Born-Infeld type modification of the gravity

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We study a non-linear modification to General Relativity in which the standard Einstein-Hilbert action is replaced by a Born-Infeld type action. Also study us stability issues to judge about viability of this modification. We establish the conditions that this modification must satisfy for to avoid the problems associated with the Dolgov-Kawasaki instability, tachyon instability and negative effective gravitational coupling. The particle content of gravitational spectrum of the linearized Born-Infeld theory shows the existence of massless gravitons plus new degree of freedom of 0-spin associated with $R^2$ term. For a toy model we proved that for appropriate values of the background curvature, this theory is free of ghost, Dolgov-Kawasaki instability and tachyon instability also it has a positive effective gravitational coupling. We find the scalar-tensor theory equivalent at this $f(R)$-theory in the Einstein frame, and we study some properties of the scalar potential.

KeyWord: Born-Infeld gravity, linearized theory, scalar-tensor theory.

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I. INTRODUCTION

Recently the cosmological observations Supernova (SNIa) $^1$, large scale structure $^2$, cosmic microwave background $^3$, the integrated Sachs-Wolfe effect $^4$, baryonic acoustic oscillations $^5$ and from gravitational lensing $^6$ have offered strong evidence that the expansion of our universe is accelerating. The General Relativity (GR) cannot explain the acceleration of late universe. In most investigations the origin of the acceleration is attributed to a mysterious component with a negative pressure, called Dark Energy (DE). The preferred candidate for this entity is a cosmological constant $\Lambda$. The Cosmological Constant problem has been a longstanding problem in fundamental theoretical physics $^7$. An alternative at Dark Energy is modified the Einstein gravity theory. Several theories of gravity of this kind have been proposed since long ago in $^8$. The modifications of gravity have been used in many contexts, perhaps the most famous is the first model of inflation introduced by Starobinsky $^9$. However, in the last few years, modifications of gravity have been attracting attention as an alternative to the Dark Energy $^{10,12}$, in order to explain the late-time acceleration at large cosmological scales $^{13,15}$.

The General Relativity is based in the Einstein-Hilbert action (EH)

$$S_{EH} = \frac{1}{2\kappa^2} \int d^4x\sqrt{-g}(R - 2\Lambda)$$

Where $\kappa^2 = 8\pi G$ is the gravitational coupling constant, $R$ is the curvature scalar, $g$ is the determinant of the metric and $\Lambda$ the cosmological constant. Numerous modifications to the classical EH action have been proposed motivated for different reasons. In particular, renormalization at one-loop demands that the EH action be supplemented by higher order curvature terms $^{16,17}$. Besides, when quantum corrections or string theory are taken into account, the effective low energy action for pure gravity admits higher order curvature invariants $^{18}$. The most well-known alternative to general relativity, are Scalar-Tensor Theories $^{19,20}$, but there are still numerous proposals for modified gravity in contemporary literature. Typical examples are the braneworld gravity of Randall-Sundrum $^{21,22}$ or Dvali-Gabadadze-Porrati $^{23}$, quantum gravity $^{24}$. But there is other way to modify the Einstein gravity theory. If the curvature is replaced in $^{11}$ by an arbitrary function $f(R)$, the gravity action is given by

$$S_g = \frac{1}{2\kappa^2} \int d^4x\sqrt{-g}f(R)$$

This kind of modification to GR has been extensively in the literature. We can consider a series expansion of $f(R)$ in the following form

$$f(R) = \ldots + \frac{\mu_2}{R^2} + \frac{\mu_1}{R} + R + \frac{R^2}{\eta_2} + \frac{R^3}{\eta_3} + \ldots$$

where $\mu_i$ and $\eta_i$ are constants with the appropriate dimensions. The expansion $^{25}$ includes phenomenologically interesting term $^{25}$. The positive power in $^{25}$ modified gravity at high energy (early universe), for example, the model $f(R) = R + \beta R^2 (\beta > 0)$ can lead to the inflation in the early universe because of the presence

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$^1$ Higher order actions are indeed renormalizable but not unitary $^{17}$.
The $f(R)$ theories have been studied in many contexts, inflation \[^9\], Dark Energy \[^33–36\], large scale structure \[^54–60\]. Even if $f(R)$ theories were not a viable alternative to explain late-time acceleration of the expansion, these modifications are of great relevance to study early-time inflation \[^9\].

