Session Types at the Mirror

Luca Padovani
Istituto di Scienze e Tecnologie dell’Informazione, Università degli Studi di Urbino “Carlo Bo”
luca.padovani@uniurb.it

We (re)define session types as projections of process behaviors with respect to the communication channels they use. In this setting, we give session types a semantics based on fair testing. The outcome is a unified theory of behavioral types that shares common aspects with conversation types and that encompass features of both dyadic and multi-party session types. The point of view we provide sheds light on the nature of session types and gives us a chance to reason about them in a framework where every notion, from well-typedness to the subtyping relation between session types, is semantically – rather than syntactically – grounded.

1 Introduction

The leitmotif in the flourishing literature on session types [14, 15, 16] is to associate every communication channel with a type that constrains how a process can use that channel. In this paper we take the opposite perspective: we define the session type associated with a channel as the projection of the behavior of the processes restricted to how that channel is used by them. As expected, this approach requires a language of session types that is more general than the ones we usually encounter in other works. But – this is in summary the contribution of this work – the language we come up with is just a minor variation of well-known value-passing process algebras that can be semantically characterized using well-known concepts and techniques.

To get acquainted with our approach, let us consider the following example written in π-calculus like language and which is a slightly simplified variant of the motivating example in [16]:

\[
\begin{align*}
\text{Seller} &= a?x.(x?\text{String}.x!\text{price}(\text{title}).x?(\text{addr} : \text{Address}).x!\text{date}(\text{title})) \\
\text{Buyer1} &= (\nu c)(a!c.c!”The Origin of Species”.c?(\text{price} : \text{Int}).(\nu d)d!d!\text{price}/2.d!c \\
\text{Buyer2} &= b?(y).y?(\text{contrib} : \text{Int}).y?(z).z!\text{address}.z?(d : \text{Date})
\end{align*}
\]

Here we have two buyers that collaborate with each other in order to complete a transaction with a seller. Buyer1 creates a local channel \( c \) that it sends to Seller through the public channel \( a \). The channel \( c \) is normally dubbed session: it is a fresh channel shared by Buyer1 and Seller on which the two can communicate privately. On \( c \), Buyer1 sends to the Seller the name of a book, and Seller answers with its price. At this stage Buyer1 asks for the collaboration of Buyer2: it creates another fresh channel \( d \) which it communicates to Buyer2 by means of the public channel \( b \), it sends Buyer2 the amount of money Buyer2 should contribute, and finally it \textit{delegates} the private channel \( c \) to Buyer2, so that Buyer2 can complete the transaction with the Seller. This implies sending the Seller a delivery address and receiving the estimated delivery date.

Let us focus on the public channels \( a \) and \( b \): the former is used by Buyer1 for sending a channel of some type, say \( \eta \), and is used by Seller for receiving a channel of the same type. In our approach we say that the type of \( a \) is \( a?\eta.1 \upharpoonright a!\eta.1 \), where \( a?\eta.1 \) is the projected behavior of Seller on \( a \), \( a!\eta.1 \) is the projected behavior of Buyer1 on \( a \), and \( \upharpoonright \) denotes the composition of these two behaviors. In a similar way, \( b \) is used by Buyer1 and Buyer2 and has type \( b?\theta.1 \upharpoonright b!\theta.1 \), assuming that the channel exchanged
between Buyer1 and Buyer2 has type $\theta$. Channel $c$ is more interesting: it is created by Buyer1, which uses it according to the type $\!\text{String}\?\text{Int}$. However, $c$ is delegated to Seller right after its creation, and to Buyer2 when Buyer1 has finished using it. So, the true type of $c$ is $\eta \mid \!\text{String}\?\text{Int}\rho$ where $\eta$ is the projection of Seller’s behavior with respect to the channel $c$ (after it has been received by Seller), and $\rho$ is the projection of Buyer2’s behavior with respect to the same channel after it has been received by Buyer2. By similar arguments, one can see that the type of $d$ is $\theta \mid \!\text{Int}\rho$.1 and the mentioned types $\eta$, $\theta$, and $\rho$ are defined as $\!\text{String}\!\text{Int}\!\text{Address}\!\text{Date}$.1, $\!\text{Int}\!\text{p}$.1, and $\!\text{Address}\!\text{Date}$.1, respectively. If we were to depict the projection we have operated for typing the channels in the example, we could summarize it as follows:

| Seller | Buyer1 | Buyer2 |
|--------|--------|--------|
| $a$: $\eta$.1 | $!\eta$.1 | $\eta$.1 |
| $b$: $!\theta$.1 | $\theta$.1 |
| $c$: $\!\text{String}\!\text{Int}\!\text{Address}\!\text{Date}$.1 | $\!\text{String}\!\text{Int}$.1 | $\!\text{Address}\!\text{Date}$.1 |
| $d$: $\!\text{Int}\!\text{p}$.1 | $\!\text{Int}\!\text{p}$.1 |

Can we tell whether the system composed of Seller and the two buyers “behaves well”? Although at this stage we have not given a formal semantics to session types, by looking at the types for the various channels involved in the example we can argue that they all eventually “reduce” to a parallel composition of 1’s. If we read the type 1 as the fact that a process stops using a channel with that type, this roughly indicates that all the conversations initiated in the example eventually terminate successfully.

The projection we have operated abstracts away from the temporal dependencies between communications occurring on different channels. This is a well-known source of problems if one is interested in global progress properties. In our approach, and unlike other presentations of session types, we do not even try to impose any linearity constraint on the channels being used, nor do we use polarities [11] or indexes [16, 1] for distinguishing different roles. For example, the process Buyer1 keeps using channel $c$ after it has been delegated, and it delegates the channel once more before terminating. As a consequence, the projection we operate may not even capture the temporal dependencies between communications occurring on the same channel. This can happen if two distinct free variables are instantiated with the same channel during some execution. Thus, we must impose additional constraints on processes only to ensure the type preservation property. Interestingly, we will see that these additional constraints are similar to those used for ensuring global progress [9, 1, 3].

We can identify three main contributions of this work: (1) we show that session types can be naturally generalized to an algebraic language of processes that closely resembles value-passing CCS; (2) as a consequence, we are able to work on session types reusing a vast toolkit of known results and techniques; in particular, we are able to semantically justify the fundamental concepts (duality, well-typedness, the subtyping relation) that are axiomatically or syntactically presented in other theories; (3) we provide a unified framework of behavioral types that encompasses features not only of dyadic and multi-party session types, but also of conversation types [2].

**Structure of the paper.** In Section 2 we define session types as a proper process algebra equipped with a labeled transition system and a testing semantics based on fair testing. This will immediately provide us with a semantically justified equivalence relation – actually, a pre-order – to reason about safe replacement of channels and well-behaving systems. In Section 3 we formally define a process language that is a minor variant of the $\pi$-calculus without any explicit construct that is dedicated to session-oriented interaction. We will show how to type processes in this language and illustrate the main features of the
type system with several examples. Finally, we will state the main properties (type preservation and local progress) of our typing relation. Section 4 concludes.

