Torsional oscillations of magnetised neutron stars with mixed poloidal-toroidal fields

Gibran H. de Souza,1⋆ and Cecilia Chirenti,2†

1 “Gleb Wataghin” Institute of Physics, UNICAMP, Campinas - SP, 13083-859, Brazil
2 Center for Mathematics, Computation and Cognition, UFABC, Santo André - SP, 09210-580, Brazil

ABSTRACT

The quasiperiodic oscillations found in the three giant flares of soft gamma-ray repeaters observed to date have been interpreted as crustal oscillations caused by a starquake following a dramatic rearrangement of the stellar magnetic field. Motivated by these observations, we study the influence of the magnetic field geometry in the frequencies of the torsional oscillations of magnetised neutron stars. We use realistic tabulated equations of state for the core and crust of the stars and model their magnetic field as a dipole plus a toroidal component, using the relativistic Grad-Shafranov equation. The frequencies of the torsional modes are obtained by the numerical solution of the eigenvalue problem posed by the linear perturbation equations in the Cowling approximation. Our results show how the asteroseismology of these stars becomes complicated by the degeneracy in the frequencies due to the large relevant parameter space. However, we are able to propose a testable scenario in which the rearrangement of the magnetic field causes an evolution in the frequencies. Finally, we show that there is a magnetic field configuration that maximizes the energy in the perturbation at linear order, which could be related to the trigger of the giant flare.

Key words: dense matter – gravitational waves – stars: neutron – stars: oscillations

1 INTRODUCTION

Magnetars are neutron stars powered by their usually very strong magnetic fields, that can reach over $10^{15} \text{ G}$ (Turolla et al. 2015). Observations of soft gamma-ray repeaters (SGRs) seem to indicate that these objects are magnetars, with high magnetic fields and low rotation rates (Duncan & Thompson 1992; Thompson & Duncan 1995). There have been three giant flares associated with these objects observed so far, SGR 0526-66 in 1979 (Mazets et al. 1979; Barat et al. 1983), SGR 1900+14 in 1998 (Hurley et al. 1999) and SGR 1806-20 in 2004 (Terasawa et al. 2005; Palmer et al. 2005).

In the late-time tail of these events some quasiperiodic oscillations (QPOs) were detected. The QPOs observed in the giant flares of SGR 0526-66 (43.5 Hz) (Barat et al. 1983), SGR 1900+14 (28, 54, 84 and 155 Hz) (Strohmayer & Watts 2005) and SGR 1806-20 (18, 26, 29, 92.5, 150, 625.5 and 1837 Hz) (Israel et al. 2005; Watts & Strohmayer 2006; Strohmayer & Watts 2006) (see also a new analysis of the giant flares done by Pumpe et al. 2018 and a study of some short recurring bursts performed by Huppenkothen et al. 2014b) seem to indicate that these could be characteristic modes of oscillation of the stars that were excited by some catastrophic event, probably connected to the strong magnetic fields of these neutron stars (see Timokhin et al. 2008).

However, the precise origin of these oscillations is still unclear, and many different mechanisms have been proposed in the literature. The initial simple model of crustal torsional oscillations (Duncan 1998) was met with difficulties related to the coupling with Alfvén modes in the core (Levin 2006; Sotani et al. 2008; Gabler et al. 2011), and more recently it has been speculated that the geometry of the magnetic field could play an important role in the understanding of this problem (Link & van Eysden 2016).

Further complicating the problem, details of the crust can also change the analysis, see for instance Deibel et al. (2014). Other works have also included effects such as a superfluid or a pasta phase in the stellar core, see Gabler et al. (2018); Sotani et al. (2018) and other references therein.

Moreover, even the observational situation is not clear. Although claims have been made for QPOs at a variety of frequencies, the strength, significance and duration of each particular QPO remain uncertain. As an example, the 625 Hz QPO of SGR 1806-20 was studied in detail in Huppenkothen et al. (2014a), where it was concluded that...
it was probably present in the signal for only 0.5 s. More recently, a reanalysis of the data performed by Miller et al. (2018) presented evidence for no long lived QPOs, favouring instead a scenario with several re-excitations of the oscillations.

