Muon Anomalous Magnetic Moment in the Left-Right Symmetric Model

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Abstract. The measurement of muon anomalous magnetic moment provides a test of the standard model and of the physics that lies beyond it. Currently, there is a deviation of 2.6σ between the standard model prediction and the experimental results. In this work, the contribution of heavy gauge bosons from the left right symmetric model (LRSM) is calculated. We find that the LRSM can give a relatively small but non-negligible extra weak contribution to the muon anomalous magnetic moment and can reduce the deviation of \( \Delta a_\mu \) from 2.6σ for the SM to 2.5σ for the LRSM model.

Introduction

1. Introduction
The SM has been so far in excellent agreement with experiment. However, some problems do not have any explanation in its minimal version. Among them, the hierarchy problem (the stability of the higgs mass under radiative corrections) [1,2], neutrino masses [3] and the anomalous magnetic moment of the muon [4], which is the subject of this work.

The left right symmetric model based on the gauge group \( SU(2)_L \times SU(2)_R \times U(1)_{R-L} \) is the most natural extension of the standard model. It can explains parity violation at low energies [5,6] and provide a seesaw mechanism that give masses to neutrinos in a natural way.

In our work, we use the LRSM to explain the anomalous magnetic moment of the muon (AMM). The study of AMM represents a very sensitive test of the SM at the quantum loop level and permits the investigation of physics that lie beyond it. The magnetic moment is defined as \( \mu = g(e/2m) \), where \( g \) is the gyromagnetic ratio. The deviation of the magnetic moment from the value of the point-like Dirac particle (\( g = 2 \)) is induced by the interactions of leptons with virtual particles which couple to electromagnetic field. Whereas the electron anomaly provides the most precise measurement of the fine structure constant, the muon anomaly is more sensitive to virtual gauge bosons. In this paper, we investigate the effects of the left right symmetric model on anomalous magnetic moment of the muon.

We consider all possible contributions from extra gauge bosons at the one loop level. Our purpose is to get a better interpretation of the experimental results of the muon anomaly. In section 2, we give a short review of the LRSM, its theoretical basis and the structure of the gauge sector. In section 3, we give the calculation of muon g-2 in the LRSM. In section 4, we give a numerical estimation of the
value of the anomalous magnetic moment of the muon. Finally, a short summary and conclusion are
given.

2. Review of the left right symmetric model
The main motivation for the left right symmetric model is the treatment of the right-handed particles
and their interaction on an equal footing with the left ones. In this model the weak interaction is based
on the gauge group $\mathbb{S}^2 \times \mathbb{S}^2 \times \mathbb{S}(1)_{B-L}$, where $B - L$ stand for the difference in baryon
and lepton numbers. The symmetry breaking (SB) of the LRSM is achieved in two steps via the
vacuum expectation value (VEV) of scalar triplets and multiples.

$$\Delta_{L,R} = \left( \frac{\Delta^+}{\sqrt{2}} - \frac{\Delta^0}{\sqrt{2}} \right)_{L,R}, \quad \phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \quad (1)$$

The symmetry-breaking scheme is as follows

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(2)_L \times U(1)_Y \quad (2)$$

At this stage the right scalar develop a vev $\langle \Delta_R \rangle = V_R$ and break the left right symmetry to the
electroweak symmetry giving masses to heavy electroweak gauge bosons. After the first stage of SM,
the kinetic term of the higgs sector become

$$\text{Tr} |D_\mu \Delta_R|^2 = \frac{g_R^2}{2} W^\mu_R - W^\mu_R + \frac{v_R^2}{2} (g_R W^\mu_R - g_{B-L} V^\mu_R)(g_R W^\mu_R - g_{B-L} V^\mu_R) \quad (3)$$

The physical heavy field $Z_R$, and the $U(1)_Y$ gauge field $B_\mu$ are derived by applying unitary
transformation characterized by the mixing angle $\phi$

$$\begin{pmatrix} Z_{R\mu} \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{pmatrix} \begin{pmatrix} W_{R\mu} \\ V_\mu \end{pmatrix} \quad (4)$$

Where $\phi$ is defined by

$$\cos(\phi) = \frac{g_R}{\sqrt{g_R^2 + g_{B-L}^2}}, \quad \sin(\phi) = \frac{g_{B-L}}{\sqrt{g_R^2 + g_{B-L}^2}} \quad (5)$$

