Density wave probes cuprate quantum phase transition

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In cuprate materials, the strong correlations in proximity to the antiferromagnetic Mott insulating state give rise to an array of unconventional phenomena beyond high temperature superconductivity. Developing a complete description of the ground state evolution is crucial to decoding the complex phase diagram. Here we use the structure of broken translational symmetry, namely d-form factor charge modulations in \((\text{Bi,Pb})_2(\text{Sr,La})_2\text{CuO}_6\delta^+,\) as a probe of the ground state reorganization which occurs at the transition from truncated Fermi arcs to a large Fermi surface. We use real space imaging of local electronic inhomogeneity as a tool to access a range of dopings within each sample, and we firmly establish the spectral gap \(\Delta\) as a proxy for local hole doping. From the \(\Delta\)-dependence of the charge modulation wavevector, we discover a commensurate to incommensurate transition that is coincident with the Fermi surface transition from arcs to large hole pocket, demonstrating the qualitatively distinct nature of the electronic correlations governing the two sides of this quantum phase transition. Furthermore, the doping dependence of the incommensurate wavevector on the overdoped side is at odds with a simple Fermi surface driven instability.

I. INTRODUCTION

In cuprates, high-temperature superconductivity lies between an undoped antiferromagnetic (AFM) insulator and a metal at high hole doping \((p)\). In proximity to the AFM insulator, the strong electronic correlations give rise to a complex phenomenology, including a large spectral gap \(\Delta\) that opens above \(T_c\), and a \(k\)-space structure lacking a conventional Fermi surface (FS) but described by open arcs [1, 2]. Both gap and arcs are widely considered hallmarks of this underdoped region of the phase diagram, and have drawn significant attention aimed at uncovering their origin(s) [3]. However, at a doping near optimal superconductivity, the Fermi arcs undergo an abrupt transition to a ‘large’ pocket consistent with a conventional volume proportional to \(1 + p\) [4–7]. A crucial challenge remains to identify the appropriate ground state(s) that underlie the theoretical framework on both sides of this transition.

On the overdoped side, long thought to be a Fermi liquid, several recent reports of anomalous behavior call into question the conventional interpretation [4, 17–21]. The observations of resistivity linear in temperature [17, 18] in Bi-based and La-based families challenge the expectations of Fermi liquid theory, and in MBE-grown La\(_{2−x}\)Sr\(_x\)CuO\(_4\) compounds, there are reports of mysterious symmetry breaking [20] and anomalous scaling of the superfluid density with critical temperature [19, 21], although the latter remains controversial [22, 23]. Furthermore, resonant inelastic x-ray experiments on Ti-, Y- [24], and La-based [25] compounds revealing spin-fluctuations have been interpreted in terms of significant electron correlations, and there are now theoretical proposals [26, 27] for how overdoped compounds may retain certain characteristic features of Fermi-liquid like behavior, while exhibiting fractionalization in the presence of strong stripe fluctuations. Additional experiments are necessary to understand the overdoped compounds, and in particular to clarify the extent to which the effects of strong correlations may persist through the FS transition.

In nearly all cuprate families, charge modulations have been reported in underdoped compounds, with detection terminating at [28] or before [6, 29] the doping where the FS transition occurs. In \((\text{Bi,Pb})_2(\text{Sr,La})_2\text{CuO}_6\delta^+,\) Bi2201, however, charge modulations extend into the overdoped regime [8, 11]. These modulations, reflecting an ordering instability of the electronic system, therefore serve as a doping-dependent fingerprint of underlying electronic interactions, not just in the underdoped regime, but across the FS transition (Fig. 1a). In the Bi2201 phase diagram, the FS transition occurs just below optimal doping [30], while the spectral gap persists in the presence of the large FS [8–10]. Peng et al. [11] recently investigated the charge modulation structure across the closing of the gap in the far-overdoped region, but it remains crucial to clearly define how the wavevec-

