Critical current of a S/F-I-N/S tunnel structure 
in a parallel magnetic field

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Abstract

We study the critical current of a S/F-I-N/S tunnel structure - a proximity coupled superconductor (S) with thin ferromagnetic (F) and nonmagnetic (N) metals separated by an insulating (I) barrier - in an external magnetic field. The dependence of the current on exchange and external magnetic fields, and their mutual orientation is analyzed within the microscopic theory of the proximity effect. We find that the S/F-I-N/S contact with strong magnetism of the F layer is in a ground state with superconducting phase difference on the electrodes about \( \pi/2 \). Low external magnetic field reverses the critical current sign of such a contact (a \( \pi \)-state of the junction) for parallel field orientation. For antiparallel alignment the dc Josephson current behavior is essentially nonmonotonic: by increasing an external magnetic field the current passes though a maximum, while the 0-phase state is held, and then the junction gets into the \( \pi \)-phase state with an opposite current direction. We demonstrate that the hybrid S/F-I-N/S metal structures possess new effects of the superconductivity in the presence of spin splitting, while an experimental set-up seems to be feasible and it may lead to further understanding of superconducting proximity effects in ferromagnetic materials.

I. INTRODUCTION

The problem of superconductivity in the presence of spin splitting is currently heavily investigated. In fact, this question was already addressed experimentally and theoretically a while ago for the simplest case of a very thin superconducting film in a parallel magnetic field. For such geometry the quasiparticle energy states in a superconductor are splitted due to the interaction of the field with spin magnetic moments of the quasiparticles, while the orbital effects are negligible. This splitting has been earlier demonstrated by extensive studies of tunnel junctions formed by thin superconducting films (see, e.g., [1-3] and references therein).

A more sophisticated case is the proximity effected hybrid systems consisting of a superconductor (S) and a ferromagnet (F). In a ferromagnet, the difference in the energy between spin-up and spin-down bands is an internal property, and that is the physical reason why a singlet Cooper pair, injected from a superconductor into a ferromagnet, acquires a finite momentum. The resulting proximity induced superconducting state of the F layer is qualitatively different from the zero-momentum state: it is spatially inhomogeneous and the order parameter contains nodes where the phase changes by \( \pi \) (see recent review [4] and references therein). Particularly, transport properties of the S/F structures have turned out to be quite unusual. Experiments that have been performed on SFS weak links [5-7] and SIFS tunnel junctions [8] directly prove the so-called \( \pi \)-phase superconductivity. Planar tunneling spectroscopy also reveals a \( \pi \)-phase shift in the order parameter, when superconducting correlations coexist with the ferromagnetic order [9]. An inverse phenomenon, the modification of the BCS density of states in mesoscopic S layer under the influence of a ferromagnet is observed in tunnel spectroscopy experiment [10] as well. For S/F-I-F/S tunnel structures with very thin F layers, it was also predicted [11,12] that one can, if there is a
parallel orientation of the F layers magnetization, turn the junction into the $\pi$-phase state with the critical current inversion, or, if there is an antiparallel orientation of the F layers internal fields, save the 0-phase junction state but enhance the tunnel current. Experimentally, however, the enhancement of the dc Josephson current has not been detected until now. One of the main problems here is the requirement to fix one layer magnetization of the sample, while preserving a possibility to reorient the magnetization of another layer by a weak magnetic field.

In this report we draw attention to another experimental set-up where, as we show below, there is the critical current enhancement, as well as other new effects of the superconductivity in the presence of spin splitting, while the experimental set-up admits of simple variations of an applied field direction. Namely, we consider the dc Josephson effect for tunnel structure where there is spin splitting for two reasons - due to an external magnetic field and due to an internal exchange field of the F layer. The ground state and critical current dependence on the applied field is analyzed for a system of proximity coupled bulk superconductor with thin ferromagnetic and thin normal nonmagnetic metals separated by an insulating barrier (S/F-I-N/S tunnel junction). Based on a microscopic theory of the superconducting proximity effect for S/F and S/N bilayers we demonstrate that the S/F-I-N/S contact with strong enough magnetism of the F layer is in a ground state with $\pm \pi/2$ superconducting phase difference on the electrodes. In such a contact low external magnetic field reverses the critical current sign (the $\pi$-state of the junction) if the fields (external and internal) have parallel orientation. For the antiparallel fields alignment the critical current behavior is much more complicated: a monotonic increase of the external field causes an enhancement of the critical current, which is then changed by a reverse of current sign.

The article is organized as follows. In Sec. II we outline the model of the junction. In Sec. III the outcomes of the theory of proximity coupled S/F and S/N bilayers have been generalized to include external magnetic field effects. The main results of the paper are presented in Sec.IV. In Sec. V we evaluate the approximations that have been made in the theory and briefly discuss a possible realization of the S/F-I-N/S systems to explore the predicted effects.

