Quasi-Particle Behavior of Composite Fermions in the Half-Filled Landau Level

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We calculate the effect of infrared fluctuations of the Chern-Simons gauge field on the single-particle Green’s function of composite fermions in the half-filled Landau level via higher-dimensional bosonization on a curved Fermi surface. We find that composite fermions remain well-defined quasiparticles, with an effective mass given by the mean-field value, but with anomalously large damping and a spectral function that contains considerable weight away from the quasi-particle peak.

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There exists growing experimental evidence\cite{1} that the fractional quantum Hall state at Landau level filling $\nu = 1/2$ is metallic. However, the single-particle excitations are not dressed electrons, but so-called composite fermions\cite{2}, which are born out of the highly correlated motion of the two-dimensional electron gas in a strong external magnetic field. Composite fermions in the half filled Landau level can be viewed as spin-polarized electrons that are bound to a flux tube carrying two flux quanta. Theoretically the flux attachment can be modeled by coupling the electrons to a Chern-Simons gauge field\cite{3}. At the mean-field level, where fluctuations of the gauge field are ignored, the magnetic field generated by the Chern-Simons field exactly cancels the external magnetic field $B$, so that mean-field theory predicts that composite fermions in the half filled Landau level should behave like free spinless fermions without magnetic field, with Fermi wave-vector $k_F = (4\pi n_e)^{1/2}$. Here $n_e$ is the areal density of the electron gas.

The existence of a well-defined Fermi surface has been confirmed by several experimental techniques, such as surface acoustic wave-experiments, measurements of Shubnikov-de Haas oscillations, and magnetic focusing experiments\cite{4}. Although the energy dispersion of the composite fermions as function of momentum $k$ is not known, and a number of experiments have found a strong enhancement of the effective mass $m^*\geq m$, there exists general agreement that the composite fermions are well-defined quasi-particles.

Theoretically the situation is less clear. In their seminal paper Halperin, Lee, and Read\cite{5} showed that the leading self-energy correction due to fluctuations of the Chern-Simons field (Eq.(1) below) completely destroys the quasi-particle peak in the spectral function predicted by mean-field theory. There have been various attempts to calculate non-perturbatively the effect of gauge-field fluctuations on the single-particle Green’s function\cite{6,7}, but the results are contradictory. This is possibly due to the gauge-dependence of the single-particle Green’s function; yet, we expect physical quantities derived from it to be gauge-invariant. Until now, the experimental fact that composite fermions manifest themselves as well-defined quasi-particles could not be justified theoretically. In this Letter we shall give such a justification, using our recently developed higher-dimensional bosonization approach for curved Fermi surfaces\cite{8}.

Specifically, we show that composite fermions remain well-defined quasi-particles even if the infrared fluctuations of the Chern-Simons gauge field are taken into account. Remarkably, we also find that these fluctuations do not lead to a divergence of the effective mass.

To introduce our notation and to set the stage for the calculations that follow, we first evaluate the leading self-energy correction (the so-called GW self energy) due to fluctuations of the Chern-Simons field in the coordinate system shown in Fig.1. As we shall see, such a coordinate system plays a crucial role in our bosonization approach.

![Wave-vectors k and q measured with respect to an arbitrary point k' on the Fermi surface.](image)

To leading order in the RPA (random phase approximation) gauge-field propagator $g_{RPA,\alpha}^{\alpha\beta}$, the imaginary frequency self-energy of the composite fermions due to fluctuations of the gauge field can be written as\cite{3}

$$\Sigma_{GW}(\tilde{k}) = \frac{1}{\beta V} \sum_\tilde{q} g_{RPA,\alpha}^{\alpha\beta} G_{0}^{\alpha}(\tilde{k} + \tilde{q}) , \quad \text{(1)}$$

where $\beta$ is the inverse temperature, $V$ is the volume of the system, and $G_{0}^{\alpha}(\tilde{k}) = [i\tilde{\omega}_{n} - \xi_{k}^{\alpha}]^{-1}$ is the mean-field Green’s function. We use the notation $\tilde{k} = [k, i\tilde{\omega}_{n}]$, $\tilde{q} = [q, i\omega_{n}]$, with $\tilde{\omega}_{n} = 2\pi(n + 1/2)/\beta$ and $\omega_{n} = 2\pi n/\beta$. The superscript $\alpha$ indicates that wave-vectors are measured with respect to $k^\alpha$. Assuming a spherical Fermi surface, the mean-field energy dispersion is $\xi_{k}^{\alpha} = v^\alpha \cdot k + k_0^{2}/2m$, with

