Dressing a Naked Singularity: an Example

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Abstract

Considering the evolution of a perfect fluid with self-similarity of the second kind, we have found that an initial naked singularity can be trapped by an event horizon due to collapsing matter. The fluid moves along time-like geodesics with a self-similar parameter $\alpha = -3$. Since the metric obtained is not asymptotically flat, we match the spacetime of the fluid with a Schwarzschild spacetime. All the energy conditions are fulfilled until the naked singularity.

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I. INTRODUCTION

One of the most important problems in the relativistic astrophysics today is the final fate of a massive star, which enters the state of an endless gravitational collapse once it has exhausted its nuclear fuel. What will be the end state of such a continual collapse which is entirely dominated by the force of gravity? The conjecture that such a collapse, under physically realistic conditions, must end into the formation of a black hole is called the Cosmic Censorship Conjecture (CCC), introduced by Penrose and Hawking [1]. Despite numerous attempts over the past three decades, such a conjecture remains unproved and continues to be a major unsolved problem, lying at the foundation of the theory and applications in black hole physics.

A considerable debate has continued on the validity or otherwise of the CCC, which effectively states that any singularities that arise from gravitational collapse from a regular initial data must not be visible to far away observers in the spacetime and are always hidden within black holes. Such an assumption has been used extensively and is fundamental to the theory as well as applications of black hole physics. On the other hand, if a naked singularity results as collapse end state, it is no longer necessarily covered by the event horizon and could communicate, in principle, with outside observers. Such a scenario is of physical interest because a naked singularity may have theoretical and observational properties quite different from a black hole end state, and communications from extreme strong gravity regions dominated by quantum gravity may be possible.

Although there is no satisfactory proof or mathematical formulation of CCC available despite many efforts, there are many examples of dynamical collapse models available which lead to a black hole or a naked singularity as the collapse end state, depending on the nature of the initial data (see e.g. [1]-[8] and references therein). In particular, pioneering analytic models by Christodoulou, and Newman, and numerical work by Eardley and Smarr [9]-[11], established the existence of shell-focusing naked singularities as the end state of a continual collapse, where the physical radius of all collapsing shells vanishes. In these models the matter form was taken to be marginally bound dust, assuming that the initial data functions are smooth and even profiles. Newman generalized these models for nonmarginally bound class, thus covering the entire class of dust collapse solutions. A general treatment for dust collapse with generic initial data was developed in [12].
Here we present an example an initial naked singularity can evolve to a black hole by the collapse of matter. We consider a perfect fluid which moves along time-like geodesics with a self-similar parameter \( \alpha = -3 \) and satisfies all the energy conditions.

The paper is organized as follows. In Section II we present the Einstein field equations. In Section III we present the exact solution that represents a perfect fluid moving along time-like geodesics for the self-similar parameter \( \alpha = -3 \). The ingoing, outgoing null congruence scalar expansions and the energy conditions [13] are analyzed. In section IV we match the fluid with Schwarzschild space time to obtain an asymptotically flat condition garanting the final structure of a black hole or a naked singularity. Finally, in Section V we present the conclusions.

II. THE FIELD EQUATIONS

The general metric of spacetimes with spherical symmetry can be cast in the form,

\[
ds^2 = r_1^2 \left[ e^{2\Phi(t,r)} dt^2 - e^{2\Psi(t,r)} dr^2 - r^2 S^2(t, r) d\Omega^2 \right],
\]

where \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \), and \( r_1 \) is a constant with the dimension of length. Then, we can see that the coordinates \( t, r, \theta \) and \( \phi \), as well as the functions \( \Phi, \Psi \) and \( S \) are all dimensionless.

