\(N = 2\) Supersymmetric QED equivalence
of \(N = 2\) Volkov-Akulov model

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Abstract

We show explicitly in two dimensional spacetime \((d = 2)\) that the \(N = 2\) Volkov-Akulov model is equivalent to the spontaneously broken linear supersymmetry (LSUSY) interacting gauge theory for \(N = 2\) vector and \(N = 2\) scalar supermultiplets. The local gauge interaction of LSUSY is induced by the specific composite structure of the auxiliary fields and the consequent transformations.

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Supersymmetry (SUSY) is a promising notion to search some unified theories beyond the Standard Model, which is realized as linear (L) representations or as nonlinear (NL) ones. The relation between the L and NL representations of SUSY is derived by linearizing of NLSUSY Volkov-Akulov (VA) model [1] which describes the dynamics of Nambu-Goldstone (NG) fermions [2]-[4] indicating spontaneous supersymmetry breaking (SSB). Indeed, by means of the linearization of NLSUSY [8]-[13] the VA model is equivalent to the various renormalizable L supermultiplet [5]-[7] with the Fayet-Iliopoulos (FI) terms which lead to the SUSY breaking mass (gaps). The components of the equivalent LSUSY theories to the VA model are expanded as composites of the NG fermions according to SUSY invariant relations.

Recently, we have shown [14] the vanishing of the ordinary LSUSY Yukawa interaction terms (and mass terms) in NLSUSY representation by means of the SUSY invariant relations, namely, we have found that these terms can be added to a LSUSY free action without violating the equivalence between the VA model and the LSUSY theory. From the viewpoint of NLSUSY general relativity (NLSUSYGR) [15, 16], this result sheds light on the way to discuss the mediation of the mass generation in the SSB from a spacetime origin in NLSUSY to the (Yukawa-)Higgs origin in LSUSY [17]. Because NLSUSYGR in asymptotic flat spacetime is essentially the VA model which is equivalent to the LSUSY theory with the mass gaps depending on the cosmological constant as well as the Newton gravitational constant. These arguments suggest that gauge interaction terms can be also discussed in the framework of NLSUSYGR through the linearization of NLSUSY.

Therefore, we study in this paper the gauge interaction terms in the linearization of NLSUSY and show the SUSY QED equivalence of VA model. We consider the (realistic) $N = 2$ NLSUSY VA model and linearize it in $d = 2$ for simplicity of calculations by means of SUSY invariant relations to a (free) LSUSY theory which is described by minimally provided $SO(2)$ vector and scalar supermultiplets. The LSUSY and local $U(1)$ gauge invariant action is constructed in the LSUSY theory and the explicit form of the gauge interaction terms in NLSUSY representation is derived, which becomes nonvanishing four fermion self-interaction terms in contrast with the vanishing Yukawa interactions under the SUSY invariant relations. Furthermore we redefine the original SUSY invariant relations by adding terms depending on a gauge coupling constant such that the nontrivial four fermion self-interaction terms corresponding to the gauge interaction terms are cancelled out from the LSUSY and gauge invariant action in NLSUSY representation. According to the new definition of the SUSY invariant relations, we show explicitly that the $N = 2$ VA model is equivalent to the spontaneously broken LSUSY gauge theory containing the gauge interaction terms.

Let us first introduce the $N = 2$ NLSUSY VA model [1] in terms of two (Majo-
rana) NG fermions \( \psi^i \) \((i = 1, 2)\). The VA action for \( \psi^i \) in \( d = 2 \) is written as the following spacetime volume form,

\[
S_{VA} = -\frac{1}{2\kappa^2} \int d^2x \, |w|,
\]

where \( \kappa \) is a constant whose dimension is \((\text{mass})^{-1}\) and

\[
|w| = \det(w^a_b) = \det(\delta^a_b + t^a_b), \quad t^a_b = -i\kappa^2 \bar{\psi}^i \gamma^a \partial_b \psi^i. \tag{2}
\]

