Reflective mathematical thinking process and student errors: an analysis in learning style

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Abstract. Mathematical reflective thinking is one of the important thinking skills trained to students, but it has less attention. Student learning style is one of the factors that influence the mathematical reflective thinking process. The purpose of this study is to explore errors and reflective mathematical thinking processes seen from learning styles according to Honey and Mumford. This research was conducted at one of the junior high schools in Indonesia. This study uses a survey method, students with activist, reflector, theorist, and pragmatist learning styles. Data were collected through learning style questionnaires, mathematical reflective tests, and observations. All research instruments will first be tested for eligibility by validation by 2 experts. Data analysis techniques include data reduction, data presentation, and conclusion. The results of the analysis can be concluded that the mathematical reflective thinking processes of students with activist, pragmatist, and theorist learning styles had gone through the stages of reacting, elaboration, and contemplating appropriately. While students with reflector learning styles are only at the stages of reacting, and elaboration, they have not reached contemplating. This study provides teachers with an understanding of the importance of errors and thought processes of students in reflective thinking problems.

1. Introduction
Various higher-order thinking skills of mathematical thinking skills are given to students [1], such as critical thinking skills, creative, logical, communication, connections, problem solving, metacognitive, and reflective [2,3], but the reality is often ignored. Reflective thinking is important for students, but in reality, reflective thinking lacks serious attention, the teacher generally only cares about the final answers obtained by students without regarding to how the answers of students are obtained [4–6]. This is very contrary to the expectations of the curriculum in force in Indonesia. Mathematical creative thinking skills of students are still low, this is because students are still focused on formulas and memorization, as the result students do not understand the concepts studied [7].

Various studies on reflective mathematical thinking have been conducted, they are research on teacher perceptions in reflective thinking [8], reflective thinking in terms of the beliefs-efficacy of pre-service mathematics teachers [4], research on intelligence in terms of Reflective thinking [9], study of reflective thinking skills with attitudes in mathematics [10], and [11] analysis of reflective mathematical thinking on Problem Based Learning viewed from visual, auditory and kinesthetic learning styles. The reflective thinking process is a stage or reflective thinking activity to solve a problem. Three stages of reflective thinking [12] are the initial response of students based on personal understanding of the problem by focusing on the nature of the natural situation, elaboration or comparison that is students do...
analysis and clarification as well as meaning and information to evaluate what is believed by comparing reaction experiences with other experiences, contemplating that is students prioritize deep personal understandings such as describing, informing, considering, and reconstructing a situation or problem.

Mathematical reflective thinking is a thinking ability that connects the knowledge gained in analysis problems and evaluating problems, by linking old knowledge to get a conclusion in solving mathematical problems. The thought process that a person goes through, to be able to solve mathematical problems has a relationship with the ability to recite and recognize the relationship between mathematical concepts, can bring up new ideas in making conclusions precisely [13]. The process of reflective thinking does not depend on knowledge of the students, but the process of how to utilize the knowledge they already have to solve the problem at hand. That means, if students can find ways to solve the problems they face so that they can achieve their goals, then these students have undertaken reflective thinking processes [5]. Individual differences can be used as a reference to recognize each learning style of the students because not all students have the same learning style and the same ability to participate in learning [14]. How a person learns will affect the structure of his brain, because sometimes someone will feel better if they listen a lot, some feel comfortable with reading, and some even feel the results will be more leverage if they practice what is being learned [15]. The learning process undertaken by each student is certainly different, and learning styles according to Honey and Mumford affect the ability of students to solve mathematical problems [16].

Reflective thinking is a process of directing and appropriate activities where individuals analyze evaluate, motivate, gain deep meaning and use appropriate learning strategies [17]. The ability of reflective thinking is the ability to think with attention to assumptions and their implications based on reason or evidence to support conclusions [18]. Five components relating to reflective thinking ability [19] are recognize the difficulty that is students recognize the problem and identify it, location and definition of the problem that is students look at the problem and there will be efforts to sharpen the problem, suggestion of possible solutions that is students develop various possibilities solutions to solve problems that have been limited and formulated and try to solve, rational elaboration of an idea that is students looking for information, thinking about and formulating problem solving by collecting data that supports, and test and formation of a conclusion that is students test possibilities by applying it to solving problems so students find their conclusions.