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where \( \mathbf{v}^\alpha = \mathbf{k}^\alpha /m \), and \( m \) is the mean-field mass of the composite fermions. Throughout this work we shall set \( \hbar = 1 \). In Coulomb gauge the propagator of the transverse Chern-Simons field is at long wave-lengths and for frequencies \( |\omega_n| \lesssim v_F |q| \) of the form

\[
h_{\text{RPA}, \alpha}^{\text{RPA}} = -\frac{2\pi}{m} \left( 1 - \frac{(\mathbf{k}^\alpha \cdot \hat{q})^2}{(q/m)^2 + |\omega_n|/v_F |q|} \right),
\]

where \( v_F = |\mathbf{v}^\alpha| \) and \( \mathbf{k}^\alpha = \mathbf{k}^\alpha /k_F \). The momentum scale \( q_c \) and the exponent \( \eta \) depend on the nature of the density-density interaction between the electrons. For long-range Coulomb forces \( q_c = k_F^2 m/e^2 \) and \( \eta = 1 \), while \( \eta = 2 \) if the Coulomb interaction is screened by metal plates. For simplicity we shall assume throughout this work that \( 1 < \eta \leq 2 \), and that \( q_c \ll k_F \). The mean-field mass \( m \) should be considered as an effective mass which takes into account Landau level mixing and short wave-length fluctuations not explicitly considered here. Simple physical arguments \([3]\) and approximate microscopic calculations \([3]\) yield \( m \propto (e^2/\ell)^{-1} \propto \sqrt{B} \), where \( \ell = \sqrt{e/(c B)} \) is the magnetic length. In this work we are interested in the infrared regime. We shall then impose an ultraviolet cutoff \( \kappa \) on all loop-integrations. For \( \kappa \gg q_c \), the infrared physics is independent of \( \kappa \).

To perform the integrations in Eq.\((3)\), we may restrict ourselves to external wave-vectors of the form \( \mathbf{k} = k || k^\alpha \), as shown in Fig.\(2\). The spherical symmetry implies that the usual self-energy \( \Sigma(k, i\omega_n) \) (where \( \mathbf{k} \) is measured with respect to the origin of the Fermi sphere) can be obtained by replacing \( k || \to |k| - k_F \) in \( \Sigma^\text{a}(k || k^\alpha, i\omega_n) \). The energy dispersion on the right-hand side of Eq.\((3)\) can then be written as \( \xi^\alpha_{k+q} = \xi_{k} + (1 + k^2_F/m) \mathbf{v}^\alpha \cdot \mathbf{q} + q^2_F/2m \), where \( \xi_{k} = v_F k || + k^2_F/(2m) \). Using the circular coordinates in Fig.\(2\), the \( \theta \)-integration yields

\[
\Sigma_{\text{GW}}^\alpha(k ||, i\omega_n) = \frac{1}{2\pi m (1 + k^2_F/m)} \int_0^\kappa dq \int_{-v_F q}^{v_F q} d\omega \frac{1}{|\omega| + \Gamma} Z(W(q, \omega)),
\]

with \( \Gamma_q = v_F q^{1+\eta}/q^2 \) and \( Z(W) = W - \sqrt{W^2 - 1} \), the root to be taken such that \( |Z| < 1 \). Here

\[
W(q, \omega) = \frac{G^{-1}_0 + i\omega - q^2}{v_F q (1 + k^2_F/m)},
\]

