Fake Massive Black Holes in the Milli-Hertz Gravitational-wave Band

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Abstract

In gravitational-wave (GW) astronomy accurate measurement of the source parameters, such as mass, relies on accurate waveform templates. Currently, templates are developed assuming that a source, such as a stellar-mass binary black hole (BBH), is residing in a vacuum. However, astrophysical models predict that BBHs could form in gaseous environments, such as common envelopes, stellar cores, and accretion disks of active galactic nuclei. Here we revisit the impact of gas on the GW waveforms of BBHs with a focus on the early inspiral phase when the GW frequency is around milli-Hertz. We show that for these BBHs, gas friction could dominate the dynamical evolution and hence duplicate chirp signals. The relevant hydrodynamical timescale, \( \tau_{\text{gas}} \), could be much shorter than the GW radiation timescale, \( \tau_{\text{gw}} \), in the above astrophysical scenarios. As a result, the observed chirp mass is higher than the real one by a factor of \( (1 + \tau_{\text{gw}}/\tau_{\text{gas}})^{1/2} \) if the gas effect is ignored in the data analysis. This kind of error also results in an overestimation of the source distance by a factor of \( (1 + \tau_{\text{gw}}/\tau_{\text{gas}}) \). By performing matched-filtering analysis in the milli-Hertz band, we prove that the gas-dominated signals are practically indistinguishable from the chirp signals of those more massive BBHs residing in a vacuum environment. Such fake massive objects in the milli-Hertz band, if not appropriately accounted for in the future, may alter our understanding of the formation, evolution, and detection of BBHs.

Unified Astronomy Thesaurus concepts: Gravitational wave sources (677); Accretion (14); Active galactic nuclei (16); Hydrodynamics (1963)

1. Introduction

The majority of black holes (BHs) detected by ground-based gravitational-wave (GW) observatories (i.e., LIGO and Virgo) have turned out to be several times more massive than those previously detected in X-ray binaries (The LIGO Scientific Collaboration & the Virgo Collaboration 2019). This discrepancy has important implications for the formation and evolution of stellar-mass binary BHs (BBHs, Abbott et al. 2016). However, mass is not a direct observable in GW astronomy. It is inferred either from the chirp signal, i.e., an increase of the GW frequency with time, or from the merger and ringdown signals. If the signal gets distorted, either at the moment of generation or during the propagation, an error is induced in the measurement of the mass.

Redshift is one such disturbing factor. It stretches the signal during its propagation. As a result, mass \( (m) \) is degenerate with redshift \( (z) \) such that one can only measure from GW signals the redshifted mass \( m(1+z) \) (Schutz 1986). In two astrophysical scenarios, the mass-redshift degeneracy could lead to an significant overestimation of the masses of BBHs. In the first scenario, a BBH is at a high cosmological redshift, and nevertheless is detected because the GW signal is magnified due to gravitational lensing (Broadhurst et al. 2018; Smith et al. 2018). So far, there is no convincing evidence supporting this scenario (Hannuksela et al. 2019). This scenario also has difficulties explaining the positive correlation between the apparent masses and distances of the detected BBHs (The LIGO Scientific Collaboration & the Virgo Collaboration 2019). In the second scenario, the BBH is captured by a supermassive black hole (SMBH) to a small distance, such that Doppler and gravitational redshifts become significant, which then leads to the mass-redshift degeneracy (Chen & Han 2018; Chen et al. 2019). In this case the GW signal of the BBH could also be gravitationally lensed by the SMBH (D’Orazio & Loeb 2019). The uncertainty of this latter scenario lies mainly in the poor understanding of the event rate. In both scenarios the signal in the LIGO/Virgo band (centered around \( 10^2 \) Hz) is similar to that of a not-redshifted but more massive BBH. Distinguishing them would be difficult partly because the signal is short, normally lasting no more than one second, which is too brief to reveal any signature of gravitational lensing (Hannuksela et al. 2019) or a nearby SMBH (Chen et al. 2019).

That difficulty would be alleviated if the BBH could be detected by a space-borne GW observatory, such as the Laser Interferometer Space Antenna (LISA, Amaro-Seoane et al. 2017). LISA is sensitive to milli-Hertz (mHz) GWs and thus can capture a BBH at the early inspiral phase, weeks to millennia before it enters the LIGO/Virgo band (Miller 2002; Sesana 2016; Moore et al. 2019). Using the chirp signal, LISA can also measure the (redshifted) mass of the BBH. In this way, LISA may detect hundreds of massive BBHs during its mission duration of 4–5 yr (Kytouk & Seto 2016; Sesana 2016; Kremer et al. 2018; Lamberts et al. 2018) and compare their masses with those from LIGO/Virgo observations.

Moreover, by tracking the BBHs with a decent signal-to-noise ratio \( (S/N, \text{e.g., } >10) \) for several months to years, LISA may reveal multiple images of a GW source if it is strongly lensed (Seto 2004; Sereno et al. 2011). The long signal may also reveal a shift of the GW phase caused by the wave effect of gravitational lensing (Nakamura 1998; Takahashi & Nakamura 2003). Moreover, if a BBH is close to a SMBH, the long waveform should also contain imprints of the orbital motion of the binary around the SMBH (Inayoshi et al. 2017; Meiron et al. 2017; Robson et al. 2018; Chamberlain et al. 2019; Wong et al. 2019; Tamanini et al. 2020; Torres-Orjuela et al. 2020) or the
perturbation of the binary orbit by the tidal force of the SMBH (Meiron et al. 2017; Fang et al. 2019; Hoang et al. 2019; Randall & Xianyu 2019). These signatures can help us identify the BBHs affected by redshift effects.

