Experimental determination of plate parameters with an air coupled instrument

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Abstract. The present work deals with an experimental determination of acoustical properties of a viscoelastic homogeneous plate using an air coupled equipment. Usually, these properties are determined separately through several measurements, while our paper provides a quick inverse method which permits to measure them simultaneously and requires only the knowledge of the sound velocity in air and its density. The experimental transmission through a face parallel plate, at normal incidence, is investigated and compared with the theoretically predicted one. Measurements are conducted using a chirp signal having 1.2 MHz central frequency with 800 kHz bandwidth and 100 µs duration. Two signals are being exploited: one only through air path and another one through the sample inserted between the transducers. The FFTs of these signals provide the complex transmission coefficient as a function of frequency. The identification of parameters is carried out using the real and imaginary parts of the transmission coefficient. Then, assuming the wave velocity and density of air are known, we deduce the four properties of the plate (thickness, density, longitudinal velocity and attenuation coefficient) as well as the attenuation coefficient of air. A variety of viscoelastic materials, whose impedances have weak values, has been studied such as Polyethylene, Plexiglas and carbon/epoxy composite. The physical properties for each plate are obtained with a good accuracy in the frequency range of investigation.

1. Introduction
For a long time, traditional ultrasonic non-destructive inspection has widely proved itself in the investigation and characterization of materials. Most of these inspection methods are based on the generation and detection of sound waves that propagate through a test sample which is placed between the source and detector of ultrasound. In most cases, a liquid coupling medium, such as water or gel is required to ensure the good transfer of acoustical energy between the transducers and the test material.

The most efficient and popular coupling method is the immersion one in which the test sample is being completely immersed in a water liquid which has an acoustic impedance of the same order of magnitude as the sample. However, there are many cases for which the use of water may not always be suitable for instance in presence of porous and composite materials that could be easily damaged or polluted through absorption of the liquid.

In another method using contact transducers, a physical contact with the specimen is required and the inspection is done by applying a gel to the surface of the structure. This method has the inconvenience of maintaining the contact pressure between the transducer and the test
piece and sometimes, the use of this type of inspection becomes impossible for example at high temperatures or if the test piece is inaccessible.

Among various techniques proposed to overcome such problems, techniques that employ air-coupled\textsuperscript{[2]} ultrasonic inspections appear most promising, largely due to their ability to perform measurements directly in atmospheric air. However, air-coupled systems have also many disadvantages compared with liquid-coupled systems: First, because of the large difference in acoustic impedance between air and most solid materials, the ultrasound is almost entirely reflected at the air/sample boundary. Second, the amplitude of the transmitted wave is extremely reduced due to the high absorption of ultrasound in air, especially at high frequencies. Finally, additional difficulties arise from the properties of air which are found to be considerably dependent on temperature, pressure and humidity environment.

Fortunately, to overcome these drawbacks, our equipment enables the generation of ultrasonic waves with high levels and allows their detection with sufficient sensitivity in order to improve the transfer of acoustic energy in air. Moreover, all measurements are made in a constant temperature, pressure and humidity environment.

This paper presents a non-contact ultrasonic inspection method for characterizing viscoelastic, homogeneous materials using piezoelectric air-coupled transducers. This method consists in studying the ultrasonic, plane wave transmission through an isotropic face parallel plate, in the same way as in the immersion method, but replacing the water medium with air. The aim is to determine the velocity and attenuation of ultrasonic waves in the material and the density and thickness of the sample, using a non-linear least-squares algorithm which minimizes the difference between experimental and calculated transmission coefficient data.

Kinra and Iyer\textsuperscript{[3]} have developed a new technique for ultrasonic non-destructive evaluation of the acoustic properties of a thin plate through normal transmission coefficient measurement. The analysis was performed for the determination of any one of the four acoustic quantities characterizing the plate, assuming the other three are known by using the spectrum of normally transmitted ultrasonic signal and without any prior knowledge of the ambient medium properties.

In this work, we describe a novel inverse method for simultaneous determination of all properties of the plate: density, thickness, and longitudinal velocity and attenuation using two ultrasonic measurements at normal incidence: with and without the sample. This method requires only the knowledge of the sound velocity in air and its density. In addition, A preliminary analysis of the effect of variations in parameters on the model has shown that the sensitivity of identification to parameters is higher for the real and imaginary parts than for the modulus. Therefore, in the algorithm used for the identification, we use the complex (real and imaginary parts) form of the transmission coefficient instead of using its modulus which is inadequate.

