Performance Improvement for Wireless Sensors Networks by Adopting Hybrid Subcarrier Intensity Modulation Over Exponentiated Weibull Turbulence Channels

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ABSTRACT We conduct a research on free space optical (FSO) communication system that is applied for transmission in wireless sensor networks (WSN), which is based on the hybrid pulse position modulation-binary phase shift keying-subcarrier intensity modulation (PPM-BPSK-SIM) undergoes novel Exponentiated Weibull fading channels. When battery-charged sensor transmission nodes with limited energy is transmitted, it is vital to study the performance of the transmission link. We have derived the exact joint probability function of the transmission link when the atmospheric turbulence and pointing errors are considering. On the basis of them, the unconditional average bit error rate (BER) for hybrid PPM-BPSK-SIM is derived, then the closed expression of the outage probability and the average channel capacity are also derived. Research results indicates that increasing the receiver aperture by aperture averaging effect can distinctly improve the performance of the link. Additionally, for any circumstances of atmospheric turbulence and pointing errors, by combining the utilization of hybrid PPM-BPSK-SIM modulation and symbol with average length greater than eight, the average BER performance can be conspicuously enhanced. The outage probability and average channel capacity of the FSO link are jointly affected by atmospheric turbulence and pointing error, and optimization strategies in different scenarios are given by our research.

INDEX TERMS Wireless sensor networks (WSN), free space optical (FSO), exponentiated Weibull fading channels, pointing errors, PPM-BPSK-SIM, BER, average symbol length.

I. INTRODUCTION

Wireless sensor networks (WSN) have raised extensive research in recent years due to their advantages of small size and low cost. However, WSN has inherent problems such as low bandwidth and limited battery supply energy. Meanwhile, the increase in number of sensor nodes has brought serious problems such as shortage of spectrum resources [1], [2]. The application of free space optical (FSO) communication to WSN transmission will bring considerable benefits, such as larger bandwidth, higher security, smaller size of transmitting and receiving antennas, and higher transmission efficiency [3]–[6]. However, when the WSN is deployed in actual application scenarios, the FSO link as its transmission medium will be affected by atmospheric turbulence, which causes problems such as fluctuations in the signal intensity of the receiving information center, as well as interference generated by the pointing error between the receiving antenna and beam center caused by incomplete coincidence [7], [8]. The high bit error rate (BER) and other problems caused by the above two phenomena deteriorate the transmission efficiency of WSN, and the limited battery supply life is seriously affected [9], [10]. Therefore, it is necessary to study the performance indicators such as the
average BER of FSO communication links under interference in WSN in order to improve the operating efficiency of the entire networks.

Malaga model mentioned in many studies is relatively applicable for effects on weak, moderate and strong turbulent conditions [11], [12]. In the actual transmission link, the aperture average effect will be an economical and simple technique to make up the fading created by atmospheric turbulence, more specifically by increasing the aperture of the receiver, the power fluctuations created by atmospheric turbulence and other factors may be averaged to various apertures to compensate the transmission loss of the FSO link, which can also be seen as an ordinary method of spatial diversity when the receiver aperture is greater than the fading correlation length [13]–[15]. Especially in the short-distance transmission commonly used in WSN, the Gaussian beam wave model with limited beam width is more suited to describe the propagation characteristics of the transmitted light wave, and the receiving aperture is not negligible for the beam width at the destination. At this time, using the traditional Malaga model, the accuracy of it is verified by the simulation data of infinite plane waves and spherical waves and consequently, the receiving aperture on the WSN cannot be well discussed. Therefore, the FSO link model used in this study adopts the newly proposed Exponentiated Weibull (EW) distribution model, which is defined by simulation of Gaussian beam wave as well as comprehensive experimental data over various turbulence conditions. Besides EW distribution model also can better fit the actual experimental data from weak to strong atmosphere turbulence under the aperture averaging effect than the Malaga models mentioned in [16]–[18], particularly for the dot-shaped aperture. Especially in the WSN communication link with high accuracy requirements, it is attached great importance to select the EW model to study the average SNR, outage probability and the channel capacity of the link.

