Modeling and analysis of mesh stiffness for straight beveloid gear with parallel axes based on potential energy method

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Abstract
In this paper, a potential energy based slice grouping method was proposed to calculate the mesh stiffness for straight beveloid gears with parallel axes. The mathematical mesh stiffness model was derived. The finite element tooth contact model was developed and the loaded tooth contact analysis was conducted to calculate the mesh stiffness. The verification for the mesh stiffness was conducted with the error 3 %, which proves the feasibility and accuracy. Then, the effects of parameters such as pressure angle, pitch cone angle, and profile shift coefficient on the mesh stiffness were investigated. Results show that the normal pressure angle and the tooth width have obvious effects both on the single tooth and synthesized mesh stiffness. When pressure angle is less than 20°, mesh stiffness will be increased with the increase of pressure angle. However, it decreases rapidly when the pressure angle exceeds 20°. Both the single tooth and synthesized mesh stiffness increase obviously as the tooth width increases. The increase of the cone angle and addendum coefficient have a little effect on the single tooth mesh stiffness, but have the obvious incremental effects on the synthesized mesh stiffness. The contact ratio increases obviously with the increase of the addendum coefficient. The profile shift coefficient and the clearance coefficient have unsubstantial effects both on the single and synthesized mesh stiffness.

Keywords : Straight beveloid gear, Mesh stiffness, Slice grouping method, Potential energy method, Parallel axes

1. Introduction

Beveloid gears, which have the general involute tooth with varying profile shift coefficients along the tooth width direction, can be used to transit motion and power between shafts with arbitrary angle. For parallel axes condition, the degree in axial direction can be released to realize the backlash-free transmission, which can be used in in steering transmission and planetary gears for robots. However, due to the tapered gear tooth, the engagement procedure is different from the cylindrical involute gear and it’s difficult to directly calculate the mesh stiffness of beveloid gear using existed cylindrical gear meshing stiffness calculation methods. Although, the finite element method is often used to deal with complex tooth contact analysis, and the mesh stiffness can be calculated considering various deformations such as bending and shearing. But, its modeling process is cumbersome, empirical and very time-consuming. Therefore, an efficient method to solve the mesh stiffness calculation for straight beveloid gears with parallel axes is urgently needed.

In recent years, numerous studies have been conducted on the mesh stiffness models. Finite element method is an effective way to develop the mesh stiffness model and two-dimensional finite element approach was used to calculate the mesh stiffness (Maclennan, 2002, Pandya, et al., 2013, 2014, Xu, et al., 2016). Then, three-dimensional finite element approach is widely used for calculating the meshing stiffness though performing the loaded tooth contact analysis (Lin et al., 2002, Wang et al., 2009, Kiekbusch et al., 2011). For the analytical method, in 1987, a so-called potential energy method which was first presented by Yang and Lin can calculate the mesh stiffness conveniently and effectively. Based on Yang’s theory, the shear potential energy of gear teeth was considered and this method was applied to the investigation of gear dynamics (Tan, 2004). The effect of tooth base deformation was considered and the wheel body was assumed as a half-plane cantilever beam to improve the calculation of gear mesh stiffness (Zhou et al., 2012). Then, a general analytical mesh stiffness model was proposed considering the effect of the gear tooth errors and this method establishes the relationship between the gear tooth errors and the synthesized mesh stiffness, load sharing among different tooth pairs in...
mesh and loaded static transmission errors (Chen, et al. 2013). A new method was presented for calculating the gear mesh stiffness for a propagating crack in the tooth root and the influence of gear mesh stiffness on the vibration-based fault detection indicators, the RMS, kurtosis and the crest factor, was investigated (Mohammed, et al., 2013). The potential energy method was applied to evaluate the time-varying mesh stiffness of a planetary gear set without considering any modification of the gear tooth involute curve (Liang, et al., 2014). Approximate formulae were presented which give the time-varying mesh stiffness function for ideal solid spur and helical gears and the corresponding results compared well with those obtained by using two-dimensional (2D) finite element (FE) models and specific benchmark software codes (Gu, et al., 2015). The differences between the rectangular stiffness and its approximate form which is the first harmonic approximate term of rectangular stiffness were analyzed in detail (Chen, et al., 2015). The comparison of the average slope method and the approach which calculates the local slope of the force-deflection curve about a nominal deflection for the gear tooth mesh stiffness calculation were performed (Cooly, et al., 2016). The previous analytical work on gear mesh stiffness mainly focuses on spur gears and helical gears. However, the existing mesh stiffness calculation method can not be applied to the beveloid gears due to the varying profile shift coefficient along the axial direction.