2. Theory

Ultrasonic through transmission measurements are frequently used for the characterization of the acoustical properties of a thin plate. The purpose of this section is to review briefly the theoretical plane wave transmission coefficient for a homogeneous, isotropic plate.

Figure 1 shows schematically an incident longitudinal plane wave $p_{\text{inc}}$ at normal incidence, propagates in the $x$ direction through a parallel face plate of thickness $E$ and density $\rho_2$, placed in air whose density is $\rho_1$. The reflected and transmitted waves are denoted as $p_r$ and $p_t$, respectively. The diagram shows also the waves propagating downwards $p_-$ and upwards $p_+$ inside the plate.

To describe the dispersion and attenuation behavior of waves one can define a complex longitudinal propagation constant $\gamma_j(\omega)$, where the real part is an attenuative part related to the frequency dependent attenuation, $\alpha_j(\omega)$, while the imaginary part is a dispersive part
related to the frequency dependent phase velocity, $V_j(\omega)$. These relations can be written as

$$\gamma_j(\omega) = \alpha_j(\omega) + i\frac{\omega}{V_j(\omega)} \quad (j = 1 \text{ or } 2)$$ \hspace{1cm} (1)

where $\omega$ is the circular frequency of the incident wave and the subscripts 1 and 2 refer to air and the plate, respectively. The attenuations of longitudinal waves in air and the plate are assumed to be frequency dependent as follows\(^4\)

$$\alpha_1(f) = \mu_1 f^2$$ \hspace{1cm} (2)

$$\alpha_2(f) = \mu_2 f$$ \hspace{1cm} (3)

where $\mu_1$ and $\mu_2$ are the longitudinal attenuation coefficients in air and the plate, respectively. $f = \frac{\omega}{2\pi}$ is the frequency.

For a viscoelastic homogeneous plate, the harmonic plane wave transmission coefficient, at normal incidence, can be expressed in complex form as

$$T_{th}(f) = \frac{2e^{\gamma_1E}}{\left(\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1}\right) \sinh (\gamma_2E) + 2 \cosh (\gamma_2E)}$$ \hspace{1cm} (4)

where

$$\gamma_1E = \frac{Ef}{V_1} (\mu_1 V_1 f + i2\pi)$$ \hspace{1cm} (5)

$$\gamma_2E = \frac{Ef}{V_2} (\mu_2 V_2 + i2\pi)$$ \hspace{1cm} (6)

$$\frac{Z_1}{Z_2} = \frac{\rho_1 V_1}{\rho_2 V_2} \times \frac{\mu_2 V_2 + i2\pi}{\mu_1 V_1 f + i2\pi}$$ \hspace{1cm} (7)

$Z_1$ and $Z_2$ are the complex acoustic impedances of air and the plate and are given by

$$Z_1 = \frac{i\mu_1 \omega}{\gamma_1}$$ \hspace{1cm} (8)

$$Z_2 = \frac{i\mu_2 \omega}{\gamma_2}$$ \hspace{1cm} (9)

![Figure 1](image_url)

**Figure 1.** A schematic diagram of a sound wave $p_{inc}$ normally incident on a plate of thickness $E$. This wave propagates in the $x$ direction and gives rise to reflected and transmitted waves denoted as $p_r$ and $p_t$, respectively.
In addition, it is assumed that air behaves in a non dispersive manner. This means that the phase velocity $V_1$ is a constant. It depends only on temperature, humidity and atmospheric pressure. Whereas, the dispersion in the plate is taken into account by using the following dispersion relationship [5,6] linking the phase velocity and attenuation of the plate

$$\frac{1}{V_2(f)} = \frac{1}{V_0} + \frac{\mu_2}{\pi^2} \ln \left( \frac{f_0}{f} \right)$$

where $V_0$ is the phase velocity at a reference frequency $f_0$.

3. Experiment

In this work, only normally incident waves are used for the characterization of the acoustical properties of a thin plate. A simplified drawing of the experimental set-up is shown in figure 2. Two identical ultrasonic air-coupled transducers (Ultran) are placed to face each other in air and are separated by a distance of $L = 8$ cm. In order to perform normal through transmission measurements, the sample is mounted between the transducers so that it is perpendicular to the incident wave as shown in figure 2b. The transducers are piezoelectric devices with a nominal central frequency of 1 MHz and an active diameter of 25 mm. The sample is a parallelepipedal plate of nominal thickness $E$, made up of a homogeneous, isotropic viscoelastic material. In the present work, three samples are tested. The first is a Polyethylene plate of nominal thickness 6 mm, the second is a 3-mm-thick Plexiglas plate and also tested is a carbon fiber epoxy sample of thickness 2.35 mm.

