Unitarity and Higher-Order Corrections in Neutralino Dark Matter Annihilation into Two Photons

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ABSTRACT

The neutralino pair annihilation into two photons in our galactic halo gives a robust dark matter signal, since it would give a quasi-monotonic gamma ray. This process is radiatively-induced, and the full-one loop calculation was done previously. However, for the heavy wino-like or Higgsino-like neutralino, the one-loop cross section violates unitarity, therefore the higher-order corrections may be important. We construct a non-relativistic theory for chargino and neutralino two-body states, and estimate all-order QED corrections and two-loop corrections by $Z$ and/or $W$ exchange. We find that the critical mass, above that the two-loop contribution is larger than one-loop one, is about 8 TeV ($O(10)$ TeV) in the limit where neutralino is wino (Higgsino)-like, respectively. Around and above the critical mass, the all-order $Z$ and/or $W$ exchange must be included to estimate the cross section. On the other hand, the QED corrections depend on the mass difference between the neutralino and chargino. In the wino-like limit where neutralino is highly degenerate with chargino in mass, we find that QED corrections enhance the pair annihilation cross section by 1.5-2.
I Introduction

Present observation of cosmological and astrophysical quantities allows a precise determination of the mean density of matter ($\Omega_M$) and baryon density ($\Omega_B$) in the Universe, and the existence of non-baryonic dark matter (DM) is established now [1]. However, the constituent of the DM is still an unresolved problem. The supersymmetric (SUSY) models provides the candidates of the DM, since the lightest SUSY particle (LSP) is stable due to the conserved R parity. The LSP may be the lightest neutralino in minimal supersymmetric standard model (MSSM). It is a linear combination of gauginos (bino and wino) and Higgsinos, which are superpartners of gauge and Higgs bosons, respectively. Thermal [2] or non-thermal processes [3] in the early Universe may produce the lightest neutralino enough to explain the DM in the Universe.

The detection of exotic cosmic rays is feasible technique to search for the dark matter particles, since a pair of the lightest neutralino could annihilate into the SM particles with significant cross section [4]. Among those, excess of monochromatic gamma ray due to the neutralino annihilation into two photons is a robust signal if observed, because the diffused gamma-ray background must have a continuous energy spectrum [5].

The neutralino annihilation to two photons is a radiative process. The dominant contribution to the cross section comes from the process where a neutralino pair is converted into a virtual chargino pair by $W$-boson exchange and then the chargino pair annihilates into two photons. The full one-loop cross section is calculated in Ref. [6]. The surprising fact is that the cross section is suppressed only by the $W$-boson mass ($m_W$), not by the neutralino mass ($m$), as $\sigma v \sim \alpha^2 \alpha_s^2 / m_W^2$, if the lightest-neutralino mass eigenstate is very close to wino or Higgsino. Since other pair-annihilation cross sections are proportional to $1/m^2$, the potential of search for the monochromatic gamma ray in cases of the wino- or Higgsino-like neutralino DM then can be understood as follows; as the neutralino mass increases, the signal rate of the monochromatic gamma ray reduces less quickly compared with the other DM signatures.

On the other hand, this cross section should be bounded from above by the unitarity limit $\sigma v < 4\pi/(vm^2)$ [8]. Thus, for the extreme heavy neutralino, this one-loop result must fail [7], and the higher-order corrections should be included. Indeed, the wino- or Higgsino-like neutralino is accompanied with chargino in the
same SU(2) multiplet, and their masses are almost degenerate. In these cases, the intermediate chargino-pair state in the process is almost on-shell, since the neutralino DM is non-relativistic (NR). When \( m \) is large, the diagrams are enhanced by a factor of \( \alpha_2 m/m_W \) for each \( W \) boson exchange, and the higher-order loop diagrams become more and more important. Then, the one-loop result is not valid anymore, and we need to sum the contributions from ladder diagrams of the weak-boson exchange to all orders.

This failure of the perturbative expansion is similar to the threshold singularity in pair creation or annihilation processes in QCD or QED [9]. In the quark-pair annihilation process at the threshold (NR) region, the higher-order corrections are non-negligible since nearly on-shell quark is dominated in the loop integration due to the small relative velocity. In other words, the deformation of the quark wave function by the QCD potential is not negligible at the threshold region. In order to get the reliable cross section, we need to sum ladder diagrams by gluon exchange to all orders, or to use the wave function for quarks under the QCD potential. In the neutralino annihilation to two photons, we need to include the Yukawa potential of the weak boson. When \( 1/(\alpha_2 m) \) is larger than the effective range of the Yukawa potential \( 1/m_W \), the Yukawa potential is point-like in the coordinate space, and does not deform the wave functions of neutralino and chargino. However, if \( m \) is heavier, the wave functions of chargino and neutralino are deviated from plane waves inside \( \sim 1/(\alpha_2 m) \). In this case, the perturbative expansion at the threshold region should be broken.

The NR field theory is useful to investigate these higher-order corrections at the threshold region in QCD or QED [10]. In this technique, we can factorize the short-distance physics, such as annihilation or production, from the long-distance physics related to the wave function [11]. The evaluation of the contribution from the long-distance physics is possible either by systematical resummation of the ladder diagrams in diagrammatic methods or by evaluating the wave function under the potential. This formalism is also useful to evaluate the higher-order corrections in the neutralino annihilation to two photons. From a viewpoint of the NR field theory, the short-distance physics comes from the chargino-pair annihilation to two photons, and the long-distance physics from the weak-boson exchanges, in addition to the photon exchange between charginos.

In this paper, the higher-order corrections to the annihilation cross section of the neutralino to two photons is studied in cases of the wino- or Higgsino-like neutralino
DM. First, we construct the two-body effective action of non-relativistic neutralino and chargino pairs. Using this formula, we estimate two-loop corrections by weak-boson exchange in a diagrammatic method. The two-loop contribution becomes important when the neutralino is as heavy as $m_W/\alpha_2$. From the explicit calculation, we found that the critical mass, above that the two-loop contribution is larger than one-loop one, is about 8 TeV for the wino-like neutralino, and is about $O(10)$ TeV for the Higgsino-like one. The resummation of the weak-boson exchange contributions to all orders is possible by solving numerically the wave functions of chargino and neutralino under the Yukawa potentials of the weak bosons [12]. We also calculate all-order corrections by photon exchange, by using the wave function deformed by Coulomb potential. The QED corrections depend on the mass difference between neutralino and chargino, $\delta m$. The wino-like neutralino is highly degenerate with the chargino, and $\delta m$ is typically $\sim 0.1$ GeV. For this case, we found that the QED corrections enhance the cross section by 1.5~2 when the neutralino mass is a few TeV.