Redshift is not the only factor in GW astronomy that could affect the measurement of mass. Gas, for example, can exert a frictional force on a binary and hence lead to a faster orbital decay (Ostriker 1999; Kim & Kim 2007; Kim et al. 2008). The resulting GW signal is expected to differ from the real chirp signal due to GW radiation only. The impact on the mass measurement deserves further investigation since a large fraction of BBHs may form in gaseous environments. For example, BBHs can be produced by binary-star evolution, and in this case the mergers may happen inside a common envelope (Ivanova et al. 2013; MacLeod et al. 2017; Ginat et al. 2020) or the fallback material from the previous supernovae (Tagawa et al. 2018). Moreover, some BBHs may form in the accretion disks of active galactic nuclei (AGNs; McKernan et al. 2012; Bartos et al. 2017; Stone et al. 2017). When these BBHs merge, it is likely that they are surrounded by dense gas. Furthermore, the dense cores of massive stars may also produce BBHs, hence the mergers would also be accompanied by gas (Loeb 2016; Fedrow et al. 2017; D’Orazio & Loeb 2018). The density of the gas can reach $10^{16} - 10^{18}$ cm$^{-3}$ in the case of AGN disks: $10^{16} - 10^{19}$ cm$^{-3}$ in common envelopes, and even higher in stellar cores (see Antoni & MacLeod 2019, for a summary).

Several earlier works studied the impact of gas on the GW signal of merging BHs, including stellar-mass BHs (Fedrow et al. 2017; Cardoso & Maselli 2019; Caputo et al. 2020; Ginat et al. 2020) and SMBHs (Kocsis et al. 2011; Yunes et al. 2011; Barausse et al. 2014; Derdinski et al. 2019). They focused on the final evolutionary stage when the semimajor axes ($a$) of the binaries are only $10^{-12}$ times the Schwarzschild radius ($r_S$) of the bigger BHs. At this stage, GW radiation predominates the dynamics and gas plays a minor role. Nevertheless, these works showed that gas could induce a small phase shift to the GW signal. The phase shift is incompatible with a standard vacuum wave. From the power and the frequency evolution of the GW signal, one can derive a characteristic mass scale

$$M = \frac{c^3}{G \left(\frac{5f^{-11/3}}{96\pi^{8/3}}\right)^{3/5}} = \frac{(m_1m_2)^{3/5}}{(m_1 + m_2)^{1/5}},$$

which is known as the “chirp mass.” It uniquely determines the time evolution of $f$.

From the chirp signal one can also derive the distance $d$ of the BBH (Schutz 1986). This is because from $f$ and $\dot{f}$ one can infer the energy-loss rate of the orbit, $F \propto f^{-3} \dot{f}^2$, which also equals the GW power. In addition, the frequency $f$ and the GW amplitude $h$, together, determine a flux $S \propto h^2 f^2$, which is the GW flux. From the power and the flux, one can derive the distance of the source,

$$d = \frac{4G M}{c^2 h} \left(\frac{G}{c^3 \pi f M}\right)^{2/3}.$$  

If the BBH is at a cosmological distance, the mass and distance encoded in the chirp signal will have slightly different meanings. First, both $f$ and $\dot{f}$ will be distorted by the redshift such that the observed frequency becomes $f_o = f(1 + z)^{-1}$ and the chirping rate appears to be $\dot{f}_o = \dot{f}(1 + z)^{-2}$. As a result, the chirp mass that one will derive from the redshifted GW signal becomes

$$M_o = \frac{c^3}{G} \left(\frac{5f_o^{-11/3}}{96\pi^{8/3}}\right)^{3/5} = M(1 + z).$$

This apparent chirp mass is larger than the intrinsic one by a redshift factor $1 + z$. Second, the GW amplitude will be determined by the transverse comoving distance $d_C$ in the

