Wilson Renormalization Group Equations for the Critical Dynamics of Chiral Symmetry

Ken-Ichi Aoki, Keiichi Morikawa, Jun-Ichi Sumi, Haruhiko Terao, and Masashi Tomoyose

Institute for Theoretical Physics, Kanazawa University
Kanazawa 920–1192, Japan

†Department of Fundamental Sciences,
Faculty of Integrated Human Studies, Kyoto University,
Kyoto 606-8501, Japan

Abstract

The critical dynamics of the chiral symmetry breaking induced by gauge interaction is examined in the Wilson renormalization group framework in comparison with the Schwinger-Dyson approach. We derive the beta functions for the four-fermi couplings in the sharp cutoff renormalization group scheme, from which the critical couplings and the anomalous dimensions of the fermion composite operators near criticality are immediately obtained. It is also shown that the beta functions lead to the same critical behavior found by solving the so-called ladder Schwinger-Dyson equation, if we restrict the radiative corrections to a certain limited type.
1. Introduction

The chiral symmetry breaking phenomena has been one of the key issues to be understood in the non-perturbative dynamics of gauge theories. The analytical study of this problem has been initiated by the Nambu-Jona-Lasinio (NJL) model [1], which was introduced as the effective theory with four-fermi interactions. For gauge theories particularly the Schwinger-Dyson (SD) equations in the ladder approximation with Landau gauge [2, 3] have been intensively studied and applied not only to QCD but also to the various models of dynamical electroweak symmetry breaking [4, 5]. In QCD, the ladder SD equation with the running gauge coupling constant, the improved ladder [6], was found to give good results even quantitatively [7]. However the ladder SD equations are known to suffer from some serious problems, specially the strong gauge dependence [8] and the difficulty to proceed beyond the ladder approximation [9].

On the other hand the Wilson renormalization group (RG) [10] has been known to offer the powerful method to analyze critical phenomena and has been applied to the various dynamical problems mainly in the statistical mechanics. The so-called exact RG equations [10, 11, 12], which are the concrete formulation of the Wilson RG in the momentum space, has been recently applied to numerical study of non-perturbative dynamics in field theories. The application to the QCD dynamics has been also considered in this framework [13]. The advantageous features of this method, compared with the SD approach, are that the critical behavior is analyzed directly form the RG equations, and that it admits the systematic improvement of approximation by the derivative expansion and truncation of the Wilsonian effective action [14, 15]. Interestingly it is rather recent that the RG method has been applied to the fermi liquid theory of superconductivity [16], which the NJL model was considered in analogy with. Also it should be noted that the fermi liquid theory of high density QCD was studied by the RG analyses [17].

In this paper we examine the chiral critical dynamics in gauge theories by using the exact RG equations, specially putting our attention to the comparison with the SD approach. There have been known several formulations of the exact RG. Here we simply employ the Wegner-Houghton RG equations [11], which are derived with sharp momentum cutoff, in the so-called local potential approximation for our present purpose. The analyses with the exact RG equations with smooth cutoff may be performed as well [18]. It will be found that the critical behavior is determined from the beta-functions of the effective four-fermi couplings induced by gauge interaction with remarkably simple calculation. The phase boundary and also the anomalous dimensions of the composite operators of fermions near the criticality will be evaluated. Our approximation scheme adopted here is even better than the ladder approximation performed in the SD equations on the critical behavior. Actually, as is seen later, if we make further approximation so as to pick up only a few types of the radiative corrections, then the critical behavior is reduced to be identical to that obtained by solving the ladder SD equation.

2. Scheme of the RG equations

The ladder SD equation for the fermion mass function is given in the form of an integral equation, where the momentum integration is carried out with sharp cutoff. In order to see the direct relation between the critical dynamics obtained by the two methods; the
SD equation and the RG equation, we consider the exact RG equation defined with sharp cutoff in this paper. There have been known the several formalisms for the exact RG [10, 12]. Here we shall adopt the so-called Wegner-Houghton RGE [11] derived as follows.

