Investigating different baryon and antibaryon polarizations in relativistic heavy-ion collisions

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We have investigated the different spin polarizations of baryons and antibaryons observed in relativistic heavy-ion collisions, based on an extended multiphase transport model with the partonic evolution described by a 3-flavor Nambu-Jona-Lasinio (NJL) transport model. Incorporating the spin-orbit coupling induced by the vector potentials from the NJL interaction and from the electromagnetic field, we observe different spin polarizations for various quarks species as a result of their different baryon and electric charges. Consequently, antiquarks (quarks) have a positive (negative) spin polarizations on average, and this presumably leads to a stronger polarization for antibaryons than for baryons at midrapidities once the coupling to the vorticity field is further incorporated.

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Understanding the properties of the quark-gluon plasma (QGP) is one of the main purposes of relativistic heavy-ion collision experiments. In noncentral heavy-ion collisions, QGP is expected to be polarized perpendicular to the reaction plane \textsuperscript{1,3} due to the large angular momentum as well as the strong magnetic field. Theoretical studies predict that the strong vorticity and magnetic field lead to a series of chiral effects (see Ref. \textsuperscript{4} for a review) as well as the spin polarization of hyperons and vector mesons \textsuperscript{5–8}, which are experimentally measurable through their weak decays. On the experimental side, continuous efforts have been made on measuring the spin polarization of these particles \textsuperscript{9,10}. In the collision systems at higher energies with nearly zero baryon chemical potential, shorter duration of the magnetic field, and smaller angular velocity, the spin polarizations of Λ and ¯Λ are found to be negligibly small \textsuperscript{9}. Very recently, the finite spin polarizations of Λ and ¯Λ have been observed experimentally \textsuperscript{11}, with the Λ spin polarization slightly larger than that of ¯Λ. Considerable efforts have been devoted to understanding the polarization of Λ \textsuperscript{12–16} but few of them try to address the different spin polarizations of baryons and antibaryons.

The studies in Refs. \textsuperscript{12–16} attribute the hyperon polarization to the coupling to the vorticity field of QGP, and the spin polarization of quarks and antiquarks are affected in a similar way. On the other hand, the vector potentials, including those from the baryon-antibaryon vector interactions and the electromagnetic field, are expected to be responsible for the different polarizations for baryons and antibaryons at lower collision energies. Due to the finite baryon chemical potential, quarks and antiquarks are affected by different spin-orbit couplings in the baryon-rich matter. In addition, the maximum value of the magnetic field is smaller while its life time is longer in heavy-ion collisions at lower energies. In the present study, we investigate the different spin polarizations of baryons and antibaryons from the vector potentials within the framework of a multiphase transport (AMPT) model \textsuperscript{17}, with the partonic phase described by a 3-flavor Nambu-Jona-Lasinio (NJL) transport model \textsuperscript{18,19}. In order to explore the vector potential effect, we kept the dominating contribution of the vector spin-orbit coupling from the NJL Hamiltonian as well as that from the magnetic field. We found that various quarks and antiquarks are polarized differently according to their different baryon and electric charges, and a larger antiquark spin polarization on average is observed compared to that of quarks, presumably leading to a larger spin polarization of antibaryons than that of baryons at midrapidities. The analysis was based on about 140,000 events of Au+Au collisions at \sqrt{s_{NN}} = 39 GeV, a system with considerable numbers of antiquarks and finite baryon chemical potential as well as the suitable duration of the electromagnetic field within calculation efforts.

