Collective Nuclear Vibrations and Initial State Shape Fluctuations in Central Pb + Pb Collisions: Resolving the $v_2$ to $v_3$ Puzzle

B. G. Zakharov*
Landau Institute for Theoretical Physics, Russian Academy of Sciences, Moscow, 117334 Russia
*e-mail: bgz@itp.ac.ru
Received August 17, 2020; revised August 17, 2020; accepted September 1, 2020

We have studied, for the first time, the influence of the collective quantum effects in the nuclear wavefunctions on the azimuthal anisotropy coefficients $v_2, v_3$ in the central Pb + Pb collisions at the LHC energies. With the help of the energy weighted sum rule, we demonstrate that the classical treatment with the Woods–Saxon nuclear density overestimates the mean square quadrupole moment of the $^{208}$Pb nucleus by a factor of $\sim 2.2$. The Monte Carlo Glauber simulation of the central Pb + Pb collisions accounting for the restriction on the quadrupole moment allows to resolve the $v_2$-to-$v_3$ puzzle.

DOI: 10.1134/S0021364020190029

1. The results of experiments on heavy ion collisions at the RHIC and LHC give a lot of evidences for formation of the quark–gluon plasma (QGP) in the initial stage of nuclear collisions. The hydrodynamic simulations of the hadron production show that the QGP undergoes early thermalization (at the proper time $\tau_0 \sim 0.5–1$ fm) and flows as an almost ideal fluid (the ratio of the shear viscosity to the entropy density is of the order of the theoretical quantum lower limit $\eta/s = 1/4\pi$) [1–3]. The most effective constraints on the QGP viscosity come from the hydrodynamic analysis of the azimuthal dependence of the hadron spectra which is characterized by the Fourier coefficients $v_n$

$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left[ 1 + \sum_{n=1}^{\infty} 2v_n \cos \left[ n(\phi - \Psi_n) \right] \right],$$

(1)

where $N$ is the hadron multiplicity in a certain $p_T$ and rapidity bin, $\Psi_n$ are the event reaction plane angles. For smooth initial conditions at midrapidity ($y = 0$) in the Fourier series (1) only the terms with $n = 2k$ survive. In addition, the azimuthal anisotropy appears only for noncentral collisions due to the almond shape of the overlap region of the colliding nuclei in the transverse plane. The event plane (for each $n$) in this case coincides with the true reaction plane and $\Psi_n = 0$. However, in the presence of fluctuations of the initial QGP entropy distributions, all the flow coefficients $v_n$ become nonzero and the event plane angles $\Psi_n$ fluctuate around the true reaction plane. The fluctuations of the initial fireball entropy distribution is a combined effect of the fluctuations of the nucleon positions in the colliding nuclei and fluctuations of the entropy production for a given geometry of the nuclear positions. The most popular method for evaluation of the initial entropy distribution for event-by-event simulation of AA-collisions is the Monte Carlo (MC) wounded nucleon Glauber model [4, 5]. The even-by-event hydrodynamic modeling with the MC Glauber (MCG) model initial conditions has been quite successful in description of a vast body of experimental data on the flow coefficients in AA-collisions obtained at RHIC and LHC. However, in the last years it was found that the hydrodynamical models fail to describe simultaneously $v_2$ and $v_3$ flow coefficients in the ultra-central ($c \rightarrow 0$) Pb + Pb collisions at the LHC energies.\(^1\) For central collisions, at $b = 0$, the anisotropy of the initial fireball geometry originates completely from the fluctuations. The hydrodynamic calculations show [7, 8] that for small centralities in each event the $v_n$ for $n \leq 3$ to good accuracy satisfy the linear response relation

$$v_n = k_n \epsilon_n,$$

(2)

where $\epsilon_n$ are the Fourier coefficients characterizing the anisotropy of the initial fireball entropy distribution, $\rho_s(\mathbf{p})$, in the transverse plane defined as [9, 10]

$$\epsilon_n = \frac{\int d\mathbf{p} p^n e^{i\mathbf{p}\cdot\mathbf{r}_s(\mathbf{p})}}{\int d\mathbf{p} p^n \rho_s(\mathbf{p})}.$$