In this paper, a potential energy based slice grouping method was proposed to calculate the mesh stiffness for straight beveloid gears with parallel axes. The mathematical mesh stiffness model was derived. To verify the proposed mesh stiffness model, the finite element tooth contact model was developed and the results of the mesh stiffness extracted from it are compared with the results of potential energy method. Then, the effects of parameters such as pressure angle, pitch cone angle, and profile shift coefficient on the mesh stiffness were investigated.

2. Calculation model of mesh stiffness for straight beveloid gear with parallel axes

In the potential energy method, the total potential energy stored in the mesh gear system was assumed to include four components: Hertzian energy $U_h$, bending energy $U_b$, shear energy $U_s$ and axial compressive energy $U_a$. They can be used to calculate Hertzian mesh stiffness $k_h$, bending mesh stiffness $k_b$, shear mesh stiffness $k_s$ and axial compressive stiffness $k_a$, respectively. Through the knowledge of elastic mechanics, the four parts are separately calculated for the stiffness components. And then, the four stiffness components are combined to obtain the total mesh stiffness. Due to the special tapered tooth shape of beveloid gears, the potential energy method can not be used to calculate the mesh stiffness directly. However, the slicing method provides an effective way to solve this problem for beveloid gear. Based on the slicing method, the straight beveloid gear is divided into a certain number of slices with the same thickness along the axial direction, as shown in Fig.1. In the figure, $O_1 O_1$ and $O_2 O_2$ represent the gear axes and the pinion axes, respectively. $a$ and $L$ represent the center distance and the tooth width, respectively. Each piece is regarded as a profile shifted cylindrical gear, which has the same geometry parameters as the transverse plane of the straight beveloid gear. Then the formula for calculating the mesh stiffness for each slice can be derived using the potential energy method. Finally, the mesh stiffness for all the slices along the axial direction are grouped based on the actual meshing procedure of the straight beveloid gear pair.

![Fig.1 Slicing diagram of straight beveloid gear with parallel axes](image)

The geometric diagram of straight beveloid gear is shown in Fig. 2. $R_{f-toe}$ indicates the root radius of toe and $R_{f-heel}$ indicates the root radius of heel. $R_b$ indicates the base radius of straight beveloid gear. $h_{x-toe}$ indicates the distance between the contact point and the central line of toe and $h_{x-heel}$ indicates the distance between the contact point and the central line of heel. $x_{toe}$ indicates the distance between the contact point and the base circle of toe and $x_{heel}$ indicates the distance between the contact point and the base circle of heel. $\alpha_2$ indicates half of central angle corresponding to the
base circle pitch. $\beta_x$ represents half of the central angle corresponding to tooth thickness of the contact point. $\alpha_{\text{tip-toe}}$ and $\alpha_{\text{tip-heel}}$ are the tip pressure angle for the toe and heel, respectively. $\alpha_x$ represents the angle between meshing force and axes in arbitrary locations.

Fig. 2 Geometry diagram of a straight beveloid gear

Any section with thickness $dL$ in the direction of tooth width of beveloid gear can be regarded as a profile shifted cylindrical gear, the geometric diagram is shown in Fig. 3.

