Closeness Centrality Algorithms For Multilayer Networks

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Abstract. Centrality measures for simple graphs are well-defined and several main-memory algorithms exist for each. Simple graphs are not adequate for modeling complex data sets with multiple entities and relationships. Multilayer networks (MLNs) have been shown to be better suited, but there are very few algorithms for centrality computation directly on MLNs. They are converted (aggregated or collapsed) to simple graphs using Boolean AND or OR operators to compute centrality, which is not only inefficient, but incurs loss of structure and semantics.

In this paper, we propose algorithms that compute closeness centrality on a MLN directly using a novel decoupling-based approach. Individual results of layers (or simple graphs) of a MLN are used and a composition function developed to compute the centrality for the MLN. The challenge is to do this accurately and efficiently. However, since these algorithms do not have complete information of the MLN, computing a global measure such as closeness centrality is a challenge. Hence, these algorithms rely on heuristics derived from intuition. The advantage is that this approach lends itself to parallelism and is more efficient compared to the traditional approach. We present two heuristics for composition and experimentally validate – accuracy and efficiency – on a large number of synthetic and real-world graphs with diverse characteristics.

Keywords: Multilayer Networks · Heuristics-Based · Closeness Centrality · Decoupling Approach · Accuracy and Efficiency.

1 Introduction

Closeness centrality measure, a global graph characteristic, defines the importance of a node in a graph with respect to its distance from all other nodes. Different centrality measures have been defined, local and global, such as degree centrality [4], closeness centrality [7], eigenvector centrality [22], stress centrality [21], betweenness centrality [3], harmonic centrality [2], and PageRank centrality [10]. Closeness centrality defines the importance of a node based on how close it is to all other nodes in the graph. Closeness centrality can be used to identify nodes from which communication with all other nodes in the network can be accomplished in least number of hops. Most of the centrality measures are defined for simple graphs or monographs. Only page rank centrality has been extended to multi-layer networks [9,13].
A multilayer network consists of layers, where each layer is a simple graph consisting of nodes (entities) and edges (relationships) and optionally connected to other layers through inter-layer edges. If one were to model the three social networks Facebook, LinkedIn, and Twitter, a MLN is a better model as there are multiple edges between two nodes (see Figure 1). This type of MLN is categorized as homogeneous MLNs (or HoMLNs) as the entities in each layer is the same, but relationships in each layer are different. It is also possible to have MLNs where each layer has different types of entities and relationships within and across layers. Modeling the DBLP data set [1] with authors, papers, and conferences need this type of heterogeneous MLNs (or HeMLNs) [15].

This paper presents two heuristic-based algorithms for computing closeness centrality nodes (or CC nodes) on HoMLNs with high accuracy and efficiency. The challenge is in computing a global measure using partitioned graphs (layers in our case) and composing them with minimal additional information to compute a global measure for the combined layers. Boolean AND composition of layers is used for ground truth in this paper (OR is another alternative.) Our MLN algorithms use the decoupling-based approach proposed in [18,19]. Based on this approach, we compute centrality on each layer once and keep minimal additional information from each layer for composing. With this, we can efficiently estimate the CC nodes of the MLN. This approach has been shown to be application independent, efficient, lends itself to parallel processing (of each layer), and is flexible for computing centrality measure on any subset of layers.

1.1 Contributions and Paper Outline

Contributions of this paper are:

– Algorithms for computing closeness centrality nodes of MLNs
– Two heuristics to improve accuracy and efficiency of computed results
– Decoupling-based approach to preserve structure and semantics of MLNs
– Experimental analysis on large number of synthetic and real-world graphs with diverse characteristics.
– Accuracy and Efficiency comparisons with ground truth and naive approach

The rest of the paper is organized as follows: Section 2 discusses related work. Section 3 introduces the decoupling approach for MLN analysis. Section 4 provides the challenges of decoupling-based approach for a global metric. Section 5 describes proposed heuristics for computing closeness centrality of a MLN. Section 6.1 describes the experimental setup and the data sets. Section 6.2 discusses result analysis followed by conclusions in Section 7.
2 Related Work

