Textured Minimal and Extended Supergravity

Unification and Implications for Proton Stability

Pran Nath

Institute for Theoretical Physics
University of California
Santa Barbara, CA 93106-4030

Department of Physics, Northeastern University
Boston, MA 02115

Abstract

We construct a class of textured supergravity unified SU(5) models using Planck scale corrections. We show that the texture constraints in the Higgs doublet sector are insufficient in general to fully determine the textures in the Higgs triplet sector. A classification of textured minimal parameter models is given and their Higgs triplet textures computed under the constraint that they possess the Georgi-Jarlskog textures in the Higgs doublet sector. It is argued that additional dynamical assumptions are needed to remove the ambiguity. The recently proposed extension of supergravity unification to include a minimal exotic sector is free of this ambiguity and leads to unique textures in the Higgs triplet sector. Implications for proton stability are discussed.

*Permanent address
The concept of quark-lepton textures at the GUT scale [1,2] has played a key role recently in the understanding of the hierarchy of mass scales at the electro-weak scale [3]. Without the textures GUT models make poor predictions for the quark lepton mass ratios. Thus, for example, while the supersymmetric SU(5) model makes acceptable predictions for $m_b/m_\tau$, the predictions of the model for the light quark-lepton mass ratios, i.e., $m_s/m_\mu$ and $m_d/m_e$ are in poor agreement with experiment. Supergravity grand unification [4,5] currently provides a successful framework for the breaking of supersymmetry. Recently, the framework of supergravity unification was extended to include textures [6]. The extension was based on the inclusion of a new sector which contains exotic matter, which couples to matter in the visible sector and in the hidden sector. After spontaneous breaking of supersymmetry, exotic matter becomes superheavy and its elimination leads to a well defined set of higher dimensional operators scaled by $\Sigma/M_P$, where $\Sigma$ is the 24-plet of SU(5). Textures are created when SU(5) breaks to SU(3)xSU(2)xU(1) at the GUT scale. It is then shown that if one fixes the textures in the Higgs doublet sector, then the textures in the Higgs triplet sector are uniquely determined.

In this Letter we consider a more general approach. Here instead of generating higher dimensional operators via the exotic sector, we add in a phenomenological fashion a set of higher dimensional operators. That higher dimensional operators can generate hierarchies in quark-lepton mass matrices has been known for some time [7] and further one expects such operators to arise quite naturally in string compactified models [8,9]. We shall show that in this case the constraints that fix the textures in the Higgs doublet sector leave a considerable degree of arbitrariness in the textures in the Higgs triplet sector. We then classify the minimal parameter solutions and find that there are at least $4 \times 5 \times 17$ textured models of this type (which we label by $A_i B_j C_k$ ($i=1,..,4$; $j=1,..,5$; $k=1,..,17$) which possess the same Georgi-Jarlskog(GJ) textures in the Higgs doublet sector but have distinct textures in the Higgs triplet sector. We compute the textures in the Higgs triplet sector for these $4 \times 5 \times 17$ minimal parameter models. They are given by eqs(9), (12) and (13) and tables 1,2 and 4. These results have important implications for p-decay lifetimes [10,11].
We give now the details of the analysis. As discussed above textures in the quark-lepton sectors can arise via higher dimensional operators. For the minimal SU(5) theory these higher dimensional operators are scaled by $\Sigma/M_P$. The hierarchy of mass scales arises when the 24-plet of $\Sigma$ field develops a VEV generating the ratio $M/M_P$, where $M$ is the GUT scale and $M_P$ is the Planck/string scale. As is commonly done we shall assume that the $(33)$ element of the up quark texture arises from a dimension four operator in the Lagrangian (or dimension 3 in the superpotential) while the remaining parts of the up quark texture and all parts of the down quark and lepton textures arise from interactions with dimensionalities higher than four. To generate the full hierarchical structure one has to include up to dimension six operators in the up quark sector and up to dimension seven operators in the down quark and lepton sector. As discussed above we shall take a phenomenological approach and write down the general set of interactions at each level of dimensionality with only the constraint of R-parity invariance. In general, the interaction structure will have the form

$$W = W_3 + W_4 + W_5 + W_6 + ..$$ (1)

