Kaluza-Klein gauge and minimal integrable extension
of \( OSp(4|6)/(SO(1, 3) \times U(3)) \) sigma-model

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Abstract

Basing upon experience from performing double-dimensional reduction of the \( D = 11 \) supermembrane on \( AdS_4 \times S^7 \) background to Type IIA superstring on \( AdS_4 \times \mathbb{CP}^3 \) we introduce Kaluza-Klein (partial) \( \kappa \)-symmetry gauge as a vanishing condition of the contribution to the \( D = 11 \) supervielbein components tangent to \( D = 10 \) space-time proportional to the differential of the coordinate parametrizing compact 11-th space-time dimension, that is identified with the supermembrane world-volume compact dimension. For \( AdS_4 \times S^7 \) supermembrane Kaluza-Klein gauge removes half Grassmann coordinates associated with 8 space-time supersymmetries, broken by the \( AdS_4 \times \mathbb{CP}^3 \) superbackground, by imposing \( D = 3 \) (anti-)Majorana condition on them. The consideration relies on the realization of \( osp(4|8) \) isometry superalgebra of the \( AdS_4 \times S^7 \) superbackground as \( D = 3 \ N = 8 \) superconformal algebra. Requiring further vanishing of the \( D = 10 \) dilaton leaves in the sector of broken supersymmetries just two Grassmann coordinates organized into \( D = 3 \) (anti-)Majorana spinor that defines minimal \( SL(2, \mathbb{R}) \)-covariant extension of the \( OSp(4|6)/(SO(1, 3) \times U(3)) \) sigma-model. Among 4 possibilities of such a minimal extension we consider in detail one, that corresponds to picking out \( D = 3 \) Majorana coordinate related to broken Poincare supersymmetry, and show that the \( AdS_4 \times \mathbb{CP}^3 \) superstring equations of motion in this partial \( \kappa \)-symmetry gauge are integrable. Also the relation between the \( OSp(4|6)/(SO(1, 3) \times U(3)) \) sigma-model and the \( AdS_4 \times \mathbb{CP}^3 \) superstring is revisited.

1 Introduction

AdS/CFT correspondence \cite{1}, being explicit realization of the idea of duality between non-Abelian gauge theories and strings, gives valuable information on non-perturbative dynamics of a class of conformal field theories that is hard to obtain using other approaches. To date the best explored instance of AdS/CFT correspondence provides dual description of the \( D = 4 \ N = 4 \) supersymmetric Yang-Mills theory with \( U(N) \) gauge symmetry in terms of \( D = 10 \) Type IIB string theory on the \( AdS_5 \times S^5 \) superbackground with \( N \) units of the RR 5-form flux.

In \cite{2} another explicit example of AdS/CFT duality was suggested stating that in the 't Hooft limit \( D = 3 \ N = 6 \) superconformal Chern-Simons-matter theory, or ABJM theory for short, can be described in terms of \( D = 10 \) Type IIA string theory on the \( AdS_4 \times \mathbb{CP}^3 \) superbackground. This supergravity background, known since the mid of 80-s \cite{3}-\cite{5}, preserves 24 of 32 Type IIA supersymmetries that together with the \( SO(2, 3) \times SU(4) \) symmetry of \( AdS_4 \times \mathbb{CP}^3 \) space-time form \( OSp(4|6) \) isometry supergroup of the \( AdS_4 \times \mathbb{CP}^3 \) superspace. The fact that this superspace is non-maximally supersymmetric, as opposed to the \( AdS_5 \times S^5 \) one, makes the ABJM correspondence more difficult to investigate \cite{6}.

Maximal supersymmetry of the \( AdS_5 \times S^5 \) superspace, combined with the \( SO(2, 4) \times SO(6) \) symmetry of bosonic background into \( PSU(2, 2|4) \) supergroup, played a crucial role.

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in construction of the Type IIB superstring action on that background. In [7], based on
the isomorphism between the $AdS_5 \times S^5$ superspace and the $PSU(2,2|4)/(SO(1,4) \times SO(5))$
supercoset manifold, a group-theoretic approach was proposed for constructing the super-
string action as a $PSU(2,2|4)/(SO(1,4) \times SO(5))$ supercoset sigma-model. This approach
was further developed in [8]-[11], in particular the role of the discrete $Z_4$ automorphism of
the background isometry superalgebra and the integrability of equations of motion have been
revealed.

Applicability of the supercoset approach to the $AdS_4 \times \mathbb{CP}^3$ superstring construction
appears to be limited to the subspace of $AdS_4 \times \mathbb{CP}^3$ superspace parametrized by all 10
space-time coordinates and only 24 of 32 anticommuting ones that are in one-to-one corre-
respondence with 24 supersymmetries of the background. This subspace is isomorphic to the
$OSp(4|6)/(SO(1,3) \times U(3))$ supercoset manifold and the sigma-model action on it was
constructed out of the $osp(4|6)/(so(1,3) \times u(3))$ Cartan forms in [12], [13]. Although this
sigma-model action describes requisite number of physical degrees of freedom, has correct
bosonic limit and corresponding equations of motion are manifestly classically integrable one
cannot use it for the description of all string configurations [12], [15]. This is traced back to
the limitations that arise because the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model action is
obtained by fixing 8 of 16 $\kappa$-symmetries in the Type IIA superstring action on $AdS_4 \times \mathbb{CP}^3$
superspace constructed in [15].

The main difficulty in constructing the $AdS_4 \times \mathbb{CP}^3$ superstring action [15] was the pres-
ence in complete superspace of 8 Grassmann coordinates associated with the broken supers-
ymmetries that is the distinctive feature of ABJM correspondence [2], as compared to the
$AdS_5/CFT_4$ one [1]. The $AdS_4 \times \mathbb{CP}^3$ superspace is not isomorphic to a supercoset manifold
so another way of recovering the superstring action dependence on those 8 'broken' fermionic
coordinates had to be taken.

It is known [4], [5] that the $AdS_4 \times \mathbb{CP}^3$ background geometry can be obtained by di-
imensional reduction of the $AdS_4 \times S^7$ one basing on the Hopf fibration realization of the
7-sphere $S^7 = \mathbb{CP}^3 \times S^1$. The $AdS_4 \times S^7$ superbackground supported by the non-zero 4-form
flux is maximally supersymmetric and on it the supermembrane can propagate. So it was
suggested in [12] that one can obtain the full-fledged $AdS_4 \times \mathbb{CP}^3$ superstring action by per-
foming double-dimensional reduction of the $AdS_4 \times S^7$ supermembrane action. Since the
$AdS_4 \times S^7$ superspace is isomorphic to the $OSp(4|8)/(SO(1,3) \times SO(7))$ supercoset manifold
the supermembrane action on it was constructed in Ref. [10] by generalizing the approach of [7].

General prescription how to perform the double-dimensional reduction of $D = 11$ super-
membrane action on a curved superbackground was elaborated in [17] (see also [18]). It relies on identifying one of the target-space compact dimensions as the supermembrane
world-volume compact dimension, remaining 10 space-time dimensions then correspond to the
bosonic body of $D = 10$ Type IIA superspace. $D = 11$ Supervielbein and 3-form poten-
tial are required to be of Kaluza-Klein (KK) ansatz form, i.e. not to depend on the coordinate $y$
of the selected 11-th dimension and also the $D = 11$ supervielbein components
tangent to $D = 10$ manifold, that are identified with the $D = 10$ Type IIA supervielbein
bosonic components, should not contain $dy$ dependent terms. Whenever the latter condition
does not hold (as is the case for the $AdS_4 \times S^7$ superspace), i.e. the $D = 11$ supervielbein
bosonic components tangent to $D = 10$ space-time acquire summands $dy G^\alpha_0$, $\hat{m'} = 0,...,9$, it is prescribed to perform the $SO(1,10)$ tangent-space Lorentz rotation to remove such

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2 Alternative way to describe the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model using the pure spinor approach was followed in [14].
contributions. Its parameters are determined by $G_{y}^{\hat{n}'}$ making the structure of the resulting Type IIA superstring action rather involved. That is why one can try to get rid of $G_{y}^{\hat{n}'}$ by partially fixing, whenever possible, the $\kappa-$symmetry gauge freedom

$$G_{y}^{\hat{n}'}|_{\text{KK gauge}} = 0$$ (1.1)

that defines the KK gauge.

As we shall see for the $AdS_{4} \times S^{7}$ supermembrane, where $D = 10$ vector $G_{y}^{\hat{n}'}$ has non-zero components only in directions tangent to the $AdS_{4}$ space-time and depends on the Grassmann coordinates related to 8 broken supersymmetries, such a gauge amounts to imposing $D = 3$ (anti-)Majorana condition on those coordinates organized into a pair of $SL(2, \mathbb{R})$ spinors and their conjugates. Requiring further vanishing of the $D = 10$ dilaton leaves just two out of 8 Grassmann coordinates from the broken supersymmetries sector that are described by $D = 3$ (anti-)Majorana spinor.

Closer look on the $\kappa-$symmetry gauge fixing in the sector of broken supersymmetries of the $AdS_{4} \times \mathbb{C}P^{3}$ superspace is also motivated by the question whether integrable structure of the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset sigma-model can be extended to incorporate 8 ’broken’ fermionic coordinates? Study of the integrable structure associated with the full-fledged $AdS_{4} \times \mathbb{C}P^{3}$ superstring was initiated in [19], where Lax representation of the equations of motion for quadratic in all 24+8 fermions part of the $AdS_{4} \times \mathbb{C}P^{3}$ superstring action was found. Since the integrable structure of the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model [12], [13] is well explored it is justified to consider corresponding Lax connection, that is expressed in terms of the $osp(4|6)$ Cartan forms, as the leading contribution to a candidate Lax connection for the $AdS_{4} \times \mathbb{C}P^{3}$ superstring in its expansion in the fermions associated with the broken supersymmetries. In other words, in searching for the integrable structure of complete $AdS_{4} \times \mathbb{C}P^{3}$ superstring one can try to take into account 24 fermionic coordinates of the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset as a whole by assuming that the Lax connection of $AdS_{4} \times \mathbb{C}P^{3}$ superstring can be expressed as a series in remaining 8 ’broken’ fermions with the coefficients given either by the $osp(4|6)$ Cartan forms or the differentials of those 8 fermions. Realization of this line of reasoning has been started in [26], where the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset Lax connection was extended by linear and quadratic terms in all 8 ’broken’ fermions. It, however, appears rather difficult technically to recover the whole series expansion. More feasible task is to trace the dependence of the $AdS_{4} \times \mathbb{C}P^{3}$ superstring Lax connection on a part of ’broken’ fermions. From this viewpoint it is natural first to identify what would be a minimal ‘quantity’ in the broken supersymmetries sector. Clearly this can be one of 8 ’broken’ fermionic coordinates. The disadvantage of this identification is the lack of covariance. Hence, because the broken supersymmetries sector is parametrized by two $SL(2, \mathbb{R})$ spinor coordinates and their conjugates related to the generators of broken super-Poincare and superconformal symmetries that enter $D = 3 \mathcal{N} = 8$ superconformal algebra, it is possible to impose (anti-)Majorana condition on one of those coordinates and gauge away another one. This allows to single out 4 minimal $SL(2, \mathbb{R})$—covariant $\frac{1}{4}$ fractions of broken supersymmetry. Then one can examine extension

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3Geometry and integrable structures in this sector of the $AdS_{4} \times \mathbb{C}P^{3}$ superstring model were previously investigated in Refs. [15], [19]. However, $\kappa-$symmetry gauge conditions for the full $AdS_{4} \times \mathbb{C}P^{3}$ superstring action considered so far [20], [21] resembled those for the $AdS_{5} \times S^{5}$ superstring [22], [23] and treated on equal footing Grassmann coordinates associated with both unbroken and broken space-time supersymmetries.

