Increased asymmetry of pit-over-peak statistics with landscape smoothing

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Abstract
The local extremes (i.e., peaks and pits) of the landscape-elevation field play a critical role in the energy, water, and nutrient distribution of a region, but their statistical distributions in relation to landscape evolution have received limited research attention. In this work, we first explain how the spatial correlation structure of the elevation field affects the counts and frequency distributions of local extremes. We then analyze local extremes statistics for eight mountainous landscapes worldwide with diverse hydroclimatic forcings and geologic histories using 24 digital elevation models and compare them with complex terrain of the Erythraeum Chaos region on Mars. The results reveal that the spherical covariance structure captures the observed spatial correlation in these cases, with the peak frequency distribution agreeing well with the elevation frequency distribution. The ratio of the pit-over-peak (POP) count is linked to the degree to which the pit and peak frequency distributions match, and carries the mark of landscape aging. The relationship between the geomorphic development stage (quantified by the reduced fatness of the slope-distribution tail) and the deviation of POP values from unity in old mountainous landscapes confirms that the evolution towards smoother topographies is atypically accompanied by reduced pit counts distinctive of organized valley and ridge patterns.

KEYWORDS
hillslope smoothing, landscape evolution, local extremes, relaxation phase, spatial correlation, topographic slope

INTRODUCTION

Mountain peaks have intrigued humankind since time immemorial, while the less frequent topographic depressions mostly have captured human attention when filled with water, like lakes and ponds (Fowler, 2011; Price, 2013). The small-scale extremes (local maxima or peaks and local minima or pits) of the elevation field affect several ecohydrological and geomorphological processes, acting as hubs for ridge and valley lines and, in turn, influencing the hydrologic partitioning (infiltration, runoff, etc.), local weathering, sediment transport, and so forth (Kiesel et al., 2010; Le & Kumar, 2014; Moser et al., 2007; Thompson et al., 2010). From an ecological perspective, the statistics of these points are related to landscape ruggedness. The latter is a critical factor in resolving the habitat selection of species (Beasom et al., 1983; Reilly et al., 1999; Sappington et al., 2007) and has been quantified by different methods (Berti et al., 2013; Grohmann et al., 2011; Smith, 2014).

Mountainous landscapes comprise a quarter of the land surface. After formation, their relaxation phase is regulated by the interaction of erosion and sedimentation (Bonetti & Porporato, 2017; Hack, 1960; Tucker & Bras, 1998; Willett & Brandon, 2002). The combined effect of these transport mechanisms, due to the inherent direction induced by gravity, acts differently on peaks and pits (as well as on ridges and valleys), so that old mountain ranges in the later stage of geomorphic development (e.g., the Smoky Mountains) not only have smoother hillslope profiles but also have different pit-over-peak statistics, with some pits either filled or worn away, as opposed to young ranges like the Alps. The effect of landscape aging on extreme statistics is expected to be affected by land-surface properties (vegetation cover, soil properties, etc.), underlying geology, climatic forcing, and anthropogenic disturbances as well (Bonetti et al., 2019; Brecheisen et al., 2019; Hooke, 2000). The precise meaning of these observations as well as a quantitative link between the perceived trends of these topographic features and the geomorphological
development stage are largely unexplored in the literature and form the motivation for this study.

Here we focus on quantifying and analyzing the difference between the statistical distributions of local extremes in diverse mountainous landscapes. We first show using idealized synthetic landscapes that the spatial correlation structure of the elevation field holds first-order information about the counts and distribution shapes of local extremes. Although in these idealized cases the impact of the hillslope erosion and sedimentation with the inherent directionality due to gravity is absent, a significant difference in pit and peak distributions may exist in natural landscapes.

To quantitatively evaluate these differences, we define the pit-over-peak ratio (POP) for 24 digital elevation models (DEMs) from eight mountainous landscapes over the Earth and one from the Erythraeum Chaos region on Mars. The deviation of POP from unity shows the decrease of local pit counts compared to the peaks and explains the dissimilarity between pit and peak distributions. Comparison of POP values with the fatness of the slope distribution tails hints at the possible linkage between the geomorphic development stage and the local extremes statistics. This result indicates that the peak and pit distributions evolve asymmetrically as the hillslopes get smoothened by the interplay of erosion and sedimentation during the relaxation phase of the mountainous landscapes.