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FIG. 1. Using local electronic inhomogeneity and charge modulations to probe the Bi2201 phase diagram. (a) Phase diagram for Bi2201, zoomed in on the superconducting dome (green shading). At low temperature, the transition from Fermi arcs to a large Fermi surface (insets) occurs just below optimal doping (dashed line [8]), while the spectral gap persists into overdoped compounds (blue shading), with boundary marking the closing of the gap, as measured by ARPES [9] and NMR [10]). The data points mark existing sample wide average measurements of the charge modulation wavevector, $Q_{\text{DW}}$, in Bi2201 from x-ray scattering (green [11], blue [12] and black [13] open symbols) and STM (blue [12], orange [14] and purple [15] filled symbols). Red triangles and circles are the sample average measurements of the $d$-form factor charge modulations in the $x$ and $y$ directions, respectively, from this work, with $p$ determined from Ando’s conversion [16], as described in the supplementary material. The green squares and diamond are the annealed and as-grown samples, respectively, from Peng et al. [11]. (b) The local spectral gap $\Delta$ (shown for UD32K), measured as the position of the peak at positive bias in the differential conductance spectrum measured at each point. The white arrow marks the same location as in Fig. 2h to highlight a re-occurrence (QPI), and eliminates artifacts associated with the scan. Previous work has shown that smaller $\Delta$ corresponds to higher hole concentration, both locally within each sample [32, 34] and globally from sample to sample [35]. Thus, the same spectrum can be found locally in samples with different global $p$, and the overlapping $\Delta$ distributions from the four samples studied in this work, UD25K, UD32K, OPT35K and OD15K (Fig. 1d), allow us to move continuously from underdoped (UD) to overdoped (OD) in the phase diagram, using spatial masking [31] to hone in on a single doping (determined by the local concentration of dopants [32, 36]) within a larger field of view (Appendix B).

To validate the use of local doping to construct the Bi2201 phase diagram, we demonstrate that the evolution of the FS with $\Delta$ mimics that of bulk samples with $p$. We calculate the ratio map $Z(r, E) = g(r, E)/g(r, -E)$ [37], which enhances Bogoliubov quasiparticle interference (QPI), and eliminates artifacts associated with the tip-sample junction setup. The FS structure is apparent in the Fourier transform QPI patterns (Fig. S1) as the $\mathbf{q}_4$ scattering channel traces out an arc following the FS but with twice the size, $\mathbf{q}_4(E) = 2k(E)$, where $k$ parameterizes the Bogoliubov quasiparticle at energy $E$ [8, 38] (Fig. 2a). The $\mathbf{q}_4$ wavevectors – extracted as a function of $\Delta$ by selecting a range of $\Delta$ values in $\Delta(r)$ data (Appendix B) – together describe a single evolution of the momentum-space electronic structure extending across all samples (Fig. 2c-e). Regions with a small gap ($\Delta = 19$ meV, Fig. 2e) exhibit QPI from a

II. INHOMOGENEOUS FERMI SURFACE TRANSITION

Within a single Bi-based cuprate crystal, an average doping of $p$ holes per unit cell produces a highly inhomogeneous spatial distribution, resulting in large variations in the local electronic properties [31–33]. In a scanning tunneling microscope, the local electronic density of states is typically measured by the spatially-resolved differential conductance, $g(r, E = eV) = dI/dV(r, V)$, where $V$ is the sample bias. Binning and averaging the local spectra by gap size, $\Delta(r)$, as shown in Fig. 1b and c, demonstrates the variation of the spectrum over a large field of view. Previous work has shown that smaller $\Delta$ corresponds to higher hole concentration, both locally within each sample [32, 34] and globally from sample to sample [35]. Thus, the same spectrum can be found locally in samples with different global $p$, and the overlapping $\Delta$ distributions from the four samples studied in this work, UD25K, UD32K, OPT35K and OD15K (Fig. 1d), allow us to move continuously from underdoped (UD) to overdoped (OD) in the phase diagram, using spatial masking [31] to hone in on a single doping (determined by the local concentration of dopants [32, 36]) within a larger field of view (Appendix B).

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charge modulation structure in this range relates to the commensurate modulation observed in a lightly doped compound, near the insulating state [15].
large FS: the sections of the pocket near the antinodes at the edge of the Brillouin zone, $k_{x,y} = \pm \pi/a$ (Fig. 2a), generate scattering with wavevectors that extend out to $q_{x,y} = \pm 2\pi/a$. Moving to larger $\Delta$ (Fig. 2d), the $q_4$ trajectory shrinks, consistent with decreasing hole concentration, and the full evolution of the FS size, extracted from the QPI, (Fig. S2) concretely confirms that $\Delta$ is well-correlated to doping, in agreement with previous observations [31, 32, 39].

