New Class of Optimal Frequency-Hopping Sequences by Polynomial Residue Class Rings

Wenping Ma¹ and Shaohui Sun²

¹ National Key Lab. of ISN, Xidian University, Xi’an 710071, P.R.China  
wp_ma@mail.xidian.edu.cn  
² Datang Mobile Communications Equipment Co.,Ltd, Beijing 100083, P.R.China

Abstract. In this paper, using the theory of polynomial residue class rings, a new construction is proposed for frequency hopping patterns having optimal Hamming autocorrelation with respect to the well-known Lempel-Greenberger bound. Based on the proposed construction, many new Peng-Fan optimal families of frequency hopping sequences are obtained. The parameters of these sets of frequency hopping sequences are new and flexible.

Index Terms: Autocorrelation Functions, Cross-correlation functions, Frequency Hopping sequences, Hamming Correlation, lower bounds.

1 Introduction

Let \( \mathcal{F} = \{f_0, f_1, \cdots, f_{l-1}\} \) be a set of available frequencies, called an alphabet. Let \( S \) be the set of all sequences of length \( \nu \) over \( \mathcal{F} \). Any element of \( S \) is called a frequency-hopping sequence of length \( \nu \) over \( \mathcal{F} \). Given any two frequency hopping sequences \( X, Y \in S \), we define their Hamming correlation \( H_{X,Y} \) to be

\[
H_{X,Y}(t) = \sum_{i=0}^{\nu-1} h[x_i, y_{i+t}], 0 \leq t < \nu,
\]

where \( h[a, b] = 1 \) if \( a = b \), and 0, otherwise, and all operations among the position indices are performed modulo \( \nu \). For any distinct \( X, Y \in S \), we define

\[
H(X) = \max_{1 \leq t < \nu} \{ H_{X,X}(t) \}
\]

\[
H(X, Y) = \max_{0 \leq t < \nu} \{ H_{X,Y}(t) \}
\]

\[
M(X, Y) = \max \{ H(X), H(Y), H(X, Y) \}.
\]

Lempel and Greenberger² developed the following lower bound for \( H(X) \). **Lemma 1:** For every frequency hopping sequence \( X \) of length \( \nu \) over an alphabet of size \( l \), we have

\[
H(X) \geq \frac{(\nu - \varepsilon)(\nu + \varepsilon - l)}{l(\nu - 1)}
\]
where $\varepsilon$ is the least nonnegative residue of $\nu$ modulo $l$.

**Corollary 1(6):** For any single frequency hopping sequence of length $\nu$ over an alphabet of size $l$, we have

$$H(X) \geq \begin{cases} k, & \text{if } \nu \neq l \\ 0, & \text{if } \nu = l \end{cases}$$

where $\nu = kl + \varepsilon$, $0 \leq \varepsilon < l$.

Let $\Gamma$ be a subset of $S$ containing $N$ sequences. We define the maximum nontrivial Hamming correlation of the sequence set $\Gamma$ as

$$M(\Gamma) = \max \{ \max_{X \in \Gamma} H(X), \max_{X,Y \in \Gamma, X \neq Y} H(X,Y) \}$$

$$H_a(\Gamma) = \max_{X \in \Gamma} H(X)$$

$$H_c(\Gamma) = \max_{X,Y \in \Gamma, X \neq Y} H(X,Y)$$

Throughout this paper, we use $(\nu, N, l, \lambda)$ to denote a set of $N$ frequency hopping sequences $\Gamma$ of length $\nu$ over an alphabet of size $l$, where $\lambda = M(\Gamma)$.

Peng and Fan developed the following bound on $H_a(\Gamma)$ and $H_c(\Gamma)$, which take into consideration the number of sequences in the family.

**Lemma 2:** For any family of frequency hopping sequences $\Gamma$, with length $\nu$, an alphabet of size $l$, and $|\Gamma| = N$, we have

$$(\nu - 1)NH_a(\Gamma) + (N - 1)\nu H_c(\Gamma) \geq 2\nu N - (I + 1)l$$

where $I = \lfloor \nu N \rfloor / l$.

**Lemma 3(6):** For any pair of distinct frequency hopping sequences $X, Y$, with $|\mathcal{F}| = l$, we have

$$M(X,Y) \geq \frac{4\nu - (I + 1)l}{4\nu - 2}$$

where $2\nu = Il + r$ and $0 \leq r < l$.