As the most generalized model for the FSO link, Malaga model has been widely studied, the performance of Malaga link considering pointing error is studied [19], [20] has investigated average capacity of Malaga link with pointing errors when adaptive transmission is adopted [21] proposed a novel indirect diffused light FSO system in vehicular networks, high speed data transmission is realized. A balanced detector in FSO link with optical fiber is presented to describe the propagation characteristics of the transmitted light wave, and the receiving aperture is not negligible for the beam width at the destination. At this time, using the traditional Malaga model, the accuracy of it is verified by the simulation data of infinite plane waves and spherical waves and consequently, the receiving aperture on the WSN cannot be well discussed. Therefore, the FSO link model used in this study adopts the newly proposed Exponentiated Weibull (EW) distribution model, which is defined by simulation of Gaussian beam wave as well as comprehensive experimental data over various turbulence conditions. Besides EW distribution model also can better fit the actual experimental data from weak to strong atmosphere turbulence under the aperture averaging effect than the Malaga models mentioned in [16]–[18], particularly for the dot-shaped aperture. Especially in the WSN communication link with high accuracy requirements, it is attached great importance to select the EW model to study the average SNR, outage probability and the channel capacity of the link.

II. SYSTEM AND CHANNEL MODELS

The system model of transmission from the sensor collection source to the destination information center in a certain area is shown in Fig. 1. The data information of the sensor collection source is modulated by PPM encoder and transmission by parallel-to-serial conversion to BPSK subcarrier modulator, after adding the DC bias and digital-to-analog, the subcarrier modulated signal is got, which will be loaded onto the light wave through laser deriver. The light wave carrying the information is transmitted to the Exponetiated Weibull turbulence channel through an optical antenna, which will be received by a certain aperture optical received. Then the received signal is converted into the electrical signal and processed by the filter and sampler, the signal is divided into parallel low rate signal by serial-to-parallel conversion, finally the output information can be obtained in destination. When the information of the sensor collection source
is transmitted by different modulation methods, we use the simple structure and more practical IM / DD technology at the receiver end. The signal generated at the destination is:

$$y = Rhx + n.$$  \hspace{1cm} (1)

where $h$ defined as the accumulated channel gain, which is indicated as $h = h_0h_1h_2$. $h_a$ is expressed as the fading caused by turbulence, and $h_b$ indicates the fading caused by the pointing error. $h_1$ stands for a deterministic fading caused by atmospheric attenuation, which can be set to a constant of $1$ without loss of generality. $R$ is the corresponding coefficient of the photodetector. $x$ is a BPSK subcarrier intensity modulated signal, the average power of which is $P$. $n$ is independent of the Gaussian white noise with a mean of $0$ and a variance of $\sigma_n^2$. Here we can give the definition of the instantaneous signal-to-noise ratio (SNR) at the receiving end as:

$$\gamma = \frac{R^2P^2h^2}{2\sigma_n^2} = \overline{\gamma}h^2.$$  \hspace{1cm} (2)

where $\overline{\gamma} = \frac{R^2P^2}{2\sigma_n^2}$ is the averages SNR in FSO link.

Considering the aperture average effect, the channel of the FSO link follow the Exponential Weibull (EW) distribution can be given by [16]:

$$f_{h_a}(h_a) = \frac{\alpha\beta}{\eta} \left( \frac{h_a}{\eta} \right)^{\beta-1} \exp \left[ - \left( \frac{h_a}{\eta} \right) ^\beta \right] \left[ 1 - \exp \left[ - \left( \frac{h_a}{\eta} \right) ^\beta \right] \right] ^{\alpha-1}.$$  \hspace{1cm} (3)

where $\alpha, \beta > 0$ are defined as the shape parameters, $\gamma > 0$ is defined as scale parameter, both of them can be calculated by the (20)-(22) in [18]. By the relation of $f_{y_a}(\gamma_a) = \frac{1}{\gamma \sqrt{\pi}} f_{h_a}(\sqrt{\gamma})$, the Probability Density Function (PDF) of the instantaneous SNR is obtained by:

$$f_{y_a}(\gamma_a) = \frac{\alpha\beta}{\gamma \sqrt{\pi}} (\eta \gamma)^{\beta-2} \exp \left( -i \gamma \right) \left[ 1 - \exp \left( -i \gamma \right) \right] ^{\alpha-1}. \hspace{1cm} \text{ (4)}$$

where $\gamma = \sqrt{\gamma / (\overline{\gamma} \eta^2)}$.

