Wavelet transform-based cross-correlation in the time-delay estimation applications

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Abstract. The article discusses the application of wavelet analysis for the time-frequency time-delay estimation. The proposed algorithm is wavelet transform-based cross-correlation time delay estimation that applies discrete time wavelet transform to filter the input signal prior to computation of cross-correlation function. The distinguishing feature of the algorithm that it uses the variation of continuous wavelet transform to process the discrete signals instead of dyadic wavelet transform that is normally applied to the case. Another feature that the implication of convolution theorem is used to compute coefficients of the wavelet transform. This makes possible to omit redundant discrete Fourier transforms and significantly reduce the computational complexity. The principal applicability of the proposed method is shown in the course of a computational experiments with artificial and real-world signal. So the method demonstrated expected selectivity for the signals localized in the different frequency bands. The application of the method to practical case of pipeline leak detection was also successful. However, the study concluded that this method provides no specific advantages in comparison with the conventional one. In the future, alternative applications in biological signal processing will be considered.

1. Introduction
The creation of a large number of automated systems is connected with the necessity to determine the time lag between the registration of similar characteristic signals by a system of spatially distributed sensors. Such systems can have different functions and work with different signals. One can lay emphasis on telecommunication fiber-optic systems [1], biophysiological sensor systems [2], industrial control systems [3, 4], local positioning systems [5], and others. Despite the differences in the functional purpose as well as the received and processed signals' physical properties, all these systems implement similar signal processing algorithms.

A general place for the implementation of such systems is the synchronous reception of the signals with a plurality of sensors spread in space and their subsequent processing in order to estimate the delay time. The obtained delay values between pairs of sensors are used to determine the position of the signal source in a given coordinate system.

In this case, from the point of view of signal processing, there exists a so-called passive scenario time delay estimation (TDE) [6]. This problem has been well researched in various applications. However, sometimes the variety of known methods turns out to be insufficient for solving practical problems, for examples, such as detecting leaks in metal [2] and plastic [7] pipes, local positioning of a person [8], biophysiological studies [2, 9] and medical diagnostics [10]. Adequate solution of these
and some other problems often requires the use of more complex mathematical methods, in particular, time-frequency transformations. In the course of recent decades, the mathematical apparatus of wavelet transforms (WT) has received wide practical application. This WT apparatus is analysed in this article.

2. Materials and methods

The method called wavelet transform-based cross-correlation (WTCC) [10] is used and investigated. A similar approach to signal processing also has other names, such as wavelet cross-correlation (WCC) [11] or wavelet domain cross-correlation (WDCC) [12]. The preference in this paper is given to the first option.

The given approach is based on the discrete signals sequential application of two operations — the wavelet transform and the calculation of the cross-correlation function for the obtained sets of wavelet coefficients [10]. Any WT, that meets the time invariance requirements, can be used to perform the first operation [13]. Thus, non-decimated discrete wavelet transform (NDWT) [12] or continuous wavelet transform (CWT) [13] have their application. In practice, time-series processing is carried out and computational operations are performed using microprocessor devices. That is why, a discrete-time wavelet transform (DTWT) [14] is applied in the second case, which is actually a CWT implementation adapted for computer implementation. The well-known cross-correlation functions (CCF) are used to perform the second operation [15].

Further, the algorithmic implementation of the considered approach is studied in detail.

2.1. Discrete-time wavelet transform

Let us suppose that there is a set of signal samples \( s(t) \), measured and recorded at times \( t_0, t_1, t_2, \ldots, t_{N-1} \) with a sampling interval \( \Delta (t_i = \Delta \cdot i) \). Thus, the signal \( s(t) \) is represented by \( N \) samples \( s(t_1), s(t_2), \ldots, s(t_{N-1}) \). To be definite, let us assume that the number of samples \( N \) is a power of 2, compiling the relation \( N = 2^n \), where \( n \) is an integer value.

