Distributed algorithms for tracking the trajectories of many objects by the set of mobile sensors

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Abstract. We consider a dynamic network system of multiple heterogeneous sensors collaborating among themselves to estimate an unknown signal of interest. That communication capability is essential because individual sensors cannot fulfill the task due to the lack of necessary information. This paper considers the distributed estimation or tracking problem and focuses on combining two algorithms: Simultaneous Perturbation Stochastic Approximation (SPSA) and consensus Local Voting Protocol (LVP). The new distributed algorithm is proposed for network systems under wide range of uncertainties including unknown-but-bounded drift of parameters and noise in observations. The suggested approach is illustrated by an application of the proposed algorithm to the solving the multisensor-multitarget problem.

1. INTRODUCTION
Recently there has been an increasing interest in distributed cooperative control of networked systems. That interest is caused by many real-world problems and different areas of application. For example, cooperative control of multi-robot systems including but not limited to the swarms of unmanned aerial vehicles and autonomous underwater vehicles has been extensively studied improving the overall performance of the whole system. These trends are actualizing the development and usage of the multi-agent approach due to low effectiveness of centralized control of such systems.

The theoretical basis of decentralized adaptive multi-agent control has been improved in the last few decades, and due to that many possible applications have been investigated [1, 2, 3]. The fundamental concept which is used in network systems [1], as an example of a system with multi-agent cooperative control, is consensus. Among network systems there are opinion dynamics [3], multi-vehicle networks [4], flocking dynamics [5, 6, 7] and sensor networks [8].

Stochastic approximation (SA) was introduced by Robbins and Monro [9]. It is a recursive method that can be used to find optimal points or zeros of a function if only noisy parameters were received. Due to the stochastic nature of these methods, it is hard to analyse their convergence and convergence rates. The applications of SA method are very popular in such areas as adaptive signal processing, adaptive resource allocation, and artificial intelligence. Usually, a solution of a stochastic optimization problem under uncertainties reduces to finding a set of system parameters (which differ over time in practical applications) that deliver a minimum
(or maximum) value to a certain mean-risk functional. In this paper, such a problem is called the minimum point tracking of a non-stationary mean-risk functional. The maximum likelihood estimator and SA algorithms with decreasing to zero stepsize are actively used to optimize mean-risk functionals [9, 10, 11]. In gradient-free conventional stochastic approximation algorithms, two measurements are used to approximate each component of the vector-gradient of the cost function (implying 2d measurements for the d-dimension state space). Simultaneous perturbation stochastic approximation (SPSA) was proposed by Spall [12]. It can be used instead of the SA algorithms to solve optimization problems in the case where it is difficult or impossible to obtain a gradient of the loss function with respect to the parameters being optimized. SPSA requires only two measurements of a loss function at each iteration. That approach is similar to a random search algorithm, because in this algorithm the current estimate is updated along a randomly chosen direction.

The distributed optimization is studied when we are interested in the minimum of some loss function \( \bar{F}(x) = \sum_{i=1}^{n} F_i(x) \), and we try to find it via interaction between agents. Here, \( x \in \mathbb{R}^d \) and \( F_i(x) : \mathbb{R}^d \rightarrow \mathbb{R} \) is the loss function of agent \( i \), which is typically known only to the agent itself. Studies of consensus and distributed optimization algorithms can be traced back to the papers from the 1970-80s [13, 14]. Distributed asynchronous stochastic approximation algorithms were studied in [15]. To date, there exists a number of approaches for the case when functions \( F_i(x) \) are convex. In particular, the Alternating Direction Method of Multipliers [16], as well as the subgradient method [17, 18] were proposed. The distributed tracking problem is closely related to the distributed optimization problems. It was considered in [19, 20].

In [21, 20] the SPSA algorithm is applied to unlimited optimization in the context of the minimum tracking problem. One of the main limitations is the property of strong convexity of the minimized mean-risk functional. In [22] this assumption was weaken by combining SPSA with the consensus algorithm from [24]. The consensus algorithm or local voting protocol was considered in [23] for more general case of stochastic network dynamics with application to load balancing. In [25] the differentiated consensuses problem was considered where the network processes the tasks of different priorities and the consensus in loads is reached for each priority level separately. The differentiated consensuses problem for the network with randomized priorities, in which tasks are chosen for execution with probability proportional to the task priority, was considered in [27]. The paper [26] proposes the way to choose the optimal step size for the local voting protocol for the differentiated consensuses problem.

