Thermodynamic constraints on matter creation models

R. Valentim

Departamento de Física, Instituto de Ciências Ambientais, Químicas e Farmacêuticas - ICAQF,
Universidade Federal de São Paulo (UNIFESP) Unidade José Alencar,
Rua São Nicolau No. 210, 09913-030 – Diadema, SP, Brazil

J. F. Jesus

Universidade Estadual Paulista (UNESP), Câmpus Experimental de Itapeva
Rua Geraldo Alckmin 519, 18409-010,
Vila N. Sra. de Fátima, Itapeva, SP, Brazil and
Universidade Estadual Paulista (UNESP), Faculdade de Engenharia de Guaratinguetá
Departamento de Física e Química,
Av. Dr. Ariberto Pereira da Cunha 333,
12516-410 - Guaratinguetá, SP, Brazil

Abstract

Entropy is a fundamental concept from Thermodynamics and it can be used to study models on context of Creation Cold Dark Matter (CCDM). From conditions on the first ($\dot{S} \geq 0$) and second order ($\ddot{S} < 0$) derivatives of total entropy in the initial expansion of Sitter through the radiation and matter eras until the end of Sitter expansion, it is possible to estimate the intervals of the parameters of the Creation of Cold Dark Matter (CCDM) models. The total entropy is calculated with the sum of the matter entropy plus the entropy of the event horizon. This term derives from the Holographic Principle where it suggests that all information is contained on the observable horizon. The main feature of this method for these models are that thermodynamic equilibrium is reached in a final de Sitter era. Total entropy of the universe is calculated with three terms: apparent horizon ($S_h$), entropy of matter ($S_m$) and entropy of radiation ($S_r$). This analysis allows to estimate intervals of parameters of CCDM models.

PACS numbers:
Keywords: Entropy, Holographic Principle and CCDM models

*Electronic address: valentim.rodolfo@unifesp.br
†Electronic address: jfjesus@itapeva.unesp.br
I. INTRODUCTION

Physical systems tend spontaneously to reach thermodynamic equilibrium when energy is exchanged between a physical system and its neighborhoods. This is the empirical basis of the second law of Thermodynamics. The Laws of Thermodynamics state that the entropy (S) for closed systems remain constant or increase (S ≥ 0). The second order derivative (S < 0), at least roughly, leads to thermodynamic equilibrium [1, 2]. One way of imposing the condition on second order derivatives in cosmic expansion is through the Holographic Principle proposed by [3] and [4], with direct application in Cosmology [5, 6]. This principle assumes that all information is on the universe horizon.

We will explore in this work the calculations of the total entropy (St) from holographic principle for five models of matter creation. These models were studied by [7] and it assumes that the creation rate Γ is a function of the Hubble parameter H. Each dark matter creation rate leads to a different cosmic evolution [8–13]. A common feature of these models is that the Universe starts in an inflationary, de Sitter phase, then it passes through the ages of radiation and matter, where it finally enters the final de Sitter stage. Total entropy (S) at each phase is equal to the sum of each entropy contribution for these different ages. S is the direct sum of the contribution of entropy to radiation, matter and the apparent horizon of the Holographic Principle [3, 4]:

\[ S = S_\gamma + S_m + S_h; \]

where \( S_h = \frac{k_B A}{4\pi_\ell^2} \) is the entropy of the apparent horizon, \( S_m \) is entropy of pressureless matter and \( S_\gamma \) is entropy of radiation. \( A \) and \( \ell_\ell \) denote the area of the horizon and Planck’s length, respectively. In an ever expanding universe, the conditions \( \dot{S}(t) > 0 \), \( \ddot{S}(t \to \infty) < 0 \) are equivalent to the conditions \( S'(a) > 0 \), \( S''(a \to \infty) < 0 \). Restricting our analysis to this class of models, we shall consider the entropy as a function of the scale factor from now on.

In this work, entropy evolution will be considered, initially based on the model proposed by [8, 9] and the models analyzed by [7]. We can use the conditions on the derivatives of the total entropy to estimate the intervals of validity of free parameters for each model [2]. We shall assume a FRW metric for a spatially flat universe, which is in agreement with the Cosmological Principle, predictions from inflation and recent Cosmic Microwave Background (CMB) observations.