Taking into account the introduced designations, in accordance with [16, 17] DTWT is defined as

\[
W(m,k) = DTWT [s] = \sum_{i=0}^{N-1} s(t_i) \cdot \psi_m(t_i - \tau_k),
\]

where \( W(m,k) \) a set of wavelet transform coefficients obtained using a given mother wavelet \( \psi(x) \); \( \tau_k \) — time shift which is determined by the value of the \( k \) parameter and is a multiple of the sampling interval \( \Delta (\tau_k = \Delta \cdot k) \); \( \psi_m(x) \) is the sampled wavelet function built on the basis of the mother wavelet \( \psi(x) \) with the scale parameter \( m \).

Following the introduced notation, the wavelet functions \( \psi_m(t_i) \) are formed as follows [16]

\[
\psi_m(x_j) = \frac{1}{\sqrt{a_0^m}} \cdot \psi \left( \frac{j \cdot \Delta}{a_0^m} \right), \quad j = 0,1,2,...,N-1.
\]

In (2), the parameter \( m \) determines the time scale of the signal which also depends on the value of the scaling coefficient \( a_0 \). Thus, when (2) is substituted into (1), the change \( x_j = t_i - \tau_k \) or its equivalent \( j = i - k \) is carried out.

The choice of particular WT method is based on a compromise between reversibility and redundancy. In this work, we use the discrete-time wavelet transform with scale factors corresponding to dyadic transformations [17]. All used scales \( a_0^n \) are located on the logarithmic scale grid with the base \( a_0 \). Since the number of \( N \) samples of the analyzed signal was previously defined as \( 2^n \), it is convenient to take the base scale equal to two \( (a_0 = 2) \). In this case, the scale parameter takes values \( m \) from the series \( m = 0, 1, \ldots, n-1 \). The time shift parameter takes the values \( k = -N/2, -N/2+1, \ldots, 0, 1, \ldots, N/2-1 \).
The expression on the right-hand side of (1), up to a constant for a fixed $N$ multiplier, is a formula for calculating the $k$-th tick of the convolution $c(t_k)$ of the analyzed $s$ signal and the basis function of a given scale $\psi_m(t)$ [11]

$$c(t_k) = \sum_{i=0}^{N-1} s_i \cdot \psi_m(t_i + t_k).$$

For the calculation in accordance with (3) of all $N$ samples of the function, an effective computational scheme [15] is based on the convolution theorem

$$c(t_k) = F^{-1} \left[ F(s(t)) \times F(\psi_m(t)) \right],$$

where $F$ – forward discrete Fourier transform (DFT), $F^{-1}$ – inverse DFT, $*$ - element-wise complex conjugation, $\times$ - element-wise product. It should be noted that (4) is to calculate circular convolution [11] and assumes that both $s$ and $\psi_m$ can be continued periodically. However, this circumstance is not an obstacle to the use of (4) in (1) in most practical TDE problems.

Thus, substituting (4) into (1), we have

$$W(m,k) = W^{(m)}(k), \quad m = 0,1,2,...,n-1$$

$$W^{(m)}(k) = F^{-1} \left[ F(s(t)) \times F(\psi_m(t)) \right], \quad k = -\frac{N}{2}, -\frac{N}{2} + 1,..., \frac{N}{2} - 1.$$ (5)

Therefore, the result of the DTWT operation described in (5) is a set of coefficients $W^{(m)}$ which represents by itself an $n \times 2^n$ matrix.

It is convenient to use the well-known Fast Fourier Transform (FFT) algorithm to perform the DFT when implementing the mentioned above action. Calculation of the signals wavelet image requires $3n$ real FFT operations, so the computational complexity is $O(N \cdot \log(N))$.

### 2.2. Windowed cross-correlation

The estimation of the lag time, as it was noted earlier, is of great practical importance. The mathematical problem formulation depends on the specific application and can differ significantly [6]. The so-called passive scenario for lag time estimating is considered further in this work.

Let us assume that there is a set of signal samples $s_A(t)$, $s_B(t)$ measured with a sampling interval $\Delta$ ($t_i = \Delta t$) at equally spaced times $t_0, t_1, t_2, ..., t_{L-1}$. We should note that the integer $L$ value is determined by the analyzed signals duration and, in general, is a large value. In this case, the signals $s_A(t)$ and $s_B(t)$ are related like

$$s_A(t) = u(t) + n_A(t),$$

$$s_B(t) = u(t - T_{AB}) + n_B(t),$$

where $u(t)$ is a non-random signal recorded by the sensors; $n_A(t)$ and $n_B(t)$ are random additive noises; $T_{AB}$ is the value of the time shift. The key of the task is to determine the unknown value $T_0$. It should be noted that (6) is an adequate problem description only if $u(t)$ component is not distorted or faded both in $s_A(t)$ and $s_B(t)$. Such problem formulation is common despite this simplification mentioned above.