Fig. 3 Slice geometry diagram of a straight beveloid gear
As shown in Fig 3, $F_b$ provides bending and shear effects, while $F_a$ cause both axial compressive and bending effect.

$h(L)$ is the distance between the contact point and the central line of the tooth, which can be obtained by

$$h(L) = r_p \sin \frac{s_p}{2r_p}$$

Where $r_p$ is the polar radius of the contact point, $s_p$ is the arc tooth thickness of the contact point.

According to the geometry of the involute tooth, in Fig 3, the distance $d(L)$ between the contact point and the base circle can be expressed as

$$d(L) = r_p \cos \frac{s_p}{2r_p} - r_b \cos \frac{s_b}{2r_b}$$

The arc tooth thickness in arbitrary locations can be calculated as

$$s_k = s \frac{r_k}{r} - 2r_k (\text{inv} a_k - \text{inv} a_n)$$

Where $a_k$ is the pressure angle, $a_n$ is the normal pressure angle, $r_k$ is the polar radius in arbitrary locations, $s$ is the reference circle arc tooth thickness, and $r$ is the reference radius.

The tooth thickness at reference diameter of the straight beveloid can be calculated as

$$s = \frac{m_n}{2} + 2x_m\tan a_n$$

Half of the tooth thickness for contact point corresponding to the central angle $\beta_x$ can be calculated as

$$\beta_x = \frac{s_x}{2r} = \frac{s}{2r} - [\text{inv} a_k - \text{inv} a_n]$$

Substituting $a_k = a_x + \beta_x$, then we can obtain

$$\tan (a_x + \beta_x) = \frac{s}{2r} + \tan a_n - a_n + a_x$$

Thus, the bending potential energy of each piece of a profile shifted cylindrical gear can be written as

$$dU_b = \frac{F^2}{2d_{kb}} = \int_0^d \left[ \left( \frac{d(f_b(L-x(L)) - P_b h(L))^2}{2d(L)} \right) dx(L) \right]$$

Where $dL_x = \frac{2}{3} [h_x(L)]^3 dL$ represents the area moment of inertia of the cross section where the distance from the dedendum circle is $x(L)$, the other parameters are shown in Fig.3.

Through proper simplification, the bending mesh stiffness of each piece of a profile shifted cylindrical gear is

$$dK_b = \int_0^d \frac{1}{2d(L)} \left[ (d(L-x(L)) \cos a_x - h(L) \sin a_x)^2 \right] dx(L)$$

Therefore, the bending mesh stiffness of beveloid gear can be written as

$$K_b = \sum_{i=1}^N \int_0^d \frac{1}{2d(L)} \left[ (d(L-x(L)) \cos a_x - h(L) \sin a_x)^2 \right] dx(L)$$

Where $N$ represents the number of slices in meshing.

According to the geometric relation of tooth profile of the straight beveloid gear, $x_i(L), \alpha_k, r_k, h_x(L)$ and $x(L)$ can be expressed as

$$\begin{cases}
  x_i(L) = \frac{L}{m_z} \tan \gamma + x_{t, toe} \\
  \alpha_k(L, \alpha_x) = \arctan \left( \frac{x_i(L)}{2r} + \tan a_n - a_n + a_x \right) \\
  r_k(L, \alpha_x) = \frac{r_b}{\cos a_k(L, \alpha_x)} \\
  h_x(L, \alpha_x) = r_k(L, \alpha_x) \sin \frac{s_k(L, \alpha_x)}{2r_k(L, \alpha_x)} \\
  x(L, \alpha_x) = r_k(L, \alpha_x) \cos \frac{s_k(L, \alpha_x)}{2r_k(L, \alpha_x)} - r_b \cos \frac{s_k(L, \alpha_x)}{2r_b}
\end{cases}$$

