Mathematical modeling of impact of two metal plates using two-fluid approach

P S Utkin\textsuperscript{1,2} and S V Fortova\textsuperscript{1}

\textsuperscript{1} Institute for Computer-Aided Design of the Russian Academy of Sciences, Vtoraya Brestskaya 19/18, Moscow 123056, Russia
\textsuperscript{2} Moscow Institute of Physics and Technology, Institutskiy Pereulok 9, Dolgoprudny, Moscow Region 141700, Russia
E-mail: pavel_utk@mail.ru

Abstract. The paper is devoted to the development of the two-fluid mathematical model and the computational algorithm for the modeling of two metal plates impact. In one-dimensional case the governing system of equations comprises seven equations: three conservation laws for each fluid and transfer equation for the volume fraction of one of the fluids. Both fluids are considered to be compressible and equilibrium on velocities. Pressures equilibrium is used as fluids interface condition. The system has hyperbolic type but could not be written in the conservative form because of nozzling terms in the right-hand side of the equations. The algorithm is based on the Harten–Lax–van Leer numerical flux function. The robust computation in the presence of the interface boundary is carried out due to the special pressure relaxation procedure. The problem is solved using stiffened gas equations of state for each fluid. The parameters in the equations of state are calibrated using the results of computations using wide-range equations of state for the metals. In simulations of metal plates impact we get two shocks after the initial impact that propagate to the free surfaces of the samples. The characteristics of shock waves are close (maximum relative error in characteristics of shocks is not greater than 7\%) to the data from the wide-range equations of states computations.

1. Introduction

The problem of high-speed impact of metal plates is important from fundamental and practical points of view. The problem has a long history of investigations and for the theoretical studies different numerical approaches were used each with its own pros and cons. In the early studies of S K Godunov with co-authors the problem was considered in two-dimensional Lagrangian statement \cite{1, 2}. In \cite{3, 4} as a result of two- and tree-dimensional gas-dynamics modeling the multidimensional structures at the contact surface were obtained and the assumptions about the mechanisms of the structures formation were done. In the recent paper \cite{5} the numerical simulations of wave formation under an oblique impact of metal plates during explosion welding on the basis of Maxwell relaxation model and molecular dynamics method were carried out. It was shown that the numerical simulation correctly reproduces the formation and evolution of waves on the contact boundary. The problem was also considered in \cite{6} using powerful Euler-Lagrangian three-dimensional computer code taking into account wide-range equations of state (EOS) of the metals.

The problem of high-speed impact of two plates is actually two-fluid problem at least at the initial stages of the process when the metals behave as non-mixing compressible fluids. At
the same time there is a lack of the works with consideration of the problem using the models of heterogeneous media mechanics. The goal of the paper is the formulation of the two-fluid mathematical model, computational algorithm and fitting of the parameters of the numerical technology for both qualitative and quantitative description of the initial stage of two metal plates impact in the statement of [7].

2. Statement of the problem
Consider the interaction of the lead plate with the thickness 2 mm and the initial density 11300 kg/m³ with the steel plate with the thickness 3 mm and the initial density 7900 kg/m³ (figure 1). The lead plate is thrown to the direction of steel one with the velocity 500 m/s. It is assumed that at the initial stage of the impact during first 10 μs the metals behave as pseudo-fluids [7] so the gas dynamics approach is valid. The initial pressure is 10⁵ Pa everywhere.

So the computational area is the interval [0 mm; 5 mm]. We set the inflow conditions at the boundaries x = 0 mm and x = 5 mm. The inflow values at x = 0 mm corresponds to the initial conditions for the steel plate and at x = 5 mm to the lead one. As a result of metal plates interaction two shock waves (SW) are formed. The computation lasts up to the moment of SW arrival to the boundaries x = 0 mm or x = 5 mm. Computational grid is uniform with the cell size Δx = 10⁻³ mm and contains 5 000 cells.