The pointing error is due to the slight shaking that will cause the center of the beam spot and the receiver aperture to be at different points. The PDF of intensity scintillation fading affected by the point error can be given by [36]:

$$f_{h_p}(h_p) = \frac{\rho^2}{A_0^2} h_p^{\rho^2-2} \; 0 \leq h_p \leq A_0.$$  \hspace{1cm} (5)

where $A_0$ is received optical power when radial displacement is $0$, given as $A_0 = [\text{erf}(\nu)]^2$, where erf (●) is error function. $\rho$ denotes the ratio of the equivalent beam waist radius and jitter standard deviation at the receiving end which represented as $\rho = \omega_{eq} / (2\sigma_j)$. $\sigma_j$ denotes the jitter standard deviation, the larger the $\sigma_j$ is, the greater the beam jitter is, and means the stronger the pointing error is.

$\omega_{eq}$ stands for the equivalent beam width that can be expressed as $\omega_{eq} = \omega \sqrt{\pi} \text{erf}(\nu) / 2\nu \exp (-\nu^2)$ and $\nu = \sqrt{\pi / 2} (a/\omega)$, $a$ denotes the radius of optical receiver, $\omega$ is the beam width at distance $z$.

The joint error model is solved as follows:

$$f_{h}(h) = \frac{1}{h_a} \int f_{h_p}(h_p) \cdot f_{h_a}(h_a) \; dh_a$$

$$= \frac{\rho^2}{A_0^2} \left( \frac{h}{h_a} \right)^{\rho^2-2} f_{h_a}(h_a). \; \; 0 \leq h \leq A_0h_a. \hspace{1cm} \text{ (6)}$$

where $\Gamma (\bullet)$ stands for the Gamma function, $G (\bullet)$ represents the Meijer’s G function [37]. Utilizing Formula of Gamma Function $\int e^{-x} x^{a-1} \; dx = \sum \frac{\Gamma (\delta + 1)}{i! \Gamma (\delta - i + 1)} \exp \left[ -i \left( \frac{h_a}{\eta} \right)^\beta \right]. \hspace{1cm} \text{ (7)}$

The PDF of the instantaneous SNR in FSO link combines turbulence fading and pointing error is derived as follows:

$$f_{y}(\gamma) = \frac{\alpha\beta^2}{(\eta A_0)^{\rho^2}} \left( \sqrt{\gamma} \right)^{\rho^2} \sum_{i=0}^{\infty} \frac{(-1)^i \Gamma (\alpha)}{i! \Gamma (\alpha - i) (1 + i)^{1 - \frac{\rho^2}{\beta}}} G \left[ 2, 1 \right] \left[ \frac{1 + i}{(\eta A_0)^{\beta}} \sqrt{\gamma} \right] \left[ 0; 1 - \frac{\rho^2}{\beta} \right]. \hspace{1cm} \text{ (8)}$$

The Cumulative Probability Function (CDF) of the instantaneous SNR is given by integration [16]:

$$F_{y}(\gamma)$$

$$= \int_0^\gamma f_{y}(\gamma) \; d\gamma$$

$$= \frac{\alpha\beta^2}{\beta (\eta A_0)^{\rho^2}} \left( \sqrt{\gamma} \right)^{\rho^2} \sum_{i=0}^{\infty} \frac{(-1)^i \Gamma (\alpha)}{i! \Gamma (\alpha - i) (1 + i)^{1 - \frac{\rho^2}{\beta}}} G \left[ 2, 1 \right] \left[ \frac{1 + i}{(\eta A_0)^{\beta}} \sqrt{\gamma} \right] \left[ 0; 1 - \frac{\rho^2}{\beta} \right]. \hspace{1cm} \text{ (9)}$$

