Almost Readily Detectable  
Time Delays from Gravity Waves ?  

Redouane Fakir  

Moroccan Center for Scientific Research  
52 Ave. Omar Ibn Khattab  
BP 1364 Agdal, Rabat  
MOROCCO  

Cosmology Group, Department of Physics  
University of British Columbia  
6224 Agriculture Road  
Vancouver, B. C. V6T 1Z1  
CANADA  

April 1993  

Abstract  

When a source of gravity waves is conveniently placed between the Earth and some source of light, preferably a pulsating source, the magnitude of time delays induced by the gravity waves could, in optimal situations, be not too far out of the reach of already existing technology. Besides the odd case of near-to-perfect alignment one might be lucky enough to encounter in the Galaxy, there exists several astronomical sites where good alignment occurs naturally. A good example is when the light source and the gravity-wave source are, respectively, a high-frequency pulsar and a neutron star, locked together in a tight binary. We are lead to believe that neutron-star gravity waves might be directly observable in timing data of systems such as PSR B1913+16.
I. Introduction

It is now well established that any direct, unequivocal detection of gravity waves would have momentous consequences on both observational astronomy and fundamental physics[1-19]. However, due (so we think,) to the extreme weakness of the waves when they reach the Earth, no such detection has yet occurred, despite almost thirty years of vigorous efforts and considerable progress on both the theoretical and the technological fronts. It is particularly unfortunate that some of the most copious and most interesting gravity waves, the low-frequency waves from astrophysical sources such as binary stars, seem bound to remain out of the reach of the few developed detectors, chiefly because of seismic noise. It is hoped, nevertheless, that space-based experiments will be able to detect low frequency waves by the beginning of the next century.

Detection prospects are no brighter for the shorter waves expected from neutron stars. The extreme weakness of these puts them hopelessly out of detectability range for most of the techniques envisaged so far.

Recently, however, the case was made that there were ways, after all, of detecting astrophysical gravity waves (such as those from binary stars and neutron stars) sooner than previously thought [20-24]. The basic idea is that there are situations where one can observe gravity waves where they are still relatively strong, that is, very near the source. If it is ultimately confirmed that this is indeed doable, then the observational situation could be suddenly improved by over several orders of magnitude. Of course, even if ultimately successful, this approach cannot replace the vigorous and long-term effort necessary for detecting gravity waves in the generic case, where the waves can come from arbitrary directions and with the usual extremely small amplitudes.

In [21], it was shown that a gravitational “pulse with memory of position” that happens to go past the Earth, could be detected and thereafter “followed” in its receding journey, through its refraction of starlight. The expected angular shifts were estimated at $\Delta \phi \approx h$, where $h$ is the gravity wave’s strength when it hits the Earth. Although this effect comes with some neat features, it could not be of more than a purely academic interest, since quantitatively the shifts should be smaller than $10^{-15}$arcsec for typical sources. Since the best angular resolutions one can hope for at the present are of the order of $10^{-7}$arcsec (for radio-waves,) the experimental situation
for this effect seemed as bad as (if not worse than) for previously considered approaches.

In [22], it was shown how in certain, not uncommon astronomical configurations, a similar (though not identical) refraction effect could be at work, with the difference that the expected shifts in apparent positions of light sources are many orders of magnitude larger than in [21].

The desirable configurations here are those that have the gravity wave source be situated between (and nearly aligned with) the Earth and the light source. The optimal angular shifts are shown to be $\Delta \phi \sim \pi h(\Lambda)$. But here $h(\Lambda)$ is the strength of gravity waves at a distance of one reduced gravitational wavelength from the source, which is many orders of magnitude larger than the strength at Earth’s level.

It is not too difficult to find astronomical sites, amidst the rich diversity of the galactic zoo, where these apparent shifts should be near the limits of observability (see below and [22-24].) For example, one can get an extremely good “alignment” if one takes as the gravitational and the electromagnetic sources (hereafter dubbed SG and SL respectively,) the two components of a binary system (fig.(1)). Once every orbital period, the light from one member of the binary (SL) passes very near the other member (SG) before starting its journey towards the Earth. Then, the constraint on the alignment becomes here a constraint on the inclination with respect to the Earth of the binary’s orbital plane. Given the extreme proximity of the members of many known tight binaries, this constraint turns out to be actually satisfied for several known candidates.

