Prediction of an exotic state around 4240 MeV with $J^{PC} = 1^{−+}$ as C-parity partner of Y(4260) in molecular picture

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The possibility of the Y(4260) being the molecular state of $D\bar{D}_{1}(2420) + c.c.$ is investigated through one boson exchange model. It turns out that the potential of $J^{PC} = 1^{−+}$ state formed by $D\bar{D}_{1}(2420) + c.c.$ is attractive and it is strong enough to bind them together when the momentum cutoff $\Lambda \gtrsim 1.5$ GeV. To produce the Y(4260) with correct binding energy, we need $\Lambda \approx 2.3$ GeV. Besides, $D\bar{D}_{1}(2420) + c.c.$ can also form a state with exotic quantum numbers, $J^{PC} = 1^{−+}$, and its potential is more attractive than that of the $J^{PC} = 1^{−−}$ state. Therefore, an exotic state with mass around 4240 MeV is expected to be exist. Our estimation of the mass of the $J^{PC} = 1^{−+}$ state in charmonium region is in agreement with that predicted by the chiral quark model and the lattice QCD.

I. INTRODUCTION

In 2005 a vector charmoniumlike state, the Y(4260), was reported by BaBar Collaboration [1] in the initial-state radiation process $e^+e^- \rightarrow \gamma_{ISR} J/\psi\pi^+\pi^−$, which was confirmed by CLEO Collaboration [2] and Belle Collaboration [3] later. It is clear that the Y(4260) contains $c\bar{c}$ quarks and is above the thresholds for $D\bar{D}$, $D\bar{D}^* + c.c.$ and $D^*\bar{D}^*$. However, no signals of the Y(4260) appear in these channels [4,6], which indicates that it is not a conventional charmonium. Besides, there seems no room for the Y(4260) in the $1^{−−} c\bar{c}$ spectrum [7]. As a candidate for the exotic meson, its nature still remains controversial and has been attracting much attention.

Several models were proposed to account for the peculiar behaviors of the Y(4260) (see Ref. [8] for a detailed discussion), among which a molecular state of $D\bar{D}_{1}(2420) + c.c.$ seems to be a good choice since the Y(4260) is just below the threshold of $D\bar{D}_{1}(2420) + c.c.$ and they can couple in S-wave. The mechanisms of the formation of the molecular Y(4260) was discussed in Refs. [9,10]. Although it was argued in Ref. [11] that the production of $D\bar{D}_{1}(2420) + c.c.$ in the electron-positron collisions is forbidden in the heavy quark limit due to the heavy quark spin symmetry (HQSS) and in turn suppressed in the real world, Ref. [12] showed that the HQSS breaking is strong enough so that the molecule interpretation of the Y(4260) does not contradict the current experimental data. From the light quark perspective, it is claimed that the Y(4260) has a sizeable $D\bar{D}_{1}(2420)+c.c.$ component, which is, however, not completely dominant [13]. By assuming the Y(4260) being the $D\bar{D}_{1}(2420) + c.c.$ molecule, its properties have been discussed in Refs. [12,14,17,34]. Furthermore, such interpretation is supported by the new experimental data: an enhancement at the $D\bar{D}_{1}(2420) + c.c.$ threshold in the $J/\psi\pi\pi$ channel [19] and the observations of $Z_u(3900)\pi$ [20,21] and $X(3872)\gamma$ [22] in the mass region of the Y(4260). We refer to Ref. [23] for more details of this molecule picture.

The $D\bar{D}_{1}(2420) + c.c.$ can also form a system with positive C-parity, which is definitely an exotic state, if exists, since $J^{PC} = 1^{−+}$ is not allowed for tradititional $q\bar{q}$ mesons. Within the chiral quark model, Ref. [10] showed that the $D\bar{D}_{1}(2420) + c.c.$ with $J^{PC} = 1^{−+}$ can form a bound state with a mass of 4253 $\sim$ 4285 MeV. Besides, it is predicted by using the lattice QCD [24] that the $J^{PC} = 1^{−+}$ state in the charmonium region has a mass of $m(1^{−+}) = m_{\eta_c} + 1233 \pm 16$ MeV $= 4217 \pm 16$ MeV, just below the Y(4260), which gives us more confidence in the existence of the $D\bar{D}_{1}(2420) + c.c.$ bound state with $J^{PC} = 1^{−+}$. On the other hand, the production and the decay of such exotic state were discussed in Ref. [25] under the assumption of the Y(4260) being a molecule of $D\bar{D}_{1}(2420) + c.c.$ where some guidance for the experiments was given.