**II. THE MODEL OF A JUNCTION**

The system we are interested in is the S/F-I-N/S tunnel structure in a parallel magnetic field shown in Fig.1. We assume that the first (left) electrode of the junction is formed by a proximity coupled S/F bilayer while the second (right) is formed by a proximity coupled S/N bilayer. The transparency of the insulating layer is small enough to neglect the effect of a tunnel current on the superconducting state of the electrodes. The S/F and S/N interfaces may have arbitrary finite transparency, but it is large compared to the transparency of the junction barrier. The F layer will be treated as a single domain ferromagnet. We also expect the F layer magnetization to be aligned parallel to the interface, so that it does not create a spontaneous magnetic flux penetrating into the S layer. The general relation between the Curie temperature of the F metal and the superconducting transition temperature of the S metal is considered. Transverse dimensions of the junction are supposed to be much less then the Josephson penetration depth, so that (a) all quantities depend only on a single coordinate $x$ normal to the interface surface of the materials, and (b) a flux quantum can
not be trapped by the junction.

We assume that the applied field penetrates and mostly concentrates into the "weak" F-I-N part of the junction. We will also neglect the effect of the magnetic field on electron orbits as compared with the effect of the field on electron spins. This assumption is well satisfied for films thinner than their superconducting coherence lengths \( d_{F,N} \ll \xi_{F,N} \). Here \( d_{F,N} \) is the thickness of the F and N layers, respectively; \( \xi_{F} \sim \sqrt{D_{F}/2\pi T_{C}} \) and \( \xi_{N} \sim \sqrt{D_{N}/2\pi T_{C}} \) are superconducting coherence lengths for F and N metals; \( D_{F,N} \) is a diffusion coefficient in the F, N metal; \( H_{e} \) is the exchange energy. For films of such thicknesses one can also be sure that there is a fairly uniform penetration of a parallel field. (The evaluation of these approximations will be made below, in Sec. V). We consider the "dirty" limit for all metals, i.e., we assume that \( l_{S} \ll \xi_{S} \) and \( l_{F(N)} < d_{F(N)} \), and that the condition \( H_{e} < \tau^{-1} \) is fulfilled [13], where \( l_{S,F,N} \) is the electron mean free path for the S, F, and N layers, respectively; \( \xi_{S} \) is the superconducting coherence length in the S metal; and \( \tau \) is the momentum relaxation time. (We have taken the system of units with \( \hbar = \mu_{B} = k_{B} = 1 \).

We restrict ourselves to the case when the spin-orbit scattering is absent and spin "up" and spin "down" electron subbands do not mix with each other. In equations below, we will use the effective coherence length \( \xi \sim \sqrt{D_{F}/2\pi T_{C}} \) for the F layer, thus providing the regular crossover to both limits \( T_{C} \gg H_{e} \rightarrow 0 \) and \( H_{e} >> T_{C} \) (see Refs. 14, 15 for details); the relation on film thickness is read as \( d_{F} \ll \xi_{F} \) or \( \xi _{\text{SN}} \), depending on the relation between \( T_{C} \) and \( H_{e} \).

III. S/F AND S/N SANDWICHES IN THE MAGNETIC FIELD

As a first step, we should generalize results of the theory for a proximity coupled S/F (S/N) bilayer of a massive, \( \xi_{S} \ll d_{S} \), superconductor and a thin ferromagnetic (nonmagnetic) metal taking into account the influence of the applied magnetic field \( H_{e} \). The most direct way to do it is to use the results for a S/F proximity coupled bilayer (Refs. [12,14,15]).

As is well known, the properties of "dirty" metals can be described by the Usadel equations [16]. Following [17], we introduce the modified Usadel functions \( \Phi \), used to take into account the normalized confinement on the normal \( G \) and anomalous \( F \) Green’s functions. Due to small thickness of the normal (magnetic and nonmagnetic) metal, the proximity effect problem can be reduced to a boundary problem for the S layer and a relation for determining modified Usadel function \( \Phi \) of a nonsuperconducting layer (see Refs. [14,15,17,18] for details). The boundary conditions at the SF (SN) interface are modelled by two parameters: \( \gamma_{M} \) and \( \gamma_{B} \). The \( \gamma_{M} \) plays the role of an effective pairbreaking parameter near the SF (SN) interface and its value is mainly determined by electron densities of S and F (N) metals. A large value of \( \gamma_{M} \) corresponds to high density of quasiparticles in the F (N) metal compared to that in the S near the SF (SN) boundary. In this case the diffusion of quasiparticles into the superconductor leads to a strong suppression of the order parameter in the S region. The scale at which superconductivity reaches the bulk value is \( \xi_{S} \) from the SF (SN) interface. In the opposite case, \( \gamma_{M} \ll 1 \), the influence of the F (N) layer on the superconductivity of the S metal is weak, and even vanishes if \( \gamma_{M} = 0 \). The parameter \( \gamma_{B} \sim R_{B} \), where \( R_{B} \) is the SF (SN) boundary resistance, describes the electrical quality of the SF (SN) interface. The case \( \gamma_{B} \ll 1 \) corresponds to a vanishing interlayer resistance, i.e., the S and F (N) metals are in good electric contact. The case \( \gamma_{B} >> 1 \) corresponds to low transparency of the SF
(SN) barrier. There is also one more parameter, \( H_e \), that is the value of the exchange spin splitting in the F layer. In the general case, the problem needs self-consistent numerical calculations of the Usadel equations. However, at some simplifying assumptions, the model admits of analytic solutions that hold all new physics we are interested in.

