EMAT generation of bulk forces in a ferromagnetic plate and their equivalent surface stresses

C Rouge\textsuperscript{1}, A Lhémery\textsuperscript{1} and C Aristégui\textsuperscript{2}

\textsuperscript{1}CEA, LIST, F-91191 Gif-sur-Yvette cedex, France
\textsuperscript{2}Université de Bordeaux, UMR CNRS 5295, I2M/Acoustique Physique, F-33405 Talence cedex, France

E-mail: alain.lhemery@cea.fr

Abstract. Electro-magnetic acoustic transducers (EMAT) are successfully used in many NDE applications, despite their low efficiency: they do not require a coupling medium and can easily generate elastic waves that standard piezoelectric transducers cannot, such as shear horizontal guided waves. There are all sorts of EMAT designs, so much so that dedicated simulation tools are necessary to optimally conceive an EMAT for a given application. EMAT performances also strongly depend on material properties of the piece under test. Here, ferromagnetic materials are considered. In such a material, an EMAT is the source of three forces resulting from three distinct and generally nonlinear phenomena: in addition to the Lorentz’s force generated in all conductive media, the magnetization and magnetostriction forces take place. All these forces are modelled as vector fields in the volume of the specimen. However, wave generation is more efficiently predicted by considering sources of surface stress than sources of body force. Thus, a general model is derived for transforming body forces into surface stresses; this approach is used to express the 2D modal amplitudes of Lamb waves generated by an EMAT in a ferromagnetic plate as quasi-closed form solutions.

1. Introduction

Electro-magnetic acoustic transducers (EMAT) are a common way to generate and receive guided waves, especially shear horizontal waves in plate and torsional waves in cylinders which are not easily generated by piezoelectric transducers. Counterbalancing both their low signal-to-noise ratio and the fact that they only work on conductive media, EMAT allow high speed inspections since they do not require a coupling medium. They can also be beneficially used in hostile environments (high temperature and/or high pressure), in which piezoelectric transducers fail.

The versatility of EMAT design is very attractive too. By choosing adequately the excitation frequency and the arrangement of the electric coil relatively to the magnetization direction provided by one (or several) permanent magnet(s), one mode and its associated polarization among the various propagative guided modes can be selected [1]. Simulation tools are therefore helpful to optimize the various parameters in the EMAT design.

In conductive media, EMAT generate ultrasonic waves through the well known Lorentz’s force. In ferromagnetic media, two supplementary forces are created and must be taken into account when modelling the wave generation: these are the magnetization and magnetostriction forces. These forces

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result from the complex behaviour of ferromagnetic material under magnetization and may constitute
the predominant phenomena of wave generation mechanism. Considering a ferromagnetic medium
implies to take into account magnetic non linear behaviours such as hysteresis, saturation and double
frequency effects. Many phenomena occur, which possibly interact between each other. For example,
magnetostriction couples two effects: when a magnetic field is applied to a ferromagnetic material, the
medium changes its physical dimensions; at the same time, this change in dimensions modifies its
magnetic field.

All these three electromagnetic forces, Lorentz, magnetization and magnetostriction, appear as
sources of body forces in the equation of wave motion. Therefore, solving the equation requires
performing volume integrations to predict the waves they generate. To avoid time-consuming
calculations and to reuse previously developed models of guided wave generation by a distribution of
surface stresses [2], body forces are thus transformed into surface stresses. The transformation follows
a method described by Thompson [3]; it also ensures the mixing of electromagnetic phenomena
(generated forces) with wave generation and propagation phenomena through the Green’s integral
formulation of the elastic wave equation.