In this paper we use the vector meson exchange interaction between $D\bar{D}_{1}(2420) + c.c.$ to investigate whether it is possible for them to form the $J^{PC} = 1^{−−}$ and $J^{PC} = 1^{−+}$ molecules. In addition, we also discuss the influence of $\sigma$ exchange on the potential. Note that there are two $D_{1}$ states with similar masses while quite different decay widths, the narrow $D_{1}(2420)$ and the wide $D_{1}(2430)$. We only use the narrow one (denoted by $D_{1}$ throughout the rest of the paper) since $D_{1}(2430)$ is too wide to form a molecular state. We assume that the potential between the components of the $D\bar{D}_{1} + c.c.$ molecule is dominant by the vector meson exchange interactions since the pseudoscalar meson exchange between $D\bar{D}_{1} + c.c.$ is forbidden by parity conservation [26]. It is different from the assumption in Ref. [9] where the...
Y(4260) was considered as the molecule of $DD_1$ or $D_0D^*$ through pseudoscalar mesons exchange (off-diagonal potential) and $\sigma$ exchange (diagonal potential). The vector mesons exchange was not included because some of the related coupling constants were not available. We emphasize that it is not advisable because $D_0$ is too wide to be the component of the Y(4260).

This paper is organized as follows: In section II, the vector meson exchange potential between $DD_1 + c.c.$ is derived; Numerical results and discussions are shown in Section III.

II. FRAMEWORK

The potential between $D(\bar{D})$ and $\bar{D}_1(D_1)$ is related to the corresponding scattering amplitude. For the state with $J^{PC} = 1^{--}$, the element of $S$ matrix reads

$$
\langle \bar{D}D_1 - D\bar{D}_1 | S | \bar{D}D_1 - D\bar{D}_1 \rangle = \langle \bar{D}D_1 | S | \bar{D}D_1 \rangle + \langle D\bar{D}_1 | S | D\bar{D}_1 \rangle - (\langle \bar{D}D_1 | S | \bar{D}D_1 \rangle + \langle D\bar{D}_1 | S | D\bar{D}_1 \rangle)
$$

(1)

while for $J^{PC} = 1^{-+}$,

$$
\langle \bar{D}D_1 + D\bar{D}_1 | S | \bar{D}D_1 + D\bar{D}_1 \rangle = \langle \bar{D}D_1 | S | \bar{D}D_1 \rangle + \langle D\bar{D}_1 | S | D\bar{D}_1 \rangle + (\langle \bar{D}D_1 | S | \bar{D}D_1 \rangle + \langle D\bar{D}_1 | S | D\bar{D}_1 \rangle).
$$

(2)

Note that we have adapted the following charge conjugation conventions,

$$
C|D\rangle = |\bar{D}\rangle,
$$

(3)

$$
C|D_1\rangle = |\bar{D}_1\rangle.
$$

(4)

and in turn the flavor wave functions of positive and negative C-parity states now read

$$
C = + : \frac{1}{\sqrt{2}} (|D\bar{D}_1\rangle + |\bar{D}D_1\rangle),
$$

(5)

$$
C = - : \frac{1}{\sqrt{2}} (|D\bar{D}_1\rangle - |\bar{D}D_1\rangle).
$$

(6)

There are four Feynman diagrams for $DD_1 + c.c.$ elastic scattering by one boson (vector mesons and $\sigma$) exchange, shown in Fig. 1. Note that the scattering amplitudes of u-channel processes in the positive and negative C-parity cases carry opposite signs and in turn yield opposite potentials.

