Searching for Strange Quark Matter Objects Among White Dwarfs

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Abstract

The ground state of matter may be strange quark matter (SQM), not hadronic matter. A whole sequence of SQM objects, ranging from strange quark stars and strange quark dwarfs to strange quark planets, can stably exist according to this SQM hypothesis. A strange dwarf has a mass similar to that of a normal white dwarf, but could harbor an extremely dense SQM core (with the density as large as $\sim 4 \times 10^{14} \text{ g cm}^{-3}$) at the center so that its radius can be correspondingly smaller. In this study, we try to search for strange dwarfs among the observed “white dwarfs” by considering their difference in the mass-radius relation. Seven strange dwarf candidates are identified in this way, whose masses are in the range of $\sim 0.02 - 0.12 M_\odot$, with the radii narrowly distributed in $\sim 9,000 - 15,000 \text{ km}$. The seven objects are LSPM J0815+1633, LP 240-30, BD+20 5125B, LP 462-12, WD J1257+5428, 2MASS J13453297+4200437, and SDSS J085557.46+053524.5. Comparing with white dwarfs of similar mass, these candidates are obviously smaller in radius. Further observations with large radio/infrared/optical telescopes on these interesting candidates are solicited.

Keywords: Compact objects, Strange quark matter, Strange quark stars

1. Introduction

It has long been argued that the ground state of matter may be quark matter \cite{1,2,3} rather than hadronic form, because its energy per baryon could be less than that of the most stable atomic nucleus ($^{56}\text{Fe}$). The existence of more exotic states such as strange quark matter (SQM – an approximately equal mixture of u, d, s quarks) in the core of compact stars was also speculated \cite{1,2,3,4}. According to this SQM hypothesis, there could exist a whole sequence of SQM objects, ranging from strange stars (SSs), with masses similar to those of neutron stars (NSs) \cite{5,6,7}, to strange dwarfs (SDs) – stellar objects composed of a small SQM core and a thick normal matter crust which could be regarded as the counterpart of normal white dwarfs (WDs) \cite{8,9,10}, and even to strange quark planets \cite{8,9,11,12,13}.

According to previous studies \cite{3,6,8,9,10}, SQM stars may be bare SQM objects, but they also may be covered by a crust of normal hadronic matter. From this point of view, crusted SQM objects share many common features with ordinary compact objects. The similarity between SQM stars and normal NSs makes it difficult to discriminate these two internally different compact stars observationally \cite{5}. Anyway, a great effort has been made to try to reveal the difference between them. For example, they may have different mass-radius ($M - R$) relations, cooling rates, maximum masses, gravitational wave patterns, etc. It was even suggested that SSs can host very close-in planets so that pulsar planets with an orbit period less than $\sim 6100 \text{ s}$ can be safely regarded as an indication of the existence of SQM planetary systems \cite{11,12,13}.

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There are also some potential problems associated with the existence of SSs. First, the coexistence of SSs and NSs in our Galaxy was questioned by Madsen [15] that if SSs exist then NSs cannot also exist due to the pollution by strangelets produced during the merger of two SSs. However, the results of general relativistic simulations of SS mergers [17] show that the flux of strangelets is very sensitive to the bag constant of the MIT bag model. When the bag constant is large enough, the flux of strangelets may even disappear so that other NSs would not be polluted by strangelets and would not be converted to SSs. In other words, it is still possible for NSs and SSs to coexist in the Galaxy. Many other studies [18–23] have been conducted based on the coexistence of both SSs and NSs. The second issue concerns the oscillation of pulsars. Watts and Reddy [24] claimed that magnetar oscillations are not compatible with objects not having a normal crust, i.e. bare SSs, but fit perfectly well with NSs. The key point to solving this problem is to consider the possibility that SSs are not bare but are covered by a normal matter crust. Mannarelli et al. [25] studied the oscillations of non-bare SSs and showed that the breaking of the ionic crust might generate some phenomena such as gamma-ray bursts and quasi-periodic oscillations. Thirdly, it has been argued that SSs may cool too rapidly to be compatible with observations [26]. But note that the cooling rate of SSs could be affected by many complicated factors, such as the uncertainties in the strange quark mass and the quark gap energy, the effect of normal matter crust, the uncertain rates of the quark direct/modified Urca processes and bremsstrahlung (see the review of Weber [27] and references therein). At the same time, the measuring of the ages and surface temperatures of pulsars is also troubled by large uncertainties in observations. As a result, it is still too early to draw a firm conclusion in this aspect [28]. In short, we see that more studies are still needed to understand the internal composition of dense compact stars.