$$d_C = \frac{r_S}{\sqrt{1 + z}}.$$
following way,
\[ h_o = \frac{4G M}{c^2} \pi f_o \mathcal{M}^{2/3} \cdot (5) \]
If one uses the observed \( f_o, \dot{f}_o, \) and \( h_o \) to infer a distance \( d_o \), one will get
\[ d_o = \frac{4G M_o}{c^2} \pi f_o \mathcal{M}_o^{2/3} = d_C(1 + z). \quad (6) \]
This distance is identical to the luminosity distance \( d_L \) in a \( \Lambda \)CDM cosmology.
Gas will accelerate the orbital shrinkage, and affect the chirp signal. Because of the gas friction (or viscosity, e.g., Haiman et al. 2009), a BBH would shrink at a faster rate of \( \dot{a} = \dot{a}_{\text{gw}} + \dot{a}_{\text{gas}}, \) where the additional term, \( \dot{a}_{\text{gas}} < 0, \) is due to gas. Correspondingly, \( \dot{f}_o \) increases more rapidly,
\[ \frac{\dot{f}_o}{f_o} = -\frac{3}{2} \left( \frac{\dot{a}_{\text{gw}} + \dot{a}_{\text{gas}}}{a(1 + z)} \right). \quad (7) \]
The apparent increase of \( \dot{f}_o \) will lead to an overestimation of the mass of the BBH, as well as the distance. To see this effect, it is useful to first define an acceleration factor:
\[ \Gamma := \frac{a_{\text{gas}}}{a_{\text{gw}}}. \quad (8) \]
From this definition, it follows that \( \dot{f}_o = (1 + \Gamma)(1 + z)^{-\frac{3}{2}} \dot{f}_o \), i.e., the chirp signal evolves faster by a factor of \( 1 + \Gamma \). Finally, by revisiting Equations (4) and (6), we find that the apparent mass and distance become
\[ \mathcal{M}_o = (1 + \Gamma)^{3/5} \mathcal{M}(1 + z), \quad (9) \]
\[ d_o = (1 + \Gamma) d_C(1 + z) = (1 + \Gamma) d_L. \quad (10) \]
The factor \( \Gamma \) in general is a function of \( a \), because both \( \dot{a}_{\text{gw}} \) and \( \dot{a}_{\text{gas}} \) depend on \( a \). This dependence has two consequences. (i) If the semimajor axis changes substantially during the observational period, there is a significant variation of \( \mathcal{M}_o \) and \( d_o \) with time. This result is inconsistent with the dynamical evolution of a BBH in a vacuum, and hence can be used to prove the presence of an environmental factor, such as gas. (ii) Otherwise, if \( a \) evolves very slowly, i.e., the corresponding evolutionary timescale \( a/\dot{a}_{\text{gw}} + \dot{a}_{\text{gas}} \) is much longer than the observational period \( T_{\text{obs}} \), the acceleration factor \( \Gamma \) is more or less a constant. In this case, the measurement of \( \mathcal{M}_o \) and \( d_o \) is relatively consistent during the observational period, and both values are greater than the intrinsic ones.
For LIGO/Virgo, the relevant BBHs are normally in the first case because the signal is typically less than a second but each LIGO/Virgo observing run lasts several weeks to several months. This is why previous studies found that the gas effect could be discerned in the LIGO/Virgo waveforms (see Section 1). For LISA, however, the majority of the in-band BBHs belong to the latter case because a BBH could dwell in the band for as long as millions of years but the canonical mission duration of LISA is only 4–5 yr. During such a short observing time, \( \Gamma \) is almost constant such that discriminating the gas effect is more difficult. An overestimation of the mass and distance becomes more likely.
A small fraction of the LISA BBHs could be compact enough to evolve rapidly on a timescale of years (Sesana 2016). They also fall in the first scenario. This is the reason that Caputo et al. (2020) found a large mismatch between the gas-dominated waveforms and the vacuum (effectual) templates for their BBHs.

### 3. Hydrodynamics versus GW Radiation
To evaluate the effectiveness of the hydrodynamical drag, we compare the GW radiation timescale, defined as \( \tau_{\text{gw}} := [\dot{a}/\dot{a}_{\text{gw}}] \), and the hydrodynamical timescale, defined as \( \tau_{\text{gas}} := [\dot{a}/\dot{a}_{\text{gas}}] \). Following these definitions, the acceleration factor \( \Gamma \) equals \( \tau_{\text{gw}}/\tau_{\text{gas}} \). Because the most sensitive band of LISA is around 3 mHz, the corresponding BBHs have a typical semimajor axis of
\[ a = \left( \frac{G m_2}{\pi f^2} \right)^{1/3} \approx 0.0021 \left( \frac{m_2}{20 \, M_\odot} \right)^{1/3} \left( \frac{f}{3 \, \text{mHz}} \right)^{-2/3} \text{au}. \quad (11) \]
According to Equation (1), without gas these binaries have a typical evolutionary timescale of
\[ \tau_{\text{gw}} = \frac{5}{64 G^3 m_1 m_2 m_3^2 c^4} \]
\[ \approx 9.1 \times 10^3 \frac{q}{(1 + q)^{1/3}} \left( \frac{m_1}{10 \, M_\odot} \right)^{-1/3} \left( \frac{f}{3 \, \text{mHz}} \right)^{-8/3} \text{yr}. \quad (12) \]
where \( q \) denotes the mass ratio \( m_2/m_1 \) of the two BHs (we assume \( m_1 \geq m_2 \)).
As for \( \tau_{\text{gas}} \), we first use the hydrodynamic drag derived in Ostriker (1999) and Sánchez-Salcedo & Brandenburg (1999) to estimate its value. The drag force on the secondary BH \( m_2 \) is calculated with \( F \approx 4\pi \rho c^2 (m_2/v)^2 \), where \( \rho \) is the mass density of the background gas and \( v \) is the Kepler velocity of the secondary. For circular orbits, we have \( v \propto a^{-1/2} \), such that \( a/\dot{a}_{\text{gas}} = -v/(2v) \). Moreover, since \( |v| = F/m_2 \), we derive
\[ \tau_{\text{gas}} = \frac{m_2 v}{2F} \approx \frac{1.1 \times 10^4}{q(1 + q)^2} \left( \frac{n}{10^{16} \, \text{cm}^{-3}} \right)^{-1} \left( \frac{f}{3 \, \text{mHz}} \right)^{1/3} \text{yr}. \quad (13) \]
where \( n \) is the number density of hydrogen atoms in the gas background. It follows that
\[ \Gamma \approx 4.3 \left( \frac{1 + q}{2} \right)^{1/3} \left( \frac{n}{10^{16} \, \text{cm}^{-3}} \right)^{-1/3} \left( \frac{f}{3 \, \text{mHz}} \right)^{-11/3}. \quad (15) \]
In the above derivation of \( \tau_{\text{gas}} \), it is assumed that the gas background is homogeneous and the small body is moving in a straight line. However, for the BHs in binaries, which move along Keplerian orbits, it has been shown that the formula for the drag force will be modified, because the shape of the density wake is different (Sánchez-Salcedo & Brandenburg 2001; Escala et al. 2004; Kim & Kim 2007; Kim et al. 2008). More recently, Antoni & MacLeod (2019) showed that when \( a \) is smaller than the Bondi accretion radius \( R_{\text{acc}} = G m_2/c^2 \) (\( c \) being the sound speed of the gas medium), the gas density close to the binary will be much higher than the background density due to accretion. This is normally the case for BBHs embedded in common envelopes and AGN accretion disks. If we use \( n \approx n(R_{\text{acc}}/a)^{3/2} \) to correct the gas density around the binary (Bondi 1952; Antoni & MacLeod 2019), we
find that the timescale due to hydrodynamical drag becomes

