The Pentaquark Candidates in the Dynamical Diquark Picture

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The recent observation by LHCb [1] of prominent exotic structures \( P_c^+ (4380) \) and \( P_c^+ (4450) \) in the \( J/\psi p \) spectrum of \( \Lambda_c \rightarrow J/\psi K^- p \) has rekindled hopes that the long-sought pentaquark states have finally been observed. The reported properties are \( m_1 = 4380 \pm 8 \pm 29 \text{ MeV} \), \( \Gamma_1 = 205 \pm 18 \pm 86 \text{ MeV} \) (at 9 standard deviations) and \( m_2 = 4449.8 \pm 1.7 \pm 2.5 \text{ MeV} \), \( \Gamma_2 = 39 \pm 5 \pm 19 \text{ MeV} \) (at 12 standard deviations), respectively, while the preferred \( J^P \) assignments are correlated, with the most likely combinations in decreasing order being \( (3^-, \frac{5}{2}^+) \), \( (3^+, \frac{5}{2}^-) \), and \( (\frac{5}{2}^+, \frac{3}{2}^-) \).

Should at least one of the states be confirmed by another experiment, it will join the famed tetraquarks—whose best-studied member [the \( J^{PC} = 1^{++} \, X(3872) \)] was discovered over a decade ago [2]—as a second class of exotic hadrons. Since the valence structure of \( J/\psi p \) is \( c\bar{c}ud \), the minimal quark content of such states is that of a pentaquark.

We note several interesting phenomenological facts. First, the two charged five-quark hidden-charm states \( P_c^+ (4380) \) and \( P_c^+ (4450) \) observed at LHCb are both lighter than the four-quark state \( Z^- (4475) \) [formerly \( Z^- (4340) \)] observed by the same group [3]. Since pentaquarks carry baryon number and therefore must have a baryon in their decay products, while tetraquarks are bosons and therefore can decay entirely to (generically lighter) mesons, less phase space is typically available to \( P_c^+ \) decays. In particular, \( Z^- (4475) \) decays dominantly to \( \psi (2S) \pi^- \), even though plenty of phase space is available for the \( J/\psi \pi^- \) mode, while neither of the \( P_c^+ \) states is kinematically allowed to decay to \( \psi (2S) p \).

Second, both of the \( P_c^+ \) states lie well above the thresholds for decay into \( \Lambda_c^+ D^{(*)0} \), which also has the valence-quark structure \( c\bar{c}ud \). Since these two-body decays are not forbidden by any obvious quantum number, the relatively small \( P_c^+ \) widths suggest interesting internal structure sufficient to suppress this immediate rearrangement, as well as that into \( J/\psi p \). The \( P_c^+ (4450) \) lies only about 10 MeV below the \( \Sigma_c^+ D^{(*)0} \) threshold, which can be used to argue for a molecular interpretation [4, 5]. Alternate molecular assignments for the \( P_c^+ \) states are presented in Refs. [6–9], and the origin of the \( P_c^+ (4450) \) as a threshold rescattering effect via \( \chi_{cJ} p \) is discussed in Ref. [10], and through additional channels in Ref. [11].

Third, all of the preferred fits from LHCb demand that the two \( P_c^+ \) states carry opposite parities. A system of four quarks and one antiquark in a relative \( S \) wave has negative parity, while positive parity requires the introduction of at least one unit of relative orbital angular momentum (\( P \) wave or higher). And yet, the two \( P_c^+ \) states are separated by only \( m_2 - m_1 = 70 \text{ MeV} \). One may argue that the figure of merit, as would be the case for Regge trajectories, should be \( m_2^2 - m_1^2 = (790 \text{ MeV})^2 \), which is a much more natural hadronic scale; however, attempting to discern a trajectory when only two points are available seems absurdly premature. Still, if the \( J^{PC} = 1^{--} \) state \( Y(4008) \) seen by Belle with a mass \( 3891 \pm 42 \text{ MeV} \) [12] is confirmed, then small mass splittings between hidden-charm exotics of opposite parities [i.e., only 20 MeV from the \( X(3872) \)]—and hence relative orbital excitations—will not appear unusual.