We assume that the particle spectrum is that of the minimal supersymmetric SU(5) model, and consists of quarks and leptons in three generations of $\bar{5}(M_x) + 10(M^{xy})$ plets of SU(5), Higgs in $\bar{5}(H_{1x}) + 5(H^2_x)$, and a field $\Sigma^x_y$ that breaks the SU(5) GUT symmetry in the 24-plet of SU(5). In the computation of textures in the up quark sector it is found sufficient to include only the first three terms of the expansion on the right hand side of eq(1), i.e., the terms $W_3, W_4, W_5$, to generate the desired hierarchies and $W_6$ and higher terms make small contributions and can be neglected. In the down quark and lepton sector we assume that $W_3$ makes no contribution and $W_4, W_5, W_6$ are then found sufficient to generate the desired hierarchies and $W_7$ and higher terms can be neglected. Under the above conditions the desired interactions are given by

$$W_3 = -\frac{1}{8} \epsilon_{uwxyz} H^u_2 M^i_{vw} h_{ij} M^{xy}_j + H_{1x} M_{yi} k_{ij} M^{xy}_j$$ (2)

$$W_4 = -\frac{1}{8 M_P} \epsilon_{uwxyz} \Sigma^u_q H^q_2 M^i_{vw} h_{1ij} M^{xy}_j - \frac{1}{8 M_P} \epsilon_{uwxyz} \Sigma^u_q M^aq_i H^w_2 h_{2ij} M^{xy}_j$$
\[ W_5 = -\frac{1}{8M_p^2} \epsilon_{u v w z} \Sigma_q^2 \Sigma_q^2 M_i^{h v} h_{5 i j} M_j^{x y} - \frac{1}{8M_p^2} \epsilon_{u v w z} \Sigma_q^2 \Sigma_q^2 M_i^{h v} h_{4 i j} M_j^{x y} \]

\[ W_6 = \frac{1}{M_p^2} H_{1 x} \Sigma_y^{3 y} M_{z i} k_{1 i j} M_j^{y z} + \frac{1}{M_p^2} H_{1 x} M_{y l} \Sigma_l^{3 y} \Sigma_Z k_{8 i j} M_j^{x z} + \frac{1}{M_p^2} H_{1 x} \Sigma_y^{2 x} M_{u l} \Sigma_Z k_{9 i j} M_j^{y z} \]

After spontaneous breaking of the GUT symmetry when \( < \Sigma >= M(2, 2, 2, -3, -3) \), eqs(1-5) create textures. Thus at the GUT scale one has

\[ W_{e f f} = (-M_{H 3} H_{1 a} H_2^a + H_{a b c} H_{a b} B_{a b}^c B_{a b}^c + \epsilon_{a b c} H_{a b} B_{a b}^c B_{a b}^c + \epsilon_{a b c} H_{a b} B_{a b}^c B_{a b}^c) \]

Here \( A^E, A^D, A^U \) are the textures in the Higgs doublet sector and \( B^E, B^D, B^U \) and \( C^U \) are the textures in the Higgs triplet sector. They contain a hierarchy of mass scales since \( W_n \) contributes terms of \( O(M/M_P)^{n-3} \) to the textures. Next we impose on eq(2-5) the condition that \( A^E, A^D, \) and \( A^U \), be the GJ textures, i.e.,

\[ A^E = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix}, \quad A^D = \begin{pmatrix} 0 & F & 0 \\ F^{-i \phi} & E & 0 \\ 0 & 0 & D \end{pmatrix}, \quad A^U = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix} \]

The texture zeros of eq(7) are generated provided,
\[ h_{ij} = h\delta_{i3}\delta_{j3}, h_{kij} = h_k(\delta_{i2}\delta_{j3} + \delta_{i3}\delta_{j2}); k = 1, 2 \]
\[ h_{lij} = h_l(\delta_{i1}\delta_{j2} + \delta_{i2}\delta_{j1}); l = 3, 4, 5, 5', 6 \]
\[ k_{ij} = k\delta_{i3}\delta_{j3}; k_{qij} = k_q\delta_{i3}\delta_{j3}, q = 1, 2; k_{pij} = k_p\delta_{i2}\delta_{j2}, p = 3, ..., 6 \]
\[ k_{rij} = (k_r\delta_{i1}\delta_{j2} + k_r^*\delta_{i2}\delta_{j1}); r = 7, ..., 13 \]

