Mean-line Modeling of an Axial Turbine

A Yu Tkachenko¹, Ya A Ostapyuk² and E P Filinov³
Department of Aircraft Engine Theory, Institute of Engine and Power Plant Engineering, Samara National Research University, 34, Moskovskoye shosse, Samara, 443086, Russia
¹tau@ssau.ru, ²oya92@mail.ru, ³filinov.evg@gmail.com

Abstract. The article describes the approach for axial turbine modeling along the mean line. It bases on the developed model of an axial turbine blade row. This model is suitable for both nozzle vanes and rotor blades simulations. Consequently, it allows the simulation of the single axial turbine stage as well as a multistage turbine. The turbine stage model can take into account the cooling air flow before and after a throat of each blade row, outlet straightener vanes existence and staggering angle controlling of nozzle vanes. The axial turbine estimation method includes the loss estimation and thermogasdynamic analysis. The single stage axial turbine was calculated with the developed model. The obtained results deviation was within 3% when comparing with the results of CFD modeling.

1. Introduction
The turbomachine development is very complex process that includes thermogasdynamic, structural, technological and economic investigations. Thermogasdynamic designing is iterative process, in which the calculations results performed at the conceptual design stage are subsequently improved with application the higher fidelity models [1, 2]. During the conceptual design stage the simple models such as the model of the one-dimensional estimation along the flow path mean line is used for performance prediction and preliminary optimization of the turbomachine design. These models are very important, because they allow fast estimation of velocity triangles, losses and parameters of the blades at mean line with acceptable accuracy. In most cases stations of the turbine flow path are normal to the axis of the engine [3, 4]. This paper presents the method in which these stations are normal to the mean line (figure 1). Thus the radial velocity component can be excluded from consideration. The main advantage is capability for both axial and radial turbines modeling.

2. One-dimensional estimation of an axial turbine

2.1. Model description
The one-dimensional (1D) mathematical model is reasonable for the turbomachines performance calculation along the flow path mean line. This model (figure 2) describes the relation between thermodynamic and kinematic parameters and geometric parameters of blade row at various operation conditions.

A model of the axial turbine stage (figure 3) can be draw up using the turbine blade row model. The model of the axial turbine stage also includes the calculation elements of nozzle vanes, blades,
stage overall performance as well as inlet and outlet data interface elements for an external model. The turbine stage model can take into account the cooling air flow before and after a throat of each blade row, outlet straightener vanes existence and stagger angle controlling of nozzle vanes, etc.

![Meridional cross section of a turbine blade row.](image)

**Figure 1.** Meridional cross section of a turbine blade row.

**2.2. Method for loss estimation in an axial turbine blade row**

Loss coefficients at turbine blade row reflect the deviation of real process of the gas expansion from its ideal (isentropic) instance. Losses in a turbine blade row are usually expressed in a velocity coefficient or a thermodynamic loss coefficient. The velocity coefficient $\varphi$ represents deviation of the gas flow real relative velocity at blade row outlet from ideal velocity at the same pressure ratio. The thermodynamic loss coefficient $\zeta$ represents loss of enthalpy:

$$\zeta = \frac{i_x - i_{x*}}{i_{w*} - i_x}$$ (1)
where

- $i_X$ – static enthalpy at outlet from blade row;
- $i_{Xs}$ – total isentropic process enthalpy at outlet from blade row;
- $i_{x}$ – total relative flow enthalpy at outlet from blade row.

**Figure 2.** Data streams of the 1D-model of an axial turbine blade row.

The velocity coefficient is related with the thermodynamic loss coefficient:

$$\varphi = \sqrt{1 - \zeta}$$

Total loss has complex structure and includes profile, secondary and cooling losses:

$$\zeta = \zeta_{pr.\,w} + \zeta_s + \zeta_c$$

The structure of the total loss is represented in figure 4, where $\overline{G_a}$ – relative cooling air mass flow rate, $K_{kw}, K_v, K_c$ – auxiliary coefficients. The cooling losses and other kind of losses are described in detail in [5, 6, 7].
Figure 4. The structure of total loss taken into account in the 1D-model of an axial turbine blade row.