In the present paper, we suggest that gravity waves could soon be detected directly through yet another effect. The astronomical configurations exploited here are similar to those considered in [22], but the effect itself has little to do (experimentally, of course,) with apparent shifts in angular positions. What will be calculated here is the Shapiro time delay experienced by a photon that grazes a gravity-wave source SG, while on its journey to the Earth. The gravity-wave induced variations in this time delay will be shown to fall not far from (if not within) current time resolution limits. Experimentally, this latter approach may well be easier to implement than that of [22], which relied on the more complicated technology of high angular resolution power.

There is more than one type of astronomical site where one can hope to detect gravity waves in this fashion. In real life, several factors have to be
weighed in the selection of candidate sites, including whether the magnitude of the light source SL is large enough in regard of the available integration time. More detailed predictions for specific sites will be dealt with elsewhere [23-24]. Here, we shall first derive the main formulae for the effect (Section II), and then estimate the orders of magnitude involved by focusing on the two specific illustrations of [22].

In the first illustration, the light source SL and the gravity-wave source SG form a tight binary system (fig.(1)). SL is a high frequency pulsar (say a millipulsar,) while SG is a substantially slower neutron star. Once every binary rotation period, the light pulses from SL pass very near SG before traveling on towards the Earth. The pulsar’s frequency, as observed from the Earth, should vary on (at least) two different time scales: one related to the binary’s rotation (variation of the impact parameter of SL’s photons with respect to SG,) the other, the object of our scrutiny, related to the relatively strong curvature fluctuations radiated by the neutron star SG.

In the second illustration, SL is again a pulsar, but SG is some much closer, unrelated gravity-wave source, such a binary of ordinary or giant stars, that happens to lie not far from the direction of the distant pulsar. Because, as shows the calculation below, the constraint on the alignment of SG and SL is not too stringent, and because such a large proportion (close to 50%) of the stars in the Galaxy are in binaries or multiple-star systems, the chances of a successful hunt for good candidates seem fairly high for this second case as well. Note that we have just come across an earlier reference where the possibility of using pulsar-timing data to detect gravity waves from binary stars was envisaged [25].
II. Equations for the Effect

Take then a spherical transverse-traceless coordinate system centered on SG. Consider, for simplicity, the effect of only one of the two polarization components of the gravity waves radiated by SG. (These two components are expected to be, in most cases, of comparable strengths.) Consider also that the problem is completely contained in the plane defined by SL, SG and the Earth. This implies choosing the axes of projection of the wave so that only deformations of geodesics in that plane are considered.

With these simplifications, a null geodesic stretching from the light source SL to the Earth, and passing near the gravitational source SG, can be described by the line-element

$$\frac{dt^2}{2} - dr^2 - (1 + h)r^2 d\phi^2 = 0 \quad .$$

(1)

\(h\) is the gravity-wave amplitude, which can be cast in the form

$$h = \frac{H}{r} \exp\{i\Omega(r - t + t_{ph})\} \quad ,$$

(2)

where \(H\) is a constant that is characteristic of the source SG. \(\Omega\) is the waves frequency component under consideration. (A source such as a neutron star could emit gravity waves at various frequencies due to different physical phenomena. See [1] and references therein.) \(t_{ph}\) fixes the phase of the wave at one end of the geodesic. Thus, in the following, \(t_{ph}\) could be the time that a terrestrial observer reads on her clock at the arrival of a given photon, while \(t\) is just an internal parameter (e.g. expressible in terms of \(r\) or \(\phi\)) for the trajectory. The quantity we are ultimately interested in, the rate of change in the gravity-wave induced time delay, is the variation, with respect to \(t_{ph}\), of the integral of \(t\) from the light source SL to the Earth.

