Searching for Doubly-Charged Higgs Bosons at Future Colliders*

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ABSTRACT

Doubly-charged Higgs bosons (Δ−−/Δ+++) appear in several extensions to the Standard Model and can be relatively light. We review the theoretical motivation for these states and present a study of the discovery reach in future runs of the Fermilab Tevatron for pair-produced doubly-charged Higgs bosons decaying to like-sign lepton pairs. We also comment on the discovery potential at other future colliders.

I. Introduction

Doubly-charged Higgs bosons (Δ−−) appear in exotic Higgs representations such as found in left-right symmetric models. The current experimental bound is \(m_{\Delta^{-}} > 45\) GeV [1] from a search for \(Z^{0} \rightarrow \Delta^{-}\Delta^{++}\) at LEP.

At the Tevatron, the two production mechanisms with potentially large cross section are pair production, \(p\bar{p} \rightarrow \gamma/Z^{0}X \rightarrow \Delta^{-}\Delta^{++}X\) or single production via \(WW\) fusion, \(p\bar{p} \rightarrow W^{-}W^{-}X \rightarrow \Delta^{-}\Delta^{-}X\). However, existing phenomenological and theoretical constraints are only easily satisfied if the \(W^{-}W^{-} \rightarrow \Delta^{-}\Delta^{-}\) coupling is vanishing (or very small). Therefore, in this analysis we will consider the discovery reach for detecting \(\Delta^{-}\Delta^{++}\) pair production at the Tevatron.

In many models, it is possible for the \(\Delta^{-}\) to couple to like-sign lepton pairs, \(\ell\bar{\ell}^{-}\). If the \(W^{-}W^{-} \rightarrow \Delta^{-}\Delta^{-}\) coupling is vanishing, it is then very likely that the doubly-charged Higgs will decay to \(\ell\bar{\ell}^{-}\) via the lepton-number-violating coupling. We will therefore concentrate upon \(\Delta^{-} \rightarrow e^{-}e^{-}\), \(\Delta^{-} \rightarrow \mu^{-}\mu^{-}\) and \(\Delta^{-} \rightarrow \tau^{-}\tau^{-}\).

Alternatively, if the \(\Delta^{-} \rightarrow \ell^{-}\bar{\ell}^{-}\) and \(\Delta^{-} \rightarrow W^{-}W^{-}\) couplings are both vanishing or very small, then the \(\Delta^{-}\) can have a sufficiently long lifetime that it will decay outside the detector. Identification of the \(\Delta^{-}\Delta^{++}\) pair via the associated \(dE/dx\) distributions in the tracking chamber would then be possible.

II. Theoretical Motivation

Doubly-charged scalar particles abound in exotic Higgs representations and appear in many models [2, 3, 4]. For example, a Higgs doublet representation with \(Y = -3\) contains a doubly-charged \(\Delta^{-}\) and a singly-charged \(\Delta^{-}\). If part of a multiplet with a neutral member, a \(\Delta^{-}\) would immediately signal the presence of a Higgs representation with total isospin \(T = 1\) or higher. Most popular are the complex \(Y = -2\) triplet Higgs representations, such as those required in left-right symmetric models, that contain a \(\Delta^{-}\), a \(\Delta^{-}\) and a \(\Delta^{0}\).

In assessing the attractiveness of a Higgs sector model containing a \(\Delta^{-}\) many constraints need to be considered. For triplet and higher representations containing a neutral member, limits on the latter’s vacuum expectation value (vev) required for \(\rho \equiv m_{W}^{2}/[\cos^{2}\theta_{W}m_{Z}^{2}] = 1\) at tree-level are generally severe. (The first single representation beyond \(T = 1/2\) for which \(\rho = 1\) regardless of the vev is \(T = 3, Y = -4\), whose \(T_{3} = 0\) member is doubly-charged.) Models with \(T = 1\) and \(T = 2\) can have \(\rho = 1\) at tree-level by combining representations. However, such models generally require fine-tuning in order to preserve \(\rho = 1\) at one-loop. The simplest way to avoid all \(\rho\) problems is to either consider representations that simply do not have a neutral member (for example, a \(Y = -3\) doublet or a \(Y = -4\) triplet representation), or else models in which the vev of the neutral member is precisely zero. We will only consider models of this type in what follows.

