Quasi-oppositional Multi-objective Antlion Optimizer Based on Differential Evolution

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Abstract: In order to solve the multi-objective problem, an improved quasi-oppositional multi-objective antlion optimization algorithm based on differential evolution (DEQOMALO) is proposed. This algorithm overcomes the defect that antlion algorithm is easy to fall into local optimum. On the one hand, this algorithm uses the idea of differential evolution to make full use of the information of the ant and the elite antlion to improve the position updating method of the original algorithm. On the other hand, the population is optimized by quasi-opposite learning strategy, and the original population and its quasi-opposite individuals are mixed and then selected as the new population, which greatly increases the diversity of the population. Finally, typical benchmarks are selected to compare the algorithm with the original antlion algorithm and other MALO algorithms with traditional evolution strategies. Experimental results show that both convergence and distribution of the improved algorithm are greatly improved. The proposed DEQOMALO algorithm has good adaptability and effectiveness in solving the two-objective optimization problem.

Key words: multi-objective optimization, Antlion Optimizer, differential evolution, quasi-oppositional learning strategy

1. Introduction

Multi-objective optimization problem (MOP) exists widely in real world practice. In order to solve this kind of problem, we need to balance several goals which often conflict with each other. Traditional mathematical programming methods are very restrictive in solving non-convex optimization problems, so intelligent optimization methods are favored by many scholars as effective tools. At present, multi-objective optimization algorithms can be categorized in Pareto based methods, decomposition based methods and index based methods [1]. The Pareto-based methods select the non-dominated solution as the optimal solution by comparing the dominating relations between different solutions. The most common used Pareto-based methods include the non-dominated sorting genetic algorithm II (NSGA-II) [2], strength Pareto evolutionary algorithm 2 (SPEA2) [3], and Pareto envelope-based selection algorithm II (PESA-II) [4], and so on. The decomposition-based methods aggregate the objectives of an MOP by a scalarizing function such that a single scalar value is generated. In these algorithms, the diversity of a population is maintained by specifying a set of well-distributed reference points to guide its individuals to search simultaneously towards different optima. The decomposition based multi-objective evolutionary algorithm (MOEA/D) is one of the most common decomposition based algorithm [5]. The idea of indicator-based algorithms is to apply performance indicator to guide the search during the evolutionary process. The most common used indicators are ε indicator [6], inverted generational distance (IGD) [7], and the hyper volume (HV) indicator [8].

Current intelligent optimization methods can also be divided into swarm intelligence and evolutionary computing. The swarm intelligence is inspired by biological group behavior, while the evolutionary algorithm is inspired by biological evolutionary mechanism. Antlion algorithm, as a swarm intelligence algorithm, was put forward by S Mirjalili [9]. In order to solve the multi-objective optimization problem by using antlion algorithm, Mirjalili himself proposed a multi-objective antlion algorithm, and a large number of experiments
proved that the performance of multi-objective antlion algorithm is better than that of traditional algorithms such as MOPSO, NSGA-II and so on [10]. Because of the superiority of the antlion algorithm, the algorithm has been applied and popularized in lots of real world engineering problems such as parameter optimization [11], path planning problem [12], and so on.

The basic antlion algorithm is still prone to fall into local optimum and convergence slowly in the later period. Therefore, this paper tries to improve its ability of global search and local exploration in solving multi-objective problems by proposing an improved algorithm named quasi-oppositional multi-objective antlion optimization algorithm based on differential evolution.

The remaining sections of this article are organized as follows. In section 2, the basic antlion optimizer and related works are introduced. Section 3 gives a thorough description of the new proposed DEQOMALO algorithm. The performance of the proposed algorithm is tested by numerical simulation in section 4. We summarize the paper with suggestions for future improvement in section 5.

2 Related Works

2.1 Antlion optimizer

The way that antlion optimizer find the optimal solutions is simulating the process of antlions build trap and prey ants in nature. So there are two populations in the antlion algorithm, that is, the antlions and the ants. The hunting process consists of five basic steps: random walk of ants, ants fall into traps, ants slide toward antlions, antlions catch ants and antlions reconstruct traps. The basic steps of the antlion optimizer simulate the above process, and the mathematical description is as follows:

(1) The random walk of ants is formulated as follows:

\[ x(t) = \left[0, \text{cumsum}(2r(t_1) - 1), \text{cumsum}(2r(t_2) - 1), ..., \text{cumsum}(2r(t_n) - 1)\right] \quad (1) \]

where \( \text{cumsum} \) calculates the cumulative sum, \( t \) is the number of iterations, and \( n \) is the maximum number of iterations. \( r(t) = \begin{cases} 1 & \text{if } \text{rand} > 0.5 \\ 0 & \text{if } \text{rand} \leq 0.5 \end{cases} \) is a random function related to \( t \), and \( \text{rand} \) satisfies the uniform distribution of \([0 \sim 1]\).