Let us summarize the contributions of the paper. We propose a new modification of the SPSA-based consensus algorithm for distributed tracking by the heterogeneous sensors, which develops our previous results in [22]. In this paper we consider sensors with different noise scales and that allows us to illustrate the performance of the new approach in another important practical problem.

The rest of this paper is organized as follows. Section 2 provides notations used in the paper. The formal problem is stated in Section 3. The modified SPSA-based consensus algorithm for tracking with different stepsizes is introduced in Section 4. The main result concerning stability properties of the proposed algorithm is shown in Section 5. In Section 6, we consider a simulation which illustrates the operability of the algorithm. Section 7 concludes the paper.

### 2. MATHEMATICAL PRELIMINARIES

In subsequent sections, we use the following notations.

Consider a dynamic network system of \( n \) agents, which collaborate among themselves. Without loss of generality, agents in the network system are numbered. Let \( \mathcal{N} = \{1, \ldots, n\} \) be the set of agents, and \( i \in \mathcal{N} \) be the number of an agent. \( \forall i \in \mathcal{N} \) let \( \mathcal{N}_i^t \) be a subset of all agents: \( \mathcal{N}_i^t \subset \mathcal{N} \), which are able to send information to agent \( i \). Here and after, an upper index
of agent $i$ is used as the corresponding number of an agent (while not as an exponent).

Let the network topology be modeled by a digraph $(\mathcal{N}, E)$, where $E$ denotes the set of edges of topology graph $(\mathcal{N}, E)$. The corresponding adjacency matrix is denoted as $A = [a^{ij}]$, where $a^{ij} > 0$ if agent $j$ is connected to agent $i$ (i.e., if there is an arc from $j$ to $i$) and $a^{ij} = 0$ otherwise. Denote $\mathcal{G}_A$ the graph corresponding to adjacency matrix $A$.

To introduce some properties of the network topology, the following definitions from the graph theory will be used. Define the weighted in-degree of node $i$ as the sum of $i$-th row of matrix $A$: $\deg_i^+(A) = \sum_{j=1}^n a^{ij}$; $\deg_{\max}^+(A)$ is the maximum node in-degree in graph $\mathcal{G}_A$; $D(A) = \text{diag} \{\deg_1^+(A), \ldots, \deg_n^+(A)\}$ is the corresponding diagonal matrix. Here and further, $\text{col}\{x_1, \ldots, x^n\}$ denotes a vector obtained by stacking the specified vectors; $\text{diag}_n(b)$ is a square diagonal matrix with vector $b$ as the main diagonal. Let $\mathcal{L}(A) = D(A) - A$ denote the Laplacian of graph $\mathcal{G}_A$. $A^T$ is a vector or matrix transpose operation; $\|A\|$ is the Frobenius norm: $\|A\| = \sqrt{\sum_{i=1}^n \sum_{j=1}^n (a^{ij})^2}$; $\text{Re}(\lambda_2(A))$ is the real part of the second eigenvalue of matrix $A$ ordered by the absolute magnitude; $\lambda_{\max}(A)$ is the eigenvalue of matrix $A$ with maximum absolute magnitude; $\mathbf{1}_n = (1, \ldots, 1)^T$ is the vector of $n$-times replication of ones; $I_n$ is the identity matrix $d \times d$. $A \otimes B$ is the Kronecker product for any $m \times n$ and $p \times q$ matrices $A$ and $B$.

3. PROBLEM STATEMENT

Let $(\Omega, \mathcal{F}, P)$ be the underlying probability space corresponding to the sample space $\Omega$ with $\sigma$-algebra of all events $\mathcal{F}$ and the probability measure $P$, and $\Xi$ be a mathematical expectation symbol.

Let $\Xi$ be a set, $\forall i \in \mathcal{N} \{f_i^t(\theta)\}_{\xi \in \Xi}$ be a family of differentiable functions: $f_i^t(\theta) : \mathbb{R}^d \to \mathbb{R}$, and let $x_1^t, x_2^t, \ldots$ be a sequence of measurement points chosen by the experimenter (observation plan), where the values $y_1^t, y_2^t, \ldots$ of functions $f_i^t(\cdot)$ are accessible to observations at every time instant $t = 1, 2, \ldots$, with additive external noise $v_i^t$

$$y_i^t = f_i^t(x_i^t) + v_i^t,$$

where $\{\xi_t\}$ is a non-controllable sequence: $\xi_t \in \Xi$ (e.g., $\Xi = \mathbb{N}$ and $\xi_t = t$, or $\Xi \subset \mathbb{R}^p$ and $\{\xi_t\}$ is a sequence of some random elements).