4This quadratic in the Grassmann coordinates part of the superstring action is in fact known for arbitrary superbackground [24]. Such an action for the $AdS_{4} \times \mathbb{C}P^{3}$ superbackground was used [25] to calculate one-loop corrections to the energies of classical spinning strings before the complete action of Ref. [13] was constructed.
of the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model Lax connection by each of those $\frac{1}{4}$ fractions of broken supersymmetry and gradually include interactions between them.

Above discussion advocates utility of the realization of $osp(4|6)$ isometry superalgebra of $AdS_4 \times \mathbb{CP}^3$ superbackground as $D = 3 \mathcal{N} = 6$ superconformal algebra that is the on-shell symmetry algebra of the ABJM gauge theory action \[27\]. In Ref. \[28\] we examined the $OSp(4|6)/(SO(1,3) \times U(3))$ supermodel in conformal basis for constituent Cartan forms, there was also obtained explicit expression for the sigma-model action by choosing the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset representative parametrized by coordinates associated with the generators of $D = 3 \mathcal{N} = 6$ superconformal algebra. In particular, anti-commuting coordinates have been organized into a pair of $SL(2,\mathbb{R})$ 2-component spinors $\theta_\mu^A$ and $\eta_{\mu a}$, $\mu = 1,2$, $a = 1,2,3$ transforming in the fundamental representation of $SU(3)$ that correspond to super-Poincare $Q_\mu^A$ and superconformal symmetry $S^{\mu a}$ generators and their conjugates.\[^5\] Analogously the $osp(4|8)$ isometry superalgebra of $AdS_4 \times S^7$ superbackground (and hidden strong coupling symmetry of ABJM gauge theory \[2\]) admits realization as $D = 3 \mathcal{N} = 8$ superconformal algebra with $SO(8)$ $R$–symmetry generators in one of 8-component spinor representations so that the fermionic generators $O_{\alpha A'}$ carry spinor indices of $Spin(1,3)$ ($\alpha = 1, \ldots, 4$) and $Spin(8)$ ($A' = 1, \ldots, 8$). These generators decompose into a pair of $SL(2,\mathbb{R})$ 2-component spinors $Q_\mu^A$, $S^{\mu A}$, $A = 1, \ldots, 4$ in $SU(4)$ and their conjugates following the decomposition of the spinor representation of $Spin(8)$ as $4 \oplus \bar{4}$. It is these generators that are identified with the super-Poincare and superconformal symmetry generators of $D = 3 \mathcal{N} = 8$ superconformal algebra. Further decomposition of $SU(4)$ fundamental representation as $4 = 3 \oplus 1$ of $SU(3)$ introduces 24 generators $Q_\mu^3$, $S^{\mu 3}$ and c.c. from the $D = 3 \mathcal{N} = 6$ superconformal algebra. Remaining 8 fermionic generators $Q_\mu^4$, $S^{\mu 4}$ and c.c. correspond to super-Poincare and superconformal symmetries broken by the $AdS_4 \times \mathbb{CP}^3$ superbackground[\(^3\)] and associated Grassmann coordinates $\theta_4^\mu \equiv \theta^\mu$, $\eta_4 a \equiv \eta^\mu$ extend the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset space to the $AdS_4 \times \mathbb{CP}^3$ superspace. This is the group-theoretic setting of our consideration \[21\].

The outline of the paper is the following. In the next section we discuss features of the double-dimensional reduction of $D = 11$ supermembrane on a curved superbackground to the Type IIA superstring. Then we analyze consequences of the KK gauge imposition for the $AdS_4 \times S^7$ supermembrane and in Section 4 we introduce minimal extension of the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset sigma-model. Section 5 contains conclusions and Appendixes supply necessary data on $osp(4|6)$ and $osp(4|8)$ superalgebras and equations of motion for the minimal extension the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model by $D = 3$ Majorana coordinate related to the broken part of $D = 3 \mathcal{N} = 8$ super-Poincare symmetry.

2 From supermembrane to superstring: Kaluza-Klein gauge

$D = 11$ Supermembrane action on a curved superbackground \[33\]

\[
S_{\text{membrane}} = - \int_V d^8 \xi \sqrt{- \det \hat{E}_{ij}^m} \hat{E}_{jim} + S_{WZ\text{membrane}} \tag{2.1}
\]

\[^5\] Previously such conformal-type parametrizations were used to examine the string/brane models involved into the higher-dimensional examples of $AdS/CFT$ correspondence \[29\]–\[31\], \[33\].

\[^6\] Such a 24+8 decomposition of the $osp(4|8)$ fermionic generators and associated coordinates is fulfilled by two projectors introduced in \[4\] (see also \[13\]) that are determined by Kähler 2-form of the $\mathbb{CP}^3$ manifold. We consider particular convenient realization for this tensor such that these projectors diagonalize \[32\].
is given by the sum of Nambu-Goto and Wess-Zumino (WZ) terms

\[ S_{WZ membrane} = \int_{\mathcal{M}_4} H(4), \quad (2.2) \]

where the integration of the 4-form field strength \( H(4) = dA_{(3)} \) is carried out over auxiliary 4-manifold \( \mathcal{M}_4 \), whose boundary coincides with the supermembrane world volume \( V \). The procedure of double-dimensional reduction \[17, 18\] consists in selecting one compact space-like dimension in the target-space, that is labeled as 11-th one, and identifying it with the compact dimension on the world volume parametrized by \( y \in [0, 2\pi) \). To fulfill the reduction \( D = 11 \) supervielbein components are required not to depend on \( y \) and their dependence on \( dy \) is restricted by the KK ansatz

\[ \hat{E}^{\hat{m}'}(d) = \left( \hat{E}^{\hat{m}'}_{11} \right) = \left( E^{\hat{m}'} \right) \Phi(dy + A); \quad \hat{F}^{\hat{\alpha}}(d) = E^{\hat{\alpha}} + \Phi^{-1} \hat{E}^{11} \chi^{\hat{\alpha}}, \quad (2.3) \]

where \( \hat{m} = (\hat{m}', 11) \) is the \( D = 11 \) vector index, \( \hat{m}' = 0, 1, ..., 9 \) is the \( D = 10 \) vector index and Greek letters \( \hat{\alpha}, \hat{\beta} = 1, ..., 32 \) label \( D = 10 \) spinors. \( E^{\hat{m}'}(d) \) and \( E^{\hat{\alpha}}(d) \) are identified with the \( D = 10 \) Type IIA supervielbein, \( \Phi = e^{2\phi/3} \) with the dilaton \( \phi \), \( A(d) \) with the RR 1-form potential and \( \chi^{\hat{\alpha}} \) with the dilatino.

In practice it may happen, as is the case with the \( AdS_4 \times S^7 \) superbackground isomorphic to the \( OSp(4|8)/(SO(1, 3) \times SO(7)) \) supercoset manifold, that \( \hat{E}^{\hat{m}'} \) contains additional summand proportional to \( dy \)

\[ \hat{E}^{\hat{m}'}(d) = G^{\hat{m}'}(d) + dyG^{\hat{m}'}_y. \quad (2.4) \]

Then it is prescribed to bring supervielbein to the Kaluza-Klein ansatz form \( (2.3) \) by a tangent-space Lorentz rotation

\[ (L\hat{E})^{\hat{m}'}(d) = L^{\hat{m}'}_{\hat{m}'} L^{\hat{m}'}_{11} \hat{E}^{\hat{m}'} + L^{\hat{m}'}_{11} \hat{E}^{11} = E^{\hat{m}'}(d), \]

\[ (L\hat{F})^{\hat{\alpha}}(d) = E^{\hat{\alpha}} + \Phi_L^{-1} (L\hat{E})^{11} \chi^{\hat{\alpha}}, \]

where

\[ (L\hat{E})^{11}(d) = L^{11}_{\hat{m}'} L^{11}_{11} \hat{E}^{\hat{m}'} + L^{11}_{11} \hat{E}^{11} = \Phi_L(dy + A_L) \quad (2.6) \]

and now \( (L\hat{E})^{\hat{m}'} \) is identified with the bosonic components \( E^{\hat{m}'} \) of \( D = 10 \) supervielbein, \( \Phi_L = e^{2\phi/3} \) is related to the dilaton field \( \phi \), while \( A_L \) is the RR 1-form potential. The Lorentz rotation matrix

\[ ||L|| = \left( \begin{array}{cc} L^{\hat{m}'}_{\hat{m}'} & L^{\hat{m}'}_{11} \\ L^{11}_{\hat{m}'} & L^{11}_{11} \end{array} \right) \in SO(1, 10), \quad L^{\hat{\alpha}}_{\hat{\beta}} \in Spin(1, 10) \quad (2.7) \]

is defined by the requirement of removing the \( dy \)-dependent summand in \( (L\hat{E})^{\hat{m}'} \): \( L^{\hat{m}'}_{\hat{m}'} G^{\hat{m}'}_y + L^{\hat{m}'}_{11} \Phi = 0 \). So the entries of the matrix \( ||L|| \) are expressed in terms of \( G^{\hat{m}'}_y \) and \( \Phi \)

\[ L^{\hat{m}'}_{\hat{m}'} = \delta^{\hat{m}'}_{\hat{m}'} + \frac{\Phi - \sqrt{\Phi^2 + (G_y G_y)}}{\sqrt{\Phi^2 + (G_y G_y)}}, \quad L^{\hat{m}'}_{11} = \frac{G^{\hat{m}'}_y}{\Phi^2 + (G_y G_y)}, \quad L^{11}_{\hat{m}'} = \frac{1}{\sqrt{G^{\hat{m}'}_y}}, \quad L^{11}_{11} = \frac{\Phi}{\sqrt{\Phi^2 + (G_y G_y)}} \quad (2.8) \]

