Impacts of Dust Feedback on a Dust Ring Induced by a Planet in a Protoplanetary Disk

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Abstract

When a planet forms a deep gap in a protoplanetary disk, dust grains cannot pass through the gap. As a consequence, the density of the dust grains can increase up to the same level of the density of the gas at the outer edge. The feedback on the gas from the drifting dust grains is not negligible in such a dusty region. We carried out two-dimensional two-fluid (gas and dust) hydrodynamic simulations. We found that when the radial flow of the dust grains across the gap is halted, a broad ring of dust grains can be formed because of the dust feedback and the diffusion of the dust grains. The minimum mass of the planet needed to form the broad dust ring is consistent with the pebble-isolation mass in the parameter range of our simulations. The broad ring of dust grains is a good environment for the formation of the protoplanetary solid core. If the ring is formed in the disk around a Sun-like star at \( \sim 2 \) au, a massive, solid core \( (\sim 50 \, M_\oplus) \) can be formed within the ring, which may be connected to the formation of hot Jupiters holding a massive, solid core, such as HD 149026b. In the disk of a dwarf star, a number of Earth-sized planets can be formed within the dust ring around \( \sim 0.5 \) au, a phenomenon that potentially explains a planet system made of multiple Earth-sized planets around a dwarf star such as TRAPPIST-1.

Key words: accretion, accretion disks – planet–disk interactions – planets and satellites: formation – protoplanetary disks

1. Introduction

In a protoplanetary disk, a giant planet such as Jupiter can form a density gap that is due to strong disk–planet interaction along with its orbit (e.g., Lin & Papaloizou 1979, 1986; Goldreich & Tremaine 1980). Recent observations have discovered bright rings and dark gaps of the dust grains (e.g., Fukagawa et al. 2013; van der Marel et al. 2013; Pérez et al. 2014; Akiyama et al. 2015, 2016; ALMA Partnership et al. 2015; Momose et al. 2015; Muto et al. 2015; Isella et al. 2016; Kataoka et al. 2016; Nomura et al. 2016; Tsukagoshi et al. 2016; Fedele et al. 2017; van der Plas et al. 2017; Dong et al. 2018). These structures can be associated with the planets embedded within the protoplanetary disk (e.g., Díaz et al. 2015; Dong et al. 2015; Kanagawa et al. 2015; Picogna & Kley 2015; Jin et al. 2016; Rosotti et al. 2016; Díaz et al. 2018b).

A planet is formed by an accumulation of dust grains in the protoplanetary disk. Because of the importance of the dust grains, the evolution of the dust grains in the disk has been investigated (e.g., Nakagawa et al. 1986; Youdin & Shu 2002; Tanaka et al. 2005; Birnstiel et al. 2012; Okuzumi et al. 2012; Drążkowska et al. 2016; Ida & Guillot 2016). When the planet forms the density gap, relatively large dust grains (so-called pebbles) are trapped by the pressure bump that is formed at the outer edge of the gap. As a result, the ring structure in which the dust grains are highly concentrated can be formed at the outer edge (e.g., Paardekooper & Mellema 2004; Muto & Inutsuka 2009; Zhu et al. 2012; Dong et al. 2015; Pinilla et al. 2015, 2016). The dust grains move inward because of friction with the surrounding disk gas, and at the same time, the disk gas feels feedback from the dust grains. Although the feedback is negligible if the dust grains are small and do not pile up, we need to consider the effects of feedback if the dust grains are relatively large or highly concentrated. In this case, the dust feedback can significantly influence the structure of the disk gas (Fu et al. 2014; Gonzalez et al. 2015; Taki et al. 2016; Díaz et al. 2017; Gonzalez et al. 2017; Kanagawa et al. 2017; Díaz et al. 2018a; Weber et al. 2018). Recently, by performing two-fluid two-dimensional hydrodynamic simulations, Weber et al. (2018) have found that although the dust feedback does not significantly affect the total amount of dust transported through the planetary gap, it substantially changes the location at which the dust accumulates outside of the planet’s orbit.

Within the ring structure of dust grains in which the dust grains are highly concentrated, planetesimals and a solid protoplanetary core can be effectively produced. The formation of the solid core in the ring structure with the high concentration of dust grains has been investigated by several previous studies (e.g., Kobayashi et al. 2012; Pinilla et al. 2015). However, the effects of the dust feedback have not been considered in previous studies. The effects of dust feedback on the ring structure need to be investigated; they might be connected to the formation of some strange planets that are difficult to understand in the current theoretical framework. For instance, compact, hot Jupiters that have a small radius compared with their mass (that is, with a high mean density), like HD 149026b, have been observed (Sato et al. 2005; Hebrard et al. 2013). Such a hot Jupiter is thought to have a massive, solid core of \( \sim 50 \, M_\oplus \) (Fortney et al. 2006; Ikoma et al. 2006). Only in an environment with a very high density of dust grains can these hot Jupiters be formed (Ikoma et al. 2006). Moreover, recent observations have revealed several Earth-sized planets orbiting around the dwarf star named...
TRAPPIST-1 (Gillon et al. 2016, 2017). Since the mass of the solid materials would be small in the protoplanetary disk around the dwarf star, it is difficult to understand the formation of a multitude of planets in such a disk. Recently, Ormel et al. (2017) have shown that if the disk of the dwarf star extends to the same extent as that of a Sun-like star (∼100 au), several Earth-sized planets can be formed around the water–snow line. Alternatively, Haworth et al. (2018) have shown that it is possible if the mass of the disk is massive. If a wide ring of dust grains is formed outside the gap, however, a multitude of Earth-sized planets may be able to form within this ring, even when the disk is compact and less massive.

In this paper, we investigate the effects of dust feedback on the ring structure of dust grains that is formed at the outer edge of the planet-induced gap, by using two-fluid (gas and dust) two-dimensional hydrodynamic simulations. In Section 2, we describe the basic equations. We show the result of the hydrodynamic simulations in Section 3. In Section 4, we discuss the effects of planetesimal formation on the ring structure and implications for planet formation within the dust ring and observations. We also discuss the validity of our simulations in this section. Our summary is included in Section 5.

2. Basic Equations and Numerical Method

2.1. Basic Equations for Dust Grains and Disk Gas

We simulate the structures of disk gas and dust grains in the protoplanetary disk with the planet. In this paper, we assume a geometrically thin and non-self-gravitating disk. We do not consider the detail of the vertical structure of the disk, and we deal with the vertically averaged values of physical quantities, such as the surface density defined by $\Sigma = \int_{-\infty}^{\infty} \rho dz$, where $\rho$ is the density of gas or dust grains. We use two-dimensional $(R, \phi)$ coordinates, and their origin is set on the position of the central star. The gas and dust velocities are expressed by $V = (V_R, V_\phi)$ and $v = (v_R, v_\phi)$, respectively, and the surface densities of the gas and the dust grains are written by $\Sigma_g$ and $\Sigma_d$, respectively. In the following, the subscripts $g$ and $d$ indicate values for gas and dust grains, respectively. We treat dust grains as a pressureless fluid.

