Fine-tuned vs. natural supersymmetry: what does the string landscape predict?

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ABSTRACT: A vast array of (metastable) vacuum solutions arise from string compactifications, each leading to different 4-d laws of physics. The space of these solutions, known as the string landscape, allows for an environmental solution to the cosmological constant problem. We examine the possibility of an environmental solution to the gauge hierarchy problem. We argue that the landscape favors softly broken supersymmetric models over particle physics models containing quadratic divergences, such as the Standard Model. We present a scheme for computing relative probabilities for supersymmetric models to emerge from the landscape. The probabilities are related to the likelihood that the derived value of the weak scale lies within the Agrawal et al. (ABDS) allowed window of values leading to atoms as we know them. This then favors natural SUSY models over unnatural (SUSY and other) models via a computable probability measure.

KEYWORDS: String Models, Supersymmetry, String and Brane Phenomenology

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1 Introduction

Supersymmetry is a key ingredient in superstring theory constructs. An advantage of compactification of 10-d string theory on a Calabi-Yau manifold [1] is that it preserves some remnant spacetime supersymmetry in the 4-d theory. Likewise, compactification of 11-d M-theory on a manifold of $G_2$ holonomy preserves some remnant spacetime supersymmetry [2]. Acharya [3] argues for the proposal that the landscape of all geometric, stable, string/M theory compactifications to Minkowski spacetime at leading order are supersymmetric. Non-SUSY preserving compactifications would lead to bubble of nothing instabilities and presumably lie within the swampland [4].

Making contact with 4-d physics at the TeV scale (which is currently being explored at the CERN LHC), it is apparent that $N = 1$ spacetime SUSY must be broken.\footnote{For a recent review of the status of SUSY after LHC Run 2, see [5].} But the question is: broken at which scale? The gauge hierarchy problem (GHP) suggests SUSY which is broken at or around the weak scale, thus providing a “natural” solution to the GHP wherein all quadratically divergent contributions to the Higgs boson mass cancel. Weak scale supersymmetry is also supported experimentally via the measured value of gauge couplings, whose values unify at a scale $m_{GUT} \simeq 2 \times 10^{16}$ GeV [6] under renormalization group evolution [7] within the Minimal Supersymmetric Standard Model [8] (MSSM) while they do not within the context of the Standard Model (SM). In addition, the measured value of the Higgs boson mass falls directly within the narrow range of values allowed by
the MSSM [9]. Unfortunately, superpartners have so far failed to appear at LHC leading to an apparent naturalness crisis [10].

An apparent alternative to naturalness has emerged from the string landscape [11]. Under flux compactifications [12], an enormous number of different compactification possibilities are available [13, 14], each leading to different 4-d laws of physics. Each of these possibilities can be accessed within the context of an eternally-inflating multiverse [15]. This scenario provides the proper setting to realize Weinberg’s anthropic solution to the cosmological constant problem [16]: we find ourselves in a (pocket) universe with a tiny cosmological constant $\Lambda_{cc} \sim 10^{-123} m_{Pl}^2$, because if $\Lambda_{cc}$ was much larger, the universe would expand so fast that structure (galaxies, stars, etc.) would not be able to form and life as we know it would not arise. Such a solution to the CC problem is known as an environmental (or anthropic) solution: environmental selection of a tiny cosmological constant within the plenitude of pocket universes within the greater multiverse can select a highly fine-tuned value for one (or more) of our fundamental physical constants. Such a solution may stand in apparent opposition to a natural solution to the CC problem.

Can the GHP also be explained via environmental/anthropic reasoning rather than naturalness? Maybe. In the seminal papers by Agrawal, Barr, Donoghue and Seckel (ABDS) [17, 18], the scenario of the Standard Model emerging from the multiverse via an anthropic solution to the hierarchy problem is investigated. The authors consider the SM as the low energy effective field theory (LE-EFT), but with a variable magnitude for the weak scale. If the weak scale were a factor $\sim 2 - 5$ times larger than its actual value, then up-down quark mass differences would increase, leading to nuclear instability: one enters a domain of the universe where only protons exist, with no complex nuclei. For the weak scale reduced by a factor two from its measured value, then protons become unstable and beta decay to neutrons: there would be no Hydrogen, just neutron rich matter. In terms of the Higgs vacuum expectation value $v$, one finds $0.5 \lesssim v/v_0 \lesssim (2 - 5)$ (where $v_0$ is the Higgs vev in our universe). This narrow range of values for the weak scale has been dubbed the ABDS window in that values of $v$ outside this range would not lead to a universe with life as we know it. The anthropic requirement for $v$ to lie within the ABDS window could allow for a tuning of the weak scale within the wider multiverse. It also selects out a narrow range of allowed values: namely $m_{\text{weak}} \simeq m_{W,Z,h} \sim 100$ GeV and can explain the magnitude of the weak scale rather than just accommodate it. The requirement for the magnitude to lie within the ABDS window is sometimes also referred to as the atomic principle in that it is required in order for any pocket universe to contain complex atoms which seem necessary for a rich chemistry and for life as we know it.

