Observer-Based Fixed-Time Consensus Control for Nonlinear Multi-Agent Systems Subjected to Measurement Noises

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ABSTRACT This article investigates the fixed-time consensus control problem for nonlinear second-order multi-agent systems subjected to measurement noises and in absence of the second-order state measurements of the leader and the followers under directed communication topology graphs. For each follower, an innovative augmented fixed-time disagreement error observer (AFTDEO) is first designed to reconstruct the disagreement error within fixed time. On one hand, the proposed AFTDEO can not only operate well only depending on the noisy measurements of the first-order states of the leader and the followers, but also suppress the measurement noises of the leader and the followers. On the other hand, the proposed AFTDEO can guarantee that the observation errors are convergent to the origin in fixed time independent of the initial observation errors. Based on the proposed AFTDEO, a novel fixed-time consensus control scheme under a directed graph having a spanning tree is constructed to address the consensus problem without the second-order measurements of the leader and the followers by utilizing fixed-time nonsmooth backstepping technique. The fixed-time convergence of the tracking error is guaranteed through Lyapunov stability analysis. Finally, simulation results demonstrate the effectiveness of the proposed consensus control scheme.

INDEX TERMS Fixed-time control, consensus control, measurement noises, augmented disagreement error observer, multi-agent systems.

I. INTRODUCTION
During the past few decades, cooperative control for multi-agent systems has received considerable attention and achieved abundant accomplishments from various scientific communities [1]–[4]. One of the most significant and fundamental issues of cooperative control is the consensus control problem, which requires all agents to converge to a common value by communicating with their neighbors [5]–[7].

Obviously, settling time is a vital performance specification for the consensus control problem of multi-agent systems as fast convergence rate exhibits excellent flexibility and strong robustness [8]. It is well known that finite-time control is one of the most effective control methods to obtain a fast convergence rate of the closed-loop system. However, the settling time of finite-time stability depends on the initial states. Thus, to solve the problem, fixed-time stability emerges and is first proposed in [9] for linear control systems. As an extension of finite-time stability, fixed-time stability guarantees that the settling time function derived from stability analysis is independent of initial conditions and uniformly bounded [10]. Due to the outstanding properties of fixed-time control, a robust fixed-time consensus control law with a directed topology for first-order multi-agent systems with external disturbances is proposed in [11]. Then, growing results of fixed-time cooperative consensus control for multi-agent systems are obtained [12]–[16].

It is worth noting that the majority of the existing fixed-time consensus control methods, including the aforementioned results, are heavily dependent on the entire accurate states of the leader and the followers. Nevertheless, measurements may be contaminated by noises in practical. It is no doubt that noisy measurements of the leader and the followers can lead to the deterioration of the consensus control performance. For the sake of solving the above problem, linear low-pass filters are adopted to suppress the...
measurement noises in industrial systems. However, due to the introductions of the low-pass filters, large phase lag is brought out to the consensus control system. In addition, the corresponding closed-loop stability is very difficult to be analyzed [17]. Furthermore, in some situations, it is very hard to get the accurate velocity (or other high-order state) measurements of the leader and the followers. To deal with this issue, some velocity observers of the leader are investigated in [18]–[20]. However, they fail to filter out the position measurement noises of the leaders. Besides, it should be pointed out that the majority of the existing observers are asymptotically stable or finite-time stable. Actually, the settling time of the observers is also of great significance to the consensus control performance. In the work of [20], a fixed-time leader state observer is designed to reconstruct the leader’s states. However, the precise position measurements without noises of the leader are required, and the velocity measurements of each follower are also indispensable in the control law. Therefore, it remains challenging to design a fixed-time observer to deal with the measurement noises of the leader and the followers, where only the first-order state measurements with noises are required.

It is well known that backstepping technique is effective in nonlinear systems with unmatched nonlinear terms. However, traditional backstepping technique suffers from “explosion of complexity” caused by the repeated derivations of virtual control inputs. To deal with this problem, dynamic surface control and command filtered backstepping control are proposed in [21], [22]. In spite of these results, the filtering errors based on the aforementioned backstepping techniques are merely convergent asymptotically rather than in fixed time. To the best of our knowledge, for second-order nonlinear multi-agent systems under directed topologies, the fixed-time consensus control design without the second-order state measurements utilizing backstepping technique in presence of measurement noises is still an open problem.

Motivated by the above observations, in this article, we propose a novel fixed-time consensus control scheme for second-order multi-agent systems subjected to measurement noises based on an augmented fixed-time disagreement error observer (AFTDEO) without the second-order state measurements of the leader and followers under directed interaction topologies. Firstly, a novel AFTDEO is designed such that each follower can get the estimates of disagreement errors in fixed time and filter out the measurement noises depending on the noisy first-order state measurements. Secondly, a fixed-time consensus control scheme under directed graphs with the designed AFTDEO based on nonsmooth backstepping technique is constructed to address the consensus control problem in presence of measurement noises and in absence of leader and followers’ second-order state measurements. The fixed-time convergence of the tracking error is obtained through Lyapunov approach, and the simulation results verify the effectiveness of the developed consensus control scheme.

