Experimental study of nonclassical teleportation beyond average fidelity

Gonzalo Carvacho,1 Francesco Andreoli,1 Luca Santodonato,1 Marco Bentivegna,1 Vincenzo D’Ambrosio,2,3 Paul Skrzypczyk,4 Ivan Šupić,2 Daniel Cavalcanti,2 and Fabio Sciarrino1,∗

1Dipartimento di Fisica - Sapienza Università di Roma, P.le Aldo Moro 5, I-00185 Roma, Italy
2ICFO-Institut de Ciencies Fotòniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain
3Dipartimento di Fisica, Università di Napoli Federico II, Complesso Universitario di Monte S. Angelo, 80126 Napoli, Italy
4H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol, BS8 1TL, United Kingdom

Quantum teleportation establishes a correspondence between an entangled state shared by two separate parties that can communicate classically and the presence of a quantum channel connecting the two parties. The standard benchmark for quantum teleportation, based on the average fidelity between the input and output states, indicates that some entangled states do not lead to channels which can be certified to be quantum. It was recently shown that if one considers a finer-grained witness, then all entangled states can be certified to produce a non-classical teleportation channel. Here we experimentally demonstrate a complete characterization of a new family of such witnesses, of the type proposed in Phys. Rev. Lett. 119, 110501 (2017) under different conditions of noise. We report non-classical teleportation using quantum states that cannot achieve average fidelity of teleportation above the classical limit. We further use the violation of these witnesses to estimate the negativity of the shared state. Our results have fundamental implications in quantum information protocols and may also lead to new applications and quality certification of quantum technologies.

Introduction. – The role of entanglement in quantum information processing is of utmost importance, but it is also subject of debate. Entanglement is today the core of many key discoveries ranging from quantum teleportation [?], to quantum dense coding [?], quantum computation [?] and quantum cryptography [?]. Quantum communication protocols such as device-independent quantum key distribution [?] are heavily based on entanglement to reach nonlocality-based communication security [?].

The prototype for quantum information transfer using entanglement as a communication channel is the quantum teleportation protocol [?], where a sender and a receiver share a maximally entangled state which they can use to perfectly transfer an unknown quantum state. This protocol represents a milestone in theoretical quantum information science [?] and lies at the basis of many technological applications such as quantum communication via quantum repeaters [?] or gate teleportation [?]. It has been implemented over hundreds of kilometers in free-space [?] and more recently in a ground-to-satellite experiment [?]. Employed platforms include mainly photonic qubits [??], atomic ensembles [??] and solid-state systems [??].

To fully understand the role of entanglement in quantum teleportation, it is necessary to gauge what is the actual entanglement content (if any) that must be involved in order to upgrade a classical channel to a quantum channel. Indeed, it is known that not all entangled states allow a average fidelity of teleportation to be reached which is higher than the one achievable using only classical communication [?], with the notable example of bound entangled states [?]. This mismatch between the nonclassicality of the shared state and nonclassicality of teleportation can be resolved by taking into account the full available information instead of only the average fidelity of teleportation. Very recently it has been theoretically shown [?] that it is possible to use a more informative witness for teleportation scenarios which detects nonclassicality of a teleportation channel whenever any amount of entanglement is present in the shared state. It was also shown that these witnesses allow for the entanglement of the shared state to be estimated [?].

In this article, we experimentally test, using pairs of polarization-entangled photons, the properties of a teleportation witness by exploiting a photonic teleportation setup able to generate a family of channel states (shared between the sender and the receiver) characterized by tunable amount of noise γ. We study the behavior of the teleportation witness for various values of γ and check its agreement with theoretical predictions. We demonstrate experimentally that the teleportation witness is able to detect the presence of a quantum channel even in conditions where the average fidelity of teleportation is below the classical limit. Finally, we use the teleportation witness to estimate the entanglement – in the form of negativity [?] – of the shared state. Our experimental re-
sults show that the analyzed teleportation witnesses represents a novel tool which is able to certify the presence of a nonclassical teleportation channel beyond the possibilities of the previously adopted benchmark, and to estimate entanglement.

**Teleportation witnesses.**—In a teleportation protocol two parties, Alice and Bob, share a quantum state $\rho_{AB}$, which they want to use to teleport arbitrary (possibly unknown) states $\ket{\psi_A^0}$ (see Fig.2). Alice applies a measurement $M$ with measurement operators $M_{aA}^A$ on the systems $A_0A$ in her possession, leaving Bob’s system in the states

$$\rho_{A|\psi_x}^B = \frac{\text{tr}_{A_0A}[(M_{aA}^A \otimes I^B) \cdot (\ket{\psi_x} \bra{\psi_x}^A_0 \otimes \rho_{AB}^B)]}{p(a|\psi_x)},$$  

where $p(a|\psi_x) = \text{tr}[(M_{aA}^A \otimes I^B) \cdot (\ket{\psi_x} \bra{\psi_x}^A_0 \otimes \rho_{AB}^B)]$ is the probability of the particular outcome $a$.

