Correlation between Instantons and QCD-monopoles in the Abelian Gauge

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ABSTRACT
The correlation between instantons and QCD-monopoles is studied both in the lattice gauge theory and in the continuum theory. From a simple topological consideration, instantons are expected to live only around the QCD-monopole trajectory in the abelian gauge. First, the instanton solution is analytically studied in the Polyakov-like gauge, where $A_4(x)$ is diagonalized. The world line of the QCD-monopole is found to be penetrate the center of each instanton inevitably. For the single-instanton solution, the QCD-monopole trajectory becomes a simple straight line. On the other hand, in the multi-instanton system, the QCD-monopole trajectory often has complicated topology including a loop or a folded structure, and is unstable against a small fluctuation of the location and the size of instantons. We also study the thermal instanton system in the Polyakov-like gauge. At the high-temperature limit, the monopole trajectory becomes straight lines in the temporal direction. The topology of the QCD-monopole trajectory is drastically changed at a high temperature. Second, the correlation between instantons and QCD-monopoles is studied in the maximally abelian (MA) gauge and/or the Polyakov gauge using the SU(2) lattice with $16^4$. The abelian link variable $u_\mu(s)$ is decomposed into the singular (monopole-dominating) part $u_\mu^{Ds}(s)$ and the regular (photon-dominating) part $u_\mu^{Ph}(s)$. The instanton numbers, $Q(Ds)$ and $Q(Ph)$, are measured using the SU(2) variables, $U_\mu^{Ds}(s)$ and $U_\mu^{Ph}(s)$, which are reconstructed by multiplying the off-diagonal matter factor to $u_\mu^{Ds}(s)$ and $u_\mu^{Ph}(s)$, respectively. A strong correlation is found between $Q(Ds)$ in the singular part and the ordinary topological charge $Q(SU(2))$ even after the Cabibbo-Marinari cooling. On the other hand, $Q(Ph)$ quickly vanishes by several cooling sweeps, which means the absence of instantons in the regular part. Such a monopole dominance for the topological charge is found both in the MA gauge and in the Polyakov gauge.
1. Topological Consideration

Recently, essential roles of monopole condensation [1-4] to the nonperturbative QCD are strongly suggested by the studies based on the lattice gauge theory [5-15]. As ’t Hooft pointed out [4], the nonabelian gauge theory as QCD is reduced to an abelian gauge theory with magnetic monopoles (QCD-monopoles) by the abelian gauge fixing, which is defined by the diagonalization of a gauge-dependent variable $X(x)$. The QCD-monopole appears from the hedgehog configuration on $X(x)$ [16-18] corresponding to the nontrivial homotopy group $\pi_2(SU(N_c)/U(1)^{N_c-1}) = \mathbb{Z}^{N_c-1}_{\infty}$, and its condensation leads to the color confinement through the dual Meissner effect. The crucial role of QCD-monopole condensation to the chiral-symmetry breaking is also supported by recent lattice studies [8,12,13] and the model analyses [16-20]. The instanton [21] is also an important topological object in QCD relating to the $U_A(1)$ anomaly, and appears in the Euclidean 4-space $\mathbb{R}^4$ corresponding to $\pi_3(SU(N_c)) = \mathbb{Z}_{\infty}$. 

Recent lattice studies [5-15] indicate the abelian dominance [4,22] for the nonperturbative quantities in the maximally abelian (MA) gauge and/or in the Polyakov gauge. If the system is completely described only by the abelian field, the instanton would lose the topological basis for its existence, and therefore it seems unable to survive in the abelian manifold. However, even in the abelian gauge, nonabelian components remain relatively large around the QCD-monopoles, which are nothing but the topological defects, so that instantons are expected to survive only around the world lines of the QCD-monopole in the abelian-dominating system. The close relation between instantons and QCD-monopoles are thus suggested from the topological consideration. In this paper, we study the correlation between instantons and QCD-monopoles both in the lattice theory [12,14,23] and in the analytical framework [18].
2. Analytical Calculation

2.1. Abelian Gauge Fixing and Monopole Charge

First, the abelian gauge fixing is studied with attention to the ordering condition [14], which is closely related to the magnetic charge of QCD-monopoles. In general, the abelian gauge fixing consists of two sequential procedures.

