A Study on a Membrane Ceilings Business under Ambiguity*

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Abstract: This paper selects an actual company and conducts a case study with respect to a new business for the company. We observe that the company is starting a membrane ceilings business alongside its existing business selling iron. We apply a real options approach to it and provide valuable implications with regard to managerial decision-making under uncertainty. In this paper, in addition to the standard Net Present Value method and the option pricing theory under risk, the potential model risk is explicitly analyzed. For this reason, this paper analyzes the business under ambiguity. In order to analyze the actual business, we first develop a systematic procedure for deriving managerial flexibility under uncertainty. This includes specifying important risk factors, seeking real options, parameter estimation, handling ambiguity, and deriving the optimal strategy. It reveals that the company is subject to four sources of uncertainty and has two types of real options. Furthermore, we formulate the business as a multiplier robust control problem to evaluate the project value under ambiguity. Our quantitative analysis shows that the real options have a significant impact on the project value of the membrane ceilings business. It further shows that, although the presence of ambiguity changes the optimal exercise strategy, the difference between the optimal strategy under ambiguity and that under risk is not significant from a practical viewpoint.

Keywords: Real options; Ambiguity; Optimal exercise strategy; Case study

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1 Introduction

Real options have been actively investigated since their introduction in the 1970s, and their application to business practice has been intensively discussed since the 1990s. In theory, real options can be applied to a wide range of problems. Among these, consideration of starting a new business, and research and development (R&D) are important areas, particularly because they involve significant managerial flexibility under high level of uncertainty. For this reason, a large number of theoretical studies have analyzed these areas, including decisions relating to an entry into a new market and R&D valuation.

Let us review some of the related studies although there are many other studies that are not referred to in this paper. The work of [11] is one of the earliest studies that applied the real options approach for examining entry and exit decisions under uncertainty. Other early studies are summarized in the book of [32], while recent studies are found in [38]. In the work of [25] the authors applied the real options approach to analyze entering and exiting new businesses. Regarding real options analyses for R&D investment, [10] analyzed behaviors of companies that avoid investing in R&D project, and found that it depends on whether they have internal or external knowledge resources. [39] examined conditionality of R&D investments from the real option’s perspective. [33] and [7] discussed early-stage biotechnology investment. We refer the readers to [30] for comprehensive study. More recently, [23] analyzed capital investment in innovative capacity. Based on the fact that innovation-related activities could be sources of new ideas and opportunities that will create growth options, they proposed a model particularly for the innovative capacity and conducted an empirical test.

In addition to the theoretical interest in academics, practitioners pay special attention to their applications since they recognized the importance of real options in actual business, especially in an uncertain environment. Thus, the demand for empirical studies and case studies that discuss the validity or applicability of the real options has been increased. The first case study for the application of the real option approach was reported in [29]. An empirical study in the pharmaceutical industry was conducted in [20], in which they chose top 500 companies to examine the applicability of the real option approach. In the work of [27], the authors argued the importance of the real options reasoning in R&D management for pharmaceutical firms and conducted an empirical investigation based on more than thirty thousand R&D projects in US, Japan and Europe. Human resource options were examined in [4], in which they applied the simple Black-Scholes option pricing model to obtain the value of human capital. In addition, [2], [28], [1] are examples in which the authors adopted the real options approach for analyzing particular cases.

In comparison to those theoretical studies, it is fair to state that the number of studies that pay main attention to practical application is limited due to the following reasons. First, the framework for applying real options to business practice is not sufficiently systematic for non-expert business people, without which it is difficult for them to put a real options approach into practice. The second point, which is often pointed out among practitioners, is mathematical complexity for evaluating real options. Finally, the company does not always have suitable data available in the market. In particular, this is the case for a company in entering a new market or in considering a new business. For further discussion, see [24], and reference therein.

In this paper, we select an actual company and conduct a case study with respect to a business
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currently considered, whereby the company is contemplating starting a membrane ceilings business alongside sale of iron as its existing business. We discuss optimal investment decisions under uncertainty. There are two important aspects in analyzing the business opportunity. On the one hand, as membrane ceilings are relatively new products, the market is not well established. Furthermore, the company needs to consider other possibilities that could affect its business profitability, such as a surge in demand and the entry of new competitors. This means that this business is highly uncertain. On the other hand, we have observed that the company can create many types of flexibility. Therefore, the real options approach is promising for analyzing this new business for the company.

In this paper, we first discuss a systematic procedure for identifying important risk factors, seeking real options, and estimating parameter values. With our procedure, we pick up four sources of underlying risk and two important real options. We refer to the approach in [34] and [37].

Then, we construct a lattice model to derive the optimal exercise strategy of the real options and evaluate the project values under the multiple sources of risk. The lattice model has become one of the standard approaches in real options analysis, especially for a practical use. Well-known multinomial lattice models were developed in [6], [22]. A recent study in [21] referred to many of the existing lattice models. The lattice procedure used in this paper is multi-dimensional in which each risk process is described as a trinomial tree. It can be an extension of the model in [36]. Although the lattice procedure is fairly standard, we show that it is suitable and useful for our analysis, particularly in the presence of ambiguity. Two sequential real options are evaluated as a switching option with three stages. For a comparison, Net Present Values (NPVs) under a couple of specious scenarios are calculated and used as benchmarks. A Monte Carlo Discounted Cash-Flow (DCF) method is also utilized for analyzing the distribution of the project values.

It is crucially important for the chief manager of the company to figure out the optimal exercise strategy, because the company cannot obtain the maximized project value without the optimal strategy. However, it is not easy to clarify it as we consider the four sources of uncertainty. To that end, we propose to use a simple yet useful way to roughly understand the optimal exercise strategy that helps the manager make reasonable decisions in this complex environment. To be concrete, we adopt a logistic regression analysis that enables us to visualize the optimal exercise boundaries.

Another distinct feature of the paper is that we explicitly consider the presence of ambiguity. It is crucial for the company to recognize the manager’s misspecification regarding the parameter values of the underlying risks. This paper formulates it as a multiplier robust control problem to analyze the value of the project under ambiguity. Accordingly, the optimal solution of the problem can be derived by solving a maxmin optimization problem.

The concept of ambiguity was first mentioned in the seminal work of [13]. Since then, a number of related studies have been published in the field of modern decision theory, as many researchers recognized that decision makers usually do not know the precise probabilities of potential outcomes. For further discussion of ambiguity, see [16], [8], [12], [18], [35] and the references therein.

Toward the quantitative analysis under ambiguity, [17] first analyzed multiple prior preferences in which they considered a set of equivalent probability measures to take ambiguity into account. Sequently, [19] introduced a notion of multiplier preferences. In their formulation they also introduced the relative entropy as a penalty factor in the optimization. [26] generalized the ideas and considered variational preferences. [3] applied this approach to optimal portfolio decisions for financial institu-
In this paper, we develop a numerical procedure for solving the robust control problem on the proposed lattice model. It should be emphasized that for the classical lattice models ([6], [22]) we need to change the structure of the lattice when we consider the stochastic process under a different probability measure. Note, however, that under the lattice model developed in this paper, the change of probability measure can be represented by simply adjusting transition probabilities whereas the structure of the lattice remains unchanged. This feature enables us to properly compare the optimal exercise policy under different levels of uncertainty.

Results of the analysis in this paper are summarized as follows. First of all, our quantitative analysis reveals that the company has valuable real options and it can significantly increase the project value by properly exercising them. Second, when we take ambiguity into account, the optimal exercise policy changes and the project value that includes the penalty term significantly decreases. However, the difference of the optimal exercise policy has little effect on the realized project value without the penalty term. Hence we can conclude that the optimal exercise policy is practically robust for the company.

The remainder of the paper is organized as follows. Section 2 presents the case study, including the features of the company and the proposed new project. We also identify the risks and embedded options in the project. Section 3 details a valuation model to be used, including NPV analysis, the real options approach under risk, and that under ambiguity. Section 4 provides the results and a discussion. Section 5 concludes the paper.

2 Description of the case study

2.1 Background

The company considered in this paper has engaged in the construction industry in Japan for more than half a century and is exposed to material cost and demand changes. Currently, the entire construction industry is booming due to significant demands of the upcoming Tokyo 2020 Olympic Games. Nevertheless, the chief manager of the company realizes potential risk to the existing business of decreasing demand after the games and also pays much attention to market risk in iron prices. As a result, he has decided to explore a new business that deals in membrane ceilings during the current boom so that the company can better prepare for any potential downside in the future. We select this new business as a case project and evaluate its profitability and risk.

It is well known that Japan is heavily exposed to the risk of earthquakes. This requires the prevention of secondary damage, of which the collapse of buildings during an earthquake is one of the most serious dangers. For example, the collapse of ceilings in a shelter poses a physical threat to many residents when an earthquake takes place. To prepare for this, the Ministry of Land, Infrastructure, Transport and Tourism implemented a number of government ordinances in 2014, for which one of the promising ways to resolve the danger of collapsing ceilings was to require membrane ceilings. Because of their lightweight, these can prevent ceilings from falling, and the damage can be less serious even if they do collapse. Membrane ceilings also have a number of other advantages, including that they are shock resistant and solid. In summary, membrane ceilings have many attractive features for Japanese
consumers.

