Critical state of phantom universe *

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Abstract

The late-time evolution behavior of the autonomous system in the SO(1, 1) dark energy model with power-law potential is studied. Big Rip may be a critical point of the autonomous system. This means that such a Big Rip may be considered as the middle state between the expanding and contracting phases of phantom universe. This result is also valid for some special interactions between matter and dark energy.

I Introduction

The observations [1, 2, 3] indicates that the universe is expanding acceleratedly and spatially flat, this requires the existence of the dark energy [4, 5, 6, 7, 8, 9]. Some dark energy models may be related to the quintessence [10, 11, 12]. The scalar phantom model [13] may be obtained from the quintessence model by adopting a pure imaginary scale field. For the expanding universe dominated by phantom (phantom universe), the energy density increases with time. Phantom universe possesses some appealing properties, which has greatly been studied [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. Some authors are interested in the dark energy models of two or more scalar fields [26, 27, 28, 29, 30, 31, 32], for which the state of equation $w$ may change from $w > -1$ to $w < -1$. We supposed the $SO(1, 1)$ dark energy model [26, 31], which is relevant to the spintessence model having the $U(1)$ symmetry [10]. Nevertheless, they are quite different in the description for dark part in universe (dark energy or dark matter). The former model may be considered as either a quintessence-like or phantom model, while in the latter model the state of

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equation is confined to be bigger than $-1$. We have discussed the cases of the exponential and inverse power law potentials, and find for both potentials there exist the late-time phantom phases \[31\,32\]. For a phantom universe, there can be the three different evolution consequences. First, if the phantom equation of state tends rapidly to $-1$, the phantom universe will approach the de Sitter universe. Secondly, it encounters the future Big Rip. Thirdly, it has the future singularity occurring at a finite scale factor studied in the generalized phantom Chaplygin gas \[33\], in which the energy density is given by $\rho = [A + Ba^{3(1+\alpha)}]^\frac{1}{1+\alpha}$ with $B < 0$.

We will focus on the phantom universe having a Big Rip. For it, one can ask the following question. Is Big Rip the end of phantom universe? Big Rip is a consequence of expanding phantom universe. Generally, it is supposed to be a doomsday when all things will be destroyed because the scale factor of universe becomes infinite \[18\]. In this paper, by analyzing the behavior of phantom universe in the $SO(1,1)$ model we find that Big Rip may correspond to the critical point of the autonomous system. Considering that such a critical point is unstable, we suggest that the Big Rip be the final state of an expanding phantom universe and at the same time the initial state of the corresponding contracting phantom universe.

II Critical point corresponding to Big Rip

It was shown that in the $SO(1,1)$ model the phantom universe for the inverse power law potential has a future Big Rip \[32\]. Here, we will further analyze the behavior of the phantom universe near the Big Rip and infer that the Big Rip is a critical point of the autonomous system.

Let us consider a spatially flat, isotropic and homogeneous universe consisting of matter and dark energy. For this background, the field equations read

\[
H^2 = (\frac{\dot{a}}{a})^2 = \frac{\kappa^2}{3}(\rho_{DE} + \rho_m),
\]

\[
\dot{H} = -\frac{\kappa^2}{2}(\rho_{DE} + \rho_m + p_{DE} + p_m),
\]

where $\rho_{DE}$ and $p_{DE}$ are energy and pressure density of dark energy, $\rho_m$ and $p_m$ are those of matter, $H = \frac{\dot{a}}{a}$ is the Hubble parameter, and a dot denotes the derivative with respect to time. In the $SO(1,1)$ model \[26\], the energy and pressure density of dark energy are marked as $\rho_\Phi$ and $p_\Phi$, which are given by

\[
p_\Phi = \rho_k + \rho_c - V, \quad \rho_\Phi = \rho_k + \rho_c + V,
\]

where

\[
\rho_k = \frac{1}{2} \dot{\Phi}^2, \quad \rho_c = -\frac{1}{2} \frac{\dot{Q}^2}{\dot{\Phi}^2 a^6},
\]
with $\bar{Q}$ being the $SO(1,1)$ charge, and $V$ is a potential. From Eq. (3), one can see that for $\rho_k < |\rho_c|$ dark energy behaves as a phantom. Generically, for the matter part one can consider the equation of state $p_m = (\gamma_m - 1)\rho_m$ with $0 < \gamma_m \leq 2$ the barotropic index. For the universe composing of dark energy and matter, the energy-momentum conserved equation may be split into

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -C,$$  \hfill (4)

$$\dot{\rho}_m + 3H(\rho_m + p_m) = C,$$  \hfill (5)

where $C$ denotes the interaction between matter and dark energy.

