$F(R)$ gravity inflationary model with $(R + R_0)^{3/2}$ term

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Abstract

The proposed inflationary model, which is a one-parametric generalization of the Starobinsky $R + R^2$ model, includes the $(R + 3m^2\beta^2)^{3/2}$ term, where the parameter $m$ is the inflaton mass, defined in the same way as in the Starobinsky model, and $\beta$ is a dimensionless constant. Using the conformal transformation and the Einstein frame potential, we obtain the inflationary parameters of the model proposed. The value of the tensor-to-scalar ratio $r$ is bigger than in the Starobinsky model. The considered inflationary model produces a good fit to current observation data.

1 Introduction

The simplest and the most popular extension to the General relativity (GR) is the $F(R)$ gravity [1, 2, 3, 4, 5], in which $F$ is an arbitrary differentiable function of the Ricci scalar $R$. Inflationary scenarios in context of $F(R)$ gravity are actively studied [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25].

The historically first $F(R)$ gravity inflationary model is the Starobinsky $R + R^2$ model [6], described by the following action:

$$S_{\text{Star}}[g_{\mu\nu}] = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} \left( R_J + \frac{1}{6m^2} R_J^2 \right),$$

where the reduced Planck mass $M_{\text{Pl}}$ and the inflaton mass $m$ are introduced. The Ricci scalar curvature $R_J$ is defined by the metric $g_{\mu\nu}$, with the spacetime signature $(-, +, +, +)$.

The Starobinsky model is known as the excellent model of large-field slow-roll cosmological inflation with a good agreement to the Planck measurements of the Cosmic Microwave Background (CMB) radiation [26, 27]. The inflaton mass $m$ is fixed by CMB measurements of the amplitude of scalar perturbations $A_s$. Note that the values of the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$ do not depend on $m$. This model is just the simplest model of inflation that has the maximal predictive power. However, the value of tensor-to-scalar ratio $r$ is not known, and maybe will be observed by future experiments. If the observed value of the tensor-to-scalar ratio $r$ will be different from its value in the Starobinsky model, then some corrections of this model will be required.

There are two main ways to generalize the Starobinsky model without adding scalar fields or other matter. One can either add string theory inspired terms [28, 29, 30, 31, 32, 33, 34] or construct

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a new $F(R)$ gravity model, continuously connected to the Starobinsky model [9, 10, 11, 12, 13, 14, 16, 17, 18, 22, 23, 24, 35]. In this paper, we choose the second way.

In Ref. [24], a few one-parameter generalizations of the Starobinsky $R + R^2$ inflationary model have been proposed. In particular, it has been shown that the adding of $R^{3/2}$ term allows to construct viable inflationary model with the tensor-to-scalar ratio $r$ in four times more than in the original Starobinsky model. Cosmological models with $R^3$ and $(R - R_0)^2$ terms, where $\gamma$ is not an integer number are actively investigated [13, 14, 25, 36, 37, 38, 39], in particular, models with $R^{3/2}$ term have been investigated in context of integrability and symmetries [5, 40, 37]. The $R^{3/2}$-term can appear in an approximate description of the Higgs field with a small cubic term in its scalar potential and a large non-minimal coupling to $R$ [41] and in the (chiral) modified supergravity models [28, 29].

At the same time, any model with the $R^{3/2}$ term is ill-defined at $R < 0$, whereas the Starobinsky model is well defined and has no ghost for all $R > -3m^2$. In this paper, we propose a new inflationary model by adding the $(R + R_0)^{3/2}$-term with a positive constant $R_0$. The propose model is well-defined for all $R > -R_0$, where $R_0 > 0$.

2 $F(R)$ models and the corresponding scalar potentials

The generic $F(R)$ gravity theories have the following action

$$S_F[g_{\mu\nu}] = \int d^4x \sqrt{-g} F(R_J)$$  \hspace{1cm} (2)

with a differentiable function $F$.

