Evidences for bouncing evolution before inflation in cosmological surveys

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Inflationary cosmology with a preceding nonsingular bounce can lead to changes on the primordial density fluctuations. One significant prediction is that the amplitude of the power spectrum may undergo a jump at a critical scale. In this Letter we propose a phenomenological parametrization of the primordial power spectrum in this scenario and confront the jump feature with latest cosmological data. Performing a global fitting, we utilize this possibility to derive a novel method for constraining bounce parameters via cosmological measurements. Combining the CMB, LSS and SNIa data, our result interestingly reveals that a nonsingular bounce, if exists, should be a fast bounce which happens at a very high energy scale, as we get an upper limit on the bounce parameters.

I. INTRODUCTION

Astronomical observations favor an adiabatic and nearly scale-invariant power spectrum of primordial perturbation, which can be realized in an inflation model by requiring a cosmological scalar field slowly rolling along the plateau of its potential. Based on this, it is usually assumed that the primordial power spectrum of cosmological perturbation is a fixed power function of the comoving wavenumber $k$ over the range of observable scales in the detailed methods of data fitting. However, as advocated by physics of the very early universe, namely the trans-Planckian physics\textsuperscript{1,2}, bounce cosmologies\textsuperscript{3,4} and so on, it is possible that the primordial spectrum shows local features which cannot be described by the usual parameterizations\textsuperscript{5,6}.

As well-known, an inflation model suffers from the problem of initial singularity and thus one cannot use the effective field approach to describe the universe at the beginning of its evolution\textsuperscript{7}. This problem can be circumvented in the framework of bounce cosmology, where the big bang singularity is replaced by a nonsingular bounce. By virtue of the effective field description, one can realize a bounce through an effective violation of certain energy condition, and thus obtain an inflationary scenario with a preceding bounce\textsuperscript{8}. In this scenario, however, the cosmological evolution is usual asymmetric with respect to the bounce point since of back-reaction of primordial perturbation and radiation\textsuperscript{9}. Therefore, there are two plausible scenarios for primordial perturbations to exit the Hubble radius with one being during a phase of matter-dominated contraction\textsuperscript{10,11} and the other being inflation after the bounce. If the initial perturbation originate from a Bunch-Davies vacuum and the contracting phase connects to an expanding one via a nonsingular bounce, the spectra produced in both stages are almost scale-invariant without changes as long as the scales we are interested are much larger than the duration of the bounce. However, they are different in their amplitudes, and thus their combination yields a nearly scale-invariant primordial power spectrum with a jump feature at a critical scale.

In the current Letter, we phenomenologically propose a parametrization of primordial power spectrum with a jump feature motivated by the bounce inflation scenario and study the constraints on this feature from current astronomical observations. Compared to the standard inflationary paradigm, this model involves two additional parameters, $k_B$ characterizing the comoving wavelength of the universe at the bounce point, and $T$ describing the slope of the jump in the power spectrum and thus corresponding to the measurement of the duration of the bouncing phase. In the literature, the technique of Markov Chain Monte Carlo (MCMC) global analysis has been widely generalized to constrain non-standard inflationary cases, namely non-canonical inflation models\textsuperscript{11,12}, trans-Planckian physics\textsuperscript{13,14}, and so on. It has proven to be very powerful to probe the parameter space beyond the standard inflationary paradigm. As a consequence, we employ the MCMC technique to do a global fitting to constrain parameters of bounce inflation and give a comparison to current data.

The letter is organized as follows. In Section II we briefly review the cosmological perturbations generated in a bounce model with a matter contraction connecting to an inflation, and present a smoothed parametrization of the primordial power spectrum of this model with a jump feature. In Section III we perform a numerical calculation and compare the result with the CMB and LSS data. We give our numerical results in Section IV before concluding with a discussion in Section V.

II. FORMALISM IN A BOUNCE-INFLATION SCENARIO

We begin with a brief discussion of the cosmological evolution of primordial perturbation in the framework of...
a flat FRW Universe. A standard process of generating primordial power spectrum suggests that, cosmological fluctuations should initially emerge inside a Hubble radius, then leave it in the primordial epoch, and finally reenter at late times. In usual, this process can be realized by stretching the physical wavelength $\frac{\lambda}{a}$ longer than the Hubble radius $1/H$ as in inflation; while an alternative is to suppress the comoving Hubble radius $1/aH$ shorter than the comoving wavelength $1/k$ as in matter bounce.