Due to the rise in popularity and availability of complex and large real-world data sets, there is a critical need for modeling them as graphs and analyzing them in different ways. The centrality measure of MLN provides insight into different aspects of the network. Though there have been a plethora of studies in centrality detection for simple graphs, not many studies have been done on detecting central entities in multilayer networks. Existing studies conducted on detecting central entities in multilayer networks are use-case specific and no common framework exists which can be used to address the issue of detecting central entities in a multilayer network.

In [7], proposes an approach to find top k closeness centrality nodes in large graphs. This approximation-based approach has higher efficiency under certain circumstances. Even though the algorithm works on large graphs, unfortunately, it does not work in the case of multi-layer networks.

In [9], authors capitalize on the tensor formalism, recently proposed to characterize and investigate complex topologies, to show how closeness centrality and a few other popular centrality measures can be extended to multiplexes. The authors in this study [23] also rely on tensor formalism to investigate and analyze complex multilayer networks. They also extend the random walk closeness centrality to identify nodes in a multilayer network.

In [8], authors propose a sampling-based approach to estimate the closeness centrality nodes with acceptable accuracy. The proposed approach takes linear time and space but it is a main memory-based algorithm. The proposed approach works on both undirected and directed graphs. The authors of [20] propose an incremental algorithm that can dynamically update the closeness centrality nodes of a graph in case of edge insertion and deletion. The algorithm has a lower memory footprint compared to traditional closeness centrality algorithms and provides massive speedup when tested in real-world graph data sets. In [10], the authors propose closeness centrality algorithms where they use effective distance instead of the conventional geographic distance and binary distance obtained by Dijkstra’s shortest path algorithm. This approach works on directed, undirected, weighted, and unweighted graphs. In [17], authors propose an approach to compute the exact closeness centrality values of all nodes in dynamically evolving directed and weighted networks. The proposed approach is parallelizable and achieves a speedup of up to 33 times.

Most of the methods to calculate closeness centrality are main memory-based and not suitable for large graphs. In [19], the authors propose a decoupling-based approach where each layer can be analyzed independently and in parallel and calculate graph properties for a HoMLN using the information obtained for each layer. Our proposed algorithms are based on the network decoupling approach which has been shown to be efficient, flexible, scalable as well as accurate.
3 The Decoupling Approach For MLNs

Most of the algorithms available to analyze simple graphs for centrality, community, and substructure detection cannot be used for MLN analysis directly. There have been some studies that extend existing algorithms for centrality detection (e.g., page rank) to MLNs \[9\], but they try to \textbf{work on the MLN as a whole.} The network decoupling approach \[18,19\] used in this paper not only uses extant algorithms for simple graphs, but also uses a partitioning approach for efficiency, flexibility, and new algorithm development.

Briefly, existing approaches for multilayer network analysis convert or transform a MLN into a single graph. This is done either by aggregating or projecting the network layers into a single graph. For homogeneous MLNs, edge aggregation is used to aggregate the network into a single graph. Although aggregation of a MLN into a single graph allows one to use extant algorithms (and there are many of them), due to aggregation, \textbf{structure and semantics of the MLN is not preserved resulting in information loss.}

The network decoupling approach is shown in Figure 2. It consists of identifying two functions: one for analysis(\(\Psi\)) and one for composition(\(\Theta\)). Using the analysis function, each layer is analyzed independently (and in parallel). The results (which are termed partial from the MLN perspective) from each of the two layers are then combined using a composition function/algorithm to produce the results for the two layers of the HoMLN. This binary composition can applied to MLNs with more than two layers. Independent analysis allows one to use existing algorithms on smaller graphs. The decoupling approach, however, adds efficiency, flexibility, and scalability.