\[ T_{\text{gas}} \simeq 8.5 \times 10^4 q^{-1}(1 + q)^{-3} \left( \frac{n}{10^{11} \text{ cm}^{-3}} \right)^{-1} \]

\[ \times \left( \frac{m_1}{10 M_\odot} \right)^{-2/3} \left( \frac{c_\text{s}}{10^2 \text{ km s}^{-1}} \right)^{-1} \left( \frac{f}{3 \text{ mHz}} \right)^{8/3} \text{ yr.} \]  

(16)

Note that the new timescale does not depend on \( a \) or \( f \). A similar result can be found in Bartos et al. (2017, see their Figure 2). Moreover, in the last equation we have rescaled \( n \) with \( 10^{11} \text{ cm}^{-3} \). Given this timescale, we derive that

\[ \Gamma \simeq 1.1 \left( \frac{1 + q}{2} \right)^{10/3} \left( \frac{n}{10^{11} \text{ cm}^{-3}} \right)^{-1} \]  

\[ \times \left( \frac{m_1}{10 M_\odot} \right)^{-2/3} \left( \frac{c_\text{s}}{10^2 \text{ km s}^{-1}} \right)^{-1} \left( \frac{f}{3 \text{ mHz}} \right)^{8/3}. \]  

(17)

For illustrative purposes, we show in Figure 1 the dependence of the \( \Gamma \) computed in the last equation on \( m_1 \) and \( n \). The black dashed curve marks the location where \( \Gamma = 1 \). Above it, gas dominates the dynamical evolution of a BBH, hence the chirp signal is determined by gas dynamics, not GW radiation.

Equations (15) and (17) suggest that for LIGO/Virgo BBHs, which typically have \( m_1 \sim 10 M_\odot \), and \( f \sim 10^2 \text{ Hz} \), the gas effect is negligible unless the gas density \( n \) is orders of magnitude higher than \( 10^{10} \text{ cm}^{-3} \). This is the reason that for LIGO/Virgo sources, a significant gas effect is expected only in stellar cores, where \( n \) could be as high as \( (10^{28} - 10^{31}) \text{ cm}^{-3} \) (e.g., Fedrow et al. 2017). For LISA sources with \( f \sim 3 \text{ mHz} \), however, the gas effect is already important when \( n \sim 10^{10} \text{ cm}^{-3} \), according to Equation (15), or \( n \sim 10^{12} \text{ cm}^{-3} \), according to Equation (17). These two characteristic densities can be found, respectively, in common envelopes and AGN accretion disks (e.g., Antoni & MacLeod 2019). Therefore, the gaseous environments common for BBHs would affect the LISA signals more than the LIGO/Virgo ones.

4. Faking a Chirp Signal

Having considered the effect and the relative importance of gas, we now compute the chirp signal of a BBH embedded in a gaseous environment. In the following, we assume \( z = 0 \) for simplicity (\( f_0 = f \)). When there is no gas, we calculate the time derivative of the GW frequency (\( \dot{f}_\text{gw} \)) using a 3.5 post-Newtonian (PN) approximation presented in Sathyaprakash & Schutz (2009). When gas is present, we have \( \dot{f}_0 = \dot{f}_\text{gw} + \dot{f}_\text{gas} \), where \( \dot{f}_\text{gas} \) is calculated with \( \dot{f}_\text{gas} = -3f(2\tau_\text{gas})^{-1} \) according to Equation (7). In this way, the gas effect is included in the model through a parameter \( \tau_\text{gas} \).

Figure 2 compares the long-term evolution of the chirp signal in the cases with and without the gas drag. The blue solid curve corresponds to a \( 10M_\odot - 10M_\odot \) BBH (\( M \simeq 8.7 M_\odot \)) embedded in a gaseous medium. The two BHs coalesce at the time \( t = t_0 \). The hydrodynamical timescale \( \tau_\text{gas} \) is computed according to Equation (16) and the model parameters are chosen such that \( \tau_\text{gw}/\tau_\text{gas} = 10 \) when \( f = 3 \text{ mHz} \). This chirp signal around \( f = 3 \text{ mHz} \), according to Equation (9), should resemble a more massive binary with a chirp mass of \( (1 + \Gamma)^{3/5}M \simeq 37 M_\odot \) in a vacuum. The chirp signal of the latter more massive BBH is shown as the dotted–dashed curve, and we can see that, indeed, at \( f = 3 \text{ mHz} \) it matches the signal of the smaller binary embedded in gas. Eventually, the two signals diverge, since \( \Gamma \) is decreasing, but the divergence appears more than one hundred years later. If we focus on the LISA observational window of five years (marked by the two vertical lines), the two signals are almost identical. Finally, during the coalescence, the blue curve recovers the chirp signal of a \( 10M_\odot - 10M_\odot \) binary in a vacuum (red dashed curve), because GW radiation dominates during the merger.

