The Condition for the Onset of High Temperature Superconductivity.

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In this work the long-wave limit of electron-electron interaction arising from the exchange of virtual phonons in an approximation close to "jelly" model is considered. It is shown that the interaction through the exchange of virtual phonons is actually not screened in contrast to the Coulomb one; this just leads to instability relative to the formation of pairs near the Fermi surface. The consequences of this approach are examined with respect to high-temperature superconducting materials that have recently been synthesized. An approximate relationship connecting sound and Fermi velocities for these materials is obtained.

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I. INTRODUCTION

A considerable number of fundamental works have been devoted to tackling the superconductivity problem and, in particular, a high temperature superconductivity one (HTSC). The opinion on the present state of the problem may be formed considering the works [1–3]. Most relevant publication are to some extent associated with the BCS theory [4]. Now, for metals enumerated in the Periodic System of elements (PSE), the superconductivity theory is considered to be more or less completed and the validity of the BCS theory leaves no room for doubt on the part of the majority of authors. At the same time in [5–8] a conclusion was drawn that the BCS theory may not be the only possible way to explain the phenomenon of superconductivity (SC).

Outstanding results [9] obtained by Bednortz J.G. and Muller A.K. in 1986 for YBa$_2$Cu$_3$O$_{7-\delta}$ caused the great sensation among chemists, physisists and material researchers. Since that time the highest value for $T_s$ at about 164K was observed in HgCa$_2$Ba$_2$Y$_3$Cu$_8+\delta$ ceramics. Rather high $T_s$ values later were observed in fullerenes [10] and their derivatives fullrides C$_{60}$M [11]. Some speculations show that in intercalates [12] and nanotubes [13] high $T_s$ values may be observed. The idea of producing HTSC in simple composition compounds such as Li$_3$P and Li$_3$N is presently in a research state [14–16].

These experimental results led to the conclusion [3,17] that the applicability of the BCS theory for HTSC is not so obvious as for convenient SC and the theorists encountered many difficult problems which are not yet solved. From our opinion the development of the SC theory was stopped at the level determined by [2,3]. It should be noted here, that the electron-phonon (el-ph) interaction in the majority of publications, referred to in [2,3], is considered for the short wave or optical phonons. This becomes obvious from the fact, that the polaron problem in these works is solved separately from the rest part of the problem, not taking into account Coulomb electron-electron (el-el) interaction. It may be easily proved, that such kind of renormalization procedures are unacceptable in the long-wave limit limit investigation due to the singular behaviour of the initial el-ph interaction matrix elements. From the other hand, the consistent accounting for Coulomb el-el interaction in the polaron problem leads to the appearance of the "screening effect" for the el-ph interaction matrix elements and subsequently reasonable expression [13] for the parameters of the operator transformations involved in
the renormalization procedure is derived.

The object of the present work is to calculate the long-wave range component of el-el interaction arising from the exchange of virtual phonons accounting for the correlation effects in an electronic plasma. An approximation close to "jelly" model was assumed to describe the el-ph interaction. Authors believe, that the proposed microscopic model may help to predict new promising systems for observing HTSC.

II. DESCRIPTION OF THE MODEL

For simplification, let us consider the Hamiltonian $H_{tot}$ of an infinite (finite-size effects are ignored) monoatomic metal with a parabolic conductivity zone as an initial one (here and further we use units with $\hbar = 1$):

$$H_{tot} = H_0 + H_{pp} + H_{ee} + H_{ep} \, ,$$

$$H_0 = \sum_{\mu} \int \epsilon_p c_{\mu p}^+ c_{\mu p} d\mathbf{p} + \int \omega_q b_{\mathbf{q}}^+ b_{\mathbf{q}} d\mathbf{q} \, , \quad \left( \epsilon_p = \frac{p^2}{2m}, \, \omega_q = sq \right) \tag{2}$$

$$H_{pp} = A \int \left[ 2 b_{\mathbf{q}}^+ b_{\mathbf{q}} - b_{\mathbf{q}}^+ b_{\mathbf{q}}^+ - b_{\mathbf{q}} b_{\mathbf{q}} \right] \frac{d\mathbf{q}}{q} \, , \tag{3}$$

$$H_{ee} = G_{ee} \sum_{\mu} \sum_{\nu} \int \int \int c_{\mu p+q}^+ c_{\nu q-k}^+ c_{\nu q} c_{\mu p} d\mathbf{p} d\mathbf{k} d\mathbf{q} \, , \tag{4}$$

$$H_{ep} = -i G_{ep} \sum_{\mu} \int \int c_{\mu p+q}^+ c_{\mu p} b_{\mathbf{q}}^+ b_{\mathbf{q}} - b_{\mathbf{q}} b_{\mathbf{q}} \right) \frac{d\mathbf{p} d\mathbf{q}}{q^2} \, , \tag{5}$$

where $A = \frac{\pi^2}{2 \frac{2}{M \Omega}}$, $G_{ee} = \frac{\pi^2}{2 e \sqrt{\pi M \Omega}}$, $G_{ep} = \frac{\pi^2}{e \sqrt{\pi M \Omega}}$, $c_{\mu p}^+$ and $c_{\mu p}$ are operators of creation and annihilation of electrons with momentum $p$, respectively, $\mu, \nu$ are spin indices, $b_{\mathbf{q}}^+, b_{\mathbf{q}}$ are operators of creation and annihilation of longitudinally polarized phonons with momentum $q$, respectively, $s, \epsilon$ are sound velocity and dielectric permittivity of the valence skeleton, respectively, $M$ is the mass of ion, $m$ is the mass of zone electron, $\Omega$ is the volume of an elementary cell, $z$ is the number of conductivity electrons per cell, and $i$ is an imaginary unit.

Eqs.\(1\)\(3\) are derived under the following assumptions. It is possible to assume that the valence skeleton creates average positively charged background with a density $\rho_0 = ze/\Omega$. The density deviations from the mean value are determined by a relative change in the crystal volume resulting from the deformation of the lattice, when a longitudinal sound wave propagates through it. At the same time operators of creation and annihilation of longitudinally polarized phonons are associated with the local shift of ions $u(r, t)$ via a relationship

$$u_{\alpha} = \frac{1}{4\pi^{3/2}} \int \sqrt{\frac{\Omega}{M \omega_q}} e^{-i \mathbf{r} \cdot \mathbf{q}} e_{\alpha q} b_{\mathbf{q}}^+ d\mathbf{q} + \text{H.c.} \, , \tag{6}$$

$$e_{\alpha q} = \frac{q_{\alpha}}{q} , \quad \alpha = x, y, z .$$

The choice of a factor in Eq.\(6\) is determined by need to fulfill the relationship $E = \frac{M^2}{2 \Omega} \int \sum_{\alpha} \left( \frac{\partial u_{\alpha}}{\partial t} \right)^2 d\mathbf{r} + \frac{M}{2 \Omega} \int \left( \frac{\partial u_{\alpha}}{\partial t} \right)^2 d\mathbf{r} = \int \omega_q b_{\mathbf{q}}^+ b_{\mathbf{q}} d\mathbf{q}$, where $E$ is the energy of longitudinally polarized phonons. Thus, the local charge density is determined by the expression $\rho_v = \rho_0 \det^{-1} \left| \frac{\partial (r+\mathbf{u})}{\partial t} \right|$. Taking into account only the terms $\sim \text{div}(\mathbf{u}(r))$ one can get for deviation of local density from its mean value $\rho = \rho_v - \rho_0$

$$\rho = -\frac{ze}{\Omega} \sum_{\alpha} \frac{\partial u_{\alpha}}{\partial t} = i \frac{ze}{4\Omega^{3/2}} \int \sqrt{\frac{\Omega q}{Ms}} e^{-i \mathbf{r} \cdot \mathbf{q}} b_{\mathbf{q}}^+ d\mathbf{q} + \text{H.c.} \, \tag{7}$$
The derived expression for $\rho$ determines the potential due to a phonon subsystem:

$$V_\rho = \frac{i z}{\epsilon} \frac{1}{\sqrt{\pi M \Omega}} \int b^\dagger_q e^{-i q \cdot r} \frac{dq}{q^{3/2}} + \text{H.c.}$$  \hspace{1cm} (8)

It is this potential that causes in the suggested model the existence of an electron-phonon interaction (5) and a specific self-action of the phonon subsystem (3).

It should be noted here that in PSE metals the division of electron states between conductivity and valence ones encounters difficulties due to hybridization effects when these states belong to the same energy interval. Nevertheless in the majority of HTSC materials with low concentration of electro-active effects these difficulties do not exist and the model developed may be useful in consideration of el-ph and phonon-phonon interactions.

In Eqs. (1)-(5) a dielectric permittivity of the valence framework $\epsilon$ was introduced. We note here that, as will become apparent below, the static long-wave limit of this value is of current significance.

As regards the sound velocity in the valence skeleton we will derive an expression (see Eq. (29)) relating $s$ to the experimentally observed sound velocity in metal. Therefore, in our opinion, the problem of $s$ determination does not involve serious difficulties. Moreover, from Eq.(29) it becomes apparent that the suggested model transforms into a so-called "jelly" model as $s \to 0$ (see Eq.(30)), that must be obvious, from our point of view, without relationship (29), too.

Actually, as compared with the "jelly" model, Eqs.(1)-(5) take into account the rigidity of the valence skeleton. This is of current significance in considering, for example, doped semiconductors.

Finally, let us consider the problem of taking into consideration transversely polarized phonons. It is believed that the interaction of these phonons with electrons should be considerably smaller than that of longitudinally polarized ones. This is associated with the fact that the elementary volume in shear deformations changes only in the second order with respect to the vector of elementary shift $u(r)$. That is why in the framework of the proposed investigation their existence may be ignored, considering the subsystem of transversely polarized phonons not interacting both conductivity electrons and longitudinally polarized phonons.

