AN ANALYTICAL APPROACH FOR SOLVING THE NONLINEAR JERK OSCILLATOR CONTAINING VELOCITY TIMES ACCELERATION-SQUARED BY AN EXTENDED ITERATION METHOD

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https://doi.org/10.26782/jmcms.2021.02.00004

(Received: December 12, 2020; Accepted: January 29, 2021)

Abstract

The technique to evade jerk from a dynamical system is to reduce the rate of acceleration or deceleration. It is an important issue for our real life. In motion control systems the term “jerk” is the main topic. The jerk equation containing velocity times acceleration-squared describes the characteristics of chaotic behaviour in many nonlinear phenomena, cosmological analysis, kinematical physics, pendulum analysis etc. Thus, the mentioned equation is important in its own right. An extended iteration method, based on Haque’s approach has been applied to find the analytical solution of the oscillator. The recently various method has been developed for finding analytical solutions of the nonlinear equation but; modified extended iteration method based on Haque’s approach is faster and straightforward than others.

Keywords: Jerk equation; Nonlinear oscillator; Extended iteration technique; Truncated Fourier series

I. Introduction

The prodigious usefulness of mathematics in the usual sciences draws from the fact that it is probable to formulate numerous properties prevailing natural phenomenon with the aid of the unequivocal words of mathematics. The science of differential equations is concerned with the behaviour of moving particles or body which represents the dynamical systems. All materials exhibit some kind of deformations or oscillations under the action of forces. Therefore, the differential equation has its application in many branches of the human lifecycle. Jerk indicates the deviation of acceleration of an object with corresponding variation of time. That is its mathematical modeling represents us a third order nonlinear differential equation.
Jerk is also named by Jolt and can be considered a subclass of three-dimensional dynamical systems. It was first introduced by Schot in 1978.

Thus, the solution of Jerk equations lies originally in solving the related differential equations. Among the establish solution methods of nonlinear differential equations the method of Perturbation [VI], [XXI]; Standard and modified Linstedt-Poincare [XXII]; Harmonic Balance [I], [III], [IV], [V], [XVII], [XIX], [XXIII]; Homotopy [II], [XVIII]; Iterative [VII], [VIII], [IX], [X], [XI], [XII], [XIII], [XIV], [XV], [XVI], [XX], [XXIV] are important.

The generalized form of jerk equations is

$$\frac{d^3x}{dt^3} = J(x, \frac{dx}{dt}, \frac{d^2x}{dt^2}) = J(x, \dot{x}, \ddot{x}) \quad (1)$$

where $J(x, \dot{x}, \ddot{x})$ denotes the jerk functions.

Here we have considered the Jerk function containing velocity times acceleration-squared.

$$-(\dot{x} + \dddot{x}^2) \quad (2)$$

In this article, we present Haque’s extended iteration approach [X] owing to the determination of approximate solutions of the jerk equation containing the mentioned jerk function.

It is remarkable that the executed modification is legitimate for higher-order iterations and provides wide-ranging solutions of the oscillator.

The modified extended solution technique is compatible, effective and provides ample way to find the analytical solution of many nonlinear equations in dynamical systems.

II. The Method

Let us consider the equation for nonlinear Jerk oscillator generalized by

$$\dddot{x} = J(\dot{x}, \ddot{x}) \quad (3)$$

where $t$ is the time, $x$ is the displacement, $J(\dot{x}, \ddot{x})$ denotes the jerk functions, $\Omega$ is the frequency and $A$ is the amplitude of this system. Here over dots denote differentiation concerning time, $t$.

Then using the shifting operator, equation (3) reduces to a second-order standard equation.

According to Haque’s extended iteration law [X], the scheme of iteration can be written as

$$\dddot{x}_{k+1} + \Omega_k^2 x_{k+1} = G(\Omega_k, x_{k+1}, \dot{x}_{k+1}) + (x_k - x_{k-1})G_1(\Omega_k, x_{k-1}, \dot{x}_{k-1}) + (\dot{x}_k - \dot{x}_{k-1})G_2(\Omega_k, x_{k-1}, \dot{x}_{k-1}) \quad (4)$$

with the same initial guess and same initial condition as [X].

