Exploiting Channel Memory for Multi-User Wireless Scheduling without Channel Measurement: Capacity Regions and Algorithms

Chih-ping Li, Student Member, IEEE and Michael J. Neely, Senior Member, IEEE

Abstract—We study the fundamental network capacity of a multi-user wireless downlink under two assumptions: (1) Channels are not explicitly measured and thus instantaneous states are unknown, (2) Channels are modeled as ON/OFF Markov chains. This is an important network model to explore because channel probing may be costly or infeasible in some contexts. In this case, we can use channel memory with ACK/NACK feedback from previous transmissions to improve network throughput. Computing in closed form the capacity region of this network is difficult because it involves solving a high dimension partially observed Markov decision problem. Instead, in this paper we construct an inner and outer bound on the capacity region, showing that the bound is tight when the number of users is large and the traffic is symmetric. For the case of heterogeneous traffic and any number of users, we propose a simple queue-dependent policy that can stabilize the network with any data rates strictly within the inner capacity bound. The stability analysis uses a novel frame-based Lyapunov drift argument. The outer-bound analysis uses stochastic coupling and state aggregation to bound the performance of a restless bandit problem using a related multi-armed bandit system. Our results are useful in cognitive radio networks, opportunistic scheduling with delayed/uncertain channel state information, and restless bandit problems.

Index Terms—stochastic network optimization, Markovian channels, delayed channel state information (CSI), partially observable Markov decision process (POMDP), cognitive radio, restless bandit, opportunistic spectrum access, queuing theory, Lyapunov analysis.

I. INTRODUCTION

Due to the increasing demand of cellular network services, in the past fifteen years efficient communication over a single-hop wireless downlink has been extensively studied. In this paper we study the fundamental network capacity of a time-slotted wireless downlink under the following assumptions: (1) Channels are never explicitly probed, and thus their instantaneous states are never known, (2) Channels are modeled as two-state ON/OFF Markov chains. This network model is important because, due to the energy and timing overhead, learning instantaneous channel states by probing may be costly or infeasible. Even if this is feasible (when channel coherence time is relatively large), the time consumed by channel probing cannot be re-used for data transmission, and transmitting data without probing may achieve higher throughput. In addition, since wireless channels can be adequately modeled as Markov chains, we will take advantage of channel memory to improve network throughput.

Specifically, we consider a time-slotted wireless downlink where a base station serves \( N \) users through \( N \) (possibly different) positively correlated Markov ON/OFF channels. Channels are never probed so that their instantaneous states are unknown. In every slot, the base station selects at most one user to which it transmits a packet. We assume every packet transmission takes exactly one slot. Whether the transmission succeeds depends on the unknown state of the channel. At the end of a slot, an ACK/NACK is fed back from the served user to the base station. Since channels are either ON or OFF, this feedback reveals the channel state of the served user in the last slot and provides partial information of future states. Our goal is to characterize all achievable throughput vectors in this network, and to design simple throughput-achieving algorithms.

We define the network capacity region \( \Lambda \) as the closure of the set of all achievable throughput vectors. We can compute \( \Lambda \) by locating its boundary points. Every boundary point can be computed by formulating a partially observable Markov decision process (POMDP) with information states defined as, conditioning on the channel observation history, the probabilities that channels are ON. This approach, however, is computationally prohibitive because the information state space is countably infinite (which we will show later) and grows exponentially fast with \( N \).

The first contribution of this paper is that we construct an outer and an inner bound on \( \Lambda \). The outer bound comes from analyzing a fictitious channel model in which every scheduling policy yields higher throughput than it does in the real network. The inner bound is the achievable rate region of a special class of randomized round robin policies (introduced

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1 One quick example is to consider a time-slotted channel with state space \( \{B, G\} \). Suppose channel states are i.i.d. over slots with stationary probabilities \( \Pr[B] = 0.2 \) and \( \Pr[G] = 0.8 \). At most 1 and 2 packets can be successfully delivered in a slot, respectively. Packet transmissions beyond the capacity will fail and need retransmissions. Channel probing can be done on each slot, which consumes 0.2 fraction of a slot. Then the policy that always probes the channel yields throughput \( 0.8 (2 \cdot 0.8 + 1 - 0.2) = 1.44 \), while the policy that never probes the channel and always sends packets at rate 2 packets/slot yields throughput \( 2 \cdot 0.8 = 1.6 > 1.44 \).
in Section [IV-A]. These policies are simple and take advantage of channel memory. In the case of symmetric channels (that is, channels are i.i.d.) and when the network serves a large number of users, we show that as data rates are more balanced, or in a geometric sense as the direction of the data rate vector in the Euclidean space is closer to the 45-degree angle, the inner bound converges geometrically fast to the outer bound, and the bounds are tight. This analysis uses results in [6, 7] that derive an outer bound on the maximum sum throughput for a symmetric system.

The inner capacity bound is indeed useful. First, the structure of the bound itself shows how channel memory improves throughput. Second, we show analytically that a large class of intuitively good heuristic policies achieve throughput that is at least as good as this bound, and hence the bound acts as a (non-trivial) performance guarantee. Finally, supporting throughput outside this bound may inevitably involve solving a much more complicated POMDP. Thus, for simplicity and practicality, we may regard the inner bound as an operational network capacity region.

In this paper we also derive a simple queue-dependent dynamic round robin policy that stabilizes the network whenever the arrival rate vector is interior to our inner bound. This policy has polynomial time complexity and is derived by a novel variable-length frame-based Lyapunov analysis, first used in [3] in a different context. This analysis is important because the inner bound is based on a mixture of many different types of round robin policies, and an offline computation of the proper time average mixtures needed to achieve a given point in this complex inner bound would require solving $\Theta(2^N)$ unknowns in a linear system, which is impractical when $N$ is large. The Lyapunov analysis overcomes this complexity difficulty with online queue-dependent decisions.

The results of this paper apply to the emerging area of opportunistic spectrum access in cognitive radio networks (see [9] and references therein), where the channel occupancy of a primary user acts as a Markov ON/OFF channel to the secondary users. Specifically, our results apply to the important case where each of the secondary users has a designated channel and they cooperate via a centralized controller. This paper is also a study on efficient scheduling over wireless networks with delayed/uncertain channel state information (CSI) (see [10]–[12] and references therein). The work on delayed CSI that is most closely related to ours is [11], [12], where the authors study the capacity region and throughput-optimal policies of different wireless networks, assuming that channel states are persistently probed but fed back with delay. We note that our paper is significantly different. Here channels are never probed, and new (delayed) CSI of a channel is only acquired when the channel is served. Implicitly, acquiring the delayed CSI of any channel is part of the control decisions in this paper.

This paper is organized as follows. The network model is given in Section II; inner and outer bounds are constructed in Sections III and IV; and compared in Section V in the case of symmetric channels. Section VI gives the queue-dependent policy to achieve the inner bound.

**II. NETWORK MODEL**

Consider a base station transmitting data to $N$ users through $N$ Markov ON/OFF channels. Suppose time is slotted with normalized slots $t$ in $\{0, 1, 2, \ldots\}$. Each channel is modeled as a two-state ON/OFF Markov chain (see Fig. 1). The state evolution of channel $n \in \{1, 2, \ldots, N\}$ follows the transition probability matrix

$$
P_n = \begin{bmatrix} P_{n,00} & P_{n,01} \\ P_{n,10} & P_{n,11} \end{bmatrix},
$$

where state ON is represented by 1 and OFF by 0, and $P_{n,ij}$ denotes the transition probability from state $i$ to $j$. We assume $P_{n,11} < 1$ for all $n$ so that no channel is constantly ON. Incorporating constantly ON channels like wired links is easy and thus omitted in this paper. We suppose channel states are fixed in every slot and may only change at slot boundaries. We assume all channels are positively correlated, which, in terms of transition probabilities, is equivalent to assuming $P_{n,11} > P_{n,01}$ or $P_{n,01} + P_{n,10} < 1$ for all $n$.

We suppose the base station keeps $N$ queues of infinite capacity to store exogenous packet arrivals destined for the $N$ users. At the beginning of every slot, the base station attempts to transmit a packet (if there is any) to a selected user. We suppose the base station has no channel probing capability and must select users oblivious of the current channel states. If a user is selected and its current channel state is ON, one packet is successfully delivered to that user. Otherwise, the transmission fails and zero packets are served. At the end of a slot in which the base station serves a user, an ACK/NACK message is fed back from the selected user to the base station through an independent error-free control channel, according to whether the transmission succeeds. Failing to receive an ACK is regarded as a NACK. Since channel states are either ON or OFF, such feedback reveals the channel state of the selected user in the last slot.

Conditioning on all past channel observations, define the $N$-dimensional information state vector $\omega(t) = (\omega_n(t) : 1 \leq n \leq N)$ where $\omega_n(t)$ is the conditional probability that channel $n$ is ON in slot $t$. We assume initially $\omega_n(0) = \pi_{n,ON}$ for all $n$, where $\pi_{n,ON}$ denotes the stationary probability that channel $n$ is ON. As discussed in [5] Chapter 5.4, vector $\omega(t)$ is a sufficient statistic. That is, instead of tracking the whole system

\footnote{Assumption $P_{n,11} > P_{n,01}$ yields that the state $s_n(t)$ of channel $n$ has auto-covariance $\mathbb{E}[(s_n(t) - \mathbb{E}s_n(t))(s_n(t+1) - \mathbb{E}s_n(t+1))] > 0$. In addition, we note that the case $P_{n,11} = P_{n,01}$ corresponds to a channel having i.i.d. states over slots. Although we can naturally incorporate i.i.d. channels into our model and all our results still hold, we exclude them in this paper because we shall show how throughput can be improved by channel memory, which i.i.d. channels do not have. The degenerate case where all channels are i.i.d. over slots is fully solved in [2].}
history, the base station can act optimally only based on $\omega(t)$. The base station shall keep track of the $\{\omega(t)\}$ process.

We assume transition probability matrices $P_n$ for all $n$ are known to the base station. We denote by $s_n(t) \in \{\text{OFF}, \text{ON}\}$ the state of channel $n$ in slot $t$. Let $n(t) \in \{1, 2, \ldots, N\}$ denote the user served in slot $t$. Based on the ACK/NACK feedback, vector $\omega(t)$ is updated as follows. For $1 \leq n \leq N$,

$$
\omega_n(t+1) = \begin{cases} 
P_{n,01}, & \text{if } n = n(t), s_n(t) = \text{OFF} \\
P_{n,11}, & \text{if } n = n(t), s_n(t) = \text{ON} \\
(1 - \omega_n(t))P_{n,01}, & \text{if } n \neq n(t).
\end{cases}
$$

(1)

If in the most recent use of channel $n$, we observed (through feedback) its state was $i \in \{0, 1\}$ in slot $(t-k)$ for some $k \leq t$, then $\omega_n(t)$ is equal to the $k$-step transition probability $P_{n,i}^{(k)}$. In general, for any fixed $n$, probabilities $\omega_n(t)$ take values in the countably infinite set $\mathcal{W}_n = \{P_{n,01}^{(k)}, P_{n,11}^{(k)} : k \in \mathbb{N}\} \cup \{\pi_{n,\text{ON}}\}$. By eigenvalue decomposition on $P_n$ [13, Chapter 4], we can show the $k$-step transition probability matrix $P_n^{(k)}$ is

$$
P_n^{(k)} = \left[ \begin{array}{cc} P_{n,00}^{(k)} & P_{n,01}^{(k)} \\
P_{n,10}^{(k)} & P_{n,11}^{(k)} \end{array} \right] = (P_n)^k
$$

(2)

where we have defined $x_n = P_{n,01} + P_{n,10}$. Assuming that channels are positively correlated, i.e., $x_n < 1$, by Fig. 2 we have the following lemma.

**Lemma 1.** For a positively correlated $(P_{n,11} > P_{n,01})$ Markov ON/OFF channel with transition probability matrix $P_n$, we have

1. The stationary probability $\pi_{n,\text{ON}} = P_{n,01}/x_n$.
2. The $k$-step transition probability $P_n^{(k)}$ is nondecreasing in $k$ and $P_{n,11}^{(k)}$ nonincreasing in $k$. Both $P_{n,01}^{(k)}$ and $P_{n,11}^{(k)}$ converge to $\pi_{n,\text{ON}}$ as $k \to \infty$.

