$B_s$-$\bar{B}_s$ mixing interplay with $B$ anomalies

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Abstract

After reviewing the theoretical uncertainties entering the Standard Model determination of the mass difference of the neutral $B_s$-$\bar{B}_s$ meson system, $\Delta M_{\text{SM}}^s$, we discuss the implications of its updated value for new physics models addressing the experimental anomalies in semi-leptonic $B$ decays. Using the most recent FLAG average of lattice results for the non-perturbative matrix elements and the CKM-fitter determination of $V_{cb}$ points to a $1.8 \sigma$ discrepancy in $\Delta M_{\text{SM}}^s > \Delta M_{\text{exp}}^s$. Extending the analysis in Ref. [1] we show that the latter tension cannot be easily accommodated within single mediator models, whenever the same mediator is also responsible for the $b \to s\ell\ell$ anomalies.

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1. Introduction

While awaiting the LHCb Run-2 updates about the tantalizing hints of new physics (NP) in semi-leptonic $B$-meson decays [2–14] it would be natural to expect possible deviations from the Standard Model (SM) also in 4-quark and 4-lepton effective operators. In fact, it is almost a theorem that a NP contribution say in $b \rightarrow s \ell \ell$ will eventually feed into a $(bs^\dagger)^2$ operator. The latter are very well constrained by the measurement of the mass difference of the neutral $B_s$–$\bar{B}_s$ meson system, $\Delta M_s$, which provides a severe constraint for any NP model aiming at an explanation of the $B$-physics anomalies. For quite some time the SM value for $\Delta M_s$ was in perfect agreement with experimental results, see e.g. [15, 16]. Taking however, the most recent lattice inputs, in particular the new average provided by the Flavour Lattice Averaging Group (FLAG) one gets a SM value considerably above the measurement. In this note, which is based on Ref. [1], we briefly review the SM prediction of $\Delta M_s$ and discuss its impact on NP models addressing the $B$ anomalies. We also complement Ref. [1] with an analysis of simplified $Z'$ models featuring either complex couplings or general left-handed (LH) and right-handed (RH) chirality structures, having in mind the possibility of fitting simultaneously both the $b \rightarrow s \ell \ell$ anomalies and the $\Delta M_s$ tension. We conclude, however, that the two latter observables cannot be straightforwardly accommodated within a single-mediator simplified model.

2. $\Delta M_s$ in the Standard Model

The mass difference of the mass eigenstates of the neutral $B_s$ mesons is given by

$$\Delta M_s \equiv M_H^s - M_L^s = 2 |M_{12}^s|.$$  \hfill (1)

![SM diagrams](image)

Figure 1: SM diagrams for the transition between $B_s$ and $\bar{B}_s$ mesons. The contribution of internal off-shell particles is denoted by $M_{12}^s$. 
The calculation of the box diagrams in Fig. 1 gives the SM value for $M_{12}^s$, see e.g. [15] for a brief review, and one gets

$$M_{12}^s = \frac{G_F^2}{12\pi^2} \lambda_t^2 M_W^2 S_0(x_t) B f_{B_s}^2 M_{B_s} \hat{n}_B,$$

(2)

with the Fermi constant $G_F$, the masses of the $W$ boson, $M_W$, and of the $B_s$ meson, $M_{B_s}$. Using CKM unitarity one finds only one contributing CKM structure $\lambda_t = V_{ts}^* V_{tb}$. The CKM elements are the only place in Eq. (2) where an imaginary part can arise. The result of the 1-loop diagrams given in Fig. 1 is denoted by the Inami-Lim function [17] $S_0(x_t = (\bar{m}_t(\bar{m}_t))^2/M_W^2) \approx 2.36853$, where $\bar{m}_t(\bar{m}_t)$ is the MS-mass [18] of the top quark. Perturbative 2-loop QCD corrections are encoded in the factor $\hat{n}_B \approx 0.83798$ [19]. In the SM calculation of $M_{12}^s$ one four quark $\Delta B = 2$ operator arises

$$Q = \bar{s}^\alpha \gamma_\mu (1 - \gamma_5) b^\alpha \times \bar{s}^\beta \gamma_\mu (1 - \gamma_5) b^\beta.$$  