Eq.(1) is expanded in terms of \( t^a_b \) or \( \psi^i \) as

\[
S_{VA} = \frac{1}{2\kappa^2} \int d^2x \left \{1 + t^a_a + \frac{1}{2!}(t^a_a t^b_b - t^b_b t^a_a) \right \}
\]

\[
= \frac{1}{2\kappa^2} \int d^2x \left \{1 - i\kappa^2 \bar{\psi}^i \partial \psi^i - \frac{1}{2} \kappa^4 \left( \bar{\psi}^i \gamma^a \partial_b \psi^i \bar{\psi}^j \gamma^b \partial_a \psi^j - \bar{\psi}^i \gamma^a \partial_b \psi^i \bar{\psi}^j \gamma^b \partial_a \psi^j \right) \right \}
\]

\[
\frac{1}{2} \kappa^4 \epsilon^{ab} \left( \bar{\psi}^i \psi^j \partial_a \bar{\psi}^i \gamma_5 \partial_b \psi^j + \bar{\psi}^j \gamma_5 \psi^i \partial_a \bar{\psi}^i \partial_b \psi^j \right), \tag{3}
\]

with the second term, \(-\frac{1}{2\kappa^2} t^a_a = \frac{i}{2} \bar{\psi}^i \partial \psi^i \), being the kinetic term for \( \psi^i \). The VA action (1) is invariant (becomes a surface term) under NLSUSY transformations of \( \psi^i \) parametrized by constant (Majorana) spinor parameters

\[
\delta_{\zeta} \psi^i = \frac{1}{\kappa} \zeta^i - i\kappa \bar{\zeta}^j \gamma^a \psi^j \partial_a \psi^i, \tag{4}
\]

which correspond to the supertranslations of \( \psi^i \) and Minkowski coordinates \( x^a \). Eq.(4) satisfies a closed off-shell commutator algebra,

\[
[\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a), \tag{5}
\]

where \( \delta_P(\Xi^a) \) means a translation with a generator \( \Xi^a = 2i\bar{\zeta}^i \gamma^a \zeta^i \).

In order to construct a LSUSY action with gauge interaction terms in \( N = 2, d = 2 \), we next minimally provide \( SO(2) \) vector and scalar supermultiplets, \( \mathbf{V} \) and

\[\text{(Minkowski spacetime indices in } D = 2 \text{ are denoted by } a, b, \cdots = 0, 1 \text{ and } SO(2) \text{ internal indices are } i, j, \cdots = 1, 2. \text{ The Minkowski spacetime metric is } \frac{1}{2} \{\gamma^a, \gamma^b\} = \eta^{ab} = \text{diag}(+,-) \text{ and } \sigma^{ab} = \frac{i}{2} \{\gamma^a, \gamma^b\} = i\epsilon^{ab} \gamma_5 \text{ (} \epsilon^{01} = 1 = -\epsilon_{01} \text{), where we use the } \gamma \text{ matrices defined as } \gamma^0 = \sigma^2, \gamma^1 = i\sigma^1, \gamma_5 = \gamma^0 \gamma^1 = \sigma^3 \text{ with } \sigma^I(I = 1, 2, 3) \text{ being Pauli matrices.} \]
The $N = 2$ LSUSY free action for $N = 2$ off-shell vector supermultiplet $V$ with a FI term is given by \[ S_{V0} = \int d^2 x \left\{ -\frac{1}{4} (F_{ab})^2 + \frac{1}{2} \lambda^i \partial \lambda^i + \frac{1}{2} (\partial_a A)^2 + \frac{1}{2} (\partial_a \phi)^2 + \frac{1}{2} D^2 - \frac{1}{\kappa} \xi D \right\}, \tag{6} \]

where the component fields $(v^a, \lambda^i, A, \phi, D)$ $(F_{ab} = \partial_a v_b - \partial_b v_a)$ mean a $U(1)$ vector field, doublet (Majorana) fermions and a scalar field in addition to another scalar field and an auxiliary scalar field, respectively. Note that the off-shell fermionic and bosonic degrees of freedom of these component fields are balanced as $4 = 4$.

