Accelerating universe in brane gravity with confining potential

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Abstract

We construct the Einstein field equations on a 4-dimensional brane embedded in an $m$-dimensional bulk where the matter fields are confined to the brane by means of a confining potential. As a result, an extra term in the Friedmann equation in an $m$-dimensional bulk appears that may be interpreted as the X-matter, providing a possible phenomenological explanation for the acceleration of the universe. The study of the relevant observational data suggests good agreement with the predictions of this model.

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1 Introduction

There has been considerable activity in the recent past in the area of higher dimensional gravity where the classical 4D space-time of General Relativity is recovered as an effective theory \cite{1}. The basic idea in these theories is the existence of a higher dimensional bulk in which our universe, called the brane, is sitting as a hypersurface. Physical matter fields are confined to this hypersurface, while gravity can reside in the higher dimensional space-time as well as on the brane. This paradigm was first proposed as a means to reconcile the mismatch between the scales of particle physics and gravity \cite{2}. The cosmological evolution of such a brane universe has been extensively investigated. Exact solutions have been found by several authors \cite{3, 4, 5}. These solutions reduce to a generalized Friedmann equation on the brane which contains a quadratic term related to matter energy density. This term arises from the imposition of the Israel junction conditions which is a relationship between the extrinsic curvature and energy-momentum tensor of the brane and results from the singular behavior in the energy-momentum tensor. The main difficulty in applying a junction condition is that it is not unique. Other forms of junction conditions exist, so that the different conditions may lead to different physical results \cite{6}. Furthermore, these conditions cannot be used when more than one non-compact extra dimension is involved. An interesting higher-dimensional model was introduced in \cite{7} where particles are trapped in a 4-dimensional hypersurface by the action of the confining potential $\mathcal{V}$. Also, in \cite{8}, the dynamics of test particles confined to a brane by the action of a confining potential, at classical and quantum levels have been studied and the effects of small perturbations along the extra dimensions investigated. Within the classical limits, the particle remains stable under small perturbations and the effects of the extra dimensions are not felt by the test particle, hence making them undetectable in this way. The quantum fluctuations of the brane cause the mass of a test particle to become quantized and, interestingly, the Yang-Mills fields appear as quantum effects.

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In this paper, we follow [8] and consider an \( m \)-dimensional bulk space without imposing the \( \mathbb{Z}_2 \) symmetry. To localize the matter on the brane, a confining potential is used rather than a delta-function in the energy-momentum tensor. As a result, the extrinsic curvature becomes independent of the matter content of the brane. The Friedmann equation in this model is modified by the appearance of an extra term which behaves like the X-matter; the phenomenological model proposed to fit the data explaining accelerated expansion of the universe. We should emphasize here that there is a difference between the model presented in this work and models introduced in [9, 10] in that in the latter no mechanism is introduced to account for the confinement of matter to the brane.

2 The model

In this section we present a brief review of the model proposed in [8]. Consider the background manifold \( \mathbb{V}_4 \) isometrically embedded in a pseudo-Riemannian manifold \( \mathbb{V}_m \) by the map \( \mathcal{Y} : \mathbb{V}_4 \to \mathbb{V}_m \) such that

\[
G_{AB}(\bar{g}_{\mu\nu}) = g_{\mu\nu}, \quad G_{AB}Y^A_{\mu}N^B_{a} = 0, \quad G_{AB}N^A_{a}N^B_{b} = g_{ab} = \pm 1.
\]

(1)

where \( G_{AB}(\bar{g}_{\mu\nu}) \) is the metric of the bulk (brane) space \( \mathbb{V}_m (\mathbb{V}_4) \) in arbitrary coordinates, \( \{Y^A\} \) (\( \{x^\mu\} \)) is the basis of the bulk (brane) and \( N^A_a \) are \( (m - 4) \) normal unite vectors, orthogonal to the brane. The perturbation of \( \mathbb{V}_4 \) in a sufficiently small neighborhood of the brane along an arbitrary transverse direction \( \xi \) is given by

\[
Z^A(x^\mu, \xi^a) = Y^A + (L_\xi Y)^A,
\]

