Fermionic Casimir Effect on the Topological Insulator Boundary

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Abstract

In this paper we study the Casimir effect on the conducting surface of a topological insulator characterized by both $Z_2$ topological index and time reversal symmetry, subject to the action of a static and spatially homogeneous magnetic field perpendicular to that surface, at zero temperature. To do this, we consider modifications in the Gauss’ law that arise due to the nonzero gradient of the axion-like pseudoscalar factor coupled to the constant magnetic field, which occur in a term that must be added to the electromagnetic Lagrangian in order to account for the topological properties of the system. Such term allows to find an effective point-like charge that changes the quantum vacuum of a spinor field in 1+2 dimensions confined on the edge under analysis. Since that the Casimir energy found depends on a length defined on the boundary, we show that there is a tangential density of force or a shear stress associated to the surface, tending to shrink or stretch it depending on the magnetic field direction. These results are extended for the case in which the surface forms a interface between two TI’s.

Key words: Casimir effect; Axion electrodynamics; Topological insulator

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I. INTRODUCTION

A lot of theoretical work has been made since the original formulation by H. Casimir [1], concerning the macroscopic effect which is caused by modifications in the fluctuations of the quantum vacuum energy of the electromagnetic field and that bears his name. The effect originally described consists of an attraction force that arises between two ideal conducting plates closely spaced, parallel and uncharged, placed in a perfect vacuum at zero temperature. This phenomenon was experimentally confirmed ten years later with some level of precision. Numerous papers published since then have addressed various types of geometrical settings, quantum fields, materials, space-times, topologies and thermal conditions. These and the latest developments both theoretical and experimental, are described in detail in [2].

The Casimir effect can also be present in empty spaces with non-Euclidean topology including cosmological models, in General Relativity or alternative theories of gravity [3]. In these space-times there are no material borders, but the identification of conditions imposed on the quantum fields can play the same role as the one presented by these boundaries. Since the Casimir effect is a phenomenon of quantum nature, we should expect that it plays an important role in the early stages of the formation of the universe [4]. Thus, it is necessary to correctly take into account the Casimir effect in order to get an accurate description of a particular physical system, from the structure and evolution of the known universe to condensed matter systems, even those considered exotic, as the so-called topological insulators.

Topological insulators (TI’s) are electronic systems with a nonzero energy gap to the excited (conducting) states in the bulk, nevertheless possessing a gapless mode on their boundary-surfaces (edges) [5]. The meaning of the term “topological” comes from the fact that the wave functions describing their electronic states span a Hilbert space with a nontrivial topology [6]. Such systems are also characterized by a topological number that reveals how robust they are against external perturbations. It is interesting to verify how the Casimir effect operates over these systems in which the nontrivial topology is in their spaces of quantum states instead of in their physical spaces. In this investigation line, the literature presents some works [7–9].

In the present work we will study the Casimir effect regarding a very special class of
TI’s, labeled by a topological number - the Z$_2$ index, which expresses the number of times that a 1D edge electronic state crosses the Fermi level between 0 and $\pi/a$, where $a$ is the lattice parameter, besides exhibiting time reversal (TR) symmetry. In such systems, there is a term in the electromagnetic Lagrangian proportional to $\theta \mathbf{E} \cdot \mathbf{B}$, derived from the axion electrodynamics [10], which, inside the TI, does not modify the usual Maxwell’s laws valid in material media by virtue the $\theta$ factor being a constant, despite changing the electromagnetic constitutive relations. This leads, for example, to the appearance of magnetization (polarization) induced by the electric (magnetic) field [11]. It is interesting point out that, in ref. [12] is possible study the breakdown of TR symmetry via magnetic fluctuations in order to $\theta$ acquire dynamics, becoming a field variable.

In despite $\theta$ in our approach is not a dynamical field, on the TI conducting surface the Maxwell’s laws are modified since there $\nabla \theta \neq 0$. Thus, we will consider the variation in the Gauss’ law due to the nonzero gradient of $\theta$ coupled to a static and spatially homogeneous magnetic field perpendicular to the TI boundary in order to find an effective point-like charge that modifies the Casimir energy on that edge, energy that was found in [13] from analysis of the quantum vacuum of a spinor field in 1+2 dimensions. Since that this new Casimir energy depends on a length defined on the boundary, there is a tangential force or a shear stress associated to it. This results will be extended for the case in which the surface forms a interface between two TI’s

The paper is organized as follows: in section 2, we present the electrodynamics of TI’s with $Z_2$ topology and TR symmetry, obtaining modifications in the elementary electric charge on its boundary. In section 3, we reobtain the Casimir energy density of a spinor field interacting with a constant magnetic field on that boundary. Finally, in section 4, we discuss the results.

