Gravitational wave detectors with broadband high frequency sensitivity

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Gravitational waves from the neutron star coalescence GW170817 were observed from the inspiral, but not the high frequency postmerger nuclear matter motion. Optomechanical white light signal recycling has been proposed for achieving broadband sensitivity in gravitational wave detectors, but has been reliant on development of suitable ultra-low loss mechanical components. Here we show demonstrated optomechanical resonators that meet loss requirements for a white light signal recycling interferometer with strain sensitivity below $10^{-24} \text{ Hz}^{-1/2}$ at a few kHz. Experimental data for two resonators are combined with analytic models of interferometers similar to LIGO to demonstrate enhancement across a broader band of frequencies versus dual-recycled Fabry-Perot Michelson detectors. Candidate resonators are a silicon nitride membrane acoustically isolated by a phononic crystal, and a single-crystal quartz acoustic cavity. Optical power requirements favour the membrane resonator, while thermal noise performance favours the quartz resonator. Both could be implemented as add-on components to existing detectors.
Since the detection of gravitational waves (GW) from binary black holes and neutron stars \(^1\)–\(^3\), there is increasing interest in improving the sensitivity and bandwidth of detectors to allow better characterization of gravitational wave sources. Detectors such as the proposed Einstein Telescope \(^5\) and Cosmic Explorer \(^6\) aim for improved low frequency sensitivity to dramatically increase the number of observable cycles from compact binary coalescence events. Other detectors focus on multi-messenger astronomy from neutron star coalescences, targeting a strain sensitivity of \(h \sim 10^{-24} \text{ Hz}^{-1/2}\) in the 1–5 kHz band. Observation of the normal modes of newborn hypermassive neutron stars will provide insight into the complex hydrodynamics of nuclear matter moments before its collapse into a black hole. \(^4\) Other sources of GWs in the range 1–5 kHz include the final moments of black hole coalescence, normal modes of newborn black holes with mass 5–20 \(M_\odot\) and core collapse supernovae.

The current sensitivity of GW detectors has so far been insufficient to characterise binary neutron star postmerger remnants. \(^8\) High frequency sensitivity in interferometric GW detectors is currently limited by quantum shot noise \(^-11\) with strain sensitivity of a few times \(10^{-23} \text{ Hz}^{-1/2}\). A straightforward way to reduce the quantum shot noise level is to increase the laser power inside the detector. In addition, configurations based on detuning and strongly coupled signal recycling \(^12\) can produce a broadband response at high frequency, but achieving target sensitivity \(h \sim 10^{-24} \text{ Hz}^{-1/2}\) still requires arm power levels an order of magnitude higher than the best attained to date.

In general, signal recycling improves detector sensitivity by resonant enhancement of the signal rather than suppression of shot noise at the detection port. However, in conventional systems the resonance response creates a trade-off between sensitivity gain and bandwidth. \(^13\) In principle, the sensitivity–bandwidth tradeoff can be overcome by the method of white light signal recycling (WLSR). While travelling along the long interferometer arms, the GW signal sidebands experience a phase delay relative to the carrier. A negative dispersion medium inside the signal recycling cavity can compensate for the signal sideband phase delay, creating a broadband resonance called a white light cavity. \(^14\), \(^15\) The energetic quantum limit of the cavity is lowered via quantum amplification \(^16\)–\(^19\), indicating that the interferometer supports a non-classical state and physical laws are not violated.

A succession of papers \(^20\)–\(^23\) have shown that WLSR can be implemented by using an optomechanically coupled negative dispersion filter. The filter consists of a mechanical resonator placed inside the Fabry-Perot cavity with optical resonance \(\omega_0\) equal to the interferometer carrier frequency. The cavity pump light has blue detuning equal to the mechanical resonance \(\omega_m\), and is stabilized by feedback. \(^20\) The negative dispersion filter can be seen as a blue-detuned analogue of optomechanically-induced transparency, where the GW signals act as the near-resonant probe. Parametric interaction between the signal, pump light, and mechanical resonator stores the signal with a frequency dependent phase compensation, creating the negative dispersion effect.

To maintain quantum amplification, noises introduced by the mechanical resonator must be kept low. It has been shown that the mechanical resonator must have a quality factor \(Q_m\) and operate at temperature \(T\) such that \(T/Q_m < 10^{-9} \text{ K}\), in order for thermal noise not to dominate the detector noise budget. \(^20\) In addition, vacuum noise sidebands at \(\omega_0 \pm 2\omega_m \pm \Omega\) are present inside the detuned cavity and create extra quantum noise at GW signal sideband frequencies. High \(\omega_m\) is required to keep the extra sidebands far detuned from the interferometer, and their impact can be further mitigated using a high finesse filter cavity. \(^23\)

Further elaboration on the origin of the WLSR parameter space is given in Supplementary Note 1.

