Leptogenesis in an $S_3$ model

Arturo Álvarez Cruz and Myriam Mondragón
Instituto de Física, Universidad Nacional Autónoma de México, Apdo. Postal 20-360, 04515 México D.F., México
E-mail: aalvarez@fciencias.unam.mx

Abstract. We study the possibility of having leptogenesis in an $S_3$ extension of the Standard Model with three Higgs doublets and three right-handed neutrinos. In the leptonic sector we introduce also a $Z_2$ symmetry to reduce further the number of parameters. The characteristics of the model allow in a very natural way for resonant leptogenesis, which is highly dependent on the values of the Dirac and Majorana phases. The associated baryogenesis is also maximal when the neutrino masses are almost degenerate.

1. Introduction

It is well known that there are more baryons than antibaryons in the Universe. Big Bang Nucleosynthesis is a solid and consistent model of the creation of the nuclei in the early Universe, and predicts the following baryonic density

$$\eta = \frac{\eta_B - \eta_\gamma}{\eta_\gamma} = \eta = (2.6 - 6.2) \times 10^{-10}.$$  \hspace{1cm} (1)

Measurements of the cosmic background radiation [1–3] show a density of

$$\eta = (6.1 \pm 0.3) \times 10^{-10},$$

in full agreement with the baryon density obtained from nucleosynthesis [2, 4]. This baryonic density can be express in terms of the entropy of the Universe,

$$Y_B \equiv \frac{\eta_B - \eta_\gamma}{s} \bigg|_0 = (8.77 \pm 0.24) \times 10^{11}.$$  \hspace{1cm} (2)

Baryogenesis, which comprises all the processes that could lead to an abundance of matter over anti-matter, cannot be achieved with the amount of CP violation present in the Standard Model (SM). For this reason it is necessary to consider models beyond the SM that include new ways to increase either the production of baryons or the amount of CP violation, or both. An increasingly popular idea to achieve this is leptogenesis. Leptogenesis is a mechanism by which a leptonic asymmetry is generated, and then by sphaleron processes, this becomes a baryon asymmetry [5]. Several things are needed for the occurrence of leptogenesis: heavy right-handed neutrinos, that the neutrinos be of Majorana nature, and a possible decay of the right-handed neutrinos to the left-handed ones.

There have been many attempts to extend the SM [6, 7], one way to do it is through flavour symmetries, that relate the three families of particles known. Among the smallest more common
The Majorana mass terms for the right-handed neutrinos are given by

with $S\,2$. The right handed neutrinos are related to the left ones through the see-saw mechanism (type I).

2. The $S_3$ model with three Higgs doublets

In the Standard Model analogous fermions in different generations have identical couplings to all gauge bosons of the strong, weak and electromagnetic interactions [11]. Prior to the electroweak symmetry breaking, the Lagrangian is chiral and invariant regarding to permutations of the left and right fermionic fields. The group $S_3$ consists of the six possible permutations of three objects $(f_1, f_2, f_3)$, and is the smallest, discrete, non-abelian group. Its irreducible representations (irreps) are the two-dimensional one, and two singlet irreps, a symmetric and an anti-symmetric one. We will only consider the doublet and the symmetric singlet here, given by

$$ f_S = \frac{1}{\sqrt{3}} (f_1 + f_2 + f_3), \quad f_D^T = (\frac{1}{\sqrt{2}} (f_1 - f_2), \frac{1}{\sqrt{6}} (f_1 + f_2 - 2f_3)). $$

The direct product of two doublets $p_D^T = (p_{D1}, p_{D2})$ and $q_D^T = (q_{D1}, q_{D2})$ may be decomposed into the direct sum of two singlets $r_s$ and $r_s'$, and one doublet $r_D$. In this way we can associate the known particles to doublets or to singlets with the following rules: we accommodate the first two families in a doublet flavour representation, and the third one in the symmetric singlet one. We follow this same pattern for the Higgs fields and the right handed neutrinos. The quark and Higgs fields are denoted as

$$ Q^T = (u_L, d_L), \quad u_r, d_r, \quad L^T = (\nu_L, e_L), \quad e_R, \nu_R \quad \text{and} \quad H. \quad (3) $$

