LYOT CORONAGRAPHY ON GIANT SEGMENTED-MIRROR TELESCOPES

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ABSTRACT

We present a study of Lyot-style (i.e., classical, band-limited, and Gaussian occulter) coronagraphy on extremely large, highly segmented telescopes. We show that while increased telescope diameter is always an advantage for high dynamic range science (assuming wave-front errors have been corrected sufficiently well), segmentation itself sets a limit on the performance of Lyot coronagraphs. Diffraction from intersegment gaps sets a floor to the achievable extinction of on-axis starlight with Lyot coronagraphy. We derive an analytical expression for the manner in which coronagraphic suppression of an on-axis source decreases with increasing gap size when the segments are placed in a spatially periodic array over the telescope aperture, regardless of the details of the arrangement. A simple Lyot stop masking out pupil edges produces good extinction of the central peak in the point-spread function (PSF) but leaves satellite images caused by intersegment gaps essentially unaffected. Masking out the bright segment gaps in the Lyot plane with a reticulated mask reduces the satellite images’ intensity to a contrast of \(5 \times 10^{-9}\) on a 30 m telescope with 10 mm gaps, at the expense of an increase in the brightness of the central peak. The morphology of interesting targets will dictate which Lyot stop geometry is preferable: the reticulated Lyot stop produces a conveniently unimodal PSF, whereas a simple Lyot stop produces an extended array of satellite spots. A cryogenic reticulated Lyot stop will also benefit both direct and coronagraphic mid-IR imaging.

Subject headings: instrumentation: adaptive optics — instrumentation: high angular resolution — methods: analytical — telescopes

1. INTRODUCTION

Current conventional wisdom deems it to be easier to construct segmented 30–100 m diameter ground-based, orbiting, or lunar telescopes than monolithic ones. Here we inspect the consequences of the segmented design choice for diffraction-limited high dynamic range Lyot-style coronagraphy, which is one of the techniques used for faint-companion searches and extrasolar planetary and disk science (Lyot 1939; Golimowski et al. 1995; Nakajima et al. 1995; Sivaramakrishnan et al. 2001; Aime & Soummer 2003, 2004; Oppenheimer et al. 2003). Just as high dynamic range coronagraphy is very sensitive to the amount of residual aberration in a system (Angél 1994), we show that Lyot coronagraphy is similarly sensitive to the amount of nonreflective area in a highly segmented telescope. Other forms of high dynamic range imaging (such as phase-mask coronagraphy, interfero-coronagraphy, or apodized and shaped pupils) may be less affected by mirror segmentation, but the simplicity and robustness of Lyot coronagraphy makes this topic relevant to the planning of astronomical instrumentation on future telescopes. Giant segmented-mirror telescope (GSMT) design must take the interaction between aperture geometry and coronagraphic performance into account if planetary-companion and other high dynamic range science is a goal for the telescope.

We describe the point-spread function (PSF) of a Lyot coronagraph on a perfect segmented-mirror telescope using the techniques of Fourier optics in the highly segmented regime. There are two scales of relevance in this problem. A Lyot coronagraph with an occulting spot that is \(s\) resolution elements in diameter has a natural scale length \(D/s\) in its Lyot pupil plane (as we explain in § 3), where \(D\) is the aperture diameter (see, e.g., Sivaramakrishnan et al. 2001 and references therein for an introductory exposition on Lyot coronography). When the segment spacing \(b\) is considerably smaller than this scale, it is easy to calculate the on-axis coronagraphic PSF (PSF\(_c\)). When the inequality \(b \ll D/s\) holds, we describe the telescope as being “highly segmented,” from a coronagraphic perspective. Makidon et al. (2000) and Sivaramakrishnan et al. (2001) demonstrated that high dynamic range, stellar Lyot coronagraphy on unapodized apertures is only interesting when the spot diameter exceeds about 4 resolution elements. A 0.034 occulting spot on the 36-segment Keck aperture (\(s = 4\) in \(H\) band) does not constitute a highly segmented telescope from the coronagraphic point of view—Lyot pupil diffraction is on a scale of \(D/4\), and segments are \(D/7\) across. There is no clear separation of these two scales. Sivaramakrishnan et al. (2004) treats the Keck case.

Coronagraphs with multiple occulting spots that occult all the satellite PSFs in the first image plane have been studied by Aime et al. (2001) in the one-dimensional, sparse aperture array case, with a dispersed image plane to address chromaticity problems. This approach looks unpromising for almost-filled GSMT apertures with anything but narrow bandpass filters.

