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The merits of privileged parking

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A R T I C L E   I N F O

Article history:
Received 12 February 2020
Revised 11 August 2020
Accepted 16 August 2020
Available online 6 September 2020

JEL classification:
H42
R41
R48

Keywords:
Privileged parking
Loading zones
Congestion pricing
Cruising for parking

A B S T R A C T

The paper investigates under what conditions it is optimal to exclude some motorists searching for parking from occupying a vacant parking bay. Privileged parking is found to be optimal if motorists are heterogeneous and can steer their search towards or away from such parking. The socially optimal allocation of privileges and search strategies are characterized. The second-best pricing policies in the presence of cruising-for-parking externality are described; short-term parkers should always be allowed to take the first vacant bay they find. A model extension studies technologically modified “special needs” parking. Unlike existing policies that make such parking exclusive for special-needs motorists, the optimal policy makes it available for an extra fee to anyone, while increasing the number of special-needs bays so that even the initial users are better off.

1. Introduction

Many governments practice discriminatory policy for geographically homogenous curbside parking. A large number of municipalities around the world have dedicated “loading zones”. Most municipalities in developed countries have exclusive parking for the disabled, with or without special access design. With the advent of electric vehicles (EV henceforth) that require lengthy periods of battery charging, cities introduce exclusive EV parking equipped with charging infrastructure. Many municipalities also designate parking for “permit holders” only, where permits (i.e. privileges) are given to local residents or other special groups.

Policies regarding privileged parking vary across countries and cities, but typically aim to exclude the majority of motorists searching for parking (searchers henceforth) in favor of certain special groups. For example loading zones in Melbourne, Australia can be used only by commercial vehicles such as delivery trucks, buses, and taxis; private passenger cars are not allowed to stop there even briefly. The “yellow zones” in San Francisco, California have similar regulations, while the “white zones” do allow any vehicle to stop for passenger unloading, as long as the parking session does not exceed 5 min. Handicapped, EV parking, and taxi stands are typically reserved for the corresponding types of vehicles, excluding all others.

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This paper aims to understand theoretically the merits of privileged parking. To fix ideas, we will say that a parking bay is privileged if some kind of government regulation prevents some searchers from parking at that bay. The questions investigated in the paper include

- Can it be optimal to segregate technologically and geographically identical parking by introducing privileges?
- Should privilege allocation be exclusive (some bays are exclusive for searcher of some type A while others are exclusive for type B) or hierarchical (some bays are exclusive for type A while others are available to anyone)?
- What policy can implement, precisely or approximately, socially optimal allocation of parking?
- How do changes in overall demand for parking (e.g. on weekends) affect optimal privilege policy?
- If some searchers require technologically modified bays (accessible, having a charging station, etc.), how should access of other searchers to such bays be regulated? And what is the socially optimal amount of such parking supply?

To answer these questions, this paper formulates a dynamic model of parking demand and search in a geographically homogenous area with motorists that differ in desired parking duration and the value of search time. Throughout most of the paper, any parking bay is technologically accessible by any searcher. All parking supply is exogenously divided into two types to which different policies can be applied; the paper focuses on whether such heterogeneity of policies is useful, and how exactly the policies should differ.

As preview of results, I find that it is optimal to make one type of parking more costly per-hour than the other. This causes a welfare-improving segregation of motorists: those with low value of search time and/or long parking durations avoid the more expensive type of parking; those with high value of time and/or short parking durations, vice versa, aim towards the more expensive parking due to higher chance of finding vacancy there. Although such discrimination of parking policy increases the total amount of cruising for parking, it is welfare-improving because it favors (i) those with high cost of search and (ii) those with minimal impact on parking occupancy due to short duration of parking session.

As simple mathematical illustration, suppose there are two parallel streets, labeled 1 and 2, each having 100 parking bays. Suppose also there are two types of searching motorists, labeled A and B. Type-A park for 5 hours; their arrival rate is 19 vehicles per hour. Type-B motorists park for 1 hour, and their arrival rate is 85 per hour. All motorists have equal value of search time and can choose the street to search for parking. Then, the expected number of parked vehicles on both streets (i.e. total demand for parking) is $19 \times 5 + 85 \times 1 = 180$. If parking policy is non-discriminatory, all motorists will choose the street with lower expected occupancy, meaning that both streets will have the same expected occupancy of 90% in the steady state. If a motorist’s cost of search is inversely related to vacancy and is equal to some $C$ in non-discriminatory steady state, the expected social cost of search in such steady state is $(19 + 85)C = 104C$ per hour. If, on the contrary, the government somehow excludes type-A motorists from street 2, they will all search on street 1, resulting in $19 \times 5/100 = .95$ occupancy of the latter and doubling the search cost of type-A motorists. At the same time, type-B motorists will search on less occupied street 2, resulting in 85% occupancy there and reducing type-B search cost by factor $\frac{5}{8}$. Then, the per-hour social search cost is reduced to $19 \times 2C + 85 \times \frac{5}{8}C = 94\frac{1}{2}C$. Hence exclusion of long-term parkers from a fraction of parking supply is socially optimal.

In Section 7, some searchers are special needs, i.e. require specially designed parking infrastructure. The amount of special needs parking supply is endogenized. I find that making special-needs parking exclusive for the respective group of motorists is not optimal. The best policy is to make such parking available for an extra fee to anyone, while increasing the share of such parking at the same time. The former policy change hurts the special-needs motorists while the latter benefits; it is shown that the latter welfare effect dominates so even the special-needs motorists benefit from optimal policy.

To the best of my knowledge, there is no existing scientific analysis of policy discrimination in otherwise homogenous parking. There exist papers studying interaction between parking policy of different suppliers, e.g. municipally-owned curbside parking and privately owned garage parking, as in Arnott (2006), Inci and Lindsey (2015), or Gragera and Albalate (2016). Like this paper, Arnott and Rowe (2013) argue that separating short-term from long-term parkers may be socially optimal, although their method of sorting is based on curbside time limits which redirect long-term parkers into private garages.

Another line of research is policy heterogeneity caused by spatial inequality of parking demand, as in Anderson and de Palma (2004), Inci and Lindsey (2015) have a model with heterogenous searchers and allow for different per-hour parking fees for different types of searchers, but in their setting the socially optimal per-hour fees are the same for all.

Zakharenko (2019) in Section 4.1 mentions that if all parking supply can be separated into two isolated but geographically identical areas, and if searchers are heterogenous, it might be optimal to set different prices per hour in the two areas. This observation however is not the main focus of that paper and is not studied in detail. Moreover, the paper assumes that searchers can fully control which of the two parking types they will inspect for parking, therefore they will never pass by a bay from which they can be excluded by regulation. The current paper offers a more general model where searchers sometimes have to pass by vacant bays at which they cannot park, which is empirically relevant.

Section 7 of the current paper is also probably the first theoretical study of the economics of special-needs parking. In particular, there is no known research on how the accessible parking should optimally be supplied. Given the fact that such parking constitutes about 10% of all parking supply in developed countries, and plays an important role in everyday life of people with disabilities, its socially optimal provision is an important question of urban economics.
2. Model

2.1. Parking supply

Consider a continuous-time environment with a continuum of parking of exogenous amount normalized to unity. A continuum of parking, by the law of large numbers, rules out any aggregate stochasticity of parking congestion. Ignoring such stochasticity in decision-making also has empirical relevance, as motorists, first, usually have limited information about the actual conditions and, second, make decisions to travel and park some time before they actually search for parking and thus have to base their decisions on expected rather than actual congestion.

All parking is divided into two types, which we will denote 1 and 2. All parking bays of all types are geographically equally attractive to all motorists. Throughout most of the paper, the two types of parking are technologically identical (i.e. any searcher can use bays of both types), and the share \( S_i \) of type-\( i \) parking is exogenously given.\(^1\) Such exogenous division of parking can be driven, for example, by the street pattern of the area in question. Section 7 studies an alternative setting where type-2 parking is “special needs”, i.e. is technologically modified for the “special needs” subset of parkers who are unable to use type-1; the share of each parking type is endogenized.

2.2. Parking demand

The motorists searching for parking differ in their desired parking duration \( t \) and in cost \( c \) per unit of parking search time; the pair \( (c, t) \) is referred to as the searcher’s type. The desired parking duration can range from zero to infinity; for simplicity, we assume that it is exogenous for each searcher and does not respond to policy or aggregate equilibrium conditions. The cost \( c \) per unit of search time may include:

1. private value of resources (time, fuel) spent on search per se;
2. value of time spent on roundtrip walk between parking location and final destination;\(^2\)
3. “cruising for parking” external effects, including delays of other traffic, pollution, and accident risks, caused by the searching vehicle.

The last, external, component of the search cost is assumed to be the same for all, while the first two may vary from zero to infinity. In particular, searchers with disabilities are assumed to have high walking component of the search cost.

The inflow of searchers of type \( (c, t) \) that initiate the search during a representative unit of time is \( a(c, t) \). To keep the model simple, we assume that such inflow is exogenous.\(^3\) It is also assumed to be time-invariant, which implies that all equilibrium aggregate parameters will be time-invariant, as well. Because all arriving searchers will end up being parked, the outflow of motorists of type \( (c, t) \) from the search mode into the parked mode is \( a(c, t) \), as well.

