2-D Steady-State Heat Transfer Prediction in Rotating Electrical Machines Taking into account Materials Anisotropy: Thermal Resistances Network, Exact Analytical and Hybrid Methods

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Abstract—This paper presents two-dimensional (2-D) thermal resistances network (TRNM), exact analytical (AM) and hybrid (HM) methods for calculating steady-state temperature and heat flux distribution in rotating electrical machines considering materials anisotropy (i.e., different thermal conductivities in both directions). They are based on the thermal equivalent circuit (TEC), the improved exact subdomain (SD) technique where the solution and thermal conductivities depend on both directions \( r, \theta \) and the coupling between the two methods. TRNM is known as a semi-analytical method that can predict the heat transfer in the machine in less time than finite element method (Fem). The implementation of TRNM by considering the difference between the thermal conductivities in \( r, \theta \) using its equivalence with Fem is presented. The SD technique is improved to consider the difference between thermal conductivities in the directions \( r, \theta \). It is known that the SD technique with non-homogeneous boundary conditions (BCs) is very sensitive to the dimensions of SDs where the harmonics number and the accuracy are lower in small subdomains. Hence, the HM from the TRNM and AM is given to answer these inaccuracies especially in electrical machines with a high number of stator slots and rotor poles. The heat sources are volumetric power losses due to hysteresis, eddy-current, Joule losses and windage losses in all the regions of the machine obtained by simplified method. The studied problem is conductive with conductive interface conditions (ICs) and convective heat transfer between the machine and the external air and at the rotor internal air. The semi-analytical results are compared between them as well as with those obtained by Fem.

Keywords—Anisotropic materials, conductive heat transfer, convection, exact subdomain technique, thermal resistances network.

**NOMENCLATURE**

- **TRNM** Thermal Resistances Network Method.
- **AM** Analytical Method.
- **HM** Hybrid Method.
- **TEC** Thermal Equivalent Circuit.
- **Fem** Finite element method.
- **SD** Subdomain.
- **BCs** Boundary Conditions.
- **PM** Permanent Magnet.
- **PDEs** Partial Differential Equations.
- **ICs** Interface Conditions.

**I. INTRODUCTION**

Thermal modeling is used to design the insulation system of electrical machines. Currently, the Fem and TEC are the most used methods [1]-[6]. Some of them take into account materials anisotropy especially in the \( z \)-direction with a three-dimensional (3-D) study. Recently, Boughrara et al. (2018) [7] introduced a new 2-D exact SD technique able to predict steady-state heat transfer in rotating electrical machines without considering the anisotropy of thermal conductivities. This model is based on the Dubas’ superposition technique [8]-[9] developed for the prediction of the magnetic field in air- or iron-cored coil. This method is very accurate and can be used for different topologies of synchronous and asynchronous machines.

The thermal modeling of electrical machines using TEC in steady-state and/or transient is fast with acceptable accuracy compared to Fem and AM, especially when the number of nodes and thermal resistances is low. TRNM is more accurate than TEC with higher time consumption [10]-[11]. However, in reality, it is not widely used for thermal design. The numerical thermal model is used in a second stage of the design to verify the temperature distribution given by semi-analytical methods. In various methods, the power losses are used as heat sources or coupled directly to electromagnetic analysis. Fem is also used with computational fluid dynamics to model convective problems inside the electrical machine and the different type of cooling. Recently, there are a few references that used an HM. TRNM of the stator and rotor are coupled to an exact AM only in the air-gap [10]-[11].

In this paper, we present three semi-analytical methods, viz., i) TRNM, ii) AM and iii) HM. TRNM is considered as Fem where the steps of meshing, materials definition with different thermal conductivities in the directions \( r, \theta \), elementary matrix/vector and introduction of BCs exist. The model presented in [7], based on the exact SD technique, is improved to consider the materials anisotropy in the directions \( r, \theta \) for the prediction of

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steady-state temperature and heat flux in rotating electrical machines. AM is suitable for the SDs with high dimensions where the harmonics number can be high to achieve very good accuracy.

![Fig. 1: Studied inset-PM machine [7.](image)](image)

This is the case of electrical machines with a low number of stator slots and rotor poles. In electrical machines with a high number of poles and stator slots, it is interesting to use the HM (i.e., the coupling between TRNM and AM) to simplify the thermal model and improve the accuracy. The used HM also considers the materials anisotropy. TRNM is used to model the stator slots and teeth SDs and rotor permanent-magnets (PMs) and teeth SDs while the AM is used to model the stator, rotor and air-gap. The coupling between TRNM and AM is achieved by using the discrete Fourier series at the ICs to satisfy the temperature continuity [12].

The developed semi-analytical methods are used to determine the heat transfer in inset-PM machines. Although, it is valid for most rotating electrical machines [7].

To determine the heat sources, a simple method is used in this paper to determine the power losses for the studied inset-PM machine [13]. Although, AM and magnetic resistances network method (MRNM) can be used [14]-[16]. The semi-analytical results are compared between them and with those obtained by Fem [17].

II. TEMPERATURE CALCULATION METHODS

The steady-state heat transfer studied in this paper is conductive with volumetric power sources and convective at the ambient air and the rotor shaft. The model is adopted with the following assumptions:

- The materials are considered anisotropic having different constant thermal conductivities in both directions without any variation with temperature;
- The stator and rotor slots have radial sides;
- The heat sources are volumetric, uniform and constant in each SD;
- The radiation outside stator and inside the rotor is ignored;
- The interfaces between all regions are considered perfect without any contact resistance.

The analyzed inset-PM machine has 6-slots/4-poles [see Fig. 1].

A. Thermal Resistance Network Method (TRNM)

The studied inset-PM machine is modeled using TRNM with the following steps:

1- Meshing

The step of meshing is performed in the same way as in Fem with the use of circular elements having a node in the middle of each element. First, we define \( w_{rs} + 1 \) radii \( w_{rs}(i) \) with \( i \) vary from 1 to \( w_{rs} + 1 \) and \( n_{s} + 1 \) angles \( w_{t}(j) \) with \( j \) vary from 1 to \( n_{s} + 1 \). The number of nodes which is equal to the number of elements is \( m_{s} \times n_{1} \). The numbering and coordinates of nodes are given by

\[
\text{For } i \text{ from 1 to } m_{s} \text{ do}
\]

\[
\text{For } j \text{ from 1 to } n_{s} \text{ do}
\]

\[
\text{nods}(i, j) = j + (i - 1) \cdot n_{s} \; ;
\]

\[
l = \text{nods}(i, j) \; ;
\]

\[
x(l) = \frac{w_{rs}(i + 1) + w_{rs}(i)}{2} \cos\left(\frac{w_{t}(j + 1) + w_{t}(j)}{2}\right) \; ;
\]

\[
y(l) = \frac{w_{rs}(i + 1) + w_{rs}(i)}{2} \sin\left(\frac{w_{t}(j + 1) + w_{t}(j)}{2}\right) \; ;
\]

\[
\text{end do}
\]

\[
\text{end do}
\]

In Fig. 2, we can show an example of mesh with 32 elements and nodes (i.e., \( m_{s} = 4 \) and \( n_{s} = 8 \)). In this example, each element has a thickness of \( w_{rs}(i + 1) - w_{rs}(i) \) and opening of \( w_{t}(j + 1) - w_{t}(j) \). This example is introduced for clarity in the implementation of TRNM:

\[
w_{rs} = 10^{-3} \begin{bmatrix} 30 & 60 & 90 & 120 & 150 \end{bmatrix}
\]

\[
w_{t} = \begin{bmatrix} 0 & \frac{\pi}{4} & \frac{\pi}{2} & \frac{3\pi}{4} & \pi & \frac{5\pi}{4} & \frac{3\pi}{2} & \frac{7\pi}{4} & \frac{2\pi}{} \end{bmatrix}
\]

As it can be shown in (1), the numbering is done starting with 1 in the \( \theta \)-direction than after in the \( r \)-direction. From the two first radii, we have 8 elements and 8 nodes numbered from 1 to 8 from right to left. The next two radii are numbered from 9 to 16 and the same for the other radii.