The equations of motions for dust grains in the radial and azimuthal directions are given by

$$\frac{\partial v_R}{\partial t} + (V \cdot \nabla) v_R - \frac{v_R^2}{R} = -\frac{\partial \Psi}{\partial R} - \frac{v_R - v_R^{\text{stop}}}{\tau_{\text{stop}}},$$  
$$\frac{\partial v_\phi}{\partial t} + (V \cdot \nabla) v_\phi + \frac{v_R v_\phi}{R} = -\frac{1}{\tau_{\text{stop}}} \left( \frac{v_R - v_\phi}{\rho} \right).$$

The last terms in the right-hand sides of Equations (1) and (2) represent drag forces between disk gas and dust grains. The stopping time of dust grains, $\tau_{\text{stop}}$, depends on the size of the dust grains. For simplicity, we consider only dust grains in the Epstein regime, which is reasonable for the range of dust grains considered in this paper. In this case, the stopping time is written by Takeuchi & Lin (2005) as

$$\tau_{\text{stop}} = \frac{\pi s_4 \rho_p}{2 \Sigma_g \Omega_K},$$

where $s_4$ and $\rho_p$ are the size and internal density of the dust grains, and $\Omega_K = \sqrt{GM_p/R^3}$ is the Keplerian angular velocity, where $G$ is the gravitational constant and $M_p$ is the mass of the central star. For convenience, we define the Stokes number of the dust grains as

$$St = \tau_{\text{stop}} \Omega_K.$$

The gravitational potential $\Psi$ is given by

$$\Psi = -\alpha \frac{GM_p}{R} \left[ R^2 + 2RR_p \cos(\phi - \phi_p) + R_p^2 \right] + \alpha \frac{GM_p}{R} \frac{1}{2} \left( 2 + \epsilon^2 \right)^{1/2},$$

where $M_p$ is the mass of the planet, which is located at $(R_p, \phi_p)$, where $R_p$ and $\phi_p$ are the orbital radius of the planet and the azimuthal angle of the planet, respectively. The softening parameter is denoted by $\epsilon$. For disk gas, the equations of motion in the radial and azimuthal directions are

$$\frac{\partial V_R}{\partial t} + (V \cdot \nabla) V_R - \frac{V_R^2}{R} = -\frac{\partial \Psi}{\partial R} - \frac{\partial \psi}{\partial R} + \frac{f_R}{\Sigma_g} - \frac{\Sigma_d}{\Sigma_g} \frac{v_R - v_R}{\tau_{\text{stop}}},$$

where we adopted a simple isothermal equation of state, in which a vertically averaged pressure is given by $\epsilon \Sigma_g \nu$, where $\epsilon$ is an isothermal sound speed. The viscous forces in the radial and azimuthal directions are represented by $f_R$ and $f_\phi$, respectively (see Nelson et al. 2000). We adopt an $\alpha$ prescription (Shakura & Sunyaev 1973), and hence the kinetic viscosity $\nu$ is expressed by $\alpha \Sigma_g \Omega_K$, where $\alpha$ is a disk scale height of the gas. In this paper, we assume a constant $\alpha$ throughout the disk.

The continuity equation for the gas in the two-dimensional disk is written by

$$\frac{\partial \Sigma_g}{\partial t} + \nabla \cdot (\Sigma_g V) = 0.$$

The turbulence in the gas disk drives random motion of dust grains, which induces diffusion of the dust grains (e.g., Cuzzi et al. 1993; Youdin & Lithwick 2007). Hence, considering this turbulence diffusion of the dust grains, we obtain the vertically averaged continuity equation of dust grains as

$$\frac{\partial \Sigma_d}{\partial t} + \nabla \cdot F_{M,d} = 0,$$

where a mass flux of the dust grains is

$$F_{M,d} = \Sigma_d v + j,$$

where $j$ represents a diffusive mass flux. Assuming diffusion caused by the spatial gradient (the gradient diffusion hypothesis), we can describe the diffusive mass flux as (e.g., Cuzzi et al. 1993;...
where $D$ is the turbulent diffusivity of the dust grains. The gradient diffusion hypothesis is appropriate for dust grains with small $St$ (e.g., Fromang & Papaloizou 2006; Okuzumi & Hirose 2011). This hypothesis may be inappropriate if the dust grains have the size of $St \sim 1$ in a highly dust-rich region where $\Sigma_d \sim \Sigma_g$. In this case, we should directly calculate an interaction between the dust grains and the turbulence in the gas. However, the motion of the dust grains in such a situation is still poorly understood. Hence, in this paper, we adopt the model of the diffusion of dust grains given by Equations (10) and (11), regardless of the values of $St$ and $\Sigma_d/\Sigma_g$. In this case, using the model of Youdin & Lithwick (2007), we obtain the turbulent diffusivity of the dust grains as

$$D = \frac{1}{\nu} + \frac{4S^2}{(1 + S^2)^2}. \tag{12}$$

### 2.2. Size Evolution of the Dust Grains

In this paper, we consider two cases: one is the case in which the size of the dust grains is constant during the simulations, and another is the case in which the growth of dust grains is considered. Here we describe the model of dust growth that we adopted in our simulations. The size of the dust grains is varied by, for example, coagulation and collisional fragmentation (e.g., Birnstiel et al. 2010; Okuzumi et al. 2012). Although the dust grains have a size distribution, Birnstiel et al. (2012) have provided a simple model of dust size evolution with a single representative size of the dust grains. Their model does not consider the situation of a planet and neglects some physical processes, such as feedback from the gas, detailed vertical dynamics, and the turbulent statistics of growing grains (see also Section 4.4). However, their model is useful for us as a first step to include the size evolution of the dust grains by considering the single representative size of the dust grains, instead of directly calculating the coagulation and fragmentation of the dust grains. Recently, Tamfal et al. (2018) have considered dust growth in two-dimensional hydrodynamic simulations using a similar method. In the following, we briefly summarize the size evolution model of the dust grains provided by Birnstiel et al. (2012).

