Double–lepton polarization asymmetries in $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay in universal extra dimension model

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Abstract

Double–lepton polarization asymmetries in $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay are calculated in universal extra dimension (UED) model. It is obtained that numerous double–lepton polarization asymmetries are very sensitive to the UED model and therefore can be very useful tool for establishing new physics predicted by the UED model.

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1 Introduction

Despite the impressive success of the standard model (SM) in describing all existing experimental data, it is commonly believed that SM is the low energy limit of a more fundamental theory. There are two different ways in looking the evidence for new physics beyond the SM:

- direct production of new particles at high energy colliders like LHC;
- signals of new interactions and particles can be obtained indirectly through the analysis of rare decays.

Rare B meson decays induced by the \( b \to s(d) \) transitions play a special role, since they are forbidden at at tree level in the SM and appear only at quantum (one–loop) level. Moreover, these decays are the most promising ones for establishing new physics. New physics in these decays can appear either through the differences in the Wilson coefficients from the ones existing in the SM or through the new operator structures in the effective Hamiltonian which are absent in the SM.

Among all decay channels of B mesons, semileptonic ones receive a special interest. These decays are theoretically, more or less, clean and they have relatively larger branching ratio. These decays contain many physically measurable quantities, like forward–backward asymmetry \( A_{FB} \), lepton polarization asymmetries, etc., which are very useful and serve as a testing ground for the SM and looking for new physics beyond the SM [1]. From experimental side, BELLE [2, 3] and BaBar [4, 5] collaborations provide recent measurements of the branching ratios of the semileptonic decays due to the \( b \to s\ell\ell^- \) transitions, which can be summarized as:

\[
B(B \to K^*\ell^+\ell^-) = \begin{cases} 
(16.5^{+2.3}_{-2.2} \pm 0.9 \pm 0.4) \times 10^{-7} & [2], \\
(7.8^{+1.9}_{-1.7} \pm 1.2) \times 10^{-7} & [4], 
\end{cases}
\]

\[
B(B \to K\ell^+\ell^-) = \begin{cases} 
(5.5^{+0.75}_{-0.70} \pm 0.27 \pm 0.02) \times 10^{-7} & [2], \\
(3.4 \pm 0.7 \pm 0.3) \times 10^{-7} & [4]. 
\end{cases}
\]

\[
B(B \to X_s\ell^+\ell^-) = \begin{cases} 
(4.11 \pm 0.83^{+0.85}_{-0.81}) \times 10^{-6} & [3], \\
(5.6 \pm 1.5 \pm 0.6 \pm 1.1) \times 10^{-6} & [5]. 
\end{cases}
\]

Another exclusive decay which is described at inclusive level by the \( b \to s\ell^+\ell^- \) transition is the baryonic \( \Lambda_b \to \Lambda\ell^+\ell^- \) decay. Unlike mesonic decays, the baryonic decays could maintain the helicity structure of the effective Hamiltonian for the \( b \to s \) transition [6]. Radiative and semileptonic decays of \( \Lambda_b \) such as \( \Lambda_b \to \Lambda\gamma, \Lambda_b \to \Lambda\ell\bar{\nu}_\ell, \Lambda_b \to \Lambda\ell^+\ell^- \) \( (\ell = e, \mu, \tau) \) and \( \Lambda_b \to \Lambda\nu\bar{\nu} \) have been extensively studied in the literature [7] (see also [1] and references therein). More about heavy baryons, including the experimental prospects, can be found in [3–5].
It is noted in [10] that some of the single lepton polarization asymmetries might be too small to be observed and therefore might not provide sufficient number of observables for checking the structure of the effective Hamiltonian. In order to obtain more observables, London et. al., proposed to take polarizations of both leptons into account [10] which are simultaneously measurable. Along these lines maximum number of independent polarization observables are constructed in [10].

Among the various models of physics beyond the SM, extra dimensions attract special interest, because they include gravity in addition other interactions, giving hints on the hierarchy problem and a connection with string theory. The model of Appelquist, Cheng and Dobrescu (ACD) [11] with one universal extra dimension (UED), where all the SM particles can propagate in the extra dimension, are very attractive (see also [12]). Compactification of the extra dimension leads to Kaluza–Klein model in the four–dimension. In this model the only additional free parameter with respect to the SM is $1/R$, i.e., inverse of the compactification radius.

The restrictions imposed on UED are examined in the current accelerators, for example, Tevatron experiments put the bound about $1/R \geq 300 \text{ GeV}$. Analysis of the anomalous magnetic moment [13], and $Z \rightarrow \bar{b}b$ vertex [14] also lead to the bound $1/R \geq 300 \text{ GeV}$.

Possible manifestation of UED models in the $K_L-K_S$ mass difference, parameter $\varepsilon_K$, $B-\bar{B}_0$ mixing, $\Delta M_{d,s}$ mass difference, and rare decays $K^+ \rightarrow \pi^0 \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K_L \rightarrow \mu^+ \mu^-$, $B \rightarrow X_s d \nu \bar{\nu}$, $B_s \rightarrow \mu^+ \mu^-$, $B \rightarrow X_s \gamma$, $B \rightarrow X_s \text{ gluon}$, $B \rightarrow X_s \mu^+ \mu^-$ and $\varepsilon'/\varepsilon$ are comprehensively investigated in [15] and [16]. Exclusive $B \rightarrow K^* \ell^+ \ell^-$, $B \rightarrow K^* \nu \bar{\nu}$ and $B \rightarrow K^* \gamma$ decays are studied in the framework of the UED scenario in [17], and $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ in UED in [18].

In the present work we study the double–lepton polarization asymmetries for the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay in the UED model. The plan of the paper is as follows. In section 2 we briefly discuss the main ingredients of ACD model and calculate all possible double–lepton polarization asymmetries for the rare $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay. Section 3 is devoted to the numerical analysis and conclusions.

