On the intrinsically cyclic nature of space-time in elementary particles

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Abstract. We interpret the relativistic and quantum behavior of elementary particles in terms of elementary cycles. This represents a generalization of de Broglie hypothesis of intrinsically “periodic phenomenon”. Similarly to a “particle in a box” or to a “vibrating string”, the constraint of intrinsic periodicity can be used as semi-classical quantization condition, with remarkable matchings to ordinary relativistic quantum mechanics. In this formalism the retarded and local variations of four-momentum characterizing relativistic interactions can be equivalently expressed in terms of retarded and local variations of “de Broglie internal clock” space-time periodicity.

1. Introduction
In this paper we present the possibility that, similarly to the harmonics of a “vibrating string” or to the semi-classical quantization of a “particle in a box”, relativistic bosonic fields can be quantized by imposing their characteristic de Broglie space-time periodicity as dynamical constraint [1, 2, 3], see also [4, 5, 6, 7, 8]. Such an assumption of intrinsic periodicity can be regarded as the natural realization of the de Broglie hypothesis at the base of undulatory mechanics (wave-particle duality) [9, 10]. By quoting the introduction of de Broglie’s famous PhD thesis, the formalism described in this paper is based on the fundamental assumption “of existence of a certain periodic phenomenon of a yet to be determined character, which is to be attributed to each and every isolated energy parcel [elementary particle]” [10, 9]. This so-called “de Broglie periodic phenomenon” [11] has been implicitly tested by 80 years of QFT and indirectly observed in a recent experiment [12]. Among others, a similar assumption was also used, for instance, by Schrödinger in his formulation of the zitterbewegung model of the electron, by Bohr in his description of the Hydrogen atom, by Sommerfeld for his quantization condition, etc. Moreover, as Galileo taught with his study on the isochronism of the pendulum, a consistent formalization of the concept of time in physics requires an assumption of intrinsic periodicity for isolated elementary systems. Time can only be defined by counting the number of cycles of isolated phenomena supposed to be periodic (roughly speaking this guarantees that a unit of time is invariant in time). Indeed the operative definition of a second is “the duration of 9,192,631,770 characteristic cycles of the Cs atom”, where $T_{Cs} \sim 10^{-10}$ s. Therefore, according to the assumption of de Broglie “periodic phenomenon”, every elementary system (elementary particle) can be regarded as a reference clock, the so-called “de Broglie internal clock” [12].

To illustrate the idea we consider in this introduction only periodicity in time. According to the de Broglie assumption of “periodic phenomenon”, to an elementary “parcel” of energy
\( \bar{E}(\bar{p}) \) observed in a generic reference frame denoted by \( \bar{p} \) there is associated a de Broglie time periodicity \( T_{t}(\bar{p}) = h/\bar{E}(\bar{p}) \). Relativistic causality is preserved by the assumption of intrinsic periodicity because every retarded and local variation of energy \( \bar{E}(\bar{p}) \) can be equivalently described in terms of retarded and local variation of the dynamical periodicity \( T_{t}(\bar{p}) \) of the “de Broglie intrinsic clock” of the particle. The definition of relativistic clock given by A. Einstein [14] is: “by a clock we understand anything characterized by a phenomenon passing periodically through identical phases so that we must assume, by the principle of sufficient reason, that all that happens in a given period is identical with all that happens in an arbitrary period”. Thus the whole information of a relativistic clock, an thus of the “internal clock” of a particle, is contained in a single (space-time) period. Therefore, by using the language of extra dimensional theories, we formalize de Broglie assumption of the intrinsic time periodicity (“periodic phenomenon”) by assuming the field embedded in a compact time dimension of relativistic length \( T_{t}(\bar{p}) \) and Periodic Boundary Conditions (PBCs) [1]. For the sake of simplicity we consider only the bosonic case. Thus the intrinsically periodic field solution of our bosonic theory is actually similar to a vibrating string embedded in a cyclic time dimension or to a particle in a box. Through discrete Fourier transform, to a compact variable corresponds a quantized conjugate variable, i.e. a variable which takes discrete values. Considering the relation \( \bar{E}(\bar{p}) = h\bar{\omega}(\bar{p}) \), to the specific case of an elementary isolated system with intrinsic periodicity \( T_{t}(\bar{p}) \) is associated the quantized energy spectrum \( E_{n}(\bar{p}) = nh\bar{\omega}(\bar{p}) = nh/T_{t}(\bar{p}) \). Since periodicity conditions mean that the only possible energy eigenmodes are those with an integer number of cycles, we have a correspondence with the Bohr-Sommerfeld quantization condition (for instance, it can be shown that the periodicity condition \( E_{n}T_{t} = nh \) can be more in general written as \( \int E_{n}dt = nh \) for interacting systems). This allows one to solve many non-relativistic quantum problems [11][4].

