Experimental analysis of heat conduction in a high diameter ratio annular gap filled with a rarefied gas

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Abstract. A first experimental attempt has been realized in order to measure the heat transfer between a heated platinum wire having a diameter (d) of 0.15 mm, disposed along the axis of a cylindrical shell in stainless steel having an inner diameter (D) of 100 mm, and air from atmospheric conditions down to 10⁻³ mbar. Temperature differences between the wire and the external stainless steel cylindrical shell in the range of 50-125 K were imposed and the heat power transferred from the wire to the surround was measured as a function of the gas pressure. The experimental results demonstrates that for an accurate measurement of the heat conduction when the pressure goes down to 0.05 mbar is very important to be able to quantify accurately the radiative contribution which becomes predominant at low pressure. The main limitations of the test rig described in this paper have been analysed in order to highlight the modifications which can be suggested to obtain experimental results comparable with theoretical models.

1. Introduction
The reduction of the dimensions of the pressure sensors has found various interests in a large spread of technological fields. The miniaturisation of the pressure gauges make possible to reduce their influence on the measured system, the power consumptions of the active components, their characteristic response time and, in many cases, their cost. In particular, microsensors based on silicon with on-chip circuitry fabricated by using the integrated circuit technology have demonstrated that it becomes possible to conjugate low costs with high level of reliability and the compatibility with the modern signal processing circuitry. A variety of microsensors and microactuators have been fabricated by standard semiconductor technologies in the last ten years: an important field of application of these miniaturized sensors is related to the measurement of the gas pressure (micro Pirani sensors [1]), especially in the range between 1 and 10⁻³ mbar; the miniaturization of the Pirani sensors has enabled the extension of their dynamic range measurement in comparison to the classical Pirani gauges. The micro Pirani sensors are sometimes based on silicon sheets realizing plate geometries and sometimes based on a hot wire with a diameter of the order of hundreds of microns inserted in a system having a characteristic dimension of the order of some centimetre with a large scale difference between the wire and its surround. The heat transfer between the wire and its surround can be modelled by considering

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the problem of heat conduction through a rarefied gas contained between two concentric cylinders. This kind of geometry has been extensively investigated in the past in order to measure thermal conductivities and thermal accommodation coefficients of gases. One of the first experimental work in this field is due to Bomelburg [2] on 1959, who measured heat conduction from 1.25, 5 and 10 \( \mu \text{m} \) diameters Wollaston wires placed in 10 and 25 cm inner diameter bell jars. In these experiments the author assumed thermal radiation and end losses as negligible. Dybbs and Springer [3] evidenced as the conditions of the experiments of Bomelburg had to be improved. Their experimental tests were conducted by using helium, neon and argon over a pressure range that included free molecular, transitional and temperature jump regimes. Special care was dedicated by the authors in order to design a test rig in which the effects due to end losses can be minimized and in order to eliminate the effect related to thermal radiation between the heated wire and surround. The experimental results obtained by Dybbs and Springer were in a good agreement with the numerical results obtained by using the momentum method proposed by Lees and Liu [4] over the entire Knudsen number range covered in the experiments (0.02<Kn<50), assuming that the thermal accommodation coefficient at the inner cylinder remained constant over this range at its free molecular value. Yu et al. [5] proposed a transient hot-wire technique in order to measure the heat flux between a fine metallic filament (platinum, nickel, tungsten, tantalum) and a rarefied gas under different heat transfer conditions ranging from a molecular to a continuum regime. Their data have been compared with the theoretical ones obtained from the solution of the BGK model by the variational method and the comparison evidenced a very good agreement between experimental and numerical values. More recently, O’Shea and Collins [6] measured the total heat losses due to conduction heat transfer through rarefied polyatomic gas (benzene and n-hexane) in a concentric cylindrical geometry in order to study the heat transfer in temperature limited evacuated solar thermal collectors. Their experimental data have been compared with the results obtained by means of a Monte Carlo simulation in the range of Knudsen numbers between 0.1 and 100 showing a good agreement for annular gaps of the order of 2.65 mm and 2.1 mm and for an imposed temperature difference between the inner cylinder and the outer cylinder equal to 60 K. The work presented in this paper is addressed to investigate the conduction heat transfer in a mixture of polyatomic gases as air by increasing the temperature difference between the wire and the surround up to 125 K and by adopting a cylindrical geometry characterised by a large value of the D/d ratio (equal to 667). These preliminary results will be extended and validated in the short future in order to be used as a benchmark for the test rig that will be further used in order to investigate the behaviour of this system under transient conditions.

