An Evaluation Method of Availability of Naval Gun Weapon Systems Based on Palm’s Theorem

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Abstract. Availability is the embodiment of the combat effectiveness of the equipment, of which the level determines the ability of the equipment to perform tasks to a certain extent. In this paper, an evaluation method of availability of naval gun weapon systems based on Palm’s theorem is developed. This evaluation method, based on maintenance strategy, takes into consideration factors that make the system unavailable, such as repair, backorders, etc.

1. Introduction
The naval gun weapon systems is a weapon system for tracking targets, firing calculations, aiming, supply and launch control. With the advent of the worldwide military revolution, high-tech is constantly integrated into the equipment, which greatly enhances its performance. However, how to make high-tech equipment have high availability, so that the equipment can be as effective as possible, has become the primary problem of navies around the world [1].

Therefore, we should pay more attention to the availability level of naval gun weapon systems. However, no in-depth research has been conducted on the evaluation method of the availability of naval gun weapon systems.

Systems efficiency is the function of availability, credibility and capacity [2]. In addition to designs, the level of credibility and capacity is not influenced by spares and inventory levels, while the level of availability is, which is determined by time between failures, repair time, support resources and delay caused by human or management factors [3]. The factors which influence availability are shows in Fig.1.

It is important to describe the maintenance strategy, before we attempt to develop the method. In the occasion of this paper, the maintenance strategy of naval gun weapon system is described as follows. When a failure occurs, the malfunctioning item is removed from the naval gun weapon system and brought into base supply. If a spare is available, it is issued and installed on the naval gun.
weapon system; otherwise a backorder is established for that user, which means systems are unavailable. The repair time is defined to be the time from when failure occurs until the time installing a spare, including delay caused by human or management factors. The malfunctioning item is taken to a base maintenance shop and scheduled into base repair, and at some later time, when fixed, it is sent to base supply, where it is used to satisfy an outstanding backorder, if any, or is added to serviceable supply on the shelf. The item repair time is defined to be the time from when the malfunctioning item is taken to a base maintenance shop until the time when it is sent to base supply. There is a probability distribution for the item repair time, depending on the complexity of the repair and the availability of personnel, shop equipment, spare parts. The maintenance strategy is show in Fig.2.

![Figure 2. The maintenance strategy.](image)

2. Evaluation Method

2.1. Assumptions
For developing the method, some assumptions are developed according to the maintenance strategy, which are showed as follows:

- The naval gun weapon system is a series system, and will not stop working until a failure occurs and the system is unavailable.
- For an item \( i \), the time between failures has a exponent distribution (also called a Poisson process) with mean \( \mu \), and annual mean of demand for an item is \( m_i \).
- The repair time for any item has a normal distribution with mean \( t \), where the failure detected, the malfunctioning item removed and the spare installed.
- The item repair time is independently and identically distributed with mean \( T_i \) years.
- The naval gun weapon system works 8 hours a day.

2.2. Palm’s Theorem
The cornerstone of this paper is a queueing theorem of Palm’s (1938). If demand for an item is a Poisson process with annual mean \( m_i \) and the item repair time is independently and identically distributed according to any distribution with mean \( T_i \) years, then the steady-state probability distribution for the number of units in repair has a Poisson distribution with mean \( m_i T_i \). The Poisson distribution, \( p(x) \), is given by:

\[
p(x) = \frac{(m_i T_i)^x}{x!} \quad x = 0, 1, 2, \ldots
\]

The importance of the theorem is that it enables us to evaluate the steady-state probability distribution of the number of units in repair from the probability distribution of the demand process and the mean of the repair time distribution, which means that there is no need to collect data on the shapes of the repair distributions.
2.3. Evaluation Method of Availability

A stock balance equation, the basis for our method, is written by Craig C. Sherbrooke [4], which is shown in (2):

\[ BO_i = OH_i + DI_i - S_i \]  

(2)

where for an item \( i \), \( S_i \), the stock level, is a constant. The number of units of stock on hand, \( OH_i \), the number of units of stock due in from repair and resupply, \( DI_i \), and the number of backorders, \( BO_i \), are non-negative random variables. Any change in one of these random variables is accompanied by a simultaneous change in another. For example, when a demand occurs, the number due in from repair increases by one. If the stock on hand is positive, it is decreased by one; otherwise, the backorders increase by one. In either case the equality is maintained. When a repair is completed, reducing \( DI_i \) by one, the backorders are reduced by one or, if there are no backorders, the on hand balance is increased by one. Again the equality is preserved.