In order to ensure that ants move randomly in search space and prevent them from crossing the boundary, it is necessary to normalize the position of ants:

\[ X'_i = \frac{(X'_i - a_i) \times (d'_i - c'_i)}{b_i - a_i} + c'_i \quad (2) \]

where \( c'_i \) and \( d'_i \) indicates the minimum value and the maximum value of the variable \( i \) of one ant at the \( t \)-th iteration respectively; \( a_i \) and \( b_i \) indicates the minimum value and the maximum value of the variable \( i \) of the ant respectively.

(2) By simulating the process of random walk around the antlion and falling into a trap, the formula is as follows:

\[ c'_j = \text{Antlion}_j' + c'; d'_j = \text{Antlion}_j' + d' \quad (3) \]

where \( c' \) and \( d' \) is the minimum and maximum values of all variables at the \( t \)-th iteration; \( \text{Antlion}_j' \) is the position of antlion \( j \) at the \( t \)-th iteration.

(3) By adaptively reducing the random walk range of the ants, the process of the ants to slide into the antlion is simulated as follows:

\[ c' = \frac{c'}{I}, d' = \frac{d'}{I} \quad (4) \]

Where \( I = 1 + 10^\omega t \) is gradually increased as the number of iterations increases, \( t \) is the number of iterations, \( T \) is the maximum number of iterations, \( \omega \) is dynamically adjusted with the number of iterations.

(4) The simulated antlions’ capture process is as follows:

\[ \text{Antlion}_j = \text{Ant}_j' \quad \text{if } f(\text{Ant}_j') < f(\text{Antlion}_j) \quad (5) \]
where Antlion\textsuperscript{j} is the position of antlion \textit{j} at the \textit{t}-th iteration, Ant\textsuperscript{i} is the position of ant \textit{i} at the \textit{t}-th iteration.

Select the antlions with optimal fitness as elite individual which decides the position of ants. Ants move randomly towards both a randomly selected antlion and the elite antlion, with the following formulas:

\[ Ant\textsuperscript{i}_t = \frac{R\textsuperscript{i}_t + R\textsuperscript{c}_t}{2} \]  

(6)

where \( R\textsuperscript{i}_t \) is a random movement around a antlion selected by roulette wheel selection mechanisms. \( R\textsuperscript{c}_t \) is a random movement around the elite antlion.

(5) Reconstruction the trap. Use an external archive to store the non-dominant solution set. The niche technique is used to measure the distribution of solutions in archives. The distribution of the solution is measured by the number of other solutions in the neighborhood of each solution. The more the solution in the neighborhood, the denser the individual is.

2.2 Quasi-opposition based learning strategy

Traditional meta-heuristic algorithms usually begin with a set of random initial solutions, but the randomly generated initial solutions may be far from the optimal solution. So it takes a long time for the algorithm to converge to the optimal solution. Therefore, in this paper, the quasi-opposition based learning (QOBL) strategy is used to optimize the initial population [13]. In this way, the convergence rate of the algorithm can be improved, as well as, the diversity of the population can be increased.

For a real number \( x \) with search interval \([a,b]\), its opposite number \( ox \) is defined as follows:

\[ ox = a + b - x \]  

(7)

Its quasi-opposites number is defined as follows:

\[ qox = rand\left(\frac{a+b}{2}, ox\right) \]  

(8)

Then, for the \( D \) dimensional individual \( X = [X_1, X_2, \ldots, X_D] \), its quasi-opposites individual is \( QOX = [qox_1, qox_2, \ldots, qox_D] \). The value of \( i \)-th variable in \( QOX \) is \( qox_i = rand\left(\frac{a_i+b_i}{2}, ox_i\right) \).

After the quasi-opposite population is obtained, it is mixed with the individual of the original population as the new population.

