Next-to-leading order QCD corrections to differential distributions of Higgs boson production in hadron-hadron collisions

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Abstract

We present the full next-to-leading order corrected differential distributions $d^2\sigma/dp_Tdy$, $d\sigma/dp_T$ and $d\sigma/dy$ for the semi-inclusive process $p+p\rightarrow H+^{'}X'$. Here $X$ denotes the inclusive hadronic state and $p_T$ and $y$ are the transverse momentum and rapidity of the Higgs-boson $H$ respectively. All QCD partonic subprocesses have been included. The computation is carried out in the limit that the top-quark mass $m_t\rightarrow\infty$ which is a very good approximation as long as $m_H, p_T < 200$ GeV. Our calculations reveal that the dominant subprocess is given by $g+g\rightarrow H+^{'}X'$ but the reaction $g+q(\bar{q})\rightarrow H+^{'}X'$ is not negligible. Another feature is that the $K$-factor representing the ratio between the next-to-leading order and leading order

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differential distributions is large. It varies from 1.4 to 1.7 depending on the
kinematic region and choice of parton densities. We show that a reliable
determination of the differential cross sections requires good knowledge of
the gluon density in the region where $x < 10^{-3}$. Further we study whether
the differential distributions are dominated at large transverse momentum by
soft-plus-virtual gluon contributions. This is of interest for the resummation
of large corrections which occur near the boundary of phase space. We also
compare our results with those previously reported in the literature.

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1 Introduction

The Higgs boson, which is the corner stone of the standard model, is the only particle which has not been discovered yet. Its discovery or its absence will shed light on the mechanism how particles acquire mass as well as answer questions about supersymmetric extensions of the standard model or about compositeness of the existing particles and the Higgs boson. The LEP experiments [1] give a lower mass limit of about $m_H \sim 114$ GeV/c$^2$ and fits to the data using precision calculations in the electro-weak sector of the standard model indicate an upper limit $m_H < 200$ GeV/c$^2$ with 95 % confidence level. After the end of the LEP program the search for the Higgs will be continued at hadron colliders in particular at the TEVATRON and the LHC. If the Higgs mass is in the above range the principal production mechanisms at hadron colliders are gluon-gluon fusion $g + g \rightarrow H + X'$ or $W^+W^-$-fusion appearing in the reaction $q + \bar{q} \rightarrow q + \bar{q} + H + X'$. In these processes 'X' denotes an inclusive hadronic state. For $m_H < 135$ GeV/c$^2$ the process $q + \bar{q} \rightarrow V \rightarrow H + V + X'$ with $V = \gamma, Z, W$ has also to be taken into consideration. The Higgs boson will be observed via its decay products among which the channels $H \rightarrow b + \bar{b}$ and $H \rightarrow \gamma + \gamma$ are the most prominent ones although $H \rightarrow \tau^+ + \tau^-$ should also be considered. However the large backgrounds make the observation of these decays very difficult and it will take a lot of experimental and theoretical effort to detect the Higgs boson provided it is there.

In this paper we concentrate on Higgs production channels where the lowest order reaction proceeds via the gluon-gluon fusion mechanism. In the standard model the Higgs boson couples to the gluons via heavy quark loops among which the top-quark loop is the most prominent since the coupling of the Higgs to a fermion loop is proportional to the mass of the fermion (for a review see [2]). In lowest order (LO) the gluon-gluon fusion process $g + g \rightarrow H$, represented by the top-quark triangle graph, was computed in [3]. The next-to-leading (NLO) processes given by gluon bremsstrahlung $g + g \rightarrow g + H$ and $g + q(\bar{q}) \rightarrow q(\bar{q}) + H$ were presented in [4], [5] and [6] from which one can derive the transverse momentum ($p_T$) and rapidity ($y$) distributions of the Higgs boson. The total integrated cross section, which also involves the computation of the QCD corrections to the top-quark loop, has been calculated in [7]. This calculation is rather cumbersome since it involves the computation of two-loop triangular graphs with massive quarks. Recently...
also the next-to-next-to-leading (NNLO) processes involving all two-to-three parton processes have been computed in [8] using the helicity method which means that the matrix elements are presented in four dimensions. From the experience gained from the NLO corrections it is clear that it will be very difficult to obtain the NNLO one-particle inclusive distributions from these calculations let alone the total cross sections.

Fortunately one can simplify the calculations if one takes the large top-quark mass limit $m_t \to \infty$. In this case the Feynman graphs are obtained from an effective Lagrangian describing the direct coupling of the Higgs boson to the gluons. The LO and NLO contributions to the total cross section in this approximation were computed in [9] and they found that the error introduced by taking the $m_t \to \infty$ limit is less than about 5% provided $m_H \leq 2 m_t$. A similar investigation was done for the differential distributions of the processes $g + g \to g + H$ and $g + q(\bar{q}) \to q(\bar{q}) + H$ in [10]. Here the approximation is valid as long as $m_H$ and $p_T$ are smaller than $m_t$. This is corroborated by the recent calculations in [8] which show that for $m_H = 120 \text{ GeV}/c^2$ the approximation is valid for jets with transverse momenta smaller than 200 GeV/c. The NNLO matrix elements using the effective Lagrangian were computed in [11], [12] albeit in four dimensions. The one-loop corrections to the two-to-two parton subprocesses were presented in [13], where the computation of the loop integrals was performed in $n$-dimensions but the matrix element was still presented in four dimensions. The results in [11], [12] and [13] were used to compute the transverse momentum and rapidity distributions of the Higgs boson up to NLO [14]. The effective Lagrangian method was also applied to obtain the NNLO total cross section by the calculation of the two-loop corrections to the Higgs-gluon-gluon vertex in [15], the soft-plus-virtual gluon corrections in [16] and the computation of the two to three body processes in [17].

In this paper we present the full NLO computation of the double differential distributions $d^2\sigma/dp_T/dy$ for Higgs boson production in hadron-hadron collisions using the gluon-gluon fusion mechanism in the $m_t \to \infty$ approximation. Here we have included all partonic subprocesses. A similar calculation has been performed in [14] but our approach differs from it in various aspects. First our calculation is purely analytical and follows the calculation carried out for the Drell-Yan process describing vector boson production in hadron-hadron collisions (see [18]). The approach in [14] was mainly numerical and based on the methods explained in [19]. Moreover it used the
two-to-three particle matrix elements in [11], [12] and the two-to-two particle matrix elements including virtual corrections in [13] which were all presented in four dimensions. In our calculation the matrix elements as well as the loop integrals and phase space integrals are computed in $n$ dimensions. The advantage of the analytical approach is that one gets more insight into the structure of the radiative corrections. This is particularly important for the large corrections, due to soft gluon radiation and collinear fermion pair production, which arise near the boundary of phase space, where the $p_T$ of the Higgs boson gets large. Resummation of this type of corrections has been carried out for the total cross section in [20]. Resummation of small $p_T$ contributions due to the Sudakov effect has been done in [21]. In view of the experimental problems to observe the Higgs boson, a recalculation of all the NLO corrections is necessary to be sure that the theoretical predictions are correct. We find that the contribution due to the (anti-) quark gluon subprocess is substantial in particular at large transverse momentum and that the important region of $x$ in the gluon density is $x < 10^{-3}$. We also investigate the region of applicability of the soft-plus-virtual (S+V) approximation for the calculation of the differential cross sections. Finally we mention that another paper has just appeared on the NLO corrections to the $g + g \rightarrow H + g$ channel, using the helicity framework [22].

Our paper will be organized as follows. In section 2 we give an outline of the kinematics and present the Born contributions. In section 3 we present the virtual contributions. In section 4 the gluon bremsstrahlung corrections to the Born reactions are computed and the soft gluon cross sections are explicitly shown. In section 5 we show how the mass factorization is carried out. In section 6 we give differential cross sections for proton-proton collisions at the LHC and make comparisons with results obtained earlier in the literature. Some particular expressions are given in appendices A and B. Other formulae, in particular those for the two-to-three body cross sections, are too long to be published. They are available upon request as files written in the algebraic manipulation program FORM [23].
2 Lowest order contributions to Higgs production

In the large top-quark mass limit the Feynman rules (see e.g. [12]) can be derived from the following effective Lagrangian density

\[ \mathcal{L}_{\text{eff}} = G \Phi(x) O(x) \quad \text{with} \quad O(x) = -\frac{1}{4} G_{\mu\nu}(x) G^{\mu\nu}(x) \quad (2.1) \]

where \( \Phi(x) \) represents the Higgs field and \( G \) is an effective coupling constant given by

\[ G^2 = 4 \sqrt{2} \left( \frac{\alpha_s(\mu_r^2)}{4\pi} \right)^2 G_F \tau^2 F^2(\tau) C^2 \left( \alpha_s(\mu_r^2), \frac{\mu_r^2}{m_t^2} \right) . \quad (2.2) \]

In the expression above \( m \) and \( m_t \) denote the masses of the Higgs boson and the top quark respectively. The running coupling is given by \( \alpha_s(\mu_r^2) \) where \( \mu_r \) denotes the renormalization scale and \( G_F \) is the Fermi constant. Further \( C \) is the coefficient function which originates from the QCD corrections to the top-quark triangle graph describing the process \( H \to g + g \) in the limit \( m_t \to \infty \). On the Born level the width of this decay process is given by

\[ \Gamma(H \to g + g) = \frac{(N^2 - 1) m^3 G_F}{128 \pi^3 \sqrt{2}} \alpha_s^2(\mu_r^2) \tau^2 F^2(\tau) , \quad (2.3) \]

where \( N \) denotes the number of colours. Further the function \( F(\tau) \) occurring in Eqs. (2.2), (2.3) is defined by

\[ F(\tau) = 1 + (1 - \tau) f(\tau) , \quad \tau = \frac{4 m_t^2}{m^2} , \]

\[ f(\tau) = \frac{1}{\sqrt{\tau}} \left( \ln \frac{1 - \sqrt{1 - \tau}}{1 + \sqrt{1 - \tau}} + \frac{\pi i}{2} \right)^2 \quad \text{for} \quad \tau < 1 . \quad (2.4) \]

In the large \( m_t \)-limit \( F(\tau) \) behaves as

\[ \lim_{\tau \to \infty} F(\tau) = \frac{2}{3 \tau} . \quad (2.5) \]
The coupling $G$ in Eq. (2.2) is presented for general $m_t$ on the Born level only whereas $C$ is computed in higher order for $m_t \to \infty$. In order to keep some part of the top quark mass dependence we take for $G$ the expression in Eq. (2.2). This is an approximation because the gluons which couple to the Higgs boson via the top-quark loop in the partonic processes describing Higgs-production are often virtual. The virtual-gluon momentum dependence is neither described by $F(\tau)$ in Eq. (2.4) nor by $C$. For on-mass-shell gluons the latter quantity has been computed in the large top-quark mass limit up to order $\alpha_s$ in [7], [9], [11] and up to $\alpha_s^2$ in [20], [24]. In second order it reads

$$C \left( \alpha_s(\mu^2), \frac{\mu^2}{m_t^2} \right) = 1 + \frac{\alpha_s^{(5)}(\mu^2)}{4\pi} \left( 11 + \left( \frac{\alpha_s^{(5)}(\mu^2)}{4\pi} \right)^2 \left[ \frac{2777}{18} + 19 \ln \frac{\mu^2}{m_t^2} \right] 
+ n_f \left( -\frac{67}{6} + \frac{8}{3} \ln \frac{\mu^2}{m_t^2} \right) \right). \quad (2.6)$$

Here $n_f$ denotes the number of light flavours and $\alpha_s^{(5)}$ is presented in a five flavour number scheme.

In this paper we study the semi-inclusive reaction with one Higgs-boson $H$ in the final state. It will be denoted by

$$H_1(P_1) + H_2(P_2) \to H(-p_5) + X', \quad (2.7)$$

where $H_1$ and $H_2$ denote the incoming hadrons and $X$ represents an inclusive hadronic final state. In lowest order the partonic reactions contributing to Eq. (2.7) are denoted by

$$a(p_1) + b(p_2) \to c(-p_3) + H(-p_5), \quad p_1 + p_2 + p_3 + p_5 = 0,$$

$$a, b, c = q, \bar{q}, g, \quad (2.8)$$

and the partonic cross sections in $n$-dimensions are given by

$$\sigma_{ab\to c H}^{(1)} = K_{ab} G^2 g^2 \mu^{4-n} \frac{1}{2s} \int \frac{d^n p_3}{(2\pi)^{n-1}} \int \frac{d^n p_5}{(2\pi)^{n-1}} \delta^+(p_3^2) \delta^+(p_5^2 - m^2) \times |M_{ab\to c H}^{(1)}(\theta_1)|^2. \quad (2.9)$$

Here $M^{(1)}$ denotes the amplitude of the process and the strong coupling constant is given by $g$ with $g^2 = 4\pi \alpha_s$. The scale $\mu$ originates from the fact that
the coupling constant acquires a dimension in \( n \) dimensions. The quantity \( K_{ab} \) represents the spin and colour average over the initial states including the statistical factor \( 1/m! \) if one integrates over \( m \) identical particles in the final state. Further we have assumed that the Higgs boson is mainly produced on-mass-shell. However one can also use the narrow width approximation. This can be achieved by replacing

\[
\delta(p_5^2 - m^2) \rightarrow \frac{1}{\pi} \frac{m \Gamma}{(p_5^2 - m^2)^2 + m^2 \Gamma^2},
\]

(2.10)

where \( \Gamma \) is the total width of the Higgs boson which is dominated by the decay \( H \rightarrow g + g \). Choosing the C.M. frame of the incoming partons we have the following parametrization

\[
p_1 = \frac{1}{2} \sqrt{P_{12}} (1, 0, \cdots, 0, 1),
\]

\[
p_2 = \frac{1}{2} \sqrt{P_{12}} (1, 0, \cdots, 0, -1),
\]

\[
-p_3 = \frac{P_{12} - m^2}{2 \sqrt{P_{12}}} (1, \cdots, -\sin \theta_1, -\cos \theta_1),
\]

\[
-p_5 = \frac{1}{2 \sqrt{P_{12}}} (P_{12} + m^2, \cdots, (P_{12} - m^2) \sin \theta_1, (P_{12} - m^2) \cos \theta_1),
\]

(2.11)

with

\[
P_{ij} = (p_i + p_j)^2, \quad P_{12} = s, \quad P_{15} = t, \quad P_{25} = u, \quad s + t + u = m^2,
\]

\[
\cos \theta_1 = \frac{t - u}{s - m^2}.
\]

(2.12)

From Eq. (2.9) we infer that

\[
s^2 d^2 \sigma_{ab \rightarrow c H}^{(1)} \frac{d^2}{d t d u} = K_{ab} G^2 \frac{\pi S_{\varepsilon}}{\Gamma(1 + \varepsilon/2)} \frac{\alpha_s(\mu_r^2)}{\mu_r^2} \left( \frac{t u}{\mu_r^2 s} \right)^{\varepsilon/2} \delta(s + t + u - m^2)
\]

\[
\times |M_{ab \rightarrow c H}^{(1)}|^2, \quad \text{with} \quad n = 4 + \varepsilon.
\]