Throughout the paper we use function \( D(k) = \int_{t=0}^{\infty} e^{-kt} a(c, t) dt \), equal to the mass of motorists who are parked in a steady state and whose ratio \( \frac{k}{t} \) is below some \( k \geq 0 \). Then, the total mass of motorists parked at any time is \( D(\infty) \), which should be strictly less than parking supply for any vacancy to exist: \( D(\infty) < 1 \). Denote by \( q_i \) the occupancy, i.e. the proportion of occupied bays of type \( i \); the average across both types of parking occupancy must be equal to \( D(\infty) \),

\[
q_1 S_1 + q_2 S_2 = D(\infty).
\]

We also assume that function \( D(k) \) is Lipschitz-continuous, that is, its derivative \( H(k) \equiv \frac{\partial D}{\partial k} = \int t a(c, t) dt \) exists and is bounded from above.

2.3. Search technology

We use a classical model of parking search, nicknamed the “binomial approximation” by Arnott and Williams (2017), with some modifications. We will assume that parkers randomly sample one parking bay at a time, such that every new try is independent from previous ones. Time is normalized so that one bay per unit of time is sampled. While the occupancy status of the drawn parking bay is completely random, searchers can somewhat influence the type of parking (1 or 2) that they draw. We will nickname such model of search by imperfect steering. The term steering is motivated by the idea that searchers, by optimizing their search route, can increase the fraction of their most desired type of parking that they encounter during the search. Steering is imperfect because, despite their best effort, the searchers sometimes still encounter the type of parking they do not desire or from which they are excluded.

\(^1\) Appendix B contains a glossary of notation used in the paper.

\(^2\) Assuming that the distance of such walk increases in proportion with the search time, every extra minute of search entails a constant walking cost. This relationship is consistent with searching along a predetermined route without loops, starting from an optimally chosen location ahead of final destination. In practice, some motorists may choose alternative strategies, such as circling around a loop, which would alter the relationship between total search cost and search time. This paper focuses on linear relationship for tractability reasons.

\(^3\) A model with endogenous arrivals is developed in Zakharenko (2016b); it balances an extra utility from a parking session and the extra search cost, internal and external, associated with that session. Exogeneity of arrivals and parking durations, assumed in the current paper, means that total utility from all parking sessions is exogenous and can be dropped from the objective of the social planner, who can focus entirely on minimization of search costs.
3. Specifically, the searchers are able to choose the probability of drawing type-1 parking from the range \([s, \bar{s}]\) such that \(s \leq \bar{s} \leq \bar{s}\). Lack of choice \(\bar{s} = \bar{s} = \bar{s}\) implies that they have no control over the search process; Section 3.4 shows that in this case, privileged parking is not optimal. In contrast, the case \(\bar{s} = 0, \bar{s} = 1\) corresponds to full control of the search process and excludes the chance of drawing an undesired type of parking. By \(s_1(c, t) \in [s, \bar{s}]\) we will denote the steering strategy of a type \([c, t]\) searcher, i.e. their chosen probability of drawing type-1 parking. Similarly, \(s_2(c, t) = 1 - s_1(c, t)\) is the probability of the alternative.

If the drawn parking bay is occupied, search must continue. If the drawn bay of type \(i \in \{1, 2\}\) is vacant, the searcher with parameters \((c, t)\) parks there with probability \(p_i(c, t)\) which is driven by the government policy introduced below. We will nickname \(p_i(c, t)\) as the parking strategy of searcher \((c, t)\) at vacant bay of type \(i\).

Given these assumptions, the chance of success of a searcher of type \((c, t)\), i.e. the probability that she ends up being parked after each draw, is

\[
r(c, t) = \sum_{i=[1,2]} s_i(c, t)(1 - q_i)p_i(c, t).
\]

2.4. Steady state occupancy

Define by \(E_i\) the entry rate into a vacant type-\(i\) bay, i.e. the rate at which a vacant bay becomes occupied. Define by \(X_i\) the rate of exit from an occupied bay. In the steady state, the total mass of bays being occupied per unit of time, \((1 - q_i)E_i\) must equal the total mass of bays being vacated, \(q_iX_i\), hence

\[
q_i = \frac{E_i}{E_i + X_i} = \frac{R_i}{1 + R_i},
\]

where \(R_i = \frac{E_i}{X_i}\). From (3), we also have that \(R_i = \frac{q_i}{1 - q_i}\), hence \(R_i\) can be interpreted as the steady-state ratio of occupancy to vacancy for type-\(i\) parking.

The entry rate \(E_i\) into type-\(i\) parking is the integral of entry rates of different searcher types. The entry rate \(E_i(c, t)\) of type \((c, t)\) into type-\(i\) parking is equal to \(\frac{a(c, t)S_i(c, t)p_i(c, t)}{r(c, t)S_i}\). Specifically,

- \(\frac{a(c, t)S_i(c, t)}{r(c, t)S_i}\) is the total mass of type-\(i\) bays drawn by type-\((c, t)\) searchers, per unit of time. Here \(a(c, t)\) is the rate of entry of such searchers into the model, \(\frac{1}{r(c, t)}\) is the average number of parking bays drawn by a searcher before a parking session begins, and \(S_i(c, t)\) is the chance of drawing type-\(i\) bay.
- \(S_i\) is type-\(i\) supply, hence \(\frac{a(c, t)S_i(c, t)}{r(c, t)S_i}\) is the rate at which a type-\(i\) bay is drawn by type-\((c, t)\) searchers.
- \(p_i(c, t)\) is probability that a vacant type-\(i\) bay is taken if sampled.

Hence, \(E_i = \int_{c,t} E_i(c, t)\,dcdt = \int_{c,t} \frac{a(c, t)S_i(c, t)p_i(c, t)}{r(c, t)S_i}\,dcdt\).

The rate of exit from type-\(i\) parking \(X_i\) is the inverse of the expected duration of parking, which in turn equals \(\int_{c,t} E_i(c, t)\,dcdt\). Hence, \(X_i = \frac{E_i}{\int_{c,t} \frac{a(c, t)S_i(c, t)p_i(c, t)}{r(c, t)S_i}\,dcdt.\) and therefore

\[
R_i = \int_{c,t} \frac{a(c, t)S_i(c, t)p_i(c, t)\,dcdt}{r(c, t)S_i}.
\]

3. Socially optimal search

3.1. Planner’s objective

In this section, we solve the problem of a social planner who aims to maximize social welfare by choosing steering strategy \(s_i(c, t)\) and parking strategy \(p_i(c, t)\) on behalf of the motorists. As common in these models (e.g. Zakharenko, 2016b), we assume the social planner has all information available to motorists (i.e. their type \([c, t]\) but does not know the actual status of every parking bay, which is treated as a shock of nature. Because the inflow of searchers is exogenous, maximization of social welfare amounts to minimization of social search costs induced by all motorists searching at a representative moment of time. The social search cost induced by motorists of type \([c, t]\) is the product of

- The rate of searcher entry into the model, \(a(c, t)\);
- The expected duration of search, \(\frac{1}{r(c, t)}\);
- The social search cost per unit of time, \(c\).

Hence, the social planner’s objective is to minimize

\[
J = \int_{c,t} \frac{a(c, t)c}{r(c, t)}\,dcdt.
\]
subject to constraints (2,3,4). The problem is solved with standard techniques for nonlinear constrained optimization, e.g. as described in chapter 1.6 of Chachuat (2007). The corresponding Lagrangian can be written as follows:

\[
\mathcal{L}(s_i(c,t), p_i(c,t), r(c,t), q_i) = \int_{c,t} \frac{a(c,t)c}{r(c,t)} \, dc \, dt + \int_{c,t} \left[r(c,t) - \sum_{i=1,2} s_i(c,t) p_i(c,t)(1 - q_i)\right] \mu(c,t) \, dc \, dt \\
+ \sum_{i=1,2} \left[\int_{c,t} \frac{a(c,t) t s_i(c,t) p_i(c,t)}{r(c,t) S_i} \, dc \, dt - \frac{q_i}{1 - q_i}\right] \lambda_i \rightarrow \min.
\]  

Prior to formal derivation of optimal search strategy, we point out the following useful observation.

**Proposition 1.** For any given searcher type \{c, t\}, optimal parking strategy for at least one type of parking is maximized: \(\max\{p_1(c,t), p_2(c,t)\} = 1\).

In simple words, for any searcher, there exists a type of parking where she will certainly park once a vacancy is found.

**Proof.** Suppose for some \{c, t\} both \(p_1(c,t) < 1\) and \(p_2(c,t) < 1\). According to the system (2,3,4), a proportionate increase in \(p_1(c,t)\) and \(p_2(c,t)\) will increase \(r(c,t)\) by the same proportion, while keeping \(q_i\) unchanged and thus having no effect on expected outcomes of other searchers. But that makes the objective function (5) smaller, improving social welfare. \(\square\)

### 3.2. First-order conditions

We now characterize the first-order conditions of the social optimum. First, for a given searcher type \{c, t\}, we minimize (6) with respect to the steering strategy \(s_1(c,t)\), accounting for the fact that \(ds_2(c,t) = -ds_1(c,t)\):

\[
\frac{d\mathcal{L}}{ds_1(c,t)} = [-p_1(c,t)(1 - q_1) + p_2(c,t)(1 - q_2)] \mu(c,t) \\
+ \frac{a(c,t)t}{r(c,t)} \left[\lambda_1 p_1(c,t) \frac{s_1(c,t)}{S_1} - \lambda_2 p_2(c,t) \frac{s_2(c,t)}{S_2}\right] \begin{cases} 
\geq 0, & s_1(c,t) = \bar{s} \\
= 0, & s_1(c,t) \in (\bar{s}, \bar{s}) \\
\leq 0, & s_1(c,t) = \underline{s} \end{cases}
\]  

In (7), the first component \([-] \mu(c,t)\) is the effect of a motorist’s steering strategy on the social objective function via changed chance of success, \(r(c,t)\), of that motorist. For example, increased steering toward more occupied type of parking lowers the chance of success and thus increases the social search cost. This effect occurs while the motorist is question is searching. The second component of (7) is the effect via changing occupancy/vacancy ratio: more steering toward type-1 parking increases its occupancy while decreasing occupancy of the other type. This is the effect via changing chance of success of subsequent motorists, due to the motorist in question being parked here rather than there.