If we devide the freedom of the quantum field $\phi(p)$ into the higher frequency modes with $|p| > \Lambda$ and the lower frequency modes with $|p| < \Lambda$ by introducing the cutoff scale $\Lambda$ in the Euclidean formalism, then the Wilsonian effective action at this scale, $S_{\text{eff}}[\phi; \Lambda]$, may be defined by integrating out the higher frequency modes in the partition function. Namely

$$Z = \int \prod_{|p| < \Lambda_0} d\phi(p) \ e^{-S_0[\phi; \Lambda_0]} = \int \prod_{|p| < \Lambda} d\phi(p) \ e^{-S_{\text{eff}}[\phi; \Lambda]},$$

where $S_0$ denotes the bare action with the bare cutoff $\Lambda_0$. This effective action contains the general operators invariant under the original symmetries in the bare action, for example the chiral symmetry of our present concern.

The Wegner-Houghton RGE determines the variation of the Wilsonian effective action under the infinitesimal change of the cutoff $\Lambda$. For example, the RGE for the $D$-dimensional scalar field theory is found to be given exactly as

$$\frac{\partial S_{\text{eff}}}{\partial t} = DS_{\text{eff}} - \int \frac{d^D p}{(2\pi)^D} \phi_p \left( \frac{2 - D - \eta}{2} - p^\mu \frac{\partial'}{\partial p^\mu} \right) \frac{\delta S_{\text{eff}}}{\delta \phi_p}$$

$$- \frac{1}{2} \int \frac{d^D p}{(2\pi)^D} \delta(|p| - 1) \left\{ \frac{\delta S_{\text{eff}}}{\delta \phi_p} \left( \frac{\delta^2 S_{\text{eff}}}{\delta \phi_p \delta \phi_{-p}} \right)^{-1} \frac{\delta S_{\text{eff}}}{\delta \phi_{-p}} - \text{tr} \ln \left( \frac{\delta^2 S_{\text{eff}}}{\delta \phi_p \delta \phi_{-p}} \right) \right\},$$

where $t = \ln(\Lambda_0/\Lambda)$ is introduced as the scale parameter. The 1st line of the RGE represents nothing but the canonical scaling of the effective action. While the 2nd line comes from the radiative corrections which correspond to the tree and the 1-loop Feynman diagrams including only the propagators with the momentum of the scale $\Lambda$.

In the practical analysis it is inevitable to simplify this RGE by some approximation. Here we shall examine the RGE in the so-called local potential approximation [11, 19]. In this approximation the radiative corrections to any operators containing derivatives are ignored in the RGE (2). Therefore solely the potential part of $S_{\text{eff}}$, $V_{\text{eff}}$, may be evolved with the shift of $\Lambda$. It should be noted that the wave function renormalization is ignored in this scheme. The RGE for the scalar theory is given explicitly by

$$\frac{\partial V_{\text{eff}}}{\partial t} = DV_{\text{eff}} - \frac{D - 2}{2} \phi \frac{\partial V_{\text{eff}}}{\partial \phi} + \frac{A_D}{2} \ln \left( 1 + \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2} \right),$$

where $A_D = 2/(4\pi)^{D/2} \Gamma(D/2)$ is the factor from the momentum integration. This equation offers us a set of the infinitely many beta functions for the general couplings appearing in $V_{\text{eff}}$. However it should be noted here that each beta function may be evaluated through just one loop corrections with the general effective interactions. The non-perturbative nature of the RGE is supposed to be maintained by solving the infinitely many coupled renormalization equations. Actually it has been known that this approximated RGE is quite effective in the case of the scalar theories [14, 15]. The generalization of this RGE to include fermions has been also studied in the relation with the triviality-stability bound for the Higgs boson mass in the standard model [20].
Now let us consider to apply this formulation to the massless fermions coupled by gauge interaction. For example we may take the action of the massless QED as the bare action

\[ S_0 = \int d^4x \left\{ \bar{\psi} \partial \psi + e \bar{\psi} A \psi + \frac{1}{4} F_{\mu \nu}^2 + \frac{1}{2\alpha} (\partial_\mu A_\mu)^2 \right\}, \]  

(4)

where \( \alpha \) is the gauge parameter. As is well known, the gauge invariance is not maintained anymore, once the momentum cutoff is performed. Therefore the generic gauge non-invariant operators are generated in the Wilsonian effective action. Then we must encounter the rather complicated problem to pick up the special RG flows corresponding to the gauge invariant theories in the infinite dimensional coupling space. Recently it has been discussed how to deal with the gauge theories in the framework of the Wilson RG by using the modified Slavnov-Taylor identities [21].