In the second stage of symmetry breaking, the other higgs fields $\phi$ and $\Delta_L$ get vev and give masses to
the SM gauge bosons $W_L$ and $Z_L$

$$\langle \phi \rangle = \begin{pmatrix} \kappa_1 \\ 0 \end{pmatrix}, \quad \langle \Delta_L \rangle = \begin{pmatrix} 0 \\ V_L \end{pmatrix} \quad (6)$$

The gauge bosons masses are given by

$$M_A = 0, \quad (7)$$

$$M_{W_L}^2 = \frac{g_R^2}{2 \cos^2(\theta_W)} \left( \kappa_1^2 + \kappa_2^2 + 4V_L^2 \right), \quad (8)$$

$$M_{Z_R}^2 = 2(g_R^2 + g_{B-L}^2)V_R^2 \quad (9)$$
3. Calculation of $a_\mu$ in the left right symmetric model

3.1. Values of $a_\mu$ in the SM

The muon anomaly in the SM is the summation of three contributions

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{Weak} + a_\mu^{Had}$$

These contributions have been determined precisely in previous works. The QED contribution is the dominant one, and it has been calculated up to the fourth order $\alpha^4$. The weak contribution has been calculated up to 2 and 3 loop level and it has not changed much in the last years.

We present below the best results of the muon anomaly calculation in the SM [7,8,9]

$$a_\mu^{QED} = 11658471.958(0.143) \times 10^{-10}$$

$$a_\mu^{Weak} = 15.4(0.2) \times 10^{-10}$$

$$a_\mu^{Had} = 697.2(5.9) \times 10^{-10}$$

The total SM value for $a_\mu$ is

$$a_\mu^{SM} = 11659184.56(5.9) \times 10^{-10}$$

and the present experimental value for $a_\mu$ is

$$a_\mu^{Exp} = 11659208.56(6) \times 10^{-10}$$

thus, the deviation of the experimental value of the anomalous magnetic moment of the muon from the SM prediction is

$$\Delta a_\mu = a_\mu^{Exp} - a_\mu^{SM} = 23.4(9.0) \times 10^{-10}$$

3.2. Calculation of $a_\mu$ in the LRSM

The LRSM contribution to the muon anomaly is calculated using the diagrams of fig 1. In our calculation, we use the t’Hooft Feynman gauge for the propagator of gauge bosons. The total amplitude for the diagrams of fig1 can be written as

$$M_{LR} = -e e_\mu(q)\bar{u}(p)\Gamma^\mu u(p)$$

Where $q$ is the four-momentum of the photon, $p$ and $p'$ are momentum of the incoming and outgoing muon respectively, $\Gamma^\mu$ is the vertex function which has the general Lorentz structure

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^\mu\nu q_\nu}{2m_\mu} + F_3(q^2)\frac{i\sigma^\mu\nu q_\nu y_5}{2m_\mu}$$
Because $F_1(q^2), F_2(q^2)$ and $F_3(q^2)$ are related to the electric charge, the anomalous magnetic moment and the electric dipole moment respectively, we must calculate only the form factor $F_2(q^2)$. The LRSM contribution to the muon anomaly is derived as

$$a_{LR}(1\text{loop}) = \frac{g - 2}{2} = F_2(0)$$  \hspace{1cm} (21)$$

In our calculation we find that the most important contribution to muon anomaly came from the extras gauge bosons $W_R, Z_R$ related to the gauge group $SU(2)_R$. For $W_R$, we find the following result for the vertex function

$$\Gamma^\mu = \frac{ie^2m_\mu}{6\pi^2m_W^2} \times \frac{7}{6} (p + p) \gamma^\mu$$  \hspace{1cm} (22)$$

using the Gordon identity $(p + p) \gamma^\mu = 2m_\mu - i\sigma^{\mu\nu}q_\nu$ and the definition of the AMM (eq(21)), we find the following result

$$\alpha_{W_R}^\mu \approx \frac{\alpha}{16\pi^2 \sin^2(\theta_W)} \left( \frac{m_\mu}{M_{W_R}} \right)^2 \frac{7}{3} \times O \left( \frac{m_\mu}{M_{W_R}} \right)^4 \int$$  \hspace{1cm} (23)$$

where $\alpha$ is the fine structure constant.