III. POLARON PROBLEM IN PROPOSED MODEL

The Hamiltonian of interaction (5) contains nondiagonal terms with structures $c^\dagger b^\dagger$ and $c^\dagger b$, corresponding to transitions without conservation of energy. Therefore Fock states of the system, constructed on the basis of operators $c^\dagger$ and $b^\dagger$, turn out to be states with indefinite energy. It is this circumstance that creates difficulties in investigating low-energy excitations of the system, which current interest is obvious in describing the system at low temperatures.

Let us consider a transformation by introducing new operators $C^\dagger$, $C$, $B^\dagger$, and $B$, being typical in solving polaron problems:

$$c^\dagger_{\mu p} = C^\dagger_{\mu p} - \int \phi^p_{\mu p} \phi^\dagger_{\mu p-q} B^\dagger_q dq + \int \phi^p_{\mu p+q} C^\dagger_{\mu p+q} B^\dagger_q dq - \frac{1}{2} \int \phi^p_{\mu p} \phi^p_{\mu p} C^\dagger_{\mu p} - \frac{1}{2} \int \int [\phi^p_{\mu p-q} \phi^\dagger_{\mu p-q} - \phi^p_{\mu p+q} \phi^\dagger_{\mu p+q}] C^\dagger_{\mu p-q} C^\dagger_{\mu p+q} B^\dagger_k B^\dagger_q dkdq$$

$$+ \frac{1}{2} \int \int [\phi^p_{\mu p-q} \phi^\dagger_{\mu p-q} - \phi^p_{\mu p+q} \phi^\dagger_{\mu p+q}] C^\dagger_{\mu p-q} C^\dagger_{\mu p+q} B^\dagger_k B^\dagger_q dkdq - \frac{1}{2} \int \int [\phi^p_{\mu p-k+q} \phi^\dagger_{\mu p-k+q} + \phi^p_{\mu p-k+q} \phi^\dagger_{\mu p-k+q}] C^\dagger_{\mu p-k+q} C^\dagger_{\mu p-k+q} B^\dagger_k B^\dagger_q dkdq$$

$$- \frac{1}{2} \int \int [\phi^p_{\mu p-k+q} \phi^\dagger_{\mu p-k+q} + \phi^p_{\mu p-k+q} \phi^\dagger_{\mu p-k+q}] C^\dagger_{\mu p-k+q} C^\dagger_{\mu p-k+q} B^\dagger_k B^\dagger_q dkdq$$

(9)
\[ + \frac{1}{4} \int\int [\varphi_{q-k}^{p-k} \varphi_{k}^{p} + \varphi_{k}^{p-k} \varphi_{q}^{p}] C_{\mu p-k-q}^{\dagger} B_{q}^{\dagger} B_{k}^{\dagger} dkdq \]
\[ + \frac{1}{4} \int\int [\varphi_{q}^{p+q} \varphi_{k}^{p+k+q} + \varphi_{k}^{p+k} \varphi_{q}^{p+q+k+q}] C_{\mu p+k+q}^{\dagger} B_{q}^{\dagger} B_{k}^{\dagger} dkdq ; \]
\[ \hat{\delta}_{q}^{\dagger} = B_{q}^{\dagger} + \int \varphi_{q}^{p} \sum_{\mu} C_{\mu p}^{\dagger} C_{\mu p-q}^{\dagger} dp \]
\[ + \frac{1}{2} \int\int [\varphi_{q}^{p+k+q} \varphi_{k}^{p-k} - \varphi_{q}^{p+q} \varphi_{k}^{p+k}] \sum_{\mu} C_{\mu p}^{\dagger} C_{\mu p-k-q}^{\dagger} B_{k}^{\dagger} dkd\]
\[ + \frac{1}{2} \int\int [\varphi_{q}^{p-k} \varphi_{k}^{p+q} - \varphi_{q}^{p+q-k} \varphi_{k}^{p}] \sum_{\mu} C_{\mu p}^{\dagger} C_{\mu p-k-q}^{\dagger} B_{k}^{\dagger} dkd\]

where \( \varphi \) stands for small parameter.

It is easy to see (Appendix 1) that this transformation does not break commutation relations between the creation and annihilation operators with an accuracy up to terms \( \sim |\varphi|^3 \).

The parameters \( \varphi_{q}^{p} \) are involved to satisfy a requirement that the Hamiltonian expressed in terms of operators \( C^{\dagger}, C, B^{\dagger}, \) and \( B \) should not contain nondiagonal terms corresponding to virtual transitions, i.e., to transitions without conservation of energy.

The necessity of fulfilling this requirement for the terms with the structure \( C^{\dagger} CB^{\dagger} \) and \( C^{\dagger} CB \) leads to the following equation for \( \varphi_{q}^{p} \)

\[-iG_{ep}q^{-3/2} + (\epsilon_{p} - \epsilon_{p} - \omega_{q}) \varphi_{q}^{p} + q \frac{A}{2} (\varphi_{q}^{p} - \varphi_{q}^{*p-q}) = 0. \tag{11} \]

Taking into account filling of the conductivity zone by electrons leads us, in a sense of Hartree–Fock approximation, to consider the terms with structures \( C^{\dagger} C^{\dagger} CCB^{\dagger} \) and \( C^{\dagger} C^{\dagger} CCB \) and to realize their linearization with respect to the occupation density \( \rho_{\mu p} \). The linearization procedure is based on the relation

\[ \langle C_{\mu p}^{\dagger} C_{\nu k} \rangle = \delta_{\mu \nu} \rho_{\mu p} \delta(p - k), \]

where \( \langle \ldots \rangle \) means vacuum expectation value (vacuum corresponds to the ground state of the system), \( \delta_{\mu \nu} \) is Kronecker symbol, \( \rho_{\mu p} \) is occupation density, \( \delta(p - k) \) is Dirac \( \delta \)-function and is carried out according to usual rules

\[ \langle C_{1}^{\dagger} C_{2}^{\dagger} C_{3} C_{4} B^{\dagger}(B) \rangle_{L} = \langle C_{1}^{\dagger} C_{4} C_{2}^{\dagger} C_{3} - (C_{1}^{\dagger} C_{3}) C_{2}^{\dagger} C_{4} + (C_{2}^{\dagger} C_{3}) C_{1}^{\dagger} C_{4} - (C_{2}^{\dagger} C_{4}) C_{1}^{\dagger} C_{3} \rangle B^{\dagger}(B) \]

where \( \langle A \rangle_{L} \) means the result of the linearization procedure for the operator expression \( A \).

Linearization results in reducing of the corresponding Hamiltonian expressions to the forms \( \rho C^{\dagger} CB^{\dagger} \) and \( \rho C^{\dagger} CB \). As a consequence, one obtains a possibility to include them into the equation for \( \varphi_{q}^{p} \), which, with taking into account additional terms, appears to be as follows

\[-iG_{ep}q^{-3/2} + (\epsilon_{p} - \epsilon_{p} - \omega_{q}) \varphi_{q}^{p} + q \frac{A}{2} (\varphi_{q}^{p} - \varphi_{q}^{*p-q}) + 2G_{ee}q^{-2} \sum_{\nu} \int (\varphi_{k}^{p} - \varphi_{q}^{*p+q}) \rho_{\nu p} dkd\]

\[ - iG_{ep}q^{-3/2} + (\epsilon_{p} - \epsilon_{p} - \omega_{q}) \varphi_{q}^{p} + q \frac{A}{2} (\varphi_{q}^{p} - \varphi_{q}^{*p-q}) + 2G_{ee}q^{-2} \sum_{\nu} \int (\varphi_{q}^{k} - \varphi_{q}^{k+q}) \rho_{\nu p} dkd = 0. \tag{12} \]

The process of including the terms of a higher order can be continued, for example, by bilinearization of the \( C^{\dagger} C^{\dagger} C^{\dagger} C^{\dagger} B^{\dagger}(B) \) terms according to \( \rho_{\nu p} \). Therefore, it is necessary to introduce hierarchy of arising terms in order to single out the main ones in the equation for \( \varphi_{q}^{p} \).

We will assume that
The analysis of the first terms of the Hamiltonian $H_{tot}$ in variables $C^{\dagger}$, $C$, $B^{\dagger}$, and $B$, included in the equation for $\varphi^P_q$, shows that the $n$-linearization leads to the appearance of a factor $J_q^n$ in corresponding equation terms. So, it follows that $J_q$, or $q^{1/2}$, in this sense plays the role of a small parameter. This conclusion does not contradict the remark made with regard to Eqs. (9)-(10). The next from Eq. (13) it is easy to see that the following equation for $\varphi^P_q$ accounts for the main terms at $q \to 0$

$$-iG_{ep}q^{-3/2} + 2iG_{ee}q^{-2}J_q + (\epsilon_{p-q} - \epsilon_p + \omega_q)\varphi^P_q + 2Aq(\varphi^P_q - \varphi^{*P-q}_q)$$

$$-\frac{3}{2}G_{ep}q^{-3/2}J_q(\varphi^P_q - \varphi^{*P-q}_q) + G_{ee}q^{-2}J_q^2(\varphi^P_q - \varphi^{*P-q}_q) = 0 .$$

Derivation of solution for $\varphi^P_q$ according to Eq. (14) is not a very complicated procedure. Therefore, it seems to be advisable simply to present the expression for $\varphi^P_q$, whose validity may be checked by a direct substitution into Eq. (14):

$$\varphi^P_q = iG_{ep}\frac{q^{1/2}}{q^2 + \lambda^2} \frac{\epsilon_p - \epsilon_{p-q} + \omega_q}{S^2q^2 - (\epsilon_{p-q} - \epsilon_p)^2} .$$

where $S^2 = s^2 + (zm/6M)V_F^2$, $V_F = K_F/m$, $\lambda^2 = (4\epsilon^2/\pi r)mK_F$, $K_F$ is Fermi wave vector, $K_F = (3\pi^2 z/\Omega)$.

