Analytical Solution for Monopile Supported Offshore Wind Turbines Considering Soil-Pile-Water Interaction

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Abstract. Offshore wind turbines (OWTs) are dynamically sensitive systems and vulnerable to external excitations such as aerodynamic and hydrodynamic loadings. These loadings may lead to excessive oscillation for the support structure of OWTs. Therefore, it is essential to evaluate the natural frequencies of the system to keep them away from excitation frequencies of external vibration sources. This paper investigates the natural frequencies and seismic responses of the OWT system. The soil-structure interaction (SSI) is modelled by three coupled springs and fluid-structure interaction (FSI) is simulated by the added mass. Then a closed-form solution of the natural frequencies is derived to consider the variation of cross-section. Furthermore, seismic responses of the system are obtained by mode superposition method. Effects of SSI, substructure mass and axial loading are finally investigated. The results indicate that substructure mass can significantly decrease the frequencies of the structure while axial loading plays only a small part in the process. SSI has an important effect on the seismic responses of OWTs. It is essential to consider the cross-coupling spring to analyze the seismic responses for accuracy.

Keywords. Offshore wind turbine, monopile foundation, natural frequency, soil-structure interaction, seismic response.

1. Introduction

With the aggravation of energy shortage and environmental pollution, renewable energy has been attracting more and more attention all over the world in recent years. As clean and renewable energy, wind power plays an important part in improving the energy structure and environment. Nowadays OWTs supported on monopile foundations have been widely used [1].

OWTs are sensitive and vulnerable to aerodynamic and hydrodynamic loadings induced by wind and wave. Moreover, wind turbines in high seismic hazard regions are under the threat of earthquakes as well. These excitations can lead to excessive vibrations which may result in fatigue damage or structural collapse. Therefore, it is essential to accurately estimate the natural frequencies of the OWT system to avoid resonance. Up to now, model test, numerical simulation and analytical methods have been widely used to explore this issue.

Some experimental approaches have been done to measure the natural frequencies of OWT system. Bhattacharya et al. summarized the results from 1:100 scaled test of a wind turbine model [2]. Yu et al. conducted a scaled monopile wind turbine model subjected to cyclic dynamic loadings. The experimental outcome shows that the natural frequency of the OWT system increases as the number of cycles increase [3].

In studying the response of OWTs, numerical simulation was always used in dynamic analysis. Zuo et al. [4] studied the dynamic responses of OWT system under combined wave and wind action by
taking SSI into account. It is found that SSI reduced the vibration frequencies and affected the tower vibrations substantially. Wang et al. [5] developed a 3D numerical model of the OWT system, where the nonlinear Winkler foundation is used to model the SSI. Feyzollahzadeh et al. [6] determined the responses of OWT system using the analytical transfer matrix method (TMM), where three models including apparent fixity length models (FL), distributed springs (DS) and coupled springs (CS) are employed.

The current analytical methodology is mostly based on the Euler-Bernoulli beam theory. The tapered tower with varying wall thickness is idealized as a constant column with equivalent diameter and wall thickness, and the hub, nacelle and blades are simplified as a lumped mass. A stiffness matrix is introduced by Zaaijer to simulate the influence of foundation accurately [7]. Along this line, Adhikari and Bhattacharya investigated the free vibration of the structure by taking SSI into account. The lateral spring and rotation spring are used and the spring stiffness are measured by experimental methods. Then, the factors affecting natural frequencies such as soil stiffness, axial loading were studied in several cases [8-9]. Arany et al. [10] found that the CS has considerable effect on the natural frequencies of OWTs and should be included in the SSI model. Furthermore, Arany et al. [11] derived the analytical solution for the natural frequencies of the OWT system by using Timoshenko beam.

Until now, the analytical solution for predicting the natural frequencies of OWT considering geometry of tower and SSI has not been provided. Taking NREL 5-MW wind turbine as a prototype, the present study investigated the natural frequencies and seismic responses of OWT system considering variation of the cross section and SSI modeled with three coupled springs.

2. Wind Turbine Model

The analysis model of the monopile OWT structure is shown in figure 1. The total length of the pile is \( h_i \), where the length embedded in the water and sea bed are \( h_w \) and \( h_s \) respectively. The total length of the tower is denoted by \( h_t \). The rotor-nacelle assembly is modeled as top mass attached with inertia moment. The relevant parameters are summarized in table 1 [12].

