P-delta effect analysis of continuous distribution system based on generalized differential quadrature method

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Abstract: Based on the generalized differential quadrature method, an analysis method of p-delta effect for continuous distribution system is proposed. Firstly, the structural system with continuous distribution of mass and stiffness is taken as the research object. Based on the principle of virtual work, the axial and transverse vibration equations considering the effect are given. Secondly, the partial differential equations are discretized by using the generalized differential quadrature method, and the dynamic ordinary differential equations are obtained to analyze the p-delta effect of the continuous distribution system. Finally, an example based on 5MW wind turbine parameters is analyzed, and the results show that the GDQM has high calculation accuracy, which can be obtained by dividing only a few nodes; This method can not only consider the second-order effect of gravity, but also consider the time-varying p-delta effect caused by vertical earthquake, and can reveal the influence of vertical earthquake on the lateral dynamic characteristics of structures in real time; No matter the gravity second-order effect or the time-varying p-delta effect, this method does not need iteration and has high computational efficiency.

1. Introduction

In recent years, high flexible structures have developed rapidly, such as Guangdong new TV Tower, wind turbine tower, high-rise chimney. The horizontal response is often affected by the axial force caused by the vertical action which can not be ignored, that is, the p-delta effect. Under the action of two-way strong earthquake, the p-delta effect may pose a serious threat to the seismic safety of the structure, which needs to be considered in the structural design.

At present, the calculation method of the p-delta effect is that the additional bending moment formed by the axial force is equivalent to the transverse force applied on the structure [1]. As far as the calculation method is concerned, it can be divided into iterative method [2] and direct method. The iterative method has high accuracy, but it is time-consuming. Therefore, most of the current methods are direct methods without iteration, such as amplification factor method [3], direct solution method [4] and finite element geometric stiffness method [5]. Generally, the above methods assume the vibration mode of the structure first, so the calculation results may not be accurate enough. In addition, the above methods are also cumbersome to deal with the time-varying effect caused by vertical earthquake.

At present, based on the differential quadrature method, Li Hongjing[6] proposed a high-precision method for effect analysis under time-varying axial force, and used the weight coefficient correction method to deal with the boundary conditions with one end fixed and one end free, but did not explain more complex boundary conditions (such as concentrated mass at the top, considering soil structure interaction, etc.). If the complex boundary conditions mentioned above are considered, the differential
quadrature method is mostly used in [7], that is, the redundant boundary conditions are imposed on the interior points very close to the boundary points. Obviously, this method can not strictly meet the boundary conditions, which may lead to the distortion of the calculation results. Therefore, in this paper, the generalized differential quadrature method is used to discretize the partial differential equations of motion. This method can avoid the above shortcomings and is suitable for any boundary conditions.

2. Partial differential equations of motion for continuous systems considering P-delta effect

2.1. Partial differential equations of motion for continuous systems

Considering a variable cross-section beam, x is the axial direction of the beam before deformation, the beam cross-sectional area is $A(x)$, the elastic modulus is $E$, and the density is $\rho$. According to the principle of virtual work, the partial differential equations of axial and transverse motion are obtained as follows:

\[ \rho A\ddot{u} - EA'u' - EA''u'' = p_x \]  \hfill (1a)
\[ \rho A\ddot{v} - \rho l\ddot{v} - \rho l\ddot{v} + E(Iv'' + 2lv'' + lv'') - Nv'' - Nv' = p_y \]  \hfill (1b)

Where $u$ and $v$ are the displacement components of the centroid of the section, $'$ denotes the derivative to the coordinate, $\ldots$ denotes the second derivative to the time variable, $I(x)$ is the moment of inertia of the section changing along the direction, and $p_x$ and $p_y$ are the distributed loads in two directions respectively.

The axial force and its derivative to coordinates are:

\[ \dot{u} = EAN \] \hfill (2a)
\[ \ddot{u} = EAN + \dot{u} \] \hfill (2b)

The reaction forces at beam end $(x=0,l)$ are:

\[ R_b = \rho l\ddot{v} + Nv' - E(Iv'' + Iv'') \] \hfill (2c)
\[ M_b = Elv'' \] \hfill (2d)

2.2. Boundary condition

All types of boundary conditions can be written in the following unified form $(x=0,l)$:

\[ \alpha_1 u(x,t) + \alpha_2 N(x,t) = \alpha_3 \] \hfill (2a)
\[ \beta_1 v(x,t) + \beta_2 R_b(x,t) = \beta_3 \] \hfill (2b)
\[ \gamma_1 \ddot{v}(x,t) + \gamma_2 M_b(x,t) = \gamma_3 \] \hfill (2c)

By giving appropriate $\alpha_j, \beta_j, \gamma_j (j=1,2,3)$, the above equations can express any kind of boundary conditions. For example, fixed is $\alpha_1 = \beta_1 = \gamma_1 = 1$, $\alpha_2 = \alpha_3 = \beta_2 = \beta_3 = \gamma_2 = \gamma_3 = 0$.