\(^1\) Experimentally, the centrality, c, of an event is defined in terms of the charged particle multiplicity. To a very good accuracy (except for the most peripheral collisions) $c$ in terms of the impact parameter $b$ reads $c = \pi b^2 / \sigma_{im}^{AA}$ [6].
Here, it is assumed that the transverse vector $\mathbf{p}$ is calculated in the transverse c.m. frame, i.e., $\int dp_{pp}(p) = 0$. The hydrodynamic calculations give $k_2/k_1 > 1$, and this ratio grows with increase in the QGP viscosity. On the other hand, the MCG calculations show that at $b = 0$ $\epsilon_2$ and $\epsilon_3$ are close to each other (and are ~0.1 for Pb + Pb collisions). This leads to prediction that $v_2/v_1 > 1$. However, experimentally it was observed that $v_2$ is close to $v_1$ in the ultra-central 2.76 and 5.02 TeV Pb + Pb collisions [11, 12]. Since the hydrodynamic prediction for $k_2/k_1$ seems to be very reliable, this situation looks very puzzling (it is called in the literature $v_2$-to-$v_2$ puzzle). This leads to a serious tension for the hydrodynamic paradigm of heavy ion collisions.

There were several attempts to resolve the $v_2$-to-$v_2$ puzzle by modifying: the initial conditions [13, 14], the viscosity coefficients [15], and the QGP equation of state of [16]. However, these attempts have not been successful. The common feature of all previous analyses devoted to the $v_2$-to-$v_2$ puzzle is the use of the Woods–Saxon (WS) nuclear distribution for sampling the nucleon positions in the MC simulations of Pb + Pb collisions. In fact, this is a universal choice in the physics of high-energy heavy ion collisions. However, the MC sampling of nucleon positions with the WS distribution completely ignores the collective nature of the long-range fluctuations of the nucleon positions. It is well known that the long-range 3D fluctuations of the nuclear density have a collective nature and are closely related to the giant nuclear resonances [17, 18] (for more recent reviews see [19, 20]). The major vibration mode of the spherical $^{208}\text{Pb}$ nucleus corresponds to excitation of the isoscalar giant quadrupole resonance [17, 18]. These collective quantum effects are completely lost if one samples the nuclear configurations with the WS distribution. It is clear that an inappropriate description of the 3D long-range fluctuation of the nucleon positions in the colliding nuclei will translate into incorrect long-range fluctuations of the 2D initial fireball entropy density, which are crucial for $\epsilon_{2,3}$ in the central $AA$-collisions, when they are driven by fluctuations.

In this work, we demonstrate that the WS distribution overestimates considerably the mean square nuclear quadrupole moment of the $^{208}\text{Pb}$ nucleus as compared to that obtained in the quantum treatment of the quadrupole vibrations. We calculate the azimuthal anisotropy coefficients $\epsilon_{2,3}$ in Pb + Pb collisions in the MCG model by sampling the nuclear configurations for ordinary WS distribution and a modified one which reproduces the quantum mean square nuclear quadrupole moment of the $^{208}\text{Pb}$ nucleus. Our results show that for the quantum version the ratio $\epsilon_2/\epsilon_3$ becomes substantially smaller than that for ordinary WS distribution. The magnitude of the obtained $\epsilon_2/\epsilon_3$ is small enough to resolve the $v_2$-to-$v_2$ puzzle.

Note that the ordinary MC simulation is also inadequate for the isovector dipole mode, which plays an important role in fluctuations of electromagnetic fields in $AA$-collisions at the RHIC and LHC energies [21] (the classical treatment overestimates the mean square dipole moments for $^{197}\text{Au}$ and $^{208}\text{Pb}$ by a factor of $\sim 5$). However, the effect of the isovector dipole vibrations on the entropy fluctuations and the ratio $\epsilon_2/\epsilon_3$ in $AA$-collisions turns out to be very small. For the first time, the problem with description of the mean square quadrupole nucleus moments in the ordinary MC simulations with the WS nuclear distribution and its importance for the event-by-event analyses of $AA$-collisions was noted in [22].