The first-order condition for optimal parking strategy is as follows:

\[
\frac{d\mathcal{L}}{dp_1(c,t)} = -s_1(c,t)(1 - q_1) \mu(c,t) + \frac{a(c,t)t s_1(c,t)}{r(c,t) S_1} \lambda_1 \begin{cases} 
\geq 0, & p_1(c,t) = 0 \\
= 0, & p_1(c,t) \in (0, 1) \\
\leq 0, & p_1(c,t) = 1 \end{cases}
\]  

The interpretation of (8) is similar to that of (7). The first, negative, component is the reduction of social search cost due to admission of a type \{c, t\} motorist to type-i parking; it includes private welfare gain as well as reduction of cruising externalities. The second component is the effect on subsequent searching motorists via increased occupancy \(q_i\) while the motorist in question is parked.

Optimal chance of success \(r(c,t)\) must be inside the unit interval, hence we have

\[
\frac{d\mathcal{L}}{dr(c,t)} = -\frac{a(c,t)c}{r^2(c,t)} + \mu(c,t) - \frac{a(c,t)t s_1(c,t)}{r^2(c,t) S_1} \sum_i s_i(c,t) p_i(c,t) \lambda_i = 0,
\]

from which we can derive the shadow value of the chance of success:

\[
\mu(c,t) = \frac{a(c,t)c}{r^2(c,t)} \left[c + t \sum_i s_i(c,t) p_i(c,t) \lambda_i \right].
\]  

Finally, from the first-order condition for optimal occupancy \(q_i\), which is inside the unit interval in a steady state, we can derive the shadow value of occupancy/vacancy ratio:

\[
\frac{\lambda_i}{(1 - q_i)^2} = \int_{c,t} s_i(c,t) p_i(c,t) \mu(c,t) \, dc \, dt.
\]
3.3. Optimal parking strategy

If a motorist never draws a parking bay of type \(i\), i.e. \(s_i(c, t) = 0\), then her parking strategy \(p_i(c, t)\) is immaterial. For \(s_i(c, t) > 0\), combination of (8) with (9) yields the following optimality condition for the parking strategy:

\[
\frac{\partial c}{\partial p_i(c, t)} = \frac{(1 - q_i)s_i(c, t)a(c, t)}{r(c, t)} \left( \frac{1}{i} + \frac{s_j(c, t)p_j(c, t)\lambda_j}{\lambda_i} \right) - \frac{\lambda_i}{S_i(1 - q_i)} t
\]

\begin{align*}
\geq 0, & \quad p_i(c, t) = 0 \\
= 0, & \quad p_i(c, t) \in (0, 1), \\
\leq 0, & \quad p_i(c, t) = 1
\end{align*}

(11)

In (11), the term preceding parentheses is the total mass of vacant type-\(i\) bays drawn by type-\(\{c, t\}\) motorists per unit of time; specifically, it is the product of the total mass of type-\(\{c, t\}\) searching motorists, \(\frac{a(c, t)}{r(c, t)}\); and the chance of every such motorist to draw a vacant type-\(i\) bay, \(s_i(c, t)(1 - q_i)\). The term in parentheses in (11) is the effect of every type-\(\{c, t\}\) motorist taking a vacant type-\(i\) bay on social welfare, per unit of their parking time. That effect consists of two channels: \(-\frac{1}{i}\) is the effect of reduction of own search time, while \(-\frac{\lambda_i}{S_i(1 - q_i)}\) is the effect of increased search time of subsequent motorists due to increased occupancy. Note that the welfare effect via reduction of own search time is the same for both types of parking, while the effect via increased occupancy does not depend on the motorist’s type. Because labeling of parking types was arbitrary, suppose without loss of generality that the welfare effect via elevated occupancy is weakly greater for the second type of parking:

\[
\frac{\lambda_1}{S_1(1 - q_1)} \leq \frac{\lambda_2}{S_2(1 - q_2)}.
\]

(12)

Then, (11) implies that

\[
p_1(c, t) \geq p_2(c, t), \quad \forall c, t.
\]

(13)

But then, Proposition 1 also implies that \(p_1(c, t) = 1\) for all searcher types \(\{c, t\}\). Therefore, we can conclude that parking privileges, if they exist, should cover only part of all parking (type-2 in our notation); the remaining parking (type-1) should be available to any searcher who finds vacancy there. In the notation used in the introduction, parking privileges should be hierarchical rather than exclusive.

To find optimal strategy for type-2 parking, rewrite (11) for \(i = 2\), substituting \(p_1(c, t) = 1\), as follows:

\[
\frac{c}{t} - \frac{s_1(c, t)\lambda_1}{S_1} - \frac{s_2(c, t)p_2(c, t)\lambda_2}{S_2} + r(c, t)\frac{\lambda_2}{S_2(1 - q_2)} \geq 0, \quad p_2(c, t) = 0 \\
= 0, \quad p_2(c, t) \in (0, 1), \\
\leq 0, \quad p_2(c, t) = 1
\]

which can be rewritten, recalling (2), as

\[
\frac{c}{t} + s_1(c, t)(1 - q_1) \left[ \frac{\lambda_2}{S_2(1 - q_2)} - \frac{\lambda_1}{S_1(1 - q_1)} \right] \geq 0, \quad p_2(c, t) = 0 \\
= 0, \quad p_2(c, t) \in (0, 1), \\
\leq 0, \quad p_2(c, t) = 1
\]

(14)

Denote by

\[
L_0 = \frac{\lambda_2}{S_2(1 - q_2)} - \frac{\lambda_1}{S_1(1 - q_1)}
\]

(15)

the excess of a motorist’s welfare effects while parked at type-2 bay, per unit of time, relative to those of type-1. Also denote by \(k_1 = s_1(c, t)(1 - q_1)L_0\) the chance of drawing a vacant type-1 parking times \(L_0\). Both \(L_0\) and \(k_1\) are non-negative by assumption (12). But then, we conclude that a searcher of type \(\{c, t\}\) should never use type-2 parking, \(p_2(c, t) = 0\). If their cost-to-duration ratio \(\xi\) is above \(k_1\): a searcher should always use vacant type-2 parking they find, \(p_2(c, t) = 1\). If such ratio is above \(k_1\). Those with \(\xi = k_1\) are indifferent between various values of \(p_2(c, t)\), but assumptions of Section 2.2 imply that the population share of such searchers is zero hence they can be ignored. Intuitively, the social planner has to weigh two options for a motorist who found a vacant type-2 bay. If she takes the bay, the excess social cost generated by her parking session will be \(L_0\). If she decides to search for a type-1 bay instead, she will have to do so for \(\frac{1}{s_1(c, t)(1 - q_1)}\) units of time, incurring the social cost \(c\) per every such unit of time. The two costs are balanced when \(\xi = k_1\).

3.4. No-steering case

Consider a special case when no steering is possible, i.e. \(s_1(c, t)\) can take only one possible value \(S_1\), meaning that draws of parking bays are completely random. In such environment, a searcher \(\{c, t\}\) who uses the fast parking strategy, i.e. always takes the first available bay \(p_i = 1, i \in \{1, 2\}\), has the chance of success equal to \(1 - D(\infty)\), which is also equal to the fraction of unoccupied bays in steady state. By assumptions of the model, such probability does not depend on parking strategy of
other searchers. A searcher \([c, t]\) pursuing any other parking strategy, i.e. \(p_2(c, t) < 1\), will have a smaller chance of success and will therefore search longer. Thus, the aggregate search cost is minimized when everyone uses the fast strategy of search, meaning that any parking privilege cannot be optimal.

3.5. Optimal steering strategy

The rest of the paper except Section 7 assumes that searchers can indeed steer their search, i.e. \(\tilde{s} < \bar{s}\). In the current section, given optimal parking strategy \(p(c, t)\), we calculate the optimal steering strategy \(s_1(c, t)\). The searchers that are excluded from type-2 parking, i.e. those with \(c < k_1 t\) and using the parking strategy \(\{p_1(c, t) = 1, p_2(c, t) = 0\}\), have the chance of success of \(r(c, t) = s_1(c, t)(1 - q_1)\). A comparison of the latter equation with the definition of \(R_i = \frac{q_i}{1-q_i}\) in (4) implies that an increase in \(s_1(c, t)\) increases \(r(c, t)\) by the same proportion without changing \(q_1\), which reduces the social search cost (5). Therefore, for such searchers the optimal steering strategy is to maximize the chance of sampling type-1 parking, \(s_1(c, t) = \bar{s}\). Which is very intuitive given the fact that they can use only this type of parking.

