A new scheme for fault location of three-terminal parallel transmission lines without transposer

Gonggui Chen1,2 | Daiying Cai1,2 | Hongyu Long1,2 | Yi Long3 | Xianjun Zeng3 | Ping Zhou3,4 | Peng Kang3,4

1 Key Laboratory of Industrial Internet of Things and Networked Control, Ministry of Education, Chongqing University of Posts and Telecommunications, Chongqing, China
2 Chongqing Key Laboratory of Complex Systems and Bionic Control, Chongqing University of Posts and Telecommunications, NO. 2 Chongwen Road, Chongqing 400065, China
3 State Grid Chongqing Electric Power Company, Chongqing, China
4 Economic and Technology Research Institute, State Grid Chongqing Electric Power Company, Chongqing, China

Correspondence
Hongyu Long, Key Laboratory of Industrial Internet of Things and Networked Control, Ministry of Education, Chongqing University of Posts and Telecommunications, Chongqing 400065, China.
Email: longhongyu20@163.com

Abstract
This paper proposes a new scheme for locating the faults of three-terminal double-circuit transmission line without transposer in the phase domain that is not affected by type of fault. Synchronous voltage and current were measured by phasor measurement units (PMUs) installed at three busbars. Considering the coupling effect of parallel lines, the distribution line model was used to obtain more accurate results and third-order Taylor series expansion to decouple distributed line model. The phase and branch of all faults on the transmission lines were effectively identified, and various types of faults were accurately located including normal shunt fault, evolving fault and cross-country fault. The proposed algorithm was tested under different fault locations, resistances, initial angles and types (including cross-country faults and evolving faults). In addition, the effects of different sampling rates, measurement errors, earth resistivities and transmission line parameter errors were also considered. The simulation calculation was performed in MATLAB software and three-terminal transmission lines were simulated in PSCAD/EMTP software. The results indicate that maximum estimated error was less than 1.8631%.

1 | INTRODUCTION

The transmission system failure will damage equipment, thus resulting in power outage and reducing power quality. The fault of transmission line accounts for the largest [1, 2]. Therefore, it is necessary to develop a technology to accurately locate the fault of transmission line so as to restore power system as soon as possible.

A lot of algorithms for locating the faults on the transmission lines have been proposed in recent decades. For example, in [3], an impedance-based location algorithm using asynchronous signals was proposed, but the lumped parameter model without line capacitance was used in the experiment. However, there are very few studies on the fault location for three-terminal transmission lines. A number of algorithms for the fault location on the three-terminal parallel transmission lines have been proposed in [4], which can be divided into three categories. The first category is to locate defaults by detecting the arrival time of traveling wave and reasonable fault branch selection rule [5]. The second is to introduce artificial neural network (ANN) and input the three-terminal voltage and current after feature extraction into ANN for fault location [6]. The last category is impedance-based algorithm, an algorithm based on a lumped parameter model is proposed in [7], which uses the voltage and current signals of one end and the current amplitude data of the other two buses. In addition, PMUs are used to measure three-terminal synchronous voltage and current and calculate fault distance [8, 9], and asynchronous voltage and current for fault location [10].
In addition, it is quite important to focus on cross-country fault. The cross-country fault contains two phases in different locations at the same or different fault inception time and thus its location is difficult to identify [11], while evolving fault includes ground faults occurring at the same location in two phases of one circuit or two phases of different circuits at different fault inception time [12]. Because of cross-country fault with two different locations, evolving fault with different inception time, they are more complicated than normal shunt fault. In order to accurately locate cross-country faults, it is necessary to identify secondary faults [11]. However, they are difficult to identify by traditional fault location methods. Recent researches have introduced a number of methods for detecting and classifying cross-country faults [12–14] and evolving researches have introduced a number of methods for detecting difficult to identify by traditional fault location methods. Recent faults, first-zone distance relaying techniques for double-circuit transmission lines using single-ended signals was proposed. In [19], combining discrete wavelet transforms (DWT) and back propagation neural network, different types of faults including normal shunt fault, evolving fault, and cross-country fault were located. In this technique, neural network cannot be directly applied to the new transmission line and requires manual retraining. In [20], directional relaying and fault location scheme based on DWT and Support Vector Machine (SVM) including inter-circuit fault, cross-country fault and high resistance fault was proposed. In [17], a high impedance fault (HIF) protection scheme was proposed on the basis of wavelet packet transform (WPT) and extreme learning machine (ELM) and was applied to the protection of transmission and distribution lines considering evolving and cross-country faults.

This paper proposes a new algorithm for locating faults including cross-country and evolving faults of parallel lines without transposer that is not affected by type of fault. The proposed algorithm is based on the transmission lines theory and Taylor series expansion of the distributed line model. The synchronous current and voltage measured by PMU are used to identify the phase and branch of fault and locate faults. This method can accurately locate all types of faults including normal shunt fault, evolving fault and cross-country fault. And different fault types, resistances and inception angles are considered to verify the effectiveness of the proposed algorithm. In addition, the factors affecting the accuracy of the proposed algorithm, such as sampling rate, measurement error, earth resistivity and transmission line parameter error are discussed.

The rest of this paper is organized as follows. The fault location algorithm is proposed in Section 2. The test results of the proposed algorithm in the three-terminal transmission line system are presented in Section 3. Section 4 makes a summary of this paper.

## 2 | PROPOSED FAULT LOCATION ALGORITHM

### 2.1 | Two-terminal line fault location algorithm

Three-terminal double-circuit line with transposer is shown in Figure 1. The line consisted of three independent branches (S-G, R-G, T-G) and could be converted into two-terminal line to estimate the fault location. Due to the influence of mutual coupling caused by the asymmetric spatial distribution of transmission lines, this paper only considers the lines without transposer, so symmetric component transformation is not applicable to this study.

