Stability of Neutron Stars with Dark Matter Core Using Three Crustal Types Impacts Mass-Radius Relations

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We investigate the effects of dark matter (DM) on the nuclear equation of state (EoS) and neutron star structure, in the relativistic mean field theory, both in the absence and presence of a crust. The \( \sigma - \omega \) model is modified by adding a WIMP-DM component, which interacts with nucleonic matter through the Higgs portal. This model agrees well with previous studies which utilized either a more complicated nuclear model or higher-order terms of the Higgs potential, in that DM softens the EoS, resulting in stars with lower maximum masses. However, instabilities corresponding to negative pressure values in the low-energy density regime of the DM-admixed EoS are present, and this effect becomes more prominent as we increase the DM Fermi momentum. We resolve this by confining DM in the star’s core. The regions of instability were replaced by three types of crust: first by an ideal gas EoS, then by the Friedman-Pandharipande-Skyrme EoS, and finally the Skyrme Lyon EoS. While the DM-admixed neutron star increases only slightly in its maximum mass with the addition of the crust, the entire mass-radius relation of the neutron star is significantly affected, with an observed increase in the radius of the star corresponding to the mass.

I. INTRODUCTION

Neutron stars are good testing grounds for predictions of theories beyond the standard model, since they are compact enough to provide conditions necessary for exotic physics to occur \([1-4]\). Furthermore, they are a staple in the studies of nuclear physics, quantum chromodynamics (QCD), and general relativity (GR) \([5-7]\).

One area of research that is currently very active in theoretical and observational astrophysics are neutron star interiors, especially with the advent of gravitational and electromagnetic wave observations among neutron star mergers \([1-4]\). The description of static, nonrotating neutron stars is achieved by solving the Tolman-Oppenheimer-Volkoff (TOV) equations of GR \([8-10]\), which are completed by an equation of state (EoS) \([1, 2, 11, 13]\). This yields the mass-radius relations for neutron stars which can be analyzed \([14]\). Realistic models of neutron stars utilize nuclear field theory in the context of the relativistic mean field theory (rMFT) in obtaining the EoS for nuclear structure, particularly at the core of the star \([15, 16]\). Moreover, several semi-empirical approaches have also been developed to describe the outer layers, such as the crust and/or the atmosphere \([12, 13]\).

Another factor that we can consider in the studies of neutron stars are the observations and measurements of the mass-energy density of the universe which shows that majority of its mass-energy content does not come from matter that is well-described by the standard model; about 25\% is of the form now known as dark matter (DM) \([17, 18]\). Strong evidence for the existence of DM using galactic rotation curves was provided by Vera Rubin, Kent Ford and Ken Freeman in the 1960s and 1970s \([19, 20]\). A favored dark matter candidate is the weakly interacting massive particle (WIMP), which is predicted by supersymmetric extensions to the standard model, and at the same time supported by N-body cosmological simulations \([21, 22]\). Reviews on DM can be found in Ref. \([18, 23, 24]\).

The effects of DM on neutron star structure, and other properties such as tidal deformability have been investigated in the literature, using different assumptions on the nature of the DM involved \([25, 31]\). Some of these used the relativistic mean field theory (rMFT) in quantum hadrodynamics (QHD) \([27, 30]\). In particular, the DM particle is assumed to be fermionic, captured and trapped inside the neutron star \([27, 30, 32]\). The result of this approach is that DM softens the nuclear equation of state, yielding neutron stars of lower masses than neutron stars without DM \([27, 30]\). This effect of reducing neutron star masses is also supported by studies assuming that there is a DM core, together with a nuclear EoS in the middle of the star \([28]\).

A nuclear EoS, however is only dominant at the core of the neutron star, with densities \( \rho > \rho_c \approx 10^{14} \text{ g/cm}^3 \), while an actual neutron star can have a crust or atmo-
sphere [12, 13]. The neutron star can then be thought of as having a crust, with density \( \rho_c \), surrounding the core, beginning with density \( \rho_c \), such that \( \rho < \rho_c \) [12, 13]. In Ref. [30], the DM-admixed nuclear EoS was added with a Baym-Pethick-Sutherland (BPS) crust [33] by using a polytropic formula. In this paper we extend these studies by admixing DM at the nuclear core, and by adding three types of crust on top of the core: an ideal neutron gas (ING), the Friedman-Pandharipande-Skyrme (FPS) crust, and finally, the Skyrme Lyon (SLy) crust.