2.3 Differential evolution strategy

Differential evolutionary algorithm (DE) is proposed on the basis of genetic algorithm (GA). It constantly updates individuals and gradually finds the optimal individual population by simulating genetic variation operations, crossover operations and selection operations. Different from the traditional genetic algorithm, the mutation individual of the differential evolution algorithm is generated by the parent difference, and then the mutation individual is crossed with the other parent individuals to create a new crossover individual. Finally, the crossover individual is compared with the parent individual. The superior individual is selected as the final offspring individual. The current mutation strategies of differential evolution are as follows [13]:

1. DE/rand/1 strategy: \( v_i = x_i + F \times (x_{2i} - x_{3i}) \)
2. DE/rand/2 strategy: \( v_i = x_i + F \times (x_{2i} - x_{3i}) + F \times (x_{4i} - x_{5i}) \)
3. DE/best/1 strategy: \( v_i = x_{best} + F \times (x_{1i} - x_{2i}) \)
4. DE/best/2 strategy: \( v_i = x_{best} + F \times (x_{1i} - x_{2i}) + F \times (x_{3i} - x_{4i}) \)
5. DE/current-to-best/1 strategy: \( v_i = x_i + F \times (x_{best} - x_i) + F \times (x_{1i} - x_{2i}) \)
6. DE/current-to-rand/1 strategy: \( v_i = x_i + rand \times (x_{1i} - x_{2i}) + F \times (x_{2i} - x_{3i}) \)

Where, \( F \) is fixed value as the control parameter, \( x_{ri} \) is a randomly selected individual from the population, \( x_i \) is mutated individuals, and \( x_{best} \) is individuals with optimal fitness function satisfied with \( x_i \neq x_{best} \neq x_j \).

These mutation strategies can use the information of parent population to generate new individuals.
Strategy 3, 4, 5 also uses the information of the optimal individual in the population to control the direction of mutation of the individual. However, it can be found from the experiments (see the experiment section of this article) that, the performance of the antlion algorithm does not improve just by introducing these mutation strategies into the algorithm. Therefore, it is necessary to use the idea of differential evolution to propose a mutation strategy adapted to the antlion algorithm.

3 The proposed DEQOMALO algorithm

In order to apply the traditional differential evolution theory to the multi-objective antlion optimization algorithm, there are two difficulties need to be solved. Firstly, the mutation strategy of the traditional differential evolution is aimed at solving the single objective optimization problem, so it is necessary to transfer the strategy from single objective to multi-objective. In the single-objective optimization problem, the individual can be ranked by comparing the fitness function value. In multi-objective optimization, multiple objectives of individual should be considered at the same time, so the optimal individual can’t be determined by comparing the fitness value. Therefore, the idea of non-dominated sorting is cited in this article. Firstly, the non-dominant solution is found according to the Pareto dominance relation of ants, and the non-dominant individual is placed into an external archive as the antlion population. Secondly, the elite antlion individual is selected according to the sort of antlion population in the archive by using niche technique. In this way, the elite antlion is superior to other antlion individual, and all antlion individual is superior to the ant individual.

Secondly, the ant position update formula of the original antlion algorithm only uses the information of the antlion in the external archive, and discards the information of the ant population. This may lead to premature convergence of the algorithm, which is not conducive to global exploration. The mutation strategy of traditional differential evolution emphasizes the information of ant population, and does not give full play to the information of antlion, which is not conducive to the convergence of the algorithm. So it is necessary to consider how to make full use of both the location information of ant population and the antlion at the same time. In this paper, a new ant location updating formula is proposed as follow:

\[
Ant_i = Antlion_{3} + F_1 \times (Antlion_{elite} - Antlion_{rand}) + F_2 \times (Antlion_{elite} - Ant_i) + F_3 \times (Antlion_{rand} - Ant_i)
\]

where, \(Antion_{ elite} = p\times Antlion_{ elite} + p\times Antlion_{ rand} + p\times Ant_i \), \(Antlion_{ elite} \) and \(Antlion_{ rand}\) are antlions selected from the external archive, \(Antlion_{ elite}\) is the elite antlion selected by niche, \(Antlion_{ rand}\) is the random antlion selected from the archive randomly, \(Antlion_{ elite} \neq Antlion_{ rand}\) \(\omega_i > 0\) \(\sum_{i=1}^{3}\omega_i = 1\) \(i = 1, 2, 3\).

Formula (9) can make full use of the information of the current population and the location of the antlion to ensure the movement of the population to the elite antlion.

At the end of each iteration, the quasi-opposite population is obtained and mixed with the population of the previous generation. A new population is selected as the offspring population to enter the next iteration according to the dominance relationship.

