Development of competencies for teaching geometry through an analysis learning obstacle

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Abstract. Teaching and learning process that is carried out often results in learning obstacles. Based on the facts teachers rarely learn the thinking behind students' learning obstacles. Through the analysis process of learning obstacles, is expected to develop the competency of a pre-service mathematics teacher to teach geometry. The design used in this study is a qualitative research design. This study uses the methodological framework of the Didactical Design Research (DDR). DDR or didactic design research that developed uses the philosophical foundation of hermeneutics, phenomenology, and ethnomet hodology. The procedure of research through the examination of pre-service mathematics teachers in the implementation of Program Pengenalan Lapangan (PPL). Participants in the course are pre-service teachers who practice in their classrooms with various activities, such as geometric analysis and construction, presentation and resolution of geometrical problems, and the development of teaching units for geometry. Each of these activities allows learning obstacles to occur. The task of the pre-service teacher is to be able to analyze the learning obstacles faced by students. With the learning obstacle analysis process, it is the beginning of a pre-service mathematics teacher compiling in-depth knowledge related to the geometry concepts being studied.

1. Introduction
The purpose of learning mathematics is developed on four basic points namely work with numbers and operations, development of geometric thinking, algebraic thinking and work with data [1]. Geometric thinking is more than the ability to do geometry tasks [2]. Geometry has been the basis of human visual practices. It was the first mathematical subject represented as a universal axiomatic deductive system [3]. The focus of geometry learning objectives involves considering the nature of geometry and the range of its applications. based on this all geometry learning activities consider spatial thinking, visualization, and proof. [4]. The teacher's ability to prepare geometry learning activities requires content knowledge and pedagogical knowledge so that the objectives of learning can be achieved. The teacher not only has expert skills in pedagogy and content but has the ability to a unique combination of these two knowledge [5], [6]. Pedagogical content knowledge represents a fertile area for expanding the knowledge base for teaching [7]. The teacher's ability to create geometrical learning situations that make students learn well needs to be supported by the teacher's content knowledge and pedagogical knowledge about geometry.

The balance between the process of adaptation and acculturation illustrates that the learning process occurs ideally so that goals can be optimally achieved from the situation designed [8]. The design of
new situations in mathematics learning can be achieved by students when they can use prior knowledge [9]. Prior knowledge is the basis for discovering new knowledge [8]. In the process of development of knowledge, an individual often encounters obstacles [10]. Learning difficulties are not to be eliminated but analyzing them will provide information to improving the learning process [11], [12]. For teachers, the ability to manage learning obstacles becomes a means for teacher professional development [13]. The teacher experiences collaborating with other teachers to plan or discuss learning obstacles are ways to strengthen their mathematical understanding and mathematics knowledge for teaching [14]. Thus, a teacher must be able to reflect on learning to strengthen their mathematical understanding and mathematical knowledge and also of course help students create learning situations based on learning obstacles.

Three types of learning barriers according to Brousseau: ontogenic barriers, epistemological barriers, and didactic barriers [15]. The ontogenic obstacles occur because the teaching and learning process is not following the readiness of students. The didactical obstacles are obstacles that occur due to errors in the learning process that comes from the learning system at school or comes from the teacher's strategy used in learning certain mathematical ideas. Epistemological obstacles are essentially student knowledge which is only limited to certain contexts. If the student faced different contexts, the knowledge possessed cannot be used or he has difficulty using it. A teacher in carrying out his professional assignments must have the competence that is always maintained. Likewise, a pre-service mathematics teacher must have these competencies before they become real teachers. Each educator has a diversity of potentials, learning experiences, and competencies in the field of education [16]. So, every college needs to prepare pre-service mathematics teachers who can achieve the objectives of learning geometry. Through the activity of analyzing learning obstacles, pre-service teachers experience a process of learning reflection that serves as one of their professional development tools.

Pre-service mathematics teachers with a background in geometry played a very important role in their learning in college courses especially the methods course [17]. Professional development has recently come to be viewed as a long term process, covering different types of opportunities and experiences that are systematically planned to stimulate the development and evolution of the teacher [18]. Through the Program Pengenalan Lapangan (PPL), is the beginning of their professional development before becoming a real teacher and of course, pre-service teachers have directed to experience the process of learning reflection by analyzing each learning obstacle faced by students in the teaching and learning process. Only a few teachers learn the thinking behind the student learning obstacle [19]. Only a few teachers learn the thinking behind the mistakes students make. The purpose of this study through the analysis process of learning obstacles pre-service teachers can develop their mathematical knowledge, by understanding mathematical knowledge is expected to have an impact on how they teach better.