2. We assume that the $^{208}\text{Pb}$ nucleus is spherical, and the nuclear density is given by the ordinary WS nuclear density

$$\rho_A(r) = \frac{\rho_0}{1 + \exp[(r - R_A)/a]}$$

with the parameters $R_A = (1.12 A^{1/3} - 0.86/ A^{1/3}) = 6.49$ fm, and $a = 0.54$ fm [5]. Let us first consider classical calculation of the nuclear mean square multipoles. We define the isoscalar $L$-multipole operator as (see, e.g., [17, 18, 20]) in terms of the spherical harmonics

$$F_L = \sum_{i=1}^{A} r_i^L Y_{L0} \hat{\rho}_i$$

with $\hat{\rho}_i = \mathbf{p}/|\mathbf{p}|$. Assuming that the many-body nuclear density factorizes into a product of the single nucleus WS densities, one can easily obtain (we ignore a very small effect of the c.m. correlations)

$$\langle F_L^* F_L \rangle_{WS} = \frac{A(2L + 1)(2i^L)}{4\pi}.$$ 

Of course, this formula becomes invalid in the presence of the nucleon correlations. Usually, in the MC simulations of $AA$-collisions, the effect of the nuclear correlations is included in the approximation of a hard-core repulsion. The short range $NN$-expulsion somewhat suppresses the mean square quadrupole moment. However, this suppression is not very strong. More important effect on the multipoles moments may come from the long-range correlations due to quantum collective nuclear excitations.

The quantum calculation of the mean square quadrupole moment of the $^{208}\text{Pb}$ nucleus can be performed with the help of the energy weighted sum rule (EWSR) (for a review, see [23]) for strength function $S(o)$ of the isoscalar quadrupole operator. For the nuclear
ground state the strength function of an operator \( F \) reads
\[
S(\omega) = \sum_n \langle n\mid F\mid 0 \rangle^2 \delta(\omega - \omega_n),
\]
(7)

where \( \omega_n = E_n - E_0 \) and \( E_n \) are the energies of the nuclear states. The ground state expectation value of the operator \( F^+F \) can be written as
\[
\langle 0\mid F^+F\mid 0 \rangle = m_0,
\]
(8)

where \( m_0 \) is the zeroth order moment of the strength function. For an arbitrary \( k \) value, the moment \( m_k \) is defined as
\[
m_k = \int_0^\infty d\omega \omega^k S(\omega).
\]
(9)

The ratio \( m_1/m_0 \) characterizes the typical energy of the states excited by the action of the operator \( F \) on the ground state, which is usually called the centroid energy \( E_c \). Then, in terms of \( E_c \) we can write
\[
\langle 0\mid F^+F\mid 0 \rangle = \frac{m_1}{E_c}.
\]
(10)

For the case of interest \( F = F_L \), the moment \( m_1 \) can be evaluated accurately using the EWSR, which gives for \( L \geq 2 \) [20, 23]
\[
m_1 = \frac{AL(2L + 1)^2 \langle r^{2L-2} \rangle}{8\pi m_N},
\]
(11)

where \( m_N \) is the nucleon mass. Then, from (6) and (10), we obtain for the ratio of the classical-to-quantum mean square moments
\[
r = \frac{\langle 0\mid F^+F\mid 0 \rangle_c}{\langle 0\mid F^+F\mid 0 \rangle_q} = \frac{2m_N E_c \langle r^{2L-2} \rangle}{L(2L + 1)^2 \langle r^{2L-2} \rangle}.
\]
(12)

In the case of the isoscalar \( L = 2 \) operator, the EWSR is exhausted by the isoscalar giant quadrupole resonance (ISGQR) with \( \omega_q = 10.89 \text{ MeV} \) and \( \Gamma_q = 3 \text{ MeV} \) [24]. Calculation with the Breit–Wigner parametrization of the quadrupole strength function gives the ISGQR centroid energy \( E_c \approx 11.9 \text{ MeV} \). Using this centroid energy, we obtain from (12) for the quadrupole mode \( r = 2.2 \). This says that the simple probabilistic treatment of the \(^{208}\text{Pb} \) nucleus with the factorized WS many-body nuclear density considerably overestimates the 3D-quadrupole fluctuations. One can expect that this can lead to incorrect predictions for the 2D-fluctuations of the QGP fireball in the MC simulation of \( AA \)-collisions as well. One of the ways to cure this problem is to use in the MC sampling of the nucleon positions the nuclear configurations that have the distribution function in the square quadrupole moment (we denote it \( Q^2 \)) of the form
\[
P_{sq}(Q^2) = rP_0(rQ^2),
\]
(13)

where \( P_0 \) is the native distribution function of the squared quadrupole moment for the WS nuclear distribution (i.e., it is calculated without imposing any filter on the nucleon positions). The MC sampling of the nucleon positions with the squeezed distribution \( P_{sq} \) automatically guarantees that the colliding nuclei will have correct mean square quadrupole moments.

