SOLUTION OF THE PROBLEM OF STABILITY OF 40X STEEL SHELL

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Abstract. Discusses the solution of the problem of a circular elastic-plastic bifurcation of thin-walled cylindrical shell under the proportional subcritical processes loading shell axial compressive force and torque in deviatoric space of deformations of A. A. Ilyushin. The solution is based on the theory of stability of inelastic systems under complex loading V. G. Zubchaninov. The incompressibility condition of the material and the homogeneity condition of the stress state in the shell up to the moment of loss of stability are used. The problem is solved in a geometrically linear formulation [1-7].

1. Introduction
Nowadays the active development of all industries and the economy, especially the construction sector, sets new goals and objectives for specialists. One of the main tasks facing engineers - is to reduce labor and material costs for the construction of a structure. Currently, much attention is paid to the structures of particularly dangerous or unique buildings and structures, some parts of which are thin-walled shell-type structures. When developing individual parts of projects based on calculations of thin-walled shells, the most difficult issues are their buckling and stability.

The results of various experiments confirm that the process of buckling is accompanied by complex deformation of the material. Experimental studies of the buckling process of shells under simple subcritical loading have become fundamental in verifying the validity of different versions of the theory of plasticity [1].

2. Methods
Under the assumed assumptions, the solution of the stability problem is reduced to the solution of the eigenvalue problem on the basis of which it is possible to calculate the value of the shell flexibility depending on the magnitude of the stress vector modulus and wave formation parameters realized at the loss of stability. In calculations based on the theory of stability A. A. Ilyushin, in which the relationship of stresses and strains is accepted in accordance with the defining relations of the theory of quasi-simple processes [8, 9], which is a private version of the hypothesis of complementarity, the system of algebraic equations of the eigenvalue problem takes the form of (1) and (2):
\[
\frac{i^2}{E g_1} \left(-K_s - \frac{E N^*_s}{2 \sigma \theta} \lambda^4_m\right) + \frac{3}{2} \frac{N^*_s \Phi^*}{g_1 \theta} \lambda^2_m K_s, S_i = \left(\lambda^2_m + n^2\right)^2 \frac{1}{2} \left(1 - \frac{g_2}{g_1}\right) K_s^2 + \frac{9}{8} \frac{N^*_s \Phi^2}{g_1 \theta} K_s^2 S_i^2.
\]

\[C \theta = \frac{N^*_s \lambda^2_m}{2} - \frac{3}{4} \frac{N^*_s \Phi^*}{i} K_s S_i,
\]

\[N_m = \int_{-h/2}^{h/2} N z^{m-1} dz, \quad P_m = \int_{-h/2}^{h/2} P z^{m-1} dz
\]

For the defining features of plasticity is taken piecewise-linear approximation of (3)

\[
\begin{cases}
N = 2G \left(1 - \omega\right), \quad P = 2G \left(1 - \lambda\right), \quad 0 \leq \theta_1 \leq \pi / 2 \\
N = P = 2G, \quad \pi / 2 \leq \theta_1 \leq \pi
\end{cases}
\]

where \(\theta_1\) is the parameter of the plasticity of A. A. Ilyushin; \(\lambda\) parameter softening.

Breaking into (3) integration by zones and assuming that the unloading zone is adjacent to the surface, we have \([1]\)

\[N^*_m = \frac{1}{m} \left[1 - (-1)^n - \omega \left(1 - Z_p^n\right)\right],
\]

\[P^*_m = \frac{1}{m} \left[1 - (-1)^n - \lambda \left(1 - Z_p^n\right)\right], \quad (m = 1, 2, 3).
\]

To determine the coordinates of the boundary of the elastic unloading and plastic loading zones of the material, we have the equations

\[4Z_p^* + \lambda \left(1 - Z_p^*\right)^2 = 2f_1, \quad f_1 = P^*_p z_p^* - P^*_z, \quad \tau = \cos \theta_1 = \hat{\sigma} \cdot \hat{\sigma} = \frac{\hat{\sigma} \cdot \hat{\mathcal{J}}}{\hat{s}},
\]

where \(\theta_1\) - the angle of convergence, \(-1 \leq z_p^* \leq +1\).

If the discharge zone is formed, then believe. In this case \(N^*_2 = P^*_2 = \Phi^* = 0\).

The modified theory of stability of shells beyond the elastic limit Of V. G. Zubchaninov \([1-7]\) is based on the assumption that an infinitesimal continuation of the process associated with bifurcation is simple or locally simple. In this case

\[\hat{\sigma} = \hat{P} \hat{\mathcal{J}}, \quad \hat{S}_y = \hat{P} \hat{\mathcal{J}}_y
\]

where the zone of plastic loading \(P = 2G\), and unloading areas \(P = 2G\).

