Rolling bearing fault diagnosis method based on local tangent space arrangement and VMD decomposition

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Abstract. In this paper, a method based on LTSA and VMD is proposed to effectively extract fault feature information of rolling bearing. The selection of reduced dimension $d$ and neighborhood selection parameter $k$ in LTSA adopts grid search method. Taking the fault characteristic energy ratio (FER) as the objective function, a set of optimal parameters is determined as the input parameter of the manifold learning algorithm. Variational modal decomposition (VMD) is used to denoise signal. The kurtosis value, correlation value and envelope entropy value are comprehensively evaluated, and the optimal component is selected for reconstruction. The optimal component of reconstruction is analyzed by envelope spectrum to realize fault diagnosis. The proposed method is applied to the fault measurement signal. The results show that the proposed LTSA-VMD fault diagnosis method has obvious advantages in signal denoising and fault feature extraction.

1. Introduction
Rolling bearings are the key components of rotating machinery and one of the main sources of failure in rotating machinery. Their working status is directly related to the safety of rotating machinery. Therefore, it is important to remove the noise from the original noisy signals obtained by the sensors and accurately extract the fault features [1].

The LTSA algorithm is applied to the signal of the rolling bearing, which can effectively realize the noise reduction processing of the signal [2]. The LTSA algorithm has two parameter selection problems: neighborhood selection $k$ and reduction dimension $d$. Different parameter choices have a different impact on the decomposition results. In this paper, an adaptive LTSA algorithm is proposed. In this method, the fault feature energy ratio is used as the objective function, and the grid search method is used to determine the optimal parameter.

Variational mode decomposition is a quasi-orthogonal, adaptive, multi-component signal decomposition method. It was proposed by Pro. Dragomiretskiy in 2014 [3]. The method is used to search the mode optimal solution within the variational mode framework and adaptively determines the center frequency and bandwidth of each component. In this paper, the VMD algorithm is used to decompose the signal denoised by adaptive LTSA algorithm. The value of kurtosis, envelope entropy and correlation of each mode are used as the mode selection standard. The modes with the largest kurtosis value, the smallest envelope entropy value and the highest correlation value are reconstructed as optimal reconstructed components. The optimal reconstruction component is analyzed by the envelope spectrum to extract the fault information.
2. Adaptive LTSA algorithm

2.1. Local Tangent Space Arrangement (LTSA) Algorithm Theory

Set the original fault signal as a one-dimensional time series signal, the m-dimensional phase space matrix is obtained by phase space reconstruction [4]: \( P = [X_1^m, X_2^m, \cdots X_m^m] \). The low dimensional manifold \( d < m \), in which the embedding dimension is calculated by Cao algorithm [5]. LTSA mainly includes the following four steps:

Step1. Selection of local domains. For each point in phase space, \( k \) neighborhood points of data points \( X_i \) are determined by Euclidean distance, \( i = 1, 2, \ldots, m \), and the neighborhood space is:

\[
[ X_i, X_{i+k}, \ldots X_{i+2k} ]
\]

Step2. Local linear fitting. A set of orthogonal base vectors \( Q_i \) are selected to construct the d-dimensional tangent space of \( X_i \), and the orthogonal projection

\[
\Theta_i^{(l)} = Q_i^T (x_i - \bar{x})
\]

from each point in the neighborhood to the tangent space is calculated. \( \bar{x} \) is the mean of the neighborhood data of the sample point \( X_i \). \( Q_i = X_i (I - ee^T/k) \), which is the top d largest left singular vector. Therefore, the local vector coordinates of the projection \( X_i \) in the tangent space are \( \Theta_i = [\theta_1^{(i)}, \theta_2^{(i)}, \ldots, \theta_d^{(i)}] \).

Step3. Global arrangement of local coordinates. Using \( B = SWW^T S^T \) to construct matrix, where

\[
W_i = (I - ee^T/k)(I - \Theta_i^T \Theta_i) ,
\]

\( Si \) is 0-1 selection matrix.

Step4. Low-dimensional global coordinate extraction. \( e \) is the eigenvector corresponding to the zero eigenvalue of matrix B. Therefore, the d eigenvectors corresponding to the second to \( d + 1 \) minimum eigenvalues are the optimal \( T \), which is the corresponding orthogonal low-dimensional global coordinate mapping matrix in the high-dimensional data set.