Differentiate formulas (10), $dx(L, \alpha_x)$ can be calculated as

$$dx(L, \alpha_x) = \left[ \frac{\partial s_k(L, \alpha_x)}{\partial a_x} \cos \frac{s_k(L, \alpha_x)}{2r_k(L, \alpha_x)} - r_k(L, \alpha_x) \right] \frac{\partial \alpha_k(L, \alpha_x)}{\partial a_x}$$

Here $\alpha_k(L, \alpha_x)$ and $r_k(L, \alpha_x)$ are calculated as

$$\partial s_k(L, \alpha_x) \cos \frac{s_k(L, \alpha_x)}{2r_k(L, \alpha_x)} - r_k(L, \alpha_x)$$

$$= \frac{r_b}{\cos a_k(L, \alpha_x)} \left[ \sin \frac{s_k(L, \alpha_x)}{2r_k(L, \alpha_x)} + \frac{\partial r_k(L, \alpha_x)}{\partial a_x} \sin \frac{s_k(L, \alpha_x)}{2r_k(L, \alpha_x)} \right]$$

$$= \frac{r_b}{\cos a_k(L, \alpha_x)} \left[ \sin \frac{s_k(L, \alpha_x)}{2r_k(L, \alpha_x)} + \frac{\partial r_k(L, \alpha_x)}{\partial a_x} \sin \frac{s_k(L, \alpha_x)}{2r_k(L, \alpha_x)} \right]$$

$$= \frac{r_b}{\cos a_k(L, \alpha_x)} \left[ \sin \frac{s_k(L, \alpha_x)}{2r_k(L, \alpha_x)} + \frac{\partial r_k(L, \alpha_x)}{\partial a_x} \sin \frac{s_k(L, \alpha_x)}{2r_k(L, \alpha_x)} \right]$$

$$= \frac{r_b}{\cos a_k(L, \alpha_x)} \left[ \sin \frac{s_k(L, \alpha_x)}{2r_k(L, \alpha_x)} + \frac{\partial r_k(L, \alpha_x)}{\partial a_x} \sin \frac{s_k(L, \alpha_x)}{2r_k(L, \alpha_x)} \right]$$
Similarly, the shear stiffness and axial compression stiffness of the straight beveloid gear can be calculated as

\[ k_s = \sum_{i=1}^{N} \int_{0}^{d(L)} \frac{1}{2\pi(1+\nu)s_i^2a_i} \frac{1}{\pi(Ls_i)} \ dx(Ls_i) \]

(12)

\[ k_a = \sum_{i=1}^{N} \int_{0}^{d(L)} \frac{1}{2\pi(1+\nu)s_i^2a_i} \frac{1}{\pi(Ls_i)} \ dx(Ls_i) \]

(13)

By substituting equation (11) into equation (9), (12) and (13), respectively, the bending stiffness, shear stiffness and axial compression stiffness of the straight beveloid gear can be obtained.

In this study, it is assumed that the mating teeth are two isotropic elastic bodies (Yang, et al., 1987), the Hertzian-contact stiffness of a pair of meshing teeth made of the same material is a constant along the entire action line. It is independent of the contact position. This constant can be expressed as

\[ k_h = \frac{\pi EL}{4(1-\nu^2)} \]

(14)

Where \( E \), \( L \) and \( \nu \) represent Young’s modulus, the width of tooth, and Poisson’s ratio, respectively.

The sum of Hertzian-contact stiffness, bending stiffness, shear stiffness, and axial compression stiffness makes up the total stiffness of a single tooth meshing of the piece straight beveloid gear, as follows

\[ k = \frac{1}{k_h + k_{h_1} + k_{h_2} + k_{h_3} + k_{h_4}} \]

(15)

Where subscripts “1” and “2” denote the pinion and gear, respectively.

