Sudden- or Frozen-density approximation in Semi-classical extended Thomas Fermi model for Wong formula used in \( ^{64}\text{Ni} + ^{64}\text{Ni} \) reaction

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Abstract

The \( \ell \)-summed Wong expression is applied in the semiclassical extended Thomas Fermi (ETF) method of Skyrme energy density formalism (SEDF), using both the sudden and frozen-density approximations, to the case of \( ^{64}\text{Ni} + ^{64}\text{Ni} \) reaction data. We find that a very much improved fit (almost exact, point to point) to the data is obtained for the frozen-density approximation with \( \ell_{\text{max}} \) (E.c.m.) varying smoothly. On the other hand, the same could not be achieved in the sudden case, still requiring a modification of the barrier at both the below- and above-barrier energies. This result means to say that the phenomenon of hindrance, observed in coupled channel calculations for \( ^{64}\text{Ni} + ^{64}\text{Ni} \) reaction, could be explained simply on the basis of the \( \ell \)-summed Wong expression, at least for the frozen-density approximation in ETF method of SEDF.

1. Introduction

The semiclassical extended Thomas Fermi (ETF) method of Skyrme energy density formalism (SEDF) provides a convenient basis for the calculation of nucleus-nucleus interaction potential. Here, both the kinetic energy density \( \tau \) and spin-orbit density \( \bar{J} \) are functions of the nucleon density \( \rho_q, \ q=n,p \). For the composite system, in ETF, the densities can be added either in sudden or frozen approximation [1]. Sudden-density contains the exchange terms due to anti-symmetrization whereas the frozen-density has no such effects in it. The exchange effects arise since, for the composite system, \( \tau(\rho) \) and \( \bar{J}(\rho) \) are expressed as functions of the \( \rho_i \ (i=1,2 \ \text{for two nuclei}) \), which, in turn, are the sums of their nucleon densities \( \rho_i=\rho_{i\nu}+\rho_{ip} \), with \( \rho=\rho_1+\rho_2 \). On the other hand, in frozen-density, the composite nucleus densities are simply the sums of the densities of two incoming nuclei.
In a recent paper [2], using Wong's approximate $\ell=0$ barrier-based formula [3] in ETF with sudden-densities, we were able to fit reasonably well (not exactly) the fusion-evaporation cross-sections for at least the $^{64}\text{Ni} + ^{64}\text{Ni}$ reaction with the barrier modified (lowered) by varying the half-density radius $R_0$ and surface thickness $a_0$ parameters of the two-parameter Fermi density (see Fig. 1(a), dotted line). An exactly similar fit is also obtained for the frozen-densities (also, see Fig. 1(a), dashed line), using another $R_0$ and $a_0$ parameter set. In fact, the barrier modification effects are also shown [4] to be contained in the Wong's $\ell$-summed expression, neglected in its $\ell=0$ barrier-based formula [3].

In this paper, we apply the $\ell$-summed Wong expression for the first time in the semiclassical ETF method of SEDF, using both the sudden and frozen-density approximations, to the $^{64}\text{Ni} + ^{64}\text{Ni}$ reaction data [5]. The paper is organized as follows: Sections 2 and 3 give, respectively, the details of the semi-classical ETF model and the Wong formula. The calculations and results are presented in Section 4.

2. The semiclassical ETF model

The interaction potential in SEDF is

$$V^N_N(R) = \left\{ H(\rho, \tau, J) - \left[ H_1(\rho_1, \tau_1, J_1) + H(\rho_2, \tau_2, J_2) \right] \right\} dR$$

with $H$ as the Skyrme Hamiltonian density, given by

$$H(\rho, \tau, J) = \frac{\hbar^2}{2m} \tau + \frac{1}{2} \int \left[ \left( 1 + \frac{1}{2} \alpha_n \right) \rho^2 - \left( \frac{1}{2} \alpha_s \rho_s^2 + \rho_n^2 \right) \right] + \frac{1}{12} \int \left[ \left( 1 + \frac{1}{2} \alpha_n \right) \rho^2 - \left( \frac{1}{2} \alpha_s \rho_s^2 + \rho_n^2 \right) \right]$$

$$+ \frac{1}{4} \int \left[ t_2 \left( 1 + \frac{1}{2} \alpha_n \right) \rho^2 - \left( \frac{1}{2} \alpha_s \rho_s^2 + \rho_n^2 \right) \right] + \frac{1}{16} \int \left[ 3 \left( 1 + \frac{1}{2} \alpha_n \right) \rho^2 - \left( \frac{1}{2} \alpha_s \rho_s^2 + \rho_n^2 \right) \right] \left( \nabla \rho \right)^2$$

$$+ \frac{1}{4} \int \left[ t_2 \left( 1 + \frac{1}{2} \alpha_n \right) \rho^2 - \left( \frac{1}{2} \alpha_s \rho_s^2 + \rho_n^2 \right) \right] \left( \nabla \rho \right)^2$$

$$- \frac{1}{2} \int \left( \rho \nabla \cdot \rho \right)^2 + \left( \rho \nabla \cdot \rho \right)^2 + \left( \nabla \cdot \rho \right)^2.$$

Here $\rho_q$, $\tau_q$ and $J_q$ ($q=n,p$) are the nucleonic, kinetic energy and spin-orbit densities, respectively. $m$ is the nucleon mass. $x_i$, $t_i$, $\alpha_0$ and $W_0$ are the Skyrme force parameters, fitted by different authors to obtain better descriptions of various ground state properties of nuclei.

