Construction and Elicitation of a Black Box Model in the Game of Bridge

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Abstract We address the problem of building a decision model for a specific bidding situation in the game of Bridge. We propose the following multi-step methodology: i) Build a set of examples for the decision problem and use simulations to associate a decision to each example ii) Use supervised relational learning to build an accurate and interpretable model iii) Perform a joint analysis between domain experts and data scientists to improve the learning language, including the production by experts of a handmade model iv) Use insights from iii) to learn an almost self-explaining and accurate model.

1 Introduction

Our goal is to model expert decision processes in Bridge. To do so, we propose a methodology involving human experts, black box decision programs, and relational supervised machine learning systems. The aim is to obtain a global model for this decision process, that is both expressive and has high predictive performance.

Following the success of supervised methods of the deep network family, and a growing pressure from society imposing that automated deci-
sion processes be made more transparent, a growing number of AI researchers are (re)exploring techniques to interpret, justify, or explain “black box” classifiers (referred to as the Black Box Outcome Explanation Problem [Guidotti et al., 2019]). It is a question of building, a posteriori, explicit models in symbolic languages, most often in the form of rules or decision trees that explain the outcome of the classifier in a format intelligible to an expert. Such explicit models extracted by supervised learning can carry expert knowledge [Murdoch et al., 2019], which has intrinsic value for explainability, pedagogy, and evaluation in terms of ethics or equity (they can help to explain biases linked to the learning system or to the set of training examples). The vast majority of works focus on local interpretability of decision models: simple local explanations (linear models/rules) are built to justify individual predictions of black box classifiers (see for instance the popular LIME [Ribeiro et al., 2016] and ANCHORS [Ribeiro et al., 2018] systems). Such local explanations do not give an overview of the black box model behaviour, they moreover rely on the notion of neighborhood of instances to build explanations and may be highly unstable. Our approach is to build post-hoc global explanations by approximating the predictions of black box models with interpretable models such as decision lists and rule sets.

We present here a complete methodology for acquiring a global model of a black box classifier as a set of relational rules, both as explicit and as accurate as possible. As we consider a game, i.e. a universe with precise and known rules, we are in the favorable case where it is possible to generate, on demand, data that i) is in agreement with the problem specification, and ii) can be labelled with correct decisions through simulations. The methodology we propose consists of the following elements:

1. Problem modelling with relational representations, data generation and labelling.
2. Initial investigation of the learning task and learning with relational learners.
3. Interaction with domain experts who refine a learned model to produce a simpler alternative model, which is more easily understandable for domain users.
4. Subsequent investigation of the learning task, taking into account the concepts used by experts to produce their alternative model, and the proposition of a new, more accurate model.

The general idea is to maximally leverage interactions between experts and engineers, with each group building on the analysis of the other.

We approach the learning task using relational supervised learning methods from Inductive Logic Programming (ILP) [Muggleton and Raedt, 1994]. The language of these methods, a restriction of first-order logic, allows learning compact rules, understandable by experts in the domain. The logical framework allows the use of a domain vocabulary together with domain
knowledge defined in a domain theory, as illustrated in early work on the subject [Legras et al., 2018].

The outline of the article is as follows. After a brief introduction to bridge in Section 2, we describe in Section 3 the relational formulation of the target learning problem, and the method for generating and labelling examples. We then briefly describe in Section 4 the ILP systems used, and the first set of experiments run on the target problem, along with their results (Section 4.2). In Section 5 bridge experts review a learned model’s output and build a powerful alternative model of their own, an analysis of which leads to a refinement of the ILP setup and further model improvements. Future research avenues are outlined in the conclusion.

2 Problem Addressed

Bridge is played by four players in two competing partnerships, namely, North and South against East and West. A classic deck of 52 playing cards is shuffled and then dealt evenly amongst the players \[\frac{52}{4} = 13\] cards each. The objective of each side is to maximize a score which depends on:

- **The vulnerability** of each side. A *non-vulnerable* side loses a low score when it does not make its contract, but earns a low score when it does make it. In contrast, a *vulnerable* side has higher risk and reward.

- **The contract** reached at the conclusion of the auction (the first phase of the game). The contract is the commitment of a side to win a minimum of \(l_{\text{min}} \in \{7, \ldots, 13\}\) tricks in the playing phase (the second phase of the game). The contract can either be in a Trump suit (♣, ♥, ♦, ♠), or No Trumps (NT), affecting which suit (if any) is to gain extra privileges in the playing phase. An opponent may Double a contract, thus imposing a bigger penalty for failing to make the contract (but also a bigger reward for making it). A contract is denoted by \(pS\) (or \(pS^X\) if it is Doubled), where \(p = l_{\text{min}} - 6 \in \{1, \ldots, 7\}\) is the level of the contract, and \(S \in \{♣, ♥, ♦, ♠, NT\}\) the trump suit. The holder of the contract is called the declarer, and the partner of the declarer is called the dummy.

- **The number of tricks won** by the declaring side during the playing phase. A trick containing a trump card is won by the hand playing the highest trump, whereas a trick not containing a trump card is won by the hand playing the highest card of the suit led.

For more details about the game of bridge, the reader can consult [ACBL, 2019].

Two concepts are essential for the work presented here:

- **Auction**: This allows each player (the first being called the dealer) the opportunity to disclose coded information about their hand or game plan
to their partner\footnote{The coded information given by a player is decipherable by both their partner and the opponents, so one can only deceive their opponents if they’re also willing to deceive their partner. In practice, extreme deception in the auction is rare, but for both strategic and practical reasons, the information shared in the auction is usually far from complete.}. Each player bids in turn, clockwise, using as a language the elements: \textit{Pass}, \textit{Double}, or a contract higher than the previous bid (where $\spadesuit < \diamondsuit < \heartsuit < \clubsuit < NT$ at each level). The last bid contract, followed by three Passes, is the one that must be played.

- \textbf{The evaluation of the strength of a hand:} Bridge players assign a value for the highest cards: an Ace is worth 4 HCP (High Card Points), a King 3 HCP, a Queen 2 HCP and a Jack 1 HCP. Information given by the players in the auction often relate to their number of HCP and their distribution (the number of cards in one or more suits).

\section{Problem Statement}

After receiving suggestions from bridge experts, we chose to analyse the following situation:

- West, the dealer, bids 4\spadesuit, which (roughly) means that they have a minimum of 7 spades, and a maximum of 10 HCP in their hand.
- North Doubles, (roughly) meaning that they have a minimum of 13 HCP and, unless they have a very strong hand, a minimum of three cards in each of the other suits (\diamondsuit, \heartsuit and \clubsuit).
- East passes, which has no particular meaning.

