Research on Power Flow Calculation of Distribution Network with Distributed Power Based on Optimal Multiplier

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Abstract. With a large number of distributed power connected to the grid, there are many new node types in the distribution network. The traditional power flow calculation method cannot be applied to the distribution network with multiple distributed generations. Three kinds of distributed power grid model such as synchronous machine, asynchronous machine and inverter are established, according to the characteristics of different distributed power. The P0 node model is proposed for nodes with reactive power output as active power output and terminal voltage function. To improve the condition number of Jacobian matrix included P0 node model, the optimal multiplier is introduced to correct Jacobian matrix. Through the simulation of a variety of operation mode in certain district grid, the results show that the introduction of distributed generation has effects on the process of power flow calculation. The optimal multiplier can effectively improve the ill-posed problem of power flow caused by P0 node model, and increase the fast convergence speed of power flow calculation.

1. Introduction

With the development of economy and the increasing capacity of power grid, the hidden danger of power supply of large power grid is becoming more and more prominent. For example, the large-scale blackout in southern China caused by the snow and ice disaster in 2008 and the "8.14" blackout in the United States in 2003 all illustrate this point. In these accidents, distributed power generation plays an important role in the power supply of important loads and the restoration of the power grid. It has become the development trend of today's power to take distributed power generation as a supplement to the large power grid [1-3].

A large number of grid-connected operation of distributed power supply is a beneficial supplement to large power grid, but it also brings many negative effects to power grid and increases the difficulty of power grid analysis. At present, a series of researches have been carried out on power flow calculation of distribution network with distributed power supply. In literature [4], the node model of distributed power supply is established, but the model is not perfect, and the solution is solved by multiple iteration. The convergence speed is linear, and the convergence speed of the flat method is not exerted. According to literature [5], distributed power supply is a controllable dynamic load. Under rated conditions, active power and reactive power are almost unchanged, and it is used as a PQ node in power flow calculation. Literature [6] studies and analyzes that distributed power supply is PV node. Sensitivity matrix is used to eliminate voltage deviation value, and forward and back substitution method is used to solve power flow. Literature [7] analyzed whether distributed power supply is PV or PQ type node according to the specific situation. PV node and the connected bus are also regarded as PV node, and PQ node is equal to the load model with constant power. In literature [8], the instability characteristics of photovoltaic and wind power are studied and analyzed, and the
stochastic mathematical model of distributed power generation is established. Literature [9] establishes different models according to the operation characteristics of different distributed power sources.

Most of the above studies are based on PV and PQ node models with distributed power supply, and the calculation is carried out through power flow calculation methods, such as the backward and forward substitution method and the Newton-Roughson method. In view of this, the influence of distributed power generation on power flow calculation of distribution network is analyzed, the corresponding node model of distributed power generation is established, and a new node model of P0 is proposed. Finally, the optimal multiplier is used to modify the Newton-Raphson method, which increases the convergence speed of power flow calculation in distribution network.

2. Node model analysis of distributed power
In this paper, distributed power generation is divided into three grid-connected models, synchronous generator, asynchronous generator and inverter, according to its power generation principle and control mode.

2.1. Synchronous generator grid-connected model
The adjustable excitation synchronous generator can control the terminal voltage by adjusting the excitation, and can be treated as PV node [10]. If the influence of stator on the rotor is ignored, the synchronous generator with unregulated excitation has the same magnetic coupling characteristics as the permanent magnet synchronous generator. When implicit pole generator model is adopted, its reactive power output is shown in Eq (1).

\[ Q = \frac{E_0^2 U_0^2}{x_d^2} - \frac{P^2}{x_d} \]  

Where: \( E_0 \) is no-load electromotive force; \( U \) is terminal voltage amplitude; \( x_d \) is the direct axis synchronous reactance.

It can be seen from Eq (1) that reactive power output is a function of active power output and terminal voltage amplitude. P0 node is defined in this paper to describe this characteristic. Specifically: if the active power output of the generator is controllable, the terminal voltage and reactive power output are not controllable. And the reactive power output is a function of the active power output and terminal voltage, then this node is called P0 node. Its mathematical expression is as follows:

\[
\begin{cases}
  P = c \\
  Q = f(P, U, \theta)
\end{cases}
\]  

(2)

2.2. Induction generator grid-connected model
Constant speed and constant frequency wind power generation system generally adopts asynchronous generator grid-connection, and the terminal is equipped with capacitors. As the reactive power emitted by the capacitor is affected by terminal voltage, the excitation of the asynchronous generator is uncontrollable and will change with the running state of the power grid [11]. Its simplified equivalent circuit model is shown in Fig 1.