![Fig. 2: Overview of the network decoupling approach](image)

The network decoupling method preserves structure and semantics which is critical for drill-down and visualization of results. As each layer is analyzed independently, the analysis can be done in parallel reducing overall response time. Due to the MLN model, each layer (or graph) is likely to be smaller, requires less memory than the entire MLN, and provides clarity. The results of the analysis functions are saved and used for the composition. Each layer is analyzed only once. Typically, the composition function is less complex and is quite efficient as we shall show. Any of the existing simple graph centrality algorithms can be used for the analysis of individual layers. \textit{Also, this approach is application-independent.}
When compared to similar single network approaches, achieving high accuracy with a decoupling approach is the challenge, especially for global measures. While analyzing one layer, identifying minimal additional information needed for improving accuracy due to composition is the main challenge. For many algorithms we have investigated, it is a trade off between additional information used and accuracy gained.

4 Benefits and Challenges of Decoupling-Based Approach

For analyzing MLNs, currently, HoMLNs are converted into a single graph using aggregation approaches. Given two vertices \( u \) and \( v \), the edges between them are aggregated into a single graph. The presence of an edge between vertices \( u \) and \( v \) depends on the aggregation function used. In Boolean AND composed layers, if an edge is present between the same vertex pairs \( u \) and \( v \) in both layers, then it will be present in the AND composed layer. Similarly, in Boolean OR composed layers, if an edge is present between the same vertex pairs \( u \) and \( v \) in one of the layers in HoMLN, then the edge will be present in the OR composed layer. In the case of HoMLN with weighted edges, linear functions can be used for aggregation.

Both HoMLNs and HeMLNs are a set of layers of single graphs. Hence, the MLN model provides a natural partitioning of a large graph into layers of a MLN. The layer-wise analysis as the basis of the decoupling approach has several benefits. First, the entire network need not be loaded into memory, only a smaller layer. Second, the analysis of the individual layers can be parallelized decreasing the total response time of the algorithm. Finally, the computation used in the composition function (\( \Theta \)) is based on intuition which is embedded into the heuristic and requires significantly less computation than \( \Psi \).

When analyzing a MLN, the accuracy depends on the information we are keeping (in addition to the output) during the analysis of individual layers. In terms of centrality measures, the bare minimum information we can keep from each layer is the high centrality nodes of that layer along with their centrality values. Retaining the minimal information, local centrality measures such as degree centrality can be calculated relatively easily with high accuracy. However, in calculating global measures, such as closeness centrality nodes require information of the entire MLN. This compounds the difficulty of computation of closeness centrality of a MLN in the decoupling approach as the change in the information used for estimation of the result will greatly impact the accuracy of the result. Identification of the useful minimal information and the intuition behind that are the primary challenges.

5 Closeness Centrality Computation

The closeness centrality value of a node \( v \) describes how far are the other nodes in the network from \( v \) or how fast or efficiently a node can spread information through the network. For example, when an internet service provider considers
choosing a new geo-location for their servers, they might consider a city that is geographically closer to most cities in the region. An airline is interested in identifying a city for their hub that connects to other important cities with a minimum number of hops or layovers. For both, computing the closeness centrality of the network is the answer.

The closeness centrality score/value of a vertex \( u \) in a network is defined as,

\[
CC(u) = \frac{n - 1}{\sum_v d(u, v)}
\]

where \( n \) is the total number of nodes and \( d(u, v) \) is the shortest distance from node \( u \) to some other node \( v \) in the network. The higher the closeness centrality score of a node, the more closely (distance-wise) that node is connected to every other node in the network. Nodes with a closeness centrality score higher than the average are considered closeness centrality nodes (or CC nodes). This definition of closeness centrality is only defined for graphs with a single connected component. The closeness centrality is defined for both directed and undirected graphs. In this paper, for an algorithm based on the decoupling approach, we focus on the problem of finding high (same or above average) closeness centrality nodes of Boolean AND aggregated layers of a MLN for undirected graphs. Even though closeness centrality is not well defined for graphs with multiple connected components, our heuristics work for networks where each layer could consist of multiple connected components or the AND aggregated layer has multiple connected components. The proposed heuristics consider the normalized closeness centrality values over the connected component in the layers [25].

