The rapidity dependence of the chemical freeze-out thermal parameters $T$ and $\mu_B$ are determined at the highest RHIC and SPS energies. These show a systematic behavior towards an increase in $\mu_B$ away from mid-rapidity and a corresponding decrease in the temperature $T$. 

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Data from the BRAHMS collaboration [1] show that antiparticle to particle ratios have a maximum at mid-rapidity and slowly decrease as the rapidity becomes larger. It was emphasized very eloquently by Röhrich [2] that the particle ratios at large rapidities are consistent with those measured at the SPS energies. This opens the possibility to compare measurements for e.g. the $K^+/\pi^+$ ratio at high rapidities and check them with the corresponding values measured in the energy scan at the SPS, thus complementing information about the rapid variation of this ratio as a function of beam energy. The sharp variation in this ratio with beam energy remains a mystery for most models and might indicate interesting dynamics possibly even linked to the appearance of the critical point at intermediate values of the baryon chemical potential in lattice quantum chromodynamics [3, 4].

A change in particle - anti-particle ratios with longitudinal momentum has been observed in many collisions, it was first discussed two decades ago in an analysis of $p-p$ and $p-N$ data [5].

The analysis of particle multiplicities in heavy ion collisions has shown overwhelming evidence for chemical equilibrium in the final state except for particles carrying strangeness which are suppressed at lower beam energies; however, their relative yields fulfill statistical equilibrium. A summary as of 2007, combining the results from many different groups, is shown in Fig. 1. Except

![Figure 1: Temperature vs. $\mu_B$ as determined from heavy ion collisions at different beam energies. The lower AGS points are based on a preliminary analysis of $4\pi$ data. The RHIC point at 62.4 GeV was obtained by J. Takahashi [6]. See [7, 8] for more details.](image)

for particle multiplicities at RHIC energies, all data in Fig. 1 use integrated particle yields, the very systematic change of thermal parameters over the full range of beam energies is one of the most impressive features of relativistic ion collisions to date. It is now possible to use the thermal model to make reliable predictions for particle multiplicities at LHC energies [9] and to determine which beam energy will lead to the highest baryon density at freeze-out [10]. With chemical equilibrium thus firmly established, we focus on other properties, in particular, since the rapidity distributions of identified particles is now becoming available also at RHIC energies [1], it is now possible to determine the rapidity dependence of thermal parameters. A first analysis was done by Stiles and Murray [11] for the data obtained by the BRAHMS collaboration at 200 GeV [1]. This shows a clear dependence of the baryon chemical potential on rapidity due to the changing $\bar{p}/p$ ratio. A thorough analysis of the rapidity dependence was recently done in Ref. [12, 13] using a model
based on a single freeze-out temperature. A first report of our results was presented elsewhere [14].

The general procedure is as follows: the rapidity axis is populated with fireballs following a
gaussian distribution function given by \( \rho(y_{FB}) \) where \( y_{FB} \) is the rapidity of the fireball.

\[
\rho(y_{FB}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{y_{FB}^2}{2\sigma^2}\right).
\] (1)

Particles will appear when the fireball freezes out and will follow a thermal distribution cen-
tered around the position of the fireball The momentum distribution of hadron \( i \) is then calculated
from the distribution of fireballs as given by Eq. [1] along the rapidity axis as follows

\[
E_i \frac{d^3N_i}{d^3p} = \int_{-\infty}^{\infty} \rho(y_{FB}) E_i \frac{d^3N_i}{d^3p}(y - y_{FB}) \ dy_{FB}
\] (2)

where \( E_i \frac{d^3N_i}{d^3p} \) is the distribution of hadrons from a single fireball. The temperature \( T \) and the baryon
chemical potential \( \mu_B \) will depend on the rapidity of the fireball and are not assumed to be constant.

We have included resonance decays in the final distribution and assumed they decay isotropi-
cally.