The proposed solution to the mechanical resonator thermal and quantum noise issues has been to use optical dilution to increase the resonant frequency and \(Q\)-factor of mechanically soft micro-pendulums. \(^21\)–\(^24\) However, optical dilution for the purpose of GW detection is technically demanding. The mechanical resonators need to be very small, and yet able to operate at high optical power densities. The trapping power required to achieve sufficient \(Q_m\) results in coupling to other loss mechanisms, placing an upper limit on the viable \(\omega_m\). Optical dilution must be balanced against thermoelectric loss, acceleration loss, and beam size. This leaves a very small volume of parameter space in which the necessary performance might be achieved. \(^21\)

Here we show the possibility of optomechanical negative dispersion using existing resonator structures, without having to resort to optical dilution. The two candidate resonators are (i) a silicon nitride membrane isolated from the external environment by a phononic crystal (henceforth referred to as "phononic crystal") resonator or PNC and (ii) a plano convex lens constructed from single-crystal quartz, known as a bulk acoustic wave (BAW) resonator due to its characteristic of bulk longitudinal phonons with extremely high quality factor. In combination with 10 dB frequency dependent squeezing of quantum noise, the resulting WLSR interferometer can achieve a sensitivity floor of \(h \sim 5.0 \times 10^{-25} \text{ Hz}^{-1/2}\), with the BAW configuration being slightly lower. The design and resulting sensitivity spectrum are shown in Fig. 1. Compared to a specialised high frequency dual-recycled Fabry–Perot Michelson interferometer, \(^25\) the WLSR interferometer produces a broader band of frequency enhancement in the neutron star GW regime 1–5 kHz, and can maintain \(h \leq 10^{-24} \text{ Hz}^{-1/2}\) for GWs up to 4.9 kHz. For the PNC resonator, the required blue-detuned pump light is 42.2 mW. The BAW resonator requires in excess of 10 kW, but the very low optical losses of quartz mean that the dissipated power could be manageable.

**Results**

**White light signal recycling interferometer configuration.** In our WLSR interferometer design we use the PNC resonator characterised by Mason, et al., who have maintained a \(\omega_m/(2\pi) = 1.135 \text{ MHz out-of-plane vibrational mode at } Q_m = 1.03 \times 10^9\) and \(T = 10 \text{ K for a } 20 \text{ nm thick Si}_3\text{N}_4\) membrane shielded with an acoustic bandgap of 1.07–1.28 MHz. \(^26\) The PNC resonator can be optomechanically coupled by using it in a "membrane-in-the-middle" (MIM) configuration as characterised by Thompson et al. \(^27\)

For the BAW resonator, we refer to Galliou, et al., who have measured \(\omega_m/(2\pi) = 204 \text{ MHz and } Q_m = 8 \times 10^9\) at 4 K for the 65th longitudinal mode of a 30 mm diameter, 1 mm thick plano-convex quartz crystal. \(^28\) Kharel et al. have demonstrated strong optomechanical coupling in BAW resonators using Brillouin scattering. \(^29\) However, Brillouin scattering using near-infrared light in quartz requires a mechanical mode of 18 GHz, which would have surface scattering losses that exceed the strict thermal noise requirements for WLSR. \(^29\) Instead, we use optomechanical coupling to the antiphase surface motion of the 204 MHz mode. We also estimate that \(Q_m = 1.5 \times 10^{10}\) is possible by operating at 1 K, and the resulting thermal noise reduction is indicated in Fig. 1. Details of the temperature scaling of the BAW resonator \(Q_m\) are given in Supplementary Note 2.

The WLSR interferometer layout is shown in Fig. 1. In the interferometer, impedance matching of the arm cavity to the signal extraction mirror allows for enhanced transmission of GW sidebands into the signal recycling cavity. The signal recycling
mirror couples the interferometer dark port, negative dispersion filter, and output photodiode. Frequency dependent squeezing may be applied by injecting squeezed vacuum at the output Faraday isolator31. The negative dispersion filter is cooled to temperatures of 1 K and contained inside a radiation shield to minimise radiation heating and scattered light noise. A model of the PNC resonator is shown in Fig. 1, embedded in a 2-dimensional phononic lattice. High Qm has been demonstrated for silicon nitride PNC resonators of size 87–346 μm32. Other resonator properties relevant to WLSR are shown in Table 1.