2. The experimental test rig
In order to study the heat transfer between the wire and a surrounded rarefied gas an experimental campaign has been conducted at the Laboratory of ENEA in Rome (Italy) with the cooperation of the DIENCA Microfluidics Lab (Bologna, Italy). A specific test rig (RAGA) has been designed and realized in order to test the heat conduction between a heated platinum wire and rarefied air within a stainless steel cylinder (Figure 1).
In Figure 1 is shown the cylindrical test section used in this experience. A stainless steel cylindrical capsule having an inner diameter \((D)\) equal to 100 mm and a total length of 400 mm; it contains a platinum wire \((l, \text{Fig.1})\) fixed between two Teflon blocks \((2, \text{Fig. 1})\). The wire has a diameter \((d)\) of 0.15 mm and a length \((L)\) of 97 mm.
The platinum wire is fixed to a rectangular support, shown in Figure 2a, which maintains the wire on the axis of the capsule. The wire is placed in the central region of the capsule in order to minimize the effects related to the ends of the cylindrical chamber. The wire is soldered to two copper ends, having the first a diameter of 1.45 mm ($d_1$), a length of 10.55 mm ($L_1$, Fig.1) and the second a diameter of 1.7 mm ($d_2$) and a length of 9.75 mm ($L_2$, Fig.1); on these copper ends the electrodes used to heat by Joule effect the wire are attached (see Figure 2). The Teflon blocks reduce the heat losses at the ends of the heated section. The geometry of the cylindrical test rig is characterized by a very large ratio between the outer and inner diameter of the annular region filled by the gas ($D/d=667$).

The cylindrical capsule is filled with air, initially at the atmospheric pressure. By connecting the capsule to the backing pump of a turbo-molecular pump (Alcatel ATS-100) it is possible to reduce the air pressure from atmospheric value down to $10^{-3}$ mbar. The pressure inside the capsule was monitored by means of a vacuum pressure gauge (MKS type 925).

The platinum wire is heated by Joule effect using a programmable DC supply (HP6032A) which provides a constant electrical current on the wire. The electrical resistance of the Pt wire varies with the average wire temperature between 0.12 and 0.39 ohm; the maximum value of electrical power which has been imposed on the wire during the tests is of the order of 700 mW.

The programmable DC supply was managed by using LabView in order to impose a fixed value of the average temperature along the wire surface. The temperature along the platinum wire was measured by means of 4 K-type thin thermocouples, having a diameter of 25 μm, glued directly on the wire surface (Figure 2). By means of Joule heating, the wire temperature was varied up to 150°C; on the contrary the external tube is maintained at the room temperature (25-27°C). The maximum value of temperature of the wire was selected by taking
into account the working range of the glue used to fix the thermocouples on the wire. The temperature of the external stainless steel tube was monitored by means of 3 K-type thermocouples (Figure 3). During the experience large values of the temperature difference between the wire and the external stainless steel wall of the capsule (ΔT) were imposed ranging between 50 K to 125 K.

Figure 3. 3 K-type thermocouples attached on the inner surface of the stainless steel external capsule.

In Table 1 the typical uncertainty of the sensors used in these tests is quoted.

|                | Range       | Accuracy               |
|----------------|-------------|------------------------|
| K-type thermocouples | 0-200°C     | ±0.1 K                 |
| Pressure gauge   | 10^{-4}-10^{-3} mbar | ±10% R (10^{-4}-10^{-3} mbar) |
|                 |             | ±5% R (10^{-3}-10^{-2} mbar) |
| Electrical current | 0-17 A     | ± 0.36% R + 15 mA       |

Before making measurements the system was evacuated, backed out and the main filament was flashed. After that, the tests were conducted by following these steps:

