A spatially-VSL gravity model
with 1-PN limit of GRT

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In the static field configuration, a spatially-Variable Speed of Light (VSL) scalar gravity model with Lorentz-Poincaré interpretation was shown to reproduce the phenomenology implied by the Schwarzschild metric. In the present development, we effectively cover configurations with source kinematics due to an induced sweep velocity field. The scalar-vector model now provides a Hamiltonian description for particles and photons in full accordance with the first Post-Newtonian (1-PN) approximation of General Relativity Theory (GRT). This result requires the validity of Poincaré’s Principle of Relativity, i.e. the unobservability of ‘preferred’ frame movement. Poincaré’s principle fixes the amplitude of the sweep velocity field of the moving source, or equivalently the ‘vector potential’ $\xi$ of GRT (e.g.; S. Weinberg, Gravitation and cosmology, 1972), and provides the correct 1-PN limit of GRT. The implementation of this principle requires acceleration transformations derived from gravitationally modified Lorentz transformations. A comparison with the acceleration transformation in GRT is done. The present scope of the model is limited to weak-field gravitation without retardation and with gravitating test bodies. In conclusion the model’s merits in terms of a simpler space, time and gravitation ontology —in terms of a Lorentz-Poincaré–type interpretation— are explained (e.g. for ‘frame dragging’, ‘harmonic coordinate condition’).

1 Introduction

Modeling relativistic gravitation in flat, “unrenormalized”, space-time has a long tradition starting with pre-general relativistic approaches of Lorentz, Poincaré, Einstein, Abraham and Nordström [37] and finalized in the field-theoretic approach of Fierz, Rosen, Gupta and many, more recent, authors [27] (see references in Cavalleri and Spinelli’s work [14]). In these latter theories the physically observed space-time —using “measuring rods and clocks” — still turns out to be Riemannian. This is because the measurements are modified by the “universal” action of gravitation, which leads to the “renormalized” curved space. It is well known that a second rank tensor theory is required to describe all current gravitational experiments adequately and, have convergence with Einstein’s theory (e.g. [14]). We have shown previously however that a 0-rank (scalar) gravitation model, with spatially-variable speed of light and gravitationally modified Lorentz transformation is still able to reproduce the four ‘basic’ experiments of GRT [10, 8]. At present we will expand the model to encompass the full first Post-Newtonian dynamics of GRT. We briefly recall that one of the basic premisses of the model is Poincaré’s conventionalism of geometry [28]. This principle purports the compensation of the adopted basic geometry by a conjugated gravitational dynamics and congruence relation [50, 26, 29] while retaining empirical indistinguishability with GRT. A second premis is to express the congruence relation explicitly by maintaining the Lorentz-Poincaré interpretation of SRT in GRT. I.e. we render explicit the

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gravitational affecting of physical space and time observations — the contraction and dilation of “measuring rods and clocks”. This leads to the use of two types of gravitationally modified Lorentz transformations, relating both; the affected (or normalized) and unaffected (or unrenormalized) perspective and, by composition of the previous, non-identical observers in the affected perspective. We emphasize that naturally the unaffected perspective cannot be realized by any physical observer, it merely corresponds to the coordinate space description. Finally, these modifications are due to both; the incorporated spatially variable speed of light and, position and time dependent scaling functions. While this latter feature is similar to some precursor theories of gravitation [37], our Lorentz-Poincaré type (“L-P”) model still maintains the invariance of the locally observed velocity of light and local Lorentz symmetry.

The term “variable speed of light” is a historic concept already occurring in the precursory work of Einstein [24] in 1907 where the velocity of light depends on the gravitational field (or potential). The term was used in tempore non suspecto by Wei-Tou Ni [46] in order to classify a number of alternative gravitational theories. The same term — now written by its acronym — “VSL” is used in some recent work where it is understood as an epochal phenomenon; i.e. exposing dependence on a cosmological time scale [45] [1] [39]. In the present L-P model however we take ab initio what in GRT is found to be the case in photon dynamics; namely that the velocity of light — or a photon on its geodesic — in coordinate space is not constant, but that it is depending on the gravitational field, implying dependence on its location and instant in coordinate space and time. Excerpts revealing this interpretation of spatial-VSL in mainstream GRT works are given in the bibliography [70].

The main aim of the present work is to extend the scalar model which previously only covered the static field configuration (short recapitulation of [8] in Section 3). Now configurations with proper source kinematics are included, e.g. Earth rotation. Our model thus assigns to a moving source a scalar and a vector potential \( \{ \varphi, \mathbf{w} \} \). The vector potential will describe the ‘effective’ velocity of a test particle in the gravitational field due to source movement. The source’s resultant sweep velocity field expresses the test particle’s source-relative movement in contrast to ‘preferred’ frame-relative movement (Section 3.1). A Hamiltonian expression for particles and photons till first order in the sweep velocity is obtained by considering — relative to the unaffected observer — a Galilean composition of the energy and momentum. The equations of motion result in a dynamical acceleration which formally is the first Post-Newtonian expression of the acceleration in GRT (Subsection 3.2). Subsequently this correspondence is rendered numerically exact by assuming the validity of the Poincaré Principle of Relativity (PPR) (Section 4). Its implementation requires an adequate interpretation of the acceleration transformation which will be compared to its equivalent in GRT (Section 4.1). Finally some examples of simpler ontological interpretations in line with the Lorentz-Poincaré interpretation in gravitation are discussed in the conclusion (Section 5).

Many alternative approaches to GRT have been formulated in the literature, some of which share aspects with our approach, e.g. interaction through gravitational potential in variable speed of light initiated by Einstein and Abraham [37], polarizable vacuum models for gravitation starting with Wilson [68] and Dicke [20] [6] and more recently in Lightman and Lee [34] and Puthoff [52], physical substrate interpretation of analogue gravity by Visser et. al. [8], scalar field gravitation models Coleman, Sjödin [16] [57] and Arminjon, Evans and Nandi [2] [55], nonsingular ether gravitation models by Cavalleri and Tonni [15] and, field conceptions of gravitation, e.g. Weinberg and others [60] [14] [47].
2 Gravitationally modified Lorentz transformations

The congruence relations, which express the gravitational and kinematical affecting of space and time observations, have been formulated as gravitationally modified Lorentz Transformations (GMLT). This is done by applying isotropic scaling functions \( \Phi (x) \), \( 0 \leq \Phi (x) \leq 1 \), to the considered physical quantities \( (n\text{-specific}) \), and also to the velocity of light \( (n = 2) \) [10]. The first type of these transformations relate quantities observed by gravitationally affected (natural, “renormalized” or “local” geometry) and unaffected (coordinate, “unrenormalized” or “Newtonian” geometry) “observers” and, these transformation explicitly expose a spatially variable speed of light \( [5, 16] \). The second type transformations shunts the unaffected perspective and relates affected observers, recovering i) the invariance of the locally observed velocity of light, and ii) the local Minkowski metric (appendix in [8]).

For a space and a time interval, observed by the gravitationally unaffected \( S_0 \) observer, and the same infinitesimal space and time interval as observed by gravitationally affected observer \( S' \) we have set:

\[
\delta x = \delta x' \Phi (x) \quad (\text{contraction}) \quad , \quad \delta t = \frac{\delta t'}{\Phi (x)} \quad (\text{dilation})
\]

These interval relations are related to a constrained differential of the diffeomorphism \( x^\mu = x^\mu (x') \) that transforms the coordinate space of \( S' \) into the space of \( S_0 \). The contraction of rulers and the dilation of the periods of clocks are local gravitational effects; their space and time extension should therefore be restricted to domains over which \( \Phi (x) \) can be considered sufficiently invariable to the required accuracy, and as such the intervals have to be infinitesimal. The specific scaling nature of the gravitational effect — as in transformation eqs. (1) — and observation conditions impose constraints on the interval relation which preempts the coincidence with a complete differential relation (cfr the Appendix):

\[
\lim_{\delta x_i \to 0} \left( \delta t' - dt' \big|_{dx_i=0} \right) = 0 \quad , \quad \lim_{\delta t', \delta x_j \to 0} \left( \delta x_i' - dx_i \big|_{dx_j \neq 0, dx_i=0, d\tau = 0} \right) = 0
\]

The congruence relations do not allow invariance of the velocity of light in both the affected \( S' \) and unaffected \( S_0 \) perspectives. In the physical perspective, of the affected observer \( S' \), the invariance of the locally observed velocity of light \( c' \) is secured by imposing specific spatial variability in the unaffected perspective of observer \( S_0 \):

\[
c(x) = c' \Phi (x)^2 \quad (c', \text{ the observed constant velocity of light})
\]

\( i.e. \ n = 2 \) as mentioned previously.