III. PERFORMA NAL ANALYSIS

A. OUTAGE PROBABILITY

Outage probability represents the probability that the instantaneous SNR is lower than the preset threshold value, $\gamma_{th}$, that is $P_{out} = P(\gamma \leq \gamma_{th})$. The outage probability of the link is
According to (2) and (8) in [31], the deterministic relation of LPPM-BPSK-SIM can be given by

\[
P_{\text{out}} = \frac{\alpha \rho^2}{\beta (\eta A_0)^2} \left( \sqrt{\frac{\gamma_i}{\gamma_0}} \right)^{\rho^2} \sum_{i=0}^{\infty} \frac{(-1)^i \Gamma (\alpha)}{i! \Gamma (\alpha - i) (1 + i)^{1 - \frac{\rho}{\beta}}}\]

\[G = \frac{2}{3} \left[ \frac{1 + i}{(\eta A_0)^2} \left( \sqrt{\frac{\gamma_i}{\gamma_0}} \right)^{\rho} \left( 1 - \frac{\rho^2}{\beta} \right) \right]. \tag{10}\]

**B. AVERAGE BIT ERROR RATE**

There are many kinds of modulation methods that can be applied to transmission system of WSN, which can generate a considerable difference in performance of system due to their different properties, for example, spectrum efficiency and bandwidth efficiency. However, it is certain that the average BER of the system is the simplest and the most intuitive criterion to assess the quality of different modulation schemes. As mentioned above, BPSK, as a simple and low BER modulation method, has been widely concerned in WSN system. Reference [38] studied the performance of BPSK, and we take it as a comparative basis.

The newly proposed PPM-BPSK-SIM combines two traditional modulation methods, PPM and BPSK-SIM. That is, the information symbols are modulated to parallel signal by the PPM encoder, after being converted, the high rate is, the information symbols are modulated to parallel signal with additional modulation methods, PPM and BPSK-SIM. That is a sequence denoted as \(n, \ldots, \frac{n+m-1}{m}\). As an example, \(4, 2\) can be expressed as \(\Delta(m, n) = \frac{n+1}{m}, \frac{n+2}{m}, \ldots, \frac{n+m}{m}\).

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Using the relationship of Meijer’s G function: \(\text{erfc}(x) = \frac{1}{\sqrt{\pi}} G_{1,1}^{0,2}(x^2, 1, 0, 0, 0, 0)\) and (2.24.1.1) in [43], the closed form formula for LPPM can be obtained:

For hybrid LPPM-BPSK-SIM empathically, the unconditional BER can be derived by:

\[
P_{\text{LPPM-BPSK-SIM}} = \int_{0}^{\infty} f_{\gamma}(\gamma)P_{e, \text{LPPM-BPSK-SIM}} d\gamma \]

\[
= \int_{0}^{\infty} \frac{\alpha \rho^2}{2 (\eta A_0)^2 \gamma} \left( \sqrt{\frac{\gamma_i}{\gamma_0}} \right)^{\rho^2} \sum_{i=0}^{\infty} \frac{(-1)^i \Gamma (\alpha)}{i! \Gamma (\alpha - i) (1 + i)^{1 - \frac{\rho}{\beta}}} \\left[ 1 - \frac{\rho^2}{\beta} \right] \right] * \text{erfc} \left( \frac{1}{2} \sqrt{\frac{\gamma L \log_2 L}{L}} \right) d\gamma. \tag{11}\]

\[
P_{\text{LPPM}} = \int_{0}^{\infty} f_{\gamma}(\gamma)P_{e, \text{LPPM}} d\gamma \]

\[
= \int_{0}^{\infty} \frac{\alpha \rho^2}{2 (\eta A_0)^2 \gamma} \left( \sqrt{\frac{\gamma_i}{\gamma_0}} \right)^{\rho^2} \sum_{i=0}^{\infty} \frac{(-1)^i \Gamma (\alpha)}{i! \Gamma (\alpha - i) (1 + i)^{1 - \frac{\rho}{\beta}}} \right] * \text{erfc} \left( \frac{1}{2} \sqrt{\frac{\gamma L \log_2 L}{L}} \right) d\gamma. \tag{11}\]

