Fate of the Hebel-Slichter peak in superconductors with strong antiferromagnetic fluctuations

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We show that magnetic fluctuations can destroy the Hebel-Slichter peak in conventional superconductors. The Hebel-Slichter peak has previously been expected to survive even in the presence of strong electronic interactions. However, we show that antiferromagnetic fluctuations suppress the peak at \( q = 0 \) in the imaginary part of the magnetic susceptibility, \( \chi'_M(q, \omega) \), which causes the Hebel-Slichter peak. This is of general interest as in many materials superconductivity is found near a magnetically ordered phase, and the absence of a Hebel-Slichter peak is taken as evidence of unconventional superconductivity in these systems. For example, no Hebel-Slichter peak is observed in the \( \kappa-(\text{BEDT-TTF})_2\text{X} \) organic superconductors but heat capacity measurements have been taken to indicate \( s \)-wave superconductivity. If antiferromagnetic fluctuations destroy the putative Hebel-Slichter peak in organic superconductors then the peak should be restored by applying a pressure, which is known to suppress antiferromagnetic correlations.

I. INTRODUCTION

Unconventional superconductivity, and the identification of the underlying mechanism, remains one of the most active areas of research in modern physics [1–5]. The challenge of understanding unconventional superconductivity is compounded by the fact that macroscopic probes of the superconducting state are sensitive only to the emergent superconducting order parameter, or gap, and not to the microscopic mechanism responsible for it [6,9]. Any attempt to explain the microscopic origin of unconventional superconductivity must also explain how the resultant gap influences experiments.

Understanding the exact form of the superconducting gap is of considerable importance for developing an understanding of the microscopic basis for unconventional superconductivity, however, it is often far from straightforward in practice. The Josephson interference experiments responsible for unambiguously identifying the ‘\( d_{x^2-y^2} \)-wave’ symmetry of the cuprates [10,11] have not been possible in many materials. The interpretation of other experimental results can be ambiguous, making a conclusive determination of the gap difficult.

In most experiments, the bulk of the insight comes from the low temperature behavior of experimental probes, which reflect the density of states of the superconductor. As such, the temperature dependence of these probes can be used to infer both the presence of nodes in the gap function and the form of these nodes (i.e. point or line nodes). Such probes cannot, however, identify the positioning of any nodes on the Fermi surface, and therefore cannot be used to differentiate between different gap functions with the same form of nodes. One notable exception to this rule is the spin-lattice relaxation rate \( 1/T_1 \) measured in nuclear magnetic resonance, which shows a peak below the superconducting transition temperature known as the Hebel-Slichter peak [12,13]. The existence of this peak was one of the earliest confirmations of the Bardeen-Cooper-Schrieffer theory of conventional superconductivity [14,15], indicating the presence of a coherent state. Experimentally, the presence of such a peak has long been taken as a key signature of superconductivity with an isotropic, nodeless gap. Here, we seek to understand the influence of magnetic fluctuations on \( 1/T_1 \) in general, and the form of the Hebel-Slichter peak in particular.

In the most interesting superconductors, both strong antiferromagnetic fluctuations and the absence of a Hebel-Slichter peak in \( 1/T_1 \) are ubiquitous [2,7,10]. Additionally, the form of the gap function in many such materials remains contentious. One significant example is the organic superconductor, \( \kappa-(\text{BEDT-TTF})_2\text{Cu[N(CN)$_2$]Br} \) (\( \kappa\text{-Br} \)). This material has the highest critical temperature (at ambient pressure) of the BEDT-TTF based superconductors [9]. \( \kappa\text{-Br} \) has been subjected to a wide variety of experimental probes over the last three decades. Despite this, the symmetry of the superconducting order parameter in this material remains a matter of considerable disagreement [4,8,17–28]. Strong antiferromagnetic fluctuations are observed in \( \kappa\text{-Br} \) [29–30]; indeed the closely related material \( \kappa-(\text{BEDT-TTF})_2\text{Cu[N(CN)$_2$]Cl} \) (\( \kappa\text{-Cl} \)) is an antiferromagnetic insulator at ambient pressure that can be driven superconducting by moderate hydrostatic pressures [4]. The superconducting states [30] and magnetic fluctuations [4] in metallic/superconducting \( \kappa\text{-Cl} \) and \( \kappa\text{-Br} \) are extremely similar.