For F (N) films thinner than their superconducting coherence lengths, \( d_{F(N)} <\ll \xi_{F(N)} \), the critical current of the S/F-I-N/S tunnel contact can be represented through the S layer Green’s functions (see Eq. (3), below). The explicit expressions for these functions can be obtained on the basis of the microscopic theory of proximity effects for S/F and S/N bilayers for a weak, \( \gamma_M <\ll 1 \), and a strong, \( \gamma_M >> 1 \), proximity effect. We generalize these results assuming the external magnetic field penetrates into the F and N layers. Namely, in the case of a bulk S layer proximity coupled with a thin F layer with a weak proximity effect \( \gamma_M <\ll 1 \) and an arbitrary value of the boundary resistance \( \gamma_B \) (quite a realistic experimental case) for the function \( \Phi_{SF} \) we have the analytical solution (see Eq. (14), Ref. 14 or Eq. (10), Ref. 15):

\[
\Phi_{SF}(\omega) \equiv \Phi_{SF}(\omega, x = 0) = \Delta_S \left( 1 - \frac{\gamma_M \beta \omega_F}{\gamma_M \beta \omega_F + \omega A_F} \right),
\]

where \( \Delta_S \) is the absolute value of the superconducting order parameter in the bulk of the S layer, \( \omega_F = \omega + i \sigma (H_e + H) \), \( \omega \equiv \omega_n = \pi T (2n + 1), n = 0, \pm 1, \pm 2, \ldots \) are Matsubara frequencies, \( \sigma = \pm 1 \) is the spin sign; \( \beta^2 = (\omega^2 + \Delta_S^2)^{1/2} / \pi T C \), and \( A_F = (1 + \gamma_B \omega_F (\gamma_B \omega_F + 2\omega / \beta^2) / (\pi T C)^2)^{1/2} \). The sign of the external magnetic field depends on the relative orientation to the internal exchange field. To be definite, we will suppose that \( H_e > 0 \) and there will be the case of the parallel external fields orientation if \( H > 0 \), while for \( H < 0 \) the fields will have antiparallel orientation.

For the S/F bilayer with a strong proximity effect \( \gamma_M >> 1 \) and free value of the boundary resistance \( \gamma_B \) our calculations yield (see Eq. (12) Ref. 15):

\[
\Phi_{SF}(\omega, x = 0) \equiv \Phi_{SF}(\omega) = B(T) (\pi T C + \gamma_B \omega_F) / \gamma_M \omega_F
\]

where \( B(T) = 2T_C [1 - (T/T_C)^2] [7 \zeta(3)]^{-1/2} \) and \( \zeta(3) \) is the Riemann \( \zeta \) function.

Qualitatively similar results are obtained for the S/N bilayer. If the influence of the N layer on the superconductivity of the S metal is weak, \( \gamma_M <\ll 1 \) and the value of interface resistance is arbitrary, for \( \Phi_{SN}(\omega) \) we have the expression (1) where the substitutions should be made: \( \omega_F \rightarrow \omega_N = \omega + i \sigma H \) and \( A_F \rightarrow A_N = [1 + \gamma_B \omega_N (\gamma_B \omega_N + 2\omega / \beta^2) / (\pi T C)^2]^{1/2} \). In the opposite case of a strong proximity effect \( \gamma_M >> 1 \), for \( \Phi_{SN}(\omega) \) we have the relation (2) where \( \omega_F \) is changed by \( \omega_N \), and, as before, there is an arbitrary value of the SN boundary resistance.