II. CREATION OF COLD DARK MATTER MODELS (CCDM)

Models of CCDM used in this work it were statistically analyzed by [7] and have a natural dependence of H (Γ ≡ Γ(H)), where Γ as function of Hubble parameter represents a relation between the matter creation and expansion rates. All the CCDM models used here have also free parameters. The models studied here were analyzed by [7] using three statistical criteria: Bayesian Information Criterion (BIC), Akaike Information Criterion (AIC) and Bayesian Evidence (BE) using SNe Ia dataset. Most of these models can be
described by a function $\Delta = \beta E + \alpha E^{-n}$, where $\Delta \equiv \frac{\Gamma}{3H}$ and $E \equiv \frac{H}{H_0}$. So, it corresponds to a creation rate $\Gamma = 3\beta H + 3\alpha H_0 \left(\frac{H_0}{H}\right)^n$.

Another model analyzed in [7] is LJO [14] with $\Gamma = 3\alpha \frac{\rho_{dm}}{\rho_{dm}} H$. LJO model has the same dynamics as $\Lambda$CDM concordance model. In LJO, the cosmological constant is exactly mimicked by particle creation. Due to this mimicking, we choose not to analyze this model here, as $\Lambda$CDM has already been thoroughly analyzed on [15]. In all models analyzed in this work we have neglected the contribution of baryons. Baryonic contribution is small, $\sim 5\%$ of Universe content and our results can be more dependent on the assumptions made here in order to estimate entropy rather than baryonic influence. Another important assumption is that Universe is spatially flat as indicated from CMB and preferred by inflation, i.e. $\Omega_k \equiv 0$ in our analysis. The models studies here are described on Table I.

| Model | Creation rate | Reference | Parameters |
|-------|---------------|-----------|------------|
| $M_1$ | $\Gamma = \frac{3\alpha H_0^2}{H}$ | [13] (JO) | $\beta = 0$, $n = 1$ |
| $M_2$ | $\Gamma = 3\alpha H_0$ | [16] | $\beta = 0$, $n = 0$ |
| $M_3$ | $\Gamma = 3\beta H$ | | $\alpha = 0$ |
| $M_4$ | $\Gamma = 3\alpha H_0 \left(\frac{H_0}{H}\right)^n$ | | $\beta = 0$ |
| $M_5$ | $\Gamma = 3\alpha \frac{H_0^2}{H} + 3\beta H$ | [16] | $n = 1$ |

Table I: Models and parameters.

III. METHODOLOGY

The methodology adopted here consists on analyzing total entropy of the Universe in the context of matter creation models. This analysis allows to estimate the validity interval for free parameters for each model. This idea is based on [2], where authors analyzed first and second order derivatives. It assumes the Second Law of Thermodynamics jointly with the idea that thermodynamic equilibrium must be achieved at some future time. An important aspect of this method is that it takes into account the horizon entropy that came from Holographic Principle [3–5] where all the information about Universe is on horizon. The total entropy is given by equation (1) and it is defined as sum of radiation, matter and apparent horizon. Restricting our analysis to CCDM models [7, 9, 16–18], entropy was considered as a function of the scale factor. In CCDM, expansion acceleration can be achieved through an effective creation pressure:

$$p_c = -\frac{(\rho + p)\Gamma}{3H} = -\frac{\rho \Gamma}{3H};$$

where $p_c$ is creation pressure, $\rho$ is dark matter (DM) density (pressure $p$ vanishes for DM), $\Gamma$ is creation rate and $H$ is Hubble parameter. Relation between Hubble parameter and $\rho$ is the Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\rho.$$
for spatially flat Universe \((k = 0)\). The equation of continuity for dark matter now reads:

\[
\dot{\rho} + 3H\rho = \Gamma\rho, \tag{4}
\]

That is \(\Gamma\rho\) is a source \((\Gamma > 0)\) or sink \((\Gamma < 0)\) for dark matter. Note that we use the dot notation to express the derivative with respect to time : \(\equiv d/dt\). The Hubble parameter corresponds to the expansion rate, that is, \(H = \dot{a}/a\), so, writing it as a function of scale factor, we have:

\[
\rho'(a) = \frac{\rho(a)}{aH}(\Gamma - 3H). \tag{5}
\]

where we denoted the derivative with respect to \(a\) with a prime. The relation between matter density \(\rho\) and particle number density \(n\) is \(\rho = nm\), where \(m\) is mass of DM particle, so, we have:

\[
n' = \frac{n}{aH}(\Gamma - 3H). \tag{6}
\]