As a corollary of Lemma 1 it follows that

$$
P_{n,11} \geq P_{n,11}^{(k_2)} \geq \pi_{n,\text{ON}} \geq P_{n,01}^{(k_3)} \geq P_{n,01}^{(k_4)} \geq P_{n,01}
$$

(3)

for any integers $k_1 \leq k_2$ and $k_3 \geq k_4$ (see Fig. 2). To maximize network throughput, 4 has some fundamental implications. We note that $\omega_n(t)$ represents the transmission success probability over channel $n$ in slot $t$. Thus we shall keep serving a channel whenever its information state is $P_{n,11}$, for it is the best state possible. Second, given that a channel was OFF in its last use, its information state improves as long as the channel remains idle. Thus we shall wait as long as possible before reusing such a channel. Actually, when channels are symmetric ($P_n = P$ for all $n$), it is shown that a myopic policy with this structure maximizes the sum throughput of the network 7.

III. A ROUND ROBIN POLICY

For any integer $M \in \{1, 2, \ldots, N\}$, we present a special round robin policy RR($M$) serving the first $M$ users

$$
\{1, 2, \ldots, M\}
$$

in the network. The $M$ users are served in the circular order $1 \to 2 \to \cdots \to M \to 1 \to \cdots$. In general, we can use this policy to serve any subset of users. This policy is the fundamental building block of all the results in this paper.

A. The Policy

**Round Robin Policy RR($M$):**

1. At time 0, the base station starts with channel 1. Suppose initially $\omega_n(0) = \pi_{n,\text{ON}}$ for all $n$.
2. Suppose at time $t$, the base station switches to channel $n$. Transmit a data packet to user $n$ with probability $P_{n,01}^{(M)}/\omega_n(t)$ and a dummy packet otherwise. In both cases, we receive ACK/NACK information at the end of the slot.
3. At time $(t+1)$, if a dummy packet is sent at time $t$, switch to channel $(n \mod M) + 1$ and go to Step 2. Otherwise, keep transmitting data packets over channel $n$ until we receive a NACK. Then switch to channel $(n \mod M) + 1$ and go to Step 2. We note that dummy packets are only sent on the first slot every time the base station switches to a new channel.
4. Update $\omega(t)$ according to (1) in every slot.

Step 2 of RR($M$) only makes sense if $\omega_n(t) \geq P_{n,01}^{(M)}$, which we prove in the next lemma.

**Lemma 2.** Under RR($M$), whenever the base station switches to channel $n \in \{1, 2, \ldots, M\}$ for another round of transmission, its current information state satisfies $\omega_n(t) \geq P_{n,01}^{(M)}$.

**Proof of Lemma 2.** See Appendix A.

We note that policy RR($M$) is very conservative and not throughput-optimal. For example, we can improve the throughput by always sending data packets but no dummy ones. Also, it does not follow the guidelines we provide at the end of Section 4 for maximum throughput. Yet, we will see that, in the case of symmetric channels, throughput under RR($M$) is close to optimal when $M$ is large. Moreover, the underlying analysis of RR($M$) is tractable so that we can mix such round robin policies over different subsets of users to form a non-trivial inner capacity bound. The tractability of RR($M$) is because it is equivalent to the following fictitious round robin policy (which can be proved as a corollary of Lemma 3 provided later).

**Equivalent Fictitious Round Robin:**

1. At time 0, start with channel 1.
2) When the base station switches to channel \( n \), set its current information state to \( P_{n,01}^{(M)} \). Keep transmitting data packets over channel \( n \) until we receive a NACK. Then switch to channel \( (n \mod M) + 1 \) and repeat Step 2.

For any round robin policy that serves channels in the circular order \( 1 \to 2 \to \cdots \to M \to 1 \to \cdots \), the technique of resetting the information state to \( P_{n,01}^{(M)} \) creates a system with an information state that is worse than the information state under the actual system. To see this, since in the actual system channels are served in the circular order, after we switch away from serving a particular channel \( n \), we serve the other \((M - 1)\) channels for at least one slot each, and so we return to channel \( n \) after at least \( M \) slots. Thus, its starting information state is always at least \( P_{n,01}^{(M)} \) (the proof is similar to that of Lemma 2). Intuitively, if information states represent the packet transmission success probabilities, resetting them to lower values degrades throughput. This is the reason why our inner capacity bound constructed later using \( RR(M) \) provides a throughput lower bound for a large class of policies.

B. Network Throughput under \( RR(M) \)

Next we analyze the throughput vector achieved by \( RR(M) \).

1) General Case: Under \( RR(M) \), let \( L_{kn} \) denote the duration of the \( k \)th time the base station stays with channel \( n \). A sample path of the \( \{ L_{kn} \} \) process is

\[
(L_{11}, L_{12}, \ldots, L_{1M}, L_{21}, L_{22}, \ldots, L_{2M}, L_{31}, \ldots)
\]

round \( k = 1 \)

round \( k = 2 \)

The next lemma presents useful properties of \( L_{kn} \), which serve as the foundation of the throughput analysis in the rest of the paper.

Lemma 3. For any integer \( k \) and \( n \in \{1, 2, \ldots, M\} \),

1) The probability mass function of \( L_{kn} \) is independent of \( k \), and is

\[
L_{kn} = \begin{cases} 1 & \text{with prob. } 1 - P_{n,01}^{(M)} \\ j \geq 2 & \text{with prob. } P_{n,01}^{(M)} (P_{n,11})^{(j-2)} P_{n,10}. \end{cases}
\]

As a result, for all \( k \in \mathbb{N} \) we have

\[
\mathbb{E}[L_{kn}] = 1 + \frac{P_{n,01}^{(M)}}{P_{n,10}} = 1 + \frac{P_{n,01}(1 - (1 - x_n)^M)}{x_n P_{n,10}}.
\]

2) The number of data packets served in \( L_{kn} \) is \( (L_{kn} - 1) \).

3) For every fixed channel \( n \), time durations \( L_{kn} \) are i.i.d. random variables over all \( k \).

Proof of Lemma 3

1) Note that \( L_{kn} = 1 \) if, on the first slot of serving channel \( n \), either a dummy packet is transmitted or a data packet is transmitted but the channel is OFF. This event occurs with probability

\[
\left(1 - \frac{P_{n,01}^{(M)}}{\omega_n(t)}\right) + \frac{P_{n,01}^{(M)}}{\omega_n(t)} (1 - \omega_n(t)) = 1 - P_{n,01}^{(M)}.
\]

Next, \( L_{kn} = j \geq 2 \) if in the first slot a data packet is successfully served, and this is followed by \((j-2)\) consecutive ON slots and one OFF slot. This happens with probability \( P_{n,01}^{(M)} (P_{n,11})^{(j-2)} P_{n,10} \). The expectation of \( L_{kn} \) can be directly computed from the probability mass function.

2) We can observe that one data packet is served in every slot of \( L_{kn} \) except for the last one (when a dummy packet is sent over channel \( n \), we have \( L_{kn} = 1 \) and zero data packets are served).

3) At the beginning of every \( L_{kn} \), we observe from the equivalent fictitious round robin policy that \( RR(M) \) effectively fixes \( P_{n,01}^{(M)} \) as the current information state, regardless of the true current state \( \omega_n(t) \). Neglecting \( \omega_n(t) \) is to discard all system history, including all past \( L_{k'k} \) for all \( k' < k \). Thus \( L_{kn} \) are i.i.d.. Specifically, for any \( k' < k \) and integers \( l_{k'} \) and \( l_k \) we have

\[
\Pr[L_{kn} = l_k | L_{k'k} = l_{k'}] = \Pr[L_{kn} = l_k].
\]

Now we can derive the throughput vector supported by \( RR(M) \). Fix an integer \( K > 0 \). By Lemma 3 the time average throughput over channel \( n \) after all channels finish their \( K \)th rounds, which we denote by \( \mu_n(K) \), is

\[
\mu_n(K) \triangleq \frac{\sum_{k=1}^K (L_{kn} - 1)}{\sum_{k=1}^K \sum_{n=1}^M L_{kn}}.
\]

Passing \( K \to \infty \), we get

\[
\lim_{K \to \infty} \mu_n(K)
\]

\[
= \lim_{K \to \infty} \frac{\sum_{k=1}^K (L_{kn} - 1)}{\sum_{k=1}^K \sum_{n=1}^M L_{kn}}
\]

\[
= \lim_{K \to \infty} \frac{(1/K) \sum_{k=1}^K (L_{kn} - 1)}{\sum_{k=1}^M \sum_{n=1}^M L_{kn}}
\]

\[
= \frac{1}{M} \mathbb{E}[L_{1n}] - 1
\]

\[
= \frac{\sum_{n=1}^M \mathbb{E}[L_{1n}]}{M} - 1
\]

\[
= \frac{\sum_{n=1}^M P_{n,01}(1 - (1 - x_n)^M)/(x_n P_{n,10})}{M + \sum_{n=1}^M P_{n,01}(1 - (1 - x_n)^M)/(x_n P_{n,10})},
\]

where (a) is by the Law of Large Numbers (noting Lemma 3 that \( L_{kn} \) are i.i.d. over \( k \)), and (b) is by Lemma 3.

2) Symmetric Case: We are particularly interested in the sum throughput under \( RR(M) \) when channels are symmetric, that is, all channels have the same statistics \( P_n = P \) for all \( n \). In this case, by channel symmetry every channel has the same throughput. From (5), we can show the sum throughput is

\[
\sum_{n=1}^M \lim_{K \to \infty} \mu_n(K) = \frac{P_{01}(1 - (1 - x)^M)}{x P_{10} + P_{01}(1 - (1 - x)^M)},
\]

where in the last term the subscript \( n \) is dropped due to channel symmetry. It is handy to define a function \( c_\alpha : \mathbb{N} \to \mathbb{R} \) as

\[
c_M \triangleq \frac{P_{01}(1 - (1 - x)^M)}{x P_{10} + P_{01}(1 - (1 - x)^M)}, \quad x \triangleq P_{01} + P_{10}.
\]
and define $c_{\infty} \triangleq \lim_{M \to \infty} c_M = P_{01}/(xP_{10} + P_{01})$ (note that $x < 1$ because every channel is positively correlated over time slots). The function $c_{\infty}$ will be used extensively in this paper. We summarize the above derivation in the next lemma.

**Lemma 4.** *Policy RR(M) serves channel $n \in \{1, 2, \ldots, M\}$ with throughput*

$$P_{n,01} (1 - (1 - x_n)^M)/(x_nP_{0,10})$$

*In particular, in symmetric channels the sum throughput under RR(M) is $c_M$ defined as*

$$c_M = \frac{P_{01} (1 - (1 - x)^M)}{xP_{10} + P_{01} (1 - (1 - x)^M)}, \quad x = P_{01} + P_{10},$$

*and every channel has throughput $c_M/M$.*

We remark that the sum throughput $c_M$ of RR(M) in the symmetric case is nondecreasing in $M$, and thus can be improved by serving more channels. Interestingly, here we see that the sum throughput is improved by having *multuser diversity* in the network, even though instantaneous channel states are never known.

C. How good is RR(M)?

Next, in symmetric channels, we quantify how close the sum throughput $c_M$ is to optimal. The following lemma presents a useful upper bound on the maximum sum throughput.

**Lemma 5** ([6], [7]). *In symmetric channels, any scheduling policy that confines to our model has sum throughput less than or equal to $c_{\infty}$.*

By Lemma 4 and 5, the loss of the sum throughput of RR(M) is no larger than $c_{\infty} - c_M$. Define $\overline{c}_M$ as

$$\overline{c}_M \triangleq \frac{P_{01} (1 - (1 - x)^M)}{xP_{10} + P_{01}} = c_{\infty} (1 - (1 - x)^M)$$

and note that $\overline{c}_M \leq c_M \leq c_{\infty}$. It follows

$$c_{\infty} - c_M \leq c_{\infty} - \overline{c}_M = c_{\infty} (1 - x)^M. \quad (7)$$

The last term of (7) decreases to zero geometrically fast as $M$ increases. This indicates that RR(M) yields near-optimal sum throughput even when it only serves a moderately large number of channels.

