Unconventional properties of high-temperature superconducting cuprates

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Abstract. In spite of much interest in various unconventional properties of superconducting cuprates such as the anisotropic superconductivity, the pseudogap state, the competing state in the underdoped region, strange metal in normal state, and anomalies in the optical sum rules etc, its microscopic mechanism still remains unsolved issues. Here these properties are considered using our recently proposed theory emphasizing that the electronic state of superconductors can be described by doping-dependent composite fermions. It is found that the anisotropy of the superconductive gap and the pseudogap are evaluated, the competing states is derived by mixed states model, and that \( T \)-linearity of resistivity and the anomaly of optical sum rules in the nearly optimal doping can be derived from considering the interplay between the composite fermion bands. It is also suggested that a crossover separating the strange metal and the normal metal in the overdoped region can be explained by the coupling effect of composite fermions.

1. Introduction
There has been an extraordinary number of studies about high-temperature superconductors such as cuprates, but its microscopic understanding still remains unsatisfactory [1, 2]. Recently, the author proposed a theory to explain generally the properties of high-temperature superconducting cuprates and iron pnictides etc. [3-6]. The theory is based on the extended \( d-p \) model emphasizing that the electronic state of superconductors can be described by doping-dependent composite fermions constructed with \( 3d \) and \( 2p \) holes, and is indicated to provide a consistent explanation about the superconducting and normal properties of these superconductors. In this paper unconventional properties of high-temperature superconducting cuprates will be investigated and then evaluated comparing with the experimental data.

2. Composite fermions theory
2.1 Effective Hamiltonian
In order to consider the unconventional properties of cuprates superconductors, we summarize the composite fermions theory briefly [4, 5]. The Hamiltonian is assumed to be an extended \( d-p \) model for a single layer of square planar
\[ H = \sum_{\sigma} \varepsilon_{d} d_{i}^{\sigma} d_{i}^{\sigma} + \sum_{\sigma} \varepsilon_{p_{i}} p_{i}^{\sigma} p_{i}^{\sigma} + \sum_{\sigma} \delta \sum_{\sigma} (\varepsilon_{j} d_{j}^{\sigma} p_{j}^{\sigma} + \text{H.c.}) + U \sum_{\sigma} d_{i}^{\sigma} d_{i}^{\sigma} d_{j}^{\sigma} d_{j}^{\sigma} + \sum_{\sigma} V_{\sigma} d_{i}^{\sigma} d_{j}^{\sigma} p_{i}^{\sigma} p_{j}^{\sigma}, \tag{1} \]

where the operator \( d_{i}^{\sigma} \) creates Cu \((3d_{x^2-y^2})\) holes at site \( i \), \( p_{j}^{\sigma} \) creates O \((2p_{x}, 2p_{y})\) holes at site \( j \), \( \varepsilon_{d} = 0 \), and \( \varepsilon_{p_{i}} \) is the nearest-neighbor hopping integral. \( U \) is the on-site Coulomb repulsion at a Cu site, and \( V_{\sigma\sigma'} \) is the interaction between neighboring Cu and O sites. In Eq. (1), the vacuum is defined as filled Cu \( d^{10} \) and O \( p^{6} \) states. Considering the \( d-p \) covalency effect, the operator combining the O states around Cu ion is defined as \( \tilde{\rho}_{\sigma} = 0.5 \sum_{\sigma} \varepsilon_{\sigma} |\varepsilon_{\sigma}| p_{\sigma}^{\sigma} \), where \( \varepsilon_{\sigma} \) is given by \( \varepsilon_{\sigma} = -\varepsilon < 0 \) \((j = 1, 2, \varepsilon > 0 \) \((j = 3, 4) \) [7]. To the second order in perturbation theory on the condition of \( \varepsilon < U \), the effective Hamiltonian of Eq. (1) is derived as following as