It is also attractive for the company to install membrane ceilings as a new business. One merit is that the company can handle interior design in addition to its existing business of selling construction steel to clients. Hence, selling membrane ceilings can expand the capacity of the business beyond material sales. Another merit is that the membrane ceilings business is highly promising in that the ceilings are not only light but also have superior design properties. It also seems reasonable to assume that the business could ultimately be combined with a lighting business because they are permeable.

2.2 Four steps for analyzing the new business

In order to start the analysis, this paper develops a procedure that is illustrated in Figure 1. The idea in the figure is based on the discussion in [37], but we extend it for our case study. First of all, it is important to recognize that before the evaluation of the project in Step 4, three pre-evaluation steps are required: the identification of important risk factors (Step 1: Identify Risk), the identification of important managerial flexibility (Step 2: Identify Options), and the estimation of the model parameter values (Step 3: Estimation).

Let us first explain how we proceeded pre-evaluation steps. The chief manager of the company initially did not know anything about real options, although he did have some basic knowledge about Net Present Value (NPV) analysis. However, even if he had little idea about real options, he may still have possessed the necessary way of thinking about real options. Therefore, we interviewed him without using overly technical terminology to better grasp the present situation and help him identify factors required for our analysis.

In Step 1, to identify risks exposed in the new business, we refer to the approach detailed in [34]. Table 1 summarizes representative risks in the membrane business. These are summarized in the column labeled “Representative risks”. For example, we categorize the entry of new competitors in the row labeled “Source of risks” and the row labeled “Social”. Such risk may result in a decrease in price and demand for the company. Note, however, that uncertainty is not always unfavorable. For instance, the policies of government agencies such as the Ministry of Land, Infrastructure, Transport and Tourism, would be changed to prevent the sudden collapse of conventional ceilings. The policy change could increase the demand for membrane ceilings.

In Step 2, we identify real options associated with the risks identified in Step 1, which is also summarized in Table 1. The column labeled “Real options” is classified as follows. The first type is
to build a processing plant mainly for producing new types of products. The plant also enables the business to reduce the costs of transportation and manufacturing products. By building the plant, the company can also change the membrane ceilings to fit a desirable size and different shapes, such as rectangle, triangle, and circular. As the prices of the value-added membrane ceilings are higher, the company can increase its revenue by selling these membrane ceilings instead of conventional ones. Namely, by installing the new business, the company can differentiate itself from other companies and increase its profitability. It is also expected that the plant will eventually change shapes of ceilings, including circular and toroidal, thereby further increase demand for its products over that of other ceiling firms. Note that the company already has space available for building the new plant.

The second type of flexibility corresponds to an option that can connect the current business with new one. For instance, the membrane ceilings business is strongly related to the lighting business. Therefore, once a lighting business is commenced, it is possible for the company to exploit the synergies between the preexisting business, the membrane ceiling business, and a lighting business. Moreover, while membrane ceilings are currently used for mainly public facilities, it could be utilized in various nonpublic places such as exhibition rooms for private companies.

As the third type of flexibility, the company can change its sales structure in respond to Japanese standards. The company provided only fireproof ceilings when it started the membrane business. As time went on, the company knew that nonflammable ceilings were required in Japan. Thus, selling membrane ceilings made of nonflammable materials is paramount.

Although we have found many real options under different sources of risk as summarized in Table 1, not all of them are practically important. Some of them are economically less significant, or hard to implement in practice. After a series of intensive interviews with the chief manager of the company, we picked up the following four sources of uncertainty: surface area per unit order, price for the existing products per unit area, price for the value-added products per unit area, and ratio of the value added products to the total products. Furthermore, we focus on two types of real options for a practical application because they are the most critical for the quantitative analysis of the new business. They are, an option to sell nonflammable products, and an option to build a processing plant. Our reasoning is as follows. The company has already recognized the importance of nonflammable products, and is ready to sell them from year 2017. In addition, the manager believes that by building a new processing plant the sales of membrane ceilings can be enhanced. Considering the instability of the present situation and the feasibility of these options, we have concluded that the other possible options including starting a lighting business, building a laboratory, and increasing human resources are less important for the analysis.

In Step 3, we discuss estimation of the parameters. When we apply a Monte Carlo DCF method and the real options approach, we express risks as stochastic processes, hence we need to estimate the parameters. However, there is no existing reliable data available for membrane ceilings. Therefore, we employ the method of extracting the information from the manager’s foresight. However, there is a possibility of model risk, that is, the manager’s prediction contains bias and misspecification. To deal with these, we evaluate the project value in the presence of ambiguity.
| Basic elements of risks | Details | Representative risks | Representative flexibility | Real options |
|------------------------|---------|----------------------|--------------------------|--------------|
| Source                 | Physical | Delayed operation due to natural disasters | Expand production | To build a processing plant to enhance production |
|                        |         | Sudden demand increase due to earthquakes | | |
| Social                 | Customer apathy toward ceilings | Seek diversification | To start a lighting business |
|                        | Entry of new competitors | Product differentiation | To sell the circular membrane ceilings |
| Political              | Sudden policy changes of government agencies to ceilings | | |
| Operational            | Opportunity risk to lose customers | Expand production | To build a processing plant |
| Economic               | Increase of imported goods | Replace imported goods by the company’s products | To build a processing plant |
| Legal                  | Differences of laws between foreign countries and Japan | To quantify the strength | To build a laboratory |
|                        | | To provide what is suitable in Japan | To sell the nonflammable membrane ceilings |
| Cognitive              | Ambiguity of the manager forecast | | |
| Hazard factors         | Growing membrane ceilings market | Prepare for a huge demand | To build a processing plant |
|                        | Seek for innovation | To expand the current business to a new direction | To start a lighting business |
| Perils                 | Late delivery of orders by accident | | |
| Resources exposed to risk | Physical resource | (No dedicated staff) | | |
|                        | Human resource | Increase in the number of employees | To choose either experienced or inexperienced employees | To increase human resources |
|                        | Financial resource | (No impact on the entire management) | | |

This table summarizes representative risks and corresponding flexibility in the membrane business. It is created based on a series of interviews with the chief manager of the company. We refer to [34] to categorize types of risks that are listed in the first and second column of the table. Blank spaces for representative flexibility mean that risks related to flexibility do not have significant effects on the business.
3 A Valuation model

In Step 4, we evaluate the new project about the membrane ceilings and derive the optimal exercise strategy that maximizes the project value. We employ a Net Present Value (NPV) method, a Monte Carlo method, and a real options approach for the valuation. In the real options approach, we evaluate the two real options specified in the previous section. Furthermore, we examine the real options value under ambiguity to explicitly take the manager’s misspecification into consideration.

Data that we obtained for the quantitative analysis are as follows: number of orders per year, surface area per unit order, ratio of the existing products to the total products, ratio of the value-added products to the total products, profit rate, labor cost, tax rate, capital expenditure, expected growth rate. They are provided by the company or obtained by the interviews with the chief manager of the company.

The two types of real options have different impact on the profitability of the project. In the model we assume that a non-flammable ceiling is a substitute for existing products. Hence even if the total number of products remains unchanged, the profitability can be increased by selling non-flammable ceilings. In contrast, the company will be able to obtain three main advantages through constructing a new plant: an increase in production, a decrease in production time, and the supply of the circular membrane ceilings. The plant enables the company to increase supply of circular membrane ceilings while still continues to sell the same amount of existing products and non-flammable ceilings. According to the feasibility evaluation made by the company, the manager can exercise the option to sell non-flammable ceilings from 2017, whereas the option to build a process plant from 2020, on the condition that the company already starts selling non-flammable ceilings.

3.1 An NPV method and a Monte Carlo DCF method

We first apply a standard NPV method to the project valuation. Based on a series of interviews with the chief manager of the company, we pick up the following four sources of uncertainty: aggregated surface area per unit order ($Y_1$), ratio of the value added products to the total products ($Y_2$) price for the existing products per unit area ($Y_3$), and price for the value-added products per unit area ($Y_4$).

Note that the project horizon is six years, and the terminal value of the project is estimated based on the cash-flow at the terminal period. To estimate the discount rate we use comparable multiple valuation method along with past data of the company, data about ceilings, market data of iron prices, and managers foresight. In addition to calculating the project, we also examine the distribution of the project using a Monte Carlo DCF method. It enables us to figure out potential return and risk that the company is facing under the uncertainty.

3.2 A real options approach

We now explain how to evaluate a new business with the embedded real options. Note that one of the main advantages for the real options approach is that we can explicitly derive the optimal investment strategy under risk to maximize the project value.

For the purpose of modelling the underlying risks as stochastic processes, we change the variables as illustrated in Table 2, that is, Surface area for existing products per unit order ($X_1$), Surface area for value-added products per unit order ($X_2$), Price for the existing products per unit area ($X_3$), and...
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Table 2: Parameter values for the underlying risks.

| i | Parameter                                      | Initial | Drift   | Volatility |
|---|------------------------------------------------|---------|---------|------------|
| 1 | Surface area for existing products per unit order | 16.59   | 0.1340  | 0.19499    |
| 2 | Surface area for value-added products per unit order | 0.3595  | 0.5018  | 0.87123    |
| 3 | Price for the existing products per unit area    | 3.984   |         |            |
| 4 | Price for the value-added products per unit area | 4.204   | 0.1733  | 0.2666     |

This table summarizes estimated values for the four underlying risks. The underlying risks $X_i$ modeled in this paper are transformed from those obtained from the company manager $Y_i$ as follows. $X_1 = Y_1(1-Y_2)$, $X_2 = Y_1Y_2$, $X_3 = Y_3$, $X_4 = Y_4$. They are estimated by combining the past data with the manager’s prediction, and the best and worst case scenarios of the company’s forecast. In the paper, we assume that they independently follow geometric Brownian motions with the drift and the volatility coefficients shown in the table.