IV. RANDOMIZED ROUND ROBIN POLICY, INNER AND OUTER CAPACITY BOUND

A. Randomized Round Robin Policy

Lemma 4 specifies the throughput vector achieved by implementing RR(M) over a particular collection of $M$ channels. Here we are interested in the set of throughput vectors achievable by randomly mixing RR(M)-like policies over different channel subsets and allowing a different round-robin ordering on each subset. To generalize the RR(M) policy, first let $\Phi$ denote the set of all $N$-dimensional binary vectors excluding the all-zero vector $(0, \ldots, 0)$. For any binary vector $\phi = (\phi_1, \phi_2, \ldots, \phi_N)$ in $\Phi$, we say channel $n$ is active in $\phi$ if $\phi_n = 1$. Each vector $\phi \in \Phi$ represents a different subset of active channels. We denote by $M(\phi)$ the number of active channels in $\phi$.

For each $\phi \in \Phi$, consider the following round robin policy RR($\phi$) that serves active channels in $\phi$ in every round.

**Dynamic Round Robin Policy** RR($\phi$):

1) **Deciding the service order in each round:**

At the beginning of each round, we denote by $\tau_n$ the time duration between the last use of channel $n$ and the beginning of the current round. Active channels in $\phi$ are served in the decreasing order of $\tau_n$ in this round (in other words, the active channel that is least recently used is served first).

2) **On each active channel in a round:**

a) Suppose at time $t$ the base station switches to channel $n$. Transmit a data packet to user $n$ with probability $P_{\phi_{0,1}}(t)/\omega_n(t)$ and a dummy packet otherwise. In both cases, we receive ACK/NACK information at the end of the slot.

b) At time $(t + 1)$, if a dummy packet is sent at time $t$, switch to the next active channel following the order given in Step 1. Otherwise, keep transmitting data packets over channel $n$ until we receive a NACK. Then switch to the next active channel and go to Step 2a. We note that dummy packets are only sent on the first slot every time the base station switches to a new channel.

3) Update $\omega(t)$ according to (1) in every slot.

Using RR($\phi$) as building blocks, we consider the following class of randomized round robin policies.

**Randomized Round Robin Policy** RandRR:

1) Pick $\phi \in \Phi$ with probability $\alpha_\phi$, where $\sum_{\phi \in \Phi} \alpha_\phi = 1$.

2) Run policy RR($\phi$) for one round. Then go to Step 1.

Note that active channels may be served in different order in different rounds, according to the least-recently-used service order. This allows more time for OFF channels to return to better information states (note that $P_{\phi_{0,1}}(k)$ is nondecreasing in $k$) and thus improves throughput. The next lemma guarantees the feasibility of executing any RR($\phi$) policy in RandRR (similar to Lemma 2). Whenever the base station switches to a new channel $n$, we need $\omega_n(t) \geq P_{\phi_{0,1}}(t)$ in Step 2a of RR($\phi$).

**Lemma 6.** *When RR($\phi$) is chosen by RandRR for a new round of transmission, every active channel $n$ in $\phi$ starts with information state no worse than $P_{\phi_{0,1}}(t)$.*

**Proof of Lemma 6.** See Appendix B.

Although RandRR randomly selects subsets of users and serves them in an order that depends on previous choices, we can surprisingly analyze its throughput. This is done by using the throughput analysis of RR(M), as shown in the following corollary to Lemma 3.

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*Footnote: We note that the throughput analysis in [6] makes a minor assumption on the existence of some limiting time average. Using similar ideas of [6] in Theorem 1 of Section IV-A, we will construct an upper bound on the maximum sum throughput for general positively correlated Markov ON/OFF channels. When restricted to the symmetric case, we get the same upper bound without any assumption.*

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*References:* [6], [7].
Corollary 1. For each policy RR(ϕ), ϕ ∈ Φ, within time periods in which RR(ϕ) is executed by RandRR, denote by \( L^\phi_{kn} \) the duration of the \( k \)th time the base station stays with active channel \( n \). Then:

1. The probability mass function of \( L^\phi_{kn} \) is independent of \( k \), and is

\[
L^\phi_{kn} = \begin{cases} 
1 & \text{with prob. } 1 - P_{n,01}^{M(\phi)} \\
j & \text{with prob. } P_{n,01}^{M(\phi)} (P_{n,11})^{j-2} P_{n,10} 
\end{cases}
\]

As a result, for all \( k \in \mathbb{N} \) we have

\[
\mathbb{E}[L^\phi_{kn}] = 1 + P_{n,01}^{M(\phi)} P_{n,10}.
\] (8)

2. The number of data packets served in \( L^\phi_{kn} \) is \( \lfloor L^\phi_{kn} \rfloor - 1 \).

3. For every fixed \( \phi \) and every fixed active channel \( n \) in \( \phi \), the time durations \( L^\phi_{kn} \) are i.i.d. random variables over all \( k \).

B. Achievable Network Capacity — An Inner Capacity Bound

Using Corollary 1 next we present the achievable rate region of the class of RandRR policies. For each RR(ϕ) policy, define an \( N \)-dimensional vector \( \eta^\phi = (\eta_1^\phi, \eta_2^\phi, \ldots, \eta_N^\phi) \) where

\[
\eta_n^\phi \triangleq \begin{cases} 
\mathbb{E}[L^\phi_{kn}] - 1 & \text{if channel } n \text{ is active in } \phi, \\
0 & \text{otherwise},
\end{cases}
\]

where \( \mathbb{E}[L^\phi_{kn}] \) is given in (8). Intuitively, by the analysis prior to Lemma 1 round robin policy RR(ϕ) yields throughput \( \eta_n^\phi \) over channel \( n \) for each \( n \in \{1, 2, \ldots, N\} \). Incorporating all possible random mixtures of RR(ϕ) policies for different \( \phi \),RandRR can support any data rate vector that is entrywise dominated by a convex combination of vectors \( \{\eta^\phi\}_{\phi \in \Phi} \) as shown by the next theorem.

Theorem 1 (Generalized Inner Capacity Bound). The class of RandRR policies supports all data rate vectors \( \lambda \) in the set \( \Lambda_{\text{int}} \) defined as

\[
\Lambda_{\text{int}} \triangleq \left\{ \lambda \mid 0 \leq \lambda \leq \mu, \mu \in \text{conv} \left( \{\eta^\phi\}_{\phi \in \Phi} \right) \right\},
\]

where \( \eta^\phi \) is defined in (9), conv(\( A \)) denotes the convex hull of set \( A \), and \( \leq \) is taken entrywise.

Proof of Theorem 1. See Appendix C.

Applying Theorem 1 to symmetric channels yields the following corollary.

Corollary 2 (Inner Capacity Bound for Symmetric Channels). In symmetric channels, the class of RandRR policies supports all rate vectors \( \lambda \in \Lambda_{\text{int}} \) where

\[
\Lambda_{\text{int}} = \left\{ \lambda \mid 0 \leq \lambda \leq \mu, \mu \in \text{conv} \left( \left\{ \frac{c_{M(\phi)}}{M(\phi)} \eta^\phi \right\}_{\phi \in \Phi} \right) \right\},
\]

where \( c_{M(\phi)} \) is defined in (6).

An example of the inner capacity bound and a simple queue-dependent dynamic policy that supports all data rates within this nontrivial inner bound will be provided later.

C. Outer Capacity Bound

We construct an outer bound on \( \Lambda \) using several novel ideas. First, by state aggregation, we transform the information state process \( \{\omega_n(t)\} \) for each channel \( n \) into non-stationary two-state Markov chains (in Fig. 4 provided later). Second, we create a set of bounding stationary Markov chains (in Fig. 5 provided later), which has the structure of a multi-armed bandit system. Finally, we create an outer capacity bound by relating the bounding model to the original non-stationary Markov chains using stochastic coupling. We note that since the control of the set of information state processes \( \{\omega_n(t)\} \) for all \( n \) can be viewed as a restless bandit problem [14], it is interesting to see how we bound the optimal performance of a restless bandit problem by a related multi-armed bandit system.

We first map channel information states \( \omega_n(t) \) into modes for each \( n \in \{1, 2, \ldots, N\} \). Inspired by [3], we observe that each channel \( n \) must be in one of the following two modes:

M1 The last observed state is ON, and the channel has not been seen (through feedback) to turn OFF. In this mode the information state \( \omega_n(t) \in [\pi_{n,\text{ON}}, P_{n,11}] \).

M2 The last observed state is OFF, and the channel has not been seen to turn ON. Here \( \omega_n(t) \in [P_{n,01}, \pi_{n,\text{ON}}] \). On channel \( n \), recall that \( \mathcal{W}_n \) is the state space of \( \omega_n(t) \), and define a map \( f_n : \mathcal{W}_n \rightarrow \{M1, M2\} \)

\[
f_n(\omega_n(t)) = \begin{cases} 
M1 & \text{if } \omega_n(t) \in (\pi_{n,\text{ON}}, P_{n,11}], \\
M2 & \text{if } \omega_n(t) \in [P_{n,01}, \pi_{n,\text{ON}}].
\end{cases}
\]

This mapping is illustrated in Fig. 3.

![Fig. 3. The mapping \( f_n \) from information states \( \omega_n(t) \) to modes \{M1, M2\}.](image-url)
To upper bound throughput, we compare Fig. 4 to the mode transition diagrams in Fig. 5 that corresponds to a fictitious model for channel $n$. This fictitious channel has constant information state $\omega_n(t) = P_{n,11}$ whenever it is in mode $M1$, and $\omega_n(t) = \pi_{n,ON}$ whenever it is in $M2$. In other words, when the fictitious channel $n$ is in mode $M1$ (or $M2$), it sets its current information state to be the best state possible when the corresponding real channel $n$ is in the same mode. It follows that, when both the real and the fictitious channel $n$ are served, the probabilities of transitions $M1 \rightarrow M1$ and $M2 \rightarrow M1$ in the upper chain of Fig. 5 are greater than or equal to those in Fig. 4, respectively. In other words, the upper chain of Fig. 5 is more likely to go to mode $M1$ and serve packets than that of Fig. 4. Therefore, intuitively, if we serve both the real and the fictitious channel $n$ in the same infinite sequence of time slots, the fictitious channel $n$ will yield higher throughput for all $n$. This observation is made precise by the next lemma.

**Lemma 7.** Consider two discrete-time Markov chains \{X(t)\} and \{Y(t)\} both with state space \{0, 1\}. Suppose \{X(t)\} is stationary and ergodic with transition probability matrix

$$
P = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix},$$

and \{Y(t)\} is non-stationary with

$$
Q(t) = \begin{bmatrix} Q_{00}(t) & Q_{01}(t) \\ Q_{10}(t) & Q_{11}(t) \end{bmatrix}.
$$

Assume $P_{01} \geq Q_{01}(t)$ and $P_{11} \geq Q_{11}(t)$ for all $t$. In \{X(t)\}, let $\pi_X(1)$ denote the stationary probability of state 1; $\pi_X(1) = P_{01}/(P_{01} + P_{10})$. In \{Y(t)\}, define

$$
\pi_Y(1) \triangleq \limsup_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} Y(t)
$$
as the limiting fraction of time \{Y(t)\} stays at state 1. Then we have $\pi_X(1) \geq \pi_Y(1)$.

**Proof of Lemma 7.** Given in Appendix E.

We note that executing a scheduling policy in the network is to generate a sequence of channel selection decisions. By Lemma 7 if we apply the same sequence of channel selection decisions of some scheduling policy to the set of fictitious channels, we will get higher throughput on every channel. A direct consequence of this is that the maximum sum throughput over the fictitious channels is greater than or equal to that over the real channels.

**Lemma 8.** The maximum sum throughput over the set of fictitious channels is no more than

$$
\max_{n \in \{1, 2, \ldots, N\}} \left\{ c_{n, \infty} \right\}, \quad c_{n, \infty} \triangleq \frac{P_{n,01}}{x_n P_{n,10} + P_{n,01}}.
$$

**Proof of Lemma 8.** We note that finding the maximum sum throughput over fictitious channels in Fig. 5 is equivalent to solving a multi-armed bandit problem with each channel acting as an arm (see Fig. 5), and note that a channel can change mode only when it is served, and one unit of reward is earned if a packet is delivered (recall that a packet is served if and only if mode $M1$ is visited in the upper chain of Fig. 5). The optimal solution to the multi-armed bandit system is to always play the arm (channel) with the largest average reward (throughput). The average reward over channel $n$ is equal to the stationary probability of mode $M1$ in the upper chain of Fig. 5 which is

$$
\frac{\pi_{n,ON}}{P_{n,10} + \pi_{n,ON}} = \frac{P_{n,01}}{x_n P_{n,10} + P_{n,01}}.
$$

This finishes the proof.