\( A^E, A^D, A^U \) constructed in the above fashion contain the desired hierarchies in powers of \( \epsilon \equiv M/M_P \). In the up quark sector \( A \sim h, B \sim \epsilon h_k, C \sim \epsilon^2 h_l \), and we find that \( A, B, C \) have the correct hierarchical orders when \( \epsilon \sim O(1/50) \) and \( h, h_k, h_l \sim O(1) \). In the down quark lepton sector we have \( D \sim k + \epsilon k_q, E \sim \epsilon^2 k_p \), and \( F \sim \epsilon^3 k_r \), and we find that \( D, E, F \) have the correct hierarchical orders with \( k = 0, k_q, k_p, k_r \sim O(1) \).

There is a weakness, however, in the above approach which we now illustrate. It resides in the lack of a full determination of the the coupling parameters that appear in the higher dimensional operators of eqs(3-5) even with the imposition of the GJ texture constraints. Consider the up quark sector first. Here the \((33)\) elements of \( B^U \) and \( C^U \) are uniquely fixed since there is one parameter \( (h) \) and one GJ texture constraint. For the \((23+32)\) elements there are two parameters \( (h_1, h_2) \) and one GJ texture constraint. However, fortuitously \( (h_1, h_2) \) enter in the exact same combination both in \( A^U \) and in \( B^U, C^U \), and so the \((23+32)\) elements of \( B^U \) and \( C^U \) are again uniquely determined. However, for the \((12+21)\) elements, one has five coupling constants \( (h_3, h_4, h_5, h_5', h_6) \) and one GJ texture constraint. Thus there is a four parameter arbitrariness here. In the down quark and lepton sector, the determination of the \((33)\) element in \( A^E, A^D \) involves the parameters \( k_1, k_2 \). However, the \( k_2 \) term spoils the \( b/\tau \) unification at the GUT scale, so we set \( k_2 = 0 \) in conformity with the GJ texture constraints of eq(7). With this constraint the down quark lepton system is uniquely determined in the \((33)\) element and thus the elements \( B^E_{33}, B^D_{33} \) are uniquely determined. In the \((22)\) element, there are four coupling constants \( (k_1, k_2, k_3, k_4) \) and two constraints, one from \( A^E \) and the other from \( A^D \), which leave us with a two parameter arbitrariness. Finally, in the \((12+21)\) elements, one has seven parameters \( (k_7, ..., k_{13}) \) and two constraints, one each from \( A^E \) and \( A^D \). Thus there is a five parameter arbitrariness in the system at
this level. The textures in the Higgs triplet sector are given by

\[
B^U = \begin{pmatrix}
0 & \frac{4}{9} C + \Delta_{12}^U & 0 \\
\frac{4}{9} C + \Delta_{21}^U & 0 & -\frac{2}{3} B \\
0 & -\frac{2}{3} B & A
\end{pmatrix},
\]

\[
C^U = \begin{pmatrix}
0 & \frac{4}{9} C + \Delta_{12}''^U & 0 \\
\frac{4}{9} C + \Delta_{21}''^U & 0 & -\frac{2}{3} B \\
0 & -\frac{2}{3} B & A
\end{pmatrix}
\]

where \(\Delta_{12}^U, \Delta_{12}''^U\) are given by

\[
\Delta_{12}^U = \epsilon^2 \left( \frac{25}{6} h_4 + \frac{50}{3} h_6 - \frac{50}{9} h_5' \right)
\]

\[
\Delta_{12}''^U = \epsilon^2 \left( \frac{25}{6} h_4 + \frac{50}{3} h_6 + \frac{25}{36} h_5' \right)
\]

and

\[
B^E = \begin{pmatrix}
0 & (-\frac{19}{27} + e^{i\phi}) F + \Delta_{12}^E & 0 \\
(-\frac{19}{27} + e^{-i\phi}) F + \Delta_{21}^E & \frac{16}{3} E + \Delta_{22}^E & 0 \\
0 & 0 & \frac{2}{3} D
\end{pmatrix}
\]

