Neutron Stars as Dense Matter Laboratories

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Abstract.
I describe how recent observations of precession (nutation) in neutron stars can be used to probe fundamental properties of matter at densities currently inaccessible in terrestrial laboratories.

One of the central questions in nuclear physics is: what is the ground state of matter above nuclear density ($\approx 2.8 \times 10^{14} \text{ g cm}^{-3}$)? Stable matter at supra-nuclear densities is not available in the laboratory. Its properties cannot be calculated from first principles as we must extrapolate the relevant interaction potentials from nucleon phase shift data and the properties of nuclei. The basic interactions to include in a description of dense matter are not known; additional degrees of freedom, such as pions, hyperons and even quarks, are likely to come into play at 2-3 times nuclear density. The situation is fraught with technical difficulties. A truly first-principles approach, computational lattice QCD, is currently not feasible for addressing the fundamental question, and the calculation of the properties of dense matter (with potentials that must be assumed) in the context of mean field theory is a formidable problem.

Whatever the state of matter is above nuclear density, it exists in abundance in the estimated $10^9$ neutron stars in our galaxy. A typical neutron star has a mass of 1.4 times that of the Sun and a radius of $\sim 10$ km. The average density, therefore, exceeds twice nuclear density, and most of the stellar volume exceeds this density. The density at the core could be much higher - five to 10 times nuclear density.

Here I describe how neutron stars can be used as laboratories for studying fundamental particle physics. I will begin with a description of the current understanding of the neutron star interior. I will then describe how observations of neutron star precession (nutation) allow new probes of the interior and challenge the standard picture. I will conclude with a discussion of deformed neutron stars as promising sources of detectable gravitational waves.

1. The Neutron Star Interior

The standard picture of the neutron star interior is depicted in Fig. 1. Beneath an atmosphere is an outer crust consisting of a lattice of heavy nuclei and degenerate, relativistic electrons. Above a density of $\approx 4 \times 10^{11} \text{ g cm}^{-3}$, in the inner crust, neutrons “drip” out of the nuclei and occupy continuum states that coexist with heavy nuclei and relativistic electrons. At about half nuclear density ($\approx 2 \times 10^{14} \text{ g cm}^{-3}$) the nuclei dissolve into a liquid of neutrons, protons, electrons and a small fraction of muons, giving way to the outer core. At a density of about twice nuclear density begins the inner core, of essentially unknown composition. In the inner core, other particles probably appear (e.g., hyperons, pions, kaons and possibly quarks). The
Neutron stars have their name because they are made of mostly neutrons, at least in their outer layers. Though a lone neutron is unstable to beta decay, bulk matter above a density of \( \simeq 4 \times 10^{11} \text{ g cm}^{-3} \) allows inverse beta decay, in which protons and relativistically degenerate electrons combine to form neutrons. The ground state at these high densities is about 95% neutrons by mass, 5% protons and enough electrons (and muons somewhat above nuclear density) to ensure charge neutrality.\(^1\)

A crucial feature of the neutron star interior, originally predicted by Migdal in 1959 [2], is that part or most of it should consist of superfluid neutrons, neutrons that pair into bosons (through Cooper pairing via the strong force) and condense into a macroscopic ground state. Superfluid protons, which by virtue of their charge are also superconducting are also expected. Since superfluidity has been well-studied in atomic nuclei and liquid helium, the basic features of superfluidity in neutron stars are thought to be rather well-understood up to about twice nuclear density. Moreover, superfluidity in neutron stars is expected to have a number of interesting consequences for the cooling (\emph{e.g.}, [3]) and rotational dynamics of neutron stars, and has played a central role in most models of glitches (\emph{e.g.}, [4–7]), sudden spin jumps observed in many neutron stars. Numerous first-principles calculations of the pairing states of nucleons in neutron stars have borne out Migdal’s prediction, cementing superfluidity as a central ingredient in the standard picture of the neutron star interior, however, the details of the pairing - which nucleons

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\(^1\) I should mention the theoretical possibility that what we call neutron stars are not neutron stars at all. Quark matter, stabilized by finite strangeness, might have a lower energy per baryon than \(^{56}\text{Fe} [1]\) in which case proto-neutron stars, after their formation in supernovae, could “burn” to strange stars. There is no compelling observational evidence for this intriguing possibility, and I will assume that neutron stars are just that.
pair, where and how strongly - is far from clear. Consequently, the question of the observational consequences of superfluidity in neutron stars has been an active area of research for over three decades.