(see \[15, 21\]). This Lorentz rotation significantly complicates the form of \( D = 10 \) supervielbein in the KK frame and the superstring action resulting from the reduction

\[ S = -\int \sum d^2 \xi \Phi_L \sqrt{-\det E_{\hat{i}\hat{m}'} E_{\hat{j}\hat{m}'} + S_{WZ}}, \quad (2.9) \]
where the WZ term

\[ S_{WZ} = \int_{\mathcal{M}_3} H_{(3)} \]  

is determined by the field-strength \( H_{(3)} = dB_{(2)} \) of NS-NS 2-form integrated over the auxiliary 3-manifold \( \mathcal{M}_3 \) with the boundary given by the string world sheet \( \Sigma \). The form of the Type IIA superstring action (2.9) will simplify if we impose, whenever admissible, KK (partial) \( \kappa \)-symmetry gauge

\[ G_{y}^{m'} |_{\text{KK gauge}} = 0. \]  

Thus the contribution of this summand to the NS-NS 3-form field strength is given by

\[ H_{(4)} = \frac{i}{8} \hat{F}^\alpha \wedge g^{m'n}_\alpha \beta \hat{F}_\beta \wedge \hat{E}_{m'} + \frac{1}{4} \varepsilon_{k'l'm'n} \hat{E}^{k'} \wedge \hat{E}^{l'} \wedge \hat{E}^{m'} \wedge \hat{E}^{n'}, \quad k', l' = 0, ..., 3 \]  

where \( g^{m'n}_\alpha \beta \) are Spin(1, 10) generators realized by \( D = 11 \) gamma-matrices, that generalizes corresponding flat-background expression, for them there is no necessity to perform tangent-space Lorentz rotation before the supermembrane reduction. This argument can be extended to the whole \( AdS_4 \times S^7 \) supermembrane action, for which

\[ H_{(4)} = \frac{i}{8} \hat{F}^\alpha \wedge g^{m'n}_\alpha \beta \hat{F}_\beta \wedge \hat{E}_{m'} + \frac{1}{4} \varepsilon_{k'l'm'n} \hat{E}^{k'} \wedge \hat{E}^{l'} \wedge \hat{E}^{m'} \wedge \hat{E}^{n'}, \quad k', l' = 0, ..., 3 \]  

acquires the only additional summand proportional to the components of \( D = 4 \) Levi-Civita tensor in the tangent space to \( AdS_4 \) and the structure of the osp(4|8) isometry superalgebra is such that only tangent to the \( AdS_4 \) components of \( D = 11 \) supervielbein have \( dy \)-dependent parts

\[ \hat{E}^{m'}(d) = G^{m'}(d) + dy G_{y}^{m'}. \]  

Thus the contribution of this summand to the NS-NS 3-form field strength is given by

\[ \frac{1}{4} \varepsilon_{k'l'm'n} \hat{E}^{k'} \wedge \hat{E}^{l'} \wedge \hat{E}^{m'} \wedge \hat{E}^{n'} = \frac{1}{2} \varepsilon_{k'l'm'n} G^{k'} \wedge G^{l'} \wedge G^{m'} \wedge dy G_{y}^{n'} + ... \rightarrow \frac{1}{2} \varepsilon_{k'l'm'n} G^{k'} \wedge G^{l'} \wedge G^{m'} G_{y}^{n'}. \]  

Although this observation can simplify to certain extent the process of derivation and the form of the resulting \( AdS_4 \times \mathbb{CP}^3 \) superstring action, its structure, depending on \( G_{y}^{m'} \), remains complicated highly non-linear, so that proper \( \kappa \)-symmetry gauge choice is required to simplify it (see [20], [21]). As far as action functionals for other point-like and extended objects in the \( AdS_4 \times \mathbb{CP}^3 \) superspace are concerned, complete, i.e. Lorentz-rotated, expressions for the supervielbein and RR forms should be used for their construction [15], [20].

3 Kaluza-Klein gauge for \( AdS_4 \times S^7 \) supermembrane

To analyze consequences of the KK gauge imposition (2.11) for the \( AdS_4 \times S^7 \) supermembrane and the \( AdS_4 \times \mathbb{CP}^3 \) superstring explicit form of \( G_{y}^{m'}. \) (2.14) is needed that can be derived upon specifying the \( OSp(4|8)/(SO(1,3) \times SO(7)) \) representative compatible with the Hopf fibration realization of the 7-sphere. Possible choice is provided by the \( OSp(4|6)/(SO(1,3) \times U(3)) \) representative \( \mathcal{G} \) 'dressed' by the \( S^1 \) fiber generator \( H \) and the generators of broken supersymmetries \( Q_\mu^4, \bar{Q}_{\mu 4} \) and \( S_{\mu 4}, \bar{S}_{\mu 4} \)

\[ \mathcal{G} = g e^{3H} e^{\theta^\nu Q_\nu^4 + \bar{\theta}^\nu \bar{Q}_{\nu 4}} e^{\eta^\mu S_{\mu 4} + \bar{\eta}^\mu \bar{S}_{\mu 4}} \in OSp(4|8)/(SO(1,3) \times SO(7)). \]
Let us note that $\hat{G}$ is the $OSp(4|8)/(SO(1,3) \times SO(7))$ representative considered in [21] to derive the $AdS_4 \times CP^3$ superstring action in AdS-light-cone gauge.

The representative $G$ defines left-invariant $osp(4|6)$ Cartan forms that in the conformal basis decompose as follows

$$G^{-1}dG = \omega^m(d)P_m + \epsilon^m(d)K_m + \Delta(d)D + \Omega_a(d)T^a + \Omega^a(d)T_a$$

+ $\omega_\mu^a(d)Q_\mu + \tilde{\omega}_\mu a(d)\tilde{Q}_\mu + \chi_\mu a(d)S^{\mu} + \tilde{\chi}_\mu a(d)\tilde{S}_m$  

+ $G^{mn}(d)M_{mn} + \tilde{\Omega}_a b(d)\tilde{V}_b a + \tilde{\Omega}_b a(d)\tilde{V}_a b$.  

Bosonic generators $D$, $P_m$, $K_m$, $M_{mn}$ belong to the $conf_3$ algebra, while $T_a$, $T^a$, $\tilde{V}_b a$ generate the $su(4) \sim so(6)$ $R$-symmetry subalgebra of $osp(4|6)$ superalgebra. Fermionic generators $Q_\mu a$, $\tilde{Q}_\mu a$ and $S^{\mu} a$, $\tilde{S}_m a$ are $D = 3 \ N = 6$ super-Poincare and superconformal symmetry generators respectively. Then the left-invariant $osp(4|8)$ Cartan forms can be presented as

$$\hat{G}^{-1}d\hat{G} = \frac{1}{2}(\omega^m(d) + \epsilon^m(d))(P_m + K_m) + \Delta(d)D$$

+ $\frac{1}{2}(\Omega_a(d) + \tilde{\Omega}_a a(d))(T^a + \tilde{T}_a) + \frac{1}{2}(\Omega^a(d) + \tilde{\Omega}^a d)(T_a + \tilde{T}_a)$

+ $(h(d) + \tilde{\Omega}_b b(d))\frac{1}{4}H + \tilde{V}_a a$  

+ $\omega_\mu^a(d)Q_\mu + \omega_\mu^a(d)Q_\mu + \tilde{\omega}_\mu a(d)\tilde{Q}_\mu + \tilde{\omega}_\mu a(d)\tilde{Q}_\mu$  

+ $\chi_\mu a(d)S^{\mu} + \chi_\mu a(d)S^{\mu} + \tilde{\chi}_\mu a(d)\tilde{S}_m + \tilde{\chi}_\mu a(d)\tilde{S}_m$  

+ $G^{mn}(d)M_{mn} + \frac{1}{2}(\omega^m(d) - \epsilon^m(d))(P_m - K_m)$

+ $\tilde{\Omega}_a b(d)\tilde{V}_b a - h(d)\tilde{V}_a a + \frac{1}{4}(3h(d) - \tilde{\Omega}_b b(d))H + \Omega_a dV_4 + \Omega^a dV_4$  

+ $\frac{1}{2}(\Omega_a(d) - \tilde{\Omega}_a a(d))(T_a - \tilde{T}_a) + \frac{1}{2}(\Omega^a(d) - \tilde{\Omega}^a d)(T_a - \tilde{T}_a)$.  

The first line contains generators $P_m + K_m = 2M_{0|3}$, $D = -2M_{0|3}$ from the coset $so(2,3)/so(1,3)$ and the sixth line - generators $M_{mn}$, forming the $so(1,3)$ subalgebra of $so(2,3)$. Similarly the $so(8)$ generators have been grouped according to the decomposition on $so(8)/so(7) = \{T_a + \tilde{T}_a, T^a + \tilde{T}_a, \frac{1}{4}H + \tilde{V}_a a\}$ (the second and third lines) and $so(7) = \{\tilde{V}_b a - \frac{1}{2}\delta_b c V_c, \frac{1}{8}\delta_b dH, T_a - \tilde{T}_a, T^a - \tilde{T}_a, V_4, V_4\}$ (two last lines) pertinent to the $AdS_4 \times S^7$ supermembrane description. We have expressed them in terms of $su(4) = \{T_a, T^a, V_a a\}$ and $u(1) = \{H\}$ generators forming the $su(4) \oplus u(1)$ isometry algebra of $CP^3 \times S^1$, as well as $\tilde{T}_a, \tilde{T}_a, V_4, V_4$ generators that belong to the coset $so(8)/(su(4) \times u(1))$. Altogether they provide the realization of $so(8)$ algebra as $su(4) \oplus u(1) \oplus so(7)/(su(4) \times u(1))$ compatible with the Hopf fibration of $S^7$-sphere relevant for the $AdS_4 \times S^7$ supermembrane reduction to the $AdS_4 \times CP^3$ superstring. Details of the relation between both realizations of the $so(8)$ algebra, as well as commutation relations of $osp(4|6)$ and $osp(4|8)$ superalgebras are given in Appendix A (see also [21]). The first five lines in (3.3) include Cartan forms that will be identified with the $AdS_4 \times S^7$ supervielbein components, while remaining Cartan forms correspond to the $so(1,3) \oplus so(7)$ connection. In (3.3) there have been underlined those of the $osp(4|6)$ Cartan forms (3.2) that, in addition to 24 ’unbroken’ Grassmann coordinates of the supercoset $OSp(4|6)/(SO(1,3) \times U(3))$, acquire dependence on 8 ‘broken’ coordinates from (3.1).