South must then make a decision: pass and let the opponents play 4\spadesuit\textsuperscript{X}, or bid, and have their side play a contract. This is a high stakes decision that bridge experts are yet to agree on a precise formulation for. Our objective is to develop a methodology for representing this problem, and to find accurate and explainable solutions using relational learners. It should be noted that Derek Patterson was interested in solving this problem using genetic algorithms \cite{Patterson2008}.

In the remainder of the article, we describe the various processes used in data generation, labelling, supervised learning, followed by a discussion of the results and the explicit models produced. These processes use relational representations of the objects and models involved, keeping bridge experts in the loop and allowing them to make adjustments where required.

In the next section we consider the relational formulation of the problem, and the data generation and labelling.
3 Automatic Data Generation and Modelling Methodology

The methodology to generate and label the data consists of the following steps:

- Problem modelling
- Automatic data generation
- Automatic data labelling
- ILP framing

These steps are the precursors to running relational rule induction (Aleph) and decision tree induction (Tilde) on the problem.

3.1 Problem Modelling

The problem modelling begins by asking experts to define, in the context described above, two rule based models: one to characterize the hands such that West makes the 4♦ bid, and another to characterize the hands such that North makes the Double bid. These rule based models are submitted to simulations allowing the experts to interactively validate their models. With the final specifications, we are able to generate examples for the target problem. For this section we introduce the terms:

- \textit{nmpq exact distribution} which indicates that the hand has \(n\) cards in ♠, \(m\) cards in ♥, \(p\) cards in ♦ and \(q\) cards in ♣, where \(n + m + p + q = 13\).
- \textit{nmpq distribution} refers to an exact distribution sorted in decreasing order (thus ignoring the suit information). For instance, a 2533 exact distribution is associated to a 5332 distribution.
- \(|c|\) is the number of cards held in the suit \(c \in \{♠, ♥, ♦, ♣\}\).

3.1.1 Modelling the 4♦ bid

Experts have modeled the 4♦ bid by defining a disjunction of 17 rules that relate to West hand:

\[
4♦ \leftarrow \bigvee_{i=1}^{17} R_i, \text{ where } R_i = C_0 \land V_i \land C_i
\]

(1)

in which

- \(C_0\) is a condition common to the 17 rules and is reported in Listing ?? in the Appendix.
- \(V_i\) is one of the four possible vulnerability configurations (no side vulnerable, both sides vulnerable, exactly one of the two sides vulnerable).
• $C_j$ is a condition specific to the $R_j$ rule.

For instance, the conditions for rules $R_2$ and $R_5$ are:

• $V_2 = \text{East-West not vulnerable, North-South vulnerable.}$
• $C_2 = \text{all of:}$
  - $|\spadesuit| = 1$,
  - 2 cards exactly among Ace, King, Queen and Jack of $\spadesuit$,
  - a 7321 distribution.
• $V_5 = \text{East-West not vulnerable, North-South vulnerable.}$
• $C_5 = \text{all of:}$
  - $|\clubsuit| \geq 4$ or $|\heartsuit| \geq 4$,
  - 2 cards exactly among Ace, King, Queen and Jack of $\spadesuit$,
  - a 7mpq distribution with $m \geq 4$.

To generate boards that satisfy these rules, we randomly generated complete boards (all 4 hands), and kept the boards where the West hand satisfies one of the 17 rules. The experts were able to iteratively adjust the rules as they analysed boards that either passed through the filter, or failed to pass through the filter (but perhaps should have). After the experts were happy with the samples, 8,200,000 boards were randomly generated to analyse rule adherence, and 10,105 of them contained a West hand satisfying one of the 17 rules. All rules were satisfied at least once. Of the times where at least one rule was satisfied, $R_2$, for example, was satisfied 16.2% of the time, and $R_3$ was satisfied 15% of the time.

### 3.1.2 Double Modelling

Likewise, the bridge experts also modeled the North Double by defining a disjunction of 3 rules relating to the North hand:

$$\text{Double} \leftarrow \bigvee_{i=1}^{3} R'_i, \text{ where } R'_i = C'_0 \land C'_i$$

This time, the conditions do not depend on the vulnerability. The common condition $C'_0$ and the specific conditions $C'_i$ are as follows:

• $C'_0$ - for all $c, c_1, c_2 \in \{\heartsuit, \spadesuit, \clubsuit\}, |c| \leq 5$ and not ($|c_1| = 5$ and $|c_2| = 5$).
• $C'_1$ - HCP $\geq 13$ and $|\heartsuit| \leq 1$.
• $C'_2$ - HCP $\geq 16$ and $|\clubsuit| = 2$ and $|\heartsuit| \geq 3$ and $|\spadesuit| \geq 3$ and $|\heartsuit| \geq 3$.
• $C'_3$ - HCP $\geq 20$.

The same expert validation process was carried out as in Section 3.1.1 A generation of 70,000,000 boards resulted in 10,007 boards being satisfied by at least one $4\heartsuit$ bid rule for the West hand and at least one Double rule for
the North hand. All rules relating to Double were satisfied at least once. Of the times that at least one rule was satisfied, $R'_1$, for example, was satisfied 69.3% of the time, and $R'_2$ was satisfied 24.8% of the time.

### 3.2 Data Generation

The first step of the data generation process is to generate a number of South hands in the context described by the $4\spadesuit$ and Double rules mentioned above. Note, again, that East’s Pass is not governed by any rules, which is close to the real situation.

We first generated 1,000 boards whose West hands satisfied at least one $4\spadesuit$ bid rule and whose North hands satisfied at least one Double rule. One such board is displayed in Example 1:

**Example 1** A board (North-South vulnerable / East-West not vulnerable) which contains a West hand satisfying rules $R_0$ and $R_5$ and a North hand satisfying rules $R'_0$ and $R'_1$:

```
♠ 6
♥ A Q 9 3
♦ K Q 10 4 2
♣ A 9 4

♠ A K J 8 7 5 2
♥ 7
♦ J 8 6 3
♣ 3

E W N S

♠ 10 3
♥ J 8 4
♦ A 7
♣ Q J 8 7 6 2

♠ Q 9 4
♥ K 10 6 5 2
♦ 9 5
♣ K 10 3
```

For reasons that become apparent in Section 3.3, for each of the 1,000 generated boards, we generated an additional 999 boards. We did this by fixing the South hand in each board, and randomly redistributing the cards of the other players until we found a board satisfying Equations 1 and 2. As a result of this process, 1,000 files were created, with each file containing 1,000 boards that have the same South hand, but different West, North and East hands.

