An approach to electromagnetism from the general relativity

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Abstract
Classical gravitation is so similar to the electrostatic that the possible unification has been investigated for many years. Although electromagnetism is formulated now successfully by quantum field theory, this paper proposes a simple approach to describe the electromagnetism from the macroscopic perspective of general relativity. The hypothesis is based on two charged particles that cause disturbance energy sufficient to disrupt the space-time and explain approximately Maxwell's equations. Therefore, with such this simple idea, we suggest the possibility that the geometric relationship between electromagnetism and gravitation is not yet fully exhausted.

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I. INTRODUCTION

The possible relationship between the electromagnetic interaction and gravitational interaction is of great interest to theoretical physics. Most of the studies explores purely quantum perspectives to develop the proposals for unification. An early theory that tried this purpose was the Kaluza-Klein, which introduced a 5th dimension at microscopic scale (Wuensch, 2003). This theory led to other theoretical problems, as the existence of the magnetic monopoles and the compactification of the fith dimension (Bailin and Love, 1987).

Kaluza demonstrated that the five-dimensional general relativity in vacuum contained four-dimensional general relativity in the presence of an electromagnetic field (Overduin and Wesson, 1997). Other examples of unified fields are the M theory (Duff, 1996; Gibbin, 2000) and the Loop Quantum Gravity (Ashtekar, 1986; Ashtekar 1989) based on geometric developments of Cartan (Cartan, 1922; Cartan, 1923; De Andrade et al., 2004; Petti, 2006). In the case of the M theory, a total of 11 dimensions are required to explain simultaneously the gravitation and electromagnetism. In all cases, only the classical four dimensions are macroscopic.

However, some similitudes are found between both interactions at macroscopic scale (Weinberg, 1972). In fact, if the gravitational and electric potential reach large distances with a similar behavior, both could arise in a similar way. And since the mass-energy is capable of generating a curvature of spacetime, it is logical to think that energy caused by the electromagnetic interaction is also capable of this. Therefore, we wonder if this curvature may explain Maxwell’s equations.

In this paper we propose to approach the classical equations of electromagnetism from the disturbance of the four-dimensional general relativity, i.e., by a curvature of spacetime.

According to the classical physics, the origin of the energy potentials is a priori arbitrary. Usually, the zero point is chosen for the infinite distance, but another hypothesis is assumed in this study: The zero point of the energy potential of a particle is defined to a minimum radius.

Finally, the General Relativity conventions used for this study are based on the flat limite, i.e., the regions tend locally to the Minkowski space ($\mathbb{R}^4$). For this limit, the metric tensor is taken as $\eta^{\mu\nu} = \{ \eta^{00} = +1, \eta^{ii} = -1 \}$, where the index '0' refers to the time and index 'i' to the space. With such a criterion, the square of the velocity, $u^\mu \equiv dx^\mu / d\tau$ (derived of the spatial coordinate, $x^\alpha$, on its modul, $\tau \equiv |x|$) equals to $u^\mu u_\mu = \eta_{\alpha\beta} u^\alpha u^\beta = +1$. 
II. THE ENERGY DISTURBANCE

Suppose that a particle 1 of mass \( m_1 \) and charge \( q_1 \) is subjected to an energy perturbation generated by a source particle 2 of mass \( m_2 \) and charge \( q_2 \). Then, a hypothesis on energy disturbance for the particle 1 is considered: the energy disturbance \( m_1 \Delta \zeta \) can be written as the product between the charge \( q_1 \) and the scalar electric field generated by the source particle 2, \( \Delta \zeta_2 \):

\[
m_1 \Delta \zeta = q_1 \Delta \zeta_2
\]

(2.1)

Then, by symmetry, the energy of the particle 2 will be affected by a similar disturbance, generated by the particle 1:

\[
m_2 \Delta \zeta = q_2 \Delta \zeta_1
\]

(2.2)

According to the classical physics, the origin of the energy potentials is a priori arbitrary. Usually, the zero point is chosen for the infinite distance, but another hypothesis is assumed in this study: The zero point of the energy potential of a particle with mass \( m \) is defined to a minimum radius, \( r_m \), therefore:

\[
\Delta \zeta (r) = \zeta (r) - \zeta (r_m)
\]

(2.3)

It must be borne in mind that \( r_m \) should be very small, and therefore generally \( \zeta_m \) is much larger than \( \zeta \). In addition, we can define the associated vector potential \( A^\alpha \) as the product between a scalar potential \( \Delta \zeta \) and the speed of source \( u^\alpha \), i.e.:

\[
A^\alpha = \Delta \zeta \ u^\alpha
\]

(2.4)

The mass-energy \( w \) of the particle 2 can be broken down into the self-energy \( m_2 \) and an additional energy \( m_2 \Delta \zeta_1 \), which is from the external electric potential \( \Delta \zeta_1 \) generated by the particle 1 (according to Eq. 2.2):

\[
w = m_2 + m_2 \Delta \zeta_1
\]

(2.5)

Thus, the stress–energy vector of the particle 2 is defined as the product of its energy \( w \) and its speed \( u^\alpha \):

\[
P^\alpha = wu^\alpha = m_2 u^\alpha + q_2 A^\alpha
\]

(2.6)

where \( A_1^\alpha \) is the vector potential of the external field \( \Delta \zeta_1 \). Therefore, a generalized stress–energy tensor \( P^\alpha \beta \) is defined for a source of particles as:

\[
P^\alpha \beta = \sum_a \int \delta^4(x - x_a) P_a^\alpha dx_a^\beta
\]

(2.7)

where \( P_a \) is the stress–energy vector of each source particle in the position \( x_a \). To check that \( P^\alpha \beta \) is well defined, it can be recalled that the stress–energy tensor and electric current defined by:

\[
P_m^\alpha \beta = \sum_a \delta^4(\vec{x} - \vec{x}_a(t)) m_a u_a^\alpha \frac{dx_a^\beta}{dt}
\]

(2.8)

\[
\dot{j}_a^\alpha = \sum_a \delta^4(\vec{x} - \vec{x}_a(t)) q_a \frac{dx_a^\alpha}{dt}
\]

(2.9)

Which can be written as:

\[
P_m^\alpha \beta = \sum_a \int \delta^4(x - x_a) p_a^\alpha dx_a^\beta
\]

(2.10)

\[
\dot{j}_a^\alpha = \sum_a \int \delta^4(x - x_a) q_a dx_a^\alpha
\]

(2.11)
where \( p_a \) is the classical stress–energy vector, i.e. \( p_a = m_a u_a \). Then, the stress–charge tensor \((J^{\alpha\beta})\) is defined as:

\[
J^{\alpha\beta} \equiv \sum_u \delta^3[\vec{x} - \vec{x}_a(t)] q_a \Delta \zeta^a \mu_a^{\alpha} \frac{dx^\beta}{dt} = \sum_u \delta^3[\vec{x} - \vec{x}_a(t)] q_a (\zeta^1(\vec{x}) - \zeta^m) \mu_a^{\alpha} \frac{dx^\beta}{dt} \tag{2.12}
\]

where \( \Delta \zeta^i \) is the external scalar field. Applying Eq. 2.3 to the Eq. 2.12, the term \( \zeta^m(x_m) \) prevails over \( \zeta^i(x) \) because it is much larger. Moreover, the term \( \zeta^m(x_m) \) does not depend on the space \( x \), and therefore it may be extracted as a common factor.