The transmission coefficient can be obtained by measuring the through transmission signals without and with the sample using the same experimental set-up. Therefore, two successive independent measurements are carried out. In the first, without the sample, the chirp waveform produced by the transmitter propagates through air and is directly acquired by the receiver. This received signal is called the reference signal $f_r(t)$. Next, with the specimen placed between the transducers, the ultrasonic signal transmitted through the plate is acquired as well and recorded as the sample signal $f_s(t)$.

![Figure 2. Measurement configuration: (a) through air path; (b) through the sample.](image-url)
For each measurement, the transmitting transducer is excited by a chirp signal having 1.2 MHz central frequency with 800 kHz bandwidth and 100 µs duration, produced by a transmitting/receiving generator (SIA 7-VN Instruments). As the transmitted signal is extremely weak, the receiving transducer is connected to an amplifier in order to increase the signal level. The amplified signal is then digitized using a digital oscilloscope (Accura 100

Figure 3. Time domain signals using 1 MHz air-coupled transducers: (a) reference signal; signals transmitted through a (b) 6-mm-thick Polyethylene; (c) 3-mm-thick Plexiglas; (d) 2.35-mm-thick composite plate.

Figure 4. (a) FFT modulus of the reference signal; FFT modulus of signals transmitted through a (b) 6-mm-thick Polyethylene; (c) 3-mm-thick Plexiglas; (d) 2.35-mm-thick composite plate.
Nicolet-12 bits), and averaged the required number of times to improve the signal-to-noise ratio (almost 30 dB of gain) before being transferred to a computer for further processing. Note that, compared to the reference signal, the sample signal level drops considerably, with a lower signal-to-noise ratio that requires a higher amplification and more signal averaging to obtain a useful signal. Frequency spectral analysis of these signals using Fast Fourier transform (FFT) algorithms then allows the calculation of the transmission coefficient as a function of frequency. Indeed, if the FFTs of \( f_r(t) \) and \( f_s(t) \) are denoted as \( F_r(f) \) and \( F_s(f) \), respectively, the measured transmission coefficient is given by

\[
T_{\text{exp}}(f) = \frac{F_s(f)}{F_r(f)} \quad (11)
\]

Corresponding to the reference signal shown in figure 3a, the transmitted waves through three plates made of Polyethylene, Plexiglas and composite material with nominal thickness of 6.3 and 2.35 mm are shown, respectively, in figures 3b, c, d. The corresponding frequency spectra (modulus) are shown in figures 4a, b, c and d.

As the velocity of sound in air depends significantly on temperature, all measurements mentioned above were made in a constant temperature environment in order to avoid the sound velocity fluctuations in air caused by small ambient temperature changes. Consequently, the temperature as well as humidity and pressure of air have been controlled and continuously recorded, during experiments, to assure the stability of the sound velocity in air.

4. Identification

In principle, the objective is to determine the four unknown real quantities characterizing the plate (thickness \( E \), mass density \( \rho_2 \), longitudinal phase velocity \( V_0 \) and attenuation coefficient \( \mu_0 \)). Substitution of Eq. (10) into Eqs. (6) and (7) yields

\[
\gamma_2 E = \frac{E}{V_0} f \left\{ \mu_2 V_0 \left[ 1 + \frac{2i}{\pi} \ln \left( \frac{f_0}{f} \right) \right] + i2\pi \right\} \quad (12)
\]

\[
\frac{Z_1}{Z_2} = \frac{\rho_1 V_1}{\rho_2 V_0} \times \frac{\mu_2 V_0 \left[ 1 + \frac{2i}{\pi} \ln \left( \frac{f_0}{f} \right) \right] + i2\pi}{\mu_1 V_1 f + i2\pi} \quad (13)
\]

Then by introducing Eqs. (5), (12) and (13) in Eq. (4), it can be observed that seven physical quantities appear,

\[
E, \ V_0, \ \mu_2, \ \rho_2, \ V_1, \ \mu_1, \ \rho_1. \quad (14)
\]

