Adaptive Output Feedback Tracking of Nonlinear Time-Delay Systems Subject to Nonsymmetric Dead-Zone Input

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ABSTRACT In this paper, an output feedback control scheme was proposed to achieve global adaptive output tracking by prescribing arbitrarily small error for a class of dead-zone nonlinear systems including time-delay and uncertain nonlinear functions with unknown output-polynomial growth rate. Two dynamic gains based respectively on identification and non-identification mechanism were introduced in the observer and controller. Picking the appropriate Lyapunov-Krasovskii functional, the constructive proof was given by combining high-gain scaling technique and one-step backstepping method. Finally, two simulation examples showed the effectiveness of the control scheme.

INDEX TERMS Adaptive tracking, dead-zone input, output feedback, time-delay systems.

I. INTRODUCTION
In the past decades, the problem of output tracking has received considerable attention. Due to existing of modeling errors or external disturbance, it is often impossible to achieve asymptotical tracking for a prescribed reference signal. Thus, practical tracking (i.e., λ-tracking), as a smaller ambition, is pursued in general, which requires that the tracking error converges to a specified accuracy. In the early stages, [1] and [2] considered the universal λ-tracking problem. For a class of highly nonlinear systems, [3] and [4] achieved global practical output tracking by using an adding a power integrator method proposed in [5]. In the case of nonlinear systems with unknown control direction, [6] presented a Nussbaum-gain controller to achieve adaptive practical tracking. In [7], a backstepping tracking control strategy was developed for nonlinear systems with unknown delay in disturbance nonlinear functions.

In [8]–[14] and references therein, output feedback control problems have been extensively investigated. In particular, with the help of homogeneous domination method proposed in [15], [16], the practical tracking problem was solved successfully in [8]. Based on universal control idea, [9] and [17] solved the adaptive practical tracking problem by output feedback. Reference [10] considered the output-polynomial function growth rate. In the presence of unknown control coefficients in nonlinear systems, [11] achieved output feedback practical tracking by using backstepping method. Recently, [13] focused on more general control coefficients allowing the upper/lower bounds to be unknown, and obtained an output feedback controller by integrating dead-zone idea and backstepping method. Simultaneously, in [14], the output feedback practical tracking was handled for nonlinear systems with unknown constant and output-polynomial growth rate.

When there exists dead-zone nonlinearity in actuator, several control problems have been considered in [18]–[24]. For example, the dead-zone inversion based method was initiated by [21], [22], and further developed to achieve asymptotical cancellation of unknown dead-zone by assuming the dead-zone output to be available in [18]. Later, smooth inverse functions of the actuator dead-zone were constructed in [19], [20]. By modeling actuator dead-zone as a linear-like controller plus bounded disturbance, an alternative controller design method, robust adaptive control method, has also been studied. For instance, [25] considered symmetric dead-zone input. Subsequently, the non-symmetric dead-zone nonlinearity in actuator was investigated in [26], [27]. Also, simple adaptive control schemes were proposed for unknown non-symmetric dead-zone input in [28], [29]. In [30]–[32], robust adaptive control schemes were generalized to several classes of nonlinear time-delay systems with dead-zone input.

As is well-known, it is difficult to measure all states of practical systems in general. Hence, the output feedback control becomes one of the most effective methods such as [33], [34]. Naturally, it is an interesting subject to develop...
observer-based controller design methods for nonlinear systems with dead-zone input. This paper would consider practical tracking problem by output feedback for more general time-delay nonlinear systems with nonsymmetric dead-zone nonlinearity. The main difficulties lie in two-folds:

- In the presence of nonsymmetric dead-zone nonlinearity with parameters being unknown, the existing output feedback control schemes are not applicable to system (1), because nonlinear functions grow nonlinearly in unmeasurable states with the output-polynomial growth rate including unknown constant and time-delay.

- To construct output feedback controller, [19] proposed different control schemes with large computational complexity. In particular, the backstepping method has the problem of “explosion of computing complexity”.

To this end, an attempt would be made to propose a controller combined design method. The main innovations are summarized as follows.

[i] An extended high-gain observer and controller were proposed, where two adaptive gains were introduced based on non-identification and identification, respectively. Simultaneously, this paper used the high-gain scaling approach (see [35], [36]) together with one-step backstepping method in the process of designing the output controller to avoid the “explosion of computing complexity”.

[ii] Compared with the existing results such as [19], [25]–[27], this paper considered more general nonlinear time-delay systems with uncertain growth rate dependent on unknown constant and polynomial function of output. The dead-zone parameters could be time-varying in some unknown compact set. Moreover, our control scheme has a good robustness to time delay and unknown bounded disturbance.

