Quantum Chaos: An introduction via chains of interacting spins 1/2

Aviva Gubin and Lea F. Santos

Department of Physics,
Yeshiva University
What is quantum chaos?

• Classical chaos
  ▫ Hypersensitivity to initial conditions
  ▫ Dynamical billiard: billiard table with no friction and elastic collisions.
  ▫ Depending on the shape: chaos
  ▫ In phase space, the trajectories of two particles with very close initial conditions will diverge exponentially in time (rate=Lyapunov exponent)
What is quantum chaos?

- **Classical chaos**
  - Hypersensitivity to initial conditions

- **Quantum chaos**
  - Cannot use hypersensitivity due to Heisenberg’s Uncertainty Principle
  - Classical systems are a limit of quantum systems
  - Quantum billiards: distribution of neighboring energy levels depends on the billiard’s classical counterpart.
Level spacing distribution

Histogram of the spacings between neighboring energy levels

| Energy levels | Energy spacings |
|---------------|-----------------|
| $E_1$        | $s_1$           |
| $E_2$        | $s_2$           |
| $E_3$        | $s_3$           |
| $E_4$        | $s_4$           |
Level spacing distribution

• When classical billiard was chaotic, the energy levels of the quantum billiard are highly correlated and repel each other.
• The distribution is given by the Wigner-Dyson distribution, \( P_{WD}(s) = \frac{\pi s}{2} e^{-\pi s^2/4} \).
• In an integrable (non-chaotic) system, the energy levels may cross \( P(s) = e^{-s} \).
• The distribution is Poissonian,
Level spacing distribution

- When classical billiard was chaotic, the energy levels of the quantum billiard are highly correlated and repel each other.
- The distribution is given by the Wigner-Dyson distribution, $P_{WD}(s) = \frac{\pi s}{2} e^{-\pi s^2/4}$.
The system

- We study a 1D system of spins-1/2 with L sites
- Each site contains either a spin up or a spin down
- We use a chain with L/3 spins up (excitations)
The Hamiltonian

• Our system is described by the Hamiltonian

\[ H = H_z + H_{NN} \]

\[ H_z = \sum_{n=1}^{L} \epsilon_n S_n^z \]

**Clean** \( \epsilon_n = \epsilon \)

**Defect** -- site with different Zeeman splitting:

\[ \epsilon_{n\neq m} = \epsilon \]
\[ \epsilon_m = \epsilon + d_m \]

\[ H_{NN} = \sum_{n=1}^{L-1} \left[ J_z S_n^z S_{n+1}^z + J \left( S_n^x S_{n+1}^x + S_n^y S_{n+1}^y \right) \right] \]

\[ H_{zz} = \sum_{n=1}^{L-1} J_z S_n^z S_{n+1}^z \]

\[ H_{XY} = \sum_{n=1}^{L-1} \left[ J \left( S_n^x S_{n+1}^x + S_n^y S_{n+1}^y \right) \right] \]

Ising Interaction

Flip-Flop Term
Level spacing distribution

Histogram of the spacings between neighboring energy levels

Energy levels

\[ E_0 \quad E_1 \quad E_2 \quad E_3 \quad E_4 \]

Energy spacings

\[ S_1 \quad S_2 \quad S_3 \quad S_4 \]
Level spacing distribution

• In chaotic systems, the energy levels are highly correlated and repel each other.
• The distribution is given by the Wigner-Dyson distribution,
\[ P_{WD}(s) = \frac{\pi s}{2} e^{-\pi s^2/4} \]
• In an integrable (non-chaotic) system, the energy levels may cross.
• The distribution is Poissonian, \( P(s) = e^{-s} \)
Level spacing distribution

- When the defect is placed on site 1, we obtain the Poissonian distribution, corresponding to an integrable system.
- When the defect is placed on site L/2, we obtain the Wigner-Dyson distribution, corresponding to a chaotic system.

$$L = 15$$
5 spins up
$$J_z = 0.5 \text{ J}$$
$$\text{Eps}_1, \text{Eps}_L/2 = 0.5 \text{ J}$$
Number of Principal Components

- NPC is a measure of the delocalization of eigenstates
  - It gives the number of basis vectors $\Phi$ which contribute to each eigenstate

$$\psi_j = \sum_{k=1}^{\text{dim}} a_{jk} \phi_k$$

$$\text{NPC}_j = \frac{1}{\sum_{k=1}^{\text{dim}} |a_{jk}|^4}$$

Small NPC – localized state
Large NPC – delocalized state
Number of Principal Components

- Chaotic systems are significantly more delocalized
- Chaotic systems NPCs have smaller fluctuations
Symmetries

- We also study chaos in a system with no defect
- To drive it to chaos, we add next-nearest-neighbor couplings
- Parity
- Spin reversal
- Total spin

\[ H = H_{NN} + \alpha H_{NNN} \]

\[ H_{NN} = \sum_{n=1}^{L-1} \left[ J_z S^z_n S^z_{n+1} + J (S^x_n S^x_{n+1} + S^y_n S^y_{n+1}) \right] \]

\[ H_{NNN} = \sum_{n=1}^{L-2} \left[ J_z S^z_n S^z_{n+2} + J (S^x_n S^x_{n+2} + S^y_n S^y_{n+2}) \right] \]
Symmetries

A: L=14, 7 spins up
B: L=15, 5 spins up
Both: alpha = .5, J=Jz
Acknowledgements

• Henry Kressel Research Scholarship, for funding this project

Thank you!