1. A value of pressure is fixed inside the cylindrical shell, starting from atmospheric value.

2. By means of the DC programmable supply (HP6032A) a value of electrical current is imposed through the platinum wire in order to maintain the average temperature of the wire equal to the set-point ($T_{sp}$). A LabView code was used in order to tune the supply by using the temperature measurements on the wire. The LabView system is set in order to verify that the measured average value of the wire temperature (calculated as average value of the 4 values of temperature measured by the K-type thermocouples ($T_{w,i}$) glued on the wire surface (Fig. 2)), is maintained by the DC programmable supply within 2% from the imposed set-point ($T_{sp}$):

$$\frac{\delta T}{T_{sp}} = \frac{\sum_{i=1}^{4} T_{w,i} - 4T_{sp}}{4T_{sp}} \leq 0.02$$

3. The value of the electrical power dissipated through the platinum wire for Joule heating is calculated as product between the square value of the imposed electrical current (range: 0.2-0.7 A) and the wire electrical resistance ($R$). In steady-state conditions the power $Q$ is equal to the heat transferred from the wire to the surround by radiation and conduction:
\[ Q = I^2 R = g(p_{gas})Q_{\text{cond}} + Q_{\text{rad}} \]  
where \( g(p_{gas}) \) is a correction factor of the heat conduction term depending on the rarefaction level of the air within the vacuum chamber.

The radiative term is independent on the gas pressure within the vacuum chamber and it is evaluated by considering the formula for two concentric cylindrical surfaces [8], simplified for the limiting case where the surface of the internal body (outer diameter of the platinum wire is 0.15 mm) is much smaller than the external, concave surface (inner diameter of the vacuum chamber is 100 mm):

\[ Q_{\text{rad}} = \sigma_0 A(T_{sp}^4 - T_{ext}^4) \]  
where \( T_{sp} \) is the imposed wire temperature, \( T_{ext} \) is the temperature of the inner surface of the stainless steel capsule (obtained as average value of the temperature measured by the 3 K-type thermocouples of Fig.3b) and \( A \) the radiative total surface of the heated section calculated as follows:

\[ A = \varepsilon_w \pi d L + \varepsilon_c \left( \pi d_1 L_1 + \pi d_2 L_2 \right) + \varepsilon_c \frac{\pi}{4} \left( d_1^2 + d_2^2 - 2d^2 \right) \]  
\( \varepsilon_w \) is the emissivity of the platinum wire (\( \varepsilon_w = 0.05 \)) and \( \varepsilon_c \) is the emissivity of the oxidized copper ends (\( \varepsilon_c = 0.53 \)). The values of the emissivity of the platinum wire and of the copper ends have been checked before to start the experimental tests by comparing the surface temperature of the components measured by an infrared camera (AVIO TVS 200EX) and by means of a thermocouple glued on the surface: the value of the emissivity has been tuned in order to obtain the same value of temperature indicated by the thermocouple. The emissivity values were checked before each experimental test. The accuracy linked to the estimation of the emissivity obtained with this method is not very high and can be estimated of the order of \( \pm 10\% \); however, the estimated emissivity of platinum and copper are in agreement with the literature values [9].

The heat loss due to conduction through the air is written in Eq.(5) as a product between the classical conduction term \( Q_{\text{cond}} \) and a correction factor \( g(p_{gas}) \), depending on the rarefaction level of the air, which reduces the conduction heat transfer when the gas pressure decreases:

\[ g(p_{gas})Q_{\text{cond}} = g(p_{gas}) \lambda_0(T_m) \frac{\Delta T}{\ln(D/d)} \]  
where \( \Delta T = T_{sp} - T_{ext} \) and \( \lambda_0 \) is the air thermal conductivity at atmospheric conditions and at the average temperature between the wire and the capsule \( (T_m = (T_{ext} + T_{sp})/2) \), calculated as a function of temperature by means of the following correlation \( (T_m \text{ in K}) \):

\[ \lambda_0(T_m) = -0.00039333 + 1.0184 \cdot 10^{-4} T_m - 4.8574 \cdot 10^{-8} T_m^2 + 1.5207 \cdot 10^{-11} T_m^3 \left[ \frac{W}{mK} \right] \]  
Eq.(6) gives values of the air thermal conductivity in the range 0-200°C with a maximum overestimation of the values quoted in Raznjevic [10] equal to +4.2%.

Since the radiative term is independent on the gas pressure for a fixed wire temperature and capsule inner temperature, the weight of thermal radiation is not negligible especially at low gas pressure where the conduction heat transfer is strongly reduced by rarefaction.

From Eq.(2) it is evident that, for a fixed wire temperature \( (T_{sp}) \) and capsule temperature \( (T_{ext}) \), \( Q \) is a function of the air pressure within the capsule.