Self-similar solutions of the second kind are given by [18]

\[
\Phi(t, r) = \Phi(x), \quad \Psi(t, r) = \Psi(x), \quad S(t, r) = S(x),
\]

where

\[
x \equiv \ln \left[ \frac{r}{(-t)^{1/\alpha}} \right],
\]

and \( \alpha \) is a dimensionless constant. The energy-momentum tensor of the perfect fluid is written in the form

\[
T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}
\]

where \( u^\mu \) denotes the four-velocity of the fluid, while \( \rho \) and \( p \) stands for the energy density and the pressure of the fluid. Then, we can see that \( \rho \) is the energy density of the fluid measured by observers comoving with the fluid. In the comoving coordinates, we have

\[
u_\mu = e^{\Phi(x)} \delta^t_\mu.
\]
Defining

\[ y \equiv \frac{\dot{S}}{S}, \] (6)

where the symbol dot over the variable denotes differentiation with respect to \( x \). The non-null components of the Einstein tensor for the metric given by equation (1) with equations (2)-(6) can be written as

\[ G_{\mu\nu} = -\frac{1}{r^2} e^{2(\Phi - \Psi)} \left[ 2\dot{y} + y(3y + 4) + 1 - 2(1 + y)\dot{\Psi} - S^{-2}e^{2\Psi} \right] \]

\[ + \frac{1}{\alpha^2 t^2}(2\dot{\Psi} + y)y, \] (7)

\[ G_{tr} = \frac{2}{\alpha tr} \left[ \dot{y} + (1 + y)(y - \dot{\Psi}) - y\dot{\Phi} \right], \] (8)

\[ G_{rr} = \frac{1}{r^2} \left[ 2(1 + y)\Phi + (1 + y)^2 - S^{-2}e^{2\Psi} \right] \]

\[ - \frac{1}{\alpha^2 t^2} e^{2(\Psi - \Phi)} \left[ 2\dot{y} + y \left( 3y - 2\dot{\Phi} + 2\alpha \right) \right], \] (9)

\[ G_{\theta\theta} = S^2 e^{-2\Psi} \left[ \Phi + \dot{y} + \Phi \left( \Phi - \dot{\Psi} + y \right) + (1 + y) \left( y - \dot{\Psi} \right) \right] \]

\[ - \frac{r^2 S^2}{\alpha^2 t^2} e^{-2\Psi} \left[ \dot{\Psi} + \dot{y} + y^2 - \left( \dot{\Psi} + y \right) \left( \Phi - \dot{\Psi} - \alpha \right) \right], \] (10)

where in writing the above expressions we have set \( r_1 = 1 \).

In the next section we have solved the Einstein’s equations, \( G_{\mu\nu} = T_{\mu\nu} \).

III. GEODESIC MODEL

We study now the solution of the perfect fluid with self-similarity in a geodesic model, that is, a situation in which the acceleration \( \ddot{\Phi} = 0 \), and in particular we made \( \Phi = 0 \). Thus, we have

\[ G_{tt} = -\frac{1}{r^2} e^{-2\Psi} \left[ 2\dot{y} + y(3y + 4) + 1 - 2(1 + y)\dot{\Psi} - S^{-2}e^{2\Psi} \right] \]

\[ + \frac{1}{\alpha^2 t^2}(2\dot{\Psi} + y)y, \] (11)

\[ G_{tr} = \frac{2}{\alpha tr} \left[ \dot{y} + (1 + y)(y - \dot{\Psi}) \right], \] (12)

\[ G_{rr} = \frac{1}{r^2} \left[ (1 + y)^2 - S^{-2}e^{2\Psi} \right] \]

\[ - \frac{1}{\alpha^2 t^2} e^{2\Psi} \left[ 2\dot{y} + y \left( 3y + 2\alpha \right) \right], \] (13)

\[ G_{\theta\theta} = S^2 e^{-2\Psi} \left[ \dot{y} + (1 + y) \left( y - \dot{\Psi} \right) \right] \]

\[ - \frac{r^2 S^2}{\alpha^2 t^2} e^{2\Psi} \left[ \dot{\Psi} + \dot{y} + y^2 - \left( \dot{\Psi} + y \right) \left( -\dot{\Psi} - \alpha \right) \right], \] (14)
The Einstein field equations yield