**Definition 1.** (1) A sequence $X \in S$ is called optimal if the Lempel-Greenberger bound in Lemma 1 is met.

(2) A subset $\Gamma \subset S$ is an optimal set if the Peng-Fan bound in Lemma 2 is met.

(3) Any pair of distinct frequency hopping sequence $\{X, Y\} \subset S$ constitute a Lempel-Greenberger optimal pair of frequency hopping sequences if the bound in Lemma 3 is met.

Lempel and Greenberger defined optimality for both single sequences and sets of sequences in other ways. A set of frequency hopping sequences meeting the Peng-Fan bound in Lemma 2 must be optimal in the Lempel and Greenberger sense.
In modern radar and communication systems, frequency hopping spread-spectrum techniques have been popular, such as frequency hopping code division multiple access and “Bluetooth” technologies [7,8].

The objective of this paper is to present a new method to construct new families of frequency hopping sequences. Both individual optimal frequency-hopping sequences and optimal families of frequency hopping sequences are presented.

2 Polynomial Residue Class Rings Preliminary

In the following, we introduce in brief polynomial residue class rings preliminary. For details on polynomial residue class rings, we refer to [1].

**Definition 2.** Let $p$ be a prime, $GF(p)$ be a finite field, $GF(p)[\xi]$ be the ring of all polynomials over $GF(p)$, and $\omega(\xi)$ be an irreducible polynomial of degree $m$ over $GF(p)$, where $m \geq 1$. Then $\mathbb{R}$ is defined as the quotient ring generated by $\omega(\xi)^k$ in $GF(p)[\xi]$, $k \geq 1$.

$$\mathbb{R} = GF(p)[\xi]/(\omega(\xi)^k)$$

We have a natural homomorphic mapping, $\mu$ from $\mathbb{R}$ to its residue field $F = GF(p)[\xi]/(\omega(\xi))$. Define $\mu : \mathbb{R} \to F$ by $\mu(a) = a \mod \omega(\xi)$. It is easy to verify that the elements in the set $\{1, \omega(\xi), \omega^2(\xi), \ldots, \omega^{k-1}(\xi)\}$ are linearly independent over $F$ and hence constitute a basis of $\mathbb{R}$ over $F$. Thus any element $a \in \mathbb{R}$ can be represented uniquely as

$$a = a_0 + a_1 \omega(\xi) + \cdots + a_{k-1} \omega^{k-1}(\xi), a_i \in F, i = 0, 1, 2, \ldots, k - 1.$$ 

Thus $\mathbb{R}$ can be written as

$$\mathbb{R} = F + F\omega + F\omega^2 + \cdots + F\omega^{k-1} \quad (1)$$

The group of units $\mathbb{R}^*$ of $\mathbb{R}$ is given by the direct product of two group $G_{PRC}$ and $G_{PRA}$, $\mathbb{R}^* = G_{PRC} \times G_{PRA}$, where $G_{PRC}$ is a cyclic group of order $p^m - 1$ and $G_{PRA}$ is an Abelian group of order $p^{m(k-1)}$.

**Lemma 4:** The set $\{G_{PRC}, 0\}$ is isomorphic to residue field $F$ and is also a subspace of $\mathbb{R}$. Thus the set $\{G_{PRC}, 0\}$ is a subring of $\mathbb{R}$.

From now on, we will omit the indeterminate $\xi$ from the representation.

Let $\mathbb{R}[x]$ be the ring of polynomials over $\mathbb{R}$. We extend the homomorphic mapping $\mu$ on $\mathbb{R}$ to polynomial reduction mapping:

$$\hat{\mu} : \mathbb{R}[x] \to F[x]$$

in the obvious way

$$f(x) = \sum_{i=0}^{r} a_i x^i \xrightarrow{\hat{\mu}} \sum_{i=0}^{r} \mu(a_i)x^i$$

A polynomial $f(x) \in \mathbb{R}[x]$ is a basic irreducible if $\mu(f(x))$ is irreducible in $F[x]$; it is monic if its leading coefficient is 1.
The Galois ring of $\mathbb{R}$ denoted as $GR(\mathbb{R}, r)$ is defined as $\mathbb{R}[x]/(f(x))$, where $f(x)$ is a basic monic irreducible polynomial of degree $r$ over $\mathbb{R}$.