where \( G^{-1}_0 = i\omega_n - \xi_{k} \). For our purpose it is sufficient to know the asymptotic behavior of \( Z(W) \) for small \( W \), which is given by \( Z(W) \sim -\text{sgn}(\text{Im} W) + O(W) \). Because \( |\omega|/(v_F q) \lesssim 1 \) and \( q^2_F/2m \lesssim v_F q \) in the domain of integration in Eq.\((3)\), the condition \( |W| \ll 1 \) is equivalent with \( q \lesssim k_0 \), where \( k_0 = \max \{|k||, |\omega_n|/v_F| \} \). It is now easy to see that the leading singular behavior of \( \Sigma_{\text{GW}}^\alpha(k) \) for small wave-vectors and frequencies is due to the regime \( q \lesssim k_0 \) and \( \omega \lesssim |\omega_n| \). In this regime we may approximate \( Z(W(q, \omega)) \approx -\text{sgn}(\omega_n + \omega) \). Now the frequency integration in Eq.\((3)\) can be performed, and the dependence of the remaining one-dimensional \( q \)-integral on the external frequency can be scaled out. For convenience we define dimensionless momenta \( \bar{k}_n = \bar{k}_|| /q_c \) and energies \( \bar{\omega}_n = \omega_n / (v_F q_c) \). If \( \bar{k}_|| \) is not much larger than \( \bar{\omega}_n \), it is easy to show that to leading order \( \bar{k}_||^\alpha \)

\[
(v_F q_c)^{-1} \Sigma_{\text{GW}}^\alpha(\bar{k}_||, i\bar{\omega}_n) \sim -\text{sgn}(\bar{\omega}_n)|\bar{\omega}_n|^{-\eta/2} c_n g, \tag{5}
\]

where \( c_n \) is a positive numerical constant of the order of unity \([4]\) and \( g = q_c / k_F \). From Eq.\((3)\) it is obvious that, after analytic continuation \( i\bar{\omega}_n \to \omega + i\tau \), \( \text{Im} \Sigma_{\text{GW}}^\alpha \) has the same order of magnitude as \( \text{Re} \Sigma_{\text{GW}}^\alpha \), so that the spectral function does not exhibit a well-defined quasi-particle peak. Thus, lowest order perturbation theory suggests that composite fermions are \textit{not} well-defined quasi-particles, in contradiction with the experiments \([4]\). If one uses the maximum of the spectral function to define the energy of the renormalized quasi-particle \([4]\) (ignoring the fact that perturbation theory suggests that such a quasi-particle really cannot propagate because the damping is not small), one finds that the energy of the composite fermion vanishes as \( \bar{k}_|| \to \bar{k}_||^\alpha \) for \( \bar{k}_|| \to 0 \), indicating a divergence of the effective mass \([10]\). Although an enhanced \( m^* \) has been observed experimentally \([4]\) (see, however, Ref. \([3]\)), the observed enhancement is much stronger than the above theoretical prediction.

We now use our non-perturbative higher-dimensional bosonization formalism for a curved Fermi surface \( \bar{r} \) to gain insight into this paradox between experiment and lowest order perturbation theory. Our approach is most accurate for \( q_c / k_F \approx g \ll 1 \), and resums self-energy and vertex corrections in a systematic way to all orders in perturbation theory. Note that for Fermi surfaces with constant curvature it is not necessary to introduce the geometric patching construction used in earlier formulations of higher-dimensional bosonization \([11]\). Within the Gaussian approximation one obtains the following expression for the Matsubara Green’s function \( G^\alpha(k) \equiv G(k^\alpha + \mathbf{k}, i\omega_n) \),

\[
G^\alpha(k) = \int d\mathbf{r} \int_0^\beta d\tau e^{-i(k \cdot r - \omega_n \tau)} G^\alpha(\mathbf{r}, \tau), \tag{6}
\]

where \( G^\alpha(\mathbf{r}, \tau) = G^\alpha(\mathbf{r}, \tau + i\tau \eta) \). The Debye-Waller factor is of the form \( Q^\alpha(\mathbf{r}, \tau) \), with \( R^\alpha = S^\alpha(0,0) \) and

\[
S^\alpha(\mathbf{r}, \tau) = \frac{1}{\beta V} \sum_q h_{q \mathbf{k}}^{\text{RPA}, \alpha} \cos(\mathbf{q} \cdot (\mathbf{r} - \omega_n \tau) \beta) \frac{1 + Y^\alpha(\mathbf{k})}{|\bar{\omega}_n - \xi_{k}^\alpha - \Sigma_{\text{GW}}^\alpha(\mathbf{k})|}, \tag{7}
\]