\[ H_{\text{eff}} = 2 \sum_{\sigma} \varepsilon_{\sigma} \tilde{\rho}_{\sigma}^{\sigma} \tilde{\rho}_{\sigma}^{\sigma} + \sum_{\sigma} (1-n_{\sigma})(\varepsilon d_{\sigma}^{\sigma} \tilde{\rho}_{\sigma}^{\sigma} + \text{H.c.}) + 2 \sum_{\sigma} V d_{\sigma}^{\sigma} \tilde{\rho}_{\sigma}^{\sigma} \tilde{\rho}_{\sigma}^{\sigma}, \tag{2} \]

where \( V = V_{\sigma} + 0.5V_{\sigma'} \) \((V_{\sigma} : \text{Coulomb interaction between the nearest neighboring sites in (1), } V_{\sigma'} : \text{antiferromagnetic interaction} \) \( V_{\sigma} = -(1-n_{\sigma})\langle n_{\sigma} \rangle + \langle 1-n_{\sigma} \rangle \langle n_{\sigma} \rangle J < 0, \sigma \neq \sigma' \) \( \langle \rangle \) is averaging of number operators, \( J = \varepsilon^{2}(U - 2\varepsilon_{\sigma} - 2V)^{1} \). Using the effective Hamiltonian, the ground state will be determined in next section before considering several unconventional properties.

2.2 Two types of the ground states

Now the ground state of the Hamiltonian (2) will be generally considered in two states depending on the doping conditions. One of these is the states in the neighborhood of the insulator (so-called Mott insulator) and the other is the superconducting state based on the band picture. First let us consider the non-superconducting region in the neighborhood of the insulator. The effective Hamiltonian (2) is appropriate in this case because its representation in real space is very useful in the treatment of this region. In order to enable to apply the mean field approximation, the operators \( d_{\sigma}^{\sigma}, \tilde{\rho}_{\sigma}^{\sigma} \) are changed by unitary transformations of \( b_{\sigma} = \alpha b_{\sigma}^{\dagger} + \beta \tilde{\rho}_{\sigma}^{\sigma}, \ c_{\sigma} = \beta b_{\sigma}^{\dagger} - \alpha \tilde{\rho}_{\sigma}^{\sigma} \), \( \alpha^{2} + \beta^{2} = 1 \). Here \( b_{\sigma}^{\dagger}, c_{\sigma}^{\dagger} \) are defined as the mixing operators of \( 3d \) and \( 2p \) holes. Assuming that there is no \( p \)-holes in half-filling, the relations of \( \langle 1-n_{\sigma} \rangle \approx 0, \langle 1-n_{\sigma} \rangle \approx (1-0.5\delta) \) are derived. For the condition of \( 2\alpha\beta\varepsilon_{\sigma} = (1-0.5\delta)\varepsilon(\beta^{2} - \alpha^{2}) \), the Hamiltonian (2) is represented as

\[ H = 2 \sum_{\sigma} \varepsilon_{\sigma}(\beta^{2} - \alpha^{2})^{\dagger}(\beta^{2} b_{\sigma}^{\dagger} b_{\sigma} - \alpha^{2} c_{\sigma}^{\dagger} c_{\sigma}) + 2 \sum_{\sigma} V(\alpha^{2} \beta^{2} b_{\sigma}^{\dagger} b_{\sigma}^{\dagger} b_{\sigma}^{\dagger} b_{\sigma} + \alpha^{2} b_{\sigma}^{\dagger} b_{\sigma}^{\dagger} c_{\sigma}^{\dagger} c_{\sigma} + \beta^{2} c_{\sigma}^{\dagger} c_{\sigma} b_{\sigma}^{\dagger} b_{\sigma} + \alpha^{2} \beta^{2} c_{\sigma}^{\dagger} c_{\sigma} c_{\sigma}^{\dagger} c_{\sigma}^{\dagger}) \tag{3} \]