To see more clearly the chirp signal in the observation window of LISA, we show in Figure 3 the evolution of \( f \) during...
a period of 1–2 yr, around the moment when \( f \) is approximately 3 mHz. Now the chirp signals look like straight lines because the observational period is orders of magnitude shorter than the evolutionary timescales of the BBHs. Although the variation of \( f \) is small during the observation period, it is detectable by LISA because LISA’s resolution is approximately \( 10^{-8}(1 \text{ yr}/T_{\text{obs}})(10/S/N) \) Hz (Seto 2002). Comparing the blue solid and the red dashed lines, we see that the presence of gas increases the slope the chirp signal. The steeper line resembles the chirp signal of a more massive BBH in a vacuum with a chirp mass of 37 \( M_\odot \).

We have seen that with or without gas, the chirp signals in the LISA band are almost straight lines and are relatively featureless compared to those in the LIGO/Virgo band. This is the main reason that in this band a BBH in a gaseous environment could be misidentified as a more massive binary in a vacuum.

### 5. Misidentification

LISA uses a technique called “matched-filtering” to search for BBHs in the data stream (Finn 1992; Cutler & Flanagan 1994). In this section, we will show that this method cannot distinguish light BBHs embedded in certain gaseous environments from massive ones in vacuum.

#### 5.1. Matched Filtering

Given two waveforms, \( h_1 \) and \( h_2 \), their similarity is quantified by a “fitting factor” (FF), which is defined as

\[
\text{FF} = \frac{\langle h_1 | h_2 \rangle}{\sqrt{\langle h_1 | h_1 \rangle \langle h_2 | h_2 \rangle}}.
\]

The term \( \langle h_1 | h_2 \rangle \) means an inner product of

\[
\langle h_1 | h_2 \rangle = 2 \int_0^\infty \tilde{h}_1(f) \tilde{h}_2^*(f) + \tilde{h}_1^*(f) \tilde{h}_2(f) S_n(f) df,
\]

where the tilde symbols stand for the Fourier transformation, the stars stand for the complex conjugation, and \( S_n(f) \) is the spectral noise density of LISA (Klein et al. 2016). Identical waveforms have FF = 1.

In our problem, \( h_1(t) \) is the chirp signal of a BBH embedded in a gaseous environment, and \( h_2(t) \) is the waveform of an inspiraling BBH in a vacuum. By tuning the parameters of \( h_2 \), we want to maximize the FF. We follow Cutler & Flanagan (1994) and compute the waveforms using

\[
h(t) = \frac{Q(\theta, \varphi, \psi, \iota) M}{d_L a(t)} \cos \left( \int 2\pi f dt \right),
\]

where \( Q(\theta, \varphi, \psi, \iota) \) is a function depending on the sky location and orientation of the BBH. In the integrand, the frequency \( f \) is a function of a. It is computed using the 3.5 PN approximation for \( h_2 \) (Sathyaprakash & Schutz 2009) and using the gas model described in Section 4 for \( h_1 \).

Because the two evolutionary timescales \( \tau_{\text{gas}} \) and \( \tau_{\text{gw}} \) are both much longer than the observational period of LISA, \( f \) is almost a constant in our model. In this case the computation of the inner product \( \langle h_1 | h_2 \rangle \) can be performed in the time domain and the calculation of the FF can be simplified. First, the noise curve \( S_n(t) \) can be taken out of the integration because of the small variation of \( f \), such that

\[
\langle h_1 | h_2 \rangle \approx \frac{2}{S_n} \int_0^\infty [\tilde{h}_1^*(f) \tilde{h}_2(f) + \tilde{h}_1(f) \tilde{h}_2^*(f)] df.
\]

Second, using Parseval’s theorem, we further derive

\[
\langle h_1 | h_2 \rangle \approx \frac{4}{S} \int_0^\infty h_1(t) h_2(t) dt.
\]

Finally, the FF can be written as

\[
\text{FF} = \frac{\int_0^\infty h_1(t) h_2(t) dt}{\sqrt{\int_0^\infty h_1(t) h_1(t) dt \int_0^\infty h_2(t) h_2(t) dt}}.
\]

Note that there is no more dependence on \( d_L, Q(\theta, \varphi, \psi, \iota) \), or \( S_n(t) \), because they all cancel out. Given \( h_1 \), i.e., the signal, we want to find a template \( h_2 \) that maximizes the FF. The parameter space in which we conduct this search is \( (M_c, q, \phi) \), where \( \phi \) is the initial phase.

#### 5.2. Examples

Figure 4 shows one example of our search. The signal is generated using a BBH with \( M_c = 8.7 M_\odot, q = 0.7 \), and a hydrodynamical timescale of \( \tau_{\text{gas}} = 10^3 \) yr. Initially, the GW frequency is \( f = 3 \) mHz, and the corresponding GW radiation timescale is about 104 yr. In this particular example the observational period is set to \( T_{\text{obs}} = 1.25 \) yr, but later we will show the FF for different \( T_{\text{obs}} \). We match the signal using the templates developed for vacuum BBHs, i.e., the 3.5 PN approximation described above. We use a simulated annealing algorithm to search for the highest FF in the parameter space of \( (M_c, q, \phi) \). The best FF is found at a chirp mass of \( M_c \approx 37 M_\odot \). It is offset from the real chirp masses by a factor of about 4.2, which is consistent with Equation (9). This result confirms our prediction that ignoring the gas effect could result in a significant overestimation of the mass of a LISA BBH.