The doublets carry capital subindices $I$ and $J$, which run from 1 to 2, and the singlets are denoted by $Q_3$, $u_{3R}$, $d_{3R}$, $L_3$, $e_{3R}$, $\nu_{3R}$ and $H_S$. Notice that the subscript 3 denotes the singlet representation and not the third generation. The most general renormalizable Yukawa interactions of this model are given by

$$ L_Y = L_{Y_d} + L_{Y_u} + L_{Y_e} + L_{Y_\nu} + L_{Y_M} \quad (4) $$

where

$$ L_{Y_\nu} = -Y_1^\nu \bar{\nu}_{1R}(i\sigma_2 H_5^\nu \nu_{1R}) - Y_3^\nu \bar{\nu}_{3R}(i\sigma_2 H_5^\nu \nu_{3R}) $$

$$ -Y_2^\nu \bar{\nu}_{2R}(i\sigma_2 H_1^\nu \nu_{2R}) - Y_3^\nu \bar{\nu}_{3R}(i\sigma_2 H_1^\nu \nu_{3R}) + \text{h.c.,} \quad (5) $$

with

$$ \kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. $$

The Majorana mass terms for the right-handed neutrinos are given by

$$ L_M = -M_1 \nu_{1R}^T C \nu_{1R} - M_3 \nu_{3R}^T C \nu_{3R}. \quad (6) $$

Due to the presence of three Higgs fields, the Higgs potential $V_H(H_S, H_D)$ is more complicated than that of the Standard Model. We assume that all the vacuum expectation values (vev’s) of the Higgs fields are real and that $\langle H_1 \rangle = \langle H_2 \rangle$, which implies that besides the $S_3$ symmetry, it has a permutational symmetry $Z_2 : H_1 \leftrightarrow H_2$, which is not a subgroup of the flavour group
The corresponding mass matrix is given by

\[ M = \begin{pmatrix}
\mu_1 + \mu_2 & \mu_2 & \mu_5 \\
\mu_2 & \mu_1 - \mu_2 & \mu_5 \\
\mu_4 & \mu_4 & \mu_3
\end{pmatrix}. \]  

(7)

The Majorana mass for the left handed neutrinos \( \nu_S \) since there is no restriction coming from the flavour symmetry \( M \)

\[ M = M_{\nu L} M^{-1} (M_{\nu L})^T, \]

(8)

where \( M = \text{diag}(M_1, M_1, M_3) \). In principle, all entries in the mass matrices can be complex since there is no restriction coming from the flavour symmetry \( S_3 \). The mass matrices are diagonalized by bi-unitary transformations as

\[ U_{d(u,e)}^T M_d(u,e) U_{d(u,e)} = \text{diag}(m_d(u,e), m_s(u,e), m_b(u,e)) \]

\[ U_{\nu}^T M_{\nu} U_{\nu} = \text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}) . \]

The entries in the diagonal matrices may be complex, so the physical masses are their absolute values. The mixing matrices are, \( V_{CKM} = U_{\nu L} U_{d L} \), \( V_{PMNS} = U_{eL} U_{\nu} K \), where \( K \) is the diagonal matrix of the Majorana phase factors.

The theoretical mixing matrix, \( V_{PMNS} \), is given by

\[ V_{PMNS}^{th} = \begin{pmatrix}
O_{11} \cos \eta + O_{31} \sin \eta e^{i\delta} & O_{11} \sin \eta - O_{31} \cos \eta e^{i\delta} & -O_{21} \\
-O_{12} \cos \eta + O_{32} \sin \eta e^{i\delta} & -O_{12} \sin \eta - O_{32} \cos \eta e^{i\delta} & O_{22} \\
O_{13} \cos \eta - O_{33} \sin \eta e^{i\delta} & O_{13} \sin \eta + O_{33} \cos \eta e^{i\delta} & O_{33}
\end{pmatrix} \times K, \]

(9)

with

\[ \sin^2 \eta = \frac{m_{\nu 3} - m_{\nu 1}}{m_{\nu 2} - m_{\nu 1}}, \quad \cos^2 \eta = \frac{m_{\nu 2} - m_{\nu 3}}{m_{\nu 2} - m_{\nu 1}}, \quad \delta = \delta_\nu - \delta_e. \]