In principle, the confrontation between models of the interior with observed neutron star phenomena and extrinsic stellar properties can be used to learn something of the nature of the interior. Hence, in the face of our current inability to answer many questions about the neutron star interior from first principles, many researchers have turned to using observations of neutron star radii, masses, spin variations and cooling behavior to indirectly study the interior. The success of these efforts, however, has been rather limited. For example, the field of neutron star cooling (comparison of cooling simulations with temperature measurements), was developed with the hope of constraining the character of matter at supra-nuclear densities. Instead, many new cooling scenarios have emerged. New kinds of constraints would be invaluable.

Recently there was a breakthrough discovery in neutron star spin dynamics. An otherwise ordinary radio pulsar (radio pulsars are the most common manifestation of neutron stars), PSR B1828-11, whose spin parameters have been monitored since the 1980’s, was realized to be undergoing what appears to be long-period precession [8] (see Fig. 2). PSR B1828-11 shows strongly-periodic timing residuals with highly-correlated changes in pulse duration, as one would expect from a wobbling radio beam. Another radio pulsar, PSR B1642-03, also shows strong evidence for precession in the form of highly-periodic timing residuals, though clear changes in pulse duration have not been identified [9]. Many other pulsars show quasi-periodic timing residuals over times scales of months to years [10], more evidence for possible precession. As I describe below, long-period precession, if confirmed, provides a new kind of probe of the neutron star interior.

2. Precession of the Neutron Star Crust
Precession is a mode of a rotating rigid body. As an example, suppose a biaxial, rigid body of moment of inertia $I$ is slightly oblate by an amount $\epsilon = \Delta I/I$ about its major principal axis of inertia. If it is then set rotating about any axis other than a principal axis, it will precess with a period $p_{\text{prec}} \simeq \epsilon^{-1}p$, where $p$ is the spin period. The motion is a superposition of a fast wobble about the angular momentum axis at period $\simeq p$, and a slow (retrograde) roll about the symmetry axis with period $p_{\text{prec}} (\gg p)$. If the object emits a beam, which sweeps past an observer as in a radio pulsar, the observer will see periodic modulation of the pulse arrival times, correlated with variations in the pulse duration. Fig. 2 shows 13 years of timing data.

**Figure 2.** Evidence for precession in PSR B1828-11 [8]. Highly-periodic period residuals (top panel) are seen with correlated changes in beam width (bottom panel). The curve in the top panel is a fit from the theoretical model described in the text.
from PSR B1828-11. The variations in pulsar period are highly periodic, though non-sinusoidal; the data are well-fit by a sum of sinusoids with a fundamental period of 511 d and a harmonic at 256 d. The reason this star is believed to be precessing is that the observed changes in pulse duration are highly-correlated with the timing data, and with exactly the same Fourier content.

It is important to interpret these data carefully. Do these data represent precession, or something else? To be more quantitative, consider a simple model in which the star is approximated as a deformed, rigid body. Though most of the interior is a liquid, over the long time scales of precession (~1 yr) the liquid is expected to corotate with the crust. This model can provide good fits to the timing data of PSR 1828-11 for many possible triaxial figures [11, 12]. An example fit which gives good agreement with the timing data is shown as the curve in the top panel of Fig. 2. This example solution has a wobble angle of ~ 3° (the characteristic angle between the star's symmetry axis and the angular momentum), consistent with the observed beam width variations of similar magnitude. That a simple, physically-motivated model can reproduce the main features of the timing data of PSR B1828-11 makes the precession interpretation compelling. Assuming we are, in fact, observing precession, then the deformation of the star inferred from the precession period is \(\epsilon \approx 10^{-8}\), which corresponds to a "mountain" on the star of height ~ 0.1 mm. It is an extraordinary accomplishment in observational astronomy to measure the deformation of a neutron star to this degree of precision.