Similarly to the relativistic Doppler effect we must consider that a de Broglie “periodic phenomenon” appears to have different periodicities if observed from different reference frames denoted by spatial momentum \( \bar{p} \). The periodicity of a “periodic phenomenon” varies with the energy \( \bar{E}(\bar{p}) \) associated to the corresponding particle. According to \( T_{t}(\bar{p}) = h/\bar{E}(\bar{p}) \), the de Broglie time periodicity must be regarded as local and dynamical as the energy. Therefore it varies according to relativistic causality. Since the time periodicity \( T_{t}(\bar{p}) \) transforms in a relativistic way, the proper-time intrinsic periodicity \( T_{\tau} = T_{t}(0) \) fixes the upper bond of the time periodicity \( T_{\tau} \geq T_{t}(\bar{p}) \). In fact the mass \( M = h/T_{\tau}c^{2} \) is the lower bond of the energy, \( Mc^{2} \leq \bar{E}(\bar{p}) \). The proper time periodicity \( T_{\tau} \) is the time for light to travel across the Compton wavelength \( \lambda_{s} = T_{\tau}c \) of the particle. The heavier the mass the faster the proper-time periodicity of the de Broglie “periodic phenomenon”. Hence, even a light particle such as the electron has intrinsic time periodicity equal or faster than \( \sim 10^{-20} \) s. It should be noted that this periodicity is about ten orders of magnitude away from the characteristic time periodicity of the cesium atomic clock, \( T_{Cs} \sim 10^{-10} \) s (the difference between these two periodicities is of the order of the difference between a solar year and the age of the universe), and that it is extremely fast even if compared with the present experimental resolution in time \( \sim 10^{-17} \) s. Thus, for every known matter particle (except the neutrino) we are in the case of too fast periodic dynamics. This aspect provides a link to one of the main motivations for our assumption of a cyclic nature of space-time in elementary particles, i.e. the ’t Hooft determinism [14][15]. It states that there is a “close relationship between the quantum harmonic oscillator” with angular frequency \( \bar{\omega} = 2\pi/T_{t} \), that is the mode of an ordinary quantum field with energy \( \bar{E} = h\bar{\omega} \), “and a classical particle moving along a circle of periodicity \( T_{t} \)”. In fact, if the periodicity is too fast with respect to our resolution in time, it turns out that at every observation the system appears in an aleatoric phase of its cyclic evolution. This is like observing a “clock under a stroboscopic light” [15]. The idea of these deterministic models is that, due to the extremely fast cyclic dynamics, we loose information about the underlying classical theory and we observe a statistical theory
— thus described by fields — that matches QM. For this reason we speak about deterministic or pre-quantum theories. The “de Broglie intrinsic clock” of elementary particles can also be imagined as a “de Broglie deterministic dice” [2], that is a dice rolling with extremely fast time periodicity \( T_t \) with respect to our resolution in time. Similarly to the ’t Hoofs determinism we will see that the statistic description of such a fast cyclic behavior has remarkable correspondences to ordinary QM.

We may also note that, on a cyclic geometry such as that of the de Broglie “periodic phenomenon”, there are many possible classical paths characterized by different winding numbers between every initial and final point. That is, a field with PBCs can self-interfere. Its evolution is naturally described by a sum over the classical paths associated to its cyclic behavior. This gives rise to a remarkable formal correspondence to the ordinary Feynman Path Integral of QM. Moreover, because of the cyclic nature of the ”periodic phenomenon”, the theory has implicit commutation relations. The reader interested in more details or in the mathematic proofs may refers to [1, 4]. In this paper we will also introduce some new aspects of the theory recently published [2], i.e. the generalization of the results of [1] to gauge interaction.

The idea is that every local and retarded variation of four momentum associated to a relativistic interaction scheme can be equivalently described as local and retarded variations of the space-time periodicity of a corresponding de Broglie ”intrinsic clock”. In turn, the variations of periodicity can be encoded in corresponding deformations of the space-time of the theory. Such a geometrodynamical description of interaction arising from this formalism has interesting analogies with general relativity. In fact, linearized gravitational interaction can be equivalently described as variations of (space-time) periodicity of reference clocks (time dilatation and length contraction), which in turn are encoded in corresponding deformations of the metric [10].

### 2. Compact Space-Time formalism

In classical-relativistic mechanics every isolated elementary system is described by a persistent four-momentum \( \vec{p}_\mu = \{E/c, -\vec{p}\} \). On the other hand, wave mechanics prescribes that a four-momentum must be associated to the de Broglie four-angular-frequency of a corresponding field, according to the relation \( \dot{\omega}_\mu \hbar = \vec{p}_\mu c \). Here we will apply (literaly) the de Broglie assumption of “periodic phenomenon” by describing elementary systems of (classical) four-momentum \( \vec{p}_\mu \) in terms of intrinsically periodic fields whose periodicity is the usual de Broglie space-time periodicity \( T^\mu = \{T_t, \vec{\lambda}_x/c\} \). As noticed by de Broglie, a “periodic phenomenon” with mass \( \bar{M} \) is fully characterized by the corresponding proper-time periodicity \( T_\tau = \hbar/\bar{M}c^2 \) [10, 9]. This means that it has topology \( S^1 \). In fact, in a generic frame, the spatial and temporal components of the de Broglie four-periodicity \( T^\mu \) are obtained through Lorentz transformations from the proper-time periodicity: \( cT_\tau = c\gamma T_t - \gamma\vec{\beta} \cdot \vec{\lambda}_x \). The energy and momentum of a particle with mass \( \bar{M} \) in the new reference frame is \( E = \gamma \bar{M}c^2 \) and \( \vec{p} = \gamma \vec{\beta} \bar{M}c \), respectively. Therefore, in a generic reference frame, we have de Broglie-Planck relation (de Broglie phase harmony)