Further constraints on Higgs representations arise if we require unification of the coupling constants without intermediate scale physics. In the Standard Model, unification is possible for a relatively simple Higgs sector that includes a single \(|Y| = 2\) triplet in combination with either one or two \(|Y| = 1\) doublets (the preferred number of doublets depends upon the precise value of \(\alpha_{s}(m_{Z})\)). In the case of the minimal supersymmetric extension of the Standard Model, precise unification requires exactly two doublet Higgs representations (plus possible singlet representations); any extra doublet representations (including ones with a doubly-charged boson) or any number of triplet or higher representations would destroy unification. However, by going beyond the minimal model and including appropriate intermediate-scale physics, supersymmetric models

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(in particular, supersymmetric left-right symmetric models [5]) with triplet and higher representations can be made consistent with unification.

In short, the popular two-doublet MSSM need not be nature’s choice. We should be on the look-out for signatures of exotic Higgs representations, the clearest of which would be the existence of a doubly-charged Higgs boson. Thus, it is important to consider how to search for and study such a particle.

The phenomenology of the \( \Delta^- \) derives from its couplings. Tri-linear couplings of the type \( W^- W^- \rightarrow \Delta^- \) are not present in the absence of an enabling non-zero vev for the neutral member (if present) of the representation, and \( q \bar{q} \Delta^{-} \) couplings are obviously absent. There are always couplings of the form \( Z, \gamma \rightarrow \Delta^- \Delta^{++} \). In addition, and of particular interest, there is the possibility of lepton-number-violating \( \ell^- \ell^- \rightarrow \Delta^- \) couplings in some models. For \( Q = T_3 + Y_T = -2 \) the allowed cases are:

\[
\begin{align*}
\ell_R^- \ell_R^- & \rightarrow \Delta^- \quad (T = 0, T_3 = 0, Y = -4), \\
\ell_L^- \ell_R^- & \rightarrow \Delta^- \quad (T = \frac{1}{2}, T_3 = -\frac{1}{2}, Y = -3), \\
\ell_L^- \ell_L^- & \rightarrow \Delta^- \quad (T = 1, T_3 = -1, Y = -2).
\end{align*}
\]

Note that the above cases do not include the \( T = 3, Y = -4 \) representation that yields \( \rho = 1 \), nor the \( T = 1, Y = -4 \) triplet with no neutral member, but do include the \( T = 1/2, Y = -3 \) doublet representation with no neutral member, and the popular \( T = 1, Y = -2 \) triplet representation. In left-right symmetric models there is a ‘right-handed’ and a ‘left-handed’ Higgs triplet, both with \(|Y| = 2\). Our analysis applies to the left-handed triplet (whose neutral member must have a very small vev to preserve \( \rho = 1 \)); the phenomenology of the right-handed triplet is completely different.

In the case of \(|Y| = 2\) triplet representation (to which we now specialize) the lepton-number-violating coupling to \( \ell^- \ell^- \) leptons is specified by the Lagrangian form:

\[
\mathcal{L}_V = i h_{ij} \bar{\psi}^T_{\ell j} C \tau_2 \Delta \psi_{\ell L} + \text{h.c.,}
\]

where \( i, j = e, \mu, \tau \) are generation indices, the \( \psi \)'s are the two-component left-handed lepton fields \( (\nu, \ell^-) \), and \( \Delta \) is the \( 2 \times 2 \) matrix of Higgs fields:

\[
\Delta = \begin{pmatrix}
\Delta^- / \sqrt{2} & \Delta^0 \\
\Delta^0 & -\Delta^- / \sqrt{2}
\end{pmatrix}.
\]