Price for the value-added products per unit area ($X_4$). These transformations are done for the following three reasons. First, modelling prices and areas is easier and more straightforward than modelling a ratio of a product. Second, the symmetry of the four risks helps us analyze the effect of each risk on the decision. Third, it is also natural to set correlation coefficients among the four risks.

We assume that these four risks follow independent geometric Brownian motions. The estimated drift and volatility terms are summarized in Table 2. They are estimated by combining the past data with the manager’s prediction, and the best and worst case scenarios of the company’s forecast. Note that these four underlying risks could take all positive values after the change of variables.

We utilize a lattice model to express a four-dimensional geometric Brownian motion:

$$dX_i(t) = \mu_i X_i(t)dt + \sigma_i X_i(t)dW_i, \quad i = 1, \ldots, 4. \quad (1)$$

Although the lattice procedure is well-known in the field of financial engineering, we briefly discuss it because there are several ways of constructing it, and the one developed in this paper is particularly useful in deriving and analyzing the optimal exercise strategy in the presence of ambiguity.

Let $z_i(m, k_i)$ represent the node of the trinomial model for the $i$-th standard Brownian motion at time point $t_m = m\Delta t$, $m = 0, 1, \ldots, M$, $\Delta t = T/M$ where $k_i$ represents the state of the $i$-th process, $T$ is the project horizon, and $M$ is the number of time points for the lattice model. To approximate a standard Brownian motion with a trinomial tree, we set

$$z_i(m, k_i) = k_i\Delta z; m = 0, \ldots, M; k_i = 0, \pm 1, \ldots, \pm m. \quad (2)$$

Let $\Delta z = \alpha\sqrt{\Delta t}$ with $\alpha > 0$, thus, the transition probabilities from node $k_i$ to node $k'_i$ is given as follows:

$$p_i(m; k_i, k'_i) = \begin{cases} \frac{1}{2\alpha^2}, & k'_i = k_i + 1, \\ 1 - \frac{1}{\alpha^2}, & k'_i = k_i, \\ \frac{1}{2\alpha^2}, & k'_i = k_i - 1, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

Applying Ito’s lemma to Eq. (1), the node of a geometric Brownian motion, denoted by $x_i(m, k_i)$, can be described as

$$x_i(m, k_i) = x_i(0, 0) \exp\left\{ \left( \mu_i - \frac{1}{2} \sigma_i^2 \right) \Delta tm + \sigma_i k_i \Delta z \right\}. \quad (4)$$
As each underlying risk is modeled as a trinomial tree, in total we generate a four-dimensional, 81-nominal lattice model. Let $z(m, k_1, k_2, k_3, k_4)$ be the node of the four-dimensional lattice model at time point $t_m$. The transition probability from node $z(m, k_1, k_2, k_3, k_4)$ to node $z(m+1, k'_1, k'_2, k'_3, k'_4)$ is given by the product of each transition probability, i.e., $\prod_{i=1}^{4} p_i (m; k_i, k'_i)$. We omit the details but the lattice model can be easily extended to a correlated geometric Brownian motion. Note that the logarithmic rate of return for each $x_i$ is given by

$$y_{\text{up}}^i = \log \frac{x_i (m+1, k_i+1)}{x_i (m, k_i)},$$
$$y_{\text{mid}}^i = \log \frac{x_i (m+1, k_i)}{x_i (m, k_i)},$$
$$y_{\text{down}}^i = \log \frac{x_i (m+1, k_i-1)}{x_i (m, k_i)}.$$

For valuing the sequential real options we regard them as switching options. A switching option is a general option which enables the holder of the option to switch from a stage to another by paying switching cost. To be concrete, we implement it a switching option with three stages: the current stage (Stage 0), a stage in selling nonflammable ceilings (Stage 1), and a stage in building the processing plant (Stage 2). Let us assume that the company can switch from one stage to another at discrete time points $t_m$ ($m = 0, 1, 2, \cdots, M$). Let $k = (k_1, k_2, k_3, k_4)$ denote a state of the underlying risks that are indexed as a node in the lattice model. The company obtains different amounts of cash-flows at time $m$ at each Stage $j$ at state $k$, denoted by $\text{CF}(m, j, k)$, since the company produces and sells different products in each stage. Let $C_{i,j}$ denote the switching cost from Stage $i$ to Stage $j$ with $i, j = 0, 1, 2$, assuming that possible switching is limited as shown in Figure 2 in practice.

For the evaluation, let $V_A(m, j, k)$ and $V_B(m, j, k)$ denote the project values after and before the decision making at time $t_m$, respectively. Using the dynamic programming principle, we can derive the following equations:

$$V_A(m, j, k) = e^{-r \Delta t} \mathbb{E}_m[V_B(m+1, j, \cdot)], \quad (j = 0, 1, 2),$$

where $r$ is the instantaneous discount rate, $\Delta t$ is the length of each time point and $\mathbb{E}_m[\cdot]$ represents the conditional expected value at time $t_m$. Based on the possibility of switching shown in Figure 2,

$$V_B(m, 0, k) = \text{CF}(m, 0, k) + \max \{V_A(m, 0, k), V_A(m, 1, k) - C_{0,1}\},$$

if the switching is possible, whereas

$$V_B(m, 0, k) = \text{CF}(m, 0, k) + V_A(m, 0, k),$$
otherwise. From Eq. (6), the business stays at Stage 0 when $V_A(m,0,k) \geq V_A(m,1,k) - C_{0,1}$ and it switches to Stage 1 when $V_A(m,0,k) < V_A(m,1,k) - C_{0,1}$.

Similarly, if the company is currently operating at Stage 1, it has an option to choose between remaining at Stage 1 and switching to Stage 2. Hence, the following equation is satisfied.

$$V_B(n,1,k) = CF(n,1,k) + \max(V_A(n,1,k), V_A(n,2,k) - C_{1,2}),$$
in case the switching is possible, and

$$V_B(m,1,k) = CF(m,1,k) + V_A(m,1,k),$$
otherwise. Lastly, in the case of Stage 2, $V_B(m,2,k)$ is written as

$$V_B(m,2,k) = CF(m,2,k) + \max\{V_A(m,2,k), V_A(m,1,k) - C_{2,1}\},$$
in case the switching is possible, and

$$V_B(m,2,k) = CF(m,2,k) + V_A(m,2,k),$$
otherwise.

### 3.3 Real options under ambiguity

For explicitly taking the manager’s misspecification into account, this paper considers a multiplier robust control model under ambiguity. For a further instruction of the multiplier robust control model, we refer the reader to [17] and [19], as mentioned in the introduction.

In this model, the company attempts to maximize the project value, while the Nature tries to minimize it. Consequently, it can be formulated as a maxmin problem, described as below:

Eq. (5) is replaced by

$$V_Q^A(m,j,k) = \min_{Q \in \mathcal{P}} e^{-\tau \Delta t} \left\{ E_{m}^{Q}[V_Q^B(m+1,j,\cdot)] + \theta R_{m+1}(Q||\mathcal{P}) \right\}, \quad (j = 0, 1, 2),$$

(7)

where

$$V_Q^B(m,0,k) = CF(m,0,k) + \max(V_Q^A(m,0,k), V_Q^A(m,1,k) - C_{0,1}),$$
if the switching is possible, whereas

$$V_Q^B(m,0,k) = CF(m,0,k) + V_Q^A(m,0,k),$$
otherwise. The notation $Q$ indicates an equivalent probability measure in the set $\mathcal{P}$, where $\mathcal{P}$ represents a set of some equivalent probability measures. In the formulation of Eq. (7), we introduce Kullback–Leibler divergence or relative entropy, denoted by $R_{m+1}(Q||\mathcal{P})$, a measure of the difference between two probability distributions. By introducing the additional term in Eq. (7), we can add the penalty that prevents us from going far away from the reference model under the original probability measure $\mathcal{P}$. The ambiguity parameter $\theta$ is introduced to represent the manager’s confidence level for the reference model, namely, a larger $\theta$ indicates a stronger confidence, as it forces us to choose the parameters near
the reference model, while a smaller \( \theta \) indicates that the manager is less confident in the reference model. In the similar manner,

\[
V_B^Q(n, 1, k) = CF(n, 1, k) + \max \left( V_A^Q(n, 1, k), V_A^Q(n, 2, k) - C_{1.2} \right),
\]
in case the switching is possible, and

\[
V_B^Q(m, 1, k) = CF(m, 1, k) + V_A^Q(m, 1, k),
\]
otherwise, and

\[
V_B^Q(m, 2, k) = CF(m, 2, k) + \max \left( V_A^Q(m, 2, k), V_A^Q(m, 1, k) - C_{2.1} \right),
\]
in case the switching is possible, and

\[
V_B^Q(m, 2, k) = CF(m, 2, k) + V_A^Q(m, 2, k),
\]
otherwise.