We note that the best FF and the corresponding best matched \( M_c \) are both functions of \( T_{\text{obs}} \). Figure 5 shows this dependence on time. The model parameters are the same as those in
Figure 4. Dependence of the FF on the chirp mass $M_c$ and initial phase $\phi$ of the template $h_2$. The dependence on $q$ is relatively weak and is not shown here. The signal $h_1$ is generated using the parameters $M_c = 8.7 M_\odot$, $q = 0.7$, and $\tau_{\text{gas}} = 10^3$ yr. The plus symbol marks the location of the maximum FF, which is offset from the real chirp mass ($8.7 M_\odot$).

Figure 5. Variation of the maximum FF and the best-fit chirp mass with the observational period. The real chirp mass of the BBH is $8.7 M_\odot$.

Figure 6. Variation of the maximum FF with time assuming different values of $\tau_{\text{gas}}$.

According to this criterion, the gas and vacuum waveforms used in Figure 5 are indistinguishable during the first 1–2 yr of observation. Only in the third year could one start to tell the difference and prove that the signal is not produced by a massive BBH of $M_c \approx 37 M_\odot$ residing in a vacuum environment.

For completeness, we show in Figure 6 the FF derived assuming different values for $\tau_{\text{gas}}$. The other model parameters are the same as those in Figure 4. As $\tau_{\text{gas}}$ increases, the FF gets better at later times because the gas effect becomes weaker. We note that when $\tau_{\text{gas}} \gtrsim 5000$ yr, the FF is better than 0.995 for almost five years, which is equivalent to the canonical mission duration of LISA. As a result, LISA may identify our BBH of a chirp mass of $8.7 M_\odot$ as a more massive binary. The measured chirp mass is approximately $18 M_\odot$ when $\tau_{\text{gas}} = 5000$ yr and $14 M_\odot$ when $\tau_{\text{gas}} = 10^4$ yr. In both cases, the overestimation of the mass, and hence the distance (see Equation (10)), is substantial.

6. Discussions

We have seen that those BBHs with $\Gamma = \tau_{\text{gas}}/\tau_{\text{gas}} \gtrsim 1$ could be misidentified by LISA as more massive binaries residing in vacuum environments. To what distance could LISA detect such fake massive binaries? The standard way of addressing this question is to derive the maximum distance at which the S/N drops to a threshold, say $S/N = 10$. This distance is known as the “detection horizon.” In the case without gas, the detection horizon has been derived in several works assuming different LISA configurations. For example, Kyutoku & Seto (2017) showed that

$$d_L \approx \frac{13 \left( \frac{M_c}{10 M_\odot} \right)^{5/3} \left( \frac{T_{\text{obs}}}{5 \text{ yr}} \right)^{1/2} \left( \frac{S/N}{10} \right)^{-1}}{\left( \frac{S_c(f)}{10^{-40} \text{ Hz}^{-1}} \right)^{-1/2} \left( \frac{f}{3 \text{ mHz}} \right)^{2/3} \text{ Mpc}}$$

for the N2A5 configuration of LISA. Here $M_c$ and $d_L$ refer to the real chirp mass and real distance of the source.

In fact, the last equation is also valid in the case with gas. This is the case for the following reasons. (i) The stationary
phase approximation (Thorne 1987) in which the last equation is derived remains valid, since the evolutionary timescale \( f / f \) for the frequency is much longer than the GW period \( 1 / f \). We note that \( f \) here stands for the observed frequency. We dropped the subscript \( o \) in \( f \), for simplicity. (ii) In this approximation, the characteristic amplitude defined as \( h_c(f) = 2\tilde{h}(f) \) becomes proportional to \( A f^{-1/2} \), where \( A \) is a function of \( M_c \), \( q \), \( f \), and \( d_L \), as well as the sky location and orientation of the binary. Noting that \( f = f_{gw} + f_g > f_{gw} \), we find that gas in general reduces the characteristic amplitude at any frequency. The reduction is due to a faster drift of the signal in the frequency domain. (iii) Using \( h_c \), we can rewrite the S/N defined in Equation (24) as

\[
S / N^2 = \int_{f_1}^{f_2} \frac{|h_c(f)|^2}{f^2 S_n(f)} df, \tag{27}
\]

where \( f_1 \) and \( f_2 \) denote the minimum and maximum frequencies during the observational period. Because \( \Delta f = f_2 - f_1 \ll f \), the integration becomes proportional to \( |h_c|^2 \Delta f / (f^2 S_n) \). (iv) In our problem we have \( T_{obs} \ll |f / f| \), because \( T_{obs} \) is typically five years but \( |f / f| \) due to gas friction is about \( 10^{5} \)–\( 10^{4} \) yr. Because of their long lifetimes (long \( |f / f| \)), the BBHs we are interested in, unlike those in Sesana (2016), are not suitable for joint observations by LIGO/Virgo and LISA. For the same reason, throughout this work we have adopted the conventional threshold \( S / N = 10 \) for LISA BBHs. For joint LIGO/Vigo/LISA observations, it is known that a more stringent threshold (\( S / N \approx 15 \)) is needed (Moore et al. 2019). Therefore, we can write \( \Delta f \approx \hat{f} T_{obs} \). Again, by noting that \( \hat{f} > f_{gw} \), we find that \( \Delta f \) is broader when gas is present. (v) Finally, the \( \hat{f}^{-1} \) from the term \( |h_c|^2 \) cancels the \( \hat{f} \) from the term \( \Delta f \), such that the S/N does not depend on \( \hat{f} \). Physically, this means the reduction of the characteristic amplitude is compensated by the larger frequency drift.