2 | SPATIAL CORRELATION AND EXTREMES IN SYNTHETIC LANDSCAPES

We begin by studying how the spatial correlation structure influences the counts and frequency distributions of local extremes (i.e., peaks and pits) for synthetic landscapes by employing both 1D and 2D numerically generated elevation fields.

2.1 | 1D Elevation Profiles

For 1D elevation profiles, the homogeneous elevation field \( z \) was characterized by a standard Gaussian probability distribution with \( \sigma \) as the standard deviation. The frequency distribution of peak \( (p^+ (z)) \) can be obtained analytically for this case (Soong, 1973; Vanmarcke, 2010) as

\[
p^+ (z) = \frac{\sqrt{1 - \lambda^2}}{2\pi \sigma^2} \exp\left( \frac{-z^2}{2\sigma^2(1 - \lambda^2)} \right) \left[ 1 + \operatorname{erf}\left( \frac{\lambda z}{\sqrt{2(1 - \lambda^2)}} \right) \right],
\]

where \( \operatorname{erf}(\cdot) \) is the error function (Zill et al., 2011) (a detailed derivation is provided in the Supporting Information). The parameter \( \nu \) represents the ratio of average number of up-crossings (crossings with positive slope) at the (zero) mean elevation to the expected number of peaks per unit length of the elevation profile. Using the definition of spectral moments \( \lambda_1 = \int_{-\infty}^{\infty} |\omega|^2 S_{\nu} d\omega \), with \( \omega \) as the associated frequency and \( S_{\nu} \), as the spectral density function), \( \nu \) can be written as

\[
\nu = \frac{\lambda_1^2}{2\sigma^4}.
\]

The limiting case, \( \nu \to 1 \), can be interpreted as there is one likely peak on average for each zero up-crossing, resulting in a Rayleigh distribution. For the other limit \( \nu \to 0 \), we can expect a very high number of peaks for every up-crossing at zero, giving the same Gaussian distribution of the elevation field. As Equation (2) indicates, the value of \( \nu \) as well as the count and frequency distribution shapes of local extremes, is controlled by the spread of the spectral density function.

This information in the frequency domain can also be translated into the spatial domain as spatial correlation and spectral density are related to one another (Fourier transform pairs), suggesting that different correlation structures of the elevation field modify the frequency distributions of local extremes in the domain (Stoica & Moses, 1997).

In this analysis, we focused on the Gaussian and spherical covariance structures, which have contrasting forms near small lag/spatial separation. The Gaussian covariance structure as a function of lag distance has a (twice-differentiable) parabolic shape at the zero-lag origin, ensuring short-scale spatial continuity for nearby elevation values. This short-scale spatial continuity is absent from the spherical covariance structure that starts linearly from the zero-lag distance, forming a cusp at the origin (Pycz & Deutsch, 2014). A 1D elevation field for both cases was realized, where peaks/pits were categorized as the points in the series having values higher/lower than the two adjacent neighbors. Results for the Gaussian covariance structure with a length scale of 10 units and a spherical covariance structure with a length scale of 100 units are shown in Figure 1a,b. The black dashed curve displays the frequency distribution of the elevation field and the filled step graph shows the numerically obtained distribution of the peak matching with the (solid) curve for the analytical expression given by Equation (1).

The peak frequency distribution does not match the elevation frequency distribution for the Gaussian covariance structure since peaks are found largely at high elevation values (Figure 1a). For the spherical covariance structure, a good match in the shape of the peak distribution (analytically and numerically) and the frequency distribution of the elevation field occurs. The pit frequency distributions for both covariance structures appears as a reflection of the respective peak distribution across the y-axis (insets in Figure 1a,b). For the Gaussian covariance structure, peak and pit distributions remain to the right and left of the elevation frequency distribution, while three distributions match for the case of the finite length scale of the spherical covariance structure. Another crucial piece of information is about the peak and pit counts (shown alongside the respective distribution), which are high for spherical structure as opposed to the Gaussian covariance structure. This confirms that changing the correlation structure from the Gaussian to spherical not only alters the distribution of local extremes but also increases the number of local extremes found in the domain.

