Gravitational Waves and the Scale of Inflation

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Does a measurement of $r$ uniquely fix $H_{\text{inf}}$?

Yes, if tensor modes are in vacuum during inflation:

$$\gamma_s k \gamma_s' k'_{\text{vac}} = (2\pi)^3 \delta^3(k + k') \delta_{ss'} k^3 H_{\text{inf}}^2 M_{\text{pl}}^2$$

What if $\gamma$ is not in vacuum?
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$$\langle \gamma_s^k \gamma_s'^{k'} \rangle_{\text{vac}} = (2\pi)^3 \delta^3(k + k') \delta^{ss'} \frac{1}{k^3} \frac{H_{\text{inf}}^2}{M_{\text{pl}}^2}$$

What if $\gamma$ is not in vacuum?
Gravitational Waves in Solar System

Jupiter
Gravitational Waves in Solar System

Jupiter

Bremsstrahlung at center of Sun (Weinberg '65).
Examples of Tensor Emission During Inflation

1) Particle Production:

\[ M_X^2 = M^2 \sin^2 \frac{\phi}{f} \]

Scattering of \( X \) particles emits gravity waves \( \gamma_X \).  
Senatore et.al '11

2) Pseudo-scalar Inflaton:

\[ \mathcal{L}_{\phi A} = \frac{\alpha}{f} \phi F \tilde{F} \]

Growing helical gauge field \( A \) excites the metric.  
Sorbo '11, Barnaby et.al. '12, Mukohyama et.al. '14
Can $\gamma_X$ be Larger than $\gamma_{\text{vac}}$?
\[ \gamma_X > \gamma_{\text{vac}}? \]

1. Available energy density:

\[ \frac{1}{2} \dot{\phi}^2 = M_{\text{pl}}^2 H^2 \epsilon \]

2. The energy in the auxiliary sector

\[ \rho_X \ll M_{\text{pl}}^2 H^2 \epsilon \]

3. Estimate emission by \( \partial^2 \gamma \sim \rho_X / M_{\text{pl}}^2 \) at frequency \( \omega \sim H \)

\[ \gamma_X \sim \frac{\rho_X}{M_{\text{pl}}^2 H^2} \ll \epsilon \]
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There is a lot of room to outperform vacuum

\[ \gamma_{\text{vac}} \sim \frac{H}{M_{\text{pl}}} \ll \gamma_X \ll \epsilon. \]
Punch Line

Scalars are emitted during energy transfer:

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\[
\frac{1}{2} \phi^2 \rightarrow \rho_X \implies \delta \phi_X
\]

Large tensor emission \textit{generically} leads to \textbf{Very Large} scalar emission

\[
\gamma_X > \gamma_{\text{vac}} \implies \delta \phi_X \gg \delta \phi_{\text{vac}}
\]

\[
 r_{\text{max}} \sim \epsilon^2
\]
1. Exponential Expansion

Suppose emission is at a physical frequency \( k/a \sim \omega_{\text{phys}} \).

1.a. Then each mode can be excited in a period

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1.b. The waves redshift before horizon crossing $\gamma_{\text{freeze-out}} = \frac{H}{\omega} \gamma_{\omega}$. It also takes more energy to excite $\gamma_{\omega}$ at higher $\omega$. With fixed energy per Hubble volume:

$$N_\gamma \sim \frac{E_\gamma H^3}{\omega^4}.$$
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Most efficient emission is at \( \omega \sim H \)
2. Weak Gravity

Tensor emission is governed by linearized Einstein equation

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2.a. \( N_X \) Incoherent Localized Events of mass \( M_X \) per Hubble volume:

\[ \langle \gamma_X^2 \rangle \sim N_X \frac{M_X^2}{M_{\text{pl}}^2} \frac{H^2}{M_{\text{pl}}^2} \]

Comment 1) This is an upper bound. Comment 2) This can exceed \( \langle \gamma^2 \rangle_{\text{vac}} \).
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2.b. Coherent emission by extended configurations (e.g. gauge field \( A \)):

\[ N_X \rightarrow \text{Number of Species} \]

\[ M_X \rightarrow \text{Energy} / \text{Hubble volume} \]
3. Scalar Emission from Energy Conservation

Scalar fluctuations $\delta \phi_X$ lead to: $\delta \rho_\phi = \dot{\phi} \delta \dot{\phi}_X$.

