Efficient Global Optimization of Non-differentiable, Symmetric Objectives for Multi Camera Placement

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Abstract—We propose a novel iterative method for optimally placing and orienting multiple cameras in a 3D scene. Sample applications include improving the accuracy of 3D reconstruction, maximizing the covered area for surveillance, or improving the coverage in multi-viewpoint pedestrian tracking. Our algorithm is based on a block-coordinate ascent combined with a surrogate function and an exclusion area technique. This allows to flexibly handle difficult objective functions that are often expensive and quantized or non-differentiable. The solver is globally convergent and easily parallelizable. We show how to accelerate the optimization by exploiting special properties of the objective function, such as symmetry. Additionally, we discuss the trade-off between non-optimal stationary points and the cost reduction when optimizing the viewpoints consecutively.

Index Terms—NETW networked sensor fusion and decisions; OPTO imaging sensors; APPL environmental monitoring and control, robotics and automation; OTHER global optimum, multiple camera placement, non-differentiable objective, symmetric objective;

I. INTRODUCTION

Camera network applications range from validating robot swarms [1] over object tracking [2] to 3D reconstruction [3]. Cameras are often applied for critical tasks, such as ensuring the integrity of a human being in industrial robotics [4] or in surveillance systems [5] or supporting a decision with considerable financial consequences at professional sport events [6]. Thus, the deployment of cameras often comes with the responsibility to build a failure-resistant system. This system will be more precise and the relevant regions of the environment are better covered, the better the cameras are placed and oriented. The purpose of this work is to find globally optimal positions and orientations of the cameras in a given 3D environment.

Optimal camera placement can be phrased as an optimization problem. The parameters of the cameras which need to be adjusted, such as positions and orientations, are the \( \text{variables} \) of the optimization. These are constrained by the mounting area of the environment and possible mounting directions. The \( \text{domain} \) \( \mathcal{D} \) is the variable space for all the cameras of the network. A \( \text{solver} \) iterates over the domain to find the best camera constellation. In each iteration step of the solver, the quality of the current camera constellation (i.e. positions and orientations of multiple cameras) needs to be measured. In general, the quality of a camera constellation is encoded in the \( \text{objective function} \) \( f: \mathcal{D} \rightarrow \mathbb{R} \), a real-valued function such as the volume of the camera’s united field of view (FOV) or a reconstruction error. In this work, we propose an iterative, numerical algorithm (Fig. 1) that efficiently produces a set of optimal camera positions and orientations, i.e. a solver for a non-linear, real-world problem optimizing \( f \).

A. Properties of the Problem

1) Positive: The objective function \( f \) that measures the quality of a camera constellation has some properties that help to accelerate the optimization:
a) Symmetry: Most of the time, cameras of the same type are installed, so the objective function is invariant under the permutation of cameras. $f$ is said to be symmetrical.
b) Prior: The field of view of a camera behaves just as in human vision, so relatively good, non-occluded camera positions may be found to initialize the optimization.
c) Subspaces: Once the quality has been evaluated for the first camera constellation, readjusting cameras consecutively (greedy method) reduces the costs of the subsequent evaluations since the field of view of the rest of the cameras will not change. In terms of optimization, the parameters of a single camera of the network lie in a *subspace* of the domain.

![Fig. 3. Example of non-optimal stationary point: Top: The initial adjustment for camera 1 is $a_1 \in P$ and for camera 2 is $a_2 \in P$. Both cameras are placed in the domain $D \in P^2$ (on dashed line always directed to $M$). Our goal is to maximize the volume of the intersection of the camera’s field of view (FOV, grey). The global maximum is adopted when both cameras are all the way to the left or right $((a_1, a_1)$. Bottom – greedy method: First optimize camera 1 (move the left camera to $a_2$), then optimize camera 2. After the first step, both cameras are in $a_2$, since this is the optimum on the subspace. Even after changing the subspace and optimizing camera 2, the optimal solution is again $(a_2, a_2)$ (since the function is symmetric). However, moving both cameras at the same time left or right $((a_2, a_2) + \epsilon \cdot (1, 1)$ with any $\epsilon > 0$ or $< 0$) will increase the volume of the FOV. Thus, the stationary point $(a_2, a_2)$ is neither a local nor a global maximum.]