Compared with the previous relevant results, the outstanding features of the control scheme proposed in this article can be summarized as follows:

1) Compared to the existing state observers in [13], [15], [18]–[20], [23]–[25], the novel AFTDEO proposed in this article can not only suppress the effects brought from the measurement noises of the leader and followers to a great extent, but also reconstruct the disagreement error for each follower in fixed time. Moreover, it is exactly because of the designed disagreement observer, which is different from the common state observers, the whole consensus control scheme can operate well independent of the second-order state measurements of the leader and followers.

2) The proposed consensus control scheme for multi-agent systems under directed interaction topologies based on AFTDEO without second-order state measurements can achieve the fixed-time convergence of tracking error in presence of measurement noises. Strict stability analysis is given, and the simulation results verify the validity and effectiveness of the consensus control scheme. In comparison with the works in [5], [7], the fixed-time consensus control scheme based on the AFTDEO achieves that the tracking errors converge to a small region around the origin in fixed time rather than UUB. Compared to the results in [24], [26], the proposed consensus control scheme can operate well in absence of the second-order state measurements of the leader and followers and in presence of measurement noises. Therefore, the proposed control scheme is really novel and practical.

3) Compared to the traditional backstepping technique, the designed nonsmooth backstepping technique can not only cope with the problem “explosion of complexity”, but also achieve the convergence of the filtering error in fixed time.

The rest of the paper is organized as follows. The problem formulation is introduced in Section II. The consensus control scheme design and analysis are studied in Section III. Numerical simulation results are presented in Section IV, after which the conclusions are drawn.

II. PROBLEM FORMULATION

In this article, we consider a multi-agent system with N followers and one leader. The dynamics of the i-th follower \((i = 1, 2, \cdots, N)\) can be described as follows:

\[
\begin{align*}
\dot{x}_i(t) &= x_i(2)(t) + f_1(x_i(1)(t)) \\
\dot{y}_i(t) &= x_i(1)(t),
\end{align*}
\]

(1)

where \(x_i(t) = [x_i(1)(t), x_i(2)(t)]^T \in \mathbb{R}^2\) is the state vector of the i-th follower; \(u_i(t) \in \mathbb{R}\) denotes the control input of the i-th follower; \(f_1(x_i(1)(t)) : \mathbb{R} \to \mathbb{R}\) and \(f_2(x_i(2)(t)) : \mathbb{R}^2 \to \mathbb{R}\) are both continuous functions; \(y_i\) is the output of the i-th follower.

Moreover, the dynamics of the leader are described as follows:

\[
\begin{align*}
\dot{x}_0(1) &= x_0(2) \\
\dot{x}_0(2) &= u_0(t) \\
y_0(t) &= x_0(1),
\end{align*}
\]

(2)

Compared to the existing state observers in [13], [15], [18]–[20], [23]–[25], the novel AFTDEO proposed in this article can not only suppress the effects brought from the measurement noises of the leader and followers to a great extent, but also reconstruct the disagreement error for each follower in fixed time. Moreover, it is exactly because of the designed disagreement observer, which is different from the common state observers, the whole consensus control scheme can operate well independent of the second-order state measurements of the leader and followers.
where \( x_0(t) = [x_{0,1}(t), x_{0,2}(t)]^T \in \mathbb{R}^2 \) is the state vector of the leader; \( u_0(t) \in \mathbb{R} \) represents the control input of the leader, which is supposed to be accessible to the \( i \)th follower who can receive the information from the leader; \( y_0 \) is the output of the leader.

As a matter of fact, the states of the \( i \)th follower \( x_{i,k} \) in the multi-agent system and the leader \( x_{0,k} \) are obtained by inaccurate measurements and contaminated by noises, \( k = 1, 2 \). Specifically, the state measurements of the \( i \)th follower \( x_{i,k} \) and the leader \( x_{0,k} \) are given as follows:

\[
x_{i,k}^m(t) = x_{i,k}(t) + \bar{n}_{i,k}(t)
\]

and

\[
x_{0,k}^m(t) = x_{0,k}(t) + \bar{n}_{0,k}(t)
\]

where \( x_{0,k}^m \) and \( x_{i,k}^m \) are the measurements of the states \( x_{0,k} \) and \( x_{i,k} \), respectively; \( \bar{n}_{i,k} \) and \( \bar{n}_{0,k} \) represent the measurement noises brought by the sensors of the followers and the leader, which satisfy the zero-mean-value condition.

To derive the main results of this article, the following reasonable assumption is made.

**Assumption 1:** The topology graph \( G \) among the followers and the leader is directed and has a spanning tree with the leader being the root node.