Together with the fact that throughput over any real channel $n$ cannot exceed its stationary ON probability $\pi_{n,ON}$, we have constructed an outer bound on the network capacity region $\Lambda$ (the proof follows the above discussions and thus is omitted).

**Theorem 2.** (Generalized Outer Capacity Bound): Any supportable throughput vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_N)$ necessarily satisfies

$$
\lambda_n \leq \pi_{n,ON}, \quad \text{for all } n \in \{1, 2, \ldots, N\},
$$

$$
\sum_{n=1}^{N} \lambda_n \leq \max_{n \in \{1, 2, \ldots, N\}} \left\{ c_{n, \infty} \right\} = \max_{n \in \{1, 2, \ldots, N\}} \left\{ \frac{P_{n,01}}{x_n P_{n,10} + P_{n,01}} \right\}.
$$

These \((N + 1)\) hyperplanes create an outer capacity bound $\Lambda_{out}$ on $\Lambda$.

**Corollary 3** (Outer Capacity Bound for Symmetric Channels). In symmetric channels with $P_n = P$, $c_{n, \infty} = c_\infty$, and $P_{01}/(P_{01} + P_{10})$. In \{Y(t)\}, define

$$
\pi_Y(1) \triangleq \limsup_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} Y(t)
$$
as the limiting fraction of time \{Y(t)\} stays at state 1. Then we have $\pi_X(1) \geq \pi_Y(1)$.
\( \pi_{n, ON} = \pi_{ON} \) for all \( n \), we have
\[
\Lambda_{out} = \left\{ \lambda \geq 0 \mid \sum_{n=1}^{N} \lambda_n \leq c_{\infty}, \lambda_n \leq \pi_{ON} \text{ for } 1 \leq n \leq N \right\},
\]

where \( \geq \) is taken entrywise.

We note that Lemma 5 in Section III-C directly follows Corollary 3.

**D. A Two-User Example on Symmetric Channels**

Here we consider a two-user example on symmetric channels. For simplicity we will drop the subscript \( n \) in notations. From Corollary 3, we have the outer bound
\[
\Lambda_{out} = \left\{ \begin{array}{l}
[\lambda_1] \\
[\lambda_2]
\end{array} \mid 0 \leq \lambda_n \leq P_{01}/x, \text{ for } 1 \leq n \leq 2,
\lambda_1 + \lambda_2 \leq P_{01}/(xP_{10} + P_{01}),
\right\}\,.
\]

For the inner bound \( \Lambda_{int} \), we note that policy RandRR can execute three round robin policies RR(\( \phi \)) for \( \phi \in \Phi = \{(1, 1), (0, 1), (1, 0)\} \). From Corollary 2, we have
\[
\Lambda_{int} = \left\{ \begin{array}{l}
[\lambda_1] \\
[\lambda_2]
\end{array} \mid \begin{array}{c}
[\mu_1] \\
[\mu_2]
\end{array} \in \text{conv} \left( \left\{ \begin{array}{c}
\left[ \frac{c_2}{2} \right] \\
\left[ \frac{c_2}{2} \right]
\end{array} \right| \begin{array}{c}
\left[ c_1 \right] \\
\left[ 0 \right]
\end{array} \right\} \right\},
\right\}.
\]

Under the special case \( P_{01} = P_{10} = 0.2 \), the two bounds \( \Lambda_{int} \) and \( \Lambda_{out} \) are shown in Fig. 6.

![Fig. 6. Comparison of rate regions under different assumptions.](image)

In Fig. 6, we also compare \( \Lambda_{int} \) and \( \Lambda_{out} \) with other rate regions. Set \( \Lambda_{ideal} \) is the ideal capacity region when instantaneous channel states are known without causing any (timing) overhead [16]. Next, it is shown in [6] that the maximum sum throughput in this network is achieved at point \( A = (0.325, 0.325) \). The (unknown) network capacity region \( \Lambda \) is bounded between \( \Lambda_{int} \) and \( \Lambda_{out} \), and has boundary points \( B, A, \) and \( C \). Since the boundary of \( \Lambda \) is a concave curve connecting \( B, A, \) and \( C \), we envision that \( \Lambda \) shall contain but be very close to \( \Lambda_{int} \).

![Fig. 7. Comparison of our inner bound \( \Lambda_{int} \), the unknown network capacity region \( \Lambda \), and a heuristically better inner bound \( \Lambda_{heuristic} \).](image)

Finally, the rate region \( \Lambda_{blind} \) is rendered by completely neglecting channel memory and treating the channels as i.i.d. over slots [3]. We observe the throughput gain \( \Lambda_{int} \setminus \Lambda_{blind} \), as much as 23% in this example, is achieved by incorporating channel memory. In general, if channels are symmetric and treated as i.i.d. over slots, the maximum sum throughput in the network is \( \pi_{ON} = c_1 \). Then the maximum throughput gain of RandRR using channel memory is \( c_N - c_1 \), which as \( N \to \infty \) converges to
\[
c_{\infty} - c_1 = \frac{P_{01}}{xP_{10} + P_{01}} - \frac{P_{01}}{P_{01} + P_{10}},
\]

which is controlled by the factor \( x = P_{01} + P_{10} \).

**E. A Heuristically Tighter Inner Bound**

It is shown in [7] that the following policy maximizes the sum throughput in a symmetric network:

Serve channels in a circular order, where on each channel keep transmitting data packets until a NACK is received.

In the above two-user example, this policy achieves throughput vector \( A \) in Fig. 7. If we replace our round robin policy RR(\( \phi \)) by this one, heuristically we are able to construct a tighter inner capacity bound. For example, we can support the tighter inner bound \( \Lambda_{heuristic} \) in Fig. 7 by appropriate time sharing among the above policy that serves different subsets of channels. However, we note that this approach is difficult to analyze because the \( \{L_{kn}\} \) process (see [4]) forms a high-order Markov chain. Yet, our inner bound \( \Lambda_{int} \) provides a good throughput guarantee for this class of heuristic policies.

**V. PROXIMITY OF THE INNER BOUND TO THE TRUE CAPACITY REGION — SYMMETRIC CASE**

Next we bound the closeness of the boundaries of \( \Lambda_{int} \) and \( \Lambda \) in the case of symmetric channels. In Section III-C by choosing \( M = N \), we have provided such analysis for
the boundary point in the direction \((1, 1, \ldots, 1)\). Here we generalize to all boundary points. Define
\[
\mathcal{V} \triangleq \left\{ (v_1, v_2, \ldots, v_N) \mid v_n \geq 0 \text{ for } 1 \leq n \leq N, \quad v_n > 0 \text{ for at least one } n \right\}
\]
as a set of directional vectors. For any \(v \in \mathcal{V}\), let \(\Lambda_{\text{int}} = (\lambda_{1\text{int}}, \lambda_{2\text{int}}, \ldots, \lambda_{N\text{int}})\) and \(\Lambda_{\text{out}} = (\lambda_{1\text{out}}, \lambda_{2\text{out}}, \ldots, \lambda_{N\text{out}})\) be the boundary point of \(\Lambda_{\text{int}}\) and \(\Lambda_{\text{out}}\) in the direction of \(v\), respectively. It is useful to compute \(\sum_{n=1}^{N} (\lambda_{n\text{out}} - \lambda_{n\text{int}})\), because it upper bounds the loss of the sum throughput of \(\Lambda_{\text{int}}\) from \(\Lambda\) in the direction of \(v\).\footnote{Note that \(\sum_{n=1}^{N} (\lambda_{n\text{out}} - \lambda_{n\text{int}})\) also bounds the closeness between \(\Lambda_{\text{out}}\) and \(\Lambda\).} We note that computing \(\Lambda_{\text{int}}\) in an arbitrary direction is difficult. Thus we will find an upper bound on \(\sum_{n=1}^{N} (\lambda_{n\text{out}} - \lambda_{n\text{int}})\).

A. Preliminary

To have more intuitions on \(\Lambda_{\text{int}}\), we start with a toy example of \(N = 3\) users. We are interested in the boundary point of \(\Lambda_{\text{int}}\) in the direction of \(v = (1, 2, 1)\). Consider two RandRR-type policies \(\psi_1\) and \(\psi_2\) defined as follows.

For \(\psi_1\), choose
\[
\begin{align*}
\phi_1 &= (1, 0, 0) \\
\phi_2 &= (0, 1, 0) \\
\phi_3 &= (0, 0, 1)
\end{align*}
\]
with prob. \(1/4\)

For \(\psi_2\), choose
\[
\begin{align*}
\phi_4 &= (1, 1, 0) \\
\phi_5 &= (0, 1, 1)
\end{align*}
\]
with prob. \(1/2\)

Both \(\psi_1\) and \(\psi_2\) support data rates in the direction of \((1, 2, 1)\). However, using the analysis of Lemma \ref{lem:basic} and Theorem \ref{thm:main}, we know \(\psi_1\) supports throughput vector
\[
\frac{1}{4} \begin{bmatrix} c_1 \\ 0 \\ 0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 \\ c_1 \\ 0 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 0 \\ 0 \\ c_1 \end{bmatrix} = \begin{bmatrix} c_1/4 \\ 1/2 \\ 1/2 \end{bmatrix},
\]
while \(\psi_2\) supports
\[
\frac{1}{2} \begin{bmatrix} c_2/2 \\ c_2/2 \\ 0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 \\ c_2/2 \\ c_2/2 \end{bmatrix} = \begin{bmatrix} c_2/4 \\ 1/2 \\ 1/2 \end{bmatrix},
\]
where \(c_1\) and \(c_2\) are defined in \(\psi_1\). We see that \(\psi_2\) achieves data rates closer than \(\psi_1\) does to the boundary of \(\Lambda_{\text{int}}\). It is because every sub-policy of \(\psi_2\), namely \(\text{RR}(\phi^4)\) and \(\text{RR}(\phi^5)\), supports sum throughput \(c_2\) (by Lemma \ref{lem:basic}), where those of \(\psi_1\) only support \(c_1\). In other words, policy \(\psi_2\) has better \textit{multiuser diversity gain} than \(\psi_1\) does. This example suggests that we can find a good lower bound on \(\Lambda_{\text{int}}\) by exploring to what extent the multiuser diversity can be exploited. We start with the following definition.

\begin{definition}
For any \(v \in \mathcal{V}\), we say \(v\) is \(d\)-user diverse if \(v\) can be written as a positive combination of vectors in \(\Phi_d\), where \(\Phi_d\) denotes the set of \(N\)-dimensional binary vectors having \(d\) entries be \(1\). Define
\[
d(v) \triangleq \max_{1 \leq d \leq N} \{d \mid v \text{ is } d\text{-user diverse}\},
\]
and we shall say \(v\) is maximally \(d(v)\)-user diverse.
\end{definition}

The notion of \(d(v)\) is well-defined because every \(v\) must be 1-user diverse.\footnote{The set \(\Phi_1 = \{e_1, e_2, \ldots, e_N\}\) is the collection of unit coordinate vectors where \(e_n\) has its \(n\)th entry be \(1\) and \(0\) otherwise. Any vector \(v \in \mathcal{V}\), \(v = (v_1, v_2, \ldots, v_N)\), can be written as \(v = \sum_{v_n > 0} v_n e_n\).}

Definition \ref{def:multiuser} is the most useful to us through the next lemma.

\begin{lemma}
The boundary point of \(\Lambda_{\text{int}}\) in the direction of \(v \in \mathcal{V}\) has sum throughput at least \(c_d(v)\), where
\[
c_d(v) \triangleq \frac{P_{01}(1 - (1 - x)^d(v))}{x P_{10} + P_{01}(1 - (1 - x)^d(v))}, \quad x \triangleq P_{01} + P_{10}.
\]
\end{lemma}

Proof of Lemma \ref{lem:bound}. If direction \(v\) can be written as a positive weighted sum of vectors in \(\Phi_d(v)\), we can normalize the weights, and use the new weights as probabilities to randomly mix \(\text{RR}(\phi)\) policies for all \(\phi \in \Phi_d(v)\). This way we achieve sum throughput \(c_d(v)\) in every transmission round, and overall the throughput vector will be in the direction of \(v\). Therefore the result follows. For details, see Appendix \ref{app:proof}.