The gravitational congruence relation [11], with kinematical affecting factor \( \gamma \) and light synchronization (e.g. [40]), lead to the full congruence relations in the form of gravitationally modified Lorentz Transformations of the first type, \( i.e. \) between affected and unaffected observers [10] [8].

The space-time GMLT, \( S_0 \) to \( S' \), with relative frame velocity \( u \) is given by:

\[
\delta x' = \left( (\delta x \| - u(x) \delta t) \gamma (u, x) + \delta x \perp \right) \frac{1}{\Phi (x)}
\]

\[
\delta t' = \left( \delta t - \frac{u(x) \cdot \delta x}{c(x)^2} \right) \gamma (u, x) \Phi (x)
\]

and the inverse GMLT, \( S' \) to \( S_0 \), is:

\[
\delta x = \left( (\delta x \| - u' \delta t') \gamma (u') + \delta x \perp \right) \Phi (x)
\]

\[
\delta t = \left( \delta t' - \frac{u' \cdot \delta x'}{c'^2} \right) \frac{\gamma (u')}{\Phi (x)}
\]
with \( u = u(x) \) satisfying:

\[
u(x) = -u' \Phi(x)^2
\]

and \( \gamma(u') \) and \( \gamma(u, x) \), by eqs. \([3] \), satisfy:

\[
\gamma(u, x) \equiv \left(1 - u(x)^2/c(x)^2\right)^{-1/2} = \left(1 - u^2/c^2\right)^{-1/2} \equiv \gamma(u')
\]

We notice thus that the values of \( \gamma(u') \) and \( \gamma(u, x) \) are equal.
The velocity \( v' = \delta x'/\delta t' \) in affected and, \( v = \delta x/\delta t \) in unaffected perspective is related according:

\[
v' = \frac{v|| - u(x) + v_\perp/\gamma(u, x)}{(1 - u(x).v/c(x)^2) \Phi(x)^2}
\]

From the space time GMLT \([1,5]\) the contravariant space-time GMLT, \( S' \) to \( S_0 \) for gradient operators can be derived:

\[
\nabla = u' \frac{1}{\Phi(x)} \left( (\gamma(u') - 1) \frac{u'.\nabla'}{u^2} + \frac{1}{c^2} \gamma(u') \partial v' \right) + \frac{1}{\Phi(x)} \nabla'
\]

\[
\partial_t = \gamma(u') \Phi(x) (\partial v' + u'.\nabla')
\]

These will be applied in the next sections.

By adequately developing the space-time GMLT towards energy and momentum quantities, distinct —first type— momentum-energy GMLT’s were obtained:

\[
p' = \left( \left( p|| - \frac{E}{c(x)^2} u(x) \right) \gamma(u, x) + p_\perp \right) \Phi(x)
\]

\[
E' = \left( E - p.u(x) \right) \frac{\gamma(u, x)}{\Phi(x)}
\]

and its inverse:

\[
p = \left( \left( p||' - \frac{E'}{c^2} u' \right) \gamma(u') + p'_\perp \right) \frac{1}{\Phi(x)}
\]

\[
E = \left( E' - p'.u' \right) \gamma(u') \Phi(x)
\]

Now letting a spatially and kinematically coincident affected observer \( S' \), with a particle with rest mass \( m'_0 \), attribute standard special relativistic energy, \( E' = m'_0c^2 \), and zero momentum, the corresponding expressions by the unaffected observers \( S_0 \) can be obtained. The spatial and kinematical coincidence of the \( S' \) frame and the particle at \( x \) fixes the frame velocity \( u(x) \) to the particle velocity \( v \), both attributed by \( S_0 \). Then momentum, energy and mass expressions attributed by \( S_0 \) are readily obtained from the energy momentum GMLT:

\[
p \equiv m(x, v)v , \quad E \equiv m(x, v)c(x)^2 , \quad m(x, v) \equiv m_0(x)\gamma(v, x) \equiv m'_0 \frac{\gamma(v, x)}{\Phi(x)^2}
\]

where now velocity \( v \) of the particle at \( x \) replaces the frame velocity \( u(x) \) in the \( \gamma \) function;

\[
\gamma(v, x) = \left(1 - v^2/c(x)^2\right)^{-1/2}
\]
eq. (17)c shows that in \( S_0 \) mass depends on location in the gravitational field, in line with a Machian effect.

The momentum equation (17)a can be easily reverted using mass expression (17)c to yield the kinematic affecting \( \gamma \) as a function of momentum \( p \):

\[
\gamma(p, x) = \left( 1 + \frac{p^2}{m_0(x)^2 c^2(x)^2} \right)^{1/2}
\]

(19)

Using the expression \( \gamma(p, x) \) in the expression for the energy eq. (17)b, we can obtain the energy in terms of the canonical Hamiltonian variables; \( E = E(x, p) \). Then \( S_0 \) attributes to particles and photons respectively the Hamiltonian energy expressions (static fields):

\[
E = \left( m_0(x)^2 c(x)^4 + p^2 c(x)^2 \right)^{1/2}, \quad E = pc(x)
\]

(20)

These Hamiltonian expressions for static source will be completed for kinematical source effects in Section (3.1).

The transformation relation between two affected observers —denoted second type GMLT— were extensively developed in previous work [8]. Specifically, the second type GMLT’s are obtained by composing two first type GMLT’s, \( S'_1 \) to \( S_0 \) and \( S_0 \) to \( S'_2 \), with elimination of the common \( S_0 \) perspective. The second type GMLT have an unspecified group symmetry along the frame-kinematical plane \( \{ u_{12}, u_{21} \} \), and orthogonal to it cause a simple isotropic scaling. When \( S'_1 \) and \( S'_2 \) are locally coincident the second type GMLT trivially reduces to a standard Lorentz transformation (\( \Phi_1 = \Phi_2 \)). The local Minkowskian metric is thus recovered in the affected perspective.

The second type GMLT can be applied in the verification of the gravitational effects in the affected perspective, i.e. in terms of observable variables. These relations also describe the resulting ‘curved’ metric of the renormalized space, which is due to the affecting of measuring rods and clocks.

We note that solving a mechanical problem does not necessitate the affected observer’s perspective. Solutions can be obtained in the unaffected \( S_0 \) perspective and subsequently transformed by a first type GMLT into the affected perspective \( S' \). In fact this corresponds to the practice in GRT, where the dynamics is expressed in coordinate space perspective, corresponding to \( S_0 \). While these results should be transformed again to local coordinates (corresponding to \( S' \)), this is usually not done because local coordinates do only interfere at higher order relative to the post-Newtonian approximation (e.g. [53], p 142 or [44], eq. 40.14). Adopting that same strategy to our present aim, the 1-PN equivalence of the present L-P model with GRT will be established in the coordinate perspective of \( S_0 \). Exceptionally the implementation of the Poincaré Principle of Relativity will require a transformation to the affected perspective (Section 4).

3 Sources and gravitational fields.

The physical content of the model is fixed by adjusting the scaling function \( \Phi \) to the limit of Newtonian gravitation. In that limit, it was shown in previous work [8], that \( \Phi \) obeys a static field equation. Formally the equation was obtained from the Newtonian energy fit by \( S_0 \), with potential energy increment according \( dE = \Phi dU_{\text{Newt}} \). This means that for \( S_0 \) the static energy change —amounting to an affected increment of Newtonian potential energy— is described identically as the affecting of rest mass itself. The latter leading to \( dE = m'_0 c^2 d\Phi \) and subsequently the identification of \( \Phi \). This fitting procedure leads to the static field equation:

\[
\Delta \Phi = \frac{4\pi G'}{c^2} \rho(x) \Phi + \frac{(\nabla \Phi)^2}{\Phi}, \quad \Phi \equiv \exp(\varphi), \quad \varphi = -G' \int_{S} \frac{\rho(x^*)}{|x-x^*|} d^3 x^*
\]

(21)
For a Schwarzschild configuration of a spherically symmetric source of radius $R$ and mass $M$ the field is given by:

$$\Phi(r) = \exp\left(-\frac{\kappa}{r}\right), \quad r > R, \quad \kappa \equiv -\frac{G'M}{c^2}$$

The closed form $\Phi(r)$, developed at $O(\kappa^3/r^3)$, was shown to adequately reproduce the GRT result for i) the deflection of light by the Sun, ii) the precession of orbital perihelia, iii) the gravitational delay of radar echo, and in $S'$ perspective; iv) the gravitational redshift of spectral lines.

