A THEORY OF AVAILABLE-BY-DESIGN COMMUNICATING SYSTEMS

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Abstract. Choreographic programming is a programming-language design approach that drives error-safe protocol development in distributed systems. Starting from a global specification (choreography) one can generate distributed implementations. The advantages of this top-down approach lie in the correctness-by-design principle, where implementations (endpoints) generated from a choreography behave according to the strict control flow described in the choreography, and do not deadlock. Motivated by challenging scenarios in Cyber-Physical Systems (CPS), we study how choreographic programming can cater for dynamic infrastructures where not all endpoints are always available. We introduce the Global Quality Calculus ($GC_q$), a variant of choreographic programming for the description of communication systems where some of the components involved in a communication might fail. $GC_q$ features novel operators for multiparty, partial and collective communications. This paper studies the nature of failure-aware communication: First, we introduce $GC_q$ syntax, semantics and examples of its use. The interplay between failures and collective communications in a choreography can lead to choreographies that cannot progress due to absence of resources. In our second contribution, we provide a type system that ensures that choreographies can be realized despite changing availability conditions. A specification in $GC_q$ guides the implementation of distributed endpoints when paired with global (session) types. Our third contribution provides an endpoint-projection based methodology for the generation of failure-aware distributed processes. We show the correctness of the projection, and that well-typed choreographies with availability considerations enjoy progress.

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1. Introduction

Choreographies are a well-established formalism in concurrent programming, with the purpose of providing a correct-by-construction framework for distributed systems [8, 12]. Using Alice-Bob’s style protocol narrations, they provide the structure of interactions among components in a distributed system. Combined with a behavioral type system, choreographies are capable of deriving distributed (endpoint) implementations. Endpoints generated from a choreography ascribe all and only the behaviors defined by it. Additionally, interactions among endpoints exhibit correctness properties, such as liveness and deadlock-freedom. In practice, choreographies guide the implementation of a system, either by automating the generation of correct deadlock-free code for each component involved, or by monitoring that the execution of a distributed system behaves according to a protocol [8, 39, 3].

In this paper we study the role of availability when building communication protocols. In short, availability describes the ability of a component to engage in a communication. Insofar, the study of communications using choreographies assumed that components were always available. We challenge this assumption on the light of new scenarios. The case of Cyber-Physical Systems (CPS) is one of them. In CPS, components become unavailable due to faults or because of changes in the environment. Even simple choreographies may fail when including availability considerations. Thus, a rigorous analysis of availability conditions in communication protocols becomes necessary, before studying more advanced properties, such as deadlock-freedom or protocol fidelity.

Practitioners in CPS take availability into consideration, programming applications in a failure-aware fashion. First, application-based QoS policies replace old node-based ones. Second, one-to-many and many-to-one communication patterns replace peer-to-peer communications. Still, programming a CPS from a component viewpoint such that it respects an application-based QoS is difficult, because there is no centralized way to ensure its enforcement.

This work reports initial steps towards a methodology for the development of failure-aware communication protocols, as exemplified by CPS. We depart from choreographic programming as a reference model, extending it in order to cope with the intrinsic characteristics present in communication protocols for CPS. This resulted in a novel language for choreographies, the Global Quality Calculus \( (GC_q) \). The novel characteristics of \( GC_q \) include a multiparty, asynchronous model of communication, including collective message-passing operators, such as broadcast, collective message aggregators, and collective method selections. Communication is rarely perfect in CPS, and successful communications depend on the availability of components. \( GC_q \) plays important consideration on this aspect, by including component availability as a first-class consideration to deem a communication successful.

We present the following contributions:

**First: A formal model for Failure-aware Choreographies:** We present the Global Quality Calculus \( (GC_q) \), a process calculus aimed at capturing the most important aspects of CPS, such as variable availability conditions and multicast communications. It is a generalization of the Global Calculus [12], enriched with collective communication primitives and explicit availability considerations. Central to \( GC_q \) is the inclusion quality predicates [41] and optional datatypes, whose role is to allow for communications where only a subset of the original participants is available. Models in \( GC_q \) can accommodate in this way application-based QoS policies, instead of a node-centric approach.
Second: A Type system to control progress in failure-aware choreographies. Our second contribution relates to the verification of failure-aware protocols. We focus on progress. As an application-based QoS, a progress property requires that at least a minimum set of components is available before firing a communication action. Changing availability conditions may leave collective communications without enough required components, forbidding the completion of a protocol. We introduce a type system, orthogonal to session types, that ensures that well-typed protocols with variable availability conditions do not get stuck, preserving progress.

Third: A Choreographic programming methodology for available-by-design distributed systems. In our third contribution, we propose a methodology to generate distributed implementations from failure-aware choreographies. To do so, we resort on previous works on choreographic programming [10, 12]. Starting from a specification in GC_q, one can generate the distributed implementation in terms of interacting processes. The language of endpoints used is an extension of standard session π calculi with quality-based input/output processes, asynchronous and queue-based communication. Quality choreographies are paired with a session-type system, taking inspiration on previous works on session types for collective communications [32]. As such, session types for quality choreographies guarantee that the specification can follow a given protocol. Moreover, they are important in that session types guide the projection to correct behavior. In this paper, we are interested in availability-by-design, a variant of deadlock-freedom that ensures communication under minimal set of available components.

This paper is a revised and extended version of [33]. In particular, the syntax, semantics and type system controlling progress capabilities in GC_q have been revised and simplified. In addition, in this paper we provide novel sections detailing the development based on session types and endpoint projection (c.f. §5, §6), that was absent in the original presentation.

Document Structure: In Section 2 we introduce the design considerations for a calculus with variable availability conditions and we present a minimal working example to illustrate the calculus in action. Section 3 introduces syntax and semantics of GC_q. The progress-enforcing type system is presented in Section 4. Section 5 presents the session type methodology for quality choreographies. The Endpoint model, and the projection from quality choreographies is presented in Section 6. Section 7 discusses related work. Finally, Section 8 concludes. Appendix A includes additional lemmata and proofs of the main results in the paper.

2. Towards a Language for CPS Communications

The design of a language for CPS requires a technology-driven approach, that answers to requirements regarding the nature of communications and devices involved in CPS. Similar approaches have been successfully used for Web-Services [11, 14, 38], and Multicore Programming [32, 14]. The considerations on CPS used in this work come from well-established sources [2, 33]. We will proceed by describing their main differences with respect to traditional networks.
2.1. Unique Features in CPS Communications. Before defining a language for communication protocols in CPS, it is important to understand the taxonomy of networks where they operate. CPS are composed by sensor networks (SN) that perceive important measures of a system, and actuator networks that change it. Some of the most important characteristics in these networks include asynchronous operation, sensor mobility, energy-awareness, application-based protocol fidelity, data-centric protocol development, and multicast communication patterns. We will discuss each of them.

Asynchrony. Depending on the application, deployed sensors in a network have less accessible mobile access points, for instance, sensors deployed in harsh environmental conditions, such as arctic or marine networks. Environment may also affect the lifespan of a sensor, or increase its probability of failure. To maximize the lifespan of some sensors, one might expect an asynchronous operation, letting sensors remain in a standby state, collecting data periodically.

Sensor Mobility. The implementation of sensors in autonomic devices brings about important considerations on mobility. A sensor can move away from the base station, making their interactions energy-intensive. In contrast, it might be energy-savvy to start a new session with a different base station closer to the new location.

Energy-Awareness. Limited by finite energetic resources, SN must optimize their energy consumption, both from node and application perspectives. From a node-specific perspective, a node in a sensor network can optimize its life by turning parts of the node off, such as the RF receiver. From a application-specific perspective, a protocol can optimize it energy usage by reducing its traffic. SN cover areas with dense node deployment, thus it is unnecessary that all nodes are operational to guarantee coverage. Additionally, SN must provide self-configuration capabilities, adapting its behavior to changing availability conditions. Finally, it is expected that some of the nodes deployed become permanently unavailable, as energetic resources ran out. It might be more expensive to recharge the nodes than to deploy new ones. The SN must be ready to cope with a decrease in some of the available nodes.

Data-Centric Protocols. One of the most striking differences to traditional networks is the collaborative behavior expected in SN. Nodes aim at accomplishing a similar, universal goal, typically related to maintaining an application-level quality of service (QoS). Protocols are thus data-centric rather than node-centric. Moreover, decisions in SN are made from the aggregate data from sensing nodes, rather than the specific data of any of them \[42\]. Collective decision-making based in aggregates is common in SN, for instance, in protocols suites such as SPIN \[22\] and Directed Diffusion \[28\]. Shifting from node-level to application-level QoS implies that node fairness is considerably less important than in traditional networks. In consequence, the analysis of protocol fidelity \[25\] requires a shift from node-based guarantees towards application-based ones.
Multicast Communication. Rather than peer-to-peer message passing, one-to-many and many-to-one communications are better solutions for energy-efficient SN, as reported in [21, 13]. However, as the number of sensor nodes in a SN scales to large numbers, communications between a base and sensing nodes can become a limiting factor. Many-to-one traffic patterns can be combined with data aggregation services (e.g., TAG [35] or TinyDB [21, 15]). However, as the number of sensor nodes in a SN scales to large numbers, communications between a base and sensing nodes can become a limiting factor. Many-to-one traffic patterns can be combined with data aggregation services (e.g., TAG [35] or TinyDB [21, 15]).

2.2. Model Preview. We will illustrate how the requirements for CPS communications have been assembled in the our calculus through a minimal example in Sensor Networks (SN). The syntax of our language is inspired on the Global Calculus [8, 12] extended with collective communication operations [32].

Example 2.1. Figure 1 portrays a simple SN choreography for temperature measurement. Line 1 models a session establishment phase between sensors \( t_1, t_2, t_3 \) (each of them implementing the indexed role \( S_i \)) and a monitor \( t_m \) with role \( M \). In Line 2, \( t_m \) executes a collective selection of method \( \text{measure} \) at each node. In Line 3, an asynchronous many-to-one communication (e.g., reduce) is performed between sensors and the monitor. Quality predicates \( q_1, q_2 \) model application-based QoS, established in terms of availability requirements for each of the nodes. For instance, \( q_1 = q_2 = \forall \) only allows communications with all sensors in place, and \( q_1 = \forall, q_2 = 2/3 \) tolerates the absence of one of the sensors in data harvesting. Once nodes satisfy applications’ QoS requirements, an aggregation operation will be applied to the messages received, in this case computing the average value.

Considerations regarding the impact of available components in a communication must be tracked explicitly. Annotations \( \{X; Y\} \) (in blue font) define capabilities, that is, control points achieved in the system. The \( X \) in \( t\{X; Y\} \) denotes the required capability for \( t \) to act, and \( Y \) describes the capability offered after \( t \) has engaged in an interaction. No preconditions are necessary for establishing a new session, so no required capabilities are necessary in Line 1. After a session has been established, capabilities \( \{\text{Acc}_i\}_{i \in \{0...3\}} \) are available in the system. Lines 2 and 3 will modify which capabilities are present in the system depending on the number of available threads. For example, a model with \( q_1 = 2/3 \) can advance from \( \text{Acc}_0, \text{Acc}_1, \text{Acc}_2, \text{Acc}_3 \) to \( M_{S_0}, \text{Acc}_1, M_{S_2}, M_{S_3} \). There may be cases in which an execution of the protocol will not generate necessary capabilities for a communication operation to be engaged, leaving a protocol stuck. One case will be if \( q_1 = 2/3, q_2 = \forall \), since not enough \( M_{S_i} \) capabilities can be provided. We will defer the discussion about the interplay of capabilities and quality predicates to Section 7.

3. The Global Quality Calculus (GC_q)

In the following, \( C \) denotes a choreography; \( p \) denotes an annotated thread \( t[A]\{X; Y\} \), where \( t \) is a thread, \( X, Y \) are atomic formulae and \( A \) is a role annotation. We will use \( t \) to
denote \( \{t_1, \ldots, t_j\} \) for a finite \( j \). Variable \( a \) ranges over service channels, intuitively denoting the public identifier of a service, and \( k \in \mathbb{N} \) ranges over a finite, countable set of session (names), created at runtime. Variable \( x \) ranges over variables local to a thread. We use terms \( t \) to denote data and expressions \( e \) to denote optional data, much like the use of option data types in programming languages like Standard ML [20]. Expressions include arithmetic and other first-order expressions excluding service and session channels. In particular, the expression some\((t)\) signals the presence of some data \( t \) and none the absence of data. In our model, terms denote closed values \( v \). Names \( m, n \) range over threads and session channels. For simplicity of presentation, all models in the paper are finite.

**Definition 3.1 (GC_q syntax).**

\[
(\text{Choreographies}) \quad C ::= \eta; C \mid \text{if } e \triangleright p \text{ then } C \text{ else } C \mid 0 \mid (\nu r) C
\]

\[
(\text{Annotated threads}) \quad p ::= t[A]\{X;Y\}
\]

\[
(\text{Interactions}) \quad \eta ::= \tilde{p}_r \quad \textbf{start} \quad \tilde{p}_a : a(k) \quad \text{(init)}
\]

\[
| p_r.e \rightarrow &^q(p_s : x_s) : k \quad \text{(bcast)}
\]

\[
| &^q(p_r.e_r) \rightarrow p_s : x : \{k, op\} \quad \text{(reduce)}
\]

\[
| p_r \rightarrow &^q(\tilde{p}_a) : k[l] \quad \text{(select)}
\]

A novelty in this variant of the Global calculus is the addition of quality predicates \( q \) binding vectors in a multiparty communication. Essentially, \( q \) determines when sufficient inputs/outputs are available. For example, \( q \) can be \( \exists \), meaning that one sender/receiver is required in the interaction, it can be \( \forall \) meaning that all of them are needed, or it can be \( m/n \), describing that \( m \) out of \( n \) components are needed. The syntax of \( q \) and other examples can be summarised in Figure 2. We require \( q \) to be monotonic (in the sense that \( q(t_r) \) implies \( q(t_s) \) for all \( t_s \subseteq t_r \)) and satisfiable.

Choreographies are composed by standard operators of restriction, if-then choice and inaction, as standard in the literature. We will focus our discussion on the novel interactions. First, **start** defines a (multiparty) session initiation between active annotated threads \( \tilde{p}_r \) and annotated service threads \( \tilde{p}_a \) over a (shared) service channel \( a \). Each active thread (resp. service thread) implements the behaviour of one of the roles in \( \tilde{A}_r \) (resp. \( \tilde{A}_s \)), sharing a new session name \( k \). We assume that a session is established with at least two participating processes, therefore \( 2 \leq |\tilde{p}_r| + |\tilde{p}_a| \), and that threads in \( \tilde{p}_r \cup \tilde{p}_a \) are pairwise different.

The language features broadcast, reduce and selection as collective interactions. A broadcast implements a one-to-many communication pattern, where a session channel \( k \) is used to transfer the evaluation of expression \( e \) (located at \( p_r \)) to threads in \( \tilde{p}_a \), with the resulting binding of variable \( x \) at \( p_t \), for each \( p_t \in \tilde{p}_a \). A reduce combines one-to-many communications as well as aggregation functions. In \( &^q(p_r.e_r) \rightarrow p_s : x : \{k, op\} \), each
annotated thread \( p_i \) in \( \tilde{p}_r \) evaluates an expression \( e_i \), and the aggregate of all receptions is evaluated using \( \text{op} \) (an operator defined on multisets such as \( \max, \min, \) etc.) Interaction \( p_r \rightarrow &^q(\tilde{p}_s) : k[l] \) describes a collective label selection: \( p_r \) communicates the selection of label \( l \) to peers in \( \tilde{p}_s \) through session \( k \). In order to simplify the technical development, we will assume that \( q = \forall \) in \( p_r \rightarrow &^q(\tilde{p}_s) : k[l] \) (that is, we require all receiving participants to perform a collective selection).

Central to our language are progress capabilities. Pairs of atomic formulae \( \{X;Y\} \) at each annotated thread state the necessary preconditions for a thread to engage \((X)\), and the capabilities provided after its interaction \((Y)\). As we will see in the semantics, there are no associated preconditions for session initiation (i.e. threads are created at runtime), so we normally omit them. Explicit \( x@p \) (resp. \( e@p \)) indicate the variable/boolean expression \( x \) (resp. \( e \)) is located at \( p \). Term \((ur)C\) represents the restriction of a name \( r \) in \( C \), and it will be only used at runtime. The same notation standard will be used for all the terms written surrounded by boxes (as in \( \boxed{(ur)C} \)). We often omit \( 0 \), empty vectors and atomic formulae \( \{X;Y\} \) from annotated threads when unnecessary.

The set of free term variables \( f_v(C) \), free names \( f_n(C) \), free threads \( f_t(C) \), service channels \( f_{sc}(C) \) and roles \( \text{roles}(C) \) are inductively defined as usual for \( C \) and for \( \eta \). An interaction \( \eta \in \eta.C \) can bind session channels, choreographies and variables. In \((\text{init})\), variables \( \{\tilde{p}_r,a\} \) are free while variables \( \{\tilde{p}_s,k\} \) are bound (since they are freshly created). In \((\text{bcast})\), variables \( \tilde{x}_s \) are bound. The \((\text{reduce})\) interaction binds \( \{x\} \).

3.1. Expressivity. The importance of roles is only crucial in a \textbf{start} interaction. Technically, one can infer the role of a given thread \( t \) used in an interaction \( \eta \) by looking at the \textbf{start} interactions preceding it in the abstract syntax tree. \( GC_\eta \) can still represent unicast message-passing patterns as in [8]. Unicast communications \( p_1.e \rightarrow p_2 : x : k \) can be encoded in multiple ways using broadcast/reduce operators. For instance, \( p_1.e \rightarrow \&^q(p_2 : x) : k \) and \( \&^q(p_1.e) \rightarrow p_2 : x : (id, k) \) are just a couple of possible implementations. Similar considerations apply also for unicast selection \( A \rightarrow B : l[k] \).

3.2. Semantics. Choreographies are considered modulo standard structural and swapping congruence relations (resp. \( \equiv, \simeq_C \)). Structural congruence \( \equiv \) is defined as the least congruence relation on \( C \) supporting \( \alpha \)-renaming, such that rules \((\nu x) (\nu y) C \equiv (\nu y) (\nu x) C \) and \((\nu x) 0 \equiv 0 \) hold.

The swap congruence [12] provides a way to reorder non-conflicting interactions, allowing for a restricted form of asynchronous behavior. Non-conflicting interactions are those involving sender-receiver actions that do not conform a control-flow dependency. For instance, \( t_A.e_A \rightarrow \&^q_1(t_B : x_B) : k_1 ; t_C.e_C \rightarrow \&^q_2(t_D : x_D) : k_2 \simeq_C t_C.e_C \rightarrow \&^q_2(t_D : x_D) : k_2 ; t_A.e_A \rightarrow \&^q_1(t_B : x_B) : k_1 \). Formally, let \( T(C) \) be the set of threads in \( C \), defined inductively as \( T(\eta; C) \equiv T(\eta) \cup T(C) \), and \( T(\eta) \equiv \bigcup_{i \in [1..j]} t_i \) if \( \eta = t_i[A_1].e \rightarrow \&^q(t_j[A_2] : x_2, \ldots, t_j[A_j] : x_j) : k \) (similarly for \( (\text{init}) \), \( (\text{reduce}) \) and \( (\text{select}) \), and standardly for the other process constructs in \( C \)). The swapping congruence rules are presented in Figure [3] where we use the shorthand notation \( A \# B \) to denote set disjointness, \( A \cap B = \emptyset \).

