Refined Swampland conjecture in deformed Starobinsky gravity

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In order to validate or invalidate a large class of low energy effective theories, the Swampland conjecture has attracted significant attention, recently. It can be stated as inequalities on the potential of a scalar field which is conjectured to satisfy certain constraints. In this work, we discuss the theoretical viability of deformed Starobinsky gravity in light of the refined Swampland conjectures. We consider the deformation of the form $f(R) \sim R^{2(1-\alpha)}$ with $\alpha$ being a constant. We then constrain $\alpha$ using the spectral index of curvature perturbation $n_s$ and the tensor-to-scalar ratio $r$. We demonstrate that the model under consideration is in strong tension with the refined swampland conjecture. However, regarding our analysis with proper choices of parameters $a$, $b = 1 - a$ and $q$, we discover that the model can always satisfy this new refined swampland conjecture. Therefore, the model might be in “landscape” since the “further refining de Sitter swampland conjecture” is satisfied.

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I. INTRODUCTION

In string theory, it has been suggested that a landscape of vacua is vast and consistent quantum theory of gravity is believed to be formulated with consistent low-energy effective field theories (EFTs). Even more recently, the authors of Refs.\textsuperscript{1–3} suggested that the landscape is possibly surrounded by an even more vast neighborhood called “swampland” where consistent EFTs, which are coupled to gravity, are inconsistent with quantum theory of gravity. To be more concrete, the swampland can be formulated by the set of consistent effective field theories which cannot be completed into any quantum gravity in the high energy regime. Hence, it is desirable for consistent EFTs not to lie in the swamplands. See a comprehensive review on the Swampland \textsuperscript{4}.

In light of recently proposed swampland conjectures which can be translated to inequalities on

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the potential for the scalar field driving inflation. To start our discussion, we first follow a setup given in Ref. [47], and consider a four-dimensional (4D) theory of real scalar field \( \phi^i \) coupled to gravity, and its dynamics is governed by a scalar potential \( U(\phi^i) \). In this case, the action is given by

\[
S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} M_p^2 R + \frac{1}{2} g^{\mu\nu} h_{ij} \partial_\mu \phi^i \partial_\nu \phi^j - U \right],
\]

where \( h_{ij}(\phi^k) \) is the field space metric, \( M_p \) is the 4d Planck mass, and the 4D space-time indexes \((\mu, \nu)\) are raised and lowered with the 4D metric \( g_{\mu\nu} \) with a signature \((+, -, -, -)\), \( U \) is a potential of an canonical normalized field. Very recently, the “refined” version of the swampland conjecture has been suggested [3, 6]. It was found that the refined conjecture imposes a slightly weaker criterion on the scalar field potential in inflation, and is consistent with the existence of a tachyonic instability. In light of the refined swampland conjecture, a scalar field potential associated with a self-consistent UV-complete effective field theory must satisfy one of the two conditions:

\[
|\nabla U| \geq \frac{c_1}{M_p} U,
\]

or

\[
\min(\nabla_j \nabla_j U) \leq -\frac{c_2}{M_p^2} U,
\]

where \( c_1 \) and \( c_2 \) are both positive constants of the order of \( \mathcal{O}(1) \) and \( |\nabla V| = \sqrt{g^{ij} \nabla_i V \nabla_j V} \). The first condition corresponds to the original “swampland conjecture” proposed in Ref. [1]. The conjectures require the above inequalities to be satisfied by any EFT which has a self-consistent ultraviolet (UV) completion. Notice that single-field inflation is apparently satisfied by the first condition, and the field variation is in excellent agreement with the well-known Lyth Bound for single-field inflation [3]. However, the second condition poses difficulties for single-field paradigm [2, 45, 46]. This tension has been explored in a number of recently related works, see for example Refs. [9–18]. Interestingly, however, more complex models, e.g., multifield inflation [10] and warm inflation [16], are still allowed. Additionally, swampland criteria was also investigated of alternative theories of gravity, see for instance [19–21]. In [22], the viability of \( f(R) \) and Brans-Dicke theories of gravity was also discussed. For any \( U \), the standard slow-roll parameters can be rewritten using the inequalities to yield

\[
\sqrt{2\epsilon_U} \geq c_1, \quad \text{or} \quad \eta U \leq -c_2.
\]

In the present work, we consider the implications of this slightly weaker constraint on a deformation of Starobinsky gravity. It is worth noting that a different model of deformed Starobinsky inflation
was so far studied in Ref. [23], while log corrections to $R^2$ gravity were investigated in Refs. [24, 25].