**C. AVERAGE CHANNEL CAPACITY**

The accurate definition of the average channel capacity can be given in [44], [45] which is \(C = E \left( \log_2 (1 + \gamma) \right)\). \(E(\bullet)\) refers to the mathematical expectation operator. Based on this, the normalized average channel capacity of the FSO link can be obtained as:

\[
< C_{\text{ave}} > = \frac{1}{\ln 2} \int_{0}^{\infty} \ln (1 + \gamma) f_{\gamma}(\gamma) d\gamma \]

\[
= \frac{1}{\ln 2} \int_{0}^{\infty} \ln (1 + \gamma) f_{\gamma}(\gamma) d\gamma. \tag{15}\]

Using the relationship of Meijer’s G function: \(\text{erfc}(x) = \frac{1}{\sqrt{\pi}} G_{1,1}^{0,2}(x^2, 1, 0, 0, 0, 0)\) and (2.24.1.1) in [43], the mathematical closed analytical formula of the above formula can be obtained (16) as shown at the bottom of the next page.

**IV. NUMERICAL RESULTS AND DISCUSSION**

This Section will use the mathematical closed formulas obtained in Section III to analyze the performance of the FSO link under the hybrid various PPM-BPSK-SIM, such as outage probability, BER, and average channel capacity.
Therefore, we use the novel EW distribution model to quantitatively describe atmospheric turbulence, the interference caused by pointing errors is considered, and the receiver employs IM/DD to obtain the performance of the FSO transmission link. Therefore, taking the average aperture effect into account, we set the receiver aperture to \( D = 200 \text{mm}, 100 \text{mm}, 50 \text{mm}. \) Besides, we set turbulence model parameter \( \sigma_R = 0.32, 2.22, 15.9 \) in correspondence with weak, moderate and strong turbulence intensity, and the wavelength is 1550nm. In (7)-(15), the closed form is given by the form of infinite series, when \( i = 100 \) is selected in the calculation, the closed type basically converges, meanwhile the truncation error of the closed type is less than \( 10^{-10}. \) Considering the similarity of the formulas, we choose some different cases in each picture for Monte Carlo simulation, the simulation results are in closely approximate with the analytic results, which validate the correctness of our formulas. The parameters of EW distribution under different conditions are listed in TABLE 1 [16], and TABLE 2 shows the parameters of the link [47], [48].

Setting the SNR threshold as \( \gamma_0 = 10 \text{dB}, \) the outage probability of the FSO transmission link against average SNR is displayed in Fig. 2 and Fig. 3 when considering turbulent fading and pointing error. Clearly, the descent in the turbulence intensity causes the decrease of the outage probability, and when \( \alpha \) decreases, which means the pointing error decreases, the outage probability drops significantly. When the pointing

\[
P_{\text{LPPM}} = \frac{-\alpha \sigma \tau}{2 \sqrt{\pi} \mu^{\frac{\alpha}{2}}} k^{-1/2 - \frac{\alpha}{2}} \sum_{i=0}^{\infty} \frac{(1 + i)^{\alpha}}{(i + 1)^{1 - \tau}} \left[ (1 + i)^{1/2} \right] \frac{\Delta(k, 1 - \tau)}{\Delta(k, 0)} \frac{\Delta(l, 1 - \rho)}{\Delta(k, 1 - \tau)} \frac{\Delta(l, 1 - \rho)}{\Delta(l, -\rho)} \frac{\Delta(k, 1)}{\Delta(l, -\rho)} \right].
\]