(2.13)
The spherical factor $S_\varepsilon$ is defined by

$$S_\varepsilon = \exp\left(\frac{\varepsilon}{2}(\gamma_E - \ln 4\pi)\right).$$

On the Born level we have the following subprocesses

$g + g \rightarrow g + H,$

$$|M_{gg\rightarrow gH}^{(1)}|^2 = N(N^2 - 1) \frac{1}{stu} \left[ \left(1 + \frac{\varepsilon}{2}\right) \left\{ s^4 + t^4 + u^4 + m^8 \right\} + 2 \varepsilon m^2 stu \right],$$

$$K_{gg} = \frac{1}{4(1 + \varepsilon/2)^2} \frac{1}{(N^2 - 1)^2},$$

$q + \bar{q} \rightarrow g + H,$

$$|M_{q\bar{q}\rightarrow gH}^{(1)}|^2 = C_A C_F \frac{1}{s} \left[ \left(1 + \frac{\varepsilon}{2}\right) \left\{ t^2 + u^2 \right\} + \varepsilon tu \right],$$

$$K_{q\bar{q}} = \frac{1}{4 N^2},$$

$q(\bar{q}) + g \rightarrow q(\bar{q}) + H,$

$$|M_{qg\rightarrow qH}^{(1)}|^2 = C_A C_F \frac{1}{u} \left[ - \left(1 + \frac{\varepsilon}{2}\right) \left\{ s^2 + t^2 \right\} - \varepsilon st \right],$$

$$K_{qg} = \frac{1}{4(1 + \varepsilon/2)} \frac{1}{N(N^2 - 1)},$$

with

$$C_A = N, \quad C_F = \frac{N^2 - 1}{2N}. \tag{2.24}$$

The $n$-dimensional matrix element $|M_{gg\rightarrow gH}^{(1)}|^2$ is proportional to $n - 2$ provided $m = 0$ so that it vanishes in two dimensions. The same feature also
appears in the lowest order matrix element for the decay $H \rightarrow g + g$ derived from the effective Lagrangian in Eq. (2.1). Notice that the factor $n - 2 = 2(1 + \varepsilon/2)$ also shows up in the spin average quantities $K_{ab}$. 
3 One-loop corrections to the $2 \to 2$-body reactions

The one-loop corrections to the gluon-gluon subprocess in Eq. (2.15) entails the computation of forty-two graphs. Fourteen graphs lead to independent expressions whereas the remaining ones can be obtained via crossing from the $s$- to $t$-channel or from the $s$- to $u$-channel. The Feynman integrals show ultraviolet, infrared and collinear singularities which will be regularized using $n$-dimensional regularization. Hence the matrix element and the Feynman integrals have to be computed in $n$ dimensions for which we used the algebraic manipulation program FORM (version 3.0) [23]. Since the Feynman integrals also contain the integration momentum in the numerator we have to apply tensorial reduction. Here we followed the procedure in [25] which was extended to $n$-dimensions in [26]. The scalar integrals for the two-, three- and four-point functions can be found in [27]. The one-loop correction to the gluon-gluon differential distribution (2.15) can be written as

\[
\frac{s^2}{d^2} \delta_{Virt}^gg \to gH \propto \pi \delta(s + t + u - m^2) S^2 \varepsilon G^2 \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2 \frac{1}{(N^2 - 1)^2}
\]

\[
\times \left[ N \left\{ -\frac{6}{\varepsilon^2} + \left( 2 \ln \frac{t}{\mu^2} - 4 \ln \frac{-t}{\mu^2} - 4 \ln \frac{-u}{\mu^2} + 6 \right) \frac{1}{\varepsilon} \right. 
\]

\[
+ \text{Li}_2 \left( \frac{t}{m^2} \right) + \text{Li}_2 \left( \frac{u}{m^2} \right) + \text{Li}_2 \left( \frac{s - m^2}{s} \right) 
\]

\[
-3 \ln \frac{-t}{\mu^2} \ln \frac{-u}{\mu^2} + \ln \frac{-t}{\mu^2} \ln \frac{s}{\mu^2} + \ln \frac{-u}{\mu^2} \ln \frac{s}{\mu^2} - \ln^2 \frac{-t}{\mu^2} 
\]

\[
- \ln^2 \frac{-u}{\mu^2} - \frac{1}{2} \ln^2 \left( \frac{t - m^2}{t} \right) - \frac{1}{2} \ln^2 \left( \frac{u - m^2}{u} \right) 
\]

\[
+ \frac{1}{2} \ln^2 \left( \frac{m^2 - t}{m^2} \right) + \frac{1}{2} \ln^2 \left( \frac{m^2 - u}{m^2} \right) - 2 \ln \frac{s}{\mu^2} 
\]

\[
+ 4 \ln \frac{-t}{\mu^2} + 4 \ln \frac{-u}{\mu^2} + \frac{11}{2} \zeta(2) - \frac{9}{2} \right] |M_{gg\to gH}|^2
\]
\[ + \left( N - n_f \right) \left\{ \frac{1}{4} \right\} |MB_{gg\rightarrow H}^{(1)}|^2, \]  
(3.1)

\[ |MB_{gg\rightarrow H}^{(1)}|^2 = \frac{2}{3} N (N^2 - 1) \frac{m^2}{s \ t \ u} \left[ s \ t \ u + m^2 \ (s \ t + s \ u + t \ u) \right]. \]  
(3.2)

Here the pole terms \( \frac{1}{\varepsilon^k}, \ k = 1, 2 \) (see Eq. (2.13)) represent the three types of divergences mentioned above and \( \text{Li}_2(x) \) denotes the dilogarithmic function defined in [28]. The virtual corrections to the quark-anti-quark reaction in Eq. (2.18) require the computation of fourteen graphs. The procedure is the same as outlined above Eq. (3.1) and yields

\[ s^2 d^2 \delta_{gg\rightarrow H}^{\text{VIRT}} = \pi \delta(s + t + u - m^2) S^2 \varepsilon G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \]

\[ \times \frac{1}{N^2} \left\{ n_f \left( \frac{2}{3\varepsilon} + \frac{1}{3} \ln \frac{-t}{\mu^2} + \frac{1}{3} \ln \frac{-u}{\mu^2} - \frac{5}{9} \right) \right. \]

\[ + C_A \left( -\frac{2}{\varepsilon^2} + \left( 2 \ln \frac{s}{\mu^2} - 2 \ln \frac{-t}{\mu^2} - 2 \ln \frac{-u}{\mu^2} - \frac{11}{3} \right) \frac{1}{\varepsilon} \right) \]

\[ + \text{Li}_2 \left( \frac{s - m^2}{s} \right) - \ln \frac{-t}{\mu^2} \ln \frac{-u}{\mu^2} + \ln \frac{-t}{\mu^2} \ln \frac{s}{\mu^2} \]

\[ + \ln \frac{-u}{\mu^2} \ln \frac{s}{\mu^2} - \ln^2 \frac{-t}{\mu^2} - \ln^2 \frac{-u}{\mu^2} - \frac{11}{6} \ln \frac{-t}{\mu^2} \]

\[ - \frac{11}{6} \ln \frac{-u}{\mu^2} - \frac{7}{2} \zeta(2) + \frac{38}{9} \]

\[ + C_F \left( -\frac{4}{\varepsilon^2} + \left( -2 \ln \frac{-t}{\mu^2} - 2 \ln \frac{-u}{\mu^2} + 3 \right) \frac{1}{\varepsilon} + \text{Li}_2 \left( \frac{t}{m^2} \right) \right) \]

\[ + \text{Li}_2 \left( \frac{u}{m^2} \right) - 2 \ln \frac{-t}{\mu^2} \ln \frac{-u}{\mu^2} - \frac{1}{2} \ln^2 \left( \frac{t - m^2}{t} \right) \]

\[ - \frac{1}{2} \ln^2 \left( \frac{u - m^2}{u} \right) + \frac{1}{2} \ln^2 \left( \frac{m^2 - t}{m^2} \right) + \frac{1}{2} \ln^2 \left( \frac{m^2 - u}{m^2} \right) \]
\[ + \frac{3}{2} \ln \frac{-t}{\mu^2} + \frac{3}{2} \ln \frac{-u}{\mu^2} + 9 \zeta(2) - 4 \} |M_{q\bar{q} \rightarrow g H}|^2 \]

+ \left( C_A - C_F \right) \left\{ \frac{1}{2} \right\} |MB_{q\bar{q} \rightarrow g H}|^2 , \quad (3.3) \]

\[ |MB_{q\bar{q} \rightarrow g H}|^2 = C_A C_F (-t - u) . \quad (3.4) \]

The square of the matrix element for the virtual corrections to the quark-gluon subprocess in Eq. (2.21) can be obtained from the one calculated for the quark-anti-quark subprocess in Eq. (2.18) via crossing from the s-channel to the u-channel. This yields

\[ s^2 \frac{d^2 \hat{\sigma}_{q\bar{q} \rightarrow g H}^{\text{VIRT}}}{d t d u} = \pi \delta(s + t + u - m^2) S^2 C^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \frac{1}{N(N^2 - 1)} \]

\[ \times \left\{ n_f \left( \frac{2}{3\varepsilon} + \frac{1}{3} \ln \frac{-t}{\mu^2} + \frac{2}{3} \ln \frac{-u}{\mu^2} - \frac{1}{3} \ln \frac{s}{\mu^2} - \frac{8}{9} \right) \right\} \]

\[ + C_A \left( - \frac{2}{\varepsilon^2} + \left( - 2 \ln \frac{t}{\mu^2} - \frac{8}{3} \right) \frac{1}{\varepsilon} + \text{Li}_2 \left( \frac{u}{m^2} \right) \right) \]

\[ - \ln \frac{-t}{\mu^2} \ln \frac{-u}{\mu^2} + \ln \frac{-t}{\mu^2} \ln \frac{s}{\mu^2} - \ln \frac{-u}{\mu^2} \ln \frac{s}{\mu^2} - \ln \frac{-t}{\mu^2} \ln \frac{-u}{\mu^2} \]

\[ + \ln^2 \frac{-u}{\mu^2} - \frac{1}{2} \ln^2 \left( \frac{u - m^2}{u} \right) + \frac{1}{2} \ln^2 \left( \frac{m^2 - u}{m^2} \right) \]

\[ - \frac{5}{6} \ln \frac{-t}{\mu^2} - \frac{11}{3} \ln \frac{-u}{\mu^2} + \frac{11}{6} \ln \frac{s}{\mu^2} + \frac{9}{2} \zeta(2) + \frac{50}{9} \]

\[ + C_F \left( - \frac{4}{\varepsilon^2} + \left( - 2 \ln \frac{-t}{\mu^2} - 4 \ln \frac{-u}{\mu^2} + 2 \ln \frac{s}{\mu^2} + 5 \right) \frac{1}{\varepsilon} \right) \]

\[ + \text{Li}_2 \left( \frac{s - m^2}{s} \right) + \text{Li}_2 \left( \frac{t}{m^2} \right) - 2 \ln \frac{-t}{\mu^2} \ln \frac{-u}{\mu^2} . \]
\[ +2 \ln \frac{-u}{\mu^2} \ln \frac{s}{\mu^2} - 2 \ln^2 \frac{-u}{\mu^2} - \frac{1}{2} \ln^2 \left( \frac{t - m^2}{t} \right) \]

\[ + \frac{1}{2} \ln^2 \left( \frac{m^2 - t}{m^2} \right) + \frac{5}{2} \ln \frac{-t}{\mu^2} + 5 \ln \frac{-u}{\mu^2} - \frac{5}{2} \ln \frac{s}{\mu^2} \]

\[ + \zeta(2) - \frac{13}{2} \right) \} |M_{qg \to qH}^{(1)}|^2 \]

\[ + \left( C_A - C_F \right) \left\{ \frac{1}{2} |MB_{qg \to qH}^{(1)}|^2 \right\} , \quad (3.5) \]

\[ |MB_{qg \to qH}^{(1)}|^2 = C_A C_F (s + t) . \quad (3.6) \]

Notice that the \( MB^{(1)} \) in Eqs. (3.2), (3.4) and (3.6) are presented in four dimensions and higher order terms in \( \varepsilon \) are ignored. This is sufficient because the factors with which they are multiplied are finite in the limit \( \varepsilon \to 0 \). Our results agree with those presented in [13]. Notice that in the latter paper the Born amplitudes \( M_{ab \to cH}^{(1)} \) are presented in four dimensions because they are constructed using the helicity method. We have computed them in \( n \) dimensions. Apparently this does not affect the factors that multiply the Born matrix elements and which contain all the pole terms in \( 1/\varepsilon^k \).
4 Gluon bremsstrahlung and other
2 → 3-body processes

In this section we give an outline of the computation of the two-to-three parton processes which show up in NLO. The calculation proceeds in an analogous way as in the case for heavy flavour production presented in [29]. The processes under consideration will be denoted by

\[ a(p_1) + b(p_2) \rightarrow c(-p_3) + d(-p_4) + H(-p_5), \quad \sum_{i=1}^{5} p_i = 0, \]

\[ a, b, c, d = q, \bar{q}, g, \]

\[ p_2^2 = 0, \quad i = 1 - 4, \quad p_5^2 = m^2. \]  \(4.1\)

The cross section corresponding to the reaction above can be expressed in \(n\) dimensions as follows

\[ \hat{\sigma}^{(2)}_{ab \rightarrow cd H} = K_{ab} \frac{1}{2s} G^2 g^4 \int \frac{d^m p_3}{(2\pi)^{n-1}} \int \frac{d^m p_4}{(2\pi)^{n-1}} \int \frac{d^m p_5}{(2\pi)^{n-1}} \int \delta^+(p_3^2) \delta^+(p_4^2) \delta^+(p_5^2 - m^2)(2\pi)^n \delta^{(n)}(p_1 + p_2 + p_3 + p_4 + p_5)
\]

\[ \times |M^{(2)}_{ab \rightarrow cd H}(\theta_1, \theta_2)|^2, \] \(4.2\)

where \(K_{ab}\) is defined below Eq. (2.9). Choosing the C.M. frame of the outgoing partons 3 and 4 we get the following parametrization in \(n\) dimensions

\[ p_1 = \omega_1 (1, 0, \cdots, 0, 0, 1), \]

\[ p_2 = (\omega_2, 0, \cdots, 0, |\vec{p}_5| \sin \psi, |\vec{p}_5| \cos \psi - \omega_1), \]

\[ -p_3 = \omega_4 (1, 0, \cdots, \sin \theta_1 \sin \theta_2, \sin \theta_1 \cos \theta_2, \cos \theta_1), \]

\[ -p_4 = \omega_4 (1, 0, \cdots, -\sin \theta_1 \sin \theta_2, -\sin \theta_1 \cos \theta_2, -\cos \theta_1), \]

\[ -p_5 = (\omega_5, 0, \cdots, 0, |\vec{p}_5| \sin \psi, |\vec{p}_5| \cos \psi), \] \(4.3\)

where the energies \(\omega_i\) are given by

\[ \omega_1 = \frac{P_{12} + P_{15} - m^2}{2 \sqrt{P_{34}}}, \quad \omega_2 = \frac{P_{12} + P_{55} - m^2}{2 \sqrt{P_{34}}}, \quad \omega_3 = \omega_4 = \frac{1}{2} \sqrt{P_{34}}, \]
\[ \omega_5 = \frac{P_{15} + P_{25}}{2\sqrt{P_{34}}}, \quad \cos \psi = \frac{P_{15} - m^2 + 2\omega_1 \omega_5}{2\omega_1 |\vec{p}_0|}. \] (4.4)

