Gluino Axion, Neutrino Seesaw, Dynamical Gaugino Mass, and $A \simeq 0$ Supersymmetry

Ernest Ma

Department of Physics and Astronomy, University of California, Riverside, California 92521, USA

Abstract

In the axionic solution of the strong CP problem, fermions which transform under quantum chromodynamics (QCD) are required. In supersymmetry, by equating $U(1)_{PQ}$ with $U(1)_R$, the natural candidates are the gluinos, as pointed out some years ago. A new specific implementation of this idea is proposed, linking the gluino axion scale to that of the canonical seesaw mechanism for neutrinos. Gaugino masses are generated dynamically and the $A$ term is predicted to be very small.
The axion is a nearly massless pseudoscalar particle postulated to solve the strong CP problem \[1\]. As such, it must be related to the mass-generation mechanism of a colored fermion multiplet. Instead of quarks, it was first pointed out by Demir and Ma \[2,3\] that gluinos may also be used. In this paper, a new specific implementation of this idea is proposed, where the gluino axion scale and that of the canonical seesaw mechanism for neutrinos in supersymmetry are one and the same \[4,5,6\]. As a consequence, gaugino masses and the $A$ term in supersymmetry are forbidden at tree level. New singlet heavy quarks (such as those available in the $27$ representation of $E_6$) are introduced to allow the gluino mass to be generated in one loop. The $A$ term is also radiatively generated but remains negligible.

The axion to be discussed is a singlet under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group of the Standard Model (SM). It comes from the spontaneous breaking of an anomalous global symmetry, i.e. $U(1)_{PQ}$, the choice of which defines the model. If it is identified \[2,3,4\] with the $U(1)_R$ of supersymmetric transformations, then the resulting axion couples to gluinos, not quarks. Under $U(1)_R$, the scalar components of a chiral superfield transform as $\phi \rightarrow e^{i\theta R} \phi$, whereas the fermionic components transform as $\psi \rightarrow e^{i\theta (R-1)} \psi$. For the Lagrangian to be invariant under $U(1)_R$, the superpotential $\hat{W}$ should have $R = 2$. In the Minimal Supersymmetric Standard Model (MSSM), this is explicitly broken by the term $\mu \hat{H}_u \hat{H}_d$, resulting in the conservation of only its well-known discrete remnant, i.e. $R$ parity.

The first task is to devise a mechanism for having the axion scale \[7\] at $10^{11}$ GeV or so and yet for it to be related to the gluino mass at the electroweak scale. Following Ref. \[3\], consider three singlet superfields $\hat{S}_2$, $\hat{S}_1$, $\hat{S}_0$ with $U(1)_R$ charges 2, 1, 0 and transforming under an additional discrete $Z_3$ symmetry as $\omega^2$, $\omega$, $\omega$ where $\omega = \exp(2\pi i/3)$. The most general $R = 2$ superpotential is given by

$$\hat{W} = m_2 \hat{S}_2 \hat{S}_0 + f_1 \hat{S}_1 \hat{S}_1 \hat{S}_0 + \Lambda \hat{S}_2 + m_1 \hat{S}_1 \hat{S}_1,$$  

\[1\]
where $Z_3$ is broken only by the soft terms $\Lambda \hat{S}_2$ and $m_1 \hat{S}_1 \hat{S}_1$. The resulting scalar potential

$$V = |m_2 S_2 + f_1 S_1|^2 + |2m_1 S_1 + 2f_1 S_1 S_0|^2 + |\Lambda + m_2 S_0|^2$$  \hspace{1cm} (2)

has a minimum at $V = 0$ if

$$v_2 = -\frac{f_1 v_1^2}{m_2},$$

$$v_0 = -\frac{m_1}{f_1} = -\frac{\Lambda}{m_2},$$  \hspace{1cm} (4)

where $v_{2,1,0}$ are the vacuum expectation values of $S_{2,1,0}$ respectively. Therefore, if $\Lambda$ is set equal to $m_1 m_2 / f_1$, $U(1)_R$ may be broken spontaneously without breaking the supersymmetry. This is of course fine tuning, but once it is done, soft supersymmetry breaking terms at the TeV scale will not change the basic quadratic relationship between $v_1$ and $v_2$ in the above. This allows $v_2$ to be much smaller than $v_1$ and is also the key to equating the axion scale to the neutrino seesaw mass scale, as shown below.