Furthermore, the tracking error vector is defined as \( \delta(t) = [\delta_1(t), \delta_2(t), \ldots, \delta_N(t)]^T \) with \( \delta_i(t) = y_{i,1}(t) - y_{0,1}(t) \). The control objective of this article is to design an observer-based fixed-time consensus control scheme based on the first-order state measurements with noises such that the tracking error \( \delta(t) \) can converge to a small region around the origin within fixed time, which leads to a successful consensus for the multi-agent system (1) and (2) subjected to measurement noises under directed graphs. For simplicity, we omit \( (t) \) for all the variables in the rest of this article.

### III. LEADER-FOLLOWER CONSENSUS CONTROL SCHEME DESIGN AND ANALYSIS

In this section, we will first construct a novel AFTDEO to deal with the measurement noises of the leader and followers with the first-order state measurements. Then, we will construct a fixed-time control scheme to address the consensus tracking problem for the multi-agent systems under directed topologies using nonsmooth backstepping technique. The block diagram of the overall consensus control scheme is depicted in Fig. 1.

#### A. AUGMENTED FIXED-TIME DISAGREEMENT ERROR OBSERVER

First of all, we introduce the following disagreement error \( e_{i,k} \) (known as local neighborhood consensus tracking error) for the \( i \)th follower:

\[
e_{i,k} = \sum_{j=1}^{N} a_{ij}(x_{i,k} - x_{j,k}) + b_i(x_{i,k} - x_{0,k}) \quad (5)
\]

where \( k = 1, 2; i, j = 1, 2, \ldots, N \). \( a_{ij} \) is the element of the associated adjacency matrix. \( a_{ij} > 0 \) if and only if the \( j \)th follower can deliver the state information to the \( i \)th follower; otherwise \( a_{ij} = 0 \). In addition, \( b_i > 0 \) if and only if the
ith follower can receive the state information from the leader; otherwise \(b_i = 0\).

Taking the differentiation of (5) with respect to time, the dynamics of the disagreement error are obtained:

\[
\begin{align*}
\dot{e}_{i,1} &= e_{i,2} + \bar{f}_{i,1}, \\
\dot{e}_{i,2} &= (l_{ii} + b_i)u_i + \sum_{j=1,j\neq i}^{N} l_{ij}u_j + \bar{f}_{i,2},
\end{align*}
\]

where \(l_{ij}\) is the \(i\)th row and \(j\)th column element of the Laplacian matrix \(L\) of the communication topology graph among the followers, and \(l_{ii}\) is the \(i\)th element on the principal diagonal of matrix \(L\), where \(L = D - A\) with \(A = [a_{ij}]\) and \(D = \text{diag}(\sum_{j=1,j\neq i}^{N} a_{ij})\). \(\bar{f}_{i,1} = (l_{ii} + b_i)f_{i,1} + \sum_{j=1,j\neq i}^{N} l_{ij}f_{j,1}\).

Then the measurement of the disagreement error is

\[
e_{i,k} = e_{i,k} + \sum_{j=1}^{N} a_{ij}(\bar{n}_{i,k} - \bar{n}_{j,k}) + b_j(\bar{n}_{i,k} - \bar{n}_{0,k}) \tag{7}
\]

In fact, the measurement noises \(\bar{n}_{i,k}\) and \(\bar{n}_{0,k}\) are fast time-varying fluctuation which are centered by zero. In addition, their integrals with respect to time tend to be zero if the measurement frequency tends to infinity [27]. Therefore, \(\int_{0}^{t} e_{i,k}(\sigma) d\sigma = \int_{0}^{t} e_{i,k}(\sigma) d\sigma + n'\) holds for a small enough and bounded \(n'\) whose derivative can be ignored in real-time control [27], [28].

In order to suppress the measurement noises, the variable \(e_{i,0} = \int_{0}^{t} e_{i,k}(\sigma) d\sigma\) is introduced for the \(i\)th follower. Then by letting \(e_{i,0}\) as the input, a novel AFTDEO for the \(i\)th follower is designed as follows:

\[
\begin{align*}
\dot{\hat{e}}_{i,0} &= \hat{e}_{i,1} - a_0[\hat{e}_{i,0} - e_{i,0}]^{p_0} - \beta_0[\hat{e}_{i,0} - e_{i,0}]^{q_0}, \\
\hat{e}_{i,1} &= \hat{e}_{i,2} - a_1[\hat{e}_{i,0} - e_{i,0}]^{p_1} - \beta_1[\hat{e}_{i,0} - e_{i,0}]^{q_1}, \\
\hat{e}_{i,2} &= -\alpha_2[\hat{e}_{i,0} - e_{i,0}]^{p_2} - \beta_2[\hat{e}_{i,0} - e_{i,0}]^{q_2}, \\
\end{align*}
\]