The prefactor Green’s function \( \tilde{G}^\alpha(\mathbf{r}, \tau) \) is

\[
\tilde{G}^\alpha(\mathbf{r}, \tau) = \frac{1}{\beta V} \sum_q e^{i(k \cdot (\mathbf{r} - \omega_n \tau) \beta)} \frac{1 + Y^\alpha(\mathbf{k})}{|\bar{\omega}_n - \xi_{k}^\alpha - \Sigma_{\text{GW}}^\alpha(\mathbf{k})|}, \tag{8}
\]
with the prefactor self-energy
\[
\Sigma^\alpha_1(\tilde{k}) = -\frac{1}{\beta V} \sum_q \hbar^{\text{RPA},\alpha} G^\alpha(q) (\tilde{k} + \tilde{q}) \\
\times \frac{\mathbf{k} \cdot \mathbf{q}^2 + (\mathbf{k} \cdot \mathbf{q})^2}{m^2 |\omega_n - \xi^\alpha_q| |\omega_n + \xi^\alpha_q|}, \tag{9}
\]
and the vertex function
\[
Y^\alpha(\tilde{k}) = \frac{1}{\beta V} \sum_q \hbar^{\text{RPA},\alpha} G^\alpha(q) (\tilde{k} + \tilde{q}) \\
\times \frac{\mathbf{q}^2 + 2\mathbf{k} \cdot \mathbf{q}}{m |\omega_n - \xi^\alpha_q| |\omega_n + \xi^\alpha_q|}. \tag{10}
\]

Here \(G^\alpha_0(\tilde{k})\) is related to the self-energy in Eq. (6) via the Dyson equation \([G^\alpha_0(\tilde{k})]^{-1} = [G^\alpha_0(\tilde{k})]^{-1} - \Sigma^\alpha_1(\tilde{k})\). To leading order in \(g\) we may replace \(G^\alpha_0 \to G^\alpha_0\) on the right-hand sides of Eqs. (6) and (8). We shall further comment on this approximation below. Note that \(\Sigma^\alpha_1\) and \(Y^\alpha\) vanish for linearized energy dispersion, i.e., for \(1/m = 0\). If we expand Eq. (6) to first order in \(\hbar^{\text{RPA},\alpha}\), we exactly recover the leading term in perturbation theory [3].

Consider first the constant part \(R^\alpha\) of the Debye-Waller factor. The integrations can be done with the same strategy as above: we first perform the angular integration exactly, approximate the result in the regime \(|\omega| \lesssim v_F q\), and then perform the frequency integration. For \(\eta > 1\) we obtain \(R^\alpha = -r_n g\), where \(r_n\) is a positive constant of the order of unity. Because by assumption \(g \ll 1\), we conclude that \(R^\alpha\) is finite and small. Obviously the extra factor of \(\cos(\mathbf{q} \cdot \mathbf{r} - \omega_n \tau)\) in the expression for \(S^\alpha(\mathbf{r}, \tau)\) given in Eq. (6) cannot lead to new divergencies. In fact, according to the Fourier integral theorem the existence of \(S^\alpha(0, 0)\) implies that \(S^\alpha(\mathbf{r}, \tau)\) vanishes for large \(r\) or \(\tau\). Thus, the factor \(\cos(\mathbf{q} \cdot \mathbf{r} - \omega_n \tau)\) is always bounded and can be replaced by \(\cos(\mathbf{r} \cdot \mathbf{q})\) in the large-distance and long-time limit. If we had linearized the energy dispersion in Eq. (6), we would have obtained a Debye-Waller factor which diverges for large \(r\) or \(\tau\) [2]. Thus, the curvature of the Fermi surface is relevant in the sense that the asymptotic Ward-identity [2] leading to an exponentiation of the perturbation series for the real space Green’s function cannot be used to resum the dominant singularities. This is a consequence of the transversality of the gauge fields.