The group of units of $GR(\mathbb{R}, r)$ denoted by $GR^*(\mathbb{R}, r)$ is given by a direct product of two groups:

$$GR^*(\mathbb{R}, r) = G_C \times G_A$$

where $G_C$ is a cyclic group of order $p^{mr} - 1$ and $G_A$ is an Abelian group of order $p^{m(k-1)r}$. On the lines of Lemma 4, it is easy to show that the set $\{G_C, 0\}$ is a field of order $p^{mr}$. This is denoted by $GF(p^{mr})$. Thus like the representation (1) for $\mathbb{R}$, we have

$$GR(\mathbb{R}, r) = GF(p^{mr}) + \omega GF(p^{mr}) + \omega^2 GF(p^{mr}) + \cdots + \omega^{k-1} GF(p^{mr}),$$

hence, any element $\alpha \in GR(\mathbb{R}, r)$ can be uniquely expressed as

$$\alpha = \alpha_0 + \omega \alpha_1 + \omega^2 \alpha_2 + \cdots + \omega^{k-1} \alpha_{k-1}, \alpha_i \in GF(p^{mr}), i = 0, 1, \cdots, k-1. \quad (2)$$

The elements of $G_A$ are of the form $1 + \omega(x)A'$, where $A' \in GR(\mathbb{R}, r)$. From (2), the elements of $G_A$ are given by the set

$$\{(1 + \omega \gamma), \gamma = \gamma_0 + \omega \gamma_1 + \cdots + \omega^{k-2} \gamma_{k-2}, \gamma_i \in GF(p^{mr})\} \quad (3)$$

The Galois automorphism group of $GR(\mathbb{R}, r)$ over its intermediate subring $GR(\mathbb{R}, s)$, where $s$ divides $r$ is cyclic of order $(r/s)$ generated by the Frobenius map $\sigma^s$ defined by

$$\sigma^s(\alpha) = (\alpha_0)^{p^s} + (\alpha_1)^{p^s} \omega + \cdots + (\alpha_{k-1})^{p^s} \omega^{k-1}$$

where $\alpha$ is as in (2). When $s = 1$, the above Frobenius map generates Galois group over $\mathbb{R}$. Using the automorphisms given above, we define below generalized trace functions which map elements of $GR(\mathbb{R}, r)$ to its intermediate subrings $GR(\mathbb{R}, s)$ where $s$ divides $r$. They are given by

$$Tr^s_r(\alpha) = \sum_{i=0}^{(r/s)-1} [(\alpha_0)^{p^{si}} + (\alpha_1)^{p^{si}} \omega + (\alpha_2)^{p^{si}} \omega^2 + \cdots + (\alpha_{k-1})^{p^{si}} \omega^{k-1}]$$

where $\alpha \in GR(\mathbb{R}, r).$ The above trace function is the generalization of trace function defined for finite fields. Like their counterparts in finite fields, the trace functions satisfy the following properties:

$$Tr^s_r(\alpha) = Tr^s_r(\sigma^s(\alpha)), \text{ for all } i$$

$$Tr^s_r(aa + b\beta) = aTr^s_r(\alpha) + bTr^s_r(\beta); \forall a, b \in GR(\mathbb{R}, s) \text{ and } \forall \alpha, \beta \in GR(\mathbb{R}, r).$$

For any fixed $b$ of $GR(\mathbb{R}, s)$, the equation $Tr^s_r(\alpha) = b$, has exactly $p^{mk(r-s)}$ solutions in $GR(\mathbb{R}, r)$.

$$Tr^1_r(\alpha) = Tr^1_r(Tr^s_r(\alpha)).$$
3 New Optimal Frequency Hopping Sequences from Residue Class Rings

Let \( q = p^m \), \( z \) is a positive integer satisfying \( \frac{n}{z} + 1, r \) is a positive integer, in this paper, we suppose \( \text{gcd}\left(\frac{q-1}{z}, n\right) = 1 \), \( \alpha \) be a primitive generator of \( G_C \) present in \( GR^*(\mathbb{R}, r) \), \( \gamma \in G_A \) with \( \kappa(\gamma) = \rho \).

Let \( s \) be an integer with \( \text{gcd}(s, q^r - 1) = 1 \), and define \( \beta = \alpha^{z^s} \). It is easy to check that the minimal positive integer \( d \) satisfying \( \beta^{d-1} = 1 \) is \( r \), thus \( 1, \beta, \beta^2, \cdots, \beta^{r-1} \) is linear independent over \( G_{PRC} \).