In this paper, we deal with the QHD model, the \( \sigma - \omega \) or the Walecka model [15] and include the Higgs fields up to order \( h^2 \). In the Standard Model, the Higgs fields are small fluctuations about the vacuum and higher orders of \( h \) can be ignored. Given the simplicity of the Walecka model, we are able to extract the implications of putting a crust on top of the core of the star. We then extend the analysis of Ref. [27] by investigating instabilities in the DM-admixed EoS, and we fix these instabilities by replacing those unstable regions, which happen to be at the low density-end of the EoS with that of crust EoS, notably first with an ING EoS, and then with the FPS EoS, and finally the SLy EoS; the latter two can be represented by semi-analytical models that describe the neutron star crust realistically [12]. The effects of these modifications to the DM-admixed EoS are then compared and studied.

We summarize the structure of this paper as follows. In Section II we discuss the modification of the Walecka model with DM. Section III then deals with adding the ING crust, the FPS crust, and the SLy crust to the DM-admixed EoS. The consequences of these modifications to the neutron star structure are discussed in Section IV. Finally, we conclude by giving some recommendations in Section V. In this paper, we work with natural units \( h = c = 1 \) unless otherwise explicitly stated.

II. THE WALECKA MODEL EQUATION OF STATE WITH DM

The simplest QHD model is the \( \sigma - \omega \) or Walecka model [3, 15]. It is a model describing nucleon-nucleon interaction that is mediated by exchanging \( \sigma \) and \( \omega \) mesons. The fields in this model are based on four particles: the nucleons (neutrons and protons) \( \psi \), the scalar meson \( \sigma \), and the omega vector mesons \( \omega \), with a Lagrangian density given by

\[
\mathcal{L}_{\text{had}} = \bar{\psi} \left( i \gamma_\mu \left( \partial_\mu + ig_\omega \omega_\mu \right) - \left( m_\omega - g_\sigma \sigma \right) \right) \psi
+ \frac{1}{2} \left( \partial_\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2 \right)
- \frac{1}{4} \omega_\mu \omega_\nu - \frac{1}{2} m_\omega^2 \omega_\mu \omega_\mu,
\]

(1)

where \( \omega_\mu \nu = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \), \( m_\sigma \approx 1 \) GeV is the mass of the nucleon (or neutron), \( m_\omega = 520 \) MeV is the mass of the \( \sigma \) meson, \( m_\omega = 783 \) MeV is the mass of the \( \omega \) meson, and the dimensionless coupling constants are \( g_\sigma^2 = 190.4 \) for the \( \omega \) meson coupled to the four-current \( \psi \gamma_\mu \psi \) and \( g_\omega^2 = 109.6 \) for the \( \sigma \) meson coupled with the baryon scalar density \( \bar{\psi} \psi \) [16, 27].

Let us now consider a DM particle with mass \( M_h = 200 \) GeV which would be the lightest supersymmetric neutralino [34]. The fermionic DM Lagrangian density is given by

\[
\mathcal{L}_{DM} = \bar{\chi} \left[ m_\chi \right] \chi + \bar{\psi} \left( m_\psi - g_\chi \psi \right) \chi
\]

(2)

where we have the Higgs boson \( h \) with mass \( M_h = 125 \) GeV, a DM-Higgs Yukawa coupling \( y \), and a nucleon-Higgs Yukawa coupling \( f m_n / v \), where \( v = 246 \) GeV is the Higgs vacuum expectation value, and \( f = 0.3 \) parametrizes the Higgs-nucleon coupling [24, 29, 34]. Very stringent constraints on the DM-nucleon interaction for DM masses above 6 GeV are given by recent DM direct detection experiments [30, 38]. We then consider a negligible DM-nucleon coupling and did not include this term in Eq. (2) [39, 40]. The total Lagrangian density for the DM-admixed system is then

\[
\mathcal{L} = \mathcal{L}_{\text{had}} + \mathcal{L}_{DM}.
\]

(3)

