Percolation of Color Sources and the Shear Viscosity of the QGP in Central A-A Collisions at RHIC and LHC Energies

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(Dated: June 22, 2011)

The Color String Percolation Model (CSPM) is used to determine the shear viscosity to entropy density ratio \( \eta/s \) of the Quark-Gluon Plasma (QGP) produced in Au-Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV at RHIC and Pb-Pb at \( \sqrt{s_{NN}} = 2.76 \) TeV at LHC. The relativistic kinetic theory relation for \( \eta/s \) is evaluated using CSPM values of the temperature and the mean free path of the QGP constituents. The experimental charged hadron transverse momentum spectrum is used to determine the percolation density parameter \( \xi \) in Au-Au collisions (STAR). For Pb-Pb at \( \sqrt{s_{NN}} = 2.76 \) TeV \( \xi \) values are obtained from the extrapolation at RHIC energy. The value of \( \eta/s \) is 0.204±0.020 and 0.262±0.026 at the CSPM initial temperatures of 193.6±3 MeV (RHIC) and 262.2±13 MeV (LHC) respectively. These values are 2.5 and 3.3 times the AdS/CFT conjectured lower bound 1/4\( \pi \).

We compare the CSPM \( \eta/s \) analytic expression with weak coupling (wQGP) and strong coupling (sQGP) calculations. This indicates that the QGP is a strongly coupled fluid in the phase transition region.

PACS numbers: 12.38.Mh; 25.75.Nq

The observation of the large elliptic flow at RHIC in non-central heavy ion collisions suggest that the matter created is a nearly perfect fluid with very low shear viscosity [1–4]. Recently, attention has been focused on the shear viscosity to entropy density ratio \( \eta/s \) as a measure of the fluidity [5–8]. The observed \( \eta/s \) based on viscous hydrodynamics analyses at RHIC, is suggestive of a strongly coupled plasma [3, 10]. The effect of bulk viscosity is expected to be negligible. It has been conjectured, based on infinitely coupled supersymmetric Yang-Mills (SYM) gauge theory using the correspondence between Anti de-Sitter (AdS) space and conformal field theory (CFT), that the lower bound for \( \eta/s \) is 1/4\( \pi \) and is the universal minimal viscosity to entropy ratio even for QCD [11]. However, there are theories in which this lower bound can be violated [12].

In this letter, we explore the color string percolation model (CSPM) [13, 14] to obtain \( \eta/s \) as a function of the temperature above and below the hadron to QGP transition. The experimental values are for Au-Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV at the RHIC and for Pb-Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV at the LHC.

The CSPM describes the initial collision of two heavy ions in terms of a dense partons of interacting colored longitudinal strings formed in the collisions and \( F(\xi) \) is the color suppression factor [13, 14]

\[
F(\xi) = \sqrt{\frac{1 - e^{-\xi}}{\xi}} \tag{3}
\]

We note that CSPM is a saturation model where \( \langle p_T^2 \rangle_1/F(\xi) \) is similar to the saturation momentum scale \( Q_s^2 \) in the Color Glass Condensate (CGC) model [13].

In two dimensional percolation theory the relevant quantity is the dimensionless percolation density parameter given by [13, 14]

\[
\xi = \frac{N_s S_1}{S_N} \tag{4}
\]

where \( S_1 \) is the transverse area of the single string and \( S_N \) is the transverse nuclear overlap area. The critical cluster which spans \( S_N \), appears for \( \xi_c \geq 1.2 \) [16]. To evaluate the initial value of \( \xi \) from data for Au-Au collisions, a parameterization of \( pp \) events at 200 GeV is used to compute the \( p_t \) distribution [17–19].

\[
dN_c/dp_t^2 = a/(p_0 + p_t)^\alpha \tag{5}
\]

where \( a, p_0, \) and \( \alpha \) are parameters used to fit the data. This parameterization also can be used for nucleus-nucleus collisions to take into account the interactions of the strings [14].