Students tend to have different learning styles, depending on the situation and level of experience as they move between the four learning styles, rather than dominating one learning style. Learning styles according to Honey and Mumford consist of activists, reflectors, theorists, and pragmatists [20]. Characteristics of activist learning styles like challenges, doing experiments, having an open mind and easily accepting ideas, always trying and enthusiastic to do everything new. Reflector learning style prefers discussion, likes to choose opinions that are considered suitable and correct, this learning style takes into consideration experience and views from several different perspectives. Whereas theorist learning styles tend to be very critical theorists, like to analyze, think rationally using reasoning, full of consideration, not easy to accept ideas or ideas unless he can prove the truth and do not like opinions or subjective judgements. Pragmatist learning styles tend to emphasize practical actions, do not like the theories of concepts, long theorems. Noting the mistake of the students in answering questions is very important, research on Newman error analysis in pre-service teachers has been conducted [21], the results of the research show that mastery of basic concepts of pre-service is low, process errors, answers are incorrect. Mathematical pre-service error patterns have been investigated [22] on conceptual, procedural, application, and careless errors. The results of the study indicated that pre-service errors were found in understanding concepts, applying concepts, procedures, and mathematical symbols. The most dominant error is in understanding the concept compared to other errors.

Noting the research that has been done about mathematical reflective thinking, no one has researched mathematical reflective thinking in terms of processes and errors of students in answering questions. Though the thought process is very important, it is not only the result that is considered. Likewise, exploring student errors is very important as an input for feedback. Learning styles can also affect the mindset of students, but no one has researched the processes and errors of mathematical reflective thinking seen from learning styles according to Honey and Mumford. The purpose of this study is to explore and analyze the processes and errors of mathematical reflective thinking seen from student
learning styles according to Honey and Mumford. This research was conducted at one of the junior high schools in Indonesia on the material to build flat side spaces.

2. Research Method
The method used in the study is a survey method, data collected from the Honey and Mumford learning style questionnaire, reflective mathematical thinking tests, and observations of student answers. Data analysis techniques include reduction, presentation, and conclusions. The research instrument consisted of a mathematical reflective thinking test item that measures indicators: recognize the difficulty, location, and definition of the problem, suggestion of possible, rational elaboration of an idea test and formation of conclusion. The Learning Style Questionnaire (LSQ) refers to the Honey and Mumford learning style, consisting of 40 statements to detect students having an activist, reflector, theorist, and pragmatist learning style. Both instruments were tested for eligibility. This study involved 4 students as participants who had activist, reflector, theorist, and pragmatist learning styles. The stages of the study began with the Honey and Mumford learning style questionnaire test, then the results were grouped into 4 learning styles, then each of the learning styles took turns given a mathematical reflective thinking test using think-aloud [2] students answered the questions while they were spoken. Also, student responses were observed and analysis to obtain data on mathematical processes and reflective thinking errors.

3. Results and Discussion
This research was conducted at Indonesian junior high schools, the Honey and Mumford learning style questionnaire was given to 29 students, the results were 6 students tended to activist learning styles, 11 students tended reflector learning styles, 5 students tended theorist learning styles, and 7 students tended pragmatist learning. The grouping is based on the answers students choose the option, most choose to agree. From each learning style, a person with the highest score of the purposive score was chosen, S23 students were selected from the activist learning style, S4 from the reflector learning style, S26 from theorist learning styles, and S22 from pragmatist learning styles. To each of the participants given mathematical reflective test questions on the material to build a flat side space, using the technique of thinking aloud, students answer questions while saying, researchers observe. The test is carried out in turns each. Then the answers of the students to the mathematical reflective questions include indicators of recognizing the difficulty of the problem, location, and definition of the problem, suggestion of possible solutions, rational elaboration of an idea, test and formation of conclusion. The results of the analysis of the answers from the participants on each indicator, obtained reflective mathematical thinking process can be seen in Table 1.
Table 1. Mathematical reflective thinking process.