In rMFT, the system is assumed to be uniform in its ground state, and the fields in the Lagrangian are replaced by their mean values [3], that is, \( \sigma \rightarrow \langle \sigma \rangle \), \( \omega_\mu \rightarrow \langle \omega_\mu \rangle \), and \( h \rightarrow \langle h \rangle \). The equations of motion then become

\[
\begin{align*}
\frac{m_\sigma^2}{m} \langle \sigma \rangle &= g_\sigma \langle \bar{\psi} \psi \rangle \\
\frac{m_\omega^2}{m} \langle \omega_\mu \rangle &= g_\omega \langle \bar{\psi} \gamma_\mu \psi \rangle \\
\left[ \gamma_\mu (i \partial_\mu - g_\omega \langle \omega_\mu \rangle) - m_\sigma^2 \right] \langle \psi(x) \rangle &= 0 \\
M_h^2 \langle h \rangle &= y \langle \bar{\chi} \chi \rangle + f \frac{m_n}{v} \langle \bar{\psi} \psi \rangle
\end{align*}
\]

(4)

where the effective masses are given by

\[
\begin{align*}
M_\chi^2 &= M_h^2 - y^2 \\
m_n^* &= m_n - g_\sigma \langle \sigma \rangle - f \frac{m_n}{v} \langle h \rangle.
\end{align*}
\]

(5)

Defining the following dimensionless quantities to increase the efficiency of our numerical calculations:

\[
\begin{align*}
\tilde{p} &= \frac{p}{m_n}; \quad \varphi &= \frac{p_F}{m_n}; \quad \phi &= \frac{p_{DM}}{m_n}; \\
\tilde{\sigma} &= \frac{g_\sigma \langle \sigma \rangle}{m_n}; \quad \tilde{\omega}_0 &= \frac{g_\omega \langle \omega_\mu \rangle}{m_n}; \quad \tilde{h} &= \frac{Y \langle h \rangle}{m_n}; \quad Y &= \frac{f m_n}{v},
\end{align*}
\]

(6)

(7)

\[
\tilde{\epsilon} = \epsilon \epsilon_0; \quad \tilde{P} = \frac{P}{\epsilon \epsilon_0}; \quad \text{where} \quad \epsilon_0 = \frac{m_n^4}{3\pi^2},
\]

(8)

where \( p \) is the particle momentum, \( p_F \) is the nucleon Fermi momentum, \( p_{DM} \) is the DM Fermi momentum, \( \epsilon \)
is the energy density, and $P$ is the pressure, then the mean fields for the DM-admixed Walecka model become

$$\tilde{\sigma} = \frac{g_\sigma^2 m_n^2}{m_\sigma^2 \pi^2} \int_0^\phi dp\tilde{p}^2 \frac{1 - \tilde{\sigma} - \tilde{h}}{\sqrt{\tilde{p}^2 + (1 - \tilde{\sigma} - \tilde{h})^2}}, \quad (9)$$

$$\tilde{\omega}_0 = \frac{g_\omega^2 m_n^2}{m_\omega^2 \pi^2} \frac{\phi^3}{3}, \quad (10)$$

$$\tilde{h} = \frac{Y m_n^2}{M_h^2 \pi^2} \int_0^\phi dp\tilde{p}^2 \frac{\left(M_{m_n} - \frac{m_n}{h} \tilde{h}\right)}{\sqrt{\tilde{p}^2 + \left(M_{m_n} - \frac{m_n}{h} \tilde{h}\right)^2}}$$

$$+ \frac{Y^2 m_n^2}{M_h^2 \pi^2} \int_0^\phi dp\tilde{p}^2 \frac{(1 - \tilde{\sigma} - \tilde{h})}{\sqrt{\tilde{p}^2 + (1 - \tilde{\sigma} - \tilde{h})^2}}, \quad (11)$$

The dimensionless, parametric, DM-admixed Walecka (or $\sigma$-$\omega$-DM) EoS of the form $\epsilon(P)$ is then written as

$$\tilde{\epsilon} = \frac{1}{c_0} \left[ \frac{1}{2} \frac{m_s m_n}{g_\sigma} \right]^2 \tilde{\sigma}^2 + \frac{1}{2} \left(\frac{m_s m_n}{g_\sigma}\right)^2 \tilde{\omega}_0^2$$

$$+ \frac{1}{2} \left(\frac{M_{m_n}}{Y}\right)^2 \tilde{h}^2 + \frac{m^4}{\pi^2} \int_0^\phi dp\tilde{p}^2 \sqrt{\tilde{p}^2 + (1 - \tilde{\sigma} - \tilde{h})^2}$$

$$+ \frac{m^4}{\pi^2} \int_0^\phi dp\tilde{p}^2 \sqrt{\tilde{p}^2 + \left(M_{m_n} - \frac{m_n}{h} \tilde{h}\right)^2}], \quad (12)$$