2.2. Fault feature energy ratio

The fault signal of rolling bearings is a periodic pulse sequence, which appears in the form of fault characteristic frequency and frequency doubling in its envelope spectrum. Therefore, the energy ratio of fault characteristic in envelope spectrum is defined as the objective function of adaptive manifold learning algorithm. Let the envelope spectrum amplitude sequence of one-dimensional time series fault signal \( X = [x_1, x_2, \cdots, x_n] \) be \( Y = [y_1, y_2, \cdots, y_L] \), \( L \) is the length of the envelope spectrum amplitude sequence, and the calculation formula of FER can be described as follows:

\[
\eta = \frac{\sum_{i=1}^{L} y_i^2}{\sum_{j=1}^{L} y_j^2}
\]

In Eq.1, \( y_j \) denotes the sequence of envelope spectrum amplitude, \( y_i \) denotes the amplitude of envelope spectrum at the i times frequency. The higher the fault characteristic frequency multiple, the lower the amplitude, takes \( K = 4 \) for example.

Taking FER value as the objective function of LTSA algorithm, the main steps of adaptive manifold learning algorithm designed by grid search are as follows:

Step1. Calculate the minimum embedding dimension \( m \) of the original signal using the Cao algorithm.

Step2. The original signal uses the phase space reconstruction theory to obtain the m-dimensional phase space matrix \( P \).

Step3. Set the initial parameters of the neighborhood selection range \( k \) and dimension reduction \( d \). In this paper, the range of \( d \) is \([1, m] \), the range of \( k \) is \([1, 40] \), and the search step size is one.

Step4. Using the grid search method to calculate the FER value of each \((k, d)\). After the grid search, the set of \((k, d)\) corresponding to the maximum FER value is the input of optimal parameter of the LTSA algorithm.

\[\text{(1)}\]
3. VMD decomposition based on adaptive LTSA algorithm

3.1. VMD algorithm
The basic idea of variational mode decomposition is the construction of variational problems and the solution of variational frames. Solving variational problems mainly includes: (1) introducing quadratic penalty factor $\alpha$ and Lagrange multiplier operator $\lambda(t)$ (2) transforming constrained variational problems into non-constrained variational problems. (3) updating each mode and center frequency by alternating direction multiplier algorithm. The specific implementation algorithm of VMD can be found in reference [6].

3.2. VMD algorithm modal component selection method
The kurtosis formula is shown in Eq.2, is usually used to reflect the impulse component in the signal and to describe the peak degree of the signal. It is a dimensionless parameter. The correlation coefficient formula is shown in Eq.3, which reflects the degree of correlation between the two signals. The range of correlation coefficient is $[-1, 1]$. The minimum envelope entropy formula is shown in Eq. 4, which reflects the sparsity of the signal. The smaller the envelope entropy, the higher the sparsity of the signal, the less noise of the component.

$$K = \frac{E(X - \mu)^4}{\sigma^4}$$  \hspace{1cm} (2)

$\sigma$ is the standard deviation of the signal, is the mean of the signal, $E(\cdot)$ is expected.

$$Corr(X, Y) = \frac{Cov(X, Y)}{\sqrt{D(X)}\sqrt{D(Y)}}$$  \hspace{1cm} (3)

$Cov(X, Y)$ is the covariance of signal X and Y. $D(X)$, $D(Y)$ is the variance of signal X and Y.

$$E_p = -\sum_{i=1}^{N} p_i \log p_i$$  \hspace{1cm} (4)

$p_i = a(i)/\sum_{i=1}^{N} a(i)$, $a(i)$ is the Hilbert envelope signal of signal X, and $p_i$ is the normalized representation of $a(i)$.

3.3. The steps of VMD decomposition based on adaptive LTSA algorithm
The LTSA algorithm can filter out most of the noise in the signal, highlight the impact components of the fault. As an adaptive variational modality algorithm, VMD algorithm can decompose the signal into a combination of multiple modal components and can suppress the modal aliasing phenomenon. The adaptive LTSA algorithm proposed in this paper is combined with the VMD algorithm. Using the kurtosis, correlation, and envelope entropy comprehensive evaluation indicators, the optimal mode of the VMD decomposition is selected for reconstruction and envelope spectrum analysis of reconstructed components.

The steps of the fault diagnosis method for rolling bearing combined with adaptive LTSA learning and VMD decomposition proposed in this paper are as follows:

Step1. Noise reduction of the original signal use adaptive LTSA algorithm
Step2. The denoised signal is decomposed by VMD.
Step3. The optimal modal component is selected by using kurtosis, correlation and envelope entropy, and reconstructing selected mode.
Step4. The reconstructed optimal modal component is analyzed by envelope spectrum, and compared with the theoretical fault characteristic frequency of bearing fault signal and judging the fault signal.