II. PROBLEM STATEMENT

Consider nonlinear time-delay system

\[
\dot{x}_i = x_{i+1} + f_i(d, x, x(t - \tau)), \quad i = 1, \ldots, n - 1, \\
y = x_1, \quad u = N(v), \tag{1}
\]

where \(x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n\) and \(y \in \mathbb{R}\) are the states and the measured output, respectively. \(x(t - \tau) = [x_1(t - \tau), \ldots, x_n(t - \tau)]^T \in \mathbb{R}^n\) denotes time-delay states with unknown constant time delay \(\tau > 0\). Vector \(d\) denotes unknown bounded parameter and is contained in uncertain functions \(f_i, \ i = 1, \ldots, n\). \(u = N(v)\) represents an actuator dead-zone of the form

\[
N(v) = \begin{cases} 
    m_r(v - b_r), & v \geq b_r \\
    0, & -b_l < v < b_r \\
    m_l(v + b_l), & v \leq -b_l 
\end{cases} \tag{2}
\]

where parameters \(m_r, m_l\) and \(b_r, b_l\) represent respectively the dead-zone slope and the dead-zone breakpoint, as shown in [21], [22].

For system (1), our goal is to design an adaptive output feedback controller such that the closed-loop system output tracks some reference signal under any pre-given error \(\lambda > 0\).

In what follows, several key lemmas are listed, which have been extensively used in [35]–[39].

Lemma 1 [37]: For \(x \in \mathbb{R}, y \in \mathbb{R}, p \geq 1\) is a constant, the following inequality hold:

\[
|x + y|^p \leq 2^{p-1}|x|^p + y^p. \tag{3}
\]

Lemma 2 [37]: For any positive integers \(m, n\) and any real-valued function \(\alpha(x, y) > 0\), then

\[
|x^m y^n| \leq \frac{m}{m + n} \alpha(x) |x|^{m+n} + \frac{n}{m + n} \alpha^{-\frac{m}{n}}(y) |y|^{m+n}. \tag{4}
\]

Lemma 3 [35]: If matrices \(A\) and \(B\) are Hurwitz, \(D = \text{diag}(\sigma, \sigma + 1, \ldots, \sigma + n - 1), D' = \text{diag}(\sigma, \sigma + 1, \ldots, \sigma + n - 2)\), then there exist \(P = P^T > 0, Q = Q^T > 0\) satisfying

\[
\begin{cases}
    A^T P + PA \leq -I_n, \quad DP + PD \geq I_n \\
    B^T Q + QB \leq -2I_{n-1}, \quad D'Q + QD' \geq I_{n-1}.
\end{cases}
\]

where identity matrix of dimension \(n \times n\) is represented as \(I_n\), \(\sigma > 0\) being any constant, and the matrices \(A\) and \(B\) are described as

\[
A = \begin{pmatrix} -h_1 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
-h_{n-1} & 0 & \cdots & 1 \\
-h_n & 0 & \cdots & 0
\end{pmatrix} \in \mathbb{R}^{n \times n},
\]

\[
B = \begin{pmatrix} 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 \\
-k_1 & -k_2 & \cdots & -k_{n-1}
\end{pmatrix} \in \mathbb{R}^{(n-1) \times (n-1)},
\]

where \(h_i > 0\) and \(k_j > 0\) are design parameters.

III. MAIN RESULTS

In order to achieve practical tracking, the following assumptions are made for system (1).

Assumption 1 [18]: \(u\) is available.

Assumption 2 [39]: For constants \(b > a > 0, m_r, m_l, b_r\) and \(b_l\) belong to the compact set \([a, b]\).

Assumption 3 [9]: \(y_r\) and \(\dot{y}_r\) are continuous and bounded.

Assumption 4: There exists a known constant \(p \geq 1\) and two unknown positive constant \(\theta_1\) and \(\theta_2\), for \(i = 1, \ldots, n\), such that

\[
|f_i| \leq \theta_1(1 + |x_i|^p) \sum_{j=1}^{i} (|x_j| + |x_j(t - \tau)|) + \theta_2. \tag{5}
\]

Theorem 1: For uncertain system (1) satisfying Assumptions 1–4, the problem of global practical tracking can be solved by a dynamic output feedback controller of the form

\[
\dot{x}_i = \dot{x}_{i+1} + L^i (y - y_r - \dot{x}_i), \quad i = 1, \ldots, n - 1, \\
\dot{x}_n = N(v) + L^n h_n (y - y_r - \dot{x}_i), \\
v = -(L^{a+n} \dot{\hat{h}} + 1)(K \hat{\dot{z}} + \hat{z}_n) \tag{6}
\]
with dynamic gains $L$ and $\hat{\theta}$ being updated by

$$\begin{align*}
\dot{L} &= \max \left\{ \beta(L, x_1), \frac{(y - y_r - \hat{x}_1)^2 + \hat{x}_1^2 - \omega^2}{L^{2\sigma}} \right\}, \\
\dot{\hat{\theta}} &= L^2(\hat{K}_0^2 + \hat{z}_\infty^2) - \hat{\theta}, \\
(\hat{L}(0) = 1, \quad \hat{\theta}(0) = 0),
\end{align*}$$

(7)

where $\hat{z} = [\hat{z}_1, \ldots, \hat{z}_{n-1}]^T$ and $\hat{z}_i = \frac{\hat{y}_i}{L^{\sigma+i-1}}$ for $i = 1, \ldots, n$.