We can add matter fields to $f(R)$ gravity action \[^2\], in this case the total action is given by

$$ S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m(g_{\mu\nu}, \xi) $$

(4)

where $\xi$ represent the matter fields, i.e. electromagnetic field. Varying the action (4) with respect to the metric, we find the following field equation

$$ f'(R)R_{\mu\nu} - \nabla_\mu \nabla_\nu f'(R) + g_{\mu\nu} \Box f'(R) - \frac{1}{2} g_{\mu\nu} f(R) = \kappa^2 T_{\mu\nu}^{(m)} $$

(5)

where

$$ T_{\mu\nu}^{(m)} = -\frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g_{\mu\nu}} $$

(6)

is the momentum-energy tensor of the matter. The field equation (5) can be written in the following form

$$ G_{\mu\nu} = \kappa_{eff}^2 \left( T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(cur)} \right) $$

(7)

where $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}^{(cur)}$ is the momentum energy tensor of curvature

$$ T_{\mu\nu}^{cur} = \frac{1}{\kappa^2} \left( \frac{f(R) - R f'(R)}{2} g_{\mu\nu} + \nabla_\mu \nabla_\nu f'(R) \right) $$

(8)

and effective gravitational coupling

$$ \kappa_{eff}^2 = \frac{\kappa^2}{f'(R)} $$

(9)

The trace of the field equation (11)

$$ f'(R)R - 2f(R) + 3\Box f'(R) = \kappa^2 T $$

(10)

The equation (10) relates $R$ with $T$ differentially and not algebraically as occur in the GR. This means that the field equation in $f(R)$-theories admit a larger variety of solutions than GR.

The $f(R)$ theories are excellent candidates to gravity theory. The $f(R)$ theories include some of the basic characteristic of higher-order gravity theories. Also there are several reasons to believe that $f(R)$ theories are only of the higher-order gravity theories, that it can avoid the well-known Orstrodski instability \[^38\].

### II. BORN-INFELD TYPE MODIFICATION

There are several ways to modify the EH action. In this paper, we consider a modification based on the gravitational analogues of non-linear Born-Infeld electrodynamics \[^39\], modification for smoothing out singularities \[^12, 40, 41\]. Through this modification the Einstein-Hilbert Lagrangian density $L_{EH} = \sqrt{-g} L$ can be replaced by one of the Born-Infeld form:

$$ L_{EH} \rightarrow L_{BI} = \sqrt{-g} \lambda \left( 1 - \sqrt{1 + \frac{2f(R)}{\lambda}} \right) $$

(11)

where the constant $\lambda$ is associated with the curvatures scale accessible to the theory. Also we can replace the Lagrangian $L$ in (11) by a $f(R)$ function within the square root (see \[^42\])

$$ L_{BI} = \sqrt{-g} \lambda \left( 1 - \sqrt{1 + \frac{2f(R)}{\lambda}} \right) $$

(12)

Let us consider a function $f(R) \propto R^n$ in BI-type Lagrangian density \[^12\] for gravity, in this case the action of the pure gravity \[^2\] can be rewritten as

$$ S_{BI} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left( 1 - \sqrt{1 + \epsilon \alpha R^n} \right) $$

(13)

where $\epsilon = \pm 1$, the constant $\alpha$ is a constant with mass dimension and $(n > 0)$.

Other variants of Born-Infeld type gravity was considered in \[^43–46\]. The BI gravity has been intensively studied in many context, as cosmological framework proves to lead to interesting behavior \[^47–49\]. In particular has been analyzed the black hole properties \[^50, 51\], the astrophysical phenomenology \[^52, 53\], massive gravity \[^54\] and primordial inflation \[^59\].
Born-Infeld gravity can be phenomenologically viable, if this modification satisfy several physically motivated requirements.

- Reduction to Einstein-Hilbert action at small curvature.
- Ghost-free
- Regularization of singularities, i.e. Schwarzschild singularity
- Supersymmetrizability

The function \( f(R) \) that appear in the action (13) also satisfies the condition \( \lim_{R \to 0} f(R) \to 0 \) which means that there is a flat space-time solution.

A. Born-Infeld Lagrangian expansion

In this section we study the expansion around maximally symmetric vacuum spaces of constant curvature of the \( f(R) \)-theory that appear in (13). In the next section we will analyze the stability issues in these constant curvature spaces.

The equation (10) for \( R = Const \) and \( T = 0 \) reduce to

\[
f'(R)R - 2f(R) = 0
\]

which is an algebraic equation in \( R \).