In this Letter, we obtain approximate expressions for on-axis PSF’s using analytical methods that are valid for Lyot-style coronographs on GSMTs. This enables us to assess the dynamic range accessible to simple coronographs on GSMTs and engenders a sound physical understanding of Lyot coronagraphy on GSMTs being planned today (e.g., Nelson 2000; Dierickx & Gilmozzi 2000). We do not treat phase-mask or apodized-pupil Lyot coronagraphy here.
2. THE APERTURE ILLUMINATION FUNCTION

In this introductory work, we describe the aperture of a segmented-mirror telescope in a somewhat more general way than is typically done for particular telescope design studies such as those of Chanlan & Troy (1999) and Yaitskova et al. (2003). Using a more general formalism enables us to identify analytically and quantify limitations of Lyot coronagraphs on highly segmented apertures with arbitrary but periodic segment geometry.

The unsegmented full aperture of the telescope is described by a function $A(x)$, which, for unapodized apertures, is unity over the aperture and zero elsewhere. Here $x = (u, v)$ is the location in the aperture, in units of the wavelength of the light. We develop the expression for a monochromatic PSF with perfect optics first and then calculate PSF, (we assume that the scalar-wave approximation and Fourier optics provide an adequate description of PSF formation).

Segment gaps are described by some lattice-like function $L(x)$, which takes the value of unity where gaps exist and zero over the segments themselves. The function $L(x)$ possesses whatever two-dimensional crystallographic symmetry suits the problem. We illustrate our arguments with the simple case of a circular aperture with square segments, although our results hold for the general case of arbitrary spatially periodic segmentation on obscured apertures.

The aperture illumination function—the complex amplitude in the aperture in response to a unit-strength (field rather than power) incoming wave—for the segmented aperture can be written as

$$A_r(x) = A(x)[1 - L(x)].$$

In the particular case of tetrad symmetry describing square segments,

$$L(x) = L_1(u; b, e) + L_*(v; b, e) - L_1(u; b, e)L_*(v; b, e),$$

where

$$L_1(u; b, e) \equiv \frac{1}{b} \text{III}(u/b) \ast \Pi(u/e)$$

and the asterisk denotes convolution. We make use of Fourier analytic techniques and results that can be found in, for example, Bracewell (1986). Here $b$ is the segment spatial periodicity and $e$ is the intersegment gap width (all pupil-plane dimensions are stated in units of the wavelength of the monochromatic light being considered). The one-dimensional shah or comb function III is defined by

$$\text{III}(u) = \sum_{n=-\infty}^{\infty} \delta(u - n),$$

where $\delta(u)$ is the Dirac delta function. Normalization by $1/b$ is required to ensure the correct “impulse strength” in each delta function in equation (3). The top-hat function $\Pi$ is given by

$$\Pi(x) = \begin{cases} 1, & \text{for } |x| < \frac{1}{2}, \\ 0, & \text{elsewhere}. \end{cases}$$

A scientifically relevant example of a highly segmented aperture coronagraph might be a 30 m telescope with 1 m segment sizes and 5–10 mm gaps, with a hard-edged or apodized occulting spot size of $6\lambda/D$ at the $H$-band central wavelength (1.63 $\mu$m).

3. A SUMMARY OF LYOT CORONOGRAPHY

In order to understand coronagraphy, it is essential to have a thorough understanding of the field strength (i.e., the wave’s complex amplitude) in the Lyot pupil plane. Sivaramakrishnan et al. (2001) discusses the fundamentals of Lyot coronography in detail: here we summarize a few key points that we use in our analysis of segmented-mirror coronagraphy.

Given an aperture illumination function $A$ (no matter whether it is segmented, filled, obstructed, or apodized), the “amplitude-spread function” or ASF (the field strength in the first image plane) is given by the Fourier transform of $A$,

$$|m(\xi, \eta)| = |A(\xi, \eta)|,$$

where $A(\xi, \eta)$ is the complex amplitude in the Lyot pupil plane. Sivaramakrishnan et al. (2001) discusses the fundamental concept of Lyot coronagraphy in detail. Here we summarize a few key points that we use in our analysis of segmented-mirror coronagraphy.

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As a concrete example, let us consider a hard-edged mask 6 $\lambda/D$ radians in diameter, which corresponds to a $w$ that is unity within a disk 6 resolution elements across. $w$, whose domain is the Lyot pupil plane, is the well-known Airy pattern with a scale size of $D/6$. Since $w$ has unit area [because $w(0, 0) = 1$], in the interior of the Lyot pupil the field strength is approximately zero. Various forms of “hard limited” coronagraphs (Kuchner & Traub 2002), which have mask shape functions that are band limited in the Fourier sense [e.g., $w(\xi, \eta) = \text{sinc}^2(\xi D/6) \text{sinc}^2(\eta D/6)$] produce Lyot pupil interior fields that are identically zero on monolithic, filled apertures.