The design parameters $K = [k_1, \ldots, k_{n-1}]$, $h^i$s and $\sigma$ are chosen to satisfy Lemma 3. $\beta(L, x_1) = 2L(1 + |x_1|^2) - L^2$; the design parameters $i$ and $\sigma$ satisfy $0 < i < 0.5$ and $0 < 2\sigma < 1$, respectively.

**Remark 1:** In the above controller, we adopt two types of dynamic gains based on universal adaptive control idea in [17]. Different from [9], [10], [33], one introduces an additional parameter estimation $\hat{\theta}$ in this controller to handle the effect of the unknown dead-zone nonlinearity in actuator.

**Proof:** Without loss of generality, one assumes that $[0, t_f]$ is maximally extended interval of the solution of the closed-loop system consisting of (1) and (6)-(7).

To simplify the controller design, we redescribe the dead-zone model (2) as

$$u = \rho(v)v + \sigma(v),$$

(8)

where

$$\begin{align*}
\rho(v) &= \begin{cases} 
mr, & \text{if } v \geq b_r, \\
\frac{mv + ml}{2}, & \text{if } -b_l < v < b_r, \\
lm, & \text{if } v \leq -b_l,
\end{cases} \\
\sigma(v) &= \begin{cases} -m_mr, & \text{if } v \geq b_r, \\
\frac{-mv - ml}{2}, & \text{if } -b_l < v < b_r, \\
ml, & \text{if } v \leq -b_l,
\end{cases}
\end{align*}$$

(9)

Together (9) with Assumption 2, we can easily know that $a \leq \rho(v) \leq b$ and $\sigma(v)$ is bounded.

According to the (7), one has

$$L^{2\sigma} \dot{L} \geq (y - y_r - \hat{x}_1)^2 + \hat{x}_1^2 - \omega^2, \\
\dot{L} \geq \beta(L, x_1), L \geq 1, \dot{L} \geq 0.$$ 

(10)

Next, we introduce scaling change of the form

$$\begin{align*}
\varepsilon_1 &= \frac{x_1 - y_r - \hat{x}_1}{L^{\sigma}}, \\
\varepsilon_i &= \frac{x_i - \hat{x}_i}{L^{\sigma+i-1}}, \quad i = 2, \ldots, n, \\
\hat{z}_i &= \frac{\hat{x}_i}{L^{\sigma+i-1}}, \quad i = 1, \ldots, n,
\end{align*}$$

(11)

and get

$$\begin{align*}
\dot{\varepsilon}_1 &= L\varepsilon_1 - \frac{\dot{L}}{L}D\varepsilon_1 + \Phi, \\
\dot{\varepsilon}_i &= LB\varepsilon_i - \frac{\dot{L}}{L}D\varepsilon_i + LH\varepsilon_1 + LC(K_{0}^2 + \hat{z}_\infty), \\
\dot{\hat{z}}_n &= \frac{N(\varepsilon)}{L^{\sigma+n-1}} - \frac{\dot{L}}{L}(\sigma + n - 1)\hat{z}_n + LH_n\varepsilon_1, \\
\end{align*}$$

(12)

where all matrices have been given in lemma 3, $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_n]^T$, $C = [0, \ldots, 0]^T \in \mathbb{R}^{(n-1) \times 1}$, $H = [h_1, \ldots, h_{n-1}]^T$, $\Phi = [\phi_1, \ldots, \phi_n]^T$ with

$$\begin{align*}
\phi_1 &= \frac{1}{L^{\sigma}f_1 - \frac{1}{L^{\sigma}}}y_r, \\
\phi_i &= \frac{1}{L^{\sigma+i-1}l_i}, \quad i = 2, \ldots, n.
\end{align*}$$

(13)

Next, choose the Lyapunov-like function

$$V = \kappa_1 V_1 + V_2,$$

(14)

for nonlinear system (1), where $\kappa_1 = 2\|QH\|^2 + 2, V_1 = \varepsilon^T P\varepsilon, V_2 = \hat{z}_n^T \hat{z}_\infty$, and $P, Q$ are given in Lemma 3.

