Purely Triplet Seesaw and Leptogenesis within Cosmological Bound, Dark Matter and Vacuum Stability

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ABSTRACT: In a novel standard model extension it has been suggested that, even in the absence of right-handed neutrinos and type-I seesaw, purely triplet leptogenesis leading to baryon asymmetry of the universe can be realised by two heavy Higgs triplets which also provide type-II seesaw ansatz for neutrino masses. In this work we discuss this model predictions for hierarchical neutrino masses in concordance with recently determined cosmological bound and oscillation data including $\theta_{23}$ in the second octant and large Dirac CP phases. We also address the issues on dark matter and vacuum stability of the scalar potential in a minimal extension of this model. We find that for both normal and inverted orderings, the model fits the oscillation data with the sum of the three neutrino masses well below the cosmological bound determined by Planck satellite data. The model also successfully predicts the observed value of baryon asymmetry for lighter triplet mass $M^\Delta_2 = 10^{12}$ GeV and its trilinear coupling $\mu_2 = 6 \times 10^{10} (7.5 \times 10^{10})$ GeV for normal (inverted) ordering. With additional $Z_2$ discrete symmetry, a minimal extension of this model is shown to be capable of predicting a scalar singlet WIMP dark matter in concordance with direct and indirect observations. Whereas in the original model, the renormalization group running of the scalar potential renders it negative for the Higgs field values $|\phi| = 5 \times 10^9 - 10^{13}$ GeV leading to vacuum instability, the presence of this scalar singlet dark matter in the minimally extended model is found to ensure stability. Although the combined constraints due to relic density and direct detection cross section allow this scalar singlet dark matter mass to be $m_\xi = 750$ GeV, the additional vacuum stability constraint pushes this limiting value to $m_\xi = 1.3$ TeV which is verifiable by ongoing experiments.

KEYWORDS: Standard Model Extension, Purely Triplet Seesaw and Leptogenesis, Neutrino Mass, Cosmological Bound, Dark Matter, Vacuum Stability
1 Introduction

Neutrino oscillation [1–3], baryon asymmetry of the universe [4, 5] and dark matter [6] are the three most prominent physics issues which can not be explained within the purview of the standard model (SM). However, seesaw mechanisms have been widely recognised as possible origin of tiny neutrino masses where leptogenesis caused by the decay of associated heavy particles are believed to be the underlying mechanism of baryon asymmetry through sphaleron interactions [7]. Large number of leptogenesis models using type-I seesaw [8–10] or other seesaw mechanisms have been proposed for successful baryon asymmetry generation and only a partial list of such extensive investigations is given in [11, 12]. Since SM itself does not have RHNs among its fermionic representations, it has to be extended to include them for implementation of type-I seesaw. But in a novel interesting proposal against the conventional lore and without using any RHNs, and supersymmetry, realisation of neutrino masses and baryon asymmetry have been also shown to be possible [13] through the SM extension by two heavy Higgs triplets, $\Delta_1$ and $\Delta_2$, each of which generates
neutrino mass by another popular mechanism, called type-II seesaw [14]. As a novel outcome towards necessary quantum correction in the process, the tree level dilepton decay of any one of these triplets combined with loop contribution generated by their collaboration predicts the desired CP-asymmetry formula for leptogenesis leading to observed baryon asymmetry of the Universe (BAU). In an implementation of this leptogenesis idea [13] in non-supersymmetric SM extension in the scalar sector, quasi-degenerate (QD) neutrino masses of order $\sim 1$ eV combined with solution to Boltzmann equation have been used to predict the baryon asymmetry of the Universe $Y_B \simeq 6.11 \times 10^{-10}$. Leptogenesis with or without RHNs has been been implemented including or excluding supersymmetry, or dark matter [12, 15–22]. The QD neutrino mass hypothesis used in [13] has also a very interesting outcome of predicting neutrinoless double beta decay rates saturating the current experimental bounds [23] and/or unifying quark and neutrino mixings at high scales [24]. On the other hand recent Planck satellite data [5] have set the cosmological upper bound on the sum of three light neutrino masses

$$\sum m_i \leq 0.23 \text{ eV} \equiv \Sigma_{\text{Planck}},$$

which is consistent with the standard $\Lambda$CDM big bang cosmology of the universe [5]. Although Planck satellite data [5] also permits a much larger value $\Sigma_C \simeq 0.71$ eV, this latter type of solution has been shown to be possible only in the absence of the cosmological constant ($\Lambda$) which appears to be necessary for the big-bang cosmology of the Universe. Compared to Planck satellite data [5] somewhat lower value of the cosmological bound $\Sigma_{\text{new}} = 0.12$ eV in the $\Lambda$CDM model has been also noted [25]. Sensitivity of non-standard interactions to neutrino masses have been also investigated [26]. As against such cosmological bound of eq.(1.1), KATRIN Collaboration [27] has recently set the upper limit on the neutrino mass scale $m_0 \leq 1$ eV which being 100% improvement over its previous limit ($m_0 \leq 2$ eV) predicts for QD neutrinos

$$\sum m_i \leq 3 \text{ eV} \equiv \Sigma_{\text{KATRIN}}.$$  

For a QD neutrino mass scale as low as $m_0 \simeq 0.2$ eV or heavier, the neutrino would manifest in the direct experimental detection of neutrinoless double beta decay establishing Majorana nature of the particle which has remained elusive so far. In any case it is quite important to investigate the impact of the cosmological bound [5] and the recently measured neutrino oscillation data [1–3] on the two-Higgs triplet seesaw and purely triplet leptogenesis [13].

Quite recently certain new features have been revealed in the neutrino oscillation data [1–3] which have to be explained in any theoretical model. Apart from being consistent with finite $\theta_{13} \sim 8^\circ$, the new data reveal the values on atmospheric neutrino mixing angle to be in the second octant with $\theta_{23} \simeq 49.6^\circ$ and large Dirac CP phase $\delta \sim 214^\circ$. The impact of new cosmological bound [5] or the new oscillation data [1–3] have not been examined on the purely triplet leptogenesis model [13, 28]. This question acquires considerable importance in view of recent observation that the Type-I leptogenesis in popular SO(10) grand unified theory is hardly compatible with $\theta_{23}$ in the second octant [29, 30] whereas type-II seesaw
dominance in SO(10) is capable of providing excellent representation of neutrino data [31] where RHN loop mediated triplet leptogenesis explains the baryon asymmetry of the universe. In particular it has been noted that the type-II seesaw dominant mass matrix elements have one-to-one correspondence with the mass matrix constructed using the most recent oscillation data, a fact which undisputes the importance of type-II over type-I. The model under discussion [13] has the interesting property of predicting two different type-II seesaw mass matrices mediated by the two heavy triplets and has the ability that one of them can dominate over the other. In fact this type-II seesaw dominance property has been utilised in the original model [13] with quasi-degenerate neutrino masses. It is, therefore, quite pertinent to examine whether the seesaw model [13] can fit the current neutrino data [1–3] for hierarchical neutrino masses satisfying the cosmological bound [5] while successfully predicting the observed baryon asymmetry of the universe as before. As a byproduct of this approach we also expect sensitivity of normally ordered (NO) and invertedly ordered (IO) mass hierarchies to the model parameters.

Supersymmetric type-I seesaw leptogenesis with RHN mass scale \( M_N \geq 10^9 \) GeV [32] is known to predict over-production of gravitinos in the early universe affecting relic abundance of light elements. Resolution of gravitino problem [33] in the supersymmetric triplet seesaw model has been discussed [15]. A clear advantage of non-supersymmetric leptogenesis models including [13] is the absence of the gravitino problem ensuring cosmologically safe light elements relic abundance.

As the purely triplet seesaw model [13, 28] does not have dark matter prediction to explain observed relic density and mass bounds determined by direct and indirect detection experiments [34–38] it would enhance the model capabilities if the dark matter phenomena can be accommodated in its simple minimal extension as suggested in this work.

Despite the presence of two heavy Higgs triplets, we note that the renormalisation group running renders the Higgs quartic coupling in the model [13, 28] to acquire negative values in the interval \( |\phi| = (5 \times 10^9 - 10^{13}) \) GeV leading to vacuum instability [39, 40]. Noting that it is a natural compulsion to guarantee vacuum stability of the scalar potential in any of the model applications such as neutrino masses, baryon asymmetry and dark matter, we have shown in this work how the the minimally extended model with scalar singlet dark matter ensures such a stability.