For the action (13) the roots of the equation (14) are

\[
R_0 = 0 \quad R_0 = \sqrt{\frac{8(n-2)}{\epsilon\alpha(n-4)^2}} \sqrt[n]{x}
\]

For \( R = 0 \) the equation (5) reduced to \( R_{\mu\nu} = 0 \) and the maximally symmetric solution is Minkowski space-time. The second root (15) reduced to \( R_{\mu\nu} = \frac{\kappa}{R_0} g_{\mu\nu} \) the equation (15) and the maximally symmetric solution is de Sitter space if \( R_0 > 0 \) or Anti de Sitter (AdS) space if \( R_0 < 0 \).

In an empty space of constant curvature \( R_0 \) all \( f(R) \) theories admit Schwarzschild-de Sitter(or AdS) solution [62]. We can find static solutions with spherical symmetric of the theory (13) in the Schwarzschild form

\[
ds^2 = -A(r)dt^2 + A^{-1}(r)dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)
\]

where

\[
A(r) = 1 - \frac{2GM}{r} - \frac{1}{12} \left[ \frac{8(n-2)}{\epsilon \alpha (n-4)^2} \right]^{\frac{1}{4}} r^2
\]

In above solution we can see that AdS-Schwarzschild black holes in Einstein gravity are also solutions of Born-Infeld gravity [13].

Let us consider the expansion of the action (13) around maximally symmetric vacuum spaces of constant curvature \( R = R_0 \), in the neighborhood of the point \( R = R_0 \),

\[
f(R) = f(R_0) + \sum_{n=1}^{\infty} \frac{f^{(n)}(R_0)}{n!} (R - R_0)^n
\]

If we consider until second order term \((R - R_0)^2\) in the series (13), one has

\[
f(R) = f(R_0) + f'(R_0)(R - R_0) + \frac{1}{2} f''(R_0)(R - R_0)^2 + O(R^3)
\]

the above expansion can be written as

\[
f(R) = \beta \left( -2\Lambda + R + \frac{R^2}{6m_0^2} \right)
\]

where we have considered the following notation

\[
-2\Lambda = \frac{f_0 - R_0 f_0'}{f' - R_0 f_0''} \quad \beta = f' - R_0 f_0''
\]

and

\[
f_0 = f(R_0) \quad f_0' = \frac{df}{dR} \bigg|_{R_0}.
\]

From the definition of the cosmological constant (21) we find the expected value \( \Lambda = \frac{R_0}{2} \). The linearized action is given by

\[
S = \frac{\beta}{2\kappa^2} \int d^4x \sqrt{-g} \left( -2\Lambda + R + \frac{R^2}{6m_0^2} \right)
\]

Where the linearized theory is defined by the \( f(R) \)-theory

\[
\tilde{f}(R) = R + \frac{R^2}{6m_0^2} - 2\Lambda
\]

For small curvatures the action (23) reduce to EH action (1). The model (24) was motivated to be renormalizable, results in the self-consistent inflation. Also the \( R^2 \) term in (23), prevents from the singular behavior in the past and in the future [63]. The gravitational spectrum of the theory (23) shows the existence of massless gravitons relate with the \( R \)-term in the linearized action (23) plus new degree of freedom of 0-spin and mass \( m_0 \) associated with the \( R^2 \)-term in (23). The mass of the scalar degree of freedom \( m_0^2 \) is obtained in the weak-field limit, if we consider small, spherically symmetric, perturbation on a Sitter space of constant curvature.

4 This requirement results quite stringent and is probably implemented if gravity descends from String Theory [60, 61].

5 This new scalar degree of freedom also is called scalaron.
The exchange of this scalar field between two test particles change the Newtonian potential (see [64])

\[
\frac{1}{r} \rightarrow \frac{1}{r} \left( 1 + \frac{1}{3} e^{-m_0 r} \right)
\]  

(25)

For large values of \(m_0\) the exponential term vanish in [25] and we recover the Newtonian potential, however for small values of \(m_0\) the exponential term do not vanish and the corrections to Newton’s law are not negligible.

### B. Stability criteria

The study of the stability in higher order modification of GR is very important to judge about viability of these modifications. These theories are plagued by several kinds of instabilities, which lead to destabilize the theory\(^6\). For example, we have the Ostrogradski instability, based on Ostrogradski’s theorem [38]. This instability is associate with Lagragian that depend higher order time derivate. In this case that dependence cannot be eliminated by partial integration. In [38] proved that \(f(R)\) theories are the only Ostrogradski-stable higher order modification to GR.