**Example 2** One of the 999 other boards generated:
- **West hand:** ♠AK108653 ♥4 ♦832 ♣84
- **North hand:** ♥J2 ♥AJ73 ♠AK106 ♥AJ9
- **East hand:** ♠7 ♥Q98 ♦QJ74 ♣Q7652
• South hand: ♠Q94 ♥K1062 ♦95 ♣K103.

Note that the South hand is identical to the one in Example 1, the West hand satisfies rules \( R_0 \) and \( R_2 \), and the North hand satisfies rules \( R'_0 \) and \( R'_2 \).

### 3.3 Data Labelling

The goal is to label each of the 1,000 South hands associated to the 1,000 sample files, with one of following labels:

- **Pass** when the best decision of South is to pass (and therefore have West play \( 4♠X \)).
- **Bid** when the best decision of South is to bid (and therefore play a contract on their side).
- **?** when the best decision is not possible to be determined. We exclude these examples from ILP experiments.

These labels were assigned by computing the score at \( 4♠X \) and other possible contracts by simulation.

#### 3.3.1 Scores Computation

During the playing phase, each player sees their own hand, and that of the dummy (which is laid face up on the table). A Double Dummy Solver (DDS) is a computation (and a software) that calculates, in a deterministic way, how many tricks would be won by the declarer, under the pretext that everyone can see each other’s cards. Though unrealistic, it is nevertheless a good estimator of the real distribution of tricks amongst the two sides [Pavlicek, 2014]. For any particular board, a DDS can be run for all possible contracts, so one can thus deduce which contract will likely yield the highest score.

For each example file (which each contain 1,000 boards with the same South hand), a DDS software [Haglund et al., 2014] was used to determine the score for the following situations:

- \( 4♠X \) played by the East/West side.
- All available contracts played by the North/South side, with the exclusion of 4NT, 5NT, 5♣, 6♣ and 7♠ (experts deemed these contracts infeasible to be reached in practice).

#### 3.3.2 Labels Allocation

The labels are assigned to each unique South hand by applying the following tests sequentially.
1. Take the DDS score from the best North/South contract on each of the 1,000 boards (the optimal contract may not always be the same), and average them. If the average score of defending $4\Spadesuit^X$ is higher than this score, assign the label of Pass to the South hand.

2. Take the North/South contract with the highest mean DDS score over all of the boards (this may not be the best contract on each board, but merely the contract with the highest mean score across all boards). If the mean score of this contract is greater than the mean score at $4\Spadesuit^X$, assign Bid to the South hand. If the mean score of the contract is at least 30 total points lower than the mean score at $4\Spadesuit^X$, assign Pass. Otherwise, assign the label ?.

The initial dataset containing 1,000 sample files is reduced to a set $S$ of 961 examples after elimination of the 39 deals with the ? label. The resulting dataset consists of:

- 338 Bid labels, and
- 623 Pass labels.

### 3.4 Relational data modelling

Now that we have generated and labeled our data, the next step is to create the relational representations to be used in relational learners. The learning task consists in using what is known about a given board in the context described above, and the associated South hand, and predicting what the best bid is. The best bid is known for the labeled examples generated in the previous section, but, of course, unknown for new boards.

We first introduce the notations used in the remainder of the paper.

#### 3.4.1 Logic Programming Notations

Examples are represented by terms and relations between terms. Terms are constants (for instance the integers between 1 and 13), or atoms whose identifiers start with a lower-case character (for instance west, east, spade, heart, ...) or variables, denoted by an upper-case character (X, Y, Suit, ...). Variables may instantiate to any constant in the domain. Relations between objects are described using predicates applied to terms. A literal is a predicate symbol applied to terms, and a fact is a ground literal, i.e. a literal without variables, whose arguments are ground terms only. For instance, \texttt{decision([sq, s9, s4, hk, h10, h6, h5, h2, d9, d5, ck, c10, c3], 4, n, west, bid)} is a ground literal of predicate symbol \texttt{decision}, with four arguments, namely, a hand described as a list of cards, and 4 constant arguments (see Section 3.4.2 for further details).
In order to introduce some flexibility in the problem description, the background knowledge (also known as the domain theory) is described as set of definite clauses. We can think of a definite clause as a first order logic rule, containing a head (the conclusion of the rule) and a body (the premises of the rule). For instance, the following clause:

\[ nb(\text{Hand}, \text{Suit}, \text{Value}) \leftarrow \text{suit} (\text{Suit}), \text{count} \text{cards of suit} (\text{Hand}, \text{Suit}, \text{Value}) \]

has the head \( nb(\text{Hand}, \text{Suit}, \text{Value}) \) where \( \text{Hand} \), \( \text{Suit} \) and \( \text{Value} \) are variables and two literals in the body \( \text{suit} (\text{Suit}) \) and \( \text{count} \text{cards of suit} (\text{Hand}, \text{Suit}, \text{Value}) \). The declarative interpretation of the rule is that given that variable \( \text{Suit} \) is a valid bridge suit, and given that \( \text{Value} \) is the number of cards of the suit \( \text{Suit} \) in the hand \( \text{Hand} \), one can derive, in accordance with the logical consequence relationship \( \models [\text{Lloyd, 1987}] \) that \( nb(\text{Hand}, \text{Suit}, \text{Value}) \) is true. This background knowledge will be used to derive additional information concerning the examples, such as hand and board properties.

Let us now describe the \textit{decision} predicate, the target predicate of the relational learning task.

### 3.4.2 Target Predicate

Given a hand, the position of the hand, vulnerability, and the dealer's position, the goal is to predict the class label. The target predicate for the bidding phase is \( \text{decision} (\text{Hand}, \text{Position}, \text{Vul}, \text{Dealer}, \text{Class}) \) where the arguments are variables.

- \( \text{Hand} \): the 13 cards of the player who must decide to \textit{bid} or \textit{pass}.
- \( \text{Position} \): the relative position of the player to the dealer. In this task, \( \text{Position} \) is always 4, the South hand.
- \( \text{Vul} \): the vulnerability configuration (\( b = \) both sides, \( o = \) neither side, \( n = \) North-South or \( e = \) East-West).
- \( \text{Dealer} \): the cardinal position of the dealer among \textit{north}, \textit{east}, \textit{south} or \textit{west}.
  
  In this task, \( \text{Dealer} \) is always \textit{west}.
- \( \text{Class} \): label to predict (\textit{bid} or \textit{pass}).

