Review

Conductance anomalies in quantum point contacts and 1D wires*

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Abstract

Over the last decade, interest in 1D charge transport has progressed from the seminal discovery of Landauer quantization of conductance, as a function of carrier density, to finer-scale phenomena at the onset of quantization. This has come to be called the ‘0.7 anomaly’, rather connoting a theoretical mystery of some profundity and universality, which remains open to date. Its somewhat imaginative appellation may tend to mislead, since the anomaly manifests itself over a range of conductance values: anywhere between 0.25–0.95 Landauer quanta. In this paper we offer a critique of the 0.7 anomaly and discuss the extent to which it represents a deep question of physics.

Keywords: conductance quantisation, conductance anomalies, 0.7 anomaly, quantum point contact, 1D wire, Kondo effect

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1. Introduction

The live topic of the ‘0.7 anomaly’ in quantized quasi-one dimensional (1D) conductance has occupied both experimentalists and theorists already for some time, and continues to do so. The effect, or at least effects very like it, has been studied over a wide spectrum of mesoscopic material structures and fabrication techniques. So far, no conclusive theoretical reason has been adduced to explain the origin of the anomaly, any more than the universality implicit in its title.

Electronic transport in quasi-1D systems is essentially studied in quantum-point contact (QPC) and quantum-wire structures. The quantized nature of their conductance, known as Landauer quantization, is a genuinely generic 1D phenomenon, first reported in QPCs of high-electron-mobility devices in GaAs/GaAlAs heterostructures. As a function of carrier density (modulate by a transverse gate potential), distinct steps in the differential conductance ($G$ is normalized to the quantum value $G_{0} = \frac{2e^2}{h}$) were observed [1, 2].

Various explanations for the quantized Landauer steps have been given over the years. Those comprise the simpler Landauer–Büttiker method, the more rigorous Kubo linear-response formula and finally the so-called nonequilibrium-Green-function technique. We cite two of our relevant papers [3, 4] presenting an appraisal of quantized-conductance physics.

From 1988 onwards, a much wider set of different 1D systems, from Si metal-oxide field-effect transistors [5], other III–V heterostructures [6], and constricted graphenes [7] etc, have been shown to exhibit these quantized steps. However, apart from the quantized steps now well established as universal in nature (occurring at integral values $n = 1, 2, 3$ for $G/G_{0} = n$), there also seem to be anomalies observed in many...
of the above systems. These involve peaks and shoulders at non-integral values of $G$, located on the threshold regions of gate voltage, at the onset of a new quantized ‘Landauer’ step. Generally there are two types of such anomalies.

- Thomas et al [8] originally identified an anomalous structure in low-temperature conductance below the first quantized step in GaAs QPCs at about 0.7$G_0$. It has been argued that this perceived anomaly is an intrinsic property caused by some many-body effect independent of the details of the material system. The authors of [8] argued that, since the effect appears to occur in the majority of 1D conduction channels, it may be universal; this is what the majority of the literature, subsequent to [8], likewise suggests. Although the anomaly has been discussed abundantly ever since, it should be pointed out that many such features have now been subsumed into the 0.7 rubric even when it is abundantly clear that anomalies occur in all kinds of temperature ranges and at nonzero magnetic fields.

- A further, and very different, anomalous peak has been identified in the nonlinear transport regime at low temperatures—known as the zero-bias anomaly (ZBA) in the differential conductance—as a function of source–drain voltage along the channel. Unlike the range of so-called 0.7 features, this anomaly is well understood and we shall not pursue it here.

So far, published explanations for the physics of the 0.7 anomalies follow two principal ideas: (1) the assumption of spontaneous spin polarization (SSP) of the carrier population, and (2) the presence of a many-body state induced by Kondo physics.