Let $\mathcal{F}_{t-1}$ be the $\sigma$-algebra of all probabilistic events which happened up to time instant $t = 1, 2, \ldots$, $\mathbb{E}_{\mathcal{F}_{t-1}}$ is a symbol of the conditional mathematical expectation with respect to the $\sigma$-algebra $\mathcal{F}_{t-1}$.

**Non-stationary problem formulation.** The time-varying point of minimum $\theta_t$ of the distributively computed mean-risk functional

$$\hat{F}_t(\theta) = \sum_{i \in \mathcal{N}} F_i^t(\theta) = \mathbb{E}_{\mathcal{F}_{t-1}} \sum_{i \in \mathcal{N}} f_{\xi_i}^t(\theta) \to \min_{\theta},$$

needs to be estimated.

More precisely, based on the observations $y_1^t, y_2^t, \ldots, y_i^t$ and inputs $x_1^t, x_2^t, \ldots, x_i^t$, $i \in \mathcal{N}$, we consider the problem of constructing an estimate $\hat{\theta}_i$ of an unknown vector $\theta_i$ minimizing the time-varying mean-risk functional (2) which is a conditional expectation of the sum of distributed sub-functions $f_{\xi_i}^t(\theta)$.

Minimization of the functional $F_t(\theta)$ is usually studied with simpler observation models

$$y_i^t = F_i^t(x_i) + v_i^t \quad \text{or} \quad y_i^t = f_{\xi_i}^t(x_i), \quad i \in \mathcal{N}.$$

The generalization used in model (1) allows separation of any uncertainties and observation disturbances with “good” (e.g., zero-mean and independent and identically distributed — i.i.d.)
statistical properties \(\{\xi_t\}\) and arbitrary additive external noise \(\{v_t^i\}\). Of course, this separation is not needed when we can assume that \(\{v_t^i\}\) is a random zero-mean and independent and identically distributed as well.

Centralized algorithms usually require the distributed agent network to transmit the whole system information \(y_1, y_2, \ldots, y_i, x_1, x_2, \ldots, x_i, i \in \mathcal{N}\), into a fusion center to estimate the unknown vector \(\theta_t\), which may lack robustness at the fusion center and needs strong communication capability over the agent networks. In many practical situations, agents may only have the capability to exchange information locally with their neighbors with noise and delays, and the network topology may switch over time. Moreover, a lot of practical reasons lead to the problem setting with cost constraints for using network topology. In sensor networks, the set of agents \(\mathcal{N}\) is a set of \(n\) nodes distributed over the geographic region.

We assume that to form its current estimates \(\hat{\theta}_t^i\) at time instant \(t\) agent \(i\) has its own noisy observation \((1)\) and, if the set \(\mathcal{N}_t^i\) is not empty, information about its neighbors’ current estimates \(\hat{\theta}_t^j, j \in \mathcal{N}_t^i\). Assumptions. First, let us formulate assumptions about the functions \(F_t^i(x), f_t^i(x), \forall i \in \mathcal{N}\), noise, disturbances, and network topology.

1: Functions \(F_t^i(\cdot)\) are convex and there is a common minimum point \(\theta_t\) and

\[
\forall x \in \mathbb{R}^d \quad \langle x - \theta_t, \mathbb{E}_{F_{t-1}} \nabla f_t^i(x) \rangle \geq 0.
\]

Here and further \(\langle \cdot, \cdot \rangle\) is a scalar product of two vectors.

2: \(\forall \xi \in \Xi\) the gradient \(\nabla f_t^i(x)\) satisfies the Lipschitz condition: \(\forall x', x'' \in \mathbb{R}^d\)

\[
\|\nabla f_t^i(x') - \nabla f_t^i(x'')\| \leq C_L \|x' - x''\|
\]

with the same constant \(C_L > 0\).

