The velocities of QED bound states such as positronium, muonium or Hydrogen depend on $\ln \alpha$. For example, the Lamb shift of Hydrogen contains the famous Bethe logarithm of order $\alpha^5 \ln \alpha$. In relativistic scattering at high energy $E \gg m$, logarithms of $E$ can be determined from renormalization group equations. The leading logarithmic series of the form $(\alpha \ln m/E)^n$ can be summed by integrating the one-loop anomalous dimension, the subleading series $\alpha (\alpha \ln m/E)^n$ by integrating the two-loop anomalous dimension, and so on. In this letter, we show how to predict logarithms of $\alpha$ for non-relativistic QED bound states using the velocity renormalization group (VRG) [3]. Our approach can also be used for other non-relativistic systems.

The leading order (LO) anomalous dimension generates the Bethe logarithm of order $\alpha^5 \ln \alpha$ for the Lamb shift, and is the only term in the series. Integrating the next-to-leading order (NLO) anomalous dimension determines the $\alpha^6 \ln \alpha$, $\alpha^7 \ln^2 \alpha$ and $\alpha^8 \ln^3 \alpha$ terms, and simultaneously gives the Lamb shift and hyperfine splitting for Hydrogen, muonium and positronium, as well as the decay widths for ortho- and para-positronium. Here we derive all these terms except the $\alpha^7 \ln \alpha$ Lamb shift and $\alpha^6 \ln \alpha$ Lamb shift and hyperfine splitting, which depend on terms in the NLO anomalous dimension for which values are not presented here. The $\alpha^8 \ln^3 \alpha$ term for Hydrogen has been the subject of a recent debate in the literature. An analytic calculation by Karshenboim [4] and a numerical calculation by Goidenko et al. [5] agree on terms in the NLO anomalous dimension for which values are not presented here. The $\alpha^8 \ln^3 \alpha$ term of the form $\ln^3 (\alpha \ln m)$ is suppressed for $m \gg m_2$ and $m_1$. The leading order logarithmic correction to terms in the $\alpha^7 \ln \alpha$ Lamb shift and $\alpha^6 \ln \alpha$ Lamb shift and hyperfine splitting is logarithmic in $\alpha$.

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\( U_r(1) = -4\pi Z\alpha \), \( U_s(1) = \frac{\pi^2 Z^2 \alpha^2 m_R}{m_1 m_2} \), \( U_\Lambda(1) = \pi Z\alpha \left( \frac{\sigma_1}{m_1^2} + \frac{\sigma_2}{m_2^2} + 4S \right) / m_1 m_2 \), \( U_v(1) = 4\pi Z\alpha m_R / m_1 m_2 \).

The reduced mass \( m_R = m_1 m_2 / (m_1 + m_2) \). Terms not needed have been omitted. Let \( U_{2+s} = U_2 + U_s S^2 \). Positronium has additional contributions from one-photon annihilation, \( U_{2+s+1}(1) = \pi \alpha^2 S'^2 / m_2^2 \), and from two-photon and three-photon annihilation graphs which give the imaginary terms (where \( m_e \) is the electron mass):

\[
U_{2+s}^\gamma = \frac{i\alpha^2 (S^2 - 2)}{m_e^2}, \quad U_{2+s}^{\gamma\gamma} = -\frac{i\alpha^3 (4\pi^2 - 9)S^2}{9\pi m_e^2}.
\]

Evaluating the matrix element of a potential of order \( \alpha^v \phi \) gives a contribution to the energy at order \( \alpha^{v+1} \).

The iteration of two potentials of order \( \alpha^v \phi \) and \( \alpha^v \phi' \) gives a potential of order \( \alpha^{v+2} \phi \phi' \). Thus, \( V(-1), V(0), V(1), V(2) \), and \( V(3) \) contribute at order of \( \alpha^4, \alpha^5, \alpha^6 \), and \( \alpha^7 \) respectively. The products \( V(0) V(0), V(0) V(1) \), and \( V(1) V(1) \) contribute starting at order \( \alpha^6 \).

In evaluating matrix elements \( \alpha \sim v, V(-1) \sim 1 \), so the Coulomb potential cannot be treated as a perturbation.