Compared to purely triplet seesaw ansatz [13, 28], in this work we have fitted the most recent neutrino data [1–3] including \( \theta_{23} \) in the second octant and large Dirac CP-phase \( (\delta \simeq 214^\circ) \). We have further exposed the success of the model potential to be compatible with recent cosmological bound from Planck satellite data [5]. For the purpose of fitting neutrino oscillation data and baryon asymmetry prediction, we follow the approach of [13] that enables us to make explicit correlations between physical observables and model parameters.

We have also shown how a simple extension of the original model [13] can account for dark matter and vacuum stability which were lacking in the earlier models [13, 28]. Highlights of the present work are

- The two-Higgs triplet seesaw model [13] is found to fit the most recent neutrino
data including $\theta_{23}$ in the second octant and large Dirac CP-phase $\delta \simeq 214^\circ$ for both normally ordered (NO) and invertedly ordered (IO) neutrino masses.

- The model also accommodates the sum of three neutrino masses significantly less than 0.23 eV of eq. (1.1) in concordance with the current cosmological bound [5]. This analysis further unravels the model potential to confront future improved measurements, if any, over the Planck satellite data.

- Successful leptogenesis leading to observed value of baryon asymmetry is predicted for the lighter triplet mass $M_{\Delta_2} = 10^{12}$ GeV and its trilinear coupling $\mu_2 = 6 \times 10^{10}(7.5 \times 10^{10})$ GeV for NO (IO) ordering whereas the second triplet mass and its trilinear coupling are one order heavier.

- Whereas the original model [13] does not have dark matter (DM), a simple extension of the model is found to predict a real scalar singlet WIMP [41] dark matter in concordance with observed relic density and direct detection measurements which set the lower bound $m_\xi = 750$ GeV.

- This real scalar DM is also found to remove the vacuum instability of the scalar potential existing in the original model.

- When the vacuum stability constraint is combined with those due to relic density and direct detection measurements, this real scalar singlet mass limit is pushed from $m_\xi = 750$ GeV to $m_\xi = 1.3$ GeV which is verifiable by ongoing experiments.

- Using the two-Higgs triplet model [13] and its further simple extension, we have thus successfully addressed four important issues confronting the standard model: neutrino masses and mixings within cosmological bound, baryon asymmetry of the universe, dark matter, and vacuum stability of the scalar potential.

This paper is organised in the following manner. In Sec. 2 we discuss the triplet leptogenesis model [13]. In Sec. 3 we discuss how the current neutrino data is fitted by two-triplet generated type-II seesaw formula with NO or IO masses consistent with cosmological bound where we also derive possible values of the scalar triplet masses and trilinear couplings for leptogenesis. Prediction of baryon asymmetry in the model is discussed in Sec. 4. Extension of the model to accommodate dark matter is discussed in Sec. 5, Sec. 5.1, and Sec. 5.1.1. In Sec. 5.1.3 we discuss the issue of vacuum instability of the scalar potential and its resolution. Our predictions on DM mass is summarized in Sec. 5.1.4. We summarise and conclude the contents of this work in Sec. 6. Relevant Higgs potentials and renormalisation group equations for gauge, scalar and top-quark Yukawa couplings have been discussed in Sec. 8 of the Appendix.

## 2 The Model

Whereas in majority of models the RHNs have been found instrumental in theories of neutrino masses and leptogenesis, in an interesting suggestion, along with type-II seesaw
ansatz for neutrino mass, possible realisation of leptogenesis in multi-triplet extensions of
standard model (without any RHNs) has been proposed in [13]. The minimal number of
additional scalars has been noted to be two triplets $\Delta_1$ and $\Delta_2$ to which we confine in
the present work. In other triplet seesaw and leptogenesis models [17, 28, 31], heavy RHNs are
needed for loop mediation even though the triplet in collaboration with SM Higgs doublet
is capable of explaining neutrino masses. But this model [13] does not need any RHN to
implement both the phenomena: neutrino mass and leptogenesis for successful prediction
of baryon asymmetry of the Universe. The resulting scalar potential in this model has
different terms depending upon the SM Higgs field values $\mu = |\phi|$ as discussed in Sec.5.
The charges of fermions and scalars have been also defined in Sec.5 in Table 2. Thus, in
addition to the usual SM interactions and their modifications, the nonstandard part of the
Lagrangian that contributes to type-II seesaw and leptogenesis are

$$-L_{\text{ext}} = \sum_{k=1}^{2} \left( (D_\mu \bar{\Delta}_k) \cdot (D^\mu \Delta_k) - M_{\Delta_k}^2 Tr(\Delta_k^\dagger \Delta_k) + \frac{1}{2} y_{ij}^{(k)} L_i^T C i \tau_2 \Delta_k L_j - \mu_k \phi^T i \tau_2 \Delta_k \phi + h.c. \right).$$

Here $i,j = 1,2,3$ denote the three lepton flavors represented by the lepton doublets $L_i$ but
$k = 1,2$ denote the two scalar triplets. $M_{\Delta_k}$ is the mass of the triplet $\Delta_k$, $y_{ij}^{(k)}$ is Majorana
coupling of $\Delta_k$ with $L_i$ and $L_j$ and $\mu_k$ is trilinear coupling of $\Delta_k$ with standard Higgs
doublet $\phi$.

Defining the induced triplet VEVs $V_{Lk}, k = 1,2$

$$V_{Lk} = -2v^2 \frac{\mu_k}{M_{\Delta_k}^2}, (k = 1,2)$$

the formula for neutrino mass matrix $m_\nu$ is

$$m_\nu = y^{(1)} V_{L1} + y^{(2)} V_{L2},$$

$$\equiv m_\nu^{(1)} + m_\nu^{(2)}.$$

Here $v = 174$ GeV, the standard Higgs vacuum expectation value (VEV). The Feynman
diagram for CP-asymmetry generation is shown in Fig.1.

The CP asymmetry formula is [13]

$$\epsilon_i = \frac{\text{Im} \left[ \mu_1 \mu_2^* \sum y_{nl}^{(1)} y_{nl}^{(2)*} \right]}{8\pi^2 \left( M_{\Delta_1}^2 - M_{\Delta_2}^2 \right) |\Gamma_i|},$$

The following set of parameters have been used for the model predictions [13]

$$y^{(1)} = 1, y^{(2)} = 0.1, |\mu_1| = 10^{13}\text{GeV}, |\mu_2| = 2 \times 10^{12}\text{GeV},$$

$$M_{\Delta_1} = 3 \times 10^{13}\text{GeV}, M_{\Delta_2} = 10^{13}\text{GeV},$$

consistent with QD neutrino mass eigen values

$$m_1 \sim m_2 \sim m_3 \sim 1.2\text{eV}$$
This choice leads to an interesting prediction for observable neutrinoless double beta decay close to the current experimental limit \[ m_0 < 0.1 \text{ eV}. \] However, recent estimations derived from cosmological measurements appear to constrain the QD spectrum considerably \[ \text{[5]} \]. As stated through eq.(1.1) the Planck satellite data limits \[ \text{[5]} \] the sum of the three neutrino masses to be \( \leq 0.23 \text{ eV}. \) In addition, the recent neutrino data has revealed certain significant interesting changes over previous results with the atmospheric neutrino mixing angle in the second octant \( \theta_{23} \geq 45^\circ \) and the Dirac CP phase \( \delta \approx 214^\circ \). Moreover, as the Type-I seesaw and leptogenesis does not seem to be compatible with these features of the recent neutrino data \[ \text{[29, 30]} \], it is necessary to investigate capabilities of type-II seesaw fit and triplet leptogenesis. Success of a class of type-II seesaw dominant models but with RHN loop mediated triplet leptogenesis has been investigated along with predictions of new CP-asymmetry formulas in concordance with the recent oscillation data \[ \text{[31]} \]. It is thus quite important to examine whether purely triplet seesaw and leptogenesis predictions without RHNs could be compatible with the recent data, cosmological bound \[ \text{[5]} \], baryon asymmetry \[ \text{[5]} \] and dark matter while ensuring vacuum stability of the scalar potential.