2) Negative: The objective $f$ has some particular properties that are challenging for most of the existing solvers:

a) Stationary solutions: Alternately placing single cameras (greedy method) has been adopted tolerating the fact that it also leads to *stationary solutions* which are neither local nor global optima (Fig. 3).
b) Expensive: When measuring the reconstruction error or the field of view of several cameras in a CAD environment, $f$ comprises geometrical operations and is expensive.
c) Black-box: The geometrical simulation renders $f$ to be a black box, i.e. operations other than function evaluations are hardly possible. Thus, $f$ cannot be decomposed in its additively separable parts or derived analytically. Approximations such as the difference quotient lead to increased costs.
d) Non-smooth: In [7], $f$ is piecewise non-differentiable and non-linear, a gradient does not always exist.
e) Non-convex: It is also non-convex with multiple local optima [7]. Searching simply in direction of the gradient will only yield a local but not global optimum.
f) Quantized: In [8], $f$ is even discontinuous and in [9] the discretized voxel set leads to a quantized $f$. This means that even if the gradient existed, it might be zero almost everywhere.

**B. Contributions and Content**

Due to these complications, the problem is usually simplified by the reduction of the domain’s dimensions or constraints, e.g. by the choice of camera positions out of predefined sets, by arranging the mounting spots around the interesting objects, by choosing a two-dimensional surveillance space, or accepting stationary solutions. Instead of simplifying the objective to overcome these difficulties, a suitable solver evaluates the objective function along subspaces of the domain, smoothens the objective $f$, recognizes the symmetric counterparts, caches the function values $f(x)$ as to not evaluate them again, and finds a *globally* optimal solution in the end. We achieve this by combining the following methods:

a) Subspace decomposition/BCA: The domain of the problem is decomposed into the subspaces, each including only the parameters of a single camera. This is called *domain decomposition*. The subset of variables of a single camera is called a *block* of variables. We will use a method in which each block is optimized in turns and is called *block coordinate ascent/descent* (BCA) and we will discuss suboptimal stationary points.
b) Surrogate/RBF: A non-smooth, quantized or black-box function still needs to be smoothened or approximated such that it delivers a gradient. In optimization, a *surrogate* of the objective function is a differentiable function which interpolates previous evaluation points and whose function value and derivative are cheaply evaluated. As soon as a high quality iterate according to the surrogate is found, the real objective is evaluated and the surrogate is updated with the new evaluation point. This way we cache function values of previous evaluations and establish a gradient.
c) Exclusion Areas/EAM: With a BCA the optimization may yield suboptimal stationary solutions. In this work, we adopt a strategy to exclude areas with already evaluated sample pairs in order to force to avoid stationary points. This solver can be computed in parallel and is globally convergent on a continuous domain for various objectives. We will demonstrate the originality of our method and a general demand for an efficient solver for optimal camera placement in Section II. Section III elaborates on the design of the solver. Elaborate synthetic and realistic experiments in Section IV show that the solver is faster than global solvers and more accurate than local solvers by covering the complete domain.

**II. RELATED WORK**

We will first discuss previous camera placement approaches and their objective functions in Section II-A. Then, an overview on useful solvers for these problems follows in Section II-B.

**A. Optimal Camera Placement**

The optimal camera constellation for one application is not necessarily the optimal solution in a different field, which is why research has been conducted for the following objective functions:
1) **General problems:** In case the application is unknown, a general form of objective function needs to be used. The *Art Gallery Problem* is concerned with minimizing the number of cameras such that a set of objects or an area of the environment is observed completely [10]–[19]. The reverse problem, the Maximum Set Covering Location Problem maximizes the area or the number of objects/RBFs observed by a given number of cameras, [8], [20]–[23].

2) **Application specific problems:** In case the camera network needs to satisfy a specific quality function, a general objective function would miss some particulars. There are online methods that dynamically adjust the cameras while performing an (robotic) task, such as eye-in-hand movement [24]–[26], scene modelling [27], object reconstruction [28], active stereo tracking [29], decentralized tracking [30], visual servoing [31], or multi-robot formation [32]. In contrast to these, offline methods are designed for optimizing and attaching the cameras before executing the task without [33]–[35] and with regarding obstacles [7], [9], [15], [16], [36], [37] or risk maps [38] or uncertainty [8] in the environment.

3) **Typical optimal placement:** The represented authors give an extended problem or visibility analysis and experiments for optimal camera placement and use popular optimization techniques including greedy methods [8], [19], [39], [40] or hillclimbing [8] or stochastic methods [7], [9], [41]. For global optimality, some authors formulate these problems as a binary program with different types of visibility and connectivity constraints [42]–[45] and solve it with Branch and Bound [11], [46]. Nonlinear constraints such as the visibility of a camera are typically precomputed and they choose camera positions out of a predefined set. In order to compute the objective function in this work and use a continuous domain, we combine non-linear optimization methods to increase the efficiency of the optimization of a symmetrical, expensive, black-box objective which is easier evaluated on camera specific subspaces. Some authors have optimized a three-dimensional problem on subspaces but they either provide gradient information [47] or do not discuss non-optimal, stationary points [8], [26], [48], [49].