$$\tilde{\rho} = \frac{1}{c_0} \left[ - \frac{1}{2} \left(\frac{m_s m_n}{g_\sigma}\right)^2 \tilde{\sigma}^2 + \frac{1}{2} \left(\frac{m_s m_n}{g_\sigma}\right)^2 \tilde{\omega}_0^2$$

$$- \frac{1}{2} \frac{M_{m_n}}{Y} \tilde{h}^2 + \frac{m^4}{3\pi^2} \int_0^\phi dp\tilde{p}^2 \sqrt{\tilde{p}^2 + (1 - \tilde{\sigma} - \tilde{h})^2}$$

$$+ \frac{m^4}{3\pi^2} \int_0^\phi dp\tilde{p}^2 \sqrt{\tilde{p}^2 + \left(M_{m_n} - \frac{m_n}{h} \tilde{h}\right)^2}], \quad (13)$$

To numerically solve the EoS, we first solve simultaneously for the mean fields Eqs. (9)-(11), for a range of hadron Fermi momenta $p_F$, and for a given DM Fermi momentum $p_{DM}^F$, before substituting these to the EoS. We take the values of the DM Fermi momenta to be $p_{DM}^F = 0.02 \text{ GeV}, 0.04 \text{ GeV}, 0.06 \text{ GeV}$, in accordance with existing literature [30], which also evades constraints from DM search experiments.

Figure 1 shows the $\sigma$-$\omega$-DM EoS plots for different values of the DM Fermi momentum. The effect of DM indeed is to “soften” the EoS, that is, to shift the EoS towards higher energy density values corresponding to pressure values, albeit very slightly.

The EoS can also be visualized in different, equivalent forms, and these are shown in Figure 2. We now analyze the EoS based on Figure 2. Notice that the $\sigma$-$\omega$-DM EoS has negative values of pressure, corresponding to unstable regions in the low-pressure regime. We see that the original $\sigma$-$\omega$ EoS is akin to the van der Waals EoS [41]. The EoS is unstable because for a system in thermodynamic equilibrium to be stable, the pressure should not increase with the volume. One can get rid of the instabilities in the original Walecka EoS (corresponding to $p_{DM}^F = 0$) via Maxwell construction [41], as done in Ref. [12], by finding a critical pressure at which the “well” and “hill” regions have equal areas. However, the same procedure cannot be done for nonzero $p_{DM}^F$, since the wells and hills, even at a critical pressure of zero, cannot have equal areas. We therefore seek for another way of fixing these unstable regions.

![Figure 1](image1.png)

**FIG. 1.** $\sigma$-$\omega$-DM $P(\epsilon)$ EoS. This plot confirms the results of Ref. [27].

![Figure 2](image2.png)

**FIG. 2.** $\sigma$-$\omega$-DM $P(V)$ EoS. The plot labeled with $p_{DM}^F = 0 \text{ GeV}$ corresponds to the unmodified Walecka EoS.
III. THE DM-ADMIXED EOS WITH CRUST

A remedy for the instability problem presented in Section II is to replace the unstable regions in the EoS. This can be done by replacing the low-pressure (low-density) regions with another EoS that better describes it, similar to an atmosphere or crust. The underlying assumption for this is that our DM is trapped only inside the core of the neutron star; this means that the crust contains a negligible amount of DM particles.

We first deal with the DM-admixed Walecka EoS with the addition of an ideal neutron gas (ING) crust, a simple model for a neutron star atmosphere. The EoS $\tilde{\epsilon}_{\text{ING}}(\tilde{p}_{\text{ING}})$ of the ING EoS, in dimensionless form, is given by

$$\tilde{\epsilon}_{\text{ING}} = 3 \int_{0}^{\varphi} \tilde{p} \sqrt{1 + \tilde{p}^2} d\tilde{p}, \quad (14)$$

$$\tilde{P}_{\text{ING}} = 3 \int_{0}^{\varphi} \tilde{p}^2 \left( \sqrt{1 + \varphi^2} - \sqrt{1 + \tilde{p}^2} \right) d\tilde{p}. \quad (15)$$

FIG. 3. $\sigma$-$\omega$-DM Dimensionless Equation of State

We now plot the ING EoS, superimposed on the $\sigma$-$\omega$-DM EoS. The result is shown in Figure 3. We see that the ideal neutron gas EoS crosses all the $\sigma$-$\omega$ EoS early on at very low pressures. Since the nuclear EoS is only dominant in the star’s core, at high densities, we replace the unstable region of the $\sigma$-$\omega$-DM EoS at low pressure and energy density with the ING EoS up until the point of intersection between the two equations of state, to ensure continuity in the EoS. The resulting ING-$\sigma$-$\omega$-DM EoS is shown in Figure 4. We can then use this resulting EoS to model our neutron star.