We first briefly review the main structure of the AMPT model \textsuperscript{17} and how we extend it \textsuperscript{18,19}. The initial phase-space information of partons is from melting hadrons produced by the Heavy-Ion Jet INteraction Generator (HIJING) model. The evolution of the partonic phase, including that of u, d, and s quarks as well as their antiquarks, is described by a 3-flavor NJL transport model with the spin-orbit coupling induced by the vector potentials to be detailed in the following. When the chiral symmetry is approximately broken, i.e., the quark mass in the central region of the system is half of that in vacuum, the partonic phase ends and the system begins to hadronize. In the present work we neglect the hadronic evolution and study the spin polarization of various quark species as well as that of baryons and antibaryons from a spin-dependent dynamical coalescence approach.
We employ the NJL Lagrangian as
\[
\mathcal{L}_{\text{NJL}} = \bar{\psi}(i\partial - M)\psi + \frac{G}{2} \sum_{\alpha=0}^{8} \left[ (\bar{\psi} \lambda^\alpha \psi)^2 + (\bar{\psi} i\gamma_5 \lambda^\alpha \psi)^2 \right]
- \frac{G_N}{2} \sum_{\alpha=0}^{8} \left[ (\bar{\psi} \gamma_\mu \lambda^\alpha \psi)^2 + (\bar{\psi} i\gamma_5 \gamma_\mu \lambda^\alpha \psi)^2 \right]
- K \left\{ \det_f [\bar{\psi}(1 + \gamma_5)\psi] + \det_f [\bar{\psi}(1 - \gamma_5)\psi] \right\},
\]
(1)
where \( \psi = (\psi_u, \psi_d, \psi_s)^T \) is the quark field with \( \psi_{u,d,s} \) for \( u, d, \) and \( s \) quarks, respectively, \( M = \text{diag}(m_u, m_d, m_s) \) is the current quark mass matrix, \( \lambda^\alpha \) are the Gell-Mann matrices in SU(3) flavor space with \( \lambda^0 = \sqrt{2/3}I \), and \( G \) and \( G_N \) are, respectively, the scalar and vector coupling constant. It was shown in Ref. \[20\] that \( G_V = 1.1G \) gives a better description of the vector meson mass spectrum. However, the strength of the vector coupling constant is still quite uncertain, and it affects the critical point of chiral phase transition in the phase diagram \[21\]. The \( K \) term, with \( \det_f \) denoting the determinant in the flavor space, is the Kobayashi-Maskawa-'t Hooft interaction \[22\] which breaks the axial \( U_A(1) \) symmetry.

We neglect the pseudoscalar and pseudovector interactions (terms with \( \gamma_5 \) in Eq. (1)) in the present study. Considering only the flavor singlet vector interaction in the second interaction term and taking the mean-field approximation \[21\], \[26\], the Lagrangian becomes
\[
\mathcal{L} = \bar{\psi} \gamma_\mu (i\partial \mu - \frac{2}{3} G_V \langle \bar{\psi} \gamma^\mu \psi \rangle) \psi - \bar{\psi} M^* \psi + \ldots
\]
(2)
with \( M^* = \text{diag}(M_u, M_d, M_s) \) and the quark effective masses given by
\[
M_u = m_u - 2G\langle \bar{u}u \rangle + 2K\langle \bar{d}d \rangle \langle \bar{s}s \rangle,
\]
(3)
\[
M_d = m_d - 2G\langle \bar{d}d \rangle + 2K\langle \bar{s}s \rangle \langle \bar{u}u \rangle,
\]
(4)
\[
M_s = m_s - 2G\langle \bar{s}s \rangle + 2K\langle \bar{u}u \rangle\langle \bar{d}d \rangle.
\]
(5)
The rest part in Eq. (2) denotes constant terms such as \( \langle \bar{\psi} \psi \rangle^2 \). The quark condensate in Eqs. (3), (4), and (5) and the vector density in Eq. (2) can be expressed respectively as
\[
\langle \bar{q} q_i \rangle = -2M_i N_c \int \frac{d^3 p}{(2\pi)^3} E_i (1 - f_i - \tilde{f}_i) \quad (i = u, d, s),
\]
(6)
\[
\langle \bar{\psi} \gamma^\mu \psi \rangle = 2N_c \sum_{i=u,d,s} \int \frac{d^3 p}{(2\pi)^3} E_i p^\mu (f_i - \tilde{f}_i),
\]
(7)
where \( N_c = 3 \) is the color degeneracy, \( E_i = \sqrt{M^2_i + p^2} \) is the single-quark energy, and \( f_i \) and \( \tilde{f}_i \) are respectively the phase-space distribution functions of quarks and antiquarks of flavor \( i \), which are calculated from the test-particle method \[23\] in the dynamical simulation. Because the NJL model is not renormalizable, a cut-off \( \Xi = 750 \text{ MeV} \) is introduced in the momentum integration in Eqs. (6) and (7). The values of other parameters are \( m_u = m_d = 3.6 \text{ MeV}, m_s = 87 \text{ MeV}, G\Xi^2 = 3.6, K\Xi^5 = 8.9 \) taken from Refs. \[20\] \[24\].