For larger $\Delta$, the QPI from antinodal (AN) states is not visible (Fig. S1), leaving only the signal from near-nodal states, as expected for Fermi arcs [8, 28]. Quantitatively, the intensity of the AN QPI in the UD32K sample (Fig. 4a) decreases with increasing $\Delta$, then near $\Delta = 50$ meV settles at a constant value, indistinguishable from the background, indicating the location of the transition, which we label $\Delta^*$. Spatially dividing the data into regions of $\Delta > \Delta^*$ and $\Delta < \Delta^*$ meV (Fig. 2f, g) shows that QPI from a large FS, and from Fermi arcs both exist within the sample, such that one can move on the phase diagram from one side of the transition to the other spatially. This emphasizes that the electronic structure is determined on length scales similar to the $\Delta$ correlation length. The FS $p$-evolution of the cuprate phase diagram, is therefore reproduced locally as a function of $\Delta$, with the transition from Fermi arcs to large FS occurring at $\Delta^* \approx 50$ meV.

### III. Commensurate to Incommensurate Transition

We now determine the doping dependence of the charge modulation wavevector, $Q_{DW}$, as a function of $\Delta$ to look for signatures of the change in ground state at $\Delta^*$. We examine $D(r)$, the $d$-form factor (dFF) component [40] of $\sum_{E>0} g(r, E)/\sum_{E<0} g(r, E)$, where the integration over an energy range larger than the typical $\Delta$ for each sample enhances the DW signal [41]. In all four samples, the amplitude of the Fourier transform, $D(q)$, has broad peaks at $(\pm Q_{DW},0)$ and $(0, \pm Q_{DW})$, near the charge modulation wavevectors that have been observed by previous experiments, indicating clearly that a dFF DW exists in all (Fig. S6). We employ the demodulation phase residue minimization technique of Mesaros et al. [42], to make $Q_{DW}$ measurements robust against the strong disorder apparent from the broad shape of the DW peaks.

Comparing Fig. 3 a-c to d-f reveals that the dFF DW exhibits distinctly different evolutions for small and large...
no significant increase is observed (Fig. 3) in the spectral gap regions. From 16 meV to 48 meV, \( \Delta \) increases from 0.15 r.l.u. to 0.23 r.l.u., matching the change in wavevector that has been measured by resonant x-ray techniques from \( p \approx 0.20 \) to near-optimal doping \([11–13]\), and consistent with an evolving incommensurate wavevector. However from \( \Delta \approx 50 \text{ meV} \) to \( \Delta \approx 85 \text{ meV} \), no significant increase is observed (Fig. 3a-c). The constant value of \( Q_{\text{DW}} \) near 0.25 r.l.u. suggests a dominant commensurate instability. In fact, the entire \( Q_{\text{DW}}(\Delta) \) evolution (Fig. 4b) is consistent with a commensurate to incommensurate transition at a location indistinguishable from \( \Delta^* \). The dashed line, showing the expected broadening of the underlying commensurate (yellow) to incommensurate (purple) trend, describes the data accurately (supplementary material Sec. SVII). Furthermore, the coincident changes in Fermiology and DW commensurability strongly suggest the presence of a quantum phase transition at \( \Delta^* \).

A. Commensurate density wave

The observed wavevector in the Fermi arc state (\( \Delta > \Delta^* \)) is consistent with a commensurate 4 unit cell charge modulation. The average \( Q_{\text{DW}} \) from UD25K, which lies almost entirely on the underdoped side of the transition, is 0.25(3) r.l.u. and 0.24(3) r.l.u. for \( x \) and \( y \) directions, respectively, where the errors represent the estimated standard deviation of spatial variations across the entire field of view. Resonant x-ray experiments \([12]\) have reported a doping dependent \( Q_{\text{DW}} \) in this same doping range, down to \( p = 0.115 \). However, this apparent discrepancy, can be understood by considering: (1) the FS transition occurs near \( p = 0.14 \) \([8]\) and beyond this point, we also observe an incommensurate wavevector, (2) the local inhomogeneity could plausibly induce a doping dependence of \( Q_{\text{DW}} \) even in samples with average \( p \) below the transition, and (3) the strongly disordered structure of charge modulations limits the precision with which the value of \( Q_{\text{DW}} \) (and \( Q_{\text{DW}}^* \)) can be determined unambiguously \([42]\).

