Effect of nanoparticles on natural convection in fluids

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Abstract. The paper investigates the impact of nanoparticles on the onset of convection currents in a fluid layer under small temperature gradient. The partial differential equations derived from conservation laws of mass, momentum, nanoparticles and energy are considered which include the nano effects (Brownian motion and thermophoresis). Nanoparticle volume fraction is considered to be small in the fluid and hence initial solution of equations assumes it to be constant while temperature and pressure vary in vertical direction. Small perturbations are added to steady state solution and the obtained partial differential equations are converted into ordinary by seeking solution which is exponential in time and periodic in space. To get the complete insight of the problem, variables are not combined or replaced by new ones at any stage leading to more significant expressions. Further, the equations are solved to get thermal Rayleigh number which establishes the influence of physical properties and diffusion effects of nanoparticles on the onset of convection cells. A detailed analysis of the problem is done numerically which lead to significant results and hence affect the convective nature of the system broadly. Copper in spite of having higher conductivity has lesser impact on stability than aluminium because of its higher density.

Keywords: Brownian motion, Thermophoresis, Conservation equations, Rayleigh number, Metallic and non-metallic nanofluids.

1. Introduction
The innovative idea of enhancing the thermal conductivity of liquids by adding metallic particles was anticipated by Maxwell [1]. Further, Choi [2] focused on smaller world to overcome the serious issues related to the size of particles and conferred the idea of nanofluids. The studies related to significant enhancement in heat transfer efficiency of nanofluids in comparison to other fluids has been reviewed by many researchers [3, 4]. To understand the phenomenon of unusual behaviour of nanofluids, Buongiorno [5] formed partial differential equations with the help of conservation laws. Tzou [6,7] investigated the convective nature of nanofluids using equations formulated by Buongiorno [5] and established the destabilizing impact of nanoparticles on fluid layer. Analytical study was made to explore the instability of nanofluids by finding expression of Rayleigh number [8]. The problem was extended to consider the impact of magnetic field and rotation [9, 10]. Both the parameters were found to delay the onset of convection currents in the fluid layer. Solutal, nanoparticles and thermal effects were investigated on a fluid layer considering it as a triple diffusive phenomenon [11]. The work was revisited numerically for alumina-water nanofluid to make it a practically meaningful problem [12]. The magnetic field and rotation parameters postpone the onset of convection currents in a nanofluid layer under solutal effects [13, 14].
All the literature surveyed above consider a top or bottom-heavy distribution of nanoparticles in the layer but it is not possible to control nanoparticle volume fraction on the boundaries physically. Nield and Kuznetsov [15] revised the model with a more realistic approach and considered zero nanoparticle’s flux across the layers. Presence of nanoparticles was found to hasten the instability of the fluid through stationary mode. Revised boundary conditions were used to study porosity effects on convection in nanofluids with Darcy-Brinkman model [16]. Further, the studies to explore the effects of Coriolis and Lorentz forces on the convective behaviour of nanofluids in a porous medium for revised model were conducted ([17] and [18]). Sharma et al. [19] modified the model again by assuming the initial nanoparticle volume fraction constant in the fluid layer leading to a more realistic approach and the problem was solved numerically for different nanofluids.

The present work considers conservation equations for nanofluids with the assumption of basic state nanoparticle volume fraction to be constant. Unlike earlier studies, to get complete insight of the problem variables are not combined or replaced by new ones at any stage leading to more significant and complete expressions. Equations are simplified wherever needed without violating the necessary physics for the analytic progress of the method to achieve the objective. Further, results are discussed in detail analytically as well as numerically. Impact of various parameters on thermal Rayleigh number is shown graphically with the help of software Mathematica.

2. Conservation equations for nanofluids
Consider a nanofluid layer in xy-plane between \( y = 0 \) and \( y = d \). The layer is heated from below.

The conservation equations for the system are (refer: [20] and [5])

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_{\phi} \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + \frac{D_{\phi}}{T_0} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \tag{2}
\]

\[
\rho_0 \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \tag{3}
\]

\[
\rho_0 \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \left[ \phi_{0 \rho} (1 - \phi) \rho_0 (1 + \beta_T (T - T_0)) \right] g, \tag{4}
\]

\[
(\rho_0 c) \left[ \rho \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right). \tag{5}
\]

Here, \( v = (u, v), \phi, \rho_0, \rho_T, T, p, D_{\phi}, \mu, D_T, c, \beta_T, t, \kappa, g \) respectively, are velocity, particle volume fraction, density, nanoparticle density, temperature, pressure, coefficient of Brownian diffusion, viscosity, coefficient of thermophoretic diffusion, specific heat, thermal volumetric coefficient, time, thermal conductivity of fluid and gravitational acceleration.