A. The vector exchange potential

1. The Langrangian

The coupling of heavy mesons and light vector meson nonet can be described by the effective Lagrangians, which satisfies the hidden gauge symmetry [27]. For $D$ and $D_1$ mesons, the Lagrangians read explicitly [9]

$$
\mathcal{L}_{DDV} = ig_{DDV} \left( D_\mu \partial_\nu D_\lambda \partial_\sigma D_\mu \right) V_{ba}^\mu
$$

(7)

$$
\mathcal{L}_{D_1D_1V} = ig_{D_1D_1V} \left( \bar{D}_\mu \partial_\nu D_a \partial_\sigma D_b \right) V_{ba}^\mu
$$

(8)

where

$$
D = (D^0, D^\pm)
$$

(9)

$$
D_1 = (D_1^+, D_1^-)
$$

(10)

$$
V = \left( \begin{array}{cc} \frac{\rho}{\sqrt{2}} & \frac{\omega}{\sqrt{2}} \\ \frac{\omega}{\sqrt{2}} & -\frac{\rho}{\sqrt{2}} \end{array} \right)
$$

(11)

and

$$
\mathcal{L}_{DDV} = -g_{DDV} = \frac{1}{\sqrt{2}} \beta g_V
$$

(12)

$$
\mathcal{L}_{D_1D_1V} = -g_{D_1D_1V} = \frac{1}{\sqrt{2}} \beta_2 g_V
$$

(13)

$$
\mathcal{L}_{D_1D_1V} = -g'_{D_1D_1V} = \frac{\lambda_2 g_V}{3\sqrt{2}} M_{D_1}
$$

(14)
FIG. 2. Feynman diagram for $K_1 \to K\rho$ decay. The angular momentum can be $L = 0, 2$ and we only consider $L = 0$.

$$g_{D_1D_1V} = \frac{1}{\sqrt{3}} g_{V} \sqrt{M_{D_1}M_{D_1}}$$

$$g_{D_1D_1V} = -\frac{1}{\sqrt{3}} g_{V} \mu_1 g_{V}.$$  

It can be easily verified that the charge conjugation invariance of the above Lagrangians, Eqs. (7) and (9), is consistent with the conventions, Eqs. (3) and (4), noting that $C V_c C^{-1} = -V^T$

2. Estimation of coupling constants

There are several parameters in the effective Lagrangians introduced in the last subsection. The already known ones are collected in the following,

$$g_{V} \approx 5.8,$$  

$$\beta \approx 0.9,$$  

$$\lambda_1 \approx 0.1 \text{ GeV}^2,$$

see Refs. [25, 29] and [27], respectively. These lead to $g_{DDV} \approx 3.7$. The rest constants $\beta_2$, $\mu_1$ and $\zeta_1$ are not available now.

The $L_{DD_1V}$ contains two types of interaction, which are denoted by $g_{D_1D_1V}$ and $g'_{D_1D_1V}$ in Eq. (9). The second type vanishes in the nonrelativistic limit since $\partial_{\mu} V_{\nu} \sim q_{\mu} V_{\nu}$ and the exchanged four-momentum $q_{\mu} = (0, \mathbf{q})$ vanishes. Therefore, we only consider the first interaction, which has nothing to do with the angular momentum of $D_1$. As a rough estimation, we take $g_{D_1D_1V} \approx g_{DDV}$ since they all describe the P-wave coupling of heavy mesons and the light vector meson. $D$ and $D_1$ have the same behaviors in such case where the spin of $D_1$ does not participate in.

The $L_{DD_1V}$ also contains two types of interaction, denoted by $g_{DD_1V}$ and $g'_{DD_1V}$ in Eq. (9). The first one describes the S-wave coupling, which dominates the interaction and hence the second one is neglected. We assume that the coupling of $K K_1 V$ is approximately the same as that of $D D_1 V$ because $s$ quarks in $K$ and $K_1$ and $c$ quarks in $D$ and $D_1$ are all spectators during the interactions. We use the decay of $K_1$ into $K\rho$ to estimate $g_{K K_1 V}$ and in turn $g_{DD_1V}$.