![Figure 1. A monopile supported offshore wind turbine system.](image)

2.1. SSI

The simplified OWT model is shown in figure 1 Three springs, (i) Lateral spring \( (k_L) \); (ii) Rotational spring \( (k_R) \); (iii) Cross coupling spring \( (k_{LR}) \) are used to model the SSI. These springs are located at the mudline \( (z=0) \). The SSI in the vertical direction is neglected in this study.

2.2. FSI

The interaction between monopile and fluid is modelled by added mass [13]. The simplified expressions of the added mass due to the outer and inner water for the hollow circular pile are written as follows [14]
\[ m_{\text{out}} = \rho_w \pi a_1^2 \left(0.6e^{-0.93} + 0.403e^{-0.156} \right) \left( \frac{2a_1}{h_w} \right) \leq 2 \]  
\[ m_{\text{m}} = \rho_w \pi h^2 \left(0.228 + 0.994 \right) \left( \frac{2h}{h_w} + 0.994 \right) \leq 2 \]

where, \( \rho_w \) is water density, \( a_1 \) and \( b_1 \) are the exterior and interior radius of the pile, respectively.

### Table 1. Monopile properties.

| Property                  | Value          |
|---------------------------|----------------|
| Monopile length           | 75 m           |
| Monopile length in soil   | 45 m           |
| Tower height              | 87.6 m         |
| Tower diameter            | 3.87-6 m       |
| Tower wall thickness      | 0.019-0.027 m  |
| Rotor mass                | 110 ton        |
| Nacelle mass              | 240 ton        |
| Density of monopile       | 7850 kg/m³     |
| Density of tower          | 8500 kg/m³     |
| Young’s Modulus of steel  | 210 GPa        |
| Poisson’s ratio           | 0.3            |
| Water depth               | 20 m           |

### 3. Loads on OWT

The equations of motion can be described as follows

\[
\begin{align*}
\frac{\partial^2 V}{\partial z^2} + P \frac{\partial^2 u(z,t)}{\partial z^2} - \rho I_t(z) \frac{\partial^4 u(z,t)}{\partial z^4 \partial t^2} + \rho A_t(z) \frac{\partial^2 u(z,t)}{\partial t^2} &= 0, \quad 0 < h_f \leq z \leq H_s \\
EI_t \frac{\partial^4 v(z,t)}{\partial z^4} + P \frac{\partial^2 u(z,t)}{\partial z^2} - \rho I_f \frac{\partial^4 u(z,t)}{\partial z^4 \partial t^2} + \left( \rho_j A_f + m_j \right) \frac{\partial^2 u(z,t)}{\partial t^2} &= 0, \quad 0 < z \leq h_f
\end{align*}
\]

where \( \rho_t \) and \( \rho_f \) are the density of tower and foundation material, respectively; \( m_a \) is the hydrodynamic added mass; \( I_t(z) \) and \( I_f \) are the cross sectional moment of inertia of tower and pile, respectively; \( A_t(z) \) and \( A_f \) are the cross section area of the tower and pile, respectively. The boundary conditions are given as:

1. Bending moment at \( z=0 \)
\[
EI_t \frac{\partial^2 u(z,t)}{\partial z^2} - k_R \frac{\partial u(z,t)}{\partial z} - k_{LM} u(z,t) \bigg|_{z=0} = 0
\]  
(4a)

2. Shear force at \( z=0 \)
\[
EI_t \frac{\partial^3 u(z,t)}{\partial z^3} + P \frac{\partial u(z,t)}{\partial z} + k_{LM} u(z,t) + k_{LR} \frac{\partial u(z,t)}{\partial z} - m \frac{\partial^2 u(z,t)}{\partial z^2} \bigg|_{z=0} = 0
\]  
(4b)

3. Bending moment at \( z=H_s \)
\[ EI_i \frac{\partial^2 u(z,t)}{\partial z^2} + J_i \frac{\partial^2 u(z,t)}{\partial z \partial t^2} \big|_{z=0} = 0 \]  