In Eq.\((6)\) the last FI term with the real parameter $\xi$ gives the vacuum expectation value $\langle D \rangle = \xi \kappa$ indicating SSB. The free action \((6)\) is invariant under $N = 2$ LSUSY transformations parametrized by $\zeta^i,$

\begin{align*}
\delta \zeta v^a &= -i \epsilon^{ij} \zeta^i \gamma^a \lambda^j, \\
\delta \zeta \lambda^i &= (D - i \bar{\phi} A) \zeta^i + \frac{1}{2} \epsilon^{ab} \epsilon^{ij} F_{ab} \gamma_5 \zeta^j - i \epsilon^{ij} \gamma_5 \bar{\phi} \zeta^j, \\
\delta \zeta A &= \bar{\zeta}^i \lambda^i, \\
\delta \zeta \phi &= -\epsilon^{ij} \bar{\zeta}^i \gamma_5 \lambda^j, \\
\delta \zeta D &= -i \bar{\zeta}^i \bar{\phi} \lambda^i. \tag{7} \end{align*}

Eqs.\((7)\) satisfy the following closed off-shell commutator algebra,

\[ [\delta Q_1, \delta Q_2] = \delta P (\Xi^a) + \delta g (\theta), \tag{8} \]

where $\delta g (\theta)$ is the $U(1)$ gauge transformation only for $v^a$ with a generator $\theta = -2 (i \bar{\zeta}_1 \gamma^a \zeta_2 v_a - \epsilon^{ij} \bar{\zeta}_1 \zeta_2 \lambda^i - \bar{\zeta}_1 \gamma_5 \zeta_2 \phi).$

On the other hand, for $N = 2$ off-shell scalar supermultiplet $\Phi$ we take $(\chi, B^i, \nu, F^i),$ where $(\chi, \nu)$ for two (Majorana) fermions, $B^i$ for doublet scalar fields and $F^i$ for auxiliary scalar fields. $N = 2$ LSUSY free action for $\Phi$ including FI terms with the real $SO(2)$ ($U(1)$) parameters $\xi^i$ is given by

\[ S_{\Phi0} = \int d^2 x \left\{ \frac{i}{2} \bar{\chi} \partial \chi + \frac{1}{2} (\partial_a B^i)^2 + \frac{i}{2} \bar{\nu} \partial \nu + \frac{1}{2} (F^i)^2 - \frac{1}{\kappa} \xi^i F^i \right\}, \tag{9} \]

which is invariant under the following $N = 2$ LSUSY transformations,

\begin{align*}
\delta \zeta \chi &= (F^i - i \bar{\phi} B^i) \zeta^i, \\
\delta \zeta B^i &= \bar{\zeta}^i \chi - \epsilon^{ij} \zeta^i \nu, \\
\delta \zeta \nu &= \epsilon^{ij} (F^i + i \bar{\phi} B^i) \zeta^j, \\
\delta \zeta F^i &= -i \bar{\zeta}^i \bar{\phi} \chi - i \epsilon^{ij} \zeta^j \bar{\phi} \nu. \tag{10} \end{align*}
Eqs. (10) also satisfy the closed off-shell commutator algebra of Eq. (5).

Here we discuss the relation between the NLSUSY VA model and the LSUSY theories described by the free actions (6) and (9) for $V$ and $\Phi$. In the linearization of $N = 2$ NLSUSY, SUSY invariant relations between the NG fermions $\psi^i$ and the component fields in Eqs. (6) and (9) are constructed such that the NLSUSY transformations (4) reproduce the LSUSY ones of Eqs. (7) and Eqs. (10). Indeed, for $V$ the $SO(2)$ and $N = 2$ SUSY invariant relations of $(v^a, \lambda^i, A, \phi, D)$ as composites of $\psi^i$ are given in all orders as follows [14]:

$$v^a = -\frac{i}{2} \xi \epsilon^{ij} \bar{\psi}^i \gamma^a \gamma^j \psi^j |w|,$$

$$\lambda^i = \xi \left[ \psi^i |w| - \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j (1 - i \kappa^2 \bar{\psi}^k \partial \psi^k) \} \right],$$

$$A = \frac{1}{2} \xi \kappa \bar{\psi}^i \psi^i |w|,$$

$$\phi = -\frac{1}{2} \xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j |w|,$$

$$D = \frac{\xi}{\kappa} |w| - \frac{1}{8} \xi \kappa^3 \square (\bar{\psi}^i \psi^i \bar{\psi}^j \psi^j).$$

