A multistage optimal placement approach of monitors considering voltage sag and fault position observability

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Abstract. Reasonable placement of limited monitors is beneficial to reduce the cost of monitoring for voltage sags assessment. However, existing methods only consider the observability of voltage sags, leading to information loss. To capture all the voltage sags and the corresponding fault positions in distribution network, a multistage optimal placement approach of power quality monitors considering voltage sag and fault position observability is proposed. The initial requirement of voltage sag monitoring, that is, voltage sag observability is ensured by monitoring reach area method. Then, based on a simple fault location method, the fault location observability index is proposed for describing monitoring effects for fault position observability. A series of placement schemes with different costs and monitoring effects are given, which is convenient for engineers to choose the proper placement scheme according to the budget and demand. The simulation results carried on the IEEE 69-bus system show that the proposed approach can is correct and available.

1. Introduction
As the most frequently occurring power quality disturbances, voltage sag, mainly caused by faults, has caused enormous economic losses to customers in various industries [1]. Installation of power quality monitors (PQMs) is the most effective way to monitor and assess voltage sags in the distribution network [2]. The goal of monitoring is to achieve fully voltage sag observability by reasonably placement of limited PQMs, which save cost and reduce the redundancy of monitoring data [3]. Voltage sag observability means that voltage sag events caused by any type of fault at any location in the system can be effectively recorded [4-5]. The goal of assessment is to analyze the influence of voltage sag based on the recorded data by PQMs, which demand the precise fault position. In other words, imprecise fault position causes inaccurate estimation of voltage sags and the mitigation of voltage sags is affected. Therefore, optimal placement of monitors approach considering voltage sag and fault position observability is required.

In general, the uppermost method for PQMs optimal placement is monitor reach area (MRA) [6-8] method, aiming to minimize number of monitors. Based on MRA and integer linear programming, the placement scheme can guarantee voltage sag observability, which means that the voltage sag caused by any type of fault can be monitored by at least one monitor. However, due to the singleness of optimization objective, the multiple placement schemes determined by MRA method cannot fulfill some other significant requirements such as fault location observability [9, 10]. In this context, as to acquire the placement scheme considering more information, indexes such as the largest sag observability index [11], the largest sag weight coefficient [12], and the smallest uncertainty area...
index [13] are introduced by the subsequent MRA-based studies, which solve the problem of multiple solution as well. Another important goal of monitors placement is to locate the fault position [13-16], by which the ability of fault location is greatly enhanced. However, the fault location method in [13, 16] is only suitable for transmission system; the disturbance source observability matrix in [14] is difficult to obtain in practice; Reference [15] require at least 2 monitors triggered at the same time, which greatly increases the costs. To sum up, since the above fault position location methods are complicated and inaccurate in distribution system, those kind of placement approaches are not available.

In this regard, by taking advantage of the fault location method in [17], a multistage optimal placement approach of monitors is presented, which makes the voltage sag occurred in the whole power grid and the fault position leading to the voltage sag observable. Firstly, the optimal placement model of stage one is established by taking the voltage sag observability as the constraint condition and the minimum number of monitors as the objective function. Then the concept of fault location observability index (FLOI) is introduced to calculate the localizable coverage of the short-current fault under certain placement scheme. Next, the objective function of the second stage, maximum FLOI is solved to obtain the optimal placement scheme. The process of stage two is repeated to procure maximum FLOI with constraints of number of monitors in last stage plus one until FLOI equals to 100%. At last, the method of choosing the optimal scheme from schemes with different monitoring costs and FLOI is proposed to keep a balance between fault location observability and economy. The simulation results carried on the IEEE 69-bus system show that the proposed method is correct and available.

2. Fault location observability analysis

If any fault can be uniquely and accurately located by one or more monitors, the corresponding placement scheme fulfill the fault location observability. In this paper, the scope is limited to the distribution network. According to Reference [17], the fault location can be accurately located by using the voltage measurement data, which is also recorded by PQMs. Compared to other fault location method, the method in Reference [17] requires fewer monitors and is applicable for system with distributed generation (DG). In the following sections, the method is briefly reviewed first. Then the concept of fault location observability index based on this method is presented.

2.1. Fault location utilizing voltage measurement at terminal buses [17]

A 6-bus distribution system with three PQMs equipped at the terminal bus shown in Figure 1 is adopted to introduce the fault location method. And the system is divided into two subsystems as indicated in Figure 2.