**Optomechanical coupling in negative dispersion filters.** The interaction Hamiltonian inside the negative dispersion filter suggests correlated two-photon exchange20:

\[
\hat{H}_{\text{int}} = -\hbar g \left( \hat{a} \hat{b} + \hat{a}^\dagger \hat{b}^\dagger \right),
\]

where \(\hat{a}\) and \(\hat{b}\) are the annihilation operators of the optical and mechanical modes inside the filter cavity, respectively, and \(g\) is the optomechanical coupling rate. Using this Hamiltonian, the negative dispersion filter is shown to have the following input–output relation:

\[
\hat{a}_{\text{out}}(\omega_0 + \Omega) = \exp \left[ -2i \frac{\Omega}{\gamma_{\text{opt}}} \right] \hat{a}_{\text{in}}(\omega_0 + \Omega),
\]

where \(\hat{a}_{\text{in}}\) and \(\hat{a}_{\text{out}}\) are the annihilation operators of the input and output optical fields, respectively, and \(\gamma_{\text{opt}}\) is the optomechanical anti-damping. Across the detector’s arm length \(L_{\text{arm}}\), the GW signal sidebands acquire phase delay \(\Omega L_{\text{arm}}/c\), which can be compensated when \(\gamma_{\text{opt}} = c/L_{\text{arm}}\) so long as we remain in the linear negative dispersion regime \(\Omega \ll \gamma_{\text{opt}}\). For a 4 km interferometer, \(\gamma_{\text{opt}}(2\pi) = 12\) kHz. The pumping power necessary to achieve \(\gamma_{\text{opt}}\) is determined by \(\gamma_{\text{opt}} = g^2/\gamma_f\), where \(\gamma_f\) is the filter cavity bandwidth. The optomechanical coupling rate can be expanded into \(g = \frac{d\omega}{dq} x_{\text{opt}} a\), where \(\frac{d\omega}{dq}\) is the optical frequency shift per unit of generalised mechanical displacement \(q\). \(x_{\text{opt}}\) is the mechanical zero-point displacement fluctuation and \(a^2\) is the mean intracavity photon number. The circulating power requirement for negative dispersion becomes:

\[
P_L = \frac{c M_{\text{eff}} \omega_p \omega_m \gamma_{\text{opt}} \gamma_f^2}{(d\omega/dq)^2},
\]

where \(M_{\text{eff}}\) is the effective mass of the mechanical resonator, \(L_f\) the filter cavity length and \(\omega_p = \omega_0 + \omega_m\) the pump frequency of the filter.

The optomechanical coupling for the PNC resonator is found from the relation of optical resonant frequency versus membrane
displacement for a MIM cavity, given as ~ 0.0075 GHz nm. We take $d = \frac{c}{L_4}$ as a desirable target. Previously reported measurements of silicon nitride absorption indicate that $10 \text{ ppm}$.. Further details are presented in "Methods" and Supplementary Table S1.

The WLSR interferometer is seen to reach strain sensitivity levels below $h = 10^{-24}$ Hz$^{-1/2}$ at GW signal frequencies up to 4.9 kHz, with a peak sensitivity $h > 5 \times 10^{-25}$ Hz$^{-1/2}$ across a broad band, as seen in Fig. 1. The peak sensitivity is limited by thermal noise coupling, set by the experimentally measured $Q_m$ of each resonator. At frequencies of 1–5 kHz, we are also concerned with optical loss from the filter cavity. The PNC sensitivity curve is set with $10 \text{ ppm}$ filter cavity round trip loss as a desirable target. Previously reported measurements of silicon nitride absorption indicate that 1–4 ppm or lower absorption is possible in the case of a 20 nm thick resonator and 1064 nm wavelength light. The relative contributions of the aforementioned noise sources to the total sensitivity are shown in Supplementary Note 6.

In a dual-recycled Fabry-Perot Michelson interferometer it has been shown that by tuning the transmissivity and length of the signal recycling cavity, an optical "sloshing" resonance at 2.5 kHz with bandwidth 1.5 kHz could be created in order to amplify neutron star signals. Figure 1 compares our WLSR scheme with the sloshing resonance design at similar levels of interferometer optical loss, arm cavity power, and optical squeezing, showing superior gain/bandwidth enhancement of quantum noise limited sensitivity in the 1–5 kHz range. The WLSR interferometer has an additional advantage of being able to maintain a short signal recycling cavity of much less than 100 m.

