Identification of a Scalar Glueball

M. Albaladejo and J. A. Oller

Departamento de Física, Universidad de Murcia, E-30071 Murcia, Spain

We have performed a coupled channel study of the meson-meson S-waves involving isospins (I) 0, 1/2 and 3/2 up to 2 GeV. For the first time the channels \( \pi \pi, K \bar{K}, \eta \eta, \sigma \sigma, \eta' \eta', \rho \rho, \omega \omega, \phi \phi, a_1 \pi + \tau \tau \) and \( \pi^+ \pi^- \) are considered. All the resonances with masses below 2 GeV for \( I = 0 \) and 1/2 are generated by the approach. We identify the \( f_0(1710) \) and a pole at 1.6 GeV, which is an important contribution to the \( f_0(1500) \), as glueballs. This is based on an accurate agreement of our results with predictions of lattice QCD and the chiral suppression of the coupling of a scalar glueball to \( \bar{q}q \). Another nearby pole, mainly corresponding to the \( f_0(1370) \), is a pure octet state not mixed with the glueball.

PACS numbers: 11.80.Gw, 12.39.Fe, 12.39.Mk

1. QCD, the present theory of strong interactions, is a non-abelian Yang Mills theory so that gluons carry colour charge and interact between them. It is generally believed that QCD predicts the existence of mesons without valence quarks, the so called glueballs. Its confirmation in the spectrum of strong interactions is then without valence quarks, the so called glueballs. Its confirmation in the spectrum of strong interactions is then

2. The multipion states, which play an increasing role for energies above \( \sigma \sigma \approx 2 \text{ GeV} \), are mimicked through the \( \eta \eta \) channel we follow a novel method including any new free parameter. This can be done because the \( \sigma \) corresponds to a pole due to the interactions between two pions in the \( I = 0 \) S-wave, \((\pi\pi)_0\).
For the interaction kernel $N_{i,4}$ one starts by calculating from the Lagrangians $L_2$ and $L_3$ the tree level amplitude $T_{i,4}^{2+S}$ for $i \rightarrow (\pi \pi)_0 (\pi \pi)_0$. To take into account the pion final state interactions, $T_{i,4}^{2+S}$ is multiplied by the factor $\prod_{k=1}^m 1/D(s_k)$, with $m$ the number of $\sigma$'s in the scattering process (2 or 4) and $s_k$ the total centre of mass (CM) energy squared of the $k_{th}$ pair. We use here that the rescattering of two $I = 0$ S-wave pions from a production kernel is given by the factor $1/D = 1/(1 + V_2 g_1)$, with $V_2 = (s - m_\pi^2/2)/f^2$ [10]. To isolate $N_{i,4}$ one takes the limit (for definiteness $i \neq 4$)

$$\lim_{s_1, s_2 \rightarrow -s} \frac{T_{i,4}^{2+S}}{D_{II}(s_1)D_{II}(s_2)} = \frac{N_{i,4}g_{\pi\pi}^2}{(s_1 - s_\sigma)(s_2 - s_\sigma)}.$$  (1)

Where the subscript $II$ indicates that the corresponding function is calculated on the second Riemann sheet (with the sign reversed in the definition of the pion three-momentum), $s_\sigma$ is the pole position and $g_{\pi\pi}$ is its coupling to $\pi\pi$. Performing the Laurent expansion around $s_\sigma$ of $1/D_{II}(s) = \alpha_0/(s - s_\sigma) + \ldots$ the evaluation of $N_{i,4}$ from eq.(1) requires the ratio $(\alpha_0/g_{\pi\pi})^2$. Since $g_{1,II}(s_\sigma) = -f^2/(s_\sigma - m_\pi^2/2)$ at $s_\sigma$, where $1 + V_2 g_1 = 0$, and taking $T_{II} \simeq V_2/(1 + V_2 g_1)$, appropriate for these energies [10], then $(\alpha_0/g_{\pi\pi})^2 = f^2/(1 - \frac{dg_{1,II}}{ds_\sigma}|_{s_\sigma = (s_\sigma - m_\pi^2/2)^2}) \approx f^2$. In this way, $N_{i,4} = T_{i,4}^{2+S} f^2$, $i \neq 4$, and $N_{i,4} = T_{i,4}^{2+S} f^4$. Using $N_{i,4}$ evaluated with $s_k = s_\sigma$ violates unitarity because $s_\sigma$ is complex and $N_{i,4}$ must be real. Instead, we interpret the width of the $\sigma$ resonance as a Lorentzian mass distribution around its nominal mass value $\sim 450$ MeV with a width $\sim 500$ MeV. In this way the $\sigma$ masses ($\sqrt{s_k}$) used to calculate the functions $N_{i,4}$ and $g_4$ are folded with the previous mass distribution. Similarly, for the $\rho\rho$ state $g_7$ is also convoluted with a $\rho$ mass distribution.