As proposed in [? ], a particular witness function which exploits the full information obtained from teleportation can be computed in order to certify the presence of entanglement in the channel state $\rho_{AB}^B$. In particular, we focus here on the situation where the channel state belongs to the family of quantum states

$$\rho_{AB}^B(\gamma) = \gamma \ket{\psi^-} \bra{\psi^-} + (1 - \gamma) \ket{11} \bra{11},$$  

where $\ket{\psi^-} = (\ket{01} - \ket{10})/\sqrt{2}$, and as input to the teleportation experiment we consider the six eigenvectors of the Pauli matrices $\{\psi_{x_{a_i}}^a\}_{i=0}^5 = \{(0| + 1))/\sqrt{2}, (0| - 1))/\sqrt{2}, (i|0) + (i|1))/\sqrt{2}, (i|0) - (i|1))/\sqrt{2}, (0|1), (1|1)\}$. We denote the resulting set of states prepared for Bob $\rho_{A|\psi}\rho_{B|\psi}$.

Using the methods discussed in [? ], one can obtain operators $F_{a|\psi}(\theta)$, defining a one-parameter family of teleportation witnesses:

$$W(\gamma, \theta) = \sum_{a, x} p(a|\psi_x) \text{tr}[F_{a|\psi}(\theta) \rho_{a|\psi}(\gamma)],\tag{3}$$

where the operators $F_{a|\psi}(\theta)$ are given in Table ?? and the parameter $\theta$ identifies a single witness of the family. By construction $\sum_{a, x} p(a|\psi_x) \text{tr}[F_{a|\psi}(\theta) \rho_{a|\psi}] \geq 0$ for all sets $\{\rho_{a|\psi}\}$ that come from teleportation processes using only classical communication (or separable states). Thus, $W(\gamma, \theta) < 0$ is a certificate of nonclassical teleportation. It was moreover shown in [? ] that the entanglement of the shared state can be estimated based upon the violations of a teleportation witness. In particular, the negativity [? ] $N(\rho_{AB}^B)$ of the shared state can be bounded

$$N(\rho_{AB}^B) \geq f(W(\gamma, \theta)),\tag{4}$$

where $f(W(\gamma, \theta))$ is a function that can be computed by semidefinite programming (See Supplemental Material).

We compare this witness with the average fidelity of teleportation, which is commonly adopted as the benchmark for
the quality of quantum teleportation. The average fidelity between the input and output states of the process \([? \, ?]\), is given by \(F_{\text{tel}} = 1/N_x \sum_{x} p(a|\psi_x) \langle \psi_x | U_a \rho_{AB} | \psi_0 \rangle \), where \(N_x\) is the number of states to be teleported and \(U_a\) is fixed (i.e., independent of the input states) unitary operators conditioned to Alice’s result \(a\). In the case of perfect teleportation (using a maximally entangled state), \(F_{\text{tel}} = 1\), while in experimental situations it is always the case that \(F_{\text{tel}} < 1\). If one defines \(F_{\text{cl}}\) as the maximal fidelity that can be obtained given a classical channel (i.e., in the absence of entanglement between Alice and Bob), the observation of \(F_{\text{tel}} > F_{\text{cl}}\) implies that the teleportation process has no classical counterpart \([? \, ?]\). On the other hand, some entangled states cannot lead to a \(F_{\text{tel}}\) greater than the classical bound. This is the case for some states in the family \((?)\), which lead to an average fidelity lower than the classical bound of \(2/3\) when \(\gamma \leq 1/2\) (see Supplemental Material), while the channel state is still entangled. Notably, for these states, the capability to achieve nonclassical teleportation can be still certified through the witness \(W(\gamma, \theta)\).

In the following, we experimentally study the behavior of \(W(\gamma, \theta)\), characterizing it with respect to the experimental imperfections arising in a photonic setup, and then we finally compare its application with the average fidelity of teleportation for detecting non-classical teleportation and estimating the negativity of the shared state.

