Hermiticity-tunable beam reshaping: localization, recurrence and group velocity oscillation

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Abstract

We propose a novel method to reshape the light beam in a Hermiticity-tunable photonic lattice with PT symmetric perturbations. By adjusting the PT perturbations, the Hermiticity of the system varies alternatively, further resulting in the periodic oscillations of the group velocity ($v_g$) as well as the group velocity dispersion (GVD). Once the values of $v_g$ and its corresponding GVD recover and the averaged group velocity ($\bar{v}_g$) is zero, the light beam will be localized and recur at these Hermitian points. Further study indicates that GVD plays a key role in the evolution of the light beam, larger positive dispersion $\Re(\text{GVD})$ and lower magnitude of $\Im(\text{GVD})$ lead to higher peak energy of the light field. Our work provides a non-resonant way to control the light propagation and to reshape the group velocity distribution, which may have promising applications in all-optical circuit, optical sensor and optical communications.

1. Introduction

In recent years, constant attention has been paid to the non-Hermitian systems to study the system-environment interaction. In such systems, the eigenvalues are generally complex [1–3] and plenty of new phenomena have been investigated, such as non-Hermitian localization in a disordered lattice [4] and non-reciprocal gain in a non-Hermitian time-Floquet system [5]. In the field of optics, non-Hermitian is usually achieved by introducing controllable loss and gain into a system, such as a particular case, PT symmetric system, in which the loss and gain are balanced [6–8]. One can observe many intriguing light wave dynamics, like unidirectional propagation [9–11], PT symmetric laser [12, 13] and photonic zero mode [14, 15]. On the other hand, group velocity controlling, an important technique in the field of slow and fast light, has been a focus for many years [16, 17]. In the conventional Hermitian system, atomic system based on the electromagnetically induced transparency effect has been used to slow and store light [18, 19] or stop light [20, 21]. Some other systems, such as photonic crystal or lattice [22–25], optical fibers [26, 27] and optomechanical system [28] are also employed to control the group velocity. In a system with loss, directional dependence of the group velocity has been investigated in periodic systems [29]. Currently, it is reported that light stops at the exceptional point in the PT symmetric waveguide [30]. Tunable slow and fast light can be achieved in PT symmetric optomechanical systems with phonon pump [31]. Such works bring new ideas for group velocity controlling in the non-Hermitian system. However, the study on group velocity controlling and corresponding beam reshaping in a Hermiticity tunable photonic lattice is still absent.
We propose a type of Hermitian-tunable photonic lattice with perturbation of PT symmetry. Such perturbations will introduce an imaginary phase factor $\Phi$ into the coupling constant. In consequence, the dispersion relations of such a system is $\Phi$ dependent and the Hermiticity of such a system will be tunable. Further study indicates that the perturbation will result in the oscillations of the group velocity $v_g$ as well as the group velocity dispersion (GVD) along the propagation direction. Light will be dynamically localized and recur at certain Hermitian points, at which $v_g$ and GVD are recovered and averaged group velocity ($\bar{v}_g$) is zero.

\section{2. Model and analysis}

We first construct a one-dimensional photonic lattice with perturbation sites, as shown in figure 1. In the tight-binding model, the Hamiltonian of such a system with perturbations can be expressed as

$$
H(z) = \begin{pmatrix}
0 & c(1 + e^{-2ik}) & \delta(z) + \delta'(z)e^{-ik} \\
c(1 + e^{2ik}) & 0 & \delta'(z) + \delta(z)e^{ik} \\
\delta(z) & \delta'(z) & \beta_\rho(z)
\end{pmatrix},
$$

(1)

where $k$ is the Bloch wave number, $\beta_\rho(z)$ is the propagation constant for the perturbations, $c$ is the coupling constant between the ordinary waveguides, $\delta(z)$ and $\delta'(z)$ represent the coupling strengths between the ordinary and perturbation sites ($\delta(z)$ and $\delta'(z)$ are real and positive). Similar to the case in \cite{32}, we assume that the refractive index of the perturbation site is much smaller than the ordinary site ($n_\rho \ll n_o$), accordingly, the coupling between the ordinary waveguide and the perturbation is weaker than the coupling between the ordinary sites, which can be regarded as $\delta(z)$ or $\delta'(z) \ll c$. Therefore, it is reasonable to assume $\delta(z) \approx \delta'(z) \approx \delta$ which suggests the perturbations are with slowly varying amplitudes.