We propose two heuristic-based algorithms for computing CC nodes for Boolean AND aggregated layers using the decoupled approach. We test the accuracy and performance of our algorithms against the ground truth. We have done extensive experiments on data sets with varying graph characteristics to show that our solutions work for any graph and has much better accuracy than the naive approach. Also, the efficiency of our algorithm is significantly better than ground truth computation.

For closeness centrality discussed in this paper, the ground truth is calculated as follows: i) Two layers of the MLN are aggregated into a single graph using the Boolean AND operator and ii) Closeness centrality nodes of the aggregated graph are calculated using an existing algorithm. The same algorithm is also used on the layers for calculating CC nodes of each layer.

For finding the ground truth CC nodes and identifying the CC nodes in the layers, we use the NetworkX package’s [12] implementation of closeness centrality.
The implementation of the closeness centrality algorithm in this package uses breadth-first search (BFS) to find the distance from each vertex to every other vertex. For disconnected graphs, if a node is unreachable, a distance of 0 is assumed and finally, the obtained scores are normalized using Wasserman and Faust approximation \cite{25} which prioritizes the closeness centrality score of vertices in larger connected components. For a graph with $V$ vertices and $E$ edges, the time complexity of the algorithm is $O(V(V + E))$.

For two-layer networks, the naive composition (as a base composition algorithm for comparison) amounts to taking the CC nodes of each layer (using the same algorithm used for the ground truth), followed by their intersection. The resultant set of nodes will be the estimated CC nodes of the MLN layers using the composition approach. The naive approach is the simplest form of composition (using the decoupling approach) and does not use any additional information other than the CC nodes from each layer. Our hypothesis is, the naive approach is going to perform poorly when the topology of the two layers are very different. If both layers are have similar topology, the naive approach will give accuracy close to the ground truth. Comparing against the naive approach will give us an idea of how well our heuristic performs as well as how much additional information is needed for the heuristic to gain accuracy.

We compare the CC nodes computed by our heuristic-based composition algorithm against the ground truth for accuracy. We use Jaccard’s coefficient as the measure to compare the accuracy of our solutions with the ground truth. We have also used precision and recall as evaluation metrics to compare the accuracy of our solutions (results not shown due to space constraints.)

Our goal is to design algorithms using the network decoupling approach so that our accuracy is significantly better than the naive approach, close to the ground truth, and the performance better than the ground truth computation. Even though our solution can be extended to MLNs with any number of layers, for our experiments we use two-layer networks in this initial work.

For performance, we compare our solution’s execution time against the execution time of the ground truth. The ground truth execution time is computed as: time required to aggregate the layers using AND composition function + time required to identify the CC nodes on the combined graph. The time required for our algorithm using the decoupling approach is: max(layer 1, layer 2 analysis times) + composition time.

For any two layers, the naive approach can be thought of as the approach with the lowest possible accuracy. The naive approach does not keep any additional information from the layers apart from the CC nodes. The accuracy of the naive approach can be improved by keeping and using additional information from the layers. The challenge is to identify the minimum amount of information to gain the highest possible accuracy over the naive approach.

\subsection{Closeness Centrality Heuristic 1 (CC1)}

Intuitively, CC nodes in single graphs have high degrees (more paths go through it and likely more shortest paths) or have neighbors with high degrees (similar
reasoning.) In the ANDed graph (ground truth graph), CC nodes that are common among both layers have a high chance of becoming a CC node. Moreover, if a common CC node has high overlap of neighborhood nodes from both layers with above average degree and low average sum of shortest path (SP) distances, there is a high chance of that node becoming a CC node in the ANDed graph. Using this intuition and observation, we propose heuristic 1 for identifying CC nodes for composing two layers.