\[
J^{\alpha\beta} \approx -\zeta^m \sum_u \delta^3[x - \vec{x}_a(t)] q_a \mu_a^{\alpha} \frac{dx^\beta}{dt} \approx -\zeta^m \sum_u \delta^3[x - \vec{x}_a] q_a \mu_a^{\alpha} dx^\beta \tag{2.13}
\]

where \( \Delta \zeta^i \) is the external scalar field. And generally, the stress–energy tensor becomes:

\[
P^{\alpha\beta} = p_m^{\alpha\beta} + J^{\alpha\beta} \tag{2.14}
\]

For a perfect fluid (without viscosity), the stress–energy tensor (Wald, 1984) can be written as:

\[
P^{\alpha\beta} = (\rho + \sigma) u^{\alpha} u^{\beta} + g^{\alpha\beta} \sigma \tag{2.15}
\]

where \( u \) is the velocity of the source, \( g \) is the metric tensor with a Minkowski limit of signature \((1, 3)\), \( \sigma \) is the pressure, and \( \rho \) is the total energy density of the particle \( 2 \). This energy density is the sum of the mass-energy density \((\rho_{m2})\) and the electric-energy density, which is approximately the product between the charge density \((\rho_{q2})\) and the external electric potential \((\Delta \zeta^i)\):

\[
\rho \approx \rho_{m2} + \Delta \zeta^i \rho_{q2} \approx \rho_{m2} - \zeta^m \rho_{q2} \tag{2.16}
\]

In addition, if it is assumed that the pressure is much smaller than the electric energy density, then the stress–charge tensor has the form:

\[
J^{\alpha\beta} \approx -\zeta^m \rho_{q2} u^{\alpha} u^{\beta} \tag{2.17}
\]

Finally, remember that if the determinant of the metric, \( g \), is significantly different from \(-1\), then the momentum-energy tensor general (Wald, 1984) becomes:

\[
\hat{p}^{\alpha\beta} = \sum_u \int \delta^3[x - \vec{x}_a] p^{\alpha} dx^\beta \tag{2.18}
\]

\[
\hat{p}^{\alpha\beta} = \frac{1}{\sqrt{-g}} p^{\alpha\beta} \tag{2.19}
\]

where \( p^{\alpha\beta} \) is the stress–energy tensor in Minkowski space. However in this work we assume a slightly perturbed space and thus the determinant of the metric is approximately \(-1\).

### III. ELECTROMAGNETIC PART OF EINSTEIN'S EQUATIONS

As a consequence of the energy disturbance caused by an electrical charge, a contribution to the space-time curvature is expected. Thus, the metric tensor \( g_{\alpha\beta} \) of Minkowski space can be written in terms of a gravitational \((\Phi_{\alpha\beta})\) and an electromagnetic \((A_{\alpha\beta})\) contribution. A priori it is assumed that these contributions are small enough vs the metric, so that we can divide it into three parts:

\[
g_{\alpha\beta} \approx \eta_{\alpha\beta} + \Phi_{\alpha\beta} + A_{\alpha\beta} \tag{3.1}
\]

With this, we must expect that the Riemann tensor (O'Neill, 1983) can also be written separately the two contributions. Recall that the tensor Riemman takes the form:
\[ R_{\alpha\beta\gamma\delta} = \left( \partial_\gamma \Gamma^\lambda_{\alpha\beta} + \Gamma^\lambda_{\alpha\gamma} \Gamma^\rho_{\beta\delta} - \Gamma^\lambda_{\beta\delta} \Gamma^\rho_{\alpha\gamma} + \Gamma^\lambda_{\beta\rho} \Gamma^\rho_{\alpha\gamma} \right) \]  

where \( \Gamma \) are the Christoffel symbols:

\[ \Gamma^\alpha_{\beta\gamma} = \frac{1}{2} g^{|\alpha}_{\rho\sigma} \left( \partial_\rho g_{\beta\sigma} + \partial_\sigma g_{\beta\rho} - \partial_\beta g_{\rho\sigma} \right) \]  

Assuming that the disturbance is small, i.e.:

\[ |\Phi_{\alpha\beta}| \ll 1 \quad i \quad |A_{\alpha\beta}| \ll 1 \]  

Then we can approximate the Christoffel symbols according to:

\[ \Gamma^\alpha_{\beta\gamma} \approx \partial_\gamma \Gamma^\lambda_{\alpha\beta} - \partial_\beta \Gamma^\lambda_{\alpha\gamma} \propto \Phi_{\alpha\gamma} \]  

and therefore the greatest contribution to the Riemann tensor is given by:

\[ R^\lambda_{\alpha\beta\gamma} \approx \partial_\gamma \Gamma^\lambda_{\alpha\beta} - \partial_\beta \Gamma^\lambda_{\alpha\gamma} \]  

With this, we see that the Ricci tensor (O'Neill, 1983) can be approximated by:

\[ R_{\alpha\beta} \approx g^{|\alpha}_{\rho\sigma} R_{\alpha\beta\gamma\sigma} \approx g^{|\alpha}_{\rho\sigma} \left( \partial_\gamma \Gamma^\lambda_{\alpha\beta} - \partial_\beta \Gamma^\lambda_{\alpha\gamma} \right) \]  

If the electric term of Eq. 3.1 is taken, Eq. 3.8 becomes as:

\[ \bar{R}_{\alpha\beta} \approx \frac{1}{2} \eta^{|\alpha}_{\rho\sigma} \eta^{|\beta}_{\mu\nu} \eta^{|\mu}_{\gamma\nu} \left( \partial_\gamma \partial_\nu A_{\alpha\rho} - \partial_\rho \partial_\nu A_{\alpha\gamma} + \partial_\gamma \partial_\rho A_{\alpha\nu} - \partial_\nu \partial_\rho A_{\alpha\gamma} \right) \]  

\[ 2\bar{R}_{\alpha\beta} \approx \partial_\beta \partial_\gamma \partial_\nu A_{\alpha\rho} - \partial_\rho \partial_\nu \partial_\gamma A_{\alpha\alpha} + \partial_\beta \partial_\gamma \partial_\nu A_{\alpha\rho} - \partial_\rho \partial_\nu \partial_\gamma A_{\alpha\alpha} \]  

For the same reason, the equation of Einstein can be separated into a contribution of the mass and another of the charge:

\[ R_{\alpha\beta} \approx R_{m\alpha\beta} + R_{q\alpha\beta} = -8\pi S_{\alpha\beta} \]  

where \( R_m \) is the curvature due to mass and \( R_q \) is the contribution of the charge, while \( S \) is a tensor constructed from the stress–energy tensor (natural units with \( G = 1 = c \)) as:

\[ S_{\alpha\beta} \equiv P_{\alpha\beta} - \frac{1}{2} P g_{\alpha\beta} \]  

where, according to Eq. 2.14, stress–charge tensor \( J_{\alpha\beta} \) can be included, according to:

\[ P_{\alpha\beta} \equiv P_{m\alpha\beta} + J_{\alpha\beta} \]  