![Figure 1. 6-bus distribution system.](image1)

![Figure 2. Divided subsystems.](image2)

When a fault occurs at section 2-3, the fault current ($I_f = I_1 + I_3$) is given by the sum of both elements in the current vector, as indicated in (1).

$$\begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \text{inv} \begin{bmatrix} Z_{T1} & Z_T \\ Z_T & Z_T + 2Z \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_3 \end{bmatrix}$$

(1)

Where $\Delta V_1$ and $\Delta V_3$ is the voltage sags at buses 1 and 5. Assuming bus-$k$ in subsystem 2 as the faulted bus, a sparse current vector is determined containing the calculated fault current at bus-$k$
position and the other elements set to zero. By multiplying the impedance matrix of subsystem 2 by the determined sparse current vector, a voltage sag vector is obtained. The same process is performed for all buses of subsystem 2. Consequently, different voltage sags at bus 5 are obtained with fault current injected into the different bus, as indicated in (2).

\[
\begin{bmatrix}
\Delta V_{5,1}^2 \\
\Delta V_{5,2}^2 \\
\Delta V_{5,5}^2
\end{bmatrix} = 
\begin{bmatrix}
Z_{ii} & 0 & 0 \\
0 & Z_{12} & 0 \\
0 & 0 & Z_{55}
\end{bmatrix} 
\begin{bmatrix}
I_f \\
I_f \\
I_f
\end{bmatrix}
\]

(2)

Where \( Z_{ij} \) is the transfer impedance between bus \( i \) and \( j \). \( \Delta V_{5,i}^2 \) is the voltage sag calculated at terminal-bus-5 due to a fault current injection at bus-\( i \) under the analysis of subsystem 2. Suppose the module of the voltage sag measured at the terminal-bus-5 as a reference, an error is attributed for each voltage sag calculated at the terminal-bus-5, as described in (3).

\[
\begin{bmatrix}
\text{Error}_{5,1}^2 \\
\text{Error}_{5,2}^2 \\
\text{Error}_{5,5}^2
\end{bmatrix} = 
\begin{bmatrix}
\Delta V_{5}^2 \\
\Delta V_{5,2}^2 \\
\Delta V_{5,5}^2
\end{bmatrix} - 
\begin{bmatrix}
\Delta V_{5,1}^2 \\
\Delta V_{5,2}^2 \\
\Delta V_{5,5}^2
\end{bmatrix}
\]

(3)

This process, from (2) to (3), is performed for all subsystems. \( \text{Error}_{5,i}^2 \) is the error attributed for the voltage calculated at terminal-bus-5 of subsystem 2 when a fault current is infected at bus-\( i \).

The bus associated with the lowest error of each subsystem is selected and called as candidate bus. After that, the bus closest to the fault should be selected as the candidate bus in all subsystems by a weight vector. After locating the bus closest to the fault, the fault section and fault point can also be located. For this, virtual buses are added to all sections connected to the bus closest to the fault, and the impedance matrix based on the new topology is recalculated. Then the process of (2) - (3) is repeated. Finally, the virtual bus corresponding to the minimum error is considered as the fault point.

2.2. Fault location observability index

According to the fault location method in [17], when monitors are installed at substations and all terminal-bus in the distribution network, the fault location can be identified precisely, in other words, the requirements of fault location observability can be completely fulfilled. However, as the size of the system and the number of branches increase, the monitoring cost will be immense. For cost savings, it is acceptable to reduce the number of monitors in practice, though the fault location observability cannot be perfectly fulfilled.

In this paper, the fault location observability index (FLOI) is defined to describe the coverage that can be located when a fault occurs in the grid, as indicated in (4), where \( D_{\text{total}} \) is the total length of lines in the distribution network, \( D_{\text{monitor}(i)} \) represents the length of lines between the substation and the monitor \( i \). The larger the FLIO, the larger the coverage that can be located. When FLIO under a certain placement scheme is equal to 100%, the fault location observability is consummately fulfilled.

\[
\text{FLOI} = \frac{\bigcup_{i=1}^{m} D_{\text{monitor}(i)}}{D_{\text{total}}} \times 100\%
\]

(4)