In longer interferometers, quantum shot noise scales with $1/\sqrt{L_{arm}}$ whereas classical displacement noises scale with $1/L_{arm}$. The potential strategy to bring quantum noise down to the level of classical noises in future detectors that plan to use arms of 10 km or longer. However, the filter cavity thermal noise requirement is proportional to the unmodified interferometer bandwidth, and is thus stricter for longer arms. Also, disregarding free spectral range effects, the high frequency strain sensitivity contribution from the beamsplitter cavity loss is length-independent, setting a limit on the length increase benefit. Further exploration of WLSR in longer and shorter detectors is given in Supplementary Note 7.
The beam waist in the Advanced LIGO output mode cleaner is from a selection of PNC resonators and reduces signal recycling cavity loss. While the noise budgets shown contribute will be significant, the contribution of mode matching and angular alignment control which is the power required to achieve negative dispersion as per Eq. (3). This is useful for tuning the location of peak sensitivity in the neutron star detection band without changing interferometer hardware or detuning the signal recycling cavity.

but we note that a 10 km configuration is still beneficial even with increased levels of arm, signal recycling and filter cavity optical loss.

When the pumping power is not perfectly matched to $\gamma_{\text{opt}}$, the quantum noise spectrum exhibits a small region of enhanced high frequency sensitivity at the expense of some lower frequency sensitivity, as shown in Fig. 3. This provides an additional advantage for the detection of binary neutron star gravitational waves, since the exact frequency of the kilohertz ringdown is unknown. WLSR presents the possibility of shifting the optimal detection frequency of the interferometer without changing the interferometer hardware or detuning the signal recycling cavity. However, this is contingent on maintaining a low contribution of filter cavity optical loss.

Discussion

There are several technical concerns not directly considered above, but will be important for implementing WLSR.

Parametric interaction between the two light fields and mechanical motion in the filter cavity results in optomechanical instability that must be controlled. A state space demonstration of filter cavity controllability has been shown, but it was later seen that the effect of time delay would hinder the global control system. Local sensing control can eliminate the time delay issue. Readout noise may arise from the addition of local control, but it is indicated that this can be cancelled by optimal postprocessing combination of the unstable filter and interferometer control signal. Another possibility is to use coherent quantum feedback with parity-time symmetry to eliminate the instability at a fundamental level.

Mode mismatch between the interferometer and filter introduces signal recycling cavity loss. While the noise budget values shown in Fig. 1 account for general signal recycling loss, the specific contribution of mode matching and angular alignment control have yet to be investigated. However, it is expected that the contribution will be significant given the beam size in the filter. From a selection of PNC resonators sufficient for WLSR, the largest is $350 \mu$m wide, requiring a beam waist of 80–100 $\mu$m to reduce optical diffraction losses. For the BAW resonator, the effective radius of the 204 MHz longitudinal mode is 260 $\mu$m. The beam waist in the Advanced LIGO output mode cleaner is $500 \mu$m, and Advanced LIGO target 1–2% mode matching losses for the next generation of 4 km detectors. The BAW resonator’s larger beam size requirement is advantageous in reducing potential mode matching loss. In an A+ type detector with 800 kW arm cavity power, signal recycling loss of ~1% contributes roughly equal to unsqueezed WLSR quantum noise in the 100–5000 Hz band. Supplementary Notes 6–7 illustrate that bandwidth broadening is still possible for detectors similar to the current generation with higher optical loss, low levels of squeezing and arm cavity power < 1 MW. In Fig. 1 we use 0.1% signal recycling loss, which contributes below the limiting noises of filter thermal, filter cavity optical loss and beamsplitter cavity noise.

Scattered light rejoining the interferometer beam can contaminate the signal with phase noise acquired from moving objects. An estimate can be obtained by analysing the degree of freedom along the optical axis. The mechanical resonator motion must be controlled, so the dominant contribution is from the filter cavity vacuum enclosure. Assuming that the motion is typical of LIGO isolated tables, the maximum tolerable light power rejoining the interferometer beam is approximately 0.5 ppm of light power incident upon the filter cavity. In Fig. 1, a large window to the cryogenic component is avoided for this reason.