Therefore, we can write that:

\[ R_{\alpha\beta} = R_{m\alpha\beta} + R_{q\alpha\beta} = -8\pi \left( P_{m\alpha\beta} - \frac{1}{2} P m g_{\alpha\beta} \right) - 8\pi \left( J_{\alpha\beta} - \frac{1}{2} J g_{\alpha\beta} \right) \]  

Separating the different contributions, we obtain:

\[ R_{m\alpha\beta} \approx -8\pi \left( P_{m\alpha\beta} - \frac{1}{2} P m g_{\alpha\beta} \right) \]  

\[ \bar{R}_{\alpha\beta} \equiv R_{q\alpha\beta} \approx -8\pi \left( J_{\alpha\beta} - \frac{1}{2} J g_{\alpha\beta} \right) \]  

This is the electromagnetic part of Einstein's equation. Moreover, considering the Eq. 3.10 and 2.17 it follows that the order of magnitude of the tensor \( A_{\alpha\beta} \) is given by:

\[ \partial^3 A_{\alpha\beta} \sim 4\pi \zeta m P_{\alpha\beta} u^\alpha u^\beta \]
IV. ELECTROMAGNETIC FIELD EQUATIONS

A. Poisson Equation

From the Eq 3.17 we expect that the tensor of electric disturbance, \( A_{\alpha\beta} \), depends on the disturbing source, \( \zeta_\alpha \), and it must also depend on field associated with the affected particle \( \Delta \zeta_i \). So suppose that the tensor \( A_{\alpha\beta} \) must be similar to:

\[
A_{\alpha\beta} \approx 2\zeta_{1\alpha\beta} u^\gamma u^\beta 
\]

(4.1)

With this, equation 3.10 becomes the following:

\[
2\tilde{R}_{\mu\nu} \approx 2\zeta_{\mu\nu} \left[ \partial_\alpha \partial^\alpha (\zeta_{\mu\nu} u_\beta) - \partial_\beta \partial^\beta (\zeta_{\mu\nu} u_\gamma) + \partial_\alpha \partial^\alpha \left( \zeta_{\mu\nu} u_\gamma \right) \right] 
\]

(4.2)

Multiplying Eq 4.2 for \( u^\gamma \) (see Appendix A), it is transformed in:

\[
\zeta_{\mu\nu} u^\gamma \tilde{R}_{\mu\nu} \approx \zeta_{\mu\nu} F_{\mu\nu} + \Delta \zeta_{\mu\nu} - \partial_\gamma \zeta_{\mu\nu} \partial^\gamma \left( \zeta_{\mu\nu} u_\beta \right) - \zeta_{\mu\nu} \left( \partial_\gamma \partial^\gamma \right) \left( \zeta_{\mu\nu} u_\beta \right) - \zeta_{\mu\nu} \left( \partial_\gamma \zeta_{\mu\nu} \right) \partial^\gamma \left( u_\beta \right)
\]

(4.3)

where \( F_{\mu\nu} \) has been defined as:

\[
F_{\mu\nu} \equiv \partial_\gamma \zeta_{\mu\nu} - \partial_\gamma \zeta_{\mu\nu} 
\]

(4.4)

Assuming that the field is stationary relative to a comoving coordinate system, then:

\[
\zeta_{\mu\nu} u^\gamma \tilde{R}_{\mu\nu} \approx \zeta_{\mu\nu} F_{\mu\nu} - \zeta_{\mu\nu} \partial_\gamma \zeta_{\mu\nu} \left( u_\beta \right) - \partial_\gamma \zeta_{\mu\nu} \left( u_\gamma \right) - \zeta_{\mu\nu} \partial_\gamma \zeta_{\mu\nu} \left( u_\beta \right) 
\]

(4.5)

Neglecting the quadratic terms with the derivative of speed, \( (\partial u)^2 \), we have:

\[
\zeta_{\mu\nu} u^\gamma \tilde{R}_{\mu\nu} \approx \zeta_{\mu\nu} F_{\mu\nu} \approx \zeta_{\mu\nu} \partial_\gamma \zeta_{\mu\nu} \left( u_\beta \right) 
\]

(4.6)

This is the largest contribution to the electrical part of the Ricci tensor. Moreover, the Eq. 3.16 and 2.17, we have:

\[
\tilde{R}_{\mu\nu} = -8\pi \left( J_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \right) = 8\pi \left( \zeta_{\alpha\beta} \rho_{\alpha\beta} \right) 
\]

(4.7)

Thus, contracting equation 4.6 with the speed \( u_{\alpha\beta} \), we have:

\[
\zeta_{\mu\nu} u^\gamma \tilde{R}_{\mu\nu} \approx 4\pi \zeta_{\mu\nu} \rho_{\alpha\beta} u_\beta 
\]

(4.8)

Multiplying the equation 4.7 for the constant \( \zeta_{\mu\nu} \) and equating to equation 4.5:

\[
\partial_\gamma F_{\mu\nu} - \zeta_{\mu\nu} \partial_\gamma \zeta_{\mu\nu} \left( u_\beta \right) \approx 4\pi \rho_{\alpha\beta} u_\beta 
\]

(4.9)

Neglecting the term \( \partial_\gamma \zeta_{\mu\nu} \), then the equation 4.8 becomes as:

\[
\partial_\gamma F_{\mu\nu} \approx 4\pi \rho_{\alpha\beta} u_\beta 
\]

(4.10)

Finally, if we select the gauge so that \( \partial_\gamma \zeta_{\mu\nu} = 0 \), and recalling the definition of equation 4.4, then:

\[
\partial_\gamma \zeta_{\mu\nu} \left( u_\beta \right) \approx 4\pi \rho_{\alpha\beta} u_\beta 
\]

(4.11)

Which is a Poisson equation with d’Alembert operator (remember that \( \partial_\gamma \partial^\gamma = \partial_\gamma^2 - \partial^2 \equiv \square \)). If the density \( \rho_{\alpha\beta} \) is integrable, Green's theorem can be applied at equation 4.10, with which the general solution of that equation is estimated (McDonald, 1997; Stewart, 2004):
where \( r = |x - x'| \) is the distance at the source and therefore \( \delta(x_0' - r) \) is the delta of delay, where \( x_0' \) is the temporal component of the position of the source, \( x' \). Note that Eq. 4.4 can be written using the left definition of the Eq. 4.11:

\[
F_{\alpha\beta} = \partial_\alpha \widetilde{A}_\beta - \partial_\beta \widetilde{A}_\alpha = \nabla_\alpha \widetilde{A}_\beta - \nabla_\beta \widetilde{A}_\alpha
\]  
(4.12)

**B. Scalar potential**

From equation 4.11, the electric potential \( \Delta \zeta \) can be written with a similar shape to the gravitational potential \( \Delta \phi \):

\[
\left\{ \begin{array}{l}
\Delta \zeta \equiv \zeta(r) - \zeta_m(r_m) \approx \frac{q}{r} - \frac{q}{r_m} \\
\Delta \phi \equiv \phi(r) - \phi_m(r_m) \approx - \frac{m}{r} + \frac{m}{r_m}
\end{array} \right.
\]  
(4.12)