For this purpose we note that eq.(2.4) can be written as

\[
\epsilon_i = \frac{M_{\Delta_1}^2 M_{\Delta_2}^2 \text{Im} \left[ \sum m_{\nu,m}^{(1)} m_{\nu,n}^{(2)*} \right]}{8\pi^2 v^4 \left( M_{\Delta_1}^2 - M_{\Delta_2}^2 \right)} |M_{\Delta_i}|, \tag{2.7}
\]

In order to fit the recent neutrino data by the present formulation we use the approximation that the type-II seesaw formula generated by lighter of the two triplets with \( M_{\Delta_2} \ll M_{\Delta_1} \).
has dominant contribution.

\[ m_\nu \simeq m_\nu^{(2)} = m_\nu^{(DATA)} \]  

(2.8)

In order to ensure this dominance we assume

\[ |m_\nu^{(1)}| = \frac{1}{10} |m_\nu^{(2)}|, \]

\[ F = \frac{|m_\nu^{(1)}|}{|m_\nu^{(2)}|} = \frac{1}{10}. \]

(2.9)

Under this approximation the factor responsible for imaginary part in eq.(2.7) is

\[ \text{Im} \left[ \sum m_{\nu,\nu,nl}^{(1)} \right] = \frac{1}{F} \sum_{nl} \left[ \sin(\phi_{nl}^{(1)} - \phi_{nl}^{(2)}) \right] |m_{\nu,nl}|^2, \]

(2.10)

In the right-hand side (RHS) of eq.(2.10), the modulus square of every matrix element

\[ |m_{\nu,nl}^{(2)}|^2 \simeq |m_{\nu,nl}|^2 \]

and the corresponding value of phase \( \phi_{nl} \) is expected to be determined from the current neutrino data. But the unknown phase \( \phi_{nl}^{(1)} \) of \( m_{\nu,nl}^{(1)} \) is left unconstrained.

For the sake of simplicity in order to predict maximal CP-asymmetry, using similar approximation made between \( \mu_1 \) and \( \mu_2^* \) in [13], we equivalently presume maximal phase difference

\[ \phi_{nl}^{(1)} - \phi_{nl}^{(2)} = \pi/2 \quad (n, l = 1, 2, 3). \]

(2.11)

In other words eq.(2.11) can predict the unknown phases of every element of \( m_\nu^{(1)} \) from the value of phases determined from neutrino data while its modulus is governed by eq.(2.9). With the subdominant part thus determined, the \( m_\nu^{(2)} \) dominance hypothesis can be verified using neutrino data. Using eq.(2.7), eq.(2.8),eq.(2.9),eq.(2.10) and eq.(2.11) in eq.(2.7) gives a simplified form

\[ \epsilon_i = \frac{M_\Delta_2^2}{8\pi^2v^4F} \frac{\sum |m_{\nu,nl}|^2}{M_\Delta_2^2 - M_\Delta_1^2} \frac{M_\Delta_2}{\Gamma}, \]

(2.12)

where the neutrino matrix \( m_\nu \) in the above equation represents actual neutrino data due to \( m_\nu^{(2)} \) dominance over \( m_\nu^{(1)} \). Further simplification of this formula is discussed below following neutrino data fitting.

For the purpose of deriving analytic expression for baryon asymmetry, the following parameters are defined [13]: \( x = M_\Delta_2/T \), \( Y_i = n_i/s(i = 1, 2) \) = the triplet scalar number densities per unit entropy, \( H = 1.66\sqrt{(g_*)T^2/M_{Planck}} \) = the Hubble parameter, \( t = x^2/2H(x = 1) \), and \( K = \Gamma_2(x = 1)/H(x = 1) \) = the factor responsible for deviation of lepton asymmetry from its equilibrium value. It has been shown [13] that for \( K \ll 1 \) and \( T \simeq M_2 \), the relevant Boltzmann equations are

\[ \frac{dY_1}{dx} = (Y_2 - Y_1^{eq})\epsilon_2 K x, \]

\[ \frac{dY_2}{dx} = (Y_2 - Y_2^{eq})K x, \]

(2.13)
which yield an asymptotic solution for \( Y_2 = \epsilon_2/g_* \). But for \( K \gg 1 \) this solution is suppressed compared to its nearly equilibrium value

\[
Y_B \equiv Y_2 \simeq \frac{\epsilon_2}{3g_*K(\ln K)^{0.6}}.
\]  

(2.14)

Using eq.(2.12) and eq.(2.14) gives

\[
Y_B = \frac{M^2_{\Delta_1} M^2_{\Delta_2}}{24\pi^2 F v^4 g_*^{-1}} \frac{|m_{\nu,m}|^2}{K(\ln K)^{0.6}} \frac{M_{\Delta_2}}{\Gamma^2}.
\]

(2.15)

Although baryon asymmetry prediction of this model has been also discussed as a part of an extensive investigation covering other models [28], our present work also illustrates this model capability to address fits to the recent neutrino data within cosmological bound. Moreover, we have also shown here how the simplest extension of the model [13] can answer issues on dark matter and vacuum stability not shown in any previous work.

3 Model Fitting of Neutrino Data within Cosmological Bound

In this section at first we show the model capability to fit the most recent neutrino data satisfying the constraint imposed by cosmological bound [5]. Using the PDG convention [43] we parameterize the PMNS mixing matrix

\[
U_{\text{PMNS}} = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \text{diag}(e^{i\alpha_M}, e^{i\beta_M}, 1)
\]

(3.1)

where \( s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij} \) with \( (i, j = 1, 2, 3) \), \( \delta \) is the Dirac CP phase and \( (\alpha_M, \beta_M) \) are Majorana phases. We use the best fit values of the oscillation data [2, 3] as summarised below in Table 1.

Table 1. Input data from neutrino oscillation experiments [2, 3]

| Quantity | best fit values | 3σ ranges |
|----------|----------------|-----------|
| \( \Delta m_{21}^2 \) [10^{-5}eV^2] | 7.39 | 6.79 – 8.01 |
| \( |\Delta m_{31}^2| \) [10^{-3}eV^2](NO) | 2.52 | 2.427 – 2.625 |
| \( |\Delta m_{32}^2| \) [10^{-3}eV^2](IO) | 2.51 | 2.412 – 2.611 |
| \( \theta_{12}/^\circ \) | 33.82 | 31.61 – 36.27 |
| \( \theta_{23}/^\circ (NO) \) | 49.6 | 40.3 – 52.4 |
| \( \theta_{23}/^\circ (IO) \) | 49.8 | 40.6 – 52.5 |
| \( \theta_{13}/^\circ (NO) \) | 8.61 | 8.22 – 8.99 |
| \( \theta_{13}/^\circ (IO) \) | 8.65 | 8.27 – 9.03 |
| \( \delta/^\circ (NO) \) | 215 | 125 – 392 |
| \( \delta/^\circ (IO) \) | 284 | 196 – 360 |
Important among new interesting salient features of this set of data points are (i) the best fit value of atmospheric mixing angle $\theta_{23}$ is in the second octant, (ii) large values of Dirac CP phases exceeding $\delta = 200^\circ$, and (iii) the reactor neutrino mixing angle is consistent with $\theta_{13} = 8.6^\circ$.

Using the mass-squared differences from Table 1 and choosing the lightest mass eigen value $m_1 = 0.001$ eV, we at first determine the other two mass eigen values. Then using the three mass eigen values, mixing angles and phases given in Table 1 we derive neutrino mass matrix consistent with best fit to the data through the standard relation

$$m_{\nu} = U_{PMNS} \text{diag}(m_1, m_2, m_3)U_{PMNS}^T$$ (3.2)

For NO and IO cases we get the following results:

**Normal Ordering (NO):**

$$m_1 = 0.001\text{ eV}, m_2 = 0.0086\text{ eV}, m_3 = 0.0502\text{ eV},$$

$$\sum_i m_i = 0.0598\text{ eV} \ll \Sigma_{Planck}.$$ (3.3)

where $\Sigma_{Planck} = 0.23$ eV is the Planck satellite value of the cosmological bound [5] given in eq.(1.1). We thus find that the present neutrino data predicts the sum of the three neutrino masses to be nearly 25% of the cosmological bound [5] in the NO case.

$$m_{\nu}^{NO}(\text{eV}) = \begin{pmatrix} 0.00367 - 0.00105i & -0.00205 + 0.00346i & -0.00634 + 0.00294i \\ -0.00205 + 0.00346i & 0.03154 + 0.00034i & 0.02106 - 0.0001i \\ -0.00634 + 0.00294i & 0.02106 - 0.0001i & 0.02383 - 0.00027i \end{pmatrix}.$$ (3.4)

This gives

$$\sum_i |m_{\nu,ni}^{NO}|^2 = 2.595 \times 10^{-3}\text{eV}^2.$$ (3.5)

Its close vicinity with $\Delta m_{31}^2$ value of Table 1 in the NO case is noteworthy.