Silicon nitride phononic crystal resonators provide a first realistic means of creating a white light signal recycling interferometer, using experimentally demonstrated values of mechanical loss, optical absorption and incident laser power. Single crystal bulk acoustic wave resonators also have promising thermal noise properties, but the required levels of optical power are untested. Proposed long-arm detectors such as Einstein Telescope and Cosmic Explorer will relax the optical power requirement, giving us more flexibility in future designs. The University of Western Australia is developing optomechanical negative dispersion filters using both mechanical resonator types described in this paper. The properties of silicon nitride phononic crystal resonators make them ideal for promptly achieving practical broadband enhancement of GW detector sensitivity, allowing greater investigation of neutron star coalescence.

Methods

Calculation of optomechanical coupling of the bulk acoustic wave resonator to the filter cavity. Optomechanical coupling of BAW resonators has been demonstrated using three-mode Brillouin scattering of optical waves to coherent acoustic phonons. In order to achieve three-mode Brillouin scattering in the BAW resonator, two energy transfer conditions must be met. First, the optical frequencies must be separated by the mechanical frequency, which is achieved through design of the filter cavity. Second, the phase matching requirement states that $\omega_m = k_1 + k_2$, where $\omega_m$ is the refractive index of quartz and $v_g$ is the speed of sound in quartz. Using the properties shown in Table 1, the resulting mechanical frequency requirement of $\omega_m/(2\pi) = 18$ GHz is a factor of 89 higher than the 204 MHz mode with optimal $Q_m$. Since we wish to formulate a means of negative dispersion using the experimentally demonstrated Q-factor and frequency shown by Galliou et al., we must resort to another mechanism of optomechanical coupling.

In our calculations, the negative dispersion filter couples the optical modes of the cavity to the antiphase surface motion of the planar and convex faces of the BAW resonator crystal. This allows access to the highest measured Q-factor mode at 204 MHz. To model coupling of the light field to the surface vibrations of the BAW resonator, we consider a BAW resonator crystal situated inside a Fabry–Perot cavity, as shown in Fig. 4. Using the boundary conditions of electromagnetic fields at the cavity end mirrors and crystal surfaces we obtain equations of the electric field as functions of crystal surface position. For given values of the cavity length $L$, and crystal center position $x_p$, we can obtain the cavity resonance frequency as a function of generalised displacement $q$, which in this case is the crystal thickness. Using electric and magnetic boundary conditions at the interface points $x = 0, x_p, x_3, L_3$ we then construct a system of equations in terms of $n_p, x_3, L_3$, the electric fields $E_{1,2,3,4}$ optical wavenumber $k$.

$$E_1 \sin(kx_1) = E_2 \sin(n_p kx_1) + E_3 \cos(n_p kx_1)$$
$$E_2 \cos(kx_2) = n_p E_1 \cos(n_p kx_1) - n E_2 \sin(n_p kx_1)$$
$$E_3 \sin(kx_3 - L_3) = E_4 \sin(n_p kx_3) + E_2 \cos(n_p kx_3)$$
$$E_4 \cos(kx_3 - L_3) = n_p E_4 \cos(n_p kx_3) - n_2 E_3 \sin(n_p kx_3)$$

$$E_1 \sin(kx_1) = E_2 \sin(n_p kx_1) + E_3 \cos(n_p kx_1)$$
$$E_2 \cos(kx_2) = n_p E_1 \cos(n_p kx_1) - n E_2 \sin(n_p kx_1)$$
$$E_3 \sin(kx_3 - L_3) = E_4 \sin(n_p kx_3) + E_2 \cos(n_p kx_3)$$
$$E_4 \cos(kx_3 - L_3) = n_p E_4 \cos(n_p kx_3) - n_2 E_3 \sin(n_p kx_3)$$

Fig. 3 Demonstration of tunable quantum noise spectrum in white light signal recycling interferometer. Adjustment of the quantum noise curve is achieved by changing the filter cavity pump power $P_{\text{pump}}$ relative to $P_r$, which is the power required to achieve negative dispersion as per Eq. (3).
The coordinates \( x_1 \) and \( x_2 \) are rearranged into crystal center position \( x_c = (x_2 + x_1)/2 \) and crystal thickness \( q = x_2 - x_1 \), and the system of equations reduces to:

\[
\eta_q \tan[k(x_c - q/2)] - \tan[\eta_0 k(x_c - q/2)] = \frac{\eta_q}{1 + \eta_0} \left[ \tan[k(x_c + \frac{q}{2} - L_1)] - \tan[\eta_0 k(x_c + \frac{q}{2})] \right] + \eta_0 \tan[k(x_c + \frac{q}{2})] - \tan[\eta_0 k(x_c + \frac{q}{2})]
\]  

