All mechanical mixing by means of orthogonally coupled cantilevers

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Abstract. An all mechanical signal mixing device is described which uses only linear springs as fundamental building blocks. Two input oscillators are coupled to an orthogonally vibrating output oscillator via a linear spring, which results in an effective nonlinear multiplicative mixing function. Numerical simulations have been performed for studying the performance characteristics of the mixer. A modified version of the mixer acting as a mechanical modulator is also numerically investigated for application as a mechanical power amplifier. The viability of the concept is demonstrated in a centimeter scale experimental setup and a micromechanical scale polymer embodiment has been built using two-photon polymerization lithographic methods.

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1. Introduction

Mechanical oscillators have unsurpassed sensing qualities. Using a scheme very similar to dynamic force microscopy, Rugar et al recently demonstrated the detection of a single electron spin by mechanical means [1]. Moreover, mechanical resonators acting as filters and/or mixers in the radio frequency regime have attracted increasing interest in recent years [2], as they hold promise to be superior to complementary metal-oxide-semiconductor (CMOS) technology in terms of noise and power consumption. Combining mechanical oscillators with nonlinear coupling elements expands the scope of potential applications even further. Such devices exploit the parametric effect to achieve mixing, amplification or noise squeezing [3]–[7].

The parametric effect is based on a nonlinear reactive element which allows to transfer energy between oscillators. Specifically, we consider here the mechanical mixing that occurs between two input oscillators which are nonlinearly coupled to an output oscillator. The nonlinear reactive element is implemented in a purely mechanical fashion using a linear spring and exploiting the fact that a spring is stretched nonlinearly if it is sheared orthogonal to the spring axis. Thus, the nonlinearity is of geometrical origin. Specifically, a system with dimensions that are typical of a micromechanical implementation is studied numerically. The mechanical mixer can also be configured to act as a mechanical amplifier. The feasibility of the concept is demonstrated in macro-scale experimental implementation validating the numerical modeling. Furthermore, a practical micro-scale realization of mechanical mixer is shown using two-photon lithographic techniques which allows the fabrication of complex arbitrary three-dimensional structures in polymeric materials.

2. Nonlinear mechanical coupling of harmonic oscillators

Nonlinear effects in a system of coupled oscillators are usually obtained by driving the system at large amplitudes such that nonlinearities of the coupling potential become significant. An
alternative, purely geometric way of realizing nonlinear coupling between oscillators is shown in figure 1. The oscillators vibrate in orthogonal directions, and a conventional spring couples the two motions of the oscillators. In contrast to the nonlinearities obtained for large vibration amplitudes, the nonlinear coupling between the orthogonal motions is already effective at small amplitudes.

The coupling potential $U_c$ between the oscillators can be written as

$$U_c(z, y) = \frac{1}{2}k_c \left( (z^2 + (l_c + y)^2)^{1/2} - l_c \right)^2. \quad (1)$$

The force component in the $y$-direction, $F_y$, is given by the $y$-component of the potential gradient $\nabla U_c$:

$$F_y = \frac{\partial U_c}{\partial y} = k_c(l_c + y) \left( 1 - \frac{l_c}{(z^2 + (l_c + y)^2)^{1/2}} \right). \quad (2)$$

The lowest-order series expansion of the above expression (equation (2)) is

$$F_y = k_c \left( y + \frac{1}{2l_c} z^2 \right). \quad (3)$$

The quadratic dependence of the force $F_y$ on the $z$-position of the input oscillator reflects the nonlinear part of the coupling potential. A sinusoidal variation of $z$ generates an oscillatory force in the $y$-direction at twice the frequency of the $z$-motion. The oscillator motion $z = A \sin(\omega t)$ of the input generates a second harmonic force $F_{yz} = k_c/4l_c \cos(2\omega t)$ acting on the output oscillator. Thus, in order to obtain a large vibration amplitude of the output oscillator, its resonance frequency has to be adjusted to twice the input frequency, i.e. at $2\omega$.

3. Mechanical mixer

The nonlinear coupling between oscillators can be used to implement a mechanical mixer. Two parallel input oscillators are linearly coupled by springs, and the resulting ‘sum’ vibration is
nonlinearly coupled to a third oscillator that vibrates perpendicularly to the input oscillators. A schematic of the mixer is shown in figure 2.

The input oscillators Osc1 and Osc2 are excited to the vibration amplitudes $z_1$ and $z_2$, respectively. They are coupled to a common point denoted as $z_s = z_1 + z_2$ by means of two springs $k_{c_1}$ and $k_{c_2}$, respectively, which model the mechanical compliance of the coupling structure (not shown in figure 1). For equal springs, i.e. $k_{c_1} = k_{c_2}$, the motion $z_s$ of the coupling point is proportional to the sum $z_1 + z_2$ of the two input amplitudes (see below). The motion $z_s$ excites the output oscillator Osc3 through the nonlinear coupling generated by the spring having its axis perpendicular to the $z$-direction. The coupling spring has a stiffness $k_{c_3}$ and a length $l_{c_3}$ at zero force. The total coupling potential is

$$U_c(z_1, z_2, z_s, y_3) = \frac{1}{2}k_{c_1}(z_1 - z_s)^2 + \frac{1}{2}k_{c_2}(z_2 - z_s)^2 + \frac{1}{2}k_{c_3}\left((z_s^2 + (l_{c_3} + y_3)^2)^{1/2} - l_{c_3}\right).$$