For the samples studied here, ignoring local doping variations, and taking the average \( Q_{\text{DW}} \) value for each sample, hides the kink at the FS transition, and produces a trend of decreasing \( Q_{\text{DW}} \) with doping, similar to previous reports (Fig. 1a). Furthermore, while our data do not rule out a small doping dependence, recent experiments on Bi2212 \([42, 43]\) and very underdoped Bi2201 \([15]\) are also consistent with a \( Q = 0.25 \) r.l.u. commensurate DW within the Fermi arc regime, arising from proximity to the Mott insulating state.

B. Incommensurate density wave

What is the mechanism for the incommensurate \( Q_{\text{DW}} \) observed for \( \Delta < \Delta^* \)? Previous work interpreted the monotonically decreasing \( Q_{\text{DW}} \) as evidence of a FS instability that follows the growing FS hole pocket \([31]\). In this picture, there are two natural candidates for \( Q_{\text{DW}} \): (1) \( Q_{\text{AN}} \) that connects nested antinodal segments of the FS, and (2) \( Q_{\text{AFZB}} \) that connects the points where the FS crosses the antiferromagnetic zone boundary, the hotspots for \( (\pi, \pi) \) spin fluctuations. Both \( Q_{\text{AN}}(\Delta) \) and \( Q_{\text{AFZB}}(\Delta) \) are shown in Fig. 4b. For the largest \( p \) (smallest \( \Delta \)), \( Q_{\text{DW}} \) becomes similar to these Fermiology-derived wavevectors. However, upon decreasing \( p \) towards the transition, \( Q_{\text{DW}} \) grows more rapidly than the FS evolves. This unexpected discrepancy between \( Q_{\text{DW}} \) and Fermiology constitutes our second major finding.

IV. DISCUSSION

Whereas most recent theories for charge modulations in the cuprates have aimed to explain an incommensurate DW in the presence of Fermi arcs, here we are discussing (1) a \textit{commensurate} DW in the presence of Fermi arcs and (2) an incommensurate DW occurring in the presence of the \textit{large FS}. This leads to two important distinctions. First, in the Fermi arc regime, strong interactions are expected, and the associated renormalization can affect a hotspot wavevector, \( Q_{\text{HS}} \). For an instability of the large FS, however, it is not a priori clear that there should be any influence of correlations on such a \( Q_{\text{HS}} \).
FIG. 4. Simultaneous DW and FS transitions. (a) Intensity of AN QPI in UD32K, determined at five locations (red lines in the insets (iii, iv)), and normalized to the intensity of near-nodal QPI (Appendix E). $\Delta^*$ indicates the approximate location where the AN QPI becomes indistinguishable from the background noise. The thick shaded lines are guides to the eye. Insets (i,ii) show the approximate Fermi surface structure extracted from two examples of the gap-masked $Z(q)$ (iii, iv) without and with AN QPI. (b) Wavevector ($Q_{DW}$) of the dFF charge modulations in the $x$ (triangles) and $y$ (circles) directions, extracted from gap-masked $D(r)$. Refer to Fig. S4 for the standard deviation of $Q_{DW}$ for each value of $\Delta$. The thick shaded lines indicate the commensurate to incommensurate trend underlying the dashed line, which includes the expected effect of Gaussian smoothing ($\sigma = 12\text{meV}$) in $\Delta$, due to the resolution of the masking technique (Figs. S5, S7). The gray and black lines indicate the Fermiology-driven candidate wavevectors, as indicated schematically in the insets, and based on the circular Fermi surface models shown in Fig. 2a-e.

The first distinction reconciles our conclusion with Ref. [12]’s interpretation that $Q_{DW}$ is Fermiology driven [44]. Second, theoretical studies have found that the dominant charge density wave (CDW) instability of the large FS in the presence of exchange interactions has a wavevector along the $(\pm q, \pm q)$ direction rather than $(0, \pm q)$ or $(\pm q, 0)$, and the presence or absence of antinodal states is important in stabilizing the former or latter orientation, respectively [45]. It is therefore significant that the orientation of the charge modulations does not change at $\Delta^*$.