\[
c^2\bar{M}T_\tau \equiv \hbar \quad \rightarrow \quad c\bar{p}_\mu T^\mu \equiv \hbar .
\]

The variation of four-momentum occurring during interaction implies a variations of the intrinsic space-time periodicity of the fields. As the Newton’s law of inertia doesn’t imply that every point particle moves on a straight line, our assumption of intrinsic periodicities does not mean that a system of elementary particles should appear to be periodic. This dynamical relation between space-time periodicity and energy-momentum guarantees time ordering and relativistic causality in the theory: the local and retarded variation of four-momentum occurring during interaction and characterizing relativistic causality is equivalently described in terms of local and retarded variations of the de Broglie space-time periodicity. A combination of “periodic phenomena” in general forms an ergodic system. Moreover, if also interaction is considered, the system results to have a chaotic evolution.
By separation of variables the free cyclic field $\Phi(x,t)$ can be written as a tower of energy eigenmodes $\phi_n(x)$

$$\Phi(x,t) = \sum_n N_n \alpha_n(p) \phi_n(x) u_n(t), \quad \text{where: } u_n(t) = e^{-\frac{i}{\hbar}E_n(p)t}, \quad E_n(p) = n\hbar\omega(p).$$

In this simplified description $N_n$ is the normalization factor and $\alpha_n(p)$ is the coefficients of the Fourier expansion. By bearing in mind the relation $\omega$ comes from the harmonic frequency spectrum $E_n(p) = n\hbar\omega(p)$ of a vibrating string with time periodicity $T_i(p)$. This quantization is the field theory analogous of the semiclassical quantization of a “particle” in a box, it also shares deep analogies with the Matsubara theory, the Kaluza-Klein (KK) theory or with the Boltzmann kinetic theory. As discussed in the introduction the whole physical information of the system is contained in a single space-time period $T^\mu$. Hence, the intrinsically space-time periodic of a free field describing the de Broglie “periodic phenomenon” can be represented as the field solution of a bosonic action in compact four dimensions with PBCs

$$S = \oint_0^{T^\mu} d^4x \mathcal{L}(\partial_{\mu}\Phi, \Phi).$$

In this notation the circle in the volume integral symbols represents the assumption of PBCs. It is important to note that the PBCs minimize the action at the boundaries. Therefore they have the same formal validity of the usual (Synchronous) BCs assumed in ordinary field theory— in particular along the time dimension. This is an essential feature because it guarantees that all the symmetries of the relativistic theory are preserved as in usual field theory, $\text{[1]}$. In particular it guarantees that the theory is Lorentz invariant.

For instance we may consider a generic global Lorentz transformation

$$dx^\mu \rightarrow dx'^\mu = \Lambda^\mu_\nu dx^\nu, \quad \quad p_\mu \rightarrow p'^\mu_{\nu} = \Lambda^\nu_\mu p_\nu.$$  \tag{4}

By definition the phase of the field is invariant under periodic translations $T^\mu$ and is a Lorentz scalar (phase harmony $\text{[11]}$). According to (1), in case of global transformations we have

$$e^{-\frac{i}{\hbar}E_n(p)T^\mu} = e^{-\frac{i}{\hbar}(E_n(p)+cT^\nu)p_\nu} = e^{-\frac{i}{\hbar}(E_n(p)+cT^\nu)p_\nu} = e^{-\frac{i}{\hbar}E_n(p')p'_\mu}.$$  \tag{5}

In this way we see that the space-time periodicity is actually a (space-like, tangent) contravariant four-vector. It transforms under global Lorentz transformations as

$$T^\mu \rightarrow T'^\mu = \Lambda^\mu_\nu T^\nu.$$  \tag{6}

This can be also inferred by noticing that after the transformation of variables $\text{[1]}$, the integration region of the free action $\text{[3]}$ turns out to be

$$S = \oint_0^{T^\mu} d^4x' \mathcal{L}(\partial_{\mu}'\Phi, \Phi').$$  \tag{7}

Therefore, according to (1) in the new reference system, the new four-periodicity $T'^\mu$ of the field is actually given by $\text{[5]}$. Thus $\text{[3]}$ describes the same “periodic phenomenon” in the new reference frame where actually the resulting four-momentum $p'_\mu$ is given by $\text{[3]}$.