### 3.1 Kinematic gravitational source

In the static configuration the affected fixed observer is considered at rest to the gravitational source, and the velocity of the test particle is considered relative to the source. In the kinematic configuration the source has a proper movement relative to some ‘preferred’ frame and —in our model— causes a sweep velocity field, denoted $w$, relative to that frame. (We consider ‘preferred’ the attribute of any given frame relative to which the gravitational source has a significant or attributable velocity.) A test particle at location $x$ has an associated effective velocity composed of its velocity $v_w$ relative to the ‘preferred’ frame, now denoted $S_w$, and, the local sweep velocity $w(x)$. Evidently the description of dynamics of a test particle by the affected observer $S'$, uniformly co-moving with the unaccelerated source, should coincide with a stationary field description. I.e. that is what the Poincaré Principle of Relativity —which Poincaré originally intended for “absolute” motion— should imply for the jointly moving source and affected observer system. Foremostly we should verify whether, for kinematic sources, the sweep velocity field correctly renders the PN acceleration expressions of GRT. Subsequently these expressions should be transformed from the preferred frame $S_w$ to an affected observer $S'$, relatively at rest to the source. Then, following the Poincaré Principle of Relativity, we need to verify whether the affected acceleration $a'$ of the particle is retrieved by the uniformly co-moving observer $S'$. This expression should then corresponds to the unaffected acceleration $a$ for static source, as the observer remains relatively at rest to the source.

First we make an educated consideration about the nature of the sweep velocity field and its effect on the velocity of a test particle. We consider the following situation: an extended source with variable velocity distribution causes one single resultant gravitational field, and thus also one resultant affecting potential and gamma factor on a given test particle. What is the effective velocity —relative to the frame in which the movement of the sources are described— that should be considered in the kinematical description of the test particle? The velocity relative to different source parts will in general contribute differently. Each part will contribute i) proportionally to local source density $\rho$ (no contribution for $\rho \to 0$), ii) proportionally to local source velocity $v_\rho$ (particle moves relatively at $-v_\rho$), iii) attenuated by distance (source contributes proportional to relative potential energy, at $1/r$). Then the resultant velocity relative to the sources is the $\varphi$-average over the source’s velocity distribution, with an as yet to be fixed parameter $\lambda$ for amplitude fitting:

$$w(x, t) \approx \lambda \left\langle \rho(x^*) \frac{v_\rho(x^*)}{|x - x^*|} \right\rangle$$

We must now fix the effect of the induced velocity $w$ on a moving particle at $(x, t)$. To order $O(w)$, we can claim that the unaffected observers $S_w$ (with kinematic source) and $S_0$ (without kinematic source), due to their nature of being unaffected, relate their respective observed velocities by adding up classically the sweep velocity to the latter. I.e. $S_w$ and $S_0$ relate the
observed velocities formally by Galilean composition with the sweep velocity field. Because the sweep velocity enters as an external (potential) field, this component will not be available as a kinematical variable of the particle’s mechanics. It will therefor be the velocity \( v_0 \) by the unaffected observer \( S_0 \) which appears as the effective velocity in the dynamics (Subsection 3.2):

\[
v_0 \equiv v_w \odot w = _{\text{Galilei}} v - w, \quad (\text{photon } v = c)
\]  

(24)

The precise nature of the dynamically effective velocity \( v_0 = v_w \odot w \) to higher order will depend on the cosmological model and the gravitational field equation: closer to the source its effect dominates, and the test mass is moving increasingly relative to the rest frame of the source instead of the rest frame of the remainder matter of the universe \([20], \text{eqs. } (5, 6))\). Till \( O(w^2) \) this competition between the frame of the source and “background” should be correctly quantified by tuning the parameter \( \lambda \) in expression \([23], \text{this will be done in Section 4}\). In the present work there will be no development beyond \( O(w) \). Notice that this relation \([24]\) does not imply a change of frame —as its Galilean form would suggest—, it describes a change of physical configuration.

We will thus hypothesize that for moving gravitational source:

i) An induced velocity field, \( w \), follows the field equation:

\[
\Delta w = 4\pi\lambda \frac{G'}{c^2} \rho \varphi(x, t)
\]

(25)

which includes solutions of the type eq. \([23]\). This basic equation is considered approximative; without gravitational self-field of the source nor propagation retardation. Also the contribution of the field’s momentum and energy density is ignored in the present approximation. The numerical parameter \( \lambda \) will be fixed by the Poincaré Priciple of Relativity (Section 4). For a rigid source, with velocity \( u \) relative to \( S_w \), the field equation \([25]\) for \( w \) solves to:

\[
w(x) = \lambda u \varphi(x)
\]

(26)

where \( \varphi \) is given by eq. \([21]c. \) (which can be easily seen by setting the ansatz, \( w = uf \), in eq. \([25]\))

In the given approximation, a linearly moving source is thus indeed enveloped by an instantaneous and parallel induced velocity field which attenuates inversely proportional to distance.

ii) The space and time intervals of the unaffected observer \( S_w \) (with sweep field), are related to the intervals of the unaffected observer \( S_0 \) (without sweep field), at \( O(w^2) \), by a “Galilean” space and time transformation:

\[
\delta x_0 = \delta x_w - w \delta t_w
\]

(27)

and

\[
\delta t_0 = \delta t_w
\]

(28)

This results in the required \( \delta x_0/\delta t_0 = v_0 = v_w - w \) velocity relation eq. \([24]\). From the space and time relation between \( S_0 \) and \( S_w \) an energy and momentum relation can be fixed as well. The Hamiltonian expression of a particle in the sweep velocity field can then be considered to be the energy attributed by \( S_0 \), kinematically boosted by the sweep velocity. The composition of energy and momentum in the velocity sweep field will again be in Galilean approximation \([24]\):

\[
p_0 \equiv p_w \odot mw = _{\text{Galilei}} p_w - mw
\]

(31)

\[
E_0 \equiv E_w \odot p_w \cdot w = _{\text{Galilei}} E_w - p_w \cdot w
\]

(32)
3.2 Dynamics relative to moving source

The time-dependent induced velocity field must appear in the dynamics of a particle. Moreover, the particle’s energy is now directly dependent on the induced velocity field. And, as is well known from standard theory of mechanics with time-variable potentials, the energy of the particle is not conserved.

We let the unaffected observer, $S_w$, attribute form-invariant expressions of energy and momentum according the static field expressions:

$$E_w = m_0(x)\gamma(v_w, x)c_w(x, w)^2, \quad p_w = m_0(x)\gamma(v_w, x)v_w$$

with

$$\gamma(v_w, x, w) = \left(1 - \frac{v_w^2}{c_w(x, w)^2}\right)^{-1/2}.$$

While for photons:

$$E_w = p_w c_w(x, w), \quad p_w = \frac{E_w}{c_w(x, w)} c_w$$

The $S_w$ expressions do not render explicit the role of the induced velocity field in the particle’s velocity $v_w$. In the description the sweep field acts as a ‘vectorial potential’, not a dynamical variable. All expressions by $S_w$ must therefor be recast on $S_0$ expressions using eqs. (28, 32).