\[ G_{tr} = \frac{2}{\alpha tr} \left[ \dot{y} + (1 + y)(y - \dot{\Psi}) \right] = 0, \quad (15) \]

and

\[ \rho = -\frac{1}{r^2} e^{-2\Psi} \left[ 2\dot{y} + y(3y + 4) + 1 - 2(1 + y)\dot{\Psi} - S^{-2} e^{2\Psi} \right] + \frac{1}{\alpha^2 t^2} (2\dot{\Psi} + y) y. \quad (16) \]

As we study the perfect fluid in this work, then we should have \( G^r_r = G^\theta_\theta \), that furnishes

\[ p = \frac{e^{-2\Psi}}{r^2} \left[ (1 + y)^2 - S^{-2} e^{2\Psi} \right] - \frac{1}{\alpha^2 t^2} \left[ 2\dot{y} + y (3y + 2\alpha) \right] \]

\[ = \frac{e^{-2\Psi}}{r^2} \left[ \dot{y} + (1 + y) \left( y - \dot{\Psi} \right) \right], \]

\[ - \frac{1}{\alpha^2 t^2} \left[ \ddot{\Psi} + \dot{y} + y^2 - \left( \dot{\Psi} + y \right) \left( -\dot{\Psi} - \alpha \right) \right]. \quad (17) \]

Thus, we can obtain from (17) the following equations

\[ (1 + y)^2 - S^{-2} e^{2\Psi} = \dot{y} + (1 + y) \left( y - \dot{\Psi} \right) = 0, \quad (18) \]

because of equation (15) and

\[ 2\dot{y} + y (3y + 2\alpha) = \ddot{\Psi} + \dot{y} + y^2 - \left( \dot{\Psi} + y \right) \left( -\dot{\Psi} - \alpha \right). \quad (19) \]

Substituting the equation (15) into (19) we get the following differential equation

\[ \ddot{y} + 3y\dot{y} + \alpha \dot{y} = 0, \quad (20) \]

for \( y \neq -1 \) and \( \alpha = -3 \), we have the particular solution

\[ S(X) = S_0 X^{2/3} e^X, \]

\[ e^{\Psi(X)} = S_0 e^X \left[ \frac{6X + 2}{3X^{1/3}} \right] \quad (21) \]

with \( S_0 > 0 \) and \( X \equiv ln(r(-t)^{1/3}) \). The resulting metric can be written as

\[ ds^2 = dt^2 - S_0^2 X^{4/3} e^{2X} \left[ 4 \left( 1 + \frac{1}{3X} \right)^2 dr^2 - r^2 d\Omega^2 \right]. \quad (22) \]

The geometric radius is given by

\[ R = rS = S_0 r^{2} (-t)^{1/3} [ln(r(-t)^{1/3})]^{2/3}. \quad (23) \]
Note that the first time derivative of the geometric radius is negative, indicating that the fluid is collapsing.

Considering the solution (21) in (16) we obtain the energy density and pressure, given by

\[ \rho = \frac{1}{3t^2} \left[ \frac{2 + 3X}{1 + 3X} \right] \left[ \frac{1}{X} + 1 \right], \]  
(24)

and

\[ p = \frac{1}{3t^2}. \]  
(25)

In order to analyze the physical singularities we get the Kretschmann scalar, given by

\[ K = R_{\alpha\beta\gamma\lambda}R^{\alpha\beta\gamma\lambda} = \frac{4}{2187} \frac{3645X^6 + 3618X^5 + 5265X^4 + 2268X^3 + 441X^2 + 48X + 16}{t^4X^4(3X + 1)^2}. \]  
(26)

Observing the energy density (24), we identify three singularities, which are \( t = 0, \quad X = 0, \quad X = -1/3 \). The singularity \( X = 0 \) is the outermost, which in terms of the \( t \) and \( r \) coordinates we have (see figure 1)

\[ (-t_{\text{sing}}) = \frac{1}{r^3}. \]  
(27)

From the Kretschmann scalar we can see that this singularity, \( X = 0 \) (corresponding to \( R = 0 \)), represents a physical one, although it does not imply in the divergence of the pressure.

