Walking at the drip line

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Abstract. Among exotic nuclei those at the drip line which are unstable against neutron emission are particularly interesting because they convey information on the nuclear force in the most extreme situations. Strictly speaking they are not "nuclei" but they exist thanks to long living resonances between a neutron and a bound "core" nucleus. Adding one more neutron they become bound and are called "borromean". Being particularly exotic they have attracted much attention in past years, see for example Refs.[1, 2, 3]. One very challenging example is \(^{13}\)Be whose level ordering has been discussed in a large number of papers in which it has been studied by transfer [4] and fragmentation experiments [5]-[11], or it has been discussed theoretically[12]-[19]. Although projectile fragmentation spectra show evident similarities, the interpretations of data all differ from each other. In this paper we argue that a way trough the problem could be to try to establish first, or at the same time, the quite elusive "nature" of the second s-state in the Beryllium isotopes with A=9-14. On the other hand there are other recent neutron removal experiments leading to nuclei unstable by one or more proton emissions [20], and thus somewhat mirror to borromean nuclei, performed with nuclei close to the proton drip line. It has been shown that by taking in coincidence all (charged) particles but the removed neutron, reconstructing the invariant mass and gating on the ground state peak, it is possible to obtain the longitudinal momentum distribution of the unbound "core". One can link it to the original wave function of the bound orbital and thus determine the initial neutron angular momentum from the shape of the distribution and the initial occupation probability from the absolute removal cross section. Then it is clear that modern experiments and theories are able to study unstable nuclei with the same degree of accuracy as stable nuclei. Such a line of research offers a great potential for numerous further studies beyond the drip line.

1. Introduction

Some time ago we thought we had established the level ordering of \(^{13}\)Be and \(^{14}\)Be on a firm basis using a time dependent projectile fragmentation model [3] [17], comparing to experimental data [9] and using structure inputs obtained from a semi-phenomenological core-vibration coupling model of two-neutron halo nuclei [18]. Thus from the shell ordering the neutron-core interaction was determined which is needed by any three body model of borromean nuclei. As a consistency test we showed that the energy spectra of some Beryllium isotopes obtained using pp-RPA predict the same shell ordering as extracted from the reaction model vs. data analysis [3] [17] [18].

Projectile fragmentation is used here to study a two neutron halo projectile \(^{14}\)Be, as in Refs.[5]-[16]. The two-neutron emission is supposed to be due to the nuclear interaction with the target. The mechanism is such that one neutron is stripped by the target, taken away from the elastic channel and not measured in coincidence with the core. The second neutron however re-interacts with the core which, is going to be \(^{12}\)Be in the case of the projectile fragmentation.
of $^{14}$Be, since $^{13}$Be is not bound. For a decaying s-state this mechanism has also been described by the sudden approximation in Ref. [19]. The sudden method leads to a transition probability independent of the impact parameter.

Another type of experiment consists in neutron removal leading to nuclei unstable by one or more proton emissions [20], performed with nuclei close to the proton drip line. It has been shown that by taking in coincidence all (charged) particles but the removed neutron, reconstructing the invariant mass and gating on the ground state peak, it is possible to obtain the longitudinal momentum distribution of the “unbound” core. One can link it to the original wave function of the bound orbital and thus determine the initial neutron angular momentum from the shape of the distribution and the initial occupation probability from the absolute removal cross section. We will show in the following some preliminary experimental spectra and the relative theoretical description. Comparison between experimental and theoretical cross sections can provide information on the single particle occupation in the initial state. These reactions will be described by the transfer-to-the-continuum model [21]-[24], using a recently developed n-$^9$Be optical potential [24] and a core-$^9$Be folding potential [25].