The parallel double-circuit transmission lines (S-R) are shown in Figure 2. If it is assumed that the length of the line is $l$, the fault point is $f$ and the distance away from bus $S$ is $x$, then the voltage and current vectors at $f$ point are expressed as:

$$
\begin{bmatrix}
V_f \\
I_f
\end{bmatrix} =
\begin{bmatrix}
\cosh(y_x) & Z_x \sinh(y_x) \\
\frac{1}{Z_x} \sinh(y_x) & \cosh(y_x)
\end{bmatrix}
\begin{bmatrix}
V_s \\
I_s
\end{bmatrix}
$$

(1)

In order to establish the subsequent fault location function $f(x, x)$ needs to be separated from Equation (1), so third-order Taylor series expansion is adopted to obtain Equations (2)–(5). $Z_x$ and $y$ are line characteristic impedance and line propagation constant, respectively.

$$
A(x) = \cosh(y_x) = 1 + \frac{ZYx^2}{2!} + \frac{(ZY)^2x^4}{4!}
$$

(2)

$$
B(x) = Z_x \sinh(y_x) = Z_x + \frac{ZYZ x^3}{3!} + \frac{(ZY)^2Z x^5}{5!}
$$

(3)

$$
C(x) = \frac{1}{Z_x} \sinh(y_x) = YD + \frac{Z^{-1}(ZY)^2x^3}{3!} + \frac{Z^{-1}(ZY)^3x^5}{5!}
$$

(4)

$$
D(x) = - \cosh(y_x) = 1 + \frac{Z^{-1}(ZY)^{-1}Zx^2}{2!} + \frac{Z^{-1}(ZY)^2Zx^4}{4!}
$$

(5)

where $Z$ and $Y$ are the series impedance matrix and the parallel admittance matrix per unit length, respectively.

As shown in Figure 2, the distance between bus $S$ and fault point $f$ is $x$. The voltage phasor $V_f$ and $V_p$ at fault point $f$ is obtained as:

$$
V_f = A(x)V_s + B(x)I_l
$$

(6)

$$
V_p = A(l - x)V_s + B(l - x)I_l
$$

(7)
Transmission line. Obviously, where

\[ \begin{align*}
V_{fs} & = \mathbf{V}_{fr} \\
I_{fs} & = \mathbf{I}_{fr}
\end{align*} \]

are complex numbers \((a + bj)\). According to the nature of the complex number, the product of a complex number and its conjugated complex number are real numbers, so the fault distance estimation function \(f(x)\) can be defined as:

\[ f(x) = \text{Im}(V_{f}^T \times \mathbf{V}_{fr}) = 0 \]  

where

\[ \begin{align*}
V_{f} & = \begin{bmatrix} \mathbf{V}_{fs} \mathbf{V}_{fr} \mathbf{V}_{fs} \mathbf{V}_{fr} \end{bmatrix}^T \\
I_{f} & = \begin{bmatrix} \mathbf{I}_{fs} \mathbf{I}_{fr} \mathbf{I}_{fs} \mathbf{I}_{fr} \end{bmatrix}^T
\end{align*} \]  

(8)

The fault distance estimation function \(f(x)\) can be translated to Equation (11), \(V_f\) and \(V_r\) can be obtained by Equations (6) and (7), respectively. \(x\) is the only unknown variable in \(f(x)\), so \(x\) can be obtained by solving Equation (11).

\[ f(x) = \text{Im}(V_{fa1} \mathbf{V}_{fr1} + \mathbf{V}_{fr1} \mathbf{V}_{fr2} + \mathbf{V}_{fr2} \mathbf{V}_{fr2}) = 0 \]  

(11)

\[ \begin{align*}
\begin{bmatrix} V_{f1} \\
I_{f1} \end{bmatrix} & = \begin{bmatrix} A(x_1) \quad B(x_1) \\
C(x_1) \quad D(x_1) \end{bmatrix} \begin{bmatrix} V_f \\
I_f \end{bmatrix} \\
\begin{bmatrix} V_{f2} \\
-I_{f2} \end{bmatrix} & = \begin{bmatrix} A(l - x_2) \quad B(l - x_2) \\
C(l - x_2) \quad D(l - x_2) \end{bmatrix} \begin{bmatrix} V_r \\
-I_r \end{bmatrix}
\end{align*} \]  

(12)

(13)

where

\[ \begin{align*}
V_{f1} & = \begin{bmatrix} V_{f1a1} \mathbf{V}_{fr1} + V_{f1a2} \mathbf{V}_{fr2} + V_{f1a3} \mathbf{V}_{fr3} \end{bmatrix}^T \\
V_{f2} & = \begin{bmatrix} V_{f2a1} \mathbf{V}_{fr1} + V_{f2a2} \mathbf{V}_{fr2} + V_{f2a3} \mathbf{V}_{fr3} \end{bmatrix}^T \\
I_{f1} & = \begin{bmatrix} I_{f1a1} \mathbf{I}_{fr1} + I_{f1a2} \mathbf{I}_{fr2} + I_{f1a3} \mathbf{I}_{fr3} \end{bmatrix}^T \\
I_{f2} & = \begin{bmatrix} I_{f2a1} \mathbf{I}_{fr1} + I_{f2a2} \mathbf{I}_{fr2} + I_{f2a3} \mathbf{I}_{fr3} \end{bmatrix}^T
\end{align*} \]  

(14)

There are two cases, as shown in Figure 3. One case is that \(S-R\) line is divided into two sections \((S-F_1, F_1-R)\) by \(F_1\) and the other is that \(S-R\) line is divided into two sections \((F_2-R, F_2-R)\) by \(F_2\). And there are different calculation formulas for these two cases.
2.2 Fault location algorithm for cross-country fault

Figure 3 shows cross-country fault in a1 and b2. From the voltage and current of both terminals of S-R, the voltage and current at F1 and F2 can be expressed as Equations (12) and (13), respectively.