Related work. Theories of dyadic session types can be traced back to the works of Honda [14] and Honda et al. [15]. Since then, the application of session types has been extended to functional languages [20, 12] and object-oriented languages (see [10, 8] for just a few examples). A major line of research is the one dealing with so-called multi-party session types, those describing sessions where multiple participants interact simultaneously [16, 1]. An in depth study of a subtyping relation for session types can be found in [11], while [19] provides an incremental tutorial presentation of the most relevant features of dyadic session types.

Conversation types [2] are a recently introduced formalism that aims at generalizing session types for the description of the behavior of processes that interact within and across the scope of structurally organized communications called conversations. Conversation types are very similar to the language of session types we propose here, for example they embed a parallel composition operator for representing the composed behavior of several processes simultaneously accessing a conversation. The difference with our approach mainly resides in the semantics of types: we treat session types as terms of a proper process algebra with a proper transition relation and all the relevant notions on types originate from here. In [2], the semantics of conversation types is given in terms of syntactically-defined notions of subtyping and merging. Also, [2] uses a process language that incorporates explicit constructs for dealing with conversations, while we emphasize the idea of projected behavior by working with the naked \( \pi \)-calculus.

Elsewhere [3] we have been advocating the use of a testing approach in order to semantically justify session types. Unlike [3], here we disallow branch selection depending on the type of channels. This reduces the expressiveness of types for the sake of a simplification of the technicalities in the resulting theory. Another difference is that in the present paper we adopt a fair testing approach [18].

Finally, it should be mentioned that the use of processes as types has already been proposed in the past, for example in [5, 17]. In particular, [17] uses a language close to value-passing CCS for defining an effect system for Concurrent ML.

2 Syntax and semantics of session types

Let us fix some conventions: \( \eta, \theta, \rho, \ldots \) range over session types; \( \alpha, \ldots \) range over actions; \( t, s, \ldots \) range over types; \( v, \ldots \) range over an unspecified set \( \mathcal{V} \) of basic values; \( B, \ldots \) range over an unspecified set of basic types such as \( \text{Int}, \text{Bool}, \text{String}, \) and so on. The syntax of session types is defined by the grammar in Table 1. Types represent sets of related values: \( \emptyset \) is the empty type, the one inhabited by no value; basic types are arbitrary subsets of \( \mathcal{V} \); for every \( v \in \mathcal{V} \) we write \( v \) for the singleton type whose only value is \( v \) itself. We will write \( v : t \) to state that \( v \) inhabits type \( t \) and we will sometimes say that \( v \) is of type \( t \).

Actions represent input/output operations on a channel. An action \( !t \) represents the sending of an arbitrary value of type \( t \); an action \( ?t \) represents the receiving of an arbitrary value of type \( t \); actions \( !\eta \) and \( ?\eta \) are similar but they respectively represent the sending and receiving of a channel of type \( \eta \).

Although session types are used to classify channels, they describe the behavior of processes using those channels. Consistently with this observation, we will often present session types as characterizing processes rather than channels. In the explanation that follows, it is useful to keep in mind that, when a process uses a channel according to some protocol described by a session type, it expects to interact with other processes that use the same channel according to other protocols. For a communication to occur, the process must perform an action on the channel (say, sending a value of some type), and another
Table 1: Syntax of session types.

| \( \eta \) ::= | session type | \( \alpha \) ::= | action | \( t \) ::= | type |
|----------------|-----------------|--------------|--------|--------|
| 0              | (failure)       | \(?t\)       | (value input) | 0      | (empty) |
| 1              | (success)       | \(!t\)       | (value output) | \(v\)  | (singleton) |
| \( \alpha.\eta \) | (action prefix) | \(?\eta\)     | (channel input) | \(B\)  | (basic type) |
| \( \eta + \theta \) | (external choice) | \(!\eta\)    | (delegation) |
| \( \eta \oplus \theta \) | (internal choice) |
| \( \eta | \theta \) | (composition) |

process must perform the corresponding co-action (say, receiving a value of the same type). The session type \( 0 \) classifies a channel on which a communication error has occurred. No correct system should ever involve channels typed by \( 0 \), and we will see that it is useful to have an explicit term denoting a static error. The session type \( 1 \) describes a process that performs no further action on a channel. The session type \( \alpha.\eta \) describes a process that performs the action \( \alpha \), and then behaves according to the protocol \( \eta \). The session type \( \eta + \theta \) is the *external choice* of \( \eta \) and \( \theta \) and describes a process that offers interacting processes to behave according to one of the branches. Dually, the session type \( \eta \oplus \theta \) is the *internal choice* of \( \eta \) and \( \theta \) and describes a process that internally decides to behave according to one of the branches. The session type \( \eta | \theta \) describes the simultaneous access to a shared channel by two processes behaving according to \( \eta \) and \( \theta \). If we have \( n \) processes sharing a common channel and each process behaves according to some protocol \( \eta_i \), then \( \eta_1 | \cdots | \eta_n \) describes the overall protocol implemented by the processes on the channel.

We do not rely on any explicit syntax for describing recursive behaviors. We borrow the technique already used in [3] and define the set of session types as the set of possibly infinite syntax trees generated by the productions of the grammar in Table 1 that satisfy the following conditions:

1. the tree must contain a finite number of different subtrees;
2. on every infinite branch of the tree there must be infinite occurrences of the action prefix operator;
3. the tree must contain a finite number of occurrences of the parallel composition operator.

The first condition is a standard *regularity condition* imposing that the tree must be a *regular tree* [6]. The second one is a *contractivity condition* ruling out meaningless regular trees such as those generated by the equations \( X = X + X \) or \( X = X \oplus X \). Finally, it can be shown that the last condition enforces that the protocol described by a session type is “finite state”.

To familiarize with session types consider the following two examples:

\[ ?\text{Int}.!\text{String}.1 + ?\text{Bool}.!\text{Real}.1 \]

describes a process that waits for either an integer number or a Boolean value. If the process receives an integer number, it sends a string; if the process receives a Boolean value, it sends a real number. After that, in either case, the process stops using the channel. Instead, the session type \(!\text{Int}.1 \oplus !\text{Bool}.1\) describes a process that internally decides whether to send an integer or a Boolean value.