Recent discovery of several ~ 2 $M_\odot$ pulsars [29–31] greatly stimulated the study of the internal composition of these enigmatic compact stars. Annala et al. [32] proposed some evidence for the existence of quark-matter cores in such massive NSs. Especially, an interesting compact object of 2.6 $M_\odot$ was reported to be associated with the gravitational wave event GW190814 [33]. Some authors argued that this massive object could be an SS [34–39] or a rapidly rotating NS with an exotic SQM core [40,41]. This further attracts researchers’ interests of trying to find SQM in compact stars and related explosive phenomena. One interesting idea is that the collapse of the crust of SSs may produce FRBs, which will serve as indirect evidence for the SQM hypothesis [42,43].

Here we will mainly focus on strange dwarfs (SDs). The normal matter crust of strange dwarfs makes them similar to normal white dwarfs (WDs), but they still have different features in the $M - R$ relation. Following Glendenning et al. [8,9], many other authors have investigated the properties of SDs and tried to differentiate them from WDs in both theoretical [27,44–51] and observational [52–57] aspects. However, note that while Glendenning et al. [8,9] suggested that strange dwarfs are stable with respect to radial oscillation due to the action of the density discontinuity between the SQM core and the crust, Alford et al. [51] recently questioned the stability of these objects after numerically solving the Sturm-Liouville equations for the lowest-energy modes of the star. They argued that strange dwarfs are not stable. In Alford et al.’s calculations, the bottom density of the crust is typically taken as $4.3 \times 10^{11}$ g cm$^{-3}$, which corresponds to the largest possible density for the crust, i.e. the neutron drip density. Note that a detailed study on the mechanical equilibrium of the crust suggests that the actual density at the bottom of the crust should be much less [58,59], i.e. no larger than $\sim \rho_{\text{drip}} / 5 \approx 8.3 \times 10^{10}$ g cm$^{-3}$. Such a thinner crust might be beneficial to the stability. Additionally, other factors such as the equation of state (EOS) of strange quark matter can also affect the stability of SDs. For example, Jiménez and Fraga [60] recently investigated the effect of strange quark mass and the interaction between quarks on the effect of EOS by means of the perturbative quantum chromodynamics (pQCD) method. Using their new EOS, they calculated the radial oscillations of quark stars. It is found that the oscillation properties are very different from that calculated under the MIT bag model. Including the effects of the strange quark mass and quark interactions can lower the period of the fundamental oscillation mode. Considering all these factors, the stability of strange dwarfs may still need further studies. Anyway, in this study, we will assume that strange dwarfs can stably exist, and will try to search for them among white dwarfs.

Identifying strange dwarfs observationally is an important method to reveal the enigma of these interesting objects. Mathews et al. [52,53] identified seven objects as candidates of strange dwarfs because their measured masses and radii are consistent with the expected mass-radius relation of strange quark dwarfs with a carbon crust. The progress in observational technology leads to a drastic increase in the number of white dwarfs being detected in the past decades. These vast amounts of data inspire us to re-examine all of them to identify more strange dwarf candidates. In this study, we will study the observed parameters of the dwarf objects systematically and try to identify possible strange dwarfs by comparing the observational data with theoretical modeling.
2. Structure of strange dwarfs

The internal structure of compact objects can be calculated by solving the general relativistic form of Tolman-Oppenheimer-Volkoff (TOV) equation [61][62],

\[
\frac{dp(r)}{dr} = -\frac{G[p(r) + p(r)/c^2][m(r) + 4\pi r^3 p(r)/c^2]}{r^2[1 - 2Gm(r)/rc^2]},
\]

(1)

\[
\frac{dm(r)}{dr} = 4\pi r^2 p(r),
\]

(2)

where \( m(r) \) is the mass within the radial coordinate \( r \), \( \rho(r) \) and \( p(r) \) are mass density and pressure at \( r \), respectively; \( G \) is the gravitational constant, and \( c \) is the speed of light. The relation between \( \rho(r) \) and \( p(r) \) is determined by the EOS which itself depends on the internal composition of the compact object.