The partial wave amplitude [30] for $L = 0$ can be expressed as

$$M = \sqrt{3/2} g_{K_1K\rho} K^{*\mu}(m_1) \rho_\mu(m_2)$$  

where $\sqrt{3/2}$ accounts for the isospin factor.

Sum of the polarizations of vector mesons leads to

$$\frac{1}{3} \sum_{m_1, m_2} |M|^2 = \frac{|g_{K_1K\rho}|^2}{2} \sum_{m_1} K^{*\mu}_1(m_1) K^{\nu}_1(m_1) \times \sum_{m_2} \rho_\mu(m_2) \rho_\nu^*(m_2)$$

$$= \frac{|g_{K_1K\rho}|^2}{2} \left( g^{\mu\nu} - \frac{p^{\mu} p^{\nu}}{p^2} \right) \left( g^{\mu\nu} - \frac{p^{\mu}_1 p^{\nu}_1}{p^2_1} \right)$$

where the $K^{*\mu}_1$ is the polarization of $K_1$ and $\rho^{\mu}$ is the polarization of $\rho$. The decay width reads

$$\Gamma = \frac{|g_{K_1K\rho}|^2}{16 \pi M_{K_1}} \left( 3 + \frac{|p^2_1|}{m^2_1} \right)$$

In PDG [31], there are two different $K_1$ states, $K_1(1270)$ and $K_1(1400)$. The decay widths are

$$\Gamma_{K_1(1270)K\rho} \approx 36 \text{ MeV},$$

$$\Gamma_{K_1(1400)K\rho} \approx 5 \text{ MeV},$$

respectively, which lead to

$$g_{K_1(1270)K\rho} \approx 3.9 \text{ GeV},$$

$$g_{K_1(1400)K\rho} \approx 0.6 \text{ GeV}.$$  

In our calculation, $g_{DD_1V}$ will vary from 0.6 GeV to 3.9 GeV.

3. The potential in position space

For the vector exchange in the first two diagrams in Fig. (11), the scattering amplitude reads

$$i M_1 = (i \cdot i g_{DDV})(-i p_1 - i p_1) \frac{-i}{q^2 - m^2_V + i \epsilon}$$

$$\times (i \cdot i g_{DD_1V})(-i p_2 - i p_2) \frac{-i_1 \cdot \epsilon_2}{q^2 - m^2_V}$$

$$= i g_{DDV} g_{DD_1V} \frac{4m_Dm_{D_1}}{|q^2 + m^2_V|}$$

where $\epsilon_1$ and $\epsilon_2$ are the polarizations of initial and final $D_1$’s, respectively. Note that $\epsilon_1 \cdot \epsilon_2 = 1$ for the S-wave to S-wave scattering.

The corresponding potential in momentum space reads

$$\hat{V}_{c_1}(q, m_V) = -\frac{M_1}{4m_D m_{D_1}}$$

$$= -g_{DDV} g_{DD_1V} \frac{1}{|q^2 + m^2_V|}.$$  

(30)

After Fourier transformation we obtain the potential in position space

$$V_{c_1}(r, m_V) = -g_{DDV} g_{DD_1V} m_V Y(m_V r)$$

(31)
where
\[ Y(x) = \frac{1}{4\pi} \frac{e^{-x}}{x} \] (32)
is the Yukawa potential.