The more realistic equations of state that can model the neutron star crust are the FPS and SLy EoS. Both models are comparable in their modelling of the crust EoS, with the primary difference being their modelling of the crust-core interface. The FPS EoS model takes into account exotic nuclear shapes near the interface, while the SLy EoS models the interface as a small phase transition [11, 12]. The semi-analytical representations of FPS and SLy EoS can model all the regions of the neutron star interior [12]. In this study, we use the FPS and SLy EoS to model the crust, as our nuclear core is DM-admixed, which was not considered in Ref. [11, 12]. The parametrization for nonrotating stars [12] is given by

$$\log P = a_1 + a_2 \log \epsilon + a_3 (\log \epsilon)^3 \frac{f_0(a_5(\log \epsilon - a_6))}{1 + a_4 \log \epsilon} + (a_7 + a_8 \log \epsilon) f_0(a_9(a_{10} - \log \epsilon))$$

$$+ (a_{11} + a_{12} \log \epsilon) f_0(a_{13}(a_{14} - \log \epsilon)) + (a_{15} + a_{16} \log \epsilon) f_0(a_{17}(a_{18} - \log \epsilon)), \quad (16)$$

where the units for $P$ and $\epsilon$ are in dyne/cm$^2$ and g/cm$^3$, respectively, the $a_i$ are fitting constants, and the function $f_0(x)$ is defined as

$$f_0(x) = \frac{1}{e^x + 1}. \quad (17)$$

The values of $a_i$, taken from Ref. [12], are given in Table I.

| $i$ | $a_1$ FPS | $a_1$ SLy | $a_2$ FPS | $a_2$ SLy |
|-----|-----------|-----------|-----------|-----------|
| 1   | 6.22      | 6.22      | 10        | 11.8421   |
| 2   | 6.121     | 6.121     | 11        | -22.003   |
| 3   | 0.006004  | 0.005925  | 12        | 1.5552    |
| 4   | 0.16345   | 0.16326   | 13        | 9.3       |
| 5   | 6.50      | 6.48      | 14        | 14.9      |
| 6   | 11.8440   | 11.4971   | 15        | 23.73     |
| 7   | 17.24     | 19.105    | 16        | -1.508    |
| 8   | 1.065     | 0.8938    | 17        | 1.79      |
| 9   | 6.54      | 6.54      | 18        | 15.13     |

Similar with the procedure done in the case of the ING EoS, we replace the unstable regions of the $\sigma$-$\omega$-DM EoS
with the FPS or SLy EoS, right until the points of intersections of the EoS. The resulting plots are shown in Figure 5 for the FPS-\(\sigma\)-\(\omega\)-DM EoS and Figure 6 for the SLy-\(\sigma\)-\(\omega\)-DM EoS.

**FIG. 5.** FPS-\(\sigma\)-\(\omega\)-DM Dimensionless Equation of State

**FIG. 6.** SLy-\(\sigma\)-\(\omega\)-DM Dimensionless Equation of State

We observe from our “crust” equations of state (ING, FPS, and SLy) that the ING EoS is the softest among the three, and so intersects with the \(\sigma\)-\(\omega\)-DM EoS at lower values of pressure, than with the FPS and SLy EoS. Meanwhile, the FPS and SLy EoS produce comparable EoS when combined with the \(\sigma\)-\(\omega\)-DM EoS. In the next section, we will compare the mass-radius relations for the neutron stars obtained from all of these EoS.