The basic equations of the problem take the form \([1]\)

\[
\begin{cases}
\dot{i}^2 = \frac{E g_2}{\sigma} \left(\lambda^2_m + n^2\right)^2 \left[-K_s - \frac{E P^*_1 \lambda^4_m}{2 \sigma \theta}\right]^{-1}
\end{cases}
\]

\[C \theta = \frac{P^*_2 \lambda^2_m}{2}, \quad \theta = \left(\lambda^2_m + n^2\right)^2
\]

3. Results

Earlier, when solving the problems of bifurcation of cylindrical shells, the graphs of the coordinates of the boundaries of the elastic unloading and plastic loading zones of the material were not constructed. Meanwhile, as shown by recent studies \([2, 6, 7]\), the study and analysis of this issue significantly contribute to the justification of the use of different versions of the theory of plasticity to obtain
physically reliable results of solving the problem of stability in both simple and complex processes of combined subcritical loading of structures.

Two batches of material were considered. Chemical analysis was carried out in the laboratory of the Tver carriage works on the equipment of the Foundry Master 01H0076 (Optik 01H0077). The results of the chemical analysis are given in tables 1 and 2. In accordance with GOST 4543-71, both parties are low-alloy steel 40X.

|            | Table 1 | Table 2 |
|------------|---------|---------|
| Fe         | 97.1    | 97.2    |
| C          | 0.374   | 0.44    |
| Si         | 0.153   | 0.267   |
| Mn         | 0.721   | 0.745   |
| P          | 0.0173  | 0.0176  |
| S          | 0.0266  | 0.0117  |
| Cr         | 0.924   | 0.995   |
| Mo         | 0.0251  | 0.9119  |
| Ni         | 0.197   | 0.0544  |
| Al         | 0.0033  | 0.0344  |
| Co         | 0.0027  | 0.0032  |
| Cu         | 0.175   | <0.002  |
| Nb         | 0.0153  | <0.002  |
| Ti         | 0.0042  | 0.0041  |
| V          | 0.0191  | 0.0246  |
| W          | 0.0455  | 0.0455  |
| Pb         | 0.0873  | <0.05   |
| Sn         | 0.002   | 0.038   |
| B          | 0.0139  | >0.0007 |
| Ca         | 0.0007  | 0.0045  |
| Zr         | 0.0298  | 0.0127  |
| As         | 0.0535  | <0.09   |
| Bi         | 0.09    |         |

The test specimens are thin-walled circular cylindrical shells. Thin-walled tubular specimens have a wall thickness $h = 1$ mm, the radius of the median surface $R = 15.5$ mm and, accordingly, $R = 15$ mm. Diagrams of material deformation during compression are shown in figure 1, where $\sigma$ – stress intensity; $\varepsilon$ – the intensity of the deformation; 1 - Party 1; 2 - Party 2.

As follows from figure 1, the material of the second batch is characterized by significantly greater hardening. Thus, when the ratio of shear modules of the material.

Also from the presented results it is seen that when the developed plastic deformation ratios of the transverse deformations is close to the value that allowed to use the condition of incompressibility of the material and equations to calculate the components of the vector of stresses and strains present in the form:

$$\mathfrak{f}_1 = \sqrt{3}/2\varepsilon_{11}; \mathfrak{f}_2 = \sqrt{2}(\varepsilon_{22} + \varepsilon_{11}/2); \mathfrak{f}_3 = \sqrt{2}\varepsilon_{12}; \mathfrak{f}_4 = \mathfrak{f}_5 = 0;$$

$$S_1 = \sqrt{3}/2(\sigma_{11} + \sigma_{22}/2); S_2 = \sigma_{22}/\sqrt{2}, S_3 = \sqrt{2}\sigma_{12}; S_4 = S_5 = 0.$$
Figures 2 – 5 shows plots of the values of critical stresses depending on the flexibility of the shell, built with various combinations of half waves \( m, n \) proportional loading processes (\( \alpha_1 = 45, \alpha_2 = 0 \)) for two batches of material casings made of low-alloy steel 40X, with different chemical properties of the material.

The graphs show the curves of the wave formation parameters realized in case of loss of stability in the axial and circumferential directions, respectively. The presented graphical values are the result of calculation according to different versions of the theory of plasticity and with different methods of accounting for the angle of fracture of the deformation trajectory at the time of bifurcation [10-15].

On the graphs, a bold solid line indicates the limit curves obtained as envelopes of the curves of stable States constructed at different values of the wave formation parameters: \( m \) from 1 to 50; \( n \) from 0 to 25.

**Figure 2.** Results of calculation of critical stress parameters according to the stability theory of A. A. Ilyushin taking into account unloading (\( \alpha_1 = 45, \alpha_2 = 0 \)).

**Figure 3.** The results of the calculation of critical stresses based on the modified theory of stability V. G. Zubchaninov including unloading (\( \alpha_1 = 45, \alpha_2 = 0 \)).
Figure 4. Results of calculation of critical stress parameters according to the stability theory of A. A. Ilyushin taking into account unloading ($\alpha_1=45$, $\alpha_2=0$).

Figure 5. Results of calculation of critical parameters of stresses on the modified theory of stability V. G. Zubchaninov taking into account unloading ($\alpha_1=45$, $\alpha_2=0$).

