Extended Dyer-Roeder Approach Improves the Cosmic Concordance Model

J. A. S. Lima, V. C. Busti and R. C. Santos

1Departamento de Astronomia, Universidade de São Paulo, 05508-900 São Paulo, SP, Brazil
2Departamento de Ciências Exatas e da Terra, Universidade Federal de São Paulo (UNIFESP), Diadema, 09972-270 SP, Brazil

A new interpretation of the conventional Dyer-Roeder (DR) approach by allowing light received from distant sources to travel in regions denser than average is proposed. It is argued that the existence of a distribution of small and moderate cosmic voids (or “black regions”) implies that its matter content was redistributed to the homogeneous and clustered matter components with the former becoming denser than the cosmic average in the absence of voids. Phenomenologically, this means that the DR smoothness parameter (denoted here by $\alpha_E$) can be greater than unity, and, therefore, all previous analyses constraining it should be rediscussed with a free upper limit. Accordingly, by performing a statistical analysis involving 557 type Ia supernovae (SNe Ia) from Union2 compilation data in a flat ΛCDM model we obtain for the extended parameter, $\alpha_E = 1.26_{-0.54}^{+0.68}$ (1$\sigma$). The effects of $\alpha_E$ are also analyzed for generic ΛCDM models and flat XCDM cosmologies. For both models, we find that a value of $\alpha_E$ greater than unity is able to harmonize SNe Ia and cosmic microwave background (CMB) observations thereby alleviating the well known tension between low and high redshift data. Finally, a simple toy model based on the existence of cosmic voids is proposed in order to justify why $\alpha_E$ can be greater than unity as required by Supernovae data.

PACS numbers: Dark energy, cosmic distance, supernovae, inhomogeneities

I. INTRODUCTION

The accelerating cosmic concordance model (flat ΛCDM) is in agreement with all the existing observations both at the background and perturbative levels. However, while more data are being gathered, there is an accumulating evidence that a more realistic description beyond the “precision era” requires a better comprehension of systematic effects in order to have the desirable accuracy.

Local inhomogeneities are not only possible sources of different systematics, but may also be signaling for an intrinsic incompleteness of the cosmic description. This occurs because the Universe is homogeneous and isotropic only on large scales ($\gtrsim 100$ Mpc). However, on smaller scales, a variety of structures involving galaxies, clusters, and superclusters of galaxies are observed. Permeating these structures there are also voids or “black regions” (as dubbed long ago by Zel’dovich [1]) where galaxies are almost or totally absent as recently suggested by the N-body Millenium simulations [2]. This means that statistically uniform cosmologies are only coarse-grained representations of what is actually present in the real Universe. As a consequence, the description of light propagation by taking into account such richness of structures is a challenging task to improve the cosmic concordance model, but the correct method still remains far from a consensus [3, 4, 5, 6, 7, 8].

Zel’dovich [9], followed by Bertotti [10], Gunn [11] and Kantowski [12] were the first to investigate the influence of small-scale inhomogeneities in the light propagation from distant sources. Later on, Dyer and Roeder (DR) [13] assumed explicitly that only a fraction of the average matter density must affect the light propagation in the intergalactic medium. Phenomenologically, the unknown physical conditions along the path, associated with the clumpiness effects, were described by the smoothness parameter:

$$\alpha = \frac{\rho_h}{\rho_h + \rho_{cl}}.$$  

where $\rho_h$ and $\rho_{cl}$ are the fractions of homogeneous and clumped densities, respectively. This parameter quantifies the fraction of homogeneously distributed matter within a given light cone. For $\alpha = 0$ (empty beam), all matter is clumped while for $\alpha = 1$ the fully homogeneous case is recovered, and for a partial clumpiness the smoothness parameter is restricted over the interval $[0, 1]$. The reader should keep in mind that such a restriction clearly excludes the possibility of light rays traveling in regions denser than average. In principle, it should be very interesting to see how the presence of cosmic voids - a key entity nowadays - could be considered in the above prescription.