3. Method for optimal placement of monitors in the distribution network

3.1. Voltage sag observability

The voltage sag observability of the entire network refers to the ability of monitors to capture all voltage sag events in the network under a pre-determined threshold [8], usually described by monitor reach area (MRA), which is defined as the area of the network that can be observed from a given monitor position. If a small number of buses can be determined so that the union of MRA can cover
the whole system, the voltage sag observability of the whole system can be guaranteed without installing monitors at each bus. MRA is represented with a binary matrix as shown in (5), where \( w \) is fault type, \( m_{ij}^{w} \) is given by (6), where \( V_{ij} \) is the remaining voltage magnitude at bus \( j \) when a fault occurs at position \( i \).

\[
M^{w} = \begin{bmatrix}
m_{11}^{w} & m_{12}^{w} & \cdots & m_{1p}^{w} \\
m_{21}^{w} & m_{22}^{w} & \cdots & m_{2p}^{w} \\
\vdots & \vdots & \ddots & \vdots \\
m_{n1}^{w} & m_{n2}^{w} & \cdots & m_{np}^{w}
\end{bmatrix}
\]  

(5)

\[m_{ij}^{w} = \begin{cases}
1 & V_{ij} \leq V_{th} \\
0 & V_{ij} > V_{th}
\end{cases}
\]  

(6)

The physical meaning of \( M^{w} \) is that the element in row \( i \) represents the MRA range of bus \( i \), a value of 1 indicates that the fault point is within the MRA, and a value of 0 indicates that it is outside the MRA. For the four types of short-circuit faults, the corresponding MRA matrices \( M^{0}, M^{1}, M^{2} \) and \( M^{3} \) are calculated respectively to form an observability matrix (7) reflecting four types of short-circuit faults

\[
M = [M^{0} \ M^{1} \ M^{2} \ M^{3}]
\]  

(7)

3.2. Placement scheme of stage one
A binary decision vector \( X \) is defined, indicating the position of monitors at the \( n \) buses of the system, as shown in (8), the elements of \( x_{i} \) are formed in (9).

\[
X = [x_{1} \ x_{2} \ \cdots \ x_{n}]
\]  

(8)

\[
x_{i} = \begin{cases}
1; \text{ equipped monitor at bus } i \\
0; \text{ otherwise}
\end{cases}
\]  

(9)

The main purpose of the first stage placement is to obtain the minimum number of monitors to fulfill the observability of voltage sag of the whole network. For this, the objective function is constructed to minimize the number of monitors, as indicated in (10):

\[
\min \sum_{i=1}^{n} x_{i}
\]  

(10)

The voltage sags caused by any type of fault at any position of the whole network should be monitored by at least one device, so \( X \) must be subject to the following Constraint (11):

\[
\sum_{i=1}^{n} x_{i} m_{ij} \geq 1
\]  

(11)

Equations (10) - (11) can be solved by genetic algorithm. If multiple placement schemes are obtained, the placement scheme with the maximal FLOI is selected as the optimal placement scheme under this number of monitors.

3.3. Placement scheme of stage two
The second stage placement is based on the number of monitors required in the first stage placement scheme. By increasing the number of monitors one by one, the placement scheme corresponding to the maximal FLOI is to be obtained under the condition of fulfilling the voltage sag observability.

The given optimal placement model in stage two aims to provide the best placement of monitors, the number of which depends on the minimum number of monitors solved in stage one. So the constraint 1 can be described as (12)

\[
\sum_{i=1}^{n} x_{i} = \text{sum}^{i-1} + 1
\]  

(12)
where \( \text{sum}^{-1} \) is the minimum number of monitors corresponding to the last placement scheme.

The voltage sag observability constraint in stage one is also necessary at this stage.

\[
\sum_{i} x_{i,j} \geq 1
\]

Taking the maximal fault location observability index as the optimization objective, the objective function (14) is constructed.

\[
\max \text{FLOI}(X)
\]

Genetic algorithm is implemented to solve Equations (12) - (14) to determine the placement scheme and the number of monitors \( \text{sum}^{-1} \). Then \( \text{sum}^{-1} \) in Equation (12) is updated with \( \text{sum}^{-1} \), the model can be ceaselessly solved until FLOI is equal to 100%. Through the above methods, a series of placement schemes under different FLOI and the number of monitors can be determined.

4. Case study

4.1. Multistage placement scheme of monitors

The proposed approach is verified by the IEEE 69-bus radial distribution network [18] shown in Figure 3, which comprises 69 buses, 8 branches 68 lines, and 1 transformer with YNyn connection mode. The voltage sag threshold of PQMs is set to 0.9 p.u. The results of multistage placement schemes are shown in Table 1.