(3)

The hadronic matrix element of this operator is parametrised in terms of a decay constant $f_{B_s}$ and a bag parameter $\hat{B}$:

$$\langle Q \rangle \equiv \langle B_s^0 | \bar{B}_s^0 \rangle = \frac{8}{3} M_{B_s}^2 f_{B_s}^2 \hat{B}(\mu),$$  

(4)

We also indicated the renormalisation scale dependence of the bag parameter; in our analysis we take $\mu = \bar{m}_b(\bar{m}_b)$.

Sometimes a different notation for the QCD corrections and the bag parameter is used in the literature (e.g. by FLAG [20]), $(\eta_B, \hat{B})$ instead of $(\hat{n}_B, B)$ with $\hat{n}_B B \equiv \eta_B \hat{B}$ and $\hat{B} = 1.51926 B$. The parameter $\hat{B}$ has the advantage of being renormalisation scale and scheme independent.

A commonly used SM prediction of $\Delta M_s$ was given by [15]

$$\Delta M_{s,2015}^{SM} = (18.3 \pm 2.7) \text{ ps}^{-1},$$  

(5)

that agreed very well with the experimental measurement [21]

$$\Delta M_{s}^{Exp} = (17.757 \pm 0.021) \text{ ps}^{-1}.$$  

(6)

In 2016 Fermilab/MILC presented a new calculation [22], which gave considerably larger values for the non-perturbative parameter, resulting in values around 20 ps$^{-1}$ for the mass difference [22–26] and being thus larger than the experimental measurement. An independent confirmation of these large
values would of course be desirable; a first step in that direction has been
done by the HQET sum rule calculation of [27]. In that work they calcu-
late only the bag parameters for \(B_d\)-mixing, however these should be close
to those for \(B_s\)-mixing – a preliminary result for the \(B_s\)-mixing parameters
was presented at CKM2018 [28]. Their results for the bag parameters agree
(within uncertainties) with Fermilab/MILC.
Using the most recent numerical inputs we predict the mass difference of the
neutral \(B_s\) mesons to be \[\Delta M_s^{\text{SM}, 2017} = (20.01 \pm 1.25) \text{ ps}^{-1}.\] (7)
Here the dominant uncertainty still comes from the lattice predictions for
the non-perturbative parameters \(B\) and \(f_{B_s}\), giving a relative error of 6%.
The uncertainty in the CKM elements (determined assuming unitarity of
the CKM matrix) contributes 2% to the error budget. Other uncertainties
can be safely neglected at the current stage. The new central value for
the mass difference in Eq. (7) is 1.8 \(\sigma\) above the experimental one given in
Eq. (6). This difference has profound implications for NP models that predict
sizeable positive contributions to \(\Delta M_s\). The new value for the SM prediction
depends strongly on the non-perturbative input as well as the values of the
CKM elements (in particular the element \(V_{cb}\)). We use the averages that are
provided by the lattice community (web-update of FLAG [20]) and by the
CKMfitter group (web-update of [29] – similar values can be taken from the
UTfit group [30]). For further details we refer the reader to Ref. [1].

3. \(\Delta M_s\) beyond the Standard Model
To determine the allowed space for NP effects in \(B_s\)-mixing we compare the
experimental measurement of the mass difference with the prediction in the
SM plus NP:
\[
\Delta M_s^{\text{Exp}} = 2 \left| M_{12}^{\text{SM}} + M_{12}^{\text{NP}} \right| = \Delta M_s^{\text{SM}} \left| 1 + \frac{M_{12}^{\text{NP}}}{M_{12}^{\text{SM}}} \right|. \tag{8}
\]
In the following, we will assume that NP effects do not involve sizeable shifts
in the CKM elements.