Introducing the dimensionless variables

$$x \equiv \pm \kappa \sqrt{\rho_k} \sqrt{3H},$$

$$y \equiv \kappa \sqrt{V} \sqrt{3H},$$

$$z \equiv \kappa \sqrt{\rho_m} \sqrt{3H},$$

$$w \equiv \kappa \Phi \sqrt{6},$$

$$v \equiv \kappa \sqrt{-\rho_c} \sqrt{3H},$$

then Eqs. (1)-(5) can be rewritten as an autonomous system

$$x' = 3x(x^2 - v^2 + \frac{\gamma_m}{2} z^2 - 1) - w^{-1}v^2 - \frac{\kappa V_\phi \dot{\phi}}{\sqrt{6}H^2} - C_1,$$

$$y' = 3y(x^2 - v^2 + \frac{\gamma_m}{2} z^2) + \frac{\kappa}{2\sqrt{3H}} \sqrt{\dot{\phi}} \frac{\dot{V}_\phi}{\sqrt{V}} \sqrt{H},$$

$$z' = 3z(x^2 - v^2 + \frac{\gamma_m}{2} z^2 - \frac{\gamma_m}{2}) + C_2,$$  \hfill (7)

$$w' = x,$$

$$v' = 3v(x^2 - v^2 + \frac{\gamma_m}{2} z^2 - 1) - xw^{-1}v,$$

where $C_1 = \frac{\kappa C_1}{\sqrt{6}H^2 \Phi}$ and $C_2 = \frac{\kappa C_2}{2\sqrt{3\rho_m} H^2}$, a prime denotes the derivative with respect to $N = \ln a$ and a dot the derivative with respect to $\phi$.

On the basis of work in Ref. [32], we will proceed to discuss the $SO(1,1)$ model with the inverse power law potential

$$V = V_0 \Phi^{-n},$$  \hfill (8)
where $0 < n < 2$ and $V_0$ are two constants. For this potential, the autonomous system (7) reduces to

\begin{align*}
x' &= 3x(x^2 - v^2 + \frac{\gamma_m}{2} z^2 - 1) + w^{-1}(\frac{n}{2} y^2 - v^2) - C_1,
\end{align*}

\begin{align*}
y' &= 3y(x^2 - v^2 + \frac{\gamma_m}{2} z^2) - \frac{n}{2} w^{-1} xy,
\end{align*}

\begin{align*}
z' &= 3z(x^2 - v^2 + \frac{\gamma_m}{2} z^2 - \frac{\gamma_m}{2}) + C_2,
\end{align*}

\begin{align*}
w' &= x,
\end{align*}

\begin{align*}
v' &= 3v(x^2 - v^2 + \frac{\gamma_m}{2} z^2 - 1) - xw^{-1} v.
\end{align*}

For the autonomous system (9)–(13), we find the following critical point

\begin{align*}
(\bar{x}, \bar{y}, \bar{z}, \bar{w}, \bar{v}) &= (0, \sqrt{\frac{2}{2 - n}}, 0, 0, \sqrt{\frac{n}{2 - n}}),
\end{align*}

with $\bar{w} = 0$ and $\bar{x} = 0$ satisfying

\begin{align*}
\bar{w}^{-1} \bar{x} &= -6 \frac{n}{n} v^2,
\end{align*}

where a bar denotes the values of the dimensionless variables at critical point.