We define the following sequence:

\[
 s_i^{(\gamma, g)} = Tr_i^r(\gamma g^{\beta^i}), i = 0, 1, \cdots, k, \cdots, g \in G_C
\]

It is easy to check that \( s_i^{(\gamma, g)} = s_i^{(\gamma, g)} \), then \( \left( s_i^{(\gamma, g)} \right)_0^n \) is a sequence of period \( n \).

We define the following sequences set:

\[
 \Gamma = \{ (s_i^{(\gamma, g^k)})_0^n : 0 \leq k < z \} \tag{4}
\]

It is obvious that \( |\Gamma| = z \).

Definition 7. Two sequences \( \left( s_i^{(\gamma, g)} \right)_0^n \) and \( \left( s_i^{(\gamma, g')} \right)_0^n \) are called projectively cyclically equivalent if there exist an integer \( t \) and a nonzero scalar \( \lambda \in G_{PRC} \)

\[
 s_i^{(\gamma, g)} = \lambda s_{i+t}^{(\gamma, g')}, i = 0, 1, 2, \cdots \tag{5}
\]
We wish to count the number of inequivalent in $\Gamma$ using (5) as the definition of equivalence.

**Theorem 3.** For any two sequences $(s_i^{(\gamma\cdot g)})_0^\infty$ and $(s_i^{(\gamma\cdot g')})_0^\infty$ belonging to $\Gamma$, they are projectively cyclically equivalent.

**Proof:** Formula (5) can be written as

$$Tr_1^* (\gamma g ^{\beta^i}) = Tr_1^* (\gamma \lambda g' ^{(t+i)}), i \geq 0$$

$$Tr_1^* [\gamma (g - \lambda g')^{\beta^i}] = 0, i \geq 0$$

It follows that Formula (5) is equivalent to

$$\frac{g}{g'} = \lambda \beta^i$$

(6)

The set of elements in $G_C$ of the form $\lambda \beta^i$ where $\lambda \in G_{PRC}$ is a subgroup of the multiplicative group of nonzero elements of $G_C$. What (6) says is that $g$ and $g'$ are equivalent if and only if $g$ and $g'$ lie in the same coset of this subgroup. It follows that the number of inequivalent $g$’s is equal to the number of such cosets, viz.

$$N_1 = \frac{(q^r - 1)}{|G|},$$

where $G$ is the subgroup of elements of the form $\{\lambda \beta^i\}$. It remains to calculate $|G|$. Now $G$ is the direct product of the two groups $G_{PRC}$ and $A = \{1, \beta, \ldots, \beta^{n-1}\}$. From elementary group theory we have

$$|G| = \frac{|A| \cdot |G_{PRC}|}{|G_{PRC} \cap A|}.$$ 

To calculate $|G_{PRC} \cap A|$ we note that this number is just the number of distinct powers of $\beta$, which are elements of $G_{PRC}$. But $\beta^i \in G_{PRC}$ if and only if $\beta^{i(q-1)} = 1$. Since $ord(\beta) = n$, this is equivalent to $n|i(q-1)$, i.e.,

$$\frac{n}{gcd(n, q-1)} | i$$

Thus if we define

$$e = gcd(n, q - 1)$$

$$d = \frac{n}{e}.$$ 

Because $e = gcd(n, q - 1) = gcd(\frac{q^r - 1}{q-1}, \frac{q-1}{z})$ and $gcd(\frac{q^r - 1}{q-1}) = 1$, then

$$e = \frac{q-1}{z}.$$
We see that $\beta^i \in G_PRC$ iff $i = 0, d, 2d, \ldots, (e - 1)d$, hence $|G_PRC \cap A| = e$, and we have

$$|G| = n(q - 1)/e = q^r - 1,$$

$$N_1 = 1.$$

**Theorem 4.**

$$W_0 \left( (s_i^{(r,g)}) \right) = \frac{q^r - 1}{z}.$$

**Proof:** Let $1, \alpha^s, \ldots, \alpha^{s(z-1)}$ be a complete set of representatives for the cosets of

$$\{1, \beta, \ldots, \beta^{n-1}\}$$

in the multiplicative group $G_C$. Every nonzero element $\theta \in G_C$ can be written as $\theta = \alpha^i \beta^j$ for a unique pair $(i, j)$, $0 \leq i \leq z - 1, 0 \leq j \leq n - 1$.

