Passivity-Based Control for Two-Wheeled Robot Stabilization

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Abstract. A passivity-based control system design for two-wheeled robot (TWR) stabilization is presented. A TWR is a statically-unstable non-linear system. A control system is applied to actively stabilize the TWR. Passivity-based control method is applied to design the control system. The design results in a state feedback control law that makes the TWR closed loop system globally asymptotically stable (GAS). The GAS is proven mathematically. The TWR stabilization is demonstrated through computer simulation. The simulation results show that the designed control system is able to stabilize the TWR.

1. Introduction
A two-wheeled robot (TWR) is a kind of mobile robots. The TWR is basically structured by a robot body and two wheels. The wheels are supporting the body. The two wheels support makes the robot to have high maneuverability which becomes the advantage of TWR compared to other mobile robots, for examples: three-wheels robot, four-wheeled robot, and legged robot. The high maneuverability makes TWR to be potential for many applications, for examples: transportation, surveillance, logistic, exploration, rescue, and leisure. Segway and hover board are two commercial product example of the TWR.

On another hand, the two wheels support provides a problem to the TWR. The two wheels support makes the TWR is statically unstable, where the robot is not able to stand by self. A stabilization is required to make the TWR to be stable such that it can stand by self. An active stabilization systems is commonly applied to stabilize the TWR. The active stabilization system is a feedback control system which utilizes deviation angle between actual position and vertical position to generate control torque for stabilizing the TWR. The active stabilization system consists of a controller, an actuator, and sensors. The actuator is to generate a control torque for the stabilization. The control torque is calculated by a controller base on a control law and states feedback. The states feedback is obtained trough states measurement using sensors.

Control law for the TWR stabilization can be designed using the available control theories. For model-based control design, dynamics model of the TWR is required. The dynamics model is a mathematics equation system which represents the TWR dynamics. Dynamic model of the TWR can be obtained through system modeling. The system modeling applies physical laws to derive dynamics equation of a plant. System modeling of a TWR can be done by applying the Newton’s laws of motion as presented in [1,2]. TWR modeling by applying the Euler-Lagrange
method is presented in [3, 4]. The TWR system modelings result in a non-linear dynamics system.

Control system design of a non-linear system is a challenge work. It has more complexity than control system design of a linear system. Control system design using non-linear control theories is not straightforward as using linear system control theories. Simplifying the complexity is commonly done through approximating the non-linear system by a linear system. This approximation is known linearization. Through the linearization, linear control theories can be applied in control system design of a non-linear system but with a limitation [5].

Linear quadratic regulator (LQR) is one of the most popular linear control theories. Studies of applying LQR for TWR stabilization have been presented [6–8]. A comparison of TWR stabilization using PID control and LQR control has been presented in [6]. The study results show that both PID and LQR are able to stabilize the TWR, but the LQR controller has better control performance than the PID controller. LQR is a full states feedback control such that measurements of all states are required. However, not all of the states are measurable because limitation of the sensors availability. Observer provides a solution for the unmeasured states. The observer can be used to estimate the unmeasured states. Study on applying observer for states estimation and applying the estimated states in the states feedback control of TWR stabilization has been presented in [8]. Linear control method is applicable for stabilizing a TWR through linearization. However, the linearization is only valid for a limited region around an equilibrium point. Therefore, a control system for TWR stabilization which is design through linearization is only able to stabilize the TWR for a limited operating area and the TWR closed-loop system is locally asymptotically stable.

The TWR is stabilized for the whole operating area if the closed loop system globally asymptotically stable (GAS). The GAS is achieved by satisfying the Lyapunov’s stability theorem [9]. The Lyapunov’s stability theorem is fundamental theorem of non-linear control theories. Therefore, applying non-linear control theories for TWR stabilization will a GAS of the closed loop system. Several studies on applying non-linear control theories for TWR stabilization has been presented [2, 10, 11]. Backstepping control method offer a systematically procedure for obtaining a control law and stability proof simultaneously [12]. Accomplishing the backstepping procedure will result in a states feedback control law which guarantees the closed loop system to be GAS. However, the backstepping method may result in a complex states feedback control law which makes difficulty in real-time system implementation [13]. Another study on TWR stabilization using Lyapunov-based control method has been presented in [14]. All of the non-linear control system designs for TWR stabilization resulted in GAS such that the TWR closed-loop system is stabilized for the whole operating area.