From Eqs. (6) and (8) we thus conclude that within higher-dimensional bosonization the Green’s function is
\[
G^\alpha(\tilde{k}) \approx \frac{e^{R^\alpha}}{i\omega_n - \xi^\alpha_k - \Sigma^\alpha_1(\tilde{k})}. \tag{11}
\]

The most singular contributions to \(\Sigma^\alpha_1(\tilde{k})\) and \(Y^\alpha(\tilde{k})\) come from the regime of large wave-vectors \(q \sim |\mathbf{k}_0|\) and small frequencies \(|\omega| \sim |\omega_n|\). So, the necessary integrations can be done in precisely the same manner as in the case of \(\Sigma^\alpha_{GW}\) discussed above. Retaining only the dominant terms for small \(\tilde{k}_0\) and \(\tilde{\omega}_n\), we find after some tedious but straightforward manipulations
\[
\Sigma^\alpha_1(k_\parallel, i\tilde{\omega}_n) \approx \frac{\text{sgn}(\tilde{\omega}_n)}{\pi m} \int_{k_0}^{\infty} dq q \ln \left(1 + \frac{|\tilde{\omega}_n|}{\Gamma_q}\right) \\
\times \frac{q^2}{G_0^{-1} - \frac{\omega^2}{m} - i\Gamma_q \text{sgn}(\tilde{\omega}_n) \frac{k_\parallel}{k_F}} \tag{12}
\]
\[
Y^\alpha(k_\parallel, i\tilde{\omega}_n) \approx \frac{\text{sgn}(\tilde{\omega}_n)}{\pi m} \int_{k_0}^{\infty} dq q \ln \left(1 + \frac{|\tilde{\omega}_n|}{\Gamma_q}\right) \\
\times \frac{q^2}{G_0^{-1} + \frac{k_\parallel}{k_F} \frac{\omega^2}{2m} - i\Gamma_q \text{sgn}(\tilde{\omega}_n)} \tag{13}
\]

Having performed the frequency integrations, we may now analytically continue these expressions to the real frequency axis. Defining \(\Sigma^\alpha_1(k_\parallel, \omega + i0^+) = \Sigma^\alpha_1 + i\Sigma^\prime_1\) and \(Y^\alpha(k_\parallel, \omega + i0^+) = Y^\prime + iY''\), the spectral function is according to Eq. (11) given by
\[
A(k_\parallel, \omega) = -\frac{e^{R^\alpha}}{\pi} \left[\frac{1 + Y^\prime + Y'' (\omega - \xi_{k_\parallel} - \Sigma^\alpha_1)}{(\omega - \xi_{k_\parallel} - \Sigma^\alpha_1)^2 + (Y'')^2}\right] . \tag{14}
\]

We are interested in the behavior of \(A(k_\parallel, \omega)\) for small \(k_\parallel = \tilde{k}_0/q_\xi\), and \(\tilde{\omega} = \omega/(v_F q_\xi)\), but for arbitrary \(\tilde{\omega}/\tilde{k}_0\). Although the precise form of \(A(k_\parallel, \omega)\) can only be calculated numerically [3], the qualitative behavior can be inferred from Eqs. (12) and (13), and is shown in Fig. 2.
FIG. 2. Qualitative behavior of the spectral function for small \( k_\parallel \) and \( \tilde{\omega} \). The subscript of \( k_\parallel \) is omitted, i.e. \( k_\parallel \) stands for \( |k_\parallel| - k_F \). Only the leading power laws are shown and logarithmic corrections are ignored. The quasi-particle damping \( \gamma_\parallel \) is given in Eq. (13). The curves with the arrows indicate the leading behavior for \( \omega \ll \xi_\parallel \) and \( \omega \gg \xi_\parallel \).