We now explain how to solve the optimization problem of Eq. (7) under ambiguity in the existing lattice model. In this paper, we assume that the chief manager has a set of priors \( \mathcal{P} \), indicating that the model allows us to choose a different set of parameter values for the underlying stochastic process. Girsanov Theorem clearly states that under the assumption that the reference model follows a geometric Brownian motion, a change of measure ends up with a change of drift term while the volatility term is unchanged. In other words, choosing a prior corresponds to choosing a different drift term \( \mu^c_i \in [\mu^{\min}_i, \mu^{\max}_i] \). Consequently, choosing the optimal \( Q \) in Eq. (7) can be interpreted as choosing the optimal \( \mu^c_i \). Note that the domain of each drift parameter is reasonably determined.

We set \( \mu^c_i \in [\mu_i - \delta \sigma_i, \mu_i + \delta \sigma_i] \), where \( \delta = 2.57583 \) which represents a 99% quantile of standard normal. On our lattice procedure, a change of the drift term is implemented by changing the transition probabilities of Eq. (3), whereas the structure of the nodes in Eq. (4) remains unchanged. To be precise, let \( \mu^c_i \) denote a candidate drift term that corresponds to a choice of a probability measure \( Q^c \). Then, the new transition probabilities to go up, go middle, and go down can be derived as follows:

\[
p^c_i(m; k_i, k_i + 1) = \frac{\Delta V_i - \Delta \nu_i (y_{i, \text{up}} - y_{i, \text{mid}})}{(y_{i, \text{up}} - y_{i, \text{mid}})^2},
\]

\[
p^c_i(m; k_i, k_i) = \frac{-\Delta V_i + \Delta \nu_i (y_{i, \text{up}} + y_{i, \text{down}})}{(y_{i, \text{up}} - y_{i, \text{mid}})^2},
\]

\[
p^c_i(m; k_i, k_i - 1) = \frac{\Delta V_i - \Delta \nu_i (y_{i, \text{down}})}{(y_{i, \text{down}} y_{i, \text{mid}})},
\]

where \( \Delta V_i = \Delta \nu_i^2 + \Delta \sigma_i^2 \) with \( \Delta \nu_i = (\mu^c_i - \frac{1}{2} \sigma_i^2) \Delta t \) and \( \Delta \sigma_i = \sigma_i \sqrt{\Delta t} \). Finally, the optimization problem in Eq. (7) is solved numerically.

Let us address the following remarks. The idea of this construction is simple and easily executed. More importantly, because the structure of the lattice is unchanged, we can reasonably compare the optimal exercise strategy under ambiguity with that under risk. Note that it is not possible for other lattice models, nor a simulation-based approach.
Table 3: NPV of the first scenario.

| Year | Number of orders | Surface area per unit order (m²) | Total area (m²) | Price for the existing products per unit area (JPY) | Sales (JPY) | Profit rate (%) | Gross profit (JPY) | Labor cost (JPY) | Profit after tax (JPY) | Capital expenditure (JPY) | CF (JPY) | Discount rate (%) | Expected growth rate (%) | Terminal value (JPY) | NPV (JPY) |
|------|------------------|---------------------------------|----------------|-----------------------------------------------|-------------|----------------|-------------------|-------------------|---------------------|--------------------------|-----------|-----------------|------------------------|-----------------------|----------|
| 2015 | 3                | 16.95                           | 50.85          | 39,835                                         | 2,025,610   | 50%            | 1,012,805         | 7,000,000         | -5,987,195                    | 1,000,000                             | -6,987,195 | 5%              | 3%                     | 390,572,792           | 294,666,889|
| 2016 | 6                | 19.80                           | 118.78         | 37,639                                         | 4,470,819   | 49%            | 2,190,701         | 7,500,000         | -5,309,299                    | 1,500,000                             | -5,309,299 | 4%              | 4%                     | 29,840,400            |
| 2017 | 15               | 23.12                           | 346.83         | 35,564                                         | 12,334,692  | 48%            | 5,920,652         | 8,000,000         | -2,079,348                    | 2,000,000                             | -2,079,348 | 47%             | 47%                    | 6,803,695             | 11,797,432|
| 2018 | 30               | 27.01                           | 810.18         | 33,603                                         | 27,224,480  | 47%            | 12,795,506        | 8,500,000         | 2,491,393                     | 2,500,000                             | -8,607    | 46%             | 46%                    | 3,803,695             | 8,297,432|
| 2019 | 45               | 31.54                           | 1419.39        | 31,750                                         | 45,066,322  | 46%            | 20,730,508        | 9,000,000         | 6,803,695                     | 3,000,000                             | -3,803    | 45%             | 45%                    | 9,500,000             | 11,797,432|
| 2020 | 60               | 36.84                           | 2210.40        | 30,000                                         | 66,312,000  | 45%            | 29,840,400        | 9,500,000         | 11,797,432                    | 5,000,000                             | -5,987    | 44%             | 44%                    | 13,797,432            | 294,666,889|

This table summarizes the first scenario from year 2015 to year 2020 for the Net Present Value analysis. Note that some numbers in the table are partly approximated due to classified information of the company.

### 4 Results

#### 4.1 NPV analysis

In this subsection, we consider three possible scenarios and calculate NPVs for our benchmarks. All scenarios are generated based on the discussions with the chief manager of the company. The first scenario assumes that the company remains at Stage 0 throughout the project horizon as summarized in Table 3. In the table, Number of orders, Profit rate, Labor cost and Capital expenditure are provided by the company, while Surface area per unit order, Price for the existing products per unit area are estimated by expected growth rates. We assume that both labor costs and capital expenditure are gradually increasing, hence the profit rate is decreasing. This reflects the fact that additional labor will require costs to build the equipment, e.g., building a showroom is important for growing the business quickly. Total area is calculated as the product of Number of orders and Surface area per unit order, Sales is the product of Total area and Price for the existing products per unit area, and Gross profit is the product of Sales and Profit rate. Free cash-flow, labeled as CF, is calculated by the Profit after tax minus Capital expenditure, where the tax rate is 42%. The discount rate and the expected growth rate are five percent and three percent, respectively. Table 3 shows that the NPV of the project under the first scenario is about 29,467 × 10⁴ yen.

As the second scenario, we assume that the company will start selling nonflammable (value-added) products from 2017. Note that it is not a real option, since time for selling the value-added products are predetermined. The switching cost from Stage 0 to Stage 1 is one million yen. Table 4 shows results regarding the second scenario. The following additional items are listed in the table. Price of the value-added products per unit area, Ratio of the existing products to the total products, Ratio of the value-added products to the total products are estimated based on discussions with the manager. In comparing Table 4 with Table 3, we can see that selling the value-added products at Stage 1 is a profitable opportunity for the company. For example, note that the terminal value in Stage 1 is much larger than in Stage 0. Although switching from Stage 0 to Stage 1 is costly, the company should commence selling the new product to capture additional revenue.

Finally, we consider the third scenario, in which the company commences selling the value-added
### Table 4: NPV of the second scenario.

| Year | Number of orders | Surface area per unit order (m²) | Total area (m²) | Price for the existing products per unit area (JPY) | Price of the value-added products per unit area (JPY) | Ratio of the existing products to the total products (%) | Ratio of the value-added products to the total products (%) | Sales (JPY) | Profit rate (%) | Gross profit (JPY) | Profit after tax (JPY) | Capital expenditure (JPY) | CF (JPY) | Discount rate (%) | Expected growth rate (%) | Terminal value (JPY) | NPV (JPY) |
|------|-----------------|----------------------------------|----------------|-----------------------------------------------|-----------------------------------------------|--------------------------------------------------|--------------------------------------------------|------------|----------------|-------------------|-------------------|-------------------------|---------|----------------|----------------------|----------------------|---------|
| 2015 | 3               | 16.95                            | 50.85          | 39,835                                       | 50,000                                        | 100%                                             | (3.00%)                                          | 2,025,610  | 50%            | 1,012,805         | -5,987,195               | 1,000,000                  | -6,987,195         | 5%          | 24.00%           | 618,685,222         | 47,601,474          | 47,601,474 |
| 2016 | 6               | 19.80                            | 118.78         | 37,639                                       | 75,000                                        | 100%                                             | 4.24%                                            | 4,470,819  | 49%            | 2,190,701         | -5,309,299               | 1,500,000                  | -6,809,299         | 4%          | 16.97%           | 111,404,160         | 47,601,474          | 47,601,474 |
| 2017 | 15              | 23.12                            | 346.83         | 35,564                                       | 70,711                                        | 96%                                              | 6.00%                                            | 12,686,329 | 48%            | 6,089,438         | -1,910,562               | 3,000,000                  | 483,117            | 4%          | 12.00%           | 111,404,160         | 47,601,474          | 47,601,474 |
| 2018 | 30              | 27.01                            | 810.18         | 31,603                                       | 84,090                                        | 94%                                              | 8.49%                                            | 29,028,309 | 47%            | 13,643,305        | 2,983,117                | 8,000,000                  | 5,483,521           | 4%          | 8.00%            | 111,404,160         | 47,601,474          | 47,601,474 |
| 2019 | 45              | 31.75                            | 1,419.39       | 31,750                                       | 100,000                                       | 92%                                              | 12.00%                                           | 51,370,020 | 46%            | 23,630,209        | 8,485,521                | 9,000,000                  | -59,933,514         | 3%          | 3%              | 614,560,121          | 47,601,474          | 47,601,474 |
| 2020 | 60              | 36.84                            | 2,210.40       | 30,000                                       | 16,643,513                                   | 88%                                              | 12.00%                                           | 84,879,360 | 45%            | 38,195,712        | 16,643,513               | 14,199,000                 | -59,933,514         | 4%          | 12.00%           | 614,560,121          | 47,601,474          | 47,601,474 |

This table summarizes the second scenario for the Net Present Value analysis. In the third scenario, the company starts selling the value-added product from year 2017. The switching cost for selling the new product is one million yen. Note that some numbers in the table are partly approximated due to classified information of the company.