The function for determination of the loss coefficient $\zeta$ can be presented in the following general form:

$$\zeta = f \left( \{G\}, T_0, T_X, V_X, \lambda_{X,Y}, \theta, \kappa_{O, \kappa_{Xef}}, H_X, \chi, s, \tau_X, \zeta_c \right).$$  \hspace{1cm} \text{(4)}$$

where $G$ – working fluid parameter array;
$P_0, T_0$ – static pressure and static temperature at blade row inlet;
$T_X$ – static temperature at blade row discharge;
$V_X$ – absolute velocity at discharge;
$\lambda_{X,Y}$ – corrected relative velocity at discharge;
$\theta$ – incidence angle;
$\kappa_O, \kappa_{Xef}$ – blade inlet angle and efficient blade outlet angle;
$H_X$ – blade height at discharge;
$\chi$ – chord profile length;
$s$ – blade spacing;
$\tau_X$ – trailing edge radius;
$\zeta_c$ – cooling losses coefficient.

2.3. Method for the thermogasdynamic analysis of an axial turbine blade row

The method for the thermogasdynamic analysis of an axial turbine blade row (nozzle vane as well as rotor blade) along the flow path mean line allows to calculate the gas flow parameters at inlet and discharge stations and the cycle parameters depending on the gas flow parameters at inlet, rotational speed and $M_{CX}$ parameter. The $M_{CX}$ parameter describes the flow state at outlet.

Since the methods of the loss coefficients estimation and thermogasdynamic analysis needs the detail description, they will be published in future papers.

3. Model validation
The software modules for computer-aided environment for thermogasdynamic calculations and analysis ASTRA [8] were implemented to perform validation of proposed models. The testing was performed on high pressure turbine of the small-scale engine. The results of calculation are shown in the table 1.

| $G_O$  | kg/s | 2.527 | Gas flow rate |
|--------|------|-------|---------------|
| $p^*_O$ | kPa  | 332,526 | Total pressure at inlet to nozzle vane |
| $T^*_O$ | K    | 1175  | Total temperature at inlet to nozzle vane |
| $n$    | rpm  | 40000 | Rotational speed |
| $\theta_{O,av}$ | °  | 0     | Taper angle at inlet to nozzle vane |
| $R_m.O_{av}$ | m | 0.094 | Mean line radius at inlet to nozzle vane |
| $\theta_{O,av}$ | °  | 0     | Angle of entrance at inlet to nozzle vane |
| $\theta_{X,av}$ | °  | 0     | Taper angle at outlet from nozzle vane |
| $R_m.X_{,av}$ | m | 0.094 | Mean line radius at outlet from nozzle vane |
| $r_{X,ns}$ | m | 0.001 | Trailing edge radius of nozzle vane |
| $\kappa_{O,av}$ | °  | 0     | Blade inlet angle of nozzle vane |
| $\kappa_{X,eff,av}$ | °  | -66   | Efficient blade outlet angle of nozzle vane |
| $\theta_{X,av}$ | °  | 0     | Taper angle at outlet from rotor blade |
| $R_m.X_{,i}$ | m | 0.09375 | Mean line radius at outlet from rotor blade |
| $r_{X,i}$ | m | 0.00035 | Trailing edge radius of rotor blade |
| $\kappa_{O,i}$ | °  | 23    | Blade inlet angle of rotor blade |
| $\eta$    | —    | 0.8828 | — |
| $N$ | kW  | 431.1 | — |

**Table 1. Results of the test calculation**

| Parameter | Value |
|-----------|-------|
| $\zeta_e$ | 0     |
| $\zeta_{fr}$ | 0.028 |
| $\zeta_{pr,th}$ | 0.042 |
| $\zeta_{pr,i}$ | 0.059 |
| $\zeta_{pr,i,W}$ | 0.0603 |
| $\zeta_{pr,W,th}$ | 0.0634 |
| $\zeta_{pr,th}$ | 0.0282 |
| $\zeta_{th}$ | 0.0214 |
| $\zeta_{th}$ | 0.0496 |
| $\zeta$    | 0.1114 |
| $\varphi$  | 0.9426 |
| $\pi$      | 1.5343 |
| $\pi^*$    | 1.046 |
| $\phi$     | 0.8828 |
| $\zeta_{fr}$ | 0.028 |
| $\zeta_{pr,th}$ | 0.042 |
| $\zeta_{pr,i}$ | 0.059 |
| $\zeta_{pr,i,W}$ | 0.0603 |
| $\zeta_{pr,W,th}$ | 0.0634 |
| $\zeta_{pr,th}$ | 0.0282 |
| $\zeta_{th}$ | 0.0214 |
| $\zeta_{th}$ | 0.0496 |
| $\zeta$    | 0.1114 |
| $\phi$     | 0.9426 |
| $\pi$      | 1.5343 |
| $\pi^*$    | 1.046 |

