Lifetimes of the $b$–flavored baryons in the light–front quark model

I.M. Narodetskii

ITEP, 117218 Moscow, Russia

The calculation of lifetimes of heavy–flavored baryons in the light–front quark model approach is briefly reviewed.

1. INTRODUCTION

The lifetimes of the $b$–flavored hadrons $H_b$ are related both to the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$ and to dynamics of $H_b$ decays. Weak decays of heavy hadrons involve three fundamental scales, the weak interaction scale $M_W$, the $b$–quark mass $m_b$, and the QCD scale $\Lambda_{QCD}$, which are strongly ordered: $M_W \gg m_b \gg \Lambda_{QCD}$. The first inequality means that the decay width of the $b$ quark rises so rapidly with $m_b$ that the spectator process dominates for large $m_b$. The second inequality means that for large $m_b$ $b$–quark decays before it can hadronize. Thus in the limit $m_b \to \infty$ the lifetimes of all $b$ hadrons must be equal: $\tau_{H^+} = \tau_{B^0_d} = \tau_{B^0_s} = \tau_{\Lambda_b}$.

Inclusive $H_b$ decays can be treated with the help of an operator product expansion (OPE) combined with the heavy quark expansion. The OPE approach predicts that all corrections to the leading QCD improved parton terms appear at the order $1/m_b^2$ and beyond. Thus mesons and baryons containing $b$ quark are expected to have lifetimes differing by no more than a few per cent. The result of this approach for the $\Lambda_b$ lifetime is puzzling because it predicts that $\tau_{\Lambda_b}/\tau_B = 0.98 + O(1/m_b^2)$, whereas the experimental findings suggest a very much reduced fraction

$$\frac{\tau_{\Lambda_b}/\tau_B}{\text{exp.}} = 0.78 \pm 0.04,$$

or conversely a very much enhanced decay rate.

Application of OPE to decays of $b$–hadrons has potentially two caveats. One is that OPE is used in Minkowsky domain, and therefore relies on assumption of quark–hadron duality at the energies involved in decays. Another uncertainty arises from poor knowledge of matrix elements of four–quark operators arising as terms of OPE. The decay rates of $B$ and $\Lambda_b$ are $\Gamma(B) = 0.63 \pm 0.02$ ps$^{-1}$ and $\Gamma(\Lambda_b) = 0.83 \pm 0.05$ ps$^{-1}$ differing by $\Delta \Gamma(\Lambda_b) = 0.20 \pm 0.05$ ps$^{-1}$. The four–fermion processes of weak scattering and Pauli interference calculated using the factorization approach and the description of the baryon relying on quantum mechanics of only the constituent quarks could explain, under certain conditions, only $(13 \pm 7)\%$ of $\Delta \Gamma(\Lambda_b)$. However, the ”spectator effects”, which formally appear at $O(1/m_b^3)$ in OPE can be considerably larger. A piloting lattice study of Ref. suggests that the effects of weak scattering and interference can eliminate $\sim 50\%$ of the discrepancy between the theoretical prediction for $\tau_{\Lambda_b}/\tau_B$ and the experimental result. It also appears that not all of the discrepancy can be accounted for by spectator effects. The OPE prediction, given all the uncertainties involved, is unlikely to produce a number lower than $\sim 0.9$.

It is not clear whether the present contradiction between the theory and the data on $\tau_{\Lambda_b}/\tau_B$ is a temporary difficulty or an evidence of fundamental flaws in theoretical understanding. In spite of great efforts of experimental activity the $\Lambda_b$ lifetime remains significantly low which continues to spur theoretical activity. In this respect, the use of phenomenological models, like the constituent quark model, could be of interest as a complimentary approach to the OPE resummation method. In this talk I shall consider the preasymptotic effects for the $\Lambda_b$ lifetime in the framework of the
light–front (LF) quark model. My talk is preceded by that of Dr. Kalman [8], covering, in part, the same topic. Therefore, I will avoid those aspects that have been already discussed. Instead, I will concentrate on the calculation of lifetimes of $\Lambda_b$ and other $b$–flavored baryons.