Building upon the SM and ABDS, Arkani-Hamed and Dimopoulos [19–21] proposed a model known as Split Supersymmetry wherein the natural SUSY solution to the GHP is eschewed in exchange for an environmental solution. This then allows the possibility of a highly fine-tuned supersymmetric model. The authors then investigate the consequences of scalar masses $\tilde{m}$ far beyond the naturalness limit, taking $\tilde{m}$ as high as $\sim 10^9$ GeV. SUSY fermions, higgsinos and gauginos, may be protected by a chiral or $R$-symmetry and may still live around the EW scale. This set-up maintains the successful gauge coupling unification and WIMP dark matter of SUSY models, but enlists the vast number of landscape solutions
to effectively tune the weak scale to lie within the ABDS window as required by the atomic principle. The advantage of very heavy scalars (especially first/second generation matter scalars), as noted much earlier by Dine et al. [22] and others [23, 24] is that they provide a decoupling solution to the SUSY flavor and CP problems and may also suppress proton decay. In addition, under gravity mediation wherein scalars get mass of order the gravitino mass, this provides a solution to the cosmological gravitino and moduli problems.2

Thus, Split SUSY and a variety of successor models [26, 27] have been considered as legitimate expressions of what sort of SUSY models are expected to emerge from the string landscape. In the literature, it is sometimes claimed that a rather heavy Higgs mass and no sign of SUSY scalars at LHC might be construed as evidence for finetuning within the multiverse as opposed to a natural solution to the GHP, wherein there is no finetuning. Split SUSY, and the other high-scale SUSY models considered here, are motivated by the expectation that the soft SUSY breaking terms are statistically favored to occur at large as opposed to small values on the landscape via a power law relation \( P(m_{\text{soft}}) \sim m_{\text{soft}}^{2n_F+n_D-1} \) which obtains if the complex-valued SUSY breaking \( F \)-term fields and real-valued SUSY breaking \( D \)-term fields are distributed uniformly on the landscape [28–30]. (Here, \( n_F \) is the number of hidden sector \( F \)-term fields and \( n_D \) is the number of hidden sector \( D \)-term fields contributing to the overall SUSY breaking scale.) This landscape draw to large soft terms must be balanced by the anthropic/environmental condition that the derived value of the weak scale in each pocket universe lies within the ABDS window of allowed values [31, 32].

In this paper we survey a variety of finetuned models (both the SM and SUSY), and compare these to natural SUSY models, all within the context of the string landscape. What we find is somewhat at odds with the literature: natural SUSY models are more likely to emerge from the string landscape than finetuned models. We advance a particular probability measure \( P_\mu \) which quantifies these probabilities. By taking ratios, we are able to evaluate the relative probabilities for different models to emerge from the landscape.

In our deliberations, weak scale naturalness plays a key role, and we must define what we mean by naturalness. We adopt the definition of so-called practical naturalness [33]: an observable \( O \) is natural provided that all independent contributions to \( O \) are comparable to or less than \( O \). For the case of the SM, where the Higgs potential is given by

\[
V = -\mu_{SM}^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2
\]

a vacuum expectation value \( v = \sqrt{\mu_{SM}^2/\lambda} \) develops and the tree-level Higgs boson mass is given by \( m_h^2 = 2\mu_{SM}^2 \). The loop-corrected Higgs mass is quadratically divergent up to some cutoff scale \( \Lambda_{SM} \) where

\[
m_h^2 = 2\mu_{SM}^2 + \delta m_h^2
\]

where at one loop

\[
\delta m_h^2 \simeq \frac{3}{4\pi^2} \left( -\sum_i \lambda_i^2 + \frac{g_i^2}{4} + \frac{g^2}{8\cos^2 \theta_W} + \lambda \right) \Lambda_{SM}^2
\]

2For a recent overview of the cosmological moduli problem, see e.g. [25].
where the $\lambda_i$ are Yukawa couplings for the $i$th fermion, $g$ is the SU(2)$_L$ gauge coupling and $\lambda$ is the Higgs quartic coupling [34]. Requiring practical naturalness then leads to $\Lambda_{SM} \lesssim 1 \text{ TeV}$ whilst finetuning is required for much higher values of $\Lambda_{SM} \gg 1 \text{ TeV}$.

