Thermal convection in a closed cavity in zero-gravity space conditions with stationary magnetic forces

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Abstract. The paper deals with the investigation of thermo-magnetic convection in a paramagnetic liquid subjected to a non-uniform magnetic field in weightlessness conditions. Indeed, in zero-g space conditions such as realized in International Space Station (ISS), or in artificial satellite, or in free-flight space vessels, the classical thermo-gravitational convection in fluid disappears. In any cases, it may be useful to restore the convective thermal exchange inside fluids such as liquid oxygen. In this paper, the restoration of heat exchange by the way of creation of magnetic convection is numerically studied.

1. Introduction

An inhomogeneous magnetic field \( \mathbf{B} \) distribution give volumic forces proportional to \( \nabla (\mathbf{B}^2) \) and to the magnetic susceptibility \( \chi \):

\[
\mathbf{f} = \frac{1}{2\mu_0} \chi \nabla (\mathbf{B}^2) \tag{1}
\]

where \( \mu_0 \) is the vacuum permeability.

The magnetic susceptibility is temperature dependent, in a way different for diamagnetic fluids (where \( \chi \) varies as the mass per unit volume \( \rho \)), or for the paramagnetic fluids obeying to a Curie law (\( \chi \) varies as \( 1/T \)). This explains that a thermal convection may be induced by a convenient field distribution. This fact has been used for promoting [1] or controlling [2] classical gravity convection in fluids.

The \( \nabla (\mathbf{B}^2) \) field obeys to general constraints because \( \mathbf{B} \) obeys to Maxwell equations. As an example, \( \nabla (\mathbf{B}^2) \) field cannot be a constant vector in any point of a 3D domain [3]. This is very important, thus for example a constant vector such as gravity cannot be perfectly compensated in any point of a 3D domain. That means also there exists a fundamental difference between classical terrestrial, gravity induced, convection, and the magnetic convection in zero-gravity space: the force distribution is constant (in little domains) for terrestrial gravity induced convection, and never constant in every point for magnetic forces.

Equation (1) shows that magnetically induced convection in zero-gravity space is directly depending on the magnetic field distribution. A first particular cylindrical distribution had been
considered in a preceding paper [4], and applied to the modification of natural convection of water (diamagnetic fluid). This field distribution was characterized by a radial force field, with a constant value of force modulus inside the channel containing the fluid.

In the present paper, another field distribution is considered, leading to a greatly inhomogeneous force field in the channel. This force field is applied to thermal convection in a paramagnetic fluid such as liquid oxygen in space zero-gravity conditions.

2. Problem formulation. Governing equations

Figure 1. Problem formulation

Paramagnetic fills a long cylindrical channel of square cross-section \((h \times h)\). Two opposite walls of the cylinder parallels to the \(x\)-axis are maintained at constant different temperatures. At the two remaining cylinder walls \(x = \pm h/2\), the temperature varies according to the linear law. Magnetic field is produced by a line of permanent magnets the size of which is small in comparison with \(h\), or by a bifilar line of conductors parallel to \(y\)-axis and fed by currents in opposite direction (fig.1). The magnets or the currents are located on the axis \(z\) at a distance \(L\) from the cylinder and their magnetic moment \(m\) per unit length is parallel to \(z\)-axis.

The field distribution is of the well-known cylindrical dipolar type, giving for the two components of \(\nabla(B^2)\) in the plane \((x,z)\):

\[
\frac{\partial(B^2)}{\partial x} = -4\Phi_0\frac{x}{r^2}, \quad \frac{\partial(B^2)}{\partial z} = -4\Phi_0\frac{z}{r^2},
\]

where \(x^2 + z^2 = r^2\), and \(\Phi_0\) is a magnetic constant of the line (unit T.m\(^2\)) easily calculable, either for a line of permanent magnets or for a bifilar line of currents. It appears that the force field is central in the plane \((Ox,Oz)\), and it varies rapidly (at the power -5) with the distance \(r\) from \(O\). This quick variation explains that convection lines, when \(L\) is low, may be very different than observed in the classical gravitational convection.

The temperature dependence of the magnetic susceptibility of paramagnetic makes ponderomotive force to be non-gradient such that in the presence of both temperature gradient and non-uniform magnetic field the liquid cannot remain motionless. The governing equations for the flow are:

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{v} + \frac{1}{2\mu_0} \mathbf{v} \times \nabla (B^2),
\]

\[
\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \kappa \Delta T = \text{div} \mathbf{v} = 0.
\]

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We chose the following scales: for the length $h$, for the velocity $v/h$, for time $h^2/v$, for the temperature the difference in hot and cold wall temperatures $\vartheta$, for the magnetic field induction $B_0 = B(z = L) = \mu_0 \Phi$, for the magnetic susceptibility $\chi_0 = \chi(z = L)$; the temperature $T_0$ at lower wall is taken as the reference temperature. Then, we obtain for two-dimensional problem applying $\text{curl}_y$ operation to the first equation of (3), introducing vorticity $\varphi$ and streamfunction $\psi$ by the relations $\varphi = \text{curl}_y v$, $v_x = -\partial \psi / \partial z$, $v_z = \partial \psi / \partial x$:

$$
\frac{\partial \varphi}{\partial t} + \frac{\partial \varphi}{\partial x} \frac{\partial \varphi}{\partial z} - \frac{\partial \psi}{\partial z} \frac{\partial \varphi}{\partial x} = \Delta \varphi + \text{Tm} \left( \frac{\partial \chi}{\partial x} \frac{\partial B^2}{\partial z} - \frac{\partial \chi}{\partial z} \frac{\partial B^2}{\partial x} \right),
$$

