Design of financial products for Stored Value Card balance

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Abstract. In our daily life, there are still some funds that are slow to circulate and the amount is relatively large, that is, the balance of stored-value cards, such as public transport cards, subway cards, etc. We can see that the stored value card plays an increasingly important role in people's lives, so the research on that card is forward-looking. In this article, the author intends to design a new type of financial product for the funds that flows relatively slow. It will not only integrate social resources and create considerable returns, but will also generate a certain percentage of income. In return, let each stored value card holders obtain profit. The specific mechanism of this financial product will be designed in this article. In order to find the experimental object, the author decided to investigate the campus card, collect and analyze the data, use professional statistical software in research, and plan to design a financial product suitable for students.

1. Introduction

The author will use the "Campus Card" as an experimental sample, and proposes to establish a financial product based on the balance of the card savings, investing in the balance of the card, giving back to the user and gaining income. At the same time, the author collects data on campus and conducts a detailed survey on the use of all users' cards. Furthermore, we learned about the habits of students using campus cards. After clarifying the general characteristics of the surveyed groups, we can start from the data and design a suitable financial product operation mechanism.

2. Research process

2.1 data processing

Here we use SPSS software for linear regression analysis. The analysis results are as follows:

Based on the following results, we have these following conclusions:

(1) Taking the minimum balance of the usual campus card as the dependent variable $y$, the current campus card balance (the amount of campus card cash) as the independent variable $x$, the linear model that can be derived is:

$$y = 7.895 + 0.103x$$

Since $0.103>0$, the minimum balance of the campus card is more for users who often have more balance on the campus card. However, the absolute value of $0.103$ is small, indicating that even if the user's campus card amount is different, the minimum balance of the campus card often does not change significantly. This is also in line with our common sense. $[1]$  

(2) We found that in the coefficient table, the values of sigma are 0.01 and 0.02, which are significantly less than 0.05, so the null hypothesis should be rejected: the coefficient of $x$ is 0, and the coefficient of $x$ is considered to be significantly zero, that is, $x$ and $y$ have clear linear relationship. $[2]$
(3) On the other hand, we see that in the coefficient table, the t values are 3.495 and 3.113, and the significance level is $\alpha=0.05$, and the degree of freedom is 122. In this study, since $n$ is large, we can approximate the freedom. The degree is 125. Check the t distribution table and find the critical value with a degree of freedom of 125, $\alpha/2=0.025$. The critical value is 1.980. Since 3.485>1.980, 3.113>1.980, the null hypothesis should be rejected: the coefficient of $x$ is 0, which should be considered as $x$. The coefficient is significantly non-zero, as described in (2). It should be considered that the minimum balance $y$ of the campus card is generally significant for the linear regression of the current campus card balance $x$. [3]

After the above discussion, we initially believe that the linear regression equation is effective, but it does not guarantee that the data fits very well, so we will analyze the residuals next. We first analyze the residual estimator, and use the graphical test method to draw the residual graph with the current campus card balance $x$ as the horizontal axis and the residual as the vertical axis. [4]

Fig. 1. The X axis is the minimum balance of the campus card as usual, and the Y axis is unstandardized residual.

Here we find that there is an abnormality in the residual graph. The residuals are not randomly distributed above and below the line with zero residual, but become larger as the current campus card balance $x$ becomes larger, and it shows a strong one-dimensional linear relationship. Since the balance of the campus card cannot be negative, the data cannot be distributed in the second quadrant and the third quadrant. If the data falling on the vertical axis cannot be ignored, most of the data is concentrated in the first quadrant, indicating that the residual and random perturbation terms have a significant positive sequence correlation.

The residuals should be randomly distributed in a certain area above and below the line $y=0$. If there is a case of linear growth, according to the definition of the residual, if the residual has obvious linear characteristics, it means that the residual also contains the quantity related to the explanatory variable. And because the residuals and $y$ have a closely related relationship with $x$, we next consider finding $(x-y)$ first, then making it equal to $z$, and then performing a linear regression on $y$ and $z$.

After linear regression, we found that the value of sigma is large. However, this is acceptable because we can analyze the entire user community without having to delete the abnormal sample points. Fortunately, after the above operation, we again use the vertical axis as the residual, and the horizontal axis is the difference between the current campus card balance and the usual minimum balance of the campus card, that is, $z$, the residual graph:
Here, we define $z$: the actual disposable amount of the user. Based on the results of our research above, if you want to finance, you should base your campus card on the minimum balance. If we raise the benchmark and exceed the minimum balance of the campus card, although it can benefit the students with more campus card balances, the students who have relatively less campus card balances will get less income and they are more likely to touch. Or break through the minimum balance of the usual campus card, which means that this part of the class will take more risks. On the other hand, if we use the minimum balance of the usual campus card as the bottom line, then our risk pool capacity can be set smaller and the mechanism is more safe.