2.2.1 Case 1

Assuming F1 and R as endpoints, F1-R segment can be regarded as a double-terminal transmission system, so the conclusion in Section 2.1 can be used to substitute Equations (12) and (13) into Equation (9) to obtain \( f(x) \).

\[
f(x) = \text{Im}.(V_{f1}^T V_{rf1}) = 0
\]  

(15)

\( V_{rf1} \) can be derived from point F1, which is expressed as:

\[
V_{f1} = A(x_2 - x_1)V_{f1} + B(x_2 - x_1)I_{f1}
\]  

(16)

\( V_{rf1} \) can be derived from bus R, which is expressed as:

\[
V_{rf1} = A(l - x_2)V_r - B(l - x_2)I_r
\]  

(17)

Substituting Equations (16) and (17) into Equation (15) to obtain \( f_1(x_1, x_2) \).

\[
f_1(x_1, x_2) = \text{Im}.[A(x_2 - x_1)V_{f1} + B(x_2 - x_1)I_{f1}]^T * A(l - x_2)V_r - B(l - x_2)I_r
\]  

(18)

In Equation (18), \( I_{f1} \) is equal to \( I_{rf1} \) except the current corresponding to phase \( a1 \) (\( I_{g1,a1} \)). If the stray shunt capacitance of the transmission line between two fault points is ignored, then \( I_{g1,a1} = I_{g2,a1} \). \( I_{f1} \) can be approximated as: \( [I_{g1,a1} I_{g1,b1} I_{g1,c1} I_{g1,a2} I_{g1,b2} I_{g1,c2}]^T \).

\( x_1 \) and \( x_2 \) are the only unknown variables in Equations (18) and (22). Combining these two equations, two fault distances \( x_1 \) and \( x_2 \) can be solved. When the difference between the obtained values \( x_1 \) and \( x_2 \) is less than 0.01, the fault is regarded as a normal shunt fault or an evolving fault; otherwise, it is a cross-country fault [11].

2.2.2 Case 2

Assuming \( F_2 \) and \( S \) as endpoints, \( S-F_2 \) segment can be regarded as a double-terminal transmission system. Similar to Case 1, \( f(x) \) is defined as:

\[
f(x) = \text{Im}.(V_{f2}^T V_{rf2}) = 0
\]  

(19)

\( V_{rf2} \) can be derived from bus S, which is expressed as:

\[
V_{f2} = A(x_2 - x_1)V_{f2} + B(x_2 - x_1)I_{f2}
\]  

(20)

\( V_{rf2} \) can be derived from point \( F_2 \), which is expressed as:

\[
V_{rf2} = A(x_2 - x_1)V_{f2} + B(x_2 - x_1)I_{f2}
\]  

(21)

Substituting Equations (20) and (21) into Equation (19) to obtain \( f_2(x_1, x_2) \).

\[
f_2(x_1, x_2) = \text{Im}.[A(x_2 - x_1)V_f + B(x_2 - x_1)I_f]^T * A(x_2 - x_1)V_{f2} + B(x_2 - x_1)I_{f2}
\]  

(22)

In Equation (22), \( I_{f2} \) is equal to \( I_{rf2} \) except the current corresponding to phase \( b2 \) (\( I_{g2,b2} \)). If the stray shunt capacitance of the transmission line between two fault points is ignored, then \( I_{g2,b2} = I_{g1,b2} \). \( I_{f2} \) can be approximated as: \( [I_{g2,a1} I_{g2,b1} I_{g2,c1} I_{g2,a2} I_{g2,b2} I_{g2,c2}]^T \).

2.3 Fault branch and phase identification

2.3.1 Fault branch identification

In order to locate the fault of the three-terminal transmission line system, it is firstly necessary to identify fault branch. The three-terminal parallel transmission line is shown in Figure 1. Three PMUs were installed on the buses S, R and T. The synchronous voltage and current phasors of the buses were directly measured by PMUs, and then all measurement data were
introduced into central protection system through the communication system by GPS technology. The lengths of the branches S-G, R-G and T-G are \( I_S, I_R \) and \( I_T \) respectively, where point G is the connection point of the three-terminal transmission line.

According to the available data of buses S, R and T combined with the line parameters of corresponding branches, the voltage vectors \((V_{GS}, V_{GR}, \text{ and } V_{GT})\) and the current vectors \((I_{GS}, I_{GR} \text{ and } I_{GT})\) at point G when there is no fault are calculated by Equations (23)–(25).

\[
[V_{GS} - I_{GS}]' = \begin{bmatrix} A(l_S) & B(l_S) \\ C(l_S) & D(l_S) \end{bmatrix} [V_S - I_S]' = (23)
\]
\[
[V_{GR} - I_{GR}]' = \begin{bmatrix} A(l_R) & B(l_R) \\ C(l_R) & D(l_R) \end{bmatrix} [V_R - I_R]' = (24)
\]
\[
[V_{GT} - I_{GT}]' = \begin{bmatrix} A(l_T) & B(l_T) \\ C(l_T) & D(l_T) \end{bmatrix} [V_T - I_T]' = (25)
\]

Equations (26)–(29) are defined to identify the faulty branch. For example, if \([\Delta V_{SR}, \Delta V_{ST}, \Delta V_{RT}]\) is assumed to be [0.5, 0.5, 0.001], \(\Delta V\) is equal to \(\Delta V_{RT}\), indicating that T-G and R-G are healthy, while S-G is faulty.

\[
\Delta V_{SR} = \text{sum}(V_{GS}) - \text{sum}(V_{GR}) \quad (26)
\]
\[
\Delta V_{ST} = \text{sum}(V_{GS}) - \text{sum}(V_{GT}) \quad (27)
\]
\[
\Delta V_{RT} = \text{sum}(V_{GR}) - \text{sum}(V_{GT}) \quad (28)
\]
\[
\Delta V = \min(\{\Delta V_{SR}, \Delta V_{ST}, \Delta V_{RT} \}) \quad (29)
\]

where \(\text{sum}(V)\) is the sum of all the elements of vector \(V\), and \(\min()\) is the minimum value.