**Inverted Ordering (IO):**

$$m_1 = 0.04938\text{ eV}, m_2 = 0.0501\text{ eV}, m_3 = 0.001\text{ eV},$$

$$\sum_i m_i = 0.060\text{ eV} < \Sigma_{Planck}.$$ (3.6)

where $\Sigma_{Planck}$ is the Planck data [5] of the cosmological bound given in eq.(1.1). It is clear that in the IO case also the sum of three neutrino masses is nearly 25% of the Planck bound [5].

$$m_{\nu}^{IO}(\text{eV}) = \begin{pmatrix} 0.0484 - 0.00001i & -0.001122 + 0.0055i & -0.00137 + 0.00471i \\ -0.001122 + 0.0055i & 0.02075 - 0.00025i & -0.02459 - 0.00026i \\ -0.00137 + 0.00471i & -0.02459 - 0.00026i & 0.02910 - 0.00026i \end{pmatrix}.$$ (3.7)
The manifestly hierarchical nature of mass eigen values are evident from eq.(3.3) and eq.(3.6). This gives
\[ \sum |m_{\nu,nl}^{\text{IO}}|^2 = 4.9 \times 10^{-3} \text{ eV}^2 \] (3.8)

which is nearly 2 times larger than the \( \Delta m_{32}^2 \) value of Table 1 in the IO case. In both the NO and IO cases, the sum of the three neutrino masses are also consistent with the upper bound 0.12 eV [25].

4 Estimation of Baryon Asymmetry

Currently the standard approach towards understanding baryon asymmetry of the Universe (BAU) requires fulfillment of Sakharov[44] conditions: (i) baryon number violation, (ii) C and CP violations, and (iii) departure from thermal equilibrium. Baryon asymmetry is defined as
\[ Y_B = \frac{n_B - n_{\overline{B}}}{s}. \] (4.1)

where \( n_B, n_{\overline{B}} \) are number densities of baryons and anti-baryons, respectively, and \( s \) is the entropy density. Another equivalent definition of BAU is
\[ \eta_B = \frac{n_B - n_{\overline{B}}}{n_\gamma}. \] (4.2)

where \( n_\gamma = \text{photon density} \). Planck satellite experimental values are[5]
\[ Y_B = 8.66 \pm 0.11 \times 10^{-11}, \] (4.3)
\[ \log_{10}(Y_B) = [-10.027, -10.086]. \] (4.4)

At first noting that the total decay width of \( \Delta_2 \) is the sum of its partial width \( (\Gamma_2^\phi) \) due to \( \Delta_2 \to \phi^\dagger \phi \) and all other widths \( (\Gamma_2^L) \) due to the dileptonic modes \( \Delta_2 \to l \bar{l} \)
\[ \Gamma_2 = \Gamma_2^\phi + \Gamma_2^L, \] (4.5)
\[ \Gamma_2^\phi = \frac{\mu_2^2}{8\pi M_{\Delta_2}}, \] (4.6)
\[ \Gamma_2^L = \frac{M_{\Delta_2} \sum_{n,l} |y_{nl}^{(2)}|^2}{8\pi}. \] (4.7)

Using these relations in the K-parameter contributing to suppression of BAU, we get
\[ K = \frac{\Gamma_2}{H(x = 1)} \]
\[ = \frac{M_{\text{Planck}}}{1.66g_s^{1/2}8\pi M_{\Delta_2}} \left[ \frac{\mu_2^2/M_{\Delta_2}^2 + \sum_{n,l} |y_{nl}|^2}{\sum_{n,l} |y_{nl}|^2} \right]. \] (4.8)

In the NO and IO cases in order to determine \( \sum_{n,l} |y_{nl}|^2 \) we at first determine the corresponding Majorana type dilepton Yukawa couplings from the neutrino mass matrices of eq.(3.4) and eq.(3.7)
\[ y_{nl} \simeq y_{nl}^{(2)} \simeq m_{\nu,nl}/VL2 \] (4.9)
With $VL2 = 3.6$ eV, the dominant factors in the NO case are $|y_{33}|^2 = 4.3 \times 10^{-5}$, $|y_{22}|^2 = 7.7 \times 10^{-5}$, $|y_{23}|^2 = |y_{32}|^2 = 3.3 \times 10^{-5}$ leading to

$$\sum_{nl} |y_{nl}|^2 \simeq 10^{-5} \quad (4.10)$$

On the other hand $M_{\Delta_2} = 10^{12}$ GeV and $\mu_2 = 6 \times 10^{10}$ GeV gives

$$\frac{\mu_2^2}{M_{\Delta_2}^2} = 3.6 \times 10^{-3} \quad (4.11)$$

Comparing eq.(4.10) and eq.(4.11) we find that the first term in eq.(4.8) dominates over the second term. Then, up to a good approximation, an analytic expression for $K$

$$K = \frac{\mu_2^2 M_{\text{Planck}}}{8\pi 1.66 g_*^{1/2} M_{\Delta_2}^3} \quad (4.12)$$

Then using eq.(4.12) and the mass hierarchy $M_{\Delta_1} \gg M_{\Delta_2}$ in eq.(2.15) gives

$$Y_B = \frac{3.32 M_{\Delta_2}^2 \sum_{nl}|m_{\nu, nl}|^2}{3 F g_4^{1/2} v^4 M_{\text{Planck}} \mu_2^2 (\ln K)^{1/2}} \quad (4.13)$$

We have further checked the validity of eq.(4.12) in the IO case leading to the approximate analytic formula of eq.(4.13).

Through the explicit dependence of $Y_B$ on $\Delta_2$ mass $M_{\Delta_2}$ and its lepton number violating coupling $\mu_2$, this formula in eq.(4.13) is consistent with the well known hypothesis that leptogenesis generated by the heavier triplet ($\Delta_1$) is erased due to the lighter ($\Delta_2$)-decay that controls the baryon asymmetry of the universe[13]. For fixed $M_{\Delta_2}$ it predicts decrease of BAU with increasing value of $\mu_2$. Since

Using our numerical estimations of $|\sum m_{\nu, nl}|^2$ given in eq.(3.5) and eq.(3.8) for the NO and IO type neutrino masses, respectively, in eq.(4.13) we now predict the variation of baryon asymmetry prediction as a function of $|\mu_2|$ for fixed $M_{\Delta_2} = 10^{12}$ GeV. We have plotted the predicted value of $Y_B$ as a function of $\mu_2$ in the NO and IO cases as shown in Fig.2 where the Planck satellite data have been shown by the the horizontal band.

For $M_{\Delta_2} = 10^{12}$ GeV, the model predicts the observed BAU $Y_B = 8.6 \times 10^{-11}$ in the NO case for $\mu_2 = 6.0 \times 10^{12}$ GeV and for $\mu_2 = 7.5 \times 10^{10}$ GeV in the IO case.

Thus, we find that the two Higgs triplet seesaw model [13] can account for the most recent neutrino data including $\theta_{23}$ in the second octant and large Dirac CP-phase in concordance with cosmological bound [5] for normal ordering (NO) as well as inverted ordering (IO) of mass hierarchies. It is quite significant to note that this type-II seesaw model is capable of accommodating the sum of three neutrino masses significantly less than the Planck bound [5] revealing the model potential to confront possible future improvements on the cosmological bound measurements consistent with $\Lambda$CDM theory of the Universe [5, 25]. Also the model successfully explains the Planck satellite data on baryon asymmetry for $\Delta_2$. 

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mass and triplet scalar coupling values nearly one order lighter than the previous estimations [13]. From different values of of $\mu_2 = 6 \times 10^{10}$ GeV in the NO case and $\mu_2 = 7.5 \times 10^{10}$ GeV in the IO case, we further note that the NO or IO mass hierarchy is also able to differentiate in triplet Higgs coupling values up to 25%. We further note that, instead of using the approximate expression for $K$ as in eq.(4.12), if we also include contribution due to leptonic decay width of $\Delta_2$ in eq.(4.8), then numerical estimation predicts similar values $M_{\Delta_2} \simeq 10^{12}$ GeV and $\mu_2 \simeq (6 - 9) \times 10^{10}$ GeV while successfully matching the observed value of the baryon asymmetry [5] of the Universe.

