Dynamic shear band evolution by the numerical manifold method and the Drucker Prager model

Zibo Fan*, Hong Zheng, Wenan Wu, Ning Zhang and Yichen Wang

Key Laboratory of Urban Security and Disaster Engineering, Ministry of Education, Beijing University of Technology, Beijing 100124, China

* Corresponding author: 406722079@emails.bjut.edu.cn

Abstract. To simulate the dynamic evolution of shear band in soil, the numerical manifold method (NMM) and the Drucker Prager model (DP) are used. Considering that the soil does not bear tension, the tensile part of DP is cut off. Then the constitutive integration could be reduced to mix complementarity problem, which is solved by the Gauss Seidel projection contraction method (GSPC). The discretization sequence is altered to resolve the transfer issue of degrees of freedom when dealing with dynamic discontinuities evolving. Different with the traditional discretization approach, the discretization of temporal variable is set prior to that of spatial variable so that all the degrees of freedom are valid only in a current step without the necessity to be transferred to the next time step. In this way, the shear band tip could stop anywhere naturally, without being improperly forced to stay on the element edge. For simplicity, the shear band is treated as a strong discontinuity with zero width. The interface contact is described by the NMM contact approach where open-close iteration is involved. Finally, a soil dam is studied to check the capability of the proposed method to capture the dynamic evolution of shear band.

1. Introduction

The shear band refers to a strip area with highly localized deformation which gradually accumulates from a smoothly varying deformation field. Such phenomenon can be treated as material instabilities or the general form of strain localization. Once fully developed, the shear band causes tangential relative sliding leading to the failure of geotechnical materials. For rate-independent elasto-plastic solids, the detection of the onset of shear band, has accorded with the loss of ellipticity of the governing equations [1].

To account for the fact that soil generally does not bear tension, the tensile part of the DP model is cut off by adding three yield functions. The integration of multiple yield surfaces could be reduced to mixed complementarity problem, which could be solved by the Gauss Seidel projection contraction method (GSPC) [2]. GSPC is an extension of the projection contraction algorithm (PCA), which has been applied to solve some challenging complementarity related problems [3,4] in geotechnical engineering.

In this paper, the numerical manifold method (NMM)[5] is used to model the shear band evolution. NMM, invented by Shi [6], compared with XFEM, NMM can simulate the discontinuities naturally without the use of level set method and the generalized Heaviside function. The update of level sets method is not satisfactory when dealing with discontinuity bifurcation.
Frictional contact law of NMM is adopted to model the shear band interface. The contact states are determined by the open-close iteration [7] which refers to a process of repeatedly applying or removing contact springs until the stable contact states are reached. This process is always convergent in the dynamic analysis thanks to inertial matrices. The convergence poses a challenge because of the coupling of material nonlinearity and contact nonlinearity. In this paper, for each iteration in material nonlinearity, open-close iteration must be invoked so that contact state is convergent.

In the field of dynamic analysis of solids and structures with FEM, the discretization order of the weak form of the momentum conservation law has always been that the spatial discretization is set prior to temporal discretization. This routine works well for the engineering problems on the condition that all the degrees of freedom are valid in all the time steps and the number of them keeps constant. However, this condition is not satisfied in the simulation of discontinuity propagation because more degrees of freedom will be generated after the discontinuity tip moves to another location. Although this poses no difficulties in linear static solutions, the transfer issue of degrees of freedom is not readily handled in dynamic discontinuous propagation analysis.

This paper solves the transfer issue of degrees of freedom by a simple and efficient strategy that the temporal discretization is set prior to the spatial discretization [8]. The transfer issue of degrees of freedom is avoided because the degrees of freedom are valid only in a current time step. Instead of transferring the degrees of freedom, such physically meaningful variables as speed and stresses at integration points are needed to be saved and then transferred to the next time step.

This paper is organized in the following way. In section 2, the governing equations are provided. Section 3 expounds the constitutive model. In section 4, a soil dam is studied respectively to test the ability of the proposed method to capture the shear band. Some conclusions are drawn in the last section 5.

