Grand unification models from SO(32) heterotic string

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Grand unification groups (GUTs) are constructed from SO(32) heterotic string via $\mathbb{Z}_{12-I}$ orbifold compactification. This invites the SO(32) heterotic string very useful for future phenomenological studies. Here, spontaneous symmetry breaking by Higgsing is achieved by the anti-symmetric tensor representations of $SU(N)$. We obtain these fields. We realize chiral representations: $36 \oplus 5 \cdot 9$ for a $SU(9)$ GUT and $3\{10_L \oplus 5_L\}$ for a $SU(5)'$ GUT. The details for the spectra calculation are present without any computer help, which is possible in the simplest $\mathbb{Z}_{12-I}$ orbifold.

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I. INTRODUCTION

Grand unified theories (GUTs) attracted a great deal of attention aesthetically because they provided unification of gauge couplings and charge quantization [1–3]. But there seems to be a fundamental reason leading to GUTs at the standard model (SM) level. With the electromagnetic and charged currents (CCs), the leptons need representations which are a doublet or bigger. A left-handed (L-handed) lepton doublet ($\nu_e, e$) alone is not free of gauge anomalies because the observed electromagnetic charges are not $\pm \frac{1}{2}$. The anomalies from the fractional electromagnetic charges of the $u$ and $d$ quarks add up to make the total anomaly from the first family vanish [4, 5]. In view of this necessity for jointly using both leptons and quarks to cancel gauge anomalies in the SM, we can view that GUTs are fundamentally needed beyond the above aesthetic viewpoints.

In the SM, the largest number of parameters is from the Yukawa couplings which form the bases of the family structure. Repetition of fermion families in 4-dimensional (4D) field theory or family-unified GUT (family-GUT) was formulated by Georgi [6], requiring un-repeated chiral representations while not allowing gauge anomalies. Some interesting family-GUT models are the spinor representation of $SO(14)$ [7, 8] and $84 \oplus 9 \cdot 9$ of $SU(9)$ [9].

While Refs. [7–9] do not provide interesting non-vanishing flavor quantum number, the $SU(11)$ model [6] allows a possibility for non-vanishing flavor quantum number such as $U(1)_{\mu - \tau}$ [10].

On the other hand, the standard-like models from string have been the main focus of phenomenological activities for the ultraviolet completion of the SM in the last several decades [12–34]. These models use the chiral spectrum from the level–1 construction which leads to unification of gauge couplings [35]. So, the standard-like models from string compactification achieved the goal of gauge coupling unification and GUT theories from string have not attracted much attention. Nevertheless, GUTs from strings [35, 36] have been discussed sporadically for anti-$SU(5)$ [39] (or flipped $SU(5)$) [40], dynamical symmetry breaking [41, 42], and family unification [43, 44]. In fact, family-GUTs are much easier in discussing the family problem, in particular on the origin of the mixing between quarks/leptons, guiding to the progenitor mass matrix [46] because the number of representations in family-GUTs is generally much smaller than in their (standard-like model) subgroups.

In this paper we study family-GUTs from string compactification. So far, most string compactification models used the $E_8 \times E_8'$ heterotic string in which a GUT with rank greater than 8 is impossible. In Ref. [10], to assign some non-vanishing $L_\mu - L_\tau$ family quantum number, only the family-GUT $SU(11)$ is chosen among the known family-GUT models. The group $SU(11)$ has rank 10 which cannot arise from compactification of $E_8 \times E_8'$. Therefore, firstly we formulate the orbifold compactification [47, 48] of SO(32) heterotic string [11] whose rank is 16. The SO(32) string compactification has been studied before [33] but it did not include the GUTs. The GUT study is here for the first time. Then, we also attempt to accompany a hidden sector nonabelian group such that provides a confining force toward breaking supersymmetry (SUSY) [42].

Among compactification schemes, we adopt the orbifold method. Among 13 possibilities listed in Ref. [47], we employ $Z_{12-I}$ orbifold because it has the simplest twisted sectors. Twisted sectors are distinguished by Wilson lines [49]. The

1 For more attempts of family-GUTs, see references in [10].
Wilson line in $Z_{12-I}$ distinguishes three fixed points at a twisted sector. Therefore, it suffices to consider only three cases at a twisted sector. In all the other orbifolds of Ref. [47], consideration of various possibilities of Wilson lines and the accompanying consistency conditions are much more involved. So, as the first step, in this paper we work with the $Z_{12-I}$ orbifold.

In Sec. III we obtain the SU(16) subgroup of SO(32). In Sec. III we recapitulate the orbifold methods used in this paper for an easy reference to Sec. IV. Even though the computer program is put in Ref. [35], the GUT families are lacking from these programs. To our experience, there are not many possible working GUTs and it is not possible to obtain them except from $Z_{12}$. In Sec. IV we list all possible massless SU(9) and SU(5)’ SUSY spectra. In Sec. IV we discuss symmetry breaking. Firstly, we comment on breaking the Georgi-Glashow SU(5) to the SM is possible by $\langle f \rangle$. Many possibilities of SO(32) where however we do not lose any chiral representation.

II. SU(16) SUBGROUP

To discuss the family number in SU(5), the easiest way is to count the number of un-paired 10’s which is equal to the family number. Theory of families in GUTs does not allow repetition of the representation and gauge anomalies. The number of un-paired 10’s automatically determines the number of un-paired 5’s from the anomaly freedom. The anomaly unit of $m$ completely anti-symmetric tensor representation in SU(N) is

$$A([m]) = \frac{(N-3)!(N-2m)!}{(N-m-1)!(m-1)!},$$

(1)

where $[m] = [N - m]$, i.e.

$$A([1]) = -1, \quad A([2]) = -N + 4, \quad A([3]) = \frac{(N-3)(N-6)}{2}, \quad \text{etc.}$$

(2)

Except the anti-symmetrized $[m]$, we do not use higher dimensional representations for matter fields, not to allow beyond 3 and 5 as quarks and anti-quarks of color SU(3).

The adjoint representation 496 of SO(32) suggested in the heterotic string [11] branches to the following SU(16) representations,

$$\Phi^a \oplus \Phi^{[ab]} \oplus \Phi_{[ab]}, \quad (a, b = 1, 2, \ldots, 16), \quad (n = 16)$$

(3)

whose dimensions are $n^2 = 255 \oplus 1, \frac{n(n-1)}{2} = 120$, and $\frac{n(n-1)}{2} = 120$, respectively. In the orbifold compactification of SO(32), it will be easy to realize the representation $\Phi^{[ab]}$ and $\Phi_{[ab]}$ even at level 1 because they are anti-symmetric representations, and the key breaking pattern of family-GUT, i.e. the separation of color SU(3)c and weak SU(2)W, to the SM is possible by $\langle \Phi^{[45]} \rangle$ and $\langle \Phi_{[45]} \rangle$ of Eq. (3). By restricting to the SU(16) subgroup of SO(32), we exclude many possibilities of SO(32) where however we do not lose any chiral representation.

Representations [1] and [2] have the following matrix forms,

$$[1] \equiv \Phi^{[A]} = \begin{pmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3 \\
\alpha_4 \\
\alpha_5 \\
f_6 \\
\vdots \\
f_N
\end{pmatrix}, \quad [2] \equiv \Phi^{[AB]} = \begin{pmatrix}
0, & \alpha_{12}, & \cdots, & \alpha_{15} & \epsilon_{16}, & \cdots, & \epsilon_{1N} \\
-\alpha_{12}, & 0, & \cdots, & \alpha_{25} & \epsilon_{26}, & \cdots, & \epsilon_{2N} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
-\alpha_{15}, & -\alpha_{25}, & \cdots, & 0 & \epsilon_{56}, & \cdots, & \epsilon_{5N} \\
-\epsilon_{16}, & -\epsilon_{26}, & \cdots, & -\epsilon_{56} & 0, & \cdots, & \beta_{6N} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
-\epsilon_{1N}, & -\epsilon_{2N}, & \cdots, & -\epsilon_{5N} & -\beta_{6N}, & \cdots, & 0
\end{pmatrix}$$

(4)

where [1] contains one 5, and [2] contains one 10 of SU(5). The number of the SU(5)GG families, i.e. that of 10 plus 5, is counted by the number of 10 minus the number of 10. The anomaly-freedom condition chooses the matching
number of $\mathbf{5}$'s. The numbers $n_1$ and $n_2$ for the vectorlike pairs $n_1(\mathbf{5} \oplus \mathbf{5}) + n_2(\mathbf{10} \oplus \mathbf{10})$ are not constrained by the anomaly freedom. Thus, we count the number of families just by the net number of two index fermion representations in the SU(5)$_{\text{GG}}$ subgroup. Because we allow only the SM fields, the fundamental representations $\Phi^{[A]}$ and $\Phi^{[A]}$, at the locations $f_6, \cdots, f_N$ of Eq. (4), are also used to reduce the rank further by these VEVs.

