The Continuous Variable Quantum Teleportation Controversy

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I argue that the objections by Rudolph and Sanders \[1\] to performing continuous variable quantum teleportation experiments using lasers, as well as the various rebuttals to their paper, are based on a misunderstanding of the Partition Ensemble Fallacy.

A quantum mechanical mixed state described by a density matrix $\rho$ can always be decomposed into a variety of ensembles of pure states $\{p_i^j, \psi_i^j\}$ such that

$$\forall j \sum_i p_i^j |\psi_i^j\rangle \langle \psi_i^j| = \rho .$$

Quantum mechanics tells us that no choice of ensemble is preferred over any other. To commit the so-called partition ensemble fallacy (PEF) \[2\] is to retrodict a particular ensemble from one’s experimental results, claiming to have determined “what really happened.” This is not to say, however, that one cannot reasonably interpret results as having arisen from one ensemble or another. In fact, this is precisely what quantum mechanics allows us to do.

A cottage industry has appeared recently disputing the claim of a demonstration of continuous variable quantum teleportation (CVQT) by Furusawa et al. \[3\]. Rudolph and Sanders (RS) assert \[1\] that the PEF has been committed, and that CVQT has not in fact been demonstrated, while several rebuttals of their refutation have appeared \[4, 5, 6, 7\], which mutually disagree to various extent. Though this author believes that many interesting, useful, and correct results have been promulgated in this endeavor, it is the purpose of this paper to argue that the whole controversy is misguided, being founded on a misunderstanding of the PEF.

The state of a laser is normally considered as a coherent state:

$$|\psi\rangle = |\alpha e^{i\phi}\rangle |\alpha e^{i\phi}\rangle,$$

with a definite phase $\phi$ and a mean photon number $|\alpha|^2$. This is not quite correct in reality, since a real laser produces a state

$$\rho = \frac{1}{2\pi} \int d\phi |\alpha e^{i\phi}\rangle \langle \alpha e^{i\phi}|,$$

an equal mixture of all phases. Experiments are typically performed considering phase relative to a reference beam taken off at a beam splitter and shared by all aspects of the experiment. This is what is done in \[3\]. All experimental outcomes relating to the relative phase will behave as if the state had been a pure coherent state all along. It is this retrodiction that is cited in \[1\] as being the commission of the PEF.

This author’s view is that there is no PEF to commit. No claim needs to be made that “what really happened” was that there was a coherent state, if only we could know its phase. Instead, the claim is about what would have happened had a particular coherent state been used. Quantum mechanics tells us, and indeed we’d be committing the PEF to think otherwise, that because we can write $\rho$ in the form (4), we are entitled to exactly this claim. It is important to stress that this is valid provided one believes quantum mechanics. Experiments in CVQT are testing our ability to perform teleportation, not testing quantum mechanics itself. It is the failure to draw this distinction that I would have called the “partition ensemble fallacy fallacy,” had that name not already been taken \[1\], so I will refer to it instead as the partition ensemble fallacy fallacy type 2 (PEF2).

Let us now examine RS’s objections, as well as the various responses to RS in light of this viewpoint:

A. Rudolph and Sanders \[1\]

As RS point out, since $\rho$ (3) is a mixed state, it is valid to consider other ensembles leading to the same density matrix, and we can write $\rho$ in terms of number states using the identity

$$\rho = \frac{1}{2\pi} \int d\phi |\alpha e^{i\phi}\rangle \langle \alpha e^{i\phi}| = \sum_n e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} |n\rangle \langle n| .$$

In this decomposition, diagonal in the number state basis, no teleportation can occur, since $\rho$ has no well defined phase at all. RS go so far to avoid the PEF that they do not give this ensemble a more special place than (3); their point is that no decomposition is privileged. As I have already argued, I consider this a non issue according to the PEF2.

RS have several more specific complaints, related to the three criteria for successful CVQT \[8\].

• a) The states to be teleported should be unknown to Alice and Bob (AB), and supplied by “an actual” third party “Victor.”

Since in the actual experiment Victor shares a reference laser with AB, his state is not truly independent. Written in the number basis the resulting joint density matrix shows significant correlation between AB and Victor. This is hardly a surprise as the point of sharing the reference laser with Victor was so his phase would be correlated with AB’s. On the other hand, working in the basis of coherent states it is simple to see that if victor inserts a phase shift into his beam, AB can have no
information about the shift, even if their overall phases are correlated with Victor’s. Appealing to the PEFF2 confirms that this is acceptable.

Still, the spirit of quantum teleportation is for Victor to share nothing with AB, save the state he hands to Alice to be teleported, and the reference laser violates that spirit. A better experiment would have Victor use his own independent laser, though this should not in principle change the results. It is not clear that everyone agrees with this point, and we shall return to it by and by.