![Diagram of the 69-bus, 12.66 kV distribution system](image)

**Figure 3.** Diagram of the 69-bus, 12.66 kV distribution system.

**Table 1.** Results of multistage placement schemes.

| Stage | Buses equipped with monitors | Number of monitors | FLOI/% |
|-------|-----------------------------|--------------------|--------|
| 1     | 1, 27, 35                   | 3                  | 50.00% |
| 2     | 1, 27, 35, 65               | 4                  | 69.12% |
| 3     | 1, 27, 35, 65, 46           | 5                  | 85.29% |
| 4     | 1, 27, 35, 65, 46, 50       | 6                  | 91.18% |
| 5     | 1, 27, 35, 65, 46, 50, 52   | 7                  | 94.12% |
| 6     | 1, 27, 35, 65, 46, 50, 52, 67 | 8              | 97.06% |
| 7     | 1, 27, 35, 65, 46, 50, 52, 67, 69 | 9          | 100.00% |

Table 1 shows the corresponding relation between the number of monitors and the FLOI. All the placement schemes consist the placement schemes set. In the first stage, buses 1, 27, and 35 are placed with monitors to ensure that voltage sag events caused by any type of fault in the whole network can be recorded by at least one monitor, but only 50% of the lines are locatable. With the increasing number of monitors, the fault location observability index also monotonically increases. When the placement scheme is 1, 27, 35, 65, 46, 50, 52, 67, and 69, the fault location observability of the entire network is implemented.

4.2. Selection method of optimal placement scheme

If it is not necessary to consider the economy, the optimal selection is the seventh stage placement scheme which completely fulfills the fault location observability. However, in order to achieve a large fault location coverage with lower cost, it is significant to explore the optimal placement scheme.
which keeps a balance between fault location observability and economy. The relation between the needed number of monitors in kinds of placement schemes and the corresponding fault observability index is shown in Figure 4.

![Figure 4](image-url)

**Figure 4.** The relation between FLOI and the number of monitors.

It can be seen from Figure 4, the curve is a convex function, which means that the fault location observability index increases as the number of monitors increasing, but the change rate gradually decreases. For further analysis, the first-order change rate and second-order change rate of each point are calculated, as presented in Table 2. As indicated in Table 2, when the number of monitors is equal to 7, the second-order change rate of FLOI is zero, indicating that if the monitors are constantly added, the increment of fault location coverage will no longer change, and the marginal effect of location effect decreases. Therefore, the optimal number of monitors is 6, and the corresponding placement scheme is 1, 27, 35, 65, 46, and 50, which can achieve 91.18% of the fault location coverage.

**Table 2.** The change rate of FLOI under different placement schemes.

| Number of monitors | FLOI/% | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|--------------------|--------|-----|-----|-----|-----|-----|-----|-----|
| First-order rate of change | 0.1912 | 0.1618 | 0.0588 | 0.0294 | 0.0294 | 0.0294 | \   |
| Second-order rate of change | \     | -0.0294 | -0.1029 | -0.294 | 0   | 0   | \   |

The methods in [13-16] that only provide one placement scheme, which is not suitable in engineering practice. As contrast, when the curve between FLOI and the number of monitors in Figure 4 is determined through this proposed method, the engineers can decide the placement scheme according to the budget and the corresponding monitoring effects by the curve. If it is not decided by engineers, the recommended placement scheme is given as well. In addition, it is convenient to update the monitoring systems by installing extra monitors according to the placement schemes set, because the buses equipped with monitors of the last stage is included in the next stage, which was not considered in [13-16]. The ability of fault location has been verified in Reference [17], and simulation results indicated that the fault location can be accurately located by using the voltage measurement data.

**5. Conclusions**

In this paper, a multistage placement approach of PQMs applied to the radial distribution network is presented, which fulfills the voltage sag observability of the whole network and the fault location observability. The main conclusions are as follows.

1) The fault location observability index is introduced to describe the localizable coverage of the short-current fault, which makes up for the lack of monitors configuration scheme in Reference [17].

2) Considering the contradiction between the monitoring effect and monitoring cost of different monitoring schemes, the determination method of the optimal monitoring scheme is given, which is greatly valuable to engineering practice.
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