1. The diagonalization of a gauge-dependent variable \( X(x) \) by a suitable gauge transformation : \( X(x) \rightarrow X_d(x) \) [16]. The gauge group \( \text{SU}(N_c)_{\text{local}} \) is reduced to \( \text{U}(1)^{N_c-1}_{\text{local}} \times P_{\text{global}}^{N_c} \) by the diagonalization of \( X(x) \).

2. The ordering on the diagonal elements of \( X_d(x) \) by imposing the additional condition, for instance,

\[
X_1^d(x) \geq X_2^d(x) \geq ... \geq X_{N_c}^d(x). \tag{2.1}
\]

The residual gauge group \( \text{U}(1)^{N_c-1}_{\text{local}} \times P_{\text{global}}^{N_c} \) is reduced to \( \text{U}(1)^{N_c-1}_{\text{local}} \) by the ordering condition on \( X_d(x) \) [14].

The magnetic charge of the QCD-monopole is closely related to the ordering condition in the diagonalization in the abelian gauge fixing [14]. For instance, in the \( \text{SU}(2) \) case, the hedgehog configuration as \( X(x) = (x \cdot \tau) \) and the anti-hedgehog one as \( X(x) = -(x \cdot \tau) \) provide a QCD-monopole with an opposite magnetic charge, because they are connected by the additional gauge transformation by

\[
\Omega = \exp\{i\pi(\frac{\tau_1}{2} \cos \alpha + \frac{\tau_2}{2} \sin \alpha)\} \in P_{\text{global}}^2 \tag{2.2}
\]

with an arbitrary constant \( \alpha \). Here, \( \Omega \) physically means the rotation of angle \( \pi \) in the internal \( \text{SU}(2) \) space, and it interchanges the diagonal elements of \( X_d(x) \), which leads a minus sign in the \( \text{U}(1)_3 \) gauge field, \( A_\mu^3(x) \). Thus, the magnetic charge of the QCD-monopole is settled by imposing the ordering condition on \( X_d(x) \).
$P_{\text{global}}^{N_c}$-symmetry is also important for the argument of gauge dependence. If a variable holds the residual gauge symmetry in the abelian gauge, it is proved to be $\text{SU}(N_c)$ gauge invariant [7]. However, one should carefully examine the residual gauge symmetry, which often includes not only $U(1)^{N_c-1}_{\text{local}}$ but also $P_{\text{global}}^{N_c}$. For instance, the dual Ginzburg-Landau theory [16] is, strictly speaking, an effective theory holding $U(1)^{N_c-1}_{\text{local}} \times P_{\text{global}}^{N_c}$ symmetry. Hence, gauge dependence of a physical variable should be carefully checked in terms of the residual gauge symmetry $U(1)^{N_c-1}_{\text{local}} \times P_{\text{global}}^{N_c}$ instead of $U(1)^{N_c-1}_{\text{local}}$ [7]. As a result, the dual gauge field $\vec{B}_\mu$ is not $\text{SU}(N_c)$-invariant, because $\vec{B}_\mu$ is $U(1)^{N_c-1}_{\text{local}}$-invariant but is changed under the global $P_{\text{global}}^{N_c}$ transformation.

\subsection*{2.2. QCD-monopoles in the Polyakov-like Gauge}

We demonstrate a close relation between instantons and QCD-monopoles in the Euclidean $\text{SU}(2)$ gauge theory in continuum [16-19]. Since there is an ambiguity on the gauge-dependent variable $X(x)$ to be diagonalized in the abelian gauge fixing [4,16], it would be a wise way to choose a suitable $X(x)$ so that the instanton configuration can be simply described. Here, we adopt the Polyakov-like gauge [14], where $A_4(x)$ is diagonalized. The Polyakov-like gauge has a large similarity to the Polyakov gauge, because the Polyakov loop $P(x)$ is also diagonal in this gauge.