Therefore, it is assumed that at least some cases there may be an analogy between the electric contribution and the mass contribution on the metric:

\[
\left\{ \begin{array}{l}
A^{00} \approx 2 \zeta_{ml} \zeta_2 \\
\Phi^{00} \approx -2 \phi_{ml} \phi_2
\end{array} \right.
\]  
(4.13)

Even so, the element \( g_{00} \) must remain similarities with Newtonian approximation (Wald, 1984), namely:

\[
g_{00} \approx 1 + 2 \phi_2 \approx \left( 1 - \frac{2m_2}{r} \right)
\]  
(4.14)

Therefore, the minimum radius, \( r_m \), is found as:

\[
\Phi^{00} \approx - \frac{2m_2}{r} \approx -2 \phi_{ml} \phi_2 = -2 \left( \frac{m_1}{r_{ml}} \right) = \frac{m_2}{r} \rightarrow r_{ml} = m_1
\]  
(4.15)

which is as the “space-time-mass” of the theory of Wesson (1983), linked with the Schwarzschild radius. Consequently, the constant \( \zeta_{ml} \) can be obtained as:

\[
A^{00} \approx 2 \zeta_{ml} \zeta_2 = 2 \left( \frac{q_1}{r_{ml}} \right) = \frac{2d_1 q_2}{m_1 r} \rightarrow \zeta_{ml} = \frac{q_1}{m_1}
\]  
(4.16)

**C. First Bianchi identity**

Using the Eq. 4.12, tensor \( F^{\alpha\beta} \) can be expanded as:

\[
\nabla_\gamma F_{\alpha\beta} + \nabla_\alpha F_{\gamma\beta} + \nabla_\beta F_{\gamma\alpha} = \nabla_\gamma \nabla_\alpha \widetilde{A}_\beta - \nabla_\beta \nabla_\gamma \widetilde{A}_\alpha + \nabla_\alpha \nabla_\beta \widetilde{A}_\gamma - \nabla_\gamma \nabla_\alpha \widetilde{A}_\beta + \nabla_\beta \nabla_\gamma \widetilde{A}_\alpha - \nabla_\alpha \nabla_\beta \widetilde{A}_\gamma
\]  
(4.17)

where the terms have been grouped as ‘\( a \)’, ‘\( b \)’ and ‘\( c \)’ to apply the following property of the Riemann curvature tensor:

\[
\widetilde{A}_\alpha R^{\gamma\alpha}_{\gamma\beta} = \nabla_\alpha \nabla_\gamma \widetilde{A}_\beta - \nabla_\beta \nabla_\gamma \widetilde{A}_\alpha
\]  
(4.18)

it is obtained:
\[ \nabla_\gamma F_{\alpha\beta} + \nabla_\alpha F_{\beta\gamma} + \nabla_\beta F_{\gamma\alpha} = \tilde{A}_\alpha R^\gamma_{\beta\gamma\alpha} + \tilde{A}_\beta R^\gamma_{\alpha\gamma\beta} + \tilde{A}_\gamma R^\gamma_{\alpha\beta\gamma} \]  

(4.19)

regrouping terms:

\[ \nabla_\gamma F_{\alpha\beta} + \nabla_\alpha F_{\beta\gamma} + \nabla_\beta F_{\gamma\alpha} = \tilde{A}_\alpha R^\gamma_{\beta\gamma\alpha} + \tilde{A}_\beta R^\gamma_{\alpha\gamma\beta} + \tilde{A}_\gamma R^\gamma_{\alpha\beta\gamma} \]  

(4.20)

Finally, applying the first Bianchi identity (O'Neill, 1983), which is:

\[ R_{\alpha\beta\gamma} + R_{\gamma\alpha\beta} + R_{\beta\gamma\alpha} = 0 \]  

(4.21)

the Eq. 4.20 is become the following identity:

\[ \nabla_\alpha F_{\beta\gamma} + \nabla_\beta F_{\gamma\alpha} + \nabla_\gamma F_{\alpha\beta} = 0 \]  

(4.22)

### D. Freefall in an electromagnetic field

Given a source of energy that perturbs the metric tensor \( g \), the equation of the geodesics is:

\[ u^\mu \nabla_\mu u^\nu = \frac{d}{d\tau} u^\nu = \frac{d^2}{d\tau^2} u^\nu + \Gamma^\nu_{\alpha\beta} u^\alpha u^\beta \]  

(4.22)

where \( \Gamma^\nu_{\alpha\beta} \) are Christoffel symbols, \( u_1 \) is the velocity of a ‘particle 1’ and \( u \) is the velocity of the source.

\[ \Gamma^u_{\alpha\beta} = \frac{1}{2} g^{\nu\rho} \left( \partial_\alpha g_{\nu\rho} + \partial_\rho g_{\nu\alpha} - \partial_\nu g_{\alpha\rho} \right) \]  

(4.23)

From Eq. 3.1, the electromagnetic part of the Christoffel symbols is approximately:

\[ \left( \Gamma^u_{\alpha\beta} \right)_{elec} \approx \frac{1}{2} \eta^{\nu\rho} \left( \partial_\alpha A_{\nu\rho} + \partial_\rho A_{\nu\alpha} - \partial_\nu A_{\alpha\rho} \right) \]  

(4.24)

\[ \left( \Gamma^u_{\alpha\beta} \right)_{elec} \approx \frac{1}{2} \left( \partial_\alpha A^\nu_{\beta} + \partial_\beta A^\nu_{\alpha} - \partial^\nu A_{\alpha\beta} \right) \]  

(4.25)

Moreover, considering the Eq. 4.1 and 4.11, the electric contribution to the perturbed metric tensor can be rewritten as:

\[ A^{\alpha\beta} \approx 2\zeta_{m\ell} \tilde{A}_{2}^{\nu} u^\nu = 2\zeta_{m\ell} \tilde{A}_{2}^{\nu} u^\nu \]  

(4.26)

Replacing the tensor \( A \) of Eq. 4.25 using the Eq. 4.26, the partial derivative of each tensor element has a partial derivative of the speed, i.e.

\[ \partial_\gamma A^{\alpha\beta} \approx 2\zeta_{m\ell} u^\beta \partial_\gamma \tilde{A}_{2}^{\nu} + 2\zeta_{m\ell} \tilde{A}_{2}^{\nu} \partial_\gamma u^\beta \]  

(4.27)

But the last term of Eq. 4.27 is needless due to the contraction of the Christoffel symbols of speed that appears in the Eq. 4.22; i.e., this term is canceled by the property \( u_\gamma \partial_\gamma u^\beta = 0 \). Thereby, using Eq. 4.27 in Eq. 4.25, it is obtained:

\[ \left( \Gamma^u_{\alpha\beta} \right)_{elec} u^\nu u^\beta \approx \zeta_{m\ell} u^\nu u^\beta \left( \partial_\gamma \tilde{A}_{2}^{\nu} + u_\nu \partial_\gamma \tilde{A}_{2}^{\nu} \right) \]  

(4.28)

where it is considered \(|A| \ll 1\) and thus \( \eta \approx \eta \). In addition, the Eq. 4.28 can be written in terms of the tensor \( F \) using Eq. 4.12, thus:

\[ \left( \Gamma^u_{\alpha\beta} \right)_{elec} u^\nu u^\beta \approx \zeta_{m\ell} u^\nu u^\beta \left( u_\beta F^\nu_{\alpha} + u_\alpha u^\beta \partial_\gamma \tilde{A}^\nu_{\gamma} \right) \]  