Passivity-based control (PCB) is another non-linear control method. The PCB is basically to make the closed loop system passive. Passivity is one of the most physically interesting concepts in systems and control theory [15]. It is inspired by nature that a passive system is asymptotically stable. Passivity is related to stability property in an input output sense, where a system is stable if a bounded input energy is supplied to the system, it results in a bounded output energy [16].

This paper is presenting a study of non-linear control system design for TWR stabilization using passivity-based control. The study is concerned on longitudinal mode only. Presentation of the paper is organized as follows. Section I provides introduction, motivation, and related works of this study. Model of TWR is described in Section II. Passivity-based control design for stabilizing the TWR is described in Section III. Performance of the TWR stabilization is evaluated through computer simulations and is presented in Section IV. Finally, Section V concludes the work.
2. Two Wheels Robot Model

A two-wheeled robot (TWR) is basically consisted of a robot body and two wheels. The wheels support the robot body. A TWR model is shown in Figure 1a. The robot body is represented by a linkage where the center of mass is assumed at the middle linkage. It is assumed that both wheels are connected by a shaft and driven by a motor. When a torque disturbance is given to the TWR, it makes the TWR to pitch or rotate such that robot position deviates from the vertical axis. Figure 1a shows a TWR at the pitching position with pitching angle $\theta$. At the pitching position, weight of the robot gives a moment to the robot and the robot is to fall down. It is a reason why the robot is statically unstable. In order to keep the robot stable, a torque is required to counter the torque disturbance and the moment due to the weight. This required torque is called the control torque. When the motor is operating, it provides torque to rotate the wheels. Friction of the wheel and the floor results in reaction torque to the body. The reaction torque can be applied as the control torque for stabilizing the robot. Therefore, the motor has two functions: driving the wheels and an actuator for robot stabilization.

Figure 1b shows free body diagram of the robot. There are two working moments on the robot body, i.e.: moment due to the body weight and the reaction torque. The working moments on the robot body are evaluated at the wheel axis. Applying the Newton’s second law results in the following dynamic equations:

\[
\sum M = I \ddot{\theta} \\
\tau - \frac{1}{2} mgl \sin \theta = I \ddot{\theta}
\]

where $M$ is moment, $I$ is inertia of the robot body, $\theta$ is the pitching angle which is the robot body deflection with respect to vertical axis, $m$ is the mass of robot body, $g$ is the gravity acceleration, $l$ is the length of the robot body, $\ddot{\theta}$ is the pitching acceleration of the robot body, and $\tau$ is the control torque. Defining $I_r = \frac{1}{2} mgl$ and rearranging (2) result in:

\[
I \ddot{\theta} + I_r \sin \theta = \tau.
\]
3. Passivity-Based Control Design

Passivity based control is purposed to make a closed loop system to be passive. Theory of the passivity-based control is given as follows [9]:

**Theorem 1** If a system

\[
\dot{x} = f(x, u) \\
y = h(x, u)
\]

is passive with a radial unbounded positive definite storage function and zero-state observable, then the origin \( x = 0 \) can be globally stabilized by \( u = -\phi(y) \), where \( \phi \) is any locally Lipschitz function such that \( \phi(0) = 0 \) and \( y^T \phi(y) > 0 \) for all \( y \neq 0 \).