In this short paper, we propose a structure for the \( P_c^+ \) states based upon a mechanism recently proposed for understanding the tetraquark states [13], wherein a diquark \( \delta \) and an antidiquark \( \bar{\delta} \) form in the attractive color-3 and color-\( 3 \) representations, respectively. This configuration is prevented from instantly reorganizing into two \( q\bar{q} \) pairs because the diquarks are forced to separate rapidly due to the large energy release in the production mechanism (e.g., a \( B \)-meson decay via \( b \rightarrow c\bar{c}s \)). However, since the diquarks are colored, they are confined and hence cannot separate indefinitely; kinetic energy is converted into the potential energy of a color flux tube connecting them. Hadronization finally occurs through the overlap of the long-distance tails of hadronic wave functions that stretch from the quarks in \( \delta \) to the antiquarks in \( \bar{\delta} \). This picture was used to explain, for example, the preference for \( Z^- (4475) \rightarrow \psi (2S) \pi^- \) over \( Z^- (4475) \rightarrow J/\psi \pi^- \), simply due to the final spatial separation of the \( c \) in \( \delta \) and

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the $\bar{c}$ in $\delta$ allowing a much greater wave function overlap with the larger $\psi(2S)$ than with the more compact $J/\psi$.

We argued implicitly in Ref. [13], and explicitly in a subsequent paper Ref. [14], that the diquarks formed from one heavy and one light quark are somewhat smaller than diquarks formed from two light quarks, and that such heavy-light ($Qq$) diquarks are not expected to be much larger than the meson ($Q\bar{q}$) formed from the same flavors. In yet another subsequent paper Ref. [15], we proposed that the preferential coupling of $3 \otimes 3 \rightarrow 3$ (and its conjugate) does not end with diquarks, but can continue sequentially to more complex structures like pentaquarks and even octoquarks. These ideas will be used to develop the pentaquark picture described in detail below. As in Ref. [13], we use the term “picture” because many distinct models could be constructed that support this dynamics, and want to emphasize that this discussion is not limited to any particular choice of potential or wave functions, for example.

The concept that pentaquarks might be formed from two compact colored constituents rather than molecules of mesons was first expressed in Ref. [10], which sought to describe the $\Theta^+(1535)$ pentaquark candidate $u\bar{s}udd$ as a molecule of a $(ud)$ diquark in a color-$3$ and a $(\bar{s}ud)$ “triquark” (a term coined in that paper) formed from a $(ud)$ diquark in a color-$6$ coupled to the $\bar{s}$ quark into an overall color-$3$. The separation of the two components of the molecule is stabilized by the centrifugal barrier introduced by relative orbital angular momentum $\ell = 1$, which as discussed above was necessary should the $\Theta^+$ have been found to carry positive parity. In comparison, the famous $\Theta^+$ pentaquark model of Ref. [17] proposed a structure of two light $(ud)$ diquarks and one exceptional $(\bar{s})$ quark. BothRefs. [10] and [17], make use of diquarks consisting of light quarks in the “good” (spin-0) combinations, so named because they are believed to be more tightly bound than the “bad” (spin-1) combination through hyperfine couplings (although in Ref. [10] the diquark inside the triquark has spin 1). In the case of heavy-light diquarks, the hyperfine couplings are proportional to $1/m_q$, and therefore the mass difference between “good” and “bad” is greatly reduced. Alternate compositions through colored components have been very recently discussed in Refs. [18, 19].

This paper is organized as follows: In Sec. IV we discuss the diquark picture used to describe the exotic states. Section V presents the diquark picture as relevant to the production of the $P^+_c$ states. In Sec. VI we present the basic phenomenology for the $P^+_c$ states provided by this picture, and in Sec. VII we summarize.