The invariants are denoted by

\[ P_{ij} = (p_i + p_j)^2, \quad P_{12} = s, \quad P_{15} = t, \quad P_{25} = u, \] (4.5)

\[ P_{34} = s_4 = s + t + u - m^2. \]

From \(|\cos \psi| \leq 1\) and \(s_4 = P_{34} \geq 0\) we derive the boundary conditions

\[ \frac{sm^2}{m^2 - t} \leq m^2 - u \leq s + t, \quad m^2 \leq m^2 - t \leq s. \] (4.6)

In the subsequent part of this paper it is more convenient to choose \(s_4\) as integration variable instead of \(u\). In this case the integration boundaries are

\[ 0 \leq s_4 \leq \frac{t(m^2 - s - t)}{m^2 - t}, \quad m^2 \leq m^2 - t \leq s. \] (4.7)

From the above kinematics the differential cross section in Eq. (4.2) becomes equal to

\[ s^2 \frac{d^2 \hat{\sigma}^{(2)}_{ab \rightarrow cd \ H}}{dt \ du} = \frac{1}{2} K_{ab} \ \frac{S_\varepsilon^2}{\Gamma(1 + \varepsilon)} G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \left( \frac{t \ u - m^2 s_4}{\mu^2 s} \right)^{\varepsilon/2} \left( \frac{s_4}{\mu^2} \right)^{\varepsilon/2} \times |M^{(2)}_{ab \rightarrow cd \ H}|^2, \] (4.8)

where \(|M^{(2)}_{ab \rightarrow cd \ H}|^2\) is the second order matrix element integrated over the polar angle \(\theta_1\) and the azimuthal angle \(\theta_2\). It is given by

\[ |M^{(2)}_{ab \rightarrow cd \ H}|^2 = \int_0^\pi d\theta_1 (\sin \theta_1)^{1+\varepsilon} \int_0^\pi d\theta_2 (\sin \theta_2)^\varepsilon |M^{(2)}_{ab \rightarrow cd \ H}(\theta_1, \theta_2)|^2 \]

\[ \equiv \int d\Omega_{n-1} |M^{(2)}_{ab \rightarrow cd \ H}(\theta_1, \theta_2)|^2. \] (4.9)

As in the calculation of the lowest order matrix elements in Eqs. (2.16), (2.19) (2.22) the next-to-leading order expressions \(|M^{(2)}(\theta_1, \theta_2)|^2\) have to be computed in \(n\) dimensions because we use \(n\)-dimensional regularization for
the collinear and infrared singularities which arise from the integration over the momenta of the final state partons. We use the algebraic manipulation program FORM (version 3.0) as in the earlier computation of the virtual corrections. Further we choose the Feynman gauge for the gluon propagators and the sum over the physical polarizations of the external gluons is given by

$$\sum_{\alpha=L,R} \epsilon^\mu(p, \alpha) \epsilon^\nu(p, \alpha) = P^{\mu\nu}(l, p), \quad \text{with} \quad l_\mu P^{\mu\nu} = P^{\mu\nu} l_\nu = 0, \quad l^2 = 0,$$

(4.10)

with

$$P^{\mu\nu} = -g^{\mu\nu} + \frac{l^\mu p^\nu + l^\nu p^\mu}{l \cdot p},$$

(4.11)

where $l$ is an arbitrary lightlike vector. There are three types of matrix elements depending on the number of gluons and (anti-) quarks appearing in the initial and final state. They will be denoted by $ggggH$, $ggqqH$ and $qqqqH$. Notice that the quark $q$ can also represent an anti-quark $\bar{q}$ depending on the type of process. The Feynman graphs can be found in [12]. The matrix element for $ggggH$ involves the computation of twenty-six graphs whereas eight graphs contribute to $ggqqH$. In the case of $qqqqH$ one has to distinguish between identical and non-identical quarks. For identical quarks we have two Feynman diagrams whereas the non-identical quarks are represented by one graph only. In the case of $ggggH$ the amplitude is denoted by $M^{(2)\mu\nu\kappa\rho}$ and the matrix element squared is obtained by contraction over the Lorentz indices as follows

$$|M^{(2)}(\theta_1, \theta_2)|^2 = P^{\mu\alpha} P^{\nu\beta} P^{\kappa\lambda} P^{\rho\sigma} M^{(2)\mu\nu\kappa\rho}_{\alpha\beta\lambda\sigma} M^{(2)\mu\nu\kappa\rho}_{\alpha\beta\lambda\sigma}.$$

(4.12)

We checked that the dependence on $l \cdot p_i$ disappears for arbitrary $l$. However this was only possible by writing the expression in Eq. (4.12) over a common denominator and expressing the result into five independent kinematical invariants. It turns out that Eq. (4.12) has the same property as the square of the Born matrix element in Eq. (2.16) namely that it is proportional to $n - 2$ provided $m = 0$. In the case of $ggqqH$ the amplitude is given by $M_{\mu\nu}^{(2)}$. Here we checked that the dependence on $l \cdot p_i$ of the matrix element squared given by

$$|M^{(2)}(\theta_1, \theta_2)|^2 = P^{\mu\alpha} P^{\nu\beta} M_{\mu\nu}^{(2)} M^{(2)}_{\alpha\beta},$$

(4.13)
already disappears after a simple partial fractioning. We made a comparison with the results in [12] which are presented in four dimensions only because they are computed using the helicity method. After corrections for some misprints the expressions obtained from Eqs. (4.12) and (4.13) are in agreement with those presented in Appendix A of [12]. The procedure to bring the matrix element over a common denominator, as discussed below Eq. (4.12), is not suitable for partial fractioning (see below) because it leads to high powers in the angular dependent kinematical variables, which show up in the numerator and denominator. To avoid this complication we proceed in a different way. Since the \( l.p_i \) terms are linear independent for \( i = 1 - 4 \) they have to cancel among themselves. This we checked by deleting them and comparing the truncated matrix element squared with the one obtained by bringing all terms over a common denominator, which is manifestly independent on \( l.p_i \). The difference between these two expressions turned out to be zero. Moreover the total power of the angular dependent kinematical invariants appearing in the numerator and denominator of the truncated matrix element does not exceed four so that it becomes amenable for partial fractioning. After this procedure the integration over the angles is performed in the following way. The expression \(|M^{(2)}(\theta_1, \theta_2)|^2\) depends on the variables \( P_{ij} \) defined in Eq. (4.5) among which five are linearly independent. Because of the parametrization in Eq. (4.3) the variables \( P_{12}, P_{15}, P_{25}, P_{34} \) are independent of the angles \( \theta_1, \theta_2 \) whereas the remaining ones \( P_{13}, P_{23}, P_{14}, P_{24}, P_{35} \) and \( P_{45} \) are angular dependent. One can distinguish two types of integrals

\[
J(P^{-k}_{ij}, P^{-l}_{m5}) = \int d\Omega_{n-1} P^{-k}_{ij} P^{-l}_{m5}, \quad i, j = 1 - 4 \quad m = 3, 4, \quad -4 \leq k \leq 2 \quad 0 \leq l \leq 2, \tag{4.14}
\]

\[
J(P^{-k}_{ij}, P^{-l}_{mn}) = \int d\Omega_{n-1} P^{-k}_{ij} P^{-l}_{mn}, \quad i, j, m, n \neq 5, \quad -3 \leq k \leq 2 \quad -3 \leq l \leq 2. \tag{4.15}
\]

The basic integrals of the first class are given by

\[
J(P^{-k}_{13}, P^{-l}_{45}) = 2^k (P_{25} - P_{34})^{-k} I_{k,1}(A, B, C),
\]

\(^3\)We found additional misprints in [12]. The \( S_{34} \) in the denominators of the two terms in (A.12) should read \( S_{24} \). In (A.16) one has instead of \( n_{13} = -S_{24} \) \( n_{12}(2 \leftrightarrow 4) \) and \( n_{23} = -S_{23} \) \( n_{13}(3 \leftrightarrow 4) \) the expressions \( n_{13} = -S_{24} \) \( n_{12}(1 \leftrightarrow 4) \) and \( n_{23} = n_{13}(3 \leftrightarrow 4) \).
\[ J(P_{13}^{-k}, P_{35}^{-l}) = 2^k (P_{25} - P_{34})^{-k} I_{k,l}(A, -B, -C), \] (4.16)

with
\[ I_{k,l}(A, B, C) = \int d\Omega_{n-1} (1 - \cos \theta_1)^{-k} (A + B \cos \theta_1 + C \sin \theta_1 \cos \theta_2)^{-l}. \] (4.17)

The basic integrals of the second class are represented by
\[ J(P_{13}^{-k}, P_{24}^{-l}) = 2^{k+l} (P_{23} - P_{34})^{-k} (P_{15} - P_{34})^{-l} I_{k,l}(\chi), \] (4.18)

with
\[ I_{k,l}(\chi) = \int d\Omega_{n-1} (1 - \cos \theta_1)^{-k} (1 - \cos \theta_1 \cos \chi - \sin \theta_1 \cos \theta_2 \sin \chi)^{-l} \]
\[ = \pi 2^{1-k-l} \frac{\Gamma(n/2 - 1 - k) \Gamma(n/2 - 1 - l) \Gamma(n - 3)}{\Gamma^2(n/2 - 1) \Gamma(n - 2 - k - l)} \]
\[ \times F_{1,2} \left( k, l, \frac{n}{2} - 1, \cos^2 \frac{\chi}{2} \right), \] (4.19)

where \( F_{1,2}(a, b; c; x) \) denotes the hypergeometric function which can be found in [30]. The other integrals can be derived by interchanging the inclusive momenta \( p_3 \) and \( p_4 \) or interchanging the parametrization for \( p_1 \) and \( p_2 \). The angular integrals above are presented in Appendix C of [29] for the case \(-2 \leq k \leq 2\). Because the square of the matrix element for Higgs production contains two extra powers of \( P_{ij} \) in the numerator the previous computation has to be extended to cover the cases \( k = -3, -4 \). These extra powers can be attributed to the momentum dependence in the effective Higgs-gluon-gluon coupling described by the Lagrangian in Eq. (2.1). The colour decompositions of the integrated expressions read as follows
\[ g + g \to g + g + H, \] (4.20)
\[ |M^{(2)}_{gg \to gg H}|^2 = N^2(N^2 - 1)|M^{(2)}_{gg \to gg H}|^2 N, \] (4.21)
\[ g + g \to q_i + \bar{q}_i + H, \] (4.22)
\[ |M_{gg \rightarrow q_i q_i}^{(2)} H|^2 = n_f C_A C_F \left[ C_A |M_{gg \rightarrow q_i q_i}^{(2)} H|^2_A + C_F |M_{gg \rightarrow q_i q_i}^{(2)} H|^2_F \right] \] (4.23)

\[ q + \bar{q} \rightarrow g + g + H, \] (4.24)

\[ |M_{q_i q_i \rightarrow gg}^{(2)} H|^2 = C_A C_F \left[ C_A |M_{q_i q_i \rightarrow gg}^{(2)} H|^2_A + C_F |M_{q_i q_i \rightarrow gg}^{(2)} H|^2_F \right] , \] (4.25)

\[ q + g \rightarrow q + g + H, \] (4.26)

\[ |M_{q_{ij} \rightarrow q_{ij}}^{(2)} H|^2 = C_A C_F \left[ C_A |M_{q_{ij} \rightarrow q_{ij}}^{(2)} H|^2_A + C_F |M_{q_{ij} \rightarrow q_{ij}}^{(2)} H|^2_F \right] . \] (4.27)

The colour and spin average factors \(K_{gg}, K_{q_i q_i}\) and \(K_{q_{ij}}\) are given in Eqs. (2.17), (2.20) and (2.23) respectively. If two identical particles in the processes above or below appear in an inclusive final state a factor \(1/2\) is implicitly understood. In NLO we encounter some new subprocesses. The first one is given by the reaction in Eq. (4.22) where a sum over all light flavours \(q_i\) with \(i = 1 \cdots n_f\) is understood. The next one is quark-quark scattering (non-identical and identical quarks) represented by

\[ q_1 + q_2 \rightarrow q_1 + q_2 + H, \quad q_1 \neq q_2, \] (4.28)

\[ |M_{q_1 q_2 \rightarrow q_1 q_2}^{(2)} H|^2 = C_A C_F |M_{q_1 q_2 \rightarrow q_1 q_2}^{(2)} H|^2_A , \] (4.29)

\[ q + q \rightarrow q + q + H, \] (4.30)

\[ |M_{q_{ij} q_{ij}}^{(2)} H|^2 = C_A C_F |M_{q_{ij} q_{ij}}^{(2)} H|^2_A + C_F |M_{q_{ij} q_{ij}}^{(2)} H|^2_F , \] (4.31)

which have the colour and spin average factors

\[ K_{q_1 q_2} = K_{q_i} = \frac{1}{4} \frac{1}{N^2} . \] (4.32)

The second subprocess is quark-anti-quark scattering

\[ q_1 + \bar{q}_2 \rightarrow q_1 + \bar{q}_2 + H, \quad q_1 \neq q_2, \] (4.33)

\[ |M_{q_1 \bar{q}_2 \rightarrow q_1 \bar{q}_2}^{(2)} H|^2 = C_A C_F |M_{q_1 \bar{q}_2 \rightarrow q_1 \bar{q}_2}^{(2)} H|^2_A , \] (4.34)

\[ q + \bar{q} \rightarrow q_i + \bar{q}_i + H, \quad q_i \neq q, \] (4.35)
\[
|M_{q\bar{q} \rightarrow q_1 \bar{q}_1 H}|^2 = (n_f - 1) C_A C_F |M_{q_1 \bar{q}_2 \rightarrow q_2 H}|^2_A, \quad q_a \neq q, \quad (4.36)
\]

\[
q + \bar{q} \rightarrow q + \bar{q} + H, \quad (4.37)
\]

\[
|M_{q\bar{q} \rightarrow q\bar{q} H}|^2 = C_A C_F \left( |M_{q_1 \bar{q}_2 \rightarrow q_1 \bar{q}_2 H}|^2_A + |M_{q_1 \bar{q}_1 \rightarrow q_2 H}|^2_A \right)
\]

\[
+ C_F |M_{q\bar{q} \rightarrow q\bar{q} H}|^2, \quad (4.38)
\]

and the colour and spin average factor reads

\[
K_{q_1 \bar{q}_2} = K_{q\bar{q}} = \frac{1}{4} \frac{1}{N^2}. \quad (4.39)
\]

Further we have summed over all quark flavours \( q_i \) in the final state provided they are not equal to the flavour of the quark \( q \) in the initial state (see Eq. (4.35)). Notice that the colour decomposition given above also holds before the squares of the matrix elements are integrated over the angles. Some of the expressions in Eqs. (4.20)-(4.38) are singular in the limit \( s_4 \rightarrow 0 \), where \( s_4 \) is defined in Eq. (4.7), which is due to soft gluon radiation or soft (collinear) fermion pair production. This will lead to infrared singularities when the differential cross section in Eq. (4.8) is convoluted with the parton densities even after mass factorization is carried out. The convolution involves an integration over \( s_4 \) so that one has to split up the partonic cross sections into hard and soft gluonic parts as follows

\[
s^2 d^2 \hat{\sigma}_{ab \rightarrow cd H} = s^2 d^2 \hat{\sigma}_{ab \rightarrow cd H}^{\text{HARD}} (s_4 > \Delta) + s^2 d^2 \hat{\sigma}_{ab \rightarrow cd H}^{\text{SOFT}} (s_4 \leq \Delta), \quad (4.40)
\]

where \( \Delta \) is chosen in such a way that it satisfies the inequalities

\[
\Delta \ll s, \quad \Delta \ll -t, \quad \Delta \ll -u. \quad (4.41)
\]