Choosing $\xi = \hat{K}_0^2 + \hat{z}_\infty$, the derivative of $V$ on $[0, t_f]$ is

$$\dot{V} \leq -\kappa_1 L \|\varepsilon\|^2 - \kappa_1 \frac{\dot{L}}{L} \|\varepsilon\|^2 + 2\kappa_1 \varepsilon^T P\Phi - 2L \|\hat{z}\|^2
\begin{align*}
&= -\frac{\dot{L}}{L} \|\varepsilon\|^2 + 2L \varepsilon^T QH\varepsilon + 2L \varepsilon^T QC\xi.
\end{align*}$$

(15)

Constant $\theta_3 > 0$ is introduced to simplify the estimation of equation (15), which is sufficiently large. Using Lemmas 1-2 and Assumptions 3-4, it follows from (13) and $L \geq 1$ that

$$\begin{align*}
2\kappa_1 \varepsilon^T P\Phi &\leq (1 + |x_1|^2)\|\varepsilon\|^2 + L \|\varepsilon\|^2 + \theta_3(\|\varepsilon\|^2 + \|\hat{z}\|^2)
\begin{align*}
&+ \|\varepsilon\|^2 + \|\hat{z}\|^2 + \theta_3(\|\varepsilon\|^2 + \|\hat{z}\|^2)
\end{align*}$$

(16)

$$\begin{align*}
2L \varepsilon^T QH\varepsilon &\leq \frac{1}{2} L \|\varepsilon\|^2 + 2L \|QH\|^2 \xi_1^2, \\
2L \varepsilon^T QC\xi &\leq \frac{1}{2} L \|\varepsilon\|^2 + 2L \|QC\|^2 \xi_2^2.
\end{align*}$$

(17)

Substituting (16-18) into (15) yields

$$\dot{V} \leq -[L - \theta_3 - (1 + |x_1|^2)](\|\varepsilon\|^2 + \|\hat{z}\|^2)
\begin{align*}
&+ \frac{\dot{L}}{L} \|\varepsilon\|^2 + \|\hat{z}\|^2 + \theta_3(\|\varepsilon\|^2 + \|\hat{z}\|^2)
\end{align*}$$

(19)

Now, select a Lyapunov-Krasovskii functional

$$\dot{V} = V + \theta_3 \int_{t-\tau}^{t} \left( (\|\varepsilon(s)\|^2 + \|\hat{z}(s)\|^2 + \xi_2(s))ds \right).$$

(20)

According to $\beta(L, x_1) = 2L(1 + |x_1|^2) - L^2 \leq \dot{L}$, we get

$$\dot{V} \leq \frac{L}{2L}(\|\varepsilon\|^2 + \|\hat{z}\|^2) + 2\theta_3 \xi
\begin{align*}
&- \frac{\dot{L}}{2L} \|\varepsilon\|^2 + \|\hat{z}\|^2 + \theta_3 + 2L \|QC\|^2 \xi_2^2.
\end{align*}$$

(21)

Observing $\xi = \hat{K}_0^2 + \hat{z}_\infty$, one has

$$\dot{\xi} = K\xi^2 - \frac{\dot{L}}{L} \varepsilon^T D\varepsilon + LH\varepsilon_1 + LC\hat{z}_\infty$$

(22)
where

\[ A' = B + CK = \begin{bmatrix} 0 & I_{n-2} \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{(n-1) \times (n-1)}, \]

which leads to

\[
\frac{1}{a} \dot{\xi} = L_1 \frac{KA' \xi}{a} + \frac{\dot{\xi}}{L} + \frac{L_1 \xi}{a} + L_2 \xi \rho + \bar{\xi} \epsilon_1
\]

\[
+ \frac{\dot{\xi}}{L} \sigma + n - 1 \frac{\dot{x} \xi}{a} + \frac{\dot{\xi}}{a} \xi + L_1 \frac{L h_n}{a} \xi \epsilon_1
\]

\[
\leq \theta_4 L(\xi^2 + \|\hat{\xi}\|^2) + \theta_a \frac{\dot{\xi}}{L} \|\hat{\xi}\|^2 + \frac{\rho}{\sigma + n - 1} \|\hat{\xi}\|^2
\]

\[
+ \frac{\dot{L} \sigma + n - 1}{a} \xi^2 + \theta_4 L_2 \xi^2 + \frac{\theta_6}{L_2^{2\sigma + 1} + 1}
\]

(23)

where \( \theta_4 > 0, \theta_5 > 0, \theta_6 > 0 \) are unknown suitable constants.

Choosing

\[
\tilde{V} = \tilde{\kappa} \tilde{V} + \frac{1}{2a} \xi^2,
\]

(24)

with \( \tilde{\kappa} = 2(\theta_4 + 1) \), we get

\[
\dot{\tilde{V}} \leq -\frac{\tilde{\kappa} L}{2} (\|x\|^2 + \|\tilde{\xi}\|^2) - \frac{\tilde{\kappa} L}{2} (\|\tilde{x}\|^2 + \|\tilde{\xi}\|^2)
\]

(25)

where \( \bar{\theta} = \frac{\kappa}{2} \|x\|^2 + \theta_3 \) and \( \theta_3 = \kappa \theta_2 + \theta_6 \).

Here, we introduce a dynamic \( \bar{\theta} \) as an estimate of \( \theta \) and the error \( \bar{\theta} = \theta - \bar{\theta} \).