In this representation, a labeled example is a grounded positive literal containing the decision predicate symbol. Example 1 given in section 3.2 is therefore represented as:

\[ \text{decision} ([\text{sq}, \text{s9}, \text{s4}, \text{hk}, \text{h10}, \text{h6}, \text{h5}, \text{h2}, \text{d9}, \text{d5}, \text{ck}, \text{c10}, \text{c3}], 4, \text{n}, \text{west}, \text{bid}) \].

Such a ground literal contains all the explicit information about the example. However, in order to build an effective relational learning model of the target predicate as a set of rules, we need to introduce abstractions of this information, in the form of instances of additional predicates in the domain theory. These predicates are described below.
3.4.3 Target concept predicates

In order to learn a general and explainable model for the target concept, we need to carefully define the target concept representation language by choosing predicate symbols that can appear in the model, more specifically, in the body of definite clauses for the target concept definition. We thus define new predicates in the domain theory, both extensional (defined by a set of ground facts) and intensional (defined by a set of rules or definite clauses). The updated domain theory allows to complete the example description, by gathering/deriving true facts related to the example. It also implicitly defines a target concept language that forms the search space for the target model which the Inductive Logic Programming algorithms will explore using different strategies. Part of the domain theory for this problem was previously used in another relational learning task for a bridge bidding problem [Legras et al., 2018], illustrating the reusability of domain theories. The predicates in the target concept language are divided in two subsets:

- Predicates that were introduced in [Legras et al., 2018]. These include both general predicates, such as \( \text{gteq}(A,B)\) (\(A \geq B\)), \( \text{lteq}(A,B)\) (\(A \leq B\)) and predicates specific to bridge, such as \( \text{nb}(\text{Hand},\text{Suit},\text{Number})\), which tests that a suit \(\text{Suit}\) of a hand \(\text{Hand}\) has length \(\text{Number}\) (e.g. \(\text{nb}(\text{Hand},\text{heart},3)\) means that the hand \(\text{Hand}\) has 3 cards in \(\text{heart}\)).
- Higher level predicates not used in [Legras et al., 2018]. Among which:
  - \(\text{hcp}(\text{Hand},\text{Number})\) is the number of High Card Points (HCP) in \(\text{Hand}\).
  - \(\text{suit}\_\text{representation}(\text{Hand},\text{Suit},\text{Honors},\text{Number})\) is an abstract representation of a suit. \(\text{Honors}\) is the list of cards strictly superior to 10 of the suit \(\text{Suit}\) in hand \(\text{Hand}\). \(\text{Number}\) is the total number of cards in \(\text{Suit}\) suit.
  - \(\text{distribution}(\text{Hand},[N,M,P,Q])\) states that the \(\text{nmpq}\) distribution (see Section 3.1) of hand \(\text{Hand}\) is the ordered list \([N,M,P,Q]\).

For instance, in Example 1 from Section 1, we may add to the representation the following literals:

- \(\text{hcp}([\text{sq},\text{s9},\text{s4},\text{hk},\text{h10},\text{h6},\text{h5},\text{h2},\text{d9},\text{d5},\text{ck},\text{c10},\text{c3}],8)\).
- \(\text{suit}\_\text{representation}([\text{sq},\text{s9},\text{s4},\text{hk},\text{h10},\text{h6},\text{h5},\text{h2},\text{d9},\text{d5},\text{ck},\text{c10},\text{c3}],\text{spade},[\text{q}],3)\).
- \(\text{distribution}([\text{sq},\text{s9},\text{s4},\text{hk},\text{h10},\text{h6},\text{h5},\text{h2},\text{d9},\text{d5},\text{ck},\text{c10},\text{c3}],\text{spade},[\text{q}],3)\).

Note again, that a fact (i.e. ground literal) can be independent of the rest of the domain theory, such as \(\text{has\_suit}(\text{hk},\text{heart})\) which states that \(\text{hk}\) is a \(\text{hearts}\) card, or it can be inferred from existing facts and clauses of the domain theory, such as \(\text{nb}([\text{sq},\text{s9},\text{s4},\text{hk},\text{h10},\text{h6},\text{h5},\text{h2},\text{d9},\text{d5},\text{ck},\text{c10},\text{c3}],\text{heart},5)\), which derives from:

- \(\text{suit}(\text{heart})\): \text{heart} is a valid bridge suit.
- \(\text{has\_suit}(\text{hk},\text{heart})\): \text{hk} is a \text{heart} card.
\[ nb(\text{Handlist}, \text{Suit}, \text{Value}) \leftarrow \text{suit}(\text{Suit}), \text{count card of suit}(\text{Handlist}, \text{Suit}, \text{Value}) \]

where \( \text{count card of suit} \) has a recursive definition within the domain theory.

### 4 Learning Expert Rules

#### 4.1 Inductive Logic Programming Systems

The ILP systems used in our experiments were Aleph and Tilde. These are two mature state of the art ILP systems of the symbolic SRL paradigm\(^2\) Recent work showed that such ILP systems, Tilde in particular, are competitive with recent distributional SRL approaches that first learn a knowledge graph embedding to encode the relational datasets into score matrices that can then fed to non-relational learners. Given a range of some relational datasets, [Dumancic et al., 2019] shows that there is no clear winner on classification tasks when comparing distributional and symbolic learning methods, and that both exhibit strengths and limitations. Tilde on the other hand outperforms KGE based approaches on Knowledge Base Completion tasks (namely concerning the ability to infer the status (true/false) for missing facts in relational datasets).

As our work aims to build explainable models in a deterministic context we only consider here ILP systems. The most prominent difference between Aleph and Tilde is their output: a set of independent relational rules for Aleph and a relational decision tree for Tilde. One important declarative parameter of both systems – and in symbolic SRL systems in general – is the so-called language bias that specifies which predicates defined in the background knowledge may be used and in what form these predicates appear in the learned model (the rule set for Aleph and the decision tree for Tilde)

#### 4.1.1 Aleph

Given a set of positive examples \( E^+ \), a set of negative examples \( E^- \), each as ground literals of the target concept predicate and a domain theory \( T \), Aleph [Srinivasan, 1999] builds a hypothesis \( H \) as a logic program, i.e. a set of definite clauses, such that \( \forall e^+ \in E^+ : H \land T \models e^+ \) and \( \forall e^- \in E^- : H \land T \not\models e^- \). In other words, given the domain theory \( T \), and after learning hypothesis \( H \), it is possible from each positive example description \( e^+ \) to derive that \( e^+ \) is indeed a positive example, while such a derivation is impossible for any

\(^2\) SRL stands for Statistical Relational Learning and is a recent development of ILP [Raedt et al., 2016] that integrates learning and reasoning under uncertainty about individuals and actions effect.
of the negative examples $e^-$. The domain theory $T$ is represented by a set of facts, i.e. ground positive literals from other predicates and clauses, possibly inferred from a subset of primary facts and a set of unground clauses (see Section 3.4). $H$ is a set of definite clauses where the head predicate of each rule is the target predicate. An example of such a hypothesis $H$ is given in Listing 5.