### B. Optimization

In order to compute a global optimum, two main strategies generally exist, iterating through each local optimum II-B.1 or, in case the local optima cannot be found deterministically, randomly sampling the domain II-B.2. Both II-B.1 and II-B.2 present weaknesses for optimal camera placement. Therefore, we suggest the use of a BCA II-B.3 in combination with a surrogate II-B.4 instead in a combination II-B.5:

1) **Deterministic methods:** Some methods such as the Nelder-Mead-Simplex [50], [51], Interior Point Filter Line Search [52], Method of Moving Asymptotes [53], or the Sequential Quadratic Program [54], [55] use the objective function’s convexity, separability, or derivative information to prove the convergence to a local optimum. Properties which may not be adopted by the objective function or information which may not be known, compare Section I-A.

2) **Randomized methods:** Randomized solvers use the fact that a global optimum can be found by sampling an arbitrary continuous function dense in a compact domain [56]. Two examples of dense sampling in randomized methods are the Ant Colony Algorithm [57] and Simulated Annealing [58], which have been used in global optimal camera placement in [9] and [7], respectively. However, for dense iterates a huge number of iterations is needed and the cost of the objective is multiplied by this number. For instance, in [7] cameras are placed in a 2D environment and for “very high dimensional spaces (> 8 < parameters>), although the algorithm provided reasonably good solutions very quickly, it sometimes took several hours to jump to a better solution”.

3) **Subspace decomposition/BCA:** The convergence of a coordinate search, block coordinate ascent/descent (BCA), block-nonlinear Gauss Seidel, alternating minimization, and domain decomposition – as the optimization with a subspace decomposition is called often – has been studied for both overlapping [59], [60] and non-overlapping subspaces [61] and under various assumptions, e.g., for strictly convex, quadratic or separable functions [62]. The problem that the subspaces may not lie in the gradient direction of the objective [63] has been addressed by rules that choose the order and direction of the subspaces [64], [65] the latter is parallelized for large data [66]. In optimal camera placement, the subspaces need to be parallel to the coordinate axes of the blocks, orthogonal to each other, and non-overlapping (according to the variables of each camera). Thus, a rule for the choice of the subspace direction cannot be applied. Moreover, there are points at which the choice of the subspace is irrelevant due to the symmetry of the objective function (Fig. 3).

4) **Surrogate/RBF:** For non-linear optimization, the surrogate needs to interpolate scattered evaluation points, i.e. points which are not arranged on a grid, compare [67], [68] for multivariate interpolation and approximation methods. Some methods approximate a subset of evaluation points in a piecewise manner (subsets may be achieved by Delaunay triangulation [69]). Alternatively, a radial basis function interpolant (RBF) [70]–[72] interpolates the whole set of evaluation points without triangulation (Fig. 4). The authors of [73], [74], have developed optimization methods with RBFs as surrogates. Powell [71] developed a method to add evaluation points subsequently to the RBF, like Newton’s subsequent interpolation method.
However, in optimal camera placement, each block/subspace (corresponding to the variables of one camera) is equally relevant and usually contains a similar number of variables.

C. Problem Definition

Let \( P \) denote the variable space of a single camera, for instance one can use \( P = \{(p,o) \mid p \in \mathbb{R}^3, o \in [0,2\pi] \times [0,\pi]\} \) consisting of the position \( p \) and the yaw and pitch of the camera \( o \). For \( M \in \mathbb{N} \) cameras \( a_1 \in P_1, \ldots, a_M \in P_M \), we call \( x := (a_1, \ldots, a_M) \) a camera constellation or the variable vector and the space \( D := P_1 \times \ldots \times P_M \) the domain.

The problem of finding the optimal camera constellation in the domain with a fixed number of cameras is denoted by

\[
\text{Find } \arg\max_{x \in D} f(x). \tag{1}
\]

In this work, the objective function \( f \) is decomposed by (2) into a fusion function \( g \) and into \( M \) camera specific functions \( \sigma_m \) which merely depend on the parameters of a single camera \( a_m \in P_m \). The observation set \( A_m \), \( m \in \{1, \ldots, M\} \) resembles the set of all objects, features, voxels, etc. which are detected by the \( m \)-th camera \( \sigma_m \).

\[
f(x) = g \circ (\sigma_1, \ldots, \sigma_M)^T(x) \tag{2}
\]

\[
g : (A_1 \times \ldots \times A_M) \to \mathbb{R} \tag{3}
\]

\[
\sigma_m : P_m \to A_m \text{ for all } m \in \{1, \ldots, M\}. \tag{4}
\]