SSP is a semi-phenomenological description [9, 10]. Spin-polarized density-functional calculations by Berggren and coworkers [9, 10] have shown that exchange interactions can induce a large sub-band splitting whenever the Fermi energy passes through the sub-band threshold energies. Non-monotonic variations in the densities of states, as the gate potential varies, leads to different degrees of spin polarization and are liable to produce conductance anomalies.

In the current literature both 0.7 and ZBA features have been explained [11, 12] by either: amplification of interaction effects when a smeared van-Hove singularity of the local density of states (DOS) (at the bottom of the lowest 1D sub-band) crosses the chemical potential, or by an emergent localized state contributing to the formation of a collective (many-body) state arising from the Kondo effect in electron transport through the localized state. Another work, [13], develops a similar idea based on Kondo physics. Here we note that the latter mechanisms both run counter to the alternative models assuming SSP, and shall return to these questions below.

In the next section we give a pedagogic picture of the principal Landauer conductance steps for a long quantum wire or quantum point contact. In section 3 we discuss various non-integral features of conductance as a function of split-gate voltage, for a variety of 1D systems. We present a brief review in section 4, with full reference to the current literature as to how one attempts to understand the anomalous features. In section 5 we conclude our study with a critical discussion study of the presumed universality of the so-called conductance anomalies, arguing that they do not meet any reasonable measure of what a truly generic phenomenon should be. In doing so we refute claims [14, 15] that these anomalies are fundamentally intrinsic to 1D electronic transport. They are, by contrast, material specific not at all ‘universal’.

2. Quantum wires and QPCs: conductance

Quantum wires and QPCs are two examples of real quasi-1D electronic systems [16]. They are fabricated by a variety of established techniques: cleaved-edge overgrowth (CEO) for the former [17], and split-gate geometry for the latter. To interpret the observed conductance behaviour of these devices we go to Landauer’s model.

The schematic of figure 1 shows how a device is connected to its external current source by two large reservoirs, which are assumed to acquire two unequal chemical potentials when biased, being mutually offset by the applied electrostatic energy eV. The 1D system has a long (ideally infinite) length supporting several electronic conduction sub-bands, depending on the width of constriction or, alternatively, the strength of side-gate potential which modulates the 1D carrier confinement. A top gate normally supplies a transverse bias potential that modulates the 1D carrier density within the channel.

The channel in a ‘perfect’ device is long and uniform and perfectly transmissive. The net electron current per channel per spin is assumed to be the difference in integrated flux $-ev_k$ between unidirectional ‘right-movers’ with distribution $f_1 = f(\varepsilon_n - \varepsilon_F - \mu_r)$ and ‘left-movers’, $f_2 = f(\varepsilon_n(k) - \varepsilon_F - \mu_l)$. Here $\varepsilon_n(k) = \varepsilon_n + \varepsilon(k)$ is the sub-band energy with $\varepsilon_n$ the threshold and $\mu_r, \mu_l$ are the left and right reservoir electro-chemical potential respectively. With Pauli blocking of the complementary final states, the result is

$$\mathcal{J}_n = \int_0^\infty \frac{dk}{2\pi}(-ev_k)(f_1(1 - f_2) - f_2(1 - f_1))$$

$$= -\frac{e}{2\pi \hbar} \int_0^\infty d\varepsilon \delta \mu \frac{\partial}{\partial \varepsilon} f(\varepsilon_n - \varepsilon_F)$$

$$= \frac{e^2}{\hbar} f(\varepsilon_n - \varepsilon_F) V, \quad (1)$$

where one also assumes the offset of reservoir electro-chemical potentials $\delta \mu = \mu_r - \mu_l$ to be the electrostatic energy per carrier, $eV$, gained from the externally applied potential difference.

In the zero-temperature limit $f(\varepsilon_n - \varepsilon_F)$ is zero if the Fermi level falls below the sub-band threshold, and unity if the level is above threshold and the sub-band is populated. With spin degeneracy and with $N$ sub-bands populated, one then obtains the total conductance as $G = (2e^2/\hbar)N = G_0 N$.