For a perfect fluid, the energy conditions are given by the following expressions [13]:

a) weak energy condition:

\[ \rho \geq 0, \quad \rho + p \geq 0, \]  
(28)

b) dominant energy condition:

\[ \rho + p \geq 0, \quad \rho - p \geq 0, \]  
(29)

c) strong energy condition:

\[ \rho + 3p \geq 0. \]  
(30)

For the energy density given by the equation (24) and the pressure given by equation (25), these expressions take the forms:

\[ \rho + p = \frac{1}{3t^2} \left[ \left( \frac{2 + 3X}{1 + 3X} \right) \left( \frac{1 + X}{X} \right) + 1 \right], \]
\[
\rho - p = \frac{1}{3t^2} \left[ \frac{2 + 4X}{(1 + 3X)X} \right],
\]
\[
\rho + 3p = \frac{1}{3t^2} \left[ \frac{2 + 8X + 12X^2}{X(1 + 3X)} \right],
\] (31)

which are all satisfied when the expression in brackets of the last formula is positive, i.e., for \((-t) \geq \frac{1}{r^3}\).

Note that by (24), (25) and (26) we have that the pressure does not diverge on the physical singularity at \(X = -1/3\), but the energy conditions diverge.

The congruence of the outgoing and ingoing null geodesic expansion are given by \([14, 15, 16, 17]\)

\[
\theta_l = \frac{f}{rS} \left[ -\frac{S_0 r^2}{3(-t)^{2/3} X^{1/3}} \left( \frac{2}{3} + X \right) + 1 \right],
\]
\[
\theta_n = \frac{g}{rS} \left[ -\frac{S_0 r^2}{3(-t)^{2/3} X^{1/3}} \left( \frac{2}{3} + X \right) - 1 \right],
\] (32)

respectively. Although we can not solve analytically the equation \(\theta_l = 0\) (which defines the apparent horizon), we can find graphically the curve \(r_{AH}(t)\) shown in figure 1, which intercepts the singularity at \(t = t_c\). The outgoing expansion is positive outside of the apparent horizon and negative inside of it, while \(\theta_n\) is always negative, characterizing the structure of trapped surface. Note that for \(t > t_c\) we have black hole, while for \(t < t_c\) we get a marginally naked singularity (see figure 1). These results suggest that this solution can represent a scenario where a naked singularity can be dressed by the collapse of a perfect fluid.

IV. MATCHING WITH ASYMPTOTICALLY FLAT SPACETIME

Since self-similar spacetimes are not asymptotically flat, then it is necessary to make a junction with a static and asymptotically flat spacetime, i.e., the Schwarzschild metric.

We use the coordinates \(\tau, \theta, \phi\) for the hypersurface junction in a comoving framework.

Thus, we can write now equation (22) as the interior metric in the following form

\[
ds^2_- = dt^2 - 4S_0^2 X^{4/3} e^{2X} \left( 1 + \frac{1}{3X} \right)^2 dr^2 - r^2 S_0^2 X^{4/3} e^{2X} d\Omega^2
\] (33)

and the Schwarzschild metric is

\[
ds^2_+ = \left( 1 - \frac{2m}{R} \right) dT^2 - \left( 1 - \frac{2m}{R} \right)^{-1} dR^2 - R^2 d\Omega^2.
\] (34)
We can describe the junction hypersurface in the interior and exterior coordinates by the following expressions

\[ H_- = r - r_\Sigma = 0 \text{ in } \nu^- \]
\[ H_+ = R - R_\Sigma(t) = 0 \text{ in } \nu^+, \quad (35) \]