2- Connectivity Matrix

From Figs. 2 ~ 4, we can show that the node 9 is connected to 4 nodes (10, 1, 16, 17). This is the case for the entire mesh with elements having 4 thermal resistances. Supplementary nodes are added to the first and last radii to represent the BC between the stator and the external air and the rotor with the shaft. The connection between the nodes permits to obtain a matrix called connectivity matrix \( isks(5, k) \) as in Fem. For internal elements that are not situated in the boundaries, we define the matrix \( isks \) as

\[
\text{For } i \text{ from 2 to } m_{s} - 1 \text{ do}
\]

\[
\text{For } j \text{ from 2 to } n_{s} - 1 \text{ do}
\]

\[
k = \text{nods}(i, j) \; ;
\]

\[
isks(1, k) = \text{nods}(i, j - 1) \; ;
\]

\[
isks(2, k) = \text{nods}(i, j) \; ;
\]

\[
isks(3, k) = \text{nods}(i, j + 1) \; ;
\]

\[
isks(4, k) = \text{nods}(i - 1, j) \; ;
\]

\[
isks(5, k) = \text{nods}(i + 1, j) \; ;
\]

\[
\text{end do}
\]

\[
\text{end do}
\]
\[ R_{w} = \frac{1}{2} \frac{\Delta \theta}{\lambda_{w} L_{w} \ln \left( \frac{R_{j}}{R_{i}} \right)} = \frac{1}{2} \frac{\text{wt}(j+1) - \text{wt}(j)}{\lambda_{w} L_{w} \ln \left( \frac{\text{wrs}(i+1)}{\text{wrs}(i)} \right)} \]  

(12)

where \( \lambda_{r} \) and \( \lambda_{\theta} \) are the thermal conductivity in the \( r \)- and \( \theta \)-direction; \( R_{i} \), \( R_{s} \), and \( \Delta \theta \) are respectively the internal, the external radii and the opening of element; and \( L_{w} \) the axial length of the machine.

The conductivity in anisotropic material is a tensor with

\[
[\lambda] = \begin{bmatrix} \lambda_{rr} & 0 \\ 0 & \lambda_{\theta\theta} \end{bmatrix}
\]

(13)

It is interesting to note that the conductivities \( \lambda_{\theta\theta} \) and \( \lambda_{rr} \) are considered null for the studied machine. In polar coordinates, the thermal conductivity tensor \([\lambda]\) can be obtained from the thermal conductivity tensor in Cartesian coordinates as [18]

\[
\begin{bmatrix} \lambda_{rr} & \lambda_{\theta\theta} \\ \lambda_{\theta\theta} & \lambda_{\theta\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \lambda_{xx} & \lambda_{xy} \\ \lambda_{xy} & \lambda_{yy} \end{bmatrix}
\]

(14)

It is important to note that Fem uses the Cartesian coordinate’s tensor for the thermal conductivity [17] and [19]. For this, the validation of the results considering the materials anisotropy in polar coordinates is done with TRNM and AM.

The boundary elements [see Fig. 4] with 1 node at fixed temperature (viz., 70°C) represent the thermal convection resistance as follows

\[ R_{a} = \frac{1}{h_{c} L_{c} \left( \text{wt}(j+1) - \text{wt}(j) \right) \text{wrs}(ms+1)} \]  

(15)

\[ R_{a} = \frac{1}{h_{c} L_{c} \left( \text{wt}(j+1) - \text{wt}(j) \right) \text{wrs}(1)} \]  

(16)

where \( h_{c} \) and \( h_{r} \) are respectively the convection coefficients at the external radius and at the shaft of the machine.

In the proposed method, the Dirichlet’s condition (without convection) can be introduced by setting the thermal resistance of convection to zero.

For the mesh example in Fig. 4, there are 48 equations corresponding to 48 elements and nodes. Each conductive element has an internal volumetric source of heat \( P_{i} \) and connected to four nodes. All elements can be represented similarly to equations of elements 1 and 2 by

\[ P_{1} = \frac{T_{1} - T_{s}}{R_{10} + R_{10}} + \frac{T_{1} - T_{2}}{R_{10} + R_{20}} + \frac{T_{1} - T_{3}}{R_{10} + R_{30}} + \frac{T_{1} - T_{s}}{R_{10} + R_{s0}} \]  

(17)

\[ P_{2} = \frac{T_{2} - T_{1}}{R_{20} + R_{10}} + \frac{T_{2} - T_{3}}{R_{20} + R_{30}} + \frac{T_{2} - T_{s}}{R_{20} + R_{s0}} + \frac{T_{2} - T_{10}}{R_{20} + R_{10}} \]  

(18)
The 16 equations of convection at boundaries are similar to those of elements 41 and 34, e.g.,

\[
0 = \frac{T_{41} - T_{25}}{R_{25} + R_{41}} \quad (19)
\]

\[
0 = \frac{T_{34} - T_{25}}{R_{25} + R_{34}} \quad (20)
\]

where \( T_{41} = T_{34} = 70 \, ^\circ\text{C} \).

In the case of Dirichlet’s conditions, the thermal resistances of convection are fixed at infinity (i.e., \( R_{25} = R_{34} = \infty \)). A fixed heat flux BC can be considered. In this case, (16) and (17) can be modified as follows

\[
P_{41} = \frac{T_{41} - T_{25}}{R_{25}} \quad (21)
\]

\[
P_{34} = \frac{T_{34} - T_{25}}{R_{25}} \quad (22)
\]

where \( P_{34} \) and \( P_{41} \) are imposed heat fluxes. The periodicity condition is satisfied by connecting nodes 1 and 8, 9 and 16, 17 and 24, 25 and 32.

3- Global Matrix

The 48 equations are represented in matrix form \( gm \) without replacing the known temperatures at the BCs. The second member of the system represents the entire values of \( P \) with a vector \( f \) which represent the power losses in the machine. We begin the assembly \( gm \) with the internal elements of the conductive problem, then the equations of thermal convection at the rotor shaft and the stator ambient air. The introduction of BCs by convection is given by the thermal resistances and the fixed temperature (viz., 70°C) at the rotor shaft and the external air of the machine. The fixed temperatures are introduced into the global matrix and vector using high diagonal numbers in the global matrix and vector. These steps for constructing the global matrix are given in Appendix A.

The global matrix and vector for the 48 unknown temperatures are solved by direct method.

\[
T = gm^{-1}.f
\quad (23)
\]

The obtained solution vector \( T(48) \) allows us to calculate the density of heat flux. The radial density of heat flux for each element can be calculated by

\[
q\left(isk\right.,(4,k),k) = \frac{T_k - T_{isk(4,k)}}{S_{kr} \left( R_{kr} + R_{isk(4,k)} \right)} \quad (24)
\]

\[
q\left(isk\right.,(5,k),k) = \frac{T_k - T_{isk(5,k)}}{S_{kr} \left( R_{kr} + R_{isk(5,k)} \right)} \quad (25)
\]

The tangential density of heat flux for each element is obtained by

\[
q_t\left(isk\right.,(1,k),k) = \frac{T_k - T_{isk(1,k)}}{S_{kr} \left( R_{kr} + R_{isk(1,k)} \right)} \quad (26)
\]

\[
q_t\left(isk\right.,(3,k),k) = \frac{T_k - T_{isk(3,k)}}{S_{kr} \left( R_{kr} + R_{isk(3,k)} \right)} \quad (27)
\]

where \( S_{kr} \) and \( S_{kr} \) are respectively the surface of an element in the \( r- \) and \( \theta- \) direction.