It may seem that the syntax of session types is overly generic, and that external choices make sense only when they are guarded by input actions and internal choices make sense only when they are guarded

---

1We use the word “shared” to highlight the fact that two (or more) processes simultaneously act on the same channel. This should not be confused with the terminology used in different session type theories, where “shared channels” are publicly known channels on which sessions are initiated.
by output actions. As a matter of facts, this is a common restriction in standard session type presentations. In our approach, this generality is actually necessary: a session type \( \eta = \! \text{Int}. \! \text{1} | \! \text{Bool}. \! \text{1} \) describes two processes trying to simultaneously send an integer and a Boolean value on the same channel. A process interacting with these two parties is allowed to read both values in either order, since both are available. In other words, the session type \( \eta \) is equivalent to \( \! \text{Int}. \! \text{1} + \! \text{Bool}. \! \text{1} \), that is the interleaving of the actions in \( \eta \). Had we expanded \( \eta \) to \( \! \text{Int}. \! \text{Bool}. \! \text{1} + \! \text{Bool}. \! \text{1} + \! \text{Int}. \! \text{1} \) instead, no interacting process would be able to decide which value, the integer or the Boolean value, to read first. The ability to express parallel composition in terms of choices is well studied in process algebra communities where it would be able to decide which value, the integer or the Boolean value, to read first. The ability to unconstrained process behaviors; second, it clearly separates communications (represented by actions) from choices, thus yielding a clean, algebraic type language with orthogonal features.

We equip session types with an operational semantics that mimics the actions performed by processes behaving according to these types. The labeled transition system of session types is defined by the rules in Table 2 plus the obvious symmetric rules of those concerning choices and parallel composition. Transitions make use of labels ranged over by \( \mu \) and generated by the grammar:

\[
\mu ::= \checkmark | ?\nu | !\nu | ?\eta | !\eta
\]

Strictly speaking, the transition system is defined by two relations: a labeled one describing external, visible actions and an unlabeled one describing internal, invisible actions. Thus, the transition system is an extension of the one of CCS without \( \tau \)'s to a value-passing calculus. Rule (R1) states that the session type 1 emits a single action \( \checkmark \) denoting successful termination of the protocol, and reduces to itself. By rule (R2), the session type \( \eta \oplus \theta \) can perform an internal transition to either \( \eta \) or \( \theta \).

Table 2: Transitions of session types.

| Rule | Transition |
|------|------------|
| (R1) | \( 1 \overset{\checkmark}{\rightarrow} 1 \) |
| (R2) | \( \eta \oplus \theta \rightarrow \eta \) |
| (R3) | \( !\nu. \eta \rightarrow \eta \) |
| (R4) | \( !\rho. \eta \rightarrow \eta \) |
| (R5) | \( ?\rho. \eta \rightarrow \eta \) |
| (R6) | \( \nu : t \rightarrow \nu.t \) |
| (R7) | \( \eta \rightarrow \eta' \) |
| (R8) | \( \eta \rightarrow \eta' \) |
| (R9) | \( \eta \rightarrow \eta' \) |
| (R10) | \( \eta \rightarrow \eta' \) |
| (R11) | \( \eta \rightarrow \eta' \) |
| (R12) | \( \eta \rightarrow \eta' \) |
| (R13) | \( \eta \rightarrow \eta' \) |
| (R14) | \( \eta \rightarrow \eta' \) |
| (R15) | \( \eta \rightarrow \eta' \) |
Rules (R3) and (R4) deal with output actions. The session type \( !v.\eta \) emits the value \( v \) and reduces to \( \eta \). Similarly, \( !p.\eta \) emits a signal \( !p \) (the output of a channel of type \( p \)). Rule (R5) is the dual of rule (R4) and states that \( ?p.\eta \) emits a signal \( ?p \) (the input of a channel of type \( p \)). Rule (R6) states that a process behaving according to \( !t.\eta \) internally chooses a value \( v \) of type \( t \) to send, and once has committed to such a value it reduces to \( !v.\eta \). Rule (R7) is the dual of rule (R3), but because of rule (R6) observes that a process behaving according to \( ?t.\eta \) commits to sending one particular value of type \( t \), whereas a process behaving according to \( ?t.\eta \) is able to receive any value of type \( t \). Rule (R8) states that + is indeed an external choice, thus internal choices in either branch do not preempt the other branch. This is a typical reduction rule for those languages with two different choices, such as CCS without \( \tau \)’s [7]. Rules (R9) and (R10) state obvious reductions for external choices, which offer any action that is offered in either branch, and parallel compositions, which allow either component to internally evolve independently.

Rule (R11) states that any action other than \( \checkmark \) is offered by a parallel composition whenever it is offered by one of the components; rule (R12) states that a parallel composition has successfully terminated only if both components have; rule (R13) states the obvious synchronization between components offering dual actions. Rule (R14) states that a process sending a channel of type \( p \) can synchronize with another process willing to receive a channel of type \( p' \), but only if \( p \leq p' \). Here \( \leq \) is a subtyping relation meaning that any channel of type \( p \) can be used where a channel of type \( p' \) is expected. We shall formally define \( \leq \) in a moment; for the time being we must content ourselves with this intuition. Rule (R15) states that if the relation \( p \leq p' \) is not satisfied, the synchronization occurs nonetheless, but it yields an error.

Before we move on to the subtyping relation for session types, we should point out a fundamental design decision that relates communication and external choices. On the one hand, values other than channels may drive the selection of the branch in external choices. For example, we have \( ?\Int.\eta + ?\Int.\eta \to \eta \) while \( ?\Int.\eta + ?\Int.\eta \to \theta \) while \( \theta \). The type of the value determines the branch, and this feature allows us to model the label-driven branch selection that is found in standard session types theories. On the other hand, the last two rules in Table 2 show that branch selection cannot be affected by the type of the channel being communicated. It is true that \( ?p.\eta + ?p'.\theta \to \eta \) and \( ?p.\eta + ?p'.\theta \to \theta \), but when we compose \( ?p.\eta + ?p'.\theta \) with \( !\rho''.\theta \) either reduction is possible, and the residual may or may not be \( \theta \) depending on the relation between \( \rho, \rho' \), and \( \rho'' \):

\[
\begin{align*}
\rho'' \leq \rho : & \quad {?p.\eta + ?p'.\theta | !\rho''.\theta} \to \eta | \theta' \\
\rho'' \nleq \rho : & \quad {?p.\eta + ?p'.\theta | !\rho''.\theta} \to \theta
\end{align*}
\]

To be sure that the residual is not \( \theta \), it must be the case that \( \rho'' \leq \rho \) and \( \rho'' \nleq \rho' \). In summary, we do not allow dynamic dispatching according to the type of a channel, namely all channels are treated as if they had the same type. This is not the only possible choice (see [3] for an alternative), but is one that simplifies the theory.

In the following we adopt standard conventions regarding the transition relations: we write \( \Rightarrow \) for the reflexive, transitive closure of \( \to \); we write \( \eta \mu \theta \) (respectively, \( \eta \mu \theta \)) if there exists \( \theta \) such that \( \eta \mu \theta \) (respectively, \( \eta \mu \theta \)); we write \( \eta \to, \mu \to, \Rightarrow \) for the usual negated relations; for example, \( \eta \to \) means that \( \eta \) does not perform internal transitions.