Using the 't Hooft symbol $\bar{\eta}^{a\mu\nu}$, the multi-instanton solution is written as [21]

$$A^\mu(x) = i\bar{\eta}^{a\mu\nu} \frac{\tau^a}{2} \partial^\nu \ln \phi(x) = -i\bar{\eta}^{a\mu\nu} \frac{\tau^a}{\phi(x)} \sum_k a_k^2 (x - x_k)^\nu \left| x - x_k \right|^4, \quad \phi(x) \equiv 1 + \sum_k \frac{a_k^2}{|x - x_k|^2}$$

(2.3)

where $x_k^\mu \equiv (x_k, t_k)$ and $a_k$ denote the center coordinate and the size of $k$-th instanton, respectively. Near the center of $k$-th instanton, $A_4(x)$ takes a hedgehog configuration around $x_k$,

$$A_4(x) \simeq i \frac{\tau^a (x - x_k)^a}{|x - x_k|^2},$$

(2.4)

like a single-instanton solution. In the Polyakov-like gauge, $A_4(x)$ is diagonalized
by a singular gauge transformation, which provides a QCD-monopole on the center of the hedgehog, $x = x_k$. Thus, the center of each instanton is inevitably penetrated by a QCD-monopole trajectory along the temporal direction in the Polyakov-like gauge [14,18]. In other words, instantons exist only along the QCD-monopole trajectories.

2.3. QCD-Monopole Trajectory in the Multi-Instanton System

For the single-instanton system, $A_4(x)$ takes a hedgehog configuration around $x_1$,

$$A_4(x) = -i a_1^2 \frac{\tau^a (x - x_1)^a}{(x - x_1)^2 \cdot ((x - x_1)^2 + a_1^2)}.$$  \hspace{1cm} (2.5)

The diagonalization of $A_4(x)$ is carried out using a time-independent singular gauge transformation with the gauge function

$$\Omega(x) = e^{i \tau_3 \phi} \cos \frac{\theta}{2} + i (\tau_1 \cos \alpha + \tau_2 \sin \alpha) \sin \frac{\theta}{2} = \begin{pmatrix} e^{i \phi} \cos \frac{\theta}{2} & i e^{i \alpha} \sin \frac{\theta}{2} \\ i e^{-i \alpha} \sin \frac{\theta}{2} & e^{-i \phi} \cos \frac{\theta}{2} \end{pmatrix}$$ \hspace{1cm} (2.6)

with $\theta$ and $\phi$ being the polar and azimuthal angles: $x - x_1 = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$. Here, $\alpha$ is an arbitrary constant angle corresponding to the residual $U(1)_3$ symmetry. Since $\Omega(x)$ is time-independent, $A_4(x)$ is simply transformed as

$$A_4(x) \rightarrow \Omega(x) A_4(x) \Omega^{-1}(x).$$ \hspace{1cm} (2.7)

After the singular gauge transformation by $\Omega(x)$, the abelian gauge field $A_\mu^3(x)$ has a singular part stemming from

$$A_\mu^{sing}(x) = \frac{1}{e} \Omega(x) \partial_\mu \Omega^{-1}(x),$$ \hspace{1cm} (2.8)

which leads to the QCD-monopole with the magnetic charge $g = 4\pi/e$ [16]. The QCD-monopole appears at the center of the hedgehog, $x = x_1$, which satisfies
\( A_4(x) = 0 \) in Eq.(2.5). Hence, the QCD-monopole trajectory \( x^\mu \equiv (x, t) \) becomes a simple straight line penetrating the center of the instanton as shown in Fig.1 (a),

\[
\mathbf{x} = \mathbf{x}_1 \quad (-\infty < t < \infty),
\]

at the classical level in the Polyakov-like gauge [16,14,18]. Similar relation for the QCD-monopole in a single instanton is found also in the MA gauge [24].

It should be noted that the singularity of \( A_\mu(x) \) at the center of the instanton can be removed easily by a gauge transformation to the non-singular gauge [21], where

\[
A_\mu(x) = i \frac{\tau^a(x - x_1)^a}{(x^2 - x_1^2) + a_1^2}
\]

provides the same QCD-monopole trajectory as mentioned above. It is also worth mentioning that the QCD-monopole trajectory is not changed by the residual \( U(1)_3 \)-gauge transformation, so that QCD-monopoles in the Polyakov-like gauge are identical to those, \( e.g. \), in the temporal gauge: \( A_4(x) = 0 \).