where function \([x]^\alpha = |x|^\alpha \text{sign}(x)\) is defined as \(|x|^\alpha\) with sign\((x)\) being the signum function of variable \(x\). \(\hat{e}_{i,0}\) and \(\hat{e}_{i,k}\) are the estimates of \(e_{i,0}\) and \(e_{i,k}\) for the \(i\)th follower, \(k = 1, 2\). The observer parameters \(p_0 \in (0, 1), p_1 \in (0, 1), q_0 > 1, q_1 > 1\) satisfy \(p_0 = kp_0 - (k - 1)\), \(q_0 = kq_0 - (k - 1)\) with \(k = 1, 2\) and \(p_0 \in (1 - \epsilon_1, 1)\) and \(q_0 \in (1, 1 + \epsilon_2)\) for sufficiently small constants \(\epsilon_1\) and \(\epsilon_2\). Other observer gains \(a_0, \alpha_2, \beta_0, \beta_2 > 0\) are chosen to satisfy the condition that the matrices \(A_1\) and \(A_2\) are Hurwitz, where \(A_1\) and \(A_2\) are described as follows:

\[
A_1 = \begin{pmatrix} -\alpha_0 & 1 & 0 \\ -\alpha_1 & 0 & 1 \\ -\alpha_2 & 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -\beta_0 & 1 & 0 \\ -\beta_1 & 0 & 1 \\ -\beta_2 & 0 & 0 \end{pmatrix}.
\]

Theorem 1: For the multi-agent systems consisting of (1) to (4), the AFTDEO (8) can achieve the convergence of the observation errors \(e_{i,0}\) and \(e_{i,k}\) to the origin within fixed time \(T_0\):

\[
T_0 \leq \frac{2(1 - p_0)\lambda_{\text{max}}(Q_1) + (q_0 - 1)\lambda_{\text{min}}(Q_2)}{(1 - p_0)\lambda_{\text{max}}(Q_1) + (q_0 - 1)\lambda_{\text{min}}(Q_2)} - 1.
\]

Remark 2: Different from the existing fixed-time observers in [12], [13], [15], [24], the designed AFTDEO can not only achieve the convergences of the observation errors to the origin in fixed time, but also simultaneously suppress the leader and followers’ measurement noises and reconstruct the disagreement error within fixed time only depending on the first-order state measurements of the leader and followers.

B. FIXED-TIME CONSENSUS CONTROL LAW DESIGN AND ANALYSIS

In this subsection, the fixed-time consensus control law will be designed by using nonsmooth backstepping technique at first. Then, the stability analysis of the whole closed-loop multi-agent system will be presented.

Step 1: For system (6), firstly, we take \(e_{i,2}\) as a virtual control input. As the measurements of \(e_{i,1}\) is contaminated by noise, the virtual control law \(\mu_i\) is designed as

\[
\mu_i = -\lambda_{i,1}[\hat{e}_{i,1}]^{p_{i,1}} - \gamma_{i,1}[\hat{e}_{i,1}]^{q_{i,2}} - \bar{f}_{i,1},
\]

where control gains \(\lambda_{i,1}, \gamma_{i,1} > 0, \phi_{i,1} > 1, 0 < \phi_{i,2} < 1\)

\(\hat{e}_{i,1}\) is obtained by the AFTDEO (8).

Choose a Lyapunov function as follows:

\[
V_1 = \frac{1}{2} e_{i,1}^T e_{i,1}.
\]
The time differentiation of (12) yields that
\[ \dot{V}_1 = e_{i,1}^T (-\lambda_{i,1} [\hat{e}_{i,1}]^{\varphi_{i,1}} - \gamma_{i,1} [\hat{e}_{i,1}]^{\varphi_{i,2}}) \]
\[ = -\lambda_{i,1} e_{i,1}^T [e_{i,1} + \hat{e}_{i,1}]^{\varphi_{i,1}} - \gamma_{i,1} e_{i,1}[e_{i,1} + \hat{e}_{i,1}]^{\varphi_{i,2}}. \]  
(13)

To show the boundedness of \( V_1 \) in any finite time interval \([t_0, t]\), the following two cases are considered:

**Case 1:** In this case, we assume that the condition \(|e_{i,1}| > |\hat{e}_{i,1}|\) is satisfied. Then, it is easy to obtain that \( \text{sign}(e_{i,1}) = \text{sign}(e_{i,1} + \hat{e}_{i,1}) \). Thus, we have
\[ \dot{V}_1 \leq -\lambda_{i,1}|e_{i,1}|e_{i,1} + \hat{e}_{i,1}|^{\varphi_{i,1}} - \gamma_{i,1}|e_{i,1}|e_{i,1} + \hat{e}_{i,1}|^{\varphi_{i,2}} \leq 0. \]
(14)

**Case 2:** In this case, the condition \(|e_{i,1}| \leq |\hat{e}_{i,1}|\) is considered. From Theorem 1, \( \hat{e}_{i,1} \) can converge to the origin in fixed time, which follows that \( \hat{e}_{i,1} \) is bounded all the time. Hence, there exists a positive constant \( z_1 \) such that \( e_{i,1}^T (-\lambda_{i,1} [e_{i,1} + \hat{e}_{i,1}]^{\varphi_{i,1}} - \gamma_{i,1} [e_{i,1} + \hat{e}_{i,1}]^{\varphi_{i,2}}) \leq z_1 \). Therefore, \( V_1 \) satisfies
\[ \dot{V}_1 \leq z_1. \]
(15)

As a result, from inequalities (14) and (15), \( V_1 \) is always bounded at any finite time interval \([t_0, t]\), which is, no finite-time escape occurs.

