Quark-model study of the hadron structure and the hadron-hadron interaction

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Abstract. Recent results of hadron spectroscopy and hadron-hadron interaction within a quark model framework are reviewed. Higher order Fock space components are considered based on new experimental data on low-energy hadron phenomenology. The purpose of this study is to obtain a coherent description of the low-energy hadron phenomenology to constrain QCD phenomenological models and try to learn about low-energy realizations of the theory.

1. Introduction

We present an overview about the status of the old dream of getting a coherent description of the low-energy hadron phenomenology in a quark model scheme [1]. We make emphasis on two different aspects. Firstly, we will address the study of meson and baryon spectroscopy in an enlarged Hilbert space considering four and five quark components. This seems nowadays unavoidable to understand the experimental data and it builds the bridge towards the description in terms of hadronic degrees of freedom. Secondly, the same scheme used to describe the hadron spectroscopy should be valid for describing the low-energy hadron-hadron scattering. We review our recent efforts to account for the strangeness $-1$ and $-2$ two- and three-baryon systems within the same scheme used for hadron spectroscopy and that has also demonstrated its validity for the nucleon-nucleon sector.

In particular we will address three different problems. In the first section we will discuss the charmonium spectroscopy above the threshold for the production of $D$ mesons. The second section is devoted to study the effect of selected S-wave meson-baryon channels to match poor baryon mass predictions from quark models with data. Finally, the third section addresses the problem of the study of two- and three-baryon systems with strangeness $-1$ and $-2$ by...
means of a baryon-baryon interaction derived from the quark model used to study the hadron spectroscopy.

2. Heavy mesons: Charmonium

Charmonium has been used as the test bed to demonstrate the color Fermi-Breit structure of quark atoms obeying the same principles as ordinary atoms [2]. Its nonrelativistic character \((v/c \approx 0.2 - 0.3)\) gave rise to an amazing agreement between experiment and simple quark potential model predictions as \(c\bar{c}\) states [3]. Close to the threshold for the production of charmed mesons quark models required of an improved interaction [4].

Since 2003, we have witnessed a growth of puzzling new charmonium mesons, like the well-established \(X(3872)\). These new states do not fit, in general, the simple predictions of the \(c\bar{c}\) schemes and, moreover, they overpopulate the expected number of states in (simple) two-body theories. This situation is not uncommon in particle physics. For example, in the light scalar-isoscalar meson sector hadronic molecules offer a reasonable explanation of the experimental data [5]. Also, the study of the \(NN\) system above the pion production threshold required new degrees of freedom to be incorporated in the theory, either as pions or as excited states of the nucleon, i.e., the \(\Delta\) [6]. This discussion suggests that charmonium spectroscopy could be rather simple below the threshold production of charmed mesons but much more complex above it. In particular, the coupling to the closest \((c\bar{c})(n\bar{n})\) system, referred to as unquenching the naïve quark model [7], could be an important spectroscopic ingredient. Besides, hidden-charm four-quark states could explain the overpopulation of quark-antiquark theoretical states.

In an attempt to disentangle the role played by multiquark configurations in the charmonium spectroscopy we have solved the Lippmann-Schwinger equation for the scattering of two 5 mesons looking for attractive channels that may contain a meson-meson molecule [8]. In order to account for all basis states we allow for the coupling to charmonium-light two-meson systems. We have consistently used the same interacting Hamiltonian to study the two- and four-quark systems to guarantee that thresholds and possible bound states are eigenstates of the same Hamiltonian. Our predictions robustly show that no deeply bound states can be expected for this system. Only a few channels, see Table 1, can be expected to present observable resonances or slightly bound states. Among them, we have found that the \(D\bar{D}^*\) system must show a bound state slightly below threshold with quantum numbers \(J^{PC}(I) = 1^{++}(0)\), that could correspond to the widely discussed \(X(3872)\).