**Nozzle vane**

- $\zeta_e$ — 0: Cooling losses coefficient
- $\zeta_{fr}$ — 0.028: Friction losses coefficient
- $\zeta_{pr,th}$ — 0.042: Profile losses coefficient (basic)
- $\zeta_{pr,i}$ — 0.059: Profile losses coefficient taked into account the angle of incidence
- $\zeta_{pr,i,W}$ — 0.0603: Profile losses coefficient taked into account the angle of incidence and flow velocity
- $\zeta_{pr,th}$ — 0.028: Secondary losses coefficient
- $\zeta_{th}$ — 0.0496: Secondary losses coefficient before the throat area
- $\zeta$, $\phi$, $\pi$, $\pi^*$: Total losses coefficient, the velocity coefficient, pressure ratio $p^*_O/p_X$

**Rotor blade**

- $\zeta_e$ — 0: Cooling losses coefficient
- $\zeta_{fr}$ — 0.042: Friction losses coefficient
- $\zeta_{pr,th}$ — 0.059: Profile losses coefficient (basic)
- $\zeta_{pr,i}$ — 0.0603: Profile losses coefficient taked into account the angle of incidence
- $\zeta_{pr,i,W}$ — 0.063: Profile losses coefficient taked into account the angle of incidence and flow velocity
- $\zeta_{pr,th}$ — 0.028: Trailing edge losses coefficient
- $\zeta_{th}$ — 0.0315: Secondary losses coefficient before the throat area
\begin{table}
\centering
\begin{tabular}{|c|c|}
\hline
\( \zeta_{pr,\text{th}} \) & 0.0433 \\
\hline
\( \zeta_{pr,\text{W.th}} \) & 0.0452 \\
\hline
\( \zeta_{s,\text{th}} \) & 0.0226 \\
\hline
\( \zeta_{th} \) & 0.0678 \\
\hline
\( \zeta \) & 0.0945 \\
\hline
\end{tabular}
\caption{Coefficient of the profile losses before the throat area taken into account the angle of incidence}
\end{table}

Basing on presented results it can be concluded that model provides the adequate solution and stable convergence in the numerical solution process. The obtained results deviation was within 3\% when comparing with the results of CFD modeling.

4. Conclusions
The blade row model of the one-dimensional estimation along the mean line was developed for more detailed analysis and preliminary optimization of an axial turbine. The blade row model allows to make up a single axial turbine stage as well as a multistage turbine. The turbine stage model can take into account the cooling air flow before and after a throat of each blade row, outlet straightener vanes existence and stagger angle controlling of nozzle vanes.

In the presented method stations are normal to the mean line. It simplifies the computation algorithm because the radial velocity component may be excluded from consideration. The main advantage is capability for both axial and radial turbines modeling.

It can be concluded that proposed model provides the adequate solution and stable convergence in the numerical solution process. The detailed comparing of the 1D and 3D CFD modelling results will be possible after models integration and creation of an identification unit.

Acknowledgments
This work was supported by the Ministry of education and science of the Russian Federation in the framework of the implementation of the Program of increasing the competitiveness of Samara University among the world's leading scientific and educational centers for 2013-2020 years.

References
[1] Popov G M, Baturin O V, Kolmakova D A, Krivcov A V Improvement results of TK-32 turbo compressor turbine with gas-dynamics and strength CAE-systems 2014 International Journal of Engineering and Technology 6(5) 2297-2303
[2] Matveev V N, Popov G M, Baturin O V, Goryachkin E S, Kolmakova D A Workflow optimization of multistage axial turbine 2015 51st AIAA/SAE/ASEE Joint Propulsion Conference 132015
[3] Ainley D G, Mathieson G C R 1951 A method of performance estimation for axial-flow turbines (London: Her Majesty's Stationery Office) p 31
[4] Schobeiri T One-dimensional methods for accurate prediction of off-design performance behavior of axial turbines 1992 ASME 1
[5] Venediktov V D 1990 Gas dynamics of a cooled turbines (Moscow: Machine Industry Press) p 240
[6] Wei N 2000 Significance of loss models in aerothermodynamic simulation for axial turbines (Doctoral thesis) (Stockholm: Royal Institute of Technology) p 164
[7] Dahlquist A D 2008 Investigation of losses prediction methods in 1D for axial gas turbines (Master thesis) (Sweden: Lund University) p 192
[8] Kuz'michev V S, Ostapyuk Y A, Tkachenko A Y, Krupenich I N, Filinov E P Comparative analysis of the computer-aided systems of gas turbine engine designing 2017 IJMERR 6 (1) 28-35