This paper is organized as follows. In next section we briefly review masses and interactions of neutralino and chargino, especially in the wino- and Higgsino-like cases. In section III, we construct the two-body effective action of the NR neutralino and chargino pairs and show the strategy for our calculation. Here, in order to show the validity for our formalism, we reproduce the previous one-loop result, and show that the unitarity bound is satisfied in an extremely heavy neutralino mass limit. In section IV, we first include all-order QED corrections to the leading cross section, and we evaluate two-loop corrections by $Z$ and $W$ boson exchange. Section V is devoted to conclusion and discussion. We give a full effective Lagrangian relevant to our study in Appendix A.

II Non-Relativistic Action of Wino- or Higgsino-like Neutralino and Chargino.

In this section, we review the mass spectrum and the low-energy interaction of the SU(2) multiplets containing the lightest neutralino. We are interested in the case where the lightest neutralino LSP ($\tilde{\chi}_1^0$) is almost degenerate with the lighter chargino ($\tilde{\chi}_1^+ \equiv \tilde{\chi}_+^1$) in mass. For simplicity, we assume that all the other SUSY particles, including the two heavier neutralino states ($\tilde{\chi}_3^0$ and $\tilde{\chi}_4^0$) and the heavier chargino ($\tilde{\chi}_2^+$), are so heavy that we can ignore their contributions to the DM annihilation.
Neutralinos are linear combinations of supersymmetric partners of gauge bosons, the bino (\(\tilde{B}\)) and the neutral wino (\(\tilde{W}^0\)), and those of Higgs bosons, the Higgsinos (\(\tilde{H}^0_1, \tilde{H}^0_2\)). While those four fields have SU(2)×U(1) invariant masses, they are mixed with each others by the SU(2)×U(1) symmetry breaking. The lightest neutralino is wino-like when \(M_2 \ll \mu, M_1\) and Higgsino-like if \(\mu \ll M_1, M_2\). Here \(M_1\) and \(M_2\) are the bino and the wino masses, respectively, and \(\mu\) is the supersymmetric Higgsino mass parameter. When \(M_1, M_2, \mu \gg m_Z\), the lightest neutralino mass is

\[ m_{\tilde{\chi}_1^0} \simeq M_2 + \frac{m_Z^2 c_W^2}{M_2} (M_2 + \mu \sin 2\beta) \]  

for the wino-like case, and

\[ m_{\tilde{\chi}_1^0} \simeq \mu + \frac{m_Z^2 (1 + \sin 2\beta)}{2(\mu - M_1)(\mu - M_2)} (\mu - M_1 c_W - M_2 s_W^2) \]

for the Higgsino-like case (\(\mu > 0\)). Here, we show the terms up to \(O(m_Z^2/m_{SUSY})\).

Charginos are linear combinations of the charged wino, \(\tilde{W}^\pm\), and the charged Higgsino (\(\tilde{H}_1^\pm, \tilde{H}_2^\pm\)). When \(M_2, \mu \gg m_W\), the lighter chargino mass is

\[ m_{\tilde{\chi}_1^\pm} \simeq M_2 + \frac{m_W^2}{M_2} (M_2 + \mu \sin 2\beta) \]  

for the wino-like case, and

\[ m_{\tilde{\chi}_1^\pm} \simeq \mu - \frac{m_W^2}{M_2 - \mu^2} (\mu + M_2 \sin 2\beta) \]  

for the Higgsino-like case. We show the terms up to \(O(m_Z^2/m_{SUSY})\) here, again.

The mass difference between chargino and the LSP, \(\delta m\), is an important parameter for the neutralino annihilation cross section into two photons. For the wino-like case, their masses are highly degenerate. The tree-level mass difference is \(O(m_Z^4/m_{SUSY}^3)\) in a case \(m_Z, M_2 \ll M_1, \mu\),

\[ \delta m_{\text{tree}} \simeq \frac{m_Z^4}{M_1 \mu^2} s_W^2 c_W^2 \sin^2 2\beta, \]

since the \(O(m_Z^2/m_{SUSY})\) corrections to the masses are SU(2)×U(1) invariant. The mass splitting receives the positive contribution from the gauge boson loops due to the custodial SU(2) breaking. The radiative mass difference in the wino limit is

\[ \delta m_{\text{rad}} = \frac{\alpha_2 M_2}{4\pi} \left( f\left(\frac{m_W}{M_2}\right) - c_W^2 f\left(\frac{m_Z}{M_2}\right) - f(0) \right), \]

where \(f(a) = \int_0^1 \frac{dx}{x} \log(x^2 + (1-x)a^2)\) \cite{13}. This correction is about 0.18GeV when \(M_2 \gg m_W\).
For the Higgsino-like LSP, the mass splitting is $O(m_Z^2/m_{SUSY})$,

$$\delta m \simeq \frac{1}{2} m_Z^2 \epsilon_W^2 \left(1 - \sin 2\beta\right) + \frac{1}{2} m_{\text{SUSY}}^2 \epsilon_W^2 \left(1 + \sin 2\beta\right).$$  

(7)

The second-lightest neutralino ($\tilde{\chi}_2^0$) also degenerates with the LSP and chargino, since they are in common SU(2) multiplets. The mass difference between the LSP and the second lightest neutralino, $\delta m_N$, is again $O(m_W^2/m_{SUSY})$,

$$\delta m_N = \frac{m_Z^2}{M_2} \epsilon_W^2 + \frac{m_Z^2}{M_1} \epsilon_W^2.$$  

(8)

This is roughly $2 \times \delta m$ when $\tan \beta \gg 1$.

Next, we consider interactions among neutralino(s) and chargino. After ignoring all SUSY particles except $\tilde{\chi}_1^{0(2)}$ and $\tilde{\chi}^+$, the interactions which we have to take into account are only the gauge interactions. The leading correction by non-vanishing $\delta m$ to the neutralino annihilation cross section to two photons is $O(\sqrt{\delta m})$. As will be shown in next section, the $O(\sqrt{\delta m})$ correction originates from an infrared behavior of the loop integrand which is controlled by $\delta m$. The next-to-leading order correction is $O(\delta m)$. To this order, the Yukawa interactions of the Higgs bosons or unphysical (NG) bosons must be included to keep the gauge invariance. In this paper, we calculate the annihilation cross section up to the leading-order correction $O(\sqrt{\delta m})$, however, it is straight-forward to include the next-to-leading order calculations.