A state \( \sigma \) keeps track of the capabilities achieved by a thread in a session, and it is formally defined as set of maps \((t,k) \mapsto X \). The rules in Figure [4] define state manipulation operations, including update \((\sigma[\sigma']) \), and lookup \((\sigma(t,k)) \).
o.w.

\[ T(\eta) \neq T(\eta') \]

\[ \eta; (\eta'\); C] \approx_C \eta'\); (\eta'\); C] \]

p \notin T(\eta) \]

\[ \begin{array}{ll}
  \text{if } e\oplus p \text{ then } \eta; C_1 \text{ else } \eta; C_2 \approx_C \eta; C_1 \text{ else } C_2 \\
  \text{if } e\oplus p \text{ then } (if e'\oplus r \text{ then } C_1 \text{ else } C_2) \FUZZY{\approx_C} \text{ else } (if e'\oplus r \text{ then } C_1 \text{ else } C_2) \\
\end{array} \]

Figure 3: Swap congruence relation, \( \approx_C \)

\[
Y = X \text{ if } (t, k, X) \in \sigma \quad Y = \emptyset \text{ o.w.}
\]

\[
\sigma(t, k) = Y
\]

\[
\delta = \left\{ (t, k, X) \mid (t, k, X) \in \sigma \land (t, k, Y) \in \sigma' \right\}
\]

\[
\sigma|\sigma' = (\sigma|\delta), \sigma'
\]

Figure 4: State lookup and update rules

A substitution \( \theta = [(p_1, \text{some}(v_1)), \ldots, (p_n, \text{some}(v_n))/x_1@p_1, \ldots, x_n@p_n] \) maps each variable \( x_i \) at \( p_i \) to optional data \( \text{some}(v_i) \) for \( 1 \leq i \leq n \). A composition \( \theta_1 \circ \theta_2(x) \) is defined as \( \theta_1 \circ \theta_2(x) := \theta_1(\theta_2(x)) \), and \( q(t_1, \ldots, t_n) = \bigwedge_{i \in 1 \leq i \leq n} t_i \) if \( q = \forall \), \( q(t_1, \ldots, t_n) = \bigvee_{i \in 1 \leq i \leq n} t_i \) if \( q = \exists \), and possible combinations therein. As for process terms, \( \theta(C) \) denotes the application of substitution \( \theta \) to a term \( C \) (and similarly for \( \eta \)).

We now have all the ingredients to understand the semantics of \( GC_q \). The set of transition rules in \( \lambda \) is defined as the minimum relation on names, states, and choreographies satisfying the rules in Figure 3. The operational semantics is given in terms of labelled transition rules. Intuitively, a transition \( \langle \sigma, C \rangle \xrightarrow{\lambda} \langle \sigma', C' \rangle \) expresses that a configuration \( \langle \sigma, C \rangle \) fires an action \( \lambda \) and evolves into \( \langle \sigma', C' \rangle \). The exchange function \( \delta(\cdot, \cdot, Y) \) returns \( (Z \setminus X) \cup Y \) if \( X \subseteq Z \) and \( Z \) otherwise. Actions are defined as \( \lambda := \{ \tau, \eta \} \), where \( \eta \) denotes interactions, and \( \tau \) represents an internal computation. Relation \( e\oplus p \downarrow v \) describes the evaluation of a expression \( e \) (in \( p \)) to a value \( v \).

We now give intuitions on the most representative operational rules. Rule \( ^{\circ}[\text{init}] \) models initial interactions: state \( \sigma \) is updated to account for the new threads in the session, updating the set of used names in the reductum. Rule \( ^{\circ}[\text{cast}] \) models broadcast: given an expression evaluated at the sender, one needs to check that there are enough receivers ready to get a message. Such a check is performed by evaluating \( q(J) \). In case of a positive evaluation, the execution of the rule will: (1) update the current state with the new states of each participant engaged in the broadcast, and (2) apply the partial substitution \( \theta \) to the continuation \( C \). Rule \( ^{\circ}[\text{set}] \) behaves in a similar way. The behaviour of a reduce operation is described via rule \( ^{\circ}[\text{red}] \). If all required threads are present, one can proceed by evaluating the operator to the set of received values, binding variable \( x \) to its results. Rule \( ^{\circ}[\text{conf}] \) allows choreographies to evolve up to swap and structural congruence. Finally, rule \( ^{\circ}[\text{if}] \) represent standard if-then-else constructs in sequential languages.

In contrast to previous works in multiparty sessions (e.g. [13]), we present an early semantics: it allows for transitions to match with distinct moves, depending on which participants are available first. We opted to favor an application-based QoS rather than a node-based QoS, as described in Section 2.

Remark 3.2 (Broadcast vs. Selection). The inclusion of separate language constructs for communication and selection takes origin in early works of structured communications [25].
Analogous to method invocation in object-oriented programming, selections play an important role in making choreographies projectable to distributed implementations. We illustrate their role with an example. Assume a session key \( s \) to behave differently based on the decisions made by \( s \).

The use of a selection operator permits s to be notified by r about which behavior to implement: \( p \rightarrow r : x : k \) if \( (x \rightarrow r) \) then \( (r, d) \rightarrow s : y : k \) else \( (s, f) \rightarrow r : z : k \) branches into two different communication flows: one from \( r \) to \( s \) if the evaluation of \( x \rightarrow r \) is true, and one from \( s \) to \( r \) otherwise. Although the evaluation of the guard in the if refers only to \( r \), the projection of such choreography to a distributed system requires \( s \) to behave differently based on the decisions made by \( r \).

Figure 5: GCq: Operational Semantics
Remark 3.3 (Broadcast vs. Reduce). We opted in favor of an application-based QoS instead of a classical node-based QoS, as described in Section 2. This consideration motivates the asymmetry of broadcast and reduce commands: both operations are blocked unless enough receivers are available, however, we give precedence to senders over receivers. In a broadcast, only one sender needs to be available, and provided availability constraints for receivers are satisfied, its evolution will be immediate. In a reduce, we will allow a delay of the transition, capturing in this way the fact that senders can become active in different instants.

The reader familiar with the Global Calculus may have noticed the absence of a general asynchronous behaviour in our setting. In particular, rule:

\[
\frac{\langle \sigma, C \rangle \xrightarrow{\lambda} \langle \sigma', (\nu \overline{\eta}) C' \rangle \quad \eta \neq \text{start} \quad \text{snd}(\eta) \subseteq \text{fn}(\lambda)}{\langle \sigma, \eta; C \rangle \xrightarrow{\text{rcv}(\eta)} \langle \sigma', (\nu \overline{\eta}) (\eta; C') \rangle}
\]

corresponding to the extension of rule \([\varepsilon|\text{async}]\) in [12] with collective communications, is absent in our semantics. The reason behind it lies in the energy considerations of our application: consecutive communications may have different energetic costs, affecting the availability of sender nodes. Consider for example the configuration

\[
\langle \sigma, (t_A[A][X;Y].e \rightarrow \&^{\overline{k}}(t_{A[B_r]} : x_r) : k); t_A[A][Y;X].e \rightarrow \&^{\overline{r}}(t_{A[B_s]} : x_s) : k \rangle
\]

with \(\overline{r} \neq \overline{s}\) and \(\sigma(t_A, k) = X\). If the order of the broadcasts is shuffled, the second broadcast may consume all energy resources for \(t_A\), making it unavailable later. Formally, the execution of a broadcast update the capabilities offered in \(\sigma\) for \(t_A, k\) to \(Y\), inhibiting two communication actions with same capabilities to be reordered. We will refrain the use rule \([\varepsilon|\text{async}]\) in our semantics.

We say that a choreography \(C\) is restriction-free if \(C\) does not contain any subterm \((\nu x) C\). We can proceed to define our notion of progress.

Definition 3.4 (Progress: Choreographies). Choreography \(C\) progresses if there exists \(C', \sigma', \lambda\) such that \(\langle \sigma, C \rangle \xrightarrow{\lambda} \langle \sigma', C' \rangle\), for all \(\sigma\).

4. Type-checking Progress

One of the challenges regarding the use of partial collective operations concerns the possibility of getting into runs with locking states. Consider a variant of Example 2.1 with \(q_1 = \exists\) and \(q_2 = \forall\). This choice leads to a blocked configuration. The system blocks since the collective selection in Line (2) continues after a subset of the receivers in \(t_1, t_2, t_3\), have executed the command. Line (3) requires all senders to be ready, which will not be the most general case. The system will additionally block if participant dependencies among communications are not preserved. The choreography in Figure 6 blocks for \(q_1 = \exists\), since the selection operator in Line (2) can execute a communication over \(t_2\), blocking the reduce operation in Line (3).

We introduce a type system to ensure progress on variable availability conditions. A judgment is written as \(\Psi \vdash C\), where \(\Psi\) is a list of formulae in Intuitionistic Linear Logic (ILL) [19]. Intuitively, \(\Psi \vdash C\) is read as the formulae in \(\Psi\) describe the program point immediately before \(C\). Formulae \(\psi \in \Psi\) take the form of the constant \(tt\), ownership types
Figure 6: Variant of Example 2.1 with locking states

of the form \( p : k[A] \triangleright X \), and the linear logic version of conjunction, disjunction and implication \((\otimes, \oplus, \rightarrow)\). Here \( p : k[A] \triangleright X \) is an ownership type, asserting that \( p \) behaves as the role \( A \) in session \( k \) with atomic formula \( X \). Moreover, we require \( \Psi \) to contain formulae free of linear implications in \( \Psi \vdash C \).

Figure 7 presents selected rules for the type system for GC\(_q\). Since the rules for inaction, restriction, conditionals and non-determinism are standard, we focus our explanation on the typing rules for communications. Rule \([\text{Init}]\) types new sessions: \( \Psi \) is extended with function \( \text{init}(t_p[A]\{X\},k) \), that returns a list of ownership types \( t_p : k[A] \triangleright X \). Conditions \( \tilde{t}_s \not\subseteq T(\Psi) \) and \( k \not\in K(\Psi) \) ensure that new names do not exist neither in the threads nor in the used keys in \( \Psi \).

The typing rules for broadcast, reduce and selection are analogous, so we focus our explanation in \([\text{Bcast}]\). Here we abuse of the notation, writing \( \Psi \vdash \psi \) to denote type checking, and \( \Psi \vdash \psi \) to denote formula entailment. The semantics of \( \forall \geq 1 J \text{ s.t. } C : D \) is given by \( \forall J \text{ s.t. } C : D \land \exists J \text{ s.t. } C \). The judgment

\[
\Psi \vdash (t_A[A]\{X_A;Y_A\}.e \rightarrow &\overline{\delta}(t_r[B_r]\{X_r;Y_r\} : x_r) : k) ; C
\]

succeeds if environment \( \Psi \) can provide capabilities for sender \( t_A[A] \) and for a valid subset \( J \) of the receivers in \( t_r[B_r] \). \( J \) is a valid subset if it contains enough threads to render the quality predicate true \((q(J))\), and judgment \( \psi_A, (\psi_j)_{j \in J}, \phi \rightarrow \phi' \vdash \phi' \) is provable. This proof succeeds if \( \psi_A \) and \( (\psi_j)_{j \in J} \) contain ownership types for the sender and available receivers with corresponding capabilities. Finally, the type of the continuation \( C \) will consume the resources used in the sender and all involved receivers, updating them with new capabilities for the threads engaged.

**Example 4.1.** In Example 2.1, \( tt \vdash C \) if \( (q_1 = \forall) \land (q_2 = \{\forall, \exists\}) \). In the case \( q_1 = \exists, q_2 = \forall \), the same typing fails. Similarly, \( tt \not\vdash C \) if \( q_1 = \exists \), for the variant of Example 2.1 in Figure 6.

A type preservation theorem must consider the interplay between the state and formulae in \( \Psi \). We write \( \sigma \models \Psi \) to say that the tuples in \( \sigma \) entail the formulae in \( \Psi \). For instance, \( \sigma \models t : k[A] \triangleright X \) iff \( (t, k, X) \in \sigma \).

**Definition 4.2 (State satisfaction).** The entailment relation between a state \( \sigma \) and an environment \( \Psi \), and the entailment relation between a state \( \sigma \) and a formula \( \psi \) are written

\[1\]We do, however, use the full set of operators when performing proof search.
Choreography Formation ($\Psi \vdash C$),

$\Psi \leftarrow \text{init}(t_{[A]}{\{Y\}}, t_{[B]}){\{Y\}}, k) \vdash C \quad \text{if } t_{[A]}{\{Y\}} \not\in T(\Psi) \quad k \notin K(\Psi)$

$\Psi \leftarrow t_{[A]}{\{Y\}} \text{ start } t_{[B]}{\{Y\}} : a(k); C$

$\forall \geq 1, \text{ s.t.} \left( J \subseteq T \land q(J) \land \Psi = \psi_{A}, (\psi_j)_{j \in J}, \Psi'$

$t_A : k \{A\} \triangleright Y_A, (t_j : k \{B_j\} \triangleright Y_j)_{j \in J}, \Psi' \vdash C \quad e@t_A : \text{opt.data} (\vdash x_i@t_i : \text{opt.data})_{i=1}^{C}$

$\Psi \leftarrow (t_A[A]\{X_A; Y_A\}, e \rightarrow & \psi(t_i[B_r]\{X_r; Y_r\}; x_r) : k) ; C$

$\forall \geq 1, \text{ s.t.} \left( J \subseteq T \land q(J) \land \Psi = \psi_{B}, (\psi_j)_{j \in J}, \Psi'$

$t_B : k \{B\} \triangleright Y_B, (t_j : k \{A_j\} \triangleright Y_j)_{j \in J}, \Psi' \vdash C \quad e@t_B : \text{opt.data} (\vdash x_i@t_i : \text{opt.data})_{i=1}^{C}$

$\Psi \leftarrow (\& \psi(t_i[A_r]\{X_r; Y_r\}; e_r) \rightarrow t_B[B]\{X_B; Y_B\} : x : \langle k, op \rangle) ; C$

$\forall \geq 1, \text{ s.t.} \left( J \subseteq T \land q(J) \land \Psi = \psi_{A}, (\psi_j)_{j \in J}, \Psi'$

$t_A : k \{A\} \triangleright Y_A, (t_j : k \{B_j\} \triangleright Y_j)_{j \in J}, \Psi' \vdash C$

$\Psi \leftarrow (t_A[A]\{X_A; Y_A\} \rightarrow & \psi(t_i[B_r]\{X_r; Y_r\}) : k[i_0]) ; C$

$\Psi \leftarrow \text{if } e@t \text{ then } C_1 \text{ else } C_2 \quad [\text{Tcond}] \quad \Psi \leftarrow C \quad [\text{Tres}] \quad \Psi \leftarrow C \quad [\text{Tinact}] \quad \Psi \leftarrow \emptyset$

Data Typing,

$[\text{TD1}] \vdash t@p : \text{data} \quad [\text{TD2}] \vdash v@p : \text{data}$

$[\text{TOD1}] \vdash e@p : \text{opt.data} \quad [\text{TOD2}] \vdash \text{some}(v@p) : \text{opt.data} \quad [\text{TOD3}] \vdash \text{none}@p : \text{opt.data}$

State Formation ($\sigma : \text{state}$),

$\emptyset : \text{state} \quad [\text{TS2}] \quad \sigma \triangleright \text{ state} \quad \sigma(t[A], k) = \emptyset \quad X \in \text{dom}(\Sigma)$

$\sigma(t[A], k, X) : \text{state} \quad [\text{TS3}] \quad \sigma \triangleright \text{ state} \quad \sigma(\{A\}, k, X) : \text{state}$

$\sigma \triangleright \text{ state} \quad \sigma(\{A\}, k, X) : \text{state}$

Formula Formation ($\Psi : \text{form}$),

$\psi \triangleright \text{ form} \quad [\text{TF1}] \quad \psi, \Psi \triangleright \text{ form} \quad [\text{TF2}] \quad \psi : \text{form} \quad [\text{TF3}] \quad \psi : \text{form} \quad [\text{TF4}] \quad \psi \triangleright \text{ form} \quad [\text{TF5}]$

Figure 7: GC_q: Type checking
σ \models \Psi \text{ and } \sigma \models \psi, \text{ respectively. They are defined as follows:}

\begin{align*}
\sigma \models \cdot & \iff \sigma \text{ is defined} \\
\sigma \models \psi, \Psi & \iff \sigma \models \psi \text{ and } \sigma \models \Psi \\
\sigma \models \texttt{tt} & \iff \sigma \text{ is defined} \\
\sigma \models t : k [A] \triangleright X & \iff (t,k,X) \in \sigma \\
\sigma \models \psi_1 \otimes \psi_2 & \iff \sigma = \sigma', \sigma'' \models \psi_1 \land \sigma'' \models \psi_2 \\
\sigma \models \psi \setminus \delta & \iff \exists \sigma' \text{ s.t. } \sigma' \models \psi \land \sigma = \sigma' \setminus \delta
\end{align*}

**Theorem 4.3** (Type Preservation). If \( \langle \sigma, C \rangle \xrightarrow{\lambda} \langle \sigma', C' \rangle \), \( \sigma \models \Psi \), and \( \Psi \vdash C \), then \( \exists \Psi'. \Psi' \vdash C' \) and \( \sigma' \models \Psi' \).

*Proof.* It follows by rule induction on the transition relation \( \langle \sigma, C \rangle \xrightarrow{\lambda} \langle \sigma', C' \rangle \). Details are presented in Appendix A.2. \( \square \)

**Theorem 4.4** (Well-typed choreographies progress). If \( \Psi \vdash C \), \( \sigma \models \Psi \) and \( C \not\equiv 0 \), then \( C \) progresses.

*Proof.* Proof by contradiction. Let us assume that \( \Psi \vdash C \), \( \sigma \models \Psi \) and \( C \not\equiv 0 \) and \( \langle \sigma, C \rangle \xrightarrow{\lambda} \).

We proceed by induction on the structure of \( C \) to show that such \( C \) does not exists. \( \square \)

The decidability of type checking depends on the provability of formulae in our ILL fragment. Notice that the formulae used in type checking corresponds to the Multiplicative-Additive fragment of ILL, whose provability is decidable [31]. For typing collective operations, the number of checks grows according to the amount of participants involved. Decidability exploits the fact that for each interaction the number of participants is bounded.