4. By means of the valve between the turbo-molecular pump and the stainless steel capsule the gas pressure was progressively reduced and the value of \( Q \), evaluated by using Eq.(1), was recorded as a function of the air pressure in the chamber.

5. All the operative procedure is repeated by changing the set-point temperature of the wire.
3. Discussion of the results

In Figure 4 the temperature distribution along the platinum wire is shown for a fixed value of the gas pressure equal to 0.1 mbar (Figure 4a) as a function of the temperature difference imposed between the platinum wire and the stainless steel external capsule, and as a function of the gas pressure for a fixed value of the temperature difference equal to 100 K (Figure 4b).

![Figure 4](image_url)

**Figure 4.** Axial distribution of the temperature along the wire: (a) for a fixed value of the gas pressure \(p=0.1\ \text{mbar}\); (b) as a function of the gas pressure for \(\Delta T=100\ \text{K}\).

It is evident that the distribution of temperature along the wire is not uniform at all, with a large temperature difference which can reach values of the order of 100 K between the first and the last thermocouple. In each test the temperature distribution along the wire was not uniform along the wire. This temperature difference increases if the gas pressure is reduced. As explained by means of Eq. (1), the wire temperature is calculated for each experimental test by calculating the mean value indicated by the 4 K-type thermocouples glued on the platinum wire. This value is found to be very close to the value measured by the third thermocouple \((z=0.046\ \text{m})\) placed close to the middle of the platinum wire.

The raw data obtained during these tests are shown in Figure 5. It is possible to distinguish 4 series of data obtained by imposing a temperature difference between the wire and the inner surface of the vacuum chamber equal to 50 K, 75 K, 100 K and 125 K.

![Figure 5](image_url)

**Figure 5.** Heat power \(Q\) as a function of the gas pressure within the capsule for different values of the imposed \(\Delta T\) (a); \(Q\) compared with the radiative and the conductive terms for an imposed \(\Delta T=100\ \text{K}\) (b).
By observing Figure 4a it is evident that $Q$ tends to saturate both for low and large values of gas pressure. When the air pressure is larger than 1 mbar the heat power $Q$ transferred from the heated wire to the surround becomes independent by the gas pressure; the heat is mainly transferred by conduction ($Q\cong Q_{\text{cond}}$). On the contrary, when the level of the gas rarefaction increases, the conduction becomes less efficient as heat transfer mode and it tends to become negligible; for this reason, the heat transfer for gas pressure less than 0.01 mbar is mainly due to the radiative component ($Q\cong Q_{\text{rad}}$) which is independent by the gas pressure (see Figure 4b).

Figure 4b highlights that, even in the case of a large temperature difference between the wire and the inner surface of the capsule, the effects due to natural convection were very low within the chamber; in fact, the heat power $Q$ is close to $Q_{\text{cond}}$ when the gas pressure increases. In the case of Figure 4b the difference between $Q$ and $Q_{\text{cond}}$ is small and entirely due to the presence of the radiative heat transfer component. The same observation can be repeated for $\Delta T=50$ K, 75 K, and 125 K. It is possible to conclude that the effects due to natural convection within the capsule in the range of temperature difference tested in this experimental campaign are very limited.

In Figure 5 is shown the trend of the conductive component of the heat power $Q-Q_{\text{rad}}$, defined as difference between the total heat power and the radiative heat transfer component (which is independent by the gas pressure) as a function of the gas pressure. As comparison, the value of the pure conductive heat power ($Q_{\text{cond}}$) is shown together with the experimental data.

In Figure 5a the experimental data determined for an imposed temperature difference between the platinum wire and the external cylindrical shell equal to 50 K. In Figure 5b,c,d are shown the experimental data obtained by increasing the temperature difference between the wire and the external shell to 75 K, 100 K and 125 K respectively.

By observing Figure 5a,b,c,d it is possible to note how the conductive heat transfer is a strong function of the gas pressure as expected. In particular, the effect of the rarefaction on the conductive heat transfer between the platinum wire and the external shell is well evident in the range of pressure lower than 2 mbar. This experience confirms that it is easy to realize a pressure sensor based on the Pirani effect.