Fig. \ref{fig:example} provides an example of Lemma \ref{lem:bound} in the two-user symmetric system in Section \ref{sec:symmetric}. We observe that direction \((1, 1)\), the one that passes point \(D\) in Fig. \ref{fig:example} is the only direction that is maximally 2-user diverse. The sum throughput \(c_2\) is achieved at \(D\). For all the other directions, they are maximally 1-user diverse and, from Fig. \ref{fig:example} only sum throughput \(c_1\) is guaranteed along those directions. In general, geometrically we can show that a maximally \(d\)-user diverse vector, say \(v_d\), forms a smaller angle with the all-1 vector \((1, 1, \ldots, 1)\) than a maximally \(d'\)-user diverse vector, say \(v_{d'}\), does if \(d' < d\). In other words, data rates along \(v_d\) are more balanced than those along \(v_{d'}\). Lemma \ref{lem:bound} states that we guarantee to support higher sum throughput if the user traffic is more balanced.
B. Proximity Analysis

We use the notion of $d(v)$ to upper bound $\sum_{n=1}^{N} (\lambda_{n}^\text{out} - \lambda_{n}^\text{int})$ in any direction $v \in V$. Let $\lambda_{n}^\text{out} = \theta \lambda_{n}^\text{int}$ (i.e., $\lambda_{n}^\text{out} = \theta \lambda_{n}^\text{int}$ for all $n$) for some $\theta \geq 1$. By (10), the boundary of $\Lambda_{n}$ is characterized by the intersection of the $(N + 1)$ hyperplanes $\sum_{n=1}^{N} \lambda_{n} = c_{\infty}$ and $\lambda_{n} = \pi_{\text{ON}}$ for each $n \in \{1, 2, \ldots, N\}$. Specifically, in any given direction, if we consider the cross points on all the hyperplanes in that direction, the boundary point $\lambda^\text{int}$ is the one closest to the origin. We do not know which hyperplane $\lambda_{n}^\text{out}$ is on, and thus need to consider all $(N + 1)$ cases. If $\lambda^\text{out}$ is on the plane $\sum_{n=1}^{N} \lambda_{n} = c_{\infty}$, i.e., $\sum_{n=1}^{N} \lambda_{n}^\text{out} = c_{\infty}$, we get

$$\sum_{n=1}^{N} (\lambda_{n}^\text{out} - \lambda_{n}^\text{int}) \leq c_{\infty} - c_{d}(v) \leq c_{\infty} (1 - x) d(v),$$

where (a) is by Lemma 9 and (b) is by (7). If $\lambda_{n}^\text{out}$ is on the plane $\lambda_{n} = \pi_{\text{ON}}$ for some $n$, then $\theta = \pi_{\text{ON}}/\lambda_{n}^\text{int}$. It follows

$$\sum_{n=1}^{N} (\lambda_{n}^\text{out} - \lambda_{n}^\text{int}) = (\theta - 1) \sum_{n=1}^{N} \lambda_{n}^\text{int} \leq \left( \frac{\pi_{\text{ON}}}{\lambda_{n}^\text{int}} - 1 \right) c_{\infty}. \tag{11}$$

The above discussions lead to the next lemma.

**Lemma 10.** The loss of the sum throughput of $\Lambda_{n}$ from $\Lambda$ in the direction of $v$ is upper bounded by

$$\min_{1 \leq n \leq N} \left\{ \left( \frac{\pi_{\text{ON}}}{\lambda_{n}^\text{int}} - 1 \right) c_{\infty} \right\} \leq c_{\infty} \min \left[ (1 - x) d(v), \frac{\pi_{\text{ON}}}{\max_{1 \leq n \leq N} \left\{ \lambda_{n}^\text{int} \right\}} - 1 \right]. \tag{11}$$

Lemma 10 shows that, if data rates are more balanced, namely, have a larger $d(v)$, the sum throughput loss is dominated by the first term in the minimum of (11) and decreases to 0 geometrically fast with $d(v)$. If data rates are biased toward a particular user, the second term in the minimum of (11) captures the throughput loss, which goes to 0 as the rate of the favored user goes to the single-user capacity $\pi_{\text{ON}}$.

VI. THROUGHPUT-ACHIEVING QUEUE-DEPENDENT ROUND ROBIN POLICY

Let $a_{n}(t)$, for $1 \leq n \leq N$, be the number of exogenous packet arrivals destined for user $n$ in slot $t$. Suppose $a_{n}(t)$ are independent across users, i.i.d. over slots with rate $\mathbb{E} \left[ a_{n}(t) \right] = \lambda_{n}$, and $a_{n}(t)$ is bounded with $0 \leq a_{n}(t) \leq A_{\text{max}}$, where $A_{\text{max}}$ is a finite integer. Let $U_{n}(t)$ be the backlog of user-$n$ packets queued at the base station at time $t$. Define $U(t) \triangleq (U_{1}(t), U_{2}(t), \ldots, U_{N}(t))$ and suppose $U_{n}(0) = 0$ for all $n$. The queue process $\{U_{n}(t)\}$ evolves as

$$U_{n}(t + 1) = \max \left\{ U_{n}(t) - \mu_{n}(s_{n}(t), t), 0 \right\} + a_{n}(t), \tag{12}$$

where $\mu_{n}(s_{n}(t), t) \in \{0, 1\}$ is the service rate allocated to user $n$ in slot $t$. We have $\mu_{n}(s_{n}(t), t) = 1$ if user $n$ is served and $s_{n}(t) = \text{ON}$, and 0 otherwise. In the rest of the paper we drop $s_{n}(t)$ in $\mu_{n}(s_{n}(t), t)$ and use $\mu_{n}(t)$ for notational simplicity. We say the network is (strongly) stable if

$$\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n=1}^{N} \mathbb{E} \left[ U_{n}(\tau) \right] < \infty.$$
\[
\sum_{n: \phi_n = 1} \left[ U_n(t) \frac{P_{n,01}^{(M)}}{P_{n,10}^{(M)}} - \left(1 + \frac{P_{n,01}^{(M)}}{P_{n,10}^{(M)}}\right) \sum_{n=1}^{N} U_n(t) \lambda_n \right],
\]

and the maximizer of \( f(U(t), \text{RR}(\phi)) \) is to activate the \( M \) channels that yield the \( M \) largest summands of the above equation.

3) Obtain \( \phi(t) \) by comparing the maximizers from the above step for different values of \( M \).

The detailed implementation is as follows.

**Polynomial time implementation of Step 2 of QRR:**

1) For each fixed \( M \in \{1, \ldots, N\} \), we do the following: Compute

\[
U_n(t) \frac{P_{n,01}^{(M)}}{P_{n,10}^{(M)}} - \left(1 + \frac{P_{n,01}^{(M)}}{P_{n,10}^{(M)}}\right) \sum_{n=1}^{N} U_n(t) \lambda_n \tag{15}
\]

for all \( n \in \{1, \ldots, N\} \). Sort these \( N \) numbers and define the binary vector \( \phi^M = (\phi_1^M, \ldots, \phi_N^M) \) such that \( \phi_n^M = 1 \) if the value \( U_n(t) \lambda_n \) is among the \( M \) largest, otherwise \( \phi_n^M = 0 \). Ties are broken arbitrarily.

Let \( \hat{f}(U(t), M) \) denote the sum of the \( M \) largest values of \( U_n(t) \lambda_n \).

2) Define \( M(t) \equiv \arg \max_{1 \leq M \leq N} \hat{f}(U(t), M) \). Then we assign \( \phi(t) = \phi^{M(t)} \).

Using a novel variable-length frame-based Lyapunov analysis, we show in the next theorem that QRR stabilizes the network with any arrival rate vector \( \lambda \) strictly within the inner capacity bound \( \Lambda_{\text{inter}} \). The idea is that we compare QRR with the (unknown) policy RandRR* that stabilizes \( \lambda \). We show that, in every transmission round, QRR finds and executes a round robin policy \( \text{RR}(\phi(t)) \) that yields a larger negative drift on the queue backlogs than RandRR* does in the current round. Therefore, QRR is stable.

**Theorem 3.** For any data rate vector \( \lambda \) interior to \( \Lambda_{\text{inter}} \), policy QRR strongly stabilizes the network.

*Proof of Theorem 3* See Appendix 1

**VII. Conclusion**

The network capacity of a wireless network is practically degraded by communication overhead. In this paper, we take a step forward by studying the fundamental achievable rate region when communication overhead is kept minimum, that is, when channel probing is not permitted. While solving the original problem is difficult, we construct an inner and an outer bound on the network capacity region, with the aid of channel memory. When channels are symmetric and the network serves a large number of users, we show the inner and outer bound are progressively tight when the data rates of different users are more balanced. We also derive a simple queue-dependent frame-based policy, as a function of packet arrival rates and channel statistics, and show that this policy stabilizes the network for any data rates strictly within the inner capacity bound.

Transmitting data without channel probing is one of the many options for communication over a wireless network. Practically each option may have pros and cons on criteria like the achievable throughput, power efficiency, implementation complexity, etc. In the future it is important to explore how to combine all possible options to push the practically achievable network capacity to the limit. It is part of our future work to generalize the methodology and framework developed in this paper to more general cases, such as when limited probing is allowed and/or other QoS metrics such as energy consumption are considered. It will also be interesting to see how this framework can be applied to solve new problems in opportunistic spectrum access in cognitive radio networks, in opportunistic scheduling with delayed/uncertain channel state information, and in restless bandit problems.

**APPENDIX A**

*Proof of Lemma 2* Initially, by \( (3) \) we have \( \omega_n(t) = \pi_{n,O,N} \geq P_{n,01}^{(M)} \) for all \( n \). Suppose the base station switches to channel \( n \) at time \( t \), and the last use of channel \( n \) ends at slot \((t - k)\) for some \( k < t \). In slot \((t - k)\), there are two possible cases:

1) Channel \( n \) turns OFF, and as a result the information state on slot \( t \) is \( \omega_n(t) = P_{n,01}^{(k)} \). Due to round robin, the other \((M - 1)\) channels must have been used for at least one slot before \( t \) after slot \((t - k)\), and thus \( k \geq M \).

By \( (3) \) we have \( \omega_n(t) = P_{n,01}^{(k)} \geq P_{n,01}^{(M)} \).

2) Channel \( n \) is ON and transmits a dummy packet. Thus \( \omega_n(t) = P_{n,11}^{(k)} \). By \( (4) \) we have \( \omega_n(t) = P_{n,11}^{(k)} \geq P_{n,01}^{(M)} \).

**APPENDIX B**

*Proof of Lemma 6* At the beginning of a new round, suppose round robin policy \( \text{RR}(\phi) \) is selected. We index the \( M(\phi) \) active channels in \( \phi \) as \( \{n_1, n_2, \ldots, n_M(\phi)\} \), which is in the decreasing order of the time duration between their last use and the beginning of the current round. In other words, the last use of \( n_k \) is earlier than that of \( n_{k'} \) only if \( k < k' \). Fix an active channel \( n_k \). Then it suffices to show that when this channel is served in the current round, the time duration back to the end of its last service is at least \((M(\phi) - 1)\) slots (that this channel has information state no worse than \( P_{n_k,01}^{(M(\phi))} \)) then follows the same arguments in the proof of Lemma 2.

We partition the active channels in \( \phi \) other than \( n_k \) into two sets \( A = \{n_1, n_2, \ldots, n_{M(\phi)}\} \) and \( B = \{n_{k+1}, n_{k+2}, \ldots, n_M(\phi)\} \). Then the last use of every channel in \( B \) occurs after the last use of \( n_k \), and so channel \( n_k \) has been idled for at least \(|B|\) slots at the start of the current round. However, the policy in this round will serve all channels in \( A \) before serving \( n_k \), taking at least one slot per channel, and so we wait at least additional \(|A| \) slots before serving channel \( n_k \). The total time that this channel has been idled is thus at least \(|A| + |B| = M(\phi) - 1\).
APPENDIX C

Proof of Theorem 7 Let $Z(t)$ denote the number of times Step 1 of RandRR is executed in $[0, t)$, in which we suppose vector $\phi$ is selected $Z_\phi(t)$ times. Define $t_i$, where $i \in \mathbb{Z}^+$, as the $(i + 1)$th time instant a new vector $\phi$ is selected. Assume $t_0 = 0$, and thus the first selection occurs at time 0. It follows that $Z(t_i) = i$, $Z(t) = i + 1$, and the $i$th round of packet transmissions ends at time $t_i$.