Varying action (2) one gets the following equations:

$$F^{(1)} R_J^{\mu\nu} - \frac{1}{2} F \delta^{\mu\nu} - (g^{\mu\rho} \nabla_{\rho} \partial_{\nu} - \delta^{\mu\nu} \Box) F^{(1)} = 0,$$  \hspace{1cm} (3)

where $F^{(1)}$ is the first derivative of $F(R_J)$ with respect to $R_J$.

To avoid graviton as a ghost and scalaron (inflaton) as a tachyon one should put the following conditions [42, 43]:

$$F^{(1)}(R_J) > 0 \quad \text{and} \quad F^{(2)}(R_J) > 0,$$  \hspace{1cm} (4)

that restrict possible values of parameters and $R_J$. For the Starobinsky model (1), the first condition in (4) is equivalent to $R_J > -3m^2$.

The $F(R)$ gravity action (2) can be rewritten as

$$S_J = \int d^4x \sqrt{-g} \left[ F_\sigma (R_J - \sigma) + F \right],$$  \hspace{1cm} (5)

where a new scalar field $\sigma$ has been introduced, and $F_\sigma(\sigma) = \frac{dF(\sigma)}{d\sigma}$. If $F_\sigma(\sigma) \neq 0$, then eliminating $\sigma$ via equation $R_J = \sigma$, one yields back the action (2).

After the Weyl transformation of the metric $g_{\mu\nu} = \frac{2F_\sigma(\sigma)}{M_{Pl}^2} g^{J\mu\nu}$ one gets the following action in the Einstein frame [44]:

$$S_E = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R_E - \frac{h(\sigma)}{2} g^{\mu\nu} \partial_{\mu} \sigma \partial_{\nu} \sigma - V_E(\sigma) \right],$$  \hspace{1cm} (6)
where
\[ h(\sigma) = \frac{3M_P^4}{2F_{\sigma}^2} F_{\sigma}\sigma \] and
\[ V_E(\sigma) = M_P^4 \frac{F_{\sigma}\sigma - F}{4F_{\sigma}^2}. \] (7)

Note that the considering Weyl transformation is well-defined for \( F,\sigma > 0 \) only.

To get the action \( S_E \) with the standard form:
\[
S_E = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R_E - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right],
\] (8)

one should use the canonical scalar field \( \phi \):
\[
\phi = \sqrt{\frac{3}{2}} M_P \ln \left[ \frac{2}{M_P^4 F_{\sigma}} \right].
\] (9)

It has been shown in [24], that the variable
\[
y = \exp \left( -\frac{2}{3} M_P \phi \right) = \frac{M_P^2}{2F_{\sigma}}
\] (10)

is useful for considering of generalization of the Starobinsky inflationary model, because it is small during inflation. As a function of \( y \) the scalar potential corresponding to the Starobinsky model has the following form:
\[
V_{\text{Star}}(y) = V_0 (1 - y)^2, \quad \text{where} \quad V_0 = \frac{3}{4} m^2 M_P^2.
\] (11)

In the slow-roll approximation, the evolution of the scalar field is defined by the following equation (see [24] for detail):
\[
y' = \frac{2y^2 V_{E,y}}{3V_E}.
\] (12)

where the prime denotes the derivatives with respect to e-folding number
\[
N_e = \ln \left( \frac{a_{\text{end}}}{a} \right).
\]

We fix \( a_{\text{end}} \) as the value of \( a \) at the end of inflation, so the exit from inflation appears at \( N_e = 0 \).

The slow-roll parameters are defined as follows:
\[
\epsilon = \frac{y^2}{3} \left( \frac{V_{E,y}}{V} \right)^2, \quad \eta = \frac{2y}{3V} \left( V_{E,y} + y V_{E,yy} \right).
\] (13)

The main cosmological parameters of inflation are given by the scalar spectral index \( n_s \) and the tensor-to-scalar ratio \( r \), whose values are constrained by the combined Planck, WMAP and BICEP/Keck observations of CMB as [26, 27]
\[
n_s = 0.9649 \pm 0.0042 \quad (68\% \text{CL}) \quad \text{and} \quad r < 0.036 \quad (95\% \text{CL}).
\] (14)

In the slow-roll approximation, these parameter are connected with the slow-roll parameters as follows [45]:
\[
n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon.
\] (15)