(4)

The motion $z_s$ of the coupling point is determined by the constraint of zero total force in equilibrium, i.e. $\partial U_c/\partial z_s = 0$. For small amplitudes $z_1, z_2$ and $y_3$, this condition approximately yields $z_s = (k_{c_1}z_1 + k_{c_2}z_2)/(k_{c_1} + k_{c_2})$. For equal coupling springs $k_{c_1} = k_{c_2}$, this expression simplifies to $z_s = \frac{1}{2}(z_1 + z_2)$, reflecting the sum of the input amplitudes. Using this expression in equation (3) yields a force component that is proportional to the product $z_1, z_2$ of the input vibration amplitudes

$$F_{y,z_1,z_2} = k_{c_3} \frac{1}{l_{c_3}} z_1 z_2.$$  

(5a)

For the input vibrations $z_1 = A_1 \cos(\omega_1 t)$ and $z_2 = A_2 \cos(\omega_2 t)$, the corresponding force has a component oscillating with the frequency $\omega_1 + \omega_2$,

$$F_{y,\omega_1+\omega_2} = k_{c_3} \frac{1}{2} A_1 A_2 \cos((\omega_1 + \omega_2)t).$$  

(5b)

By choosing the resonance frequency $\omega_3$ of the output oscillator such that it coincides with the sum frequency $\omega_1 + \omega_2$, the amplitude $y_3$ of the output oscillator becomes proportional to the

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**Figure 2.** Schematic of the mechanical mixer.
product $A_1 A_2$ of the input oscillators. Thus, the output oscillator only vibrates at its resonance frequency if there is a nonvanishing vibration amplitude at both input oscillators simultaneously. Therefore, one can identify the mixer function with a logical AND function whereby the logical states are encoded as mechanical vibrations.

4. Numerical simulation of the mechanical mixer

4.1. Mathematical model and numerical procedures

Based on the model in figure 2, the response of the mechanical mixer was numerically simulated for a variety of different coupling parameters $k_{c_1}$, $k_{c_2}$, $k_{c_3}$, and $l_{c_1}$, and input amplitudes $z_1$ and $z_2$. The differential equations describing the motion of the two input oscillators (Osc1, Osc2) and the output oscillator (Osc3) are

\[ \ddot{z}_1 + \frac{\omega_1^2}{Q_1} \dot{z}_1 + \omega_1^2 z_1 + \frac{\omega_2^2}{k_1} \frac{\partial U_c}{\partial z_1} = \frac{\omega_1^2}{k_1} F_1 \cos(\omega_{1s} t), \]  

\[ \ddot{z}_2 + \frac{\omega_2^2}{Q_2} \dot{z}_2 + \omega_2^2 z_2 + \frac{\omega_3^2}{k_2} \frac{\partial U_c}{\partial z_2} = \frac{\omega_2^2}{k_2} F_2 \cos(\omega_{2s} t), \]  

\[ \ddot{y}_3 + \frac{\omega_3^2}{Q_3} \dot{y}_3 + \omega_3^2 y_3 + \frac{\omega_3^2}{k_3} \frac{\partial U_c}{\partial y_3} = 0, \]  

where $\omega_n$ is the (angular) resonance frequency of the corresponding oscillator, $Q_n$ is the quality factor of the resonance, and $k_n$ is the spring constant describing the coupling to the external driving force with amplitude $F_n$. The frequency of the driving force matches the resonance frequency of the coupled oscillator, $\omega_{1s}$, is shifted with respect to the resonance frequency $\omega_1$ of the free oscillator according to the relation $\omega_{1s} = \omega_1(1 + k_{c_1}^{-1} (\partial^2 U_c)/(\partial z_1^2))$. A corresponding expression relates $\omega_{2s}$ to $\omega_2$ and $\omega_{3s}$ to $\omega_3$. To excite the output oscillator at its resonance frequency $\omega_{3s}$, the frequency $\omega_3$ of the free oscillator is chosen such that the relation $\omega_{3s} = \omega_{1s} + \omega_{2s}$ is met for the actual coupling potential.

Starting from the initial positions $z_1 = 0$, $z_2 = 0$ and $y_3 = 0$ at $t = 0$, the differential equations (equations (6a)–(6c)) are iteratively integrated for a time step $dt$ by means of the Runge–Kutta algorithm. Prior to each integration step, the position of the coupling point $z_s$ is calculated from the zero-force condition $\partial U_c/\partial z_s = 0$ for the actual positions $z_1$, $z_2$ and $y_3$. During the iteration process, the mean absorbed power $P_{a_1}$ and $P_{a_2}$ of the input oscillators are calculated from the instantaneous energy flows $I_{E_1}(t) = F_1 \cos(\omega_{1s} t) \dot{z}_1(t)$ and $I_{E_2}(t) = F_2 \cos(\omega_{2s} t) \dot{z}_2(t)$ by filtering $I_{E_1}(t)$ and $I_{E_2}(t)$ with a second-order Butterworth low-pass filter having a cutoff frequency $\omega_{c,p} = \omega_{1s}/20$. Similarly, the mean dissipated powers $P_{d_1}$, $P_{d_2}$ and $P_{d_3}$ are calculated using equivalent low-pass filters with the same cutoff frequency. Because the output oscillator (Osc3) is driven only by the nonlinear coupling to the motion $z_s$ of the coupling point, the absorbed power $P_a$ in Osc3 is equal to the dissipated power $P_d$. Once the vibration amplitudes have reached a steady-state value, thus, $P_d$ reflects the amount of energy transferred from the input oscillators to the output oscillator.