Fix a vector $\phi$. Within the time periods in which policy $\text{RR}(\phi)$ is executed, denote by $L_{kn}$ the duration of the $k$th time the base station stays with channel $n$. Then the time average throughput that policy $\text{RR}(\phi)$ yields on its active channel $n$ over $[0, t)$ is

$$\frac{\sum_{k=1}^{z_{\phi}(t_i)} \left( T_{\phi} - 1 \right)}{\sum_{\phi \in \Phi} \sum_{k=1}^{z_{\phi}(t_i)} \sum_{n: \phi = n} T_{kn}}. \quad (16)$$

For simplicity, here we focus on discrete time instants $\{t_i\}$ large enough so that $Z_\phi(t_i) > 0$ for all $\phi \in \Phi$ (so that the sums in (16) make sense). The generalization to arbitrary time $t$ can be done by incorporating fractional transmission rounds, which are amortized over time. Next, rewrite (16) as

$$\frac{\sum_{k=1}^{z_{\phi}(t_i)} \sum_{n: \phi = n} L_{kn}^\phi - \sum_{k=1}^{z_{\phi}(t_i)} \sum_{n: \phi = n} T_{kn}}{\sum_{\phi \in \Phi} \sum_{k=1}^{z_{\phi}(t_i)} \sum_{n: \phi = n} L_{kn}^\phi} \quad (17)$$

As $t \to \infty$, the second term $(*)$ of (17) satisfies

$$(*) = \frac{1}{\sum_{k=1}^{z_{\phi}(t_i)} L_{kn}^\phi - \sum_{k=1}^{z_{\phi}(t_i)} T_{kn}} \sum_{n: \phi = n} \sum_{k=1}^{z_{\phi}(t_i)} L_{kn}^\phi \sum_{n: \phi = n} \sum_{k=1}^{z_{\phi}(t_i)} T_{kn} \quad (a)$$

$$\sum_{n: \phi = n} \sum_{k=1}^{z_{\phi}(t_i)} \frac{L_{kn}^\phi - T_{kn}}{n} \quad (b)$$

where (a) is by the Law of Large Numbers (we have shown in Corollary 1 that $L_{kn}$ are i.i.d. for different $k$) and (b) by (9).

Denote the first term of (17) by $\beta_\phi(t_i)$, where we note that $\beta_\phi(t) \in [0, 1]$ for all $\phi \in \Phi$ and $\sum_{\phi \in \Phi} \beta_\phi(t_i) = 1$. We can rewrite $\beta_\phi(t_i)$ as

$$\beta_\phi(t_i) = \frac{\frac{z_{\phi}(t_i)}{Z(t_i)}}{\sum_{\phi \in \Phi} \frac{z_{\phi}(t_i)}{Z(t_i)}} \sum_{n: \phi = n} \frac{1}{\sum_{k=1}^{z_{\phi}(t_i)} \sum_{n: \phi = n} T_{kn}^\phi} \sum_{k=1}^{z_{\phi}(t_i)} \sum_{n: \phi = n} T_{kn}^\phi \sum_{n: \phi = n} \sum_{k=1}^{z_{\phi}(t_i)} \frac{L_{kn}^\phi}{T_{kn}}. \quad (18)$$

As $t \to \infty$, we have

$$\beta_\phi = \lim_{i \to \infty} \beta_\phi(t_i) = \frac{\alpha_\phi \sum_{n: \phi = n} \sum_{k=1}^{z_{\phi}(t_i)} \frac{L_{kn}^\phi}{T_{kn}}}{\sum_{\phi \in \Phi} \alpha_\phi \sum_{n: \phi = n} \sum_{k=1}^{z_{\phi}(t_i)} \frac{L_{kn}^\phi}{T_{kn}}}, \quad (19)$$

where by the Law of Large Numbers we have

$$Z_{\phi}(t) = \alpha_\phi, \quad \frac{1}{Z_{\phi}(t)} \sum_{k=1}^{z_{\phi}(t)} L_{kn}^\phi \to \mathbb{E}[L_{kn}^\phi].$$

From (16), (17), (18), we have shown that the throughput contributed by policy $\text{RR}(\phi)$ on its active channel $n$ is $\beta_\phi n^\phi$. Consequently, RandRR parameterized by $\{\alpha_\phi, \phi \in \Phi\}$ supports any data rate vector $\lambda$ that is entrywise dominated by $\lambda \leq \sum_{\phi \in \Phi} \beta_\phi n^\phi$, where $\{\beta_\phi, \phi \in \Phi\}$ is defined in (18) and $n^\phi$ in (9).

The above analysis shows that every RandRR policy achieves a boundary point of $\Lambda_{\text{in}}$ defined in Theorem 1. Conversely, the next lemma, proved in Appendix D, shows that every boundary point of $\Lambda_{\text{in}}$ is achievable by some RandRR policy, and the proof is complete.

Lemma 11. For any probability distribution $\{\beta_\phi, \phi \in \Phi\}$, there exists another probability distribution $\{\alpha_\phi, \phi \in \Phi\}$ that solves the linear system

$$\beta_\phi = \frac{\alpha_\phi \sum_{n: \phi = n} \sum_{k=1}^{z_{\phi}(t_i)} \frac{L_{kn}^\phi}{T_{kn}}}{\sum_{\phi \in \Phi} \alpha_\phi \sum_{n: \phi = n} \sum_{k=1}^{z_{\phi}(t_i)} \frac{L_{kn}^\phi}{T_{kn}}}, \quad \text{for all } \phi \in \Phi. \quad (19)$$

Lemma 11, proved in Appendix D, is the main result of this section. It shows that the optimal $\{\beta_\phi, \phi \in \Phi\}$ can be achieved by a RandRR policy, and the proof is complete.

APPENDIX D

Proof of Lemma 1 For any probability distribution $\{\beta_\phi, \phi \in \Phi\}$, we prove the lemma by inductively constructing the solution $\{\alpha_\phi, \phi \in \Phi\}$. The induction is on the cardinality of $\Phi$. Without loss of generality, we index elements in $\Phi$ by $\Phi = \{\phi^1, \phi^2, \ldots\}$, where $\phi^k = (\phi_1^k, \ldots, \phi_N^k)$. We define $\chi_k = \sum_{n=1}^{N^\phi_k} \mathbb{E}[L_{kn}^\phi_k]$ and redefine $\beta_{\phi_k} = \beta_k$, and $\alpha_{\phi_k} = \alpha_k$. Then we can rewrite (19) as

$$\beta_k = \frac{\alpha_k \chi_k}{\sum_{1 \leq k \leq |\Phi|} \alpha_k \chi_k}, \quad \text{for all } k \in \{1, 2, \ldots, |\Phi|\}. \quad (20)$$

We first note that $\Phi = \{\phi^1\}$ is a degenerate case where $\beta_1$ and $\alpha_1$ must both be 1. When $\Phi = \{\phi^1, \phi^2\}$, for any probability distribution $\{\beta_1, \beta_2\}$ with positive elements it is easy to show

$$\alpha_1 = \frac{\chi_2 \beta_2}{\chi_1 \beta_2 + \chi_2 \beta_1}, \quad \alpha_2 = 1 - \alpha_1.$$
It remains to show (22) and (23) are the desired solution. It is easy to observe that \( \alpha_k \in [0, 1] \) for \( 1 \leq k \leq K + 1 \), and

\[
\sum_{k=1}^{K+1} \alpha_k = \alpha_1 + (1 - \alpha_1) \sum_{k=2}^{K+1} \gamma_k = \alpha_1 + (1 - \alpha_1) = 1. 
\]

By rearranging terms in (22) and using (23), we have

\[
\beta_1 = \frac{\alpha_1 \chi_1}{\sum_{k=1}^{K+1} \alpha_k \gamma_k}, \quad (24)
\]

For \( 2 \leq k \leq K + 1 \),

\[
\frac{\alpha_k \gamma_k}{\sum_{k=1}^{K+1} \alpha_k \gamma_k} = \left( \frac{1 - \alpha_1) \gamma_k}{\sum_{k=2}^{K+1} (1 - \alpha_1) \gamma_k} \right) \left[ 1 - \frac{\alpha_1 \chi_1}{\sum_{k=1}^{K+1} \alpha_k \gamma_k} \right] \equiv (a) \left( \frac{1 - \alpha_1) \gamma_k}{\sum_{k=2}^{K+1} (1 - \alpha_1) \gamma_k} \right), \quad (b) \left( \frac{1 - \alpha_1) \gamma_k}{\sum_{k=2}^{K+1} (1 - \alpha_1) \gamma_k} \right), \quad (c) \left( \frac{1 - \alpha_1) \gamma_k}{\sum_{k=2}^{K+1} (1 - \alpha_1) \gamma_k} \right), \quad (d) \left( \frac{1 - \alpha_1) \gamma_k}{\sum_{k=2}^{K+1} (1 - \alpha_1) \gamma_k} \right),
\]

where (a) is by plugging in (23), (b) uses (24), (c) uses (21), and (d) is by \( \sum_{k=1}^{K+1} \beta_k = 1 \). The proof is complete.

\[\boxed{\text{APPENDIX E}}\]

**Proof of Lemma 12** Let \( \mathcal{N}_1(T) \subseteq \{0, 1, \ldots, T - 1\} \) be the subset of time instances in which \( Y(t) = 1 \). Note that \( \sum_{t=0}^{T-1} Y(t) = \mathcal{N}_1(T) \). For each \( t \in \mathcal{N}_1(T) \), let \( 1_{[1,0]}(t) \) be an indicator function which is 1 if \( Y(t) \) transits from 1 to 0 at time \( t \), and 0 otherwise. We define \( \mathcal{N}_0(T) \) and \( 1_{[0,1]}(t) \) similarly.

In \( \{0, 1, \ldots, T - 1\} \), since state transitions of \( \{Y(t)\} \) from 1 to 0 and from 0 to 1 differ by at most 1, we have

\[
\left| \sum_{t \in \mathcal{N}_1(T)} 1_{[1,0]}(t) - \sum_{t \in \mathcal{N}_0(T)} 1_{[0,1]}(t) \right| \leq 1, \quad (25)
\]

which is true for all \( T \). Dividing (25) by \( T \), we get

\[
\frac{1}{T} \sum_{t \in \mathcal{N}_1(T)} 1_{[1,0]}(t) - \frac{1}{T} \sum_{t \in \mathcal{N}_0(T)} 1_{[0,1]}(t) \leq \frac{1}{T}. \quad (26)
\]

Consider the subsequence \( \{T_k\} \) such that

\[
\lim_{k \to \infty} \frac{1}{T_k} \sum_{t=0}^{T_k-1} Y(t) = \pi_Y(1) = \lim_{k \to \infty} \frac{\mathcal{N}_1(T_k)}{T_k}. \quad (27)
\]

Note that \( \{T_k\} \) exists because \( \{1/T\} \sum_{t=0}^{T_k-1} Y(t) \) is a bounded sequence indexed by integers \( T \). Moreover, there exists a subsequence \( \{T_n\} \) of \( \{T_k\} \) so that each of the two averages in (26) has a limit point with respect to \( \{T_n\} \), because they are bounded sequences, too. In the rest of the proof we will work on \( \{T_n\} \), but we drop subscript \( n \) for notational simplicity.

Passing \( T \to \infty \), we get from (26) that

\[
\left( \lim_{T \to \infty} \frac{\mathcal{N}_1(T)}{T} \right) \left( \lim_{T \to \infty} \frac{1}{|\mathcal{N}_1(T)|} \sum_{t \in \mathcal{N}_1(T)} 1_{[0,1]}(t) \right) \equiv \beta, 
\]

where \( \alpha \) is by (27) and \( \beta \) is by \( |\mathcal{N}_1(T)| + |\mathcal{N}_0(T)| = T \). From (28) we get

\[
\pi_Y(1) = \frac{\gamma}{\beta + \gamma}. 
\]

The next lemma, proved in Appendix F helps to show \( \gamma \leq P_{01} \).