3: The gradient \(\nabla f_t^i\) is uniformly bounded in the mean-squared sense at the minimum points \(\theta_t\): \(\mathbb{E}\|\nabla f_t^i(\theta_t)\|^2 \leq g_2^2\), \(\mathbb{E}\langle \nabla f_t^i(\theta_t), \nabla f_t^i(\theta_{t-1}) \rangle \leq g_2^2\) (\(g_2 = 0\) if \(\xi_t\) is not a random parameter, i.e. \(f_t^i(x) = F_t(x)\)).

4: The drift is bounded: a) \(\|\theta_t - \theta_{t-1}\| \leq \delta_0 < \infty\), or \(\mathbb{E}\|\theta_t - \theta_{t-1}\|^2 \leq \delta_0^2\) and \(\mathbb{E}\|\theta_t - \theta_{t-1}\|\|\theta_{t-1} - \theta_{t-2}\| \leq \delta_0^2\) if a sequence \(\{\xi_t\}\) is random;

b) \(\mathbb{E}_{\mathcal{F}_{2k-2}} f_{2k-2}^i(x) - f_{2k-1}^i(x) \| \leq \delta_0^2\) \(g_0^2 + g_1^2\|x - \theta_{2k-2}\|\) for \(q = 1, 2\) and for any \(i \in \mathcal{N}\).

5: For \(n = 1, 2, \ldots\), the successive differences \(\tilde{v}_k^i = v_{2k}^i - v_{2k-1}^i\) of observation noise are bounded: \(\|\tilde{v}_k^i\| \leq c_v^i < \infty\), or \(\mathbb{E}(\tilde{v}_k^i)^2 \leq \langle c_v^i \rangle^2\) if a sequence \(\{\tilde{v}_k^i\}\) is random.

6: For any \(i, j \in \mathcal{N}\) a) vectors \(\Delta_k^i\) are mutually independent; b) \(\Delta_k^i\) and \(\xi_{2k-1}, \xi_{2k}\) (if they are random) do not depend on the \(\sigma\)-algebra \(\mathcal{F}_{2k-2}\); c) if \(\xi_{2k-1}, \xi_{2k}\) are random, then random vectors \(\Delta_k^i\) and elements \(\xi_{2k-1}, \xi_{2k}, \tilde{v}_n^i\) are independent.

7: Graph \(G_A\) is strongly connected.

**4. SPSA-BASED CONSENSUS ALGORITHM**

Let \(\Delta_k^i, k = 1, 2, \ldots, i \in \mathcal{N}\), be an observed sequence of independent Bernoulli random vectors from \(\mathbb{R}^d\) with each component independently taking values \(\pm \sqrt{\alpha/\gamma}\) with probabilities \(1/2\). This sequence is usually called the simultaneous test perturbation. Let us take fixed nonrandom initial vectors \(\theta_0 \in \mathcal{R}^d\), positive step-size \(\alpha\), gain coefficient \(\gamma\) and choose the scale of perturbation \(\beta > 0\). Denote \(\alpha^i = \alpha/c_v^i\), where \(c_v^i = c_v^i\) if \(c_v^i > 0\), otherwise \(c_v^i = 1\). We consider the algorithm with two observations of distributed sub-functions \(f_t^i(\theta)\) for each agent \(i \in \mathcal{N}\) for constructing
sequences of points of observations \( \{x_t^i\} \) and estimates \( \{\hat{\theta}_t^i\} \):

\[
\begin{align*}
    x_{2k}^i &= \hat{\theta}_{2k-2}^i + \beta \Delta_k^i, \quad x_{2k-1}^i = \hat{\theta}_{2k-2}^i - \beta \Delta_k^i, \\
    \hat{\theta}_{2k-1}^i &= \hat{\theta}_{2k-2}^i, \\
    \hat{\theta}_{2k}^i &= \hat{\theta}_{2k-1}^i - \alpha^i \left[ \frac{y_{2k}^i - y_{2k-1}^i}{2\beta} + \gamma \sum_{j \in N_i} a^{i,j} (\hat{\theta}_{2k-1}^j - \hat{\theta}_{2k-1}^j) \right].
\end{align*}
\]

(3)

The first part of the algorithm (3) is similar to SPSA-like algorithm from [21] and the second one coincides with a local voting protocol (LVP) from [23], where it was studied for stochastic networks in the context of load balancing problem. The SPSA part represents a stochastic gradient descent of sub-functions \( f_k(\theta) \), and LVP part is determined for each agent \( i \) by the weighted sum of differences between the information about the current estimate \( \hat{\theta}_{2k-1}^i \) of agent \( i \) and information about the estimates of its neighbors.