\[
B^D = \begin{pmatrix}
0 & -\frac{8}{27} F + \Delta_{12}^D & 0 \\
-\frac{8}{27} F + \Delta_{12}^D & -\frac{4}{3} E + \Delta_{22}^D & 0 \\
0 & 0 & -\frac{2}{3} D
\end{pmatrix}
\]

where \(\Delta_{12}^E, \Delta_{12}^D\) are given by

\[
\Delta_{12}^E = \epsilon^3 \left( -25 (k_9 + k_{10}) + \frac{350}{9} k_{11} - \frac{100}{3} (k_{12} + k_{13}) \right)
\]

\[
\Delta_{12}^D = \epsilon^3 \left( -\frac{350}{9} k_{11} + \frac{100}{3} (k_{12} + k_{13}) \right)
\]

and where \(\Delta_{22}^E, \Delta_{22}^D\) are given by

\[
\Delta_{22}^E = \epsilon^2 (25 k_5 - \frac{50}{3} k_6)
\]

\[
\Delta_{22}^D = \epsilon^2 (\frac{50}{3} k_6)
\]

We consider now solutions where all the arbitrary parameters except for those necessary to satisfy the GJ texture constraints are set to zero. We call these the minimal parameter
solutions. For $A^U$ we find 5 solutions two of which, however, are degenerate in the Higgs triplet sector leaving us with only four distinct solutions for $B^U$ and $C^U$, listed as $A_1, \ldots, A_4$ in table1 (The case $A_5$ is similar to the case $A_1$ and is not distinct). In the down quark and lepton sector there are five minimal parameter solutions that give the same (22) GJ texture element in $A^E$ and $A^D$ but lead to distinct $B^E_{22}$ and $B^D_{22}$ elements and are listed as cases $B_1, \ldots, B_5$ in table2. In the (12+21) down quark lepton sector there are seventeen minimal two parameter solutions that give the same ($A^E_{12}, A^E_{21}$), and ($A^D_{12}, A^D_{21}$) elements, and lead to distinct values for ($B^E_{12}, B^E_{21}$), and ($B^D_{12}, B^D_{21}$). These are exhibited in table 3 and the corresponding elements $B^E_{12}$ and $B^D_{12}$ are listed as cases $C_1, \ldots, C_{17}$ in table4. We find then that there are four minimal parameter solutions that lead to distinctly different textures in the $H_a^1$ color interactions and $5 \times 17$ minimal parameter solutions that lead to distinctly different textures in the $H_a^2$ color interactions, while giving the exact same GJ textures in the Higgs doublet sector. We label these models by $A_i B_j C_k$ where $i=1, \ldots, 4; j=1, \ldots, 5; k=1, \ldots, 17$. It can be easily seen from tables 1, 2 and 4 that a subset of these minimal parameter models, i.e., $A_i B_m C_n$, where $i=1, \ldots, 4; m=1,4; n=1,4,7,8,9,16,17$ satisfies the texture sum rule $A^E + B^E + B^D = A^D$, while the remaining subset violates the sum rule. The source of these violations can be traced to the couplings $k_{5ij}$ in $W_5$ of eq(4) and the couplings $k_{9ij}$ and $k_{10ij}$ in $W_6$ of eq(5). These are the couplings where the $\Sigma$-field appears at more than one location in the interaction structure.

The analysis given above shows that the textures in the the $H_a^2$ sector have a 4 parameter arbitrariness while the textures in the $H_a^1$ sector have a 3 + 6 parameter arbitrariness. If one integrates out the heavy color higgs fields, one finds as usual dimension five operators which are of the type LLLL and RRRR, where L(R) denote chiralities. The LLLL part involves the textures $B^E$ and $C^U$ while the RRRR part involves the textures $B^D$ and $B^U$. We find that each of these parts involves a 4+3+6 parameter arbitrariness. Thus the proton lifetime predictions are rendered highly ambiguous in the general case. If we make the choice of picking the minimal number of parameters to satisfy the texture constraints in the Higgs doublet sector, then one has $4 \times 5 \times 17$ different possibilities for the LLLL+RRRR dimension
five operators as exhibited in tables 1, 2 and 4. Thus each of the $4 \times 5 \times 17$ $A_i B_j C_k$ models will lead to its own set of proton decay predictions. From tables 1, 2 and 4 we see that the (12) and (22) texture elements show large variations which will translate into significant variations for proton decay lifetimes. As pointed out in ref [6] there is also the additional feature that the CP violating phase enters the Higgs triplet textures. This phase influences proton decay lifetimes and decay signatures as it enters prominently in the LLLL and in the RRRR dimension five operators.