The first semantic characterization we give is that of complete session type, namely a session type that can always reach a successful state, no matter of its internal transitions.

**Definition 2.1** (completeness). We say that \( \eta \) is complete if \( \eta \Rightarrow \eta' \) implies \( \eta' \Rightarrow \). Intuitively, a complete protocol is one implemented by processes which can always terminate successfully their interaction, without the help of any other process. Observe, as a side note, that completeness implies that no evolution of the system may yield an error or lead to a state where one process insists
on sending a message that no interacting party is willing to accept. 1 is the simplest complete session type; the session types \(?\eta.1\mid \eta.1\) and \(\eta.1\mid \text{String}\) we have seen in the introduction are also complete, since every maximal transition leads to a successfully terminated state. The simplest example of incomplete session type is \(0\), another example being \(?\text{Int}.1\mid \text{Real}.1\) because of the maximal reduction \(?\text{Int}.1\mid \text{Real}.1 \rightarrow \text{Int}.1\mid !\sqrt{2}.1 \rightarrow\). If we take \(\eta\) as the solution of the equation \(X = \text{Int}.X\) and \(\theta\) as the solution of the equation \(Y = !\text{Int}.Y\) we have that \(\eta \mid \theta\) is not complete, despite it never reaches a deadlock state. In this sense the notion of completeness embeds a fairness principle that is typically found in fair testing theories [18].

Completeness is the one notion that drives the rest of the theory. We define the subtyping relation for session types, which we call \(\eta\) is “smaller than” \(\theta\) if every session type that completes \(\eta\) completes \(\theta\) as well.

**Definition 2.2** (subsession). We say that \(\eta\) is a subsession of \(\theta\), notation \(\eta \preceq \theta\), if \(\eta \mid \rho\) complete implies \(\theta \mid \rho\) complete for every \(\rho\). We write \(\approx\) for the equivalence relation induced by \(\preceq\), namely \(\approx = \preceq \cap \succeq\).

In other words, we are defining an equivalence relation for session types based on (fair) testing [18]; we use completeness as the notion of test, and we say that two session types are equivalent if they pass the same tests. As a consequence, the equational theory generated by this definition is not immediately obvious, although a few relations are easy to check: for example, \(+\), \(\oplus\), and \(\mid\) are commutative, associative operators; \(0\) is neutral for \(+\) and \(1\) is neutral for \(\mid\); furthermore \(\eta \oplus \theta \preceq \eta\). Namely, it is safe to use a channel with type \(\eta \oplus \theta\) where another one of type \(\eta\) is expected. If the safety property mentioned here seems hard to grasp, one should resort to the intuition that the “type” of a channel actually is the behavior of a process communicating on that channel. A process that expects to receive a channel with type \(\eta\) will behave on that channel according to \(\eta\); if we send that process a channel with type \(\eta \oplus \theta\), the receiving process will still behave according to \(\eta\), which is a more deterministic behavior than \(\eta \oplus \theta\), hence no problem may arise. As a special case of reduction of nondeterminism, we have \(!\text{Real}.\eta \preceq !\text{Int}.\eta\) assuming that \(\text{Int}\) is a subtype of \(\text{Real}\). Other useful relations are those concerning failed processes: we have \(0 \approx \alpha.0\) and \(!0.\eta \approx ?0.\eta \approx 0\). More generally, the relation \(\eta \approx 0\) means that there is no session type \(\theta\) such that \(\eta \mid \theta\) is complete: \(\eta\) is intrinsically flawed and cannot be remedied. The class of non-flawed session types will be of primary importance in the following, to the point that we reserve them a name.

**Definition 2.3** (viability). We say that \(\eta\) is viable if \(\eta \mid \rho\) is complete for some \(\rho\).

**Remark 2.1.** At this stage we can appreciate the fact that subsession depends on the transition relation, and that the transition relation depends on subsession. This circularity can be broken by stratifying the definitions: a session type \(\eta\) is given weight 0 if it contains no prefix of the form \(?\rho\) or \(!\rho\): a session type \(\eta\) is given weight \(n > 0\) if any session type \(\rho\) in any prefix of the form \(?\rho\) or \(!\rho\) occurring in \(\eta\) has weight at most \(n - 1\). By means of this stratification, one can see that the definitions of the transition relation and of subsession are well founded.

It is fairly easy to see that \(\preceq\) is a precongruence with respect to action prefix, internal choice, and parallel composition. The case of the action prefix is trivial. As regards the internal choice, it suffices to observe that \((\eta \oplus \theta) \mid \rho\) complete if and only if both \(\eta \mid \rho\) and \(\theta \mid \rho\) are complete. Namely, \(\oplus\) corresponds to a set-theoretic intersection between session types that complete \(\eta\) and \(\theta\). As regards the parallel composition, the precongruence follows from the very definition of subsession, since \(\eta \mid \eta' \preceq \eta\) if \((\eta \mid \eta') \mid \rho\) complete implies \((\theta \mid \eta') \mid \rho\) complete, namely if \(\eta \mid (\eta' \mid \rho)\) complete implies \(\theta \mid (\eta' \mid \rho)\) complete, that is if \(\eta \preceq \theta\). Because all the non-viable session types are \(\approx\)-equal, however, \(\preceq\) is not a precongruence with respect to the external choice. For example, we have \(0 \preceq !\text{Int}.0\) but \(!\text{Int}.1 + 0 \not\preceq !\text{Int}.1 + !\text{Int}.0 \approx 0\). This is a major drawback of the subsession relation as it is defined,
since it prevents \( \leq \) from being used in arbitrary contexts for replacing equals with equals (note that \( \approx \) is not a congruence for the same reasons). We resort to a standard technique for defining the largest relation included in \( \leq \) that is a precongruence with respect to the external choice. We call this relation strong subsession:

**Definition 2.4** (strong subsession). Let \( \sqsubseteq \) be the largest relation included in \( \leq \) that is a precongruence with respect to \(+\), namely \( \eta \sqsubseteq \theta \) if and only if \( \eta + \rho \leq \theta + \rho \) for every \( \rho \). We write \( \approx \) for the equivalence relation induced by \( \sqsubseteq \), namely \( \approx = \sqsubseteq \cap \exists \).

We end this section with a few results about \( \leq \) and \( \sqsubseteq \). First of all, we can use \( \sqsubseteq \) for reasoning about viability and completeness of a session type:

**Proposition 2.1.** The following properties hold:

1. \( \eta \) is not viable if and only if \( \eta \sqsubseteq 0 \);
2. \( \eta \) is complete if and only if \( 1 + \eta \sqsubseteq \eta \).

