SINS of Viscosity Damped Turbulence

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Abstract. The problems with explaining the Small Ionized and Neutral Structures (SINS) appealing to turbulence stem from inefficiency of the Kolmogorov cascade in creating large fluctuations at sufficiently small scales. However, other types of cascades are possible. When magnetic turbulence in a fluid with viscosity that is much larger than resistivity gets to a viscous damping scale, the turbulence does not vanish. Instead, it gets into a different new regime. Viscosity-damped turbulence produces fluctuations on the small scales. Magnetic fields sheared by turbulent motions by eddies not damped by turbulence create small scale filaments that are confined by the external plasma pressure. This creates small scale density fluctuations. In addition, extended current sheets create even stronger density gradients that accompany field reversals in the plane perpendicular to mean magnetic field. Those can be responsible for the SINS formation. This scenario is applicable to partially ionized gas. More studies of reconnection in the viscosity dominated regime are necessary to understand better the extent to which the magnetic reversals can compress the gas.

1. Structures below Viscous Damping Scale

Turbulence can be viewed as a cascade of energy from a large injection energy scale to dissipation at a smaller scale. The latter is being established by equating the rate of turbulent energy transfer to the rate of energy damping arising, for instance, from viscosity. Naively, one does not expect to see any turbulent phenomena below such a scale.

Such reasoning may not be true in the presence of magnetic field, however. Consider magnetized fluid with viscosity $\nu$ much larger than magnetic diffusivity $\eta$, which is the case of a high magnetic Prantl number $Pr$ fluid. The partially ionized gas can serve as an example of such a fluid up to the scales of ion-neutral decoupling (see a more rigorous treatment in Lazarian, Vishniac & Cho 2004, henceforth LVC04). Fully ionized plasma is a more controversial example. For instance, It is well known that for plasma the diffusivities are different along and perpendicular to magnetic field lines. Therefore, the plasma the Prandtl number is huge if we use the parallel diffusivity $\nu_\parallel \gg \nu_\perp$. A treatment of the fully ionized plasma as a high Prandtl number medium is advocated in Schechochihin et al (2004, henceforth SCTMM).

The turbulent cascade in a fluid with isotropic $\eta$ proceeds up to a scale at which the cascading rate, which for the Kolmogorov turbulence, i.e. $v_t \sim l^{1/3}$, is determined by the eddy turnover rate $v_t/l$ gets equal to the damping rate $\eta/l^2$. Assuming that the energy is injected at the scale $L$ and the injection velocity is $V_L$, the damping scale $l_c$ is $LRe^{-3/4}$, where $Re$ is the Reynolds number $LV_L/\eta$. 

1
Figure 1. **Left:** Filaments of density created by magnetic compression in the slice of data cube of the viscosity-damped regime of MHD turbulence. **Right:** Spectra of density and magnetic field are similar, while velocity is damped. The resistive scale in this regime is not $L/R_m$ but $LR_m^{-1/2}$ (from Beresnyak & Lazarian, in preparation).

However, it is evident that magnetic fields at the scale $l_c$ at which hydrodynamic cascade would stop are still sheared by eddies at the larger scales. This should result in creating magnetic structures at scales $\ll l_c$ (LVC04). Note that magnetic field in this regime is not a passive scalar, but important dynamically.

Cho, Lazarian & Vishniac (2002) reported a new viscosity-damped regime of turbulence using incompressible MHD simulations (see an example of more recent simulations in Fig. 1). In this regime, unlike hydro turbulence, motions, indeed, do not stop at the viscosity damping scale, but magnetic fluctuations protrude to smaller scales. Interestingly enough, these magnetic fluctuations induce small amplitude velocity fluctuations at scales $\ll l_c$.

Cho & Lazarian (2003) confirmed these results with compressible simulations and speculated that these small scale magnetic fluctuations can compress ambient gas to produce SINS (see also Fig. 1b). According to the model in LVC04, while the spectrum of averaged over volume magnetic fluctuations scales as $E(k) \sim k^{-1}$, the pressure within intermittent magnetic structures increases with the decrease of the scale as $(\delta b_h^2)_k \sim k$, while the filling factor $\phi_k \sim k^{-1}$, the latter being consistent with numerical simulations. The pressure of gas confining the magnetic filaments should increase accordingly, resulting in fluctuations of density increasing with the decrease of the scale. The fact that the emerging structures are filamentary allows occasional picks in the observed column densities, which could correspond to the Heiles (1987) model of SINS.