(2)

where \( L_\xi \) represents the Lie derivative. By choosing \( \xi \) orthogonal to the brane, we ensure gauge independency [8] and have perturbations of the embedding along a single orthogonal extra direction \( \bar{N}_a \) giving local coordinates of the perturbed brane as

\[
Z^A_{\mu}(x^\nu, \xi^a) = Y^A_{\mu} + \xi^a\bar{N}^A_{a\mu}(x^\nu),
\]

(3)

where \( \xi^a \) \( (a = 1, 2, ..., m - 4) \) is a small parameter along \( N^A_a \) that parameterizes the extra noncompact dimensions. One can see from equation (2) that since the vectors \( \bar{N}^A_{a} \) depend only on the local coordinates \( x^\mu \), they do not propagate along the extra dimensions

\[
\bar{N}^A_{a}(x^\mu) = \bar{N}^A_{a} + \xi^b[\bar{N}^b_{\nu}, \bar{N}^A_{a}]^{\nu} = \bar{N}^A_{a}.
\]

(4)

The above assumptions lead to the embedding equations of the perturbed geometry

\[
G_{\mu\nu} = G_{AB}Z^A_{\mu}Z^B_{\nu}, \quad G_{\mu a} = G_{AB}Z^A_{\mu}N^B_{a}, \quad G_{AB}N^A_{a}N^B_{b} = g_{ab}.
\]

(5)

If we set \( N^A_{a} = \delta^A_a \), the metric of the bulk space can be written in the following matrix form

\[
G_{AB} = \begin{pmatrix} g_{\mu\nu} + A_{\mu c}A^c_{\nu} & A_{\mu a} \\ A_{a\nu} & g_{ab} \end{pmatrix},
\]

(6)

where

\[
g_{\mu\nu} = \bar{g}_{\mu\nu} - 2\xi^a\bar{K}_{\mu\nu a} + \xi^a\xi^b\bar{g}^{\alpha\beta}\bar{K}_{\mu\nu a\alpha\beta},
\]

(7)

is the metric of the perturbed brane, so that

\[
\bar{K}_{\mu\nu a} = -G_{AB}Y^A_{\mu}N^B_{a\nu},
\]

(8)

represents the extrinsic curvature of the original brane (second fundamental form). We use the notation \( A_{\mu c} = \xi^d A_{\mu cd} \), where

\[
A_{\mu c d} = G_{AB}N^A_{d\mu}N^B_{c} = \bar{A}_{\mu cd},
\]

(9)
represents the twisting vector fields (the normal fundamental form). Any fixed $\xi^a$ signifies a new perturbed geometry, enabling us to define an extrinsic curvature similar to the original one by

$$K_{\mu \nu a} = -G_{AB}Z^A_{\mu a}N^B_{\nu a} = \tilde{K}_{\mu \nu a} - \xi^b K_{\mu \gamma a} K_{\gamma \nu b} + A_{\mu a} A^c_{\nu b}.$$  \hfill (10)

Note that definitions (6) and (10) require

$$\tilde{K}_{\mu \nu a} = -\frac{1}{2} \frac{\partial G_{\mu \nu}}{\partial \xi^a}.$$  \hfill (11)

In geometric language, the presence of gauge fields $A_{\mu a}$ tilts the embedded family of sub-manifolds with respect to the normal vector $N^A$. According to our construction, the original brane is orthogonal to the normal vector $N^A$. However, equation (5) shows that this is not true for the deformed geometry. Let us change the embedding coordinates and set

$$X^A_{\mu} = Z^A_{\mu} - g^{ab} N_a^{\alpha} A_{\alpha \mu}.$$  \hfill (12)