Since $T^\mu$ transforms as a tangent four vector $\text{[17]}$, so that (1) can be expressed with the following mnemonic notation $c p'_\mu = \hbar/T^\mu$. In this way it is easy to see that the underlying Minkowski metric induces the following constraint on the dynamical four-periodicity

$$\frac{1}{T^\tau} = \frac{1}{T^\mu} \frac{1}{T^\nu}.$$
In fact, considering the above notation, this is nothing but the relativistic constraint
\[ \overline{M}^2 c^2 = \overline{p}^\mu \overline{p}_\mu. \]

We will denote by the bar sign the quantities related to the fundamental mode. That is
\[ \Phi(x) = \overline{N} \phi(x) = \overline{N} e^{-\frac{i}{\hbar} \overline{p}_\mu x^\mu}. \] (7)

For instance, the Lagrangian of the fundamental mode \( \Phi(x) \) is
\[ S = \frac{1}{2} \int_{T^4} d^4x \left[ \partial_\mu \Phi^*(x) \partial^\mu \Phi(x) - \overline{M}^2 \Phi^*(x) \Phi(x) \right] \] (8)

The circle in the integral symbol has been removed because this single mode solution can be retrieved by assuming ordinary Stationary BCs at the boundary \( T^4 \).

Note that the fundamental mode \( \Phi(x) \) coincides with the mode of Klein-Gordon field with energy \( \overline{E} \) and mass \( \overline{M} \). This also means that the fundamental mode describe a classical particle. Thus, neglecting the higher harmonics of the field, the fundamental mode can be always quantized through second quantization obtaining ordinary quantum field theory. However, without any explicit quantization, when all the possible energy eigenmodes allowed by the PBCs are considered, the periodic field turns out to have the same quantized energy spectrum as ordinary second quantized fields — after normal ordering. From (2) we find the dispersion relation of the harmonic energy spectrum (9)
\[ \overline{E}_n(\overline{p}) = n\hbar \overline{\omega}(\overline{p}) = n \sqrt{\overline{p}^2 c^2 + \overline{M}^2 c^4}. \]

Depending whether we want to make explicit the normalization factor of the energy eigenmodes, we write the cyclic field solution by using the following notations,
\[ \Phi(x) = \sum_n \Phi_n(x) = \sum_n N_n \alpha_n(\overline{p}) \phi_n(x) = \sum_n N_n \alpha_n(\overline{p}) e^{-\frac{i}{\hbar} \overline{p}_n \cdot x}. \] (9)

In the free case, i.e. persistent periodicity, the four-momentum spectrum is harmonic \( p_n \mu = n \overline{p}_\mu \). That is, in the phase of the field the PBCs yields to the Bohr-Sommerfeld condition for isochronic systems \( \oint dx^\mu p_n \mu = p_n T^\mu = n \hbar \). The single quantum number \( n \) is related to de fundamental periodicity \( S^1 \) of the de Broglie clock. A complete description of the intrinsically periodic field solution however should involve a further expansion in spherical harmonics. For instance, in case of isotropic symmetry the field solution must be described in terms of two additional cyclic coordinates, i.e. the angles \( \theta \to \theta + \pi \) and \( \varphi \to \varphi + 2\pi \). These further periodicities reproduce in the standard way the usual quantization of the angular momentum. As a consequence the cyclic field solution has two additional quantum numbers \( \{m, l\} \) for a resulting topology \( S^1 \otimes S^2 \). In this paper we will not consider any further the expansion in spherical harmonics or their deformations. We also note that the natural generalization of this bosonic description to fermions is given by the Schödinger’s zitterbewegung model.

Finally, it is interesting to note that in the rest frame (\( \overline{p} \equiv 0 \)) this quantized energy spectrum is dual to the harmonic KK mass tower \( M_n = \overline{E}_n(0)/c^2 = n \overline{M} \). This mass spectrum is obtained from the harmonic Bohr-Sommerfeld condition in the rest frame \( M_n c \lambda_n = n \hbar \). Indeed, for such

\[1\] From this correspondence to a Klein-Gordon mode it is possible to generalize the geometrodynamical analysis of the de Broglie periodicities that we will perform later to ordinary field theory, [2].
a massive field, the assumption of periodicity along the time dimension means that in the rest frame the proper-time $\tau$ has intrinsic periodicity

$$T_\tau = T_\tau(0) = \frac{\hbar}{M c^2}$$

The invariant mass $\bar{M}$ is fixed geometrically by the reciprocal of the proper-time intrinsic periodicity $T_\tau$ of the elementary field. In other words, by imposing intrinsic time periodicity, the world-line parameter $s = c\tau$ turns out to be compact with PBCs. It behaves similarly to the XD of a KK field with zero 5D mass and with fundamental mass $\bar{M}$\(^2\). As a consequence the world-line compactification length $\lambda_s = cT_\tau$ is the Compton wave length of the field. In order to bear in mind these analogies with an XD field theory we say that the world-line parameter play the role of a Virtual XD (VXD) with compactification length $\lambda_s$. This aspect of the theory is subject of the recent paper \cite{3}. It is interesting to note that, originally, T. Kaluza introduced the XD formalism as a “mathematical trick” to unify gravity and electromagnetism and not as a real XD \cite{15}, whereas the original O. Klein proposal was to use PBCs at the end of a compact XD (cyclic XD) in the attempt to interpret QM \cite{19}.