\[
p_0 \rightarrow p_0 \left( \frac{\langle nS_1/S_N \rangle_{Au-Au}}{\langle nS_1/S_N/pp \rangle} \right)^{1/4} \tag{6}
\]
where $S_n$ corresponds to the area occupied by the $n$ overlapping strings. The thermodynamic limit, i.e., letting $n$ and $S_n \to \infty$ while keeping $\xi$ fixed, is used to evaluate

$$dN_c/dp_t^2 = \frac{a}{(p_0 \sqrt{F(\xi) + p_t^2})} \left( \frac{\xi}{(p_0 \sqrt{F(\xi) + p_t^2})} \right)^n$$

(7)

In pp collisions $\langle nS_1/S_{1,pp} \rangle \sim 1$ at not very high energies due to the low overlap probability.

The connection between $\xi$ and the temperature $T(\xi)$ involves the Schwinger mechanism (SM) for particle production. The Schwinger distribution for massless particles is expressed in terms of $p_t^2$

$$dn/dp_t^2 \sim e^{-p_t^2/x^2}$$

(8)

with the average value of string tension, $\langle x^2 \rangle$. Gaussian fluctuations in the string tension around its mean value transforms SM into the thermal distribution

$$dn/dp_t^2 \sim e^{-(p_t^2)/2\xi^2}$$

(9)

with $\langle x^2 \rangle = \pi\langle p_t^2 \rangle_1/F(\xi)$.

The temperature is expressed as

$$T(\xi) = \sqrt{\langle p_t^2 \rangle_1/2F(\xi)}$$

(10)

The string percolation density parameter $\xi$ which characterizes the percolation clusters also determines the temperature of the system. In this way at $\xi_c = 1.2$ the percolation phase transition at $T(\xi_c)$ models the thermal deconfinement transition. We will adopt the point of view that the experimentally determined chemical freeze-out temperature is a good measure of the phase transition temperature, $T_c$ \[22\]. $\langle p_t^2 \rangle_1$ is calculated at $\xi_c = 1.2$ using the universal chemical freeze-out temperature of 167.7 ± 2.6 MeV \[23\]. This gives $\sqrt{\langle p_t^2 \rangle_1} = 207.2 \pm 3.3$ MeV which is close to $\sim 200$ MeV used in a previous calculation of the percolation transition temperature \[21\].

The relativistic kinetic theory relation for the shear viscosity over entropy density ratio, $\eta/s$ is given by

$$\frac{\eta}{s} \sim \frac{T \lambda_{mfp}}{5}, \lambda_{mfp} \sim \frac{1}{(n\sigma_{tr})}$$

(11)

where $T$ is the temperature and $\lambda_{mfp}$ is the mean free path. $n$ is the number density of an ideal gas of quarks and gluons and $\sigma_{tr}$ the transport cross section.

In CSPM the number density is given by the effective number of sources per unit volume

$$n = \frac{N_{sources}}{S_N L}$$

(12)

$L$ is the the longitudinal extension of the string $L \sim 1$ fm. The area occupied by the strings is related to the percolation density parameter $\xi$ through the relation (1–

$$e^{-\xi})S_N.$$. Thus the effective no. of sources is given by the total area occupied by the strings divided by the area of an effective string $S_1F(\xi)$.

$$N_{sources} = \frac{(1 - e^{-\xi})S_N}{S_1F(\xi)}$$

(13)

In general $N_{sources}$ is smaller than the number of single strings. $N_{sources}$ equals the number of strings $N_s$ in the limit of $\xi = 0$. The number density of sources from Eqs. (12) and (13) becomes

$$n = \frac{(1 - e^{-\xi})}{S_1F(\xi)L}$$

(14)

In CSPM the transport cross section $\sigma_{tr}$ is the transverse area of the effective string $S_1F(\xi)$. Thus $\sigma_{tr}$ is directly proportional to $1/L$, which is in agreement with the estimated dependence of $\sigma_{tr}$ on the temperature \[24, 25\]. Equations (11)-(15) give

$$\frac{\eta}{s} = \frac{1}{5\sqrt{2}(1 - e^{-\xi})^{5/4}}\xi^{1/4}L$$

(15)