Out of the systems made of a particle and its corresponding antiparticle, \(DD\) and \(D^*\bar{D}^*\), the \(J^{PC}(I) = 0^{++}(0)\) is attractive. It would be the only candidate to accommodate a wide resonance for the \(D\bar{D}\) system. For the \(D^*\bar{D}^*\) the attraction is stronger and structures may be observed close and above the charmed meson production threshold. We have shown that the \(J^{PC}(I) = 2^{++}(0,1)\) \(D^*\bar{D}^*\) channels are attractive due to the coupling to the \(J/\Psi\omega\) and \(J/\Psi\rho\) channels, respectively. The charged states reported in Ref. [9] could correspond to the \(D^*\bar{D}^*\) \(J^{PC}(I) = 2^{++}(1)\) we have predicted [10]. Its confirmation would represent a unique tool in discriminating among different theoretical models.

| System  | \(D\bar{D}\) | \(D\bar{D}^*\) | \(D^*\bar{D}\) |
|---------|--------------|----------------|--------------|
| \(J^{PC}(I)\) | 0^{++}(0) | 1^{++}(0) | 0^{++}(0) | 2^{++}(0) | 2^{++}(1) |
3. Light baryons

As we have seen above, there is nowadays compelling evidence of meson resonances containing more than quark-antiquark ($q\bar{q}$) valence components. One can guess a similar situation for some baryon resonances that may contain more than three-quark ($3q$) valence components. In these cases valence quark models based on $3q$ or $q\bar{q}$ components that describe correctly the bulk of spectral data fail systematically to reproduce their masses. Actually we can interpret the systematic failure in the description of a given resonance by valence quark models as an indication of its anomalous nature in the sense of containing more than the valence components. Here we use this interpretation as a criteria to identify possible anomalies in the Light-quark Baryon Spectrum (LqBS).

A comparative analysis of Constituent Quark Models (CQM) predictions for the LqBS has been carried out in Ref. [12]. This analysis makes clear the presence of anomalies for which $3q$ predicted masses are significantly higher than their Particle Data Group (PDG) averages [13], see Fig. 1. Moreover all the anomalies have in common the existence of $S-$ wave meson-baryon thresholds with the same quantum numbers than the anomalies (also drawn in Fig. 1) close above the PDG average masses. This suggests a possible contribution of these meson-baryon ($4q\bar{q}$) components to the anomalies to give a correct account of their masses. To make a rough estimate of this contribution we shall consider the coupling of the free meson-baryon ($mb$) channels to the corresponding $3q$ states. To make effective such a coupling we parametrize the transition matrix element, $<mb|H|3q>=a$, and diagonalize the hamiltonian matrix by assuming $<3q|H|3q>=M_{3q}$ being $M_{3q}$ the predicted $3q$ mass and $<mb|H|mb>\sim M_m+M_b$ being $M_m+M_b$ the mass of the meson-baryon threshold. The results from the diagonalization corresponding to the lowest energy states are shown in Fig. 2. Despite the shortcomings of our toy model calculation the quantitative agreement with data is spectacular (there is only one universal parameter, $a=85$ MeV, assumed for simplicity to be the same in all cases). Qualitatively our toy model implies that the anomalies correspond mostly to meson-baryon

![Figure 1](image_url)

**Figure 1.** Mass predictions for the anomalies from Ref. [11] (dashed lines) as compared to the experimental mass intervals (boxes). N.C. means non-cataloged resonance. Meson-baryon thresholds are indicated by solid lines.
Figure 2. Predicted masses for the anomalies (dashed lines) as compared to the experimental mass intervals (boxes). N.C. means non-cataloged resonance.

states but with a non-negligible three-quark probability that makes them stable against decay into \( m+b \). This interpretation could need some refinement when going to a more complete model where the meson-baryon interaction were also considered \( (\langle mb | H | mb \rangle > \sim M_m + M_b) \). Then some anomalies might appear as simply meson-baryon bound states. Keeping this in mind we may conclude that the coupling to selected \( mb \) channels acts as an effective healing mechanism to match deficient CQM predictions with data. Furthermore the implementation of these selected channels in the analyses of data might add certainty to the existence of the anomalies and at the same time help to reconcile competing and sometimes not compatible partial wave analyses.

4. Strangeness \(-1\) and \(-2\) two and three-baryon systems
In Ref. [14] we constructed different families of \( S = -1 \) interacting potentials, by introducing small variations of the mass of the effective scalar exchange potentials, that allow us to study the dependence of the results on the strength of the spin-singlet and spin-triplet hyperon-nucleon interactions. These potentials are characterized by the \( \Lambda N \) scattering lengths \( a_{i,s} \) and they reproduce the cross sections near threshold of the five hyperon-nucleon processes for which data are available. Let us analyze the three-body systems with strangeness \(-1\).