Here, however, we shall simply ignore the corrections to the operators including the gauge fields as well as imposing the local potential approximation as the first step towards the analysis of the chiral symmetry breaking phenomena. Then we may avoid the intriguing problem of the gauge invariance, since any gauge non-invariant operators do not appear in the effective action. This approximation is indeed so rough as to make the beta function of the gauge coupling vanish identically, therefore it cannot be supposed to give any picture of the real dynamics of the gauge theories. However, on the other hand, the so-called ladder approximation used in the SD approach also totally ignores the vertex corrections as well as the corrections to the gauge kinetic functions. Therefore it would be meaningful to examine the RGE in this scheme in comparison with the SDE’s in the ladder approximation. The effect of the running gauge coupling will be discussed in section 4. Nevertheless this approximation scheme is thus rather crude, it will be seen that the chiral critical behavior may well be described. Actually it will be found that this approximation is even better than the ladder approximation applied for the SD equations.

The Wilsonian effective action to be solved by the RGE in this scheme is now reduced to the form of

\[ S_{\text{eff}}[\psi, \bar{\psi}; \Lambda] = \int d^4x \left\{ \bar{\psi} \partial \psi + V_{\text{eff}}(\psi, \bar{\psi}; \Lambda) + e \bar{\psi} A \psi + \frac{1}{4} F_{\mu \nu}^2 + \frac{1}{2\alpha} (\partial_\mu A_\mu)^2 \right\}, \]  

(5)

where \( V_{\text{eff}}(\psi, \bar{\psi}) \) denotes the general potential composed of the chiral symmetric multi-fermion operators. These multi-fermion operators, which are induced by exchange of the “photon” with higher momentum. The so-called gauged NJL model is often examined in the SD approach and the phase diagram in the two parameter space of the gauge coupling and the four-fermi coupling has been examined [3]. However, in the RG point of view, this coupling space should be regarded as a subspace of the infinite dimensional coupling space of the Wilsonian effective action. It should be noted also that these multi-fermi operators are irrelevant or non-renormalizable, and therefore, are not considered in the perturbative QED. However they cannot be simply discarded in the strong coupling region. It will be seen in the next section that the four-fermi coupling turns out to be relevant near the criticality and plays a crucial role for the critical dynamics of the chiral symmetry breaking.

3. Critical dynamics of the chiral symmetry

In this section we examine explicitly the Wegner-Houghton RGE in the approximation
discussed in the previous section. The form of the effective potential \( V_{\text{eff}} \) written in terms of the fermions is found to be restricted into a polynomial composed of the following parity and chiral invariant operators, which are mutually independent;

\[
\begin{align*}
O_1 &= (\bar{\psi}\gamma_5\psi)^2 + (\bar{\psi}i\gamma_5\gamma_\mu\psi)^2 = -\frac{1}{2} \left\{ (\bar{\psi}\gamma_\mu\psi)^2 - (\bar{\psi}\gamma_5\gamma_\mu\psi)^2 \right\}, \\
O_2 &= (\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_5\gamma_\mu\psi)^2, \\
O_3 &= \left\{ (\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma_5\gamma_\mu\psi) \right\}^2.
\end{align*}
\]

Therefore the 4-fermi part of the effective potential may be written down as

\[
V_{\text{eff}}(\psi, \bar{\psi}; t) = \frac{G_S(t)}{2\Lambda^2} \left\{ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 \right\} + \frac{G_V(t)}{2\Lambda^2} \left\{ (\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_5\gamma_\mu\psi)^2 \right\}.
\]

Hereafter let us call \( G_S \) the scalar four-fermi coupling and also \( G_V \) the vector four-fermi coupling.\(^1\)