2. Projectile Fragmentation and Transfer-to-the-Continuum

We calculate breakup by a semiclassical approximation in which the dynamics is controlled by the three potentials describing nucleon-core, nucleon-target, and core-target interactions. The collision is described in terms of only the three-body variables of nucleon coordinate, projectile coordinate, and target coordinate. The projectile-target relative motion is treated semiclassically by using a trajectory of the center of the projectile relative to the center of the target. The formalism is then valid for incident energies above the Coulomb barrier. To first order such excitations can be described along this trajectory by a time dependent perturbation amplitude [21]-[26] for a transition from a nucleon state bound in the projectile, to a final continuum state. The radial part of the single particle initial state wave function is calculated in a potential $V_{WS}(r)$ which is fixed in space and chosen such as to reproduce the experimental valence particle separation energy. For the proton emitters nuclei of the second part of this paper our wave functions are also fitted to ab initio overlaps calculated with the Variational Montecarlo Method [28]. The role of the target is just to perturb the initial bound state wave function and to allow the transition to the continuum by transferring some momentum to the neutron. For the description of the neutron-core relative energy spectrum used to study $^{13}$Be it is enough to choose a simplified form of the interaction, such as a delta-function potential $V(r) = v_2 \delta(x) \delta(y) \delta(z)$. The value of the strength $v_2 \equiv [\text{MeV fm}^3]$ used in the calculation is taken equal to the volume integral of the appropriate neutron-target interaction. The coordinate system and all other details of the calculations can be found in Ref.[17]. On the other hand, the sudden approximation has been followed in Ref.[19] leading to a simplified formalism. Our formalism [17] reduces to that of [19] for a s-to-s transition in the sudden limit. However our approach is not restricted to transitions with conservation of the angular momentum.

For the inclusive transfer-to-the-continuum (TC) reactions which correspond to the measurement of the “core” longitudinal distribution a continuum final state in which the neutron interacts with the target via an optical potential [24] is taken and thus both elastic and inelastic breakup contribute. In the projectile fragmentation formalism instead only the elastic breakup contributes as neutron and core are taken in coincidence which means the neutron energy is not degraded by the interaction with the target.

3. Structure and Reactions of $^{13-14}$Be

The ground state of $^{14}$Be has spin $J^\pi = 0^+$. In a simple model assuming two neutrons added to a $^{12}$Be core in its ground state the wave function is: $|^{14}Be\rangle = |b_1(2s_{1/2})^2 + b_2(1p_{1/2})^2 + b_3(1d_{5/2})^2\rangle \otimes |^{12}Be, 0^+\rangle$. Then the bound neutron can be in a 2s, 1p$_{1/2}$ or 1d$_{5/2}$ state. However
the situation is much more complicated [12]-[14] and in particular the calculations of Ref. [15] show that there is a large component (2\text{s}_{1/2}, 1\text{d}_{5/2}) \otimes |^{12}\text{Be}, 2^+\rangle with the core in its low energy 2^+ state which can modify the neutron distribution.

To describe the valence neutron in the $^{13}\text{Be}$ continuum we assume that the single neutron hamiltonian with respect to $^{12}\text{Be}$ has the form $h = t + U$ where $t$ is the kinetic energy and $U(r) = V_{WS} + \delta V$ is the real part of the neutron-core interaction. $V_{WS}$ is a Woods-Saxon potential plus spin-orbit whose parameters are given in Ref.[17], and $\delta V$ is a correction $\delta V(r) = 16\alpha e^2 (r-R)/a(1 + e^{(r-R)/a})^4$ which originates from particle-vibration couplings [13] evaluated using Bohr and Mottelson collective model of the transition amplitudes between zero and one phonon states. Such couplings are relevant only for the low energy spectrum. Values for the parameter $\alpha$ are given in Table 1. Therefore our structure model is not a simple single-particle in a potential model but contains in it the full complexity of single-particle vs. collective couplings. If good fittings of experimental data will be obtained, then the parameters of a semi-phenomenological potential can be deduced and linked to a more microscopic model. Results obtained including the s, p and d initial states in $^{14}\text{Be}$ leading to unbound $^{13}\text{Be}$ states are shown in the LHS of Fig.1. Each curve corresponds to just one transition as indicated. The solid curve is the sum of all transitions from the s-bound state which gives the dominant contribution to the transition probability due to the long tail of the wave function. To make them visible some curves have been multiplied by a factor of five as indicated in the legenda. In the RHS figure we compare the s to s cross sections from our model with the results of the sudden formula used in [19]. Large differences are seen and the sudden formula seems to over predict the overall strength of the transitions. The s-threshold state in $^{13}\text{Be}$ has a very small scattering length and it is probably a complicated state coupled to an excited core. Finally to compare with available data, we show at the bottom of Fig. 1 the experimental points from H. Simon et al [9] for the reaction $^{14}\text{Be} + ^{12}\text{C} \rightarrow n + ^{14}\text{Be} + X$ at 250 A.MeV and our calculations. More details in the legenda and in Refs. [17].
Figure 1. Top LHS: Individual transitions from the bound s,p,d components of the initial wave function to the unbound s,p,d states. Only the contribution from the initial s-orbital are summed to give the full curve. Top RHS: n-^{12}\text{Be} relative energy spectrum; comparison of fragmentation method for an s to s transition with results of the sudden formula using the same optical model phase shifts corresponding to scattering lengths as indicated. Bottom center: Sum of all transitions from the s initial state for the reaction [8] ^{14}\text{Be}+^{12}\text{C} \to n+^{12}\text{Be}+X. Dashed line is the folding of the calculated spectrum with the experimental resolution curve.