We have not included the background Schwarzschild curvature in (1). Usually, the time delay related to that contribution can be easily subtracted from the observations. However, for extremely close encounters of SL’s photons with SG, (impact parameters smaller than one gravitational wavelength,) it is not always trivial to disentangle (dipole) Newtonian contributions from relativistic ones.
Using the radial coordinate \( u \equiv 1/r \) we can write (1) in the form

\[
u'^2 - \frac{u'^2}{u^2} = 1 + h \ ,
\]

where primes indicate derivatives with respect to \( \phi \).

For \( H = 0 \), the photon follows a straight trajectory given by

\[
u_0 = \frac{\sin \phi}{b} \ , \quad t_0 = -b \cot \phi \ ,
\]

where \( b \) is the distance of closest approach (the “impact parameter”) of the photon from SG’s position in the absence of gravity-waves.

Let us try to solve the problem perturbatively by writing

\[
u = \nu_0 + \nu_1 \ , \quad t = t_0 + t_1 \ ,
\]

where the gravity-wave-induced fluctuations \( \nu_1 \) and \( t_1 \) should be of first order in \( h \).

Using (5), (3) becomes

\[
u_0^2 t_0^2 \left( 1 + 2 \frac{\nu_1}{\nu_0} + 2 \frac{t_1'}{t_0'} \right) - \frac{\nu_0'^2}{\nu_0^2} \left( 1 + 2 \frac{\nu_1'}{\nu_0'} - 2 \frac{\nu_1}{\nu_0} \right) = 1 + h \ .
\]

Here, as throughout this paper, only first-order terms are retained in the calculations.

As can be seen from (4), the zeroth order parts of \( \nu \) and \( t \) verify

\[
u_0^2 t_0^2 - \frac{\nu_0'^2}{\nu_0^2} = 1 \ .
\]

Combining this with (6) yields

\[
\frac{t_1'}{b} = \frac{h}{2} - \frac{1 + \cos^2 \phi}{\sin^3 \phi} \nu_1 + \frac{\cos \phi}{\sin^2 \phi} \nu_1 ' \ ,
\]

where we used \( \nu_0^2 t_0 = 1 \) (see (4)).
Hence, the gravity-wave-induced perturbation of a photon’s time of flight from the light source SL to the Earth, is given by the compact expression

$$\Delta t_1 \equiv t_1(\phi_{final}) - t_1(\phi_{initial}) = b^2 \left( \frac{\cos \phi}{\sin^2 \phi} u_1 \right)_{\phi_{initial}}^{\phi_{final}} + \frac{b}{2} \int_{\phi_{initial}}^{\phi_{final}} h d\phi \quad (9)$$

Furthermore, the photon paths we are interested in are those which start and end at the fixed positions of the light source and the observer, respectively [26]. These paths have

$$u_1(\phi_{final}) = u_1(\phi_{initial}) = 0 \quad (10)$$

and so, the first term in (9) vanishes. (Of course, this holds only to first order in $h$. Corrections due, e.g., to the fluctuation of SL’s and the observer’s positions are of higher order, and contribute a negligible amount to the integral.)

We can now write the rate of change in the gravity-wave-induced time delay (see (2)):

$$\dot{\tau} \equiv \frac{d}{d\tau_{ph}} \Delta t_1 = \frac{ib\Omega}{2} \int_{\phi_{initial}}^{\phi_{final}} h d\phi \quad (11)$$

To first order, we can use (4) to write $h$ as

$$h = \frac{H}{b} \sin \phi \exp \left\{ ib\Omega \frac{1 + \cos \phi}{\sin \phi} \right\} e^{i\Omega \tau_{ph}} \quad (12)$$

Thus, we obtain

$$|\dot{\tau}| \approx \frac{1}{2} \frac{\Omega H}{b} \left| \int_{\phi_{initial}}^{\phi_{final}} e^{i\Omega \tau_{ph}} \sin \phi \exp \left\{ ib\Omega \frac{1 + \cos \phi}{\sin \phi} \right\} d\phi \right| \quad (13)$$

The variation of $|\dot{\tau}|$ with the impact parameter is plotted in fig.2 for the generic case: $\phi_{initial} \to 0$ , $\phi_{final} \to \pi$.