### 2.3.2 Fault branch identification

Similarly, the fault phase is defined by Equation (30), and \(I_{GS}, I_{GR} \text{ and } I_{GT}\) can be obtained by Equations (23)–(25). Obviously, according to Kirchhoff’s current law (KCL), when there is no fault, all values in \(I_{G}\) are zero. However, when a fault occurs, the values of fault phase in \(I_{G}\) are not equal to zero. In practice, a threshold (\(\varepsilon = 0.1\)) is defined where an element greater than 0.1 in \(I_{G}\) corresponds to the fault phase. For example, when \(I_{G}\) is [0.99, 0.01, 0.01, 0.01, 0.01, 0.01, 1.00], \(a1\) and \(a2\) are fault phases.

\[
I_{G} = \frac{\text{abs}(I_{GS} + I_{GR} + I_{GT})}{\max(\text{abs}(I_{GS} + I_{GR} + I_{GT}))} \quad (30)
\]

\(I_{G_{\text{healthy}1}}\) and \(I_{G_{\text{healthy}2}}\) represent the current flowing into point G from two healthy branches, and \(I_{G_{\text{fault}}}\) represents the current flowing into point G from the faulty branch.

It is worth noting that pseudo roots are an inevitable problem when using line impedance to estimate the fault location. The reason can be explained from the perspective of power as follows. Assuming that the fault ground impedances are pure resistances, the complex power generated by these resistances at the fault point only includes active power, and the imaginary part is equal to zero, Equations (11), (18) and (22) for fault location are derived based on Equation (9), as shown in

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**FIGURE 4** The flow chat of the proposed scheme

#### 2.4 Calculation process

The flow chart of the proposed fault location algorithm is shown in Figure 4. The faulty branch and phase are identified by Equations (26)–(30) after the voltage and current of the three buses were pre-processed. The voltage and current at both terminals of the fault branch are obtained using the proposed algorithm. The voltage and current of the three-terminal connecting point G can be obtained from the two healthy branches, and KCL is applicable to point G. The relationship between current is shown in Equation (31), and more details are given in Figure 4. Finally, if the fault is two-phase, \(x_1\) and \(x_2\) can be solved by combining Equations (18) and (22); otherwise, the fault distance \(x\) is solved by Equation (11).

\[
I_{G_{\text{fault}}} = -(I_{G_{\text{healthy}1}} + I_{G_{\text{healthy}2}}) \quad (31)
\]
FIGURE 5 Effect of increasing absolute distance $|x_1 - x_2|$ on estimated error in the fault location for line branch ($T-G$)

FIGURE 6 Block diagram of PMU phasor-measurement

3.1 Fault signal sampling

The sampling process of the measured signal and the process of converting the measured signal to complex phasor are shown in Figure 6. The sampled signal first goes through a low pass filter (LPF). The sinusoidal signal is then fed into the analog (A/D) converter, while the GPS synchronizes all the sampled PMU signals, while the GPS synchronizes all the sampled PMU signals and the time synchronization error is less than $1 \mu s$ [23]. Finally, DFT is used to calculate the fundamental frequency component [24]. LPF and DFT eliminate the influence of high frequency harmonics.

At the same time, the current and voltage pass through a low-pass second-order Butterworth filter with a cut-off frequency of 400 Hz. The sampling frequency of the signal is 3000 Hz, and the decaying DC component is removed by the digital filter [25]. In addition, the models of current transformer (CT) and capacitive voltage transformer (CVT) [26], and the model data of CT and CVT are provided in Appendix A3.

The current waveform of Case 9 in Table 1 and the sampling time window are shown in Figure 7. The sampling starting time was set as 0.02 s after the fault was detected, and the cut-off time was after one power-frequency cycle of the starting time (0.02 s). Figure 8 shows the waveform comparison between the CVT and the ideal VT in Case 9 of Table 1, in addition, the filtered waveform result of the CVT is also shown.
3.2 Test on normal shunt and evolving faults

In this section, the performance of the proposed algorithm for normal shunt faults and evolving faults are evaluated while considering two ground faults \( R_f = 0.1 \, \Omega \) and \( R_f = 100 \, \Omega \) and different initial angles.

The fault location results of normal shunt faults are shown in Table 1. Due to the limitation of scale, only the \( I_G \) of the fault phases is listed. Value in \( \Delta V \) equaling to 0 means its subscripts corresponding to healthy branches, and the last column is the relative error. For example, Case 2 in Table 1 is the two-phase ground fault \( b_1, c_1-g \) of the \( S-G \) branch, with a fault distance of 0.8 p.u., fault resistance of 100 \( \Omega \) and an initial angle of 135°. It can be seen from Table 1 that \( \Delta V_{RT} \) was equal to 0, which indicated that \( S-G \) was the faulty branch. \( I_G \) of \( b_1 \) and \( c_1 \) were 1 and 0.9872, respectively. The fault distances corresponding to \( b_1 \) and \( c_1 \) were estimated to be 0.7995 and 0.8064 p.u., respectively, and the two values were approximately equal. Therefore, it was a normal shunt fault or an evolving fault, with an average fault distance of 0.8030 p.u.

Table 2 lists the results of the transmission line parameter matrix modelled by the 3rd and 4th order Taylor series. And Table 2 lists the results obtained by previous algorithm in [19]. It can be seen that 4th order Taylor series failed to improve the accuracy of fault location. Therefore, in order to reduce calculation time, 3rd order Taylor series was adopted in the following simulations. In addition, the proposed algorithm was more accurate than previous methods.