In SUSY models with the MSSM as the LE-EFT, then the weak scale is actually predicted in terms of the weak scale soft SUSY breaking terms and superpotential $\mu$ parameter. Minimization of the Higgs potential in the MSSM leads to

\[ m_Z^2/2 = \left( m_{H_u}^2 + \Sigma^u - (m_{H_d}^2 + \Sigma^u) \tan^2 \beta \right) \frac{\tan^2 \beta - 1}{\tan^2 \beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \mu^2 - \Sigma^u(\tilde{t}_1, \tilde{t}_2) \tag{1.4} \]

where the right-hand-side approximation is obtained for moderate-to-large $\tan \beta \gtrsim 5$. Here, $m_{H_u}^2$ and $m_{H_d}^2$ are the Higgs soft breaking masses and $\tan \beta$ is the usual ratio of Higgs field vacuum expectation values $v_u/v_d$. The $\Sigma$ terms contain over 40 loop corrections (explicit formulae for the $\Sigma$ terms may be found in Ref's [35] and [36] and leading two-loop terms may be found in ref. [37]). Requiring the MSSM weak scale as given by the measured value of $m_Z$ to be natural then requires $|\mu| \lesssim 350 \text{ GeV}$ while $m_{H_u}^2$ is driven radiatively to small negative values at the weak scale (electroweak symmetry is barely broken). Also, the leading loop corrections $\Sigma^u(\tilde{t}_1, \tilde{t}_2)$ are minimized for TeV scale top squarks with large, negative $A_t$ trilinear soft terms [38] which also give rise to nearly maximal stop mixing and large values of $m_h \sim 125 \text{ GeV}$. The finetuning measure [38]

\[ \Delta_{EW} \equiv \frac{|\text{maximal term on r.h.s. of eq. (1.4)}|}{(m_Z^2/2)} \tag{1.5} \]

is then a measure of practical naturalness in the MSSM, where for natural models usually $\Delta_{EW} \lesssim 30$ is required. Notice that $\Delta_{EW}$ is closely related to the ABDS anthropic window in that requiring $\Delta_{EW} \lesssim 30$ then requires all independent contributions to the MSSM weak scale to be within the ABDS window.

2 A survey of some natural and unnatural SUSY models

2.1 CMSSM

For a long time, the mSUGRA [39] or CMSSM [40] model served as a sort of paradigm model for SUSY phenomenology. This model posits gravity-mediated SUSY breaking which induces a common scalar mass $m_0$, a common gaugino mass $m_{1/2}$ and a common trilinear soft term $A_0$ all prescribed at the GUT scale $m_G \simeq 2 \times 10^{16} \text{ GeV}$. The weak scale soft terms are determined by RGE running to the weak scale, where electroweak symmetry is radiatively broken via a large top quark Yukawa coupling. The $\mu$ term is tuned via eq. (1.4) to give the measured value of $m_Z$. In pre-LHC days, it was possible within the CMSSM model to gain accord with naturalness (low $\Delta_{EW}$) and with an acceptable thermal relic abundance of the LSP. After LHC Run 2-- while respecting the LHC measured Higgs mass and also LHC sparticle search limits-- natural CMSSM spectra are no longer possible [41, 42, 65].

For illustrative purposes, we compute the mSUGRA/CMSSM spectra using the Isasugra spectrum generator [43, 44] for a mSUGRA/CMSSM benchmark point with $(m_0, m_{1/2},$
$A_0, \tan \beta = 5000 \text{GeV}, 1200 \text{GeV}, -8000 \text{GeV}, 10$ which yields a gluino mass $m_{\tilde{g}} = 2.8 \text{TeV}$ (well above current LHC bounds) with $m_h = 124.3 \text{GeV}$ and with $\Delta_{EW} = 2641$ (highly EW finetuned). The thermal bino LSP abundance is $\Omega_{\chi} h^2 \simeq 249$ so non-thermal processes would need to be invoked to bring the relic density into alignment with the measured dark matter abundance [45].

### 2.2 PeV SUSY

PeV-scale supersymmetry [46, 47] is motivated by the possibility of SUSY breaking via “charged” SUSY breaking fields $S$. For charged SUSY breaking, scalar partners gain mass via Kähler potential terms $K \ni \frac{s^T S}{m_P} Q^T Q$ where the $Q$ are visible sector fields and $S$ are hidden sector fields which carry some charge, perhaps $R$ charge. Thus, scalar fields gain a mass $m_Q^2 \sim F_S^T F_S / m_P^2 \sim m_{3/2}^2$ whilst gaugino masses, which ordinarily gain mass via the gauge kinetic function $f \ni k S$ are forbidden. Hence, the leading contribution to gaugino masses (and also $A$-terms) are the loop-suppressed anomaly-mediated contributions $m_{\lambda} = \frac{\beta (g_\lambda)}{\beta (g)} m_{3/2}$ and we expect $M_1 \simeq m_{3/2}/120, M_2 \simeq m_{3/2}/360$ and $M_3 \simeq m_{3/2}/40$. The wino is then the LSP and can make up the dark matter. Thermally produced relic winos can make up all the missing dark matter for $m_{\text{wino}} \sim 3 \text{TeV}$. Then, with a 3 TeV wino, one expects scalar masses $m_{\tilde{m}} \sim 1000 \text{TeV}$, i.e. close to the PeV scale (1 PeV=1000 TeV). The PeV scale scalar masses provide a decoupling solution to the SUSY flavor and CP problems [22]. The $\mu$ parameter may range anywhere between $m_{\text{wino}}$ and $\tilde{m}$. The resultant light Higgs mass is expected in the range $125 \text{ GeV} < m_h < 155 \text{ GeV}$ [20].