The EMAT is modelled as being made of a permanent magnet that creates a static magnetic
induction field, and of an electric coil that creates a dynamic magnetic induction field. In a first part,
from the static and dynamic induction fields, the excitation field and the magnetization field are
expressed. The three electromagnetic forces are then written analytically. A discussion about the
modelling of the double frequency effect, the saturation and the hysteresis phenomena is also made.
These forces are then transformed into surface stresses. As an application, the modal amplitudes of
Lamb waves are obtained as quasi-closed form solutions in the 2D case. Examples of calculation are
then given and discussed. In what follows, developments are written in the Cartesian coordinate
system because only guided waves generated in a plate (Lamb waves) will be considered.

2. Theory
We develop a semi-analytical model for simulating the generation of elastic waves in a ferromagnetic
material by an EMAT. The magnetic fields generated by the EMAT and the resulting forces are first
derived; then, equivalent surface stresses are obtained through a transformation that generalizes a
transformation already used in [4] for SH guided waves.

2.1. Magnetic field created by an EMAT
An EMAT is made of a permanent magnet combined with an electrical coil: the first component
creates a static magnetic induction field in the material while the second creates a dynamic one. These
fields are denoted by \( \vec{B}_S \) and \( \vec{B}_D \), respectively. The dynamic field has the same frequency content as
that of the current injected in the electric coil. From now on, a continuous wave excitation \( e^{i\omega t} \) is
considered, where \( \omega \) denotes the angular frequency [rad.s\(^{-1}\)].

The other two magnetic fields, the excitation field \( \vec{H} \) and the magnetization field \( \vec{M} \), two necessary
quantities to predict the electromagnetic forces created by an EMAT in a ferromagnetic material, are
derived from the induction field \( \vec{B} \) through the two following relations:

\[
\vec{B} = \mu_0 \mu \vec{H} ,
\]

\[
\vec{M} = \chi \vec{H} ,
\]

where \( \mu_0 = 4\pi 10^{-7} \text{H.m}^{-1} \) is the vacuum permeability. The magnetic permeability \( \mu \) is a diagonal
second rank tensor while the magnetic susceptibility \( \chi \) is a scalar constant.

The magnetic permeability of a ferromagnetic material depends both on the amplitude of the
applied magnetic induction [5] and on its frequency [6]. In what follows, the permeability is given by
two different values. If the dynamic and static fields have a non zero component in the same direction,
the static value of the permeability is used because the static induction field is generally much higher
than the dynamic one, in standard applications of EMAT. For example, if the wires of the electric coil
are along the $\hat{y}$ direction, the dynamic field is along the $\hat{x}$ and $\hat{z}$ directions. If a static field along the $\hat{z}$ direction is superimposed, the permeability used in the $\hat{x}$ direction is the dynamic one and that used in the $\hat{z}$ direction is the static one. Assuming that a model for the dependencies of $\mu$ on the amplitude and on the frequency of the induction field exists, it could be introduced in our overall approach at no additional cost.

2.2. Body forces generated by an EMAT into a ferromagnetic medium

The forces generated by an EMAT in a ferromagnetic material can now be computed as they result from the magnetic fields described in the previous paragraph.

2.2.1. Lorentz’s force. This force is generated in all conductive media. It results from the interaction of eddy currents created by the electric coil of the EMAT and the induction field $\vec{B}$ created by both the permanent magnet and the electric coil. It is expressed by:

$$\vec{F}_L = \vec{j} \times \vec{B},$$

where $\vec{j}$ is the induced current density [A.m$^{-2}$].

2.2.2. Magnetization force. The magnetization force:

$$\vec{F}_M = \vec{M} \cdot \vec{H}$$

directly results from the Bloch wall motion and the magnetic moments rotation of a ferromagnetic material under magnetization [5].

2.2.3. Magnetostriction force. This force results from the magnetization of a ferromagnetic material. If a magnetic field is applied to a ferromagnetic medium, the medium geometry changes; the reciprocal effect has been shown to make a negligible contribution in EMAT transduction processes [7]. Magnetostriction combines magnetic and elastic phenomena; it is commonly assumed that it does not induce an overall volume change.