The results of the evolving faults of different branches are shown in Table 3. For example, in Case 5, the fault occurred to the \( R-G \) branch, the fault phases were \( c_1 \) and \( b_2 \), the actual fault distance was 0.4 p.u., the ground resistances of fault 1 and fault 2 were 100 and 0.1 \( \Omega \), respectively, and the initial angles of fault 1 and fault 2 were 45° and 135°, respectively. The fault
### TABLE 1

| Branch   | Initial Angle (°) | Estimated x (p.u.) | ΔVSR (p.u.) | ΔVST (p.u.) | ΔVRT (p.u.) | Df (p.u.) | Rf (Ω) | δ (°) | IG Estimated | Error (%) |
|----------|-------------------|--------------------|-------------|-------------|-------------|-----------|--------|-------|---------------|-----------|
| T-G b1g  | 0.5333            | 0.1                | 0.5333      | 0.1         | 0.5340      | 0.64      | 0.65   | 0.00  | -             | 1.00      |
|          | 0.8000            | 100                | 0.8024      | 0.1         | 0.8024      | 0.83      | 0.85   | 0.00  | -             | 0.99      |
| R-G c1g  | 0.5480            | 100                | 0.5498      | 0.1         | 0.55       | 0.56      | 1.00   | -     | -             | 1.00      |
|          | 0.1960            | 0.1                | 0.1969      | 0.36    | 0.56       | 0.56      | 1.00   | -     | -             | 1.00      |
| T-G a1g  | 0.2000            | 0.1                | 0.2000      | 0.36    | 0.75       | 0.75      | 1.00   | -     | -             | 1.00      |
|          | 0.4900            | 100                | 0.4905      | 0.1         | 0.73       | 0.73      | 1.00   | -     | -             | 1.00      |
| b1c1g    | 0.9800            | 100                | 0.9825      | 0.1         | 0.9825      | 0.83      | 0.82   | -     | -             | 1.00      |

Branch $R-G$ was determined by $\Delta V$. Furthermore, the $I_G$ values of $a_1$ and $b_2$ were 0.65 and 1, respectively, indicating that $R-G$ was fault branch; at the same time, the $I_G$ values corresponding to $b_1$ and $a_2$ were 0.2865 and 0.7566 p.u., respectively. They were not equal, so it was cross-country fault. In general, the maximum error was 0.5074%, which indicated that the proposed algorithm was accurate for locating cross-country faults.

3.3 Test on cross-country fault

The test results of the proposed algorithm for cross-country faults are shown in Table 4. Due to limited space, only the $I_G$ of the fault phases was displayed. In all cases, fault branch and fault phase could be accurately identified. For example, in case 9, $\Delta V$ was $[0,0.74,0.74]$, indicating that $T-G$ was fault branch; at the same time, the $I_G$ values corresponding to $b_1$ and $a_2$ were 0.65 and 1, respectively, indicating that $b_1$ and $a_2$ were fault phases. In addition, the estimated distances corresponding to $b_1$ and $a_2$ were 0.2865 and 0.7566 p.u., respectively. They were not equal, so it was cross-country fault. In general, the maximum error was 0.5074%, which indicated that the proposed algorithm was accurate for locating cross-country faults.

In order to verify the performance on locating cross-country faults at different voltage levels. Figure 5 shows the results of fault-location accuracy at different voltage level (220 and 330 KV). The branch $T-G$ with a fault resistance of 0.1 Ω, and an initial angle of $0^\circ$ was used for simulation. As expected, the error increased as the spacing between $x_1$ and $x_2$ increased, and the higher the voltage level, the greater the error. It can be explained as follows: Since the proposed algorithm utilizes approximation method, the greater the spacing between $x_1$ and $x_2$, the greater error caused by the approximation. Similarly, the higher the voltage level, the worse effect the approximation and the greater the error. However, the maximum error was less than 0.8% and still acceptable, as shown in Figure 5.

3.4 Effect of line parameters error

Since line parameters cannot be accurately estimated, it is meaningful to prove the performance of the proposed algorithm under different line parameter errors. Parameter errors of 1%, 2% and 3% were respectively considered in $Z$ and $Y$. Other faults are shown in Tables 1, 3 and 4. The mean error and max error of different types of fault are shown in Figure 9. As expected, line parameter errors will affect fault location accuracy, with mean error and max error respectively being 0.6526% and 1.3619%. In general, the accuracy of fault location is still acceptable from a practical perspective.

3.5 Effect of different sampling rate

Figure 10 shows the influence of different sampling frequencies (2.5, 5 and 10 kHz) on the accuracy of fault location. The
### TABLE 2  Comparison with previous technique and the effect of multiple expansion and line feed on the accuracy of fault location

| Branch | Type   | $D_f$ (p.u.) | 3rd order expansion | 4th order expansion | Previous technology |
|--------|--------|--------------|----------------------|---------------------|---------------------|
|        |        | Estimated distance (p.u.) | Error (%) | Estimated distance (p.u.) | Error (%) | Estimated distance (p.u.) | Error (%) |
| T-G    | blg    | 0.5333       | 0.5340 | 0.1194 | 0.5340 | 0.1194 | 0.5360 | 0.5119 |
|        | blc1lg | 0.8000       | 0.8024 | 0.2967 | 0.8024 | 0.2967 | 0.8033 | 0.4130 |
|        | acl1   | 0.2000       | 0.2000 | 0.0083 | 0.2000 | 0.0083 | 0.2006 | 0.2825 |
|        | aclb1clg | 0.2866 | 0.2867 | 0.0229 | 0.2867 | 0.0229 | 0.2875 | 0.3123 |
| R-G    | clg    | 0.5480       | 0.5498 | 0.3218 | 0.5498 | 0.3218 | 0.5506 | 0.4832 |
|        | aclbg  | 0.1960       | 0.1960 | 0.0212 | 0.1960 | 0.0212 | 0.1975 | 0.7621 |
|        | aclb1  | 0.8040       | 0.8093 | 0.4563 | 0.8093 | 0.6563 | 0.8068 | 0.3466 |
|        | aclb1clg | 0.3360 | 0.3363 | 0.0951 | 0.3363 | 0.0951 | 0.3367 | 0.2195 |
| T-G    | alg    | 0.2000       | 0.2000 | 0.0038 | 0.2000 | 0.0038 | 0.2010 | 0.5038 |
|        | aclbg  | 0.4900       | 0.4902 | 0.0471 | 0.4902 | 0.0471 | 0.4909 | 0.1805 |
|        | blc1   | 0.9800       | 0.9825 | 0.2575 | 0.9825 | 0.2575 | 0.9837 | 0.3826 |
|        | aclb1clg | 0.3000 | 0.3000 | 0.0121 | 0.3000 | 0.0121 | 0.3013 | 0.4271 |

Fault conditions are the same as Tables 1, 3, and 4. Obviously, for the three types of fault in Figure 9, minimum and maximum mean errors were 0.1533% and 0.2782%, respectively, with a very small change. Similarly, the max error was also in the range of 0.6563–0.9570%. Therefore, the change of the sampling frequency had little effect on the accuracy of fault location. Therefore, it can be concluded that the proposed algorithm has good performance in locating faults under different sampling frequencies.