For the searchers with \(c > k_1 t\) who use the fast parking strategy, optimal steering strategy is less straightforward. For them, (7) can be rewritten as

\[
\frac{dL}{ds_1(c, t)} = (q_1 - q_2)\mu(c, t) + \frac{a(c, t)t}{r(c, t)} \left[ \frac{\lambda_1}{S_1} - \frac{\lambda_2}{S_2} \right]
\]

\[
= \frac{a(c, t)}{r^2(c, t)} \left( (q_1 - q_2) \left( c + t \sum_i \frac{s_i(c, t)\lambda_i}{S_i} \right) + t \sum_i s_i(c, t)(1 - q_i) \left( \frac{\lambda_1}{S_1} - \frac{\lambda_2}{S_2} \right) \right) \geq 0, \quad s_1(c, t) = \bar{s}
\]

\[
\leq 0, \quad s_1(c, t) = \tilde{s}.
\]  \(\text{(16)}\)

In the last line of (16), the term preceding square brackets is the marginal effect of increased \(s_1(c, t)\) on the mass of motorists who indeed arrive at vacant type-1 bays; \((R_1 - R_2)c\) is the excess social cost due to these motorists searching longer; the remaining term, also equal to \(-L_{opt}\), is the reduction of search costs incurred by future motorists due to lower occupancy of type-2 bays.

We now analyze two candidate steady states.

3.5.1. No privileged parking

**Proposition 2.** If (12) is equality, then \(R_1 = R_2\).

**Proof.** If (12) is equality, then according to (14) everyone should follow the fast parking strategy \(p_1(c, t) = 1\). \(\forall i, c, t\). Furthermore, the sign of (16) is equal to the sign of \(R_1 - R_2\). Suppose \(R_1 < R_2\); then all searchers steer towards type-1 parking, \(s_1(c, t) = \bar{s}\), \(\forall c, t\). The assumptions of Section 2.3 then imply that

\[
\frac{s_1(c, t)}{S_1} \geq 1 \geq \frac{s_2(c, t)}{S_2}, \quad \forall c, t.
\]  \(\text{(17)}\)

But then, from the definition (4) of \(R_i\), it follows that \(R_1 \geq R_2\), contradicting the initial assumption. Similar logic excludes \(R_1 > R_2\). The fact that \(R_1 = R_2\) also means \(q_1 = q_2\). \(\Box\)

In simple words, (12) being equality means that the social cost of occupying both types of parking is the same, which further means that all searchers should use the fast parking strategy, and that they can choose an arbitrary steering strategy \(s_1(c, t) \in [\bar{s}, \bar{s}]\) as long as the aggregate proportion of motorists using type-1 parking equals the share of such parking in aggregate supply.

3.5.2. Privileged parking

**Proposition 3.** If (12) is strict inequality, then \(R_1 > R_2\).

**Proof.** Suppose (12) is strict inequality but \(R_1 \leq R_2\). Then (16) is strictly negative, meaning that all searchers steer towards type-1 parking, \(s_1(c, t) = \bar{s}\), \(\forall c, t\); thus (17) is true. This inequality together with (13) implies that \(R_1 \geq R_2\) according to (4). This excludes \(R_1 < R_2\).

Suppose further that \(R_1 = R_2\). From (10), we can rewrite the terms in (12) as

\[
\frac{\lambda_i}{S_i(1 - q_i)} = \int s_i(c, t)p_i(c, t)(1 - q_i)\mu(c, t)dc\,dt.
\]

But then, (13) and (17) imply that \(\frac{\lambda_1}{S_1(1 - q_1)} \geq \frac{\lambda_2}{S_2(1 - q_2)}\), contradicting the assumption of this proposition. \(\Box\)
Given $R_1 > R_2$, the optimal steering strategy can be formulated from (16) as follows:

$$s_1(c, t) = \begin{cases} \bar{s}, & \zeta < k_2 \\ \xi, & \zeta = k_2, \\ \bar{s}, & \zeta > k_2 \end{cases}$$

(18)

where $k_2 \equiv \frac{1}{R_1 - R_2} L_0$. Intuitively, a motorist steering towards type-2 (less occupied) parking reduces own search time and associated costs, but increases occupancy in that parking type and thus increases search costs incurred by subsequent motorists. As the former effect is proportional to $c$ while the latter proportional to parking duration $t$, the two costs are equated when the ratio $\xi$ is equal to $k_2$.

It is trivial to see that $k_2 > k_1$, hence searchers with $\xi = k_1$ steer towards type-1 parking and hence $k_1$ can be rewritten as $k_1 = \hat{s}(1 - q_1)L_0$.

3.6. Summary of optimal strategy

As a summary of the above discussion, all searchers can be divided into three groups.

**Searchers excluded from privileged parking** Searchers with lowest cost-to-duration ratio, $\xi < k_1$, can use only type-1 parking and should naturally steer towards that parking. We will label these searchers as “group $x$”. The mass of such motorists parked at any time is $D(k_1)$; the flow of social costs caused by searching group-$x$ motorists is $G(k_1)$, where $G(k) \equiv \int_{t=0}^{\bar{k}} a(c, t) c \, dc dt$.

In the no-privileged-parking scenario, we have that $k_1 = 0$ hence group $x$ does not exist.

**Searchers prioritizing non-privileged parking** Those with intermediate cost-to-duration ratio, $k_1 < \xi < k_2$, use the fast parking strategy while steering towards type-1 (more occupied) parking. We will denote them as group-$y$ searchers. The mass of such motorists parked in steady state is $D(k_2) - D(k_1)$; the search by these motorists causes the flow of social cost equal to $G(k_2) - G(k_1)$.

In equilibrium without privileged parking, all searchers are indifferent between various steering strategies; we can assume that those with $\xi < k_2$ behave as group-$y$ searchers, i.e. choose $s_1(c, t) = \bar{s}$. The specific value of $k_2$ is defined below in (19). The mass of parked motorists of this type is $D(\hat{k}_2)$ while the flow of social cost is $G(\hat{k}_2)$.

**Privileged searchers** Finally, searchers with highest cost-to-duration ratio, $\xi > k_2$, should use the fast parking strategy and steer towards type-2 (privileged) parking, so that $s_1(c, t) = \xi$. Denote them as “group $z$”. The mass of parked motorists of this type is $D(\infty) - D(k_2)$ while the flow of social cost is $G(\infty) - G(k_2)$.

In the scenario without parking privileges, we can assume that those with $\xi > \hat{k}_2$ behave as group-$z$ searchers. The requirement specified in Section 3.5.1, that the share of type-1 parking used by motorists equals its share in supply, can then be written as $D(\hat{k}_2)\bar{s} + D(\infty) - D(\hat{k}_2)\xi = D(\infty)\bar{s}_1$. Hence, without parking privileges, we have

$$D(\hat{k}_2) = D(\infty)\frac{\bar{s}_1 - \bar{s}}{\bar{s} - \xi},$$

(19)

which pins down the unique value of $\hat{k}_2$. Fig. 1 illustrates the three groups of searchers.

3.6.1. Reduced-form optimization problem

Section 3.1 defines the planner’s problem in an infinite-dimensional space, as optimal strategy has to be calculated for every motorist type. The above summary of optimal strategy allows us to reformulate the problem with just three unknowns: strategy cutoffs $k_1$, $k_2$, as well as occupancy $q_1$ (the other occupancy $q_2$ can be treated as function of $q_1$ given by (1)). Specifically, the chance of success $r(c, t)$ is the same within each group [$x$, $y$, $z$] and equals, respectively, $r_x(q_1) = \bar{s}(1 - q_1)$, $r_y(q_1) = \bar{s}(1 - q_1) + (1 - \bar{s})(1 - q_2(q_1))$, and $r_z(q_1) = \bar{s}(1 - q_1) + (1 - \bar{s})(1 - q_2(q_1))$. The objective (5) to be minimized then
becomes

\[ j(k_1, k_2, q_1) = \frac{G(k_1)}{r_x(q_1)} + \frac{G(k_2) - G(k_1)}{r_y(q_1)} + \frac{G(\infty) - G(k_2)}{r_z(q_1)}, \tag{20} \]

while the optimization constraint (4) on \( R_1 \) becomes

\[ \frac{D(k_1) \bar{x}}{r_x(q_1) S_1} + \frac{(D(k_2) - D(k_1)) \bar{x}}{r_y(q_1) S_1} + \frac{(D(\infty) - D(k_2)) \bar{x}}{r_z(q_1) S_1} - \frac{q_1}{1 - q_1} = 0. \tag{21} \]

### 3.6.2. Symmetric policy is not optimal

Section 3.4 finds that non-discriminatory policy with equal occupancies \( q_1 = q_2 \) and all searchers using the fast parking strategy is optimal when there is no steering, \( \bar{x} = \bar{y} \). Section 3.5 has found that such policy also meets all first-order conditions of social optimum when motorists can steer their search, \( \bar{x} < \bar{y} \). But is it indeed optimal to rule out parking privileges in the latter scenario? This section proves that the answer is negative: despite the first-order conditions of optimality being met, non-discriminatory policy is inferior to optimal privileges when motorists can steer their search.

The method of proof is to construct a small deviation from a symmetric policy point and demonstrate an improvement (decrease) in the social objective function (20). This point is defined by \( k_1 = 0 \) and \( k_2 = \hat{k}_2 \); both types of parking are equally occupied so that \( q_i = D(\infty) \), \( \forall i \). We also have that \( r_y = r_z = 1 - q_1 = 1 - D(\infty) \).