| Stages      | Thinking Process                                                                 |
|-------------|----------------------------------------------------------------------------------|
| Reacting    |                                                                                  |
| S23 (Activist) | Write down what is known and asked but do not write the data completely and clearly, do not write a calculation plan before answering. |
| S04 (Reflector) | Understand the problem by writing down what is known and asked but do not write complete data and does not write a calculation plan. |
| S26 (Theorist) | Understand the problem by writing down what is known and asked but do not write the calculation phase plan, immediate answer. |
| S22 (Pragmatist) | Do not understand the problem because it does not write what is known and asked, and does not write a calculation plan. |
| Elaboration |                                                                                  |
| S23 (Activist) | Link the problem in the matter with the Pythagorean Formula.                     |
| S04 (Reflector) | Linking known concepts to the Pythagorean Formula.                               |
| S26 (Theorist) | Answering in sequence and linking the previous concept with the Pythagorean Formula. |
| S22 (Pragmatist) | Associate with the Pythagorean Formula.                                          |
| Contemplating |                                                                                  |
| S23 (Activist) | Examine answers using other ways of answering questions, and conclude.          |
| S04 (Reflector) | Do not use other different ways of answering, and not conclude.                 |
| S26 (Theorist) | Use other ways to answer, and write conclusions.                                 |
| S22 (Pragmatist) | Answering in another way but wrong, so the conclusion is also wrong.             |

Based on Table 1 at the reacting stage, S23 is student with the most complete activist learning styles of mathematical reflective thought processes compared to S04, S26, and S22. Whereas S04 and S26 mathematical reflective thinking processes are relatively similar, and S22 answers but it is wrong. In the elaboration stage, all students think the process is relatively the same and appropriate. In the contemplating stage S23 and S26 answer in two ways and correct and make conclusions, while S04 does not use other methods, and S22 uses other methods but is wrong. Thus the mathematical reflective thinking process on the most complete and correct flat side space material is S23 students with Activist learning styles, then S26 who has theorist learning styles, S04 with reflector learning styles, and finally S22 with pragmatist learning styles. The mathematical reflective thinking process of each participant can be seen in Figure 1.

Figure 1. Mathematical Reflective Thinking Process.
To obtain data on reflective thinking processes of the students and their deeper mistakes, interviews were held with all participants. S23 students with activist learning styles at the reacting stage understand what problems are known and asked about in problems, relationships, able to explain what is known is enough to answer asked. This is similar to the results of research conducted by [16]. In the elaboration, the stage can explain how to solve problems with confidence and associate prior knowledge by writing the Pythagorean formula correctly. Whereas the contemplating stage has tried to find another way and finally can answer the problem in another way to detect the correctness of the answers that have been done. Correct and explain the occurrence of errors in the answers, and students make appropriate conclusions on the problems that exist in the problem and have high confidence with the results of the answers already obtained.

Then S23 can explain the problem clearly and easily invited to dialogue, as well as demonstrate his hand in explaining using other ways to answer questions by opening the webs. Then this student is not careful in answering so writing wrong answers but can correct them is revealed through interviews. Participants learning style reflector S04, the reacting stages write what is known and what is asked in the problem, can express the relationship between the known and those asked, explaining what is known is enough to answer the question. Similar to the results of research conducted by [16] that the subject of the reflector understands the information in the problem. Then the elaboration stage, S04 participants explained how to solve the problem, linking prior knowledge by writing down the Pythagorean theorem formula. The contemplating phase S04 does not know and apply a different way to detect the correctness of the answer. Students can deduce the problem, and can only use one method in working on the problem. This is supported by the results of [16] research that the reflector learning style has not yet reached the stage of re-checking.