\[\text{A more conservative determination of the SM value of the mass difference using only}\]
\[\text{tree-level inputs for the CKM parameters is } \Delta M_s^{\text{SM}, 2017 \text{ (tree)}} = (19.9 \pm 1.5) \text{ ps}^{-1}.\]
A simple estimate shows that the improvement of the SM prediction from Eq. (5) to Eq. (7) can have a drastic impact on the size of the allowed NP effects on $B_s$-mixing. For a generic NP model we can parametrise

$$\frac{\Delta M_{s}^{\text{Exp}}}{\Delta M_{s}^{\text{SM}}} = \left| 1 + \frac{\kappa}{\Lambda_{NP}^2} \right|,$$

where $\Lambda_{NP}$ denotes the mass scale of the NP mediator and $\kappa$ is a dimensionful quantity which encodes NP couplings and the SM contribution. If $\kappa > 0$ (this is often the case in many NP scenarios for $B$ anomalies), and since $\Delta M_{s}^{\text{SM}} > \Delta M_{s}^{\text{Exp}}$, the $2\sigma$ bound on $\Lambda_{NP}$ scales like

$$\frac{\Lambda_{NP}^{2017}}{\Lambda_{NP}^{2015}} = \left| \frac{\Delta M_{s}^{\text{Exp}}}{(\Delta M_{s}^{\text{SM}} - 2\Delta M_{s}^{\text{SM}})^{2015}} - 1 \right| \simeq 5.2,$$

where $\delta\Delta M_{s}^{\text{SM}}$ denotes the $1\sigma$ error of the SM prediction. Hence, in models where $\kappa > 0$, the limit on the mass of the NP mediators is strengthened by a factor 5. On the other hand, if the tension between the SM prediction and $\Delta M_{s}^{\text{Exp}}$ increases in the future, a NP contribution with $\kappa < 0$ would be required in order to accommodate the discrepancy.

A typical example where $\kappa > 0$ is that of a purely LH vector-current operator, which arises from the exchange of a single mediator featuring real couplings, cf. Section 3.1. In such a case, the short-distance contribution to $B_s$-mixing is described by the effective Lagrangian

$$\mathcal{L}_{\Delta B = 2} = -\frac{4G_F}{\sqrt{2}} (V_{tb} V_{ts}^*)^2 \left[ C_{bs}^{LL} (\bar{s}_L \gamma_\mu b_L)^2 + \text{h.c.} \right],$$

where $C_{bs}^{LL}$ is a Wilson coefficient to be matched with some ultraviolet (UV) model. This coefficient enters Eq. (8) as

$$\frac{\Delta M_{s}^{\text{Exp}}}{\Delta M_{s}^{\text{SM}}} = \left| 1 + \frac{C_{bs}^{LL}}{R_{\text{loop}}^{\text{SM}}} \right|,$$

where

$$R_{\text{loop}}^{\text{SM}} = \frac{\sqrt{2}G_F M_W^2 \hat{\eta}_B S_0(x_t)}{16\pi^2} = 1.3397 \times 10^{-3}.$$

In the following, we will show how the updated bound from $\Delta M_s$ impacts the parameter space of simplified models (with $\kappa > 0$) put forth for the explanation of the recent discrepancies in semi-leptonic $B$-physics data (Section 3.1) and then discuss the feasibility of some $\kappa < 0$ scenarios (Section 4).
3.1. Impact of $B_s$-mixing on NP models for $B$ anomalies

A useful application of the refined SM prediction in Eq. (7) is in the context of the recent hints of LFU violation in semi-leptonic $B$-meson decays. Focussing on neutral current anomalies, the main observables are the LFU violating ratios $R_{K^{(*)}} \equiv B(B \to K^{(*)} \mu^+ \mu^-)/B(B \to K^{(*)} e^+ e^-)$ [2, 3], together with the angular distributions of $B \to K^{(*)} \mu^+ \mu^-$ [4, 11, 31, 32] and the branching ratios of hadronic $b \to s \mu^+ \mu^-$ decays [4, 5, 33]. As hinted by various recent global fits [34–40], and in order to simplify a bit the discussion, we assume NP contributions only in purely LH vector currents involving muons. The effective Lagrangian for semi-leptonic $b \to s \mu^+ \mu^-$ transitions contains the terms