2. LF QUARK MODEL

The LF quark model [7] is a relativistic constituent quark model based on the LF formalism. In Ref. [8] this formalism has been used to establish a simple quantum mechanical relation between the inclusive decay rate of the B meson and that of a free $b$ quark. The approach of [8] relies on the idea of duality in summing over the final hadronic states. It has been assumed that the sum over all possible charm final states $X_c$ can be modeled by the decay width of an on–shell $b$ quark into on–shell $c$ quark folded with the $b$–quark distribution function $f_B^{x/p}(x, p^2_{\perp}) = |\psi_B^x(x, p^2_{\perp})|^2$. The latter represents the probability to find $b$ quark carrying a LF fraction $x$ of the hadron momentum and a transverse relative momentum squared $p^2_{\perp}$. For the partial rates $d\Gamma(B)/dq^2$ the above mentioned relation takes the form

$$\frac{d\Gamma(B)}{dq^2} = \frac{d\Gamma_b}{dq^2} R_B(q^2),$$

where $d\Gamma_b/dq^2$ is the free quark semileptonic or nonleptonic differential decay rate, with $q$ being the 4–momentum of the $W$ boson. The function $R_B(q^2)$ incorporates the non perturbative effects related to binding effect and to the Fermi motion of the heavy quark inside the hadron and is expressed in terms of the bound–state factor $|\psi_B^x(x, p^2_{\perp})|^2$. For the details see [7]. The non–perturbative corrections to the free quark decay rate vanish as $m_b \rightarrow \infty$ [8] but may be essential at finite values of the $b$ quark mass due to the difference between $m_b$ and $m_{H_b}$ and primordial motion of the $b$ quark inside $H_b$.

To define the LF wave function we use the simple operational ansatz [8] and express $\psi_B^x(x, p^2_{\perp})$ in terms of the equal time radial wave function $\psi_B(p^2)$. Explicit formulæ relating $\psi_B^x(x, p^2_{\perp})$ and $\psi_B(p^2)$ can be found e.g. in Ref. [7].

3. $B$–MESON LIFETIME

In what follows the B meson orbital wave function is assumed to be the Gaussian function as $\psi_B(p^2) = (1/\beta_{bd}\sqrt{\pi})^2 \exp\left(-p^2/2\beta^2_{bd}\right)$, where the parameter $1/\beta_{bd}$ defines the confinement scale. We take $\beta_{bd} = 0.45$ GeV. For $|\psi_B(0)|^2$, the square of the wave function at the origin, we have $|\psi_B(0)|^2 = 1.64 \times 10^{-2}$ GeV$^3$ that compares favorably with the estimation in the constituent quark ansatz [13] $|\psi_B(0)|^2 = M_B f^2_B/12 = (1.6 \pm 0.7) \times 10^{-2}$ GeV$^3$ for $f_B = 190$ MeV.

We have found that the $\tau_{\Lambda_b}/\tau_B$ ratio is rather stable with respect to the precise values of the heavy quark masses $m_b$ and $m_c$ provided $m_b - m_c \geq 3.5$ GeV. From now on we shall use the reference values $m_b = 5.1$ GeV and $m_c = 1.5$ GeV. The value of the CKM parameter $|V_{cb}|$ cancels in the ratio $\tau_{\Lambda_b}/\tau_B$, but is important for the absolute rates. Details of our calculations of $\Gamma(B)$ are given in Table 1 for the three values of the constituent mass $m_{sp}$. The values $m_{sp} \sim 300$ (200) MeV are usually used in non-relativistic (relativistic) quark models. We have also considered a very low constituent quark mass $m_{sp} = 100$ MeV to see how much we can push up the theoretical prediction for $\Gamma(\Lambda_b)/\Gamma(B)$, see below. All the semileptonic widths include the pQCD correction as an overall reduction factor equal to 0.9. Following Ref. [10] the transitions to baryon-antibaryon ($\Lambda_c N$ and $\Xi_c \bar{\Lambda}$) pairs are included. In addition we have added BR $\approx 1.5\%$ for the $b \rightarrow \nu$ decays with $|V_{ub}/V_{cb}| \sim 0.1$. The value of $|V_{cb}|$ is defined by the condition that the calculated B lifetime is 1.56 ps.

4. $\Lambda_b$ LIFETIME

We shall analyze the inclusive semileptonic and non–leptonic $\Lambda_b$ rates on simplifying assumption that $\Lambda_b$ is composed of a heavy quark and a light scalar diquark with the effective mass $m_{ud}$. Then the treatment of the inclusive $\Lambda_b$ decays is simplified to a great extent and one can apply the model considered above with the minor modifications. For the heavy–light diquark wave function $\psi_{\Lambda_b}$ we again assume the Gaussian ansatz with the oscillator parameter $\beta_{su}$. The width of $\Lambda_b$ can
be obtained from that of $B$ by the replacements $M_B \rightarrow M_{\Lambda_b}$, $m_{sp} \rightarrow m_{ud}$ and $\beta_{ud} \rightarrow \beta_{bu}$. The latter two replacements change $f^b_B$, the $b$ quark distribution function inside the $\Lambda_b$, in comparison with $f^B_B$.