On the other hand with the s and p single particle states given in Table 1, pp-RPA calculations have been performed [3] and [18]. Some of those results are reported here in Table 2 using two hypothesis for the level ordering. The best results are obtained for the s(bound)-p(unbound) inversion case, in particular notice the RPA amplitudes, given in Ref. [18] which are closely related to the spectroscopic factors and give a hint on the occupation probabilities of the orbitals. It is evident that the mixing of s-p-d states is very strong in both ^{12}\text{Be} and ^{14}\text{Be} and that these nuclei have a dynamical nature far from the traditional shell ordering and closure concepts. Furthermore, notice that ^{13}\text{Be} is the first nucleus in which a s-p inversion is shown to happen across threshold.

An alternative way of solving the problem of the level scheme of ^{13}\text{Be} would be to try to understand and establish consistently the ”nature” and evolution of the second s-state in the Beryllium isotopes with A=9-14, see also Ref. [27]. As one can see from the level schemes of ^{9,10,11}\text{Be} [29] the position and contribution of the second s-state is either unknown or not well understood. In particular some recent efforts are illustrated in Fig. [2]. We report: i) top LHS, a table from Ref. [30] with the parameters extracted for the possible (1/2)^+ resonant state in ^{9}\text{Be} from recent (\gamma,n) and (e,e') experiments, where the authors tried to established whether the state is a single particle virtual state or a many-body resonance; ii) top RHS, the effective single particle energies for the 1s and 0d_{5/2} states in the N=9 isotones presented in Ref. [32], in
blue our extrapolation for $^{13}\text{Be}$; iii) at the bottom of the figure, the bound states obtained by the dispersive optical model (DOM) for $^{9}\text{Be}$ [31]; there is no hint of s-state in this case which suggests the possibility of a very wide resonance.

4. Proton emitters
As we mentioned in the Introduction we are now studying the possibility to obtain information on the shell structure and single particle properties of very light exotic nuclei which after one neutron knockout are unbound by one or more proton emissions. The relevant experiments have been performed by the HiRA collaboration at MSU and TEXAS AM laboratories. Details on the experiment will be given elsewhere [35]. Table 3 gives the relevant structure information on one of the cases under analysis. Such inputs were used in reaction calculations done with the TC model [21, 22, 23] and the optical potentials of Refs. [24, 25]. Fig. 3 shows the invariant mass spectrum and parallel momentum distribution of the $^{9}\text{Be}(^{6}\text{Li},^{5}\text{Li})X$ reaction at 36.6 A.MeV [20]. The calculated spectrum, obtained by summing both p-contributions, and convoluted with the experimental resolution function is shown by the green curve. It is in very good agreement with the data shown in black. In particular notice both the kinematical cut and the long tail at large

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**Figure 2.** Top, from LHS to RHS: table from Ref. [30] with the parameters extracted for the possible $(1/2)^+$ resonant state in $^{9}\text{Be}$ from recent $(\gamma,n)$ and $(e,e')$ experiments; effective single particle energies for the N=9 isotones [32], in blue our extrapolation for $^{13}\text{Be}$. Bottom: bound states obtained by the dispersive optical model (DOM) for $^{9}\text{Be}$ [31].