The above conclusion that gas does not affect the S/N is derived in the scenario of LISA observations. It does not apply to LIGO/Virgo because in the latter case the assumption \( T_{obs} \ll |f / f| \) is invalid. In fact, gas will reduce the S/N for LIGO/Virgo sources by suppressing \( |h_c| \). Nevertheless, the corresponding change of S/N is small because, as has been explained in Section 3, the acceleration factor \( \Gamma \) is small when a BBH enters the LIGO/Virgo band. Therefore, we can use Equation (26) to estimate the detection horizon and subsequently discuss the detectability of the fake massive BBHs embedded in gaseous environments. For \( M_c = 10 M_\odot \), the detection horizon corresponding to an S/N of 10 is approximately 13 Mpc, assuming 5 yr of observation. If \( M_c = 30 M_\odot \), as the LIGO/Virgo observations tend to suggest (The LIGO Scientific Collaboration & the Virgo Collaboration 2019), the detection horizon increases to about 80 Mpc. To estimate the number of fake massive BBHs within the detection horizon, we start from the event rate in the LIGO/Virgo band, which is estimated to be \( O(10^5) \) Gpc\(^{-3} \) yr\(^{-1} \) (The LIGO Scientific Collaboration & the Virgo Collaboration 2019). According to this rate, the number of BBHs in the last year before their coalescence is about \( N (\tau = 1 \text{yr}) = 100 \) per Gpc\(^3 \), where \( \tau = |a/\dot{a}| \) denotes the orbital evolutionary timescale. For the other BBHs at an earlier evolutionary stage, we can follow the continuity equation (e.g., see Section V in Amaro-Seoane 2019) and derive that \( dN / d\ln a \propto \tau \). In our problem, \( \tau = (1 / f_{gw} + \tau_{gas})^{-1} \). At a frequency of \( f = 3 \text{mHz} \), where LISA is the most sensitive, \( \tau_{gw} \approx (1500–9000) \text{yr} \) when \( M_c \) varies from \( 30 M_\odot \) to \( 10 M_\odot \). (i) Without gas, \( \tau = \tau_{gw} \), and we find that the number density of BBHs at \( f \approx 3 \text{mHz} \) is about \( (1.5–9) \times 10^5 \) Gpc\(^{-3} \). The number of BBHs inside the detection horizon is 8–320. (ii) With gas, the number is smaller because \( \tau \) is shortened by gas friction. In the extreme case in which all BBHs are embedded in gas, if we assume \( \tau_{gas} = 10^3 \text{yr} \), we find that \( \tau \approx (600–900) \text{yr} \) when \( M_c \) varies from \( 30 M_\odot \) to \( 10 M_\odot \). Correspondingly, the number density of BBHs at \( f \approx 3 \text{mHz} \) is \( (6–9) \times 10^4 \) Gpc\(^{-3} \). The number of BBHs inside the detection horizon decreases to \( 0.8–130 \), but is not zero.

Note that the decay of a BBH in a gaseous medium may be more complicated than what is depicted in our model. In particular, it has been shown that the density waves in the circumbinary disk (Roedig et al. 2012; Derdzinski et al. 2019) or the accretion onto individual BBHs (Shi et al. 2012; Duffell et al. 2019; Muñoz et al. 2019) may induce a positive torque on the binary. As a result, the binary may expand, which could slow down the chirp signal and result in a smaller chirp mass. The magnitude of these additional hydrodynamical effects is sensitive to the viscosity and the thermodynamical properties of the gas (Tang et al. 2017), a tendency that deserves further exploration.

Since BBHs embedded in gaseous environments could be common, the effect of hydrodynamics should be considered more carefully in waveform modeling. Otherwise, as our results suggest, LISA may provide a biased demography of BBHs. This bias may also affect future cosmology studies, given the possibility of using BBHs as standard sirens to measure cosmological parameters.

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References
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2016, ApJL, 818, L22
Abbott, B.P., Abbott, R., Abbott, T.D., et al. 2019, PhRvX, 9, 031040
Amaro-Seoane, P. 2019, PhRvD, 99, 123025
Amaro-Seoane, P., Audley, H., Babak, S., et al. 2017, arXiv:1702.00786
Amaro-Seoane, P., Babak, S., Perri, M., et al. 2019, PhRvX, 9, 031040
Antoni, A., MacLeod, M., & Ramirez-Ruiz, E. 2019, ApJ, 884, 22
Barausse, E., Cardoso, V., & Pani, P. 2014, PhRvD, 89, 104059
Bartos, I., Kocsis, B., Haiman, Z., & Márka, S. 2017, ApJ, 835, 15
Bondi, H. 1952, MNRAS, 112, 195
Broadhurst, T., Diego, J. M., & Smoot, G. F. 2018, arXiv:1802.05273
Caputo, A., Shiba, L., Toubiana, A., et al. 2020, ApJ, 892, 90
Cardoso, V., & Masielli, A. 2019, arXiv:1909.05870
Chamberlain, K., Moore, C. J., Gerosa, D., & Yunes, N. 2019, PhRvD, 99, 024025
Chen, X., & Amaro-Seoane, P. 2017, ApJL, 842, L2
Chen, X., & Han, W.-B. 2018, CmPhy, 1, 53
Chen, X., Li, S., & Cao, Z. 2019, MNRAS, 485, L141