(4.29)

Contracting speed on the right:

\[ \left( \Gamma^u_{\alpha\beta} \right)_{elec} u^\nu u^\beta \approx \zeta_{m\ell} \left( u^\nu F^\nu_{\alpha} + u_\alpha u^\beta \partial_\gamma \tilde{A}^\nu_{\gamma} \right) \]  

(4.30)
\[ \left( \Gamma^\alpha_{\beta \mu} \right)_{\text{rec}} u^\alpha_i u^\mu_j \approx \tilde{\zeta}_{m_i} \left( u_i^\alpha F^\alpha_\gamma + u_i^\alpha u_\alpha d_\gamma \tilde{A}_\gamma \right) \]  

(3.41)

Therefore, adding the electric and mass contribution of the geodesic equation, this becomes:

\[ \frac{D}{d\tau} u^\mu_i \approx \frac{d}{d\tau} u^\mu_i + \left( \Gamma^\mu_{\alpha \beta} \right)_{\text{mass}} u^\alpha_i u^\beta_j + \left( \Gamma^\mu_{\alpha \beta} \right)_{\text{rec}} u^\alpha_i u^\beta_j \]  

(3.42)

\[ \frac{D}{d\tau} u^\mu_i \approx \frac{d}{d\tau} u^\mu_i + \left( \Gamma^\mu_{\alpha \beta} \right)_{\text{mass}} u^\alpha_i u^\beta_j + \zeta_{m_i} u^\gamma_i F^\gamma_\mu + \frac{1}{2} \zeta_{m_i} u^\alpha_i u_\alpha d_\gamma \tilde{A}_\gamma \]  

(3.43)

by the asymmetry of the electromagnetic tensor, \( F^\mu_\gamma = - F^\gamma_\mu \), Eq. 3.43 is changed to:

\[ \frac{D}{d\tau} u^\mu_i \approx \frac{d}{d\tau} u^\mu_i + \left( \Gamma^\mu_{\alpha \beta} \right)_{\text{mass}} u^\alpha_i u^\beta_j - \zeta_{m_i} u^\gamma_i F^\gamma_\mu + \frac{1}{2} \zeta_{m_i} u^\alpha_i u_\alpha d_\gamma \tilde{A}_\gamma \]  

(3.44)

and according to 4.16, the constant \( \zeta_{m_1} \) can be replaced by \( \zeta_{m_1} = q_i/m_i \), then:

\[ \frac{D}{d\tau} u^\mu_i \approx \frac{d}{d\tau} u^\mu_i + \left( \Gamma^\mu_{\alpha \beta} \right)_{\text{mass}} u^\alpha_i u^\beta_j - q_i u^\gamma_i u_\gamma F^\gamma_\mu + q_i u^\alpha_i u_\alpha d_\gamma \tilde{A}_\gamma \]  

(3.45)

In case there was an outside force \( f_\mu \), then they satisfy:

\[ f^\mu = m_i \frac{d}{d\tau} u^\mu_i \approx m_i \frac{d}{d\tau} u^\mu_i + m_i \left( \Gamma^\mu_{\alpha \beta} \right)_{\text{mass}} u^\alpha_i u^\beta_j - q_i u^\gamma_i F^\gamma_\mu + q_i u^\alpha_i u_\alpha d_\gamma \tilde{A}_\gamma \]  

(3.46)

And, isolating the classical newtonian term:

\[ m_i \frac{d}{d\tau} u^\mu_i \approx f^\mu + q_i u^\gamma_i F^\gamma_\mu - q_i u^\mu_i u_\mu d_\gamma \tilde{A}_\gamma - m_i \left( \Gamma^\mu_{\alpha \beta} \right)_{\text{mass}} u^\alpha_i u^\beta_j \]  

(3.47)

Note that this equation is the Lorentz force in a gravitational field, except the term \( q_i u^\alpha_i u_\alpha d_\gamma \tilde{A}_\gamma \), which depends on the self-time derivative of the field \( \tilde{A} \). But this term is usually zero. A link between the Lorentz motion and the gravitation is also obtained from the post-newtonian approach (Weinberg, 1972).

**V. DISCUSSION**

**A. Order of magnitude of the disturbance**

For the previous approximations to be valid, one of the tests to be performed is that the constant \( \zeta_m \) will be much larger than the ordinary scalar potential \( \zeta \). Recall that the electric contribution to the energy perturbation is given by Equation 2.5, where we can undo the natural units \( (G \equiv 1 \equiv c) \), and we have that:

\[ w = m + q(\zeta - \zeta_m) = mc^2 + \left( \frac{Kq}{r} - \frac{Kq^2}{Gm} \right) \]  

(5.1)

Choosing a typical radius of the Bohr radius, about \( 5 \cdot 10^{-11} \) m, we see that the constant \( \zeta_m \) for proton and electron is of the order of \( 10^{15} \) J/C and \( 10^{16} \) J/C respectively, while the electric potential ordinary \( \zeta \) is the order of \( 10 \) J/C for both. On the other hand, if the theory is correct, we expect that the electrical disturbance of the metric is relatively small. Undoing the natural units, it becomes:

\[ A_{\alpha \beta} \approx 2 \zeta_{m_c} u_\alpha u_\beta = \left( \frac{G}{k^2 c^3} \right) \zeta_{m_c} u_\alpha u_\beta = \frac{2q}{mc^2} \frac{Kq}{r} u_\alpha u_\beta \]  

(5.2)

And taking again typical values for the proton and electron in the hydrogen atom, the value of \( d'A_{\alpha \beta} \) is typically \( 10^{-3} \) and \( 10^{-4} \), respectively, thus accomplished the initial approach \( |A_{\alpha \beta}| \ll 1 \) (expression 3.1). However, the equation 4.11 have been written after several approaches which do not
ensure compliance in all cases, so it is possible that the most appropriate way by the tensor 5.2 is another.

B. Possible relationship between the charge and energy

The possible similarity between the curvature generated by mass energy and electric energy opens a discussion about whether this is a coincidence or if this relationship really goes beyond that. If we make the hypothesis that there is a relationship of identity between the electrical energy \( q \zeta \) and gravitational energy \( m \phi \), then we see that somehow we can replace one another using a simple transformation (equation 5.3), which also makes that metric elements (equation 4.16) are equal by the component 00:

\[
\tilde{m} \leftrightarrow i \sqrt{\frac{k}{G}} q
\rightarrow \begin{cases}
q_1 \zeta_2 = \tilde{m} \phi_1 \\
\Lambda^{00}(q_1, q_2) = \Phi^{00}(\tilde{m}_1, \tilde{m}_2)
\end{cases}
\]  

(5.3)

where \( k \) is Coulomb's constant and \( G \) is Newton's constant, which in natural units are considered equal to unity. In short, the equation 5.3 could be a mere coincidence in the way or could be the trace of something that is more than similarity. In fact, we see that this transformation is also satisfied in the equation 2.5 for the total energy \( w \) (equation 2.5), where:

\[
q_2 \Delta \zeta_1 = \tilde{m}_2 \Delta \phi_1(\tilde{m}_1) = -\frac{\tilde{m}_2 \tilde{m}_1}{r} + \frac{\tilde{m}_2 \tilde{m}_1}{r_{m_1}} \approx \frac{\tilde{m}_2 \tilde{m}_1}{r_{m_1}} \approx \tilde{m}_2
\]  