The sequence of iterative solutions is to be obtained from successive iterative steps.
III. Solution procedure

Let us suppose the following oscillator

\[ \dddot{x} + \dddot{x} = - \dddot{x} \dot{x}^2. \]  

(5)

It is known as the Jerk equation containing velocity times acceleration-squared. To make the equation (5) to second-order equation we have to insert the space variable \( y(t) \) with \( x = y \), we have

\[ \ddot{y} = -(y + yy^2). \]

(6)

For getting standard form adding \( \Omega^2 y \) on both sides of equation (6), we get

\[ \ddot{y} + \Omega^2 y = G(y, \dot{y}, \Omega) \]

(7)

where \( G(y, \dot{y}, \Omega) = \Omega^2 y - (y + yy^2) \)

Then \( G_y = \frac{\partial G}{\partial y} = \Omega^2 - (1 + y^2) \) and \( G_{\dot{y}} = \frac{\partial G}{\partial \dot{y}} = -2y \dot{y} \)

Thus, the extended iteration scheme of equation (7) is

\[ \ddot{y} + \Omega^2 y = G(y_k, \dot{y}_k, \Omega_k) + (y_k - y_{k-1}) G_y (\Omega_k, y_{k-1}, \dot{y}_{k-1}) \]

\[ + (\dot{y}_k - \dot{y}_{k-1}) G_{\dot{y}} (\Omega_k, y_{k-1}, \dot{y}_{k-1}); \quad k = 1, 2, 3, \ldots \]

(8)

where the initial approximation is \( y_0 = y_0(t) = A \cos \varphi; \varphi = \Omega t \).

The first approximation \( y_1(t) \) and the frequency \( \Omega_0 \) are obtained by using the direct iterative method, which are

\[ y_1 = \{A - \frac{1}{32} (-4 + A^2) a_{1,1}\} \cos \varphi + \frac{1}{32} (-4 + A^2) a_{1,3} \cos 3\varphi \]

(9)

and

\[ \Omega_0 = \left(\frac{4}{4 - A^2}\right)^{1/2} \]

(10)

where

\[ a_{1,1} = \frac{1}{4} (-4A + 4A\Omega_0^2 - A^3\Omega_0^2) \]

\[ a_{1,3} = \frac{1}{4} A^3\Omega_0^2 \]

(11)

Going on to the second stage of iteration \( y_2(t) \) satisfies the equation

\[ \ddot{y}_2 + \Omega_2^2 y_2 = \Omega_2^2 y_0 - y_0(1 + y_0^2) + (\Omega_2^2 - 1 - y_0^2)(y_1 - y_0) \]

(12)

Substituting \( y_1 \) from equation (9) into the equation (12) and then expanding in a truncated Fourier cosine series up to five harmonics we obtain.

\[ \ddot{y}_2 + \Omega_2^2 y_2 = b_{2,1} \cos \varphi + b_{2,3} \cos 3\varphi + b_{2,5} \cos 5\varphi \]

(13)

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where

\[ b_{21} = \frac{16A}{(4 - A^2)(4 + A^2)} - \frac{23A^3}{2(4 - A^2)(4 + A^2)} + \frac{2A^5}{(4 - A^2)(4 + A^2)} + \frac{A'}{32(4 - A^2)(4 + A^2)} + \frac{4A'\Omega_i}{\sqrt{4 - A^2}(4 + A^2) - 5A'\Omega_i} - \frac{5A'\Omega_i}{4\sqrt{4 - A^2}(4 + A^2)} \]

\[ + \frac{A'\Omega_i^2}{16\sqrt{4 - A^2}(4 + A^2)} - \frac{16A\Omega_i^2}{(4 - A^2)(4 + A^2)} + \frac{15A'\Omega_i^2}{2(4 - A^2)(4 + A^2)} - \frac{3A'\Omega_i^2}{4(4 - A^2)(4 + A^2)} - \frac{A'\Omega_i^2}{32(4 - A^2)(4 + A^2)} \]