FIG. 1: Color suppression factor $F(\xi)$ as a function of charged particle multiplicity per unit transverse area $dN/d\eta/S_N$ for Au+Au collisions at 200 GeV (STAR data) \[18\]. The solid red circles are for Au-Au collisions at 200 GeV (STAR data) \[18\]. The error is smaller than the size of the symbol. The line is fit to the STAR data. The solid blue squares are for Pb-Pb at 2.76 TeV.

In the above expression for $\eta/s$ the key quantity $\xi$ is obtained from the determination of $F(\xi)$ given in Eq. (7).

Figure 1 shows a plot of $F(\xi)$ as a function of charged particle multiplicity per unit transverse area $dN/d\eta/S_N$ for Au+Au collisions at 200 GeV for various centralities.
FIG. 2: Energy density $\varepsilon$ as a function of percolation density parameter $\xi$. The line is fit to Au-Au data. The LHC point is obtained from $\varepsilon/T^4$ value [17].

TABLE I: The percolation density parameter $\xi$, temperature $T$, the energy density $\varepsilon$ and the ratio of shear viscosity to entropy $\eta/s$ for percolation threshold ( $\xi_c$). Au-Au at 200 GeV and Pb-Pb at 2.76 TeV (estimated) [17, 18]. Au-Au is for 0-10% and Pb-Pb is for 0-5% central events.

| System  | $\xi$  | $T$ (MeV) | $\varepsilon$(GeV/fm$^3$) | $\eta/s$ |
|---------|--------|-----------|--------------------------|---------|
| Critical| 1.2    | 167.0 ±2.6| 0.94±0.07                | 0.210±0.016 |
| Au-Au   | 2.88±0.09| 193.6 ±3.0| 2.27±0.16                | 0.204±0.020 |
| Pb-Pb   | 10.56±1.05| 262.2±13.0| 8.32±0.83                | 0.260±0.026 |

The error on $F(\xi)$ is $\sim$ 3%. $F(\xi)$ decreases in going from peripheral to central collisions. The $\xi$ value is obtained using Eq. (3), which increases with the increase in centrality. The fit to the Au-Au points has the functional form

$$F(\xi) = \exp[-0.165 - 0.094 \frac{dN}{d\eta_c}/S_N]$$  (16)

Recently, ALICE experiment at LHC has published the charged-particle multiplicity density data as a function of centrality in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28]. The STAR results for Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV have been used to estimate $F(\xi)$ values for Pb-Pb collisions at different centralities using the fit function Eq.(16) for Au+Au. The ALICE data points are shown in Fig.1. For central 0-5% in Pb-Pb collisions $\xi = 10.56$ as compared to $\xi = 2.88$ for Au-Au collisions at 200 GeV. The temperature was obtained using Eq. (10). For Pb-Pb collisions the temperature is $\sim 262.2$ MeV for 0-5% centrality, which is $\sim 35$% higher than the temperature from Au-Au collisions [17]. In our previous work on the Equation of State (EOS) it was demonstrated that $\xi$ is proportional to energy density $\varepsilon$ at RHIC energies [17]. Figure 2 shows a plot of $\varepsilon$ vs $\xi$ for Au-Au collisions at $\sqrt{s_{NN}} = 19.6$, 62 and 200 GeV for 0-10% centrality [17, 19, 27]. The energy density for the LHC energy was calculated using the value of $\varepsilon/T^4= 12.5$ at $T = 262.2$ MeV [17]. This value is shown in Fig. 2. The line is fit to the STAR data. This shows that the linear relationship between $\xi$ and $\varepsilon$ is valid even at 2.76 TeV when extrapolated from RHIC energy. Table I gives the CSPM values for $\xi$, $T$, $\varepsilon$ and $\eta/s$ at $T/T_c = 1$, 1.16 and 1.57.