4. Simulation analysis of measured signals

In order to verify the effectiveness of the proposed method for noise reduction and fault feature enhancement of actual rolling bearing fault signals, the bearing data of Case Western Reserve University in the United States are selected. Single point of failure due to EDM specific fault diameter. The test uses the 6205-2RS-JEM-SKF deep groove ball bearing. Vibration data is acquired by using an accelerometer at a sampling frequency of 12 kHz, which is mounted on the drive end of the motor. In this paper, the outer ring fault signal with fault diameter (0.007 inch), motor load (0 hp) and the speed (1797 rpm) is selected as the measured data. The theoretical fault characteristic frequency of the rolling bearing outer ring fault signal is calculated to be 107.37 Hz. Figure 1 shows the original waveform of the outer ring fault of the rolling bearing. Figure 2 shows the relationship between the embedding dimension m and the E1 value calculated by the Cao algorithm. It can be seen that when m ≥ 14, the value of E1 remains basically unchanged. Therefore, the phase space embedding dimension m of the measured signal in this paper is 14.

![Figure 1. Original waveform of measured signal of rolling bearing](image1)

![Figure 2. Cao algorithm for embedding dimension of measured signal](image2)

Fig. 3 shows the three-dimensional relationship between reduction dimension d, domain selection k and fault feature energy ratio FER in adaptive LTSA algorithm. When the parameter (k, d) = (19, 1), the fault characteristic energy ratio is the largest, and the FER is 0.0155. Therefore (19, 1) is selected as the optimal parameter of LTSA algorithm for the measured signal. Fig. 4 shows the mode component diagram obtained by LTSA-VMD decomposition of measured signals.

![Figure 3. Three-dimensional relationship between reduction dimension d, domain selection k and fault feature energy ratio FER](image3)

![Figure 4. Mode component diagram obtained by LTSA-VMD decomposition of measured signals](image4)
Figure 3. Measured signal FER varying with d and k in LTSA

Figure 4. Modal components of measured signal after LTSA-VMD decomposition

The kurtosis, correlation and envelope entropy of each component of the measured signal decomposed by LTSA-VMD are shown in the table below.

It can be seen from Table 1 that the maximum component of the kurtosis is Mode1. The component with the highest correlation is Mode3, and the component with the minimum envelope entropy is Mode4. Therefore, Mode1, Mode3, and Mode4 are selected for reconstruction. The reconstructed component envelope spectrum is shown in Figure 5. It can be seen from Figure 5 that the fault frequency and its multiplication fault information are obvious. It is proved that the LTSA-VMD fault diagnosis method proposed in this paper can extract the fault characteristic frequency of the actual fault signal of the rolling bearing.

Table 1. The kurtosis, correlation, envelope entropy of each modal component of the measured signal

| Mode  | Kurtosis | Correlation | Envelope entropy |
|-------|----------|-------------|------------------|
| Mode1 | 6.4200   | 0.1193      | 3.2110           |
| Mode2 | 3.1165   | 0.6618      | 3.2632           |
| Mode3 | 2.5686   | 0.7031      | 3.2599           |
| Mode4 | 5.3236   | 0.4590      | 3.1568           |
| Mode5 | 4.3164   | 0.1381      | 3.2366           |

To prove the superiority of the modal selection method proposed in this paper, the envelope spectra of mode1, mode2 and Mode3 are analyzed respectively. Their envelope spectra are shown in Figure 6, Figure 7, Figure 8.
It can be seen from Figure 6 that the envelope spectrum fault characteristic frequency amplitude of Mode1 is low, and it is difficult to identify the fault components; The envelope spectrum of Mode3 can only extract the fundamental frequency and double frequency of the fault, and the amplitude of the interference noise frequency component before the fundamental frequency of the fault is large, which is easy to be confused with the fault frequency. Although the envelope spectrum of Mode4 can extract faults clearly, it is not clear enough in the enhancement of fault features and other high frequency features are not obvious. In summary, the proposed modal component selection method has great advantages in fault feature extraction.
5. Conclusion
In this paper, a fault diagnosis method of rolling bearing based on adaptive LTSA and VMD decomposition is studied. The concrete conclusions are as follows:

(1) The adaptive LTSA algorithm takes FER as the objective function and determines the optimal combination of input parameters by grid search algorithm. This method is more conducive to fault information extraction.

(2) In VMD decomposition, kurtosis, correlation and envelope entropy are taken as the comprehensive standard for selecting optimal mode and reconstructing optimal mode. Contrastive experiments show that this mode selection method is more conducive to extracting fault information.

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