(4c) Shear force at \( z=H_i \),

\[ EI_i \frac{\partial^4 u(z,t)}{\partial z^4} + P \frac{\partial u(z,t)}{\partial z} - \rho I_i (z) \frac{\partial^2 u(z,t)}{\partial z \partial t^2} - M_i \frac{\partial^3 u(z,t)}{\partial t^3} \big|_{z=0} = 0 \]  

(4d)

The transversal displacement can be described as

\[ u(z,t) = \phi(z) e^{i\omega t} \]  

(5)

Substituting this in the equation of motion we have

\[
\begin{align*}
\left\{ & \frac{\partial^2}{\partial z^2} \left( EI_i \frac{d^2 \phi}{dz^2} \right) + P \frac{d^2 \phi}{dz^2} + \rho I_i(z) \omega^2 \frac{d^2 \phi}{dz^2} - \rho_s A_i(z) \omega^2 \phi = 0, \quad h_f < z \leq H_i \\
& EI_i \frac{d^4 \phi}{dz^4} + P \frac{d^2 \phi}{dz^2} + \rho I_i \omega^2 \frac{d^2 \phi}{dz^2} - (\rho_s A_i + m_i) \omega^2 \phi = 0, \quad 0 < z \leq h_f
\end{align*}
\]

(6)

The boundary conditions associated can be rewritten as

(1) Bending moment at \( z=0 \)

\[ EI_j \frac{d^2 \phi}{dz^2} - k_{lr} \frac{d \phi}{dz} - k_{kr} \phi = 0 \]  

(7a)

(2) Shear force at \( z=0 \)

\[ EI_j \frac{d^3 \phi}{dz^3} + P \frac{d \phi}{dz} + k_{lr} \phi + k_{kr} \frac{d \phi}{dz} + m_i \omega^2 \frac{d \phi}{dz} = 0 \]

(7b)

(3) Bending moment at \( z=H_i \)

\[ EI_i \frac{d^2 \phi}{dz^2} - \omega^2 J_i \frac{d \phi}{dz} = 0 \]  

(7c)

(4) Shear force at \( z=H_i \)

\[ EI_i \frac{d^3 \phi}{dz^3} + P \frac{d \phi}{dz} + \rho I_i(z) \omega^2 \frac{d \phi}{dz} + M_i \omega^2 \phi = 0 \]  

(7d)

3.1. Solving Equations of Motion

Differential equation for the \( i \)th element is expressed as

\[
EI_i \frac{d^4 \phi_{(i)}}{dz^4} + P \frac{d^2 \phi_{(i)}}{dz^2} + \rho I_i \omega^2 \frac{d^2 \phi_{(i)}}{dz^2} - m_i \omega^2 \phi_{(i)} = 0
\]

(8)

\[
EI_i = \begin{cases} 
EI_j & i \leq 2 \\
\frac{1}{I_j} \int_{z_i}^{z_f} EI_j(z) dz & i = 3
\end{cases}
\]

(9a)

\[
\rho I_i = \begin{cases} 
\rho I_j & i \leq 2 \\
\frac{1}{I_j} \int_{z_i}^{z_f} \rho I_j(z) dz & i = 3
\end{cases}
\]

(9b)
\[ m_i = \begin{cases} 
\rho_i A_i + m_i & i = 1 \\
\rho_i A_i & i = 2 \\
\frac{1}{I_i} \int_{z_{i-1}}^{z_i} \rho_i A_i(z) dz & i \geq 3 
\end{cases} \quad \text{(9c)} \]

where \( l_i = (z_i - z_{i-1}) \)

It is convenient to express equation (8) by elementary rearrangements as

\[ \frac{d^4 \phi_{(i)}}{dz^4} + v_i^2 \frac{d^2 \phi_{(i)}}{dz^2} - \beta_i^4 \phi_{(i)} = 0 \quad \text{(10)} \]

where \( v_i^2 = \left( P + \rho_i I_i \omega^2 \right)/EI_i \) and \( \beta_i^4 = m_i \omega^2 /EI_i \)

Assuming the solution of the form is

\[ \phi(z) = e^{\lambda z} \quad \text{(11)} \]

Substituting in equation (8) we have the following characteristic equation

\[ \lambda_i^4 + v_i^2 \lambda_i^2 - \beta_i^4 = 0 \quad \text{(12)} \]