Now let us evaluate the radiative corrections to these four-fermi operators, since it will be found enough to see only these couplings for the purpose of understanding the critical dynamics. By using the propagator of the gauge field in the Landau gauge \((\alpha = 0)\), the RG equations for the four-fermi couplings are found to be

\[
\begin{align*}
\frac{d}{dt}g_S &= -2g_S + \frac{3}{2}g_S^2 + 4g_Sg_V + g_S\lambda - \frac{1}{6}\lambda^2, \\
\frac{d}{dt}g_V &= -2g_V + \frac{1}{4}g_S^2 - g_V\lambda - \frac{1}{12}\lambda^2,
\end{align*}
\]

where we introduced the rescaled couplings, \( g_S = G_S/(4\pi^2) \), \( g_V = G_V/(4\pi^2) \), \( \lambda = 3e^2/(4\pi^2) \). Here we should note that any multi-fermi couplings more than four do not take part in the radiative corrections for the four-fermi couplings owing to the 1-loop nature of the RGE. Therefore we may obtain the RG flows within the 2 dimensional coupling space (or 3 dimensional, if the gauge coupling is also taken into account) irrespectively to other couplings.

In Fig.1 the Feynman diagrams representing the one-loop corrections to the four fermi couplings are shown. Let us call the corrections given by the diagrams in the dashed-line box in Fig.1 the “ladder” type, and the others the “non-ladder” type hereafter. If we approximate the RGE by restricting to the “ladder type” correction, then the beta function for the scalar four-fermi coupling is found to be given by

\[
\frac{d}{dt}g_S = -2g_S + 2(g_S + \lambda/4)^2,
\]

where it is noted that the RGE for the scalar four-fermi decouples from that for the vector four-fermi coupling.

Before examining the full RGE’s (8), let us consider the RG flows in the subspace of \( \lambda = 0 \). The beta functions tell the “fixed points” at \((g_S^*, g_V^*) = (0, 0), (1, 1/8), (-4, 2)\).

\(^1\)The sign of the scalar four-fermi coupling introduced here follows the conventional one in the literatures.
Fig. 1: Feynman diagrams of the radiative corrections to the four-fermi couplings $g_S$ and $g_V$ considered in the RGE (8). The diagrams surrounded by the dashed line show the “ladder” type corrections.

$(0, 0)$ is the IR trivial fixed point, and $(1, 1/8)$ is the UV fixed point on the critical surface. The RG flows are found as is shown in Fig. 2. It is seen that there are two phases divided by a critical surface. The chiral symmetry is supposed to be spontaneously broken in the upper region in Fig. 2. Then we may realize that the chiral symmetry breaking is caused essentially by the strong scalar four-fermi interaction, not by the vector four-fermi interaction.

It is also easy to evaluate the exponents which are important physical quantities in the critical dynamics. By linearizing the RG equations around the UV “fixed point”, the dimensions of the relevant four-fermi coupling and the irrelevant four-fermi coupling are found to be 2 and $-5/2$ respectively. The relevant four-fermi operator is given by the combination of $O_{\text{rel}} = O_1 - (1/8)O_2$. The renormalized trajectory in Fig. 2 is given by the straight line passing through the non-trivial fixed point. Indeed we may deduce from the RGE’s (8)

$$\frac{d}{dt}(g_S - 8g_V) = -(2 + \frac{1}{2}g_S)(g_S - 8g_V),$$

which means that once the combination of $(g_S - 8g_V)$ is vanishing at a point, then it keeps null along the renormalization flow. Therefore the renormalized trajectory is precisely given by the line of $g_S = 8g_V$. Namely the effective four-fermi operator in the low energy is just $O_{\text{rel}}$ irrespectively to the phases.

Next let us examine the RG equations (9) given by the “ladder type” corrections. For each gauge coupling $\lambda$ there are the UV fixed point and the IR fixed point, which are found to be

$$g^*_S(\lambda) = \left(1 \pm \sqrt{1 - \lambda}\right)^2 / 4,$$

2 Strictly speaking $(g^*_S, g^*_V) = (1, 1/8)$ is not a fixed point, since the beta function for the eight-fermi coupling does not vanish there. However it turns out to be the non-trivial fixed point for the space-time dimension of $2 < d < 4$. We do not consider the point $(-4, 2)$, since it seems to be fake due to this approximation.
where + is for the UV fixed point and − is for the IR fixed point. Namely they form a fixed line in the (λ, gs) space as shown in Fig.3. The phase boundary is also shown in Fig.3. The upper region is supposed to be the chiral symmetry broken phase. The critical gauge coupling is given by

\[ \lambda_{cr} = 1. \]  

(12)

Indeed this critical surface just coincides with that obtained by solving the SDE in the ladder approximation [2, 3].

