Fourier Analysis of Ghost Imaging

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Fourier analysis of ghost imaging (FAGI) is proposed in this paper to analyze the properties of ghost imaging with thermal light sources. This new theory is compatible with the general correlation theory of intensity fluctuation and could explain some amazed phenomena. Furthermore we design a series of experiments to testify the new theory and investigate the inherent properties of ghost imaging.

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Conventionally, coherent, at least partially coherent, illumination is required to image the phase detail of an object [1], and the theoretical limit of the resolution is determined by the wavelength \( \lambda \) of the light beam. Since the coherent sources in the ultra-violation and even shorter frequency regions are still not existed, we could not obtain a higher resolution by using a coherent light with a shorter wavelength. In 1994, Belinsky and Klyshko [2] found that, by exploiting the spatial correlation between two entangled photons generated by parametric down conversion (PDC), ghost imaging could be realized with entangled incoherent light. Due to the possibility to improve the resolution, ghost imaging is bestowed with great potential in quantum metrology, quantum holography and quantum lithography, etc.

The nature of ghost imaging leads to many interesting debates about the necessity of the entanglement [3-14]. Presently, the idea that both classical thermal sources and quantum entangled beams can be used for ghost imaging and ghost diffraction is widely accepted.

In order to realize practical use of the ghost imaging, further investigation about the properties of ghost imaging is stringent. Gati et al. compared the visibility in the classical and quantum regimes, and pointed that the visibility of classical regime increases as the mean photon number of per mode in thermal state increases [9]. D’Angelo et al. compared the resolution of quantum and classical ghost imaging [15]. Meanwhile, Ferri et al. realized the high-solution of ghost imaging and ghost diffraction with thermal light [16], and they even tried ghost imaging with homodyne detection [17]. Bache et al. analyzed the spatial average technique which would make the imaging bandwidth of the reconstructed diffraction pattern virtually infinite and the correlation convergence rate faster [18]. Cheng et al. have presented theoretical analysis on the noise of ghost imaging with entangled photons [19] and pseudo-thermal light fields [20].

Most recently our group demonstrated lensless Fourier-transform ghost imaging with both amplitude only and phase only object by using classical incoherent light [21].

There are still some questions unresolved, such as what limits the complexity of the object, why the visibility decreases as the transmission increases. In this paper we propose a new theory, Fourier analysis of ghost imaging (FAGI), to answer these questions.

For an ideal infinite and uniform thermal source located at \( z = 0 \), the field at a point in the source is presented by \( U_0(x_0, y_0, t) = \int \int_{-\infty}^{\infty} dk_x dk_y \exp[i(k_x x_0 + k_y y_0 - \omega t)] \) . When \( k_x^2 + k_y^2 \geq (\frac{\omega}{2 \pi})^2 \), \( k_x \) is imaginary, and the amplitude of this evanescent component decays exponentially as the distance \( z \) increases, so the integral region can be confined in \( k_i \in (-k, k), i \in (x, y) \). Since the sources used in experiments are always finite, which means \( k_x \) and \( k_y \) are not uniformly distributed, the typical Gaussian distribution is adopted. For simplicity \( k_x \) is assumed to be independent of \( k_y \), and the amplitude \( A \) is a time variant and random distributed, so the field is written as \( U_0(x_0, y_0, t) = \int \int_{-\infty}^{\infty} dk_x dk_y \exp(-\frac{k_x^2 + k_y^2}{\sigma^2}) A_k(t) \exp[i(k_x x_0 + k_y y_0)] \).