• b) Alice and Bob share only a nonlocal entangled resource and a classical channel through which Alice transmits her measurement results to Bob.

Just as Victor gets to share in the reference laser, in the experimental setup of [8] Alice and Bob share the reference beam during the entire course of the teleportation. This in itself violates criterion b (it is something extra other than entanglement or a classical channel), but worse, it implies they in fact share a quantum channel during the teleportation. Perhaps quantum information is being sent through the channel channel, cheating on the experiment of performing true teleportation. Wiseman, as well as van Enk and Fuchs (vEF) [4, 5, 7], point out that this could be addressed by sharing the time reference before the teleportation commences, just as the entanglement must be shared first, ensuring quantum information can not be surreptitiously passed from Alice to Bob during the protocol. This is an additional shared resource without which teleportation cannot be done, but we must live with this. Teleportation as originally conceived [8] assumes such a resource—a shared basis is required even for simple qubit based protocols.

This presharing of the reference laser has not been done in actual experiments, and this author believes it should be before a complete CVQT experiment can be claimed.

• c) Entanglement should be a verifiable resource. RS show that the density matrix of the squeezed state generated starting from the mixed state \( \rho \otimes \rho \), rather than from a pure coherent state, is separable. But it is demonstrated in [8] that when one includes the reference beam and considers the entire state of AB that they do indeed share distillible entanglement.

It is not obvious that merely sharing distillible entanglement without actually distilling it into pure entanglement is sufficient for teleportation. However, it is shown [8] that the fidelity achieved in the experiment exceeds that which could be achieved by a classical measure and resend scheme. The fact that this calculation was performed using the coherent state ensemble merely reduces the argument back to the PEF and PEFF2. In a way, criterion c is not really a separate criterion at all. No one really cares if entanglement is used, only that the fidelity achieved with entanglement exceeds what could be achieved without it. Checking for entanglement is simply one way to establish whether the protocol is classical or not. This leads us to the following discussion:

### B. The privileged role of Victor

In a teleportation protocol Victor, the verifier, plays a special role. It is he who gets to decide if AB have successfully teleported the state he asks them to. Consider a situation where he as inserted AB’s teleporter into one arm of an interferometer and where he can vary the phase in that arm in a way unknown to Alice and Bob. If Victor uses a pure coherent state and the fidelity at the output is high, he will consider AB to have teleported his state correctly. He does not care at all whether AB really had entanglement, or about any of their problems with the PEF. If, on the other hand, Victor also has to use the same sort of mixed states as [8], he can still check the fidelity AB achieve in their teleportation apparatus, but he is left scratching his own head over the PEF. This leaves us with several questions:

1. If Victor has pure coherent states, but AB have to use states of the form [8], can they teleport Victor’s states with high fidelity?

Even if AB have a phase relative to Victor’s pure state that is unknown to them, the goal of teleportation is to faithfully transfer unknown states. Including an additional unknown phase shift is of no consequence.

A teleportation protocol for \( S \) for AB acts on an input pure state \( |\psi_\phi\rangle \) with phase \( \phi \) and leads to an output density matrix \( S(\psi_\phi) \) with the property that the fidelity is high for all input states:

\[
\wp_\phi(\psi_\phi S(|\psi_\phi\rangle |\psi_\phi\rangle) \geq f_{\min}
\]

Introducing an unknown phase \( \eta \) between Victor and AB means \( S \) acts on a phase-shifted state and leads to a density matrix for Victor

\[
\rho_V = \frac{1}{2\pi} \int d\eta \Phi_\eta S(|\psi_\phi\rangle |\psi_\phi\rangle \Phi_\eta^\dagger,
\]

where \( \Phi_\eta |\psi_{\phi+\eta}\rangle \equiv |\psi_\phi\rangle \). Then, using (??)

\[
\rho_V = \frac{1}{2\pi} \int d\eta \Phi_\eta f_{\min} \langle \psi_{\phi+\eta} | \langle \psi_{\phi+\eta} | + (1-f_{\min}) \rho_{\phi\eta} \Phi_\eta^\dagger
\]

\[
= \frac{1}{2\pi} \int d\eta [ f_{\min} \rho_{\phi\eta} \Phi_\eta^\dagger + (1-f_{\min}) \rho_{\phi\eta} \Phi_\eta^\dagger]
\]

where \( \rho_{\phi\eta} \) is a density matrix which in general might depend on \( \phi \) and \( \eta \). Finally, since the second term is positive, \( \rho_V \) has fidelity \( F(\rho_V, |\psi_\phi\rangle) \geq f_{\min} \).

2. If Victor also must use states of the form [8], can he really know if AB could have teleported a pure coherent state, and can we really say they have achieved teleportation? The answer is yes, by the now tiresome appeal to the PEFF2.