Regarding the refined version, there exist relevant discussions of general constraints on inflation and other cosmological/astrophysical models, see Refs. [26, 27, 29–34, 47].

This paper is organized as follows. In Sec. II, we present a summary of deformed Starobinsky gravity by following the work proposed by Ref. [40]. Here we linearize the action by introducing an auxiliary field method. We use the conformal transformations in order to transform the theory in the Jordan frame to the Einstein frame. The above transformation connects both theories and allows us to rewrite the action in terms of a propagating scalar field minimally coupled to gravity.

Sec. III is devoted to discuss the theoretical viability of deformed Starobinsky gravity in light of the refined Swampland conjectures. We further discuss the refined Swampland criteria in deformed Starobinsky gravity in Sec. IV. Our conclusions are reported in the last section.

II. DEFORMED STAROBINSKY GRAVITY: A SHORT RECAP

Among many others, an intriguing possibility is that the gravity itself can be directly responsible for the inflationary period of the universe. The examination requires us to go beyond time-honored Einstein-Hilbert (EH) action. This can be achieved by adding a $R^2$ term to the original EH theory as in the Starobinsky model [35]. The model is highly natural since gravity itself drives cosmic inflation without the need of the scalar fields. It is worth noting that the model predicts a nearly vanishing ratio of tensor to scalar modes which is in excellent agreement with the observations, e.g. PLANCK data [36, 37]. Moreover, the logarithm corrections to $R^2$ gravity of the form $M_p^2R/2 + (a/2)R^2/[1 + b\ln(R/\mu^2)]$, where $R$ is the Ricci scalar, $a$ and $b$ are constants and $\mu$ is an energy scale, suggested by asymptotic safety were recently considered in Ref. [38].

In light of the observations, a discovery of primordial tensor modes can be used to constrain the cosmological parameters at the inflationary scale, which turn out to be close or at the grand unification energy scale. In general, the effective action for gravity can be in principle derived by considering the Taylor expansion in the Ricci scalar, $R$. Here without assuming a concrete form for the function $f(R)$, we consider

$$S = \int d^4x \sqrt{-g} f(R) \equiv \int d^4x \sqrt{-g} \left[ a_0 + a_1R + a_2R^2 + \ldots \right].$$

The first term $a_0$ is like the cosmological constant and must be small. The next coefficient $a_1$ can be set to one as in general relativity. Regarding the Starobinsky gravity, we have $a_2 = 1/(6M^2)$ where a constant $M$ has the dimensions of mass, see cosmological implications of the model [39].
Here the ellipses may include the Weyl tensor $C^2$ and the Euler topological terms $E$. As mentioned in Ref. [40], the $E$ terms can be ignored since it is just a total derivative. Moreover, the Weyl terms are subleading when gravity is quantized around a flat background. Higher powers of $R$, $C^2$ and $E$ are naturally suppressed by the Planck mass. Interestingly, the authors of Ref. [40] also take into account marginal deformations of the action (5) by including logarithmic corrections. The authors consider a simple form of the gravitational action formulated in the Jordan frame:

$$S_J = \int d^4x \sqrt{-g} \left[ -\frac{M_p^2}{2} R + hM_p^{4\alpha} R^{2(1-\alpha)} \right], \quad (6)$$