2 $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay in ADC model

Let us remind the interested reader about the main ingredients of the simplest ACD model, which is the minimal extension of the SM in $4+1$ dimensions. The five–dimensional ACD model with a single UED uses orbifold compactification, namely, the fifth dimension $y$ that is compactified in a circle of radius $R$, with points $y = 0$ and $y = \pi R$ that are fixed points of the orbifolds. Generalization of the SM is realized by the propagating fermions, gauge bosons and the Higgs fields in all five dimensions. The Lagrangian in ACD can be written as

$$\mathcal{L} = \int d^4x dy \left\{ \mathcal{L}_A + \mathcal{L}_H + \mathcal{L}_F + \mathcal{L}_Y \right\},$$

where

$$\mathcal{L}_A = -\frac{1}{4} W^{MNa} W_{MN}^a - \frac{1}{4} B^{MN} B_{MN},$$
\[ \mathcal{L}_H = (\mathcal{D}^M \phi)^\dagger \mathcal{D}_M \phi - V(\phi), \]
\[ \mathcal{L}_F = \mathcal{Q}(i \Gamma^M \mathcal{D}_M) \mathcal{Q} + \bar{u}(i \Gamma^M \mathcal{D}_M) u + \bar{\mathcal{D}}(i \Gamma^M \mathcal{D}_M) \mathcal{D}, \]
\[ \mathcal{L}_Y = -\bar{Q} \gamma_\mu \phi \gamma^\mu u - \bar{Q} \gamma_\mu \phi \mathcal{D} \phi + \text{h.c.}. \]

Here \( M \) and \( N \) running over \( 0,1,2,3,5 \) are the five–dimensional Lorentz indices, \( W^a_{MN} = \partial_M W^a_N - \partial_N W^a_M + \tilde{g} \varepsilon^{abc} W^b_M W^c_N \) are the field strength tensor for the \( SU(2)_L \) electroweak gauge group, \( B_{MN} = \partial_M B_N - \partial_N B_M \) are that of the \( U(1) \) group, and all fields depend both on \( x \) and \( y \). The covariant derivative is defined as \( \mathcal{D}_M = \partial_M - i \tilde{g} W^a_M T^a - i \tilde{g}' B_M Y \), where \( \tilde{g} \) and \( \tilde{g}' \) are the five–dimensional gauge couplings for the \( SU(2)_L \) and \( U(1) \) groups. The five–dimensional \( \Gamma_M \) matrices are defined as \( \Gamma^\mu = \gamma^\mu \), \( \mu = 0,1,2,3 \) and \( \Gamma^5 = i \gamma^5 \).

In the case of a single extra dimension with coordinate \( x_5 = y \) compactified on a circle of radius \( R \), a field \( F(x,y) \) would be periodic function of \( y \), hence can be written as

\[ F(x,y) = \sum_{n=-\infty}^{+\infty} F_n(x) e^{iny/R}. \]

The Fourier expansion of the fields are

\[ B_\mu(x,y) = \frac{1}{\sqrt{2\pi R}} B^{(0)}_\mu + \frac{1}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} B^{(0)}_\mu(x) \cos \left( \frac{ny}{R} \right), \]
\[ B_5(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} B^{(n)}_5 \sin \left( \frac{ny}{R} \right), \]
\[ Q(x,y) = \frac{1}{\sqrt{2\pi R}} Q^{(0)}_L + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ Q^{(n)}_L \cos \left( \frac{ny}{R} \right) + Q^{(n)}_R \sin \left( \frac{ny}{R} \right) \right], \]
\[ U(D)(x,y) = \frac{1}{\sqrt{2\pi R}} U^{(0)}_R + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ U^{(n)}_R \cos \left( \frac{ny}{R} \right) + U^{(n)}_L \sin \left( \frac{ny}{R} \right) \right]. \]

Under parity transformation \( P_5 : y \rightarrow -y \) fields having a correspondent in the four–dimensional SM should be even, so that their zero–mode in the KK can be interpreted as the ordinary SM field, and all remaining new fields should be odd.

In ACD model the KK parity is conserved. This conservation implies that there is no tree level diagrams with exchange of KK modes in low energy processes (at the scale \( \mu \ll 1/R \)) and single KK excitation cannot be produced, i.e., they appear only in pairs. Lastly, in the ACD model there are three additional physical scalar modes \( a^{(0)}_n \) and \( a^{\pm}_n \). The zero–mode is either right–handed or left–handed.

Lagrangian of the ACD model can be obtained by integrating over \( x_5 = y \)

\[ \mathcal{L}_4(x) = \int_0^{2\pi R} \mathcal{L}_5(x,y) dy. \]

Note that the zero–mode remains massless unless we apply the Higgs mechanism. All fields in the four–dimensional Lagrangian receive the KK mass \( n/R \) on account of the derivative operator \( \partial_y \) acting on them. The relevant Feynman rules are derived in [13] and for more details about the ACD model we refer the interested reader to [16] and [17].
After this introduction, let us start discussing the main problem, namely, double–lepton polarization asymmetries for the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay.

At quark level, $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay is described by $b \to s \ell^+ \ell^-$ transition. Effective Hamiltonian governing this transition in the SM with $\Delta B = -1$, $\Delta S = 1$ is described in terms of a set of local operators

$$\mathcal{H} = \frac{4G_F}{\sqrt{2}} V_{tb}^* V_{ts} \sum_1^{10} C_i(\mu) O_i(\mu) ,$$

where $G_F$ is the Fermi constant, $V_{ij}$ are the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. Explicit forms of the operators, which are written in terms of quark and gluon fields can be found in [19].