According to Theorem 1, \( \hat{e}_{i,1} \) will converge to the origin after \( t \geq T_0 \). Therefore, for \( t \geq T_0 \), (13) can be rewritten as
\[ \dot{V}_1 = e_{i,1}^T (-\lambda_{i,1} [e_{i,1}]^{\varphi_{i,1}} - \gamma_{i,1} [e_{i,1}]^{\varphi_{i,2}}) \]
\[ = -\lambda_{i,1} V_1^{\varphi_{i,1}} - \gamma_{i,1} V_1^{\varphi_{i,2}}. \]
(16)

According to Lemma 1 in [13], \( e_{i,1} \) will converge to the origin in fixed time.

Next, we will introduce a new state \( \hat{\mu}_i \), which is obtained by the following nonlinear nonsmooth filter:
\[ \tau_i \hat{\mu}_i = [\mu_i - \hat{\mu}_i]^{\varphi_{i,1}} + [\mu_i - \hat{\mu}_i]^{\varphi_{i,2}}, \]
\[ \hat{\mu}_i(t_0) = \mu_i(t_0). \]
(17)

where the filter parameter \( \tau_i \) is a small positive constant.

**Step 2:** Define the tracking error as \( \bar{e}_i = e_{i,2} - \hat{\mu}_i \), in which \( \hat{\mu}_i \) is obtained by the nonlinear nonsmooth filter (17) in Step 1. Then the dynamics of \( \bar{e}_i \) can be written as
\[ \dot{\bar{e}}_i = (\bar{u}_i + b_i a_i) + \sum_{j=1,j \neq i}^{N} l_{ij} \bar{u}_j + \hat{f}_i - \hat{\mu}_i. \]
(18)

Therefore, the actual control law for the \( i \)th follower is designed as
\[ u_i = \frac{1}{(l_i + b_i)} (-\lambda_{i,2} [\hat{e}_{i,2}]^{\varphi_{i,1}} - \gamma_{i,2} [\hat{e}_{i,2}]^{\varphi_{i,2}}) \]
\[ - \sum_{j=1,j \neq i}^{N} l_{ij} \bar{u}_j - \hat{f}_i + \hat{\mu}_i, \quad i = 1, 2, \ldots, N. \]
(19)

where the control gains \( \lambda_{i,2}, \gamma_{i,2} > 0 \); and \( \hat{\mu}_i \) is obtained by (17); \( \hat{\mu}_i = \hat{e}_{i,2} - \bar{\mu}_i \).

Define the filtering error of the nonsmooth filter \( e_i \) as
\[ e_i = \mu_i - \hat{\mu}_i. \]
(20)

The time differentiation of (20) yields that
\[ \dot{e}_i = -([e_i]^{\varphi_{i,1}} + [e_i]^{\varphi_{i,2}}) / \tau_i + \hat{\mu}_i. \]
(21)

Substituting (19) into (18) and (11) into (6) yields
\[ \dot{\bar{e}}_i = \bar{e}_i + \bar{\mu}_i = \bar{e}_i + \mu_i - \bar{\mu}_i + \bar{f}_i \]
\[ \hat{\mu}_i = -\lambda_{i,2} [\hat{e}_{i,2}]^{\varphi_{i,1}} - \gamma_{i,2} [\hat{e}_{i,2}]^{\varphi_{i,2}}. \]
(22)

Construct the following Lyapunov function
\[ V_2 = V_1 + \frac{1}{2} e_i^T \dot{e}_i + \frac{1}{2} e_i^T e_i. \]
(23)

The derivative of \( V_2 \) with respect to time \( t \) is
\[ \dot{V}_n = e_{i,1}^T \dot{e}_{i,1} + e_{i,2}^T \dot{e}_{i,2} + e_i^T \dot{e}_i \]
\[ = e_{i,1}^T (-\lambda_{i,1} \hat{e}_{i,1})^{\varphi_{i,1}} - \gamma_{i,1} \hat{e}_{i,1})^{\varphi_{i,2}} - e_i + \hat{\mu}_i \]
\[ + \xi_i^T (-\lambda_{i,2} \hat{e}_{i,2})^{\varphi_{i,1}} - \gamma_{i,2} \hat{e}_{i,2})^{\varphi_{i,2}} + e_{i,k}^T (-([e_i]^{\varphi_{i,1}} + [e_i]^{\varphi_{i,2}}) / \tau_i + \hat{\mu}_i). \]
(24)