$$L_{NP}^{b \to s \mu^+ \mu^-} \supset \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* (\delta C_9 O_9^\mu + \delta C_{10}^\mu O_{10}^\mu) + \text{h.c.},$$

with

$$O_9^\mu = \frac{\alpha}{4\pi} (\bar{s}_L \gamma_\mu b_L)(\bar{\mu}_\gamma \mu),$$

$$O_{10}^\mu = \frac{\alpha}{4\pi} (\bar{s}_L \gamma_\mu b_L)(\bar{\mu}_\gamma \mu \gamma_5 \mu).$$

Assuming purely LH currents and real Wilson coefficients the best-fit of $R_K$ and $R_{K^*}$ yields (from e.g. [35]): $\text{Re} (\delta C_9^\mu) = -\text{Re} (\delta C_{10}^\mu) \in [-0.81, -0.48]$ ([-1.00, -0.32]) at 1$\sigma$ (2$\sigma$). Adding also the data on $B \to K^{(*)} \mu^+ \mu^-$ angular distributions and other $b \to s \mu^+ \mu^-$ observables improves the statistical significance of the fit, but does not necessarily implies larger deviations of $\text{Re} (\delta C_9^\mu)$ from zero (see e.g. [34]). For the results first presented in Ref. [1], we will stick only to the $R_K$ and $R_{K^*}$ observables and denote this benchmark as “$R_{K^{(*)}}$”, while for new results we present here a wider range of observables is used (denoted by “$b \to s \ell \ell^*$”).

3.1.1. $Z'$

A paradigmatical NP model for explaining the $B$ anomalies in neutral currents is that of a $Z'$ dominantly coupled via LH currents. Here, we focus only on the part of the Lagrangian relevant for $b \to s \mu^+ \mu^-$ transitions and $B_s$-mixing, namely

$$\mathcal{L}_{Z'} = \frac{1}{2} M_{Z'}^2 (Z'_\mu)^2 + \left( \lambda_{Q}^{L} \bar{d}_L^\alpha \gamma^\mu d_L^\alpha + \lambda_{L}^{Q} \bar{\ell}_L^\alpha \gamma^\mu \ell_L^\alpha \right) Z'_\mu,$$

where $d^i$ and $\ell^\alpha$ denote down-quark and charged-lepton mass eigenstates, and $\lambda^{Q,L}$ are hermitian matrices in flavour space. Of course, any full-fledged
(i.e. $SU(2)_L \times U(1)_Y$ gauge invariant and anomaly free) $Z'$ model attempting an explanation of $R_{K(\ast)}$ via LH currents can be mapped into Eq. (17). After integrating out the $Z'$ at tree level, we obtain the effective Lagrangian

$$\mathcal{L}^{\text{eff}}_{Z'} = -\frac{1}{2M^2_{Z'}} \left( \lambda^Q_{ij} \bar{d}_L \gamma_\mu d^i_L + \lambda^L_{\alpha\beta} \bar{\ell}^\alpha_L \gamma_\mu \ell^\beta_L \right)^2$$

$$\supset -\frac{1}{2M^2_{Z'}} \left( (\lambda^Q_{23})^2 (\bar{s}_L \gamma_\mu b_L)^2 + 2\lambda^Q_{23} \lambda^L_{22} (\bar{s}_L \gamma_\mu b_L)(\bar{\mu}_L \gamma_\mu \mu_L) + \text{h.c.} \right).$$

Matching with Eq. (14) and (11) we get

$$\delta C^\mu_9 = -\delta C^\mu_{10} = -\frac{\pi}{\sqrt{2G_F M^2_{Z'}}} \left( \lambda^Q_{23} \lambda^L_{22} \frac{V_{tb} V^*_{ts}}{M_{Z'}} \right),$$

and

$$C^{LL}_{bs} = \frac{\eta^{LL}(M_{Z'})}{4\sqrt{2G_F M^2_{Z'}}} \left( \frac{\lambda^Q_{23}}{V_{tb} V^*_{ts}} \right)^2,$$

where $\eta^{LL}(M_{Z'})$ encodes the running down to the bottom mass scale using NLO anomalous dimensions \cite{41, 42}. E.g. for $M_{Z'} \in [1, 10] \text{ TeV}$ we find $\eta^{LL}(M_{Z'}) \in [0.79, 0.75]$.