For the fundamental representation in SU(9), we choose $[1]$ as

$$\mathbf{9} = (1 \ 0 \ 8).$$

In this case, obviously we have the following $[2]

$$\mathbf{36} = (1 \ 1 \ 0 \ 7).$$

For spinors, however, it is more involved. An SO(18) spinor, which is $2^8(= 256)$ dimensional, is chiral. In terms of SU(9) representations, let us define

$$\begin{align*}
\text{SU}(9) : & \quad \begin{pmatrix} + & - \end{pmatrix}, \begin{pmatrix} + & + & + & - \end{pmatrix}, \begin{pmatrix} + & 5 & - & - & - \end{pmatrix}, \begin{pmatrix} + & 7 & - & - \end{pmatrix}, \begin{pmatrix} + & 9 \end{pmatrix} \; , \\
\text{SU}(9) : & \quad \begin{pmatrix} + & + & - \end{pmatrix}, \begin{pmatrix} + & + & + & - \end{pmatrix}, \begin{pmatrix} + & + & + & + & - \end{pmatrix}, \begin{pmatrix} + & + & + & + & + & - \end{pmatrix}, \begin{pmatrix} + & + & + & + & + & + & - \end{pmatrix}, \begin{pmatrix} + & + & + & + & + & + & + \end{pmatrix}, \begin{pmatrix} + & + & + & + & + & + & + & + \end{pmatrix}.
\end{align*}$$

The complex conjugation of (8) is

$$\begin{align*}
\text{SU}(9) : & \quad \begin{pmatrix} - & + \end{pmatrix}, \begin{pmatrix} - & + & + & - \end{pmatrix}, \begin{pmatrix} - & + & + & + & - \end{pmatrix}, \begin{pmatrix} - & + & + & + & + & - \end{pmatrix}, \begin{pmatrix} - & + & + & + & + & + & - \end{pmatrix}, \begin{pmatrix} - & + & + & + & + & + & + & - \end{pmatrix}, \begin{pmatrix} - & + & + & + & + & + & + & + & - \end{pmatrix}.
\end{align*}$$

We defined the spinors in this way such that we include only even numbers of $+$ signs inside the SU(9) spinors. We will use the definitions given in Eqs. (5,6,7,8).

We find that SU(9) is the maximal subgroup of SU(16) from string construction, allowing two indices anti-symmetric tensor fields. Its covering group is SO(18) which belongs to SO$(4n+2)$ groups allowing chiral spinors. The SO$(4n+2)$ groups were used for field theory GUTs [7, 52, 53]. But, in string construction we cannot obtain spinors. Among branching of spinors to SU$(2n+1)$ representations, we obtain at most two indices anti-symmetric tensor fields.

### III. ORBITFOLD COMPACTIFICATION

Orbifolds are manifolds with identification of space points by discrete groups. This idea was used to reduce 6D internal space from 10D string models to obtain 4D light fields. The internal 6D is so small that their details are shown up only through effective high dimensional interactions of the 4D light fields. Light fields appear in the untwisted sector $U$ and also in the twisted sector $T$ [47, 48]. Gauge groups are determined from $U$. In the untwisted sector $U$, spinors lattice points of $E_8 \times E'_8$ heterotic string satisfying $P^2 = 2$ arise also, but it is not so in the SO(32) heterotic string. In a sense, therefore, it is easier to obtain gauge groups from the SO(32) heterotic string.

In the twisted sectors, there are fixed points and the fixed points can be distinguished by the Wilson lines which circle around the fixed points [49]. In the most discussed $Z_{6-11}$ and $Z_{12-1}$ orbifolds, the number of fixed points are 12 and 3, respectively. Here, $\phi_s$ are given as $\phi_s = \frac{1}{6}(3, 2, 1)$ and $\phi_s = \frac{1}{12}(5, 4, 1)$, respectively, where each entry represents the two-dimensional torus of the internal six dimensions. The central number (in $\phi_s$) in $Z_{6-11}$ and $Z_{12-1}$ are 2 and 4, respectively, which mean that they have $Z_{6/2}$ and $Z_{12/4}$ symmetries, i.e. both have the $Z_3$ symmetry in the second torus. Except in the second torus, we calculate the multiplicities by the direct product of the multiplicities in the remaining two tori in case there is no Wilson line, i.e. the case $l = 0$ of Table I in case of $Z_{12-1}$.

Toward the SU(9) family-GUT, we note that [51]

1. Matter representations $\Psi^{[ABCD]}$ and $\Psi^{[ABC]}$ do not appear.
2. Matter $\Psi^{[AB]}$ and $\Psi^{[A]}$ can appear in the untwisted (viz. Eq. (3)) and twisted sectors.
3. Among mod integers, choose only one integer.

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2 See, for example, a book presenting toolkits for orbifold compactification [51].
In string compactifications, therefore, the number of families is counted by the number of the antisymmetric representation [2]. Matter in the untwisted sector $U_i$ occurs with $P \cdot V = \frac{N}{12}$. For example, $N_i = \frac{27}{12}(5,4,1)$ for $\phi_s$ of $Z_{12-I}$ is shown in the second column of Table I.

In the $k$-th twisted sector of $Z_N$ orbifold, multiplicities $P_k$ is

$$P_k = \frac{1}{N} \sum_{l=0}^{N-1} \tilde{\chi}(k, l)e^{i2\pi(l\Theta_0)},$$  \hspace{1cm} (12)

where $\tilde{\chi}(k, l)$ in the $Z_{12-I}$ orbifold are listed in Table I and the phase angle $\Theta_0$ will be defined later. The chirality is given by the first entry $s_0$ in $s$ with the even number of total `-'s in Eq. (13),

$$s = (s_0; \tilde{s}) = (\ominus \text{ or } \oplus ; \pm, \pm),$$  \hspace{1cm} (13)

where $s_0$ corresponds to L- or R- movers. In Table III, multiplicities in the $Z_{12-I}$ orbifold are presented [51]. Here,

$$\Theta_0 = \sum_j (N^j_L - N^j_R) \hat{\phi}^j - \frac{k}{2}(V^2_a - \phi_s^2) + (P + kV_a) \cdot V_a - (\tilde{s} + k\phi_s) \cdot \phi_s + \text{integer},$$  \hspace{1cm} (16)

$3 \tilde{\chi}(\theta^k, \theta^l)$ are presented in Ref. [51].
where \( \Delta_k \) is

\[
\Delta_k = (P + kV_a) \cdot V_a - \frac{k}{2}(V_a^2 - \phi_s^2) + \sum_j (N_{Lj} - N_{Rj}) \hat{\phi}^j
\]

(17)

\[
\equiv \Delta_k^0 + \Delta_k^N.
\]

(18)

\( V_a \) is the shift vector \( V \) distinguished by Wilson lines \( a \), and

\[
\Delta_k^0 = P \cdot V_a + \frac{k}{2}(-V_a^2 + \phi_s^2),
\]

(19)

\[
\Delta_k^N = \sum_j (N_{Lj} - N_{Rj}) \hat{\phi}^j.
\]

(20)

We choose \( 0 < \hat{\phi}^j \leq 1 \) mod integer and oscillator contributions due to \( (N_L - N_R) \) to the phase can be positive or negative with non-negative number \( N_{L,R} \geq 0 \). But each contribution to the vacuum energy \( N_{L,R} \hat{\phi}^j \) is nonnegative. One oscillation contributes one number in \( \phi_s \). With the oscillator \( \hat{\phi}^j \), the vacuum energy is shifted to

\[
(P + kV_a)^2 + 2 \sum_j N_{Lj} \hat{\phi}^j = 2\hat{c}_k
\]

