Nonlinear Dynamic Analysis of Multi-vehicle Vehicle-bridge Coupling System

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Abstract. By analyzing different nonlinear models of vehicle-bridge coupling vibration when multiple vehicles travel on a bridge, this paper proposes the nonlinear vibration equation of multiple vehicle-bridge coupling. A comprehensive analysis is conducted on the quarter vehicle model, the nonlinear motion model of the bridge, and the coupled coupling equation of the vehicle and bridge. The vehicle-bridge coupled nonlinear analysis model of a single vehicle is established, and the correctness of the results calculated by the model is verified by calculation examples. In addition, taking the three-span continuous beam bridge with equal cross section and equal span as an example, this paper studies the driving conditions of multiple vehicles respectively from the influence of vehicle number, vehicle spacing and vehicle speed on the dynamic response value of the bridge. The results show that vehicle speed, vehicle number and vehicle spacing have significant influence on the dynamic response value of the bridge.

1. Introduction

While the vehicle moving on the bridge, it will bump if the bridge deck is not smooth. The vehicle bumping has great influence on the dynamic response of bridges\cite{1}. It is an important aspect of vehicle-bridge coupling dynamics. At present, the research on vehicle-bridge coupling nonlinear dynamic analysis is usually limited to single vehicle driving condition\cite{2}\cite{3}. However, in actual operation, there are often several vehicles acting on the bridge at the same time. When multiple vehicles act on the bridge at the same time, the dynamic response value of the bridge is not a simple sum of the dynamic response value of the bridge when each vehicle runs alone, but the result of the mutual influence and interaction of various factors.

The dynamic response of the bridge is affected by the number of vehicles, running state and speed. Therefore, it is necessary to analyze the dynamic response of the bridge under the condition of multiple vehicles. Since there are many driving conditions of multiple vehicles, it is impossible and unnecessary to analyze each working condition. The dynamic response values of the bridge under the influence of different factors when multiple vehicles are running simultaneously are obtained.

2. Multi-vehicle vehicle-bridge coupling nonlinear analysis model

Compared with the single-vehicle vehicle-bridge coupling analysis model, with the increase of the number of vehicles, more factors can influence multi-vehicle vehicle-bridge coupling nonlinear analysis and its nonlinear coupling equation gets more complex. In this paper, the one-quarter vehicle model is adopted, and each vehicle is numbered in turn and included in the vehicle-bridge coupling nonlinear equation\cite{4}.
Appropriate simplification assumptions for the vehicle model are made as follows: (1) The deformation of the vehicle is ignored and the vehicle is regarded as an absolute rigid body. (2) The vehicle is simplified into a body and suspension, which are connected by springs and dampers. The mass of the body is concentrated and simplified on the axle. (3) The spring is seen as an ideal elastic system. (4) The vehicle always keeps close contact with the ground during driving. It is shown as Figure.1.

\[
\text{Figure.1 The one-quarter vehicle model}
\]

A vehicle is simplified into a nonlinear spring-mass-dashpot system with two degrees of freedom in the one-quarter vehicle model, assuming that the mass parameters of each part are respectively as follows: \(m_v\) —body quality, \(m_w\) —total mass of vehicle suspension and wheel mass, \(k_v\) —vehicle body stiffness, \(k_w\) —vehicle suspension stiffness, \(c_v\) —vehicle body damping, \(c_w\) —damping of vehicle body suspension system, \(y_v\) —vertical displacement of vehicle body, \(y_w\) —vertical displacement of the vehicle body suspension system, \(y_c\) —vertical displacement of vehicle point.

The vibration balance equation of the vehicle system can be obtained by force analysis of the vehicle body and suspension:

\[
\begin{align*}
 m_v \ddot{y}_v + c_v (\dot{y}_v - \dot{y}_w) + k_v (y_v - y_w) &= 0 \\
 m_w \ddot{y}_w + c_w (\dot{y}_w - \dot{y}_c) + k_w (y_w - y_c) &= 0 \\
 -c_v (\dot{y}_v - \dot{y}_w) - k_v (y_v - y_w) &= 0
\end{align*}
\]

Convert the system of equations into matrix form:

\[
[M_v] \{\dot{Y}_v\} + [C_v] \{\ddot{Y}_v\} + [K_v] \{Y_v\} = \{F_v\}
\]

3. **Nonlinear balance equation**

When multiple vehicles travel on the bridge, the force of vehicle \(j\) can be expressed as:

\[
P_j(t) = (m_{v_j} + m_{w_j})g + m_{v_j} \ddot{y}_{v_j} + m_{w_j} \ddot{y}_{w_j}
\]

The force of vehicle \(j\) on the bridge is:

\[
f(x, t) = -\sum_{j=1}^{N} \delta(x - v_j t) P_j(t)
\]