While Knight shift measurements on \( \kappa\text{-Br} \) consistently indicate singlet pairing [31,32], due to the vanishing of the Knight shift at zero temperature, interpretations of the results of other experiments have been inconsis-
tent. There has been evidence from the temperature dependence of low temperature specific heat measurements taken to indicate nodeless ('s-wave') superconductivity \[24, 25, 33\] while other experiments indicate the presence of nodes of the gap function \[34–36\]. Similarly, penetration depth measurements were contentious \[37, 39\] until recently, with more precise measurements showing a power law temperature dependence suggestive of a nodal superconducting state \[27\]. The density of states from the surface tunneling spectroscopy has been found to show some indication of multiple coherence peaks, which has been linked to a complicated mixed order parameter \[22\], though similar signals have been observed in other multi-band materials with a superconducting gap magnitude that varies between bands \[40\]. This ongoing disagreement has led both theorists \[8, 17, 20, 21, 31, 41, 42\] and experimentalists \[22, 43\] to discuss the possibility of a variety of superconducting gaps, including those with symmetry required or accidental nodes in organic superconductors.

Despite the lack of an observed Hebel-Slichter peak in \(1/T_1\) \[31, 32\], there are some who argue that the superconducting gap may in fact be nodeless \[9, 28\], as supported by recent thermal conductivity measurements \[26\]. In such a scenario, the absence of a Hebel-Slichter peak needs a detailed explanation. Thus, it is of significant interest to understand how magnetic fluctuations influence the \(1/T_1\) relaxation rate and whether the suppression of a peak by electronic interactions is sufficient to explain the relaxation rate in such materials. In particular, we will focus on those interactions encapsulated by vertex corrections, which are responsible for renormalization of the coupling constants, and encapsulate the strength of the antiferromagnetic fluctuations.

Early attempts to understand the unconventional superconductivity in the cuprates, found that coherence effects can, in principle, be disguised by a combination of strong-coupling and electronic interactions \[44, 45\]. It was also found, however, that these effects alone were insufficient to match experimental data with an isotropic gap \[40, 42\]. However, a detailed analysis of the influence of magnetic fluctuations on the Hebel-Slichter peak is currently lacking. For example, the absence of the Hebel-Slichter peak, and the potential role played by magnetic fluctuations has not previously been examined in the organic superconductors. These materials are of particular interest because the bandwidth, and therefore the relative strength of electronic interactions, in these materials is tunable by the application of external pressure \[9, 18\]. Therefore, it may be possible to alter the interaction strength and determine the gap structure by measuring \(1/T_1T\) and comparing both the temperature and interaction dependence of the relaxation rate to predictions. We show that in these materials the suppression of the Hebel-Slichter peak can be explained entirely due to vertex corrections, rather than gapless superconductivity, as has been discussed previously \[49\].

In a previous work \[42\], we demonstrated the potential use of the nuclear magnetic relaxation rate, \(1/T_1T\), to experimentally differentiate between those gaps with accidental nodes (i.e. nodes not required by symmetry) and those gaps with nodal positions constrained by symmetry, due to a peak arising in \(1/T_1T\) for the former case immediately below \(T_c\), similar to the well known Hebel-Slichter peak found in nodeless superconductors. In addition to considering the Hebel-Slichter peak in isotropic superconductors, we will also address the suppression of this Hebel-Slichter-like peak by antiferromagnetic fluctuations.