The amplitude of scalar perturbations is given by
\[
A_s = \frac{2V_E}{3\pi^2 M_P^4 r}.
\] (16)

Its value observed by the Planck telescope [26] is \( A_s = 2.1 \times 10^{-9} \).
The generic inflationary model with \((R + R_0)^{3/2}\) term

In Ref. [24], the model the \(R^{3/2}\) term has been proposed:

\[
F(R_J) = \frac{M_{Pl}^2}{2} \left[ R_J + \frac{1}{6m^2} R_J^2 + \frac{\delta}{m} R_J^{3/2} \right],
\]

where \(\delta \geq 0\) is a dimensionless parameter.

Let us consider the following modification of the action (17):

\[
F(R_J) = \frac{M_{Pl}^2}{2} \left[ \left( 1 - \frac{3\beta^2}{2} \right) R_J + \frac{R_J^2}{6m^2} + \frac{\delta}{m} (R_J + \beta^2 m^2)^{3/2} - \frac{m^2 \beta^3 \delta}{2} \right].
\]

This model includes two dimensionless parameters \(\delta\) and \(\beta\). To construct new one-parametric generalizations of the Starobinsky model, we plan to connect these parameters.

The first derivative

\[
F^{(1)}(R_J) = \frac{M_{Pl}^2}{4} \left( 2 - 3\beta \right) + \frac{M_{Pl}^2}{6m^2} R_J + \frac{3M_{Pl}^2 \delta}{4m} \sqrt{\beta^2 m^2 + R_J} > \frac{M_{Pl}^2}{2}
\]

for any \(R_J > 0\) and \(F^{(1)}(0) = M_{Pl}^2/2\) as in the Starobinsky model. The second derivative

\[
F^{(2)}(R_J) = \frac{M_{Pl}^2}{6m^2} + \frac{3M_{Pl}^2 \delta}{8m \sqrt{\beta^2 m^2 + R_J}} > 0
\]

for any \(R_J > -\beta^2 m^2\) and \(\delta \geq 0\). So, the model is well-defined for any values of parameters \(\beta \neq 0\) and \(\delta > 0\) at \(R_J > R_0\), where \(R_0 < 0\).

The function \(F(R_J)\) has a correct GR limit at \(R_J \ll m^2\):

\[
F = \frac{M_{Pl}^2}{2} R_J \left[ 1 + \frac{4\beta + 9\delta}{24\beta} - \frac{\delta}{16\beta^3} \tilde{\sigma}^2 + \frac{3\delta}{128\beta^5} \tilde{\sigma}^3 + O(\tilde{\sigma}^4) \right],
\]

where \(\tilde{\sigma} = R_J/m^2\).

To get inflationary parameters we construct the corresponding scalar potential (7):

\[
V_E(\tilde{\sigma}) = \frac{4V_0 \left( 6\beta^2 \delta + 3\delta \sqrt{\beta^2 + \tilde{\sigma} (\tilde{\sigma} - 2\beta^2) + \tilde{\sigma}^2} \right)}{ \left( 9\delta \beta - 9\delta \sqrt{\beta^2 + \tilde{\sigma} - 2\tilde{\sigma} - 6} \right)^2 },
\]

At \(\delta = 0\), we get the potential for the Starobinsky inflationary model:

\[
V_{\text{Star.}} = \frac{V_0 \sigma^2}{(\tilde{\sigma} + 3)^2}.
\]

To obtain \(V_E(y)\), we solve Eq. (10):

\[
y + \frac{6}{9\delta \beta - 9\delta \sqrt{\beta^2 + \tilde{\sigma} - 2\tilde{\sigma} - 6}} = 0,
\]

and get

\[
\tilde{\sigma} = \frac{3(1 - y)}{y} + \frac{9\delta}{8y} \left( 4\beta y + 9\delta y \pm s \right),
\]

where \(s = \sqrt{(72\delta \beta y + 81\delta^2 y + 16\beta^2 y - 48y + 48)} y\). To get \(\sigma = 0\) at \(y = 1\) for all nonnegative values of \(\delta\) and \(\beta\) we choose solution (24) with ".". For some values of parameters \(\delta\) and \(\beta\) the potentials \(V_E(y)\) and \(V(\phi)\) are presented in Fig. 1.
Figure 1: The potential $V_E(y)$ (left) and the corresponding $V(\phi)$ (right) in the case of the deformation of the Starobinsky model for $\delta = 0$ (red), $\delta = 1/2$ and $\beta = \sqrt{3}$ (grey), $\delta = 1$ and $\beta = \sqrt{3}$ (green), $\delta = 1$ and $\beta = 0$ (blue).