(12)

\[
P_{\text{LPPM-BPSK-SIM}} = \frac{-\alpha \sigma \tau}{4 \sqrt{\pi} \mu^{\frac{\alpha}{2}}} \sum_{i=0}^{\infty} \frac{(1 + i)^{\alpha}}{(i + 1)^{1 - \tau}} \left[ (1 + i)^{1/2} \right] \frac{\Delta(k, 1 - \tau)}{\Delta(k, 0)} \frac{\Delta(l, 1 - \rho)}{\Delta(k, 1 - \tau)} \frac{\Delta(l, 1 - \rho)}{\Delta(l, -\rho)} \frac{\Delta(k, 1)}{\Delta(l, -\rho)} \right].
\]

(14)

By the form of infinite series, when \( i = 100 \) is selected in the calculation, the closed form is given by the form of infinite series, when \( i = 100 \) is selected in the calculation, the closed type basically converges, meanwhile the truncation error of the closed type is less than \( 10^{-10}. \) Considering the similarity of the formulas, we choose some different cases in each picture for Monte Carlo simulation, the simulation results are in closely approximate with the analytic results, which validate the correctness of our formulas. The parameters of EW distribution under different conditions are listed in TABLE 1 [16], and TABLE 2 shows the parameters of the link [47], [48].

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Receive Aperture} & \text{Turbulence} & \alpha & \beta & \eta \\
\hline
\text{Weak} & 1.84 & 7.11 & 0.99 \\
D=100mm & Moderate & 3.52 & 2.15 & 0.76 \\
D=200mm & Strong & 4.82 & 1.07 & 0.47 \\
D=50mm & Moderate & 2.15 & 5.40 & 0.96 \\
\hline
\end{array}
\]

TABLE 1. The parameters of EW distribution for different conditions.

\[
\begin{array}{|c|c|c|}
\hline
\text{Parameter} & \text{Value} \\
\hline
\text{Wavelength (k=1440nm) propagating path(z)} & 5000m \\
\text{Corresponding beam width(\alpha) at z} & 0.02\text{mrad}\ast z \\
\text{Corresponding jitter standard deviation(\sigma_j)} & 20\text{cm or 30cm} \\
\hline
\end{array}
\]

TABLE 2. The parameters of the FSO link.
becomes 4dB while the pointing error is high. Such research has revealed that when the pointing error is low, the outage probability of the transmission link is more influenced by turbulence, and artificially reducing the pointing error, such as correcting the symmetry of the transmitter and receiver or reducing the jitter of the transmitted beam can weaken the influence of uncontrollable atmospheric turbulence fluctuations on the outage performance of FSO link. Besides, under strong turbulent conditions, the atmospheric fading is severe, and the impact of the change in pointing errors on the outage performance will become smaller at this time, therefore it is not cost-effective to correct the pointing error to produce such enhancement in performance. When considering the receiver aperture, larger receiving aperture brings better outage performance. When the receiver with larger aperture is adopted in the system, the influence of pointing error on the system will be overcome to some extent. When the average SNR is fixed to 60dB, the decrease in pointing error will lead to the changes in outage probability of three systems with different apertures sizes ($D = 50$mm, 100mm and 200mm), which are $1.8 \times 10^{-3}$ to $1.2 \times 10^{-4}$, $1.5 \times 10^{-5}$ to $3.4 \times 10^{-8}$ and $1.4 \times 10^{-7}$ to $2.6 \times 10^{-12}$. That is, the outage performance of the system with large receiving aperture is more sensitive to the change of pointing errors, which means it is more reasonable to adjust the alignment of the receiving and transmitting antennas to obtain the outage probability required by the actual WSN’s communication system.

**Fig. 4 (a) and Fig. 4 (b)** respectively study the performance of various LPPM and hybrid various LPPM-BPSK-SIM under weak turbulence conditions with the two pointing error strengths. We can conclude that increasing the value $L$ will significantly improve the BER performance of the link. LPPM will outperform BPSK in BER when value $L$ is greater than four, while adopting LPPM-BPSK-SIM, the required $L$ needs to be greater than eight. Through a comparative study on the change of the pointing error intensity, it is found that the two modulation methods are close to improvement brought by the traditional BPSK modulation.
However, increasing $L$ will reduce the bit error rate at the expense of band efficiency, so we focus our research on $L$ valued at 8 or 16.

