A Compounding Method for Calculating Stress Intensity Factor of Pipelines with MSD Structure

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Abstract. Multiple site damage (MSD) often occurs during the service of the nuclear pipeline. The typical structure is multiple cracks on circumferential internal surface of the pipeline. In order to evaluate the reliability of the nuclear pipeline and discuss the damage tolerance of the pipeline with multiple cracks, a compounding method for calculating stress intensity factor (SIF) of pipelines with MSD structure is proposed. Compared with the finite element results, the compounding method is accurate and reliable. The SIF of double cracks and triple cracks are studied by using crack spacing as single factor variable. The result shows that the SIF of MSD cracks can be influenced by the crack spacing of the adjacent crack.

1. Introduction
With the increase of service life of the nuclear pipeline, nuclear accidents happen frequently. The reliability assessment of safety life of nuclear pipeline has attracted wide attention in the industry [1]. In the past, the research object was limited to single crack in pipeline, however, the multiple site damage (MSD) structure on the inner surface of the pipe is a common form of damage in actual processing or use [2]. As shown in Fig. 1, the $ABC$, $A'B'C'$ and $A''B''C''$ areas are surface crack shapes.

The commonly used methods to calculate MSD stress intensity factor (SIF) are: boundary element method [3], crack growth method and crack closure method [4], finite element method [5]. The surface crack belongs to the category of non-though crack, which is difficult to discuss. Due to its complexity, the analytical result of the surface multiple crack problem can not be solved accurately in most cases. At present, three-dimensional finite element solution is considered as the most accurate SIF solution for surface cracks because it consists well with the experimental results. However, the three-dimensional finite element mesh is very dense, and the calculation work is quite heavy. In this paper, a compounding method for calculating the stress intensity factor of MSD structure of pipeline is proposed, and some changing laws of the SIF in the direction of crack depth (point $A$) in nuclear pipeline are studied.
2. **Compounding Method**

2.1. Calculation function of SIF with multiple cracks constructed by compounding method

The compounding method [6] is an approximate calculation method of MSD stress intensity factor based on the principle of constraint substitution and superposition of theoretical mechanics. Under several boundaries far away from each other, the interaction of each boundary is omitted according to the local effect, and the stress intensity factor of the research object can be obtained.

Taking the nuclear pipeline as an example, which subjects to the axial tensile stress \( \sigma \). The basic situation of MSD problem on the internal surface of the pipeline is a semi-infinite body with a single semi-elliptical crack, which subjects to the tensile load \( \sigma \) in the \( z \) direction, as shown in Fig. 2. The research object is mainly affected by two factors: the pipeline boundary and the adjacent crack. The stress intensity factor of point \( A \) of the crack is

\[
K_A = K_{A0} f_p f_l
\]

Where \( K_{A0} \) is the SIF of the basic situation, \( f_p \) is the SIF correction factor of the pipe boundary, \( f_l \) is the SIF correction factor of the adjacent crack.

By using the multiplicative compounding solution in reverse, the compounding SIF calculation function is obtained

\[
K_A = K_{Ap} f_l
\]

Where \( K_{Ap} \) is the SIF of single crack on the inner surface of the pipe, \( f_l \) is the SIF correction factor of the adjacent crack of MSD semi-infinite body. The structure diagram is shown in Fig. 3.

2.2. Solution of \( KAp \) and \( fl \)

Referring to the European Manual of stress intensity factors [7], the stress intensity factors of crack in depth direction of pipeline are

![Figure 1. MSD structure and loading mode of inner surface of nuclear pipeline.](image1)

![Figure 2. Semi-infinite body with single surface crack.](image2)

![Figure 3. Structure of Influencing Factors on MSD Crack of Pipeline.](image3)
\[ K_i = \sqrt{\pi a} \left( \sum_{j=0}^{3} \sigma_j \left( \frac{a}{l} \right) \frac{R_j}{R_i} + \sigma_{bg} f_{bg} \left( \frac{a}{l} \right) \right) \]  

(3)

Where \( \sigma_i (i=0-3) \) is the normal stress component along the crack surface, \( \sigma_{bg} \) is the axial bending stress, \( f_{bg} \left( \frac{a}{l} \right) \) and \( f_{bg} \left( \frac{a}{l} \right) \) are the corresponding shape correction factor.