For instance, the set of observations is composed of voxels, the cameras can either detect them or not, and the fusion function counts the voxels that are detected by all the cameras. In practice, the decomposition of \( f \) in (2) is a valid assumption and the use of a general observation set \( A_m \) can be applied in many optimal camera placement scenarios. For instance in Table I, the decomposition of sample problems from [7]–[9] are given. In these examples, all the cameras are allowed to be placed at the same mounting spots \( D := P_1 = \ldots = P_M \). The same sensor model is used for each camera \( \sigma := \sigma_1 = \ldots = \sigma_M \) which can make the same observations \( A := A_1 = \ldots = A_M \). Thus, their objective functions are symmetric in all the subspaces.

### Table I

#### Examples for Optimal Camera Placement Decompositions

|                        | Represent. \( \Delta_m \) | Fusion \( g \) |
|------------------------|---------------------------|----------------|
| A single cam.’s ob.- | Voxel set                 |                |
| servation \( \sigma_m \)| Real value                |                |
|                        |                          |                |
| [7] Simulation of FOV  |                          |                |
| in the CAD model       |                          |                |
| Determination of the   |                          |                |
| area of multiple       |                          |                |
| objects’ region of     |                          |                |
| occlusion Analysis of   |                          |                |
| observation of         |                          |                |
| pedestrian paths       |                          |                |

III. Proposed Solver

Our solver is customised for problems (1) in the general form (2) and (4) that fulfill the properties outlined in Sections I-A and II-C. We combine a block coordinate ascent BCA (Section III-B) and an exclusion area method EAM (Section III-C) with a surrogate (Section III-A) of the objective function as proposed in Section I-B. Although a natural choice, this has not been considered for optimal camera placement before, cf. Section II.

A. Surrogate

In optimization, a response surface model or surrogate of an objective function \( f : D \to \mathbb{R} \) with domain \( D \subset \mathbb{R}^n \) is an inexpensive function \( f : D \to \mathbb{R} \) that interpolates (possibly) scattered evaluation points \((s_1, f_1), \ldots, (s_K, f_K) \in D \times \mathbb{R} \) for which an objective value is already known \( f_k := f(s_k), \ k \in \{1, \ldots, K\} \). In this publication, a radial basis function interpolant [70], [71] as illustrated in Fig. 4 is used as a surrogate as in [77].

Let \( ||\cdot|| \) denote the Euclidean norm. Let \( \phi : \mathbb{R}^n_+ \to \mathbb{R} \) be a continuously differentiable function with \( \phi(0) = 0 \). Let \( \Pi_n^m \) be the linear space of polynomials of degree less than or equal to \( m \in \mathbb{N} \) with \( n \) variables. A real function \( f : D \to \mathbb{R} \) with

\[
f(x) := \sum_{k=1}^{K} \omega_k \phi(||x - s_k||) + p(x), \quad x \in D \tag{5}
\]

is called a radial basis function interpolant (RBF) of \( s_1, \ldots, s_K \) on \( f \) if weights \( \omega_1, \ldots, \omega_K \in \mathbb{R} \) and a polynomial \( p \in \Pi_n^m \) exist with

\[
f_k = f(s_k) \quad \text{for all } k \in \{1, \ldots, K\} \tag{6}
\]

\[
0 = \sum_{k=1}^{K} \omega_k q(s_k) \quad \forall q \in \Pi_n^m. \tag{7}
\]

Intuitively, the interpolant (5) is a polynomial surface \( p \) that interpolates the evaluation points by allowing radially symmetric dents \( \phi(||\cdot||) \) of different size \( \omega \) around each evaluation point \( s_k \). The function \( \phi \) is called the kernel of the RBF. In this work, we use a linear polynomial with a thin plate spline as a kernel:

\[
\phi(r) := r^2 \log(r), \quad p(x) := \nu \cdot x + \nu_0, \quad r, \nu \in \mathbb{R}^n, \nu_0 \in \mathbb{R} \tag{8}
\]