In the wino limit, the gauge interactions of inos are

$$L_{\text{int}} = -\frac{e}{s_W} \left(\tilde{\chi}_1^{0(2)} \gamma^\mu \tilde{\chi} - W^\mu_\mu + h.c.\right) + e \frac{c_W}{s_W} \tilde{\chi} - \gamma^\mu \tilde{\chi} - Z_\mu + e \tilde{\chi} - \gamma^\mu \tilde{\chi} - A_\mu.$$  

(9)

The NR Lagrangian can be derived by taking a NR limit of neutralino and chargino and integrating out of the gauge fields, and we find

$$L_{\text{NR}}^{(W)} = \eta_N^\dagger \left(i \partial_t + \frac{\nabla^2}{2m}\right) \eta_N + \eta_C^\dagger \left(i \partial_t - \delta m + \frac{\nabla^2}{2m}\right) \eta_C + \xi_C^\dagger \left(i \partial_t - \delta m - \frac{\nabla^2}{2m}\right) \xi_C$$

$$+ \frac{\alpha}{2} \int d^3 y \ \eta_C^\dagger(x) \xi_C(\vec{y}, t) \frac{1 - \epsilon_W^2 e^{-m_W |\vec{x} - \vec{y}|}}{|\vec{x} - \vec{y}|} \xi_C^\dagger(\vec{y}, t) \eta_C(x)$$

$$- \frac{\alpha}{2 \epsilon_W^2} \int d^3 y \ \left(\eta_N^\dagger(x) \eta_N(\vec{y}, t) e^{-m_W |\vec{x} - \vec{y}|} \frac{1}{|\vec{x} - \vec{y}|} \xi_C^\dagger(\vec{y}, t) \eta_C(x) + h.c.\right).$$  

(10)

Here, $\eta_C$, $\xi_C$ and $\eta_N$ are defined as

$$\eta_C = \frac{1 + \gamma^0}{2} \tilde{\chi}^0 e^{imt}, \quad \xi_C = \frac{1 + \gamma^0}{2} \tilde{\chi}^0 e^{-imt}, \quad \eta_N = \frac{1 + \gamma^0}{2} \tilde{\chi}_1^0 e^{imt}.$$  

(11)
We keep isospin singlet terms only, because only these terms are necessary to calculate the neutralino annihilation cross section.

The gauge interactions in the Higgsino limit are

\[
\mathcal{L}_{\text{int}} = -\frac{e}{2s_W} \left( \bar{\chi}_1^0 \gamma^\mu \chi_1^0 W_\mu^\dagger \right) - \frac{e}{s_W c_W} \left( \frac{1}{2} - c_W^2 \right) \bar{\chi}_2^0 \gamma^\mu \chi_2^0 W_\mu^\dagger + \text{h.c.} \right) - \frac{e}{2s_W c_W} \bar{\chi}_1^0 \gamma^\mu \gamma_5 \chi_2^0 Z_\mu .
\]

(12)

The NR Lagrangian is derived in the same way as the wino-like case,

\[
\mathcal{L}_{\text{NR}}^{(H)} = \eta_N^\dagger \left( i \partial_t + \frac{\nabla^2}{2m} \right) \eta_N + \eta_C^\dagger \left( i \partial_t - \delta m + \frac{\nabla^2}{2m} \right) \eta_C \\
+ \xi_C^\dagger \left( i \partial_t - \delta m - \frac{\nabla^2}{2m} \right) \xi_C + \xi_N^\dagger \left( i \partial_t - \delta m_N + \frac{\nabla^2}{2m} \right) \xi_N \\
+ \frac{1}{2} \int d^3y \ \eta_N^\dagger(x) \xi_C(\vec{y}, t) \frac{1 - (1 - 2q^2/v^2 e^{-m_N|x-y|})}{|x-y|} \xi_C^\dagger(\vec{y}, t) \eta_C(x) + \text{h.c.}
\]

(13)

Here, \( \xi_N \) is defined as \( \xi_N = (1 - \gamma^0)\chi_1^0 e^{-i\varpi t}/2 \), since the sign of the the second-lightest neutralino mass is opposite to that of the LSP.

So far we have derived the NR Lagrangian relevant to the neutralino and chargino scattering. Now we would like to include the terms relevant to the neutralino pair annihilation into two photons in the framework. The process \( \bar{\chi}_1^0 \chi_1^0 \rightarrow \gamma\gamma \) involves external photons whose momentums are of the order of the neutralino mass. Those photons cannot be described in the NR Lagrangian. However, it is possible to calculate the pair-annihilation cross section in this formalism, by introducing the a non-unitary four-Fermi terms in the NR Lagrangian. The optical theorem relates the imaginary part of neutralino forward-scattering amplitude \( \mathcal{T} \) to the pair-annihilation cross section \( \sigma \);

\[
2 \text{ Im } \mathcal{T} = s \sigma v .
\]

(14)

Here \( s \) and \( v \) are square of the center-of-mass energy and the relative velocity, respectively. The LSP-pair annihilation cross section therefore can be obtained by calculating the imaginary part of the forward-scattering amplitude \( \mathcal{T} \) if proper non-unitary
terms are incorporated. This also justifies the factorization of the short-distance physics from the the long-distance one, and it is discussed in detail in Ref. [11].

When we explicitly keep the chargino fields in the NR action, the annihilation into two photons is described by a chargino four-Fermi operator. The annihilation term is given by

\[ L_{\text{ann}} = d \eta_C^\dagger(x) \xi_C(x) \xi_C^\dagger(x) \eta_C(x) . \]  

(15)

The coupling \( d \) is matched to the chargino pair-annihilation cross section to two photons using Eq. (14), or equivalently \( v \sigma_{\text{ann}}^{(c)} = \text{Im} d/2 \). We find \( d = i \pi \alpha^2/m^2 \). We do not include the annihilation terms for the LSP or the next-lightest neutralino, since they are responsible to the processes to \( W \)- or \( Z \)-boson pairs, not to photons.