Now consider the following $z \times n$ array, which we call Array 1:

$$
\begin{array}{cccc}
1 & \beta & \beta^2 & \ldots & \beta^{n-1} \\
\alpha^s & \alpha^s \beta & \alpha^s \beta^2 & \ldots & \alpha^s \beta^{n-1} \\
\alpha^{2s} & \alpha^{2s} \beta & \alpha^{2s} \beta^2 & \ldots & \alpha^{2s} \beta^{n-1} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\alpha^{(z-1)s} & \alpha^{(z-1)s} \beta & \alpha^{(z-1)s} \beta^2 & \ldots & \alpha^{(z-1)s} \beta^{n-1} \\
\end{array}
$$

Now let $s_{ij} = Tr_1^r(\alpha^i \beta^j)$ and consider this array, which we call Array 2:

$$
\begin{array}{cccc}
s_00 & s_{01} & s_{02} & \ldots & s_{0(n-1)} \\
s_{10} & s_{11} & s_{12} & \ldots & s_{1(n-1)} \\
s_{20} & s_{21} & s_{22} & \ldots & s_{2(n-1)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
s_{(z-1)0} & s_{(z-1)1} & s_{(z-1)2} & \ldots & s_{(z-1)(n-1)} \\
\end{array}
$$

Since Array 2 is the “trace” of Array 1, and since every nonzero element of $G_C$ appears exactly once in Array 1, it follows that $0$ appears exactly $q^{(r-p)} - 1$ times in Array 2. Finally, since $N_1 = 1$, we know that every row of Array 2 can be obtained from the first row by shifting and multiplying by scalars. Thus $0$ appears the same number of times in each row of Array 2. Since there are $z$ rows in the array, and $0$ appears $q^{(r-p)} - 1$ time altogether, each row contains exactly $\frac{q^{(r-p)} - 1}{z}$ 0s.

**Theorem 5.** \( \{s_i^{(r,g)}\}_{i=0}^{\infty} \) is an optimal frequency hopping sequence with parameters \( (\frac{q^r - 1}{z}, q^p, \frac{q^{(r-p)} - 1}{z}) \).

**Proof:** Because $\frac{q^r - 1}{z} = q^p \cdot \frac{q^{(r-p)} - 1}{z} + \frac{q^p - 1}{z}$, the conclusion follows from Lemma 1 and Corollary 1.

**Theorem 6.** If $g, g'$ belong to distinct cyclotomic classes of order $z$ in $G_C$, then \( (s_i^{(r,g)})_{i=0}^{\infty} \) and \( (s_i^{(r,g')})_{i=0}^{\infty} \) constitute a Lempel–Greenberger optimal pair of frequency hopping sequences.
Proof: By Theorem 5, $H_a((s_i^{(γ,g)})^∞_0) = H_a((s_i^{(γ,g′)})^∞_0)$. Now we compute the cross-correlation values of $(s_i^{(γ,g)})^∞_0$ and $(s_i^{(γ,g′)})^∞_0$. From the definition of $s_i^{(γ,g′)}$, we know that for any $t \in \{0,1,\ldots,n-1\}$, if we cyclically shift $s_i^{(γ,g′)}$ to the left for $t$ time, we obtain $s_i^{(γ,g′)} = T_{r_i}^{(γ,g′)}(\gamma^t \beta_i^t)$, $i = 0,1,2,\ldots$, then, by noting that $s_i^{(γ,g)} - s_i^{(γ,g′)} = T_{r_i}^{(γ,g)}[\nu(g - g′ \beta_i^t)]$, $i = 1,2,\ldots$. Since $g,g′$ are in distinct cyclotomic classes of order $z$ in $G_C$, $g - g′ \beta_i^t$ can never be zero. It then follows from Theorem 4 that

$$H_{(s_i^{(γ,g)})^∞_0,(s_i^{(γ,g′)})^∞_0}(t) = \frac{q^{r-\rho} - 1}{z}.$$ 

For any $t \in \{0,1,\ldots,n-1\}$. Therefore we can conclude that $H((s_i^{(γ,g)})^∞_0,(s_i^{(γ,g′)})^∞_0) = \frac{q^{r-\rho} - 1}{z}$. We claim that $(s_i^{(γ,g)})^∞_0$ and $(s_i^{(γ,g′)})^∞_0$ constitute a Lempel–Greenberger optimal pair of frequency hopping sequences, if $g,g′$ belong to distinct cyclotomic classes of order $z \geq 2$ in $G_C$. In fact, for any two $q^e$-ary sequences $(s_i^{(γ,g)})^∞_0$ and $(s_i^{(γ,g′)})^∞_0$ of length $q^e - 1$, since $q^e - 1 = q^{r-\rho} - 1 + q^\rho - 1$, we put $d = q^{r-\rho} - 1$ and $e = q^\rho - 1$, then by Lemma 3, we have