E.g. the energy attributed by $S_0$ in terms of velocity $v_0$ is given by:

$$E_0 = m_0(x)\gamma(p_0, x)c^2 = m_0(x)\gamma(v_w \ominus w, x)c(x)^2$$

This expression allows to discern and isolate the effect of the sweep velocity. Thus $S_w$ attributes—at $O(w)$ due to the approximative relation eq. (32)b—an energy $E_w$ to a particle in the sweep velocity field, using the $E_0$ expression eq. (36) in the energy transformation eq. (32)b between $S_0$ and $S_w$:

$$E_w(v_w, x) = m_0(x)\gamma(v_w - w, x)c(x)^2 + p_w w(x)$$

Now the momentum expression eq. (19) of the $\gamma(p, x)$ function is used to express the energy $E_w(v_w, x)$ as the Hamiltonian $H(p, x)$ of the particle as (with $p \equiv p_w$) till $O(w)$:

$$H = m_0(x)\gamma(p, x)c(x)^2 + p w(x)$$

since for $\gamma(p, x)$ we find that:

$$\nabla_p \gamma(p, x) = \gamma(p, x)^{-1} \frac{p}{m_0(x)c(x)^2}$$

and by using the Hamilton equation for velocity (with $\dot{x} \equiv v \equiv v_w$);

$$v \equiv \nabla_p H = \frac{p}{m(x, p)} + w(x)$$

where we used;

$$m(x, p) = m_0(x)\gamma(p, x)$$

as in eq. (17)c. Notice thus that the momentum’s canonical conjugate velocity is $v$ while the effective dynamical velocity is $v - w$, for

$$p = m(x, p)(v - w)$$
then;

$$\gamma(p, x) = \gamma(v - w, x) \quad (43)$$

as is indeed required to express $E(v, x)$, eq. (36), as the Hamiltonian $H(p, x)$, eq. (38). We emphasize that, given the order of validity $O(w)$ of the Hamiltonian eq.(38), the ensuing results of calculations should in the end always be restricted to expressions to first order in $w$.

For photons the description in the velocity sweep field, with $c_0 = c_w \otimes w$ and $|c_0| = c$, leads to the Hamiltonian at order $O(w)$:

$$H = pc(x) + p.w(x) \quad (44)$$

and with the Hamilton equation for velocity ($\dot{x} \equiv v_c \equiv c_w$)

$$v_c = c(x)1_p + w(x) \quad (45)$$

Where we have indicated by index $c$ that $v_c$ is the velocity of light relative to $S_w$.

### 3.2.1 Particle dynamics

We study now the dynamics according the Hamiltonian, eq. (38), of a test body in the gravitational fields $\Phi$ and $w$ of a moving source. From the Hamiltonian expression we immediately obtain the classical mechanical equations of motion in coordinate space $S_w$:

$$H = mc^2 + p.w \rightarrow \dot{p} = -\nabla H, \quad \dot{x} = \nabla_p H, \quad \dot{H} = \partial_t H \quad (46)$$

where $\nabla \equiv \partial_x, 1_i$ and $\nabla_p \equiv \partial_p, 1_p$ are the usual shorthand notations for the three dimensional partial derivatives towards $x$ and $p$ respectively. For lucidity of notation we drop from now on the variable dependencies of $\gamma, w, m, \varphi, \Phi$ and $c$ in our discussion of the Hamilton equations.

The velocity Hamilton equation was explicited before, eq. (40), with $\dot{x} \equiv v$;

$$v = \frac{p}{m} + w \quad (47)$$

The power Hamilton equation gives;

$$\dot{H} = \partial_t H = mc^2 \partial_t \varphi + p.\partial_t w \quad (48)$$

As mentioned above energy conservation is only present for time invariant potentials. That is, no energy transfer between particle and sweep velocity field will occur for stationary sweep velocity fields, e.g. from an axially symmetric source with constant axial rotation (cfr. concluding section 5).

The force Hamilton equation finally is found to be, to order $O(w)$;

$$\dot{p} = -\nabla H = -mc^2 \left(2 - \frac{1}{\gamma^2}\right) \nabla \varphi - p \times (\nabla \times w) - (p.\nabla)w \quad (49)$$

where we have used eq. (11) for $m$, eq. (19) for $\gamma$, eq. (3) for $c$ and eq. (21)b for $\Phi$ in calculating;

$$- \nabla \left(mc^2\right) = -m_0' c^2 \nabla \left(\Phi \left(1 + \frac{p^2 \Phi^2}{m_0'^2 c^2}\right)^{1/2}\right) = -mc^2 \nabla \varphi - mc^2 \gamma^{-2} (\gamma^2 - 1) \nabla \varphi \quad (50)$$
and standard calculus gives ($p$ is an independent Hamiltonian variable);

$$\nabla p . w = p \times (\nabla \times w) + (p . \nabla) w + w \times (\nabla \times p) + (w . \nabla) p = p \times (\nabla \times w) + (p . \nabla) w$$  \(51\)

In order now to obtain the acceleration, $a \equiv \dot{v}$, of the particle in the coordinate space of $S_w$, we must take the complete time derivative of the velocity Hamiltonian equation (40):

$$a = \frac{\dot{p}}{m} - \frac{m}{m^2} \dot{p} + \dot{w}$$  \(52\)

using now again eq. (41) for $m$, eq. (19) for $\gamma$, eq. (3) for $c$ and eq. (21)b for $\Phi$ we obtain;

$$\dot{m} = m \left( \frac{\dot{\gamma}}{\gamma} - 3\dot{\varphi} \right)$$  \(53\)

$$\dot{\gamma} = \frac{1}{\gamma} \left( (\gamma^2 - 1)\dot{\varphi} + \frac{p . \dot{p}}{m^2 c^2} \right)$$  \(54\)

In the latter expression for $\dot{\gamma}$ the gravitational force $\dot{p}$ should be substituted by its expression eq. (49).

All required terms have been calculated now in order to explicitly render the acceleration $a$, eq. (52). Respecting the validity of the Hamiltonian at $O(w)$, all terms appearing in $a$ should finally be restricted to this order. The terms $\gamma$, eq. (43), and $p$, eq. (42), must be expressed again as $\gamma(v)$ and $m(v - w)$ and are approximated to their first post-Newtonian value. The time derivatives of the fields, occurring in the expression of $a$, are expanded $\dot{\varphi} = \partial_t \varphi + v . \nabla \varphi$ and $\dot{w} = \partial_t w + (v . \nabla) w$. Then, upon gathering all parts in $a$, eq. (52), at order $O(w, c^2 \nabla \varphi^2, v^2 \nabla \varphi, v\partial_t \varphi)$;

$$\frac{\dot{p}}{m} \approx -c^2 \nabla (\varphi + 2\varphi^2) - v^2 \nabla \varphi - v \times (\nabla \times w) - (v . \nabla) w$$  \(55\)

$$-\frac{\dot{m}}{m^2} p \approx -(v . \nabla \varphi - 3(\partial_t \varphi + v . \nabla \varphi)) v$$  \(56\)

$$\dot{w} = \partial_t w + (v . \nabla) w$$  \(57\)

the expression for the acceleration $a$ for a test body in the gravitational fields of a moving source, again at order $O(w, c^2 \nabla \varphi^2, v^2 \nabla \varphi, v\partial_t \varphi)$, is found to add up to:

$$a = -c^2 \nabla (\varphi + 2\varphi^2) - v^2 \nabla \varphi + 4vv . \nabla \varphi - v \times (\nabla \times w) + 3v\partial_t \varphi + \partial_t w$$  \(58\)

This is exactly the form of the 1-PN expression of the acceleration of test particle in the gravitational fields of a moving source in General Relativity Theory (eq. (9.5.3) till $O(\tilde{v}^6)$). Of course in GRT the modus to obtain this result is quite different; it follows from the condition for geodesic motion in curved spacetime.

Now to obtain quantitative equivalence with the acceleration expression in GRT, the “vector potential” $\xi$ from GRT (eq. (9.1.61)) must equal minus the sweep velocity field of our present L-P model, i.e. $\xi = -w$. Then the correspondence with GRT is exact. We will obtain, in section 4, this correspondence by implementation of the Poincaré Principle of Relativity in our model; from this principle the velocity sweep field $w$ will be shown to satisfy the same equation as the vector potential $\xi$ of GRT.

### 3.2.2 Photon dynamics

The Hamiltonian (44) for a photon in the gravitational fields $\Phi$ and $w$ of a moving source, will lead again to the description of its motion in coordinate space $S_w$. We follow the same line of
argument as for particles. The classical mechanical equations following the photon Hamiltonian are, with $v_c \equiv \dot{r}$:

$$H = pc + p \cdot w \to \begin{cases} \dot{p} = - \nabla H = - p \nabla c - p \times (\nabla \times w) - (p . \nabla)w \\ \dot{v}_c = \nabla_p H = c \dot{1}_p + w \\ H = \partial_t H = 2pc \partial_t \varphi + p . \partial \dot{w} \end{cases} \quad (59)$$

where $1_p = p / p$ is the direction of the photon momentum. We will denote the direction of propagation as $1_e = v_c / v_e$.