### III Two-body State Effective Action

In order to calculate the neutralino annihilation cross section, it is convenient to use the effective Lagrangian for the neutralino and the chargino two-body states than the NR Lagrangian directly. The two-body state effective action is derived by introducing auxiliary fields for \( \eta_C^\dagger \xi_C, \eta_N^\dagger \xi_N \), and their Hermitian conjugate fields. After some calculations, the two-body state effective action is found to be

\[ S_{\text{2body}} = \int \frac{d^4 P}{(2\pi)^4} \int d^3r \left[ \phi_C^{(P)}(\vec{r}) \left( E - 2\delta m + \frac{\nabla^2}{m} + \frac{2\pi i \alpha^2}{m^2} \delta(\vec{r}) \right) \phi_C^{(P)}(\vec{r}) \right. \\
+ \sum_{i=1}^2 \phi_N^{(P)}(\vec{r}) \left( E - 2\delta m_i + \frac{\nabla^2}{m} \right) \phi_N^{(P)}(\vec{r}) \\
+ \zeta_C e^{-m_Z r} \phi_C^{(P)}(\vec{r}) \phi_C^{(P)}(\vec{r}) \\
+ \sum_{i=1}^2 \omega_i e^{-m_W r} \left\{ \phi_C^{(P)}(\vec{r}) \phi_N^{(P)}(\vec{r}) + \text{h.c.} \right\} \\
+ \zeta_N e^{-m_Z r} \left\{ \phi_N^{(P)}(\vec{r}) \phi_N^{(P)}(\vec{r}) + \text{h.c.} \right\} \right]. \]  

(16)

Here, \( \phi_C^{(P)}(\vec{r}) \), \( \phi_N^{(P)}(\vec{r}) \), and \( \phi_N^{(P)}(\vec{r}) \) correspond to the chargino pair, the lightest neutralino pair and the second-lightest neutralino pair, respectively, and \( \delta m_1 = 0 \) and \( \delta m_2 = \delta m_N \). The capital \( P \) is the center of mass energy and momentum of the two-body state, and \( \vec{r} \) is the relative coordinate. The internal energy of the neutralino pair, \( E_i \), is defined as \( E = P^0 - \vec{P}^2/(4m) \). The coupling constants \( \omega_1, \omega_2, \zeta_C \) and \( \zeta_N \) in \( S_{\text{2body}} \) are

\[ \omega_1 = -\frac{\sqrt{2} \alpha}{s_W^2} , \quad \omega_2 = 0 , \quad \zeta_C = -\frac{\alpha c_W^2}{s_W^2} , \quad \zeta_N = 0 \]  

(17)
for the wino-like case, and
\[
\omega_1 = -\frac{\sqrt{2}\alpha}{4s_W^2}, \quad \omega_2 = -\frac{\sqrt{2}\alpha}{4s_W^2}, \quad \zeta_C = -\frac{\alpha(1-2c_W^2)}{4c_W^2s_W^2}, \quad \zeta_N = -\frac{\alpha}{4c_W^2s_W^2}
\] (18)
for the Higgsino-like case.

The S-wave states of the neutralino and chargino pairs must give the largest contribution to the annihilation cross section, because higher angular-momentum modes are suppressed by power(s) of the relative velocity \(v\) compared with the S-wave state. Thus, the partial wave expansion of \(S_{2\text{body}}\) is convenient for us. For this purpose, we expand \(\phi_{N_1}^{(P)}\) as
\[
\phi_{N_1}^{(P)}(\vec{r}) = \sum_{l,m} \int \frac{dp}{2\pi} N_{plm}(P)(2p) j_l(p r) Y_{lm}(\theta, \phi).
\] (19)

Here, \(j_l(p r)\) is the spherical Bessel function, and \(Y_{lm}(\theta, \phi)\) is the spherical harmonic function. The quantum number \(p\) is related to the internal energy \(E\) as \(E = p^2/m\) under the on-shell condition. In Eq. (19), \(N_{plm}\) is the annihilation operator for the positive-energy state with quantum numbers \((p l m)\).

As mentioned in Section II, the annihilation cross section of the neutralino pair can be calculated from the forward-scattering amplitude by using the optical theorem,
\[
\sigma v \left(\frac{p^2}{m}\right) = \frac{1}{2m^2} \text{Im} T_{pp} \left(\frac{p^2}{m}\right)
\] (20)
where the forward-scattering amplitude \(T_{pp}(E)\) is
\[
(2\pi)^4 \delta^{(4)}(P - P') i T_{pp}(E) = \frac{8\pi}{v^2} \langle 0|N_{p00}(P)N_{p00}^\dagger(P')|0\rangle
\] (21)
and the relative velocity \(v = 2p/m\). The amplitude \(T_{pp}\) may be obtained by solving the equation of motion for the two-body state derived from Eq. (16), as in the scattering theory in the quantum mechanics. Or, we may also calculate it by using the perturbative theory in a diagrammatic method.

The dominant contribution to \(T_{pp}\) in the each loop order comes from the ladders of photon and weak-boson exchange, as illustrated in Fig. provided that the intermediate neutralinos and charginos are almost on-shell and enhance the corresponding amplitudes. Similar phenomena are found in QED or QCD as mentioned in Section I. In the electron and positron pair annihilation/production at the threshold region, the ratio between the lowest-order amplitude and the amplitude with additional one-photon exchange between electron and positron is proportional to \(\alpha/v\) at
$\chi \sim 0$

Figure 1: Ladder diagrams in the calculation of the forward-scattering amplitude ($\mathcal{T}_{pp}$). The crossing points correspond to the annihilation term for chargino.

$v \rightarrow 0$. This is well-known as the threshold singularity. In order to evaluate the cross section, we need the resummation of the ladder diagrams since a ladder diagram with $n$-photon exchange is proportional to $(\alpha/v)^n$. The other diagrams, such as the crossed ladder diagrams, are suppressed by additional factors of $v$.

The efficient evaluation for the effect of the resummation of the ladder diagrams to the electron and positron pair annihilation/production is to use the wave functions for electron and positron pair under the QED (Coulomb) potential. When we expand the two-body state of the electron and positron pair by the wave functions under the Coulomb potential, the Coulomb potential disappears from the two-body state action, and the calculation of the annihilation cross section at the threshold region is only for a tree-level diagram by the annihilation term, and becomes extremely simple.