Below we will suppose for simplicity, that the interface parameters involved, \( \gamma_M \) and \( \gamma_B \), have the same values for both SF and SN interfaces. An extension to different SN and SF boundary parameters is straightforward, and will be made towards the end of the Sec. IV. An additional physical approximation is also the assumption that the spin discrimination by the interfaces is unimportant. Generalization for interfaces with different transmission probabilities for quasiparticles is obvious and we do not present it here.
IV. CRITICAL CURRENT

The critical current of a S/F-I-N/S tunnel contact can be represented in the form (see, e.g., Ref. [15])

$$j_C = (eR_N/2\pi T_C)I_C = (T/T_C) \text{Re} \sum_{\omega > 0, \sigma} \{G_S \Phi_S/\omega\}|_F\{G_S \Phi_S/\omega\}|_N \times$$

$$\{[1 + 2\omega_F G_S (\gamma_B/\pi T_C) + \omega_F^2 (\gamma_B/\pi T_C)^2]_F \times [1 + 2\omega_N G_S (\gamma_B/\pi T_C) + \omega_N^2 (\gamma_B/\pi T_C)^2]_N\}^{-1/2}$$

where $R_N$ is the resistance of the contact in the normal state; the subscript F (N) labels quantities referring to the left ferromagnetic and right nonmagnetic electrodes. Here we used $G_S = \omega/(\omega^2 + \Phi_S)^{1/2}$, where $\Phi_S(\omega, H_{eff}) = \Phi_S^0(\omega, -H_{eff})$ and $H_{eff}$ is $(H + H_e)$ or $H$.

A. Strong proximity effect

Let us first present results for the case of a strong proximity effect, $\gamma_M >> 1$, and a vanishing interface resistance, $\gamma_B = 0$. If there is a strong suppression of the order parameter near S/F (S/N) boundary then, using the expressions for $\Phi_{SF}$ and $\Phi_{SN}$, we can recast the Eq. (3) in the form:

$$j_C(H, H_e) \approx 2(T/T_C) B_M^2(T) \sum_{\omega} \omega^2 \frac{\omega^2 - H(H_e + H)}{\omega^2 \{[(\omega^2 - H(H_e + H)]^2 + \omega^2(H_e + 2H)^2\}}$$

where $B_M(T) = B(T)\pi T_C/\gamma_M$, and in proceeding these relations, we have taken into account that the value of $\Phi_{SF(SN)}$ is small, $\Phi_{SF(SN)}^{-1} \gamma_M << 1$. Performing the substitution $(H_e + H) \rightarrow H_{eR}$ and $H \rightarrow \pm H_{eL}$ in Eq. (4), one can obtain the results for the (S/F)$_L$-L-(F/S)$_R$ tunnel structure for (anti)parallel orientation of the $H_{eR}$ and $H_{eL}$ fields (see Eq. (15) in Ref. 15 and Eq. (5) in Ref. 12). If $H_e = 0$ and $H = 0$ one can also restore the result for S/N-I-N/S junction (see, e.g., Eq. (31) of Ref. 17 and Eq. (46) of Ref. 18). In the general case, as it is seen from Eq. (4), in the geometry with the parallel fields by increasing the external magnetic field one can inverse the supercurrent sign of the S/F-I-N/S tunnel structure. In other words, at large enough applied field, $H(H_e + H) >> \omega^2(\pi T_C)^2$, the critical current changes its sign in comparison with that when $H \rightarrow 0$. In particular, from Eq. (4) we obtain in the limit of large field values:

$$j_C(H, H_e) \approx B^2(T)/\gamma_M^2 \sum_{\omega} 1/\omega^2(H(H_e + H)) \rightarrow (T_C - T)^2/\gamma_M^2 H(H_e + H),$$

i. e., a crossover of the S/F-I-N/S junction from the 0-phase state to the $\pi$-phase state takes place.

The most interesting results have been obtained for the geometry with the opposite field orientation: $H < 0$. Simple substitution into Eq. (4) gives for $H = -H_e$ that $j_C(-H_e, H_e) = j_C(0, H_e)$, while if $H = -H_e/2$ we directly obtain that $j_C(-H_e/2, H_e) = j_C(0, H_e)$. Comparing, we also find that $j_C(-H_e/2, H_e) \neq j_C(H_e/2, H_e)$ and
\( j_C(H_e, H_e) \neq j_C(-H_e, H_e) \). As it follows from Eq. (4), for large enough values of the external field, the critical current again inverses its sign and the crossover of the S/F-I-N/S junction from the 0-phase state to the \( \pi \)-phase state takes place. However, while increasing the field in the interval \( 0 < |H| < H_e \), a nonmonotonic current versus field dependence is observed. The main features of this dependence are shown on Fig. 2, where the Josephson current amplitude of the S/F-I-N/S junction versus the applied magnetic field is given for the case of a strong proximity effect, \( \gamma_M = 10 \) and \( \gamma_B = 0 \), and different exchange field intensity. (Here and below, dashed curves are used for the results of the numerical calculations for the parallel fields orientation and full curves are used for the results in the opposite geometry). It can be seen, that the dc current greatly depends on relative orientation of the fields.