For the analyzed inset-PM machine with TRNM, \( ms = 36, ns = 360 \), the total number of elements and nodes is 12,960. The number of additional nodes to consider convection heat transfer at the external radius is 360 and at the shaft is 360. The dimensions of the global matrix and vector are respectively 13,680 \times 13,680 and 13,680.

B. Analytical Method (AM)

1- Problem Description, Assumptions and Partial Differential Equations (PDEs)

In this section, we have improved the AM developed in [7] to consider the materials anisotropy in both directions \( (r, \theta) \). The machine is subdivided into 7 regions, viz., Region I for the air-gap, Region IIj for the PMs, Region III for the stator yoke, Region IVi for the stator slots, Region V for the rotor yoke, Region VIj for the rotor teeth, and Region VIIi for the stator teeth.

In steady-state, PDEs representing the temperature distribution in each region are given by

- in Region I by

\[
\frac{\lambda_v}{r} \frac{\partial}{\partial r} TI + \frac{\lambda_v}{r^2} \frac{\partial^2}{\partial r^2} TI + \frac{\lambda_v}{r^2} \frac{\partial^2}{\partial \theta^2} TI = -p_v \quad (28)
\]

where \( \lambda_v \) and \( \lambda_m \) are respectively the thermal conductivities (in \( W/mK \)) of the air-gap in the \( r- \) and \( \theta- \) direction, and \( p_v \) is the windage loss density (in \( W/m^3 \)).

- in Regions IIj by

\[
\frac{\lambda_n}{r} \frac{\partial}{\partial r} TIIj + \frac{\lambda_n}{r^2} \frac{\partial^2}{\partial r^2} TIIj + \frac{\lambda_n}{r^2} \frac{\partial^2}{\partial \theta^2} TIIj = -pm_j \quad (29)
\]

where \( \lambda_n \) and \( \lambda_m \) are respectively the thermal conductivities (in \( W/mK \)) of PMs in the \( r- \) and \( \theta- \) direction, and \( pm_j \) the power loss density of the \( j \)th Region II (in \( W/m^3 \)).

\[
\begin{align*}
q_r &= L(r) \\
\Delta T &= 0 \\
T &= f(\theta) \\
q_\theta &= R(\theta)
\end{align*}
\]

(a) Non-homogeneous BCs.
where \( \lambda_r \) and \( \lambda_\theta \) are respectively the thermal conductivities in \( W/mK \) of the stator yoke in the \( r \)- and \( \theta \)-direction, and \( p_s \) the power loss density in the stator iron (in \( W/m^3 \)). This power loss is considered uniform and constant in the whole stator iron.

- in Regions IVi by

\[
\frac{\lambda_{sr}}{r} \frac{\partial}{\partial r} TIV_i + \frac{\lambda_{sr}}{r^2} \frac{\partial^2}{\partial r^2} TIV_i + \frac{\lambda_{sr}}{r^2} \frac{\partial^2}{\partial \theta^2} TIV_i = -Psl_i
\]

where \( \lambda_{sr} \) and \( \lambda_{sr} \) are respectively the thermal conductivities in \( W/mK \) of stator slot in the \( r \)- and \( \theta \)-direction, and \( Psl_i \) the Joule and proximity losses densities (in \( W/m^3 \)). This power is considered uniform and constant in each stator slot.

- in Region V by

\[
\frac{\lambda_r}{r} \frac{\partial}{\partial r} TV + \frac{\lambda_r}{r^2} \frac{\partial^2}{\partial r^2} TV + \frac{\lambda_{sr}}{r^2} \frac{\partial^2}{\partial \theta^2} TV = -p_r
\]

where \( \lambda_r \) and \( \lambda_{sr} \) are respectively the thermal conductivities in \( W/mK \) of rotor yoke in the \( r \)- and \( \theta \)-direction, and \( p_r \) the power loss density in the rotor iron (in \( W/m^3 \)). This power loss is considered uniform and constant in the rotor iron.

- in Regions VIi by

\[
\frac{\lambda_{ar}}{r} \frac{\partial}{\partial r} TVI_i + \frac{\lambda_{ar}}{r^2} \frac{\partial^2}{\partial r^2} TVI_i + \frac{\lambda_{sr}}{r^2} \frac{\partial^2}{\partial \theta^2} TVI_i = -Pdr_i
\]

where \( \lambda_{ar} \) and \( \lambda_{ar} \) are respectively the thermal conductivities in \( W/mK \) of rotor tooth in the \( r \)- and \( \theta \)-direction, and \( Pdr_i \) the power loss density in the rotor tooth (in \( W/m^3 \)). This power loss is considered uniform and constant in the rotor tooth.

- in Regions VIIIi by

\[
\frac{\lambda_{ar}}{r} \frac{\partial}{\partial r} TVII_i + \frac{\lambda_{ar}}{r^2} \frac{\partial^2}{\partial r^2} TVII_i + \frac{\lambda_{sr}}{r^2} \frac{\partial^2}{\partial \theta^2} TVII_i = -Pds_i
\]

where \( \lambda_{ar} \) and \( \lambda_{ar} \) are respectively the thermal conductivities in \( W/mK \) of the stator tooth in the \( r \)- and \( \theta \)-direction, and \( Pds_i \) the power loss density in the stator tooth (in \( W/m^3 \)). This power loss is considered uniform and constant in the stator tooth.

Using \( \mathbf{q} = -[\lambda] \cdot \nabla \mathbf{T} \), the heat flux density components (in W/m²) in polar coordinates are defined as

\[
q_r = -\lambda_r \frac{\partial T(r,\theta)}{\partial r} \quad (35)
\]

\[
q_\theta = -\frac{\lambda_\theta}{r} \frac{\partial T(r,\theta)}{\partial \theta} \quad (36)
\]

where \( \lambda_r \) and \( \lambda_\theta \) are respectively the thermal conductivities in the \( r \)- and \( \theta \)-direction.

### 2. Temperature Solution in each SD

The steady-state heat transfer in the inset-PM machine is studied using the improved 2-D exact SD technique presented in [7]. The general solutions of the above PDEs in non-homogeneous BCs [see Fig. 5(a)] are deduced by applying the superposition principle [8]-[9] [see Fig. 5(b)] and using the Fourier’s series as well as the separation of variables method. The Laplace’s equations in Region I, III and V have homogeneous BCs and Region II, IV, VI and VII present non-homogeneous BCs. The solution of Laplace’s equation

\[
\frac{\partial^2 T(r,\theta)}{\partial r^2} + \frac{\partial^2 T(r,\theta)}{\partial \theta^2} = 0
\]

Using the separation of variables method by replacing \( T(r,\theta) = R(r)\Theta(\theta) \) gives

\[
R(r) \frac{d^2 R(r)}{dr^2} + \frac{\lambda_r}{r} \frac{d^2 R(r)}{dr^2} \frac{\partial^2 \Theta(\theta)}{\partial \theta^2} = \mu^2 \text{ and/or } -\mu^2 \quad (37)
\]

where \( \mu^2 \) is positive constant.

