HAS A STAR ENOUGH ENERGY TO EXCITE THE THOUSAND OF MODES OBSERVED WITH CoRoT?

A. MOYA\textsuperscript{1,4} AND C. RODRÍGUEZ-LÓPEZ\textsuperscript{2,3}

\textsuperscript{1} Instituto de Astrofísica de Andalucía, IAA - CSIC, Granada E-18008, Spain; amoya@cab.inta-csic.es
\textsuperscript{2} Laboratoire d'Astrophysique de Toulouse-Tarbes, CNRS, Université de Toulouse, F-31400 Toulouse, France
\textsuperscript{3} Universidade de Vigo, Dpt. Física Aplicada, Vigo, Spain E-36210

Received 2009 September 22; accepted 2009 December 15; published 2010 January 19

ABSTRACT

The recent analyses of the light curves provided by CoRoT have revealed pulsation spectra of unprecedented richness and precision—in particular, thousands of pulsating modes and a clear distribution of amplitudes with frequency. In the community, some scientists have started doubting the validity of the classical tools to analyze these very accurate light curves. This work provides the asteroseismic community with answers to this question showing that (1) it is physically possible for a star to excite at a time and with the observed amplitudes such a large number of modes; and (2) that the kinetic energy accumulated in all those modes does not destroy the equilibrium of the star. Consequently, mathematical tools presently applied to the analyses of light curves can a priori be trusted. This conclusion is even more important now, when a large amount of space data coming from Kepler is currently being analyzed. The power spectrum of different stellar cases and the non-adiabatic code GraCo have been used to estimate the upper limit of the energy per second required to excite all the observed modes and their total kinetic energy. A necessary previous step for this study is to infer the relative radial pulsational amplitude from the observed photometric amplitude, scaling our linear pulsational solutions to absolute values. The derived upper limits for the required pulsational energy were compared with (1) the luminosity of the star; and (2) the gravitational energy. We found that both upper energy limits are orders of magnitude smaller.

\textit{Key words:} stars: individual (HD 174936, HD 49434) – stars: oscillations – stars: variables: Cepheids – stars: variables: delta Scuti

1. INTRODUCTION

The first data provided by CoRoT for \( \delta \) Scuti stars made clear that the larger the precision of the observations, the larger the number of observed modes (Poretti et al. 2009; García-Hernández et al. 2009). In the case of HD 174936, for example, (García-Hernández et al. 2009), a \( \delta \) Scuti target of the initial CoRoT run, the number of frequencies extracted from the observed light curves with the traditional tools can be on the order of thousands. The same can be said from other CoRoT \( \delta \) Scuti and \( \gamma \) Doradus targets. In such a scenario, the question if all these frequencies are real or not should be a previous step to any modeling attempt. To answer this, our aim is to check if a \( \delta \) Scuti or a \( \gamma \) Doradus star has enough energy to excite such a large amount of modes simultaneously and with the observed amplitudes for which we need to know the real energy of the modes.

To our knowledge, the only existing studies about the real energy of the modes, through characterization of their amplitudes, are for stochastic pulsators (as solar-like stars) which were done through a description of the nonlinear effects in the pulsation equations (Samadi et al. 2003; Belkacem et al. 2008). In the case of thermodynamic pulsators, as those driven by a \( \kappa \) mechanism, these kinds of studies have not yet been attempted, since the non-adiabatic studies have been done in the linear approximation (Osaki 1990). Therefore, the solutions yielded by the pulsation codes are not absolute, as they are affected by a constant indefinite factor, and only ratios of observed and theoretical quantities can be compared.

We scaled the solutions given by the non-adiabatic linear pulsational code GraCo (Moya et al. 2004; Moya & Garrido 2008) to absolute values using the observational amplitudes provided by space- and ground-based data, as described below. In this way, we were able to compare the derived absolute values of the required pulsational energy to simultaneously excite all the observed modes with the real energy available for exciting the modes.

In our study, we use one of the models best reproducing the ground-based and space observations obtained for the \( \delta \) Scuti HD 174936 (see García-Hernández et al. 2009), and a model at the center of the observed HR photometric error box for the hybrid \( \delta \) Scuti—\( \gamma \) Doradus star HD 49434. In order to check the consistency of our procedure, we use a representative model of the largest amplitude pulsator known, a classical Cepheid star presenting variations on the order of magnitudes, and verified that it is energetically stable under its pulsations.