The Wilson coefficients in (1) have been computed at NNLO in the SM in [19]. At NLO the coefficients are calculated for the ACD model including the effects of KK modes, in [15] and [16], which we have used in our calculations. It should be noted here that, there does not appear any new operator in the ACD model, and therefore, the effect of new particles leads to modification of the Wilson coefficients existing in the SM, if we neglect the contributions of the scalar fields, which are indeed very small.

At $\mu = \mathcal{O}(m_W)$ level, only $C_{7}^{(0)}$, $C_9^{(0)}$, $C_{10}^{(0)}$ are different from zero, and the remaining coefficients are all zero.

In the SM, at quark level, $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay is described with the help of the operators $C_7^{\text{eff}}$, $C_9$ and $C_{10}$ as follows:

$$\mathcal{M} = \frac{G_F}{4\sqrt{2}} V_{tb}^* V_{ts} \left\{ C_7^{\text{eff}} \bar{s}i\sigma_{\mu\nu}(1 + \gamma_5)q^\nu b \bar{\ell} \gamma^\mu \ell + C_9 \bar{s} \gamma_\mu(1 - \gamma_5)b \bar{\ell} \gamma^\mu \ell + C_{10} \bar{s} \gamma_\mu(1 - \gamma_5)b \bar{\ell} \gamma_5 \gamma^\mu \ell \right\} .$$

As has already been noted, $C_7^{\text{eff}}$, $C_9$ and $C_{10}$ are calculated in the SM in [19] (see also [20] and [21]).

Contributions coming from UED model to these Wilson coefficients are calculated in [15] and [16], which can be written as

$$C_7^{(0)}(\mu_W) = -\frac{1}{2} D'(x_t, 1/R) ,$$

$$C_9(\mu) = P_0^{NDR} + \frac{Y(x_t, 1/R)}{\sin^2 \theta_W} - 4Z(x_t, 1/R) + P_E E(x_t, 1/R) ,$$

$$C_{10} = -\frac{Y(x_t, 1/R)}{\sin^2 \theta_W} .$$

where $P_0^{NDR} = 2.60 \pm 0.25$ and the superscript $(0)$ referring to leading log approximation. Explicit expressions of the functions $D'(x_t, 1/R)$, $Y(x_t, 1/R)$ and $Z(x_t, 1/R)$ can be found in [15] [16] [17].

With these coefficients and the operators in (1) the inclusive $b \to s \ell^+ \ell^-$ transitions are studied in [15] [16].
The amplitude of the exclusive $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay is obtained by calculating the matrix element of the effective Hamiltonian for the $b \rightarrow s \ell^+ \ell^-$ transition between initial and final baryon states $\langle \Lambda | H_{eff} | \Lambda_b \rangle$. It follows from Eq. (2) that the matrix elements

$$
\langle \Lambda | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle , \\
\langle \Lambda | \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b | \Lambda_b \rangle ,
$$

(4)

are needed in order to calculate the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay amplitude.

These matrix elements parametrized in terms of the form factors are as follows (see [22][23])

$$
\langle \Lambda | \bar{s} \gamma_\mu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[ f_1 \gamma_\mu + i f_2 \sigma_{\mu\nu} q^\nu + f_3 q_\mu \right] u_{\Lambda_b} ,
$$

(5)

$$
\langle \Lambda | \bar{s} \gamma_\mu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[ g_1 \gamma_\mu \gamma_5 + ig_2 \sigma_{\mu\nu} \gamma_5 q^\nu + g_3 q_\mu \gamma_5 \right] u_{\Lambda_b} ,
$$

(6)

where $q = p_{\Lambda_b} - p_{\Lambda}$.

The form factors of the magnetic dipole operators are defined as

$$
\langle \Lambda | \bar{s} i \sigma_{\mu\nu} q^\nu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[ f_1^T \gamma_\mu + i f_2^T \sigma_{\mu\nu} q^\nu + f_3^T q_\mu \right] u_{\Lambda_b} ,
$$

(7)

$$
\langle \Lambda | \bar{s} i \sigma_{\mu\nu} \gamma_5 q^\nu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[ g_1^T \gamma_\mu \gamma_5 + ig_2^T \sigma_{\mu\nu} \gamma_5 q^\nu + g_3^T q_\mu \gamma_5 \right] u_{\Lambda_b} .
$$

Using the identity

$$
\sigma_{\mu\nu} \gamma_5 = -\frac{i}{2} \epsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta} ,
$$

the following relations between the form factors are obtained:

$$
f_1^T = -\frac{q^2}{m_{\Lambda_b} - m_{\Lambda}} f_3^T , \\
g_1^T = \frac{q^2}{m_{\Lambda_b} + m_{\Lambda}} g_3^T .
$$

(8)

Using these definitions of the form factors, for the matrix element of the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ we get

$$
\mathcal{M} = \frac{G_F}{4\sqrt{2}} V_{tb} V_{ts}^{\ast} \frac{1}{2} \left\{ \bar{\ell} \gamma_\mu (1 - \gamma_5) \ell \bar{u}_\Lambda \left[ (A_1 - D_1) \gamma_\mu (1 + \gamma_5) + (B_1 - E_1) \gamma_\mu (1 - \gamma_5) \\
+ i \sigma_{\mu\nu} q^\nu \left( (A_2 - D_2) (1 + \gamma_5) + (B_2 - E_2) (1 - \gamma_5) \right) \\
+ q_\mu \left( (A_3 - D_3) (1 + \gamma_5) + (B_3 - E_3) (1 - \gamma_5) \right) \right] u_{\Lambda_b} \\
+ \bar{\ell} \gamma_\mu (1 + \gamma_5) \ell \bar{u}_\Lambda \left[ (A_1 + D_1) \gamma_\mu (1 + \gamma_5) + (B_1 + E_1) \gamma_\mu (1 - \gamma_5) \\
+ i \sigma_{\mu\nu} q^\nu \left( (A_2 + D_2) (1 + \gamma_5) + (B_2 + E_2) (1 - \gamma_5) \right) \\
+ q_\mu \left( (A_3 + D_3) (1 + \gamma_5) + (B_3 + E_3) (1 - \gamma_5) \right) \right] u_{\Lambda_b} \right\} ,
$$

