Abstract—Learning the covariance matrices of spatially-correlated wireless channels, in millimeter-wave (mmWave) vehicular communication, can be utilized in designing environment-aware beamforming codebooks. Such channel covariance matrices can be represented on non-Euclidean Riemannian manifolds, thanks to their symmetric positive definite (SPD) characteristics. Consequently, in this paper, we propose a Riemannian-Geometric machine learning (G-ML) approach for estimating the channel covariance matrices based on unsupervised K-Means model. The proposed K-means algorithm utilizes Log-Euclidean metric (LEM) as the distance measure among channel covariance matrices over the Riemannian manifolds. We show that our proposed K-Means G-ML model can achieve up to 80% less error compared to Euclidean-based K-Means algorithm, which applies clustering on the channel vectors themselves.

Index Terms—Channel covariance matrices, Geometric machine learning, K-Means, millimeter wave, Riemannian geometry, vehicular communications.

I. INTRODUCTION

The increasing demand for high data rates to support emerging applications (e.g., vehicular communication) has induced numerous research challenges in high mobility networks [1]. Millimeter-Wave (mmWave) is a promising candidate for these applications to support high rates [2], thanks to its large bandwidth availability [3]. In a dynamic vehicular environment, the transmission from a multiple-antenna base station (or generally a relay) to a vehicular user equipment (UE) depends on the spatially correlated channels [4] which are determined by their relative positions. Since each relay’s location is different, each relay will experience different channel characteristics towards its UEs. As such, having a fixed pre-defined beamforming codebook for all relays will be inefficient. Instead, there is a need for each relay to learn its wireless environment characteristics (i.e., channel covariance matrices), then construct a matched beamforming codebook according to [5], [6], [7]. Learning the spatially correlated covariance matrices can dramatically reduce the network overhead needed to design an environment-aware beamforming codebook [7].

There are many studies in the literature that estimated the covariance matrices due to their advantages towards obtaining the channel knowledge [8], [9]. The work in [8] investigated the covariance estimation problem for hybrid mmWave systems by forming a low-rank matrix structure to minimize the complexity. The authors in [9] proposed covariance estimation by adopting a maximum-likelihood technique based on systematic allocations of pilot sequences. The work in [10] performed a compressive sensing based covariance estimation for sparse mmWave channels. Nevertheless, there is a need for leveraging machine learning (ML) algorithms that can dynamically adapt to the changes in a mmWave vehicular environment [11] for predicting the channel covariance matrices.

A recent study in [12] used a supervised ML approach that required prior training with label information to correctly estimate the covariance matrices. However, such supervised solutions may not be feasible for high dynamic vehicular scenarios, especially when the network is equipped with mobile base stations and labelled samples for prior training are not available. Therefore, we need a learning strategy without prior label information that can precisely estimate the covariance matrices to meet the high-dynamic vehicular demand. To do this, we turn our attention to unsupervised ML that requires no labelled training [13] to learn the channel covariance matrices.

The success of any unsupervised ML significantly depends on identifying the underlying structure within the dataset under consideration [13]. In that matter, we note that the mmWave multiple-antenna channels have spatial correlation characteristics and their covariance matrices are symmetric positive semi-definite ones in nature. Applying positive shifts to these semi-definite matrices transform them to symmetric positive definite (SPD) ones which can be modeled as points over the conic (i.e., non-euclidean) Riemannian manifolds [14], [15]. Making use of these manifolds can actually help in accurate estimation of the channel covariance matrices, which we will show later in this paper.

Riemannian geometry [16] has been recently considered in designing beamforming vectors [17], [18]. Additionally, non-euclidean methods have been utilized in codebook design such as non-conic Riemannian manifolds (i.e, unitary, fixed rank) as in [19], [20]. While these studies present novel non-euclidean geometric perspectives of beamforming and codebook designs, they have not been utilized in channel covariance estimation. Consequently, our goal is to make use of the underlying Riemannian-geometric structures of the spatially-correlated multiple-antenna channels for channel
covariance estimation in vehicular networks.