For a positive constant equal to \( \mu^2 \), the solutions are

\[
R_1(r) = C1.\sqrt{R_0} + C2.\frac{\sqrt{R_0}}{r}
\]

\[
\Theta_1(\theta) = C3 \sin \left( \frac{\mu \theta}{\sqrt{\lambda_\theta}} \right) + C4 \cos \left( \frac{\mu \theta}{\sqrt{\lambda_\theta}} \right)
\]

For a negative constant equal to \( -\mu^2 \), the solutions are

\[
R_2(r) = E1.\cos \left( \frac{\mu \ln(r)}{\sqrt{\lambda_0}} \right) + E2.\sin \left( \frac{\mu \ln(r)}{\sqrt{\lambda_0}} \right)
\]

\[
\Theta_2(\theta) = E3 \sinh \left( \frac{\mu \theta}{\sqrt{\lambda_0}} \right) + E4 \cosh \left( \frac{\mu \theta}{\sqrt{\lambda_0}} \right)
\]

For the constant equal to zero, the solutions are

\[
R_3(r) = B1 + B2 \ln(r)
\]
In Region I, III and V with the homogenous BCs and periodicity equal to $2\pi$, the constant $A_1=0$ and $\mu = n\sqrt{\lambda_0}$ (with $n$ a positive integer). The periodic regions II, IV, VI and VII in the $r$- and $\theta$-direction with the non-homogeneous BCs have the constant $A_1=0$ and $\mu = \frac{np}{a} \sqrt{\lambda_0}$ in the $\theta$-direction for PMs region where $a$ is the PM-opening angle and $\mu = \frac{np}{a} \sqrt{\lambda_0}$ in the $r$-direction. The particular solution of Poisson’s equations (28) to (34) in each SD is given by

$$T_p = -pr^2/4\lambda_\theta$$

where $p$ is a volumetric constant power loss in each SD.

The solution of (28) in the air-gap with the homogenous BCs is given for each harmonic $n$ by

$$T_{II}(r, \theta) = \left[ R_i(r) \Theta_i(\theta, r) + R_i(r) \Theta_i(\theta, r) \right] \cdots + T_p$$

and can be reduced as

$$T_{II}(r, \theta) = -pr^2/(4\lambda_\omega) + A_{10} + A_{20} \ln(r)$$

$$L + \sum_{n=1}^{\infty} A_{1n} \left( \frac{r}{R_n} \right)^{n\text{r}} + A_{2n} \left( \frac{r}{R_n} \right)^{n\text{r}} \sin(n\theta)$$

$$L + \sum_{n=1}^{\infty} A_{3n} \left( \frac{r}{R_n} \right)^{n\text{r}} + A_{4n} \left( \frac{r}{R_n} \right)^{n\text{r}} \cos(n\theta)$$

where $\tau_r = \sqrt{\lambda_\theta/\lambda_\omega}$, and $p_r$ the windage loss in the air-gap.

The solution of (29) in Region II with the non-homogenous BCs is given for each harmonic by

$$T_{II} = \left[ R_i(r) \Theta_i(\theta, r) + R_i(r) \Theta_i(\theta, r) \right] \cdots + T_p$$

and can be reduced using BCs presented in Fig. 5 as

$$T_{II}(r, \theta) = B_{1\alpha} + B_{2\alpha} \ln(r) - P_m r^2/(4\lambda_\omega)$$

$$+ \sum_{n=1}^{\infty} \left[ B_{1n} \left( \frac{r}{R_n} \right)^{n\text{r}} + B_{2n} \left( \frac{r}{R_n} \right)^{n\text{r}} \cos(f_{\alpha n}(\theta - \theta_{1j})) \right]$$

$$+ \sum_{j=1}^{J} \left[ \begin{array}{c} \text{sh} \left( \frac{f_{ij}}{\tau_{\alpha}} (\theta - \theta a_{1j}) \right) \\ \text{sh} \left( \frac{fr_{ij}}{\tau_{\alpha}} \right) \sin \left( fr_{ij} \ln \left( \frac{r}{R_j} \right) \right) \end{array} \right]$$

$$+ \sum_{j=1}^{J} \left[ \begin{array}{c} \text{sh} \left( \frac{f_{ij}}{\tau_{\alpha}} (\theta - \theta a_{2j}) \right) \\ \text{sh} \left( \frac{fr_{ij}}{\tau_{\alpha}} \right) \sin \left( fr_{ij} \ln \left( \frac{r}{R_j} \right) \right) \end{array} \right]$$

$$+ \frac{1}{l} \sum_{j=1}^{J} \left[ \text{sh} \left( \frac{f_{ij}}{\tau_{\alpha}} (\theta - \theta a_{2j}) \right) \sin \left( fr_{ij} \ln \left( \frac{r}{R_j} \right) \right) \right]$$

where $f_{\alpha 1j} = \frac{k_1\pi}{a}$, $f_{\alpha 2j} = \frac{k_2\pi}{a}$, $g_1 = \ln \left( \frac{R_j}{R} \right)$, $f_{\alpha n} = \frac{m_\alpha \pi}{a}$, $\theta a_{1j} = g_j + \frac{a}{2}$, and $\tau_{\alpha} = \sqrt{\lambda_{\alpha}/\lambda_{\omega}}$.

The stator yoke represented by Region III has homogenous BCs and the solution of (30) is given by

$$T_{III}(r, \theta) = A_{60} + A_{50} \ln(r) - pr^2/(4\lambda_\omega)$$

$$L + \sum_{n=1}^{\infty} A_{5n} \left( \frac{r}{R_n} \right)^{n\text{r}} + A_{6n} \left( \frac{r}{R_n} \right)^{n\text{r}} \cos(n\theta)$$

$$L + \sum_{n=1}^{\infty} A_{7n} \left( \frac{r}{R_n} \right)^{n\text{r}} + A_{8n} \left( \frac{r}{R_n} \right)^{n\text{r}} \sin(n\theta)$$

where $\tau_r = \sqrt{\lambda_\theta/\lambda_\omega}$.

The $Q_s$ stator slots represented by Region IVi has non-homogenous BCs, the solution of (31) is given by

$$T_{IV}(r, \theta) = C_{1i0} + C_{2i0} \ln(r) - pr^2/(4\lambda_\omega)$$

$$+ \sum_{n=1}^{\infty} \left[ C_{1in} \left( \frac{r}{R_n} \right)^{n\text{r}} + C_{2in} \left( \frac{r}{R_n} \right)^{n\text{r}} \cos(f_{s_{\alpha i n}}(\theta - \theta c_1)) \right]$$

$$+ \sum_{j=1}^{J} \left[ \text{sh} \left( \frac{f_{ij}}{\tau_{\alpha}} (\theta - \theta c_{1j}) \right) \sin \left( fr_{ij} \ln \left( \frac{r}{R_j} \right) \right) \right]$$

where $f_{\alpha 1i} = \frac{k_1\pi}{a}$, $f_{\alpha 2i} = \frac{k_2\pi}{a}$, $g_1 = \ln \left( \frac{R_j}{R} \right)$, $f_{s_{\alpha i n}} = \frac{m_\alpha \pi}{a}$, $\theta c_1 = \alpha_i - \frac{c_i}{2}$, $\theta c_2 = \alpha_i + \frac{c_i}{2}$, and $\tau_{\alpha} = \sqrt{\lambda_\theta/\lambda_\omega}$.