The Friedmann equation and continuity equation fully describe the CCDM background dynamics. From these equations we can derive a relation between \(H\) and \(\Gamma\):

\[
\dot{H} + \frac{3}{2}H^2 \left(1 - \frac{\Gamma}{3H}\right) = 0 \tag{7}
\]

or, in terms of scale factor,

\[
H' = -\frac{3H}{2a} \left(1 - \frac{\Gamma}{3H}\right) \tag{8}
\]

This class of models suggests that matter creation \((\Gamma > 0)\) generates a negative pressure \((p_c < 0)\) which may explain the acceleration of the Universe.

**IV. THERMODYNAMICS OF MATTER CREATION MODELS**

In our analysis we are interested only on recent and future times, so we shall restrict ourselves to the matter dominated age, as radiation becomes negligible in the past. From equation \([1]\), shown earlier, we will analyze the derivatives of each of the terms for the total entropy: entropy of the apparent horizon, matter and radiation \([2]\).

Entropy of apparent horizon is \(S_h = k_B A / (4l_{Pl}^2)\), where \(A\) denotes the area of apparent horizon and \(l_{Pl}\) is Planck’s length. The area of the apparent horizon is given by \(A = 4\pi \tilde{r}_A^2\), where \(\tilde{r}_A = \frac{1}{\sqrt{H^2 + ka^2}}\). As we are restricting our analysis to spatially flat models \((k = 0)\), this assumption yields \(\tilde{r}_A = H^{-1}\) and \(A = 4\pi H^{-2}\). In this case, the horizon entropy reads:

\[
S_h = \frac{k_B \pi}{l_{Pl}^2 H^2}; \tag{9}
\]

That is, entropy is function of Hubble parameter only. Thus, the first derivative of apparent horizon entropy with respect to scale factor is:

\[
S_h' = -\frac{2k_B \pi H'}{l_{Pl}^2 H^3}. \tag{10}
\]
The first-order derivative of the entropy results in an expression that is a function of $H$ and its first derivative. Eq. (8) yields $H' = \frac{\Gamma - 3H}{2a}$, thus we may write for $S'_h$:

$$S'_h = \frac{k_B \pi}{\ell_P^2 a H^3}(3H - \Gamma). \quad (11)$$

For the $S_m$ entropy, we may consider that every single particle contributes to the entropy inside the horizon by a single bit, $k_B$.[2]. In this case, we have:

$$S_m = k_B \frac{4\pi}{3} \rho_3 A_n = k_B \frac{4\pi n}{3H^2}, \quad (12)$$

where $n$ is the number density. By deriving this equation we find:

$$S'_m = \frac{4\pi k_B}{3H^4}(n'H - 3n'H'). \quad (13)$$

This expression is first derivative of entropy as function $H$, $H'$ and $n$. By using Eqs. (6) and (8), we may write:

$$S'_m = \frac{2\pi k_B n}{3aH^4}(3H - \Gamma) \quad (14)$$

A derivative entropy of matter is functions $H, n$ and $\Gamma$. Now combining Eqs. (10) and (14), we have

$$S' = \frac{k_B \pi}{aH^3} \left( \frac{1}{\ell_P^2} + \frac{2n}{3H} \right) (3H - \Gamma) \quad (15)$$

So, a necessary and sufficient condition for having $S' \geq 0$ is $\Gamma \leq 3H$, that is, the particle creation rate must be less or equal to the volumetric expansion rate$^1$. Let us define the dimensionless quantity $s_1$:

$$s_1 \equiv \frac{3H - \Gamma}{H_0} \quad (16)$$

Thus, $S' \geq 0$ corresponds to $s_1 \geq 0$. Now, let us impose the concavity condition $S''(a \to \infty) < 0$, that is, impose that the Universe reaches thermodynamic equilibrium in infinite future. By deriving (10):

$$S''_h = \frac{2\pi k_B}{\ell_P^2 H^4}(3H'^2 - HH'') \quad (17)$$

Using $\rho = nm$ and deriving the Friedmann equation (3), we have

$$2HH' = 8\pi G n'm = \frac{8\pi G n'm}{3} \quad (18)$$

Combining this with the Friedmann equation, we find the relation$^2$

$$2\frac{H'}{H} = \frac{n'}{n} \Rightarrow 2H'n = Hn' \quad (19)$$

---

$^1$ Any volume in the Hubble flow scales with $a^3$, thus $\dot{V} = 3H$.