Now we analyze the paradigmatic example of the imaging system given in Ref.[22], except that the source is replaced by a thermal source. Here the setup of the object arm is an f-f system with the object close to the beam splitter (BS). For simplicity the output-plane of the BS is selected as the source, the fields on these two planes are shown as

\[
U(x_0, y_0, t) = \frac{1}{2} \int \int_{-\infty}^{\infty} dk_x dk_y \exp(-\frac{k_x^2 + k_y^2}{\sigma^2}) A_k(t) \exp[i(k_x x_0 + k_y y_0)] \tag{1}
\]

\[
V(x_0', y_0', t) = \frac{1}{2} \int \int_{-\infty}^{\infty} dk_x dk_y \exp(-\frac{k_x^2 + k_y^2}{\sigma^2}) A_k(t) \exp[i(k_x x_0 + k_y y_0)] \tag{2}
\]

and the object transmitting function is \( t(x_0, y_0) \). Neglecting the finite pupil of the lens, through the Fourier Transform of the lens, the spatial frequency distribution
imaging can be realized with only a pixel detector in the illumination case, it is easy to understand why the ghost pairing with a single frequency component in the coherent scene of the thermal source, a fixed pixel detector collects a certain range of spatial frequency components. Comparing with a single frequency component in the coherent illumination case, it is easy to understand why the ghost imaging can be realized with only a pixel detector in the object arm.

In this system, the reference arm is a 2f-2f setup, and the transmitting function \( h(x_f, y_f, x_0, y_0) = \delta(x_0 - x_0', y_0 - y_0') \), so the field distribution on the detector plane is

\[
U(x_f, y_f, t) = \frac{1}{2} \int_{-\infty}^{\infty} dk_x dk_y \exp(-\frac{k_x^2 + k_y^2}{\sigma^2})
A_k(t - \frac{2f}{c}) T[(\frac{2\pi}{\lambda_f} x_f - k_x), (\frac{2\pi}{\lambda_f} y_f - k_y)]
\]

(3)

Here, the constant phase, which has no sensible influence on the result is neglected. Because of the incoherence of the thermal source, a fixed pixel detector collects a certain range of spatial frequency components.

The presentation of the Gaussian spreading function contributes a background to the object image, which explains the occurrence of the inherent noise in the second order intensity fluctuation correlation. From this equation it easy to deduce that when the transmission area increases, the background increases while the visibility decreases.

\[
\Gamma(x_f, y_f, x_2f, y_2f) = \frac{1}{4} \int_{-\infty}^{\infty} dk_x dk_y \langle A_k(t - \frac{2f}{c}) A_k(t - \frac{4f}{c}) \rangle \exp(-2\frac{k_x^2 + k_y^2}{\sigma^2}) T[(\frac{2\pi}{\lambda_f} x_f - k_x), (\frac{2\pi}{\lambda_f} y_f - k_y)] \exp[-i(k_x x_2f + k_y y_2f)]
\]

(4)

The intensity fluctuation correlation function is \( G(x_1, y_1, x_2, y_2) = |\Gamma(x_1, y_1, x_2, y_2)|^2 \), and the second order correlation function \( \Gamma(x_f, y_f, x_2f, y_2f) = \langle U(x_f, y_f, t)V^*(x_2f, y_2f, t) \rangle \). On the assumption of the independence of the amplitude of different wave vectors, \( \langle A_k(t - \frac{2f}{c}) A_k(t - \frac{4f}{c}) \rangle = 0 \), when \( k \neq k' \), the second order correlation function is simplified as

\[
\Gamma(0, 0, x_2f, y_2f) = \frac{1}{4} I \int_{-\infty}^{\infty} dk_x dk_y \exp(-2\frac{k_x^2 + k_y^2}{\sigma^2}) T(-k_x, -k_y) \exp[-i(k_x x_2f + k_y y_2f)]
\]

(5)

Within the coherent time, \( \langle A_k(t - \frac{2f}{c}) A_k(t - \frac{4f}{c}) \rangle \) is a const. When \( x_f = 0, y_f = 0 \), eq.(5) is presented as:

\[
\Gamma(0, 0, x_2f, y_2f) = \frac{\sigma^2 I}{16} \exp(-\frac{\sigma^2 x_2f^2 + y_2f^2}{8})
\]