Further, we use notation \( \bar{\theta}_t = \text{col}\{\hat{\theta}_1^t, \ldots, \hat{\theta}_n^t\} \) for the common vector of estimates of all agents at time instant \( t \). Also we introduce the following notations:

\[
    \bar{y}_t = \text{diag}_n \left( \text{col}\{y_1^t, \ldots, y_n^t\} \right), \quad \Delta_{t+2} = \text{col}\{\Delta_{t+2}^1, \ldots, \Delta_{t+2}^n\}.
\]

Using the notations introduced above, the algorithm (3) can be rewritten in the following form

\[
    \hat{\theta}_{2k} = \hat{\theta}_{2k-1} - \alpha \left[ \frac{\bar{y}_{2k} - \bar{y}_{2k-1}}{2\beta} \otimes I_d \right] \Delta_k + \gamma (\mathcal{L}(A) \otimes I_d) \hat{\theta}_{2k-1}.
\]

(4)

5. MAIN RESULT

This section presents Theorem 1 for the algorithm (3).

To analyze the quality of estimates we apply the following definition for the problem of minimum tracking for mean-risk functional (2).

**Definition.** A sequence of estimates \( \{\hat{\theta}_k\} \) has an asymptotically efficient upper bound \( \tilde{L} > 0 \) of residuals of estimation if \( \forall \varepsilon > 0 \exists \bar{k} \) such that \( \forall k > \bar{k} \)

\[
    \sqrt{E\|\hat{\theta}_k - \mathbf{1}_n \otimes \theta_{2k}\|^2} \leq \tilde{L} + \varepsilon.
\]

Denote \( \tilde{\lambda}_2 = \text{Re}(\lambda_2(\mathcal{L}(A))), \tilde{\lambda}_m = \lambda_m^2(\mathcal{L}(A))^T \mathcal{L}(A), \delta_\beta = \frac{\delta_\beta}{2\beta}, c_1 = \delta_\beta g_1 + 1, c_2 = \delta_\beta g_1^2/C_L^2 + 1, c_\mu = (\tilde{\lambda}_2 - \alpha \tilde{\lambda}_m C_L c_1)/\tilde{\lambda}_m^2, c_\delta = \sqrt{1 - 2\alpha^2 C_L^2 \tilde{\lambda}_m^2/(\tilde{\lambda}_2 - \alpha \tilde{\lambda}_m C_L c_1)^2}, C_v = \sum_{i \in N} \frac{1}{C_i}, \tilde{C}_v = \max_{i \in N} \frac{1}{C_i}, \bar{C}_v = \sum_{i \in N} \frac{1}{C_i}. \)

The following theorem shows the asymptotically efficient upper bound of estimation residuals provided by algorithm (3).

**Theorem 1:** If Assumptions 1–7 hold, positive constant \( \alpha \) is sufficiently small:

\[
    \alpha < \frac{\tilde{\lambda}_2}{\lambda_m M (c_1 + \sqrt{2c_2})} c_\mu (1 - c_d) < \alpha \gamma < c_\mu (1 + c_d)
\]

(5)

then the sequence of estimates provided by algorithm (3) has an asymptotically efficient upper bound which equals to

\[
    \tilde{L} = \frac{1}{\mu} \left( h + \sqrt{h^2 + u \mu} \right),
\]

(6)
where \( \mu = 2\gamma \lambda_2 - \alpha \tilde{C}_l(\tilde{\lambda}_m^2 - 2 + 2\alpha C_L(\gamma \tilde{\lambda}_m c_1 + C_L c_2)), \) \( \theta = \gamma (2\sqrt{n} \tilde{\lambda}_m \delta_0 + \alpha \tilde{\lambda}_m C_L \tilde{C}_v (\delta_0 \gamma_0 + 2\beta)) + C_l \tilde{C}_v (3.5 + g_1/2) \delta_0 + 2C_L^2 \tilde{C}_v \delta_0, \) \( u = 4n \tilde{\lambda}_m \gamma_0 + C_L \epsilon_2 \delta_0^2 + 4\beta \tilde{C}_v (2.25 \delta_0^2 + g_2^2 + 4C_L^2 \beta^2) + \alpha (2+ \alpha \tilde{C}_l \delta_0^2) + C_L (C_v + \tilde{C}_v \delta_0 \gamma_0) + 4C_L \tilde{C}_v \delta_0 (C_L + 0.5 + g_2)). \)

The proof of Theorem 1 is similar to the proof in [28].

