Spherical accretion onto neutron stars and black holes

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Abstract. Spectral formation in steady state, spherical accretion onto neutron stars and black holes is examined by solving numerically and analytically the equation of radiative transfer. The photons escape diffusively and their energy gains come from their scattering off thermal electrons in the converging flow of the accreting gas. We show that the bulk motion of the flow is more efficient in up-scattering photons than thermal Comptonization in the range of non-relativistic electron temperatures. The spectrum observed at infinity is a power law with an exponential turnover at energies of order the electron rest mass. Especially in the case of accretion into a black hole, the spectral power-law index is distributed around 1.5. Because bulk motion near the horizon (1-5 Schwarzschild radii) is most likely a necessary characteristic of accretion into a black hole, we claim that observations of an extended power law up to about $m_e c^2$, formed as a result of bulk motion Comptonization, is a real observational evidence for the existence of an underlying black hole.

Key words: accretion — black hole physics — radiation mechanisms: Compton and inverse Compton — radiative transfer — stars: neutron — X-rays: general

1. Introduction

Spherical accretion into black holes as a Compton upscattering problem of low frequency photons was studied by Blandford and Payne (Blandford & Payne 1981a, henceforth BP; Payne & Blandford 1981, henceforth PB). Especially in PB the above authors solved the photon transfer equation in the case of a steady state spherically symmetric supercritical accretion into a central black hole with the assumptions of power law flow velocity and cold electrons $T_e = 0$. An extension of the work of PB on accreting black holes to include the thermal Comptonization in the converging flow of the accreting gas was shown by Lyubarskij & Sunyaev (1982) in the case of a finite optically thick medium. In a extension of the work of Blandford & Payne (1981b), they studied Comptonization in a radiation-dominated shock, taking into account not only the bulk motion of the electrons but also their thermal motion.

In this paper we present the emergent spectrum from a spherical inflow by taking into account the effects of bulk motion and thermal Comptonization in addition to the boundary conditions of the inflow. For proofs and detailed discussions the reader is referred to our long paper (Titarchuk, Mastichiadis and Kylafis 1996, henceforth TMK). By extending the relevant work of Mastichiadis & Kylafis (1992, henceforth MK92) we allowed the inner boundary to be anything from fully reflective to fully absorptive assigning the former case to a neutron star and the latter to a black hole. Also, we determined the eigenvalues of the problem as functions of the mass accretion rate and we found numerically and analytically the general solution for the spectrum as well as its asymptotic forms. The obtained results in the black hole case showed a remarkable similarity with the X-ray spectra of the Galactic black hole candidates in their high states. In \S 2 we present the solution of the radiative transfer problem while in \S 3 we give some examples and draw our conclusions.

2. Radiative Transfer

Consider a spherically symmetric accretion onto a compact object with mass rate $M$, where the bulk velocity of the infalling plasma is given by

$$v(r) = c \left[ \frac{(1 - \ell) r_s}{r} \right]^{1/2},$$

(1)

with $c$ the speed of light, $r_s$ the Schwarzschild radius and $\ell$ the ratio of the escaping luminosity to the Eddington
The Thomson optical depth of the flow is defined by

$$\tau_T(r) = \int_r^\infty dr n_e(r)\sigma_T = \frac{r_s}{r(1-\ell)^{1/2}}$$

where $n_e(r)$ is the electron number density, $\sigma_T$ is the Thomson cross section, $\dot{m} = \dot{M}/\dot{M}_E$ and $\dot{M}_E$ is the Eddington accretion rate defined by

$$\dot{M}_E \equiv \frac{L_E}{c^2} = \frac{4\pi GMm_p}{\sigma c}.$$  

Here $L_E$ is the Eddington luminosity, $M$ is the mass of the central object, $m_p$ is the proton mass and $G$ is the gravitational constant.

Consider next low energy photons that find themselves in the accretion flow. As the photons diffuse outward, they scatter off the infalling electrons gaining energy on average. At some characteristic radius, called the trapping radius (Rees 1978; Begelman 1979), the conditions of optical depth and velocity of the infalling electrons are such that the photons are advected inwards at the same rate as the one with which they diffuse outwards. Photons that either are emitted inside the trapping radius or find themselves there cannot escape easily, but if they do, they gain significant amount of energy.