In this case, for $J_q$ we derive the following expression:

$$J_q = 8\pi mK_FG_{ep}\frac{q^{1/2}}{q^2 + \lambda^2} ,$$

being in accord with initial assumption (13). In deriving Eqs. (15)-(16) we assumed that

$$\rho_{\nu\nu} = \begin{cases} 1 & \text{for } p < K_F \\ 0 & \text{for } p > K_F \end{cases} ,$$

which well describes the case of a degenerated electron gas at temperatures $T < E_F/k_B$, $E_F = K_F^2/2m$, $k_B$ is the Boltzman constant. Attention should also be paid to a relationship (see Eq. (14))

$$2Aq - 3G_{ep}q^{-3/2}J_q + G_{ee}q^{-2}J_q^2 = Dq ;$$

$$D = \frac{\pi^2 z^2}{4mMK_F\Omega s} ,$$

which is fulfilled at an accuracy up to terms $\sim q^2$. We will return to it later when discussing the problems of renormalization of the sound velocity in the model under consideration.

**IV. INFLUENCE OF SCREENING ON THE PHONON SPECTRUM**

The interaction of phonons with the electrons of the conductivity zone leads to an effect of renormalization of sound velocity. The point is that experimentally observable sound velocity in metal substantially differs from sound velocity in the valence skeleton $S$, which was directly inserted into Eq. (2). In order to prove this statement, let us substitute Eqs. (3)-(14) in view of Eq. (13) into Eqs. (2) and single out terms with the structures $B^{\dagger}B$, $B^{\dagger}B^{\dagger}$, $BB$, $C^{\dagger}CB^{\dagger}B^{\dagger}$, $C^{\dagger}CB^{\dagger}B^{\dagger}$, $C^{\dagger}CBB$, $C^{\dagger}C^{\dagger}CCBBB^{\dagger}$, $C^{\dagger}C^{\dagger}CCBB^{\dagger}B^{\dagger}$ and $C^{\dagger}C^{\dagger}CCBB$. 

\[ \sum_{\nu} \int (\varphi_{q}^{k+q} - \varphi_{q}^{k}) \rho_{\nu k} dk = -iJ_q \sim q^{1/2} . \]
The terms containing $C^\dagger C$ are linearized and terms containing $C^\dagger C^\dagger CC$ are bilinearized according to a scheme described in Section 3. After singling out the main terms at $q \to 0$ at an accuracy up to the terms $\sim M^{-1/2}$ ($A$ and $s \sim M^{-1/2}$, $G_{ep}$, $J_q$, and $\varphi \sim M^{-1/4}$), it is possible to obtain the following expressions for the energy of phonon system $H_{op}$:

$$H_{op} = \int \omega_q B^\dagger_q B_q dq$$

(19)

$$+ 2A \int B^\dagger_q B_{-q} dq - 2G_{ep} \int J_q B^\dagger_q B_{-q} dq + 2G_{ee} \int J_q B^\dagger_q B_{-q} dq$$

(20)

$$- A \int B^\dagger_q B_{-q} dq + G_{ep} \int J_q B^\dagger_q B_{-q} dq - G_{ee} \int J_q B^\dagger_q B_{-q} dq$$

(21)

$$- A \int B_q B_{-q} dq + G_{ep} \int J_q B_q B_{-q} dq - G_{ee} \int J_q B_q B_{-q} dq$$

(22)

$$+ \int \left[ (\epsilon_{k+q} - \epsilon_k - \omega_q) |\varphi_{q+k}^2| - (\epsilon_k - \epsilon_{k+q} - \omega_q) |\varphi_{q-k}|^2 \right] \sum_{\nu} \rho_{\nu k} B^\dagger_q B_{-q} dqdk$$

(23)

$$+ \int \left[ (\epsilon_k - \epsilon_{k+q}) \varphi_{q-k}^2 \sum_{\nu} \rho_{\nu k} B^\dagger_{-q} B_q + (\epsilon_k - \epsilon_{k+q}) \varphi_{q+k}^2 \sum_{\nu} \rho_{\nu k} B_q B_{-q} \right] dqdk$$

(24)

From an easily verified relationship

$$\frac{A}{q} = G_{ep} q^{-3/2} J_q + G_{ee} q^{-2} J^2_q = 0,$$

(25)

which is fulfilled at an accuracy up to terms $\sim q^2$, it is possible to conclude that the terms (20)-(22) do not contribute to the renormalization of sound velocity. Thus, it is possible to state that the Hamiltonian $H_{pp}$ in Eq.(3), describing the self-action of a phonon field, turns out to be completely compensated, when the screening action of the electrons of the conductivity zone has been accounted for. Presumably the appearance of a factor $Aq^{-1}$ in the model under investigation may be considered to be the opening link in a chain $Aq^{-1} \to G_{ep} q^{-3/2} J_q \to G_{ee} q^{-2} J^2_q \to$ leading to the compensation of the singularities induced by the factor $Aq^{-1}$ in the Hamiltonian (1) (see also (18)).

After the integration of the terms (23),(24) and taking into account that $S - s \sim M^{-1/2}$, $S^2 - s^2 \sim M^{-1}$, the problem of phonon spectrum determination reduces to diagonalization of the Hamiltonian $H_{op}$:

$$H_{op} = (s + F) \int q B^\dagger_q B_q dq - \frac{F}{2} \int q \left[ B^\dagger_q B_{-q} + B_q B_{-q} \right] dq ;$$

(26)

$$F = \frac{z}{6mM s} K^2_{ext} .$$

This problem can be resolved using Bogolubov (1947) transformation:

$$B^\dagger_q = C_\theta \tilde{B}^\dagger_q + S_\theta \tilde{B}_{-q} ; \quad B_q = C_\theta \tilde{B}_{-q} + S_\theta \tilde{B}^\dagger_q ,$$

(27)

where

$$C_\theta = \frac{\cos \theta}{\sqrt{\cos 2\theta}} ; \quad S_\theta = \frac{\sin \theta}{\sqrt{\cos 2\theta}} ; \quad (\sin 2\theta = \frac{F}{s + F}) .$$

(28)

Transition to the new operators $\tilde{B}^\dagger_q$ and $\tilde{B}_q$ leads to the renormalization of sound velocity, given by the following expression

$$\tilde{S} = \sqrt{s^2 + \frac{zm}{3M} V^2_{ext}}$$

(29)
where \( V_F = K_F/m \) is the Fermi velocity, and \( \tilde{S} \) is the experimentally observed sound velocity in metal. At \( s \to 0 \) the sound velocity becomes equal to the Bohm–Steier velocity \( \tilde{S}_{BS} \) taken from the "jelly" model

\[
\tilde{S}_{BS} = V_F \sqrt{\frac{2m}{3M}}. 
\]

On the other hand, the validity of Eq.(29) in the case of a dielectric material \((K_F = 0)\) also casts no doubts. The realization of Eq.(29) in two limits proves that the derived expression (29) takes into account the main corrections in phonon spectrum associated with the filling of the conductivity zone with electrons.

V. CORRELATION EFFECTS IN THE DEGENERATED ELECTRON PLASMA

Transition to polaron operators of creation and annihilation \( \tilde{C}^{\dagger} \) and \( \tilde{C} \) leads to the renormalization of the el-el interaction. This becomes evident when the terms (1)-(5) are substituted into Eqs.(1)-(5), followed by singling out the terms with the structure \( \tilde{C}^{\dagger} \tilde{C} \). If we limit ourselves to consideration of merely el-el interaction, the Hamiltonian of the system may be reduced to

\[
H_{tot} = \sum_\mu \int \epsilon_p C_{\mu p}^{\dagger} C_{\mu p} dP + \sum_\mu \sum_\nu \int \int \int \int U \left( \begin{array}{c} p \\ k \\ q \end{array} \right) C_{\mu p+q}^{\dagger} C_{\nu k-q}^{\dagger} C_{\nu k} C_{\mu p} dP dK dQ. 
\]

where

\[
U \left( \begin{array}{c} p \\ k \\ q \end{array} \right) = G_{ee} q^{-2} + \delta V_{epe} ,
\]

\[
\delta V_{epe} = i \left[ G_{kp} q^{-3/2} \left( \varphi_{k-q}^{*} - \varphi_{k-q} \right) - G_{ee} q^{-2} J_q \left( \varphi_{k-q}^{*} - \varphi_{k-q} \right) \right]. 
\]

Note that the main terms \( \sim M^{-1/2} \) and \( \sim q^{-2} \) at \( q \to 0 \) have been singled out in Eq.(32). No renormalization of the electron kinetic energy (mass) due to the polaron effect has been done. This is associated with the fact that the kinetic energy of electron is assumed to be the highest of the values included in the present investigation. Therefore, its relative change cannot be substantial. As regards the term \( \delta V_{epe} \), in our formalism it is just the very term that is responsible for el-el interaction via virtual phonon exchange.

