A novel approach for extracting model-independent nuclear level densities far from stability

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The level density of quantum states in statistical mesoscopic systems is a critical input for various fields of physics, including nuclear physics, nuclear astrophysics, atomic physics and their applications. In atomic nuclei the level density is a fundamental measure of their complex structure at relatively high energies. Here we present the first model-independent measurement of the absolute partial nuclear level density for a short-lived unstable nucleus. For this purpose we introduce the “Shape method” to extract the shape of the $\gamma$-ray strength function. Combining the Shape method with the existing $\beta$-Oslo technique allows the extraction of the nuclear level density without the need for theoretical input. We benchmark the Shape method using results for the stable $^{76}$Ge nucleus, finding excellent agreement to previous experimental results. We apply the Shape method to new experimental data on the short-lived $^{88}$Kr nucleus. Our method opens the door for measurements of the nuclear level density and $\gamma$-ray strength function far away from stability, a pivotal input required to understand the role of exotic nuclei in forming the cosmos.

I. INTRODUCTION

Nuclei are complex quantum many-body systems. For low-excitation energies, their structure can be described using the properties (energy, spin, parity, width) of individual levels. However, moving to higher energies these properties need to be combined into a statistical description of the nucleus, originally described in the 1930s by N. Bohr, as the levels get closer together and overlap. One of the most important statistical properties is the nuclear level density (NLD) as it carries information about the structure of the nucleus, such as pair breaking, shell effects, shape changes and collectivity. In addition, the NLD is a critical input in nuclear reaction calculations, in particular for neutron-capture reaction cross sections and neutron-induced fission calculations, both pivotal input for nuclear astrophysics and applications in nuclear energy and security.

Nuclear theory efforts for describing the NLD have been ongoing for more than eight decades. In 1936, H.A. Bethe first described the nucleus as a group of non-interacting fermions and showed that the NLD increases roughly exponentially as a function of excitation energy. Since then, numerous theoretical efforts attempted to provide a description of this important quantity, leading to modern approaches which are typically (semi)microscopic, e.g., or shell-model based, e.g.,. Theoretical calculations often have difficulty reproducing the available experimental data for stable nuclei, and their predictions diverge even more when extrapolating to regions inaccessible by experiment. Differences between models can reach an order of magnitude. This is especially important when looking at systems far from stability since these are the most sensitive regions for particular applications like for the astrophysical $r$ process and for nuclear energy production.

Experimentally, measurements of the NLD are limited to stable nuclei or their closest neighbors because the experimental approaches used to date are based on stable-
beam experiments. Commonly-used techniques for extracting the NLD around stability are the Oslo method \[18\] [20] and the particle-evaporation method \[21\] [22]. New experimental techniques were developed recently that can provide NLDs on short-lived nuclei (\(\beta\)-Oslo [23], inverse-Oslo [24]). In addition, the surrogate method [24] was also developed recently for constraining neutron-capture reactions on unstable nuclei. However all aforementioned methods rely on inputs from theoretical models. While their results are still of high value since they are the only available methods to extract NLDs and neutron-capture reaction cross sections on short-lived nuclei, the dependence on theoretical models is a limitation. Here we present the first model-independent approach for extracting NLDs for a short-lived nucleus.

In this work we make use of the \(\beta\)-Oslo method [23] and populate the nucleus of interest via \(\beta\) decay, allowing experiments with secondary beam intensities as low as 1 pps far away from nuclear stability [23, 26]. A segmented total absorption spectrometer is used to simultaneously measure the excitation energy and individual \(\gamma\) rays transitions of the populated nucleus. Following the unfolding of the data with the detector response [20], an iterative subtraction process allows for the extraction of unfolding of the data with the detector response [20], an iterative subtraction process allows for the extraction of the first generation \(\gamma\) rays as a function of excitation energy, \(E_x\). The extracted first generation (primary) \(\gamma\)-ray matrix \(P(E_x, E_\gamma)\) can be factorised as [18]:

\[
P(E_x, E_\gamma) \propto T(E_\gamma) \rho(E_x - E_\gamma),
\]

where \(\rho(E_x - E_\gamma)\) is the NLD at the excitation energy after the first \(\gamma\)-ray is emitted, and \(T(E_\gamma)\) is the transmission coefficient for \(\gamma\) emission. An infinite number of solutions are possible for the above equation [18] and the physical solution is obtained when normalizing the \(\rho(E_x - E_\gamma)\) and \(T(E_\gamma)\) to known data with:

\[
\rho'(E_x - E_\gamma) = A e^{\alpha(E_x - E_\gamma)} \rho(E_x - E_\gamma),
\]

\[
T'(E_\gamma) = B e^{\nu(E_\gamma)} T(E_\gamma),
\]

where \(A\) and \(B\) are constants and \(\alpha\) is a common slope parameter. Typical normalization data used in the Oslo method are: 1) low-lying discrete levels, 2) the level density at the neutron separation energy, \(\rho(S_n)\), calculated from neutron-resonance spacing, \(D_0\), measurements where available, and 3) the average total radiative width \((\Gamma_\gamma)\) at \(S_n\). Experiments performed close to the valley of stability often have reliable data for these three quantities. Whereas Coulomb dissociation measurements in inverse kinematics have been conducted [20, 30] to gain a normalization of the absolute \(\gamma\)-ray strength in unstable nuclei, there is currently no technique to experimentally determine \(\rho(S_n)\), even a few nuclei away from stability. Consequently, the absolute value of NLDs in unstable nuclei rely on theory, alone.

In this work, we achieve absolute normalization of the NLD by developing a new approach for extracting the shape of the \(\gamma\) strength function (\(\gamma\)SF), a measure of the average reduced \(\gamma\)-ray decay probability, the so-called “Shape method”. This method is based on the “Ratio method”, which was proposed by Wiedeking et al. in 2012 [31], and successfully applied to \(^{95}\)Ge. (Right) Projections along the diagonal, \(E_x - E_\gamma\), for three different energy bins (all in keV) \(3900 < E_x < 4400\) (a), \(4700 < E_x < 5100\) (b), \(5500 < E_x < 5900\) (c). The peaks at \(563\) keV and \(1108\) keV correspond to decays into the states \(2^+_1\) and \(2^+_2\), respectively. The purple shaded areas were used to define a linear background under each peak.

The Shape method relies on the observation of statistical \(\gamma\) ray decays from the quasi-continuum into discrete, low-lying levels, \(L_{ij}\), with energies \(E_{L_{ij}}\). We assume that the primary \(\gamma\) decays into the states, \(L_j\), will be dominated by dipole transitions [35]. In this work we restrict the analysis to two discrete states, \(L_{ij} = 2^+_1, 2^+_2\), in the even-even daughter nucleus, i.e. two states with identical spin and parity. These transitions appear in our data as diagonals in a two-dimensional (2D) matrix with excitation energy on the \(y\)-axis and \(\gamma\)-ray energy on the \(x\)-axis, e.g. Fig. 1. We use projections of the 2D matrix along the diagonals \((E_x - E_\gamma)\), as shown in the same figure, for different excitation energies and extract the \(\gamma\)-ray intensities \(N_{L_{ij}}\) into the states of interest \(L_j\).

The ratio of the intensities \(N_{L_{ij}}\) along the diagonals, corrected for the detector response, is related to the ratio

\[
\frac{N_{L_{ij}}}{N_{L_{ij}^{obs}}},
\]

\[
N_{L_{ij}}^{obs} = \frac{\text{counts}_{L_{ij}^{obs}}}{A_{L_{ij}^{obs}}},
\]

where \(A_{L_{ij}^{obs}}\) and \(\text{counts}_{L_{ij}^{obs}}\) are the area under the peak and number of counts associated with that peak. We assume the assumption is that the \(\gamma\)SF is independent of the excitation energy in any given nucleus for the excitation energies of interest (Brink hypothesis [31]).

![FIG. 1: (Color online) (left) Raw matrix of excitation energy vs \(\gamma\)-ray energy in SuN after population of \(^{76}\)Ge. (right) Projections along the diagonal, \(E_x - E_\gamma\), for three different energy bins (all in keV) \(3900 < E_x < 4400\) (a), \(4700 < E_x < 5100\) (b), \(5500 < E_x < 5900\) (c). The peaks at \(563\) keV and \(1108\) keV correspond to decays into the states \(2^+_1\) and \(2^+_2\), respectively. The purple shaded areas were used to define a linear background under each peak.](image)
The γSF can be obtained via a “sewing” approach, where the pairs of data points from each excitation energy bin are normalized to each other via linear interpolation. One specific excitation energy, $E_{x,i}$, is used as a starting point of the interpolation, yielding a pair of γ SF values at energies $E_{x,i} - E_{L_1}$ and $E_{x,i} - E_{L_2}$. Values of the γ SF for the next higher (lower) bin are pairwise normalized such that $f(E_{x,i+1} - E_{L_2})$, and $f(E_{x,i-1} - E_{L_1})$ follow the linear trend given by the pair belonging to the bin $E_{x,i}$. The procedure is repeated for all energy bins $E_{x,i}$. Combined together, the $n$ pairs of values, $f(E_{γ})$, reflect the shape of the γ SF of the final nucleus. Details of the sewing approach are discussed in [36].