Geometric machine learning (G-ML) algorithms can characterize the data given in non-euclidean domain which is not possible with conventional machine learning approaches that deal with euclidean-structured data samples [22], [23]. In this paper, we propose an unsupervised approach, namely Geometric K-Means ML, to cluster the SPD variants of the channel covariance matrices with distinguishable spatial-correlation characteristics. Then Riemannian metrics, such as Log-Euclidean metric (LEM) [21], is applied to calculate the geodesic distance among these SPD covariance matrices over the given Riemannian manifold. We show that our proposed non-euclidean based G-ML approach can provide higher accuracy in clustering the channel covariance matrices compared to the euclidean-based solutions. The contributions of this paper can be highlighted as follows:

• For estimating the covariance matrices in a mmWave vehicular network, we model the problem as an unsupervised clustering problem that requires no prior label information while training to learn the covariance matrices as opposed to the existing supervised solutions.
• We propose a Geometric K-Means ML, that makes use of the underlying Riemannian-geometric characteristics of the spatially correlated multiple-antenna channels in estimating the channel covariance matrices.

The rest of the paper is organized as follows. Section II introduces the system model along with some preliminary concepts. Section III introduces the proposed solution and Section IV shows the simulation results. Finally, Section V concludes the paper.

II. SYSTEM MODEL

In this section, we first provide a brief preliminary on Riemannian geometry and then describe the system model for covariance estimation in a mmWave vehicular network.

A. Riemannian Geometry

A non-euclidean manifold $\Phi$, is illustrated in Fig. 1. The tangent space $T_\chi \Phi$ at a given point $\chi$ is composed of all the vectors $\nu$ which are tangent to the manifold $\Phi$ through the point $\chi$. We define an exponential map as the function mapping of a point $\chi$ on the tangent space $T_\chi \Phi$ of manifold $\Phi$, $\exp_\chi : T_\chi \Phi \rightarrow \Phi$. In most cases, manifolds are classified as a special category of topological manifolds, namely Riemannian manifolds [16], [20]. SPD matrices lie on the interior of such Riemannian manifolds which have a cone-shaped structure [15]. Riemannian metrics, such as LEM [21], are inner products on the tangent space $T_\chi \Phi$ and measure the length of geodesics i.e., shortest length of curves over the manifold among the given SPD matrices.

B. Vehicular Communications

We show a typical vehicular scenario in Fig. 2 where the users are located at different locations of a street. The different locations resemble different vehicular environments that have unique channel characteristics and varies fast as the vehicle moves. For clustering the covariance matrices in such a scenario, we consider a base station or road side unit (RSU) with $m$ antennas and a single antenna at each UE. Let $h_i \in \mathbb{C}^{m \times 1}$ represents an $m \times 1$ correlated wireless channel vector between the base station and one of its UE at the $i$-th location spot. A multiple-input single-output (MISO) channel between a given base station and its UE is modeled as a correlated Rayleigh fading channel vector [14]. Calculating the spatial correlation of these multi-antenna vehicular channels results in symmetric positive semi-definite covariance matrices. Applying positive shift to these matrices transforms them into SPD ones according to the following

$$Q_i = \lambda I + h_i h_i^H,$$

where $I$ is a $m \times m$ identity matrix, $\lambda$ is an arbitrary small scalar, and $H$ denotes matrix hermitian. We consider $\lambda = 0.5$ in this paper. These sample covariance matrices (or simply SPD matrices) lie on the conic Riemannian manifold over the locations $i = 1, 2, ..., N$. Consequently, $\Phi = \{Q_1, Q_2, ..., Q_N\}$ represents the set of observations for all the given SPD matrices, where each observation within $\Phi$ is an $m \times m$ matrix.
III. LEARNING THE CHANNEL COVARIANCE MATRICES

A. Problem Description

In a high-dynamic vehicular scenario, the covariance matrices of the spatially correlated channels are not known in advance as they keep changing based on the nature of the environment. Hence, designing fixed beamforming codebook in advance is not an efficient solution for a dynamic vehicular case. Consequently, the codebook design needs to be learned based on the user’s channels within each relay’s cluster, and hence the covariance matrices. We aim to partition the collected samples of the shifted covariance matrices into \( K \leq N \) disjoint groups \( R_1, R_2, \ldots, R_K \), based on their distinguishable spatial correlation characteristics. For doing so, we need to find the set of the estimated means (i.e., centroids) of each group given as \( \hat{G} = \{ \hat{G}_1, \hat{G}_2, \ldots, \hat{G}_K \} \). The set of indices for the location points that belong to any group \( R_k \) is denoted as \( l_k = \{1, 2, \ldots, n\} \), where \( k = 1, 2, \ldots, K \).