The roots are

\[ \lambda_i^2 = -0.5v_i^2 \pm \left( \beta_i^4 + 0.25v_i^4 \right)^{0.5} \quad \text{(13)} \]

The four roots can be expressed as

\[ \lambda_i = \pm i\delta_i \pm \varepsilon_i \quad (\delta_i = \sqrt{\beta_i^4 + 0.25v_i^4}^{0.5} + 0.5v_i^2, \quad \varepsilon_i = \sqrt{\beta_i^4 + 0.25v_i^4}^{0.5} - 0.5v_i^2) \quad \text{(14)} \]

So that General solution is found in the form

\[ \phi_{(i)}(z) = A_i \sin(\delta_i z_i) + B_i \cos(\delta_i z_i) + C_i \sinh(\varepsilon_i z_i) + D_i \cosh(\varepsilon_i z_i) \quad \text{(15)} \]

where \( z_i(z) = z - z_{i-1} \quad z_i \leq z \leq z_{i+1} \)

### 3.2. Obtaining the Transfer Matrix

According to the continuity conditions between \( i \) th and \( (i+1) \) th elements, we get

\[ \phi_{(i)}(z_i) = \phi_{(i)}(z_i) \quad \text{(16a)} \]

\[ \phi_{(i)}'(z_i) = \phi_{(i)}'(z_i) \quad \text{(16b)} \]

\[ (EI)_{(i+1)} \phi_{(i)}^{(n)}(z_i) = (EI)_{(i)} \phi_{(i)}^{(n)}(z_i) \quad \text{(16c)} \]

\[ (EI)_{(i+1)} \phi_{(i)}^{(n)}(z_i) = (EI)_{(i)} \phi_{(i)}^{(n)}(z_i) \quad \text{(16d)} \]

Substituting in equation (15) we have

\[ T_{(i+1)} A_{(i+1)} = T_{(i)} A_{(i)} \quad \text{(17)} \]
\[
T_{(i)} = \begin{bmatrix}
\sin \delta I_i & \cos \delta I_i & \sinh \varepsilon I_i & \cosh \varepsilon I_i \\
\delta \cos \delta I_i & -\delta \sin \delta I_i & \varepsilon \cosh \varepsilon I_i & \varepsilon \sin \varepsilon I_i \\
-EL\delta^2 \sin \delta I_i & -EL\delta^2 \cos \delta I_i & \varepsilon L \varepsilon^2 \sinh \varepsilon I_i & \varepsilon L \varepsilon^2 \cosh \varepsilon I_i \\
-EL\delta^3 \cos \delta I_i & EL\delta^3 \sin \delta I_i & \varepsilon L \varepsilon^3 \cosh \varepsilon I_i & \varepsilon L \varepsilon^3 \sin \varepsilon I_i
\end{bmatrix}
\]
(18)

\[
T_{(i+1)} = \begin{bmatrix}
0 & 1 & 0 & 1 \\
\delta_{i+1} & 0 & \varepsilon_{i+1} & 0 \\
0 & -EL\delta_{i+1}^2 & 0 & EL\varepsilon_{i+1}^2 \\
-EL\delta_{i+1}^3 & EL\delta_{i+1}^3 & 0 & EL\varepsilon_{i+1}^3
\end{bmatrix}
\]
(19)

\[
A_{(i)} = \begin{bmatrix} A, B, C, D \end{bmatrix}^T
\]
(20)

\[
A_{(i+1)} = \begin{bmatrix} A, B, C, D \end{bmatrix}^T
\]
(21)

Equation (17) can be furtherly rewritten as
\[
A_{(i+1)} = T_{(i+1)}^{-1} T_{(i)} A_{(i)} = Z_{(i)} A_{(i)}
\]
(22)