Since the wave velocity in air \( V_1 \) and its density \( \rho_1 \) will be considered as known, the number of physical properties to be found can be reduced and only five combinations of them denoted as

\[
a_1 = \frac{E}{V_1}, \ a_2 = \mu_1 V_1, \ a_3 = \frac{\rho_1 V_1}{\rho_2 V_0}, \ a_4 = \frac{E}{V_0}, \ a_5 = \mu_2 V_0, \quad (15)
\]

are used for the optimization as shown in the following equations

\[
\gamma_1 E = a_1 f \left( a_2 f + i2\pi \right) \quad (16)
\]

\[
\gamma_2 E = a_4 f \left\{ a_5 \left[ 1 + \frac{2i}{\pi} \ln \left( \frac{f_0}{f} \right) \right] + i2\pi \right\} \quad (17)
\]

\[
\frac{Z_1}{Z_2} = a_3 \times \frac{a_5 \left[ 1 + \frac{2i}{\pi} \ln \left( \frac{f_0}{f} \right) \right] + i2\pi}{a_2 f + i2\pi} \quad (18)
\]
where the constants $a_m$ ($m = 1$ to $5$) linking the different acoustic quantities of both the plate and air, are the parameters which are to be identified using Eq. (4). The five unknown physical quantities of the set (14) are defined from the set of parameters (15) by

$$V_0 = \frac{a_1 V_1}{a_4}, \quad \rho_2 = \frac{a_1 a_4}{a_1 a_3}, \quad \mu_2 = \frac{a_4 a_5}{a_1 V_1}, \quad E = a_1 V_1, \quad \mu_1 = \frac{a_2 V_1}{(19)}$$

One can observe that the precision and accuracy of the plate properties are related to those of the wave velocity in air; hence the interest in measuring it with a good precision.

To determine the acoustical properties of the plate, it is necessary to know two of the three physical quantities characterizing air. Therefore, an additional measurement of wave velocity in air is performed by moving the receiver through known distances from the transmitter and measuring the time of flight. Concerning density we refer to tables giving the density of air as a function of temperature.

4.1. Effect of parameters on transmission coefficient

For the identification problems, the sensitivity analysis can be an essential help to know the behavior of the model and to predict the effect of each parameter on this model in order to find the optimal parameters values. This effect is investigated by determining the relative variation of the complex transmission coefficient due to a few per cent change in each parameter using a relative error function

$$\eta = \sqrt{\frac{\sum_{n=1}^{N} [RT_0 (f_n) - RT_1 (f_n)]^2 + \sum_{n=1}^{N} [IT_0 (f_n) - IT_1 (f_n)]^2}{\sum_{n=1}^{N} [RT_0 (f_n)^2 + IT_0 (f_n)^2]}}.$$

where $N$ is the total number of frequency data points taken for comparison. $RT_0 (f_n)$ and $IT_0 (f_n)$ are, respectively, the real and imaginary parts of the theoretical transmission coefficient for the frequency $f_n$, calculated with standard parameters. $RT_1 (f_n)$ and $IT_1 (f_n)$ are, respectively, the real and imaginary parts of the transmission coefficient when the value of a parameter is changed.

In the case of the 6-mm-thick polyethylene plate, this study has demonstrated that the variations for the complex theoretical transmission coefficient due to a 1% change in each of parameters $a_1$, $a_2$, $a_3$, $a_4$ and $a_5$ are 108%, 0.16%, 1%, 72% and 0.69%, respectively. Thus, one can observe that the transmission coefficient is strongly dependent on parameters $a_1$ and $a_4$, while it is weakly dependent on parameters $a_2$, $a_3$ and $a_5$.

4.2. Parameters exploration

For the inverse algorithm, we begin by comparing the experimental $T_{exp}$ and calculated $T_{th}$ transmission coefficients. By sweeping all parameters one by one, the relative error between $T_{exp}$ and $T_{th}$ is investigated using Eq. (20) in which $T_0$ and $T_1$ are replaced by $T_{exp}$ and $T_{th}$, respectively. The goal is to obtain approximate values of the initial guesses with upper and lower bounds.

