Automated Geometric and Computer-aided Non-Circular Gear Formation Modeling

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Abstract. In non-circular gears of hydromachines used for transit of oil, fuel-oil residue and water, difference takes place not only between gear teeth profiles, but also between lateral profiles of the same tooth. Profiling of such gears requires application of complex mathematical apparatus. The solution to the inverse task of formation is even more complex in this case. This paper proposes a geometrical model of solution to the task of profiling a non-circular gear, the centroid of which consists of interconnected arcs. It is effective in case all the conditions of formation are met. The proposed automated solid computer-aided modeling of the direct and the inverse tasks of formation realized on virtual imitation level allows us not only to acquire the envelope of the profile, but also to observe the possible formation of transition curves and undercuts. It reveals constructive and technological conditions under which they appear, presents an opportunity to conduct the respective research in order to introduce the required corrections into the kinematic scheme of formation, which results in achieving a high-grade solution to the task of profiling. Automated solid modeling of the inverse task of formation allows us to validate the solution to the allocated task and, if necessary, to refine the initial data and therefore exclude the necessity to conduct the expensive full-scale experiments. The results of the study can be used in non-circular gear design in the field of hydromachines used for transit of oil, fuel-oil residue and water.

1. Introduction

Items comprising non-circular gears are applied in numerous fields of industry. They have found the widest application in looms, measuring devices, flow meters and various other mechanisms and machines [1-5]. The application of non-circular gears appears promising in planetary rotary hydromachines used in pumps that transit oil, fuel-oil residue and water, especially contaminated i.e. on drill sites, as well as in dosing pumps for various liquids [6,7].

The two main problems that restrain the spread of non-circular gear drives are the complexity of non-circular gear teeth machining and the complexity of tooth profile calculation. At the present time the first problem is effectively solved by the use of CNC machines, i.e. EDM machines. The solution to the second problem is acquired by the use of various approaches. One of such approaches is based on the classical theory of envelopes [1]. This approach is characterized by substantial complexity in case the centroid line is represented by an ellipse. In this case, difference takes place not only between all of the teeth of the same gear, but also between lateral profiles of the same tooth. The calculations require complex mathematical apparatus or the use of mathematical and design CAD [8-12]. In case the precision of tooth profile is not subject to strict requirements, the elliptic centroid line is approximated by arcs [6, 13-15] and the possibility of application of classical profiling methods emerges. However, in this case there are complications emerging from the fact that the centroid...
consists of a number of interconnected sections. At the present time the field of workpiece profiling based on computer-aided modeling of actual process of workpiece formation by an instrument [9-11], [16-18] with further integration into CAM system is successfully being developed. Since the teeth profiles acquired as a result of formation modeling do not always consist of the envelope of the corresponding family of profiles, but rather have undercuts and transition curves, it is required to verify the result. In this case, it is required to solve the inverse task of formation. As shown by the analysis of the existing scientific literature, such approach to non-circular gear formation has not yet been considered.

2. Problem Definition

The aim of this paper is the automation of the process of geometric and computer-aided non-circular gear teeth formation modeling. The objectives of the study are: acquiring a mathematical model of gear tooth profile as an envelope of a family of instrument profiles; development of an algorithmic kinematic formation model that would provide the solution to the direct and the inverse tasks of non-circular gear profiling as well as the possibility of acquiring their actual profile that would include the specific elements such as transition curves and undercuts; realization of said model in automated mode on the basis of computer-aided solid modeling methods in order to acquire the digital model of a workpiece adapted to CAM systems; establishment of automated solid modeling technology of solving the inverse task of formation, that would include the creation of removed stock model and allow us to exclude the necessity to conduct the expensive full-scale experiments.

3. Theory

1. Formation of initial data for modeling.

A number of applications [6, 7] provide the ability to approximate the elliptic centroid with elementary geometric primitives – circles and lines. In this case the kinematic scheme of formation of a workpiece with an instrument is reduced to rolling motion of one centroid along the other. On the first stage the two tasks are to be solved: 1) to construct out of primitives a centroid that would sufficiently closely approximate the initial centroid; 2) to select the parameters of the primitives so that the total length of the centroid would be divisible by the pitch of teeth of the generating gear.