The coordinates $X^A_{\mu}$ describe a new family of embedded manifolds whose members are always orthogonal to $N^A$. In this coordinates the embedding equations of the perturbed brane is similar to the original one, described by equations (1), so that $Y^A$ is replaced by $X^A$. This new embedding of the local coordinates are suitable for obtaining induced Einstein field equations on the brane. The extrinsic curvature of a perturbed brane then becomes

$$K_{\mu \nu a} = -G_{AB}X^A_{\mu a} X^B_{\nu a} = \tilde{K}_{\mu \nu a} - \xi^b K_{\mu \gamma a} K_{\gamma \nu b} = -\frac{1}{2} \frac{\partial g_{\mu \nu}}{\partial \xi^a},$$  \hfill (13)

which is the generalized York relation and shows how the extrinsic curvature propagates as a result of the propagation of the metric in the direction of extra dimensions. The components of the Riemann tensor of the bulk written in the embedding vielbein $\{X^A_{\mu}, N^A\}$, lead to the Gauss-Codazzi equations [11]

$$R_{\alpha \beta \gamma \delta} = 2g^{ab} K_{\alpha[\gamma a} K_{\delta]b} + R_{ABCD} X^A_{\alpha \gamma} X^B_{\beta \gamma} X^C_{\delta},$$  \hfill (14)

$$2K_{\alpha[\gamma c] \delta]b} = 2g^{ab} A_{\alpha[\gamma a} K_{\delta]b} + R_{ABCD} X^A_{\alpha c} N^B_{\gamma b} X^C_{\delta},$$  \hfill (15)

where $R_{ABCD}$ and $R_{\alpha \beta \gamma \delta}$ are the Riemann tensors for the bulk and the perturbed brane respectively. Contracting the Gauss equation (14) on $\alpha$ and $\gamma$, we find

$$R_{\mu \nu} = (K_{\mu c} K_{\nu}^{\ ac} - K_{\nu c} K_{\mu}^{\ c}) + R_{AB} X^A_{\mu} X^B_{\nu} - g^{ab} R_{ABCD} N^A_{\mu} X^B_{\nu} X^C_{\delta} N^D_{\delta},$$  \hfill (16)

which readily gives

$$G_{\mu \nu} = G_{AB} X^A_{\mu} X^B_{\nu} + Q_{\mu \nu} + g^{ab} R_{AB} N^A_{\mu} N^B_{\nu} g_{\mu \nu} - g^{ab} R_{ABCD} N^A_{\mu} X^B_{\nu} X^C_{\delta} N^D_{\delta},$$  \hfill (17)

where $G_{\mu \nu}$ is the Einstein tensor of the brane and

$$Q_{\mu \nu} = -g^{ab} \left( K_{\mu a} K_{\nu b} - K_{\nu a} K_{\mu b} \right) + \frac{1}{2} \left( K_{\alpha \beta a} K_{\gamma \delta a} - K_{\alpha \beta} K_{\gamma \delta a} \right) g_{\mu \nu}.$$  \hfill (18)

As can be seen from the definition of $Q_{\mu \nu}$, it is independently a conserved quantity, that is $Q_{\mu \nu} = 0$ [9]. Using the decomposition of the Riemann tensor into the Weyl curvature, the Ricci tensor and the scalar curvature

$$R_{ABCD} = C_{ABCD} - \frac{2}{(m - 2)} \left( G_{B[D} R_{C]A} - G_{A[D} R_{C]B} \right) - \frac{2}{(m - 1)(m - 2)} R (G_{A[D} G_{C]B}),$$  \hfill (19)
we obtain the 4D Einstein equations as

\[ G_{\mu\nu} = G_{AB} \chi^A_{\mu} \chi^B_{\nu} + Q_{\mu\nu} - \mathcal{E}_{\mu\nu} + \frac{m - 3}{2} g^{ab} R_{AB} \chi^A_{\mu} \chi^B_{\nu} g_{\mu\nu} - \frac{m - 4}{(m - 2)} R_{AB} \chi^A_{\mu} \chi^B_{\nu} + \frac{m - 4}{(m - 1)(m - 2)} R g_{\mu\nu}, \]  

(20)

where

\[ \mathcal{E}_{\mu\nu} = g^{ab} C_{ABCD} N_a^A \chi^B_{\mu} \chi^C_{\nu}, \]  