5 Minimal Model Extension for Dark Matter and Vacuum Stability

The inert scalar doublet model has radiative seesaw ansatz for neutrino masses and intrinsic capability for dark matter [45] which has been also shown to originate from SO(10) [46] with matter parity [47–49] as the stabilising discrete symmetry. More recently new possible...
origin of scotogenic dark matter stability has been also suggested from softly broken global lepton number symmetry $U(1)_L$ [50]. This inert doublet model [45] also does not have vacuum instability problem in the associated scalar potential. But the two heavy Higgs scalar triplet model [13] (or the purely triplet seesaw model [28]), as such, does not possess dark matter through which it can explain cosmological evidences including the observed relic density ($\Omega_{DM}h^2 = 0.1172 - 0.1224$) [5, 6, 38]. The expected DM mass has been also bounded from direct and indirect detection experiments [34–37]. This issue has been also addressed in a number of ways in SM extensions through a singlet scalar representing a weakly interacting massive particle (WIMP)[41] as DM candidate and the investigations have been also updated more recently in [42]. But most of the models discussed in [42] and earlier have not addressed neutrino oscillation data, cosmological bound, and baryon asymmetry via leptogenesis. Also they have not addressed the issue on the vacuum stability of the associated scalar potential [39, 40]. Using corresponding renormalisation group evolutions (RGEs) discussed in the Appendix we find that in the two heavy Higgs triplet model [13] with $M_{\Delta_i}(i = 1, 2) \geq 10^{13}$ GeV, the instability problem has been considerably improved by predicting the Higgs quartic coupling $\lambda_\phi$ to be positive in an extended region with $|\phi| \leq 5 \times 10^9$ GeV and $|\phi| \geq 10^{13}$ GeV. However, we note that in this model [13] the standard Higgs quartic coupling $\lambda_\phi$ runs negative in the interval $|\phi| \simeq 10^{10} - 10^{13}$ GeV showing the persistence of vacuum instability of the scalar potential [39, 40]. In this section we discuss how the heavy Higgs triplet model that accounts for neutrino mass and baryon asymmetry as discussed above can also be easily extended further to account for the phenomena of WIMP DM while completing vacuum stability through the same scalar DM. We add a real scalar singlet $\xi$ to the two Higgs triplet model [13] and assume an additional $Z_2$ discrete symmetry under which $\xi$ and all SM fermions are odd. All other Higgs scalars including the SM Higgs $\phi$ and the two triplets are assumed to possess $Z_2 = +1$. Thus the resulting Lagrangian after this real scalar extension has the symmetry $SU(2)_L \times U(1)_Y \times SU(3)_C \times Z_2(\equiv G_{213} \times Z_2)$. The particle content and their charges in the minimally extended model under this symmetry are shown in Table 2.

5.1 Real Scalar Singlet Dark Matter

At first noting that in [13] at lower mass scale $\mu \ll M_{\Delta_i}(i = 1, 2)$, the two heavy Higgs triplets are expected to have decoupled leading effectively to the SM scalar potential, $\mu \ll M_{\Delta_i}(i = 1, 2)$:

$$V_{SM} = -\mu^2_H\phi^\dagger\phi + \lambda_\phi(\phi^\dagger\phi)^2.$$  (5.1)

It is well known that this SM potential alone develops vacuum instability as the quartic coupling $\lambda_\phi$ runs negative at energy scales $\mu \geq 5 \times 10^9$ GeV [39, 40]. In models with type-I seesaw extensions of the SM, the negativity of $\lambda_\phi$ is further enhanced due to RHN Yukawa interactions. This latter type of enhancement due to RHN is absent in the purely triplet leptogenesis model [13]. Using renormalisation group equations discussed in the Appendix we find that the SM Higgs quartic coupling remains positive for field values $|\phi| \leq 5 \times 10^9$ GeV and $|\phi| \geq 10^{13}$ GeV where the latter limit is due to $M_{\Delta_2} = 10^{13}$ GeV in [13]. Although
Table 2. Singlet scalar extensions of the two Higgs triplet model [13] and its particle content with respective charges under $G_{213} \times Z_2$. Although only first generation fermions have been specified, the second and the third generation fermions have identical transformation properties.

| Particle | SM charges | $Z_2$ charge |
|----------|------------|--------------|
| $(\nu, e)_L^T$ | $(2, -1/2, 1)$ | $-1$ |
| $e_R$ | $(1, -1, 1)$ | $-1$ |
| $(u, d)_L^T$ | $(2, 1/6, 3)$ | $-1$ |
| $u_R$ | $(1, 2/3, 3)$ | $-1$ |
| $d_R$ | $(1, -1/3, 3)$ | $-1$ |
| $\phi$ | $(2, 1/2, 1)$ | $+1$ |
| $\Delta_1$ | $(3, -1, 1)$ | $+1$ |
| $\Delta_2$ | $(3, -1, 1)$ | $+1$ |
| $\xi$ | $(1, 0, 1)$ | $-1$ |

such positive values of Higgs quartic coupling is a considerable improvement over purely SM running, the model [13] does not resolve the vacuum insatbility issue. This is due to the fact that standard Higgs quartic coupling in the model [13] acquires negative values in the region $|\phi| \simeq 5 \times 10^9$ GeV to $|\phi| \simeq 10^{13}$ GeV. Further details of discussion of this problem has been made below in Sec. 5.1.3.

In order to resolve both the issues on DM and vacuum stability of the scalar potential we make a simple extension of the model [13] by adding a real scalar singlet $\xi$ whose mass we determine from DM relic density, direct detection experimental bounds and vacuum stability fits. For the stability of DM we impose a $Z_2$ discrete symmetry under which $\xi$ and all SM fermions are odd, but all other scalars in the extended model including the triplets are even under $Z_2$ as shown in Table 2. The standard model scalar potential is modified in the presence of $\xi$

$$V_\xi = V_SM + \mu_\xi^2 \xi^2 + \lambda_\phi \xi^4 + 2\lambda_{\phi\xi}(\phi^\dagger \phi)\xi^2.$$  \hspace{1cm} (5.2)

In eq.(5.2) $\lambda_\xi = $ dark matter self-coupling, $\lambda_{\phi\xi} =$ Higgs portal coupling and $\mu_\xi =$ mass of $\xi$. The VEV of the standard Higgs doublet redefines the DM mass parameter

$$M_{DM}^2 = 2(\mu_\xi^2 + \lambda_{\phi\xi}^2 v^2),$$
$$m_\phi^2 = 2\mu_H^2 = 2\lambda_\phi v^2.$$

For mass scales $\mu \geq M_{\Delta_2}$ the Higgs potential receives additional contributions due to $\Delta_i (i = 1, 2$) and its interactions with $\phi$ and $\xi$

$$\mu \geq M_{\Delta_2}:$$
\[ V_{\xi\Delta} = V_{\xi} + \sum_{i=1,2} \left( M_{\Delta_i}^2 \text{Tr}(\Delta_i^\dagger \Delta_i) + \lambda_{\xi}^1 \left[ \text{Tr}(\Delta_i^\dagger \Delta_i) \right]^2 + \lambda_{\xi}^2 \left[ \text{Tr}(\Delta_i^\dagger \Delta_i) \right]^2 - \text{Tr}[\Delta_i^\dagger \Delta_i]^2 \right) \]
\[ + \sum_{i=1,2} \left( \lambda_{\phi\xi}^1 (\phi^\dagger \phi) \text{Tr}(\Delta_i^\dagger \Delta_i) + \lambda_{\phi\xi}^2 [\Delta_i^\dagger \Delta_i - (\Delta_i \Delta_i^\dagger)] \phi + \left[ \frac{\mu_i}{\sqrt{2}} \phi^T i \tau_2 \Delta_i^\dagger \phi + H.c. \right] \right). \]

(5.4)

where \( V_{\xi} \) has been defined in eq.(5.2).

In order to examine the allowed values of the the Higgs portal coupling \( \lambda_{\phi\xi} \) we use two different kinds of experimental results: (i)bounds on cosmological DM relic density \([5, 38]\) \( \Omega_{DM} h^2 = 0.1172 - 0.1224 \), (ii)bounds from DM direct detection experiments such as LUX-2016\([34]\), XENON1T\([35, 36]\) and PANDA-X-II\([37]\). Using our ansatz we estimate the relic densities for different combinations of \( m_{\xi}, \lambda_{\phi\xi} \). It is then easy to restrict the values of \( m_{\xi} \) and \( \lambda_{\phi\xi} \) using the bound on relic density mentioned above. In direct detection experiments it is assumed that WIMPs passing through earth scatter elastically from the target material of the detector. The energy transfer to the detector nuclei can be measured through various types of signals. All those direct detection experiments provide DM mass vs DM-nucleon scattering cross section plot which clearly separates the allowed regions below the predicted curve from the forbidden regions above the curve.

