Subsonic choking in microchannel slip flow: Isothermal or adiabatic?

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ABSTRACT

Some experimental evidence of subsonic choking in rarefied gas flow regime is available; however, the nature of the flow (isothermal/adiabatic) close to the choking point is not known. The theoretical limiting Mach number (Ma) for isothermal choked flows is \(1/\sqrt{\gamma}\) (where \(\gamma\) is the ratio of specific heat) and that for adiabatic choked flows is unity. In this work, we perform measurements for temperature, pressure, and mass flow rate at the inlet and outlet of a microchannel of aspect ratio 0.49 in the slip flow regime (4.04 \(\times\) \(10^{-3}\) < \(Kn_o\) < 7.04 \(\times\) \(10^{-3}\), where \(Kn_o\) is the outlet Knudsen number). We see some evidence of choking at Ma close to \(1/\sqrt{\gamma}\) with a shift in the choking point to Ma of unity. The measured static temperature is observed to be constant at microchannel inlet and outlet, indicating isothermal flow behavior for lower Ma values. The stagnation temperature is calculated to be nearly constant at the microchannel outlet for higher Ma values, indicating a shift in flow behavior to adiabatic. This study emphasizes the significance of temperature measurement for understanding the choking behavior. There is no active transfer of heat during the experiments, making the present work relevant to practical and real situations. This state-of-the-art study would be immensely useful while designing microchannels and microtubes for long-distance gas transportation, microelectromechanical systems, and space applications, where one needs to be careful about any occurrence of choking.

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I. INTRODUCTION

High-performance gas flow is noteworthy in aircraft propulsion engines, power and chemical plants, vacuum appliances, and flow machineries. Transportation of natural gas over long distances is carried out by commercial pipelines, where isothermal flow with friction has great significance. In macroscopic channels, increase in pressure ratio may lead to an increase in local flow velocity achieving an outlet Mach number (Ma_o) of unity. For further increase in pressure ratio, the back conditions are not able to alter the subsonic flow character of the inner channel, and the mass flux attains a maximum value. This phenomenon is called sonic choked flow. If Ma_o < 1 appears at channel exit, even when the flow is choked, it is referred to as subsonic choking. The choking phenomenon is more significant in compressible microflows as compared to macroflows due to greater surface to volume ratio, surface roughness, and Mach numbers that can be achieved at relatively smaller Reynolds numbers (Re). The choking limits the amount of mass of gas that can be transported through a micropassage; therefore, studying this phenomenon has great practical applications.

Engineering applications where compressible gas flows occur in miniaturized channels are central processing units, consumer electronic cooling, gas sensors, microspacecraft, micronozzles, and microelectromechanical systems (MEMS). Choking is applicable in outer space for the emission of reactive gas micromass. A comprehensive investigation of the choking phenomenon would therefore aid in improving the mass transmission and efficiency of heat transfer in MEMS and forms the subject of the current investigation.

The effect of microchannel outlet conditions is prominent in a rarefied choked flow. In the continuum flow regime, the flow gets choked below a critical value of the back pressure. However, for rarefied gases, there exists no such critical value. That means, whatever the back pressure is, it will influence the rarefied flow. As the degree of rarefaction increases, the influence of back pressure is seen to be more in the upstream direction. When the value of back pressure is zero, the mass flow rate achieved is maximum (\(i_{m_{max}}\)). At a fixed pressure of the upstream tank, if the pressure of the downstream tank is decreased, the value of the mass flow rate (in) across the channel first increases and then slowly achieves a saturation value.
On reducing the pressure of the upstream tank, the difficulty in attaining the choked state increases.