(21)

is the electric part of the Weyl tensor \( C_{ABCD} \). Now, let us write the Einstein equation in the bulk space as

\[ G^{(b)}_{AB} + \Lambda^{(b)} g_{AB} = \alpha^{*} S_{AB}, \]  

(22)

where \( \alpha^{*} = \frac{1}{M^{*}} \) (\( M^{*} \) is the fundamental scale of energy in the bulk space), \( \Lambda^{(b)} \) is the cosmological constant of the bulk and \( S_{AB} \) consists of two parts

\[ S_{AB} = T_{AB} + \frac{1}{2} V g_{AB}, \]  

(23)

where \( T_{AB} \) is the energy-momentum tensor of the matter confined to the brane through the action of the confining potential \( V \). We require \( V \) to satisfy three general conditions: firstly, it has a deep minimum on the non-perturbed brane, secondly, depends only on extra coordinates and thirdly, the gauge group representing the subgroup of the isometry group of the bulk space is preserved by it [8].

Use of equation (22) results in

\[ R = -\frac{2}{m - 2} (\alpha^{*} S - m \Lambda^{(b)}), \]  

(24)

and

\[ R_{AB} = -\frac{\alpha^{*}}{(m - 2)} G_{AB} S + \frac{2}{m - 2} \Lambda^{(b)} g_{AB} + \alpha^{*} S_{AB}. \]  

(25)

Substituting \( R \) and \( R_{AB} \) from the above into equation (20), the tangent component of equation (22), also known as the “gravi-tensor equation”, becomes

\[ G_{\mu\nu} = Q_{\mu\nu} - \mathcal{E}_{\mu\nu} + \frac{(m - 3)}{(m - 2)} \alpha^{*} g^{ab} S_{ab} g_{\mu\nu} + \frac{2\alpha^{*}}{(m - 2)} S_{\mu\nu} - \frac{(m - 4)(m - 3)}{(m - 1)(m - 2)} \alpha^{*} g_{\mu\nu} \]

\[ + \frac{(m - 7)}{(m - 1)} \Lambda^{(b)} g_{\mu\nu}. \]  

(26)

On the other hand, again from (22), the trace of the Codazzi equation (15) gives the “gravi-vector equation”

\[ K_{a;\gamma,\delta} - K_{a,\gamma} - A_{b,\alpha,\gamma} K^b + A_{bo,\delta} K^{bo} + B_{a,\gamma} = \frac{3(m - 4)}{m - 2} \alpha^{*} S_{a,\gamma}, \]  

(27)

where

\[ B_{a,\gamma} = g^{mn} C_{ABCD} N_{m,\alpha}^A \chi^B_{\gamma,\alpha} \chi^C_{n,\omega} \chi^D. \]  

(28)

Finally, the “gravi-scalar equation” is obtained from the contraction of (16), (20) and using equation (22)

\[ \alpha^{*} \left[ \frac{m - 5}{m - 1} S - g^{mn} S_{mn} \right] g_{ab} = \frac{m - 2}{6} (Q + R + W) g_{ab} - \frac{4}{m - 1} \Lambda^{(b)} g_{ab}, \]  

(29)

where

\[ W = g^{ab} g^{mn} C_{ABCD} N_{m,\alpha}^A \chi^B_{n,\omega} \chi^C_{\omega,\alpha} \chi^D. \]  

(30)