In Extended Thomas-Fermi (ETF) model, the kinetic energy density $\tau(\mathbf{r})$ and spin-orbit density $J(\mathbf{r})$ are functions of the nucleon density $\rho_q$, included here up to second order,
defined as

\[
\tau_q^{ETF}(\bar{r}) = \frac{3}{5} \left( \frac{3}{2} \pi \right)^{\frac{2}{3}} \rho_q^{\frac{5}{3}} + \frac{1}{36} \left( \frac{\Delta \rho_q}{\rho_q} \right)^{\frac{2}{3}} + \frac{1}{3} \Delta \rho_q + \frac{1}{6 \rho_q} \left[ (\nabla \rho_q) \cdot (\nabla f_q) + \rho_q \Delta f_q \right] \\
- \frac{1}{12} \rho_q \left( \frac{\nabla f_q}{f_q} \right)^2 + \frac{1}{2} \left( \frac{2m}{\hbar^2} \right)^2 \rho_q \left( \frac{W_0}{f_q} \nabla \rho_q + \rho_q \right)^2 = \tau_{TF} + \tau_S
\]  

(3)

\[
\bar{J}_q(\bar{r}) = -\frac{2m}{\hbar^2} W_0 \frac{\rho_q}{f_q} \nabla \rho_q(\rho_i + \rho_q), \; q=n,p; \; i=1,2
\]  

(4)

with the effective mass form factor

\[
f_q(\bar{r}) = 1 + \frac{2m}{\hbar^2} \left[ \frac{1}{4} \left( t_1(1 + x_1) + t_2(1 + x_2) \right) \rho_q(\bar{r}) - \frac{1}{4} \left( t_1(x_1 + \frac{1}{2}) - t_2(x_2 + \frac{1}{2}) \right) \rho_q(\bar{r}) \right].
\]  

(5)

For the composite system, depending on the approximation used, in:

(a) Sudden approximation,

\[
\tau(\rho) = \tau(\rho_i + \rho_2) = \tau(\rho_{in} + \rho_{2n}) + \tau(\rho_{ip} + \rho_{2p})
\]

\[
\bar{J}(\rho) = \bar{J}(\rho_i + \rho_2) = \bar{J}(\rho_{in} + \rho_{2n}) + \bar{J}(\rho_{ip} + \rho_{2p})
\]

and, in (b) Frozen approximation,

\[
\tau(\rho) = \tau_1(\rho_i) + \tau_2(\rho_2), \quad \bar{J}(\rho) = \bar{J}_1(\rho_i) + \bar{J}_2(\rho_2) \quad \text{with}
\]

\[
\tau_1(\rho_i) = \tau_{in}(\rho_{in}) + \tau_{ip}(\rho_{ip}), \quad \bar{J}_1(\rho_i) = \bar{J}_{in}(\rho_{in}) + \bar{J}_{ip}(\rho_{ip}).
\]

Introducing slab approximation, we write Eq. (1) as nuclear proximity potential [6]

\[
V_N(R) = 2\pi R \int_{s_0}^{\infty} e(s) ds = 2\pi R \int \left[ H(\rho) - \left[ H_1(\rho_1) - H_2(\rho_2) \right] \right] dZ.
\]  

(6)

with \( R = R_{01}(\alpha_1, T) + R_{02}(\alpha_2, T) + s \). Here, \( R_{01} \) and \( R_{02} \) are the temperature (T) dependent radii of two deformed and oriented nuclei, separated by \( s \), with minimum \( s_0 \)-value. \( \bar{R} \) is the mean curvature radius given by \( \bar{R} = \frac{R_{01}R_{02}}{R_{01} + R_{02}} \), defining the geometry of the system.