These participants tend to belong in answering questions and are very careful in writing answers, very slow in drawing. Similar to the results of research [14] that reflector learning styles tend to belong and cannot do something in a hurry to understand information. This participant was hesitant in working on the problem, seen on the answer sheet there was a part of the answer that was crossed out, even though the part that was crossed out was the correct result, then when interviewed answered in a small and unclear voice. Then do not try to solve the problem, when given a stimulus linking lego toys to recall memories of the students in understanding cube nets and prisms, but this S04 cannot imagine and apply cube nets and prisms to solve problems.

Participants with theorist style S26 at the reacting stage, understand the purpose of the problem and can take a picture, write what is known and what is asked on the problem, can connect between the known with the asked and explain what is known is enough to answer the question. This is similar to the research conducted by [16] that theorist learning styles can mention information that is known and asked, can tell a problem given in their own words can make a picture with the given problem. Furthermore, at the elaboration stage, explaining how to solve problems and writing out their solutions in a systematic way and being able to link prior knowledge by writing down the Pythagorean formula. In the contemplating stage, use other means to detect the truth of the answers that have been done, and
re-examine the results of the answers that have been obtained by other means and produce the same answer, so that it can conclude.

In the reacting stage, S22 participants with pragmatist learning styles write what is known and asked but only draw the shape of the cube and the roof in the form of a triangular prism but understand the purpose of the problem in the problem. Besides, explaining the relationship between what is known and what is asked and explains what is known to what is asked is enough to answer the question. This is similar to the opinion of [20] which suggests that pragmatic individuals like practical things. Furthermore, at the elaboration stage, explaining how to solve problems, and find other ways to solve problems in the problems associated with the term 'unboxing' that exist in daily life. Linking prior knowledge by writing down the Pythagorean formula. While in the contemplating stage, do not try hard to find other ways to detect the truth of the answers, but only recalculate the answers obtained, correct errors and be able to deduce the problem.

Participants with the S23 activist learning style did not experience significant errors, their shortcomings simply did not write a calculation plan, but the concepts used, application concepts and procedures and the final results were correct. Error S04 with reflector learning style, the concept used is correct, but it is procedural wrong so the end result is wrong. Furthermore, S26 with theorist learning style, concepts and procedures are correct but careless so the final result is wrong. The most mistakes are S22 with pragmatist learning styles, errors in concepts, procedures, and final results. The errors made by S23, S04, S26, and S22 are very diverse, this is in accordance with the results of the study [21,22]. The biggest participant errors were found in answering in different ways but the results were the same or using another method. In addition, the next error lies in writing a calculation plan.

4. Conclusions
The mathematical reflective thinking process of students is seen from the learning styles according to Honey and Mumford. It can be concluded that S23 students are ones with activist learning styles in writing what is known and asked in the questions, what relationships are known and asked and explain what is known is enough to answer asked. In the elaboration stage, students explain the answers to the problems obtained and link with prior knowledge by writing the Pythagorean theorem formula correctly. Then in contemplating stage, students can detect the correctness of the answers, correct and explain the occurrence of errors in the answers, and make conclusions correctly. Students with S04 reflector learning styles, at the reacting stage, write what is known and asked, write the relationship between the known and what is asked, and explain what is known to answer what is asked. Elaboration stage, students explain the answers to the problems obtained, linking prior knowledge by writing the Pythagorean formula. In the contemplating the stage, students do not detect the truth in determining answers but can deduce the problem.

The mathematical reflective thinking process of students S26 is as theorist learning style, the reacting stage of students to write what is known and asked, write the relationship between the known and asked and can explain what is known is enough to answer what is asked. In the elaboration stage, students explain the answers to the problems obtained, can link previous knowledge by writing a formula that must be used. Contemplating stage, students detect the truth in determining answers, students correct and explain the occurrence of errors in the answers, and can make conclusions correctly. Students S22 is one with a pragmatic learning style of the thought process at the reacting stage. Students put forward what is known and what is asked in the problem, explain the relationship between the known and asked, and explain what is known is enough to answer the question. In the elaboration stage, student can explain the answer to the problem obtained and link prior knowledge by writing down the formula used. In the contemplating stage, students detect the correctness of the answers, correct and explain the occurrence of errors in the answers and can conclude the problem.

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