3. Quantum Behavior

Next we summarize the correspondence between the cyclic evolution of a periodic field with the canonical formulation of QM as well as with the Feynman Path Integral (FPI) formulation. This has been shown in \cite{1,4}. The evolution along the compact time dimension is described by the so called bulk equation of motions \((\partial^2_t + \omega_n^2)\phi_n(x, t) = 0\) — for the sake of simplicity in this section we assume a single spatial dimension $x$, avoiding the expansion in spherical harmonics. Thus the time evolution of the energy eigenmodes can be written as first order differential equations

$$i\hbar \partial_t \phi_n(x, t) = E_n \phi_n(x, t).$$

The periodic field \cite{9} is a sum of on-shell standing waves. Actually this harmonic classical system is the typical case where a Hilbert space can be defined. In fact, the energy eigenmodes form a complete set with respect to the inner product

$$\langle \phi | \chi \rangle \equiv \int_0^{\lambda_s} \frac{dx}{\lambda_s} \phi^*(x) \chi(x).$$

Therefore the energy eigenmodes can be defined as Hilbert eigenstates $|\phi_n(x)\rangle \equiv \phi_n(x)/\sqrt{\lambda_s}$. As can be easily see in the free case where the periodicity is persistent, this definition of the Hilbert space can be extended to an integral over an arbitrary large number $N_x$ of periods $\lambda_s \to V_x = N_x \lambda_s$. On this Hilbert space we can formally build a Hamiltonian operator $\mathcal{H} |\phi_n\rangle \equiv \hbar \omega_n |\phi_n\rangle$ and a momentum operator $\mathcal{P} |\phi_n\rangle \equiv -i\hbar k_n |\phi_n\rangle$, where $k_n = nk = n\hbar/\lambda_s$. Thus the time evolution of a generic Hilbert state $|\phi(0)\rangle \equiv \sum_n a_n |\phi_n\rangle$, i.e. of a generic cyclic field, is actually described by the familiar Schrödinger equation

$$i\hbar \partial_t |\phi(t)\rangle = \mathcal{H} |\phi(t)\rangle.$$  \hfill (11)

Moreover the time evolution is given by the usual time evolution operator $\mathcal{U}(t'; t) = \exp[-i\frac{\hbar}{\lambda_s} \mathcal{H}(t - t')]$ which turns out to be a Marcovian operator: $\mathcal{U}(t''; t') = \prod_{m=0}^{N-1} \mathcal{U}(t' + t_{m+1}; t' + t_m - \epsilon)\) where $N \epsilon = t'' - t'$.

From the fact that the spatial coordinate is in this theory a cyclic variable; by using the definition of the expectation value of an observable $\partial_x F(x)$ between the generic initial and final states $|\phi_i\rangle$ and $|\phi_f\rangle$ of this Hilbert space; and integrating by parts \cite{10}, we find

$$\hbar \langle \phi_f | \partial_x F(x) | \phi_i \rangle = i \langle \phi_f | \mathcal{P} F(x) - F(x) \mathcal{P} | \phi_i \rangle.$$ \hfill (12)

\(^2\) The theory can be even regarded as a particular kind of string theory. In fact the world-line parameter of the theory is compact with PBCs, but the theory can be generalized to Neumann or Dirichlet BCs. It plays the role of the compact world-sheet parameter of ordinary string theory.
Assuming now that the observable is such that \( F(x) = x \) we obtain the usual commutation relation of ordinary QM: \( [x, \hat{P}] = i\hbar \). With this result we have checked the correspondence with canonical QM. Furthermore, it is possible to prove the correspondence with the FPI formulation. In fact, it is sufficient to plug the completeness relation of the energy eigenmodes in between the elementary time evolutions of the Marcovian operator. With this elements at hand and proceeding in a complete standard way we find that the evolution of the cyclic fields turns out to be described by the usual FPI which in phase space is

\[
Z = \lim_{N \to \infty} \int_{V_x} \left( \prod_{m=1}^{N-1} \frac{dx_m}{V_x} \right) \prod_{m=0}^{N-1} \langle \phi | e^{-\frac{i}{\hbar} \left( \hat{H} \Delta \epsilon_m - \hat{P} \Delta x_m \right)} | \phi \rangle .
\]

(13)

Proceeding in complete analogy with the ordinary derivation of the FPI in configuration space we also note that the infinitesimal products in (13) can be generically written in terms of the action of the corresponding classical particle

\[
S_{cl}(t_f, t_i) \equiv \int_{t_i}^{t_f} dt L_{cl} = \int_{t_i}^{t_f} dt \left( \frac{\dot{x}}{P} - H \right)
\]

(14)