It is obvious that correlation effects associated with the filling of the conductivity zone should lead to a substantial change in the effective el-el interaction. In connection with this, let us consider a transition to new, quasi-particle electron creation and annihilation operators \( \tilde{C}^{\dagger} \) and \( \tilde{C} \):

\[
C_{\mu p}^{\dagger} = \tilde{C}^{\dagger}_{\mu p} - \sum_\nu \int \theta_{\nu}^\mu \left( \begin{array}{c} p \\ k \\ q \end{array} \right) \tilde{C}_{\mu p+q}^{\dagger} \tilde{C}_{\nu k-q}^{\dagger} \tilde{C}_{\nu k} \tilde{C}_{\mu p} dK dQ ; 
\]

\[
C_{\mu p} = \tilde{C}_{\mu p} - \sum_\nu \int \theta_{\nu}^{*\mu} \left( \begin{array}{c} p \\ k \\ q \end{array} \right) \tilde{C}_{\nu k} \tilde{C}_{\nu k-q}^{\dagger} \tilde{C}_{\mu p+q}^{\dagger} dK dQ , 
\]

where

\[
\theta_{\nu}^\mu \left( \begin{array}{c} p \\ k \\ q \end{array} \right) = \frac{1}{\epsilon_p + \epsilon_k - \epsilon_{p+q} - \epsilon_{k-q}} H_{\nu}^\mu \left( \begin{array}{c} p \\ k \\ q \end{array} \right); 
\]

\[
H_{\nu}^\mu \left( \begin{array}{c} p \\ k \\ q \end{array} \right) = 2\tilde{U} \left( \begin{array}{c} p \\ k \\ q \end{array} \right) + \delta_{\nu}^\mu \left[ 2\tilde{V} \left( \begin{array}{c} p \\ k \\ q \end{array} \right) - \tilde{U} \left( \begin{array}{c} p \\ k \\ q \end{array} \right) - \tilde{U} \left( \begin{array}{c} p \\ k \\ q \end{array} \right) \right] . 
\]
For the terms $\hat{U}$ and $\hat{V}$, without infringing on the general pattern of our arguments, we stipulate the fulfillment of the following conditions

$$
\hat{U} \left( \begin{array}{c} p \\ k \\ q \\ -n \end{array} \right) = \hat{U} \left( \begin{array}{c} k \\ p \\ q \\ -n \end{array} \right) ; \quad \hat{V} \left( \begin{array}{c} p \\ k \\ q \\ -n \end{array} \right) = \hat{V} \left( \begin{array}{c} k \\ p \\ q \\ -n \end{array} \right) ; \quad \hat{V} \left( \begin{array}{c} p \\ k \end{array} \right) = -\hat{V} \left( \begin{array}{c} p \\ k \end{array} \right) \quad (35)
$$

Let us note two remarks. Firstly, the transformation (33) does not break the commutation relations for operators $\hat{C}^\dagger$ and $\hat{C}$ at an accuracy up to the terms $\sim \theta^2$ (function $\theta$ now serves as a small parameter, see Appendix 2). Secondly, it is transformation (33) that diagonalizes in the first order of the perturbation theory in $\theta$ the Hamiltonian of the following form (see Appendix 3)

$$
\hat{H}_{tot} = \sum_{\mu} \int \epsilon_{\mu} \hat{C}_{\mu} \hat{C}^\dagger_{\mu} d\rho + \hat{H}_{ee} ; \quad (36)
$$

$$
\hat{H}_{ee} = \sum_{\mu} \sum_{\nu} \int \int \int \left[ \hat{U} \left( \begin{array}{c} p \\ k \\ q \\ -n \end{array} \right) \right] \hat{V} \left( \begin{array}{c} p \\ k \\ q \\ -n \end{array} \right) \delta_{\nu\nu} \hat{C}_{\mu} \hat{C}_{\nu} \hat{C}^\dagger_{\nu} \hat{C}^\dagger_{\mu} d\rho d\nu d\kappa \quad (37)
$$

It is easy to see that the condition (33) comes from symmetries of (37) under permutations, while the Hermiticity requirement dictates the following relation:

$$
\hat{U} \left( \begin{array}{c} p \\ k \\ q \\ -n \end{array} \right) = \hat{U}^* \left( \begin{array}{c} k \\ p \\ q \\ -n \end{array} \right) ; \quad \hat{V} \left( \begin{array}{c} p \\ k \\ q \\ -n \end{array} \right) = \hat{V}^* \left( \begin{array}{c} k \\ p \\ q \\ -n \end{array} \right) ; \quad (38)
$$

and finally the symmetry of equations of motion under time reverse $t \rightarrow -t$ gives the following relations:

$$
\hat{U} \left( \begin{array}{c} p \\ k \\ q \\ -n \end{array} \right) = \hat{U}^* \left( \begin{array}{c} -p \\ -k \\ -q \\ n \end{array} \right) ; \quad \hat{V} \left( \begin{array}{c} p \\ k \\ q \\ -n \end{array} \right) = \hat{V}^* \left( \begin{array}{c} -p \\ -k \\ -q \\ n \end{array} \right) . \quad (39)
$$

It is obvious that the potentials $\hat{U}$ and $\hat{V}$ after introduction the self-consistent procedure will play the role of an effective el-el interaction potential, accounting for the correlation effects of electron plasma. The procedure for determination of $\hat{U}$ and $\hat{V}$ matrix elements should account for, as in the case of the determination of $\hat{\varphi}^\rho_{\mu\nu}$, finite filling of the conductivity zone. For this purpose, similarly to what has been done in Section 3, let us express the Hamiltonian (31) in terms of operators $\hat{C}^\dagger$ and $\hat{C}$ using Eqs. (34).

The substitution results in the appearance of the terms with the structure $\hat{C}^\dagger \hat{C}^\dagger \hat{C} \hat{C} \hat{C} \hat{C}$ in the Hamiltonian $\hat{H}_{tot}$. These terms may be linearized with respect to occupation density $\hat{\rho}_{\mu\nu}$, producing in this way the structure $\hat{\rho} \hat{C}^\dagger \hat{C}^\dagger \hat{C} \hat{C} \hat{C}$. This enables us to include these terms in the self-consistency equation for $\hat{U}$ and $\hat{V}$ of the following form

$$
\hat{U} \left( \begin{array}{c} p \\ k \\ q \\ -n \end{array} \right) + \delta_{\nu\nu} \hat{V} \left( \begin{array}{c} p \\ k \\ q \\ -n \end{array} \right) = U \left( \begin{array}{c} p \\ k \\ q \\ -n \end{array} \right) - \sum_{\xi} \int U \left( \begin{array}{c} t \\ k \\ q \\ -n \end{array} \right) \theta_{\mu}^{\xi} \left( \begin{array}{c} t \\ -q \\ p \\ -n \end{array} \right) dt - \sum_{\xi} \int \int \left( \begin{array}{c} p \\ t \end{array} \right) \theta_{\mu}^{\xi} \left( \begin{array}{c} t \\ -q \\ k \\ -n \end{array} \right) \left( \begin{array}{c} t \\ k \\ q \\ -n \end{array} \right) dt
$$

$$
- \sum_{\xi} \int \int \left( \begin{array}{c} p \\ t \\ k \\ q \\ -n \end{array} \right) \theta_{\nu}^{\xi} \left( \begin{array}{c} t \\ k \\ q \\ -n \end{array} \right) dt - \sum_{\xi} \int \int \left( \begin{array}{c} t \\ q \\ k \\ -n \end{array} \right) \theta_{\nu}^{\xi} \left( \begin{array}{c} t \\ k \\ q \\ -n \end{array} \right) \left( \begin{array}{c} t \\ k \\ q \\ -n \end{array} \right) dt
$$

$$
+ \int \int \left( \begin{array}{c} t \\ k \\ q \\ -n \end{array} \right) \theta_{\mu}^{\nu} \left( \begin{array}{c} t \\ k \\ q \\ -n \end{array} \right) dt + \int \int \left( \begin{array}{c} t \\ k \\ q \\ -n \end{array} \right) \theta_{\nu}^{\mu} \left( \begin{array}{c} t \\ k \\ q \\ -n \end{array} \right) dt \quad (40)
$$

the integration in Eq. (40) being carried out in the region $t < K_F$, it means that the case of low temperatures is considered. Note, that in the rhs of Eq. (40) only the terms $\sim \theta$ and $\sim q^{-3}$ are presented.
It is easy to see from Eq.(40) that in order the term \( \tilde{U} \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle q \end{array} \right) \) could satisfy conditions (35) and (38), the term \( U \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle q \end{array} \right) \) must also satisfy analogous relations:

\[
U \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle q \end{array} \right) = U \left( \begin{array}{c} \scriptstyle k \\ \scriptstyle p \end{array} \right) ; \quad U \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle q \end{array} \right) = U^* \left( \begin{array}{c} \scriptstyle k + q \\ \scriptstyle -q \end{array} \right) ; \quad \ldots \tag{41}
\]

In connection with requirements of Eq.(41), symmetrization of the term \( U \) leads to the following expression for it:

\[
U \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle k \\ \scriptstyle q \end{array} \right) = \frac{G_{ee}}{q^2} + \frac{G_{ep}^2}{q^2 + \lambda^2} \frac{ms}{2qS} \left[ \pm \frac{1}{2pq + q^2 + 2mSq} \pm \frac{1}{2kq - q^2 + 2mSq} \right], \tag{42}
\]

and in this case the Hamiltonian (31) obviously is not changed.

Although it would be preferable to solve Eq.(40) numerically, from the point of view of the achievable accuracy of solution it seems to be of particular interest to obtain even a rough approximate solution without using computer technique: such an approximate solution enables one to predict the character of limit relationships in exact solution.

The essential point in the proposed solution scheme is the following approximate estimation of integrals inside the spheres with radius \( K_F \):

\[
\int_{t < K_F} F(t)S(t)dt \approx F(0) \int_{t < K_F} S(t)dt, \tag{43}
\]

where \( F(t) \) is regular, and \( S(t) \) is the singular functions of variable \( t \). It is obvious that this approximate relation is well performed when the character scale of change for \( F \) turns out to be greater than \( K_F \).