**FIGURE 5.** Average BER against average SNR under strong turbulence with high pointing error, the corresponding jitter standard deviation is $\sigma_s = 30$ cm. Turbulence parameters are $\alpha = 4.82$, $\beta = 1.07$ and $\eta = 0.47$.

Fig. 5 investigates the BER performance of high pointing error and strong turbulent fading conditions. It is discovered that in the case where the channel quality is very poor, the conclusions obtained are consistent with the case where the channel quality is better, that is, $L$ must be greater than four or eight, respectively, so that the BER performance of the two modulation methods outperforms the traditional BPSK system, which is also consistent with the conclusion in Fig. 4 (a) and Fig. 4 (b). Then Fig. 6 compares the performance of the hybrid modulation system under strong and weak turbulence with low pointing errors. When the intensity of turbulence rises, the average BER will increase significantly. In order to reduce the average BER from $10^{-3}$ to $10^{-6}$, the link adopting 16PPM-BPSK-SIM needs to increase the average SNR by 14 dB under strong turbulence, while only 9.5 dB of increase in the average SNR is needed under weak turbulence. Meanwhile, when adopting the 16PPM-BPSK-SIM in transmission link, in order to make the BER lower than $10^{-3}$, the average SNR under weak turbulent conditions will be 7 dB lower than under strong turbulent conditions, in both cases, the BER of 8PPM-BPSK-SIM is tremendously close to that of BPSK.

Considering that the aperture averaging effect has a certain impact on improving the link’s performance, the influence of the receiver aperture size on the performance of BER will be studied next. Fig. 7 depicts the average BER in a large aperture receiver ($D = 200$ mm) against the average SNR. The simulation results show that when the receiver aperture is increased to $D = 200$ mm, the link’s performance is conspicuously improved. Meanwhile, the improvement effect of LPPM and LPPM-BPSK-SIM compared with traditional BPSK is the same as when $D = 100$ mm. It is worth noting that when the pointing error is low ($\sigma_s = 20$ cm), increasing the value $L$ of the two various modulation methods will distinctly improve the performance of the link, and under the same average SNR, the BER of 64PPM-BPSK-SIM is only $6.25 \times 10^{-5}$ of 8PPM-BPSK-SIM, while under high pointing error ($\sigma_s = 30$ cm), it drops to $6.67 \times 10^{-3}$, which means high pointing error will impair the effect of the value $L$. The research results show that in the high pointing error ($\sigma_s = 30$ cm) situation, the cost-effectiveness of reducing the average BER by increasing the average symbol length will decrease, and the rate of the average BER decreasing with the increase of the average SNR will become slow, which leads to the flat decline of the average BER. Consequently, the link must be under the conditions with high average BER to obtain ideal average BER performance.
Fig. 8 compares the influence of the receiver aperture size on the system. It can be found that when the receiving aperture rises, the average BER performance of the system is significantly improved. For example, when the system adopts 8PPM-BPSK-SIM or 16PPM-BPSK-SIM, in order to achieve average BER of $10^{-6}$, the receiver aperture of $D = 200\text{mm}$ will bring a gain of 15.2dB in the average SNR over the receiver aperture of $D = 100\text{mm}$. On the other hand, under three different receiving apertures, LPPM and LPPM-BPSK-SIM have the same improvement in the average BER performance of BPSK system, but larger receiving aperture makes change in average BER more sensitive to the average SNR. When adopting 16PPM-BPSK-SIM, in order to reduce the average BER from $10^{-3}$ to $10^{-6}$, the increase needed in the average SNR is 9.4dB and 10.8dB for receiver with $D = 200\text{mm}$ and $D = 100\text{mm}$, respectively, which indicates that the large aperture receiver system is more suitable for communication under the complex channel environment. Therefore, in different turbulence, pointing errors and receiver aperture conditions, the hybrid various PPM-BPSK-SIM may conspicuously enhance the BER performance of the link. Compared with the BPSK system, the improvement of hybrid PPM-BPSK-SIM also exists even under conditions of strong turbulence and severe pointing errors.