Finally the FPI (13) can be written in the familiar form

\[
Z = \int_{V_x} \frac{dx}{V_x} e^{\frac{i}{\hbar} S_{cl}(t_f, t_i)} .
\]

(15)

This important result has been obtained without any further assumption than PBCs. Here we have only considered the case of a free “periodic phenomenon” since this case is already sufficient to show the essential aspects of the correspondence. As we will shortly summarize later this can be generalized to interaction, see [2]. The path integral description of the evolution of a “periodic phenomenon” has a simple classical interpretation which can be shown in both a mathematical and graphical way, [1, 4]. In a cyclic geometry, such as that associated to a “de Broglie periodic phenomenon” there is an infinite set of possible classical paths with different winding numbers that link every given initial and final points. If we imagine to open this cyclic geometry we obtain a lattice with period \( T^\mu \) of initial and final points linked by classical paths. Thus there are many possible classical evolutions of a field from an initial configuration to a final configuration, which can self-interfere similarly to the non-classical paths of the FPI. However there is a fundamental conceptual difference with respect to the usual Feynman formulation: all these possible paths are classical paths, that is they are classical paths with different winding numbers on \( S^1 \). This means that in this path integral formulation it is not necessary to relax the classical variational principle in order to have path interference.

The non-quantum limit of a massive field, i.e. the non-relativistic single particle description, is obtained by putting the mass to infinity. As shown in [1, 4], in an effective classical limit, only the first level of the energy spectrum must be considered. In particular this means that, in the non-quantum limit the “periodic phenomenon” has four-momentum \( \vec{p}_\mu \). This yields to a consistent interpretation of the double slit experiment [4]. The wave-particle duality in this case is enforced to a dualism between “periodic phenomena” and “particles”, which is nothing but a literal realization of the de Broglie original hypothesis [7]. In fact, as can be seen by plotting the \(|\Phi(x)|^2\) (modulo the de Broglie internal clock), the wave function of a massive cyclic is localized inside its Compton wavelength along the path of the corresponding classical particle. The description of this field in the classical-limit where \( M \to \infty \), corresponds to a delta distribution since the Compton wavelength tends to zero, and thus to a particle description.

On the other hand a massless field has infinite Compton wavelength and thus an infinite proper-time periodicity. Its quantum limit is at high frequency where, in fact, the PBCs
are important. In this limit we manifestly have a quantized energy spectrum, as in the
ordinary description of the black-body radiation (no UV catastrophe). In agreement with the
experimental observations, the corpuscular description (single photon) arises at high frequencies.
The opposite limit described by a continuous energy spectrum is when time periodicity tends
to infinity. This is the case of a wave description.

The assumption of intrinsic periodicity implicitly contains the uncertain relation of QM. The
mathematic demonstration is given in [1]. Roughly speaking, in a “de Broglie clock”, to
determine the energy $E(\vec{p}) = \hbar \omega(\vec{p})$ with good accuracy $\Delta E(\vec{p})$ we must count a large number
of cycles, that is to say we must observe the system for a long time $\Delta t(\vec{p})$, according to the
relation $\Delta E(\vec{p}) \Delta t(\vec{p}) \gtrsim \hbar(\vec{p})$.

In the original ’t Hooft toy model the period $T_{\xi}$ was assumed to be of the order of the Planck
and its cyclic dynamics associated to some sort of hidden variables [21]. Furthermore the
Hamiltonian operator was not positive defined. In our case, similarly to the KK theory where
there are no tachyons, a cyclic field can have positive of negative frequency modes but the energy
spectrum describes always positive energies and the Hamiltonian operator is positive defined.
The PBCs in our description play the role of quantization condition, that is a full relativistic
generalization of the quantization of a particle in a box. Therefore we have the remarkable
property that QM emerges without involving any hidden-variable. The Bell theorem can not be
applied to our theory because the hypothesis of local and hidden variable are not satisfied (the
assumption of intrinsic periodicity can be regarded as an element of non locality in the theory).
As suggested by the formal parallelism to QM (in particular concerning the expectation value of
an observable) described so far the theory can in principle violates the Bell’s inequality as QM.

4. Geometrodynamics
To introduce interactions we must bear in mind that the four-periodicity $T^\mu$ is fixed by the
four-momentum $\vec{p}_\mu$ according to the de Broglie-Planck relation [11]. As already said, to an
isolated elementary system (i.e. free field) can be associated a persistent fundamental four-
momentum $\vec{p}_\mu$ and a four-periodicity $T^\mu$. On the other hand, an elementary system under
a generic interaction scheme can be described in terms of corresponding variations of four-
momentum along its evolution with respect to the free case. For instance the value of the four-momentum in a given interaction point $x = X$ can be written as

$$\vec{p}_\mu \to \vec{p}'_\mu(X) = e^a_\mu(x)_{x=X} \vec{p}_a.$$  (16)

In other words we describe interactions in terms of the so called tetrad (or virebein) $e^a_\mu(x)$. Thus
the interaction scheme [11] turns out to be encoded in the corresponding variation (modulation)
of the local (instantaneous) four-periodicity, which in the interaction point $x = X$ is

$$T^\mu \to T'^\mu(x) = e^a_\mu(x)_{x=X} T^a,$$  (17)