(9)
where

\[ A_1 = \frac{1}{q^2} \left( f_1^T - g_1^T \right) (-2m_sC_7) + \frac{1}{q^2} \left( f_1^T + g_1^T \right) (-2m_bC_7) + (f_1 - g_1) C_9^{eff}, \]

\[ A_2 = A_1 \left( 1 \rightarrow 2 \right), \]

\[ A_3 = A_1 \left( 1 \rightarrow 3 \right), \]

\[ B_1 = A_1 \left( g_1 \rightarrow -g_1; \ g_1^T \rightarrow -g_1^T \right), \]

\[ B_2 = B_1 \left( 1 \rightarrow 2 \right), \]

\[ B_3 = B_1 \left( 1 \rightarrow 3 \right), \]

\[ D_1 = C_{10} \left( f_1 - g_1 \right), \]

\[ D_2 = D_1 \left( 1 \rightarrow 2 \right), \]

\[ D_3 = D_1 \left( 1 \rightarrow 3 \right), \]

\[ E_1 = D_1 \left( g_1 \rightarrow -g_1 \right), \]

\[ E_2 = E_1 \left( 1 \rightarrow 2 \right), \]

\[ E_3 = E_1 \left( 1 \rightarrow 3 \right). \]

From these expressions it follows that \( \Lambda_b \rightarrow \Lambda \ell^+ \ell^- \) decay is described in terms of many form factors. It is shown in [6] (see also [23]) that Heavy Quark Effective Theory reduces the number of independent form factors to two (\( F_1 \) and \( F_2 \)) irrelevant of the Dirac structure of the corresponding operators, i.e.,

\[ \langle \Lambda(p_\Lambda) | s \Gamma b | \Lambda(p_{\Lambda_b}) \rangle = \bar{u}_\Lambda \left[ F_1(q^2) + F_2(q^2) \right] \Gamma u_{\Lambda_b}, \]

where \( \Gamma \) is an arbitrary Dirac structure and \( u^\mu = p_{\Lambda_b}^\mu / m_{\Lambda_b} \) is the four–velocity of \( \Lambda_b \).

Comparing the general form of the form factors given in Eqs. (5)–(7) with the ones given in (11), one can easily obtain the following relations among them [22, 23],

\[ g_1 = f_1 = f_2^T = g_2^T = F_1 + \sqrt{\hat{r}_\Lambda} F_2, \]

\[ g_2 = f_2 = g_3 = f_3 = \frac{F_2}{m_{\Lambda_b}}, \]

\[ g_1^T = f_1^T = \frac{F_2}{m_{\Lambda_b}} q^2, \]

\[ g_3^T = \frac{F_2}{m_{\Lambda_b}} (m_{\Lambda_b} + m_{\Lambda}), \]

\[ f_3^T = -\frac{F_2}{m_{\Lambda_b}} (m_{\Lambda_b} - m_{\Lambda}), \]

where \( \hat{r}_\Lambda = m_{\Lambda_b}^2 / m_{\Lambda_b}^2 \).

As we have already noted, our purpose is the calculation of double–lepton polarizations in UED model.

For calculation of the double lepton polarization asymmetries, the following orthogonal unit vectors \( s_i^{\pm \mu} \) in the rest frame of \( \ell^\pm \) (\( i = L, T \) or \( N \), stand for longitudinal, transversal or normal polarizations, respectively, are chosen as:
\[ s_L^{-\mu} = \left(0, e_L^{-}\right) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right), \]
\[ s_N^{-\mu} = \left(0, e_N^{-}\right) = \left(0, \frac{\vec{p}_\Lambda \times \vec{p}_-}{|\vec{p}_\Lambda \times \vec{p}_-|}\right), \]
\[ s_T^{-\mu} = \left(0, e_T^{-}\right) = \left(0, e_N^{-} \times e_L^{-}\right), \]
\[ s_L^{+\mu} = \left(0, e_L^{+}\right) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right), \]
\[ s_N^{+\mu} = \left(0, e_N^{+}\right) = \left(0, \frac{\vec{p}_\Lambda \times \vec{p}_+}{|\vec{p}_\Lambda \times \vec{p}_+|}\right), \]
\[ s_T^{+\mu} = \left(0, e_T^{+}\right) = \left(0, e_N^{+} \times e_L^{+}\right), \]

where \( \vec{p}_- \) and \( \vec{p}_\Lambda \) are the three–momenta of the leptons \( \ell^- \) and \( \Lambda \) baryon in the center of mass frame (CM) of \( \ell^- \ell^+ \) system, respectively. Transformation of unit vectors from the rest frame of the leptons to CM frame of leptons can be done by the Lorentz boost. Boosting of the longitudinal unit vectors \( s_L^{\pm\mu} \) yields

\[ \left(s_L^{\pm\mu}\right)_{CM} = \left(\frac{|\vec{p}_\pm|}{m_\ell}, \frac{E_\ell \vec{p}_\pm}{m_\ell |\vec{p}_\pm|}\right), \]

where \( \vec{p}_\pm = -\vec{p}_- \), \( E_\ell \) and \( m_\ell \) are the energy and mass of leptons in the CM frame, respectively. The remaining two unit vectors \( s_N^{\pm\mu} \), \( s_T^{\pm\mu} \) are unchanged under Lorentz boost.