(6)

this equation is a Fourier Transform, where the object spectrum multiplies a Gaussian function, which causes a increasing loss for high frequencies. According to the properties of Fourier Transform, a convolution of the object image and a Gaussian function is obtained:

\[
\Gamma(0, 0, x_2f, y_2f) \propto \exp[-i\frac{2\pi}{\lambda_f}(x_2f y_2f)] \Gamma(0, 0, x_2f, y_2f)
\]

(7)

When \( x_f \neq 0, y_f \neq 0 \), the second order correlation function is

\[
\Gamma(x_f, y_f, x_2f, y_2f) \propto \exp[-i\frac{2\pi}{\lambda_f}(x_2f y_2f)] \Gamma(0, 0, x_2f, y_2f)
\]

(8)

If the pixel detector is replaced by a bucket detector
in the object arm, the corresponding result will be

\[
\int \int dx_f dy_f \Gamma(x_f, x_{2f}, y_{2f}) = \int \int dx_f dy_f \exp\left[-\frac{2\pi}{\lambda f}(x_f x_{2f} + y_f y_{2f})\right] \Gamma(0, 0, x_{2f}, y_{2f})
\]

(9)

FIG. 1: (a) the image of different objects and (b) the corresponding spatial frequency spectrum. (i) two slits, the width is 300 µm, and the distance is 900 µm. (ii) four slits, width 150 µm, and distance 450 µm. (iii) six slits, width 100 µm, and distance 300 µm.

Similar results can be obtained with only coefficient difference.

We have designed several experiments to demonstrate these deductions. Since the source we used is a pseudo thermal source, the best resolution is limited by the coherence length \( l_c \), simply by replacing \( \lambda \) with \( l_c \), all results obtained in previous discussions are still correct. In the experiment, the coherence length of the source is \( 75 \) µm.

The first experiment is to investigate the frequency response of ghost imaging. The objects have the same duty cycle but different periods, and we get the image and the corresponding spatial frequency distribution of each object.

In Fig.1, the black bold line in each (a) is the object, the disperse squares are the normalized intensity fluctuation correlation, and the fitting curve is the image. Each (b) is the corresponding spatial frequency distribution of (a).

In this experiment we choose the ratio of the slit width and the distance \( a : d = 1 : 3 \) to make sure the image reconstructed by low frequency has no direct current background. Moreover, comparing the first-order component of the image to the zero-order component of the object in Fig.1 (i), the ratio is bigger than 1/2, which implies no direct current background of zero-order component, so the background noise is the contribution of the convolution of Gaussian spread function. As the frequency increases, the ratio of image first-order component to object first-order component decreases, so the effective information decreases. Results are shown in Table.1 and Fig.2.

In Fig.2 the frequency response curve has an approximate Gaussian form, as given in the theoretical result Eq.(6).

In the following experiment we demonstrate the rela-

| CSL * | FFC * | RFR * |
|-------|-------|-------|
| infinity | 0 | 1 |
| 300 | 0.0625 | 0.64 |
| 150 | 0.125 | 0.449 |
| 100 | 0.1875 | 0.388 |
| 75 | 0.25 | 0 |

* Character Spatial Length.
* Frequency of First-order Component.
* Relative Frequency Response.
FIG. 3: The experiment result of objects with different transmission area. (a) the image of two-slit, (b) four-slit and (c) six-slit.

The transmission area is large enough, the image is submerged in the inherent noise. Here we design a two-slit and its reverse, the objects are shown in Fig. 4.

In experiment, the object (a) is easy to be imaged, but under the same condition, its reverse (b) can not be imaged.

From the above experiment results, we conclude that the Fourier analysis theory presented in this paper can interpret the principle of ghost imaging in different systems, and resolve some difficulties of intensity correlation function.

In summary, we propose a new theory to clarify the principle of ghost imaging, which is compatible with the generally used intensity fluctuation correlation theory, and can be used to explain the phenomena we realized in experiments. Moreover, the analysis of ghost imaging with a classical thermal source can be generalized to the case of an entangled source, which remains for the future work.

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