3. We fit our 12 free parameters to 370 data points from threshold up to 2 GeV. The data comprise the $I = 0$ S-wave $\pi\pi$ phase shifts $\eta_0$, the elasticity $\eta_0 = |S_{1,1}|$, the $I = 0$ S-wave $\pi\pi \rightarrow K\bar{K}$ phase shifts $\delta_{1,2}$ and modulus $|S_{1,2}|$, the S-wave contribution to the $\pi\pi \rightarrow \eta\eta'$, $\eta\eta$ event distributions and the phase ($\phi$) and modulus ($A$) of the $K^-\pi^+ \rightarrow K^-\pi^+$ amplitude from the LASS data. The S-matrix element $S_{i,j}$ is given by $S_{i,j} = \delta_{i,j} + 2i\sqrt{p_i^2 T_{i,j}^{(f)}}$, where $p_i = q_i/8\pi\sqrt{s}$ and $q_i$ is the CM three-momentum for channel $i$. In order, these data are shown on the first eight panels of Fig.1 from top to bottom and left to right. For $\sqrt{s} \leq m_K$ in the $\delta_{i,j}^f$ panel we have the inset showing in detail the precise data from $K_{\pi\pi}$ decays. The reproduction of the data is fair, as shown in the figure. The dashed lines on the first eight panels include the $a_1\pi$ and $\pi^+\pi$ states, while the solid ones do not. The similarity between both curves indicates that these channels give small contributions. The width of the band represents our systematic uncertainties at the level of two standard deviations, $n_\sigma = \Delta \chi^2/(2\chi^2)^{1/2}$ [11]. Compared with other works [12–14] we determine the interaction kernels from standard chiral Lagrangians, avoid ad-hoc parameterizations, include many more channels and fewer free parameters are used. For $I = 1/2$ the $\kappa$ pole is.

FIG. 1. Fit to experimental data. More details are given in the text.
located at \((708 \pm 6 - i 313 \pm 10) \text{ MeV}\), the \(K_0^*(1430)\) at \((1435 \pm 6 - i 142 \pm 8) \text{ MeV}\) and the \(K_0^*(1950)\) at \((1750 \pm 20 - i 150 \pm 20) \text{ MeV}\), similarly to ref. [8]. For \(I = 0\) one has the \(f_0^0(600)\) or \(\sigma\) at \((456 \pm 6 - i 241 \pm 7) \text{ MeV}\) and the \(f_0^0(980)\) at \((983 \pm 4 - i 25 \pm 3) \text{ MeV}\). There are poles at \((1690 \pm 20 - i 110 \pm 20) \text{ MeV}\), corresponding to the \(f_0^0(1710)\), and at \((1810 \pm 15 - i 190 \pm 20) \text{ MeV}\), with mass and width in agreement with those reported for the \(f_0^0(1790)\) by BESII. In the PDG [2] the width for the \(f_0^0(1710)\) is \(137 \pm 8 \text{ MeV}\), much smaller than \(220 \pm 40 \text{ MeV}\) from the given pole position. However, we have checked that on the real axis the value of the width corresponding to the half-maximum for the partial waves with prominent \(f_0^0(1710)\) peaks is just \(160 \text{ MeV}\) [15]. This reduction is due to the opening of several channels along the resonance region and the agreement with the PDG is restored. The other poles at \((1466 \pm 15 - i 158 \pm 12) \text{ MeV}\) and \((1602 \pm 15 - i 44 \pm 15) \text{ MeV}\), connected with the \(f_0^0(1370)\) and \(f_0^0(1500)\), are referred in the following as \(f_0^R\) and \(f_0^R\), respectively. Despite that we have included only three bare resonances in \(I = 0\) we have generated six. The poles are located on the unphysical Riemann sheets that connect continuously with the physical one for some interval along the real \(s\)-axis. Note that the pole \(f_0^R\) does not influence the physical axis beyond the \(\eta\eta'\) threshold at 1505 MeV, since above this energy a different Riemann sheet is the one that matches with the physical \(s\)-axis. This effect typically gives rise to a pronounced signal at the \(\eta\eta'\) threshold and this is the reason for the \(f_0^0(1500)\) mass, \((1505 \pm 6) \text{ MeV}\) [2]. If a physical amplitude is dominated by the \(f_0^R\) pole, then its peak at 1505 MeV has an effective width larger than the one from the pole position, 88 MeV. This is so because given a Breit-Wigner located at the position of the \(f_0^R\) pole the energy interval below \(1.5 \text{ GeV}\) at which half the modulus squared at \(1.5 \text{ GeV}\) is reached is \(\delta = 1.2 \Gamma = 105 \text{ MeV}\), the width of the \(f_0^0(1500)\) [2]. The \(f_0^0(1370)\) is mainly given by the \(f_0^R\) pole, though its precise shape is sensitive to \(f_0^R\) for those channels that couple strongly with the latter. In Fig.1 we also show in the last two rows data from \(pp\) inelastic scattering at 450 \text{ GeV/c}\) and \(\bar{p}p\) annihilation by the WA102 and Crystal Barrel (CBC) Collaborations, respectively. We have fitted the WA102 data using a coherent sum of Breit-Wigner functions and a non-resonant term, similarly as done by the WA102 Collaboration [16]:

\[
\begin{align*}
  &i) \sqrt{s} < m_\eta + m_{\eta'} \rightarrow A = \{ \sigma, f_0(980), f_0^R \}, \\
  &A(\sqrt{s})_i = NR(\sqrt{s}) + \sum_{j \in A} \frac{a_j e^{i \theta_j} g_{j;i}}{M_j^2 - s - i M_j \Gamma_j}, \\
  &ii) \sqrt{s} > m_\eta + m_{\eta'} \rightarrow B = \{ \sigma, f_0(980), f_0(1710), f_0(1790) \}, \\
  &A(\sqrt{s})_i = NR(\sqrt{s}) + \sum_{j \in B} \frac{a_j e^{i \theta_j} g_{j;i}}{M_j^2 - s - i M_j \Gamma_j}.
\end{align*}
\]

where \(a_j\) and \(\theta_j\) are the amplitude and the phase of the production vertex of the \(j\thinspace th\) resonance, \(M_j, \Gamma_j\) and \(g_{j;i}\) are, respectively, the mass, width and the coupling to channel \(i\) of the same resonance. The latter is determined from the residue of the partial waves at the pole position. In addition, \(m_k + m_l\) is the threshold for the channel \(i\) and \(\alpha, \beta, \gamma, \delta\) are real parameters. The form of the non-resonant term is taken from the WA102 Collaboration [16]. The constant \(r_i\) is fixed so as the amplitude \(A(\sqrt{s})_i\) is continuous at \(m_\eta + m_{\eta'}\). As explained above, once the \(m_{\eta'}\) threshold is crossed one has to consider other Riemann sheets which do not have the \(f_0^R\) and \(f_0^R\) poles but the \(f_0^0(1710)\) and \(f_0^0(1790)\) ones. Above \(m_{\eta'}\) the \(\sigma\) and \(f_0(980)\) give tiny contributions. \(\Gamma_j\) in eq.(2) is the largest between its value from the pole position and the one calculated by summing the partial decay widths \(\Gamma_j = \theta(\sqrt{s} - m_k - m_l)\lambda_j |g_{j;i}|^2 q_i/(8\pi M_j^2)\), with \(\lambda_j = 1/2\) for identical particles. Eq.(2) incorporates important new facts compared to the analyses of the WA102 Collaboration. First, the pole positions for the different resonances are those already determined from our study of the scattering data on the first 8 panels of Fig.1. Let us stress that these observables only involve two particles in the final state and their analysis is theoretically cleaner. Second, the couplings \(g_{j;i}\) are similarly fixed. Third, the \(a_j\) and \(\theta_j\) parameters are the same for all the WA102 reactions, that are fitted simultaneously. For the Crystal Barrel data on \(pp\) annihilation we also use eq.(2) but without \(NR(\sqrt{s})\). A good reproduction of the data results. In \(\bar{p}p \rightarrow n^0\eta\eta\) one observes a broad bump for the \(f_0(1370)\) and a prominent peak for the \(f_0(1500)\), that also gives strong signals in the WA102 data. Other peaks are observed for the \(\sigma\), \(f_0(980)\) and \(f_0(1710)\). The latter is important for the threshold in \(pp \rightarrow pp\eta\eta\) above 1.5 GeV.

| GeV  | \(f_0(1370)\) | \(f_0^R\) | \(f_0(1710)\) |
|------|----------------|----------|----------------|
| \(g_{s+s^-}\) | 3.59 ± 0.16 | 1.30 ± 0.22 | 1.21 ± 0.16 |
| \(g_{K^+K^-}\) | 2.23 ± 0.18 | 2.06 ± 0.17 | 2.0 ± 0.3 |
| \(g_{\eta\eta}\) | 1.7 ± 0.3 | 3.78 ± 0.26 | 3.3 ± 0.8 |
| \(g_{\eta\eta'}\) | 4.0 ± 0.3 | 4.99 ± 0.24 | 5.1 ± 0.8 |
| \(g_{\eta'\eta'}\) | 3.7 ± 0.4 | 8.3 ± 0.6 | 11.7 ± 1.6 |