Equations (26)-(29) represent the projections of the Einstein field equations on the brane-brane, bulk-brane, and bulk-bulk directions.
As was mentioned in the introduction, localization of matter on the brane is realized in this model by the action of a confining potential. This can simply be realized by
\[
\alpha \tau_{\mu \nu} = \frac{2\alpha^*}{(m-2)} T_{\mu \nu}, \quad T_{\mu a} = 0, \quad T_{ab} = 0,
\]  
where \( \alpha \) is the scale of energy on the brane. Now, the induced Einstein field equation on the original brane can be written as
\[
G_{\mu \nu} = \alpha \tau_{\mu \nu} - \frac{(m-4)(m-3)}{2(m-1)} \alpha \tau g_{\mu \nu} + \Lambda g_{\mu \nu} + Q_{\mu \nu} - E_{\mu \nu},
\]
where \( \Lambda = \frac{(m-7)}{(m-1)} \Lambda^{(b)} \). As was noted before, \( Q_{\mu \nu} \) is a conserved quantity which according to [9] may be considered as an energy-momentum tensor of a dark energy fluid representing the \( x \)-matter, the more common phrase being ‘X-Cold-Dark Matter’ (XCDM). This matter has the most general form of the equation of state which is characterized by the following conditions [12]: first it violates the strong energy condition at the present epoch for \( \omega_x < -1/3 \) where \( p_x = \omega_x \rho_x \), second, it is locally stable i.e. \( c_s^2 = \delta p_x / \delta \rho_x \geq 0 \), and third, causality holds good, that is \( c_s \leq 1 \). Ultimately, we have three different types of ‘matter’ on the right hand side of equation (32), namely, ordinary confined conserved matter represented by \( \tau_{\mu \nu} \), the matter represented by \( Q_{\mu \nu} \) which will be discussed later and finally, the Weyl matter represented by \( E_{\mu \nu} \).

At this point, it would be appropriate to consider the case where the bulk space metric can be written as a flat piece plus a perturbation. This worths looking at since questions like localization of gravity on the brane and corrections to the Newtonian potential stems mostly from such a linearized theory. In the usual brane models the problem of localization of gravity is discussed in several papers. For example, in [13], the authors address the localization of gravity on the Friedmann-Robertson-Walker (FRW) type branes embedded in either \( AdS_5 \) or \( dS_5 \) bulk space and show that the graviton zero mode is trapped on the brane. Non-trapped, massive Kaluza-Klein (KK) modes correspond to a correction to Newton’s law. Also in [14], the Randall-Sundrum model with localized gravity is considered, replacing the extra compact space-like dimension by a time-like one. In this way the authors show that the solution to the hierarchy problem can be reconciled with correct cosmological expansion of the visible universe. One the other hand in [15], the authors show that, in brane models with a compact extra dimension, like that of the Randall-Sandrum two brane model, or models with a universal extra dimension [16] which allow for a zero mode like that known from the KK theories, the five dimensional linearly perturbed Einstein equations are in conflict with observations.

To briefly comment on linearized gravity in our model, consider the linear expansion of the bulk metric around the \( m \)-dimensional Minkowski spacetime
\[
G_{AB} = \eta_{AB} + h_{AB}.
\]
It is straightforward to show that the linear approximation of the Einstein field equations in the bulk space, in the harmonic gauge, is
\[
\left[ \Box - \frac{4}{m-2} \Lambda^{(b)} - \alpha^* X \right] h_{AB} = \frac{4}{m-2} \Lambda^{(b)} \eta_{AB} + 2\alpha^* \left( T_{AB} - \frac{1}{m-2} T \eta_{AB} \right),
\]
where
\[
\Box \equiv \eta^{AB} \partial_A \partial_B = \Box^{(4)} + \eta^{ab} \partial_a \partial_b.
\]
One may therefore make a linear expansion of the metric of the bulk space and obtain three sets of equations that describe the gravitons, gravi-vectors and gravi-scalars respectively. The existence of the confining potential in the linearized field equations (34) may then have various effects that could be addressed in a separate work.
3 Cosmological equations

In what follows we will analyze the influence of the trace of $\tau_{\mu\nu}$ and the extrinsic curvature terms on a FRW universe, regarded as a brane embedded in an $m$ dimensional bulk. The spatially flat FRW line element is written as
\[
ds^2 = -dt^2 + a(t)^2 \left[ dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right) \right].
\] (36)