The contrasting effect of two covariance structures on the counts and frequency distribution shapes of local extremes can be explained by the computed value of \( \nu \) in both cases. For the Gaussian covariance structure, the number of peaks and the number of up-crossings at the
mean elevation are reduced by the same proportion, so that the value of $\nu$ stays high ($= 0.602$; Figure 1a) and the peak distribution does not shift leftward. In the case of spherical covariance structure, the number of peaks remains high while the zero up-crossing count gets reduced, such that the value of $\nu$ declines ($= 0.149$; Figure 1b), and the frequency distribution of peak shifts and matches the elevation frequency distribution. These alterations in up-crossings and extremes statistics can be discerned from the spatial fluctuation patterns shown in the insets for both covariance structures.

### 2.2 2D elevation profiles

We further examined the frequency distributions of peak and pit for the 2D isotropic field on a raster grid for the same two sets of covariance structures, where the nodes in the domain having values higher/lower than the eight adjacent neighbors were determined as local peaks/pits. 2D elevation field was realized from the Gaussian distribution with 1500 units mean and 100 units standard deviation on a square domain with 1500 units side length. We simulated a base case of elevation field with no spatial correlation to be compared with the Gaussian covariance structure for a length scale of 20 units and the spherical covariance structure for a length scale of 100 units (shown in Figure 1c,d,e). The black dashed curve shows the frequency distribution of the elevation field and the solid-filled/hashed histogram displays the obtained frequency distribution of peak/pit. Using isotropic Gaussian/spherical covariance structure with 20/100 length scale. The elevation field frequency distribution is displayed as a black dashed curve. Peak distribution is presented as a solid-filled step graph and pit distribution as a hashed step graph with counts mentioned for each case. The inset in each panel shows a 3D surface plot (from a square portion with a 50-unit long side) of the corresponding realization.
elevation field with the high number of peak and pit counts in this scenario (Figure 1e). The shifts in the spatial fluctuation patterns due to two sets of covariance structures can be visualized from the 3D surface plots of the square domain (50-unit long side) taken from the realized elevation fields. As the insets Figure 1c,e show, a small number of peaks/pits occur mostly above/below the mean elevation value for the Gaussian covariance structure, unlike a high count of peaks/pits scattered both above and below the mean elevation field with three frequency distributions matching for the case of spherical covariance structure.

3 | REAL LANDSCAPES

Informed by the theoretical and numerical results of the previous section on the role of the spatial correlation structure on the local extremes, we analyzed extreme distributions of the elevation field for different mountain ranges. Following the definition of extremes used for synthetic landscapes, individual pixels in the DEM having higher/lower elevations compared to the eight adjacent neighbors were identified as local peaks/pits. Besides the correlation structure impacts, we also wanted to examine the potential effect of hillslope smoothing on these local extremes statistics. Twenty-four DEMs from eight mountain ranges over the Earth (three DEMs per mountain range) were analyzed in the study (Figure 2). We used three distinct classes of DEM: 3D Elevation Program DEMs for the Rocky and Smoky Mountains in the United States; ASTER Version-3 DEMs for the remaining mountainous landscapes across the globe; and one DEM by NASA Pacific Regional Planetary Data Center for the Erythraeum Chaos region on Mars (Malin et al., 2007; Spacesystems N & Team USJAS, 2019; US Geological Survey, 2017). This was done to lessen the bias of the particular type of sensor error and the applied data-processing algorithm on the final constructed DEMs. The reader is referred to the Supporting Information for more information on the types of data sets used and the pre-processing performed before the statistical analysis.

3.1 | Pit vs peak frequency distributions

For all the DEMs considered in this study, we found good agreement between the frequency distributions of peak and elevation field, showing an effectively decoupled functional relationship between peak frequency distribution and hydroclimatic conditions. For pits, we witnessed several cases where they too obeyed the elevation frequency distribution, with all three frequency distributions matching each other, but there were mountainous landscapes where the pit frequency distributions did not reflect the same shape as peak and elevation frequency distributions. Between these two observed limits of the frequency distribution ensembles, a few landscapes were

![Figure 2](http://example.com/figure2.png)
observed to maintain some similarities between pit frequency distributions with elevation field and peak frequency distributions. We juxtapose two limits of frequency distribution ensembles (peak, pit, and elevation) in Figure 3, where the frequency distribution of the elevation field is shown as a black dashed curve, peak frequency distribution is the solid-filled step graph, and pit frequency distribution is displayed as a hashed step graph. Panels (a) and (b), from Mars and Ural Mountains, present the landscapes where three distributions follow the same shape. Panels (c) and (d) present distributions for the DEMs from the Smoky Mountains and the Rocky Mountains, where the elevation field distribution agrees with the peak frequency distribution but differs from the pit frequency distribution.