Let us investigate next the relative importance of the bulk (converging flow) and the thermal motion of the electrons to the mean photon energy change per scattering. The mean energy gain per scattering of a photon by thermal Comptonization $<\Delta E_{\text{th}}>$ is proportional to $(v/c)^2$, i.e., $<\Delta E_{\text{th}}> \approx E(4kT_e - E)/m_e c^2$. On the other hand, the mean energy gain $<\Delta E_{\text{ad}}> \in$ the presence of a converging flow is proportional to $v/c$ (e.g., BP). By using equations (1) and (2) we write

$$<\Delta E_{\text{th}}> \approx 4Edv(c)/d\tau_T = 4(1-\ell)/\dot{m}.$$  

Thus, we have

$$<\Delta E_{\text{th}}> \approx <\Delta E_{\text{ad}}> \approx (kT_e - E/4)\dot{m}/(1-\ell)m_e c^2 < 1/\delta,$$

where

$$\delta = \frac{m_e c^2 (1-\ell)}{kT_e \dot{m}}.$$ 

This can be further written as $\delta = 51.1 \times T_{10}^{-1} \dot{m}^{-1}$, with $T_{10} \equiv kT_e/(10 \text{ keV})$. Hence, the bulk motion Comptonization dominates the thermal one if $\dot{m} T_{10} < 51$.

The dominant power-law and the high-energy cutoff of the spectrum can be understood in the following way. As it was shown above, the fractional increase in energy of a low-energy photon in its collision with accreting electrons of bulk velocity $v(r)$ and temperature $T_e$ is given by

$$<\Delta E>_{\text{incr}}/E \approx 4(1-\ell)/\dot{m} + 4kT_e/m_e c^2.$$  

At the same time, the recoil effects cause a fractional decrease in the energy of the photon given by $<\Delta E>_{\text{dec}}/E \approx -E/m_e c^2$. When $E \ll m_e c^2$, the recoil effect is negligible, resulting in a pure power-law spectrum. At high energies the two effects are comparable and the turnover in the spectrum occurs at $E_c/m_e c^2 \approx 4(1-\ell)/\dot{m} + 4kT_e/m_e c^2$, or $E_c/m_e c^2 \approx 4(1-\ell)\dot{m}^{-1}$ when $kT_e \ll m_e c^2$.

The radiation spectrum observed at infinity can be found by solving the equation of radiative transfer for the photon occupation number $n(r, \nu)$ (Eqn. [18] of BP). We remind the reader that the BP equation does not take fully into account general relativistic effects; also the associated diffusion coefficient has been derived in a fully self consistent manner. Both of these issues have been recently discussed (Zanni & Titarchuk 1996, henceforth ZT96).

The two boundary conditions of the radiative transfer equation can be formulated in terms of the spectral energy flux $F$ (cf. Eqn. [21] of BP). The first is that the total spectral flux integrated over the photosphere should depend only on $E$ as $\tau_T \to 0$ or $r \to \infty$. The second boundary condition is that we have a boundary with albedo $A$ at some radius $r_b$. The net energy flux through this surface is

$$F(r_b, x) = -x^3 \left( \frac{1 - A}{1 + A} \right) n_{\nu}^{1/2},$$

where $x = h\nu/kT_e$ and $T_e$ is the electron temperature considered to be constant throughout the flow. From the setup of the problem, $r_b$ is the smallest value that the variable $r$ can obtain.

If the inner boundary is fully reflective ($A = 1$), all input photons escape. This is the problem considered in MK92 who pointed out the analogy of the fully reflective boundary to a neutron star surface. On the other hand, if the inner boundary is fully absorptive ($A = 0$), then we have the equivalent of a black-hole horizon. However, since we do not take an infinite atmosphere but we consider instead a finite flow extending up to $r_b$, our case is different from the one considered in PB and Colpi (1988). Relativistic effects close to the horizon of the compact object have been ignored.