Limits on the \( h_{ij} \) coupling strengths come from many sources. Experiments that place limits on the \( h_{ij} \) by virtue of the \( \Delta^- \rightarrow \ell^- \ell^- \) couplings include Bhabha scattering, \((g-2)_\mu\), muonium-antimuonium conversion, and \( \mu^- \rightarrow e^- e^- e^+ \). These limits [3][8][9] suggest small off-diagonal couplings (as assumed in our analysis). Writing

\[
|h_{\ell \ell}|^2 = c_{\ell \ell} m_{\Delta^-}^2 \quad (\text{GeV}),
\]

the limits imply \( c_{\ell \ell} \lesssim 10^{-5} \) and \( c_{\mu \mu} \lesssim 6 \cdot 10^{-5} \).

Regarding production mechanisms, the fusion process [3][4],[8][9], \( W^- W^- \rightarrow \Delta^- \), is absent since the required tri-linear coupling is zero if the vev of the neutral member (if there is one) of the Higgs representation is zero (as we assume so that \( \rho = 1 \) naturally). Single production of \( \Delta^-, \Delta^{++} \) is possible in \( e^+ e^- \) (ep) collisions at LEP2 (HERA) via diagrams involving the \( \Delta^- \rightarrow e^- e^- \) or \( \Delta^{++} \rightarrow e^+ e^+ \) couplings. If \( c_{ee} \) saturates its upper limit, then LEP2 and HERA will probe up to \( m_{\Delta^-} \sim 150 \text{ GeV} \) [3][9]. However, it is likely that \( c_{ee} \) is much smaller than its current bound and that these sources of single production will be negligible.

Thus, we focus on \( \gamma^*, Z^* \rightarrow \Delta^- \Delta^{++} \) pair production, the cross section for which is determined entirely by the quantum numbers of the \( \Delta^- \). For a general spin-0 boson \( B \), with weak isospin \( T_3 \) and charge \( Q \), and a fermion \( f \), with \( t_3 \) and \( q \), the \( f \bar{f} \rightarrow B \bar{B} \) pair-production cross section is:

\[
\sigma_{\text{pair}}(s) = \left( \frac{\pi a_{\ell}^2 \beta^3}{6} \right) \left\{ 2QqA(a_L + a_R) \frac{x_\ell y_W}{x_W y_W} + P_{ZZ} A^2 (a^2_L + a^2_R) \right\},
\]

where \( s \) is the \( f \bar{f} \) center of mass energy squared, \( \beta = \sqrt{1 - 4m_B^2/s} \), \( x_W = \sin^2 \theta_W \), \( y_W = 1 - x_W \), \( A = T_3 - x_W Q \), \( a_L = t_3 - x_W q \), \( a_R = -x_W q \), \( P_{\gamma} = s^{-2} \), \( P_{ZZ} = [s - m_B^2]^2 + m_B^2 T_3^2]^{-1} \), and \( P_{ZZ} = (s - m_B^2) P_{ZZ} / s \). We will consider a \( \Delta^- \) with \( T_3 = -1 \), \( Q = -2 \). An extra factor of 1/3 is required for color averaging in \( q \bar{q} \) annihilation in \( pp \) or \( pp \) collisions.

Figure 1: \( \Delta^{++}/\Delta^- \) pair production cross section as a function of \( \Delta^- \) mass for both the Tevatron and the LHC.

In \( e^+ e^- \rightarrow \Delta^- \Delta^{++} \), kinematic reach is limited to \( m_{\Delta^-} \lesssim \sqrt{s}/2 \), i.e. no more than about 230–240 GeV at a future \( \sqrt{s} = 500 \text{ GeV} \) NLC. We will find that the discovery reach at the Tevatron can cover much, if not all, of this range, depending upon the dominant \( \Delta^- \rightarrow \) decay mode. The mass reach for pair production at a \( pp \) collider increases rapidly with machine
energy. Figure 1 shows the $\Delta^{-}\Delta^{++}$ pair production cross section for both the Tevatron (at $\sqrt{s} = 2$ TeV) and the LHC. At the Tevatron, $\sigma^{\text{pair}} \sim 0.9(0.24)$ fb at $m_{\Delta^{-}} = 250(300)$ GeV. With total accumulated luminosity of 30 fb$^{-1}$ (as planned for the TeV33 upgrade) there would be about 27(7) $\Delta^{-}\Delta^{++}$ events. The marginality of the latter number makes it clear that $m_{\Delta^{-}} \lesssim 300$ GeV will be the ultimate mass reach possible at the Tevatron.