In this paper we study how the addition of a toroidal component to the dipolar magnetic field can modify the frequencies of the torsional modes of the crust. We do not aim here to present a model that can reproduce all of the observed frequencies for each star, for this would require some careful fine tuning of the stellar properties. Rather, we work with a comprehensive set of core and crust equations of state, and explore the parameter space in compactness, magnitude and geometry of the magnetic field.

This paper is organized as follows. In Section 2 we present our equilibrium models for the magnetised neutron stars. In Section 3 we discuss our analytical and numerical setup for solving the perturbation equations and obtaining the quasinormal frequencies in the relativistic Cowling approximation and we find some close-to-universal relations followed by the frequencies. In Section 4 we consider a quasi-static evolution scenario for the magnetic field of the neutron star and its consequences as a possible trigger for the giant flare. We present our final remarks in Section 5.

2 EQUILIBRIUM MODELS WITH MIXED MAGNETIC FIELD CONFIGURATIONS

The background spacetime of a rotating neutron star is given by a spherically symmetric line element of the form

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2(d\theta^2 + \sin^2 \theta (d\phi - \omega dt)^2),$$

(1)

where $\nu$, $\lambda$ and $\omega$ are functions only of the radial coordinate $r$, and a stress energy tensor given by:

$$T^{\alpha\beta} = (\rho + p)u^{\alpha}u^{\beta} + \left(\rho + \frac{H^2}{2}\right)\delta^{\alpha\beta} - H^\alpha H^\beta,$$

(2)

where $p$ is the fluid pressure, $\rho$ is the total energy density, $u^{\alpha}$ is the fluid 4-velocity and $H^{\alpha} = B^{\alpha}/4\pi$ is the magnetic field.

We describe the hydrostatic equilibrium and internal structure of our relativistic magnetised neutron stars with the Tolman-Oppenheimer-Volkoff (TOV) equations (Tolman 1939; Oppenheimer & Volkoff 1939). We use three different equations of state (EOSs) for the core and three different EOSs for the crust (each crust has also a different core-crust transition density) as summarised in Tables 1 and 2.

The mass-radius $M-R$ relation for the resulting equilibrium configurations is presented in Fig. 1. As we can see in the figure, the total mass and radius of the star are mostly determined by the core EOS. The crust EOS results in a correction on the radius of less massive configurations. For a 1.4 $M_\odot$ star, our choices for the crust EOS imply a variation of $\approx 1-2\%$ on the total radius of the star. However, the choice of the crust EOS will be the most important ingredient to determine the properties of the torsional oscillations of the star, as we will show in Section 3.

The shear modulus $\mu$ of the crusts we are considering will also be needed in our analysis in Section 3. We follow Sotani et al. (2007) and calculate the shear modulus in the zero temperature limit of eq. (13) from Strohmayer et al. (1991). Our results for the shear modulus as a function of the density $\rho$ are given in Fig. 2, where we present smooth composite polynomial fits for crusts Gs and SLy, which also describe the change of behaviour observed for $\rho \approx 3.8 \times 10^{11}$ g/cm$^3$, and for the older crust model NV we use a polynomial fit provided by Duncan (1998).

When considering possible magnetic field configurations for our models, we must remember that it has long been known that stars with purely toroidal magnetic field configurations are always unstable due to the Tayler instability (Tayler 1973), and that the purely poloidal field case is also unstable (Markay & Tayler 1973, 1974; Flowers & Ruderman 1977). A stable configuration requires a mixed poloidal-toroidal field inside the star, and the appearance of the field at the surface is a dipole with smaller contributions from higher multipoles (Braithwaite & Nordlund 2006; Braithwaite 2008).