II. REDEFINING THE ELEMENTARY CHARGE

In this section, we will study how variations in the electrodynamics laws induced by the topology of TI’s modify in turn the electric charge of the point-like carriers. The electromagnetic Lagrangian density in a $Z_2$-type TI with TR symmetry can be written as [10]

$$\mathcal{L}_E = \frac{1}{8\pi} (\mathbf{E}^2 - \mathbf{B}^2) + \frac{c\alpha}{4\pi^2} \theta \mathbf{E} \cdot \mathbf{B},$$

(1)
without source terms. The factor \( \alpha = e^2/\hbar c \) is the fine structure constant. The second term of Eq. (1) - the \( \theta \)-term, which encodes the topological features of TI’s, is quite similar to that one which occurs in the axion electrodynamics, where \( \theta(x) \) is a pseudoscalar field, for which its transformation in a temporal reversion given by \( t \to -t \), is \( \theta(x) \to -\theta(x) \), and whose coupling with the electromagnetic field generates unusual effects as axion-photon oscillations \([14]\) and shining light through walls \([15]\).

The presence of the \( \theta \)-term changes the electromagnetic constitutive relations, in form

\[
\begin{align*}
D &= E - \frac{\theta c\alpha}{\pi} B, \\
H &= B + \frac{\theta c\alpha}{\pi} E,
\end{align*}
\]

in such way that within the TI bulk these quantities lead to the so-called magnetoelectric effect, in which the polarization is proportional to the magnetic field or the magnetization is proportional to the electric field. This effect can be exploited in order to obtain other exotic phenomena such as magnetic monopole charge-image \([16]\). In that region inside TI, \( \theta \) is a constant of value equal to \( \pi \), and thus the electrodynamics laws in material media becomes unchanged because the extra term is just a total derivative \([17]\).

On the boundary of the aforementioned TI’s, the Maxwell’s equations are modified, besides the constitutive relations, because the \( \theta \) axion-like factor presents a nonzero gradient, since in regions external to the TI (vacuum or ordinary insulators), \( \theta = 0 \), and inside it, \( \theta = \pi \). Thus, the Gauss’ law changes to

\[
\nabla \cdot E = 4\pi \tilde{\rho} = 4\pi \rho - \frac{\alpha c}{\pi} \nabla \theta \cdot B, \tag{4}
\]

where \( \tilde{\rho} \) is the effective (axion) electric charge density.

Let us now consider the point-like elementary electric charge modeled by the delta function and situated on the TI boundary, such that \( \rho(x) = e\delta^3(x) \), as well as a magnetic field \( B \) perpendicular to the boundary, supposed flat, aligned in the \( z \) direction. Integrating Eq. (1) in a cylindrical volume around the charged particle, we get

\[
\tilde{e} = e - \frac{\alpha c}{4\pi^2} B \int \int dx dy \int_{z_{\text{in}}}^{z_{\text{out}}} \frac{\partial \theta}{\partial z} dz = e + \frac{\alpha c}{4\pi} \Phi, \tag{5}
\]

where \( \tilde{e} \) is the point-like effective charge, equal to \( e \) when \( \theta \) is constant, and \( \Phi \) is the magnetic flux on the surface. If we consider quanta of magnetic flux (\( \Phi \sim h/e \)), thus that effective charge can be seen as composed of an elementary electric charge and of other fractionary
one that shields a (elementary) magnetic monopole - a dyon - predicted in the description of the Witten effect [18].

III. CASIMIR EFFECT ON THE TI BOUNDARY

It is known that Casimir effect arises from the variations in the vacuum expectation value of observables associated with a quantum field, imposed by external boundary conditions or by interactions with an external field generated by a particular source, in physical spaces with trivial topology. The difference between the vacuum energies of these configurations is named Casimir energy. In this section, we will consider the configuration of a static and spatially homogeneous magnetic field acting perpendicularly on the surface of a $Z_2$-type TI with TR symmetry in order to calculate the Casimir energy of a spin 1/2 field situated on this edge.