More recently, many studies concerning the light propagation and its effects on the derived distances have been performed [3, 4, 7, 14, 15]. Current constraints on the smoothness parameter are still weak [16, 17, 18, 19], however, it is intriguing that the quoted analyses had their best fits for $\alpha$ equal to unity which corresponds to a perfectly ΛCDM homogeneous model at all scales [10, 17]. More recently, some authors have also argued for a crucial deficiency of the DR approach, and, as such, it should be replaced by a more detailed description, probably, based on the weak lensing approach [3, 12].
In this letter we advocate a slightly different but complementary point of view. It will be assumed that the DR approach is an useful tool in the sense that it provides the simplest one-parametric description of the effects caused by local inhomogeneities, but its initial conception needs to be somewhat extended. This is done in two steps: (i) by allowing $\alpha$ (here denoted by $\alpha_E$) to be greater than unity in the statistical data analyses, (ii) by interpreting the obtained results in terms of the existence of an uncompensated distribution of cosmic voids or “black regions” in the Universe (see section V). As we shall see, by performing a statistical analysis involving 557 SNe Ia from the Union2 compilation data [22], we obtain $\alpha_E = 1.26^{+0.68}_{-0.54}$ (1$\sigma$) for a flat $\Lambda$CDM model. This 1$\sigma$ confidence region shows that $\alpha > 1$ has a very significant probability. We also show that $\alpha$ greater than unity is also able to harmonize the low redshift (Supernovae Ia) and baryon acoustic oscillations (BAO) data with the observations from cosmic microwave background (CMB).

II. THE DYER-ROEDER DISTANCE

The differential equation driving the light propagation in curved spacetimes is the Sachs optical equation

$$\sqrt{A''} + \frac{1}{2} R_{\mu\nu} k^\mu k^\nu \sqrt{A} = 0,$$

where a prime denotes differentiation with respect to the affine parameter $\lambda$, $A$ is the cross-sectional area of the light beam, $R_{\mu\nu}$ the Ricci tensor, $k^\mu$ the photon four-momentum ($k^\mu k_\mu = 0$), and the shear was neglected [20].

Five steps are needed to achieve the luminosity distance in the Dyer-Roeder approach:

- the assumption that the angular diameter distance $d_A \propto \sqrt{A}$,
- the relation between the Ricci tensor and the energy-momentum tensor $T_{\mu\nu}$ through the Einstein’s field equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu},$$

where in our units $c = 1$, $R$ is the scalar curvature, $g_{\mu\nu}$ is the metric describing a FRW geometry, $G$ is Newton’s constant and $R_{\mu\nu} k^\mu k^\nu = 8\pi G T_{\mu\nu} k^\mu k^\nu$.
- the relation between the affine parameter $\lambda$ and the redshift $z$

$$\frac{dz}{d\lambda} = (1 + z)^2 \frac{H(z)}{H_0},$$

where $H(z)$ is the Hubble parameter whose present day value, $H_0$, is the Hubble’s constant,
- the ansatz $\rho_m$ goes to $\alpha \rho_m$, and, finally,
- the validity of the duality relation between the angular diameter and luminosity distances [24, 25, 26]

$$d_L(z) = (1 + z)^2 d_A(z).$$

For a general XCDM model, where the dark energy component is described by a perfect fluid with equation of state $p_X = w \rho_X$ ($w$ constant), the Dyer-Roeder distance ($d_L = H_0^{-1} D_L$) can be written as:

$$\frac{3}{2} \left[ \alpha_E(z) \Omega_m (1 + z)^3 + \Omega_X (1 + w)(1 + z)^{(1 + w)} \right] D_L(z) + (1 + z)^2 E(z) \frac{d}{dz} \left[ (1 + z)^2 E(z) \frac{dD_L(z)}{dz} (1 + z)^2 \right] = 0,$$

where $\alpha_E(z)$ denotes the extended Dyer-Roeder parameter, $\Omega_X$, $w$, are the density and equation of state parameters of dark energy while the dimensionless Hubble parameter, $E(z) = H/H_0$, reads:

$$E(z) = \sqrt{\Omega_m (1 + z)^3 + \Omega_X (1 + z)^{(1 + w)} + \Omega_k (1 + z)^2},$$

where $\Omega_k = (1 - \Omega_m - \Omega_X)$ and the limiting case ($w = -1$, $\Omega_X = \Omega_\Lambda$) of all the above expressions describe an arbitrary $\Lambda$CDM model. The above Eq.(6) must be solved with two initial conditions, namely: $D_L(z = 0) = 0$ and $\frac{dD_L}{dz}|_{z=0} = 1$. As in the original DR approach, from now on it will be assumed that $\alpha_E$ is a constant parameter (see, however, [18, 21]).