We will assume that the sought functions \( \tilde{U} \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle k \\ \scriptstyle q \end{array} \right) \) and \( \tilde{V} \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle k \\ \scriptstyle q \end{array} \right) \) have the form

\[
\tilde{U} \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle k \\ \scriptstyle q \end{array} \right) = F_1 \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle k \\ \scriptstyle q \end{array} \right) S_1(q) ; \quad \tilde{V} \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle k \\ \scriptstyle q \end{array} \right) = F_2 \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle k \\ \scriptstyle q \end{array} \right) S_2(q) ; \tag{44}
\]

where \( F_1 \) and \( F_2 \) are regular, whereas \( S_1 \) and \( S_2 \) are singular functions of their parameters. Then only those terms should be left in Eq.(40), in which the terms \( S_l(q) \) can be put outside the integral, and with the help of Eq. (43) it is possible to obtain the following approximate equations

\[
\tilde{V} \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle k \\ \scriptstyle q \end{array} \right) = 0 ; \tag{45}
\]

\[
\tilde{U} \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle k \\ \scriptstyle q \end{array} \right) = U \left( \begin{array}{c} \scriptstyle p \\ \scriptstyle k \\ \scriptstyle q \end{array} \right) + 4\tilde{U} \left( \begin{array}{c} \scriptstyle p + q \\ \scriptstyle -q \end{array} \right) \left\{ \frac{mG_{ee}}{q^3} I \left[ \frac{(p + q)q}{q} \right] \right. \\
+ \frac{G_{ep}^2 m^2 s / S}{2q^2 q^2 + \lambda^2} \left[ \frac{1}{2kq - q^2 - 2mSq} - \frac{1}{2kq - q^2 + 2mSq} \right] I \left[ \frac{(p + q)q}{q} \right] \\
+ \frac{G_{ep}^2 m^2 s / S}{2q^2 q^2 + \lambda^2} \left[ \frac{1}{2pq + q^2 + 2mSq} \right] \left( I \left[ \frac{(p + q)q}{q} \right] - I \left[ mS + \frac{q}{2} \right] \right) \\
- \frac{G_{ep}^2 m^2 s / S}{2q^2 q^2 + \lambda^2} \left[ \frac{1}{2pq + q^2 + 2mSq} \right] \left( I \left[ \frac{(p + q)q}{q} \right] + I \left[ mS - \frac{q}{2} \right] \right) \right\}.
\]
\[-4\hat{U}\left(\frac{p}{q}\right)\left\{mG_{ee}I\left[\frac{pq}{q^3}\right] + \frac{G_{ep}^2 m^2 s / S}{2q^2 q^2 + \lambda^2} \left[\frac{1}{2kq - q^2 - 2msq} - \frac{1}{2kq - q^2 + 2msq}\right] I\left[\frac{pq}{q}\right]\right.\]
\[+ \frac{G_{ep}^2 m^2 s / S}{2q^2 q^2 + \lambda^2} \left[\frac{1}{2pq + q^2 - 2msq}\right] \left(I\left[\frac{pq}{q}\right] - I\left[ms - \frac{q}{2}\right]\right)\]
\[- \frac{G_{ep}^2 m^2 s / S}{2q^2 q^2 + \lambda^2} \left[\frac{1}{2pq + q^2 + 2msq}\right] \left(I\left[\frac{pq}{q}\right] + I\left[ms + \frac{q}{2}\right]\right)\right\}\]
\[+ 4\hat{U}\left(\frac{k-q}{q}\right)\left\{mG_{ee}I\left[\frac{(k-q)q}{q}\right] + \frac{G_{ep}^2 m^2 s / S}{2q^2 q^2 + \lambda^2} \left[\frac{1}{2kq - q^2 - 2msq}\right] \left(I\left[\frac{k-q}{q}\right] - I\left[ms - \frac{q}{2}\right]\right)\right.\]
\[+ \frac{G_{ep}^2 m^2 s / S}{2q^2 q^2 + \lambda^2} \left[\frac{1}{2kq - q^2 + 2msq}\right] \left(I\left[\frac{k-q}{q}\right] + I\left[ms + \frac{q}{2}\right]\right)\]
\[- \frac{G_{ep}^2 m^2 s / S}{2q^2 q^2 + \lambda^2} \left[\frac{1}{2pq + q^2 - 2msq} - \frac{1}{2pq + q^2 + 2msq}\right] I\left[\frac{(k-q)q}{q}\right]\right\},\tag{46}\]

where

\[I[a] = q \int_{kF} dt \left[\frac{dt}{q - a}\right] = \pi K_F^2 \ln \left(\frac{K_F - a}{K_F + a}\right) - 2\pi a K_F .\tag{47}\]

Note that at $a \ll K_F$ we have $I[a] \approx -4\pi a K_F$.

The self-consistency Eq. (46) can be related to equation of recurrent type. It is solved according to the scheme

\[\hat{U}\left(\frac{-a}{q}\right) \rightarrow \hat{U}\left(\frac{p}{q}\right) \rightarrow \hat{U}\left(\frac{p}{a}\right) .\]

For the term $\hat{U}\left(\frac{-a}{q}\right)$ it is possible to derive the following expression:

\[\hat{U}\left(\frac{-a}{q}\right) = U\left(\frac{-a}{q}\right) + 8\hat{U}\left(\frac{-a}{q}\right) \left[mG_{ee} + \frac{G_{ep}^2 m^2 s / S}{2q^2 q^2 + \lambda^2} \left(\frac{1}{q^2 - 2msq} - \frac{1}{q^2 + 2msq}\right)\right] I[q]\]

and, thus, we get

\[\hat{U}\left(\frac{-a}{q}\right) = \frac{G_{ee}}{q^2 + 2\lambda^2} + \frac{zm}{3M} K_F^{2a} - 2 \frac{G_{ee}}{q^2 - \chi_1}\]

where $\chi = 4 \left(m^2 \hat{S}^2 - (zm/3M) K_F^2\right)$ (see Eqs. (15), (29)), $\chi_1 = 1 + (zm/3M)(K_F^2/\lambda^2)$, and one has $\chi_1 - \chi \ll \chi$.

The direct substitutions into Eq. (46) enables one to derive $\hat{U}\left(\frac{p+a}{q}\right) = \hat{U}\left(\frac{a}{q}\right)$. Thus, we obtain an equation for $\hat{U}\left(\frac{p}{q}\right)$:
\[
\tilde{U} \left( \frac{p}{a} \right) = U \left( \frac{p}{a} \right) + 4\tilde{U} \left( \frac{p}{a} \right) \left\{ \frac{mG_{ee}}{q^3} I \left[ \frac{(p + q)q}{q} \right] \right. \\
+ \frac{G_{ep}^2 m^2 s/S}{2q^2 q^2 + \lambda^2} \left[ \frac{1}{q^2 - 2mSq} - \frac{1}{q^2 + 2mSq} \right] I \left[ \frac{(p + q)q}{q} \right] \\
+ \frac{G_{ep}^2 m^2 s/S}{2q^2 q^2 + \lambda^2} \left[ \frac{1}{2pq + q^2 + 2mSq} \right] \left( I \left[ \frac{pq}{q} \right] - I \left[ mS - \frac{q}{2} \right] \right) \\
- \frac{G_{ep}^2 m^2 s/S}{2q^2 q^2 + \lambda^2} \left[ \frac{1}{2pq + q^2 + 2mSq} \right] \left( I \left[ \frac{pq}{q} \right] + I \left[ mS - \frac{q}{2} \right] \right) \right\} \\
- 4\tilde{U} \left( \frac{-a}{a} \right) \left\{ \frac{mG_{ee}}{q^3} + \frac{G_{ep}^2 m^2 s/S}{2q^2 q^2 + \lambda^2} \left( \frac{1}{2pq + q^2 + 2mSq} - \frac{1}{2pq + q^2 + 2mSq} \right) \right\} I[q].
\]

Now, it is necessary to make a remark. We will assume that \( p \approx K_F \). In other words, we are interested in the interaction of electrons in the vicinity of the Fermi surface. Then, at \( q \to 0 \), the fractions of the type \( \frac{1}{q^2 - 2mSq} \) actually contain a factor \( K_F^{-1} \), whereas the fractions of the type \( \frac{1}{q^2 + 2mSq} \) contain the factor \( (mS)^{-1} \). So, the fractions of the first type turn out to be small as compared to the fractions of the second type with the order of smallness \( \sim mS/K_F \ll 1 \).