There are many known methods for solving the TDE problem [18] and the correlation method is most widely used among them. In accordance with the algorithm described in [19] at the initial stage the analyzed signal is divided into $Q$ segments, while the number of samples in a segment is $N$.

Consequently, the number of segments does not exceed $Q \leq L/N$. $N = 2^n$ as was stated in the previous subsection.

Next, for each $q$-segment ($q = 0, 1 ... Q-1$), CCF is calculated using the convolution theorem [15]
\[ r_{ab}^{(q)}(\tau_j) = F^{-1} \left[ F^* \left( s_A^{(q)}(t_j) \right) \times F \left( s_B^{(q)}(t_j) \right) \right], \] \hspace{1cm} (7)

Where superscript \((q)\) indicates the segment sequence number; \(\tau_j\) is the value of the time shift multiple of the sampling interval \(\tau_j = \Delta j\) at \(j = -N/2, -N/2+1, \ldots, N/2-1;\) \(*\) - is the element wise complex conjugation. Then the correlation functions, calculated for each of the \(Q\) windows, are averaged

\[ r_{ab}(\tau_j) = \frac{1}{Q-1} \sum_{q=0}^{Q-1} r_{ab}^{(q)}(\tau_j). \] \hspace{1cm} (8)

It should be noted that the sequential application of (7) and (8) is connected with the execution of redundant inverse DFT operations. In order to avoid them you can use

\[ r_{ab}(\tau_j) = F^{-1} \left[ \sum_{q=0}^{Q-1} F^* \left( s_A^{(q)}(t_j) \right) \times F \left( s_B^{(q)}(t_j) \right) \right]. \] \hspace{1cm} (9)

The computational complexity, regardless of the calculation scheme, is \(O(Q \cdot N \cdot \log(N))\). After those steps, the TDE is carried according to

\[ \tilde{\tau}_{ab} = \arg \max \left[ r_{ab}(\tau_j) \right], \]

where \(\tilde{\tau}_{ab}\) is the delay time estimation.

### 2.3. Wavelet-based TDE technique

In general the idea of the approach comes down to the usage in (7) of not directly sampled signals \(s_A\) and \(s_B\) but the coefficients of their wavelet transform \(W_{S_{\alpha}}\), \(W_{S_{\beta}}\) [10], which can be obtained in accordance with (5). The signal segmentation is carried out as was described earlier, and the wavelet transform is applied to each of the \(Q\) segments

\[ W_{S_{\alpha}}^{(m,q)}(k) = DTWT \left[ s_{\alpha}^{(q)} \right], \]
\[ W_{S_{\beta}}^{(m,q)}(k) = DTWT \left[ s_{\beta}^{(q)} \right]. \] \hspace{1cm} (10)

In this case (7) will look like

\[ r_{ab}^{(m,q)}(\tau_j) = F^{-1} \left[ F^* \left( W_{S_{\alpha}}^{(m,q)}(k) \right) \times F \left( W_{S_{\beta}}^{(m,q)}(k) \right) \right]. \] \hspace{1cm} (11)

Taking (5) we can reproach (10) to get

\[ W_{S_{\alpha}}^{(m,q)}(k) = F^{-1} \left[ F \left( s_{\alpha}^{(q)} \right) \times F \left( \psi_m \right) \right]. \] \hspace{1cm} (12)

