Design of rotor shape to reduce torque ripple in IPM motors

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Abstract. This paper presents a shape design sensitivity analysis of the effect of geometrical parameters on torque pulsations of an interior permanent magnet (IPM) synchronous motor. The analysis was conducted in two dimensions using the finite element method (FEM). Calculation results have been compared with experimental results.

1. Introduction
An IPM synchronous motor is a highly efficient motor and can operate in a wide speed range. It is widely used in many industrial applications and electric household appliances. However, the presence of torque pulsations (i.e. cogging torque and torque ripple) in IPM motors is a major problem in some applications where speed and position control accuracy is of great importance. Several techniques for reducing the torque pulsations in IPM motors have been proposed in the literature [1-7]. Most of the techniques used for reduction of pulsating torque based either on an adequate motor design [1-4, 6-7] or on control techniques [5, 6]. Although control-based techniques for reduction of pulsating torque can be quite effective, this paper concentrates on the design-based approaches for sinusoidal drives.

Based on the design-based approaches, a decrease of the torque pulsations can be expected by drilling three small circular holes [3] and varying the geometry shape of the saliencies in the magnetic island of the rotor of the IPM motor. Therefore, this paper presents a shape sensitivity analysis of torque pulsations with respect to the holes and rotor saliency of an IPM motor using the FEM [8-11]. Experimental results to support the simulation findings are included in this paper.

2. Model of IPM motor
The initial design of the motor is a three-phase, 6.9-kW, 3900-rpm, four-pole, twelve-slot, Y-connected IPM motor as shown in Figure 1. It uses four V-shaped sintered NdFeB magnets with a remanence $B_r = 1.1$ T and coercivity $H_c = 840$ kA/m. The stator and rotor core use the nonoriented silicon steel with saturated flux density $B_s = 1.8$ T.

3. Geometry optimization using gradient method
The geometry optimization problem arises in finding the maximum performance, e.g. minimum torque pulsations of an IPM motor subject to constraints on magnetic (available magnetic saturation level) requirement and manufacturing and geometry limits. Such a constrained problem involving finite
elements may be expressed as

\[
\begin{align*}
\text{minimize } & \quad f(x) \\
\text{subject to } & \quad g_i(x) \leq 0 \quad i = 1, 2, \ldots, m \\
\text{and } & \quad h_j(x) = 0 \quad j = 1, 2, \ldots, \ell.
\end{align*}
\]

where \( x = (x_1, x_2, \ldots, x_n) \) is a column vector of \( n \) real-valued design variables that control the shape or topology of the motor. The finite element-based optimization problem in (1) is solved using gradient methods. Consider an implicit function \( q \) which can be represented an objective or constraint function in (1), as

\[
q = q(x)
\]

Let \( x^0 \) be the current design. Then the gradient of \( q \) evaluated at \( x^0 \) is

\[
\nabla q = \left[ \frac{dq}{dx_1}, \frac{dq}{dx_2}, \ldots, \frac{dq}{dx_n} \right]^T
\]

It can be computed by the direct method or the adjoint method. In this analysis, since the number of design variables is less than that of constraints, the direct method is adopted because it is more efficient than the adjoint method [11].

4. Geometry optimization of the IPM motor
To reduce the torque pulsations, one of the effective methods is to modify the distribution of flux in the rotor. In this study, three small circular holes have been drilled in the magnetic island and the saliencies of the rotor have been also created. Figure 2 shows the definition of parameters \( R, r, \) and \( \theta \) for the three circular holes and parameters \( d \) and \( r_d \) for the variation of saliencies of a rotor core. Where \( R \) and \( R+r \) are the radius of the holes, \( \theta \) is the angle between the center hole and side holes, and \( d \) is the distance from the center of shaft to the center for the radius \( r_d \) used to cut the saliencies of the rotor. The air gap length and the bridge width are fixed to 1 mm and 1 mm to allow for clearance and tolerances. Figure 3 Summarizes the steps involved in the complete shape design sensitivity analysis of the motor. To have a more realistic design, the constraints of parameters \( R, r, \) \( \theta \) and \( d \) are also shown in Figure 3.

5. Results and discussions
5.1. Cogging torque
5.1.1. Influence of angle \( \theta \)
In this part of analysis, the only variable is \( \theta \). The other parameters \( R \) and \( r \) are held constant (\( R = 0 \) mm and \( r = 2 \) mm). The results are shown in Figure 4. As noted in the figure, as the angle \( \theta \) increases, the peak-to-peak value of cogging torque (\( T_{cog} \)) decreases. It is shown that \( T_{cog} = 3.8 \) N.m with \( \theta = 27.4^\circ \).