As we discussed earlier, in the decoupling approach we use the analysis function $\Psi$ to analyze the layers once and use the partial results and additional information in the composition function $\Theta$ to obtain intermediate/final results. For $CC_1$, after the analysis phase ($\Psi$) on each layer (say, $x$) we maintain for each node (say, $u$), its degree ($deg_x(u)$) and sum of shortest path distances ($sumDist_x(u)$), and the one-hop neighbors ($NBD_x(u)$), if $u$ is a CC node. When calculating $sumDist_x(u)$, if a node $v$ is unreachable, we consider the distance $dist(u,v) = n$ where $n$ is the number of vertices in the layer (same in each layer – HoMLN).

In the composition phase ($\Theta$), for each vertex $u$ in each layer (say, $x$), we calculate the $degDistRatio_x(u)$ as $\frac{sumDist_x(u)}{\min(deg_x(u), deg_y(u))}$, where $y$ is the second layer, to estimate the likelihood of a node to be a CC node in the ANDed graph. A smaller value of this ratio (i.e. smaller sum of distances and/or higher degree) means the vertex $u$ has a higher chance of becoming a CC node in the ANDed graph. When estimating the $degDistRatio_x(u)$, we assume no change in sum of SP distances for vertex $u$ in layer $x$, but for degree of node $u$, we assume it is going to be $\min(deg_x(u), deg_y(u))$, which is the maximum possible degree of node $u$ in the ANDed graph. Instead of using the degree of the nodes in each layer, using the estimated degree of ground truth graph gives us a better approximation of the ratio value of the sum of distance and degree for the ground truth graph. Due to decrease in the edges in the ground truth graph, the average sum of distances will increase (as on an average paths will get longer). As a result we estimate $avgDegDistRatio_{x\text{AND}y}$ as $\max(avgDegDistRatio_x, avgDegDistRatio_y)$. For each CC node in each layer, we find out the set of central one-hop nodes, that is the ones that have the $degDistRatio$ less than the $avgDegDistRatio_{x\text{AND}y}$. Finally, those common CC nodes from the two layers, which have a significant overlap among the central one-hop neighbors are identified as the CC nodes of the ANDed graph. Due to space constraints, the algorithm is not shown.

The complexity of the composition algorithm is dependent on the final step where the overlaps of CC nodes and their one-hop neighborhoods is considered. The algorithm will have a worst case complexity of $O(V^2)$, if both layers are complete graphs consisting of $V$ vertices. Based on the wide variety of data sets used in experiments, we believe that the composition algorithm will have an average case complexity of $O(V)$.

The closeness centrality heuristic (or $CC_1$) discussed in this section has a couple of drawbacks. First, the composition time is quite high for large graphs. Second, keeping one-hop neighbors of the CC nodes of both layers is significant amount of additional information. The figure shows the accuracy of $CC_1$ compared
against naive approach on a subset of the synthetic data set 1 shown in Table 1. Figure 4b shows the maximum composition time against the minimum analysis time of the layers (worst case scenario). Even though the composition time takes less time than the time it takes for the analysis of layers (one-time cost), we want to further reduce the composition time and the additional information necessary for the composition without making any major sacrifice to accuracy. As a result, we developed Closeness Centrality Heuristic 2 or \( \text{CC2} \), which keeps less information than \( \text{CC1} \), has faster composition time, and provides better accuracy (or same) than \( \text{CC1} \).

### 5.2 Closeness Centrality Heuristic 2 (CC2)

For heuristic \( \text{CC1} \), if estimated value of \( \text{avgDegDistRatio}_{x \land y} \) is larger than the actual average degree-distance ratio of the AND composed layer, it will include a lot of false positives. The composition step of \( \text{CC1} \) is also computationally expensive and requires a lot of additional information. Furthermore, \( \text{CC1} \) cannot identify all CC nodes of the ground truth graph. We design closeness centrality heuristic 2 or \( \text{CC2} \) to address these issues.