**Figure 1.** The electroweak one loop Feynman diagrams of the muon anomalous magnetic moment in the left right symmetric model

In the same manner, we calculate the heavy neutral gauge boson ($Z_R$) correction to the vertex function, and we deduce the corresponding muon anomaly

$$\alpha_{Z_R}^\mu = - \frac{em_\mu}{12\pi M_{Z_R}^2} \left( \frac{1 - \tan^2(\theta)(1 + \tan^2(\theta))}{\sin^2(\theta)(1 - \tan^2(\theta))} \right)$$  \hspace{1cm} (24)$$

The calculation of the muon anomaly corresponding to the charged higgs, represented by diagrams three and four in Fig 1, show that its contribution is negligible compared to the $W$ and $Z$ contribution.
For the scalar neutral higgs $H_1^0, H_2^0$ and the neutrals pseudoscalar higgs $\varphi_1^0$ and $\varphi_2^0$, we get the following results

$$a_{H_1^0}^\mu \approx \frac{\alpha}{8\pi s^2(\theta)} \frac{\tan^3(\beta)(1+\tan^2(\beta))}{(1-\tan^2(\beta))^2} \left( \frac{m_\mu}{M_{H_1^0}} \right)^2 \left( \frac{m_\mu}{M_{H_1^0}} \right)^2 \ln \left( \frac{M_{H_1^0}^2}{m_\mu^2} \right)$$

(25)

$$a_{H_2^0}^\mu \approx \frac{\alpha}{8\pi s^2(\theta)} \frac{\tan^3(\beta)(1+\tan^2(\beta))}{(1-\tan^2(\beta))^2} \left( \frac{m_\mu}{M_{H_2^0}} \right)^2 \left( \frac{m_\mu}{M_{H_2^0}} \right)^2 \ln \left( \frac{M_{H_2^0}^2}{m_\mu^2} \right)$$

(26)

$$a_{\varphi_1^0}^\mu = \frac{\alpha}{8\pi s^2(\theta)} \frac{\tan^3(\beta)(1+\tan^2(\beta))}{(1-\tan^2(\beta))^2} \left( \frac{m_\mu}{M_{\varphi_1^0}} \right)^2 \int_0^1 dx \frac{x^3}{x^2+(1-x)\frac{m_\mu^2}{m_{\varphi_1^0}^2}}$$

(27)

$$a_{\varphi_2^0}^\mu = \frac{\alpha}{8\pi s^2(\theta)} \frac{\tan^3(\beta)(1+\tan^2(\beta))}{(1-\tan^2(\beta))^2} \left( \frac{m_\mu}{M_{\varphi_2^0}} \right)^2 \int_0^1 dx \frac{x^3}{x^2+(1-x)\frac{m_\mu^2}{m_{\varphi_2^0}^2}}$$

(28)

Where $\beta$ is a free parameter of the LRSM defined as

$$\tan(\beta) = \frac{\kappa_1}{\kappa_2}$$

(29)

3.3. Numerical results

To get an estimate of the value of muon anomalous magnetic moment in the LRSM, we use the following values for the LRSM parameters: $\alpha = \frac{1}{137}, M_{Z_L} = 90 GeV, M_{W_R} = 1 TeV, M_{Z_R} = TeV, M_{H_1^0} = M_{H_2^0} = M_{\varphi_1^0} = M_{\varphi_2^0} = 5 TeV, \tan(\beta) = 10, \sin^2(\theta) = 0.223$. After the summation of all contributions, we get the final result

$$a_{LR}^\mu = 0.137 \times 10^{-10}$$

(30)

so, the deviation of the experimental value of the AMM of the muon from the SM prediction is reduced in the LRSM to

$$\Delta a_\mu = a_{LR}^{\text{exp}} - a_{LR}^\mu = 23.26(0.9) \times 10^{-10}$$

(31)

4. Conclusions and Summary

The LRSM is an alternative candidate of new physics beyond the SM which can explain parity violation at low energies. The LRSM predicts new particles, such as heavy gauge bosons ($W_R, Z_R$), and heavy charged and neutral higgs ($H_{1,2}, H_{1,2}^0, \varphi_{1,2}^0$). In this work, we have calculated the muon anomalous magnetic moment at the one loop level in the LRSM. We find that the LRSM electroweak contribution can reduce slightly the deviation of the theoretical prediction of muon AMM from the experimental result. The total contribution of LRSM to muon $g-2$ is about $0.137 \times 10^{-10}$, so the muon anomaly decreases from $\Delta a_\mu = 23.4 \times 10^{-10}$ in the SM to $\Delta a_\mu = 23.26 \times 10^{-10}$ in the LRSM, which is $0.6 \%$ smaller. We conclude that the LRSM gives a small but non-neglegable extra contribution to muon $g-2$, and reduce the deviation $\Delta a_\mu$ from 2.6$\sigma$ in the SM to 2.5$\sigma$ in the LRSM.

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