Fig 1 The simplified equivalent circuit of asynchronous generator
In the fig 1, \( U \) represents the terminal voltage and \( x_c \) represents the impedance of the capacitor. Let \( R_s = r_s + \frac{r_e}{s} \), where \( s \) represents the slip of the asynchronous generator, then the following expression can be obtained:

\[
P = -\frac{U^2r_m}{r_m^2 + x_m^2} - \frac{R_sU^2}{r_m^2 + x_m^2}
\]

\[
Q = -\frac{U^2 - U^2x_m}{x_c} - \frac{x_cU^2}{r_m^2 + x_m^2}
\]

Since the slip \( s \) is less than 0 when the asynchronous motor is working in the generating state, the expression of \( R_s \) can be obtained according to the active power equation in Eq (3).

\[
R_s = \frac{-U^2 + \sqrt{U^4 - 4x_c^2P_s^2}}{2P_s}
\]

Among them: \( P_s = P + \frac{U^2r_m}{r_m^2 + x_m^2} \). By substituting Eq (4) into the reactive power equation of Eq (3), it can be seen that the reactive power output is a function of \( P \) and \( U \), so when the asynchronous generator is grid-connected, it can be regarded as a P0 node.

2.3. Inverter grid-connected model

The inverter can realize the decoupling control of active power and reactive power, and can conveniently realize constant power factor operation. When phase-locked loop technology is applied, the power factor can be constant as 1, so the PQ model is often used to connect the distributed power supply through the inverter. For doubly-fed wind power generation system, because the excitation is connected to the grid through the inverter, it is convenient to accurately adjust the excitation voltage, current and phase by controlling the inverter, so the PQ or PV node model can be adopted [12].

3. Optimal Multiplier Modified Newton-Roughson Flow Flow Calculation

3.1. Power flow calculation and analysis with distributed power

In PQ, PV and equilibrium node models, the power flow equation is shown in Eq (5):

\[
S = \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} p(u, \theta) \\ q(u, \theta) \end{bmatrix}
\]

Where, \( P \) is the active power injection column vector of PQ and PV nodes; \( Q \) is the reactive power injection column vector of PQ node; \( u \) and \( \theta \) are the amplitude and phase Angle column vectors of node voltage respectively. The \( k \) iteration calculation equation is shown as following:

\[
\Delta S_k = J_k \begin{bmatrix} \Delta U_k \\ \Delta \theta_k \end{bmatrix} = \begin{bmatrix} H_k & M_k \\ N_k & L_k \end{bmatrix} \begin{bmatrix} \Delta U_k \\ \Delta \theta_k \end{bmatrix}
\]

Where: \( J_k \) is the Jacobian matrix in the \( k \) iteration, and \( H_k = \frac{\partial p}{\partial u_k}, M_k = \frac{\partial p}{\partial \theta_k}, N_k = \frac{\partial q}{\partial u_k}, L_k = \frac{\partial q}{\partial \theta_k} \).

After introducing the P0 node model defined in this paper, the power flow equation of the power grid becomes Eq (7):

\[
S = \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ f(P, u, \theta) \end{bmatrix} \begin{bmatrix} p(u, \theta) \\ q(u, \theta) \end{bmatrix}
\]
Where, \( f(P,u,\theta) \) is the column vector composed of the reactive power output functions of all P0 nodes. The introduction of \( f(P,u,\theta) \) function will have two influences on the flow calculation of Newton-Raphson method. First, it affects \( \Delta S \), but it doesn't affect the convergence of Newton-Raphson; Secondly, the value of the Jacobian matrix will be changed, which may lead to the singularity of the Jacobian matrix and make the convergence of the Newton-Raphson worse or even not convergent.

3.2. Optimal Multiplier Correction of Jacobian Matrix

As can be seen from the above analysis, only node voltage amplitude changes in iteration \( f(P,u,\theta) \), so it can be rewritten as \( Q = f(u) \). Therefore, when solving Eq (7) by using the Newton-Roughson method, only \( N \) in the Jacobian matrix will be affected by the P0 node model and change the condition number of the Jacobian matrix.

In view of the influence of P0 node model on the Jacobian matrix, this paper introduces damping factor \( \mu_k \) to modify the Jacobian matrix, and increases the main diagonal element of the Jacobian matrix under the condition of \( \|J_{\mu_k}\| \leq \|J\| \), so as to ensure the convergence of the Newton-Roughson method. The optimal multiplier is calculated and taken as the damping factor in this paper. The modified \( k \) iteration equation is shown in Eq (8):

\[
\Delta S_k = [J_k + \mu_k I] \begin{bmatrix} \Delta \mu_k \\ \Delta \theta_k \end{bmatrix}
\]

4. Analysis of examples

In this paper, the influence of distributed power node model and P0 node model on power flow calculation is simulated and analyzed. The grid structure is shown in Fig 2, where G represents the large grid and C represents the capacitors. The four schemes are as follows: (1) no distributed power supply, (2) set the distributed power supply of the P0 node model at No. 233 node, (3) set the distributed power supply of the P0 node model at three nodes (203, 223 and 233), and (4) adjust the parameters of the P0 node model based on the third case. The Newton-Raphson method and the optimal multiplier modified Newton-Raphson method are used to calculate and analyze the four cases respectively, and the specific situation is shown in Tab 1, Tab 2 and Tab 3.

