Unified Analyses of Multiplicity Distributions and Bose–Einstein Correlations at the LHC using Double-Stochastic Distributions

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Abstract

We analyze data on multiplicity distributions (MD) at Large Hadron Collider (LHC) energies using a double-negative binomial distribution (D-NBD) and double-generalized Glauber–Lachs formula (D-GGL). Moreover, we investigate the Bose–Einstein correlation (BEC) formulas based on these distributions and analyze the BEC data using the parameters obtained by analysis of MDs. From these analyses it can be inferred that the D-GGL formula performs as effectively as the D-NBD. Moreover, our results show that the parameters estimated in MD are related to those contained in the BEC formula.

1 Introduction

In Large Hadron Collider (LHC) experiments, various multiplicity distributions (MD) have been measured with a pseudorapidity interval ($|\eta| < \eta_c$) in proton-proton collision \cite{1,2,3}. Additionally, the negative binomial distribution (NBD) has been used to analyze MD.

\begin{equation}
P_{\text{NBD}}(n, k, \langle n \rangle) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}\left(\frac{\langle n \rangle}{k}\right)^n \left(1 + \frac{\langle n \rangle}{k}\right)^{-k}.
\end{equation}

However, we cannot understand the behavior of the parameter: $k = 7, 8$ for lower energy (ISR) and $k = 2, 3$ for higher energy (SppS).

The double-negative binomial distribution (D-NBD) \cite{4,5} has been proposed to explain the KNO scaling \cite{6} violation observed by the UA5 \cite{7,8,9}, using $\alpha_1 + \alpha_2 = 1$, \begin{equation}
P_{\text{D-NBD}}(n, \langle n \rangle) = \alpha_1 P_{\text{NBD}_1}(n, k_1, \langle n_1 \rangle) + \alpha_2 P_{\text{NBD}_2}(n, k_2, \langle n_2 \rangle).
\end{equation}

In 2010, we analyzed the ALICE data \cite{10} by the generalized Glauber–Lachs (GGL) formula, and showed that GGL fits as well as NBD \cite{11}:

\begin{equation}
P_{\text{GGL}}(n, k, p, \langle n \rangle) = \frac{(p(n)/k)^n}{(1 + p(n)/k)^{n+k}} \exp \left[ -\frac{\gamma p(n)}{1 + p(n)/k} \right] L_n^{(k-1)} \left( \frac{\gamma k}{1 + p(n)/k} \right),
\end{equation}

where $p = 1/(1+\gamma)$ and $\gamma$ reflects the contamination ($K, p/\pi$’s) and the degree of superposition of the phase spaces of the particles. Our analyses suggested that the coherent component is necessary. Moreover, Eq. (3) has the following stochastic property:

\begin{equation}
\text{Eq. (3)} \quad \begin{cases} \quad k \to 1 \quad \rightarrow \quad \text{original GL} \quad \rightarrow \quad p \to 1 \quad \rightarrow \quad \text{Furry distribution} \\
\quad p \to 1 \quad \rightarrow \quad \text{NBD of Eq. (1)} \\
\quad \downarrow k \to \infty \\
\quad p \to 0 \quad \rightarrow \quad \text{Poisson distribution.}
\end{cases}
\end{equation}

To explain the shape of the MD with higher energy than SppS, two sources or two collision mechanisms \cite{12,13,14} are required; thus, the two-component model is necessary.

In the theoretical analysis of experimental data, MD and the Bose–Einstein correlation (BEC) have been handled as independent observables. In contrast, we have proposed a method for analyzing MD and BEC data using common parameters \cite{15,16,17}. See a review book \cite{18}.
2 Double-GGL formula (D-GGL)

Herein, we propose the D-GGL formula\[19\] for analyzing the multiplicity distribution at LHC:

\[
P_{\text{D-GGL}}(n, \langle n \rangle) = \alpha_1 P_{\text{GGL}}(n, k = 2, p_1, \langle n_1 \rangle) + \alpha_2 P_{\text{GGL}}(n, k = 2, p_2, \langle n_2 \rangle),
\]

where \(k = 2\) reflects the degree of freedom for the \((+−)\) particle ensembles (Fig. 1). Our results are presented in Fig. 2 and Table I.

To understand the KNO scaling violation, the weight factors \((\alpha_1 \text{ and } \alpha_2)\) are displayed in Fig. 3. The fluctuation of \((\alpha_1 \text{ and } \alpha_2)\) are reflecting the KNO scaling violation.