3.3. Equation of the Natural Frequency

The relationship between \( Z_{(i)} \) and \( Z_{(i)} \) can be given as follows
\[
\Gamma_i A_{(i)} = 0
\]
(23)

\[
\Gamma_i = \begin{bmatrix}
-k_R \delta I_i & -EL\delta_2^2 + (P + k_{LR} + \rho_j I_j \omega^2)\delta L_i \\
-EL\delta_2^2 - k_{LR} & \delta L_i \\
-k_R \varepsilon I_i & EL\varepsilon_2^2 + (P + k_{LR} + \rho_j I_j \omega^2)\varepsilon L_i \\
EL\varepsilon_2^2 - k_{LR} & \varepsilon L_i
\end{bmatrix}
\]
(24)

\[
\Gamma_i A_{(i)} = 0
\]
(25)

\[
A A_{(i)} = 0
\]
(27)

\[
A = \begin{bmatrix}
-\delta_2^2 \sin \delta I_i - \gamma_2 \delta I_i \cos \delta I_i & -\delta_2^3 \cos \delta I_i, h & \delta_2^3 \sin \delta I_i h & \gamma_2 \delta, L_i \cos \delta I_i L_i + \chi_2 \sin \delta I_i L_i \\
-\delta_2^3 \cos \delta I_i, L_i + \gamma_2 \delta I_i \sin \delta I_i & \gamma_2 \delta I_i \cos \delta I_i L_i - \delta_2^3 \sin \delta I_i h & \chi_2 \delta, L_i \sin \delta I_i L_i + \gamma_2 \delta, L_i \cos \delta I_i L_i \\
\varepsilon_2^2 \sin \varepsilon I_i L_i - \gamma_2 \varepsilon I_i \cos \varepsilon I_i L_i & \varepsilon_2^2 \cos \varepsilon I_i h & \chi_2 \varepsilon, L_i \cos \varepsilon I_i L_i + \gamma_2 \varepsilon, L_i \cos \varepsilon I_i L_i & \gamma_2 \varepsilon, L_i \sin \varepsilon I_i L_i + \chi_2 \varepsilon, L_i \cos \varepsilon I_i L_i
\end{bmatrix}
\]
(28)

where \( \chi_4 = \frac{\omega^2 I_j}{EI_L}, \chi_2 = \frac{M_j \omega^2}{EI_L}, \chi_3 = \frac{(P + \rho_j I_j \omega^2)}{EI_L} \)

Substituting equation (23) into equation (27), we get
\[
A A_{(i)} = \Gamma_{(i)} A_{(i)} = 0
\]
(29)
Furtherly, using equation (25) and equation (29), we get

\[ \Gamma A_{(i)} = 0 \]  
\[ \Gamma = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} \]  

Here \( \Gamma \) is the matrix whose elements are nonlinear functions. Setting the determinant of the matrix to zero and then the equation is written by

\[ \begin{vmatrix} \Gamma_1 \\ \Gamma_2 \end{vmatrix} = 0 \]  

4. Results and Discussion

4.1. Verification

In order to validate the precision of the proposed method, the relevant non-dimensional parameters were determined in table 2. The natural frequencies of OWT system derived by the analytical solution are compared with those obtained from numerical simulation in table 3. As shown that analytical solution results are quite close to those calculated by numerical simulation in ABAQUS.

| Dimensionless group | Formula |
|---------------------|---------|
| Lateral stiffness   | \( n_L = k_L H_s^3 / EI \) |
| Rotational stiffness| \( n_R = k_R H_s / EI \) |
| Cross stiffness     | \( n_{LR} = k_{LR} H_s^2 / EI \) |
| Axial force         | \( \nu = PH_s^2 / EI \) |
| Mass ratio          | \( \alpha = M_s / mH_s \) |
| Rotary inertia      | \( \beta = J_s / mH_s^3 \) |

| First frequency     | Second frequency | Third frequency |
|---------------------|------------------|-----------------|
| Analytical solution | 1.1069           | 8.2726          | 25.9727         |
| Numerical simulation| 1.1063           | 8.2277          | 25.5667         |
| Difference (%)      | 0.0814           | 0.5457          | 1.5880          |

4.2. Free Vibration

The fundamental frequencies with rigid foundation and SSI model are denoted by \( f_0 \) and \( f \), respectively. A convergence analysis was performed to obtain the element numbers for the discretization of the tower. To examine the convergence, the natural frequency versus \( N \) is plotted in figure 2. It is shown that satisfactory agreement is achieved when the number of segments is ten. As a result, the tower is divided into ten segments to achieve high accuracy. Subsequently, non-dimensional lateral, rotational and cross-coupling stiffness were defined to explore how they affect the natural frequency. The values of the main parameters can be found in the literature [10].