3. Mathematical model
Mathematical model is based on the system of equations from [8] which describes two-fluid compressible flows:

\[
\begin{align*}
\mathbf{u}_t + \mathbf{f}(\mathbf{u})_x &= \mathbf{h}(\mathbf{u})/\bar{\alpha}_x + \mathbf{p}, \\
\mathbf{u} &= \begin{bmatrix} \bar{\alpha} \\ \bar{\rho} \\ \bar{\rho} \bar{v} \\ \bar{\rho} \bar{E} \\ \alpha \rho v \\ \alpha \rho E \end{bmatrix}, \\
\mathbf{f} &= \begin{bmatrix} 0 \\ \bar{\alpha} \bar{\rho} \bar{v} \\ \bar{\alpha} \bar{\rho} \bar{v}^2 + \bar{p} \\ \bar{\alpha} \bar{v} (\bar{\rho} \bar{E} + \bar{p}) \\ \alpha \rho v \\ \alpha (\rho v^2 + p) \end{bmatrix}, \\
\mathbf{h} &= \begin{bmatrix} -\bar{v} \\ 0 \\ \bar{p} \bar{v} \\ 0 \\ -\bar{p} \bar{v} \\ -\bar{p} \bar{v} \end{bmatrix}, \\
\mathbf{p} &= \begin{bmatrix} \mu \Delta p \\ 0 \\ -\lambda \Delta v \\ 0 \\ \lambda \Delta v \\ \mu \bar{p} \Delta p + \lambda \bar{v} \Delta v \end{bmatrix}, \\
\bar{\alpha} + \alpha &= 1, \\
\bar{\alpha} \bar{\rho} &= \frac{\bar{v}^2}{2} + \bar{c} = \frac{\bar{v}^2}{2} + \bar{p} + \gamma \bar{P}_0 \frac{\bar{\gamma}}{\bar{\rho}(\bar{\gamma} - 1)}, \\
\bar{\alpha} \rho v E &= \frac{\bar{v}^2}{2} + \bar{c} = \frac{\bar{v}^2}{2} + \frac{\bar{p} + \gamma \bar{P}_0}{\rho(\gamma - 1)}, \\
\Delta p &= \bar{p} - p, \\
\Delta v &= \bar{v} - v.
\end{align*}
\]
Here \( \mathbf{u} \) is the vector of “conservative” variables (actually system of equations (1) does not have the conservative form due to the term \( \mathbf{h}(\mathbf{u})\hat{\alpha}_x \) in the right-hand side), \( \mathbf{f} \) is the differential flux vector, \( \mathbf{h}(\mathbf{u})\hat{\alpha}_x \) is the differential source term, \( \mathbf{p} \) is the algebraic source term connected with the fluids pressures and velocities relaxation. The standard notations are used, namely \( t \) is the time, \( x \) is the spatial coordinate, \( \alpha \) is the volume fraction, \( \rho \) is the real density, \( v \) is the velocity, \( p \) is the pressure. Parameters with bars correspond to the steel and without—to the lead. Velocity and pressure with the tilde correspond to the parameters at the fluids interface and are taken equal to [8]:

\[
\begin{align*}
\tilde{\rho} &= \alpha \bar{\rho} + \hat{\alpha} \rho, \\
\tilde{v} &= \frac{\alpha \rho v + \hat{\alpha} \rho \bar{v}}{\alpha \rho + \hat{\alpha} \rho}.
\end{align*}
\]

The stiffened gas EOS [2] with the parameters \( \gamma \) and \( P_0 \) is used for each fluid.

The equations (1) were originally derived in [9] for the mathematical modeling of deflagration-to-detonation transition in the heterogeneous explosives. Parameter \( \mu \) is the dynamic viscosity coefficient of compaction. The coefficient describes the fluids pressures relaxation at the interface. Vector \( \mathbf{p} \) is the correction of the one-dimensional model, which takes into account multidimensional process of the pressure relaxation that occur at the interface. In the present work, the interface condition

\[
p = \tilde{p}
\]

is used. In the numerical scheme (see section 4) condition (2) is taken into account in the special pressures relaxation procedure. It is important to note that the condition (2) is not the priori assumption about the fluids pressures equality. Such assumption can lead to the non-hyperbolicity of (1) [10]. Instead it is the additional condition which is applied at the interface. Parameter \( \lambda \) is responsible for the fluids velocities relaxation.

So the governing system of equation comprises the mass, momentum and energy conservation laws for both fluids. The specific features of the model are compressibility of fluids and the absence of conservative form. Conservation laws are supplemented by transfer equation for the volume fraction for one of the fluids. The mathematical properties of the system (1) are well-known. It is hyperbolic under the assumptions [11]:

\[
\alpha \neq 0, \quad \hat{\alpha} \neq 0, \quad (v - \bar{v})^2 \neq c^2,
\]

where \( c \) is the speed of sound. The hyperbolicity of the governing system of equations provides the opportunity to use Godunov-type methods for its numerical integration [12].