The numerical simulations were performed assuming parameters that are typical of a device with micromechanical dimensions. The resonance frequencies were in the 1 kHz, and spring constants in the 10 N m$^{-1}$ range. Specifically, the resonance frequency of the input oscillators were set to $\omega_1 = 1000$ Hz and $\omega_2 = 1400$ Hz (note that we use the symbol $\omega$ synonymously to
denote angular frequency and the frequency in periods per second, where the latter meaning is implied if Hz units are specified), the spring constants were $k_1 = 10 \text{ N m}^{-1}$ and $k_2 = 16 \text{ N m}^{-1}$, and the $Q$-factors were $Q_1 = Q_2 = 30$. The spring constants of the coupling springs were $k_{c1} = k_{c2} = 1 \text{ N m}^{-1}$, inducing a relative resonance frequency shift of $+0.049$ for input oscillator 1 and $+0.029$ for input oscillator 2 yielding $\omega_{1s} = 1049 \text{ Hz}$ and $\omega_{2s} = 1441 \text{ Hz}$. The resonance frequency of the output oscillator was set to $\omega_{3s} = \omega_{1s} + \omega_{2s} = 2490 \text{ Hz}$ by adjusting $\omega_3$ for the actual parameter of the coupling spring constant $k_{c3}$. The spring constant $k_3$ of the output oscillator is set to $10 \text{ N m}^{-1}$. The $Q$-factor of the output oscillator was set to the same value as $Q_1$ and $Q_2$, viz. $Q_3 = 30$. In a realistic setup of a mechanical mixer, the coupling to the output oscillator is implemented via a mechanical reduction, see figure 10 below. Whence, the output oscillator motion seen by the coupling spring is reduced by a factor $r_{c3}$, and correspondingly $y_3$ in the coupling potential $U_c$ is replaced by the expression $r_{c3} y_3$. For the simulations described here, the reduction factor was set to $r_{c3} = 0.2$.

**4.2. Coupling efficiency for various coupling parameters**

The coupling efficiency from the inputs to the output was explored by varying the parameters $k_{c3}$ and $l_{c3}$ from $1 \text{ N m}^{-1}$ to $1000 \text{ N m}^{-1}$ and $5 \mu\text{m}$ to $300 \mu\text{m}$, respectively. The input force amplitudes $F_1$ and $F_2$ were chosen such that the resulting vibration amplitudes $z_1$ and $z_2$ were between $1 \text{ nm}$ and $3 \mu\text{m}$. The performance of the mechanical mixer was analyzed in terms of the power transfer efficiency $\eta = P_{out} / (P_{in} + P_{out})$ and the amplitude gains $g_1 = y_3 \text{rms}/z_{1 \text{rms}}$ and $g_2 = y_3 \text{rms}/z_{2 \text{rms}}$. True rms values were calculated because of the significant nonharmonic contributions to the vibration amplitude $y_3(t)$. Both the power values and the amplitude values were calculated after an integration time $\tau_i$ of about $60 \text{ ms}$. This time period corresponds to about six times the slowest response-time constant in the system, i.e. the response time of input oscillator 1, which is $\tau_1 = 2Q_1/\omega_1 = 9.1 \text{ ms}$. The integration step width, $d\tau$, was determined by dividing the oscillation period of the fastest oscillator in the system, i.e. the output oscillator Osc3, into 50 equal steps. The resulting time-step width was $d\tau \simeq 7 \mu\text{s}$.

Figure 3 shows the power transfer efficiency $\eta$ and the output vibration amplitude $y_3 \text{ rms}$ as a function of the length scale $l_{c3}$ for various stiffness values $k_{c3}$ of the coupling spring. The input amplitudes were $z_1 = z_2 = 0.7 \mu\text{m}_\text{rms}$. The power transfer efficiency $\eta$ has a maximum of $\pm 0.1$ for $k_{c3} = 1000 \text{ N m}^{-1}$ at $l_{c3} \simeq 50 \mu\text{m}$. For these parameters, up to 10% of the input power can be transferred to the output oscillator. As can be seen from figure 3(b), the vibration amplitude $y_3 \text{rms}$ of the output oscillator has a maximum. Position and height of the maximum depend on the coupling constant $k_{c3}$. For small values of $k_{c3}$, the position of the maximum appears to be below $l_{c3} = 5 \mu\text{m}$ and the height reaches values slightly larger than $y_3 \text{rms} = 0.15 \mu\text{m}$; for $k_{c3} > 100 \text{ N m}^{-1}$, the position of the maximum moves to values of $l_{c3} > 10 \mu\text{m}$, whereas the height decreases to less than $y_3 \text{rms} = 0.1 \mu\text{m}$.