The rotor yoke represented by Region V has homogenous BCs, the solution of (32) is given by

$$T_{V}(r, \theta) = A_{10} + A_{90} \ln(r) - pr^2/(4\lambda_\omega)$$

$$L + \sum_{n=1}^{\infty} A_{9n} \left( \frac{r}{R_n} \right)^{n\text{r}} + A_{10n} \left( \frac{r}{R_n} \right)^{n\text{r}} \cos(n\theta)$$

$$L + \sum_{n=1}^{\infty} A_{11n} \left( \frac{r}{R_n} \right)^{n\text{r}} + A_{12n} \left( \frac{r}{R_n} \right)^{n\text{r}} \sin(n\theta)$$

where $\tau_r = \sqrt{\lambda_\theta/\lambda_\omega}$.
In Region VIj with the non-homogenous BCs, the solution of (33) is given by

\[
TVI_j(r, \theta) = B6_{j,n} + B5_{j,0} \ln(r) - Pdr_i \cdot r^2/\left(4\lambda_{s}\right)
\]

\[
+ \sum_{n=1}^{\infty} \left[ B7_{j,0} \frac{r}{R_2} + B6_{j,n} \frac{r}{R_2} \right] \cos \left( frb_n (\theta - \theta b_1) \right)
\]

\[
+ \sum_{l=1}^{L} \left[ B8_{j,l} \frac{r}{R_2} \right] \sin \left( frl \ln \left( \frac{r}{R_2} \right) \right)
\]

\[
\left( \frac{Br_{j,0}(\theta - \theta b_1)}{\tau_{de}} \right) + \frac{Br_{j,0}(\theta - \theta b_2)}{\tau_{de}}
\]

\[
\left( \frac{Fr_{l,1}(\theta - \theta d_1)}{\tau_{de}} \right) + \frac{Fr_{l,1}(\theta - \theta d_2)}{\tau_{de}}
\]

\[
\left( \frac{Fr_{l,1}(\theta - \theta d_2)}{\tau_{de}} \right)
\]

where \( frb_n = \frac{m\pi}{b} \), \( \theta b_1 = \beta_1 - \frac{b}{2} \), \( \theta b_2 = \beta_1 + \frac{b}{2} \) and \( \tau_{de} = \sqrt{\lambda_{s}/\lambda_{de}} \).

In Region VIIi representing the stator teeth with \( P_{ds_i} \) power losses, we have

\[
TVII_i(r, \theta) = C5_{s,0} + C6_{s,0} \ln(r) - P_{ds_i} \cdot r^2/\left(4\lambda_{s}\right)
\]

\[
+ \sum_{n=1}^{\infty} \left[ C7_{s,n} \frac{r}{R_2} \right] \cos \left( fsd_{si} (\theta - \theta d_1) \right)
\]

\[
+ \sum_{l=1}^{L} \left[ C8_{s,l} \frac{r}{R_2} \right] \sin \left( fsd_{si} \ln \left( \frac{r}{R_2} \right) \right)
\]

\[
\left( \frac{Fs_{s,0}(\theta - \theta d_1)}{\tau_{de}} \right) + \frac{Fs_{s,0}(\theta - \theta d_2)}{\tau_{de}}
\]

\[
\left( \frac{Fs_{s,0}(\theta - \theta d_2)}{\tau_{de}} \right)
\]

where \( fs_{s,0} = \frac{k1\pi}{2\pi} \), \( fsd_{si} = \frac{m1\pi}{2\pi} \), \( \theta d_1 = \gamma_i - \frac{d}{2} \), \( \theta d_2 = \gamma_i + \frac{d}{2} \) and \( \tau_{de} = \sqrt{\lambda_{s}/\lambda_{de}} \).

The anisotropy coefficient can be defined as

\[
\psi = \lambda_d / \lambda_s
\]

3- ICs and their Development

To determine the unknown coefficients of temperature in each SD, there are 18 ICs, viz., 14 ICs are in the \( \theta \)-direction and 4 ICs in the \( r \)-direction [7]. The development of ICs permits to obtain an equations system whose unknowns are the coefficients of Fourier’s series solution in each SD. The solving of this system gives the temperature and heat flux distribution in the whole machine.

For the studied inset-PM machine with the harmonics number in each SD: \( m_{ml} = 40 \), \( k_{kl} = 40 \), \( m_{mm} = 50 \), \( k_{kk} = 50 \) and \( mn = 200 \), the dimensions of global matrix and vector are respectively 5,966×5,966 and 5,966.

Fig. 7: Heat flux density IC between TRNM and AM at \( R_i \).

C. Hybrid Method (HM)

It is well known that AM is more accurate with low computational time than TRNM. However, when the number of SDs is high (i.e., the ICs number in both directions is also high) and their dimensions are small, the harmonics number can be very low and then the accuracy of AM can be very low. This situation can be found in the case of inset-PM machine with high number of stator and rotor slots. For this, we propose in this paper to model the stator and rotor slots regions (also the stator and rotor teeth) with TRNM and the other regions with AM [see Fig. 6].

To release the coupling between AM and TRNM, nodes of coupling are added to TRNM presented in Section A at the radii separating the two methods (ICs between AM and TRNM at \( R_i \), \( R_m \), \( R_n \), and \( r_j \)).

For example, at the radius \( R_i \), the continuity conditions of AM are given by

\[
TV(R_i, \theta) = TVI_i(R_i, \theta)
\]

\[
q_{V_i}(R_i, \theta) = \begin{cases} q_{II_i}(R_i, \theta) & \text{for } \theta \in [g_j + \frac{a}{2}; g_j - \frac{a}{2}] \\ q_{VI_i}(R_i, \theta) & \text{for } \theta \in [\beta_j - \frac{b}{2}; \beta_j + \frac{b}{2}] \end{cases}
\]

In the HM, the Region IIj and VIj are modeled by TRNM and Region V by AM. To satisfy the ICs (57) and (58), the nodes temperatures of TRNM at \( R_i \) are written as a \( 2\pi \) periodic function using discrete Fourier series [12] as

\[
T_{TRNM - II_j}(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\theta) + b_n \sin(n\theta)
\]

\[
a_0 = \frac{2}{2\cdot mn} \sum_{k=mod(1,1)} \cos(n\theta_k)
\]

\[a_n = \frac{2}{2\cdot mn} \sum_{k=mod(1,1)} \cos(n\theta_k) \forall n
\]

\[b_n = \frac{2}{2\cdot mn} \sum_{k=mod(1,1)} \sin(n\theta_k) \forall n
\]

where \( mn = ns/2 \), \( T_{\cdot \cdot} \) are the temperatures at the boundary nodes \( nodb(1,1,1) \) to \( nodb(1,1,ns) \) situated at \( R_i (i=1) \) and
\( \theta_j \) the angular position of boundary nodes at the radius \( R_i \). It is important to note that \( nn \) is also the total harmonics number of the AM solution in the Region V.

This Fourier series function permits to replace the ICs (57) and (58) by

\[
TV(R_i, \theta) = T_{TRNM, b0}(\theta) \quad (61)
\]

This above equation permits to get 3 equations as

\[
\frac{1}{4} q_i R_i^2 + A\theta_i \ln(R_i) + A100 = \frac{1}{2\, mn} \sum_{k=mn}^{mn+nn} T_k \quad (62)
\]

\[
A\theta_i + A100 \left( \frac{R_i}{R} \right)^n = \frac{1}{mn} \sum_{k=mn}^{mn+nn} T_k \cos(n\theta) \quad (63)
\]

\[
A1i_1 + A12\left( \frac{R_i}{R} \right)^n = \frac{1}{mn} \sum_{k=mn}^{mn+nn} T_k \sin(n\theta) \quad (64)
\]

To satisfy the heat flux density IC at \( R_i \), i.e., (59), each boundary node is considered receiving a radial heat flux \( \phi_j \) from the AM region [see Fig. 7] as [20]-[21]

\[
\phi_j = \frac{T_i - T_j}{R_j} = -\lambda_{\phi} R_i L_r \int_{\theta_j - \theta_0}^{\theta_j + \theta_0} \frac{\partial TV(r, \theta)}{\partial r} d\theta \quad (65)
\]

where \( \theta_j \) is the angular position of interface nodes and \( d\theta \) is half opening angle of an element equal to 0.5° (each element of TRNM has an angular opening equal 1° in TRNM).