### 2.3 Split SUSY

In split SUSY [19–21], the motivation is that the string landscape may provide a selection mechanism for the finetuning of the electroweak scale in that the weak scale must lie within the ABDS window in order to have a universe with complex atoms as we know them, which seem necessary for life. However, SUSY may still be needed for consistency with string theory, but the SUSY breaking scale may now be far higher than that which is usually required by naturalness. One may then allow masses of squarks and sleptons (which occur in multiplets of SU(5)) to be as high as $m_{\phi} \sim 10^9 \text{ GeV}$ while fermion masses, which are protected by chiral symmetry, can lie near the weak scale. This model then preserves the SUSY success stories of gauge coupling unification and WIMP dark matter while appealing to vacuum selection from the string landscape to “tune” the EW scale to its value as required by the atomic principle. Thus, in split SUSY, one expects both gauginos and higgsinos around the weak scale whilst squarks and sleptons decouple at some intermediate scale (e.g. $10^9 \text{ GeV}$). Such a split hierarchy of masses can arise from $D$-term SUSY breaking which maintains an approximate, accidental $R$-symmetry [21]. The very high scalar mass scale $\tilde{m}$ provides a decoupling solution to the SUSY flavor and CP problems and also alleviates the cosmological gravitino and moduli problems by making these particles sufficiently heavy and thus shortlived in the early universe. The striking signature of split SUSY models is long lived gluinos which may decay with displaced vertices or even outside of the collider detector. For scalar masses as high as $\sim 10^9 \text{ GeV}$, then the lightest Higgs scalar is expected to have mass $m_{h} \sim 130 – 145 \text{GeV}$ [48].
2.4 High-scale SUSY

In high-scale SUSY (HS-SUSY) \[49–51\], it is assumed that the underlying 4-d theory is indeed SUSY, but with a much higher SUSY breaking scale than that which is usually assumed to solve the gauge hierarchy problem. Thus, in HS-SUSY, the superpartners are typically clustered at some very high mass scale $\tilde{m} \sim 10^{-10^{13}}$ TeV. In HS-SUSY, the SM is the LE-EFT and only the light Higgs particle is expected to be produced at LHC. Indeed, by requiring the model to yield the measured Higgs mass $m_h \sim 125$ GeV, then $\tilde{m} \sim 10^4 - 10^7$ TeV \[48, 52\].

2.5 Mini-Split

Mini-Split \[27\] SUSY is a version of split SUSY wherein the scalar mass $\tilde{m}$ is lowered to the $\sim 10^{-4}$ TeV range in order to accommodate the measured Higgs mass $m_h \sim 125$ GeV while gauginos remain near the TeV scale. Several scenarios are envisaged in \[27\] including non-sequestered AMSB and $U(1)'$ mediation. These scenarios include a small $A$ parameter while $\mu$ may be either at the gaugino scale (light) or at the scalar scale (heavy).

2.6 Simply unnatural SUSY

In simply unnatural SUSY \[53\] (SUN-SUSY), the scalar mass scale $\tilde{m}$ is determined by the measured value of the Higgs mass $m_h \sim 125$ GeV to be $\tilde{m} \sim 10^2 - 10^3$ TeV where the trilinear soft terms $A_i$ are assumed to be tiny (little mixing in the stop sector). The SUSY $\mu$ term is also expected to be $\mu \sim \tilde{m}$ while the gaugino masses, which require an $R$-symmetry breaking to gain mass, are expected to be at the TeV scale. Minimally, the gaugino masses are expected to obtain the AMSB form, but the presence of heavy vector-like states could alter those relations leading to a more compressed gaugino spectrum. Typically, the wino is expected to be the LSP, and the relic abundance may be produced either thermally or non-thermally due to late-decaying TeV-scale moduli fields.