If the length $l_{c3}$ is reduced to less than $5 \mu\text{m}$ or the stiffness $k_{c3}$ of the spring increased to values above $1000 \text{ N m}^{-1}$, the amplitude $y_3$ of the output will no longer attain a steady-state value but become chaotic. Thus, its detection after a fixed integration time is meaningless for the characterization of the coupling efficiency $\eta$ and of the amplitude gains $g_1$ and $g_2$. These limits for $l_{c3}$ and $k_{c3}$ for achieving a steady-state output amplitude are only valid for the input amplitudes $z_1 = z_2 = 0.7 \mu\text{m}_\text{rms}$; for larger (smaller) amplitudes, the lower limit for $l_{c3}$ becomes larger (smaller) and the upper limit for $k_{c3}$ becomes smaller (larger).
4.3. Coupling efficiency for variable input amplitudes

Owing to the nonlinearity of the coupling, the output amplitude does not a priori scale with the input amplitudes. For this reason, the dependence of the output amplitude $y_3$ on the input amplitudes $z_1$ and $z_2$ was investigated for fixed parameters $l_{c_3}$ and $k_{c_3}$ of the nonlinear coupling. The values of these parameters were chosen such that the power transfer was maximum for the input amplitudes $z_1 = z_2 = 0.7 \, \mu m_{\text{rms}}$, i.e. $l_{c_3} + 20 \, \mu m$ and $k_{c_3} = 300 \, \text{N m}^{-1}$. Figure 4(a) shows the dependence of the power transfer efficiency $\eta = P_{d_3}/(P_{a_1} + P_{a_2})$ on $z_1$ for various values of $z_2$. As expected, the maximum output power is obtained if a large signal amplitude is applied to both inputs. If at least one of the inputs has a low amplitude, the output power is reduced significantly. Figure 4(b) shows the output/input amplitude gain $g_1 = y_{3 \, \text{rms}}/z_{1 \, \text{rms}}$ with the other input amplitude $z_2$ at a constant level. For small $z_2$, the gain $g_1$ stays constant for small $z_1$ and becomes larger as $z_1$ is increased to more than 0.5 \, \mu m. For large $z_2$, however, the gain $g_1$ decreases as the amplitude $z_1$ is increased, rendering the output signal insensitive to the input signal amplitude, which is an important stabilizing feature from a system point of view.

For the range of input vibration amplitudes investigated here, the output amplitude reaches a stable value at the end of the integration time (~60 ms) for any combination of the two amplitudes $z_1$ and $z_2$. For larger amplitudes, however, the time evolution of the output amplitude becomes chaotic.
4.4. Bias stress on coupling spring

In the model used so far, the spring of the nonlinear coupling was assumed to have zero stress when the input amplitudes \( z_1 \) and \( z_2 \) were zero. This assumption is not a necessary restriction for the nonlinear coupling. However, the coupling potential changes fundamentally when a bias stress is applied to the coupling spring which is equivalent to replacing \( y_3 \) with the expression \( y_3 + y_b \) in equation (4). Figure 5 shows the potential as a function of the position of the sum point \( z_s \) for either compressive \((y_b < 0)\) or expansive bias stress \((y_b > 0)\). For expansive stress, the curvature of the potential at \( z_s = 0 \) increases, resulting in a positive shift of the resonance frequency. For compressive stress, however, the potential splits into two equivalent minima similar to an Euler instability of a loaded rod. For small amplitudes of \( z_s \), the motion is confined to either one of the two minima where the system initially was at \( t = 0 \). For larger amplitudes, however, the energy in the system is sufficient to exceed the barrier at \( z_s = 0 \), initiating a sudden increase of the vibration amplitude \( z_s \) as the input amplitude is increased. As a consequence, the excitation of the output oscillator rises accordingly and results in a significant increase of the amplitude \( y_3 \).

This highly nonlinear behavior is reflected in the transfer efficiency \( \eta \) and the amplitude gains \( g_1 \) and \( g_2 \) of the mixer. Figure 6 shows the power transfer efficiency \( \eta \) in (a) and the amplitude gain \( g_1 \) in (b) for four different compressive stress bias values \( y_b \). For no or low compressive stress \((y_b = 0, -2 \, \mu m)\), the power transfer in (a) continuously increases and the amplitude gain in (b) becomes slightly smaller as the input amplitude \( z_1 \) is increased. Above a critical amplitude \( z_1 \approx 3 \, \mu m \), the coupling efficiency \( \eta \) drops slightly and increases again.

**Figure 4.** (a) Power transfer efficiency and (b) output vibration amplitude for varying input amplitudes \( z_1 \) and \( z_2 \).
for larger values of $z_1$. For $\gamma_b = -4 \, \mu m$, however, the power transfer to the output oscillator is almost independent of $z_1$; the output is driven by the second input oscillator only. For this specific bias stress, the input amplitude $z_2 = 1 \, \mu m$ is large enough to raise the energy of the system above the potential barrier at $z_s = 0$, see figure 5, and enables a large oscillation of $z_s$ between the two branches of the potential curve. This behavior is also reflected in the amplitude gain curve in figure 6(b). The slope of $-1$ for $\gamma_b = -4 \, \mu m$ reflects the independence of the output amplitude $y_3_{rms}$ of the input for small values of $z_1$. For a slightly larger compressive stress of $\gamma_b = -6 \, \mu m$, the output amplitude $y_3$ abruptly increases for a narrow range of input amplitudes $z_1$ above $1 \, \mu m$.