The LO anomalous dimension for a potential coefficient of order \( \alpha^y \) will be defined by terms of order \( \alpha^{y+1} \), the NLO anomalous dimension by terms of order \( \alpha^{y+2} \), etc., rather than by the conventional definition in terms of the number of loops. In bound states where the typical momentum is smaller than the electron mass, the Coulomb potential has no anomalous dimension. The first terms that have an anomalous dimension are \( V(0) \) and \( V(1) \). The LO anomalous dimension for \( V(0) \) and \( V(1) \) generate the LO series in the energy \( \alpha^y (\ln \alpha)^n \).

Their NLO anomalous dimensions generate the NLO series \( \alpha^5 (\ln \alpha)^n \). Matrix elements of \( V(2) \) or \( V(3) \) or products of \( V(0) \) and \( V(1) \) first contribute at order \( \alpha^6 (\ln \alpha)^n \), the same order as the NNLO anomalous dimensions for \( V(0) \) and \( V(1) \), and are not needed for our analysis. However, the potential \( V(2) \) mixes into \( V(1) \) at NLO, and is therefore necessary for solving the VRG equations.

The anomalous dimensions in the effective theory can be generated by soft, potential and ultrasoft loops, with energy-momentum of order \( (k^0 \sim mw, k \sim mw) \), \( (k^0 \sim mw^2, k \sim mw) \) and \( (k^0 \sim mw^2, k \sim mw^2) \) respectively. There are no potential anomalous dimensions at LO. In NRQED soft vertices are labelled by \( \sigma \geq 1 \), and a loop graph with two soft vertices is of order \( v^{\sigma_1+\sigma_2-1} \).

The one-loop soft anomalous dimension is non-zero only for \( V^{2n-1} \) with \( n \geq 1 \) at LO only the running of \( V^{1} \) is needed (from Fig. 1) with two \( \sigma = 1 \) vertices:

\[
\nu \frac{dU_2}{dv} = \frac{14Z^2\alpha^2}{3m_1 m_2},
\]

where the other coefficients in \( V^{1} \) have zero soft anomalous dimensions. The LO ultrasoft anomalous dimension is independent of the momentum structure of the potential (from Fig. 1), etc.:

\[
\nu \frac{dU}{dv} V(p, p') = \frac{2\alpha}{3\pi} \left( \frac{1}{m_1} + \frac{Z}{m_2} \right)^2 k^2 V(p, p').
\]

At LO, the VRG equations for \( V^{(0,1,2)} \) are:

\[
\nu \frac{dU_1}{dv} = 0,
\]

\[
\nu \frac{dU_2}{dv} = \frac{2\alpha}{3\pi} \left( \frac{1}{m_1} + \frac{Z}{m_2} \right)^2 U_c + \frac{14Z^2\alpha^2}{3m_1 m_2},
\]

\[
\nu \frac{dU_3}{dv} = \frac{2\alpha}{3\pi} \left( \frac{1}{m_1} + \frac{Z}{m_2} \right)^2 U_c + \gamma_1 U_c + \gamma_2 U_c^2,
\]

obtained by summing Eqs. (5) and (6). The last equation also has additional spin-independent contributions denoted by \( \gamma_{1,2} \), which are not needed here. The \( U_2 \) integration is trivial,

\[
U_2(\nu) = \gamma_0 U_c \ln \nu + U_2(1),
\]

\[
\gamma_0 = \frac{2\alpha}{3\pi} \left( \frac{1}{m_1} + \frac{Z}{4m_1 m_2} + \frac{Z^2}{m_2^2} \right),
\]

and \( U_2(1) \) has no large logarithm. The matrix element of the \( U_2 \) potential is \( (m_R Z\alpha)^3 / \pi n^3 \) for the \( nS \) state (and zero for \( L \neq 0 \) states). With the coefficient \( U_2(\nu = Z\alpha) \) from Eq. (5) this gives the logarithmic energy shift

\[
\Delta E = -\frac{8Z^4\alpha^5 m_R^3}{3\pi n^3} \left( \frac{1}{m_1^2} + \frac{Z}{4m_1 m_2} + \frac{Z^2}{m_2^2} \right) \ln Z\alpha,
\]

which is the well-known Bethe logarithm in the Lamb shift, and is valid for Hydrogen, muonium and positronium. Equation (5) has no imaginary part or spin-dependence, so there is no contribution to the decay width or hyperfine splitting at this order. Equation (5) has been computed before using an effective field theory. The VRG method makes it clear that the LO series \( \alpha^4 (\ln \alpha)^n \) has only a single term—the anomalous dimension for \( U_2 \) depends on \( \alpha \) and \( U_c \), both of which do not run, so integrating the VRG equation produces only a \( \ln \nu \) term. Below the electron mass, QED is not very efficient at generating logarithms.