### Table 5: NPV of the third scenario.

| Year | Number of orders | Surface area per unit order (m²) | Total area (m²) | Price for the existing products per unit area (JPY) | Price of the value-added products per unit area (JPY) | Ratio of the existing products to the total products (%) | Ratio of the value-added products to the total products (%) | Sales (JPY) | Profit rate (%) | Gross profit (JPY) | Profit after tax (JPY) | Capital expenditure (JPY) | CF (JPY) | Discount rate (%) | Expected growth rate (%) | Terminal value (JPY) | NPV (JPY) |
|------|-----------------|----------------------------------|----------------|-----------------------------------------------|-----------------------------------------------|--------------------------------------------------|--------------------------------------------------|------------|----------------|-------------------|-------------------|-------------------------|---------|----------------|----------------------|----------------------|---------|
| 2015 | 3               | 16.95                            | 50.85          | 39,835                                       | 50,000                                        | 100%                                             | (3.00%)                                          | 2,025,610  | 50%            | 1,012,805         | -5,987,195               | 1,000,000                  | -6,987,195         | 5%          | 24.00%           | 618,685,222         | 47,601,474          | 47,601,474 |
| 2016 | 6               | 19.80                            | 118.78         | 37,639                                       | 75,000                                        | 100%                                             | 4.24%                                            | 4,470,819  | 49%            | 2,190,701         | -5,309,299               | 1,500,000                  | -6,809,299         | 4%          | 16.97%           | 111,404,160         | 47,601,474          | 47,601,474 |
| 2017 | 15              | 23.12                            | 346.83         | 35,564                                       | 70,711                                        | 96%                                              | 6.00%                                            | 12,686,329 | 48%            | 6,089,438         | -1,910,562               | 3,000,000                  | 483,117            | 4%          | 12.00%           | 111,404,160         | 47,601,474          | 47,601,474 |
| 2018 | 30              | 27.01                            | 810.18         | 31,603                                       | 84,090                                        | 94%                                              | 8.49%                                            | 29,028,309 | 47%            | 13,643,305        | 2,983,117                | 8,000,000                  | 5,483,521           | 4%          | 8.00%            | 111,404,160         | 47,601,474          | 47,601,474 |
| 2019 | 45              | 31.75                            | 1,419.39       | 31,750                                       | 100,000                                       | 92%                                              | 12.00%                                           | 51,370,020 | 46%            | 23,630,209        | 8,485,521                | 9,000,000                  | -59,933,514         | 3%          | 3%              | 614,560,121          | 47,601,474          | 47,601,474 |
| 2020 | 60              | 36.84                            | 2,210.40       | 30,000                                       | 16,643,513                                   | 88%                                              | 12.00%                                           | 84,879,360 | 45%            | 38,195,712        | 16,643,513               | 14,199,000                 | -59,933,514         | 4%          | 12.00%           | 614,560,121          | 47,601,474          | 47,601,474 |

This table summarizes the third scenario for calculating the Net Present Value. In the third scenario, the company starts selling the value-added product from year 2017 and builds the plant in year 2020. The switching cost for selling the value-added product is one million yen, and that for building the plant is 80 million yen. Note that some numbers in the table are partly approximated due to classified information of the company.
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Figure 3: Distributions of the NPV under the first scenario.

This figure illustrates the distribution of the Net Present Value under the first scenario. The figure is created by the Monte Carlo method with one million iterations on the lattice model, assuming that all the switching costs are infinite. In the figure, the $x$-axis indicates the Net Present Values with a unit of $10^4$ (JPY), and the $y$-axis indicates the frequency of the histogram.

products from 2017 and builds the processing plant in 2020. The switching cost from Stage 1 to Stage 2 is 80 million yen. Table 5 provides results for the scenario. We can observe that, although the cash-flow for year 2020 is negative because of the switching cost to Stage 2, this investment increases the terminal value and makes the project more profitable than the second scenario. Furthermore, profitability doubles if we compare the sales of year 2020 with those of year 2019. This clearly indicates potential profitability of building the new plant.

We also implement a Monte Carlo DCF method to derive the distribution of the expected project value, which helps the manager analyze the risk profile of the business. In the Monte Carlo method, we use one million iterations to generate a histogram for project value. The simulation is implemented on the lattice model. Figure 3 illustrates the distribution of the NPVs for the first scenario. The estimated NPV on the lattice model is $26,783 \times 10^4$ (JPY), and its sample standard deviation is $35,392 \times 10^4$ (JPY).

4.2 Real options under risk

As explained in the previous section, we evaluate the membrane ceiling business with the two real options. It is important to note that the real options approach enables us to derive the optimal switching strategy. Switching cost is summarized as follows; $C_{01}$ is 1 million yen, $C_{12}$ is 80 million yen, and $C_{21}$ is 10 million yen.

First of all, we examine the impact of the number of periods of the lattice model on the project value. Table 6 summarizes the impact. In the following, note that the unit of the project value
Table 6: The effect of the number of periods.

| # periods p.a. | Values | change(%) |
|----------------|--------|-----------|
| 1              | 125,963|           |
| 2              | 126,227| 100.2     |
| 3              | 126,154| 99.9      |
| 4              | 126,081| 99.9      |
| 6              | 125,980| 99.9      |
| 12             | 125,851| 99.9      |

This table shows the numerical results that examine how the number of periods affects the project value under risk. The first row labeled ‘# periods p.a.’ indicates the number of periods in the lattice model per year, ‘Values’ indicates the project values with real options with a unit of $10^4$ (JPY), and ‘change’ indicates the percentage of change to clarify the relative change. The table clearly indicates insensitivity of the project value to the number of the periods.

The project value with real options estimated by the lattice model is estimated as $126,081 \times 10^4$ yen. The result clearly indicates that the embedded real options have significant impact on the project value. To be more precise, in comparison to the NPV of $26,783 \times 10^4$, the project value with real options is about 4.7 times larger than the one without the real options. When comparing the result with the NPV under the third scenario, it is 1.9 times greater. Figure 4 illustrates the distribution of the project value with real options. The standard deviation of the distribution in this case is $883,884 \times 10^4$. When comparing Figure 4 with Figure 3, we notice that the distribution with real options has a heavier right tail than the one without the real options. This means that the real options have a significant positive impact on the project value, whereas the negative impact remains unchanged.

Next, we examine the optimal exercise strategy. Understanding the optimal strategy is crucial for the company, as the real options value is accomplished by the optimal strategy. Table 7 summarizes transition probabilities from one stage to another. The probability of each switching is estimated by a Monte Carlo method with 10 million iterations on the lattice. Note that the company can switch from Stage 0 to Stage 1 from year 2017, Stage 1 to Stage 2 from year 2019, and Stage 2 to Stage 1 from year 2020. At the bottom row in the table, probabilities in operating at each stage at the end of the project are listed. The implication is summarized as follows:

- When we look at the transition probability at period 8 from Stage 0 to Stage 1, the company decides to switch to Stage 1 (start selling nonflammable ceilings) with a probability of more than 75%. This indicates that in many possible future scenarios, the company would be better off to enter the new business as soon as possible.

- In contrast, the probability to switch from Stage 1 to Stage 2 at period 16 is relatively small, depending on the outcome of the underlying processes. It implies that the judicious decision of the switching is more important for the project.
Figure 4: Distributions of the project value with the real options.

This figure illustrates the distribution of the project value with real options under risk. The figure is created by the Monte Carlo method with one million iterations on the lattice model on the condition that the company follows the optimal exercise policy. In the figure, the $x-$axis indicates the project values with a unit of $10^4$ (JPY), and the $y-$axis indicates the frequency of the histogram.

- Although there exists a real option to switch from Stage 2 back to Stage 1, it is never exercised.

- At the end of the project, the probability of operating in Stage 0 is only 1.9%, which means that in most cases the company will exercise the real options.

- The probability of operating in Stage 1 is almost the same as that in Stage 2. This indicates that the sequential real option is quite valuable.

Next, we analyze optimal exercise boundaries of the real options that are derived by the lattice model. They depend on both the period and values(nodes) of the underlying processes. However, it is difficult to visualize these boundaries since the company is subject to four underlying risk processes and, hence, the boundaries have four-dimensional surfaces. To capture the effect of each underlying risk on the exercise strategy, we conduct a logistic regression analysis. The purpose of the regression is to visualize the relative importance of the underlying risks on the optimal exercise boundaries.