Other instability is known as Dolgov-Kawasaki instability \(^7\). This instability was discovered by Dolgov and Kawasaki in the theory \(f(R) = R - \frac{\mu^4}{R^2}\).

Dolgov and Kawasaki discovered this instability in the theory \(f(R) = R - \frac{\mu^4}{R^2}\), this instability manifests itself on an extremely short time scale. The existence of this instability is sufficient to rule out a model \(^20\). In \(^32, 36\) proved that adding an \(R^2\) term at this theory, it can removes this instability. The study of Dolgov-Kawasaki instability was generalized for arbitrary \(f(R)\) theories in \(^26\).

Other instabilities are caused by the presence of a 0-spin tachyon degree of freedom. This instability is associated with negative values of the mass squared \(m_0^2\) defined in \(^21\). This new scalar degree of freedom (\(\phi = f'(R)\)) with mass \(m_0\) is associated to \(R^2\) term in \(^23\). From \(^20\), we can see that the constant \(\beta = f' - R_0 f''\) is an factor that multiplies the action \(^28\), and, hence, it can change the sign of this, therefore is necessary that \(\beta > 0\).

Therefore, a \(f(R)\) theory physically viable should be free of ghost, tachyon, Dolgov-Kawasaki instability, and negative effective gravitational coupling.

- **Ostrogradski Stability**
  How we said previously the \(f(R)\)-theories are Ostrogradski-stable \(^38\). Obviously the modification \(^13\) is Ostrogradski-stable.

- **Dolgov-Kawasaki Stability**
  Assuming a positive effective gravitational coupling \(^9\), we find

\[
\frac{dG_{eff}}{dR} = \frac{\kappa^2 f''(R)}{8\pi f'(R)^2}
\]

(26)

when \(f'' < 0\), the effective gravitational coupling increases with the curvature. If \(f'' < 0\) this instability can be seen as an instability in the gravity sector. In other words, this positive mechanism leads to destabilize the theory \(^25\). The modification \(^13\) avoid the Dolgov-Kawasaki instability if \(f'' > 0\).

\[
f'' = -\frac{\alpha n R_0^{n-2} [\alpha (n-2) R_0^n + 2(n-1)]}{4 (\alpha R_0^n + 1)^{3/2}} \geq 0
\]

(27)

This condition also guarantees the classical stability of Schwarzschild black hole \(^63\).

- **Positive Effective Gravitational Coupling**
  The effective gravitational coupling multiplies the linearized BI action \(^23\), is necessary that this factor do not change the sign of the action.

\[
\frac{\beta}{2\kappa^2} = \frac{\alpha n R_0^{n-1} [\alpha (n-4) R_0^n + 2(n-2)]}{8\pi (\alpha R_0^n + 1)^{3/2}} > 0
\]

(28)

- **Absence of Tachyon Instability**
  The Dolgov-Kawasaki stability condition \((f'' > 0)\) and a positive effective gravitational coupling \((f' - R_0 f'' > 0)\) together imply absence of tachyon in Born-Infeld gravity. The mass of the scalar degree of freedom for the theory \(^13\), in the linear approximation is given by

\[
m_0^2 = \frac{R_0 [4 - 2n - \alpha (n-4) R_0^n]}{3\pi (n-2) R_0^n + 6(n-1)} > 0
\]

(29)

- **Ghost free**
  Ghosts are massive state of negative norm that cause apparent absence of unitarity, it appear constantly in higher order gravity theories. A viable gravity theory should be ghost-free. The stability condition \(f'' > 0\) essentially amounts guarantee that the scalar degree of freedom is not a ghost \(^16, 66, 67\). If the BI modification is free of Dolgov-Kawasaki instability also is ghost-free.

### C. Scalar-Tensor theory equivalent to Born-Infeld gravity

The \(f(R)\) theory in the metric formalism can be cast in the form of Brans-Dicke theory \(^19\) with a potential for the effective scalar degree of freedom \(^26\). By introducing

\(m_0 \approx \frac{R_0 [4 - 2n - \alpha (n-4) R_0^n]}{3\pi (n-2) R_0^n + 6(n-1)} \geq 0\)
the auxiliary field $\chi$, we rewrite the action (13) of the \( f(R) \)-gravity in the following form:

\[
S_{BI} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left( f'(\chi)(R - \chi) + f(\chi) \right)
\]  

(30)

Varying this action with respect to $\chi$, we obtain

\[
f''(\chi)(R - \chi) = 0
\]

(31)