\[ b_{23} = \frac{7A^3}{2(4 - A^2)(4 + A^2)} - \frac{9A^5}{8(4 - A^2)(4 + A^2)} + \frac{A^7}{16(4 - A^2)(4 + A^2)} - \frac{4A'\Omega_i}{\sqrt{4 - A^2}(4 + A^2)} + \frac{7A'\Omega_i}{8\sqrt{4 - A^2}(4 + A^2)} + \frac{A'\Omega_i}{32\sqrt{4 - A^2}(4 + A^2)} \]

\[ + \frac{A'\Omega_i^2}{2(4 - A^2)(4 + A^2)} - \frac{A'\Omega_i^2}{4(4 - A^2)(4 + A^2)} + \frac{A'\Omega_i^2}{32(4 - A^2)(4 + A^2)} \]

\[ b_{25} = \frac{A^5}{8(4 - A^2)(4 + A^2)} - \frac{A^7}{32(4 - A^2)(4 + A^2)} + \frac{3A'\Omega_i}{8\sqrt{4 - A^2}(4 + A^2)} - \frac{3A'\Omega_i}{32\sqrt{4 - A^2}(4 + A^2)} \]

We know the secular terms are dominating terms that is why these terms are needed to be removing from the solutions. Applying this phenomenon in equation (13), we have

\[ \Omega_1 = (-128A^2 + 40A^4 - 2A^6 - 2\sqrt{4(4096A^4 - 2560A^6 + 528A^6 - 40A^{10} + 16384(4 - A^2) - 11264A^2(4 - A^2) + 1680A^2(4 - A^2) + 32A^6(4 - A^2) - A^8(4 - A^2)})) / (2(-128\sqrt{4 - A^2} + 28A^2\sqrt{4 - A^2} + A^4\sqrt{4 - A^2})) \]

This \( \Omega_1 \) represents the second approximate frequency of the oscillator (6).

Later solving equation (13) and satisfying the initial condition \( y_2(0) = A \), we have

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\[ y_2 = (A + \frac{1}{\Omega_2^2} \sum_{n=1}^{\infty} \frac{a_{2n+1}^2}{(2n+1)^2 - 1}) \cos \phi - \frac{1}{\Omega_2^2} \sum_{n=1}^{\infty} \frac{a_{2n+1}}{(2n+1)^2 - 1}) \cos(2n+1)\phi \]  

(16)

This \( y_2 \) represents the second approximate solution of equation (6) and corresponding frequency \( \Omega_2 \) is to be determined. The value of \( \Omega_2 \) will be achieved from the solution of

\[ \ddot{y}_3 + \Omega_2^2 y_3 = \Omega_2^2 y_2 - y_0(1 + \dot{y}_0^2) + (\Omega_2^2 - 1 - \dot{y}_0^2)(y_2 - y_0) \]  

(17)

Substituting \( y_2 \) from equation (16) into the equation (17) and then expanding in a Fourier series as even function up to eleven harmonics we obtain.

\[ \ddot{y}_3 + \Omega_2^2 y_3 = c_{3,1} \cos \phi + c_{3,3} \cos 3\phi + c_{3,5} \cos 5\phi + c_{3,7} \cos 7\phi + c_{3,9} \cos 9\phi + c_{3,11} \cos 11\phi \]  

(18)

where \( c_{3,1}, c_{3,3}, c_{3,5}, c_{3,7}, c_{3,9}, \) and \( c_{3,11} \) are known constants.

We know the secular terms are dominating terms that is why these terms are needed to be removing from the solutions. Applying this phenomenon in equation (18), we have