Two-dimensional (2D) direct simulation Monte Carlo (DSMC) method has been employed in rarefied choked gas flows by Chong.\textsuperscript{7} Roohi et al.,\textsuperscript{2} and Ilgaz and Çelenligil.\textsuperscript{1} Chong simulated the isothermal flow of channels with a pressure ratio of $p_i/p_o$ from 101 to 1010. The author claimed that the Navier Stokes equations estimate the mass flux correctly when length/width ($L/W$) $>1000$. Roohi et al.\textsuperscript{2} performed pressure-driven analysis for $p_i/p_o = 15$, $Re < 10$, $L/W = 0.66$, and $Kn_i = 0.074$. They claimed that subsonic flow appears near the exit of microchannels/ nanochannels if back pressure less than ordinary exit pressure is applied. It is advisable to impose back pressure in the buffer zone away from the actual channel outlet to obtain the correct flow physics and choked state. Ilgaz and Çelenligil\textsuperscript{1} worked with the adiabatic rarefied choked flow in parallel plates ($L/W = 214$, $Re < 10000$). They concluded that the back pressure primarily controls the rarefied gas flow and its influence is felt more strongly upstream upon increasing the amount of rarefaction.

Other numerical works include those by Shang and Wang,\textsuperscript{3} Croce et al.,\textsuperscript{4} and Lijo et al.\textsuperscript{5} Shang and Wang\textsuperscript{3} used the generalized Enskog Monte Carlo method and found the subsonic choking phenomenon to occur at an asymptotic value of $p_i/p_o$, which increases linearly with $L/W$ ratio of a microchannel. They predicted that this linear behavior would discontinue for $L/W > 100$. Croce et al.\textsuperscript{4} incorporated a 2D compressible finite volume solver with Maxwell slip boundary condition for saw-tooth roughness obstructions with relative roughness up to 2.65%, $Re < 200$, $Ma < 0.8$, and $Kn = 2.49 \times 10^{-2}$. They explicated about the dominant influence of compressibility compared to roughness and rarefaction for higher values of $p_i/p_o$ and $Ma$. Lijo et al.\textsuperscript{5} simulated for isothermally heated adiabatic walls with a no-slip condition in a microchannel of hydraulic diameter, $D_h = 300 \mu m$. They found choking to occur further upstream of microchannel outlet location on increasing $p_i/p_o$. The downstream conditions controlled the flow in microchannel and evolution of supersonic flow even after the flow was choked. The velocity gradient at the surface of the channel wall increased upon choking due to the acceleration of gas. The heat transfer rate near the outlet was directly proportional to the pressure ratio, which was accredited to the scaling effect in microchannels, thinning of the boundary layer, and higher strain rates and temperature gradients. Lijo et al.\textsuperscript{5} highlighted the need for a new scaling factor to include the influence of outlet on choked flow.

There appears to be only two experimental studies on choked rarefied flow by Yao et al.\textsuperscript{6} and Harley et al.\textsuperscript{7} Other experimental choking studies exist at high pressure flows,\textsuperscript{10} but not in the slip regime. Yao et al.\textsuperscript{6} investigated isothermal rarefied air flow with 2D microchannel of dimensions $L \times H \times W = 39900 \times 1000 \times 7 \mu m^3$, where $L$, $H$ and $W$ denote length, height and width, respectively. They defined the subchoking phenomenon as the corresponding flow, where $(m_{\text{max}} - m_{\text{max}}) / m_{\text{max}}$ is less than 5%. This subchoking phenomenon occurred at $p_i/p_o > 5.3$ due to the intense surface effect inside a microchannel.\textsuperscript{6} They concluded that subchoking critical pressure ratio is directly proportional to the surface to volume ratio of the microchannel. Yao et al.\textsuperscript{6} did not include the analysis of choking behavior (isothermal/adiabatic) in their study. Harley et al.\textsuperscript{7} also worked with 2D microchannels, and with inlet pressures greater than 1 atm. The maximum outlet Mach number achieved was about 0.85, which is close to theoretical isothermal choking limit.