In order to describe a relativistic neutron star with a general magnetic field configuration, including both poloidal

\begin{table}
\centering
\caption{The neutron star core equations of state used in this work.}
\begin{tabular}{|l|l|}
\hline
core EOS & Ref. \\
\hline
APR & Akmal et al. (1998) \\
H4 & Lackey et al. (2006) \\
SLy & Douchin & Haensel (2001) \\
\hline
\end{tabular}
\end{table}

\begin{table}
\centering
\caption{The neutron star crust equations of state used in this work, and the core-crust transition density $\rho_c$ used for each crust.}
\begin{tabular}{|l|l|l|}
\hline
crust EOS & $\rho_c$ [g/cm$^3$] & Ref. \\
\hline
Gs & 2.01 $\times 10^{14}$ & Steiner (2012) \\
NV & 2.40 $\times 10^{14}$ & Negele & Vautherin (1973) \\
SLy & 1.34 $\times 10^{14}$ & Steiner (2012) \\
\hline
\end{tabular}
\end{table}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.pdf}
\caption{Mass-radius relation for the crust+core equations of state used in our study. The core EOS determines the bulk properties of the star, but the crust EOS will be the most important ingredient for the characteristics of the torsional oscillations.}
\end{figure}
and toroidal components, we use the TOV equations to provide the metric coefficients of the spacetime and the fluid variables inside the star, and the relativistic Grad-Shafranov equation (Ioka & Sasaki 2004) to describe the magnetic field:

\[ e^{-\frac{1}{2}}a'' + \frac{1}{2}e^{-\frac{1}{2}}a'_1 + \left(2^2 - \frac{2}{r^2}\right) a_1 = 4\pi(\epsilon + p)\gamma^2 c_0, \]  

where \( \zeta \) is the ratio between the poloidal and toroidal magnetic field components, \( a_1(r) \) is the radial profile of the vector potential and \( c_0 \) is a constant. We have assumed a dipolar poloidal field, and the source of the magnetic field is a ring of current inside the star, as described by the right hand side of the equation. In this formalism we also assume that the magnetic field is not strong enough to deform the spherical symmetry of the star (Colaiuda et al. 2008; Ciolfi et al. 2009) and the magnetic field components are given by:

\[
B_r = \frac{e^{-\frac{1}{2}}(\sin \theta a_1,\theta)}{r^2 \sin \theta}, \quad B_\theta = -\frac{e^{-\frac{1}{2}}(\sin \theta a_1,\theta)}{r^2 \sin \theta}, \quad B_\phi = -\frac{\zeta}{\sin \theta}. \]

We work with a sequence of static magnetic field configurations, starting with a purely dipolar field and then supplementing it with an increasing toroidal component \( B_\phi \), obtained by increasing the parameter \( \zeta \). Typical examples of the magnetic field lines are presented in Fig. 3. As we can see in the sequence of panels with increasing toroidal field in the figure, the purely dipolar field lines are distorted by the presence of the toroidal field to the point where the field lines are defined in disjoint domains (Colaiuda et al. 2008). For each crust+core EOS used in our work, we determine the value of \( \zeta_{\text{max}} \) where this happens and restrict our range to \( \zeta < \zeta_{\text{max}} \). The influence of the toroidal field component on the torsional oscillations will come indirectly through this deformation of the poloidal field components, as we will show in our discussion of the linear perturbation equations in Section 3 below.

3 INFLUENCE OF THE TOROIDAL MAGNETIC FIELD COMPONENT IN THE TORSIONAL OSCILLATIONS OF THE CRUST

Working in the Cowling approximation (Cowling 1941) in which we neglect all metric perturbations and considering only barotropic perturbations (for a review on neutron star quasinormal modes, see Kokkotas & Schmidt 1999), the only perturbation variables for a non-rotating neutron star are \( \delta p, \delta \rho, \delta H^a \). The linear perturbation equations can be obtained by manipulations of the linearised Euler and energy conservation equations (\( i = r, \theta, \phi \)):