The size of the dust grains can be characterized by the maximum size of the dust grains. When it is determined by the fragmentation due to the turbulence, the Stokes number of the maximum-sized grains is obtained by $\approx \alpha^{-1}(u_{frag}/c_s)^2$, where $u_{frag}$ is the fragmentation threshold velocity. That is, in the Epstein regime, the maximum size of the dust grains can be given by

$$s_{frag} = f_{frag} \frac{2}{3\pi} \frac{\Sigma_g}{\rho_p} \left(\frac{u_{frag}}{c_s}\right)^2, \tag{13}$$

where $f_{frag}$ is a correction factor, which is given by 0.37.

Radial drift can limit the maximum size of the dust grains if the timescale of the dust radial drift is comparable with, or less than, the timescale of dust growth. The timescale of the dust growth can be estimated by

$$\tau_{growth} = \frac{1}{\Omega_K} \frac{\Sigma_g}{\Sigma_d}. \tag{14}$$

For dust grains with $St \leq 1$, the timescale of the dust radial drift can be estimated by

$$\tau_{drift} = \frac{R}{v_R} = \frac{1}{St} \left(\frac{h_g}{R}\right)^2 \left[\frac{\partial \ln\left(\Sigma_g^2/\Sigma_d^2\right)}{\partial \ln R}\right]^{-1} \frac{1}{\Omega_K}. \tag{15}$$

When the radial drift dominates the size of the dust grains, the maximum size of the dust grains can be given by $\tau_{growth} \approx \tau_{drift}$. That is, using the Epstein law, we obtain

$$s_{drift} = f_{drift} \frac{2}{\pi} \frac{\Sigma_g}{\rho_p} \left(\frac{h_g}{R}\right)^2 \left[\frac{\partial \ln\left(\Sigma_g^2/\Sigma_d^2\right)}{\partial \ln R}\right]^{-1}, \tag{16}$$

where the correction factor $f_{drift}$ is 0.55.

The representative size of the dust grains takes the smaller of $s_{frag}$ and $s_{drift}$. In the model developed by Birnstiel et al. (2012), the size growth of dust grains from the size of monomers is considered. However, since we assume the situations in which the planet is already formed, it could be reasonable to consider that the size of the dust grains reaches either size given by Equations (13) or (16). Hence, we describe the representative size of the dust grains as

$$s_d(R, t) = \text{min}\{s_{frag}, s_{drift}\}. \tag{17}$$

Note that in the model described above, we assume the size distribution of the dust grains is in coagulation–fragmentation equilibrium when the representative size of the dust grains is determined by the fragmentation. In the unperturbed disk, this assumption would be valid. However, when the density of the dust grains is significantly small and thus the dust growth time is very long (e.g., in the edge of the gap), this assumption may not be valid. The above situation could be realized in the inner disk of the planet when the gap is very deep. According to Equation (17), the representative size of the dust grains is very small due to $s_{drift}$ with a very small value of $\Sigma_d$. Hence, since the stopping time of the dust grains becomes short, we must choose a smaller time step. To avoid the above condition, we keep the representative size of the dust grains as $St = 0$ by using the function $St + 0.1 \exp\left(\Sigma_d/[10^{-4}\Sigma_g]\right)$ if the dust-to-gas mass ratio is smaller than $10^{-4}$. Because we focus on the structure of the outer disk in this paper, this prescription does not affect our results.

It should also be noted that we consider only the collision velocity caused by the gas turbulence, which is reasonable if the value of $\alpha$ is relatively large. However, if the situation with a very small value of $\alpha$ is considered, the collision velocity of the dust grains is dominated by the difference of the radial drift velocities, as noted by Birnstiel et al. (2012). In this case, we cannot adopt Equation (13) for $s_{frag}$. In our parameter range of $\alpha \gtrsim 10^{-3}$, the collision velocity related to the radial drift is smaller than or comparable to that related to the turbulence. Hence, we neglect this effect in this paper.\footnote{We estimated the collision velocity related to the radial drift at each time step and in each spatial mesh. If it is larger than that related to the turbulence, we determined $s_{frag}$ by the collision velocity related to the radial velocity, instead of that related to the turbulence. However, in most cases, $s_{frag}$ was given by Equation (13) in our parameter range. In the case of smaller $\alpha$, the fragmentation induced by the radial drift could be significant.} To include the size
The evolution of the dust grains precisely, a more sophisticated model (e.g., Sato et al. 2016) is required.

2.3. Numerical Method and Setup

We carried out two-fluid hydrodynamic simulations using a code based on FARGO (Masset 2000), which is an Eulerian polar grid code with a staggered mesh. The FARGO algorithm, which removes the azimuthally averaged velocity for the Courant time step, enables us to do long-term simulations. Expanding FARGO to include the dust component, we numerically solved the equations of motion and the continuity equations for gas and dust grains (Kanagawa et al. 2017).

The computational domain runs from \( R/R_p = 0.4 \) to \( R/R_p = 4.0 \) with 512 radial zones (equally spaced in logarithmic space) and 1024 azimuthal zones (equally spaced). The subscript \( p \) indicates the value at \( R = R_p \) in the following. The initial condition of the surface density of the disk gas is set as \( \Sigma_g = \Sigma_{0}(R/R_p)^{-1} \) and \( \Sigma_{0} = 6 \times 10^{-4} \), which corresponds to 53 \( \mathrm{g} \ \mathrm{cm}^{-2} \) when \( M_{d0} = 1 \ M_{\odot} \) and \( R_p = 10 \alpha \). The dust aspect ratio is assumed to be \( h_g/R = (h_{g,p}/R_p) \ (R/R_p)^{3/4} \) with \( h_{g,p}/R_p = 0.05 \). The initial angular velocity of the disk gas is given by \( \Omega_k \sqrt{1 - \eta} \), where

\[
\eta = \frac{1}{2} \left( \frac{h_g}{R} \right)^2 \frac{\partial \ln (c_s^2 \Sigma_g)}{\partial \ln R}.
\]

For the dust component, the initial distribution of the dust grains is set for the dust-to-gas mass ratio to be 0.01. The initial angular velocity of the dust grains is given by \( \Omega_k \), and the initial radial velocity is set to be zero. For simplicity, we neglect mass growth of the planet. We also neglect time variation of the planetary orbit, and the planet’s orbit is fixed to be \( R = R_p \). For convenience, we define \( \tau_0 \) as \( 2\pi/\Omega_{K,p} \). We adopt the softening parameter \( \epsilon \) as 0.6 \( h_{g,p} \). We use the orbital radius of the planet \( R_p \) as the unit of the radius, and the mass of the central star \( M_\star \) is used as the unit of mass. Hence, the surface density is normalized by the value of \( M_\star / R_p^2 \).