The ordinary Lagrangian density of a electromagnetic field without sources plus a massive spinor field interacting with it is, in covariant language, given by

\[ L_{\text{qed}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi} \gamma^\mu (i \partial_\mu - e A_\mu) \Psi - m \bar{\Psi} \Psi, \]

where $\gamma^\mu$ are the Dirac’s 2x2 matrices (in 1+2 dimensions) and $\bar{\Psi} = \Psi^\dagger \gamma^0$. The gauge to be chosen is

\[ A_\mu \equiv (0, 0, -\frac{1}{2} \rho B, 0), \]

in cylindrical coordinates $(\rho, \phi, z)$, compatible with a constant magnetic field $B$ pointing in the $z$ direction. The Casimir energy density, $\epsilon_C$, related to the spin 1/2 field in 1+2 dimensions and for this particular gauge, was found in [13] by the method of the zeta function regularization, which is given by

\[ \epsilon_C = \frac{2eB}{2\pi} \left[ \frac{m}{2} - \sqrt{2eB} \zeta \left( -\frac{1}{2}, \frac{m^2}{2eB} \right) \right] - \frac{m^3}{3\pi}, \]

in natural units ($\hbar = c = 1$), with $\zeta(s, q)$ being the Hurwitz zeta function [19]. The last term in the expression above is the vacuum energy density of the spinor field with the magnetic field switched off (free field). It is worth point out that this Casimir energy did not need to pass by the renormalization of some parameters, since it is naturally finite, at least in 1+2 dimensions. We also remark that such energy does not generate none physical influence because it does not depend on any length parameter.
By considering now the surface of a $Z_2$-type TI with TR symmetry, we have seen that one must add the axion-like term to the Lagrangian (6), given in covariant notation by
\[
\frac{c\alpha}{4\pi^2}\theta\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} = \frac{c\alpha}{4\pi^2}\theta \mathbf{E} \cdot \mathbf{B},
\]  
(9)
where $\epsilon_{\mu\nu\rho\sigma}$ is the four-dimensional completely anti-symmetric tensor. Therefore, the modified electromagnetic sector of the Lagrangian allows us, via equations of movement, redefine the elementary charge according to Eq. (5), which must be taken into account in the coupling terms. Beside this, we must do $m = 0$, provided that the effective mass of the carriers in a graphene-like conducting sheet that forms the TI surface is null, and we also must change $c \rightarrow v_F$, where this latter is the Fermi velocity, because the linear dispersion relation found around the Dirac’s points [20]. Joining all this and following the procedure made in [13], we arrive to
\[
\epsilon_C = -\frac{2eB}{2\pi} \left[ \sqrt{2\epsilon B} \left\{ -\frac{1}{2}, 0 \right\} \right] \approx \frac{0.207}{2\pi h^{1/2}v_F^{1/2}} \left[ 2 \left( e + \frac{\alpha v_F \Phi}{4\pi} \right) B \right]^{3/2},
\]  
(10)
for the same gauge (7), restoring the constants and with $\zeta(-1/2, 0) \approx -0.207$.

The above result can take into account the fact of the surface considered forming a interface between two TI’s. In this case, the more general expression for the axion-like parameter is $\theta = \pi(2n + 1), n = 0, 1, 2...$ [21], and the expression for the Casimir energy density becomes
\[
\epsilon_C \approx \frac{0.207}{2\pi h^{1/2}v_F^{1/2}} \left\{ 2 \left[ e + \frac{(m - n)\alpha v_F \Phi}{2} \right] B \right\}^{3/2},
\]  
(11)
yielding a quantization of the Casimir energy density.