With this kernel, the interpolant \( \tilde{f}(x) \) in (5) is continuously differentiable. In order to find the interpolant, you need to determine the weights of the kernel \( \omega \) and of the polynomial \( \nu \) and \( \nu_0 \). These weights are set by the system of linear equations (6) and (7). The interpolation conditions (6) are underdetermined with \( K + n + 1 \) weights and \( n \) constraints. To enforce uniqueness of the interpolant and regularity of the system’s matrix, the conditions (7) are introduced. These orthogonality conditions accept smaller dents \( \omega \) where the polynomial approximates the evaluation points better and otherwise prefers larger dents. Note that \( q \) is not the same as \( p \). In practice \( q \) resembles a base of \( \Pi_n^m \), e.g. \{1, \( x^{(1)}, \ldots, x^{(n)} \} \), the brackets \( x^{(i)} \) denoting a component of \( x \). With the kernel matrix \((\Phi)_{jk} := \phi(||s_j - s_k||)\) of two evaluation points \( s_j, s_k \) and the function values \( F = (f_1, \ldots, f_K)^T \) the system of equations in this work is

\[
(\Phi \quad Q^T \quad 0) \begin{pmatrix} \omega \\ \nu \end{pmatrix} = \begin{pmatrix} F^T \\ 0 \end{pmatrix} \quad \text{with } Q := \begin{pmatrix} 1 & s_1^{(1)} & \cdots & s_K^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & s_1^{(n)} & \cdots & s_K^{(n)} \end{pmatrix}.
\]
In our solver the surrogate is optimized instead of the expensive objective function. When a good solution for the surrogate \( s_K \) is found, the actual (costly) objective function \( f(s_K) \) is evaluated and the surrogate is updated by the new evaluation point. The update of the RBF is depicted in Algorithm 1.

**Algorithm 1** \((f, x_0, f_o) \leftarrow Update(f, s_1, \ldots, s_K, s_K)\)

**Outpus:** Update of surrogate \( f \) with each eval. point \((s_k, f(s_k)), k \in \{1, \ldots, K\}\), update of maximum pair \((x_o, f_o)\).

1: \( f_K \leftarrow f(s_K) \)  //expensive function evaluation
2: Solve eq. syst. (6) and (7) with variables \( \omega, \nu, \nu_0 \)
3: \( f_o \leftarrow \max(f_1, \ldots, f_K) \)
4: \( x_o \leftarrow \arg\max(f_1, \ldots, f_K) \)
5: return \((f(\omega, \nu, \nu_0), x_o, f_o)\)

The objective function \( f \) is only evaluated in Algorithm 1 (Line 1). This way, all previous evaluation points are cached in the surrogate and the function value as well as the gradient of intermediate variable vectors \( x \neq s_k, \forall k = 1, \ldots, K \) can be approximated easily. Moreover, symmetric points of already evaluated variable vectors can be updated with the same objective value, in Line 2 by adding an additional row and column to the equation system.

**B. Block Coordinate Ascent**

For finding the next evaluation point (a.k.a. next good camera constellation) with unknown objective value to be integrated into the surrogate we employ the following approach. Within a computational optimization procedure, the \( n_m \in \mathbb{N} \) variables of a single camera \( a_m \in \mathbb{P}_m, m \in \{1, \ldots, M\} \) represent a block of the whole variable vector \( x \). The procedure of optimizing blocks of variables in turns is called Block Coordinate Ascent/Descent (BCA). The decomposed domain \( D := \mathbb{P}_1 \times \ldots \times \mathbb{P}_M \) has the dimension \( n = n_1 + \ldots + n_M \). We decompose the identity matrix \( I_n \) into the partitions \( U_m \).

\[ I_n := (U_1, \ldots, U_M) \in \mathbb{R}^{n \times n}, U_m \in \mathbb{R}^{n \times n_m}, m = 1, \ldots, M. \]

Instead of solving the problem (1) on the whole domain \( D \), a sequence of smaller problems is solved iteratively. Choose the initial variable vector \( x(0) := U_1u_1^{(0)} + \ldots + U_Ma_m^{(0)} \in \mathbb{P}_1 \times \ldots \times \mathbb{P}_M \), e.g. \( x(0) = 0 \) and subsequently solve \((i \rightarrow i+1)\):

\[
\begin{align*}
    u_1 &:= \arg\max_{v_1 \in \mathbb{P}_1} f(U_1v_1) + x^{(i)} \quad (9) \\
    u_2 &:= \arg\max_{v_2 \in \mathbb{P}_2} f(U_2v_2) + U_1u_1 + x^{(i)} \quad (10) \\
    &\vdots \\
    u_M &:= \arg\max_{v_M \in \mathbb{P}_M} f(U_Mv_M) + \sum_{m=1}^{M-1} U_mu_m + x^{(i)} \quad (11) \\
    x^{(i+1)} &:= U_1u_1 + \ldots + U_Mu_M + x^{(i)}, \text{ and go to } (9) \quad (12)
\end{align*}
\]

The only vector parameters in each line \( m = 1 \cdots M \) are the variables of the \( m \)-th block \( v_m \in \mathbb{P}_m \), the parameters of the remaining cameras are constant. We call the optimization in each line a *subspace optimization*. Since the previous evaluation point is \( x^{(i)} = (a_1, \ldots, a_M) \), the optimal solution in this subspace is

\[
(a_1 + u_1, \ldots, a_{m-1} + u_{m-1}, a_m + u_m, a_{m+1}, \ldots, a_M).
\]

The result \( u_m \in \mathbb{P}_m \) resembles an offset between \( x^{(i)} \) and the optimal solution in the subspace \( \mathbb{P}_m \). In the last step (12), the communication between the subspaces is established by adding the offsets of the recent iteration step to get a new variable vector \( x^{(i+1)} \).