In the evaluation for the neutralino annihilation cross section, the resummation of weak-boson exchange diagrams, in addition to that of photon exchange, is required for the heavy neutralino annihilation cross section. The wave functions of chargino and neutralino(s) under both Coulomb and Yukawa potentials can be derived numerically, not analytically. Then, we also consider the behavior of the cross section in a limit of $m \rightarrow \infty$. In next section we use the wave functions under the Coulomb potential for the chargino pair and the free wave functions for the neutralino pairs so that we can incorporate the all-order ladder diagrams of photon exchange, and we derive some numerical results.

In order to demonstrate the validity of the effective action Eq. (16), first, we reproduce the previous one-loop result for the neutralino annihilation cross section to two photons in our formulation. The leading contribution to Im $\mathcal{T}_{pp}$ in the per-
turbative calculation is
\[ \text{Im } \mathcal{M}(0) = 4\pi \alpha^2 \left| (4\pi \omega_1) \int \frac{d^3 k}{(2\pi)^3} \frac{i}{\vec{k}^2 + m_W^2 |\vec{k}|^2 / m + 2\delta m} \right|^2. \] (22)

The velocity of the incident neutralinos is almost 0 in the above calculation since \( v/c \sim 10^{-3} \) for the neutralino DM in our galactic halo. By integrating the r.h.s. of Eq. (22) and using Eq. (20), we find
\[ \sigma v_{\text{1-loop}}(0) = \frac{2\pi \alpha^2 \omega_1^2}{m_W^2} \left( 1 + \sqrt{\frac{2m\delta m}{m_W^2}} \right)^{-2}. \] (23)

This agrees with the result [6] which is obtained of the full one-loop calculation in the heavy wino or Higgsino limit.

It is found from Eq. (22) that the amplitude is enhanced by a factor of \( \omega_1 m / m_W \), in which \( m \) comes from chargino propagators in Eq. (22) if \( \delta m \) can be neglected. Other factor \( 1/m_W \) comes from the weak-boson propagators and the loop integral. The same enhancement factors also appear in the higher-order calculations. Whenever one weak-boson exchange is added to the ladder graph, it gives an additional factor \( \omega_1 m / m_W \).

Note that the leading correction due to the mass difference \( \delta m \) is \( \mathcal{O}(\sqrt{\delta m}) \) in Eq. (23), not \( \mathcal{O}(\delta m) \) as mentioned in the previous section. If the correction was \( \mathcal{O}(\delta m) \), the integrand in Eq. (22) could be expanded by \( \delta m \). However, in this case, the integral is infrared-divergent.

The one-loop cross section is independent of the neutralino mass in small \( \delta m \) limit. On the other hand, the cross section should be bounded by the unitarity condition \( \sigma v < 4\pi/(vm^2) \). Therefore the higher-order corrections must dominate the cross section, so that the correct \( 1/m^2 \) behavior would be reproduced. This is explained as follow. If the neutralino mass is much larger than the weak-boson mass, we can neglect the effect of the weak-boson mass in the equation of motion. This can be seen by introducing the dimensionless coordinate \( x = \alpha_2 mr \) in Eq. (13). The factor \( e^{-m_W r} \) can be approximated to 1 in such an extremely large \( m \) case. Then, it is found that the cross section of the extremely heavy neutralino has the mass dependence \( \sigma \sim 1/m^2 \) from the dimensional analysis. In fact, we can solve the equation of motion and obtain the forward-scattering amplitude analytically in the large mass limit, because gauge interactions in this Lagrangian generate a common potential \( 1/r \), say the Coulomb force.

Now we calculate the cross section in this large mass limit. We take the wino limit for the LSP as an example, however, the result is similar for the Higgsino-like
Figure 2: The ladder diagrams of photon exchange relevant to the calculation of the forward-scattering amplitude. $A_0$ is the one-loop amplitude; $A_0 \sim \alpha \alpha_2 m / m_W$.

neutralino. It is convenient to change the basis from $(\phi_C^{(P)}, \phi_N^{(P)})$ to $(\phi_+^{(P)}, \phi_-^{(P)})$ in the two-body state effective action so that the $1/r$ potential terms are diagonalized as

\[
\begin{pmatrix}
\phi_C^{(P)} \\
\phi_N^{(P)}
\end{pmatrix}
\begin{pmatrix}
\frac{\alpha + \zeta_c}{r} & \frac{\omega_1}{r} \\
\frac{\omega_1}{r} & 0
\end{pmatrix}
\begin{pmatrix}
\phi_C^{(P)} \\
\phi_N^{(P)}
\end{pmatrix}
\rightarrow
\begin{pmatrix}
\phi_+^{(P)} \\
\phi_-^{(P)}
\end{pmatrix}
\begin{pmatrix}
\frac{\lambda_+}{r} & 0 \\
0 & \frac{\lambda_-}{r}
\end{pmatrix}
\begin{pmatrix}
\phi_+^{(P)} \\
\phi_-^{(P)}
\end{pmatrix}.
\] (24)

Here, $\lambda_\pm = 0.5 \left(\alpha + \zeta_c \pm \sqrt{(\alpha + \zeta_c)^2 + 4 \omega_1^2}\right)$. Since $\lambda_+ > 0$ and $\lambda_- < 0$, $\phi_+^{(P)}$ feels an attractive force, and $\phi_-^{(P)}$ feels a repulsive one. The neutralino annihilation cross section is derived from the $\phi_+^{(P)}$ annihilation, while the contribution from $\phi_-^{(P)}$ is exponentially suppressed due to the repulsive force. Neglecting the $\phi_-$ contribution, the forward-scattering amplitude is written as follows;

\[
\text{Im } T_{pp} \left(\frac{p^2}{m}\right) \simeq \frac{4 \pi^2 \omega_1^2 \alpha^2 \lambda_+}{v (\lambda_+^2 + \omega_1^2) (1 - \exp(-2 \pi \lambda_+ / v))} .
\] (25)

Then, the neutralino annihilation cross section to two photons in the large mass limit is

\[
\sigma v = \frac{2 \pi^2 \omega_1^2 \alpha^2 \lambda_+}{m^2 v (\lambda_+^2 + \omega_1^2) (1 - \exp(-2 \pi \lambda_+ / v))} \sim 2.8 \times 10^{-5} \frac{1}{vm^2} .
\] (26)

This cross section behaves as $\sigma v \sim 1/vm^2$, and satisfies the unitarity bound as expected.