The radiative transfer equation, satisfying the above formulated boundary conditions is solved in TMK. This solution is extended in ZT96 by taking into account relativistic corrections. The spectra are calculated numerically by using the iteration method to solve the boundary problem of the appropriate Fokker-Planck equation. The analytic approximation found by the separation of variables method describes satisfactorily the emergent spectra in the whole energy range up to the high energy turnover $E_c \sim m_e c^2 / \dot{m}$. Alternatively, as it is shown by TMK, this approximation can be written as a sum of two components: The low frequency injection is presented by a coherent solution and the high energy tail is presented by the fundamental mode. This mode can be presented by a convolution of the low-frequency source spectrum and the Green function $I(x, x_0)$ given by

$$I = \frac{b}{2\mu x_0} \left( \frac{x}{x_0} \right)^{\alpha+3+\delta},$$

where $b$ represents the injection rate at a characteristic frequency, $\mu$ is the visibility of the source at a characteristic distance, $\alpha$ is the spectral index, and $\delta$ is the energy spectral index.

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for $x \leq x_0$ and by

$$I = \frac{b}{2\mu x_0 \Gamma(2\mu)} \left(\frac{x}{x_0}\right)^{-\alpha} \int_0^\infty e^{-t} e^{(x + t)^{\alpha + \delta} t^{\alpha - 1}} dt,$$

(6b)

for $x \geq x_0$.

Here $b = \alpha (\alpha + 3 + \delta)$, $\mu = \frac{1}{2}[(\delta - 3)^2 + 4\gamma]^{1/2}$, $\gamma = 2\lambda^2 \delta$, $\lambda$ is the first eigenvalue of the appropriate boundary problem and the spectral index is determined by

$$\alpha = \mu - (3 + \delta)/2.$$

The normalization of $I(x, x_0)$ is chosen in such a way as to keep the photon number equal to $1/x_0$.

As we noted earlier, the parameter which shows the importance of the bulk Comptonization effects relative to thermal Comptonization is $\delta$, given by Eqn. (4). In the case where $kT_e \ll m_e c^2$ and $\tilde{m} \simeq 1$, then $\delta \gg 1$ and this leads immediately to an asymptotic relation for the energy spectral index $\alpha$

$$\alpha = 2\lambda^2 - 3.$$

As long as $\delta \gg 1$, bulk Comptonization effects dominate the thermal ones and the value of $\alpha$ is independent of the temperature of the electrons.

This boundary condition, along with the radiative equation implies that the eigenvalues are the roots of the equation (TMK)

$$\left(\frac{5}{2} - \frac{\tilde{\varepsilon}}{3\tau_b}\right) \Phi(-\lambda^2 + \frac{5}{2} \frac{7}{2}, \tau_b) + \frac{5 - 2\lambda^2}{7} \tau_b \Phi(-\lambda^2 + \frac{7}{2} \frac{9}{2}, \tau_b) = 0,$$

(9)

where $\tau_b = 3 \tilde{m}/2$ and

$$\tilde{\varepsilon} = \alpha + 3 - \frac{3(1 - A)}{2(1 + A)} \left[ \frac{\rho_b}{\rho_s(1 - \ell)} \right]^{1/2}.$$

(10)

By using Eqns. (9-10) it can be demonstrated (TMK) that the spectrum becomes harder as $A$ increases. The dependence of the spectral index on the albedo $A$ is displayed in Figure 1. We point out that for $A = 0$ the power-law index $\alpha$ is always larger than 1 and approaches 1.5 for $\tilde{m} \gg 1$. For the fully reflective inner boundary case ($A = 1$), $\alpha \to 0$ for $\tilde{m} \gg 1$. Thus, the hardest spectral slope ($\alpha = 0$) corresponds to the fully reflective inner boundary as it was shown in MK92.

Of special interest is the case of a fully absorptive boundary $A = 0$. The obtained in this case spectral energy power-law index $\alpha$ is shown in Figure 2 as a function of $\tilde{m}$. As it can be seen, the spectral index is a weak function of the electron temperature in the wide range of mass accretion rates $\tilde{m} = 1 - 20$ and electron temperatures $kT_e = 0 - 3$ keV. The temperature dependence of the spectral index becomes stronger for very high accretion rates $\tilde{m} > 20$ and for electron temperatures $kT_e > 10$ keV.