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Ω = 10555311626649604A \sqrt{4 - A^2} - 1507430441680896A \sqrt{4 - A^2} + 916442941751296A^4
\sqrt{4 - A^2} - 31261784573952A^6 \sqrt{4 - A^2} + 66768152297472A^8 \sqrt{4 - A^2} - 9368509612032A^{10} \sqrt{4 - A^2}
\sqrt{4 - A^2} + 825897910272A^{12} \sqrt{4 - A^2} - 29787150336A^{14} \sqrt{4 - A^2} - 1911659520A^{16} \sqrt{4 - A^2}
+ 202430208A^{22} \sqrt{4 - A^2} - 1855488A^{24} \sqrt{4 - A^2} - 339736A^{26} \sqrt{4 - A^2} + 4492A^{28} \sqrt{4 - A^2}
+ 228A^{30} \sqrt{4 - A^2} + 659706976656A^{32} \sqrt{4 - A^2} (96A^4 - 2560A^6 + 528A^8 - 40A^{10} + A^{12})
+ 16384(4 - A^2) - 11264A^4(4 - A^2) + 1680A^6(4 - A^2) + 32A^8(4 - A^2) - A^4(4 - A^2))^{1/2}
- 7215545057280A^{14} \sqrt{4 - A^2} (96A^4 - 2560A^6 + 528A^8 - 40A^{10} + A^{12}) + 16384(4 - A^2) - 11264A^4(4 - A^2)
+ 1680A^6(4 - A^2) + 32A^8(4 - A^2) - A^4(4 - A^2))^{1/2} - 3940632494080A^2 \sqrt{4 - A^2} (4096A^4 - 2560A^6 + 528A^8 - 40A^{10} + A^{12}) + 16384(4 - A^2) - 11264A^4(4 - A^2)
+ 1680A^6(4 - A^2) + 32A^8(4 - A^2) - A^4(4 - A^2))^{1/2} - 1365732491264A^2 \sqrt{4 - A^2} (4096A^4
- 2560A^6 + 528A^8 - 40A^{10} + A^{12}) + 16384(4 - A^2) - 11264A^4(4 - A^2) + 1680A^6(4 - A^2)
+ 32A^8(4 - A^2) - A^4(4 - A^2))^{1/2} + 285145563136A^2 \sqrt{4 - A^2} (4096A^4 - 2560A^6 + 528A^8
- 40A^{10} + A^{12}) + 16384(4 - A^2) - 11264A^4(4 - A^2) + 1680A^6(4 - A^2) + 32A^8(4 - A^2) - A^4
(4 - A^2))^{1/2} - 29507095712A^2 \sqrt{4 - A^2} (4096A^4 - 2560A^6 + 528A^8 - 40A^{10} + A^{12}) + 16384(4 - A^2)
+ 1680A^6(4 - A^2) + 32A^8(4 - A^2) - A^4(4 - A^2))^{1/2} + 113788288A^4 \sqrt{4 - A^2} (4096A^4 - 2560A^6
+ 528A^8 - 40A^{10} + A^{12}) + 16384(4 - A^2) - 11264A^4(4 - A^2) + 32A^8(4 - A^2)
- A^4(4 - A^2))^{1/2} - 5613344A^4 \sqrt{4 - A^2} (4096A^4 - 2560A^6 + 528A^8 - 40A^{10} + A^{12}) + 16384(4 - A^2)
- A^4(4 - A^2))^{1/2} - 11264A^4(4 - A^2) + 1680A^6(4 - A^2) + 32A^8(4 - A^2) - A^4(4 - A^2))^{1/2} - 178232A^6
\sqrt{4 - A^2} (4096A^4 - 2560A^6 + 528A^8 - 40A^{10} + A^{12}) + 16384(4 - A^2) - 11264A^4(4 - A^2)
+ 1680A^6(4 - A^2) + 32A^8(4 - A^2) - A^4(4 - A^2))^{1/2} + 8938A^8 \sqrt{4 - A^2} (4096A^4 - 2560A^6 + 528A^8
- 40A^{10} + A^{12}) + 16384(4 - A^2) - 11264A^4(4 - A^2) + 1680A^6(4 - A^2) + 32A^8(4 - A^2)
- A^4(4 - A^2))^{1/2} + 228A^10 \sqrt{4 - A^2} (4096A^4 - 2560A^6 + 528A^8 - 40A^{10} + A^{12}) + 16384(4 - A^2)
- 11264A^4(4 - A^2) + 32A^8(4 - A^2) - A^4(4 - A^2))^{1/2} + 14740327759872A^4(4096A^4 - 2560A^6 + 528A^8
- 40A^{10} + A^{12}) + 16384(4 - A^2) - 11264A^4(4 - A^2)