The design of \( \text{CC2} \) is based on estimating the sum of shortest path (SP) distances for vertices in the ground truth graph. If we know the sum of SP distances of a vertex in the individual layers, we can estimate the upper and lower limit of the sum of SP distances for that vertex in the ground truth graph. This idea can be intuitively verified. Let the sum of SP distances for vertex \( u \) in layer \( x \) (\( y \)) be \( \text{sumDist}_x(u) (\text{sumDist}_y(u)) \). Let the set of layer \( x \) (\( y \)) edges be \( e_x (e_y) \). In the ground truth graph, the upper bound for the sum of SP distances of a vertex \( u \) is going to be \( \infty \), if the vertex is disjoint in any of the layers. If \( e_x \cap e_y = e_x \), the sum of SP distances for any vertex \( u \) in the ground truth graph is going to be \( \text{sumDist}_x(u) \). If \( e_x \subseteq e_y \), then the ground truth graph will have
the same edges as layer $x$ and sum of SP distances for a vertex $u$ in the ground truth graph will be same as the sum of SP distances for that vertex in layer $x$. When $e_x \cap e_y = e_x$, for any vertex $u$, $\text{sumDist}_x(u) \geq \text{sumDist}_y(u)$ because layer $x$ will have less edges than layer $y$ and average length of SPs in layer $x$ will be higher than average length of SPs in layer $y$. Similarly, when $e_x \cap e_y = e_y$, the sum of SP distances for any vertex $u$ in the ANDed graph is going to be $\text{sumDist}_y(u)$ and $\text{sumDist}_y(u) \geq \text{sumDist}_x(u)$. From the above discussion, we can say the sum of SP distances of a vertex $u$ in the ANDed graph is between $\max(\text{sumDist}_x(u), \text{sumDist}_y(u))$ and $\infty$.

\begin{algorithm}
\begin{algorithmic}
\For{$u \in X.\text{nodes}$}
\State $\text{estSumDist}_{x \text{AND} y}(u) \leftarrow \max(\text{sumDist}_x(u), \text{sumDist}_y(u))$
\EndFor
\State Calculate $CH'_{x \text{AND} y}$ using $\text{estSumDist}_{x \text{AND} y}$
\end{algorithmic}
\caption{Procedure for Heuristic CC2}
\end{algorithm}

Algorithm 1 shows the composition step for the heuristic CC2. In this composition function, we assume the estimated sum of SP distances of vertex $u$ is $\text{estSumDist}_{x \text{AND} y}(u)$. $\text{sumDist}_x(u)$ and $\text{sumDist}_y(u)$ are the sum of SP of node $u$ in layer $x$ and layer $y$ respectively and $CH'_{x \text{AND} y}$ is the set of CC nodes of the ANDed layer. Figure 5 shows an example of how the composition function is applied on a HoMLN with two layers. For each node $u$, we estimate the sum of SP which is the maximum sum of SP of node among the two layers. In the example, node $A$ has a sum of SP as 9 in Layer-1 and 7 in layer-2. The estimated sum of SP of node $A$ is going to be 9 in the ANDed layer. Similarly, we can estimate the sum of SP of other vertices. Once the estimated sum of SP of all the vertices in the ANDed graph is completed, we can use Equation 1 to calculate the CC values of the nodes in the ANDed layer. As we calculate the
CC value for the ANDed layer using the estimated sum of SP, we can either take the CC nodes with above-average closeness centrality scores or take the top-k CC nodes. For an MLN with V nodes in each layer, the worst-case complexity of the composition algorithm for CC2 is \( O(V) \).

6 Experimental Analysis of Algorithms

6.1 Data Sets and Environment

Our implementation is in Python using the NetworkX [12] package. The experiments were run on SDSC Expanse [24] with single-node configuration. Each node in the expanse has an AMD EPYC 7742 CPU with 128 cores and 256GB of memory running the CentOS Linux operating system.