In Fig. 1 we show the results of our high resolution compressible MHD simulations that exhibit strong density fluctuations at the scales below the one at which hydrodynamic turbulence would be damped. The testing of the LVC04 model is important by itself, even without a possible connection to SINS. According to that model, the viscosity-damped regime is ubiquitous in turbulent partially ionized gas. Some of its discussed consequences, such as an intermittent resumption of the turbulence the fluid of ions as magnetic fluctuations reach the ion-neutral decoupling scale, are important for radio scintillations.
2. Current Sheets in Viscosity-Damped Regime

We note, that irrespectively of the density fluctuations arising from compressions by magnetic fields, the viscosity-damped turbulence produces current sheets with length determined by the size of the eddies at the viscous damping scale $l_e$. If we assume that the thickness of the resulting current sheets is determined by the Sweet-Parker (henceforth SP) reconnection condition (see Parker 1979) and therefore is $d \sim l_e Rm^{-1/2}$, where $Rm$ is the Lunquist number for a scale $l_e$, i.e. $\eta / V_A l_e$, where $V_A$ is the Alfvén velocity. In the SP reconnection the magnetic pressure changes across the current sheet by $(\delta b)^2 / c_s^2$, which entails the corresponding variation of density $\delta n / n \sim (\delta b)^2 / P_{gas}$. Thus for favorable observing geometry the current layer induces the maximal variation of observed column density at the scale of $d$ which is $l_e (\delta b)^2$. This value is a factor of $(l_e / d)^{5/3} \sim Rm^{5/6}$ larger than the column density of an eddy at a scale created at the scale $d$, provided that the compression is due to $\delta b$, which cascading proceeds according to the Kolmogorov law.

Assume, for the sake of simplicity, that at the injection scale the injection velocity is equal to $V_A$. This is the situation for which the Goldreich-Sridhar (1995) model of MHD turbulence has been formulated originally. In some situations the discussion of a more general case is essential (see Lazarian 2006), but this goes beyond the scope of this short communication. In terms of the magnetic Prandtl number the amplification of density perturbations at a scale $d$ is of the order $Pr^{5/6} Re^{5/18}$. If, following Goldreich & Sridhar (2006, henceforth GS06), we use for plasma $Pr \approx 3 \times 10^8 (T / 10^4 K)$, then an eddy at scale $d$ can provide a density contrast that is $\sim 10^7 (T / 10^4 K)^{5/6} Re^{5/18}$ times larger than the Kolmogorov prediction.
3. Discussion

Recently GS06 attempted to explain the extreme radio-wave scattering in the direction of the Galactic Center assuming plasma can be described with high Pr number and the folded fields in the spirit of SCTMM are present along the line of sight. They do not appeal directly to the SP reconnection process, but the arguments about the current sheets there implicitly assume that the SP-type reconnection. As the result, their estimates of the variation of the column density can be obtained from ours assuming that $L = l_c$ for the folded fields model. Compared to the GS06 the model in the paper is applicable to the situations when strong mean magnetic field is present. If attempting to explain the extreme scattering events with viscosity-damped turbulence model, one could use the corresponding estimates in GS06, modifying the column density fluctuation as discussed above. In the highly viscous fluid the values of $L$ and $l_c$ do not differ dramatically, anyhow.

GS06 and the model above when applied to plasmas share the same set of problems. As GS06 admitted, the SCTMM folded field structure could unwind rapidly due to the low kinematic viscosity in directions perpendicular to the magnetic field. In view of this, the formation of the SINS in partially ionized gas (e.g. HI) could be seen as a safer bet.

Another potential difficulty shared by both models is related to using SP model of current sheets for both the folded fields and the viscosity damped turbulence. Usually, one would expect that the reconnection happens faster, opening up the reconnection layers (see Shay et al. 2001 for collisionless reconnection and Lazarian & Vishniac 1999 for stochastic reconnection). However, the conditions for the field wandering and the outflows within both the viscosity damped turbulence at scales less than $l_c$ (see LVC04) and the folded field dynamo (see SLTMM) differ from the usual formulation of the reconnection problem. Further research should clarify what what is going on in such situations. Potentially, compression in current sheets looks as an attractive solution of the SINS phenomenon.

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