Neglecting the smalls terms proportional to \( O_{11} \) and \( O_{11}^2 \), we get

\[ \tan^2 \theta_{12} = \frac{(\Delta m_{12}^2 + \Delta m_{13}^2 + |m_{\nu 3}|^2 \cos^2 \phi_{nu})^{1/2} - |m_{\nu 3}| |\cos \phi_{nu}|}{(\Delta m_{13}^2 + |m_{\nu 3}|^2 \cos^2 \phi_{nu})^{1/2} + |m_{\nu 3}| |\cos \phi_{nu}|} . \]

(10)

From this expression, we may readily derive expressions for neutrinos masses in terms of \( \tan \theta_{12}, \phi_{\nu} \) and the differences of the squared masses. The sum of the neutrino masses is

\[ \sum_{i=1}^{3} |m_{\nu i}| \approx \frac{\Delta m_{13}^2}{2 \cos \phi_{\nu} \tan \theta_{12}} (1 + 2 \sqrt{1 + 2 \tan^2 \theta_{12}(2 \cos^2 \phi_{\nu} 1) + \tan^4 \theta_{12} \tan^2 \theta_{12})}. \]

(11)

The most restrictive cosmological upper bound for this sum is

\[ \sum |m_{\nu i}| \leq 0.17 \text{eV}. \]

(12)

The neutrino masses \( |m_{\nu i}| \) assume their minimal values when \( \cos \phi_{\nu} = 1 \). When \( \cos \phi_{\nu} \) takes values in the range \( 0.55 \leq \cos \phi \leq 1 \), the neutrino masses change very slowly with \( \cos \phi_{\nu} \). In the absence of experimental information we will assume that the Majorana phase \( \phi_{\nu} \) vanishes. Hence, setting \( \phi_{\nu} = 0 \) in our formula, we find

\[ m_{\nu 1} = 0.052 \text{ eV}, \quad m_{\nu 2} = 0.053 \text{ eV}, \quad m_{\nu 3} = 0.019 \text{ eV}. \]

(13)
The computed sum of the neutrino masses is

\[
(\sum_{i=1}^{3} |m_{\nu_i}|)^{th} = 0.13 \text{ eV}, \tag{14}
\]

below the cosmological upper bound given in eq. (12), as expected, since we used the cosmological bound to fix the bound on \(\cos \phi_\nu\). The most restrictive direct neutrino measurement involving electron type neutrinos, is based on fitting the shape of the beta spectrum. In such measurement, the quantity

\[
\bar{m}_{\nu e} = \sqrt{\sum_i |V_{ei}|^2 m_{\nu i}}
\]
is determined or constrained. A very restrictive upper bound for this sum is obtained from nucleosynthesis processes [13, 14]

\[
(m_{\nu e})^{exp} < 0.37 \text{ eV}. \tag{15}
\]

We find

\[
(m_{\nu e})^{th} = 0.053 \text{ eV}, \tag{16}
\]
again, well below the experimental upper bound given in eq. (15).

3. Leptogenesis in an \(S_3\) model

The Yukawa couplings of the neutrinos allow the decay of the right-handed neutrinos to the left-handed ones \(Y^0 I_l (i \sigma_2 H_S \nu_I R)\).

The asymmetry is defined as

\[
\epsilon_1 = \sum_{\alpha} \frac{\Gamma(N_1 \rightarrow \ell_\alpha H) - \Gamma(N_1 \rightarrow \bar{\ell}_\alpha H)}{\Gamma(N_1 \rightarrow \ell_\alpha H) + \Gamma(N_1 \rightarrow \bar{\ell}_\alpha H)}, \tag{17}
\]

where \(\Gamma\) is the decay rate, and \(N_1\) is a singlet neutrino. The possible decays up to tree level are shown in fig. 1.

**Figure 1.** Feynman diagrams up to second order: a) tree level b) second order c) self energy diagram

The asymmetry generated by these decays is,

\[
\epsilon \simeq - \frac{3}{8\pi} \frac{1}{(h_\nu h_\nu)^2} \sum_{i=2,3} \text{Im}\{(h_\nu h_\nu)_i^2\} \left[ f \left( \frac{M_i^2}{M_1^2} \right) + g \left( \frac{M_i^2}{M_1^2} \right) \right], \tag{18}
\]
and the self interactions are

\[ f(x) = \sqrt{x}[1 - (1 + x)\ln\left(\frac{1 + x}{x}\right)], \quad g(x) = \frac{\sqrt{x}}{1 - x}. \]