As far as we can tell from surveys of radio pulsars, precession in isolated neutron stars is a rare phenomenon. Precession, once excited, would eventually damp through internal friction. The rareness of precession could mean that it is rarely excited in isolated neutron stars.

The picture I have presented so far treats a neutron star as a classical rigid body, and does not account for the fact that most of the stellar interior constitutes a quantum liquid. What are the implications for the neutron star interior? I describe below how precession can be used to constrain the properties of the ground state of nuclear matter in the outer core, in particular, the possible states of hadronic superfluidity. First, I briefly review the current theoretical understanding of superfluidity and superconductivity in neutron stars.

3. Superfluidity and Superconductivity

**Nucleon pairing.** Nucleons can pair in various parts of a neutron star through the strong interaction, leading to superfluidity/superconductivity. Neutron superfluidity in the inner crust has been well-established by reliable first-principles calculations below nuclear density [15–17]. Beneath the inner crust, in the outer core, pairing calculations indicate that the protons pair to form a superconductor, but the pairing situation for neutrons is not so clear. Most calculations indicate moderately strong neutron superfluidity in the outer core [15,18–24], but there are also calculations which indicate no neutron superfluidity there at all, e.g., [14]. The difficulty lies in treating the spin-dependence of the strong interaction. It seems very likely that protons pair in the outer core in a spin zero state. This case is relatively straightforward to calculate. The neutrons, however, if they pair, do so in an \(s = 1\) state (the nuclear force becomes repulsive for neutron pairs in the \(s = 0\) state at high density). The pairing calculations for this pairing channel are not yet reliable. Above a density of \(\sim 1.7\) times nuclear density, near the entrance to the inner core, the pairing situation is essentially unknown [23] (and so is the composition).

**Superfluid rotation.** Any rotating superfluid is threaded by an array of quantized vortex lines, the distribution of which determines the superfluid's total angular momentum [25]. In a typical neutron star, the neutron vortices of the inner crust or outer core (if the neutrons are indeed superfluid there) have characteristic dimensions of 10 fm (1 fm = 10^{-13} cm) and are separated by about 0.1 mm in a triangular array. Superfluid vortices are persistent hydrodynamic structures. Their properties and dynamics have been well studied in laboratory liquid helium. In the inner core, the neutron vortices are likely to pin to the nuclei through an attractive interaction [26–28]. The pinning exists because it is energetically favorable for a vortex and a
nucleus to overlap, by an energy of order an MeV per overlap. Vortex pinning in the crust has played a central role in most models of neutron star spin jumps, or glitches \((\text{e.g.,}[4-7])\). As I describe below, pinning is generally inconsistent with long-period precession.

**Core magnetic field structure.** Neutron stars have typical magnetic fields of \(\sim 10^{12} \text{G}\). How the star accommodates the field depends on the proton pairing state in the core. The protons of the outer core are expected to pair, but the type of superconductivity, which determines the field configuration, is uncertain. Superconducting protons rotate nearly as a rigid body and accommodate the magnetic field \(\mathbf{B}\) either as a type I or type II superconductor without forming vortices [29]. In the type II case, there is a dense arrangement of flux tubes. In each flux tube \(B \approx 10^{15} \text{G}\), while between the flux tubes \(B = 0\). The flux tubes are essentially one-dimensional structures: long tubes of microscopic radii (\(\sim 50 \text{fm}\)). In the type I case, the field would be arranged in two-dimensional structures - slabs or more complicated configurations of alternating superfluid and normal regions. In the normal regions, \(B \sim 10^{15} \text{G}\), while in the superconducting regions, \(B = 0\). Usually the core is assumed to be a type II superconductor, though it is quite possible it is type I. The standard picture of the outer core consists of a rotating neutron superfluid with vortices which form a nearly rectilinear array, entangled in the far more numerous flux tubes (see Fig 3, left figure). The flux tubes have a highly complicated arrangement that froze in the core when the protons condensed several months after the star’s formation. The neutron vortices of the outer core, which are themselves magnetized through Fermi liquid effects [29], pin to the flux tubes, which are frozen to the superconducting medium [30]. The origin of the pinning is that bringing a (magnetized) vortex...
close to a flux tube raises (or lowers, depending on orientation) the magnetic energy by $\sim 5$ MeV per intersection.