**IV. THE STRUCTURE EQUATIONS AND MASS-RADIUS RELATIONS**

Using the dimensionless quantities for \(\epsilon\) and \(P\) defined in Eq. (8) as well as the following:

\[
\tilde{M} = \frac{M}{M_\odot}, \quad \tilde{r} = \frac{r}{R_0}, \quad R_0 = GM_\odot, \quad \Omega = \frac{4\pi\epsilon_0}{M_\odot R_0^3},
\]

(18)

\[
\frac{d\tilde{P}}{d\tilde{r}} = -\left[\tilde{\epsilon} + \tilde{P}\right] \frac{\tilde{M} + \Omega \tilde{r}^3 \tilde{P}}{\tilde{r}^2 - 2\tilde{M} \tilde{r}},
\]

(19)

\[
\frac{d\tilde{M}}{d\tilde{r}} = \Omega \tilde{r}^2 \tilde{\epsilon},
\]

(20)

with the conditions

\[
\tilde{P}(\tilde{r} = 0) = \tilde{P}_c, \quad \tilde{P}(\tilde{r} = R_\star) = 0
\]

\[
\tilde{M}(\tilde{r} = 0) = 0, \quad (\tilde{r} = R_\star) = M_\star,
\]

(21)

where \(P_c\) is the central pressure and \(R_\star\) is the stellar radius. The TOV equations describe static, spherically symmetric, nonrotating stars in GR [5, 7, 10]. The modified EoS from Section III are fed into the TOV equations and solved numerically using the forward Euler method over a range of central pressures \(\tilde{P}_c\). The initial condition is such that the pressure is greatest at the center of the star, and reaches zero at the star’s edge, defining the stellar radius at \(\tilde{r} = R_\star\). Meanwhile, the equation for the mass is cumulative, such that it reaches the stellar mass \(M = M_\star\) at \(R_\star\). For a range of central pressures, we can then form a parametric relation between \(M_\star\) and \(R_\star\), known as the mass-radius relation of the star. We now investigate in this section the effects of three modified EoS that we obtained in Section III to the mass-radius relations of neutron stars.

As a reference, we also obtain the mass-radius relations for the \(\sigma\)-\(\omega\)-DM model for different values of \(p_F^{DM}\), without any crust, which is similar to the results of Ref. [27]. These are shown in Figure 7. Meanwhile, the mass-radius relations for the ING-\(\sigma\)-\(\omega\)-DM EoS are shown in Figure 8. Note that the addition of the ING EoS produces significant changes to the mass-radius relations, by increasing the radii of the neutron star corresponding to the mass. However, the effect of DM remains generally the same: to “shrink” the neutron star, by producing stars of lower masses and smaller radii as the value of \(p_F^{DM}\) gets larger. We also note that the limiting/maximum mass of the neutron star increases by small amounts for the ING-\(\sigma\)-\(\omega\)-DM EoS. It is also interesting to see that for some constant radius (starting at around 15 km), the masses of the neutron stars are all the same for different \(p_F^{DM}\). For a constant given \(p_F^{DM}\), the mass then decreases with
increasing radius (starting at around 15 km, or 0.4\(M_\odot\)).

The mass-radius relations for the FPS-\(\sigma\)-\(\omega\)-DM EoS are shown in Figure 9. The changes to the mass-radius relations from the neutron star without crust are also substantial, and different from that of the ING-\(\sigma\)-\(\omega\)-DM EoS, especially for greater values of \(p_F^{DM}\). Each plot for every nonzero \(p_F^{DM}\) for the FPS-crusted mass-radius relations in Figure 9 intersects that of the original \(\sigma\)-\(\omega\)-DM EoS (\(p_F^{DM} = 0\)). The stellar masses on the right side of the intersection point are increased from that without DM as a function of increasing \(p_F^{DM}\), while the masses decrease in the left of the intersection point as a function of increasing \(p_F^{DM}\). We also note that in the absence of DM, that is, at \(p_F^{DM} = 0\), the mass-radius relation for both the FPS- and ING- crusted neutron stars yield more or less the same maximum masses.

Finally, Figure 10 shows the mass-radius relations for the SLy-\(\sigma\)-\(\omega\)-DM EoS. Because the FPS and SLy EoS are quite similar in nature, the mass-radius relations with the SLy crust are comparable with those containing the FPS crust. The changes to the mass-radius relations are also significant for large values of \(p_F^{DM}\).

From the mass-radius relations, we can then obtain the maximum or limiting masses and corresponding limiting radii for the neutron star for the different EoS, and also for varying values of \(p_F^{DM}\). The results are summarized in Table II.