Consider now the superfields of the MSSM. Under $U(1)_R \times Z_3$, the Higgs superfields $\hat{H}_u$, $\hat{H}_d$ transform as $(0, \omega^2)$; $\hat{Q} = (\hat{u}, \hat{d})$, $\hat{L} = (\hat{\nu}, \hat{e})$ as $(3/2, 1)$; $\hat{u}^c$, $\hat{d}^c$, $\hat{e}^c$, $\hat{N}^c$ as $(1/2, \omega)$. [This differs from the usual $U(1)_R$ assignment by the transformation $R \rightarrow R + (3B + L)/2$.] The resulting $R = 2$ superpotential is given by

$$\hat{W} = h_u \hat{H}_u \hat{Q} \hat{u}^c + h_d \hat{H}_d \hat{Q} \hat{d}^c + h_e \hat{H}_d \hat{L} \hat{e}^c + h_2 \hat{S}_2 \hat{H}_u \hat{H}_d + h_N \hat{H}_u \hat{L} \hat{N}^c + \frac{1}{2} h_1 \hat{S}_1 \hat{N}^c \hat{N}^c.$$  \hspace{1cm} (5)

The usual $\mu$ term is now replaced by $h_2 v_2$ and the singlet neutrino mass $m_N$ by $h_1 v_1$.

Using Eqs. (3) and (4), with the redefinition of $\hat{S}_{2,1,0} \rightarrow v_{2,1,0} + \hat{S}_{2,1,0}$, Eq. (1) can be rewritten as

$$\hat{W} = \frac{m_2}{v_1}(v_1 \hat{S}_2 - 2v_2 \hat{S}_1) \hat{S}_0 + f_1 \hat{S}_1 \hat{S}_1 \hat{S}_0,$$  \hspace{1cm} (6)

showing clearly that the linear combination

$$\hat{S} = \frac{v_1^* \hat{S}_1 + 2v_2^* \hat{S}_2}{\sqrt{|v_1|^2 + 4|v_2|^2}}$$  \hspace{1cm} (7)
is a massless superfield. Consider now the breaking of supersymmetry by soft terms at the TeV scale which preserve the $U(1)_R$ symmetry but not necessarily the $Z_3$ discrete symmetry. In the scalar sector, the important terms are

$$\mu_1^2 S_1^* S_1 + \mu_{12} S_1^* S_2^* + \mu_{12}^* (S_1^*)^2 S_2,$$

which lift the indeterminacy \[8\] of Eq. (3) and result in \[3\]

$$|v_1|^2 = \frac{\mu_1^2}{4Re(\mu_{12} f_1/m_2)}.$$  

(8)

For example, let $m_2 = 10^{16}$ GeV, $f_1 = 0.1$, $\mu_1 = 20$ TeV, $\mu_{12} = 1$ TeV, then

$$v_1 = 10^{11} \text{ GeV}, \quad v_2 = 10^5 \text{ GeV}.$$  

(9)

The neutrino seesaw mass scale $m_N = h_1 v_1$ may then be easily of order $10^8$ GeV if $h_1 \sim 10^{-3}$.

Since $v_1 >> v_2$, the scalar component (saxion) of the axion superfield is mostly $S_1$ and acquires a mass given by

$$m_S^2 = \mu_1^2 - 2Re(\mu_0 m_1/f_1),$$  

(10)

where the second term comes from $(\mu_0 S_0 + \mu_0^* S_0^*) S_1^* S_1$. As for the fermionic component (axino), since $\tilde{S}_1 \tilde{S}_1$ is an allowed term under $U(1)_R$, it can have an arbitrary Majorana mass at the scale of soft supersymmetry breaking. In Eq. (5), since $N^c$ is heavy and $h_2 v_2 = \mu$, the only term beyond those of the MSSM is

$$\frac{2\mu}{|v_1|} \hat{S} \hat{H}_u \hat{H}_d.$$  

(11)

If the axino is light enough, the would-be lightest supersymmetric particle of the MSSM will decay into it, allowing for possible collider signatures \[9\].