For the evaluation of the proposed approaches, both synthetic and real-world-like data sets were used. The synthetic data sets were generated using PaRMAT [14], a parallel version of the popular graph generator RMAT [6] which uses the Recursive-Matrix-based graph generation technique.

For diverse experimentation, for each base graph, we generate 3 sets of synthetic data sets using PaRMAT. Our generated synthetic data set consists of 27 HoMLNs with 2 layers with varying edge distribution for the layers. The base graphs start with 50K vertices with 250K edges and go up to 150K vertices and 3 million edges. The summary of synthetic data set-1 is shown in Table 1:

| Base Graph | G_{id} | Edge Dist. % | #Edges |
|------------|--------|--------------|--------|
| #Nodes, #Edges | in Layers | L1 | L2 | L1 AND L2 |
| 50KV, 250KE | 1 | 70,30 | 224976 | 124988 | 50319 |
| 2 | 60,40 | 149982 | 199983 | 50392 |
| 3 | 50,50 | 174980 | 174977 | 50422 |
| 50KV, 500KE | 4 | 70,30 | 399962 | 199986 | 51374 |
| 5 | 60,40 | 249978 | 349954 | 51458 |
| 6 | 50,50 | 299971 | 299960 | 51541 |
| 50KV, 1ME | 7 | 70,30 | 349955 | 749892 | 55158 |
| 8 | 60,40 | 649918 | 449935 | 55647 |
| 9 | 50,50 | 549933 | 549922 | 55896 |
| 100KV, 500KE | 10 | 70,30 | 249989 | 449970 | 100412 |
| 11 | 60,40 | 299986 | 399978 | 100494 |
| 12 | 50,50 | 349983 | 349981 | 100493 |
| 100KV, 1ME | 13 | 70,30 | 799937 | 399978 | 101695 |
| 14 | 60,40 | 699948 | 499969 | 101822 |
| 15 | 50,50 | 599958 | 599964 | 101998 |
| 100KV, 2ME | 16 | 70,30 | 699949 | 1499899 | 106389 |
| 17 | 60,40 | 1299914 | 899926 | 107141 |
| 18 | 50,50 | 1099924 | 1099923 | 107785 |
| 150KV, 750KE | 19 | 70,30 | 674971 | 374979 | 150398 |
| 20 | 60,40 | 449982 | 599970 | 150447 |
| 21 | 50,50 | 524978 | 524975 | 150475 |
| 150KV, 1.5ME | 22 | 70,30 | 1199942 | 599978 | 151684 |
| 23 | 60,40 | 749696 | 1049954 | 151883 |
| 24 | 50,50 | 899950 | 899956 | 152005 |
| 150KV, 3ME | 25 | 70,30 | 1049951 | 2249888 | 156501 |
| 26 | 60,40 | 1349920 | 1949906 | 157295 |
| 27 | 50,50 | 1649922 | 1649909 | 157602 |
In the first synthetic data set, one layer (L1) follows power-law degree distribution and the other one (L2) follows normal degree distribution. In the second synthetic data set, both HoMLNs layers have power-law degree distribution. In the final synthetic data set, both layers have normal degree distribution. For each of these, 3 edge distributions (70, 30; 60, 40; and 50, 50) are used for a total of 81 HoMLNs of varying edge distributions, number of nodes and edges for experimentation.

Table 1 shows the details of the different 2-layer MLNs from the first synthetic data set (L1: power-law, L2: normal) used in our experiments. The other two synthetic data sets have similar node and edge distributions.

For our real-world-like data set, the network layers are generated from real-world like monographs using a random number generator. The real-world-like graphs are generated using RMAT with parameters to mimic real world graph data sets as discussed in [5]. As a result, the graphs are not a single connected components and neither are their ground truth graph.