$^2$ It can also be found from Eqs. (6) and (8).
That is, the particle density relative variation (w.r.t. $a$) is double of Hubble parameter relative variation. We may use this to simplify Eq. (13):

$$S_m' = -\frac{4\pi k_B n H'}{3H^4}$$

(20)

Now it is easier to derive it to find $S_m''$.

$$S_m'' = \frac{4\pi k_B n}{3H^5} (2H'^2 - HH'')$$

(21)

where we have derived (20) and used the relation (19) again in order to omit $n$ derivatives. By summing (17) and (21), we find:

$$S'' = \frac{2\pi k_B}{\ell_p^2 H^4} (3H'^2 - HH'') + \frac{4\pi k_B n}{3H^5} (2H'^2 - HH'')$$

(22)

Let us define the dimensionless quantities:

$$s_{h2} \equiv \frac{3H'^2 - HH''}{H_0^2}$$

(23)

$$s_{m2} \equiv \frac{2H'^2 - HH''}{H_0^2}$$

(24)

Thus, the conditions $S''_h < 0$ and $S''_m < 0$ correspond to $s_{h2} < 0$ and $s_{m2} < 0$, respectively. A sufficient (but not necessary) condition for having $S'' < 0$ is having both $s_{h2} < 0$ and $s_{m2} < 0$. Although it may be a much stringent condition over the models, we consider it reasonable in order to achieve a result not much dependent on the choice of the contribution of each particle to the entropy ($S_m/N$).

Another interesting inference we can make from expressions (23) and (24) is that $s_{h2} = s_{m2} + \frac{H'^2}{H_0^2}$, so $s_{h2}$ $\geq$ $s_{m2}$ at all times, so every time that $s_{h2} < 0$, we have $s_{m2} < 0$. That is, $S''_h < 0$ implies $S''_m < 0$.

In the next section we will analyze a quite general model for the rate of creation of dark matter with three free parameters.

V. CASE STUDY: $\Gamma = 3\beta H + 3\alpha H_0 \left(\frac{H_0}{H}\right)^n$

We now analyze a quite general model of the matter creation rate which was derived by [16] with three free parameters: $\alpha$, $\beta$ and $n$. All the models that we will deal with here are particular cases of this model whose dependence with $H$ is given by:

$$\Gamma = 3\beta H + 3\alpha H_0 \left(\frac{H_0}{H}\right)^n$$

(25)

This model for $\Gamma$ is a combination of two important dependencies: the first term $\propto H$ and the second term $\propto H^{-n}$. In this case, Eq. (8) reads

$$\frac{dE}{da} = \frac{3}{2a} \left[\alpha E^{-n} - (1 - \beta)E\right]$$

(26)
where \( E(a) \equiv \frac{H(a)}{H_0} \). As shown by \([7]\), Eq. (26) can be solved as
\[
E(a) = \frac{H(a)}{H_0} = \left[ \frac{\alpha + (1 - \alpha - \beta)a^{-\frac{3}{2}(n+1)(1-\beta)}}{1 - \beta} \right]^{\frac{1}{n+1}},
\] (27)
in case that \( \beta \neq 1 \) and \( n \neq -1 \). Case \( n = -1 \) is equivalent to \( \alpha = 0 \). If \( \beta = 1 \), \( E(a) \) can be obtained from (26) as
\[
E = \left[ 1 + \frac{3\alpha(n+1)}{2} \ln a \right]^{\frac{1}{n+1}}
\] (28)

The eq. (27) shows \( H(a) \) as a function of scale factor \( a \), \( H_0 \), \( \alpha \), \( \beta \) and \( n \). By writing \( H(a) \) as an explicit function of the parameters, we can now impose the condition \( S' \geq 0 \).

