Mathematical model for prediction of efficiency indicators of educational activity in high school

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Abstract. The quality of high school is a current problem all over the world. The paper presents the system dedicated to predicting the accreditation indicators of technical universities based on J. Forrester mechanism of system dynamics. The mathematical model is developed for prediction of efficiency indicators of the educational activity and is based on the apparatus of nonlinear differential equations.

1. Introduction
Each year all higher education institutions in Russia are monitoring the efficiency activity using the following indicators: educational and research activity, financial and economic activity, international activity, the contingent of students and employment, and others.

Evaluation of the effectiveness of the university management becomes urgent in the conditions of developing market relations and reducing government financing with the appearance of new requirements for the market of educational services and the labour market. For a long time the main methods for assessing the quality of the educational process in the Russian Federation were licensing, certification and accreditation procedures. But gradually, public accreditation of the university acquires more and more importance, especially within the university, rankings are compiled by various magazines, newspapers, agencies and scientists.

University managers must have disposal analytical tools to predict the dynamics of changes in the main university indicator. This tool helps in the promotion of the university and its inclusion in academic ratings. Without the use of the apparatus of system dynamics, it is difficult to conduct such studies. The information and advisory system developed on the basis of this apparatus and allow the decision maker to determine the change in the institution characteristics at different time intervals.
2. Methods

The mathematical model is developed for modelling and prediction of efficiency educational activity indicators in the universities. The developed model is based on the Forrester’s and Meadows’s models and allows formalizing the complex cause-effect relationships between system variables. The system dynamics model has the form:

\[
\frac{dX_i}{dt} = \alpha_{i,0} + \sum_{j=1}^{n} \alpha_{i,j} \prod_{k=0}^{n} \omega_{i,j,k}(X_k)X_j, \quad i = 1, \ldots, n
\]

where \(X_1, \ldots, X_n\) – levels or stocks: characteristics set that determines the state of the system at any given time completely;

\(\frac{dX_i}{dt}\) – flows: rates of levels change per unit time, which are added from the rates multiplied by levels;

\(\alpha_{i,j}X_j, j = 1, \ldots, n\) – rates of flows, which change a level and include all factors causing its growth or decrease;

\(\omega_{i,j,k}, k = 1, \ldots, n\) – functional dependencies between levels.

The indicators of the educational activity effectiveness in the universities are presented in the model as simulated variables. The indicators of the World University Ranking U-Multirank were chosen as indicators for the proposed model. U-Multirank has the following advantages: it is multi-dimensional, user-driven, and stakeholder-oriented and a multi-level ranking, it shows the diversity of institutions, compares institutions with similar activity profiles and does not provide over-simplified league tables.

The model is a system of non-linear differential equations, the modelling characteristics of the educational process being determined according to the solution of this system. The graph model is used to illustrate the casual relationships between the system-levels of the mathematical model. The oriented graph was built according to analysis of cause-and-effect relationships between the selected system variables (accreditation indicators of the university). An algorithm is proposed to determine the indicators of the educational activity effectiveness in the university to solve the system of nonlinear differential equations.

The proposed approach is aimed at solving complex problems of managing the educational process in universities. The structure of the proposed model repeats the structure of cause-effect relationships in the system. The model provides the suggestion to quickly and relevantly assess the performance of the system, which could be used by a person responsible for managing quality control.

The apparatus of nonlinear differential equations has the form (1):
The results of forecasting the characteristics of the university accreditation in the time interval from 2010 to 2015 are presented below. They allow identifying the main trends in the change of these characteristics, which is necessary when making managerial decisions to ensure quality control and the functioning of the university.

Let us use the statistical data of the monitoring of the Ministry of Education of the Russian Federation during the computational experiment. Fig. 1 shows the graphs of the solutions of the system of equations (1) for some, the most significant indicators of accreditation.

\[
\begin{aligned}
\frac{dX_1(t)}{dt} &= X_1(t) \left( \frac{DP_h}{X_2(t)} \times f_1(X_3) \times f_2(X_{19}) \times f_3(X_{27}) - \frac{DP_h}{X_2(t)} \times f_1(X_{21}) \times f_2(X_{22}) \times f_3(X_{23}) \times f_4(X_{25}) \right) \\
\frac{dX_2(t)}{dt} &= X_2(t) \left( \frac{MR_h}{X_3(t)} \times f_1(X_4) \times f_2(X_{20}) \times f_3(X_{22}) - \frac{MR_h}{X_3(t)} \times f_1(X_{10}) \times f_2(X_{12}) \times f_3(X_{13}) \times f_4(X_{15}) \right) \\
\frac{dX_3(t)}{dt} &= X_3(t) \left( \frac{MV}{X_4(t)} \times f_1(X_{16}) - \frac{MV}{X_4(t)} \right) \\
\frac{dX_4(t)}{dt} &= X_4(t) \left( \frac{MV}{X_5(t)} \times f_2(X_{19}) - \frac{MV}{X_5(t)} \right) \\
\frac{dX_5(t)}{dt} &= \frac{X_5(t)}{\alpha} \times f_3(X_3) \times f_4(X_{20}) \times f_5(X_{22}) \times f_6(X_{25}) \times f_7(X_{29}) \\
\end{aligned}
\]
Figure 1. Change of university accreditation indicators at different time intervals

The total number of publications $X_d(t)$ is of particular interest among the numerical solutions obtained.

The smallest number of publications $X_d(t)$ was recorded in 2010, which is explained by the minimum number of publications of researchers, recorded in various bibliographic databases of scientific publications (Russian Science Citation Index, Scopus, Web of Science). Over time, function $X_d(t)$ increases. This fact corresponds to an increase in the intensity of publications of researchers and their subsequent registration in bibliographic databases.

Fig. 2 shows the correspondence of the learned values of function $X_d(t)$ to real statistical data for 2010-2015.

Figure 2. The actual number of publications and the number of publications identified by the model
From Fig. 2 it follows that the general trend of growth in the number of publications, determined by model (1), is consistent with the statistics of the Ministry of Education of the Russian Federation. This fact confirms the adequacy of the developed mathematical model.

4. Conclusion
A system of indicators characterizing the educational process of the university is proposed and justified when assessing the effectiveness of its functioning with the help of the rating system U-Multirank.

A mathematical model of the process of changing the indicators of the functioning of a university is developed and based on the mathematical apparatus of system dynamics and regression analysis.

The structure of the proposed model visually illustrates the structure of cause-effect relationships in the system and allows taking into account external and internal changes. The person responsible for quality control is given the opportunity to react promptly and adequately to them. But the developed mathematical model has difficulties with an accurate analysis of some interdependencies and in tracking feedback, therefore it requires checking the adequacy of the developed model with the help of the regression model.

Thus, the developed mathematical model allows one to take into account the features of the educational process as a complex system and to monitor the quality of the process at any discrete point in time.

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