$$M((s_i^{(γ,g)})^∞_0,(s_i^{(γ,g′)})^∞_0) \geq 4I\nu - (I+1)I \nu - 2 = \frac{2d\nu - \nu + 2de + e}{2\nu - 1} = \frac{d - \nu - 2de - e - d}{2\nu - 1} = \frac{d - de(z-2)}{2\nu - 1}.$$ 

This implies that

$$M((s_i^{(γ,g)})^∞_0,(s_i^{(γ,g′)})^∞_0) \geq d = \frac{q^{r-\rho} - 1}{z}.$$ 

Theorem 7. The $Γ$ of (4) is a $(q^e - 1, z, q^\rho, q^{r-\rho} - 1, z)$ set of frequency hopping sequence, meeting the Peng–Fan bound.

Proof: We apply Lemma 2, where $I = [\nu z/q^\rho] = q^{r-\rho} - 1$, 

$$(\nu - 1)zH_a(Γ) + (z-1)z\nu H_a(Γ) = (q^r - 1)\frac{q^{r-\rho} - 1}{z} + (z-1)\frac{q^r - 1 - q^{r-\rho} - 1}{z} = (q^r - 2)(q^{r-\rho} - 1)$$
and
\[ 2Iνz - (I + 1)IQ^ρ = 2(q^{r-ρ} - 1)\frac{q^{r} - 1}{z}z - q^{r-ρ}(q^{r-ρ} - 1)q^ρ \]
\[ = (q^r - 2)(q^{r-ρ} - 1). \]

We know that
\[ (ν - 1)zHa(Γ) + (z - 1)zνHc(Γ) = 2Iνz - (I + 1)IQ^ρ \]
which means that \( \{H_a(Γ) = \frac{q^{r-ρ} - 1}{z}, H_c(Γ) = \frac{q^{r-ρ} - 1}{z}\} \) is a pair of the minimum integer solutions of the inequality described in Lemma 2, that is, \( Γ \) is a Peng – Fan optimal family of frequency hopping sequences.

4 Conclusion

In this paper, new optimal frequency hopping sequences are constructed from polynomial residue class rings. When \( ρ = 1 \), our construction is same with the related constructions in [4,5,6], thus our construction can be take as an extension of the related constructions in [4,5,6]. Our construction posses the following advantages: (1) the parameters of the construction are new and flexible, (2) by choose different parameter \( γ \), one can construct many different Peng – Fan optimal frequency hopping sequence families.

References

1. P.Udaya and M.U.Siddiqi, Optimal large linear complexity frequency hopping patterns derived from polynomial residue class rings. IEEE Transactions on Information Theory, Vol.44, No.4, July 1998.
2. Abraham Lempel, and Haim Greenberger, Families of sequences with optimal Hamming correlation properties, IEEE Transactions on Information Theory, Vol.20, No.1, January 1974.
3. Daiyuan Peng and Pingzhi Fan, Lower bounds on the Hamming Auto- and Cross correlations of Frequency-Hopping sequences, IEEE Transactions on Information Theory, Vol.50, No.9, September, 2004.
4. Cunsheng Ding, Marko J. Moisio, and Jin Yuan, Algebraic constructions of optimal frequency hopping sequences, IEEE Transactions on Information Theory, Vol.53, No.7, July 2007.
5. Cunsheng Ding, Jianxing Yin, Sets of optimal frequency hopping sequences, IEEE Transactions on Information Theory, Vol.53, No.8, August 2008.
6. Gennian Ge, Ying Miao, and Zhongxiang Yao, Optimal frequency hopping sequences: Auto-and Cross correlation properties, IEEE Transactions on Information Theory, Vol.55, No.2, February 2008.
7. R.A.Scholtz, "The spread spectrum concept," IEEE Trans. Commun. Vol.25, No.8, pp.748-755, Aug.1977.
8. Specification of the Bluetooth systems-Core. The Bluetooth special interest Group. Available: http://www.bluetooth.com/

9. Robert J. Elie, Finite fields for computer scientists and engineers, Kluwer Academic Publishers, 1987.