2.7 Spread SUSY

In ref. \[26\], it is emphasized that there may exist a forbidden region on the scale of SUSY breaking $\tilde{m}$ such that if $\tilde{m} \gtrsim O(1)$ TeV, then LSP dark matter will be overproduced which can violate the anthropic bounds which disfavor/forbid DM overproduction in that the baryon-to-DM ratio may be insufficient for baryonic structure formation in the universe \[54\]. This forbidden region should persist up to $\tilde{m} \sim T_R$ where $T_R$ is the reheat temperature of the universe at the end of inflation. Higher values of $\tilde{m} > T_R$ are allowed in that SUSY particles wouldn’t be produced during the reheat process. Taking $\tilde{m} > T_R$ then leads to a very heavy SUSY spectrum (High Scale SUSY) whilst taking $\tilde{m} \sim 1$ TeV leads to Spread SUSY in the case of SUSY breaking via “charged” hidden sector fields (where scalars gain mass $\tilde{m}$ but gauginos and $A$ terms do not) or via uncharged hidden sector fields (which leads to all sparticles at $\tilde{m} \sim 10$ TeV, dubbed the “environmental MSSM”). The spread SUSY spectrum divides into two possibilities: 1. scalar masses $\tilde{m} \sim 10^5$ TeV with gauginos at $10^2$ TeV and higgsinos at $\sim 1$ TeV and 2. scalars around $10^3$ TeV with higgsinos and gravitinos $\sim 10^2$ TeV and gauginos $\sim 1$ TeV. Thus, the spread SUSY models typically have SUSY mass spectra spread across three mass scales.
2.8 G2MSSM

The G2MSSM labels the sort of SUSY spectra expected to emerge from 11-dimensional $M$-theory compactified on a manifold of $G_2$ holonomy [55, 56] which preserves $N = 1$ SUSY in the low energy 4-d effective field theory (LE-EFT). The LE-EFT then consists of the usual MSSM fields plus an assortment of moduli fields which are string remnants from the compactification. Scalar masses $\tilde{m}$ and the lightest modulus field are expected to gain masses of order the gravitino mass $m_3/2$ and in order to solve the cosmological moduli/gravitino problems then $\tilde{m} \sim 30 - 100\,\text{TeV}$. Gaugino masses are suppressed relative to scalars by a factor $\log(m_P/m_3/2) \sim 30$ so gauginos (and higgsinos) are expected at the 1-3 TeV range and may have comparable moduli/anomaly-mediated contributions. The LSP may be bino or wino-like but the relic density is seriously affected by non-thermal production via the late-decaying lightest modulus field [57]. In later renditions, the possibility of a hidden sector LSP is entertained [58].

2.9 Radiatively-driven/Stringy natural SUSY

In radiatively-driven natural SUSY (RNS) [35, 38], large high scale soft terms can be radiatively driven to small weak scale values. Then all weak scale contributions to the weak scale are of order the weak scale. This corresponds to $\Delta_{EW} \lesssim 30$. The RNS models can be generated from NUHM2 or NUHM3 models [35, 38], from generalized mirage mediation [59] and from natural anomaly-mediation [60]. As an example, we take a simple NUHM2 model with first/second/third generation GUT scale scalar masses $m_0(1, 2) = m_0(3) = 4.5\,\text{TeV}$, $m_{1/2} = 1\,\text{TeV}$, $A_0 = -7.2\,\text{TeV}$, $\tan\beta = 10$ with $\mu = 200\,\text{GeV}$ and $m_A = 2\,\text{TeV}$. The model has $m_3 \sim 2.4\,\text{TeV}$ (LHC safe) with $\Delta_{EW} = 12.8$ and $m_h = 124.3\,\text{GeV}$. The higgsino-like LSP is $m_\chi = 195.3\,\text{GeV}$ with $\Omega_\chi h^2 = 0.011$ (so room for additional axion dark matter).

While RNS models are typically slightly more natural for lower $m_0$ and $m_{1/2}$ values, we expect from the string landscape, under spontaneous SUSY breaking via a single $F$-term field distributed uniformly as a complex number throughout the landscape, a linear statistical draw to large soft terms [31]. For more SUSY breaking fields, the statistical draw goes as $f_{SUSY} \sim m_{soft}^{2n_F + n_D - 1}$ where $n_F$ is the number of hidden sector $F$ breaking fields and $n_D$ is the number of hidden sector $D$-breaking fields (the latter distributed as real numbers) [28–30].

Convolution of the statistical draw to large soft terms with the anthropic requirement that the derived weak scale lies within the ABDS window then leads to a probability distribution for $m_h$ that rises to a peak around $m_h \sim 125\,\text{GeV}$ [32] (in part because $A_0$ is also drawn to large (negative) values giving maximal stop mixing [63]) with sparticles typically beyond LHC reach. In this rendition, naturalness is replaced by what Douglas calls stringy naturalness [64], where a mode is more stringy natural if more landscape vacua lead to such a result. In stringy natural SUSY, a 3 TeV gluino is more (stringy) natural than a 300 GeV gluino [65]. The RNS benchmark given above is thus

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3 A computer code DEW4SLHA is available which computes the value of $\Delta_{EW}$ for any MSSM model listed in SUSY Les Houches Accord format [36].