The soft gluon contribution to the differential cross section is defined by

\[
s^2 d^2 \hat{\sigma}_{ab \rightarrow cd H}^{\text{SOFT}} = \frac{1}{2} K_{ab} \frac{S_\epsilon^2}{\Gamma(1+\epsilon)} G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \left( \frac{t}{\mu^2 s} \right)^{\epsilon/2} \delta(s + t + u - m^2) \int_0^\Delta ds_4 \left( \frac{s_4}{\mu^2} \right)^{\epsilon/2} |M_{ab \rightarrow cd H}|^2. \quad (4.42)
\]
Only the singular part of $|M_{ab\rightarrow cd}^{(2)} H|^2$, which behaves as $1/s_4$, contributes to the above integral whereas the non-singular terms vanish in the limit $s_4 \rightarrow 0$. This singular part is called $|M_{ab\rightarrow cd}^{SOFT} H|^2$ and behaves in two different ways

I.

$$|M_{ab\rightarrow cd}^{SOFT} H|^2 \sim \frac{1}{s_4},$$  \hspace{1cm} (4.43)

and

II.

$$|M_{ab\rightarrow cd}^{SOFT} H|^2 \sim \frac{1}{s_4} \left( \frac{s}{t} \right)^{\epsilon/2}. \hspace{1cm} (4.44)$$

The $d^2\hat{\sigma}_{ab}^{SOFT}$ for the various subprocesses are presented in Appendix A. Adding the latter to the virtual contributions presented in section 3 leads to the soft-plus-virtual differential cross sections defined by

$$s^2 \frac{d^2\hat{\sigma}_{ab}^{S+V}}{dtdu} = s^2 \frac{d^2\hat{\sigma}_{ab}^{SOFT}}{dtdu} + s^2 \frac{d^2\hat{\sigma}_{ab}^{VIRT}}{dtdu}. \hspace{1cm} (4.45)$$

The infrared singularities cancel in this expression so that the double pole terms $1/\epsilon^2$, occurring in both the soft and the virtual contributions, all vanish.
5 Renormalization and mass factorization of the partonic differential cross sections

The virtual cross sections in section 3 have two types of ultraviolet divergences. One is removed by renormalization of the operator $O(x)$ occurring in Eq. (2.1) and the other by renormalization of the strong coupling constant. The renormalization constant corresponding to the former is

$$Z_O = \left(1 + \frac{2}{\varepsilon} \beta(\alpha_s)\right)^{-1} = 1 + \frac{\alpha_s}{4\pi} \frac{2}{\varepsilon} \beta_0 + \left(\frac{\alpha_s}{4\pi}\right)^2 \left[\frac{4}{\varepsilon^2} \beta_0^2 + \frac{2}{\varepsilon} \beta_1\right] + \cdots$$

$$= 1 + Z_O^{(1)} + Z_O^{(2)} + \cdots, \quad (5.1)$$

up to order $\alpha_s^2$ [31], where $\beta_0$ and $\beta_1$ are the lowest order coefficients of the beta-function given by

$$\beta(\alpha_s) = -\frac{\alpha_s}{4\pi} \beta_0 - \left(\frac{\alpha_s}{4\pi}\right)^2 \beta_1 + \cdots,$$

$$\beta_0 = \frac{11}{3} C_A - \frac{2}{3} n_f, \quad \beta_1 = \frac{34}{3} C_A^2 - 2 n_f C_F - \frac{10}{3} n_f C_A. \quad (5.2)$$

In lowest order $Z_O$ is the same as the renormalization constant for the strong coupling constant which reads

$$Z_{\alpha_s} = 1 + \frac{\alpha_s}{4\pi} \frac{2}{\varepsilon} \beta_0 + \left(\frac{\alpha_s}{4\pi}\right)^2 \left[\frac{4}{\varepsilon^2} \beta_0^2 + \frac{1}{\varepsilon} \beta_1\right] + \cdots$$

$$= 1 + Z_{\alpha_s}^{(1)} + Z_{\alpha_s}^{(2)} + \cdots. \quad (5.3)$$

Since the operator insertion always appears twice in the differential cross sections the latter have to be multiplied by $Z_O^2$.

After renormalization one has still to perform mass factorization to remove the remaining collinear divergences. This is achieved by the formula

$$s^2 d^2 \hat{\sigma}_{ab \to H+X'} (s, t, u, \varepsilon) = \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2} \Gamma_{ca}(x_1, \varepsilon) \Gamma_{f6}(x_2, \varepsilon)$$

$$\times s^2 d^2 \frac{\sigma_{ef \to H+X'}}{d \hat{t} d \hat{u}} (\hat{s}, \hat{t}, \hat{u})$$
\[ \equiv \Gamma_{ea} \otimes \Gamma_{fb} \otimes \hat{s}^2 \frac{d^2 \sigma_{ef \rightarrow H'X'}}{d \hat{t} \, d \hat{u}} \]  
(5.4)

where an implicit summation over \( e, f \) is understood, with

\[ \hat{s} = x_1 x_2 s, \quad \hat{t} = x_1 (t - m^2) + m^2, \quad \hat{u} = x_2 (u - m^2) + m^2. \]  
(5.5)

In Eq. (5.4) \( d^2 \hat{\sigma} \) represents the singular cross section which still contains the collinear divergences indicated by \( \varepsilon \). These divergences are removed by the kernels \( \Gamma_{ab} \) so that the finite cross sections \( d^2 \sigma \) are left. Both therefore depend on the mass factorization scale \( \mu \). Since the mass factorization has to be carried out for the semi-inclusive cross section up to NLO the kernels \( \Gamma_{ab} \) have to be corrected up to order \( \alpha_s \) only. They are denoted by

\[ \Gamma_{ab}(x, \varepsilon) = \delta_{ab} \delta(1 - x) + \Gamma^{(1)}_{ab}(x, \varepsilon). \]  
(5.6)

In section 4 the cross sections were split up into hard gluon \( (H) \) (Eq. (4.40)) and soft-plus-virtual gluon parts \( (S+V) \) (Eq. (4.45)). Therefore one proceeds with the kernels in the same way so that the mass factorization can be carried out for both parts separately

\[ \Gamma^{(1)}_{ab} = \Gamma_{ab}^{\text{HARD},(1)} + \delta(1 - x) \Gamma_{ab}^{S+V,(1)}. \]  
(5.7)

The various kernels are expressed into the splitting functions \( P_{ab} \), depending on the partons \( a, b \), which represent the residues of the collinear singularities

\[ \Gamma^{(1)}_{qq} = \Gamma^{(1)}_{\bar{q}q} = \frac{\alpha_s}{4\pi} S_\varepsilon \frac{1}{\varepsilon} P^{(0)}_{qq}(x), \]

\[ \Gamma^{(1)}_{gq} = \Gamma^{(1)}_{\bar{q}g} = \frac{\alpha_s}{4\pi} S_\varepsilon \frac{1}{2\varepsilon} P^{(0)}_{q\bar{q}}(x), \]

\[ \Gamma^{(1)}_{\bar{q}g} = \Gamma^{(1)}_{g\bar{q}} = \frac{\alpha_s}{4\pi} S_\varepsilon \frac{1}{\varepsilon} P^{(0)}_{\bar{q}g}(x), \]

\[ \Gamma^{(1)}_{gg} = \frac{\alpha_s}{4\pi} S_\varepsilon \frac{1}{\varepsilon} P^{(0)}_{gg}(x). \]  
(5.8)

The splitting functions are given by

\[ P^{(0)}_{qq}(x) = 4 C_F \left[ 2 \left( \frac{1}{1 - x} \right)_+ - 1 - x + \frac{3}{2} \delta(1 - x) \right], \]
\[ P_{gg}^{(0)}(x) = 4 C_F \left[ \frac{1 + (1 - x)^2}{x} \right], \]
\[ P_{qg}^{(0)}(x) = 8 T_f \left[ x^2 + (1 - x)^2 \right], \]
\[ P_{gg}^{(0)}(x) = 8 C_A \left[ \left( \frac{1}{1 - x} \right) + \frac{1}{x} - 2 + x - x^2 + \frac{11}{12} \delta(1 - x) \right] \]
\[- \frac{4}{3} n_f \delta(1 - x). \] (5.9)

The collinearly finite cross sections can be derived from Eq. (5.4) by using the above kernels. In LO they are equal to the Born results derived in section 2. In NLO the finite order \( \alpha_s^2 \) contributions can be written as

\[ g + g \rightarrow g + g + H, \]
\[ s^2 d^2 \frac{\sigma_{gg \rightarrow gg}^{(2)} H}{d t d u} = s^2 d^2 \frac{\Delta_{gg \rightarrow gg}^{(2)} H}{d t d u}, \]
\[- \Gamma_{gg}^{(1)} \otimes s^2 d^2 \frac{\sigma_{gg \rightarrow gg}^{(1)} H}{d t d u} = \Gamma_{gg}^{(1)} \otimes s^2 d^2 \frac{\sigma_{gg \rightarrow gg}^{(1)} H}{d t d u}, \] (5.10)

\[ g + g \rightarrow q + \bar{q} + H, \]
\[ s^2 d^2 \frac{\sigma_{gg \rightarrow qq}^{(2)} H}{d t d u} = s^2 d^2 \frac{\Delta_{gg \rightarrow qq}^{(2)} H}{d t d u}, \]
\[-2 \Gamma_{gg}^{(1)} \otimes s^2 d^2 \frac{\sigma_{gg \rightarrow qq}^{(1)} H}{d t d u} = 2 \Gamma_{gg}^{(1)} \otimes s^2 d^2 \frac{\sigma_{gg \rightarrow qq}^{(1)} H}{d t d u}, \] (5.11)

\[ q + \bar{q} \rightarrow g + g + H, \]
\[ s^2 d^2 \frac{\sigma_{qg \rightarrow gg}^{(2)} H}{d t d u} = s^2 d^2 \frac{\Delta_{qg \rightarrow gg}^{(2)} H}{d t d u}, \]
\[ -\Gamma_{qq}^{(1)} \otimes s^2 \frac{d^2 \sigma_{qq \rightarrow qg}^{(1)}}{dt \, du} - \Gamma_{qg}^{(1)} \otimes s^2 \frac{d^2 \sigma_{qg \rightarrow qg}^{(1)}}{dt \, du}, \]

\[ q + g \rightarrow q + g + H, \]

\[ s^2 \frac{d^2 \sigma_{qq \rightarrow qq}^{(2)}}{dt \, du} = s^2 \frac{d^2 \delta_{qq \rightarrow qq}^{(2)}}{dt \, du} \]

\[ -\Gamma_{gg}^{(1)} \otimes s^2 \frac{d^2 \sigma_{gg \rightarrow gg}^{(1)}}{dt \, du} - \Gamma_{gq}^{(1)} \otimes s^2 \frac{d^2 \sigma_{gq \rightarrow gg}^{(1)}}{dt \, du}, \]

\[ q_1 + q_2 \rightarrow q_1 + q_2 + H, \quad q + q \rightarrow q + q + H, \]

\[ s^2 \frac{d^2 \sigma_{qg \rightarrow qq}^{(2)}}{dt \, du} = s^2 \frac{d^2 \delta_{qg \rightarrow qq}^{(2)}}{dt \, du} \]

\[ -\Gamma_{qg}^{(1)} \otimes s^2 \frac{d^2 \sigma_{qg \rightarrow qg}^{(1)}}{dt \, du} - \Gamma_{gq}^{(1)} \otimes s^2 \frac{d^2 \sigma_{gq \rightarrow qg}^{(1)}}{dt \, du}, \]

\[ q_1 + \bar{q}_2 \rightarrow q_1 + \bar{q}_2 + H, \quad q + \bar{q} \rightarrow q + \bar{q} + H, \]

\[ s^2 \frac{d^2 \sigma_{\bar{q}g \rightarrow qg}^{(2)}}{dt \, du} = s^2 \frac{d^2 \delta_{\bar{q}g \rightarrow qg}^{(2)}}{dt \, du} \]

\[ -\Gamma_{gq}^{(1)} \otimes s^2 \frac{d^2 \sigma_{gq \rightarrow qg}^{(1)}}{dt \, du} - \Gamma_{gq}^{(1)} \otimes s^2 \frac{d^2 \sigma_{\bar{q}g \rightarrow qg}^{(1)}}{dt \, du}. \]

Notice that the finite parts of these cross sections can become negative due to the above subtractions. The expressions for the finite hard gluon cross sections \( d^2 \sigma_{ab \rightarrow cd}^{\text{HARD}} \) are too long to be published. They exist as FORM [23].
files and they are available on request. Here we can only show those parts which behave like \(1/s_4\) since they are of interest later on. In the soft gluon limit \(s_4 \to 0\) they read

\[
\lim_{s_4 \to 0} s_4^2 \frac{d^2 \sigma_{\text{HARD}}^{gg \to gg \ H}}{d t \ d u} = \pi G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \frac{N}{(N^2 - 1)^2} \frac{1}{s_4} \left[ 3 \ln \frac{s_4}{\mu^2} - \ln \frac{tu}{\mu^2} - \frac{11}{12} \right] |M_{g g \to g \ H}^{(1)}|^2,
\]

(5.16)

\[
\lim_{s_4 \to 0} s_4^2 \frac{d^2 \sigma_{\text{HARD}}^{qq \to gg \ H}}{d t \ d u} = \pi G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \frac{n_f}{(N^2 - 1)^2} \frac{1}{s_4} \left[ 1 \right] \times |M_{g g \to g \ H}^{(1)}|^2,
\]

(5.17)

\[
\lim_{s_4 \to 0} s_4^2 \frac{d^2 \sigma_{\text{HARD}}^{qq \to q \ H}}{d t \ d u} = \pi G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \frac{1}{N(N^2 - 1)} \frac{1}{s_4} \left[ C_A \left\{ 2 \ln \frac{s_4}{\mu^2} + C_F \left\{ \ln \frac{s_4}{\mu^2} - \frac{3}{4} \right\} \right\} \times |M_{q g \to q \ H}^{(1)}|^2.
\]

(5.18)

\[
\lim_{s_4 \to 0} s_4^2 \frac{d^2 \sigma_{\text{HARD}}^{qq \to q \ H}}{d t \ d u} = \pi G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \frac{1}{N^2} \frac{1}{s_4} \left[ C_A \left\{ 2 \ln \frac{s_4}{\mu^2} - \ln \frac{tu}{\mu^2} - \frac{11}{12} \right\} + C_F \left\{ \ln \frac{s_4}{\mu^2} - \frac{3}{4} \right\} \right] |M_{q g \to q \ H}^{(1)}|^2.
\]

(5.19)