We have neglected here contributions from the time variation of the impact parameter $b$. In the solar (Schwarzschild-metric) case, the variation of $b$ (due to the Earth’s revolution around the Sun.) plays a central role in making
the Shapiro time delay observable (see e.g. [9]). In contrast, the effect in the present gravity-wave case is made observable mainly by the variation of the metric itself. For most conceivable configurations where the effect described here might be at work, the time scale of $b$-variations is much larger than the gravity-waves typical period (see next section.)
III. Order-of-magnitude predictions

The observational consequences of eq. (13) depend closely on the characteristics of the particular astronomical site considered. Here we derive typical orders of magnitude to be expected from the two types of astronomical configurations considered in [22]. A more detailed analysis of a few actual candidate sites is carried out elsewhere [24].

A word, first, about the additional contribution to $\dot{\tau}$ of the time variation of the impact parameter $b$, which we have not included in our formulae. As we pointed out earlier, it is that time variation which makes the time delay observable in the Schwarzschild case. In the calculation of the solar time delay, for example, the metric itself is static, so that the only time dependence in the time-delay formula comes from the $db/dt_{ph}$ produced by the Earth’s revolution around the Sun. (See (2) for the definition of $t_{ph}$.) In our radiative case, the contribution to $\dot{\tau}$ from the time variation of the metric far exceeds that from the time variation of $b$. That is to say, one usually has $db/dt_{ph} << b\Omega$. Even when SG and SL form a binary, which is a typical case where $b$’s time modulation could be expected to play a role, $db/dt_{ph}$ is of the order of 100km/sec, or about $3 \times 10^{-4}$ in geometrized units. $b\Omega$, on the other hand, comes to about 300 in the same units.

Going back to (13), the integral there is of order unity for the small impact parameters $b\Omega \approx 1$ (see fig.(2)), that is, for $b$ of the order of one reduced gravitational wavelength $\Lambda \equiv 1/\Omega$. Hence,

$$|\dot{\tau}|_{b \approx \Lambda} \sim \Omega H = |h(r = \Lambda)|. \quad (14)$$

This is, of course, several orders of magnitude larger than similar time-delay effects expected from background (essentially plane) gravitational waves [29].

For larger values of $b$, the integral in (13) decreases very roughly as $1/b\Omega$ (fig.(2)). This implies that the effect’s order of magnitude is given by the very approximate formula (see (13,14))

$$|\dot{\tau}| \sim \frac{H}{b} = |h(r = b)|. \quad (15)$$

Hence, the effect is weaker by about one order of magnitude when the rays
from SL graze SG at ten, rather than at one reduced gravitational wavelength. The observational significance of this formula will become clear when we apply it to some known astronomical sites such as tight binary pulsars (below.)

To put some numbers on the above analysis, let us consider the same two show-cases of [22].

In the first, the source of gravity waves SG is a binary star like, say, the binary of giant stars \(\mu\)-Sco in the Scorpio constellation. That system should be generating gravity waves at a frequency of about \(1.6 \times 10^{-6}\)Hz, and with a strength such that the constant \(H\) in (2,12) is about 6cm. These numbers produce a maximum time-delay variation \(\dot{\tau} \sim 10^{-15}\) to \(10^{-14}\), which is just about what technology can handle today. (Note that the gravity-wave period is typically half the dynamical time-scale of the source.)

The remaining issue, of course, is how likely or unlikely it is to find, for a given pulsar (SL), an intervening binary star (SG) lying sufficiently close to the direction of that pulsar in the sky. The constraint is that the angular separation of SL and SG should be within one order of magnitude of the ratio \(\Lambda/D\), where \(D\) is the distance from SG to the Earth. This means that binaries such as \(\mu\)-Sco, which is at \(D = 109\)pc, should be within a few seconds of arc of the pulsar SL. Note that the reduction in the probability of finding a good SG candidate for a given SL, due to the decrease of the allowed SL-SG angular separation with \(D\), is partially compensated by the fact that many of the known pulsars lie at very large distances (a few or several kpc.) Since about half the stars in the Galaxy are in double or multiple stars, a solid angle of a few seconds of arc, in most directions, is not unlikely to contain a good SG candidate. The binaries one encounters in the literature are usually, for obvious reasons, the closest ones to the Earth (e.g. \(\mu\)-Sco.) But, as is clear from all the above, the closeness of SG to the Earth is not indispensable for observing this effect. Thus, the spectrum of observationally interesting objects is widely broadened in this approach. High angular resolution space-based missions like the Hypparcos satellite (sky mapping with a resolution of about \(10^{-4}\) arc), could be instrumental in identifying cases of sufficient SL-SG alignment in the Galaxy.