We first note that on resonance (i.e. for \( \omega = \xi_\parallel \)) the leading behavior of \( \Sigma_\parallel^\alpha(k_\parallel, \xi_\parallel + i0^+) \) agrees exactly with the perturbative self-energy \( \Sigma_\parallel^\alpha_W(k_\parallel, \xi_\parallel + i0^+) \) discussed above. This is due to the fact that on resonance \( G_\parallel^0 \to 0 \), so that the last factor in Eq. (12) is proportional to \( k_F/k_\parallel \) and cancels the small prefactor \( k_\parallel/k_F \). For the same reason we obtain from Eq. (13) for \( k_\parallel \to 0 \)

\[
\gamma_\parallel(k_\parallel, \xi_\parallel + i0^+) = \frac{4k_F \ln |k_\parallel|}{k_\parallel(1 + \eta)} \left[ \frac{\eta}{4} - \frac{\text{sgn}(k_\parallel)}{\pi} \ln |k_\parallel| \right].
\]

(15)

The important point is that on resonance this vertex leads to a drastic enhancement (\( \propto k_F/k_\parallel \)) of the spectral weight, and thus produces the quasi-particle peak in the spectral function shown in Fig. 2. The width \( \gamma_\parallel \) of the peak can be estimated from the condition that the term \( G_\parallel^0 \) in the denominator of the last factor in Eqs. (12) and (13) becomes comparable with \( \frac{k_F^2}{2m} \).

Keeping in mind that \( g^2 \) scales as \( |\tilde{\omega}|^{1/2} \), it is easy to see that this leads to \( |\omega - \xi_\parallel| \gg g^2 |k_\parallel|^{1/2} \) and so that

\[
\gamma_\parallel(k_\parallel, \xi_\parallel + i0^+) \approx g^2 (|k_\parallel|/k_F)^2 |k_\parallel|^{1/2},
\]

so that the width of the peak vanishes faster than \( \xi_\parallel \) as \( k_\parallel \to 0 \). We conclude that the quasi-particle is well-defined and has the same energy dispersion as the bare particle.

The effective mass \( m^* \) of the composite fermions can be identified with the mean-field mass \( m_\parallel \). Note, however, that \( \gamma_\parallel \) vanishes slower than \( k_\parallel^2 \), so that the damping is anomalously large. Off-resonance the self-energy contribution \( \Sigma_\parallel^\alpha \) is completely negligible. Then the behavior of the spectral function is determined by the imaginary part \( Y_\parallel \) of the vertex function. This is shown in Fig. 2. Obviously there is considerable weight off resonance, and the overall shape of the spectral function cannot be approximated by a Lorentzian. More accurate numerical results will be presented elsewhere [15].

Finally, let us justify the replacement \( G_\parallel^0 \to G_\parallel^0 \) on the right-hand sides of Eqs. (12) and (13). Because the integrals are dominated by the regime \( q \gg k_\parallel \) and \( |\omega| \gg |\tilde{\omega}| \), Eqs. (12) and (13) are determined by \( G_\parallel^0(k + \tilde{q}) \) off resonance. Recall that in this regime \( \Sigma_\parallel^\alpha \) is negligible.

In summary, using a controlled non-perturbative approach we have shown that the quasi-particle picture for composite fermions in the half-filled Landau level remains valid even if the infrared fluctuations of the Chern-Simons gauge field are taken into account, in agreement with the experimental fact [10] that composite fermions behave like non-interacting particles. Moreover, the infrared fluctuations do not lead to a singularity in the effective mass \( m^* \). Thus, the experimentally observed enhancement of \( m^* \) must have a different origin. We speculate that this is related to Landau level mixing or short-wavelength fluctuations, both of which are not correctly described by the Chern-Simons approach [3]. Indeed, there exists some experimental evidence for a correlation between Landau level mixing and an enhanced \( m^* \) in the experiment by Manoharan et al. [1], where the Landau level mixing was particularly strong, the increase of \( m^* \) close to \( \nu = \frac{1}{2} \) was larger than in the other experiments [1].

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[14] In deriving Eq. (12) we have ignored sub-leading terms which possibly yield a finite correction to $m$ proportional to $g$. This is justified because by assumption $g \ll 1$. 