Notice that the expressions above satisfy a supersymmetric relation. It turns out that the expressions for \(gg \to' X' + H\) in Eqs. (5.16) plus (5.17), \(qq \to' X' + H\) in Eqs. (5.18) plus (5.20) and \(qq \to' X' + H\) in Eq. (5.19) become equal for a \(\mathcal{N} = 1\) supersymmetry where \(C_A = C_F = n_f = N\).
After renormalization of the operator $O(x)$ and the coupling constant using $Z_O$ and $Z_{\alpha_s}$ in Eqs. (5.1) and (5.3) respectively the ultraviolet divergences vanish in the soft-plus-virtual cross section $d^2\sigma_{ab}^{S+V}$ in Eq. (4.45). The remaining collinear divergences are removed via mass factorization using $\Gamma_{ab}^{S+V}$ in Eqs. (5.7)-(5.9). The combined effect of these three operations leads to the following expressions

$$s^2 \frac{d^2 \sigma_{gg\rightarrow gH}^{S+V}}{dt\,du} = s^2 \frac{d^2 \hat{\sigma}_{gg\rightarrow gH}^{S+V}}{dt\,du}$$

$$+ \left( Z_{\alpha_s}^{(1)} + 2 Z_O^{(1)} - 2 \Gamma_{gg}^{S+V,(1)} \right) s^2 \frac{d^2 \sigma_{gg\rightarrow gH}^{(1)}}{dt\,du}, \quad (5.21)$$

$$s^2 \frac{d^2 \sigma_{q\bar{q}\rightarrow gH}^{S+V}}{dt\,du} = s^2 \frac{d^2 \hat{\sigma}_{q\bar{q}\rightarrow gH}^{S+V}}{dt\,du}$$

$$+ \left( Z_{\alpha_s}^{(1)} + 2 Z_O^{(1)} - 2 \Gamma_{qq}^{S+V,(1)} \right) s^2 \frac{d^2 \sigma_{q\bar{q}\rightarrow gH}^{(1)}}{dt\,du}, \quad (5.22)$$

$$s^2 \frac{d^2 \sigma_{qg\rightarrow qH}^{S+V}}{dt\,du} = s^2 \frac{d^2 \hat{\sigma}_{qg\rightarrow qH}^{S+V}}{dt\,du}$$

$$+ \left( Z_{\alpha_s}^{(1)} + Z_O^{(1)} \right) s^2 \frac{d^2 \sigma_{qg\rightarrow qH}^{(1)}}{dt\,du} - \Gamma_{qq}^{S+V,(1)} s^2 \frac{d^2 \sigma_{gg\rightarrow gH}^{(1)}}{dt\,du}$$

$$- \Gamma_{qq}^{S+V,(1)} s^2 \frac{d^2 \sigma_{gg\rightarrow gH}^{(1)}}{dt\,du}. \quad (5.23)$$

The results for the finite soft-plus-virtual differential cross sections read

$$s^2 \frac{d^2 \sigma_{gg\rightarrow gH}^{S+V}}{dt\,du} = \pi \delta(s + t + u - m^2) G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \frac{1}{(N^2 - 1)^2}$$

$$\times \left[ \left\{ n_f \left( \frac{1}{6} \ln \frac{\Delta}{\mu^2} - \frac{5}{18} \right) \right. \right.$$ 

$$+ N \left( 3 \ln^2 \frac{\Delta}{\mu^2} + \left( - \ln \frac{tu}{\mu^2 s} - \frac{11}{12} \right) \ln \frac{\Delta}{\mu^2} + \text{Li}_2 \left( \frac{t}{m^2} \right) \right)$$
$$s^2 d^2 \frac{\sigma_{q\bar{q}\rightarrow g H}}{dtdu} = \pi \delta(s + t + u - m^2) G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \frac{1}{N^2} \times \left[ \{ n_f \left( \frac{1}{6} \ln \frac{\Delta}{\mu^2} + \frac{1}{3} \ln \frac{s}{\mu^2} - \frac{5}{6} \right) ight.$$  

$$+ C_A \left( - \frac{1}{2} \ln^2 \frac{\Delta}{\mu^2} + \left( \ln \frac{tu}{\mu^2s} - \frac{11}{12} \right) \ln \frac{\Delta}{\mu^2} + \text{Li}_2 \left( \frac{s - m^2}{s} \right) \right)$$  

$$- \frac{1}{2} \ln^2 \frac{-t}{\mu^2} - \frac{1}{2} \ln^2 \frac{-u}{\mu^2} + \frac{1}{2} \ln^2 \frac{s}{\mu^2} - \frac{11}{6} \ln \frac{s}{\mu^2} + 5 \zeta(2) + \frac{73}{12} \right]$$  

$$+ C_F \left( 2 \ln^2 \frac{\Delta}{\mu^2} - 2 \ln \frac{tu}{\mu^2s} \ln \frac{\Delta}{\mu^2} + \text{Li}_2 \left( \frac{t}{m^2} \right) + \text{Li}_2 \left( \frac{u}{m^2} \right) \right)$$  

$$- \ln \frac{-t}{\mu^2} \ln \frac{s}{\mu^2} - \ln \frac{-u}{\mu^2} \ln \frac{s}{\mu^2} + \ln^2 \frac{-t}{\mu^2} + \ln^2 \frac{-u}{\mu^2}$$  

$$- \frac{1}{2} \ln^2 \left( \frac{t - m^2}{t} \right) - \frac{1}{2} \ln^2 \left( \frac{u - m^2}{u} \right) + \frac{1}{2} \ln^2 \left( \frac{m^2 - t}{m^2} \right)$$  

$$+ \text{Li}_2 \left( \frac{u}{m^2} \right) - \frac{1}{2} \ln^2 \left( \frac{m^2 - t}{m^2} \right)$$  

$$+ \frac{1}{2} \ln^2 \frac{-t}{\mu^2} + \frac{1}{2} \ln^2 \frac{-u}{\mu^2} + \frac{1}{2} \ln^2 \frac{s}{\mu^2} - \frac{1}{2} \ln^2 \left( \frac{t - m^2}{t} \right)$$  

$$- \frac{1}{2} \ln^2 \left( \frac{u - m^2}{u} \right) + \frac{1}{2} \ln^2 \left( \frac{m^2 - t}{m^2} \right)$$  

$$+ 3 \zeta(2) + \frac{67}{36} \right\} |M_{gg\rightarrow gH}|^2$$  

$$+ (N - n_f) \left\{ \frac{1}{4} |MB_{gg\rightarrow gH}|^2 \right\}, \quad (5.24)$$
\begin{align*}
&+ \frac{1}{2} \ln^2 \left( \frac{m^2 - u}{m^2} \right) + \frac{3}{2} \ln \frac{s}{\mu^2} + 8 \zeta(2) - 4 \right) \right] |M_{qg \rightarrow g H}^{(1)}|^2 \\
&+ \left( C_A - C_F \right) \left\{ \frac{1}{2} \right\} |M_{qg \rightarrow g H}^{(1)}|^2, \\
\end{align*}

\[
\frac{s^2 d^2 \sigma_{qqg}^{S+V}}{d t d u} = \pi \delta(s + t + u - m^2) G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \frac{1}{N(N^2 - 1)} \times \left\{ \right.
\begin{align*}
&n_f \left( \frac{1}{3} \ln \frac{-u}{\mu^2} - \frac{5}{9} \right) \\
&+ C_A \left( \ln^2 \frac{\Delta}{\mu^2} - \ln \frac{t}{\mu^2 s} \ln \frac{\Delta}{\mu^2} + \ln \frac{t}{\mu^2} \right) - \ln \frac{-u}{\mu^2} \ln \frac{s}{\mu^2} \\
&+ \ln^2 \frac{-u}{\mu^2} - \frac{1}{2} \ln^2 \left( \frac{u - m^2}{v} \right) + \frac{1}{2} \ln^2 \left( \frac{m^2 - u}{m^2} \right) - \frac{11}{6} \ln \frac{-u}{\mu^2} \\
&+ 4 \zeta(2) + \frac{38}{9} \right) \\
&+ C_F \left( \frac{1}{2} \ln^2 \frac{\Delta}{\mu^2} - \frac{3}{4} \ln \frac{\Delta}{\mu^2} + \ln \frac{t}{m^2} \right) + \ln \frac{s - m^2}{s} \\
&- \ln \frac{-t}{\mu^2} \ln \frac{s}{\mu^2} + \frac{1}{2} \ln^2 \frac{-t}{\mu^2} - \frac{1}{2} \ln^2 \frac{-u}{\mu^2} + \frac{1}{2} \ln^2 \frac{s}{\mu^2} \\
&- \frac{1}{2} \ln^2 \left( \frac{t - m^2}{t} \right) + \frac{1}{2} \ln^2 \left( \frac{m^2 - t}{m^2} \right) + \frac{3}{2} \ln \frac{-u}{\mu^2} \\
&- \zeta(2) - \frac{9}{4} \right) \right\} |M_{qg \rightarrow g H}^{(1)}|^2 \\
&+ \left( C_A - C_F \right) \left\{ \frac{1}{2} \right\} |M_{qg \rightarrow g H}^{(1)}|^2, 
\end{align*}
\tag{5.25}
\end{align*}

The expressions multiplying the Born matrix elements $|M_{ab \rightarrow c H}^{(1)}|^2$ satisfy
a supersymmetry relation. If we choose a $\mathcal{N} = 1$ supersymmetry so that the quarks are put in the adjoint representation i.e. $C_A = C_F = n_f = N$ then all expressions are equal except for the rational constants which are equal to $19/12$ in Eq. (5.24), $15/12$ in Eq. (5.25) and $17/12$ in Eq. (5.26) respectively. These differences originate from the fact that we use $n$-dimensional regularization rather than $n$-dimensional reduction. The result for the latter case can be obtained via a finite renormalization. It turns out that for $n$-dimensional reduction the rational numbers become equal to $7/4$ for all three reactions which provides us with a strong check on our calculations. Finally we want to comment on the scale $\mu$. In the computation of the radiative corrections we have assumed that the renormalization scale $\mu_r$ is equal to the mass factorization scale $\mu$. If one wants to distinguish between both scales one has to substitute

$$
\alpha_s(\mu^2) = \alpha_s(\mu_r^2) \left[ 1 + \frac{\alpha_s(\mu_r^2)}{4\pi} \beta_0 \ln \frac{\mu_r^2}{\mu^2} \right],
$$

(5.27)
in all finite expressions.
6 Differential distributions for the process 

\[ p + p \rightarrow H' +'X' \]

In this section we will present the differential cross sections for Higgs-boson production in proton-proton collisions at the LHC. Here we study the dependence of the cross sections on input parameters like the QCD scale \( \Lambda \), the renormalization/factorization scale \( \mu \) and the dependence on the chosen set of parton densities. We also study which region in the gluon density is the most important. Furthermore we make a comparison with similar results presented in previous papers. Finally we also study at which values of the kinematical variables the soft-plus-virtual (S+V) gluons start to dominate the cross sections. This will give an indication about the validity of the S+V gluon approximation which is of importance when one wants to improve the perturbation series via resummation techniques leading to a better prediction for the differential cross sections.

The hadronic cross section \( d\sigma \) is obtained from the partonic cross section \( d\sigma_{ab} \) as follows

\[
S^2 \frac{d^2\sigma^{H_1 H_2}}{dT \, dU}(S,T,U,m^2) = \sum_{a,b=q,g} \int_{x_{1,\min}}^{x_1} \frac{dx_1}{x_1} \int_{x_{2,\min}}^{x_2} \frac{dx_2}{x_2} f_a^{H_1}(x_1,\mu^2) \times f_b^{H_2}(x_2,\mu^2) s^2 \frac{d^2\sigma_{ab}}{dt \, du}(s,t,u,m^2,\mu^2). \tag{6.1}
\]

In analogy to Eq. (2.12) the hadronic kinematical variables are defined by

\[
S = (P_1 + P_2)^2, \quad T = (P_1 + p_5)^2, \quad U = (P_2 + p_5)^2, \tag{6.2}
\]

where \( P_1 \) and \( P_2 \) denote the momenta of hadrons \( H_1 \) and \( H_2 \) respectively (see Eq. (2.7)). In the case parton \( p_1 \) emerges from hadron \( H_1(P_1) \) and parton \( p_2 \) emerges from hadron \( H_2(P_2) \) we can establish the following relations

\[
p_1 = x_1 P_1, \quad p_2 = x_2 P_2, \\
\begin{align*}
  s &= x_1 x_2 S, \\
  t &= x_1(T - m^2) + m^2, \\
  u &= x_2(U - m^2) + m^2,
\end{align*}
\]

\[
x_{1,\min} = \frac{-U}{S + T - m^2}, \quad x_{2,\min} = \frac{-x_1(T - m^2) - m^2}{x_1 S + U - m^2}. \tag{6.3}
\]
When the matrix element behaves as $1/s_4$, which occurs in the case of gluon bremsstrahlung or collinear fermion pair emission, it is more convenient to choose the integration variable $s_4$ instead of $x_2$ in order to get better numerical stability. The cross section becomes

$$
S^2 \frac{d^2 \sigma_{H_1H_2}}{dT \, dU}(S,T,U,m^2) = \sum_{a,b=q,g} \int_{x_{1,\text{min}}}^{1} \frac{dx_1}{x_1} \int_{0}^{s_{4,\text{max}}} \frac{ds_4}{s_4 - x_1(T - m^2) - m^2}
$$

$$
\times f_a^{H_1}(x_1, \mu^2) \, f_b^{H_2}(x_2^*(s_4), \mu^2)
$$

$$
\times s^2 \frac{d^2 \sigma_{ab}}{dt \, du}(s,t,s_4,m^2,\mu^2).
$$

(6.4)

The function $x_2^*(s_4)$ can be derived from

$$
s_4 = s + t + u - m^2 = x_1 \, x_2 \, S + x_1(T - m^2) + x_2(U - m^2) + m^2,
$$

and it reads

$$
x_2^*(s_4) = \frac{s_4 - x_1(T - m^2) - m^2}{x_1 S + U - m^2}, \quad s_{4,\text{max}} = x_1(S + T - m^2) + U.
$$

(6.5)

When parton $p_1$ emerges from hadron $H_2(P_2)$ and parton $p_2$ emerges from $H_1(P_1)$ one obtains the same expressions as in Eqs. (6.3)-(6.6) except that $T$ and $U$ are interchanged. This result has to be added to Eq. (6.4). When the partonic cross section is symmetric under $t \leftrightarrow u$ one can also use the representation in Eq. (6.4) without adding the result where $T$ and $U$ are interchanged provided one makes the replacement $f_a^{H_1} \to f_a^{H_2}$, $f_b^{H_2} \to f_b^{H_1}$.