Then, we prove that \( \leq \) and \( \sqsubseteq \) are almost the same relation, in the sense that they coincide as soon as the smaller session type is viable. This means that for all practical purposes the use of \( \sqsubseteq \) in place of \( \leq \) is immaterial, if not for the gained precongruence, since in no case we will be keen on replacing a channel with a viable type with one that is not viable.

**Theorem 2.1.** \( \eta \leq \theta \) if and only if either \( \eta \sqsubseteq 0 \) or \( \eta \sqsubseteq \theta \).

**Remark 2.2.** It is interesting to compare \( \sqsubseteq \) with the subtyping relation for session types in \([11]\). From a technical point of view, the two relations arise in completely different ways: \( \sqsubseteq \) arises semantically as a relation between session types that preserves completeness; the subtyping relation in \([11]\) is defined (co)inductively and by cases on the syntax of session types being related. The essence of this latter relation is strictly connected with the direction of the exchanged messages: when \( S \leq T \) holds, \( S \) sends more things and receives fewer, regardless of whether such things are labels or actual data. In contrast, the relation \( \sqsubseteq \) is fundamentally determined by reduction of nondeterminism, which is captured by the law \( \eta + \theta \sqsubseteq \eta \). Note that this law does not say anything about messages being sent or received. For example, we have \( !\text{Int}.1 \oplus !\text{Bool}.1 \sqsubseteq !\text{Int}.1 \) but also \( ?\text{Int}.1 \oplus ?\text{Bool}.1 \not\sqsubseteq ?\text{Int}.1 \). We can identify two other significant differences: the first one is that in our theory of session types, the successfully terminated session type \( 1 \) can be composed with actions. For example, \( 1 + ?\text{Int}.1 \) describes a process that is waiting for an integer, but is also perfectly happy to terminate the session at this time without any further communication. As another example, \( 1 \oplus !\text{Int}.1 \) describes the behavior of a process that internally decides whether to terminate the session without any further communication, or to do so only after having sent an integer. Incidentally, observe that the two examples complete each other. In \([11]\) (and in most session type theories) the terminal behavior cannot be composed with others. The type system we will describe later does not use this capability either, but this is just to keep it simple and with a reasonable number of rules. The second and last difference we want to emphasize is that the law \( \eta \sqsubseteq \eta + \theta \), which is somehow dual of \( \eta \oplus \theta \sqsubseteq \eta \), does not hold, while it is sound in \([11]\). Two main reasons justify this fact: the first is that in our theory \( + \) is an algebraic operator that can combine arbitrary session types, and for this reason the external choice sometimes is an internal choice in disguise: for example, it is possible to prove that \( ?\text{Int}.\eta + ?\text{Int}.\theta \approx ?\text{Int}.(\eta \oplus \theta) \). This cannot happen in \([11]\) because of the very syntax of session types, which prevents arbitrary compositions of behaviors. The second reason is that the synchronous communication model we are relying upon does not tolerate the introduction of interferences. For example, we have \( ?\text{Int}.1 \not\sqsubseteq ?\text{Int}.1 + ?\text{Bool}.1 \) because the session type \( !\text{Int}.1 + !\text{Bool}.0 \) completes the first session type but not the second one: the \( ?\text{Bool}.0 \) branch introduces
Table 3: Syntax of processes.

|       | process |       | action |
|-------|---------|-------|--------|
| \( P \ ::= \) | process | \( \pi \ ::= \) | action |
| \( 0 \) (idle) | \( \pi.P \) (action prefix) | \( u?(x:t) \) (value input) |
| \( \pi.P \) (action prefix) | \( \star P \) (replication) | \( u!e \) (value output) |
| \( P + P \) (external choice) | \( P \oplus P \) (internal choice) | \( u?(x) \) (channel input) |
| \( P \mid P \) (parallel composition) | \( (v_c)P \) (restriction) | \( u!v \) (delegation) |

an interference that may enable harmful synchronizations. For this and other reasons the adoption of a synchronous communication model is questionable in practice. However, one can show that by suitably restricting behaviors (for instance, by forbidding outputs in external choices such as in the example above) some instances of the law \( \eta \sqsubseteq \eta + \theta \) become sound again. Furthermore, it is possible to simulate partial forms of asynchrony by means of the session type language we have presented (the idea is not explored in detail here, but the interested reader may find some hints in [4]). In summary, the \( \sqsubseteq \) relation is both an extension and a conservative restriction of the subtyping relation in [11].

3 Processes

Processes are defined by the grammar in Table 3. We use \( P, Q, R, \ldots \) to range over processes; we use \( \pi, \ldots \) to range over action prefixes; we use \( a, b, c, \ldots \) to range over channel names and variables (\( \nu \) should not be confused with \( v \) that we used to range over elements of \( \forall \)’); we let \( e, \ldots \) range over an unspecified language of expressions. The process language is a minor variation of the \( \pi \)-calculus, so we remark here only the differences: we have four action prefixes: \( u?(x:t) \) denotes a receive action for a basic value \( x \) of type \( t \) on channel \( u \); \( u!e \) denotes a send action for the value of the expression \( e \) on channel \( u \); \( u?(x) \) denotes a receive action for a channel \( x \) on channel \( u \); \( u!v \) denotes a send action for a channel \( v \) on channel \( u \). Consistently with the language of session types, actions denoting send/receive operations of channels are “untyped”. The process \( \star P \) denotes unbounded replications of process \( P \), and \( P + Q \) and \( P \oplus Q \) respectively denote the external and internal choice between \( P \) and \( Q \). We will usually omit the \( 0 \) process; we will write \( \text{fn}(P) \) for the set of free channel names occurring in \( P \) (the only binder for channel names is restriction); we will write \( P\{m/x\} \) for the process \( P \) where all free occurrences of the variable \( x \) have been replaced by \( m \).

The transition relation of processes is defined by an almost standard relation in Table 4, so we will not provide detailed comments here. In the table, we write \( e \downarrow v \) for the fact that expression \( e \) evaluates to \( v \). Labels of the transition relation are ranged over by \( \ell, \ldots \) and are generated by the following grammar:

\[
\ell ::= \tau | c?m | c!m | c!(d)
\]

where \( m, \ldots \) ranges over messages, namely basic values and channel names. Action \( \tau \) denotes an internal computation or a synchronization. Actions of the form \( c?m \) and \( c!m \) are often called free inputs and free outputs respectively. Actions of the form \( c!(d) \) are called bound outputs and represent the extrusion of a private channel, \( d \) in this case. We use these actions to model session initiations, whereby a private channel is exchanged and subsequently used for the actual interaction. Notions of free and bound names
and if the session type associated with that does not use any channel; the latter denotes a communication error or a deadlock. Rule (\(\tau\)) should not be confused with the failed session type \(\tau\). The assumption \(x : t\) implies \(\tau\) is moved into the environment \(\theta\). This rule is fundamental in the typing of processes. We remark only two distinctive features of the transition relation: (1) the replicated process \(\ast P\) evolves by means of an internal transition to \(\ast P \mid P\); technically this makes \(\ast P\) a divergent process, but the fact that we work with a fair semantics makes this only a detail; (2) similarly to the transition relation for session types, the transition relation for processes selects branches of external choices according to the type of the basic value being communicated. This is evident in the transitions for \(c? (x : t) . P\), which are labeled by values of type \(t\).

