Convective and non-convective mixing in AGB stars

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Abstract.
We review the current state of modeling convective mixing in AGB stars. The focus is on results obtained through multi-dimensional hydrodynamic simulations of AGB convection, both in the envelope and the unstable He-shell. Using two different codes and a wide range of resolutions and modeling assumptions we find that mixing across convective boundaries is significant for He-shell flash convection. We present a preliminary quantitative analysis of this convectively induced extra mixing, based on a sub-set of our simulations. Other non-standard mixing will be discussed briefly.

1. Introduction

Our understanding of the physics of mixing in stars especially AGB stars (Iben & Renzini 1983; Herwig 2005), is not in a satisfying state. We do get many global properties right, but the list of things that models can not accurately account for is getting longer as the observations are becoming more detailed and numerous. The lack of predictive models for mixing and consequent stellar properties and nuclear yields is severely inhibiting the usefulness of this field for helping to address some general questions in astronomy; for example in the emerging field of near-field cosmology. With better simulations we could use detailed abundance observations to precisely characterize extra-galactic populations. Thus, there is a real need for improving our understanding of convective and non-convective mixing in AGB stars.

The most important physical process for mixing in stars is convection. In 1D stellar evolution models we use the mixing-length theory (Bohm-Vitense 1958) or some variant. The mixing-length parameter determines the geometric scale of convective eddies and is calibrated by using a solar model. However, we know for certain that the mixing length parameter is not constant throughout all phases and conditions in stellar evolution. Figure 5 in the important paper by Ludwig et al. (1999) shows the mixing-length parameter from their set of 2D radiation-hydrodynamic simulations of the outer convection layer in stars similar to the sun. These simulations show that the mixing-length parameter

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is very sensitive to the temperature (and thus to the depth of the convection zone), and that just within the small parameter range covered here, say within ±1000 K the mixing-length parameter changes by ±15%. Such a variation would change many important features in AGB simulations, most notably dredge-up and the hot-bottom burning efficiency. This has been known since first shown by Boothroyd & Sackmann (1988) and should be kept in mind when comparing dredge-up efficiency-dependent model predictions and observations. The hot-bottom burning dependence on the mixing-length parameter has implications in particular for super-AGB stars and massive low-metallicity AGB stars.

How can this situation be improved? Canuto & Mazzitelli (1991) and collaborators have suggested the Full-spectrum turbulence convection model as an alternative. This theory overcomes one important simplification of MLT: convection does not only consist of one blob size, but rather contains a spectrum of scales. Unfortunately, so far it seems that no convincing validation case for AGB envelope convection has been identified. Other uncertain input physics such as mass loss, other sources of mixing (rotation and gravity waves), and in some cases nuclear reaction rates have a similar, independent effect on the same astrophysical observable (e.g., certain abundances, like the $^{12}$C/$^{13}$C ratio). In addition, the variability of a convection parameter studied by Ludwig et al. (1999) see above) applies here as well (see their Fig. 6).

A resolution to these problems can only come from simulations. Several years ago Porter & Woodward (2000) presented highly resolved simulations with simplyfied input physics (Fig. 1). One of their important results was that convec-
tion settles into a dominant global dipole mode, a finding that is now confirmed for fully convective spheres in core-carbon burning WDs that are about to ignite a SN Ia ([Kuhlen et al. 2006]. Another finding was that convection itself without any other physical mechanism leads to pulsations. These may match observed pulsations of AGB stars, but a more detailed comparison needs to be done. Other AGB envelope models in 3D were developed by Freytag (see Freytag 2003; Höfner et al. 2005, and http://www.astro.uu.se/~bf for more details). These models feature a more realistic outer boundary condition but are not as highly resolved as the Porter & Woodward simulations.

Porter & Woodward derived a mixing-length parameter equivalent to the velocity field from the simulations. They find $\alpha_{\text{MLT}} \sim 2.6$. These simulations, both by Freytag and by Porter & Woodward, did not resolve the inner parts of the envelope convection, which is so crucial for nucleosynthesis. We anticipate that significant improvements of the treatment of convection properties in stellar evolution modeling can be made through hydrodynamic models with available and coming resources.