2. SCALING LINEAR SOLUTIONS TO ABSOLUTE VALUES

To determine the energy needed by the system to unstabilize all of the observed modes, we have to face the problem that most of the pulsational codes, and in particular the non-adiabatic ones, use the linear approximation to solve the system of differential equations. Aside from nonlinear physical effects, negligible for small pulsational amplitudes, this means that every eigenfunction of an oscillation mode is known except for a multiplicative constant, i.e., if \( f(x) \) are the solutions of the set of differential equations for a given mode, \( A f(x) \), \( A \) being a constant, are also solutions. Therefore, before doing any study about the global properties of the star, we have to determine the value of this constant \( A_{\text{real}} \) scaling the linear solutions of each mode to the real values.

In the linear approximation, the problem of this indefinite constant is solved by scaling all the solutions of the pulsation
equations to an arbitrary normalization condition. Most pulsation codes use as a normalization condition
\[ \frac{\xi_r}{r} = 1 \text{ at } r = R, \] (1)
where \( \xi_r / r \) is the radial eigenfunction (Dziembowski 1971).

The real pulsation amplitude on the surface is given by
\[ \frac{\delta R}{R} = A_{\text{real}} \left( \frac{\xi_r(r = R)}{R} \right), \] (2)
and following Equation (1)
\[ \frac{\delta R}{R} = A_{\text{real}}. \] (3)

Therefore, the multiplicative constant \( A_{\text{real}} \), scaling all the linear solutions to the real values of the pulsation, may be obtained through the oscillation amplitude of the relative radius at the surface of the star. This value can be deduced from the oscillation amplitude of the observed luminosity.

We use the observed amplitude spectrum for HD 174936 (Figure 1 and García-Hernández et al. 2009) as an example to illustrate the procedure presented here. The spectrum shows a maximum oscillation amplitude of about 2 mmag, which would correspond to a low-degree mode. Decreasing amplitudes are found at both sides of this highest amplitude mode. Therefore, we assume that “the mode amplitude is given by a Gaussian distribution centered in the largest amplitude mode, with the maximum amplitude of this mode, and the variance fixed by the decrease in amplitude of the modes closest to the highest one. This intrinsic amplitude is independent of the spherical degree \( \ell \) of the mode.” The oscillation amplitudes of the other modes, out of the highest amplitude one, would be affected by geometrical visibility, which is translated in amplitudes lower than the predicted Gaussian envelope.

In this case, the distribution is centered on 377 \( \mu \)Hz, with a maximum amplitude of 2.12 mmag and a square root of the variance \( \sigma = 50 \) \( \mu \)Hz. The assumed oscillation amplitudes are also shown in Figure 1 (left, plus signs), compared with the observed amplitude spectrum. We note that a significant number of observed amplitudes fall out of the range [200, 500] \( \mu \)Hz, where our distribution predicts amplitudes close to zero. Therefore, we built a modified distribution as the addition of the Gaussian already explained plus a flat background at 0.3 mmag for the modes with negligible amplitude (Figure 1, right, plus signs). There is no physical reason for this election, but only a fitting of the observations. The value of the background was chosen to take into account the amplitude of the observed mode at 158.1 \( \mu \)Hz, and the distribution is spanned in the range [100, 800] \( \mu \)Hz well comprising the canonical range of observed modes for \( \delta \) Scuti. We deliberately leave out modes under 100 \( \mu \)Hz as they lay far out of the instability range of \( \delta \) Scuti, and no model is able to predict their excitation.

The next step is to relate this intrinsic oscillation amplitude in luminosity to oscillation amplitude in relative radius. To do so, we use the definition of the apparent magnitude variation for a certain wavelength given by Balona & Stobie (1979)
\[ \Delta m_\lambda = \Delta S_\lambda - 2.5 \log e \Delta A/A, \] (4)
where \( A \) is the projected area of the photosphere, and \( \Delta S_\lambda \) is the variation in surface brightness defined as
\[ \Delta S_\lambda = -2.5 \Delta \log (\bar{F}_\lambda), \] (5)
with \( \bar{F}_\lambda \) being the projected flux at a certain wavelength.