4 A new one-parametric generalization of the Starobinsky inflationary model

Let us consider the case of

$$\beta = \sqrt{3} - \frac{9}{4} \delta. \quad (25)$$

In the case of an arbitrary $\beta$, we obtain from Eq. (24) and the condition $\sigma(1) = 0$ that

$$\sigma(y) = -\frac{3}{2y} \left( 3\delta \sqrt{3} y - 3\sqrt{3} \delta y + 2y - 2 \right). \quad (26)$$

The corresponding potential has a simple form:

$$V_E(y) = V_0(y - 1)^2 - \sqrt{3} V_0 \delta \sqrt{y} (\sqrt{y} + 2) (\sqrt{y} - 1)^2 \quad (27)$$

that is equivalent

$$V(\phi) = V_{\text{Star}} + \sqrt{3} V_0 \delta e^{-\phi/(\sqrt{6} M_{\text{Pl}})} \left( e^{-\phi/(\sqrt{6} M_{\text{Pl}})} + 2 \right) \left( e^{-\phi/(\sqrt{6} M_{\text{Pl}})} - 1 \right)^2. \quad (28)$$

The potential $V_E(y)$ has an extremum at

$$y_{\text{extr}} = \frac{\sqrt{3} - 6\delta + \sqrt{3 - 12\sqrt{3} \delta + 27\delta^2}}{2(\sqrt{3} - 3\delta)}. \quad (29)$$

It is easy to see that $0 < y_{\text{extr}} < 1$ for $\delta > 4\sqrt{3}/9$ and $y_{\text{extr}} = 1$ at $\delta = 4\sqrt{3}/9$. In Fig. 2 the potentials $V_E(y)$ and $V(\phi)$ are presented. So, to get a positive potential without any extremum for $0 < y < 1$ one should put the condition $\delta \leq 4\sqrt{3}/9$ that is equivalent to $\beta > 0$. 

5
Using Eq. (13), we get the slow-roll parameters as functions of $y$:

$$
\epsilon = \frac{y (\sqrt{y} - 1)^2 (2 y \sqrt{3} \delta + 2 \sqrt{3} \delta \sqrt{y} - \sqrt{3} \delta - 2 y - 2 \sqrt{y})^2}{3 (\sqrt{3} \delta y^2 - 3 \sqrt{3} \delta y + 2 \sqrt{3} \delta \sqrt{y} - y^2 + 2 y - 1)^2}
$$

(30)

and

$$
\eta = \frac{\sqrt{y} (8 \sqrt{3} \delta y^{3/2} - 8 y^{3/2} - 6 \sqrt{3} \delta \sqrt{y} + 4 \sqrt{y} + \sqrt{3} \delta)}{3 \left(\sqrt{3} \delta y^2 - 3 \sqrt{3} \delta y + 2 \sqrt{3} \delta \sqrt{y} - (y - 1)^2\right)}.
$$

(31)

Therefore, according to Eq. (15), the inflationary parameters are given by

$$
n_s = 1 - \frac{2}{3 (\sqrt{3} y^2 \delta - 3 \sqrt{3} \delta y + 2 \sqrt{3} \delta \sqrt{y} - y^2 + 2 y - 1)^2} \times \left(5 \sqrt{3} \delta y^{5/2} - 15 \delta^2 y^{5/2} - 8 \sqrt{3} \delta y^4 + 12 \delta^2 y^4 + 2 \sqrt{3} \delta y^{3/2} + 10 \sqrt{3} \delta y^3 - 9 \delta^2 y^{3/2} - 18 \delta^2 y^3 - 4 \sqrt{3} \delta y^2 + 27 \delta^2 y^2 + 4 y^4 + \sqrt{3} \delta \sqrt{y} - 6 \sqrt{3} \delta y + 3 \delta^2 y - 4 y^3 - 4 y^2 + 4 y\right)
$$