Since there exists no \( p \)-hole in half-filling Mott insulator, it will not be necessary to consider the interaction \( V \) in this case. Notice that in non-doping Hamiltonian (3) reduces to a \( S = 1/2 \) Heisenberg model represented as \( H_{J} = J \sum_{\sigma} S_{\sigma} \cdot S_{\sigma} \). However, in the existence of \( p \)-holes, since \( V_{\sigma}(\sigma \neq \sigma') \) is interpreted to be attractive due to \( d-p \) exchange antiferromagnetic interaction, the case of \( V < 0 \) will play an important role for determining the ground state. Considering the experimental facts that the doping can almost supply the \( p \)-holes, doping holes are expected to occupy the quantum state corresponding to this situation. For the condition \( \alpha^{2} < \beta^{2} \) which allows the reliable value of \( \varepsilon_{\sigma} \) and \( \varepsilon \), the term \( b_{\sigma} b_{\sigma} c_{\sigma} c_{\sigma} \) \( c_{\sigma} c_{\sigma} b_{\sigma} b_{\sigma} \) can be the most dominant interaction. This means that \( b-c \) or \( c-b \) pair will mainly contribute to determine the ground state. Thus, for the equivalency of sites, the normalized wave function is assumed to be

\[ \langle \Psi_{\sigma} \rangle = \prod_{\sigma}(s + t \beta \varepsilon_{\sigma})^{\dagger} \langle s + t \alpha \varepsilon_{\sigma} \rangle \langle \Psi_{m} \rangle \quad s^{2} + t^{2} = 1 \tag{4} \]
where $|\Psi_{\alpha k}\rangle$ is Mott insulator and the coefficient $t$ indicates the probability of local-antiferro pair state. On the approximation using the relation of $\langle \Psi_{\alpha k} | \sum p^*_i p_i | \Psi_{\alpha k}\rangle \approx 2N \delta^2 \epsilon$ ($\delta$: the doping ratio relative to half-filling), the ground-state energy is given by

$$E_p = \langle \Psi_{\alpha k} | H | \Psi_{\alpha k}\rangle = N \delta (\epsilon_p + V), \quad V < 0$$  (5)

Using the presumed excited state wavefunction, the excited energy is obtained as $N \delta (\epsilon_p + V) - 2V$. This indicates that there is the energy gap between the ground and the excited states. Thus, the wave function (4) will be identified as the pseudogap state in the nearly non-doped region. The pseudogap energy at $T = 0$ is then estimated to be $\Delta_p = |V|$. The critical temperature can be calculated as $T_p = 0.5 \Delta_p$. Next let us consider the case in the neighborhood of optimally doped region. In this doping region the relation of $\langle 1 - n_{\alpha k} \rangle = 0$, $\langle 1 - n_{\alpha k} \rangle = 1 - 0.5 \delta$ will be generally reasonable. On the condition of $\alpha_k \beta_k \epsilon_p = (1 - 0.5 \delta) \epsilon(s) \beta_k (\beta_k^2 - \epsilon_k^2)$, the Hamiltonian (2) is represented by

$$H = \sum_{k \mathbf{k}^\prime} (\beta_k^2 - \epsilon_k^2) \langle \beta_{\mathbf{k}^\prime}^* \beta_\mathbf{k} \rangle + N^{-1} \sum_{k\mathbf{k}^\prime} V \delta (\mathbf{k}, \mathbf{k}^\prime) (\alpha_k \alpha_{\mathbf{k}^\prime} \beta_k \beta_{\mathbf{k}^\prime} + \beta_k \beta_{\mathbf{k}^\prime} \alpha_k \alpha_{\mathbf{k}^\prime} + \beta_k^* \beta_{\mathbf{k}^\prime}^* \alpha_k^* \alpha_{\mathbf{k}^\prime}^* + \beta_k^* \beta_{\mathbf{k}^\prime}^* \alpha_k \alpha_{\mathbf{k}^\prime} + \beta_k^* \beta_{\mathbf{k}^\prime} \alpha_k^* \alpha_{\mathbf{k}^\prime} + \beta_k^* \beta_{\mathbf{k}^\prime} \alpha_k \alpha_{\mathbf{k}^\prime} + \gamma)$$