### 5.1.1 Estimation of Dark Matter Relic Density

We assume the WIMP DM particle \( \xi \) to have decoupled from the thermal bath at some early epoch which has thus remained as a thermal relic. The following conventions are used at a certain stage of evolution of the Universe. Denoting \( \Gamma = \) particle decay rate and \( H = \) Hubble parameter, a particle species is said to be coupled if \( \Gamma > H \). Similarly it is assumed to have decoupled if \( \Gamma < H \). The corresponding Boltzmann equation\([51, 52]\) are solved for the estimation of the particle relic density

\[ \frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{eq}^2) \]

(5.5)

where \( n = \) actual number density of \( \xi \) at a certain instant of time and \( n_{eq} = \) its equilibrium number density. We denote \( v_0 = \) velocity and \( \langle \sigma v_0 \rangle = \) thermally averaged annihilation cross section. Approximate solution of Boltzmann equation gives the expression for the relic density\([52, 53]\)

\[ \Omega_{DM} h^2 = \frac{1.07 \times 10^9 x_F}{\sqrt{g_* M_{pl}} \langle \sigma v_0 \rangle} \]

(5.6)

where \( x_F = \frac{m_{\xi}}{T_F} ; T_F = \) freezeout temperature, \( g_* = \) effective number of massless degrees of freedom and \( M_{pl} = 1.22 \times 10^{19} \) GeV. This \( x_F \) can be computed by iteratively solving the equation

\[ x_F = \ln \left( \frac{m_{\xi}}{2\pi^2} \sqrt{\frac{45 M_{pl}^2}{8 g_* x_F \langle \sigma v_0 \rangle}} \right). \]

(5.7)
In eq.(5.6) and eq.(5.7), the only particle physics input is the thermally averaged annihilation cross section. The total annihilation cross section is obtained by summing over all the annihilation channels of the singlet DM which are $\xi \xi \rightarrow F\bar{F}, W^+ W^-, ZZ, hh$ where the symbol $F$ represents all the associated fermions of SM. Using the expression of total annihilation cross section\[54–56\] in eq.(5.7) at first we compute $x_F$ which is then utilised in eq.(5.6) to yield the relic density. Two free parameters involved in this computation are mass of the DM particle $m_\xi$ and the Higgs portal coupling $\lambda_{\phi \xi}$. The relic density has been estimated for a wide range of values of the DM matter mass ranging from few GeVs to few TeVs while the coupling $\lambda_{\phi \xi}$ is also varied simultaneously in the range $(10^{-4} – 1)$. The parameter space $(m_\xi, \lambda_{\phi \xi})$ is thus constrained by using the bound on the relic density reported by WMAP \[38\] and Planck \[5\]. In Fig.3 we show only those combinations of $\lambda_{\phi \xi}$ and $m_\xi$ which are capable of producing relic density in the experimentally observed range.

### 5.1.2 Dark Matter Mass Bounds from Direct Detection Experiments

We get exclusion plots of DM-nucleon scattering cross section and DM mass from different direct detection experiments. The spin independent scattering cross section of singlet DM on nucleon is given by \[57\]

$$\sigma_{SI} = \frac{f_n^2 \lambda_{\phi \xi}^2 \mu_R^2 m_N^2}{4 \pi m_\xi^2 m_h^2} \text{ (cm}^2)$$

(5.8)

where $m_h =$ mass of the SM Higgs ($\sim 125$ GeV), $m_N =$ nucleon mass $\sim 939$ MeV, $\mu_R = (m_\xi m_N)/(m_\xi + m_N) =$ reduced DM-nucleon mass and the factor $f_n \sim 0.3$. Using eq.(5.8) the exclusion plots in the $\sigma - m_\xi$ plane can be easily brought to $\lambda_{\phi \xi} - m_\xi$ plane. We superimpose the $\lambda_{\phi \xi}$ vs. $m_\xi$ plots for different experiments on the plot of allowed parameter space constrained by relic density bound resulting in Fig.3. Thus the Fig.?? exhibits the parameter space ($\lambda_{\phi \xi}$ vs. $m_\xi$) constrained by both the relic density bound and the direct detection experiments.

From Fig.3 we note that the points on the yellow curve lying below the green band are allowed by both relic density and direct detection experiments. This predicts lower values of DM mass at in the region $m_\xi \simeq 59 - 63$ GeV for the Higgs portal coupling $\lambda_{\phi \xi} \leq 10^{-3}$ that is too small to be compatible with vacuum stability as discussed below in Sec.5.1.3.

All values of DM masses $m_\xi \geq 750$ GeV are also allowed by relic density and direct detection experimental constrains. But as discussed below this region will be further constrained by vacuum stability criteria.

### 5.1.3 Resolution of Vacuum Instability

We have examined vacuum stability of different scalar potentials encountered in different regions of Higgs field value $\mu = |\phi|$ starting from $\mu = m_{\text{top}} - M_{\text{Planck}}$ though the renormalisation group evolutions (RGEs) of the standard Higgs ($\phi$) quartic coupling in the respective cases \[58–61\] which have been given in Sec.8 of the Appendix. At first using RGEs for Higgs quartic coupling $\lambda_\phi$ and gauge and top quark Yukawa couplings for the SM alone in the absence of DM $\xi$ or heavy triplets we have plotted the quartic coupling against standard Higgs field values $\mu = |\phi| = m_{\text{top}} - M_{\text{Planck}}$. As already noted \[39, 40\]
Figure 3. Determination of dark matter mass from observed relic density, direct detection experiments and vacuum stability: The yellow curve denotes the values of the parameters \((\lambda_{\phi \xi}, m_\xi)\) allowed by the relic density bound \((\Omega_{\text{DM}} h^2 = 0.1172 - 0.1224)\). The green band represents overlapping exclusion plots from direct detection experiments of LUX-2016, XENON1T(2017) and PANDA-XII(2017) for which any region below (above) the green band is allowed (forbidden). The vertical line at \(\log(m_\xi) = 3.11 (m_\xi = 1.3 \text{ TeV})\) is due to limit set by vacuum stability of the scalar potential as discussed below in Sec.5.1.3.

\(\lambda_{\phi}(\mu)\) runs negative for all field values \(\mu \geq 5 \times 10^9\) GeV clearly exhibiting vacuum instability of the SM Higgs potential. This has been shown by the lower curve in Fig.4. We next examined the evolution of \(\lambda_{\phi}(\mu)\) in the two-heavy Higgs triplet extension model \([13]\) using \(M_{\Delta_2} \simeq M_{\Delta_1} \simeq 10^{13}\) GeV but in the absence of DM chi. Besides being positive for \(\mu < 5 \times 10^9\) GeV, the Higgs potential became definitely positive for field values \(\mu \geq 10^{13}\) GeV with considerable improvement on the stability. However, the quartic coupling was found to be negative for field values \(\mu = 5 \times 10^9\) GeV to \(\mu = 10^{13}\) GeV as demarcated by the two vertical green dashed lines in Fig.4). We next included the effect of DM \(\xi\) and the Higgs portal coupling \(\lambda_{\phi \xi}\) through the DM modified Higgs potential \(V_\chi\) ignoring the presence of Higgs triplets in the model extension. The quartic coupling \(\lambda_{\phi}\) was found to be positive in the entire region of Higgs field values until the Planck mass. This behaviour has been shown by the upper curve in Fig.4 excluding the threshold like enhancement at \(\mu = 10^{12}\) GeV. Finally the combined effect of DM \(\xi\) and the heavy Higgs triplets has been included on the Higgs quartic coupling where the effect of heavy Higgs triplets occurs only for \(\mu \geq 10^{12}\) GeV. In this region we have taken \(\lambda_{1}^{(2)} = \lambda_{2}^{(1)} \simeq 0.15\) and ignored the effect of all other quartic couplings by setting their starting values to be negligibly small. We have also retained small threshold effect due to \(\Delta_2\) resulting in \(\Delta \lambda_{\phi} = \mu_2^2/M_{\Delta_2^2}\). Due to allowed
heavier mass of $\Delta_1$ its threshold effect has been treated to be negligible.