In order to obtain the single-particle Hamiltonian of quark flavor \( i \), we start from the Euler-Lagrange equation
\[
[\gamma^\mu (i\partial_\mu - A_\mu) - M_i] \psi_i = 0.
\]
(8)
In the above, \( A_\mu = (A_0, -\vec{A}) \) contains the time and space components of the vector potential expressed respectively as
\[
A_0 = B_i g_V \rho_0 + Q_i e \varphi,
\]
(9)
\[
\vec{A} = B_i g_V \vec{\rho} + Q_i e \vec{A}_m,
\]
(10)
with \( g_V = \frac{2}{3} G_V, \rho_0 = \langle \bar{\psi} \gamma^0 \psi \rangle \) and \( \vec{\rho} \equiv \langle \bar{\psi} \gamma^i \psi \rangle \) being respectively the time and space components of the vector density, \( B_i = 1 \) for quarks and \( -1 \) for antiquarks, and \( Q_i \) being the charge number of quark species \( i \). \( \varphi \) and \( \vec{A}_m \) are the scalar and vector potential of the real electromagnetic field
\[
e \varphi(t, \vec{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{1}{R_n - \vec{v}_n \cdot \vec{R}_n},
\]
(11)
\[
e \vec{A}_m(t, \vec{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{\vec{v}_n}{R_n - \vec{v}_n \cdot \vec{R}_n},
\]
(12)
where \( Z_n \) is the charge number of the \( n \)-th particle, \( \vec{v}_n \) is the velocity of the \( n \)-th particle at the retarded time \( t'_n = t - |\vec{r} - \vec{r}_n| \) when the particle emits radiation, and \( \vec{R}_n = \vec{r} - \vec{r}_n \) is the relative position of the field point \( \vec{r} \) with respect to the particle position \( \vec{r}_n \). In the present study we consider the contribution of all spectator protons to the electromagnetic field, since the contribution from the partonic phase was shown to be less important \[25\].

By separating the time and space derivatives, i.e., \( \gamma^\mu \partial_\mu \psi = \gamma^0 \partial_0 \psi + \gamma^i \partial_i \psi \) where \( \gamma^i \partial_i \psi \) is generally much smaller than \( (\vec{p} - \vec{A})^2 + M_i^2 \), the eigenvalues of \( \hat{H} \) and abandoning a negative solution, we obtain the single-particle Hamiltonian as
\[
\hat{H} = \gamma^0 \partial_0 (-\vec{p} + A_k + A_k) + \gamma^0 M_i + A_0,
\]
(13)
where \( \vec{p}_k = i \partial_k \). By calculating the eigenvalue of \( \hat{H} \) and abandoning a negative solution, we obtain the single-particle Hamiltonian as
\[
H = \sqrt{(\vec{p} - \vec{A})^2 + M_i^2} - \vec{\sigma} \cdot (\nabla \times \vec{A}) + A_0.
\]
(14)
Since the spin term \( \vec{\sigma} \cdot (\nabla \times \vec{A}) \) is generally much smaller than \( (\vec{p} - \vec{A})^2 + M_i^2 \), the above Hamiltonian can be further expressed as
\[
H \approx \sqrt{M_i^2 + (\vec{p} - \vec{A})^2} + A_0 - \frac{\vec{\sigma} \cdot (\nabla \times \vec{A})}{2\sqrt{M_i^2 + (\vec{p} - \vec{A})^2}}.
\]
(15)
The first and the second terms are the same as those used in previous studies \cite{18,19}, while the last term represents the spin-orbit coupling for partons, with the strength depending on the baryon and electric charge of the parton as well as its mass.

Starting from the single-particle Hamiltonian with the spin-orbit coupling, the spin-dependent equations of motion (EOMs) can be derived consistently from the spin-dependent Boltzmann-Vlasov equation as \cite{29}

\[
\begin{align*}
\dot{\vec{r}} &= \nabla \vec{p} H, \\
\dot{\vec{p}} &= -\nabla H, \\
\dot{\vec{\sigma}} &= -i[\vec{\sigma}, H].
\end{align*}
\]

The EOMs of \(\vec{r}\) and \(\vec{p}\) are exactly the canonical equations. \(\vec{\sigma}\) are the Pauli matrices in the picture of quantum mechanics, while in the simulation a unit vector in the \(4\pi\) solid angle is assigned to each particle representing the spin expectation direction, and its time evolution is the same as that in the Heisenburg picture of quantum mechanics. The detailed EOMs are