For the vector exchange in the last two diagrams, \[ iM_2 = (g_{\Delta D}V) \epsilon_1 e_{1\nu} \left( \frac{q^{\nu} - q^{\nu}'}{m_V^2} \right) \left( q^2 - m_V^2 + i \epsilon \right) (g_{DD}V) \epsilon_{2\nu} \]
\[ \approx ig_{DD}V \left( 1 - \frac{\epsilon_1 \cdot q}{m_V^2} \right) \frac{1}{|q|^2 + m^2} \] (33)
with
\[ m^2 = m_V^2 - (m_{D1} - m_{D2})^2 \] (34)
and the potential (see e.g. Ref.\[32\] for more details of such Fourier transformation) reads
\[ \tilde{V}_V(q, m_V) = -\frac{g_{DD}V}{4m_Dm_D} \left( 1 - \frac{\epsilon_1 \cdot q}{m_V^2} \right) \frac{1}{|q|^2 + m^2} \] (35)
\[ V_V = -\frac{g_{DD}V}{4m_Dm_D} \left\{ \bar{m}Y(\bar{m}r) + \frac{\bar{m}}{3m_V^2} \left[ \bar{m}^2 Y(\bar{m}r) \epsilon_1 \cdot \epsilon_2 
+ S_{12}(\bar{r}) \bar{m}^2 \left( 1 + \frac{3}{\bar{m}r} + \frac{3}{m^2} \right) \bar{m}Y(\bar{m}r) \right) \right\} \] (36)
with \( S_{12}(\bar{r}) = 3 \epsilon_1 \cdot \bar{r} \epsilon_2 \cdot \bar{r} - \epsilon_1 \cdot \epsilon_2 \). We have used the facts that \( \epsilon_1 \cdot \epsilon_2 = 1 \) and \( S_{12} = 0 \) for the S-wave to S-wave scattering\[33\]. The delta function has been ignored since the components have finite sizes.

After taking the isospin factor into account, we obtain
\[ V_{V1}^{I=0}(r, m_V) = \frac{1}{2} \left( 3V_1(r, m_p) + V_1(r, m_n) \right), \] (37)
\[ V_{V2}^{I=0}(r, m_V) = \frac{1}{2} \left( 3V_2(r, m_p) + V_2(r, m_n) \right), \] (38)
\[ V_{V1}^{I=1}(r, m_V) = \frac{1}{2} \left( V_1(r, m_p) - V_1(r, m_n) \right), \] (39)
\[ V_{V2}^{I=1}(r, m_V) = \frac{1}{2} \left( V_2(r, m_p) - V_2(r, m_n) \right). \] (40)

For the \( J^{PC} = 1^{-\pm} \) state, the total vector exchange potential reads
\[ V_V^{C=\pm} = V_{V1}^{I=1} \pm V_{V2}^{I=1}, \] (41)
where \( I = 0, 1 \). Note that \( m_p \approx m_n \), the potentials for isovector (I=1) are very weak and we only consider the isoscalar states here.

A form factor
\[ F(q, m, \Lambda) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2} \] (42)
is introduced to the potential at each vertex in order to take into account the actual size of the mesons. The potentials in position space, Eqs.\[31\], become
\[ V_{V1}(r, m_V) = -g_{DD}V g_{DD}D \left( m_V Y(m_V - m_V r) \right) - \frac{\Lambda Y(\Lambda r)}{2} \left( \frac{1}{2} \right) (\Lambda^2 - m_V^2) r Y(\Lambda r), \] (43)
\[ V_{V2}(r, m_V) = -\frac{g_{DD}V}{4m_Dm_D} \left( 1 + \frac{m^2}{3m_V^2} \right) \left[ \bar{m}Y(\bar{m}r) \right. \left. - \frac{\bar{m}Y(\bar{m}r)}{2} \right] \] (44)
with
\[ \bar{A}^2 = \Lambda^2 - (m_{D1} - m_{D2})^2. \] (45)

B. The \( \sigma \) exchange potential

The \( \sigma \) exchange potential has been calculated in Ref.\[9\]. For the t-channel process,
\[ V_{\sigma 1}(r) = -g_{\sigma \pi} \left( m_{\sigma} Y(m_{\sigma} r) - \Lambda Y(\Lambda r) \right) \] (46)
and for the u-channel process,
\[ V_{\sigma 2} = \frac{2}{9} \left( \frac{\Lambda}{\bar{m}} \right)^2 \left[ \frac{\Lambda Y(\Lambda r)}{2} \right. \left. - \frac{\bar{m}Y(\bar{m}r)}{2} \right] \] (47)
with
\[ \bar{m}^2 = m^2 - (m_{D1} - m_{D2})^2, \] (48)
\[ \bar{A}^2 = \Lambda^2 - (m_{D1} - m_{D2})^2. \] (49)

In our calculation, the constants in the above potentials are taken to be
\[ g_{\sigma \pi} = \pm 0.58, \] (50)
\[ h_{\sigma} = 0.35, \] (51)
\[ \pi = 132, \] (52)
as in Ref.\[9\].