In order to check the critical point (14), we start from Eq. (4) which contains an interaction term $C$. For the inverse power law potential, (4) reduces to

\begin{align*}
\dddot{\Phi} + 3H \dot{\Phi} + \bar{Q}^2 \Phi^{-3} a^{-6} - nV_0 \Phi^{-(n+1)} = -C \dot{\Phi}^{-1},
\end{align*}

or

\begin{align*}
a = \left( \frac{\bar{Q}^2}{nV_0} (1 + \eta_1) \Phi^{n-2} \right)^{\frac{1}{3}},
\end{align*}

with

\begin{align*}
\eta_1 &= \bar{Q}^{-2} (\dot{\Phi} + 3H \dot{\Phi} + C \dot{\Phi}^{-1}) a^6 \Phi^3.
\end{align*}

According to Eq. (17), the Hubble parameter is written as

\begin{align*}
H = \frac{n - 2 \dot{\Phi}}{6 \Phi} + \frac{\eta_1}{6(1 + \eta_1)}.
\end{align*}
Combining Eq. (1) and (19) gives rise to

$$\frac{\dot{\Phi}}{\Phi} + \xi = \frac{2\sqrt{3}}{2-n} \frac{\kappa V}{\delta^2} \sqrt{1 + \frac{\eta_3}{\delta^2}},$$

(20)

where

$$\delta = \left[ 1 - \frac{n(1 - \eta_2)}{2(1 + \eta_1)} \right]^\frac{1}{2},$$

(21)

$$\xi = \frac{\dot{\eta}_1}{(2-n)(1+\eta_1)},$$

(22)

$$\eta_2 = \frac{\rho_k}{\rho_c}, \quad \eta_3 = \frac{\rho_m}{V}.$$  

(23)

Let us first consider the $C = 0$ case. As has been done in Refs. [31, 32], we assume the parameters $\eta_1, \eta_2$ and $\eta_3$ to be some small quantities at late times. Under this assumption, the late-time field and scale factor are the same as those given in [32]

$$\Phi_L \simeq \Phi_0 \tau^n, \quad \Phi_0 = \left[ \frac{3n^2 k^2 V_0}{2(2-n)} \right]^\frac{1}{n},$$

(24)

$$a_L \simeq A_0 [Q \tau^\frac{n-2}{n}]^\frac{1}{4}, \quad A_0 = \left( \frac{1}{nV_0} \Phi_0^{n-2} \right)^\frac{1}{4},$$

(25)

with $\tau = t_{br} - t$, where a subscript "L" denotes the meaning of late-time. From (24) and (25), follow the late-time potential $V$ and $\rho_c$

$$V_L \simeq V_0 \Phi^{-n} \simeq \frac{2(2-n)}{3n^2 \kappa^2} \tau^{-2},$$

(26)

$$\rho_{cL} \simeq -\frac{Q^2}{24\Phi^2 a^6} \simeq \frac{n-2}{3nk^2} \tau^{-2}.$$  

(27)

According to $\rho_m \sim a^{-3\gamma_m}$, then we have the late-time matter density

$$\rho_{mL} = \rho_{m0} Q^{-\gamma_m} \tau^\frac{2-n}{n} \gamma_m,$$

(28)

where $\rho_{m0}$ is a constant.

From Eq. (6) and Eqs. (24)-(28), we obtain the late-time $x, y, z, w$ and $v$ as

$$x_L \simeq -\frac{\sqrt{6k}}{2-n} \left[ \frac{3n^2 k^2 V_0}{2(2-n)} \right]^\frac{1}{n} \tau^n,$$

(29)

$$y_L \simeq \sqrt{\frac{2}{2-n}},$$

(30)

$$z_L \simeq \frac{\sqrt{3n}}{2-n} k B_0^\frac{1}{2} \left[ Q^{-\gamma_m} \tau^\frac{2-n}{2n} \gamma_m + 1 \right],$$

(31)

$$w_L \simeq \frac{k}{\sqrt{6}} \left[ \frac{3n^2 k^2 V_0}{2(2-n)} \right]^\frac{1}{n} \tau^n,$$

(32)

$$v_L \simeq \sqrt{\frac{n}{2-n}}.$$  

(33)
Eqs. (29)-(33) tell the fact that at \( \tau = 0 \) there is \((\bar{x}, \bar{y}, \bar{z}, \bar{w}, \bar{v}) = (0, \sqrt{\frac{2}{2-n}}, 0, 0, \sqrt{n})\). In what follows, we confirm it a critical point.