As described in ref.[8][9], a strange dwarf is composed of SQM in the core and normal matter in the crust. There is a strong electric field between the core and the crust, which makes it possible that the crust is suspended out of contact with the SQM core [5][7]. The EOS of such a structure is characterized by a discontinuity in density between the SQM and the normal crust mass across the electric gap where the pressure at the bottom of the nuclear crust equals the pressure at the surface of the SQM core [8][9][27]. The SQM core and the normal matter crust are two distinct regions so that each region requires a different EOS. Consequently, the property of the strange dwarf is determined by two parameters, the central density of the core and the density at the crust bottom (\( \rho_{\text{ch}} \)).

For SQM in the core, we employ an EOS derived from the simple MIT bag model [4][5], i.e. \( p(r) = (\rho(r)c^2 - 4B)/3 \), where \( B \) is the so called bag constant. In our study, we take a typical value of \( B \approx 57 \text{ MeV fm}^{-3} \). For normal matter in the crust where the density ranges in a very wide range, we adopt the EOS proposed by Baym, Pethick & Sutherland (usually called as the BPS EOS) [63]. At the surface of the crust, the density is only \( \sim 8 \text{ g cm}^{-3} \). It increases quickly when the depth increases, but \(^{56}\text{Fe} \) keeps to be the dominant nuclide as long as the density is below \( \sim 8 \times 10^{6} \text{ g cm}^{-3} \). When \( 8 \times 10^{6} \text{ g cm}^{-3} < \rho < 1 \times 10^{9} \text{ g cm}^{-3} \), the dominant nuclide becomes \(^{62}\text{Ni} \) due to the presence of heavy nuclei (\(^{84}\text{Se}, ^{82}\text{Ge}, ^{80}\text{Zn}, ^{78}\text{Ni}, ^{76}\text{Fe}, ^{124}\text{Mo}, ^{122}\text{Zr} \) ) will appear in succession. Finally, at the bottom of the crust, the nuclide will be dominated by \(^{118}\text{Kr} \) and the density will be up to \( 4.3 \times 10^{11} \text{ g cm}^{-3} \). This density is called the neutron drip density (\( \rho_{\text{drip}} \)), above which neutrons will drip out from nuclei. \( \rho_{\text{drip}} \) is believed to be the largest possible density for \( \rho_{\text{ch}} \) because free neutrons could not be supported by the electric field, but will fall directly onto the SQM core and be converted to quarks [6][8][27]. However, note that a further study by [58][59] indicates that the maximum density at the crust bottom actually should be significantly smaller, i.e. no larger than \( \rho_{\text{drip}}/5 \approx 8.3 \times 10^{10} \text{ g cm}^{-3} \).

Note that the BPS EOS covers a wide range of density. It essentially can satisfactorily describe the \( p-\rho \) relation of matter as long as \( \rho \leq \rho_{\text{drip}} \) [63][64]. It incorporates the various electron-capture and fusion reactions associated with the appearance of different nuclides, which allow matter composed of nuclei to reach the cold-catalyzed ground state.

Having specified the EOS, we can solve the TOV equation by using the Runge-Kutta method to derive the properties of SQM objects with a normal matter crust. Our calculations will be carried out by assuming different initial values for \( \rho_{\text{ch}} \) (\( \rho_{\text{ch}} = 4.3 \times 10^{11} \text{ g cm}^{-3}, 8.3 \times 10^{10} \text{ g cm}^{-3}, 10^{9} \text{ g cm}^{-3}, 10^{8} \text{ g cm}^{-3} \), etc.) to investigate the effect of crusts with various thicknesses. For each \( \rho_{\text{ch}} \) value, the \( M_{\text{core}} \) will further vary from \( 10^{-12} \text{ M}_{\odot} \) to \( 0.1 \text{ M}_{\odot} \). The results of our calculations will be presented in Section 3.