A second important consequence of the solution is that, despite the presence of the exponential in front of the integral in equation (6b), the power law can extend to $x \gg 1$ (see TMK). Thus, for $kT_e \ll m_e c^2$, the power law extends to energies of order $m_ec^2$ independently of the electron temperature. Only for energies $x > \delta$ the spectrum exhibits an exponential turnover.

Figure 3 shows a comparison between the analytical approximation where the hard tail of the spectrum is presented by Eqn. (6b) and the numerical solution. The analytical approximation represents the emergent spectrum quite reliably in the whole energy range below the exponential turnover. In this region two effects, the bulk motion upscattering and the Compton (recoil) downscattering, compete forming the hard tail. Because the analytical representation does not take into account properly the inner boundary condition at energies $E > E_c$ there is a significant deviation of the analytical curve from the numerical solution close to the cut-off. When one takes

![Fig. 1. Plot of the energy spectral index $\alpha$ versus $\tilde{m}$ for different albedo values $A$. Here $kT_e = 1$ keV, $\ell = 0$ and $A = 0$ (solid line), $A = 0.5$ (dashed line), $A = 1$ (dash-dotted line).](image1)

![Fig. 2. Plot of the energy spectral index $\alpha$ versus $\tilde{m}$ for different electron temperature values $kT_e$. Here $A = 0$, $\ell = 0$ and $kT_e = 0$ keV (solid line), $kT_e = 0.1$ keV (dashed line), $kT_e = 1$ keV (dash-dotted line) and $kT_e = 3$ keV (dotted line).](image2)
The tail of the spectrum $E > 15$ keV is described by Equation (6b). The space source distribution is given by equation (11) with $S_0 = 1$ and $\mu = 0.3$.

In Figure 4 the spectra are presented for different mass accretion rates. Here (as in Fig. 3) we have assumed external illumination of the converging flow by the low-energy black body radiation of an accretion disk having a characteristic temperature $T_{bb}$; furthermore we have assumed a space source distribution of the form (see, for example, Sobolev 1975)

$$S(r) = S_0 r^{-2} \exp(-\tau_T(r)/\mu),$$

where $S_0$ is a normalization constant and $\mu$ is the cosine of the angle of incidence at $r \sim r_s$. All spectra are pure power laws in the high energy range from 15 kev up to the exponential turnover, which occurs at energy $E_e \approx (1 - \ell) m_e c^2 / \dot{m}$. It is worth pointing out that, despite the fact that the accretion rate is taken to vary from 2 to 10 $M_E$, the resulting energy spectra have indices around 1.5.

**3. Conclusions**

We have solved numerically and analytically the radiative transfer equation in the diffusion approximation in the case of thermal electrons infalling spherically onto a compact object. We have considered a finite medium (as opposed to an infinite medium considered in the related work of PB and Colpi [1988]) taking as inner boundary a totally absorptive or reflective surface, in the case where the compact object is a black hole or a neutron star, respectively. We have shown that the escaping spectral energy flux exhibits an extended power law at high energies up to a few hundred keV and has an index that depends on the inner boundary condition. The important parameter (in addition to the albedo introduced through the inner boundary condition) which determines the spectral slope is $\delta$ (eq. 4) and as long as this remains much greater than unity the upscattering is mainly due to the bulk motion of the electrons. Therefore we find that, for a wide range of values of the dimensionless accretion rate $\dot{m}$ and of the electron temperature $T_e$, the spectral slope is insensitive to both of these quantities.

The above conclusion is especially true in the case of accretion into a black hole (absorptive boundary). The dominant energy spectral index takes values around 1.5 for a wide variety of $\dot{m}$ ranging from 1.5 to 10 (see Figs. 2-4). Furthermore, this spectral index appears to be independent of the electron temperature as long as this remains below 10 keV (see Fig. 2). Therefore we propose that our calculations may offer an explanation for the spectra observed by black-hole candidates in their high states (e.g. Ebisawa et al. 1993, Sunyaev et al. 1994 – for a recent review see Tanaka & Lewin 1995). The situation is changed only when either the temperature of the electrons is higher than 10 keV or when the escaping luminosity approaches the Eddington limit. In this last case the parameter $\delta$ is reduced due to the radiative force. In both cases the spectra are dominated by thermal Comptonization and are related to the hard spectra of black holes and neutron stars.

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