To simplify equation (1), one can eliminate perturbation site amplitudes $\rho_n$ using adiabatic approximation \cite{32}

$$
\rho_n = -\frac{\delta(a_{n+1} + a_{n})}{\beta_\rho(z)}.
$$

(2)

To obtain a Hermiticity-tunable system, we set the propagation constant of the perturbation sites as $\beta_\rho(z) = -\delta^2/(e^{ik} - c)$, we assume that $\Phi$ varies with propagation distance $z$ and has the form of $\Phi = -k_\rho z$ where $k_\rho$ has a unit of the wave vector. The real and imaginary parts of the propagation constant of the perturbations are $\text{Re}[\beta_\rho(z)] = \delta^2(c - \cos \Phi)/e^{ik} - 2c \cos \Phi + 1$ and $\text{Im}[\beta_\rho(z)] = i\delta^2 \sin \Phi / (e^{2k} - 2c \cos \Phi + 1)$, respectively. Apparently, one can control the propagation constant $\beta_\rho(z)$ through adjusting the phase term $\Phi$ and the periodicity of $\beta_\rho(z)$ is $2\pi$ (see figure 2). Further, one can notice that the real part of the propagation constant satisfies $\text{Re}[\beta_\rho(z)] = \text{Re}[\beta_\rho(z)]$ and the imaginary part satisfies $\text{Im}[\beta_\rho(z)] = -\text{Im}[\beta_\rho(z)]$, which suggests that the perturbation sites carry balanced loss and gain along the evolution direction ($z$ axis). Therefore, such a perturbation waveguide system is PT symmetric along the $z$ axis.

After making the aforementioned hypothesis, one can obtain the effective Hamiltonian for the main waveguide system as

$$
H'(z) = \begin{pmatrix}
\beta_{\rho}(z) & e^{i\Phi}(1 + e^{-2ik}) \\
e^{i\Phi}(1 + e^{2ik}) & \beta_{\rho}(z)
\end{pmatrix},
$$

(3)

where $\beta_{\rho}(z) = -2\delta^2/\beta_{\rho}(z) = 2(e^{ik} - c)$, representing the effective propagation constant of the Hermiticity-tunable lattice and an imaginary phase factor $\Phi$ is introduced into the coupling constant. Clearly, the Hermiticity of the system is decided by the phase term $\Phi$. When $\Phi = m\pi$ ($m$ is some integers), the system degenerates into a Hermitian system and the effective Hamiltonian satisfies $H' = H'^\dagger$. $H'^\dagger$ is the Hermitian transpose of the effective Hamiltonian; Otherwise, the system is non-Hermitian. Thus, we have realized a Hermiticity-tunable one-dimensional photonic lattice by introducing perturbations with PT symmetry along the evolution direction.

To theoretically study the light wave dynamics in such a Hermiticity-tunable system, we first calculate the dispersion relations of the main waveguide system, which can be read as

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{(a1) The schematic of the one-dimensional photonic lattice with PT symmetric perturbations. The dotted line box represents a unit of the periodic structure. $a_n$ is the ordinary waveguide and $\rho_n$ is the perturbations. (a2) Effective model for the Hermitian-tunable structure.}
\end{figure}
Equation (4) indicates that the dispersion relations are functions of both Bloch wave number $k$ and phase term $\Phi$. Comparing with the conventional photonic lattice, it is easy to find that these dispersion relations have been modified due to the introduction of the perturbation sites and therefore possess the imaginary energy spectrum except that the real one. The evolutions of the dispersion relations of our system are shown in figure 3. The real and the imaginary parts of the dispersion relations vary alternatively and periodically with respect to the phase term $\Phi$ (the left and right columns in figure 3) both in the first (figures 3(a1) and (a2)) and the second (figures 3(b1) and (b2)) Brillouin zones. The system evolves from one Hermitian state to another Hermitian state every one $\pi$ and the Hermiticity of the system changes a round every $2\pi$. The dotted lines in figure 3 correspond exactly to the states where $\text{Im}(E) = 0$ (see the dashed lines marked from $a \sim f$ in figure 3 corresponding to $0 \sim 5\pi$), implying that the system is Hermitian. We name these positions as the Hermitian points, where the interaction between system and environment is prohibited.