(21)

\[
(p_{vec} + k\phi_s)^2 + 2 \sum_j N_{Rj} \hat{\phi}^j = 2c_k,
\]

(22)

where \( 2\hat{c}_k \) and \( 2c_k \) in the most discussed \( Z_{6-11} \) and \( Z_{12-I} \) orbifolds are

\[
Z_{6-11} : \begin{cases} 2\hat{c}_k : & \frac{5}{36}(k = 1), \frac{11}{36}(k = 2), \frac{5}{18}(k = 3), \\ 2c_k : & \frac{1}{36}(k = 1), \frac{2}{36}(k = 2), \frac{1}{18}(k = 3), \end{cases}
\]

(23)

\[
Z_{12-I} : \begin{cases} 2\hat{c}_k : & \frac{20}{114}(k = 1), \frac{2}{114}(k = 2), \frac{2}{114}(k = 3), \frac{1}{114}(k = 4), \frac{2}{114}(k = 5), \frac{2}{114}(k = 6), \\ 2c_k : & \frac{1}{24}(k = 1), \frac{1}{8}(k = 2), \frac{1}{3}(k = 3), \frac{1}{4}(k = 4), \frac{1}{24}(k = 5), \frac{1}{24}(k = 6). \end{cases}
\]

(24)

Note that \( 2\hat{c}_k - 2c_k = 1 \) which is the required condition for \( \mathcal{N} = 1 \) supersymmetry in 4D.

The Wilson loop integral is basically the Bohm-Aharanov effect in the internal space of two-torus,

\[
\oint V^i dx_i = \frac{1}{2} \int (\vec{\nabla} \times \vec{V})^i_{0+,} e^{ijk} dx_{(jk)}.
\]

(25)

If the \( B \)-field \( \text{(i.e. } \vec{\nabla} \times \vec{V} \text{) at the orbifold singularity is present, the phase through } \Delta_0 \text{ contributes in the multiplicity. For } Z_{12-I}, \text{ this is the case in } T_1,2,3,4,5. \text{ The complication arises at the points with } 3a_3 = 0 \text{ mod. integer, i.e. at } T_3,6,8, \text{ where the Bohm-Aharanov phase has to be taken into account explicitly. At } T_3,6 \text{ and also } U, \text{ for the (internal space) gauge symmetry we must require explicitly}

\[
(P + kV_0) \cdot a_3 = 0.
\]

(26)

We distinguish \( T_3 \) by 0, + and – because the phase \( \Delta_k^0 \) of Eq. 20 contains an extra \( \frac{1}{2} \) factor. Namely, Eq. 26 is applied only at \( U, T_3^{0,+,} \text{ and } T_6. \)

A. Vacuum energy and multiplicity in the twisted sectors

In the compactification of the \( E_8 \times E_8 \) heterotic string, spinors for rank 8 can contribute. But in the compactification of the \( SO(32) \) heterotic string, spinors in \( U \) are not useful because \( P = (\pm, \pm, \cdots, \pm) \) with sixteen entries gives \( P^2 = 4 \)

\[ T_0 \text{ contains the CTP conjugate states of } T_3. \]
instead of $P^2 = 2$. Only vector types are useful. In the twisted sectors of $Z_{12-1}$ orbifold, Wilson lines distinguish three fixed points in the second torus. At the $T_k$ twisted sector, the three cases are

$$T^{0,+,−}_k : \begin{cases} \beta kV \equiv kV_0 \\ k(V + a_3) \equiv kV_+ \\ k(V - a_3) \equiv kV_−. \end{cases}$$

(27)

Because $3a_3 = 0 \mod. \text{ integer}$, in the sectors with $k = \{3, 6, 9\}$, 0, +, and − are not distinguished by the Wilson lines. But, Eq. (16) contains the factor $\frac{1}{2}$ and hence $k = \{3, 9\}$ are distinguished by Wilson lines and $k = 6$ is not distinguished by Wilson lines.

We select only the even lattices shifted from the untwisted lattices, therefore, we consider even numbers for the sum of entries of each elements of $P$.

In the $k$-th twisted sector, the masslessness condition to raise the tachyonic vacuum energy to zero is

$$\left[(P + kV_0)^2 + 2 \sum_j N^j_φ J^j - 2\epsilon_k\right] = 0,$$

(28)

$$\left[(p + kφ_s)^2 + 2 \sum_j N^j_φ \phi^j - 2\epsilon_k\right] = 0,$$

(29)

where $2\epsilon_k$ and $2\epsilon_k$ are given in Eq. (24), and the brackets must be taken into account when oscillators contribute. When the conditions (17) are satisfied, we obtain the SUSY spectra for which the chirality and multiplicity are calculated from $\Theta_0$ in the $k$-th twisted sector, from Eqs. (14) and (28)

$$\Theta_0 = -\hat{s} \cdot \phi_s + kP \cdot V_0 + \Delta^0_k + \Delta^N_k - (k p_{vec} \cdot \phi_s + 2\delta^N_k),$$

(30)

where $p_{vec}$, $p_{orb}$ and $\delta^N_k$ are given in Table IV. $p_{orb}^2$ saturates the 2nd line in Eq. 24, $p_{vec}$ in the right-moving sector mimics the lattice points $P$ in the left-moving sector, and

| Orbifold | Twisted Sector | $k\phi$ | $p_{vec}$ | $p_{orb}$ | $\delta^N_k$ |
|----------|----------------|---------|-----------|-----------|-------------|
| $Z_{6-11}$ | $T_1$ | $(\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$ | $(0, 0, 0)$ | $(\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$ | 0 |
| $Z_{12-1}$ | $T_2$ | $(\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$ | $(-1, 0, 0)$ | $(\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$ | 0 |
| $Z_{12-1}$ | $T_3$ | $(\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$ | $(-1, -1, 0)$ | $(\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$ | 0 |
| $Z_{12-1}$ | $T_4$ | $(\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$ | $(1, -1, 0)$ | $(\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$ | 0 |
| $Z_{12-1}$ | $T_5$ | $(\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$ | $(-2, -1, -1)$ | $(\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$ | 0 |
| $Z_{12-1}$ | $T_6$ | $(\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$ | $(-2, -2, 0)$ | $(\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$ | 0 |

TABLE III: $H$ momenta, $p_{orb}$, in the twisted sectors of $Z_{12-1}$, Table 10.1 of [51]. Requiring $(p_{vec} + p_{orb})^2 = (2\text{nd line in Eq. 24})$, we have $p_{vec}$ in the 3rd column. In the fourth column, $\delta^N_k$ is shown, from which we have the energy contribution from right movers $2\delta^N_k \geq 0$.

$$\Delta^0_k = \frac{k}{2}(\phi^2_s - V^2_0),$$

(31)

$$\Delta^N_k = 2 \sum_j N^j_\phi \phi^j,$$

(32)

$$\delta^N_k = 2 \sum_j N^j_R \phi^j,$$

(33)
As an example, consider the $T_3$ sector. Note that $(p_{\text{vec}} + 3\phi_s)^2 = (\frac{1}{2}, 0, \frac{3}{2})^2 = \frac{5}{2}$ with $p_{\text{vec}} = (-1, -1, -1)$, which saturates $2c_3 = \frac{5}{8}$ of Eq. (19). Hence, the $N_R$ contribution is 0. If we choose $0 < \phi^j \leq 1$ mod integer, not using $\phi^j$, oscillator contributions due to $(N_L - N_R)$ can be in principle positive or negative. We used $p_{\text{vec}} \cdot \phi_s = -\frac{10}{12}$ as shown in Table IV because $p_{\text{vec}}$ is already listed in the $k^{th}$ twisted sector.