The development of (65) gives

\[
\phi_j = \left[ -L_{\phi} \lambda_{\phi} A\theta_i + \frac{R_i^2}{2} L_{\phi} q_i \right] \int_{\theta_j - \theta_0}^{\theta_j + \theta_0} d\theta \\
\cdots + L_{\phi} \lambda_{\phi} n_{rr} \left[ -A\theta_i + A100 \left( \frac{R_i}{R} \right)^n \right] \int_{\theta_j - \theta_0}^{\theta_j + \theta_0} \cos(n\theta) d\theta \\
\cdots + L_{\phi} \lambda_{\phi} n_{rr} \left[ -A11i_1 + A12 \left( \frac{R_i}{R} \right)^n \right] \int_{\theta_j - \theta_0}^{\theta_j + \theta_0} \sin(n\theta) d\theta \quad (66)
\]

In the HM, \( ms = 21 \) and \( ns = 360 \) with 11 radii for TRNM rotor region and 11 radii for TRNM stator region. The elements and nodes number of TRNM stator region is \( nsgs = 3,600 \) and TRNM rotor region is \( nsg = 3,600 \). In TRNM rotor region, we have added 2 boundaries additional nodes at \( R_i (i = 1) \) and \( R_n (i = 11) \) represented by vectors \( nodb(1,1,j) \) and \( nodb(2,11,j) \) where \( j \) varies from 1 to \( ns \). For TRNM stator region, we have added 2 boundaries nodes at \( R_i (i = 12) \) and \( r_j (i = 22) \) represented by vectors \( nodb(3,12,j) \) and \( nodb(4,22,j) \). The nodes of TRNM connected to the 4 boundary nodes \( nodb(1,1,j), nodb(2,11,j), nodb(3,12,j), \) and \( nodb(4,22,j) \) are \( Bdrt(j) = nobs(1,j), \) \( Bdto(j) = nobs(10,j), \) \( Bdst(j) = nobs(12,j) \) and \( Bdsto(j) = nods(ms,j) \) respectively. The total elements and nodes number of TRNM mesh considering the nodes number at the 4 BCs is equal to \( nsgb = 8,640 \). There are 3 AM regions in the HM where each AM region necessitates \( 2 \times 4 \times nn \) unknowns. The total nodes number of TRNM regions and AM regions is \( nsgb1 = 10,806 \).

Table I

| Symbol | Parameters | Value |
|--------|------------|-------|
| \( B_{pm} \) | Remanence flux density of PMs | 1.3 T |
| \( \mu_{pm} \) | Relative permeability of PMs | 1.0277 |
| \( N_c \) | Number of conductors per stator slot | 23 |
| \( I_m \) | Peak phase current | 7 A |
| \( Q_s \) | Number of stator slots | 6 |
| \( c \) | Stator slot-opening | 30 deg |
| \( a \) | PM-opening | 40 deg |
| \( p \) | Number of pole pairs | 2 |
| \( R_{ext} \) | Radius of the external stator surface | 110 mm |
| \( r_e \) | Outer radius of stator slot | 97 mm |
| \( R_s \) | Radius of the stator inner surface | 80.5 mm |
| \( R_m \) | Radius of the rotor outer surface at the PM | 79.7 mm |
| \( R_r \) | Radius of the rotor inner surface at the PM | 73 mm |
| \( g \) | Air-gap length | 0.8 mm |
| \( L_a \) | Axial length | 40 mm |
| \( \Omega \) | Mechanical speed | 500 rpm |

Table II

| Symbol | Parameters | Value |
|--------|------------|-------|
| \( \lambda_{\phi} \) | Thermal conductivity of air-gap | 0.03 W/(m K) |
| \( \lambda_{a} \) | Thermal conductivity of air | 0.03 W/(m K) |
| \( \lambda_{pm} \) | Thermal conductivity of PMs | 9 W/(m K) |
| \( \lambda_{ls} \) | Thermal conductivity of stator iron | 55 W/(m K) |
| \( \lambda_{lr} \) | Thermal conductivity of rotor iron | 55 W/(m K) |
| \( \lambda_{sl} \) | Thermal conductivity of stator slot coil | 1.73 W/(m K) |
| \( p_{cr} \) | Stator core losses | 4.07 W |
| \( p_{rl} \) | Rotor core losses | 0.31 W |
| \( p_{pm} \) | PM losses | 7.94 W |
| \( p_{sl} \) | Stator slot losses | 18.12 W |
| \( p_{wl} \) | Winding losses in the air-gap | 5 W |
| \( h_r \) | Convection coefficient inside the rotor | 100 W/(m² K) |
| \( T_{in} \) | Temperature inside the rotor | 70 °C |
| \( h_s \) | Convection coefficient outside the stator | 100 W/(m² K) |
| \( T_{ext} \) | Temperature outside the stator | 70 °C |
III. TEMPERATURE AND HEAT FLUX RESULTS

The parameters and dimensions of the studied inset-PM machine are given in Table I. The machine has a simple distributed 4 poles winding. The power losses of the inset-PM machine at 500 rpm as well as the thermal conductivities, convection coefficients and ambient temperatures used in the thermal model are listed in Table II. The harmonics number of AM is $nn = 200$, $mm = 50$, $kk = 50$, $mm1 = 40$ and $kk1 = 40$. These harmonics numbers provide very good accuracy compared to Fem with a reasonable computation time. The average elements and nodes number of the Fem calculation [15] are respectively 109,168 and 55,484.

A. TRNM Results Without Materials Anisotropy and Validation with Fem

For the 6-slots/4-poles inset-PM machine, the temperature distribution at speed of 500 rpm in the whole machine without taking into account the materials anisotropy is shown in Fig. 8. We can observe that the temperature is higher inside the stator slots where power loss is higher [see Table II]. The directions of heat flux are represented with vectors oriented to inside and outside the machine. This is due to convections coefficients imposed outside and inside the machine. In the middle of the air-gap, the distribution of temperature and heat flux components calculated by the developed TRNM and Fem using the parameters and power losses in Table II are given in Fig. 9.

---

| $\psi$ | Anisotropy coefficients |
|-------|-------------------------|
|       | 0.5/1/1.5               |

---

Fig. 8: Temperature and flux distribution obtained using Fem.

Fig. 9: Temperature and heat flux components distribution in the air-gap at the radius 80.05 mm.

Fig. 10: Temperature in the middle of the first PM.
To show the ability of the TRNM to predict the temperature distribution in the PMs and the stator slots, the temperature curves in the \( \theta \)- and \( r \)-direction obtained using TRNM are shown in Figs. 10 ~ 11 and compared with Fem. The TRNM results are in good agreement with the Fem results. A small difference exists between TRNM and Fem results in the PM region. This difference is due to the meshing of the machine which can be improved as done in TRNM [22]-[23]. A parametric analysis with variation of the convective coefficients \( h_i \) and \( h_s \) is also performed. Fig. 12 shows the temperature and heat flux distribution in the inset-PM machine when the convective coefficients \( h_i = 20 \, \text{W/m}^2\cdot\text{K} \) and \( h_s = 100 \, \text{W/m}^2\cdot\text{K} \). The vectors of heat flux are oriented to inside and outside the machine. It can be seen that the heat flux oriented to inside the rotor is higher than the heat flux oriented to outside the stator. The corresponding air-gap temperature distribution is shown in Fig. 13. It can be observed that the temperature is higher than in the case with \( h_i = 100 \, \text{W/m}^2\cdot\text{K} \).