(32)

and

$$
r = \frac{16 y (\sqrt{y} - 1)^2 (2 y \sqrt{3} \delta + 2 \sqrt{3} \delta \sqrt{y} - \sqrt{3} \delta - 2 y - 2 \sqrt{y})^2}{3 (\sqrt{3} \delta y^2 - 3 \sqrt{3} \delta y + 2 \sqrt{3} \delta \sqrt{y} - y^2 + 2 y - 1)^2}.
$$

(33)

At $\delta = 1/\sqrt{3}$, that corresponds to $\beta = \sqrt{3}/4$, we obtain the potential

$$
V_{E}(y) = V_0 (1 - \sqrt{y})^2, \quad V(\phi) = V_0 \left(1 - e^{-\phi/(\sqrt{6} M_{Pl})}\right)^2
$$

(34)
and simple expressions for slow-roll and inflationary parameters:

\[
\epsilon = \frac{y}{3(1 - \sqrt{y})^2}, \quad \eta = \frac{\sqrt{y}(2\sqrt{y} - 1)}{3(1 - \sqrt{y})^2}.
\]

\[
n_s = 1 - \frac{2\sqrt{y}(\sqrt{y} + 1)}{3(\sqrt{y} - 1)^2}, \quad r = \frac{16y}{3(1 - \sqrt{y})^2}.
\]

In the case of \(\delta = 4\sqrt{3}/9\), one gets from Eq. (25) that \(\beta = 0\). The corresponding potential is

\[
V_E(y) = \frac{V_0}{3} (3 + \sqrt{y}) (1 - \sqrt{y})^3,
\]

so,

\[
V(\phi) = \frac{V_0}{3} \left( e^{\phi/(\sqrt{6}M_{Pl})} - 1 \right)^3 (1 + 3e^{\phi/(\sqrt{6}M_{Pl})}) e^{-2\sqrt{2/3}\phi/M_{Pl}}.
\]

This case has been considered in Ref. [24] in detail. Let us consider the case \(\delta = 1/\sqrt{3}\).

At \(\delta = 1/\sqrt{3}\), the conditions \(\epsilon(y_{\text{end}}) = 1\) and \(y_{\text{end}} < 1\) gives the following solution:

\[
y_{\text{end}} = \frac{1}{4} \left( 3 - \sqrt{3} \right)^2.
\]

The slow-roll evolution equation (12),

\[
y' = \frac{2y^{3/2}}{3(\sqrt{y} - 1)}
\]

allows us to express \(N_e\) via \(y\):

\[
N_e = \frac{3}{\sqrt{y}} + \frac{3}{2} \ln(y) + N_0,
\]

where the integration constant \(N_0\) fixed by the condition \(N_e(y_{\text{end}}) = 0\):

\[
N_0 = -\frac{3}{\sqrt{y_{\text{end}}}} - \frac{3}{2} \ln(y_{\text{end}}).
\]

Using Eq. (36) we calculate the value of \(y_{\text{in}}\) for suitable values of \(n_s\), namely for \(n_s = 0.961, n_s = 0.965,\) and \(n_s = 0.969\). It allows us to calculate the corresponding values of \(r\) and number of e-folding during inflation. The same procedure can be made for an arbitrary \(\delta\). The results of calculations for different values of \(\delta\) are presented in Table 1. One can see that the tensor-to-scalar ratio \(r(\delta)\) is an increasing function and \(r(1/\sqrt{3})\) is almost in four times more than in the Starobinsky model.