II. GENERALIZING THE DIQUARK PICTURE

The color algebra of QCD provides more than one way to obtain attractive channels between quarks. The combination $3 \otimes 3 \rightarrow 1$, which explains the strong binding of $q\bar{q}$ pairs in conventional mesons, is of course extremely well known. However, one other binary combination is strongly binding, the channel $3 \otimes 3 \rightarrow 3$. To quantify the effect, note that the coupling of two colored objects in the irreducible representations $R_1$ and $R_2$ is computed by the same techniques as one computes the spin coupling between objects carrying spins $S_1$ and $S_2$ combining to total spin $S = |S_1 + S_2|$, via the trick

$$S_1 \cdot S_2 = \frac{1}{2} \left[ (S_1 + S_2)^2 - S_1^2 - S_2^2 \right].$$

(1)

One generalizes to an arbitrary Lie algebra by computing the combination of quadratic Casimirs

$$g_{1 \times 2} \equiv C_2(R) - C_2(R_1) - C_2(R_2).$$

(2)

Considering all binary combinations $3 \otimes 3 = 3 \oplus 6$ and $3 \otimes 3 = 1 \oplus 8$, one computes the relative strengths

$$g_{1 \times 2} = \frac{1}{3} (-8, -4, +2, +1) \text{ for } R = (1, 3, 6, 8).$$

(3)

The diquark attractive $3$ channel $\delta$ is therefore fully half as strong as that of the singlet $q\bar{q}$ channel. In a multiquark system in which two quarks or two antiquarks happen to lie in closer proximity than to one of their antiparticles, the diquark $\delta$ (antidiquark $\bar{\delta}$) attraction is naturally expected to dominate. Unless stronger color forces intervene (e.g., the production of nearby $q\bar{q}$’s, which would create available color-singlet combinations), the $\delta$ and $\bar{\delta}$ combinations can be expected to form quasi-bound, but colored and therefore confined, states.

In Ref. [13], the diquarks containing a charm and a light quark were crudely estimated to have a comparable size to a $D$ meson, roughly $\langle r \rangle \lesssim 0.5$ fm. The subsequent paper Ref. [14] argued that diquarks formed with a heavy quark should be somewhat smaller than those formed from two light quarks: A heavier quark is more localized in space, while each lighter quark has a more diffuse wave function. A key phenomenological question in identifying whether diquarks have formed is whether or not any antiquarks appear within this radius.

This hypothesis was used in Ref. [15] to suggest a means by which multiquark exotics could be produced, particularly at threshold, where the limited phase space allows the soft heavy quark pairs such as $c\bar{c}$ to coalesce with light valence quarks moving at similar rapidities. Such multiquark states can be formed through a sequence of two-body bound-state clusters of color-$3$ diquark and color-$3$ antidiquark states. In the absence of easy opportunities for the formation of color singlets, sequential diquark formation provides the strongest channels for binding. For example, anti-de Sitter/QCD models on the light front have a universal confining potential that confirms the importance of diquarks in hadron spectroscopy. While the examples given in Ref. [15] describe literal clusters of diquarks such as a charmed, charge $Q = 4$, baryon-number $B = 2$ state $[uu]_{3C}[cu]_{3C}[uu]_{3C}$, another route of sequential color-triplet (antitriplet) formation is available, which is the hypothesis of this paper:
A pre-existing diquark $\delta'$ that subsequently encounters an antiquark $Q$ forms a bound antitriquark $\theta \equiv (Q\delta')$ via the attractive color coupling $3\Theta 3 \to 3$. This mechanism, as discussed in the next section, provides a completely analogous production channel for pentaquark states to that described for tetraquark states in Ref. [13].

To say that two quarks or antiquarks encountering one another combine only into the most attractive channel is of course a great simplification. First, the color coupling factors apply without reservations only when fundamental QCD interactions dominate the interaction. Longer-distance effects dress the interactions and can obfuscate this simple result. In reality, one expects a type of thermodynamic ensemble of states in various color combinations, in which the levels at the lowest energies are driven by diquark binding. The existence of such an ensemble assumes that the formation of overall color singlets is precluded due to the presence of a potential energy barrier, such as from a large spatial separation between the quarks needed to form the singlets. In that case, the eventual hadronization can be considered as a tunneling process. Second, the Pauli exclusion principle must be taken into account if the purported diquarks contain quarks of identical flavor, since then the flavor wave function must be antisymmetric. In the specific example discussed below, this constraint is not an issue, but it must be kept in mind for other cases.