Provided $f''(\chi) \neq 0$ it follows that $\chi = R$. Hence the action (30) recovers the action (13) in $f(R)$ gravity. Using the conformal transformation $\hat{g}_{\mu\nu} = e^{-\phi} g_{\mu\nu}$ in the action (13) where the new scalar degree of freedom is defined by

\[
\phi = -\ln f'(R)
\]

(32)

Usually in (32) is assumed $f'(R) > 0$, even if $f'(R) < 0$ can be defined $\phi = -\ln |f'(R)|$. Then the sign in front of the scalar curvature becomes negative. In other words, the regimen of anti-gravity can be realized, however for the action (13) the regimen anti-gravity $f'(R) < 0$ can be avoid if $\epsilon = -1$.

The Einstein frame action looks like

\[
S_{BI}^E = \int d^4x \sqrt{-\hat{g}} \left( \frac{1}{2\kappa^2} \hat{R} - \frac{3}{2} \left( \hat{\nabla} \phi \right)^2 - V(\phi) \right)
\]

(33)

where $\left( \hat{\nabla} \phi \right)^2 = \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ and potential is given by

\[
V(\phi) = \frac{f'(R)R - f(R)}{2\kappa^2 f'(R)^2} |R - R(\phi)|
\]

(34)

For the $f(R)$-function that appear in (13) we find

\[
e^{-\phi} = -\frac{n\alpha R^{n+1}}{\sqrt{1 + \alpha R^n}}
\]

(35)

If $R$ is large and using $\phi = -\ln |f'(R)|$ for $\epsilon = 1$ we have

\[
e^{-\phi} \approx n\sqrt{\alpha R^{n/2}}
\]

(36)

Then if $e^{-\phi} \to \infty$

\[
V(\phi) = (\beta - \sqrt{\alpha R^n}) e^{(n+1)\phi}
\]

(37)

where $\beta = (\frac{1}{n\sqrt{\alpha}})^{n/2}$. The above potential has been found under approximation $1 \ll \alpha R^n$. For $0 \leq n < 2$ and $n > 4$ the potential (37) diverge if $\phi \to \infty$ and $V(\phi) \to 0$ if $\phi \to -\infty$.

How we can see in (35) to obtain the scalar-tensor theory equivalent to (13) is still very complicated. That represent a difficulty to study the particle spectrum of the theory (68). However we can begin with the action (24), in this case the scalar degree of freedom is given $\phi = -\ln \hat{f}'(R)$

\[
e^{-\phi} = 1 + \frac{R}{3m_0^2}
\]

(38)

The above potential correspond to linearization of the original theory and not the theory itself. However, we can prove, that the scalar-tensor theory dual to linearized theory (23) and the corresponding to original theory (13) are equivalent. This affirmation rest on the compute of the effective mass of the scalar degree of freedom corresponding to theory (24)

\[
m_{eff}^2 = \frac{\hat{f}_0 - R_0 \hat{f}_0''}{3\hat{f}_0''}
\]

(40)

For the function $\hat{f}(R)$ defined in (24)

\[
\hat{f}_0 = 1 + \frac{R_0}{3m_0^2}, \quad \hat{f}_0'' = \frac{1}{3m_0^2}
\]

(41)

then the effective mass of the scalar degree of freedom coincides with the mass of the 0-spin excitation $m_0^2$ computed through (24): $m_{eff}^2 = m_0^2$.

In the Fig.1 we illustrate the potential (39), in this figure we can see that this potential when $\phi \to \infty$ $V(\phi) \to 3m_0^2$, and when $\phi \to -\infty$ the potential diverge.

In the regimen $\phi \to \infty$ the potential is nearly constant $V(\phi) \approx \frac{3m_0^2}{2}$ which can leads to slow-roll inflation. The potential (39) has a stationary point at $\phi_c = \ln 1 + \frac{4\Lambda}{3m_0^2}$, this point correspond to minimum of potential $V''(\phi_c) > 0$ if $m_0^2 > 0$. From potential (39), we obtain the mass of a scalar state

\[
m^2_{\phi} = \frac{d^2V(\phi)}{d\phi^2} = \frac{9m_0^4}{4\Lambda + 3m_0^2}
\]

(42)

Through the above equation, like $m^2_{\phi} = m_0^2$, we find that the cosmological constant $\Lambda = \frac{3}{2}m_0^2$. Observationally we
know that $\Lambda \sim (10^{-42} \text{GeV})^2$ which is a very small value and we obtain that $m_0 \sim 10^{-42} \text{GeV}$ which imply that the correction to Newton’s law (25) is very large.