\[
\delta \left( \left( u^a_{\alpha} + u_{\alpha} \right) T^{\alpha \beta}_{\phi} \right) = 0, \quad \delta \left( u_{\alpha} T^{\alpha \beta}_{\phi} \right) = 0,
\]

together with the perturbed induction equations

\[
\delta \left( u^a H^b - H^a u^b \right) = 0,
\]

and with constraints given by the ideal MHD approximation and the 4-velocity normalisation condition, which can be subsequently used to reduce the number of independent variables to 7: \( \delta p, \delta \rho, \delta H^a \) (this formalism was used for the calculation of r-modes of magnetised neutron stars in a previous work, see Chirenti & Skákala 2013).

Here we restrict our study to the torsional modes of oscillation of the crust of a magnetised neutron star, and we analyse the influence of the magnetic field geometry on the mode frequencies. Therefore we neglect the coupling of the crustal modes to the fluid modes in the core (Levin 2006) and the coupling between axial and polar modes (Colaiuda & Kokkotas 2012). It is important to remark here that a full treatment of the coupled oscillations would require the study of global magneto-elastic oscillations (Colaiuda et al. 2009; van Hoven & Levin 2011; Gabler et al. 2011; Colaiuda & Kokkotas 2011).

Our treatment results in a considerable simplification of the problem, but interesting results for the torsional crustal frequencies can be achieved, and the trends we observe are expected to be independent of the neglected couplings to other modes. Although the toroidal component of the magnetic field \( B_\phi \) does not appear explicitly in the linear perturbation equations in our treatment, it still influences the results by inducing changes in the poloidal components \( B_r \) and \( B_\theta \) of the magnetic field through the \( \zeta \) parameter in the radial function of the vector potential \( a_1 \), see eqs. (3) and (4a)-(4c).

The only perturbed fluid variable present in the torsional modes of oscillation is, to first order, \( \delta \rho \), the perturbation in the azimuthal component of the 4-velocity of the stellar fluid (Schumaker & Thorne 1983), to which we must add the \( \delta H^a \) perturbations in the magnetic field components, in the case of magnetars (Duncan 1998). The formalism used for deriving the perturbation equations is known in the literature and we refer the reader to the derivation presented by Messios et al. (2001).

In a nutshell, both the non-vanishing components of the linearised stress shear tensor (Schumaker & Thorne 1983) and the \( \delta H^a \) components of the perturbed magnetic field (given by the linearised induction equation (7)) can be writ-
ten in terms of \( \delta \omega^\theta \), which can be given as
\[
\delta \omega^\theta = e^{-\tau} \frac{Y_l (P_l (\cos \theta)) \mu}{\sin \theta},
\]
where \( Y_l (r, \theta) = e^{iu \theta} Y_l (r) \) and \( P_l (\cos \theta) \) is the Legendre polynomial of order \( l \). The final radial perturbation equation for \( Y_l (r) \) can be put in the form
\[
\left( \mu + \frac{(1 + 2\lambda_1)(a_1)^2}{\pi r^4} \right) Y_{l,r} + \left( \frac{2 + 5\lambda_1}{2\pi r^4} \right) e^{-2\phi} = (\lambda + 1) \left( \frac{2 + 5\lambda_1}{2\pi r^4} \right) \left( \frac{\lambda_1 (a_1)^2}{2\pi r^4} \right) e^{-2\phi} Y_l,
\]
where \( \lambda = (l+1) \).\( \lambda_1 = -(l+1)/(2l+3) \) and we have neglected the \( l \pm 2 \) couplings that appear when we use eq.(8) to attempt to separate variables (Sotani et al. 2007). We solve this equation with a shooting method, imposing the zero traction condition at the base of the crust, \( r = R_c \), and zero torque at the surface of the star, \( r = R \).