Consider the optimal strategy at period 8, the beginning of year 2017. Let $p_0$ be the probability in staying at Stage 0. Hence, $1 - p_0$ is the probability to switch from Stage 0 to Stage 1. Thus, a negative coefficient in the equation suggests that the company should exercise the option, whereas a positive coefficient indicates that the company should not exercise the option and remain operating at Stage 0. The total number of nodes at time point $m$ under a trinomial lattice model is equal to $2m + 1$. In our model, $k_i \in \{-m, -m + 1, \ldots, 0, \ldots, m\}$. Thus, the regression is done with $17^4$ data points, from
Table 7: Transition of switching under risk.

| Year  | Period No. | Stage 0 → Stage 1 | Stage 1 → Stage 2 | Stage 2 → Stage 1 |
|-------|------------|-------------------|-------------------|-------------------|
| 2017  | 8          | 75.9%             | —                 | —                 |
|       | 9          | 5.1%              | —                 | —                 |
|       | 10         | 2.6%              | —                 | —                 |
|       | 11         | 3.7%              | —                 | —                 |
| 2018  | 12         | 1.3%              | —                 | —                 |
|       | 13         | 1.3%              | —                 | —                 |
|       | 14         | 1.2%              | —                 | —                 |
|       | 15         | 1.3%              | —                 | —                 |
| 2019  | 16         | 0.6%              | 9.2%              | —                 |
|       | 17         | 0.7%              | 2.2%              | —                 |
|       | 18         | 0.5%              | 2.7%              | —                 |
|       | 19         | 0.7%              | 3.3%              | —                 |
| 2020  | 20         | 0.4%              | 2.9%              | 0.0%              |
|       | 21         | 0.5%              | 2.4%              | 0.0%              |
|       | 22         | 0.4%              | 3.1%              | 0.0%              |
|       | 23         | 0.6%              | 4.2%              | 0.0%              |
|       | end        | 1.3%              | 14.3%             | 0.0%              |
|       | End of project | Stage 0 | Stage 1 | Stage 2 |
|       |             | 1.9% | 53.8% | 44.3% |

This table summarizes the transition probabilities among the three stages for the project under risk. Transition probabilities from Stage 0 to Stage 1, Stage 1 to Stage 2, and Stage 2 to Stage 1 for each time period are listed. At the bottom row, we present probabilities in operating at each stage at the end of the project. The transition probabilities are estimated by the Monte Carlo method with ten million iterations on the lattice model.
node \((-8, \ldots, -8)\) to node \((8, \ldots, 8)\). The regression result is given by the following equation:

\[
\log \left( \frac{p_0}{1 - p_0} \right) = -20.5322 + .9457k_1 - 18.0611k_2 + .3363k_3 - 5.4845k_4, \tag{8}
\]

where \(k_i, i = 1, 2, 3, 4\) are indexes of nodes for the \(i\)-th underlying risk, which is defined in Eq. (2). The implications of the logistic regression analysis can be described as follows:

- The negative intercept indicates that it is optimal for the company to exercise the option when the underlying process takes an expected move. It is consistent with the fact shown in Table 7, where the company exercises the option with a probability of 75.9%.

- Among the four underlying risks, the second risk (Surface area for value-added products per unit order) has the largest impact on the determination of the optimal exercise strategy as its coefficient has the largest absolute value, followed by the fourth risk (Price for the value-added products per unit area), the first (Surface area for existing products per unit order), and the third (Price for the existing products per unit area).

- Both the surface area and the price of the value-added products have positive impacts on the early exercise, that is, larger values tend to cause an exercise of the switching option from Stage 0 to Stage 1. In contrast, larger values of the surface area and those of the price for the existing products prevent the company from the switching.

Eq. (9) shows the result of the logistic regression for specifying the boundary of the switching from Stage 1 to Stage 2 at period 16. Note that the probability of the switching at this period is 9.2% in Table 7.

\[
\log \left( \frac{p_1}{1 - p_1} \right) = 93.417 + .2552k_1 - 29.6037k_2 + .111k_3 - 9.0508k_4. \tag{9}
\]

The implications are summarized as follows:

- The intercept is positive and larger than that in Eq. (8). This implies that it is less probable, on average, for the company to exercise the option at period 16.

- The signs of the coefficients in Eq. (9) are consistent with those in Eq. (8). This indicates that an increase of both the surface area and the price of the value-added products have a positive impact on the exercise probability, while that of the existing products has a negative impact.

- The absolute values of the coefficients are in the same order as above, i.e., the order of coefficients is \(k_2, k_4, k_1, \) and \(k_3\).

We next conduct a sensitivity analysis with respect to the drift coefficient of each underlying risk. Table 8 shows the project values when the drift of each underlying risk changes. Relative percentage changes are also listed in the table. Parameter \(\alpha\) describes a relative percentage difference, from 60% to 140%. The table indicates that the project value is monotonically increasing as each drift increases. Note that the sign of \(\mu_3\) is negative. Among the four underlying risks, we can confirm a change of the second risk (Surface area for value-added products per unit order) has the largest impact on the project value, which is consistent with the regression result.
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Table 8: Sensitivity with respect to each drift term.

| $\alpha \times \mu_i$ | Project value ($i = 1$) | Project value ($i = 2$) | Project value ($i = 3$) | Project value ($i = 4$) |
|-----------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| $\alpha$              | value change(%)         | value change(%)         | value change(%)         | value change(%)         |
| 60                    | 108,470                 | 86                       | 55,264                  | 44                       | 135,358                  | 107                      | 91,517                  | 73                       |
| 70                    | 112,378                 | 89                       | 65,817                  | 52                       | 132,922                  | 105                      | 98,848                  | 78                       |
| 80                    | 116,597                 | 92                       | 80,169                  | 64                       | 130,565                  | 104                      | 106,990                 | 85                       |
| 90                    | 121,154                 | 96                       | 99,656                  | 79                       | 128,286                  | 102                      | 116,034                 | 92                       |
| 100                   | 126,081                 | 100                      | 126,081                 | 100                      | 126,081                  | 100                      | 126,081                 | 100                      |
| 110                   | 131,411                 | 104                      | 161,882                 | 128                      | 123,949                  | 98                       | 137,236                 | 109                      |
| 120                   | 137,177                 | 109                      | 210,351                 | 167                      | 121,886                  | 97                       | 149,627                 | 119                      |
| 130                   | 143,419                 | 114                      | 275,928                 | 219                      | 119,890                  | 95                       | 163,381                 | 130                      |
| 140                   | 150,176                 | 119                      | 364,611                 | 289                      | 117,958                  | 94                       | 178,609                 | 142                      |

This table shows the results of a sensitivity analysis with respect to drift changes in each underlying risk. Project values and relative percentage changes are listed, where $i$ indicates the index of the underlying risk shown in Table 2. In the table, parameter $\alpha$ is a relative percentage difference, from 60% to 140%.

Table 9: Sensitivity with respect to each volatility term.

| $\alpha \times \sigma_i$ | Project value ($i = 1$) | Project value ($i = 2$) | Project value ($i = 3$) | Project value ($i = 4$) |
|--------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| $\alpha$                 | value change(%)         | value change(%)         | value change(%)         | value change(%)         |
| 60%                      | 126,124                 | 100.03                  | 124,343                 | 98.62                   | 126,094                  | 100.01                  | 125,928                  | 99.88                    |
| 70%                      | 126,120                 | 100.03                  | 124,739                 | 98.94                   | 126,091                  | 100.01                  | 125,960                  | 99.90                    |
| 80%                      | 126,113                 | 100.02                  | 125,182                 | 99.29                   | 126,088                  | 100.01                  | 125,997                  | 99.93                    |
| 90%                      | 126,100                 | 100.02                  | 125,642                 | 99.65                   | 126,085                  | 100.00                  | 126,036                  | 99.96                    |
| 100%                     | 126,081                 | 100                      | 126,081                 | 100                      | 126,081                  | 100                      | 126,081                  | 100                      |
| 110%                     | 126,054                 | 99.98                   | 126,469                 | 100.31                  | 126,077                  | 100.00                  | 126,131                  | 100.04                   |
| 120%                     | 126,017                 | 99.95                   | 126,778                 | 100.55                  | 126,071                  | 99.99                   | 126,182                  | 100.08                   |
| 130%                     | 125,969                 | 99.91                   | 126,979                 | 100.71                  | 126,066                  | 99.99                   | 126,239                  | 100.12                   |
| 140%                     | 125,911                 | 99.86                   | 127,037                 | 100.76                  | 126,059                  | 99.98                   | 126,298                  | 100.17                   |

This table shows the results of a sensitivity analysis with respect to volatility changes in each underlying risk. Project values and relative percentage changes are listed, where $i$ indicates the index of the underlying risk shown in Table 2. In the table, parameter $\alpha$ is a relative percentage difference, from 60% to 140%.
Table 10: Sensitivity with respect to correlation.

| case | $\rho_{1,2}$ | $\rho_{3,4}$ | Project value | change(%) |
|------|---------------|---------------|---------------|-----------|
| 1    | -0.5          | -0.5          | 126,190       | 100.09    |
| 2    | -0.3          | -0.3          | 126,137       | 100.04    |
| 3    | -0.1          | -0.1          | 126,093       | 100.01    |
| 4    | 0.1           | 0.1           | 126,075       | 100.00    |
| 5    | 0.3           | 0.3           | 126,086       | 100.00    |
| 6    | 0.5           | 0.5           | 126,112       | 100.02    |
| 7    | -0.5          | 0             | 126,187       | 100.08    |
| 8    | 0             | -0.5          | 126,085       | 100.00    |
| 9    | 0.5           | 0             | 126,116       | 100.03    |
| 10   | 0             | 0.5           | 126,075       | 100.00    |

This table shows the results of a sensitivity analysis with respect to correlation changes among the underlying risks. In the table $\rho_{1,2}$ indicates a correlation between the first and the second risk, and $\rho_{3,4}$ indicates a correlation between the third and the fourth risk, where the numbers of index $i$ for the underlying risk is shown in Table 2. We consider ten sets of correlation structures and list project values and relative percentage changes.