Letting

\[
W = \tilde{V} + \frac{1}{2L} \dot{\bar{\theta}}^2,
\]

(26)

one has

\[
\dot{W} \leq -(L - 2\tilde{\kappa} \theta_3)(\|x\|^2 + \|\tilde{\xi}\|^2) + \frac{\rho}{\sigma + n - 1} \|\hat{\xi}\|^2
\]

\[
+ \frac{1}{L} (\dot{\bar{\theta}} L_2 \xi^2 + \dot{\bar{\theta}} L_2 \xi^2 + \theta_4 \xi^2)
\]

(27)

By Lemma 2, substituting \( L \geq 1, \dot{\bar{\theta}} \geq 0, \rho / a > 1 \) and (7) into (27) yields

\[
\dot{W} \leq -(L - 2\tilde{\kappa} \theta_3)(\|x\|^2 + \|\tilde{\xi}\|^2) - \xi^2
\]

\[
- \frac{1}{2L} \bar{\theta}^2 + \frac{1}{L} \dot{\bar{\theta}} (\theta_4 + \frac{1}{2} \theta_2^2)
\]

(28)

where we use

\[
\frac{1}{a} \dot{\bar{\theta}} \leq -\frac{1}{2L} \bar{\theta}^2 + \frac{1}{2L} \bar{\theta}^2.
\]

(29)

Summarizing the above proof process, the following lemma can be deduced and proved in appendix A.

**Lemma 4**: \( L(t), \hat{z}(t) = [\hat{z}_1, \ldots, \hat{z}_n]^T \) and \( \epsilon(t) \) are bounded on the maximal interval of existence of solution \([0, t_f]\).

Noting that \( L^2(\hat{z}_2^2 + \hat{z}_n^2) < +\infty \) and \( \dot{\hat{z}} = L^2(\hat{z}_2^2 + \hat{z}_n^2)^2 \) in (7), we can get \( \hat{\theta} \) is bounded on \([0, t_f]\). In addition, we can also get \( t_f = +\infty \), and \( L(t) \) is bounded on \([0, +\infty)\) and \( \int_0^{+\infty} L(s) ds < +\infty \).

From scaling change (11), one gets the boundedness of \((L, \hat{\theta}, x, \hat{x}, y, \hat{y})\), and it is easy to figure out that \( \hat{L}(t) \) is uniformly continuous on \([0, +\infty)\). Using Barbálat’s lemma, one has

\[
\lim_{t \to +\infty} \hat{L}(t) = 0.
\]

(30)

Further, by \( y - y_r = (y - y_r - \hat{x}_1) + \hat{x}_1 \), Lemma 1 and (10), we have

\[
(y - y_r)^2 \leq 2(y - y_r - \hat{x}_1)^2 + 2\hat{x}_1^2 \leq 2L^2 \theta^2 + 2\lambda \theta^2,
\]

(31)

which, together with the boundedness of \( L \), implies that the tracking error \( y - y_r \) will go into a set \((-\lambda, \lambda)\) and stay there forever by picking an arbitrarily small \( \lambda > 0 \).

**IV. EXAMPLES**

In this section, two examples are given to demonstrate the effectiveness of our control scheme.

**Example 1**: Consider the following system:

\[
\begin{align*}
\dot{x}_1 &= x_2 + c_1(x_1(x_1 - x_2(t - \tau)) + \omega_1(t)) \\
\dot{x}_2 &= \dot{u} + c_2(x_1(t - \tau) + x_2(t - \tau)) + \omega_2(t) \\
y &= x_1, \quad u = N(v)
\end{align*}
\]

(32)

where \( c_1, c_2 > 0, \omega_1(t) \) and \( \omega_2(t) \) are unknown bounded disturbance.

In fact, by choosing \( p = 2 \), we can get

\[
\begin{align*}
c_1(x_1(t - \tau)) &\leq c_1(x_1 + x_2)^2 \left( |x_1| + |x_1| \right)
\end{align*}
\]

(33)

where \( c_1, c_2 > 0, \omega_1(t) \) and \( \omega_2(t) \) are unknown bounded disturbance.

Choosing \( y_r = \cos(t) \), it is obvious that all assumptions are satisfied. Selecting \( \tau = 0.25, \sigma = 0.25, \lambda = 0.6 \), we can get the controller

\[
\begin{align*}
\dot{x}_1 &= \dot{x}_2 + L_1 (y - y_r - \hat{x}_1)
\end{align*}
\]

(34)

(35)

By Lemma 2, substituting \( L > 1, \dot{\bar{\theta}} \geq 0, \rho / a > 1 \) and (7) into (27) yields

\[
\dot{W} \leq -(L - 2\tilde{\kappa} \theta_3)(\|x\|^2 + \|\tilde{\xi}\|^2) - \xi^2
\]

\[
- \frac{1}{2L} \bar{\theta}^2 + \frac{1}{L} \dot{\bar{\theta}} (\theta_4 + \frac{1}{2} \theta_2^2)
\]