We will select only the even lattices shifted from the untwisted lattices. They form even numbers if the entries of each elements of $P$ are added. In the tables, we list SU(9) and SU(3)$'$ non-singlets and columns are ordered according to

$$\Theta_{\text{Group}} = -s \cdot \phi_s - kp_{\text{vec}}^{\text{th}} \cdot \phi_s + kp \cdot V_0 + \frac{k}{2}(\phi_s^2 - V_0^2) + \Delta_k^N - \delta_k^N,$$

where

$$\delta_k^N = 2\delta_k.$$  \hspace{1cm} (35)

From Table III, we note that non-vanishing contributions at $\theta \neq 0$ are present in $T_3, T_4$ and $T_6$. Therefore, we discuss these more complicated $T_3, T_4$ and $T_6$ sectors first.

### IV. FAMILY UNIFICATION WITH SU(9) GUT

We anticipated to achieve the anomaly-free key spectra needed for SU(9) family-GUT,

$$3\Psi^{[AB]} + 12\Psi^{[A]} + \cdots,$$

where $\cdots$ contain vectorlike pairs and singlets. Since it is impossible to obtain high dimensional representations $\psi^{[ABC]}$ and $\psi^{ABCD}$ from orbifold compactification, the family number is counted by the number of $\psi^{[AB]} \equiv 36$. We are interested in obtaining three chiral families. The chiral representations are represented by $\Psi$’s, and vectorlike rerepresentations are represented by $\Phi$ which contain candidates for the Higgs bosons.

The orbifold conditions, toward a low energy 4D effective theory, remove some weights of the original ten dimensional SU(16) weights. The remaining ones constitute the gauge multiplets and matter fields in the untwisted sector in the low energy 4D theory. Therefore, the weights in the untwisted sector $U$ must satisfy $P^2 = 2$. Because the rank of U(16) is 16, spinors with $P^2 = 2$ are not available. Orbifold conditions produce singularities. They are typically represented in three two-dimensional tori. A loop of string can be twisted around these singularities and define twisted sectors $T_k^{+,−}$ $(k = 1, 2, \cdots, 11)$. Twisting can introduce additional phases. Since $T_{12−k}$ provides the anti-particles of $T_k$, we consider only $T_k$ for $k = 1, 2, \cdots, 6$. $T_6$ contains both particles and anti-particles. $T_6$, not affected by Wilson lines, is like an untwisted sector. It contains the antiparticles also as in $U$.\(^5\)

The shift vector $V_0$ and Wilson line $a_3$ are restricted to satisfy the $\mathbf{Z}_{12−1}$ orbifold conditions,

$$12(V_0^2 - \phi_s^2) = 0 \text{ mod even integer},$$

$$12(V_0 \cdot a_3) = 0 \text{ mod even integer},$$

$$12|a_3|^2 = 0 \text{ mod even integer}.$$  \hspace{1cm} (37, 38, 39)

Here, $a_3 (= a_4)$ is chosen to allow and/or forbid some spectra, and is composed of fractional numbers with the integer multiples of $\frac{1}{12}$ because the second torus has the $\mathbf{Z}_3$ symmetry. Toward SU(9) non-singlet spectra in the $\mathbf{Z}_{12−1}$ orbifold from SO(32) heterotic string, we choose the following model,

$$V_0 = (\frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}), \hspace{1cm} V_0^2 = \frac{522}{144} \rightarrow \frac{54}{144},$$

$$V_+ = (\frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}), \hspace{1cm} V_+^2 = \frac{522}{144} \rightarrow \frac{54}{144},$$

$$V_− = (\frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}), \hspace{1cm} V_−^2 = \frac{138}{144}.$$  \hspace{1cm} (40)

\(^5\) Actually, $T_{12}$ can be viewed as $U$.\)
where
\[ a_3 = a_4 = \left( 0^8; 0, \frac{-1}{3}, 0, \frac{1}{3}, 0, \frac{1}{3}, 0, \frac{1}{3} \right). \] (41)

The R-hand weights are
\[ \phi^2_s = \frac{42}{144}. \] (42)

Shifted lattices by Wilson lines are given by \( V_+ \) and \( V_- \).

The order of presentation is \( U, T_3, T_4, T_1, T_2, \) and \( T_5 \) which contain chiral spectra. Finally, we present \( T_6 \) which contains only vector-like pairs. \( V_0 \) is the most important shift vector of \( \mathbb{Z}_{12-I} \). In this paper, we are interested in obtaining chiral spectra and hence do not discuss \( T_6 \) which gives only vector-like spectra.

We use the following notations: \( V_{0,+,-} \) represent (left-hand or gauge group) shift vectors and \( P_{\text{Group}} \) (or sometimes just \( P \) if no confusion arises) is the lattice point in the SU(16) group space.

### A. Untwisted sector \( U \)

In \( U \), we find the following nonvanishing roots of \( SU(9) \times SU(5)' \times U(1)^4 \),

\[
\begin{align*}
SU(9) & \text{ gauge multiplet : } P \cdot V = 0 \text{ mod. integer} \\
SU(9) & : \quad P = (\pm 1 - 1000000; 00; 000000) \quad (43) \\
SU(5)' & \text{ gauge multiplet : } \quad P \cdot V = 0 \text{ mod. integer and } P \cdot a_3 = 0 \text{ mod. integer} \\
SU(5)' & : \quad P = (0^9; 0^2; 1 - 10000). \quad (44)
\end{align*}
\]

For tensor notations, we use \( A \) for SU(9) representations and \( \alpha \) for SU(5)' representations. In addition, there exists U(1)^4 symmetry. The non-singlet matter fields are

\[
\text{SU(9) and/or SU(5)’ matter multiplet : } \quad P \cdot V = \frac{1}{12}, P \cdot a_3 = 0 \text{ mod. integer} \quad (45)
\]

The conditions \( (45) \) allows the \( P^2 = 2 \) lattice shown in Table IV. The four entry set \( s \) is the \( s^2 = 2 \) right-hand spin lattice, \( s = (\ominus \oplus \ominus \oplus ) \) with every entry being integer multiples of \( \frac{1}{2} \). In the following + and – represent \( \frac{1}{2} \) and \( - \frac{1}{2} \), respectively. Three entry set in the right-hand sector is also used

\[
\hat{\phi}_s = \left( \frac{5}{12}, \frac{4}{12}, \frac{1}{12} \right). \quad (46)
\]

| \( U_i \) | \( P \) | Tensor form | Chirality | \( [p_{\text{spin}}] \) (\( p_{\text{spin}} \cdot \hat{\phi}_s \)) |
|---|---|---|---|---|
| \( U_1 (p \cdot V = \frac{1}{12}) \) | – | None | – | – |
| \( U_2 (p \cdot V = \frac{1}{12}) \) | \( (\frac{1}{2}; 1 0 0; 0^5) \) | \( \Psi^A \) | L | \( [\ominus; + + -] \) (\( \frac{1}{12} \)) |
| \( U_3 (p \cdot V = \frac{1}{12}) \) | – | None | – | – |

**TABLE IV:** There is a \( SU(\Psi^A) \) in view of Eq. \( (5) \) in the twisted sector convention. Chirality is read from the circled sign in \( s = (\ominus \oplus \ominus \ominus \ominus ) \) where \( \pm \) represents \( \pm \frac{1}{2} \). \( s = (\ominus; + -) = (\ominus; p_{\text{spin}}) \) gives chirality \( L \) (\( \ominus \)) because \( P \cdot V_0 = p_{\text{spin}} \cdot \hat{\phi}_s = \frac{1}{12} \) where \( \hat{\phi}_s \) is shown in Eq. \( (46) \). The convention on the chirality in \( U \) (as the 12th twisted sector) in the twisted sector convention defined from \( T_{1,2,...,6} \) is the opposite of \( \ominus \) or \( \ominus \). In the same way, we take the opposite chirality from \( \ominus \) or \( \ominus \) in the twisted sector \( T_3 \), since one entry in \( 5\hat{\phi}_s \) exceeds 2.
B. Twisted sector $T_3 (\delta_3 = \frac{1}{12})$

In the multiplicity calculation in $\Theta_0$, there is a factor $\frac{1}{2}$ between the lattice shifts by Wilson lines. Even though the Wilson lines cannot distinguish the fixed points, we consider $V_+$ and $V_-$ also as if Wilson lines distinguish fixed points.