From the Eq. (15) and (16) it yields:
\[
s_1 = 3 \left( \frac{H}{H_0} \right)^{-n} \left[ (1 - \alpha - \beta)a^{-\frac{3}{2}(n+1)(1-\beta)} \right] \geq 0.
\] (29)

We must have \( S' \geq 0 \) at all times, so we must have \( 1 - \alpha - \beta \geq 0 \) by this analysis. According to \([17]\), \( S''_h < 0 \) implies \( 3H^2 - HH'' < 0 \), so
\[
s_{h2} = 3 \left( \frac{H}{H_0} \right)^{-2n} \left( \frac{1 - \alpha - \beta}{1 - \beta} \right) \times a^{-2-\frac{3}{2}(n+1)(1-\beta)} \left\{ 2(1 - \alpha - \beta)(2 - 3\beta)a^{-\frac{3}{2}(n+1)(1-\beta)} + \alpha [3\beta - 3n(1 - \beta) - 5] \right\} < 0
\] (30)

Now, let us impose the condition \( S''_m < 0 \). It implies, from (21) that \( 2H^2 - HH'' < 0 \). We have
\[
s_{m2} = 3 \left( \frac{H}{H_0} \right)^{-2n} \left( \frac{1 - \alpha - \beta}{1 - \beta} \right) \times a^{-2-\frac{3}{2}(n+1)(1-\beta)} \left\{ (1 - \alpha - \beta)(1 - 3\beta)a^{-\frac{3}{2}(1+n)(1-\beta)} + \alpha [3\beta - 3n(1 - \beta) - 5] \right\} < 0
\] (31)

We remind that we are interested in the sign of (30) and (31) only in the limit \( a \to \infty \). However, this limit is strongly dependent in the parameter set \( \{\alpha, \beta, n\} \), so, instead of putting limits for the general model, we shall put limits for each particular model. Let us do it in next subsections.

A. \( M_1 : \Gamma = \frac{3\alpha H^2}{H_0} \)

In this case we have the fixed parameter values \( \beta = 0 \) and \( n = 1 \), so from (29) we see that \( S' \geq 0 \) reads
\[
s_1 = 3(1 - \alpha) \left( \frac{H_0}{H} \right) a^{-3} \geq 0.
\] (32)
which implies \( \alpha \leq 1 \). From (30), the condition \( S''_h < 0 \) reads
\[
s_{h2} = 3 \left( \frac{H_0}{H} \right)^2 (1 - \alpha) a^{-5} [(1 - \alpha)a^{-3} - 2\alpha] < 0
\] (33)
Thus, for $a \to \infty$, it implies $0 < \alpha < 1$. From (31), the condition $S''_m < 0$ reads

$$s_{m2} = \frac{3}{4} \left( \frac{H_0}{H} \right)^2 (1 - \alpha)^2 \left[(1 - \alpha) a^{-5} - 8\alpha \right] < 0$$

(34)

which yields the same limit for $a \to \infty$, $0 < \alpha < 1$.

In Figure 1, we may see that $s_1 \geq 0$ for $\alpha \leq 1$ and $s_{h2}(a \to \infty) < 0$ for $0 < \alpha < 1$, in agreement with our analysis. As discussed above, $s_{h2} < 0$ implies $s_{m2} < 0$, so we choose to plot only $s_1$ and $s_{h2}$ for each model, for clarity.

**Figure 1:** Model $M_1$: $s_1$ and $s_{h2}$ as function of scale factor for some values of $\alpha$.

**B.** $M_2 : \Gamma = 3\alpha H_0$

In this case we have the fixed parameter values $\beta = 0$ and $n = 0$, so from (29) we see that $S' \geq 0$ reads

$$s_1 = 3(1 - \alpha) a^{-\frac{7}{2}} \geq 0$$

(35)

which implies $\alpha \leq 1$. From (30), the condition $S''_h < 0$ reads

$$s_{h2} = \frac{3}{4} (1 - \alpha) a^{-\frac{7}{2}} \left[4(1 - \alpha) a^{-\frac{7}{2}} - 5\alpha\right] < 0$$

(36)

Thus, for $a \to \infty$, it implies $0 < \alpha < 1$. From (31), the condition $S''_m < 0$ reads

$$s_{m2} = \frac{3}{4} (1 - \alpha) a^{-\frac{7}{2}} \left[(1 - \alpha) a^{-\frac{7}{2}} - 5\alpha\right] < 0$$

(37)

which yields the same limit for $a \to \infty$, $0 < \alpha < 1$.