Decays of a $\Delta^{-}$ are generally quite exotic $[3, 4]$. For $\sim 0 \Delta^{-} \rightarrow W^{-}W^{-}$ coupling, the only two-body decays that might be important are $\Delta^{-} \rightarrow \Delta^{-}W^{-}, \Delta^{-} \rightarrow \Delta^{-}\Delta^{-}$ and, if the lepton coupling is present, $\Delta^{-} \rightarrow \ell^{-}\ell^{-}$. Typically, the $\Delta^{-}$ and $\Delta^{+}$ have similar masses, in which case $\Delta^{-} \rightarrow \Delta^{-}\Delta^{-}$ is likely to be disallowed. Thus, we will focus on the $\Delta^{-}W^{-}$ and $\ell^{-}\ell^{-}$ final states. For a $T = 1$, $Y = -2$ triplet we find $[3, 4]$

$$
\Gamma^{\Delta^{-}W^{-}} = \frac{s^2 m_{\Delta^{-}}}{16\pi} m_{W}^{-3} \sim (1.3 \text{ GeV}) \left( \frac{m_{\Delta^{-}}}{100 \text{ GeV}} \right)^3 \beta^3, \\
\Gamma^{\ell^{-}\ell^{-}} = \frac{\left(4m_{\ell} m_{\Delta^{-}}\right)^2}{\beta s} \sim (0.4 \text{ GeV}) \left( \frac{m_{\Delta^{-}}}{100 \text{ GeV}} \right)^3 .
$$

(6)

where $\beta$ is the usual phase space suppression factor, and we used Eq. $[4]$. For example $[3]$, if $m_{\Delta^{-}} = 360$ GeV, $m_{W} = 250$ GeV we find $\Gamma(\Delta^{-} \rightarrow \Delta^{-}W^{-}) \sim 2$ GeV and $\Gamma(\Delta^{-} \rightarrow \ell^{-}\ell^{-}) = 19 \text{ GeV} \left( \frac{m_{\ell}}{\beta} \right)$. If any $c_{\ell\ell}$ is near $10^{-5}$ then $\Gamma^{\ell^{-}\ell^{-}} > \Gamma^{\Delta^{-}W^{-}}$ is likely. Since there are currently no limits on $c_{\ell\tau}$, the $\tau^{-}\tau^{-}$ channel could easily have the largest partial width and be the dominant decay of the $\Delta^{-}$. On the other hand, if all the $c_{\ell\ell}$ are very small then the $\Delta^{-}W^{-}$ mode is quite likely to be dominant if it is kinematically allowed. The implications for detection of $\Delta^{-}\Delta^{++}$ pairs will now be discussed.

### III. Simulation and Reconstruction

The signal and backgrounds are simulated with the PYTHIA Monte Carlo, which has been modified to allow the process:

$$
p\bar{p} \rightarrow Z^{0}\gamma X \rightarrow \Delta^{-}\Delta^{++}X, \tag{7}
$$

with the $\Delta^{-}$ then forced to decay to like-sign lepton pairs.

The events are then fed to a CDF detector simulation which includes the geometry of the Run I CDF detector. For the Main Injector runs of the Tevatron, the CDF and D0 detectors will both be upgraded to handle higher instantaneous luminosity. In addition, the acceptances of the upgraded detectors will improve. This simulation includes muon coverage for $|\eta| < 1$, which will be improved to $|\eta| < 1.5$ for Run II. This results in approximately a 20% improvement in acceptance for this process.