3. We consider the initial condition for the QGP fireball in \( \text{Pb} + \text{Pb} \) collisions in the central rapidity region (\( y = 0 \)). For evaluating the distribution of the entropy density in the transverse plane, we use the MCG approach developed in [25, 26]. The MCG scheme of [25, 26] allows to perform calculations describing the nucleon as a one-body state and accounting for the meson–baryon component of the physical nucleon. This model describes very well the data on the centrality dependence of the midrapidity charged particle density in 0.2 TeV Au + Au collisions at RHIC and 2.76 TeV Pb + Pb collisions [26]. The theoretical predictions for 5.02 TeV Pb + Pb and 5.44 TeV Xe + Xe collisions [27] are also in very good agreement with the data. In the present analysis, we perform calculations for the versions with and without the meson–baryon component. Both the versions lead to very close predictions for the ratio \( \epsilon_2(2)/\epsilon_2(1) \) we are interested in. Here, we briefly sketch the algorithm used in our MCG model for the version without the meson–baryon component (for this case our model is similar to the well-known MCG generator GLISSANDO [5]). The interested reader is referred to [25, 26] for the detailed description of the model and the parameters of the model.

We use two-component scheme [28] with two kinds of the entropy sources: corresponding to the wounded nucleons (WN) and to the hard binary collisions (BC). The center of each WN source coincides with the position of the WN, and the center of each BC source is located in the middle between the pair of the colliding nucleons. The suppression of the probability of hard BC for a given NN-interaction is controlled by the parameter \( \alpha \). The total event entropy density in the transverse plane is given by
\[
\rho_s(\rho) = \sum_{i=1}^{N_H} S_{wn}(\rho - \rho_i) + \sum_{i=1}^{N_H} S_{bc}(\rho - \rho'_i),
\]
(14)
where the $S_{\text{en}}$ terms corresponds to the sources for wounded constituents and $S_{\text{bc}}$ terms to the binary collisions. $N_{\text{wn}}$ and $N_{\text{bc}}$ are the number of the WNs and BCs, respectively. The entropy distribution for WN and BC sources are written as

$$S_{\text{wn}}(\rho) = \frac{(1 - \alpha)}{2} s(\rho), \quad S_{\text{bc}}(\rho) = s(\rho),$$

(15)

where for $s(\rho)$ we use a Gaussian distribution

$$s(\rho) = s_{0} \exp\left[\frac{-\rho^{2}/\sigma^{2}}{2}\right]$$

(16)

with $s_{0}$ the total entropy of the source, and $\sigma$ width of the source. We perform calculations for $\sigma = 0.4$ fm. The results for the anisotropy coefficients become sensitive to the width of smearing of the sources only for very peripheral collisions, and for the central collision, they are weakly sensitive to the values of $\sigma$.

We describe fluctuations of the total entropy for each source by the Gamma distribution. The parameters of the Gamma distribution have been adjusted to fit the experimental $pp$ data on the mean charged multiplicity and its variance in the unit pseudorapidity window $|\eta| < 0.5$ using the ratio of the entropy to the charged multiplicity $dS/dy = CdN_{\text{ch}}/dN_{\text{ch}}$, with $C = 7.67$ [29]. In the version with the meson–baryon component of our MCG generator [25, 26] the entropy sources can be produced in $BB$, $MB$, and $MM$ collisions. Both the versions of the model give similar predictions for the charged multiplicity. However, the optimal values of the parameter $\alpha$ are somewhat smaller for the version with the meson–baryon component. The fit to the data on centrality dependence of the midrapidity charged particle density gives $\alpha = 0.09 (0.14)$ for the versions with (without) the meson–baryon component (see [26, 27] for details).