(6)

We then solve the wavenumber in terms of the generalised dispersion \( q \). Selected solutions are obtained in proximity to \( \omega_0(2\pi) = 2.82 \times 10^{14} \text{ Hz} \), corresponding to a 1064 nm wavelength. The dependence of optical resonance frequency with crystal thickness is shown in Fig. 5. Over micron-scale motion, there is an approximately linear negative \( dw/dq \), and an appropriate optical mode can be selected such that there is a linear negative \( dw/dq \) within \(-50 \text{ nm} < q - q_0 < 50 \text{ nm} \). Local frequency variation is due to the sloshing between the left coupled cavity, the crystal itself and the right coupled cavity. The free spectral range between modes varies sinusoidally, which is consistent with studies on BAW resonator coupled cavities. The coupling \( dw/dq \) decreases with cavity length for a small cavity of \( L_1 = 5 \text{ mm} \), the maximum coupling is \( dw/dq = 2\pi \times 0.061 \text{ GHz nm}^{-1} \), while a longer cavity \( L_1 = 20 \text{ mm} \) results in maximum \( dw/dq = 2\pi \times 0.018 \text{ GHz nm}^{-1} \). The maximum single photon coupling rates \( \eta^{\text{single}}_{\text{mod}} \) are 2\( \times 10.0 \text{ Hz} \) and 2\( \times 0.031 \text{ Hz} \), respectively. As expected, surface optomechanical coupling is small compared to optomechanical coupling to the bulk longitudinal mechanical mode, where Kharel et al. demonstrated a near-infrared single photon coupling rate of 24 Hz in a BAW resonator at similar effective mass. Micro-pendulums used in previous proposals have estimated single photon coupling rates of \(-40 \text{ Hz} \), though at much lower effective mass (\(-10 \text{ ng} \)) and mechanical frequency (\(-100 \text{ kHz} \)).

**Calculation of effective mass of the bulk acoustic wave resonator mechanical mode.** To obtain the pumping power necessary to produce negative dispersion, we must find the effective mass of the relevant mechanical mode. We use the following formula given by Goryachev:

\[
M_{\text{p,0,0}} = \rho m \frac{q_0^2 r_c^2}{\sqrt{\eta_0}} \frac{\text{Erf}(\sqrt{m} q_0)}{\eta q_0 m},
\]

(7)

where the crystal radius \( r_c \) and density \( \rho \) are given in Table 1. The coupling factors \( \eta_0 \) quantify the trapping of the Gaussian longitudinal mode within the crystal and are given by:

\[
\eta_0 = r_c \sqrt{n_3^2},
\]

(8)

\[
\eta_0 = r_c \sqrt{n_4^2},
\]

(9)

\[
a^2 = \frac{c_0}{R_0^2} M,
\]

(10)

\[

\]

(11)

where \( R \) is the radius of curvature of the curved surface of the BAW, \( M \) and \( P \) are material dependent transverse elastic parameters which are only well known at room temperature. Goryachev estimates \( c_0 / R_0 > 0.4 \) for cryogenic quartz crystals. This results in \( \eta_q - \eta_0 = 5.08 \). The effective mass for the \( m = 65 \), \( \omega_0(2\pi) = 204 \text{ MHz} \) mode is \( M_{\text{p,0,0}} = 0.56 \text{ mg} \). In addition, the optical plane wave corresponding to this effective mass has area \( A = \pi a^2 / 2 \) giving an appropriate optical beam radius of 260 \( \mu \text{m} \) to match to the mechanical mode.

**Quantum noise matrix calculation for white light signal recycling interferometer.** Noise budgets of WLSR interferometers are calculated using the two photon formalism of Caves and Schumaker, where cavity components are represented as transfer matrices which can also incorporate optomechanical interaction. For example, a beam travelling distance \( L_{\text{free}} \) and reflected from a moving mirror in free space can be represented as:

\[
\begin{bmatrix}
\hat{P}_{\text{in}}(\Omega) \\
\hat{P}_{\text{out}}(\Omega)
\end{bmatrix} = e^{2i\omega_0 L_{\text{free}}} 
\begin{bmatrix}
1 & 0 \\
-\kappa & 1
\end{bmatrix} 
\begin{bmatrix}
\hat{a}_{\text{in}}(\Omega) \\
\hat{a}_{\text{out}}(\Omega)
\end{bmatrix},
\]