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\[ A \times (4 - A) + 1608 A (4 - A) + 32 A (4 - A) - A (4 - A) \]


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+5429375530564A^4(4096A^4−2560A^4+528A^4−40A^10+A^12+16384(4−A^1)−11264A^1(4−A^1)+1680A^1

(4−A^1)^3+32A^4(4−A^1)−A^1(4−A^1)^2)−855839342592A^4(4096A^4−2560A^4+528A^4−40A^10+A^12

+16384(4−A^1)−11264A^1(4−A^1)+1680A^1(4−A^1)+32A^4(4−A^1)−A^1(4−A^1)^2)−39463157760A^10

(4096A^4−2560A^4+528A^4−40A^10+A^12+16384(4−A^1)−11264A^1(4−A^1)+1680A^1(4−A^1)+32A^4

(4−A^1)−A^1(4−A^1)^2)−2286944256A^12(4096A^4−2560A^4+528A^4−40A^10+A^12+16384(4−A^1)

−11264A^1(4−A^1)+1680A^1(4−A^1)+32A^4(4−A^1)−A^1(4−A^1)^2)

IV. Results and Discussions

Haque’s extended iterative method [X] has been presented based on Mickens extended iteration method [XX] to obtain the approximate analytic solution of the jerk oscillator containing “velocity times acceleration-squared”. The frequency in addition to amplitude has been earned by a modified approach and compared with those earned by another existent process. The results are presented in Table 1. Earned modified first, second and third approximate frequencies denoted by \( \Omega_0 \), \( \Omega_1 \) and \( \Omega_2 \) respectively and corresponding periods are denoted by \( T_0 \), \( T_1 \) and \( T_2 \). It is noteworthy that the analytical solutions of algebraic equations produced by the executed method are very easy to calculate. So, the iteration steps can be preceded to a finite necessary level. In this modification, we have found the solution up to the third iteration step.

Here, we have presented the accuracy of the solutions obtained from the modified extended iteration procedure by comparing with the solutions obtained by Gottlieb [IV], Zheng et al. [XXIV] and Haque et al. [XVI] results of the oscillator. Gottlieb [IV] has determined the analytical solutions of the oscillator by using the harmonic balance method. But the solutions obtained by Gottlieb were not accurate enough. The harmonic balance method used by Gottlieb is a lower order method of HB. This method is not able to carry the higher-order approximation. In this situation, it is necessary to solve a set of intricate non-algebraic equations. Zheng et al. [XXIV] have examined the comparison of two iteration procedure for the jerk equation but sometimes their technique is invalid. Haque et al. [XVI] applied the direct iteration method; it is slower than the extended iteration procedure. To compare the results, we have also provided the existent outcome earned by Gottlieb [IV], Zheng et al. [XXIV] and Haque et al. [XVI] respectively in the following Table 2.

Verification of the modified solutions we have used the law of percentage errors (denoted by Error (%)), defined as follows:

\[
\text{Error(}Er\text{ (%) )} = \left| \frac{T_E - T_k}{T_E} \right| \times 100\%
\]

where \( T_k; k = 0,1,2,\cdots \) illustrate the different periods earned by the presented modified procedure and \( T_E \) represents the related exact period of the oscillator.

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Table 1

The periods obtained by present’s technique of $\dddot{x} + \dot{x} = -\ddot{x} \dot{x}^2$ with percentage errors:

| A   | \( T_0 \) Er(%) | \( T_1 \) Er(%) | \( T_2 \) Er(%) |
|-----|-----------------|-----------------|-----------------|
| 0.1 | 6.275326 1.12 e^{-2} | 6.275334 3.49 e^{-6} | 6.275334 3.43 e^{-6} |
| 0.2 | 6.251690 1.90 e^{-3} | 6.251809 4.80 e^{-6} | 6.251809 3.61 e^{-7} |
| 0.5 | 6.083668 7.85 e^{-2} | 6.088382 1.09 e^{-3} | 6.088449 3.01 e^{-6} |
| 1  | 5.441398 1.55 | 5.523412 6.85 e^{-2} | 5.527434 4.24 e^{-3} |
| 1.5 | 4.155936 11.39 | 4.683553 1.43 e^{-1} | 4.709049 4.00 e^{-4} |

\( T_0, T_1 \) and \( T_2 \) denote first, second and third periods respectively obtained by the present method.