TABLE I. Couplings of the \(f_0(1370)\), \(f_0^R\) and \(f_0(1710)\).
tified as the $f_0(1370)$), $f_0^R$ and $f_0(1710)$ poles to the two pseudoscalar channels. We observe that the couplings of the $f_0^R$ and $f_0(1710)$ are quite similar. This is so because the two poles coalesce in the same one when moving continuously from the sheet of one of them to the one of the other. They correspond to the same underlying resonance, but split in two due to the interaction in coupled channels. From the couplings of the $f_0(1710)$ one can calculate the branching ratios $\Gamma(K\bar{K})/\Gamma_{\text{total}} = 0.36 \pm 0.12(0.38^{+0.09}_{-0.19}), \Gamma(\eta\bar{\eta})/\Gamma_{\text{total}} = 0.22 \pm 0.12(0.18^{+0.03}_{-0.01})$, and $\Gamma(\pi\pi)/\Gamma(K\bar{K}) = 0.32 \pm 0.14(<0.11)$, where the values of the PDG are given between brackets. The values are compatible within one sigma. We also obtain that the $f_0(1790)$ has a small $KK$ coupling, and this is a major difference with respect to the $f_0(1710)$ as stressed by BESII. The couplings of the $f_0^R(1370)$ in table I correspond to the pure $I = 0$ octet member $(uu+dd-2ss)/\sqrt{6}$ because they are very close to the tree level ones $g_{+\pi} = 3.9, |g_{K^0\bar{K}^0}| = 2.3, |g_{\eta\bar{\eta}}| = 1.4, |g_{\eta\eta'}| = 3.7, |g_{\eta'\eta'}| = 3.8$ GeV calculated from the Lagrangian $L_S$ [7], with $c_d^{(1)}, c_m^{(1)}$ and $M_k^{(1)}$ given above. We have also checked that this is the case for the $K_0^*(1430)$ resonance which is the $I = 1/2$ member of the same octet. It follows then that the first octet is a pure one without mixing with the nearby $f_0$ and $f_0(1710)$. The $f_0^R(1370)$ couplings imply a large width to $\pi\pi$ with $\Gamma(f_0(1710) \rightarrow 4\pi)/\Gamma(f_0(1370) \rightarrow \pi\pi) = 0.30 \pm 0.12$, in good agreement with the interval $0.10-0.25$ given in the recent ref. [17]. Let us see that the pattern of sizes of the couplings of the $f_0^R$ and $f_0(1710)$ corresponds to the chiral suppression of the coupling of a scalar glueball, $G_0$, to $q\bar{q}$ [5]. According to ref. [5] this coupling is proportional to the quark mass, which then implies a strong suppression in the production of $\bar{u}u$ and $dd$ relative to $ss$ from $G_0$. With a pseudoscalar mixing angle $\sin \beta = -1/3$ one has that $\eta = -\eta_s/\sqrt{3} + \eta_u\sqrt{2}/3$ and $\eta' = \eta_s/\sqrt{2}/3 + \eta_u/\sqrt{3}$ with $\eta_s = ss$ and $\eta_u = (\bar{u}u + dd)/\sqrt{2}$. Denoting by $g_{ss}$ the production of $\eta_s\eta_s$, $g_{sn}$ that of $\eta_s\eta_u$ and $g_{nn}$ for $\eta_u\eta_u$, we obtain $\eta'\eta' = 2g_{ss}/3 + g_{nn}/3 + 2\sqrt{2}g_{ns}/3$, $g_{\eta\eta'} = -\sqrt{2}g_{ss}/3 + \sqrt{2}g_{nn}/3 + g_{ns}/3$, $g_{\eta\eta'} = g_{ss}/3 + 2g_{nn}/3 - 2\sqrt{2}g_{ns}/3$. If the chiral suppression of ref. [5] operates then $|g_{ss}| \gg |g_{nn}|$. This together with the OZI rule suppress the coupling $g_{ns}$. Taking e.g. the couplings of $f_0^R$ one obtains $g_{ss} = 11.5 \pm 0.5, g_{ns} = -0.2$ and $g_{nn} = -1.4$ GeV, and the strong suppression is clear. We now consider the $K\bar{K}$ coupling. A $K^0$ in terms of valence quarks corresponds to $\sum_{i=1}^3 s_i u_i/\sqrt{3}$, summing over the colour indices, and analogously for the $\bar{K}^0$. The production of a colour singlet $\bar{u}u$ from the $K^0\bar{K}^0$ requires then the combination $s_is_i/d_s^2 = (\bar{s}_i s_i - d_s^2)/3$, and similarly for $u_i u_i$. As the production occurs from the colour singlet $\bar{u}u$ source, only the configuration $\bar{u}u$ contributes, picking up a suppression factor of $1/3$. In addition, the coupling $g_{ss}$ has an extra factor 2 compared to that of $\bar{u}u$, because the former contains two $\bar{u}u$. One then expects that the coupling to $K^0\bar{K}^0$ has the absolute value $g_{ss}/6$. For the $f_0^R$ and $f_0(1710)$ it results $|g_{K^0\bar{K}^0}| \approx 2$ GeV, in good agreement with table I. Another resonance with a known enhanced coupling to $\bar{u}u$ is the $f_0(980)$. However, the sizes of its couplings to $\eta\eta'$ and $\eta'\eta'$ follow the opposite order to the $f_0(1790)$ and $f_0^R$ cases and all of them are much smaller than the coupling to $K\bar{K}$. Note that quenched lattice QCD [4] establishes that the couplings of the lightest scalar glueball to pseudoscalar pairs in the SU(3) limit scales as the quark mass, in support of the chiral suppression mechanism of ref. [5], that we also observe as discussed above. This mechanism also implies that the glueball should remain unmixed. This accurately fits with our previous result that both the $f_0^R$ and $f_0(1710)$ do not mix with the nearby $f_0^R$. In addition, the masses of the $f_0^R$ and $f_0(1710)$ poles are in excellent agreement with the quenched lattice QCD prediction for the mass of the lightest glueball, $(1.66 \pm 0.05)$ GeV.