**Characterization of the teleportation witness.**—We experimentally tested the proposed teleportation witness \((?)\) by implementing a teleportation scenario in which a tunable amount of noise can be introduced in the entangled pair shared between Alice and Bob (see Fig.\(\ref{fig:fig3}\)). Two photon pairs (1-2 and 3-4) are generated in two separated nonlinear crystals by means of type-II spontaneous parametric down-conversion (SPDC) process. Photon pair 3-4 will embody the quantum channel \(\rho_{AB}^{\text{AB}}\) shared by Alice and Bob. To generate the tunable amount of noise we send Bob’s photon through the interferometer depicted in the yellow rectangle of Fig.\(\ref{fig:fig3}\), which is composed by a first calcite displacer (CD), a half wave-plate (HWP) at an angle \(\alpha\), a second CD and a HWP at \(45^\circ\). When \(\alpha = 45^\circ\) the interferometer implements the identity transformation on Bob’s photon, while for \(\alpha = 0^\circ\) the state is vertically projected \(|\psi\rangle\rangle\). As a result, in the ideal case Alice and Bob will share a state of the form of \((?)\), where the parameter \(\gamma\) can be tuned by acting on the physical angle \(\alpha\). The relationship between \(\alpha\) and \(\gamma\) is explicitly derived in the Supplemental Material. This ideal situation, however, cannot be completely reproduced in any actual implementation. As discussed in the Supplemental Material, to account for this we develop a model in order to theoretically predict the main imperfections which can arise in a photonic scheme similar to the one in Fig.\(\ref{fig:fig3}\). We address the SPDC generation and the calculations.

\[
\begin{array}{c|cc|cc|cc|cc}
\hline
\alpha & |0\rangle & |1\rangle \\
\hline
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 2 \sin \theta \sigma_x & 2 \cos \theta \sigma_y & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
1 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 2 \sin \theta \sigma_y & 2 \cos \theta \sigma_x & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\hline
\end{array}
\]

Table 1. One-parameter family of teleportation witnesses operators \((\theta \in (0, \pi/2))\) used in Eq.\((?)\) that detects the nonclassicality of the teleportation process using the state in Eq.\((?)\), the input states \(\psi_x\) and a partial BSM where the outcome \(a = 0\) corresponds to a projection into a singlet state \(|\psi^{-}\rangle = (|01\rangle - |10\rangle)/\sqrt{2}\), and \(a = 1\) the orthogonal subspace.

Figure 3. **Experimental test of the non-classical teleportation witness.** Theoretical (lines in figure 3-a) and experimental (dashed lines of figure 3-b) witness function obtained assuming our noise parameter estimation and varying the free parameter \(\theta\), given different values of \(\gamma\). Shaded areas correspond to one standard deviation of uncertainty upper and lower the dashed experimental lines, due to Poissonian statistics. The bar legend on the right explains the color relationship with \(\gamma\), while black points on the curves stands for the minimum experimental value obtained for a given \(\gamma\).
We measured different basis combinations of the teleported states and we introduce two parameters (cite interferometer as the two main sources of imperfection, while blue points show the optimal experimental values of the witness. The black dashed line is the classical teleportation threshold (i.e. the maximal value of \( W_{\text{min}}(\gamma) \) necessary to certify nonclassical teleportation), while the red dashed line represents the minimal value of \( \gamma \) needed to have an entangled teleportation channel, given our noise model. Inset: The optimal estimated negativity for each value of \( \gamma \). Green line represents the theoretical prediction, while blue points show the experimental value. We stress that for each \( \gamma \) a different \( \theta \) is used to achieve the best witness violation and the best negativity estimate. b) Average fidelity of teleportation estimation when considering the outcome \( |\psi^-\rangle \). The green line shows the theoretical estimation for the mean value curve \( W_{\text{cl}}(\gamma,\theta) \) (given our noise model), while blue points depict the experimental values. The red dashed line represents the minimum value of \( \gamma \) necessary to have entanglement in the teleportation channel, while the orange curve shows \( F_{\text{cl}} \), the classical fidelity of teleportation bound. Error bars indicate one standard deviation of uncertainty, due to poissonian statistics.

As a result, given the estimated values of \( v \) and \( \delta \), for some low values of \( \gamma \), the generated quantum states can become separable, differently from the ideal one shown in Eq. (3).