D. Toy model (case $n = 1$)

The objective of this section is to illustrate the results obtain in previous section for the following toy model corresponding to $n = 1$ and $\epsilon = 1$ in the original theory (13). This model has been previously studied in [69]. Through the eq. (14) we find

$$\frac{4\alpha R - 4\sqrt{\alpha R + 1} + 4}{\alpha} - R = 0$$

which is the algebraic equation with respect to $R$, the roots of this equation are

$$R = 0 \quad R = -\frac{8}{9\alpha}$$

Like $\alpha > 0$ the second solution corresponds to $\text{AdS}$ space.

We know that these kinds of modifications are plagued of several kind of instability. In this case through the eq. (27) we find that the theory is free Dolgov-Kawasaki Instability for

$$R_0 \geq -\frac{1}{\alpha}$$

the effective gravitational coupling (28) is positive for

$$-\frac{1}{\alpha} < R < \frac{3}{2\alpha}$$

The conditions of Dolgov-Kawasaki stability (13) and positivity of the effective gravitational coupling (10) together imply absence of tachyon instability for

$$-\frac{1}{\alpha} < R < \frac{3}{2\alpha}$$

The condition $f''(R) \geq 0$ is warranty that the scalar degree of freedom is not a ghost. Through (33) for $n = 1$ and $\epsilon = 1$ we find

$$e^{-\phi} = \frac{\alpha}{2\sqrt{1 + \alpha R}}$$

The self-interaction potential of the scalar field has the following form

$$V(\phi) = \frac{e^{\phi} (\alpha e^\phi - 2) (3\alpha e^\phi + 2)}{4\alpha}$$

In the Fig 2 we show the potential (49) and we can see how $V(\phi) \to 0$ if $\phi \to -\infty$ while for $\phi \to \infty$ the potential diverge.

The potential (49) has a stationary point at $\phi_c = \ln \frac{2(2 + \sqrt{1 + 4\alpha})}{4\alpha}$, in this point (49) possesses minimum $V''(\phi_c) > 0$. From potential (49), we obtain the mass of a scalar state

$$m_\phi^2 = \frac{d^2 V(\phi)}{d\phi^2} = \frac{27}{4} \alpha e^{\phi} - \frac{e^\phi}{\alpha} - 4e^{2\phi}$$

We can consider that present universe corresponds to the minimum $\phi_c$ of $V(\phi)$. It follows from (51) that $m_\phi^2 = \frac{4(52 + 17\sqrt{13})}{81\alpha^2}$. If the constant $\alpha$ is very small the effective mass $m_\phi^2$ is large and the scalar field $\phi$ decouples, hence corrections to Newton’s law (25) are negligible. Then the obtained theory does not conflict with the cosmological observations, say, the solar system observations [70].

In [49] was studied the phase space of a theory like this with $\epsilon = -1$ and $n = 1$ and was proved that this model can produce late time acceleration, which is associated with a solution dominated by the curvature see [49].

III. DISCUSSION

In summary, in this paper we have considered a modification type Born-Infeld to Einstein-Hilbert action. We paid special attention to the issues associated with linearization of this theory around vacuum, maximally symmetric spaces of constant curvature.

The gravitational spectrum of the linearized theory show the existence massless graviton plus a new scalar degree of freedom of spin-0 which is associated with $R^2$ term in (28). The existence of this new scalar degree of freedom modified the newtonian potential, however for large values of $m_0$ this correction is negligible. We prove that the masses of the scalar degree of freedom corresponding to original theory and linearized theory are equals.

From the potential (29) corresponding to linearized theory we find that the mass of the scalar field is of order $m_0 \sim 10^{-42} \text{GeV}$ which imply that the correction to
Newton’s law \((25)\) is not negligible. The potential \((39)\) behaves almost constant when \(\phi \rightarrow \infty\) which can leads to inflation in the early universe.

The toy model is free of the Dolgov-Kawasaki instability, Tachyon instability and negative effective gravitational coupling for \(-\frac{1}{\alpha} < R < \frac{1}{2\alpha}\). The condition \(f''(R) > 0\) is warranty that the scalar degree of freedom is not a ghost. Also it was demonstrated that for small values of the constant \(\alpha\) the mass of scalar degree of freedom is large and the corrections to Newton’s law are negligible. Also in \([48]\) was proved that this model can leads to a late time acceleration state in the universe without to include Dark Energy, in this model the acceleration of the expansion is produced by nonlinear effects of the curvature.

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