Given that the unknown multiplicative constant is defined by the variation of the radius, we are only interested in the intrinsic amplitude of the radial eigenfunction. Therefore, although the relative variation of the projected surface area \( \Delta A/A \) includes dependences as the degree \( \ell \) of the modes, the viewing angle or the limb darkening, we may adopt, for our purposes, \( \Delta A \) as the radial variation of the total surface of the star. In addition, as the observed CoRoT amplitude spectrum for HD 174936 was taken in white light, we will consider \( F_\lambda \) as the total flux at the surface. Therefore, \( \Delta m_\lambda = \Delta m = \Delta M_{\text{bol}} \), since the bolometric magnitude \( (M_{\text{bol}}) \) and absolute magnitude \( (m) \) differ only in a constant. Including all this in Equation (4) yields
\[ \Delta m = -2.5 \log e \left[ \Delta (\ln \sigma T_{\text{eff}}^4) - 2 \frac{\Delta R}{R} \right]. \] (6)
where \( \sigma \) is the Stefan Boltzmann constant. Simple calculations give
\[ \Delta m = \left( -5 \frac{\Delta R}{R} - 10 \frac{\Delta T_{\text{eff}}}{T_{\text{eff}}} \right) \log e. \] (7)
Therefore, the variations in brightness are a combination of the variations of the relative radius and the relative flux at the surface of the star, measured through relative variations of the effective temperature. The observed variations of the relative radius may be obtained as

\[ \frac{\Delta R}{R} = -\frac{\Delta m}{\log e(5 + 10dT)}. \]  

(8)

where

\[ dT = \frac{\delta T_{\text{eff}}}{T_{\text{eff}}} \int \frac{\xi_r}{r} \, \text{at} \, r = R. \]  

(9)

The negative sign of this equation is related to the definition of the stellar magnitude, and it has no physical origin. We will not take it into account in the rest of this study, since it has no influence in the results. Therefore, once the variations in brightness are known, the relative radius variations can be obtained, as the variable \( dT \) is supplied by the non-adiabatic equations. We used the GraCo non-adiabatic code for this purpose, including the time-dependent convection description (TDC, Grigahcène et al. 2005; Dupret et al. 2005), although its inclusion revealed of no significance in the results. The final relative radius variations as a function of the frequency of the modes for the two distributions are displayed also in Figure 1.

3. OBTAINING THE ENERGY BALANCE OF THE MODES FOR DIFFERENT STELLAR CASES

Once the multiplicative constant \( \delta R/R \) scaling all the linear solutions to their real values is known, the absolute values of the kinetic energy and the energy required to excite the observed modes can be estimated. Then, they can be compared to the energy balancing the star and the energy available in the star, respectively.

To check if the star is able to have thousands of modes excited at a time, we have to verify (1) if the energy flux is enough to provide the energy that all these modes need per second, and (2) if the total kinetic energy accumulated in them does not destroy the equilibrium structure of the star. We note that quantitative estimations of upper limits of these energies are enough to achieve the aims of this work.

Three stellar cases have been studied: (1) a representative model of the classical Cepheid stellar type, adopting the largest pulsation amplitude reported (around 2 mag) as a falsifiability test of our main assumptions, (2) the \( \delta \) Scuti star HD 174936, and (3) the hybrid \( \delta \) Scuti—\( \gamma \) Doradus star HD 49434.

For the latter two cases, we have used CoRoT and ground-based observations. In the case of HD 49434, the publication of the data analysis is still in preparation and here we use only some general and preliminary results, which is, however, enough for our purpose.

3.1. A Classical Cepheid Star

Our first stellar case applies to the variable stellar type with the highest amplitude luminosity known to date, the classical Cepheid type. This type of stars do not show a large amount of pulsational modes; however, we are going to use it as a test of falsifiability of our main hypotheses, since for them, their high variability offsets the smaller number of pulsation modes.

We built a representative equilibrium model lying in the classical Cepheid instability strip (Sandage et al. 2009). The main physical characteristics of this model are displayed in Table 1, and are similar to one of the theoretical models studied in Bono et al. (1999). The energy balance of the pulsational modes is then compared to some absolute values of the star and with some theoretical results obtained through nonlinear resolutions, as described in Section 2.

The variation of the relative radius of the fundamental radial mode obtained following our procedure (\( \Delta R/R = 0.16 \)) is similar to the predicted for an equivalent nonlinear model described in Bono et al. (1999) (\( \Delta R/R = 0.13 \)).

3.1.1. Pulsational Energy

The energy balance between gains and losses, for a single mode, in a complete period of oscillation is given by Unno et al. (1989):

\[ W = \frac{\pi}{\sigma} \int_0^M \frac{\delta T}{T} \left( \epsilon_N - \frac{1}{\rho} \nabla \cdot \vec{F} \right) dM. \]  

(10)

This quantity is provided by GraCo and can also be regarded as the required energy for each mode to be driven during one oscillation cycle. The scaling of the linear solutions to their real values is done multiplying Equation (9) by \( A_{\text{real}}^2 \). As we are interested in an upper limit for this quantity, we assume that for every mode the energy balance is positive, i.e., that each mode removes energy from the star. This is only true for unstable modes, as stable modes transfer energy to the star. However, in this way, we make sure that the real amount of energy subtracted by the modes from the star, regardless of the theory used to obtain it, will be always lower than our estimated upper limit.