In Eqs. (11) the transformation of $v^a(\psi)$ under Eq. (4) gives the $U(1)$ gauge transformation besides the ordinary LSUSY one (7) as

$$\delta_v v^a(\psi) = -i \epsilon^{ij} \bar{\zeta}^i \gamma^a \lambda^j (\psi) + \partial^a W(\zeta; \psi)$$

(12)

with the $U(1)$ gauge transformation parameter,

$$W(\zeta; \psi) = \xi \kappa^2 \epsilon^{ij} \bar{\zeta}^i \psi^j \bar{\psi}^k \psi^k (1 - i \kappa^2 \bar{\psi}^l \partial \psi^l).$$

(13)

From Eq. (12) the commutator algebra on $v^a(\psi)$ under Eq. (4) does not induce the $U(1)$ gauge transformation term $\delta_\psi(\theta)$ in Eq. (8) [12, 14]; namely, it is closed as Eq. (5). While, for $\Phi$ the $(SO(2))$ SUSY invariant relations between $(\chi, B^i, \nu, F^i)$ and $\psi^i$ are constructed in all orders as follows:

$$\chi = \xi^i \left[ \psi^i |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j (1 - i \kappa^2 \bar{\psi}^k \partial \psi^k) \} \right],$$

$$B^i = -\kappa \left( \frac{1}{2} \xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j \right) |w|,$$

$$\nu = \xi^i \epsilon^{ij} \left[ \bar{\psi}^j |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^k (1 - i \kappa^2 \bar{\psi}^l \partial \psi^l) \} \right],$$

$$F^i = \frac{1}{\kappa} \xi^i \left[ |w| + \frac{1}{8} \kappa^4 \square (\bar{\psi}^j \psi^i \bar{\psi}^k \psi^k) \right] - i \kappa \xi^j \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|).$$

(14)
Substituting Eqs. (11) and (14) into the free actions (6) and (9) gives the equivalence of $S_{V_0}$ and $S_{\Phi_0}$ to the VA action $S_{VA}$ of Eq. (1) up to surface terms, respectively, i.e.

$$S_{VA} = S_{V_0} + \text{(surface terms)}$$

(15)
in all orders of $\psi^i$ [14] when $\xi^2 = 1$, while

$$S_{VA} = S_{\Phi_0} + \text{(surface terms)}$$

(16)
at least up to $O(\kappa^0)$ when $(\xi^i)^2 = 1$. (We can expect that from the experiences the relation (16) in all orders of $\psi^i$.) Therefore, the VA action is also equivalent to the sum of those free actions, $S_{V_0} + S_{\Phi_0}$, as

$$S_{VA} = S_{V_0} + S_{\Phi_0} + \text{(surface terms)},$$

(17)
provided that $\xi^2 + (\xi^i)^2 = 1$.

Now let us construct a LSUSY and gauge invariant interacting action by introducing a gauge interaction term for the $N = 2$ vector supermultiplet $V$ and the $N = 2$ scalar supermultiplet $\Phi$. We take the following interacting action where $e$ is a gauge coupling constant whose dimension is (mass)$^{-1}$,

$$S_e = \int d^2x \left[ e \left( i v_a \bar{\chi}^a \gamma^\nu \nu - e^{ij} v^a B^i \partial_a B^j + \bar{\chi}^i \chi B^i + e^{ij} \bar{\chi}^i \nu B^j - \frac{1}{2} D(B^i)^2 \right) 
+ \frac{1}{2} (\bar{\chi} \chi + \bar{\nu} \nu) A - \bar{\chi} \gamma_5 \nu \phi \right] + \frac{1}{2} e^2 (v_a^2 - A^2 - \phi^2)(B^i)^2, $$