In Figure 2, we may see that $s_1 \geq 0$ for $\alpha \leq 1$ and $s_{h2}(a \to \infty) < 0$ for $0 < \alpha < 1$, in agreement with our analysis.
C. $M_3$: $\Gamma = 3\beta H$

In this case we have the fixed parameter value $\alpha = 0$, so from (29) we see that $S' \geq 0$ reads

$$s_1 = (1 - \beta)a^\frac{2}{3}(\beta - 1) \geq 0.$$  \hspace{1cm} (38)

which implies $\beta \leq 1$. From (30), the condition $S''_h < 0$ reads

$$s_{h2} = \frac{3}{2}(1 - \beta)(2 - 3\beta)a^{3\beta - 5} < 0$$  \hspace{1cm} (39)

Thus, it implies $\frac{2}{3} < \beta < 1$. From (31), the condition $S''_m < 0$ reads

$$s_{m2} = \frac{3}{4}(1 - \beta)(1 - 3\beta)a^{3\beta - 5} < 0$$  \hspace{1cm} (40)

which yields the limit $\frac{1}{3} < \beta < 1$. As one may see, for all the interval that we have $S''_h < 0$ we have also $S''_m < 0$, as expected.

In Figure 3, we may see that $s_1 \geq 0$ for $\beta \leq 1$ and $s_{h2}(a \to \infty) < 0$ for $\frac{2}{3} < \beta < 1$, in agreement with our analysis.

Figure 2: Model $M_2$: $s_1$ and $s_{h2}$ as function of scale factor for some values of $\alpha$.

Figure 3: Model $M_3$: $s_1$ and $s_{h2}$ as function of scale factor for some values of $\beta$. 

9
D. $M_4 : \Gamma = 3\alpha H_0 \left(\frac{H_0}{H}\right)^n$ 

In this case we have the fixed parameter value $\beta = 0$, so from (29) we see that $S' \geq 0$ reads

$$s_1 = 3(1 - \alpha) \left(\frac{H}{H_0}\right)^{-n} a^{-\frac{3}{2}(n+1)} \geq 0.$$ 

(41)

which implies $\alpha \leq 1$. From (30), the condition $S''_h < 0$ reads

$$s_{h2} = \frac{3}{4} \left(\frac{H}{H_0}\right)^{-2n} (1 - \alpha) a^{-2-\frac{3}{2}(n+1)} \left[4(1 - \alpha)a^{-\frac{3}{2}(n+1)} - \alpha (3n + 5)\right] < 0$$ 

(42)

For $a \to \infty$, there are some subcases here, according to the sign of the exponent $-\frac{3}{2}(n+1)$, that is, if $n$ is greater than $-1$ or not. If $n > -1$, the condition can be summarized as $\alpha(\alpha - 1)(3n + 5) < 0$. As $3n + 5 > 0$, it implies $0 < \alpha < 1$. If $n < -1$, the condition is $(\alpha - 1)^2 < 0$, which is impossible, so $n < -1$ is discarded by this analysis. In the special case of $n = -1$, we recover the model $M_3$, so $\frac{2}{3} < \alpha < 1$.

From (31), the condition $S''_m < 0$ reads

$$s_{m2} = \frac{3}{4} \left(\frac{H}{H_0}\right)^{-2n} (1 - \alpha) a^{-2-\frac{3}{2}(n+1)} \left[(1 - \alpha)a^{-\frac{3}{2}(1+n)} - \alpha (3n + 5)\right] < 0$$ 

(43)

which yields the same limit for $a \to \infty$ and $n > -1$: $0 < \alpha < 1$. Just like before, $n = -1$ implies, like in $M_3$, $\frac{1}{3} < \alpha < 1$.

In Figure 4, we may see that $s_1 \geq 0$ for $\alpha \leq 1$ and $s_{h2}(a \to \infty) < 0$ for $0 < \alpha < 1$ and $n > -1$, in agreement with our analysis.