For the case with \( h_i = 20 \, \text{W/m}^2\cdot\text{K} \) and \( h_s = 100 \, \text{W/m}^2\cdot\text{K} \), Figs. 14 ~ 15 show the temperature distribution in the machine obtained using Fem and temperature distribution in the middle of the air-gap obtained using the developed TRNM and Fem. It can be seen from Fig. 14 that the heat flux oriented to outside the stator is higher than the heat flux oriented to inside the rotor. Also, in this case, the TRNM results are very close to those of Fem. The variation of the temperature in the middle of the PM and stator slot when the convective coefficients \( h_i \) and \( h_s \) varies is shown in Figs. 16 ~ 17. The comparison of the TRNM results with those obtained by Fem confirms the validity of the proposed TRNM to predict the temperature and heat flux distribution in the inset-PM machine with a very good accuracy.
Fig. 16: Temperature variation with varying $h_l$ and $h_y = 100 \text{W/}(m^2 \cdot K)$ in a point at the center of PM and stator slot.

(a) Temperature at the center of the first PM.

(b) Temperature at the center of the first stator slot.

(b) Temperature at the center of the first stator slot.

Fig. 17: Temperature variation with varying $h_l$ and $h_y = 100 \text{W/}(m^2 \cdot K)$ in a point at the center of PM and stator slot.

B. AM Thermal Results with Materials Anisotropy and Validation with TRNM

The distribution of temperature and heat flux components in the middle of the air-gap obtained with AM and TRNM taking into account the materials anisotropy is shown in Fig. 18. We can observe a very good agreement between the AM and TRNM results. The temperature distribution in the middle of the first PM and the stator slot in the $\theta$- and $r$-direction [see Figs. 19 ~ 20] obtained analytically and with TRNM confirm the accuracy of the proposed AM. Again, we can show a small difference due to the mesh size adopted in TRNM.

When the cooling outside the inset-PM machine is not sufficient, i.e., $h_l = 20 \text{W/}(m^2 \cdot K)$, the heat is not evacuated and the temperature is very high in the air-gap [see Fig. 21]. The same observation can be done in the case of insufficient cooling in the rotor shaft with $h_y = 20 \text{W/}(m^2 \cdot K)$ [see Fig. 22]. In this case, the rotor temperature is high but lower than the case of low value of $h_y$. The variation of temperature in the middle of the first PM and the stator slot with the convection coefficient $h_l$ and $h_y$ is shown in Figs. 23 ~ 24. Those curves are very important for the design of stator winding insulation and PMs whose characteristics depend on temperature.

Fig. 18: Temperature and heat flux components distribution in the middle of the air-gap.

(a) In the $\theta$-direction.

(b) In the $r$-direction.
Fig. 19: Temperature distribution in the middle of the first PM.

(a) In the $\theta$-direction.

(b) In the $r$-direction.

Fig. 20: Temperature in the middle of the first stator slot.

Fig. 21: Temperature distribution in the middle of the air-gap for $h_\theta = 20\, W/(m^2\cdot K)$ and $h_r = 100\, W/(m^2\cdot K)$.

Fig. 22: Temperature distribution in the middle of the air-gap for $h_\theta = 20\, W/(m^2\cdot K)$ and $h_r = 100\, W/(m^2\cdot K)$.

(a) Temperature at the center of the first PM.

(b) Temperature at the center of the first stator slot.

Fig. 23: Temperature variation with varying $h_\theta$ and $h_r = 100\, W/(m^2\cdot K)$ in a point at the center of PM and stator slot.

(a) Temperature at the center of the first PM.

(b) Temperature at the center of the first stator slot.

Fig. 24: Temperature variation with varying $h_\theta$ and $h_r = 100\, W/(m^2\cdot K)$ in a point at the center of PM and stator slot.

C. HM Thermal Results and Validation with TRNM and Fem

It is not easy to use AM in heat transfer prediction for rotating electrical machines with high number of stator slots and rotor poles. The ICs in the $r$- and $\theta$-direction are important and the dimensions of SDs are small, requiring a small harmonics number and thus lower accuracy. For this, it is appropriate to use HM. In this section, we apply the HM in both cases, with isotropic and anisotropic materials. For isotropic materials, the validation of results can be performed with Fem.

1. Isotropic Materials

The temperature and heat flux distribution in the middle of the air-gap is shown in Fig. 25. The HM results are very close to those from Fem. The temperature distribution in the $\theta$- and $r$-direction in the middle of the first PM and the first stator slot is shown in Figs. 26 ~ 27. The accuracy of HM is established also in those SDs where it is important to know the heat transfer for the insulation design. A small difference can be observed in the PM region between HM and Fem. The mesh size of the TRNM rotor has affected the HM results. For this, it is necessary to optimize the TRNM parts using them in HM.
The effect of cooling outside the inset-PM machine and inside the rotor shaft is represented with the convective coefficients $h_s$ and $h_r$ respectively. In Fig. 28, for $h_s = 20 \frac{W}{(m^2 \cdot K)}$ which is small, we represent the temperature distribution in the middle of the air-gap. The temperature is higher compared to $h_s = 100 \frac{W}{(m^2 \cdot K)}$.

For $h_r = 20 \frac{W}{(m^2 \cdot K)}$ compared to $h_r = 100 \frac{W}{(m^2 \cdot K)}$, the temperature distribution in the air-gap [see Fig. 29] is higher than the case with $h_r = 100 \frac{W}{(m^2 \cdot K)}$. Low values of convective coefficients represent a barrier for heat transfer outside the stator and inside the rotor.

![Fig. 25: Temperature and heat flux components distribution in the middle of the air-gap.](image)

![Fig. 26: Temperature in the middle of the first PM.](image)

![Fig. 27: Temperature in the middle of the first stator slot.](image)

![Fig. 28: Temperature distribution in the middle of the air-gap for $h_s = 20 \frac{W}{(m^2 \cdot K)}$ and $h_r = 100 \frac{W}{(m^2 \cdot K)}$.](image)
2- Anisotropic Materials

In the analysis of heat transfer in the inset-PM machine taking into account the materials anisotropy, TRNM is used for the validation of results. This is due to Fem which use Cartesian representation of thermal conductivities [17] and [19]. For the dimensions and parameters of the studied machine [see Tables I and II] and an anisotropy coefficient equal to 0.5, we represent on Fig. 30 the distribution of temperature and heat flux in the middle of the air-gap. It can be seen that the temperature in the air-gap is higher than in the case of isotropic materials and the comparison between the two methods gives very good agreement. It is important to note that the anisotropy coefficient of materials is applied for rotor and stator iron, slots and PM without air-gap.

A parametric study is performed in this section as a function of the anisotropy coefficient (viz., $\varphi = 0.5, 1, 1.5$). When $\varphi = 1$, the materials are isotropic and when $\varphi = 0.5$ the tangential value of thermal conductivity is smaller than the radial value. For $\varphi = 1.5$, the tangential thermal conductivity is higher than the radial conductivity. The temperature distribution at the middle of the air-gap, the middle of the PM and the middle of the stator slot for different values of $\varphi$ is shown in Figs. 31 ~ 33. It can be observed that the heat transfer in the machine is better when the tangential thermal conductivity of materials is higher than the radial conductivity. In this case, the temperature is lower. This remark is valid for the studied case where all materials of the machine have the same anisotropy coefficient, which is not true. A more realistic study should take into consideration consider the real values of thermal conductivity anisotropy in each region (i.e., slots, stator and rotor iron cores, PM). Moreover, it is important to note that the conductivity in the z-direction of rotating electrical machines is mostly affected by materials anisotropy and a 3-D study is appropriate.

