In a recent paper, Liddle and Ureña-Lopez suggested that to have a unified model of inflation and dark matter is imperative to have a proper reheating process where part of the inflaton field remains. In this paper I propose a model where this is possible. I found that, incorporating the effect of plasma masses generated by the inflaton products, enable us to stop the process. A numerical estimated model is presented.

PACS numbers: 98.80.Cq, 95.35.+d

I. INTRODUCTION

In a recent paper [1], Liddle and Ureña-Lopez studied the conditions under which we can have a unified description of inflation [2], dark energy [3] and dark matter [4]. The key ingredient required is to have a reheating process where not all the inflaton energy density decays into radiation. The crucial role of reheating in these type of models was found earlier in [5], where models of quintessential inflation were proposed. The authors of [1] found that the standard reheating mechanism [2] can not be used for this task, because in this scenario the scalar field decays completely. They also studied the preheating scenario [6], where it is known that the field decay can be incomplete, however as was discussed in [1] the simplest model based on a theory with a quadratic scalar field turns out inconsistent with observations.

In this work I revisited this issue. Specifically I am interested in a unified description of inflation and dark matter. The key problem is to consider an appropriate phase of reheating that both serve as a bridge between inflation and the standard model of cosmology, and also give us an observationally consistent amount of dark matter. I found that even using the standard (perturbative) reheating mechanism, we can obtain a partially decaying inflaton field. However, this observation does not help to build the model by itself, because for a large initial amplitude of the inflaton field, which is $\sim M_p$ at the end of inflation, we can not neglect the parametric resonance effects [6]. So, we have to consider the preheating scenario [7], in which the first stages of reheating occur in a regime of a broad parametric resonance, then the resonance becomes narrow, and the last stage can be described by the elementary theory of reheating [2]. However, this time the last stage is not so simple as the standard one, because the particles created at the first stages affects the evolution of the inflaton field. One way to consider this, is take into account the generation of plasma masses for the inflaton decays products [5], which can stop the particle creation process, doing it kinematically forbidden. In section II we describe the basics of the model we studied. The observational constraint is discussed in section III, and applied to the model in section IV where is made manifest the problem. The solution is described in section V where the plasma masses are taking into account.

II. THE MODEL

Let us assume a theory with a quadratic scalar potential $V(\phi) = V_0 + m^2 \phi^2 / 2$. Here $V_0$ is a small positive constant needed to explain dark energy [3]. During inflation, the friction produced by the expansion of the universe makes the field evolve slowly towards its vacuum, e.g. $m \ll H$. In this case the equations controlling the evolution are

$$H^2 \simeq \frac{4\pi m^2}{3M_p^2} \phi^2, \quad 3H \dot{\phi} + m^2 \phi \simeq 0.$$  \hspace{1cm} (1)

Inflation last until the kinetic energy of the field equals the potential energy $\phi^2 \simeq V(\phi)$. As is well known [2] the field at the end of inflation takes the value $\phi \simeq M_p$. Of course the end of inflation coincides with the condition $m \simeq H$, as can be seen clearly from the Friedman equation. After inflation the universe enters into the reheating phase, the process where almost all the particles in the universe were created. During this phase the scalar field continues rolling down the hill of the potential towards its minimum and starts to oscillate around it. In numerical estimates one realizes that during the first oscillations the expansion of the universe is still important [7] and the amplitude of the field falls down very quickly (see Figure 1). After the first oscillation, the amplitude reaches the value $0.04M_p$, indicating that the expansion is still important. Later the scalar field enters into the oscillatory regime where the amplitude decreases slowly

$$\phi(t) = \frac{M_p}{\sqrt{3}\pi m} \sin(mt) = \Phi(t) \sin(mt).$$ \hspace{1cm} (2)