4 In [61], it is found that a linear $n = 1$ soft term draw is obtained for KKLT [62] moduli-stabilization models.
highly stringy natural. Thus, under stringy naturalness, RNS models with LHC-compatible sparticle masses most commonly emerge from the landscape [66].

3 A scheme for computing relative probabilities from the landscape

The central question we wish to address is: how likely are various SUSY models (and the SM) to arise from the landscape? To answer this, we will restrict ourselves to string vacua containing the MSSM as the low energy EFT, and where SUSY breaking is mediated by gravity, i.e. spontaneous SUSY breaking in a $N = 1$ supergravity framework. [67] In such a SUGRA framework, scalar masses are generically non-universal [68–71] unless protected by some symmetry: e.g. the matter scalars of each generation fill the 16-dimensional spinor rep of SO(10) so one might expect these to have a common mass $m_0(i)$, $i = 1 - 3$ a generation index.\(^5\) Since the Higgs scalars come in split multiplets, there is no reason to expect $m_0(i) = m_{H_u,d}$ and thus we expect the LE-EFT to be a non-universal Higgs model (NUHM). This framework accommodates all of the high-scale and natural SUSY models under consideration here.\(^6\) While an absolute probability for any particular LE-EFT (including those not within the realm of the MSSM) is not possible to calculate (at least at this time), we can make estimates of relative probabilities amongst gravity-mediated MSSM models based on certain reasonable assumptions.

In table 1, we list a variety of supersymmetric models, along with the SM, and the proposed range for various first/second $m_0(1,2)$ and third generation $m_0(3)$ scalar masses, along with the expected range for gaugino and higgsino masses and the range of the light Higgs mass. In the last column we list the relative probability measure $P_\mu$ to be explained below. For the two SUSY models CMSSM and RNS, we have approximate supersymmetry extending all the way down to the weak scale. For the remainder of SUSY models, which include rather high mass scales $\tilde{m}$, we assume the heavy SUSY states are integrated out at scale $Q \simeq \tilde{m}$ which then destroys softly broken SUSY below the $\tilde{m}$ scale, so that quadratic divergences arise which are proportional to $\Lambda = \tilde{m}$ as in eq. (1.3). A pictorial comparison of the spectra from the various models is given in figure 1.

For the two SUSY models RNS and CMSSM, the dominant contribution to the weak scale can be extracted from the value of $\Delta_{EW}$. Then the pocket universe value of $m_{PU}^{Z^\ast}$ can be computed using eq. (1.4) as

$$\frac{(m_{PU}^{Z^\ast})^2}{2} = \frac{(m_{PU}^{Z^\ast}\sqrt{\Delta_{EW}})^2}{2} - \mu_{PU}^2$$

(assuming the dominant contribution dominates all other contributions to $(m_{PU}^{Z^\ast})^2$, which is usually the case.) Here, $m_{PU}^{Z^\ast} = 91.2 \text{ GeV}$, the value of $m_{Z}$ in our universe (OU). In most SUSY spectrum calculations, the value of the $\mu$ parameter is finetuned to ensure that $m_{Z}$ gains its measured value in our universe. However, in the multiverse, each pocket universe containing the MSSM as the LE-EFT will have a different value of $\mu_{PU}$ which

\(^5\)We regard the AMSB soft terms as included in the gravity-mediated soft terms.

\(^6\)For instance, in this framework, there is no known reason to favor the CMSSM model over any of the NUHM models.
the integrated distribution is indeed scale invariant. no preferred scale for String theory is the string scale, and all other scales likely arise dynamically: i.e. there is other from the string landscape. This seems reasonable in that the only scale inherent in 

\[ \mu_{KN} \]

with the measured fermion mass values. We will adopt the Donoghue et al. ansatz for the distribution uniformly across the decades of possible values, which appears to match well fact, Donoghue, Dutta and Ross \[ 75 \] make a convincing case that Yukawa couplings are

\[ R \]

discrete

has been computed in a particular well-motivated \( KN \) solution based on an anomaly-free symmetry. Thus, the \( KN \) acquires a vev of order

\[ S \]

the SUSY parameter arises in the superpotential as in the Kim-Nilles (KN) solution to

\[ \lambda \]

What is the likely distribution of SUSY \( \mu_{PU} \) parameters in the multiverse? Here, we assume the \( \mu \) parameter arises in the superpotential as in the Kim-Nilles (KN) solution to the SUSY \( \mu \) problem \[ 72 \],\(^7\) where we expect \( W \ni \lambda_\mu S^2 H_u H_d/\mu_{PU} \). The PQ charged field \( S \) acquires a vev of order \( f_\mu \sim 10^{11} \text{GeV} \) under PQ breaking so that a \( \mu \) parameter arises:

\[ \mu(KN) \sim \lambda_\mu f_\mu^2/\mu_{PU} \sim m_{\text{weak}}. \] \hspace{1cm} (3.2)