Events are passed through the normal CDF event reconstruction package. Muon candidates must have tracks in both the central tracking and the muon chambers, electron candidates must have a track and an isolated electromagnetic calorimeter cluster. The lepton momenta are determined from the central tracking chamber and, if fiducial, the silicon microvertex detector. For tracks which do not pass through the microvertex detector, the fit is performed assuming that the track originated from the interaction point. This so-called “beam-constraint” significantly improves momentum and, hence, mass resolution.

### IV. $\Delta^{-} \rightarrow e^{-}e^{-}, \Delta^{-} \rightarrow \mu^{-}\mu^{-}$

For the case where the $\Delta$ decays to like-sign leptons (excluding taus), the signature is a spectacular $4e$ or $4\mu$ final state. Here we will focus upon the $4\mu$ final state. Backgrounds are very similar for both the two channels, although the discovery reach will be slightly higher in the electron channel due to better mass resolution and larger electromagnetic calorimeter coverage.

The dominant backgrounds in the $4\mu$ mode (accepting at least 2 same-sign $\mu$’s as described below) arise from electroweak processes where real high-$p_{T}$ muons are created from $W$ or $Z$ decays along with either fake muons or muons from heavy flavor decay. The backgrounds are diboson production ($ZZ \rightarrow 4\mu, WZ \rightarrow 3\mu + \nu, WW \rightarrow 2\mu + 2\nu$); $t\bar{t}$ production ($t\bar{t} \rightarrow \mu^{+}\nu b \mu^{-}\bar{b}$); and boson plus jets ($W + \text{jets}$, $Z + \text{jets}$), where $W \rightarrow \mu\nu$, $Z \rightarrow \mu^{+}\mu^{-}$ and the jets produce real or fake muons. We use the measured cross sections for $t\bar{t}$, $W + \text{jets}$ and $Z + \text{jets}$ $[10, 11, 12]$ and the calculated cross sections for $WZ$ and $ZZ$ production $[13]$. The PDF world average branching ratios are used for $Z \rightarrow \mu^{+}\mu^{-}$ (0.03367) and $W \rightarrow \mu^{+}\nu_{\mu}$ (0.104) $[13]$.

![Figure 2: Background contributions to the same sign mass plot after all cuts. As can be seen in the inset, the dominant background above 100 GeV is from diboson production. The $N_{\text{jet}} \leq 1$ jet requirement removes most of the $t\bar{t}$ background.](image)
an issue at higher masses. We expect very little additional activity in $\Delta$ pair production other than the energy recoiling against the virtual $Z/\gamma$. For dimuon top decay backgrounds, there is additional jet activity from the two $b$ decays. The third muon is supplied by one of these $b$ decays. The background in the high mass region from top decays can be greatly suppressed by requiring that the event has no more than one jet seen in the calorimeter ($|\eta| < 2.5$) with more than 7.5 GeV of transverse energy ($E_T$).

Figure 3 shows the same-sign muon mass contributions from each of the backgrounds listed above after all cuts. Above 100 GeV in same-sign mass, the dominant background is diboson production.

Figure 3 shows the same-sign dimuon invariant mass distributions for both signal and background for three different $\Delta$ masses: 100, 200 and 300 GeV after all cuts have been applied. The signal-to-background ratio remains high at $m_{\Delta^{--}} = 300$ GeV, although the dimuon mass resolution is worsening. The dimuon mass resolution is dominated by the $p_T$ resolution of the detector, which worsens at higher momenta. For the case $\Delta^{--} \rightarrow e^- e^-$ the dielectron mass resolution does not degrade as rapidly due to the energy resolution of the electromagnetic calorimeter. In the case of a high mass search for $\Delta^{--}$ decaying to muon or electron pairs, the technique would be a counting experiment, looking for an excess of high mass pairs over the small background.