We were able to experimentally test the expected properties of the teleportation witness (3), analyzing its behavior for different values of \( \gamma \) and \( \theta \). In our scheme, Alice prepares the state to be teleported in photon 2 by means of remote state preparation (i.e., by performing projective measurement on photon 1) and teleporting it to Bob by performing a partial Bell state measurement (BSM) on photons 2 and 3. BSM is implemented with an in-fiber 50/50 beam splitter (BS) followed by polarization analysis on each of the two outputs. Bell states \( |\psi^-\rangle \) will be identified by two-fold coincidences from different output arms. We address the “robustness” of the witness by considering the six eigenstates of the Pauli matrices \( \{|\psi_{x}^{A}\rangle\}_{x=0}^{5} \) as the possible states to be teleported. We measured different basis combinations of the teleported states \( \rho_{\text{noisy}}^{B|\psi_{x}} \) and used this data to experimentally estimate the value

\[
\rho_{\text{noisy}}^{AB} = \frac{1}{4} \begin{pmatrix}
(1 - v^2)\gamma & 0 & 0 \\
0 & 2 - v^2(2 - 3\gamma) - \gamma & -2(1 - 2\delta)^2v^2\gamma & 0 \\
0 & -2(1 - 2\delta)^2v^2\gamma & (1 + v^2)\gamma & 0 \\
0 & 0 & 0 & 2 + v^2(2 - 3\gamma) - \gamma
\end{pmatrix}
\]

(5)

of \( W(\gamma, \theta) \).

Fig. 3-a shows the theoretical expectations for the same set of states and parameters for comparison. Experimental results are shown in Fig. 3-b, where the estimated value of \( W(\gamma, \theta) \) is plotted as a function of the parameter \( \theta \) and for different levels of noise \( \gamma \). As expected the witness certifies the nonclassicality of the channel even in those cases when the average fidelity of teleportation is lower than classical fidelity. This is shown in detail in figure 4 a-b.

In Fig. 4-a are reported the optimal values of \( W \) for fixed values of \( \gamma \). Blue points show the experimental results, obtained evaluating \( W(\gamma, \theta) \) for different \( \theta \)-s and numerically minimizing over the parameter \( \theta \). As expected, only those points with \( \gamma \) higher than the estimated entanglement threshold (red dot-dashed line), have \( W_{\text{min}}(\gamma) < 0 \), and thus show nonclassical teleportation behavior. The data are in good agreement with the theoretical values of \( W_{\text{min}}(\gamma) \) (green line).
which are expected given our noise model (see Supplemental Material). To have a direct comparison with the teleportation witness, we plot in Fig.4-b the estimated average fidelity of teleportation for the same set of experimental points while in the supplemental information can be found the average fidelity considering both outcomes $|ψ^+\rangle$. Finally, as an inset to Fig. 4-a we show the optimal lower bound on negativity of the underlying shared state based on the witness violations for different values of $γ$.

Discussion.– Our work experimentally demonstrates the usefulness of teleportation witnesses, recently introduced in [9], to detect non-classical teleportation in realistic situations. We started with the generation of a general family of two-qubit states unable to display a faithful regime. We tuned the amount of noise in our states in order to test the properties of the nonclassical witness for different values of $γ$ and $θ$ and establishing in this way a direct and general link between the expected properties and our experimental results. Moreover, we experimentally showed that beyond a certain noise threshold one can enter a region where the standard benchmark of the average fidelity is useless to certify a quantum channel while our witness can, thus providing an experimental tool with fundamental implications in quantum information protocols which can also lead to new applications in quantum technologies.

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* fabio.sciarrino@uniroma1.it

[1] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels, Phys. Rev. Lett. 70, 1895 (1993).

[2] C. H. Bennett, and S. J. Wiesner, Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states, Phys. Rev. Lett. 69, 2881–2884 (1992).

[3] R. P. Feynman, Simulating physics with computers, International journal of theoretical physics 21, 467–488 (1982).

[4] P. W. Shor, Scheme for reducing decoherence in quantum computer memory, Phys. Rev. A. 52, R2493–R2496 (1995).

[5] A. M. Steane, Error correcting codes in quantum theory, Phys. Rev. Lett. 77, 793–797 (1996).

[6] A. K. Ekert, Quantum cryptography based on Bell’s theorem, Phys. Rev. Lett. 67, 661-663 (1991).

[7] T. Jennewein, C. Simon, G. Weihs, H. Weinfurter, and A. Zeilinger, Quantum cryptography with entangled photons, Phys. Rev. Lett. 84, 4729–4732 (2000).

[8] A. Acín, N. Brunner, N. Gisin, S. Massar, S. Pironio and V. Scarani, Device-Independent security of quantum cryptography against collective attacks, Phys. Rev. Lett. 98, 230501 (2007).

[9] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani and S. Wehner, Bell nonlocality, Rev. Mod. Phys. 86, 419 (2014).

[10] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, 2000).

[11] C. Weedbrook et al., Gaussian Quantum Information, Rev. Mod.Phys.84, 621 (2012).

[12] S. Pirandola, J. Eisert, C. Weedbrook, A. Furusawa, and S. L. Braunstein, Advances in quantum teleportation, Nature Photon. 9, 641–652 (2015).