The CSPM $\eta/s$ values are obtained using Eq.(15). This ratio grows slowly as $\xi^{1/4}$ and remains small between RHIC and LHC energies. Figure 3 shows a plot of $\eta/s$ as a function of $T/T_c$ calculated for different values of $\xi$. The experimental value for Au-Au at 200 GeV for 0-10% centrality is marked with arrow at $T/T_c = 1.15$. The estimated value of $\eta/s$ for Pb-Pb is also shown in Fig. 3 at $T/T_c = 1.57$. The lower bound is shown in Fig.3 given by AdS/CFT [11]. The results from STAR and ALICE show that the $\eta/s$ value is 2.5 and 3.3 times the KSS bound [11]. The small $\eta/s$ in CSPM is due to the small transverse correlation length, which decreases as the energy increases compensating the increase of the effective number of sources in such a way that the $\lambda_{mfp}$ given in Eq. (11) remains almost constant [29].

The viscosity to entropy ratio has been calculated for both the weakly (wQGP) and strongly (sQGP) coupled QCD plasma [9]. In this letter the CSPM results are compared with these calculations of wQGP and sQGP [9].

$$\frac{(\eta/s)_{wQGP}}{\alpha_s(T)^2 \ln(\frac{1}{\alpha_s(T)})} = 0.071$$  (17)

where

$$\alpha_s(T) \approx \frac{4\pi}{[18\ln(4T/T_c)]}$$  (18)

In the sQGP phase the ratio is

$$\frac{(\eta/s)_{sQGP}}{\frac{1}{4\pi}[1 + w\ln(T/T_c)]^2} = 1$$  (19)

Both sQGP and wQGP $\eta/s$ values are shown in Fig. 3. It is seen that at the RHIC top energy $\eta/s$ is close to the sQGP. Even at LHC energy it follows the trend of the sQGP. Extrapolating the $\eta/s$ CSPM values it is clear that it will approach the weak coupling limit near $T/T_c \sim 5.8$ as the temperature is increased. Figure 3 shows that both Au-Au and Pb-Pb systems are close to the strongly interacting QGP calculations [9].

The CSPM model calculations have successfully described the dependence of the nuclear modification factor on the azimuthal angle and obtain a similar pt dependence of $v_2$ at RHIC and LHC energies [28, 29]. In addition CSPM coupled to a Bjorken hydrodynamic expansion determined the equation of state of the QGP...
and the bulk thermodynamic value of $\varepsilon/T^4$ and $s/T^3$ which are in good agreement with Lattice QCD simulations \cite{17,30}.

In summary the relativistic kinetic theory relation for shear viscosity to entropy density ratio $\eta/s = 1/5 T \lambda_{mfp}$ was evaluated using the Color String Percolation Model. The mean free path $\lambda_{mfp}$ is proportional to the number density of the colored sources in CSPM. The color suppression factor $F(\xi)$ was extracted from the transverse momentum spectrum of charged hadrons. We found $\eta/s = 0.204 \pm 0.020$ at $T/T_c = 1.15$ (RHIC) and $\eta/s = 0.260 \pm 0.020$ at $T/T_c = 1.57$ (LHC). The CSPM $\eta/s$ was compared to weak coupling (wQGP) and strong coupling (sQGP) calculations. The CSPM values are close to the sQGP estimates at RHIC and LHC energies but could approach the wQGP estimate near $T/T_c \sim 5.8$. In the phase transition region $\eta/s$ is 2-3 times the conjectured quantum limit for RHIC to LHC energies.

This research was supported by the Office of Nuclear Physics within the U.S. Department of Energy Office of Science under Grant No. DE-FG02-88ER40412. J.D.D. thanks the support of the FCT/Portugal project PPCDT/FIS/57568/2004. C. P was supported by the project FPA2008-01177 of MICINN the Spanish Consolider Ingenio 2010 program CPAN and Conselleria Educacion Xunta de Galicia.

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