For the single anti-instanton system, one has only to replace \( A_4(x) \rightarrow -A_4(x) \) corresponding to \( \bar{\eta}^{\alpha\mu\nu} \rightarrow \eta^{\alpha\mu\nu} \) in the above argument [21]. Since this replacement interchanges the hedgehog and the anti-hedgehog on \( A_4(x) \), it leads to the change of the QCD-monopole charge as mentioned in Section 2.1. Then, the QCD-monopole with the opposite magnetic charge, \(-g\), appears and passes through the center of the anti-instanton as shown in Fig.1 (b). In Figs.1 (a) and (b), relative difference on the QCD-monopole charge is expressed by the direction of the arrow.

For the two-instanton system, two instanton centers can be put on the \( zt \)-plane by a suitable spatial rotation in \( \mathbb{R}^3 \) without loss of generality, so that one can set \( x_1 = y_1 = x_2 = y_2 = 0 \). Owing to the axial-symmetry around the \( z \)-axis of the system, the QCD-monopole trajectory only appears on the \( zt \)-plane, and hence one has only to examine \( A_4(x) \) on the \( zt \)-plane by setting \( x = y = 0 \). In this case,
\( A_4(x) \) is already diagonalized on the \( zt \)-plane:

\[
A_4(z, t; x = y = 0) = -i \frac{\tau^3}{\phi(z, t)} \sum_{k=1}^2 a_k^2 \frac{(z - z_k)}{(z - z_k)^2 + (t - t_k)^2} \equiv A_3^3(z, t) \tau^3. \tag{2.11}
\]

Therefore, the QCD-monopole trajectory \( x^\mu = (x, y, z, t) \) is simply given by \( x = y = 0 \) and \( A_4^3(z, t) = 0 \) or

\[
\sum_{k=1}^2 a_k^2 \frac{(z - z_k)}{(z - z_k)^2 + (t - t_k)^2} = 0. \tag{2.12}
\]

Here, \( A_4(x) \) takes a hedgehog or an anti-hedgehog configuration near the QCD-monopole at each \( t \).

We show in Figs. 2 (a),(b) and (c) the typical examples of the QCD-monopole trajectory in the two-instanton system. The QCD-monopole trajectories are found to be rather complicated even at the classical level. Fig.2 (a) shows the simplest case for two instantons with the same size, \( a_1 = a_2 \), locating at the same Euclidean time, \((z_1, t_1) = -(z_2, t_2) = (z_0, 0)\). In this case, the QCD-monopole trajectory \((z, t)\) is analytically solved [18] as

\[
z = 0 \quad \text{or} \quad t^2 = (z_0^2 - z^2) + 2|z_0|\sqrt{(z_0^2 - z^2)}, \tag{2.13}
\]

and there appear two junctions and a loop in the QCD-monopole trajectory [18]. Here, the QCD-monopole charge calculated is expressed by the direction of the arrow.

Fig.2 (b) shows an example for two instantons with the same size, \( a_1 = a_2 \), but a little rotated in \( \mathbb{R}^4 \) as \((z_1, t_1) = -(z_2, t_2) = (1, 0.05)\). In this case, the QCD-monopole trajectory has a folded structure [14]. Fig.2 (c) shows an example for two instantons locating at the same time \((z_1, t_1) = -(z_2, t_2) = (1, 0)\), but with a little different size, \( a_2 = 1.1a_1 \). There appears a QCD-monopole loop in this case [14]. Thus, the QCD-monopole trajectories originating from instantons are very
unstable against a small fluctuation relating to the location or the size of instantons [14].

For a general $N$-instanton system with $N \geq 3$, it is rather difficult to find a suitable gauge transformation diagonalizing $A_4(x)$, and therefore it is hard to obtain the QCD-monopole trajectory. However, the QCD-monopole trajectory can be also obtained by $x = y = 0$ and $A_4(z, t) = 0$ as Eq.(2.12) for the multi-instantons located on the $zt$-plane. Hence, we have examined such a special case in the multi-instanton system.

In general, the QCD-monopole trajectory becomes highly complicated and unstable in the multi-instanton system even at the classical level, and a small fluctuation of instantons often changes the topology of the QCD-monopole trajectory as shown in Fig.2. In addition, the quantum fluctuation would make it more complicated and more unstable, which leads to appearance of a long twining trajectory as a result. Hence, instantons may contribute to promote monopole condensation, which is signaled by a long complicated monopole loop in the lattice QCD simulation [7,9,10].