In order to integrate the stiffness set of each slice of the straight beveloid gear pair, the area for teeth engagement at different moments should be considered according to the actual meshing condition. The meshing plane of the straight beveloid gear pair with parallel axes is shown in Fig. 4, which is the internal common tangent plane of the gear base cylinders. \( B_1B_2 \) is the intersection of the pinion addendum cone surface and the meshing plane. \( B_3B_4 \) is the intersection of the gear addendum cone surface and the meshing plane. The area enclosed by the \( B_1B_2B_3B_4 \) in the meshing plane is the spatial meshing area of the straight beveloid gear pair with parallel axes. Unlike the conventional spur gear, when the straight beveloid gear meshes, it meshes in from the toe and meshes out from the heel. The contact line gradually extends from zero to the entire tooth width. When it reaches position 2, the heel side begins to engage and the entire tooth width enters the meshing. When the position 3 is reached, the toe side ends the engagement and the heel side is still in contact. When the position 4 is reached, the whole tooth surface meshes out.

Therefore, for the initial period from position \( B_1 \) to \( B_2 \) and the last period from \( B_3 \) to \( B_4 \), the width of the contact area is not equal to the gear width and mesh stiffness grouping cannot be performed using the whole tooth width. The length of the \( L_0 \) can be obtained from the geometric relationship of the straight beveloid gear shown in Fig. 2, and the angle \( \gamma \) of the action line relative to the axes can be obtained by combining the tooth width. In this example, it is divided into \( m \) parts along the meshing line direction, and divided into \( n \) parts along the tooth width direction. The actual width for contact area along tooth width direction can be calculated from the actual meshing length and \( \gamma \), the total meshing stiffness of the straight beveloid gear at the moment can be calculated. When the full tooth along the tooth width is in contact, we can directly sum all the thin slices set.

Fig.4 The plane of action of beveloid gear pair with parallel axes
The base circles of the toe and heel is equal for a beveloid gear, the position relation of the contact points of the toe side and the heel side and its projection position on the transverse surface during meshing is shown in Fig.5 (a) and (b), respectively. \( r_{B_2} \) and \( r_{B_1} \) indicate the polar diameters of the contact points for the toe and heel in the meshing process, respectively. \( \gamma_e \) represents the difference of working pressure angle between toe and heel at the same time in the acting plane. \( L_t \) represents the projected length of the instantaneous contact line at the transvers surface during meshing. \( L_t \) is a constant (except for the limit position).

![Fig.5 Relation of contact point position of beveloid gear](image)

From geometric relation, \( L_t \) can be calculated by

\[
L_t = r_{B_2} \tan \alpha_{k_2} - r_{B_1} \tan \alpha_{k_1}
\]

According to the derivation of the above formula

\[
L_t = \Delta x_t m_t \sin \alpha_n
\]

For integrating, the entire meshing area is divided into three segments by nodes \( B_2 \), \( B_3' \), the toe side segment for mesh in, the full-tooth meshing segment and the heel side segment for mesh out. The length of the full-tooth meshing segment is \( L_3 \).

\[
L_3 = \varepsilon P_{bt} - L_t
\]

Where \( P_{bt} \) is the transverse base pitch, \( \varepsilon \) is the transverse contact ratio.

The heel side segment for mesh out

\[
L_3 = L_t
\]

Therefore, the mesh stiffness model for a straight beveloid gear pair is shown in the following.

1. When \( \Delta l = L_t \), the required number of slices and mesh stiffness is

\[
n = m \Delta l / L_t / \Delta b
\]

\[
k = \sum_{i=0}^{n} k_i
\]

2. When \( L_t < m \Delta l < L_3 \), it is the full tooth meshing phase. The required number of slices and mesh stiffness is

\[
n = n_{max}
\]

\[
k = \sum_{i=0}^{n} k_i
\]

3. When \( m \Delta l > L_3 \), the required number of slices and mesh stiffness is
\[ n = \left( m \Delta l - L_1 - L_2 \right) b / L_1 \Delta b \]  
\[ k = \sum_{n_3}^{n_{\text{max}}} k_i \]