Compressive stress on the coupling spring could be used to obtain significantly larger output vibration values for the same input levels. Furthermore, because of the pronounced nonlinearity for a specific range of input amplitudes, compressive stress could be used to set a threshold for the input amplitudes to generate a detectable output signal. However, in both cases, the bias stress has to be accurately tuned to obtain the specific properties. In a micromechanical device, this might be a difficult if not even impossible requirement to fulfill.

Figure 5. Total potential $U$ as a function of the ‘sum point’ coordinate $z_s$ for either compressive ($\gamma_b < 0$) or expansive bias stress ($\gamma_b > 0$) applied to the coupling spring.
5. Mechanical power amplification

The mixer can be viewed as an amplitude modulator. One of the inputs acts as the high-frequency carrier input, the other is the low-frequency modulator input. The nonlinear coupling generates an oscillatory force at both the sum frequency (upper sideband) and the difference frequency (lower sideband) of the input oscillators. In the case of negligible loss in the nonlinear coupling, the power transfer to the output is governed by the Manley–Rowe equations [8]. Neglecting higher-order sidebands, these equations read as follows. For the upper sideband $\omega_c + \omega_1$

\[
\frac{P_c}{\omega_c} + \frac{P_+}{\omega_c + \omega_1} = 0, \quad (7a)
\]

\[
\frac{P_1}{\omega_1} + \frac{P_+}{\omega_c + \omega_1} = 0 \quad (7b)
\]

and for the lower sideband $\omega_c - \omega_1$

\[
\frac{P_c}{\omega_c} + \frac{P_-}{\omega_c - \omega_1} = 0, \quad (8a)
\]

\[
\frac{P_1}{\omega_1} - \frac{P_-}{\omega_c - \omega_1} = 0. \quad (8b)
\]
$P_c$ and $P_l$ denote the power absorbed at the carrier input and at the modulator input, respectively, and $\omega_c$ and $\omega_l$ are the corresponding frequencies. The power transferred to the upper and lower sideband is $-P_c$ and $-P_l$, respectively (note that absorbed powers are positive). Rewriting equation (7b), the power gain $G_+ = -P_+/P_l$ in the upper sideband is given by $G_+ = (\omega_c + \omega_l)/\omega_l$. For a large carrier frequency $\omega_c \gg \omega_l$, the gain is approximately given by $G_+ = \omega_c/\omega_l$, and the output power rises linearly with the carrier frequency $\omega_c$. In physical terms, the equations describe an anti-Stokes scattering process in which one phonon of energy $\hbar \omega_l$ of the modulator input and one phonon of energy $\hbar \omega_c$ of the carrier input are absorbed and re-emitted as one phonon with energy $\hbar \omega_c + \hbar \omega_l$ at the sum frequency. Correspondingly, the power gain is a simple consequence of the frequency up-conversion.

For the lower sideband, however, the power gain is negative, i.e. $G_- = -P_-/P_l = -(\omega_c - \omega_l)/\omega_l < 0$. Because the output emits power, $P_-$ is negative by convention, and because $G_- < 0$, also the input power $P_l$ must be negative. Accordingly, there is a power flow from the carrier input via the output oscillator to the modulator input. Thus, the modulator input oscillator can be excited without any signal being applied to the modulator input. Therefore, extremely high gain figures can be realized. However, because of the regenerative nature the lower sideband amplifier is a potentially unstable device. The mode of operation corresponds to a Stokes scattering process in which a carrier frequency phonon splits up in two phonons with energy $\hbar \omega_l$ and $\hbar \omega_c - \hbar \omega_l$, which are emitted at the respective terminals. In principle, both processes, Stokes and anti-Stokes, occur simultaneously. However, the corresponding scattering probabilities depend on the coupling strengths of the scattered phonons $\hbar \omega_c \pm \hbar \omega_l$ to the environment. The coupling is strong if the mixer output is terminated with a high $Q$ resonator tuned to the appropriate frequency (termed idler frequency in the parametric amplifier literature). Hence, by tuning the idler to the sum or the difference frequency, the modulator resonator can be cooled or excited in exact analogy to laser trapping of atoms.

Because of the inherent stability problem with down conversion we simulated only the upper sideband amplifier. The resonance frequency of one of the inputs was varied to investigate the power gain for various carrier frequencies $\omega_c$. Figure 7(a) shows the power gain $G_+ = P_{d+}/P_{a+}$ as a function of the modulator input amplitude $z_1$ where the carrier amplitude has a fixed value of $z_2 = 2.2 \mu m$ and the carrier frequency ranges from $\omega_c = 4940 \text{ Hz}$ to $46700 \text{ Hz}$. For low input amplitudes $z_1$, the power gain varies as $z_1^2$, indicating a constant output amplitude, whereas the input power rises as $z_1^4$. For input amplitudes in the range $z_1 = 0.1$ to $2 \mu m$, the power gain $G_+$ is independent of $z_1$, as demanded for a linear amplifier. Above $z_1 \simeq 2 \mu m$, the carrier amplitude $z_c$ is smaller than the modulator amplitude $z_1$, and the output amplitude is dominated by the absorbed power at the modulator input. As a consequence, $G_+$ drops as $z_1$ is increased further.