The relative frequency is the ratio of the fundamental frequency considering SSI and that fixed at the base, that is, \( f/f_0 \). Figure 3 shows \( f/f_0 \) versus \( n_L \). It can be easily seen that \( f/f_0 \) increases as \( n_L \) increasing, which indicates that the natural frequencies of the structures increase when the lateral
stiffness of the foundation increases. It also can be seen that cross-coupling spring decreases the natural frequencies of the structures which will be more significant when the lateral stiffness is weak. Figure 4 shows $f/f_0$ versus $n_R$. It is found that the $f/f_0$ increases as $n_R$ increasing. We can see from the ordinates that rotational spring makes a more important role than lateral spring on the natural frequencies of the structures. It also can be seen that the cross-coupling spring could decrease the natural frequencies of the structures and it will be more obvious when the stiffness is weak.

Figure 2. Natural frequency with respect to segments.

Figure 3. The variation of ratio with respect to $n_L$.

Figure 4. The variation of ratio with respect to $n_R$.

Figure 5. The variation of ratio with respect to mass ratio.

Figure 6. The variation of ratio with respect to mass ratio.

Figure 7. The frequency with respect to $n_R$. 
The natural frequency of OWT system with substructure mass \( M_s \), axial loading \( P \) are denoted by \( f_m \) and \( f_p \). Respectively, \( f_1 \) and \( f_2 \) denote the frequency without \( M_s \) and \( P \). Figure 5 shows the variation of ratio \( f_m/f_1 \) with respect to mass ratio \( \alpha \) for different soil stiffness. We can see that the \( f_m/f_1 \) decreases rapidly with the increasing value of \( \alpha \), which means substructure mass can significantly decrease the frequency of the structure and it cannot be ignored. We can also see from the figure that substructure mass makes a little more effect on rigid foundation. Figure 6 shows the variation of ratio \( f_p/f_2 \) with respect to \( \alpha \) for different stiffness of the foundation. We can know from the figure that the ratio \( f_p/f_2 \) decreases marginally with the increasing value of \( \alpha \). So axial loading plays only a small part in the process and it makes a little more effect on rigid foundation as well. Figure 7 shows the frequency with respect to \( n_k \) for whether considering rotary inertia. We can see that the influence of the rotary inertia can be ignored. So we may consider \( J_s=0 \) and it is not a bad assumption because there is little misalignment.

5. Application

The deformation of the OWT can be expressed as

\[
u(z,t) = \sum_{j=1}^{J} \phi_j(z) q_j(t)
\]

where \( q_j(t) \) is the generalized coordinates of \( j \)th mode of vibration. Assuming the classical damping, the generalized coordinates \( q_j(t) \) can be obtained from the following equation

\[
(M_j + M_{wj}) \ddot{q}_j + C_j \dot{q}_j + K_j q_j = -M_{ij} \dddot{u}_k - M_{owj} \dddot{u}_k + F^w_j
\]

\[
M_j = \int_0^h m \phi_j(z)^2 \, dz
\]

\[
M_{ij} = \int_0^h m \phi_j(z) \phi_i(z) \, dz
\]

\[
K_j = \omega_j^2 (M_j + M_{wj})
\]

\[
C_j = 2 \xi \omega_j (M_j + M_{wj})
\]

\[
M_{wj} = \int_0^h m_w \phi_j(z)^2 \, dz
\]

\[
M_{owj} = \int_0^h m_w \phi_j(z) \phi_i(z) \, dz
\]

\[
F^w_j = \int_0^h f_w \phi_j(z) \, dz
\]

where \( f_w \) is the wave force on the structure; \( \xi \) denotes the damping ratio.

Figure 8 shows the maximum tower-tip displacement response of the OWT with respect to the lateral spring \( k_L \). The dimensionless parameter RTU denotes the ratio of peak displacement on the tip of the structure considering SSI and rigid foundation. The other two dimensionless parameters are RTV and RTA which denote the ratio of peak velocity and acceleration, respectively. The SSI reduces the tower-tip displacement amplitude, this effect is more significant as \( k_L \) decreasing. The similar phenomenon occurs on RTV and RTA as shown in figures 9-10. The relative acceleration RTA has more obvious tendency than RTV and RTU as \( k_L \) decreasing. On the whole, lateral spring will reduce the seismic response of the structure. This tendency will be more significant when the lateral soil spring is weaker.