The expression in Eq. (6.4) simplifies when the partonic cross sections correspond to the Born reactions in Eq. (2.13) or the S+V contributions in Eqs. (5.24)-(5.26). In this case $d^2\sigma \sim \delta(s_4)$ and the integral in Eq. (6.4) becomes one-dimensional. For two-to-three body processes with no soft gluons or collinear fermion pairs in the final state the integral is two dimensional as given in Eq. (6.4). If the processes are of the gluon bremsstrahlung type, or contain collinear fermion pairs in the final state we have to split the two-to-three body partonic cross section in Eq. (4.40) into two parts i.e. a hard gluon part with $s_4 > \Delta$ and a soft gluon part with $s_4 \leq \Delta$. The terms
containing the cut off parameter $\Delta$ cancel between the S+V cross sections originating from Eqs. (5.24)-(5.26) and the corresponding terms in the hard gluon hadronic cross section in Eq. (6.4), namely
\[
\int_{x_{1,\text{min}}}^{1} \frac{dx_{1}}{x_{1}} \int_{\Delta}^{s_{4,\text{max}}} ds_{4} \frac{ds_{4}}{s_{4} - x_{1}(T - m^{2}) - m^{2}} f_{a}^{H_{1}}(x_{1}, \mu^{2}) f_{b}^{H_{2}}(x_{2}^{*}(s_{4}), \mu^{2}) \times s^{2} \frac{d^{2} \sigma_{\text{HARD}}^{ab}}{d t d u}(s, t, s_{4}, m^{2}, \mu^{2}).
\]
(6.7)

In order to achieve the cancellation analytically we rewrite the expression above as follows
\[
\int_{x_{1,\text{min}}}^{1} \frac{dx_{1}}{x_{1}} \left[ \int_{\Delta}^{s_{4,\text{max}}} ds_{4} \frac{ds_{4}}{s_{4} - x_{1}(T - m^{2}) - m^{2}} f_{a}^{H_{1}}(x_{1}, \mu^{2}) f_{b}^{H_{2}}(x_{2}^{*}(s_{4}), \mu^{2}) \right.
\][ 
\times s^{2} \frac{d^{2} \sigma_{\text{HARD}}^{ab}}{d t d u}(s, t, s_{4}, m^{2}, \mu^{2}) - \int_{\Delta}^{s_{4,\text{max}}} ds_{4} \frac{ds_{4}}{-x_{1}(T - m^{2}) - m^{2}} f_{a}^{H_{1}}(x_{1}, \mu^{2})
\]
\[
\times f_{b}^{H_{2}}(x_{2}^{*}(0), \mu^{2}) \lim_{s_{4} \to 0} s^{2} \frac{d^{2} \sigma_{\text{HARD}}^{ab}}{d t d u}(s, t, s_{4}, m^{2}, \mu^{2}) \right]
\]
\[
+ \int_{x_{1,\text{min}}}^{1} \frac{dx_{1}}{x_{1}} \int_{\Delta}^{s_{4,\text{max}}} ds_{4} \frac{ds_{4}}{-x_{1}(T - m^{2}) - m^{2}} f_{a}^{H_{1}}(x_{1}, \mu^{2}) f_{b}^{H_{2}}(x_{2}^{*}(0), \mu^{2})
\]
\[
\times \lim_{s_{4} \to 0} s^{2} \frac{d^{2} \sigma_{\text{HARD}}^{ab}}{d t d u}(s, t, s_{4}, m^{2}, \mu^{2}).
\]
(6.8)

The expression between the square brackets is integrable in $s_{4}$ so we can put $\Delta = 0$ in it. One can perform the integration over $s_{4}$ analytically in the second part by using the simple expressions $d^{2} \sigma_{\text{HARD}}^{ab}$ in the limit $s_{4} \to 0$. The $\ln^{i} \Delta/\mu^{2}$ terms, which arise from the integration over $s_{4}$, then cancel those appearing in the S+V cross sections.

In practice one is not interested in the Higgs boson differential cross sections depending on $T$ and $U$ but rather in the rapidity $y$ and the transverse momentum $p_{T}$ distributions. Neglecting the masses of the incoming hadrons
we have the following relations

\[ T = m^2 - \sqrt{S \sqrt{p_T^2 + m^2}} \cosh y + \sqrt{S \sqrt{p_T^2 + m^2}} \sinh y, \]

\[ U = m^2 - \sqrt{S \sqrt{p_T^2 + m^2}} \cosh y - \sqrt{S \sqrt{p_T^2 + m^2}} \sinh y, \] (6.9)

so that the cross section becomes

\[ S \frac{d^2 \sigma_{H_1H_2}}{dp_T^2 dy} (S, p_T^2, y, m^2) = S^2 q^2 \frac{d^2 \sigma_{H_1H_2}}{dT dU} (S, T, U, m^2). \] (6.10)

The kinematical boundaries are

\[ m^2 - S \leq T \leq 0, \quad -S - T + m^2 \leq U \leq \frac{S m^2}{T - m^2} + m^2, \] (6.11)

from which one can derive

\[ 0 \leq p_T^2 \leq p_{T,max}^2, \quad \frac{1}{2} \ln \frac{S}{m^2} \leq y \leq \frac{1}{2} \ln \frac{S}{m^2}, \]

with

\[ p_{T,max}^2 = \frac{(S + m^2)^2}{4 S \cosh^2 y} - m^2, \] (6.12)

or

\[ -y_{max} \leq y \leq y_{max}, \quad 0 \leq p_T^2 \leq \frac{(S - m^2)^2}{4 S} \equiv \bar{p}_{T,max}^2, \]

with

\[ y_{max} = \frac{1}{2} \ln \frac{1 + \sqrt{1 - sq}}{1 - \sqrt{1 - sq}}, \quad sq = \frac{4 S (p_T^2 + m^2)}{(S + m^2)^2}. \] (6.13)

Since the cross section diverges for \( p_T \to 0 \) we cannot perform the integral over this kinematical variable down to zero. However we can perform the integral over the rapidity and obtain the transverse momentum distribution

\[ \frac{d \sigma_{H_1H_2}}{dp_T} (S, p_T^2, m^2) = \int_{y_{max}}^{y_{max}} d y \frac{d^2 \sigma_{H_1H_2}}{dp_T dy} (S, p_T^2, y, m^2), \] (6.14)

with \( y_{max} \) given in Eq. (6.13). There is an alternative way to obtain the distribution above. This is shown in Appendix B. We checked that both
procedures lead to the same numerical result. In the case we plot the rapidity distribution we have to impose a cut $p_{T,\text{min}}$ on the transverse momentum integration i.e.

$$
\frac{d}{dy} \sigma_{H_1H_2}(S, y, m^2) = \int_{p_{T,\text{min}}}^{p_{T,\text{max}}} dp_T \frac{d^2}{dy} \sigma_{H_1H_2}(S, p_T^2, y, m^2),
$$

(6.15)

with $p_{T,\text{max}}$ given in Eq. (6.12). Actually the differential cross section dies off so fast that it is sufficient to put $p_{T,\text{max}} = 8 \times p_{T,\text{min}}$. Finally we define what we mean by leading order (LO) and next-to-leading order (NLO). In LO the differential cross section is defined by

$$
\frac{d^2}{dp_T dy} \sigma^{\text{LO}}(S, p_T^2, y, m^2) = \frac{d^2}{dp_T dy} \sigma^{(1)}(S, p_T^2, y, m^2),
$$

(6.16)

and the gluon-gluon-Higgs coupling is given by $G$ in Eq. (2.2) with $C = 1$ (see Eq. (2.6)). We also adopt the leading logarithmic representation for the running coupling and the parton densities. The NLO corrected differential cross section reads

$$
\frac{d^2}{dp_T dy} \sigma^{\text{NLO}}(S, p_T^2, y, m^2) = \left[ 1 + 22 \left( \frac{\alpha_s(M_\text{Z})}{4\pi} \right) \right] \frac{d^2}{dp_T dy} \sigma^{(1)}(S, p_T^2, y, m^2)
$$

$$
+ \frac{d^2}{dp_T dy} \sigma^{(2)}(S, p_T^2, y, m^2),
$$

(6.17)

where the LO contribution in this formula is now multiplied with $C^2 = 1 + 22 \alpha_s/4\pi$. Furthermore the running coupling and parton densities are represented in next-to-leading order for which we have chosen the $\overline{\text{MS}}$-scheme. In this way one obtains a result which is consistently corrected up to NLO. In our computations the number of light flavours is taken to be $n_f = 5$ which holds for the running coupling, the partonic cross sections and the number of quark flavour densities. Further we have chosen for our plots the parton densities obtained from the sets MRST98 [32] CTEQ4 [33], GRV98 [34] and MRST99 [35] (see Table 1). Notice that the GRV sets do not contain charm and bottom quark densities. For simplicity the factorization scale $\mu$ is set equal to the renormalization scale $\mu_r$. For our plots we take $\mu^2 = m^2 + p_T^2$ unless mentioned otherwise. Here we want to emphasize that the magnitudes
Table 1: Various parton density sets with the values for the QCD scale $\Lambda$ and the running coupling $\alpha_s$.

| Variant  | $\Lambda$ (LO, MeV) | $\alpha_s$(LO)($M_Z$) | $\Lambda$ (NLO, MeV) | $\alpha_s$(NLO)($M_Z$) |
|----------|----------------------|-----------------------|----------------------|------------------------|
| MRST98 (LO, lo05a.dat) | $130.5$ | $0.125$ | $220$ | $0.175$ |
| MRST98 (NLO, ft08a.dat) | $181$ | $0.132$ | $202$ | $0.116$ |
| CTEQ4 (LO, cteq4l.tbl) | $131$ | $0.125$ | $173$ | $0.114$ |
| CTEQ4 (NLO, cteq4m.tbl) | $181$ | $0.125$ | $202$ | $0.1175$ |
| GRV98 (LO, grv98lo.grid) | $131$ | $0.125$ | $173$ | $0.114$ |
| GRV98 (NLO, grvnlm.grid) | $181$ | $0.125$ | $202$ | $0.1175$ |
| MRST99 (NLO, cor01.dat) | $220$ | $0.1175$ | $220$ | $0.1175$ |

of the cross sections are extremely sensitive to the choice of the renormalization scale because the effective coupling constant $G \sim \alpha_s(\mu_F)$, which implies that $d\sigma^{\text{LO}} \sim \alpha_s^3$ and $d\sigma^{\text{NLO}} \sim \alpha_s^4$. However the slopes of the differential distributions are less sensitive to the scale choice if they are only plotted over a limited range.

For the computation of the Higgs-gluon-gluon effective coupling constant $G$ given in Eq. (2.2) we choose the top-quark mass $m_t = 173.4$ GeV/c$^2$ and the Fermi constant $G_F = 1.16639$ GeV$^{-2} = 4541.68$ pb. In this paper we will only study Higgs boson production in proton-proton collisions at the center of mass energy $\sqrt{S} = 14$ TeV characteristic of the LHC. Since the hadrons $H_1$ and $H_2$ are now identical the $y$ differential cross sections are symmetric.

The effect of the NLO corrections to Higgs-boson production were already studied earlier by the authors in [14]. In order to compare with their results we present LO and NLO differential cross sections in $p_T$, integrated over $y$, for $m = 120$ GeV/c$^2$ and $\mu^2 = m^2 + p_T^2$ in Figs. 1a and 1b respectively. The MRST98 parton densities [32] were used for these plots. We note that the NLO results from the $q(\bar{q})g$ and $qq$ channels are negative at small $p_T$ so we have plotted their absolute values multiplied by 100. It is clear that the $gg$ subprocess dominates but the $q(\bar{q})g$-subprocess is also important. The corresponding results for the $y$ distributions integrated over the $p_T$ region between $p_{T,\text{min}} = 30$ GeV/c and $p_{T,\text{max}} = 240$ GeV/c are given in Figs. 2a and 2b respectively. The latter value was chosen because the cross section above 240 GeV/c is extremely small and can be neglected. Here the scale is $\mu^2 = m^2 + p_{T,\text{min}}^2$. Again we see that the $gg$-subprocess dominates. Using
the recent MRST99 set [35] we present the dependence of the NLO \( p_T \)- and \( y \)-distributions on the Higgs mass in Figs. 3 and 4 respectively.

Next we plot the scale dependence of the above differential cross sections in Figs. 5 and 6. For the \( p_T \)-distributions we have chosen the scale factors \( \mu = 2\mu_0, \mu = \mu_0 \) and \( \mu = \mu_0/2 \) with \( \mu_0^2 = m^2 + p_T^2 \). In the case of the \( y \)-distributions we adopted \( \mu_0^2 = m^2 + p_{T,\text{min}}^2 \) where \( p_{T,\text{min}} = 30 \text{ GeV}/c \) (see Eq. (6.15)). Furthermore we have again adopted the MRST98 parton density set in [32] since it contains both LO and NLO versions. In the case of the \( p_T \)-distribution in Fig. 5a one observes a small reduction in the scale dependence while going from LO to NLO. This reduction becomes more visible when we plot the quantity

\[
N \left( p_T, \frac{\mu}{\mu_0} \right) = \frac{d\sigma(p_T, \mu)/dp_T}{d\sigma(p_T, \mu_0)/dp_T}
\]  

(6.18)

in the range \( 0.1 < \mu/\mu_0 < 10 \) at fixed values of \( p_T = 30, 70 \) and 100 GeV/c. The upper set of curves at small \( \mu/\mu_0 \) are for LO and the lower set are for NLO. Notice that the NLO plots at 70 and 100 are extremely close to each other and it is hard to distinguish between them. Further one sees that the slopes of the LO curves are larger that the slopes of the NLO curves. This is an indication that there is better stability in NLO, which was expected. However there is no sign of a flattening or an optimum in either of these curves. This implies that one will have to calculate the differential cross sections in NNLO to find a better stability under scale variations. In the case of the \( y \)-distributions we adopted the scale \( \mu_0^2 = m^2 + p_{T,\text{min}}^2 \) where \( p_{T,\text{min}} = 30 \text{ GeV}/c \) (see Eq. (6.15)). As shown in Fig.6 there is hardly any reduction in the scale dependence for the \( y \)-distributions between the LO and NLO curves.

Besides the dependence on the factorization and renormalization scales there are two other uncertainties which affect the predictive power of the theoretical cross sections. The first one concerns the rate of convergence of the perturbation series which is indicated by the \( K \)-factor defined by

\[
K = \frac{d \sigma^{\text{NLO}}}{d \sigma^{\text{LO}}}
\]  

(6.19)

Another uncertainty is the dependence of the cross section on the specific
choice of parton densities, which can be expressed by the factors

\[ R^{\text{CTEQ}} = \frac{d \sigma^{\text{CTEQ}}}{d \sigma^{\text{MRST}}}, \quad R^{\text{GRV}} = \frac{d \sigma^{\text{GRV}}}{d \sigma^{\text{MRST}}}. \] (6.20)