3. Generalized differential quadrature method (GDQM)

3.1. Basic principle of generalized differential quadrature method

We consider a one-dimensional function $f(x)$, which is continuously differentiable in the interval $[a,b]$. In this interval, we take $n$ nodes $a=x_1 < x_2 < \cdots < x_{n-1} < x_n < b$, which are different from each other. There are $m_i$ constraints on the $i$ node, corresponding to unknown variables. There are $M = m_1 + \cdots + m_n$ constraints in the field. Therefore, we can construct $M$ interpolation basis functions. Then the generalized differential quadrature method can be expressed as:

\[ \frac{d^k f(x)}{dx^k} = \sum_{j=1}^{M} A_{ij} U_j \] \hfill (3)

Where $U_j$ is the $j$th independent unknown quantity. When $m=1$, it is differential quadrature. $A_{ij}$ is the derivative value of the $k$-th derivative of the $j$-th interpolation basis function at the $i$-th node, that is, the $k$-th weight coefficient. It can be seen that the essence of the generalized differential quadrature
method is to express the value of the function and its derivative at a given node by the weighted sum of the independent unknowns of all nodes in the whole field, thus changing the differential equation into a group of algebraic equations with the independent unknowns of nodes as unknowns. The quadrature weight coefficients of different order differential equations are different. The partial differential equations in this paper are of second order and fourth order respectively. For the expression of the weight coefficients, see references [9,10].

3.2. discrete mode of nodes
The weight coefficient is closely related to the discrete mode of nodes. When the generalized differential quadrature method is used to discrete the equation, the nodes with reasonable distribution must be adopted. A large number of calculations show that the selection of non-uniform nodes has faster convergence speed and higher accuracy. In this paper, the following formula is selected as the coordinate of discrete nodes [8].

\[
y_i = \frac{1}{2} l \left[ 1 - \cos \left( \frac{(i-1)\pi}{(N-1)} \right) \right] (i = 1, 2, ..., n)
\]  

(4)

4. Discrete differential equations of motion by generalized differential quadrature method
The governing equation (1a) can be discretized as:

\[
\rho A i_i - E A_i \sum_{j=1}^{n} a_{ij}^{[1]} u_j - E A_i \sum_{j=1}^{n} a_{ij}^{[2]} u_j = p_{xi} (i = 2, ..., n-1)
\]

(5a)

\[N_i\] and \[N'_i\] can be solved from (5a) and its boundary conditions. From the weight coefficient discrete control equation (1b) of the fourth order differential equation, it can be obtained that:

\[
E \left( I \sum_{j=1}^{n+2} G_{ij}^{[1]} v_j + 2 I \sum_{j=1}^{n+2} G_{ij}^{[3]} v_j + I \sum_{j=1}^{n+2} G_{ij}^{[2]} v_j \right) - N \left( t \sum_{j=1}^{n+2} G_{ij}^{[2]} v_j - N'_i(t) \sum_{j=1}^{n+2} G_{ij}^{[1]} v_j \right)
\]

\[+ \rho A v_i - \rho l \sum_{j=1}^{n+2} G_{ij}^{[2]} v_j - \rho l \sum_{j=1}^{n+2} G_{ij}^{[1]} v_j = p_{ni} (i = 2, ..., n-1)
\]

(5b)

The different boundary conditions are discretized into algebraic or differential algebraic equations, and the algebraic constraints are eliminate different boundary conditions are discretized into algebraic or differential algebraic equations, which can be written as time domain dynamic equations together with control equations.

\[M v + \left[ K + K_G(t) \right] v = F
\]

Where \(M\) is the mass matrix, \(K\) is the material stiffness matrix, \(K_G\) is the geometric stiffness matrix caused by the p-delta effect, \(v\) is the node displacement vector, and \(F\) is the node force vector. Obviously, when the vertical force is only gravity, the \(K_G\) does not change with time, and the effect is called gravity second-order effect. When vertical action is vertical earthquake, \(K_G\) changes with time, and p-delta effect is called time-varying P-delta effect caused by vertical earthquake.