The resistance of this ideal system is $G^{-1}$, which is finite, independent of length, and indeed universal. When the chemical potential crosses the bottom of each sub-band, the conductance ratio $G/G_0$ steps up from 1 to 2, and so on.
These are the well known quantized conductance steps (see figure 1).

The flux $-\psi_k$ in the integrand on the right-hand side of equation (1) above is further modified in the presence of quantum coherent scattering (unitary, hence nondissipative), accounting for partial reflection of incoming states and lowering the net transmission below ideal. The conductance in a single channel typically becomes $G = G_0 T$, where $T \leq 1$. When the transmission factor $T$ is non-ideal, the conductance will be seen to undershoot its full integer value.

The commonly accepted phenomenology of $G$ is known as the Landauer (sometimes, Landauer–Büttiker) model and is overwhelmingly applied in most analyses of real measurements (figure 2). It asserts [16] that the dissipation takes place exclusively the leads. If so, however, a further question becomes inevitable at ideal transmission $T = 1$: What relevance can the internal physics of a coherent quantum channel have, in that case, for the (universal) Joule-heating formula $P = IV = GV^2$?

In figure 3 we show the quantized conductance steps as a function of chemical potential. Differently from the Landauer model, these are calculated within a standard quantum-kinetic description of a 1D wire, in which inelastic dissipation enters naturally and concurrently with elastic scattering. The model is an application of the canonical Kubo formula for transport.

3. Anomalies at non-integer values of $G$

In the introduction we highlighted the existence of anomalies in the value of conductance at specific values of gate voltages and thus chemical potentials in the channel. The following (partial) list gives an indication of the non-integral ‘anomalous’ values of $G$ in a range of device situations (see table 1).

The actual locations and magnitudes of these anomalies are evidently many and varied. The point to be emphasised here is that $0.7G_0$ is hardly a universal fixture in the classic sense of Landauer quantization. Nevertheless, the 0.7 anomaly, which in
appears consistent with a Kondo mechanism, as we discuss below. For details, see the caption of figure 4.

4. Theoretical arguments

Attempts to understand the anomalous conductance features at non-integral values are already numerous and include, in the main: SSP; Kondo and van Hove types of singularity at/ around the Fermi level; Majorana modes; Wigner crystallization etc. There are more besides, and we discuss here the better-known theoretical models popularised in the existing literature.

4.1. Spontaneous spin polarization

Consider a 2D electron system on the interface of a GaAs/AlGaAs heterojunction material [10]. Motion in the perpendicular direction to the heterojunction interface (say in the z-direction) is strongly inhibited by transverse quantum-well confinement, whose bound-state energies are large. An additional, somewhat weaker quantum-well confinement is imposed in the y-direction by constriction either from side gates or by channel etching. In a harmonic-potential approximation the corresponding sub-band levels in the y-direction would be $E_n = \hbar(n + 1/2)$ for $n = 1, 2, 3$, etc.

The conductive channel now extends indefinitely in the x-direction, leading to a quasi-continuous band of extended, and thus current-carrying, states. Consider a specific sub-band in the heterojunction. The sub-band states’ effective Schrödinger equation, within the Kohn–Sham local-spin-density approximation (LSDA), is

$$\left[\frac{p_x^2 + p_y^2}{2m^*} + V_{\text{conf}}(y) + V_{\text{exch}}(x, y)\right] \psi(x, y) = E \psi(x, y),$$

(2)

where $\sigma$ refers to the carriers’ spin and $m^*$ is their effective mass. Here also $V_{\text{conf}}(x, y)$ represents the lateral confinement potential (as well as the carriers’ mean-field (Hubbard) electrostatic potential). In its bare form, the confinement part is taken to be

$$V_{\text{conf}}(x, y) = \frac{m^*}{2}(\omega_0^2 y^2 - \omega_0^2 x^2) + V_0$$

The exchange potential $V_{\text{exch}}(x, y)$, on the other hand, is intrinsic to interacting fermionic carriers. Exchange has a particular form within the LSDA theory, and interested readers may find the details in [10].