3. Data Collection

The fundamental parameters of white dwarfs can be determined through various observations, especially via the photometric technique [65]. Here we are mainly concerned about the mass (\( M \)) and radius (\( R \)) of white dwarfs. We briefly describe how these parameters are measured observationally. Firstly, the distance (\( D \)) can be known from parallax measurements such as those provided by Gaia [66]. Secondly, the solid angle (\( \pi R^2/D^2 \)) and the surface temperature (\( T_{\text{eff}} \)) can be obtained by fitting the observed flux and the observed continuum spectrum. From these quantities, people can easily solve for the radius of the white dwarf. Thirdly, the surface gravity (\( g \)) can also be determined by fitting the copious line features in the spectrum, since the strength and width of the lines are sensitive to the density of particles in the atmosphere, which is controlled by the surface gravity [66][67]. Note that the central
4. Identifying strange dwarf candidates

4.1. Mass-Radius Relation

Following the procedure described in Section 2, we have calculated the theoretical $M - R$ relations for both strange dwarfs and white dwarfs. The results are plotted in figure 1. In this figure, the solid curves illustrate the mass-radius relations of white dwarfs, under various approximation. The solid magenta curve represents the $M - R$ relation in Chandrasekhar’s model [70]. The solid orange line and the solid lime curve represent the $M - R$ relation in the zero temperature model for pure He and pure Mg white dwarfs [71], respectively. The solid red curve represents white dwarfs in the BPS approach.

Note that in our modeling, the strange dwarfs are all covered by a normal matter crust. The dashed curves in figure 1 are plotted for strange dwarfs with various crust-bottom densities, with the density value marked near each curve (in units of g cm$^{-3}$). In fact, the radius of a strange dwarf is mainly determined by the crust. The vertical bar ‘’ marked with a letter “b” in each of the sequences represents the lightest object, and the cross symbol “×” marked with “c” indicates the most massive strange dwarf. The cross symbol “×” marked with “d” is the endpoint of the sequence in the case of $\rho_{cb} = 4.3 \times 10^{11} \text{ g cm}^{-3}$. The SQM core shrinks to almost zero at the end point of each curve. Comparing with the white dwarf sequences, the strange dwarf sequences are generally more compact. For example, for an object with a typical mass of 0.8 $M_\odot$, the radius of strange dwarf is significantly less than that of white dwarf. It provides us a useful clue to search for candidates of strange dwarfs.

4.2. Comparison with Observations

In figure 1 we have also plotted the observational data points which represents the 39041 white dwarfs in the MWDD database. For the sake of clarity, we did not show the error bar of each point. From this figure, we see that the majority of the data points generally comply with the conventional white dwarf theory [70][72]. But there are still some data points deviating from the conventional $M - R$ relation. For example, some massive white dwarfs exceed the Chandrasekhar mass limit of 1.44 $M_\odot$. There are also some white dwarfs having a large radius. These phenomena may be caused by different composition inside the objects [71][73], or by strong magnetic field [74][77] and fast rotation [78][79].

However, among the observed 39041 “white dwarfs”, we notice that seven stars appear to be quite special. They obviously deviate from the mass-radius curves for white dwarfs, but well match the relation of strange dwarfs. Their masses are in the range of $\sim 0.02 - 0.12 M_\odot$ and their radii are $\sim 9,000 – 15,000$ km. In other words, they are too compact to be normal white dwarfs. We argue that these seven objects are good candidates for strange dwarfs. In figure 1 we have specially marked these strange dwarf candidates with blue color. Some key parameters of them are listed in Table 1. Note that among these seven dwarfs, the parameters of six objects (LSPM J0815+1633, LP 240-30, BD+20 5125B, LP 462-12, WD J1257+5428 and 2MASS J13453297+4200437) were derived based on the spectroscopic data and the Gaia observations [65]. In their calculations, H/He composition of the atmosphere has been
Figure 1: The mass-radius relations for the white dwarf and strange dwarf sequences. Different curves represent the mass-radius relations for compact dwarfs with different compositions. Observational data points corresponding to the 39041 white dwarfs in the MWDD database are also plotted. The black points represent normal white dwarfs and the blue points indicate candidates of strange dwarfs identified in this study. See the main text in Section 4 for more details of the curves and symbols.