In order to stabilize the TWR, defined position error of the robot:

\[ e = \theta - \theta_r \]  

where \( e \) is the position error, \( \theta \) is the actual position angle, and \( \theta_r \) is the desired position angle. For the TWR stabilization, the desired position is the robot at vertical position \( (\theta_r = 0) \). Time derivatives of the position error are:

\[ \dot{e} = \dot{\theta} \quad \text{and} \quad \ddot{e} = \ddot{\theta}. \]  

Substituting (6) and (7) into (3) results in error dynamics of the TWR system and is given as follows:

\[ I \ddot{e} + I_r \sin e = \tau. \]  

Define states:

\[ x_1 = e, \quad x_2 = \dot{e} \]

such that (8) can be expressed in a states equation as follows:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{I}{I} \sin x_1 + \frac{1}{I} \tau.
\end{align*}
\]

TWR stabilization is purposed to make \( e = 0 \) and \( \dot{e} = 0 \) as time goes to infinity, i.e. \( x_1(\infty) = 0 \) and \( x_2(\infty) = 0 \). It is an explicit statement that the (10) should be GAS. For achieving the GAS, define a radially positive definite function:

\[ V = \frac{1}{2} x_1^2 + \frac{1}{2} I x_2^2. \]

Time derivative of \( V \) is given as follows:

\[
\begin{align*}
\dot{V} &= x_1 \dot{x}_1 + I x_2 \dot{x}_2 \\
&= x_1 x_2 + I x_2 \left( -\frac{I}{I} \sin x_1 + \frac{1}{I} \tau \right) \\
&= x_1 x_2 + x_2 (-I \sin x_1 + \tau) \\
&= x_2 (x_1 - I \sin x_1 + \tau).
\end{align*}
\]

Let choosing \( \tau = I \sin x_1 - x_1 + v \), substituting it into (12) result in:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{I} (-x_1 + v),
\end{align*}
\]
Table 1. Simulation Parameters

| Parameter          | Symbol | Value      | Unit     |
|--------------------|--------|------------|----------|
| Mass of the rod    | $m_r$  | 0.1        | kg       |
| Length of rod      | $l$    | 0.2        | m        |
| Inertia of the rod | $I_r$  | $13 \times 10^{-4}$ | kg.m$^2$ |
| Control gain       | $\lambda$ | 2        |          |

Figure 2. TWR stabilization when the robot is disturbed by an impulse torque 15 Nm.

and substituting into (12) results in:

$$\dot{V} = x_2 v.$$  \hspace{1cm} (14)

By defining $v$ as the input and $y = x_2$ as the output, the system (13) is passive with $V$ in (11) as the storage function.

Giving a condition $v = 0$ and $y = 0$ for (13) implies $x_2 = 0$ and $x_1 = 0$. It shows that (13) is zero-state observable. Therefore, (13) is globally asymptotically stabilized by $v = -\phi(x_2)$ with any function $\phi$ such that $\phi(0) = 0$ and $y^T \phi(y) > 0$ for all $y \neq 0$. For this case, let choosing $v = cx_2$ with $c < 0$.

4. Simulation
Stabilizing the TWR using the designed control system is evaluated through computer simulation. Parameter values for this simulation is given in Table 1. It is assumed that all system states are measured and noise are neglected. In the simulation, the robot position is initially at $\theta = 0^\circ$. At $t = 0.2$ second, the robot is disturbed by an impulse torque 15 Nm. Figure 2 shows the simulation result. The torque disturbance makes the robot to pitch about $11^\circ$. As the control system is active, the controller give command signal to the motor to generate control torque such that the robot position return to $\theta = 0^\circ$. This result shows that the robot
closed loop system is globally asymptotically stable. The design controller results in fast response of stabilization such that the robot return origin in about 0.4 seconds.

5. Conclusion
A control system design for two-wheeled robot stabilization using passivity-based control has been presented. Applying the method results in a state feedback control law which makes the closed loop system of the robot globally asymptotically stable. The design results in a fast TWR stabilization system which is able to stabilize the robot in about 0.4 seconds.

6. Future Works
This work is a part of research project on autonomous robot. This work will be continued by implementing the designed controller in a real TWR.

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