2. Methods
This research is based on an interpretive paradigm to understand phenomena that occur and can contribute to problem solutions based on theoretical perspectives. This research is research using the methodology of Didactical Design Research and through three stages of analysis, namely prospective analysis, metapedia didaktik analysis, and retrospective analysis [20]. Stage 1: Prospective analysis (1) Determine the material that will become the research material, in this study the material chosen is the topic of congruence. (2) Looking for data/literature on congruence materials (3). Perform repersonalization (self-meaning of the concepts being studied) and re-contextualization (the way of self-meaning to the evolution experienced) of the material that has been determined. Stage 2 Metapedididaktik analysis, and Stage 3: Retrospective analysis. Data collection is done by interview, observation and documentation for data collection [21]. Before doing the initial didactic design of congruent material on a triangle, the first step to take is to test the instrument to get a learning obstacle for students who have learned the concept. The next step is to analyze the learning trajectory, including analyzing the sourcebook as a guide for students and teachers and analyzing the learning video. Based
on Obstacle Learning and learning trajectory obtained can be a reference to compile an initial didactic design that can overcome the learning obstacle in learning.

3. Result and Discussion

Congruence, and triangle congruence, in particular, is generally taken to be a key topic in school geometry. When comparing the two triangles in the congruence concept, the angles of the layout are equal and the sides are proportional. If choosing sides that are valued at 1, for example at the meeting AB: PQ = 1, BC: QR = 1 and AC: PR = 1, then the two triangles ABC and PQR are congruent or known as congruent. When we will discuss the concept of congruence, mathematics teacher candidates must pay attention to the prerequisite material that students must master is the concept of transformation geometry, one of which. When faced with learning obstacle problems that have the potential to be faced by students, pre-service mathematics teachers are expected to be able to analyze this. Based on a literature review and analysis that has been done. Pre-service mathematics teachers try to give questions about the concept of congruent triangles. Figure 1. below shows the students' questions and answers.

Note that $\triangle ABC, AC \neq BC$ prove that $\triangle ABC \neq \triangle BCD$

Look at the image below

If $\triangle DEF \cong \triangle HEG$, determine the value of $x!$

Figure 1. Questions and answer student

Based on Figure 1. above, mathematics teacher candidates are expected to be able to analyze the reality of the meaning written by students. When prospective mathematics teachers conduct obstacle learning analysis they will experience a didactical transposition process. It is usual to say that French didactics has three main theoretical pillars: the theory of didactical situations due to Brousseau, the theory of conceptual fields due to Vergnaud, and the anthropological theory of the didactic that emerged from the theory of didactic transposition, due to Chevallard [22]. Didactic transposition process: The object of scientific knowledge is generated, usually in a research context. These are then selected and reorganized in a social context to be part of the knowledge to be taught, for example as part of the official curriculum. Then transformed into actual knowledge taught in the context of teaching, for example, classrooms. Finally, obtained by students, knowledge is learned. Pre-service mathematics teachers undergoing a process of repersonalization and re-contextualization, re-depersonalization and re-decontextualization [23]. The process of didactic transposition found in Figure 2.
Figure 2. The process didactic transposition [24]

**THE POINT.**

1. A point is that which has position but not dimensions.

**THE LINE.**

2. A line is length without breadth.

**THE PLANE.**

5. A surface is that which has length and breadth.

**THE ANGLE.**

9. The inclination of two right lines extending out from one point in different directions is called a *rectilinear angle*.

**CONCURRENT LINES.**

18. Three or more right lines passing through the same point are called *concurrent* lines.

**THE TRIANGLE.**

20. A *triangle* is a figure formed by three right lines joined end to end. The three lines are called its *sides*.

21. A triangle whose three sides are unequal is said to be *scalene*, as $A$; a triangle having two sides equal, to be *isosceles*, as $B$; and and having all its sides equal, to be *equilateral*, as $C$.

22. A *right-angled* triangle is one that has one of its angles a right angle, as $D$. The side which subtends the right angle is called the *hypotenuse*.

23. An *obtuse-angled* triangle is one that has one of its angles obtuse, as $E$.

24. An *acute-angled* triangle is one that has its three angles acute, as $F$.

25. An *exterior* angle of a triangle is one that is formed by any side and the continuation of another side.

Hence a triangle has six exterior angles; and also each exterior angle is the supplement of the adjacent interior angle.