### II. THEORY

The spin lattice relaxation rate, \(1/T_1\), measured in nuclear magnetic resonance, is related to the transverse spin susceptibility, \(\chi_{++}(q, \omega) = \chi_0(q, \omega) + i\chi_{+-}^\prime(q, \omega)\), via

\[
\frac{1}{T_1 T} = \lim_{\omega \to 0} \frac{2k_B}{\gamma_e^2 \hbar^4} \sum_q |A_H(q)|^2 \frac{\chi_{+-}^\prime(q, \omega)}{\omega},
\]

where \(\gamma_e\) is the electronic gyromagnetic ratio, \(A_H(q)\) is the hyperfine coupling, which we will approximate by a point contact interaction, constant with respect to \(q\). In a conventional nodeless superconductor, the relaxation rate increases below \(T_c\) to a peak before decreasing rapidly as temperature is lowered. Formally, the peak arises due to a divergence in the relaxation rate that is cut off by a combination of effects due to impurities, slight anisotropy of the gap, electronic interactions or in the extreme limit, by the influence of the crystal lattice, which sets a characteristic length scale \[50, 51\]. The fact that such a divergence is absent in the majority of unconventional superconductors is typically taken as evidence of nodes in the superconducting gap \[11, 16, 52\], though in some cases it has been argued that strong electronic correlations may be responsible for the suppression of the peak \[9\].

The magnetic susceptibility in the superconducting state, in the absence of vertex corrections is given by,

\[
\chi_{+-}(q, \omega) = \chi_0(q, \omega) = \frac{1}{N} \sum_k \left[ \frac{1}{2} \left( 1 + \frac{\xi_{k+q} + \Delta_{k+q} \Delta_k}{E_{k+q} E_k} \right) \right] \frac{f(E_{k+q}) - f(E_k)}{\omega - (E_{k+q} - E_k) + i\eta} + \frac{1}{4} \left[ 1 - \frac{\xi_{k+q} + \Delta_{k+q} \Delta_k}{E_{k+q} E_k} \right] \frac{f(E_{k+q}) - f(E_k)}{\omega - (E_{k+q} + E_k) + i\eta},
\]
where $E_k = \sqrt{\xi_k^2 + \Delta_k^2}$ is the superconducting quasiparticle energy, defined in terms of the electron dispersion \(\xi_k = \varepsilon_k - \mu\) and the superconducting gap \(\Delta_k\). \(f(E)\) is the Fermi-Dirac distribution function \([\bar{f}(E) = 1 - f(E)]\), and in the absence of interactions the limit of the lifetime \(\eta \to 0^+\) is implied.

### A. Anisotropic gaps with accidental nodes

Previously \[42\], we demonstrated the possibility of a Hebel-Slichter-like peak emerging in the relaxation rate in systems where the superconducting gap is nodal, but the location of the nodes is not dictated by symmetry. In such systems, the (in general) nonzero average of the gap over the Fermi surface gives rise to a peak in the relaxation rate analogous to the Hebel-Slichter peak, even if the integral of the superconducting gap over the Brillouin zone vanishes.

Lifetime effects (via the self-energy) on \(1/T_1 T\) have already been investigated to a degree in Ref. \[42\], where a finite quasiparticle lifetime was introduced into the numerical calculations. This served the purpose of investigating the contribution of impurity effects on the Hebel-Slichter-like peaks. Including electronic interactions in the quasiparticle lifetime is not expected to alter the picture dramatically, introducing a temperature dependence to the lifetime but not significantly influencing the stability of the peak structure. In this work, we investigate the effects of antiferromagnetic fluctuations introduced by the inclusion of vertex corrections and show that they have a much more dramatic effect.

### B. Vertex Corrections and The Random Phase Approximation

In the language of quantum field theory, the interactions within a system are described by two functions.

\[
\chi_{+-}(q, \omega) = \lim_{i\omega_n \to \omega + i\eta} \sum_{k, k', \sigma \neq \sigma'} G_{k+q, \sigma}^{(0)}(i\omega_n) G_{k', \sigma}^{(0)}(i\omega_n + i\Omega_m),
\]

in which case the relaxation rate can be expressed with the influence of the two Green’s functions separated \[12\], due to a property of the convolution, \(\sum_{q, k} f(k + q) f(k) = |\sum_k f(k)|^2\), and

\[
\frac{1}{T_1 T} \propto \lim_{\omega \to 0} \frac{1}{\omega} \sum_q \chi_{+-}(q, \omega) = \lim_{\omega \to 0} \frac{1}{\omega} \left[ \sum_k G_{k, \sigma}^{(0)}(\omega) \right]^2.
\]