7In Ref. [15] such a relation was given in conventional basis for $D = 6$ vectors and without specifying explicitly the Kähler 2-form on $CP^3$. However, natural realization of the $u(3)$ stability algebra of $CP^3$ is provided by the generators $\tilde{V}_a b$ in $3 \times 3$ representation of $SU(3)$. Associated representation of $D = 6$ vectors is in $3 \oplus 3$ basis, where there is evident choice for the Kähler 2-form such that its contraction with the $su(4)$ generators constructed out of $D = 6$ chiral gamma-matrices is given by the diagonal $4 \times 4$ matrix \[32\].
Using that the tangent to $AdS_4$ components of the $AdS_4 \times S^7$ supervielbein are identified with the Cartan forms $\frac{1}{2}(\omega^m + c^m)$, $(\Delta(d))$ associated with the $so(2, 3)/so(1, 3)$ coset generators $M_{\alpha\beta}$,
\[
\hat{E}^m(d) = \left( \begin{array}{c} E^m \\ E^3 \end{array} \right) = \left( \begin{array}{c} \frac{1}{2}(\omega^m + c^m) \\ -\Delta \end{array} \right) 
\]
and recalling the form of $OSp(4|8)/(SO(1, 3) \times SO(7))$ supercoset representative (3.11), gives that the corresponding $dy-$ dependent terms (2.14) 
\[
G_{y'}^{m'} = \left( \begin{array}{c} G_{y'}^m \\ G_{y'}^3 \end{array} \right) = \left( \begin{array}{c} \frac{1}{2}(\omega^m + c^m) \\ -\Delta_{y'} \end{array} \right), 
\]
where $\omega_y$, $c_y$ and $\Delta_y$ are $dy$-dependent parts of the corresponding Cartan forms, appear to be functions of the 'broken' fermionic coordinates only. Explicit calculation yields
\[
\omega_y^m + c_y^m = 4[1 - (\eta\bar{\eta})^2](\theta\sigma^m\bar{\theta}) + 4\{\{\theta\bar{\eta}\} + \{\bar{\theta}\eta\}\}(\eta\sigma^m\bar{\eta}), \quad \Delta_y = 2[(\bar{\theta}\eta) - (\theta\bar{\eta})] \tag{3.6}
\]
with the $SL(2, \mathbb{R})$ spinor contractions defined by $(\eta\bar{\eta}) = \eta^\mu\bar{\eta}_\mu$, $(\theta\sigma^m\bar{\theta}) = \theta^\mu\sigma^m\bar{\theta}^\nu$ etc. $D = 3$ gamma-matrices $\sigma^m$ are symmetric in spinor indices so (3.6) vanishes if Grassmann coordinates satisfy (anti-)Majorana condition
\[
\bar{\theta}^\mu = s\theta^\mu, \quad \bar{\eta}^\mu = s\eta^\mu, \quad s = \pm 1.
\]
Sign factors for $\theta$ and $\eta$ coordinates are correlated to turn to zero $\Delta_y$.

Concentrating on the case $s = 1$ we use the same notation $\theta^\mu$ and $\eta^\mu$ for the Grassmann coordinates that satisfy $D = 3$ Majorana condition. Then the $AdS_4 \times S^7$ supervielbein bosonic components tangent to the $AdS_4$ space-time acquire the form
\[
E^m(d) = G^{0m} - \frac{1}{2}c^{mkl}G_{kl}[(\theta\bar{\theta}) + (\eta\bar{\eta})]
- 2i\epsilon_n(\theta\sigma^n\bar{\theta}^m\eta) - c^m(\theta\bar{\eta})(\eta\bar{\eta}) - i(d\theta\sigma^m\eta + d\eta\sigma^m\bar{\eta}), \tag{3.8}
- E^3(d) = [1 + 2i(\theta\bar{\eta})]\Delta + iG^{mn}(\theta\sigma_{mn}\eta) + 2i(d\theta\eta),
\]
where $G^{0m}(d) = \frac{1}{2}(\omega^m + c^m)$ and $-\Delta(d)$ are tangent to the $AdS_4$ components of the $OSp(4|6)/(SO(1, 3) \times U(3))$ supervielbein. $G^{mn}(d)$ represents the $so(1, 2)$ part of $so(1, 3)$ connection on the $OSp(4|6)/(SO(1, 3) \times U(3))$ supercoset manifold. Tangent to the $\mathbb{CP}^3$ components of $AdS_4 \times S^7$ supervielbein read
\[
E_\alpha(d) = i(\Omega_{\alpha} + \tilde{\Omega}_{\alpha}) = i[\Omega_{\alpha} + 2\chi_{\mu\alpha}\theta^\mu - 2\omega^\mu_{\alpha}\eta_\mu - 2i\chi^{\mu}_{\alpha}\eta_\mu(\theta\bar{\theta})], 
E^\alpha(d) = i(\Omega^\alpha + \tilde{\Omega}^\alpha) = i[\Omega^\alpha - 2\bar{\chi}^\alpha_{\mu}\theta^\mu + 2\bar{\omega}^{\alpha\mu}\eta_\mu + 2i\bar{\chi}^{\mu\alpha}_{\eta}(\theta\bar{\theta})], \tag{3.9}
\]
where the $su(4)/u(3)$ Cartan forms $\Omega_\alpha(d)$, $\Omega^\alpha(d)$ are identified with the $\mathbb{CP}^3$ part of $OSp(4|6)/(SO(1, 3) \times U(3))$ supervielbein, while Cartan forms $\omega^\mu_{\alpha}(d)$ and $\chi_{\mu\alpha}(d)$ related to $D = 3$ $N = 6$ super-Poincare and superconformal symmetry generators are identified with the $OSp(4|6)/(SO(1, 3) \times U(3))$ supervielbein fermionic components. Because (3.6) is zero in the KK gauge expressions (3.8), (3.9) coincide with the $AdS_4 \times \mathbb{CP}^3$ supervielbein

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8This justifies why to find out restrictions imposed by the KK partial $\kappa-$symmetry gauge condition we have not specified explicitly the form of $OSp(4|6)/(SO(1, 3) \times U(3))$ supercoset representative. Possible choice compatible with the realization of $osp(4|6)$ superalgebra as $D = 3$ $N = 6$ superconformal algebra is as given in [28], [21]. It results in parametrization of the $AdS_4$ space-time by the Poincare coordinates.

9Our conventions for the spinor algebra are those of Refs. [28], [21].
bosonic components and we removed hats to indicate this. The $AdS_4 \times S^7$ supervielbein component tangent to the $S^4$ fiber of $S^7 = \mathbb{CP}^3 \times S^3$ equals

$$\hat{E}^{11}(d) = h + \Omega^b = \Phi(dy + a): \Phi = 1 - 4i(\theta \eta), \ a(d) = \Phi^{-1} \Omega^b(d). \quad (3.10)$$

$\Omega^b(d)$ corresponds to the $u(1)$ part of $u(3)$ connection $\Omega^b(d)$ on the $OSp(4|6)/(SO(1,3) \times U(3))$ supermanifold and $\Phi = e^{2\phi/3}$ determines the $D = 10$ dilaton. Fermionic components of the $AdS_4 \times S^7$ supervielbein

$$\hat{F}^{\dot{a}}(d) = \begin{pmatrix} F_{\mu A}^a \\ \bar{F}_{\mu}^a \\ F_{\mu A}^a \end{pmatrix} = f^{\dot{a}}(d) + dy F^{\dot{a}}_y \quad (3.11)$$
in the KK gauge read

$$f^{\mu a}_\alpha(d) = \omega^{\mu a}_\alpha = \omega^{\mu a}_\alpha + i\chi^{\mu a}_\alpha(\theta \eta),$$

$$f^{\mu}_\mu(d) = \omega^{\mu}_\mu = d\theta^\mu - 2i d\theta^\mu(\theta \eta) + 2i d\theta^\mu(\theta \eta),$$

$$F_{y^\mu} = \omega_{y^\mu} = 2i d\theta^\mu, \quad (3.12)$$

$$f_{\mu a}(d) = \chi_{\mu a} = \chi_{\mu a} - 4i\chi_{\nu a\eta\mu}^\nu - i\omega_{\mu a}(\eta\eta) - \chi_{\mu a}(\theta(\eta\eta)),$$

$$F_{y\mu} = \chi_{y\mu} = 2i\eta_a[1 - 2i(\theta\eta)]$$

and c.c. From (3.11) one derives the $AdS_4 \times \mathbb{CP}^3$ supervielbein fermionic components and dilatino (cf. (2.3))

$$E^{\dot{a}}(d) = f^{\dot{a}} - a F^{\dot{a}}_y \quad (3.13)$$

4 Minimal extension of the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset sigma-model

To further simplify the form of $AdS_4 \times S^7$ supervielbein (3.8)-(3.12) it is possible to additionally require $\Phi = 1$, i.e. vanishing of the $D = 10$ dilaton. This can be achieved in two ways: either by setting $\eta^\mu = 0$ or $\theta^\mu = 0$ that leads to minimal super-Poincare or superconformal extension of the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset sigma-model by $D = 3$ Majorana fermions. Consider in detail the first possibility. In this case bosonic part of the $AdS_4 \times S^7$ supervielbein (3.8)-(3.10) acquires the form

$$E^m(d) = \ G^m G^m - i \varepsilon^{mkl} G_{kl}(\theta \theta) - id\theta \sigma^m \theta, \ E^3(d) = -\Delta,$$

$$E_a(d) = i(\Omega_a + 2\chi_{\mu a} \theta^\mu), \ E^a(d) = i(\Omega^a - 2\chi^a_{\mu} \theta^\mu) \quad (4.1)$$

and

$$\hat{E}^{11} = dy + \Omega^a_a. \quad (4.2)$$
Fermionic non-zero components read

\begin{align*}
 f^\mu_a(d) &= \omega^\mu_a + i\chi^\mu_a(\theta\theta), \\
 f^\mu_4(d) &= d\theta^\mu + \frac{1}{2}G^\mu_{\nu\sigma} \sigma_{\mu\nu\sigma} + \Delta\theta^\mu, \\
 F^\mu_{4} &= 2i\theta^\mu, \\
 f_{\mu a}(d) &= \chi_{\mu a}, \\
 f_{\mu 4}(d) &= c_{m}^{a} \sigma_{\mu\nu}^m \theta^\nu.
\end{align*}

Eqs. (4.1)-(4.3) are used to derive action functional for the $AdS_4 \times \mathbb{C}P^3$ superstring with the $\kappa$–symmetry gauge freedom partially fixed in such a way that in the broken supersymmetries sector there remains single $D = 3$ Majorana spinor coordinate $\theta^\mu$. In other words, one arrives at the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset sigma-model extended by $D = 3$ Majorana fermion related to the broken part of $D = 3 \mathcal{N} = 8$ Poincare supersymmetry

\[ S = \int_{\Sigma} d^3\xi (\mathcal{L}_{kin} + \mathcal{L}_{WZ}), \] (4.4)

where kinetic and WZ Lagrangians have the form

\[ \mathcal{L}_{kin} = -\frac{1}{2} \gamma^{ij} (E_{ij}^m E_{mj} + \Delta_i \Delta_j - \Delta_{ia} E_{ia}), \] (4.5)

\[ \mathcal{L}_{WZ} = -\frac{1}{2} (f_{\mu a}^\mu + 2i\Omega_a \theta^\mu) \wedge (\bar{f}_{\mu a}^\mu - 2i\Omega_{\mu a} \theta^\mu) - \frac{1}{2} f_{\mu a} \wedge \bar{f}_{\mu a} + (\Omega_a \wedge \Omega^a + 2i\tilde{\Omega}_a \wedge \Delta)(\theta^\mu). \] (4.6)