One often uses a gauge-invariant variable $\zeta$, the curvature fluctuation in comoving coordinates, to characterize the cosmological inhomogeneities. It is associated with a canonical variable $v = z\zeta$, where $z \equiv \sqrt{2a}$ with $\epsilon \equiv -\dot{H}/H^2$. The equation of motion for the Fourier mode $v_k(\eta)$ in the context of standard Einstein gravity is given by [27]

$$v''_k + \left( k^2 - \frac{z''}{z} \right) v_k = 0,$$  

where the prime denotes the derivative with respect to the comoving time $\eta \equiv \int dt/a$.

To perform a specific analysis, one may take the scale factor as $a(t) = a_B(\frac{t}{T_B})^{1/\epsilon}$, where the subscript “$B$” denotes any reference time which will be referred as the bouncing point later. For a constant background equation-of-state (EoS) $w$, one obtains

$$\frac{z''}{z} = \frac{\nu^2 - \frac{1}{2}}{\eta^2}, \quad \text{with} \quad \nu = \pm \frac{\epsilon - 3}{2(\epsilon - 1)}.$$  

We assume the cosmological perturbations originate from vacuum fluctuations, which suggests

$$v_k^i \simeq \frac{1}{\sqrt{2k}} e^{-i \int_{T_B}^{\eta} k d\tilde{\eta}},$$  

when fluctuations are born with $|k\eta| \gg 1$. This is consistent with the asymptotic solution to Eq. (3) when the last term $\frac{z''}{z}$ is negligible. Therefore, the mechanism of generating primordial perturbations requires that the absolute value of the comoving time is enough large which can only be achieved in a contracting or an inflationary setup. Another asymptotic solution to Eq. (1) can be derived by virtue of the mathematic property of the Bessel function,

$$v_k \sim \eta^{\frac{1}{2}} \left[ c(k) \eta^{1/2} \right],$$  

in the super-Hubble regime, which implies $|k\eta| \ll 1$.

Now we match the two asymptotic solutions [3] and [4] at the moment of Hubble crossing $|k\eta| \sim 1$, and thus determine the form of $v_k$ on super-Hubble scale,

$$v_k(\eta) \simeq \frac{1}{\sqrt{2k}} (k\eta)^{1/2 - |\nu|}.$$  

From the definition of the power spectrum, one learns that $\zeta \sim k^{3/2} |v_k|$ can only be scale-invariant when $|\nu| = 3/2$. As a consequence, the primordial fluctuations are nearly scale-invariant in both the matter contraction and inflationary scenarios. However, $\epsilon$ takes the value $3/2$ in the matter contraction but becomes very small during inflation. The amplitude of the primordial spectrum would undergo a jump around the scale comparable to the bounce scale. A detailed calculation gives the expression of the primordial power spectrum for a model of bounce inflation as follows

$$P_\zeta = \begin{cases} H^2/12\pi^2, & k < k_B \\ \frac{H^2}{8\pi^2}, & k \geq k_B, \end{cases}$$  

where the parameter $k_B$ characterizes the comoving wavelength of the universe at the bouncing point. Note that, when the perturbation passes through the bouncing phase, its positive and negative frequency modes could be mixed at the transfer surface and thus bring an oscillating signal around the bounce scale [4]. However, this signal strongly depends on extra parameters introduced in specific models and is not quite sensitive to current observations. In this Letter we have smoothed this signal and study its average effect directly.

For phenomenological considerations, we would like to parameterize the above spectrum by assuming a form

$$P_\zeta = P_m + \frac{P_{inf} - P_m}{2} \left( 1 + \tanh[(k - k_B)T] \right),$$  

where $P_{inf} = \frac{H^2}{12\pi^2}$ and $P_m$ is the spectrum of curvature perturbation before the bounce. Theoretically, $P_m = \frac{T_H^2}{8\pi^2}$, which is relevant to primordial tensor fluctuations, should be less than $1\%$ of $P_{inf}$ due to the upper bound of tensor-to-scalar ratio. The power spectrum after the bounce $P_{inf}$ is parameterized as the power low format via $P_{inf} = A_k k^{n_s - 1}$, in which $A_k$ and $n_s$ are the amplitude and the spectral index correspondingly. $k_B$ is a comoving wavenumber relevant to the bounce scale and denotes the occurrence moment of the jump feature in the power spectrum (in unit of $h$ Mpc$^{-1}$). The parameter $T$ characterizes the slope of the jump feature (in unit of Mpc) and thus the duration of the bouncing phase. In the bounce inflation scenario, we have $k_B T \sim H \Delta t_B$. For this description, the phenomenology of the scale-invariant primordial power spectrum will be recovered when $k - k_B > 0$ and $T \to \infty$. In the following we study observational constraints from current data on the bounce parameters introduced in Eq. (4).