Figure 3. Basic study of basic geometry pre-service mathematics teachers
The didactic transposition process with the subject of mathematical dimensions is to provide a mathematical process experience. The existence of mathematics courses in the mathematics teacher education curriculum is not only to equip the conceptual substance aspects but more than that, namely to provide the experience of thinking as a mathematician thinks. When they examine a problem related to how to study the learning obstacle, the pre-service mathematics teacher must go through this didactic transposition process. Pre-service mathematics teacher builds scholarly knowledge of congruent material based on the study of several books at the university level. The approach used to detect obstacle learning in terms of the prerequisite knowledge in a certain basic competency. Before students understand new basic competencies, they must first understand the basic competencies of the prerequisites, both vertically and horizontally [25]. Figure 3 is an example of a pre-service teacher builds scholarly knowledge of congruent based on the book The Elements Of Euclid [26].

The first stage in the Gagne learning hierarchy is to analyze a target learning ability and identify student abilities. The next stage is to identify basic abilities, namely abilities that are the basis or prerequisites for mastering the next ability. These basic abilities can be simple concepts, conventions, or algorithms, they can also be concepts that have sub-concepts. Pre-service mathematics teachers build their congruent knowledge based on references they have reviewed, one of which is based on references from the Teaching and Learning Geometry book by Dough French. Based on the book, one of the prerequisites in constructing knowledge of the concept of congruence is to explore three transformations is a translation, reflection, and rotation [27]. Study of three transformations pre-service mathematics teacher used the book An Introduction to transformational Geometry with its author Frank M Eccles. Study of three transformations pre-service mathematics teacher used the book An Introduction to transformational Geometry with its author Frank M Eccles. Figure 4 is an example of a study conducted by a pre-service mathematics teacher to explore the concept of transformation.

**Definition.** A transformation of the plane is a one-to-one function from the plane onto the plane.

**Definition.** A mapping $S$ is a translation if there exists a directed segment $\overrightarrow{AB}$ such that for every point $P$ of the plane, $S(P) = P' \text{ and } \overrightarrow{PP'} = \overrightarrow{AB}$.

**Definition.** A directed angle is an angle one ray of which has been designated the initial side and the other ray the terminal side.

**Definition.** Let $A$ be a given point and let $\theta$ be a number between $-180$ and $+180$. A rotation about $A$ through an angle $\theta$ is a mapping $R_{A,\theta}$ defined for all points of the plane as follows:

1. $R_{A,\theta}(A) = A$ and
2. for $P \neq A$, $R_{A,\theta}(P) = P^*$ such that $m \angle PAP^* = \theta$ and $AP^* = AP$.

**Corollary 3.2A.** The product of two line reflections is either a rotation or a translation.

Figure 4. Scholarly knowledge about three transformation.

After exploring the concept of three transformation researchers tried to ask the relationship between the concept of three transformations with congruency. Pre-service mathematics teacher's answer "we all know that congruence is two fields that have the same shape and size while translation is a shift of fields from point to point with a certain distance and direction, so the relationship between the two is the process of the translation does not change the shape and the size of a field, and this is the same as the concept of congruence ". Then their findings on the concept of rotation "is a change in the position of objects by turning around a certain centre and angle, the direction of the angle will be positive if it is counterclockwise and will be negative if it is clockwise. The relationship between rotation and congruence is the shape of the size of a field that undergoes the same shape and size rotation process. " The third concept is, Then their findings on the concept of rotation "is a change in the position of objects by turning around a certain centre and angle, the direction of the angle will be positive if it is
counterclockwise and will be negative if it is clockwise. The relationship between rotation and congruence is the shape of the size of a field that undergoes the same shape and size rotation process. "Understanding candidates for pre-service mathematics teachers becomes very important to the material being studied, so when they will teach they will give a positive attitude. If a teacher has a good conceptual understanding of mathematics, it positively influences the learning process in the classroom [28], [29].

4. Conclusion
The didactic transposition process, one of which is to build scholarly knowledge, is very important as an alternative to improving teacher knowledge in mastering mathematical concepts. Even when they will analyze their learning obstacle analysis of the difficulties students face will be detected properly, this will also be a solution if faced with problems in the learning process that is the problem arises one of them from the culture of mathematics education community where governance of resources human resources (HR) both in universities and schools of cultural academic thinking that tend to be imitative in the context of learning; (2) the culture of thinking of the teaching profession that tends to be procedural-administrative in the context of self-capacity development; (3) a culture of community thinking among the teaching profession that tends to be isolated. One of the keywords that can be used to initiate the process of continuous improvement in improving the educator's belief system is independence. Many students hold that the teacher is the main source of acquiring knowledge so the teacher needs to prepare a learning situation that can make students construct their knowledge. The continuous improvement here is that pre-service mathematics teachers must experience didactic transposition.

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