The amplitude gain $g_+ = y_3 \text{rms}/z_1 \text{rms}$ of the upper sideband modulator is shown in figure 7(b). The linear drop of $g_+$ for small input amplitudes indicates that the output amplitude $y_3$ is independent of the modulator input amplitude $z_1$. The amplitude gain is constant in the range $z_1 = 0.1$ to $2 \mu m$ and drops at $z_1 > 2 \mu m$, reflecting the corresponding behavior of the power gain $G_+$ at these input amplitudes $z_1$. Unlike the power gain $G_+$, however, the amplitude gain $g_+$ is almost independent of the carrier frequency $\omega_c$. The power gain $G_+$ increases with larger $\omega_c$, as shown in figure 8 for various carrier amplitudes $z_2$. The dependence of $G_+$ on $\omega_c$ is almost linear, as predicted by the Manley–Rowe equations, (7a) and (7b). For these
Figure 7. (a) Power gain $G_+$ and (b) amplitude gain $g_+$ of the upper sideband modulator for a variable modulator input amplitude $z_1$ and for various carrier frequencies $\omega_c$ and constant carrier input amplitude $z_c = 1 \mu m$. Constant power gain $G_+$ is achieved for input amplitudes in the range $z_1 = 0.1$ to $2 \mu m$.

Figure 8. Power gain $G_+$ of the upper sideband modulator for variable carrier frequency $\omega_c$ and various carrier amplitudes $z_c$. The modulator frequency is $\omega_1 = 1000$ Hz and the input amplitude is $z_1 = 1 \mu m$. $G_+$ increases linearly with $\omega_c$.

equations, the entire power output was assumed to be confined to the upper sideband. In reality, however, the nonlinear coupling transfers power into other frequency components such as the multiple harmonics of the input frequencies ($2\omega_1$, $2\omega_c$, $3\omega_1$, $3\omega_c$, ...). As a result, the magnitude of the power in the upper sideband $\omega_c + \omega_1$
is smaller than what is expected from the (simplified) Manley–Rowe equations ((7a), (7b) and (8a), (8b)).

6. Nonlinear coupling with internal resonances

In the numerical simulations above, all couplings between the oscillators were assumed to have zero mass and, consequently, no internal resonances. In a real system, however, the coupling springs have internal resonances. Yet for a well-designed system, these resonance frequencies are significantly higher than the frequencies of the oscillators. In this case, the model with no or infinitely high resonance frequencies is a good approximation.

In a more realistic model, a finite mass $m_s$ is added to the coupling springs. The mathematical model describing the dynamics of the system (equations (6a)–(6c)) is extended by a differential equation for the motion of the mass $m_s$.

$$\ddot{z}_s + \frac{\omega_s}{Q_s} \dot{z}_s + \frac{\omega_s^2}{k_{c1} + k_{c2}} z_s + \frac{\partial U_c}{\partial z_s} = 0.$$ (9)

Instead of using the not well-defined mass $m_s$ of the coupling to characterize the internal dynamics of the coupling, the resonance frequency $\omega_s$ of the coupling is used in equation (9). This equation replaces the condition $\partial U_c/\partial z_s = 0$ used in the preceding sections to determine the position $z_s$.

For the numerical simulation with variable internal resonance frequency $\omega_s$, similar parameters as in section 4 were used for the coupling springs, namely, $k_{c1} = k_{c2} = 1$ N m$^{-1}$, $k_{c3} = 500$ N m$^{-1}$, $r_{c1} = 0.2$ and $l_{c3} = 30$ µm. The resonance frequencies of the uncoupled input oscillators were $\omega_1 = 1000$ Hz and $\omega_2 = 1650$ Hz, respectively. The coupling shifted the resonance frequencies by +49 and +38 Hz, yielding $\omega_{1s} = 1049$ Hz and $\omega_{2s} = 1688$ Hz. The output oscillator frequency then was $\omega_{3s} = \omega_{1s} + \omega_{2s} = 2737$ Hz. The $Q$-factors had equal values, $Q_1 = Q_2 = Q_3 = 50$. For the internal resonance of the coupling system, negligible loss was assumed by using a large $Q$-factor $Q_s = 10^4$.

The internal resonance frequency of the coupling, $\omega_{3s}$, was varied from 200 Hz to 8200 Hz. Figure 9(a) shows the output vibration amplitude as a function of $\omega_s$ for the input amplitudes $z_1 = z_2 = 1$ µm. The output amplitude $y_3$ varies by more than a factor of ten for different resonance frequencies of the coupling. Characteristic amplitude peaks and dips occur at multiples of the resonance frequencies $\omega_1$ and $\omega_2$ of the inputs and the outputs (for the sake of simplicity, the index ‘s’ denoting the shifted resonance frequencies is omitted in this section). At reduced input amplitudes ($z_1 = 1$ µm and $z_2 = 0.1$ µm), most of the peaks and dips disappear, and for a high internal resonance frequency, the amplitude levels off at a well-defined plateau (figure 9(b)). For larger input amplitudes however ($z_1 = z_2 = 5$ µm), the motion of the output oscillator becomes chaotic and the output amplitude spectrum is very noisy (figure 9(c)).