We performed numerical calculations of the rms coefficients $\langle \varepsilon_{n}^{2}/2 \rangle$ (they are usually denoted as $\varepsilon_{n}(2)$) for $n = 2$ and 3 by MC generation of $5 \times 10^{5}$ central $(b = 0)$ Pb + Pb collisions at $\sqrt{s} = 2.76$ and 5.02 TeV. The results for both the versions, with and without the meson–baryon component, are summarized in Table 1. According to Table 1, the quantum collective effects for the quadrupole deformations do not affect $\varepsilon_{2}(2)$. However, they give a noticeable reduction of $\varepsilon_{2}(2)$. For the quantum version with the meson–baryon component, we obtain $\varepsilon_{2}(2)/\varepsilon_{3}(2) = 0.8$. For the version without the meson–baryon component we obtain a bit bigger $\varepsilon_{2}/\varepsilon_{3}$. However, the change in the ratio $\varepsilon_{2}(2)/\varepsilon_{3}(2)$ is very small (it is increased by $\sim 0.01$).

The obtained magnitude of the ratio $\varepsilon_{2}(2)/\varepsilon_{3}(2)$ allows to resolve the $v_{1s}$-to-$v_{2s}$ puzzle in the ultra-central Pb + Pb collisions. Because the hydrodynamic calculations give $k_{2}/k_{3} = 1.2$–1.4 [13, 14, 30, 31] for Pb + Pb collisions at the LHC energies for small centralities (c $\leq$ 2%). Then, using our quantum prediction for $\varepsilon_{2}(2)/\varepsilon_{3}(2)$ we obtain $v_{2s}/v_{3} = 0.096$–1.12. This is in reasonable agreement with the ALICE measurements [12] for 2.76 and 5.02 TeV Pb + Pb collisions that give in the limit $c \to 0$ $v_{2s}/v_{3} = 1.08 \pm 0.05$.

Our calculations have been performed for zero impact parameter $b$. Due to fluctuations of the multiplicity (at a given impact parameter), there is some mismatch/smearing between $b$ and $c$ which experimentally is measured via the multiplicity. We checked

```
Table 1. The rms eccentricities $\varepsilon_{2}(2)$ and the ratio $\varepsilon_{2}(2)/\varepsilon_{3}(2)$ for central 2.76 and 5.02 TeV Pb + Pb collisions obtained within the MCG model of [26] with and without (numbers in brackets) the meson–baryon component in the nucleon. For each energy the left column shows the results for the sample of nucleon configurations without restrictions on the squared quadrupole moment $Q^{2}$ (i.e., for the native distribution $P_{n}(Q^{2})$ for the WS nuclear density), and the right one for the sample corresponding to the squeezed distribution $P_{sq}(Q^{2})$ (see main text for details).

|              | Pb + Pb 2.76 TeV | Pb + Pb 5.02 TeV |
|--------------|------------------|------------------|
|              | MC with $P_{n}(Q^{2})$ | MC with $P_{n}(Q^{2})$ | MC with $P_{n}(Q^{2})$ | MC with $P_{n}(Q^{2})$ |
| $\varepsilon_{2}(2)$ | 0.107(0.112) | 0.0946(0.0983) | 0.107(0.112) | 0.0939(0.0977) |
| $\varepsilon_{3}(2)$ | 0.118(0.121) | 0.118(0.121) | 0.117(0.12) | 0.117(0.12) |
| $\varepsilon_{2}(2)/\varepsilon_{3}(2)$ | 0.907(0.926) | 0.802(0.812) | 0.915(0.931) | 0.802(0.814) |
```

4 The numbers in Table 1 were obtained for the factorized WS distribution without short range $NN$-correlations. We also performed the MCG simulation with the hard repulsion for the expulsion radius $r_{c} = 0.9$ [32] and 0.6 [33] fm. We obtained for these two versions $\varepsilon_{2}(2)/\varepsilon_{3}(2) = 0.845$ and 0.825, respectively. These values also lead to $v_{2}v_{3}$ which is in rather reasonable agreement with the data. However, one should bear in mind that from the point of view of the entropy production in AA-collisions the real situation with the contribution to the entropy density of the short range $NN$-pairs may differ from that in the picture with a big explosion volume (as, e.g., in [32]). Say, for a successful dibaryon paradigm of the short range $NN$-interaction (for reviews, see [34, 35]) the expulsion region is not empty, but occupied by a $6q$-cluster. As in the case of $hD$-scattering [36], the $6q$-states can participate in the color exchanges between the colliding nuclei and contribute to the entropy generation. For this reason, in reality the effect of the short range $NN$-configurations may be of the opposite sign.