3. Comparison and verification of mesh stiffness

The finite element method was used to verify the above calculation model. The main geometry parameters of the beveloid gear pair are shown in Table 1. Based on the proposed mathematical model of the beveloid gear tooth surface (Zhu, et al., 2012), the coding work by Matlab was performed to plot the beveloid gear tooth surface as shown in Fig.6.

| Parameters                      | Pinion | Gear  |
|--------------------------------|--------|-------|
| Centre distance \( E(\text{mm}) \) |        | 109   |
| Tooth width \( b(\text{mm}) \)    |        | 18    |
| Number of teeth \( N \)           | 20     | 50    |
| Cone angle \( \gamma \) (\(^\circ\)) | 6      |       |
| Normal module \( m_n(\text{mm}) \) | 3      |       |
| Normal pressure angle \( \alpha_n \) (\(^\circ\)) | 20 |       |
| Working cone angle \( \gamma_w \) (\(^\circ\)) | 5.14 | -5.14 |
| Addendum coefficient \( h_{an} \) | 1      |       |
| Gap coefficient \( c_n \)        | 0.25   |       |
| Profile shift coefficient \( x_t \) | 0.0100 (Toe) | 0.9013 (Heel) |

![Fig. 6 Beveloid gear tooth surface](image-url)
After obtaining the coordinates of the points on tooth surface of beveloid gear, the solid model of beveloid gear pair with parallel axes can be developed in the SolidWorks software as shown in Fig.7. Then the commercial finite element software Abaqus is used to develop the mesh model for the beveloid gear pair as shown in Fig.8. The material of the beveloid gears is defined as 17CrNiMo6 with the Young’s modulus 208GPa and the Poisson's ratio 0.298. For the simulation, the pinion was defined as the driving gear and the direction of rotation is anticlockwise when observing from the toe to the heel. Light load is taken into account in the finite element analysis since the loading effect has not been taken into account when using the potential energy method, the torque applied to the driven wheel is 80 Nm.

\[
\begin{align*}
\quad 
\end{align*}
\]

The formula of transmission error under load is as follows

\[
e_{L} = (\varphi_2 - \varphi_{20}) - (\varphi_1 - \varphi_{10})N_1/N_2
\]

(26)

Where \(\varphi_{10}\) and \(\varphi_{20}\) indicate the initial rotation angle when the driving pinion and the driven wheel come into mesh, respectively. \(\varphi_1\) and \(\varphi_2\) indicate the rotation angle of the driving pinion and the driven wheel, respectively. \(N_1\) and \(N_2\) indicate the numbers of teeth for the driving pinion and driven wheel, respectively.

The equivalent mesh stiffness can be calculated as (Song, et al., 2012)

\[
k_m = F / (\lambda e_1)
\]

(27)

Where \(M\) indicates the torque along the axes, \(F\) indicates the total mesh force, \(\lambda\) indicates the equivalent radius.

The time-varying mesh stiffness of the straight beveloid gear pair with parallel axes based on potential energy method and finite element mesh stiffness calculation model are shown in Fig. 9 and 10, respectively.

It can be seen that both methods can reflect the mutation of the gear mesh stiffness caused by the alternate from single teeth pair to double teeth pair in contact. The peak value of the mesh stiffness computed by finite element method is \(6.64 \times 10^7\) N/m while by the potential energy method is \(6.82 \times 10^7\) N/m, and the error is 3%. The mean value of mesh stiffness calculated by the finite element method is \(6.35 \times 10^7\) N/m and it is \(6.17 \times 10^7\) N/m by the potential energy method, the error
is 2.8%. All the errors above are small enough within the acceptable range to verify the feasibility and accuracy of calculation model of beveloid gear based on the potential energy method.

4. Parametric analysis of mesh stiffness

The effects of different normal pressure angles on the time-varying and mean values of single tooth and synthesized mesh stiffness of straight beveloid gear pair are shown in Fig.11.