Thus, the KN \( \mu \) parameter has the form of a (Planck-suppressed) Yukawa coupling, in accord with the other Yukawa couplings which occur in the superpotential. But the question is: what sort of distribution for \( \mu \) would we expect on the landscape? For fixed \( \lambda_\mu \), this has been computed in a particular well-motivated KN solution based on an anomaly-free discrete \( R \)-symmetry \( \mathbb{Z}_4^R \) \[ 74 \]. However, for non-fixed \( \lambda_\mu \), this may well be different. In fact, Donoghue, Dutta and Ross \[ 75 \] make a convincing case that Yukawa couplings are distributed uniformly across the decades of possible values, which appears to match well with the measured fermion mass values. We will adopt the Donoghue et al. ansatz for the KN \( \mu \) parameter as well: that no particular scale for the \( \mu_{PU} \) value is favored over any other from the string landscape. This seems reasonable in that the only scale inherent in string theory is the string scale, and all other scales likely arise dynamically: i.e. there is no preferred scale for \( \mu_{PU} \). This corresponds to a landscape distribution \( f_\mu \sim 1/\mu \) so that the integrated distribution is indeed scale invariant.

\(^7\)For a recent review of twenty solutions to the SUSY \( \mu \) problem, see ref. \[ 73 \].
In figure 2, we show on the x-axis over 15 decades of possible values for $\mu_{PU}$. For the RNS model, where the maximal contribution to the r.h.s. of eq. (1.4) is bounded to lie within a factor a few of our measured value of the weak scale, then there is a substantial range of $\mu_{PU}$ values leading to $m_Z^{PU}$ lying within the (blue-shaded) ABDS window. We will take (quite arbitrarily) the lower limit of $\mu_{PU}$ to be $\sim 10$ GeV. Values of $\mu_{PU}$ (min) higher or lower by an order of magnitude from this value lead to differences in $P_\mu$ of a factor $\sim 2$ which is inconsequential for our purposes. The probability for a random value of $\mu_{PU}$ to give rise to $m_Z^{PU}$ within the ABDS window is then

$$P_\mu \equiv \log_{10}(\mu_{PU}\text{max})/\mu_{PU}\text{min})$$

(3.3)

i.e. the length of the interval of logarithmically distributed $\mu_{PU}$ values. Using this interval, we find $P_\mu(\text{RNS}) \sim 1.4$. 

Figure 1. Mass spectra from various unnatural and natural SUSY models as depicted in table 1.
Figure 2. Values of $m_{PU}^Z$ vs. $\mu_{PU}$ or $\mu_{SM}$ for various natural (RNS) and unnatural SUSY models and the SM. The ABDS window extends here from $m_{PU}^Z \sim 50 - 500$ GeV.

For the CMSSM benchmark model with $\Delta_{EW} = 2641$, then the maximal contribution to the r.h.s. of eq. (1.4) is well beyond the ABDS window. Thus, a finely-tuned value of $\mu_{PU}$ will be needed in order for $m_{PU}^Z$ to lie within the ABDS window, in accord with the atomic principle. One will have to live in the nearly vertical portion of the red CMSSM curve, for which the interval length is $P_{\mu}(CMSSM) \sim 0.005$. While the absolute values of $P_{\mu}$ don’t have a particular meaning (we don’t know the overall normalization), the ratios of probabilities do. In this case, we would expect the RNS model to be $P_{\mu}(RNS)/P_{\mu}(CMSSM) \sim 260$ times more probable on the landscape than the CMSSM benchmark model. In this case, the “natural” value for the weak scale in the case of the CMSSM benchmark model would be $m_{\text{weak}} \sim m_Z \sqrt{\Delta_{EW}} \sim 5$ TeV.

We can also calculate a value of $P_{\mu}$ for the SM, assuming the SM is valid all the way up to a scale $Q \sim m_{GUT}$ as is assumed in estimates of the SM vacuum stability [76]. Here, we will also assume that $\mu_{SM}$ has a scale invariant distribution so that the x-axis of figure 2 pertains to $\mu_{SM}$ of eq. (1.2) as well as to $\mu_{PU}$. Taking the value of $m_{PU}^Z \sim m_h$, we can use eq. (1.2) to plot the value of the weak scale in the SM. The plot is shown in figure 2 as the SM curve. Here, we see a value of $\mu_{SM} \sim 10^{15}$ GeV is needed for $m_{PU}^Z(\text{SM})$ to lie within the ABDS window while the natural value of $m_{PU}^Z(\text{SM})$ is $\sim 10^{15}$ GeV. This shows the extreme finetuning needed by the SM in order to ensure the weak scale lies within the ABDS window. We can compute $P_{\mu}(\text{SM})$ and find it to be $\sim 7 \cdot 10^{-27}$, that is the RNS model about $10^{26}$ times more likely than the SM to emerge from the landscape.
We can now also compute $P_\mu$ values for the various high-scale SUSY models listed in table 1. The key point here is that quadratic divergences still cancel out at energy scales $Q > \tilde{m}$. But once $Q$ drops below $\tilde{m}$, then we must integrate out the heavy sparticles in the LE-EFT and the quadratic divergences no longer cancel. Then we may use the uncanceled terms in eq. (1.3) to compute corrections to the Higgs mass, again with $m^{PU}_Z \approx m_h$. For most of these models, we take $\Lambda \sim \tilde{m} = m_0(3)$ to compute the curves of $m^{PU}_Z$ vs. $\mu_{SM}$, where now the Higgs potential is that of the SM for $Q < \tilde{m}$.