To reveal the underlying physics behind the critical current behavior on field, let us consider a ground state and a phase shift across the contact at zero external field. As follows from Eq. (2), the modified Usadel function \( \Phi_{SF}(\omega) \) can be written in the form (the case \( \gamma_B = 0 \))

\[
\Phi_{SF}(\omega) = B(T) (\pi T C / \gamma_M) \frac{\omega - i H_e}{\omega^2 + H_e^2}
\]

Taking into account that the typical value \( \omega \sim \pi T C \), one can see that in the limit \( H_e >> \pi T C \) the unified correlation function of the S/F bilayer acquires an additional \( \pm \pi/2 \) phase shift in comparison with the similar function for the S/N bilayer. When the external field is applied, \( H \neq 0 \) and \( \gamma_M >> 1 \), \( H_e >> \pi T C \), we may evaluate the critical current as

\[
\tilde{j}_C = \text{Re} \sum_\omega \Phi_{SF}(\omega) \Phi_{SN}(\omega) - \text{Re} \sum_\omega \Phi_{SF}(\omega) \Phi_{SN}(\omega)/\omega^2 - \text{Re} \sum_\omega \frac{\exp\{\pm i[\pi/2 + \arctan(H/\omega)]\}}{\omega^2 (\omega^2 + H^2)^{1/2}}
\]

i.e., for \( H \rightarrow 0 \), the value of the supercurrent may be low enough - the exchange field blocks a contact. So, the S/F-I-N/S contact with strong enough magnetism of the F layer can have a ground state with the superconducting phase difference on the electrodes about \( \pi/2 \). Note, that this mechanism of the \( \pi/2 \) junction realization should not be confused with the \( \pi/2 \)-phase state induced by current fluctuations in the junction plane, the situation that has been treated earlier in Ref. [19] for SFS structures with a thick F layer. We postpone a more detailed description of the \( \pi/2 \) state of S/F-I-F/S and S/F-I-N/S hybrid systems to a forthcoming publication [20], but here discuss the effects of the external magnetic field. As one can see from expression (5), if the contact is in magnetic field, the additional phase shift of the Usadel function \( \Phi_{SN}(\omega) \) has been induced. Depending on the relative orientation, the applied field can prompt the total phase shift across the contact to a \( \pi \)-state of the junction or to a 0-phase state. For the latter case one can expect an essential enhancement of the critical current amplitude. Such type of behavior we can see on Figs. 2-7.

As seen on Fig. 2, a low external magnetic field reverses the critical current sign if the fields (external and internal) have parallel orientation. For antiparallel fields alignment the critical current behavior is much more complicated. In some interval of the applied field the
enhancement of the dc Josephson current takes place in comparison with the case of $H = 0$, while the 0-phase state of the junction is held. By increasing the magnetic field one can turn the current through a few extremum and then adopt the junction to the $\pi$-phase state superconductivity with the opposite current direction.

The results above are obtained for the case of a vanishing interface resistance, $\gamma_B = 0$. After simple but cumbersome algebra, one can also obtain expressions for $\gamma_M \gg 1$, and $\gamma_B \neq 0$. We will not extract here the formulae but illustrate our calculations on Fig. 3, where the dc current dependence versus the applied magnetic field is shown for the S/F-I-N/S junction with a strong influence of the F (N) layer on superconducting layers, $\gamma_M = 10$, and different interface transparency: $\gamma_B = 1 \div 10$. A critical current amplitude of the junction depends greatly on bilayers parameters, and on the interface transparency $\gamma_B$, in particular, decreasing if the transparency decreases, as it is seen on Fig. 3.

### B. Weak proximity effect

Using the expression (1), after simple transformations one can obtain for the dc Josephson current in the case of vanishing interface resistance, $\gamma_B = 0$, and a weak proximity effect, $\gamma_M << 1$:

$$j_C(H, H_e) \approx 2(T/T_C) \Re \sum_\omega \frac{\Delta_S^2}{\Omega^2} \times \{1 + 2i\sigma\omega\gamma_M\beta(H_e + 2H)\Omega^{-2} - \gamma_M^2\beta^2[H^2 + (H_e + H)^2 + 4\omega^2\Omega^{-2}H(H_e + H)]\Omega^{-2}\}^{-1/2},$$

where $\Omega^2 = \Delta_S^2 + \omega^2$. In proceeding these relations, we have taken into account that $\gamma_M\beta$ is small, $\gamma_M\beta << 1$. Simple analysis shows that $j_C(0, H_e) = j_C(H_e, H_e)$ as it should be from the symmetry of the fields geometry. If $\gamma_M\beta H_e << 1$, one can also obtain that

$$j_C(-H_e/2, H_e) \approx 2(T/T_C) \sum_\omega \frac{\Delta_S^2}{\Omega^2} \{1 - (\gamma_M\beta H_e)^2\frac{\Delta_S^2 - \omega^2}{2\Omega^2}\}^{-1/2} > j_C(0, H_e)$$