To further study the properties of the light wave in such a Hermiticity tunable photonic lattice, $v_g$, GVD and $\tilde{v}_k$ in the first band are introduced as [25]

$$E(\Phi, k) = \beta_n(z) \pm 2e^{i\delta} \cos k.$$ (4)

$$v_g = \frac{\partial E}{\partial k} = -2e^{-ikz^2} \sin k,$$ (5)

$$\text{GVD} = \frac{\partial^2 E}{\partial k^2} = -2e^{-ikz^2} \cos k,$$ (6)
for distribution by controlling the group velocity dispersion. Figure 6 illustrates the energy distribution as a function and the last one is the result of group velocity dispersion, which suggests that one can control the energy describes the coupling effect among the ordinary waveguides, the second term arises from the PT perturbations. Clearly, the energy distribution function consists of three parts. The

\[ \vartheta_k = \frac{1}{z} \int_0^z \frac{\partial E}{\partial k} \, dz = -\frac{2i(1 - e^{-ikp})}{z} \sin k. \]

Obviously, \( v_g \), GVD and \( \vartheta_k \) are the functions of \( k_p \) related to PT perturbation, propagation distance \( z \) and wave number \( k \). Therefore, the group velocity of such a Hermitian-tunable system is controllable with the advent of PT perturbations. Figure 4 shows the image of \( v_g \), \( \vartheta_k \) and GVD as functions of \( k_p \) and \( z \). For the case of normal incidence \( (k = 0) \), the top row in figure 4, the values of \( v_g \) and \( \vartheta_k \) are both zeros, while the real and imaginary parts of GVD both vary with respect to \( k_p \) and \( z \) (see figures 4(b1) and (b2)) and GVD experiences a process from acceleration to deceleration in one round and repeats such cycles along the \( z \) axis. Similar process occurs for the real part of \( v_g \) in the case of half Bragg incidence \( (k = \pi/2) \), the bottom row in figure 4(d1); due to a phase shift of \( \pi/2 \) between the real part, the imaginary one of \( v_g \) oscillates with a phase delay (figure 4(d2)). Similar phase delay occurs for \( \vartheta_k \) except that their amplitudes are decreased with respect to \( z \) until 0 at infinity (figures 4(f1) and (f2)). Therefore, one can control the acceleration of group velocity by adjusting the periodicity of PT symmetric perturbations.

To better understand the impacts of wave numbers \( k \) on \( v_g \) and \( \vartheta_k \), the evolutions of \( v_g \) and \( \vartheta_k \) verse \( k \) with a fixed \( k_p \) are explored as shown in figure 5. It is seen that \( v_g \) is periodic and corresponding periodicity for \( k_p = \pi \) is half as that for \( k_p = \pi/2 \). The two part of \( \vartheta_k \) are also damped both along the propagation direction \( z \) (see figure 5(b1) and (b2), (d1) and (d2)).

Until now we have shown the variation of group velocity in such a Hermiticity-tunable system. To investigate how group velocity and GVD affect the light wave distribution in the Hermitian-tunable lattice, we theoretically solve equation (3) with the ansatz solution \( a_n(z) = A(z) \exp(i\kappa n) \), one can obtain the following formula

\[ A(z) = A(0) \exp \left[ 2icz + \frac{2(\cos k + 1)e^{-ikp}}{k_p} \right], \]

where \( A(0) \) is the initial amplitude. The energy distribution is a periodic function and its corresponding periodicity is \( 2m\pi/k_p \). With the definition of GVD in equation (6), the energy distribution can be written in the term of

\[ A(z, \text{GVD}) = A(0) \exp \left[ 2icz + \frac{1}{k_p}(2e^{-ikp} - \text{GVD}) \right]. \]

Clearly, the energy distribution function consists of three parts. The first term in the exponential function describes the coupling effect among the ordinary waveguides, the second term arises from the PT perturbations and the last one is the result of group velocity dispersion, which suggests that one can control the energy distribution by controlling the group velocity dispersion. Figure 6 illustrates the energy distribution as a function of both propagation distance \( z \) and GVD for the case of \( k_p = \pi/2 \) and \( k \) with \( k = 0 \). The energy distribution of the light beam oscillates along the propagation direction due to the existence of PT perturbations (the second term in equation (9)) and the corresponding periodicity can be calculated as \( z_p = 2m\pi/k_p = 4m \) and \( z_p' = 2m \) for \( k_p = \pi/2 \) and \( k_p = \pi \), respectively. When \( k_p = \pi/2 \), the energy of the light beam reaches its maximum when \( \text{Re(GVD)} = -2 \) and the imaginary part of GVD is zero at some certain Hermitian points marked as \( z = 4 \) and 8.
Figure 5. Group velocity, averaged group velocity and group velocity dispersion as functions of $z$ with different wave number $k$. Circles mark the Hermitian points where both of the real and imaginary part of group velocity return to their initial values and averaged group velocity is zero.

Figure 6. Theoretical calculation of energy distribution $|A(z, \text{GVD})|^2$ as a function of GVD and $z$ for the case of $k_p = \pi/2$ and $k_p = \pi$. Here we take $A(0) = 1$. 