4. Computational algorithm

Computational algorithm for one time step integration consists of three stages. At first the hyperbolic step \( L_h \) is carried out with \( \mathbf{p} = 0 \) and then the velocities relaxation procedure \( L_v,relax \) and the pressures relaxation procedure \( L_p,relax \):

\[
\mathbf{u}_j^{n+1} = L_p,relax L_v,relax L_h \mathbf{u}_j^n.
\]

Here \( j \) stands for the spatial index, \( n \)—for the time index.

Hyperbolic step is carried out using the numerical scheme HLL (Harten–Lax–van Leer) taking into account the approximation of the right-hand side term \( \mathbf{h}(\mathbf{u})\hat{\alpha}_x \). The initial system (1) can be formally subdivided into the first equation and the following sub-system:

\[
\begin{align*}
\mathbf{U}_1 + \mathbf{F}(\mathbf{u})_x &= \mathbf{H}\hat{\alpha}_x, \\
\mathbf{U} &= \begin{bmatrix} \hat{\alpha} \bar{\rho} \\ \hat{\alpha} \bar{\rho} \bar{v} \\ \hat{\alpha} \bar{\rho} E \end{bmatrix}, & \mathbf{F} &= \begin{bmatrix} \hat{\alpha} \bar{\rho} \bar{v} \\ \hat{\alpha} \bar{\rho} E + \bar{p} \end{bmatrix}, & \mathbf{H} &= \begin{bmatrix} 0 & \bar{p} & \bar{v} \\ \bar{p} & 0 & 0 \\ -\bar{p} & -\bar{p} & -\bar{p} \end{bmatrix}.
\end{align*}
\]
Approximation of the first equation of (1) and the system (4) is carried out as follows:

\[ \alpha_{h,j}^{n+1} = \alpha_j^n - \frac{\Delta t^n}{\Delta x} \left[ \bar{v}_j^n \left( \frac{S_{j+1/2}^p \alpha_j^n - S_{j+1/2}^m \alpha_{j+1}^n}{S_{j+1/2}^p - S_{j+1/2}^m} \right) + \frac{\bar{v}_j^n \left( S_{j-1/2}^m \alpha_{j-1}^n - S_{j-1/2}^p \alpha_j^n \right)}{S_{j-1/2}^p - S_{j-1/2}^m} \right], \]

\[ U_{h,j}^{n+1} = U_j^n - \frac{\Delta t^n}{\Delta x} \left[ F_{j+1/2}^n(U_j^n, U_{j+1}^n) - F_{j-1/2}(U_{j-1}, U_j^n) \right] + H(U_j^n, \alpha_j^n)(\Delta_t^\alpha \alpha_x), \]

where the numerical flux \( F_{j+1/2} \) is calculated using HLL scheme:

\[ F_{j+1/2}^n = \frac{S_{j+1/2}^p F_j^n + S_{j+1/2}^m F_{j+1}^n}{S_{j+1/2}^p - S_{j+1/2}^m}, \]

\[ S_{j+1/2}^p = \max(0, v_j^n + c_j^n, v_{j+1}^n + c_{j+1}^n, \bar{v}_j^n + \bar{c}_j^n, \bar{v}_{j+1}^n + \bar{c}_{j+1}^n), \]

\[ S_{j+1/2}^m = \min(0, v_j^n - c_j^n, v_{j+1}^n - c_{j+1}^n, \bar{v}_j^n - \bar{c}_j^n, \bar{v}_{j+1}^n - \bar{c}_{j+1}^n), \]

and the gradient of \( \alpha \) in the differential non-conservative source term in (4) is approximated as:

\[ \Delta_t^\alpha \alpha_x = \frac{1}{\Delta x} \left( \frac{S_{j+1/2}^p \bar{\alpha}_j^n - S_{j+1/2}^m \bar{\alpha}_{j+1}^n}{S_{j+1/2}^p - S_{j+1/2}^m} - \frac{S_{j-1/2}^p \bar{\alpha}_{j-1}^n - S_{j-1/2}^m \bar{\alpha}_j^n}{S_{j-1/2}^p - S_{j-1/2}^m} \right). \]

The last approximation is adjusted with the approximation of the divergent part of (4) and for each fluid the property “a two fluid system, uniform in velocity and pressure at \( t = 0 \) will be uniform on the same variable during its temporal evolution” [8] is valid.