The inclusive nonleptonic channels for $\Lambda_b$ are the same as for B meson except the decays into baryon–antibaryon pairs which are missing in case of $\Lambda_b$. The absence of this decay channel leads to the reduction of $\Gamma(\Lambda_b)$ by $\approx 7\%$. This effects prolongs $\tau_{\Lambda_b}$ over $\tau_B$. The phase space enhancement in $\Lambda_b$ is marginal and can not be responsible for a shorter lifetime of $\Lambda_b$. The only distinction between the two lifetimes, $\tau_{\Lambda_b}$ and $\tau_B$, can occur due to the difference of the binding and Fermi motion effects. These effects are encoded in the parameter $x_b = m_b/m_{H_b}$ and the distributions $f^b_{\Lambda_b}$ and $f^B_B$.

The concrete amount of the deviation between $\tau_{\Lambda_b}$ and $\tau_B$ depends in essential way on $m_{ud}$ and $\beta_{ub}$. In what follows the diquark mass is taken as:

$$m_{ud} = m_* = \frac{1}{2}(m_u + m_d - m_\pi).$$

(3)

In this relation inspired by the quark model, the factor $1/2$ arises from the different color factors for $u$ and $d$ in the $\pi$–meson ( a triplet and antitriplet making a singlet) and $u$ and $d$ in the the $\Lambda_b$ (two triplets making an antitriplet).

The quantity $\beta_{ub}$ can be translated into the ratio of the squares of the wave functions determining the probability to find a light quark at the location of the $b$ quark inside the $\Lambda_b$ baryon and $B$ meson, i.e. $r = |\Psi_{\Lambda_b}(0)|^2/|\Psi_{B}(0)|^2 = (\beta_{ub}/\beta_{bd})^3$. If we take $r = 1.2 \pm 0.2$ as suggested by the lattice study of Ref. [3] we obtain $\langle \tau_{\Lambda_b}/\tau_B \rangle_{LF} = 0.88 \pm 0.02$ for $m_{sp} = 200$ MeV. Adding the contribution of the $4$–quark operators $\Delta\Gamma^{\pi \eta} = 0.08 \pm 0.02$ from [3] we reproduce the experimental result [11].

It is instructive to note that the calculated branching fractions of $\Lambda_b$ show marginal dependence on the choice of the model parameters; they are $\sim 11.5\%$ for the semileptonic $b \rightarrow c\ell\nu_\ell$ transitions, $\sim 2.8\%$ for $b \rightarrow c\tau\nu_\tau$, $\sim 50\%$ for the nonleptonic $b \rightarrow c\bar{d}u$ transitions, and $\sim 16\%$ for $b \rightarrow c\bar{e}\bar{s}$ transitions.

An exploratory study of the dependence of $\tau_{\Lambda_b}$ on $m_{ud}$ and $r$ has been performed in Ref. [12]. Two choices for diquark masses were taken: $m_{ud} = m_u + m_d$ corresponding to zero binding approximation and $m_{ud} = m_*$ from (3). The wave function ratio has been varied in the range $0.3 \leq r \leq 2.3$ that corresponds to $0.3$ GeV $\leq \beta_{bu} \leq 0.6$ GeV. The results for $\tau_{\Lambda_b}/\tau_B$ are exhibited in Table 2, which we now discuss.

We observe that to decrease the theoretical prediction for $\tau_{\Lambda_b}$ requires to decrease the value of the hadronic parameter $r$ to $0.3$-$0.5$ and the value of $m_{sp}$ to $\sim 100$ MeV. For example, assuming that $r \sim 0.3$ we find that the lifetime ratio is decreased from $0.88$ to $0.81$ if $m_{sp}$ is reduced from $300$ MeV to $100$ MeV and $m_{ud} = m_u + m_d$. For the $m_{ud} = m_*$ the ratio is almost stable ($\sim 0.8$), so that reducing of the diquark mass produces a decrease of the lifetime ratio by $1\%$, $5\%$, and $8\%$ for $m_{sp} = 100, 200$, and $300$ MeV, respectively. Varying the spectator quark mass in a similar way we find that for the "central value"

Table 1

| $m_{sp}$ [MeV] | 100 | 200 | 300 |
|----------------|-----|-----|-----|
| $b \rightarrow c\ell\nu_\ell$ | 10.65 | 10.98 | 11.46 |
| $b \rightarrow c\mu\nu_\mu$ | 10.59 | 10.93 | 11.40 |
| $b \rightarrow c\tau\nu_\tau$ | 2.47 | 2.51 | 2.57 |
| $b \rightarrow c\bar{d}u$ | 47.88 | 47.88 | 47.52 |
| $b \rightarrow c\bar{c}s$ | 14.07 | 14.31 | 14.63 |
| $b \rightarrow c\bar{s}u$ | 2.94 | 3.09 | 3.32 |
| $B \rightarrow \Xi_{cs} \Lambda_c$ | 2.22 | 1.83 | 1.43 |
| $B \rightarrow \Lambda_c N$ | 7.70 | 6.91 | 6.02 |
| $b \rightarrow u$ | 1.47 | 1.54 | 1.66 |