Subsequently, the effect of the rotational spring \( k_R \) is explored in figure 11. The rotational spring \( k_R \) will increase the displacement ratio RTU. This may account for that the weak stiffness of rotational
spring makes the angle on the mudline become larger so that the peak displacement on the top of the structure increase. This effect is more significant as $k_R$ decreasing. However, RTV and RTA on the top of the structure show the opposite trend to the relative displacement of the structure as seen in figures 12-13. Although RTV and RTA as $k_R$ increasing, the rotational spring reduces RTV and RTA of the system and it will be more obvious when the rotational stiffness is weak.

**Figure 8.** Displacement response with respect to lateral spring.  
**Figure 9.** Velocity response with respect to lateral spring.  
**Figure 10.** Acceleration response with respect to rotational spring.  
**Figure 11.** Displacement response with respect to rotational spring.  
**Figure 12.** Velocity response with respect to rotational spring.  
**Figure 13.** Acceleration response with respect to rotational spring.
Based on these investigations, the history of relative displacement, velocity and acceleration time history are plotted in figures 14-16, respectively. The relative displacement, velocity and acceleration along the height on the top of the structure at the peak time are shown in figures 17-19. We can see from these figures that the seismic responses of the offshore wind turbine structure considering lateral spring are weaker than that with rigid foundation, and this influence will be more significant when the lateral stiffness is weak. The rotational spring significantly increases the displacement response and the maximum displacement at the peak time increases by about 22% compared with the rigid foundation. The relative velocity and acceleration responses of the system considering rotational spring are weaker than that with rigid foundation. The maximum velocity and acceleration at the peak time increase by about 18% and 30% compared with the rigid foundation, respectively. The seismic responses of the OWT system considering coupled spring are reduced by nearly 10% compared to the two spring model. Considering only lateral and rotational springs may overestimate the seismic response of the structure. Therefore, the SSI makes important effect on the seismic responses of OWT system. The value of soil stiffness is of great importance for the dynamic responses of the structure. It is indispensable to consider coupled spring to analyze the seismic responses of the structure for accuracy.

**Figure 14.** The history of relative displacement, velocity and acceleration time history with respect to the lateral spring.

**Figure 15.** The history of relative displacement, velocity and acceleration time history with respect to the rotational spring.

**Figure 16.** The history of relative displacement, velocity and acceleration time history for whether considering coupled spring.
Figure 17. The relative displacement, velocity and acceleration along the height at the peak time with respect to the lateral spring.

Figure 18. The relative displacement, velocity and acceleration along the height at the peak time with respect to the rotational spring.

Figure 19. The relative displacement, velocity and acceleration along the height at the peak time for whether consider coupled spring.

6. Conclusions
A semi-analytical model has been proposed to assess the natural frequencies and seismic responses of monopile OWT system. The foundation is modelled by three coupled springs and interaction between structure and water is modelled by using added mass. The closed-form expression of the natural frequencies is derived by using TMM to take the variation of cross-section into account. And the seismic response of the system under earthquake loading is furtherly obtained by mode superposition method. Several conclusions can be drawn from this study:

1) The SSI decreases the natural frequencies of the system. Rotational spring makes a more important role than lateral spring on natural frequencies of the structure. The cross-coupling spring decreases the natural frequencies of the structure and it will be more obvious when the lateral and rotational stiffness is weak.

2) Substructure mass can significantly decrease the frequencies of the structure and it cannot be ignored while axial loading plays only a small part in the process. Effect of the rotary inertia can be ignored. So it is not a bad assumption to neglect because there is very less misalignment.
(3) The SSI makes important effect on the seismic responses of OWT system. Lateral spring will reduce the seismic response and it will be more significant when the stiffness is weak. Rotational spring will increase the relative displacement of the structure, however, the relative velocity and acceleration show the opposite trend to the relative displacement of the structure. This will be more obvious when the rotational stiffness is weak. Considering only lateral and rotational springs may overestimate the seismic responses of the structure. So it is indispensable to consider the cross-coupling spring to analyze the seismic responses of the structure for accuracy.

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