Fig. 29: Temperature distribution in the middle of the air-gap for $h_r = 20 \text{ W/m}^2\text{K}$ and $h_z = 100 \text{ W/m}^2\text{K}$.

Fig. 30: Temperature and heat flux components distribution in the middle of the air-gap.

Fig. 31: Temperature distribution in the middle of the air-gap using HM.

Fig. 32: Temperature in the middle of the first stator slot using HM.
For $k$ from 1 to 32 do

$\text{gm}(k, \text{isk}(5,k)) = \frac{1}{R_{\varphi} + R_{\text{isk}(5,k)\varphi}}$;

$\text{gm}(k, \text{isk}(2,k)) = -\text{gm}(k, \text{isk}(5,k))$;

end do

The equations of thermal convection at the rotor shaft are assembled in the global matrix $\text{gm}$ by

For $k$ from 33 to 40 do

$\text{gm}(k, \text{isk}(5,k)) = \frac{1}{R_{\varphi} + R_{\text{isk}(5,k)\varphi}}$;

$\text{gm}(k, \text{isk}(2,k)) = -\text{gm}(k, \text{isk}(5,k))$;

end do

For $k$ from 33 to 48 do

$\text{gm}(k, \text{isk}(4,k)) = \frac{1}{R_{\varphi} + R_{\text{isk}(4,k)\varphi}}$;

$\text{gm}(k, \text{isk}(2,k)) = -\text{gm}(k, \text{isk}(4,k))$;

end do

The fixed temperature of 70 °C is introduced in the global matrix as in Fem by

For $k$ from 33 to 48 do

$\text{gm}(k, k) = \text{gm}(k, k) + 10E30$;

$f(k) = 10E30(70 + 273.16)$;

end do

APPENDIX B

We start with the nodes of TRNM representing the rotor slots and teeth as

For $k$ from 1 to $n_{gr}$ do

If $k \in \text{Bdrti}$ then

$\text{gm}(k, \text{isk}(1,k)) = -\frac{1}{R_{\varphi} + R_{\text{isk}(1,k)\varphi}}$;

$\text{gm}(k, \text{isk}(3,k)) = -\frac{1}{R_{\varphi} + R_{\text{isk}(3,k)\varphi}}$;

$\text{gm}(k, \text{isk}(4,k)) = -\frac{1}{R_{\varphi} + R_{\text{isk}(4,k)\varphi}}$;

$\text{gm}(k, \text{isk}(5,k)) = -\frac{1}{R_{\varphi} + R_{\text{isk}(5,k)\varphi}}$;

$\text{gm}(k, \text{isk}(2,k)) = -(\text{gm}(k, \text{isk}(1,k)) + \text{gm}(k, \text{isk}(3,k))..$  

end if

elif $k \in \text{Bdrto}$ then

$\text{gm}(k, \text{isk}(1,k)) = -\frac{1}{R_{\varphi} + R_{\text{isk}(1,k)\varphi}}$;

$\text{gm}(k, \text{isk}(3,k)) = -\frac{1}{R_{\varphi} + R_{\text{isk}(3,k)\varphi}}$;

$\text{gm}(k, \text{isk}(5,k)) = -\frac{1}{R_{\varphi} + R_{\text{isk}(5,k)\varphi}}$;

$\text{gm}(k, \text{isk}(2,k)) = -(\text{gm}(k, \text{isk}(1,k)) + \text{gm}(k, \text{isk}(3,k))..$  

end if

else ($k \notin \text{Bdrti}$ and $k \notin \text{Bdrto}$) then

$\text{gm}(k, \text{isk}(1,k)) = -\frac{1}{R_{\varphi} + R_{\text{isk}(1,k)\varphi}}$;

$\text{gm}(k, \text{isk}(3,k)) = -\frac{1}{R_{\varphi} + R_{\text{isk}(3,k)\varphi}}$;

$\text{gm}(k, \text{isk}(5,k)) = -\frac{1}{R_{\varphi} + R_{\text{isk}(5,k)\varphi}}$;

$\text{gm}(k, \text{isk}(2,k)) = -(\text{gm}(k, \text{isk}(1,k)) + \text{gm}(k, \text{isk}(3,k))..$  

The implementation of TRNM in ENP Engineering Science Journal, Vol.1, No.1, July, 2021
gm(k, isks(3,k)) = \frac{-1}{R_{k,\theta} + R_{\theta,k})(\theta)};

\text{for } k \neq 1

\text{and } j \neq 1

gm(k, isks(4,k)) = \frac{-1}{R_{k,\theta} + R_{\theta,k})(\theta)};

\text{for } k \neq 1

\text{and } j \neq 1

gm(k, isks(5,k)) = \frac{-1}{R_{k,\theta} + R_{\theta,k})(\theta)};

\text{for } k \neq 1

\text{and } j \neq 1

gm(k, isks(2,k)) = \text{gm(k, isks(1,k)) + gm(k, isks(3,k))} \ldots + \text{gm(k, isks(4,k)) + gm(k, isks(5,k))};

\text{end if}

\text{end do}

\text{The introduction of equations (62) to (64) in the global matrix}\n
gm\text{ is done as follow. (62) is introduced by}

\text{gm(nsgb +1, nsgb +6 +9nn +n) = } \ \text{ln}\ (R_{k,\theta})

\text{For } j \text{ from 1 to ns do}

\text{ksb +1, k) = } \frac{1}{2(2nn)}\n
\text{end do}

\text{f (nsgb +1) = } \frac{-q \cdot R_{R}}{4}\n
(63)\text{ gives}

\text{For } n \text{ from 1 to nn do}

\text{gm(nsgb +1 + n, nsgb +6 +9nn +n) = } \text{ln}\ (R_{k,\theta})^{-n}\n
\text{gm(nsgb +1 + n, nsgb +5 +8nn +n) = } -1\n
\text{For } j \text{ from 1 to ns do}

\text{ksb +1 + n, k) = } \frac{\text{cos}(n\theta_{k})}{nn}\n
\text{end do}

\text{end do}

\text{(64) is added to gm as}

\text{For } n \text{ from 1 to nn do}

\text{gm(nsgb +1 + nn + n, nsgb +6 +11nn +n) = } \text{ln}\ (R_{k,\theta})^{-n}\n
\text{gm(nsgb +1 + nn + n, nsgb +6 +10nn +n) = } -1\n
\text{For } j \text{ from 1 to ns do}

\text{ksb +1 + nn + n, k) = } \frac{\text{sin}(n\theta_{k})}{nn}\n
\text{end do}

\text{end do}

\text{There are 360 equations (66) to be introduced in the global}\n
\text{matrix depending on the number of boundary nodes at } R_{k,\theta}.

\text{They are given as}

\text{For } j \text{ from 1 to ns do}

\text{ksb +1, k) = } \frac{1}{R_{ksb +1, k)(\theta)}\n
\text{gm(k, isks(5,k)) = } \text{gm(k, isks(2,k))} \ldots + \text{gm(k, isks(4,k)) + gm(k, isks(5,k))};

\text{end do}

\text{For } n \text{ from 1 to nn do}

\text{gm(k, nsgb +5 +8nn +n) = } \text{ln}\ (R_{k,\theta})^{-n}\n
\text{gm(k, nsgb +6 +9nn +n) = } \text{ln}\ (R_{k,\theta})^{-n}\n
\text{gm(k, nsgb +6 +10nn +n) = } \text{ln}\ (R_{k,\theta})^{-n}\n
\text{gm(k, nsgb +6 +11nn +n) = } \text{ln}\ (R_{k,\theta})^{-n}\n
\text{end do}

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