For parameters $a_2$, $a_3$ and $a_5$, the shape of the error function presents only a minimum while it contains many minima for the case of parameters $a_1$ and $a_4$. Therefore, the study of the location of these minima permits to distinguish between the true minimum and the false minima so that they can be easily avoided during the inverse process.
4.3. Inverse algorithm
The parameters are identified by solving an inverse identification problem which consists in optimizing the parameters values such that the theoretical transmission formula becomes as close as possible to the experimental data in the useful frequency domain. This numerical process is done by minimizing the following objective function

\[ g(a_m) = \sum_{n=1}^{N} \left[ (RT_{th}(f_n, a_m) - RT_{exp}(f_n))^2 + (IT_{th}(f_n, a_m) - IT_{exp}(f_n))^2 \right] \]  

(21)

\(RT_{th}(f_n, a_m)\) and \(IT_{th}(f_n, a_m)\) are, respectively, the real and imaginary parts of the theoretical transmission coefficient, computed with parameters \(a_m\) representing the material properties which are to be optimized. \(RT_{exp}(f_n)\) and \(IT_{exp}(f_n)\) are, respectively, the real and imaginary parts of the experimental transmission coefficient measured for frequency \(f_n\). The optimization of parameters \(a_m\) is conducted by using the Matlab® routine \(lsqnonlin\) which consists in solving non-linear least squares problems.

The inverse algorithm is supplied with an initial guess for which lower and upper bounds are specified for all the parameters. The tolerances in parameters and function values are set at \(1.0 \times 10^{-6}\) for all estimations. The parameters are estimated for \(N = 11700\) frequency points, which are included in the bandwidth of the transducers.

Because the transmission coefficient is weakly dependent on parameters \(a_2\), \(a_3\) and \(a_5\), the simultaneous research for all parameters in a five-dimensional space is not easy for finding the optimal solution.

![Figure 5](image_url)

**Figure 5.** Complex transmission (real and imaginary parts) of a plate of polyethylene of nominal thickness 6 mm: blue dotted line represents experiments and red continuous line represents the modelling transmission computed with optimized parameters values.
Consequently, a strategy is defined for optimizing parameters. First of all, we determine two parameters $a_1$ and $a_4$ considering $a_2$, $a_3$ and $a_5$ are known. Then, for the optimal values obtained, the three other parameters are optimized together by running the algorithm again. This procedure is repeated until the minimum possible between the calculated and experimental transmission coefficient is found. This gives good estimates of parameters.

Figure 5 shows an example of identification for a 6-mm-thick polyethylene plate and table 1 gives the optimized acoustical properties values for the three tested samples assuming the wave velocity in air and its density are known. The standard deviation between the measured and theoretical curves is estimated at $4 \times 10^{-5}$.

| Sample     | $V_1 (m/s)$ | $\rho_1 (kg/m^3)$ | $V_0 (m/s)$ | $\rho_2 (kg/m^3)$ | $\mu_2(N_p m^{-1} Hz^{-1})$ | $E (mm)$ | $\mu_1(N_p m^{-1} Hz^{-1})$ |
|------------|-------------|--------------------|-------------|--------------------|-----------------------------|----------|-----------------------------|
| Polyethylene | 344.9       | 1.19               | 2291        | 942                | $2.26 \times 10^{-5}$       | 5.974    | $2.16 \times 10^{-11}$     |
| Plexiglas   | 345.1       | 1.187              | 2739        | 1185               | $1.90 \times 10^{-5}$       | 3.034    | $2.25 \times 10^{-11}$     |
| Composite   | 344.7       | 1.19               | 3073        | 1511               | $2.68 \times 10^{-5}$       | 2.439    | $2.00 \times 10^{-11}$     |

Table 1. The optimized acoustical properties values identified for the three tested samples. We choose $f_0 = 1 \text{ MHz}$.

5. Conclusion

In this article, we describe an ultrasonic method for simultaneous determination of four acoustical quantities (thickness, mass density, longitudinal velocity and attenuation coefficient) of a viscoelastic plate embedded in a known fluid (air). Measurements are made locally in areas potentially inaccessible (center of the plate). This method can be applied to study the transmission (reflection) coefficient of a plate immersed in a liquid (water).

The results are found to be in good agreement with additional independent measurements (average thickness, average mass density and longitudinal wave velocity). Moreover, the values of the acoustical properties are very consistent with those obtained with a classical, ultrasonic, contact technique.

Measuring the wave velocity in air with a good accuracy, during experiments, permits to obtain uncertainties in thickness and velocity, clearly lower than 1 per cent, while attenuation coefficient and mass density are estimated with an accuracy of 1 and 2 per cent. The value of attenuation coefficient in air is in accordance with standard measurements\cite{7}.

Further work is to investigate a theoretical model taking into account the weak diffraction effects in the transmission coefficient in order to improve the accuracy of the parameters determination. The accuracy and reliability of these results justify the further study of materials with inhomogeneity in density, sound speed or thickness.

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