These arguments provide a basis to exclude from Eq. (50) terms containing of the type \( \frac{1}{2pq + q^2 + 2mSq} \) as factors. This will simplify Eq. (50), and, after proper transformation, it can be reduced to the form

\[
\tilde{U} \left( \frac{p}{a} \right) = U \left( \frac{p}{a} \right) + 4\frac{mG_{ee}}{q^3} I[q] \tilde{U} \left( \frac{-a}{a} \right) \\
+ 4\tilde{U} \left( \frac{p}{a} \right) \left[ \frac{pq}{q} \right] \left[ \frac{mG_{ee}}{q^2} + \frac{G_{ep}^2 m^2 s/S}{2q^2 q^2 + \lambda^2} \left( \frac{1}{q - 2mS} - \frac{1}{q + 2mS} \right) \right],
\]

where, within the limits of our rough approximations, it is possible to assume

\[
\left( I \left[ \frac{(p + q)q}{q} \right] - I \left[ \frac{pq}{q} \right] \right) \approx q \approx I' \left[ \frac{pq}{q} \right]; \quad I'[x] \equiv \frac{d}{dx} I[x].
\]

Leaving at \( q \to 0 \) merely the main terms and in the view of (49) it is possible to derive from Eq. (51) the following one:

\[
4\frac{mG_{ee}}{q^2} I' \left[ \frac{pq}{q} \right] \tilde{U} \left( \frac{p}{a} \right) = -\frac{G_{ee}}{2q^2} - 2\left( \frac{2m}{3M} \right)^2 \frac{G_{ee}}{q^2} \frac{K_F^2}{q^2 - \chi_1} \frac{K_F^2}{q^2 - \chi}
\]

In an analogous way for \( \tilde{U} \left( \frac{p}{k} \right) \) at \( p, k \approx K_F \) accounting for the remarks to derivation of Eq. (51) we obtain
\[
\hat{U} \left( \frac{p}{k} \right) = \frac{G_{ee}}{q^2} + 4 \frac{mG_{ee}}{q^2} I' \left[ \frac{pq}{q} \right] \hat{U} \left( \frac{p}{q} \right) + 4 \frac{mG_{ee}}{q^2} I' \left[ \frac{qk}{q} \right] \hat{U} \left( \frac{k}{-q} \right) \quad (54)
\]

and finally
\[
\hat{U} \left( \frac{p}{k} \right) = -4 \left( \frac{zm}{3M} \right)^2 \frac{G_{ee}}{q^2} K_F^2 \left( \frac{Q}{2} \right) - \chi \left( \frac{Q}{2} \right) \chi \quad (55)
\]

In a coordinate representation el-el interaction \( V(r) \) may be estimated as follows (\( Q^2 = \chi, Q_1^2 = \chi_1 \)):
\[
V(r) \approx \left( \frac{zm}{M} \right)^2 \frac{G_{ee} K_F^2}{(\chi - 1)^2} \left[ \alpha^{-1} \sin^2 \left( \frac{Qr}{2} \right) - \chi^{-1} \sin^2 \left( \frac{Qr}{2} \right) \right] \quad (56)
\]

According to (55) the specific features of the el-el interaction associated with the virtual phonon exchange result in the absence of screening of the long-wave part of this interaction component. We would like to note here that in the case of Coulomb el-el interaction the same calculation procedure (Appendix 4) gives the result, which is consistent with the consideration of electron plasma in Thomas–Fermi approximation.

The potential of the type (56) causes instability with respect to formation of pairs, which may lead to a superconductive phase transition. Without taking in consideration short-wave corrections the pair binding energy \( E_b \) can be estimated as follows:
\[
E_b \sim \left( \frac{zm}{M} \right)^2 \frac{G_{ee} K_F^4}{Q^3} \quad (57)
\]

Several remarks should be made. Firstly, it is obvious that \( E_b \sim M^{-1/2} \), that is in agreement with the observations of the isotopic effect in a series of metals [13]. Secondly, the term \( E_b \) in Eq. (55) has been explicitly overrated if we have the intention to estimate \( T_s \) from \( E_b \) according to standard procedure. In fact, Eq. (57) determines the energy gap averagely over the Fermi surface while the transition temperature is determined by its minimum value over it. This means that enhancing symmetry should lead to an increase in \( T_s \), which is consistent with the observation of superconductivity in amorphous alloys [20]. Thirdly, and it is the main point, oscillations of the potential \( V(r) \) lead to the decrease of interaction between pairs. This is due to the fact that calculating the corresponding matrix elements involves some averaging procedure of \( V(r) \) over the regions roughly equal in size to the size of a pair. This question needs detailed consideration and is postponed for further publication.

Of particular interest is the singular behavior of the binding energy \( E_b \) at \( Q \to 0 \), following from Eq. (57). In real systems this singularity should not take place in the behavior of \( T_s \) even though in virtue of the symmetry reasons stated above. Nevertheless maximum values of \( T_s \) should be observed if the following condition were fulfilled:
\[
\tilde{S}^2 \approx \frac{zm}{3M} V_F^2 \quad (58)
\]

This condition may be fulfilled with sufficient accuracy in systems with widely modified physical parameters, for instance, in solid solutions, where we believe HTSC is to be observed when the condition (58) is fulfilled. From this viewpoint the researches into HTSC in stoichiometric compounds of simple composition [14, 16] become of current interest because reliable zone calculations can be carried out just for these materials. In this case analyzing experimental data it should be kept in mind that the parameters \( M \) and \( z \) in Eq. (58) become equal to the sum of masses of ions and number of conductivity electrons respectively per cell.
APPENDIX A: INVARIANCE OF THE COMMUTATION RELATIONS FOR CREATION AND ANNHIILATION OPERATORS WITH RESPECT TO TRANSFORMATIONS (9)-(10).

For the commutator $[c^\dagger_{\mu p} ; c_{\nu k}]$ we have:

$$[c^\dagger_{\mu p} ; c_{\nu k}] = [C^\dagger_{\mu p} ; C_{\nu k}]$$

$$- \int \varphi^k_q [C^\dagger_{\mu p} ; C_{\nu k-q} B_q] \, dq + \int \varphi^{k+q}_q [C^\dagger_{\mu p} ; C_{\nu k+q} B^\dagger_q] \, dq$$

$$- \frac{1}{2} \int |\varphi^k_q|^2 [c^\dagger_{\mu p} ; C_{\nu k}] \, dq$$

$$+ \frac{1}{2} \sum_{\xi} \int \int \left( \varphi^m_{-q} \varphi^k_{-q} - \varphi^k_{q} \varphi^m_{q} \right) \left[C^\dagger_{\mu p} ; C_{\xi m} C_{\xi m+q} C_{\nu k-q} \right] \, dq \, dm$$

$$- \frac{1}{2} \int \left( \varphi^k_{-m} \varphi^k_{-m+q} + \varphi^{k+q}_m \varphi^{k+q}_m \right) \left[C^\dagger_{\mu p} ; C_{\nu k-m} B^\dagger_m B^\dagger_m \right] \, dq \, dm$$

$$+ \frac{1}{4} \int \left( \varphi^k_{-m} \varphi^k_{-m+q} \varphi^k_{-m} \varphi^k_{m+q} \right) \left[C^\dagger_{\mu p} ; C_{\nu k-m} B_m B^\dagger_m \right] \, dq \, dm$$

$$- \int \varphi^p_q \left[C^\dagger_{\mu p-q} B^\dagger_q ; C_{\nu k} \right] \, dq + \int \varphi^{p+q}_q \left[C^\dagger_{\mu p+q} B^\dagger_q ; C_{\nu k} \right] \, dq$$

$$- \frac{1}{2} \int |\varphi^p_q|^2 \left[C^\dagger_{\mu p} ; C_{\nu k} \right] \, dq$$

$$+ \frac{1}{2} \sum_{\xi} \int \int \left( \varphi^p_q \varphi^m_{-q} - \varphi^m_{q} \varphi^p_{q} \right) \left[C^\dagger_{\mu p-q} C^\dagger_{\xi m+q} C_{\xi m} ; C_{\nu k} \right] \, dq \, dm$$

$$- \frac{1}{2} \int \left( \varphi^p_{-m} \varphi^p_{-m+q} + \varphi^{p+q}_m \varphi^{p+q}_m \right) \left[C^\dagger_{\mu p-m} B^\dagger_m B^\dagger_m ; C_{\nu k} \right] \, dq \, dm$$

$$+ \frac{1}{4} \int \left( \varphi^p_{-m} \varphi^p_{-m+q} \varphi^p_{-m} \varphi^p_{m+q} \right) \left[C^\dagger_{\mu p-m} B_m B^\dagger_m ; C_{\nu k} \right] \, dq \, dm$$

$$+ \int \left\{ \varphi^p_{-m} \varphi^k_{-m+q} \varphi^k_{-q} \varphi^k_{m+q} \right\} \left[C^\dagger_{\mu p-q} B^\dagger_q B^\dagger_q C_{\nu k} \right] \, dq \, dm$$

$$= \left[C^\dagger_{\mu p} ; C_{\nu k} \right] \left( 1 - \int |\varphi^p_q|^2 \, dq + \int |\varphi^p_q|^2 \, dq \right)$$

$$+ \frac{1}{2} \int \left( \varphi^p_{-k} B^\dagger_q \varphi^k_{-p-k} - \varphi^k_{-k} B^\dagger_p \varphi^p_{-p-k} + \varphi^k_{-p-k} B^\dagger_{p-k} \right) \, dq$$

$$+ \frac{1}{2} \int \left( \varphi^k_{-p-k} B^\dagger_q \varphi^k_{-p-k} \varphi^k_{-p-k} \right) \sum_{\xi} \left[C^\dagger_{\xi k+q} C_{\xi k+q} \right] \, dq$$

$$+ \frac{1}{2} \int \left( \varphi^k_{-p-k} \varphi^k_{-p-k} \varphi^k_{-p-k} \right) \sum_{\xi} \left[C^\dagger_{\xi k+q} C_{\xi k+q} \right] \, dq$$

$$- \frac{1}{2} \int \left( \varphi^p_{-q} \varphi^k_{-q} - \varphi^k_{-q} \varphi^p_{-q} \right) \left[C^\dagger_{\mu p-q} C_{\nu k-q} \right] \, dq$$

$$- \frac{1}{2} \int \left( \varphi^p_{-q} \varphi^k_{-q} - \varphi^k_{-q} \varphi^p_{-q} \right) \left[C^\dagger_{\mu p-q} C_{\nu k-q} \right] \, dq$$
with an accuracy up to terms $\sim |\varphi|^3$. The invariance for the remaining commutation relations $[c_{\mu p}^{\dagger}; c_{\nu k}^{\dagger}]$, $[c_{\mu p}^{\dagger}; b_{k}]$, $[b_{p}^{\dagger}; b_{k}]$, and $[b_{p}^{\dagger}; b_{k}^{\dagger}]$ is proved in close analogy to what has been given above.