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**Table 1:**-$R$-matrix parameters ($E_R, \Gamma_R$) extracted in recent experiments using real and virtual photons and the corresponding positions ($E_{res}$) and the widths ($\Gamma$) of the $^{9}\text{Be} 1/2^+$ resonant state.

| Ref. | $E_R$ (keV) | $\Gamma_R$ (keV) | $E_{res}$ (keV) | $\Gamma$ (keV) |
|------|-------------|------------------|-----------------|----------------|
| [2]  | 83(6)       | 274(8)           | -31(6)          | 153(4)         |
| [3]  | 70(3)       | 225(12)          | -21             | 132            |
| [4]  | 66(2)       | 213(6)           | -21             | 124            |

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**Figure 3.** (Color online) Effective single particle binding energies and $0d_{5/2}$ orbitals as a function of Z, for N = 9 isotones. The $^{13}\text{B}$ points are from the present measurement, and the lines are to guide the eye.
Table 3. Structure parameters for the indicated overlap from shell model [33] and VMC [34]. Given are the neutron ($S_n$) separation energy, the asymptotic normalization constants (ANC) and the spectroscopic factors (SF). VMC overlaps are normalized to the SF, shell model’s to 1.

| $^6Li$ | $^5Li$ | $S_n$ [MeV] | ANC$_{WS}$ | SF$_{SM}$ | ANC$_{VMC}$ | SF$_{VMC}$ |
|-------|-------|-------------|-------------|-----------|-------------|-----------|
| $^6Li$ | $^5Li$ | 5.66        | 2.85        | 2.89      | P$_{1/2}$   | 0.3301    |
|       |       |             |             |           | P$_{3/2}$   | 0.3384    |
|       |       |             |             |           |             |           |

and small $P_\parallel$ respectively are perfectly reproduced by the reaction model. Also the calculated cross section for elastic and inelastic breakup is 38.5mb vs an experimental value of 38.1mb. The neutron wave function has been calculated in a Wood-Saxon plus spin orbit potential, fitting the separation energy such that the ANC would agree with the ANC of the VMC model. The core-target S-matrix has been parametrized by a smooth cutoff function with strong absorption radius $R_s=5.8$fm. We have excluded the possibility that the weakly bound proton ($S_p=4.6$MeV) would breakup before the neutron according to the model presented in Ref. [36], by calculating the proton breakup probability with the appropriate VMC wave function.

Figure 3. Invariant mass spectrum and parallel momentum distribution of the $^9Be(^6Li,^5Li)X$ reaction at 36.6A.MeV[20].

5. Conclusions and Outlook

From the structure point of view, in the search for the dripline position, a very important role is played by the study of nuclei unstable by neutron emission. Results of projectile fragmentation theory for elastic breakup have been presented together with structure calculations of $^{12}-^{14}Be$. We have introduced also the study of neutron removal giving rise to an unbound core which emits protons in very light nuclei. They are somewhat mirror to borromean nuclei. For such nuclei one can use both the shell model and advanced $ab\ initio$ model wave functions in reaction calculations and comparisons are in progress. This is one of the most important subjects of study within RIB physics which need to be addressed and developed further in the near future and for which some suggestions of possible structure and reaction formalisms applied to state-of-the-art experiments have been discussed.
Acknowledgements. I wish to acknowledge the collaboration with G. Blanchon, D. M. Brink, A. García-Camacho and the late N. Vinh Mau for the work presented in the first part of this contribution and with R. J. Charity and G. Salvioni for the work presented in the second part.

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