Numerical Kohn-Sham LSDA calculations based on equation (2) were carried out in a series of papers by Berggren and co-workers [9, 10]. The conductance $G$ as a function of the height of the height $V_0$ of the saddle point in the bare confinement potential is calculated (figure 4 of in [10]). In the noninteracting Landauer model the ratio $G/G_0$ goes through a step of unity in a certain range of values of $V_0$; but in the spin-polarised exchange model, that ratio acquires a Zeeman-splitting substructure at 0.5.

Several groups [21–23] have suggested that electron pairs are coupled in singlet and triplet configurations, experiencing different transmission barriers. Within a stochastic approach to the occupancy, the ratio of triplet to singlet resonances will be 3:1. This suggests that there are two structures in the plateau-to-plateau transition: at $G = 0.75G_0$ and $G = 0.25G_0$.

The calculations just cited proceed within the exchange-only LSDA theory. Later Berggren et al [24] included the short-range correlation potential (that is, beyond Hubbard) by using a parametrised correlation part from Monte Carlo data of Ceperley and Tanatar [25]. Although the exchange contribution dominates, short-range Coulomb correlations have an important role in determining the spin-split QPC potential. As a result, the calculated conductance as a function of gate voltage shows two anomalies in the first plateau-to-plateau transition, one at $0.4G_0$ for a lower gate potential and another at $0.75G_0$ at a somewhat higher gate potential.

We note here that the spin-split picture here applies for free carriers in a perfect 1D channel. This is in contrast with the picture where the presence of an impurity is pivotal. We now discuss this theory.

4.2. Kondo physics in a QPC

The Kondo effect arises from the interaction of a single magnetic-impurity atom, via its localised spin, with the surrounding conduction electrons of a nonmagnetic metal. This interaction induces a drop of electrical resistance at a particular temperature, which rises again as the temperature is further lowered. Kondo’s theory explained this anomalous behaviour in resistance for a large number of nonmagnetic
systems having dilute magnetic impurities. The key idea is the presence of the localised spin interacting in a conducting continuum.

Recently the Kondo effect has also been observed in quantum dot systems. A quantum dot with at least one unpaired electron acts as a magnetic impurity and the conduction electrons can scatter off the dot. This is analogous to the Kondo effect outlined above. In a parallel development, the possibility of a Kondo-like mechanism has been argued to have relevance to the so-called conductance anomaly as well [11–13, 19, 26, 27]. For a QPC this requires the underlying assumption that the system possesses a localised Kondo-like unpaired spin.

We examine the putative Kondo connection with the 0.7 anomaly. Its physics differs markedly from the previous spin-based idea that the anomaly is qualitatively explained by SSP, not requiring an embedded, magnetically active localised impurity. A number of papers have appeared in favour of a Kondo connection with the 0.7 anomaly.

Cronenwett et al. [28] performed conductance measurements in a GaAs/AlGaAs QPC. Apart from the regular conductance quantization, they also found an anomaly around 0.7G₀. Their argument goes against the two spin channels being simultaneously occupied. Since the 0.7 structure is observed to disappear with lower temperatures they ascribed this behaviour to the Kondo effect (in addition to the presence of a zero-bias conductance peak). Further investigations suggested that the 0.7 anomaly and ZBA share certain similarities with the Kondo effect in quantum dots [12, 13].