In $T_3$, we have

\[ 3V_0 = \left( \frac{+3}{12} \right)^9; \frac{+9}{12}; \frac{+18}{12} \right)^6, \quad V_0^2 = \frac{-54}{144}, \]

\[ 3V_+ = \left( \frac{+3}{12} \right)^9; \frac{+9}{12} \right)^6; \frac{+30}{12} \right)^5, \quad V_+^2 = \frac{-54}{144}, \]

\[ 3V_- = \left( \frac{+3}{12} \right)^9; \frac{+9}{12} \right)^6; \frac{+30}{12} \right)^5, \quad V_-^2 = \frac{132}{144}. \]

1. Two indices spinor-form from $T_3^0$

| Chirality | $\delta$ | $-\delta \cdot \phi_s$ | $-k \psi^{\text{th}} \cdot \phi_s k P_0 \cdot V_0$ | $(k/2)\phi_s^2$, $-(k/2)V_0^2$ | $\Delta_N^0$, $-\delta_N^0$ | $\Theta_0$, Mult. of SU(9) |
|-----------|---------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\Theta = L$ (\(-+-\)) | $\frac{5}{12}$ | $\frac{10}{12}$ | $63$ | $\frac{81}{12}$ | $0$ | $\frac{8}{12}$ | $\frac{9}{12}$ | $0$ |
| $\Theta = L$ (\(+\)) | $0$ | $\frac{10}{12}$ | $63$ | $\frac{81}{12}$ | $0$ | $\frac{8}{12}$ | $\frac{9}{12}$ | $1$ |
| $\Theta = L$ (\(++-\)) | $\frac{1}{12}$ | $\frac{10}{12}$ | $63$ | $\frac{81}{12}$ | $0$ | $\frac{8}{12}$ | $\frac{9}{12}$ | $0$ |
| $\Theta = L$ (\(+++-\)) | $\frac{5}{12}$ | $\frac{10}{12}$ | $63$ | $\frac{81}{12}$ | $0$ | $\frac{8}{12}$ | $\frac{9}{12}$ | $2$ |
| $\Theta = L$ (\(-+-\)) | $0$ | $\frac{10}{12}$ | $63$ | $\frac{81}{12}$ | $0$ | $\frac{8}{12}$ | $\frac{9}{12}$ | $1$ |
| $\Theta = L$ (\(-++-\)) | $\frac{1}{12}$ | $\frac{10}{12}$ | $63$ | $\frac{81}{12}$ | $0$ | $\frac{8}{12}$ | $\frac{9}{12}$ | $0$ |
| $\Theta = L$ (\(--+\)) | $\frac{5}{12}$ | $\frac{10}{12}$ | $63$ | $\frac{81}{12}$ | $0$ | $\frac{8}{12}$ | $\frac{9}{12}$ | $1$ |

TABLE V: One index spinor-form from $T_3^0$: Thus, we obtain $[\bar{\Psi}, 1)_L + 2([\bar{\Psi}, 1)_L + (\bar{\Psi}, 1)_R]$.  

- Two indices spinor-form from $T_3^0$: the spinor forms satisfying $(P + 3V_0)^2 = \frac{234}{144} = \frac{13}{8}$ and $12(P + 3V_0) \cdot a_3 = 0$ are possible for SU(9):

\[ P_9 = (++-7): -; --; --; --. \]  

(50)

from which no chiral field is obtained.

- One index spinor forms from $T_3^0$: the spinor forms satisfying $(P + 3V_0)^2 = \frac{234}{144} = \frac{13}{8}$ and $12(P + 3V_0) \cdot a_3 = 0$ are possible for SU(9):

\[ P_9 = (+-8, \frac{-3}{2}, -; --; --). \]  

(51)

for which the massless fields are shown in Table VII. Note that the entries in (\(-+-\)) and (\(+-+\)) determine the spectra. Thus, Table VI can be abbreviated to Table VII.

| Chirality | $\delta$ | $-\delta \cdot \phi_s$ | $-k \psi^{\text{th}} \cdot \phi_s k P_0 \cdot V_0$ | $(k/2)\phi_s^2$, $-(k/2)V_0^2$ | $\Delta_N^0$, $-\delta_N^0$ | $\Theta_0$, Mult. of SU(9) |
|-----------|---------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\Theta = L$ (\(-+-\)) | $\frac{5}{12}$ | $\frac{10}{12}$ | $63$ | $\frac{81}{12}$ | $0$ | $\frac{8}{12}$ | $\frac{9}{12}$ | $3$ |
| $\Theta = L$ (\(++\)) | $\frac{5}{12}$ | $\frac{10}{12}$ | $63$ | $\frac{81}{12}$ | $0$ | $\frac{8}{12}$ | $\frac{9}{12}$ | $2$ |

TABLE VI: One index spinor-form from $T_3^0$: Thus, we obtain $[\bar{\Psi}, 1)_L + 2([\bar{\Psi}, 1)_L + (\bar{\Psi}, 1)_R]$.  


• Two indices spinor form for $T^+_3$: the spinor forms satisfying $(P + 3V_+)^2 = \frac{234}{144}$ and $12(P + 3V_+) \cdot a_3 = 0$ are possible for SU(9):

$$P_0 = (++^{−7}; ++; + + − − − )$$

from which no chiral field is obtained.

• One index spinor-form from $T^+_3$: the spinor forms satisfying $(P + 3V_+)^2 = \frac{234}{144}$ and $12(P + 3V_+) \cdot a_3 = 0$ are possible for SU(9):

$$P_9 = (+^{−8}; −\frac{3}{2}; − − − − )$$

The massless fields are shown in Table XVI.

• Two indices spinor-form from $T^-_3$: the spinor form

$$P_9 = (++, −7; −; − − − 5).$$

gives $(P + 3V_0)^2 = \frac{234}{144}$. The massless fields are shown in Table VII. We follow the definition of Eq. (10).

| Chirality | $\delta$ | $\bar{s} \cdot \phi_3$ | $−k \, \phi^{kth}_{12} \cdot \phi_3$ | $k \, P_0 \cdot V_0 \cdot (k/2) \phi_3^2$, $−(k/2) V_0^2$, $\Delta^N$, $− \delta^N$, $\Theta_0$, Mult. of SU(9) |
|-----------|----------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\ominus = L$ | $(−−−)$ | $\frac{10}{12}$ | $\frac{−3}{2}$ | $\frac{5}{12}$ | 138 | 12 | 2 |
| $\oplus = L$ | $(++−)$ | $\frac{5}{12}$ | $\frac{−3}{2}$ | $\frac{5}{12}$ | 138 | 12 | 1 |

TABLE VII: Two indices spinor-form from $T^-_3$: Thus, we obtain $(36, 1)_L \oplus [(36, 1)_L + (36, 1)_R]$.

In Table VIII, GUT breaking representations $36 \oplus 3\bar{6}$ appear.

• One index spinor-form from $T^-_3$: the spinor form

$$P_0 = (+^{−8}; −\frac{3}{2}; − − − − )$$

gives $(P + 3V_0)^2 = \frac{234}{144}$. No massless field is obtained.