It is during this stage where the average over many oscillations of the scalar field can be described as non-relativistic matter $\rho_\phi \simeq a^{-3}$. Reheating occurs when the amplitude of the field decreases more rapidly than Eq.(2). Historically this process was studied first introducing an ad-hoc term in the equation of motion for the
two scalar fields and other particles. For example, if this scalar field decays in

\[ \Gamma \propto \sigma \chi \]  

where \( \Gamma \) is the rate of particle decay of the scalar field into other particles. For example, if this scalar field decays in two scalar fields \( \phi \rightarrow \chi \chi \), the rate is \[ \Gamma \approx \frac{g^4 \sigma^2}{8\pi m} \]  

where I am assuming a coupling \( g^2 \sigma \chi^2 \phi \). The decay products of the inflaton field are ultrarelativistic \( (m \gg m_\chi) \), and their energy density decreases due to the expansion of the universe much faster than the energy of the oscillating field \( \phi \). In this case reheating may end when the Hubble parameter \( H \sim 2/3t \) becomes smaller than \( \Gamma \). However, for \( \sigma \ll \Phi \), where \( \Phi \) is the amplitude of the oscillations, we can write \[ \Gamma \approx g^4 \Phi^2 / 8\pi m \], then because \( \Phi^2 \) decreases as \( t^{-2} \) (see Eq. (2)) in the expanding universe, whereas the Hubble parameter decreases only as \( t^{-1} \), the decay rate never catches up with the expansion of the universe, and reheating never completes.

So even in the perturbative regime of reheating, we have a mechanism to stop the process leaving part of the oscillating field \( \phi \) decoupled and behaving as dark matter. However, as we can see in the next sections, this model by itself can not work, because it gives a result still in contradiction with the observational constraints.

**III. OBSERVATIONAL CONSTRAINTS**

In this section we derive a constraint for the initial amplitude of the scalar field \( \phi \) oscillations, which can be identified as the dark matter component of the model. The idea is to compare the theoretical results with an observational constraint.

As we saw in the previous section, during inflation the scalar mass satisfy \( m \ll H \), meanwhile for the quadratic potential the condition \( m \approx H \) marks the end of inflation. To recover the standard dark matter scenario, the scalar mass should satisfy \( m \gg H_{eq} \), where \( H_{eq} \) is the value of the Hubble parameter at the time of radiation and matter equality. If we denote by \( t^* \) the time at which the scalar mass equals the Hubble parameter \( m = H^* \), then

\[ m^2 = \frac{8\pi}{3M_p^2} \rho_R^* \]  

We are assuming here that a large part of the scalar energy density \( \rho_\phi \) was transformed into radiation, dominating the matter content of the universe, and the rest is still in the form of an oscillating field. For \( t > t^* \) the scalar field \( \phi \) oscillate around the minimum of the potential. The energy density average behaves as dust

\[ \rho_\phi = \frac{1}{2} m^2 \sigma^2 \left( \frac{a_*}{a} \right)^3 \]  

and also the radiation component evolves

\[ \rho_R = \rho_R^* \left( \frac{a_*}{a} \right)^4 \]  

What we want to compare with observations is the dark matter mass per photon ratio \( \xi_{dm} = \rho_\phi / n_\gamma \). This quantity is constant for \( t > t^* \) apart from changes in the number of degrees of freedom of the species. We assume here an adiabatic expansion where \( S = g_S T^3 a^3 \) is constant during the evolution, where \( g_S \) is the entropic degrees of freedom, usually very similar to the total degrees of freedom \( g_s \). Because \( n_\gamma = 2\zeta(3) T^3 / \pi^2 \), then

\[ n_{\gamma,0} = n_{\gamma}(a_* / a_0)^3 \]  

where the zero subscript indicates current values. Notice that we are assuming a change in the number of entropic degrees of freedom. Then the current dark matter per photon ratio is

\[ \xi_{dm,0} = \frac{\rho_{dm}}{n_{\gamma,0}} \approx \frac{g_{S,0}}{g_{s,0}} \frac{m^{1/2}}{M_p^{3/2}} \frac{\sigma^2}{M_p^2} M_p, \]  