where $h$ is a dimensionless parameter and $\alpha$ is a real parameter which is assumed as $2|\alpha| < 1$. Note that the condition of the parameter $\alpha$ is further examined in the context of gravity’s rainbow [41]. One can linearize the above action by introducing an auxiliary field $y$ such that $S_J = \int d^4x \sqrt{-g} \left[ f(y) + f'(y)(R-y) \right]$ with $f(R) = -M_p^2 R/2 + hM_p^{4\alpha} R^{2(1-\alpha)}$ where $f'(y) = df(y)/dy$. Here the equation of motion for $y$ implies $R = y$ provided $f''(y)$ does not vanish. The explicit relation between (5) and the effective quantum-corrected nonminimally coupled scalar field theory used in Ref. [42] can be done by introducing the conformal mode $\psi = -f'(y)$ with $V(\psi) = -y(\psi)\psi - f(g(\psi))$ and having introduced the mass-dimension one real scalar field $\varphi$ via $\chi = M_p^2 \varphi$ to obtain:

$$S_J = \int d^4x \sqrt{-g} \left[ -\frac{M_p^2}{2} + \frac{\xi \varphi^2}{2} R + V(\varphi) \right], \quad (7)$$

where

$$V(\varphi) = \lambda \varphi^4 \left( \frac{\varphi}{M_p} \right)^{4\gamma} \text{ with } \alpha = \frac{\gamma}{1+2\gamma}, \quad (8)$$

and

$$h^{1+2\gamma} = \frac{\xi(1+2\gamma)^{2(1+\gamma)}}{4(1+\gamma) \lambda (1+2\gamma)} \quad (9)$$

Notice from Eq. (7) that the kinetic term for the field $\varphi$ is absent in the Jordan frame. However introducing the following conformal transformation of the metric, the kinetic term of the field can be simply generated via:

$$\tilde{g}_{\mu\nu} = \Omega(\varphi)^2 g_{\mu\nu}, \text{ with } \Omega^2 = 1 + \frac{\xi \varphi^2}{M_p^2}. \quad (10)$$

The above transformation connects both theories and allows us to rewrite the action in terms of a propagating scalar field minimally coupled to gravity. The resulting action is written in the Einstein frame and takes the form:

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{M_p^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) \right], \quad U(\chi) = \Omega^{-4} V(\varphi(\chi)). \quad (11)$$
Notice that the action is written in terms of the canonically normalized field $\chi$ which is related to $\varphi$ via

$$\frac{1}{2} \left( \frac{d\chi}{d\varphi} \right)^2 = \frac{M_p^2 (\sigma M_p^2 + (\sigma + 3\xi)\varphi^2)}{(M_p^2 + \xi\varphi^2)^2}. \quad (12)$$

It is worth noting that when setting $\sigma = 0$ a map from the Jordan frame of $f(R)$ gravity to the Einstein frame with a canonically normalized field is obtained. Throughout this work, we will set $\sigma = 0$. An explicit relation between $\chi$ and $\varphi$ can be obtained by assuming that inflation occurs at large values of the scalar field, i.e. $\varphi \gg M_p/\sqrt{\xi}$ and we obtain

$$\chi \simeq \sqrt{6} M_p \log \left[ \frac{\sqrt{\xi} \varphi}{M_p} \right]. \quad (13)$$

Substituting the above canonical normalized field into Eq.(10), therefore the Einstein frame potential takes the form

$$U(\chi) = \frac{\lambda M_p^4}{\xi^2} e^{\frac{2\sqrt{\xi} \chi}{M_p}} \left( e^{\frac{\sqrt{\xi} \chi}{M_p}} + 1 \right)^{-2} e^{\frac{4\varphi}{\sqrt{6}M_p\xi}} \xi^{-2\gamma}, \quad \gamma = \frac{\alpha}{1 - 2\alpha}. \quad (14)$$

It is worth noting that in the limit of $\gamma = 0$ (or equivalently $\alpha = 0$) one recovers the Starobinsky model [35]. It’s pointed out in Ref.[40] that for $0 < \alpha < 0.5$ the potential grows exponentially, and then an inflationary model with nonzero primordial tensor modes can be successfully obtained. Note that the exact Einstein-frame potential of $f(R) = R + \lambda R^p$ with $p$ being not necessarily an integer in general was also derived in Ref.[43].