(18)
and the total action,

$$S = S_{V_0} + S_{\Phi_0} + S_e,$$

(19)
where $S_{\Phi_0}$ means the LSUSY action (9) without the FI terms $-\frac{1}{2} \xi^i F^i$ due to the gauge invariance. The action (19) is invariant under the LSUSY transformations (7) for $V$ and the following LSUSY (gauge) transformations for $\Phi$ suggested from the superfield technology $\delta_\zeta \Phi \sim e V \Phi$ for $N = 1$ [18],

$$\delta_\zeta \chi = (F^i - i \theta B^i) \zeta^i - e \epsilon^{ij} V^i B^j,$$

$$\delta_\zeta B^i = \bar{\zeta}^i \chi - e \epsilon^{ij} \zeta^j \nu,$$

$$\delta_\zeta \nu = e^{ij} (F^j + i \theta B^j) \zeta^i + e V^i B^j,$$

$$\delta_\zeta F^i = -i \bar{\zeta}^i \theta \chi - ie \epsilon^{ij} \zeta^j \theta \nu$$

$$- e \{ e^{ij} \bar{V}^j \chi - V^i \nu + (\bar{\zeta}^j \lambda^i + \zeta^j \lambda^i) B^j - \bar{\zeta}^i \lambda^j B^i \}$$

(20)
with \( V^i = iv_a \gamma^a \zeta^i - \epsilon^{ij} A^i - \phi \gamma_5 \zeta^i \). Eqs.(20) satisfy the off-shell commutator algebra depending on \( e \),
\[
[\delta_{\zeta_1}, \delta_{\zeta_2}] \chi = \Xi^a \partial_a \chi - e \theta \nu,
\]
\[
[\delta_{\zeta_1}, \delta_{\zeta_2}] B^i = \Xi^a \partial_a B^i - ee^{ij} \theta B^j,
\]
\[
[\delta_{\zeta_1}, \delta_{\zeta_2}] \nu = \Xi^a \partial_a \nu + e \theta \chi,
\]
\[
[\delta_{\zeta_1}, \delta_{\zeta_2}] F^i = \Xi^a \partial_a F^i + ee^{ij} \theta F^j,
\]
where \( \theta \) is the generator of the \( U(1) \) gauge transformation in Eq.(8). The closure of the commutator algebra (21) as gauge covariant transformations of the fields can be seen in a manifest gauge invariant formulation of the LSUSY theory as explains below.

Here let us show explicitly the gauge invariance of the action (19). We define a complex (Dirac) spinor field \( \chi_D \) and complex scalar fields \( B, F \) in \( \Phi \) by
\[
\chi_D = \frac{1}{\sqrt{2}} (\chi + i\nu), \quad B = \frac{1}{\sqrt{2}} (B^1 + iB^2), \quad F = \frac{1}{\sqrt{2}} (F^1 - iF^2),
\]
and express the \( S'_{\Phi_0} + S_e \) in the action (19) in terms of \( (\chi_D, B, F) \) as
\[
S'_{\Phi_0} + S_e = \int d^2 x \left\{ i\bar{\chi}_D \gamma^a \partial_a \chi_D + |D_a B|^2 + |F|^2 + e(\bar{\chi}_D \lambda B + \bar{\lambda} \chi_D B^* - D|B|^2 + \bar{\chi}_D \chi_D A + i\bar{\chi}_D \gamma_5 \chi_D \phi) - e^2 (A^2 + \phi^2)|B|^2 \right\} + [ \text{surface term} ],
\]
where \( \lambda = \frac{1}{\sqrt{2}} (\lambda^1 - i\lambda^2) \) and the covariant derivative \( D_a = \partial_a - ie v_a \). Then the invariance of the action (19) under the ordinary local \( U(1) \) gauge transformations,
\[
(\chi_D, B, F) \rightarrow (\chi_D', B', F')(x) = e^{i \theta(x)} (\chi_D, B, F)(x),
\]
\[
v_a \rightarrow v'_a(x) = v_a(x) + \frac{1}{e} \partial_a \theta(x),
\]
is manifest. The commutator algebra (21) is also rewritten for the fields (22) as
\[
[\delta_{Q_1}, \delta_{Q_2}] = \delta_g(D),
\]
where \( \delta_g(D) \) means a gauge covariant transformation according to \( D = \Xi^a \partial_a + ie \theta \).