The double–polarization asymmetries are defined in the following way [10]:

\[ P_{ij}(q^2) = \frac{\left(d\Gamma(s_i^-, s_j^+) - d\Gamma(-s_i^-, s_j^+)\right)}{dq^2} - \frac{\left(d\Gamma(s_i^-, -s_j^+)\right)}{dq^2} - \frac{\left(d\Gamma(-s_i^-, -s_j^+)\right)}{dq^2}, \]

where, the first subindex \( i \) represents lepton and the second one antilepton. Using this definition of \( P_{ij} \), nine double–lepton polarization asymmetries are calculated. Their expressions are

\[ P_{LL} = \frac{16m_A^4}{3\Delta}\text{Re}\left\{-6m_A\sqrt{\hat{r}_\Lambda(1 - \hat{r}_\Lambda + \hat{s})}\left[\hat{s}(1 + v^2)(A_1A_2^* + B_1B_2^*) - 4\hat{m}_\ell^2(D_1D_3^* + E_1E_3^*)\right]
\right.\]
\[ + 6m_A^2(1 - \hat{r}_\Lambda - \hat{s})\left[\hat{s}(1 + v^2)(A_1B_2^* + A_2B_1^*) + 4\hat{m}_\ell^2(D_1E_3^* + D_3E_1^*)\right]
\right.
\[ + 12\sqrt{\hat{r}_\Lambda}\hat{s}(1 + v^2)(A_1B_1^* + D_1E_1^* + m_A^2\hat{s}A_2B_2^*)
\right.
\[ + 12m^2_A\hat{m}_\ell^2\hat{s}(1 + \hat{r}_\Lambda - \hat{s})\left(|D_3|^2 + |E_3^*|^2\right)
\right.
\[ - (1 + v^2)(1 + \hat{r}_\Lambda - \hat{r}_\Lambda(2 - \hat{s}) + \hat{s}(1 - 2\hat{s})\right)\left(|A_1|^2 + |B_1|^2\right)
\right.
\[ - \left[(5v^2 - 3)(1 - \hat{r}_\Lambda)^2 + 4\hat{m}_\ell^2(1 + \hat{r}_\Lambda) + 2\hat{s}(1 + 8\hat{m}_\ell^2 + \hat{r}_\Lambda) - 4\hat{s}^2\right]\left(|D_1|^2 + |E_1|^2\right)\]
\begin{align}
- m_{\Delta_b}^2 (1 + v^2) \hat{s} \left[ 2 + 2 \hat{r}^2 + \hat{\lambda}(1 + \hat{s}) - \hat{r}_{\lambda}(4 + \hat{s}) \right] \left( |A_2|^2 + |B_2|^2 \right) \\
- 2 m_{\Delta_b}^2 v^2 \left[ 2(1 + \hat{r}^2) - \hat{\lambda}(1 + \hat{s}) - \hat{r}_{\lambda}(4 + \hat{s}) \right] \left( |D_2|^2 + |E_2|^2 \right) \\
+ 12 m_{\Delta_b} \hat{s}(1 - \hat{r}_{\lambda} - \hat{s}) v^2 \left( D_1 E_2^* + D_2 E_1^* \right) \\
- 12 m_{\Delta_b} \sqrt{\hat{r}_{\lambda}} \hat{s}(1 - \hat{r}_{\lambda} + \hat{s}) v^2 \left( D_1 D_2^* + E_1 E_2^* \right) \\
+ 24 m_{\Delta_b}^2 \sqrt{\hat{r}_{\lambda}} \hat{s} \left( \hat{s} v^2 D_2 E_2^* + 2 \hat{m}_l^2 D_3 E_3^* \right) \right), \quad (16)
\end{align}

\begin{align}
P_{LN} &= \frac{16 \pi m_{\Delta_b}^4 \hat{m}_l \sqrt{\hat{\lambda}}}{\Delta \sqrt{\hat{s}}} \text{Im} \left\{ (1 - \hat{r}_{\lambda})(A_1^* D_1 + B_1^* E_1) + m_{\Delta_b} \hat{s}(A_1^* E_3 - A_2^* E_1 + B_1^* D_3 - B_2^* D_1) \\
&+ m_{\Delta_b} \sqrt{\hat{r}_{\lambda}} \hat{s}(A_1^* D_3 + A_2^* D_1 + B_1^* E_3 + B_2^* E_1) - m_{\Delta_b}^2 \hat{s}^2 \left( B_2^* E_3 + A_2^* D_3 \right) \right\}, \quad (17)
\end{align}

\begin{align}
P_{NL} &= - \frac{16 \pi m_{\Delta_b}^4 \hat{m}_l \sqrt{\hat{\lambda}}}{\Delta \sqrt{\hat{s}}} \text{Im} \left\{ (1 - \hat{r}_{\lambda})(A_1^* D_1 + B_1^* E_1) + m_{\Delta_b} \hat{s}(A_1^* E_3 - A_2^* E_1 + B_1^* D_3 - B_2^* D_1) \\
&- m_{\Delta_b} \sqrt{\hat{r}_{\lambda}} \hat{s}(A_1^* D_3 + A_2^* D_1 + B_1^* E_3 + B_2^* E_1) - m_{\Delta_b}^2 \hat{s}^2 \left( B_2^* E_3 + A_2^* D_3 \right) \right\}, \quad (18)
\end{align}

\begin{align}
P_{LT} &= \frac{16 \pi m_{\Delta_b}^4 \hat{m}_l \sqrt{\hat{\lambda}}}{\Delta \sqrt{\hat{s}}} \text{Re} \left\{ (1 - \hat{r}_{\lambda}) \left( |D_1|^2 + |E_1|^2 \right) - \hat{s} \left( A_1^* D_1^* - B_1 E_1^* \right) \\
&- m_{\Delta_b} \hat{s} [B_1 D_2^* + (A_2 + D_2 - D_3) E_1^* - A_1 E_2^* - (B_2 - E_2 + E_3) D_1^*] \\
&+ m_{\Delta_b} \sqrt{\hat{r}_{\lambda}} \hat{s} [A_1 D_2^* + (A_2 + D_2 - D_3) D_1^* - B_1 E_2^* - (B_2 - E_2 - E_3) E_1^*] \\
&+ m_{\Delta_b}^2 \hat{s}(1 - \hat{r}_{\lambda})(A_2 D_2^* - B_2 E_2^*) - m_{\Delta_b}^2 \hat{s}^2(D_2 D_3^* + E_2 E_3^*) \right\}, \quad (19)
\end{align}