Initially we tested our code for non-magnetised stars, generating the results presented in the upper plot of Fig. 4. Our results were compared with those of Sotani et al. (2007) for the frequency \( f_n \) of the fundamental \( (n = 0) \) mode with \( l = 2, 3, \ldots, 10 \) for their A core EOS with an SLy crust and different masses. We found that in all cases our results agreed within less than 3%. Figure 4 shows that the SLy crust always presents the highest values for the frequencies, followed by the NV and the Gs crusts; but for a given crust EOS, the SLy core presents the highest frequencies, followed by the APR and H4 cores. The difference in the frequencies can be quite large: for a \( 1.4M_\odot \) star, the difference in the frequencies across the different core+crust EOS combinations reaches \( \approx 45\% \). We note that the mode frequency, even in the simple non-magnetised case, is already degenerate with the mass and composition of the star.
To make it more difficult to and for the same stellar sequences presented in Sotani et al. (2019) that this effect persists for sequences with a fixed that this sequence of stars the total magnetic energy is also increasing. (However, we will show in Sec. 4 that this effect persists for sequences with a fixed magnetic field energy.) The fundamental $l = 2$ mode is the most sensitive to $\zeta$, presenting a variation of $\approx 50\%$ in our range of $\zeta$. The variation in the frequencies decreases with increasing $l$, and we also note that for $l > 7$ the behaviour is non-monotonic, but in these cases the variation is less than $2\%$.

Next we use our full set of 9 combinations of core+crust EOSs to explore the behaviour of the fundamental mode, and the results are reported in Fig. 6. We can see now that the crust EOS is crucial for determining the values of the frequencies. The SLy crust is the most sensitive to the variation of the magnetic field geometry, and the frequency values increase by $\approx 30\%$ in comparison with the purely dipolar case, while the variations in the case of the Gs and NV crusts are $\approx 15\%$. But we note that even if we fix the mass of the star and magnetic field at the pole, as we did in Fig. 6, the same frequency value can correspond to different EOSs with different magnetic field configurations. These results add to the those presented in Fig. 4 to make it more difficult to solve the inverse problem, that is, to obtain the parameters of the star from its asteroseismology.

In order to look for possible universal relations for these frequencies, we explored the way in which eq. (10) is modified by an increasing toroidal field component. We found that the parameter $\alpha_0$ is no longer a constant for a given stellar mass and EOS, but it is now a function of $\zeta$. As we report in Fig. 7 for the same stellar sequences presented in Fig. 6, $\alpha_0$ increases approximately linearly with $\zeta^2$. Therefore we propose a simple parametrisation of the form

$$2\alpha_0 = \alpha_1 (1 + \alpha_2 \zeta^2),$$

(11)

where for a pure dipole, $\alpha_0 = \alpha_1$, whereas $\alpha_2$ gives the coefficient of the correction due to the toroidal field component. We explored their dependence on the stellar parameters and looked for any noticeable trends, motivated by some universal relations known for neutron stars perturbations (see for instance Chirenti et al. 2015; Yagi & Yunes 2013). By inspection we found that $\alpha_1$ increases linearly with the square of the reciprocal of the compactness $M/R$, and this behaviour is approximately universal (EOS-independent), see the upper plot of Fig. 8. For $\alpha_2$, we found the opposite behaviour, although the spread due to the EOS is larger and mostly determined by the EOS of the crust, see the lower plot of Fig. 8.

4 QUASI-STATIC EVOLUTION FOR THE MAGNETIC FIELD

If the giant flare was caused by a rearrangement of the magnetic field of the star, it is plausible that it caused or was caused by a change in the magnetic field geometry. The frequencies of the QPOs could carry a signature of this process if their values are “drifting” with time. To simulate this scenario without artificially increasing the magnetic energy inside the star, we kept its constant as we increased $\zeta$ in our next analysis.