The analysis based on the Kondo approach is outlined here, with the model Hamiltonian [29]

$$H = \sum_{\sigma, k \in L, R} \epsilon_k c_{\sigma k}^\dagger c_{\sigma k} + \sum_{\sigma} \epsilon_{\sigma} d_{\sigma}^\dagger d_{\sigma} + U_{n, n}$$

$$+ \sum_{\sigma, k \in L, R} \left[ V_{1L}^{(1)} (1 - n_{\sigma} c_{\sigma k}^\dagger d_{\sigma} + V_{2L}^{(2)} n_{\sigma} c_{\sigma k}^\dagger d_{\sigma} + h.c. \right].$$

Equation (3) is a canonical Anderson Hamiltonian with \(\epsilon_k\) and \(\epsilon_{\sigma}\) the energies of conduction and localised electrons respectively, \(U\) is the on-site Coulomb interaction and the \(V_{\sigma}\) control the hybridization of conduction and localised electrons. The subsidiary notations are detailed in [23].

By performing a Schrieffer–Wolff transformation on this Hamiltonian [27] one obtains a Kondo Hamiltonian with coupling (to second order)
4.4. Other theoretical suggestions

In hybrid semiconducting superconductor-nanowire devices involving a QPC, tunnelling features are observed to manifest a ZBA along with other conductance features, such as the anomaly around $G = 0.7G_0$. With the current interest in unusual topological states of matter and non-abelian quasiparticle statistics, in this context one looks for an interpretation bringing in Majorana zero modes [30]. Nevertheless, to date these experiments cannot clearly resolve differences between Kondo and Majorana physics.

A promising and attractive prospect is to study more deeply the explicit electron correlations in QPCs, since no noninteracting-carrier model is able to produce the conductance anomalies. At low electron density, spontaneous electron localization may occur below the first plateau [31]. This resembles the occurrence of 1D Wigner crystallization [32]. The picture of electron localization may, in any case, be argued in support of the Kondo picture of 0.7-like anomalies.

5. Summary and conclusions

We have presented a brief overview of the conductance anomaly, particularly at non-integral values of conductance at or just below the first Landauer-step transition. We began by examining quasi-1D structures, built on a 1D semiconductor heterojunction surface and with the carrier density modulation by an applied gate potential.

Generally there are two standard types of 1D conductive structure: a quantum-point-contact system and a ballistic D quantum wire made by the CEO technique [17]. Both these structures show pronounced quantized steps of conductance when the gate potential modulates the carrier density in the channel.

We remarked above that a 1D electronic system will show quantized conductance plateaus corresponding to the accessing of distinct sub-bands in the channel. Once the chemical potential moves up with the applied gate voltage, at some value the next upper sub-band starts to fill. The explanation of this basic physical effect is clear within various models. The literature already contains an abundance of observations and their explanation.

Beyond this basic outcome several finer-scale anomalies appear. One is the well observed zero bias anomaly. A tunnelling picture at nearly zero source-drain bias will reproduce this anomaly. In this case explanations by quasi-particle tunnelling and topological Majorana states have brought some controversy.

Matters are quite different for the main issue we have discussed, concerning the so-called 0.7 anomaly in the value of the conductance at some threshold in the channel chemical potential. Despite many claims to universality, it is clear that the anomaly at the non-integral conductance value is observed to be non-universal and non-generic. It shows itself as material-sensitive and also sensitive to temperature and applied magnetic field.

We have discussed the received reasons for this anomaly, as reflected in its considerable literature. Of the leading models, the spin-polarised band picture requires a pure 1D electron gas, while, by contrast, the alternative Kondo picture needs the existence of a localised impurity state. Although these two explanations are mutually exclusive, published measurements can be adduced in support of both. Therefore there is no consensus, either theoretically or even in terms of any arguable ‘universality’ in observations of the 0.7 anomaly.

In our opinion the van-Hove scenario is an acceptable alternative towards an account of the anomaly, if only because every 1D electronic system must exhibit—universally—singularities at the band edges. Near band energies at the edge, the occurrence of high densities of states will influence the conductance profile. As a result an anomaly is possible at a plateau-to-plateau crossover in carrier density. This is sometimes known as the ridge-state anomaly. We foreshadow a detailed trial calculation based on this picture.

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