3. Basic solution and perturbation equations
Initially, the fluid is assumed to be at rest while pressure and temperature are functions of ‘y’ only and volume fraction of nanoparticles in the layer is assumed to be constant \( (\phi_0) \). Let us write

\[
v = 0, \phi = \phi_0, p = p_b(y), T = T_b(y). \tag{6}
\]

Using Eqs. (1)-(5) and boundary conditions
We get the value of initial temperature as

\[ T_0(y) = \left( \frac{T_1 - T_0}{d} \right) y + T_1. \]  

(8)

The initial pressure \( p_b \) can be calculated from Eq. (4).

Using perturbation theory, we write

\[ \nu = 0 + \nu', \ \phi = \phi_b + \phi', T = T_b + T', \ \rho = \rho_b + \rho', \]  

(9)

where superscript ‘‘ denote the perturbed variables. Using Eqs. (8) and (9) in Eqs. (1)-(5), we get

\[ \frac{\partial \nu'}{\partial x} + \frac{\partial \nu'}{\partial y} = 0, \]  

(10)

\[ \frac{\partial \phi}{\partial t} = D_b \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + D_L \left( \frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial y^2} \right), \]  

(11)

\[ \frac{\partial \nu'}{\partial x} = - \rho_b \frac{\partial \nu'}{\partial x} + \mu \left( \frac{\partial^2 \nu'}{\partial x^2} + \frac{\partial^2 \nu'}{\partial y^2} \right) + \phi' \left( \rho_p - \rho_b \right) + \rho_0 \beta_r T', \]  

(12)

\[ \left( \rho_b \beta_c \right) \left( \frac{\partial T'}{\partial t} + \nu' \left( \frac{T_1 - T_0}{d} \right) \right) = \kappa' \left( \frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial y^2} \right). \]  

(13)

Using Eqs. (10), (12) and (13) we get

\[ \mu \nabla^2 \nu' - \rho_b \frac{\partial \nu'}{\partial t} \nabla^2 \nu' = \left( \rho_p - \rho_b \right) \frac{\partial^2 \phi'}{\partial x^2} + \rho_0 \beta_r \frac{\partial^2 T'}{\partial x^2} g. \]  

(15)

where \( \nabla^2 = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \).

Notice that pressure term is eliminated at this stage and hence will not contribute directly in the further investigation of problem. Let us write

\[ \nu' = V(y) e^{\sigma t} \sin \alpha x, \ \phi' = \phi(y) e^{\sigma t} \sin \alpha x, \ T' = T(y) e^{\sigma t} \sin \alpha x, \]  

(16)

where \( \sigma \) is the growth-rate and \( \alpha \) is the wave-number. The Eqs. (11), (14) and (15) reduce to

\[ \mu \left( D^2 - \alpha^2 \right) V - \sigma \rho_b \left( D^2 - \alpha^2 \right) V = \left( \rho_p - \rho_s \right) \left( \frac{\partial^2 \phi'}{\partial x^2} \right) + \rho_0 \beta_r \left( -\alpha^2 \right) T g, \]  

(17)

\[ \sigma \phi = D_b \left( D^2 - \alpha^2 \right) \phi + \frac{D_L}{T_0} \left( D^2 - \alpha^2 \right) T. \]  

(18)
\[
(\rho_c c) \left[ \sigma T + \frac{(T_i - T_0)}{d} V \right] = \kappa (D^2 - \alpha^2) T. \tag{19}
\]

Put \( \sigma = 0 \) to find transition as buoyancy forces do not act in reverse directions and hence oscillatory motions are not possible. On simplifying Eqs. (17)-(19) we get (after reducing it to non-dimensional form by putting \( \alpha = \alpha_d, D = dD \))

\[
\left( D^2 - \alpha^2 \right)^3 V + Ra \alpha^2 V + \left( \frac{\rho_p - \rho_0}{\rho_0} \right) \frac{(T_i - T_0) D_T}{T_0 \alpha_f D_B} (\alpha^2)^3 Vg = 0, \tag{20}
\]

where the non-dimensional number \( Ra = \rho_o g \beta_T d^3 (T_i - T_0) / \mu \alpha_f \) is the thermal Rayleigh number and \( \alpha_f = k/\rho_0 c \) is the thermal diffusivity.