**Theorem 4.5** (Decidability of Typing). \( \Psi \vdash C \) is decidable.

## 5. Session Types

We now present a type system which allows one to specify multiparty protocols, allowing only specifications that respect causality relations between interactions.

**Definition 5.1** (Global Types: Syntax).

\begin{align*}
(Sorts) & \quad S ::= \text{ bool } | \text{ int } | \text{ string } | \ldots \\
\text{(Global Types)} & \quad G ::= A \rightarrow B : \langle S \rangle ; G \quad \text{(broadcast)} \\
& \quad \mid \bar{A} \rightarrow B : \langle S \rangle ; G \quad \text{(reduce)} \\
& \quad \mid A \rightarrow \bar{B} : \{l_i : G_i\}_{i \in I} \quad \text{(branch)} \\
& \quad \mid \text{end} \quad \text{(end)}
\end{align*}

The syntax of global types describes the flow of interactions one can have in \( GC_q \). Sorts \text{ bool, int, string, ...} \) describe basic value types. Type \( A \rightarrow \bar{B} : \langle S \rangle ; G \) dictates the presence of a one-to-many communication from role \( A \) to roles \( \bar{B} \) of sort \( S \), followed by a continuation of type \( G \). The type \( \bar{A} \rightarrow B : \langle S \rangle ; G \) describes a many-to-one communication from roles \( \bar{A} \) to role \( B \) with sort \( S \). In type \( A \rightarrow \bar{B} : \{l_i : G_i\}_{i \in I} \), role \( A \) will spawn method identified
with label $l_i$ collectively on threads implementing roles $\tilde{B}$, following with a continuation of type $G_i$ in each of the receivers. The type end indicates termination and is often omitted.

The labelled type transition relation $G \xrightarrow{\alpha} G'$ expresses the abstract execution of protocols, where $\alpha = \{A \rightarrow \tilde{B} : \langle S \rangle, A \rightarrow B : \langle S \rangle, A \rightarrow \tilde{B} : [l_i]\}$. $G \xrightarrow{\alpha} G'$ is the smallest relation on global types satisfying the rules given in Figure 8. Intuitively, the transition $G \xrightarrow{\alpha} G'$ expresses in $\alpha$ the interaction consumed. Rules $[G^{\text{Bcast}}]$ and $[G^{\text{Branch}}]$ track the one-to-many communications performed in a protocol, and rule $[G^{\text{Red}}]$ records the many-to-one patterns. Rule $[G^{\text{Swap}}]$ captures the swapping notion existing in choreographies, and it is based on a swap relation for types $G \simeq G'$. The set of rules documenting the behavior of $\simeq_G$ is presented in Figure 8.

A type judgment is written as $\Gamma; \Psi \vdash C \triangleright \Delta$. We commonly refer to $\Gamma$ and $\Delta$ as the service and session environments, respectively. The unrestricted environment $\Gamma$ contains different types of information. First, it contains maps from process variables to sorts, as in $x \cdot p : S$. Second, it contains maps from service channels to global types, as in $a : G(\tilde{A} | \tilde{B})$, where $\tilde{A}$ and $\tilde{B}$ represent the roles of the active and service processes, respectively. Furthermore, we assume that $\tilde{A}$ and $\tilde{B}$ are the only roles in $G$. Third, it contains ownership types, as in $p : k [A]$, asserting that $p$ behaves as the role $A$ in session $k$. Note that the ownership types used in this stage are a variant of the ownership types used in Section 4 with no capabilities. The environment $\Psi$ denotes a set of linear logic formulae, and it is
\[ \Gamma ::= \emptyset \quad \Delta ::= \emptyset \\
| \Gamma, a : G(\bar{A} | \bar{B}) \quad | \Delta, k : G \\
| \Gamma, x@p : S \\
| \Gamma, p : k[A] \]

Figure 10: Global Types: Typing environments

\[
\begin{align*}
\Gamma \vdash a : G(\bar{A} | \bar{B}) & \quad \Gamma, \text{init}(\{t_r[A_r], t_s[B_s], k\}) \vdash C \triangleright \Delta, k : G & \tilde{t}_s \neq \Gamma \\
\Gamma \vdash t_r[A_r]\{Y_r\} \text{ start} t_s[B_s]\{Y_s\} & : a(k); C \triangleright \Delta \\
\end{align*}
\]

Figure 11: GC\eta: Type checking - Global types

the same environment described in Section \[\text{Section}^4\] Finally, the linear environment \(\Delta\) contains maps from session variables to Global types, as in \(k : G\). Its purpose is to track the state of each running session with respect to the protocol. The set of typing rules defining the judgments for \(\Gamma; \psi \vdash C \triangleright \Delta\) is presented in Figure \[\text{Figure}^11\].

Intuitively, a choreography \(C\) is well typed with respect to \(\Gamma, \Psi\) and \(\Delta\) if its shared channels are used, its processes behave according to the global types in \(\Gamma\), and the capabilities for each collective communication in \(\Psi\) are respected. We proceed to describe the typing rules in Figure \[\text{Figure}^11\].

An important observation to make is that the information tracked by \(\Psi\) is independent from environments \(\Gamma\) and \(\Delta\). This allows us to divide the analysis into two independent
analyses, one for capabilities and one for global types [40]. Rule $\tau G$ represents this fact, dividing the analysis of $\Gamma; \Psi \vdash C \triangleright \Delta$ into separate analysis for causalities $\Psi \vdash C$ and for global types $\Gamma \vdash C \triangleright \Delta$. All other rules in Figure 11 pertain only global typing (no $\Psi$ is required).

In rule $\tau G\text{init}$ the sequence of interactions started with an (init) action is typed: each process in the term is given ownership to the role declared for it in the created session $k$ through the function $\text{init}(t_p[A], k)$, which returns a set of ownership assignments $\{t_q : k[B] \mid t_q[B] \in t_p[A]\}$. The condition $t_s \# \Gamma$ ensures that service threads are fresh.

The typing rules for communications are grouped into $\tau G\text{cast}$, $\tau G\text{red}$ and $\tau G\text{sel}$. In each of them we must check that the communication performed by the threads involved at both the sender and receivers owns their respective roles in the communication over the session in use. This is ensured by checking that the type environment contains the ownership typings $t_p : k[A]$ and $t_j : k[B_j]$ ($i \in \{2\ldots j\}$) for a broadcast operation, and similarly for reduce and selection. In $\tau G\text{cast}$, we additionally check that the type expression sent by the sender corresponds to the carried sort $\langle S \rangle$, and that the continuation is typed according to the initial type environment $\Gamma$ extended with the assignment of type $S$ to the variables used by the receiver threads in $t_2, \ldots, t_j$. Rule $\tau G\text{red}$ behaves complementary, ensuring that each of the expressions used by the sender threads behave according to the same sort $\langle S \rangle$, and ensuring that the continuation is typed according to the initial type environment $\Gamma$ extended with the assignment of type $S$ to the variable used by the receiver in the communication. Rule $\tau G\text{sel}$ types labelled selections: A selection of a label $l_h$ on session $k$ is well-typed if the label is in those allowed by the protocol of session $k$ ($h \in I$). The continuation must then implement the selected continuation $G_h$ on session $k$.

Rule $\tau G\text{cond}$ is the standard typing rule used to type conditional blocks: we must check that the expression $e@t$ is a proposition, and that each of the branches of the conditional are typed according to the session environment $\Delta$. Rule $\tau G\text{res}$ types name restriction in the standard way. Observe that if $r$ is a process identifier, then restriction only affect $\Gamma$ since $\Delta$ does not refer to processes.

Rule $\tau G\text{cast}$ types termination: $0$ is well-typed under any unrestricted environment $\Gamma$ and session environment $\Delta$ if each session $k$ typed in $\Delta$ has a type $\text{end}$, meaning that it has been successfully terminated. Predicate $\text{end}(\Delta)$ is formalized as $\{tt \mid \forall k : G \in \Delta, G = \text{end}\}$ and $\text{ff}$ otherwise.

**Example 5.2 (The typing in practice).** Consider the variant of Example 2.1:

$$
t_1\{x_1\}[S_1], t_2\{x_2\}[S_2], t_3\{x_3\}[S_3] \text{ start } t_m\{x_m\}[M] : \text{temperature}(k);
$$

$$
t_m\{x_m; y_m\}.\text{today} -\rightarrow \&^\gamma(t_1\{x_1; y_1\} : x_1, t_2\{x_2; y_2\} : x_2, t_3\{x_3; y_3\} : x_3) : k;
$$

$$
\&^\gamma(t_1\{y_1; z_1\}.\text{temp}, t_2\{y_2; z_2\}.\text{temp}, t_3\{y_3; z_3\}.\text{temp}) -\rightarrow t_m\{y_m; z_m\} : x_m : \langle k, \text{max}\rangle; 0
$$

We can show that the choreography in [5.1] is typable under environments $\Psi = \emptyset$ and $\Gamma = \text{temperature} : M -\rightarrow \langle S_1, S_2, S_3\rangle : \langle \text{date} \rangle; \langle S_1, S_2, S_3\rangle -\rightarrow M : \langle \text{float} \rangle; \text{end}$.

We can proceed to establish the technical results of the session type discipline. In the following, we write $\Delta \xrightarrow{k: \alpha} \Delta'$ to say that $k : G \in \Delta$, $G \xrightarrow{\Delta} G'$, and that $\Delta'$ corresponds to the substitution $\Delta[k : G'/k : G]$. Figure 12 formalizes the correspondence between labels in the choreography and their respective global types ($\Gamma \vdash \lambda k : \alpha$). We can now establish our type preservation theorem for global types.
Theorem 5.4 (Decidability). For any \( \Gamma; \Psi \vdash C \triangleright \Delta \), the checking of \( \Gamma; \Psi \vdash C \triangleright \Delta \) is decidable.

5.1. **Linearity.** A conflict (race condition) may be generated when implementing multiparty choreographies. While at the choreographic level one imposes a sequence of interactions among participants, the projection of a choreography into endpoints generate a set of participants acting concurrently. A poorly defined choreography may lead to implementations that do not follow the sequence imposed by the choreography. Take the following choreography:

\[
p[A] \text{ start } q[B] : a(k); r[D] \text{ start } s[E] : a(k'); C'
\] (5.2)

Here four processes start two different sessions using the same service and same session key. Once projected, threads implementing session initiation constructs \( p[A] \text{ start } q[B] : a(k) \) and \( r[D] \text{ start } s[E] : a(k') \) will compete. The race occurs when the thread implementing \( p[A] \) establishes a session with \( s[E] \). Such behavior will correspond to term

\[
\Gamma \vdash t_A : k[A] \quad \Gamma \vdash t_i : k[B_i] \quad \Gamma \vdash v @ t_A : S \quad \text{s.t. } i \in \{1...|B|\}
\]

\[
\Gamma \vdash t_A[A], v \rightarrow &q(t_r[B_r] : sb_r) : k \triangleright k : A \rightarrow B : \langle S \rangle
\]

\[
\Gamma \vdash t_B : k[B] \quad \Gamma \vdash t_i : k[A_i] \quad \Gamma \vdash v_i @ t_i : S \quad \text{s.t. } i \in \{1...|A|\}
\]

\[
\Gamma \vdash \&q(t_r[A_r], v_r) \rightarrow t_B[B] : \text{some}(v) : \langle k, op \rangle \triangleright k : A \rightarrow B : \langle S \rangle
\]

\[
\Gamma \vdash t_A[A] \rightarrow \&q(t_r[B_r]) : k[l] \triangleright k : A \rightarrow B : \langle l \rangle
\]
$p[A] \text{ start } s[E] : a(k)$, which do not exists in the original choreography. Similar considerations apply for the race between $r[D]$ and $q[B]$. We appeal to the use of linearity conditions, that we adapt to multiparty-collective interactions.

An interaction node, denoted by $n$, is an abstraction of a node in the abstract syntax tree. Node $n$ can be

- $\tilde{p}$ start $\tilde{q} : a$ for (init), with $\text{fn}(n) = \tilde{p}$,
- $p \rightarrow \tilde{q}$ for (bcast) and (select), with $\text{fn}(n) = \{\tilde{p}, \tilde{q}\}$, or
- $\tilde{p} \rightarrow q$ for (reduce), with $\text{fn}(n) = \{p, \tilde{q}\}$.

We say that $n_2$ depends on $n_1$ in $C$, written $n_1 \prec n_2 \in C$, whenever $n_1$ precedes $n_2$ in $C$ (i.e.: $n_1$ and $n_2$ cannot appear in different branches in conditional and sum statements). An interaction dependency $n_1 \prec_p n_2 \in C$ occurs whenever $n_1 \prec n_2 \in C$ and one of the following conditions hold:

- $n_1 = \tilde{p}$ start $\tilde{q} : a$ and $n_2 = p \rightarrow \tilde{q}'$ and $p \in \{\tilde{p}, \tilde{q}\}$, or
- $n_1 = \tilde{p}$ start $\tilde{q} : a$ and $n_2 = \tilde{p}' \rightarrow q$ and $p \in p'$ and $p \in \{\tilde{p}, \tilde{q}\}$, or
- $n_1 = \tilde{p}$ start $\tilde{q} : a$ and $n_2 = \tilde{r}$ start $\tilde{s} : b$, where $p \in \{\tilde{p}, \tilde{q}\}$, and $p \in \tilde{r}$, or
- $n_1 = \tilde{q} \rightarrow p$ and $p \in \text{fn}(n_2)$, or
- $n_1 = q \rightarrow \tilde{r}$ and $p \in \tilde{r}$ and $p \in \text{fn}(n_2)$.

The interaction dependency $n_1 \prec_p n_2 \in C$ says that the projection of a process $p$ for the interaction node abstracted by $n_2$ cannot occur before that for $n_1$. Interaction dependencies are the basis for establishing a linearity property.

**Definition 5.5** (Linearity [37]). Let $C$ be a choreography. We say that $C$ is linear if for all nodes $n_1 = \tilde{p}$ start $\tilde{q} : a$ and $n_2 = \tilde{r}$ start $\tilde{s} : a$ such that $n_1 \prec n_2 \in C$ we have that $\forall r \in \tilde{r}. \exists p \in \{\tilde{p}, \tilde{q}\}. n_1 \prec_p \ldots \prec_r n_2$.

Intuitively, linearity checks that for dependent nodes $n_1, n_2$ such that $n_1 \prec n_2$, if they both take the form of start nodes over a common service name $a$, then all active processes used in $n_2$ depend on some process in $n_1$. In this way, the races between active processes explained before are avoided.

In the following, we recall that bound variables are renamed apart in $C$; That means that for two dependent (start) nodes using the same service name, the session keys used will be different. The use of renaming for session keys proves useful by limiting additional races where service processes compete with active processes in the establishment of a new session.

6. THE ENDPOINT QUALITY CALCULUS ($EC_q$), AND ENDPOINT PROJECTION

The $EC_q$ calculus extends the Quality calculus [11] with session-based communication and input-output queues. In addition to the syntactic categories defined in Section 5, $P, Q, \ldots$ denote processes, $k, \ldots, r, s$ denote names, $p$ denotes an annotated thread $t[A]$, where $t$ is a thread. We will use $t$ to denote $\{t_1, \ldots, t_j\}$ for a finite $j$. 

Definition 6.1 (ECₗ Syntax).

\[ P, Q ::= \pi[\tilde{A}(k); P] \mid a[A](k); P \mid !a[A](k); P \mid P \mid Q \mid k[A]!q[\tilde{B}(\varepsilon); P] \mid k[A]?[B](x); P \mid k[A]?q[B](x); P \mid k[A]!q[\tilde{B}] \triangleright i; P \mid k[A]?[B] \triangleright \{i_i : P_i\}_{i \in I} \]
\[ \text{if } e \text{ then } P \text{ else } Q \]
\[ (\nu k) P \]
\[ h ::= \emptyset \mid m \cdot h \]
\[ m ::= (A, q : ((\tilde{B} : b) : w)) \mid (q : ((A : b : sb) : B)) \]
\[ sb ::= \text{some}(v) \mid \text{none} \]
\[ w ::= sb \mid l \]
\[ b ::= \text{tt} \]

The first three terms correspond to a session establishment phase. A process \( \pi[\tilde{A}(k); P \mid a[A](k); P \mid !a[A](k); P \mid P \mid Q \) acts as a requester for service \( a \), with roles \( \tilde{A} \). It will interact with endpoint providers implementing each of the behaviours in \( \tilde{A} \), being those replicated services (i.e.: \( !a[A](k); P \), or one-time instances (i.e.: \( a[A](k); P \)). In these cases, \( P \) denotes the continuation process. The pair \( k[A]!q[\tilde{B}(\varepsilon); P \mid k[A]?[B](x); P \) models one-to-many communications. While the sender part of a broadcast is parameterised with a quality predicate, the receiver does not require it, as receivers only communicate with one sender. The pair \( k[A]?q[B](x); P \mid k[A]!q[\tilde{B}] \triangleright i; P \mid k[A]?[B] \triangleright \{i_i : P_i\}_{i \in I} \) implements many-to-one communication patterns. Dual to broadcast, here the receiver process requires the quality predicate, while the sender process does not. The pair \( k[A]!q[\tilde{B}] \triangleright i; P \mid k[A]?[B] \triangleright \{i_i : P_i\}_{i \in I} \) implements a one-to-many method selection (where \( \{i_i\} \) should be pairwise distinct). Runtime processes \( \text{wait}^i(k, \tilde{A}, B, op, x) \mid P \) and \( \text{wait}^0(k, A, B) \mid P \) implement queue-synchronization processes, and interact directly with input/output session queues \( k : h \). Each queue contains messages with one sender and many recipients \( (A, q : ((\tilde{B} : b) : w)) \) or many recipients and one sender \( (q : ((A : b : sb) : B)) \). Other process constructs, such as parallel composition, if-then constructs, and restriction are standard. Boxed terms can only be used at runtime. The free session channels, free term variables and service channels are defined as usual over processes are denoted by \( fsc(P), fv(P) \) and \( channels(P) \) respectively.

6.1. Semantics. ECₗ is equipped with a structural congruence relation over processes.

Definition 6.2 (Structural Congruence in ECₗ). The structural congruence relation \( \equiv \) in ECₗ is the least congruence on processes supporting \( \alpha \)-renaming, such that \( (P, 0, \mid ) \) is an abelian monoid, and the following rules are satisfied:

(i) (vr) \( 0 \equiv 0 \),
(ii) \((\nu r)(\nu s) P \equiv (\nu s)(\nu r) P\),

(iii) \((\nu r)(P \mid Q) \equiv (\nu r) P \mid Q\) if \(r \notin \text{fn}(Q)\),

(iv) \(k: h \cdot (A, q: \langle \bar{B} : w' \rangle) \cdot (C, q': \langle \bar{D} : w' \rangle) \cdot h' \equiv k: h \cdot (C, q': \langle \bar{D} : w' \rangle) \cdot (A, q: \langle \bar{B} : w \rangle) \cdot h'\)

\(\text{if } C \neq A \text{ or } B \neq D\),

(v) \(k: h \cdot (q: \langle A : sb \rangle, B) \cdot (q': \langle C : sb' \rangle, D) \cdot h' \equiv k: h \cdot (q': \langle C : sb' \rangle, D) \cdot (q: \langle A : sb \rangle, B) \cdot h'\)

\(\text{if } A \# C \text{ or } B \neq D\).