Thus the textures derived from the most general expansions based on higher dimensional operators do not lead to a predictive theory for proton decay. The arbitrariness encountered arises due to the possibility of writing an operator of higher dimensionality in several different ways due to the several ways one can contract the indices. This kind of arbitrariness is not expected to be removed by the so called horizontal symmetries since the nature and number of fields in each configuration is the same for all the terms at a given level of dimensionality. One needs more constraining principles to reduce the arbitrariness in the theory.

In ref [6] a model was proposed which reduces the arbitrariness encountered above by deriving the higher dimensional operators from a dynamical postulate. The proposed model extends supergravity unification to include an exotic sector with couplings to both the visible and the hidden sectors. After spontaneous breaking of supersymmetry exotic matter becomes superheavy and its elimination leads to a well defined set of higher dimensional operators. It is then shown that the assumption of an exotic sector belonging to the simplest vector like representation leads to predictive textures in the Higgs triplet sector when the texture constraints in the Higgs doublet sector are imposed. The model of ref [6] is the case $A_1 B_1 C_1$ in the notation of tables 1, 2 and 4 (corresponding to $\Delta_U^{12}, \Delta_U^{12} ', \Delta_F^{12}, \Delta_D^{12}, \Delta_E^{22}$ and $\Delta_D^{22}$ all equal to zero) and leads to predictive proton decay lifetime and decay signatures.
ACKNOWLEDGEMENTS

This research was supported in part by NSF grant number PHY–19306906 and at the Institute for Theoretical Physics in Santa Barbara under grant number PHY94-07194.
Table 1: Evaluation of $\Delta_{12}^U$ and $\Delta'_{12}^U$ and $B_{12}^U$ and $C_{12}^U$

for minimal parameter models.

| Model | $(h_3, h_4, h_5, h'_5, h_6)$ | $\Delta_{12}^U$ | $\Delta'_{12}^U$ | $B_{12}^U$ | $C_{12}^U$ |
|-------|-----------------|-----------------|-----------------|------------|------------|
| $A_1$ | $(\frac{1}{9}, 0, 0, 0, 0)C$ | 0               | 0               | $\frac{4}{9}C$ | $\frac{4}{9}C$ |
| $A_2$ | $(0, \frac{4}{21}, 0, 0, 0)C$ | $\frac{50}{63}C$ | $\frac{50}{63}C$ | $\frac{26}{21}C$ | $\frac{26}{21}C$ |
| $A_3$ | $(0, 0, 0, -1, 0)C$ | $\frac{50}{9}C$ | $\frac{25}{36}C$ | 6$C$ | $-\frac{1}{4}C$ |
| $A_4$ | $(0, 0, 0, 0, \frac{1}{30})C$ | $\frac{5}{9}C$ | $\frac{5}{9}C$ | $C$ | $C$ |
| $A_5$ | $(0, 0, -\frac{4}{9}, 0, 0)C$ | 0               | 0               | $\frac{4}{9}C$ | $\frac{4}{9}C$ |

Table 2: Evaluation of $\Delta_{22}^E$, $\Delta_{22}^D$, $B_{22}^E$ and $B_{22}^D$

for minimal parameter models.