In the present work, measurements of mass flow rate, static pressure, and temperature are carried out in a three-dimensional (3D) microchannel ($D_h = 132.26 \mu m$) of rectangular cross section with an aspect ratio ($\alpha = W/H$) 0.49 in the slip flow regime.\textsuperscript{11} The range of inlet pressures ($p_i$) is chosen upto 1 atm, since it is not covered in the existing experimental choking studies. The aim of this work is to explore the nature of choked state in the microchannel for Reynolds number ($91.21 \leq Re < 364.84$), outlet Mach number ($0.43 < Ma < 0.99$), and outlet Knudsen number ($4.04 \times 10^{-3} < Kn_o < 7.04 \times 10^{-3}$). A thorough investigation of the gas flow behavior to be isothermal or adiabatic at the subsonic choked state is desirable. There is no active temperature control in our measurements, which is a typically accepted scenario of real systems.

II. EXPERIMENTAL SETUP

A. Fabrication of microchannel

A three-dimensional (3D) microchannel [Fig. 1(c)] of dimensions $L \times H \times W = 30000 \times 201.34 \times 98.47 \mu m^3$ is fabricated using standard soft-lithography of poly(dimethylsiloxane) due to its ease of fabrication and low cost. The aspect ratio ($\alpha = W/H$) is 0.49 similar to the microchannel numerical study of Garg and Agrawal.\textsuperscript{12} The details of fabrication can be found in Singh et al.\textsuperscript{13} T-shaped silicon connectors are placed at the channel inlet and exit for making pressure/temperature ports. The channel roughness value is 0.44 $\mu m$.

B. Method of measurement

The schematic of the mass-driven experimental setup is shown in Fig. 1. The working fluid, nitrogen gas, is driven by a vacuum pump (Hindivac ED-21) of speed 350 lpm and pumping capacity of 0.1 Pa. The gas cylinder is connected to two filters for blocking any impurities of size more than 4 $\mu m$ before entering the mass flow controller (MFC I, MKS 1179A of range 200 sccm). The gas settles down in the upstream tank, whose absolute pressure is monitored via MKS 626C gauge of 1332.22 mbar range (with accuracy, 0.25% of reading). Now the gas enters the main microchannel through long silicon tubings to avoid any sudden contraction effect at the microchannel inlet. The microchannel outlet is connected to a second mass flow controller (MFC II, MKS GE50A of range 1000 sccm) to meter the actual flow being pushed in the choked state. The output of MFC II is trusted for flow rate readings.

Inlet and outlet static pressures are measured using pressure gauges of range 1332.22 mbar and 666.61 mbar, respectively. Inlet and outlet static temperatures are monitored through K-type thermocouples. After exiting MFC II, the flow reaches the downstream tank, whose pressure is checked using a 26.66 mbar gauge. The leakage in the system is 0.14 sccm, which is 1% of the minimum flow rate passed through the channel. Therefore, three quantities are measured at the inlet and outlet of the microchannel, namely, mass flow rate, static pressure, and static temperature.

C. Data reduction

The most significant parameter to quantify the flow compressibility is Mach number (Ma) given by Eq. (1). Reynolds number
FIG. 1. (a) Schematic of the experimental setup. (b) Actual image of setup with real-time measurements. (c) Top view of the fabricated 3D microchannel. (d) Front view of the microchannel.

(Re) is calculated from Eq. (2). The gas rarefaction is quantified by Knudsen number (Kn) displayed in Eq. (3) as follows:

\[
Ma = \frac{p}{G \sqrt{\gamma RT}}^{3/2},
\]

\[
Re = \frac{\dot{m}D_h}{\mu A_c},
\]

\[
Kn = \frac{\lambda}{D_h} = \frac{\mu}{pD_h} \sqrt{\frac{\pi RT}{2}} = \sqrt{\frac{\pi \gamma Ma}{2 Re}}.
\]

Here, \( p \) is the measured absolute pressure, \( G \) is the mass velocity given by \( \dot{m}/A_c \), \( \dot{m} \) is the mass flow rate measured, \( A_c \) is area of cross section, \( \gamma \) is the ratio of specific heats for the gas, \( R \) is the gas constant, \( T \) is the absolute gas temperature, \( \mu \) is the dynamic viscosity, \( D_h \) is the hydraulic diameter of the microchannel given by \( 4A_c/P \), where \( P \) is the perimeter, and \( \lambda \) denotes the mean free path of the gas.