The expressions in (12) are as follows

\[ F \left( W_{S_{\alpha}}^{(m,q)}(k) \right) = F \left( F^{-1} \left[ F \left( s_{\alpha}^{(q)} \right) \times F \left( \psi_m \right) \right] \right) = F \left( s_{\alpha}^{(q)} \right) \times F \left( \psi_m \right). \] \hspace{1cm} (13)

\[ F^* \left( W_{S_{\beta}}^{(m,q)}(k) \right) = F^* \left( F^{-1} \left[ F \left( s_{\beta}^{(q)} \right) \times F \left( \psi_m \right) \right] \right) = F^* \left( s_{\beta}^{(q)} \right) \times F^* \left( \psi_m \right). \] \hspace{1cm} (14)
Substitution of (13) and (14) into (11) gives

\[ r_{ab}^{(m,q)}(\tau) = F^{-1}\left[ \Psi_m \times F^*(s^{(q)}_A) \times F(s^{(q)}_B) \right], \]  

(15)

where \( \Psi_m \) is the amplitude frequency response of the filter that is determined by the form of the mother wavelet \( \psi(x) \) and the scale \( m \)

\[ \Psi_m = \left| F(\psi_m) \right|^2 = F^*(\psi_m) \times F(\psi_m). \]  

(16)

By analogy with (9) instead of (15) can be used

\[ r_{ab}^{(m)}(\tau) = F^{-1}\left[ \Psi_m \times \sum_{q=0}^{Q-1} \left( F^*(s^{(q)}_A) \times F(s^{(q)}_B) \right) \right]. \]  

(17)

The set of amplitude characteristics of discrete filters \( \Psi_m (m = 0, 1, \ldots, n-1) \) can be calculated once and in advance for a given type of the mother wavelet \( \psi(x) \) and the transformation window size \( N = 2^n \). A similar idea was suggested earlier in [14]. Further application of (15) corresponds to the time-frequency correlation analysis technique described in [20]. The computational complexity for implementation of (15) is \( O(Q \cdot N \cdot (\log(N))^2) \), and for (17) is \( O((Q+\log(N)) \cdot N \cdot \log(N)) \). At the same time, the application of (17) will not allow obtaining the analysis results in real time as the input data is received and accumulated.

3. Results and discussion

A series of computational experiments was carried out in order to demonstrate the fundamental applicability and usefulness of the proposed mathematical apparatus in TDE problems. The experiments were carried out both on synthesized signals and on real vibroacoustic signals obtained during non-destructive testing of water pipes.

The purpose of the first experiment was to investigate and demonstrate the ability of the WTCC to select signals according to their spectral characteristics. The second experiment is aimed at studying the influence of the mother wavelet type on the time-frequency diagram information value.

The final experiment was carried out with real world data and was aimed at method prospective estimation regarding the engineering problem of leaks position determining in water supply pipes. Earlier, WTCC was already used to solve a similar problem in [21] but the algorithm was significantly different from the proposed one.

Continuous wavelets constructed on the basis of derivatives of the Gaussian functions of the \( n \)-th order WAVE-wavelet, at \( n = 1 \), MHAT-wavelet, at \( n = 2 \), G3-wavelet, at \( n = 3 \), and also continuous wavelet DOG-wavelet (difference of gaussians) [16] were used during the experiments. The type of wavelets is shown in Figure 1.
Figure 1. Wavelets in the time-domain: a WAVE-wavelet b MHAT-wavelet c G3-wavelet d DOG-wavelet

3.1. Dependence on scale

The test signal sets for the first experiment were synthesized from wavelets got by scaling the basic MHAT-wavelet. The signal of channel A is the same for the first and for the second set. At the same time, in both cases the signal components in channel B were shifted by a given number of samples, as shown in Figure 2.

Figure 2. Test signals composition in a and b sets

The WTCC method was applied to the synthesized signals further, as described in section 2.3. The number of samples in each of the test signals was limited by the size of the transformation window \( N = 1024 \). The MHAT-wavelet was taken as the analyzing wavelet in this experiment.

Time-scale diagrams of the functions \( s_{AB}(j) \) were constructed in Mathcad Prime in order to interpret the results. The diagrams are shown in Figure 3.
Figure 3. Time-scale diagrams for the first (a) and the second (b) test sets.

In the time-scale diagram (Figure 3 a) a shift in the signal of channel B by 120 samples of the wavelet scaled by $m = 1$ is evidenced by a significant peak located at the sample $j = 120$ and on the scales $m = 1$, $m = 2$ and $m = 3$.