In the present work the Shape Method and the β-Oslo technique are combined for the first time to extract a model-independent NLD. In parallel to this work, the Shape method has also been applied to reaction-based experiments [90]. To verify our approach, we have first applied the Shape method in β decay to previously published data for the stable nucleus $^{76}$Ge, fed from the decay of $^{76}$Ga [23, 44]. The Shape method requires knowledge of the initial excitation energy of the daughter nucleus after β decay, as well as the primary γ-ray decay into the states $L_j$ on an event-by-event basis. All data of this work make use of the Summing NaI (SuN) detector, which is a segmented total absorption γ-ray spectrometer [23] [41]. The $^{76}$Ga secondary beams were implanted into a Si surface barrier detector at the center of SuN, providing a β-decay coincidence for the measurement of γ-rays in SuN. This is a re-analysis of an already published experiment and details can be found in the original publications [29, 44].

Fig. 1(left) shows the raw 2D matrix, created from the SuN detector data by using the total deposited energy to get $E_x$ on the y-axis and the individual segments to get $E_γ$ on the x-axis (Fig. 1(left)). The diagonals from the feeding of the first two excited states from higher lying states in $^{76}$Ge, $2^+_1$ and $2^+_2$, are clearly seen in fig. 1, shifted by 563 and 1108 keV, respectively, from the 0$^+_1$ ground state. A dedicated software “ShapeIt” [40] was developed for this analysis. Projections of the 2D matrix along the diagonal were created for N bins with constant widths $\Delta E_{x,i}$ (Fig. 1(right)). Peaks belonging to decays into the $1^+_2 \rightarrow 1^+_2$ state, were fit as doublets with the close-lying first-escape peak at $E_{x} - E_{γ} = 511$ keV. For each excitation energy $E_{x,i}$, the median energy was determined and ratios of values $f(E_{γ})$ were calculated using Eq. [3]. Applying the above described linear interpolation, the shape of the γ SF was determined. The analysis was repeated for constant energy widths 400 keV $\leq \Delta E_{x,i} \leq 800$ keV in steps of 50 keV. At each iteration, a sliding window approach was used to vary the energy of the first and all subsequent bins over the entire widths, $\Delta E_{x,i}$, in steps of 50 keV. All resulting sets of values $f(E_{γ})$ were normalized to each other using χ² minimization. The resulting sets of $f(E_{γ})$ are largely independent of the choice of energy ranges, $E_{x,i}$. Details of this analysis will be discussed in [40].

Figure 2 shows the resulting γ SF of $^{74}$Ge (black dots) compared to the results of the β-Oslo method [23], as well as the Oslo results for $^{74}$Ge [37]. It should be noted that the original publication for $^{76}$Ge using the β-Oslo method used systematics and theoretical calculations to fix the slope of the NLD and consequently the shape of the γ SF. The Shape method provides here a purely experimental approach to extracting the shape of the γ SF, in excellent agreement with the previous result. The good agreement serves as a robust test of the Shape method. The statistical uncertainties of our result are significantly reduced compared to the β-Oslo results and are comparable to uncertainties reported for the stable-beam experiment on $^{74}$Ge. Note that the Shape method can only determine the shape of the γ SF and the absolute magnitude still requires external information. Here the absolute γ SF was adjusted to the β-Oslo result to guide the eye.

In the following we present first results of the Shape method on an unstable nucleus, $^{88}$Kr. The new experiment was performed at the CARIBU [47] facility at Argonne National Laboratory. A $^{88}$Br beam was implanted...
FIG. 3: (left) Experimental matrix of the $^{88}$Br $\beta$ decay into $^{88}$Kr. (middle) $\gamma$SF for various krypton isotopes. The black points represent the results of the present work. The red triangles are from Ref. [23], the blue inverse triangles from Ref. [35] and the green crosses from [39]. The data points correspond to the NLD extracted from the present work. The solid black line represents the known discrete levels, while the additional lines correspond to five NLD calculations used in the statistical model code TALYS: Constant temperature matched to the Fermi gas model (NLD1) [40], Back-shifted Fermi gas model (NLD2) [40, 41], Generalized super fluid model (NLD3) [42, 43], Hartree Fock using Skyrme force (NLD4) [41] and the Hartree-Fock-Bogoliubov Skyrme force + combinatorial method (NLD5) [41].