IV Higher-order Corrections in Neutralino Annihilation to Two Photons

In the previous section, we showed that the higher-order corrections from the weak-boson ladder diagrams must be non-negligible if the neutralino mass $m$ is larger than $m_W/\alpha_2$. In this section we estimate more carefully the scale where the
perturbative approximation is broken down. We also discuss another important higher-order corrections from the photon ladder diagrams.

We start our discussion from the effect of photon exchange. The corresponding diagrams in the perturbative calculation of $T_{pp}$ are shown in Fig. 2. In this figure, the orders of the amplitudes are also shown. The lowest-order diagram at the left side of the figure gives the amplitude $A_0 (A_0 \sim \alpha \alpha_2 m/m_W)$. The diagrams have an addition factor $\alpha \sqrt{m/\delta m}$ for each photon exchange, and then, the higher-order diagrams of photon exchange become important if the mass difference ($\delta m$) is smaller than $\alpha^2 m$.

The enhancement factor $\alpha \sqrt{m/\delta m}$ comes from the massless photon exchange at $t$-channel. For the neutralino DM pair annihilation, the initial neutralinos is almost at rest, and then the intermediate charginos just slightly deviate from on-shell states by the mass difference between chargino and neutralino, $\delta m$. The situation is similar to calculation of the pair production at threshold in QED or QCD. In this case, the intermediate particles deviate from on-shell states by $O(v^2 m)$ where $v$ is the relative velocity of the incident particles, and the diagrams have an addition factor $\alpha_s/v$ for each photon (gluon) exchange.

If neutralino is highly degenerate with chargino in mass, we need resummation of the photon ladder diagrams. Similar to the positron and electron annihilation/production explained in the previous section, the resummation of the ladder diagrams is possible analytically, by expanding the chargino-pair field ($\phi_C(\vec{r})$) in the two-body state effective action Eq. (16), in terms of the eigenfunctions of the Schrödinger equation with the Coulomb potential,

\[ \phi_C^{(P)}(\vec{r}) = \sum_{l,m} \left[ \int \frac{dk}{2\pi} C_{klm}(P) R_C^{(kl)}(r) + \sum_n \tilde{C}_{nlm}(P) \tilde{R}_C^{(nl)}(r) \right] Y_{lm}(\theta, \phi) . \]  

Here, $C_{klm}$ is the annihilation operator of the continuum state of a chargino pair with quantum numbers $(klm)$, and $\tilde{C}_{nlm}$ is one of a chargino-pair bound state with $(nlm)$. $R_C^{(kl)}(r)$ and $\tilde{R}_C^{(nl)}(r)$ are eigenfunctions of the radial direction for the states. Both $R_C$ and $\tilde{R}_C$ are given by the confluent hypergeometric function, and the S-wave parts of these functions are

\[ R_C^{(kl)}(\vec{r}) = 2 \sqrt{\frac{\pi \alpha m k}{1 - \exp(-\pi \alpha m/k)}} e^{-ikr} \, _1F_1 \left( \frac{\alpha m}{2k} i + 1, 2, 2ikr \right) , \]

\[ \tilde{R}_C^{(nl)}(\vec{r}) = 2 \left( \frac{\alpha m}{2n} \right)^{3/2} e^{-\alpha mr/2n} \, _1F_1 \left( -n + 1, 2, \alpha mr \right) , \]  

where $_1F_1$ is the (Kummer’s) confluent hypergeometric function. With this expansion and Eq. (19), we derive the S-wave effective Lagrangian of the two-body states.
We show details of the Lagrangian in appendix A.

In the following we calculate the higher-order corrections of gauge-boson exchange. The forward-scattering amplitude $T_{pp}$ is calculated from the S-wave effective Lagrangian by treating weak-boson exchange interactions perturbatively. The effect of all-order photon-exchange effects are now automatically taken into account in $\alpha$ dependent coefficients of the interactions.

We first calculate the cross section in the leading level where the effect of only one $W$-boson exchange is included. After some calculation, we find

$$
\sigma v \left( \frac{p^2}{m} \right) = \frac{2\pi \alpha^2}{m^2} \left| \frac{i \omega_1}{p} \int \frac{dk}{2\pi} \frac{imA_{NC}(p,k)}{p^2 - k^2 - 2m\delta m + i0^+ \sqrt{\frac{\pi \alpha m}{1 - \exp(-\pi \alpha m/k)}}} \right|^2 + \frac{i \omega_1}{p} \sum_n \frac{imA_N(p,n)}{p^2 + \alpha^2 m^2/4n^2 - 2m\delta m + i0^+ \left(\frac{\alpha m}{2}\right)^{3/2}}.
$$

The coefficients $A_{NC}$ and $A_N$ are defined in Appendix A. Here the first term in r.h.s. comes from the continuum states of chargino pair, while the second term from the discrete states of chargino pair.

We first investigate contributions of the chargino bound states to the cross section. We can see from Eq. (31) that their contributions are suppressed by a factor $\alpha$ compared with the continuum part. This is because the overlap between the wave functions of the neutralino pair and the chargino bound states is suppressed by $\alpha$. If energy of the neutralino pair is on the pole of the chargino bound state ($p^2/m \sim 2\delta m - \alpha^2 m/4n^2$), the cross section is enhanced beyond the suppression due to the small overlap of the wave functions. However, it is very unlikely for the pair annihilation of the neutralino DM. The average DM velocity in our galactic halo is roughly $v/c \sim 10^{-3}$. Thus, the bound state contribution is negligible as far as the neutralino mass is smaller than 100TeV, since $p^2/m \sim 10^{-6}m$ and the mass difference $\delta m$ may be larger than 100MeV.

Thus, we only have to take into account the contribution from the continuum chargino-pair states. Since the continuum part of the cross section is a smooth function around the $p \sim 0$, the cross section for the neutralino DM annihilation may be approximated by $\sigma v(0)$. The result is

$$
\sigma v(0) = \frac{2\pi \alpha^2}{m^2} |C(0)|^2
$$

where

$$
C(0) = \frac{i \omega_1}{\pi} \int \frac{dk}{2\pi} \frac{im \pi \alpha mk}{1 - e^{-\pi \alpha m/k}} \exp \left[ -\frac{\alpha m}{2k} \arctan\left( \frac{2m_W k}{m_W^2 - k^2} \right) \right] \frac{1}{-k^2 - 2m\delta m}.
$$
Figure 3: a) The leading $\tilde{\chi}^0_1\tilde{\chi}^0_1 \to 2\gamma$ cross section taking into account all-order QED effect, $\sigma v$, as a function of the neutralino mass $m$ (solid lines). For comparison, result of the one-loop calculation Eq. (23) is also shown (dashed lines). b) The contours of $\sigma/\sigma_{1\text{-loop}}$ in a $(m, \delta m)$ plane.