The tendency towards chaotic motion pointed out above imposes stringent design and operational constraints. In order to achieve a high coupling efficiency of the output, the stiffness of the spring has to be of the order of magnitude of the spring constants of the oscillators. The high stiffness is achieved by using a bulkier geometry of the springs. As a consequence of the larger mass of the spring, the internal resonance frequencies of the coupling drop, making the output amplitude critically depend on the actual internal resonance frequency at vibration amplitudes that are still below the critical values for chaotic behavior. At larger input amplitudes, the output amplitude no longer reaches a stable value.
Figure 9. Output amplitudes of the mechanical mixer with variable internal resonance frequency $\omega_s$ of the nonlinear coupling. (a) Input amplitudes $z_1 = z_2 = 1 \mu m$; (b) $z_1 = 1 \mu m$, $z_2 = 0.1 \mu m$, and (c) $z_1 = z_2 = 5 \mu m$. Multiples of the input frequencies are marked in the plots (for the sake of simplicity, the index ‘s’ has been omitted).

7. Macromechanical model of the vibrational and function

The feasibility of the nonlinear mechanical coupling scheme was tested on a macromechanical model. The experimental setup is shown in figure 10. Aluminium cantilevers with a thickness $t = 0.5 \text{mm}$ and a width $w = 8 \text{mm}$ act as input and oscillators. The lengths of the two input cantilevers were $l_1 = 45 \text{mm}$ and $l_2 = 33 \text{mm}$, respectively. The output cantilever at the higher frequency had a length of $l_3 = 25 \text{mm}$. The coupling between the inputs was V-shaped and cut from stainless steel foil having a thickness of 0.2 mm. The lengths of the two legs were $l_{c1} = 20 \text{mm}$ and $l_{c2} = 15 \text{mm}$, respectively, and the width was $2.5 \text{mm}$. The nonlinear coupling spring was made from the same foil and was S-shaped, making the spring stiff in the direction of the input vibration and soft in the direction of the output vibration. The length of the unloaded spring was $l_{c3} = 20 \text{mm}$.
Figure 10. Experimental setup of the mechanical mixer. Note that the z-axis is orthogonal to the paper plane.

The spring constants of the cantilevers were estimated from the dimensions using the formula $k = (1/4)Ew(t/l)^3$, where $E$ is Young’s modulus. For the levers we obtained $k_1 = 200$, $k_2 = 500$ and $k_3 = 1100$ N m$^{-1}$, and for the couplings $k_{c1} = 120$, $k_{c2} = 300$ and $k_{c3} = 2300$ N m$^{-1}$. The resonance frequencies of the input cantilevers were $\omega_1 = 202$ and $\omega_2 = 259$ Hz, with the $Q$-factors $Q_1 = 230$ and $Q_2 = 200$. The frequency of the output oscillator was adjusted to match the condition $\omega_3 = \omega_1 + \omega_2$, i.e. $\omega_3 = 461$ Hz. The corresponding $Q$-factor was $Q_3 = 120$.

The cantilevers were excited by the magnetic force between a small permanent magnet glued onto the cantilevers and a small coil located close to the magnet. The vibration signal of the cantilevers was detected by miniature microphones located about 2 mm away from each cantilever. The output ac voltage from the microphones was calibrated by using a large excitation of the cantilevers and measuring the resulting vibration amplitude with an optical microscope.
Figure 11. Output vibration amplitude of the macroscopic model of the mechanical mixer as one of the input amplitudes is varied and the other kept constant.

Figure 11 shows the measured output amplitude as a function of the amplitude at input 1 which is varied from about 20 µm to 400 µm and for a set of fixed vibration amplitudes at input 2. The output amplitude increases almost linearly (slope = 1) with the input amplitudes as expected for the up-conversion mode of operation implemented in the test setup (see figure 7). Furthermore, the mixer operates below the chaotic limit even for the highest vibration amplitudes of the two input resonators. For large input amplitudes on both inputs, the resulting output amplitude reaches levels of several 100 µm which demonstrates that a significant fraction of the input power is converted to the output resonator. Thus, it is demonstrated that the theoretical concept can be made to work in practice.

8. Micromechanical implementation

In a next step we discuss the implementation of the mechanical mixer on a micrometer scale. The concept implies fabrication of perpendicularly oriented cantilevers to generate the 2 second-order coupling. Additionally, the geometry of the coupling spring needs to be in part out of plane. These requirements are difficult to reconcile using standard silicon surface micro-machining. As an alternative, two photon polymerization (TPP) lithography was chosen for the fabrication. TPP is a technique to fabricate truly three-dimensional structures in a photoresist [9, 10]. Ultrashort infrared laser pulses (70 fs; 780 nm) are focused by an oil immersion microscope objective with a numerical aperture $N.A = 1.4$ into SU8, a UV-sensitive photoresist. Cross-linking of the resist takes place only if two photons are absorbed simultaneously and therefore only in the focus of the beam where the intensity is high. The sample is mounted on a piezo-scanner with a travel range of 100 µm in all directions. Structures are written line by line, by scanning the resist relative to the focus. The smallest structures obtained using this process had an extension of about 300 nm along the optical axis and about 130 nm perpendicular to this axis [11].