We give an operational semantics in terms of labeled reductions \(P \mu \rightarrow P'\), where

\[
\mu ::= \tau \quad | \quad A \text{ start } \bar{B} : a(k) \quad | \quad !A \rightarrow \bar{B} : k(v)
\]

\[
| \quad !A \rightarrow \bar{B} : k[l] \quad | \quad ?A \rightarrow \bar{B} : k[l] \quad \downarrow_\tau \quad \uparrow_\tau
\]

The operational semantics for \(EC_\varphi\) is defined by the rules given in Figure [13]. We give an intuition of the most important rules. Rule \([\varphi]_{\text{init}}\) describes session initiation: A requester process \(\pi[A, \bar{B}](k)\); \(P\) can establish a new session \(k\), if it is in interaction with active threads \(\prod_{i \in [2,...,\bar{A}]} a[A_i](k); P_i\), and replicated services \(\prod_{i \in [1,...,\bar{B}]} !a[B_i](k).P\).

Asynchronous queue-based communication is implemented by the interplay of rules \([\varphi]_{\text{bc.o}}\), \([\varphi]_{\text{bc.l}}\) and \([\varphi]_{\text{wait.o}}\). Starting with a parallel composition of a sender process and a queue, rule \([\varphi]_{\text{bc.o}}\) adds to the session queue the contents resulting of evaluating expression 2 at the sender side. In the meantime, the sender process will move into a waiting state, denoted by \(\text{wait}^\varphi(k, A, \bar{B})\); \(P\). Rule \([\varphi]_{\text{bc.l}}\) captures the interplay between receivers and the queue. Its transition updates the message on top of the session queue, generating a substitution of the communicated value on the receiver process. In order to avoid performing the substitution multiple times over the same participant, the queue will be modified to include information regarding the identity of the receiver. Finally, dequeueing occurs once the evaluation of the quality predicate over the set of performed substitutions deems satisfiable (Rule \([\varphi]_{\text{wait.o}}\)). At this point, we will have the following concurrent processes:

- A sender in its waiting state, \(\text{wait}^\varphi(k, A, \bar{B})\); \(P\).
- A queue tracking roles who have performed substitutions \((B')\), and those who have not \((B'')\).
- A parallel composition of all receiver processes who have not yet synchronised, \(\prod_{B_i \in \bar{B}'} k[B_i]?[A](x_i); Q_i\).

The consequence of this transition is the dequeueing of the top message from the queue, the activation of the sender process, and a none substitution on all receiver processes that did not synchronise with the queue. Similar considerations are given for the triad of label selection rules \([\varphi]_{\text{se.o}}\), \([\varphi]_{\text{br.o}}\), and \([\varphi]_{\text{wait.o}}\).

Rules for reduce act similarly as the ones for broadcast: A reduce process enqueues a message with placeholders for each of the senders involved, as well as the quality predicate, and blocks until enough senders have sent information (c.f.: rule \([\varphi]_{\text{red.o}}\)). The presence of a sender will update the queue, enclosing the new value \(v_i\) as an optional datatype (c.f.: rule \([\varphi]_{\text{red.o}}\)). The release of the reduce happens in Rule \([\varphi]_{\text{wait.i}}\), once enough senders have contributed with values, the substitution of \(x\) with the result of the operation \(op(sb_1, \ldots, sb_n)\) is performed on the continuation of the reduce, and the queue is updated. The remaining rules are standard in the session \(\pi\) calculus.
6.2. **Endpoint Projection.** The projection function maps the behaviours described by a choreography into endpoints. Special care must be payed when constructing the endpoint projection. In particular, an endpoint may implement different behaviours depending on the choreography made in a choreography. For instance, consider the following choreography:
if \( e @ P_A \) then \((P_A \rightarrow & (P_B) : k[l_1])\) else \((P_A \rightarrow & (P_B) : k[l_2])\) \hspace{1cm} (6.1)

When projecting the behavior of thread \( P_B \), it is not clear a priori whether the endpoint should implement the behaviour dictated by label \( l_1 \) or by \( l_2 \). We make use of a merge operator \([10][17]\) to collect all such labels into a label branching operator.

**Definition 6.3** (Merging). \( P \sqcup Q \) is a partial commutative binary operator on processes that is well-defined iff \( P \bowtie Q \) and it satisfies the following rules:

\[
(k[A]?[B] \triangleright \{ l_i : P_i \}_{i \in I}) \sqcup (k[A]?[B] \triangleright \{ l_i : Q_i \}_{i \in J}) = k[A]?[B] \triangleright \{ l_i : P_i \}_{i \in I \setminus J} \cup \{ l_i : Q_i \}_{i \in J \setminus I} \cup \{ l_i : (P_i \sqcup Q_i) \}_{i \in I \cap J}
\]

\[P \sqcup Q = P' \bowtie Q' \quad (P \equiv P', Q \equiv Q')\]

Intuitively, the merge takes branching processes with the same roles and generates a single process with all their options. Above, \( P \bowtie Q \) denotes the smallest congruence relation over endpoints processes such that:

\[\forall i \in (K \cup J).P_i \bowtie Q_i \quad \forall k \in (K \setminus J).\forall j \in (J \setminus K).l_k \neq l_j \quad (6.2)\]

**Definition 6.4** (Process Projection). \([C]^p\) is a partial function from choreographies to processes, defined on the structure of \( C \) according to the rules in Figure [14].

We provide some coments on the mechanics behind \([C]^p\). Depending on the chosen thread \( p \), a choreography term \( t_p[A]\) start \( t_p[B] : a(k) ; C \) will generate either (i) an initiating process \( \pi[A,B](k) ; [C]^p \) or (ii) an active process \( a[A_i](k) ; [C]^p \) or (iii) a service process \( !a[B_i](k) ; [C]^p \) or (iv) \([C]^p\) if \( p \) was not one of the threads involved in (start). Collective communications are projected similarly, therefore describing broadcast will suffice. The thread projection of \( t_p[A],e \rightarrow &^q(t_{s[B_s]} : x) : k ; C \) will generate either a quality broadcast \( k[A]^q[B](e) ; [C]^p \) or a receiver process \( k[B_i]^q[A](x_i) ; [C]^p \) for any of the roles \( B_i \in B_s \). In any other case, \([C]^p\) will simply continue operating over the continuation \( C \). Since the conditional construct if \( e @ t_p \) then \( C_1 \) else \( C_2 \) depends solely on the guard of one given thread \( t_p \), its thread projection will generate a conditional localized in such a thread. The projection of the conditional for any other thread will merge both branches, in order to preserve the label behaviors at each side.

A service merge operator joins the behaviour of different service processes started on the same public channel, playing the same roles. Formally, the service merge operator, denoted \([C]^p_R\), returns a set of annotated threads, and it is defined below:

\[
[t_p[A] \text{ start } t_{p[B]} : a(k) ; C]_R^p = \begin{cases} s[R] \cup [C]^a_R & \text{if } s[R] \in t_{p[B]} \\ [C]^a_R & \text{otherwise} \end{cases}
\]

\[
[\text{if } e @ t_{p} \text{ then } C_1 \text{ else } C_2]_R^p = [C_1]^a_R \cup [C_2]^a_R
\]

\[
[\eta ; C]_R^p = [C]^a_R \quad \text{if } \eta \neq (\text{init})
\]

We can finally provide a definition of the Endpoint Projection.
\[
[t_r[A] \text{ start } t_s[B] : a(k); C] \mathcal{P} = \begin{cases} \{ \pi \tilde{A}, \tilde{B}(k); [C] \mathcal{P} \} & \text{if } p = t_1[A_1] \\
\{ a[A_i](k); [C] \mathcal{P} \} & \text{if } p = t_i[A_i] \in \tilde{t}_i \setminus t_1 \\
\{ !a[B_i](k); [C] \mathcal{P} \} & \text{if } p = t_i[B_i] \in \tilde{t}_i \\
[\{ \} & \text{otherwise} 
\end{cases}
\]

\[
[t_r[A], e \to & q(t_s[B]) : x; k; C] \mathcal{P} = \begin{cases} \{ k[A] [\tilde{B}(e)]; [C] \mathcal{P} \} & \text{if } p = t_r[A] \\
k[B_i](A)(x_i); [C] \mathcal{P} & \text{if } p = t_i[B_i] \in \tilde{t}_i \setminus t_s \\
[\{ \} & \text{otherwise} 
\end{cases}
\]

\[
[\& q(t_r[A], e_r) \to t_s[B] : (k, op); C] \mathcal{P} = \begin{cases} \{ k[A] [\tilde{B}(e_i)]; [C] \mathcal{P} \} & \text{if } p = t_i[A_i] \in \tilde{t}_i[A] \\
k[A](\tilde{B}(op)(x)); [C] \mathcal{P} & \text{if } p = t_i[B_i] \in \tilde{t}_i \\
[\{ \} & \text{otherwise} 
\end{cases}
\]

\[
[t_r[A] \to & q(t_s[B]) : k[l]; C] \mathcal{P} = \begin{cases} \{ k[A] [\tilde{B}(l)]; [C] \mathcal{P} \} & \text{if } p = t_r[A] \\
k[B_i](A)(l); [C] \mathcal{P} & \text{if } p = t_i[B_i] \in \tilde{t}_i \\
[\{ \} & \text{otherwise} 
\end{cases}
\]

\[
[0] \mathcal{P} = \begin{cases} \{ \text{if } e \text{ then } [C_1] \mathcal{P} \text{ else } [C_2] \mathcal{P} \} & \text{if } p = t_r \\
[0] & \text{otherwise} 
\end{cases}
\]

**Figure 14:** Process Projection, \([C] \mathcal{P}\), where \(\widetilde{t_r[A]} = t_1[A_1], \ldots, t_n[A_n]\) and \(\widetilde{t_s[B]} = t_1[B_1], \ldots, t_m[B_m]\)

**Definition 6.5** (Endpoint Projection). Let \(C = (\nu \tilde{k}, \tilde{p}) C'\) with a restriction-free \(C'\). The projection of \(C\), denoted \([^C] \mathcal{P}\), is defined as:

\[
[^C] = (\nu \tilde{k}) \left( \prod_{p \in \text{fsc}(C')} \prod_{k \in \text{fsc}(C')} \prod_{A \in \text{roles}(C')} \left( \bigcap_{p' \in [C']^A} [\tilde{C'}] \mathcal{P} \right) \right)
\]

Recall that \(\text{fsc}(C)\) and \(\text{ft}(C)\) contain the set of free service channels and free threads in \(C\), respectively. Essentially, process \((\nu \tilde{k}, \tilde{p}) C'\) contains active sessions \(\tilde{k}\) and the set of free threads \((\tilde{p})\). The Endpoint projection of \(C\) is the parallel composition of all the active processes with associated empty queues, and the parallel composition of replicated processes resulting from merging all service processes with same service channel and same role.

The persistent nature of service processes means that they will not disappear once engaged into a session initiation phase. Recalling the definition of \([\text{init}] \mathcal{P}\), a system will evolve into:

\[
(\nu k) (P | \bigcap_{i \in [2,n]} P_i | \bigcap_{i \in [1,m]} Q_i | k; \emptyset) | \bigcap_{i \in [1,m]} !a[i](k) \mathcal{P} \]

Processes in \(!a[i](k)\); \(Q_i\) may not be used after this interaction. The role of the pruning relation \([10][12]\), is to garbage-collect replicated services that are not in use.

**Definition 6.6** (Pruning). A pruning between endpoints \(P\) and \(Q\), written \(P \prec Q\), is the relation between \(P\) and \(Q\) such that \(Q \equiv Q_0 | \bigcap_{i \in I} !a[i](k_i) \mathcal{P} \). \(R_i\) and it satisfies the following conditions:

Theorem 6.8 (Correctness of the Endpoint Projection)

Proof.

Let $\lambda = \{\lambda_1, \ldots, \lambda_n\}$ and $P \xrightarrow{\lambda_1} \ldots \xrightarrow{\lambda_n} P'$ (resp. for $\mu$). We are ready to establish the correctness of the Endpoint Projection.

Theorem 6.8 (Correctness of the Endpoint Projection). Let $C = (v, k, p) C_1$ with a linear restriction-free $C_1$, and $\Gamma; \Psi \vdash C \triangleright \Delta$, and $\sigma \models \Psi$. Then

- **(Soundness)** If $\langle \sigma, C \rangle \xrightarrow{\lambda} \langle \sigma', C' \rangle$ and $\|C\| < P$, then $\langle \sigma', C' \rangle \xrightarrow{\tilde{\lambda}} \langle \sigma'', C'' \rangle$, $P \xrightarrow{\tilde{\mu}} P'$, $\|C''\| < P'$ and $\lambda, \tilde{\lambda} \vdash \tilde{\mu}$.

- **(Completeness)** If $\|C\| \xrightarrow{\mu} P$ then there exists $P', \tilde{\lambda}$ such that $P \xrightarrow{\tilde{\mu}} P'$, $\langle \sigma, C \rangle \xrightarrow{\tilde{\lambda}} \langle \sigma', C' \rangle$, $\|C'\| < P'$, and $\lambda \vdash \mu_1, \tilde{\mu}_2$.
The judgment $\lambda \vdash \mu$ captures whether the labels in $\mu$ correspond to the choreographic behavior in $\lambda$, and it is defined as the minimal relation satisfying the rules in Figure 15.

**Proof.** The proof proceeds by rule induction on the transition rules for $\lambda \rightarrow$ in the case of Soundness, and by induction on the structure of $C_1$ in the case of Completeness. The details on the proof are presented in Appendix A.4.

We can combine Theorem 5.3 and Theorem 6.8 to derive that projections out of a well-typed choreography always progress.

**Theorem 6.9** (Availability By Design). Let $C = (v \tilde{k}, \tilde{p}) C'$ with a restriction-free $C'$ and $C$ is linear, and $\Gamma; \Psi \vdash C \triangleright \Delta$, $\sigma \models \Psi$, then either there exists $P', \mu' \triangleright \Delta$, s.t. $[C] \tilde{p} \rightarrow P$, and $P \mu' \rightarrow P'$, or $[C] \equiv 0$.

**Proof.** We proceed by structural induction on $C'$. If $C' = 0$, then the projection $[[((v \tilde{k}, \tilde{p}) 0)]] = (v \tilde{k}) \left( \prod_{p \in ft(0)} [0]^p \mid \prod_{k \in fsc(0)} k : \emptyset \mid \prod_{\alpha \in sc(0), A \in roles(0)} \left( \bigcup_{p \in \{0\} \setminus A} [0]^p \right) \right) \equiv 0$ and we are done. This reasoning also applies when $C' \equiv 0$.

If $C' \not\equiv 0$, then by applying inversion in $\Gamma; \Psi \vdash C \triangleright \Delta$, we know that $\Psi \vdash C$. By the application of Theorem 4.4 along with assumption $\sigma \models \Psi$, then we know that there exists $\lambda$ s.t. $\langle \sigma, C \rangle \rightarrow \langle \sigma', C'' \rangle$. From Lemma 6.7 we know that there exists a $P$, s.t. $[C] \prec P$. Then by the application of Theorem 6.8 we know that $P \mu \rightarrow P'$ and we are done.

## 7. Related Work

Availability considerations in distributed systems have recently spawned novel research strands in regular languages [24, 1], continuous systems [2], and endpoint languages [41]. To the best of our knowledge, this is the first work considering availability from a choreographical perspective.

A closely related work is the Design-By-Contract approach for multiparty interactions [4]. In fact, in both works communication actions are enriched with pre-/post-conditions, similar to works in sequential programming [23]. The work on [4] enriches global types with assertions, that are then projected to a session $\pi-$calculus. Assertions may generate ill-specifications, and a check for consistency is necessary. Our capability-based type system guarantees temporal-satisfiability as in [4], not requiring history-sensitivity due to the simplicity of the preconditions used in our framework. The most obvious difference with [4] is the underlying semantics used for communication, that allows progress despite some participants are unavailable.

Other works have explored the behavior of communicating systems with collective/broadcast primitives. In [27], the expressivity of a calculus with bounded broadcast and collection is studied. In [32], the authors present a type theory to check whether models for multicore programming behave according to a protocol and do not deadlock. Our work differs from these approaches in that our model focuses considers explicit considerations on availability for the systems in consideration. Also for multicore programming, the work in [14] presents a calculus with fork/join communication primitives, with a flexible phaser mechanism that allows some threads to advance prior to synchronization. The type system guarantees a node-centric progress guarantee, ideal for multicore computing, but too coarse for CPS.
Finally, the work [29], present endpoint (session) types for the verification of communications using broadcast in the Ψ-calculus. We do not observe similar considerations regarding availability of components in this work.

The work [13] presented multiparty global types with join and fork operators, capturing in this way some notions of broadcast and reduce communications, which is similar to our capability type-system. The difference with our approach is described in Section 3. On the same branch [16] introduces multiparty global types with recursion, fork, join and merge operations. The work does not provide a natural way of encoding broadcast communication, but one could expect to be able to encode it by composing fork and merge primitives.

The work by Kouzapas, Yoshida and Honda explore a session π calculus with an asynchronous semantics based on input/output queues [30]. The language presented there bears similarities with the Endpoint Calculus presented in Section 6. The use of use of message queues and the use of predicates to identify when a message has arrived to a local buffer resembles the interplay between input/output queues and quality predicates. In our model, collective operations such as broadcast and reduce imply that there can be multiple orderings on the communication events occurred (e.g.: we cannot guarantee when receivers of a broadcast will consume the message). In future work, we would like to explore how behavioral theories such as the one in [30] can be adapted for collective communications.

The current work is an extension of the Quality Choreographies work presented at [33]. As mentioned in the introduction, in this version we have provided a full methodology of choreographic programming, where choreographies can project to a novel asynchronous endpoint language. Moreover, implementations have been proven to guarantee a novel availability-by-design property, the corresponding deadlock-freedom property for failure-aware communication protocols. Technically, the choreographic language presented in this version bears differences in some of the language operators, as well as in the operational semantics: the non-deterministic choice presented in [33] proved difficult to accommodate in an endpoint projection that could respect the soundness and completeness guarantees in Theorem 6.8. Further restrictions involved limitations on the quality predicates used for collective selections. We would like to revisit such aspects in future works.