**Proposition 4.** A small increase of cutoff \( k_2 \) from symmetric policy value \( \hat{k}_2 \) reduces the social cost of search (20).

The proof is in Appendix A.

Once an increase in steering strategy cutoff \( k_2 \) is implemented, occupancies in the two types of parking become unequal: \( q_1 > q_2 \). But then, analysis of Section 3.5 implies that (12) should hold with strict inequality, which further means that \( k_1 > 0 \) and searchers with \( \bar{x} < \bar{k}_1 \) should optimally be excluded from type-2 parking.

To conclude, if motorists can steer their search, no-privilege policy is not optimal despite the fact that all first-order conditions of optimality are met. Mathematically, the no-privilege policy appears to be a saddle point in the space of parking and steering strategies. Thus, the social optimum must feature unequal occupancies \( q_1 > q_2 \) and some searchers must be excluded from type-2 parking.

### 3.7. Low demand

How should optimal policy respond to periods of low demand, e.g. at night or on weekend? To fix ideas, assume that the inflow of searchers \( a(c, t) \) proportionately decreases to zero for every \( (c, t) \). Then, it becomes increasingly unlikely that someone draws and already occupied bay, thus the externality of a parked motorist on subsequent searching motorists converges to zero, and so does the difference in such externality across the two parking types. But that means that during periods of near-zero demand, privileges are not optimal, and no searcher should be excluded from any parking bay.

Mathematically, because finding vacancy is easier, the chance of success \( r(c, t) \) rises to an upper bound of unity, which means that the quantity \( \mu(c, t) \) in (9) decreases at the same rate as \( a(c, t) \). But then, \( \lambda_i \) in (10), \( L_0 \) in (15), and \( k_1 \) also decrease to zero at the same rate.

### 4. Implementing optimal policy

This section studies how the government can implement the socially optimal policy of Section 3. Because a motorist’s duration of parking \( t \) is known to the government while the cost of search \( c \) is not, regulation of parking can depend only on \( t \). Furthermore, because entry \( a(c, t) \) of each particular type of searchers is exogenous by a model assumption and does not respond to regulation, we will focus on the difference in government regulation across the two parking types. We will assume that, for a searcher with parking duration \( t \), regulation consists of a monetary premium \( L(t) \) for using type-2, rather than type-1, parking. Outcomes of many other regulations can be achieved by selecting an appropriate value of \( L(t) \). For example a direct ban to use type-2 parking for \( t \) units of time is equivalent to \( L(t) = \infty \).

As mentioned in the introduction, the cost of search \( c \) consists of (i) private cost \( c_1 \) and (ii) external cost \( c_0 \). The former category includes passengers’ value of time and the cost of operating a vehicle; the latter includes cruising-for-parking externalities, pollution, and accidents. We will assume that \( c_1 \) is heterogenous and is private information, while \( c_0 \) is common to all searchers and can be estimated by the government. We will further assume that motorists only account for their private search cost \( c_1 \) when calculating their optimal search strategy.

A motorist with parameters \( (c_1, t) \) selects her steering strategy \( s_i(c_0 + c_1, t) \) and parking strategy \( p_i(c_0 + c_1, t) \), \( \forall i \) to minimize her total cost of parking \( C(c_1, t) \) which consists of the following elements:

- the private cost of inspecting another parking bay;
- the chance of ending up in type-2 bay, times the premium paid for parking there;
- the chance of having to continue search, times the total cost of parking:
\[
C(c_1, t) = \min_{s_i(c), p_i(t)} c_1 + s_2(c_0 + c_1, t)(1 - q_2)p_2(c_0 + c_1, t)\mathcal{L}(t) \\
+ \left(1 - \sum_{i} s_i(c_0 + c_1, t)(1 - q_i)p_i(c_0 + c_1, t)\right)C(c_1, t)
\]

By our earlier assumption, type-2 parking is less occupied (without loss of generality) which can be achieved by making it more expensive: \(\mathcal{L}(t) \geq 0\). Then, optimization of the above objective yields \(p_1(c_0 + c_1, t) = 1\) for all searchers. Equilibrium type-2 parking strategy is (cf.\((14)\))

\[
p_2(c_0 + c_1, t) \begin{cases} 
1, & c_1 > \\
\in [0, 1], & c_1 = \mathcal{L}(t)(1 - q_1), \\
0, & c_1 < 
\end{cases}
\]

while the equilibrium strategy is (cf.\((16)\))

\[
s_1(c_0 + c_1, t) \begin{cases} 
\bar{s}, & c_1 < \\
\bar{s}, & c_1 = \mathcal{L}(t) - R_1 - R_2, \\
\tilde{s}, & c_1 > 
\end{cases}
\]

Note that when \(c_0 = 0\), i.e. there are no search externalities, equilibrium search strategy is socially optimal iff \(\mathcal{L}(t) = tl_0\).

4.1. Positive external search costs

When \(0 > c_0\), we essentially have a positive lower bound on the social search cost \(c = c_0 + c_1\) which, by assumption, is common to all searchers. But then, all searchers with a sufficiently short duration of parking belong to group \(z\), i.e. they should be allowed to park at any bay and should target type-2 (privileged) parking. To achieve this in equilibrium, the government should not practice any parking discrimination for those parking short-term, i.e. \(\mathcal{L}(t) = 0\) for small \(t\). In particular, a vehicle that is loading or unloading passengers or cargo should be allowed to use all types of parking; the cost of such parking should be the same for a given (short) parking duration at all types of parking.

For a longer parking duration \(t\), the first-best policy is not achievable as the government has only one degree of freedom, \(\mathcal{L}(t)\), to meet two moments: the optimal parking strategy cutoff \(c = k_1t\) and the optimal steering cutoff strategy \(c = k_2t\). The second-best policy has to balance between the two moments, generally violating both; derivation of such \(\mathcal{L}(t)\) is tedious and lacking economic intuition. Instead, we only formulate the problem for optimization of \(\mathcal{L}(t)\), without discussing its solution.

The social cost function to be minimized can be formulated as (cf.\((5)\))

\[
J(L(t), q_1) = \int_{0}^{\infty} \int_{0}^{\mathcal{L}(t)(1-q_1)} \text{c}(c(t)) \text{d}c(t) \text{d}t + \int_{0}^{\infty} \int_{\mathcal{L}(t)(1-q_1)}^{\mathcal{L}(t)} \frac{\text{c}(c(t)) \text{d}c(t) \text{d}t}{r_{1,2}} + \int_{0}^{\infty} \int_{\mathcal{L}(t)}^{\infty} \frac{\text{c}(c(t)) \text{d}c(t) \text{d}t}{r_{1}}.
\]

where \(r_{x}, r_{y}, r_{z}\) are defined in Section 3.6.1. Here occupancy \(q_1\) is defined via \((4)\):

\[
\int_{0}^{\mathcal{L}(t)(1-q_1)} \frac{\text{c}(c(t)) \text{d}c(t) \text{d}t}{r_{1,2}} + \int_{\mathcal{L}(t)(1-q_1)}^{\mathcal{L}(t)} \frac{\text{c}(c(t)) \text{d}c(t) \text{d}t}{r_{1}} - \frac{\text{c}(c(t)) \text{d}c(t) \text{d}t}{r_{2}} - q_1 = 0,
\]

which serves as an optimization constraint. Fig. 2 illustrates the first- and the second-best search strategies.

4.2. Homogenous search cost

In a special case when all searchers have the same cost of search \(c_1\), optimal search strategy cutoffs for parking duration are determined as follows. The duration cutoff \(t_p\) between \(p_2 = 0\) (i.e. group \(x\)) and \(p_2 = 1\) (group \(y\)) is found from \(\frac{c_0 + c_1}{c_p} = \bar{s}_0(1 - q_1)\); the cutoff \(t_s\) between \(s_1 = \tilde{s}\) (group \(y\)) and \(s_1 = \bar{s}\) (group \(z\)) is determined by \(\frac{c_0 + c_1}{c_s} = \frac{l_0}{R_1 - R_2}\). It turns out that the same strategy cutoffs can be achieved by a parking premium regulation, by making \(\mathcal{L}(t) = \frac{c_1}{c_0} \bar{s}_0 - l_0 t\). It is then straightforward to verify that \((22)\) holds with equality iff \(t = t_p\), while \((23)\) is equality iff \(t = t_s\). Hence, parking premium regulation allows to achieve social optimum if searchers vary only in desired parking duration. Fig. 3 illustrates such equilibrium.
5. Empirical relevance

Suppose a government decides to adopt recommendations of this paper, divides municipal parking supply in a certain area into two types, and introduces a premium \( L(t) \) for using type-2 parking for \( t \) units of time. How to assess whether this parking premium is optimal? This section describes how to use administrative data on parking demand under given \( L(\cdot) \) for such assessment.

To make such judgements, all model parameters should be calibrated or estimated. To assess the externality of a cruising vehicle \( c_0 \), traffic flow theory can be applied. For example one can use “bathtub” models (Arnott, 2013; Fosgerau, 2015) which relate average travel speed to the density of vehicles on the road in the area of interest. As those searching for parking add to the density, they cause delays of other vehicles. Arnott et al. (2015) incorporate these effects into their model of search for parking. By calibrating parameters of a bathtub model, one can assess the value of \( c_0 \) under prevailing traffic conditions.