D. Uncertainty of measurement

The systematic errors in measurement have been taken care of by getting the mass flow controllers, capacitance manometers, and thermocouples calibrated. The errors in individual measured (independent) quantities propagate on calculating the derived (dependent) result. Hence, there is a need for uncertainty analysis to accommodate the combined effect of errors in the desired result of friction factor (\( f_{exp} \)) given as

\[
f_{exp} = \frac{D_h}{L} \left( p_i - p_o \right) \left\{ \frac{1}{G^2 RT} - \frac{1}{p_i p_o} \right\}.
\]

where \( L \) denotes the microchannel’s length. The subscripts i and o represent inlet and outlet of the channel. \( f \) is calculated by integrating pressure from the inlet to outlet based on the momentum theorem, as shown in Eq. (4). The first term in Eq. (4) reflects frictional pressure drop while the second term indicates acceleration of the flow. The analytical root-sum-square expression for finding uncertainty in the friction factor (\( \delta f \)) is as follows:

\[
\delta f \cong \sqrt{\left( \frac{\partial f}{\partial \delta p_i} \right)^2 + \left( \frac{\partial f}{\partial \delta p_o} \right)^2 + \left( \frac{\partial f}{\partial \delta \dot{m}} \right)^2 + \left( \frac{\partial f}{\partial \delta T} \right)^2 + \left( \frac{\partial f}{\partial \delta \lambda} \right)^2}.
\]

The error in the final result is more than or equal to the maximum error in any quantity used to calculate the result.
Table I shows the possible experimental uncertainties in every quantity. The major error in the uncertainty of the friction factor results from the mass flow rate measurement. As Re increases, the latter remains the dominant error source, followed by uncertainty in the hydraulic diameter. The comparative uncertainties in microchannel length, width, static pressure, and gas temperature are less, playing an insignificant role in the overall uncertainty. The uncertainty in temperature results in deviation in the value of gas dynamic viscosity ($\mu$), which has been included in the overall uncertainty. It is recommended to perform uncertainty analysis before setting up the instrumentation, so that maximum investment goes in the measuring operation to which the final result is most sensitive.

The overall uncertainty in the friction factor reduces as Reynolds number increases.

### III. VALIDATION OF RESULTS

Mass flow rate and absolute pressures at the inlet and outlet of the microchannel are measured for calculating the frictional resistance. It is observed that the pressure drop increases with the mass flow rate. The friction factor decreases monotonically with Reynolds number (Fig. 2). This trend is compared against the analytical solution by Ebert and Sparrow\cite{16} for the 2D isothermal rarefied gas solution given by

$$f(K_n) = \left[ 1 - \left( \frac{p_o}{p_i} \right)^2 \right] + 2\eta K_n \left( 1 - \frac{p_o}{p_i} \right)$$

$$+ \frac{2\eta^2 R T \chi R e^2}{p_i^2 D_h^3} \left( \eta K_n \left( 1 - \frac{p_o}{p_i} \right) + \ln \left( \frac{p_o}{p_i} \right) \right) \frac{D_h^5 p_i^2}{\mu^2 R T \chi R e^2 L^2}, \quad (6)$$

where the values of constants $\chi = 1.35$ and $\eta = 6.306$ are taken depending upon the channel’s aspect ratio.\cite{16} This theoretical solution has been used by Harley et al.\cite{17} for comparing their experimental results. The first two terms on the right hand side represent wall shear, whereas the last term represents the momentum change. There is an excellent agreement with the classical compressible theory, which gives us confidence in our measurements and validates the current experimental results.