Initial values of the Higgs quartic coupling $\lambda_{\phi}$, DM self coupling $\lambda_{\xi}$, DM Higgs portal coupling $\lambda_{\phi\xi}$, SM gauge couplings $g_Y, g_{2L}, g_{3c}$, and the top quark Yukawa coupling $h_t$ used for RG evolution have been shown in Table.3 for $m_{\xi} = 1.3$ TeV and $m_{\xi} = 2$ GeV. We find that at $m_{\xi} = 1.3$ TeV the one-loop evolution of evolution of $\lambda_{\phi}$ touches horizontal line in Fig.4 around $|\phi| = 10^{13}$ GeV. But if $m_{\xi} < 1.3$ TeV, then $\lambda_{\phi}$ tends to run negative in the region $10^{11} - 10^{12}$ GeV even in the presence of heavy triplets which have their masses $\geq 10^{12}$ GeV in the present investigation. This leads us to conclude that the vacuum stability predicts the real scalar DM mass to be $m_{\xi} \geq 1.3$ TeV. As the direct detection cross section rapidly decreases with increasing $m_{\xi}$ in this region, the predicted mass $m_{\xi} = 1.3$ TeV is expected to be more accessible to experiments compared to values $m_{\xi} \gg 1.3$ TeV, although the latter values are also allowed by three constraints: relic density, direct detection, and vacuum stability.

**Figure 4.** Renormalization group evolution of Higgs quartic coupling ($\lambda_{\phi}$) as a function of scalar field value $\mu = |\phi|$ showing presence of vacuum instability in the SM (lower red curve) for $\mu > 5 \times 10^9$ GeV. The vertical green dashed lines represent boundaries of the region within which $\lambda_{\phi}$ runs negative for $5 \times 10^9$ GeV $< \mu < M_{\Delta_2} \approx 10^{13}$ GeV in the model of [13]. The middle blue curve marked as SM+DM represents evolution of $\lambda_{\phi}$ in the presence of real scalar DM $\xi$, excluding triplets, in the present model extension. Additional RG correction in the present model extension due to triplet masses has been shown by the uppermost curve marked as SM+DM+$\Delta$ where threshold enhancement due to $\Delta_2$ mass $M_{\Delta_2} = 10^{12}$ GeV has been also included.
Table 3. Initial values of coupling constants at top quark mass $\mu = m_{\text{top}} = 173.34$ GeV [62, 63] for different values of the scalar singlet dark matter mass $m_\xi$. The initial values of gauge couplings $g_i (i = Y, 2L, 3C)$ and top-quark Yukawa coupling $h_t$ are estimated using PDG data as explained in the Appendix. The predicted values of $\lambda_{\phi \xi}$ and $m_\xi$ are obtained from the plot of constrained parameter space of Fig.3.

| $m_\chi$ (TeV) | $\lambda_{\phi \xi}$ | $\lambda_\xi$ | $\lambda_\phi$ | $g_1Y$ | $g_{2L}$ | $g_{3C}$ | $h_t$ |
|---------------|----------------------|--------------|--------------|--------|---------|---------|------|
| 0.75          | 0.075                | 0.19         |              |        |         |         |      |
| 1.3           | 0.118                | 0.22         |              |        |         |         |      |
| 1.5           | 0.140                | 0.165        | 0.129        | 0.35   | 0.64    | 1.16    | 0.94 |
| 2             | 0.158                | 0.1          |              |        |         |         |      |

5.1.4 Summary of Dark Matter Mass Prediction

We summarise below the results of theoretical and computational analyses on DM mass carried out in this section

- Although the DM mass values in the narrow region $m_\xi = 59 - 63$ GeV are permitted by both relic density and direct detection measurements, the corresponding Higgs portal coupling values $\lambda_{\phi, \xi} \simeq 1.7 \times 10^{-4} - 1.6 \times 10^{-3}$ are too small to complete vacuum stability of the scalar potential.

- All DM mass values $m_\xi \geq 750$ GeV easily satisfy both the relic density and the direct detection constraints. But for masses $0.75$ TeV < $m_\xi < 1.3$ TeV, the corresponding input values of $\lambda_{\phi, \xi}$ yield negative values for the RG evolution of $\lambda_\phi$ in the region $|\phi| \simeq 10^{10} - 10^{11}$ GeV leading to vacuum instability of the scalar potential.

- We thus find that the minimal extension of the model [13] predicts real scalar singlet DM mass $m_\xi \geq 1300$ GeV that satisfies all the three constraints: relic density, direct detection, and vacuum stability of the scalar potential. Out of these, the lowest limit $m_\xi = 1.3$ TeV is expected to be comparatively more sensitive and accessible to direct detection experiments.

6 Summary and Outlook

The original suggestion of purely triplet seesaw and leptogenesis [13] addresses the interesting possibility that both neutrino masses and baryon asymmetry of the universe can be explained using only two heavy Higgs triplets in the absence of right-handed neutrinos. If neutrinos are quasi-degenerate with relatively larger mass scale, they can predict baryon asymmetry of the universe [13] while manifesting in experimentally verifiable double beta decay. Noting that the recently determined cosmological bound due to Planck satellite measurement has severely restricted the sum of three neutrino masses to < 0.23 eV, and
the recent neutrino oscillation data have revealed $\theta_{23}$ to be in the second octant with large Dirac CP-phase ($\simeq 214^\circ$), in this work we have examined this model predictions with hierarchical neutrino masses satisfying the cosmological bound and the neutrino data. Our motivation towards the model [13] capable of exhibiting type-II seesaw dominance is driven by the recent observation that the Type-I leptogenesis in popular SO(10) grand unified theory (with strongly hierarchical RHNs) is hardly compatible with $\theta_{23}$ in the second octant [29, 30]. On the other hand it has been found that type-II seesaw dominance in SO(10) is capable of providing excellent representation of recent neutrino data [31] where RHN loop mediated triplet leptogenesis explains the baryon asymmetry of the universe.

We have also attempted to explore the model potential in addressing current issues on dark matter and vacuum stability of the scalar potential through a simple minimal extension of the model [13]. We find that the original model can explain both the recent neutrino oscillation data while successfully predicting baryon asymmetry of the universe for both normal and inverted orderings where the sum of the three neutrino masses is nearly 25% of the Planck satellite bound [5]. We have further shown explicitly that, up to a reasonable approximation, the predicted baryon asymmetry depends predominantly upon the lighter triplet mass $M_{\Delta_2}$ and its trilinear coupling $\mu_2$. In particular, for a fixed value of $M_{\Delta_2} = 10^{12}$ GeV, the recent Planck data on baryon asymmetry [5] determines $\mu_2 = 6 \times 10^{10} (7.5 \times 10^{10})$ GeV for normal (inverted) ordering of neutrino masses. We further find that a simple minimal extension of the two-Higgs triplet model [13] successfully predicts a real scalar single dark matter mass $m_\xi \simeq 1.3$ TeV in agreement with observed relic density and mass bounds set by direct and indirect detection experiments. Noting that in the the scalar potential of the original model [13], the standard Higgs quartic coupling $\lambda_\phi$ runs negative in the region $5 \times 10^9 \text{ GeV} \leq |\phi| \leq 10^{13}$ GeV, we have shown how the presence of this real scalar singlet DM also completes the vacuum stability. The minimally extended model thus predicts the lowest mass of the scalar singlet DM to be 1.3 TeV that satisfies the existing constraints due to relic density, direct and indirect detection experiments, and vacuum stability of the scalar potential.

In conclusion we note that the purely triplet seesaw model for neutrino mass and leptogenesis [13] is capable of successfully describing the most recent neutrino oscillation data including $\theta_{23}$ in the second octant and large Dirac CP-phase for both normal and inverted ordering of neutrino masses in concordance with the cosmological bound determined by Planck satellite measurement. Being non-supersymmetric the model has no gravitino problem and has a natural advantage of predicting cosmologically safe relic abundance of light elements. We further conclude that a simple minimal extension of this model [13] successfully explains the direct and indirect evidences of dark matter; it also completes vacuum stability of the scalar potential. Thus, a simple and minimal extension of the original model [13] is capable of solving current puzzles confronting the SM: neutrino oscillation and baryon asymmetry of the universe within the cosmological bound but without gravitino problem, dark matter, and vacuum stability.

Although the dark matter stabilising $Z_2$ discrete symmetry has been assumed in the present model extension based upon the symmetry $SU(2)_L \times U(1)_Y \times SU(3)_C \times Z_2$ in
the spirit of numerous other models including [19, 42, 45, 66], it would be interesting to
explore its deeper gauge theoretic origin as in [46–48, 67, 68] from unified model perspectives
[31, 49, 69].