By omitting the term \( \sum_{m=1}^{M} U_mu_m \) in each line (9)-(11), each subspace optimization has the same starting point, namely \( x^{(i)} \). Thereby, each subspace optimization does not depend on the results of the subspaces before and thus the lines (9)-(11) can be computed in parallel.

1) **Complexity of one iteration step:** For a problem of the general form (1)-(4) with the objective function \( f(x) = g \circ (\sigma_1, \ldots, \sigma_M)^T(x) \), let the cost for the determination of the camera specific function \( \sigma_m \) be denoted by \( c_\sigma \) and let \( c_g \) denote the cost for the fusion \( g \). Let \( I_m \in \mathbb{N} \) be the number of times that the function \( f(x) \) is evaluated in the optimization of subspace \( \mathbb{P}_m \). The evaluation \( f(x^{(i)}) \) with the complete set of \( M \) camera observations \( \sigma_1(a_1), \ldots, \sigma_M(a_M) \) needs to be calculated only in the first function evaluation with \( O(M \cdot c_\sigma + c_g) \). All except \( \sigma(a_m) \) are constant in the rest of the \( I_m - 1 \) subspace evaluations, resulting in \( O((I_m - 1) \cdot (c_\sigma + c_g)) \). With the maximum number of subspace optimization steps \( I_0 = \max_{m \in \{1, \ldots, M\}} I_m \), the complexity of the full iteration step \((i \rightarrow i+1) \) would already be reduced to \( O(M(Mc_\sigma + c_g + (I_0 - 1)(c_\sigma + c_g))) \) instead of the full \( M(Mc_\sigma + c_g)I_m \). In case of a parallel computation, \( f(x^{(i)}) \) is only called once in the whole iteration step. Let \( T \in \mathbb{N} \) be the number of possible parallel threades, reducing the complexity further:

\[
O\left(\frac{M}{T} c_\sigma + c_g + \frac{M}{T} (I_0 - 1)(c_\sigma + c_g)\right). \quad (13)
\]

2) **Stationary points:** Let us compare termination conditions in order to list stationary points in Table III-B.2 and classify the suboptimal stationary point from the example in Fig. 3. Let \( x_* \in D \) not be in the boundary of the domain. A possible termination criterion for the total BCA procedure is the arrival at a stationary point \( x_* \in D \) which holds

\[ x^{(i)} = x^{(i+1)} \quad \text{with } u_m = 0, \quad m = 1, \ldots, M \quad (14) \]

If the \( m \)-th offset is zero \( u_m = 0 \) in the \( m \)-th subspace optimization at the point \( x_* \) then

1) NO direction \( v = U_mv_m \) in the \( m \)-th block \( v_m \in \mathbb{P}_m \) exists with \( f(v + x_*) > f(x_*) \) or
2) \( f \) (differentiable) the \( m \)-th block of the gradient \( \nabla m = (\nabla_{x_m})m \) is zero

In the example in Fig. 3, \( x_* = (a_2, a_2) \) is a stationary point, all offsets are zero, but it is not optimal. If \( f \) was differentiable in \( x_* \), the gradient on all the subspaces would be zero, which renders the total gradient \( \nabla f(x_*) = (\nabla_1, \ldots, \nabla_M)^T \) being zero as well. This means all the blocks of \( x_* \) are local optima on their subspaces. However, even in this case, \( x_* \) may still be
both, a local optimum or a saddle point. Is the saddle point the suboptimal stationary point we are looking for? The gradient in both, a local optimum or a saddle point. Is the saddle point the maximum of each of the two subspaces coincides but is neither a local nor global maximum nor a saddle point, since the gradient is not zero.