In its turn, while analyzing the second set, when the signal of channel B was shifted by 120 samples of the original wavelet ($m = 0$) and the wavelet scaled by $m = 5$ by -100 samples, on the time-scale diagram (Figure 3 b) one can observe the symmetry breaking at the $j = 120$ sample, as well as a significant peak, the center of which is located approximately at the $j = -100$ sample. The diagram shows that this peak is located at scales $m \geq 5$.

Figure 4. A family of correlograms for the first (a) and second (b) test cases.
It is obvious that the correlation peak is narrow at small scales which makes it possible to determine impulses in time with sufficient accuracy (Figure 4). However, when interpreting time-scale diagrams, a problem related to the display of narrow peaks in the chart arises. Often such peaks are practically indistinguishable.

On the contrary, the peaks associated with large scales are poorly localized in time. Thus, there is a problem of low resolution at large scales. It can be explained as follows that, for example, for \( m = n - 1 \), the wavelet width is comparable to the data segment width.

The obtained results make it possible to reliably judge the presence of a pulse in the signal, while determining the duration and intensity of a given pulse is associated with the problems that were emphasized above.

3.2. Dependence on form of mother wavelet

The results obtained previously (Figures 3, 4) correspond to the case when the wavelet from which the test signals were constructed, namely the MHAT-wavelet, was taken as the basic wavelet for data analysis.

To study the influence of the parent wavelet type on the time-scale diagrams information content, similarly to 3.1, diagrams for the second synthesized signal through usage as mother wavelets other than the wavelet underlying the synthesized signal were constructed.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Scale-time diagrams for the second test set with different analyzing wavelets: \( a\) WAVE, \( b\) G3, \( c\) DOG.}
\end{figure}

While using WAVE and G3 as basic wavelets, the time-scale diagrams (Figure 5 a, b) are comparable to the diagram obtained earlier. Based on the new diagrams for WAVE and G3, you can draw conclusions similar to those obtained in subsection 3.2. Such results can be explained by the fact that the WAVE, MHAT and G3 wavelets belong to the same family and have a lot in common. However, the diagram obtained with the analytical DOG wavelet (Figure 5 c) does not carry information about the temporal localization of the impulses shifted in channel \( B \).
Figure 6 shows correlograms for $m = 5$ and various types of analytical wavelet. Despite the fact that the location of the largest peak in terms of intensity corresponds to $j = -100$, the two adjacent peaks are comparable in amplitude. This circumstance complicates the diagrams interpretation and makes the analysis of large scales uninformative.

3.3. Study of pipeline leak noise
To estimate the prospects of the WTCC method for detecting the leaks position in pipelines, a series of computational experiments with real world data were carried out. Two recordings of signals in *.wav format were used as input data for subsequent processing by the proposed method. The signals are obtained from survey of a water pipeline with a commercial leak detector. Both data sets represent difficult cases for analysis and subsequent interpretation of the results.

The sampling rate was $f_d = 21362$ Hz for both records. The number of ticks in the analyzed signals was $L = 131072$, the size of the data segment was taken as $N = 4096$, while the number of segments was $Q = 32$.

The proposed method for TDE based on the wavelet transform is compared with the classical correlation method, which is described in Section 2.2.

The MHAT-wavelet was chosen as the mother wavelet $\psi(x)$ for the WTCC. The number of scale factors is $n = 12$.

Figure 7 a and b shows the correlograms in pairs for the two compared methods, respectively for the first and second signal.

During the interpretation, the following results were obtained: for the first case $\hat{\tau}_{ab} = 1322\Delta$ for both methods, for the second case $\hat{\tau}_{ab} = 895\Delta$ and $\hat{\tau}_{ab} = 877\Delta$ for the classical and WTCC methods respectively. Despite the difference in the latter case, there is no way to determine which of the methods turned out to be more accurate. This is due to the relative proximity of the results and insufficient accuracy of distance measurements during performed work.
Figure 7. Correlograms of pipeline leaks acoustic signals. For the WTCC method the curves are plotted at \( m = 0 \).

Figure 8 shows the corresponding time-scale diagrams. Peaks that are not noticeable in the presented CCFs are visible in these charts. The presence of such peaks may be due to the fact that background acoustic noises corresponding to some scales are correlated. The functions corresponding to high scales (\( m > 5 \)) are almost non-informative and therefore are not presented.