2. **Multi-D hydrodynamic simulations of He-shell flash convection**

**Motivation, codes and setup**

In order to understand better nucleosynthesis in AGB stars, we have performed multi-dimensional He-shell flash convection simulations. This work has several goals. First of all, the hydrodynamics of He-shell flash convection has never been simulated before. So, we want to investigate the hydrodynamical properties and the topology of He-shell flash convection. For example, how well does MLT describe the vertical velocity profile of the convection zone? What are the dominating scales? What is the dependence on resolution and on the nuclear energy generation? Obtaining a resolved velocity distribution from hydrodynamic simulations can give a new framework to study short-lived, temperature-dependent s-process branchings fueled by the $^{22}$Ne neutron source. And, most importantly, we want to study convection induced mixing across the convective boundaries.
Several 1D stellar evolution studies using a parameterized recipe for convective overshooting (e.g. Herwig 2000; Lugaro et al. 2003) have shown that overshooting at the bottom of the He-shell flash convection zone strongly affects the pulse strength, and thus the dredge-up efficiency, as well as the intershell abundance and the temperature for the $^{22}\text{Ne}$ s-process. Some of the predictions of stellar evolution models with convective extra mixing at the bottom of the He-shell flash convection zone have been confirmed by observations of H-deficient post-AGB stars (Werner & Herwig 2006).

We have performed simulations using two codes. One set of simulations (see Herwig et al. 2006 for initial results) was done with the RAGE code, an explicit, compressible, Eulerian grid code. The other code (scPPM) is a PPM (Eulerian, compressible grid as well) code for stellar convection that solves for perturbations to a base state defined in the problem setup. This later approach takes advantage of the fact that buoyancy driving the convection in these stellar interior environments is very small and that Mach numbers of the flow are small (Ma < 0.03). We will give more details on the scPPM code in a forthcoming paper.

For the initial setup we construct a piecewise polytropic stratification with gravity that resembles closely the actual conditions in a detailed 2 $M_\odot$, Z = 0.01 thermal pulse stellar evolution model just before the He-shell flash peak luminosity. The driving of the convection results from a constant volume heating at the bottom of the convectively unstable region that injects the same amount of energy as the corresponding stellar model.

The stratification comprises approximately 11 pressure scale heights, half of which represent the convectively unstable region. This gives enough simulation space for stable layers both above and below the convection zone to avoid
artifacts in the convection boundary simulation from simulation box boundary effects.

Morphology and gravity waves

In Fig. 2 we show a snapshot from a 2D RAGE run on a 1200 × 400 grid. The same heating rate as in the stellar evolution model was applied. At the snapshot time the simulation has lasted for approximately 5 convective turn-over times and has thus reached convective steady-state. The initially imposed convective boundaries are well maintained, sharp and clearly visible in this snapshot. The convective flow is dominated by three to four large systems, which vertically span the entire unstable layer and are centered in the lower half of the unstable region. In the stable layers above and below the convection zone oscillations due to internal gravity waves can be seen. Through a systematic resolution study we find that the total number of large convective systems does not change over a wide range of grid sizes, down to a 300 × 100 grid.

Gravity waves can be identified in the top and bottom stable layers. Movies of these simulations show that the g-modes in the stable layer above the convection zone are excited even before the convective motions have reached the top boundary of the unstable layer. In contrast to shallow surface convection, for example in A-type stars (Freytag et al. 1996), coherent convective systems do not cross the convective boundaries in a fashion visible in this representation. This is not surprising, because the relative stability of convective boundaries in the stellar interior is much larger than in stellar near-surface convection.

We have performed 3D simulations with both the RAGE and scPPM code. The vertical velocities in one snapshot of the 3D RAGE run are shown in Fig. 3.
In the He-shell flash convection zone, convection originates through heating at the bottom, contrary to surface or envelope convection which is driven by cooling from the surface. It is interesting to note that here we see near the bottom of the convection zone (panel 2.23Mm) structures that resemble the inverse of solar granulation: cool granules with hot intergranular lanes.

Even outside the convection zone dynamic fluid motions exist, and their patterns again depend on the distance to the convective boundary. Panel 1.51Mm shows a very granular appearance with a lot of detail on small scales whereas both the panels above and below show fluctuations on larger scales. The layers most distant from the convection zone, both above and below, show a rather diffuse pattern of large-scale fluctuations.