Energy conservation: $\int d^3x \delta \rho_\phi = M_X \implies \delta \phi_X \propto \frac{M_X}{\dot{\phi}}$
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Converting to $\zeta_X^2 \sim N_X(H\delta\phi_X/\dot{\phi})^2$ results in:

$$\langle \zeta_X^2 \rangle \sim N_X \frac{M_X^2}{\epsilon M_{pl}^2} \frac{H^2}{\epsilon M_{pl}^2}$$

Comparison with $\gamma_X$ gives $r_{max} \sim \epsilon^2$. 
3. Scalar Emission from Energy Conservation

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In the concrete example of localized incoherent events

$$r_{\text{max}} \sim 0.3 \epsilon^2$$
These scenarios can dominate vacuum tensor fluctuations and break the relation between $r$ and $H_{\text{inf}}$, but then they dominate vacuum scalar fluctuations and $\epsilon$ must be relatively large for observable values of $r$.

However, scalar and tensor tilts are less sensitive to $\epsilon$:

\begin{align*}
    n_s - 1 &= -\frac{1}{2} \epsilon - \frac{5}{4} \frac{\dot{\epsilon}}{H \epsilon} \\
    n_t &= -\frac{1}{2} \epsilon - \frac{3}{4} \frac{\dot{\epsilon}}{H \epsilon}
\end{align*}
Non-Gaussianity

If there are $N_X$ incoherent emission events per $H^{-4}$:

$$\langle \zeta_X^3 \rangle = \frac{\zeta_X^3}{f_{NL}} \approx \frac{1}{N_X^{1/2}}$$

$f_{NL}$ can be made small by increasing $N_X$. 
Non-Gaussianity

If there are $N_X$ incoherent emission events per $H^{-4}$:

$$\frac{\langle \zeta_X^3 \rangle}{\zeta_X^3} = f_{NL} \zeta_X \sim \frac{1}{N_X^{1/2}}$$

$f_{NL}$ can be made small by increasing $N_X$. But there is an upper bound

$$\rho_X = N_X H^3 M_X \ll M_{pl}^2 H^2 \epsilon$$

Combined with $\zeta_X$ gives

$$f_{NL} \gg 1$$

Away from the squeezed limit:

$$B(k_1, k_2, k_3) \propto \frac{1}{k_1^2 k_2^2 k_3^2}$$
Conclusions

1. It is possible to have $\gamma_X \gg \gamma_{\text{vac}}$.

2. Then $\zeta_X \gg \zeta_{\text{vac}}$ such that $r \sim \epsilon^2$. Hence detectable $r$ requires relatively large $\epsilon$.

3. Tensor consistency condition $r = -2n_t$ is violated.

4. There is large non-Gaussianity $f_{NL} \gg 1$.

5. Generically, the same bound applies to multi-field models, but models in which scalar emission is suppressed can be built.
Thank you!
Exception– A two-field scenario

Consider a two filed inflationary model with both fields $\phi$ and $\psi$ slow-rolling. Suppose the energy source for the auxiliary sector $X$ is $\psi$:

$$M_X^2 = M^2 \sin^2 \frac{\psi}{f}.$$  

Energy transfer from $\frac{1}{2}\dot{\psi}^2$ to $X$ sector leads to $\delta \psi$ emission. If

$$\frac{\partial \zeta}{\partial \psi} \ll \frac{H \dot{\psi}}{\dot{\phi}^2 + \dot{\psi}^2}$$

then the contribution to scalar spectrum can be made small. This seems non-generic, but can be realized for instance if the re-heating surface is determined by $\phi$:

$$V(\phi, \psi) = \theta(\phi - \phi_0)U(\phi, \psi).$$