Quantum control of atomic systems by time-resolved homodyne detection of spontaneous emission

Holger F. Hofmann and Ortwin Hess
Institut für Technische Physik, DLR, Pfaffenwaldring 38-40, 70569 Stuttgart, Germany

Günter Mahler
Institut für Theoretische Physik und Synergetik, Pfaffenwaldring 57, 70550 Stuttgart, Germany

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Abstract
We describe the light-matter interaction of a single two level atom with the electromagnetic vacuum in terms of field and dipole variables by considering homodyne detection of the emitted fields. Spontaneous emission is then observed as a continuous fluctuating force acting on the atomic dipole. The effect of this force may be compensated and even reversed by feedback.

1 Introduction
The spontaneous emission of light from a single atom is usually described as the random appearance of a photon in the vacuum light field. However, this description is only valid if photons are actually detected. The field variables continuously evolve from the dipole dynamics of the atom according to Maxwells equations. If field variables are measured, the spontaneous emission of a single two level atom may be interpreted as the interaction of a fluctuating dipole with a noisy light field [1, 2]. Quantum jumps are avoided and the continuous evolution of the atomic system may be controlled by weak coherent feedback fields compensating the observed quantum noise. In the following we describe the evolution of the quantum state of a two level atom conditioned by projective homodyne detection and discuss some of the possible feedback scenarios.
2 Homodyne detection of weak fields

In a balanced homodyne detection setup the coherent laser field of a local oscillator interferes with the low intensity input field at a beamsplitter as shown in Figure 1. The difference between the photon numbers registered in detector 1 and detector 2 corresponds to the interference term

$$\Delta \hat{n} = \hat{a}^{\dagger} \hat{b} + \hat{a} \hat{b}^{\dagger}.$$  \hspace{1cm} (1)

where $\hat{a}$ and $\hat{b}$ represent the annihilation operators of the local oscillator mode and the source field, respectively. The field modes are emitted during the measurement time interval $\tau$ and represent wave packets of length $c\tau$. The quantum state emitted by the local oscillator may be represented by a coherent state with an average complex amplitude of $\alpha$. The photon number difference $\Delta n$ then corresponds to $2 \mid \alpha \mid$ times the quadrature component of the source field which is in phase with the local oscillator. For weak fields, the probability distribution of the measurement results $\Delta n$ is approximately given by the vacuum fluctuations of the observed quadrature component,

$$p(\Delta n) \approx \frac{\exp\left[-\frac{\Delta n^2}{2|\alpha|^2}\right]}{\sqrt{2\pi |\alpha|^2}}.$$  \hspace{1cm} (2)

Within the measurement time interval $\tau$ the dipole fluctuations of a two level atom with a spontaneous emission rate of $\Gamma$ emit an average light field energy of
\( \Gamma \tau \) times the quantum fluctuation intensity of \( \hbar \omega / 2 \). If \( \Gamma \tau \) is much smaller than one, the dipole radiation emitted by the atom is much weaker than the quantum fluctuations of the electromagnetic vacuum. Therefore, the dipole radiation is obscured by quantum noise and the information about the state of the atomic system obtained in the homodyne detection measurement is extremely small. Nevertheless some information is obtained about the most likely orientation of the atomic dipole and this observation will modify the quantum state of the system as explained below.

3 Quantum diffusion of a two level atom

The back action of continuous time-resolved homodyne detection on a quantum system results in a stochastic evolution of the wave function equivalent to a Monte Carlo wavefunction formalism \([3, 4]\). The derivation from a projective measurement base is discussed in \([5]\). It is convenient to describe the back action in terms of the Bloch vector \( \mathbf{s} \) of the atomic two level system, where \( s_x \) is the expectation value of the observed dipole component and \( s_z \) is the expectation value of the atomic inversion. The \( s_y \) component describes the expectation value of the unobserved dipole component. The back action corresponding to a measurement result of \( \Delta n \) for an arbitrary initial Bloch vector \( (s_x, 0, s_z) \) in the \( s_y = 0 \) plane reads

\[
\delta s_x = \sqrt{\Gamma \tau} \frac{\Delta n}{|\alpha|} (1 + s_z) s_z \\
\delta s_z = - \sqrt{\Gamma \tau} \frac{\Delta n}{|\alpha|} (1 + s_z) s_x.
\]

(3)

The Bloch vector is thus rotated by an angle of \( \sqrt{\Gamma \tau} \frac{\Delta n}{|\alpha|} (1 + s_z) \) around the y-axis in response to the homodyne detection measurement.

4 Controlling the quantum state by feedback

Without feedback, the ground state \( s_z = -1 \) is stationary while diffusion is at a maximum for the excited state \( s_z = +1 \). It is possible to interpret this back action effect as a sum of Rabi rotations induced by the quantum noise and an epistemological effect of the information obtained about the dipole component \( s_x \) of the atom. The ground state is stationary because the dipole emission effects compensate the absorption of vacuum fluctuations. In the excited state the dipole emission effect and the response to the vacuum fluctuations add up and cause twice the diffusion expected from classical field fluctuations.

By applying a negative feedback equal to the observed quadrature component the Rabi rotations induced by the quantum fluctuations of the measured
field component may be compensated. The diffusive back action which remains is then associated with the information gained about the atomic system. As mentioned in the previous section, it is possible to identify this back action as a weak measurement effect of the dipole variable $s_x$. Due to the weak dipole radiation emitted by the atom, $\Delta n > 0$ is more likely for $s_x = +1$ and $\Delta n < 0$ is more likely for $s_x = -1$. Coherent superpositions of dipole eigenstates diffuse because of the modified statistical weight of the dipole eigenstate components. Consequently the dipole eigenstates are stationary while the diffusion is at a maximum for both the ground state $s_z = -1$ and the excited state $s_z = +1$.

By applying twice the negative feedback necessary for compensation it is possible to invert the effects of quantum fluctuations. The excited state $s_z = +1$ becomes stationary and diffusion is at a maximum in the ground state $s_z = -1$. By inverting the sign of the observed quantum fluctuation component the roles of the excited state and the ground state are exchanged. The excited state now absorbs quantum fluctuations, thus compensating the effects of dipole emission, while the ground state amplifies the fluctuations and thus shows twice the average diffusion.

The back action effect of homodyne detection without feedback, with feedback compensating the quantum fluctuations and with feedback inverting the quantum fluctuations is shown in figure 2.
5 Conclusions

The coherent and excited states of a two level atom may be stabilized by homodyne detection and negative feedback. If the dipole eigenstates are stabilized the back action of homodyne detection corresponds to weak measurements of the dipole component which emits light in phase with the local oscillator. The irreversible nature of spontaneous emission is thus associated with a weak projective measurement of the atomic dipole.

Homodyne detection avoids the discontinuous quantum jumps associated with photon detection. Therefore the stabilization of quantum states does not require short time pulses of high intensity. The feedback amplitudes needed for the stabilization of quantum states by homodyne detection and feedback are of the same order of magnitude as the observed vacuum fluctuations. It may thus be sufficient to couple the local oscillator field to the atomic system as a function of the intensity difference $\Delta n$ by an optical nonlinearity.

References

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