Although the coherent convective systems do not cross the convective boundary on a scale that is any significant fraction, of the horizontal scale, this does not mean that there is no oscillation exitation and mixing across the boundary. Looking at the vertical velocity reveals how motions and oscillations in the stable and unstable layer do in fact correlate, and therefore communicate. Features in one slice have corresponding features in neighboring slices above and below inside the convection zone, even if the dominating scales are different in different layers of the convection zone. This is consistent with the observation in the 2D snapshots that convective system traverse the entire convection zone vertically.
However, patterns in the vertical velocity images also show correlations between the boundary planes just inside the convection zone and the neighboring planes several hundred km out in the stable layer. Thus, while convective systems do not cross the convective boundaries, they do imprint their signature on the oscillation properties of the stable layers.

But how do properties like the vertical velocities depend on numerical resolution and numerical scheme? To address this question we compare in Fig. 4 a snapshot of the velocity field in the upper layers of the convection zone from the RAGE and scPPM simulations. While the scPPM simulation shows much more small scale structure, there are similarities in the overall pattern. Downflows are ordered in lanes surrounding larger areas with upwelling flows. The more highly resolved grid of the scPPM simulation is only partly responsible for the difference. The main reason for the difference in scale distribution is the higher order of the PPM numerical scheme compared the modified Van Leer method implemented in the RAGE version used here.

The correspondence between different layers, including nearby convective and non-convective layers, can be further analysed in a series of k-ω diagrams of several vertical planes in the 3D RAGE run (Fig. 5). These diagrams show the distribution of power as a function of wavelength and frequency. In these diagrams the signature of convective motions shows up as an unstructured blob in the lower left part of the diagram. It is most prominent in the panels showing planes in the unstable region. However, even in the panels representing the stable layer a significant convective signature is evident. In addition, the stable layers show the characteristic signature of gravity waves, and all panels show the signature of p-modes.

We conclude that our simulations show that convection does influence the fluid flows in the neighboring stable layers, both through exciting g-mode os-
oscillations and by forcing motions with convective wavelengths and frequencies. We now need to look at how these correspondences between fluid flows on both sides of the convective boundary translate into actual mixing.

**Mixing for He-shell flash convection**

The scPPM simulation has been performed with multiple fluids representing the different abundance compositions in the three layers of our sandwich setup. In Fig. 6 we show the result of material from the top stable layer above the convection zone being entrained into the convection zone. Conceptually, such a situation may occur in He-shell flashes of extremely metal-poor stars when the convection zone reaches into the H-rich envelope, or in the very-late thermal pulse flash associated with the born-again evolution. In this simplified simulation, the effect of $\mu$-gradients has been neglected as all fluids have the same molecular weight. Therefore the entrainment seen in this case is probably an upper limit. However, we do see that the distribution of the fluid from the upper stable layer in the convection zone is rather isotropic. If the fluid from above would react with the material in the convection zone we would likely be in a distributed burning regime, rather than in a flame propagation situation. Using the multi-fluid scPPM code we will in the future be able to quantify mixing across the convective boundary. In this way we can take full advantage of the high-order advection scheme implemented here (Woodward et al. 2006).

With the RAGE simulations we performed a mixing analysis based on the tracer particle technique of Freytag et al. (1996). We determined diffusion coefficients at different horizontal positions, in particular near the convective boundaries. An example for this approach (full details will be given in Freytag et al., 2006, in prep) is shown Fig. 7. The final solution of the diffusion coefficient
as a function of vertical position is a composite of two approaches. Diffusion coefficients derived from the spread evolution of the entropy are more suitable for the stable layer. The evolution of the vertical coordinate is more appropriate in the convection zone. In the transition layer, both approaches give the same result. The fall-off of the diffusion coefficient has been fitted with an exponential. This approach has before been used by Freytag et al. (1996) and applied to stellar evolution calculations, first by Herwig et al. (1997). Within this framework, the convection induced mixing across the bottom convective boundary can be described by a succession of two exponential decay laws of the form

\[ D = D_0 \exp\left(-\frac{2z}{fH_p}\right) \]

where \( z \) is the distance from the location at which a baseline \( D_0 \) has been determined, and \( H_p \) is the pressure scale height (Freytag et al. 1996; Herwig 2000). The first decay starts already somewhat inside the convection zone with \( f_{\text{bot},1} = 0.01 \), while just outside the convection zone (when \( D \) has fallen to \( \sim 10^5 \text{cm}^2/\text{s} \)) the decay of mixing efficiency flattens and can be represented by \( f_{\text{bot},2} = 0.14 \). At the top boundary our initial analysis gives a single exponential decay with \( f_{\text{top}} = 0.10 \).