The anomalous dimension of the four-fermi coupling \( g_S \), \( \gamma_G = 2 + \text{dim}[g_S] \), near criticality is immediately deduced from the RGE (9) as

\[ \gamma_G = 4g_S^*(\lambda) + \lambda = 2 \left( 1 + \sqrt{1 - \lambda} \right), \]  

(13)

which is also found to coincide with the result by the ladder SDE [3]. Therefore it is seen that our approximation used to derive the RGE’s (8) is certainly better than the ladder approximation. Moreover it is easy to take the all corrections shown by Fig.1 including the “ladder type” ones in our framework. It should be noted that the sum of corrections of the “ladder” diagram and the “crossed ladder” given in the last line of Fig.1 is found to be free from gauge parameter dependence. Thus this extension of approximation beyond “ladder” is significant to obtain the gauge independent results [18]. We would like to stress here that the exact RG equations allow us to examine the critical dynamics by remarkably simple calculation, which is a clear contrast with the SD approach.

Now we are in a position to go beyond “ladder” by examining the full RG equations (8). The fixed line \((g_S^*(\lambda), g_V^*(\lambda))\), which is given by the solution of the 3rd order equation in turn, is shown in Fig.3 and also in Fig.4 by projection to the \((\lambda, gs)\)-plane and \((g_V, gs)\)-plane respectively. It is seen that the critical gauge coupling is now found to be slightly bigger than the value in the “ladder” approximation, \((\lambda_{cr} = 1.0409)\). In Fig.4 the critical surface separating the two phases is also shown by the cross sections at various gauge coupling up to the critical one. The critical surface given in the case of “ladder” type should be compared with the cross section between the critical surface and the \(g_V = 0\) plane, which is found as shown in Fig.3. It is seen that the phase boundary obtained by our scheme is shifted towards outside compared with the phase boundary, which has been known so far in the ladder SD approach.

The exponents at the fixed line also are similarly obtained. The exponent or the dimension of the relevant operator, which was found to be 2 in the previous analysis for \( \lambda = 0 \), reduces as the gauge coupling becomes larger. Then it eventually vanishes at \( \lambda = \lambda_{cr} \), which is also seen directly from the eq.(8). In Fig.5 the anomalous dimension of the relevant four-fermi coupling \( \gamma_G \) is presented in comparison with the “ladder” value given by eq.(14).

Before ending this section let us mention the anomalous dimension of the fermion mass operator, which we denote \( \gamma_m \). In order to evaluate it we may incorporate the mass term in the effective action. Then the beta function for the mass \( m \) may be derived by one-loop diagrams and is found to be

\[ \frac{d}{dt} m = m - \frac{2m}{1 + m^2}(g_S + \lambda/4). \]  

(14)
Here it should be noted that the contribution from the “non-ladder” type corrections vanishes, therefore the vector four fermi coupling does not appear in this beta function. The anomalous dimension on the fixed line is simply given by

\[ \gamma_m = 2g_S^*(\lambda) + \frac{\lambda}{2}, \]

which is shown in Fig.5 as well. In the “ladder” case it is seen that the anomalous dimensions satisfy the relation, \( \gamma_m = \gamma_G/2 \), which has been known also in the analysis of the ladder SDE [3]. In our analysis, however, \( \gamma_m \) turns out to be fairly larger than \( \gamma_G/2 \) and also than \( \gamma_m \) obtained so far in the ladder SD approach.