Our results for this quasi-static evolution scenario are presented in Fig. 9. Comparing Figs. 6 and 9, we see that the

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4.pdf}
\caption{Upper plot: Frequency $\omega_0$ of the fundamental ($n = 0$) $l = 2$ mode of torsional oscillations for non-magnetised stars with different masses. Lower plot: Same as the upper plot, but for magnetised stars with a fixed mass $M = 1.4M_\odot$ as a function of the magnetic field strength at the pole $B$, normalised by $B_\mu = 4 \times 10^{15}$ G.}
\end{figure}
behaviour is qualitatively similar: the frequencies increase with $\zeta$. However, this effect is less pronounced in 9, and the fractional variations are approximately half than those presented in Fig. 6. We believe that this is caused by the way in which the sequences of stars used in the two figures were constructed. In Fig. 6, the sequences of stars had constant magnetic field at the pole and an increasing toroidal component, which increased the total magnetic field energy, while in Fig. 9 a fixed total magnetic field energy is shared between the dipolar field and the increasing toroidal field. Therefore Fig. 9 also shows that the field configuration alone is also responsible for changing the frequencies of the torsional modes.

Such an evolution of the magnetic field geometry could be responsible for a trend in the behaviour of a long-lived QPO in the tail of a giant flare. If the field is rearranged from a more complicated mixed configuration (that could be caused by a persistent twisting of the field lines with the slow rotation of the magnetar) to a simpler pure dipole, the frequencies of the QPOs could go down in a noticeable way, if they can be determined with a resolution of 1 Hz or lower.

Lastly we performed an analysis of the energy in the perturbation, given by integral of the $\delta T^{tt}$ component of the linearised stress energy tensor over the volume of the crust. For the torsional modes of the crust, we have

$$\delta T^{tt} = 2H^{\alpha} \delta H_{\alpha} e^{-2\nu},$$

and after we substitute the magnetic field components given by eqs. (4a)-(4c) (we remind that $H^{\alpha} = B^{\alpha}/4\pi$), and the perturbations $\delta H^{\alpha}$ in the magnetic field given by the induction equations (7) in terms of $\delta \nu^{\alpha}$, given by eq. (8), we obtain the total energy of the mode with the volume integral

$$\int_V \delta T^{tt} dV = \frac{8}{3} \sqrt{\pi} \int_{R_c}^R r^3 \left[ a_1 Y_0 + 2a_1 r Y_1 \right] e^{\nu dr} dr,$$

where the only non-zero contribution comes from the $l = 2$ term. We remind here that our linear perturbation treatment of the modes does not allow us to make predictions on the amplitude of the mode. However, we can see that it depends linearly on the amplitude of the oscillation and quadratically on the magnetic field amplitude, as expected. We also stress

Figure 5. Influence of the magnetic field geometry on the frequencies of the first modes of torsional oscillations for a sequence of 1.4 $M_\odot$ SLy+SLy stars. The sequence has a fixed amplitude $10^{-15}$ G of the magnetic field at the pole and increasing toroidal field starting with a pure dipole configuration at $\zeta = 0$. The lowest modes are more sensitive to $\zeta$, but the effect of increasing the toroidal field component becomes non-monotonic for higher modes.

Figure 6. Same as Fig. 5, but only for the $l = 2$ mode and for our set of 9 combinations of core+crust EOSs. The crust EOS is the most relevant for the behaviour of the frequencies of the torsional oscillations.

Figure 7. Behaviour of the coefficient $\delta a_0$ of the magnetic field correction to the fundamental torsional frequency, defined in eq. (10), as a function of the magnetic field geometry as parametrised by $\zeta$. 

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that this lowest order contribution to the energy is present only in configurations with a toroidal field component and goes to zero as $\zeta \to 0$.

An unexpected result appears when we analyse the energy in the mode as a function of the magnetic field geometry, as we can see in Fig. 10. The change in the energy is non-monotonic with $\zeta$, and each combination of core+crust EOS has a magnetic field configuration that maximizes the energy at linear order in the amplitude of the mode.