4.1. Strangeness -1 three-baryon systems: \( \Lambda NN \) and \( \Sigma NN \)
The channels \((I,J) = (0,1/2)\) and \((0,3/2)\) are the most attractive ones of the \( \Lambda NN \) system. In particular, the channel \((0,1/2)\) has the only bound state of this system, the hypertriton. We give in Table 2 the results of the models constructed in Ref. [14] for the two \( \Lambda d \) scattering lengths and the hypertriton binding energy. We compare with the results, in parentheses, obtained in Ref. [14] including only the three-body \( S \) wave configurations. As a consequence of considering the \( D \) waves, the hypertriton binding energy increases by about 50–60 keV [15], while the \( A_{0,1/2} \) scattering length decreases by about 3–5 fm. The largest changes occur in the \( A_{0,3/2} \) scattering length where both positive and negative values appeared which means, in the case of the negative values, that a bound state is generated in the \((I,J) = (0,3/2)\) channel. Since this channel depends mainly on the spin-triplet hyperon-nucleon interaction and experimentally
Table 2. \(Ad\) scattering lengths, \(A_{0,3/2}\) and \(A_{0,1/2}\) (in fm), and hypertriton binding energy, \(B_{0,1/2}\) (in MeV), for several hyperon-nucleon interactions characterized by \(\Lambda N\) scattering lengths \(a_{1/2,0}\) and \(a_{1/2,1}\) (in fm).

| \(a_{1/2,0}\) | \(a_{1/2,1}\) | \(A_{0,3/2}\) | \(A_{0,1/2}\) | \(B_{0,1/2}\) |
|---|---|---|---|---|
| 2.48 | 1.41 | 31.9 (66.3) | -16.0 (-20.0) | 0.129 (0.089) |
| 2.48 | 1.65 | -72.8 (198.2) | -13.8 (-17.2) | 0.178 (0.124) |
| 2.48 | 1.79 | -28.5 (-62.7) | -12.9 (-16.0) | 0.207 (0.145) |
| 2.31 | 1.65 | -76.0 (198.2) | -17.1 (-22.4) | 0.113 (0.070) |
| 2.74 | 1.65 | -72.1 (198.2) | -12.0 (-14.4) | 0.244 (0.182) |

There is no evidence whatsoever for the existence of a \((I, J) = (0, 3/2)\) bound state one can use the results of this channel to set limits on the value of the hyperon-nucleon spin-triplet scattering length \(a_{1/2,1}\). As one can guess from Table 2, the three-body channel \((I, J) = (0, 3/2)\) becomes bound if \(a_{1/2,1} > 1.58\) fm (see Ref. [16]). Moreover, we found in Ref. [14] that the fit of the hyperon-nucleon cross sections is worsened for those cases where the spin-triplet \(\Lambda N\) scattering length is smaller than 1.41 fm, so that we conclude that \(1.41 \leq a_{1/2,1} \leq 1.58\) fm. To set some limits to the hyperon-nucleon spin-singlet scattering length, we have calculated in Table 3 the hypertriton binding energy using for the hyperon-nucleon spin-triplet scattering length the allowed values \(1.41 \leq a_{1/2,1} \leq 1.58\) fm and for the spin-singlet scattering length \(2.33 \leq a_{1/2,0} \leq 2.48\) fm which leads to a hypertriton binding energy within the experimental error bars \(B_{0,1/2} = 0.13 \pm 0.05\) MeV.

We show in Table 4 the \(\Sigma d\) scattering lengths \(A'_{1,3/2}\) and \(A'_{1,1/2}\). The \(\Sigma d\) scattering lengths are complex since the inelastic \(\Lambda NN\) channels are always open. The scattering length \(A'_{1,3/2}\) depends mainly on the spin-triplet hyperon-nucleon channels and both its real and imaginary parts increase when the spin-triplet hyperon-nucleon scattering length increases. The effect of the three-body D waves is to lower the real part by about 20\% and the imaginary part by about 10\%. The scattering length \(A'_{1,1/2}\) shows large variations between the results with and without three-body D waves, due to the presence of a pole very near threshold. The position of this pole in the complex plane, given in the last column of Table 4, changes very little with the model used and it lies at around \(2.8 - i 2.1\) MeV.