3. In the real experiments [8] Victor shares the reference beam. Would the experiment still work if Victor had his own independent laser, and can we really say we have tested teleportation? This question is the acid test for having achieved teleportation, and surprisingly it does not appear to be crisply answered in most of the literature [10]. Perhaps it is considered such an obvious consequence as to be not worth mentioning.
Answering question 3 is a combination of the first two answers. An additional laser with another unknown phase will not change AB’s ability to teleport a phase unknown to them. And Victor’s test of their ability is valid under the PEFF2. This view is apparently not accepted by all, some not mentioning the issue and vEF making the odd claim that Victor can use his own laser, but only if he phase locks it with AB’s reference laser. They tell us that “Alice’s claim is only that she can teleport a quantum state of a particular mode: Victor is free to choose the state to be teleported, but not the Hilbert Space.” To me this suggests that Victor must have a laser of the same frequency (the same mode) as AB’s, but the phase is exactly the variable being teleported, and therefore must be free.

Because of this disagreement and, as I have mentioned earlier, having Victor share the reference with AB goes against the spirit of teleportation, it would seem the experiment with Victor having his own laser ought to be performed.

C. van Enk and Fuchs

In this paper vEF show that the state of an ideal propagating laser field divided into packets of some duration $T$ can be thought of as the tensor product of coherent state packets of unknown phase, but all sharing the same unknown phase. They then use the quantum de Finetti Theorem to show that this is the only valid tensor product decomposition of the corresponding density matrix. This is a compelling suggestion that the decomposition plays a privileged role, at least when describing propagating lasers. It is also an extremely useful formulation, making quite clear, for example, how to calculate the fidelity of a CVQT experiment, and why there is distillable entanglement in AB’s (mixed) squeezed state.

Rudolph and Sanders do not find this convincing, however reiterating the point that $p$ remains a mixed state and no decomposition is truly privileged. They argue that while the vEF formulation makes some things easy to understand if one prefers the decomposition but that there is still no necessary reason to prefer it. I again suggest the entire debate is ill posed.

D. Wiseman

In, Wiseman argues extensively against the existence of an absolute phase and for the idea that a laser is as good a clock to use for an agreed time or phase standard as any other. His dismiss both the claim by RS that continuous-variable quantum teleportation has not been achieved (one way to state their claim is that a laser rather than an absolute clock was used to synchronize Alice and Bob) and the refutation of RS by vEF (who still implicitly appeal to the idea of absolute phase).

Wiseman says a laser is the best thing one can use as a time standard; the PEFF2 says that it doesn’t matter if a there is or is not a better time standard. These points of view are, in a sense, two sides of the same coin and if Wiseman were to argue that they are indeed the same, I would not strongly disagree—the discussion will have long since moved closer to philosophy than physics.

I doubt than anyone would seriously disagree with Wiseman’s contention that there is no better clock at optical frequencies than a laser, and that the best we can do is measure phase relative to a laser as standard. What is easy to object to is the idea of going around willy-nilly defining laser phases as 0 relative to themselves. When Alice and Bob have one laser, and Victor has another, they cannot both be defined as 0 phase (though this is OK provided they never do anything to compare phase, nor does anyone perform a calculation depending on the phase difference).

Wiseman states “[his own] arguments lead inevitably to the conclusion that in quantum optical experiments there is no necessity to consider, even hypothetically, any time-keeper beyond the laser which serves as a phase reference. No other clock is superior in any fundamental sense.” and that “it is precisely because no experiment is affected by the supposed randomness in the phase of the laser (if it is being used as a time reference) that makes it possible to describe the laser by a single state ...”

The trouble with this, of course, is in deciding who is right when there is more than one phase reference laser. When AB purport to teleport Victor’s state, Victor is well justified in saying his clock is superior to theirs in the fundamental sense of “the customer is always right.” From his point of view (relative to his clock, AB’s reference laser is in a mixed state, and they cannot claim otherwise. As I have argued, teleportation works regardless, but there certainly are experiments where AB’s randomness of phase affects the outcome. Victor’s own randomness of phase relative to the implicit but arguably unmeasurable absolute phase is unimportant for the reasons given by Wiseman, as well as for those given here. The advantage in the PEFF2 formulation over Wiseman’s is that it still makes sense whether or not a reference laser is shared by all.

A secondary point concerns Wiseman’s discussion of criterion b. He agrees that the experiment could be performed with Alice’s and Bob’s clocks synchronized before the teleportation commences, avoiding all chance of quantum information sneaking through the clock synchronization channel. He goes on to argue that a classical channel could be used to synchronize clocks, so that even if it were being used during the teleportation, no quantum information could sneak through. But then he makes a logical mistake.