**Lemma 12** (Stochastic coupling of random binary sequences). Let \( \{I_n\}_{n=1}^{\infty} \) be an infinite sequence of binary random variables. Suppose for all \( n \in \{1, 2, \ldots\} \) we have

\[
\Pr[I_n = 1 | I_1 = i_1, \ldots, I_{n-1} = i_{n-1}] \leq P_{01} \quad (29)
\]

for all possible values of \( i_1, \ldots, i_{n-1} \). Then we can construct a new sequence \( \{\tilde{I}_n\}_{n=1}^{\infty} \) of binary random variables that are i.i.d. with \( \Pr[I_n = 1] = P_{01} \) for all \( n \) and satisfy \( \tilde{I}_n \geq I_n \) for all \( n \). Consequently, we have

\[
\limsup_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} I_n \leq \limsup_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \tilde{I}_n = P_{01}. 
\]

To use Lemma 12 to prove \( \gamma \leq P_{01} \), let \( t_n \) denote the nth time \( Y(t) = 0 \) and let \( \tilde{I}_n = I_{[0,1]}(t_n) \). For simplicity assume \( \{t_n\} \) is an infinite sequence so that state 0 is visited infinitely often in \( \{Y(t)\} \). By the assumption that \( \tilde{Q}_0(t) \leq P_{01} \) for all \( t \), we know (29) holds. Therefore by Lemma 12 we have

\[
\gamma \leq \limsup_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} 1_{[0,1]}(t_n) \leq P_{01}. \]

Similarly as Lemma 12 we can show \( \beta \geq P_{10} \) by stochastic coupling. Therefore

\[
\pi_Y(1) = \frac{\gamma}{\beta + \gamma} \leq \frac{\gamma}{P_{10} + \gamma} \leq \frac{P_{01}}{P_{01} + P_{10}} = \pi_X(1). 
\]

\[\boxed{\text{APPENDIX F}}\]

**Proof of Lemma 12** For simplicity, we assume

\[
\Pr[I_n = 0 | I_1 = i_1, \ldots, I_{n-1} = i_{n-1}] > 0
\]

for all \( n \) and all possible values of \( i_1, \ldots, i_{n-1} \). For each \( n \in \{1, 2, \ldots\} \), define \( \tilde{I}_n \) as follows: If \( I_n = 1 \), define \( \tilde{I}_n = 1 \). If \( I_n = 0 \), observe the history \( I_{n-1} \) and define \( \tilde{I}_n = 0 \) for all \( n \). Then, we have

\[
\Pr[I_n = 1 | I_1 = i_1, \ldots, I_{n-1} = i_{n-1}] \geq P_{01}
\]

for all \( n \) and all possible values of \( i_1, \ldots, i_{n-1} \).
independently choose \( \hat{I}_n \) as follows:

\[
\hat{I}_n = \begin{cases} 
1 & \text{with prob. } \frac{P_{01} - P_r[I_n=1|I_{n-1}^1]}{P_{01} - P_r[I_n=1|I_{n-1}^1]}, \\
0 & \text{with prob. } 1 - \frac{P_{01} - P_r[I_n=1|I_{n-1}^1]}{P_{01} - P_r[I_n=1|I_{n-1}^1]},
\end{cases}
\]

(30)

The probabilities in (30) are well-defined because \( P_{01} \geq P_r[I_n = 1 | I_{n-1}^1] \) by (29), and

\[
P_{01} \leq 1 = Pr[I_n = 1 | I_{n-1}^1] + Pr[I_n = 0 | I_{n-1}^1]
\]

and therefore

\[
P_{01} - Pr[I_n = 1 | I_{n-1}^1] \leq Pr[I_n = 0 | I_{n-1}^1].
\]

With the above definition of \( \hat{I}_n \), we have \( \hat{I}_n = 1 \) whenever \( I_n = 1 \). Therefore \( \hat{I}_n \geq I_n \) for all \( n \). Further, for any \( n \) and any binary vector \( i_{n-1} \sim (i_1, \ldots, i_{n-1}) \), we have

\[
Pr[\hat{I}_n = 1 | I_{n-1}^1 = i_{n-1}] = Pr[I_n = 1 | I_{n-1}^1 = i_{n-1}] + Pr[I_n = 0 | I_{n-1}^1 = i_{n-1}] \\
\times \frac{P_{01} - Pr[I_n = 1 | I_{n-1}^1 = i_{n-1}]}{Pr[I_n = 0 | I_{n-1}^1 = i_{n-1}]} = P_{01}.
\]

Therefore, for all \( n \) we have

\[
Pr[\hat{I}_n = 1] = \sum_{i_{n-1}} Pr[\hat{I}_n = 1 | I_{n-1}^1 = i_{n-1}] Pr[I_{n-1}^1 = i_{n-1}] = P_{01},
\]

and thus the \( \hat{I}_n \) variables are identically distributed. It remains to prove that they are independent.

Suppose components in \( \hat{I}_n \sim (\hat{I}_1, \ldots, \hat{I}_n) \) are independent. We prove that components in \( \hat{I}_{n+1} = (\hat{I}_1, \ldots, \hat{I}_{n+1}) \) are also independent. For any binary vector \( \hat{i}_{n+1} \sim (i_1, \ldots, i_{n+1}) \), since

\[
Pr[\hat{I}_{n+1} = \hat{i}_{n+1}] = Pr[\hat{I}_{n+1} = \hat{i}_{n+1} | \hat{I}_{n} = \hat{i}_n] Pr[\hat{I}_{n} = \hat{i}_n] \\
= Pr[I_{n+1} = i_{n+1} | I_{n} = i_n] \prod_{k=1}^{n} Pr[I_k = i_k],
\]

it suffices to show

\[
Pr[\hat{I}_{n+1} = 1 | \hat{I}_n = \hat{i}_n] = Pr[I_{n+1} = 1] = P_{01}.
\]

Indeed,

\[
Pr[\hat{I}_{n+1} = 1 | \hat{I}_n = \hat{i}_n] = \sum_{i_1} Pr[I_{n+1} = 1 | I_{n} = i_n] Pr[I_{n} = i_n] \\
\times Pr[I_{n} = i_n | \hat{I}_{n} = \hat{i}_n] \\
= \sum_{i_1} Pr[I_{n+1} = 1 | I_{n} = i_n] Pr[I_{n} = i_n | \hat{I}_{n} = \hat{i}_n] \\
= \sum_{i_1} P_{01} Pr[I_{n} = i_n | \hat{I}_{n} = \hat{i}_n] = P_{01},
\]

where (a) is by (31), and the proof is complete.

\[\square\]

Appendix G

\textbf{Proof of Lemma 2} \ By definition of \( d(v) \), there exists a nonempty subset \( A \subseteq \Phi_d(v) \), and for every \( \phi \in A \) a positive real number \( \beta_\phi > 0 \), such that \( v = \sum_{\phi \in A} \beta_\phi \phi \). For each \( \phi \in A \), we have \( M(\phi) = d(v) \) and thus \( c_{M(\phi)} = c_d(v) \).

Define

\[
\beta_\phi \triangleq \frac{\beta_\phi}{\sum_{\phi \in A} \beta_\phi}
\]

for each \( \phi \in A \) and \( \{\beta_\phi\}_{\phi \in A} \) is a probability distribution. Consider a RandRR policy that in every round selects \( \phi \in A \) with probability \( \beta_\phi \). By Lemma 3, this RandRR policy achieves throughput vector \( \lambda = (\lambda_1, \ldots, \lambda_N) \) that satisfies

\[
\lambda = \frac{\beta_\phi \sum_{\phi \in A} \beta \phi M(\phi)}{d(\phi) \sum_{\phi \in A} \frac{\beta_\phi}{\sum_{\phi \in A} \beta_\phi}} = \frac{\beta_\phi c_d(v)}{d(\phi) \sum_{\phi \in A} \beta_\phi} v,
\]

which is in the direction of \( v \). In addition, the sum throughput

\[
\sum_{n=1}^{N} \lambda_n = \beta_\phi c_d(v) \sum_{n=1}^{N} \phi_n = \beta_\phi c_M(\phi) = c_d(v)
\]

is achieved.

\[\square\]

Appendix H

\textbf{Proof of Theorem 3} \ (A Related RandRR Policy) For each randomized round robin policy RandRR, it is useful to consider a renewal reward process where renewal epochs are defined as time instants at which RandRR starts a new round of transmission.\(^{10}\) Let \( T \) denote the renewal period. We say one unit of reward is earned by a user if RandRR serves a packet to that user. Let \( R_n \) denote the sum reward earned by user \( n \) in one renewal period \( T \), representing the number of successful transmissions user \( n \) receives in one round of scheduling. Conditioning on the round robin policy \( RR(\phi) \) chosen by RandRR for the current round of transmission, we have from Corollary 1

\[
E[T] = \sum_{\phi \in \Phi} \alpha_\phi E[T \mid RR(\phi)]
\]

(32)

\[
E[T \mid RR(\phi)] = \sum_{n: \phi_n = 1} E[I_1^\phi] ,
\]

(33)

and for all \( n \in \{1, 2, \ldots, N\} \),

\[
E[R_n] = \sum_{\phi \in \Phi} \alpha_\phi E[R_n \mid RR(\phi)]
\]

(34)

\(^{10}\)We note that the renewal reward process is defined solely with respect to RandRR, and is only used to facilitate our analysis. At these renewal epochs, the state of the network, including the current queue state \( U(t) \), does not necessarily renew itself.
\[ \mathbb{E}[R_n | RR(\phi)] = \begin{cases} \mathbb{E}[L_{1n}^\phi - 1] & \text{if } \phi_n = 1 \\ 0 & \text{if } \phi_n = 0. \end{cases} \] (35)

Consider the round robin policy \( RR((1, 1, \ldots, 1)) \) that serves all \( N \) channels in one round. We define \( T_{max} \) as its renewal period. From Corollary 1, we know \( \mathbb{E}[T_{max}] < \infty \) and \( \mathbb{E}[(T_{max})^2] < \infty \). Further, for any \( RR(\phi) \), including using a \( RR(\phi) \) policy in every round as special cases, we can show that \( T_{max} \) is stochastically larger than the renewal period \( T \), and \( (T_{max})^2 \) is stochastically larger than \( T^2 \). It follows that

\[ \mathbb{E}[T] \leq \mathbb{E}[T_{max}], \quad \mathbb{E}[T^2] \leq \mathbb{E}[(T_{max})^2]. \] (36)

We have denoted by \( RR^\ast \) (in the discussion after (13)) the randomized round robin policy that achieves a service rate vector strictly larger than the target arrival rate vector \( \lambda \) entrywise. Let \( T^\ast \) denote the renewal period of \( RR^\ast \), and \( R_n^\ast \) the sum reward (the number of successful transmissions) received by user \( n \) over the renewal period \( T^\ast \). Then we have

\[ \frac{\mathbb{E}[R_n^\ast]}{\mathbb{E}[T^\ast]} = \sum_{\phi \in \Phi} \alpha_\phi E[R_n^\ast | RR(\phi)] / \sum_{\phi \in \Phi} \alpha_\phi E[T^\ast | RR(\phi)] \]

\[ = \sum_{\phi \in \Phi} \frac{\alpha_\phi E[T^\ast | RR(\phi)]}{\sum_{\phi \in \Phi} \alpha_\phi E[T^\ast | RR(\phi)]} \frac{\mathbb{E}[R_n^\ast | RR(\phi)]}{\mathbb{E}[R_n^\ast | RR(\phi)]} \]

\[ = \sum_{\phi \in \Phi} \beta_\phi \phi_n^\ast \geq \lambda_n + \epsilon, \] (37)

where (a) is by (32), (b) is by rearranging terms, (c) is by plugging (3) into (19), (d) is by plugging (33) and (35) into (9) in Section IV-B, and (e) is by (13). From (37), we get

\[ \mathbb{E}[R_n^\ast] > (\lambda_n + \epsilon) \mathbb{E}[T^\ast], \quad \text{for all } n \in \{1, \ldots, N\}. \] (38)

**Lyapunov Drift** From (12), in a frame of size \( T \) (which is possibly random), we can show that for all \( n \)

\[ U_n(t + T) \leq \mathbb{E}[U_n(t) - \sum_{\tau=0}^{T-1} \mu_n(t + \tau), 0] + \sum_{\tau=0}^{T-1} a_n(t + \tau). \] (39)

We define a Lyapunov function \( L(U(t)) \equiv \frac{1}{2} \sum_{n=1}^{N} U_n^2(t) \) and the \( T \)-slot Lyapunov drift

\[ \Delta_T(U(t)) \equiv \mathbb{E}[L(U(t + T) - L(U(t))) | U(t)], \]

where in the last term the expectation is with respect to the randomness of the whole network in frame \( T \), including the randomness of \( T \). By taking square of (39) and then conditional expectation on \( U(t) \), we can show

\[ \Delta_T(U(t)) \leq \frac{1}{2} N(1 + A_{max}^2) \mathbb{E}[T^2 | U(t)] \]

\[ - \mathbb{E} \left[ \sum_{n=1}^{N} U_n(t) \left( \sum_{\tau=0}^{T-1} (\mu_n(t + \tau) - a_n(t + \tau)) \right) | U(t) \right]. \] (40)