For nuclear density \( \rho_i \), we use the T-dependent Fermi density distribution

\[
\rho_i(Z_i) = \rho_{0i}(T) \left[ 1 + \exp \left( \frac{Z_i - R_i(T)}{a_i(T)} \right) \right]^{-1}
\]  

(7)

with -\( \infty \leq Z \leq \infty \), \( Z_2 = R - Z_1 \), and \( \rho_{0i}(T) = \frac{3A}{4\pi R_{0i}^3(T)} \left[ 1 + \frac{\pi^2 a_i^2(T)}{R_i^2(T)} \right]^{-1} \) with nucleon densities \( \rho_{0q} \).
further defined as \( \rho_m = \frac{N_i}{A_i} \rho_i \), \( \rho_w = \frac{Z_i}{A_i} \rho_i \), and the half density radii \( R_0(T = 0) \) and the surface thickness parameters \( a_i(T = 0) \) obtained by fitting the experimental data to the polynomials in nuclear mass \( A (= 4–209) \), as [7]

\[
R_0(T = 0) = 0.90106 + 0.10957 A_i - 0.0013 A_i^2 + 7.71458 \times 10^{-6} A_i^3 - 1.62164 \times 10^{-8} A_i^4, \\
a_i(T = 0) = 0.34175 + 0.01234 A_i - 2.1864 \times 10^{-4} A_i^2 + 1.46388 \times 10^{-6} A_i^3 - 3.24263 \times 10^{-9} A_i^4.
\]

The temperature dependence in the above formulae are then introduced as in Ref. [8]

\[
R_{\text{th}}(T) = R_{\text{th}}(T = 0) [1 + 0.0005 T^2], \quad a_i(T) = a_i(T = 0) [1 + 0.01 T^2].
\]

Next, the \( \ell \)-dependent interaction potential \( V(R) \) is given by,

\[
V(R) = V_N(R, A_i, \beta_{di}, \theta_i, T) + V_C(R, Z_i, \beta_{di}, \theta_i, T) + \frac{\hbar^2 \ell (\ell + 1)}{2\mu R^2},
\]

where \( V_N \) is the nuclear proximity potential calculated from the ETF approach and \( V_C \) is the Coulomb potential [9]. The variables \( V_{B^\ell}, R_{B^\ell} \) and \( \hbar \omega^\ell \), entering in the Wong formula [3], are calculated from this potential with effect of deformations (up to \( \beta_4 \)) and orientations included.

3. The Wong formula

Wong [3] defines the fusion cross-section for two deformed and oriented nuclei lying in same plane, and colliding with center-of-mass (c.m.) energy \( E_{\text{c.m.}} \), in terms of angular-momentum \( \ell \) partial waves, as

\[
\sigma(E_{\text{c.m.}}, \theta_i) = \frac{\pi}{k} \sum_{\ell=0}^{\ell_{\text{max}}} (2\ell + 1) P_{\ell}(E_{\text{c.m.}}, \theta_i); \quad k = \sqrt{\frac{2\mu E_{\text{c.m.}}}{\hbar^2}}, \quad (8)
\]

and \( \mu \) as the reduced mass. Here, \( P_{\ell} \) is the transmission coefficient for each \( \ell \) which describes the penetration of barrier \( V_\ell(R, E_{\text{c.m.}}, \theta_i) \), calculated in Hill-Wheller approximation [10]. An explicit summation over \( \ell \) in Eq. (8) requires the complete \( \ell \)-dependent potentials \( V_\ell(R, E_{\text{c.m.}}, \theta_i) \), with \( \ell_{\text{max}} \) to be determined empirically.

Wong [3] carried out the \( \ell \)-summation in Eq. (8) approximately under the conditions of using only \( \ell = 0 \) quantities, and on replacing the summation by an integral, obtained

\[
\sigma(E_{\text{c.m.}}, \theta_i) = \frac{R_{B^0}^2}{2E_{\text{c.m.}}} \hbar \omega_0(\theta_i) \ln \left[ 1 + \exp \left( \frac{2\pi}{\hbar \omega_0(\theta_i)} \left( E_{\text{c.m.}} - V_{B^0}(\theta_i) \right) \right) \right], \quad (9)
\]
which on integrating over \( \theta_i \) gives \( \sigma(E_{c.m.}) \),

\[
\begin{align*}
\sigma(E_{c.m.}) &= \int_{\theta_i=0}^{\pi/2} \sigma(E_{c.m., \theta_i}) \sin \theta_1 d\theta_1 \sin \theta_2 d\theta_2 \\
&= \sigma(E_{c.m.}, \theta_i).
\end{align*}
\] (10)

4. Calculations and results

Fig. 1(a), solid line, for the case of \( \ell \)-summed Wong expression, show a point to point fit to data for the frozen-density approximation, with \( \ell_{\text{max}}(E_{c.m.}) \) varying smoothly, as illustrated in Fig. 1(b), solid line. On the other hand, the same could not be achieved for the sudden-density (dash-dot line in Fig. 1(a)), still requiring a modification of the barrier at both the below- and above-barrier energies. Similar results are obtained for \(^{58}\text{Ni}+^{58}\text{Ni}\) reaction [11], showing a similar exact fit to data with frozen-density in ETF. This means that the frozen-density gives appropriate barriers for the phenomenon of hindrance, observed in coupled channel calculations for these reactions [12], to be explained simply on the basis of \( \ell \)-summed Wong expression.
Concluding, the $\ell$-summed Wong expression, using the barriers calculated in frozen-density approximation in semi-classical ETF method based on SEDF, describe the $^{64}\text{Ni} + ^{64}\text{Ni}$ data on cross-sections without introducing any effects of barrier modifications.

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