Noting that \( x' = \dot{x}H^{-1} \) and the late-time Hubble parameter \( H_L \simeq \frac{2-n}{3n} \tau^{-1} \), from Eqs. (9)-(13) we have

\[
x'_L \simeq -\frac{6\sqrt{6k}}{(n-2)^2} \frac{3n^2k^2V_0}{2(2-n)} \frac{1}{n} \frac{2}{\tau} \frac{2}{n} \simeq 0,
\]
\[
y'_L \simeq 0,
\]
\[
z'_L \simeq C \frac{2}{2n} \frac{\gamma_m+1}{\tau} \simeq 0,
\]
\[
w'_L \simeq \frac{\sqrt{6k}}{2-n} \frac{3n^2k^2V_0}{2(2-n)} \frac{1}{n} \frac{2}{\tau} \frac{2}{n} \simeq 0,
\]
\[
v'_L \simeq 0.
\]

One may also start from Eqs. (29)-(33) with \( C_1 = 0 \) and \( C_2 = 0 \) to derive Eqs. (34)-(38). From Eqs. (34)-(38), it is deduced that at \( \tau = 0 \) there exactly is \((\bar{x}', \bar{y}', \bar{z}', \bar{w}', \bar{v}') = (0, 0, 0, 0, 0)\). A Big Rip corresponding to the critical point of the autonomous system can have an important hint for the evolution of phantom universe. A discussion for this will be given in the end of the next section.

### III Discussions

In this section, we first show that for the two cases of interactions the Big Rip will also occur, which corresponds to the critical point of the autonomous system. Then, we give a possible discussion of the phantom universe with Big Rip corresponding to the critical point.

Let us assume that (24) and (25) are still appropriate for some special interactions. Under this assumption, we may proceed to determine the three parameters \( \eta_1, \eta_2 \) and \( \eta_3 \). From (18) and (23) we get

\[
\eta_1 \simeq C \bar{Q}^{-2} \hat{\Phi}_L^{-1} \Phi_L^6 \hat{\Phi}_L^3 \sim C \tau^3,
\]
\[
\eta_2 \simeq \frac{6}{n(2-n)} \Phi_0^2 k^2 \tau_n^\frac{2}{n},
\]
\[
\eta_3 = \frac{\rho_{mL}}{V} \sim \frac{3n^2k^2}{2(2-n)} B_0 \bar{Q}^{-\gamma_m} \tau_n^{\frac{2-n}{n} \gamma_m+2}.
\]

Clearly, \( \eta_2 \) and \( \eta_3 \) tend to zero with \( \tau \to 0 \), noting \( 0 < n < 2 \) and \( \gamma_m > 0 \). The late-time behavior of \( \eta_1 \) depends on that of \( C \). Providing that \( C \) behaves as \( C \sim \tau^p \) with \( p > -3 \) at late times, then \( \eta_1 \) is a small quantity. One can ask the question that such \( \eta_1 \) doesn’t guarantee the quantity \( \xi \) in
Noting that \( \dot{\Phi}_C \) the following two cases: (I) have been derived from string theory and scalar-tensor theory \([34, 35, 36]\). Here, we will analyze

For \( \dot{\xi} \) obtaining (24) and (25) only if \( \xi \) is a relative small quantity to the term \( \dot{\Phi}/\Phi \sim \tau^{-1} \).

Next, we find the interactions that satisfy the requirement, \( p > -3 \). Some forms of interactions have been derived from string theory and scalar-tensor theory \([34, 35, 36]\). Here, we will analyze the following two cases: (I) \( C = \alpha \kappa \rho_m \dot{\Phi} \) and (II) \( C = 3 \beta H \rho_m \) with \( \alpha \) and \( \beta \) two positive constants. Noting that \( \dot{\Phi}_L \sim \tau^{\frac{2}{n} - 1} \) and \( \rho_m \sim \tau^{\frac{2}{n} - n_m} \gamma_m \) with \( 0 < n < 2 \) and \( \gamma_m > 0 \), one can see the interaction (I) satisfies the requirement for \( p \) obviously.