Define \( f(U(t), \theta) \) as the last term of (40), where \( \theta \) represents a scheduling policy that controls the service rates \( \mu_n(t + \tau) \) and the frame size \( T \). In the following analysis, we only consider \( \theta \) in the class of \( RandRR \) policies, and the frame size \( T \) is the renewal period of a \( RandRR \) policy. By (36), the second term of (40) is less than or equal to the constant \( B_1 \equiv \frac{1}{2} N(1 + A_{max}^2) \mathbb{E}[(T_{max})^2] < \infty \). It follows that

\[ \Delta_T(U(t)) \leq B_1 - f(U(t), \theta). \] (41)

In \( f(U(t), \theta) \), it is useful to consider \( \theta = RandRR^\ast \) and \( T \) is the renewal period \( T^\ast \) of \( RandRR^\ast \). Assume \( t \) is the beginning of a renewal period. For each \( n \in \{1, 2, \ldots, N\} \), because \( R_n^\ast \) is the number of successful transmissions user \( n \) receives in the renewal period \( T^\ast \), we have

\[ \mathbb{E} \left[ \sum_{\tau=0}^{T^\ast-1} \mu_n(t + \tau) | U(t) \right] = \mathbb{E}[R_n^\ast]. \]

Combining with (38), we get

\[ \mathbb{E} \left[ \sum_{\tau=0}^{T^\ast-1} \mu_n(t + \tau) | U(t) \right] > (\lambda_n + \epsilon) \mathbb{E}[T^\ast]. \] (42)

By the assumption that packet arrivals are i.i.d. over slots and independent of the current queue backlogs, we have for all \( n \)

\[ \mathbb{E} \left[ \sum_{\tau=0}^{T^\ast-1} a_n(t + \tau) | U(t) \right] = \lambda_n \mathbb{E}[T^\ast]. \] (43)

Plugging (42) and (43) into \( f(U(t), RandRR^\ast) \), we get

\[ f(U(t), RandRR^\ast) \geq \epsilon \mathbb{E}[T^\ast] \sum_{n=1}^{N} U_n(t). \] (44)

It is also useful to consider \( \theta = a \) as a round robin policy \( RR(\phi) \) for some \( \phi \in \Phi \). In this case frame size \( T \) is the renewal period \( T^\phi \) of \( RR(\phi) \) (note that \( RR(\phi) \) is a special case of \( RandRR \). From Corollary 1 we have

\[ \mathbb{E}[T^\phi | U(t)] = \mathbb{E}[T^\phi] = \sum_{n; \phi_n=1}^{N} \mathbb{E}[L_{1n}^\phi], \] (45)

where \( \mathbb{E}[L_{1n}^\phi] \) can be expanded by (8). Let \( t \) be the beginning of a transmission round. If channel \( n \) is active, we have

\[ \mathbb{E} \left[ \sum_{\tau=0}^{T^\phi-1} \mu_n(t + \tau) | U(t) \right] = \mathbb{E} \left[ L_{1n}^\phi - 1 \right], \]

and 0 otherwise. It follows that

\[ f(U(t), RR(\phi)) \]

\[ \equiv \sum_{n; \phi_n=1}^{N} U_n(t) \mathbb{E}[L_{1n}^\phi - 1] - \mathbb{E}[T^\phi] \sum_{n=1}^{N} U_n(t) \lambda_n \]

\[ \equiv \sum_{n; \phi_n=1}^{N} U_n(t) \mathbb{E}[L_{1n}^\phi - 1] - \mathbb{E} \left[ L_{1n}^\phi \sum_{n=1}^{N} U_n(t) \lambda_n \right], \] (46)

where (a) is by (45) and rearranging terms.

**Design of QRR** Given the current queue backlogs \( U(t) \), we are interested in the policy that maximizes \( f(U(t), \theta) \) over all \( RandRR \) policies in one round of transmission. Although
the RandRR policy space is uncountably large and thus searching for the optimal solution could be difficult, next we show that the optimal solution is a round robin policy RR(φ) for some φ ∈ Φ and can be found by maximizing $f(U(t), RR(φ))$ in (46) over φ ∈ Φ. To see this, we denote by φ(τ) the binary vector associated with the RR(φ) policy that maximizes $f(U(t), RR(φ))$ over φ ∈ Φ, and we have

$$f(U(t), RR((φ(τ))) ≥ f(U(t), RR(φ)), \quad (47)$$

For any RandRR policy, conditioning on the policy RR(φ) chosen for the current round of scheduling, we have

$$f(U(t), RandRR) = \sum_{φ ∈ Φ} αφ f(U(t), RR(φ)),$$

where $\{αφ\}_{φ ∈ Φ}$ is the probability distribution associated with RandRR. By (47)–(48), for any RandRR we get

$$f(U(t), RR(φ(t))) ≥ \sum_{φ ∈ Φ} αφ f(U(t), RR(φ)) = f(U(t), RandRR).$$

We note that as long as the queue backlog vector $U(t)$ is not identically zero and the arrival rate vector λ is strictly within the inner capacity bound $Λ_{int}$, we get

$$\max_{φ ∈ Φ} f(U(t), RR(φ)) \overset{(a)}{=} f(U(t), RR(φ(t))) ≥ f(U(t), RandRR) \overset{(c)}{=} 0,$$

where (a) is from the definition of φ(t), (b) from (49), and (c) from (44).

The policy QRR is designed to be a frame-based algorithm where at the beginning of each round we observe the current queue backlog vector $U(t)$, find the binary vector φ(t) whose associated round robin policy RR(φ(t)) maximizes $f(U(t), RandRR)$ over RandRR policies, and execute RR(φ(t)) for one round of transmission. We emphasize that in every transmission round of QRR, active channels are served in the order that the least recently used channel is served first, and the ordering may change from one round to another.

(Stability Analysis) Again, policy QRR comprises of a sequence of transmission rounds, where in each round QRR finds and executes policy RR(φ(t)) for one round, and different φ(t) may be used in different rounds. In the kth round, let $T_{k}^{QRR}$ denote its time duration. Define $t_{k} = \sum_{i=1}^{k} T_{i}^{QRR}$ for all $k ∈ N$ and note that $t_{k} - t_{k-1} = T_{k}^{QRR}$. Let $t_{0} = 0$. Then for each $k ∈ N$, from (41) we have

$$\Delta_{k}^{QRR}(U(t_{k-1})) \overset{(a)}{=} B_{1} - f(U(t_{k-1}), QRR) \overset{(b)}{=} B_{1} - f(U(t_{k-1}), RandRR) \overset{(c)}{=} B_{1} - \sum_{n=1}^{N} U_{n}(t_{k-1}),$$

where (a) is by (41), (b) is because QRR is the maximizer of $f(U(t_{k-1}), RandRR)$ over all RandRR policies, and (c) is by (44). By taking expectation over $U(t_{k-1})$ in (51) and noting that $E[T^{*}] ≥ 1$, for all $k ∈ N$ we get

$$E[L(U(t_{k}))] - E[L(U(t_{k-1}))] \leq B_{1} - \epsilon E[T^{*}] \sum_{n=1}^{N} E[U_{n}(t_{k-1})] \leq B_{1} - \epsilon \sum_{n=1}^{N} E[U_{n}(t_{k-1})].$$

Summing (52) over $k ∈ \{1, 2, \ldots, K\}$, we have

$$E[L(U(t_{K}))] - E[L(U(t_{0}))] \leq K B_{1} - \epsilon \sum_{k=1}^{K} \sum_{n=1}^{N} E[U_{n}(t_{k-1})].$$

Since $U(t_{K}) ≥ 0$ entrywise and by assumption $U(t_{0}) = U(0) = 0$, we have

$$\epsilon \sum_{k=1}^{K} \sum_{n=1}^{N} E[U_{n}(t_{k-1})] \leq K B_{1}. \quad (53)$$

Dividing (53) by $\epsilon K$ and passing $K → ∞$, we get

$$\limsup_{K→∞} \frac{1}{K} \sum_{k=1}^{K} \sum_{n=1}^{N} E[U_{n}(t_{k-1})] ≤ B_{1} / \epsilon < ∞. \quad (54)$$

Equation (54) shows that the network is stable when sampled at renewal time instants $\{t_{k}\}$. Then that it is also stable when sampled over all time follows because $T_{k}^{QRR}$, the renewal period of the RR(φ) policy chosen in the kth round of QRR, has finite first and second moments for all k (see (36)), and in every slot the number of packet arrivals to a user is bounded. These details are provided in Lemma 13 which is proved in Appendix I.

**Lemma 13.** Given that

$$E[T_{k}^{QRR}] ≤ E[T_{max}], \quad E[(T_{k}^{QRR})^{2}] ≤ E[(T_{max})^{2}] \quad (55)$$

for all $k ∈ \{1, 2, \ldots, K\}$, packets arrivals to a user is bounded by $A_{max}$ in every slot, and the network sampled at renewal epochs $\{t_{k}\}$ is stable from (54), we have

$$\limsup_{K→∞} \frac{1}{K} \sum_{\tau=0}^{t_{k}-1} \sum_{n=1}^{N} E[U_{n}(\tau)] < ∞.$$

**APPENDIX I**

**Proof of Lemma 13.** In $[t_{k-1}, t_{k})$, it is easy to see for all $n ∈ \{1, \ldots, N\}$

$$U_{n}(t_{k-1} + \tau) ≤ U_{n}(t_{k-1}) + \tau A_{max}, \quad 0 ≤ \tau < T_{k}^{QRR}. \quad (56)$$

Summing (56) over $\tau ∈ \{0, 1, \ldots, T_{k}^{QRR} - 1\}$, we get

$$\sum_{\tau=0}^{T_{k}^{QRR} - 1} U_{n}(t_{k-1} + \tau) ≤ T_{k}^{QRR} U_{n}(t_{k-1}) + (T_{k}^{QRR})^{2} A_{max}/2. \quad (57)$$
Summing \( t_{K-1} \sum_{\tau=0}^{t-1} U_n(\tau) = \sum_{k=1}^{K} T^{QRR}_{k} U_n(t_{k-1}) \) and noting that \( t_K = \sum_{k=1}^{K} T^{QRR}_{k} \), we have

\[
E_{(a)} \left[ \sum_{k=1}^{K} T^{QRR}_{k} U_n(t_{k-1}) + (T^{QRR}_{k})^2 A_{\text{max}} / 2 \right],
\]

where \( (a) \) is by \( \text{(57)} \). Taking expectation of \( \text{(58)} \) and dividing it by \( t_K \), we have

\[
\frac{1}{t_K} \sum_{\tau=0}^{t-1} E[U_n(\tau)] \leq \frac{1}{K} \sum_{\tau=0}^{t-1} E[U_n(\tau)]
\]

where \( (a) \) follows \( t_K \geq K \) and \( (b) \) is by \( \text{(58)} \). Next, we have

\[
E \left[ T^{QRR}_{k} U_n(t_{k-1}) \right] = E \left[ \left. T^{QRR}_{k} U_n(t_{k-1}) \right| U_n(t_{k-1}) \right]
\]

where \( (a) \) is because \( E \left[ T^{QRR}_{k} \right] \leq E \left[ T_{\text{max}} \right] \). Using \( \text{(55)} \) to upper bound the last term of \( \text{(59)} \), we have

\[
\frac{1}{t_K} \sum_{\tau=0}^{t-1} E[U_n(\tau)] \leq B_2 + E \left[ T_{\text{max}} \right] \frac{1}{K} \sum_{k=1}^{K} E[U_n(t_{k-1})],
\]

where \( B_2 \triangleq \frac{1}{2} E \left[ (T_{\text{max}})^2 \right] A_{\text{max}} < \infty \). Summing \( \text{(61)} \) over \( n \in \{1, \ldots, N\} \) and passing \( K \to \infty \), we get

\[
\lim_{K \to \infty} \frac{1}{t_K} \sum_{\tau=0}^{t-1} E[U_n(\tau)] = N B_2 + E \left[ T_{\text{max}} \right] \left( \lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} E[U_n(t_{k-1})] \right)
\]

where \( (a) \) is by \( \text{(54)} \). The proof is complete. 

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