If we consider that the magnetic flux is given in function of a linear dimension of the TI boundary, like $\Phi = B\pi R^2$ if the TI is cylindrical with basis radius $R$, thus the Casimir energy density depends on this length taken on the TI surface, which yields a density of radial force $f_C(R)$ given by
\[
f_C(R) = \frac{d\epsilon_C}{dR} \approx -\frac{0.621\alpha v_F^{1/2}B}{8\pi h^{1/2}} \left( 2B \right)^{3/2} \left( e + \frac{\alpha v_F R^2}{4B} \right)^{1/2} R.
\]  
(12)
This Casimir force density is tangential to the TI surface, and can be seen as a radial shear stress that arises due to the quantum vacuum influence. At first instance, by the signal of this force we deduct that it tends to shrink the surface.
IV. CONCLUDING REMARKS

In this paper we have investigated the Casimir effect in a very special class of TI’s, labeled by the topological index $Z_2$ and exhibiting time reversal (TR) symmetry, at zero temperature. Initially it was discussed how, on the surface of such systems, the Maxwell’s laws with source terms are modified, leading to the redefinition of the electric charge in function of the presence of a nonzero gradient of the axion-like pseudoscalar factor $\theta$ as well as of a static and spatially homogeneous magnetic field, here taken as being perpendicular to the boundary under analysis.

Thus, we have obtained an effective point-like charge, which is formed by an elementary charge and by an elementary dyon, if the magnetic flux is sufficiently weak, in order to calculate the modifications in the Casimir energy density of a spin 1/2 field confined on one of the TI conducting edges. Such energy density calculation has been presented in literature [13] regarding an ordinary surface. We have also obtained the Casimir energy density for the situation in which the surface forms a interface between two TI’s, and found a expression that points out the quantization of this energy.

It is worth notice that, differently from the ordinary fermionic Casimir energy presented in literature, the result found by us depends on a length parameter defined on the boundary, which yields a tangential force (a shear stress) associated to it. The negative signal of the force found, which is directed to the sample center, would indicate a tendency for it to shrink the surface.

However, by dealing with an electron field, we must do $e \rightarrow -e$ in Eqs. (10) and (12), and it is easy to show that the Casimir force is not observable when the magnetic flux is $\Phi \leq 4\Phi_0$, where $\Phi_0 = h/2e$ is the magnetic flux quantum, unless that one inverts the field direction. In this case, the Casimir force always tends to stretch the surface.

[1] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. 51, 793 (1948).
[2] M. Bordag et al, Advances in the Casimir Effect, Oxford University Press, 2009.
[3] C. R. Muniz, V. B. Bezerra and M. S. Cunha, Phys. Rev. D. 88, 104035 (2013).
[4] V.B. Bezerra, H. F. Mota, and C.R. Muniz, Phys. Rev. D 89, 024015 (2014).
[5] M. Z. Hasan, C. L. Kane, Rev.Mod.Phys. 82, 3045 (2010).
[6] A. P. Schnyder, S. Ryu, A. Furusaki, and A. W. W. Ludwig: Phys. Rev. B 78, 195125 (2008).
[7] A. G. Grushin and A. Cortijo, Phys. Rev. Lett. 106, 020403 (2011).
[8] P. Rodriguez-Lopez and A. G. Grushin, Phys.Rev.Lett. 112, 056804 (2014).
[9] A. G. Grushin, P. Rodriguez-Lopez, and Alberto Cortijo, Phys. Rev. B84, 045119 (2011).
[10] F. Wilczec, Phys. Rev. Lett. 58, 1799 (1987).
[11] Y. Ando, J. Phys. Soc. Jap, 82, 10, 102001 (2013).
[12] R. Li, J. Wang, X-L Qi, and S-C Zhang, Nat.Phys., 6, 284 (2010).
[13] D. H. Correa, La Plata - Th 00/8, arXiv:hep-th/0008223v1.
[14] G. Raffelt and L. Stodolsky, Phys. Rev., D37, 5, 1237 (1988).
[15] J. Redondo and A. Ringwald, DESY 10-175, MPP-2010-149, arXiv:1011.3741 [hep-ph].
[16] X. L. Qi, R. Li, J. Zang, and S. C. Zhang, Science 323, 5918, 1184 (2009).
[17] A. M. Essin, J. E. Moore, and David Vanderbilt, Phys. Rev. Let. 102, 146805 (2009).
[18] E. Witten, Phys. Lett. B86,283 (1979).
[19] M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions, Dover Publications (1964).
[20] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, Rev. Mod. Phys. 81, 109 (2009).
[21] X-L Qi, Field Theory Foundations of Topological insulators, in Topological Insulators, Ed. by M. Franz and L. Molenkamp, Elsevier (2013).