### IV. RESULTS

The working fluid is nitrogen whose mass flow rate varies from $2.43 \times 10^{-7}$ kg/s (13 sccm) to $9.73 \times 10^{-7}$ kg/s (52 sccm). The static pressures are measured at the inlet and outlet of the microchannel. As will be demonstrated later, the measured static temperature of the inlet and outlet of the microchannel has a constant value of 298 K throughout the flow. All the measurements are carried out in the early slip regime ($4.04 \times 10^{-3} < K_n < 7.04 \times 10^{-3}$).

A. Evidence of subsonic choking

The first sign of choking is obtained when the mass flow rate ($\dot{m}$) attains a maximum value. The microchannel is not able to allow mass beyond $9.73 \times 10^{-7}$ kg/s (corresponding to 52 sccm) to pass through it (Fig. 3).

Initially, the pressure drop ($\Delta p$) is directly proportional to the mass flow rate and then becomes asymptotic as the mass flow rate ($\dot{m}$) increases. If we cast the data in terms of pressure ratio and

![Fig. 2. Variation of friction factor with respect to Reynolds number for present 3D microchannel measurements compared with the 2D rarefied isothermal solution by Ebert and Sparrow.](image-url)
outlet Mach number, we find that the pressure ratio \( \frac{p_i}{p_o} \) is inversely proportional to the outlet Mach number \( (M_o) \) and eventually attains a critical value of 8.14 when the mass flow rate attains its maxima of 52 sccm, occurring at \( M_o = 0.99 \).

Note that \( \frac{p_i}{p_o} \) starts to become constant and tends to a critical value of 8.14 at \( M_o = 0.94 \). The mass flow rate at \( M_o = 0.94 \) is 49 sccm, which is within 5.7% of the maximum mass flow rate \( (\dot{m}_\text{max}) \) obtained. \( M_o \) is less than unity; this is a symptom of subsonic choking, since the mass flow rate is within the acceptable limit 5% of \( \dot{m}_\text{max} \). Yao et al.\(^9\) experimentally obtained isothermal subchoking phenomenon to occur at \( (\dot{m}_i - \dot{m}_\text{max})/\dot{m}_\text{max} \) value of 5%, very close to our observations. According to Shang and Wang,\(^3\) different \( p_i \) values resulted in different critical pressure ratios when the choked flow gets initiated and subsonic choking occurred when the critical \( p_i/p_o \) was almost greater than 6.5 and \( L/W \geq 10 \). The critical \( p_i/p_o \) value increased linearly with \( L/W \), until \( L/W = 100 \) in their case. The cause of subsonic choking was attributed to expansion wave prediction downstream and noteworthy nonequilibrium effects at the outlet since Knudsen number at the outlet is much greater than Knudsen number at the inlet.

In the present work, \( Kn_o \) is one order of magnitude greater than \( Kn_i \) throughout the measurements. In Fig. 4, for 621.10 mbar \( < p_i < 994.95 \) mbar, \( p_i/p_o \) decreases monotonically from 8.72 to 8.21 when 0.43 \( < M_o < 0.94 \). The inlet pressure \( (p_i) \) greater than 1 atm leads to critical \( p_i/p_o = 8.17 \) when \( M_o = 0.99 \). For 1002 mbar \( < p_i < 1015 \) mbar, \( p_i/p_o \) attains a constant value of about 8.16 when \( 0.94 < M_o < 0.99 \). This shows the occurrence of subsonic choking in our measurements with \( L/W = 304.64 \), which is significantly greater than the \( L/W \) values in the simulations of Shang and Wang.\(^9\) Yao et al.\(^9\) experimentally found subchoking phenomenon to occur at \( p_i/p_o > 5.3 \), which to reinforces the occurrence of subsonic choking in the present measurements.