Fig. 9 and Fig. 10 depict the average channel capacity under different turbulence intensities, pointing errors and receiving apertures, respectively. The results show that the average channel capacity increases with the decrease of turbulence intensity and pointing error and the increase of receiving apertures. Under strong turbulent conditions, when the $\sigma_s$ changes from 20cm to 30cm and the average SNR is 40dB, the average channel capacity of the FSO link increases by 0.8 bit/s/Hz, the fluctuation of the corresponding jitter standard deviation in this range seems to have a more significant impact on the average channel capacity. When the pointing error is fixed, the channel capacity will decrease slightly as the turbulence intensity increases. We have concluded that lower pointing error causes higher average capacity, then pointing errors may make the influence of turbulence on capacity performance more serious. Besides, when the receiving aperture increases, the channel capacity of the link will be significantly improved compared with the effect caused by pointing errors. When the average SNR is 40 dB, the channel capacity gain caused by the decrease of pointing error is approximately 0.7bits/s/Hz under the three sizes of receiving apertures ($D = 50\text{mm}$, $D = 100\text{mm}$ and $D = 200\text{mm}$). The above study shows that when the receiving aperture is fixed, the channel capacity of the link cannot be improved obviously by adjusting the pointing errors artificially. Specifically, in WSN applications where high transmission speed is required, the receiver with larger aperture can better meet the actual communication needs according to the averaging aperture effect.
V. CONCLUSION

In summary, we have studied the performance of the FSO link in the wireless sensor networks when the hybrid LPPM-BPSK-SIM is adopted. The channel conditions of the transmission link comprehensively consider the fading caused by the atmospheric turbulence and pointing error that undergoes novel Exponentiated Weibull model. Based on the precise Meijer’s G function, the closed mathematical form of the joint error model of the link is derived, from which the unconditional BER of the transmission link is obtained. In addition, the performance of the outage probability and channel capacity are also investigated on the basis of the joint error model above.

The results of our research prove that the average BER performance could be improved by adopting hybrid LPPM-BPSK-SIM. Moreover, the exact improvement effect has a close relationship with the average symbol length. More specifically, when the average symbol length increases, the average BER of the link will decrease significantly. Especially when it is greater than eight, the average BER of the link will be better than adopting BPSK. Even in the condition of strong turbulence and high pointing errors, adopting hybrid LPPM-BPSK-SIM can significantly improve the link’s performance stably. On the other hand, pointing error will obviously worsen the link quality, which will impair the enhancement in BER performance generated by the increase of L, as well as causing the average BER to decrease more slowly as the average SNR increases. Owing to the aperture averaging effect, when the receiver aperture is increased, it can obviously compensate the system fading caused by strong turbulence and high pointing error, so that the system performance is significantly improved. The outage probability and average channel capacity of the link seem sensitive to the pointing error, and the pointing errors will make the impact of atmospheric turbulence more serious. However, the joint utilization of hybrid LPPM-BPSK-SIM and aperture averaging effect can effectively improve the transmission efficiency of wireless sensor networks under any conditions of turbulence and pointing errors. In our work, we study the outage probability, average BER and average channel capacity of FSO link. On the premise of fully considering the uncontrollable factor of atmospheric turbulence, we propose the practical strategy, that is, to adjust the changeable pointing error parameters and receiver aperture in the actual scene, which supplies a conception for the deployment of FSO in WSN.

APPENDIX

The following abbreviations are used in this manuscript:

- **FSO**: Free space optical
- **WSN**: Wireless sensor network
- **BER**: Bit error rate
- **AA**: Aperture average
- **EW**: Exponentiated Weibull
- **OP**: Outage probability

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