Figure 8. Time-scale diagrams for the first signal (a), the second signal (b). The first half of the scales \( m = 0, 1...n/2 - 1 \) is presented.

Figure 8 shows the maxima that carry information about the delay true value. So, in the time-scale diagram obtained by analyzing the first signal (Figure 8 a), the maximum is located at the sample \( j = 1322 \), in the diagram corresponding to the second signal (Figure 8 b), the maximum is located at the sample \( j = 877 \). The other maxima presumably correspond to correlated noises due to fluid stream. Unfortunately, there is no way to test this hypothesis.

3.4. Discussion
In the course of experiments, it was shown that the given method is selective in terms of the frequency signal characteristics and can be used for time-frequency analysis. The dataset obtained as a result of processing is well suited for visualization in the form of time-scale diagrams [22]. The diagrams clearly reflect the time delays between the signal scale components and carry information about the pulse duration.

In general, the WTCC TDE is well suited for examining pulsed signals. The best informational analysis content can be achieved when the unit impulses shape is similar to the mother wavelet. The more significant the differences between impulses and the basic wavelet are the less informative and reliable the result is. The practical application complexity of the method is that it is often difficult to choose a suitable basic wavelet. Partially because of this reason, the results of the analysis of the
vibroacoustic signal cannot be accurately characterized as positive. On the one hand, in most cases, using WTCC, it was possible to correctly estimate the delay time. On the other hand, the advantages of this time-frequency method in comparison with the conventional correlation method [4] are not obvious. We should note that similar conclusions were made in [21]. Despite the fact that the authors were able to show an advantage over the basic correlation method, it cannot be considered a significant one.

Another problem with WT-based TDE techniques is that time resolution is directly related to scale. On large scales ($m > 5$), the duration of the analytical wavelet is comparable to the duration of the data window. This feature makes it difficult to analyze signals where the noise component is more often low-frequency. This is also typical for background noise in pipelines [23].

4. Conclusion
The algorithmic solution to the TDE problem based on DTWT with scale factors defined on a logarithmic grid is proposed and investigated in this article. The main idea of the approach is that the obtained WT coefficients are used as input signals of the classical TDE algorithm based on the calculation and analysis of CCF.

A peculiarity of the proposed algorithm is the use of the CWT-based WT technique, while many alternative algorithms use the dyad-based NDWT. This solution combines the visual expression of the results and ease of interpretation, as well as the implementation simplicity. However, at the same time, the method has a relatively high computational complexity.

The implementation simplicity is because a well-optimized and highly parallel FFT algorithm can be used to implement the WT. The disadvantage associated with computational complexity was partially neutralized by eliminating redundant DFTs arising at the stage of calculating the CCF for the coefficients obtained as a result of DTWT. In addition, computations can be optimized due to preliminary computation and storage of a discrete filters bank in the memory. As a result, the computational complexity of the WTCC TDE algorithm per segment of input signals of size $N$ was $O(N\cdot(\log(N))^2)$, which is a rather good indicator for the time-frequency method.

The applicability of the suggested algorithm was demonstrated in a series of computational experiments with synthesized signals. During them, it was found out that the suggested approach makes it possible not only to estimate the time shift but also to select signals that differ in duration. In addition, it was shown that the best analysis information content is achieved when the information signal $u(t)$ reproduces the form of the mother wavelet $\psi(t)$.

The final experiment showed the applicability of the approach to the study of vibroacoustic signals acquired in the course of nondestructive testing of water supply pipes. Despite the fact that it was not possible to show significant advantages of the proposed approach in comparison with the traditional one, with its application it was still possible to localize the leak position in most cases. The practical difficulties that have arisen happen due to both the characteristic features of the signal and the non-optimal choice of the analysis parameters.

Wavelet analysis techniques are potentially useful in signal processing problems related to the estimation of the lag time in various applications. The study carried out in this work was not of a systemic nature and on its basis, it is impossible to draw a clear conclusion regarding the lack of prospects for the use of WTCC in the tasks of passive vibroacoustic diagnostics. However, in the course of further research, it is planned to focus on other applications, in particular, on the analysis of biological signals.

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