2.4. QCD-MONOPOLE TRAJECTORY IN THE THERMAL-INSTANTON SYSTEM

We also study the thermal instanton system in the Polyakov-like gauge. The multi-instanton solution at finite temperature $T$ is given by

$$A^\mu(x) = i\tilde{\eta}^{\alpha\mu\nu} \frac{\tau^a}{2} \partial^\nu \ln \phi(x) = i\tilde{\eta}^{\alpha\mu\nu} \frac{\tau^a}{2} \partial^\nu \frac{\phi(x)}{\phi(x)} / \phi(x),$$

$$\phi(x) = 1 + \sum_k a_k^2 \sum_{n=-\infty}^{\infty} \frac{1}{(x - x_k)^2 + (t - t_k - n/T)^2} = 1 + \pi T \sum_k \frac{a_k^2}{|x - x_k|} \cdot \frac{\sinh(2\pi T|x - x_k|)}{\cosh(2\pi T|x - x_k|) - \cos\{2\pi T(t - t_k)\}}.$$ (2.14)
In this system, $A_4(x)$ is given by

$$
A_4(x) = -i \frac{\pi T \tau^a}{2\phi} \sum_k a_k^2 (x - x_k)^a \left( \frac{\sin(2\pi T |x - x_k|)}{\cosh(2\pi T |x - x_k|) - \cos\{2\pi T (t - t_k)\}} \right) - 2\pi T |x - x_k| \cdot \frac{1 - \cosh(2\pi T |x - x_k|) \cos\{2\pi T (t - t_k)\}}{\left[\cosh(2\pi T |x - x_k|) - \cos\{2\pi T (t - t_k)\}\right]^2} \right) + O(2.15)
$$

At the high-temperature limit $T \to \infty$,

$$
A_4(x) \simeq -i \frac{\pi T \tau^a}{2\phi} \sum_k a_k^2 (x - x_k)^a \left( \frac{\sin(2\pi T |x - x_k|)}{\cosh(2\pi T |x - x_k|) - \cos\{2\pi T (t - t_k)\}} \right)
$$

becomes time-independent, so that $A_4(x)$ can be diagonalized using a time-independent gauge transformation by $\hat{\Omega}(x)$,

$$
A_4(x) \to \hat{\Omega}(x) A_4(x) \hat{\Omega}^{-1}(x) = A_4^d(x),
$$

where QCD-monopoles appear at the points $x_s$ satisfying $A_4(x_s) = 0$. These points $x_s$ includes all the centers of instantons, $x_k$, and become the centers of the (anti-) hedgehog configuration on $A_4(x)$. Thus, the QCD-monopole trajectory is reduced to simple straight lines

$$
x = x_s \quad (-\infty < t < \infty),
$$

where each instantons are penetrated in the temporal direction. Such a simplification of the QCD-monopole trajectory may corresponds to the deconfinement phase transition through the vanishing of QCD-monopole condensation [5-11,17,25].

For the thermal two-instanton system, all instanton centers can be put on the $zt$-plane by a suitable spatial rotation in $\mathbb{R}^3$ like the two-instanton system at $T = 0$, so that one can set as $x_k = y_k = 0 \ (k = 1, 2)$. Owing to the axial-symmetry around the $z$-axis of the system, the QCD-monopole trajectory only appears on the $zt$-plane, where $A_4(x)$ in Eq.(2.15) is already diagonalized. Hence, the QCD-monopole trajectory $x^\mu = (x, y, z, t)$ is simply given by $x = y = 0$ and
$A_4(z,t; x = y = 0) = 0$. Here, $A_4(x)$ takes a hedgehog or an anti-hedgehog configuration near the QCD-monopole at each $t$.

We show in Fig.3 the typical examples of the QCD-monopole trajectory in the thermal two-instanton system. As temperature goes high, the trajectory tends to be straight lines in the temporal direction. There also appears the QCD-monopole with the opposite magnetic charge at the point satisfying $A_4(x) = 0$. The topology of the QCD-monopole trajectory is drastically changed at $T_c \simeq 0.6d^{-1}$, where $d$ is the distance between the two instantons. If one adopts $d \sim 1$fm as a typical mean distance between instantons, such a topological change occurs at $T_c \sim 120$MeV [14].