8. Conclusions and Future Work

We have presented a process calculus aimed at studying protocols with variable availability conditions, as well as a type system to ensure their progress. Paired with session types, choreographies guide the correct implementation of distributed systems with failure conditions, on a communication model based on synchronous and collective communications. This constitutes the first step towards a methodology for the safe development of communication protocols in CPS. Some important considerations have been left out for future work. First, linearity considerations require each participant to implement one unique behavior. This is not natural in failure-aware communication, that requires several copies of the same component to be deployed, all of them implementing the same behavior. A possible extension will be to integrate parameterized or index-based multiparty session types in our analysis, taking inspiration from the works of [17] [32]. Other possible efforts include the modification of the type theory to cater for recursive behavior, non-determinism, and considerations of compensating [7] [9] [34] and time [6] [5]. Type checking is computationally expensive, because for each collective interaction one must perform the analysis on each subset of participants involved. The situation will be critical once recursion is considered.
We believe that the efficiency of type checking can be improved by modifying the theory so it generates one formulae for all subsets.

Traditional design mechanisms (including sequence charts of UML and choreographies) usually focus on the desired behavior of systems. In order to deal with the challenges from security and safety in CPS it becomes paramount to cater for failures and how to recover from them. This was the motivation behind the development of the Quality Calculus that not only extended a $\pi$-calculus with quality predicates and optional data types, but also with mechanisms for programming the continuation such that both desired and undesired behavior was adequately handled. In this work we have incorporated the quality predicates into choreographies and thereby facilitate dealing with systems in a failure-aware fashion. However, it remains a challenge to incorporate the consideration of both desired and undesired behavior that is less programming oriented (or EndPoint Projection oriented) than the solution presented by the Quality Calculus. This may require further extensions of the calculus with fault-tolerance considerations.

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A.1. Results related to states.

**Lemma A.1** (Validity: States). If $\sigma \models \Psi$, then $\Psi : \text{form}$ and $\sigma : \text{state}$.

*Proof.* By rule induction on the first hypothesis.

**Lemma A.2** (Weakening: States). If $\sigma \models \Psi$, then $\sigma, (t [A], k, X) \models \Psi, t : k [A] \triangleright X$.

*Proof.* It follows by induction on the hypothesis.

**Lemma A.3** (Update: States). If $\sigma \models \Psi$ and $\sigma' = (\Psi \setminus \delta), \Psi'$, where $\delta = \{(t, k, X) | (t, k, X) \in \sigma \land (t, k, Y) \in \sigma'\}$.

*Proof.* It follows directly from the hypotheses, the definition of $\sigma[\sigma']$ and Definition 4.2.

A.2. Results related to choreographies.

**Lemma A.4** (Weakening: choreographies). Let $\psi : \text{form}$. If $\Psi \vdash C$, then $\Psi, \psi \vdash C$.

*Proof.* By rule induction on the hypothesis.

**Lemma A.5** (Strengthening: choreographies). If $\Psi, t : k [A] \triangleright X \vdash C$ and $X \notin \text{f}_\text{form}(C)$, then $\Psi \vdash C$.

*Proof.* By rule induction on the first hypothesis.

**Lemma A.6** (Substitution). Let $t$ be a term, and $x@p \in \text{Vars}(C)$. If $\Psi \vdash C$, then $\Psi \vdash \theta(C)$.

*Proof.* By rule induction on the hypothesis. Note that $x \notin \Psi$.

**Lemma A.7** (Subject Congruence). If $C \equiv C'$ and $\Psi \vdash C$, then $\Psi \vdash C'$.

*Proof.* It proceeds by induction on the depth of the first premise.

**Lemma A.8** (Inversion Lemma). Let $\Psi \vdash C$ then either:

- $C = \eta; C'$, and:
  - $\eta = t_A[A_r] [Y_r] \vdash \text{start} t_s[B_r] [Y_r] : a(k)$ and $\Psi, \text{init}(t_r[A_r] [Y_r], t_s[B_r] [Y_s], k) \vdash C'$, and $\{t_s, k\} \#(T(\Psi) \cup K(\Psi))$, or
  - $\eta = t_A[A_r] [X_A; Y_r] \to e \to \& t_r[B_r] [X_r; Y_r] : x_r : k$ and $\forall \geq 1, J : t.s.(J \subseteq B \land \phi(J)) \vdash \psi_A, (\psi_j)_{j \in J}, \psi' \land \phi = t_A : k[A] \triangleright X_A \otimes_{j \in J} (t_j : k[B_j] \triangleright X_j) \land \phi' = t_A : k[A] \triangleright Y_A \otimes_{j \in J} (t_j : k[B_j] \triangleright Y_j) \land \psi_A, (\psi_j)_{j \in J}, \phi \to \phi' \vdash \phi') : t_A : k[A] \triangleright Y_A, (t_j : k[B_j] \triangleright Y_j)_{j \in J}, \psi' \vdash C'$, and $\vdash e @ t_A : \text{opt.data}, \text{and}(\in \phi \in i \in \varnothing, \text{or})$
  - $\eta = & t_r[A_r] [X_r; Y_r] \to e \to t_B[B] [X_B; Y_B] : x : (k, op)$ and $\forall \geq 1, J : t.s.(J \subseteq A \land \phi(J)) \land \psi = \psi_B, (\psi_j)_{j \in J}, \psi' \land \phi = t_B : k[B] \triangleright X_B \otimes_{j \in J} (t_j : k[A_j] \triangleright X_j) \land \phi' = t_B : k[B] \triangleright Y_B \otimes_{j \in J} (t_j : k[A_j] \triangleright Y_j) \land \psi_B, (\psi_j)_{j \in J}, \phi \to \phi' \vdash \phi') : t_B : k[B] \triangleright Y_B, (t_j : k[A_j] \triangleright Y_j)_{j \in J}, \Psi' \vdash C'$, and $\vdash e @ t_B : \text{opt.data}, \text{and}(\in \phi \in i \in \varnothing, \text{or})$.
Proof. By case analysis on the type formation rules.

Lemma A.9 (Subject Swap). If $C \simeq_C C'$ and $\Psi \vdash C$, then $\Psi \vdash C'$.

Proof. It proceeds by induction on the depth of the first premise. Most of the cases are straightforward except $\eta; (\eta'; C) \simeq_C \eta'; (\eta; C)$, which requires the application of Lemma A.8 and case analysis.

Theorem 4.3 (Type Preservation). If $\langle \sigma, C \rangle \xrightarrow{\lambda} \langle \sigma', C' \rangle$, $\sigma \models \Psi$, and $\Psi \vdash C$, then $\exists \Psi'. \Psi \vdash C'$ and $\sigma' \models \Psi'$.

Proof. It follows by rule induction on the first hypothesis. We have eight cases.

**Case** [BCast], [Sel] rules: Standard Inversion/formation rules. Process typing requires substitution (Lemma A.6) and state typing requires validity (Lemma A.1) and state update (Lemma A.3). We proceed to show the case for [BCast].

1. Inversion $J' \subseteq \bar{t}_r$
2. Inversion $\forall i \in \{A\} \cup J' : X_i \subseteq \sigma(t_i, k)$ and $\sigma'(t_i, k) = [X_i; Y_i](\sigma(t_i, k))$
3. Inversion $\forall i \in C : \theta(x_i) = \begin{cases} \text{some}(v) & i \in J' \\ \text{none} & \text{otherwise} \end{cases}$
4. Inversion $\forall \geq 1 J$, s.t.
   \[
   \begin{aligned}
   & J \subseteq \bar{t}_r \\
   & \land q(J) \land \Psi = \psi_A, (\psi_j)_{j \in J}, \Psi' \land \phi = t_A : k [A] \triangleright X_A \bigotimes_{j \in J} (t_j : k [B_j] \triangleright X_j)
   \end{aligned}
   \]
   \[
   \begin{aligned}
   \land \phi' = t_A : k [A] \triangleright Y_A \bigotimes_{j \in J} (t_j : k [B_j] \triangleright Y_j) \land \psi_A, (\psi_j)_{j \in J}, \phi \vdash \phi' \end{aligned}
   \]
   \[
   t_A : k [A] \triangleright Y_A, (t_j : k [B_j] \triangleright Y_j)_{j \in J}, \Psi' \vdash C
   \]
From the definition of \( \text{fv}(\cdot) \) we know that \( x_i \in \text{fv}(t_A[A]\{X_A;Y_A\};e \Rightarrow \& q(t_r[B_r]\{X_r;Y_r\} : x_r) : k; C), i \in \{1, \ldots, 16\} \). From this, Equations 9 and 11 and the application of Lemma \( A.6 \) we can conclude

\[
\forall \geq J. \text{ s.t.} \begin{cases}
J \subseteq t_r \land q(J) \land \Psi = \psi_A, (\psi_j)_{j \in J}, \Psi' \\
\land \phi = t_A: k[A] \triangleright X_A \otimes_{j \in J} (t_j: k[\{B_j\} \triangleright X_j]) \\
\land \phi' = t_A: k[A] \triangleright Y_A \otimes_{j \in J} (t_j: k[\{B_j\} \triangleright Y_j]) \\
\land \phi, (\psi_j)_{j \in J}, \phi \rightarrow \phi' \rightarrow \phi'
\end{cases} \\
\sigma[\sigma'] \models t_A: k[A] \triangleright Y_A, (t_j: k[\{B_j\} \triangleright Y_j])_{j \in J}, \Psi' \vdash \theta(C)
\]

For state typing we need to show that

\[
\forall \geq J. \text{ s.t.} \begin{cases}
J \subseteq t_r \land q(J) \land \Psi = \psi_A, (\psi_j)_{j \in J}, \Psi' \\
\land \phi = t_A: k[A] \triangleright X_A \otimes_{j \in J} (t_j: k[\{B_j\} \triangleright X_j]) \\
\land \phi' = t_A: k[A] \triangleright Y_A \otimes_{j \in J} (t_j: k[\{B_j\} \triangleright Y_j]) \\
\land \psi_A, (\psi_j)_{j \in J}, \phi \rightarrow \phi' \rightarrow \phi'
\end{cases} \\
\sigma[\sigma'] \models t_A: k[A] \triangleright Y_A, (t_j: k[\{B_j\} \triangleright Y_j])_{j \in J}, \Psi'
\]

By the application of state update rule and state satisfaction we get:

\[
6, 7, 11 \quad J' \subseteq J \quad (A.15)
\]

From Eq. \( A.8 \) we know that \( \sigma \) contains all triples \((t_i, k, X_i)_{i \in \{A\} \cup J'}\) and possibly more. We denote with \( \sigma'' \) the set of additional tuples in \( \sigma \). From Eq. 3 we know that

\[
\sigma'' \models \Psi' \quad (A.16)
\]

\[
\text{A.8 state satisfaction} \quad \exists R. \sigma \models (t_i: k[R] \triangleright X_i)_{i \in \{A\} \cup J'} \quad (A.17)
\]

\[
\text{A.8 state satisfaction} \quad \exists R. \sigma' \models (t_i: k[R] \triangleright Y_i)_{i \in \{A\} \cup J'} \quad (A.18)
\]

\[
3, 16, \text{lem. A.3} \quad \exists R. \sigma[\sigma'] \models (\Psi \setminus \delta), (t_i: k[R] \triangleright Y_i)_{i \in \{A\} \cup J'} \quad (A.19)
\]

\[
3, 16, \text{lem. A.3} \quad \delta = \{(t, k, X) \mid (t, k, X) \in \sigma \land (t, k, Y) \in \sigma'\} \quad (A.20)
\]

Moreover, we know that \( \delta \cup \sigma'' = \sigma \), and from the definition of store update, we know that \( \sigma[\sigma'] = (\sigma \setminus \delta), \sigma' \). We can rewrite Eq. 19 as

\[
\exists R. (\sigma'', \delta) \setminus \delta, \sigma' \models (\Psi \setminus \delta), (t_i: k[R] \triangleright Y_i)_{i \in \{A\} \cup J'} \quad (A.21)
\]

By replacing \( \Psi \setminus \delta \) by \( \Psi' \) in Eq. 21, the definition of state update, and by applying simple formula exchange, we get:

\[
\exists R. \sigma[\sigma'] \models (t_i: k[R] \triangleright Y_i)_{i \in \{A\} \cup J'}, \Psi' \quad (A.22)
\]

**Case \([\text{init}]\text{Red}\):** It corresponds to the same equivalence class as the case for \([\text{init}]\text{Bcast}\). Its proof is analogous.

**Case \([\text{init}]\text{rule}\):** Standard inversion/formation rules. It requires state validity (Lemma \( A.1 \)), state weakening (Lemma \( A.2 \)) and state update lemma (Lemma \( A.3 \)).

Hypothesis \( (\nu m) \left< \sigma, t_r[A_r]\{Y_r\} \right. \left. \text{ start } t_s[B_s]\{Y_s\} : a(k); C \right> \overset{2}{\rightarrow} (\nu m, \overline{n}) \left< \sigma[\sigma'], C \right> \quad (A.1)

Hypothesis \( \Psi \vdash t_r[A_r]\{Y_r\} \left. \text{ start } t_s[B_s]\{Y_s\} : a(k); C \right> \quad (A.2)

Hypothesis \( \sigma \models \Psi \quad (A.3)\)
We proceed by induction on the structure of $C$ (Well-typed choreographies progress).

It follows by rule induction on the first hypothesis.

Lemma A.10 (Substitution). If $\Gamma, x@p : S ; \Psi \vdash C \triangleright \Delta$ and $\Gamma \vdash v : S$ then $\Gamma ; \Psi \vdash \theta(C) \triangleright \Delta$.

Proof. It follows by rule induction on the first hypothesis.

Lemma A.11 (Subject Congruence). If $\Gamma ; \Psi \vdash C \triangleright \Delta$ and $C \equiv C'$, then $\Gamma ; \Psi \vdash C' \triangleright \Delta$.

Proof. It follows by induction on the depth of the premise $C \equiv C'$.
We write $\Delta \simeq_G \Delta'$ to denote that $\text{dom}(\Delta) = \text{dom}(\Delta')$ and for all $k \in \text{dom}(\Delta)$, $\Delta(k) \simeq_G \Delta'(k)$. Similarly, we say $\Delta' \subseteq \Delta$ when $k: G' \in \Delta'$, implies that $\exists a, k; k: G \in \Delta$ and $G \overset{\alpha}{\to} G', \forall k: G' \in \Delta'$.

**Lemma A.12** (Subject Swap). If $\Gamma; \Psi \vdash C \triangleright \Delta$ and $C \simeq_C C'$, then there exists $\Delta'$ s.t. $\Gamma; \Psi \vdash C' \triangleright \Delta'$ and $\Delta \simeq_G \Delta'$.

**Proof.** It follows by induction on the depth of the premise $C \simeq_C C'$, as well as the swap relation rules for global types in Figure 9.

**Theorem 5.3** (Type Preservation for Global Types). If $\Gamma; \Psi \vdash C \triangleright \Delta$, $\sigma \models \Psi$, and $\langle \sigma, C \rangle \overset{\lambda}{\to} \langle \sigma', C' \rangle$, then there exists $\Gamma'$ s.t. $\Gamma'; \Psi' \vdash C' \triangleright \Delta'$, $\sigma' \models \Psi'$, and

- if $\lambda = \text{init}$, $\text{start}$ $t_s[B] : a(k)$, then $\Delta' = \Delta$,
- otherwise $\Delta \overset{k; \alpha}{\to} \Delta'$ and $\Gamma \vdash \lambda \triangleright \alpha$.

**Proof.** The proof follows by rule induction on $\langle \sigma, C \rangle \overset{\lambda}{\to} \langle \sigma', C' \rangle$, using Theorem 4.3 to guarantee the type preservation of judgments $\Psi \vdash C$. We have seven cases:

**Case Rule $[\ell\text{[init]}$:**

| Hypothesis                                                                 | $\langle \sigma, \text{init}_A \{Y_1\} \text{ start } t_s[B] \{Y_2\} \rangle : a(k); C'$ |
|---------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| (A.1)                                                                     | $\langle \sigma, \text{init}_A \{Y_1\} \text{ start } t_s[B] \{Y_2\} \rangle : a(k); C'$ |

Moreover, we know that $\bar{t}_s \not\in \Delta$ since $\Delta$ only contain information regarding session variables. Also, recall that function $\text{init} \{\{t_r[A], t_s[B]\}, k\}$ returns a list of ownership types $t_p: k [A]$ where $\forall t_p \in \bar{t}_p, t_p \in \{t_r, t_s\}$. We can conclude by the sequence of applications of $\text{TGres}$ to type the redex. Let $\Gamma' = \Gamma; \text{init} \{\{t_r[A], t_s[B]\}, k\} \bar{t}_s$, then

| Hypothesis                                                                 | $\Psi \vdash \text{init}_A \{Y_1\} \text{ start } t_s[B] \{Y_2\} \rangle : a(k); C'$ |
|---------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| (A.8)                                                                     | $\Psi \vdash \text{init}_A \{Y_1\} \text{ start } t_s[B] \{Y_2\} \rangle : a(k); C'$ |

| Hypothesis                                                                 | $\Psi \vdash \langle \nu \bar{t}_s, k \rangle C'$                                      |
|---------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| (A.10)                                                                    | $\Psi \vdash \langle \nu \bar{t}_s, k \rangle C'$                                      |

A.6, $\bar{t}_s \# \Delta$, rule $\text{TGres} \times [\bar{t}_s]$ times $\Gamma' \vdash (\nu \bar{t}_s) C' \triangleright \Delta, k: G$ (A.11)

A.11, rule $\text{TGres}$ $\Gamma' \vdash (\nu \bar{t}_s, k) C' \triangleright \Delta$ (A.12)

A.10, A.12, Rule $\text{TG}$ $\Gamma' \vdash (\nu \bar{t}_s, k) C' \triangleright \Delta$ (A.13)
Case Rule $[^6]$Bcast:

Hypothesis

\[
\left( \sigma, \left( t_A[A]\{X_A; Y_A\}.e \rightarrow &^q(t_r[B_r]\{X_r; Y_r\} : x_r) : k \right); C' \right)
\]

(A.1)

\[
\theta(t_A[A]\{X_A; Y_A\}.v \rightarrow &^q(t_r[B_r]\{X_r; Y_r\} : x_r) : k) \rightarrow (\sigma[\sigma'], \theta(C'))
\]

Hypothesis

\[
\Gamma; \Psi \vdash \left( t_A[A]\{X_A; Y_A\}.e \rightarrow &^q(t_r[B_r]\{X_r; Y_r\} : x_r) : k \right); C' \triangleright \Delta
\]

(A.2)


A.2, inversion

\[
\Gamma \vdash \left( t_A[A]\{X_A; Y_A\}.e \rightarrow &^q(t_r[B_r]\{X_r; Y_r\} : x_r) : k \right); C' \triangleright \Delta', k: \left( A \rightarrow \overline{B}: \langle S \rangle ; G \right)
\]

(A.4)


A.1, inversion

\[
J \subseteq \overline{\tilde{t}}_r
\]

(A.5)


A.1, inversion

\[
q(J)
\]

(A.6)


A.1, inversion

\[
\forall i \in \{t_A\} \cup J : X_i \subseteq \sigma(t_i, k) \land \sigma'(t_i, k) = [X_i; Y_i](\sigma(t_i, k))
\]

(A.7)


A.1, inversion

\[
e@t_A \downarrow v
\]

(A.8)


A.1, inversion

\[
\forall i \in \{1, \ldots, |\overline{\tilde{t}}_r|\} : \theta(x_i) = \begin{cases} 
\text{some}(v) & t_i \in J \\
\text{none} & \text{o.w.}
\end{cases}
\]

(A.9)


A.4, inversion

\[
\Gamma \vdash t_A: k[A]
\]

(A.10)


A.4, inversion

\[
\Gamma \vdash t_i: k[B_i] \quad \forall i \in \{1, \ldots, |\overline{\tilde{t}}_r|\}
\]

(A.11)


A.4, inversion

\[
\Gamma \vdash e@t_A: S
\]

(A.12)


A.4, inversion

\[
\Gamma, x_r@t_r[B_r] : S \vdash C' \triangleright \Delta', k: G
\]

(A.13)


A.8, A.9, A.12, A.13, lem[A.10]

\[
\Gamma \vdash \theta(C') \triangleright \Delta', k: G
\]

(A.14)


A.4, A.14, rule $[^6]$Bcast

\[
\Delta', k: \left( A \rightarrow \overline{B}: \langle S \rangle ; G \right) \xrightarrow{k: \alpha} \Delta', k: G \quad \alpha = A \rightarrow \overline{B}: \langle S \rangle
\]

(A.15)


A.1, A.9, A.11, A.12, rule $[^4]$Bcast

\[
\Gamma \vdash t_A[A].v \rightarrow &^q(t_r[B_r]: sb_r) : k \triangleright k: A \rightarrow \overline{B}: \langle S \rangle
\]

(A.16)

Case Rule $[^6]$Sel: The case is analogous to the one above (excluding substitutions)

Case Rule $[^6]$Red: The case is analogous to the $[^6]$Bcast.