The photon acceleration $a$, $a \equiv \ddot{v}_c$, is obtained by time derivation of the velocity Hamilton equation:

$$a = \frac{c}{p} \ddot{p} + c \frac{\dot{p}}{p} - c^2 \frac{\dot{p}}{p} p + \dot{w} \quad (60)$$

In order now to express the acceleration in terms of $1_c$, instead of $1_p$, we must relate both. From the velocity Hamilton equation we obtain by the scalar product with $1_c$:

$$v_c = c + w . 1_c \quad (61)$$

The velocity Hamilton equation can then be reverted:

$$1_p = 1_c - \frac{w}{c} + \frac{w}{c} 1_c 1_c \quad (62)$$

We develop all terms required in $a$, eq. (60), at order $O(w, \partial_t \varphi, v_c \nabla \varphi, v_c \nabla \varphi^2)$ using again eq. (3) for $c$ and eq. (62) for $1_p$:

$$\dot{c} = 2c'(\partial_t \varphi + (1 + 2\varphi)(v_c . \nabla)\varphi) \quad (63)$$

$$\dot{p} \over p = 1_p . \ddot{p} = -2c'1_c . \nabla \varphi - 1_c . (1_c . \nabla)w \quad (64)$$

$$\dot{w} = \partial_t w + (v_c . \nabla)w \quad (65)$$

Then the required terms can be written to explicitly render the acceleration $a$, eq. (60), of the photon in the gravitational field, to order $O(w, \partial_t \varphi, v_c \nabla \varphi, v_c \nabla \varphi^2)$:

$$a = -2c^2 \nabla (\varphi + 2\varphi^2) + 4c^2 1_c 1_c . \nabla (\varphi + 2\varphi^2)$$

$$-c'1_c \times (\nabla \times w) + c'1_c 1_c . (1_c . \nabla)w + 2c'1_c \partial_t \varphi + \partial_t w \quad (66)$$

The first part of the expression is exactly the 1-PN photon acceleration of GRT — till order $O(\nabla \varphi^3)$ — in the statical configuration ([65], Eq 9.2.6 and Eq 9.2.7). What about the correspondence with the sweep velocity dependent terms?

In order to compare the full correspondence with the PN expression of GRT for photon acceleration in general, with $w$, we calculate — following Weinberg’s approach — the post-newtonian development ([65], eq. (9.2.6)) by including in the acceleration equation ([65], eq. (9.1.2)) the higher order terms ([65], eq. (9.1.68), eq. (9.1.69), eq. (9.1.71), eq. (9.1.73)):

$$\frac{d u}{dt} = -(1 + u^2) \nabla \phi + 4u(u. \nabla)\phi$$

$$- (u. \nabla)\zeta + u.(\nabla)\zeta - uu.(u. \nabla)\zeta + 3u \partial_t \phi - uu^2 \partial_t \phi - \nabla (2\phi^2 + \psi) - \partial_t \zeta \quad \text{(GRT)} \quad (67)$$

In Weinberg’s notation; $c = 1$, the photon velocity is $u$, and has amplitude $|u| = 1 + 2\phi + O(\tilde{v}^3)$. It can be easily shown that for $w = \lambda u \varphi$ (our notation) and $\zeta = \alpha v \psi$ (Weinberg’s notation, $\alpha \equiv 4$), both expressions ([66] and [67]) precisely correspond, e.g. for the rotation term of $w$:

$$c \times (\nabla \times w) = + \lambda u (c. \nabla)\varphi - \lambda u . c \nabla \varphi \quad \text{(LP term)} \quad (68)$$

$$- (u. \nabla)\zeta + u.(\nabla)\zeta = - \alpha v (u. \nabla)\phi + \alpha v . u \nabla \phi \quad \text{(GRT term)} \quad (69)$$
Thus $\lambda = -\alpha$, or $w = -\zeta$, must be satisfied for precise correspondence between GRT and the GMLT approach.

As mentioned in the previous subsection the requirement of the validity of the Poincaré Principle of Relativity, by fixing the amplitude $\lambda$ of the sweep velocity will be shown to assure the 1-PN correspondence with GRT, for particles and photons alike.

As a final note on the photon dynamics, we mention that we obtain—as at order $O(w)$— using the time derivative of the photon Hamilton equation (59)\textsuperscript{a,b}, expanded time derivative of $w$ (65) and, momentum and velocity relation (62), the unit acceleration vector obeys;

$$\ddot{1}_c = - (\nabla c)_{\perp 1_c} + \frac{1}{c} (\partial_t w)_{\perp 1_c} - 1_c \times (\nabla \times w),$$

consistent with its orthogonality requirement $1_c.\dot{1}_c = 0$.

4  Affected perspective dynamics and the Poincaré Principle.

Let an affected observer at rest relative to a \textit{uniformly} moving gravitational source describe the gravitational mechanics of a test body in this field. This description should \textit{not} involve the common velocity —of source and observer— relative to a preferred frame. This property in the context of relativistic movement was set forward by Poincaré as the Principle of Relativity of movement; it can not be observed whether one is moving, or not, \textit{uniformly} relative to a preferred frame. This property in the context of relativistic movement was set forward by Poincaré as the Principle of Relativity of movement; it can not be observed whether one is moving, or not, \textit{uniformly} relative to a preferred frame. [48] considered it one of the main fundamental principles of physics. Note that a \textit{non}-uniform motion of a source should of course be detectable—as in GRT for rotation and linear acceleration [60] [61] [33] [38]. [32] has analysed the meaning of the Relativity Principle prior to 1905 and, [63], sec. 3.8, summarizes shortly its context. In practice Poincaré’s approach to the Relativity Principle is mainly referenced to in philosophy and history of physics, some works by Sjödin and Ivert applied the principle in [58] [56] in transformation modeling.

In order to verify the absence of preferred frame effects, the mechanics of a test body should be expressed in terms of an affected observer “locked at a fixed distance” to the moving source. Thus after transformation from observer $S_w$ to the affected observer $S'$, at rest relative to the moving source, all kinematical reference to a preferred frame —as $v$ and $w(r)$ in particle acceleration eq. (58)—should disappear from the mechanical description of the test body. The invisibility of a preferred frame for this observer $S'$ is now settled by $\lambda$-adjustment in the field equation, eq. (25), of the sweep velocity of the source. For the present purpose we must thus suppose the source is not accelerating relative to $S_w$; we set $\dot{u} = 0$ in eq. (20). Since $S'$ is at rest relative to the source, the frame velocity of $S'$ is equal to the source velocity relative to $S_w$, both are $u$. The acceleration $a'$ of a particle relative to $S'$, should not depend on $w$ anymore eq. (58), for to $S'$ the seemingly static source should not be attributed a sweep velocity field.

In the particle mechanics description by $S_w$, the mass $m$, the momentum $p$ and $\gamma$-factor are dependent on $v_0$, eqs. (58) [63] [22]. In order to have this dynamical dependency in $\gamma$, it is required to use the GMLT with effective frame velocity $u - w$, i.e. $u_0$. It is thus the $S_0$ to $S'$ GMLT which assures the kinematic affecting and for $\delta x' = 0 \rightarrow \delta x/\delta t = u_0$, we have $\delta x = \delta x_0$ and $\delta t = \delta t_0$. The space time GMLT between $S'$ and $S_0$ is:

$$\delta x' = ((\delta x_0 || - u_0 \delta t_0) \gamma_0 (u_0) + \delta x_0 \perp) \frac{1}{\Phi(x)}$$

$$\delta t' = (\delta t_0 - \frac{u_0 \delta x_0}{c_0^2}) \gamma_0 (u_0) \Phi(x)$$

with $\gamma_0 = (1 - u_0^2/c_0^2)^{-1/2}$ —for $S_0'$; $c_0 \equiv c$— precisely as $\gamma$ in eq. (13).