As the source we take the perfect fluid given in co-moving coordinates by
\[
\tau_{\mu\nu} = \rho u_{\mu} u_{\nu} + ph_{\mu\nu}, \quad u_{\mu} = -\delta_{\mu}^0, \quad p = (\gamma - 1)\rho,
\] (37)
where
\[
h_{\mu\nu} = g_{\mu\nu} + u_{\mu} u_{\nu}.
\]
The Weyl tensor $\mathcal{E}_{\mu\nu}$, appearing in equation (32) is given by
\[
\mathcal{E}_{\mu\nu} = -\mathcal{U} \frac{\alpha^*}{4} \left( u_{\mu} u_{\nu} + \frac{1}{3} h_{\mu\nu} + P_{\mu\nu} + Q_{\mu\nu} + Q_{\nu\mu} \right),
\] (38)
where $\mathcal{U}$ is an effective nonlocal energy density on the brane which arises from the gravitational field in the bulk and is negative for localizing the gravitational field near the brane and reads
\[
\mathcal{U} = -\frac{4}{\alpha^*} \mathcal{E}_{\mu\nu} u^\mu u^\nu.
\] (39)

Since $\mathcal{E}_{\mu\nu}$ is traceless, its effective local pressure is $P = \frac{1}{3} \mathcal{U}$. On the other hand, an effective nonlocal anisotropic stress is given by
\[
P_{\mu\nu} = -\frac{4}{\alpha^*} \mathcal{E}_{[\mu\nu]},
\] (40)
while an effective energy flux on the brane is
\[
Q_{\mu} = -\frac{4}{\alpha^*} (E_{\mu\nu} u^\nu + E_{\nu\mu} u^\mu).
\] (41)
The contracted Bianchi identities in the bulk space $G_{A:B} = 0$, using equation (22), imply
\[
\left( T_{AB} + \frac{1}{2} \mathcal{V} G_{AB} \right)_{;A} = 0.
\] (42)

Since the potential $\mathcal{V}$ has a minimum on the brane, the above conservation equation reduces to
\[
\tau_{\mu\nu} = 0,
\] (43)
and gives
\[
\dot{\rho} + \frac{3}{a} \gamma \rho = 0.
\] (44)

This is the conservation equation for the matter fields on the brane. Taking the covariant derivative of both sides of the induced Einstein equation (32) and taking into account the conservation of the matter represented by $Q_{\mu\nu}$ and using equation (43), we find
\[
\mathcal{E}_{\mu\nu;\nu} = \frac{\alpha}{4} g^{\mu\nu} \tau_{\nu}.\]
(45)

As we can see from the above equation, $\tau_{\mu\nu}$ is the source for $\mathcal{E}_{\mu\nu}$. After substituting from equation (37) and considering the isotropic form of $\mathcal{E}_{\mu\nu}$ from equation (38), we obtain
\[
\dot{U} + 4 \frac{\dot{a}}{a} U + \rho \frac{\alpha}{4} (3\gamma - 2) \dot{a} a^{3\gamma + 1} = 0,
\] (46)
with solution
\[ U = \frac{\alpha \gamma \rho}{3a^2} + \frac{c}{a^4}. \] (47)

From hereon, we consider an AdS\textsubscript{m}, dS\textsubscript{m} or flat bulk, so that \( E_{\mu\nu} = 0 \). For late times this assumption seems reasonable because the effects of such a term is negligible. The Codazzi equations (15) with the assumption of vanishing twisting vector fields to make the problem at hand simpler, reduce to

\[ K_{\alpha\gamma\alpha\sigma} - K_{\alpha\sigma\alpha\gamma} = 0. \] (48)

Using the Yorks relation
\[ K_{\mu\nu a} = \frac{1}{2} \frac{\partial g_{\mu\nu}}{\partial \xi^a}, \] (49)
we realize that in the FRW space-time (diagonal metric), \( K_{\mu\nu a} \) is diagonal. After separating the spatial components, the Codazzi equations reduce to

\[ K_{\mu\nu a,\rho} - K_{\nu\sigma a} \Gamma_{\mu\rho}^{\sigma} = K_{\mu\rho a,\nu} - K_{\rho\sigma a} \Gamma_{\mu\nu}^{\sigma}. \] (50)

The first equation gives \( K_{11a,\nu} = 0 \) for \( \nu \neq 1 \), since \( K_{11a} \) does not depend on the spatial coordinates. After defining \( K_{11a} = b_a(t) \), where \( b_a(t) \) are arbitrary functions of \( t \), the second equation gives

\[ K_{00a} = -\frac{1}{a} \frac{d}{dt} \left( \frac{b_a}{a} \right). \] (52)