We examined the spatial correlation of the elevation field for selected DEMs by estimating the variogram and performing the covariance structure fit (Müller & Schüler, 2021). The results for two DEMs are presented in the insets of Figure 3b,d: UR2 (the three frequency distributions match) and RC3 (peak and elevation frequency distributions coincide, different from pit distribution). The variograms ($\gamma$) of the elevation field as the function of lag distance ($r$) along the $x$- and $y$-axes are displayed using diamond and circle symbols, respectively. The spherical covariance structure fits well for both DEMs along two axes (dashed/dotted curve for $x/y$-axis) and captures well the spatial structure of the elevation field with correlation coefficient $r = 0.997$ for the $x$-axis and $r = 0.988$ for the $y$-axis in RC3, and $r = 0.997$ for the $x$-axis and $r = 0.983$ for the $y$-axis in UR2 DEM.

3.2 Pit-over-peak ratio

The spherical correlation structure of mountainous landscapes elucidates the overlap between the peak and elevation frequency distributions (similar to the synthetic landscapes). However, it does not explain the dissimilarity between elevation and pit distributions observed in some cases. To explore the reasons for this, we examined the POP—i.e., the ratio of pit counts to peak counts. POP is around 1 for synthetic surfaces, with both local extremes frequency distributions resembling the elevation frequency distribution (Figure 1e).

We quantified the differences between shapes of frequency distributions using the Kantorovich–Rubinstein (KR) distance function, which represents the cost associated with the most optimal mapping that shapes one probability distribution into another (Panaretos & Zemel, 2020). For 1D distributions, this metric equals the area between cumulative distributions of two frequency distributions (Villani, 2003). We defined $\Delta_{DKR}$ as a measure of dissimilarity of pit frequency distribution compared to peak and elevation frequency distributions. The metric $\Delta_{DKR}$ was computed by subtracting the KR

![Figure 3](https://example.com/figure3.png)
distance of peak and elevation frequency distributions from the KR distance of pit and elevation frequency distributions with a lower value where three distributions match as opposed to the higher value of \( \Delta \text{DKR} \) for cases where pit distribution does not conform with the other two frequency distributions (refer to the Supporting Information for more details).

The relationship between POP and \( \Delta \text{DKR} \) is shown in Figure 4, where each circle symbol represents a DEM considered in this study. A strong negative correlation between these two variables indicates that, as POP reduces, the distance of pit frequency distribution increases from the elevation and peak frequency distributions. The DEMs in Figure 3 can be mapped to the different regions of the phase space based on POP and \( \Delta \text{DKR} \). Panels (a,b) of Figure 3 (MA, UR2) correspond to the top-left region of Figure 4, where the count ratio of the pit to the peak is close to unity, and consequently a good match among peak, pit, and elevation frequency distributions is observed. For DEMs in the bottom-right region (e.g., SM2 and RC3 from Figure 3), there is a mismatch between the pit frequency distribution and the peak and elevation frequency distributions. DEMs like AP3, EH3 belong to the middle region, where the shape of pit frequency distribution only partly agrees with peak and elevation frequency distributions.

### 3.3 Loss of pits and smoothing of landscapes with age

The behavior observed in the previous section has revealed a landscape evolution characterized by a decline in pits while leaving peak statistics relatively unchanged. This asymmetric evolution of extremes likely arises due to the directionality of sediment transport mechanisms set by gravity. In fact, the interplay of erosion and sedimentation processes going from the top of the ridges to the base of the hillslopes and the related valley aggradation influence local pits and peaks differently. Local pits define flow-convergence zones of the elevation field as opposed to local peaks where the flow diverges (Bonetti et al., 2018). Consequently, considering the so-called stream-power law for fluvial erosion, \( K_a a^m z^n \) (where \( a \) is the specific contributing area, \( K_a \) is the erosion coefficient, and \( m, n \) are positive constants) (Bonetti et al., 2020; Royden & Taylor Perron, 2013; Whipple & Tucker, 1999), it appears that erosion is more effective in eroding away a pit’s surroundings given the larger contributing area as opposed to peak erosion where the contributing area is zero (Cayley, 1859; Florinsky, 2016). Moreover, at the base of the hillslopes, both colluvial (mass movement of sediment due to gravity) and alluvial (overland or channelized water flow under the action of gravity) processes supply and redeposit the material from the upper parts of the hillslopes with a high chance of filling a local topographic depression (pit).