III. PENTAQUARK PRODUCTION MECHANISM

We propose that the states $P_\delta^+$ observed at LHCb are pentaquarks consisting of a confined but rapidly separating pair of a color-3 diquark $\delta = (cu)$ and a color-3 antitriquark $\theta = \bar{u}(ud)$, in which the $(ud)$ subsystem of $\theta$ is a color-3 diquark $\delta'$, as depicted in Fig. 1.

This picture is completely analogous to (the charge conjugate of) that for the $Z^+ (4475)$ presented in Ref. [13], except that the diquark $\delta' = (ud)$ in Fig. 1 is replaced by the single quark $\bar{d}$. The parent hadron for the $P_\delta^+$ is the $\Lambda_b$ baryon, while the parent hadron for the $Z_c^+$ is the $B^0_s$. In either case, the composite state is not a molecule in the traditional sense of the word, because it lasts only as long as the $\delta-\theta$ pair (or $\delta-\bar{\delta}$ in Ref. [13]) possesses positive kinetic energy to continue separating. The colored $\delta$ and $\bar{\delta}$ constituents create a color flux tube between them, losing their energy to the color field and slowing down. For this picture to be physically meaningful, the $\delta-\bar{\delta}$ pair must be sufficiently compact that their wave function overlap becomes insignificant. As in the standard Wentzel-Kramers-Brillouin approximation, the probability of a transition—in this case, hadronization—increases as the components approach the classical turning point.

Let us emphasize the differences between this and other pictures for the pentaquark. First, it is clearly different from the hadronic molecule picture, in which the constituent baryon and meson are color singlets treated as forming a static molecule and are held together by weak color van der Waals forces. It is also different from the diquark-triquark model of Ref. [13], not only because the diquarks are always assumed here to form color triplets, but also because the diquark-triquark state in that case is again a static molecule, stabilized by a centrifugal barrier; in our picture, such states are expected to last only as long as the components continue to separate, and also would exist in $S$ waves as well as higher partial waves. The recent work of Ref. [19] describes the $P_\delta^+$ states as being formed of diquarks, via the composition $c\bar{u}(cq)\bar{3}(q'q'')\bar{3}$; while the color structure is the same as ours, we emphasize the importance of the $c$ belonging to a compact component of the overall state.

The new ingredient for the $P_\delta^+$ as compared to the $Z^+ (4475)$ is the use of the intrinsic diquark $\delta' = (ud)$ originating in the $\Lambda_b$. Note from Fig. 1 that $\delta'$ acts as a spectator in the $P_\delta^+$ production process. $\Lambda_Q$ baryons have always had a special place in the history of diquark models, because these baryons by definition are isosinglets, and since the heavy quarks $Q = s, c, b$ are also isosinglets, the remaining quark pair $(ud)$ also forms an isosinglet, with a wave function that is antisymmetric under flavor exchange. Since $Q$ is a color-3, $(ud)$ is a color 3, again an antisymmetric combination. The Pauli principle therefore demands an antisymmetric spin wave function for $(ud)$. Since both the ground-state $\Lambda_Q$ baryons and the heavy quarks $Q$ have $J^P = \frac{1}{2}^+$, the $(ud)$ is therefore expected to live in the antisymmetric spin-0 combination. The $(ud)$ pair in $\Lambda_Q$ baryons is frequently termed a diquark, and indeed it has exactly the color structure
we want; it differs from the ones we have previously discussed only by consisting solely of light quarks, and is in the “good” diquark combination only.