4 Simulation experiment and result analysis

4.1 Experimental setup

For multi-objective optimization algorithm, the performance of the algorithm is mainly reflected in convergence and the distribution of the results [14].Convergence describes the degree of approximation of the result obtained by the algorithm to the true Pareto front (PF). The stronger the convergence of the algorithm, the closer the solution set is to the true optimal solution, and the more accurate the result is. The distribution describes the distribution characteristics of the obtained result in the objective space. On the one hand, the results should be distributed as much as possible on the whole PF, and on the other hand, the results should be distributed as evenly as possible. The stronger the distribution of the algorithm represents a better the global exploration ability of the algorithm.

To measure the performances of the algorithm, among the many metrics, inverted generation distance (IGD) and hyper volume (HV) were selected because they can reflect the convergence and distribution of the algorithm at the same time [15].
In order to study the properties of the improved algorithm, the test functions of ZDTs are selected to verify the algorithm [16]. The experimental parameter settings, formulations of the mutation strategy and the value of $F$ are present at Table 1 [17].

**Table 1** Variation strategy and parameter setting

| mutation strategy | formulations | parameter |
|-------------------|--------------|-----------|
| Classical         | $A_{n_t} = F \times (\text{Antlion}_{\text{elite}} - \text{Antlion}_{\text{rand}})$ | $F = 0.5$ |
| DE/rand/1         | $A_{n_t} = A_{n_t} + F \times (A_{n_t} - A_{n_{t-1}})$ | $F \sim N(0.5,0.3)$ |
| DE/best/1         | $A_{n_t} = A_{\text{elite}} + F \times (A_{n_t} - A_{n_{t-1}})$ | $F \sim N(0.5,0.3)$ |
| DE/current-to-best/1 | $A_{n_t} = A_{n_t} + F \times (A_{\text{elite}} - A_{n_{t-1}}) + F \times (A_{n_t} - A_{n_{t-1}})$ | $F_i \sim N(0.5,0.3)$ |
| Proposed method   | $A_{n_t} = A_{\text{elite}} + F_i \times (A_{\text{elite}} - A_{n_{t-1}}) + F \times (A_{n_t} - A_{n_{t-1}})$ | $F_i \sim N(0.5,0.3)$ |

4.2 The effect of the evolutionary strategy on the algorithm

In order to test the effect of the proposed differential evolution strategy, the proposed algorithm is compared with the classical antlion algorithm with traditional differential evolution strategy optimization algorithm. Improve the original multi-objective antlion algorithm by different mutation strategies without Quasi-Opposition strategy. The algorithms are tested on each standard test function, and the experimental results are shown in Figure 1.

![Figure 1. Optimal solution distribution of improved differential evolutionary strategies without quasi-opposites learning strategies](image)

In Figure 1, from left to right, represent the results of the original MALO algorithm, the DE/rand/1 strategy improvement algorithm, the DE/best/1 strategy improvement algorithm, the DE/current-to-best/1 strategy improvement algorithm and the DEQOMALO algorithm proposed in this paper. Five test functions are represented from top to bottom. The distribution of the results of each algorithm on each test function can be observed intuitively from the diagram. The optimal solution set of MALO are almost fall on the true PF, but the distribution of MALO is poor. The convergence and distribution of the algorithm under DE/rand/1 strategy are very poor. This is mainly because the strategy adopts a completely random mutation strategy and completely ignores the role of elite antlions. The distribution of the algorithm under DE/best/1 strategy and DE/current-to-best/1 strategy in the objective space are also poor. The optimal solution of the proposed algorithm can be uniformly and widely distributed on the true PF in the objective space of all the test functions.
functions, and the convergence and distribution of the algorithm are both optimal.

5. Conclusion
In order to improve the precision and global search ability of antlion algorithm in solving multi-objective problem, an improved multi-objective antlion algorithm is proposed. The following improvements are made.

Firstly, by introducing the idea of differential evolution, an individual updating mechanism, which can make full use of both population information and elite antlion information, is proposed to replace the original algorithm. Secondly, the quasi-opposite learning strategy is used to optimize the population. On the one hand, it can increase the diversity of populations, on the other hand, it can improve the convergence speed of the algorithm. In order to verify the performance of the proposed algorithm, the DEQOMALO is compared with the multi-objective antlion algorithm with traditional evolutionary strategy. Simulation results show that the improved algorithm has better convergence and distribution than other algorithms. The improved multi-objective antlion algorithm has higher stability, as well. Therefore, the quasi-oppositional multi-objective antlion optimization algorithm based on differential evolution proposed in this paper has good adaptability and robustness in solving multi-objective optimization problems.

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