Table 1: Strange dwarf candidates in our sample.

| MWDD ID       | $T_{\text{eff}}$ (K) | $\log g$ (cm s$^{-2}$) | $M$ ($M_\odot$) | $R$ (km) | Ref. |
|---------------|---------------------|------------------------|-----------------|---------|------|
| LSPM J0815+1633 | 4655 ± 35           | 6.772 ± 0.076          | 0.082 ± 0.031   | 13563.23 ± 1024.76 | 65   |
| LP 240-30     | 4680 ± 25           | 6.768 ± 0.039          | 0.081 ± 0.016   | 13542.5 ± 626.6   | 65   |
| BD+20 5125B   | 4395 ± 90           | 6.795 ± 0.097          | 0.08 ± 0.038    | 13046.72 ± 1124.18 | 65   |
| LP 462-12     | 4800 ± 20           | 6.697 ± 0.054          | 0.054 ± 0.024   | 11999.23 ± 1552.78 | 65   |
| WD J1257+5428 | 7485 ± 85           | 6.441 ± 0.068          | 0.032 ± 0.03    | 12403.13 ± 3561.25 | 65   |
| 2MASS J13453297+4200437 | 4270 ± 75 | 6.688 ± 0.086          | 0.031 ± 0.04    | 9186.23 ± 3405.51 | 65   |
| SDSS J085557.46+053524.5 | 10670 ± 1677 | 6.09 ± 1.078          | 0.02 ± 0.245    | 14688.29 ± 767.07 | 80   |

Table 1: Strange dwarf candidates in our sample.

5. Comparison with previous studies

It is interesting to note that Mathews et al [52] have also suggested a few compact objects as candidates of strange dwarfs. To give a direct comparison with their results, we present a zoomed-in plot of the mass-radius relation for
strange dwarfs in figure 3. From this figure, we see that candidates of [52] are mainly in the mass range of 0.4 – 0.8 $M_\odot$ and the radii are 6,000 – 9,000 km. In such a high-mass region, the difference between strange dwarfs and white dwarfs on the mass-radius plot is actually very small. In fact, all the candidates of [52] lie only slightly above the dashed curves, which means they still seem to be not compact enough. On the contrary, our candidates are in the low-mass region, with masses between 0.02 – 0.12 $M_\odot$ and radii between 9,000 – 15,000 km. In this region, the difference between strange dwarfs and white dwarfs is quite significant. So, it is a more appropriate region for discriminating between these two kinds of dwarfs. From figure 3 we clearly see that our candidates can only be fitted by the strange dwarf curves. They obviously deviate from the white dwarf sequences even when the error bars are considered (c.f. figure 1). Figure 3 also shows that these strange dwarfs generally should have a crust bottom density of $1.0 \times 10^9$ g cm$^{-3} \leq \rho_{cb} \leq 1.0 \times 10^{11}$ g cm$^{-3}$, which is significantly smaller than the neutron drip density ($\rho_{drip}$). From figure 3 we argue that our sample are more credible strange dwarf candidates.