Consider a centroid of a non-circular gear outlined by four coupled arcs of radiuses $R_1$ and $R_2$ (fig.1). In order to acquire the points $A$ of coupling of circles of radiuses $R_1$ and $R_2$ let us put down the equation of a circle with center in point $O_2$ and radius equal to $R_1 - R_2$ (fig.2):

$$\left( x - \frac{d}{2} \right)^2 + y^2 = (R_1 - R_2)^2.$$  \hspace{1cm} (1)

The intersection of the circle (1) with the axis $0Y$ defines the point $B$, which is the center of the circle $R_1$ with coordinates $x=0, y = -\sqrt{(R_1 - R_2)^2 - \frac{d^2}{4}}$. Then angle $\beta$ can be acquired, for example, from the following correlation:

$$\beta = \arctg \left( \frac{d}{2 \sqrt{(R_1 - R_2)^2 - \frac{d^2}{4}}} \right).$$  \hspace{1cm} (2)

This angle is further used to define boundaries of rolling motion of the centroid of the circular gear along the non-circular gear. Furthermore, the point of $A$ coupling of two arcs is defined. Its coordinates are acquired from the following correlations:

$$x_A = -\frac{d}{2} - R_1 \cdot \sin \beta, y_A = R_1 \cdot \cos \beta.$$  \hspace{1cm} (3)
2. Geometric modeling of formation.

Geometric modeling constitutes realization of the algorithm of kinematic formation scheme, in which the centroid of the generating gear is rolling along the centroid of a workpiece without slipping. One of such algorithms is performed in the following sequence: first, the generating gear turns around the axis of the modeled gear on the angle \( \varphi_2 \), then around its own axis on the angle \( \varphi_1 \). The angles \( \varphi_1 \) and \( \varphi_2 \) are linked with a the following correlation: \( \varphi_1 = \frac{D_2}{D_1} \cdot \varphi_2 \). This geometric formation modeling algorithm is described by formulas of transition from the moving coordinate system of the generating gear to the coordinate system of the workpiece. As a result of the mentioned turns, the family of instrument profiles appears. Its envelope represents the sought profile of the workpiece. The analytical solution of gear profiling comes to determination of correlation between the parameters of the initial profile and its family, which for the case under consideration is of the form of equation \([19]\)

\[
 k \cdot x(t) \cdot \frac{\partial x(t)}{\partial t} + k \cdot y(t) \cdot \frac{\partial y(t)}{\partial t} + A \cdot x(t) \cdot \sin \left( \frac{R_2}{R_1} \cdot \varphi_1 \right) + A \cdot y(t) \cdot \cos \left( \frac{R_2}{R_1} \cdot \varphi_1 \right) = 0,
\]

(4)

where the initial profile is defined by parametric equations of the form \( x = x(t), y = y(t) \); \( \frac{\partial x(t)}{\partial t}, \frac{\partial y(t)}{\partial t} \) represents partial derivatives with respect to parameter \( t \); \( k = \frac{R_1 + R_2}{R_1} \); \( R_1 \) represents the centroid radius of the instrument; \( R_2 \) represents the centroid radius of the workpiece; \( \varphi_1 \) represents the family parameter.

3. Computer-aided modeling of formation.

In the present paper, following from the essence of solid modeling, the technical sequence of solution of the formation task under consideration is developed and realized in automated mode in the
AutoCAD environment in AutoLISP programming language. The algorithm of this technical sequence includes the following stages:
1) Initialization of the solid models of the generator instrument and the workpiece.
2) Execution of the direct task of formation in accordance with the developed modeling programs, which constitutes acquiring non-circular gear profile using the initial model of a cylindrical gear with intermittent teeth.
3) Solution of the inverse task of formation, which constitutes acquiring the generating gear model given the acquired non-circular gear model; this task is solved in case it is necessary to confirm the significance of the result of direct task solution.
4) In case of necessity, execution of removed stock modeling in order to appoint optimal technological parameters of formation.
Contents of the suggested automated computer-aided modeling are demonstrated by the conducted experiments.