To obtain superstring equations of motion it is convenient to consider as independent variation parameters the following combinations of the $osp(4|6)/(so(1,3) \times u(3))$ Cartan forms (3.2)

\[ G^{\tau m}(d) = \frac{1}{2} (\omega^m + c^m), \quad \Delta(d), \] (4.7)

related to the $so(2,3)/so(1,3)$ generators $M^{\tau m}$ that belong to the eigenspace with eigenvalue $-1$ under the $\mathbb{Z}_4$ automorphism of $osp(4|6)$ superalgebra, and

\begin{align*}
 \omega_{(1)a}^{\mu}(d) &= \frac{1}{2} (\omega^a_{\mu} + i\chi^a_{\mu}), \\
 \omega_{(3)a}^{\mu}(d) &= \frac{1}{2} (\omega^a_{\mu} - i\chi^a_{\mu})
\end{align*}

(4.8)

associated with the supergenerators $Q_{(1)a}^{\mu}$ and $Q_{(3)a}^{\mu}$ that have the $\mathbb{Z}_4$ eigenvalues $i$ and $i^3 = -i$ respectively. Details on the $\mathbb{Z}_4$–graded representation of the $osp(4|6)$ superalgebra are transferred to Appendix A. Motivation for choosing this basis for Cartan forms and $osp(4|6)$ generators is that the Lax connection of the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset sigma-model [12], [13] takes concise form there, generic to sigma-models on other supercoset manifolds with $\mathbb{Z}_4$-invariant isometry superalgebras [11], [34]-[36]. It is reasonable to assume that it plays the role of leading contribution to the Lax connection of the $AdS_4 \times \mathbb{C}P^3$ superstring viewed as a series expansion in the ‘broken’ Grassmann coordinates. Thus equations
that can be conveniently arranged as the sum of three terms. The first takes value in the linear and quadratic contributions in $\theta$ for the $\text{OSp}(U)$ supercoset sigma-model. Hence they should be added 'manually' to the equations of motion $\delta G$ does not become trivial and at the same time they cannot be derived from the $V$ where $\omega(1)_{\mu}^a \wedge \omega(1)_{\mu}^a + 2 \omega(3)_{\mu}^a \wedge \omega(3)_{\mu}^a + ... = 0,$

\[
\delta G = \partial_i (\gamma^{ij} G_i^m G_j^0 n) + 2 \gamma^{ij} G_i^m G_j^0 n + 2 \gamma^{ij} G_i^3 m \Delta_i
\]

where $V_{\pm} = \gamma^{ij} \pm \epsilon^{ij}$ are world-sheet projectors and $G^3 m (d) = \frac{1}{2} (e^n - w^n)$ are Cartan forms related to the generators $M_{3m}$ from the $\text{so}(1,3)$ stability algebra of $\text{AdS}_4$. Eqs. (4.9) represent the $\text{OSp}(4|6)/(SO(1,3) \times U(3))$ supercoset sigma-model equations [12], [13] deformed by linear and quadratic contributions in $\theta^\mu$ and its differential. Their form is rather complicated so we relegated complete expressions to Appendix B. Similarly equations of motion for the fermionic coordinates from the broken supersymmetries sector can be presented as a series in $\theta^\mu$ and its differential (see Appendix B). In the main text we only reproduce leading $\theta$-independent terms

\[
\delta G \Big|_{\delta s} = \gamma^{ij} (\Omega_{ia} \tilde{\omega}^{ia} - \Omega_{a}^i \chi_{j}^{ia} + i (\Omega^a \wedge \omega_{ia}^a + \Omega_a \wedge \omega^{ia}) + ... = 0,
\]

\[
\delta G \Big|_{\delta s} = i \gamma^{ij} (\Omega_{ia} \chi_{j}^{ia} + \Omega_i^a \chi_{j}^{ia} + \Omega^a \wedge \omega_{ia}^a - \Omega_a \wedge \omega^{ia} + ... = 0,
\]

\[
\delta G \Big|_{\delta s} = \gamma^{ij} (\Omega_{ia} \tilde{\omega}^{ia} - \Omega_{a}^i \chi_{j}^{ia} + i (\Omega^a \wedge \omega_{ia}^a + \Omega_a \wedge \omega^{ia}) + ... = 0,
\]

\[
\delta G \Big|_{\delta s} = \gamma^{ij} (\Omega_{ia} \chi_{j}^{ia} + \Omega_i^a \chi_{j}^{ia} + \Omega^a \wedge \omega_{ia}^a - \Omega_a \wedge \omega^{ia} + ... = 0,
\]

where coordinate $\xi^\mu = -\frac{i}{2} (\theta^\mu - \bar{\theta}^\mu)$ satisfies Majorana condition corresponding to the choice $s = -1$ in (3.7). Eqs. (4.10) deserve a comment. When 8 'broken' fermions are put to zero, that is the partial $\kappa$-symmetry gauge condition used to obtain the $\text{OSp}(4|6)/(SO(1,3) \times U(3))$ sigma-model Lagrangian [12], [13] from the complete one of the $\text{AdS}_4 \times \mathbb{CP}^3$ superstring [15], Eqs. (4.10) do not become trivial and at the same time they cannot be derived from the supercoset sigma-model. Hence they should be added 'manually' to the equations of motion for the $\text{OSp}(4|6)/(SO(1,3) \times U(3))$ sigma-model in complete analogy with the Virasoro constraints that complement equations derivable from the (super)string action in conformal gauge for $2d$ auxiliary metric.

Above equations of motions (4.9), (4.10) (see also Appendix B) turn to zero curvature 2-form

\[
dL - L \wedge L = 0
\]

of the Lax connection

\[
L(d) = L_{\text{so}(2,3)} + L_{\text{su}(4)} + L_F \in \mathfrak{osp}(4|6)
\]

that can be conveniently arranged as the sum of three terms. The first takes value in the $\text{so}(2,3)$ algebra

\[
L_{\text{so}(2,3)} = l^{mn}(d) M_{mn} + b^{3m}(d) M_{3m} + a^0 m(d) M_0 m + f(d) D \in \text{so}(2,3)
\]
with the 1-form coefficients
\[
\begin{align*}
l^{mn} &= G^{mn} + i \ell_2 \varepsilon^{mnk}(\ell_2 G^0_k + \ell_1 * G^0_k)(\theta \theta), \\
l^3m &= 2G^{3m}, \\
a^{\alpha m} &= 2\ell_1 G^{\alpha m} + 2\ell_2 * [G^{\alpha m} - i(d\theta \sigma^m \theta) - \frac{i}{2}\varepsilon^{mnk}G_{nk}(\theta \theta)], \\
f &= \ell_1 \Delta + \ell_2 * \Delta + 2i\ell_2 \tilde{\Omega}_a(\theta \theta).
\end{align*}
\]

The star * denotes 2d Hodge dual of a 1-form \(*a_i = \varepsilon_{ijk} \gamma^j a_k\). The second summand in (4.12) belongs to the su(4) isometry algebra of \(\mathbb{C}P^3\) manifold
\[
\mathcal{L}_{su(4)} = w\alpha_b(d)\tilde{\Omega}_b^a + w\beta_b(d)\tilde{\Omega}_a^b + y^a(d)T_a + \bar{y}_a(d)\bar{T}^a \in su(4),
\]
where
\[
\begin{align*}
w\alpha_b &= \tilde{\Omega}_b - i\ell_2 \delta_b^a(\ell_1 \Delta + \ell_2 * \Delta)(\theta \theta), \\
y^a &= \ell_1 \Omega^a + \ell_2 * \Omega^a - 2i\ell_2 \tilde{\Omega}_a(\theta \theta) - 2\ell_2 * \tilde{\chi}_a(\theta \theta) + 2\ell_2 \Omega^a(\theta \theta).
\end{align*}
\]

The last term is the linear combination of the supergenerators of osp(4|6) superalgebra divided according to their \(\mathbb{Z}_4\) eigenvalues
\[
\mathcal{L}_F = \varepsilon^{(1)}(d)Q^{(1)}_a + \varepsilon^{(1)}(d)\bar{Q}^{(1)}_a + \varepsilon^{(3)}(d)Q^{(3)}_a + \varepsilon^{(3)}(d)\bar{Q}^{(3)}_a
\]
with the 1-form coefficients equal
\[
\begin{align*}
\varepsilon^{(1)}(d) &= \ell_3 \omega^{(3)}(d) - i\ell_2 \ell_4 \Omega^a (\theta \theta) - i\ell_2 \ell_4 \Omega (\theta \theta) - \frac{i}{2}\ell_2 \ell_4 \omega^{(1)}(\theta \theta) - \frac{i}{2}\ell_2 \ell_4 \chi^a(\theta \theta), \\
\varepsilon^{(3)}(d) &= \ell_4 \omega^{(3)}(d) - i\ell_2 \ell_3 \Omega (\theta \theta) - i\ell_2 \ell_3 \Omega (\theta \theta) - \frac{i}{2}\ell_2 \ell_3 \omega^{(3)}(\theta \theta) - \frac{i}{2}\ell_2 \ell_3 \chi^a(\theta \theta). (4.18)
\end{align*}
\]

Parameters \(\ell_1, \ell_2, \ell_3\) and \(\ell_4\) are the same as those entering Lax connection of the \(OSp(4|6)/(SO(1,3) \times U(3))\) sigma-model [12, 13]. They satisfy the constraints
\[
\ell_1^2 - \ell_2^2 = \ell_3 \ell_4 = 1, \quad (\ell_1 - \ell_2)\ell_4 = \ell_3, \quad (\ell_1 + \ell_2)\ell_3 = \ell_4
\]
that can be solved to recover dependence on a single spectral parameter, e.g. as follows
\[
\ell_1 = \frac{1}{2} \left( \frac{1}{z^2} + z^2 \right), \quad \ell_2 = \frac{1}{2} \left( \frac{1}{z^2} - z^2 \right), \quad \ell_3 = z, \quad \ell_4 = \frac{1}{z}
\]
At \(z = 1\) Lax connection (4.12) reduces to the definition of Cartan forms (3.2) analogously to the supercoset Lax connection. So the Lax connection of the minimal extension of \(OSp(4|6)/(SO(1,3) \times U(3))\) sigma-model by the Majorana fermion associated with the broken part of Poincare supersymmetry includes linear and quadratic terms in \(\theta^\mu\) and its differential and appears to correspond to the part of Lax connection [20] that takes into account all 8 ‘broken’ fermions up to quadratic order. As a result of gauging away 6 of the fermionic coordinates from the broken supersymmetries sector Eq. (4.12) turns to be the complete expression for the Lax connection with its curvature strictly equal zero.