At the inner and outer boundaries, the gas and dust velocities are set to be these in a steady state during the simulations. The surface densities of the gas and dust are also set so that the mass flux is constant. To avoid an artificial wave reflection, the damping is used in a so-called wave-killing zone near the boundary layers, as in de Val-Borro et al. (2006). In this region, we force the physical quantities to be azimuthally symmetric. For details, see Kanagawa et al. (2017). The wave-killing zones are located from \( R_{\text{in}} = 0.2 R_p \) to \( R_{\text{out}} \) for the outer boundary and from \( R_{\text{in}} = R_{\text{in}} + 0.1 R_p \) for the inner boundary, where \( R_{\text{out}} \) and \( R_{\text{in}} \) are the radii of the outer and inner boundaries, respectively.

We carried out two kinds of simulations. One considers the size evolution of the dust grains using the model described in Section 2.2; another does not consider the size evolution of the dust grains. When the size evolution of the dust grains is not considered, the size of dust grains is constant throughout the disk during the simulations. For convenience, we define a characteristic size of the dust grain as \( s_{d,0} = 2 \Sigma_{g,0}/(\pi \rho_p) \), and that is

\[
s_{d,0} = 34.0 \left( \frac{\Sigma_{g,0}}{53 \ \mathrm{g} \ \mathrm{cm}^{-2}} \right) \left( \frac{\rho_p}{1 \ \mathrm{g} \ \mathrm{cm}^{-3}} \right)^{-1} \mathrm{cm},
\]

where \( \Sigma_{g,0} \) is the gas surface density of the unperturbed disk. The size of \( s_{d,0} \) corresponds to the size of the dust grain when \( St = 1 \) at \( R = R_p \) in the unperturbed disk. We usually adopt \( s_d = 0.1 s_{d,0} \) when the size evolution of the dust grains is not considered.

When the size evolution of the dust grains is considered, the size of the dust grains varies as described by the model discussed in Section 2.2. In this case, the maximum size of the dust grains is associated with the threshold velocity of the fragmentation \( u_{\text{frag}} \) if it is determined by the fragmentation. The threshold velocity depends on the composition of the dust grains. For icy grains, the threshold velocity of the fragmentation could be as high as 50 m s\(^{-1}\) (e.g., Wada et al. 2009) when the size of the monomer is 0.1 \( \mu \mathrm{m} \). For rocky grains, the threshold velocity of the fragmentation is about \( \sim 1 \) m s\(^{-1}\), which is much smaller than that for the icy grains. However, since the threshold velocity depends on the size of the monomers (e.g., Dominik & Tielens 1997), the threshold velocity could be at the same level as for the icy grains, if nanograins are assumed (Arakawa & Nakamoto 2016). In this paper, therefore, we consider \( u_{\text{frag}} = 30 \) m s\(^{-1}\) as a fiducial value.

3. Results of Hydrodynamic Simulations

3.1. Ring Structures Induced by Jupiter-sized Planets

3.1.1. Density Distributions

First we show the two-dimensional distributions of the surface density of the dust grains in the case with a constant size of dust grains \( (s_d = 0.1 s_{d,0}) \) in Figure 1. In this subsection, we show the results of the simulations with a Jupiter-sized planet \( (M_p/M_\star = 10^{-3}) \), and we adopt \( h_{g,p}/R_p = 0.05 \) and \( \alpha = 4 \times 10^{-3} \) as fiducial values. In the upper panel of the figure, the dust feedback is not considered. In the lower panel, we consider the dust feedback. In both cases, with and without dust feedback, the radial flow of the dust grains across the gap is halted because the gap of the gas is deep (the ratio of the minimum surface density of the gap to the unperturbed surface density is about \( \sim 0.01 \)). Hence, the dust grains are completely swept out from the region within the gap and inside the planet orbit. As a consequence, a ring structure of dust grains is formed in the outer disk of the planet orbit. In the earlier stage as in the case of \( t = 1000 \tau_0 \), the widths of the dust ring are almost the same in both cases, in which the dust feedback is considered and it is ignored. For the case in which the dust feedback is considered, however, the dust ring gradually becomes wider with time, whereas the width of the dust ring in the case in which feedback is ignored is much narrower and no longer changes after 1000 \( \tau_0 \).

We should note that when dust feedback is considered, the distribution of the gas surface density is slightly modified from that in the case without dust feedback, as in the distribution of the dust grains. The dust feedback reduces the gas surface density by a factor at the outer edge of the gap, as shown in Appendix A. We also examine the resolution dependence of the gas and dust structures in this case (see Appendix B), and we find that with our fiducial resolution, the structures of the gas and the dust grains are well converged.

In Figure 2, we show the two-dimensional distribution of the surface density of dust grains when the size evolution of the dust grains is considered \( (u_{\text{frag}} = 30 \) m s\(^{-1}\) is adopted). The planet mass and the disk parameters are the same as in Figure 1.
When the size evolution of the dust grains is considered, the size of the dust grains is not constant, and it varies as described by Equation (17). In Figure 3, we show the Stokes number for the representative size of the dust grains at $t = 5000t_0$, in the case of Figure 2. As can be seen in Figure 3, the representative size of the dust grains is determined by the fragmentation in the outer disk. In this case, the representative size is quite similar to the size of the grains in the case of constant grain size. Hence, the structure and width of the dust ring are also similar to each other in the lower panels of Figures 1 and 2. At the outer edge of the gap ($R \sim 1.2R_0$), the dust-to-gas mass ratio sharply decreases, and then $s_{\text{drift}}$ becomes much smaller than $s_{\text{tag}}$. Therefore, the representative size is given by $s_{\text{drift}}$ in this region. Note that, as mentioned in Section 2.2, the assumption of coagulation–fragmentation equilibrium would not be valid, and the model of Birnstiel et al. (2012) may not be appropriate in this region. Also note that in the region where almost no dust grains stay (i.e., $\Sigma_d/\Sigma_g < 10^{-4}$), the dust size is fixed to be $\Sigma = 0.1\alpha$ to avoid a too-small time step (see Section 2.2).

Due to the rapid change in the representative size of the dust grains at the outer edge of the gap, the edge of the dust ring can be swung as in the right panel of Figure 2. Such a swing may be related to the dust supply to the vicinity of the planet. However, in this region, the size distribution of the dust grains could not reach the coagulation–fragmentation equilibrium. To address this issue correctly, we have to consider a more sophisticated model for the dust evolution.