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8 Appendix: Renormalisation group equations for gauge and scalar cou-
plings

We use the following electroweak precision data at $\mu \simeq m_{top}$ and the Higgs mass [62–64]
as inputs in the bottom-up approach

$$m_{top} = 173.34 \pm 0.77 \text{GeV}$$
$$\sin^2 \theta_W = 0.23129 \pm 0.00005$$
$$\alpha_s = 0.1182 \pm 0.0005$$
$$\frac{1}{\alpha} = 127.9 \pm 0.02$$
$$m_h = 125.09 \pm 0.237 \text{GeV}$$

(8.1)

These values determine the initial boundary values $\lambda_\phi = 0.129$, the SM gauge couplings
$g_1 Y = 0.35, g_2 L = 0.64, g_3 C = 1.16$ and the top-quark Yukawa coupling $h_t = 0.94$. In
addition we use the Higgs triplet masses, their trilinear couplings, and scalar singlet DM
mass as discussed in Sec. 3, Sec. 4, Sec. 5, and Sec. 5.1,

$$M_{\Delta_2} = 10^{12} \text{GeV}$$
$$\mu_2 = 6 \times 10^{10} \text{GeV}$$
$$m_\xi = 1.3 \text{ TeV}$$
$$M_{\Delta_1} = 10^{13} \text{GeV}$$
$$\mu_1 = 10^{12} \text{GeV}.$$  

(8.2)
The beta functions for desired quartic couplings are given by

\[
\beta_{Y_t} = \lambda_t \left[ 12 \lambda_t - \left( \frac{9}{2} g_{1Y}^2 + 9 g_{2L}^2 \right) h_t^2 + \frac{9}{4} g_{1Y}^4 + \frac{2}{5} g_{1Y} g_{2L} + g_{2L}^2 \right] + \sum_{(i=1,2)} (6 \lambda_i^3)^2 + 4 (\lambda_i^4)^2 - 12 h_t^4, \tag{8.6}
\]

where \( g_{2L}, g_{1Y}, g_{3C} \) are the gauge couplings of \( SU(2)_L, U(1)_Y, SU(3)_C \), respectively, and \( h_t \) is the top quark Yukawa coupling. The RG equations for the scalar quartic couplings up to one loop level are

\[
\frac{d\lambda_\phi}{d \ln \mu} = \frac{1}{16\pi^2} \left[ (12 h_t^2 - 3 g_{1Y}^2 - 9 g_{2L}^2) \lambda_\phi - 6 h_t^4 + \frac{3}{8} \{ 2 g_{2L}^4 + (g_{1Y}^4 + g_{2L}^4)^2 \} + 24 \lambda_\phi^2 + 4 \lambda_\phi^2 \right],
\]

\[
\frac{d\lambda_\xi}{d \ln \mu} = \frac{1}{16\pi^2} \left[ \frac{1}{2} (12 h_t^2 - 3 g_{1Y}^2 - 9 g_{2L}^2) \lambda_\phi \xi + 4 \lambda_\phi \xi (3 \lambda_\phi + 2 \lambda_\xi) + 8 \lambda_\phi \xi \right],
\]

\[
\frac{d\lambda_\xi}{d \ln \mu} = \frac{1}{16\pi^2} \left[ 8 \lambda_\phi \xi + 20 \lambda_\xi^2 \right]. \tag{8.4}
\]

For mass scale \( \mu \geq M_{\Delta_1} \approx 10^{12} \text{ GeV} \), the scalar potential is defined through eq.(5.4) of Sec.5.1.

We define the respective beta functions through

\[
16\pi^2 \frac{dC}{dt} = \beta_C \quad (C = \lambda_\phi, \lambda_\phi \xi, \lambda_\xi, \lambda_1, \lambda_2, \lambda_3, \lambda_4, (i = 1, 2)). \tag{8.5}
\]
For $i = 1, 2$, the RGEs for respective quartic couplings are

$$
\beta_{\lambda_1} = \lambda_1^2 \left[ 14\lambda_1^4 + 4\lambda_1^3 - \left( \frac{36}{5} g_{1Y}^2 + 24 g_{2L}^2 \right) + 4 \text{Tr} [T] \right] + \frac{108}{25} g_{1Y}^4 + \frac{72}{5} g_{1Y}^2 g_{2L}^2 + 18 g_{2L}^4
+ 2(\lambda_1^2)^2 + 4(\lambda_1^3)^2 - 8 \text{Tr} [T^2],
$$

$$
\beta_{\lambda_2} = \lambda_2^2 \left[ 12\lambda_1^4 + 3\lambda_2^3 - \left( \frac{36}{5} g_{1Y}^2 + 24 g_{2L}^2 \right) + 4 \text{Tr} [T] \right] - \frac{144}{5} g_{1Y}^2 g_{2L}^2 + 12 g_{2L}^4
- 8(\lambda_1^2)^2 + 8 \text{Tr} [T^2],
$$

$$
\beta_{\lambda_3} = \lambda_3^2 \left[ 6\lambda_3 + 8\lambda_1^4 + 2\lambda_2^3 + 4\lambda_3 - \left( \frac{9}{2} g_{1Y}^2 + \frac{33}{2} g_{2L}^2 \right) + 6 h_i^2 + 2 \text{Tr} [T] \right]
+ \frac{27}{25} g_{1Y}^4 + 6 g_{2L}^4 + 8(\lambda_3^2)^2 - 4 \text{Tr} [T^2],
$$

$$
\beta_{\lambda_4} = \lambda_4^2 \left[ 2\lambda_1^4 + 2\lambda_1^3 - 2\lambda_2^3 + 8\lambda_3 - \left( \frac{9}{2} g_{1Y}^2 + \frac{33}{2} g_{2L}^2 \right) + 6 h_i^2 + 2 \text{Tr} [T] \right]
- \frac{18}{5} g_{1Y}^2 g_{2L}^2
+ 4 \text{Tr} [T^2],
$$

where $T$ is defined as $T = y^{(2)\dagger} y^{(2)}$ where $y^{(2)} \simeq m_\nu / V L^2$. and its beta function is expressed through the relation

$$
\beta_T = T \left[ 6 T - 3 \left( \frac{3}{5} g_{1Y}^2 + 3 g_{2L}^2 \right) + 2 \text{Tr} [T] \right].
$$

We have examined how vacuum stability of the scalar potential in this minimally extended model is ensured by the presence of the scalar singlet DM even with its lowest mass $m_\xi \simeq 1.3$ TeV and its associated Higgs portal coupling. We have estimated RG evolution of standard Higgs quartic coupling $\lambda_0$ in the presence of the DM as well as the heavy scalar triplets in the appropriate ranges of mass scales and Higgs field values. When the DM and the triplets are excluded we get the lowermost red curve [39, 40] of Fig.4 of Sec. 5.1.3 where $\lambda_0$ runs negative for all values of Higgs field $|\phi| > 5 \times 10^9$ GeV showing unstable SM vacuum. When we exclude the scalar DM but include the two heavy triplets as in the original model of [13], the negativity of the quartic coupling persists only in the interval $|\phi| = 5 \times 10^9 - 10^{13}$ GeV after which the quartic coupling has the ability to be positive due to the additional contribution of the triplets. Here a major compensation is caused by the $\Delta_2$-threshold enhancement at $M_{\Delta_2} = 10^{13}$ GeV not shown in Fig. 4. In Fig.4 the negative part of the red coloured curve bounded by vertical green dashed lines is also predicted by the original model [13] signifying vacuum instability in the model. Excluding the triplets but including DM, the solution is given by the upper blue curve of Fig.4 marked as SM+DM (excluding threshold enhancement). When effects of heavy triplets are also included along with DM in the present model extension, the RG evolution for the quartic coupling develops threshold enhancement at $M_{\Delta_2} = 10^{12}$ GeV (rather than $10^{13}$ GeV of [13]) which has been predicted by matching the baryon asymmetry data in the present analysis. This threshold enhancement is $\Delta \lambda_0 \simeq \mu_2^2 / M_{\Delta_2}^2 \simeq 0.005 - 0.01$. In addition we have also included the effects small triplet portal couplings using $\lambda_{\xi}^{(2)} \simeq \lambda_{\psi}^{(2)} \simeq 0.1$. The resulting corrections have been shown by the uppermost curve for $\mu > 10^{12}$ Gev in Fig.4 of Sec.5.1.3. This part of the curve has been marked as SM+DM+\Delta.
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