Time step is chosen dynamically during the calculation from the condition:

\[ \Delta t^n = \text{CFL} \cdot \min_j \left( \frac{\Delta x}{v_j^n + c_j^n}, \frac{\Delta x}{v_j^n + \bar{c}_j} \right), \quad c = \sqrt{\frac{\gamma p + P_0}{\rho}}, \quad \bar{c} = \sqrt{\frac{\gamma \bar{p} + \bar{P}_0}{\bar{\rho}}}, \]

where CFL is the coefficient from 0 to 1.

Velocities relaxation procedure is realized according to [8].

For the pressures relaxation across the interface boundaries the following system of ordinary differential equations should be solved at the third stage of the algorithm:

\[
\begin{align*}
\frac{d\alpha}{dt} &= \mu(p - \bar{p}), \\
\frac{d(\alpha p)}{dt} &= 0, \\
\frac{d(\alpha \rho v)}{dt} &= 0, \\
\frac{d(\alpha p^E)}{dt} &= -\mu\bar{p}(p - \bar{p}), \\
\frac{d(\bar{\alpha} \bar{\rho} v)}{dt} &= 0, \\
\frac{d(\bar{\alpha} \bar{\rho} v^E)}{dt} &= \mu\bar{p}(p - \bar{p}).
\end{align*}
\]

Actually the system (5) is solved in all computational cells. As the initial conditions values from the previous velocities relaxation stage are used. The second and the third equations of (5) give \( v = \text{const} \) at the third stage of the algorithm. Similarly the fifth and the sixth equations provide the condition \( \bar{v} = \text{const} \). We can get the following consequences of (5) then:

\[
\frac{d\bar{p}}{dt} = \bar{p} \frac{d}{dt} \left( \frac{\rho}{\bar{\rho}} \right), \quad \frac{d\bar{\rho}}{dt} = -\bar{p} \frac{d}{dt} \left( \frac{\rho}{\bar{\rho}} \right).
\]

The following approximation of (6) was proposed in [13]:

\[ e^{n+1} - e^n = -\frac{1}{2} \left( p^{n+1} + \bar{p}^n \right) \left( \frac{1}{\rho^{n+1}} - \frac{1}{\rho^n} \right), \quad e^{n+1} - e^n = -\frac{1}{2} \left( p^{n+1} + \bar{p}^n \right) \left( \frac{1}{\rho^{n+1}} - \frac{1}{\rho^n} \right). \]
Pressures relaxation procedure implies the search of \( \rho^{n+1}, \bar{p}^{n+1} \) and \( p \) values that provide the condition:
\[
p^{n+1} = \bar{p}^{n+1} = p, 
\]
as a result of solution of the system:
\[
\begin{align*}
\frac{p + \gamma P_0}{\rho^{n+1}(\gamma - 1)} - \frac{p^n + \gamma P_0}{\rho^n(\gamma - 1)} &= -\frac{1}{2}(p + \bar{p})\left(\frac{1}{\rho^{n+1}} - \frac{1}{\rho^n}\right), \\
\frac{p + \bar{\gamma} \bar{P}_0}{\bar{\rho}^{n+1}(\bar{\gamma} - 1)} - \frac{p^n + \bar{\gamma} \bar{P}_0}{\bar{\rho}^n(\bar{\gamma} - 1)} &= -\frac{1}{2}(p + \bar{p})\left(\frac{1}{\bar{\rho}^{n+1}} - \frac{1}{\bar{\rho}^n}\right), \\
\alpha^n \rho^n + \bar{\alpha}^n \bar{\rho}^n &= 1.
\end{align*}
\tag{8}
\]
The first and the second equations of (8) are the equations (7). The last one is the consequence of the second and the fifth equations of (5) and the condition \( \alpha + \bar{\alpha} = 1 \). Let us denote:
\[
p \equiv x, \quad \rho^{n+1} \equiv y, \quad \bar{p}^{n+1} \equiv z,
\]
and rewrite (8) as:
\[
\begin{align*}
A_1 x + B_1 y + C_1 z &= D_1, \\
A_2 x + B_2 y + C_2 z &= D_2, \\
A_3 x + B_3 y + C_3 z &= 0,
\end{align*}
\tag{9}
\]
where
\[
A_1 = \frac{\gamma + 1}{2} \rho^n, \quad B_1 = -(p^n + \gamma P_0) - \frac{\gamma - 1}{2} \bar{p}^n, \quad C_1 = -\frac{\gamma - 1}{2}, \\
D_1 = -\gamma P_0 \rho^n - \frac{\gamma - 1}{2} \bar{p}^n \rho^n, \quad A_2 = \frac{\bar{\gamma} + 1}{2} \bar{\rho}^n, \quad B_2 = -(\bar{p}^n + \bar{\gamma} \bar{P}_0) - \frac{\bar{\gamma} - 1}{2} \bar{p}^n, \\
C_2 = \frac{1 - \bar{\gamma}}{2}, \quad D_2 = -\bar{\gamma} \bar{P}_0 \bar{\rho}^n - \frac{\bar{\gamma} - 1}{2} \bar{p}^n \bar{\rho}^n, \quad A_3 = \bar{\alpha}^n \bar{\rho}^n, \quad B_3 = \alpha^n \rho^n, \quad C_3 = -1.
\]
Solution of (9) leads to the quadratic equation with the unknown \( y \) for example:
\[
G y^2 + H y + I = 0, \quad \tag{10}
\]
where
\[
G = -A_2 B_1 C_3 + A_3 B_2 C_1 + A_3 B_1 C_2 - C_1 C_3 D_2, \\
H = -A_2 B_1 C_3 + A_2 B_3 D_1 - A_1 A_3 B_2 - A_3 C_2 D_1 - B_3 C_1 D_2 - A_1 C_3 D_2, \\
I = A_2 B_3 D_1 - A_1 B_3 D_2.
\]
With known roots of (10) the rest unknowns of (9) could be found:
\[
x = \frac{D_1 - B_1 y}{A_1 + C_1 y}, \quad z = -\frac{A_3 y}{B_3 + C_3 y}. \quad \tag{11}
\]
Among two sets \((x, y, z)\) of roots of (9) the physically meaningful one is chosen: the densities and pressure are positive, the volume fractions for fluids are greater than 0 and less than 1.