C. Twisted sector $T_4$ ($\delta_4 = 0$)

For $4V^0_{0, +, −}$ in $T_4$, we have

$$4V_0 = \left(\frac{−4}{12}^9; 1; \frac{+8}{12}; \left(\frac{+8}{12}\right)^5\right), \quad V_0^2 = \frac{−54}{144}.$$  

$$4V_+ = \left(\frac{−4}{12}^9; 1; \frac{+8}{12}; \left(\frac{+40}{12}\right)^5\right), \quad V_+^2 = \frac{−54}{144}.$$  

$$4V_- = \left(\frac{−4}{12}^9; 1; \frac{+40}{12}; \left(\frac{+8}{12}\right)^5\right), \quad V_-^2 = \frac{+138}{144}.$$  

• One index spinor-form from $T^0_4$: the spinor-form

$$P = (+^{−8}; −\frac{3}{2}; −; − − − 5).$$

satisfies $(P + 4V_0)^2 = \frac{156}{144}$ which is short by $\frac{36}{144}$ from the target value of $\frac{192}{144}$. Therefore, $\Delta^N_4 = \frac{3}{12}$. Thus, we obtain Table VIII.
| Chirality | \( \hat{s} \) | \( -\hat{s} \cdot \hat{\phi}_a \) | \( -k \tilde{p}_{\text{vac}}^a \cdot \hat{\phi}_a \) | \( k \mathbf{P}_9 \cdot \mathbf{V}_9 \cdot (k/2) \phi_0^a, -(k/2) \phi_0^a, \Delta^N_{1/4}, -\delta^N_1, \Theta_0, \text{Mult. of SU}(9) \) |
|-----------|-----------------|-----------------|-----------------|-------------------------------------------------|
| \( \ominus = L \) (---) | 0/72 | 0/72 | 0/72 | 144 |
| \( \oplus = L \) (+++) | 5/72 | 0/72 | 4/72 | 63 |

**TABLE VIII:** One index spinor-form from \( T^0_4 \): Thus, we obtain \( 7(5,1)_L \).

| Chirality | \( \hat{s} \) | \( -\hat{s} \cdot \hat{\phi}_a \) | \( -k \tilde{p}_{\text{vac}}^a \cdot \hat{\phi}_a \) | \( k \mathbf{P}_9 \cdot \mathbf{V}_9 \cdot (k/2) \phi_0^a, -(k/2) \phi_0^a, \Delta^N_{1/4}, -\delta^N_1, \Theta_0, \text{Mult. of SU}(9) \) |
|-----------|-----------------|-----------------|-----------------|-------------------------------------------------|
| \( \ominus = L \) (---) | 5/72 | 0/72 | 4/72 | 63 |
| \( \oplus = L \) (+++) | 5/72 | 0/72 | 4/72 | 63 |

**TABLE IX:** One index spinor-form from \( T^+_4 \): Thus, we obtain \( 7(5,1)_L \).

- One index spinor form for \( T^+_4 \): the spinor-form

\[
P = (\pm 8; -\frac{3}{2}; -5).
\]

satisfies \((P + 4V_\perp)^2 = \frac{156}{144}\) which is short by \(\frac{36}{144}\) from the target value of \(\frac{192}{144}\). Therefore, \(\Delta^N_4 = \frac{3}{12}\). The massless spectra are presented in Table IX.

- One index spinor-form for \( T^-_4 \): the spinor form satisfying \((P + 4V_\perp)^2 = \frac{192}{144}\) and \(12(P + 4V_0) \cdot a_3 = 0\) are possible for SU(9):

\[
P_9 = \left(\pm 8; -, + \mid - - - - - \right). \tag{61}
\]

The massless spectra are presented in Table XVIII.

### D. Twisted sector \( T_1 (\delta_1 = \frac{1}{12}) \)

In \( T^{0,+,-}_1 \), we use Eq. [40].

- One index vector-form for \( T^+_1 \): the vector

\[
P_9 = (0^3; 0, 0; -1, -1, -1, 0), \tag{62}
\]

satisfies \((P + V_\perp)^2 = \frac{186}{144}\) which is short by \(\frac{24}{144}\) from the target value of \(\frac{210}{144}\), and the spectra are shown in Tabe XIX.

In addition, the lattice point

\[
P_9 = (-1; 0^5; 0, 0; -1, -1, -1, -1), \tag{63}
\]

satisfies \((P + V_\perp)^2 = \frac{186}{144}\) which is short by \(\frac{24}{144}\) from the target value of \(\frac{210}{144}\), and the spectra are shown in Tabe XIX even though \(k \mathbf{P} \cdot \mathbf{V}_\perp \) is changed to \(\frac{3}{12}\).

- One index vector-form for \( T^-_1 \): the vector

\[
P_9 = (-1; 0^5; 0, -1; 0^5), \tag{64}
\]

satisfies \((P + V_\perp)^2 = \frac{186}{144}\) which is short by \(\frac{24}{144}\) from the target value of \(\frac{210}{144}\), and the spectra are shown in Tabe X.
E. Twisted sector $T_2$ ($\delta_2 = 0$)

For $T_2^{0,+,-}$, we have

$$2V_0 = \left( +\frac{2}{12}, \frac{6}{12}, \frac{12}{12}, \frac{12}{12}, \frac{12}{12} \right), \quad V_0^2 = -\frac{54}{144},$$

$$2V_+ = \left( +\frac{2}{12}, \frac{6}{12}, \frac{4}{12}, \frac{20}{12}, \frac{12}{12} \right), \quad V_+^2 = +\frac{138}{144},$$

$$2V_- = \left( +\frac{2}{12}, \frac{6}{12}, \frac{20}{12}, \frac{4}{12}, \frac{5}{12} \right), \quad V_-^2 = -\frac{54}{144}.$$ (65) (66) (67)

- One index vector-form from $T_2^0$: the vector

$$P_0 = (0^9; -1, -1; -1, -1, -1, 0),$$

satisfies $(P + 2V_0)^2 = \frac{172}{144}$. Thus, we need the oscillator contribution $\frac{44}{144}$, and there is no massless fields.

- One index vector-form from $T_2^\pm$: the vector

$$P_\pm = (0^9; -1, 0; -1, -2, -2, -2, -2) \equiv (0^9; -1, 0; -1, 0^5),$$

satisfies $(P + 2V_0)^2 = \frac{216}{144}$. $\Theta_9$ turns out to be $\frac{8}{12}$, and there is no massless fields. For the shifted lattice,

$$V_2^{+ \pm} = \left( +\frac{9}{12}, \frac{6}{12}, \frac{4}{12}, \frac{12}{12}, \frac{12}{12} \right),$$

one index spinor-form

$$P_9 = (+-8; -+; +5),$$

satisfies $(P + 2V_+)^2 = \frac{216}{144}$, but there but results no massless field.

- One index vector-form from $T_2^-$: the vector

$$P_0 = (0^9; -1, -2; -1, 0, 0, 0, 0),$$

satisfies $(P + 2V_0)^2 = \frac{240}{144}$, which gives Table XII.

| Chirality | $\tilde{s}$ | $-\tilde{s} \cdot \phi_s$ | $-k P_{v_{\infty}}^k \cdot \phi_s$ | $k P_0 \cdot V_0$ | $(k/2)\phi_s^2,$ | $-(k/2)\phi_s^2$ | $\Delta_N^N,$ | $-\delta_1^N$ | $\Theta_9$, Mult. of SU(9) |
|-----------|-------------|-----------------|-----------------|-------------------|----------------|-----------------|----------------|----------------|------------------|
| $\ominus = L$ (---) | 0 | $\frac{5}{12}$ | $\frac{5}{12}$ | $\frac{15}{12}$ | $\frac{42}{144}$ | $\frac{54}{144}$ | $\frac{0}{17}$ | $\frac{0}{17}$ | $\frac{0}{12}$ | 3 |
| $\oplus = L$ (+++) | 0 | $\frac{5}{12}$ | $\frac{5}{12}$ | $\frac{15}{12}$ | $\frac{42}{144}$ | $\frac{54}{144}$ | $\frac{0}{17}$ | $\frac{0}{17}$ | $\frac{0}{12}$ | 0 |

| TABLE XI: One index vector-form from $T_2^{-}$: Thus, we obtain 3(1,5)$_L$. |
F. Twisted sector $T_5 (\delta_5 = \frac{1}{12})$

For $T_5^{0,+,-}$, we have

$$5V_0 = \left(\frac{+5}{12}, \frac{+15}{12}, \frac{+30}{12}, \frac{+30}{12}\right), \quad V_0^2 = \frac{-54}{144},$$

(73)

$$5V_+ = \left(\frac{+5}{12}, \frac{+15}{12}, \frac{+10}{12}, \frac{+50}{12}\right), \quad V_+^2 = \frac{-54}{144},$$

(74)

$$5V_- = \left(\frac{+5}{12}, \frac{+15}{12}, \frac{-10}{12}, \frac{-50}{12}\right), \quad V_-^2 = \frac{+138}{144},$$

(75)

- One index spinor form for $T_5^0$:

$$P_9 = (\underline{+8}, \underline{-3}, \underline{+}; \underline{-5}),$$

(76)

gives $(P_3 + 5V_-)^2 = \frac{162}{144}$ which is short of $\frac{48}{144} = \frac{4}{12}$ from $\frac{210}{144}$, thus, we need $\Delta^N = \frac{3}{12}$, and obtain Table XIII.