The relation of $S'$ with $S_w$ must then be obtained by composition with the Galilean relation.
The velocity relation obtained from the $S_0$ to $S'$ space and time GMLT —relation (10) but with effective velocities— is now;

$$v' = v_0 - u_0 + \left(\gamma^{-1} - 1\right)v_{0\perp} \frac{1}{1 - u_0v_0c_0^{-2}} \frac{1}{\Phi(x)^2}$$  \hspace{1cm} (73)

where the orthogonality is relative to $u_0$. The link with the velocity $v$ is provided by Galilean $O(w)$ relation between $S_w$ and $S_0$;

$$v_0 = v - w$$  \hspace{1cm} (74)

These velocity relations will be used subsequently to establish relations between observed accelerations in $S'$ and $S_w$. (We recall that the velocity and acceleration $v$ and $a$, as in (58, 66) are relative to $S_w$, the index $w$ on these quantities has been dropped for clarity.)

4.1 Relativistic acceleration transformations.

The acceleration of particles or photons in curved space-time is seldom used in GRT because it depends on the chosen metric of the coordinate system. However a transformation of acceleration is well known for Fermi and Schwarzschild coordinates. A number of specific acceleration transformation laws in GRT were described by Mishra and Rindler [54, 43] and more generally using generic coordinates by [5] and, [12] for static observers (a coordinate-free space-time decomposition of a covariant expression):

$$a = g - v(v.g)$$  \hspace{1cm} (75)

where $a$ is the general local 3-proper-acceleration of a particle in a static gravitational field and $g$ is this same acceleration but with the “physical” relative velocity $v = 0$. Following the choice of adapted coordinates in which to transform the acceleration (e.g. Fermi coordinates) this equation amounts to a kinematic decomposition of the observed gravitational acceleration. The specific case of the description of “physical” acceleration relative to a static observer in the Schwarzschild metric has been developed in the literature as well. For the (local) affected radial acceleration observed in a static frame $S'$, McGruder gives in GRT [42]:

$$a_t = g \left(v_R^2 - v_t^2 - 1\right) + O(r^{-3})$$  \hspace{1cm} (76)

with $v_R$ the affected radial velocity and $v_t$ the affected transversal velocity (here $g = c^2\kappa/r^2$).

In our model, the particular appearance of the scaling function $\Phi$ in the space-time GMLT [71, 72] requires comparison to GRT-descriptions in isotropic Schwarzschild coordinates. We will therefore only need to find correspondence with the affected radial acceleration, eq. (76).

We recall that in GRT the acceleration of a test particle in a local inertial free frame is obtained by a coordinate transformation of the covariant derivative of the velocity; this renders the metric connections null and the local relative acceleration vanishes [7].

At present we verify the local acceleration expressions $i)$ for the static case, as in eq. (76) and, $ii)$ for the case of the observer tracking the uniformly moving source as requied for checking the Poincaré Relativity principle.

In these cases —with $\dot{u} = 0$— the acceleration transformation is obtained by applying the standard time derivation to the velocity transformation (73) [8].

The acceleration transformation with $\dot{u} = 0$ and with frame acceleration attributed by $S_0$;
\[ \dot{u}_0 = -\dot{w}, \] is then given by:

\[ a' = \left(1 - \frac{u_0 \cdot v_0}{c_0^2}\right)^{-2} \frac{1}{\gamma_0 c_0^2} \left\{ a_{0||} + \frac{1}{\gamma_0} a_{0\perp} + v' \frac{\Phi^2}{c_0^2} u_0 \cdot a_0 - 2v' \frac{\Phi^2}{c_0^2} \dot{\varphi}_0\right\} \]

\[ -2v' \Phi^2 \left(1 + \frac{u_0 \cdot v_0}{c_0^2}\right) \dot{\varphi}_0 + 2v_0 \cdot \gamma_0 \frac{u_0^2}{c_0^2} \dot{\varphi}_0 \]

\[ -\dot{w} + \frac{1}{\gamma_0 - 1} \left( \frac{\dot{\varphi} \times v_0}{u_0^2} + \frac{(u_0 \times v_0) \times \dot{w}}{u_0^2} - 2 \frac{\dot{w} \cdot u_0}{u_0^2} v_{0\perp}\right) \]

\[ -v_{0\perp} \frac{\dot{w} \cdot u_0}{c_0^2} + v' \frac{\Phi^2}{c_0^2} \dot{w} \cdot v_0 \right\} \]

Which is complemented with the Galilean relation between \( S_0 \) and \( S_w \):

\[ a_0 = a - \dot{w} \]

Taking into account the validity of order \( O(w) \) the composition of both transformation, with the use of the velocity decomposition (74), gives for \( a' \):

\[ a' = \frac{1}{\Phi^2} \left\{ \left(1 + 2\frac{u \cdot v}{c^2}\right) a + v' \frac{\Phi^2}{c^2} u \cdot a - 2v' \Phi^2 \dot{\varphi}\right\} \]

notice that no supplementary introduction of \( w \)-terms occurs at this order. In order to express the component with \( \dot{\varphi}_0 \equiv \partial_{t_0} + v_0 \cdot \nabla_0 \varphi \) in terms of \( S' \), the gradient transformation eq. (11) must be used and, the relativistic first-order approximated velocity relation \( S_0 \) to \( S' \);

\[ v_0 \approx v' \left(1 + \frac{u' \cdot v'}{c'^2}\right) \Phi^2 - u' \Phi^2 \]

leading to the expression \( \dot{\varphi}_0 \) with first relativistic correction;

\[ \dot{\varphi}_0 \approx \Phi \left(1 + \frac{u' \cdot v'}{c'^2}\right) \left(\partial_{t'} \varphi + v' \cdot \nabla' \varphi\right) \]

In order to obtain the full expression of \( a' \), eq. (79), in \( S' \), the velocity relations approximated to relativistic first-order between \( S' \) and \( S_w \) must be used:

\[ v' \approx \frac{1}{\Phi^2} \left(v \left(1 + \frac{u \cdot v}{c^2}\right) - u\right) + O(v^2/c^2 u) \]

while for the photon case an additional appearance of \( w \) needs to be retained:

\[ c' \approx \frac{1}{\Phi^2} \left(c \left(1 + \frac{(u - w) \cdot c}{c^2}\right) - u\right) + O(u) \]

Finally we can insert the acceleration expressions which were obtained form the Hamiltonian mechanics in the perspective of \( S_w \). For easy reference, we apply the transformation first in the case of a static source.

### 4.2 Static source — Endorsement of the acceleration transformation

The particle acceleration (58) in \( S_w \), now with \( w = 0 \), can be transformed to the fixed observer \( S' \). The acceleration transformation, eq. (79), \( u' = 0, w = 0 \), and gradient-GMLT eq. (11) gives in the affected observer’s perspective to first Post-Newtonian order:

\[ a' = -c'^2 \nabla' \varphi - v'^2 \nabla' \varphi + 2v' \nabla' \varphi \cdot \nabla' \varphi \]
Similarly, the photon acceleration in affected and unaffected perspective are related by the acceleration transformation, eq. (79), with \( u' = 0 \), \( v' = c' \), \( w = 0 \). In the affected perspective of \( S' \) the photon acceleration (66) becomes to first Post-Newtonian order:

\[
a' = 2c'c'.\nabla'\varphi - 2c'^2 \nabla'\varphi = 2c' \times (c' \times \nabla'\varphi)
\]  

(85)

and

\[
1'_{c'} = 2c' (1_{c'} 1_{c'}.\nabla'\varphi - \nabla'\varphi) = \frac{a'}{c'}
\]  

(86)

endorsing the view that in the affected perspective of \( S' \) light curves but does not change velocity amplitude in a gravitational field.

In order to compare the L-P acceleration with the GRT term, eq. (76), we specify radial and lateral velocity \( v = v_r + v_l \), and set \( g' = -c'^2 \nabla'\varphi \), in the acceleration \( a'_r \), eq. (84);

\[
a' = (1 + v_r^2/c^2 + v_l^2/c^2)g' - 2(v'_r + v'_l)(v'_r + v'_l)g'/c^2
\]  

(87)

With \( v'_l g'/c^2 = 0 \) we have the lateral and radial components of the affected acceleration

\[
a'_r = (1 - v_r^2/c^2 + v_l^2/c^2)g'
\]

(88)

\[
a'_l = -2v'_l v'_r g'/c^2
\]

(89)

Which endorses the radial acceleration expression, eq. (76), of GRT in Schwarzschild metric.