Figure 4 shows the azimuthally averaged value of $\eta$ (Equation (18)) at $t = 5000t_0$ in the case of Figure 1. A pressure bump is formed at the location where $\eta = 0$ and $\partial \eta / \partial R > 0$. The “primary” pressure bump is formed at $R/R_p \simeq 1.4$, due to the gap formed by the planet in both cases, in which the dust feedback is considered and it is ignored. The locations of the primary pressure bump are quite similar in both cases. It is remarkable that in the case in which the dust feedback is considered, the value of $\eta$ is very small compared to that in the case in which the dust feedback is ignored, in the broad region within the dust ring. Moreover, the secondary pressure bump is formed at $R/R_p = 1.6$, in the case in which the dust feedback is considered. Figure 5 shows the radial mass flux of the dust grains at $t = 5000t_0$ in the case of Figure 1. When the dust feedback is ignored, the mass flux due to advection is so effective as to not be canceled out by the diffusive mass flux outside the pressure bump. Hence, the dust grains are accumulated within the narrow annulus near the pressure bump. On the other hand, when the dust feedback is considered, the mass flux due to the advection is weakened because $\eta$ becomes small, as shown in Figure 4. Because of it, the dust grains can diffuse from the dust ring and modify the gas structure of the outer edge of the ring to let $\eta$ be small. As a consequence, the dust grains can further diffuse from the edge of the ring, and the width of the ring increases with time, due to the effects of dust feedback. Even when the growth of the dust grains is considered, the above picture does not change. Note that although we adopt the model of turbulent diffusion given by Equation (11), this model may be appropriate when $\Sigma_d > \Sigma_g$ and $\Sigma = 1$. The structure of the dust ring would depend on the model of the turbulent diffusion of the dust grains, which is discussed below again.

3.1.2. Widths of the Dust Ring

In this subsection, we consider the width of the dust ring. First, we define the width of the ring as the radial width of the region where the azimuthally averaged value of the dust-to-gas mass ratio is larger than the threshold value ($\Sigma_d/\Sigma_g \eta_i$). For convenience, we also define the radii of the inner and outer edges of the ring as $R_{i,i}$ and $R_{o,o}$, and $R_{o} = (R_{i,i} + R_{o,o})/2$.

The width of the dust ring may be estimated as follows. When the radial flow of the dust grains is almost halted, the mass of the dust ring would increase as the mass flux of the dust grains from the outside. Hence, we can obtain $2\pi d(R_{m} \Delta \Sigma_{\text{d,mean}}/dt \simeq -F_{\text{M,d}}(R_{\text{out}})$, where $\Sigma_{\text{d,mean}}$ and $F_{\text{M,d}}$ are the mean surface density of the dust grains in the ring and the mass flux of dust grains from the outside, respectively.
If $\Sigma_{\text{d,mean}} \sim \Sigma_g \sim \Sigma_0$, then

$$\frac{d}{d(t/t_0)} \left( \frac{\Delta_{\text{ring}} R_m}{R_p^2} \right) = -\frac{F_{\text{M,d}}(R_{\text{out}}) t_0}{2\pi \Sigma_0}. \quad (20)$$

Here, $F_{\text{M,d}}(R_{\text{out}})$ is given by $2\pi R_\Sigma v_R$ at $R = R_{\text{out}}$, and $v_R$ is

$$v_R(R) = \frac{2(s_d/s_d,0) \eta_p}{1 + (s_d/s_d,0)^2 (R/R_p)^2} \left( \frac{R}{R_p} \right)^{2f+3/2}. \quad (21)$$

Hence,

$$f_{\text{M,d}} = -\frac{F_{\text{M,d}}(R_{\text{out}}) t_0}{2\pi \Sigma_0} = \frac{4 \pi (s_d/s_d,0) \eta_p (\Sigma_d/\Sigma_g)_{\text{out}} (R_{\text{out}}/R_p)^{2f+3/2}}{1 + (s_d/s_d,0)^2 (R_{\text{out}}/R_p)^2} \left( \frac{R_{\text{out}}}{R_p} \right)^{2f+3/2}. \quad (22)$$

Note that the value of $\Delta_{\text{ring}} R_m$ indicates the area of the ring region. Hence the value of $\Delta_{\text{ring}} R_m$ is described by a linear function of time as

$$\frac{\Delta_{\text{ring}} R_m}{R_p^2} = \frac{f_{\text{M,d}}}{t_0} (t - t_0), \quad (23)$$

where in $t > t_r$, the ring of dust grains could be formed.
density of the dust grains in the inner disk is significantly small, and the inner disk of the dust grains is almost depleted. In these cases, the radial mass flow of the dust grains across the gap is almost halted. Moreover, only in these cases, a wide ring of dust grains can be formed.

The minimum mass of the planet to form the dust ring should correspond to the pebble-isolation mass (e.g., Morbidelli & Nesvorny 2012; Lambrechts et al. 2014; Bitsch et al. 2018). Recently, Bitsch et al. (2018) have obtained a formula of the pebble-isolation mass by carrying out three-dimensional hydrodynamic simulations. The pebble-isolation mass given by their formula (Equation 26 of that paper) is about $1.3 \times 10^{-4} M_p$ when $\alpha = 4 \times 10^{-3}$ and $h_{zp}/R_p = 0.05$. Their pebble-isolation mass reasonably agrees with our minimum mass needed of the planet to form the broad ring of dust grains.

Figure 7 shows the time variation of the area of the dust rings for various planet masses. When $M_p/M_\ast = 2 \times 10^{-4}$ and $M_p/M_\ast = 5 \times 10^{-4}$, the width of the dust ring increases with time, as in the case of $M_p/M_\ast = 10^{-3}$. However, the increase rates of the dust ring area in the cases with $M_p/M_\ast = 2 \times 10^{-4}$ and $M_p/M_\ast = 5 \times 10^{-4}$ are slightly smaller than that in the case with $M_p/M_\ast = 10^{-3}$. This discrepancy could originate from the different surface densities of the gas within the dust ring. As shown in Appendix A, the surface density of the gas within the dust ring is slightly larger when the planet mass is smaller (or the gap is shallower). Hence, since the dust-to-gas mass ratio is approximately unity within the dust ring, the surface density of the dust grains is larger with the smaller mass of the planet. Therefore, the increase rate of the dust ring is smaller when the planet mass is relatively small. Nonetheless, we can roughly estimate the width of the dust ring by Equation (20) because the discrepancy from Equation (20) is not more than a factor of two.