III. DETERMINING $\alpha_E$ FROM SUPERNOVA DATA

In order to show the physical interest of the approach proposed here we have performed a statistical analysis involving 557 SNe Ia from the Union2 compilation data [22]. Following standard lines, we have applied the maximum likelihood estimator [we refer the reader to Ref. 10, 22 for details on statistical analysis involving Supernovae data].

In Fig. 1(a) we display the results obtained by assuming a flat $\Lambda$CDM model. The contours correspond to 68.3% (1$\sigma$) and 95.4% (2$\sigma$) confidence levels. The best fits are $\Omega_m = 0.25$ and $\alpha_E = 1.26$. As we can see from Figs. 1(b) and 1(c) the matter density parameter is well constrained, being restricted over the interval $0.21 \leq \Omega_m \leq 0.29$ (1$\sigma$), while the smoothness parameter is in the interval $0.72 \leq \alpha_E \leq 1.94$ (1$\sigma$). Although $\alpha_E$ being poorly constrained, we see that the probability peaks in $\alpha_E > 1$, and, therefore, denser than average regions in the line of sight are fully compatible with the data. It is interesting to compare the bounds over $\Omega_m$ with our previous analysis with the restriction $\alpha \leq 1.0$ [10]. The interval $0.24 \leq \Omega_m \leq 0.35$ (2$\sigma$) was obtained. As should be expected, by dropping the restriction $\alpha \leq 1.0$ lesser values of $\Omega_m$ are allowed by data.
IV. SUPERNOVAE-CMB TENSION AND $\alpha_E$

The tension between low and high redshift data has been reported by many authors (see, for instance, [23]). A numerical weak lensing approach to solve this problem was recently discussed by Amendola et al. [4] based on a meatball model. Can such a tension be alleviated by our extended DR approach?

In order to answer that, let us consider an arbitrary $\Lambda$CDM model and plot the bounds on the $(\Omega_m, \Omega_\Lambda)$ plane by fixing three different values of $\alpha_E$. By selecting $\alpha_E = 0.7, 1.0$ and $1.3$ we may study what happens with the $(\Omega_m, \Omega_\Lambda)$ contours when higher values are considered. In Fig. 2(a) we show the contours obtained for the chosen values of $\alpha_E$. Note that when $\alpha_E$ grows from 0.7 to 1.3 the best fit moves of around 1$\sigma$ towards lower values of the pair $(\Omega_m, \Omega_\Lambda)$ thereby becoming more compatible with the cosmic concordance flat $\Lambda$CDM model. This is a remarkable result since it improves the agreement with independent constraints coming from baryon acoustic oscillations (BAO) and the angular power spectrum of the cosmic microwave background (CMB), and, more important, maintaining the same reduced $\chi^2_{red}$.

In Table I the basic results are summarized. Note that the greatest value of $\alpha_E$ yields the minimum reduced $\chi^2_{red} = \chi^2/\nu$ ($\nu$ is number of d.o.f).

TABLE I: Best fits for $\Omega_m$ and $\Omega_\Lambda$.

| $\alpha_E$ | $\Omega_m$ | $\Omega_\Lambda$ | $\chi^2_{red}$ |
|-----------|------------|------------------|---------------|
| 0.7       | 0.39       | 0.83             | 0.978         |
| 1.0       | 0.30       | 0.78             | 0.977         |
| 1.3       | 0.24       | 0.74             | 0.977         |

In Fig. 2(b), we display the statistical results for a flat XCDM model and the same values for $\alpha_E$ adopted in the previous $\Lambda$CDM analysis. Again, we see that for higher values of $\alpha_E$, the contours are displaced towards regions with higher values for $w$ and smaller values for $\Omega_m$, again contributing to cancel the tension between the low and high redshift data.

In Table II, we summarize the best fits for $\Omega_m$ and $w$ along with their respective minimum reduced $\chi^2_{red}$.