\(N\) —vehicle number

The nonlinear balance equation of the bridge is:

\[
m \int_{l}^{l} \sum_{i=1}^{n} \varphi_i(x) \ddot{q}_i(t) \varphi_n(x) dx + \int_{l}^{l} EI \sum_{i=1}^{n} \varphi_i^4(x) q_i(t) \varphi_n(x) dx + \int_{l}^{l} \eta EI \sum_{i=1}^{n} \varphi_i^4(x) \ddot{q}_i(t) \varphi_n(x) dx - \int_{l}^{l} P \sum_{i=1}^{n} \dddot{q}_i(x) q_i(t) \varphi_n(x) dx
\]
When multiple vehicles pass the bridge, the displacement coordination equation of vehicle j is as follows:

\[ y_{cj} = w(x_j, t) + r(x_j) \]

\[ y_{cj} = \sum_{i=1}^{n} \varphi_i(x_j)q_i(t) + r(x_j) \]  

(7)

\[ y_{cj} \] — the vertical displacement of vehicle j’s wheel-bridge interaction point

\[ w(x_j, t) \] — the vertical displacement of the bridge at the wheel point of vehicle j

\[ r(x_j) \] — the unevenness value of the bridge deck where vehicle j is

4. Equation solving

Assuming that the spacing between vehicles is \( l_0 \), the speed of vehicle j is \( v_j \), vehicles drive into the bridge one after another and keep a constant speed, and the spacing between vehicles remains unchanged, then the vertical displacement of wheel-bridge interaction point at time t is:

\[ y_{c1} = w(x_1, t) + r(x_1) \]

\[ = w(v_1 t, t) + r(v_1 t) \]

\[ y_{c2} = w(x_2, t) + r(x_2) \]

\[ = w(v_2 t - l_0, t) + r(v_2 t - l_0) \]

\[ = \sum_{i=1}^{n} \varphi_i(v_2 t - l_0)q_i(t) + r(v_2 t - l_0) \]

\[ \ldots \]

\[ y_{cj} = w(x_j, t) + r(x_j) \]

\[ = w(v_j t - (j-1)l_0, t) + r(v_j t - (j-1)l_0) \]

\[ = \sum_{i=1}^{n} \varphi_i(v_j t - (j-1)l_0)q_i(t) + r(v_j t - (j-1)l_0) \]  

(8)

Taking the nonlinear vibration analysis equation of vehicle-bridge coupling of simply supported beam as an example, assume that vehicle j travels on the bridge at speed \( v_j \), that is, \( x_j = v_j t \)

Substitute the free vibration function of simply supported beam into the above equation, say \( \omega_n^2 = \frac{El}{ml} \left(\frac{n\pi}{l}\right)^4 \), then it can be drawn that

\[ \frac{ml}{2} \ddot{q}_n(t) + \eta EI \left(\frac{n\pi}{l}\right)^4 \varphi \cdot \frac{1}{2} \cdot \dot{q}_n(t) \]

\[ + EI \left(\frac{n\pi}{l}\right)^4 \varphi \cdot \frac{1}{2} \cdot q_n(t) - p \left(\frac{n\pi}{l}\right)^4 \frac{1}{2} \cdot q_n(t) \]

\[ = - \int_0^l \sum_{j=1}^N \delta(x - v_j t) \cdot P_j(t) \cdot \varphi_n(v_j t) dx \]  

(9)

Divide everything by \( ml/2 \),

\[ \ddot{q}_n(t) + \eta \omega_n^2 \dot{q}_n(t) + \omega_n^2 q_n(t) + \left(\frac{P}{m}\right) \left(\frac{n\pi}{l}\right)^4 \frac{1}{2} \cdot q_n(t) \]

\[ = - \frac{2}{ml} \left[ \sum_{j=1}^N P_j(t) \varphi_n(v_j t) \right] \]  

(10)

Vibration of vehicle-bridge coupling system:

\[ M(t)\ddot{Y} + C(t)\dot{Y} + [K(t) + H(t)]Y = Q(t) \]  

(11)
In the study of vehicle-bridge coupled nonlinearity, the first few modes of vibration are often selected for calculation to reduce the order of nonlinear differential equations. As long as the vibration mode function and natural frequency of the bridge are known, the nonlinear equation of the bridge can be solved.

The nonlinear multi-vehicle vibration equation of three-span continuous beam bridge is similar to that of simply supported beam-bridge. However, since the vibration mode function of continuous beam is quite different from that of simply supported beam, it is necessary to replace the latter with the former, and then use the vehicle balance equation to solve multi-vehicle nonlinear vibration of three-span continuous bridge.

The dynamic response of the bridge is influenced by the bridge span and vehicle position. The larger span of the bridge means the greater dynamic response. From the peak value data of the bridge, when
the dynamic response value of the bridge reaches the peak value, the position of the vehicle is not in the mid-span, but in the positions on both sides of the mid-span.