Table 9 shows the project values when the volatility term of each underlying risk changes. Relative percentage changes are also listed in the table. Parameter $\alpha$ describes a relative percentage difference, from 60% to 140%. We confirm that, unlike the valuation via the Net Present Value method, volatility changes affect the project value with real options, although the effect of the volatility changes is relatively small, as compared with the results in Table 8. The direction of the effect for the second and the fourth volatility is consistent with the standard option pricing theory, that is, an increase of the volatility term increases the project value. It is important to observe the reverse effect for the first and the third underlying risks, that is, an increase of the volatility term decreases the project value. Accordingly, we could interpret that the difference of volatility term between the existing products and the value-added products, $\sigma_2 - \sigma_1$ and/or $\sigma_4 - \sigma_3$, are essentially critical factors for the project value.

Finally, we examine the sensitivity of the project value with respect to correlation structure. In the original analysis we assume that all the underlying risks are independent. It is reasonable to consider nonzero correlation between the surface areas of the two products and their prices. For example, negative correlation between the existing products and value-added products indicates that they are substitute goods. On the other hand, positive correlation implies that in the business for both products are subject to the same macro factor, such as economic trends in the construction business. Table 10 summarizes the results. In the table, ten sets of correlation structures are examined. Comparatively, the correlation with respect to the surface areas of the two products are a little more sensitive to that with respect to the prices of them, but, in general, the results indicate that the project values are insensitive to the correlation structure.

4.3 Real options under ambiguity

In this subsection we show various quantitative results in the presence of ambiguity. In order to discuss the effect of ambiguity on the optimal decision and project value, it is critical to properly estimate the value of the ambiguity parameter $\theta$ in Eq. (7). It is difficult to get visible information
Figure 5: Project values under ambiguity with respect to values of the ambiguity parameters.
This figure illustrates sensitivity of the real options values with respect to values of the ambiguity parameter $\theta$. In the figure, the $x$-axis represents $\log(\theta)$, while the $y$-axis represents corresponding project values.

Figure 5: Project values under ambiguity with respect to values of the ambiguity parameters.

from the manager. In order to resolve the difficulty, we first conduct a sensitivity analysis with respect to the ambiguity parameter. It enables us to analyze the effect of the ambiguity parameter. Figure 5 illustrates the real options value with different values of the ambiguity parameter. In the figure, the $x$-axis represents $\log(\theta)$, while the $y$-axis represents the corresponding project values.

It is important to notice that the ambiguity parameter $\theta$ represents the confidence level regarding parameter estimation. To be precise, a smaller value of $\theta$ implies that the manager is not confident in estimating the parameter $\mu_i$ of the reference model, whereas a large $\theta$ implies that he is confident in the reference model. As a limit case, a model under ambiguity with $\theta = \infty$ is equivalent to a model under risk. From the figure we confirm that the project values are monotonically increasing with respect to $\theta$, and they tend to converge to the one under risk.

As an alternative approach, we calculate the value of the ambiguity parameter under the assumption that the manager can obtain a locally robust estimate for the NPV on the first scenario. To be concrete, the ambiguity parameter $\theta$ is chosen in a way that the sum of the NPV and the penalty term used in the robust control problem becomes insensitive to drift shifts. We use this approach to fix the parameter value.

For preparation, Table 11 shows Net Present Values, the corresponding KL divergence with different drift shifts, and square roots of them. In the table, we consider ten sets of negative drift shifting; from $-1\%$ to $-10\%$. Values in the first row represent the original set of parameters used in the standard real option analysis. Figure 6 illustrates the NPVs with respect to square roots of the KL divergence. The figure clearly shows that the NPVs are linearly dependent on the square root of the KL divergence. By regression analysis, the slope of the line is estimated as $-121.258$ with $R^2 = 0.9998$. Consequently, after adjusting the discount factor for the quarterly time interval, we determine that
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Table 11: NPV and KL-divergence.

| drift shift $\alpha$ | $\mu_1$   | $\mu_2$   | $\mu_3$   | $\mu_4$   | $KL$      | $\sqrt{KL}$ | NPV     |
|----------------------|-----------|-----------|-----------|-----------|-----------|-------------|---------|
| 0%                   | 0.1340    | 0.5018    | -0.0567   | 0.1733    | 0         | 0           | 26,783  |
| -1%                  | 0.1326    | 0.4968    | -0.0573   | 0.1716    | 3.030e-5  | 5.505e-3    | 26,655  |
| -2%                  | 0.1313    | 0.4918    | -0.0578   | 0.1698    | 1.212e-4  | 1.101e-2    | 25,335  |
| -3%                  | 0.1300    | 0.4868    | -0.0584   | 0.1681    | 2.727e-4  | 1.651e-2    | 24,622  |
| -4%                  | 0.1286    | 0.4818    | -0.0590   | 0.1664    | 4.849e-4  | 2.202e-2    | 23,917  |
| -5%                  | 0.1273    | 0.4767    | -0.0595   | 0.1646    | 7.576e-4  | 2.752e-2    | 23,220  |
| -6%                  | 0.1259    | 0.4717    | -0.0601   | 0.1629    | 1.091e-3  | 3.303e-2    | 22,530  |
| -7%                  | 0.1246    | 0.4667    | -0.0607   | 0.1612    | 1.485e-3  | 3.853e-2    | 21,848  |
| -8%                  | 0.1233    | 0.4617    | -0.0612   | 0.1594    | 1.939e-3  | 4.404e-2    | 21,173  |
| -9%                  | 0.1219    | 0.4567    | -0.0618   | 0.1577    | 2.455e-3  | 4.954e-2    | 20,505  |
| -10%                 | 0.1206    | 0.4517    | -0.0624   | 0.1560    | 3.030e-3  | 5.505e-2    | 19,844  |

This table summarizes Net Present Values under the first scenario and corresponding KL-divergence with different sets of drift terms. We consider eleven cases where the drift terms of all four underlying risks take a downside shift $\alpha$ with $\alpha = -1\%, \ldots, -10\%$. The first column of the table shows the drift shift rate $\alpha$, the second to five columns show the drift rate for each underlying risk. The column labeled ‘KL’ represents the KL-divergence, or relative entropy between the reference model and the model with modified drift rates. The last column represents Net Present Values under the first scenario with unit of $10^4$ (JPY).

The NPV-robust ambiguity parameter is $\theta^* = 24,314$. This means that with the NPV-robust $\theta^*$, the Net Present Value is locally equivalent to that under the reference model, regardless of the choice of the small drift shifts listed in Table 11. This is because, if the Nature chooses larger downside drift shifts, the decrease of the Net Present Value driven by the small drift terms are compensated by a larger penalty term. In the following analysis, we replace values of KL-divergence with its square root to reflect the linear relations.

Figure 7 illustrates the distribution of the project value with real options under ambiguity. The optimal strategy is derived by solving the multiplier robust control model with $\theta^* = 24,314$. The histogram is generated by the Monte Carlo method based on the optimal exercise decision given by the lattice model. In this case, the optimized drift term is utilized in generating Monte Carlo samples. In other words, we assume that the underlying risk process does not follow the reference model but follows the model with the optimized drift term obtained by solving Eq. (7). Furthermore, the penalty terms $\theta^* R_{n+1}(Q|P)$ are included in calculating the cash-flows along each sample path. Therefore, we can expect that the sample mean obtained by the Monte Carlo method converges to the expected project value obtained by the lattice model.

The project value obtained by the lattice model is $51,607 \times 10^4$ (JPY) and sample standard deviation by the Monte Carlo method is $59,219 \times 10^4$ (JPY). Compared with the project value under risk, the estimated project value is significantly decreased, by the factor of 0.46. Although the histogram under ambiguity in Figure 7 looks very similar to that under risk in Figure 4, the right tail of the distribution in Figure 7 is less heavy than that in Figure 4. In fact, the minimum values of realized outcomes with one million simulations for both cases are almost the same (around $-45,000 \times 10^4$), but the probability in which the project value goes beyond $45000 \times 10^4$ is 0.1% in the ambiguity case, whereas it is as large as 4.0% without ambiguity. This means that, although the distribution in the risk case is right-heavy tailed, the presence of ambiguity significantly reduces the possibility to obtain...
This figure illustrates the Net Present Values under the first scenario with respect to the square roots of the KL divergences shown in Table 11. It clearly shows that the NPVs are linearly dependent on the square root of the KL divergence. With a regression analysis, the slope of the line is estimated as \(-121,258\) with \(R^2 = 0.9998\). From this figure we determine \(\theta^* = 24,314\) as a locally robust estimate for the ambiguity parameter.

large project values.

\[
\log \left( \frac{p_0'}{1 - p_0'} \right) = -0.8369 + 0.0644k_1 - .9291k_2 + 0.0247k_3 - 0.2399k_4, \tag{10}
\]

\[
\log \left( \frac{p_1'}{1 - p_1'} \right) = 7.7440 + 0.0408k_1 - 2.1100k_2 + 0.0156k_3 - 0.6535k_4. \tag{11}
\]

Eqs. (10) and (11) represent the boundary of the optimal switching under ambiguity from Stage 0 to Stage1, and from Stage 1 to Stage 2, respectively. By comparing (10) with (8), and (11) with (9), we can observe the following.