| Model | $(k_3, k_4, k_5, k_6)$ | $\Delta_{22}^E$ | $\Delta_{22}^D$ | $B_{22}^E$ | $B_{22}^D$ |
|-------|-----------------|-----------------|-----------------|------------|------------|
| $B_1$ | $(\frac{7}{15}, -\frac{4}{5}, 0, 0)E$ | 0               | 0               | $\frac{16}{3}E$ | $-\frac{4}{3}E$ |
| $B_2$ | $(-\frac{1}{15}, 0, -\frac{4}{15}, 0)E$ | $-\frac{20}{3}E$ | 0               | $-\frac{4}{3}E$ | $-\frac{4}{3}E$ |
| $B_3$ | $(0, -\frac{1}{10}, -\frac{7}{30}, 0)E$ | $\frac{35}{6}E$ | 0               | $-\frac{1}{2}E$ | $-\frac{4}{3}E$ |
| $B_4$ | $(0, -\frac{4}{5}, 0, \frac{7}{30})E$ | $-\frac{7}{3}E$ | $\frac{7}{3}E$ | 3$E$ | $E$ |
| $B_5$ | $(0, 0, -\frac{4}{15}, -\frac{1}{30})E$ | $-\frac{19}{3}E$ | $-\frac{1}{3}E$ | $-E$ | $-\frac{5}{3}E$ |
Table 3: Evaluation of $\Delta E_{12}$ and $\Delta D_{12}$ for minimal parameter models.

| $(k_7, k_8, k_9, k_{10}, k_{11}, k_{12}, k_{13})$ | $\Delta E_{12}$ | $\Delta D_{12}$ |
|-------------------------------------------------|-----------------|-----------------|
| $\left( -\frac{1}{27} + \frac{1-\epsilon^{i\phi}}{35} \right), -\frac{1-\epsilon^{i\phi}}{35}, 0, 0, 0, 0, 0 \right) F$ | $0$ | $0$ |
| $\left( -\frac{1}{27} + \frac{1-\epsilon^{i\phi}}{45} \right), 0, -\frac{1-\epsilon^{i\phi}}{45}, 0, 0, 0, 0 \right) F$ | $\frac{5}{9}(1 - \epsilon^{i\phi})$ | $0$ |
| $\left( \frac{1}{15}\left( \frac{12}{27} - \epsilon^{i\phi} \right), 0, 0, -\frac{1}{15}\left( 1 - \epsilon^{i\phi} \right), 0, 0, 0 \right) F$ | $\frac{5}{3}(1 - \epsilon^{i\phi})F$ | $0$ |
| $\left( -\frac{2}{135} + \epsilon^{i\phi}, 0, 0, 0, 0, 0, -\frac{(1-\epsilon^{i\phi})}{150} \right) F$ | $\frac{2}{3}(1 - \epsilon^{i\phi})F$ | $-\frac{2}{9}(1 - \epsilon^{i\phi})F$ |
| $\left( 0, -\frac{1}{15} + \frac{1}{10}\epsilon^{i\phi}, \frac{8}{15}(\frac{1}{27} + \frac{\epsilon^{i\phi}}{8}), 0, 0, 0, 0 \right) F$ | $-(\frac{20}{27} + \frac{5\epsilon^{i\phi}}{2})F$ | $0$ |
| $\left( 0, \frac{1}{45} + \frac{\epsilon^{i\phi}}{20}, 0, -(\frac{2}{135} + \frac{\epsilon^{i\phi}}{20}), 0, 0, 0 \right) F$ | $(\frac{10}{27} + \frac{5\epsilon^{i\phi}}{4})F$ | $0$ |
| $\left( 0, -\frac{(1-\epsilon^{i\phi})}{35}, 0, 0, -(\frac{4}{225} + \frac{9\epsilon^{i\phi}}{350}), 0, 0 \right) F$ | $-(\frac{8}{27} + \epsilon^{i\phi})F$ | $(\frac{8}{27} + \epsilon^{i\phi})F$ |
| $\left( 0, -\frac{1}{35}(1 - \epsilon^{i\phi}), 0, 0, 0, -(\frac{4}{1575} + \frac{3\epsilon^{i\phi}}{350}), 0 \right) F$ | $(\frac{16}{189} + \frac{2\epsilon^{i\phi}}{7})F$ | $-(\frac{16}{189} + \frac{2\epsilon^{i\phi}}{7})F$ |
| $\left( 0, -\frac{1}{15} + \frac{\epsilon^{i\phi}}{10}, 0, 0, 0, 0, (\frac{2}{225} + \frac{3\epsilon^{i\phi}}{100}) \right) F$ | $-(\frac{8}{27} + \epsilon^{i\phi})F$ | $(\frac{8}{27} + \epsilon^{i\phi})F$ |
| $\left( 0, 0, \frac{-2}{135} + \frac{\epsilon^{i\phi}}{30}, -\frac{1}{45} + \frac{\epsilon^{i\phi}}{30}, 0, 0, 0 \right) F$ | $\frac{25}{27}F$ | $0$ |
| $\left( 0, 0, -\frac{(1-\epsilon^{i\phi})}{45}, 0, -(\frac{1}{75} + \frac{\epsilon^{i\phi}}{50}), 0, 0 \right) F$ | $\left( \frac{1}{27} - \frac{4\epsilon^{i\phi}}{3} \right)F$ | $(\frac{14}{27} + \frac{7\epsilon^{i\phi}}{9})F$ |
| $\left( 0, 0, -\frac{(1-\epsilon^{i\phi})}{45}, 0, 0, -(\frac{1}{225} + \frac{\epsilon^{i\phi}}{150}), 0 \right) F$ | $\left( \frac{19}{27} - \frac{\epsilon^{i\phi}}{3} \right)F$ | $-(\frac{4}{27} + \frac{2\epsilon^{i\phi}}{9})F$ |
| $\left( 0, 0, 0, -\frac{(1-\epsilon^{i\phi})}{15}, (\frac{2}{75} - \frac{3\epsilon^{i\phi}}{50}), 0, 0 \right) F$ | $(\frac{137}{27} - 4\epsilon^{i\phi})F$ | $-(\frac{28}{27} + \frac{7\epsilon^{i\phi}}{3})F$ |
| $\left( 0, 0, 0, -\frac{(1-\epsilon^{i\phi})}{15}, 0, (\frac{2}{225} - \frac{\epsilon^{i\phi}}{50}), 0 \right) F$ | $\left( \frac{37}{27} - \epsilon^{i\phi} \right)F$ | $(\frac{8}{27} - \frac{2\epsilon^{i\phi}}{3})F$ |
| $\left( 0, 0, 0, -(\frac{1}{45} + \frac{\epsilon^{i\phi}}{30}), 0, 0, -(4 - \frac{9\epsilon^{i\phi}}{900}) \right) F$ | $\left( \frac{19}{27} + \frac{\epsilon^{i\phi}}{2} \right)F$ | $(\frac{-4}{27} + \frac{\epsilon^{i\phi}}{3})F$ |
| $\left( 0, 0, 0, 0, -(\frac{1}{75} + \frac{\epsilon^{i\phi}}{30}), 0, -(\frac{1-\epsilon^{i\phi}}{150}) \right) F$ | $-(\frac{8}{27} + \epsilon^{i\phi})F$ | $(\frac{8}{27} + \epsilon^{i\phi})F$ |
| $\left( 0, 0, 0, 0, 0, -(\frac{1}{225} + \frac{\epsilon^{i\phi}}{150}), -(\frac{1-\epsilon^{i\phi}}{150}) \right) F$ | $\frac{10}{27}F$ | $-\frac{10}{27}F$ |
Table 4: Evaluation of $B_{12}^E$ and $B_{12}^D$ for minimal parameter models.