where $\delta(k) = \sin k_x + \sin k_y$, $\theta(k, k^\prime) = \cos (k_x - k_x^\prime) + \cos (k_y - k_y^\prime)$, and the Cu-O distance is used as the length unit. Using BCS-like wave function, the gap equation is obtained as

$$\Delta_k = -0.5 \sum_{\mathbf{k}} \Delta_{\mathbf{k}} V_{\mathbf{k} \mathbf{k}^\prime} (\Delta_{\mathbf{k}}^2 + \epsilon_k^2)^{-1/2}, \quad V_{\mathbf{k} \mathbf{k}^\prime} = N^{-1} V \alpha_\mathbf{k} \beta_{\mathbf{k}^\prime} \beta_\mathbf{k} \theta_{\mathbf{k} \mathbf{k}^\prime}$$  (7)

where the relation $\epsilon_k = 0.5 (\epsilon_p + [\epsilon_p^2 + 4(1 - 0.5 \delta)^2 \epsilon(s) \beta_k (\beta_k^2 - \epsilon_k^2)]^{1/2}) - \epsilon_k$ is defined, measuring the energy relative to the Fermi level $\epsilon_p$. Replacing the sum in gap equation by an integral, the solution which is even in $k$ is given by $\Delta_k = \Delta_0 b_k (\cos k_x - \cos k_y)$. The solution is $\Delta_k \propto (\cos k_x - \cos k_y)$ which is almost consistent with the experimental facts about the anisotropic gap of high-temperature cuprate superconductors [8]. Using several approximations on the band structure, the superconductive maximum gap energy and the superconductive critical temperature are given by

$$\Delta_0 = 2 \epsilon_p m \delta [1 + 4(1 - 0.5 \delta) \epsilon^2 / \epsilon_p^2]^{1/2} \exp (1 / N \delta V), \quad \Delta_s = \gamma k_b T_c$$  (8)

where $m$ is an enhanced factor contributing to Cooper pairs, $N_\uparrow$ is the density of states at Fermi level and $V = V + V'(1 - 0.5 \delta)^2 (V' = \epsilon V / \epsilon_p^2), \gamma \approx 2$. Denote that the energy relative to Cu 3d level is measured and the contribution of the $\epsilon_{3d}$ fermions to the total system energy is considered.

3. Unconventional properties

There has been much interest in various unconventional properties of cuprates superconductors [2]. Here we shall discuss about the several topics such as the anisotropic superconductivity, the pseudogap state and the competing state in the underdoped region, strange metal and anomalies in the optical sum rules in the nearly optimally doped region, and the overdoped region.

3.1 Anisotropic superconductivity

First let us consider the anisotropy of superconductive gap energy. It is by now well known that cupraes superconductors indicate the $d_{x^2-y^2}$ anisotropic superconductivity and its gap energy is given by the relation of $\Delta_k \propto (\cos k_x - \cos k_y)$ ($k$ is the wave vector in the Brillouin zone). However, the
relation of \( \Delta_k \approx (\cos 0.5k_x - \cos 0.5k_y) \) is obtained in our theory, because the Cu-O distance is used as the length unit. Then, rewriting the anisotropic gap energy by the length unit of Cu-Cu distance,

\[
\Delta_k \approx \frac{\cos k_x - \cos k_y}{2(\cos 0.5k_x + \cos 0.5k_y)} \tag{9}
\]

The relation (9) indicates that in plotting \( \Delta_k \) by \( \cos k_x - \cos k_y \) there exists a kink at the position of \( \cos k_x - \cos k_y \approx 0.6 \), and it agrees with the recent experimental data of ARPES [9]. This result can also be interpreted as a prediction of composite fermions theory.