$|V_{bc}| = 38.3$ | 39.3 | 40.7 |

$R^d_{\Lambda_b}$
Table 2
The LF quark model results for the ratio $\tau_{\Lambda_b}/\tau_B$ calculated for different values of $r$ and $m_{sp}$. $m_{sp}$ are in units of MeV. $m_{ud}$ are given by (1). The results corresponding to $m_{ud} = m_u + m_d$ are given in parentheses.

| $r$ | $m_{sp}$ | 100  | 200  | 300  |
|-----|-----|-----|-----|-----|
| 0.3 | 0.80 | 0.81 | 0.79 | 0.80 | 0.89 |
| 1.0 | 0.87 | 0.88 | 0.86 | 0.90 | 0.86 |
| 2.3 | 0.93 | 0.94 | 0.91 | 0.95 | 0.91 |

$r \sim 1$ the lifetime ratios are reduced from 0.93 to 0.88 for $m_{ud} = m_u + m_d$ and remain almost stable ($\sim 0.86$) for $m_{ud} \sim m_s$. For the largest possible value of $r$ suggested in (1), $r \sim 2.3$, the lifetime ratios are reduced from 0.97 to 0.94 in the former case and remain almost stable $\sim 0.91 - 0.93$ in the latter case.

The lifetimes of other $b$–flavored baryons are reported in Table 3. The results are obtained using $r = 1$ and adding the marginal contribution of $\Delta \Gamma^{4q}$ from Ref. (3). As we have discussed earlier this contribution may be strongly enhanced by the QCD effects in which case the lifetime predictions should be decreased by $\sim 10\%$.

5. CONCLUSIONS

If the current value of $(\tau(\Lambda_b))_{\exp}$ persists, the most likely explanation is that some hadronic matrix elements of four–quark operators are larger than the naive expectation (2). If a significant fraction $\sim 50\%$ of the discrepancy between the theoretical prediction for $\tau_{\Lambda_b}/\tau_B$ and the experimental result can be accounted for the spectator effects (2) then the remainder of the discrepancy can be naturally explained by the preasymptotic effects. We have found that the binding effect produces for $r = 1.2 \pm 0.2$ $m_{sp} = 200$ MeV and $m_{ud} = 250$ MeV an additional increase of $\Delta \Gamma(\Lambda_b)/\Gamma(B)$ by $12 \pm 2\%$ (3).

$\frac{3}{3}$Note that for these values of quark and diquark masses the Fermi motion effects in $\Lambda_b$ and $B$ do not differ significantly.

Table 3
The predicted lifetimes $\tau_{\Lambda_b}$ (in units of ps). $r = 1$.

| $H_Q$ | $\tau(H_Q)$ | $\tau_{\exp}(H_Q)$ |
|-----|-----|-----|
| $B^0$ | 1.56 | 1.56 $\pm$ 0.04 |
| $B_s$ | 1.53 | 1.54 $\pm$ 0.07 |
| $\Lambda_b$ | 1.31 | 1.24 $\pm$ 0.08 |
| $\Xi^0_b$ | 1.31 | 1.39 $^{+0.24}_{-0.28}$ |
| $\Xi_b$ | 1.41 | |
| $\Omega_b$ | 1.32 | |

ACKNOWLEDGMENTS

I would like to thank Calvin Kalman, Pepe Salt and Miguel–Angel Sanchis–Lozano for organizing an excellent Conference with a stimulating scientific program.

REFERENCES

1. I.Bigi et al., Ann. Rev. Nucl. Part. Sci, 47 (1997) 591
2. M.Neubert and C.T.Sachrajda, Nucl. Phys. B483 (1997) 339
3. T.Junk, talk at the 2nd Int. Conf. on B–Physics and CP–Violation, Honolulu, Hawai, March 1997
4. J.L.Rosner, Phys. Lett. B379 (1996) 267
5. M.Di Pierro, C.T.Sachrajda, and C.Michael, hep–lat/9906031
6. I.D’Souza, C.S.Kalman et al., these Proceedings
7. see e.g. W.Jaus, Phys. Rev. D60 (1999) 054026 and references therein
8. S.Ya.Kotkovsky et al., Phys. Rev. D60 (1999) 114024
9. F.Coester, Prog. Part. Nucl. Phys. 29 (1992) 1
10. I.L.Grach et al., Nucl. Phys. B502 (1997) 227
11. E.V.Shuryak, Nucl. Phys. B198 (1982) 83
12. P.Yu.Kulikov et al., Pis’ma v ZhETF, 71 (2000) 528 [JETP Letters, 71 (2000) 362]