The second show-case used to illustrate [22] had SL again be a rapid pulsar, but SG was now some more slowly rotating neutron star, tied with SL in a binary system (fig.(1)). Take SG to be a neutron star like Vela, with a frequency of 22Hz, and a gravity-wave strength such that \(H \sim 10^{-4}\)cm, sup-
posing that the mechanism of gravity-wave generation is something like the so-called CFS (Chandrasekhar-Friedman-Schutz) instability [27,28]. Than the maximum time-delay variation observed in SL's data comes to a few times $10^{-14}$.

The alignment problem is almost automatically solved, in this case. (We name below a known astronomical site where sufficient alignment occurs naturally.) We recall that the alignment constraint, for this type of site, is really a constraint on the orbital inclination of the SG-SL binary with respect to the observer's line of sight: the closer the system is to being an “eclipsing” binary, the smaller a minimal impact parameter the light from SL will reach as SL goes around SG (see fig.(1).) The numbers used above and in [22] actually allow for orbital inclinations that are many degrees away from the eclipsing value, which leaves room for statistically many potential candidates (see Introduction and below.)

We close this paper with a name and a celestial address of a site where, according to this study, a truly direct observation of gravity waves could already be within the realm of possibility. Take, for instance, the well documented Taylor pulsar (PSR B1913+16. See e.g. [30]). This is a 17Hz pulsar which is in very close, highly eccentric orbit around a slightly lighter and much darker neutron star companion. The gravitational wavelength of the latter should not be much smaller than about one light-second. Every $7^h45'$ or so, the pulsar comes to within only half a solar radius of its companion. That is also a distance of just about one light-second. Hence, we have here a known case (and there are a few more [24],) where $b/\Lambda$ might be close to unity. It seems therefore safe to predict that, before long, neutron-star gravity waves could be seen directly in this system and alike. (To give a precise estimate of $\dot{\tau}$ for this system one would need, of course, to take into account the precise disposition in space of the orbit. But this is unlikely to spoil order-of-magnitude estimates.)

In the ongoing search for the effects of background gravity-waves in pulsar-timing data, the focus has been, so far, on stochastic, and in particular on cosmological gravity waves [29]. Could part of the large data set already collected bare, hidden, the signature of monochromatic sources such as neutron
stars or binary systems?

Acknowledgements

I benefited greatly from discussions with S. Braham, F. Gaitan and W. Shuter. I am particularly indebted to my teacher W.G. Unruh for his relentless advice and support, as well as for refusing to buy into earlier versions of this paper, which lead me to discover some nasty mistakes therein.

This research was supported, in part, by the Cosmology Group in the Department of Physics, University of British Columbia.
References

[1] K.S. Thorne, in 300 Years of Gravitation, S.W. Hawking and W. Israel, Eds.(Cambridge University Press, Cambridge, 1987.)

[2] K.S. Thorne, Science 256, 325 (1992).

[3] K.S. Thorne, in Recent Advances in General Relativity, A. Janis and J. Porter, Eds.(Birkhauser, Boston, 1992.)

[4] L.P. Grishchuk, Annals of the New York Academy of Sciences, vol.302, 439 (1977).

[5] A. Einstein, Preuss. Akad. Wiss. Berlin, Sitzungsberichte der physicalisch-mathematischen Klasse, p.688 (1916).

[6] A. Einstein, Preuss. Akad. Wiss. Berlin, Sitzungsberichte der physicalisch-mathematischen Klasse, p.154 (1918).

[7] H. Weyl, Space-Time-Matter. Methuen:London (1922).

[8] A.S. Eddington, The Mathematical Theory of Relativity, 2nd edn, Cambridge University Press, Cambridge (1924).