4. Constraining Precession Physics and Stellar Properties

The existence of vortices in a neutron star has important consequences for how a star would precess. A vortex array responds to torques exactly as does a gyroscope. As noted by Shaham [31], for a neutron star to precess slowly, the vortices in the crust must be free to follow the instantaneous rotation axis of the crust (plus any parts of the star tightly coupled to it); if instead the vortices of the inner crust are immobilized by pinning, the gyroscopic nature of the pinned vortex array drives the star to precess at very high frequency (the precession period in PSR B1828-11 would be less than 100 s). Hence, in any star that is precessing with a long period (hundreds of days), there cannot be significant pinning in the crust. Vortices could still pin in stars that are not precessing. Similar considerations apply to the core.

Constraining the inner-crust fluid. Precession gives a direct constraint on vortex/nuclei interactions in the inner crust. In particular, precession of PSR B1828-11 with a wobble angle of $\sim 1^\circ$ would make vortex pinning in the inner crust unstable [32]. Suppose for a moment that vortices are pinned to the crust and cannot unpin. As the star precesses, the superfluid flow past pinned vortices exerts a lift force - the Magnus force. The Magnus force that the vortices exert against the crustal lattice is what drives the precession at high frequency. The Magnus forces in a precessing neutron star, however, are strong enough to pull the vortices off of the nuclei to which they are pinned, so the pinned state cannot persist as long as the star is precessing. This fact alone does not guarantee that long-period precession is possible. The vortices must also be able to move with sufficiently little dissipation through the inner crust to be able to follow the angular velocity vector of the crust; otherwise the vortices behave almost as if pinned, and the star will precess at high frequency [33]. The theoretical case for neutron pairing and the required existence of vortices in the inner crust is quite strong, and so it seems necessary to conclude that, in PSR B1828-11, the vortices of the inner crust are unpinned and moving with little dissipation. The vortices are kept unpinned by the forces exerted on them as the star precesses. Exactly how vortices may move with respect to nuclei with little dissipation merits further study.

Constraining the core fluid. Long-period precession presents a similar but more serious problem in understanding the outer core. Here again, the neutron vortices are thought to pin, but now to a tangled arrangement of type II proton flux tubes (Fig. 3, left figure). It is extremely difficult to move the flux tubes through the superconducting core; the core is a perfect conductor between the flux tubes, and any movement of the flux must occur through dissipation occurring in the flux tube cores, where the protons are normal. The flux tube cores occupy a very small fraction of the volume of the fluid, the dissipation rate per volume is therefore low, and the local flux is nearly constant. Consequently, over the time scale of PSR 1828-11’s putative precession, the flux tubes are essentially immobile against vortices being driven against them by Magnus forces [30]. The charged components of the star, the protons and electrons, to which the crust is frozen by magnetic stresses, act effectively as a single body to which is affixed an enormous gyroscope (the vortex array). In this configuration of magnetic flux and fluid vorticity, precession would be $\sim 10^7$ times faster than observed [13]. Hence, if the precession interpretation of PSR B1828-11 is correct, the neutron vortices and proton flux tubes of the core cannot coexist anywhere. In principle, the same hydrodynamic forces that would unpin vortices in the crust could drive vortices through flux tubes in the core, but this process is so dissipative that the precession would be highly over-damped. After less than an hour, the precession would damp to the extent that vortices repin against flux tubes, and the precession would become very fast and of low amplitude ($<< 1^\circ$). For this problem to be evaded, the vortices must be able to follow the instantaneous spin vector of the rest of the star, which requires them to cut through
the flux tubes at a speed of $\sim 10^{-2}$ cm s$^{-1}$ with little dissipation. But for the cutting to happen, the differential velocity of the superfluid past the pinned vortices must be $\sim 10^7$ cm s$^{-1}$. At this differential velocity, the Magnus force against the flux tubes is so large that the precession will be very fast (comparable to the star’s spin frequency).