The total energy during a cycle of oscillation is given by the sum of all the energy interchanged by all the modes in the observed frequency range. We compared this energy to the luminosity of the star, as a measure of the available radiation energy per time unit. For classical Cepheid stars, we have used the fundamental radial mode and its overtones. Therefore, the sum of \( W \cdot \sigma \) for each mode (\( \sigma \) being the frequency of the mode in Hz) yields a total energy per time unit needed by the modes to be overstable of \( 10^{35} \text{ erg s}^{-1} \). The luminosity of the star is on the order of \( 10^{36} \text{ erg s}^{-1} \); that is, one order of magnitude higher. As we expected, the real energy value to be even lower than the one estimated here, we conclude that this extreme case has enough energy per time unit to unstabilize all observed modes. This result is used only to legitimize our main assumptions.

3.1.2. Total Kinetic Energy

For the second comparison, we have calculated the kinetic energy of each mode as (Unno et al. 1989)

\[ E_{\text{kin}} = \frac{1}{2} \sigma^2 \int_0^M \xi_r^2 dM. \]  

(11)

Once the kinetic energy of each mode is scaled by its variation of the relative radius (multiplying by \( A_{\text{real}}^2 \)), the total sum of the kinetic energies of the selected modes gives a total kinetic energy of the order of \( 10^{42} \text{ ergs} \). Bono et al. (1999) give the same order of magnitude for the kinetic energy of the model we
are comparing with. This value also has to be compared with the total energy of the star, i.e., the sum of the internal energy of the star, mass, rotational kinetic energy, turbulent energy, gravitational energy, etc. The estimation of all these parameters is a complex task, but the gravitational energy that holds the star together is already on the order of $10^{49}$ ergs. Therefore, we can infer that the total kinetic energy of pulsations of the radial modes for the case of a classical Cepheid star is low enough for the star to bear it maintaining the hydrostatic equilibrium. Again, our assumptions are validated with this test.

### 3.2. A $\delta$ Scuti Star Observed by CoRoT (HD 174936)

The $\delta$ Scuti HD 174936 was the first CoRoT target of this stellar type (together with HD 50844) presenting unexpected space data. The observational spectrum and the relative variation of the radius have already been presented and explained in Section 2 and depicted in Figure 1. The physical characteristics of the model used are displayed in Table 1.

The total energy per oscillation cycle is given by the sum of all the energy interchanged by all the modes in the frequency range $[100, 800] \mu$Hz (a generous gamut for $\delta$ Scuti, as we said above) and with degrees $\ell = [0, 7]$, to be able to account for the thousands of observed modes. The observation of spherical degrees up to $\ell = 7$ with high-precision photometric time series was predicted by Daszyńska-Daszkiewicz et al. (2006).

The sum of $W \cdot \sigma$ for each mode yields a total energy per time unit needed by the modes to be overstable of $10^{39}$ erg s$^{-1}$ for both proposed amplitude distributions (with and without TDC). The luminosity of the star is of the order of $10^{31}$ erg s$^{-1}$; that is, five orders of magnitude higher. Consequently, as the real energy value is even lower than the one estimated here, we conclude that the star has enough energy per time unit to unstabilize all the observed modes.

On the other hand, the total sum of the kinetic energies of the selected modes gives a total kinetic energy in the range $[10^{39}, 10^{41}]$ ergs, depending on the amplitude distribution. The gravitational energy holding the star together is of the order of $10^{48}$ ergs. Therefore, as in the previous case, the total kinetic energy of pulsations of the thousands of modes is low enough for the star to bear it maintaining the hydrostatic equilibrium.

### 3.3. A Hybrid $\gamma$ Doradus—$\delta$ Scuti Star Observed by CoRoT (HD 49434)

The last stellar case studied is a hybrid $\delta$ Scuti—$\gamma$ Doradus star. HD 49434, which has also been observed by CoRoT. The analysis of its light curve is still in progress, and the results have not yet been published. Hence, we use only the preliminary result that thousands of modes have been detected in the $\gamma$ Doradus region, tens of them reported as “undoubtedly” real (E. Rodríguez 2009, private communication).

