The static analysis of arch dam-foundation system by using scaled boundary finite element method

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Abstract: The scaled boundary finite element (SBFEM) as a semi-analytical numerical method has been used in many fields. SBFEM has the following advantages: reducing the spatial dimension by one, rigorously modelling the unbounded domain and involving no fundamental solution. The computational model based on SBFEM will be conducive to the reliability and efficiency of safety evaluation of arch dam. However, owing to the requirement of scaling inherently, the SBFEM cannot be immediately applied in the analysis of arch dam which belongs to irregular shell structure. This limitation can be overcome by employing the sub-structure method to subdivide the arch dam into several sub-structures which satisfy the scaling requirement. An example of static analysis of the arch dam-foundation system is presented to verify the effectiveness of the SBFEM sub-structure method. The result is helpful to establish the calculating model of arch dam-reservoir-foundation system based on scaled boundary finite element method.

1. Introduction

The scaled boundary finite element method as a semi-analytical numerical method was proposed by Wolf and Song[1, 2]. The scaled boundary finite element method combines the advantages of finite element method and boundary element method. Compared to the FEM, scaled boundary finite element method reduces the spatial dimension by one because only the boundary need to be discretized. Compared to BEM, scaled boundary finite element method involves no fundamental solution. Scaled boundary finite element method has been applied in many fields. In Reference[3], a super-element for the dynamic analysis of two-dimensional crack problems is proposed based on scaled boundary finite element method. In Reference[4], a novel method coupling scaled boundary finite element method and FEM is developed for linear elastic fracture modelling. In Reference[5], a NURBS enhanced scaled boundary finite element method is firstly exploited to solve electrostatic problems. In Reference[6], dynamic fracture analysis of the soil-structure interaction system by using scaled boundary finite
element method is presented. In Reference[7], a NURBS enhanced scaled boundary finite element method is presented for the numerical solution of seepage problems in the unbounded domain.

The basic theory of the scaled boundary finite element method is based on the scaled boundary transformation between the global coordinates and the local coordinates. The arrangement of scaling center has an important influence on the results of analysis and calculation. The scaling requirement includes: in order to obtain regular element mesh and avoid the appearance of distorted element, the scaling center should be chosen at the centroid; in order to avoid element overlapping, scaling centers should satisfy boundary visibility. As shown in Figure 1, the scaled boundary finite element method cannot be immediately applied in the analysis of arch dam which belongs to irregular shell structure. Therefore, it is necessary to employ the sub-structure method to divide the arch dam into several sub-structures which satisfy the scaling requirement, as shown in Figure 2.

In this paper, the static analysis of arch dam-foundation system by using scaled boundary finite element method and sub-structure method is presented. The results of the SBFEM sub-structure method is verified by comparing with the results of finite element method.

![Figure 1. The sketch of crown cantilever and arch ring](image)

![Figure 2. The sub-structuring of crown cantilever and arch ring](image)

2. Scaled boundary finite element method

By using SBFEM to discretize 3D static bounded problem, the original question will be converted into a 2D problem. Based on coordinate system transformation, the governing equation can be obtained by using weighted residual method.

\[
E_0 \frac{\partial^2 u}{\partial \xi^2} + \left(2E_0 + (E^1)^T - E^1\right) \frac{\partial u}{\partial \xi} + \left((E^1)^T - E^2\right) u(\xi) + F(\xi) = 0
\]  

Eq.(1) is the weak governing equation of the system which can be solved analytically. Eq.(2) is the boundary conditions on the outer boundary $\xi=1$.

\[
E_0 \frac{\partial^2 u}{\partial \xi^2} + (E^1)^T u = P
\]
The equilibrium equation (3) on the boundary is obtained by using the matrix function solution\cite{8}.

\[ Ku = P \] (3)

3. Computing model
The arch dam-foundation system is shown as Figure 3. The height of the arch dam is 240m. As shown in Figure 4, eight-node surface element is employed in the SBFEM model. The arch dam-foundation system is subdivided into 26 sub-structures. Each sub-structure satisfies the scaling requirement. The total number of surface element is 1954. The total number of node is 5523.

![Figure 3. The diagram of arch dam-foundation system](image)

(a) View from downstream to upstream  (b) View from upstream to downstream
![Figure 4. The discretization of the boundary surface for the arch dam-foundation system for SBFEM analysis](image)

As shown in Figure 5, eight-node solid element is employed in the FEM model. The total number of solid element is 122578. The total number of node is 177743.

![Figure 5. The meshes of the arch dam-foundation system for FEM analysis](image)

The dam body and foundation are assumed as linear elastic materials and have the same physical and mechanics parameters in the SBFEM model and FEM model. The mass density, elasticity modulus and Poisson's ratio is 2400kg/m\(^3\), 2.1e10MPa and 0.167, respectively. The arch dam-foundation system subject to hydrostatic pressure and body load.
4. Results

Figure 6. The displacement distribution in the transverse direction calculated by using SBFEM

Figure 7. The displacement distribution in the river direction calculated by using SBFEM

Figure 8. The displacement distribution in the vertical direction calculated by using SBFEM

Figure 9. The displacement distribution in the transverse direction calculated by using FEM

Figure 10. The displacement distribution in the river direction calculated by using FEM
Figure 11. The displacement distribution in the vertical direction calculated by using FEM

Table 1 The displacement extreme values calculated by SBFEM and FEM

| Method | transverse direction | river direction | vertical direction |
|--------|----------------------|-----------------|--------------------|
|        | to the left bank     | to the right bank| at the central crest| on both sides of the dam body | absolute maximum | absolute minimum |
| SBFEM  | 0.015                | -0.015          | -0.011             | 0.005 | -0.065 | -0.035 |
| FEM    | 0.016                | -0.016          | -0.012             | 0.006 | -0.059 | -0.036 |

As shown in Figure 6, 7, 8, 9, 10, 11 and Table 1, the displacement distribution and extreme values calculated by using SBFEM and FEM, is almost the same in the transverse, river and vertical direction of the arch dam. Because the element size of SBFEM model is greater than that of FEM Model, the displacement distribution calculated by using SBFEM and FEM has a small difference.

5. Conclusion

The static analysis of arch dam-foundation system by using scaled boundary finite element method and sub-structure method is presented. The arch dam-foundation system is divided into several substructures which satisfy the scaling requirement. Only each substructure surface is discretized by using SBFEM. Compared to FEM, SBFEM reduces the workload of mesh discretization and achieves the same level accuracy with less degree of freedom. Therefore, SBFEM has a broad prospect in simulating the dynamic response of arch dam-reservoir-foundation system.

Reference

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