Here, we first consider the case of a real coupling $\lambda^Q_{23}$, so that $C^{LL}_{bs} > 0$ and $\delta C^\mu_9 = -\delta C^\mu_{10}$ is also real. This assumption follows the standard approach of nearly all the groups performing global fits \cite{34, 39, 43}. The case of complex $Z'$ couplings will be considered in Section 4.1.

The impact of the improved SM calculation of $B_s$-mixing on the parameter space of the $Z'$ explanation of $R_{K(\ast)}$ is displayed in Fig. 2, for the reference value $\lambda^L_{22} = 1$\footnote{For $m_{Z'} \lesssim 1 \text{ TeV}$ the coupling $\lambda^L_{22}$ is bounded by the $Z \rightarrow 4\mu$ measurement at LHC and by neutrino trident production \cite{44}. See for instance Fig. 1 in \cite{45} for a recent analysis.}. Note that the old SM determination, $\Delta M_{s,\text{SM},2015}$, allowed for $M_{Z'}$ as heavy as $\approx 10 \text{ TeV}$ in order to explain $R_{K(\ast)}$ at $1\sigma$. In contrast, $\Delta M_{s,\text{SM},2017}$ implies now $M_{Z'} \lesssim 2 \text{ TeV}$. Even for $\lambda^L_{22} = \sqrt{4\pi}$, which saturates the perturbative unitarity bound \cite{46, 47}, we find that the updated limit from $B_s$-mixing requires $M_{Z'} \lesssim 8 \text{ TeV}$ for the $1\sigma$ explanation of $R_{K(\ast)}$. Whether a few TeV $Z'$ is ruled out or not by direct searches at LHC depends however on the details of the $Z'$ model. For instance, the stringent constraints from di-lepton searches \cite{48} are tamed in models where the $Z'$ couples mainly to
third generation fermions (as e.g. in [49]). This notwithstanding, the updated limit from $B_s$-mixing cuts dramatically into the parameter space of the $Z'$ explanation of the $b \to s\mu^+\mu^-$ anomalies.

4. Model building directions for $\Delta M_s^{\text{NP}} < 0$

Given that $\Delta M_s^{\text{SM}} > \Delta M_s^{\text{exp}}$ at about 2$\sigma$, it is worth to investigate possible ways to obtain a negative NP contribution to $\Delta M_s$, thus relaxing the tension between the SM and the experimental measurement.

Sticking to the simplified model of Section 3.1 ($Z'$ coupled only to LH currents), an obvious solution in order to achieve $C_{bs}^{\text{LL}} < 0$ is to allow for complex couplings (cf. Eq. (20)). For instance, in $Z'$ models this could happen as a consequence of fermion mixing if the $Z'$ does not couple universally in the gauge-current basis (see e.g. [50]). Extra phases in the couplings are constrained by CP-violating observables, which we will discuss in Section 4.1.
An alternative way to achieve a negative contribution for $\Delta M_s^{NP}$ is to go beyond the simplified models of Section 3.1 and contemplate generalised chirality structures. Let us consider for definiteness the case of a $Z'$ coupled both to LH and RH down-quark currents

$$\mathcal{L}_{Z'} \supset \frac{1}{2} M_{Z'}^2 (Z'_\mu)^2 + \left( \lambda_{ij}^Q \bar{d}_i^L \gamma^\mu d_j^L + \lambda_{ij}^d \bar{d}_i^R \gamma^\mu d_j^R \right) Z'_\mu.$$  

Upon integrating out the $Z'$ one obtains

$$\mathcal{L}_{Z'}^{\text{eff}} \supset -\frac{1}{2 M_{Z'}^2} \left[ (\lambda_{23}^Q)^2 (\bar{s}_L \gamma_\mu b_L)^2 + (\lambda_{23}^d)^2 (\bar{s}_R \gamma_\mu b_R)^2 
+ 2 \lambda_{23}^Q \lambda_{23}^d (\bar{s}_L \gamma_\mu b_L)(\bar{s}_R \gamma_\mu b_R) + \text{h.c.} \right].$$

The LR vector operator can clearly have any sign, even for real couplings, and we take up this possibility in Section 4.2.

