Kinematic Arguments Against Single Relativistic Shell Models for GRBs

E. E. Fenimore, E. Ramirez, and M. C. Sumner

NIS-2, MS D436, Los Alamos National Laboratory, Los Alamos NM 87545

Abstract. Two main types of models have been suggested to explain the long durations and multiple peaks of Gamma-Ray Bursts (GRBs). In one, there is a very quick release of energy at a central site resulting in a single relativistic shell that produces peaks in the time history through its interactions with the ambient material. In the other, the central site sporadically releases energy over hundreds of seconds forming a peak with each burst of energy. We show that the average envelope of emission and the presence of gaps in GRBs are inconsistent with a single relativistic shell. We estimate that the maximum fraction of a single shell that can produce gamma-rays in a GRB with multiple peaks is $10^{-3}$, implying that single relativistic shells require $10^3$ times more energy than previously thought. We conclude that either the central site of a GRB must produce $\sim 10^{51} \text{ erg s}^{-1}$ for hundreds of seconds, or the relativistic shell must have structure on a scale the order of $\sqrt{\epsilon} \Gamma^{-1}$, where $\Gamma$ is the bulk Lorentz factor ($\sim 10^2$ to $10^3$) and $\epsilon$ is the efficiency.

INTRODUCTION

Two classes of models have arisen that explain different (but not all) aspects of the duration of GRBs. In the “external” shock model [1], the release of energy is very quick and a relativistic shell forms that expands outward for a long period of time ($10^5$ to $10^7$ sec). At some point, interactions with the external medium (hence the name) cause the energy of the bulk motion to be converted to gamma-rays. The alternative theory is that a central site releases energy in the form of a wind or multiple shells over a period of time commensurate with the observed duration of the GRB [2]. The gamma-rays are produced by the internal interactions within the wind, hence these scenarios are often referred to as internal shock models.

In Fenimore, Madras, & Nayakshin [3], we used kinematics to demonstrate that a single relativistic shell has extreme difficulties explaining the observed GRB time structure. We have made direct comparisons to the observations for three of the most potent arguments: the average envelope [4,5], gaps in the time history [3], and the maximum active fraction of the shell [6]. In this paper, we summarize those arguments.
Argument 1: Average Envelope

If a single relativistic shell with high bulk Lorentz factor ($\Gamma$) expands outward from a central site towards an observer, the observed time structure is dominated by two effects. First, although the shell might produce gamma-rays for a long period of time (say $t_0$ to $t_{\text{max}}$), the shell keeps up with the photons such that they arrive at a detector over a short period of time. If the shell has velocity $v = \beta c$ such that the Lorentz factor, $\Gamma$ is $(1 - \beta^2)^{-1/2}$, then photons emitted over a period $t$ arrive at a detector over a much shorter period, $T = (1 - \beta)t \approx t/(2\Gamma^2)$. Second, the curvature causes regions of the shell off-axis to arrive later at the detector. The additional distance that photons must travel is $\sim R(1 - \cos \theta)$ where $R$ is the radius of the shell ($\sim ct$). At a typical observable angle of $\theta = \Gamma^{-1}$, the delay due to the curvature is the same order as the time scale of arrive for on-axis photons: $t/(2\Gamma^2)$.

In [3,4], we showed that a single symmetric shell produces a “FRED”-like shape (fast rise, rapid decay):

$$V(T) = V_0 - \frac{T_0^\omega - T_{\text{max}}^\omega}{T_{\alpha+1}}$$

if $T_0 < T < T_{\text{max}}$;

$$= V_0 - \frac{T_{\text{max}}^\omega - T_0^\omega}{T_{\alpha+1}}$$

if $T > T_{\text{max}}$  \hspace{1cm} (1)

where $V_0$ is a constant, $\omega = \alpha + 3 - \nu$, $\alpha$ is the spectral number index (e.g., 1.5), and $\nu$ is a power law index for the intrinsic variation of the shell’s emissivity as a function of time. The expansion effects occur in the rise of the envelope and the curvature dominates in the fall. We have also shown [4] that during the decay phase, the spectra should evolve as $T^{-1}$.

To test this, we have added together 32 bright BATSE bursts with durations between 16 and 40 s. We align each burst by scaling it to a standard duration defined to be $T_{100} = (T_{90} + T_{50})/0.7$ where $T_{90}$ and $T_{50}$ are the durations that contain 90% and 50% of the counts. Figure 1 is from ref [5] which should be consulted for compete details. The average envelope and the average spectral evolution are linear whereas a single relativistic shell predicts that they should be power laws with indexes $-\alpha - 1$ and -1, respectively. We conclude that the average envelope of GRBs is not consistent with a single relativistic shell.

Argument 2: Gaps in Time History

Gaps or precursors in GRBs produce the strongest evidence against a single relativistic shell. The sharp rise in the average profile indicates that the shell emits for a short period of time (i.e., $t_0$ to $t_{\text{max}}$ is short relative to the duration of the event), so that the shape of the overall envelope is dominated by photons delayed by the curvature. During the decay phase, the region that can contribute photons to a given section of the time history is an annulus oriented about the line of sight to the observer (see Fig. 2). Gaps in the time history indicate that some annuli...
FIGURE 1. The average temporal and spectral evolution of bright events with intermediate durations ($T_{90}$ between 16 and 40 s) based on the BATSE MER data. (a) The average time history. The decay phase starting 20% after the beginning of the $T_{100}$ period is linear rather than the expected power law. (b) The average evolution of the the peak of the $\nu F_\nu$ distribution. The peak energy is also a linear function rather than the expected $T^{-1}$. These patterns are inconsistent with that expected from a single relativistic shell.

emit while others do not. These annuli are causally disconnected, making it difficult to achieve this large scale coherence. (See Figure 7 in [3] for attempts to fit the emission of shells to bursts with gaps and precursors.)