(5.4)

where again we have used natural units (\( G = 1 = k \)), and where we have taken the form of potential according to equation 4.15. Therefore, it is reasonable to assume that the total energy involved in the process really is:

\[
w = m_1 \Delta \phi_1 + q_2 \Delta \zeta_1 \approx m_2 + q_2 \Delta \zeta_1
\]  

(5.5)

From the equation 5.5 can be defined more generally the momentum-energy vector that arises from our hypothesis:

\[
P^{\alpha} = w u^\alpha = m_2 U_1^\alpha + q_2 A_1^\alpha
\]  

(5.6)

where we defined:

\[
U_1^\alpha = \Delta \phi u^\alpha
\]  

(5.7)

With this, the stress–energy tensor (equations 2.7 to 2.17) would be like:

\[
P^{\alpha \beta} \approx \left( \Delta \phi \rho_{m_2} + \Delta \zeta_1 \rho_{q_2} \right) u^\alpha u^\beta \approx \left( \phi_{m_2} \rho_{m_2} - \zeta_{m_1} \rho_{q_2} \right) u^\alpha u^\beta
\]  

(5.8)

and applying equations 4.9 and 4.15, whew \( \partial_\zeta \zeta = \rho_q \) and \( \partial_\zeta \phi = -\rho_m \), obtain that:

\[
P^{\alpha \beta} \approx \frac{1}{4\pi} \left( \phi_{m_2} \partial_\zeta \phi_1 - \zeta_{m_1} \partial_\zeta \phi_1 \right) u^\alpha u^\beta
\]  

(5.9)

Moreover, the equations 4.7 to 4.10 show that:

\[
u^\beta \tilde{R}_{\gamma \beta} \approx \zeta_{m_1} \partial_\zeta \zeta \left( \zeta_1 u_\beta \right) \approx 4\pi \zeta_{m_1} \rho_{q_2} u_\beta
\]  

(5.10)

Therefore, generalizing based on equations 4.13, 5.3 and 5.8, we have:

\[
u^\beta \tilde{R}_{\gamma \beta} \approx \zeta_{m_1} \partial_\zeta \zeta \left( \zeta_1 u_\beta \right) - \phi_{m_2} \partial_\zeta \phi_1 \left( \phi_{m_2} u_\beta \right) \approx 4\pi \left( \zeta_{m_1} \rho_{q_2} + \phi_{m_2} \rho_{m_2} \right) u_\beta \approx -4\pi P^{\alpha \beta} u^\alpha
\]  

(5.11)

where \( P^{\alpha \beta} = g_{\gamma \alpha} g_{\beta \mu} P^{\mu \nu} \). This seems to be consistent with the transformation defined in equation 5.3 and with the metric proposed above. But we must investigate to what extent such a transformation is valid, i.e., we should see if there is a gravitational field that it meets the relationship \( \Lambda^{\alpha \beta}(q_1, q_2) \leftrightarrow \Phi^{\alpha \beta}(m_1, m_2) \). In fact, in recent decades some authors have investigated the possible analogy of the magnetic field in gravity, known as gravitomagnetism (Ciufolini et al., 2003; Bini et al., 2003), and
have sought empirical evidence that, for now, does not appear to rule out such a possibility (Ashby et al., 2007).

C. Equations of Klein-Gordon and Dirac

The equation 2.6 is compatible with the equations of Klein-Gordon and Dirac with electromagnetic interaction. To verify this, we must develop the square of momentum vector. Recall that the limit signature of the metric is taken equal to (1, 3), and therefore the square of momentum vector \( P_\alpha \) is equal to \(+w^2\):

\[
P_\alpha = wu_\alpha \quad \Rightarrow \quad g^{\alpha \beta} P_\alpha P_\beta = w^2 g^{\alpha \beta} u_\alpha u_\beta = w^2
\]  

(5.12)

If both members of the equality is contracted using a set of matrices \( \gamma^\mu \) (4-vector), then:

\[
\gamma^\alpha P_\alpha = w \gamma^\alpha u_\alpha \quad \Rightarrow \quad \gamma^\alpha \gamma^\beta P_\alpha P_\beta = w^2 \gamma^\alpha \gamma^\beta u_\alpha u_\beta = w^2
\]  

(5.13)

where \( \gamma^\mu \) is a matrix that necessarily satisfies that:

\[
\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2 g^{\alpha \beta} I
\]  

(5.14)

where \( g^{\alpha \beta} \) is the metric tensor, \( I \) is the identity matrix and 5.13 is governed by the Clifford algebra \( Cl(1,3) \) (Lounesto, 2001). So if the equation 5.13 is transformed in operators agree with the principle of correspondence of quantum field theory, then we can write:

\[
\left( \gamma^\alpha P_\alpha - \gamma^\alpha wu_\alpha \right) \psi = 0 \tag{5.15}
\]

where \( \psi \) is a field, and \( P_\alpha \) and \( wu_\alpha \) are operators. The operator \( P_\alpha \) is identically equal to \( i\nabla_\alpha \), while the operator \( wu_\alpha \) can be broken down into two, one multiplies the mass \( (u_\alpha) \) and another that multiplies the charge \( (A_\alpha) \):

\[
\left( i \gamma^\alpha \nabla_\alpha - \gamma^\alpha mu_\alpha - \gamma^\alpha q A_\alpha \right) \psi = 0 \tag{5.16}
\]

Multiplying the above equation by a similar term of operator but changing the sign of mass, then we see that we recover Klain-Gordon equation with the presence of electromagnetic field (Mandl and Shaw, 1993):

\[
\left( i \gamma^\alpha \nabla_\alpha - q A_\alpha \right) \psi + \frac{1}{2} q \sigma^{\alpha \nu} F_{\mu \nu} - m^2 \psi = 0 \tag{5.17}
\]

However, as already noted Dirac, the term appears on the operator of the mass can be replaced directly by the mass scalar, i.e., with a metric of signature \((1, 3)\), the equation 5.16 can be written as:

\[
\left( i \gamma^\alpha \nabla_\alpha - m - \gamma^\alpha q A_\alpha \right) \psi = 0 \tag{5.18}
\]

Thus, multiplying again by the same term but with the sign changed for the mass, it recovers the equation 5.17. Equation 5.18 is known as the Dirac equation in presence of electromagnetic field (Mandl and Shaw, 1993). But note that if we assume equations 5.6 and 5.7 are valid, then equation 5.15 becomes as:

\[
\left( i \gamma^\alpha \nabla_\alpha - \gamma^\alpha m U_\alpha - \gamma^\alpha q A_\alpha \right) \psi = 0 \tag{5.19}
\]

where:

\[
U_\alpha = \Delta \phi_u = \left( \frac{m}{r} + \frac{m}{r_m} \right) u_\alpha = \left( \frac{m}{r} + 1 \right) u_\alpha = \phi u_\alpha + u_\alpha = V_{1\alpha} + u_\alpha \tag{5.20}
\]

Therefore, we can rewrite equation 5.19 as:

\[
\left( i \gamma^\alpha \nabla_\alpha - \gamma^\alpha m U_\alpha - \gamma^\alpha q A_\alpha \right) \psi = 0 \tag{5.21}
\]

And using the transformation of Dirac (equations 5.16 and 5.18), we obtain:
\[
(i\gamma^a \nabla_a - m - \gamma^a m V_a - \gamma^a q A_a)\nu = 0
\]
(5.22)

Note that if the charge is zero, the quadratic form of equation 5.22 would be an expression similar to equation 5.17 but referred to gravitation.