This argument hinges on the fact that the susceptibility, \(\chi_{+-}(q, \omega)\), depends on the momentum \(q\) solely through the convolution of the two Green’s function. In the presence of vertex corrections, the susceptibility can no longer be written in this way, due to the momentum...
dependence of the renormalized vertex. In order to investigate these effects, we turn to the random phase approximation (RPA), as the simplest treatment of vertex correction. Within the RPA, the susceptibility is given by a series of ladder diagrams \[57\], which may be evaluated to give

\[
\chi_{\text{RPA}}(q, \omega) = \frac{\chi_0(q, \omega)}{1 - U\chi_0(q, \omega)},
\]

where \(\chi_0(q, \omega)\) is the bare transverse magnetic (superconducting) susceptibility, and \(U\) is a Hubbard interaction parameter (longer range interactions may be included by introducing a momentum dependence in this interaction parameter). The imaginary part of the susceptibility, which enters into \(1/T_1 T\), is then given by

\[
\chi''_{\text{RPA}}(q, \omega) = \frac{\chi_0''(q, \omega)}{[1 - U\chi_0(q, \omega)]^2 + [U\chi_0'(q, \omega)]^2}.
\]

Within the framework of the RPA, the transition to a magnetically ordered state is described by a divergence in the static (\(\omega = 0\)) susceptibility. The real and imaginary parts of the susceptibility are related by a Kramers-Kronig transformation, as a result of a fluctuation-dissipation theorem \[53, 58, 59\]. One of the consequences of this relationship is that, at low frequencies, the imaginary part of the susceptibility varies linearly with frequency, vanishing in the static limit, while the real part tends to a constant value. The divergence of the susceptibility in the RPA then must occur when \(U\chi'(q, 0) = 1\). This corresponds to a magnetic instability in the material and the RPA predicts long-range antiferromagnetic order for \(U > U_c\). Thus, \(U_c\) sets an upper limit for the interaction strength in numerical calculations investigating the superconducting phase of \(U_c = 1/\text{max}\{\chi'(q, 0)\}\).

### III. Numerical Results

In order to explore the behavior of \(1/T_1 T\) in the presence of vertex corrections, we numerically calculate the

\[
B_q = \chi_0(q, \omega = 0) = \sum_k \left\{ \frac{1}{2} \left[ 1 + \frac{\xi_k \xi_{k+q} + \Delta_k \Delta_{k+q}}{E_k E_{k+q}} \right] \frac{f(E_{k+q}) - f(E_k)}{E_k - E_{k+q}} + \frac{1}{4} \left[ 1 - \frac{\xi_k \xi_{k+q} + \Delta_k \Delta_{k+q}}{E_k E_{k+q}} \right] \frac{f(E_{k+q}) - f(E_{k+q})}{E_k + E_{k+q}} \right\},
\]

and the structure of \(B_q\) can be seen to depend in a complicated manner on the band structure and gap symmetry, particularly with regards to approximate nesting of the Fermi surface (which may enhance the first term).

The real part of the susceptibility can, in principle, enhance the features dominating the relaxation rate (i.e. if \(C_q\) and \(B_q\) have similar momentum-dependence, large contributions to the relaxation rate will be enhanced while smaller contributions will be unaffected). There is, however, no a priori reason to expect such enhancement, as the momentum-dependences of \(C_q\) and \(B_q\) may differ drastically.