The magnetostriction is modelled from the theory proposed by Hirao and Ogi [8]. The associated force:

$$\vec{F}_{MS} = \vec{\nabla} \cdot \vec{\sigma}_M$$

depends on the magnetostriction stress $\vec{\sigma}_M$,

$$\sigma_{ij}^M = -e_{ijm}H_m,$$

which is defined from the piezomagnetic stress tensor $\vec{e}$,

$$e_{ijm} = C_{ijkl}d_{klm},$$

where $\vec{C}$ is the stiffness tensor and $\vec{d}$ is the piezomagnetic strain coefficient tensor given by:

$$d_{klm} = \frac{\partial \sigma_{kl}}{\partial H_m}.$$  

$\vec{e}$ denotes the magnetostrictive strain tensor. In this model, the values of the last tensor may be deduced from experimental data. For example, Hirao and Ogi [8] use only one measured curve of $\varepsilon$ as a function of $H$, assuming that no magnetic anisotropy occurs, so that a magnetic microscopic description is not necessary. Their magnetostriction curve corresponds to the deformation along the total field. Then, they use a rotation of the strain tensor in order to express the magnetostrictive strain tensor $\vec{e}$ as a function of the interpolated experimental magnetostriction curve and its derivative, in a two dimensions configuration.
2.3. Magnetic non linear effect modelling: double frequency and saturation effects

2.3.1. Double frequency effect. This effect is generally negligible in classical EMAT applications and not modelled. Experiments [9] confirmed that the doubling frequency effect is negligible when the static field in the material is much larger than the dynamic one. This can be easily explained as follows.

First, considering the static and dynamic inductions, equations (1) and (2) yield:

\[ \vec{B}_S = \mu_0 \vec{H}_S , \]
\[ \vec{B}_D e^{i\omega t} = \mu_0 \vec{H}_D e^{i\omega t} , \]
\[ \vec{M}_S = \chi \vec{H}_S , \]
\[ \vec{M}_D e^{i\omega t} = \chi \vec{H}_D e^{i\omega t} , \]

where the induction field is simply decomposed as the sum:

\[ \vec{B} = \vec{B}_S + \vec{B}_D e^{i\omega t} . \]  (13)

The Lorentz’s force is then expressed by:

\[ \vec{F}_L = j e^{i\omega t} \wedge (\vec{B}_S + \vec{B}_D e^{i\omega t}) = j \wedge \vec{B}_S e^{i\omega t} + j \wedge \vec{B}_D e^{2i\omega t} . \]  (14)

The current density is a pure dynamic quantity resulting from the dynamic current injected in the electric coil. In this expression, it is obvious that if the static field dominates the dynamic field, the double frequency effect is negligible.

For the magnetization force, the doubling frequency is a bit more hidden in the equations. We have:

\[ \vec{F}_M = \mu_0 (\vec{M}_S + \vec{M}_D e^{i\omega t}) \cdot (\nabla \vec{H}_S + \nabla \vec{H}_D e^{i\omega t}) 
= \mu_0 \vec{M}_S \cdot \nabla \vec{H}_S + \mu_0 (\vec{M}_S \cdot \nabla \vec{H}_D + \vec{M}_D \cdot \nabla \vec{H}_S) e^{i\omega t} + \mu_0 \vec{M}_D \cdot \nabla \vec{H}_D e^{2i\omega t} . \]  (15)

Here, the gradient of the excitation field is involved. If the static field can be considered as being uniform, as it is the case in many EMAT designs, the magnetization force reduces to:

\[ \vec{F}_M = \mu_0 \vec{M}_S \cdot \nabla \vec{H}_D e^{i\omega t} + \mu_0 \vec{M}_D \cdot \nabla \vec{H}_D e^{2i\omega t} , \]  (16)

and again, the double frequency effect can be neglected if the amplitude of the static field is much higher than that of the dynamic one. Under this assumption, the case of periodic permanent magnet EMAT cannot be treated.