Now we discuss the gauge interaction terms (18) in NLSUSY representation based on the SUSY invariant relations (11) and (14). Substituting Eqs.(11) and (14) into Eq.(18) gives the following nonvanishing four fermion self-interaction terms,
\[
S_e(\psi) \equiv \int d^2 x \left\{ \frac{1}{4} e \kappa (\xi i)^2 \bar{\psi}_j \psi^j \bar{\psi}_k \psi^k \right\}.
\]
(The terms proportional to $e^2$ vanish due to $(\psi^i)^5 \equiv 0$.) In order to reduce the action (19) to the VA action (1) from the NLSUSYGR (SGM [15]) scenario for everything, the terms (26) corresponding to the action (18) should be cancelled out by other four fermion self-interaction terms, e.g. those which appear from other parts of Eq.(19) in NLSUSY representation. For this purpose we redefine the SUSY invariant relations (14) for the scalar supermultiplet $\Phi$, in particular, we add four fermion self-interaction terms depending on the gauge coupling constant $e$ to the original $F^i = F^i(\psi)$ as follows:

$$
\tilde{F}^i = \frac{1}{\kappa} \xi^i \bigg\{ |w| + \frac{1}{8} \kappa^3 \Box (\bar{\psi}^j \gamma^a \psi^k \gamma^a \psi^k) \bigg\} - i \kappa \xi^i \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|) \\
- \frac{1}{4} e \kappa^2 \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k.
$$

(27)

The new set of SUSY invariant relations is denoted by $\tilde{\Phi} = \tilde{\Phi}(\psi)$, which corresponds to $(\chi(\psi), B^i(\psi), \nu(\psi), \tilde{F}^i(\psi))$ in Eqs.(14) and (27), while the original set of the SUSY invariant relations (11) for the vector supermultiplet are maintained and denoted as $V = V(\psi)$ below. By substituting the $\tilde{\Phi}(\psi)$ and $V(\psi)$ into Eq.(19), the $S(\psi)$ reduces to the $N = 2$ VA action (1) by means of the cancellations among the four fermion self-interaction terms as follows:

$$
S(\psi) = (S_{V0} + S'_{\tilde{\Phi}0} + S_e)(\psi) \\
\equiv S_{VA} + \int d^2 x \left\{ -\frac{1}{4} e \kappa \xi (\xi^i)^2 \bar{\psi}^k \psi^k \bar{\psi}^j \psi^j + \frac{1}{4} e \kappa \xi (\xi^i)^2 \bar{\psi}^k \psi^k \bar{\psi}^j \psi^j \right\} \\
+ [surface terms] \\
= S_{VA} + [surface terms],
$$

(28)

where the terms $-\frac{1}{4} e \kappa \xi (\xi^i)^2 \bar{\psi}^k \psi^k \bar{\psi}^j \psi^j$ in the second line appear from $\frac{1}{2}(\tilde{F}^i)^2$ in $S'_{\Phi 0}$. Now $S_{V0} + S'_{\tilde{\Phi}0} + S_e$ is equivalent to $N = 2$ VA action provided $\xi^2 - (\xi^i)^2 = 1$ for the real parameters $\xi$ and $\xi^i$ in Eq.(28) due to the absence of the FI terms $-\frac{1}{\kappa} \xi^i \tilde{F}^i$ from $S'_{\tilde{\Phi}0}$.

