Impedance Reshaping and Stability Analysis of the Unbalanced Three-phase System Considering the Grid-tied Inverter based Imbalance Compensation

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Abstract—Grid-tied inverters have been adopted both as the interface of renewable energy resources and as a solution to address power quality issues. To compensate the imbalance distortion caused by asymmetric local load, DQ transformation based compensation method is introduced into the control scheme of the grid-tied inverter. Taking both the inverter control scheme and circuit topology into account, the space vector based impedance modelling and stability analysis approaches are proposed in this paper to analyse the dynamic of the asymmetric system with active imbalance compensation. The proposed impedance models indicate that the compensation control method can reshape the output impedance features, and affect the stability of the system. The stability analysis results are verified in this paper by experimental results.

1. Introduction

Three-phase imbalance is a common power quality problem in the electric power system. Under normal conditions, the imbalance of the power system is predominantly caused by the asymmetry of the load, especially in the distribution grid [1]. To attenuate the imbalance introduced by the load, three-phase grid-tied voltage source inverter (VSI) with additional compensation control scheme is considered as a highly cost-efficient solution, which has already been widely adopted as grid-interface module of distributed generation (DG) [2]. By adopting grid-tied VSI based imbalance compensation, the system with asymmetric load exhibits a three-phase balanced feature under a grid voltage at the positive fundamental frequency. However, recent studies indicate that harmonic oscillation can be caused due to the interaction between the equivalent impedance of the VSI and the grid-impedance, especially in a weak grid condition [3].

To investigate the small-signal stability of the unbalanced three-phase system with grid-tied VSI based imbalance compensation, an approach to build the impedance model for the compensated system is proposed in this paper. Based on the proposed impedance model, a stability analysis method for the unbalanced system is also proposed, taking the coupling between the positive-sequence and negative-sequence components into account. Thus, a small-signal stability analysis of the unbalanced system is carried out, considering the adoption of imbalance compensation. According to the stability analysis results and the experimental results, the compensation method is proven to have highly important influence on the stability of the unbalanced system.
2. System Configuration and Complex Space Vector-Based Modelling

The topology of the three-phase system with the grid-tied VSI and asymmetric local load is shown in Fig. 1. The local load is connected to the grid at the same point of common coupling (PCC) with the VSI. Delta-connected three-phase admittance is adopted to represent the local load, where $Y_{ab}$, $Y_{bc}$, and $Y_{ca}$ are the phase-to-phase admittances. As shown in Fig. 1, the current control scheme in the $\alpha\beta$ stationary frame is adopted in the grid-tied VSI. The current controller is $G_i(s)$. To compensate the unbalanced current of the load, a power quality compensation unit is adopted to generate the compensation current references.

In a three-phase three-wire system, complex space vectors can be used to represent the three-phase quantities [4]. For instance, a symmetric three-phase voltage can be represented as complex space vector $\hat{U} = U e^{j\omega t}$, where $\hat{U} = e^{jUU\theta}$. The magnitude of the three-phase voltage is $U$, $\theta$ is the initial phase, and the angular frequency of the space vector is denoted by $\omega$. To a positive-sequence complex space vector, the angular frequency meets $\omega>0$. To a negative-sequence complex space vector, the angular frequency meets $\omega<0$. The relationship between the complex space vector and the associated phase quantities is presented in Fig. 2.

According to Fig. 2, $U$ can be expressed in different stationary frames as [5]

$$U = u_\alpha + j u_\beta = \frac{2}{3} \left( u_\alpha + hu_b + h^* u_c \right) \quad h = e^{j2\omega t} \quad (1)$$

where $u_\alpha$, $u_b$ and $u_c$ are the scalar projections of $U$ on $abc$ stationary frame. $u_\alpha$ and $u_\beta$ are the scalar projections of $U$ on $\alpha\beta$ stationary frame. The transformation between the $abc$ frame and the $\alpha\beta$ frame can be realized by the commonly adopted Clarke and inverse Clarke transformation. With the complex space vector, the three-phase quantities can be analyzed as one variable and the expressions in the $abc$
frame and the αβ frame are unified. Moreover, the positive-sequence and negative-sequence space vectors in the three-phase system can also be expressed in the same form by introducing a negative angular frequency, which can simplify system modeling.

By adopting complex space vectors, the model of current control scheme of the VSI, unbalanced load, and the compensation current reference calculation scheme of the VSI is illustrated in Fig. 3.