Several investigations following the study by Strahler (1956) described the relationship between the statistical properties of the landscape (elevation, slope, gradient, etc.) and its systematic progress towards maturity (Hurst et al., 2013; Vico & Porporato, 2009). Particularly, the slope frequency distribution parameters were shown to provide a compact metric of aging while comparing landscapes in different stages of the relaxation phase (Bonetti & Porporato, 2017; Bonetti et al., 2019). These works explained that the landscape age tends to be linked to the slope \( S \) distribution tails via a power law as

\[
\rho(S) \propto S^\beta,
\]

for large values of slope in the distributional tails. The exponent \( \beta \) is typically higher for young ranges but decreases with age and reaches...
lower values for old landscapes with smoother hillslopes in the relaxation phase of the geomorphic evolution (Bonetti & Porporato, 2017; Bonetti et al., 2019).

Building on these results, we calculated the exponent $\beta$ of the slope distribution tails for DEMs considered in this study to investigate the modified pit statistics in relation to landscape smoothing with age. Linear regression of the slope distribution tail on the logarithmic scale was performed to compute the exponent value for each DEM. The power-law region for the tails of the slope distributions were considered to extend between distribution values $p(S) = 10^{-5.5} \text{ and } p(S) = 10^{-2.5}$ (see Supporting Information for more details). The value of the exponent is rather insensitive to the specified limit values as long as the linear regression is employed to the region of the slope distribution tail that obeys the power-law form (Bonetti and Porporato, 2017).

For the studied DEMs, we compared the metric $\Delta D_{\text{DKR}}$ with the corresponding value of $\beta$. The results are shown in Figure 5a, where each symbol represents the position of a DEM in the space formed by $\beta$ (x-axis) and $\Delta D_{\text{DKR}}$ values (y-axis). This analysis presents a strong correlation between the reduced slope-tail fatness and increased distance of the pit distribution from peak and elevation frequency distributions (gray line showing the linear trend). A similar comparison of POP values with $\beta$ for 25 DEMs is presented in Figure 5b, where the data display the connection between the reduction in pit counts relative to the local peak counts as landscapes attain smoother topography. Although there exists a scatter in the trend, it still reveals an evident agreement between modified pit statistics and the smoother hillslope profiles as landscapes age. The presented results extend the linkage between the landscape development stage and the behavior of slope distribution tails to the enhanced asymmetry of local pit-over-peak statistics. As landscapes evolve over time, there is a systematic progression of extreme statistics beyond the reduced hillslope steepness with pits getting reduced, unlike robust peaks, which increases the statistical distance between the two distributions.

Landscapes positioned in different parts of this space have distinctive hillslope morphology as well as pit-over-peak statistics. For example, DEMs like SM1, SM2, and SM3 are situated on the west side of the Smoky Mountains with a humid climate. In these ranges, hillslopes have been eroded over the past $\sim$360 million years (Hibbard, 2000), resulting in gentle rolling ridges and valleys (average $\beta = -13.47$) with fewer pit counts in the region (average POP = 0.45) and a high degree of mismatch between pit and peak frequency distributions (average $\Delta D_{\text{DKR}} = 0.143$). On the contrary, DEMs like HR1, HR2, and HR3 with POP $= 1$, which belong to the northwest rugged part of the Hamersley Range from Western Australia, have high topographic unevenness with thin ridges and steep hillslopes (average values of $\beta = -7.22$, $\Delta D_{\text{DKR}} = 0.036$). Local studies confirm these features while describing the region as mountainous deserts with an arid climate (Byrne et al., 2017; Department of Water WA, 2016; van Etten & Fox, 2017). Consequently, less favorable conditions for sediment transport and smooth hillslope morphology have resulted in unaltered pit-over-peak statistics.

The correlation of reduction of pit counts with age also suggests that this process is associated with the so-called ‘relaxation phase’ of landscape evolution, which follows the faster ‘freezing phase’ (Banavar et al., 2001; Sinclair & Ball, 1996). During the freezing time phase, rapid development of the drainage network occurs, after which its form is reasonably fixed. Once the initial rapid network-forming phase of an evolving landscape passes, the subsequent slow relaxation phase is dominated by the gradual smoothing of hillslopes (Bonetti & Porporato, 2017; Fernandes & Dietrich, 1997). The interaction of erosion and sedimentation controls the hillslope...

