This workshop is meant to help you review the main topics covered so far in the course, in preparation for the upcoming midterm exam.

Work out each of the following problems on the board. Make sure you discuss your solution with your TAs before you erased the board and start a new problem.

1. Find the domain and range of $F(x) = f(f(x))$ where
   
   \[ f(x) = \frac{1}{x} \]

   Sketch the graph of $F(x)$ and determine whether $F$ is invertible.

2. Sketch the graph of a function $f(x)$ whose domain is all real numbers except 0, and such that
   
   \[ \lim_{x \to -1^+} f(x) = 1, \quad \lim_{x \to -1^-} f(x) = 3, \quad \lim_{x \to 0^+} f(x) = -\infty, \quad \lim_{x \to 2} f(x) = 1, \text{ and the slope of the tangent line to the curve at (3,2) is } -1. \]

3. Evaluate the following limits, if they exist.
   
   (i) \( \lim_{x \to 1} \frac{3x + 1}{2x - 1} \)
   
   (ii) \( \lim_{x \to 2} f(x) \) where
   
   \[ f(x) = \begin{cases} 
   x^2 - 3x + 4, & 0 \leq x \leq 2 \\
   8 - 3x, & 2 < x \leq 5 
   \end{cases} \]

   (iii) \( \lim_{x \to 1^+} \frac{x^2 - 2}{x^2 - 4x + 3} \)

4. Find a value of the constant $a$ such that \( \lim_{x \to a} f(x) \) exists if
   
   \[ f(x) = \begin{cases} 
   x^2 - 3x - 18, & x < a \\
   0, & x = a \\
   -2 - 3x, & x > a 
   \end{cases} \]

   Is $f$ continuous at $x = a$ (for $a$ equal to the value you found above)?

5. Carefully prove there is at least one solution to the equation $x^4 - x^3 + 2x^2 - 1 = 0$ between $x = -1$ and $x = 1$. Is there more than one solution?

6. Find the equation of the tangent line to the curve $y = x^2 - 2x + 2$ at the point with $x$-coordinate $x = 3$.

7. Sketch the parabolas $y = x^2$ and $y = x^2 - 2x + 2$. Is there a line that is tangent to both curves? If so, find its equation. If not, why not?