2. Governing equations
Given that the contact between the discontinuous interfaces is reduced to the contact of distinct contact pairs, the weak form of the momentum conservation law can be expressed as

\[
\int_\Omega (\delta \sigma)^T \sigma d\Omega = \int_\Omega (\delta \mathbf{u})^T (\mathbf{b} - \bar{\rho} \dot{\mathbf{u}}) d\Omega + \int_{S_p} (\delta \mathbf{u})^T \bar{\rho} \mathbf{d} S_p + \sum_{\nu} (\delta \mathbf{u}_\nu - \delta \mathbf{u}_{\nu'}) \mathbf{p}^{\nu'}
\]  

which holds at any time \( t \). Here \( \delta \) is the variational operator, \( \sigma \) the total stress vector, \( \mathbf{u} \) the total displacement vector, \( \mathbf{b} \) the total volume force vector, \( \bar{\rho} \) the mass density, \( \bar{\rho} \) the traction vector loaded on the surface \( S_p \) of \( \Omega \), \( \dot{\mathbf{u}} \) the second order time derivative of \( \mathbf{u} \). And the last component in equation (1) refers to the virtual work of all the contact pairs.

It is important to know that the time discretization precedes the space discretization in the dynamic evolution of discontinuities in NMM so that the transfer issue of degree of freedom could be circumvented [8].

It should be mentioned that the displacement controlled method (DCM) proposed by Zheng [9] is applied to solve the equations above. The critical fact about the DCM by Zheng is that this algorithm is derived based on the Sherman-Morrison theorem and only elastic tangent matrix is needed. Hence a huge amount of labor for calculating the elastoplastic tangent matrix could be saved, especially in the case of the multiple yield surfaces, which will be elaborated later.

Note that for each iteration in DCM, the open-close iteration must be invoked reach the convergence in contact. When both DCM and the open-close iteration reach convergence, the next loading step is imposed.

3. Constitutive models
The Drucker-Prager (D-P) model, which is a simple non-associative pressure sensitivity plasticity
model, is chosen here to account for the behavior of soil. Even though the model is isotropic in the \( \Pi \) plane and not able to properly explain the yield strength of soil, it is useful to test a new method formulated in the paper.

The yielding function of D-P model is [10]

\[
 f_i = \sqrt{\frac{1}{2} s : s + \frac{3 \tan \phi}{\sqrt{9 + 12 \tan^2 \phi}}} p - \frac{3}{\sqrt{9 + 12 \tan^2 \phi}} c
\]

(2)

Here, \( c \) is the cohesion, \( \phi \) the friction angle, \( p \) the hydrostatic stress, \( s \) the deviatoric stress tensor. And \( c \) is described by

\[
 c = c_0 + H \tilde{e}^p
\]

(3)

Here, \( c_0, H \) and \( \tilde{e}^p \) are respectively the initial cohesion, hardening modulus and equivalent plastic strain.

The plastic potential function of D-P model is

\[
 g_i = \sqrt{\frac{1}{2} s : s + \frac{3 \tan \psi}{\sqrt{9 + 12 \tan^2 \psi}}} p
\]

(4)

with \( \psi \) being the dilation angle.

Taking into account the fact the soil does not bear tension, three new yield functions are introduced,

\[
 f_2 = \sigma_1; f_3 = \sigma_2; f_4 = \sigma_3;
\]

(5)

And associated tensile flow rule is adopted by \( g_2 = \sigma_1, g_3 = \sigma_2, g_4 = \sigma_3 \). Note that the constitutive integration induced by the multiple yield surfaces could be easily tackled by the Gauss Seidel projection contraction (GSPC) method [11]. Owing to the DCM method applied in this paper, no attention is needed in solving the elastoplastic tangent matrix for the multiple yield surfaces, because DCM method just adopts the elastic tangent matrix. This paper adopts the GSPC approach to solve the constitutive integration of multiple surfaces. It is worth attention that GSPC could easily reach the same accuracy as the implicit return mapping method, without the labor of calculation for the Jacobian matrix of the multiple yield surfaces[2].