Based on the above research, we have a deeper understanding of the use of the user's campus card. This laid the foundation for us to establish financing methods and the scale of financing later: as we found in the research, although some users will have a lot of money in the campus stored value card, but those who do not have a lot of money in the campus card. As for users, the minimum balance of their campus cards is similar. That is to say, if we want to finance, we must take the minimum balance into account, and even the amount of financing obtained from each user cannot exceed the usual minimum balance of the campus card. Although it is intuitively analyzed, this will greatly reduce the final financing scale, which will affect the final income. However, if we take the minimum balance of the usual campus card of each user as a reference amount, all of them will be taken out and accumulated for investment. Significantly reduce the size of the risk pool (the funds in the risk pool are used to balance risks and not participate in the investment), and thus the benefits are improved.

2.2 Establish a mechanism

Based on the above findings, the following mechanisms have been established:

(1) By studying the user's campus card usage, we found that in the case of more users, it is more reasonable to extract the minimum balance of each person's stored-value card for financing. This part of the funds is taken out and accumulated, called initial financing, and this amount is called the initial financing scale.

(2) After obtaining initial financing, we will set up a risk pool. Since the balance of the user's campus card is usually not lower than the usual minimum balance of the campus card, the risk pool may not be built to a large extent, as long as the user's normal use is ensured, and no risk is assumed. It is easy to find that the smaller the risk pool capacity, the larger the amount that can be used for investment, and the higher the return, so we expect the risk pool to be as small as possible, so it is necessary to study the minimum amount of the risk pool.

(3) After the establishment of the risk pool, the remaining funds are used to invest in safe and stable financial products, and the proceeds are obtained. We believe that this income is fixed and the rate of
return is fixed. This part of the income is called total income. The time spent on investment is called the investment duration.

(4) After the income is obtained, we will give back part of the income to the user, which is called user income; part of the income is put into the risk pool, which is called the risk pool expansion income; part of the income is used to balance the management of the system, called management loss. The last part of the proceeds will be taken out as the income of the school or institution, which is called real income.

(5) According to the stability of the user's balance and whether the user has a clear relationship with the school, that is, the use of campus cards during the school period, the use of campus cards is not frequent during the holiday, all the above variables will change accordingly, so the new mechanism and the holiday period should be analyzed separately.

Then we evaluate this mechanism and initially estimate how much revenue the user can earn after the mechanism is running, and how much real income the school or institution can get.

Some symbols will be used below, and will be uniformly labeled here:

- \( i \): Yield. Refers to the interest rate of funds used for investment in financial markets.
- \( a \): The minimum amount of campus card usually. It is generally believed that the balance of the user will not be lower than the minimum amount of the usual campus card.
- \( b \): Current campus card balance. Refers to the remaining amount of the campus card.
- \( P \): The number of people. Refers to the total number of users in this user group.
- \( p \): The number of participants. Refers to the number of people who frequently use campus cards, which is proportional to the number of people.
- \( u \): The proportion of participants.
- \( s \): Average per capital amount. Refers to the amount of investment per capital.
- \( S \): Initial financing scale. The average of the summed available amount.
- \( t \): Investment time. Refers to the length of time that investment funds invest in financial products.
- \( C \): Risk pool capacity. Refers to the size of the funds in the risk pool.
- \( \alpha \): Risk pool scale factor. Refers to the ratio of the risk pool to the initial financing scale.
- \( G \): Investment amount. Refers to the difference between the initial financing scale and the risk pool capacity, that is, the funds actually used for investment.
- \( I \): Total income. Refers to the income of the investment amount obtained within a certain period of time.
- \( D \): Management loss. Refers to the total cost of management over a period of time.
- \( d \): Average daily management loss. Refers to the average daily management cost.
- \( I_1 \): User revenue. Refers to the revenue of all users.
- \( I_2 \): Real income. Refers to the income earned by the school or institution.
- \( I_3 \): Risk pool expands revenue. See note (2) for meaning.