3. Instanton and Monopole on Lattice

3.1. Framework

We study the correlation between instantons and QCD-monopoles in the maximally abelian (MA) gauge [12] and in the Polyakov gauge using the SU(2) lattice with $16^4$ and $\beta = 2.4$. All measurements are done every 500 sweeps after a thermalization of 1000 sweeps using the heat-bath algorithm. After generating the gauge configurations, we examine the monopole dominance [5-15] for the topological charge using the following procedure [12,14].

1. The abelian gauge fixing is carried out by diagonalizing $R(s) = \sum_\mu U_\mu(s) \tau^3 U^{-1}_\mu(s)$ in the MA gauge, and/or the Polyakov loop $P(s)$ in the Polyakov gauge.

2. The SU(2) link variable $U_\mu(s)$ is factorized into the abelian link variable $u_\mu(s) = \exp\{i\tau_3 \theta_\mu(s)\}$ and the ‘off-diagonal’ factor $M_\mu(s)$ [6-14],

$$U_\mu(s) = \begin{pmatrix} \sqrt{1 - |c_\mu(s)|^2} & -c_\mu(s) \\ c_\mu^*(s) & \sqrt{1 - |c_\mu(s)|^2} \end{pmatrix} \begin{pmatrix} e^{i\theta_\mu(s)} & 0 \\ 0 & e^{-i\theta_\mu(s)} \end{pmatrix} \equiv M_\mu(s) u_\mu(s). \quad (3.1)$$

where $c_\mu(s)$ transforms as the charged matter field.
3. The abelian field strength \( \theta_{\mu\nu} \equiv \partial_\mu \theta_\nu - \partial_\nu \theta_\mu \) is decomposed as

\[
\theta_{\mu\nu}(s) = \bar{\theta}_{\mu\nu}(s) + 2\pi M_{\mu\nu}(s)
\]

with \(-\pi < \bar{\theta}_{\mu\nu}(s) < \pi\) and \(M_{\mu\nu}(s) \in \mathbb{Z}\). Here, \(\bar{\theta}_{\mu\nu}(s)\) and \(2\pi M_{\mu\nu}(s)\) correspond to the regular photon part and the Dirac string part, respectively [26].

4. The U(1) gauge field \(\theta_\mu(s)\) is decomposed as \(\theta_\mu(s) = \theta^{Ph}_\mu(s) + \theta^{Ds}_\mu(s)\) [5-14,26], where the regular part \(\theta^{Ph}_\mu(s)\) and the singular part \(\theta^{Ds}_\mu(s)\) are obtained from \(\bar{\theta}_{\mu\nu}(s)\) and \(2\pi M_{\mu\nu}(s)\), respectively,

\[
\theta^{Ds}_\mu(s) = 2\pi \sum_{s'} G(s - s') \partial^\lambda M^\lambda_\mu(s'), \quad \theta^{Ph}_\mu(s) = \sum_{s'} G(s - s') \bar{\partial}^\lambda \bar{\theta}^\lambda_\mu(s'),
\]

using the lattice Coulomb propagator \(G(s)\) in the Landau gauge, which satisfies \(\partial^2 G(s - s') = \delta^4(s - s')\). The singular part carries almost the same amount of the magnetic current as the original U(1) field, whereas it scarcely carries the electric current. The situation is just the opposite in the regular part. For this reason, we regard the singular part as ‘monopole-dominating’, and the regular part as ‘photon-dominating’ [12,14].

5. The corresponding SU(2) variables are reconstructed from \(\theta^{Ph}_\mu(s)\) and \(\theta^{Ds}_\mu(s)\) by multiplying the off-diagonal factor \(M_\mu(s)\):

\[
U^{Ds}_\mu(s) = M_\mu(s) \exp\{i\tau_3 \theta^{Ds}_\mu(s)\}, \quad U^{Ph}_\mu(s) = M_\mu(s) \exp\{i\tau_3 \theta^{Ph}_\mu(s)\},
\]

6. By using \(U_\mu(s), U^{Ph}_\mu(s)\) and \(U^{Ds}_\mu(s)\), we calculate the topological charge \(Q\) and the integral \(I_Q\) of the absolute value of the topological density,

\[
Q = \frac{1}{16\pi^2} \int d^4x \text{Tr}(G_{\mu\nu} \tilde{G}_{\mu\nu}), \quad I_Q = \frac{1}{16\pi^2} \int d^4x |\text{Tr}(G_{\mu\nu} \tilde{G}_{\mu\nu})|,
\]
and the action divided by $8\pi^2$,

$$S = \frac{1}{16\pi^2} \int d^4x \text{Tr}(G_{\mu\nu}G_{\mu\nu}).$$  \hspace{1cm} (3.6)