4.1. Complex Couplings

In this section we consider the case of complex couplings, first from a model independent perspective (Section 4.1.1) and then in a specific $Z'$ model (Section 4.1.2).

4.1.1. Fit to complex $\delta C^\mu_9$

So far the focus of global fits has been on NP coefficients with the same phase as the SM contributions (which are essentially real as only a very small phase is generated by $\text{Arg}(V_{tb} V_{ts}^*) = -3.12 \approx -179^{\circ}$), barring however few exceptions [51, 52]. Here, we extend the study in Ref. [1] by performing our own fit to a specific scenario where NP only arises in $\delta C^\mu_9 = -\delta C^\mu_{10}$, using flavio [53]. The result is shown in Fig. 3 – we see that while there is a relatively narrow range for the real part to explain the flavour anomalies, the imaginary part has much more freedom. (The shape of the allowed region in the complex $\delta C^\mu_9$ space matches that found by [51].) This can be qualitatively understood from the fact that the imaginary part only arise quadratically in the expressions for $R_{K^{(*)}}$ since the leading interference term with the SM amplitude is real. Hence the imaginary part is relatively unconstrained by the fit unless $\text{Im} \delta C^\mu_9 \gtrsim \text{Re} \delta C_9$. 

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4.1.2. Complex $Z'$

Once we allow the $Z'$ quark coupling to be complex, there are extra constraints to be considered, in the form of CP-violating observables that arise from $B_s$-mixing. The most relevant here is the mixing-induced CP asymmetry \cite{15,54}, arising from interference between B meson mixing and decay. The semi-leptonic CP asymmetries for flavour-specific decays, $a_{s\ell\ell}^a$, are not competitive here since the experimental errors are still too large \cite{15}. Defining

\[
\phi_\Delta = \text{Arg} \left( 1 + \frac{C_{bs}^{LL}}{F_{\text{SM}}^{\text{loop}}} \right),
\]

(23)

the mixing-induced CP asymmetry is given by

\[
A_{\text{CP}}^{\text{mix}}(B_s \to J/\psi\phi) = \sin (\phi_\Delta - 2\beta_s),
\]

(24)

where $A_{\text{CP}}^{\text{mix}} = -0.021 \pm 0.031$ \cite{21}, $\beta_s = 0.01852 \pm 0.00032$ \cite{29}, and we neglected penguin contributions \cite{15}.
Figure 4: Fit to complex $Z'$ couplings. The darker (lighter) shaded regions show the 1σ (2σ) allowed regions respectively.

Including this extra observable in our fit, we display our results in Fig. 4, for the reference values $M'_{Z} = 5$ TeV and $\lambda_{22} = 1$. While there are regions in which both $b \to s\ell\ell$ and $\Delta M_s$ can be accommodated at 1σ, the additional constraint from $A_{\text{mix}}^{\text{CP}}$ precludes this possibility by setting a strong a limit on the imaginary part of the $Z'$ coupling.

4.2. Fit with RH quark coupling

As discussed above, if we extend the minimal model to include both LH and RH down-quark currents, there arises an interference term in $\Delta M_s$ with arbitrary sign. Moreover, since this term gets enhanced by renormalisation-group effects compared to LL and RR vector operators [55], it can easily dominate the contribution to $\Delta M_s^{\text{NP}}$. However, while there are no extra constraints to be taken into account as for the case of a complex coupling, this scenario brings in its own problem – namely that the contribution to $R_{K^{(*)}}$ via RH quark currents must be sizable. Current global fits disfavour a purely RH quark current, as this breaks the experimentally observed relation.
$R_K \approx R_{K^*}$ (see e.g. [36] for further details). The question then is whether a combined explanation of $R_{K^(*)}$ and $\Delta M_s$ is possible within the framework of current experimental results.