5. Example analysis
In this paper, the parameters of a 5MW wind turbine with three blades are analyzed. The engine room, rotor, blade and hub of wind turbine are simplified as lumped mass 403220kg, tower elastic modulus and height. The outer radius of the bottom of the tower is 4.215m, the wall thickness of the bottom
tower is 0.048m, the outer radius of the top tower is 1.935m, and the wall thickness of the top tower is 0.025m. It is assumed that the outer radius of the tower and the wall thickness change linearly along the height direction of the tower. The bottom of the tower is completely fixed. In this paper, nine nodes are divided. According to the mass and material stiffness matrix, the natural frequency of wind turbine can be calculated.

Table 1  The first five natural frequencies/Hz of 5MW wind turbine structure

| Second order effect of gravity | NO  | ABAQUS | YES  | difference |
|-------------------------------|-----|--------|------|------------|
| 1st                           | 0.317 | 0.315 | 0.313 | 1.2%       |
| 2st                           | 2.140 | 2.110 | 2.135 | 0.2%       |
| 3st                           | 6.084 | 5.847 | 6.047 | 0.6%       |
| 4st (axial vibration)         | 7.283* | 7.283* | 7.283* | 0%         |
| 5st                           | 12.186 | 11.456 | 12.170 | 0.08%      |

Note: * indicates axial vibration

It can be seen from table 1 that the natural frequencies of wind power generation structures are reduced after considering the second-order effect of gravity. Because the axial force caused by gravity reduces the overall stiffness of the wind turbine structure, and the mass distribution of the structure does not change, the natural frequency decreases slightly. In this table, the natural frequency of wind turbine structure based on ABAQUS finite element software analysis is also given. It can be seen that the results obtained by the two analysis methods are almost the same, while the finite element method is divided into 800 solid elements. Here, the generalized differential quadrature method only uses 9 nodes to calculate the natural frequency, which shows that the calculation accuracy and efficiency of the generalized differential quadrature method are very high.

In order to verify the effectiveness of the method, Kobe seismic wave in 1995 is selected as the excitation, and its acceleration time history curve is shown in Figure 1. The vertical seismic wave is set as 0.65 times of the seismic wave.

![Fig. 1 Kobe seismic wave, Japan](image1)

![Fig. 2 displacement time history of top concentrated mass](image2)

**Note:** * indicates axial vibration.
Fig. 3 variation of first-order frequency with time of wind turbine structure

Fig. 2 is the displacement time history curve of the concentrated mass at the top of the wind turbine structure. It can be seen from the diagram that whether considering only gravity or vertical earthquake, the dynamic response is different from that without considering the p-delta effect at all. Fig. 3 shows the change of wind turbine structure and first-order frequency with time. It can be seen from the figure that the natural frequency of the structure does not change with time when p-delta effect is not considered. When the second-order effect of gravity is considered, the natural frequency will decrease to a certain value without changing with time. When gravity and vertical earthquake are considered at the same time, the natural frequency of the structure will oscillate with time at a certain value caused by the second-order effect of gravity. In other words, the second-order effect of gravity is a time invariant effect. When the dominant frequency of transverse seismic wave is close to or equal to the above determined value, the p-delta effect will be more significant, while the vertical seismic action is a time-varying p-delta effect, which may increase or decrease the amplitude, which is related to structural parameters, vertical and lateral earthquakes.

6. Conclusion
This paper proposes a method for analyzing p-delta effect of continuous distribution system based on generalized differential quadrature method. The main conclusions are as follows:

(1) The analysis method based on generalized differential quadrature method can not only consider the second-order effect of gravity, but also the time-varying p-delta effect caused by vertical earthquake. In addition, the method can reveal the dynamic characteristics of continuous distribution in real time without iteration;

(2) The generalized differential quadrature method can satisfy all kinds of boundary conditions with high accuracy and efficiency. Generally, it only needs more than 9 nodes to obtain the results meeting the accuracy requirements;

(3) The second-order effect of gravity caused by the gravity of the continuous distribution system forms the axial pressure in the system, which leads to the decrease of the lateral bending stiffness of the system. The axial force caused by earthquake makes the lateral natural vibration characteristics of the system change with time, which will affect the lateral dynamic response. Considering the p-delta effect, it may increase the amplitude or decrease the amplitude.

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