5.1.2. Influence of radius \( r \)
In this part of analysis, the only variable is \( r \). The other parameters \( R \) and \( \theta \) are held constant (\( R = 0 \) mm and \( \theta = 27.4^\circ \)). The effects of length \( d \) on cogging torque are illustrated in Figure 5. As the radius \( r \) increases, the peak-to-peak value of cogging torque (\( T_{cog} \)) decreases. It is shown that \( T_{cog} = 2.6 \) N.m with \( r = 2.5 \) mm.
5.1.3. Influence of length $d$

In this part of analysis, the only variable is $d$. The other parameters $R$, $r$ and $\theta$ are held constant ($R = 1$ mm, $r = 2.5$ mm and $\theta = 27.4^0$). The results are shown in Figure 6. As noted in the figure, the minimum value of cogging torque is 0.8 N.m with $d = 5.5$ mm. From the above analysis, the best parameter set for lower level of cogging torque is $R = 1$ mm, $r = 2.5$ mm, $\theta = 27.4^0$ and $d = 5.5$ mm.
5.2. Torque ripple
Torque ripple is defined as a ratio of the difference between the values of maximum and minimum torque to the average torque ($T_{avg}$) multiplied by 100%. In this part of analysis, the only variable is $d$. The other parameters $R$, $r$ and $\theta$ are held constant ($R = 1$ mm, $r = 2.5$ mm and $\theta = 27.4^\circ$). Sinusoidally excited currents are applied to the motor. The sensitivity of $d$ on the torque ripple ($T_{rip}$) is plotted in Figure 7. As the figure shows, increasing the value of $d$, decreasing the torque ripple. It is found that the minimum value of torque ripple is 6.48% with $d = 7.5$ mm. Based on the above analysis, varying the saliencies of the rotor core is more effective in reducing the magnitude of both cogging torque and torque ripple.


![Figure 6. Sensitivity of $d$.](image)

![Figure 7. Sensitivity of $d$.](image)

5.3. Summary of results
The effect of drilling three small circular holes and varying the saliencies of the rotor on the torque pulsations of the IPM motors has been studied. The simulation results for the optimal design motor with $\theta = 27.4^\circ$, $r = 2.5$ mm, $R = 1$ mm, and $d = 5.5$ mm are investigated. Figure 8(a) and (b) compare the cogging torques and average torques. It is seen that the values of predicted and measured cogging torque of the optimized machine are good agreement with each other. Also, it can be seen that the peak to peak values of cogging torque and torque ripple reduce from the initial design of 5.16 N.m and 51.7 % to the optimal design of 0.8 N.m and 6.48 %, respectively, and the average torque increases from the initial of 19.45 N.m to the optimal design of 20.14N.m. In particular, the reduction of torque pulsations does not result in any reduction of the average torque. Furthermore, the core loss reduces from the initial design of 226.303 W to the optimal design of 197.860 W, results in the enhancement of efficiency from 95.05% to 95.5%.

![Figure 8. Comparison of simulation and test results between initial and optimal design of (a) cogging torques and (b) average torques.](image)
To drill three circular holes in the magnetic island and shape the saliency of the rotor may affect the inductances of d-axis \((L_d)\) and q-axis \((L_q)\). Figure 9 compares the values of \(L_d\) and \(L_q\) between initial and optimal design. As noted in the figure, initially, the value of \(L_q\) drops rapidly with increasing current, but the value of \(L_d\) reduces slightly, and then they almost do not change. The holes and saliency of the rotor do affect the flux path of q-axis then d-axis around rated operation condition. The saliency ratio (e.g. \(L_q/L_d\)) reduces from 2.586 of initial design to 2.185 of optimal design at rated current of 68 A. Figure 10 shows the optimized cross section of the IPM motor.

6. Conclusion
In this paper, the influence of drilling three small circular holes in the magnetic island and varying the geometry shape of the saliencies of the rotor on the torque pulsations of three-phase IPM motors has been studied. Experimental results confirm a good correlation between torque pulsations and the proposed finite element sensitivity analysis. It was shown that the technique presented in this paper is effective for minimizing the production of pulsating torque in IPM motors.

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7. References
[1] Kim D H, Park I, Lee J H and Kim C E 2003 *IEEE Trans. Magn.* 3 1456
[2] Zhu Z Q, Ruangsinchaiwanich S, Schofield N and Howe D 2003 *IEEE Trans. Magn.* 5 3238
[3] Kioumarsi A, Moallem M and Fahimi B 2006 *IEEE Trans. Magn.* 11 3706
[4] Kang G H, Son Y D and Kim G T 2007 *Proc. IEEE IAS Annu. Meeting* (New Orleans) 558-565
[5] Lee G H, Kim S I, Hong J P and Bahn J H 2008 *IEEE Trans. Magn.* 6 1582
[6] Parasa L and Hao L 2008 *IEEE Trans. Ind. Electroics* 2 602
[7] Han S H, Jahns T M and Zhu Z Q 2010 *IEEE Trans. Ind. Appl.* 1 187
[8] Biedinger J M and Lemoine D 1997 *IEEE Trans. Magn.* 3 2309
[9] Wang S and Kang J 2000 *IEEE Trans. Magn.* 4 1119
[10] Lee J H, Kim D H and Park I H 2003 *IEEE Trans. Magn.* 3 1269
[11] Belegundu A D and Chandrupatla T R 1999 *Optimization Concepts and Applications in Engineering* (New Jersey: Prentice Hall, Inc.)