In the LSUSY theory (28) equivalent to the VA model, the component fields $(\chi, B^i, \nu, \tilde{F}^i)$ in $\tilde{\Phi}$ constitute the LSUSY scalar multiplet, where the LSUSY transformations are realized as Eqs.(20) and their commutator algebra is closed as Eq.(21) (or Eq.(25)). On the other hand, the variations of $\tilde{\Phi} = \tilde{\Phi}(\psi)$ by means of the NL-SUSY transformations (4) can be computed straightforwardly, and by comparing these variations with the LSUSY transformations (20) for the component fields in $\tilde{\Phi}$, they (except $\delta \zeta B^i(\psi)$) are recasted as follows:

$$
\delta \zeta \chi(\psi) = (LSUSY(20))(\psi) + X(\zeta; \psi), \\
\delta \zeta \nu(\psi) = (LSUSY(20))(\psi) + Y(\zeta; \psi), \\
\delta \zeta \tilde{F}^i(\psi) = (LSUSY(20))(\psi) + Z^i(\zeta; \psi),
$$

(29)
where \((\text{LSUSY}(20))(\psi)\) means Eqs.(20) in terms of \(\tilde{\Phi} = \tilde{\Phi}(\psi)\) and \(V = V(\psi)\). \((X, Y, Z^i)(\zeta; \psi)\) are defined as

\[
X(\zeta; \psi) = \frac{1}{4} e\kappa^2 \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \zeta^i ,
\]

\[
Y(\zeta; \psi) = \frac{1}{4} e\kappa^2 \xi^i \epsilon^{ij} \bar{\psi}^k \psi^k \bar{\psi}^l \psi^l \zeta^j ,
\]

\[
Z^i(\zeta; \psi) = e\kappa \xi \left\{ \xi^j (\bar{\zeta}^i \psi^j - \bar{\zeta}^j \psi^i) \bar{\psi}^k \psi^k (1 - 2 i\kappa^2 \bar{\psi}^l \phi) \right. \\
\left. - i\kappa^2 \xi^j \bar{\zeta}^i \bar{\psi}^k \psi^k \bar{\psi}^l \phi \right\} .
\] (30)

(Note that \(-e\epsilon^{ij} V^i(\psi) B^j(\psi)\) and \(e V^i(\psi) B^i(\psi)\) of \(\delta_\zeta \chi(\psi)\) and \(\delta_\zeta \nu(\psi)\) in Eq.(29) vanish respectively.) Equation (29) shows that the variations of \(\tilde{\Phi} = \tilde{\Phi}(\psi)\) by means of the NLSUSY transformations (4) become the sum of the LSUSY transformations (20) for \(\tilde{\Phi}\) and \((X, Y, Z^i)(\zeta; \psi)\) defined as Eqs.(30).

In the above arguments, we find that \(\tilde{\Phi} = \tilde{\Phi}(\psi)\) is the set of SUSY invariant relations in a sense that the commutator algebra under the NLSUSY transformations (4) is closed as \([\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a)\) of Eq.(5). In particular, this is the case for the new \(\tilde{F}^i = \tilde{F}^i(\psi)\) of Eqs.(27) due to

\[
[\delta_{\zeta_1}, \delta_{\zeta_2}] \left( -\frac{1}{4} e\kappa^2 \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \right) = \Xi^a \partial_a \left( -\frac{1}{4} e\kappa^2 \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \right) .
\] (31)

Namely, in Eq.(28) the original definition of the SUSY invariant relations (14) is relaxed to the \(\tilde{\Phi} = \tilde{\Phi}(\psi)\) with respect to their variations under the NLSUSY transformations (4) as in Eqs.(29) with the \((X, Y, Z^i)(\zeta; \psi)\). In appearance, these situations are similar to Eqs.(12), (13) and (5) for \(\delta_\zeta \psi^a(\psi)\). Therefore, we conclude that the \(N = 2\) VA action (1) is equivalent to the LSUSY and local U(1) gauge invariant action (19) with the FI term, i.e.

\[
S_{VA} = S_{V0} + S_{\Phi0}' + S_c + \text{[surface terms]},
\] (32)

under the new definition of the SUSY invariant relations \(\tilde{\Phi} = \tilde{\Phi}(\psi)\) in addition to \(V = V(\psi)\).