### 3.3. The Case of a Low Viscosity

According to Equation (22), the increase rate of the ring area is independent of the viscosity. Figure 11 shows the two-dimensional distributions of the dust surface density at $t = 5000 t_0$ when $\alpha = 10^{-3}$ and $h_{zp}/R_p = 0.05$. As in the case of $\alpha = 4 \times 10^{-3}$ shown in Figures 1 and 2, the broad dust ring is formed in the outer disk in both cases where the size evolution of the dust grains is ignored and it is considered. In this case, since the collision velocity due to the turbulence is
smaller than that when \( \alpha = 4 \times 10^{-3} \), the broad dust ring can be formed even if \( u_{\text{frag}} = 15 \text{ m s}^{-1} \). When \( u_{\text{frag}} \) is small, the representative size of the dust grains also becomes small (see Equation (13)). Hence, the increase rate of the ring area decreases with a smaller \( u_{\text{frag}} \). Figure 12 shows the time variations of the area of the dust ring in the case shown in Figure 11. In both cases, in which the dust size is constant and in which the dust growth is considered with \( u_{\text{frag}} = 30 \text{ m s}^{-1} \), the increase rates of the ring area are quite similar to each other, and they are also similar to that in the case of \( \alpha = 4 \times 10^{-3} \) and \( M_p/M_* = 10^{-4} \). When \( u_{\text{frag}} = 15 \text{ m s}^{-1} \), the increase rate of the ring area is about 0.5 times that in the case of \( u_{\text{frag}} = 30 \text{ m s}^{-1} \), as mentioned above.

When the viscosity is small, the minimum mass of the planet needed to form the broad dust ring is also small, since even a small planet can form the gas gap. Figure 13 illustrates the two-dimensional distributions of the dust surface density at \( t = 5000t_0 \) when \( \alpha = 10^{-3} \) and constant-size dust grains are adopted. As can be seen in the figure, when \( M_p/M_* = 8 \times 10^{-5} \), the inner disk of the dust grains still remains, and the dust grains can pass through the gap. In this case, a relatively broad ring is formed at \( t = 5000t_0 \), but it does not increase anymore, while when \( M_p/M_* = 10^{-4} \), the inner disk of the dust grains is almost depleted, and the dust grains cannot pass through the gap. In this case, the area of the dust ring increases with time. Hence, in this case, we can consider that the minimum mass of the planet needed to form the broad dust ring is \( M_p/M_* \approx 10^{-4} \). This minimum mass also reasonably agrees with the pebble-isolation mass given by Bitsch et al. (2018).

It should be noted that as shown in Section 3.1, the dust ring enlarges as the diffusing dust grains from the edge of the ring modify the gas structure at the edge of the ring. Hence, the
growth timescale of the dust ring becomes longer as the turbulent diffusivity of the dust grains becomes ineffective. As shown above, the dust ring can grow wider during the simulation time (i.e., $10,000\,t_0$) in our parameter range as $\alpha \gtrsim 10^{-3}$. However, if the turbulent diffusivity of the dust grains is extremely low, this timescale would be significantly longer. In addition, in such a low-viscosity case, the collision velocity induced by the radial drift could be superior to that induced by the turbulence. Hence, we need to adapt a more sophisticated model of the dust grains in this case.

4. Discussion

4.1. Formation of Planetesimals via the Streaming Instability

For simplicity, we do not include the formation of planetesimals in our simulations. In the region in which the dust grains are accumulated such as the dust ring, however, the planetesimals can be effectively formed via the streaming instability (e.g., Youdin & Goodman 2005; Johansen & Youdin 2007; Youdin & Johansen 2007; Drążkowska & Dullemond 2014; Carrera et al. 2015;
Auffinger & Laibe 2018). If most of the dust grains are instantaneously converted to the planetesimals by the streaming instability, the dust ring could not grow anymore. However, not all of the dust grains may be converted to planetesimals (Johansen et al. 2012; Simon et al. 2016). If the efficiency of the formation of planetesimals is moderate, it is possible that the dust ring can grow while the planetesimals are formed. Moreover, the value of \( \eta \) becomes very small within the dust ring, as shown in Figure 4, which affects the spatial scale and the growth timescale of the streaming instability (Youdin & Goodman 2005; Youdin & Johansen 2007). With a smaller value of \( \eta \), the spatial scale of the streaming instability becomes smaller and the growth timescale is longer. The small value of \( \eta \) could prevent an effective formation of planetesimals by the streaming instability. When the timescale of the planetesimals is comparable with or longer than the growth timescale of the dust ring, a broad dust ring could be formed.

### 4.2. Implications for Planet Formation

Planetesimals can be produced within the ring with the high dust density formed outside the planet-induced gap, as discussed by previous studies (e.g., Kobayashi et al. 2012; Pinilla et al. 2015). In the previous section, we have shown that the very wide ring of dust grains can be formed outside the planet-induced gap when the dust feedback is considered. This fact implies that the planetesimal formation can be triggered in a wide region outside the planet-induced gap.

The maximum area of the dust ring can be constrained by the total mass of the dust grains within the protoplanetary disk. If the radius of the protoplanetary disk holding the dust grains is given by \( R_{d,\text{out}} \) and the initial dust-to-gas mass ratio is \( \Sigma_d/\Sigma_g \) throughout the disk, the total dust mass can be roughly estimated by \( \Sigma_d/\Sigma_g \) \( \times \) \( \Sigma_g \) \( R_{d,\text{out}}^2 \). Considering \( \Sigma_d/\Sigma_g \approx 1 \) within the dust ring, we can roughly estimate the maximum area of the ring as

\[
\left( \frac{\Delta_{\text{ring}} R_p}{R_p^2} \right)_{\text{max}} = \left( \frac{\Sigma_d}{\Sigma_g} \right) \left( \frac{R_{d,\text{out}}}{R_p} \right)^2.
\]

Assuming \( R_{c,\text{in}} = R_p \), we obtain

\[
\Delta_{\text{ring}} = R_p \left( 1 + 2 \left( \frac{\Sigma_d}{\Sigma_g} \right) \left( \frac{R_{d,\text{out}}}{R_p} \right)^2 - 1 \right).
\]

When \( \Sigma_d/\Sigma_g \) \( \approx \) 0.01, \( R_{d,\text{out}} \) = 100 au, and \( R_p \) = 2 au, we can estimate \( \Delta_{\text{ring}} \approx 10 \) au. Similarly, if in a compact disk we have \( R_{d,\text{out}} = 10 \) au and \( R_p = 1 \) au, we can estimate \( \Delta_{\text{ring}} \approx 1 \) au.