As mentioned above, one may expect the \((ud)\) diquark to be slightly larger than the heavy-light diquarks, but even so, its binding to the heavy quark \(b\) in \(\Lambda_b\) and \(c\) in \(\bar{\theta}\) restricts the full spatial extent of the wave function. For example, using heavy-quark symmetry and a variational approach, Ref. [21] calculate root-mean square matter radii for \(\Lambda_b\) and \(\Lambda_c\) to be no more than 0.22 fm and 0.31 fm, respectively. Treating the \(\bar{\theta}\) antitriquark as a “would-be” \(\Lambda_c\) baryon (i.e., differing by \(c \leftrightarrow \bar{c}\) but otherwise bound by essentially the same nonperturbative physics), one expects \(\bar{\theta}\) to be not much larger than \(\Lambda_c\).

In principle, the \(u\) quark created from a gluon can mix with the one in the \(\Lambda_c\) diquark. However, inasmuch as this initial \((ud)\) diquark is expected to be fairly tightly bound, one expects it to propagate as an undisturbed spectator quasiparticle through the process; otherwise, the most likely outcome would be a dissociation of the diquark, leading to a different intermediate state than described in Fig. 1.

In the case of the observed decay \(P_c^+ \rightarrow J/\psi p\), the decay rate is suppressed due to the final separation of the \(c\) quark (in \(\delta\)) and \(\bar{c}\) quark (in \(\bar{\theta}\)) compared to the typical size of the \(J/\psi\) wave function, and to a lesser extent due to the separation of the \((ud)\) diquark \(\delta'\) in \(\bar{\theta}\) from the \(u\) quark in \(\delta\) compared to the typical size \(\langle r_p \rangle \simeq 0.88\) fm of the proton wave function.

IV. PHENOMENOLOGY OF THE \(P_c^+\) STATES

The first interesting point one notes from Fig. 1 is the peripheral role played in the process by the \(u\bar{u}\) pair created by gluodynamics. Certainly, creation instead of a \(d\bar{d}\) pair would give a nearly identical scenario. One therefore predicts isodoublet partners \(P_c^0\) to be produced via \(\Lambda_b \rightarrow P_c^0 \bar{K}^0 \rightarrow J/\psi n \bar{K}^0\) at masses just a few MeV higher (from \(u \rightarrow d\)) than those for the \(P_c^+\) states. That the \(P_c\) states should form isospin doubles is of course guaranteed by \(P_c^+ \rightarrow J/\psi p\) being a strong decay and hence conserving isospin, while \(I_{J/\psi} = 0\) and \(I_p = 1/2\); nevertheless, it is useful to see the charge symmetry process explicitly in the context of a particular decay mechanism.

We now obtain a crude estimate of the separation of the diquark and antitriquark using the same technique as in Ref. [13]: Since the two components transform as a color-(3,3) pair, one may describe them using the well-known linear-plus-Coulomb “Cornell” nonrelativistic potential [22]. In the most thorough recent analysis [23], the central part of the potential for \(cc\) systems is given by

\[
V(r) = \frac{4}{3} \frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_c^2} \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma r} S_c \cdot S_{\bar{c}},
\]

with \(\alpha_s = 0.5461\), \(b = 0.1425\) GeV\(^2\), \(m_c = 1.4797\) GeV, and \(\sigma = 1.0946\) GeV. The \(-4/3\) color factor is the same one as in Eq. (3), and \(c(\bar{c})\) now refer to the components containing these quarks, in our case \(\delta\) and \(\bar{\theta}\), respectively.

The calculation of Ref. [13] further exploited the fact that at least one of the components \(\delta\) and \(\bar{\theta}\) is in a state of zero spin for each state of interest \([X(3872)\) and \(Z^+ (4475)\)], so that \(S_c \cdot S_{\bar{c}} = 0\), and that both are expected to have all quarks in a relative \(S\) wave, so that noncentral contributions to \(V(r)\) are not needed. In the case of the \(P_c^+\) states, we have seen that at least one of them must have an orbital excitation, so this calculation strictly applies only to the \(S\)-wave state. Furthermore, the antitriquark \(\bar{\theta}\) necessarily carries half-integer spin; nevertheless, the \(S_c \cdot S_{\bar{c}}\) term has little effect on \(V(r)\) except at the smallest values of \(r\), so we neglect it here. Lastly, we use the QCD sum-rule based estimate [24] \(m_\delta = 1.860\) MeV (note its nearness to the \(D^0\) mass 1.865 GeV) and the antitriquark mass estimate \(m_\bar{\theta} = 2.286\) GeV. Using these assumptions, the diquark-antitriquark separations \(R\) obtained from Eq. (4) are

\[
\begin{align*}
R &= 0.64\text{ fm for } P_c^+(4380),
R &= 0.70\text{ fm for } P_c^+(4450).
\end{align*}
\]