It is also shown that the agreement between the prediction of $Q_{\text{cond}}$ (Eq.5) and the experimental data becomes good when the gas pressure is larger than 5 mbar for all the values of $\Delta T$ tested. This fact seems to underline that the convection is not significant in the annular gap also by increasing the temperature difference between the platinum wire and the external shell. In fact, in the presence of natural convection within the annular gap a strong increase of the total heat power is expected by increasing the gas pressure; the experimental data seems to indicate the absence of natural convection under the operative conditions tested in this work.

In Figure 5a,b,c,d the error bars of each experimental data are shown. The uncertainty on the measured pressure is linked to the characteristics of the pressure sensor used in these tests (given in Table 1). More complicated is the evaluation of the total uncertainty on the conductive thermal power $Q-Q_{\text{cond}}$.

It is evident that the uncertainty tends to increase dramatically when the gas pressure decreases; this is due especially to the increase of the weight of the radiative component on the total heat power exchanged between the wire and the surround when the gas pressure is reduced and also to the low values of electric current measured. In particular, the estimation of the emissivity with an uncertainty of the order of 10% is too large to obtain accurate experimental results in terms of thermal power when the gas pressure is lower than 0.05 mbar. The analytical calculation of the total uncertainty on Q highlights that the radiative heat transfer is strongly linked in this experience to the presence of the copper ends attached to the platinum wire. In particular, since the copper emissivity is larger than the emissivity of platinum the presence of these ends increases the radiative heat transfer during this experience. Moreover, the axial distribution of the wire temperature shown in Figure 4 highlights that the contribution of the two copper ends is different due to the strong variation of the temperature along the wire axis. This fact underlines that, if the goal is to reduce the uncertainty of the experimental data for low gas pressure (where the radiative component of heat transfer becomes predominant), in the
future the test rig must be improved in order to limit the weight of the radiative component on the total thermal power; this can be done by removing the copper ends and by changing the method of heating the platinum wire.

As a general observation, one can note that all the choices made during the realisation of this experience were finalized to simplify the realization of the tests; this simplification is a great disadvantage if the goal is to try to build a theoretical model of this experience. In fact, in order to compare the experimental results presented in this paper with the theoretical results it is important to take into account that this experience has been conducted by using air, a mixture of poli-atomic gases, and not a pure gas. In the modelling of this experience must be taken into account that the vacuum pump can have a selective behaviour with respect to the different gases contained in the air; it means that in this case, when the total air pressure is reduced by means of the vacuum pump, the concentration of the single gas varies determining a variation of the composition of the air in the annular gap. In addition, since in this experimental work the humidity of air was not checked and measured, also the role of the water content must be taken into account in order to simulate the situation investigated experimentally in this paper. It is also important to bear in mind that during the experience the temperature difference between the wire and the external shell varies: the analysis of the experimental data has evidenced that this temperature difference had a variation which was for a large part of data within 3% during the tests but, in some case a maximum variation equal to ±5% is reached. This variation is due to the choice to leave free the temperature of the external shell which is exposed to the variation of the room thermal conditions. The consequence of this choice has a minor impact on the accuracy of the results.
All these observations, put in evidence how, at the present stage, the test rig needs to be improved in order to obtain more accurate experimental results. For this reason, further experimental tests have been planned in the short future in order to improve the test rig and generate new and more accurate experimental data under operative conditions which can be easily modelled.

5. Conclusions
In this paper an experimental and numerical analysis of the heat conduction between a heated platinum wire having a diameter ($d$) of 0.15 mm, disposed along the axis of a cylindrical shell in stainless steel having an inner diameter ($D$) of 100 mm filled with air is described. Temperature differences between the wire and the external stainless steel wall in the range 50-125 K were imposed and the heat power transferred from the wire to the surround was measured as a function of the gas pressure starting from air at atmospheric conditions down to $10^{-3}$ mbar. The analysis of the experimental results evidenced that many things can be done in order to improve the accuracy of the results in terms of total heat power. It has been demonstrated that, at the present stage, the radiative component plays a predominant role when the gas pressure is reduced below 0.05 mbar and this fact contributes to the increase of the value of the uncertainty on the total thermal power exchanged by the platinum wire with the surround up to 50%. These results suggest the need of a modification of the experimental test rig in order to obtain a reduction of the radiative component of the heat transferred from the wire to the surround and a modification of the heating method of the wire in order to obtain an axial distribution of temperature along the wire more uniform.

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