3 Bose–Einstein correlations (BEC)

By using Eq. (5), we obtain the moments as

\[
\langle n \rangle = \alpha_1 \langle n_1 \rangle + \alpha_2 \langle n_2 \rangle,
\]

\[
\langle n(n-1) \rangle = \alpha_1 \langle n_1(n_1-1) \rangle + \alpha_2 \langle n_2(n_2-1) \rangle.
\]

For BEC, we need the ratio \(N^{(2+; 2−)}/N^{\text{BG}}\), where \(N\) stands for the number of pairs. The numerator \(N^{(2+; 2−)}\) is the number of the pairs of the same charged particles. \(N^{\text{BG}}\) is that of different charged particles. There are
Table 1: Analyses results of analyses of ATLAS data on the MD \((p_t > 100\text{ MeV}, |\eta| < 2.5, n_{ch} \geq 2)\) using Eqs. (2) and (5).

| \(\sqrt{s} [\text{TeV}]\) | \(\alpha_1\) | \(k_1\) | \(\langle n_1 \rangle\) | \(k_2\) | \(\langle n_2 \rangle\) | \(\chi^2/\text{ndf}\) |
|---|---|---|---|---|---|---|
| 0.9 | 0.23±0.03 | 7.52±0.77 | 32.9±1.3 | 2.70±0.12 | 12.6±0.4 | 70.2/46 |
| 7.0 | 0.60±0.02 | 2.61±0.10 | 39.4±0.6 | 2.72±0.09 | 11.0±0.1 | 129/80 |

Two possibilities for calculating \(N_{BG}^1\). Whole events can be categorized as two ensembles labeled \(\alpha_1\) and \(\alpha_2\). Each \(N_{BG}^1\) and \(N_{BG}^2\) are calculated inside those ensembles: \(N_{BG}^1 = \langle n_1^{(+)} \rangle C_1 \times \langle n_1^{(-)} \rangle C_1 = \langle n_1^{(+)} \rangle \langle n_1^{(-)} \rangle \) and \(N_{BG}^2 = \langle n_2 \rangle^2\). This procedure needs the complete separation of two ensembles, for this aim.

Conversely, the second \(N_{BG}^{II}\) is computed from two ensembles, such as \(N_{BG}^{II} = (\alpha_1 \langle n_1 \rangle^2 + \alpha_2 \langle n_2 \rangle^2)\). The denominator is the sum of numerators of pairs come from the first and second ensembles. Thus, we have two formulas for BEC, which depend on \(N_{BG}^1\) and \(N_{BG}^{II}\).

\[
N^{(2+:2–)}_{BG} = \alpha_1 \left[ 1 + \frac{2}{k_1} f(p_1, E_{BE_1}) \right] + \alpha_2 \left[ 1 + \frac{2}{k_2} f(p_2, E_{BE_2}) \right],
\]

\[
N^{(2+:2–)}_{BG}^{II} = \frac{1}{1+\Gamma} \left[ 1 + \frac{2}{k_1} f(p_1, E_{BE_1}) \right] + \frac{1}{1+\Gamma} \left[ 1 + \frac{2}{k_2} f(p_2, E_{BE_2}) \right],
\]

where, \(\Gamma = (\alpha_2 \langle n_2 \rangle^2)/(\alpha_1 \langle n_1 \rangle^2)\) and the normalization \((1 + \Gamma)^{-1} + (1 + \Gamma^{-1})^{-1} = 1\), and

\[
f(p, E_{BE}) = 2p(1-p)E_{BE} + p^2E_{BE}^2.
\]

The exchange function between the same charged particles is expressed by the exponential function \((E, n = 1)\) or the Gaussian function \((G, n = 2)\).

\[
E_{BE}^2 = \exp(-|RQ|^n),
\]

where \(R\) is the magnitude of the interaction range and \(Q = \sqrt{-(p_1 - p_2)^2}\) is the momentum transfer. For BEC based on D-NBD, we obtain the formula by adapting \(p = 1.0\) in Eq. (9).

Our results are based on the parameters presented in Table 1 and Eqs. (7)~(10). The results are displayed in Table 2 and Fig. 3.

4 Concluding remarks

1) From the analysis of MD with \(|\eta| < 2.5-3.0\), we obtained the energy dependence of the weight factors \((\alpha_1, \alpha_2)\) (Fig. 3). This behavior denotes the degree of the KNO scaling violation.