(28)

where we use

\[
\frac{1}{a} \dot{\bar{\theta}} \leq -\frac{1}{2L} \bar{\theta}^2 + \frac{1}{2L} \bar{\theta}^2.
\]

(29)
The dead-zone parameters are chosen as

\[ u = \begin{cases} 
1.5(v - 0.3), & v \geq 0.3 \\
0, & -0.5 < v < 0.3 \\
2(v + 0.5), & v \leq -0.5 
\end{cases} \]

Figures 1-4 are the simulation results by picking \( k = c_1 = c_2 = 2, \omega_1 = 0.5 \sin(t), \tau = 0.3, \omega_2 = h_1 = h_2 = 1 \) and \( [x_1(0), x_2(0), \dot{x}_1(0), \dot{x}_2(0), L(0), \dot{\theta}(0)] = [1, -2, -1, 2, 1, 0] \).

**Example 2:** The two-stage chemical reactor system with delayed recycle streams [40]:

\[
\begin{align*}
\dot{x}_1 &= -\frac{1}{\theta_1}x_1 - k_1x_1 + \frac{1 - R_2}{V_1}x_2 + \delta_1(d, x(t - \tau)), \\
\dot{x}_2 &= -\frac{1}{\theta_2}x_2 - k_2x_2 + \frac{R_1}{V_2}x_1(t - \tau)
\end{align*}
\]

where parameters \( \theta_1 = \theta_2 = 2, k_1 = k_2 = 0.3, R_1 = R_2 = 0.5, V_1 = V_2 = 0.5, F_2 = 0.5, \) and the nonlinear functions

\[
\begin{align*}
\delta_1(\cdot) &= c_3 \sin(t)x_1(t - \tau), \\
\delta_2(\cdot) &= c_4 \sin(t)x_2(t - \tau),
\end{align*}
\]

with \( c_3, c_4 > 0 \). Then, (37) can be rewritten as

\[
\begin{align*}
\dot{x}_1 &= x_2 + f_1(\cdot), \\
\dot{x}_2 &= u + f_2(\cdot),
\end{align*}
\]

where

\[
\begin{align*}
f_1(\cdot) &= c_3 \sin(t)x_1(t - \tau) - 0.8x_1, \\
f_2(\cdot) &= c_4 \sin(t)x_2(t - \tau) - 0.8x_2 + x_1(t - \tau).
\end{align*}
\]

It is easy to get

\[
\begin{align*}
f_1(\cdot) &\leq c_3(1 + x_1^2(x_1 + |x_1(t - \tau)|)), \\
f_2(\cdot) &\leq c_4(1 + |x_2|^2|x_2| + |x_1(t - \tau)| + |x_2(t - \tau)|).
\end{align*}
\]

Choose \( c_3 = c_4 = 1 \), and the reference signal \( y_r = \sin(t) \), other parameters are same as example 1, the initial states \( [x_1(0), x_2(0), \dot{x}_1(0), \dot{x}_2(0), L(0), \dot{\theta}(0)] = [1, -3, -1, 0, 1, 0] \).
nonsymmetric dead-zone in actuator. By improving high-gain delay systems with uncertain polynomial growth rate and Lemma by output feedback for a broad class of nonlinear time-

This paper investigated the adaptive practical tracking problem by output feedback for a broad class of nonlinear time-delay systems with uncertain polynomial growth rate and nonsymmetric dead-zone in actuator. By improving high-gain observer, the output feedback control scheme was proposed, which contained two types of new dynamic gains. Instead of constructing inverse function of dead-zone input, an additional parameter estimation \( \hat{\theta} \) was used to deal with the unknown dead-zone nonlinearity in actuator. The effectiveness of scheme proposed was showed by the last two numerical examples.

**APPENDIX A**

The proof of Lemma 4 is divided into three parts. A generic constant \( \vartheta \geq 0 \) is used to represent a finite positive constant value in the following proof, and \( \vartheta \) may be implicitly changed in various places.

**Claim 1:** \( L(t) \) is Bounded on \( [0, t_f] \).

First, we can know from the previous process of proof that \( \dot{L}(t) \geq 0 \) and \( L(t) \) is monotone non-decreasing function on \( [0, t_f] \). By supposing \( L(t) \) to be unbounded on \( [0, t_f] \), then, we have

\[
\lim_{t \to t_f} L(t) = +\infty,
\]

(A.1)

which indicates that there is a time \( t_1 \in (0, t_f) \), when \( t_1 \leq t < t_f \), such that

\[
L - 2\kappa_2 \vartheta_3 \geq 1.
\]

(A.2)