then the on the hypersurface, making \( r = \text{constant} \), the interior and exterior metrics becomes

\[ ds_\Sigma^2 = dt^2 - r_\Sigma^2 S_0^2 X^{4/3} e^{2X} d\Omega^2, \quad (36) \]
\[ ds_+^2 = dT^2 \left[ \left( 1 - \frac{2m}{R_\Sigma} \right) - \left( 1 - \frac{2m}{R_\Sigma} \right)^{-1} \left( \frac{dR_\Sigma}{dT} \right)^2 \right] - R_\Sigma^2 d\Omega^2. \quad (37) \]

Comparing the first fundamental form furnished by the above metrics, we have

\[ R_\Sigma = r_\Sigma^2 S_0 (-t)^{1/3} \ln (r_\Sigma (-t)^{1/3})^{2/3}, \]
\[ dt^2 = d\tau^2 = dT^2 \left[ \left( 1 - \frac{2m}{R_\Sigma} \right) - \left( 1 - \frac{2m}{R_\Sigma} \right)^{-1} \left( \frac{dR_\Sigma}{dT} \right)^2 \right] , \quad (38) \]

from which we obtain an expression for \( \frac{dR}{dT} \), that is, considering the hypersurface (dropping the subscript \( \Sigma \))

\[ \frac{dR}{dT} = -1 \pm \sqrt{1 + 4r^2 S_t^2 \left( 1 - \frac{2m}{R} \right)} . \quad (39) \]

The interior e exterior extrinsic curvature are given by

\[ K_{\tau\tau}^- = 0, \quad (40) \]
\[ K_{\theta\theta}^- = \frac{9}{2} S_0 X^{2/3} e^X \left( 1 + \frac{1}{3X} \right) \frac{r}{(X + 3)^2}, \quad (41) \]
\[ K_{\tau\tau}^+ = \frac{d^2 R}{dT^2} - \frac{m}{R^3} (2m - R) \left( \frac{dT}{d\tau} \right)^2 + \frac{m}{R(2m - R)} \left( \frac{dR}{dT} \right)^2, \quad (42) \]
\[ K_{\theta\theta}^+ = -\left( 1 - \frac{2m}{R} \right) - \left( \frac{dR}{dT} \right)^2 \left( 1 - \frac{2m}{R} \right)^{-1}. \quad (43) \]

From the continuity of \( K_{\tau\tau} \) we obtain

\[ \frac{d^2 R}{dT^2} = \frac{m}{R} \left[ \frac{2m - R}{R} - \frac{1}{2m - R} \left( \frac{dR}{dT} \right)^2 \right]. \quad (44) \]
Since the radius of hypersurface junction is diminishing with the time and radius of Schwarzschild event horizon is constant, it is reasonable to expect that the junction radius will cross the horizon some time, forming a black hole. We can note from equations (39) and (44) that, when $R \to 2m$, we have $\frac{dR}{dT} = \frac{d^2R}{dT^2} = 0$.

In the following we show that the junction is also possible if we consider a thin shell separating the two spacetimes.

In this case, the expressions for the energy density and the pressure on the shell are, respectively

$$
\sigma = - \{[K_{\tau \tau}] - [K]\},
$$

$$
\eta = - \frac{1}{R^2} \left\{[K_{\theta \theta}] + R^2 [K] \right\}
$$

(45)

where $[K]$ and $[K_{ab}]$ are

$$
[K] = g^{ab}[K_{ab}],
$$

$$
[K_{ab}] = K^+_{ab} - K^-_{ab}
$$

(46)

and $K_{ab}$ is the extrinsic curvature, given by

$$
K_{ab} = -n^i \left[ \frac{\partial^2 x^i}{\partial \xi^a \partial \xi^b} + \Gamma^i_{jk} \frac{\partial x^j}{\partial \xi^a} \frac{\partial x^k}{\partial \xi^b} \right],
$$