**Theorem 2:** Consider the multi-agent system with \( N \) followers (1) and one leader (2) subjected to noisy measurements (3) and (4). Suppose Assumption 1 holds. Then under the control law (19), the multi-agent system can achieve the expected consensus in fixed time \( T_1 \leq \frac{2}{\gamma_{i,1} - \lambda_{i,1} \phi + \frac{2}{\phi_2} (1 - \psi_{i,2})} + \frac{2}{\phi_2} (1 - \psi_{i,2}) + T_0 \) with \( c_1 = \min \left\{ \frac{\psi_{i,1} + 1}{\phi_1 - \lambda_{i,1} - 2}, \frac{\psi_{i,1} + 1}{\phi_1 - \lambda_{i,2} - 1} \right\} \), \( c_2 = \min \left\{ \frac{\psi_{i,2} + 1}{\phi_2 - \gamma_{i,1} - 2}, \frac{\psi_{i,2} + 1}{\phi_2 - \gamma_{i,2} - 1} \right\} \), and \( 0 < \phi < 1 \), if the following condition is satisfied:
\[ \begin{align*}
2 & \psi_{i,1} + 1 \\ 2 & \psi_{i,2} + 1 \\ \frac{\psi_{i,1} + 1}{\phi_1 - \lambda_{i,1} - 2} & > 0 \\ \frac{\psi_{i,1} + 1}{\phi_1 - \lambda_{i,2} - 1} & > 0 \\ \frac{\psi_{i,2} + 1}{\phi_2 - \gamma_{i,1} - 2} & > 0 \\ \frac{\psi_{i,2} + 1}{\phi_2 - \gamma_{i,2} - 1} & > 0 \\
\frac{1}{\phi_1} & > 0 \\
\frac{1}{\phi_2} & > 0.
\end{align*} \]
(25)

**Proof:** The proof contains two steps. First, we prove \( e_{i,1}, \bar{e}_i \) and \( e_i \) are bounded in any finite time interval \([t_0, t]\). Then, we prove that \( e_{i,1}, \bar{e}_i \) and \( e_i \) can converge to a small region around the origin in fixed time \( T_1 \).

**Step 1:** Invoking Young’s inequality, (24) can be rewritten as
\[ \dot{V}_2 \leq e_{i,1}^T (-\lambda_{i,1} \hat{e}_{i,1})^{\varphi_{i,1}} - \gamma_{i,1} \hat{e}_{i,1})^{\varphi_{i,2}} + e_{i,1}^T \dot{e}_i + \frac{1}{2} e_i^T \dot{e}_i + \frac{1}{2} e_i^T \dot{e}_i \]
\[ + \xi_i^T (-([e_i]^{\varphi_{i,1}} + [e_i]^{\varphi_{i,2}}) / \tau_i + \hat{\mu}_i). \]
(26)
Similar to [30, 31], we assume that there exists a positive constant \( \varepsilon_M \) such that \( |\dot{\xi}_i| \leq \varepsilon_M \). Then we can get
\[
e^{T}_t \dot{\xi}_i \leq \frac{1}{2} \varepsilon^2 \xi_t + \frac{1}{2} \varepsilon^2 \xi_t.
\]
(27)

In addition, let \( \tilde{\xi}_i = \xi_i - \dot{\xi}_i = \dot{\xi}_i \). Then we obtain that
\[
\dot{V}_2 \leq e^{T}_t (\lambda_i \xi_i) + \gamma_i (\tilde{\xi}_i)^{\psi_i} - \gamma_i (\tilde{\xi}_i)^{\psi_i} + e^{T}_t e_{t,1} + \frac{1}{2} \varepsilon^2 \xi_t + \frac{1}{2} \varepsilon^2 \xi_t
\]
(28)

To show the boundedness of \( V_2 \) in any finite time interval, the following two cases are also considered:

Case 1: In this case, we assume that the condition \( |\xi_i| > |\dot{\xi}_i| \) is satisfied. Then, it is easy to obtain that \( \text{sign}(\xi_i) = \text{sign}(\tilde{\xi}_i + \dot{\xi}_i) \). Thus, we have
\[
\xi^T_i (\lambda_i \xi_i)^{\psi_i} - \gamma_i (\tilde{\xi}_i)^{\psi_i} \leq 0.
\]
(29)

Case 2: In this case, the condition \( |\xi_i| \leq |\dot{\xi}_i| \) is considered. From Theorem 1, \( \tilde{\xi}_i \) can converge to the origin in fixed time, which follows that \( \xi_i \) is bounded all the time. Hence, there exists a positive constant \( z \) such that
\[
\xi^T_i (\lambda_i \xi_i)^{\psi_i} - \gamma_i (\tilde{\xi}_i)^{\psi_i} \leq z.
\]
(30)

Combining the above two cases, we can get
\[
\xi^T_i (\lambda_i \xi_i)^{\psi_i} - \gamma_i (\tilde{\xi}_i)^{\psi_i} \leq z + e^2_M.
\]
(31)

Therefore, it can be derived that
\[
\dot{V}_2 \leq e^{T}_t e_{t,1} + e^{T}_t e_{t,1} + \frac{1}{2} \varepsilon^2 \xi_t + e^2_M
\]
(32)

where \( z = z + z \).