The f-value determination varies as a function of resolution within a factor of a few. However, we could not find any apparent trend, indicating that resolution may be a limiting aspect within the chosen range of simulations. Also we did not find a systematic difference between simulations with the realistic heating rate compared to those with 30 times larger heating rate. This indicates that within a range of driving energies, mixing properties will depend predominantly on the background stratification. This means that the f-value for mixing across the convective boundary remains the same for a given convective boundary until the conditions change drastically.

From the few 3D results we have, we find that the 2D/3D difference is of the same order as the dependence on resolution. Overall the 3D runs confirm the results we found in 2D.

Initial stellar evolution test calculations over a couple of thermal pulses implementing these hydrodynamic convective mixing results indicate that we recover the increased O and C intershell abundance that we found in earlier calculations using the f-overshoot for this convection zone. The simulated C and O abundances in the intershell are consistent with observations of H-deficient PG1159 and [WC]-CSPN stars (Werner & Herwig 2006).

3. **Non-convective mixing**

Rotation has an important effect on the production of elements in AGB stars but is unfortunately not yet studied very well. Models of rotating AGB stars (Langer et al. 1999) show that with the present physics model rotation induced mixing at the convective boundary is insufficient for the formation of the \( ^{13}\text{C} \)-pocket for the \( s \) process. Even worse, Herwig et al. (2003) showed that rotationally induced mixing may be even harmful to the \( s \)-process nucleosynthesis. In models in which a \( ^{13}\text{C} \) pocket was introduced using the convective overshooting paradigm, rotation induced mixing of \( ^{14}\text{N} \) into the \( ^{13}\text{C} \)-pocket during the interpulse phase. The neutron exposure is well below \( \tau = 0.1\text{mbarn}^{-1} \) which is far too low to reproduce well established \( s \)-process observables. \( ^{14}\text{N} \) acts as a neutron poison due to its large \( (n,p) \) cross section. Overall, there are indications that angular
momentum transport from the core to the envelope is underestimated in current models of rotating AGB stars. Therefore angular velocity gradients at the core-envelope interface, and consequently rotation induced mixing at this interface are overestimated. The $^{13}$C-pocket is located exactly at this interface. Another indication for missing angular momentum transport is that core rotation rates of rotating AGB models are too large compared to observed rotation rates from white dwarf pulsations (Kawaler 2003). Magnetic fields could add angular momentum transport and reduce angular velocity gradients that cause shear mixing. However, there is another likely possibility, in particular in light of what we see in our hydrodynamic simulations. Internal gravity waves could have a similar desired effect of angular momentum transport (Talon & Charbonnel, this volume). We see them plentifully in the hydrodynamic simulations, and their effect on mixing in AGB stars has not yet been studied in sufficient detail. The only work on gravity waves in AGB stars besides the new results by Talon & Charbonnel is that by Denissenkov & Tout (2003) who show that mixing induced by these waves may provide the conditions needed for the formation of a $^{13}$C-pocket. However, more detailed studies in this area are needed.

4. Conclusions

Hydrodynamic simulations of convection in AGB stars which give meaningful insight for stellar evolution models, both in the envelope and in the intershell are feasible and offer an exciting tool to study mixing. We can obtain a detailed picture of flow structures both within the unstable zone as well as in the neighboring layers. We will study in greater detail the variation of the averaged mixing efficiency for AGB envelopes that determines important evolutionary properties, such as the dredge-up efficiency as well as the hot-bottom burning efficiency. Our initial results on He-shell flash convection allow a first quantitative glimpse at mixing at and across the convective boundaries. The simulations emphasize the need to study the role of gravity waves in much greater detail. The issue of rotationally induced mixing is much more difficult to approach with multi-dimensional simulations, mainly because the flow velocities are significantly smaller than in convection. This is especially unfortunate as rotating models currently do not reproduce observables of AGB stars although these stars obviously rotate, at least their cores.

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