The results of calculations of dimensionless coordinates of the boundary of the material zones are presented in figures 6, 7. The numbers mark: 1 - the Party 1, with $\alpha_1=0$; 2 - Party 2 with $\alpha_1=0$; 3 - Party 1, with $\alpha_1=45^\circ$; 4 - Party 2 with $\alpha_1=45$; 5 - the Party 1, with $\alpha_1=90$; 6 - the Party 2 with $\alpha_1=90^\circ$. 
Figure 6. Coordinates of zone boundaries based on the modified theory of stability V. G. Zubchaninov.

Figure 7. Coordinates of zone boundaries based on the stability theory of A. A. Ilyushin.

4. Conclusions

From the obtained results it follows that for all considered processes the unloading zone as a whole is not large. In the greatest degree of unloading is shown for processes of pure compression. As the flexibility of the shell decreases, the coordinate of the zone boundary approaches the value, which corresponds to a purely plastic loss of stability. The influence of the angle of inclination of the trajectory beam in the plane on the values in the calculations on the theory of stability of A. A. Ilyushin is manifested to a lesser extent than in the calculations on the modified theory of stability of V. G. Zubchaninov [16-20].

According to the modified stability theory of V. G. Zubchaninov, for example, the ratio of the coordinates of the boundaries of the samples of batch 1 and batch 2 is:

\[
\frac{Z_{p1}}{Z_{p2}} = 2.6\% \text{, while } \alpha = 0; \quad \frac{Z_{p1}}{Z_{p2}} = 4.0\% \text{, while } \alpha = 45^0; \quad \frac{Z_{p1}}{Z_{p2}} = 3.7\% \text{, while } \alpha = 90^0.
\]

On the theory of stability of A. A. Ilyushin at we have:

\[
\frac{Z_{p1}}{Z_{p2}} = 4.4\% \text{, while } \alpha = 0; \quad \frac{Z_{p1}}{Z_{p2}} = 3.5\% \text{, while } \alpha = 45^0; \quad \frac{Z_{p1}}{Z_{p2}} = 3.6\% \text{, while } \alpha = 90^0.
\]

For shells of low flexibility, the difference does not exceed 1% both in the calculations based on the stability theory of A. A. Ilyushin and on the modified stability theory of V. G. Zubchaninov. It is noted that with increasing degree of hardening the unloading area of the material becomes larger.
5. References

[1] Zubchaninov V G 2007 *Ustoichivost’ i plastichnost’* (Moscow: Fizmatlit) p 448
[2] Zubchaninov V G 2002 *Matematicheskaya teoriya plastichnosti* (Tver’: TGTU) p 300
[3] Ilyushin A A 1948 *Plastichnost’. Uprugoplasticheskie deformacii* (Moscow: Gostekhizdat) p 376
[4] Zubchaninov V G 2003 *Ekperimental’naya plastichnost’* (Tver’: TGTU) p 172
[5] Zubchaninov V G 2010 *Mekhanika processov plasticheskikh sred* (Moscow: Fizmatlit) p 352
[6] Zubchaninov V G, Alekseev A A and Gul’tyaev V I 2015 *Problemy prochnosti i plastichnosti* 77 113
[7] Zubchaninov V G, Alekseev A A and Gul’tyaev V I 2015 *Problemy prochnosti i plastichnosti* 77 18
[8] Zubchaninov V G, Alekseev A A, Alekseeva E G and Gul’tyaev V I 2017 *Materials physics and mechanics* 32 298
[9] Zubchaninov V G, Alekseev A A and Gul’tyaev V I 2017 *Vestnik permskogo nacional’nogo issledovatel’skogo politekhnicheskogo universiteta. Mekhanika* 3 203
[10] Zubchaninov V G 2011 *Izvestiya rossijskoj akademii nauk. Mekhanika tverdogo tela* 1 27
[11] Zubchaninov V G 2000 *Strength of materials* 32 13
[12] Garanikov V V, Zubchaninov V G and Ohlopkov N L 1999 *Problemi prochnosti* 5 36
[13] Vasin R A 2011 *Izvestiya rossijskoj akademii nauk. Mekhanika tverdogo tela* 1 19
[14] Vesnin V A, Vasin R A, Kozicin A S and Shundeev A S 2005 *Avtomatika i telemekhanica* 7 171
[15] Ermachenko A G, Vasin R A and Enikeev F U 2001 *Vestnik permskogo gosudarstvennogo tekhnicheskogo universiteta. Matematicheskoie modelirovanie sistem processov* 9 20
[16] Bondar V S, Abashev D R and Petrov V K 2017 *Vestnik permskogo gosudarstvennogo tekhnicheskogo universiteta. Mekhanika* 3 53
[17] Bondar V S 2017 *Materials physics and mechanics* 23 1
[18] Temis U M 2018 *Propulsion and power research* 7 277
[19] Zubchaninov V G, Alekseev A A and Alekseeva E 2016 *Materials physics and mechanics* 29 150
[20] Garanikov V V, Zubchaninov V G and Ohlopkov N L 1999 *Strength of materials* 31 336