Our results motivate us to propose a possible mechanism for the still unknown trigger of the magnetar giant flare. If the magnetic field configuration evolves over time, as oscillation modes could be constantly present in the crust (excited by magnetic stresses, for example), the energy in these modes would peak for a certain contribution of the toroidal field component, at which point the flare could be triggered. A higher order analysis of this effect should be performed, of course, before any strong statements can be made. For instance, the increase in the energy of the modes should come from the magnetic field of the star, but the linear perturbation calculation does not include this backreaction. However, our analysis already points to a non-trivial behaviour of the energy as a function of $\zeta$, which should be further explored.

5 CONCLUSIONS

The interiors of neutron stars, from their crusts to their cores, represent states of matter that cannot be accessed on Earth. Their study therefore allows us to glimpse physics in a realm that constructively challenges our understanding of nuclear and condensed matter physics. Since the first re-
port of QPOs from SGR giant flares it has been thought that a full understanding of the QPOs could give us unique insight into this mysterious realm. There are also some hopes that these giant flares, and possibly also the QPOs might eventually be detectable as gravitational wave events (Corsi & Owen 2011; Quitzow-James et al. 2017), but see also Levin & van Hoven (2011), perhaps by third-generation detectors such as the Einstein Telescope (Punturo et al. 2010) or the Cosmic Explorer (Abbott et al. 2017).

We have presented here a careful analysis of the frequencies of torsional modes of oscillation of the crust for magnetised neutron stars. Motivated by the observations of QPOs in the tail of the observed giant flares of SGRs and the prospect of doing neutron star asteroseismology, we explored the influence of the magnetic field geometry, encoded in the variable ζ in the values of the frequencies. We found that the frequency of the fundamental l = 2 torsional mode can change by up to ≈ 30% with the variation of the magnetic field geometry. This variation would be detectable if the frequencies can be determined with a resolution of 1 Hz or lower, and could point to an overall evolution of the magnetic field geometry.

The solution of the inverse problem, that is, the calculation of the parameters of the star from its observed frequencies of oscillation becomes complicated by the multidimensionality of the parameter space in this case. However, we were able to make considerable progress towards an EOS-independent relation for the frequency of the l = 2 fundamental mode in the case when the magnetic field is (close to) a pure dipole.

Finally, we examined the energy in the mode and found a dominant contribution in the linear order, that appears only in magnetic fields with a toroidal component. This energy presents a maximum for a specific (EOS-dependent) magnetic field configuration, and we proposed that it could be linked to the still unknown mechanism behind the triggering of the giant flare.

ACKNOWLEDGEMENTS

We thank Hajime Sotani for sharing the A EOS table, which we used in our initial code tests, and Andrew Steiner for providing the Gs and SLy crust EOS tables used in this work. We also thank Kostas Kokkotas and Cole Miller for useful comments and suggestions. This work was supported by CAPES and the São Paulo Research Foundation (FAPESP Grant 2015/20433-4).