\begin{align}
P_{TL} &= \frac{16 \pi m_{\Delta_b}^4 \hat{m}_l \sqrt{\hat{\lambda}}}{\Delta \sqrt{\hat{s}}} \text{Re} \left\{ (1 - \hat{r}_{\lambda}) \left( |D_1|^2 + |E_1|^2 \right) + \hat{s} \left( A_1^* D_1^* - B_1 E_1^* \right) \\
&+ m_{\Delta_b} \hat{s} [B_1 D_2^* + (A_2 - D_2 + D_3) E_1^* - A_1 E_2^* - (B_2 + E_2 - E_3) D_1^*] \\
&- m_{\Delta_b} \sqrt{\hat{r}_{\lambda}} \hat{s} [A_1 D_2^* + (A_2 - D_2 - D_3) D_1^* - B_1 E_2^* - (B_2 + E_2 + E_3) E_1^*] \\
&- m_{\Delta_b}^2 \hat{s}(1 - \hat{r}_{\lambda})(A_2 D_2^* - B_2 E_2^*) - m_{\Delta_b}^2 \hat{s}^2(D_2 D_3^* + E_2 E_3^*) \right\}, \quad (20)
\end{align}

\begin{align}
P_{NT} &= \frac{64 m_{\Delta_b}^4 \hat{\lambda} v}{3 \Delta} \text{Im} \left\{ (A_1 D_1^* + B_1 E_1^*) + m_{\Delta_b}^2 \hat{s} (A_2^* D_2^* + B_2^* E_2) \right\}, \quad (21)
\end{align}
\[ P_{TN} = -\frac{64m_{\Lambda_b}^4 \lambda v}{3\Delta} \text{Im}\left\{ (A_1D_1^* + B_1E_1^*) + m_{\Lambda_b}^2 s (A_2^*D_2 + B_2^*E_2) \right\}, \quad (22) \]

\[ P_{NN} = \frac{32m_{\Lambda_b}^4}{3\tilde{s}\Delta} \text{Re}\left\{ 24\tilde{m}_{\tilde{t}}^2 \sqrt{\tilde{r}_A} \tilde{s} (A_1B_1^* + D_1E_1^*) \right\}
- 12m_{\Lambda_b} \tilde{m}_{\tilde{t}}^2 (1 - \tilde{r}_A + \tilde{s})(A_1A_2^* + B_1B_2^*)
+ 6m_{\Lambda_b} \tilde{m}_{\tilde{t}}^2 \tilde{s} [m_{\Lambda_b} \tilde{s} (1 + \tilde{r}_A - \tilde{s}) (|D_3|^2 + |E_3|^2) + 2\sqrt{\tilde{r}_A} (1 - \tilde{r}_A + \tilde{s})(D_1D_3^* + E_1E_3^*)]
- 12m_{\Lambda_b} \tilde{m}_{\tilde{t}}^2 \tilde{s} (1 - \tilde{r}_A - \tilde{s})(A_1B_2^* + A_2B_1^* + D_1E_3^* + D_3E_1^*)
- [\tilde{\lambda s} + 2\tilde{m}_{\tilde{t}}^2 (1 + \tilde{r}_A^2 - 2\tilde{r}_A + \tilde{r}_A \tilde{s} + \tilde{s} - 2\tilde{s}^2)] (|A_1|^2 + |B_1|^2 - |D_1|^2 - |E_1|^2)
+ 24m_{\Lambda_b} \tilde{m}_{\tilde{t}}^2 \sqrt{\tilde{r}_A} \tilde{s}^2 (A_2B_2^* + D_3E_3^*) - m_{\Lambda_b} \tilde{s}^2 \tilde{v}^2 (|D_2|^2 + |E_2|^2)
+ m_{\Lambda_b} \tilde{s} \{ \tilde{\lambda s} - 2\tilde{m}_{\tilde{t}}^2 [2(1 + \tilde{r}_A^2) - \tilde{s}(1 + \tilde{s}) - \tilde{r}_A (4 + \tilde{s})] \} (|A_2|^2 + |B_2|^2) \right\}, \quad (23) \]

\[ P_{TT} = \frac{32m_{\Lambda_b}^4}{3\tilde{s}\Delta} \text{Re}\left\{ -24\tilde{m}_{\tilde{t}}^2 \sqrt{\tilde{r}_A} \tilde{s} (A_1B_1^* + D_1E_1^*) \right\}
- 12m_{\Lambda_b} \tilde{m}_{\tilde{t}}^2 \sqrt{\tilde{r}_A} (1 - \tilde{r}_A + \tilde{s})(D_1D_3^* + E_1E_3^*) - 24m_{\Lambda_b} \tilde{m}_{\tilde{t}}^2 \sqrt{\tilde{r}_A} \tilde{s}^2 (A_2B_2^* + D_3E_3^*)
- 6m_{\Lambda_b} \tilde{m}_{\tilde{t}}^2 \tilde{s} [m_{\Lambda_b} \tilde{s} (1 + \tilde{r}_A - \tilde{s}) (|D_3|^2 + |E_3|^2) - 2\sqrt{\tilde{r}_A} (1 - \tilde{r}_A + \tilde{s})(A_1A_2^* + B_1B_2^*)]
- 12m_{\Lambda_b} \tilde{m}_{\tilde{t}}^2 \tilde{s} (1 - \tilde{r}_A - \tilde{s})(A_1B_2^* + A_2B_1^* + D_1E_3^* + D_3E_1^*)
- [\tilde{\lambda s} - 2\tilde{m}_{\tilde{t}}^2 (1 + \tilde{r}_A^2 - 2\tilde{r}_A + \tilde{r}_A \tilde{s} + \tilde{s} - 2\tilde{s}^2)] (|A_1|^2 + |B_1|^2)
+ m_{\Lambda_b} \tilde{s} \{ \tilde{\lambda s} + \tilde{m}_{\tilde{t}}^2 [4(1 - \tilde{r}_A)^2 - 2\tilde{s}(1 + \tilde{r}_A) - 2\tilde{s}^2] \} (|A_2|^2 + |B_2|^2)
+ \{ \tilde{\lambda s} - 2\tilde{m}_{\tilde{t}}^2 [5(1 - \tilde{r}_A)^2 - 7\tilde{s}(1 + \tilde{r}_A) + 2\tilde{s}^2] \} (|D_1|^2 + |E_1|^2)
- m_{\Lambda_b} \tilde{s}^2 \tilde{v}^2 (|D_2|^2 + |E_2|^2) \right\}. \quad (24) \]