The observational measure of this ratio is \( \xi_{dm,0} = 2.2 \times 10^{-28} M_p \) using values from WMAP3 \[8\], which for typical values \( g_s \sim 100, g_{S,0} = 3.9 \) gives the following constraint

\[ \frac{m^{1/2}}{M_p^{3/2}} \frac{\sigma^2}{M_p^2} \approx 4 \times 10^{-29}. \]  

Using considerations from structure formation, we get an upper bound for the scalar field mass \( m/M_p = 10^{-52} \) or \( m > 10^{-28} \text{eV} \). Obtaining the correct amplitude for scalar

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**FIG. 1**: Oscillations of the inflaton field after inflation. The value of the scalar field is measured in units of \( M_p \), and time in units of \( m^{-1} \).
perturbations requires $m/M_p \sim 10^{-6}$, then the condition \cite{10} imposes that $\phi_\ast \sim 10^{-13} M_p$, which is in contradiction with the initial statement for a value of $\phi \sim M_p$ at the end of inflation. In this case to get an observable viable model we need an incomplete reduction of the scalar field amplitude during reheating \cite{4}, reducing the energy density a factor of $10^{26}$.

As I stressed at the end of section II, even in the standard picture of reheating we can have a partial decay of the inflaton field. In this case, we find that at the beginning of the oscillations the field amplitude is already $\sim 10^{-2} M_p$. So, the constraint mentioned in the last paragraph implies that during reheating the energy density must decays in 22 orders of magnitude.

IV. THE PROBLEM

So, what we need, is to have an incomplete reheating phase after inflation, in agreement with the observational constraint: that of having at the beginning of the oscillations (which is identified with dark matter) an scalar field amplitude $\phi_\ast \sim 10^{-13} M_p$. Clearly for a model where the amplitude of oscillations is very large, we can not neglect the parametric resonant effects during reheating \cite{6}.

Using preheating, the authors of \cite{1} found that, for an interaction term $g^2 \chi^2 \phi^2$ where $g$ is the coupling constant, the decay is incomplete, stopping until the amplitude of the scalar field falls below $m/g$. Using the required amplitude of the scalar field at the end of inflation (see eq. \cite{10}), $\phi_\ast \sim 10^{-7} m$ implies a coupling constant value of $g \approx 10^7$, which is clearly incompatible with the model.

However the restriction can not be applied at the end of preheating, because after the broad and narrow parametric resonance phases \cite{4}, the reheating process follows through the standard mechanism. So, these conditions have to be applied after the entire reheating phase ends. In particular, the fact that the amplitude falls below $m/g$ only means that resonant production stops.

For a range $g \sim 10^1 - 10^{-3}$ of the coupling constant, the amplitude of the field after preheating is $\Phi \sim 10^{-5} M_p - 10^{-3} M_p$ respectively. These are the initial values for the standard reheating phase, so the observational constraint implies that during particle decay the energy density has to decrease 16 orders of magnitude and not 26 as was settled in \cite{1}.

So, to get a viable observational model we have to consider the process of reheating once the resonant phase has finished. In this new context, the study of reheating is clearly different from the standard scenario: now the universe not only has the contribution of the coherent oscillations of the inflaton field but also the particles created during preheating. Therefore, our problem is to find a mechanism that not only allows to avoid that the field inflaton decays completely, but also take into account the presence of particles created in the previous phase.