To search for information about the nature of the transition at $\Delta^*$, we consider that generically in CDW systems, a discontinuity or sharp jump in $Q_{DW}$ occurs at a commensurate to incommensurate transition [46]. Due to the experimental resolution, the data cannot distinguish between a continuous $Q_{DW}$ or one with a small jump, as shown by the yellow and purple trends in Fig. 4b (see also Fig. S5j). To extract this information, further experiments would be necessary to determine $\Delta^*$ and the incommensurate $Q_{DW}(\Delta)$ with reduced uncertainties. However, if present, a continuous $Q_{DW}$, which generically requires fine tuning, would imply a mechanism-derived constraint on the position of $\Delta^*$, not explained by existing theoretical models of the Fermi surface transition.

Ubiquity across underdoped compounds has widely been cited to motivate studying charge modulations as a route to understanding Fermi arc physics and the mechanism behind high $T_c$ superconductivity in cuprates. However, the relationship among these three phenomena has remained an open question. The coincidence of the DW and FS transitions, observed here, establishes an intimate link between the DW and the presence of Fermi arcs, and furthermore suggests that the same interactions which generate the commensurate instability may also be responsible for the arc phenomenology.

In summary, we have reported three concrete observations: (1) there is a commensurate to incommensurate transition of $Q_{DW}$ at a doping consistent with that of the FS transition; (2) on the underdoped side, $Q_{DW}$ is consistent with a commensurate 4 unit cell modulation; and (3) on the overdoped side, the doping-dependence of $Q_{DW}$ is stronger than that of the FS size.

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Appendix A: STM data

Differential conductance maps were collected in scanning tunneling microscopes at 6 K with the following tip-sample junction setup conditions: 100 mV and 100 pA for OD15K, -100 mV and 400 pA for OPT35K, -200 mV and 400 pA for UD32K, and -150 mV and 400 pA for UD25K. Measurements used a lock-in technique with 2 mV, 10 mV, 5 mV, and 5 mV bias modulations, respectively. Data were corrected for artificial distortions due to instrument drift to register the simultaneously recorded topography to a perfect lattice [47].
Appendix B: Gap-masking technique

The gap map \( \Delta(r) \) is calculated by finding the position of the peak at positive sample bias for the spectrum at each pixel, \( r \), in \( g(r,E) \), where Gaussian smoothing in energy reduces errors from noise in the data. The gap-masks are then generated by dividing the values of \( \Delta \) into bins with equal counts. The mask for bin \( b \), \( M_b(r) \), has a value of 1 if \( \Delta(r) \) is in \( b \), or zero otherwise. In order to reduce periodic structure in the masks arising from the atomic corrugation or the charge modulations [31], a bilateral filter is applied to \( \Delta(r) \) before generating the masks.

To obtain clear images in momentum transfer, \( q \), space, we use an additive masking technique, where for each bin \( b \), we look for the change in Fourier transform amplitude, when adding \( b \) into the field of view. The additive masks therefore include bins summed up to a bin \( b \): \( M_{1,b}(r) = \sum_{j=1}^{b} M_j(r) \). To avoid introducing artifacts from spatial structure of the masks, we apply Gaussian smoothing to the mask, with a spatial resolution of \( 1/w \). The filtered masks \( M_{1,b}(r) = G_w(r) * M_{1,b}(r) \), can now have any value in the range [0,1], where \( G_w \) is the Gaussian filter, and * indicates convolution. This filtering imposes a spatial resolution of the masks, and implies a spread in the \( \Delta \) distribution within each mask (Fig. S5).

To explain the additive masking technique, we describe how the images in Fig. 3 are generated. The additive masks are applied to the \( D(r) \) map in real space: \( D_{a,b}(r) = M_{a,b}(r)D(r) \). To visualize the charge order peaks from bin \( b \), we take the difference of the absolute values of the Fourier transforms, including and not including bin \( b \). The additive masking can either add up \( \Delta \) values from small to large (forward, fw) or in the reverse direction (backward, bw). For these two cases, the gap-masked \( D(q) \) is

\[
D^{fw}_b(q) = D_{1,b}(q) - D_{1,b-1}(q) \tag{B1}
\]

\[
D^{bw}_b(q) = D_{b,N}(q) - D_{b+1,N}(q) \tag{B2}
\]

where \( N \) is the total number of bins and \( D_{a,b}(q) \) is a real number, the amplitude of the Fourier transform. Unless otherwise specified, functions of \( q \) refer to amplitudes of Fourier transforms. The panels of Fig. 3 have Gaussian smoothing applied with width 0.01 r.l.u., to reduce the appearance of noise.