The steering parameters \( \bar{s}, \bar{s} \) depend on the street pattern and location of the different types of parking. For example to assess \( \bar{s} \), one should study what is the maximal share of type-1 parking that a searching vehicle can encounter while cruising within a designated search zone. Section 6 provides an example of calibration of \( \bar{s} \) and \( \bar{s} \).

Occupancy levels \( q_i \) can be observed directly from the data on use of both parking types. Such data is collected using various methods: parking occupancy sensors; payment-for-parking records; parking citations; manual measurements of occupancy. Recently, Nsosie et al. (2020) use satellite imagery of Wuhan hospital parking lots to measure the timing of healthcare demand in the wake of Covid-19 epidemic; their approach may pave the way for other parking demand studies. Given this data, equilibrium choice of parking \( p_i(\cdot, t) \) and steering \( s_i(\cdot, t) \) strategies can be assessed from (22) and (23), respectively. Given all above information, the chance of success (2) can be calculated.

To find the remaining unknowns, the distribution of searchers by type \( a(c, t) \) is needed. While the distribution of parking durations can be estimated from data on parking demand, the cost of search \( c \) is private information and its distribution can be recovered only partly, through the fact that those with higher \( c \) are more likely to end up in type-2 parking. To address the problem, \( a(c, t) \) can be parameterized as follows: \( a(c, t) = b(t)g(c, \alpha(t)) \). Here \( b(t) \) is the inflow, per unit of time, of all searchers who will park for \( t \) units of time, so that \( \int_0^\infty b(t)dt = D(\infty) \). Function \( b(\cdot, \cdot) \) can be estimated from administrative data on parking demand. The function \( g(c, \alpha(t)) \) is the probability distribution function of search costs for those who park for \( t \) units of time, with lower bound \( c_0 \) and mean \( c_0 + \alpha(t) \). The shape of the function cannot be derived from the data and should be assumed, e.g. exponential \( g(c, \alpha) = \frac{1}{c} \exp(-\frac{c}{\alpha}) \), \( c \geq c_0 \). The predicted share of searchers with duration \( t \) who end up in privileged parking increases with mean search cost \( \alpha(t) \), hence the latter can be estimated by matching such predicted share to empirically observed share, for every duration \( t \).
The shadow costs $\mu(c, t)$ and $\lambda_i$ can be found by solving the system (9,10), using the above described estimates of all other ingredients.

With all model parameters estimated, one can calculate the value of $L_0$ from (15) and then make judgements whether or not the existing policy $\lambda(\cdot)$ is close to the optimal one formulated in Section 4. If not, comparison of existing and optimal policies can also suggest in what direction the existing policy $\lambda(\cdot)$ should be changed.

### 6. Calibrated example

Consider the following stylized example. Suppose a city center consists of North-South “avenues” and East-West “streets”, all of which are one-way. Suppose further that the streets are three times more proximate to each other than the avenues (think of street pattern in Manhattan) and thus have three times more curbside. Also suppose that, because the avenues are more busy, the government decides to make parking there privileged. Hence, street curbs offer type-1 parking such that $S_1 = 0.75$, while the avenue curbs have type-2 parking.

Because both streets and avenues are one-way, a searcher who prioritizes type-1 (street) parking is best-off using a snake search pattern, gradually drifting south, as illustrated on the left panel of Fig. 4. Given such search pattern, 6 out of 7 parking bays encountered by the searcher will be type-1, hence $\bar{\varphi} = \frac{6}{7}$.

A motorist prioritizing type-2 parking can use a similar pattern, starting at the easternmost avenue of the search area and gradually drifting west. Because the streets are much more frequent, we will assume that a searcher travels for four blocks on a given avenue before turning west. Then, the proportion of street bays to avenue bays encountered by the searcher is 3:4, hence $\bar{\varphi} = \frac{3}{4}$.

We will assume that demanded parking duration $t$ and search cost $c$ are each exponentially distributed and independent from each other, such that the p.d.f. of the distribution is $\frac{1}{\mu} \exp \left( -\frac{\mu}{\mu} \right)$ and $\exp \left( -\frac{\lambda_i}{\lambda_i} \right)$. The mean duration of parking is assumed to be one hour; Zakharenko (2016b) estimates that 1500 parking bays can be sampled during this time, hence we assume $Et = 1500$. The mean cost of search, which also includes the time cost of walking between the parked automobile and the final destination, is assumed to be $S1 per minute of cruising; the cost is assumed to be entirely private. As we assume that a searcher can inspect 25 bays per minute of cruising, we have that $Ec = 1/25 per draw of a parking bay.

For comparison purposes, we calculate optimal policy for two distinct values of overall inflow of searchers and corresponding levels of congestion. In example 1, searcher inflow is such that the steady-state occupancy is 90%. In example 2, it is 98%. Table 1 reports the results.

The two experiments with different mean occupancy levels produce comparable results, after adjusting for a five-fold change in the vacancy rate. In both cases, the vacancy rate in the privileged type-2 parking is approximately double that of non-privileged type-1 parking. Both experiments conclude that only one-third of all searchers, those with lowest $\bar{\varphi}$ ratio, should be excluded from the privileged parking. Thus, we can conclude that currently practiced privileged parking policies which exclude the vast majority of searchers (e.g. loading zones, taxi stands, etc.) are excessively restrictive.

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4 van Ommeren and McIvor (2018) come up with a similar estimate of 1600.

5 Zakharenko (2016b) uses a similar estimate. van Ommeren and McIvor (2018) use a much smaller estimate of 25 Australian dollars per hour, but they emphasize that they exclude the walking costs.
The experiments also find that optimal privilege policy reduces the social costs of search by about 10%, regardless of overall congestion levels.

7. Special needs parking

7.1. Problem description

Some motorists cannot use standard parking bays and require specially designed ones to park their vehicles. For example, accessible parking designed for people with disabilities usually has larger dimensions, allowing users to unload their mobility devices. Additional features may include proximity of a ramp to get on a sidewalk, and removal of curbside storm drains for safety of the disabled. Another example of special needs parking is the one for electric vehicles. As these vehicles require lengthy charging periods, they typically combine charging with parking and therefore can only park at bays equipped with relevant infrastructure.

This section assumes that a given fraction of all motorists have such special needs. We seek answers to the following questions. Assuming special needs parking is more costly, what share of all parking should be modified? Which ordinary motorists, if any, should be allowed to use the special needs parking?

As the model of this paper assumes spatially homogenous parking, the results below are most suitable for curbside public unassigned parking in busy urban areas, where potential destinations of the special-needs motorists are spread along the street. The model below is less suitable for off-street parking near lone-standing buildings such as hospitals or shopping malls, where parking bays differ by their proximity to the single destination (building entrance) and where the special-needs parking is usually placed nearest to the destination.

7.2. The model

To answer these questions, we modify the model of this paper as follows. The unitary parking supply is endogenously divided into type-1 regular parking with share \( S_1 \) and type-2 special needs parking with remaining share \( S_2 = 1 - S_1 \). As type-2 parking is technologically more sophisticated, each bay costs extra \( f \) dollars per unit of time to the society.

All searchers are also divided into regular and special needs. Regular searchers are able to use any parking bay. The inflow of regular motorists with search cost \( c \) and parking duration \( t \) is \( a(c, t) \); the stock of such motorists parked in steady state is \( D(\infty) \), as before. Special needs searchers, in turn, can use only type-2 parking. As their search strategy is trivial, their distribution across \( c \) and \( t \) is immaterial; we will only assume that their inflow per unit of time is \( A_n \), mean parking duration \( t_n \), and mean search cost \( c_n \). Thus, the total mass of vehicles parked at any time is \( D(\infty) + A_n t_n \).

Contrary to Section 3.4, this section finds that special needs parking can be privileged even if all motorists cannot steer their search. To make the paper shorter, we will make that simplifying assumption. As many governments aim to disperse accessible parking across all urban blocks, rather than concentrate them in some blocks, increasing or decreasing the share of such parking along any search route is indeed problematic. Given such exogenous search strategy \( s_i(c, t) = S_i \), optimal policy consists of finding the optimal parking strategy \( p_i(c, t) \) for regular searchers, and of the optimal share of each type of parking \( S_i \). We now reiterate the steps of Section 3 to find the optimal policy in the new environment.

The chance of success of a regular searcher is redefined from (2) as \( r(c, t) = \sum_{i \in \{1, 2\}} S_i (1 - q_i) p_i(c, t) \), while the chance of success of a special-needs searcher is \( r_n = S_2 (1 - q_2) \). Occupancy-vacancy ratio is redefined from (4) as \( R_1 = \int_{c \in t} a(c, t) p_1(c, t) r(c, t) \, dc \, dt + \frac{A_n c_n}{T_n} \) and \( R_2 = \int_{c \in t} a(c, t) p_2(c, t) r(c, t) \, dc \, dt + \frac{A_n t_n}{T_n} \). The objective function to be minimized now includes the social cost of search, redefined from (5), as well as the additional capital cost of special needs parking:

\[
J = \int_{c \in t} \frac{a(c, t) c}{r(c, t)} \, dc \, dt + \frac{A_n c_n}{T_n} + f S_2. \tag{24}
\]

7.3. Optimal parking strategy

To find optimal parking strategy \( p_i(c, t) \), we minimize the Lagrangian equal to (cf.6)

\[
\mathcal{L} = \int_{c \in t} \frac{a(c, t) c}{r(c, t)} \, dc \, dt + \frac{A_n c_n}{S_2 (1 - q_2)} + \int_{c \in t} \left[ r(c, t) - \sum_i S_i (1 - q_i) p_i(c, t) \right] \mu(c, t) + \lambda_1 \left[ \int_{c \in t} \frac{a(c, t) p_1(c, t) t}{r(c, t)} \, dc \, dt - \frac{q_1}{1 - q_1} \right] + \lambda_2 \left[ \int_{c \in t} \frac{a(c, t) p_2(c, t) t}{r(c, t)} \, dc \, dt + \frac{A_n t_n}{S_2 (1 - q_2)} - \frac{q_2}{1 - q_2} \right]. \tag{25}
\]

while keeping \( S_i \) constant. Proposition 1 remains valid in the environment of this section, hence we have \( \max_i (p_i(c, t)) = 1, \forall c, t \).