**APPENDIX B: INVARIANCE OF THE COMMUTATION RELATIONS FOR CREATION AND ANNihilation OPERATORS WITH RESPECT TO TRANSFORMATIONS (33).**

We have:

$$[c_{\mu p}^{\dagger}; c_{\nu k}] = \left[\tilde{c}_{\mu p}^{\dagger} - \sum_{\xi} \int \theta_{\xi}^{\mu} \left( n_{\xi}^{p} \right) \tilde{C}_{\xi \nu k}^{\dagger} \tilde{C}_{\xi \nu k}^{\dagger} d\mathbf{m} d\mathbf{q} \right] ;$$

$$= \left[\tilde{c}_{\mu p}^{\dagger}; \tilde{c}_{\nu k}^{\dagger} - \sum_{\xi} \int \theta_{\xi}^{\mu} \left( n_{\xi}^{k} \right) \tilde{C}_{\xi \nu k}^{\dagger} \tilde{C}_{\xi \nu k}^{\dagger} d\mathbf{m} d\mathbf{q} \right] ;$$

$$= \left[\tilde{c}_{\mu p}^{\dagger}; \tilde{c}_{\nu k}^{\dagger} - \sum_{\xi} \int \theta_{\xi}^{\mu} \left( n_{\xi}^{n} \right) \tilde{C}_{\xi \nu k}^{\dagger} \tilde{C}_{\xi \nu k}^{\dagger} d\mathbf{m} d\mathbf{q} \right] ;$$

$$- \sum_{\xi} \int \theta_{\xi}^{\mu} \left( n_{\xi}^{n} \right) \tilde{C}_{\xi \nu k}^{\dagger} \tilde{C}_{\xi \nu k}^{\dagger} d\mathbf{m} d\mathbf{q} ;$$

Opening the commutators $[\tilde{c}_{\mu p}^{\dagger}; \tilde{c}_{\nu k}^{\dagger}]$ and $[\tilde{c}_{\mu p}^{\dagger}; \tilde{c}_{\nu k}^{\dagger}]$ we derive

$$[c_{\mu p}^{\dagger}; c_{\nu k}] = \left[\tilde{c}_{\mu p}^{\dagger}; \tilde{c}_{\nu k}^{\dagger} \right]$$

$$- \delta_{\mu}^{\nu} \sum_{\xi} \int \theta_{\xi}^{\mu} \left( n_{\xi}^{k} \right) \tilde{C}_{\xi \nu k}^{\dagger} \tilde{C}_{\xi \nu k}^{\dagger} d\mathbf{m} d\mathbf{q}$$

$$+ \int \left[ \theta_{\nu}^{\mu} \left( n_{\nu}^{p} \right) + \theta_{\nu}^{\mu} \left( n_{\nu}^{q} \right) \right] \tilde{C}_{\nu k}^{\dagger} \tilde{C}_{\nu k}^{\dagger} d\mathbf{m} d\mathbf{q} ;$$

$$+ O(\bar{\theta}^{2}) .$$
Substituting the term $\theta$ from Eq. (34) in view of Eqs. (35) and (36) we definitely have
\[
\left[ C_{\mu \nu}^\dagger : C_{\nu k} \right] = \left[ \tilde{C}_{\mu \nu}^\dagger : \tilde{C}_{\nu k} \right]
\]
with an accuracy up to terms $\sim \theta^2$.

The invariance for the commutation relations $\left[ \tilde{C}_{\mu \nu}^\dagger : \tilde{C}_{\nu k}^\dagger \right]$ and $\left[ \tilde{C}_{\mu \nu} : \tilde{C}_{\nu k} \right]$ is proved in close analogy to what has been given above.

**APPENDIX C: TESTING DIAGONALIZATION OF THE HAMILTONIAN (36) BY THE TRANSFORMATION (33).**

Expressing Eq. (33) via operators $\tilde{C}^\dagger$ and $\tilde{C}$ it is possible to obtain with an accuracy up to terms $\sim \theta^2$
\[
\int \epsilon_p \left[ \tilde{C}^\dagger_{\mu \nu} - \sum_{\nu} \gamma_{\nu} \left( \frac{p}{k} \right) \tilde{C}^\dagger_{\mu \nu} + \tilde{C}^\dagger_{\nu k - q} \tilde{C}_{\nu k} dkdq \right] \left[ \tilde{C}_{\mu \nu} - \sum_{\nu} \gamma_{\nu} \left( \frac{p}{k} \right) \tilde{C}_{\mu \nu} + \tilde{C}_{\nu k - q} \tilde{C}_{\nu k} dkdq \right] dp
\]
\[
= \int \epsilon_p \tilde{C}^\dagger_{\mu \nu} \tilde{C}_{\mu \nu} - \sum_{\nu} \epsilon_p \gamma_{\nu} \left( \frac{p}{k} \right) \tilde{C}^\dagger_{\mu \nu} \tilde{C}_{\nu k - q} \tilde{C}_{\nu k} dkdq dp + O(\theta^2) .
\]

(C.1)

Exchanging variables $p' = p + q$, $k' = k - q$, and $q' = -q$ we obtain
\[
\int \int \int \epsilon_p \gamma_{\nu} \left( \frac{p}{k} \right) \tilde{C}^\dagger_{\mu \nu} \tilde{C}_{\nu k - q} \tilde{C}_{\nu k} dkdq dp = \int \int \int \epsilon_p \gamma_{\nu} \left( \frac{p + q}{k - q} \right) \tilde{C}^\dagger_{\mu \nu} \tilde{C}_{\nu k - q} \tilde{C}_{\nu k} dkdq dp
\]
and then $H_{\theta}$, the sum of linear in $\theta$ terms in Eq. (C.2), is
\[
H_{\theta} = - \int \int \int \sum \epsilon_p \gamma_{\nu} \left( \frac{p + q}{k - q} \right) \left( \frac{p}{k} \right) \tilde{C}^\dagger_{\mu \nu} \tilde{C}_{\nu k - q} \tilde{C}_{\nu k} \tilde{C}_{\mu \nu} dkdq dp .
\]

The second substitution of variables $k' = p$, $p' = k$, and $q' = -q$ gives the following expression
\[
H_{\theta} = - \frac{1}{2} \int \int \int \sum \epsilon_p \gamma_{\nu} \left( \frac{p + q}{k - q} \right) \left( \frac{p}{k} \right) \tilde{C}^\dagger_{\mu \nu} \tilde{C}_{\nu k - q} \tilde{C}_{\nu k} \tilde{C}_{\mu \nu} dkdq dp .
\]

Substituting $\theta$ by the term (34) in view of Eqs. (35) and (36) leads to the following relation
\[
H_{\theta} = - \int \int \int \sum \epsilon_p \gamma_{\nu} \left( \frac{p}{k} \right) \left( \frac{p}{k} \right) \tilde{C}^\dagger_{\mu \nu} \tilde{C}_{\nu k - q} \tilde{C}_{\nu k} \tilde{C}_{\mu \nu} dkdq dp .
\]

On the other hand, by separate considering cases $\mu = \nu$ and $\mu \neq \nu$ it is possible to obtain the following expression for the term $H_{ee}$ in Eq. (17):
\[
H_{ee} = \int \int \int \sum \sum \epsilon_p \gamma_{\nu} \left( \frac{p}{k} \right) \left( \frac{p}{k} \right) \tilde{C}^\dagger_{\mu \nu} \tilde{C}_{\nu k - q} \tilde{C}_{\nu k} \tilde{C}_{\mu \nu} dkdq dp .
\]

Thus, it is the transformation (33) that diagonalizes the term $H_{tot}$ (36) with an accuracy up to terms $\sim \tilde{U}^2, \tilde{V}^2$. 
Let us assume in this case that the effective interaction $\hat{U}\left(\frac{p}{k}, \frac{q}{a}\right)$ is described by quite a regular function of all three parameters $p$, $k$, and $q$. Then the equation determining the term $\hat{U}\left(\frac{p}{k}, \frac{q}{a}\right)$ has the following form:

$$
\hat{U}\left(\frac{p}{k}, \frac{q}{a}\right) = \frac{G_{ee}}{q^2} + \frac{mG_{ee}}{q^3}\left\{ \left[ 2\hat{U}\left(\frac{q}{p}\right) - 4\hat{U}\left(\frac{p}{q}\right) \right] I\left[\frac{pq}{q}\right] + \left[ 4\hat{U}\left(\frac{p+q}{-q}\right) - 2\hat{U}\left(\frac{-q}{p+q}\right) \right] I\left[\frac{pq}{q} + q\right] \\
+ \left[ 4\hat{U}\left(\frac{-q}{k}\right) - 2\hat{U}\left(\frac{-q}{k}\right) \right] I\left[\frac{kq}{q}\right] + \left[ 2\hat{U}\left(\frac{k-k}{q}\right) - 4\hat{U}\left(\frac{k-k}{q}\right) \right] I\left[\frac{kq}{q} - q\right] \right\} .
$$