4.1.2 Tilde

Tilde [Blockeel and Raedt, 1998] relies on the learning by interpretation setting. Tilde learns a relational binary decision tree in which the nodes are conjunctions of literals that can share variables with the following restriction: a variable introduced in a node cannot appear in the right branch below that node (i.e. the failure branch of the test associated to the node). In the decision tree, each example, described through a set of ground literals, is propagated through the current tree until it reaches a leaf. In the final tree, each leaf is associated to the decision bid/pass, or a probability distribution on these decisions. An example of a Tilde decision tree output is given in Listing 4.

4.2 Learning setup

In our problem, the target predicate is decision, as defined in Section 3.4.2 and exemplified in Section 3.4.2. Regarding Aleph, in positive examples the last argument of the target predicate has value bid while in negative examples it has value pass. The Aleph standard strategy induce learns the logic program as a set of clauses using a hedging strategy: at each iteration it selects a seed positive example $e$ not entailed by the current solution, find a clause $R_e$ which covers $e$, and then remove from the learning set the positive examples covered by $R_e$. The learning procedure is repeated until all positive examples are covered. The induce strategy is sensitive to the order of the learning examples. We also used Aleph with induce max, which is more expensive but insensitive to the order of positive examples: it constructs a best clause, in terms of coverage, for each positive example, then selects a clause subset. induce max has proved beneficial in some of our experiments.

3 Available as part of the ACE Ilp system at https://dtai.cs.kuleuven.be/ACE/
4.2.1 Experimental setup

The performance of the learning system with respect to the size of the training set is averaged over 50 executions \( i \). The sets \( \text{Test}_i \) and \( \text{Train}_i \) are built as follows:

- \( \text{Test}_i = 140 \) examples randomly selected from \( S \) using seed \( i \).
- Each \( \text{Test}_i \) is stratified, i.e. it has the same ratio of examples labeled \( \text{bid} \) as \( S \) (35.2 %).
- \( \text{Train}_i = S \setminus \text{Test}_i \) (820 examples).

For each execution \( i \), we randomly generate from \( \text{Train}_i \) a sequence of increasing number of examples \( n_k = 10 + 100 \times k \) with \( k \) between 0 and 8. We denote each of these subsets \( T_{i,k} \), so that \( |T_{i,k}| = n_k \) and \( T_{i,k} \subset T_{i,k+1} \). Each model trained on \( T_{i,k} \) is evaluated on \( \text{Test}_i \).

4.2.2 Evaluation criteria

As we can interpret our methodology as a post-hoc agnostic black-box explanation framework, we evaluate this framework in terms of fidelity, unambiguity and interpretability [Lakkaraju et al., 2019].

- A high fidelity explanation should faithfully mimic the behavior of the black box model. To measure fidelity we use the accuracy of the surrogate relational model with respect to the \( \text{he} \) labels assigned by the black box model.
- Unambiguity evaluates if one or multiple (potentially conflicting) explanations are provided by the relational surrogate model for a given instance. While we do not explicitly report on unambiguity in our experiments, we note that decision trees by nature are non ambiguous models. Rule sets, on the other hand, may indeed display some overlap. However, but as our Aleph experiments only consider one target label (see Listing 5) even if two rules apply for some instance, they conclude on the same label and the model again displays no ambiguity. That does not mean that the notion is useless in our context, rather that we have to find more sophisticated ways to evaluate in which sense such models include some level of ambiguity.
- Interpretability quantifies how easy it is to understand and reason about the explanation. This dimension of explanation is subjective and as such, maybe prone to the expert/user bias. We therefore choose a crude but objective metric: the model complexity, i.e. the number of rules/nodes of the models. We will also see that in our experiments we obtain similar decision tree accuracies with \( L_2 \), a language containing only predicates appearing in a human-made model, as with the much larger language \( L_1 \). Clearly, the \( L_2 \) models have better interpretability.
4.3 First experiments

The first experiments involve a baseline non relational data representation, together with the $L_0$ language mentioned earlier as well as a smaller languages $L_1$. Table 1 and Figures 1 and 2 display the results of our experiments as well as results obtained using a different language $L_2$ further introduced in Section 5.2.

We have first learned Random Forest models, using the `sklearn` python package [Pedregosa et al., 2011], with default parameters including using forests made of 100 trees. The data representation is as follows: each hand is described by 54 attributes: vulnerability, target class, highest card, second highest card, ..., 13th highest card in each suit (ordered as ♣ < ♥ < ♠). If a player has $n$ cards in a given suit, the corresponding $n+1,...,13$th attributes for this suit are set to 0.

The relational models are learned with Tilde and Aleph where Aleph is trained on both the `induce` and `induce_max` settings. Figure 1-left presents the average accuracy, over all $i = 1...50$ executions, of the models trained on the sets $T_i,k$ and therefore requiring building 450 models for each curve. Figure 2 displays the corresponding model complexity, expressed in average number of nodes for Tilde and Random Forest. Table 1 displays the average accuracies and complexities of each experiment for learning set sizes 310 ($k=4$) and 810 ($k=9$). An experiment represents 450 Tilde or Aleph executions, and the resulting CPU cost of these executions is also recorded. As the experiments were performed on similar machines with a different number of cores (either 8 or 16) we report the CPU cost as the number of cores of the machine times the total CPU time, i.e. in core hours units.

Aleph failed to cope with the large $L_0$ language, and couldn’t return a solution in reasonable time. As can be seen in Figure 1 and Table 1, the $L_0$-Tilde models had rather low accuracy and a large number of nodes (which grew linearly with the number of training examples used).

After an analysis of the Tilde output trees, it became apparent that Tilde was selecting many nodes with very specific conditions, in particular, with the suit_representation and distribution predicates. While this improved performance in training, the specificity hindered the performance in testing. To counter this, we removed from $L_0$ the suit_representation and distribution predicates, creating the more concise language $L_1$.

As expected, the average number of nodes in $L_1$-Tilde reduced dramatically. $L_1$-Tilde had the best accuracy of all models, which continued to increase between 610 and 810 examples. Its complexity was relatively stable,

---

4 For Random Forest, the reported number of nodes is an average over the 100 trees of a model.
5 For instance, 16 core-hours represents 1 hour of CPU time on a 16-core machine or 2 hours CPU time on a 8-core machine.
6 Aleph was able to learn models after we removed the $nb$ predicate from $L_0$, albeit rather slowly ($\approx 384$ core hours).
Fig. 1 Average model accuracies of $L_0$, $L_1$ models (left) and $L_2$ models (right).