| Model | $B_{12}^E$ | $B_{12}^D$ |
|-------|-------------|-------------|
| $C_1$ | $(-\frac{19}{27} + e^{i\phi})F$ | $-\frac{8}{27}F$ |
| $C_2$ | $(-\frac{4}{27} + \frac{4}{9}e^{i\phi})F$ | $-\frac{5}{27}F$ |
| $C_3$ | $(\frac{26}{27} - \frac{2}{3}e^{i\phi})F$ | $-\frac{8}{27}F$ |
| $C_4$ | $(-\frac{13}{27} + \frac{7}{9}e^{i\phi})F$ | $(-\frac{14}{27} + \frac{2}{9}e^{i\phi})F$ |
| $C_5$ | $(-\frac{13}{9} + \frac{3}{2}e^{i\phi})F$ | $-\frac{8}{27}F$ |
| $C_6$ | $(-\frac{1}{3} + \frac{9}{4}e^{i\phi})F$ | $-\frac{8}{27}F$ |
| $C_7$ | $-F$ | $e^{i\phi}F$ |
| $C_8$ | $(-\frac{13}{21} + \frac{9}{7}e^{i\phi})F$ | $-(\frac{8}{21} + \frac{2}{7}e^{i\phi})F$ |
| $C_9$ | $-F$ | $e^{i\phi}F$ |
| $C_{10}$ | $(\frac{2}{3} + e^{i\phi})F$ | $-\frac{8}{27}F$ |
| $C_{11}$ | $(-\frac{2}{3} + \frac{1}{7}e^{i\phi})F$ | $(\frac{2}{7} + \frac{7}{7}e^{i\phi})F$ |
| $C_{12}$ | $\frac{2}{3}e^{i\phi}F$ | $-(\frac{4}{9} + \frac{2}{9}e^{i\phi})F$ |
| $C_{13}$ | $(2 - 3e^{i\phi})F$ | $(-\frac{4}{3} + \frac{7}{3}e^{i\phi})F$ |
| $C_{14}$ | $\frac{2}{3}F$ | $-\frac{2}{3}e^{i\phi}F$ |
| $C_{15}$ | $\frac{3}{2}e^{i\phi}F$ | $(-\frac{4}{9} + \frac{1}{3}e^{i\phi})F$ |
| $C_{16}$ | $-F$ | $e^{i\phi}F$ |
| $C_{17}$ | $(-\frac{1}{3} + e^{i\phi})F$ | $-\frac{2}{3}F$ |
REFERENCES