- Signs of all coefficients in the regression models are consistent, that is, the factors for the existing products have a negative impact on the exercise, whereas those of the value-added products have a positive impact on the exercise.

- The order of impact under ambiguity is also consistent with that under risk, i.e., the absolute value of the coefficient of \(k_2\) is the largest, followed by that of \(k_4, k_1, k_3\).

- However, the estimated Eq. (10) is clearly different from Eq. (8), and Eq. (11) is different from Eq. (9). This means that the optimal exercise policy under ambiguity is not equivalent to that under risk. More detailed analysis reveals that although the exercise policy under ambiguity is similar to that under risk, the company refrains from switching to the new stage in some cases due to the presence of ambiguity.
Figure 7: Distributions of the project value under ambiguity.

This figure illustrates the distribution of the project values with real options under ambiguity. It is created by the Monte Carlo method with one million iterations on the lattice model on the condition that the company follows the optimal exercise policy under ambiguity. The optimal strategy is derived by solving the multiplier robust control model with $\theta^* = 24,314$. Note that the penalty term $\theta^* R_{t+1}(Q|P)$ is included in calculating the cash-flows along each sample path. In the figure, the $x$–axis indicates the project values with a unit of $10^4$ (JPY), and the $y$–axis indicates the frequency of the histogram.
Table 12: Frequency of switching under ambiguity.

| Year | Period No. | Stage 0 → Stage 1 | Stage 1 → Stage 2 | Stage 2 → Stage 1 |
|------|------------|-------------------|-------------------|-------------------|
| 2017 | 8          | 75.5%             | —                 | —                 |
|      | 9          | 4.5%              | —                 | —                 |
|      | 10         | 2.8%              | —                 | —                 |
|      | 11         | 3.3%              | —                 | —                 |
| 2018 | 12         | 1.6%              | —                 | —                 |
|      | 13         | 1.3%              | —                 | —                 |
|      | 14         | 1.0%              | —                 | —                 |
|      | 15         | 1.2%              | —                 | —                 |
| 2019 | 16         | 0.7%              | 7.6%              | —                 |
|      | 17         | 0.7%              | 2.5%              | —                 |
|      | 18         | 0.6%              | 2.5%              | —                 |
|      | 19         | 0.7%              | 3.1%              | —                 |
| 2020 | 20         | 0.01%             | 2.7%              | 0.0%              |
|      | 21         | 0.6%              | 2.6%              | 0.0%              |
|      | 22         | 0.7%              | 2.8%              | 0.00001%          |
|      | 23         | 0.8%              | 5.1%              | 0.00003%          |
|      | 24         | 1.4%              | 15.0%             | 0.0%              |
|      | End of project | Stage 0 | Stage 1 | Stage 2 |
|      | 2.0%       | 54.3%             | 43.7%             |

This table summarizes transition probabilities among the three stages for the project under ambiguity. The optimal strategy is derived by solving the multiplier robust control model with \( \theta^* = 24,314 \). Transition probabilities from Stage 0 to Stage 1, Stage 1 to Stage 2, and Stage 2 to Stage 1 for each time period are listed. At the bottom row we present probabilities in operating at each stage at the end of the project. The transition probabilities are estimated by the Monte Carlo method with ten million iterations on the lattice model. In the Monte Carlo method, each sample path is generated by the underlying model under \( Q \).

To further examine the optimal exercise behaviors of the company, we estimate transition probabilities under ambiguity. Table 12 summarizes them that are estimated by the Monte Carlo method. Note that the optimized drift term is utilized in generating Monte Carlo samples. One of the notable observations from the table is that although the presence of ambiguity does change the optimal exercise policy, most of the transition probabilities are very similar to the case under risk. In other words, although the project value under ambiguity is much smaller than that under risk, the difference of the actual exercise behavior is not that significant. This could happen when the probability to reach the different boundary is relatively small, even if the optimal exercise boundary is changed.

Table 13 also shows the transition probabilities from Stage 0 to Stage 1, from Stage 1 to Stage 2, and Stage 2 to Stage 1. The main difference from Table 12 is that each sample path is not generated by the model under the changed measure \( Q \) but by the reference model, in which the drift term is fixed as the originally estimated values. Figure 8 depicts the estimated distribution. It is based on the optimal exercise strategy under ambiguity, but the Monte Carlo samples are generated based on the reference model. Furthermore, in this simulation, we ignore the penalty term for calculating realized project values. It is interesting to compare the distribution with that in Figure 4 to examine how the difference of the optimal exercise policy could affect project values. We can confirm that the results given in Table 13 are close to those in Table 7. We also confirm that the histograms of Figure 4 and
Figure 8: Distributions of the project value under ambiguity without penalty.

This figure illustrate the distribution of the project value with real options under ambiguity. It is created by the Monte Carlo method with one million iterations on the lattice model on the condition that the company follows the optimal exercise policy under ambiguity. The optimal strategy is derived by solving the multiplier robust control model with $\theta^* = 24,314$. Note that the penalty terms $\theta^* R_{n+1}(Q|P)$ are excluded in calculating the cash-flows along each sample path. In the figure, the $x$–axis indicates the project values with a unit of $10^4$ (JPY), and the $y$–axis indicates the frequency of the histogram.
| Year | Period No. | Stage 0 $\rightarrow$ Stage 1 | Stage 1 $\rightarrow$ Stage 2 | Stage 2 $\rightarrow$ Stage 1 |
|------|------------|-------------------------------|-------------------------------|-------------------------------|
| 2017 | 8          | 75.4%                         | —                             | —                             |
|      | 9          | 4.6%                          | —                             | —                             |
|      | 10         | 2.8%                          | —                             | —                             |
|      | 11         | 3.3%                          | —                             | —                             |
| 2018 | 12         | 1.6%                          | —                             | —                             |
|      | 13         | 1.3%                          | —                             | —                             |
|      | 14         | 1.1%                          | —                             | —                             |
|      | 15         | 1.2%                          | —                             | —                             |
| 2019 | 16         | 0.7%                          | 7.6%                          | —                             |
|      | 17         | 0.7%                          | 2.4%                          | —                             |
|      | 18         | 0.5%                          | 2.5%                          | —                             |
|      | 19         | 0.7%                          | 3.1%                          | —                             |
| 2020 | 20         | 0.6%                          | 2.7%                          | 0.0%                          |
|      | 21         | 0.6%                          | 2.6%                          | 0.0%                          |
|      | 22         | 0.6%                          | 3.0%                          | 0.0%                          |
|      | 23         | 0.8%                          | 5.4%                          | 0.0%                          |
|      | 24         | 1.4%                          | 14.9%                         | 0.0%                          |
| End of project | Stage 0 | Stage 1 | Stage 2 |
|               | 2.0%     | 53.9%  | 44.1%   |

This table summarizes the transition probabilities among the three stages for the project under ambiguity. The optimal strategy is derived by solving the multiplier robust control model with $\theta^* = 24,314$. Transition probabilities from Stage 0 to Stage 1, Stage 1 to Stage 2, and Stage 2 to Stage 1 for each time period are listed. At the bottom row we present probabilities in operating at each Stage at the end of the project. The transition probabilities are estimated by the Monte Carlo method with ten million iterations on the lattice model. In the Monte Carlo method, each sample path is generated by the reference model.
Figure 8 look similar. In fact, if the penalty term is excluded, the estimated average of 100 million Monte Carlo iterations without ambiguity is 125,959, while that under ambiguity is 125,952, which is slightly smaller. Note that this comparison is reasonable because in order to compare the difference of the two optimal exercise policies, we use exactly the same sample paths.

In summary, the company has valuable real options and it can significantly increase the project value by properly exercising them. When the company takes ambiguity into account, the optimal exercise policy is changed and the project value that includes the penalty term significantly decreases. However, the difference of the optimal exercise policy has little effect on the realized project value without the penalty term. Hence, we can conclude that the optimal exercise policy is practically robust for the company.

5 Concluding Remarks

In this paper, we estimated the value of a membrane ceilings business for an actual company. We first applied a real options approach to deriving the firm’s optimal exercise strategy under four sources of risk. We constructed a four-dimensional lattice model to express the underlying risks. The sequential real options were modelled as switching options with three stages. Furthermore, to explicitly take ambiguity into account, we defined a multiplier robust control problem, since we recognized that it was difficult for the chief manager of the company to properly estimate the parameter values for the underlying model.

The quantitative analysis in this paper revealed that the embedded real options have significant economic value if the company exercises them properly. We showed that ambiguity could significantly decrease the value of the company’s business, and delay the optimal exercise in comparison to the case under risk. However, our analysis also indicated that even in the presence of ambiguity the company’s optimal exercise policy is relatively robust, that is, we can expect that the actual exercise policy under ambiguity is close to the case under risk.

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