The simulated data shown in Figures 2 and 3 represent the response of the Run I CDF detector. The product of the efficiency and acceptance for a signal event to produce at least one entry in the same-sign mass plot depends upon $m_{\Delta^{--}}$, but is typically 50%. If we assume that the efficiency will be the same for the Run II detector, scaling the acceptance to the improved muon coverage ($|\eta| < 1.5$) brings this number up to 60%, an improvement factor in the accepted signal of 1.2. The corresponding scaling of the acceptance for the background is found to be $\sim 2$ over the entire mass region above 50 GeV.

In addition to the significance of a bump in the same-sign mass distribution, there is additional information in the number of high-$p_T$ muons in the event. With the Run I CDF detector used in this simulation, approximately 20% of the signal events have four found muons, yielding two entries in the mass distribution, while $\sim 6\%$ of the background events have more than three muons such that both same-sign pairs have mass greater than 50 GeV. For the Run II detector, these numbers go up to approximately 40% for signal and 11% for the background. The probability that a background event would have four muons and both same-sign combinations near one-another in mass is exceedingly small. We therefore conclude that production of enough events so that two events are measured to have four muons (in addition to the other same-sign dimuon mass entries from 2 and 3 muon events) will be more than adequate to establish a signal for the $\Delta^{--}$. As an example using the numbers above: if 10 events are produced, 6 events would produce at least one same-sign mass pair. Of those 6 events, 2 (from 2.4) would have four found muons and 4 (from 3.6) would have 2 or 3 found muons, yielding 8 entries in the same-sign mass plot on a background of approximately one same-sign dimuon mass pair and zero four muon events. Tri-muon events offer little additional evidence for $\Delta^{--}$ production, since background events often have two real, opposite-signed leptons in addition to one lepton from either a fake or heavy flavor decay.

Using the criteria that 10 pair-produced events would lead to an unambiguous discovery of the $\Delta^{--}$, we conclude that a reach of approximately 200, 250, 300 GeV in the mass of the $\Delta^{--}$ could be achieved in 2, 10, 30 fb$^{-1}$ of Tevatron running, respectively, for the cases where $\Delta^{--} \rightarrow e^- e^-$ and $\Delta^{--} \rightarrow \mu^- \mu^-$.

V. $\Delta^{--} \rightarrow \tau^+ \tau^-$

Unlike the electron and muon channels, reconstructing an invariant mass in the tau channel is problematic because of the neutrinos involved in their decay. We therefore use a counting method to estimate the reach of a doubly charged Higgs search for Run II.

Tau lepton identification is not trivial at a hadron collider. Identification efficiencies are much lower than for electrons or muons ($\sim 50\%$) and fake rates from QCD jets are significant ($\sim 0.5\%$). Nonetheless, searching the tau lepton channel is worthwhile because the doubly charged Higgs may preferentially couple to the taus, and the tau lepton offers the possibility of measuring the spin of its parent.

Selection of tau lepton candidates which decay into hadrons is detailed in [4]. The algorithm begins by looking for jets in the calorimeter. The tau candidate must have one or three charged particles in a $10^\circ$ cone about the jet axis and no additional charged particles in a cone of $30^\circ$. In addition, the tau
Table I: Expected $\Delta^{--} \rightarrow \tau\tau$ events passing all cuts in 10 fb$^{-1}$.

| $M_{\Delta^{--}}$ (GeV) | events |
|--------------------------|--------|
| 50                       | 19. ±4 |
| 100                      | 8.8 ±0.6 |
| 150                      | 3.07±0.20 |
| 200                      | 0.72±0.11 |
| 250                      | 0.23±0.03 |

(continues)
Phenomenological considerations for Higgs triplet representations, especially in left-right symmetric models, are outlined in J.F. Gunion, J. Grifols, B. Kayser, A. Mendez, and F. Olness, Phys. Rev. D40 (1989) 1546; N.G. Deshpande, J.F. Gunion, B. Kayser, and F. Olness, Phys. Rev. D44 (1991) 837.

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