D. Critical issues of theory

The main weakness of the theory is that, to reproduce the approximate equations of classical electromagnetism, it is necessary to assume the prior existence of the electromagnetic interaction, so that energy is initially defined as \( m_{\text{q1}} = q_1 \Delta \zeta_2 \) (Equation 2.1). This initial premise should be deduced from a more general form.

In the same way, the geometric interpretation of the electrical disturbance (\( A^{\text{eb}} \)) of the metric tensor is not trivial. This is because it also depends on the two charges (o els dos camps, \( \zeta_1 \) i \( \zeta_2 \)).

Regarding the possible correspondence between charge and energy, this suggests deeper in relations between the two metric perturbations: the electric and mass perturbation. But this can lead to too daring results, such that there may be an interaction between charge and energy because the total energy of a particle would \( m = m_{\text{real}} + im_{\text{imag}} \).

Therefore, the energy of interaction between two particles would logically be of the order \( m_{\text{int}} = m_1 \phi_2 = G m_1 m_2 / r \), where it would provide an imaginary part of \( m_{\text{int}} \), \( m_{\text{int}} \) nonzero, \( \text{Im}\{m_{\text{int}}\} \neq 0 \). This is not a problem for two simple particles, because the electric part is so dominant that effect would be negligible. However, the interaction between a charged particle and an intense gravitational should be so important that the charge is not entirely conserved:

\[
\tilde{m}\left(1 - \frac{GM}{rc^2}\right) = i \sqrt{\frac{k}{G}} q \left(1 - \frac{GM}{rc^2}\right) = i \sqrt{\frac{k}{G}} \rightarrow q' = q \left(1 - \frac{GM}{rc^2}\right)
\]
(5.23)

For example, if the fundamental electron charge has been measured as \( q' = -1.602176487(40) \times 10^{-19} \text{C} \) (Mohr et al., 2008) on the Earth, then its value would be \( q \approx -1.602176488 \times 10^{-19} \text{C} \) on the flat space. In this case, both values are compatible with each other, but it is expected that other values will be observed in areas with higher gravitational effects, as in the biggest stars.

And apparently, variations of the fine structure constant (\( \alpha \equiv ke^2/\hbar c \), see Table 1) have been measured, which could be explained partly by the phenomenon described in equation 5.23. Specifically, depending on the region of space, it observed values of \( \Delta \alpha/\alpha \) ranging between \( \pm 10^{-7} \) and \( \pm 10^{-3} \) (Table 1). This regional fluctuation does not seem to be fully explained by the possible temporal variation, since this is more empirically small, with an order of magnitude between \( 10^{-17} \) and \( 10^{-15} \) (Damour and Dyson, 1996; Murphy et al., 2003; Gutiérrez and López-Corredoira, 2010).
TABLE I. Relative variation of the fine structure constant for different areas (measured according to the redshift). Significant differences are marked with (*).

| redshift | Δα/α | Reference | Methodology |
|----------|------|-----------|-------------|
| z = 0    | + (0.1 ± 1.0) × 10⁻⁷ | Damour and Dyson (1996) | Oklo phenomenon |
| 0.035 < z < 0.281 | + (1.5 ± 0.7) × 10⁻³ (*) | Grupe et al. (2005) | Many-multiplier method |
| 0.16 < z < 0.80 | + (7 ± 14) × 10⁻⁵ | Bahcall et al. (2004) | Quasars emission lines |
| z < 0.8  | + (2.4 ± 2.5) × 10⁻⁵ | Gutiérrez and López-Corredoian (2010) | QSQ absorption |
| z = 0.2467 | + (0.51 ± 1.26) × 10⁻⁵ | Darling (2004) | Molecular absorption systems |
| z = 0.6847 | − (0.08 ± 0.27) × 10⁻⁵ | Murphy et al. (2001a) | Molecular absorption systems |
| 1.59 < z < 2.32 | + (0.15 ± 0.44) × 10⁻⁵ | Chand et al. (2005) | QSO absorption |
| z = 1.15 | − (0.07 ± 0.84) × 10⁻⁶ | Levshakov et al. (2006) | SIDAM E0515–4414 |
| z = 1.15 | − (0.12 ± 1.79) × 10⁻⁶ | Molaro et al. (2008) | QSQ absorption |
| z = 1.84 | + (5.4 ± 2.5) × 10⁻⁶ (*) | Levshakov et al. (2007) | SIDAM Q1101–264 |
| 2.8 ≤ z ≤ 3.1 | + (0.2 ± 0.7) × 10⁻⁴ | Varshalovich et al. (1996) | Fine-structure splittings |
| 0.5 < z < 1.0 | − (0.2 ± 0.4) × 10⁻⁵ | Webb et al. (1999) | QSQ absorption |
| 0.5 < z < 1.6 | − (1.1 ± 0.4) × 10⁻⁵ (*) | Webb et al. (1999) | QSO absorption |
| 1.0 < z < 1.6 | − (1.9 ± 0.5) × 10⁻⁵ (*) | Webb et al. (1999) | QSO absorption |
| 2 < z < 3  | − (0.5 ± 1.3) × 10⁻⁵ (*) | Webb et al. (2001) | QSQ absorption |
| 1.8 < z < 3.5 | − (0.76 ± 0.28) × 10⁻⁵ (*) | Webb et al. (2001) | QSQ absorption |
| 0.5 < z < 1.8 | − (0.7 ± 0.23) × 10⁻⁵ (*) | Webb et al. (2001) | QSQ absorption |
| 1.0 < z < 1.8 | − (1.2 ± 0.3) × 10⁻⁵ (*) | Murphy et al. (2001b) | QSO absorption |
| 0.5 ≤ z ≤ 3.5 | − (0.72 ± 0.18) × 10⁻⁵ (*) | Murphy et al. (2001b) | Many-multiplier method |
| 0.2 < z < 3.7 | − (0.574 ± 0.102) × 10⁻⁵ (*) | Murphy et al. (2003) | Optical absorption lines |
| 0.2 < z < 4.2 | − (0.57 ± 0.11) × 10⁻⁵ (*) | Murphy et al. (2004) | Many-multiplier method |
| 0.4 < z < 2.3 | − (0.06 ± 0.06) × 10⁻⁵ | Srianand et al. (2004) | Mg II absorption lines |
| z = 10³ | − (3.5 ± 5.5) × 10⁻² | Avelino et al. (2001) | CMB |
| z = 10⁸ | − (2.24 ± 3.75) × 10⁻⁴ | Ichikawa and Kawasaki (2002) | BBN |

VI. CONCLUSIONS

This work is based on the hypothesis that the electromagnetic interaction can be understood as a direct result of its associated energy disturbance, which would cause a curvature of spacetime. The problem therefore reduces to finding the most appropriate expression for the electric contribution of the metric tensor. Specifically, we propose that the metric element must be similar to:

\[ A^\alpha\beta \approx 2\zeta_{1m}\zeta_2 u^\alpha u^\beta \] (6.1)

where \( \zeta_2 \) is the external electric field, \( \zeta_{1m} \) is the constant of electric field of the particle concerned. Thus, it is found that the disturbance represented in the equation 6.1 generates a small curvature so that approximately reproduces the fundamental equations of classical theory of Maxwell and also reproduces the Lorentz force in freefall. The approaches developed in this study are consistent with the orders of magnitude obtained for the value of the disturbance of spacetime. For example, in the hydrogen atom, the value of \( A^\alpha\beta \) is typically \( 10^{-7} \) and \( 10^{-8} \) respectively for proton and electron.