Another way to analyze the mass-radius relations is to plot the stellar mass \(M_*\) as a function of the DM Fermi momentum \(p_F^{DM}\), for some constant stellar radius \(R_*\). The result for the SLy-crusted star is shown in Figure 11. Each line in Figure 11 corresponds to one fixed radius \(R_*\) corresponding to different masses \(M_*/M_\odot\) as the \(p_F^{DM}\) increases. The radii are separated by an interval of
0.1 km, and each line changes shape for every value of $R_*$. Lines that start from the left at $M_*/M_\odot \lesssim 1.0$ correspond to different radii greater than 16 km. Lines that start at $M_*/M_\odot \gtrsim 1.5$ correspond to different radii lower than 15 km. The different lines of constant radius tend to approach a value of $M_{\text{con}}/M_\odot \simeq 1.3$, as $p_F^{\text{DM}}$ increases, and this mass corresponds to radius $R_* \simeq 15$ to 16 km, and central pressure $P_c \simeq 0.01$ to 0.03. From this, we can speculate that a DM-admixed compact object, which may not necessarily be a neutron star, could potentially exist, with size (mass and radius) that is conducive to a wide range of values of the DM Fermi momentum $p_F^{\text{DM}}$. We emphasize that the DM in the star, as previously mentioned, does not interact with the nucleons, and only interacts with the Higgs particle. This star may be comprised mostly of DM and Higgs particles but not yet detectable by current observational means.

The same behavior is observed for the FPS-crusted star, albeit with slightly different values of $M_{\text{con}}$ and corresponding $R_*$. Finally, for the ING-crusted star, the masses seemingly converge at $M_{\text{con}}/M_\odot \lesssim 0.4$ with increasing $p_F^{\text{DM}}$; this is readily observed in Figure 8. This mass is way below that of neutron stars but may possibly indicate a compact object which accommodates a wide range of $p_F^{\text{DM}}$ whose radius is around 15 km and larger.

FIG. 11. $M_*/M_\odot$ as a function of $p_F^{\text{DM}}$, SLy-$\sigma$-$\omega$-DM EoS

V. CONCLUSIONS

In this work, we extended the investigation of DM-admixed neutron stars by confining the DM in the star’s core and by adding a crust on the core. This simulates a neutron star with a crust, and with a core dominated by the nuclear equation of state, which, in this case, was the Walecka model added with DM, which was obtained via relativistic mean field theory.

Three types of crust were considered: the ideal neutron gas, FPS, and SLy crusts. These crust equations of state were used to replace the instabilities in the $\sigma$-$\omega$-DM EoS, corresponding to negative values of the pressure at the lower density regime. The resulting mass-radius relations are markedly different from neutron stars without crust. DM effects are primarily responsible for decreasing the star mass, while the main effect of the crust is to increase the star radius. We also note that, with or without the crust, both the maximum mass and limiting radius of the neutron star progressively decreases as the DM Fermi momentum $p_F^{\text{DM}}$ increases in value (see Table II). We also speculate the possibility of a compact object, containing a DM core and a crust, existing with a mass and radius that accomodate a wide range of values for $p_F^{\text{DM}}$.

One can then extend this study by using more complicated models for the nuclear equation of state, to address the limitations of the Walecka model. The effects of DM may also be investigated on the star’s crust, taking into account the relative amounts of DM that must be present in either the core or the crust. Observations of neutron stars and neutron star mergers may also give constraints on the parameters of DM and the models used in this study.

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TABLE II. Maximum Masses and Limiting Radii among various EoS

| $p_F^{\text{DM}}$ (GeV) | $\sigma$-$\omega$-DM | ING-$\sigma$-$\omega$-DM | FPS-$\sigma$-$\omega$-DM | SLy-$\sigma$-$\omega$-DM |
|------------------------|----------------------|------------------------|------------------------|------------------------|
| $M_{\text{lim}}/M_\odot$ | $R_{\text{lim}}$ (km) | $M_{\text{lim}}/M_\odot$ | $R_{\text{lim}}$ (km) | $M_{\text{lim}}/M_\odot$ | $R_{\text{lim}}$ (km) |
| 0                      | 2.827                | 13.039                 | 2.829                  | 13.454                 | 2.827                  | 13.512                 |
| 0.02                   | 2.785                | 12.836                 | 2.786                  | 13.279                 | 2.785                  | 13.567                 |
| 0.04                   | 2.541                | 11.490                 | 2.544                  | 11.970                 | 2.542                  | 12.532                 |
| 0.06                   | 2.124                | 9.447                  | 2.128                  | 10.006                 | 2.138                  | 10.899                 |

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