Once the planetesimal forms, its mass increases by capturing dust grains within its feeding zone until its mass reaches the core-isolation mass (e.g., Kokubo & Ida 1995, 2000). The core-isolation mass \( (M_{\text{pl,iso}}) \) can be estimated by \( 2 \pi \Sigma_d R_c \Delta_t \), where \( \Delta_t \) is the width of the feeding zone. The width of the feeding zone is approximately given by \( 10 R_p (M_{\text{pl,iso}}/3M_p)^{1/3} \), where \( M_{\text{pl,iso}} \) is the core-isolation mass and the width of its feeding zone can be obtained by

\[
M_{\text{pl,iso}} = 3.6 \left( \frac{\Sigma_d}{100 \text{ g cm}^{-2}} \right)^{3/2} \left( \frac{M_p}{1 \text{ M}_\oplus} \right)^{-1/2} \left( \frac{R_p}{1 \text{ au}} \right)^3 \text{ M}_\oplus. \tag{26}
\]

### First we consider the disk around a Sun-like star

First we consider the disk around a Sun-like star \( (M_{\text{disk}} \approx 10^{-2} \text{ M}_\odot; R_{\text{d,\text{out}}} \approx 100 \text{ au}; \text{ and } M_p \approx 1 \text{ M}_\oplus) \). When \( R_p = 10 \text{ au} \) and \( \Sigma_d \approx 10 \text{ g cm}^{-2} \), the maximum width of the dust ring can be estimated as 7 au, which is comparable with the width of the feeding zone \( (\approx 5 \text{ au}) \). In this case, a massive core \( (M_{\text{pl,iso}} \approx 100 \text{ M}_\oplus) \) can be formed in the dust ring. However, since this core-isolation mass is much larger than the critical core mass of the runaway gas accretion (typically \( \approx 10 \text{ M}_\oplus \); e.g., Mizuno 1980; Kanagawa & Fujimoto 2013), most of the dust grains would accrete onto the gaseous envelope after the runaway gas accretion, rather than onto the core. Hence, the solid core of the giant planet may not be so massive in this case, instead of a metal-rich gas envelope. On the other hand, when the ring is formed in the inner region of the disk, a giant planet holding a massive, solid core might be formed. When we consider \( R_p = 2 \text{ au} \) and \( \Sigma_d \approx 200 \text{ g cm}^{-2} \), \( \Delta_t \) is about 1 au. The maximum width of the dust ring estimated by Equation (25) is about 10 au, which is much wider than the width of the feeding zone. In this case, as shown by Ikoma et al. (2006), since the critical core mass for the runaway gas accretion increases because of the high core accretion rate, the solid core can grow to be the isolation mass \( (M_{\text{pl,iso}} \approx 80 \text{ M}_\oplus) \) before the runaway gas accretion. After strong gravitational scattering and giant impact events, a gas giant planet holding a massive, solid core can possibly be formed. Observations have found compact, hot Jupiters (giant planet with a very close orbit) with massive, solid cores of \( M_p \approx 50 \text{ M}_\oplus \) (Sato et al. 2005; Fortney et al. 2006; Ikoma et al. 2006), and WASP-59b is also suggested to have a massive, dense core (Hébrard et al. 2013). Such a giant planet holding a massive, solid core might be formed within the ring of the dust grains.

Next, let us consider the dust ring in the disk around the dwarf star. Recently, seven Earth-sized planets have been found around the dwarf star named TRAPPIST-1 \( (M_\ast \approx 0.08 \text{ M}_\odot \text{; Gilon et al. 2016, 2017}) \). All seven planets orbit within 0.1 au from the host star, and all planets have masses similar to Earth. Here we consider the formation of the planetary system around a small-mass star like TRAPPIST-1. Hence, we consider \( M_p = 0.1 \text{ M}_\oplus \) and a compact and less massive disk \( (R_{d,\text{out}} = 10 \text{ au} \text{ and } M_{\text{disk}} \approx 10^{-3} \text{ M}_\odot) \). Here we consider \( R_p = 0.5 \text{ au} \). The surface density of disk gas around 0.5 au can be considered as 100 g cm\(^{-2}\). In the dust ring, the surface density of the dust grains is the same level of \( \Sigma_g \). According to Equation (24), in this case, the width of the dust ring can reach up to 1 au. The width of the feeding zone is about 0.12 au, as can be seen in Equation (27), and the mass of the solid core is about 1.4 \text{ M}_\oplus (\text{Equation (26)}). Hence, we can consider that several (up to eight) Earth-sized planets can be formed in the dust ring in this case. In addition, since the mass of the central star is small, even when its mass is small, the planet can form the dust ring in the inner region of the planet. According to the formula given by Bitsch et al. (2018), when \( h_{p,\text{iso}} / R_p = 0.03 \) and \( \alpha = 10^{-3} \), the pebble-isolation mass is about \( M_p/M_\ast = 1.6 \times 10^{-5} \), which corresponds to 0.5 \text{ M}_\oplus \text{ when } M_p = 0.1 \text{ M}_\oplus \). The above discussion implies that once the small planet is formed in the small disk hosted by the dwarf
star, then a multiple-planet system of Earth-sized planets is formed. A system of multiple Earth-sized planets may be common around a dwarf star like TRAPPIST-1.

4.3. Implications for Observations

As shown in Figure 3, the size of the dust grains trapped in the dust ring is approximately given by $s_d = 0.1d_{0.0}$, which corresponds to $\sim 3$ cm in the fiducial disk as $\Sigma_d = 50$ g cm$^{-2}$ at $R_p = 10$ au. Here we consider the observations in the millimeter and submillimeter range by, for example, the Atacama Large Millimeter/submillimeter Array (ALMA). In this range of wavelength, the opacity of dust grains with $s_d \sim 1$ cm is 0.1 times smaller than that of millimeter-sized dust grains (e.g., Miyake & Nakagawa 1993). Hence, almost the whole region of the dust ring is observable as a bright ring. Note that if we assume micrometer-sized grains, we would underestimate the mass of the dust grains.

Until now, broad dust rings in which the width is comparable to their distance from the central star have been observed in several protoplanetary disks (e.g., Pérez et al. 2014; van der Plas et al. 2017). Such a broad ring could be induced by the planet when the dust feedback is considered. As discussed above, once the broad ring is formed, a multitude of planets can be formed. After the formation of the planets, the broad structure could be destroyed by each planet. Hence, in a protoplanetary disk holding multiple rings and gaps, such as the disks of HL Tau, TW Hydra, and MWC 758 (ALMA Partnership et al. 2015; Nomura et al. 2016; Dong et al. 2018), it would be difficult to identify the broad structure of the dust ring, even if it was formed before.