(47)

with the energy momentum tensor of the shell given by the following expression

$$
S_{ab} = - \left\{ K_{ab} - g_{ab}[K] \right\}.
$$

(48)

Using the solution (21) and the expressions for the energy density and the pressure for the shell (45), we obtain

$$
\sigma = \frac{2}{r^2 S_0^2 X^{4/3} e^{2X}} \left[ \frac{2m - R}{(1 - 2m/R) - \left( \frac{dR}{dT} \right)^2 (1 - 2m/R)^{-1}} \right] + \frac{9}{r S_0 X^{2/3} e^X (3 + X)^2} \left( 1 + \frac{1}{3X} \right),
$$

(49)

and

$$
\eta = \frac{1}{(1 - 2m/R) - \left( \frac{dR}{dT} \right)^2 (1 - 2m/R)^{-1}} \left[ \frac{dR}{dT} \left[ - \frac{\partial^2 T}{\partial \tau^2} + \frac{2m}{R(R + 2m)} \frac{\partial T}{\partial \tau} \frac{\partial R}{\partial \tau} \right] \right]
$$

and

$$
\int 1
$$
\[ + \left( \frac{\partial^2 R}{\partial \tau^2} - \frac{m(2m - R)}{r^3} \left( \frac{\partial T}{\partial \tau} \right)^2 + \frac{m}{R(2m - R)} \left( \frac{\partial R}{\partial \tau} \right)^2 \right) \right] \\
- \frac{1}{r^2 S_0^2 X^{4/3} e^{2X}} \left[ \frac{(2m - R)}{\left( 1 - \frac{2m}{R} \right) - \left( \frac{dR}{dT} \right)^2 \left( 1 - \frac{2m}{R} \right)^{-1}} \right] \\
+ \frac{9}{2} S_0 X^{2/3} e^{X} \left( 1 + \frac{1}{3X} \right) \frac{r}{(3 + X)^2} \right) \right]. \] (50)

We require for the energy tensor of the shell that the energy density must be positive, then we should have

\[- \left[ 1 - \left( -\frac{1 \pm \sqrt{1 + 4r^2 S_0^2}}{2rS_0} \right)^{27} \right]^{-1} + \frac{9}{2} \left( 1 + \frac{1}{3X} \right) \frac{1}{(3 + X)^2} > 0, \] (51)

which means, if we choose the minus sign in the first term of the above expression, it is always positive. Thus, it was possible to do the matching with a shell and the exterior spacetime is Schwarzschild one, characterizing a black hole as the final structure of the collapse process.

V. CONCLUSION

In this work we have studied the evolution of a perfect fluid which collapses into an initial naked singularity.

We have used the self-similar general solution of the Einstein field equations, for \( \alpha = -3 \), by assuming that the fluid moves along time-like geodesics. The energy conditions, geometrical and physical properties of the solution was studied.

The analysis of the congruence of the outgoing and ingoing null geodesic expansion have shown that the initial naked singularity evolve to a black hole after a particular time \( t = t_c \). This can be clearly seen in figure 1, where the spacetime around the singularity is initially untrapped \( (\theta_t > 0, \theta_n < 0) \) and after \( t = t_c \) it has become trapped \( (\theta_t < 0, \theta_n < 0) \).

In order to be sure that the final structure is really a black hole, we have shown that the matching with the asymptotically and static spacetime, the Schwarzschild one, is possible, either considering a thin shell or not.

We have presented an example where an initial naked singularity can be dressed and it becomes a black hole by collapsing of standard matter. A similar result was shown by Brandt et al. [7].
Finally, this result and the fact that the pressure does not diverge on the naked singularity suggests that there can exist a connection between naked singularities and some kind of weakness of the gravitational field, compared to that associated to black holes.

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FIG. 1: This plot shows the main structures of the interior spacetime, before the matching with the exterior Schwarzschild spacetime. The dashed curve \((-t) = 1/r^3\) represents the outermost singularity considered.