The quantities in Eqs. (6.19) and (6.20) are plotted for the \( p_T \) distributions in Figs. 7a and 7b respectively where we have chosen the same parameters and scales \( (\mu = \mu_0) \) as in Figs. 5a,b and 6. Depending on the parton density set the \( K \)-factors shown in Fig. 7a are pretty large and vary from 1.4 at \( p_T = 30 \) GeV/c to 1.7 at \( p_T = 150 \) GeV/c. Here the CTEQ4 parton densities lead to the smallest \( K \)-factor whereas the MRST98 set provides us with the largest one with the results from the GRV98 set in between. In Fig. 7b we show the dependences of the ratios defined in Eq. (6.20) as a function of \( p_T \). From this figure we infer that both the GRV98 and the CTEQ4 densities lead to larger cross sections than those computed from MRST98. The difference between the results obtained from the latter set with respect to the other ones is smaller in NLO than in LO. Furthermore In LO there is a small decrease in the \( R \) values as a function of \( p_T \) which is in contrast to NLO where we observe an increase. In the case of the \( y \)-distributions in Fig. 8a again the CTEQ4 parton densities yield the smallest \( K \)-factor which does not vary much as a function of \( y \). The latter also holds for the GRV98 set where however the \( K \)-factor is larger than the one obtained from CTEQ4. The largest \( K \)-factor is obtained from the the MRST98 densities which show a much stronger dependence on the rapidity \( y \) than the ones obtained from the other sets. It becomes maximal at \( y = 0 \) and decreases at larger absolute values of the rapidity. In Fig. 8b we show the ratio \( R \) as a function of the rapidity. Like in Fig. 7b the difference between the cross sections obtained from the MRST set and the two other sets becomes smaller while going from LO to NLO. At \( y = 0 \) the discrepancy between the cross sections attains a maximum in the case of LO whereas it reaches a minimum for NLO. Notice that our results for the \( K \)-factors computed in NLO in Figs. 7a, 8a agree with those shown in the corresponding Figs. 2a, 2b in [14]. The authors of [14] informed us that they did not adopt a fixed scale at \( \mu_0^2 = p_{T,\text{min}}^2 + m^2 \) but they used a variable scale \( \mu_0^2 = p_T^2 + m^2 \) when integrating over \( p_T \) to calculate the rapidity \( y \)-distributions. We have also run our programs with the latter scale, which yields different rapidity-distributions. The rapidity cross sections become smaller due to the smaller
running coupling constant but the $R$-ratios hardly change w.r.t. the ones shown in Fig. 8b. Therefore the agreement between our results and those obtained in [14] are not spoiled. We also made a comparison between our results obtained for the differential cross section $d\sigma/dp_T$ and those shown in Fig. 1a of [14]. To compare we have read off the central values of the bins from their Fig. 1a and, after changing bin sizes, have replotted their values versus ours in Fig. 9 for the MRST98 set and the same input parameters. We only show the case where the scale is $\mu = \mu_0$. Fig. 9 shows a small difference in LO and NLO between our cross sections (indicated in Fig. 9 by RSN(1)) and the those given by [14] (indicated by FGK). However the slopes are exactly the same. The difference might be due to a different choice of the effective Higgs-gluon-gluon coupling constant in Eq. (2.2). We have chosen $m_t = 173.4 \text{ GeV}/c^2$ whereas the authors in [14] took the limit $m_t \to \infty$ (see Eq. (2.5)). If we take the value $m_t = 10^4 \text{ GeV}/c^2$, which is rather close to an infinite top quark mass, our curves (indicated in Fig. 9 by RSN(2)) approach the ones given in Fig. 1a of [14]. For further checks we also made a comparison with the LO $p_T$-distribution in Fig. 6 of [5] by reading off their values. Choosing the same input parameters and parton density set we completely obtain the same results as in [5]. Finally we checked several of the figures in [22]. In the latter reference only the NLO corrections to the $gg$-subprocess have been completed. Using the same parton densities and parameters we found excellent agreement between our results for this subprocess and those obtained in [22].

We now analyse why the MRST98, GRV98 and CTEQ4 parton densities yield different results for the differential cross sections. This can be mainly attributed to the small $x$-behaviour of the gluon density because gluon-gluon fusion is the dominant production mechanism. To investigate the small $x$-behaviour we consider the product of the gluon flux with the corresponding partonic cross section

$$F_{gg}(xS,p_T^2,\mu^2) = \Phi_{gg}(x,\mu^2) \frac{d\sigma_{gg}(xS,p_T^2,\mu^2)}{d\sigma_{gg}(xS,p_T^2,\mu^2)}$$ (6.21)

In this case we have removed the factor $G^2 \alpha_s/4\pi$ from the LO (Born) cross section and the factor $G^2 (\alpha_s/4\pi)^2$ from the NLO contribution to the partonic cross section. This was done to suppress the dependence on the strong coupling constant $\alpha_s$ so that the differences between the several $F_{gg}$ can be only attributed to the various parton density sets. In Fig. 10a we have
plotted the LO partonic cross section $d \sigma_{gg}^{(1)}/dp_T$ versus $\log_{10} x$ for different values for $p_T$. For $x < 10^{-3}$ this cross section rises steeply whereas it flattens out for $x > 10^{-3}$. The latter behaviour is changed when we look at the NLO partonic cross section $d \sigma_{gg}^{(2)}/dp_T$ plotted versus $\log_{10} x$ in Fig. 10b. Here the rise at small $x$ becomes even steeper whereas for $x > 10^{-3}$ the flatness shown by the Born cross section is replaced by a steep decrease when $x \to 1$ except at high $p_T$. Notice that the NLO partonic cross section becomes negative (here for $x \geq 8.0 \times 10^{-2}$) due to mass factorization as explained below Eq. (5.15).

To show the effect of the gluon flux we have plotted $F_{gg}(xS, p_t^2, m^2)$ as a function of $\log_{10} x$ in Fig. 11 for $p_T = 100$ GeV/c using three parton density sets in table 1. From Fig. 11 we infer that the GRV98 set contains the steepest gluon density whereas the gluon densities from the other sets are about equal. From this observation one can understand the relative ordering of the plots in Figs. 7a and 7b. Another feature is that the hadronic cross section, which is represented by the integral of $F_{gg}$ over $x$, receives its main support from the small $x$-region. Using the representation for the hadronic cross section $d\sigma/dp_T$ as given in Eq. (B.7) we have computed this quantity where now all subprocesses are included. Further we have chosen two different integration regions. The first one is the full range $x_{\text{min}} \leq x \leq x_{\text{max}}$ with $x_{\text{max}} = 1$ (see Eq. (B.8) and the second range is given by $x_{\text{max}} = 5 \times x_{\text{min}}$. In Fig. 12 we have shown the $p_T$-distributions due to the two different integration regions. They do not differ by more than 10% which shows that the whole differential cross section is dominated by the small $x$-region $x < 5 \times x_{\text{min}}$. Hence apart from the different coupling constants the difference between the cross sections is due to the different gluon densities. Future HERA data will have to provide us with unique gluon densities before we can make more accurate predictions for the Higgs differential distributions. Note that we have chosen the axes in Fig. 12 to coincide with those in Fig. 1b so that one can see the $p_T$ dependence of the $K$-factor for these two densities by overlaying the plots.

After having studied the small $x$-region we now investigate the large $x$-region of the partonic cross sections where S+V gluons and collinear quark anti-quark pairs dominate the radiative corrections. The S+V gluon part of the partonic cross section is obtained by omitting the hard contributions which are regular at $s_4 = 0$ so that we only keep the singular parts presented in Eqs. (5.16)-(5.20). Furthermore we include the S+V partonic cross
sections in Eqs. (5.24)-(5.26). These two contributions constitute the S+V gluon approximation. To study its validity we compute in NLO the ratio
\[ R_{S+V} = \frac{d\sigma^{S+V}}{d\sigma^{\text{EXACT}}}, \]
for the \( p_T \) and \( y \) distributions which are given in Figs. 13 and 14 respectively. Here we use the MRST99 parton density set. One expects that the approximation becomes better at larger transverse momenta where \( p_T = p_{T,\text{max}} \) in Eq. (6.13) at the boundary of phase space, which leads to \( x = 1 \) using Eq. (B.8). However in Fig. 13 the highest value of \( p_T \), given by \( p_T = 150 \text{ GeV/c} \), is still very small with respect to \( p_{T,\text{max}} \sim \sqrt{S}/2 = 7 \times 10^5 \text{ GeV/c} \). Therefore it is rather fortuitous that the approximation works so well for \( p_T > 100 \text{ GeV/c} \) where one obtains \( R_{S+V} < 1.2 \). The boundary of phase space is also approached when the Higgs mass increases. This can also be inferred from Eq. (B.8) where at fixed \( p_T \), \( x_{\text{min}} \rightarrow 1 \) when \( m^2 \rightarrow S - 2\sqrt{S} p_T \). Therefore we have plotted \( R_{S+V} \) for various Higgs boson masses in Fig. 13. This figure reveals that the largest Higgs mass leads to the worst approximation contrary to our expectations which means that the kinematics is not in the large \( x \)-region. In Fig. 14 we plot the ratio \( R_{S+V} \) for the \( y \)-distributions for the same mass range as in Fig. 13 where we have chosen the cut \( p_{T,\text{min}} = 30 \text{ GeV/c} \) in Eq. (6.15). Here the approximation is less good than for the \( p_T \) distributions in Fig. 13 and like in the latter figure it becomes better when the mass of the Higgs gets smaller. At the lowest value chosen for the mass i.e. \( m = 120 \text{ GeV/c}^2 \) we observe that \( R_{S+V} \sim 1.3 \). Furthermore we see no variation in \( R_{S+V} \) with respect to the values of \( y \). The approximation becomes better, see Fig. 15, if \( p_{T,\text{min}} \) is chosen to be larger than 30 GeV/c so that it becomes closer to the values given in Fig. 13. Finally the figures discussed above show that the S+V gluon approximation overestimates the exact NLO result. However this overestimate becomes smaller when the transverse momentum gets larger. In particular for \( p_T > 200 \text{ GeV/c} \) the S+V approximation is good enough so that resummation techniques can be used to give a better estimate of Higgs boson production corrected up to all orders in perturbation theory. This statement will also hold when Higgs boson production is described according to the standard model approach where the boson is coupled via top-quark loops to the gluons without taking \( m_t \rightarrow \infty \) as we did above. Although the \( p_T \) distributions will change for the exact and S+V gluon approximation at large \( p_T \), the ratio \( R_{S+V} \) in Fig. 13 will be
less affected by the large top quark mass approach because of cancellations
between the numerator and denominator in Eq. (6.22). Notice that the S+V
gluon approach in the case of the exact cross section can be obtained from
Eqs. (5.16)-(5.20) and (5.24)-(5.26) by replacing the approximate Born ma-
trix elements $M_{ij \to k H}^{(1)}$, obtained from the effective Lagrangian in Eq. (2.1),
by the exact expressions presented in [4], [5], [6].

Summarizing the above we have calculated the NLO corrections to the
differential cross section for Higgs boson production in the large top quark
mass approach which can be obtained from an effective Lagrangian. The
calculation was carried out in standard $n$-dimensional regularization where
the $\overline{\text{MS}}$ scheme was chosen for renormalization and mass factorization. We
have presented some of the NLO results in the region where the effe-
cctive Lagrangian should be trustworthy. It turns out that the gluon-gluon subprocess
dominates the hadronic cross section but the (anti-)quark-gluon subprocess
is certainly not negligible. For the transverse momentum distributions there
is a small reduction in scale dependence while going from the LO to the
NLO cross section which is not observed for the rapidity distributions. Fur-
ther there is still a large uncertainty in our predictions because the $K$-factor
varies from approximately 1.4 to 1.7 depending on the parton density set.
Also the dependence on the running coupling and the parton density set is
appreciable. The latter is mainly due to the small $x$ behaviour of the various
gluon densities since both the partonic cross sections and the gluon densities
increase very steeply at decreasing $x$. Finally we have shown that the S+V
approximation is quite reasonable provided $p_{T,\text{min}} > 100 \text{ GeV}/c$ in spite of
the fact that $x$ is still too small to belong to the large $x$-region. This means
that this approximation can be used to resum the large corrections due to
S+V gluons in order to obtain a better estimate of the all order corrected
cross section.

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Appendix A

Here we list the soft contributions to the partonic cross sections as defined in Eq. (4.42) which were not explicitly given in section 4. For the various processes we obtain

\[
s^2 \frac{d^2 \sigma^{\text{SOFT}}_{gg \rightarrow g H}}{d t d u} = \pi \delta(s + t + u - m^2) S^2 \varepsilon G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \frac{N}{(N^2 - 1)^2} \times \left[ \frac{6}{\varepsilon^2} + \left( 4 \ln \frac{\Delta}{\mu^2} + 2 \ln \frac{tu}{\mu^2 s} - \frac{47}{6} \right) \frac{1}{\varepsilon} + \ln \frac{\Delta}{\mu^2} \ln \frac{tu}{\mu^2 s} \\
+ \frac{3}{2} \ln^2 \frac{\Delta}{\mu^2} + \frac{1}{2} \ln \left( \frac{tu}{\mu^2 s} \right) - \frac{59}{12} \ln \frac{\Delta}{\mu^2} - \frac{35}{12} \ln \frac{tu}{\mu^2 s} \right] \left| M^{(1)}_{gg \rightarrow g H} \right|^2, \quad (A.1)
\]

\[
s^2 \frac{d^2 \sigma^{\text{SOFT}}_{qq \rightarrow q H}}{d t d u} = \pi \delta(s + t + u - m^2) S^2 \varepsilon G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 n_f \frac{1}{(N^2 - 1)^2} \times \left[ \frac{1}{3\varepsilon} + \frac{1}{6} \ln \frac{\Delta}{\mu^2} + \frac{1}{6} \ln \frac{tu}{\mu^2 s} - \frac{11}{18} \right] \left| M^{(1)}_{gg \rightarrow g H} \right|^2, \quad (A.2)
\]

\[
s^2 \frac{d^2 \sigma^{\text{SOFT}}_{q\bar{q} \rightarrow gg H}}{d t d u} = \pi \delta(s + t + u - m^2) S^2 \varepsilon G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \frac{1}{N^2} \times \left[ C_A \left\{ \frac{2}{\varepsilon^2} + \left( 2 \ln \frac{tu}{\mu^2 s} - \frac{11}{6} \right) \frac{1}{\varepsilon} + \ln \frac{\Delta}{\mu^2} \ln \frac{tu}{\mu^2 s} - \frac{1}{2} \ln^2 \frac{\Delta}{\mu^2} \\
+ \frac{1}{2} \ln^2 \frac{tu}{\mu^2 s} - \frac{11}{12} \ln \frac{\Delta}{\mu^2} - \frac{11}{12} \ln \frac{tu}{\mu^2 s} - \frac{3}{2} \xi(2) + \frac{67}{36} \right\} \\
+ C_F \left\{ \frac{4}{\varepsilon^2} + \left( 4 \ln \frac{\Delta}{\mu^2} \right) \frac{1}{\varepsilon} + 2 \ln^2 \frac{\Delta}{\mu^2} - \zeta(2) \right\} \right] \left| M^{(1)}_{q\bar{q} \rightarrow g H} \right|^2, \quad (A.3)
\]
\[
\frac{s^2 d^2 \hat{\sigma}^{\text{SOFT}}_{qg\rightarrow q\bar{g} H}}{d t d u} = \pi \delta(s + t + u - m^2) S_\varepsilon^2 G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \frac{1}{N(N^2 - 1)}
\times \left[ C_A \left\{ \frac{2}{\varepsilon^2} + \left( 2 \ln \frac{\Delta}{\mu^2} - 1 \right) \frac{1}{\varepsilon} + \ln^2 \frac{\Delta}{\mu^2} - \ln \frac{\Delta}{\mu^2} \right. \right.
\left. - \frac{1}{2} \zeta(2) + \frac{1}{2} \right] \right.
\left. + C_F \left\{ \frac{4}{\varepsilon^2} + \left( 2 \ln \frac{\Delta}{\mu^2} + 2 \ln \frac{t u}{\mu^2 s} - \frac{7}{2} \right) \frac{1}{\varepsilon} + \ln \frac{\Delta}{\mu^2} \ln \frac{t u}{\mu^2 s} \right. \right.
\left. + \frac{1}{2} \ln^2 \frac{\Delta}{\mu^2} + \frac{1}{2} \ln^2 \frac{t u}{\mu^2 s} - \frac{7}{4} \ln \frac{\Delta}{\mu^2} - \frac{7}{4} \ln \frac{t u}{\mu^2 s} \right. \right.
\left. - 2\zeta(2) + \frac{7}{2} \right] \left| M_{qg\rightarrow q H}^{(1)} \right|^2 , \quad (A.4)
\]

\[
\frac{s^2 d^2 \hat{\sigma}^{\text{SOFT}}_{q\bar{q} \rightarrow q\bar{q} H}}{d t d u} = \pi \delta(s + t + u - m^2) S_\varepsilon^2 G^2 \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 n_f \frac{1}{N^2}
\times \left[ \frac{1}{3\varepsilon} + \frac{1}{6} \ln \frac{\Delta}{\mu^2} + \frac{1}{6} \ln \frac{t u}{\mu^2 s} - \frac{5}{18} \right] \left| M_{q\bar{q} \rightarrow q H}^{(1)} \right|^2 . \quad (A.5)
\]
Appendix B