V. REHEATING AND THERMAL MASSES

As we can see below, both features are connected; the presence of radiation during the perturbative reheating phase enable us to stop the process. Let us assume that during the process of preheating the inflaton decay products scatter and thermalize to form a thermal background \cite{8}. This thermalized particle species acquires a plasma mass $m_\phi(T)$ of the order of $\nu T$ where $\nu$ is the typical coupling governing the particle interaction. The presence of thermal masses imply that the inflaton zero mode cannot decay into light states if its mass $m$ is smaller than about $\nu T$. If we expressed the inflaton zero-temperature decay width as $\Gamma_\phi = \alpha_\phi m$, at finite temperature it becomes

$$\Gamma_\phi(T) = \alpha_\phi m \sqrt{1 - \frac{\nu^2 T^2}{m^2}}.$$  \hspace{1cm} (11)

The system of equations to be solved is then

$$\rho_\phi + (3H + \Gamma_\phi)\rho_\phi = 0,$$
$$\rho_R + (4H \rho_R - \Gamma_\phi)\rho_\phi = 0.$$ \hspace{1cm} (12)

When the plasma mass $m_\phi(T) = \nu T$ becomes comparable to the inflaton mass $m$, the temperature reaches the value $T \sim m/2\nu$, remaining constant for a while, indicating that particle creation stopped; $\Gamma_\phi = 0$. At this time $\rho_R$ stays constant and $\rho_\phi$ decays as $a^{-3}$. In the absence of plasma masses, right after inflation the temperature increases until $T_{max}$, and then decreases as $a^{3/2}$ until it reaches the reheating temperature $T_{rh}$ \cite{10}

$$T_{rh} = 0.2 \left( \frac{100}{g_\ast} \right)^{1/4} \alpha^{1/2} \sqrt{m M_p}.$$ \hspace{1cm} (13)

In the case $m < T_{rh}$, and using the values $m \approx 10^8$GeV and $g_\ast \sim 100$ (the effective number of relativistic degrees of freedom) we obtain

$$\alpha_\phi \geq 3 \times 10^{-10}.$$ \hspace{1cm} (14)

To estimate the duration of this $T$ constant phase, I use the fact that decays are not possible for $T \geq m/2g$. The scalar field energy density $\rho_\phi \sim a^{-3}$ until it becomes smaller than $\rho_R$. So the condition is simply

$$V \left( \frac{a_i}{a_f} \right)^3 \leq \frac{\pi^2}{30} g \left( \frac{m}{2g} \right)^4$$ \hspace{1cm} (15)

where $a_i$ indicates the end of inflation and $a_f$ the time when the $\phi$ energy density equals $\rho_R$. So, the universe evolves

$$\frac{a_f}{a_i} \approx \left( \frac{V}{m^4} \right)^{1/3},$$ \hspace{1cm} (16)

which is equivalent to a number of e-foldings $N_e = 14$ after the standard particle decay process started, using the values $V^{1/4} \sim 10^{13}$GeV and $m \sim 10^8$MeV \cite{8}. After
the universe becomes radiation dominated and the rest of the inflaton energy density $\rho_\phi$ continue its evolution as a coherently oscillating field, behaving as dark matter. We can estimate the order of magnitude of the decrease in energy density using (16)

$$\rho_f \simeq \rho_i \left( \frac{a_i}{a_f} \right)^3 \sim \rho_i 10^{-18},$$

which is in perfect agreement with the calculations of section IV.

VI. SUMMARY

In this paper, I have investigated the possibility to build a theoretical model which describe both, the inflationary phase and also the dark matter, using the inflaton field. The crucial ingredient is the period of reheating after inflation, where almost all the energy stored in the inflaton field is converted into relativistic particles. In fact, as the authors of [1] have stressed, the main condition that must be satisfied is that not all the energy density of the inflaton decays into radiation. I found that, even in the standard (perturbative) reheating scenario, the process may end before all the inflaton energy density transformed into radiation. However, this observation does not help to build the model by itself, because for a large initial amplitude of the inflaton field (with a typical value $\phi_i \sim M_p$ at the end of inflation), we cannot neglect the parametric resonance effects [6]. Taking into account the entire reheating phase, starting with the broad and narrow resonance phases, and after that, the standard perturbative one, I found that this can be accomplished. The key element in this description is that inflaton decay products acquire plasma masses [8], which may cause inflaton decay to be kinematically forbidden, stopping the process.

Acknowledgments

VHC was supported by DI-UNAB Grant 14-06/R.

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