5 Conclusion

Derivation of the \(AdS_4 \times \mathbb{C}P^3\) superstring action from the \(AdS_4 \times S^7\) supermembrane [15, 20, 21] provides important ‘practical’ instance of application of the general scheme of double-dimensional reduction [17, 18]. The structure of the \(AdS_4 \times S^7\) superspace, isomorphic to
the $OSp(4|8)/(SO(1, 3) \times SO(7))$ supercoset manifold, is such that the left-invariant Cartan forms, identified with the tangent to $AdS_4$ part of $D = 11$ supervielbein, acquire non-trivial contributions $G^m_y$, proportional to the differential $dy$ of the coordinate parametrizing compactification dimension in the space-time given by $S^1$ fiber of the 7-sphere Hopf fibration $\mathbb{CP}^3 \times S^1$ [1, 5]. This essentially complicates the form of $AdS_4 \times S^7$ supervielbein and hence the $AdS_4 \times \mathbb{CP}^3$ superstring action.

That is why we have introduced KK (partial) $\kappa$—symmetry gauge as a condition of vanishing of such contributions $G^m_y$ and analyzed its consequences for the $AdS_4 \times S^7$ superbackground. For this purpose it is convenient to realize the $osp(4|8)$ isometry superalgebra of $AdS_4 \times S^7$ superbackground as $D = 3 \mathcal{N} = 8$ superconformal algebra. As a result KK gauge amounts to imposing (anti-)Majorana condition on $D = 3$ spinor coordinates $\theta^\mu$, $\bar{\theta}^\mu$ and $\eta^\mu$, $\bar{\eta}^\mu$ associated with 8 fermionic generators from the $osp(4|8)$ superalgebra corresponding to Poincare and conformal supersymmetries broken by the $AdS_4 \times \mathbb{CP}^3$ superbackground. Further simplification of the $AdS_4 \times \mathbb{CP}^3$ superstring action is attained by setting $D = 10$ dilaton field to unity that retains in the broken supersymmetries sector single (anti-)Majorana fermion associated with either Poincare or conformal supersymmetry. Among four possibilities for choosing such a $SL(2, \mathbb{R})$ spinor coordinate we have considered in detail one that corresponds to $D = 3$ Majorana fermion $\theta^\mu$ related to broken Poincare supersymmetry. This yields minimal extension of the $OSp(4|6)/(SO(1, 3) \times U(3))$ supercoset sigma-model [12, 13]. Equations of motion of the $AdS_4 \times \mathbb{CP}^3$ superstring in such a partial $\kappa$—symmetry gauge are integrable and can be obtained from the zero curvature condition for the Lax connection that includes linear and quadratic terms in $\theta^\mu$ and its differential. This connection coincides with the part of Lax connection found in [26] that takes into account all 8 ‘broken’ fermions up to quadratic order with the curvature turning to zero up to quadratic order also.

Residual $\kappa$—symmetry gauge freedom, remained upon imposing proposed gauge conditions in the broken supersymmetries sector, may be used to remove also a part of the ‘unbroken’ fermions. Then the $AdS_4 \times \mathbb{CP}^3$ superstring action, including only the gauge-fixed physical fermions, can be of use in studying the semiclassical quantization around particular solutions to the equations of motion. The point is that the results, obtained shortly after the ABJM conjecture [2] was put forward, on the one-loop corrections to spinning string energies [25] revealed mismatch with the calculations based on the conjectured Bethe equations [37]. Further study [38]-[41] have not resulted in a completely satisfactory resolution, however, more recently additional arguments have been given that support the Bethe ansatz based calculation from the stringy perturbation theory [42]-[45]. The computation of the higher-order corrections, that requires knowledge of the fermionic sector of $AdS_4 \times \mathbb{CP}^3$ superstring action beyond the quadratic order provided in [24] for a general background, might finally settle the matter.

Quite analogously to the treated in detail case of the $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-model extended by the Majorana fermion related to broken Poincare supersymmetry, it is possible to consider partial $\kappa$—symmetry gauges that single out three other $D = 3$ Majorana spinor coordinates that can be viewed as various $SL(2, \mathbb{R})$—covariant $\frac{1}{4}$ fractions of the broken supersymmetry. This introduces certain hierarchy of the broken supersymmetries that can be used to study integrability in various sectors of the $AdS_4 \times \mathbb{CP}^3$ superstring model. Namely, integrable structure associated with $\frac{1}{4}$ fractions of the broken supersymmetry is presumably described by the Lax connection of Ref. [26] quadratic in the ’broken’ fermions. Then one can examine the possibility of its extension to the case of $\frac{1}{2}$ fractions of the broken supersymmetry, when a half of the Grassmann coordinates related to broken supersymmetry is retained upon partial $\kappa$—symmetry gauge fixing. There arise 6 options. One can take either two Majorana fermions or two anti-Majorana fermions associated with the broken Poincare and conformal
supersymmetries. Both options are covered by the KK gauge (3.7). Alternatively it is possible to choose Majorana fermion related to the broken Poincare supersymmetry and anti-Majorana one related to the broken conformal supersymmetry and vice versa. Remaining two options, namely, unconstrained spinor coordinates $\theta^\mu$ and $\bar{\theta}^\mu$ or $\eta^\mu$ and $\bar{\eta}^\mu$ (cf. (3.1)) correspond to the $\kappa-$symmetry gauge conditions of Ref. [20] restricted to the sector of broken supersymmetries. If succeeded in proving integrability in these 6 subsectors, one can 'switch on' extra $\frac{1}{4}$ fraction of broken supersymmetry to examine the case of $\frac{3}{4}$ fractions and so on. This will hopefully allow to learn more on the integrable structure of the superstring beyond the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model.

6 Acknowledgements

The author is grateful to A.A. Zheltukhin for stimulating discussions.

A $osp(4|8)$ and $osp(4|6)$ superalgebras

Both orthosymplectic superalgebras share bosonic subalgebra $sp(4) \sim so(2,3)$

$$[M_{kl}, M_{mn}] = \eta_{km} M_{ln} - \eta_{kn} M_{lm} - \eta_{lm} M_{kn} + \eta_{ln} M_{km}, \quad k, l = 0', 1, 2, 3. \quad (A.1)$$

Picking out the $so(2,3)/so(1,3)$ coset generators $M_{0'\ell', m'}$, $m' = 0, 1, 2, 3$, leads to the realization of $so(2,3)$ algebra as Anti-de Sitter algebra $ads_4$

$$[M_{0'\ell'}, M_{0'\ell'}] = M_{\ell'\ell'}, \quad [M_{m'n'}, M_{0'k'}] = \eta_{n'k'} M_{0'm'} - \eta_{m'k'} M_{0'n'},$$

$$[M_{k'l'}, M_{m'n'}] = \eta_{k'n'} M_{l'm'} - \eta_{k'm'} M_{l'n'} - \eta_{l'm'} M_{k'n'} + \eta_{l'n'} M_{k'm'}. \quad (A.2)$$

To consider another useful realization of the $so(2,3)$ algebra as $D = 3$ conformal algebra introduce conformal group generators

$$D = -2 M_{0'3}, \quad P_m = M_{0'm} - M_{3m}, \quad K_m = M_{0'm} + M_{3m}, \quad m = 0, 1, 2. \quad (A.3)$$

Then Eq. (A.1) acquires the form of conf$_3$ algebra commutation relations

$$[D, P_m] = 2 P_m, \quad [D, K_m] = -2 K_m, \quad [P_m, K_n] = \eta_{mn} D + 2 M_{mn},$$

$$[M_{mn}, P_l] = \eta_{ml} P_m - \eta_{ml} P_n, \quad [M_{mn}, K_l] = \eta_{ml} K_m - \eta_{ml} K_n,$$

$$[M_{kl}, M_{mn}] = \eta_{kn} M_{lm} - \eta_{km} M_{ln} - \eta_{ln} M_{km} + \eta_{lm} M_{kn}. \quad (A.4)$$

Commutation relations of the $so(8)$ algebra, that is another bosonic subalgebra of the $osp(4|8)$ superalgebra, in the vector form read

$$[V_{IL}, V_{JK}] = \delta_{IJ} V_{LK} - \delta_{IK} V_{JL} - \delta_{IL} V_{JK} + \delta_{JK} V_{IL}, \quad I, J = 1, \ldots, 8. \quad (A.5)$$

The $so(8)$ generators admit decomposition on the $so(8)/so(7)$ coset generators $V_{8I'}$, $I' = 1, \ldots, 7$ and the generators $V_{I'J'}$ of the $so(7)$ stability group of the 7-sphere

$$[V_{8I'}, V_{8J'}] = -V_{I'J'}, \quad [V_{I'J'}, V_{8K'}] = \delta_{I'K'} V_{8J'} - \delta_{I'J'} V_{8K'},$$

$$[V_{I'J'}, V_{K'L'}] = \delta_{I'J'} V_{K'L'} - \delta_{I'K'} V_{J'L'} - \delta_{I'L'} V_{J'K'} + \delta_{I'K'} V_{J'L'}. \quad (A.6)$$
Further one can present the so(8) algebra commutation relations retaining manifest only so(6) covariance

\[ [V_{87}, V_{7I}] = V_{8I}, \quad [V_{87}, V_{8I}] = -V_{7I}, \quad [V_{7I}, V_{7J}] = [V_{8I}, V_{8J}] = -V_{IJ}, \]

\[ [V_{7I}, V_{8J}] = \delta_{IJ}V_{87}, \quad [V_{IJ}, V_{7(8)K}] = \delta_{JK}V_{7(8)I} - \delta_{IK}V_{7(8)J}, \]

\[ [V_{IJ}, V_{KL}] = \delta_{IL}V_{JK} - \delta_{IK}V_{JL} - \delta_{JL}V_{IK} + \delta_{JK}V_{IL}, \]

(A.7)

where \( V_{IJ} \) are generators of the so(6) R–symmetry subalgebra of \( osp(4|6) \) superalgebra. Generators \( V_{7I}, V_{8I} \) and \( V_{IJ} \) can be written in the form corresponding to the decomposition of \( SO(6) \) representations under \( SU(3) \) as

\[ V_A^B = \frac{i}{4} \delta^J_J A^B V^{IJ} = \left( \begin{array}{ccc} V_a^b & V_a^4 \\ V_4^a & V_4^4 \end{array} \right), \quad V_4^4 = -V_a^a, \quad a = 1, 2, 3. \]  