This function depends strongly on the hierarchy of light neutrino masses. It can lead to a strong enhancement of the CP asymmetries if the masses \( M_2 \) and \( M_3 \) are nearly degenerate. The asymmetry for the \( S_3 \) model is given by

\[ \epsilon = \frac{\text{Im}[e^{2i\delta} M_2 m_3 \sqrt{M_2(m_3 - m_1)}(f\left[M_2^2\right] + g\left[M_3^2\right])]}{8\pi |M_2 m_3|} \]  

(19)

The relation between the lepton and baryon asymmetry is given through the sphaleron process

\[ Y_B = a Y_{B-L} = \frac{a}{a - 1} Y_L, \]  

(20)

where \( a = (8N_f = 4NH)/(22N_f + 13NH) \) and \( Y_L \equiv \frac{n_L - n_{\bar{L}}}{s} \). We can express the lepton asymmetry in terms of the CP asymmetry

\[ Y_L = \kappa \frac{\epsilon_i}{g^*}, \]

where \( g \) is 110, the number of relativistic degrees of freedom, \( \kappa \) is obtained from solving the Boltzmann equations, and it can be reparametrized in terms of \( K \), defined as the ratio of \( \Gamma_1 \), the tree-level decay width of \( N_1 \) to \( H \) the Hubble parameter at temperature \( M_1 \). Thus, \( \kappa = \Gamma_1/H < 1 \) describes processes out of thermal equilibrium and \( \kappa < 1 \) describes washout effects

\[ \kappa \approx \begin{cases} 0.3 & \text{for } 10 < K < 10^6 \\ \frac{1}{2\sqrt{K^2 + 9}} & \text{for } 0 < K < 10. \end{cases} \]

The decay width of \( N_1 \) by the Yukawa interaction at tree level and Hubble \( H \) parameter in terms of the temperature \( T \) and Planck scale \( M_{Pl} \) are

\[ \Gamma_1 = (m_D^{\dagger} m_D)_{11} T / (8\pi v^2) \quad \text{and} \quad H = 1.66g^{*1/2}T^2/M_{Pl}, \]

respectively. At temperature \( T = M_1 \) the ratio \( K \) is

\[ K = \frac{M_{Pl}}{1.66\sqrt{g^*}(8\pi v^2)} \frac{(m_D^{\dagger} m_D)_{11}}{M_1} \]  

(21)

The value of the baryon asymmetry on the \( S_3 \) model has a dependence on \( \phi_\nu \) and on the real masses of the neutrinos \( m_1, M_1, m_2, M_2, m_3, M_3 \), where the masses of the right-handed neutrinos are considered real.

We can calculate the dependence of the baryon asymmetry on the phases \( \delta \) and \( \phi_\nu \) independently of the masses. Knowing the value of the masses would give the actual value of the baryon asymmetry. The maximum value is found on \( \delta = 3/4\pi \) and \( \phi = \pi \), this values are the same that make the best fit to the neutrino masses. As fig. 3 shows, leptogenesis in this model is highly dependent on values of the phases.

The value of the baryon asymmetry is determined by the masses of the light neutrinos and the difference of the masses of the right-handed neutrinos, the see-saw mechanism relates the masses of the right handed neutrinos making the right-handed neutrino masses larger than \( 10^{12} \text{ GeV} \). The maximal contribution to baryogenesis occurs in the case where the three masses are nearly degenerate, basically almost equal.
Figure 2. Baryon asymmetry in the \( S_3 \) flavour symmetry model. It can be seen the strong dependence not only on the Dirac phase \( \delta \), but also on the neutrino phase \( \phi_\nu \).

Figure 3. Baryogenesis dependence on the right-handed neutrino masses in the \( S_3 \) flavour symmetry. For leptogenesis to contribute to the explanation of the baryon asymmetry, an almost degenerate heavy \( (M \sim 10^{12} \text{ GeV}) \) right-handed neutrinos mass is required.

4. Conclusions
The \( S_3 \) extension of the SM with three Higgs doublets allows a definite structure of the Yukawa couplings, and we have studied its consequences regarding leptogenesis. The theoretical predictions obtained from the \( S_3 \otimes Z_2 \) symmetry are consistent with the experimental observations made to this time and the model allows leptogenesis as a mechanism to solve in part the baryonic asymmetry.

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