[1] H. Georgi and C. Jarlskog, Phys. Lett. B86 (1979)297.

[2] J. Harvey, P. Ramond and D. Reiss, Phys. Lett. B92(1980)309.

[3] G. Anderson, S. Raby, S. Dimopoulos, L. Hall and G.D. Starkman, Phys.Rev. D49 (1994) 3660; P. Ramond, R. G. Roberts and G. G. Ross, Nucl. Phys. B406(1993)19; L.Ibanes and G.G.Ross, Phys.Lett. B332(1994)100;P.Binetruy and P.Ramond, Phys.Lett B350(1995)49; V. Jain and R. Shrock, Phys. Lett. B35 (1995)83;Stony Brook Report ITP- SB-95-22(1995); K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 74 (1995)2418; K. S. Babu and S. M. Barr, Bartol Preprint BA-95-21; N. Arkani-Hamed, H-C. Cheng and L. J. Hall, Lawrence Berkeley Laboratory Preprint LBL-37343(1995); R. D. Peccei and K. Wang, UCLA Preprint, UCLA/95/TEP/29.

[4] A. H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 29 (1982)970; R. Barbarieri, S. Ferrara and C. A. Savoy, Phys. Letts. B119 (1983) 343; L. Hall, J. Lykken and S. Weinberg, Phys. Rev. D22(1983) 2359; P. Nath, R. Arnowitt and A. H. Chamseddine, Nucl.Phys.B322(1983)121.

[5] For a review see, P. Nath, R. Arnowitt and A. H. Chamseddine, ”Applied N=1 Supergravity”, ( World Scientific, Singapore 1984); H. Nilles, Phys.Rep.110 (1984)1;R.Arnowitz and P. Nath, VII J.A. Sweica Summer School( Sinagapore: World Scientific) 1994.

[6] P.Nath, NSF-ITP-95-138/ NUB-TH-3129.

[7] C.D.Froggatt and H.B.Nielsen, Nucl. Phys. B147 (1979)277.

[8] S.Choudhuri,S.W.Chung, G.Hockney,and J.Lykken, FERMILAB- PUB-94/413-T.

[9] K.Dienes and E. Farraga,IAASSNS-HEP-94/113;G.Cleaver,OHSTPY- HEP-T-95-001-1.

[10] S. Weinberg, Phys.Rev. D26(1982)287; N. Sakai and T. Yanagida, Nucl.Phys. B197
(1982)533; S.Dimopoulos, S.Raby and F.Wilczek, Phys.Lett.112B(1982)133; J.Ellis, D.V.Nanopoulos and S.Rudaz, Nuc. Phys.B202(1982)43;

[11] R. Arnowitt, A.H. Chamseddine and P. Nath, Phys. Letts. 156B (1985)215; P.Nath, R. Arnowitt, and A.H.Chamseddine,Phys. Rev.32 D(1985)2348; J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B402(1993)46, and the references quoted therein.