\[
\begin{align*}
\frac{dr_{k}}{dt} &= \frac{p_{k}^{i}}{E_{i}} + \frac{1}{2} \frac{p_{k}^{j}}{E_{i}^{3}} [\vec{\sigma} \cdot (\nabla \times \vec{A})], \\
\frac{dp_{j}^{i}}{dt} &= -M_{j} \frac{\partial M_{i}}{\partial r_{k}} + \frac{p_{j}^{i}}{E_{i}} \frac{\partial A_{j}}{\partial r_{k}} - \frac{\partial A_{j}}{\partial t} - \frac{\partial A_{j}}{\partial t} - \frac{1}{2} [\vec{\sigma} \cdot (\nabla \times \vec{A})] \frac{M_{j}}{E_{i}} \frac{\partial M_{i}}{\partial r_{k}}, \\
&+ \frac{1}{2} \frac{[\vec{\sigma} \cdot (\nabla \times \vec{A})]}{E_{i}^{3}} \frac{\partial A_{j}}{\partial r_{k}}, \\
&+ \frac{\vec{\sigma}}{2E_{i}} \cdot (\nabla \times \vec{A}), \\
\frac{d\vec{\sigma}}{dt} &= \frac{\vec{\sigma} \cdot (\nabla \times \vec{A})}{E_{i}}.
\end{align*}
\]

with \(k,j = x,y,z\) obeying the contraction rule, \(i\) denoting the quark species, and \(E_{i}^{3} = q^{2} + p^{\mu} p^{\nu} M_{i} + \vec{p} \cdot \vec{p} - \vec{A}\). It can be seen that particles with different spins are affected by their different mean-field potentials, and their spins are processed around the total magnetic field \(\nabla \times \vec{A}\), including the contribution of the effective magnetic field \(\nabla \times q \vec{v} \vec{p}\) and the real magnetic field \(\nabla \times e \vec{A}_{m}\).

Figure 1 displays the distributions of various densities and fields at different times in the reaction plane (x-o-z plane), with x the direction for the impact parameter and z the beam direction. The first row gives the general picture how quarks are evolved with time, and the density evolution of antiquarks is similar to that of quarks but with a smaller magnitude. As shown in the second row, although the central magnetic field decreases dramatically with time, the areas where the magnetic field is strong move with the spectators towards \(\pm z\) directions. We found that the real magnetic field calculated from \(e\vec{B} = \nabla \times e\vec{A}_{m}\) gives almost the same value as that calculated from \[\vec{B}(t,\vec{r}) = \frac{e}{4\pi} \sum_{n} \frac{\vec{v}_{n} \times \vec{R}_{n}}{(\vec{R}_{n} - \vec{v}_{n} \cdot \vec{R}_{n})^{3}} (1 - v_{n}^{2}). \]

On the other hand, using \(\vec{A}_{m}\) has the advantage of reducing the possibility of divergence at \(R_{n} - \vec{v}_{n} \cdot \vec{R}_{n} \sim 0\), due to the lower power of it in the denominator of Eq. (12). Compared with the real magnetic field, the distribution of the effective magnetic field from the curl of the net quark flux is more diffusive, while in the central region it is positive (negative) at \(x \cdot z > 0 (x \cdot z < 0)\), as shown in the third row of Fig. 1. The real and the effective magnetic field lead to local spin polarizations of various quark species according to their electric and baryon charges. Due to the \(-\vec{\sigma} \cdot (\nabla \times \vec{A})\) term in the single-particle Hamiltonian, the spin \(\vec{\sigma}\) tends to follow the direction of \(\nabla \times \vec{A}\) in order to minimize the energy. \(u (\bar{u})\) quarks have the positive (negative) baryon number and +2/3 (−2/3) charge number, and are most strongly affected by the spin-orbit coupling due to the additive effects from the real and the effective magnetic field. This can be observed from the distribution of their spin densities defined as \(\tilde{s} = \sum_{c=q,(\bar{u})} \vec{\sigma}_{c}\) at each local cell, with its y component for \(u\) and \(\bar{u}\) quarks respectively shown in the fourth and the fifth row of Fig. 1. Particularly, the midrapidity region of \(u (\bar{u})\) quarks is expected to be polarized in the +y (−y) direction, while the polarization is inverse at large rapidities. For \(d\) and \(s\) quarks as
and this leads to the situation that the \( \Lambda (\bar{\Lambda}) \) spin is of its flavor and spin wave function must be symmetric. The left part of Fig. 2 displays how the spin polarizations for various quark species at midrapidities are developed during the partonic evolution in midcentral Au+Au collisions at \( \sqrt{s} = 39 \) GeV. Generally, the polarization is built quickly in the early stage of the partonic evolution but slowly at the later stage. Quarks have the strongest polarization while those for d and s quarks are much weaker. The opposite spin polarization for d and s quarks compared to u quarks is due to the overwhelming effect of the real magnetic field over the effective magnetic field. For each flavor, quarks and antiquarks have the opposite spin polarization. If we average over all flavors, quarks have a net negative polarization while antiquarks have a net positive polarization. The value of \( G_V \) slightly affects the polarization while the difference is within the statistical error, showing that the polarization is dominated by the real magnetic field.