Fig.2  Wiring diagram of a district grid
Table 1 Converging process of Newton-Raphson method

| Iterations | Verge distance | Condition number |
|------------|----------------|------------------|
|            | 0   | 1   | 3   | 0   | 1   | 3   |
| 1          | 0.90| 0.90| 0.82| 1853| 2112| 2632|
| 2          | 0.18| 0.17| 0.20| 1701| 2131| 2648|
| 3          | 0.00| 0.05| 0.03| 1690| 2098| 2637|
| 4          | 3.1e-7| 7.4e-3| 3.0e-3| 1691| 2095| 2629|
| 5          | 2.3e-14| 8.3e-4| 5.1e-4| 1691| 2095| 2623|
| 6          | -   | 2.8e-5| 3.2e-5| -   | 2095| 2619|
| 7          | -   | 1.0e-7| 7.2e-6| -   | 2095| 2619|
| 8          | -   | 8.1e-9| 3.6e-6| -   | 2095| 2619|
| 9          | -   | -   | 7.5e-7| -   | -   | 2619|
| 10         | -   | -   | 1.5e-8| -   | -   | 2619|

As can be seen from Tab 1, when there is no P0 node, the Newton-Raphson method converges for 5 iterations. When there are 1 and 3 P0 nodes, the Newton-Raphson method iterates 8 times and 10 times of financial convergence, respectively. The convergence becomes worse with the increase of the number of nodes in P0, because the condition number of the Jacobian becomes larger with the increase of the number of nodes in P0.

Table 2 Converging process of Newton-Raphson method corrected by optimal multiplier

| Iterations | Verge distance | Condition number |
|------------|----------------|------------------|
|            | 0   | 1   | 3   | 0   | 1   | 3   |
| 1          | 0.90| 0.99| 0.62| 1853| 2311| 2461|
| 2          | 0.18| 0.26| 0.59| 1701| 2115| 2469|
| 3          | 2.4e-3| 5.1e-3| 0.13| 1690| 2018| 2396|
| 4          | 3.1e-7| 4.5e-5| 5.2e-3| 1691| 2002| 2395|
| 5          | 2.3e-14| 2.9e-7| 8.8e-5| 1691| 2001| 2395|
| 6          | -   | 2.4e-7| 2.4e-7| -   | 1992| 2391|
| 7          | -   | 1.2e-8| 1.2e-8| -   | -   | 2390|
| 8          | -   | -   | -   | -   | -   | -   |
| 9          | -   | -   | -   | -   | -   | -   |
| 10         | -   | -   | -   | -   | -   | -   |

It can be seen from Tab 2 that the optimal multiplier can change the condition number of the Jacobian. The effect of P0 node model on the convergence of Newton-Raphson method is overcome effectively, and the convergence speed is accelerated. And the convergence speed of the Newton-Raphson method does not slow down with the increase of P0 nodes.

Table 3 Converging process after modifying parameter of P0 node

| Iterations | Newton-Raphson method | Optimal multiplier method |
|------------|------------------------|---------------------------|
|            | Verge distance | Condition number | Verge distance | Condition number |
| 1          | 3.83       | 3096           | 2.36       | 1923           |
| 2          | 1.93       | 451            | 4.22       | 1978           |
| 3          | 7.29       | 2045           | 2.14       | 1967           |
| 4          | 10.45      | 1212           | 1.03       | 1956           |
| 5          | 5.32       | 14.7           | 1.7e-2     | 1955           |
| 6          | 39.81      | 4049           | 3.3e-3     | 1953           |
| 7          | 4.75       | 1125           | 6.1e-5     | 1950           |
| 8          | 1.55       | 21.3           | 2.1e-7     | 1950           |
| 9          | 0.73       | 2314           | 6.7e-8     | 1950           |
| 10         | 2.46       | 67.8           | -          | -              |

As can be seen from Tab 3, after adjusting the parameters of P0 nodes, the condition number of Jacobian matrix changes disorderly and the calculation does not converge when the Newton-Raphson
method is used to solve the problem. The results show that the optimal multiplier correction can improve the convergence performance of the Newton-Raphson method.

5. Conclusions
According to the output characteristics of distributed power generation, the P0 node model is proposed in this paper. This model will make the condition of Jacobian matrix worse and the convergence of power flow calculation worse. In view of the influence of the P0 node model on the power flow equation and the Newton-Raphson method, the optimal multiplier is used to correct it. The convergence speed is improved obviously, and the convergence has little relation with the number of P0 nodes, which can effectively improve the ill-posed power flow caused by the P0 node model. In addition, the research content of this paper still needs to be further studied. For example, in the large-scale distributed power distribution network, how to use the relevant results of this paper to further optimize the power flow calculation method and improve the accuracy and speed of power flow calculation will be the focus of the next research.

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