Here, defining the function \( V^* = \kappa_2 V + \frac{1}{2\sigma} \xi^2 + \frac{1}{2L} \tilde{\vartheta}^2 \), and combining with (28) leads to

\[
\dot{W} \leq - (\|\xi\|^2 + \|\tilde{z}\|^2) - \xi^2 - \frac{1}{2L} \tilde{\vartheta}^2 + \frac{\Theta}{L}
\]

\[
\leq -\min \left\{ \frac{\kappa_1 \kappa_2}{\lambda_{\text{max}}(P)} \sigma, \frac{\kappa_2}{\lambda_{\text{max}}(Q)} \frac{1}{2\sigma}, 1 \right\} V^* + \frac{\Theta}{L},
\]

(A.3)

where \( \Theta = \theta_1 + \frac{1}{2} \vartheta^2 \). It is not difficult to get from (A.3) that there exists a constant such that

\[
W \leq \vartheta, \quad \forall t \in [0, t_f),
\]

(A.4)

which implies \( \varepsilon_1 \) and \( \tilde{z}_1 \) are bounded.

Noting the fact that \( 0 < 2\sigma \sigma + 1 < 2 \), and \( x_1 = L^\sigma (\varepsilon_1 + \tilde{z}_1) + y_r \), we deduce that there exists a time \( t_2 \in [t_1, t_f) \), when \( t_2 \leq t < t_f \), such that

\[
\beta(L, x_1) = 2L(1 + |x_1|^\sigma) - L^2 < 0.
\]

(A.5)

By the definition of (7) and (A.5), we have

\[
\dot{L} = 2L \times \frac{(y - y_r - \hat{x}_1)^2 + \hat{x}_1^2 - \xi_1^2}{2L^2\sigma + 1} \leq \varepsilon_1^2 + \xi_1^2 \leq \vartheta.
\]

(A.6)

Now, define two functions

\[
\Lambda_1 = LV^*, \quad \Lambda_2 = LW,
\]

(A.7)

satisfying \( \Lambda_2 \geq \Lambda_1 \).

By (A.3)-(A.6) and (10), taking the derivative of \( \Lambda_2 \) on \([t_1, t_f)\), one yields

\[
\dot{\Lambda}_2 \leq -v\Lambda_1 + \Theta + \vartheta^2 \leq -v\Lambda_1 + \tilde{\vartheta},
\]

(A.8)

where \( \tilde{\vartheta} > 0 \) is chosen as a suitable constant.

**V. CONCLUSION**

Simulation results are given in Figures 5-8.

The dead-zone nonlinearity is

\[
u = \begin{cases} 
\nu - 0.7, & \nu \geq 0.7 \\
0, & -0.7 < \nu < 0.2 \\
\nu + 0.2, & \nu \leq -0.2
\end{cases}
\]

(A.3)

The dead-zone nonlinearity is

\[
u = \begin{cases} 
\nu - 0.7, & \nu \geq 0.7 \\
0, & -0.7 < \nu < 0.2 \\
\nu + 0.2, & \nu \leq -0.2
\end{cases}
\]

(A.4)

The response of input and output of actuator.

FIGURE 6. The response of gain \( L \) and \( \dot{\vartheta} \).

FIGURE 7. The response of input and output of actuator.

FIGURE 8. The tracking error trajectory.
From (A.8), we can conclude that $A_1$ is bounded on $[t_2, t_f]$ and

$$A_1 = LV^* \geq \min(\lambda_{\text{min}}(P), \lambda_{\text{min}}(Q))L(e_{t_f}^2 + \hat{x}_{1_f}^2)$$

$$= \min(\lambda_{\text{min}}(P), \lambda_{\text{min}}(Q))L^{1-2\sigma}((y - y_r - \hat{x}_1)^2 + \hat{x}_{1_f}^2) < +\infty,$$  \hspace{1cm} (A.9)

which and $\lim_{t \to t_f} L^{1-2\sigma} = +\infty$, leads to

$$\lim_{t \to t_f} [(y - y_r - \hat{x}_1)^2 + \hat{x}_{1_f}^2] = 0.$$

Thus there is a finite time $t_3$, when $t \in [t_3, t_f)$, such that

$$\text{which shows that } \dot{L}(t) = 0 \text{ when } t_3 \leq t < t_f. \text{ This contradicts with the hypothesis } (A.1). \text{ Therefore, the dynamic gain } L(t) \text{ is bounded on } [0, t_f).$$

**Claim 2:** $\dot{z}(t)$ is Bounded on $[0, t_f)$.

In combination with the boundedness of $L$, one chooses the following Lyapunov-like function

$$V_2^* = \kappa_2 V_2 + \frac{1}{2a} \xi^2 + \frac{1}{2L} \bar{\theta}^2.$$  \hspace{1cm} (A.12)