JETP LETTERS Vol. 112 No. 7 2020

ZAKHAROV
that the effect of this smearing on our predictions is very small. In addition, to understand the sensitivity of the results to the form of the squeezed distribution $P_{\text{sq}}(Q^2)$ used for filtering the nucleon configurations in our MCG simulations, we also performed calculations for sampling the nuclear configurations with a sharp cutoff in $Q^2$. The cutoff on the squared quadrupole moment has been adjusted to fit the EWSR mean square quadrupole moment of the $^{208}$Pb nucleus. This ansatz leads to the value of $\epsilon_2(2)/\epsilon_2(2)$ which is in perfect agreement with that for the ansatz given by (13). This test demonstrates high stability of our predictions for $\epsilon_2(2)/\epsilon_2(2)$ against the changes of the $P_{\text{sq}}(Q^2)$ distribution. It means that for $\epsilon_2(2)/\epsilon_2(2)$ the only crucial quantity is the total mean square quadruple moment of the colliding nuclei.

In this preliminary study, we have ignored possible inadequacy of the MCG simulation with the WS density for the octupole ($L = 3$) vibrations of the $^{208}$Pb nucleus. The mean square octupole moment can be defined via the EWSR in the same way as for the quadrupole mode using the ratio of the moments $m_q/m_h$ calculated via the experimental data on the strength function. However, for the octupole mode the strength function is not exhausted by a single resonance, but it gets contribution from a broad range of $\omega$.

It has peaks at $\omega \sim 2.6$ MeV and $\omega \sim 20$ MeV (see, e.g., [24, 37, 38]). From the available experimental data [24, 37, 38] one can conclude that for $L = 3$ the ratio $r$ given by (12) is close to one or a bit smaller. The octupole strength function calculated in [39] within the random phase approximation for the Skyrme interaction also leads to $r = 1$. However, of course the determination of $r$ from the experimental data seems to be preferable. However, experimental uncertainties for contribution of the low and high energy $\omega$-regions to the EWSR for the octupole mode are rather large. This renders difficult an accurate calculation of the ratio (12). We checked that the scenario with $r < 1$ (for $L = 3$) leads an increase in the value of $\epsilon_3(2)$, and the ratio $\epsilon_3(2)/\epsilon_3(2)$ will be somewhat smaller than that obtained in the present analysis. We leave a detailed MC simulation for this scenario with accounting for filtering for both the quadrupole and octupole moments for future work.

4. In summary, we have studied, for the first time, the influence of the collective quantum effects in the nuclear wavefunctions on the azimuthal anisotropy coefficients $\epsilon_{2,3}$ in the central Pb + Pb collisions at the LHC energies. We have compared the predictions for the mean square quadrupole moment of $^{208}$Pb obtained in the classical probabilistic treatment with the WS nuclear distribution with that obtained from the quantum analysis using the EWSR and the experimental data on the isoscalar giant quadrupole resonance. This analysis shows that the classical treatment overestimates the mean square quadrupole moment of the $^{208}$Pb nucleus by a factor of $r = 2.2$. In our MCG simulations of Pb + Pb collisions, we cure this problem by sampling the nucleus configurations for the distribution in the squared quadrupole moment squeezed by the factor $r$. This guarantees that the colliding nuclei have the mean square quadrupole moment predicted by the quantum EWSR. We have found that the EWSR version of the MCG simulation leads to a noticeable reduction of the azimuthal asymmetry $\epsilon_2$, as compared to the ordinary MC sampling without restrictions on the quadrupole moments of the colliding nuclei. The values of $\epsilon_2$ for two versions of the MCG simulations are practically the same. For the EWSR version, we obtained $\epsilon_2(2)/\epsilon_2(2) \approx 0.8$. This leads to $v_2(2)/v_2(2) \approx 0.096-1.12$ (if one adopts the hydrodynamic linear response coefficients $k_{\omega}$ from [13, 14, 30, 31]), which is in rather good agreement with the data from ALICE [12].