Note that the isospin factor is trivial in this case. For the \( J^{PC} = 1^{-\pm} \) state, the total \( \sigma \) exchange potential reads
\[ V_{\sigma}^{C=\pm} = V_{\sigma 1} \pm V_{\sigma 2}. \] (53)

III. NUMERICAL RESULTS AND DISCUSSIONS

We use the following values from PDG \[31\],
\[ m_D = 1.867 \text{ GeV}, \] (54)
Besides, we take $g_{D_1D,V} \approx g_{DDV} \approx 3.7$, as analyzed above. Using the decay of $K_1$ we estimate that $g_{D_1D,V}$ is in the range from 0.6 GeV to 3.9 GeV.

The vector and $\sigma$ exchange potentials are shown in Fig. 3 and Fig. 4, respectively. From Fig. 3 we see that $v^+_{D} < v^0_{D} < 0$ and in turn $V_{C}^{++} < V_{C}^{--} < 0$. (59)

Meanwhile, the $\sigma$ exchange potentials are much smaller than the vector ones and therefore, it is expected that the $D\bar{D}_1$ bound state with $J^{PC} = 1^{--}$ exists if the Y(4260) can be interpreted as a $D\bar{D}_1$ bound state with $J^{PC} = 1^{--}$.

The Schrödinger equations for both $V_{C}^{++}$ and $V_{C}^{--}$ are solved and the binding energies are shown in Fig. 3. The coupling constant $g_{DD,V}$ is estimated in the range from 0.6 GeV to 3.9 GeV and it is adjustable in our calculation. For each value of $g_{DD,V}$ in this range, we use the fact that Y(4260), as a bound state of $D\bar{D}_1$ with $J^{PC} = 1^{--}$, has a binding energy of around 59 MeV to determine the momentum cutoff $\Lambda_0$. With the same $g_{DD,V}$ and $\Lambda_0$, we obtain the binding energy of the bound state of $D\bar{D}_1$ with $J^{PC} = 1^{--}$, around 60 ~120 MeV. We also include the $\sigma$ exchange potential and its influence turns out to be insignificant. The results for different $g_{DD,V}$ and $g_{DD,\sigma}$ are listed in Table I. If we assume that the Y(4260) is a pure $D\bar{D}_1 + c.c.$ bound state with $\Lambda_0 \approx 2.3$ GeV, its $1^{--}$ partner should has a mass around 4200 MeV. Since the Y(4260) may be a mixture of $D\bar{D}_1 + c.c.$ molecule and $\psi(nD)$, then a more commonly used $\Lambda_0 \approx 1.5$ GeV leads to a $1^{--}$ $D\bar{D}_1 + c.c.$ molecule with a mass around 4280 MeV. Therefore, we expected this exotic $1^{--}$ $D\bar{D}_1 + c.c.$ molecule to be around 4240 MeV.

In summary, we have used the one boson exchange potential between the $D\bar{D}_1 + c.c.$, for both $J^{PC} = 1^{--}$ and $J^{PC} = 1^{--}$ systems, to investigate if it is possible for them to form bound states. We use the effective Lagrangians, which satisfy the heavy quark symmetry, to describe the interaction between $D$ and $D_1$. First, we only consider the vector exchange. The coupling constant $g_{DD,V}$ is taken to be in the range from 0.6 GeV to 3.9 GeV, which is estimated from the decay width of $K_1 \rightarrow K\rho$. It turns out that with a momentum cutoff $\Lambda = 2.20 \sim 2.44$ GeV, the attractive force between the $D\bar{D}_1 + c.c.$ bound state with $J^{PC} = 1^{--}$ is strong enough to form a bound state, corresponding to the Y(4260). The C-parity partner of the Y(4260), i.e. the exotic $D\bar{D}_1 + c.c.$ bound state with $J^{PC} = 1^{--}$, is predicted to be exist. Its mass is expected to be less than the Y(4260), which is consistent with the prediction by lattice QCD and chiral quark model. The $\sigma$ exchange potential is then included and it turns out to have little influence on the binding energies. The possible decay modes of the predicted exotic $1^{--}$ state include $\eta\eta'$ and $\chi_{c1}\eta$. (55)