At NLO, one needs the anomalous dimension of \( V^{(0)} \) to order \( \alpha^4 \), and of \( V^{(1)} \) to order \( \alpha^3 \). The possible terms that

![Fig. 1. Graphs contributing to the soft (a) and ultrasoft (b,c,...) anomalous dimensions at leading order. The potential is denoted by ⊗, and one sums over all possible ultrasoft exchanges (including wavefunction renormalization).](image_url)
The dimension needed is: \( \frac{\nu dU_{2+s}}{d\nu} \)\( \) NLO \( = \rho_{cc} U_{c}^{3} + \rho_{cc2} U_{c}^{2}(U_{2+s} + U_{c}) + \rho_{c2} U_{c}(U_{2+s} + U_{c}) + \rho_{c2} U_{c}(U_{2+s} + U_{c}) + \rho_{c3} U_{c}(U_{3} + \ldots) + \rho_{s} Z^{\alpha^{3}/m_{1}m_{2}}, \)

where \( U_{2+s} = U_{2} + U_{s} S^{2} \) and

\[
\begin{align*}
\rho_{cc} &= \frac{-m_{4}^{2}}{64\pi^{2}} \left( \frac{1}{m_{1}^{2}} + \frac{1}{m_{2}^{2}} \right), \\
\rho_{cc2} &= \frac{-m_{4}^{2}}{4\pi^{2}} \left( \frac{1}{m_{1}^{2}} + \frac{1}{m_{2}^{2}} \right), \\
\rho_{c3} &= \frac{2m_{4}^{2}}{\pi^{2}}, \\
\rho_{ck} &= \frac{m_{4}^{2}}{2\pi^{2}} \left( \frac{1}{m_{1}^{2}} + \frac{1}{m_{2}^{2}} \right), \\
\rho_{k2} &= \frac{2m_{4}^{2}}{\pi^{2}}.
\end{align*}
\]

The potential \( V^{(3)} \) does not mix into \( V^{(1)} \) when the basis is chosen to avoid the \( p^{3}p^{2}/k^{2} \) term. Equation \( \) depends on \( V^{(2)} \), so we also require its LO VRG equation in Eq. \( \). The ellipses in Eq. \( \) denote terms other than \( U_{3} \) in the \( V^{(2)} \) potential (e.g., \( U_{3a} \)), which contribute to the running of \( U_{2} \). However, they do not have a LO anomalous dimension, and thus contribute with one less logarithm and are not required for our analysis.

Integrating Eq. \( \) gives an infinite series of terms, since it is non-linear in \( U_{2+s} \), including \( \alpha^{2}(\ln \nu)^{n} \) terms with \( n = 1, 2, 3 \). Thus, the NLO \( \alpha^{2}(\ln \nu)^{n} \) series terminates after three terms. Integrating the \( U_{c} U_{2} \) term in Eq. \( \) gives these three possible terms, depending on whether \( U_{2} \) is replaced by the \( \gamma_{0} U_{c} \ln \nu \) or \( U_{2}(1) \) term in Eq. \( \).
in agreement with Refs. [2, 3].

The positronium width is \( \Gamma = -2\text{Im} E \). The matching coefficients \( U_{2+s} \) have imaginary parts from the two- and three-photon annihilation graphs, of order \( \alpha^2 \) and \( \alpha^3 \) respectively. They are the lowest order contributions to the imaginary part, so we can use Eq. (12) for the widths, even though real parts of the same order have been neglected:

\[
\frac{\Delta \Gamma}{\Gamma_0} = \gamma_0 \rho_{c22} U_c(1)^2 \ln^2 \nu = -\frac{3}{2\pi} \alpha^3 \ln^2 \alpha,
\]

for the ortho- and para-positronium widths in agreement with Ref. [4]. The \( \alpha^7 \ln^2 \alpha \) Lamb shift depends on \( \gamma_{1,2} \), and will be discussed elsewhere.