...into the SuNTAN tape transport system [48] at the center of the SuN detector decaying into the nucleus $^{88}$Kr. Isobar contaminants and daughter activity were removed from the data by using appropriate tape cycles due to their different half-lives compared to $^{88}$Br. Surrounding the implantation point, a 3 mm-thick plastic scintillator barrel was used to detect the emitted $\beta$ particles. The signals from the plastic barrel were collected by 32 wavelength-shifting optical fibers and read by two photomultiplier tubes outside of SuN.

The 2D matrix of the populated nucleus, $^{88}$Kr, is shown in Fig. 3 (left). The $\beta$-decay Q-value of the $^{88}$Br decay is 8.97 MeV, however the data in the matrix is exhausted at roughly $E_x = 7$ MeV which corresponds to the neutron separation energy of $^{88}$Kr ($S_n = 7.05$ MeV). The same analysis procedure that was outlined for $^{76}$Ge was applied here, once again using the diagonals corresponding to $L_j = (2^+_1, 2^-_2)$ at energies 775 and 1577 keV, respectively. The resulting $\gamma$SF is shown in Fig. 3 (middle) and is compared to other measurements of neutron-rich krypton isotopes [24, 35, 39]. Within the limitations of the large uncertainties in the previous measurements, the general shape of the $\gamma$SF is in excellent agreement. The observed fluctuations below about 3.5 MeV $\gamma$-ray energy are likely caused by the non-statistical behaviour of $^{88}$Kr in this low excitation energy regime. Once again, we can only compare the shape of the $\gamma$SF as the absolute scale is not constrained by our method.

As described in the case of $^{76}$Ge, the $\gamma$SF slope $\alpha$ is used to constrain the slope of the NLD. The results are shown in Fig. 3 (right). Here we do not apply any spin corrections and instead show the model-independent experimental result for the partial NLD populated in the experiment. Assuming that the ground state of $^{88}$Br is $1^-$ [40] and including allowed $\beta$ transitions and dipole $\gamma$ transitions, we expect to populate spins $0 - 3$ of both parities. In Fig. 3 the NLD experimental results are compared to five NLD models that are commonly used in nuclear reactions applications and that are available in the open source code TALYS [50, 51]. The same spin range is used in the models as in the experiment. The agreement between experiment and theory is remarkable, showing the validity of the newly-developed method. This analysis can also be used to exclude NLD models that might not reproduce the data well enough, for example NLD5 in the case of Fig. 3.

$^{88}$Kr is only two neutrons away from the last stable isotope of krypton, and therefore the available NLD models are expected to be well constrained by previous experiments. However, moving into more neutron-rich isotopes, which are more critical for some of the applications mentioned earlier, the NLD models diverge significantly. This can be seen in Fig. 4 which shows the ratios of the predicted NLD for the five NLD models versus model NLD4, taken at 4.5 MeV for neutron-rich isotopes of krypton in the mass range 85-97. In addition, strong differences are observed in the model predictions with respect to odd-even effects. With the technique introduced in this work, we will be able to distinguish between models when applied to more neutron-rich isotopes.

In summary, we introduced in this work and in [36] the Shape method, which is a new technique that can provide the shape of the $\gamma$SF in a model-independent way. In combination with the $\beta$-Oslo method, we were able to extract a model-independent NLD for the partial spin range populated in $^{76}$Ge and unstable $^{88}$Kr. Thanks to the sensitivity of the $\beta$-Oslo technique in combination with the Shape method our error bars are comparable to...
those achieved in stable beam experiments. This opens up a new avenue to study the partial NLDs of thousands of unstable nuclei far away from stability, with major impacts on our understanding of nuclear structure, nuclear astrophysics and nuclear applications. Specifically, our results will allow for significantly reduced model-dependence and uncertainties in constraining neutron-capture rates of neutron-rich nuclei via the $\beta$-Oslo and inverse-Oslo technique. This will allow the acquisition of highly-demanded information on neutron-capture rates in the r process \[17\]. We also plan to investigate the possible application of the Shape method to atomic physics of heavy elements where level densities were found to behave statistically and similarly to NLDs towards the ionization energy \[52\], with impact on e.g. the development of the nuclear $^{229}$Th clock \[53\].

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