#### 3.6 Effect of measurement error

In the process of signal preprocessing, measurement error is inevitable. In order to verify the effect of measurement error on the accuracy of fault location, the measurement error was set to 3% under the transient conditions with reference to IEEE standard [23]. The other fault conditions are the same as Tables 1, 3, and 4. Figure 11 shows the mean errors and max errors for different types of fault. As can be clearly seen from the figure, the measurement error has a significant effect on the accuracy of fault location. The max error and mean error reached 1.8631% and 0.5643%, respectively, as shown in Figure 11. However, these errors are still objectively acceptable.

#### 3.7 Effect of earth resistivity change

The earth resistivity depends on external conditions such as soil moisture and temperature. It is well known that earth resistivity affects transmission line parameters, so it is important to pay attention to the effect of earth resistivity. The fault conditions are the same as Tables 1, 3, and 4. The different earth resistivities (100, 150, and 200 ohm.km) were applied to the calculation of line parameters. The results of the accuracy of
fault location are shown in Figure 12. Obviously, the mean errors and max errors were in the range of 0.2024–0.2481%, and 0.4851–0.5517%, respectively. It can be concluded that changes in earth resistivity has little effect on the accuracy of fault location.

3.8 Algorithm performance near connecting point G

An outstanding location algorithm can accurately locate faults at any position in the line. Therefore, Table 5 shows the experimental results of different branches of the three-terminal transmission network at a distance of 1% from the connecting point G. In Table 5, for all branches, $\Delta V$ could accurately identify the faulty branch, and $I_G$ could accurately detect the faulty phase. Furthermore, the maximum error in all cases shown in Table 5 is 0.5356%, indicating that fault location is accurate.

3.9 Effect of source impedances

When the power transmission system is running, changes in source impedance may change the magnitude of voltage or current, thereby affecting the accuracy of fault location. Therefore, the branch S-G was used for verification. While ensuring that other conditions were the same as those in Tables 1, 3 and 4, the source impedance of bus S was changed to 0.5, 1 and 1.5 times the original value, respectively. The source impedance can
### TABLE 3  Results of different fault cases for evolving faults

| Branch | $D_f$ | Type | $R_f$ (Ω) | $\delta$ (°) | Fault 1 condition | $\Delta V_{SR}$ | $\Delta V_{ST}$ | $\Delta V_{RT}$ | $I_G$ | $x_1$ | $Error_1$ (%) | $x_2$ | $Error_2$ (%) |
|--------|-------|------|-----------|--------------|-------------------|----------------|--------------|--------------|------|------|----------------|------|----------------|
| S-G    | 0.6800| a1   | 0.1       | 45           | b2                | 0.73           | 0.75          | 0.00         | 1.00 | 0.6821| 0.2120           | 0.6779| 0.2130         |
|        | 0.2600| c1   | 100       | 135          | a2                | 0.47           | 0.47          | 0.00         | 1.00 | 0.2602| 0.0201           | 0.2598| 0.0175         |
|        | 0.9200| b1   | 100       | 90           | c1                | 0.89           | 0.94          | 0.00         | 1.00 | 0.9238| 0.3813           | 0.9162| 0.3813         |
|        | 0.1533| b2   | 0.1       | 0            | c2                | 0.39           | 0.39          | 0.00         | 1.00 | 0.1534| 0.0038           | 0.1534| 0.0080         |
| R-G    | 0.4000| c1   | 100       | 0            | b2                | 0.46           | 0.46          | 0.00         | 1.00 | 0.3985| 0.1489           | 0.3985| 0.1500         |
|        | 0.4920| a2   | 0.1       | 135          | b2                | 0.52           | 0.52          | 0.00         | 1.00 | 0.4945| 0.2504           | 0.4895| 0.2502         |
|        | 0.0600| a1   | 0.1       | 90           | c1                | 0.22           | 0.00          | 0.22         | 1.00 | 0.0600| 0.0035           | 0.0601| 0.0094         |
|        | 0.9320| a1   | 100       | 0            | b2                | 0.85           | 0.88          | 0.00         | 1.00 | 0.9244| 0.7568           | 0.9396| 0.4568         |
|        | 0.6500| a1   | 0.1       | 0            | b2                | 0.00           | 0.83          | 0.83         | 1.00 | 0.6510| 0.0986           | 0.6490| 0.0985         |
|        | 0.2000| b1   | 100       | 45           | a2                | 0.55           | 0.56          | 0.00         | 1.00 | 0.2000| 0.0038           | 0.2000| 0.0038         |
|        | 0.7200| b2   | 100       | 135          | c1                | 0.00           | 0.87          | 0.87         | 1.00 | 0.7213| 0.1273           | 0.7187| 0.1273         |
|        | 0.5500| b1   | 0.1       | 0            | c1                | 0.00           | 0.77          | 0.77         | 1.00 | 0.5506| 0.0639           | 0.5506| 0.0640         |