Our results are shown in Fig. 5 – while a negative contribution to $\Delta M_s$ favours the LH and RH quark couplings to have the same sign, the small region favoured by the semi-leptonic $B$ anomalies has no overlap with the $\Delta M_s$ region at $1\sigma$.

5. Conclusions

In this note, we have restated our update [1] of the SM prediction for the $B_s$-mixing observable $\Delta M_s$ (Eq. (7)) using the most recent values for the input parameters, in particular the latest lattice results from FLAG. Our update shifts the central value of the SM theory prediction upwards and implies a $1.8\sigma$ discrepancy from the SM.

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$^4$Note that the matrix element of the vector LR operator is negative, while that of the LL and RR operators is positive.
We further discussed an important application of the $\Delta M_s$ update for NP models aimed at explaining the recent anomalies in semi-leptonic $B$ decays. The latter typically predict a positive shift in the NP contribution to $\Delta M_s$, thus making the discrepancy with respect to the experimental value even worse. As a generic result we have shown that, whenever the NP contribution to $\Delta M_s$ is positive, the limit on the mass of the NP mediator that must be invoked in order to explain the anomalies is strengthened by a factor of five (for a fixed coupling) compared to using the 2015 SM calculation for $\Delta M_s$ – a representative example of a simplified model of this type is a $Z'$ featuring purely LH and real couplings in order to accommodate $R_{K^*}$. The improvement in the upper bound on the $Z'$ mass is shown in Fig. 2.

Here we extended our study [1] to investigate potential “loopholes” to those results, whereby a negative contribution to $\Delta M_s$ could arise that would lessen the tension in $B_s$-mixing while still providing a good fit to the currently observed $B$ anomalies. Two cases were investigated – one where we allowed the quark coupling in our minimal $Z'$ model to be complex and another where we extended the minimal model with $Z'$ couplings to RH down quarks.

For the case of complex coupling, we showed that despite the fact that a relatively large imaginary part for $\delta C_9^{\mu}$ is compatible with the $b \to s \ell \ell$ data, any extra phase present in $B_s$-mixing is tightly constrained by the measurement of $A_{CP}^{\text{mix}}$ and this prevents an improvement of the overall fit (see Fig. 4).

In the other extended case study, the results are again negative. While it is known that adding a RH quark coupling is disfavoured by the $R_{K^*}$ fit (assuming NP in muons), it is also true that chirality-mixed LR vector operators give an RG enhanced contribution to $B_s$-mixing. However, as we see from Fig. 5 the fit to $\Delta M_s$ and $b \to s \ell \ell$ data favours respectively the same and opposite sign combination for the $Z'$ couplings to LH and RH quarks.

Although other ways to accommodate $\Delta M_s$ together with the $B$ anomalies could certainly exist, we conclude that the simplest possibility of a single-mediator simplified model is disfavoured.

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5Here we mention two notable possibilities: i) sticking only to $R_K$ and $R_{K^*}$, these can be accommodated via NP in electrons featuring sizeable contributions from RH quark currents, thus allowing also for negative contributions to $\Delta M_s$ (see e.g. [56]) and ii) as pointed out in [57], in UV complete models of the vector leptoquark $U_\mu \sim (3,1,2/3)$ addressing both $R_{D^*(\mu)}$ and $R_{K^*(\mu)}$, the fermion couplings of extra $Z'/G'$ states not directly related to the anomalies can naturally have a large phase in order to accommodate a negative $\Delta M_s$, without being in tension with CP violating observables.
We finally reiterate the importance of an independent confirmation of the FNAL/MILC lattice result for the four-quark matrix elements, given the central role of $B_s$-mixing in constraining NP models for $B$ anomalies.

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