The numerous parametric numerical studies the results of which are presented in the following section demonstrate that the described method for pressures relaxation works excellent for the problem in consideration and provides the robust calculation in the presence of interface boundary between two distinct metals. Although the initial systems (1) and (5) contain the coefficient \( \mu \) the final formulas (10) and (11) do not demand its value. The reason for that is the condition (2). The value of \( \mu \) becomes important for more difficult conditions at the interface boundary. In this case the pressures relaxation procedure implies the solution the system of non-linear equations.
Table 1. Reference values courtesy of professor I V Lomonosov (IPCP RAS).

| SW    | $p$ (GPa) | $\rho$ (kg/m$^3$) | $v$ (m/s) | $D$ (km/s) |
|-------|-----------|-------------------|-----------|------------|
| In steel | 7.99      | 8 246             | 211       | 4.72       |
| In lead  | 7.99      | 12 830            | 211       | 1.97       |

Table 2. Parameters of the stiffened gas EOS.

| No. | $\bar{\gamma}$ | $P_0$ (GPa) | $\gamma$ | $\bar{P}_0$ (GPa) |
|-----|-----------------|-------------|----------|-------------------|
| 1   | 3.9             | 65.6        | 3.0      | 30.0              |
| 2   | 3.9             | 65.6        | 3.0      | 60.0              |
| 3   | 3.9             | 65.6        | 4.0      | 30.0              |
| 4   | 3.9             | 65.6        | 3.0      | 15.0              |
| 5   | 3.0             | 65.6        | 3.0      | 15.0              |
| 6   | 2.9             | 65.6        | 3.0      | 15.0              |
| 7   | 2.7             | 65.6        | 3.0      | 15.0              |

Table 3. Predicted characteristics of SW.

| No. | $p_{cont}$ (GPa) | $v_{cont}$ (m/s) | $\rho_L$ (kg/m$^3$) | $D_L$ (km/s) | $\rho_R$ (kg/m$^3$) | $D_R$ (km/s) |
|-----|------------------|------------------|---------------------|-------------|---------------------|-------------|
| 1   | 10.1 (+26%)      | 215 (+2%)        | 8 195 (+1%)         | 6.0 (+27%)  | 12 431 (-3%)       | 2.7 (+38%)  |
| 2   | 11.9 (+49%)      | 251 (+19%)       | 8 245 (+0%)         | 6.0 (+27%)  | 12 001 (-7%)       | 3.8 (+94%)  |
| 3   | 11.0 (+38%)      | 232 (+10%)       | 8 219 (-0%)         | 6.1 (+29%)  | 12 203 (-5%)       | 3.2 (+64%)  |
| 4   | 8.5 (+6%)        | 182 (-14%)       | 8 151 (-1%)         | 6.0 (+27%)  | 13 054 (+2%)       | 3.2 (+4%)   |
| 5   | 8.0 (+0%)        | 196 (-7%)        | 8 210 (-0%)         | 5.2 (+11%)  | 12 981 (+1%)       | 1.8 (-7%)   |
| 6   | 8.0 (+0%)        | 198 (-6%)        | 8 219 (-0%)         | 5.1 (+8%)   | 12 972 (+1%)       | 1.8 (-7%)   |
| 7   | 7.8 (-2%)        | 202 (-4%)        | 8 238 (+0%)         | 5.05 (+7%)  | 12 951 (+1%)       | 1.9 (-4%)   |