Fig. 2 Average model complexities of rule-based (left) and tree-based (right) models.

| Model            | Accuracy-310 | Complexity-310 | Accuracy-810 | Complexity-810 | Core Hours |
|------------------|--------------|----------------|--------------|----------------|------------|
| L0-Tilde         | 80.6%        | 23 nodes       | 83.7%        | 49 nodes       | 93.5       |
| L1-Tilde         | 88.1%        | 15 nodes       | 89.8%        | 22 nodes       | 2.0        |
| L1-Aleph-ind     | 85.8%        | 17 rules       | 85.6%        | 16 rules       | 55.5       |
| L1-Aleph-ind_max | 84.5%        | 48 rules       | 84.4%        | 47 rules       | 256.2      |
| L2-Tilde         | 85.5%        | 12 nodes       | 87.3%        | 26 nodes       | 2.3        |
| L2-Aleph-ind     | 82.7%        | 6 rules        | 83.5%        | 6 rules        | 9.7        |
| L2-Aleph-ind_max | 89.2%        | 20 rules       | 88.2%        | 20 rules       | 25.5       |
| Random-Forest    | 84.3%        | 101 nodes      | 86.8%        | 242 nodes      | <0.1       |

Table 1 Summary of average accuracy and complexity for all models when trained on 310 and 810 examples, together with total CPU cost of the 450 Tilde or Aleph executions in CPUtime×hours. Reported complexity of Random Forest only concerns one tree among the 100 trees in the forest.
increasing only slightly between 510 and 810 examples, to a total of 21 tree nodes. With the smaller $L_1$ language, $L_1$-Aleph-induce was able to find solutions in reasonable time. It’s accuracy was worse than that of $L_1$-Tilde (85.6% compared to 89.8% when training on 810 examples). After 310 examples, the $L_1$-Aleph-induce model size remained stable at 16 rules (its accuracy also showed stability over the same training set sizes).

We also used Aleph’s `induce_max` strategy with $L_1$. Compared to `induce`, this search strategy has a higher time cost and can produce rules with large overlaps in coverage of learning examples, but can generalize better due to the fact that it saturates every example in the training set, not just a subset based on the coverage of a single seed example. This search strategy was of no benefit in this case, with $L_1$-Aleph-induce_max showing slightly worse accuracy (84.4% compared to 85.6%) and a much higher complexity (47 rules compared to 15) when compared with $L_1$-Aleph-induce.

The CPU cost of learning the Random Forest models is negligible (less than 10 seconds) compared to those of relational models. The accuracies lie between those of $L_1$-Tilde and of $L_1$-Aleph models, while the number of nodes per tree is much higher (see Figure 2-right). This indicates that there is no gain in using ensemble learning on non-relational decision trees, although the data representation is much simpler and needs only limited Bridge expertise to define. The Random Forest models are, as expected, difficult to interpret, apart from variable importance: It appears that the first three highest cards in each suit and vulnerability are the most important variables in this task. It would be possible to use post-hoc formal methods to \textit{a posteriori} explain those random forest models \cite{Izza and Marques-Silva, 2021}, however the whole process would be expensive, and, after \cite{Rudin et al., 2021}, \textit{Explaining black boxes, rather than replacing them with interpretable models, can make the problem worse by providing misleading or false characterizations.}

Following these experiments, we had bridge experts do a thorough analysis of one of the successful $L_1$-Tilde outputs. Their goal was to create a submodel which was much easier to interpret than the large decision trees\footnote{The experts noted that, in general, the Tilde trees were more difficult to read than the rule-based output produced by Aleph.} and had an accuracy comparable to that seen by $L_1$-Tilde. We also used the experts’ insights to further revise the domain theory, in the hope of learning a simpler model with Aleph and Tilde. We describe this process in the next section.
5 Expert Rule Analysis and new Experiments

5.1 Expert Rule Analysis

To find an $L_1$-Tilde output tree for the experts to analyse, we chose a model with good accuracy and low complexity (a large tree would be too hard to parse for a bridge expert). A subtree of the chosen $L_1$-Tilde output is displayed in Listing 1 (the full tree is displayed in Listing 4 in the Appendix). Despite the low complexity of this tree (only 15 nodes), it remains somewhat difficult to interpret.

Listing 1 Part of the L1-tree learned by Tilde

```
decision(¬A, ¬B, ¬C, ¬D, ¬E)
nb(A, ¬F, ¬G), gteq(G, 6) ?
  +--yes: hcp(A, 4) ?
    |    +--yes: [pass]
    |    +--no: [bid]
  +--no: hcp(A, ¬H), gteq(H, 16) ?
    +--yes: [bid]
    +--no: nb(A, spade, ¬I), gteq(I, 1) ?
      +--yes: lteq(I, 1) ?
        |        +--yes: hcp(A, ¬J), lteq(J, 5) ?
        |        |    +--yes: [pass]
        |        |    +--no: nb(A, ¬K, 5) ?
        |        +--yes: hcp(A, 6) ?
        |        |    +--yes: [pass]
        |        |    +--no: [bid]
        +--no: [pass]
```

Simply translating the tree to a decision list does not solve the issue of interpretability. Thus, with the help of bridge experts, we proceeded to do a manual extraction of rules from the given tree, following these post-processing steps:

1. Natural language translation from relational representations.
2. Removal of nodes with weak support (e.g. a node which tests for precise values of HCP instead of a range).
3. Selection of rules concluding on a bid leaf.
4. Reformulation of intervals (e.g. combining overlapping ranges of HCP or number of cards).
5. Highlighting the specific number of cards in ♠, and specific $nmpq$ distributions.

The set of five rules resulting from this post-processing, shown Listing 3 in the next section, are much more readable than $L_1$-Tilde trees. Despite its low complexity, this set of rules has an accuracy of 90.6% on the whole of $S$. 
5.2 New experiments

Following this expert review, we created the new $L_2$ language, which only used the predicates that appear in the post-processed rules above. This meant the reintroduction of the distribution predicate, the addition of the $nbs$ predicate (which is similar to $nb$ but denotes only the number of ♠ in the hand), and the omission of several other predicates. We display Table 2 in the Appendix the predicates involved in the languages $L_0$, $L_1$ and $L_2$. Results are displayed together with those regarding $L_0$ and $L_1$ on Figure 1, Figure 2 and Table 1.