In the present analysis, we have addressed only the case of the spherical $^{208}$Pb nucleus. However, it is clear that for high-energy collisions of the non-spherical nuclei, like $^{197}$Au + $^{197}$Au and $^{238}$U + $^{238}$U, the MCG simulations with the ordinary WS density may be inadequate as well. This may be important for interpretation of the results of the event shape engineering, which uses the event multiplicity to select the events with a certain initial system geometry (e.g., the tip–tip collisions of the prolate $^{238}$U nuclei as in the STAR experiment [40]).

The quantum collective effects discussed in the present analysis may be important for analyses of the data on the flow effects in Au + Au collisions in future experiments at NICA. In the NICA energy region, the critical point effects may influence the medium evolution, and accurate treatment of the initial state geometry becomes especially important.

ACKNOWLEDGMENTS

I am grateful to S.P. Kamerdzhiyev for helpful communications on the physics of giant resonances and to N.N. Nikolaev for discussing the results.

FUNDING

This work was supported in part by the Russian Foundation for Basic Research, project no. 18-02-40069mega.

REFERENCES

1. T. Hirano, P. Huovinen, K. Murase, and Y. Nara, Prog. Part. Nucl. Phys. 70, 108 (2013); arXiv: 1204.5814.
2. R. Derradi de Souza, T. Koide, and T. Kodama, Prog. Part. Nucl. Phys. 86, 35 (2016); arXiv: 1506.03863.
3. P. Romatschke and U. Romatschke, arXiv:1712.05815.
4. M. L. Miller, K. Reygers, S. J. Sanders, and P. Steinberg, Ann. Rev. Nucl. Part. Sci. 57, 205 (2007); nucl-ex/0701025.
5. M. Rybczynski, G. Stefanek, W. Broniowski, and P. Bozek, Comput. Phys. Commun. 185, 1759 (2014); arXiv:1310.5475.
6. W. Broniowski and W. Florkowski, Phys. Rev. C 65, 024905 (2002); nucl-th/0110020.
7. M. Luzum and H. Petersen, J. Phys. G 41, 063102 (2014); arXiv:1312.5503.
8. H. Niemi, G. S. Denicol, H. Holopainen, and P. Huovinen, Phys. Rev. C 87, 054901 (2013); arXiv:9411.0051.
9. D. Teaney and L. Yan, Phys. Rev. C 83, 064904 (2011); arXiv:1102.0737.
10. E. Retinskaya, M. Luzum, and J.-Y. Ollitrault, Nucl. Phys. A 926, 152 (2014); arXiv:1401.3241.
11. S. Chatrchyan et al. (CMS Collab.), J. High Energy Phys. 1402, 088 (2014); arXiv:1312.5503.
12. S. Acharya et al. (ALICE Collab.), J. High Energy Phys. 1807, 03 (2018); arXiv:1804.02944.
13. C. Shen, Z. Qiu, and U. Heinz, Phys. Rev. C 92, 014901 (2015); arXiv:1502.04636.
14. P. Carzon, S. Rao, M. Luzum, M. Sievert, and J. Noronha-Hostler, arXiv:2007.00780.
15. J.-B. Rose, J.-F. Paquet, G. S. Denicol, M. Luzum, B. Schenke, S. Jeon, and C. Gale, Nucl. Phys. A 931, 926 (2014); arXiv:1408.0024.
16. P. Alba, V. Mantovani Sarti, J. Noronha, J. Noronha-Hostler, P. Parotto, I. Portillo Vazquez, and C. Ratti, Phys. Rev. C 98, 034909 (2018); arXiv:1711.05207.
17. A. Bohr and B. R. Mottelson, Nuclear Structure (W. A. Benjamin, New York, 1975).
18. W. Greiner and J. A. Maruhn, Nuclear Models (Springer, Berlin, 1996).
19. S. Kamerzhiev, J. Speth, and G. Tertychny, Phys. Rep. 393, 1 (2004); nucl-th/0311058.
20. X. Roca-Maza and N. Paar, Prog. Part. Nucl. Phys. 101, 96 (2018); arXiv:1803.06256.
21. B. G. Zakharov, JETP Lett. 105, 785 (2017); arXiv:1703.04271.
22. B. G. Zakharov, JETP Lett. 108, 723 (2018); arXiv:1810.08942.