For \( \mu, \nu = 2, 3 \) we obtain \( K_{22a} = b_a(t) r^2 \) and \( K_{33a} = b_a(t) r^2 \sin^2 \theta \) and generally \( \mu, \nu \neq 0 \)

\[ K_{\mu\nu a} = \frac{b_a}{a^2} g_{\mu\nu}. \] (53)

We find from (18) that

\[ Q_{\mu\nu} = -\frac{1}{a^4} \left( 2 \frac{b a^2}{H} \right) g_{\mu\nu}, \quad Q_{00} = \frac{3b a^2}{a^4}. \] (54)

Assuming that the functions \( b_a \) are equal and Denoting \( b_a = b, h = \frac{b}{a} \) and \( H = \frac{\dot{a}}{a} \), the components of \( Q_{\mu\nu} \) become

\[ Q_{\mu\nu} = \frac{n b^2}{a^4} \left( 2 \frac{h}{H} - 1 \right) g_{\mu\nu}, \quad Q_{00} = \frac{3n b^2}{a^4}, \] (55)
where \( n = m - 4 \). It would how be interesting to see how the above geometrical interpretation is compared with the X-matter explanation, a phenomenological candidate for dark energy. To this end we consider \( Q_{\mu\nu} \) as a perfect fluid and write

\[ Q_{\mu\nu} = -\frac{1}{\alpha} \left[ \rho_x u_{\mu} u_{\nu} + p_x h_{\mu\nu} \right], \quad p_x = (\gamma_x - 1) \rho_x. \] (56)

Use of the above equations leads to an equation for \( b(t) \)

\[ \frac{\dot{b}}{b} = \frac{1}{2} (4 - 3\gamma_x(t)) \frac{\dot{a}}{a}. \] (57)

It is interesting to note that this equation resembles one of the phenomenological candidates for dark energy, the x-matter [12], but in our case this field has a fundamental geometrical justification for the equation of state, having been derived from the term \( Q_{\mu\nu} \) in the Einstein equation (26), itself a result of the extrinsic curvature. If \( \gamma_x \) is taken as a constant, the solution for \( b(t) \) is

\[ b(t) = b_0 a(t)^{\frac{1}{2}(4-3\gamma_x)}, \] (58)
where \( b_0 \) is an integration constant. With this solution the energy density of XCDM becomes
\[
\rho_x = \frac{3nb_0^2}{\alpha} a^{-3\gamma_x}. \tag{59}
\]
Using this density for \( Q_{\mu\nu} \), the Friedman equations become
\[
\frac{\dot{a}^2}{a^2} = \frac{\alpha \gamma}{4} \rho_0 a^{-3\gamma} - \frac{\Lambda}{3} + nb_0^2 a^{-3\gamma_x}, \tag{60}
\]
\[
\frac{\ddot{a}}{a} = - \frac{\alpha \gamma}{4} \rho_0 a^{-3\gamma} - \frac{\Lambda}{3} + nb_0^2 a^{-3\gamma_x} \left( 1 - \frac{3}{2} \gamma_x \right). \tag{61}
\]
A qualitative classification of the solutions on the basis of different values of the parameter \( \gamma_x \) can be achieved without solving these equations. If one defines the potential
\[
V(a) = - \frac{\alpha \gamma}{4} \rho_0 a^{-3\gamma+2} - nb_0^2 a^{-3\gamma_x+2}, \tag{62}
\]
then equation (60) may be written as
\[
\dot{a}^2 + V(a) = 0. \tag{63}
\]
The qualitative behavior of the scale factor \( a(t) \) for different values of \( \gamma_x \) may be realized from the above equation by noting that \( \dot{a}^2 \) is positive. This behavior is much dependent on the range of the values that \( \gamma_x \) can take. We distinguish the following possibility for having an accelerating universe
\[
0 < \gamma_x < \frac{2}{3}. \tag{64}
\]

The behavior of the potential \( V(a) \) and the corresponding evolution of the scale factor \( a(t) \) are illustrated in figure 1.