Compared to (I), the interaction (II) is stronger and the analysis for it is slightly complicated. For this interaction, Eqs. (4) and (5) may be rearranged as

\[
\dot{\rho} + 3H\dot{\rho}_\Phi = 0, \quad \gamma_{\Phi} = \gamma + \beta r
\]  

(42)

with \( r = \frac{\rho_m^*}{\dot{\rho}_m} \) and the late-time \( \gamma_{\Phi} = 1 + \omega_{\Phi} \simeq -\frac{2n}{2-n}, \) and

\[
\dot{\rho}_m + 3H\dot{\rho}_m = 0, \quad \gamma_m = \gamma_m - \beta,
\]  

(43)

which is the same as the one satisfied by the noninteracting matter, with only \( \gamma_m \) being replaced by \( \gamma_m \). For the parameter \( r \), it is a small quantity at late times, only if \( \gamma_m > \gamma_\Phi \). For \( z \) being a small quantity at late times, it is required that \( \gamma_m > -\frac{2n}{2-n}. \) If one further requires matter density to decrease with time, i.e., \( \gamma_m > 0 \), then the lower limit of \( \gamma_m \) should be modified to \( \beta \) with \( \beta < 2 \). In the following we will consider this case.

In the case of interaction (II), we need to reanalyze Eqs. (9) and (11) to determine whether the Big Rip is proceed to be a critical point. Defining \( \tilde{C} = w^{-1}(\frac{n}{2}y^2 - v^2) \), then directly starting from the definitions of \( w, y \) and \( v \) gives rise to

\[
\tilde{C}_L = \frac{k_n V_0}{\sqrt{6}H^2} \Phi_L^{-(n+1)} \eta_1 (1 + \eta_1)^{-1}.
\]  

(44)

Substituting \( \eta_1 \simeq CQ^{-2} \dot{\Phi}_L^{-1} a_L^2 \Phi_L^3 \simeq \frac{1}{m} \Phi^{-1} L_n \Phi_L^{n+1} \) into the numerator of the factor \( \frac{m}{1+\eta_1} \) on the right hand-side of Eq. (44) yields \( \tilde{C}_L \simeq \frac{1}{1+\eta_1} C_{1L} \) with \( C_{1L} = C_L = \frac{k_n C}{\sqrt{6}H^2 L_n \Phi_L} \), which gives

\[
\tilde{C}_L - \tilde{C}_{1L} \simeq \frac{\kappa}{\sqrt{6}n V_0} H_L^{-2} \Phi_L^{-2} \Phi_L^{n+1} C^2.
\]  

(45)

Noting that \( C \sim \tau^{-1 + \frac{2}{n} - n_m} \gamma_m \) at late times, then there is \( \tilde{C}_L - \tilde{C}_{1L} \sim \tau^{4 - \frac{2}{n} + \frac{2(2-n)}{n} \gamma_m} \) near Big Rip time. Setting \( \tilde{C}_L - \tilde{C}_{1L} = 0 \) at \( \tau = 0 \) leads to the following inequality

\[
n(2 - \gamma_m) - 1 + 2\gamma_m > 0.
\]  

(46)

For \( \gamma_m \geq \frac{1}{2} \) inequality (46) can always be realizable, while for \( \gamma_m < \frac{1}{2} \) the constraint on \( n \) should be imposed, \( n > \frac{1-2\gamma_m}{2-\gamma_m} \). Noting that \( 0 < n < 2 \) and \( 0 < \gamma_m \leq 2 \), then one can easily find \( C_{2L} = 0 \).
at $\tau = 0$. As a result, for the interaction (II) with $n$ and $\tilde{\gamma}_m$ satisfying (46), the Big Rip can still be the critical point. When (46) isn't satisfied, the Big Rip will be no longer a critical point. This means there are the two different kinds of Big Rip, which may be distinguished from whether they are a critical point. Here, we are interested in the case that Big Rip corresponding to the critical point and attempt to give a possible interpretation for it.

The critical point discussed here is obviously unstable since Big Rip is only an evolution consequence of expanding phantom universe. What does a Big Rip mean? Generically, it is assumed to be the final state of an expanding phantom universe. In Ref. [32], we have demonstrated the symmetric properties of the field and scale factor of phantom universe about the Big Rip. This suggests that the solution of the phantom universe having a Big Rip should be extended to the range $\tau < 0$. In this paper, we have shown that the Big Rip in the $SO(1,1)$ model can be a critical point. For such phantom universe that the Big Rip is a critical point we conjecture the Big Rip may only be a middle point in its evolving. In other words, such a phantom universe should undergo the two evolution phases, the accelerating expansion and decelerating contraction which transit at the Big Rip.

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