The \( \ln \nu \) term that depends on the matching value of \( U_{2+s} \) is

\[
U_{2+s}[\rho_{c22} U_c (U_{2+s} + 2 U_r) + \rho_{c32} U_c^2 + \rho_{2k} U_k] \ln \nu,
\]

where the \( U_i \)'s are evaluated at \( \nu = 1 \). Using Eqs. (11) and (3), the imaginary part gives the positronium width:

\[
\frac{\Delta \Gamma}{\Gamma_0} = \left( \frac{m^2}{2\pi} \text{Re} U_{2+s} - 2 \right) \ln \nu = \left( \frac{7S^2}{6} - 2 \right) \alpha^2 \ln \alpha,
\]

so for \( S^2 = 2 \) and \( S^2 = 0 \) we have:

\[
\left( \frac{\Delta \Gamma}{\Gamma_0} \right)_\text{ortho} = \frac{\alpha^2}{3} \ln \alpha, \quad \left( \frac{\Delta \Gamma}{\Gamma_0} \right)_\text{para} = -2\alpha^2 \ln \alpha,
\]

in agreement with Ref. [4]. The \( \alpha^6 \ln \alpha \) Lamb shift and hyperfine splitting have contributions from \( V(\nu = 1) \), \( \gamma_3 \) and \( \rho_5 \).

There are two infinite series of logarithmic terms that are easily identified. Neither of them gives the complete contribution at a given order, but they do show that there are logarithmic terms of arbitrarily high order. The ultrasoft anomalous dimension Eq. (3) generates the potential

\[
V(\nu = Z \alpha) = \exp \left[ \frac{2\alpha}{3\pi} \left( \frac{1}{m_1} + \frac{Z}{m_2} \right)^2 \frac{k^2 \ln Z \alpha}{\kappa^2} \right] \frac{U_c(1)}{k^2},
\]

where the exponential is related to the Sudakov form factor. In position space, the modified Coulomb potential is

\[
V(Z \alpha) = -\frac{Z \alpha \text{Erf}}{r} \left[ \sqrt{\frac{3\pi}{8\alpha \ln[1/(Z \alpha)]}} \left( \frac{1}{m_1} + \frac{Z}{m_2} \right)^{-1} \right],
\]

where Erf is the error function. This gives a series in the energy of the form \( \alpha^5 \ln Z \alpha (\alpha^3 \ln Z \alpha)^n / n! \), \( n \geq 0 \). The second infinite series of logarithmic terms is obtained by integrating the VRG equation for \( U_2 \) retaining only the \( U_2 \) terms in Eqs. (3) and (4),

\[
\frac{\nu}{\nu} \frac{d U_2}{d \nu} = \gamma_0 U_c + \rho_{c22} U_c U_2^2.
\]

Solving this equation gives

\[
U_2(\nu) = \frac{U_2(1) + \sqrt{\gamma_0} [\rho_{c22}] \tanh \left( \frac{\sqrt{\gamma_0} [\rho_{c22}] U_c(1) \ln \nu}{1 + [\rho_{c22}] U_2(1) \tanh \left( \sqrt{\gamma_0} [\rho_{c22}] U_c(1) \ln \nu \right)} \right)}{1 + [\rho_{c22}] U_2(1) \tanh \left( \sqrt{\gamma_0} [\rho_{c22}] U_c(1) \ln \nu \right)},
\]

which has an expansion of the form \( \alpha^2 \ln \alpha (\alpha^3 \ln^2 \alpha)^n \). The first two terms are the \( \alpha^3 \ln \alpha \) and \( \alpha^8 \ln^3 \alpha \) Lamb shifts. The difference between the exact expression and the first two terms is less than 1 Hz in the Lamb shift.

We have derived logarithmic terms in QED bound states using the VRG. The results are in agreement with results computed previously using other methods. The computation of the \( \alpha^8 \ln^3 \alpha \) Hydrogen Lamb shift by the VRG supports the value computed by Karshenboim [4]. We have also computed the \( \alpha^6 \ln \alpha \) positronium Lamb shift. The VRG approach makes the universal nature of the \( \ln \alpha \) terms clear, and is also an efficient way of computing these terms. The computations in this paper support a key feature of the VRG method proposed in Ref. [4]: the simultaneous running of soft and ultrasoft effects from \( m \) using a subtraction velocity. We have also identified two infinite series of logarithms in QED.

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