3.2 Anisotropic pseudogap state
According to our theory \([4, 5]\), pseudogap energy \( \Delta_{pc} \) is given by

\[
\Delta_{pc} = 0.5\langle \Psi_{\alpha \beta} | H | \Psi_{\alpha \beta} \rangle - \langle \Psi_{\alpha} | H | \Psi_{\alpha} \rangle = 0.5N\delta(\varepsilon_{\alpha} + V) - V - 0.5N\delta(\varepsilon_{\beta} + V) \tag{10}
\]

Notice that \( |\Psi_{\alpha \beta} \rangle \) does not show the excited state but show a single particle excitation corresponds to the surrounding oxide 4p holes \((fb_{\alpha}^\dagger - \alpha c_{\beta}^\dagger = \tilde{p}_{\alpha}^\dagger)\). The anisotropic pseudogap will be then determined by the anisotropy of \( \langle \tilde{p}_{\alpha}^\dagger | \tilde{p}_{\alpha} \rangle \). Transforming \( \tilde{p}_{\alpha} \) into momentum representation,

\[
p_{k_x} = N^{-1/2}(\sin 0.5k_x + \sin 0.5k_y) \sum \tilde{p}_{\alpha} e^{-ik_x k_y} \tag{11}
\]

As a result the anisotropic pseudogap is obtained as

\[
\Delta_{pc} \propto \langle \tilde{p}_{\alpha}^\dagger | \tilde{p}_{\alpha} \rangle \propto \Delta \left( (\sin 0.5k_x + \sin 0.5k_y)^2 - C \text{[groundstate]} \right) \tag{12}
\]

where \( C \) is a coefficient for the contribution of ground state to pseudogap. This is consistent with the ARPES measurements \([10]\).

3.3 Competing state in the underdoped region
Two types of electronic states considered here were solely limited to the cases of a pure pseudogap and superconductive states. However, it is well known that there exists the unsolved region between these states. Let us consider this case in the underdoped superconductive region. The antiferromagnetic long-range order between copper sites disappears by holes doping, but there still exists the short-range order interaction which plays an important role in the presence of holes. As shown here, the ground states in the neighborhood of the insulator and the superconducting region can be calculated. However, the doping dependency of these states is not clear. Here an overall wavefunction in the underdoped region is assumed to be

\[
|\Psi\rangle = \prod (s + tb_{\alpha}^\dagger c_{\beta}^\dagger)(s + tc_{\alpha}^\dagger b_{\beta}^\dagger)\prod (u_{\alpha} + v_{\alpha} b_{\alpha}^\dagger b_{\alpha}^\dagger) |0\rangle \tag{13}
\]

Then free energy of the ground state is represented as

\[
F = \langle \Psi | H | \Psi \rangle = E_x (\delta_\alpha) + E_x (\delta - \delta_\beta) \tag{14}
\]
where $\delta_c$ and $\delta_s$ are doping quantity contributing to pseudogap state and superconductive state, respectively, and $\delta = \delta_c + \delta_s$ (total doping). For the fixed $\delta_c$, minimizing $F$ respect to $\delta_s$,

$$\frac{dF}{d\delta_s} = 0 \rightarrow \frac{d[E_c(\delta_s) + E_s(\delta - \delta_c)]}{d\delta_s} = 0 \quad (15)$$

Substituting $E_c(\delta_s)$ and $E_s(\delta - \delta_c)$ into (15), the following equation is obtained

$$V_c + V(1 - 2\delta_c + 0.75\delta_c^2) + 2\epsilon_c(\delta - \delta_c)\exp[2IN_fV(1 - 0.5(\delta - \delta_c))]$$

$$+ \epsilon_s(\delta - \delta_c)\exp[2IN_fV(1 - 0.5(\delta - \delta_c))] |2IN_fV(1 - 0.5(\delta - \delta_c))| = 0 \quad (16)$$