As a result, the above inequality indicates that \( V_2 \) is always bounded at any finite time interval \( [t_0, t] \).

Step 2: From Theorem 1, the observation error \( \tilde{e}_{i,k} \) will converge to the origin when \( t \geq T_0 \). Hence, for \( t \geq T_0 \), the following inequality can be obtained:
\[
\dot{V}_2 \leq e^{T}_t (\lambda_i \xi_i) + e^{T}_t e_{t,1} + \frac{1}{2} \varepsilon^2 \xi_t + e^2_M
\]
(28)

The last inequality is obtained based on the fact that
\[
x \leq x + \frac{x^{\psi_i}}{2} \quad \text{holds for a positive variable } x.
\]
Define \( q_1 = 3 - \frac{1}{\xi_t} \), \( q_2 = \xi_t \), and \( \Delta = e^2_M \). Invoking Lemmas 3 and 4 in [13], then (33) can be rewritten as
\[
\dot{V}_2 \leq -c_1 (e^{T}_{t,1} e_{t,1})^{\psi_i} - c_2 (e^{T}_{t,1} e_{t,1})^{\psi_i} - c_1 (\xi_t)^{\psi_i} - c_2 (\xi_t)^{\psi_i} - \frac{1}{2} \xi_t^2 + e^2_M.
\]
(34)

According to Lemma 2.2 in [32], we can obtain that \( X_i = [e_{i,1}, \xi_i, e_i] \) is practical fixed-time stable and will converge to the following compact set \( \Omega_i \) within fixed time \( T_1 \).

\[
\Omega_i = \{ X_i | V_2(X_i) \leq \text{min}[(\frac{\Delta}{(1 - \phi)q_1} {\frac{1}{2}}^{\frac{x_1 - 1}{x_1}}), \frac{\Delta}{(1 - \phi)q_2} {\frac{1}{2}}^{\frac{x_2 - 1}{x_2}}] \}
\]
(35)

Since Assumption 1 holds, then matrix \( L + B \) is non-singular, where \( L \) is the Laplacian matrix of the topology graph among the followers, \( B = \text{diag}(b_1, b_2, \ldots, b_N) \). Thus, due to \( e_1 = (L + B) \delta \) with \( e_1 = [e_{1,1}, e_{2,1}, \ldots, e_{N,1}] \), we can derive that \( \delta \) is practical fixed-time stable since \( e_1 \) is practical fixed-time stable. Therefore, we can conclude that \( \delta \) will converge to a small compact set by appropriately selecting the control parameters within fixed time \( T_1 \). Therefore, the multi-agent system can achieve the expected consensus within fixed time \( T_1 \).

This completes the proof.

Remark 3: Due to the designed AFTDEO, which reconstructs the disagreement error for each follower instead of the states of the leader or each follower, the proposed control scheme can achieve the consensus without the second-order state measurements of the leader and the followers. As a matter of fact, the second-order state measurements of the leader or the followers are still needed in the most observer-based consensus/formation control, such as [13, 19, 20, 33]. Therefore, the proposed novel consensus control scheme is much more practical compared to the most existing results.

Remark 4: From Theorem 2, the tracking error can converge to a small region around the origin in fixed time in presence of measurement noises and depending on the first-order state measurements of the leader and followers under the proposed control scheme. Consequently, the proposed novel consensus control scheme can improve the control performance compared to the aforementioned results.
Remark 5: By proposing the nonlinear nonsmooth filter, the singular problem brought from the item $[\mu_i - \bar{\mu}_i]^\phi_{i,2}$ is well solved, as well as the "explosion of complexity" problem. In addition, compared to the first-order traditional filters, the proposed filter can ensure the fixed-time convergence of the filtering error.

Remark 6: Based on the proposed AFTDEO which reconstructs the disagreement error for each follower in fixed time, the constructed consensus control scheme can achieve consensus for the second-order nonlinear multi-agent systems under directed topology graphs. As a matter of fact, most existing researches on fixed-time consensus/formation control for second-order multi-agent systems are conducted under undirected topology graphs [13], [24], [26], [34]. Since directed topologies are more practical than undirected topologies, the obtained results are superior to these results. In addition, some fixed-time consensus control laws are derived for second-order multi-agent systems under directed graphs in [12], [15], [23], [35]. However, they fail to filter out the measurement noises and cannot work only depending on the first-order state measurements of the leader and followers. Therefore, on the whole, the obtained control scheme can achieve the expected fixed-time consensus and in the meanwhile overcome the drawbacks of the existing results.