Note:

(1) In order to indicate that the model is different depending on whether the user is attending school or during the holiday period, we will use the subscript to distinguish between the representation of a certain amount during school days and during the holiday. For example, during school days, the minimum amount of campus card is usually \( a_1 \), and during the holiday period, the minimum amount of campus card is usually \( a_2 \). All variables related to the school days are marked with a lower corner mark 1 and the variables related to holiday are marked with 2.

(2) User income related to the total income of all users within a certain period of time. According to the different input of users, the income distributed will be measured and distributed. That is to say, the more inputs, the higher the income. Due to the weighted distribution of proceeds, it is often the case that the benefits are not integers. In order to avoid such a thing happening, we subtract the total loss from the management loss and the real income, and then divide the remaining funds into integers according to the method of integer, accurate to 0.01 yuan, and distribute the proceeds to all users, and the remaining change is put back. In the risk pool, this not only ensures that the user's balance can be accurate to 0.01
yuan, but also expands the risk pool to further reduce the risk. This part of the funds is called the risk pool expansion income. That is to say, part of the total revenue is used to consolidate the risk pool so that this mechanism can run smoothly for a long time.

After making the above annotations, we can make a rough estimate of the benefits of the model. At the same time, in order to be able to estimate the approximate amount of revenue, we will make the following assumptions:

1. Due to the fact that during the school period, the user will use the campus card and recharge when the balance is insufficient. Therefore, we believe that the total amount of the campus card balance of the whole school has not changed greatly during the school. During the holiday, because the frequency of use of campus cards is almost zero, we also think that the total amount of campus card balances of the whole school is unchanged during the holidays. Let us assume that within one year, the total amount of campus card balances of the whole school will remain unchanged.

2. The rate of return is calculated according to the balance. Because the income of Yu'e Bao is stable and flexible, we temporarily calculate it with its seven-day annualized rate of return. Here we always assume that the annualized rate of return on the seven days is the rate of return i, first assume i = 4%, which means: assuming one year of investment, the amount of investment is 10,000 yuan, and the income of 400 yuan can be obtained after one year.

3. \( P = 60000, p = 0.84P \) (0.84 is determined by the data), \( s = 13 \) (yuan/person), \( s \) is obtained by adding and averaging the minimum amount of investment a. The duration is \( t = 1 \), and the risk pool is determined according to a certain percentage, which is 10% of the initial financing scale. The average management cost is 2 yuan/day. So:

\[
G = S - c = Ps - c = Ps - 0.1C = Ps - 0.1Ps = 0.9Ps
\]

\[
I = Git - 365dt = 0.9Psit - 365dt = 27350
\]

Through the above calculations, in the case of ignoring \( I_3 \), the revenue that can be obtained by running the mechanism for one year is 27,350 yuan. It can be seen that after preliminary theoretical analysis, the mechanism can run and earn revenue.

Next we will make further assumptions:

1. In order to simplify the calculation, the whole year is divided into teaching period and holiday, the teaching period is 8 months, the holiday is 4 months, and the year is 360 days. Since the income is difficult to be accurate to the days, we will follow the income settlement period is performed in the month, and it is assumed that 30 days per month, time \( t = 1 \) (year).

2. The rate of return during school is \( i_1 \), the rate of return on holidays is \( i_2 \), \( \alpha_1 = 0.1, \alpha_2 = 0.02 \), and 10 students who do not often use campus cards are removed, the ratio is 10/124 = 0.08; \( s_1 = 26 \) (yuan/person), \( s_2 = 45 \) (yuan/person). Since the annualized rate of return on the 7th is the nominal rate of return, if the annual rate of return is constant, the monthly rate of return is \( i/12 \), in other words, \( i_{\text{month}} = 1/12 \).

\[
p = 0.92P,
\]

From the above analysis, the formula can be:

\[
I = ps_1(1 - \alpha_1) * i_1 + ps_2(1 - \alpha_2) * i_2 - 360dt
\]

\[
= ps_1(1 - \alpha_1) * 8i^{(12)} + ps_2(1 - \alpha_2) * 4i^{(12)} - 360dt
\]

\[
= 66182.4
\]

After calculation, the total return is 66182.4 yuan per year, and the income is considerable. At the same time, we find that in the same time, the benefits of the holiday are nearly double that of the school. Next we will distribute half of the revenue to the user and risk pool.

\[
I_1 + I_3 = 0.5I = 33091.2
\]

Then the average revenue per user is:

\[
I_2 = 0.5I / p = 0.59
\]

Because the rounding up, the remaining funds were collected into the risk pool, \( I_3 = 523.2 \) (yuan).
Because of the income we calculate the average value obtained by each user, and the user's investment is often not enough, usually not more than 100 yuan, so the income is 0.59 yuan is reasonable.