The typing rules for the process language are inductively defined in Table 5. Judgments have the form \(\Gamma \vdash P : \Delta\), where \(\Gamma\) is a standard environment mapping variables to basic types and \(\Delta\) is an environment mapping channel names and channel variables to session types. We write \(\text{dom}(\Delta)\) for the domain of \(\Delta\). Rule (T-WEAK) allows one to enrich \(\Delta\) with assumptions of the form \(u : I\), indicating that a process does not use the channel \(u\). The premise \(u \notin \text{dom}(\Delta)\) implies \(u \notin \text{fn}(P)\) since it is always the case that \(\text{fn}(P) \subseteq \text{dom}(\Delta)\). Rule (T-SUB) is an almost standard subsumption rule regarding the type of a channel \(u\). The peculiarity is that it works “the other way round” by allowing a session type \(\theta\) to become a smaller session type \(\eta\). The intuition is that \(P\) behaves according to \(\theta\) on the channel \(u\). Thus, it is safe to declare that the session type associated with \(u\) is even less deterministic than \(\theta\). This rule is fundamental in the type system since many other rules impose equality constraints on session types that can only be satisfied by finding a lower bound to two or more session types. It should also be appreciated the importance of using \(\subseteq\), which is a precongruence, since this allows us to apply rule (T-SUB) in arbitrary contexts. Rule (T-RES) types restrictions, by requiring the session type associated with the restricted channel to be of the form \(I + \eta\). In light of rule (T-SUB) and of Proposition 2.12, this requirement imposes that the session type of a restricted channel \(c\) must be complete. Namely, there must not be communication errors on \(c\). Rule (T-NIL) types the idle process 0 with the empty session environment. The process 0 should not be confused with the failed session type \(0\); the former is the successfully terminated process that does not use any channel; the latter denotes a communication error or a deadlock. Rule (T-INPUT) types an input action for basic values of type \(t\). The assumption \(x : t\) is moved into the environment \(\Gamma\) and if the session type associated with \(u\) in the continuation \(P\) is \(\eta\), then the overall behavior of \(P\) on \(u\) is described by \(? x . \eta\). Rule (T-OUTPUT) is similar, but regards output actions of basic values. We assume an
unspecified set of deduction rules for judgments of the form \( \Gamma \vdash e : t \), denoting that the expression \( e \) has type \( t \) in the environment \( \Gamma \). Rule (T-INPUTS) types an input action for a channel \( x \). The continuation \( P \) must be typed in a session environment of the form \( \{x : \rho\} \), requiring that \( P \) must not refer to (free) channels other than the received one. Consequently, the whole process behaves according to the session type \( ?\rho.1 \). The severe restriction on the continuation process is necessary for type preservation, as we will see in Example 3.5 below. Rule (T-OUTPUTS) types delegations, whereby a channel \( v \) is sent over another channel \( u \). This rule expresses clearly the idea of projection we are pursuing in our approach: the delegated channel \( v \) is used in the continuation \( P \) according to the session type \( \theta \) (which may be \( 1 \) in case rule (T-WEAK) is applied); at the same time, the channel \( v \) is delegated to another process which will behave on it according to \( \rho \). As a consequence, the overall behavior on \( v \) is expressed by the composition of \( \theta \) and \( \rho \), namely by \( \theta | \rho \). If \( u \) is used in the continuation \( P \) according to \( \eta \), then its type is \( !\rho.\eta \) in the conclusion. Rule (T-EXT) types external choices. These are well typed only when each branch of the choice is guarded by an action whose subject is \( u \) (we write \( \text{subj} (\pi) \) for the subject of action \( \pi \)). For this reason the rule is only applicable to processes of the form \( \pi_1.P_1 + \cdots + \pi_n.P_n \), which we abbreviate as \( \sum_{i \in \{1, \ldots, n\}} \pi_i.P_i \), and the resulting behavior on \( u \) is the sum of \( \eta_1 + \cdots + \eta_n \) of the individual behaviors on \( u \) of each branch, which we abbreviate as \( \sum_{i \in I} \eta_i \). Rule (T-IN) types internal choices, but only when the two branches do have the same session environment. This can be achieved by repeated applications of rules (T-WEAK) and (T-SUB). Rule (T-PAR) types the parallel composition of processes. Again this rule shows the idea of projection and, unlike other session type systems, allows (actually requires) both processes to use exactly the same channels, whose corresponding session types are composed with \( | \). In this context rule (T-WEAK) can be used to enforce that the session environments for \( P \) and \( Q \) are exactly the same, recalling that \( 1 \) is neutral for \( | \). Finally, rule (T-BANG) types replicated processes: the basic
idea is that a replicated process $\star P$ is well typed if any channel it uses is “unlimited” (in the terminology of 11), which in our case translates to the property that it must be smaller than two copies of itself. 1 is the simplest session type with this property, but there are others as we will see in Example 3.1.

**Remark 3.1.** Thanks to our setting, we have the opportunity to make some interesting connections between the subtyping relations used in type theories for programming languages and the behavioral preorders that arise in many testing theories for process algebras. According to Definition 2.2 if $\eta \leq \theta$, then it is safe to replace a process behaving according to $\eta$ with another process behaving according to $\theta$. This is because, by definition of $\leq$, every context that completes $\eta$ will also complete $\theta$. Note in particular that the safe substitution regards the larger object. This contrasts with the subtyping relations where it is safe to replace an object of type $T$ with another object of type $S$ if $S$ is a subtype of $T$. In fact, this is exactly the notion of safe substitutability we are using in rule (T-SUB). This mismatch can be source of confusion: recall that in our view a session type is not the type of channel, but rather is the allowed behavior of a process on a channel. Thus, if a channel has type $\mathtt{!Int.1}$, that means that the process using it behaves according to $\mathtt{!Int.1}$. Now, it is safe to replace that channel with another one with type $\mathtt{!Int.1} \oplus \mathtt{!Bool.1}$: since we are replacing the channel, and not the process, the process will still behave according to $\mathtt{!Int.1}$, but this time on a channel that allows more behaviors. Since $\mathtt{!Int.1} \oplus \mathtt{!Bool.1} \sqsubseteq \mathtt{!Int.1}$, we are assured that the substitution is safe.