### B. Static temperature measurement and calculation of stagnation temperature

The temperature measurement will provide useful clues about the nature of choking. Also, the effect of temperature on choking has not been discussed in the earlier studies on choking. K-type thermocouples are utilized for measuring the static temperatures at the inlet \( (T_{in}) \) and outlet \( (T_{out}) \) of the microchannel. The raw data of measured inlet and outlet temperatures with respect to the mass flow rate is displayed in Fig. 5. Both the locations indicate a temperature \( (T) \) of 298 K with a maximum deviation of 0.7 K. The value of room temperature is 298 K. Note that there is no attempt to actively control the temperature of the gas or microchannel walls during the experiments.

The stagnation temperature is also calculated from the value of static temperature \( (T) \) as

\[
T^{stag} = T \left(1 + \frac{V^2}{2C_pT}\right),
\]

where \( V \) is the flow velocity, and \( C_p \) is specific heat at constant pressure.

Figure 5 also shows the stagnation temperatures \( (T^{stag}) \) with respect to the mass flow rate. It is observed that inlet \( T^{stag} \) is virtually constant, almost overlapping with the static temperature. However, the outlet \( T^{stag} \) increases initially because of the increase in the microchannel’s outlet velocity from 152.52 m/s to 350.09 m/s. From Eq. (7), if the measured static temperature \( (T) \) and \( C_p \) are constants, the only parameter influencing stagnation temperature is velocity. Therefore, an increase in the channel’s outlet velocity would lead to an increase in the channel’s outlet stagnation...
mass flow rate greater than $8.99\times 10^{-7}$ kg/s. This phenomenon will be further explicated after nondimensionalizing the axes as shown in Fig. 6 (see Sec. V).

![Fig. 5](image_url)

**Fig. 5.** Raw data of stagnation and static temperatures at the microchannel’s inlet and outlet with respect to the mass flow rate. The vertical blue (dashed-dotted) and brown (dashed) lines represent theoretical choking limiting Mach number for isothermal ($Ma_o = 0.845$) and adiabatic flow ($Ma_o = 1$), respectively. Here, the room temperature is 298 K.

Later, $T_{stag}$ shows a tendency to become constant at mass flow rates greater than $8.99\times 10^{-7}$ kg/s. This phenomenon will be further explicated after nondimensionalizing the axes as shown in Fig. 6 (see Sec. V).

![Fig. 6](image_url)

**Fig. 6.** Normalized stagnation temperature ($T_{stag}^{stag} - T_{stag}^{in}$) with respect to normalized mass flow rate ($\dot{m}_{norm}$). The vertical blue (dashed-dotted) and brown (dashed) lines represent theoretical choking limiting Mach number for isothermal ($Ma_o = 0.845$) and adiabatic flow ($Ma_o = 1$), respectively.

V. DISCUSSION

The isothermal and adiabatic are two extreme situations of compressible flows. There is enough time available for heat transfer through channel walls due to low Reynolds number, which results in flow nearly of isothermal nature. For highly compressible flows, Re is too high for providing sufficient time for heat transfer to occur, leading to a flow behavior that is rather close to adiabatic. The actual situation could be a polytropic process in between these two extremes. It is difficult to determine with surety if the choked state is isothermal or adiabatic. Nonetheless, we will carefully examine both the scenarios in Sec. V A.

A. Choked flow behavior: Isothermal or adiabatic?

We check for the choked flow behavior at the exit of the microchannel in this subsection. Toward this, we nondimensionalize the stagnation temperature to calculate $T_{stag}^{stag}$ as

$$T_{stag}^{stag} = \frac{T_{stag} - T_o}{T_i - T_i}. \quad (8)$$

The normalized stagnation temperature is plotted against the normalized mass flow rate ($\dot{m}_{norm}$) calculated as

$$\dot{m}_{norm} = \frac{24\dot{m}_{stag}LRT}{H^3 W_p \gamma (\frac{h}{\gamma})^2 - 1 + 12 \sqrt{\frac{\gamma}{\alpha_o}} \gamma \mu (\frac{h}{\gamma} - 1)}, \quad (9)$$