**Argument 3: Active Fraction of Shell**

Each dot in Fig. 2 is a causally connected region. Note region 3 has more dots so it produces a smoother time history (the intensity is less because the emission is off axis so fewer photons are beamed towards the observer). We have shown [7] that the volume of the annulus that contributes at any time is a constant so all sections of a time history should have about the same smoothness. We assume that the “peaks” in a time history represent Poisson fluctuations in the number of entities contributing at any time. We determine the total number of entities ($N_N = \mu_N(T/\Delta T)$) up to time $T$ by determining the rate of entities: $\mu_N = N^2/\delta N^2$ where $N$ and $\delta N$ are the mean and root-mean-square of the profile. The fraction of the shell that became active is $\epsilon = N_N A_N/A_S$. Here, $A_N$ is the size of each entity ($= \pi c^2 \Gamma^2 \Delta T^2/k$, where $k$ is 13 for entities arising from entities that grow at the speed of sound and is 1 for interactions with interstellar matter (ISM) clouds, see [6]). The total area of the shell is $A_S = 4\pi c^2 \Gamma^2 T^2 f$ where $f$ is the fraction of the shell out to $\theta = \Gamma^{-1}$ that contributes up to time $T$. For FRED-like bursts, Eq. 1 usually fits the profile such that $f$ is unity at $T = 0.8T_{50}$. For non-FRED like bursts, we simply assume that $f = 1$ at $T = 0.8T_{50}$. Figure 3 gives the efficiency for 6 FRED-like bursts and 46 bright, long complex BATSE bursts based on

$$\epsilon = N_N \left[\frac{\Delta T}{2T}\right]^2 \frac{1}{k_f} = \frac{N^2}{(\delta N)^2} \frac{\Delta T}{3.2kT_{50}} \quad (2)$$
where we have used the case of shocks growing at the speed of sound. For complete details see reference [6]. Thus, the spikiness of GRB time histories implies that only $\sim 10^{-3}$ of the surface of a shell becomes active. This is lower than previously estimated [3,8], and implies that models require $\epsilon^{-1} \sim 10^3$ times more energy than previously thought. Of course, reducing the fraction of the sky into which each shell expands can compensate for low efficiency for the small price of requiring a higher density (by $\epsilon^{-1}$) of GRBs in the universe.

A common misconception is that one can just use ISM clouds that cover most of the shell’s surface. Each cloud could cause a relatively large peak while efficiently utilizing the area of the shell. This does not work because the curvature of the expanding shell prevents the shell from engaging the cloud instantaneously. Rather, the portion of the shell at $\theta \sim \Gamma^{-1}$ requires $R(1-\cos\theta)/v$ longer before it reaches the cloud. Even if the cloud happens to have a concave shape such that the shell reaches the cloud simultaneously over a wide range of angles, the resulting photons at $\theta \sim \Gamma^{-1}$ must travel farther to the detector resulting in emission that is delayed by $R(1-\cos\theta)/c$. Since the speed is weakly dependent on $\Gamma$ or the ambient material, there is no reason to believe variations in the ambient material could cause the shell to develop into a plane wave oriented towards the observer such that the photons produced by an interaction with an ISM cloud or a shock would arrive as a short flare. Only the instantaneous interaction of two plain parallel surfaces oriented perpendicular to the line of sight can produce a short peak from large surfaces.

These three arguments make a strong case against single, symmetric relativistic shells that undergo variations either due to shocks or interactions with the ISM. There are two alternative explanations. First, one can accept the internal shock
FIGURE 3. Typical values of the fraction of a relativistic shell that becomes active during a GRB as a function of the duration of the emission ($T_{50}$). The six solid squares are FRED-like BATSE bursts for which direct estimates of the size of the shell can be made. The 46 open squares are long complex BATSE bursts where we estimate the size in a manner similar to the FRED-like estimate. Under most conditions, the efficiency is $\sim 0.1 \Delta T/T$. These low values imply that either only a small fraction of the shell converts its energy into gamma-rays or that GRBs consist of very fine jets with angular sizes much smaller than $\Gamma^{-1}$.

models [2]. These models have two weaknesses: there is a concern that internal shocks are rather inefficient, and the long, complex time history of a GRB requires the central site to produce $10^{51}$ erg s$^{-1}$ for hundreds of seconds. Second, one can retain the quick energy release associated with the single shell but break the spherical symmetry of the shell by having the emitting material confined to fine jets with angular width the order of $\sim \sqrt{\Gamma^{-1}}$.

REFERENCES

1. Mészáros, P., and Rees, M. J., *ApJ* **405**, 278 (1993).
2. Rees, M. J., and Mészáros, P., *ApJ* **430**, L93 (1994).
3. Fenimore, E. E., Madras, C. D., and Nayakshin, S., *ApJ* **473**, 998 (1996) astro-ph/9607163.
4. Fenimore, E. E., and Sumner, M. C., *All-Sky X-Ray Observations in the Next Decade*, Eds. M. Matsuoka, N. Kawai, in press, (1997), astro-ph/9705052.
5. Fenimore, E. E., *ApJ* submitted.
6. Fenimore, E. E., Cooper, C., Ramirez, E., and Sumner, M. C., *ApJ* to be submitted.
7. Sumner, M. C., and Fenimore, E. E., these proceedings, astro-ph/9712302.
8. Sari, R., and Piran, T., *ApJ* in press.