(a) Current control scheme of the grid-tied VSI

(b) Impedance model of the unbalanced load

(c) Compensation current reference calculation scheme

The current control loop of the VSI can be equivalent to the inner-impedance of the VSI. \( K \) is the pulse width modulation (PWM) gain of the inverter bridge. \( D(s) \) is the transfer function of the sampling delay. \( G_i(s) \) is the current controller. Therefore, the impedance can be derived as:

\[
Y_i(\omega) = \frac{1}{U} = \frac{C L_i s^2 + C R s + 1}{C L_i s^2 + C R s + 1 + (L_e + L) s + K G_i(s) D(s)(C R s + 1)}
\]

Under the voltage of PCC \( U \), the unbalanced load current \( I_L \) is composed of the component \( I_{L1} \) at the frequency \( \omega \), and the component \( I_{L2} \) at the frequency \(-\omega\). The \( I_{L1} \) is considered to be produced by the equivalent balanced admittance \( Y_{i1}(\omega) \) and can be expressed as \( I_{L1} = Y_{i1}(\omega)U \). The load current component \( I_{L2} \) at the frequency \(-\omega\) is considered to be introduced by a voltage-controlled-current-source. The coupling admittance \( Y_{n2}(\omega) \) is defined to calculate \( I_{L2} \) under \( U \). Thus, the unbalanced load current component can be expressed as \( I_{L2} = Y_{n2}(\omega)U^* \), where \( U^* \) is the conjugate vector of \( U \). The expression of \( Y_{i1}(\omega) \) and \( Y_{n2}(\omega) \) can be derived as:

\[
Y_{i1}(\omega) = Y_{iab}(\omega) + Y_{iac}(\omega) + Y_{abc}(\omega) \quad Y_{n2}(\omega) = -Y_{nab}(\omega) - Y_{nac}(\omega) - h^2 Y_{abc}(\omega) \quad h = e^{j 2 \pi / 3}
\]

To analyze the small-signal response of the system, the perturbation voltage \( U_e \) is considered in series with the fundamental voltage of the grid \( U_1 \) at the PCC. The load current can be expressed as:

\[
I_L = I_1 + I_{n1} + I_c + I_{nc} = \hat{I}_1 e^{j \omega t} + \hat{I}_{n1} e^{-j \omega t} + \hat{I}_c e^{j \omega t} + \hat{I}_{nc} e^{-j \omega t}
\]

According to the impedance model of the asymmetric load, the components of \( I_L \) can be derived as \( I_1 = Y_{i1}(\omega)U_1 \), \( I_{n1} = Y_{n1}(\omega_1)U_1^* \), \( I_c = Y_{c1}(\omega)U_c \) and \( I_{nc} = Y_{nc1}(\omega)U_c^* \).

As shown in Fig.2(c), the DQ method is commonly adopted to calculate the compensation current reference in harmonic or imbalance compensation devices. According to the DQ method, the sampling value of the load current is transformed into the synchronous reference frame to extract the unbalanced
component as the compensation current reference. To perform DQ transformation, the synchronous reference frame phase-locked loop (SRF-PLL) is adopted to obtain the instantaneous phase of the grid voltage. Considering the influence of perturbation voltage $U_e$, the compensation current reference $I_{cr}$ can be derived following the harmonic linearization principle. The main components of $I_{cr}$ can be expressed as:

$$
I_{cr} = \begin{cases}
I_{cr1} = H(-2\omega_1)I_{al}

I_{cr} = H(\omega_e - \omega_1)D(\omega_1)I_{al} - 0.5F(\omega_e - \omega_1)H(\omega_e - \omega_1)D(\omega_1)Y_e(\omega_1)U_e

I_{crne} = H(-\omega_e - \omega_1)D(-\omega_1)I_{ne} + 0.5F(-\omega_e + \omega_1)[H(-\omega_e - \omega_1) - H(-2\omega_1)]D(-\omega_1)Y_e(\omega_1)U^*_e
\end{cases}
$$

As shown in Fig.2(c), $H(\omega) = 1 - H_{LPF}(j\omega)$, which can be considered as a high-pass-filter. $F(\omega)$ can be derived by the scheme of the SRF-PLL, which can be expressed as:

$$
F(\omega) = \frac{U_KK_r s + U_KK_i}{s^2 + U_KK_r s + U_KK_i}
$$

According to (5), DQ method based compensation reference calculation can extract the unbalanced load current components $I_{al}$ at the frequency $-\omega_1$ and $I_{ne}$ at the frequency $-\omega_e$. $I_{cr1}$ and $I_{crne}$ denote the components of the compensation current reference corresponding to $I_{al}$ and $I_{ne}$. Additionally, the balanced perturbation component $I_e$ is also introduced into the reference signal as $I_{cr}$ at the frequency $\omega_e$.