4.2. Strangeness -2 two-baryon systems: \(\Lambda \Lambda, \Sigma \Sigma\) and \(N \Xi\)

In this section, we derive the strangeness \(-2\) baryon-baryon interactions: \(\Lambda \Lambda, \Lambda \Sigma, \Sigma \Sigma\) and \(N \Xi\). We use these two-body interactions to calculate two-body elastic and inelastic scattering cross sections [17] and we compare to experimental data and other theoretical models. We also

Table 3. Hypertriton binding energy (in MeV) for several hyperon-nucleon interactions characterized by \(\Lambda N\) scattering lengths \(a_{1/2,0}\) and \(a_{1/2,1}\) (in fm) which are within the experimental error bars \(B_{0,1/2} = 0.130 \pm 0.050\) MeV.

| \(a_{1/2,1} = 1.41\) | \(a_{1/2,1} = 1.46\) | \(a_{1/2,1} = 1.52\) | \(a_{1/2,1} = 1.58\) |
|---|---|---|---|
| \(a_{1/2,0} = 2.33\) | 0.080 | 0.087 | 0.096 | 0.106 |
| \(a_{1/2,0} = 2.39\) | 0.094 | 0.102 | 0.112 | 0.122 |
| \(a_{1/2,0} = 2.48\) | 0.129 | 0.140 | 0.152 | 0.164 |
would like to emphasize the agreement of our results with the Ξ mb. The upper limit of the cross section was derived as 12 mb at 90% confidence level. We evaluated in Ref. [17] the inelastic Ξ − p cross section compared to the in-medium experimental Ξ − p cross section around \( p_{\text{lab}}^2 \approx 550 \text{ MeV}/c \), where \( \sigma_{\Xi^- p} = 30 \pm 6.7 \pm 3.7 \text{ mb} \) [18]. Another analysis using the eikonal approximation gives \( \sigma_{\Xi^- p} = 20.9 \pm 4.5 \pm 2.5 \text{ mb} \) [19]. A more recent experimental analysis [20] for the low energy Ξ − p elastic and Ξ − p → ΛΛ total cross sections in the range 0.2 GeV/c to 0.8 GeV/c shows that the former is less than 24 mb at 90% confidence level and the latter of the order of several mb, respectively. We also evaluated in Ref. [17] the inelastic Ξ − p → ΛΛ cross section. It has been recently estimated at a laboratory momentum of \( p_{\text{lab}}^2 \approx 500 \text{ MeV}/c \), see Ref. [20], assuming a quasi-free scattering process for the reaction \(^{12}\text{C}(\Xi^-, \Lambda\Lambda)X\) obtaining a total cross section \( \sigma(\Xi^- p \rightarrow \Lambda\Lambda) = 4.3^{+5.3}_{-2.7} \text{ mb} \). The upper limit of the cross section was derived as 12 mb at 90% confidence level. We would like to emphasize the agreement of our results with the Ξ − p → ΛΛ conversion cross section. This reaction is of particular importance in assessing the stability of Ξ − quasi-particle states in nuclei.

The scattering lengths for the different spin-isospin channels are given in Table 5. As mentioned above, since the observation of the Nagara event [24] it is generally accepted that the ΛΛ interaction is only moderately attractive. Our result for the \(^1S_0\) ΛΛ scattering length is compatible with such event. A rough estimate of \( B_{\Lambda\Lambda}(B_{\Lambda\Lambda} \approx (\hbar c)^2/2\mu_{\Lambda\Lambda}a_{^1S_0}^{\Lambda\Lambda}) \) drives a value of 5.41 MeV, below the upper limit extracted from the Nagara event, \( B_{\Lambda\Lambda} = 7.25 \pm 0.19 \text{ MeV} \). Moreover, although from the ΛΛ scattering length alone one cannot draw any conclusion on the magnitude of the two-Λ separation energy, recent estimates [25] have reproduced the two-Λ separation energy, defined as \( \Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda}(\Lambda\Lambda) - 2B_{\Lambda}(\Lambda\text{He}) \), with scattering lengths of -1.32 fm.

### Table 4.\( \Sigma d \) scattering lengths, \( A_{^1S_0}^{1/3/2} \) and \( A_{^1S_0}^1 \) (in fm), and position of the quasi-bound state \( B_{^1S_0}^1 \) (in MeV) for the hyperon-nucleon interactions of Table 2.