Since fabricated structures consists of cross-linked polymeric material they are electrically non-conducting. In order to be able to excite mechanical resonators using electrostatic actuation titanium/gold (3 nm/100 nm) electrodes were deposited by means of directed evaporation from the top. The electrode areas were defined within the geometry of the structure by e.g. writing T-shaped walls for separating the electrodes along a line, as shown in an example in figure 12.
A scaled down MEMS implementation of the mechanical mixer is shown in figure 13(a). Two input oscillators (Osc1 and Osc2) are coupled nonlinearly to an output oscillator (output). The lengths of the input oscillators are 65 and 55 µm at a width of 1.7 µm and a height of 7 µm. The output oscillator is a double clamped beam with a length of 89 µm, a width of 1 µm and a height of 7 µm. The mechanical coupling is similar to the coupling shown in the macroscopic device in figure 10. A meander-like spring between the two inputs provides the summation of the input signals. The bow-shaped structure to the output oscillator provides the nonlinear second-order coupling to the output. Capacitive actuation is provided by the structures written in parallel to the input oscillators. Their surface is connected to the ground electrode in the center, whereas Osc1 is connected to electrode 1 and Osc2 to electrode 2. The bridge above the input resonators acts as a shadow mask to prevent electrical shorts between inputs and output.

Figure 13(b) shows the optically measured amplitude response to an electrostatic actuation of the input resonators as a function of excitation frequency. Curves are shown for the input oscillators with and without the coupling structure. All measured data can be fitted by Lorentzians, shown as red lines in figure 13. As expected, the addition of the coupling structure stiffens the effective springs of the input oscillators, and their resonance frequency is shifted to higher values. The shift is 3% for the smaller and 4% for the larger oscillator. However, no disturbance of the resonator responses due to the higher order coupling modes can be detected which is important for the proper functioning of the mixer (see section 6). Q values for all resonators are close to 10. Given the dimensions $l$ and $t$ and the resonance frequencies $f_R$ of the oscillators, we can calculate Young’s modulus $E$ of the SU8/metal combination:

$$E = \left( \frac{f_R l^2}{0.162 t} \right)^2 \rho,$$

(10)

where $\rho$ is the density approximated by the density of SU8 as 1200 kg m$^{-3}$. The resulting modulus of 3.5 ± 0.5 GPa is in the range of the literature values for such cured SU8 [12]. The influence of the thin metal layer on the mechanical properties of the levers resonators is negligible.

At this point the full mixing operation of the MEMS device could not be successfully demonstrated. There are a few technical obstacles that need to be further addressed. In the device shown in figure 13 sensing of the output signal is accomplished by measuring the voltage induced by the mechanical oscillation, parallel to the ground plane, in a static magnetic field.
Figure 13. (a) MEMS implementation of a mechanical mixer in SU8. Two input oscillators Osc1 and Osc2 are coupled to an output oscillator ‘output’. (b) Optically measured mechanical responses of the input levers with and without coupling elements. The red lines indicate Lorentzian fits to the data. From [11], reproduced with permission.

oriented perpendicular to the ground plane. So far, sensing of the output resonator motion could not be achieved. It turned out that the fabrication of a conducting path along the narrow resonator beam was challenging and actual devices were jeopardized by short-circuits between sense and ground electrodes and/or a broken electrical path along the thin output oscillator of $1\,\mu m$ thickness. Optical detection was not successful because of inadequate sensitivity. In addition, the use of a double clamped structure for the output resonator turned out to be challenging. The resonance frequency must be tuned to the sum frequency of the input oscillators which requires accurate control of the mechanical dimensions. As can be seen in figure 13 the beam resonator is bowed due to compressive stress along the beam which makes the prediction of the resonance frequency almost impossible. Therefore, a single ended beam appears to be a better solution for the output resonator whereby the motion is sensed electrostatically. This has the additional advantage that the resonance frequency can be fine tuned by exploiting the force gradient imposed by the field acting between the resonator and the reference electrode.

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9. Summary and conclusion

A concept for mechanical signal mixing is proposed which uses only linear springs as fundamental building blocks. Two input oscillators are coupled to a orthogonally vibrating output oscillator via a coupling spring, which results in an effective nonlinear multiplicative mixing function. The equations describing the system have been solved numerically for parameters typical of a mechanical realization with micro-mechanical dimensions. The viability of the concept has been verified in a centimeter scale test embodiment. In particular, it is shown that vibrational energy can be efficiently transferred from the input oscillators to the output oscillator which is tuned to the sum frequency of the input stimuli. A MEMS scale implementation has been built by means of TPP lithography in a polymer matrix. Mechanical mixing can also be seen as the fundamental mechanism for realizing an AND function for logical states which are represented as presence or absence of vibrational oscillator modes. Furthermore, the concepts discussed in this paper can be readily ported to the molecular level which would open entirely new prospects in the THz range for which more traditional electronic devices are extremely difficult to conceive.

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