Chen, Xuan, & Peng
Chen, X., & Shen, Z. 2019, Proc. of Recent Progress in Relativistic Astrophysics, 17, 4
Cutler, C., & Flanagan, É. É. 1994, PhRvD, 49, 2658
D’Orazio, D. J., & Loeb, A. 2018, PhRvD, 97, 083008
D’Orazio, D. J., & Loeb, A. 2019, arXiv:1910.02966
Derdzinski, A. M., D’Orazio, D., Duffell, P., et al. 2019, MNRAS, 486, 2754
Duffell, P. C., D’Orazio, D., Derdzinski, A., et al. 2019, arXiv:1911.05506
Escala, A., Larson, R. B., Coppi, P. S., & Mardones, D. 2004, ApJ, 607, 765
Fang, Y., Chen, X., & Huang, Q.-G. 2019, ApJ, 887, 210
Fedrow, J. M., Ott, C. D., Sperhake, U., et al. 2017, PhRvL, 119, 171103
Finn, L. S. 1992, PhRvD, 46, 5236
Ginat, Y. B., Glanz, H., Perets, H. B., Grishin, E., & Desjacques, V. 2020, MNRAS, 493, 4861
Haiman, Z., Kocsis, B., & Menou, K. 2009, ApJ, 700, 1952
Hannuksela, O. A., Haris, K., Ng, K. K. Y., et al. 2019, ApJL, 874, L2
Hoang, B.-M., Naoz, S., Kocsis, B., Farr, W. M., & McIver, J. 2019, ApJL, 875, L31
Inayoshi, K., Tamanini, N., Caprini, C., & Haiman, Z. 2017, PhRvD, 96, 063014
Ivanova, N., Justham, S., Chen, X., et al. 2013, A&ARv, 21, 59
Kim, H., & Kim, W.-T. 2007, ApJ, 665, 432
Kim, H., Kim, W.-T., & Sánchez-Salcedo, F. J. 2008, ApJL, 679, L33
Klein, A., Barausse, E., Sesana, A., et al. 2016, PhRvD, 93, 024003
Kocsis, B., Yunes, N., & Loeb, A. 2011, PhRvD, 84, 024032
Kremer, K., Chatterjee, S., Breivik, K., et al. 2018, PhRvL, 120, 191103
Kyutoku, K., & Seto, N. 2016, MNRAS, 462, 2177
Kyutoku, K., & Seto, N. 2017, PhRvD, 95, 083525
Lamberts, A., Garrison-Kimmel, S., Hopkins, P. F., et al. 2018, MNRAS, 480, 2704
Lindblom, L., Owen, B. J., & Brown, D. A. 2008, PhRvD, 78, 124020
Loeb, A. 2016, ApJ, 819, L21
MacLeod, M., Antoni, A., Murguia-Berthier, A., Macias, P., & Ramirez-Ruiz, E. 2017, ApJ, 838, 56
McKernan, B., Ford, K. E. S., Lyra, W., & Perets, H. B. 2012, MNRAS, 425, 460
Meiron, Y., Kocsis, B., & Loeb, A. 2017, ApJ, 834, 200
Miller, M. C. 2002, ApJ, 581, 438
Moore, C. J., Gerosa, D., & Klein, A. 2019, MNRAS, 488, L94
Muñoz, D. J., Miranda, R., & Lai, D. 2019, ApJ, 871, 84
Nakamura, T. T. 1998, PhRvL, 80, 1138
Ostriker, E. C. 1999, ApJ, 513, 252
Peters, P. C. 1964, PhRv, 136, 1224
Randall, L., & Xianyu, Z.-Z. 2019, arXiv:1902.08604
Robson, T., Cornish, N. J., Tamanini, N., & Toonen, S. 2018, PhRvD, 98, 064012
Roedig, C., Sesana, A., Dotti, M., et al. 2012, A&A, 545, A127
Sánchez-Salcedo, F. J., & Brandenburg, A. 1999, ApJL, 522, L35
Sánchez-Salcedo, F. J., & Brandenburg, A. 2001, MNRAS, 322, 67
Sathyaprakash, B. S., & Schutz, B. F. 2009, LRR, 12, 2
Schutz, B. F. 1986, Natur, 323, 310
Sereno, M., Jetzer, P., Sesana, A., & Volonteri, M. 2011, MNRAS, 415, 2773
Sesana, A. 2016, PhRvL, 116, 231102
Seto, N. 2002, MNRAS, 333, 469
Seto, N. 2004, PhRvD, 69, 022002
Shi, J.-M., Krolik, J. H., Lubow, S. H., et al. 2012, ApJ, 749, 118
Smith, G. P., Jauzac, M., Veitch, J., et al. 2018, MNRAS, 475, 3823
Stone, N. C., Metzger, B. D., & Haiman, Z. 2017, MNRAS, 464, 946
Tagawa, H., Saitoh, T. R., & Kocsis, B. 2018, PhRvL, 120, 261101
Takahashi, R., & Nakamura, T. 2003, ApJ, 595, 1039
Tamanini, N., Klein, A., Bonvin, C., Barausse, E., & Caprini, C. 2020, PhRvD, 101, 063002
Tag, Y., MacFadyen, A., & Haiman, Z. 2017, MNRAS, 469, 4258
Thorne, K. S. 1987, in Gravitational Radiation, ed. S. W. Hawking & W. Israel (Cambridge: Cambridge Univ. Press), 330
Torres-Orjuela, A., Chen, X., & Amaro-Seoane, P. 2020, PhRvD, 101, 083028
Wong, K. W. K., Balbi, V., & Berti, E. 2019, MNRAS, 488, 5665
Yunes, N., Kocsis, B., Loeb, A., & Haiman, Z. 2011, PhRvL, 107, 171103