**Example 3.1** (persistent service provider). Consider the process

$$Q \equiv \star \text{server}?\!(x).P$$

which accepts an unbounded number of connection requests on the channel server and processes them in the process $P$. Assume we can type the non-replicated process as follows:

$$\Gamma \vdash P : \{x : \rho\}$$

$$\Gamma \vdash \text{server}?\!(x).P : \{\text{server} : ?\rho{.1}\}$$

To apply rule (T-BANG) for $Q$ we need server to have a type $\eta$ such that $\eta \sqsubseteq \eta \mid \eta$, and $?\rho{.1}$ clearly does not have this property. Consider the session type $\eta$ that is solution of the equation $X = 1 \oplus ?\rho{.}X$. We have $\eta \sqsubseteq ?\rho{.}1$ and furthermore $\eta \sqsubseteq \eta \mid \eta$. Hence we can now type $Q$ with an application of rule (T-SUB) followed by (T-BANG).

**Example 3.2** (multi-party session). Intuitively, a multi-party session is a conversation taking place on a restricted channel that is shared between three or more participants. Consider a system $(va)(P | P | Q)$ where

$$P \equiv a?(x).x?(y : \mathtt{Int}).(x!\mathtt{isprime}(y) + x?(z : \mathtt{abort}))$$

$$Q \equiv (vc)(a!c.c!n | a!c.c!n | c?(x : \mathtt{Bool}).c!\mathtt{abort})$$

the idea being that the two instances of $P$ represent two servers checking whether a number is prime. The process $Q$ establishes a connection by sending the two servers a fresh channel $c$ and sending on this channel some integer number $n$. The two servers are thus able to process the number in parallel and the first one that succeeds sends the result back to $Q$. Upon reception of the result from one of the servers, $Q$ notifies the other server by sending a dummy value $\mathtt{abort}$, which we assume is a singleton type inhabited only by $\mathtt{abort}$ itself.

It is easy to verify that, within $P$, the channel $x$ has type $\eta = \mathtt{?Int.(!\mathtt{Bool.1} + ?\mathtt{abort.1})}$ and $a$ is used according to the type $\mathtt{?\eta.1}$. In $Q$, $a$ is used according to the type $\mathtt{!\eta.1} | \mathtt{?\eta.1}$ and $c$ is used according to the type $\eta | \mathtt{!\mathtt{Int.1} | \mathtt{?\mathtt{Bool.1}!abort.1}}$. Hence, the overall type of $a$ is $\mathtt{!\eta.1} | \mathtt{?\eta.1}$ and the whole system is well typed since both $a$’s type and $c$’s type are complete.
The type system permits to find type derivations for processes using channels with a non-viable session type. Examples of such processes are $c?\langle x : 0 \rangle . 0$. A non-viable session type indicates an intrinsic flaw in the process. For this reason viability is really the one notion that characterizes well-typedness of processes. We say that a session environment $\Delta$ is viable if so is every session type in its codomain.

**Theorem 3.1** (subject reduction). Let $\Gamma \vdash P : \Delta$ and $P \xrightarrow{\tau} Q$ and $\Delta$ viable. Then $\Gamma \vdash Q : \Delta$.

**Example 3.3.** If compared with more standard session type theories, the notion of viability looks as an additional complication of our more general setting. Actually, the rules in Table 5 project the behavior following processes:

On one hand, $P$ sends a fresh channel $c$ to $Q$, but does not use it anymore. On the other hand, $Q$ pretends to send an integer on the channel it receives from $P$. According to the rules in Table 5 we have $\vdash P \mid Q : \{a : !(\text{Int.1}) \mid ?!(\text{Int.1}).1\}$. In particular, the session type associated with $a$ is not viable, because $1 \not\preceq !\text{Int.1}$ does not hold. Indeed, we have the reduction

$$P \mid Q \xrightarrow{\tau} (\text{vc})(0 \mid c!3)$$

where the residual process is ill typed, since $c$ is associated with the session type $1 \mid !\text{Int.1}$ which is not complete, hence it does not satisfy the premise of rule (T-RES).

Before addressing type safety, we justify by means of examples the two main constraints imposed by the type system in order to guarantee type preservation.

**Example 3.4.** To justify rule (T-EXT), consider the process

$$P \overset{\text{def}}{=} a?(x : \text{Int}) . b?(y : \text{Bool}) + b?(x : \text{Int}). a?(y : \text{Bool})$$

and suppose it well typed, where $a : ?\text{Int.1} + ?\text{Bool}$ and $b : ?\text{Bool.1} + ?\text{Int.1}$. Apparently, both $a$ and $b$ are able to receive either an integer or a Boolean value and a system such as $(va)(vb)(P \mid a!3 \mid b!3)$ would be well typed. Alas, the external choices in the types of $a$ and $b$ do not take into account the fact that any synchronization of $P$ with another process may actually disable one branch in these choices. The reduction

$$(va)(vb)(P \mid a!3 \mid b!3) \xrightarrow{\tau} (va)(vb)(b?(y : \text{Bool}) \mid 0 \mid b!3)$$

leads to an ill-typed process, since $b$ has type $?\text{Bool.1} \mid !\text{Int.1}$ which is not complete.

**Example 3.5.** The severe constraint in the premise of rule (T-INPUTS) can be justified by looking at the following processes:

$$P \overset{\text{def}}{=} a!c.alc.c?(x : \text{Int}).c?(y : \text{Bool})$$

$$Q \overset{\text{def}}{=} a?(x).a?(y).y!true.x!3$$

where $P$ can be typed with a derivation like the following:

$$\Gamma \vdash c?(x : \text{Int}).c?(y : \text{Bool}) : \{a : 1, c : ?\text{Int.1}.?\text{Bool.1}\}$$

$$\Gamma \vdash a!c.alc.c?(x : \text{Int}).c?(y : \text{Bool}) : \{a : !(\text{Int.1}) \mid !\text{Bool.1}.1, c : !\text{Bool.1} \mid ?\text{Int.1} \mid ?\text{Int.1}.?\text{Bool.1}\}$$
The process $P$ delegates the channel $c$ twice on $a$. The first time, the delegated behavior is $\texttt{!Int.1}$, while the second time the delegated behavior is $\texttt{!Bool.1}$. Each time $c$ is delegated, $P$ assumes that the receiving process will implement the delegated behavior. However, as it can be clearly seen in the conclusion of the typing derivation above, the overall delegated behavior of $c$ is $\texttt{!Int.1} | \texttt{!Bool.1}$, namely the parallel composition of the two behaviors that were separately delegated. This is fundamental for the completeness of $c$’s type, since the input operations performed by the residual of $P$ at the top of the typing derivation occur in a specific order.