The various curves are shown in figure 2 for the assorted high scale SUSY models of table 1. We can then extract the values of $P_\mu$ for each case. As an example, Split SUSY with $m_0(3) \sim 10^6$ TeV gives $P_\mu \sim 7 \cdot 10^{-12}$ so that RNS is $\sim 10^{12}$ times more likely than Split SUSY to emerge from the landscape. Lest one be dismayed by the low relative probability for Split SUSY to emerge from the landscape, it is worth noting that the Split SUSY benchmark is $\sim 10^{15}$ times more likely to emerge from the landscape than the SM (when the SM is valid up to $Q = m_{GUT}$). Scaling $\tilde{m}$ to lower values in order to accommodate the measured value of $m_h$ as in mini-split helps matters somewhat: in this case, mini-split with a wino LSP and $\tilde{m} \sim 10^2$ TeV has $P_\mu \sim 4 \cdot 10^{-4}$, so the RNS benchmark is more likely to emerge than the mini-split benchmark by a factor $\sim 3000$.

4 Conclusions

In this paper we examined the question: are finetuned or natural models of particle physics more likely to emerge from selection of string vacua within the multiverse? We required as the anthropic condition that the atomic principle be fulfilled in that the derived value of the weak scale lies within the ABDS window. We also assumed a scale invariant distribution for the superpotential $\mu$ parameter in that for string theory with only a single mass scale, the string scale, all other scales are equally likely. We assumed the same distribution for $\mu_{SM}$. For natural models with $\Delta_{EW}$ below say 30, then all contributions to the weak scale lie within the ABDS window. In this case, then a wide range of $\mu_{PU}$ values still lead to $m^{PU}_Z$ within the ABDS window (since no finetuning of $\mu_{PU}$ is required). For unnatural models with large contributions to the weak scale, then only tiny ranges of $\mu_{PU}$ or $\mu_{SM}$ are allowed in order for $m^{PU}_Z$ to lie within the ABDS window.

Basically, particle physics models which require electroweak finetuning may be possible on the landscape, but for a uniform distribution of the tuning parameters, they are likely to be rare because the finetuned solutions should be rare on the landscape relative to natural models. This seems like common sense, but apparently contradicts the common wisdom in the literature which asserts that the string landscape provides motivation to take finetuned models as a plausible possibility since the cosmological constant also appears to be finetuned. The origin of the discrepancy may be traced to how the anthropic condition is implemented. In many early works (e.g. [19, 28]), the anthropic penalty (finetuning factor) is given as a factor $(m_{weak}/m_{soft})^2$ which favors $m_{soft} \sim m_{weak}$ but also allows for $m_{soft} \gg m_{weak}$. This finetuning factor can be overwhelmed by a landscape draw to large

\footnote{For the SM parameter values entering eq. (1.3) in the case of high scale SUSY models with scale boundary $\tilde{m}$, we use FlexibleSUSY and FlexibleEFTHiggs to extract the appropriate values [77–79].}
soft terms $m_{n}^{soft}$ with $n \geq 3$, thus favoring high scale SUSY breaking. In contrast, we require as the anthropic condition that the derived value of the weak scale lies within the ABDS window. In our method, if any contribution to the weak scale in a given model is far beyond the measured value of $m_{weak}$, then only a teensy range of $\mu_{PU}$ values are allowed to regain the ABDS window. This is in contrast to natural models where a wide range of $\mu_{PU}$ values lead to $m_{weak}$ within the ABDS window (since no finetuning is needed).

We examined the relative probabilities of various natural and finetuned SUSY models, and the SM, to emerge from the landscape via a computable measure of relative probabilities. A summary of our results is shown in figure 3 where we present a bar chart of the relative probabilities of the various benchmark models considered here in relation to the radiatively-driven natural SUSY benchmark where a wide range of $\mu_{PU}$ values lie within the ABDS window. Basically, the more finetuned a model is, the less likely it is to emerge from the landscape in comparison to particle physics models with electroweak naturalness.

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