Remarks. 1. The observation noise \( v_t \) in Theorem 1 can be said to be almost arbitrary since it may either be nonrandom but bounded or it may also be a realization of some stochastic process with arbitrary internal dependencies. In particular, to prove the results of Theorem 1, there is no need to assume that \( v_t \) and \( \mathcal{F}_{t-1} \) are independent.

2. The result of the Theorem 1 shows that for the case without drift (\( \delta_0 = 0 \)) we have \( \delta_\theta = 0, c_1 = c_2 = 1 \) and the asymptotic upper bound is \( L = (2C_L \tilde{C}_v \beta (\gamma \tilde{\lambda}_m + C_L) + (4C_L^2 \tilde{C}_v \beta (\gamma \tilde{\lambda}_m + C_L)^2 + (\alpha (C_L \epsilon_2 + 4C_L \tilde{C}_v \delta_0 g_2 + 2n \tilde{\lambda}_m \gamma_0) + 4\beta C_L (C_v + \tilde{C}_v \beta)) + \tilde{C}_v g_2^2) (2\gamma \lambda_2 - \alpha \tilde{C}_l (\tilde{\lambda}_m^2 - 2 + 2\alpha C_L(\gamma \tilde{\lambda}_m + C_L)))) \). Under any noise level \( \epsilon \), this bound can be made infinitely small by choosing sufficiently small \( \alpha \) and \( \beta \). At the same time, in the case of drift, the bigger drift norm \( \delta_0 \) can be compensated by choosing a bigger step-size \( \alpha \). This leads to a tradeoff between making \( \alpha \) smaller because of noisy observations and making \( \alpha \) bigger due to the drift of optimal points.

6. MULTISENSOR-MULTITARGET ESTIMATION

In this section, we show the numerical experiment, which illustrates the performance of the suggested algorithm (3) in distributed estimation problem setting described in Section 3.

To illustrate the general problem setting, consider a distributed network of \( n \) planar sensors that have in their zone of visibility \( m \) planar targets whose state vectors are to be estimated. Let \( N = \{1, 2, ..., n\} \) be the set of sensors, \( M = \{1, 2, ..., m\} \), the set of targets, \( s_i^l \in \mathbb{R}^2 \), \( s_i^l = \begin{bmatrix} r_{i,1}^l \\ r_{i,2}^l \end{bmatrix} \).

the vector of the current state of sensor \( i, l \in N \), at time \( t, r_t^l \in \mathbb{R}^2 \), \( r_t^l = \begin{bmatrix} r_{t,1}^l \\ r_{t,2}^l \end{bmatrix} \) is the state of target \( l, l \in M \), at time \( t \). We denote that for time instant \( t \) each sensor \( i \) is able to observe one of the targets. Let \( l \) be its number and define \( b_{i,l}^t = 1 \) for this case and \( b_{i,l}^t = 0 \) otherwise. In such a way we define the assignment matrix \( B, b : N \times M \rightarrow K \). We assume that the node \( i \) can estimate either squared distance \( \psi (s_i^l, r_t^l) \) or the squared distance \( \rho (s_i^l, r_t^l) \) to a moving point \( l \). The state \( r_t^l \) of the target is accessible to observers through measurements

\[
 z_{t,l}^{i,\psi} = \psi (s_i^l, r_t^l) + z_{t,l}^{i,d,\psi}, \quad z_{t,l}^{i,d,\rho} = \rho (s_i^l, r_t^l) + z_{t,l}^{i,d,\rho}, \tag{7}
\]

where \( z_{t,l}^{i,d,\psi} \) and \( z_{t,l}^{i,d,\rho} \) are the first and second coordinates of noisy measurements \( z_{t,l}^{i,d} \in \mathbb{R}^2 \). about target \( l \) available to sensor \( i \) at time \( t \), \( \psi (s_i^l, r_t^l) = \arctg \frac{r_{t,1}^l - r_{i,1}^l}{r_{t,2}^l - r_{i,2}^l} \) is the angle between the direction from the sensor to the north and direction to the observed object, also called the azimuth angle, or directional angle, and \( \rho (s_i^l, r_t^l) = \sqrt{(r_{t,1}^l - r_{i,1}^l)^2 + (r_{t,2}^l - r_{i,2}^l)^2} \) is the distance from the location of the sensor to the target. \( z_{t,l}^{i,d,\psi} \) is an unknown-but-bounded noise in the measurements.