There is an alternative way to obtain the transverse momentum distribution as presented in Eq. (6.14). First one computes the partonic cross section

\[
\frac{d \sigma_{ab}}{d p_T^2} (s, p_T^2, m^2, \mu^2) = \int_0^{s_{4,\text{max}}} ds_4 \frac{d^2 \sigma_{ab}}{d p_T^2 d s_4} (s, p_T^2, s_4, m^2, \mu^2),
\]

with

\[
\frac{d^2 \sigma_{ab}}{d p_T^2 d s_4} (s, p_T^2, s_4, m^2, \mu^2) = \frac{s}{\sqrt{(s + m^2 - s_4)^2 - 4 s (p_T^2 + m^2)}}
\times \left[ \frac{d^2 \sigma_{ab}}{d t d u} (s, t, u, m^2, \mu^2) + \frac{d^2 \sigma_{ab}}{d t d u} (s, u, t, m^2, \mu^2) \right],
\]

and

\[
s_{4,\text{max}} = s + m^2 - 2 \sqrt{s(p_T^2 + m^2)},
\]

The reason is that if one changes the variables \(t\) and \(u\) into \(s_4\) and \(p_T^2\) one has two possibilities

\[
t = \frac{1}{2} \left[ s_4 + m^2 - s + \sqrt{(s + m^2 - s_4)^2 - 4 s (p_T^2 + m^2)} \right] \equiv t_1,
\]

\[
u = \frac{1}{2} \left[ s_4 + m^2 - s - \sqrt{(s + m^2 - s_4)^2 - 4 s (p_T^2 + m^2)} \right] \equiv u_1,
\]

or

\[
t = \frac{1}{2} \left[ s_4 + m^2 - s - \sqrt{(s + m^2 - s_4)^2 - 4 s (p_T^2 + m^2)} \right] \equiv t_2 = u_1,
\]

\[
u = \frac{1}{2} \left[ s_4 + m^2 - s + \sqrt{(s + m^2 - s_4)^2 - 4 s (p_T^2 + m^2)} \right] \equiv u_2 = t_1,
\]

which follows from the substitution

\[
\sinh y = \pm \frac{\sqrt{(s + m^2 - s_4)^2 - 4 s (p_T^2 + m^2)}}{2 \sqrt{s \sqrt{p_T^2 + m^2}}}, \quad \cosh y = \frac{s + m^2 - s_4}{2 \sqrt{s \sqrt{p_T^2 + m^2}}},
\]

\[(B.6)\]
in Eq. (B.2). Hence one has to compute the sum $d\sigma(t_1, u_1) + d\sigma(t_2, u_2)$ which is equal to $d\sigma(t_1, u_1) + d\sigma(u_1, t_1)$. Notice that when the partonic cross section is symmetric in $t$ and $u$ one can replace the sums above by $2\, d\sigma(t_1, u_1)$. This does not apply to the quark-gluon subprocess because here the cross section is asymmetric in $t$ and $u$. The hadronic cross section is now obtained from

$$
\frac{d\sigma^{H_1H_2}}{dp_T}(S, p_T^2, m^2) = \sum_{a,b=q,g} \int_{x_{\min}}^{x_{\max}} dx \, \Phi_{ab}^{H_1H_2}(x, \mu^2) \frac{d\sigma_{ab}}{dp_T}(x, S, p_T^2, m^2, \mu^2),
$$

with

$$
x_{\min} = \frac{m^2 + 2p_T^2 + 2\sqrt{p_T^2(p_T^2 + m^2)}}{S}, \quad x_{\max} = 1, \quad (B.7)
$$

and $\Phi_{ab}$ denotes the partonic flux defined by

$$
\Phi_{ab}^{H_1H_2}(x, \mu^2) = \int_0^1 dx_1 \int_0^1 dx_2 \, \delta(x - x_1 x_2) f_a^{H_1}(x_1, \mu^2) f_b^{H_2}(x_2, \mu^2). \quad (B.9)
$$

The lower boundary $x_{\min}$ follows from the inequality in Eq. (6.13) where $s = x \cdot S$. We have checked that expressions Eqns. (6.14) and (B.7) lead to the same result.
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Figure Captions

Fig. 1a. The differential cross section $d\sigma/dp_T$ integrated over the whole rapidity range (see Eq. (6.14)) with $m = 120$ GeV/$c^2$ and $\mu^2 = m^2 + p_T^2$. The LO plots are presented for the subprocesses $gg$ (long-dashed line), $q(\bar{q})g$ (dot-dashed line) and $100 \times (q\bar{q})$ (dotted line) using the parton density set MRST98(lo05a.dat).

Fig. 1b. Same as Fig. 1a in NLO except for $100 \times \text{abs}(q\bar{q})$ (dotted line) and the additional subprocess $100 \times \text{abs}(qq)$ (short-dashed line) using the parton density set MRST98(ft08a.dat).

Fig. 2a. The differential cross section $d\sigma/dy$ for $p_T,_{\text{min}} = 30$ GeV/$c$ and $p_T,_{\text{max}} = 240$ GeV/$c$ (see Eq. (6.15)) with $m = 120$ GeV/$c^2$ and $\mu^2 = m^2 + p_T^2,_{\text{min}}$. The LO plots are presented for the subprocesses $gg$ (long-dashed line), $q(\bar{q})g$ (dot-dashed line) $100 \times q(\bar{q})$ (dotted line) using the parton density set MRST98(lo05a.dat).

Fig. 2b. Same as Fig. 2a in NLO with the additional subprocess $100 \times qq$ (short-dashed line) using the parton density set MRST98(ft08a.dat).

Fig. 3. The mass dependence of $d\sigma^{\text{NLO}}/dp_T$ (Eq. (6.14)) using the set MRST99 with $\mu^2 = m^2 + p_T^2$ for Higgs masses $m = 120$ GeV/$c^2$ (solid line), $m = 160$ GeV/$c^2$ (dashed line) and $m = 200$ GeV/$c^2$ (dot-dashed line).

Fig. 4. The mass dependence of $d\sigma^{\text{NLO}}/dy$ for $p_T,_{\text{min}} = 30$ GeV/$c$ (Eq. (6.15)) using the set MRST99 with $\mu^2 = m^2 + p_T^2,_{\text{min}}$. The notation is the same as in Fig. 3.

Fig. 5a. The scale dependence of $d\sigma^{\text{LO}}/dp_T$ integrated over the whole rapidity range (see Eq. (6.14)) with $m = 120$ GeV/$c^2$, $\mu_0^2 = m^2 + p_T^2$ using the MRST98 parton density sets. The results are shown for $\mu = \mu_0/2$ (dashed line), $\mu = \mu_0$ (solid line) and $\mu = 2 \times \mu_0$ (dot-dashed line). The upper three curves are the NLO results.

Fig. 5b. The quantity $N(p_T, \mu/\mu_0)$ (see Eq. (6.18)) plotted in the range $0.1 < \mu/\mu_0 < 10$ at fixed values of $p_T$ with $m = 120$ GeV/$c^2$ and $\mu_0^2 = m^2 + p_T^2$ using the MRST98 parton density sets. The results are
shown for \( p_T = 30 \) GeV/c (solid line), \( p_T = 70 \) GeV/c (dashed line), \( p_T = 100 \) GeV/c (dot-dashed line). The upper three curves on the left hand side are the LO results whereas the lower three curves refer to NLO.

**Fig. 6.** The scale dependences of \( d \sigma^\text{LO} / dy \) and \( d \sigma^\text{NLO} / dy \) integrated over the region \( p_{T,\text{min}} = 30 \) GeV/c and \( p_{T,\text{max}} = 240 \) GeV/c (see Eq. (6.15)). The notation and parameters are as in Fig.5.

**Fig. 7a.** The K-factors in Eq. (6.19) for \( d \sigma / dp_T \) integrated over the whole rapidity range (see Eq. (6.14)) with \( m = 120 \) GeV/c\(^2\) and \( \mu^2 = m^2 + p_T^2 \) for the MRST98 sets (solid line), the GRV98 sets (dot-dashed line), and the CTEQ98 sets, (dashed line).

**Fig. 7b.** The ratios \( R \) in Eq. (6.20) for the same differential cross sections as in Fig. 7a, \( R^{\text{GRV}} \) in LO (solid line), \( R^{\text{GRV}} \) in NLO (dot-dashed line), \( R^{\text{CTEQ}} \) in LO (dashed line) and \( R^{\text{CTEQ}} \) in NLO (dotted line).

**Fig. 8a** The K-factors in Eq. (6.19) for \( d \sigma / dy \) integrated over the region \( p_{T,\text{min}} = 30 \) GeV/c and \( p_{T,\text{max}} = 240 \) GeV/c (see Eq. (6.15)) with \( m = 120 \) GeV/c\(^2\) and \( \mu^2 = m^2 + p_{T,\text{min}}^2 \). The parton density sets and notation are as in Fig. 7a.

**Fig. 8b.** The ratios \( R \) in Eq. (6.20) for the same differential cross sections as in Fig. 8a. The notation is the same as in Fig. 7b.

**Fig. 9.** Comparisons of \( d \sigma / dp_T \) in LO and NLO obtained from our calculation (RSN) versus those in Fig. 1a in [14] (FGK), for \( m_t = 173.4 \) GeV/c\(^2\) RSN(1) (solid line), \( m_t = 10^4 \) GeV/c\(^2\) RSN(2) (long dashed line) and \( m_t = \infty \) FGK (dot-dashed line). The upper curves are for NLO and the lower ones for LO.

**Fig. 10a.** The partonic differential cross section \( d \sigma^{(1)}_{gg} / dp_T \) (Born) with \( m = 120 \) GeV/c\(^2\) and \( \mu^2 = m^2 + p_T^2 \) for \( p_T = 200 \) GeV/c (solid line), \( p_T = 150 \) GeV/c (long-dashed line), \( p_T = 100 \) GeV/c (dot-dashed line), \( p_T = 75 \) GeV/c (short-dashed line) and \( p_T = 50 \) GeV/c (dotted line).

**Fig. 10b.** Same as in Fig. 10a but now for the NLO differential cross section \( d \sigma^{(2)}_{gg} / dp_T \).
**Fig. 11.** The flux multiplied by the partonic cross section represented by the quantity $F_{gg}(xS, p_T^2, m^2)$ in Eq. (6.21) with $p_T = 100$ GeV/c, $m = 120$ GeV/c$^2$ and $\mu^2 = m^2 + p_T^2$ plotted as a function of $\log_{10} x$ for MRST98 (solid lines), GRV98 (dot-dashed lines) and CTEQ4 (dashed lines). The upper curves are for NLO and the lower ones for LO.

**Fig. 12.** The NLO hadronic cross section $d\sigma/dp_T$ obtained from Eq. (B.7) (all subprocesses included) with $m = 120$ GeV/c$^2$ and $\mu^2 = m^2 + p_T^2$. The parton density set is GRV98 with $x_{\text{max}} = 1$ (solid line) and with $x_{\text{max}} = 5 x_{\text{min}}$ (dashed line).

**Fig. 13.** The ratio $R^{S+V}$ in Eq. (6.22) for the $p_T$ distributions using the set MRST99 with $\mu^2 = m^2 + p_{T,\text{min}}^2$ and various Higgs masses given by $m = 120$ GeV/c$^2$ (solid line), $m = 160$ GeV/c$^2$ (dashed line) and $m = 200$ GeV/c$^2$ (dot-dashed line).

**Fig. 14.** The ratio $R^{S+V}$ in Eq. (6.22) for the $y$ distributions using the MRST99 set with $\mu^2 = m^2 + p_{T,\text{min}}^2$ for $p_{T,\text{min}} = 30$ GeV/c and various Higgs masses given by $m = 120$ GeV/c$^2$ (solid line), $m = 160$ GeV/c$^2$ (dashed line) and $m = 200$ GeV/c$^2$ (dot-dashed line).

**Fig. 15.** The ratio $R^{S+V}$ in Eq. (6.22) for the $y$ distributions using the MRST99 set with $\mu^2 = m^2 + p_{T,\text{min}}^2$ and $m = 120$ GeV/c$^2$, for various values of $p_{T,\text{min}}$, namely $p_{T,\text{min}} = 100$ GeV/c (solid line), $p_{T,\text{min}} = 200$ GeV/c (dashed line), $p_{T,\text{min}} = 250$ GeV/c (dot-dashed line) and $p_{T,\text{min}} = 300$ GeV/c (dotted line).
NLO(sum)

$gg$

$q(q)g$

$100*\text{abs}(qq)$

$100*\text{abs}(q\bar{q})$
The diagram shows the differential cross-section $d\sigma/dy$ (in pb) as a function of $y$, with various processes contributing:

- **NLO(sum)**
- **gg**
- **q\(\bar{q}\)g**
- **100*q\(\bar{q}\)**
- **100*q\(\bar{q}\)**

The processes are represented by different line styles and colors, allowing for a clear comparison of their contributions to the total cross-section.
\[ \frac{d\sigma}{dp_T} (\text{pb/GeV}) \]

- \( \mu = \mu_0 / 2 \)
- \( \mu = \mu_0 \)
- \( \mu = 2 \mu_0 \)

\( p_T (\text{GeV}) \)
\begin{align*}
N(p_T, \mu/\mu_0) &= \text{LO}(100) \\
N(p_T, \mu/\mu_0) &= \text{LO}(70) \\
N(p_T, \mu/\mu_0) &= \text{LO}(30) \\
N(p_T, \mu/\mu_0) &= \text{NLO}(100) \\
N(p_T, \mu/\mu_0) &= \text{NLO}(70) \\
N(p_T, \mu/\mu_0) &= \text{NLO}(30)
\end{align*}
\[ \frac{d\sigma}{dy}\text{(pb)} \]

\[ \mu = \frac{\mu_0}{2} \]

\[ \mu = \mu_0 \]

\[ \mu = 2\mu_0 \]
\[
\frac{d\sigma}{dp_T}\text{(pb/GeV)}
\]

- RSN(1)
- FGK
- RSN(2)
The graph shows the differential cross-section $d\sigma^{(1)}_{gg}/dp_T$ (pb/GeV) as a function of $\log_{10} x$ for different values of $p_T$. The lines represent different $p_T$ values:

- Solid line: $p_T = 200$
- Dashed line: $p_T = 150$
- Dot-dashed line: $p_T = 100$
- Dotted line: $p_T = 75$
- Dot-dashed line: $p_T = 50$
\[ \frac{d\sigma}{dp_T} (pb/GeV) \]

\[ \log_{10} x \]

- \( p_T = 200 \)
- \( p_T = 150 \)
- \( p_T = 100 \)
- \( p_T = 75 \)
- \( p_T = 50 \)
\[ \frac{d\sigma}{dp_T} (\text{pb}/\text{GeV}) \]

- \( x_{\text{max}} = 1 \)
- \( x_{\text{max}} = 5 \times x_{\text{min}} \)