How the proton spin is determined by its inside structure is a famous unsolved problem called “proton spin crisis” (see, e.g., Ref. [34] for a review). The European Muon Collaboration have been investigating this problem since 1980s [35]. The proton spin may come from not only the spin of quarks but also their orbital angular momentum [35]. Recently it was realized that gluons may contribute significantly to the proton spin [37]. The situation is even more uncertain for arbitrary baryons. Despite the uncertain relation between the quark and baryon spin, we estimate the productions of baryons and antibaryons at different spin states by extending the dynamical coalescence model [38–40] to include spin degree of freedom [41, 42]. In the dynamical coalescence model, the probability of three quarks (antiquarks) to form a baryon (an antibaryon) is expressed by a Wigner function with Gaussian form in both coordinate and momentum space, and the width of the Gaussian function is fitted by the charge radius of the baryon (antibaryon). With the s-wave coordinate wave function and the antisymmetric color wave function for \( \Lambda \) or \( \bar{\Lambda} \), the direct product of its flavor and spin wave function must be symmetric, and this leads to the situation that the \( \Lambda (\bar{\Lambda}) \) spin is determined by that of its constituent s (\( \bar{s} \)) quark [3].

Technically, this can be done by selecting combinations of an s (\( \bar{s} \)) quark with a fixed spin and a pair of u and d (\( \bar{u} \) and \( \bar{d} \)) quarks with opposite spins. This naturally results in a positive spin polarization for \( \Lambda \) and a negative spin polarization for \( \bar{\Lambda} \), differently from the larger \( \bar{\Lambda} \) polarization than \( \Lambda \) observed experimentally [11]. However, since the quark (antiquark) spin is negative (positive) on average, if we do such spin-dependent quark coalescence regardless of the quark flavor, i.e., a spin-up (spin-down) baryon is formed by two spin-up (spin-down) quarks and one spin-down (spin-up) quark, we naturally get positive polarization for antibaryons and negative polarization for baryons at midrapidities, as shown in the right part of Fig. 2.

Figure 3 further displays the rapidity distribution of average spin polarizations of quarks and antiquarks as well as those of baryons and antibaryons. It is found that antibaryons (baryons) have a positive (negative) spin polarization at midrapidities but a negative (positive) spin polarization at large rapidities, consistent with the different spin polarizations of quarks or antiquarks at smaller and larger \( z \) seen from Fig. 1. It would be of great interest to confirm the larger spin polarizations for baryons than for antibaryons at large rapidities on the experimental side.

To summarize, based on an extended multiphase transport model with the partonic evolution described by a 3-flavor Nambu-Jona-Lasinio (NJL) transport model, we have investigated the different spin polarizations of quarks and antiquarks. The spin-orbit couplings induced by the vector potentials, which originate from the NJL...
extended AMPT model with $R_b$ baryons and antibaryons from quark coalescence in midcentral collisions at $\sqrt{s} = 39$ GeV from the extended AMPT model with $R_V = G_V/G = 1.1$.

vector interaction and the electromagnetic field, are different for various quark species, as a result of their different baryon and electric charges. An positive spin polarization is observed for antiquarks but negative for quarks on average at midrapidities. The spin polarizations of baryons and antibaryons also depend on their internal structures, and based on a spin-dependent dynamical coalescence model a positive spin polarization is observed for antibaryons but negative for baryons at midrapidities.

In the future study, we will incorporate the coupling to the vorticity of QGP into the NJL transport model, and this is expected to enhance the spin polarization of both quarks and antiquarks simultaneously. It will also be interesting to investigate the collision energy dependence of the spin polarization based on the present framework. In addition, the different spin polarizations of baryons and antibaryons may also be attributed to their different spin-orbit couplings in the baryon-rich hadronic phase. These questions need further investigations.

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![Graph showing rapidity distributions of the average spin polarizations of quarks and antiquarks as well as those of baryons and antibaryons from quark coalescence in midcentral Au + Au collisions at $\sqrt{s} = 39$ GeV from the extended AMPT model with $R_V = G_V/G = 1.1$.](image-url)
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