Case Rule $[^6]$Res: This case follows straightforwardly after application of the induction hypothesis.

Case Rule $[^6]$If:

Hypothesis

\[
\langle \sigma, \text{if } e@t \text{ then } C_1 \text{ else } C_2 \rangle \xrightarrow{r} \langle \sigma, C_i \rangle
\]

(A.1)

Hypothesis

\[
\Gamma; \Psi \vdash \text{if } e@t \text{ then } C_1 \text{ else } C_2 \triangleright \Delta
\]

(A.2)

Hypothesis

\[
\sigma \models \Psi
\]

(A.3)

A.2, inversion

\[
\Gamma \vdash \text{if } e@t \text{ then } C_1 \text{ else } C_2 \triangleright \Delta
\]

(A.4)

A.2, inversion

\[
\Psi \models \text{if } e@t \text{ then } C_1 \text{ else } C_2
\]

(A.5)

A.4, inversion

\[
\Gamma \vdash C_1 \triangleright \Delta
\]

(A.6)

A.4, inversion

\[
\Gamma \vdash C_2 \triangleright \Delta
\]

(A.7)
Assume \(e@t \downarrow tt\) (the other case is analogous).

\[
\text{case } \langle \sigma, \text{if } e@t \text{ then } C_1 \text{ else } C_2 \rangle \xrightarrow{\tau} \langle \sigma, C_1 \rangle \tag{A.8}
\]

A.3, A.5, A.8, thm. 13

\[
\Psi' \vdash C_1 \tag{A.9}
\]

A.3, A.5, A.8, thm. 13

\[
\sigma' \models \Psi' \tag{A.10}
\]

A.6, A.9, rule \(\Lceil TG\)

\[
\Gamma; \Psi' \vdash C_1 \triangleright \Delta \tag{A.11}
\]

A.4. Results Related to the Endpoint Projection.

**Lemma A.13** (Substitution: Process Projection). \([C[v/x@p]]^p = [[C]^p[v/x@p]]\).

**Proof.** It follows directly from Definition 6.4.

**Lemma A.14** (Substitution: Projection Locality). Let \(C = (\nu \bar{p}, \bar{k}) C', q \in \text{ft}(C')\), and

\[
[[C]] = (\nu \bar{k}) \left( \prod_{p \in \text{ft}(C')} [C']^p \mid \prod_{k \in \text{sc}(C')} k : \emptyset \right) \mid \prod_{a \in \text{roles}(C')} \left( \bigcup_{p \in [C']^a} [C'^p] \right).
\]

Then

\[
[[C[v/x@q]]] = (\nu \bar{k}) \left( [C'[v/x@q]]^q \mid \prod_{p \in \text{ln}(C') \setminus q} [C'^p] \mid \prod_{k \in \text{ln}(C')} k : \emptyset \right) \mid \prod_{a \in \text{sc}(C')} \left( \bigcup_{p \in [C'^a]} [C'^p] \right).
\]

**Proof.** From Definition 6.5 we know that there is only one thread projection for \(q\) in \([[C]]\). The rest follows directly from Lemma A.13.

**Lemma A.15** (\(\equiv\) Preserves Linearity). If \(C \equiv C'\) and \(C\) is linear, then \(C'\) is linear.

**Proof.** Follows by rule induction on the derivation of the hypothesis.

**Lemma A.16** (Projection Congruence). If \(C \equiv C'\) then \([[C]] \equiv [[C']]\).

**Proof.** It follows by rule induction on the rules for \(C \equiv C'\).
Lemma A.17 (Session Linearity). If \( \Gamma; \Psi \vdash C \triangleright \Delta \), and there exists \( P, Q \), s.t. \( \{[C]\} \equiv (\nu k)(P \vdash Q) \). We have either

- \( P = k[A]!q[\overline{B}](e); P' \), and \( Q \) does not contain actions with free subject \( k[A]!q[\overline{B}] \), or
- \( P = k[A]!q[\overline{B}] \triangleright \{l\}; P' \), and \( Q \) does not contain actions with free subject \( k[A]!q[\overline{B}] \), or
- \( P = k[A]?q[\overline{B}]\text{op}(x); P' \), and \( Q \) does not contain actions with free subject \( k[A]?q[\overline{B}] \).

Proof. We analyze each of the cases separately. From \( \Gamma; \Psi \vdash C \triangleright \Delta \) and \( P = k[A]!q[\overline{B}](e); P' \) we know that \( C \) corresponds to \( t_A[A]\{X_A;Y_A\}.e \rightarrow \&q(t_r[B_r]\{\overline{X_r};\overline{Y_r}\}:x_r):k;C' \). Moreover, performing inversion in the typing rule for broadcast allow us to conclude that there is only one judgment for \( \Gamma \vdash t_A : k[A] \). A similar reasoning is performed for Reduce and Collective selections.

Lemma A.18 (\( \simeq_C \) Preserves Linearity). If \( C \simeq_C C' \) and \( C \) is linear, then \( C' \) is linear.

Proof. It follows by rule induction on the rules for \( C \simeq_C C' \).

We say that \( a[A] \) is enabled in \( P \) if \( P \) contains a sub-term \( a(k) \); \( P \) or \( !a(k) \); \( P \).

Lemma A.19 (Projections of linear choreographies do not introduce races). Let \( C \) be a linear choreography, and \( \{[C]\} \xrightarrow{\tilde{\mu}} P \) for a finite \( \tilde{\mu} \). If \( P \) contains a sub-term \( \overline{\pi}(k) \); \( Q \), then there exists at most one sub-term \( P' \) in \( P \) s.t. \( a[A] \) is enabled in \( P' \) for any \( A \).

Proof. Since \( P \) contains a sub-term \( \overline{\pi}(k) \); \( Q \) then we know that \( C \) contains a sub-term \( \tilde{p}\text{start} \tilde{q} : a(k) \). We proceed by induction on the length of \( \tilde{\mu} \).

Case \( |\tilde{\mu}| = 0 \): then \( \{[C]\} = P \). The interaction dependencies imposed by the linearity of \( C \) say that \( \forall r \in \tilde{r} \) there is an interaction dependency with threads in \( \tilde{p}, \tilde{q} \). \( n_1 = \tilde{p}\text{start} \tilde{q} : a(k) \), \( n_2 = \tilde{r}\text{start} \tilde{s} : a(k) \) in \( C \). Then the thread projection of \( r \in \tilde{r} \) is indeed a continuation of one of the projections of threads in \( \tilde{p}, \tilde{q} \). Finally, the sub-term \( \overline{\pi}(k) \); \( Q \) corresponding to the thread projection of \( r \in \tilde{r} \) are disabled until the terms generated by the projection of \( \tilde{p}\text{start} \tilde{q} : a(k) \) evolve.

Case \( |\tilde{\mu}| > 0 \): We assume that \( \{[C]\} \xrightarrow{\tilde{\mu}} P' \xrightarrow{\tilde{\mu}'} P \), and that there exists at most one sub-term \( P'' \) in \( P' \) s.t. \( a[A] \) is enabled in \( P'' \) for any \( A \). We check now all the possible derivation sequences for \( P' \xrightarrow{\tilde{\mu}'} P \). The interesting one refers to rule \( [\text{init}] \), that will consume term \( \overline{\pi}(k) \); \( Q \) when reducing to \( P \). The existence of at most one sub-term \( P'' \) in \( P' \) s.t. \( a[A] \) is enabled in \( P'' \) from the same argument regarding linearity conditions as the case for \( |\tilde{\mu}| = 0 \).

Lemma A.20 (Swapping: Endpoint Invariance). If \( C \simeq_C C' \), then \( \{[C]\} = \{[C']\} \).

Proof. The proof proceeds by rule induction on the swapping relation rules in Figure 3 followed by case analysis on the shape that \( \eta \) can take. Here we present the case where \( \eta = p_r.e \rightarrow \&q(p_s : x_s) : k \). The other cases are similar.

Case \( \eta = p_r.e_1 \rightarrow \&q(p_s : x_s) : k \).

Hypothesis

\[
\text{if } e_1 @ p \text{ then } p_r.e_1 \rightarrow \&q(p_s : x_s) : k; C_1 \text{ else } p_r.e_1 \rightarrow \&q(p_s : x_s) : k; C_2
\]

\[
\simeq_C p_r.e_1 \rightarrow \&q(p_s : x_s) : k; \text{ if } e_1 @ p \text{ then } C_1 \text{ else } C_2
\]

(A.1)
A.1, inversion \( p \notin T(p_r.e_1 \rightarrow \&q(\tilde{p}_s : x_s) : k) \) \( \text{(A.2)} \)

A.1, def. \( \| \cdot \| \) \( \| C \| = (\nu \overline{k}) \left( \prod_{p \in ft(C')} [C']^p \mid \prod_{k \in fsc(C')} k : \emptyset \right) \)

\( \mid \prod_{a \in sc(C'), A \in roles(C')} \left( \bigsqcup_{p \in [C']_A^p} [C']^p \right) \) \( \text{(A.3)} \)

A.3, def. \( \| \cdot \| \) \( \| C' \| = (\nu \overline{k}) \left( \begin{array}{l}
\text{if } e \text{ then } [p_r.e_1 \rightarrow \&q(\tilde{p}_s : x_s) : k; C_1]^p \\
\text{else } [p_r.e_1 \rightarrow \&q(\tilde{p}_s : x_s) : k; C_2]^p \\
\prod_{p' \in (ft(C_1), ft(C_2)) \mid p}(p_r.e_1 \rightarrow \&q(\tilde{p}_s : x_s) : k; (C_1 \sqcup C_2))^{p'} \\
\prod_{k \in fsc(C)} k : \emptyset \\
\prod_{a,A} \left( \bigsqcup_{p \in [C']_A^p} [C']^p \right)
\end{array} \right) \) \( \text{(A.4)} \)

A.3, def. \( \| \cdot \| \) \( \| C' \| = (\nu \overline{k}) \left( \begin{array}{l}
k[A]^{1q}(\tilde{B}) (x) ; ([C_1]^t \sqcup [C_2]^t) \\
\prod_{i \in [1, i]} k[B_i]^t[A] (x_i) ; ([C_1]^t \sqcup [C_2]^t) \\
\prod_{t_h \in ft(C')} k[t_h] if e_1 \cdot p then C_1 else C_2 \mid k_1 \\
\prod_{k' \in fsc(C')} k' : \emptyset \mid k : \emptyset \\
\prod_{a,A} \left( \bigsqcup_{p \in [C']_A^p} [C']^p \right)
\end{array} \right) \) \( \text{(A.5)} \)

From A.4 and A.5 it suffices to observe that for \( t_h \in ft(C') \setminus \{ \xi_1, C_1 \} \) the projection do not change, and that for all the other processes, the merge between continuations \( C_1 \) and \( C_2 \) is maintained. \( \square \)

**Lemma A.21** (Passive Process Pruning Invariance). Assume a restriction-free \( C \). If \( \langle \sigma, C \rangle \xrightarrow{\Delta} \langle \sigma', C' \rangle \) then \( \| C' \|^p \prec \| C \|^p, \forall p \in fn(C) \setminus fn(\lambda) \).

**Proof.** The proof proceeds by rule induction on \( \langle \sigma, C \rangle \xrightarrow{\Delta} \langle \sigma', C' \rangle \). Cases \( [^c] \text{init}, [^c] \text{bcast}, [^c] \text{sel} \), \( [^c] \text{red} \) are straightforward. In the conditional case, we have that \( \langle \sigma, if e_1 \cdot p then C_1 else C_2 \rangle \xrightarrow{.\Delta.} \langle \sigma, C_1 \rangle \) (the other case is analogous). Then

\[
\left( \prod_{p \in ft(C_1')} [C_1]^p \mid \prod_{k \in fsc(C_1')} k : \emptyset \right)
\mid \prod_{a \in sc(C_1'), A \in roles(C_1')} \left( \bigsqcup_{p \in [C_1']_A^p} [C_1']^p \right)
\prec \left( \begin{array}{l}
\text{if } e \text{ then } [C_1]^p \text{ else } [C_2]^p \\
\prod_{p' \in (ft(C_1), ft(C_2)) \mid p}(p_r.e_1 \rightarrow \&q(\tilde{p}_s : x_s) : k; (C_1 \sqcup C_2))^{p'} \\
\prod_{k \in fsc(C')} k : \emptyset \\
\prod_{a,A} \left( \bigsqcup_{p \in [C']_A^p} [C']^p \right)
\end{array} \right)
\]

Follows directly from the definition of pruning in Def. 6.6 The case for \( [^c] \text{Cong} \) follows from application of Lemma A.20 for the case of swapping, and of Lemma A.16 for structural congruence. \( \square \)

**Theorem 6.8** (Correctness of the Endpoint Projection). Let \( C = (\nu \overline{k}, \overline{p}) C_1 \) with a linear restriction-free \( C_1 \), and \( \Gamma; \Psi \vdash C \triangleright \Delta, \) and \( \sigma \vdash \Psi. \) Then

- **(Soundness)** If \( \langle \sigma, C \rangle \xrightarrow{\Delta} \langle \sigma', C' \rangle \) and \( \| C \| \prec P \), then \( \langle \sigma', C' \rangle \xrightarrow{\Delta} \langle \sigma'', C'' \rangle \), \( P \xrightarrow{\Delta} P' \), \( \| C'' \| \prec P' \), and \( \lambda, \lambda' \vdash \overline{\mu} \).
- **(Completeness)** If \( \| C \| \xrightarrow{\mu_1} P \) then there exists \( P', \overline{\lambda} \) such that \( P \xrightarrow{\Delta} P' \), \( \langle \sigma, C \rangle \xrightarrow{\Delta} \langle \sigma', C' \rangle \), \( \| C' \| \prec P' \), and \( \lambda, \lambda' \vdash \mu_1, \overline{\mu}_2 \).
Proof. On Soundness:
The proof proceeds by rule induction on the transition rules for $\xrightarrow{\lambda}$ in Figure 5. We have seven cases:

**Case Rule $[^{c}]^{\text{init}}$:**

Hypothesis \[ C = (\nu k,\tilde{p}) \text{ start } t_s[B_s] : a(k) ; C_1 \] (A.1)

Hypothesis \[ t_r[A_r] \text{ start } t_s[B_s] : a(k) ; C_1 \text{ is restriction-free} \] (A.2)

Hypothesis \[ t_r[A_r] \text{ start } t_s[B_s] : a(k) ; C_1 \text{ is linear} \] (A.3)

Hypothesis \[ \Gamma; \Psi \vdash (\nu k,\tilde{p}) t_r[A_r] \text{ start } t_s[B_s] : a(k) ; C_1 \triangleright \Delta \] (A.4)

Hypothesis \[ \sigma \models \Psi \] (A.5)

Hypothesis \[ \langle \sigma, (\nu k,\tilde{p}) t_r[A_r] \{Y_r\} \text{ start } t_s[B_s] \{Y_s\} : a(k) ; C_1 \rangle \xrightarrow{\lambda} \langle \sigma[\sigma'[\sigma'']], C'' \rangle \] (A.6)

**A.6** \[ C'' = (\nu k,\tilde{p}) ((\nu k,\tilde{t}_s) (C_1)) \] (A.7)

**A.6, inversion** \[ \lambda = t_r[A_r] \{Y_r\} \text{ start } t_s[B_s] \{Y_s\} : a(k) \] (A.8)

**A.6, inversion** \[ \sigma' = [t_s, k] \mapsto Y_i |_{i=1}^{[\tilde{t}_s]} \] (A.9)

**A.6, inversion** \[ \sigma'' = [(t_s, k) \mapsto Y_i |_{i=1}^{[\tilde{t}_s]}] \] (A.10)

**A.1, def. $\{[]\}$** \[ \{C\} = (\nu \tilde{k}) \left( \prod_{p \in \Pi(C')} [C'_p]^{p} \prod_{k \in \text{fsc}(C')} k : \emptyset \right) \left( \bigcup_{p \in C'_p} [C'_p]^{p} \right) \] (A.11)

**A.10, exp.** \[ \{C\} = (\nu \tilde{k}) \left( \prod_{p \in \Pi(C')} [C'_p]^{p} \prod_{k \in \text{fsc}(C')} k : \emptyset \right) \left( \bigcup_{p \in C'_p} [C'_p]^{p} \right) \] (A.11)

**P_1 = (\nu \tilde{k}) \left( \prod_{p \in \Pi(C')} [C'_p]^{p} \prod_{k \in \text{fsc}(C')} k : \emptyset \right) \left( \bigcup_{p \in C'_p} [C'_p]^{p} \right) \] (A.12)

**A.11, exp. $\lfloor . \rfloor^{a}_A$** \[ \lfloor C \rfloor_A = \left( \nu \tilde{k} \right) \left( \prod_{j \in \Pi(C')} [C'_p]^{p} \prod_{k \in \text{fsc}(C')} k : \emptyset \right) \left( \bigcup_{p \in C'_p} [C'_p]^{p} \right) \] (A.13)