The process $Q$, which receives both delegations, is unaware that $x$ and $y$ will be instantiated with the same channel. So, $Q$ is well typed and $x$ and $y$ have respectively type $\texttt{!Bool.1}$ and $\texttt{!Int.1}$, as requested by $P$, but $Q$ uses these channels in a specific order that is not captured by the projections. The process $P \mid Q$ deadlocks in two steps:

$$P \mid Q \xrightarrow{\tau} c?(x : \texttt{Int}).c?(y : \texttt{Bool}) \mid c! \texttt{true}.c!3$$

where in the final state we have $c : \texttt{!Int.}? \texttt{Int.1} | \texttt{!Int.1}$ and $\theta = \texttt{!Bool.1} | \texttt{!Bool.1}$ which is not complete. By requiring, in the premise of rule (T-INPUTS), that the receiving process cannot use any channel other than the received one, we are basically imposing that the receiving process must handle every received channel in a thread of its own.

In judgments of the form $\Gamma \vdash P : \Delta$ the environment $\Delta$ is an approximation of $P$ insofar as it describes the projections of $P$’s behavior with respect to the channels it uses and delegates. It is well known that this approximation is unable to capture situations where well-typed processes deadlock because the interdependence between communications occurring on different channels are lost. Our approach is no exception, as shown by the following example.

**Example 3.6 (deadlock).** Consider the system

$$(va)(vb)(a!3.b?(x : \texttt{Bool}) \mid b! \texttt{true}.a?(x : \texttt{Int}))$$

where the channels $a$ and $b$ have respectively type $\eta = \texttt{!Int.1} | \texttt{?Int.1}$ and $\theta = \texttt{!Bool.1} | \texttt{!Bool.1}$. In both cases we have $1 + \eta \sqsubseteq \eta$ and $1 + \theta \sqsubseteq \theta$, hence the system is well typed but deadlock.

The safety property we are able to state guarantees that, if all the processes sharing some channel $c$ are immediately ready to communicate on $c$, then they will eventually synchronize. Since in our transition relation for processes synchronization is triggered not just by the channels on which messages are exchanged, but also by the type of the exchanged messages, the eventual synchronization translates to the fact that there is no communication error: it is never the case that there is a process willing to send a message of some type, and no other process is ever willing to receive messages of that particular type. The notion of “readiness” we mentioned is captured by the following definition:

**Definition 3.1 (readiness).** We say that $P$ is ready on $c$ if $P \downarrow c$ is derivable by the rules:

$$\pi.P \downarrow \text{subj}(\pi) \quad \frac{c \notin \text{fn}(P)}{P \downarrow c} \quad \frac{P \downarrow c \quad Q \downarrow c}{P + Q \downarrow c} \quad \frac{P \downarrow c \quad Q \downarrow c}{P \mid Q \downarrow c} \quad \frac{P \downarrow c}{(vd)P \downarrow c} \quad \frac{c \neq d}{P \downarrow c}$$

Intuitively, $P$ is ready on $c$ if either it does not use $c$, in which case it plays no role in any synchronization on $c$, or if $P$ is prefixed by an action whose subject is $c$, or if every branch of $P$ is ready on $c$. Observe that when $P \equiv P_1 + P_2$, both branches are required to be ready on $c$. This is not overly restrictive because, by rule (T-EXT), if either branch is prefixed by an action whose subject is $c$, so must be the other branch.

**Theorem 3.2.** If $\Gamma \vdash P : \Delta \cup \{c : \eta\}$ and $\eta$ complete and $P \downarrow c$, then either $c \notin \text{fn}(P)$ or $P \xrightarrow{\tau}$. 


4 Concluding remarks

It may sound obvious to state that session types are behavioral types. Yet, although session types are normally associated with channels, channels do not expose any behavior. The solution of this apparently innocuous paradox lays in the equally obvious observation that the session type associated with a channel reflects the behavior of a process concerning the input/output operations that the process performs on that channel. By taking this mirrored point of view we have been able to define a simple and, in our opinion, elegant theory of session types that generalizes, unifies, and semantically justifies many concepts that can be found scattered in the current literature: (multi-party) session types are terms of a suitably defined process algebra closely based on value-passing CCS; completeness expresses the property that a session is well-formed and never yields a communication error; duality $\eta \bowtie \theta$ is the special case where $\eta | \theta$ is complete; viability captures the concept of well-typed process, namely of process that can be composed with others in order to implement complete sessions; the subtyping relation between session types arises semantically by relating those session types that preserve completeness in arbitrary contexts.

The adoption of a fair testing semantics [18] for session types is original to the best of our knowledge. In fact, most presentations of session types rely on notions of duality or well-formed composition where the only concern is the absence of communication errors, while the fairness principle we adopt imposes an additional constraint: that at any time a conversation is always able to reach a so-called successful state. Whether or not this is desirable in practice, from a technical point of view there are both pros and cons: on the one hand, the fair subsession relation is more difficult to characterize coinductively and axiomatically because fairness escapes the mere structure of types; on the other hand, the subsession relation is an all-in-one tool that incorporates safe substitutability (rule (T-SUB)), viability, and completeness (Proposition 2.1). We have been unable to fully characterize completeness in terms of a non-fair subsession relation (see [4] for an attempt in the context of behavioral contracts).

The type system we have provided as a proof-of-concept in Section 3 may look excessively restrictive, in particular with respect to the rule (T-INPUTS) which demands that the continuation cannot use any (known) session if not the received one. We have three observations regarding this point: (1) this is a direct consequence of our focus on the idea of projected behavior, which allows a more liberal use of channels; (2) similar restrictions can be found in type systems guaranteeing global progress [9][1][3]; (3) the provided type system is very natural and simple, considering the freedom it leaves in the use of channels; this simplicity suggests that it can be smoothly extended with features such as polarities or roles which would likely help relaxing the constraints. We leave this extension as future work.

References

[1] Lorenzo Bettini, Mario Coppo, Loris D’Antoni, Marco De Luca, Mariangiola Dezani-Ciancaglini & Nobuko Yoshida (2008): Global Progress in Dynamically Interleaved Multiparty Sessions. In: CONCUR’08, LNCS 5201. Springer, pp. 418–433.

[2] Luis Caires & Hugo Vieira (2009): Conversation Types. In: ESOP’09, LNCS 5502. Springer, pp. 285–300.

[3] Giuseppe Castagna, Mariangiola Dezani-Ciancaglini, Elena Giachino & Luca Padovani (2009): Foundations of Session Types. In: PPDP’09. ACM, pp. 219–230.

[4] Giuseppe Castagna & Luca Padovani (2009): Contracts for Mobile Processes. In: CONCUR’09, LNCS 5710. Springer, pp. 211–228.

[5] Sagar Chaki, Sriram K. Rajamani & Jakob Rehof (2002): Types as models: model checking message-passing programs. SIGPLAN Not. 37(1), pp. 45–57.
Acknowledgments. I wish to thank Giuseppe Castagna and Mariangiola Dezani for having provided comments on early versions of this paper. The anonymous referees of the ICE workshop have contributed with invaluable feedback and insight, not only with their reviews but also on the forum associated with the workshop Web site, where they asked several intriguing questions.