Explicit expression of \( \Delta \) appearing in \( P_{ij} \) can be found in [24].

### 3 Numerical results

In this section we present our numerical results for the double–polarization asymmetries. The values of the input parameters we need in performing the numerical calculations are: \(|V_{tb}V_{ts}^*| = 0.0385, m_\tau = 1.77\text{ GeV}, m_\mu = 0.106\text{ GeV}, m_b = 4.8\text{ GeV} \text{[25]}, m_t = 172.7\text{ GeV} \text{[26]}, \text{ and } \tau_{B_s} = (1.527 \pm 0.008)\text{ ps}.\)

The \( \Lambda_b \to \Lambda \) transition form factors are the main input parameters in performing the numerical analysis, which are embedded into the expressions of the double–lepton polarization asymmetries. The analysis of all form factors responsible for the \( \Lambda_b \to \Lambda \) transition has not been accomplished so far. Therefore, for the form factors we will use the results coming
from QCD sum rules in corporation with HQET \cite{27,28}, which reduce the number of independent form factors to two, and their $q^2$ dependence are given in terms of three–parameter fit as follows:

$$F_i(\hat{s}) = \frac{F(0)}{1 - a_F \hat{s} + b_F \hat{s}^2}.$$  

The values of the parameters $F(0)$, $a_F$ and $b_F$ are given in table 1.

|   | $F(0)$  | $a_F$   | $b_F$   |
|---|---------|---------|---------|
| $F_1$ | 0.462   | -0.0182 | -0.000176 |
| $F_2$ | -0.077  | -0.0685 | 0.00146 |

Table 1: Form factors for $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay in a three parameter fit.

The analysis of the double–lepton polarization asymmetries leads to the following results:

- $P_{LL}$ in UED for the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay practically coincides with the SM result, for all values of $q^2$.

- For the $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ case the difference between the predictions of SM and UED is substantial at $q^2 = 12 \text{ GeV}^2$, i.e., $(P_{LL})_{UED} = 2(P_{LL})_{SM}$ at $1/R = 200 \text{ GeV}$; with increasing $q^2$ the difference between the two models decreases, but they never coincide (see Fig. 1).

- For the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay, starting from $q^2 = 1 \text{ GeV}^2$ up to the end of the spectrum, the value of $P_{TT}$ in the SM is larger compared to that of the one predicted by the UED model. Especially, up to $q^2 = 10 \text{ GeV}^2$, $(P_{TT})_{SM} = 2(P_{TT})_{UED}$ (see Fig. 2).

Therefore measurement of the values of $P_{LL}$ for the $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay and $P_{TT}$ for the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay can give quite important information about the presence of new physics beyond the SM.

- For the $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay, the difference between the predictions of the SM and UED is maximally about 60%, i.e., in terms of modulo, $|(P_{LT})_{UED}| > |(P_{LT})_{SM}|$, which can also be very useful for establishing new physics (see Fig. 3).

- The maximum value of the difference between the SM and UED models concerning $P_{TN}$, $P_{NT}$, $P_{LN}$, $P_{NL}$, $P_{TL}$ (excluding $q^2 = 1 \text{ GeV}^2$ region for the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ channel) for both decay channels, is about 10%. Note that at $q^2 = 1 \text{ GeV}^2$, $(P_{LT})_{UED} = (P_{TL})_{UED} \approx 2(P_{LT})_{SM} = 2(P_{TL})_{SM}$, for the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay.

- When $2 \text{ GeV}^2 \leq q^2 \leq 10 \text{ GeV}^2$, the prediction of the UED model on $P_{NN}$ is maximally two times larger than the SM prediction for the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay (see Fig. 4).
• As far as $P_{NN}$ for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay is concerned, the situation is more promising. When the momentum transfer square $q^2$ varies in the region $14 \text{ GeV}^2 \leq q^2 \leq 18 \text{ GeV}^2$, the difference between the results of the two models on $P_{NN}$ is quite large, about four times, i.e., $(P_{NN})_{UED} \simeq 4(P_{NN})_{SM}$, and the magnitude of $P_{NN}$ is larger more than 10% in the UED model compared to that in the SM, which can be measurable in the experiments.

From the above–presented discussion we conclude that measurement of various double–lepton polarization asymmetries can be very useful for establishing new physics predicted by the UED model. Here we should note that single–lepton polarization is not a suitable tool for discrimination of the UED model and SM (see [10]).

In conclusion, we study the double–lepton polarization asymmetries in the UED model. We find that various double–lepton polarization asymmetries are very sensitive to the UED model and the results are substantially different compared to the ones obtained in the SM, and hence can serve as a promising tool for establishing new physics beyond the SM.

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References

[1] T. M. Aliev, M. Savcı, JHEP 05 (2006) 1.

