 3s_{23} & 3s_{24} & 3s_{34} & 3s_{44} & 3s_{44} & 3s_{44} & 3s_{44} & 3s_{44}
\end{pmatrix} \begin{pmatrix}
\varphi_1 \\
\varphi_2 \\
\varphi_3 \\
\varphi_4
\end{pmatrix} \\
&+ \frac{m l}{2} \begin{pmatrix}
5\cos \varphi_1 \\
3\cos \varphi_3 \\
\cos \varphi_4
\end{pmatrix} = \begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix} \begin{pmatrix}
\tau_1 \\
\tau_2 \\
\tau_3
\end{pmatrix} + \frac{\lambda_1}{\lambda_2} \begin{pmatrix}
\lambda_3 \\
\lambda_4
\end{pmatrix} R
\end{align*}
\]

Assuming a Coulomb friction model and using the Lagrange multiplier method, considering the stress equation, the normal component of the reaction, i.e. \( R \), calculations can be performed from the last row of the matrix form equation (191) given for the sub-motions \( M_1 \) and \( M_2 \) respectively, where we need to be replaced by:

\[
\lambda_i = l_i (\cos \varphi_i + \mu \text{sgn} \left( v_{tip} \cdot \hat{x} \right) \sin \varphi_i)
\]

where, \( \mu \) is the kinetic friction coefficient, and \( v_{tip} \) is the velocity vector of the tip of the last link in each sub-motion scuffing on the ground.

3. Conclusions

In the recent past, Automated Guided Vehicles (AGVs) have been the subject of a research effort to improve vehicle intelligence in different applications. Path tracking has been seen as a major challenge in autonomous mobile systems. Currently, some researchers are studying this problem under uncertain conditions such as dynamic obstacles and some known environments. This paper reviewed the different mathematical models for various AGV and service robots. Comparing the different AGV structures shows that the robot platform is mostly categorized based on its wheels or motion mechanism. To summarize, the AGV can be categorized as Differential type with Four, Six and Caterpillar, Independent’s type, Ackerman, Legs and other structures as shown in figure (12). The following figure shows the types of service robots whose dynamic models are accumulated in this paper.

![Figure 9. Types of service robot surveyed](image)

Based on the robots structures, the two Four- and six-wheel robots are typically used to carry higher payloads and / or to traverse rough terrain and they can offer good mobility. Compared to the four-wheel robot, the six-wheel robot has more redundancy in the event of wheel failure or loss. For the same tire size and vehicle mass, six wheels will exert less ground pressure than four wheels. However, six-wheeled vehicles of the same size may be heavier. Six wheels require more complex steering, drivetrain and suspension arrangements than four. To apply the same tractive force, the friction between the six-wheel drive and the ground is less than that of the four-wheel drive [42]. The applications of robotic could be extended on various potential works such as environmental, energy, hybrid materials, biomaterials, etc [43-52].

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