Also, we denote the channel vectors within group \( R_k \) as \( \mathbf{h}_{i,k} = \mathbf{h}_i, (\forall i \in l_k) \). For clustering the SPD covariance matrices, we first propose our non-euclidean based scheme that clusters the given samples over Riemannian manifolds. Then, to compare the performance of our proposed solution with the benchmark approaches, we clustered the covariance matrices using euclidean-based clustering technique.

B. Non-Euclidean Based Geometric Solution

We apply an unsupervised G-ML approach, namely Geometric K-Means ML, that considers the spatially correlated samples over the Riemannian space. Since, the set of observations can be modeled on the Riemannian manifold, learning these covariance matrices can be also conducted using Riemannian metrics. We consider LEM, which is a valid metric to calculate the geodesic distance between points [14], [21] over the Riemannian manifolds. For the given samples \( \mathbf{Q}_i \in \Phi \), the mean (i.e., centroid) of the channel covariance matrices over the Riemannian manifold is defined as the minimizer of sum of squared distances [24]

\[
\hat{G} = \arg\min_{G \in \Phi} \frac{1}{N} \sum_{i=1}^{N} \rho^2(\mathbf{Q}_i, G),
\]

where \( \rho \) is the geodesic distance calculated using LEM. The minimizer is obtained with a gradient descent procedure. From an initial value \( t = 0 \), the optimal centroid for group \( R_k \) is reached by repeatedly applying the following update formula according to [25]

\[
\hat{G}_{k+1} = \exp_{\hat{G}_k} \left( \frac{1}{n_k} \sum_{i \in l_k} \log_{\hat{G}_k} (\mathbf{Q}_i) \right), k = 1, 2, \ldots K
\]

(3)

where \( n \) is the number of samples within group \( R_k \).

Finally, we subtract the induced positive shift from the estimated centroid \( \hat{G}_{k+1} \) to achieve the desired original estimated centroid of the covariance matrices for cluster \( R_k \) according to given formula

\[
\hat{G}_{k+1} = \hat{G}_{k+1} - \lambda I.
\]

(4)

The channel covariance matrices having the minimum LEM distances (i.e., in similar environment) from \( \hat{G}_{k+1} \) fall in the same group, while the samples collected from two different vehicular environments should lie in disjoint groups. Fig. 3 shows an illustration of the proposed covariance clustering scheme where the SPD covariance matrices are clustered into different groups over a conic Riemannian manifold. We use different shapes for the points to represent the distinguishable spatial correlation characteristics of the given SPD matrices in Fig. 3.

C. Euclidean Based Solution

An euclidean-based K-Means scheme takes the channel vectors \( \mathbf{h}_i \) as the inputs and applies the euclidean distance to calculate the estimated centroid. Unlike our proposed solution that clusters the channel covariance matrices on Riemannian manifold, the euclidean-based K-Means attempt to make \( K \)-partitions of the channel vectors by applying the following minimization formula [26]

\[
\min_{\mathbf{h}_k} \sum_{k=1}^{K} \sum_{i \in l_k} ||\mathbf{h}_i - \hat{h}_k||^2,
\]

(5)

where \( \hat{h}_k \) is the centroid obtained by the euclidean-based scheme for group \( R_k \). The solution then calculates the covariance matrices \( \mathbf{Q}_{i,k} \) of those classified channels for each group \( R_k \) as

\[
\mathbf{Q}_{i,k} = \mathbf{h}_{i,k} \mathbf{h}_{i,k}^H, (\forall i \in l_k).
\]

(6)

To compare the performance of our proposed G-ML solution with these benchmark euclidean-based schemes [12], [27], we also applied euclidean-based clustering of the channel vectors and computed their covariance matrices by applying (6). Finally, the covariance matrices are predicted from these matrices of the classified samples and the estimated mean (i.e., centroid) for each group.
IV. SIMULATION RESULTS

In this section, we describe the different datasets that we considered along with the simulation results for learning the covariance matrices in a vehicular environment.

A. Simulation Setup

Our work assumes a vehicular network operating at 60 GHz with 0.5 GHz bandwidth and the base station is equipped with $4 \times 4$ phased arrays. We adopt three different vehicular datasets from the “O1−60 ray tracing scenario” [28] with their channel models. We provide brief description of these datasets before explaining the simulation results.

The area of interest under Dataset-1 considers a total of 5460 user location points from user grid-1 of the “O1−60 ray tracing scenario” [28]. The area of interest for Dataset-1 is shown in Fig. 4 (a). For Dataset-2, our work assumes a total of 10860 location points from the considered user grid-1. The area of interest for Dataset-2 is illustrated in Fig. 4 (b). The considered area under Dataset-3 is the largest one which consists of a total of 16800 samples from user grid-1 as shown in 4 (c). The distance between any two location points are same for all the considered scenarios. The objective for considering different datasets is to observe the efficiency of the proposed scheme for different vehicular environments.