### III. CONSTRAINTS FROM OBSERVATIONAL DATA

#### A. signal in CMB and LSS

To test our model we first consider the current astronomical observations from the Wilkinson Microwave
Anisotropy Probe 7-year data (WMAP)[10] and the Sloan Digital Sky Survey (SDSS)[17]. Similar to the trans-Planckian physics, the effect brought by a bounce is most sensitive to the modes of primordial perturbation exiting the Hubble radius at earliest time. Therefore, the bounce induced jump feature of the primordial power spectrum will imprint its effects on the CMB at very large length scales, and correspondingly depress the CMB anisotropies at large angular scales. Fig. 1 illustrates this impact on the CMB temperature power spectrum. We compare a ΛCDM power spectrum (black solid line) with \( \omega_b = 0.023, \omega_\Lambda = 0.11, h = 0.71, \tau = 0.09, A_s = 2.2 \times 10^{-9}, n_s = 0.97 \) and four bounce models with different power spectra, namely \( P_m = 5 \times 10^{-12}, T = 2000, k_B = 3 \times 10^{-4} \) (red dashed line), \( P_m = 10^{-11}, T = 4, k_B = 8 \times 10^{-6} \) (green dotted line), \( P_m = 10^{-10}, T = 2000, k_B = 8 \times 10^{-6} \) (blue dash dotted line) and \( P_m = 10^{-11}, T = 2000, k_B = 10^{-3} \) (cyan short dashed line).

At large angular scale, the initial condition is given by \( P_m \) rather than \( P_{inf} \), so the reduction in the amplitude of the power spectrum happens beyond the bounce scale where \( k < k_B \). This effect can be seen by comparing the black solid line with the blue dashed line. Moreover, the smaller value of \( P_m \) we choose, the effect of this reduction is more obvious at large scale. The parameter \( k_B \) is determined by the energy scale of the bounce, and thus the minimal size of the universe in this scenario. From Fig. 1, we can find that the amplitude of primordial power spectrum would obtain an increase around the scale of \( k_B \). In numerical computation, we illustrate this effect by choosing \( k_B = 3 \times 10^{-4} \) and \( k_B = 10^{-3} \) respectively. Additionally, the parameter \( T \) influences the shape of the temperature power spectrum obviously. If \( T \) is large enough, the jump feature of the power spectrum is shown to be very apparent. However, if we choose a small value of \( T \), the whole shape of the power spectrum becomes smooth relatively and a quite sizable regime of this spectrum could be suppressed manifestly. It can be seen by the green dotted line with \( T = 4 \), which is too small to accommodate the WMAP data, and this feature could lead to a constraint on \( T \) as will be analyzed later.

The jump feature in bounce inflation can also leave its signature on the matter power spectrum on large scales as shown in the lower panel of Fig. 1. Since the signature of the bounce effect occurs only at the edge of the observable regime of the current data of LSS and accordingly we can only obtain a bound instead of an accurate constraint. Our global analysis provides a powerful approach towards a future detection of bounce cosmologies making use of data from more accurate astronomical experiments.

### B. MCMC Likelihood Analysis

The MCMC technique is widely applied to give multi-dimensional parameter constraints from observational data. In the current Letter, we employ MCMC to generate a random sample from the posterior distribution \( P(\theta|x) \) of a set of parameters \( \theta \) given an event \( x \) (for us, it is the total data set that used), and obtain

\[
P(\theta|x) = \frac{P(x|\theta)P(\theta)}{\int P(x|\theta)P(\theta)d\theta}, \quad (8)
\]

via Bayes' Theorem, where \( P(x|\theta) \) is the likelihood of event \( x \) given the model parameters \( \theta \) and \( P(\theta) \) is the prior probability distribution of obtaining a model parameter value \( \theta \). The simulated random observations from the posterior distribution are of the likelihood surface, and from this sample, we can estimate the posterior distribution of the parameter of interest.

