MAI statistics estimation and analysis in a DS-CDMA system

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Abstract. A primary limitation of Direct Sequence Code Division Multiple Access DS-CDMA link performance and system capacity is multiple access interference (MAI). To examine the performance of CDMA systems in the presence of MAI, i.e., in a multiuser environment, several works assumed that the interference can be approximated by a Gaussian random variable. In this paper, we first develop a new and simple approach to characterize the MAI in a multiuser system. In addition to statistically quantifying the MAI power, the paper also proposes a statistical model for both variance and mean of the MAI for synchronous and asynchronous CDMA transmission. We show that the MAI probability density function (PDF) is Gaussian for the equal-received-energy case and validate it by computer simulations.

1. Introduction

The Direct Sequence Code Division Multiple Access (DS-CDMA) scheme is robust to frequency selective fading, offers the advantages of interferer diversity and flexibility and has been successfully introduced in commercial cellular mobile communications systems such as (3G, 4G, LTE and LTE-Advanced) as the key technology to improve the spectrum efficiency and flexible user resource allocation [1-3]. However, it suffers from multiple access interference (MAI) caused by the non-orthogonality of spreading codes, particularly for heavily loaded systems [4].

In CDMA systems, direct-sequence spread-spectrum (DS-SS) signals must be power controlled to mitigate multiuser interference access or (MAI), and to properly perform this, signal strengths, or signal “quality,” must be estimated. Also, for call handoff between cells in cellular systems, and for rapid performance estimation, signal strength-quality estimation is essential [5] even for more advanced reception techniques such as multiuser detection [6-8] and often require signal quality estimation for proper operation.

Thus, multiple access interference (MAI) is a factor that limits the capacity and performance of DS-CDMA systems [9]. MAI refers to the interference between direct-sequence users. This interference is the result of the random time offsets between signals, which make it impossible to design the code waveforms to be completely orthogonal. While the MAI caused by any one user is generally small, as the number of interferers or their power increases, MAI becomes substantial [10]. Theoretical tools for evaluating the performance in terms of bit error rate (BER) are important in simplifying the system design and deployment tasks. In the recent past, such theoretical evaluations of the BER of various systems have been reported under different conditions.
Single user in AWGN channel was considered in [11] and [12]. Under these conditions, the problem is straightforward and the BER can be represented by the Q-function (or the Gaussian tail probability function) exactly. System performance in AWGN channels considering MAI was addressed in [12-15], where the MAI is modeled as a Gaussian random variable (Gaussian Approximation (GA)). With the GA assumption, the problem was simplified and became tractable with a simple closed form solution. In multi-user multi-path fading conditions, either the GA was used in deriving the average BER [12], [16] or the performance evaluation was based entirely on Monte-Carlo simulations [17].

In this paper, we present an exact, simple and more realistic method to model the statistics of MAI for a DS-CDMA system and DS-BPSK scheme in AWGN channel. Using this model, the characteristic function of the total interference is derived. Simulations are undertaken to prove the analytical work.

The rest of this paper is organized as follows. A description of system model is presented in section 2. In section 3, the statistical model of the MAI in AWGN channel is developed where simulations are used to determine the characteristic function of the MAI.

2. System description

We here consider a DS-CDMA system consisting of K active users who employ coherent BPSK and share a flat fading multiple access channel with AWGN.

![Transmitter and receiver block diagram for DS-CDMA and BPSK modulation](image)

The transmitted signal (spread spectrum signal) of the kth user can be expressed as:

$$s_k(t) = \sqrt{2P_k} \cdot d_k(t) \cdot c_k(t) \cos(\omega_0 t)$$

(1)

Where $P_k$ denotes the kth user's transmitted signal power, $c_k(t)$ and $d_k(t)$ are the spreading code and data waveforms respectively (we assume rectangular pulses for both), $\omega_0$ is the carrier frequency. Each symbol $c_t$ (usually called a chip) has duration $T_c = T_b/N$, $T_b$ is the bit width and $N$ is the bandwidth expansion factor, or the spreading gain.

In the conventional DS-CDMA detector, and according to [18] the received signal can be modeled as:

$$r(t) = \sum_{k=1}^{K} s_k(t - \tau_k) + n(t)$$

(2)

Where $n(t)$ is an AWGN with zero mean and power spectral density $\frac{n_0}{2}$, and $\tau_k$ is the random phase offset due to propagation, $\tau_k$ represents the propagation delay for the $k^{th}$ user.

The received signal can be written in the form:

$$r(t) = \sum_{k=1}^{K} \sqrt{2P_k} \cdot d_k(t - \tau_k) \cdot c_k(t - \tau_k) \cos(\omega_0 t + \theta_k) + n(t)$$

(3)

Where $\theta_k$ is the random phase, of user k. $\tau_k$ and $\theta_k$ are assumed to be independent random variables uniformly distributed respectively over $[0,T_b]$ and $[0,2\pi]$. 