Regarding fill rate, there will be a fill if the number due in is \( S_i - 1 \) or less, because that implies there is stock on hand. Whenever the number \( DI_i \) is \( s \) or more, there is no stock on hand. According to Palm’s theorem, the steady-state probability distribution for the number of units in repair has a Poisson distribution. Without regarding to the transportation time, the number \( DI_i \) is equal to the number of units in repair, so that we can designate the expected fill rate as \( EFR(S_i) \):

\[ EFR(S_i) = p(D_i|D_i = 0) + ... + p(D_i|D_i = S_i - 1) \]

(3)

And we can easily designate the expected unfilled rate for an item as \( EUR(S_i) \):

\[ EUR(S_i) = 1 - EFR(S_i) \]

(4)

According to the assumptions developed, the naval gun weapon system will not stop working until failure occurs in any item. So it is evident that periods where systems are unavailable caused by different failures are not overlapping. Thus, the expected probability of unavailable systems in a year caused by backorders can be designated as \( EUR(S) \):

\[ EUR(S) = \sum_{i=1}^{N} EUR(S_i) \]

(5)

where \( N \) is amount of item variety. The equation of inherent availability, given by logisticians, is showed as follows:

\[ \text{Inherent Availability} = \frac{MTBF}{MTBF + MTTR} \times 100\% \]

(6)

where \( MTBF \), the abbreviation of mean time between failures, can be evaluated by (6) with mean \( \mu \):

\[ \mu = \frac{[1 - EUR(S)] \times 8 \times 365 - \sum_{i}^{N} m_i}{\sum_{i}^{N} m_i} \]

(7)
and MTTR, the abbreviation of mean time to repair, can be evaluated with mean $t$, which can be given by statistical data of repair time from use or tests. Thus, availability of the naval gun weapon system can be designated as $Ao$, and evaluated by the equation as follows:

$$Ao = \left[1 - EUR(S)\right] \frac{\mu}{\mu + t} \times 100\% = \left[1 - \sum_i^{\infty} EUR(S_i)\right] \frac{\mu}{\mu + t} \times 100\%$$ \hspace{2cm} (8)

### 3. Case Application and Analysis

In this chapter, a case of availability of the naval gun weapon system is given to illustrate the application of the evaluation method. For the sake of calculations, the naval gun weapon system, satisfying assumptions developed in this paper, is a series system with two items and the stock levels $S_i$ for any item equal two. The data for availability evaluation is given in Table 1.

**Table 1.** The data for availability evaluation.

| Item No. | Year | Demand | Date When Failure Occurred | Date When Item Repaired | $t$/h |
|----------|------|--------|-----------------------------|-------------------------|-------|
| 1        | 2016 | 4      | 2016. 8. 15                | 2016. 10. 20            | 25    |
| 2        | 2016 | 1      | 2016. 5. 13                | 2016. 8. 20             | 27    |
| 1        | 2017 | 2      | 2017. 6. 10                | 2017. 8. 5              | 24    |
| 2        | 2017 | 1      | 2017. 9. 18                | 2017. 12. 11            | 24    |

The data is analyzed, so that the annual mean of demand $m_i$ and the mean item repair time $T_i$ years are calculated in Table 2.

**Table 2.** The analysis results of the annual mean of demand and the mean item repair time.

| Item No. | $m_i$ | $T_i$/year | $m_iT_i$ | $t$/h |
|----------|-------|------------|----------|-------|
| 1        | 3     | 0.1667     | 0.5000   | 25    |
| 2        | 1     | 0.2500     | 0.2500   |       |

According to Palm’s theorem, the steady-state probability distribution for the number of units in repair has a Poisson distribution with mean $m_iT_i$. Thus, for different items with different stock levels (if they can be changed), the $EFR(S_i)$ and $EUR(S)$ can be calculated by (1), (3), (4) and (5), and mean time between failures $\mu$ can be calculated by (7), and availability evaluation value $Ao$ can be calculated by equation 8. The results are shown in Table III.

**Table 3.** The results of the EUR(S) and Ao.

| No. | $S_1$ | $EFR(S_1)$ | $S_2$ | $EFR(S_2)$ | $EUR(S)$ | $\mu$ | $Ao$  |
|-----|-------|------------|-------|------------|----------|-------|-------|
| 1   | 1     | 0.6065     | 1     | 0.7788     | 0.6147   | -     | 37.08%|
| 2   | 1     | 0.6065     | 2     | 0.9735     | 0.4200   | -     | 55.82%|
| 3   | 2     | 0.9098     | 1     | 0.7788     | 0.3114   | -     | 66.27%|
| 4   | 2     | 0.9098     | 2     | 0.9735     | 0.1167   | 640   | 85.01%|
| 5   | 2     | 0.9098     | 3     | 0.9978     | 0.0924   | -     | 87.35%|
| 6   | 3     | 0.9856     | 2     | 0.9735     | 0.0409   | -     | 92.30%|
| 7   | 3     | 0.9856     | 3     | 0.9978     | 0.0165   | -     | 94.65%|

One thing to note is that, the mean time between failures $\mu$ is influenced by accumulated working time, while it is not supposed to be influenced by stock levels $S_i$. Thus, mean time between failures $\mu$ can only be calculated in No.4, which represents the real situation. And the three-dimensional surface charts with color filled and with color mapped are shown in Fig.3 and Fig.4.
From the results we can know that, first, as the stock levels $S_i$ increased, the availability evaluation value $Ao$ increased; second, when stock levels $S_i$ of different items are the same, if we want to increase the availability by adding only one unit of one of items, we should choose the item of which $m_iT_i$ is the largest among items.

4. Conclusion
An evaluation method of availability of naval gun weapon systems based on Palm’s theorem is developed. This evaluation method, based on the maintenance strategy, takes into consideration factors that make the system unavailable, such as repair, backorders, etc. A case of availability of the naval gun weapon system is given to illustrate the application of the evaluation method, and preliminary recommendations for optimizing stock levels are proposed in results analysis.

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