The QPI shown in the insets of Fig. 4, are \( Z^{fw}_b(q) \), generated from \( Z(r) \) using this same technique. Additional gap-masked QPI from UD32K in Fig. S1 shows the evolution of the Fermi surface structure across the full range of \( \Delta \) within the sample. These QPI images have Gaussian smoothing applied with width 0.015 r.l.u., and have been four-fold symmetrized.

For analysis of the Fermi surface structure, we used overlapping bins, i.e.

\[
Z^{fw}_b(q) = Z_{1,b}(q) - Z_{1,b-n}(q) \tag{B3}
\]

\[
Z^{bw}_b(q) = Z_{b,N}(q) - Z_{b+n,N}(q) \tag{B4}
\]

in order to measure the QPI wavevectors at finer spaced intervals of \( \Delta \), with more variations in the masks. The integer \( n \) determines the overlap between subsequent bins.

The average value of the gap for bin \( b \) is then:

\[
\Delta^{fw}_b = \frac{\sum r [M_{1,b}(r) - M_{1,b-n}(r)]}{\sum r [M_{1,b}(r) - M_{1,b+n}(r)]}, \tag{B5}
\]

\[
\Delta^{bw}_b = \frac{\sum r [M_{b,N}(r) - M_{b+n,N}(r)]}{\sum r [M_{b,N}(r) - M_{b+n,N}(r)]}, \tag{B6}
\]

where \( n = 1 \) for distinct, as opposed to overlapping, bins.

Here, we used \( N = 19 \) for UD25K, 31 for UD32K, 31 for OPT35K and 21 for OD15K, with \( n = 4 \), and \( w = 0.20 \) r.l.u.

For \( D(q) \) in Fig. 3, we used \( N = 9 \) with \( n = 1 \) for UD25K, UD32K and OPT35K, and \( N = 21 \) with \( n = 4 \) for OD15K. For OD15K, only the data for bins \( b = 5, 9, 13, 17, \) and 21 are plotted in Fig. 4b, to present data from distinct bins, as were used on the other 3 samples. The mask smoothing parameter, \( w \), was chosen to include the resolution, \( \Lambda \), of determining \( Q_{DW} \) (Extracting \( Q_{DW} \) from dFF charge modulation, below): \( w = \Lambda = 0.10 \) r.l.u. for UD25K, \( w = \Lambda = 0.10 \) r.l.u. for UD32K, \( w = \Lambda = 0.04 \) r.l.u. for OPT35K and \( w = (\Lambda^2 + 0.20 \text{r.l.u.}^{-2})^{-1/2} \) with \( \Lambda = 0.04 \) r.l.u. for OD15K.

Appendix C: Fermi surface structure from gap-masked QPI

\( Z(q) \) was integrated over the low energy layers to capture the full dispersion of the Bogoliubov quasiparticles, from 1.5 meV to 9 meV for OD15K, 5 meV to 5 meV for OPT35K, 10 meV to 25 meV for UD32K, and 5 meV to 15 meV for UD25K.

For each \( \Delta \) bin, the QPI wavevectors, as shown in Fig. 2, were extracted from the positions of peaks in one-dimensional cuts through \( Z^{fw}_b(q) \) and \( Z^{bw}_b(q) \), and are shown in Fig. S1 for UD32K. To quantitatively determine the size of the Fermi surface, the wavevectors from all samples were binned together, as shown in Fig. 2. A circular hole pocket was determined from the average radius of the data in the range \( \theta = 0.105 \pi \) to 0.145 \( \pi \), where \( \theta \) is defined in Fig. S1, and the range was selected because this near-nodal QPI is consistently measured across the Fermi surface transition (unlike the antinodal QPI), and is least influenced by nearby scattering channels or the DW signal. The evolution of the Fermi surface radius is shown in Fig. S2, and explicitly demonstrates that \( \Delta \) tracks the local doping.