### TABLE 4  Results of different fault cases for cross-country

| Branch | $D_f$ | Type | $R_f$ (Ω) | $\delta$ (°) | Fault 2 condition | $\Delta V_{SR}$ | $\Delta V_{ST}$ | $\Delta V_{RT}$ | $I_G$ | $x_1$ | $Error_1$ (%) | $x_2$ | $Error_2$ (%) |
|--------|-------|------|-----------|--------------|-------------------|----------------|--------------|--------------|------|------|----------------|------|----------------|
| S-G    | 0.9067| b1   | 0.1       | 45           | c2                | 0.54           | 0.55          | 0.00         | 1.00 | 0.9116| 0.4874           | 0.9048| 0.5074         |
|        | 0.9600| c1   | 100       | 135          | a2                | 0.69           | 0.71          | 0.00         | 1.00 | 0.9642| 0.4206           | 0.3510| 0.4306         |
|        | 0.4267| a1   | 100       | 90           | c1                | 0.48           | 0.48          | 0.00         | 1.00 | 0.4249| 0.1792           | 0.1652| 0.1492         |
|        | 0.6400| a1   | 0.1       | 0            | b1                | 0.80           | 0.84          | 0.00         | 1.00 | 0.6423| 0.2255           | 0.5624| 0.2755         |
| R-G    | 0.2360| c1   | 0.1       | 135          | b2                | 0.49           | 0.50          | 0.41         | 1.00 | 0.2401| 0.4142           | 0.8195| 0.4142         |
|        | 0.3520| a1   | 100       | 0            | c2                | 0.49           | 0.50          | 0.72         | 1.00 | 0.3503| 0.1665           | 0.5893| 0.1665         |
|        | 0.0880| a1   | 100       | 90           | b1                | 0.29           | 0.29          | 0.71         | 1.00 | 0.0894| 0.1395           | 0.2626| 0.1195         |
|        | 0.3360| b1   | 0.1       | 45           | c1                | 0.51           | 0.52          | 0.59         | 1.00 | 0.3386| 0.2605           | 0.6970| 0.3105         |
| T-G    | 0.2900| b1   | 100       | 45           | a2                | 0.00           | 0.74          | 0.74         | 0.65 | 1.00  | 0.2865           | 0.3517| 0.7566         |
|        | 0.8600| c1   | 0.1       | 0            | b2                | 0.00           | 0.79          | 0.79         | 0.64 | 1.00  | 0.8637           | 0.3716| 0.3716         |
|        | 0.2000| b1   | 100       | 135          | c1                | 0.00           | 0.63          | 0.64         | 0.73 | 1.00  | 0.2020           | 0.2010| 0.4875         |
|        | 0.9100| a1   | 0.1       | 90           | c1                | 0.00           | 0.97          | 0.95         | 1.00 | 0.9088| 0.1248           | 0.8188| 0.1348         |
TABLE 5  Results at 1% of the distance between different branches to G point

| Branch | Fault type   | ΔV_{SR}  | ΔV_{ST}  | ΔV_{RT}  | I_G       | Error (%) |
|--------|--------------|----------|----------|----------|-----------|-----------|
| S-G    | a1-g         | 0.7129   | 0.7307   | 0        | 1.0000    | 0.0030    | 0.0710    | 0.0930    | 0.0820    | 0.0660    | 0.3123    |
|        | b1c1-g       | 0.6845   | 0.9857   | 0        | 0.0200    | 1.0000    | 0.9853    | 0.0120    | 0.0820    | 0.0120    | 0.3851    |
|        | a1b1c1-g     | 0.9836   | 0.9852   | 0        | 0.9953    | 1.0000    | 0.9877    | 0.0780    | 0.0620    | 0.0620    | 0.2161    |
| R-G    | b1-g         | 0.7400   | 0        | 0.7214   | 0.0660    | 1.0000    | 0.0540    | 0.0330    | 0.0260    | 0.0110    | 0.4879    |
|        | a1c1-g       | 0.6871   | 0        | 0.6844   | 1.0000    | 0.0920    | 0.9987    | 0.0280    | 0.0510    | 0.0610    | 0.5356    |
|        | a1b1c1-g     | 0.7211   | 0        | 0.7296   | 0.9875    | 0.9832    | 1.0000    | 0.0710    | 0.0220    | 0.0210    | 0.5252    |
| T-G    | c1-g         | 0        | 0.6977   | 0.6994   | 0.0357    | 0.0187    | 1.0000    | 0.0760    | 0.0187    | 0.0187    | 0.4198    |
|        | a1b1-g       | 0        | 0.7298   | 0.7231   | 1.0000    | 0.9878    | 0.0230    | 0.0220    | 0.0540    | 0.0340    | 0.2316    |
|        | a1b1c1-g     | 0        | 0.6741   | 0.6752   | 0.9325    | 0.9457    | 1.0000    | 0.0320    | 0.0160    | 0.0640    | 0.3387    |

FIGURE 13  Mean and max errors in different source impedances

be found in the appendix, and the result is shown in Figure 13, which shows that the fault location algorithm proposed in this paper is not affected by the source impedance.

4.1 CONCLUSION

This paper proposed a new scheme for locating the faults of three-terminal double-circuit transmission line without trans-poser in the phase domain that is not affected by type of fault. Synchronous voltage and current measured by the PMUs at three buses were used to identify the faulty phases and branch, and locate the fault. Third-order Taylor series was used to model transmission line parameter matrix. The coupling effect between parallel lines was considered in the derivation of this algorithm. Simulation results demonstrate the robustness of the algorithm under different fault types, fault resistances and inception angles. In addition, the proposed algorithm has good performance for the fault location on the transmission line under parameter errors, measurement errors, earth resistivity variations and different sampling frequencies. For all transmission line faults, the faulty phase and branch can be efficiently identified, and various types of faults can be accurately located, including normal shunt fault, evolving fault and cross-country fault. And max estimation error of fault location is less than 1.8631%.