4.3 Kinematic source — Implementation of the Poincaré Principle of Relativity

In the configuration required for checking Poincaré’s Principle of Relativity, the source moves by with a velocity \( u \) relative to \( S_w \). Thus to \( S_w \) the field \( \Phi \) varies in time; following the time-independent field equation, eq. (21), \( \Phi \) changes by uniform translation with the source at velocity \( u, \varphi = \varphi(r_i - u_w(t_w - t_{w_i})) \). The observer \( S' \) is moving with frame velocity \( u \) along with the source at a distance. Therefor the observer \( S'_w \) remains in a fixed position relative to the source, and thus to its field \( \varphi \):

\[
\partial_t \varphi = \gamma (u_0) \frac{1}{\Phi} (\partial_{t_w} + u_w \nabla_w) \varphi(r_i - u_w(t_w - t_{w_i})) = 0
\]  

(90)

where we used the gradient transformations, eqs. (111 30), and velocity relations of \( S', S_0 \) and \( S_w \) to first Post-Newtonian order.

— Particle dynamics

The particle acceleration \( a' \), in the perspective of \( S' \) fixed relative to the moving source, is obtained using acceleration transformation, Eqs. (79), with \( a \), eq. (58), reformulated in terms of \( S' \), using eqs. (82 81 26). The acceleration \( a' \) to first Post-Newtonian order is given by:

\[
a' = -c'^2 \nabla'(\varphi + 2\varphi^2) - v'^2 \nabla'\varphi + 2v'v'.\nabla'\varphi
\]

\[+(4 + \lambda) (u'.v'\nabla'\varphi - u'v'.\nabla'\varphi) + O(u^2, uv, v^2/c^2)
\]  

(91)

where the time-independence of \( \varphi \), eq. (90), relative to the particular observer \( S' \), was used.

All reference to movement relative to the ‘preferred’ frame should disappear: the terms containing the frame velocity \( u' \) should vanish. The Poincaré Principle thus requires;

\[
\lambda = -4
\]  

(92)
The affected observer $S'$ then attributes to the particle the acceleration as if in the stationary case eq. (84). With the Poincaré Principle implemented, the sweep velocity is “normalized” at $w = -4u\varphi = 4u\kappa/r$. The 1-PN acceleration expressions of the L-P model, eqs. (58, 66), now correspond exactly with the acceleration expressions of GRT, because $w$ formally and quantitatively satisfies the same field equation (25) as the one for the “vector potential” $\zeta$ of GRT (65, Eq 9.1.161).

— Photon dynamics
In the affected perspective of $S'$, uniformly co-moving with the source, we should retrieve the static description, eq. (85), of the photon acceleration. The acceleration transformation (79) will render the photon acceleration $a'$ in the perspective of $S'$. With the time-independence of $\varphi$ relative to $S'$, eq. (90), and the $S_w$ acceleration $a$ (66) for photons in terms of $S'$, using (83, 81, 26), $a'$ is given by:

$$a' = -2c'2\nabla'\varphi + 2c'c'.\nabla'\varphi + (\lambda + 4) (u'c'.\nabla'\varphi - c'u'.\nabla'\varphi) + O(u^2)$$

(93)

Thus, consistent with the previous particle case, again the normalization $\lambda = -4$ leads to the elimination of the $u'$-terms with reference to the preferred frame.

We conclude that the implementation of the Poincaré Principle of Relativity, by eq. (92), establishes, till order $O(u^2)$, and both for particles and photons, the equivalence of the GMLT-based description and GRT.

5 Interpretation and conclusions
In terms of interpretation the presented gravitation model has the advantage of retaining a classical ontology of Hamiltonian mechanics with a gravitational field in a flat-metric space. While the construction of the model requires a number of consistent hypotheses —GMLT, VSL, form-invariance, PPR— their nature is proper to a Lorentz-Poincaré approach to gravitation.

Two supplementary examples of geometric features interpreted physically in terms of the present scalar–vector model show the interpretational merits of our model.

Ex. 1) The derivation of PN expressions in GRT requires the posing of the harmonic coordinate conditions in order to constrain the degrees of liberty of the GRT field equations. The time-component of the harmonic coordinate condition (in weak field approximation, $O(\nabla\varphi^2)$) (65, eq. 9.1.66) in $S_0$ perspective becomes, in view of the relation between $\zeta$ and $w$:

$$4\frac{\partial \varphi}{\partial t} + \nabla \zeta = 0 \quad \rightarrow \quad 4\dot{\varphi} \bigg|_{v=u} = 0 \quad \rightarrow \quad \partial_t' \varphi = 0$$

(94)

The harmonic coordinate condition expresses precisely that the observer $S'$, with frame-velocity $u$, is at rest relative to the moving source. Or, taking into account the transformation for gradient operators (11), the time-component of the harmonic coordinate condition is satisfied in the affected perspective when $\partial_t' \varphi = 0$.

Ex. 2) The Lense-Thirring effect [33], or ‘frame dragging’, of an orbiting system due a rotating source is interpreted in GRT as a local deformation of the space-time continuum. In the present model the equivalent effect is easily obtained. Let $\rho = \rho(r')$ and $\Omega'$ fixed, $r > R$, and $J_s$ be the angular momentum of the source, then the resultant sweep velocity field according (25) (cfr [65], eq. 9.5.18):

$$w = -4\frac{G'}{2c'2} \frac{r}{r^3} \times J_s, \quad J_s = \frac{8\pi}{3} \int_R \Omega' \rho(r') r'^4 dr'$$

(95)
With orbital momentum $\mathbf{L} \equiv \mathbf{r} \times \mathbf{p}$, the Hamiltonian energy expression \[H \equiv (m_0^2 c^4 + p^2 c^2)^{1/2} + 4 \frac{G' c^2 J_s \cdot \mathbf{L}}{r^3}, \] can be re-expressed with coupling of source angular momentum $J_s$ and orbital momentum of testbody $\mathbf{L}$ till $O(J_s^2)$:

$$H = (m_0^2 c^4 + p^2 c^2)^{1/2} + 4 \frac{G' c^2 J_s \cdot \mathbf{L}}{r^3} \quad (96)$$

Straightforward development of the Hamilton-Lagrange principle then leads to the first acceleration correction $a_1$ of GRT at $O(v \nabla \varphi J_s/r^2, vv^2/c^2 J_s/r^3)$:

$$a_1 = 4 \frac{G'}{2c'^2 r^3} (v \times J_s - 3v \times J_{sr}), \quad J_{sr} \equiv J_s \cdot 1_r 1_r$$

These terms precisely cause the ‘frame dragging’ of the particle frame in geometrodynamic interpretation, while in the present scalar–vector model the effect is merely caused by the different relative velocities with respect to the parts of the rotating source. Each part of the source therefore exerts a different kinematical $\gamma$-contraction and time dilation on the particle. These examples and the description of particle and photon dynamics show that the interpretation of the gravitational effects extends conservatively the Lorentz-Poincaré interpretation of SRT.

We recall that we used Poincaré’s principle of Geometric Conventionalism as a fundamental premiss to develop this model. This principle demands that the gravitational phenomenology of GRT should be reproduced regardless the choice of the fundamental metric. The main result of this present work is that we realized this till the first Post-Newtonian order of GRT. Whether the Poincaré Principle of Relativity can be successfully implemented beyond $O(u^2)$ needs to be studied in later work: is the mutual elimination of terms at higher order rigorous and absolute, as was hypothetically advanced by Poincaré? (\[48\], p. 183) In the present work the model ignored gravitation propagation, source acceleration, etc. Some of these matters will be covered in upcoming work.

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Appendix: The constrained differential nature of the space and time GMLT

Since GRT is a differential geometric theory of gravitation, the space time GMLT transformations, eqs. \[57\] b, beg the comparison with a differential diffeomorphic relation connecting $S_0$ and $S'$. The Lorentz-Poincaré type approach to gravitation however clearly specifies the Gravitationally Modified Lorentz Transformations to intervals $\delta x'$ which are gravitationally scaled infinitesimal intervals constrained by conditions for observation by $S'$.