| Chirality | $\tilde{s}$ | $-\tilde{s} \cdot \phi_s$ | $-kp_{vec}^{1h} \cdot \phi_s$ | $kP_9 \cdot V_+ (k/2)\phi_s^* - (k/2)V_+^2$ | $\Delta^N_3$ | $-\delta^N_k$ | $\Theta_9$, Mult. of SU(9) |
|-----------|-------------|-----------------|-----------------|-------------------------------|-----------------|-----------------|-------------------|
| $\Theta = L$ | $(- - -)$ | $\frac{5}{12}$ | $\frac{19}{12}$ | $\frac{46}{144}$ | $\frac{135}{144}$ | $\frac{4}{12}$ | $\frac{2}{12}$ | $\frac{4}{12}$ | $3$ |
| $\Theta = L$ | $(++ +)$ | $\frac{5}{12}$ | $\frac{19}{12}$ | $\frac{46}{144}$ | $\frac{135}{144}$ | $\frac{4}{12}$ | $\frac{2}{12}$ | $\frac{4}{12}$ | $0$ |

**TABLE XII:** One index vector form from $T_5^0$: $3(\overline{3}, 1)_R$. Here we changed the chirality because we used $T_5$ instead of $T_7$.

- One index spinor form for $T_5^+$:

$$P_3 = (\underline{+8}, +, +; -5),$$

(77)

gives $(P_3 + 5V_+)^2 = \frac{162}{144}$ which is short of $\frac{48}{144} = \frac{4}{12}$ from $\frac{210}{144}$, and obtain Table XIV.

| Chirality | $\tilde{s}$ | $-\tilde{s} \cdot \phi_s$ | $-kp_{vec}^{1h} \cdot \phi_s$ | $kP_3 \cdot V_+ (k/2)\phi_s^* - (k/2)V_+^2$ | $\Delta^N_3$ | $-\delta^N_k$ | $\Theta_9$, Mult. of SU(9) |
|-----------|-------------|-----------------|-----------------|-------------------------------|-----------------|-----------------|-------------------|
| $\Theta = L$ | $(- - -)$ | $\frac{5}{12}$ | $\frac{19}{12}$ | $\frac{46}{144}$ | $\frac{135}{144}$ | $\frac{4}{12}$ | $\frac{2}{12}$ | $\frac{4}{12}$ | $3$ |
| $\Theta = L$ | $(++ +)$ | $\frac{5}{12}$ | $\frac{19}{12}$ | $\frac{46}{144}$ | $\frac{135}{144}$ | $\frac{4}{12}$ | $\frac{2}{12}$ | $\frac{4}{12}$ | $0$ |

**TABLE XIII:** One index vector form from $T_5^+$: $3(\overline{3}, 1)_R$. Here we changed the chirality because we used $T_5$ instead of $T_7$.

- One index spinor form for $T_5^-$:

$$P_9 = (\underline{+8}, +, -; -5),$$

(78)

gives $(P_3 + 5V_-)^2 = \frac{162}{144}$ which is short of $\frac{48}{144} = \frac{4}{12}$ from $\frac{210}{144}$, and obtain Table XV.

In addition, the vector

$$P_5 = (\underline{9}; -, +, +, +, -),$$

(79)

gives $(P_5 + 5V_-)^2 = \frac{198}{144}$ which is short of $\frac{12}{144}$. Thus, we obtain Table XV.
TABLE XIV: One index spinor-form from $V_5^-; 3(1, 10)_R$. Here we changed the chirality because we used $T_5$ instead of $T_7$.

G. Twisted sector $T_6(\delta_0 = \frac{9}{12})$

Twisted sectors $T_6^{0,+,-}$ are not distinguish by Wilson lines. So, in $T_6$ we just calculate the spectra whose multiplicity should be 3.

$$6V_6 = \left( \frac{+6}{12}, \frac{+18}{12}, 0, 0^5 \right), \quad V_6^2 = \frac{234}{144} \rightarrow \frac{-54}{144}. \quad (80)$$

- Two indices spinor-form from $T_6^0$: The spinor with the even number of +'s

$$P_5 = (-^9, -, --, ++, --) \quad (81)$$

satisfies $(P_5 + 6V_0)^2 = \frac{216}{144}$ which satisfies the massless condition. Since $P_5 \cdot 6V_0 = \frac{-45}{12} \rightarrow \frac{-3}{12}$, we obtain Table XVI where there result five vector-like pairs of $10'$s of SU(5).

- One index spinor-form from $T_6^0$:

$$P_5 = (-^9, --, +++, --) \quad (82)$$

satisfies $(P_5 + 6V_0)^2 = \frac{216}{144}$ which satisfies the massless condition. Since $P_5 \cdot 6V_0 = \frac{-45}{12} \rightarrow \frac{-3}{12}$, we obtain Table XV again where there result five vector-like pairs of $5'$s (due to Eq. (82)) of SU(5)$'$.

TABLE XV: Two indices and one index spinor-forms from $V_0^0$. There result five vector-like pairs. Since the multiplicity is three, we obtain $15 \cdot [(1, 10')_L + (1, 10')_R] \oplus 15 \cdot [(1, \overline{5})_L + (1, \overline{5})_R]$.

V. SYMMETRY BREAKING

Breaking GUT groups are classified into two. These are done by two indices tensors, $\Phi_{\theta}^a(=\text{adjoint representation})$ and $\Phi_{\theta}^{ab}(=\text{anti-symmetric representation given in Eq. (4)})$ [39]. In string compactification, at the level-1 construction there is no adjoint representation. In our construction, there are U(1)$_X$ symmetry out of which we can pick up U(1)$_X$ of flipped-SU(5)/anti-SU(5). With this choice of U(1)$_X$ for any SU($N$) with $N \geq 5$, U(1)$_{em}$ preserving direction, $\alpha_{g5}$ and $-\alpha_{14}$, which separates color and the rest is possible in Eq. (4). Now, we interpret SU(5)$'$ as the subgroup of the anti-SU(5)/flipped-SU(5) GUT [33, 40] and there exist the needed spectra for the anti-SU(5) symmetry breaking $15(10' \oplus \overline{10}')$ in $T_6$.

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6 The + and - represent $\frac{1}{2}$ and $\frac{1}{2}$, respectively.

7 For the rank 4 GUT SU(5), the F-theory introduces an adjoint representation, which is not arising from a ten dimensional string theory. For SU(9), it is impossible to obtain an adjoint representation.
For SUSY breaking, chiral spectra is needed. For SU(5), one family was hinted to break SUSY \cite{67}, and it was shown that the idea can be realized in a model from string compactification \cite{42}. In our case, the confining force SU(9) is the source for SUSY breaking due to the condensate

\[ S^{ij} \equiv \Psi^{AB}_A \Psi^{j}_{B}, \]  

(83)

where \( \Psi^{AB} \) is the \( 36 \) of Table VIII and \( \Psi^{j}_{A} \) are the remaining (after removing vector-like pairs) \( \mathcal{F} \)'s from Tables VII, IX, and X. Since there remain five \( \mathcal{F} \)'s, there are 10 independent SU(9) singlets formed below the confining scale. Since we consider SUSY, we can construct a superpotential in terms of some \( S^{ij} \) below the confining scale

\[ W \sim M^2_S + M'_SS' + \cdots \]  

(84)

Since we have the effective term above the confining scale, with an O(1) coupling,

\[ W_0 \sim \Psi^{AB}_A \Psi^{j}_{B} \rightarrow M^2_S \]  

(85)

where \( S \) is defined at the scale \( M \). Comparing (84) and (85), we have \( M_c \sim M \). For \( M'_c \), we have \( M'_c \sim M^4/M^3 \) which is \( M^3/M^3 \) factor smaller than \( M_c \), where \( M_P \) is the Planck mass. Below the GUT scale, there are complications due to the GUT symmetry breaking. So, if we take \( M_c \) somewhat below the GUT scale, \( M'_c \) is at least \( 10^6 \) times smaller than \( M_c \). In this approximation, we consider \( W \sim M^2 S \) which does not satisfy the SUSY condition: \( \partial W/\partial S = M^2 \neq 0 \). \( M_c \) is nonzero because it was given by Eq. (83) \cite{42}.