TABLE II: Best fits for $\Omega_m$ and $w$.

| $\alpha_E$ | $\Omega_m$ | $w$ | $\chi^2_{red}$ |
|------------|------------|-----|----------------|
| 0.7        | 0.35       | -1.18 | 0.978       |
| 1.0        | 0.29       | -1.06 | 0.978       |
| 1.3        | 0.23       | -0.96 | 0.977       |

V. WHY IS $\alpha_E$ BIGGER THAN UNITY?

Here we propose a simple toy model based on the existence of cosmic voids in order to explain why $\alpha_E$ can be bigger than unity. Recent studies have pointed out that cosmic voids not only represent a key constituent of the cosmic mass distribution, but, potentially, may become one of the cleanest probes to constrain cosmological parameters [27]. The idea is to consider that very large voids are relatively rare entities, i.e. their formation suffer from the same kind of size (mass) segregation ‘felt’ by the largest galaxies and clusters. By assuming that the 3 basic entities filling the observed Universe are: (i) matter homogeneously distributed ($\rho_h$), (ii) the clustered component ($\rho_{cl}$) and (iii) voids ($\rho_{vd}$) of small and moderate sizes, we define the extended DR parameter (see Eq.(1)):

$$\alpha_E = \frac{\rho_h}{\rho_h + \rho_{cl} + \rho_{vd}}.$$  (8)
The important task now is to quantify the contribution of voids representing the local underdensities in the Universe. The presence of a void means that its matter was somehow redistributed to the clustered and the homogeneous components. The gravitational effect of a void in an initially homogeneous distribution is equivalent to superimpose a negative density (for small densities the nonrelativistic superposition principle is approximately valid). For simplicity, it will be assumed here that the overall contribution of the void component can be approximated by the linear expression, \( \rho_{vd} = -\delta(\rho_h + \rho_{cl}) \), where \( \delta \) is a positive number smaller than unity. Therefore, \( \alpha_E \) given Eq. (8) can be rewritten as:

\[
\alpha_E = \frac{\rho_h}{(\rho_h + \rho_{cl})(1 - \delta)} \equiv \frac{\alpha}{1 - \delta},
\]

which clearly satisfies the inequality \( \alpha_E \geq \alpha \), where \( \alpha \) is the standard DR parameter. In particular, when the clustered component does not contribute we find \( \alpha_E = \frac{1}{1 - \delta} \geq 1 \). The previous analyses using supernovae data implies that we have effectively constrained the extended parameter, \( \alpha_E \). How to roughly estimate the void contribution from this crude model? By applying the standard DR approach to the Union2 sample, the best fit is \( \alpha = 1 \), and combining with the result for a flat ΛCDM model (section III), one may check that the void contribution has a best fit of \( \delta \approx 0.2 \). It should be important to search for a possible connection between the present approach and more sophisticated methods from weak lensing.

VI. CONCLUSIONS

In this Letter we have discussed the role played by local inhomogeneities on the light propagation based on an extended Dyer-Roeder approach. In the new interpretation light can travel in regions denser than average, a possibility phenomenologically described by a smoothness parameter \( \alpha_E > 1 \).

In order to test such a hypothesis we have performed a statistical analysis in a flat ΛCDM model and the best fit achieved was \( \alpha_E = 1.26 \) and \( \Omega_m = 0.25 \), the parameters being restricted to the intervals \( 0.72 \leq \alpha_E \leq 1.94 \) and \( 0.21 \leq \Omega_m \leq 0.29 \) within the 68.3% confidence level. Although \( \alpha_E \) being poorly constrained, the results are fully compatible with the hypothesis of light traveling in denser than average regions. We have also analyzed how different values for the smoothness parameter affect the bounds over \((\Omega_m, \Omega_\Lambda)\) in an arbitrary ΛCDM model. Interestingly, \( \alpha_E > 1 \) improves the cosmic concordance model since it provides a better agreement between low and high redshift data (Supernovae, CMB and BAO). The same happened when a flat XCDM model was considered with the assumption that \( \alpha_E > 1 \).

Such results suggest that the hypothesis of light traveling in regions denser than the cosmic average seems to be quite realistic. A toy model justifying why this may occur with values of \( \alpha_E \) greater than unity was also discussed by taking into account the possible influence of cosmic voids on the Dyer-Roeder approach. The simplicity of the model and the obtained results reinforce the interest on the influence of local inhomogeneities and may pave the way for a more fundamental description.
Acknowledgments

JASL is partially supported by CNPq and FAPESP while VCB and RCS are supported by CNPq and INCT-Astrofísica, respectively.