Optimization of (25) yields the first-order condition of optimal parking strategy similar to (11):

\[
\begin{align*}
\frac{1}{r(c, t)} \left[ c + \sum_j p_j(c, t) \lambda_j \right] - \frac{\lambda_i}{S_i (1 - q_i)} & \leq 0, \quad p_i(c, t) = 0 \\
0 & \leq p_i(c, t) \in (0, 1), \\
\geq 0, \quad p_i(c, t) = 1
\end{align*} \tag{26}
\]
hence the sign of \( p_2(c, t) - p_1(c, t) \) is, again, inversely related to the sign of \( L_0 = \frac{\lambda_2}{s_2(1-q_2)} - \frac{\lambda_1}{s_1(1-q_1)} \). In Section 3, we could assume that \( L_0 \geq 0 \) without loss of generality, because the two parking types were ex-ante identical. This is no longer the case, so the sign of \( L_0 \) has to be proven.

**Proposition 5.** Under socially optimal policy, we have that \( L_0 \geq 0 \).

The proof is in Appendix A.

Given \( L_0 \geq 0 \), those with \( \zeta < k_1 \equiv L_0 S_1(1 - q_1) \) will use type-1 parking only (\( p_2(c, t) = 0 \)) so we label them “group-x”. Their chance of success is

\[
r_x = S_1(1 - q_1).
\]

(27)

Note that the chances of success of group x and of special needs parkers add up to exogenously given population-average vacancy (cf. [A.2]) \( r_x + r_n = r_y \). This means that a rise in cutoff \( k_1 \), by increasing the share of group-x searchers and thus making type-1 parking more congested, will reduce \( r_x \) while increasing \( r_n \) by the same amount.

### 7.4. Solving for social optimum

Using the function \( G(\cdot) \) defined in Section 3.6 and accounting for the dependence of \( r_x \) and \( r_n \) on \( k_1 \) and \( S_1 \), we can rewrite the objective (24) as follows:

\[
\min_{k_1, S_1} \frac{G(k_1)}{r_x(k_1, S_1)} + \frac{G(\infty) - G(k_1)}{r_y} + \frac{A_n c_n}{r_n(k_1, S_1)} + f(1 - S_1).
\]

The equilibrium value of \( r_x \) can be found from (27) and from the fact that \( \frac{q_1}{1-q_1} = \frac{D(k_1)}{r_x} + \frac{D(\infty)-D(k_1)}{r_y} \), as follows:

\[
r_x(k_1, S_1) = \frac{r_y(S_1-D(k_1))}{r_y+D(\infty)-D(k_1)} = \frac{r_y(S_1-D(k_1))}{1-D(k_1) - A_n t_n}.
\]

(29)

Note that, if \( S_1 = 0 \) (there is no type-1 parking), then \( r_x = 0 \) (those targeting type-1 parking will search forever), which also means \( k_1 = 0 \) (no one will pursue such strategy). We also have that the first term in (28) is zero (there are no group-x searchers hence no associated cost), while \( r_n = r_y \) (the chance of success of special-needs parkers is maximized). Then, the first-order condition of optimality of \( S_1 = 0 \), accounting for \( dr_x = -dr_n \), reads \( \frac{\partial}{\partial S_1} = \frac{A_n c_n}{r_y(1-A_n t_n)} - f \geq 0 \).

The share of regular parking \( S_1 \) must also be less than unity so that there is room to accommodate special-needs searchers; the first-order condition for optimal \( S_1 > 0 \) then reads

\[
\left( -\frac{G(k_1)}{r_x^2} + \frac{A_n c_n}{r_n^2} \right) \frac{r_y}{1-D(k_1) - A_n t_n} = f.
\]

(30)

The optimal chance of success \( r_x \) of group-x searchers must take an interior value, hence the first-order condition for optimal \( k_1 \) reads (noting that \( \frac{\partial G(k_1)}{\partial k_1} = k_1 \frac{\partial^2 G(k_1)}{\partial k_1^2} = kH(k_1) \))

\[
k_1 H(k_1) \left( \frac{1}{r_x} - \frac{1}{r_y} \right) - \left( -\frac{G(k_1)}{r_x^2} + \frac{A_n c_n}{r_n^2} \right) H(k_1) \frac{r_y(1-S_1-A_n t_n)}{(1-D(k_1) - A_n t_n)^2} = 0.
\]

(31)

Equations (30) and (31) determine the optimal values of threshold cost-duration ratio \( k_1 \) and share of type-1 parking \( S_1 \).

When the search cost \( c \) consists of both external component \( c_0 \) and privately known individual component \( c_1 \), the optimal policy can still achieve the first-best (30) and (31). This is because, for every parking duration \( t \), the planner has one degree of freedom - special-needs parking premium \( L(t) \) - to meet one requirement that cutoff \( c \) for special-needs parking use equals \( k_1 t \). Specifically, the first-best requirement \( c_0 + c_1 = k_1 t = L_0 S_1(1-q_1) t \) and the incentive-compatibility requirement (cf. (14)) \( c_1 = L(t) S_1(1-q_1) \) can both be satisfied by setting \( L(t) = L_0 t + \frac{\lambda_0}{S_1(1-q_1)} \).

### 7.5. Exclusive policy is bad even for special-needs searchers

Currently, governments practice an exclusive policy for special-needs parking, banning all regular searchers to use it. In the language of this paper, governments impose \( k_1 = \infty \). The previous section has proven that such policy does not maximize social welfare. The following proposition demonstrates that, if the shares of different parking types respond optimally to changes in \( k_1 \), admission of a small fraction of regular searchers to special-needs parking makes even special-needs searchers better off.

**Proposition 6.** Decreasing \( k_1 \) from infinity to large finite number, while adjusting \( S_1 \) according to (30), increases the chance of success \( r_n \) of the special-needs searchers.

Intuitively, admission of some regular searchers to special-needs parking increases occupancy of the latter, making special-needs searchers worse off. At the same time, such policy change makes the optimal share \( S_2 = 1 - S_1 \) of special-needs parking to increase, improving welfare of special-needs searchers. Proposition 6 states that, if policy changes are small, the positive welfare effect dominates the negative one.
Proof. At the initial policy \( k_1 = \infty \), no regular searcher is admitted to type-2 parking, hence (30) reads (cf. (A.2))

\[
-G(k_1) \frac{d^2 c_n}{r_n^2} + A_n c_n = f.
\]

We have that \( \frac{dc}{dk} = k_1 \frac{dp}{dk} \). This means that, if \( k_1 \) is decreased from infinity to a large finite value so that \( G(k_1) \) marginally changes, the change in \( D(k_1) \) will be of smaller order of magnitude and thus can be ignored. Intuitively, when \( k_1 \) is large, then regular parkers who are admitted to type-2 parking have high cost of search \( c \) and low parking duration \( t \), which means that their impact on occupancies is negligible relative to their impact on social cost of search. But that means that the optimality condition for \( S_1 \), (30), can still be approximated by (32) for large finite \( k_1 \). As the change in \( k_1 \) from infinity to a finite value decreases \( G(k_1) \), formula (32) combined with \( dr_n = -dr_x \) implies that the chance of success \( r_n \) of special-needs searchers must increase. Intuitively, admission of high-value regular searchers to type-2 parking increases its social value, leading to increase in the optimal amount of such parking and thus improving \( r_n \). \( \square \)

7.6. Numerical example

We illustrate the optimal special-needs parking policy by calibrating the model to a large city deciding how much accessible parking to supply and who should be allowed to use it.

The primary cost \( f \) of accessible parking is its increased size, which is necessary for safe exit of disabled passengers from the vehicle. We will assume that additional 10 square meters are needed for an accessible parking bay. Using the land value estimate of 16 million USD per acre (as in downtown Los Angeles), the extra cost of a type-2 parking bay is then approximately $40000. Assuming the interest rate of 3.5%, the annuity equivalent of such cost is $1400 per year, or $26.92 per week. Assuming further that parking is only demanded 40 hours per week, we arrive at \( f = 0.67 \) per hour of parking.

For simplicity, the cost of search is assumed to be the same for all regular searchers and equal to $1 per minute of cruising, as in Section 6. The mean search cost of special-needs motorists is five times higher, $5 per minute of cruising, primarily because cruising time is strongly correlated with the walking distance from vehicle to destination, and such walking is much more costly for the disabled. The mean duration of parking is 1 h for both types of searchers, and is exponentially distributed for the regular ones. The inflow of searchers is such that 80% of all parking bays are occupied by regular motorists, and another 5% by special-needs motorists.