5 Conclusion

It is well-known [9, 23, 24] that adding of \(R^0_J\) term to the Starobinsky model with \(n > 2\) results to a maximum of the corresponding scalar potential at some positive value of the scalar field \(\phi\). In this case, inflation demands a fine-tuning of initial value of \(\phi\) that makes such inflationary models unrealistic. In the Starobinsky model, this potential is monotonically increasing function at positive \(\phi\). The generalization with \(R^{3/2}\) term proposed in [24] has the same property, but is not well-defined for negative values of \(R_J\).
Table 1: The values of $y$, $N_e$ and $r$ corresponding to $n_s = 0.961$, $n_s = 0.965$ and $n_s = 0.969$, respectively, and the values of $y_{\text{end}}$ for some values of the parameter $\delta$.

| $\delta$ | 0     | 0.1   | $\frac{\sqrt{3}}{9}$ | $\frac{2\sqrt{3}}{9}$ | $\frac{1}{\sqrt{3}}$ | $\frac{4\sqrt{3}}{9}$ |
|-----------|-------|-------|------------------------|------------------------|------------------------|------------------------|
| $y_{\text{end}}$ | 0.464 | 0.460 | 0.455                  | 0.439                  | 0.402                  | 0.299                  |
| $y_{n_s=0.961}$ | 0.0140 | 0.0114 | 0.00895               | 0.00479                | 0.00253                | 0.00146                |
| $y_{n_s=0.965}$ | 0.0126 | 0.0101 | 0.00777               | 0.00402                | 0.00209                | 0.00120                |
| $y_{n_s=0.969}$ | 0.0112 | 0.00878 | 0.00660             | 0.00329                | 0.00168                | 0.000968               |
| $N_e,n_s=0.961$ | 49.3 | 45.2 | 44.0                  | 45.3                  | 47.4                  | 48.1                  |
| $N_e,n_s=0.965$ | 55.0 | 50.4 | 49.1                  | 50.7                  | 53.0                  | 53.8                  |
| $N_e,n_s=0.969$ | 62.3 | 57.0 | 55.6                  | 57.6                  | 60.1                  | 60.9                  |
| $r_{n_s=0.961}$ | 0.0043 | 0.0074 | 0.010                | 0.014                 | 0.015                 | 0.015                 |
| $r_{n_s=0.965}$ | 0.0035 | 0.0061 | 0.0084              | 0.0114                | 0.012                 | 0.012                 |
| $r_{n_s=0.969}$ | 0.0027 | 0.0049 | 0.0068            | 0.0091                | 0.0097                | 0.0098                |

In this paper, a new one-parameter generalization of the Starobinsky model has been constructed, describing by

$$F = \frac{M_{Pl}^2}{2} \left[ \frac{1}{8} \left( 8 - 3\delta(4\sqrt{3} - 9\delta) \right) R_J + \frac{R_J^2}{6m^2} + \frac{\delta}{m} \left[ R_J + \left( \frac{3}{4} - \frac{3}{4}\delta \right)^2 m \right]^{3/2} - m^2 \delta \left( \frac{3}{4} - \frac{3}{4}\delta \right)^3 \right],$$

For any $0 \leq \delta \leq 4\sqrt{3}/9$, this model is consistent with cosmological observations and the corresponding potential is monotonically increasing function at $\phi > 0$. At $\delta = 0$, the proposed model coincides with the Starobinsky inflationary model, at $\delta = 4\sqrt{3}/9$, it coincides with model considered in Ref. [24]. Note that the parameter both $n_s$ and $r$ do not depend on the scalaron mass $m$, so this parameter is fixed by the observable values of $A_s$ as well as in the Starobinsky inflationary model.

The most simple form of the scalar field potential

$$V(\phi) = V_0 \left( 1 - e^{-\phi/(\sqrt{3}M_{Pl})} \right)^2,$$

corresponds to $\delta = 1/\sqrt{3}$, when

$$F = \frac{M_{Pl}^2}{2} \left[ \frac{5}{8} R_J + \frac{R_J^2}{6m^2} + \frac{\sqrt{3}}{3m} \left[ R_J + \frac{3}{16} m^2 \right]^{3/2} - \frac{3}{64} m^2 \right]. \quad (42)$$

In this case, the tensor-to-scalar ratio $r$ is almost in four times more than in the Starobinsky model (see Table 1).

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