The dependence of the Josephson current amplitude of S/F-I-N/S junction on the applied magnetic field for a weak proximity effect, $\gamma_M = 0.1$, and different exchange field values of the F layer is shown on Fig. 4. As in the case of strong proximity effect, the current greatly depends on the relative orientation of the fields. For parallel geometry, a ground state with a phase shift across the contact about $\pi/2$ is found at low field, while with the field increasing the ground state is changed to the $\pi$ phase superconductivity. For the antiparallel orientation the current behavior is essentially nonmonotonic and the increasing of the field turns the current through a few extrema until the junction crossovers into the $\pi$-phase state with opposite current direction. Shown on Fig. 5. are representative dc current dependences versus the applied magnetic field for the junction with $\gamma_M = 0.1$, $H_e = 1.5\pi T_C$, and different interface transparency. As is expected, the critical current amplitude decreases with increasing the interface resistance.

As one can see on Figs. 2-5, for antiparallel geometry the $\pi$ phase superconductivity takes place when the external field value is about the internal one, $-H \gtrsim H_e$. This result, far
from being surprising, is related to the quasiparticle spin splitting in the effective magnetic fields in bilayers. Namely, the S/F-I-N/S structure in antiparallel magnetic field \( H = -H_e \) has the same spin splitting as S/N-I-F/S one in zero magnetic field. In other words, for \(-H > H_e \) we should have the critical current versus field dependence as those we have for \( H > 0 \). That is what we see on Figs. 2-5.

C. Asymmetric bilayer interface parameters

To illustrate the main features of the Josephson effect for the S/F-I-N/S tunnel structure caused by an applied magnetic field, we took, for simplicity, the same interface parameters for both SF and SN interfaces. As the nonsuperconducting metals, ferromagnetic and nonmagnetic ones, should be different, this approximation may be unrealizable experimentally. A further step consists in extending the theory to a real system, where the boundary conditions for S/F and S/N bilayers can be different.

Generalization of the results to SF and SN interfaces with different transmission probabilities for quasiparticles is obvious. We present the results for two main cases: (a) the structure with a strong proximity effect for the SN boundary and a weak proximity effect for the SF boundary, and vice versa: (b) a strong proximity effect for the SF boundary and a weak proximity effect for the SN boundary. Let us first discuss the analytical results for vanishing interface resistance \( \gamma_B = 0 \). We obtain

\[
j_C(H, H_e) \sim \frac{\Delta S}{\Omega(\omega^2 + H^2)} \left\{ 1 - \gamma_{MN} \beta H(H_e + H) \Omega^{-2} \right\}
\]

if \( \gamma_{MN} >> 1 \), and \( \gamma_{MF} << 1 \); here \( B_{MN}(T) = B(T)\pi T_C/\gamma_{MN} \). As before, in the geometry with the parallel fields, the increase of external magnetic field inverts the supercurrent direction of the S/F-I-N/S tunnel structure at large enough applied field, \( \gamma_{MF} H(H_e + H) >> \omega^2 (\pi T_C)^2 \). In the geometry with the opposite field orientation, simple substitution into Eq. (7) gives that \( j_C(-H_e, H_e) = j_C(0, H_e) \), while at \( H = -H_e/2 \) we again obtain that \( j_C(-H_e/2, H_e) > j_C(0, H_e), j_C(-H_e, H_e) \). At low values of the field, \( |H| < H_e \), the nonmonotonic current versus field dependence can be observed; for large enough values of the external field, \( |H| > H_e \), the critical current inverts its sign and the crossover of the S/F-I-N/S junction from the 0-phase state to the \( \pi \)-phase state takes place.

Qualitatively similar results are obtained for a junction with \( \gamma_{MN} << 1, \gamma_{MF} >> 1 \). If \( \gamma_B = 0 \) our analytical calculations give

\[
j_C(H, H_e) \sim \frac{\Delta S}{\Omega(\omega^2 + (H + H_e)^2)} \left\{ 1 - \gamma_{MN} \beta H(H_e + H) \Omega^{-2} \right\}
\]

where \( B_{MF}(T) = B(T)\pi T_C/\gamma_{MF} \).