In a limit of vanishing $\alpha$, the $\sigma v$ reduces to $\sigma v_{1\text{-loop}}$ in Eq. (23).

In Fig. 3 a), solid lines are $\sigma v(0)$ for $\delta m = 0.1\text{GeV}$, 1 GeV and 10 GeV. The left axis corresponds to the wino-like neutralino annihilation cross section, and the right axis is for the Higgsino-like one. For comparison, $\sigma v_{1\text{-loop}}$ in Eq. (23) are also shown by dashed lines. Note that influences of the QED potential always work to enhance the cross section. This is because the Coulomb force acts as an attractive force between the chargino pair. In Fig. 3b), contours of $\sigma v(0)/\sigma v_{1\text{-loop}}$ is shown in a $(m, \delta m)$ plane. The enhancement is large if the mass difference $\delta m$ is small and $m$ is large as expected. For the wino-like neutralino, which is highly degenerate with chargino as $\delta m \sim \mathcal{O}(100)\text{MeV}$, the enhancement by factor of 2 is possible when the neutralino mass is $\mathcal{O}(1)\text{TeV}$.

Next, we discuss the next-to-leading order corrections arising from the diagrams of two weak-boson exchange. The detail of the calculation will be given elsewhere [12], and we only show the ratio between the next-to-leading order corrections and the leading amplitude, $R$, in Fig. 4.

In the wino-like case, the $Z$ exchange between the chargino pair, shown in Fig. 5, leads to the next-to-leading order corrections. In Fig. 4 a), the ratio $R$ in the wino-like case is plotted as a function of $m$ with $\delta m = 0.1$ GeV and 1 GeV. We call the value of $m$ where $|R| = 1$ as the critical neutralino mass $m_{\text{crit}}$. Above and around the mass, the higher-order corrections is larger than the leading-order one, therefore the perturbation is not reliable any more. For the wino-like case, $m_{\text{crit}} \sim 10\text{TeV}$,
Figure 4: The ratio between the next-to-leading order amplitude and the leading-order amplitude as a function of the neutralino mass $m$. a) and b) are for the wino- and the Higgsino-like neutralino, respectively.

$$\chi_{0}^0 \xrightarrow{\tilde{\chi}^0} \tilde{\chi}^+ \xrightarrow{\tilde{\chi}^+} \tilde{\chi}^+ \xrightarrow{\tilde{\chi}^+} \tilde{\chi}^+$$

Wino-like

$$\chi_{0}^0 \xrightarrow{\tilde{\chi}^0} \tilde{\chi}^+ \xrightarrow{\tilde{\chi}^+} \tilde{\chi}^+ \xrightarrow{\tilde{\chi}^+} \tilde{\chi}^+$$

Higgsino-like

Figure 5: Diagrams which contribute to next-to-leading order correction. a) and b) are for the wino- and Higgsino-like neutralino DM, respectively.

and for the Higgsino-like case $m_{\text{crit}} \sim O(10)\text{TeV}$.

V Conclusion and Discussion

Although the existence of DM is established, we do not yet know the DM nature. The neutralino DM predicted in MSSM is actively searched for by various experiments, trying to solve this myth. The search for gamma-ray signal from the center of our galaxy is a feasible way to discover the neutralino DM. Especially the monochromatic gamma ray from the neutralino DM pair annihilation is a robust signal.

The reliable estimation of the signal rate is important to either set limit of the neutralino DM or interpret the observed signals. The major systematic error of the signal rate comes from the distribution of the dark matter in our galaxy. However, the pair annihilation cross section $\sigma(\chi_{1}^0\chi_{1}^0 \rightarrow 2\gamma)$ has not been also understood
completely. The problem arises when $\tilde{\chi}_1^0$ is wino- or Higgsino-like. In that case, the one-loop cross section $\sigma_{1-\text{loop}}$ is approximately constant, and does not scale as $1/m^2$. The one-loop cross section obviously breaks the unitarity bound in $m \to \infty$ limit. The enhancement comes from chargino in the loop, which is degenerate with the LSP neutralino.

In this paper we study the higher-order loop corrections to the pair annihilation process. We find that the dominant contribution comes from the ladder diagram of weak bosons and photon exchange. We study the corrections using NRQED technique, which allows us to incorporate ladder-type QED corrections analytically to all orders, while including weak boson exchange corrections perturbatively. We find the QED corrections enhance the cross section up to a factor of two. We also find the critical scale $m_{\text{crit}}$, around and above which the next-to-leading order corrections by weak-boson exchange are larger than the leading contribution, is $\sim O(10)$ TeV for the Higgsino-like LSP and $\sim 10$ TeV for the wino-like LSP. The corrections reduce the cross section for $m < m_{\text{crit}}$. We also find the pair-annihilation cross section satisfies the unitarity bound in a limit of $m \to \infty$. We note that our formulation may be used for any other DM candidate if the DM comes from SU(2) multiplets and the DM is degenerate with other components in mass.

The relic density of the Higgsino- or wino-like LSP with the mass above 1 TeV may be consistent to the matter density of the Universe $\Omega_M$. The monochromatic gamma ray in TeV region may be searched for the future atmospheric Cherenkov telescopes (ACT) \cite{5}. The signal flux may be above the sensitivity, if the DM density distribution is singular at the center of our galaxy. Therefore it is important to estimate the neutralino pair-annihilation cross section reliably. For such a heavy Higgsino- or wino-like neutralino, one has to sum the weak corrections to all orders as we discussed in this paper. The resummation is possible by the solving the NR equation of motion of neutralino \cite{12}.