### 1. Effects on the \(1/T_1\) Relaxation Rate

To fully understand the effects of vertex corrections on \(1/T_1 T\), it is necessary to resort to numerical calculations (see Section \[III\]), but some insight may still be gained analytically. In the low frequency limit, the susceptibility may be approximated by

\[
\begin{align*}
\chi_0'(q, \omega) &\approx \chi_0'(q, 0) = B_q, \\
\frac{\chi_0''(q, \omega)}{\omega} &\approx \frac{\chi_0''(q, \omega)}{\omega \rightarrow 0} = C_q,
\end{align*}
\]

in which case the relaxation rate is given by

\[
\frac{1}{T_1 T} \propto \lim_{\omega \rightarrow 0} \sum_q \frac{1}{\omega} \left[ \frac{\chi_0''(q, \omega)}{[1 - U\chi_0'(q, \omega)]^2 + [U\chi_0'(q, \omega)]^2} \right] \frac{C_q}{1 - UB_q}^2.
\]

In the absence of vertex corrections the relevant features are given by the form of \(C_q\), which are influenced, when \(U \neq 0\), by features of the static real part of the susceptibility. In particular, since \(UB_q \leq 1\), whenever \(B_q \approx 1\), the contribution to the relaxation rate is enhanced, and when \(B_q\) is large and negative, features of \(C_q\) are suppressed.

The static part of the susceptibility in a superconductor is given by

\[
B_q = \chi_0'(q, \omega = 0) = \sum_k \left\{ \frac{1}{2} \left[ 1 + \frac{\xi_k \xi_{k+q} + \Delta_k \Delta_{k+q}}{E_k E_{k+q}} \right] \frac{f(E_{k+q}) - f(E_k)}{E_k - E_{k+q}} + \frac{1}{4} \left[ 1 - \frac{\xi_k \xi_{k+q} + \Delta_k \Delta_{k+q}}{E_k E_{k+q}} \right] \frac{f(E_{k+q}) - f(E_{k+q})}{E_k + E_{k+q}} \right\},
\]
relaxation rate for various interaction strengths.

A. Effective models

To highlight the generality of our results, we consider two concrete examples of models with strong anisotropy and in both cases consider fully gapped superconducting states and those with accidental nodes.

The first example is a toy model with anisotropic hopping parameters along the two axes,

$$\xi_k = t_x \cos (k_x) + t_y \cos (k_y).$$

(10)

Such a model is useful in demonstrating effects arising in a d-wave superconducting state with accidental nodes. For example, for $t_x \neq t_y$ the nodes in a superconducting state have accidental nodes, giving rise to a Hebel-Slichter peak, where the average hopping along the $x$ and $y$ directions, and $\delta_t = (t_1 - t_2) / 2$ is the difference between the alternating hopping strengths (which are dependent on the dimer orientation).

Due to the anisotropy of this model, the ‘d$_{xy}$-wave’ state has accidental nodes, giving rise to a Hebel-Slichter-like peak in $1/T_1 T$, and is given by

$$\Delta_k^{(xy)} = \Delta_0 \sin (k_x) \sin (k_y).$$

(13)

where $t'$ and $t'_2$ are (anisotropic) hopping parameters between next-nearest-neighbor dimers, $t = (t_1 + t_2) / 2$ is the average hopping along the $x$ and $y$ directions, and $\delta_t = (t_1 - t_2) / 2$ is the difference between the alternating hopping strengths (which are dependent on the dimer orientation).

The suppression of the Hebel-Slichter peak is shown for a purely isotropic s-wave gap function in Fig. 2 for both the orthorhombic model with $t_y = 0.4 t_x$ at quarter filling and for the effective model of $K$-Br, with hopping magnitudes parametrized by density functional theory [60]. This model offers the opportunity to understand the resilience of the Hebel-Slichter peak in a more realistic band structure. The BEDT-TTF dimers are treated as sites, and the tight-binding parameters are displayed schematically in Fig. 1 with dispersion given by

$$\xi_{k_\pm} = t' \cos (k_c) + t'_2 \cos (k_a) \pm t' \left[ \cos \left( \frac{k_a + k_c}{2} \right) + \cos \left( \frac{k_a - k_c}{2} \right) \right]^2 + \delta_t \left[ \sin \left( \frac{k_a + k_c}{2} \right) + \sin \left( \frac{k_a - k_c}{2} \right) \right]^2,$$

(12)

where $t'$ and $t'_2$ are (anisotropic) hopping parameters between next-nearest-neighbor dimers, $t = (t_1 + t_2) / 2$ is the average hopping along the $x$ and $y$ directions, and $\delta_t = (t_1 - t_2) / 2$ is the difference between the alternating hopping strengths (which are dependent on the dimer orientation).