We can infer from above that there exist two unknown constants $\vartheta_1 > 0$ and $\vartheta_2 > 0$ such that

$$V_2^* \leq -\vartheta_1 V_2^* + \frac{\vartheta_2}{2} (e_{t_f}^2 + \hat{x}_{1_f}^2) + \vartheta_2$$

$$\leq -\vartheta_1 V_2^* + \vartheta_2 \hat{L} + \vartheta_2,$$  \hspace{1cm} (A.13)

which results in

$$\dot{V}_2^* e^{\vartheta_1 t} + \vartheta_1 V_2^* e^{\vartheta_1 t} \leq \vartheta_2 \hat{L} e^{\vartheta_1 t} + \vartheta_2 e^{\vartheta_1 t},$$  \hspace{1cm} (A.14)

and

$$\frac{d(V_2^*)}{dt} \leq \frac{d(\vartheta_2 \hat{L} e^{\vartheta_1 t})}{dt} - \vartheta_1 \vartheta_2 L(t) e^{\vartheta_1 t} + \vartheta_2 e^{\vartheta_1 t}.$$  \hspace{1cm} (A.15)

Recalling $L(0) = 1$ and integrating both the sides of (A.15), one has

$$V_2^* \leq e^{-\vartheta_1 t} (V_2^* |_{t=0} - \frac{\vartheta_2}{\vartheta_1}) + \vartheta_2 L(t) - \vartheta_2 L(0) + \frac{\vartheta_2}{\vartheta_1} \leq +\infty.$$  \hspace{1cm} (A.16)

This shows that $\dot{z}(t)$ is bounded on $[0, t_f)$.

**Claim 3:** $\dot{z}(t)$ is Bounded on $[0, t_f)$.

According to the previous rescaling transformation, one chooses a suitable constant $L^*$ satisfying $L^* > \max\{L(t_f), 2 + \vartheta + \lambda_{\text{max}}(P)\}$. Using coordinate change of the form

$$e_i = \frac{x_{i} - y_r - \hat{x}_1}{(L^*)^\sigma}, \quad i = 2, \ldots, n,$$  \hspace{1cm} (A.17)

we get

$$\dot{e} = L^* A e + L^* H^* e_i - L^* T^* H^* e_i + \Phi^*,$$  \hspace{1cm} (A.18)

where $e = [e_1, \ldots, e_n]^T$, $\Gamma = \text{diag}(1, \frac{1}{L^*}, \ldots, \frac{1}{L^n})$, $H^* = [h_1, \ldots, h_n]^T$, and $\Phi^* = [\phi_1^*, \ldots, \phi_n^*]^T$ with

$$\phi_i^* = \frac{f_i}{(L^*)^\sigma}, \quad i = 2, \ldots, n.$$  \hspace{1cm} (A.19)

The following analysis is the same as previous. Differentiating the quadratic function $V_e = e^T P e$ on $[0, t_f)$ along (A.18), we can get

$$\dot{V}_e \leq -L^* \|e\|^2 + 2L^* e^T PH^* e_1 - 2L^* H^* H^* e_1 + 2e^T P \Phi^*.$$  \hspace{1cm} (A.20)

Using Lemmas 1-2, one has

$$2L^* e^T PH^* e_1 - 2L^* P H^* e_1 \leq \|e\|^2 + \bar{\theta} \epsilon_i^2.$$

Combining

$$\left| \frac{\hat{x}_i}{(L^*)^{\sigma+i-1}} \right| = \left| \frac{L}{L^*} \right|^\sigma \hat{z}_i \leq |\hat{z}_i|,$$

with Assumptions 4, we have

$$2 \epsilon^T P \Phi^* \leq \|e\|^2 + \bar{\theta} \|\hat{z}(t - \tau)\|^2$$

$$+ \|\epsilon(t - \tau)\|^2 + \|\hat{\epsilon}(t)\|^2.$$  \hspace{1cm} (A.23)

It follows from Claim 2 that

$$\dot{V}_e \leq \bar{\theta} \left( \|\hat{z}(t - \tau)\|^2 + \|\epsilon(t - \tau)\|^2 \right) - (L^* - 2\|e\|^2 + \bar{\theta} \epsilon_i^2 + \bar{\theta}.$$  \hspace{1cm} (A.24)

Further, by $L^* > 2 + \bar{\theta} + \lambda_{\text{max}}(P)$, we deduce

$$\dot{V}_e \leq - (L^* - 2 \|e\|^2 + \bar{\theta} \epsilon_i^2 + \bar{\theta}.$$  \hspace{1cm} (A.26)

and

$$\hat{V}_e \leq - \bar{V}_e + \bar{\theta} \epsilon_i^2 + \bar{\theta} \epsilon_i^2 + \bar{\theta} - \frac{\bar{\theta}}{\bar{\theta}} \hat{L} + \bar{\theta},$$  \hspace{1cm} (A.27)

where $\bar{\theta}$ is a suitable constant.

(A.27) indicates the boundedness of $\hat{V}_e$, thus $\epsilon(t)$ and $\epsilon(t)$ are bounded on $[0, t_f)$.
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