The complete differential of the diffeomorphism $x = x(x')$ relating the coordinate systems of $S_0$ and $S'$ gives:

$$dx_i = \frac{\partial x_i}{\partial x'_j} dx'_j + \frac{\partial x}{\partial t'} dt', \quad dt = \frac{\partial t}{\partial x'_j} dx'_j + \frac{\partial t}{\partial t'} dt' \quad (1)$$
which for consistency requires commutativity of the second derivatives ($\mu = 0, \ldots, 3$);
\[
\frac{\partial^2 x_\mu}{\partial x_\alpha \partial x_\gamma} = \frac{\partial^2 x_\mu}{\partial x'_\alpha \partial x'_\gamma}.
\] (2)

The space and time GMLT, eqs. (7) b, can be written as:

\[
\left( \begin{array}{c} \delta t \\ \delta \mathbf{x} \end{array} \right) = \Lambda^{-1}(\mathbf{u}, \mathbf{r}) \left( \begin{array}{c} \delta t' \\ \delta \mathbf{x}' \end{array} \right), \quad \Lambda^{-1}(\mathbf{u}, \mathbf{r}) \equiv \left( \begin{array}{cc} \gamma \Phi^{-1} & -\mathbf{u'}c\gamma \Phi^{-1} \\ -\mathbf{u'}\gamma \Phi & 1\Phi + \frac{\mathbf{u'}\mathbf{u'}c^2}{u^2} (\gamma - 1)\Phi \end{array} \right)
\] (3)

or in factorized form;

\[
\left( \begin{array}{c} \delta t \\ \delta \mathbf{x} \end{array} \right) = \left( \begin{array}{cc} \Phi^{-1} & 0 \\ 0 & \Phi \end{array} \right) \Lambda^{-1}(-\mathbf{u'}) \left( \begin{array}{c} \delta t' \\ d\mathbf{x}' \end{array} \right) = \left( \begin{array}{cc} \Phi^{-1} & 0 \\ 0 & \Phi \end{array} \right) \left( \begin{array}{c} \delta \tau' \\ d\xi' \end{array} \right)
\] (4)

where the coordinates ($\delta \tau', \delta \xi'$) were defined as $\Lambda(u')(\delta t', \delta \mathbf{x}')$ and $\Lambda(u')$ is a standard Lorentz Transformation, with $\Lambda^{-1}(-\mathbf{u'}) = \Lambda(\mathbf{u'})$.

We can now concisely write the GMLT as;

\[
\delta t = \Phi^{-1}\delta \tau', \quad \delta \mathbf{x} = \Phi \delta \xi'
\] (5)

which simplifies the discussion; i.e. as if the affected observer was in relative rest to $S_0$.

The gravitational field is claimed to affect an infinitesimal ruler as a local isotropical scaling, and the periods of clocks are also locally dilated. The local spatial measurements of $S'$ and $S_0$ are therefore related by a local scaling transformation. We let the observer $S'$ and $S_0$ relate to Cartesian coordinates. An observation of an infinitesimal spatial interval $\delta x'$ in direction $1_{x'}$, spanned by a material ruler, will extend over a different length along $1_x$ of $S_0$ and because of the scaling effect no other orthogonal interval in $S_0$ is covered by this ruler: $\delta y = 0$ and $\delta z = 0$.

Moreover the measurement is done by simultaneously marking the ends of the infinitesimal ruler, leading to $\delta t = 0$. For the orthogonal rulers, $\delta y'$ and $\delta z'$, similar constraints are imposed. For the observation of an infinitesimal time interval $\delta t'$ the clock is supposed to be stationary, i.e. with no displacement occurring: $\delta x = \delta y = \delta z = 0$.

We compare these intervals to similar observations in GRT in local coordinates of the observer using;

\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu
\] (6)

with similar constraints we have (we use primed symbols for the local coordinates);

\[
d\tau'|_{dx_i=0} = (g_{00})^{1/2} dx^0, \quad d\xi_i'|_{dx_{j\neq i}=0, dr=0} = (g_{ii})^{1/2} dx^i
\] (7)

From which we obtain the relation between the GMLT intervals and the constrained differentials of GRT in the limit of $\delta \tau', \delta \xi_i' \to 0$;

\[
\delta \tau' = d\tau'|_{dx_i=0}, \quad \delta \xi_i' = d\xi_i'|_{dx_{j\neq i}=0, dr=0}
\] (8)

The GMLT does therefor not correspond to the differential of the diffeomophism relating the coordinate systems of $S_0$ and $S'$. That relation of course exists, but is not deployed in the formalism. Only a constrained form of the differentials are used. The constrained form of the differential evidently is not required to satisfy the full relation (1) and commutation of the second derivatives (2).

Finally, the gravitational field is claimed to affect the material constituents so that the ruler is isotropically scaled, $(g_{ii})^{1/2} = \Phi$. The assumption of isotropy is not stringent, but we have
shown that in this simpler model —in terms of symmetry— such a constraint on the scaling is in concordance with 1-PN mechanics in GRT. Moreover in that restricted case we have till that order successfully used Φ = (g_{00})^{-1/2}. It is however possible to generalize the GMLT, eq. 5, with higher order distinct effects on space and time:

\[ \delta t = \Omega^{-1} \delta t' , \quad \delta x = \Phi \delta \xi' \quad (9) \]

This type of generalization —or anisotropic variants— could be invoked to fit higher order PN terms.

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\[ c = \frac{v_n}{n} \] (where \( n = -g_{44}^{-1/2} \)); Longair [35], p. 453, eq. 17.62, “the apparent variability of the speed of light in the radial direction according an observer at infinity... in terms of coordinate time \( t \) and the distance measure \( r \) is”\
\[ c(r) = \frac{dr}{dt} = c(1 - 2GM/rc^2) \]; Kenyon [31], p. 95, eq. 8.15 and next, “The tangential and radial coordinate velocities are obtained...”\
\[ \frac{rd\varphi}{dt} = c\sqrt{Z} \] and \[ \frac{dr}{dt} = cZ \] “showing that as light approaches the origin its coordinate velocity falls.” (here \( Z = 1 - 2GM/rc^2 \) and \( \varphi \) is the angular variable of the orbital); Will [67], p. 144, eqs. 6.14, 615, “... post-Newtonian equations for the deviation \( x_p \) of the photon’s path from uniform, straight line motion”\
\[ \frac{d^2x_p}{dt^2} = (1 + \gamma)[\nabla U - 2n(\nabla U)] \] and \[ n.dx_p/dt = -(1 + \gamma)\nabla U \] (where \( \gamma = 1 \) in GRT and \( U \) is minus the Newtonian potential.); Weinberg [65], p. 222, eq. 9.2.5, “... note that the photon speed is ...”\
\[ |u| = 1 + 2\varphi + O(v^3) \]; Eddington [23], p. 93, eq. 43.4, “At a distance \( r_1 \) from the origin the velocity of light is accordingly”\
\[ (1 - m/2r_1)/(1 + m/2r_1)^3 \]; Moeller [41], pp. 239-240, eqs. 69, 69’, 70, “...we see that the velocity of light \( w \) depends on the direction of propagation \( n \) of the signal if \( \gamma_i \neq 0 \) in the system of coordinates considered.....”\
\[ w(n^i) = c\sqrt{-g_{44}/(\gamma_in^i + 1)} \] (where \( \gamma_i \equiv g_{44}/\sqrt{-g_{44}} \)); Einstein [26], p. 93, eq. 107, “...velocity of light \( L \) is...”\
\[ \sqrt{dx^2_i/dt} = 1 - \frac{k}{c^2} \int \sigma/rdV_0 \]; Eddington [22], p. 107 and Chapter VI. “...[an] alternative way of viewing this effect on light...[the] velocity of light in the gravitational field is not a constant...[however] if he performed Fizeaus experiment the velocity of light would be exactly the same as that of a terrestrial observer... . It is the coordinate velocity that is here referred to...” and “for light ... in radial propagation” \( (dr/dt)^2 = \gamma^2 \) “... in transversal propagation” \( (d\theta/dt)^2 = \gamma \) (where \( \gamma = 1 - 2m/r \)); Einstein [25], p. 906, Eq 3, \( c = c_0(1 + \Phi/c^2) \) “The principle of constancy of of the velocity of light holds good according to this theory on a different form from that which usually underlies the ordinary theory of relativity” (prior to establishing GRT in 1915 Einstein derived half the value of the coordinate velocity of light; leading to half the deflection angle); Einstein [24], following Eq 32 b , p 461, “...[Eq 32 b], hier tritt aber an die Stelle von \( c \) der Wert” \( c(1 + \gamma\xi/c^2) = c(1 + \Phi/c^2) \) (where \( \Phi \) is the gravitational potential, \( \gamma \) is the acceleration —“beschleunigung”— and \( \xi \) the coordinate of translation in the accelerated system.)