Table 2

Comparison of the periods with corresponding exact periods \( T_E \) [IV] of $\dddot{x} + \dot{x} = -\ddot{x} \dot{x}^2$ :

| A   | \( T_E \) Er(%) | \( T_G \) Er(%) | \( T_Z \) Er(%) | \( T_{ID} \) Er(%) | \( T_E \) Er(%) |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.1 | 6.275333 1.18 e^{-4} | 6.275326 1.12 e^{-2} | 6.275338 2.75 e^{-7} | 6.275334 2.39 e^{-7} | 6.275334 3.43 e^{-6} |
| 0.2 | 6.251809 1.90 e^{-3} | 6.251690 1.90 e^{-3} | 6.251808 2.68 e^{-6} | 6.251809 3.39 e^{-7} | 6.251809 3.61 e^{-7} |
| 0.5 | 6.088449 7.85 e^{-2} | 6.083668 7.85 e^{-2} | 6.088416 5.41 e^{-4} | 6.088451 3.24 e^{-5} | 6.088449 3.01 e^{-6} |
| 1  | 5.527200 1.55 | 5.441398 1.55 | 5.525570 2.95 e^{-2} | 5.527511 5.62 e^{-3} | 5.527434 4.24 e^{-3} |
| 1.5 | 4.690247 11.39 | 4.155936 11.39 | 4.672129 3.86 e^{-1} | 4.683187 1.51 e^{-1} | 4.709049 4.00 e^{-4} |

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$T_E$ denote the modified approximate periods for direct and extended iteration methods respectively, $T_G$ denotes the approximate periods obtained by Gottlieb [IV], $T_Z$ denotes the approximate periods obtained by Zheng et al [XXIV] and $T_{ID}$ denotes the approximate periods obtained by Haque et al [XVI].

V. Convergence and Consistency Analysis

Test of convergence: The iteration method will be convergent if the set of solutions $x_k$ (or frequencies $\Omega_k$ or amplitudes $T_k$) in ascending order satisfy the following property

$$x_k = \lim(x_k) \quad \text{or,} \quad \Omega_k = \lim(\Omega_k) \quad \text{or,} \quad T_k = \lim(T_k), \quad k \to \infty$$

Here $X_E$ is considered as the exact solution, $\Omega_k$ denotes the frequencies and $T_k$ denotes the corresponding periods of the nonlinear oscillator.

In the obtained solutions, it has been indicating that there is less error to iterative steps in ascending order and finally it has been shown that $|T_2 - T_E| < \epsilon$, where $\epsilon$ is a small positive number.

Hence the presented extended iteration method is convergent.

Test of consistency: The iterative method will be consistent if the set of solutions $x_k$ (or frequencies $\Omega_k$ or amplitudes $T_k$) in ascending order satisfy the following property

$$\lim|x_k - x_E| = 0 \quad \text{or,} \quad \lim|\Omega_k - \Omega_E| = 0 \quad \text{or,} \quad \lim|T_k - T_E| = 0, \quad k \to \infty$$

In the obtained solutions, it has been indicating that there is less error to iterative steps in ascending order and finally it has been shown that $|T_k - T_E| = 0, \quad k \to \infty$ as $|T_2 - T_E| = 0$.

Hence the presented extended iteration method is consistent.

VI. Conclusion

In this research, it has been seen that the largest part of solutions has been enhanced drastically. The modified solutions show that this modification is more precise than other existing methods of solution and is valid for the large amplitude of oscillation for the jerk system. We have seen that it is not always true that the extended iteration method yields better results than the direct iteration method. It can be accomplished that the adopted modification is steadfast, effectual and conformable also it present sufficient well-suited solutions to the nonlinear jerk equations arise in mathematical physics, applied mathematics and different field of engineering specially in Mechanical, Electrical and Space engineering.

Conflict of Interest

There was no relevant conflict of interest regarding this paper.

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