[1] Ya. B. Zel’dovich, Astrofizika 6, 164 (1973). See also, Ya. B. Zel’dovich and S. F. Shandarin, Sov. Astron. Lett. 8, 67 (1982).
[2] V. Springel et al., Nature 435, 629 (2005), arXiv:0504097[astro-ph].
[3] S. Rasanen, J. Cosmol. Astropart. Phys. 2, 11 (2009), arXiv:0812.2872[astro-ph].
[4] L. Amendola, K. Kainulainen, V. Marra and M. Quartin, Phys. Rev. Lett. 105, 121302 (2010), arXiv:1002.1232[astro-ph].
[5] K. Bolejko, Mon. Not. R. Astron. Soc. 412, 1937 (2011), arXiv:1101.3338[astro-ph]; J. Cosmol. Astropart. Phys. 1102, 025 (2011), arXiv:1102.1232[astro-ph].
[6] E. W. Kolb, Class. Quant. Grav. 28, 1 (2011).
[7] C. Clarkson et al., Mon. Not. R. Astron. Soc. 426, 1121 (2012), arXiv:1109.2484[astro-ph].
[8] V. C. Busti and J. A. S. Lima, Mon. Not. R. Astron. Soc. 426, L41 (2012), arXiv:1204.1083[astro-ph]; C. Clarkson, B. Bassett and T. Hui-Ching Lu, Phys. Rev. Lett. 101, 011301 (2008), arXiv:0712.3457[astro-ph].
[9] Ya. B. Zel’dovich, Sov. Astron. 8, 13 (1964).
[10] B. Bertotti, Proc. R. Soc. London A 294, 195 (1966).
[11] J. E. Gunn, Astrophys. J. 150, 737 (1967).
[12] R. Kantowski, Astrophys. J. 155, 89 (1969).
[13] C. C. Dyer and R. C. Roeder, Astrophys. J. 174, L115 (1972); Astrophys. J. 180, L31 (1973).
[14] C. Bonvin, R. Durrer and M. A. Gasparini, Phys. Rev. D 73, 023523 (2006), arXiv:0511183[astro-ph]; K. Kainulainen and V. Marra, Phys. Rev. D 80, 123020 (2009), arXiv:0909.0822[astro-ph]; T. Mattsson, Gen. Relativ. Gravit. 42, 567 (2010), arXiv:0911.4264[astro-ph].
[15] S. Rasanen, J. Cosmol. Astropart. Phys. 1003, 018 (2010), arXiv:0912.3370[astro-ph].
[16] V. C. Busti, R. C. Santos and J. A. S. Lima, Phys. Rev. D 85, 103503 (2012), arXiv:1202.4449[astro-ph].
[17] J. S. Alcaniz, J. A. S. Lima, R. Silva, Int. J. Mod. Phys. D 13, 1309 (2004); R. C. Santos, J. V. Cunha and J. A. S. Lima, Phys. Rev. D 77, 023519 (2008), arXiv:0709.3679[astro-ph].
[18] R. C. Santos and J. A. S. Lima, Phys. Rev. D 77, 083505 (2008), arXiv:0803.1865[astro-ph].
[19] V. C. Busti and R. C. Santos, Research in Astronomy and Astrophysics 11, 637 (2011), arXiv:1103.0244[astro-ph].
[20] P. R. K. Sachs, Proc. R. Soc. London A 264, 309 (1961); P. Jordan, J. Ehlers, and R. K. Sachs, Akad. Wiss. Mainz 1, 1 (1961).
[21] E. V. Linder, Astron. Astrophys. 206, 190 (1988); Astrophys. J. 497, 28 (1998), arXiv:9707349[astro-ph].
[22] R. Amanullah et al., Astrophys. J. 716, 712 (2010), arXiv:1004.1711[astro-ph].
[23] A. Shafieloo, V. Sahni, and A. A. Starobinsky, Phys. Rev. D 80, 101301(R) (2009), arXiv:0903.5141[astro-ph].
[24] I. M. H. Etherington, Phil. Mag. 15, 761 (1933); B. A. Bassett and M. Kunz, Phys. Rev. D 69, 101305 (2004), arXiv:0312443[astro-ph].
[25] R. F. L. Holanda, J. A. S. Lima and M. B. Ribeiro, Astrophys. J. 722, L233 (2010), arXiv:1005.4458[astro-ph]; Astron. and Astrophys. 528, L14 (2011), arXiv:1003.5906[astro-ph].
[26] G. Ellis, R. Poltis, J.-P. Uzan, and A. Weltman, arXiv:1301.1312[astro-ph].
[27] P. M. Sutter, G. Lavaux, B. D. Wandelt, and D. H. Weinberg, Astrophys. J. 761, 44 (2012), arXiv:1207.2524[astro-ph].