III. THE REFINED SWAMPLAND CRITERIA IN DEFORMED STAROBSKY GRAVITY

In the present work, we test a model of inflation of deformed Starobinsky gravity in general scalar-tensor theories of gravity. For our analysis below, we first define two new parameters for any scalar field $U(\chi)$:

$$F_1 = \frac{|dU(\chi)/d\chi|}{U(\chi)}, \quad (15)$$

and

$$F_2 = \frac{d^2U(\chi)/d\chi^2}{U(\chi)}. \quad (16)$$

Considering Eq.(14), the above parameters can be recast in terms of the slow-roll parameters to yield

$$F_1 = \sqrt{2\varepsilon_U}, \quad F_2 = \eta_U. \quad (17)$$
We see that $F_1$ and $F_2$ are written in terms of the slow-roll parameters. Therefore, they can be related to the spectrum index of the primordial curvature power spectrum $n_s$ and tensor-to-scalar ratio $r$. In the present case, it is rather straightforward to show that

$$F_1 = \sqrt{2\varepsilon_U} = \sqrt{\frac{r}{8}},$$

and

$$F_2 = \eta_U = \frac{1}{2}(n_s - 1 + \frac{3}{8}r).$$

Below we examine if deformed Starobinsky inflation does satisfy this new refined swampland conjecture, or not. In establishing the connection among the swampland conditions and the parameters of the model considered, we consider the well-known inflationary parameters, i.e., the scalar spectral index $n_s$, and tensor to scalar ratio $r$, and from the standard formulation we write

$$n_s = 1 - 6\varepsilon_U + 2\eta_U, \quad r = 16\varepsilon_U,$$

where in terms of the potential the slow-roll parameters can be defined as

$$\varepsilon_U = \frac{M_p^2}{2} \left(\frac{U'}{U}\right)^2, \quad \eta_U = \frac{M_p^2}{2} \left(\frac{U''}{U}\right),$$

where primes denote derivatives with respect to the field $\chi$. It is useful to express $\varepsilon_U$ and $\eta_U$ in terms of the number of e-foldings. Substituting Eq.(14) into Eq.(21), we find up to the first order of $\gamma$:

$$\varepsilon_U = \frac{3}{4N^2} + \frac{\gamma}{N} + O(\gamma^2),$$

$$\eta_U = -\frac{1}{N} + \frac{2\gamma}{3} + O(\gamma^2).$$

We notice that when setting $\gamma = 0$ the results recover Starobinsky inflation. Using the above expressions, we can rewrite Eq.(20) in terms of the the number of e-foldings, $N$, as

$$n_s \approx 1 - 6 \left(\frac{3}{4N^2} + \frac{\gamma}{N}\right) + 2 \left(-\frac{1}{N} + \frac{2\gamma}{3}\right),$$

$$r \approx 16 \left(\frac{3}{4N^2} + \frac{\gamma}{N}\right),$$

where we have used the potential from Eq.(14). Note here that the very small tensor-to-scalar ratio $r = 12 N^2$ for the Starobinsky $R + R^2$ inflationary model was first (and quantitatively correctly) presented in Ref.[44]. Interestingly, usual $\phi^4$ inflation refers to the results when setting $\gamma = 0$, that is, non-minimally coupled $\phi$ inflation. Here a term with $\gamma$ is used to clarify how the results deviate.
from $\phi^4$ inflation. An expansion is, however, justified for tiny values of $\gamma$. We then constrain values of $\gamma$ using a condition of $r < 0.06$

$$r \simeq 16 \left( \frac{3}{4N^2} + \frac{\gamma}{N} \right) < 0.06,$$

(26)

to obtain

$$\gamma < \frac{0.002 \left( 2N^2 - 375 \right)}{N} \quad \text{or} \quad \alpha < \frac{2N^2 - 375}{4N^2 + 500N - 750}.$$

(27)

Assuming $N = 55$ and $\alpha = 0.1461$, this model predicts $n_s \simeq 0.965$ and $r \simeq 0.00464$ which are consistent with the observed data [49]. Inserting these values into Eq.(15) and Eq.(16), we obtain

$$F_1 = \sqrt{2\epsilon_U} = \sqrt{\frac{r}{8}} = 0.02410,$$

(28)

$$F_2 = \eta_U = \frac{1}{2} (n_s - 1 + 3r/8) = -0.02596.$$

(29)

Considering the refined swampland conjecture (31), we find

$$c_1 \leq 0.02410 \quad \text{or} \quad c_2 \leq 0.02596.$$

(30)

Clearly, neither $c_1$ nor $c_2$ are of the order of $O(1)$ implying that the model under consideration is in strong tension with the refined swampland conjecture.