From the above results, with the increase of normal pressure angle, the distribution trend of mesh stiffness in one mesh cycle is affected unsubstantially. However, for both the single tooth mesh stiffness and synthesized mesh stiffness, the meshing stiffness increases gradually when the normal pressure angle is less than 20 °, and it decreases rapidly when the pressure angle exceeds 20 °. The proportion of single tooth region increases in one mesh cycle, which indicates that the contact ratio is reduced. It may affect the smoothness of gear transmission and may causes vibration and noise problems. When the cone angle increases, the tooth thickness difference between the toe and heel will become larger and larger, which may cause the topping phenomenon in the heel side. Therefore, the cone angle should be properly designed. The effects of different cone angles on the time-varying and mean values of single tooth and synthesized mesh stiffness of straight beveloid gear pair are shown in Fig.12.
Fig. 12 Effect of cone angle on mesh stiffness

Similar to the pressure angle, the variation of the cone angle does not change the distribution trend of mesh stiffness in one mesh cycle obviously. In terms of the amplitude of mean values, both single and synthesized mesh stiffness increase gradually. For the synthesized mesh stiffness, the cone angle increases by 1° each, the mean value increases by approximately 1.8%, which means cone angle has little effect on the mesh stiffness. However, as the cone angle increases, the proportion of single tooth areas decreases significantly, which indicates that the contact ratio is increased and the mesh will become more smooth. Fig. 13 shows the time-varying and mean values of single and synthesized mesh stiffness of straight beveloid gear when the profile shift coefficient of toe are 0.01, 0.02 and 0.03, respectively.
It shows that both the distribution shape and the mean values of the single and synthesized mesh stiffness of straight beveloid gear change unsubstantially with the increase of the profile shift coefficient. The effect of tooth width on the mesh stiffness of straight beveloid gear pair is shown in Fig.14.

It shows that both the single tooth and synthesized mesh stiffness increase obviously as the tooth width increases. For each increment 2 mm in tooth width, the mean value of single tooth mesh stiffness increases by approximately 9% and the synthesized mesh stiffness increases by approximately 11%. The increase of the tooth width will not affect the transverse
contact ratio but will increase the axial contact ratio, so that the total contact ratio will be increased and the meshing will become more smooth. The effects of the different addendum coefficients on the time-varying and mean value of single and synthesized mesh stiffness of beveloid gear are shown in Fig.15. The effects of clearance coefficient on the time-varying and mean value of mesh stiffness of straight beveloid gear are shown in Fig.16.

![Fig.15 Effect of addendum coefficient on mesh stiffness](image-url)
From the results, the increase of the addendum coefficient from 0.7 to 1.1 has little effect on the mean value of single tooth mesh stiffness, but it tends to increase the mean value of synthesized mesh stiffness obviously. The mean value of synthesized mesh stiffness increases by about 4% with each increment 0.1 for the addendum coefficient. The contact ratio increases obviously with the increase of the addendum coefficient. For the clearance coefficient, it has little effects both on the distribution shape and mean value of the single tooth and synthesized mesh stiffness.

5. Conclusion

(1) A potential energy based slice grouping method was proposed to calculate the mesh stiffness for straight beveloid gears with parallel axes. The mathematical mesh stiffness model was derived. The verification for the mesh stiffness was conducted using the finite element tooth contact model with the error 3 %, which proves the feasibility and accuracy.

(2) The normal pressure angle and the tooth width have obvious effects both on the single tooth and synthesized mesh stiffness. When pressure angle is less than 20°, mesh stiffness will be increased with the increase of pressure angle. However, it decreases rapidly when the pressure angle exceeds 20°. Both the single and synthesized mesh stiffness increase obviously as the tooth width increases.

(3) The increase of the cone angle and addendum coefficient have a little effect on the single tooth mesh stiffness, but have the obvious incremental effects on the synthesized mesh stiffness. The contact ratio increases obviously with the increase of the addendum coefficient. The profile shift coefficient and the clearance coefficient have unsubstantial effects both on the single and synthesized mesh stiffness.

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