For our implementation, we use a generalized version of the CosmoMC package[15], in which the cosmological parameters are:

\[
\theta \equiv (\omega_b, \omega_\Lambda, \Theta_s, \tau, n_s, A_s, P_m, k_B, T), \quad (9)
\]

where \( \omega_b \equiv \Omega_b h^2 \) and \( \omega_\Lambda \equiv \Omega_\Lambda h^2 \), in which \( \Omega_b \) and \( \Omega_\Lambda \) are the baryon and cold dark matter densities relative to the critical density, \( \Theta_s \) is the ratio (multiplied by 100) of the sound horizon to the angular diameter distance at decoupling, and \( \tau \) is the optical depth to re-ionization. The remaining parameters are related to the primordial power spectrum given by Eq.(4). For simplicity, we assume a purely adiabatic spectrum of fluctuations and a flat universe with a cosmological constant whose EoS is \( w = -1 \) at the initial moment.
In our analysis, we have included the WMAP7 temperature and polarization power spectra with the routine for computing the likelihood supplied by the WMAP team, the matter power spectrum from the “Luminous Red Galaxies” sample from the SDSS [17] as well as the SNIa “Union” compilation (307 sample) [19]. In calculating the likelihood from SNIa we have marginalized over the “nuisance parameter” [20]. Furthermore, we make use of the Hubble Space Telescope (HST) measurement of the Hubble parameter $H_0 \equiv 100h \, \text{km s}^{-1}\text{Mpc}^{-1}$ by applying a Gaussian likelihood function centered around $h = 0.742$ with standard deviation $\sigma = 0.038$ [21], and take the total likelihood to be the products of the separate likelihoods of CMB, SNIa and LSS. Alternatively defining $\chi^2 = -2 \log L$, we get

$$\chi^2_{\text{total}} = \chi^2_{\text{CMB}} + \chi^2_{\text{SNIa}} + \chi^2_{\text{LSS}}.$$  

IV. COMBINED CONSTRAINTS

From numerical computations, we find that the present data are far from putting explicit constraint on bounce parameters on currently observationally accessible scales. However, the situation will improve greatly with the appearance of more and more accurate CMB experiments in the near future. With the current data we can obtain upper bounds on $P_m$, $k_B$ and a constraint on $T$. From the numerical results, we find that $P_m < 7.03 \times 10^{-11}$, $k_B < 2.44 \times 10^{-4}$, and $2.63 \times 10^2 < T < 7.98 \times 10^5$ at 2\sigma C.L.. The result also shows that a bounce model gives a slightly better (lower) $\chi^2$ for the dataset than the $\Lambda$CDM model by 1.4. Accordingly one may conclude that a bounce inflation model is quite efficient to explain cosmological observations at large scale.

Finally, we calculated the combined constraints on the inflationary Hubble parameter $H$ and the duration of bouncing phase $\Delta t_B$, present the contour plot at 1\sigma and 2\sigma respectively in Fig. 2. The yellow star represents a non-vanishing best-fit point. We also studied the combined constraints on the inflationary spectral index $n_s$ and its amplitude $A_s$ using global fitting. Although the derivation is very limit, we find that a nonsingular bounce before inflation leads to a larger red tilt for the spectral index and to more power for primordial fluctuations compared with the $\Lambda$CDM model.

V. SUMMARY

Since a nonsingular bounce happened at an extremely high energy scale, it can hardly be tested directly by experiments. To find evidence for it, we need to know its observational consequences. This issue has been discussed widely in the literature, and one potential clue is to study primordial perturbations. For example, in the context of the Pre-Big-Bang scenario [22] and in the cyclic/Ekpyrotic model [23], the resulting cosmological perturbation was found to strongly depend on the physics at the epoch of thermalization, and thus an uncertainty for prediction is involved [21, 25]. In this Letter, motivated by a combined scenario of matter bounce and inflation, we introduce a parametrization of the primordial power spectrum in a class of bounce cosmologies, and introduce two parameters to characterize the relevant physics. By performing a global analysis, we show that the present observations are quite consistent with a featureless power spectrum, and consequently put a strong constraint on bounce cosmologies, which suggests a fast bounce model at very high energy scale. Although far from being conclusive, our numerical method can be extended to other bounce cosmologies, and thus is relevant in light of forthcoming astronomical observations, such as PLANCK.

Note added: while this Letter was being finalized, a paper appeared pointing out that the scenario of bounce inflation could be embedded into loop quantum cosmology [24], and the data analysis in that work shows an abnormal enhancement at the first peak of CMB anisotropy. We find it more generic to consider the bouncing physics in a model-independent way as introduced in the current Letter.

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