B. Performance Analysis

We use the normalized mean square error (NMSE) as the considered loss function to evaluate the performance of the proposed solution for predicting the mmWave channel covariance matrices according to the following

$$
NMSE_k = \frac{||\hat{G}_{k,z} - G_{k,z}||_F^2}{||G_{k,z}||_F^2}, \text{ (where } k = 1, 2, ..., K) \tag{7}
$$

where $G_{k,z} = \mathbb{E}[h_{i,k}h_{i,k}^H]$ is the expected value of the covariance matrices over the location $i$ for group $R_k$, and $||.||_F$ denotes the matrix Frobenius norm operator. Consequently, the average NMSE over all the given clusters is denoted as

$$
NMSE = \frac{1}{K} \sum_{k=1}^{K} NMSE_k. \tag{8}
$$

The proposed LEM-based Geometric K-Means ML was applied over the Riemannian manifold using the “geomstats” python package available in [29]. On the other hand, the euclidean-based conventional K-Means model can be applied using any of the available resources like [30]. We simulated the performance of the proposed non-euclidean based Geometric K-Means ML scheme and compared that with the euclidean-based solutions for all the three considered datasets.

Fig. 5 shows the average NMSE comparison of our proposed non-euclidean based solution with the euclidean-based scheme considering Dataset-2 only. The proposed solution outperforms the euclidean scheme significantly for any given number of clusters. For example, while the euclidean-based approach provides an average NMSE value around $3.5 \times 10^{-3}$ for having 4 clusters, the corresponding non-euclidean scheme achieves an NMSE around $7 \times 10^{-4}$, which is 80% less than the euclidean-based solution. In other words, the proposed Geometric K-Means ML solution can provide up to 80% higher accuracy compared to the euclidean-based one. Also, the lowest NMSE value is achieved at 4 clusters for both the solutions.

Fig. 5 shows the NMSE comparison for the non-euclidean and euclidean based solution for Dataset-2.

![Fig. 4: Datasets representing different portions of a street.](image)

![Fig. 5: NMSE comparison for the non-euclidean and euclidean based solution for Dataset-2.](image)
Fig. 6: NMSE comparison for having different data size.

Fig. 7: Number of optimal clusters versus data size.

Fig. 8: Average NMSE for different regions of Dataset-2 having 4 clusters.

Our approach always performs better compared to its corresponding euclidean-based scheme for any given data size. For instance, the NMSE of the proposed solution considering 5460 samples at 4 clusters is 50% less than that of its corresponding euclidean solution. These results indicate that clustering the channel covariance matrices directly on Riemannian manifolds based on LEM among covariance matrices is more accurate compared to clustering the channel vectors based on their euclidean distances. Note that both of our simulated cases apply unsupervised learning and require no prior labelled training in clustering the channel covariance matrices.

One more information can be extracted from Fig. 6. It can be seen that the lowest NMSE is achieved for having different clusters for the three different scenarios. While the least NMSE for 5460 samples is achieved for 2 clusters, the least NMSE is obtained for having 6 clusters when considering Dataset-3 with 16800 location spots. We plot the number of optimal clusters versus the data size in Fig. 7. For considering 5460 samples representing Dataset-1, we found 2 disjoint clusters would suffice for the area of interest. But as we increase the number of observations in Dataset-2 with more location points, the area of interest demands 4 disjoint clusters to provide the optimal result. Further extending the region with 16800 samples, the proposed solution requires 6 optimal clusters that provides the lowest average NMSE. These results suggest that the required number of optimal clusters may change according to the size of the dataset and the considered environment. Generally, dividing the data samples into more clusters is equivalent to adding more codewords (or simply beams) in the codebook design for the given location points. As such, we are required to estimate the optimal number of clusters to design an efficient codebook. Our results suggest that the adopted codebook for Dataset-1 should have two unique codewords, while Dataset-2 and Dataset-3 require 4 and 6 codewords, respectively. With these results for learning the covariance matrices, a matched environment-aware beamforming codebook can be designed accordingly by adopting any of the existing techniques in [5], [7].