[9] C.W. Misner, K.S. Thorne, J.A. Wheeler, Gravitation (Freeman, San Fransisco,1973.)

[10] M. Zimmerman and K.S. Thorne, in Essays in General Relativity, F.G. Tipler, ed. (Academic Press: New York,1980.)

[11] J.R. Bond and B.J. Carr, Monthly Notices of the Royal Astronomical Society 207, 585 (1984).

[12] L. Smarr, Sources of Gravitational Waves (Cambridge University Press, Cambridge, 1979.)

[13] see reviews in N. Deruelle and T. Piran, Gravitational Radiation (North Holland: Amsterdam, 1983.)
[14] J. Weber, *Physical Review* 117, 306 (1960).

[15] see for example J.A. Tyson and R.P. Giffard, *Annual Reviews of Astronomy and Astrophysics* 16, 521 (1978), E. Amaldi and G. Pizella, in *Relativity, Quanta and Cosmology in the Development of the Scientific Thought of Einstein*, vol.1, 96 (1979), and references in D.G. Blair, *The Detection of Gravitational Waves* (Cambridge University Press, Cambridge, 1991) and P.F. Michelson, J.C. Price and R.C. Taber, *Science* 237, 150 (1987).

[16] D. Christodoulou, *Physical Review Letters* 67, 1486 (1991).

[17] K.S. Thorne, *Physical Review D* 45, 520 (1992).

[18] R.E. Vogt, *The U.S. LIGO Project* in *Proc. of the Sixth Marcel Grossmann Meeting on GRG, MGC, Kyoto, Japan, 1991*.

[19] C. Bradaschia et. al., *Nucl. Instrum. & Methods*, A289, 518 (1990).

[20] R. Fakir (1991), *Gravity Waves and Hipparcos*, proposal presented in *Table Ronde on Moroccan-European Collaboration in Astronomy*, Moroccan Center for Scientific Research Archives, Rabat, Morocco.

[21] R. Fakir (1992), *Gravitational-Wave Detection, a Non-Mechanical Effect*, The Astrophysical Journal, vol.418, 20.

[22] R. Fakir (1993), *Gravity-Wave Watching*, submitted for publication.

[23] R. Fakir (1993), *Early Direct Detection of Gravity Waves*, UBC preprint UBCTP-93-016.

[24] R. Fakir (1993), *Gravity Waves and Light*, in preparation.

[25] M.V. Sazhin, *Sov. Astron.-AJ* 22, p36 (1978).

[26] I thank W.G. Unruh for pointing this out to me.

[27] S. Chandrasekhar, *Physical Review Letters*, **24**, 611 (1970).

[28] J.L. Friedman and B.F. Schutz, *The Astrophysical Journal*, **222**, 281 (1978).
[29] See, for example, S. Detweiler, *Astrophys. J.* 234, 1100 (1979); R.W. Romani and J.H. Taylor, *Astrophys. J. Lett.* 265, L65 (1983); R.W. Hellings and G.S. Downs, *Astrophys. J. Lett.* 265, L39 (1983); D.R. Stinebring, M.F. Ryba, J.H. Taylor and R.W. Romani, *Phys.Rev.Lett.* 65-3, 285 (1990) and references therein.

[30] J.H. Taylor, *Phil. Trans. R. Soc. Lond. A* 341, 117-134 (1992).
Figure captions

Figure 1: A pulsating light source SL (e.g. a centi- or milli-second pulsar) and a gravity wave source SG (e.g. a slower neutron star) are locked in a tight binary system (with a possibly highly excentric orbit.) Viewed from the Earth, the two companions are indistinguishable and SG is invisible for most cases of interest. But, once every few or several hours, the frequency of light pulses should be modulated by SG’s gravity waves (see caption of fig.(2).)

Figure 2: The magnitude of the effect is shown to decrease roughly as $1/b$. This implies that the modulation mentioned in the caption of fig.(1) is of the order of the gravity-wave amplitude at a distance from the source that is equal to the impact parameter (i.e. $|\tau| \sim h(r = b)$; see eqs.(17-19).) Hence, this effect is many orders of magnitude larger than the effects the same waves may cause in the vicinity of the Earth.