Using $L_2$ with Aleph-induce results in a much simpler model than $L_1$-Aleph-induce (6 rules compared to 16 when trained on 810 examples), at the cost of somewhat lower accuracy (83.5% compared to 85.6%). The complexity of $L_2$-Tilde was also smaller than its counterpart $L_1$-Tilde (12 nodes compared to 15), also at the expense of accuracy (87.3% compared to 89.8%). In both cases, the abstracted and simplified $L_2$ language produced models which were far less complex (and thus more easily interpretable), but had a slight degradation in accuracy, when compared with $L_1$.

When using $L_2$ the more expensive induce_max search strategy proved useful, with $L_2$-Aleph-induce_max showing an accuracy of 88.2%, second only to $L_1$-Tilde (89.8%). This was at the cost of complexity, with $L_2$-Aleph-induce_max models having an average of 20 rules compared to 6 for $L_2$-Aleph-induce.

We selected a $L_2$-Aleph-induce_max model, displayed Listing 5 in the Appendix. The rules of this model are relatively easy to apprehend, and the model achieves an accuracy of 89.7% on the whole example set $S$. We provide Listing 6 the experts translation $M$ of this model and compare it to the rewriting $H$ displayed Listing 3 of the $L_1$-Tilde tree model which led to the $L_2$ language definition. The translation is much simpler than the previous one and required only steps 1, 4 and 5 of the original translation process. An in-depth expert explanation of how the translated models differ is provided in the Appendix. It is noticeable that the two translated models are very similar: each rule in $M$ matches, with only slight differences, the rule in $H$ with same number. The only apparent exception is the overly specific rule $M2b$. A closer look to rule $H2$ reveals that rule $M2b$ completes rule $M2$ by considering the case in which the player has 6 cards, leading to a closer match to human rule $H2$. 
Listing 2  Translation of the $L_2$-Aleph-induce-max model

We bid:
M1 − With 0♠.
M2 − With 7+ cards in a non-spade suit OR 14+HCP.
M2b − With 6+ cards in a non-spade suit AND
  (1♠ AND 5−13HCP) OR
  (2♠ AND 7−13HCP) OR
  (3♠ AND 10−13HCP)
M3 − With 1♠ AND (5−4−3−1♠ OR 5−5−2−1♠) AND 8−13HCP.
M4 − With 2♠ AND 5−5−2−1
M5 − With 2♠ AND (5−4−2−2♠ OR 5−3−3−2♠) AND 13HCP.

Listing 3  Rules extracted by the experts from the L1-Tilde tree

We bid:
H1. − With 0♠.
H2 − With 6+ cards in a non-spade suit OR 16+HCP.
H3 − With 1♠ AND (5−4−3−1♠ or 5−5−2−1♠) AND 6−15HCP.
H4 − With 2♠ AND 5−5−2−1 AND 7−15HCP.
H5 − With 2♠ AND 5−4−2−2♠ AND 11−15HCP.

6 Conclusion

We conducted an end-to-end experiment to acquire expert rules for a bridge bidding problem. We did not propose new technical tools, our contribution was in investigating a complete methodology to handle a learning task from beginning to end, i.e. from the specifications of the learning problem to a learned model, obtained after expert feedback, who not only criticise and evaluate the models, but also propose hand-made (and computable) alternative models. The methodology includes generating and labelling examples by using both domain experts (to drive the modelling and relational representation of the examples) and a powerful simulation-based black box for labelling. The result is a set of high quality examples, which is a prerequisite for learning a compact and accurate relational model. The final relational model is built after an additional phase of interactions with experts, who investigate a learned model, and propose an alternative model, an analysis of which leads to further refinements of the learning language. We argue that interactions between experts and engineers are made easy when the learned models are relational, allowing experts to revisit and contradict them. Indeed, a detailed comparison between the models, as translated by experts, shows that the whole process has led to quite a fixed-point. Clearly, learning tasks in a well defined and simple universe with precise and deterministic rules, such as in games, are good candidates for the interactive methodology we proposed. The extent to which experts and learned models
cooperating in this way can be transferred to less well defined problems, including, for instance, noisy environments, has yet to be investigated. Note that complementary techniques can also be applied, such as active learning, where machines or humans dynamically provide new examples to the system, leading to revised models. Overall, our contribution is a step towards a complete handling of learning tasks for decision problems, giving domain experts (and non-domain experts alike) the tools to both make accurate decisions, and to understand and scrutinize the logic behind those decisions.