(A.8)

and

\[ V_{7I} = (V_{7a}, V_{7}^{a}), \quad V_{8I} = (V_{8a}, V_{8}^{a}). \]  

(A.9)

To describe the Hopf fibration realization of the 7-sphere at the level of so(8) algebra one cannot directly identify the so(8)/so(7) coset generators \( V_{8I'} = (V_{8a}, V_{8}^{a}, V_{87}) \) with the \( su(4)/u(3) \) generators from the \( \mathbb{C}P^3 \) manifold isometry algebra and \( u(1) \) generator of the \( S^1 \) fiber of \( S^7 = \mathbb{C}P^3 \times S^1 \) because the commutator of \( V_{8I} \) with itself closes on the so(6) generators rather than \( u(3) \) generators \( V_a^b \) and \( V_{87} \) does not commute with \( V_{8I} \) (see (A.7)). That is why it is necessary to make the basis change for the so(8) generators:

\[ T_a = \frac{1}{2}(V_7^a - i V_8^a), \quad T^a = -\frac{1}{2}(V_7^a + i V_8^a), \quad \widetilde{V}_a^b = V_a^b - \frac{1}{2} \delta^b_c V_c^a + \frac{1}{4} \delta^b_a V_{87}. \]  

(A.10)

Commutation relations of the generators (A.10) reproduce the \( su(4) \) isometry algebra of \( \mathbb{C}P^3 \)

\[ [T_a, T^b] = i(\tilde{V}_a^b + \delta^b_c \tilde{V}_c^a), \quad [T_a, \tilde{V}_b^c] = -i \delta_a^c T_b, \quad [T^a, \tilde{V}_b^c] = i \delta_a^c T^c, \]

\[ [\tilde{V}_a^b, \tilde{V}_c^d] = i(\delta_a^b \tilde{V}_c^d - \delta_a^d \tilde{V}_c^b) \]

(A.11)

and the \( S^1 \) generator

\[ H = V_{87} + 2V_a^a \]  

(A.12)

commutes with them. Remaining so(8) generators

\[ \tilde{T}_a = -\frac{1}{2}(V_7^a + i V_8^a), \quad \tilde{T}^a = \frac{1}{2}(V_7^a - i V_8^a) \]  

(A.13)

and \( V_4^4, V_4^a \) are associated with the coset so(8)/(su(4) \times u(1)). Their commutation relations with the \( su(4) \oplus u(1) \) generators (A.10), (A.12) can be found in Appendix B of [21].

Inverse transformation allows to express in terms of the generators (A.10), (A.12), (A.13) those of the so(8)/so(7) coset

\[ V_{8a} = i(T_a + \tilde{T}_a), \quad V_8^a = i(T^a + \tilde{T}^a), \quad V_{87} = \tilde{V}_a^a + \frac{1}{4} H \]  

(A.14)

and the so(7) stability algebra of the 7-sphere

\[ V_{7a} = T_a - \tilde{T}_a, \quad V_7^a = -T^a + \tilde{T}^a, \quad V_a^b = \tilde{V}_a^b - \frac{1}{2} \delta^b_c \tilde{V}_c^a + \frac{1}{8} \delta^b_a H. \]  

(A.15)
Among 32 fermionic generators of the \(osp(4|8)\) superalgebra, realized as \(D = 3\) \(\mathcal{N} = 8\) superconformal algebra, 24 generators \(Q^a_{\mu}, \bar{Q}^a_{\nu}\) and \(S^{\mu a}, \bar{S}^a_{\nu}\) belong to \(D = 3\) \(\mathcal{N} = 6\) superconformal algebra and satisfy the following non-trivial anticommutation relations

\[
\{Q^a_{\mu}, \bar{Q}^b_{\nu}\} = 2i\delta^a_b \sigma^m_{\mu \nu} P_m, \quad \{S^{\mu a}, \bar{S}^a_{\nu}\} = 2i\delta^a_b \sigma^{m \mu \nu} K_m, \\
\{Q^a_{\mu}, \bar{S}^b_{\nu}\} = -i\delta^a_b \delta_{\mu \nu} D + i\delta^a_b \sigma^{mn \mu \nu} M_{mn} - 2\delta^a_b (\bar{V}^a_{\mu} - \delta^a_b \bar{V}^c_c), \\
\{\bar{Q}^a_{\mu}, S^{\nu b}\} = -i\delta^a_b \delta_{\mu \nu} D + i\delta^a_b \sigma^{mn \mu \nu} M_{mn} + 2\delta^a_b (\bar{V}^b_{\mu} - \delta^b_a \bar{V}^c_c), \\
\{Q^a_{\mu}, S^{\nu b}\} = 2\delta^a_b \varepsilon_{abc} T_c, \quad \{\bar{Q}^a_{\mu}, \bar{S}^b_{\nu}\} = -2\delta^a_b \varepsilon_{abc} T_c.
\]

(A.16)

Remaining 8 generators \(Q^4_{\mu}, \bar{Q}^4_{\nu}\) and \(S^{4 a}, \bar{S}^4_{\nu}\) correspond to \(D = 10\) IIA supersymmetries broken by the \(AdS_4 \times \mathbb{CP}^3\) superbackground. Their anticommutation relations between themselves and with the above 24 generators are given in Appendix B of [21].

To study integrability of the \(AdS_4 \times \mathbb{CP}^3\) superstring it is convenient to manifestly exhibit \(\mathbb{Z}_4\)-graded structure of the \(osp(4|6)\) isometry superalgebra, in which the Lax connection takes value. Commutation relations of the \(so(2, 3)\) algebra presented in the form of \(ads_4\) algebra (A.2) already have \(\mathbb{Z}_2\)-graded structure

\[
ads_4 = g^{ads}_{(0)} \oplus g^{ads}_{(2)}, \quad g^{ads}_{(0)} = \{M_{m'm'}\}, \quad g^{ads}_{(2)} = \{M_{0'\nu m'}\}.
\]

(A.17)

The \(g^{ads}_{(0)}\) generators are invariant under the \(\mathbb{Z}_4\) automorphism \(\Upsilon\), while those from the \(g^{ads}_{(2)}\) eigenspace change the sign

\[
\Upsilon(g^{ads}_{(0)}) = i^0 g^{ads}_{(0)}, \quad \Upsilon(g^{ads}_{(2)}) = i^2 g^{ads}_{(2)}.
\]

(A.18)

Analogously \(\mathbb{Z}_2\) eigenvalues can be assigned to the \(su(4)\) generators (A.10)

\[
su(4) = g^{su(4)}_{(0)} \oplus g^{su(4)}_{(2)}, \quad g^{su(4)}_{(0)} = \{\bar{V}^b_{\mu}\}, \quad g^{su(4)}_{(2)} = \{T_a, T^a\}.
\]

(A.19)

To bring anticommutation relations (A.16) between the fermionic generators of \(osp(4|6)\) superalgebra to the \(\mathbb{Z}_4\)-graded form introduce their following combinations

\[
Q_{(1)\mu}^a = Q^a_{\mu} + i\bar{S}^a_{\mu}, \quad \bar{Q}_{(1)\mu} = \bar{Q}^a_{\mu} - i\bar{S}^a_{\mu}; \quad Q_{(3)\mu}^a = Q^a_{\mu} - iS^a_{\mu}, \quad \bar{Q}_{(3)\mu} = \bar{Q}^a_{\mu} + i\bar{S}^a_{\mu}
\]

(A.20)

that belong to the eigenspaces \(g_{(1)}\) and \(g_{(3)}\) under the automorphism \(\Upsilon\)

\[
g_{(1)} = \{Q_{(1)\mu}^a, \bar{Q}_{(1)\mu}^a\}: \quad \Upsilon(g_{(1)}) = i^1 g_{(1)}; \quad g_{(3)} = \{Q_{(3)\mu}^a, \bar{Q}_{(3)\mu}^a\}: \quad \Upsilon(g_{(3)}) = i^3 g_{(3)}.
\]

(A.21)

Then (A.16) acquires the form

\[
\{Q_{(1)\mu}^a, \bar{Q}_{(1)\nu}^b\} = 4i\delta^a_b \sigma^{mn}_{\mu \nu} M_{0'm} + 2\delta^a_b \varepsilon_{\mu \nu} D, \quad \{Q_{(3)\mu}^a, \bar{Q}_{(3)\nu}^b\} = 4i\delta^a_b \sigma^{m \mu \nu} M_{0'm} - 2\delta^a_b \varepsilon_{\mu \nu} D, \\
\{Q_{(1)\mu}^a, \bar{Q}_{(1)\nu}^b\} = -4i\varepsilon_{\mu \nu \varepsilon_{abc}} T_c, \quad \{Q_{(1)\mu}^a, \bar{Q}_{(1)\nu}^b\} = -4i\varepsilon_{\mu \nu \varepsilon_{abc}} T_c, \\
\{Q_{(3)\mu}^a, \bar{Q}_{(3)\nu}^b\} = 4i\varepsilon_{\mu \nu \varepsilon_{abc}} T_c, \quad \{Q_{(3)\mu}^a, \bar{Q}_{(3)\nu}^b\} = 4i\varepsilon_{\mu \nu \varepsilon_{abc}} T_c, \\
\{Q_{(1)\mu}^a, \bar{Q}_{(3)\nu}^b\} = -4i\delta^a_b \sigma_{\mu \nu} M_{3m} - 2\delta^a_b \sigma^{mn}_{\mu \nu} M_{mn} + 4i\varepsilon_{\mu \nu} (\bar{V}^a_{\nu} - \delta^a_b \bar{V}^c_c), \\
\{Q_{(3)\mu}^a, \bar{Q}_{(1)\nu}^b\} = -4i\delta^a_b \sigma_{\mu \nu} M_{3m} + 2\delta^a_b \sigma^{mn}_{\mu \nu} M_{mn} - 4i\varepsilon_{\mu \nu} (\bar{V}^a_{\nu} - \delta^a_b \bar{V}^c_c).
\]