Then the real benefits that schools and institutions ultimately receive are:

\[ I_2 = 0.5I = 33091.2 \] (7)

After our research, this mechanism has been successfully established. It currently considers that the user balance may exceed the minimum balance of the usual campus card.

3. **Evaluates of the mechanism**

Next we will evaluate the mechanism in details. Programming in R language, it is the same as our previous calculations. Next, we will experiment on the control variable method for some variables (the charts are omitted):

3.1 **Change in number of people**

We assume that the total number of users varies from 60,000 to 100,000. The resulting real income changes \(^5\). We found that the real income has a significant change, and for every 8,000 people added, the real income (finally obtained) will increase by 4,463.31 yuan, and the total income will increase by 8920.62 yuan. Increasing the number of users will increase the revenue relatively clearly.

3.2 **Changes in the proportion of regular use of campus cards**

We assume that the proportion of people who use campus cards frequently ranges from 100\% to 20\%, with an interval of 2\% by running R code. The conclusion is that by increasing the proportion of users who frequently use campus stored value cards, we find that for every 2\% increase in active users, real income will change. The school or institution can encourage more users to use the campus stored value card to create convenience for the users who lost the campus stored value card, so as to enhance the final real income, the effect will be improved.

3.3 **The minimum balance of the campus card is usually changed during school**

Assuming that the minimum average balance of the campus card is usually changed from 10 to 80, and the result is calculated every more 3.5 yuan, we found that the real income of the final school or institution has changed significantly. At the same time, users have invested a lot of money, and the benefits per user have been greatly improved. It can be said that this is a very effective way to improve the income of users, schools and institutions.

3.4 **Changes in the minimum balance of campus cards during the holidays**

Then we assume that during the holidays, more users tend to leave more balances in the campus card during the holidays. We assume that during the holiday, the average card balance of each user varies from 10 yuan to 100 yuan, and the interval is 4.5 yuan. We find that, like (3), there is a clear growth trend in real income, that is, encourage users to store more money on campus stored value cards during the holidays can make the final benefits obviously increase.

3.5 **Change in risk pool ratio during school**

We assume that the risk pool ratio during the school period is reduced from 50\% to 5\%, and is calculated every 5\%. We found that as the risk pool capacity shrinks, real income has a significant increase. However, during the school period, users use campus cards more frequently, and there may be some risks in the market. There is a potential risk of lowering the proportion of risk pools during school. In short, even if the risk pool capacity during school is reduced, the risk can be significantly improved, and the risk caused by this behavior cannot be ignored.

3.6 **Change in risk pool ratio during holidays**
We assume that the risk pool ratio is reduced from 30% to 0 during the holidays and is calculated every 5%. Unlike (5), the risk pool ratio during the holiday period will not increase the risk too much, and as the risk pool decreases, the real income will increase significantly. In other words, we can reduce the risk pool during the holidays and gain more improvement of revenue.

### 3.7 Yield change

The rate of return directly affects the real income. Next we use R software to program, yields from 3% to 7%, with an interval of 0.1%. We find that with the increase of the rate of return, the real income has obvious changes. Even if the rate of return has a little change, the real income will have obvious changes. Here we believe that the rate of return is a very important reference factor. In order to obtain higher returns, increasing the rate of return is the most effective means.

### 3.8 Changes in the number of days that generate management losses

We assume that the maintenance time is from one day to 360 days and is calculated every 10 days. We have found that changes in the number of days that generate management losses have little effect on the final benefit. As the number of days in which management losses occur, the reduction in real returns can be ignored.

### 3.9 Changes in average daily management loss

We assume that the average daily management fee varies from 0 yuan to 20 yuan, and the result is calculated every 1 yuan. We observed that with the increase in daily average management loss costs, real revenues have declined. Protecting the safety of the system and reducing the occurrence of faults can avoid losses to some extent.

### 4. Summary

According to the research above, the mechanism can be operated and can generate certain benefits. When it can successfully run:

1. The most effective way to increase the total return and real income is to increase the rate of return. A very effective method is to let the user have more balance in the stored value card.
2. Other factors will also have a certain impact on real income. As long as it is as beneficial as possible, it will not have an excessive impact on the overall situation of the mechanism.

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