**FIGURE 5** Linkage between local extremes statistics and smoothing of hillslopes. (a) Plot of $\beta$ and $\Delta D_{\text{DKR}}$ for 25 DEMs with the linear trend shown as a gray line (correlation coefficient $r = 0.781$). (b) Space formed by $\beta$ (x-axis) and POP (y-axis) with the linear trend shown as a gray line (correlation coefficient $r = 0.662$). The observed relationship indicates that younger landscapes (large $\beta$ with fatter tails of slope distributions) tend to have POP values closer to 1 and a higher degree of similarity of pit distribution shapes with peak and elevation frequency distributions. The statistical distance of the pit distribution from peak and elevation distributions increases with a reduction in their counts, unlike local peaks, as landscapes age and reach smoother geometries. [Color figure can be viewed at wileyonlinelibrary.com]
evolution towards quasi-equilibrium forms in this phase, as long as underlying geological and hydroclimatic conditions remain the same (Hack, 1960; Kirkby, 1971). As landscape systematically progresses through this stage towards maturity, asymmetric evolution of pit-over-peak statistics is likely to emerge by the sediment transport mechanisms with reduced hillslope steepness and smoother morphology.

4 | CONCLUSIONS

A statistical analysis of extremes of the elevation field performed in this study showed the enhanced asymmetry of local pit-over-peak statistics as mountainous hillslopes smooth and advance towards geomorphic maturity. We first determined that the spherical covariance structure captures well the spatial correlation of the elevation field in complex mountainous landscapes. The covariance structure explains a good match between peak and elevation frequency distribution shapes, a feature observed in most mountainous landscapes.

The matching degree between pit frequency distribution shapes compared to peak and elevation frequency distributions varied for landscapes analyzed in this study. To explain these changes in pit distributions quantitatively, we defined a metric given by the ratio of pit counts to peak counts (POP). Its relationship with ΔDKR (a measure of dissimilarity of pit distribution to peak and elevation frequency distributions) reveals that the reduction in local pit counts caused the mismatch between frequency distribution shapes. During the long timescale of relaxation phase, the peak statistics remain practically unaltered under diverse hydroclimatic forcing, while the directionality inherent in sediment transport over the hillslope due to gravity reduces the pit counts as the landscape advances towards maturity. To support this reasoning, we analyzed the slope distribution tails for the selected DEMs, which tend to be heavier in young landscapes compared to the mature ranges. The relationship between ΔDKR and the fatness of the slope-distribution tails determines the preferential reduction of pits along with smoother hillslopes.

These results have notable implications for landscape-evolution modeling as well as hydrologic response studies (Bonetti et al., 2020; Chen et al., 2014; Tucker & Hancock, 2010; Walker et al., 2021). In particular, current start-of-the-art models that examine water/sediment transport, transient dynamics, and channel-forming instabilities for landscape evolution exclude topographic pits from the model by removing or filling pits during the initial condition of the simulations (Garbrecht & Martz, 1997; Martz & De Jong, 1988; Soille et al., 2003). The POP value is well above zero even for old smoothened mountainous landscapes, which indicates a restricted scope of these models on simulating and analyzing the mountainous landscape morphology with local extremes during the relaxation phase. Hence a genuine scope exists for future hydrogeomorphic modeling efforts that include the presence of local extremes during the water flux movement and landscape evolution (Barnes et al., 2020; Callaghan & Wickert, 2019; Li et al., 2011).

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CONFLICT OF INTEREST

The authors declare that they have no known conflict of interest.

AUTHOR CONTRIBUTIONS

Both authors contributed equally to designing the study, S.K.A. performed the analysis; Both authors discussed the results and wrote the manuscript.

DATA AVAILABILITY STATEMENT

3D Elevation Program (3DEP) 1 arc-second resolution DEMs were made available by the National Map Data Download and Visualization Services (managed by USGS). ASTER Global Digital Elevation Model V003 DEMs were made available by EARTHDATASEARCH webservice (managed by NASA, Japan’s Ministry of Economy, Trade and Industry (METI), and Japan Space Systems). A DEM of grid resolution 5.19 m obtained by the stereo pairs of MRO CTX was made possible by NASA Pacific Regional Planetary Data Center (PRPDC).

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