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Appendix 1

7 Tree-based and rule-based models

Listing 4 A complete L1-tree learned by Tilde

decision(−A,−B,−C,−D,−E)
    nb(A,−F,−G),gteq(G,6)?
        ++−yes: ∧ hcp(A,4)?
            | ++−yes: [pass] 2.0
            | ++−no: [bid] 57.0
        ++−no: ∧ hcp(A,−H),gteq(H,16)?
            | ++−yes: [bid] 6.0
            | ++−no: ∧ nb(A, spade,−I),gteq(I,1)?
                | ++−yes: ∧ lteq(I,1)?
                |           | ++−yes: ∧ hcp(A,−J),lteq(J,5)?
                |           |             | ++−yes: [pass] 11.0
                |           |             | ++−no: ∧ nb(A,−K,5)?
                |           |             |                 | ++−yes: ∧ hcp(A,6)?
                |           |             |                 |             | ++−yes: [pass] 3.0
                |           |             |                 |             | ++−no: [bid] 27.0
                |           |             | ++−no: [pass] 3.0
            | ++−no: ∧ hcp(A,15)?
                | ++−yes: [bid] 3.0
                | ++−no: ∧ nb(A,−L,3)?
                    | ++−yes: [pass] 146.0
                    | ++−no: ∧ hcp(A,11)?
                        | ++−yes: [bid] 3.0
                        | ++−no: ∧ hcp(A,−M),lteq(M,6)?
                            | ++−yes: [pass] 15.0
                            | ++−no: ∧ lteq(I,2)?
                                | ++−yes: ∧ nb(A,−N,1)?
                                    | ++−yes: [bid] 3.0
                                    | ++−no: ∧ hcp(A,−O),gteq(O,11)?
                                        | ++−yes: [bid] 5.0
                                        | ++−no: ∧ [pass] 16.0
                                    | ++−no: [pass] 5.0
                                | ++−no: ∧ lteq(I,2)?
                                    | ++−yes: ∧ nb(A,−N,1)?
                                        | ++−yes: [bid] 3.0
                                        | ++−no: ∧ hcp(A,−O),gteq(O,11)?
                                            | ++−yes: [bid] 5.0
                                            | ++−no: ∧ [pass] 16.0
                                    | ++−no: [pass] 5.0
                            | ++−no: ∧ lteq(I,2)?
                                | ++−yes: ∧ nb(A,−N,1)?
                                    | ++−yes: [bid] 3.0
                                    | ++−no: ∧ hcp(A,−O),gteq(O,11)?
                                        | ++−yes: [bid] 5.0
                                        | ++−no: ∧ [pass] 16.0
                                    | ++−no: [pass] 5.0
                        | ++−no: ∧ hcp(A,−M),lteq(M,6)?
                            | ++−yes: [pass] 15.0
                            | ++−no: ∧ lteq(I,2)?
                                | ++−yes: ∧ nb(A,−N,1)?
                                    | ++−yes: [bid] 3.0
                                    | ++−no: ∧ hcp(A,−O),gteq(O,11)?
                                        | ++−yes: [bid] 5.0
                                        | ++−no: ∧ [pass] 16.0
                                    | ++−no: [pass] 5.0
                    | ++−no: ∧ hcp(A,11)?
                        | ++−yes: [bid] 3.0
                        | ++−no: ∧ hcp(A,−M),lteq(M,6)?
                            | ++−yes: [pass] 15.0
                            | ++−no: ∧ lteq(I,2)?
                                | ++−yes: ∧ nb(A,−N,1)?
                                    | ++−yes: [bid] 3.0
                                    | ++−no: ∧ hcp(A,−O),gteq(O,11)?
                                        | ++−yes: [bid] 5.0
                                        | ++−no: ∧ [pass] 16.0
                                    | ++−no: [pass] 5.0
                    | ++−no: [bid] 5.0
                | ++−no: [bid] 5.0
            | ++−no: [bid] 5.0
        ++−no: [bid] 5.0
    ++−no: [bid] 5.0
Listing 5 A $L_2$-Aleph-induce_max model

```prolog
decision(A,4,B,C,bid) :-
    nbs(A,0).

decision(A,4,B,C,bid) :-
    hcp(A,D), gteq(D,5), nb(A,E,F), gteq(F,6), nbs(A,1).

decision(A,4,B,C,bid) :-
    hcp(A,D), gteq(D,9), nbs(A,1).

decision(A,4,B,C,bid) :-
    hcp(A,D), gteq(D,8), nb(A,E,F), gteq(F,5), nbs(A,1).

decision(A,4,B,C,bid) :-
    hcp(A,D), gteq(D,4), distribution(A,[6,4,2,1]).

decision(A,4,B,C,bid) :-
    hcp(A,D), gteq(D,8), nb(A,E,F), gteq(F,6), nbs(A,2).

decision(A,4,B,C,bid) :-
    hcp(A,D), gteq(D,13), nb(A,E,F), gteq(F,5), nbs(A,2).

decision(A,4,B,C,bid) :-
    hcp(A,D), gteq(D,10), nb(A,E,F), gteq(F,6).

decision(A,4,B,C,bid) :-
    nb(A,D,E), gteq(E,7).

decision(A,4,B,C,bid) :-
    hcp(A,D), gteq(D,14), nb(A,E,F), gteq(F,5).

decision(A,4,B,C,bid) :-
    nb(A,D,E), lteq(E,1), nbs(A,2).

decision(A,4,B,C,bid) :-
    hcp(A,D), nb(A,E,D), nb(A,F,G), gteq(G,6).

decision(A,4,B,C,bid) :-
    hcp(A,D), gteq(D,12), nb(A,E,F), lteq(F,1).

decision(A,4,B,C,bid) :-
    hcp(A,D), lteq(D,7), gteq(D,7), nb(A,E,F), gteq(F,6).

decision(A,4,B,C,bid) :-
    hcp(A,D), gteq(D,14), nbs(A,2).
```

Appendix 2

8 Expert comparison of models $M$ and $H$

From an expert point of view, the model $H$ from the $L_1$-Tilde model (see Listing 3) and the model $M$ from the $L_2$-Aleph-InduceMaxrules (see Listing 2), both obtained after a human post-processing, are very similar. They differ only on transition zones of certain characteristics, which are of low support. A detailed analysis follows.
Table 2 Predicates in the domain theories.

| predicate                  | L₀                      | L₁                      | L₂                      |
|----------------------------|-------------------------|-------------------------|-------------------------|
| suit_rep(H,Suit,Hon,Num)   |                         |                         |                         |
| distribution(H, [N,M,P,Q]) |                         |                         | distribution(H, [N,M,P,Q]) |
| nb(H,Suit,Num)             | nb(H,Suit,Num)          |                         |                         |
| hcp(H,Num)                 | hcp(H,Num)              |                         |                         |
| semibalanced(H)            | semibalanced(H)         |                         |                         |
| unbalanced(H)              | unbalanced(H)           |                         |                         |
| balanced(H)                | balanced(H)             |                         |                         |
| vuln(Pos,Vul,Dir,NS,EW)    |                         |                         |                         |
| lteq(Num,Num)              |                         |                         |                         |
| gteq(Num,Num)              |                         |                         |                         |
| longest_suit(H,Suit)       |                         |                         |                         |
| shortest_suit(H,Suit)      |                         |                         |                         |
| major(Suit)                |                         | major(Suit)             |                         |
| minor(Suit)                |                         | minor(Suit)             |                         |
| nbs(H,Num)                 |                         |                         |                         |

8.1 Transition zones within HCP

The models differ on the range 14-15 HCP (M2 vs H2). Note that with very unbalanced hand distributions, some parameters will be of a greater influence than a deviation of 1 or 2 HCP to take a decision: we can mention for example the number of HCP outside the ♠ suit or the number of HCP in the 4-card-suits (or more). An increase in the associated values would be an indication that, for the same value of HCP, the position of honors in the hand of the player is more favorable to the decision bid.

8.2 Transition zones within distribution

A M2b rule deals with hands holding a 6-card-suit exactly while those which have a 7-card-suit generate the same decision bid. Within L₁ and L₂ models hands with 5-card-suits also trigger similar rules. L₁ rule H2 always generate a bid while the L₂ rules of model M lead to a distinction according to the number of cards in ♠ and the number of HCP: the more cards you have in ♠, the more HCP you need to make the bid decision. If we look more closely, Tilde before post-processing had an embryo of similar rule since it predicted a bid with a 6-card-suit (or more) and exactly 4 HCP. The support being very small (2 examples), it had been ignored.
8.3 Summary

The existence of these transition zones is beyond doubt for experts, but their very identification is likely to advance knowledge in the field. New experiments conducted on these transition zones, with a more specific vocabulary devised with experts in the loop, could lead to a more accurate elicitation of the black box model.