5. Results of modeling

To satisfy the hyperbolic conditions (3) all computational cells should contain both fluids so on the interval [0 mm; 3 mm] we set $\alpha = 10^{-6}$ and on the interval [3 mm; 5 mm] we set $\bar{\alpha} = 10^{-6}$. CFL number is equal to 0.6.

To reproduce quantitatively reasonable characteristics of metal plates impact the parameters of stiffened gas EOS should be fitted using either experimental data or the results of simulations taking into account the wide-range EOS for metals. We use the results of simulations courtesy of professor I V Lomonosov (IPCP RAS) obtained using wide-range EOS [14] and the numerical approach [6]. The results of I V Lomonosov will be referred as reference values further, table 1. Here and elsewhere SW in steel plate will be called the “left” one and in lead plate—the “right” one in accordance with the statement in figure 1.

A series of numerical experiments was carried out in order to fit the results of the calculations with stiffened gas EOS to the results of modeling using wide-range EOS. The parameters $\gamma$, $\bar{\gamma}$,
\( P_0 \) and \( \bar{P}_0 \) were varied (table 2) and the shock speeds \( (D_L \text{ and } D_R) \), contact velocity \( (v_{\text{cont}}) \) and pressure \( (p_{\text{cont}}) \), post-shock densities \( (\rho_L \text{ and } \rho_R) \) were compared with the reference values (table 3). For each parameter the relative error in comparison with the reference value was measured.

We started with the EOS parameters for the steel plate from [2] and some default parameters for the lead one. The largest error was obtained for the contact surface pressure and SW speeds. In calculations 2 and 3, the values of \( \gamma \) and \( P_0 \) were increased. The errors for mentioned parameters became larger. The post-shock densities remained almost the same and it is the general tendency for the following numerical experiments. The positive dynamics for the contact pressure and right SW speed was obtained with the decrease of \( P_0 \) in calculation 4. At the same time the left SW speed was insensitive to the variation of the EOS parameters in calculations 1–4. In calculations 5–7, we successively decreased the value of \( \gamma \) and obtained the error 7\% for the left SW speed. Figures 2–6 illustrate the predicted distributions of steel volume fraction and density, lead density and contact pressure and velocity for the EOS parameters 7 in table 2. Note

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{figure2.png}
\caption{Distributions of steel volume fraction.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{figure3.png}
\caption{Distributions of steel density.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{figure4.png}
\caption{Distributions of velocity.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{figure5.png}
\caption{Distributions of pressure.}
\end{figure}
that the numerical scheme is quite diffusive and should be improved but at same time provides the robust calculation for the huge pressure gradients up to the 5 orders of magnitude. The vertical dashed lines in figure 2 show the “exact” locations of the interface boundary between the plates obtained with the use of contact surface velocity about 200 m/s (see figure 4).

6. Conclusions

The paper presents the parametric numerical study of two metal plates impact using two-fluid approach in comparison with the simulations taking into account wide-range EOS for the metals. In calculations the formation of two SW is obtained with the characteristics with maximum relative error 7% in comparison with the reference values.

The determinative system of equations [8] is used. Both metals are considered to be weakly compressible media with stiffened gas equations of state. The computational algorithm is based on the operator splitting approach and HLL method for the approximation of the numerical flux and the non-conservative right-hand side terms. The key point of the algorithm is the pressures relaxation procedure to provide the robust calculation at the interface boundaries.

The mathematical model is extendable by the third fluid—ambient air to describe the free surfaces of the metal plates. The proposed numerical approach is suitable for direct generalization for multidimensional cases for simple geometries by means of spatial splitting procedure.

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