3. Small-signal Stability Analysis of the System

Considering the adoption of DQ method based active imbalance compensation, a small-signal impedance model of the compensated system can be obtained which is shown in Fig.4.

As shown in Fig.4(a), when the compensation currents are considered, the equivalent admittances for the compensated load can be derived based on the parallel connection of the original load admittances and the compensation current components. Thus, the equivalent balanced admittance of the compensated load can be derived as:

$$
Y_{Le}(\omega_e) = [1 - H(\omega_e - \omega_1)G(\omega_1)D(\omega_1)]Y_L(\omega_e) + 0.5F(\omega_e - \omega_1)H(\omega_e - \omega_1)G(\omega_1)D(\omega_1)Y_e(\omega_1)
$$

(a) Impedance model of the system without grid-impedance

(b) Impedance model of the system with grid-impedance

Fig.4 Impedance model of the system considering active imbalance compensation
Due to the introduction of \( I_{\text{cne}} \), the coupling admittance of the compensated load can be derived as:

\[
Y_{Lc}(\omega) = \left[ 1 - H(-\omega_c - \omega_c)G(-\omega_c)D(-\omega_c) \right] Y_{Lc}(\omega) - 0.5F(-\omega_c + \omega_c) \left[ H(-\omega_c - \omega_c) - H(-2\omega_c) \right] G(-\omega_c)D(-\omega_c)Y_{Lc}(\omega)
\]  
(8)

Grid impedance is one of the major issues affecting the small-signal stability of grid-tied VSIs. Considering the introduction of grid-impedance \( Z_g \), the coupling between the equivalent circuits at the frequency \( \omega_c \) and \(-\omega_c\) under the perturbation voltage \( U_e \) is illustrated in Fig. 4(b).

Because of grid-impedance, both the circuit at the frequency \( \omega_c \) and the circuit at the frequency \(-\omega_c\) contain current components generated by the coupling admittance. To simplify the stability analysis process, a single-input-single-output (SISO) equivalent to the coupling system can be adopted, in which \( U_e \) is considered as input signal, and \( I_{\text{pe}} \) is considered as output signal [6]. Thus, the small-signal stability of the system can be analysed based on the Nyquist curve of the open-loop transfer function \( H(\omega_c) \) which can be expressed as:

\[
H(\omega_c) = Y_p(\omega_c)Z_g(\omega_c) - \frac{Y_{pm}(\omega_c)Y_{pm}^*(\omega_c)Z_g^*(\omega_c)}{1 + Z_g^*(\omega_c)Y_p(\omega_c)}Z_g(\omega_c)
\]  
(9)

To show the usage of the proposed modeling and stability analysis method, a test case is presented. In the test case, the grid-impedance is considered as pure inductance in the worst condition, and can be expressed as \( Z_g(\omega_c) = j\omega_cL_g \). The multi-resonant PR controller is used as current controller of the VSI, the expression of \( G_i(\omega) \) can be shown as:

\[
G_i(\omega) = K_{pi} + \sum_{n=1,3,5} \frac{K_{hi} \omega_c \omega}{s^2 + 2\omega_c \omega s + \omega_n^2} + \sum_{n=-j\omega_c} \frac{K_{hi} \omega_c \omega}{s^2 + 2\omega_c \omega s + \omega_n^2}
\]  
(10)

The specific parameters of the controller are \( K_{pi} = 0.01; \ K_{h1} = 0.2; \ \omega_1 = 100\pi; \ \omega_3 = 2; \ K_{h3} = 0.1; \ \omega_5 = 300\pi \text{ rad/s}; \ \omega_3 = 2; \ K_{h5} = 0.1; \ \omega_7 = 500\pi \text{ rad/s}; \ \omega_5 = 2; \ K_{h7} = 0.1; \ \omega_7 = 700\pi \text{ rad/s}; \ \omega_7 = 2. \) Other parameters are illustrated in Table 1.