In the actual bridge operation process, multiple vehicles often act on the bridge at the same time, and the number, running state and speed of vehicles will have an impact on the dynamic response of the bridge, so it is necessary to analyze the dynamic response of the bridge under the condition of multiple vehicles.

Since there are many driving conditions of multiple vehicles, it is impossible and unnecessary to analyze every condition. It is necessary to classify and simplify various conditions into groups, then contrast and discuss simplified models in groups. The dynamic response value of the bridge under the influence of different factors when multiple vehicles travel simultaneously is obtained.

Take three-span continuous beam bridge with equal section as an example. Let two vehicles pass through the bridge at a speed of 20m/s at the same time, and take the actual safety distance between vehicles as the basis, then the dynamic response values of the bridge under three vehicle distance conditions of 10m, 20m and 30m respectively can be shown as Figure.2.

![Figure.2](image-url)

**Figure.2** the dynamic response of the bridge span with different distance between vehicles

| TABLE 1. Dynamic response peak under different driving conditions |
|---------------------------------------------------------------|
| **Single Vehicle**                                          |
| **The first peak** | **The second peak** | **The third peak** |
| Time(s)     | 0.49               | 1.97            | 2.86 |
| Peak(mm)    | -2.10              | 9.44            | -2.27 |
| **Vehicle distance 10 m**                                  |
| **The first peak** | **The second peak** | **The third peak** |
| Time(s)     | 0.37               | 1.37            | 2.49 |
| Peak(mm)    | -5.14              | 8.73            | -4.01 |
| **Vehicle distance 20 m**                                  |
| **The first peak** | **The second peak** | **The third peak** |
| Time(s)     | 0.46               | 0.71            | 0.97 |
| Peak(mm)    | 3.24               | 0.54            | 1.24 |
| **Vehicle distance 30 m**                                  |
| **The first peak** | **The second peak** | **The third peak** |
| Time(s)     | 0.31               | 1.19            | 1.89 |
| Peak(mm)    | 5.12               | -5.11           | 6.73 |

With the increase of vehicle spacing, the dynamic response curve amplitude of the bridge first increases and then decreases, so does the peak number of span deflection of the bridge. According to the comparison between the peak data of two vehicles and that of a single vehicle, the dynamic response values of the bridge generated by two vehicles are superimposed on each other during driving, so the
influence of multiple vehicles on the bridge is not greater than that of a single vehicle in all cases. The peak values are shown as TABLE I.

Select two vehicles, assume that the distance between them is 10m, and both vehicles keep a constant speed of 20m/s, 30m/s, and 40m/s respectively, then the dynamic responses of the bridge is solved based on these three speeds and the calculation results are shown as Figure 3.

![Figure 3](image)

**Figure 3** the dynamic response of the bridge span with different speed

| TABLE 2. Dynamic response peak under different driving conditions |
|---------------------------------------------------------------|
| **speed** | **Peak value** | **The first peak** | **The second peak** | **The third peak** |
|-----------|----------------|-------------------|-------------------|-------------------|
| 20m/s     | Time(s)        | 0.24              | 0.96              | 2.47              |
|           | Peak(mm)       | -5.30             | 8.23              | -5.21             |
| 30m/s     | Time(s)        | 0.26              | 0.76              | 1.51              |
|           | Peak(mm)       | -5.87             | 8.65              | -5.95             |
| 40m/s     | Time(s)        | 0.21              | 0.63              | 1.21              |
|           | Peak(mm)       | -6.14             | 13.32             | -7.85             |

When vehicles pass the bridge at different speeds, the trend of dynamic response curves of the bridge is basically the same, but the peaks and the peak positions differ. It can be seen from the peak data that the dynamic response value of the bridge does not change linearly with the increase of speed, and the change trend of each peak is different. The peak values are shown as TABLE II.

The dynamic response of the bridge is greatly influenced by the running state of the vehicle. The influence of the running state of the vehicle should be fully considered in the design of the bridge. When vehicles travel at different speeds, the dynamic response value of the bridge will increase, and the position with the greatest influence will also change. Therefore, the influence of such factors should be considered in the design of the bridge, and the bridge in this area should be strengthened.

5. Conclusion
The dynamic response value of the bridge span increases first and then decreases with the increase of vehicle spacing. Therefore, the most unfavorable vehicle spacing should be considered in bridge design.

With the increase of vehicles, the dynamic response value of the bridge increases first and then decreases. Therefore, in bridge design, not only the most unfavorable load arrangement should be considered, but also the mutual influence between multiple vehicles should be taken into account in order to achieve the maximum economic benefit.
The increase of vehicle speed leads to the increase of dynamic response value of the bridge. In bridge design, the dynamic response value should be calculated according to the design speed of different bridges and the fundamental frequency of bridges.

Acknowledgment
Science and Technology Foundation of Jiang’xi Educational Committee (GJJ170667)
Jiangxi Science & Technology Normal University project (JGYB-16-78-16) & (S202011318069)

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