**IV. REFINING DE SITTER SWAMLAND CONJECTURE**

Very recently, the authors of [47] have proposed a single condition on both $\epsilon_U$ and $\eta_U$ has been proposed. The approach is called a further refining de Sitter swampland conjecture. The statement of an alternative refined de Sitter conjecture is suggested that a low energy effective theory of a quantum gravity that takes the form (12) should verify, at any point in field space where $U > 0$

$$\left( \frac{M_p |\nabla U|}{U} \right)^q - a M_p^2 \min(\nabla_j \nabla_j U) \geq b \quad \text{with} \quad a + b = 1, a, b > 0, q > 2,$$

(31)

in which a combination of the first and second derivatives of the scalar potential is achieved. In terms of the slow-roll parameters, the conjecture implies [47]

$$(2\epsilon_U)^{q/2} - a \eta_U \geq b.$$

(32)

Interestingly, the authors of Ref.[48] have discovered that Higgs inflation model, Palatini Higgs inflation, and Higgs-Dilaton model can always satisfy this new swampland conjecture if only they
adjust the relevant parameters \( a, b = 1 - a \) and \( q \). Substituting the above results into Eq. (32), we have

\[
0.02408^q + 0.01663 a \geq 1 - a \quad \text{or} \quad 0.02408^q \geq 1 - 1.01663 a .
\]  \( \text{(33)} \)

If we can find \( a \) to satisfy the condition

\[
\frac{1}{1.01663} (1 - 0.02408^q) \leq a < 1 , \quad q > 2 ,
\]  \( \text{(34)} \)

then the further refining swampland conjecture can be satisfied. In this case, when \( a = \frac{1}{1.01663} \), we have \( 1 - 1.01663 a = 0 \). Therefore, we can examine that when \( a < \frac{1}{1.01663} \), we can always find a \( q \) whose value is larger than 2. It is possible to give an example of values of the parameters \( a, b, q \), which work for this model. From Eq. (34), we use \( q = 2.2 \) which is satisfied by a condition \( q > 2 \). We find for this particular case that \( 0.983371 \leq a < 1 \) and choose \( a = 0.9834 < \frac{1}{1.01663} = 0.98364 \) and \( 1 - a = 1 - 0.9834 = b = 0.0166 > 0 \).

\[ \text{V. CONCLUSION} \]

Recently, the Swampland conjecture has attracted significant attention. The conjecture allows us to validate or invalidate a large class of low energy effective theories. It can be formulated as inequalities on the potential of a scalar field which is satisfied by certain constraints. In this work, we first considered the deformation of the form \( f(R) \sim R^{2(1-\alpha)} \) with \( \alpha \) being a constant. We then constrained a parameter \( \alpha \) using the spectral index of curvature perturbation \( n_s \) and the tensor-to-scalar ratio \( r \). Here we linearized the original action by introducing an auxiliary field method and used the conformal transformations in order to transform the theory in the Jordan frame to the Einstein frame. We rewrote the action in terms of the canonical normalized scalar field minimally coupled to gravity.

We discussed the theoretical viability of deformed Starobinsky gravity in light of the recent refined Swampland conjectures. Our analysis showed that the model under consideration is in strong tension with the refined swampland conjecture. However, regarding our analysis with proper choices of parameters \( a, b = 1 - a \) and \( q \), we discovered that the model can always satisfy this new refined swampland conjecture. Therefore, the model might be in “landscape” since the “further refining de Sitter swampland conjecture” is satisfied.
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