By direct substitution into Eq. (D.2) it is possible to show that $\hat{U}\left(\frac{p+a}{-q}, \frac{q}{a}\right) = \hat{U}\left(\frac{q}{a}\right)$ and $\hat{U}\left(\frac{k}{-q}, \frac{q}{a}\right) = \hat{U}\left(\frac{k}{-q}, \frac{q}{a}\right)$. Therefore, Eq. (D.1) is reduced to the form

$$
\hat{U}\left(\frac{p}{k}, \frac{q}{a}\right) = \frac{G_{ee}}{q^2} + \frac{mG_{ee}}{q^3}\left\{ 4 \left[ I\left[\frac{pq}{q} + q\right] - I\left[\frac{pq}{q}\right] \right] \hat{U}\left(\frac{p}{q}\right) + 4 \left[ I\left[\frac{kq}{q}\right] - I\left[\frac{kq}{q} - q\right] \right] \hat{U}\left(\frac{k}{-q}, \frac{q}{a}\right) \\
+ 2I\left[\frac{pq}{q}\right] \hat{U}\left(\frac{a}{p}, \frac{p}{q}\right) - 2I\left[\frac{pq}{q} + q\right] \hat{U}\left(\frac{-q}{p+q}, \frac{q}{a}\right) - 2I\left[\frac{kq}{q}\right] \hat{U}\left(\frac{-q}{k}, \frac{k}{a}\right) + 2I\left[\frac{kq}{q} - q\right] \hat{U}\left(\frac{q}{k-k}, \frac{k}{a}\right) \right\} .
$$

Let us consider the calculation of the term $\hat{U}\left(\frac{-q}{a}, \frac{q}{a}\right)$:

$$
\hat{U}\left(\frac{-q}{a}, \frac{q}{a}\right) = \frac{G_{ee}}{q^2} + \frac{mG_{ee}}{q^3} I[|q|] \hat{U}\left(\frac{-q}{a}, \frac{q}{a}\right) ;
\hat{U}\left(\frac{-q}{a}, \frac{q}{a}\right) = \frac{G_{ee}}{q^2 - 4mG_{ee} I[|q|]} .
$$

Note that at $q \to 0$ we have

$$
\hat{U}\left(\frac{-q}{a}, \frac{q}{a}\right) = \frac{G_{ee}}{q^2} + \frac{mG_{ee}}{q^3} I[|q|] \hat{U}\left(\frac{-q}{a}, \frac{q}{a}\right) ,
$$

whereas at $q \to K_F$ this term is logarithmically small: $\hat{U}\left(\frac{-q}{a}, \frac{q}{a}\right) \sim \ln^{-1} |q - K_F|$.

Using Eqs. (D.3) and (D.4) it is possible to obtain

$$
\left[ 1 + 4 \frac{G_{ee}}{q^2} \left( I\left[\frac{pq}{q}\right] - I\left[\frac{pq}{q} + q\right] \right) \right] \hat{U}\left(\frac{p}{q}\right) = \frac{G_{ee}}{2q^2} + \frac{1}{2} \hat{U}\left(\frac{-q}{a}, \frac{q}{a}\right) + 2mG_{ee} \times \\
\left( I\left[\frac{pq}{q}\right] \hat{U}\left(\frac{-q}{a}, \frac{q}{a}\right) - I\left[\frac{pq}{q} + q\right] \hat{U}\left(\frac{-q}{a}, \frac{q}{a}\right) \right) ;
$$

$$
\left[ 1 + 4 \frac{G_{ee}}{q^2} \left( I\left[\frac{kq}{q} - q\right] - I\left[\frac{kq}{q}\right] \right) \right] \hat{U}\left(\frac{k}{a}, \frac{-q}{a}\right) = \frac{G_{ee}}{2q^2} + \frac{1}{2} \hat{U}\left(\frac{-q}{a}, \frac{q}{a}\right) - 2mG_{ee} \times \\
\left( I\left[\frac{kq}{q} \right] \hat{U}\left(\frac{k}{a}, \frac{-q}{a}\right) - I\left[\frac{kq}{q} - q\right] \hat{U}\left(\frac{k}{a}, \frac{-q}{a}\right) \right) .
$$
Comparing Eqs. (D.2) and (D.5)-(D.6) makes it evident that

\[ \tilde{U} \left( \begin{array}{c} p \\ k \\ q \\ -q \end{array} \right) = \tilde{U} \left( \begin{array}{c} p \\ q \\ q \\ -q \end{array} \right) + \tilde{U} \left( \begin{array}{c} k \\ -q \\ -q \\ -q \end{array} \right) - \tilde{U} \left( \begin{array}{c} -q \\ -q \\ -q \\ -q \end{array} \right). \]  

(D.7)

So, the determination of the interaction potential reduces to the calculation of the terms \( \tilde{U} \left( \begin{array}{c} p \\ q \\ q \\ 0 \end{array} \right) \) and \( \tilde{U} \left( \begin{array}{c} k \\ -q \\ -q \\ 0 \end{array} \right) \). Further we will limit our investigation to the case of \( p, k \approx K_F \).

Considering Eq. (D.5) ensures us that the terms \( \tilde{U} \left( \begin{array}{c} q \\ p \\ p \\ p \end{array} \right) \) and \( \tilde{U} \left( \begin{array}{c} -q \\ p \\ p \\ p \end{array} \right) \) are logarithmically small as well as the term \( \tilde{U} \left( \begin{array}{c} -p \\ p \\ p \\ p \end{array} \right) \). This permits us to exclude the terms \( \tilde{U} \left( \begin{array}{c} q \\ p \\ p \\ p \end{array} \right) \) and \( \tilde{U} \left( \begin{array}{c} -q \\ p \\ p \\ p \end{array} \right) \) from Eq. (D.5). Then we have:

\[ \left[ 1 - 4 \frac{m_{ee}}{q^2} \right] \tilde{U} \left( \begin{array}{c} p \\ q \\ q \\ p \end{array} \right) = G_{ee} \frac{2}{2q^2} + \frac{1}{2} \tilde{U} \left( \begin{array}{c} -q \\ q \\ q \\ q \end{array} \right). \]  

(D.8)

For the term \( I' \) we use the expression

\[ I'[x] = -\pi K_F^2 \left( \frac{1}{K_F - x} + \frac{1}{K_F + x} \right) - 2\pi K_F = -2\pi K_F \frac{2K_F^2 - x^2}{K_F^2 - x^2}. \]

Singling out the main terms in Eq. (D.8) at \( q \to 0 \), we come to the following equation for the term \( \tilde{U} \left( \begin{array}{c} p \\ q \\ q \\ q \end{array} \right) \)

\[ \left[ 1 + \frac{\lambda^2}{2q^2} \frac{2q^2 K_F^2 - (pq)^2}{q^2 K_F^2 - (pq)^2} \right] \tilde{U} \left( \begin{array}{c} p \\ q \\ q \\ q \end{array} \right) = G_{ee} \frac{2}{2q^2}, \]  

(D.9)

and consequently

\[ \tilde{U} \left( \begin{array}{c} p \\ q \\ q \\ q \end{array} \right) = G_{ee} \frac{q^2 K_F^2 - (pq)^2}{2q^2 K_F^2 (q^2 + \lambda^2) - (pq)^2 (2q^2 + \lambda^2)}. \]  

(D.10)

Analogously we get

\[ \tilde{U} \left( \begin{array}{c} k \\ -q \\ -q \\ -q \end{array} \right) = G_{ee} \frac{q^2 K_F^2 - (kq)^2}{2q^2 K_F^2 (q^2 + \lambda^2) - (kq)^2 (2q^2 + \lambda^2)}. \]  

(D.11)

A set of Eqs. (D.4), (D.7), (D.10), and (D.11) defines the solution of the problem when \( p, k \approx K_F \) and \( q \to 0 \).
[1] J. G. Bednorz, K. A. Muller, "Perovskite-type Oxides - the New Approaches to High - $T_c$ superconductivity" (The Nobel Foundation, New York, 1988).

[2] A. S. Alexandrov, A. B. Krebs, Uspehi Fiz. Nauk (Rus) 162, N 5, 1 (1992).

[3] V. L. Ginzburg, Uspehi Fiz. Nauk (Rus) 170, N 6, 619 (2000); ibid. Conf.: "Major Trends in Superconductivity in New Millenium", 31m.-6 apr. Klostering, Switzerland (2000).

[4] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 106, 162 (1957); ibid. 108, 1175 (1957).

[5] V. L. Ginzburg, Progr. Low Temp. Phys. 12, 1 (1959).

[6] N. N. Bogolubov, J. Exp. Theor. Phys. (Rus) 34, 41 (1958).

[7] J. Valatin, Nuovo Cimento 7, 843 (1958).

[8] C. Kuper, Adv. Phys. 8, 29 (1959).

[9] J. G. Bednorz, K. A. Muller, Z. Phys. K1.B Bd64, 189 (1986).

[10] H. V. Crespi, Phys. Rev. B 60, 100 (1999).

[11] W. Neuberger, V. B. Kalinin, Pat. USA accepted, (Appl. N 09/450.633.PCT 11/22/00).

[12] I. Bozovich, J. Ekstein, "Phys. Properties of HTSC”, Ser. Singapore 5, (1996).

[13] B. I. Tsebro et al., Pis’ma v JETP (Rus) 70, 457 (1999).

[14] V. B. Kalinin et al., Materialovedeniye 2, 40 (1999).

[15] V. B. Kalinin and A. M. Golubev, Materialovedeniye 4, 26 (1999).

[16] Kao et al., Pat. USA US5580703 (1995).

[17] P. W. Anderson. "Theory of SC in high $T_c$ Cuprates" (Princeton, New York Univ. Press 1997).

[18] D. Bohm, D. Pines, Phys. Rev. 82, 625 (1951).

[19] C. A. Reynolds, B. Serin, W. A. Wright, and L. B. Nesbit, Phys. Rev. 78, 487 (1950).

[20] V. V. Brashkin, V. V. Glushkova, and S. V. Demichev, J. Phys. Condens. Mat. 5, 5933 (1993).