The SUSY breaking discussed in the above paragraph needs a qualification in string compactification. The essential point is the appearance of chiral spectra containing two indices representation \( \Psi^{[AB]} \). But, a chiral spectra containing \( \Psi^{[AB]} \) in SU(\( N \)) with \( N \geq 5 \), in addition to three visible sector families, was appeared previously only in Ref. \cite{41}. The present model \cite{40} is the second example even with \( N \) as large as 9. All standard-like models so far considered have not addressed this question.

\[ 36_L(T_3) + 9_L(U) + \mathcal{F}_L(T_3^0) + 3 \cdot 9_L(T_1^-) + 7 \cdot \mathcal{F}_L(T_4^0) + 7 \cdot \mathcal{F}_L(T_4^+) + 3 \cdot \mathcal{F}_R(T_5^0) + 3 \cdot \mathcal{F}_R(T_5^+), \]  

(86)

that lead to \( 36_L + 5 \cdot \mathcal{F}_L \), and

\[ 3 \cdot \mathcal{F}_L(T_2^-) + 3 \cdot 10'_L(T_5^+). \]  

(87)

These spectra do not lead to non-Abelian gauge anomalies.

The observable sector with three families can be interpreted as the SU(\( 5' \)) GUT, and the fields \( 10' \oplus 10' \) needed for separating color and flavor \cite{39} of the anti-SU(\( 5' \)) appear from the sector \( T_6 \). Breaking of SUSY is provided by the confining force SU(9). The key chiral spectrum needed for SUSY breaking \( \Phi^{[AB]}(36) \) appears from \( T_3 \).

In addition, we could confirm the entries (9 1 1 9 1 1 9 1 1 1 1) in the sector \( T_4 \) of \( \mathbb{Z}_{12-1} \). It was possible to confirm it because we have an SU(9) GUT which needs a nontrivial number (five) of \( \mathcal{F} \)'s accompanying a \( 36 \) to cancel the gauge anomaly.

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\(^8\) Even without chiral spectra, SUSY breaking is possible with an extra confining force if the intervention of gravity is considered \cite{63, 66}. However, we disregard it here.
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Appendix A

| Chirality | \( \tilde{s} \) | \(-\tilde{s} \cdot \phi_5\) | \(-k k_{P_{10}}^* \cdot \phi_5\) | \(k/2 \phi^2_5\) | \((-k/2) V^2_0\) | \(\Delta^N_2\) | \(\delta^N_3\) | \(\Theta_9\), Mult. of SU(9) |
|-----------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \(\oplus = L\) \((- - -)\) | \(\mp \frac{5}{12}\) | \(\mp \frac{10}{12}, \frac{4}{12}\) | \(63, \frac{6}{144}, \frac{0}{12}, \frac{3}{12}\) | \(\frac{3}{12}\) | \(\frac{3}{12}\) | \(3\) |
| \(\oplus = L\) \((+++)\) | \(\mp \frac{5}{12}\) | \(\mp \frac{10}{12}, \frac{4}{12}\) | \(63, \frac{6}{144}, \frac{0}{12}, \frac{3}{12}\) | \(\frac{3}{12}\) | \(\frac{3}{12}\) | \(3\) |

TABLE XVI: One index spinor-form from \(T^+_3\): Thus, we obtain 3[\(\Phi, 1\)_L + \(\Phi, 1\)_R].

| Chirality | \( \tilde{s} \) | \(-\tilde{s} \cdot \phi_5\) | \(-k k_{P_{10}}^* \cdot \phi_5\) | \(k/2 \phi^2_5\) | \((-k/2) V^2_0\) | \(\Delta^N_2\) | \(\delta^N_3\) | \(\Theta_9\), Mult. of SU(9) |
|-----------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \(\oplus = L\) \((- - -)\) | \(\mp \frac{5}{12}\) | \(\mp \frac{10}{12}, \frac{4}{12}\) | \(63, \frac{6}{144}, \frac{0}{12}, \frac{3}{12}\) | \(\frac{3}{12}\) | \(\frac{3}{12}\) | \(0\) |
| \(\oplus = L\) \((+++)\) | \(\mp \frac{5}{12}\) | \(\mp \frac{10}{12}, \frac{4}{12}\) | \(63, \frac{6}{144}, \frac{0}{12}, \frac{3}{12}\) | \(\frac{3}{12}\) | \(\frac{3}{12}\) | \(0\) |

TABLE XVII: One index spinor-form from \(T^+_3\): Thus, we obtain 3[\(\Phi, 1\)_L + \(\Phi, 1\)_R].

| Chirality | \( \tilde{s} \) | \(-\tilde{s} \cdot \phi_5\) | \(-k k_{P_{10}}^* \cdot \phi_5\) | \(k/2 \phi^2_5\) | \((-k/2) V^2_0\) | \(\Delta^N_2\) | \(\delta^N_3\) | \(\Theta_9\), Mult. of SU(5)^{'} |
|-----------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \(\oplus = L\) \((- - -)\) | \(\mp \frac{5}{12}\) | \(\mp \frac{10}{12}, \frac{4}{12}\) | \(63, \frac{6}{144}, \frac{0}{12}, \frac{3}{12}\) | \(\frac{3}{12}\) | \(\frac{3}{12}\) | \(0\) |
| \(\oplus = L\) \((+++)\) | \(\mp \frac{5}{12}\) | \(\mp \frac{10}{12}, \frac{4}{12}\) | \(63, \frac{6}{144}, \frac{0}{12}, \frac{3}{12}\) | \(\frac{3}{12}\) | \(\frac{3}{12}\) | \(0\) |

TABLE XVIII: One index spinor-form from \(T^+_3\): Thus, we obtain 2[\(9, 1\)_L + \(9, 1\)_R].

| Chirality | \( \tilde{s} \) | \(-\tilde{s} \cdot \phi_5\) | \(-k k_{P_{10}}^* \cdot \phi_5\) | \(k/2 \phi^2_5\) | \((-k/2) V^2_0\) | \(\Delta^N_2\) | \(\delta^N_3\) | \(\Theta_9\), Mult. of SU(9) |
|-----------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \(\oplus = L\) \((- - -)\) | \(\mp \frac{5}{12}\) | \(\mp \frac{10}{12}, \frac{4}{12}\) | \(63, \frac{6}{144}, \frac{0}{12}, \frac{3}{12}\) | \(\frac{3}{12}\) | \(\frac{3}{12}\) | \(0\) |
| \(\oplus = L\) \((+++)\) | \(\mp \frac{5}{12}\) | \(\mp \frac{10}{12}, \frac{4}{12}\) | \(63, \frac{6}{144}, \frac{0}{12}, \frac{3}{12}\) | \(\frac{3}{12}\) | \(\frac{3}{12}\) | \(0\) |

TABLE XIX: One index spinor-form from \(T^+_3\): None.

| Chirality | \( \tilde{s} \) | \(-\tilde{s} \cdot \phi_5\) | \(-k k_{P_{10}}^* \cdot \phi_5\) | \(k/2 \phi^2_5\) | \((-k/2) V^2_0\) | \(\Delta^N_2\) | \(\delta^N_3\) | \(\Theta_9\), Mult. of SU(9) |
|-----------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \(\oplus = L\) \((- - -)\) | \(\mp \frac{5}{12}\) | \(\mp \frac{10}{12}, \frac{4}{12}\) | \(63, \frac{6}{144}, \frac{0}{12}, \frac{3}{12}\) | \(\frac{3}{12}\) | \(\frac{3}{12}\) | \(0\) |
| \(\oplus = L\) \((+++)\) | \(\mp \frac{5}{12}\) | \(\mp \frac{10}{12}, \frac{4}{12}\) | \(63, \frac{6}{144}, \frac{0}{12}, \frac{3}{12}\) | \(\frac{3}{12}\) | \(\frac{3}{12}\) | \(0\) |

TABLE XX: One index vector-form from \(V^\pm_5\): None.