IV. SIMULATION RESULTS
Construct a multi-agent system with six followers whose dynamics are described by (1) and one leader whose dynamics are described by (2). Choose the graph $G_0$ depicting the information flow among the followers and the leader, as shown in Fig. 2, which is a directed graph having a spanning tree.

The initial states of the six followers and the leader are set as $x_1(0) = [4, 0]^T$, $x_2(0) = [2, 0]^T$, $x_3(0) = [3, 0]^T$, $x_4(0) = [5, 0]^T$, $x_5(0) = [4, 0]^T$, $x_6(0) = [5, 0]^T$, $x_0(0) = [0, 0]^T$. The trajectory of the leader is $x_{0,1}(t) = 5\cos(0.15t)$. The initial values of the states of the AFTDEO (8) are all set as zero. In addition, the terms $f_{1,1}$ and $f_{1,2}$ are set as $f_{1,1} = 0.5\cos(0.1x_{1,1})$ and $f_{1,2} = 4\sin(0.2x_{1,2})$, respectively. The selected values of the observer and the control parameters are shown in Table 1 and Table 2, respectively. Moreover, the maximum values of Gaussian measurement noises $\bar{n}_{i,k}$ and $\bar{n}_{0,k}$, $k = 1, 2$ are set as shown in Table 3.

The simulation results are presented by Figs. 3-7. Fig. 3 shows the curves of the outputs of the six followers and the leader. It is observed from the results that the
the tracking error $\delta_i$ converges to a small neighbourhood of the origin within 10s. Fig. 5 presents the observation errors $\tilde{e}_{i,1}$ and $\tilde{e}_{i,2}$, respectively. From Fig. 5, we can see that the observation errors $\tilde{e}_{i,1}$ and $\tilde{e}_{i,2}$ also converge to the origin within 10s. The fast convergence rates show the superiority of fixed-time control scheme. Fig. 6 depicts the curves of measured disagreement error $e_{m,1}$ (blue line) and the estimated disagreement error $\hat{e}_{i,1}$ (red line), which shows that the designed AFTDEO can guarantee high estimation accuracy and alleviate the adverse effects of measurement noises to a great extent. Fig. 7 displays the curves of the control inputs of the six followers.

V. CONCLUSION

For the purpose of realizing consensus for the second-order nonlinear multi-agent systems under directed graphs without the second-order measurements of the leader and followers subjected to measurement noises, an innovative observer-based fixed-time consensus control scheme has been constructed in this article. A novel AFTDEO is established to guarantee that each follower can obtain the estimates of disagreement errors in fixed time only depending on the first-order measurements and to filter out the measurement noises. A fixed-time consensus control scheme under directed graphs based on the designed AFTDEO without the second-order measurements of the leader and followers is constructed to achieve the fixed-time convergence of the tracking error. The fixed-time convergence of the tracking error is proved by Lyapunov approach, and the simulation results verify the effectiveness of the developed control scheme in this article. In the future, we will further focus on event-triggered consensus control problem and try our best to design an event-triggered fixed-time consensus control scheme in presence of measurement noises.

REFERENCES

[1] R. Wang, X. Dong, Q. Li, and Z. Ren, “Distributed time-varying formation control for multiagent systems with directed topology using an adaptive output-feedback approach,” IEEE Trans. Ind. Informat., vol. 15, no. 8, pp. 4676–4685, Aug. 2019.
[2] X. Dong, Y. Zhou, Z. Ren, and Y. Zhong, “Time-varying formation control for unmanned aerial vehicles with switching interaction topologies,” Control Eng. Pract., vol. 46, pp. 26–36, Jan. 2016.
[3] Y. Chen, Z. Wang, Q. Han, and J. Hu, “Synchronization control for discrete-time-delayed dynamical networks with switching topology under actuator saturations,” IEEE Trans. Neural Netw. Learn. Syst., early access, Jun. 10, 2020, doi: 10.1109/TNNLS.2020.29960942.
[4] T. Zhu, Y. Guo, C. Wang, and C. Ni, “Inter-hour forecast of solar radiation based on the structural equation model and ensemble model,” Energies, vol. 13, no. 17, p. 4534, Sep. 2020.
[5] J. Qin, G. Zhang, W. X. Zheng, and Y. Kang, “Neural network-based adaptive consensus control for a class of nonaffine nonlinear multiagent systems with actuator faults,” IEEE Trans. Neural Netw. Learn. Syst., vol. 30, no. 12, pp. 3633–3644, Dec. 2019.
[6] Y. Chen, Z. Wang, B. Shen, and H. Dong, “Exponential synchronization for delayed dynamical networks via intermittent control: Dealing with actuator saturations,” IEEE Trans. Neural Netw. Learn. Syst., vol. 30, no. 4, pp. 1000–1013, Apr. 2019.
[7] Y. Zhang, J. Sun, H. Liang, and H. Li, “Event-triggered adaptive tracking control for multiagent systems with unknown disturbances,” IEEE Trans. Cybern., vol. 50, no. 3, pp. 890–901, Mar. 2020.