We will consider two policies. The exclusive policy, introduced in Section 7.5, makes type-2 parking available only to special-needs searchers, i.e. \( k_1 = \infty \). Then, according to (29), \( r_x(\infty, S_1) = S_1 - D(\infty) \) and thus \( r_n = r_y - r_x = 1 - S_1 - A_n t_n \). For both quantities to be positive, \( S_1 \) should belong to \( (D(\infty), 1 - A_n t_n) \), which is (0.8,0.95) in our example. The optimality condition for \( S_1 \), (30), is then given by (32) with a unique solution at \( S_1 = 89.9\% \) in our example.

The optimal policy of minimizing (28) with respect to \( S_1 \) and \( k_1 \) yields quite different results. Optimal \( S_1 \) turns out to be 85.2%, i.e. the share \( S_2 \) of special-needs parking is increased by approximately 50%. At the same time, optimal \( k_1 \) is low enough so that those with parking duration less than \( \frac{r}{k_1} = 38 \) min should be allowed to use special-needs parking. Table 2 compares the two policies.

Several results are worth noting. Under both policies, the special needs group has the highest duration of search, despite having very high cost of such search and despite lower occupancy of special-needs parking. This is because special-needs parking is rare and thus takes more time to find. Consistently with Proposition 6, the optimal policy benefits the special-needs group by decreasing the duration of search by 10%. Essentially, optimal policy benefits everyone except the long-term “group-x” regular searchers who remain excluded from type-2 parking and face a smaller supply of parking bays. The aggregate objective function, i.e. expected search cost of all motorists plus the additional cost of special-needs parking supply, is decreased by about 5% by the optimal policy.

Table 2

| Parameter | Exclusive policy | Optimal policy |
|-----------|-----------------|----------------|
| Share of special-needs parking, % | 10.1 | 14.8 |
| Type-1 “regular” occupancy, % | 89.0 | 89.1 |
| Type-2 “special needs” occupancy, % | 61.5 | 61.5 |
| Share of group-x among regular searchers, % | 52.85 | 52.85 |
| Group-x search duration, sec | 24 | 26 |
| Group-y search duration, sec | 47 | 42 |
| Special-needs group search duration, sec | 0 | N/A |
| Type-2 parking premium for regular searchers, $ per hour | 0.674 | 0.674 |
| Average social cost per parking bay, $ per hour | 0.557 | 0.557 |
8. Conclusion

This paper is perhaps the first study that focuses on the economic benefits of discrimination in supply of parking. The main takeaway message is that discrimination can be useful in many cases; at the same time, optimal discrimination is less discriminating than many existing privileged parking policies, such as loading zones, handicapped parking, and EV parking, which squareclude everyone except a minority of privileged motorists. In particular, the calibrated example of Section 7 demonstrates that admission of non-privileged short term parkers to special-needs (e.g. handicapped) parking bays benefits not only the newly admitted, but even the original users, because such reform will be complemented by an increase in supply of special-needs parking.

In the environment where all parking is technologically identical, discrimination that excludes some searchers from some “privileged” vacant bays may be optimal as long as newly arriving motorists can steer their search towards most appropriate type of parking. Such discrimination, by partially segregating short-term or high-search-cost motorists, decreases social search cost by about 10% in calibrated examples. To maximize the ability of motorists to steer their search, privileged and non-privileged parking should optimally be located on different blocks. In case of Manhattan, for example, parking at avenues can be made more expensive than parking at streets.

This last finding of the paper is also its main limitation: the ability of motorists to steer their search depends strongly on the street pattern, which is unique to each city, meaning that there is no one-for-all optimal policy. The ability of city governments to divide all curbside into two types, such that both types are proximate to any destination, while motorists are able to avoid the type they don’t want, is key to success.

The paper (except Section 7) assumes that all bays are equally attractive to everyone; empirically, demand for parking may change rapidly over space. Zakharenko (2016b) in a model without parking discrimination but with heterogenous aggregate arrivals rate finds that, given the same occupancy level, optimal price is proportional to the motorist arrivals rate, making parking at “hotspots” much more expensive. Combining the results of two papers for a setting with parking discrimination and spatially non-homogenous demand, it seems that “hotspots” are natural candidates for privileged (i.e. more expensive) parking intended for short-duration and/or high-value-of-time users.

Zakharenko (2016a) has argued that introduction of self-driving technology may dramatically change how and where vehicles are parked, as autonomous “self-driving” cars can drop off their passengers before searching for parking in an automated mode. In the language of the current paper, fully autonomous vehicles have no walking component of the search cost c, thus bias the distribution of c toward its lower end. Thus, in the world where self-driving and traditional vehicles coexist, heterogeneity of the search costs will be higher than today, increasing the importance of the results of this paper.

Credit Author Statement

Roman Zakharrenko: all stages of production.

Appendix A. Proofs

A1. Proof of proposition 4

From (21), one can show that an increase in $k_2$ from the symmetric policy point (SPP) causes occupancy $q_1$ to increase by the same order of magnitude.\footnote{The proof is available from author upon request.} Intuitively, more motorists steering towards type-1 parking increases occupancy of that type. But then, because $r_y = r_z$ at the SPP, the direct effect of $k_2$ on objective (20) is zero so we can focus on the effect via increasing $q_1$. Observe that, at the SPP, \[ \frac{d\gamma}{dq_1} = -\frac{-S_1}{2} \] \[ \frac{d\gamma}{dq_1} = \frac{S_1}{2}. \] Hence, \[
\frac{dJ(0, \hat{k}_2, D(\infty))}{d\gamma_1} = \frac{G(\hat{k}_2)}{(1 - D(\infty))^2} \frac{\hat{S} - S_1}{2} \] \[ = \frac{G(\hat{k}_2) - G(\hat{k}_2)}{(1 - D(\infty))^2} \frac{S_1 - \hat{S}}{2}. \] \[ \text{(A.1)} \]

From (19), we have that \[ D(\hat{k}_2)(\hat{S} - S_1) = (D(\infty) - D(\hat{k}_2))(S_1 - \hat{S}). \] By dividing (A.1) by the latter equality we obtain that the sign of (A.1) is the same as the sign of \[ \frac{G(\hat{k}_2)}{D(\hat{k}_2)} - \frac{G(\infty) - G(\hat{k}_2)}{D(\infty) - D(\hat{k}_2)}. \] Recalling the definitions of $G$ and $D$, the ratio \[ \frac{G(\hat{k}_2)}{D(\hat{k}_2)} \] can be interpreted as the ratio of average cost $c$ to the average parking duration $t$ for group $y$, while \[ \frac{G(\infty) - G(\hat{k}_2)}{D(\infty) - D(\hat{k}_2)} \] is the same ratio for group $z$. But because, for any given $t$, those in group $z$ have greater $\hat{S}$ ratios by definition of the group, we have that \[ \frac{G(\hat{k}_2)}{D(\hat{k}_2)} < \frac{G(\infty) - G(\hat{k}_2)}{D(\infty) - D(\hat{k}_2)} \] and thus (A.1) is negative. Which means that a small increase in $k_2$ and related increase in $q_1$ decrease the social cost of search and thereby improve social welfare.

A2. Proof of proposition 5

Suppose the converse, that $L_0 < 0$. Then, from (26) combined with Proposition 1, regular searchers with $\hat{S} \leq S_1(1 - q_1)|L_0|$ will take only type-2 parking: $p_1(c, t) = 0, p_2(c, t) = 1$; denote them “group-$\nu$”. The remaining regular searchers will
use the fast parking strategy, \( p_1(c, t) = 1 \forall i \), which makes them group-y in the language of Section 3.6. Because the probability of drawing each parking type is exogenous, the chance of success of group-y searchers is exogenous too and is equal to population share of vacant bays,

\[
r_y = 1 - D - A_n t_n. \tag{A.2}
\]

Group-x searchers have a lower chance of success. Consider all group-x searchers adopting group-y strategy (i.e. increasing \( p_1(c, t) \) to the maximum). The welfare of these searchers is increased as they find vacancy faster; the welfare of initial group-y searchers is unchanged as their chance of success does not depend on strategies of others. As some of group-x searchers now end up in type-1 parking, the welfare of special-needs searchers also increases, as their chance of success \( r_n \) rises due to type-2 parking becoming less congested. Thus, we have indicated a Pareto-improvement, which contradicts optimality of initial allocation.

Appendix B. Mathematical glossary

Notation used within a single section is omitted.

| Notation | Description |
|----------|-------------|
| \( a \) | Motorist arrival rate |
| \( c \) | Social cost of search, per unit of time |
| \( D(k) \) | Mass of motorists with \( \frac{j}{c} < k \) parked in steady state |
| \( G(k) \) | Flow of search cost by motorists with \( \frac{j}{c} < k \) |
| \( H(k) \) | Derivative of \( D(k) \) |
| \( f \) | Aggregate social search cost |
| \( k_3 \) | Slope of type-2 parking strategy cutoff |
| \( k_2 \) | Slope of steering strategy cutoff |
| \( L_0 \) | Excess social cost of type-2 occupancy |
| \( L(t) \) | Monetary premium for type-2 parking, relative to type-1 |
| \( p_i \) | Parking strategy: probability of taking vacant bay |
| \( q_i \) | Steady-state occupancy of type-i parking |
| \( R_i \) | Occupancy/vacancy ratio of type-i parking |
| \( r \) | Chance of successful draw of parking bay |
| \( s_1 \) | Share of type-i parking in total supply |
| \( s_i \) | Steering strategy: chance of drawing type-i parking |
| \( t \) | Duration of parking |

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