The basis of the global convergence of randomized solvers is a theorem by [56, Theorem 1.3] adjusted to our notation: “Let the ‘optimization’ region \( D \) be compact. Then, a global ‘optimization’ algorithm converges to the global ‘optimum’ of any continuous function iff the sequence of iterates of the algorithm is everywhere dense in \( D \).” The randomized strategy (Algorithm 2) has been used by [77] in combination with a surrogate \( f \). The vicinity of each previous variable vector defines an exclusion area which is excluded from further sampling in the domain in Line 3. Hereby, the size of the exclusion areas \( \beta \Delta \) can be adjusted by \( \beta \in [0,1) \). The radius \( \Delta \in \mathbb{R} \) in Line 1 is the largest distance between the previously chosen variable vectors \( s_1, ..., s_K \in D \) and the furthest point in the search space \( V \subset D \). Under these constraints, the new variable vector maximizes the surrogate \( f \) in the search space \( V \).

The method of Exclusion Areas (EAM) is repeating the search and update strategies (Algorithms 2 and 1). By decreasing the size of the exclusion areas, the solver determines the next variable vector more locally in the vicinity of previous variable vectors, the choice is rather global when increasing the size. In order to use both local and global strategies, [77] has chosen to vary \( \beta \) in cycles of \( L \in \mathbb{N} \) different sizes: \( \beta_1, ..., \beta_L \).

D. RBF-BCA
A combination of the BCA, the EAM, and the RBF as a surrogate is depicted in Algorithm 3 (RBF-BCA). The solver stores the previous evaluation points \( s_1, ..., s_K \in D \) and the furthest point in the search space \( V \subset D \). Under these constraints, the new variable vector maximizes the surrogate \( f \) in the search space \( V \).

The next evaluation point \( s_{K+1} \) is chosen outside the exclusion areas in Line 4 and integrated into the surrogate in Line 5. The sequence from Line 6 until Line 11 constitute the BCA iteration steps (9)-(11). Beginning at the last evaluation point \( s_{K+1} \), the solver searches the surrogate for the best solution in one subspace of the domain. After having found the maximum in the subspace, the subspace is changed. The expensive objective function is evaluated at only such a changing point. The Line 12 handles the BCA update (12). The evaluation point is then incorporated with its symmetrical evaluation points into the surrogate in Line 13.

The RBF-BCA is terminated if a desired density of recent variable vectors \( \Delta_0 \in \mathbb{R} \) is achieved. The global convergence of the evaluation points to an optimal solution is established since \( \Delta \to 0, (K \to \infty) \) with Lines 5 and 6 [78] and therefore
Algorithm 3 RBF-BCA
BCA using a surrogate-update Update() and an exclusion area strategy Search() with termination criterion $\Delta_0 \in \mathbb{R}$.

1: $(f, x_o, f_o) \leftarrow \text{Update}(f, s_1, \ldots, s_K)$
2: while $\Delta \leq \Delta_0$ do
3:     for all $\beta \in <\beta_1, \ldots, \beta_L>$ do
4:         $s_{K+1} \leftarrow \text{Search}(\mathcal{D}, f, s_1, \ldots, s_K, \beta)$
5:         $(f, x_o, f_o) \leftarrow \text{Update}(f, s_1, \ldots, s_{K+1})$
6:     while $s$ is not stationary point do
7:         for all subspaces $\mathcal{P}_m$, $m = 1, \ldots, M$ do
8:             $t_m \leftarrow \text{Search}(\mathcal{P}_m, f, s_1, \ldots, s_{K+1}, \beta)$
9:         $(f, x_o, f_o) \leftarrow \text{Update}(f, s_1, \ldots, s_{K+1}, t)$
10:     end for
11: end while
12: $s_{K+2} \leftarrow U_1 t_1 + \ldots + U_M t_M$
13: $(f, x_o, f_o) \leftarrow \text{Update}(f, s_1, \ldots, s_{K+1}, s_{K+2})$
14: end for
15: end while

the candidates are generated dense in the domain (recall the earlier mentioned [56, Theorem 1.3]).

IV. EXPERIMENTS

We compare the efficiency and accuracy of our solver to the efficiency and accuracy of several optimization methods in synthetic (Sec. IV-B) and realistic experiments (Sec. IV-C). In the synthetic examples, the solver parameters are set such, that none of the solver iterations exceeded the maximum number of function evaluations, while in the realistic examples the solvers were purposely interrupted to record the time consumption of the expensive objective function. Let us start by consulting on the implementation of the Algorithms (Sec. IV-A).

A. Implementation:

In our algorithm, a single global surrogate $\hat{f}$ is used to cache all the evaluation points. Experiments (Fig. 6) showed that a global search in Line 4 and update to a global surrogate in Lines 5 and 13 are necessary for the convergence of the surrogate. Using a surrogate on each subspace with the same dimension as the global surrogate are difficult: Rarely, linear dependent evaluation points from the subspaces yield a singular matrix (6). More probable is the independent case, in which all the evaluation points in a subspace still shape a hyperline in the domain. The surrogate shapes a smooth hill on the hyperline even though the expensive objective $f$ does not have an extremum there.