The lower panel of figure 3 shows the SQM fraction versus the stellar radius of strange dwarfs. Here, the SQM fraction is defined as the mass percentage of the SQM core with respect to the whole dwarf star ($M_{\text{core}}/M$). We see that the SQM fraction of low-mass strange dwarfs are higher than that of high-mass ones on each curve (see Section 4.1 for the variation of $M_{\text{core}}$). The SQM fraction is about 3.58\% ($M = 9.1 \times 10^{-3} M_\odot$, $M_{\text{core}} = 3.26 \times 10^{-4} M_\odot$) for the largest radius object in the case of $\rho_{cb} = 10^8$ g cm$^{-3}$, and is about 19.2\% ($M = 8.75 \times 10^{-2} M_\odot$, $M_{\text{core}} = 1.68 \times 10^{-2} M_\odot$) for the largest radius object in the case of $\rho_{cb} = 4.3 \times 10^{11}$ g cm$^{-3}$. Since the candidates of [52] are generally more massive, their SQM fractions are typically much smaller than 0.001. However, the SQM fraction of our candidates are larger than 0.001, which means the SQM core plays a more important role inside the star so that its observational effects could be more significant. For example, it may affect the cooling process of the star (see discussion in the next section). In fact, the strange dwarf candidates in our sample generally have a core mass of $M_{\text{core}} \sim 10^{-4} - 10^{-2} M_\odot$. The relatively larger SQM fraction of less massive strange dwarfs further supports our scheme of trying to search for SQM objects among smaller white dwarfs.
6. Discussion and Conclusions

In this study, we have tried to search for strange quark dwarf candidates among the observed white dwarfs. The selection of candidates is conducted by comparing the mass and radius of observed dwarfs with the theoretical mass-radius relation of strange dwarfs. It is found that the masses and radii of seven objects are consistent with that of strange dwarfs. The mass of the SQM core of these candidates ranges in \( \sim 10^{-4} - 10^{-2} M_{\odot} \). The seven objects are
How strange dwarfs are produced is an interesting question. It has been suggested that strange dwarfs may be formed either from the capture of strange nuggets by stellar objects (main-sequence stars, brown dwarfs, etc.), or from the growth of strange clumps by accreting from ambient medium. Below are several detailed formation channels for strange dwarfs. Firstly, the Universe once went through a quark era in its expansion history according to the big bang theory. At that time, a large amount of SQM clumps may be formed and survive up to now [3]. They can be captured by small main-sequence stars and fall into the center of the star, and convert the normal matter into SQM [5]. Secondly, explosive events such as supernovae, merging of two compact stars, phase transition of massive NSs may be responsible for the formation of SSSs, and a large amount of SQM nuggets may be ejected during these processes. The flux of SQM nuggets produced in this way in a typical galaxy is estimated as ~ 0.1 cm$^{-2}$ s$^{-1}$ [6][16][82]. These SQM nuggets can contaminate surrounding objects, and convert them into strange dwarfs [33]. Thirdly, a newborn SS is quite hot and highly turbulent. It may directly eject a large clump of SQM due to the joint effect of fast-spinning and turbulence [11][12][84]. The SQM clump then can accrete ambient matter throughout its life and evolve to its current form.

A remarkable feature of the strange dwarf candidates in our sample is that their temperatures and masses are very low (see table[1]). Generally, their temperatures are cooler than 13,000 K and their masses are in the range of 0.02 – 0.12 $M_\odot$. Many authors have discussed this feature in the framework of white dwarfs. Low-mass white dwarfs are likely formed in binary systems because it is impossible for a star with a mass below 0.45 $M_\odot$ to become a white dwarf through single-star evolution within the age of the universe [85][86]. On the contrary, in a binary system, the progenitor star can lose mass through binary interactions [47][76][87][88]. This opinion is supported by the observations of several extremely low-mass white dwarfs in compact binaries involving NSs/pulsars [47][87][89][90] and WDs [91][92]. However, there is no evidence for binarity for most of our candidates according to present observations. As for the low temperature, Blouin et al. [65] suggested that these ultra-cool objects may be polluted by rocky debris, but they did not provide any information about the origin of these rocky debris around such low-mass objects. In this study, we have identified these objects as strange dwarf candidates. The low temperature might be self-consistently explained as due to the effect of the SQM core inside them. In fact, SQM objects generally cool faster so that their surface temperature is usually lower than their hadronic matter counterparts [26]. It is interesting to note that more candidates of small SQM stars up to planetary SQM objects have been suggested recently by Huang and Yu [14], Kuerban et al. [15][93]. Especially, the planet PSR 1719-14 b is generally regarded as an appropriate candidate because it has a density at least larger than 23 g cm$^{-3}$ [12][14][15][93][94]. In the future, these objects deserve being extensively examined by using large optical/infrared telescopes and radio facilities.

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