REFERENCES

Abbott B. P., et al., 2017, Classical and Quantum Gravity, 34, 044001
Aknal A., Pandharipande V. R., Ravenhall D. G., 1998, Phys. Rev. C, 58, 1804
Barat C., et al., 1983, A&A, 126, 400
Braithwaite J., 2008, MNRAS, 386, 1947
Braithwaite J., Nordlund Å., 2006, A&A, 450, 1077
Chirenti C., Skákala J., 2013, Phys. Rev. D, 88, 104018
Chirenti C., de Souza G. H., Kastaun W., 2015, Phys. Rev. D, 91, 044034
Ciolfi R., Ferrari V., Gualtieri L., Pons J. A., 2009, MNRAS, 397, 913
Colaiuda A., Kokkotas K. D., 2011, MNRAS, 414, 3014
Colaiuda A., Kokkotas K. D., 2012, MNRAS, 423, 811
Colaiuda A., Ferrari V., Gualtieri L., Pons J. A., 2008, MNRAS, 385, 2689
Colaiuda A., Beyer H., Kokkotas K. D., 2009, MNRAS, 396, 1441
Corsi A., Owen B. J., 2011, Phys. Rev. D, 83, 104014
Cowling T. G., 1941, MNRAS, 101, 367
Deibel A. T., Steiner A. W., Brown E. F., 2014, Phys. Rev. C, 90, 025802
Douchin F., Haensel P., 2001, A&A, 380, 151
Duncan R. C., 1998, ApJ, 498, L45
Duncan R. C., Thompson C., 1992, ApJ, 392, L9
Flowers E., Ruderman M. A., 1977, ApJ, 215, 302
Gabler M., Cerdá-Durán P., Font J. A., Müller E., Stergioulas N., 2011, MNRAS, 410, L37
Gabler M., Cerdá-Durán P., Stergioulas N., Font J. A., Müller E., 2018, MNRAS, 476, 4199
Huppenkothen D., Watts A. L., Levin Y., 2014a, ApJ, 793, 129
Huppenkothen D., Heil L. M., Watts A. L., Gökşen E., 2014b, ApJ, 795, 114
Hurley K., et al., 1999, Nature, 397, 41
Ioka K., Sasaki M., 2004, Astrophys. J., 600, 296
Israel G. L., et al., 2005, ApJ, 628, L53
Kokkotas K. D., Schmidt B. G., 1999, Living Reviews in Relativity, 2, 2
Lackey B. D., Nayyar M., Owen B. J., 2006, Phys. Rev. D, 73, 024021
Levin Y., 2006, MNRAS, 368, L35
Levin Y., van Hoven M., 2011, MNRAS, 418, 659
Link B., van Eysden C. A., 2016, ApJ, 823, L1
Markey P., Taylor R. J., 1973, MNRAS, 163, 77
Markey P., Taylor R. J., 1974, MNRAS, 168, 505
Mazets E. P., Golentskii S. V., Ilnitskii V. N., Aptekar R. L., Guryan I. A., 1979, Nature, 282, 587
Messios N., Papadopoulos D. B., Stergioulas N., 2001, MNRAS, 328, 1161
Miller M. C., Chirenti C., Strohmaier T., 2018
Negele J. W., Vautherin D., 1973, Nuclear Physics A, 207, 298
Oppenheimer J. R., Volkoff G. M., 1939, Physical Review, 55, 374
Palmer D. M., et al., 2005, Nature, 434, 1107
Pumpe D., Gabler M., Steininger T., Enßlin T. A., 2018, A&A, 610, A61
Punturo M., et al., 2010, Classical and Quantum Gravity, 27, 194002
Quitzow-James R., et al., 2017, Classical and Quantum Gravity, 34, 164002
Schumaker B. L., Thorne K. S., 1983, MNRAS, 203, 457
Sotani H., Kokkotas K. D., Stergioulas N., 2007, MNRAS, 375, 261
Sotani H., Kokkotas K. D., Stergioulas N., 2008, MNRAS, 385, L5
Sotani H., Iida K., Oyamatsu K., 2018, MNRAS, 479, 4735
Steiner A. W., 2012, Phys. Rev. C, 85, 055804
Strohmaier T. E., Watts A. L., 2005, ApJ, 632, L111
Strohmaier T. E., Watts A. L., 2006, ApJ, 653, 593
Strohmaier T., Ogata S., Iyetomi H., Ichimaru S., van Horn H. M., 1999, ApJ, 517, 679
Taylor R. J., 1973, MNRAS, 161, 365
Terasawa T., et al., 2005, Nature, 434, 1110
Thompson C., Duncan R. C., 1995, MNRAS, 275, 255
Timokhin A. N., Eichler D., Lyubarsky Y., 2008, ApJ, 680, 1398
Tolman R. C., 1939, Physical Review, 55, 364
Turolla R., Zane S., Watts A. L., 2015, Reports on Progress in Physics, 78, 116901
Watts A. L., Strohmaier T. E., 2006, ApJ, 637, L117
Yagi K., Yunes N., 2013, Phys. Rev. D, 88, 023009
van Hoven M., Levin Y., 2011, MNRAS, 410, 1036