This relation determines the electric state of underdoped superconductor. Let us consider two special cases of pure pseudogap and pure superconductive states. The pure pseudogap state corresponds to the case of $\delta = \delta_c$ and its condition is given by

$$V_c + V(1 - 2\delta_c + 0.75\delta_c^2) = 0 \quad (17)$$

The solution $\delta_{c0}$ of Eq. (17) presents the maximum doping quantity of the pure pseudogap state. The pure superconductive state corresponds to the case of $\delta = \delta_s$ (and $\delta_c = 0$) and its condition is given by

$$V_c + V + 2\epsilon_c(\delta - \delta_s)\exp[2IN_fV(1 - 0.5\delta_s)] + \epsilon_s(\delta - \delta_s)\exp[2IN_fV(1 - 0.5\delta_s)] |2IN_fV(1 - 0.5\delta_s)| = 0 \quad (18)$$

Similarly, the solution $\delta_{s0}$ of Eq. (18) presents the minimum doping quantity of the pure superconductive state. Considering the effective range of two representations in real and momentum spaces, it is reasonable that the pure pseudogap state is in the region of $0 < \delta \leq \delta_{c0}$, the pure superconductive state is in the region of $\delta \geq \delta_{s0}$ and the competing state is in the region of $\delta_{c0} < \delta < \delta_{s0}$. In the competing state the values of $\delta_c$ and $\delta_s$ for fixed $\delta$ can be determined by Eq. (16). Thus the superconductor in the underdoped region can be interpreted to the mixed state constructed with the pseudogap and superconductive states.

3.4 Anomaly of optical sum rules

It has been reported in the nearly optimal doping that there is an interesting phenomenon such as optical sum rule anomaly for $T_c > T > 0$ [11]. The experimental data indicate that in decreasing temperature the spectrum weight in the lower energy band increases, while the weight in the higher energy band decreases. It has been discussed until now whether the carries relevant to superconductivity can behave like Fermi-liquid or non-Fermi liquid. This phenomenon has been considered as to suggest non-Fermi liquid like in cuprates superconductors. Here we discuss about the possibility that these phenomena originate from the $b$-$c$ coupling terms (attractive interaction) of Hamiltonian (6). The $b$-$c$ coupling terms are represented as

$$I(b,c) = \sum_{kk'\nu\nu'} V_{kk'}(\alpha_{kk'}\alpha_{kk'}\alpha_{kk'}\alpha_{kk'}b^\dagger_{k\nu}c_{k\nu'}b^\dagger_{k'\nu'}c_{k'\nu'}) + \beta_{kk'}\beta_{k'k'}\beta_{kk'}\beta_{k'k'}c_{kk'}b_{k\nu}b_{k'\nu'} \quad (19)$$

Since $b$-$c$ coupling contributes to the matrix element of the interaction term, it is necessary to include $c_{kk'}^\dagger$ fermions in the excited state. The excited state is then given by

$$|\Psi_{ee}\rangle = \prod_k \phi^\dagger_{kk'} \prod_k (u_k + v_k b^\dagger_{kk'} b_{kk'}) \prod_p c_{pp'}^\dagger |0\rangle$$

$$|\Psi_{ee}\rangle = \prod_k \phi^\dagger_{kk'} \prod_k (u_k + v_k b^\dagger_{kk'} b_{kk'}) \prod_p c_{pp'}^\dagger |0\rangle \quad (20)$$
where \( \gamma_{\nu}^* \) is a creation operator of quasiparticles destructing Cooper pairs. The free energy is then obtained as

\[
F = \sum_{k\nu} \epsilon_k \langle b_k^* b_k \rangle + \sum_{k\nu} V_{\nu\sigma} \langle b_k^* b_k \rangle \langle h_{k\nu}^* h_{k\nu} \rangle + \langle I(b,c) \rangle - k_B ST
\]

where \( S \) is the entropy of quasiparticles. Minimizing \( F \) with respect to \( \langle \gamma_{\nu}^* \gamma_{\nu} \rangle \), the distribution of quasiparticles is obtained as