We have now a second constraint on the neutron star interior: the standard picture of the outer core (Fig. 3, left figure), is incorrect. How might this difficulty be resolved? One possibility is that the outer core is a type I superconductor, rather than type II as is usually assumed (Fig. 3, middle figure). In this case the neutron vortices would be able to move through the two-dimensional magnetic field structures without the pinning impediment they would suffer if the protons were in the type II state. The dissipation rate associated with this motion is low enough to allow long-period, weakly-damped precession [34]. Another possibility (Fig. 3, right figure) is that the outer core is superconducting (either type I or type II), but the neutrons are normal. As mentioned above, the theoretical case for outer core superconductivity is strong, but the question of the existence of neutron superfluidity is rather less certain. Interestingly, recent cooling calculations, in confrontation with surface temperature measurements of neutron stars, favor superfluid protons in the outer core and normal neutrons [3]. The constraint imposed by precession, in light of these considerations, suggests an outer core consisting of normal neutrons. A third, extreme possibility, is that the neutron and protons of the outer core are both normal. This possibility seems unlikely, as the conclusion from pairing calculations that the protons pair is on firm footing. This possibility should nevertheless be kept in mind.

Constraining the material properties of the crust. Precession also allows constraints on the poorly-known material properties of the inner crust. Using data from PSR B1828-11, a lower limit on the strength of the neutron star crust can be obtained [35]. While the shear modulus of the crust has been calculated from first principles, the strain angle at which the crust yields to stress is unknown. (Even in terrestrial solids, this parameter is determined experimentally). The crust must be strong enough to support sufficient deformation to precess with a period of $\sim 1$ yr; to satisfy this requirement, the critical strain angle must exceed $\sim 10^{-5}$. This number is well below the theoretical maximum of $\sim 0.1$, indicating that the crust is likely strong enough to sustain the required deformation. This result has interesting implications for neutron stars as gravitational wave sources.

5. Gravitational Waves from Neutron Stars
The LIGO experiment is an interferometer designed to detect high-frequency gravitational waves from astrophysical sources (such as supernovae, gamma-ray bursts, coalescing binaries). A rotating neutron star with a non-zero mass quadrupole moment would also be a gravitational wave source. The strain induced in a detector by a passing gravitational wave is proportional to $\epsilon \omega^2 / d$, where $\epsilon$ is the fractional (quadrupolar) deformation of the star, $\omega$ is the spin rate and $d$ is the distance. Hence, the best candidate neutron stars for detectable gravitational waves are rapidly rotating, close and highly deformed. The “millisecond pulsars” are very promising in this respect. Millisecond pulsars are neutron stars that have been spun up to rotation rates of nearly a kilohertz by accretion torques from a companion star. Many are relatively close-by, at distances of about $3 \times 10^4$ light-years. In the future, LIGO will be tunable to have high sensitivity at the frequencies at which typical millisecond pulsar would emit gravitational waves (twice the spin frequency).

If the precession interpretation of PSR 1828-11 is correct, then we know of at least one neutron star that can support a deformation of $\epsilon \sim 10^{-8}$. If some millisecond pulsars are deformed to this extent, they might be detectable by LIGO-II [36]. (These stars need not precess. Incidentally, PSR 1828-11 spins too slowly to be a detectable source). There is no reason to expect $\epsilon \sim 10^{-8}$ to represent an upper limit. If millisecond pulsars can support quadrupolar deformations of $\epsilon \sim 10^{-6}$ or larger, they could be among the strongest sources of gravitational waves in the sky.
6. Conclusions
Precession of neutron stars provides a new probe of the properties of matter at supra-nuclear densities. In particular, the pairing states of the outer core can be constrained. I have argued that long-period precession is inconsistent with the standard picture of an outer core consisting of coexisting superfluid neutrons and type II, superconducting protons. Precession data might be telling us that the outer core protons are paired in a type I state, or that both the neutron and proton fluids are normal there. These constraints are robust provided that precession in neutron stars is real. They do not depend on the state of matter in the inner core.

The precession interpretation of PSR B1828-11 requires the star to be deformed to $\epsilon \simeq 10^{-8}$. If millisecond pulsars can support similar or larger quadrupolar deformations, they would be interesting gravitational wave sources for LIGO-II.

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