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(13)

are not symmetry required (for example, adding a small s-wave component does not change the symmetry or cause a phase transition) and the average of the order parameter over the Fermi surface is non-zero [42].

The second model we consider is a two-band effective tight-binding model for $κ$-Br, with hopping magnitudes parametrized by density functional theory [60]. This model offers the opportunity to understand the resilience of the Hebel-Slichter peak in a more realistic band structure. The BEDT-TTF dimers are treated as sites, and the tight-binding parameters are displayed schematically in Fig. 1 with dispersion given by

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(13)
FIG. 2. The temperature dependence of $1/T_1T$, and suppression of the Hebel-Slichter peak, with increasing interaction strength in (a) the orthorhombic model with $t_x = 0.4t_y$, and (b) a model for $\kappa$-Br, with an isotropic nodeless gap. In the limit of strong antiferromagnetic fluctuations, the peak narrows and eventually vanishes entirely. While the peak only disappears for $U \geq 0.95U_c$, the narrowing and suppression of the peak at lower interaction strengths may be sufficient to disguise the Hebel-Slichter peak in experiments. At low temperatures, $1/T_1T$ has an exponential temperature dependence, even in the presence of strong antiferromagnetic fluctuations. In these calculations, $U_c \sim 12.4t$, $\eta = 5 \times 10^{-3}t$, $\omega = 10^{-3}t$, and $\Delta_0/2 = 2.5k_BT_c$.

In order to better understand the origin of this suppression of the peak we examine, in Fig. 3, the properties of the transverse susceptibility for the model of $\kappa$-Br close to $T_c$. In the absence of vertex corrections, the Hebel-Slichter peak results from a peak in the imaginary part of the susceptibility at $q = 0$, which is present in both bands and only at higher temperatures. In the real part of the susceptibility, a corresponding negative peak in the same region is responsible for the suppression of this feature in the RPA susceptibility.

The peak in the real susceptibility is responsible, at strong interactions, for masking the corresponding peak in the imaginary part of the susceptibility, as can be seen in Fig. 3, which displays the imaginary part of the dressed susceptibility. While vertex corrections introduce some structure in the susceptibility away from $q = 0$, these features do not protect the Hebel-Slichter peak from suppression.

In Fig. 4, we examine the dressed susceptibility at an intermediate interaction strength, not sufficient to suppress the Hebel-Slichter peak entirely. In this case, there is clearly still a large peak in the imaginary part of the susceptibility around $q = 0$, which is suppressed entirely for large interaction strengths. Additionally, the features away from $q = 0$ have not yet reached the magnitude seen in Fig. 3, highlighting that both the suppression of the $q = 0$ peak, and therefore the Hebel-Slichter peak, and the enhancement of the other features, arise due to the vertex corrections.

Finally, we wish also to understand how this suppression influences the Hebel-Slichter like peak expected in superconductors with accidental nodes [42]. Fig. 5 displays the suppression of the Hebel-Slichter-like peak for the orthorhombic and $\kappa$-Br models with $d_{x^2-y^2}$-wave and $d_{xy}$-wave superconducting gaps, respectively, each with accidental nodes. The peak is suppressed in the same manner as in the previous case, though much more rapidly with increasing interaction strength.

The low temperature behavior for the gaps with accidental nodes in Fig. 5 does not show the exponential suppression of quasiparticle states seen for the isotropic gap, but is again qualitatively unchanged by the increasing interaction strength. It may be necessary, in general, to examine the low temperature behavior of $1/T_1T$, and not just the presence or absence of a peak near $T_c$ to infer the superconducting gap symmetry.