We are now interested to see how fast our proposed solution can cluster the given covariance matrices. To investigate this, we consider our adopted Dataset-2 and divided that into 4 disjoint groups. First, we consider 500 location points in each group (i.e., 4 × 500 = 2000 samples from Dataset-2), and then gradually increased the group size to 2715 location points which is equivalent to considering the entire Dataset-2 with 10860 samples. Fig. 8 shows the average NMSE having 4 clusters for considering different group size from Dataset-2. Considering 2715 samples in each clustered group, the proposed solution provides an NMSE of $7 \times 10^{-4}$, but it increases to $8 \times 10^{-4}$ while considering 2000 samples in each group (i.e., 8000 location points). As a result, the NMSE of the proposed solution increases by almost 15% for reducing the data size by a total of 2860 samples. In other words, our solution can make the estimation around 26% faster by sacrificing only 15% in learning the channel covariance matrices. Thus, the proposed learning scheme balances a trade-off between fast estimation and higher accuracy.
V. CONCLUSIONS

In this paper, we proposed a novel unsupervised G-ML scheme for learning the covariance matrices in a mmWave vehicular environment. The proposed solution does not require any prior labelled data for training and makes use of the underlying Riemannian-geometric characteristics of the spatially correlated wireless channels while learning the covariance matrices. Simulation results suggest that the proposed non-euclidean based approach can provide up to 80% higher accuracy compared to the existing euclidean-based schemes. Also, our proposed solution can efficiently determine the required number of codewords for a given vehicular scenario. In addition, the proposed G-ML strategy is capable to act 26% faster with minimal sacrifice in the learning process.

REFERENCES

[1] J. Wu and P. Fan, “A Survey on High Mobility Wireless Communications: Challenges, Opportunities and Solutions,” IEEE Access, vol. 4, pp. 450-476, 2016.

[2] Vutha Va; Takayuki Shimizu; Gaurav Bansal; Robert W. Heath Jr., Millimeter Wave Vehicular Communications: A Survey, now, 2016.

[3] T. S. Rappaport et al., “Millimeter Wave Mobile Communications for 5G Cellular: It Will Work!,” IEEE Access, vol. 1, pp. 335-349, 2013, doi: 10.1109/ACCESS.2013.2260813.

[4] A. Adhikary, J. Nam, J. Ahn and G. Caire, “Joint Spatial Division and Multiplexing—The Large-Scale Array Regime,” IEEE Transactions on Information Theory, vol. 59, no. 10, pp. 6441-6463, Oct. 2013.

[5] D. J. Love and R. W. Heath, “Limited Feedback Diversity Techniques for Correlated Channels,” IEEE Transactions on Vehicular Technology, vol. 55, no. 2, pp. 718-722, March 2006.

[6] J. Choi, V. Va, N. Gonzalez-Prelcic, R. Daniels, C. R. Bhat and R. W. Heath, “Millimeter-Wave Vehicular Communication to Support Massive Automotive Sensing,” IEEE Communications Magazine, vol. 54, no. 12, pp. 160-167, December 2016.

[7] V. Raghavan, J. Nam, J. Ahn and G. Caire, “Systematic Codebook Designs for Quantized Beamforming in Correlated MIMO Channels,” IEEE Journal on Selected Areas in Communications, vol. 25, no. 7, pp. 1298-1310, September 2007.

[8] R. Hu, J. Tong, J. Xi, Q. Guo, and Y. Yu, “Channel Covariance Matrix Estimation via Dimension Reduction for Hybrid MIMO MMwave Communication Systems,” Sensors, vol. 19, no. 15, p. 3368, Jul. 2019.

[9] D. Neumann, M. Joham and W. Utschick, “Covariance Matrix Estimation in Massive MIMO,” IEEE Signal Processing Letters, vol. 25, no. 6, pp. 863-867, June 2018, doi: 10.1109/LSP.2018.2827323.

[10] S. Park and R. W. Heath, “Spatial Channel Covariance Estimation for the Hybrid MIMO Architecture: A Compressive Sensing-Based Approach,” IEEE Transactions on Wireless Communications, vol. 17, no. 12, pp. 8047-8062, Dec. 2018.

[11] Y. Zhang, M. Alrabeiah, and A. Alkhateeb, “Learning Beam Codebooks with Neural Networks: Towards Environment-Aware mmWave MIMO,” arXiv online, arXiv:2002.10663v1, 2020.