One measure of Lyot coronagraph mask width is the equivalent width of the mask shape function (see, e.g., Bracewell 1986). This is a measure of the occulting power of the mask given a uniform intensity in the image plane. The wider the mask width in image space, the narrower the edge effects are in the Lyot plane, and the larger the zero or low field strength area in the Lyot plane interior is (see Lloyd & Sivaramakrishnan 2005 for further details and a comparison of hard-edged and graded image-plane mask coronagraphs). The distance $D/6$ is the most important length scale in the pupil plane of a Lyot coronagraph.

4. THE LYOT PUPIL PLANE FIELD

We can now estimate how much energy is diffracted into the pupil by segment gaps. In a Lyot coronagraph on a segmented-telescope, the field in the Lyot plane is given by

$$E_{\text{Lyot}} = A_r(x, y) - A_r(x, y) \ast W(x, y),$$

where $A_r(x, y)$ is the complex amplitude in the Lyot pupil plane. Sivaramakrishnan et al. (2001) discusses the fundamental concept of Lyot coronagraphy. Here we summarize a few key points that we use in our analysis of segmented-mirror coronagraphy.

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where \( A \) is the aperture illumination function from equation (1) and \( W \) is the Fourier transform of the mask shape function \( w \), which describes the occulting mask’s departure from complete transparency (see Fig. 1, left). Equation (6) expands to

\[
E_{\text{Lyot}} = A - AL - W \ast A + W \ast (AL).
\]

In the interior of the Lyot plane, the quantity \( A - W \ast A \) (which is the field strength for the coronagraphic ASF of a monolithic mirror telescope with the same overall aperture geometry as our segmented telescope, but without segment gaps) is negligibly small or zero, depending on the coronagraph design. Dropping this term in the interior of the Lyot pupil, we obtain

\[
E_{\text{Lyot\,interior}} = -AL + W \ast (AL).
\]

The equivalent width of the function \( W \) is \( D/s \). For \( D \sim 30 \text{m} \), and a typical segment size \( b \sim 1 \text{m} \), the condition \( D/s \gg b \) holds (in N. Yaitskova & A. Sivaramakrishnan [2005, in preparation], we show that a less restrictive condition, \( D/s \approx 3b \), is in fact good enough for the following calculation to be valid). In the highly segmented telescope case this inequality holds, and the cited work demonstrates that the convolution \( W \ast (AL) \) does not depend on the segmentation geometry, but only on the ratio of the gap width to the segmentation periodicity, \( g = eb \):

\[
W \ast (AL) = 1 - (1 - g)^2 \approx 2g.
\]

An unapodized, unsegmented, undersized Lyot stop with an aperture function \( A(x) \) is used to remove the bright annulus of light on the borders of the Lyot pupil (Fig. 1, middle). We note that \( A / A = A \), that is, the Lyot pupil on this monolithic mirror telescope is contained within the aperture itself.

We also introduce a reticulated mask that blocks out the bright segment gaps in the Lyot pupil (Fig. 1, right). This corresponds to a mask function \( 1 - L \). Our Lyot stop is therefore written as \( A(1 - L) \). The field immediately after our Lyot stop then becomes

\[
E_{\text{Lyot\,stop}} = A(1 - L)[W \ast (AL)] = 2gA(1 - L).
\]

Comparing this equation with the expression in equation (1) for the segmented aperture illumination itself, we note that \( E_{\text{Lyot\,stop}} \) coincides with the complex amplitude for the segmented telescope with the field strength reduced by a factor of \( 2g \) (in the interior of the Lyot stop). If the initial illumination of the aperture is uniform and there are no residual phase errors (i.e., the segments are cophased and wave-front correction by adaptive optics is perfect), we can apply the techniques of Yaitskova et al. (2003) to describe PSF.

5. THE CORONOGRAPHIC PSF

The Lyot field described by equation (10) produces a segmented-telescope coronagraphic ASF

\[
d_{sc} = a' \ast (\delta - l) \ast [w(a \ast l)],
\]

\( a' \) being the ASF corresponding to the unsegmented Lyot stop \( A \), \( \delta \) the two-dimensional Dirac delta function, and \( l \) the Fourier transform of \( L \).

The on-axis coronagraphic image, PSF, (Fig. 2, right), which results from using a reticulated Lyot stop that masks out the segment gaps, is reduced by a factor of \( 4g^2 \) from the PSF of the segmented telescope (Fig. 2, left) on which the Lyot coronagraph is applied. The PSF of the segmented telescope with gaps (Fig. 2, left) contains higher order “satellite” diffraction peaks (in addition to the central dominant peak) located on a grid with an angular periodicity that is inversely proportional to the segment center-to-center separation (i.e., the segment spatial periodicity). In PSF, there is also a broadening of the central and higher order diffraction peaks due to the undersized Lyot stop diameter relative to the original entrance aperture diameter. The intensity of the higher order PSF, diffraction peaks with the reticulated Lyot stop is reduced by a factor of \( 4g^2 \) from those same satellite peaks without the reticulated Lyot stop. The intensity of the central peak of the PSF before the coronagraph (the Strehl ratio, if a monolithic aperture is the reference unaberrated PSF) is \( (1 - g)^2 = 1 - 4g^2 \). Hence the intensity of the central peak after the coronagraph is \( (1 - 4g)4g^2 = 4g^2 \) relative to the monolithic aperture’s PSF peak. For a 10 mm intersegment gap, the value of this expression is \( 4(0.01/1.5)^2 = 2 \times 10^{-4} \). The intensity of the brightest peaks for a hexagonally segmented aperture is \( 0.68g^2 \) relative to the peak of the unsegmented aperture’s PSF.