In Fig. 6 we again examine the origin of the peak suppression for the gap with accidental nodes in $\kappa$-Br, finding a situation at high temperatures ($T = 0.98T_c$) that is qualitatively the same as the nodeless gap. The peak in $1/T_1T$ is caused by a peak in $\chi''(q,\omega)/\omega$ near $q = 0$, which is suppressed by a corresponding peak in $\chi'(q,\omega)$, the influence of which is increased in the RPA as the interaction increases. Additionally, the RPA-dressed susceptibilities in both cases are qualitatively indistinguishable, with the only apparent difference the magnitude of the susceptibility, despite the significant reduction in the magnitude of the peak in $1/T_1T$. This further solidifies the similarities between the two gap functions, despite the presence of line nodes in the second case, and the corresponding alteration of the density of states.

**IV. CONCLUSIONS**

Vertex corrections, described by the random phase approximation, suppress the Hebel-Slichter peak in a fully gapped superconductor, and the similar peak found for gaps with accidental nodes, only for significant antiferromagnetic fluctuations. Even when the peak is suppressed by vertex corrections, near $U/U_c \approx 1$, the low tempera-
The transverse susceptibility of $\kappa$-Br with an isotropic superconducting gap, at $T = 0.98T_c$, with $\omega = 10^{-3} t$. The imaginary (a) and real (b) parts of the susceptibility in the absence of vertex corrections, both show divergent behavior at $q = 0$ which, in the case of the imaginary part, is responsible for the Hebel-Slichter peak. The dominant term in the denominator of the RPA dressed susceptibility (c) is shown for an interaction strength of $0.9U_c$, for which the Hebel-Slichter peak is suppressed. The denominator of the RPA shows a peak that grows noticeably as the interaction strength increases, masking the divergence of the bare imaginary part, as can be seen in (d), which shows the imaginary part of the RPA dressed susceptibility. Interestingly, though features away from $q = 0$ are significantly enhanced, beyond the magnitude of the peak in the bare susceptibility, for both bands, these features do not contribute to the Hebel-Slichter peak, which is strongly suppressed at this interaction strength.

The temperature behavior of the nuclear magnetic relaxation rate remains qualitatively unchanged by the interactions. This is because the peaks in both the real and imaginary parts of the susceptibility near $q = 0$ decrease in magnitude as the temperature is lowered. And so, just as the Hebel-Slichter peak is only evident near $T = T_c$, the influence of the vertex corrections is less significant at low temperatures.

In the organic superconductors, the application of pressure can be used to decrease the effective interaction strength, which will increase the magnitude of any peak in $1/T_1 T$. Therefore, we propose an additional experimental probe of the superconducting gap in these materials, by measuring the temperature and pressure (and therefore $U/U_c$) dependence of $1/T_1 T$ to give further insight into the gap symmetry. In particular, for a nodeless gap, or one with accidental nodes, a Hebel-Slichter peak should appear as pressure is increased. For a gap with symmetry required nodes, no such peak will emerge under pressure.

[1] V. P. Mineev and K. V. Samokhin, Introduction to Unconventional Superconductivity (Gordon and Breach, Amsterdam, 1999).
[2] D. J. Scalapino, Reviews of Modern Physics 84, 1383 (2012).
[3] M. Sigrist and K. Ueda, Reviews of Modern Physics 63,
FIG. 4. The RPA-dressed transverse susceptibility of $\kappa$-Br with an isotropic superconducting gap, at $T = 0.98T_c$ and $\omega = 10^{-3}t$, with an intermediate interaction strength of $0.6U_c$, for which the Hebel-Slichter peak is only partially suppressed. Unlike the dressed susceptibility in Fig. 3, the peak in the denominator of the RPA susceptibility is not sufficient to fully screen the peak in the imaginary part of the bare susceptibility, and its influence is still present in the dressed susceptibility.