4. Results of Experiments.
In a computer experiment the parameters of the generating gear and the non-circular gear centroid have been selected according to fig. 3. The diameter of the centroid of the gear with intermittent teeth has been calculated so that the length of non-circular gear centroid is divisible by the pitch of teeth of the generating gear. In order to achieve this, the points of coupling of arcs of non-circular gear centroid have been calculated beforehand using the correlations (1) – (3), and their lengths have been acquired subsequently. Solid models of the generating gear and the workpiece with non-circular centroid have been created with said parameters. The solid models constitute the initial data for the automated computer-aided modeling of the direct task of formation.

![Figure 3. Centroids of circular and non-circular gears and their parameters](image)

On the subsequent stage of task solution, in accordance with developed programs, formation modeling of the workpiece upon rolling motion of the generating gear centroid along the non-circular gear is performed on the basis of boolean operations. First, the rolling motion occurs along the arc of radius $R_1$, then along the arc of radius $R_2$.
Since in the considered example the length of the non-circular gear centroid is divisible by four lengths of instrument centroid, the modeling is performed only for one quarter of its length. In order to compare the acquired teeth profile of various sections of non-circular gear, fig. 4 depicts its fragment, while fig. 5 depicts teeth profiles scaled up. As follows from fig. 5, pitch of teeth along the centroid is
equal on all sections, which confirms that the calculations were performed correctly. However, tooth top on section 2 is insignificantly sharpened. Undercut of dedendum is present on both sections.

Figure 4. Non-circular gear workpiece fragment after its formation

Figure 5. Teeth profile fragments of various sections of workpiece centroid

The result of solid modeling of a non-circular gear by means of an instrument with intermittent teeth is rendered on fig. 6. The acquired digital model of workpiece can be used on a CNC-operated machine without further preparation in order to produce non-circular gears of required quality.

Figure 6. Computer solid models of the generating and the non-circular gears after formation

The developed software allows us to solve the inverse task of formation, i.e. acquiring the instrument tooth model given the acquired non-circular gear model. Fig. 7 depicts a fragment of modeling of inverse formation task. It validates the acquired results and drawn conclusions.

The developed software executes removed stock modeling in the process of gear teeth formation modeling, which allows us to assign optimal tool advance and number of passes as well as detect cutting edge workload on the basis of its attribute-based and quantitative parameters.
5. Consideration of the Results

The more complex the form of non-circular gear centroid, the more complex the geometry of teeth profile formation of such gear. In general, difference takes place not only between all the teeth of the non-circular gear, but also between lateral profiles of the same tooth. The solution to such task by means of analytic methods requires complex mathematical apparatus and presents significant difficulties. The reverse task of formation is even more complex in this case. Computer-aided solid modeling realized on a virtual imitation level in automated mode allows us to observe the formation of transition curves and undercut. It presents an opportunity to envision design-manufacturing conditions under which they appear, allows us to conduct the respective research and introduce the required corrections into the kinematic scheme of formation. Since complex form of non-circular gear centroids corresponds to complex kinematic formation scheme, it is essential to refine and develop new approaches and algorithms of solid computer-aided formation.

6. Conclusions

1. A geometrical model of solution to a task of profiling of non-circular gear, the centroid of which comprises interconnected arcs, is proposed. A calculation of generating wheel with intermittent teeth parameters as well as workpiece centroid parameters is performed. Such calculation is effective only in case the sought teeth profile consists exclusively of envelopes of families of circular gear profile.
2. The automated solution to the task of non-circular gear profiling is acquired on the basis of solid modeling. The profiling is executed on virtual imitation level on the basis of algorithms, which model the formation of a workpiece with intermittent teeth by means of a gear cutter. Such solution presents an opportunity not only to acquire the sought profile as an envelope of a family of curves, but also to model the undercut and transition curves, which allows us to achieve a high-grade solution to the task of profiling.
3. The proposed solid modeling also executes the solution to the inverse task of formation in automated mode. The inverse task of solution constitutes acquiring a profile of a gear with intermittent teeth given the previously acquired model of a non-circular gear. This modeling presents an opportunity to validate the solution of the assigned task, to adjust the initial data if needed and to exclude the necessity to conduct the expensive full-scale experiments.
4. The developed software presents an opportunity to create solid models of removed stock, analyzing which it is possible to solve technological tasks including the assignment of optimal tool advance and number of passes.

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