Note that consistency with the quantum theory seems to be guaranteed because the same initial disturbance energy hypothesis is supported by the additional term of electric field that appears in the equations Klain-Gordon and Dirac. In addition, this paper gives a hypothetical relationship between
the electrical and gravitational contribution that could lead to an additional term in the equations Klain-Gordon and Dirac in the presence of strong gravitational fields.

However, the theory presents some critical points such as that based on a previous electromagnetic interaction (electrostatic energy) that depends on the vector of generalized momentum, which is the source of the metric perturbation. One of the bolder predictions of this theory is about the possibility that the fundamental value of the charge is affected by gravity, which must be seen, or otherwise justified. Therefore, we suggest the possibility that the geometric relationship between electromagnetism and gravitation is not fully exhausted, but we need to delve further into this issue.

**Appendix A. Contraction of the Ricci tensor with the speed**

Develop the Eq. 4.2, with the aim of contracting it with the speed:

\[ \tilde{R}_{\beta\gamma} \approx \xi_m \left[ \frac{\partial^\gamma}{\partial \xi^\gamma} \left( \xi^\alpha u_{\alpha \beta} \right) - \frac{\partial^\beta}{\partial \xi^\beta} \left( \xi^\alpha u_{\alpha \gamma} \right) \right] + \partial^\beta \partial^\gamma \xi^\gamma - \partial^\gamma \partial^\beta \xi^\beta \left( \xi^\gamma u_{\gamma \beta} \right) \]  \hspace{1cm} (A.1)

The term \( u_\gamma \xi_m \) can be isolated:

\[ \xi_m^{-1} \tilde{R}_{\beta\gamma} \approx \xi_m \left[ \frac{\partial^\gamma}{\partial \xi^\gamma} u_{\gamma \beta} \left( \xi^\alpha u_{\alpha \gamma} \right) - u_{\gamma \beta} \partial^\gamma \left( \xi^\alpha u_{\alpha \gamma} \right) \right] + \partial^\beta \partial^\gamma \xi^\gamma - \partial^\gamma \partial^\beta \xi^\beta \left( \xi^\gamma u_{\gamma \beta} \right) \]  \hspace{1cm} (A.2)

\[ \xi_m^{-1} \tilde{R}_{\beta\gamma} \approx \xi_m \left[ \frac{\partial^\gamma}{\partial \xi^\gamma} u_{\gamma \beta} \left( \xi^\alpha u_{\alpha \gamma} \right) - u_{\gamma \beta} \partial^\gamma \left( \xi^\alpha u_{\alpha \gamma} \right) \right] + \partial^\beta \partial^\gamma \xi^\gamma - \partial^\gamma \partial^\beta \xi^\beta \left( \xi^\gamma u_{\gamma \beta} \right) \]  \hspace{1cm} (A.3)

Following with the isolating of the speed \( u_\beta \) by the remarked term:

\[ \xi_m^{-1} \tilde{R}_{\beta\gamma} \approx \xi_m \left[ \frac{\partial^\gamma}{\partial \xi^\gamma} u_{\gamma \beta} \left( \xi^\alpha u_{\alpha \gamma} \right) - u_{\gamma \beta} \partial^\gamma \left( \xi^\alpha u_{\alpha \gamma} \right) \right] + \partial^\beta \partial^\gamma \xi^\gamma - \partial^\gamma \partial^\beta \xi^\beta \left( \xi^\gamma u_{\gamma \beta} \right) + O_\beta (u_\lambda) \]  \hspace{1cm} (A.4)

where the operator \( O_\beta \) is:

\[ O_\beta = \left[ \xi_m \left[ \right] \right] + \partial^\beta \partial^\gamma \xi^\gamma - \partial^\gamma \partial^\beta \xi^\beta \left( \xi^\gamma u_{\gamma \beta} \right) \]  \hspace{1cm} (A.5)

Note that operator \( O_\beta \) has differential terms of first order (1, 2 and 3) and of second order (4 and 5). Therefore, if the Eq. A.5 is connected with speed, we have that:

\[ \xi_m^{-1} u^\beta \tilde{R}_{\beta\gamma} \approx \xi_m \left[ \right] + u^\beta \partial^\gamma \xi^\gamma - u^\beta \partial^\gamma \xi^\gamma + u^\beta O_\beta (u_\lambda) \]  \hspace{1cm} (A.6)

with:

\[ u^\beta O_\beta (u_\lambda) = \frac{u^\beta \partial^\gamma \xi^\gamma - u^\beta \partial^\gamma \xi^\gamma}{\partial^\gamma \partial^\beta \xi^\beta} \]  \hspace{1cm} (A.7)

where it was considered that:

\[ 0 = \partial^\beta \left( u_\beta u^\beta \right) = 2u_\lambda \partial^\beta \xi^\beta \]  \hspace{1cm} (A.8)

\[ 0 = \partial_\beta \partial^\beta \left( u_\beta u^\beta \right) = 2u_\lambda \partial_\beta \partial^\beta \xi^\beta + 2 \partial_\beta u_\lambda \partial^\beta u^\beta \]  \hspace{1cm} (A.9)

13
Given that \( u, \partial' = d_n \), and making the quasi-minkowskian approximation for the continuity \( \partial' u_r \approx 0 \), then equation A.7 becomes:

\[
\zeta_m^{-1} u^a \tilde{R}_{ab} = \partial' \left( \zeta_m u^b \right) - \partial' \left( \zeta_m u^b \right) + d_c \partial' \zeta_c - d_c \left( \zeta_m u^b \right) + u^d O_d \left( u_\lambda \right)
\]  

(A.10)

Taking the definition \( F_{ab} = \partial' \left( \zeta_m u^b \right) - \partial' \left( \zeta_m u^b \right) \):

\[
\zeta_m^{-1} u^a \tilde{R}_{ab} \approx \partial' F_{ab} + d_c \partial' \zeta_c - d_c \left( \zeta_m u^b \right) + u^d O_d \left( u_\lambda \right)
\]  

(A.11)

Finally, replacing Eq. A.8 in A.11:

\[
\zeta_m^{-1} u^a \tilde{R}_{ab} \approx \partial' F_{ab} + d_c \partial' \zeta_c - d_c \left( \zeta_m u^b \right) - \zeta_m \left( \partial' u_\lambda \right) \left( d_c u^d \right) - \zeta_m u^a \left( \partial' u_\lambda \right) \left( \partial' u^b \right)
\]  

(A.12)

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