**A.13, rules $[^{k}]^{\text{init}},[^{k}]^{\text{res}}$**

\[ \left( \nu \tilde{k} \right) \left( \prod_{t_k \in \Pi(C')} [C'_p]^{p} \prod_{k \in \text{fsc}(C')} k : \emptyset \right) \left( \bigcup_{p \in C'_p} [C'_p]^{p} \right) \] (A.11)

\[ \prod_{k \in \Pi(C')} [C'_p]^{p} \prod_{k \in \text{fsc}(C')} k : \emptyset \right) \left( \bigcup_{p \in C'_p} [C'_p]^{p} \right) \] (A.13)

\[ P_2 = (\nu \tilde{k}) \left( \prod_{p \in \Pi(C')} [C'_p]^{p} \prod_{k \in \text{fsc}(C')} k : \emptyset \right) \left( \bigcup_{p \in C'_p} [C'_p]^{p} \right) \] (A.14)
Moreover, from Lemma A.21 we know that \( \forall p \in \text{fn}(t_r[A_r]) \) start \( t_s[B_s] : a(k) \); \( C_1 \) \( \{t_r, \tilde{t}_s, k\}, [C_1]^p \prec [t_r[A_r]] \) start \( t_s[B_s] : a(k) \); \( C_1 \) \( p \). The projection of \( C'' \) up to reordering and alpha conversion in eq. A.7 is

\[
\llbracket (\nu \tilde{k}, \tilde{p}) (C_1) \rrbracket = (\nu \tilde{k}, \tilde{k}') \left( \prod_{p \in \text{fn}(C_1)} [C_1]^p \mid \prod_{k \in \text{fsc}(C_1)} k : \emptyset \right) \prod_{a \in \text{sc}(C_1), A \in \text{roles}(C_1)} \left( \bigcup_{p \in [C_1]^p \delta_A} [C_1]^p \right) \tag{A.16} \]

That corresponds to eq. A.15. It is easy to see that \( t_r[A_r]\{X_r; Y_r\}, e_r \rightarrow t_B[B] \{X_B; Y_B\} : x : \langle k, op \rangle ; C_1 \)

Case Rule \([\text{aRed}]\):

Hypothesis
\[
C = (\nu \tilde{k}, \tilde{p}) &^{q}(t_r[A_r]\{X_r; Y_r\}, e_r) \rightarrow t_B[B] \{X_B; Y_B\} \rightarrow x : \langle k, op \rangle ; C_1 \quad \text{(A.1)}
\]

Hypothesis
\[
&^{q}(t_r[A_r]\{X_r; Y_r\}, e_r) \rightarrow t_B[B] \{X_B; Y_B\} \rightarrow x : \langle k, op \rangle ; C_1 \quad \text{is restriction-free} \quad \text{(A.2)}
\]

Hypothesis
\[
&^{q}(t_r[A_r]\{X_r; Y_r\}, e_r) \rightarrow t_B[B] \{X_B; Y_B\} \rightarrow x : \langle k, op \rangle ; C_1 \quad \text{is linear} \quad \text{(A.3)}
\]

Hypothesis
\[
\Gamma ; \Psi \vdash (\nu \tilde{k}, \tilde{p}) &^{q}(t_r[A_r]\{X_r; Y_r\}, e_r) \rightarrow t_B[B] \{X_B; Y_B\} \rightarrow x : \langle k, op \rangle ; C_1 \triangleright \Delta \quad \text{(A.4)}
\]

Hypothesis
\[
\sigma \models \Psi \quad \text{(A.5)}
\]

Hypothesis
\[
\llangle \sigma, &^{q}(t_r[A_r]\{X_r; Y_r\}, e_r) \rightarrow t_B[B] \{X_B; Y_B\} \rightarrow x : \langle k, op \rangle ; C_1 \rrangle \triangleright \llangle \sigma[\alpha'], C'' \rrangle \quad \text{(A.6)}
\]

A.6, inversion
\( J \subseteq t_r \quad \text{(A.7)} \)

A.6, inversion
\( q(J) \quad \text{(A.8)} \)

A.6, inversion
\( (e_i \circ \eta_i) \downarrow v_i \) \( \in J \quad \text{(A.9)} \)

A.6, inversion
\( \forall t_i \in \{t_B\} : \llbracket X_i \rrbracket \subseteq \sigma(t_i, k) \land \sigma'(t_i, k) = [X_i; Y_i] \llbracket \sigma(t_i, k) \rrbracket \quad \text{(A.10)} \)

A.6, inversion
\( \forall i \in \{1, \ldots, |\tilde{t}_r|\} : \theta(x_i) = \begin{cases} \text{some}(v_i) & t_i \in J \text{ some}(v) \text{ o.w.} \end{cases} \quad \text{(A.11)} \)

A.6, inversion
\( \text{op}(\theta) \downarrow \text{some}(v) \quad \text{(A.12)} \)

A.6, inversion
\( C'' = (\nu \tilde{k}, \tilde{p}) C_1[\text{some}(v)/x] @ t_B \quad \text{(A.13)} \)

A.6, inversion
\( \lambda = &^{q}(t_r[A_r]\{X_r; Y_r\}, v_r) \rightarrow t_B[B] \{X_B; Y_B\} \rightarrow \text{some}(v) : \langle k, op \rangle \quad \text{(A.14)} \)
A.1, def. \$\{\cdot\}\$: \$\{C\} = (\nu k,k) \left( \left( \prod_{p \in \text{fl}(C')} [C'_p]^p \mid \prod_{k \in \text{fsc}(C')} k : \emptyset \right) \right. \\
\left. \mid \prod_{a \in \text{sc}(C')} A \in \text{roles}(C') \left( \bigcup_{p \in [C'_a]} [C'_p]^p \right) \right) = P_1 \tag{A.15} \]

A.15, exp. \$P_1 = (\nu k,k) \left( \prod_{i \in [1,|\overline{C}|]} k[A_i]^i[B] \langle e_i \rangle ; [C_1]^u \mid k[\overline{A}]?q[B] \langle op(x) \rangle ; [C_1]^uB \right) \mid k : \emptyset \right) \left( \prod_{p \in \text{fl}(C') \setminus \overline{t}_B} [C_1]^p \mid \prod_{k' \in \text{fsc}(C') \setminus k} k' : \emptyset \right) \right) \\
\left. \mid \prod_{a \in \text{sc}(C')} A \in \text{roles}(C') \left( \bigcup_{p \in [C'_a]} [C'_p]^p \right) \right) = P_2 \tag{A.16} \]

Then we can proceed by moving the reduce into a blocking state, updating the session queue with information regarding expected senders:

\$P_2 \xrightarrow{\uparrow r} \$

A.16, rules \$[\text{E][Rd.1]}, [\text{E][Res]} \$

\$ (\nu k,k) \left( \prod_{i \in [1,|\overline{C}|]} k[A_i]^i[B] \langle e_i \rangle ; [C_1]^u \mid \text{wait}^i(k, \overline{A}, B, op, x) ; [C_1]^uB \right) \mid k : \emptyset \right) \left( \prod_{p \in \text{fl}(C') \setminus \overline{t}_p} [C_1]^p \mid \prod_{k' \in \text{fsc}(C') \setminus k} k' : \emptyset \right) \\
\left. \mid \prod_{a \in \text{sc}(C')} A \in \text{roles}(C') \left( \bigcup_{p \in [C'_a]} [C'_p]^p \right) \right) = P_3 \tag{A.17} \]

From eq. A.7 we know that for a subset \$J\$ of threads the quality predicate \$q\$ is deemed to be satisfied. We proceed by firing enough sender to guarantee \$q(J)\$:

\$P_3 \xrightarrow{!A_1 \triangleright B : k \langle \text{some}(v_1) \rangle} \ldots \xrightarrow{!A_n \triangleright B : k \langle \text{some}(v_n) \rangle} \$

A.9, A.17, rules \$[\text{E][Rd.1]}, [\text{E][Res]} \$

\$ J \times \$

\$ (\nu k,k) \left( \prod_{i \in [1,|J|]} [C_1]^u \mid \prod_{i \in \overline{U}_J} k[A_i]^i[B] \langle e_i \rangle ; [C_1]^u \right) \mid k : \emptyset \right) \left( \prod_{p \in \text{fl}(C') \setminus \overline{t}_p} [C_1]^p \mid \prod_{k' \in \text{fsc}(C') \setminus k} k' : \emptyset \right) \right) \\
\left. \mid \prod_{a \in \text{sc}(C')} A \in \text{roles}(C') \left( \bigcup_{p \in [C'_a]} [C'_p]^p \right) \right) = P_s \tag{A.18} \]

Where \$\overline{A} = \overline{A'}\$ and \$\forall (A : \text{tt} : sb) \in \langle A' : \text{tt} : sb \rangle\$ has the form of \$\text{some}(v_i)\$. Now we are in position to unpause the reduce process and apply the continuation to senders not used in the reduce.

\$P_s \xrightarrow{?q A \triangleright B : k \langle \text{some}(v) \rangle} \$

A.9, A.12, A.18, rules \$[\text{E][Wait]}, [\text{E][Res]} \$

\$ (\nu k,k) \left( \prod_{i \in [1,|J|]} [C_1]^u \mid \prod_{i \in \overline{U}_J} k[A_i]^i[B] \langle e_i \rangle ; [C_1]^u \right) \mid k : \emptyset \right) \left( \prod_{p \in \text{fl}(C') \setminus \overline{t}_p} [C_1]^p \mid \prod_{k' \in \text{fsc}(C') \setminus k} k' : \emptyset \right) \right) \\
\left. \mid \prod_{a \in \text{sc}(C')} A \in \text{roles}(C') \left( \bigcup_{p \in [C'_a]} [C'_p]^p \right) \right) \tag{A.19} \]

By applying the endpoint projection function on the redex of eq. A.6:
That corresponds to Eq. A.19 up to pruning. To prove behavioral implementation, we just need to check that the labels generated in eq. A.17, A.18 and A.19 correspond to the \( \lambda \) generated in eq. A.6. This follows after application of rule \([\|$Red].\\n\\n**Case** Rule \([\|$Cast]:** This case corresponds to the same equivalent class as the one from \([\|$Red]. Its proof is shown above.\\n\\n**Case** Rule \([\|$Sel]:** This case is analogous to broadcast. Recall that \( q \) for collective selection has been restricted to \( \forall \). Such restriction is fundamental to guarantee that after all endpoints have chosen their branch, the selector can continue.\\n\\n**Case** Rule \([\|$If]:**

\[
\begin{align*}
\text{Hypothesis} & \quad C = (\nu k, \bar{p}) \text{ if } e \oplus t \text{ then } C_1 \text{ else } C_2 \quad (A.1) \\
\text{Hypothesis} & \quad \text{if } e \oplus t \text{ then } C_1 \text{ else } C_2 \text{ is restriction-free} \quad (A.2) \\
\text{Hypothesis} & \quad \text{if } e \oplus t \text{ then } C_1 \text{ else } C_2 \text{ is linear} \quad (A.3) \\
\text{Hypothesis} & \quad \Gamma; \Psi \vdash (\nu k, \bar{p}) \text{ if } e \oplus t \text{ then } C_1 \text{ else } C_2 \triangleright \Delta \quad (A.4) \\
\text{Hypothesis} & \quad \sigma \models \Psi \quad (A.5) \\
\text{Hypothesis} & \quad \langle \sigma, \text{if } e \oplus t \text{ then } C_1 \text{ else } C_2 \rangle \xrightarrow{\tau} \langle \sigma, C_1 \rangle \quad (A.6) \\
\text{Hypothesis} & \quad \{(\nu k, \bar{p}) \text{ if } e \oplus t \text{ then } C_1 \text{ else } C_2 \} \prec P \quad (A.7)
\end{align*}
\]

Let us assume \( e \oplus t \downarrow \text{tt} \) (the other case is analogous).

A.1, \( e \oplus t \downarrow \text{tt} \)

\[
\langle \sigma, (\nu k, \bar{p}) \text{ if } e \oplus t \text{ then } C_1 \text{ else } C_2 \rangle \rightarrow \langle \sigma, (\nu k, \bar{p}) C_1 \rangle \quad (A.8)
\]

A.7, def. \([\cdot]\)

\[
\{C]\) = \begin{cases}
(\nu k) \left( \prod_{p \in \text{ft}(C')} [C']^p \mid \prod_{k \in \text{fsc}(C')} k : \emptyset \right) \\
\prod_{a \in \text{sc}(C'), A \in \text{roles}(C')} \left( \bigcup_{p \in [C']^A} [C']^p \right)
\end{cases} = P_1 \quad (A.9)
\]

A.9, def. \([\cdot]^p\)

\[
P_1 = \begin{cases}
(\nu k) \left( \text{if } e \text{ then } [C_1]^p \text{ else } [C_2]^p \mid \prod_{p' \in (\text{fn}(C_1), \text{fn}(C_2))} [C_1]^p \cup [C_2]^p \right) \\
\prod_{a,A} \left( \bigcup_{p \in \text{if } e \oplus p \text{ then } c_1 \text{ else } c_2} [C_1]^p \cup [C_2]^p \right)
\end{cases} = P_2 \quad (A.10)
\]

A.10, rules \([\|$If\], \([\|$Res\])

\[
P_2 \xrightarrow{\tau} \begin{cases}
(\nu k) \left( [C_1]^p \mid \prod_{p \in (\text{fn}(C_1), \text{fn}(C_2))} [C_1]^p \cup [C_2]^p \right) \\
\prod_{a,A} \left( \bigcup_{p \in \text{if } e \oplus p \text{ then } c_1 \text{ else } c_2} [C_1]^p \cup [C_2]^p \right)
\end{cases} \quad (A.11)
\]

From Lemma A.21 we know that \( \forall p \in \text{fn}(C), \{(\nu k, \bar{p}) C_1\} \prec \{(\nu k, \bar{p}) \text{ if } e \oplus t \text{ then } C_1 \text{ else } C_2\} \), hence, by the application of the pruning Lemma (lemma 6.7) along hypothesis A.7 then we can conclude \( \{(\nu k, \bar{p}) C_1\} \prec P. \quad \tau \models \tau \) follows after application of Rule \([\|$Tau\]).
Case Rule \([c]_{\text{Cong}}\): The case has two sub-cases, one for structural congruence and another for the swapping congruence. The first sub-case follows after application of Lemma \([A.16]\) and the second after application of Lemma \([A.20]\)

**On Completeness:**
The proof proceeds by induction on the structure of \(C'\). We have six cases:

**Case** \(C' = \bar{p}_r\) start \(\bar{p}_s : a(k); C_1\):

Hypothesis \(C = (\nu k, \bar{p})\) t\(_r\)[\(A_r\)] start \(t_s[B_s] : a(k); C_1\) \(\tag{A.1}\)

Hypothesis \(t_r[A_r]\) start \(t_s[B_s] : a(k); C_1\) is restriction-free \(\tag{A.2}\)

Hypothesis \(t_r[A_r]\) start \(t_s[B_s] : a(k); C_1\) is linear \(\tag{A.3}\)

Hypothesis \(\Gamma; \Psi \vdash (\nu \bar{k}, \bar{p})\) t\(_r\)[\(A_r\)] start \(t_s[B_s] : a(k); C_1 \triangleright \Delta\) \(\tag{A.4}\)

Hypothesis \(\sigma \models \Psi\) \(\tag{A.5}\)

Hypothesis \[\{(\nu k, p)\} t_r[A_r] \text{ start } t_s[B_s] : a(k); C_1\] \(\mu_1 \rightarrow P\) \(\tag{A.6}\)

On Completeness:

The proof proceeds by induction on the structure of \(C'\). We have six cases:

**Case** \(C' = \bar{p}_r\) start \(\bar{p}_s : a(k); C_1\):

Hypothesis \(C = (\nu k, \bar{p})\) t\(_r\)[\(A_r\)] start \(t_s[B_s] : a(k); C_1\) \(\tag{A.1}\)

Hypothesis \(t_r[A_r]\) start \(t_s[B_s] : a(k); C_1\) is restriction-free \(\tag{A.2}\)

Hypothesis \(t_r[A_r]\) start \(t_s[B_s] : a(k); C_1\) is linear \(\tag{A.3}\)

Hypothesis \(\Gamma; \Psi \vdash (\nu \bar{k}, \bar{p})\) t\(_r\)[\(A_r\)] start \(t_s[B_s] : a(k); C_1\) \(\triangleright \Delta\) \(\tag{A.4}\)

Hypothesis \(\sigma \models \Psi\) \(\tag{A.5}\)

Hypothesis \[\{(\nu k, p)\} t_r[A_r] \text{ start } t_s[B_s] : a(k); C_1\] \(\mu_1 \rightarrow P\) \(\tag{A.6}\)

6, def. \(\{\cdot\}\)

on Completeness:

The proof proceeds by induction on the structure of \(C'\). We have six cases:

**Case** \(C' = \bar{p}_r\) start \(\bar{p}_s : a(k); C_1\):

Hypothesis \(C = (\nu k, \bar{p})\) t\(_r\)[\(A_r\)] start \(t_s[B_s] : a(k); C_1\) \(\tag{A.1}\)

Hypothesis \(t_r[A_r]\) start \(t_s[B_s] : a(k); C_1\) is restriction-free \(\tag{A.2}\)

Hypothesis \(t_r[A_r]\) start \(t_s[B_s] : a(k); C_1\) is linear \(\tag{A.3}\)

Hypothesis \(\Gamma; \Psi \vdash (\nu \bar{k}, \bar{p})\) t\(_r\)[\(A_r\)] start \(t_s[B_s] : a(k); C_1\) \(\triangleright \Delta\) \(\tag{A.4}\)

Hypothesis \(\sigma \models \Psi\) \(\tag{A.5}\)

Hypothesis \[\{(\nu k, p)\} t_r[A_r] \text{ start } t_s[B_s] : a(k); C_1\] \(\mu_1 \rightarrow P\) \(\tag{A.6}\)

7, def. \([\cdot]\)

on Completeness:

The proof proceeds by induction on the structure of \(C'\). We have six cases:

**Case** \(C' = \bar{p}_r\) start \(\bar{p}_s : a(k); C_1\):

Hypothesis \(C = (\nu k, \bar{p})\) t\(_r\)[\(A_r\)] start \(t_s[B_s] : a(k); C_1\) \(\tag{A.1}\)

Hypothesis \(t_r[A_r]\) start \(t_s[B_s] : a(k); C_1\) is restriction-free \(\tag{A.2}\)

Hypothesis \(t_r[A_r]\) start \(t_s[B_s] : a(k); C_1\) is linear \(\tag{A.3}\)

Hypothesis \(\Gamma; \Psi \vdash (\nu \bar{k}, \bar{p})\) t\(_r\)[\(A_r\)] start \(t_s[B_s] : a(k); C_1\) \(\triangleright \Delta\) \(\tag{A.4}\)

Hypothesis \(\sigma \models \Psi\) \(\tag{A.5}\)

Hypothesis \[\{(\nu k, p)\} t_r[A_r] \text{ start } t_s[B_s] : a(k); C_1\] \(\mu_1 \rightarrow P\) \(\tag{A.6}\)
We have two cases: \( \langle \sigma, C \rangle \overset{\text{start } a(k)}{\rightarrow} \langle \sigma, C'_1 \rangle \), or there exists \( C_s \) s.t. \( C \rightarrow^* C_s \) and \( \langle \sigma, C_s \rangle \overset{\text{start } a(k)}{\rightarrow} \langle \sigma, C'_1 \rangle \), and \( \vec{X} = \vec{X}_1, \vec{X}_2 \). From Lemma \( \text{[A.20]} \) we know that \( \{C\} \) and \( \{C_s\} \) are the same, so we consider only one of them. We perform case analysis on the labels \( \mu_1 \) that our \( \{C\} \) in eq. \( \text{[A.8]} \) can generate.