(A.22)
The $Z_4$-graded representation for the commutators of $so(2,3)$ generators with the fermionic ones reads

$$
[D, Q_{(1)}^a] = Q_{(3)}^a, \quad [D, Q_{(3)}^a] = Q_{(1)}^a, \quad [D, Q_{(1)}^a] = Q_{(1,3)}^a, \quad [D, Q_{(1,3)}^a] = Q_{(1,3)}^a,
$$

$$
[M_{0^m}, Q_{(1)}^a] = \frac{i}{2} \sigma_{m\mu} Q_{(3)}^{a\nu}, \quad [M_{0^m}, Q_{(3)}^a] = -\frac{i}{2} \sigma_{m\mu} Q_{(1)}^{a\nu},
$$

$$
[M_{0^m}, Q_{(1)}^a] = -\frac{i}{2} \sigma_{m\mu} Q_{(3)}^{a\nu}, \quad [M_{0^m}, Q_{(3)}^a] = \frac{i}{2} \sigma_{m\mu} Q_{(1)}^{a\nu},
$$

$$
[M_{mn}, Q_{(1)}^a] = \frac{i}{2} \sigma_{mn\mu} Q_{(3)}^{a\nu}, \quad [M_{mn}, Q_{(3)}^a] = -\frac{i}{2} \sigma_{mn\mu} Q_{(1)}^{a\nu},
$$

$$
[M_{3m}, Q_{(1)}^a] = -\frac{i}{2} \sigma_{mn\mu} Q_{(3)}^{a\nu}, \quad [M_{3m}, Q_{(3)}^a] = \frac{i}{2} \sigma_{mn\mu} Q_{(1)}^{a\nu},
$$

$$
[M_{3m}, Q_{(1)}^a] = \frac{i}{2} \sigma_{mn\mu} Q_{(3)}^{a\nu}, \quad [M_{3m}, Q_{(3)}^a] = -\frac{i}{2} \sigma_{mn\mu} Q_{(1)}^{a\nu}.
$$

Similarly it is possible to present in the $Z_4$-graded form commutators of the $su(4)$ generators ($A.10$) with the fermionic generators ($A.20$)

$$
[T^a, Q_{(1)}^b] = i \varepsilon^{abc} Q_{(3)c}, \quad [T^a, Q_{(3)c}] = i \varepsilon^{abc} Q_{(1)c},
$$

$$
[T_a, Q_{(1)c}] = -i \varepsilon^{abc} Q_{(3)c}, \quad [T_a, Q_{(3)c}] = -i \varepsilon^{abc} Q_{(1)c},
$$

$$
[\bar{V}_a^b, Q_{(1)c}] = i \varepsilon^{abc} Q_{(3)c} - i \delta^b_a Q_{(1)c}, \quad [\bar{V}_a^b, Q_{(3)c}] = i \varepsilon^{abc} Q_{(1)c} - i \delta^b_a Q_{(3)c},
$$

$$
[\bar{V}_a^b, Q_{(1)c}] = i \varepsilon^{abc} Q_{(3)c} + i \delta^b_a Q_{(1)c}, \quad [\bar{V}_a^b, Q_{(3)c}] = -i \varepsilon^{abc} Q_{(1)c} + i \delta^b_a Q_{(3)c}.
$$

All in all, Eqs. ($A.17$), ($A.19$) and ($A.21$) correspond to the $Z_4$-graded form of the $osp(4|6)$ superalgebra

$$
osp(4|6) = g(0) \oplus g(1) \oplus g(2) \oplus g(3) : \quad \mathcal{Y}(g(k)) = i^k g(k),
$$

such that (anti)commutation relations ($A.2$), ($A.11$), ($A.22$)-($A.24$), used to calculate curvature 2-form of the Lax connection ($4.12$), can be written in concise form as

$$
[g(j), g(k)] = g(j+k \mod 4).
$$

Non-trivial commutators of the bosonic and fermionic generators from the $osp(4|8)$ superalgebra that have not been presented here can be found in Appendix B of [21].

**B  Equations of motion for the minimal extension of $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-model by the Majorana fermion associated with broken Poincare supersymmetry**

This Appendix contains complete form of the equations of motion ($4.9$) that are used (together with the Maurer-Cartan equations [28], [21]) to prove vanishing of the curvature of the Lax connection ($4.12$). They can be conveniently arranged as a series expansion in the coordinate $\theta^\mu$ and its differential $d\theta^\mu$

$$
\frac{\delta S}{\delta G^m_{\alpha \beta \nu} (\theta)} = \partial_\theta (\gamma^{ij} G_j^{0\alpha}) + 2 \gamma^{ij} G_i^{m\nu} G_j^{0 \alpha} + 2 \gamma^{ij} G_i^{3m} \Delta_j
$$

$$
+ 2i \omega_1^{(1)} \wedge \sigma_{m\mu} \omega_1^{(1)} - 2i \omega_3^{(3)} \wedge \sigma_{m\mu} \omega_3^{(3)}
$$

$$
+ \gamma^{ij} [\Omega^{a\nu}(\sigma_{m\mu} \theta) - \Omega^{a\nu}(\sigma_{m\mu} \theta)] - i [\gamma^{a\nu} \wedge (\chi_a \sigma^{m\nu} \theta) + \Omega^{a\nu} \wedge (\chi_a \sigma^{m\nu} \theta)]
$$

$$
- i \partial_\theta [\gamma^{ij} ((\partial_\theta \sigma^{m\nu} \theta) + \frac{1}{2} \sigma_{mnp} G_{j^{np}} (\theta))] - 2i \gamma^{ij} G_{i}^{m} [(\partial_\theta \sigma_{m\nu} \theta)]
$$

$$
+ \frac{i}{2} \varepsilon_{npr} G_j^{3(p} (\theta) + \gamma^{ij} [\chi_{ia} \sigma^{m\nu} \omega_j^{a} - (\omega_{ia} \sigma^{m\nu} \chi_j^{a}) (\theta)] - 4i \tilde{\Omega}_{a}^{a} \wedge G^{3m} (\theta) = 0;
$$

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\[
\frac{\delta S}{\delta \Delta (\delta)} = \frac{1}{2} \partial_i (\gamma^{ij} \Delta_j) - 2 \gamma^{ij} G_i^m G_j^n \omega_m \omega_n - \frac{1}{2} \gamma^{ij} \omega_i \omega_j - 2 \epsilon \gamma^{ij} \frac{\partial}{\partial \Delta_j} \epsilon \gamma^{ij} \frac{\partial}{\partial \Delta_j} + 4 \Omega_a \mu \chi_a \chi_a \mu + \frac{1}{2} \gamma^{ij} \Delta_j \gamma^{ij} \omega_i \omega_j = 0;
\]  
(B.2)

\[
\frac{\delta S}{\delta \Omega_a (\delta)} = \frac{1}{2} \partial_i (\gamma^{ij} \Omega_{ja}^b) + 2 \gamma^{ij} \Omega_{ia}^b - \frac{1}{2} \gamma^{ij} \epsilon \gamma^{ij} \sigma_{m} \sigma_{m} - \frac{1}{2} \gamma^{ij} \epsilon \gamma^{ij} \Delta_j \partial_i \epsilon \gamma^{ij} \Delta_j \partial_i + 2 \epsilon \gamma^{ij} \epsilon \gamma^{ij} \Omega_{ia}^b - \chi_a \chi_a \mu + \chi_a \chi_a \mu = 0;
\]  
(B.3)

\[
\frac{\delta S}{\delta \omega_1 (\delta)} = \frac{1}{2} \partial_i (\gamma^{ij} \omega_1 (\delta)) + 2 \gamma^{ij} \omega_1 (\delta) - \frac{1}{2} \gamma^{ij} \epsilon \gamma^{ij} \sigma_{m} \sigma_{m} - \frac{1}{2} \gamma^{ij} \epsilon \gamma^{ij} \Omega_{ia}^b = 0;
\]  
(B.4)

In a similar form one can present fermionic equations from the broken supersymmetries sector

\[
\frac{\delta S}{\delta \Omega_a (\delta)} = \frac{1}{2} \partial_i (\gamma^{ij} \Delta_j) - 2 \gamma^{ij} G_i^m G_j^n \omega_m \omega_n + \frac{1}{2} \gamma^{ij} \omega_i \omega_j + \frac{1}{2} \gamma^{ij} \omega_i \omega_j + \frac{1}{2} \gamma^{ij} \omega_i \omega_j = 0;
\]  
(B.6)
$$\frac{\delta S}{\delta \sigma^\mu} = i \gamma^{ij} (\Omega^a_i \tilde{\chi}_j^\mu + \Omega^a_i \chi_j^\mu) + \Omega^a_i \omega^\mu - \Omega_i \wedge \tilde{\omega}^{\mu a}$$
$$- 4 i \gamma^{ij} \tilde{\Omega}^a_i G_{j}^{0m} \tilde{\sigma}_m^\mu \theta^\mu + 2 i \gamma^{ij}(\chi_{ia_l} \tilde{\chi}_{l}^\nu + \chi_{ia_l} \tilde{\chi}_{l}^\nu) \theta^\nu$$
$$- 2 i G^{0m} \wedge G^{3n} \theta^\nu \sigma_{mn}^\mu - 2 i G^{0m} \wedge G^{m} \theta^\nu$$
$$- 2 i \Delta \wedge (d \theta^\mu + \frac{1}{2} G^{0m} \theta^\nu \sigma_{mn}^\mu) - 2 (\omega_a^\mu \wedge \tilde{\chi}^\nu + \chi_a^\nu \wedge \tilde{\omega}^{\mu a}) \theta^\nu$$
$$+ i (\Omega^a_i \chi_a^\mu - \Omega_i \wedge \tilde{\chi}^{\mu a}) (\theta^\mu)$$
$$+ 6 \gamma^{ij} \tilde{\Omega}^a_i \sigma_{ij} \theta^\mu (\theta^\mu) - 4 G^{0m} \wedge \tilde{\sigma}_m^\mu \theta^\nu (\theta^\mu) + c^m \wedge \tilde{\sigma}_m^\mu \theta^\nu (\theta^\mu) = 0;$$

$$\frac{\delta S}{\delta \sigma^\mu} + \frac{\delta S}{\delta \sigma^\nu} = \gamma^{ij} (\Omega^a_i \omega_{ja}^\mu + \Omega_{ia} \tilde{\omega}^a_j^\mu) + i (\Omega^a_i \chi_a^\mu - \Omega_i \wedge \tilde{\chi}^{\mu a})$$
$$+ 4 \gamma^{ij} \Delta (\tilde{\Omega}^a_i a \theta^\mu + 2 \gamma^{ij} (\omega_{ia_l} \tilde{\chi}_{l}^\nu + \chi_{ia_l} \tilde{\chi}_{l}^\nu) \theta^\nu$$
$$- 2 G^{0m} \wedge \tilde{\sigma}_m^\mu (d \theta^\mu - \frac{1}{2} G^{kl} \sigma_{kl}^\lambda \theta^\lambda) + 2 \Delta \wedge G^{3m} \tilde{\sigma}_m^\mu \theta^\nu - 2 i (\chi_{ia}^\nu + \chi_{ia}^\nu \wedge \tilde{\omega}_{ja}^{\mu a}) \theta^\nu$$
$$+ i \gamma^{ij} (\Omega^a_i \chi_a^\mu + \Omega_{ia} \tilde{\chi}_j^{\mu a}) (\theta^\mu)$$
$$- 2 i d \theta^\mu \wedge (d \theta^\mu) + \frac{1}{2} \epsilon_{lmn} G_{mn} \wedge \tilde{\sigma}_i^\mu \theta^\nu (\theta^\mu) - 3 i \Delta \wedge d \theta^\mu (\theta^\mu) = 0.$$
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