[12] X. Li, A. Alkhateeb and C. Tepedelenlioglu, “Generative Adversarial Estimation of Channel Covariance in Vehicular Millimeter Wave Systems,” 52nd Asilomar Conference on Signals, Systems, and Computers, pp. 1572-1576, Pacific Grove, CA, USA, 2018, doi: 10.1109/ACSSC.2018.8645463.

[13] “Unsupervised Learning: How Machines Learn on Their Own”, Accessed: Apr. 15, 2021. [Online]. Available: https://learn.g2.com/unsupervised-learning.

[14] A. S. Ibrahim, “Rethinking Maximum Flow Problem and Beamforming Design Through Brain-inspired Geometric Lens,” IEEE Global Communications Conference, Taiwan, pp. 1-6, 2020, doi: 10.1109/GLOBECOM2020.9321983.

[15] S. Sra and R. Hosseini, “Conic Geometric Optimization on the Manifold of Positive Definite Matrices,” SIAM Journal on Optimization, vol. 1, pp. 713-739, 2015.

[16] S. Gudmundsson,”An Introduction to Riemannian Geometry,” Lund University, 2014.

[17] W. Fan, C. Zhang, and Y. Huang, “Flat Beam Design for Massive MIMO Systems via Riemannian Optimization,” IEEE Wireless Communications Letters, vol. 8, no. 1, pp. 301-304, Feb. 2019.

[18] J. Chen, “Low-PAPR Precoding Design for Massive Multiuser MIMO Systems via Riemannian Manifold Optimization,” IEEE Communications Letters, vol. 21, no. 4, pp. 945-948, Apr. 2017.

[19] M. Arvanitidis, S. Hauberg, P. Hennig, and M. Schober, “Fast and Robust Shortest Paths on Manifolds Learned from Data,” in proc. of Machine Learning Research, vol. 89, pp. 1506–1515, 2019.

[20] X. Yu, J. Shen, J. Zhang, and K. B. Letaief, “Alternating Minimization Algorithms for Hybrid Precoding in Millimeter Wave MIMO Systems,” IEEE Journal of Selected Topics in Signal Processing, vol. 10, no. 3, pp. 485-500, 2016.

[21] V. Arsigny, P. Fillard, and N. Ayache, “Log-Euclidean Metrics for Fast and Simple Calculus on Diffusion Tensors,” Magn. Reson. Med. vol. 56, no. 2, pp. 411-421, 2006. doi: 10.1002/mrm.20965.

[22] Monti et al., “Geometric deep learning on graphs and manifolds using mixture model CNNs,” arXiv online, arXiv:1611.08402v3, 2016.

[23] M. M. Bronstein, J. Bruna, Y. LeCun, A. Szlam and P. Vandergheynst, “Geometric Deep Learning: Going beyond Euclidean data,” in IEEE Signal Processing Magazine, vol. 34, no. 4, pp. 18-42, July 2017.

[24] Y. Shinohara, T. Masuko and M. Akamine, “Covariance Clustering on Riemannian Manifolds for Acoustic Model Compression,” IEEE International Conference on Acoustics, Speech and Signal Processing, 2010, pp. 4326-4329, doi: 10.1109/ICASSP.2010.5495661.

[25] X. Pennec, P. Fillard, X. Pennec, and N. Ayache, “Log-Euclidean Metrics for Fast and Simple Calculus on Diffusion Tensors,” Magn. Reson. Med. vol. 56, no. 2, pp. 411-421, 2006. doi: 10.1002/mrm.20965.

[26] M. Alkhateeb, “DeepMIMO: A Generic Deep Learning Dataset for Millimeter Wave and Massive MIMO Applications,” in Proc. of Information Theory and Applications Workshop (ITA), San Diego, CA, Feb. 2019.

[27] A. Alkhateeb, “DeepMIMO: A Generic Deep Learning Dataset for Millimeter Wave and Massive MIMO Applications,” in Proc. of Information Theory & Applications Workshop (ITA), San Diego, CA, Feb. 2019.

[28] A. Alkhateeb, “DeepMIMO: A Generic Deep Learning Dataset for Millimeter Wave and Massive MIMO Applications,” in Proc. of Information Theory & Applications Workshop (ITA), San Diego, CA, Feb. 2019.

[29] N. Miolane, J. Mathe, C. Donnat, M. Jorda, and X. Pennec, “Geomstats: A Python Package for Riemannian Geometry in Machine Learning,” 2019.

[30] N. Miolane, J. Mathe, C. Donnat, M. Jorda, and X. Pennec, “Geomstats: A Python Package for Riemannian Geometry in Machine Learning,” 2019.