Thus, the variation of periodicity occurring during interactions can be thought of as stretching
of the compact dimensions of the theory. This suggests that the interaction scheme [19] can be
equivalently encoded into a corresponding curved space-time background

$$\eta_{\mu\nu} \to g_{\mu\nu}(x) = e^a_\mu(x) e^b_\nu(x) \eta_{ab}.$$  (18)

This description can be double checked by considering the following local transformation of
space-time variables

$$x^\mu \to x'^\mu(x) = x^a \Lambda^\mu_a(x), \text{ where: } e^a_\mu(x) = \left(\frac{\partial x^a}{\partial x'^\mu}\right), \ x^a \Lambda^\mu_a(x) \simeq \int x^a dx^a e^a_\mu(x).$$  (19)
We are working in the approximation $e^a_\mu(x') \simeq e^a_\mu(x)$ (for the sake of simplicity we neglect Christoffel symbols). In the last relation we have used the fact that $x'^a\Lambda^a_\mu(x')$ is the primitive of the tetrad $e^a_\mu(x)$ (omitting prime indexes in the integrand). Indeed, by using (19) as a substitution of variables in the compact 4D action (20), we find that the interaction scheme (19) is described by the following action in deformed compact 4D

$$S \simeq \int T^a\Lambda^a_\mu|_{X}(T) \, d^4x \sqrt{-g(x)} \mathcal{L}(e^a_\mu(x) \partial_{\mu}\Phi'(x), \Phi'(x)).$$

The transformation (19) relates locally the inertial frame $x \in S$ of the free cyclic field solution $\Phi(x)$ to the generic frame $x' \in S'$ associated with the interacting cyclic field solution $\Phi'(x)$. Its Jacobian is $\sqrt{-g(x)} = \det[e^a_\mu(x)]$. The Latin letters describe the free field in an inertial frame $S$ while the Greek letters refer to the locally accelerated frame $S'$ of the interacting field $\Phi'$ (22).

It is important to point out that (19) induces the local deformation (or stretching) of the boundary

$$T^a\Lambda^a_\mu|_{X}(T) \simeq \int_{X}^{X+T^a} dx^a e^a_\mu(x).$$

This actually is the local deformation of the boundary associated with the variation of local periodicity $T^a(x)$, (17), i.e. with the interaction scheme (16). With this local transformations of variable we see that the boundary (21) transforms as an ordinary four-vector ($\propto x^\mu$) whereas the related periodicity (17) transforms as a tangent four vector ($\propto dx^\mu$, 17).

This geometrodynamical approach to interactions is interesting because it actually mimics the usual geometrodynamical approach of GR. Actually gravitational interaction can be described in terms of variation of periodicity of reference clocks. In fact, considering only the fundamental mode (i.e. neglecting quantum corrections), if we suppose a weak Newton potential $V(x) = \frac{-GM_\odot}{|x|} \ll c^2$, we find that the energy of a de Broglie “internal clock” on a gravitational well varies (with respect to the free case) as $E \rightarrow E' \sim (1 + GM_\odot/|x|c^2) E$. According to (17) or (1), this means that the de Broglie clocks in a gravitational well are slower with respect to the free clocks $T_i \rightarrow T'_i \sim (1 - GM_\odot/|x|c^2) T_i$. Thus we have a gravitational redshift $\tilde{\omega} \rightarrow \tilde{\omega}' \sim (1 + GM_\odot/|x|c^2) \tilde{\omega}$. With this schematization of interactions we have retrieved two important predictions of GR. Besides the time periodicity we must also consider the analogous variation of spatial momentum and the corresponding contraction of spatial wavelength of the particles (18). According to the relation (18) the weak newtonian interaction turns out to be encoded in the usual linearized Schwarzschild metric. We have found that the geometrodynamical approach to interactions of a “periodic phenomenon” actually mimics linearized gravity. Here we only mention that, as well known, see for instance (10), it is possible to retrieve ordinary GR from a linear formulation by including self-interactions. Nevertheless it is important to mention that it is not uniquely defined “what is fixed at the boundary of the action principle of GR” (23). SR and GR fix the differential framework of the 4D without giving any particular prescription about the BCs. The only requirement for the BCs is to minimize a relativistic action at the boundary. For this aspect both PBCs have the same formal validity and consistency with relativity of the Stationary BCs of ordinary QFT.