| Parameter                        | Value       | Parameter                        | Value       |
|----------------------------------|-------------|----------------------------------|-------------|
| Inverter side inductance of the LCL filter \( L_1 \) | 0.9mH       | Magnitude of the grid phase-voltage with fundamental frequency | 110V        |
| Grid side inductance of the LCL filter \( L_2 \) | 0.1mH       | Admittance between A to B \( Y_{ph}(\omega) \) | 0.1S        |
| Capacitance of the LCL filter \( C \) | 100uF       | Admittance between B to C \( Y_{ch}(\omega) \) | 0           |
| Damping resistance of the LCL filter \( R \) | 1Ω          | Admittance between A to B \( Y_{ph}(\omega) \) | 0           |
| Grid inductance \( L_g \) | 1.8mH       | Control delay \( T_d \) | 10^{-4}s    |
| PWM gain \( K_{\text{pwm}} \) | 225         | Cut-off frequency of LPF \( \omega_{\text{PF}} \) | 10 \pi \text{ rad/s} |
| Fundamental frequency | 50Hz        | Cut-off frequency of PLL \( \omega_{\text{PLL}} \) | 54 \pi \text{ rad/s} |

Based on the Nyquist criterion for the space-vector based model, the system is unstable if the Nyquist curve of \( H(\omega_c) \) encircles the \((-1, 0)\) for \(-\infty < \omega_c < \infty \) [4]. Therefore, the stability of the uncompensated and compensated unbalanced system can be shown in Fig. 5.

Scenario 1: The unbalanced system is not compensated. In this scenario, \( Y_i(\omega) = Y_i(\omega_c) + Y_i(-\omega) \) and \( Y_{pm}(\omega_{pe}) = Y_{pm}(\omega) \), where \( Y_i(\omega_c), Y_i(-\omega) \) and \( Y_{pm}(\omega_{pe}) \) can be calculated based on (2), and (3). Therefore, based on the Nyquist curve of \( H(\omega_c) \) shown in Fig. 5(a), the system is stable.
Scenario 2: The unbalanced system is compensated by the DQ method. In this scenario, $Y_p(\omega_k) = Y_i(\omega_k) + Y_{Lc}(\omega_k)$ and $Y_{pn}(\omega_k) = Y_{Lcn}(\omega_k)$ can be obtained based on (8) and (9). According to the Nyquist curve of $H(\omega_k)$ shown in Fig.5(b), the system is unstable because the curve encircles the $(-1, 0)$. Additionally, the resonant frequencies are 355 Hz and -355 Hz.

In summary, the VSI based imbalance compensation can affect the stability of the system. In the presented case, adopting the DQ method can destabilize the system.

4. Experimental Results

To verify the stability analysis, experimental results are presented in this section. The laboratory test setup is built based on the topology shown in Fig. 1. The control scheme of the grid-tied VSI is implemented in a TMS320F28335 processor. The parameters of the test platform are presented in Table 1. The waveforms of the inverter output current $I_s$, the PCC current $I_p$ and the PCC voltage $U_p$ are shown in Fig. 6 under different compensation conditions.

As shown in Fig. 6(a), due to the asymmetric load, $I_p$ includes the component at the negative fundamental frequency. The waveforms of $I_s$, $I_p$ and $U_p$ have little harmonic distortion, proving that the system is small-signal stable.

As shown in Fig. 6(b), when DQ method based imbalance compensation is adopted, the component of $I_p$ at the negative fundamental frequency is offset by the compensation current in $I_s$. However, harmonic contents of the current and voltage waveforms are significantly higher, indicating that the system is destabilized.

As shown in Fig. 6(c), $I_p$ of the uncompensated system contains the $-1$th harmonic which is caused by the asymmetric load. When DQ method based imbalance compensation is adopted, the $-1$th harmonic current is attenuated. However, $\pm7$th harmonic components arise because of the resonance introduced by the compensation (According to the theoretical analysis shown in Section 3, the resonant frequencies are close to $\pm7$th harmonic frequencies).
5. Conclusion
In this paper, a grid-connected VSI was used to compensate the three-phase imbalance introduced by asymmetric local load. The impedance modeling approach was proposed to analyze the small-signal stability issues of the compensated system, considering the influence of different compensation control methods.

Based on the proposed the stability analysis method, the theoretical analysis and experimental results indicate that the reshaping of the output impedance feature caused by active imbalance compensation can affect the small-signal stability of the system. In general, the proposed modeling and stability analysis method can assist in selecting a more robust compensation algorithm and control parameters, considering the load and main grid condition."

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