(12)

where \( \alpha \) and \( \beta \) respectively represent the input and output beams, and subscripts \( a \) and \( p \) the amplitude and phase quadratures. The amplitude and phase quadratures of light are related to the sideband creation and annihilation operators by:

\[
\begin{bmatrix}
\hat{a}_{\text{in}}(\Omega) \\
\hat{a}_{\text{out}}(\Omega)
\end{bmatrix} = \begin{bmatrix}
1 & 1 \\
\sqrt{2} & -1
\end{bmatrix} 
\begin{bmatrix}
\hat{\alpha}_{\text{in}}(\Omega) \\
\hat{\alpha}_{\text{out}}(\Omega)
\end{bmatrix},
\]

(13)

where \( \hat{\alpha}_{\text{in}}(\Omega) \) is the annihilation operator of the upper sideband at frequency \( +\Omega \) with respect to the reference and \( \hat{\alpha}_{\text{out}}(\Omega) \) is the creation operator of the lower sideband at frequency \( -\Omega \) with respect to the reference. As such the two-photon formalism is naturally used in cases where modulation produces paired sidebands. Optomechanical coupling is incorporated in the frequency of mirror rotation \( \Omega \) and the coupling factor \( \kappa = 8P_0\omega_0/\left(MCL_{\text{sub}}\right) \), where \( P_0 \) is the incident power and \( M \) is the mass of the mirror. 2 \( \times \) 2 transfer matrices in the two-photon basis can also be built up for tuned and detuned optomechanical cavities in a similar manner to Eq. (12). We obtain an overall sensitivity spectrum by looking at the input-output relation at the output photodetector. For this transfer matrix method it is simple to calculate the sensitivity spectrum for the measurement of any linear combination of amplitude and phase quadrature, though for the purpose of this paper we only require measurement of the phase quadrature. Additional sideband noises are produced by the filter cavity, as illustrated by Fig. 6. The calculation considers the GW signal sidebands at optical frequencies of \( \omega_0 \pm \Omega \) along with the noise sidebands \( \omega_0 + 2\omega_{\text{in}} \pm \Omega \) that arise as a result of...
radiation pressure interactions of the GW sidebands within the detuned filter cavity. Doing so requires expanding the optomechanical transfer matrix from $2 \times 2$ to $4 \times 4$, and the detuned filter cavity, we also switch from the two-photon picture to the sideband creation/annihilation picture. The basis vector incorporates each of the sidebands shown in Fig. 6. As such, we produce a transfer function:

$$b_{\alpha} (\omega) = M_{\alpha \alpha} (\omega) \cdot \mathcal{O}_{\alpha} (\omega),$$

where $M_{\alpha \alpha}$ is the transfer matrix of an optomechanical cavity detuned from $\omega_0$ by $\omega_{\text{arm}}$, and the argument $\omega$ represents the separation in sidebands that the two-photon interaction causes to be centered around $\omega_0 + \omega_{\text{arm}}$ as per Fig. 6. For example, sideband 1 in Fig. 6, the lower GW signal sideband, is separated from the center frequency by $\omega = \omega_0 + \Omega$. An appropriate transfer matrix can be constructed by taking the two-photon transfer matrix of an optomechanical cavity detuned by $\omega_{\text{arm}}$, with GW sidebands occurring at $\omega = \Omega$, transforming to the sideband basis using the matrix in Eq. (13), and arranging the appropriate entries into a $4 \times 4$ matrix according to the following basis:

$$\begin{bmatrix} \hat{a}_\Omega \\ \hat{a}_{2\omega_0 - \Omega} \\ \hat{a}_{2\omega_0 + \Omega} \end{bmatrix} = M_{\alpha \alpha} (\omega) \cdot \begin{bmatrix} \hat{a}_\Omega \\ \hat{a}_0 \\ \hat{a}_{2\omega_0 - \Omega} \\ \hat{a}_{2\omega_0 + \Omega} \end{bmatrix}. \tag{15}$$