[2] K. Abe et al., BELLE Collaboration, prep: hep-ex/0410006 (2004).

[3] M. Iwasaki et al., BELLE Collaboration, Phys. Rev. D 72, 092005 (2005).

[4] B. Aubert et al., BaBar Collaboration, prep: hep-ex/0507005 (2005).

[5] B. Aubert et al., BaBar Collaboration, Phys. Rev. Lett. 93, 081802 (2004).

[6] T. Mannel and S. Recksiegel, J. Phys. G 24, 979 (1998).

[7] P. Bialas, J. G. Körner, M. Krämer, and K. Zalewski, Z. Phys. C 57, 115 (1993); F. Hussain, J. G. Körner, and R. Migneron, Phys. Lett. B 248, 406 (1990); 252 723 (E) (1990); J. G. Körner and M. Krämer, Phys. Lett. B 275, 495 (1992); T. Mannel and G. A. Schuler, Phys. Rev. D 279, 194 (1992); M. Gronau and S. Wakaizumi, Phys. Rev. D 47, 1262 (1993); M. Tanaka, Phys. Rev. D 47, 4969 (1993); M. Gronau, T. Hasuike, T. Hattori, Z. Hioki, T. Hayashi, and S. Wakaizumi, J. Phys. G 19, 1987 (1993); Z. Hioki, Z. Phys. C 59, 555 (1993); B. König, J. G. Körner, and M. Krämer, Phys. Rev. D 49, 2363 (1994); M. Gremm, G. Köpp, and L. M. Sehgal, Phys. Rev. D 52, 1588 (1995); C. S. Huang and H. G. Yan, Phys. Rev. D 56, 5981 (1997); S. Balk, J. G. Körner, and D. Pirjol, Eur. Phys. J. C 1, 221 (1998); J. G. Körner and D. Pirjol, Phys. Lett. B 334, 399 (1994); Phys. Rev. D 60, 014021 (1999).

[8] J. G. Körner, M. Krämer, and D. Pirjol, Prog. Part. Nucl. Phys. 33, 787 (1994).

[9] Z. Zhao et al., Report of Snowmass 2001 working group E2: Electron Positron Colliders from the φ to the Z, in Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001) ed. R. Davidson and C. Quigg, hep-ex/0201047 (2002).

[10] W. Bensalem, D. London, N. Sinha and R. Sinha, Phys. Rev. D 67, 034007 (2003).

[11] T. Appelquist, H. C. Cheng and B. A. Dobrescu, Phys. Rev. D 64, 035002 (2001).

[12] I. Antoniadis, Phys. Lett. B 246, 317 (1990).

[13] K. Agashe, N. G. Deshpande and G. H. Wu, Phys. Lett. B 511, 85 (2001); T. Appelquist and B. A. Dobrescu, Phys. Lett. B 516, 85 (2001).

[14] J. F. Oliver, J. Papavissiliou and A. Santamaria, Phys. Rev. D 67, 056002 (2002).

[15] A. J. Buras, M. Sprangler and A. Weiler, Nucl. Phys. B 660, 225 (2003).

[16] A. J. Buras, A. Poschenrieder, M. Spranger Nucl. Phys. B 678, 455 (2004).

[17] P. Colangelo, F. De Fazio, R. Ferrandes, T. N. Pham, prep: hep-ph/0604029 (2006).

[18] T. M. Aliev, M. Savcı, prep: hep-ph/0606225 (2006).
[19] C. Bobeth, M. Misiak and J. Urban, Nucl. Phys. B 574, 291 (2001); A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, Nucl. Phys. B 685, 351 (2004); C. Bobeth, P. Cambino, M. Gorbahn and U. Haisch, JHEP 0404, 071 (2004).

[20] A. Buras, M. Misiak, M. Münz and S. Pokorski, Nucl. Phys. B 424, 374 (1994).

[21] M. Misiak, Nucl. Phys. B 393, 23 (1993); Erratum ibid B 439, 161 (1995); B. Buras, M. Münz, Phys. Rev. D 52, 186 (1995).

[22] T. M. Aliev, A. Özpıneci, and M. Savcı, Nucl. Phys. B 649, 1681 (2003).

[23] C. H. Chen and C. Q. Geng, Phys. Rev. D 64, 074001 (2001).

[24] T. M. Aliev, A. Özpıneci, and M. Savcı, Phys. Rev. D 67, 035007 (2003).

[25] S. Eidelman et al., Particle data Group, Phys. Lett. B 592, 1 (2004).

[26] CDF Collaboration, prep: hep-ex/0507091.

[27] T. Mannel, W. Roberts, and Z. Ryzak, Nucl. Phys. B 355, 38 (1991).

[28] C. S. Huang, H. G. Yan, Phys. Rev. D 59, 114022 (1999).
Figure captions

Fig. 1 The dependence of $P_{LL}$ on $q^2$ for the $\Lambda_b \rightarrow \Lambda \tau^+\tau^-$ decay at fixed values of the compactification parameter $1/R$. For completeness, here and in all following figures, SM results are also given.

Fig. 2 The dependence of $P_{TT}$ on $q^2$ for the $\Lambda_b \rightarrow \Lambda \mu^+\mu^-$ decay at fixed values of the compactification parameter $1/R$.

Fig. 3 The same as in Fig. 1, but for $P_{LT}$.

Fig. 4 The same as in Fig. 2, but for $P_{NN}$.

Fig. 4 The same as in Fig. 1, but for $P_{NN}$.
Figure 1:

Figure 2:
Figure 3:

\[ P_{LT}(\Lambda_b \to \Lambda^+ \tau^-) \]

\[ q^2 \text{ (GeV}^2\text{)} \]

Figure 4:

\[ P_{NN}(\Lambda_b \to \Lambda^+ \mu^- \bar{\mu}) \]

\[ q^2 \text{ (GeV}^2\text{)} \]