In the CCR (Fig. 2), the received data sequences are correlated with the signature code to generate correlation functions. Data sequences arriving at the decoder with unmatched signature code result in cross-correlation functions, which in turn, are considered as interference [19]. After a coherent demodulation, the correlated signal is [20]:

\[
r_{corr}(t) = r(t)c_1(t) \cos(\omega_0 t)
\] (4)

Substituting (3) into (4) gives

\[
r_{corr}(t) = c_1(t) \cos(\omega_0 t) \\
\times \left( n(t) + \sum_{k=1}^{K} \sqrt{2P_k} d_k(t - \tau_k)c_k(t - \tau_k) \cos(\omega_0 t + \theta_k) \right)
\] (5)

Which can be rewritten as:

\[
r_{corr}(t) = c_1(t) \cos(\omega_0 t) \\
\times \left( n(t) + \sqrt{2P_1} d_1(t - \tau_1)c_1(t - \tau_1) \cos(\omega_0 t + \theta_1) \right) \\
+ \sum_{k=2}^{K} \sqrt{2P_k} d_k(t - \tau_k)c_k(t - \tau_k) \cos(\omega_0 t + \theta_k)
\] (6)

Assuming perfect phase coherence, bit timing, and chip timing at the receiver (\(\theta_1 = 0\) et \(\tau_1 = 0\)), under a synchronous environment, the correlated signal becomes:

\[
r_{corr}(t) = n(t). c_1(t). \cos(\omega_0 t) + \sqrt{2P_1}. d_1(t). c_1^2(t). \cos^2(\omega_0 t) \\
+ \sum_{k=2}^{K} \sqrt{2P_k} d_k(t - \tau_k)c_k(t - \tau_k) c_1(t) \cos(\omega_0 t) \cos(\omega_0 t + \theta_k)
\] (7)

The DS-CDMA output of the matched filter for the recovery of the user 1 can be expressed as

\[
Z_i^{(1)} = \int_{0}^{T_b} r_{corr}(t) dt
\] (8)

Where \(Z_i^{(1)}\) is the decision variable for the \(i^{th}\) data bit \(d_i^{(1)}\) for user \(n = 1\).

The resulting decision variable, as shown below, comprises three components, namely, the AWGN \(V\), the desired information bit \(A\), and the MAI term \(I\) which will be analyzed in more detail in the section below.
3. MAI analysis

The total MAI incurred by the reference user is given by the sum of individual interferer MAI contributions as:

\[ I = \sum_{k=2}^{K} I_k^{(1)} \]  

The expression for the individual interferer MAI contributions were given in equation (9):

\[ I = \int_0^{T_b} \sum_{k=2}^{K} \sqrt{2P_k} \cdot d_i^{(k)}(t - \tau_k). c_k(t - \tau_k). c_i(t). \cos(\omega_0 t) \cos(\omega_0 t + \theta_k) dt \] 

This expression can be written as:

\[ I = \int_0^{T_b} \left( \sum_{k=2}^{K} \sqrt{\frac{P_k}{2}} \cdot d_i^{(k)}(t - \tau_k). c_k(t - \tau_k). c_i(t). \cos(\phi_k) \right) dt \]

\[ = \sum_{k=2}^{K} \sqrt{\frac{P_k}{2}} \int_0^{T_b} d_i^{(k)}(t - \tau_k). c_k(t - \tau_k). c_i(t). \cos(\phi_k) dt \]  

where \( \phi_k = \theta_k - \omega_0 \tau_k \)

\( \tau_k \) and \( \theta_k \) are modelised as independent random variables uniformly distributed respectively over \([0,T_b]\) and \([0,2\pi]\) so \( \cos(\phi_k) \) is independent from spreading code and data. \( E[\cos(\phi_k)] = 0 \), and \( E[\cos^2(\phi_k)] = \frac{1}{2} \). The MAI is therefore a zero-mean random variable with \( E[I] = 0 \) and the variance of this term of interference can be shown to be:

\[ \text{var}[I] = E[(I)^2] \]
is the variance of the MAI contribution from user k on the reference user 1. To model $E[(I)^2]$, several expressions have been proposed [22,23], all agree that it’s proportional to the power $P$ and the square of symbol duration $T_b$. Furthermore, for the same period sequences $N$, $I_k^{(1)^2}$ is expressed as [22]

$$I_k^{(1)^2} = \frac{P_k}{4} E \left( \left( \int_0^{T_b} d_1^{(k)}(t - \tau_k). c_k(t - \tau_k). c_1(t) dt \right)^2 \right)$$

(14)