It must be noted here that the detection and evolution approaches of the shear band is based on those in the literature[12], which is not repeated here. Once the strong discontinuity shear band is introduced, the interface behavior is governed by the contact treatment of NMM where open close iteration would be used.

4. Numerical example

As depicted in Figure 1, a soil dam is subjected to the displacement controlled loading by the rigid foundation. The shape of the soil dam is square with the edge length of 12.5m. The right side of the soil dam is constrained in horizontal direction. \( \delta \) is the ultimate compressive displacement. The bottom side of the soil dam is constrained in both horizontal and vertical directions. The soil is assumed to be under plane strain condition. The material properties are defined by: Young’s modulus \( E = 30MPa \), Poisson’s ratio \( \nu = 0.2 \), initial cohesion \( c = 63kPa \), friction angle \( \phi = 24^\circ \) and dilation angle \( \psi = 5^\circ \), mass density \( \rho = 2000kg/m^3 \). Hardening modulus is \( H = 0kPa \). The average velocity of loading is about \( 10^{-3} \text{mm/s} \). By DCM, the incremental displacement is specified on the degree of freedom in the middle of the side which is under the rigid foundation.
As illustrated in Figure 2, three types of meshes, ranging from coarse meshes in the left to the fine meshes in the right, are used to simulate the growth of shear bands whose final evolution curves are represented by the red lines. As seen in Figure 2, three evolution paths are highly similar. Hence, the evolution path of shear band is mesh independent. It should be noted that no initial imperfection is incorporated in the numerical model to trigger the onset of shear band. The stress distribution induced by the loading is not uniform because of the force boundary conditions of the problem is not symmetric. The final shear band obtained is along a curve path, which were also observed in [13] and [14]. However, the shear band in [15] evolves along a straight line which is not satisfactory, because the simulation is mesh dependent. It should be noted that the simulation results in this paper cannot be directly compared with those in [13–15], because this paper hasn’t allowed for the effect of fluid, which was considered in [13–15].

Figure 2. The evolution paths with three different types of meshes which are coarse, medium and fine respectively.

Figure 3 depicts the four contours of effective plastic strain corresponding to four representative locations where shear band tip has stopped during the whole evolution process. The effective plastic strain band, similar to the shear band, initiates in the element adjacent to the right end of the rigid foundation, then evolves along a curve until the band cuts through the soil dam. We observe that the band of equivalent plastic strain evolves faster than that of the strong discontinuity shear band, because being plastic is precondition of initiation of shear band.

When material nonlinearity and contact nonlinearity are coupled, it poses a challenge for DCM to reach converge. Therefore, it is noticed that when the shear band is introduced, the number of plastic iterations is much bigger than that before the occurrence of shear band.

Figure 3. Contours of effective plastic strain.
It should be noted that for better accuracy of the simulation, not only the loading speed must be low enough, but also the evolving step length must keep short enough, so that the shear band has enough time to go through the localized deformation area around the tip.

5. Conclusions
Contrast to the conventional discretization order in the dynamic analysis, the time discretization is carried out before the space discretization in this paper so that the transferring issue of degrees of freedom could be easily solved in the problem of the dynamic evolution of shear band. In this way, instead of the degrees of freedom at nodal points, which are valid only in the current time step, the total stress and the speed at the integration points are saved and transferred to the next step.

The non-associated Drucker-Prager model is incorporated into the NMM to account for the behavior of soil. The tensile part of the DP model is cut off, giving rise to multiple yield surfaces, of which the constitutive integration is reduced to mixed complementarity problem, which is solved by the GSPC method.

The proposed approach could simulate the evolution of shear band in a mesh independent manner. The equivalent plastic strain band initiates and evolves earlier than the strong discontinuity shear band. Both bands evolve along curve paths.

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