The results obtained in this paper are summarized as follows. Firstly, we have linearized \(N = 2\) NLSUSY VA model in \(d = 2\) and shown by using the free SUSY invariant relations (11) and (14) it is equivalent to the LSUSY (free) theory which is described by \(N = 2\) (SO(2)) vector and scalar supermultiplets as depicted in Eq.(17). Next, we have constructed the (covariant) gauge interaction terms (18) for these LSUSY multiplets and studied the relation between the VA action (1) and the
LSUSY local $U(1)$ gauge invariant action (19). Contrary to the vanishments of the NLSUSY representation of the Yukawa and the mass terms (14) the LSUSY gauge interaction terms produce the nonvanishing four fermion self-interaction terms (26) when the free SUSY invariant relations (14) and (11) are adopted. We have shown that such a nontrivial (and harmful from the NLSUSYGR scenario) terms (26) are cancelled out in the action (19) in the NLSUSY representation by means of the new SUSY invariant relations (14), in particular, by means of the redefinition of the new $\tilde{F}^i(\psi)$ of Eq.(27) where the four fermion self-interaction terms depending on the gauge coupling constant $e$ are added to the free case SUSY invariant relations $F^i(\psi)$.

Then the equivalence of the action (19) to the $N = 2$ VA action (1) has been shown as in Eq.(28), where the component fields in $\tilde{\Phi}$ constitute the LSUSY scalar multiplet. For the proof of this equivalence we have changed (relaxed) the initial definition of the SUSY invariant relations (14) valid for the free case into the new interacting case $\tilde{\Phi} = \tilde{\Phi}(\psi)$ with the consequent NLSUSY transformations (29) containing the $(X,Y,Z^i)(\zeta;\psi)$. In spite of the relaxation, the closure of the commutator algebra for the $\tilde{\Phi} = \tilde{\Phi}(\psi)$ under the NLSUSY transformations (4) has been proved as Eq.(5), in particular, for the new $\tilde{F}^i(\psi)$ due to Eq.(31). Therefore, we have concluded in Eq.(32) that the $N = 2$ VA model is equivalent to the spontaneously broken $N = 2$ LSUSY gauge theory with the action (19) in a sense that the new definition of the SUSY invariant relations $\tilde{\Phi} = \tilde{\Phi}(\psi)$ including Eqs.(27) holds.

Since the (free) SUSY invariant relations (11) for the vector supermultiplet $V$ are maintained in the linearization, the LSUSY Yukawa interaction terms $S_{Vf} = \int d^2x \{f(A\bar{x}^i\lambda^i + e^{ij}\phi\bar{x}^i\gamma_5\lambda^j + A^2D - \phi^2D - \epsilon^{ab}A\phi F_{ab})\}$ and the mass terms $S_{Vm} = \int d^2x \{-\frac{1}{4}m (\bar{x}^i\lambda^i - 2AD + \epsilon^{ab}\phi F_{ab})\}$ in $V$ [14] can be added to the equivalent LSUSY gauge theory without violating the equivalence (32). Namely,

$$S_{VA} = S_{V0} + S'_{\Phi0} + S_e + S_{Vf} + S_{Vm} + \{\text{surface terms}\} \quad (33)$$

holds for $V = V(\psi)$ and $\tilde{\Phi} = \tilde{\Phi}(\psi)$ as one possible configuration of the basic component fields, which is equivalent to the $N = 2$ VA model. Once the gauge and Yukawa interaction terms, $S_e$ and $S_{Vf}$, are added to the free action $S'_{\Phi0}$ and $S_{V0}$ containing the FI term, they become nontrivial in the action, break SUSY spontaneously and produce mass for the composite fields automatically as demonstrated in Ref.[17]. These results are very favourable for the SGM [15] scenario where all particles except graviton are the composite of NG-fermion (called superons) of the NLSUSY VA model. It is interesting that the four-fermion self-interaction term (i.e. the condensation of $\psi^i$) which contributes to the auxiliary field is the origin of the familiar local $U(1)$ gauge symmetry of LSUSY theory. As for the SUSY QED equivalence of $N = 2$ VA model in $d = 4$, the similar results are anticipated but further investigations are needed.
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