More general cases, \( \gamma_B \neq 0 \), are illustrated by numerical calculations, shown on Figs. 6 and 7. In Fig.6 the dependence of the Josephson current amplitude of the junction on the applied magnetic field is given for a strong proximity effect for the SN boundary, \( \gamma_{MN} >> 1 \), a weak proximity effect for the SF boundary, \( \gamma_{MF} << 1 \), and different interface transparency \( \gamma_B = 0 \div 2 \). (Here and for Fig. 7 the numerical calculations have been made for simplicity for SF and SN interfaces with the same resistivity, i.e., \( \gamma_{BF} = \gamma_{BN} = \gamma_B \).) Opposite
situation is illustrated by the results on Fig. 7. As is seen, the main features of the current-field behavior are robust enough and are held for the S/F-I-N/S junction with asymmetric bilayer parameters, too. However, one can also see, that the current amplitude depends on the conditions on the SF and SN interfaces.

In an experiment, the exchange and applied fields may be noncollinear. When these is an angle $\theta$ between the exchange and applied fields directions, the expression for the critical current can be written in the form

$$j_C(\theta) = j_C^P \cos^2(\theta/2) + j_C^A \sin^2(\theta/2)$$

where $j_C^{P(A)}$ is the current for parallel (antiparallel) fields alignment. The dependence of the critical current on the angle between the fields is illustrated by Fig. 8 for S/F-I-N/S structure with $\gamma_M = 0.1$, $\gamma_B = 5$ for the SF and SN boundaries and $H_e = \pi T_C$.

V. CONCLUSION

The question of superconductivity in the presence of spin splitting is currently heavily investigated, both theoretically and experimentally. However, in spite of a larger number of theoretical predictions, most of them have not been detected experimentally until now. There are some set-up problems, and one of them is the requirement to fix one layer magnetization, while preserving the possibility to reorient the magnetization of another layer by a weak magnetic field. In this report we have analyzed a superconducting tunnel structure where there is spin splitting for two reasons: spin splitting caused by an external magnetic field and by an internal exchange field. The advantage of such structure is the possibility to get a convenient governing and adjustable parameter - an external magnetic field. We demonstrate that in the contacts under consideration the critical current can reverse its sign (the $\pi$-state of the junction) for the parallel field orientation. For the antiparallel field alignment the critical current behavior is much more complicated: for $|H| < H_e$ a monotonic increase of the external field causes an enhancement of the critical current; further field’s increase $|H| > H_e$, reverses the current direction. We reveal the physics behind such critical current versus external field behavior. Namely, we show that in the case of a strong enough magnetism of the F layer a superconducting phase difference on the junction electrodes is about $\pi/2$. If such a contact is in magnetic field, an additional phase shift on the electrodes has been induced, and, depending on the relative orientation, the applied field prompts the junction to the $\pi$ - or to the $0$ - phase state. From the fundamental point of view, these results provide new effects of the superconductivity in the presence of a spin splitting in hybrid superconductor-ferromagnet-normal metal structures and may lead to further understanding of the proximity effects in ferromagnetic materials. The feature important for practical applications is that the superconducting properties of S/F-I-N/S junctions can be simply varied by changing the external magnetic field direction.

Some simplifying assumptions have been made, such as the model admitted of analytical solutions. We consider the dirty limit. However, when the thickness of the F and N layers is much less then its coherence length, $d_{F(N)} << \xi_{F(N)}$, the results are, in most cases, applicable for clean F and N metals, too. Indeed, if scattering centers are on average separated by a distance smaller than the coherence length, then the dirty limit may be
applied. The physical presence of interfaces with atomic roughness ensures that there are such scattering processes for very thin layers.

Let us estimate the orbital effects. The magnetic field $H$ induces screening currents and leads to some suppression of the condensate function. For ferromagnetic and nonmagnetic layers in the dirty limit the depairing rate due to the Meissner current is determined by the energy $D_{p_S}^2$, where $p_S \approx H d_{F(N)}/\phi_O$ is the condensate momentum (here $H d_{F(N)}$ is the magnitude of the vector potential into the F (N) layer and $\phi_O$ is the flux quantum). So, the orbital effects can be neglected if

$$D_{p_S}^2 < H, H_e,$$

where $H_{C2} \phi_O/\xi_{F(N)}^2$ is the upper critical field of F (N) layer. We see, that the nanoscale nonsuperconducting layers thickness, $d_{F(N)} \ll \xi_{F(N)}$, ensures that there is a fairly uniform penetration of an applied field and that orbital depairing effects are minimized. However, for nanoscale hybrid structures a strong electric field arising near metal-metal boundaries may be a source of spin-flip processes \[21\]. Decreasing the average (ferro)magnetic field experienced by the Cooper pair, the spin-flip scattering is much more destructive than the orbital effect \[22,23\].