5. In summary, we have presented a coupled channel study of the $I = 0, 1/2$ meson-meson $S$-waves from $\pi\pi$ threshold up to 2 GeV with $13$ coupled channels. All the $I = 0$ and $1/2 0^{++}$ resonances with masses below 2 GeV have been generated. The $f_0(1710)$ and a pole at 1.6 GeV, which is an important contribution to the $f_0(1500)$, are identified as glueballs. Another pole at $(1.466 - i0.157)$ GeV, mainly corresponding to the $f_0(1370)$, is shown to be a pure octet member.

We thank C. Piqueras for his collaboration. Financial support from the grants MEC FPA2007-6277 and Fundación Séneca 02975/PI/05 and 05113/FPI/06 is acknowledged.

[1] C.J. Morningstar and M. Peardon, Phys. Rev. D60, 034509 (1999); A. Vaccarino and D. Weingarten, Phys. Rev. D60, 114501 (1999); Y. Chen et al., Phys. Rev. D73, 014516 (2006).
[2] W.-M. Yao et al., Journal of Physics, G33, 1 (2006).
[3] C. Amsler and F.E. Close, Phys. Lett. B353, 385 (1995); D.V. Bugg, M.J. Peardon and B.S. Zou, Phys. Lett. B486, 49 (2000).
[4] J. Sexton, A. Vaccarino and D. Weingarten, Phys. Rev. Lett. 75, 4563 (1995).
[5] M.S. Chanowitz, Phys. Rev. Lett. 95, 172001 (2005); idem. 98, 149104 (2007).
[6] J.A. Oller and E. Oset, Phys. Rev. D60, 074023 (1999).
[7] A. Pich, Rept. Prog. Phys. 58, 563 (1995).
[8] M. Jamin, J.A. Oller and A. Pich, Nucl. Phys. B587, 331 (2000).
[9] U.-G. Meißner, Phys. Rept. 161, 213 (1988).
[10] J. A. Oller, Phys. Rev. D71, 054030 (2005).
[11] A. Etkin et al., Phys. Rev. D25, 1786 (1982).
[12] D.V. Bugg, A.V. Sarantsev and B.S. Zou, Nucl. Phys. B471, 59 (1996).
[13] W.M. Kloet and B. Loiseau, Z. Phys. A353, 227 (1995).
[14] S.J. Lindenbaum and R.S. Longacre, Phys. Lett. B274, 492 (1992).
[15] M. Albaladejo and J.A. Oller, arXiv:0711.1977; Talk at Scadron70, http://www.um.es/oller/talks/al.pdf
[16] D. Barberis et al. [WA102 Collaboration], Phys. Lett. B462, 462 (1999).
[17] D.V. Bugg, Eur. Phys. J. C52, 55 (2007).