| \( a_{1/2,0} \) | \( a_{1/2,1} \) | \( A_{^1S_0}^{1/3/2} \) | \( A_{^1S_0}^1 \) | \( B_{^1S_0}^1 \) |
|---|---|---|---|---|
| 2.48 | 1.41 | 0.14 + i 0.24 (0.20 + i 0.26) | 19.82 + i 16.94 (19.28 + i 25.37) | 2.92 − i 2.17 |
| 2.48 | 1.65 | 0.28 + i 0.27 (0.36 + i 0.29) | 12.08 + i 38.98 (−1.55 + i 42.31) | 2.84 − i 2.14 |
| 2.48 | 1.79 | 0.36 + i 0.29 (0.44 + i 0.31) | −8.00 + i 42.58 (−17.33 + i 35.01) | 2.79 − i 2.10 |
| 2.31 | 1.65 | 0.28 + i 0.27 (0.36 + i 0.29) | 19.01 + i 23.21 (14.95 + i 31.61) | 2.88 − i 2.14 |
| 2.74 | 1.65 | 0.28 + i 0.27 (0.36 + i 0.29) | −26.01 + i 17.95 (−23.29 + i 13.32) | 2.73 − i 2.09 |

Table 5. Two-body singlet and triplet scattering lengths, in fm, for different models as compared to our results. The results between squared brackets indicate the lower and upper limit for different parametrizations used in that reference.

| Model | Ref. [21] | Ref. [22] | Ref. [23] | Ref. [15] | Ours [17] |
|---|---|---|---|---|---|
| \( a_{^1S_0}^{\Lambda\Lambda} \) | [−0.27, −0.35] | [−1.555, −3.804] | [−1.52, −1.67] | −0.821 | −2.54 |
| \( a_{^1S_0}^{\Sigma^0} \) | [0.40, 0.46] | [0.144, 0.491] | [0.13, 0.21] | 0.324 | −3.32 |
| \( a_{^3S_0}^{\Sigma^+} \) | [−0.030, 0.050] | − | [0.0, 0.03] | −0.207 | 18.69 |
| \( a_{^1S_0}^{\Sigma^0} \) | [6.98, 10.32] | − | [−6.23, −9.27] | −85.3 | 0.523 |
5. Summary
To summarize, we have reviewed some recent results of hadron spectroscopy and hadron-hadron interaction in terms of the constituent quark model. The main goal of our presentation was to highlight the complementarity of a simultaneous study of both problems in order to constrain low-energy realizations of the underlying theory, QCD. We have seen how the enlargement of the Hilbert space when increasing energy, that was seen to be necessary to describe the nucleon-nucleon phenomenology above the pion production threshold, seems to be necessary to understand current problems of hadron spectroscopy. We have also tried to emphasize that spectroscopy and interaction can and must be understood within the same scheme when dealing with quark models. Any other alternative becomes irrelevant from the point of view of learning about properties of QCD.

In our analysis we have shown the first systematic study of four-quark hidden-charm states as meson-meson molecules. Our predictions robustly show that no deeply bound states can be expected for charmonium. Only a few channels can be expected to present observable resonances or slightly bound states. Among them, we have found that the $D^*D^*$ system must show a bound state slightly below the threshold for charmed mesons production with quantum numbers $J^{PC}(I) = 1^{++}(0)$, that could correspond to the widely discussed $X(3872)$. We have also shown that the $J^{PC}(I) = 2^{++}(1) D^*D^*$ channel is attractive and may then represent a charge state contributing to charmonium spectroscopy.

We have seen how the $4q1\bar{q}$ components, in the form of $S$ wave meson-baryon channels which we identify, play an essential role in the description of the anomalies, say baryon resonances very significantly overpredicted by three-quark models based on two-body interactions. Considering a simplified description of the anomalies as systems composed of a free meson-baryon channel interacting with a three-quark confined component we have shown they could correspond mostly to meson-baryon states but with a non-negligible $3q$ state probability which makes their masses to be below the meson-baryon threshold.

We also presented results for three-baryon systems with strangeness $-1$ using the hyperon-nucleon and nucleon-nucleon interactions derived from a constituent quark model with full inclusion of the $\Lambda \leftrightarrow \Sigma$ conversion. In the case of the $\Lambda NN$ system, our calculation permits to constrain the $\Lambda N$ scattering lengths to: $1.41 \leq a_{1/2,1} \leq 1.58$ fm and $2.33 \leq a_{1/2,0} \leq 2.48$ fm. In the case of the $\Sigma NN$ system there exists a narrow quasibound state near threshold in the $(I, J) = (1, 1/2)$ channel.

We have presented results for the doubly strange two-baryon interactions. Our predictions are consistent with the recently obtained doubly strange elastic and inelastic scattering cross sections. In particular, our results are compatible with the $\Xi^- p \rightarrow \Lambda\Lambda$ conversion cross section, important in assessing the stability of $\Xi^-$ quasi-particle states in nuclei.

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