![Figure 4: Model $M_4$: $s_1$ and $s_{h2}$ as function of scale factor for some values of ($\alpha, n$).](image)

E. $M_5 : \Gamma = 3\alpha \frac{H_0^2}{H} + 3\beta H$

In this case we have the fixed parameter value $n = 1$, so from (29) we see that $S' \geq 0$ reads

$$s_1 = 3(1 - \alpha - \beta) \left(\frac{H_0}{H}\right) a^{-3(1-\beta)} \geq 0.$$ 

(44)
which implies \(1 - \alpha - \beta \geq 0\). From (30), the condition \(S''_h < 0\) reads
\[
s_{h2} = \frac{3}{2} \left( \frac{H_0}{H} \right)^2 \frac{1}{(1 - \beta)} \left[(1 - \alpha - \beta)(2 - 3\beta)a^{-3(1-\beta)} + \alpha (3\beta - 4)\right] < 0
\]
(45)

To analyze the behaviour for \(a \to \infty\) we have to make assumptions about the scale factor exponent, \(-3(1 - \beta)\). If \(\beta < 1\), \(S''_h < 0\) implies \(\alpha(1 - \alpha - \beta)(3\beta - 4) < 0\). If we combine with the condition from \(s_1\), we must have \(1 - \alpha - \beta > 0\), thus it simplifies to \(\alpha(3\beta - 4) < 0\). Thus, \(\alpha > 0\) and \(\beta < \frac{4}{3}\) or \(\alpha < 0\) and \(\beta > \frac{4}{3}\).

For \(\beta > 1\), \(S''_h < 0\) would imply \(\beta < \frac{2}{3}\), so \(\beta > 1\) is not allowed by this analysis.

If \(\beta = 1\), Eq. (28) with \(n = 1\) yields
\[
E = [1 + 3\alpha \ln a]^{1/2},
\]
from which we find
\[
s_{h2} = \frac{3\alpha}{2a^2} \left( \frac{H_0}{H} \right)^2 (1 + 6\alpha + 3\alpha \ln a)
\]
(47)

In this case, in the limit \(a \to \infty\), \(S''_m < 0\) implies \(\alpha^2 < 0\), that is, \(\beta = 1\) is not allowed by this analysis.

From (31), the condition \(S''_m < 0\) reads
\[
s_{m2} = \frac{3}{4} \left( \frac{H_0}{H} \right)^2 \frac{1}{(1 - \beta)} \left[(1 - \alpha - \beta)(2 - 3\beta)a^{-3(1-\beta)} + 2\alpha (3\beta - 4)\right] < 0
\]
(48)

In this case, in the limit \(a \to \infty\), for \(\beta < 1\), \(S''_m < 0\) implies \(\alpha(3\beta - 4)(1 - \alpha - \beta) < 0\). Combining it with the condition from \(s_1\), we have \(1 - \alpha - \beta > 0\), thus \(\alpha(3\beta - 4) < 0\). So, if \(\alpha > 0\), \(\beta < \frac{4}{3}\) and if \(\alpha < 0\), we have \(\beta > \frac{4}{3}\).

For \(\beta > 1\), \(S''_m < 0\) would imply \(\beta < \frac{1}{3}\), so \(\beta > 1\) is not allowed by this analysis.

For \(\beta = 1\), \(s_{m2}\) is written:
\[
s_{m2} = \frac{3\alpha}{4a^2} \left( \frac{H_0}{H} \right)^2 (2 + 9\alpha + 6\alpha \ln a)
\]
(49)

In this case, in the limit \(a \to \infty\), \(S''_m < 0\) implies \(\alpha^2 < 0\), that is, \(\beta = 1\) is not allowed by this analysis. The limits for model \(M_5\) can be viewed on Fig. 5.

In Figure 6 we may see that \(s_1 \geq 0\) for \(\alpha \leq 1\) and \(s_{h2}(a \to \infty) < 0\) for \(0 < \alpha < 1\) and \(n > -1\), in agreement with our analysis.

The results of all models from Table I can be seen on Table II.

VI. DISCUSSION AND CONCLUDING REMARKS

We have analyzed the Thermodynamics of 5 spatially flat CCDM models, taking into account a contribution from the horizon entropy, based on Holographic Principle.

In principle, the initial state of de Sitter age should be stable (\(H\) and \(S\) constants when \(t \to \infty\)) but particle creation (\(\Gamma\)), according to [2], can be seen as an external agent acting
on the system. Before the thermodynamic equilibrium was reached, the Universe needed to self-adjust to allow the ultimate expansion of de Sitter through the ages of radiation and matter.