2) The formula based on \(N_{BG}^{II}\), i.e., Eq. (8), explains the BEC data more reasonably than the formula based on \(N_{BG}^1\), i.e., Eq. (7) (Table 2). At present, the second denominator seems to be available for the study on the BEC. Detailed calculations will be provided elsewhere [21].

3) In 1986, the violation of KNO scaling was discovered by UA5 Collaboration. To explain this phenomenon, the UA5 collaboration proposed the D-NBD, i.e., Eq. (2). Moreover, in experiments at LHC energies, a remarkable violation of the KNO scaling has been found. In such situation, we proposed the unified analyses of MD and BEC at the LHC by making use of D-GGL as well as D-NBD. For BEC, we have derived Eqs. (7) and (8). They are unified as follows:

\[
CF_{II} = 1.0 + \lambda_1 E_{BE1}^2 + \lambda_2 E_{BE2}^2,
\]
Table 2: Analyses results of ATLAS data on BEC ($p_t > 100$ MeV, $|\eta| < 2.5$, $n_{ch} \geq 2$) [20] using Eqs. (7), (8), and (11). The BEC formulas contain the normalization factor and the long range correlation as $(1 + \varepsilon Q)$. The conventional formula (CF$_1$) is given as $\text{CF}_1 = 1.0 + \lambda E^2_{BE}$.

| $\sqrt{s}$ [TeV] | Eq. (7) | D-NBD | $R_1$ [fm] | $R_2$ [fm] | $\chi^2$/ndf | Eq. (8) | D-GGL | $R_1$ [fm] | $R_2$ [fm] | $\chi^2$ | CF$_1$ | $\lambda$ | $\chi^2$ |
|-----------------|---------|--------|-------------|-------------|-------------|---------|--------|-------------|-------------|-----------|--------|--------|--------|
| 0.9             | Eq. (7) | D-NBD  | 1.7±0.4 (E) | 1.7±0.1 (E) | 98.6/75     | Eq. (8) | D-GGL | 2.8±0.2 (E) | 2.7±0.1 (E) | 148       | 1.8±0.1(E) | 0.62±0.01 | 86.0 |
| 7.0             | Eq. (7) | D-NBD  | 1.8±0.0 (E) | 3.1±0.1 (E) | 743/75      | Eq. (8) | D-GGL | 3.6±0.1 (E) | 3.2±0.1 (E) | 629       | 2.1±0.0(E) | 0.62±0.01 | 919  |

| $\sqrt{s}$ [TeV] | Eq. (11) | CF$_{II}$ ($\lambda_1$ and $\lambda_2$ : free) |
|-----------------|---------|----------------------------------|
| 0.9             | 0.9     | 0.87±0.03 (G) 0.26±0.02 2.8±0.3 (G) 0.47±0.07 79.8 |
| 7.0             | 0.9     | 1.85±0.02 (E) 0.59±0.01 3.5±0.1 (G) 0.28±0.01 466 |

| $\sqrt{s}$ [TeV] | Eq. (12) | CF$_{II}$ (\lambda_1 and \lambda_2 : fixed) |
|-----------------|---------|----------------------------------|
| 0.9             | 0.9     | 0.85±0.02 (G) 0.39 2.55±0.10 (G) 0.24 81 |
| 7.0             | 0.9     | 1.87±0.01 (E) 0.59 3.05±0.05 (G) 0.18 519 |

Figure 3: The energy dependence of the ($\alpha_1$, $\alpha_2$) parameters in Eqs. (2) and (5).

where $\lambda_2$ is the second degree of coherence. Eq. (11) can be named a second conventional formula as CF$_{II}$. Our result by Eq. (11) is also given in Table 2.

4) Analyses of ATLAS BEC by the triple-NBD (T-NBD) [21] are added. The T-NBD is the extensive formula of Eq. (2):

$$P_{T-NBD}(n, \langle n \rangle) = \sum_{i=1}^{3} \alpha_i P_{\text{NBD},i}(n, k_i, \langle n_i \rangle).$$

For T-NBD, see also [22]. The similarities with results by CF$_{II}$ seem to be better those by D-NBD and D-GGL: Results by CF$_{II}$ and those by Eq. (12) with the extensive formula of Eq. (8) are almost the same, except for $\lambda$’s and $\chi^2$’s.

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