Wiseman claims that since time synchronization channel could be dephased and the experiment would still work, that this is just as good as performing the experiment with a dephased channel and that the onus is on the
debunker to dephase the channel if he wants to disprove the experiment. But this is wrong. The onus is always on the one who claims to have performed and experiment to convince the skeptics not the other way around. It is not enough to claim that the experiment would have worked, if only we hadn’t cheated. This does sound dangerously like commission of the PEFF2, which allows us to make claims about “what would have happened.” The difference is that in one case we are arguing whether an apparatus has been shown to teleport; in the other the argument is over whether the apparatus should be called a teleporter at all.

Wiseman also wants to claim “by fiat” that the synchronization laser can be considered classical because other classical systems (marbles are his example) are really quantum mechanical and only called classical by fiat in some approximation. But marbles typically have wavefunctions like $\psi = \frac{1}{\sqrt{2\sigma^2}} e^{-(x-x_0)^2/(2\sigma^2)}$. If one were to create a superposition of these states $\frac{1}{\sqrt{2}}(|\psi_0\rangle + |\psi_1\rangle)$, it would almost immediately collapse to $|\psi_{0,1}\rangle$. This natural decoherence due to the environment is why marbles are classical, it is not merely an arbitrary choice. Coherent states of lasers do not suffer the same decoherence, or at least do so only at a vastly different timescale. Indeed if they did, we wouldn’t be able to even consider using them for CVQT.

E. The Partition Ensemble Fallacy Fallacy Fallacy

Nemoto and Braunstein state (NB) that the choice of a flat distribution over phases in $\frac{1}{\sqrt{2}}(|\psi_0\rangle + |\psi_1\rangle)$ is unfounded, and that any choice of distribution is in fact unfalsifiable. Since choosing a single phase is also unfalsifiable, we (they argue) may do so thus invalidating the PEF and RS’s argument as applied to CVQT. This is a quite similar point of view to that of Wiseman, and has the same flaw. NB call this the “partition ensemble fallacy” (PEFF). The PEFF is itself fallacious, or at least NB’s application of it is.

Their mistake is in believing that just because absolute phase is unobservable (itself contentious) that the distribution over phases of a laser state is also unobservable. If one were to measure the phases of many different lasers relative to a single reference laser, one could experimentally determine the distribution of their phases, even without knowing the absolute phase of the reference. In other words, imagine Victor measuring $AB$’s phase relative to his own—we expect it will be random with the distribution.

Because this distribution over phases is measurable, there is no “freedom of religion” over choice of distribution as there is over choice of ensembles that lead to the same density matrix. The PEFF2 does not have this same weakness—it explains why laser CVQT experiments are acceptable independently of whether either absolute phase or the distribution over phases are falsifiable.

F. Conclusions

It is useful to consider whether regular qubit based teleportation protocols share the same conceptual difficulties as CVQT. The analogous situation is on in which Alice, Bob, and Victor each have a qubit bases modified by a random unitary operators each time they perform the experiment. The reason their bases must be randomized each time is to ensure they employ states of the form $\rho^\otimes n$ rather than $\int d\psi (|\psi\rangle \langle \psi|)^\otimes n$. This is in analogy to a laser having a new random phase each time it is turned on (and for real lasers, any time past their coherence time).

For their teleportation to succeed, Alice and Bob will have to coordinate bases before attempting to send Victor’s state. Victor, on the other hand, need not contrive to align his basis with some outside observer’s basis (be it some fourth party Carmela, or a preferred basis of the universe). I don’t think anyone would question that Victor is testing $AB$’s teleportation experiment despite the fact that when he creates a pure state to give to Alice, Carmela considers it a completely mixed state. If Victor chooses to think of his state as a pure state, that convenient fiction is well justified by the PEFF2.

To sum up: I have argued that CVQT is possible with traditional laser sources, despite their lack of absolute phase. The actual experiments are slightly unsatisfactory in that Alice and Bob share the reference laser during the teleportation, and Victor shares the reference rather than having his own laser beam, but neither of these weaknesses are fundamental.

The objections to these experiments appealing to the PEF are based on a misunderstanding of the PEF that we are retrodicting “what really happened” rather than predicting what would given a particular state to teleport.

Some of the objections to RS’s objections are based on a (appropriate) denial of the reality of absolute phase, but become confused (or at least confusing) by presence of more than one laser in the world, not all of which can simultaneously be defined as 0 phase. It is also confusing that teleportation is a protocol that always works, and indeed is designed to always work, on inputs of any relative phase to Alice and Bob. Because of this their absolute phase and phase relative to anyone aside from each other is unimportant. This fact tends to obscure the already subtle situation of phase in CVQT experiments.

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[14] By “classical channel” Wiseman means a completely dephasing channel in some basis. This is called in some contexts a quantum-classical-quantum channel.