4. RG flow with the running gauge coupling

So far the effect of the renormalization of the gauge coupling has been totally ignored, therefore the obtained phase diagrams do not reflect the realistic ones for gauge theories. In the SD approach, the so-called improved ladder approximation [6], in which the gauge coupling is simply replaced by the running coupling subject to the perturbative RGE apart from the SD framework, has been often used. However this prescription cannot be regarded as systematic improvement of the approximation. On the other hand, in the Wilson RG framework it is possible to include the correction to the gauge coupling naturally by improvement of the previous approximation. This makes a clear contrast to...
If we try to treat the non-perturbative dynamics by strong gauge interactions faithfully in the Wilson RG framework, we must encounter the hard problems such as extraction of the gauge invariant theories, development of simple approximation scheme, incorporation of the topological excitations and so on. However, as a primitive approximation, we may evaluate the Wilson beta function of the gauge coupling by the 1st order correction, namely the perturbative one. Then it is enough to solve the RG equations given by (8) in turn coupled with the perturbative RG equation for the gauge coupling $\lambda$. In Fig.6 the RG flows for QED obtained in this manner are shown in the $\left(\lambda, g^s\right)$-plane. The critical surface separating the spontaneously broken and the unbroken phases is maintained, while the non-trivial fixed points turn out disappear. Note that the point $\left(\lambda, g^s, g^V\right) = (0, 1, 1/8)$ is not a fixed point in 4 dimensions.

The RG flows for the QCD like asymptotically free gauge theory is shown in Fig.7. These results should be compared with those obtained by solving the SD equations [22]. It is seen that the phase structure is completely swept off. The entire region is supposed to be in the broken phase of the chiral symmetry, since the 4-fermi coupling keeps growing in the infrared. The effective theories on the attracting line coming out from the trivial fixed point correspond to the continuum limit of QCD. Namely this line gives the so-called renormalized trajectory of QCD. However other flows of the gauged NJL models, specially starting at the critical point of the NJL model in the UV limit, do not converge on the renormalized trajectory. Therefore it may be supposed that these flows show other renormalized trajectories. If this is the case, the gauged NJL model offers non-perturbatively

**Fig.4:** Cross sections of the critical surface at $\lambda = 0, 0.1, 0.2, \cdots, 1.0, \lambda_{cr}$ and the fixed line projected on the plane.

**Fig.5:** Anomalous dimensions $\gamma_m$ and $\gamma_G$ obtained in our LPA scheme and in the ladder approximation.
renormalizable theories different from QCD. Indeed, existence of the non-trivial continuum limit other than QCD, or renormalizability of some kinds of the gauged NJL models has been claimed so far [22, 23]. In our framework of the non-perturbative RG, renormalizability of the gauged NJL model may be shown by examining whether these flows are really the renormalized trajectories or not. Such studies will be reported separately [24].

Fig. 6: RG flow diagram for QED. There appears the two phase structure. The upper (lower) region is supposed to be (un)broken phase.

Fig. 7. RG flow diagram for the QCD like gauge theory. There appears no phase boundary.

5. Discussions

In this paper we examined the chiral critical behavior of the gauge theories in the Wilson RG framework. We considered evolution of the effective potential composed of the chirally invariant multi-fermi operators by the exact RG equation with sharp cutoff in the local potential approximation. The RG flow of the four-fermi couplings were found to determine the phase structure. It is straightforward and remarkably easy to find the critical surface and the anomalous dimensions of the composite operators of fermions in this framework. Moreover the critical dynamics obtained by solving the ladder SD equations are exactly reproduced by restricting the radiative corrections taken in the beta functions to the “ladder” type. While our RG equations contain also the “non-ladder” type corrections, which are necessary to obtain gauge independent physical results [18], though we have considered only the case of the Landau gauge in this paper.

However we cannot assert with this analysis that the chiral symmetry is indeed spontaneously broken in the region supposed to be the broken phase. In order to clarify it we need to evaluate the order parameters such as the dynamical mass of the fermion, the
condensation of the composite operator of fermion, and so on. Evaluation of these order parameters is important especially in QCD, because they are the physical quantities to show the dynamical chiral symmetry breaking. While, for example, the spontaneous generation of the fermion mass itself seems to be even non-trivial in the Wilson RG picture, since the Wilsonian effective action remains chiral symmetric in evolution, that is, there is no room for the mass term to show up. These issues as well as the method to evaluate the order parameters in the Wilson RG framework will be also reported in a separate publication [25].

In our analyses the RG equation for the gauge coupling was approximated by the perturbative one. Of course the fully non-perturbative treatment for the RGE is required to see the dynamics of strong gauge interaction in infrared. It would be still an open question whether the Wilson RG approach gives a useful framework in this non-perturbative region. However it may be said that the non-perturbative RG has a good chance to seek for the dynamical chiral symmetry breaking phenomena in gauge theories further by going beyond the level examined so far in the SD approach.

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