By analyzing the crack shape and the loading mode, as shown in Fig. 3(a), the stress intensity factor of the crack \( K_i \) is

\[ K_i = \sqrt{\pi a} \sigma_0 f_0 \left( \frac{a}{l} \right) \frac{R_i}{R_i} \]  

(4)

the shape correction factor \( f_0 \left( \frac{a}{l} \right) \) can be solved by the corresponding analytical formula or obtained by looking up the table according to the size parameters. By plugging \( \sigma_0 = \sigma \) and shape coefficient \( f_0 \), the analytical solution of SIF of cracks can be obtained. The result is the stress intensity factor \( K_{Ap} \) of the pipe with single surface crack

\[ K_{Ap} = \sqrt{\pi a} \sigma_0 f_0 \left( \frac{a}{l} \right) \frac{R_i}{R_i} \]  

(5)

SIF of coplanar double cracks as Fig. 3(b) shown is solved by the 1/4 node method in the finite element method [8]. The hexahedral 20 node isoparametric element is selected to analyze the semi-infinite body with coplanar surface cracks. By changing the crack center distance \( d \), a series of SIF values of point \( A \) at the front edge of the crack are obtained [9]. The normalized SIF value of point \( A \) is the SIF correction factor of the adjacent crack \( f_i \). In order to make the solution of the coefficient more universal, the spacing ratio \( \lambda \) is used as the dependent variable of \( f_i \), \( \lambda = 2a/d \).

Fig. 4 shows the FEM value of the correction coefficient and its fitting curve. By using the polynomial fitting function of Origin software, the finite element fitting solution of the SIF correction factor \( f_i \) can be obtained as follows

\[ f_i = 1.01729 + 0.01722 \lambda + 0.04991 \lambda^2 \]  

(6)
2.3. Accuracy verification of the method
In order to discuss the calculation results, the analysis is carried out under the condition of load \( \sigma = 50 \text{MPa} \), pipe inner diameter \( \sigma = 50 \text{MPa} \) and pipe wall thickness \( t = 10 \text{mm} \). Select the fixed crack \( a = 5 \text{mm}, c = 5 \text{mm} \), and the adjacent crack \( a' = 5 \text{mm}, c' = 5 \text{mm} \). By changing the crack spacing \( l \) (the arc length between \( BC' \) in Fig.1), a series of SIF calculation values are obtained. The solution results are compared with the FEM results in reference [10], as shown in Table 1. The results show that the calculation results are accurate and reliable, which verifies the accuracy of the compounding method.

**Table 1.** Comparison of SIF between calculation results and FEM results at different distance

| \( L \) (mm) | \( K_d \) (MPa·\( \text{mm}^{1/2} \)) | Calculation result | FEM result | Error(%) |
|-------------|---------------------------------|-------------------|------------|---------|
| 15          | 141.3372                        | 136.1203          | 3.8326     |
| 12          | 141.7844                        | 136.6945          | 3.7235     |
| 10          | 142.1881                        | 137.2244          | 3.6172     |
| 7           | 143.0524                        | 138.373           | 3.3817     |
| 5           | 143.9100                        | 139.512           | 3.1524     |
| 3           | 145.1583                        | 141.1413          | 2.8461     |
| 2           | 146.0116                        | 142.3265          | 2.5892     |

3. Analysis of MSD structure on inner surface of pipeline
In the discussion of method accuracy, it has been proved that the interaction of MSD crack stress intensity factors is related to crack spacing. Under the premise that the length and depth of fixed crack and adjacent crack do not change, calculate the stress intensity factor of crack depth direction (point \( A \)) under different crack spacing. The SIF calculation results of double cracks and triple cracks are shown in Fig. 5. and Fig. 6. In each case, a group of crack pairs will get a data line. To make the calculation results show more information, select five groups of crack pairs to get a series of data lines.

**Figure 5.** SIF calculation results of double cracks under different crack spacing.

**Figure 6.** Calculation results of triple cracks under different crack spacing.

4. Conclusion
The stress intensity factors of MSD cracks on the inner surface of pipes can be affected by self cracks and adjacent cracks. When the crack spacing is constant, the stress intensity factor increases with the increase of the number of adjacent cracks. When the crack spacing is a single factor, the SIF in the direction of crack depth decreases with the increase of crack spacing. With the increase of the crack...
spacing, the influence of the crack itself gradually becomes the dominant factor of SIF, while the influence of the adjacent crack will gradually weaken.

Acknowledgments
This work was financially supported by Liaoning Province Doctoral Research Initiation Fund Guidance Project (20170520125); Scientific Research Fund Project of Liaoning Provincial Department of Education (JL-1909).

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