Pulsations in the $\delta$ Scuti region of the frequency spectrum have also been reported (Uytterhoeven et al. 2008). We assume that in this region the situation is similar to that found for HD 174936 in order to study a possible future situation. Uytterhoeven et al. (2008) did ground-based multisite observations for this star: we use the highest amplitude mode reported to fix the center and maximum amplitude of our Gaussian distribution. The mode has a photometric amplitude of 2.0 mmag at a frequency of 20 $\mu$Hz. The observed frequencies in the $\delta$ Scuti region present values around 90 $\mu$Hz. We have proceed in a way similar to the previous case, but using a double Gaussian, one for each region presenting pulsational modes. A flat background of 0.3 mmag has also been used for a second possible distribution, as explained in Section 2.

We built a model fulfilling the physical characteristics shown in Uytterhoeven et al. (2008; see Table 1). For this study, frequencies in the range $[3, 900] \mu$Hz, to well cover the $\delta$ Scuti and $\gamma$ Dor range, and degrees in the range $\ell = 0, 7$ have been used.

We find that the total energy per time unit needed by the modes to be overstable is on the order of $10^{30}$ erg s$^{-1}$ (only using TDC, more suitable for $\gamma$ Dor targets). The luminosity of the star is on the order of $10^{34}$ erg s$^{-1}$, that is, four orders of magnitude higher. We conclude again that the star has enough energy per time unit to unstabilize all the possible modes. The total sum of the kinetic energies of the modes selected is in the range $[10^{22}, 10^{43}]$ ergs, depending on the amplitude distribution. The gravitational energy that holds the star together is of the order of $10^{49}$ ergs. Again, we conclude that the total kinetic energy of pulsation of the thousands of modes is low enough for the star to bear it maintaining the hydrostatic equilibrium.

### 4. CONCLUSIONS

Using a non-adiabatic linear pulsational code, we have related the observed variation in brightness of classical Cepheid stars, a $\delta$ Scuti, and a hybrid $\delta$ Sct-$\gamma$ Dor star with the intrinsic variation of their relative radius. This has allowed us to scale theoretical quantities from this linear code of oscillations to real values of the star, and to make the first comparison to date, up to our knowledge, of absolute pulsational quantities with absolute parameters of non-stochastic pulsating stars.

The subsequent analysis of the total kinetic energy of the thousands of observed modes, and of the required energy to excite them, shows that all of them have enough energy to sustain their oscillation. Our study shows that the results yielded by the analyses of the light curves with the classical tools (standard sine wave fitting) do not come into conflict with what it is physically possible.

A.M. acknowledges financial support from a Juan de la Cierva contract of the Spanish Ministry of Science and Innovation. C.R.L. acknowledges an Ángeles Alvariño contract under Xunta de Galicia.

### REFERENCES

Balona, L. A., & Stobie, R. S. 1979, MNRAS, 187, 217
Belkacem, K., Samadi, R., Goupil, M. J., & Dupret, M.-A. 2008, A&A, 478, 163
Bono, G., Marconi, M., & Stellingwerf, R. F. 1999, ApJS, 122, 167
Daszyńska-Daszkiewicz, J., Dziembowski, W. A., & Pamyatnykh, A. A. 2006, Mem. Soc. Astron. Ital., 77, 113
Dupret, M.-A., Grigahcène, A., Garrido, R., Gabriel, M., & Scuflaire, R. 2005, A&A, 435, 927
Dziembowski, W. A. 1971, Acta Astron., 21, 289
García-Hernández, A., et al. 2009, A&A, 506, 79
Grigahcène, A., Dupret, M.-A., Gabriel, M., Garrido, R., & Scuflaire, R. 2005, A&A, 434, 1055
Moya, A., & Garrido, R. 2008, Ap&SS, 316, 129
Moya, A., Garrido, R., & Dupret, M.-A. 2004, A&A, 414, 1081
Osaki, Y. 1990, Prog. Seismol. Sun Stars, 367, 75
Poretti, E., et al. 2009, A&A, 506, 85
Samadi, R., Nordlund, Å., Stein, R. F., Goupil, M. J., & Roxburgh, I. 2003, A&A, 403, 303
Sandage, A., Tammann, G. A., & Reindl, B. 2009, A&A, 493, 471
Unno, W., Osaki, Y., Ando, H., Sato, H., & Shihabashi, H. 1989, Nonradial Oscillations of Stars (2nd ed.; Tokyo: Univ. of Tokyo Press)
Uytterhoeven, K., et al. 2008, A&A, 489, 1213