Appendix D: Fermi arc and large Fermi surface QPI

To generate Fig. 2f and g, we locate the bin \( b^* \), which has the largest average gap value below 49 meV, the average of \( \Delta^* \) estimates based on forward and backward
masked data shown in Fig. 4a and Fig. S3b. Bins \( b \leq b^* \) primarily have \( \Delta < \Delta^* \), and \( b > b^* \) primarily \( \Delta > \Delta^* \). This division is only approximate due to the Gaussian smoothing of the masks (Fig. S5). To probe QPI from the two regimes divided by \( \Delta^* \), panels f and g show \( Z_{b^*+1,N}(q) \), and \( Z_{1,b^*}(q) \), respectively. Note that in this case, there is no subtraction after applying the Fourier transform. Fig. 2h shows \( \mathcal{M}_{1,b^*}(r) \), where the colorscale interpolates between 0.0 (copper) and 1.0 (purple). Because the masks used to obtain f and g are related by \( \mathcal{M}_{b^*+1,N}(q) = 1 - \mathcal{M}_{1,b^*}(q) \), the copper and purple indicate the regions primarily contributing to the Fermi arc and large Fermi surface QPI, respectively.

**Appendix E: Intensity of AN QPI in UD32K**

In order to quantify the disappearance of the antinodal (AN) QPI, which is apparent directly in the data (Fig. S1), we measure the intensity at cuts spaced at regular angular intervals, as shown in Fig. S3. The cuts are averaged over a transverse width of 0.07 r.l.u., and a length determined by the QPI radius (twice the FS radius) ± 0.06 r.l.u. To compare QPI intensities from different bins, the intensities of the cuts are normalized by the average intensity of five cuts closer to the nodal QPI (dashed red lines in Fig. S3a). Fig. 4a tracks this normalized intensity for five cuts near the antinode (red lines in Fig. 4a-iii and -iv).

**Appendix F: Extracting QDW from dFF charge modulation**

We follow the procedure described by Mesaros [42], where \( Q_{DW} \) is determined as the wavevector which minimizes the demodulation residue, \( R_Q = \sqrt{|R_Q^2| + |R_Q^2|} \), over the field of view. In the case of a strongly disordered density wave, this measurement has a more clearly defined interpretation than fitting peaks of the Fourier transformed data. Both techniques are compared in Fig. S4. A detailed explanation of demodulation residue can be found in Ref. 42. In this section, we extend the technique for application to masked regions of the data’s field of view.

\[ D(r) \text{ is demodulated by the reference wavevector } Q \text{ in} \]

\[ \tilde{\Psi}_Q(q) = \exp \left( \frac{-q^2}{2 \Lambda^2} \right) \tilde{\psi}(q + Q), \]  

where \( \tilde{\psi}(q) = \tilde{D}(q) \) over a domain which isolates the charge order peak, \( D \) is the complex-valued Fourier transform of \( D(r) \), and \( \tilde{\Psi}_Q, \tilde{\psi} \) are complex-valued functions. The Gaussian cutoff imposes a spatial resolution of \( 1/\Lambda \).

The \( Q_{DW} \) measurement proceeds in each bin, b, by integrating the residue only over the masked region:

\[ R^\alpha_{Qb}(\Psi) = \int d^2r M_b(r) \text{Re} \left[ \Psi_Q^\alpha(-i\partial_\alpha)\Psi_Q \right], \]

\[ \sigma^2 = \frac{\int d^2r M_b(r) \sum_\alpha \left| \Psi_Q^\alpha(-i\partial_\alpha)\Psi_Q \right|^2}{\int d^2r M_b(r) \left| \Psi_Q \right|^2}, \]

where \( \alpha \) is either \( x \) or \( y \), \( \Psi_Q(r) \) is the inverse Fourier transform of \( \tilde{\Psi}_Q(q) \), and with \( Q = Q_{DW} \), \( \sigma \) estimates the standard deviation of spatial fluctuations in the modulation wavevector.

For this analysis, we used \( w = 0.0 \) as the mask smoothing parameter for UD25K, UD32K and OPT35K, with \( \Lambda = 0.10 \) r.l.u., 0.10 r.l.u. and 0.04 r.l.u., respectively. For OD15K, we used \( w = 0.20 \) r.l.u. and \( \Lambda = 0.04 \) r.l.u. The samples with smaller wavevector require a smaller \( \Lambda \) for \( Q_{DW} \) to be robust against the choice of \( \Lambda \).

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