We denote by \( \theta_t = \text{col} (r_1^l, ..., r_m^l) \) the general state vector of all targets. Let \( \hat{r}_t^l \) be an estimate for the state of the target \( l \) at time \( t \), \( \hat{\theta}_t = \text{col} (\hat{r}_1^l, ..., \hat{r}_m^l) \), the cumulative total vector of estimates. In a sufficiently general case, the problem of estimation the unknown target states can be
Figure 1. Network topology graph. Different colors represent different types of measuring devices.

formulated as the problem of minimizing the mean-risk functional

$$F_t(\hat{\theta}_t) = \mathbb{E}_{F_{t-1}} \sum_{i \in M} \sum_{l \in N} b_{i,l}^t ((\psi(s_i^t, \hat{r}_l^t) - z_{i,l,\psi}^t)^2 + (\rho(s_i^t, \hat{r}_l^t - z_{i,l,\rho}^t)^2) =$$

at observations

$$y_{i,\psi}^t = f_{i,\psi}^t + v_{i,\psi}^t,$$

$$y_{i,\rho}^t = f_{i,\rho}^t + v_{i,\rho}^t,$$

where $v_{i,\psi}^t; v_{i,\rho}^t$ is the observation noise.

Dynamics of the points movement is as follows: $\hat{r}_l^t = \hat{r}_l^{t-1} + \chi_{l,t-1}^t$. Let $\chi_{l,t-1}^t$ be a random vector uniformly distributed on the ball of radius equal to 1.

Simulation

We consider $n = 3$ sensors tracking $m = 5$ moving targets whose states are changing with time (drift of states), i.e., $\delta_\theta = 1$. Each sensor consists of two parts measuring distance to the target and its azimuth angle with respect to the sensor. The parts exchange information within the sensor and with corresponding parts of the neighbor sensors (i.e. distance measuring parts exchange distance estimates and angle parts communicate the angle estimates.) We assume sensors are located near each other and the distance to the targets is much larger than the distance between sensors. For the described application, Assumptions 2, 3, and 4 hold if the corresponding constants are as follows: $C_L = 2$, $g_0 = 3$, $g_1 = 2$, $g_2 = 0$, $c_v = 1$. Let the communication graph $G_A$ be full, i.e. all nodes are connected to each other and there are no self-loops. In this case, $\lambda_2 = 3$ and $\lambda_m = 3$.

Algorithm (3) working on each node has the following parameters: $\alpha = 0.18$, $\beta = 4$, $\gamma = 0.285$. We consider three types of noise: uniformly distributed random variable falling within the
Figure 2. Estimates of angles to the targets $r_t^r$ obtained by nodes.

Figure 3. Estimates of distances to the targets $r_t^s$ obtained by nodes.
interval $[-1; 1]$, periodic oscillation (e.g., sine or cosine), and an unknown constant. In the simulation presented in the paper $v_{i,\psi}^t$ is uniformly distributed in $[-0.1; 0.1]$ and $v_{i,\rho}^t$ is uniformly distributed in $[-1; 1]$. Observation noises $\zeta_{i,l,\psi}^t$ and $\zeta_{i,l,\rho}^t$ are random values uniformly distributed in intervals $[-0.1; 0.1]$ and $[-1; 1]$ correspondingly.

Fig. 2 illustrates the estimates of the angles evolution with time. Sensors via LVP reach consensus about the angles value. Five resulting lines correspond to five moving targets. The duration of the experiment is 300 discrete time steps. The initial angle estimate was chosen randomly from $[-\pi; \pi]$. The points $r_l^t$, $l = 1 \cdots m$ start their movements at the position consisting of randomly chosen components from the interval $[0; 100]$.

Fig. 3 shows how the distances from the sensors to the targets evolve over time. The initial estimate on each sensor was chosen randomly from the interval $[0; 10]$.

Figures show that there exists the time instant $t$ starting with which the estimations converge to the real value and move next to it.

7. CONCLUSIONS

In this paper, we propose SPSA-based consensus algorithm for state estimation in networked systems. We expand that approach on heterogenous sensors with different noise scales. The multisensor-multitarget problem is considered as an example which shows the practical applicability of the general scheme. The theoretical investigation is illustrated by the simulation.

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