It was also assumed that the applied field is concentrated into the ”weak” F-I-N part of the junction. In a real situation, an external magnetic field can partly penetrate into the S layers and cause some suppression of the order parameter $\Delta_S$. This issue was already addressed a while ago and well understood (see, e.g., de Gennes [24], Chap.VII). While, in the general case, it is necessary to take into account self-consistently corrections to the pair potential of the S layer, this fact, however, does not qualitatively influence the results of the paper, if the magnetic flux that penetrates into the contact is much less that a flux quantum. The condition is fulfilled when the transverse dimensions of the junction are much less than the Josephson penetration depth.

Let us briefly comment on an experimental set-up, as well. Several recent works (see, e.g., [5-7,25]) have found that dilute ferromagnetic $Cu_xNi_{1-x}$ alloys (with $x <0.6$) are most adopted as materials for the F layers, since their weak ferromagnetism is less devastating to superconductivity. Magnetic hysteresis measurements show for $Cu_xNi_{1-x}$ films with $x \approx 0.4 \div 0.5$ the coercive field of $\sim 100$ Oe (see, e.g., [25] and references therein). One can also pin the magnetization of the $CuNi$ layer by adjacent $FeMn$ layer via exchange bias, and in such a way enlarge the interval of an applied magnetic field, where the magnetization remains fixed. On the other hand, various niobium oxides, one of them NbO, possess the properties of a normal nonmagnetic metal. Such an N layer is usually formed during the fabrication process (see, e.g., [26] and references therein). So, conventional, e.g., magnetron sputtering technique can be used to grow the tunnel superconductor-ferromagnet-insulator-nonmagnetic metal-superconductor structures.

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Fig. 1. The S/F-I-N/S system in a parallel magnetic field.

Fig. 2. Critical current of the S/F-I-N/S tunnel junction vs external magnetic field at high interface transparency, $\gamma_B = 0$, and strong proximity effect, $\gamma_M = 10$, for different values of the exchange field of the F layer: $H_e/\pi T_C = 0.5, 0.7, 0.8$ and $1.0$ (curves 1, 2, 3 and 4, respectively). Here and on Figs. 3-7, solid curves illustrate the case of antiparallel orientation of the internal F layer exchange field and the external magnetic field; dashed curves illustrate the case of parallel orientation of these fields. The numerical results where obtained for $T = 0.1 T_C$. 

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Fig. 3. Critical current of the S/F-I-N/S tunnel junction vs external magnetic field at strong proximity effect, $\gamma_M = 10$, $H_e = \pi T_C$ and different values of SF (SN) boundary transparency: $\gamma_B = 1, 2, 7, \text{ and } 10$ (curves 1, 2, 3 and 4, respectively).

Fig. 4. Critical current vs external magnetic field for the S/F-I-N/S tunnel junction with $\gamma_B = 5$ and weak proximity effect $\gamma_M = 0.1$ for different values of the exchange field of the F layer: $H_e/\pi T_C = 1.0, 1.5, \text{ and } 2.0$ (curves 1, 2, and 3, respectively).

Fig. 5. Critical current of an S/F-I-N/S tunnel junction vs exchange energy and weak proximity effect $\gamma_M = 0.1$, $H_e = 1.5\pi T_C$ and different values of SF (SN) boundary transparency: $\gamma_B = 3, 4, \text{ and } 5$ (curves 1, 2, and 3, respectively).

Fig. 6. Critical current for nonsymmetric S/F-I-N/S tunnel structure with a strong proximity effect for the SN boundary, $\gamma_{MN} = 10$, and a weak proximity effect for the SF boundary, $\gamma_{MF} = 0.1$, $H_e = \pi T_C$ and different values of SF (SN) interface transparency: $\gamma_B = 0.5, 1, \text{ and } 2$ (curves 1, 2 and 3, respectively).

Fig. 7. Critical current of nonsymmetric S/F-I-N/S tunnel structure with a strong proximity effect for the SF boundary, $\gamma_{MF} = 10$, and a weak proximity effect for the SN boundary, $\gamma_{MN} = 0.1$, $H_e = \pi T_C$ and high values of SF (SN) interface transparency: $\gamma_B = 0.2, 0.5, 0.7$ (curves 1, 2 and 3, respectively).

Fig. 8 The dependence of the critical current on the angle $\theta$ between the fields for S/F-I-N/S structure with $\gamma_M = 0.1$, $\gamma_B = 5$ for SF and SN boundaries and $H_e = \pi T_C$; $\theta = 0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 180^\circ$ (curves 1 – 7, respectively).
Fig. 2
Fig. 3
Fig. 5
Fig. 6
Fig. 7

\[ j_C \]

\[ H/\pi T_C \]
Fig. 8