\[
f_{\nu}(E_k) = \langle \gamma_{\nu}^* \gamma_{\nu} \rangle = \frac{1}{\exp[(E_k - \mu)/kT] + 1} \quad (E_k = \sqrt{(\epsilon_k + V)^2 + \Delta_k^2}, \ V < 0)
\]

where \( E_k \) is the energy of quasiparticles, \( V = \langle I(b,c) \rangle \) and \( \mu \) is the chemical potential. This relation suggests that the energy of quasiparticles decreases due to the attractive interaction \( V \). On the other hand, since there exists the thermal excitation from \( c_k \) band to \( b_k \) band, its distribution will be generally given by

\[
f_{\nu} = \frac{1}{\exp[(\epsilon_\nu + V - \mu)/kT] + 1}
\]

where \( \epsilon_\nu \) is the average energy between the Fermi level of \( b_k \) band and \( c_k \) bands. Thus considering the excitation from \( c_k \) band to \( b_k \) band, the distribution of \( b_k \) band is given by

\[
f(E_k) = f_{\nu}(E_k) + (1 - f_{\nu}(E_k))f_{\nu} > f_{\nu}(E_k)
\]

Eq. (24) suggests that the occupancy of the excited states increases more than Fermi-Dirac distribution. Now let us try to explain Molegraaf et al’s experimental data based on the relations obtained here. It is well known that the optical conductivity of superconductor has both a regular \( (\omega > 0) \) part and a singular part, \( D(\omega) \). The conservation of spectral weight can expressed as the Ferrell-Glover-Tinkham sum rule [12, 13]

\[
D = A_F^+ - A_F^- + A_0^+ - A_0^-
\]

where \( l \) and \( h \) present the spectral weight of the lower and higher energy bands, respectively, and \( s \) and \( n \) present the superconducting and the normal states, respectively. In BCS theory the interband spectral weight \( A_h \) is not affected by the superconducting phase transition. However, Molegraaf et al reported that there is the spectral weight transferred from \( A_h \) to \( D + A_l \) with decreasing of temperature. On the other hand, in the presence of the distribution \( f_{\nu} \), the relation (24) derived by the composite fermions theory indicates that the occupancy of excitates decreases with decreasing of temperature. Since for \( V \geq 0 \), the distribution \( f_{\nu} \) becomes a negligible term, there will be no temperature dependence of this effect. The attractive interaction \( V (\omega < 0) \) is indispensable for this phenomenon. Thus it is concluded that as Molegraaf et al insist, this is the property peculiar to superconducting copper oxides. [11].

3.5 Strange metal

Another interesting phenomenon is the resistivity-linearity \( (T\text{-linearity}) \) at optimal doping in cuprates superconductors, which has been called the strange metal playing an important role in early thinking. In the late 1980s, Varma suggested the phenomenological marginal Fermi liquid theory that is agreement with the \( T\text{-linearity} \) [14]. In the 1990s it was reported that the quantum criticality could explain the property of the strange metal. A quantum phase transition occurs at quantum critical point \( (QCP) \) which defines the boundary between the ordered and disorder quantum phase. \( T\text{-linearity} \) has
been explained by relating the QCP [15–17]. However, the former has not still the corresponding microscopic theory, the latter indicates that the relation between the $T$-linearity and QCP is still not clear. Here, based on the two band model adopted in composite fermions theory, we propose a model that $T$-linearity originates from the coupling effect between two bands. Since $T$-linear resistivity in the strange metal is a phenomenon for $T > T_c$, the distribution function in the normal metal is represented as $\Delta \rightarrow 0$ limit in Eq. (22). The distribution is given by

$$f_0(\epsilon_k) = \langle b_k^\dagger b_k \rangle = \frac{1}{\exp[(\epsilon_k + V - \mu)/kT] + 1} \quad (V < 0)$$

(26)

In the case of conventional metal, the resistivity $\rho$ is given by

$$\rho \propto \frac{1}{\tau} = \frac{4\pi^2}{\hbar} \sum_q |V(q)|^2 S(q, w_m)(1 - n_{qp})$$