**Subcase** \( \mu_1 = \vec{A} \text{ start } \vec{B} : a(k) \)

\[
\{C\} \overset{\vec{A} \text{ start } \vec{B} : a(k)}{\rightarrow} \{C'\}
\]

A.8, rules [\text{[Par]}, [\text{[Struct]}]

\[
(\nu \vec{k}, \vec{p}) \left( \begin{array}{l}
(\nu \vec{k}) \left( \prod_{i \in [2, |r|]} [C_i]^h_i \mid \prod_{j \in [1, |r|]} [C_i]^b_j \mid k : \emptyset \right) \\
\mid \prod_{i \in [1, |r|]} \sigma, \sum_i C_i \end{array} \right)
\]

A.15, def. \([\cdot]\)

\[
\{[\nu \vec{k}, \vec{p}] C_i\} = (\nu \vec{k}) \left( \prod_{p \in \mathcal{P}(C_i)} [C_i]^p \mid \prod_{k \in \mathcal{F}(C_i)} k : \emptyset \right)
\]

A.14, A.9

\[
\langle \sigma, (\nu \vec{k}, \vec{p}) C' \rangle \overset{\text{start } a(k)}{\rightarrow} \langle \sigma', (\nu \vec{k}, \vec{p}) \langle \nu \vec{k}, \vec{p} \rangle C_i \rangle \overset{\vec{X}}{\rightarrow} \langle \sigma'', C'_1 \rangle
\]

A.16, A.18

\[
\text{start } \vec{B} : a(k) ; \mu_1, \mu_2
\]

**Subcase** \( \mu_1 \neq \vec{A} \text{ start } \vec{B} : a(k) \): From this case we know that the reduction has been performed by threads outside the session establishment phase.

\[
\{C\} \overset{\mu_1}{\rightarrow} \{C'\}
\]

A.8, rules [\text{[Par]}, [\text{[Res]}]

\[
(\nu \vec{k}, \vec{p}) \left( \begin{array}{l}
(\nu \vec{k}) \left( \prod_{i \in [2, |r|]} [C_i]^h_i \mid \prod_{j \in [1, |r|]} [C_i]^b_j \mid k : \emptyset \right) \\
\mid \prod_{i \in [1, |r|]} P_s \sigma, \sum_i C_i \end{array} \right)
\]

A.18

Where \( P_s \) captures the evolution of \( (\nu \vec{k}, \vec{p}) \left( \prod_{i \in [2, |r|]} [C_i]^h_i \mid \prod_{k' \in \mathcal{F}(C_i)} k' : \emptyset \right) \).

We can split the reduction chain to one that considers the evolutions of \( P_{11} \), followed by the execution of the start action.

\[
\text{A.18, sub-case} \quad \{C\} \overset{\mu_1}{\rightarrow} P_{11} \overset{\mu_2}{\rightarrow} P_{12} \overset{\vec{A} \text{ start } \vec{B} : a(k)}{\rightarrow} P_{13} \overset{\vec{A} \text{ start } \vec{B} : a(k)}{\rightarrow} P_{14}
\]
This thesis now follows from the induction hypothesis.

**Case** $C' = p_r.e \to \&^q(p_s : x_s) : k; C_1$:

- Hypothesis $C = (\nu\tilde{k}, \tilde{p}) p_r.e \to \&^q(p_s : x_s) : k; C_1$  \hspace{1cm} (A.1)
- Hypothesis $p_r.e \to \&^q(p_s : x_s) : k; C_1$ is restriction-free  \hspace{1cm} (A.2)
- Hypothesis $p_r.e \to \&^q(p_s : x_s) : k; C_1$ is linear  \hspace{1cm} (A.3)
- Hypothesis $\Gamma; \Psi \vdash (\nu\tilde{k}, \tilde{p}) p_r.e \to \&^q(p_s : x_s) : k; C_1 \triangleright \Delta$  \hspace{1cm} (A.4)
- Hypothesis $\sigma \models \Psi$  \hspace{1cm} (A.5)
- Hypothesis $\{((\nu\tilde{k}, \tilde{p}) p_r.e \to \&^q(p_s : x_s) : k; C_1) \} \overset{\mu_1}{\rightarrow} P$  \hspace{1cm} (A.6)

**I.H.** $\forall \mu_1', \{((\nu\tilde{k}, \tilde{p}) C_1)\} \overset{\mu_1'}{\rightarrow} P_1$  \hspace{1cm} (A.7)

**I.H.** $P_1 \overset{\mu_2'}{\rightarrow} P_1'$  \hspace{1cm} (A.8)

**I.H.** $\langle \sigma_1, (\nu\tilde{k}, \tilde{p}) C_1 \rangle \overset{\lambda'}{\leadsto} \langle \sigma_2, C'_1 \rangle$  \hspace{1cm} (A.9)

**I.H.** $\{C'_1\} \sim P_1'$  \hspace{1cm} (A.10)

**I.H.** $\lambda' \vdash \mu_1', \mu_2'$  \hspace{1cm} (A.11)

The case is similar to the reasoning for the start case. Notice that although there cannot be in-session linearity (receivers may implement the same role), its impact only increases size and partitions of the reduction chain needed to execute the dequeuing of an action in the session queue. In particular, the projection $\{((\nu\tilde{k}, \tilde{p}) (t_r[A].e \rightarrow \&^q(t_s[B_s] : x_s) : k; C_1))\}$ generates the following endpoints:

$\overset{(\nu\tilde{k})}{\{C\}} \overset{?}{\rightarrow} (\nu\tilde{k}) \left( k[A]^q \tilde{B} \langle e \rangle ; [C'_1]^{t_1} \mid \Pi_{i \in [1..|\tilde{r}|]} k[B_i]^q[A](x_i); [C_1]^{t_1} \right)$

$\overset{\text{I.H.}}{\mid \Pi_{h \in \Pi[C']\cup t_r}[C_1]^{t_1} \mid \Pi'_{k' \in \Pi[C'] \cup t_r} k' : \emptyset \mid k : \emptyset}$

$\overset{\text{I.H.}}{\mid \Pi'_{a,A} \left( \bigcup_{p \in [t_r[A].e \rightarrow \&^q(t_s[B_s] : x_s) : k; C_1]^p} t_r[A].e \rightarrow \&^q(t_s[B_s] : x_s) : k; C_1 \right)^p} P_1$  \hspace{1cm} (A.10)

The cases related to the reduction chains generated by processes in $\Pi_{h \in \Pi[C'] \cup t_r}[C_1]^{t_1} \mid \Pi'_{k' \in \Pi[C'] \cup t_r} k' : \emptyset$, as well as the transitions generated by the swapping congruence relation are similar to the analysis in the start case. We will focus then on the behavior generated by remaining processes. Applying rules $[\text{\textsf{I}}^\text{\textsf{I}}\text{\textsf{B}}\text{\textsf{E}}\text{\textsf{C}}\text{\textsf{O}}, \text{\textsf{I}}^\text{\textsf{I}}\text{\textsf{P}}\text{\textsf{A}}\text{\textsf{R}}]$ with $e \downarrow v$ to the projected process leads to

$\overset{\text{I.H.}}{\mid \Pi'_{a,A} \left( \bigcup_{p \in [t_r[A].e \rightarrow \&^q(t_s[B_s] : x_s) : k; C_1]^p} t_r[A].e \rightarrow \&^q(t_s[B_s] : x_s) : k; C_1 \right)^p} P_1$  \hspace{1cm} (A.11)

With a blocked output, and receivers ready to interact. Their interaction cannot be assumed to happen in a given order. In general, each of the receive actions can be preceded or succeeded by a sequence of actions $\mu'$ generated from the interaction of processes outside session $k$. After a finite sequence of reductions $P_1 \overset{\mu_1}{\rightarrow} P_2 \overset{?A \triangleright B_1;k(some(v))}{\rightarrow} P_3 \overset{\mu_2}{\rightarrow} \ldots \overset{\mu_{j-1}}{\rightarrow} P_j$.
\[
P_j \xrightarrow{?A \geq B_1; k \langle \text{some}(v) \rangle} P_{j+1} \xrightarrow{\mu_j+1} P_n, \text{ with a given } j = |J|. \quad P_n \text{ has now the form:}
\]
\[
\begin{align*}
P_n = (\nu \tilde{k}) & \left( \prod_{i \in [1,|\tilde{k}\rangle - j]} [C_1^{\tilde{k}} | \prod_{i \in [1,|\tilde{k}\rangle - j]} [C_1^{\tilde{k}} | \text{some}(v)/x_j] | P_s \right) \\
& | k: h_1 \cdot (A, q : (\tilde{B}\langle \vec{t}, \tilde{B}'' \langle \vec{f}, \text{some}(v) \rangle : h_2) \right)
\end{align*}
\]

With \( P_s \) the result of the interactions of processes and queues not involved in session \( k \), \( \tilde{B} = \tilde{B}', \tilde{B}'' \), and \( h_1, h_2 \) the result of messages on the same session. From Eq. A.4 and Lemma A.17 we know that \((A, q : (\tilde{B}\langle \vec{t}, \tilde{B}'' \langle \vec{f}, \text{some}(v) \rangle : h_2) \notin h_1, h_2) \). Inversion on Eq. A.4 guarantees that predicate \( q(B') \) is satisfiable. From the application of structural congruence rules to rearrange \( h_1, h_2 \), the application of rules \([\text{Wait}_8],[\text{Res}]\) and \([\text{Par}]\) on \( P_n \), and Lemma A.13 we get:

\[
P_n \xrightarrow{\text{A.3, def. (\nu \tilde{k})}} (\nu \tilde{k}) \left( \prod_{i \in [1,|\tilde{k}\rangle - j]} [C_1^{\tilde{k}} | \prod_{i \in [1,|\tilde{k}\rangle - j]} [C_1^{\tilde{k}} | \text{some}(v)/x_j] | P_s \right) \\
& | k: h' \left( \prod_{p \in [\text{tl}[A], e \rightarrow \& q(t_s[B_s] : x_s) ; k; C_1]_A} [\text{tl}[A], e \rightarrow \& q(t_s[B_s] : x_s) : k; C_1]_A \right)
\]

With \( h' = h_1 \cdot h_2 \). The thesis now follows from the application of the induction hypothesis.

**Case** \( C' = \& ^q(\tilde{p}, \tilde{e}) \rightarrow p_s : x : (k, q p); C_1 \): This case corresponds to the same equivalence class as the one for broadcast. They differ on the fact that we must account that for each asynchronous output, a subsequence of actions due to concurrent sessions can occur. Moreover, we must guarantee a session linearity condition requiring that there are no other reduce operations under the same roles. This is guaranteed by Lemma A.17.

**Case** \( C' = p_r \rightarrow \& ^q(\tilde{p}_s) : k[\tilde{f}]; C_1 \): Recall that according to the syntactic restrictions for collective selections introduced in Section 3 predicate \( q \) must correspond to a \( \forall \) operator. The case follows in a similar way as the case for broadcast.

**Case** \( C' = \text{if } e \& p \text{ then } C_1 \text{ else } C_2 \):

Hypothesis \( C = (\nu \tilde{k}, \tilde{p}) \) if \( e \& p \) then \( C_1 \) else \( C_2 \) \hspace{1cm} (A.1)

Hypothesis if \( e \& p \) then \( C_1 \) else \( C_2 \) is restriction-free \hspace{1cm} (A.2)

Hypothesis if \( e \& p \) then \( C_1 \) else \( C_2 \) is linear \hspace{1cm} (A.3)

Hypothesis \( \Gamma; \Psi \vdash (\nu \tilde{k}, \tilde{p}) \) if \( e \& p \) then \( C_1 \) else \( C_2 \) \( \triangleright \Delta \) \hspace{1cm} (A.4)

Hypothesis \( \sigma \models \Psi \) \hspace{1cm} (A.5)

Hypothesis \( \{\{\langle \nu \tilde{k}, \tilde{p} \rangle \text{ if } e \& p \text{ then } C_1 \text{ else } C_2 \}\} \xrightarrow{\mu_1} P \) \hspace{1cm} (A.6)

\[
\{\{\langle \nu \tilde{k}, \tilde{p} \rangle \text{ if } e \& p \text{ then } C_1 \text{ else } C_2 \}\} =
\]

A.6, def. \( \{[\ldots]\} \) \hspace{1cm} (A.7)
Assume $e \downarrow \top$ (the opposite case is analogous). We have the following induction hypotheses:

I.H. \[ \forall \mu_1', \{ (\nu \tilde{k}, \tilde{p}) C_1 \} \xrightarrow{\mu_1'} P_1 \] (A.9)

I.H. \[ P_1 \xrightarrow{\tilde{\mu}_1} P'_1 \] (A.10)

I.H. \[ \langle \sigma, (\nu \tilde{k}, \tilde{p}) C_1 \rangle \xrightarrow{X} \langle \sigma', C'_1 \rangle \] (A.11)

I.H. \[ \{ C'_1 \} \prec P'_1 \] (A.12)

I.H. \[ \tilde{\lambda} \vdash \mu_1', \tilde{\mu}_2' \] (A.13)

By the application of $[\cdot]_{\Gamma}$ along with eq. A.11, we form the following reduction chain:

\[ \langle \sigma, (\nu \tilde{k}, \tilde{p}) \text{ if } e \oplus p \text{ then } C_1 \text{ else } C_2 \rangle \xrightarrow{X} \langle \sigma', C'_1 \rangle \] (A.14)

With $\tilde{\lambda}' = \tilde{\lambda}'_1, \tau, \tilde{\lambda}'_2$ and $\tilde{\lambda} = \tilde{\lambda}'_1, \tilde{\lambda}'_2$.

According to the use of the swap congruence, we must consider whether in the reduction chain $\{ C \} \xrightarrow{\tilde{\mu}_1} P$ the projection of $p$ executes or not the tau action associated to the conditional.

**Subcase** $\{ C \} \xrightarrow{\tilde{\mu}_1} P$:

A.8, rule $[\cdot]_{\Gamma}$ \[ \{ C \} \xrightarrow{\tilde{\mu}_1} (\nu \tilde{k}') (\{ C_1 \} | P_s) \]

\[ \mid \prod_{k \in \text{fsc}(C')} k : \emptyset \]

\[ \mid \prod_{\mu_1, A} \left( \bigcup_{p \in \{ \text{if } e \oplus p \text{ then } C_1 \text{ else } C_2 \}^A} [C_1]^\mu \cup [C_2]^\mu \right) \] (A.15)

Where $P_s$ denotes processes in $(\nu \tilde{k}) (\prod_{p \in \{ \text{if } e \oplus p \text{ then } C_1 \text{ else } C_2 \}^A} [C_1]^\mu \cup [C_2]^\mu) \mid \prod_{k \in \text{fsc}(C')} k : \emptyset$.

We proceed by the application of induction hypotheses in eq. A.9, A.10 for $P_s$.

A.15, eq. A.9, A.10 \[ \{ C \} \xrightarrow{\tilde{\mu}_1} (\nu \tilde{k}') (\{ C_1 \} | P_s) \]

\[ \mid \prod_{\mu_1, A} \left( \bigcup_{p \in \{ \text{if } e \oplus p \text{ then } C_1 \text{ else } C_2 \}^A} [C_1]^\mu \cup [C_2]^\mu \right) \]

\[ \xrightarrow{P_2} P' \] (A.16)

The thesis now follows from the application of induction hypotheses in A.12 and A.13, where $\mu_1 = \tau$ and $\tilde{\lambda} = \tilde{\lambda}'$.

**Subcase** $\{ C \} \xrightarrow{\mu_1} P, \mu_1 \neq \tau$:

A.9, $\mu_1 = \mu_1'$ \[ \{ C \} \xrightarrow{\mu_1} (\nu \tilde{k'}) (\{ C_1 \} | P_s) \]

\[ \mid \prod_{\mu_1, A} \left( \bigcup_{p \in \{ \text{if } e \oplus p \text{ then } C_1 \text{ else } C_2 \}^A} [C_1]^\mu \cup [C_2]^\mu \right) = P \] (A.15)
Where $P_s$ denotes new processes and session queues generated from the evolution of processes in $(\nu \tilde{k})(\prod_{p \in \{\text{ft}(C_1),\text{ft}(C_2)\}\setminus \mu}[C_1]^p \sqcup [C_2]^p | \prod_{k \in \text{fsc}(C')}(k: \emptyset))$.  

A.15, rule $[^{E}|if|]$.  $P \xrightarrow{\tau} (\nu \tilde{k})([C_1]^p | P_s) \mid \prod_{a,A}(\bigoplus_{p \in \{if \circ \bar{e} \circ p\} then C_1 else C_2}_{|A})([C_1]^p \sqcup [C_2]^p) = P'$ (A.16)

The thesis now follows by the application of induction hypotheses in eq. A.12, A.13

A.15, eq. A.9, A.10  

$\{[C]\xrightarrow{\mu_1} P \xrightarrow{\tau} P' \xrightarrow{\tilde{\mu}_2} P''\}$ (A.17)

where $\tilde{\mu}_2 = \tau, \tilde{\mu}_2$.

**Case $C' = C_1 + C_2$:** This case is essentially the same as the deterministic choice explained above. When proving $\langle \sigma, (\nu \tilde{k}, \bar{p}) C_1 + C_2 \rangle \xrightarrow{\lambda} \langle \sigma', C' \rangle$, we are reminded that the reduction chain $\xrightarrow{\lambda}$ can be given from the $\langle \sigma, (\nu \tilde{k}, \bar{p}) C_1 \rangle \xrightarrow{\lambda_1} \langle \sigma', C'_1 \rangle$ or from $\langle \sigma, (\nu \tilde{k}, \bar{p}) C_2 \rangle \xrightarrow{\lambda_2} \langle \sigma', C'_2 \rangle$. They correspond to the possible evolutions in  

$\{(\nu \tilde{k}, (C_1 + C_2))\} = (\nu \tilde{k})(\prod_{p \in \{\text{ft}(C_1),\text{ft}(C_2)\}}([C_1]^p + [C_2]^p) \mid \prod_{k \in \text{fsc}(C_1),\text{fsc}(C_2)}(k: \emptyset))$ 

$\prod_{a,A}(\bigcup_{p \in \{C_1 + C_2\}_{|A}}([C_1 + C_2]^p))$

**Case $C' = 0$:** This case is vacuously true.

\qed