Conjugating the second and third rows of the transfer matrix $M_{\alpha \alpha}$ represents changing the second entry of the basis vector to an annihilation operator and the third entry to a creation operator. This allows us to use the following transformation matrix:

$$\begin{bmatrix} \hat{a}_\Omega \\ \hat{a}_{2\omega_0 - \Omega} \\ \hat{a}_{2\omega_0 + \Omega} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 \\ i & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & i \end{bmatrix} \begin{bmatrix} \hat{a}_\Omega \\ \hat{a}_0 \\ \hat{a}_{2\omega_0 - \Omega} \\ \hat{a}_{2\omega_0 + \Omega} \end{bmatrix}, \tag{16}$$

where the frequency of the argument is now written with respect to the carrier frequency $\omega_0$ instead of the blue-detuned pumping frequency $\omega_0 - \Omega$. The first two rows represent the amplitude and phase quadrature of signal sidebands generated about $\omega_0 \pm \Omega$, while the third and fourth rows represent the quadratures of light generated about $\omega_0 + 2\omega_0 \pm \Omega$. The transfer matrix $M_{\alpha \alpha}$ can thus give the two-photon transfer function for the quantum noise from sidebands at $\omega_0 + 2\omega_0 \pm \Omega$. For simplicity, we assume that the sidebands at $\omega_0 - 2\omega_0 \pm \Omega$ are far enough detuned from the interferometer resonance to be simply reflected back into the signal recycling optics.

As a consequence of keeping propagation phase factor such as that shown in Eq. (12), the calculation also takes into effect cavity free spectral range, which has a significant impact on the audio band sensitivity of GW detectors 10 km and above in length.

An initial impression dictates that $\nu_{\text{rms}}(2\Omega)$ is set to 12 kHz in order to cancel the phase delay accumulated by GW signals in the 4 km interferometer arms. However, by slightly offsetting the optomechanical anti-damping, the calculated quantum noise response curve extends further into the 1–5 kHz band at a slight cost in peak sensitivity, as indicated by Fig. 3. For the main results in this paper, we actually use $\nu_{\text{rms}}(2\Omega) = 11.5$ kHz.

**Optical loss calculation for white light signal recycled interferometer.** Optical losses from various sources introduce uncorrelated vacuum noise to the GW signal sidebands. Optical loss from the negative dispersion filter is treated as transmission of uncorrelated vacuum through the end mirror of the filter cavity. Likewise, optical loss in the interferometer arms is introduced as transmission of uncorrelated vacuum through the end test mass. Loss from the output optics to the photodiode is introduced between the signal recycling mirror (SRM) and output Faraday isolator. It behaves similar in frequency dependence to the quantum noise curve, but is actually caused by the homodyne detection process as described by Kibble et al. Similarly to Kibble et al., we consider the effect of resonantly enhanced optical losses inside the interferometer beam splitter cavity, which is dominated by power-dependent thermal lensing noise. This is due to absorption of optical power onto the input test mass (ITM) and beam splitter, causing heat gradients that distort the carrier wave from its desired shape. These losses are then resonantly enhanced inside the beam splitter cavity. In the WLSR configuration of Fig. 1, this resonant enhancement of arm power-dependent optical loss occurs inside the cavity formed by the ITM and signal extraction mirror (SEM). The wavefront distortion contributions from the ITM and beam splitter scale approximately as:

$$\epsilon_{\text{ITM}} = \frac{p}{\kappa_{\text{ITM}}} \frac{\alpha_{\text{ITM}}}{1000 \text{ ppm}}, \tag{17}$$

$$\epsilon_{\text{BS}} = \frac{p_{\text{BS}}}{\kappa_{\text{BS}}} \frac{\alpha_{\text{BS}}}{500 \text{ ppm}}. \tag{18}$$

where $\alpha_{\text{ITM}}$ represent optical absorption, $\alpha_{\text{BS}}$ the compensation factor from various systems that reduce thermal lensing and $\kappa_{\text{ITM}}$ the incident power on the beam splitter. The total signal extraction loss $\kappa_{\text{ITM}} = \kappa_{\text{BS}}$ is introduced as uncorrelated vacuum between the main beam splitter and SEM. Resonant enhancement causes significant contribution of signal extraction loss in the 1–5 kHz band.

Introducing the SEM also causes impedance matching of losses between the signal recycling cavity and the arm cavity. As such, losses occurring in the signal recycling cavity (SRC) are combined with arm cavity losses into one total loss $\kappa_{\text{SE}}$, which is introduced as uncorrelated vacuum inside the SRC. The shot noise amplitude spectrum of this loss behaves similarly to that of quantum noise in a simple Michelson at high frequency, scaling inversely proportional to $\kappa_{\text{SE}}$. As such, the contribution of impedance matched arm losses is reduced in longer interferometers.

**Data availability**

Datasets are used in Supplementary Figs. S1 and S2 and are available at Figshare repository. Calculations regarding the noise budget of WLSR interferometers as described in Methods were performed using Mathematica. Annotated code is available from the corresponding author upon request.

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