For free-free boundaries

\[
V = D^3 V = 0 \text{ at } y = 0, 1. \tag{21}
\]

Let \( V = \sin \pi y \) (satisfying Eq. (21)) in Eq. (20), we get

\[
Ra = \left( \frac{\pi^2 + \alpha^2}{\pi^2} \right)^3 - \frac{D_T (T_i - T_0)}{\alpha_f T_0} \left( \frac{\rho_p - \rho_0}{\rho_0} \right) g d^3 \frac{\rho_0}{D_B \mu}. \tag{22}
\]

In the absence of nanoparticle parameters \((D_T = 0)\), Eq. (22) reduces to the one given by Chandrasekhar [20], which validates the result. Also, presence of nanoparticles reduces the value of Rayleigh number as \( T_i > T_0 \) and \( \rho_p > \rho_0 \). It is interesting to note that nano-effects; thermophoretic and Brownian diffusion coefficients have contradictory impact on onset of instability.

4. Numerical results and Discussions

Using the fact that the value of Brownian diffusion coefficient \((D_B)\) is constant for most of the nanoparticles [5] and

\[
D_T = 0.26 \frac{k}{2k + k_p} \frac{\mu}{\rho_0} \phi_p. \tag{23}
\]

Let us further simplify Eq. (22) using Eq. (23), we get

\[
Ra = \left( \frac{\pi^2 + \alpha^2}{\pi^2} \right)^3 - \frac{\left( \rho_p - \rho_0 \right)}{2k + k_p} A, \tag{24}
\]

where the constant ‘A’ does not depend on density and conductivity of nanoparticles. Thus, increase in density of nanoparticles lead to less stable system while nanoparticles conductivity has reverse impact on stability. It is noteworthy that ratio of density to conductivity of nanoparticles decides the convective stability of nanofluid. Table 1 shows the ratio of desired properties of some metallic and
non-metallic nanoparticles which establishes the stability pattern as: Aluminium > copper >> silicon oxide > copper oxide.

Table1. Properties of different metallic and non-metallic nanoparticles.

| Properties | Al  | Cu  | CuO | SiO₂ |
|------------|-----|-----|-----|------|
| ρ (kg/m³)  | 2700| 9000| 6510| 2600 |
| k (W/mK)   | 237 | 401 | 18  | 10.4 |
| ρp / kₚ    | 11.3| 22.4| 361.6| 250  |

Thus, non-metallic nanoparticles make the fluid less stable than metallic. Interestingly, both density and conductivity influence the convective instability of the system and more importantly it’s their ratio (density/conductivity) which decides the convective nature of the system. It is worth mentioning that copper in spite of having maximum conductivity among the elements of Table1 has lesser impact on stability than aluminium because of its higher density.

Let us explore the problem further and rewrite the Eq. (22) as

\[
Ra = \frac{R₀}{1 + \frac{Dᵣ}{β Dₖ T₀} \left( \frac{ρₚ}{ρ₀} - 1 \right)},
\]

(25)

where \( R₀ = \frac{(π² + α²)³}{α²} \).

Equation (25) is analyzed to see the effects of variables for fixed values of \( Dᵣ = 6 \times 10^{-11}, β = 0.006, Dₖ = 4 \times 10^{-11}, T₀ = 300, ρₚ = 3000, ρ₀ = 1000 \).

Figure 1. Effect of temperature

Figure 2. Effect of density of nanoparticles

Figure 1 depicts the impact of temperature (at the top of the layer) on the onset of convection currents in the fluid. As temperature increases, convective instability delays and hence more amount of heat is transferred through conduction. The effect of density of nanoparticles is shown in Fig. 2. Denser nanoparticles lead to lesser stability of nanofluids for fixed conductivity.
Figures 3 and 4 show the nano effects on the convective instability of the layer. Thermophoresis has destabilizing effect (Fig. 3) while Brownian diffusion has shown stabilizing impact (Fig. 4) on the system. Stabilizing influence of Brownian motion is more as compared to destabilizing impact of thermophoretic diffusion for same rate of increment in nano variables. The critical wave number (at which tangent line becomes horizontal) is same for all the figures and hence doesn’t get affected by the addition of nanoparticles in the fluid.

5. Conclusions
The paper analyzes the differential equations for instability of a horizontal nanofluid layer which is under small temperature gradient. As nanoparticle concentration is assumed to be small in the fluid, initial solution considers the nanoparticle volume fraction to be constant. Two-dimensional set of equations are solved to get expression for thermal Rayleigh number. It is found that the presence of nanoparticles affects the stability of the layer significantly. Non-metallic nanofluids are less stable than metallic. A ratio of density to conductivity of nanoparticles affect the convective nature of the fluid layer and higher is the said ratio lesser is the stability of the system. Copper in spite of having higher conductivity has lesser impact on stability than aluminium because of having higher density. The stabilizing impact of Brownian diffusion and destabilizing influence of thermophoresis i.e. contrary nature of nano effects on the stability of the system is established.

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