$$
\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial z} - \frac{\partial \psi}{\partial z} \frac{\partial T}{\partial x} = \frac{1}{\text{Pr}} \Delta T, \quad \varphi = -\Delta \psi \quad \text{curl}_y v.
$$

where $\text{Tm} = \frac{\chi_0 B_0^2 h^2}{(2 \mu_0)^2}$

Substituting the expressions for the derivative of $B^2$ from (2) and taking into account that the dimensionless magnetic susceptibility is $\chi = 1/(1 + \alpha T)$ where $\alpha = \vartheta / T_0$ we have

$$
\frac{\partial \varphi}{\partial t} + \frac{\partial \varphi}{\partial x} \frac{\partial \varphi}{\partial z} - \frac{\partial \psi}{\partial z} \frac{\partial \varphi}{\partial x} = \Delta \varphi + 4 \text{Tm} \frac{1}{\text{Pr}} \alpha \left( \frac{\partial T}{\partial z} z \frac{\partial T}{\partial x} \right),
$$

$$
\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial z} - \frac{\partial \psi}{\partial z} \frac{\partial T}{\partial x} = \frac{1}{\text{Pr}} \Delta T, \quad \varphi = -\Delta \psi \quad \text{curl}_y v.
$$

The boundary conditions at the rigid wall of container are no-slip conditions

$$
\psi \Big|_0 = 0, \quad \frac{\partial \psi}{\partial n} \Big|_0 = 0
$$

The thermal boundary conditions correspond to the perfectly conductive boundaries, the horizontal boundaries $z = l, \ z = l + 1$ are maintained at fixed different temperatures and at the vertical boundaries $x = \text{ml} / 2$ the linear temperature distribution is fixed:

$$
T(x, z) = A(z - l)
$$

where $A = -1$ for the heating from below and $A = 1$ for the heating from above. Most of the calculations were performed for the fixed values of Prandtl number and parameter $\alpha$: $\text{Pr} = 1, \ \alpha = 0.03$. The parameters $\text{Tm}$ and $l$ were varied.

The problem was solved numerically by finite difference method. Uniform mesh with 41x41 nodes was used in most of the calculations.

### 3. Numerical results

The calculations show that at small values of the parameter $\text{Tm}$, independently of the direction of imposed vertical temperature gradient and on the distance $l$, there arises convective flow of two-vortex structure (see, fig.2). The boundary between the vortices coincides with $z$-axis. The flow direction is such that in the central part of the cylinder the fluid moves from the cold wall to the hot one.
Figure 2. Streamlines of convective flow at $T_m = 10^3$, $l = 1$ for the cases of heating from below (left) and above (right).

Figure 3. Kinetic energy of steady flow versus $T_m$ for $l = 1$ in the cases of heating from below and above.

Fig.3 presents the dependences of the convective flow intensity on the parameter $T_m$ at $l = 1$ for the cases of the heating from below and from above. As one sees in both case the flow intensity grows monotonically with $T_m$. At small values of $T_m$ the flow intensities are nearly the same, however with the growth of $T_m$, at some value of $T_m = T_m^*$ the flow intensity for heating from below demonstrates sharp increase and for heating from above the behaviour remains the same which results in the large difference in the flow intensities for $A = -1$ and $A = 1$ at $T_m > T_m^*$. The flow structure for two cases at $T_m > T_m^*$ is presented in fig.4.
Figure 4. Streamlines of convective flow at $T_m = 10^6$, $l = 1$ for the cases of heating from below (left) and above (right).

Figure 5. Kinetic energy of steady flow versus $T_m$ for $l = 4$ in the case of heating from below.

Figure 6. The sum of streamfunction over the fluid volume versus $T_m$ for $l = 4$ in the case of heating from below.

At larger distances between the magnets and the cylinder with the increase of $T_m$ spontaneous breakdown of the symmetry occurs accompanied by the branch of two asymmetric solutions (see, figs. 5 and 6).

4. Conclusions
We have theoretically and numerically studied the behavior of a paramagnetic fluid filling a square cylindrical channel, for zero-gravity space conditions. A particular magnetic field distribution is applied to the fluid, allowing the initiation of thermal convection when a temperature gradient is present. Magnetic dipolar sources are located “below” the channel, at a variable distance of it.

The analysis show that the streamlines of convective flow are related to a specific number $T_m$, the value of which depends on the magnetic field, the magnetic susceptibility and the viscosity of the fluid, and the size of the channel.
Numerical results show that for low value of $T_m$, and low distance between the channel and the magnetic line, in the central part of the channel the fluid moves from the cold wall to the hot one, with a two-vortex structure. Behavior is the same, either for heating from below (in the side of the magnetic sources) or for heating from above. Increasing $T_m$ above a particular value leads to a net dissymmetry, with a sharp increase of the flow intensity for heating from below, instead the flow increase monotonically for heating from below.

For higher values of the distance between the channel and the magnetic line, leading to a better homogeneity of the magnetic force in the channel, asymmetric solutions for the flow appear, with another critical value for $T_m$ number.

These results show that in zero gravity space conditions, the magnetic force distributions in the channel are highly influential on the shape of streamlines. Further work includes taking into account other realistic magnetic sources.

References
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