(27)

where $\tau$ is the lifetime of quasiparticles, $|V(q)|$ is the Fourier component of Coulomb interaction, and $S(q, w_m)$ is the structure function. Notice that $S(q, w_m)$ is proportional to Fermi-Dirac distribution $f_0(\epsilon_k)$. It is well known that this relation leads to the resistivity of $\rho \propto T^2$. Here as shown in previous section, let us consider the thermal excitation from $c_k$ band to $b_k$ band. This thermal excitations will restrict the density of fermions contributing to transport in the $b_k$ band. Then, regarding this restriction as the decreasing of the scattering rate, the following relation is assumed

$$\frac{1}{\tau} \propto \left(1 - \frac{1}{1 + e^{(\epsilon_p + V)/kT}}\right)$$

(28)

Here $V (= I(b, c))$ is negative due to the attractive interaction and is assumed to be effective even at $T > T_c$. Since positive $V$ can be neglected as in the situation of the conventional Fermi liquid, this effect will not contribute to $1/\tau$ at all. But in the presence of $V < 0$, it will affect the transport properties because the thermal excitation from $c_k$ band to $b_k$ band can be substantially allowed. For higher temperature, the relation of $1/(1 + e^{(\epsilon_p + V)/kT}) \approx e^{(\epsilon_p + V)/kT}$ is obtained. If $|V| \rightarrow 0$, $e^{(\epsilon_p + V)/kT} \approx 0$, the relation of $\rho \propto T^2$ is obtained. In the condition of $(\epsilon_p + V)/kT << 1$, $e^{(\epsilon_p + V)/kT} \approx 1 - (\epsilon_p + V)/kT$. Thus the temperature dependence of the resistivity is finally modified as follow as

$$\rho \propto T^2 \times (\epsilon_p + V)/kT \propto T$$

(29)

This can be interpreted as a microscopic derivation of Varma’s marginal Fermi liquid model [14].

3.6 Overdoped region

According to our theory, the attractive interaction $V$ decreases with the doping beyond optimal doping. Also, Eq. (23) indicates that the thermal excitation from $c_k$ band to $b_k$ band decreases with decreasing of $V$. As shown in previous section 3.5, the temperature dependence of the resistivity will gradually become from $\rho \propto T$ to $\rho \propto T^2$ and will finally show Fermi liquid like property. This is supported by ARPES measurements in which the sharp peaks are observed nearly Fermi surface through the Brillouin zone [18]. If the overdoped region is a real Fermi liquid, it would maintain Fermi liquid like even at higher temperature. However, recent inelastic X-ray scattering data have demonstrated that strong electron correlations persist even in highly overdoped cuprates superconductors [19, 20]. This implies that there exists the strange metal in higher temperature of overdoped region, and also suggests
that the cuprates superconductor is not a real Fermi liquid in the normal state. How can the composite theory explain this phenomenon? It says that increasing of the temperature leads to increase the scattering from $c_k$ band to $b_k$ band and finally the relation of $\rho \propto T$ appears again. In other words, for small $V$ the higher temperature can satisfy the relation of $(e^-_r + V)/kT \ll 1$. This means that increasing of the attractive $V$ and temperature $T$ has the same effect on the transport properties in the normal state. It is thus concluded that our theory can give an explanation on the mysterious property of strange metal.

4. Conclusions
Unconventional properties of cuprates superconductors are investigated using composite fermions theory. This theory reveals that the anisotropy of the superconductive gap is different from the previous model, but is in agreement with the recent ARPES results. It is found that the anisotropy of pseudogap is similar to the superconductive gap, the competing states in the underdoped region is revealed by mixed states model, and that the anomaly of optical sum rules and $T$-linearity of resistivity in the nearly optimal doping can be derived from considering the interplay between the composite fermion bands. It is also suggested that a crossover separating the strange metal and the normal metal in the overdoped region can be explained by the coupling effect of composite fermions.

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