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FIG. 3. The vector exchange potentials with $\Lambda = 1.5$ GeV and $g_{DD_1V} = 0.6$ GeV for the left figure while $g_{DD_1V} = 3.9$ GeV for the right one. The “t-channel” represents the potential for the first two diagrams in Fig. (1) and the “u-channel” for the second two diagrams. “$C = +$” and “$C = -$” represent the total potentials for positive and negative $C$-parity states, respectively.

FIG. 4. The $\sigma$ exchange potentials with $\Lambda = 1.5$ GeV and $g_\sigma g_\sigma' = +0.58$ for the left figure while $g_\sigma g_\sigma' = -0.58$ for the right one. The legends are the same as those in Fig. (3).

FIG. 5. Dependence of binding energies on the cutoff $\Lambda$. Here we take $g_{DD_1\sigma} = 0$ as an illustration. $g_{DD_1V} = 0.6$ GeV for the left figure while $g_{DD_1V} = 3.9$ GeV for the right one.
TABLE I. The binding energy of $D\bar{D}_1 + \text{c.c.}$ bound state. We choose $g_{DD_1\sigma} = 0$ or $\pm 0.58$, $g_{DD_1V} = 0.6$ or $3.9$ and $\Lambda = 1.5, 2.0$ or $2.5$ GeV to investigate how the binding energy of the $D\bar{D}_1$ bound state with $J^{PC} = 1^{--}$ or $1^{--}$ changes. In the last two columns, $\Lambda_0$ means the cutoff, which, together with other specified couplings, yields the experimental binding energy of the $Y(4260)$, $59$ MeV and $E_0$ means the expected corresponding binding energy of the $J^{PC} = 1^{--}$ state.

| $g_{DD_1\sigma}$ | $g_{DD_1V}$(GeV) | $\Lambda$(GeV) | $\frac{E_B}{\text{MeV}}$ | $\Lambda_0$(GeV) | $E_0$(MeV) |
|------------------|------------------|-----------------|---------------------|-----------------|-----------|
|                  |                  | $C = -$ | $C = +$ |                  |           |           |
| 0                | 3.9              | 1.5       | 0.04 | 7.2 | 2.44 | 118.0 |
|                  |                  | 2.0       | 22.7 | 37.8 |
|                  |                  | 2.5       | 64.8 | 127.1 |
| 0.6              |                  | 1.5       | 2.3 | 2.5 | 2.20 | 60.1 |
|                  |                  | 2.0       | 38.5 | 39.3 |
|                  |                  | 2.5       | 93.6 | 95.1 |
| $+0.58$          | 3.9              | 1.5       | 0.7 | 8.1 | 2.34 | 110.3 |
|                  |                  | 2.0       | 28.2 | 61.7 |
|                  |                  | 2.5       | 74.7 | 134.1 |
| 0.6              |                  | 1.5       | 4.0 | 3.1 | 2.13 | 56.4 |
|                  |                  | 2.0       | 45.0 | 42.6 |
|                  |                  | 2.5       | 104.7 | 101.4 |
| $-0.58$          | 3.9              | 1.6       | 1.2 | 11.3 | 2.50 | 114.7 |
|                  |                  | 2.0       | 20.1 | 50.8 |
|                  |                  | 2.5       | 59.4 | 115.3 |
| 0.6              |                  | 1.5       | 1.8 | 1.2 | 2.24 | 56.3 |
|                  |                  | 2.0       | 35.3 | 33.1 |
|                  |                  | 2.5       | 87.5 | 84.3 |