We conclude that a elementary system under the interaction scheme (16) is described by the modulated solutions of the bulk equations of motion on the deformed compact background (18) and intrinsic periodicity (17). Intuitively we can mention that, (2), this variation of periodicity for a single energy eigenmode of periodic field solution can be described with the formalism of modulated signals, e.g. in phase of the free fundamental mode the frequency must be modulated: $-\frac{i}{\hbar}x^\mu p_\mu \rightarrow -\frac{i}{\hbar} \int x^\mu dx^\mu p_\mu(x)$. As discussed in detail in (2), in doing this we are assuming that the normalization factor is invariant. Similarly to the Bohr-Sommerfeld condition we find that, in a given interaction point $x = X$, the quantization condition of the the modulated solution coming
5. Generalization to gauge interaction
This section is a short introduction to the results of the recent study on the “boundary geometrodynamics of gauge interaction” [2]. For the sake of simplicity the geometrodynamics investigated are approximated to local isometries, $\sqrt{-g} \approx 1$. Since a local isometry is a local transformation of flat space-time, the equation of motions (KG equation in ordinary field theory) are left invariant. Thus, in ordinary field theory local isometries have no effect on the field solution. The KG field used for practical computations is the most generic solution of the KG equation so that BCs have a marginal role in ordinary field theory. However, local transformations of flat metric can affect the boundary of the theory. For instance, in the case of local isometries we have local rotations of the boundary of the theory described by (21), whereas the structure of the Lagrangian is left invariant, see [20]. Actually, one of the interesting characteristics of the formalism of compact space-time dimensions is that it allows us to control the variations of the boundary [21] and the consequent variations of field solution associated with transformations of the reference frame [19]. That is, the resulting local rotation of BCs of the field implies a local variation of field solution. In [2] we have studied the possibility that the theses local variations of field solution associated to local rotations of the boundary can be used to describe the internal transformations of ordinary gauge theory. In particular we have shown that in this isometries (the case of local scale or conformal invariance has been partially investigated through the dualism with XD theories in [3]), the resulting interaction scheme (19) can be formally written as minimal substitution. In this description the resulting gauge field encodes the local modulations of periodicity of a matter field under the local transformation of variables. We also show that the only possible way to write fields with different periodicities in an action with persistent boundary is to use gauge invariant terms. Gauge transformations allows one to tune the periodicity of the different fields of a theory in order to minimize the action at the common boundary. For the same reason we have noted that only particular types of local isometries, which we will call polarized, are allowed by the variational principle. These polarized local isometries reproduce Maxwell dynamics for the gauge field. Indeed, the geometrodynamics associated with these particular local isometries can be used to interpret gauge interaction [2]. This can be regarded as in the spirit of Kaluza’s and Weyl’s original proposals.

As already discussed in this paper, the formalism of field theory in compact 4D also provides a semi-classical quantization method [1, 3]. We have shown that the PBCs at the geometrodynamical boundary of the theory represent a semi-classical quantization condition, relativistic generalization of the quantization of a particle in a box. Generalizing the description of sec. [3] to gauge interaction, when the PBCs are explicitly consider at the geometrodynamical boundary of the theory, we find that the variation of four-periodicity of all the energy eigenmodes constituting an interacting cyclic field is formally described in local Hilbert spaces by the Scattering Matrix of QM. Similarly the evolution associated to such a locally transforming “periodic phenomenon” has a remarkable formal correspondence with the FPI of scalar QED.
6. Conclusions
In this paper we have discussed the possibility of a semi-classical interpretation of quantum field theory in terms of de Broglie’s postulate of “the existence of periodic phenomena allied with each parcel of energy” \cite{9}. Such an assumption of intrinsic “periodic phenomenon” is implicitly at the base of the ordinary undulatory formulation of QM and has been indirectly observed in a recent experiment \cite{12}. Here we have formalized intrinsic periodicity by using the formalism of field theory in compact space-time dimensions and PBCs \cite{1}. The main requirement for the BCs is that they must fulfill the variational principle. Actually PBCs minimize the relativistic bosonic action at the boundary of the compact space-time dimensions. Therefore the assumption of intrinsic periodicity is fully consistent with a relativistic bosonic theory. Relativistic causality and time ordering can be seen from the fact that the space-time periodicity $T^\mu$ of a de Broglie “periodic phenomenon” and the classical four-momentum $\bar{p}^\mu$ of a corresponding elementary system are related by the de Broglie-Planck relation (1). Indeed, the retarded and local variations of four-momentum characterizing relativistic interactions in our formalism are equivalently described by corresponding retarded and local variation of the de Broglie space-time periodicity. In the theory this is represented by the fact that the boundary varies in a relativistic way. This also means that our formalism interactions can be described in terms of the geometrodynamics of the boundary of the theory. As shown in \cite{2}, this formalism shows the possibility of a geometrodynamical interpretations of gauge interactions. The gauge field turns out to modulate the periodicity of an interacting ”periodic phenomenon”. With the assumption of intrinsic periodicity the field can be regarded as a vibrating string and we have a relativistic generalization of the semi-classical quantization of the particle in a box. Actually we observe that, without introducing any hidden variable in the theory, the theory reproduces remarkable correspondences with the ordinary quantum behavior. ”This hypothesis [of periodic phenomenon] is at the base of our theory: it is worth as much, like all hypotheses, as can be deduced from its consequences”, \cite{9}. We conclude that, after nearly 90 years, de Broglie’s ideas can play a renewed role to address conceptual and computational problems of modern physics.

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