An important parameter is the width of the distribution. For the RHIC data at 200 GeV this
was determined from the \( \pi^+ \)'s as these are very sensitive to the value of \( \sigma \) and less to variations
in the baryon chemical potential. The width of the distribution \( \sigma = 2.183 \) is compatible with the
values quoted by the BRAHMS collaboration [1], e.g. \( \sigma_{\pi^+} = 2.25 \pm 0.02 \) and \( \sigma_{\pi^-} = 2.29 \pm 0.02 \).
This is shown in Fig. [3]. The hadrons described by Eq. [2] are mainly hadronic resonances. Only a
fraction of these are stable under strong interactions and most of them decay into stable hadrons at
chemical freeze-out, hence the need to implement multi-particle decays.

We assume that the temperature \( T \) and the chemical potential \( \mu_B \) are always related via the
freeze-out curve as given in Fig. [1]; if the temperature varies along the rapidity axis, then also the
chemical potential will vary. Thus a decrease in the temperature of the fireball will be accompanied
by an increase in the baryon chemical potential. In other words, we assume a universality of
the chemical freeze-out condition. This relationship between temperature and baryon chemical
potential is very reasonable since all particle abundances measured so far follow it. Once the width
of the distribution of fireballs has been fixed, we can go on with the dependence of the baryon
chemical potential on the rapidity of the fireball (units are in GeV) at RHIC:

\[
\mu_B = 0.025 + 0.011 y_{FB}^2
\] (3)

This is shown graphically in Fig. [3].

A comparison of the resulting net proton distribution \( p - \bar{p} \) with RHIC data at 200 GeV is
shown in Fig. [4].

The variation of the temperature along the rapidity axis is shown in Fig. [5]. The temperature
is maximal at mid-rapidity and gradually decreases towards higher (absolute) values of the rapidity.
Note that the temperature varies less than 2 MeV over the rapidity interval.

The situation at the highest SPS energy is more difficult to describe because the changes of
the thermal variables with rapidity are much larger. First of all we need the distribution of fireballs
at SPS energies. This is determined by the variable \( \sigma \) appearing in Eq. [1] which we fixed using
Rapidity Variation.

J. Cleymans

\[ \sigma = 2.183 \]

**Figure 2:** Fit to the pion distribution as measured by the BRAHMS collaboration.

\[ \mu_B = 0.025 + 0.011 y^2 \]

**Figure 3:** Values of the baryon chemical potential as a function of rapidity at the highest RHIC energy.

The results obtained at various beam energies are shown in Fig. [6]. The baryon chemical potential can approximately be described by:

\[ \mu_B = 0.237 + 0.011 y^2 \]

(4)

This is shown graphically in Fig. [8]. The corresponding change in the temperature as a function of rapidity is shown in Fig. [9]. Whereas there is almost no change in the temperature in the rapidity interval under consideration (the change is less than 2 MeV at RHIC), at the highest SPS energy the change in temperature is much larger and ranges from about 160 MeV down to about 120 MeV. The comparison between the resulting net proton distribution \( p - \bar{p} \) with SPS data at 17.2 GeV is shown in Fig. [7].

In summary, particle yields measured in heavy ion collisions show an overwhelming evidence for chemical equilibrium at all beam energies. The rapidity dependence of the thermal parameters \( T \) and \( \mu_B \) can now be determined over a wide range of rapidities and show a systematic behavior towards an increase in \( \mu_B \) as the rapidity is increased.
Rapidity Variation.

Figure 4: The $p - \bar{p}$ distribution as a function of rapidity.

Figure 5: Values of the chemical freeze-out temperature function of rapidity at the highest RHIC energy.

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Figure 6: Width of the Gaussian distribution, $\sigma$, used to describe the distribution of fireballs for various beam energies.

Figure 7: The $p - \bar{p}$ distribution as a function of rapidity at SPS.

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Figure 8: Values of the baryon chemical potential as a function of rapidity at the highest SPS energy.

\[ \mu_b = 0.237 + 0.05 y_{\text{rap}}^2 \]

Figure 9: Values of the chemical freeze-out temperature as a function of rapidity at the highest SPS energy.