Here, $I_Q$ is introduced to get information on the instanton and anti-instanton pair [12,14]. Three sets of quantities are thus obtained:

$$U_\mu(s) \rightarrow \{Q(\text{SU}(2)), I_Q(\text{SU}(2)), S(\text{SU}(2))\}$$

$$U_{\mu}^{Ds}(s) \rightarrow \{Q(\text{Ds}), I_Q(\text{Ds}), S(\text{Ds})\}$$

$$U_{\mu}^{Ph}(s) \rightarrow \{Q(\text{Ph}), I_Q(\text{Ph}), S(\text{Ph})\}$$ \hspace{1cm} (3.7)

Of course, $\{Q(\text{SU}(2)), I_Q(\text{SU}(2)), S(\text{SU}(2))\}$ defined with the full SU(2) link variable is a set of the ordinary quantities. On the other hand, $\{Q(\text{Ds}), I_Q(\text{Ds}), S(\text{Ds})\}$ and $\{Q(\text{Ph}), I_Q(\text{Ph}), S(\text{Ph})\}$ provide the information on the singular (monopole-dominating) part and the regular (photon-dominating) part, respectively.

7. The correlations among these quantities are examined using the Cabibbo-Marinari cooling method.

We prepare 40 samples for the MA gauge and the Polyakov gauge, respectively. These simulations have been performed on the Intel Paragon XP/S(56node) at the Institute for Numerical Simulations and Applied Mathematics of Hiroshima University. Since quite similar results have been obtained in the MA gauge [12] and in the Polyakov gauge, only latter case is shown below.

3.2. MONOPOLE DOMINANCE FOR TOPOLOGICAL CHARGE ON LATTICE

Fig.4 shows the correlation among $Q(\text{SU}(2))$, $Q(\text{Ds})$ and $Q(\text{Ph})$ at various cooling sweeps in the Polyakov gauge. A strong correlation is found between $Q(\text{SU}(2))$ and $Q(\text{Ds})$, which is defined in singular (monopole-dominating) part. Such a strong correlation remains even at 80 cooling sweeps. On the other hand, $Q(\text{Ph})$ quickly vanishes only by several cooling sweeps, and no correlation is seen between $Q(\text{Ph})$ and $Q(\text{SU}(2))$. 

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We show in Fig.5 the cooling curves for $Q$, $I_Q$ and $S$ in a typical example with $Q(\text{SU}(2)) \neq 0$ in the Polyakov gauge. Similar to the full SU(2) case, $Q(\text{Ds})$, $I_Q(\text{Ds})$ and $S(\text{Ds})$ in the singular (monopole-dominating) part tends to remain finite during the cooling process. On the other hand, $Q(\text{Ph})$, $I_Q(\text{Ph})$ and $S(\text{Ph})$ in the regular part quickly vanish by only several cooling sweeps. Therefore, instantons seems unable to live in the regular (photon-dominating) part, but only survive in the singular (monopole-dominating) part in the abelian gauges.

We show in Fig.6 the cooling curves for $Q$, $I_Q$ and $S$ are examined in the case with $Q(\text{SU}(2)) = 0$ in the Polyakov gauge. Similar to the full SU(2) results, $I_Q(\text{Ds})$ and $S(\text{Ds})$ decrease slowly and remain finite even at 70 cooling sweeps, which indicates the existence of the instanton and anti-instanton pair in the singular (monopole-dominating) part. On the other hand, $I_Q(\text{Ph})$ and $S(\text{Ph})$ quickly vanish, which indicates the absence of such a topological pair excitation in the regular (photon-dominating) part [14].

In conclusion, the monopole dominance for the topological charge is found both in the MA gauge and in the Polyakov gauge. In particular, instantons would survive only in the singular (monopole-dominating) part in the abelian gauges, which agrees with the result in our previous analytical study.

4. Summary and Concluding Remarks

We have studied the relation between instantons and monopoles in the abelian gauge. Simple topological consideration indicates that instantons survive only around the QCD-monopole trajectory, which is the topological defect.