The rate of particle production is irreversible, in this case for the five models treated in this work. In practice, irreversibility directly implies the generation of entropy \([17]\), as well as the increase in volume in the phase space. In our analysis, the particle production rate \(\Gamma\) for the five models analyzed, was implicitly or explicitly included in the expressions for \(S'\) and \(S''\), as can be seen in equations \([15]\) and \([22]\). For easy of analysis we defined the quantities \(s_1\) for the first derivative and \(s_{h2}\) and \(s_{m2}\) for the second order derivatives. All models discussed in this work are particular cases of the general model with three free
parameters: $\alpha$, $\beta$ and $n$. The $M_1$ model has only one free parameter $\alpha$ and the analysis of the derivatives suggests that it is between $0 < \alpha < 1$. $M_2$ is a model similar to $M_1$ but with constant $\Gamma$, the limits for $\alpha$ is $0 < \alpha < 1$. The $M_3$ model has $\beta$ as a free parameter and $\Gamma$ varies linearly with $H$ and $\frac{2}{3} < \beta < 1$. $M_4$ has two free parameters: $\alpha$ and $n$, $\Gamma$ is a power law over $\dot{H}$: $\Gamma \propto H^{-n}$. The validity interval was $0 < \alpha < 1$ with $n > -1$, for $n = 0$ $M_4$ corresponds to $M_2$ and if $n = 1$ it becomes $M_1$. For the model $M_5$, which is a combination of $M_1$ and $M_3$, $\beta \leq 1 - \alpha$ and $\alpha(3\beta - 4) < 0$.

The limits over the parameters $\alpha$ and $\beta$ could be seen in figure 5.

Further analysis of matter creation models may include the conserved baryonic contribution and spatial curvature. Other creation rates not considered here could also be analyzed.

**Acknowledgments**

JFJ is supported by Fundação de Amparo à Pesquisa do Estado de São Paulo - FAPESP (Process no. 2017/05859-0) and R. V. has been supported by Fundação de Amparo à Pesquisa do Estado de São Paulo - FAPESP (Process no. 2013/26258-4 and 2016/09831-0). This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

---

[1] Callen, H. B. “Thermodynamics and an Introduction to Thermostatistics”, 2nd Edition, pp. 512. Wiley-VCH,1985.
[2] Mimoso, J. P., & Pavón, D. 2013, Phys. Rev. D , 87, 047302.
[3] 't Hooft, G. 1993, arXiv:gr-qc/9310026.
[4] Susskind, L. 1995, Journal of Mathematical Physics, 36, 6377.
[5] Fischler, W., & Susskind, L. 1998, arXiv:hep-th/9806039.
[6] Bak, D., & Rey, S.-J. 2000, Classical and Quantum Gravity, 17, L83.
[7] J. F. Jesus, R. Valentim and F. Andrade-Oliveira, JCAP **1709** (2017) no.09, 030 [arXiv:1612.04077 [astro-ph.CO]].
[8] Lima, J. A. S., Basilakos, S., & Costa, F. E. M. 2012, Phys. Rev. D , 86, 103534.
[9] Lima, J. A. S., Basilakos, S., & Solà, J. 2013, Monthly Notices of Royal Astronomical Society, 431, 923.
[10] Freaza, M. P., de Souza, R. S., & Waga, I. 2002, Phys. Rev. D, 66, 103502.
[11] Lima, J. A. S., Jesus, J. F., & Oliveira, F. A. 2010, Journal of Cosmology and Astroparticle Physics, 11, 027.
[12] Lima, J. A. S., Silva, F. E., & Santos, R. C. 2008, Classical and Quantum Gravity, 25, 205006.
[13] J. F. Jesus and F. Andrade-Oliveira, JCAP 1601, 014 (2016) arXiv:1503.02595 [astro-ph.CO]]
[14] J. A. S. Lima, J. F. Jesus and F. A. Oliveira, JCAP 1011, 027 (2010) arXiv:0911.5727 [astro-ph.CO]]
[15] N. Radicella and D. Pavon, Gen. Rel. Grav. 44 (2012) 685 arXiv:1012.0474 [gr-qc]]
[16] L. L. Graef, F. E. M. Costa and J. A. S. Lima, Phys. Lett. B 728, 400 (2014) arXiv:1303.2075 [astro-ph.CO]]
[17] M. O. Calvão, J. A. S. Lima and I. Waga, Phys. Lett. A 162 (1992) 223.
[18] J. A. S. Lima, S. Basilakos and F. E. M. Costa, Phys. Rev. D 86 (2012) 103534 arXiv:1205.0868 [astro-ph.CO]].