[13] H.-J. Briegel, W. Dur, J. I. Cirac, and P. Zoller Quantum repeaters: The role of imperfect local operations in quantum communication, Phys. Rev. Lett.81, 5932 (1998).

[14] W. Dur, H.-J. Briegel, J. I. Cirac and P. Zoller, Quantum repeaters based on entanglement purification, Phys. Rev. A 60, 725 (1999).

[15] D. Gottesman, and I. L. Chuang, Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations, Nature402, 6390-393 (1999).

[16] J. Ying et al,Quantum teleportation and entanglement distribution over 100-kilometre free-space channels, Nature 488, 185-188 (2012).

[17] X.S. Ma et al Quantum teleportation over 143 kilometres using active feed-forward, Nature 489, 269-273 (2012).

[18] J.G. Ren et al. Ground-to-satellite quantum teleportation, Nature 549, 70–73 (2017).

[19] D. Bouwmeester, J.-W. Pan, K. Mattle, M.Eibl, H. Weinfurter, and A. Zeilinger, Experimental quantum teleportation, Nature 390, 575-579 (1997).

[20] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Experimental realisation of teleporting an unknown pure quantum state via dual classical and Einstein–Podolski–Rosen channels, Phys. Rev. Lett. 80, 1121–1125 (1998).

[21] D. Bouwmeester, J.-W. Pan, K. Mattle, M.Eibl, H. Weinfurter, and A. Zeilinger, Experimental quantum teleportation, Nature 390, 575-579 (1997).

[22] F. Lombardi, F. Sciarrino, S. Popescu, and F. De Martini, Teleportation of a Vacuum–One-Photon Qubit, Phys. Rev. Lett. 88, 070402 (2002).

[23] R. Ursin et al., Quantum teleportation across the Danube, Nature 430, 849 (2004).

[24] I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden and N.Gisin, Long-distance teleportation of qubits at telecommunication wavelengths, Nature 421, 509–513 (2003).

[25] de Riedmatten, H. et al., Long-distance quantum teleportation in a quantum relay configuration, Phys. Rev. Lett. 92, 047904 (2004).

[26] X.-L. Wang et al., Quantum teleportation of multiple degrees of freedom in a single photon, Nature 518, 516–519 (2015).

[27] Nielsen, M. A., Knill, E. and Laflamme, R., Complete quantum teleportation using nuclear magnetic resonance, Nature 396, 52 (1998).

[28] J. F. Sherson et al., Quantum teleportation between light and matter, Nature 443, 557–560 (2006).

[29] H. Kräuter et al., Deterministic quantum teleportation between distant atomic objects, Nature Phys. 9, 400–404 (2013).

[30] M. D Barrett et al., Deterministic quantum teleportation of atomic qubits, Nature 429, 737–739 (2004).

[31] M. Riebe et al., Deterministic quantum teleportation with atoms, Nature 429, 734–737 (2004).

[32] W. B. Gao et al., Quantum teleportation from a propagating photon to a solid-state spin qubit, Nature Communications 4, 2744 (2013).

[33] J. Steffen et al., Deterministic quantum teleportation with feed-forward in a solid state system, Nature 500, 319–322 (2013).

[34] W. Pfaff et al., Unconditional quantum teleportation between distant solid-state quantum bits, Science 345, 532–535 (2014).
[34] M. Horodecki, P. Horodecki, and R. Horodecki, *General teleportation channel, singlet fraction, and quasi-distillation*, Phys. Rev. A **60**, 1888 (1999).

[35] R. Horodecki, P. Horodecki, M. Horodecki and K. Horodecki, *Quantum entanglement*, Rev. Mod. Phys. **81**, 865 (2009).

[36] Michał Horodecki, Paweł Horodecki, and Ryszard Horodecki *Mixed-State Entanglement and Distillation: Is there a “Bound” Entanglement in Nature?*, Phys. Rev. Lett. **80**, 5239 (1998).

[37] D. Cavalcanti, P. Skrzypczyk, and I. Šupić, *All entangled states can demonstrate nonclassical teleportation*, Phys. Rev. Lett. **119**, 110501 (2017).

[38] I. Šupić, P. Skrzypczyk, D. Cavalcanti, *Quantifying non-classical teleportation*, arXiv:1804.10612.

[39] G. Vidal, R.F. Werner, *A computable measure of entanglement*, Phys. Rev. A **65**, 032314 (2002).

[40] S. Massar and S. Popescu, *Optimal Extraction of Information from Finite Quantum Ensembles*, Phys. Rev. Lett. **74**, 1259 (1995).