These distances are not especially large for light hadronic systems. However, inasmuch as diquarks, and especially triquarks, containing heavy quarks may be rather smaller as discussed above, these components may be considered as well separated for the purpose of computing quantum-mechanical wave function overlaps. This separation, particularly of the \(c\) and \(\bar{c}\) quark, explains the suppressed decay rate to \(J/\psi\), since the potential Eq. (1) gives \(\langle r_{J/\psi} \rangle = 0.39\) fm. The similarly small size for the \(\Lambda_c\) also predicts slow transitions to \(\Lambda_c^+ D^{(*)0}\), and hence overall widths that are suppressed compared to naive expectations.

Finally, let us consider the quantum numbers of the allowed states in this picture. Orbital excitations can occur not only along the flux tube, but within the diquark and antitriquark as well. Nevertheless, let us for simplicity ignore the latter. Inasmuch as the diquark \(\delta'\) inherited from the \(\Lambda_b\) has spin zero, the set of allowed quantum numbers is even simpler, since then the spin of \(\bar{\theta}\) is \(\frac{1}{2}\). For \(S\) waves, one has the \(J^P\) possibilities

\[
\left(\frac{1}{2} \otimes 0^+ \otimes \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} \right) = \left\{ \begin{array}{c} \frac{1}{2} \\ \frac{3}{2} \end{array} \right\}
\]

and for \(P\) waves, one has

\[
\left(\frac{1}{2} \otimes 1^- \otimes \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} \right) = \left\{ \begin{array}{c} \frac{1}{2} \\ \frac{3}{2} \end{array} \right\}
\]

where the three numbers on the left-hand side are the spin of the \((\bar{c}(ud))\) antitriquark, the orbital excitation, and the spin of the \((cu)\) diquark.

In this simplified picture, we find only one state with \(J = \frac{5}{2}^-\), namely, the \(P\)-wave \(\frac{5}{2}^+\). It is natural to identify this state with the higher-mass \(P_c^+(4450)\), since it
has a narrower width that can be explained by the near-threshold phase-space suppression of $P$ waves. Then the broader $P_C^+$ (4380) must have $J^P = \frac{3}{2}^-$ and lie in an $S$ wave.

Note that both of these states have the (cu) diquark in a spin-1 configuration. It is an interesting phenomenological fact of the tetraquark sector that no $J^P = 0^+$ state has yet been confirmed; in the context of the diquark-antidiquark picture, the simplest such states would have both diquarks in $S = 0$ combinations, with $L = 0$ in the color flux tube as well. It is plausible that such states are much broader due to the absence of any angular momentum barriers impeding rapid decays. Likewise, the lower-spin states in Eqs. (6) and (7) might be more difficult to discern experimentally. In any case, the discovery of two new states and the possibility of numerous others left to find will certainly spur on further experimental examination.

V. CONCLUSIONS

We have seen that the recently observed charmoniumlike pentaquark candidates $P_C^+$ may have a common dynamical origin with the charmoniumlike tetraquark states. Both are proposed to occur as systems of rapidly separating color-3 and -3 component pairs, and in particular in the $P_C^+$ pentaquarks through the sequential preferential formation of color-triplet combinations $\bar{c}(ud)_3(cu)_3$. The diquark (cu) and antitriquark $\bar{c}(ud)$ achieve a substantial separation before hadronization must occur, providing a qualitative explanation for the suppression of the measured widths compared to available phase space. The $P_C^+$ states in this picture form isospin doublets with neutral, as-yet undiscovered partners. States with the observed $J^P$ quantum numbers can easily be accommodated in this scheme, and suggest the potential for discovery of numerous additional related states in the future.

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