We have found a close relation between instantons and the QCD-monopole trajectory in the Polyakov-like gauge, where $A_4(x)$ is to be diagonalized. Every instantons are penetrated by the world lines of QCD-monopoles inevitably. The QCD-monopole trajectory in $\mathbb{R}^4$ tends to be folded and complicated in the multi-instanton system, although it becomes a simple straight line in the single-instanton
solution. The QCD-monopole trajectory is very unstable against a small fluctuation on the location and the size of instantons.

We have also studied the thermal instanton system in the Polyakov-like gauge. At the high-temperature limit, the QCD-monopole trajectory becomes straight lines in the temporal direction. The QCD-monopole trajectory drastically changes its topology at a high temperature.

We have studied the correlation between instantons and QCD-monopoles in the maximally abelian (MA) gauge and/or the Polyakov gauge on SU(2) lattice with $16^4 (\beta=2.4)$. The abelian link variable $u_\mu(s)$ is decomposed into the singular (monopole-dominating) part $u_\mu^{D_s}(s)$ and the regular (photon-dominating) part $u_\mu^{P_h}(s)$. We have measured the instanton numbers, $Q(Ds)$ and $Q(Ph)$, using the SU(2) variables, $U_\mu^{D_s}(s)$ and $U_\mu^{P_h}(s)$, which are reconstructed by multiplying the off-diagonal matter factor to $u_\mu^{D_s}(s)$ and $u_\mu^{P_h}(s)$, respectively. Topological charge $Q(Ds)$ in the singular (monopole-dominating) part remains to have a finite number during the cooling process. On the other hand, $Q(Ph)$ quickly vanishes by several cooling sweeps, which indicates the absence of instantons in the regular (photon-dominating) part. Thus, instantons cannot live in the regular (photon-dominating) part, but survive in the singular (monopole-dominating) part. We have found a strong correlation between $Q(Ds)$ and the ordinary topological charge $Q(SU(2))$ during the cooling process. We have found such a monopole dominance for the topological charge both in the MA gauge and in the Polyakov gauge.
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**Figure Captions**

*Fig.1* The QCD-monopole trajectory (a) in the single-instanton system, (b) in the single anti-instanton system. The (anti-)instanton is denoted by a small circle.

*Fig.2* Examples of the QCD-monopole trajectory in the two-instanton system with (a) \( (z_1, t_1) = -(z_2, t_2) = (1,0), a_1 = a_2; \) (b) \( (z_1, t_1) = -(z_2, t_2) = (1,0.05), a_1 = a_2; \) (c) \( (z_1, t_1) = -(z_2, t_2) = (1,0), a_2 = 1.1a_1. \)

*Fig.3* The QCD-monopole trajectory in the thermal two-instanton system with \( (z_1, t_1) = -(z_2, t_2) = d/2 \cdot (1,0) \) and \( a_1 = a_2 \) (a) at \( T^{-1} = 2d; \) (b) at \( T^{-1} = 1.5d. \) The same with \( (z_1, t_1) = -(z_2, t_2) = d/2 \cdot (1,0.05) \) and \( a_1 = a_2 \) (c) at \( T^{-1} = 2d; \) (d) at \( T^{-1} = 1.5d. \)

*Fig.4* (a) Correlations between \( Q(Ds) \) and \( Q(SU(2)) \) at various cooling sweeps. (b) Correlations between \( Q(Ph) \) and \( Q(SU(2)) \) at various cooling sweeps.
Fig. 5 Cooling curves for (a) $Q(\text{SU}(2)), I_Q(\text{SU}(2)), S(\text{SU}(2))$; (b) $Q(\text{Ds}), I_Q(\text{Ds}), S(\text{Ds})$; (c) $Q(\text{Ph}), I_Q(\text{Ph}), S(\text{Ph})$ in the case with $Q(\text{SU}(2)) \neq 0$.

Fig. 6 Cooling curves for (a) $Q(\text{SU}(2)), I_Q(\text{SU}(2)), S(\text{SU}(2))$; (b) $Q(\text{Ds}), I_Q(\text{Ds}), S(\text{Ds})$; (c) $Q(\text{Ph}), I_Q(\text{Ph}), S(\text{Ph})$ in the case with $Q(\text{SU}(2)) = 0$. 

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