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Appendix A S-wave Lagrangian

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In this appendix, we list the full S-wave Lagrangian, which is used to calculate the leading and the next-to-leading order corrections of the cross section. The S-wave Lagrangian is obtained by substituting Eq. (19) and Eq. (27) to the two-body state effective action Eq. (16), and extracting the S-wave parts. We find

\[ \mathcal{L}_S = \mathcal{L}_0 + \mathcal{L}_W + \mathcal{L}_Z + \mathcal{L}_a , \]

where

\[ \mathcal{L}_0 = \int \frac{dp}{2\pi} \left( E - \frac{p^2}{m} \right) N_p^{(1)^\dagger} N_p^{(1)} + \int \frac{dp}{2\pi} \left( E - 2\delta m_N - \frac{p^2}{m} \right) N_p^{(2)^\dagger} N_p^{(2)} \]

\[ + \int \frac{dk}{2\pi} \left( E - 2\delta m - \frac{k^2}{m} \right) C_k^\dagger C_k + \sum_n \left( E - 2\delta m + \frac{m\alpha^2}{4n^2} \right) \tilde{C}_n^\dagger \tilde{C}_n , \]

\[ \mathcal{L}_W = \sum_{i=1}^2 \omega_i \int \frac{dp}{2\pi} A_{NC}(p, k) \left[ C_k^\dagger N_p^{(i)} + N_p^{(i)^\dagger} C_k \right] \]

\[ + \sum_{i=1}^2 \omega_i \sum_n \int \frac{dp}{2\pi} A_{NC}(p, n) \left[ \tilde{C}_n^\dagger N_p^{(i)} + N_p^{(i)^\dagger} \tilde{C}_n \right] , \]

\[ \mathcal{L}_Z = \zeta_C \int \frac{dk}{2\pi} A_{CC}(k, k') C_{k'}^\dagger C_k + \zeta_C \int \sum_{n, n'} A_{C\tilde{C}}(n, n') \tilde{C}_n^\dagger \tilde{C}_{n'} \]

\[ + \zeta_C \sum_n \int \frac{dk}{2\pi} A_{C\tilde{C}}(k, n) \left[ \tilde{C}_n^\dagger C_k + C_k^\dagger \tilde{C}_n \right] \]

\[ + \zeta_N \int \frac{dp}{2\pi} A_{NN}(p, p') \left[ N_p^{(1)^\dagger} N_{p'}^{(2)} + N_{p'}^{(2)^\dagger} N_p^{(1)} \right] , \]

\[ \mathcal{L}_a = i \frac{2\alpha^2}{m^2} \left[ \int \frac{dk}{2\pi} \sqrt{\frac{\pi \alpha m k}{1 - \exp(-\pi \alpha m/k)}} C_k^\dagger + \sum_n \left( \frac{\alpha m}{2n} \right)^{3/2} \tilde{C}_n^\dagger \right] \]

\[ \cdot \left[ \int \frac{dk'}{2\pi} \sqrt{\frac{\pi \alpha m k'}{1 - \exp(-\pi \alpha m/k')}} C_{k'} + \sum_{n'} \left( \frac{\alpha m}{2n'} \right)^{3/2} \tilde{C}_{n'} \right] . \] (30)

Here, we omit the arguments \( P, l, \) and \( m \) for simplicity, which are the center of mass energy and momentum, and orbit-angular momentums, respectively. Coupling constants \( \omega_1, etc, \) are defined by the Eqs. (17,18). \( N_p^{(1)} \) and \( N_p^{(2)} \) are annihilation operators of the lightest and next-lightest neutralino pairs, respectively, and \( C_k \) and \( \tilde{C}_n \) are for the continuum and discrete chargino pair states. The coefficients \( A_{NC} \) and \( \tilde{A}_{N\tilde{C}}, etc \), are given by a little complicated functions,

\[ A_{NC}(p, k) = -4 \frac{\pi m}{\alpha m} \sqrt{\frac{\pi \alpha m k}{1 - \exp(-\pi \alpha m/k)}} \Im \left[ \frac{m_W - i(p + k)}{m_W - i(p - k)} \right]^{-i\alpha m/2k} , \]
\begin{align*}
A_{N\tilde{C}}(p, n) &= -\frac{4}{\alpha m} \left(\frac{\alpha m}{2n}\right)^{3/2} \text{Im} \left[ \left( \frac{m_W - \alpha m/2n - ip}{m_W + \alpha m/2n - ip} \right)^n \right], \\
A_{CC}(k, k') &= \frac{4}{m_z^2 + (k - k')^2} \left[ \frac{\pi \alpha m k}{1 - \exp(-\pi \alpha m/k)} \sqrt{1 - \exp(-\pi \alpha m/k')} \right] \\
&\times \left( \frac{m_z + i(k + k')}{{m_z - i(k - k')}} \right)^{\alpha m/2k} \left( \frac{m_z + i(k + k')}{{m_z - i(k' - k)}} \right)^{\alpha m/2k'} \\
&\times 2F_1 \left( \frac{\alpha m}{2k}, 1 + \frac{\alpha m}{2k'} i + 1, 2, -\frac{4kk'}{m_z^2 + (k - k')^2} \right), \\
A_{\tilde{C}\tilde{C}}(n, n') &= 2 \frac{\alpha^3 m^3}{4m_z^2 - \alpha^2 m^2(1/n - 1/n')^2 (nn')^{3/2}} \\
&\times \left( \frac{2m_z - \alpha m(1/n - 1/n')}{2m_z + \alpha m(1/n + 1/n')} \right)^n \left( \frac{2m_z - \alpha m(1/n' - 1/n)}{2m_z + \alpha m(1/n + 1/n')} \right)^{n'} \\
&\times 2F_1 \left( 1 - n, 1 - n', 2, \frac{4\alpha^2 m^2/(nn')}{4m_z^2 + \alpha^2 m^2(1/n - 1/n')^2} \right), \\
A_{\tilde{C}C}(k, n) &= \frac{4}{m_z^2 + (ik - \alpha m/(2n))^2} \left[ \frac{\pi \alpha m k}{1 - \exp(-\pi \alpha m/k)} \left( \frac{\alpha m}{2n} \right)^{3/2} \right] \\
&\times \left( \frac{m_z + \alpha m/(2n) + ik}{{m_z + \alpha m/(2n) - ik}} \right)^{\alpha m/2k} \left( \frac{m_z + \alpha m/(2n) + ik}{{m_z - \alpha m/(2n) + ik}} \right)^{-n} \\
&\times 2F_1 \left( \frac{\alpha m}{2k}, 1 + 1 - n, 2, \frac{2ik\alpha m/n}{m_z^2 - (\alpha m/(2n) - ik)^2} \right), \\
A_{NN}(p, p') &= \log \left( \frac{m_z^2 + (p + p')^2}{m_z^2 + (p - p')^2} \right). \quad (31)
\end{align*}

Here, \(2F_1\) is the (Gauss’s) hypergeometric function.

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