On a coronagraph with a matched reticulated Lyot stop, this ratio is \( 0.68g^24g^2 = 2.7g^4 \). For the same example as before (a
Fig. 2.—Direct and coronagraphic images (shown on the same logarithmic stretch) from the coronagraph whose Lyot pupil is shown in Fig. 1. Left: Direct image showing a bright core and a grid of satellite images typical of segmented apertures. The satellite images along the principal symmetry axes are brighter than the other satellite images, by a factor of the square of the ratio of the gap width to the segment periodicity. Middle: Coronagraphic image with the simple Lyot stop shown in the middle panel of Fig. 1. The central peak has been suppressed almost completely by the perfect coronagraph. Right: Masking out the bright intersegment gaps in the Lyot pupil (Fig. 1, right) produces significant reduction in the satellite images’ brightness but creates a bright central image. Such a unimodal on-axis PSF is suited to disk studies around adaptive optics targets.

30 m telescope with 1.5 m segments and a 10 mm intersegment gap, $2.7 g^4 = 5 \times 10^{-9}$. Analogous estimates can be made for the diffraction peaks of any order. The intensity around the peaks can also be obtained as the intensity of the PSF for the segmented telescope but, again, reduced by the factor $4g^4$.

The coronagraphic PSF of a segmented telescope is $a_s a_r^s$.

A detailed analysis of the contrast and morphology of coronagraphic PSFs on telescopes with hexagonal segmentation will be presented in N. Yaitskova & A. Sivaramakrishnan (2005, in preparation).

As Figure 2 shows, the coronagraphic PSF with the reticulated Lyot stop has a central core and faint satellite spots. This PSF is suitable for studying extended objects such as protoplanetary disks. In contrast, using a simple Lyot stop creates better suppression of the core of the PSF but transmits the satellite spots caused by primary-mirror segmentation.

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REFERENCES

Aime, C., & Soummer, R. 2003, in Astronomy with High Contrast Imaging, ed. C. Aime & R. Soummer (Les Ulis: EDP Sci.), 79
———. 2004, in Astronomy with High Contrast Imaging II, ed. C. Aime & R. Soummer (Les Ulis: EDP Sci.), 65
Aime, C., Soummer, R., & Lopez, B. 2001, A&A, 370, 680
Angel, J. R. P. 1994, Nature, 368, 203
Bracewell, R. N. 1986, The Fourier Transform and Its Applications (2nd rev. ed.; London: McGraw-Hill)
Chanan, G., & Troy, M. 1999, Appl. Opt., 38, 6642
Diericks, P., & Gilmozzi, R. 2000, Proc. SPIE, 4004, 290
Golimowski, D. A., Nakajima, T., Kulkarni, S. R., & Oppenheimer, B. R. 1995, ApJ, 444, L101
Kuchner, M. J., & Traub, W. A. 2002, ApJ, 570, 900
Lloyd, J. P., & Sivaramakrishnan, A. 2005, ApJ, 621, 1153
Lyot, B. 1939, MNRAS, 99, 580
Makidon, R. B., Sivaramakrishnan, A., Koersko, C. D., Berkefeld, T., Kuchner, M. J., & Winsor, R. S. 2000, Proc. SPIE, 4007, 989
Nakajima, T., Oppenheimer, B. R., Kulkarni, S. R., Golimowski, D. A., Matthews, K., & Durrance, S. T. 1995, Nature, 378, 463
Nelson, J. E. 2000, Proc. SPIE, 4004, 282
Oppenheimer, B. R., Sivaramakrishnan, A., & Makidon, R. B. 2003, in The Future of Small Telescopes in the New Millennium, Vol. 3, ed. T. D. Oswalt (Dordrecht: Kluwer), 155
Sivaramakrishnan, A., Koersko, C. D., Makidon, R. B., Berkefeld, T., & Kuchner, M. J. 2001, ApJ, 552, 397
Sivaramakrishnan, A., et al. 2004, Proc. SPIE, 5490, 535
Yaitskova, N., Dohlen, K., & Dierickx, P. 2003, J. Opt. Soc. Am., 20, 1563