where $\alpha_o$ is the tangential momentum accommodation coefficient (assumed to be unity here). Note that the mass flow rate is being normalized by the analytical expression of mass flux for rarefied gas flow given by Arkilic et al. Figure 6 shows that the stagnation temperature initially decreases on incrementing the mass flow rate, but later tends to achieve an almost constant value. The outlet choking Mach number ($Ma_o$) for isothermal flow in a long duct is theoretically $1/\sqrt{\gamma} = 0.845$ for the 1D channel (shown by the dashed-dotted vertical blue line in figures). $Ma_o = 1$ is the outlet choking Mach number for adiabatic flow (dashed vertical brown line). The adiabatic behavior of the flow is characterized by a nearly constant value of stagnation temperature as observed in Fig. 6. Figure 5 shows an isothermal behavior, which is characterized by a constant temperature of gas throughout the flow. These two pieces of evidence taken together, therefore, indicate a shift in flow behavior from isothermal to adiabatic.

According to Shapiro, upon proceeding toward the isothermal choking limit ($Ma_o = 1/\sqrt{\gamma}$), there is a rapid change in the gas properties with distance. The nature of flow is likely to be more close to adiabatic as compared to isothermal, if there is no active transfer of heat. On substituting Mach number value of $1/\sqrt{\gamma}$ in

$$\frac{dT_{stag}}{T_{stag}} = \frac{\gamma (\gamma - 1) Ma^4}{2(1 - \gamma Ma^2) (1 + \frac{\gamma - 1}{2} Ma^2)} \frac{dx}{D_h}, \quad (10)$$

we obtain $dT_{stag}$ as infinite, which upon further substituting in

$$dQ = c_p dT + \frac{1}{2} V^2 = c_p dT_{stag} \quad (11)$$
gives $dQ$ (heat transfer per unit length) as infinite. (Here, $x$ is the Cartesian coordinate in the streamwise direction.) Therefore, Shapiro claimed that the isothermal choking Mach number limit...
of $1/\sqrt{T}$ as nonphysical. The present measurement indicates some tendency for isothermal choking, but since this is not achievable, a shift to adiabatic choking is seen with an increase in the mass flow rate.

Lijo et al.\textsuperscript{22} stated from a numerical model that it is very hard to obtain adiabatic conditions at the microchannel’s inlet and exit in real scenarios. Roohi et al.\textsuperscript{21} performed 2D microchannel DSMC simulations on choked subsonic flow. They compared velocity profiles at $x/L = 0.3$ and $x/L = 1$ (channel exit) with the isothermal analytical solutions. The velocity profile at $x/L = 0.3$ agreed with the isothermal solution, but the velocity profile at $x/L = 1$ did not match with the analytical solution, due to which they concluded that the flow at the microchannel’s exit is not isothermal. These earlier reported results provide further supporting evidence that the flow can deviate from isothermal flow behavior in the choked flow regime.

### B. Supporting evidence of adiabatic choked state

Theoretical calculation of static temperature at the microchannel outlet ($T_o$) is estimated by substituting the theoretical Mach numbers limits of isothermal flow ($M_{ao} = 0.845$) and adiabatic flow ($M_{ao} = 1$) in the following equation:

$$T_o = \left( \frac{M_{ao} A_p p_o}{m} \right)^2 \frac{1}{\gamma R}$$

(12)

On substituting $M_{ao} = 0.845$ in Eq. (12), $T_o$ comes out to be 211.12 K, which is rather far from the measured temperature value of 298 K at the outlet. On substituting $M_{ao} = 1$, $T_o$ comes out to be 301.17 K with only 3.17 K deviation from the measured temperature value.