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APPENDIX

A. Three-terminal line parameter

\[
Z_{SG} = \begin{bmatrix}
0.902 + 0.5973i0.0620 + 0.2979i0.0612 + 0.2562i0.0612 + 0.2409i0.0620 + 0.2527i0.0635 + 0.2706i
0.0620 + 0.2979i0.0875 + 0.5978i0.0601 + 0.2970i0.0600 + 0.2527i0.0607 + 0.2519i0.0620 + 0.2527i
0.0612 + 0.2562i0.0601 + 0.2970i0.0862 + 0.5978i0.0595 + 0.2711i0.0600 + 0.2527i0.0612 + 0.2409i
0.0612 + 0.2409i0.0600 + 0.2527i0.0595 + 0.2711i0.0862 + 0.5978i0.0601 + 0.2970i0.0612 + 0.2527i
0.0620 + 0.2527i0.0607 + 0.2519i0.0600 + 0.2527i0.0601 + 0.2970i0.0875 + 0.5978i0.0620 + 0.2979i
0.0635 + 0.2706i0.0620 + 0.2527i0.0612 + 0.2562i0.0620 + 0.2979i0.0902 + 0.5973i
\end{bmatrix}
\]

\[
Y_{SG} = 1.0e - 05 \begin{bmatrix}
0.3079i - 0.0662i - 0.0221i - 0.0132i - 0.0242i - 0.0432i
-0.0662i0.3199i - 0.0584i - 0.0176i - 0.0187i - 0.0224i
-0.0224i - 0.0584i0.3247i - 0.0305i - 0.0176i - 0.0132i
-0.0132i - 0.0176i - 0.0305i0.3247i - 0.0584i - 0.0221i
-0.0224i - 0.0187i - 0.0176i - 0.0584i0.3199i - 0.0662i
-0.0432i - 0.0224i - 0.0132i - 0.0221i - 0.0662i0.3079i
\end{bmatrix}
\]

\[
Z_{RG} = \begin{bmatrix}
0.0887 + 0.5976i0.0613 + 0.3329i0.0609 + 0.2964i0.0609 + 0.2520i0.0612 + 0.2501i0.0619 + 0.2606i
0.0613 + 0.3329i0.0873 + 0.5979i0.0603 + 0.3313i0.0603 + 0.2500i0.0606 + 0.2442i0.0612 + 0.2501i
0.0609 + 0.2964i0.0603 + 0.3313i0.0867 + 0.5978i0.0600 + 0.2608i0.0603 + 0.2500i0.0609 + 0.2520i
0.0609 + 0.2520i0.0603 + 0.2500i0.0600 + 0.2608i0.0867 + 0.5978i0.0603 + 0.3313i0.0609 + 0.2964i
0.0612 + 0.2501i0.0606 + 0.2442i0.0603 + 0.2500i0.0603 + 0.3313i0.0873 + 0.5979i0.0613 + 0.3329i
0.0619 + 0.2606i0.0612 + 0.2501i0.0609 + 0.2520i0.0609 + 0.2964i0.0613 + 0.3329i0.0887 + 0.5976i
\end{bmatrix}
\]

\[
Y_{RG} = 1.0e - 05 \begin{bmatrix}
0.3282i - 0.0926i - 0.0439i - 0.0167i - 0.0157i - 0.0289i
-0.0926i0.3463i - 0.0865i - 0.0129i - 0.0098i - 0.0157i
-0.0439i - 0.0865i0.3362i - 0.0223i - 0.0129i - 0.0167i
-0.0167i - 0.0129i - 0.0223i0.3362i - 0.0865i - 0.0439i
-0.0157i - 0.0098i - 0.0129i - 0.0865i0.3463i - 0.0926i
-0.0289i - 0.0157i - 0.0167i - 0.0439i - 0.0926i0.3282i
\end{bmatrix}
\]

\[
Z_{TG} = \begin{bmatrix}
0.0885 + 0.5975i0.0615 + 0.3788i0.0608 + 0.2965i0.0608 + 0.2451i0.0615 + 0.2441i0.0618 + 0.2517i
0.0615 + 0.3788i0.0880 + 0.5977i0.0605 + 0.2971i0.0605 + 0.2392i0.0612 + 0.2373i0.0615 + 0.2441i
0.0608 + 0.2965i0.0605 + 0.2971i0.0866 + 0.5979i0.0599 + 0.2520i0.0605 + 0.2520i0.0608 + 0.2451i
0.0608 + 0.2451i0.0605 + 0.2392i0.0599 + 0.2520i0.0866 + 0.5979i0.0605 + 0.2971i0.0608 + 0.2965i
0.0615 + 0.2441i0.0612 + 0.2373i0.0605 + 0.2392i0.0605 + 0.2971i0.0880 + 0.5977i0.0615 + 0.3788i
0.0618 + 0.2517i0.0615 + 0.2441i0.0608 + 0.2451i0.0608 + 0.2965i0.0615 + 0.3788i0.0885 + 0.5975i
\end{bmatrix}
\]

\[
Y_{TG} = 1.0e - 05 \begin{bmatrix}
0.3663i - 0.1536i - 0.0479i - 0.0153i - 0.0136i - 0.0200i
-0.1536i0.3640i - 0.0498i - 0.0108i - 0.0097i - 0.0136i
-0.0479i - 0.0498i0.3196i - 0.0215i - 0.0108i - 0.0153i
-0.0153i - 0.0108i - 0.0215i0.3196i - 0.0498i - 0.0479i
-0.0136i - 0.0097i - 0.0108i - 0.0498i0.3640i - 0.1536i
-0.0200i - 0.0136i - 0.0153i - 0.0479i - 0.1536i0.3663i
\end{bmatrix}
\]
### B. Voltage source data

| Quantity       | Source S   | Source R   | Source T   |
|----------------|------------|------------|------------|
| Ermf (KV)      | 225∠0°     | 2.30∠10°   | 220∠5°     |
| Pos. Seq. Impedance (Ω) | 1.125 + j15.773 | 1.042 + j14.110 | 1.027 + j13.974 |
| Zero. Seq. Impedance (Ω) | 6.421 + j39.562 | 5.447 + j35.54 | 5.321 + j34.38 |

### C. Current and voltage transformers data

|                     | Current transformer | Voltage transformers |
|---------------------|---------------------|----------------------|
| Ratio               | 2 × 600/5 (A)       | 220√3:0.1√3:0.1      |
| Burden              | 50VA                | 10/50/50VA           |
| Accuracy            | SP20                | 3P                   |