4.4. Validity of the Model

In Section 3.1, we have shown that the diffusion of the dust grains plays a critical role in the formation of the broad dust ring. In this paper, we adopt the model of the turbulent diffusion of dust grains described by Equation (11). In Equation (11), we assume that the dust grains diffuse only because of gas turbulence. However, when $\Sigma_d \sim \Sigma_g$, the turbulence could also be driven by the instability of the dust layer (e.g., Sekiya 1998; Sekiya & Ishitsu 2000; Johansen et al. 2006; Chiang 2008; Lee et al. 2010a, 2010b). This additional turbulence may lead to diffusion in addition to that described by Equation (11). Moreover, Equation (11) is derived from the gradient diffusion hypothesis (Cuzzi et al. 1993; Takeuchi & Lin 2002). However, the validity of this hypothesis is not confirmed yet when $\Sigma_d \gtrsim \Sigma_g$. If the turbulent diffusion of the dust grains is significantly different from that described by Equation (11), the structure of the dust ring may be different from that obtained in this paper. A more accurate formulation for dust and gas would be required to investigate this in the future.

We do not consider the formation of the planetesimals in our calculations. However, within the ring of the dust grains, the planetesimals can be formed via the streaming instability, as discussed in Section 4.1. Only if the efficiency of the formation of the planetesimals is moderate can a wide ring of dust grains be formed. The efficiency of planetesimal formation after the streaming instability significantly occurs is not fully understood. In addition, within the dust ring, the value of $\eta$ is much smaller than that of the unperturbed disk. In such a situation with small $\eta$, the properties of the streaming instability are also not fully understood. We also note that the onset of the streaming instability has been investigated in inviscid situations, but it may depend on the viscosity (or turbulent diffusion). Further investigations are required to understand the properties of the streaming instability in the dusty ring at the outer edge of the planet-induced gap.

We also adopt the simple model of the size evolution of the dust grains provided by Birnstiel et al. (2012). In this model, we assume that the size distribution of the dust grains is in coagulation–fragmentation equilibrium when the representative size of the dust grains is limited by the fragmentation. However, at the outer edge of the gap, this assumption may not be valid, because the dust growth timescale is long, which is due to the small dust-to-gas mass ratio. In this case, it is possible that some small grains do not grow to the representative size and penetrate into the gap. This nonequilibrium effect may suppress the growth of the dust ring and prevents the depletion of the inner disk of the dust grains. Moreover, as Dipierro et al. (2018b) have shown recently, the feedback from larger dust grains compared to that from the representative size may not be negligible, depending on the distribution of the grain size. The feedback would be more effective if the size of the dust grains is widely distributed, because of the contribution from the larger grains. To address these issues, simulations considering multiple sizes of dust grains would be required.

5. Summary

In this paper, we have investigated the effects of dust feedback on the structure of the dust ring induced by the planet. Considering the dust feedback and the turbulent diffusion of the dust grains, we have carried out the two-dimensional gas–dust two-fluid hydrodynamic simulations. Our results are summarized below:

1. When the gap is sufficiently deep, the radial flow of the dust grains is almost halted. In this case, we found that a very wide ring of dust grains can be formed in the outer disk at the outer edge of the gap (Figure 1). The dust feedback from the diffusing dust grains from the ring region modifies the structure of the gas to reduce the gas pressure gradient (and $\eta$; Figure 4). As a result, the dust grains can diffuse from the outer edge of the ring, and the area of the dust ring gradually increases with time. Then the broad ring of dust grains can be formed when the dust feedback is considered. Even when the size evolution of the dust grains is considered, the structure of the ring is quite similar to that in the case where the size evolution is not considered if $\alpha \sim 10^{-2}$ and $\mu_{\text{gas}} \sim 10$ m s$^{-1}$. In this case, the broad ring can also be formed (Figure 2).

2. The increase rate of the area of the dust ring can be estimated by the mass flux of the dust grains from the outside of the disk (Figures 6, 10, and 12).

3. The minimum mass of the planet needed to form the dust ring is consistent with the pebble-isolation mass provided by Bitsch et al. (2018) in the case of $\alpha \sim 10^{-3}$.

4. As discussed in Section 4.2, if the dust ring is formed by the planet at $\sim 2$ au in the less massive disk around a Sun-like star, a massive, solid core may be able to be formed within the dust ring. This may be connected to the formation of hot Jupiters holding a massive, solid core, like HD 149026b. In the disk held by the dwarf star with
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Appendix A
Gas Structures

When the dust feedback is considered, the structure of the disk gas is modified from that without the dust feedback. Here we show the gas structure of the disk. Figure 14 shows the two-dimensional distributions of the gas surface density when \( M_p/M_s = 10^{-3} \), \( h_{g,p}/R_p = 0.05 \), and \( \alpha = 4 \times 10^{-3} \) (the same case as Figures 1 and 2). No matter whether or not the dust feedback and the dust growth are considered, the distributions of the gas surface density are similar to each other, as different in the structure of the dust grains. However, the structure of the outer edge of the gap is slightly different. Figure 15 shows the azimuthally averaged surface density of the gas at \( t = 5000t_0 \) in the case of Figure 14. As can be seen in the figure, because of the dust feedback, the surface density of the gas at the outer edge of the gap \( (1.0 \lesssim R/R_p \lesssim 2.0) \) is smaller by a factor of 2–3 than that in the case without dust feedback, whereas the dust growth does not significantly influence the structure of the gas.

Figure 16 shows the azimuthally averaged surface density of the gas at \( t = 5000t_0 \) for the various planet masses when \( \alpha = 4 \times 10^{-3} \) and \( h_{g,p}/R_p = 0.05 \). As the planet mass increases, the gap becomes deep, and the surface density of the gas at the outer edge also decreases. The surface density of the gas at the

\[ \sim 0.1 M_\oplus, \] a multitude of Earth-sized planets could be formed within the ring around 0.5 au, which may explain the formation of a planetary system like TRAPPIST-1.
outer edge of the gap is larger when the planet mass is smaller or the gap is shallower, though the difference is at most a factor of two.

Appendix B
Resolution Convergence

In Figure 17, we show the azimuthally averaged surface densities of the gas and the dust grains, in the same case as that shown in Figure 1, with different resolutions: \((N_r, N_{\phi}) = (256, 512)\) (low-resolution case), \((512, 1024)\) (fiducial case), and \((1024, 2048)\) (high-resolution case). As can be seen in the upper panel, the structure of the gas hardly depends on the resolution. For the surface density of the dust grains, on the other hand, the surface density of the dust grains within the dust ring around \(R \simeq 1.5R_0\) is smaller than those in the fiducial and high-resolution cases, because the numerical diffusion is stronger than that in the other cases. However, in the fiducial and high-resolution cases, the surface densities of the dust grains are quite similar. Hence, our fiducial resolution is sufficient to resolve the gas structure and the formation of the dust ring.

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