### 4 The Accelerating universe

It is well known that in FRW cosmology accelerated expansion of the universe may only be obtained in the case when the universe is filled with some exotic form of matter giving rise to a negative pressure, e.g. a cosmological constant. However, if we look at figure 2 we note that, within the context of the present model, the universe can also exhibit accelerated expansion in the case of a vanishing or positive pressure. Unfortunately, the Friedmann equation \( \dot{a}^2 + nb_0^2 a = \frac{\alpha \rho_0}{4a} \) (derived from (60) for \( \gamma = 1 \) and \( \gamma_x = \frac{1}{3} \)) cannot be solved in closed form. However, in two extreme cases corresponding to small and large \( a(t) \), exact solutions may easily be found
\[
a(t) = \left( \frac{9\alpha \gamma \rho_0}{16} \right)^{\frac{1}{4}} t^{\frac{3}{4}}, \quad \text{for small } a, \tag{65}
\]
The first solution is of the Einstein de-Sitter type, while the second represents an evidently inflationary of the power-law type. This means that in our model the universe starts as decelerating and finally ends up as accelerating. In the simplest FRW cosmological models with a one-component fluid filling up the universe such behavior is not possible. The declaration parameter for the model reads

\[ q = -\frac{\ddot{a}}{a^2} = (3\gamma_x - 2) \frac{\Omega_x}{2} + \frac{3}{4}\Omega_m, \]

(67)

where \( \Omega_m = \frac{\rho_m}{3H^2} \) and \( \Omega_x = \frac{\rho_x}{3H^2} \). For \( \Omega_m \sim 0.3 \) and \( \Omega_x \sim 0.7 \), as has been suggested by recent observations, the present epoch requires \( \gamma_x < 0.52 \) as in the x-matter scenarios [12]. In the next section we discuss the observational parameters of the model.

5 The Age of universe

Long existing discrepancy between a relativity large value of the Hubble parameter \( H \sim 701 \text{cm/sec/mpc} \) and the large universe age is nicely resolved if it existed \( \Omega_x \sim 0.7 \). If we compare two regimes of cosmological expansion, acceleration and deceleration, then with the same value of the hubble parameter at the present epoch, expansion was slower in the past for the accelerating regime. It means that to reach the same magnitude of \( H_0 \) more time was necessary and the accelerating universe should be older. We find the age of the universe by direct integration of the Freidmann equation (60),

\[ t_B^0 = \frac{1}{H_0} \int_0^1 \frac{dx}{\left( \frac{\Omega_m}{x} + \Omega_x x \right)^{1/2}}, \]

(68)

Analogously, the calculated age of the universe in FRW models reads

\[ t_F^0 = \frac{1}{H_0} \int_0^1 \frac{dx}{\left( \frac{\Omega_m}{x} + (1 - \Omega_0) \right)^{1/2}}, \]

(69)

where \( H_0^{-1} = 9.8 \times 10^9 \text{h}^{-1} \text{years} \) and the dimensionless parameter \( \Omega_x \), according to modern data, is about 0.7. Hence, in the flat matter dominated universe with \( \Omega_0 = 1 \) the age of the universe would be only 9.3 Gyr, whereas the old globular clusters indicate much larger age 12-15 Gyr. On the other hand in our model for flat universe with \( \Omega_m = 0.3 \) and \( \Omega_x = 0.7 \) the age of the universe, according to equation (68), is 12 Gyr, in good agreement with the range quoted above. We have plotted the age of the universe in both models as a function of the energy density parameter in figure 2.
6 Conclusions

In this paper we have studied a brane world model in which the matter is confined to the brane through the action of a confining potential, rendering the use of any junction condition redundant. This has provided the ground for presenting a scenario in which a FRW universe is embedded in an $m$ dimensional bulk where the extrinsic curvature causes the universe to accelerate. This result could be of interest since we have shown that the existence of exotic matter (given as having a negative pressure) is not necessary to drive an accelerated expansion. Finally, we have found that the age of the universe in this model is remarkably larger than the FRW models.

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