The requirement of $U(1)_R$ symmetry forbids all gaugino masses at tree level as well as the trilinear scalar $A$ terms of the MSSM. Whereas $A = 0$ is not a problem phenomenologically, the absence of gaugino masses is not acceptable. Indeed, a mass for the gluino is necessary for the axion to couple to it. The direct coupling $S_2^* \tilde{g}\tilde{g}$ is not allowed because it is a hard term (of dimension four) which breaks supersymmetry. Hence new singlet heavy quarks $h, h^c$ of
charge \( \mp 1/3 \) are proposed, transforming under \( U(1)_R \times Z_3 \) as \((3/2, 1), (1/2, \omega)\) respectively. [They can come from the 27 representation of \( E_6 \) for example.] Since \( d^c \) also transforms as \((1/2, \omega)\), there are two more terms in the superpotential, i.e.

\[
m_h \hat{h}\hat{h}^c + h_d \hat{H}_d \hat{Q} \hat{h}^c.
\]

The first term serves to define \( \hat{h}^c \) and the second mixes \( d \) and \( h \), thus allowing \( h \) to decay. For \( m_h \) large compared to the electroweak scale, this mixing is also small enough to be acceptable phenomenologically. Now there can be a soft supersymmetry breaking trilinear scalar term \( \lambda_h S_2^* \hat{h}\hat{h}^c \), which allows the gluino to acquire a mass in one loop as shown in Fig. 1. The same mechanism also works for the \( U(1)_Y \) gaugino.

![Figure 1: One-loop generation of gluino mass.](image)

Assuming the mass eigenvalues of the \((\hat{h}, \hat{h}^c)\) sector to be \( m_h^2 \pm |\lambda_h v_2^*| \), the gluino mass is given by

\[
m_{\tilde{g}} = \frac{\alpha_s m_h}{16\pi} \left[ \ln \left( \frac{1 - x^2}{x} \right) + \ln \left( \frac{1 + x}{1 - x} \right) \right],
\]

where \( x = |\lambda_h v_2^*|/m_h^2 \). Let \( m_h = 1.1 \times 10^5 \) GeV, \( \lambda_h = v_2 = 10^5 \) GeV, \( \alpha_s = 0.12 \), then \( m_{\tilde{g}} = 253 \) GeV.

As for the Higgs sector, the important \( B \) term is allowed by \( U(1)_R \) symmetry, i.e. \( \mu_{12}^2 H_u H_d + H.c. \). Together with the induced \( \mu \) term, the \( SU(2)_L \) and \( U(1)_Y \) gauginos also receive radiative mass contributions as shown in Fig. 2. Note that there are additional heavy
Higgs superfields beyond those of the MSSM which are available for example in $E_6$, allowing these masses to be also of order $m_{\tilde{g}}$. Once the gauginos are massive, the $A$ term is also radiatively generated, but it is a two-loop effect, hence $A \simeq 0$ is expected.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.pdf}
\caption{One-loop generation of $SU(2)_L$ gaugino mass.}
\end{figure}

Because of the necessity of generating realistic gaugino masses, the scale of soft supersymmetry breaking as well as $v_2$ and the mass of new particles should be of order $10^5$ GeV. However, just as the SM allows a wide range of Yukawa couplings, some of the supersymmetry breaking parameters may be as low as $10^2$ GeV. The experimental tests of this model are Eq. (11) and the prediction $A \simeq 0$. Without the $A$ term, the mixing matrix linking left sfermions with right sfermions is automatically proportional to the corresponding fermion mass matrix. This solves the usual problem of flavor changing neutral currents in supersymmetry.

I thank V. Barger, M. Frigerio, and K. Hagiwara for discussions during the 2007 Neutrino Workshop at the Aspen Center for Physics. This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.
References

[1] For a recent review, see for example R. D. Peccei, hep-ph/0607268.

[2] D. A. Demir and E. Ma, Phys. Rev. D62, 111901(R) (2000).

[3] D. A. Demir and E. Ma, J. Phys. G27, L87 (2001).

[4] D. A. Demir, E. Ma, and U. Sarkar, J. Phys. G26, L117 (2000).

[5] E. Ma, Phys. Lett. B514, 330 (2001).

[6] E. Ma, J. Phys. G29, 313 (2003).

[7] See for example G. Raffelt, hep-ph/0611350.

[8] E. Ma, Mod. Phys. Lett. A14, 1637 (1999).

[9] See for example B. Feldstein, L. J. Hall, and T. Watari, Phys. Lett. B607, 155 (2005).