With

$$I_k^{(1)^2} = \frac{P_k}{4} E \left( \int_0^{T_b} d_1^{(k)}(t - \tau_k). c_k(t - \tau_k). c_1(t) dt \right)^2$$

(15)

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$$I_k^{(1)^2} = \frac{N T_b^2}{6} P_k$$

(16)

Thus

$$E[(I)^2] = \frac{N T_b^2}{6} \sum_{k=2}^{K} P_k$$

(17)

Assuming that all users exhibit equal power $P_k = P_{\text{moy}}$, then:

$$E[(I)^2] = \frac{N T_b^2 P_{\text{moy}}}{6} (K - 1)$$

(18)

3.2. Simulation results

To determine the MAI properties (mean and variance), theoretical analysis are compared to simulation results which have been performed for both synchronous and asynchronous DS-CDMA systems.

3.2.1. MAI mean. We first determined, by simulation, MAI mean versus the number of users in both synchronous and asynchronous cases. For the asynchronous case, offsets between active users and desired user codes are generated randomly. The results are shown on the following Fig. 3.

![Figure 3. MAI mean versus number of active users](image-url)

The means are viewed as a random process around the value null in both the synchronous and asynchronous cases. These results are obtained whatever the value of $N$ (63, 127, 255) is.

To confirm this result, we undertake a statistical study in which we determine the frequency of appearance of the values obtained for different numbers of active users, for different lengths of the spreading sequence, for different powers (same powers for all users) and this in both the synchronous and asynchronous cases. Examples of results are presented in the graphs on Fig. 4 below.
3.2.2. MAI variance: synchronous case.

Since the variance is depending on the number of users, we have simulated this variance according to $K$. The result is shown in Fig. 5.

![Figure 5. MAI variance versus number of active synchronous users, N=63 and $P=3.17 \times 10^{-3}\text{ (ua)}$](image)

We observe a random distribution, and the variance values are very low ($10^{-36}$) for a user average power $3.17 \times 10^{-3}\text{ (AU)}$. We can see an increasing of the variance with the number of users.

To determine the type of variance, the same statistical study made previously for MAI mean is made for the MAI variance. The graphs in Fig. 6 show some examples of the simulations results.
We infer that the distributions are Gaussian, but the values are very low. Indeed, in a synchronous case, bit of each user’s data stream are temporally aligned at the receiver and thus the spreading sequences retain their desired low cross-correlation properties. As a result, the MAI incurred upon reception is minimal (if not totally prevented), hence the very negligible obtained values.

3.2.3. MAI variance: asynchronous case. In the asynchronous case, spreading codes of different users are shifted randomly compared with the desired user one. The MAI variance versus the active users number is shown in Fig. 7.

![Figure 6. MAI variance distribution for different values of P, N and K in synchronous case](image)

![Figure 7. MAI variance versus number of active asynchronous users, N=63 and P=3.17 \times 10^{-3}(ua)](image)

We notice the random behavior the evolution. The MAI variance increases when the number of users increases. These variance values are not negligible compared to the user power value and to the variance in the synchronous case. We will then determine the MAI average variance using a statistical study. The results are shown on graphs in Fig. 8.
Figure 8. MAI variance distribution in the asynchronous case

The graphs show MAI variances histograms performed for different sequences lengths, different powers and different active users numbers. Distributions are Gaussian with large values compared to the users power.

Using these simulation results, we can plot the MAI variance (average values of the Gaussians) versus active users number or users power.

The MAI variance versus average users power for different values of K, Tc and N are shown in Fig. 9.

Figure 9. MAI variance versus average users power (in Watt) for K=10, N=63, Tc=1 (unit) in the asynchronous case

We note a perfectly linear behavior, the data adjustment provides a slope of 95. The theoretical slope (Eq 18) $\frac{NT^2}{6}(K-1)=94.5$ is in good agreement with the simulation results. Other simulations with different parameters values confirm this result.

Fig. 10 shows an example of the MAI variance versus active users number K plot for N=127, Tc=0.5 and average power $P_{moy}=3.17*10^{-3}$(W). All obtained results for different values of N, Tc and $P_{moy}$ are qualitatively the same.
We notice the perfectly linear behavior with a slope of 0.012. The theoretical slope (Eq 18) is also in good agreement with the simulation result.

4. Conclusion
Through this study, we developed an analytical model of the MAI, we showed that MAI can be considered as a noise with zero mean and variance expressed:

$$E[(J)^2] = \frac{N\cdot P_{moy} T_{c}^{2}}{6} (K - 1)$$

$$= \frac{P_{moy} T_{c}^{2}}{6N} (K - 1)$$

The simulation study confirms these results.

The MAI variance allows the contribution estimation of interference (MAI) in the signal-to-noise-plus-interference ratio (SNIR) in the CDMA system. So In a future work, we will use it to present performance analysis of CCR by evaluating the instantaneous SNIR and the BER.

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