6.2 Result Analysis and Discussion

In this section, we present the results of our experiments. We have tested our two proposed heuristics over synthetic and real-world-like data sets with diverse characteristics. As a measure of accuracy, we use Jaccard’s coefficient. As a performance measure, we compare the time taken by the decoupling approach with the time taken to compute the ground truth (as defined earlier in Section 5.) In addition, we also highlight the significance of the decoupling approach by comparing the maximum composition time of our algorithms with the minimum analysis time of the layers. We compare the accuracy of our algorithms against the naive approach that serves as the baseline for comparison.

Figure 6 illustrates the accuracy of both the heuristics and the naive approach against the ground truth for the synthetic data set-1. As we can see, the accuracy of the heuristics is better than the naive approach in all cases. In most cases, CC2 performs better than CC1. The accuracy of CC1 increases with the graph density. Similar trend has been observed in other synthetic data sets as well, where our proposed CC1 and CC2 heuristics perform better than the naive approach.

| Data Set   | Degree Distribution | Mean Accuracy |
|------------|---------------------|---------------|
|            |                     | CC1 | CC2 | CC1 vs. Naive | CC2 vs. Naive |
| Synthetic-1| Power law, Normal   | 43.56% | 46.77% | +52.57% | +63.83% |
| Synthetic-2| Power law, Power law| 55.95% | 55.20% | +9.77% | +8.30% |
| Synthetic-3| Normal, Normal      | 48.87% | 50.90% | +47.55% | +53.65% |

We could not run graph sizes larger than this on a single node due to the number of hours allowed and other limitations of the XSEDE environment.
Table 2 shows the mean accuracy and average percentage gain in accuracy over the naive approach for the synthetic data sets. For all synthetic data sets, our heuristics significantly outperform the naive approach as shown. Only when both layers have power-law degree distribution, the naive approach has an accuracy which is closer to our algorithms.

Figure 7 shows the accuracy of the algorithms on real-world-like data sets (distributions mimic real world networks[5]). Across all data sets, both our heuristics have more than 80% accuracy. Accuracy of our heuristics does not go below the naive approach even for disconnected graphs.
The ground truth graph obtained from Boolean AND operation on layers of HoMLN will always have same or less number of edges than the individual layers as an edge will appear in the ground truth graph only if it is connected between the same nodes in both the layers. The package we are using, NetworkX \cite{12} uses BFS to calculate the summation of distances from a node to every other node while calculating the normalized closeness centrality of the nodes. As the complexity of BFS depends on the number of vertices and edges in a graph, our ground truth will always require same or less time than the analysis time for the largest layer. Although the sum of the analysis time of the layers may be more than that of the ground truth, we need to only consider the maximum analysis time of layers as they can be done in parallel. Further, our composition time is drastically less than the analysis time of any layer. Hence, we compare the minimum analysis time for layers with the maximum composition to show the worst case scenario. As can be seen from Figure \ref{fig:worst-case-performance-comparison}, the maximum CC1 composition time is at least 80\% faster and the maximum CC2 time close to 99.99\% faster than the minimum analysis time! In addition, we perform the layer analysis once and used for all subset CC node computation of n layers (which is exponential on n).

Both proposed heuristics are better than the naive approach in terms of accuracy and way more efficient than ground truth computation. CC2 is better than CC1 if overall accuracy and efficiency are considered, but CC1 performs better than CC2 for high-density graphs and has better precision. Availability of multiple heuristics and their efficacy on accuracy and efficiency allows one to choose appropriate heuristics based on graph/layer characteristics.
7 Conclusions and Future Work

In this paper, we have addressed the challenges of the decoupling approach for computing a global graph metric (closeness centrality) directly on a MLN. Two heuristics were developed to improve accuracy over the naive approach. CC2 gives significantly higher accuracy than naive for graphs on a large number of synthetic graphs generated with varying characteristics using RMAT and real-world datasets.

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