In order to find the furthest distance in Line 1 of the Search-Algorithm 2 an MMA implemented by the nlopt library is used. To pick an initial differentiable point for Line 1, a point is randomly chosen in the domain excluding the previous variable vectors. The constraints of the search (Line 3) need to be modified before each optimization, which is done with the IPOPT library. In Line 2 of Alg.1, the equation system is solved by an LU-decompos. of the gsl library.

As termination criterion of Algorithm 3 in Line 2 we use $\Delta_0 = 5$ in case of the synthetic examples and $\Delta_0 = 0.7$ in case of the realistic examples. The number of evaluations of the actual objective function (in the Update-Algorithm 1 in Line 1) are set to 2000 in the synthetic examples, and 50 in the realistic examples. The size $n \in \mathbb{N}$ of the problem was 2,3,4, and 5 in the synthetic examples and 24 in the realistic examples. The cycle pattern $< \beta_1, \ldots, \beta_L > = < 0.98, 0.6, 0.75, 0.2, 0.01 >$ is used in Line 3 of Algorithm 3.

B. Synthetic Examples:

In order to solve the optimal camera placement problem, [41] has used an evolutionary method, [8] has used hill climbing, [39], [40] have used greedy methods. We compare the results of our method to the results of solvers similar to these implemented by the nlopt library. For local hill climbing some do not use derivative information (LN_NELDERMEAD), some more sophisticated do (LD_MMA and LD_SLSQP). As a greedy method, a local method is used solving Nelder Mead on subspaces of the domain (LN_SBPLX) and as a global evolutionary method the solver GN_ISRES is used. We compare both quantized (Fig. 4, left) and piecewise differentiable functions (Fig. 7, (3)) in several dimensions $n = 2, \ldots, 5$.

The proposed methods have randomized components. The output could therefore differ even if the same solver parameters are used. This is the reason why we called each optimization method in groups of 20 solver calls with the same solver configuration but different initial variable vectors randomly put could therefore differ even if the same solver parameters are used. Please note the logarithmic scale in the following plots which is why the results of the other local solvers are even less accurate in reality, the number of function evaluations of the other global solver is even higher and values near 0 (GN_ISRES) are not shown. The deviation of the result of the local solvers from the actual maximum value exceeds the final deviation of the RBF-BCA result by about 30 on average (Fig. 7 (1)). The number of function evaluations of the global solver exceeds the number of evaluations during the RBF-BCA about 30 times on average. (Fig. 7 (2)).

The RBF-BCA was also compared to the RBF-solver in [77] combining the RBF and EAM without BCA (Fig. 7 (4)-(5)): The incorporation of symmetric sample pairs in the surrogate increases the efficiency of our EAM linearly. Additionally,
the number of sequential function evaluations of the RBF-Solver according to the complexity (13) exceeds the number of sequential function calls of the parallelized RBF-BCA (3 evaluations of \(n+2\) were sequential).

C. Realistic Examples

Six cameras were placed and oriented with our RBF-BCA, a greedy BCA solver without surrogate (LN_SBPLX, nlopt library), a surrogate solver without BCA (RBF-Solver after [77]), a heuristic to place the cameras in the corners of the room, and a random placement. The aim is to compare the computation times and the voxel detection rates, so we have intentionally stopped the iterations at exactly 50 function calls. For each camera, two parameters determine the position at the ceiling, two parameters are used to direct it towards the rectangle \([0, 2] \times [0, 2.85]\), where \((0, 0, 0)\) is the attachment of the robot model to the floor, \((0, 2, 0)\) is the human foot and 2.85 is the height of the human model.

The quantized, expensive black-box objective function measures the volume of the voxels that are not in the visual hull by rendering images \((320 \times 240\) pixels, OpenGL 2.2.0, NVIDIA 340.46) and simulating a visual hull algorithm in the robot cell (Fig. 1, 85k human faces, 35k robot faces, and 0.3k environment faces, \(80 \times 90 \times 75\) voxels). The experiments have been performed on an “Intel(R) Core(TM)2 Duo CPU E8400” with 1999.0MHz and 6144 KB cache size using 64-bit “openSUSE 13.1 (x86_64)” and 2GB RAM.

V. CONCLUSION

The manual placement of multiple cameras in environments with several rooms or complicated obstacles takes consideration time and becomes less intuitive the more complex the environment becomes. We strongly recommend an iterative method, in particular the combination of a BCA to accelerate the objective function, a surrogate for fast, smooth optimization, and an EAM for global convergence.
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