239 (1991)

[4] B. J. Powell and R. H. McKenzie, Journal of Physics: Condensed Matter 18, R827 (2006)

[5] M. R. Norman, Science 332, 196 (2011)

[6] J. F. Annett, Advances in Physics 39, 83 (1990)

[7] J. F. Annett, Physica C: Superconductivity 317-318, 1 (1999)

[8] B. J. Powell, Journal of Physics: Condensed Matter 18, L575 (2006)

[9] J. Wosnitza, Journal of Low Temperature Physics 146, 641 (2007)

[10] D. A. Wollman, D. J. Van Harlingen, W. C. Lee, D. M. Ginsberg, and A. J. Leggett, Physical Review Letters 73, 593 (1994)

[11] C. C. Tsuei, J. R. Kirtley, C. C. Chi, L. S. Yu-Jahnes, A. Gupta, T. Shaw, J. Z. Sun, and M. B. Ketchen, Physical Review Letters 73, 593 (1994)

[12] L. C. Hebel and C. P. Slichter, Physical Review 107, 901 (1957)

[13] L. C. Hebel and C. P. Slichter, Physical Review 113, 1504 (1959)

[14] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Physical Review 106, 162 (1957)

[15] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Physical Review 108, 1175 (1957)

[16] B. J. Powell and R. H. McKenzie, Reports on Progress in Physics 74, 056501 (2011)

[17] J. Schmalian, Physical Review Letters 81, 4232 (1998)

[18] K. Kuroki, T. Kimura, R. Arita, Y. Tanaka, and Y. Matsuda, Physical Review B 65, 100516 (2002)

[19] B. J. Powell and R. H. McKenzie, Physical Review B 69, 024519 (2004)

[20] B. J. Powell and R. H. McKenzie, Physical Review Letters 98, 027005 (2007)

[21] D. Guterding, M. Altmeier, H. O. Jeschke, and R. Va-
FIG. 6. The bare (a) and RPA-dressed (b) transverse susceptibility of α-Br with a superconducting gap with accidental nodes, at $T = 0.987\hbar$ and $\omega = 10^{-3}\hbar t$, are distinguishable from the case with isotropic gap only by the magnitude. The imaginary part of the susceptibilities in the absence of vertex corrections again diverges near $q = 0$, while the RPA dressed susceptibility for $U = 0.9U_c$, shows no such divergence. The reduced magnitude of the peak in this case compared to the Hebel-Slichter peak for the isotropic gap in the absence of vertex corrections is related to a reduction in the width of the $q = 0$ peak, rather than its magnitude, which is slightly increased.

[22] D. Gutering, S. Diehl, M. Altmeyer, T. Methfessel, U. Tutsch, H. Schubert, M. Lang, J. Müller, M. Huth, H. O. Jeschke, R. Valentí, M. Jourdan, and H.-J. Elmers, Physical Review Letters 116, 237001 (2016)
[23] K. Zantout, M. Altmeyer, S. Backes, and R. Valentí, Physical Review B 97, 014530 (2018).
[24] J. Wosnitza, S. Wanka, J. Hagel, M. Reibelt, D. Schweitzer, and J. A. Schlueter, Synthetic Metals Proceedings of the Yamada Conference LVI. The Fourth International Symposium on Crystalline Organic Metals, Superconductors and Ferromagnets (ISCOM 2001), 133-134, 201 (2003).
[25] H. Elsinger, J. Wosnitza, S. Wanka, J. Hagel, D. Schweitzer, and W. Strunz, Physical Review Letters 84, 6098 (2000)
[56] H. Monien and D. Pines, Physical Review B 41, 6297 (1990).

[57] S. Doniach and E. H. Sondheimer, Green’s Functions for Solid State Physicists (Imperial College Press, London, 1998).

[58] P. Coleman, An Introduction to Many-Body Physics (Cambridge University Press, Cambridge, 2015).

[59] G. B. Arfken and H. J. Weber, Mathematical Methods for Physicists (6e), 6th ed. (Academic Press, Oxford, 2005).

[60] T. Koretsune and C. Hotta, Physical Review B 89, 045102 (2014).