Apparently, this is different from the conditions for the dynamic localization in curved waveguides. Here we propose a method to fabricate lattices in Fe-doped LiNbO3 crystal. The ordinary waveguides are deeply induced while the perturbations are shallowly modulated. The PT symmetry along the propagation distance can be realized by applying another pump beam with uneven distribution of light energy, where the low intensity can form loss for the signal wave due to the excitation of electrons from Fe2⁺ to the conduction and high intensity will result in the optical gain through two-wave mixing using the materials photorefractive nonlinearity. (figures 6(a1) and (a2)). In other words, larger positive dispersion and lower magnitude of \( \text{Im(GVD)} \) can lead to higher energy of the light beam. Notice that \( v_g \) and GVD return to their initial value and \( \text{Im}(\text{GVD}) = 0 \) (see the circles in figures 3(a1) and (b1)), suggesting that the light beam will be dynamically localized and recur at those special Hermitian points. The similar intensity distribution is observed for \( k_p = \pi/2 \) with half periodicity compared with \( k_p = \pi \) (figures 6(b1) and (b2)). Therefore, GVD plays a crucial role in the energy distribution, one can manipulate the energy pattern by controlling the group velocity via PT symmetric perturbations. For the cases of \( k = 0 \), one can obtain a similar evolution of \( |A(z, \text{GVD})|² \) except that the range of GVD is smaller.

To verify our analysis in the above, we numerically explore the dynamical localization of a light beam in such a Hermitian-tunable system with a Gaussian profile given by \( n_g(\bar{z} = 0) = \exp\left[-(n - n_0)²/\omega_0² - ik\bar{z}\right] \), where \( n_0 \) is the incident waveguide number, \( \omega_0 \) is the width of the Gaussian beam. Figure 7 illustrates the numeric results of the propagation for a broad beam in such a system (\( \omega_0 = 2 \)). Several waveguides are excited at the input facet (see the profiles at \( z = 0 \) in figure 7). In both of the cases of \( k_p = \pi/2 \) and \( k_p = \pi \), most of the light energy will be focused at the Hermitian points for all four wave vectors (see the top row for \( k_p = \pi/2 \) and bottom row for \( k_p = \pi \) in figure 7). When \( k = 0 \), perfect dynamic localizations occur with separations of \( z_p = 4 \) (figure 7(a1)) and \( z_p = 2 \) (figure 7(b1)). When \( k = \pi/2 \) or \( 3\pi/2 \), modified dynamic localization still occur with same separations as \( k = 0 \) (figures 7(a2) and (b2)). Obviously, the directions of tilt are consistent with the coordinates in the dispersion spectra (see figure 3). When \( k = \pi \), most of the light energy will be trapped in the incident direction with very few dispersion except that light will still be focused at these particular Hermitian points due to the low gradient along \( z \) as shown in figures 3(b1) and (b2). Apparently, \( \text{Im}(E) \) are equal to zeros and \( \text{Re}(E) \) reach their maximum at these Hermitian points (see dashed lines \( c \) and \( e \) in figure 3), corresponding to the positions where \( v_g \) and GVD recover their initial value and \( \text{Re}(\text{GVD}) \) are zeros (see the circles in figure 5). Apparently, this is different from the conditions for the dynamic localization in curved waveguides [33, 34].

An applicable proposal of such a structure might be achieved in atomic medium or photonic lattice [35]. Here we propose a method to fabricate lattices in Fe-doped LiNbO3 crystal [36]. The ordinary waveguides are deeply induced while the perturbations are shallowly modulated. The PT symmetry along the propagation distance can be realized by applying another pump beam with uneven distribution of light energy, where the low intensity can form loss for the signal wave due to the excitation of electrons from Fe2⁺ to the conduction and high intensity will result in the optical gain through two-wave mixing using the materials photorefractive nonlinearity.
3. Summary and conclusion

We have proposed a new method to reshape the light beam in a Hermiticity-tunable photonic lattice with PT symmetric perturbations. Such perturbations can introduce an imaginary phase $\Phi$ into the coupling constant. Both of the Hermiticity and the dispersion relations of the system vary with $\Phi$. Consequently, $v_g$ and its corresponding GVD vary alternatively and periodically with the PT perturbations. Further study suggests that GVD can directly affect the energy distribution of the light beam, the larger positive dispersion and lower magnitude of the imaginary part of GVD can lead to higher peak energy of the light field. Recurrence and dynamical localization can be achieved at some special Hermitian points, where $v_g$, GVD return to their initial values and $v_g = 0$. Our work proposed a new method to realize the dynamic localization and group velocity oscillation of the light wave, which may bring some promising insights to the field of optical communication and all-optical circuit.

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