Hence, the theoretical calculation of exit temperature matches with adiabatic choked state and not with isothermal conditions. This is because, to maintain isothermal choking conditions, the flow would need an infinite amount of heat transfer, which is nonphysical.\textsuperscript{1,17} In our measurements, the mass flow rate continues to increase until 52 sccm, where $M_{ao}$ becomes 0.99 (almost 1) after which the mass flow rate cannot be increased further at all. Moreover, the choked state symptoms first occur at subsonic $M_{ao} = 0.94$; the limiting adiabatic $M_{ao}$ may have been reduced from unity due to the three-dimensionality of our microchannel. 1D flow has uniform velocity in all the flow directions, whereas 2D flow has a parabolic velocity resulting in less mass flow rate compared to 1D flow. The 3D velocity profile results in even less mass flow compared to that in 2D velocity profile. The decrease in the mass flow rate might lead to a decrease in experimental choking Mach number at the channel’s exit.

The claim of adiabatic behavior is also strengthened by the nearly constant normalized stagnation temperature values (Fig. 6) achieved at the maximum normalized flow rate at the outlet, where subsonic choking takes place. Figure 5 shows that the inlet exhibits a nearly constant normalized stagnation temperature value. Hence, we are able to fortunately attain adiabatic choked conditions at the microchannel’s inlet as well as the outlet, which is very hard to attain experimentally.\textsuperscript{9,17} Ilgaz and Çelenligil\textsuperscript{9} assumed the wall temperature to be equal to room temperature because there was negligible heat transfer to the walls due to which, they obtained air flow temperature inside parallel plates to be almost equal to the room temperature. Hence, the adiabatic conditions were applicable to their flow. Our study also has gas flow temperature the same as room temperature of 298 K, as shown in Fig. 5. Hemadri et al.\textsuperscript{20} performed local temperature measurements in a stainless steel tube for rarefied gas flows and proved that the Nusselt number has an extremely low magnitude in the slip flow regime. In practical applications of gas transport in pipelines, space-shuttle maneuvering in rarefied upper atmospheric regions, MEMS, etc., it is difficult to actively control the temperature. Our work would be highly beneficial in such real applications of compressible high speed flows, where choking might act as a hindrance to the quantity of mass flow rate allowed to pass.

### VI. CONCLUSIONS

Mass flow rate, static pressure, and temperature measurements have been carried out in a microchannel ($D_h = 132.26 \mu m$) for $91.21 < Re < 364.84, 4.04 \times 10^{-3} < Kt_n < 7.04 \times 10^{-3}, 0.43 < M_{ao} < 0.99$, and $8.72 < p/p_0 < 8.72$. For the first time, the evidence of subsonic choking in a more likely adiabatic than isothermal choked flow is justified experimentally in a microchannel for the slip regime. Yao et al.\textsuperscript{17} and Harley et al.\textsuperscript{19} being the only experimental studies of subsonic choked flow involved microchannels, which were not three-dimensional. Yao et al.\textsuperscript{17} did not include the analysis of choking behavior (isothermal/adiabatic) in their study. Their range of inlet pressures is 1300, 2500, and 3200 mbar, which is much greater than 1 atm, whereas the present work covers much lower inlet pressures ranging from 621 mbar to 1015 mbar. The other experimental study by Harley et al.\textsuperscript{19} involved maximum $M_{ao} = 0.85$, close to the theoretical isothermal choking limit. Their range of inlet pressures was also greater than 1 atm. This makes the current three-dimensional work with maximum $M_{ao} = 0.99$, close to theoretical adiabatic choking limit unique to the rarely existing experimental work on rarefied subsonic choking. The study is significant for slip flows in MEMS, long-distance gas transportation on earth as well as space applications.

The future scope of this work could be making local pressure measurements via pressure sensors mounted along the length of the microchannel. This would aid in finding out the actual location of choking of the microchannel, which could be upstream of the outlet.\textsuperscript{9} More refined measurements would be necessary near the theoretical isothermal choking Mach number limit to adiabatic choking Mach number limit for a variety of aspect ratio and pressure ratio. The effect of roughness and microridges on choked gas flow could also be investigated.\textsuperscript{11}

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