The same analysis cannot be made for the magnetostriction force because the analytical expression of the magnetostriction curve was derived from the approximation of an experimental curve. This curve depends on static and dynamic field amplitudes so that the piezomagnetic stress coefficients matrix \( \tilde{\varepsilon} \) also depends on both these magnetic fields. The frequency-dependent behaviour is therefore complex for low amplitude of the static field as \( \tilde{\varepsilon} \) shows strong variations in this domain. At higher static amplitude, \( \tilde{\varepsilon} \) varies almost linearly so that its derivative is constant; as a consequence, the frequency-dependency of the force becomes proportional to that of the induction and varies as \( e^{i\omega t} \).

2.3.2. Saturation effect. The non linear effect of magnetic field saturation, modelled here, is directly linked to the hysteresis effect. When the magnetization field in the material reaches a certain value, given for example by Bozorth [5], this field cannot increase beyond this saturation value and becomes stationary. In the calculations, if the saturation value, denoted by \( M_{\text{sat}} \), is exceeded, so that \( ||\vec{M}|| > M_{\text{sat}} \), each component of the magnetization vector field is then proportionally modified, so that \( ||\vec{M}|| = M_{\text{sat}} \).
2.4. Transformation of body forces into surface stresses

Let us consider the coordinate system shown in Figure 1a and used for the demonstration.

As said in the introduction, the three electromagnetic forces appearing as source terms in the elastic wave equation:

\[ -\mu_L \dddot{u} - (\lambda_L + \mu_L) \ddot{\nabla} u + \rho \frac{\partial^2 \dddot{u}}{\partial t^2} = F_L + F_M + F_{MS}, \tag{17} \]

have to be transformed into surface stresses. In equation (17), \( \mu_L \) and \( \lambda_L \) are the Lamé’s coefficients [Pa], \( \dddot{u} \) is the particle displacement [m] and \( \rho \) the material mass density [kg.m\(^{-3}\)].

As explained by Thompson [3], this transformation starts from a general formula for the particle displacement written as a spatial convolution of the Green’s tensor \( \hat{G} \) with the sum of the body forces denoted by \( f \):

\[ u_i(\vec{r}) = \sum_j \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_{ij}(\vec{r}, \vec{r}_0) f_j(\vec{r}_0) d\vec{x}_0 d\vec{y}_0 d\vec{z}_0. \tag{18} \]

From this formula, results are sought to be rewritten as a spatial convolution of the Green’s functions with a distribution of equivalent surface stresses, as:

\[ u_i(\vec{r}) = \sum_j \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{G}_{ij}(\vec{r}, \vec{r}_0) \sigma_j(\vec{r}_0| z_0=0) G_{ij}(\vec{r}, \vec{r}_0| z_0=0) d\vec{x}_0 d\vec{y}_0, \tag{19} \]

where \( \vec{r} = (x, y, z) \) and \( \vec{r}_0 = (x_0, y_0, z_0) \) are the observation point and the source point, respectively. For convenience, the notation \( \vec{r}_0| z_0=0 \) reduces to \( \vec{r}_0' \) in what follows.

The first transformation step consists in decomposing the Green’s function into a Taylor series at the surface of a half-space, Figure 1b. This can be done because the spatial variations of the Green’s functions are weak compared to those of the body forces: the former are related to the ultrasonic wavelength, typically of the order of few millimetres, while the latter are related to the skin depth, typically not exceeding few hundredths of a millimetre in ferromagnetic materials at frequencies used in EMAT applications. The Taylor series decomposition is stopped at the second order. Equation (18) becomes:

\[ u_i(\vec{r}) = \sum_j \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ G_{ij}(\vec{r}, \vec{r}_0) T_j(\vec{r}_0) + \frac{\partial G_{ij}(\vec{r}, \vec{r}_0)}{\partial z_0} M_j(\vec{r}_0) + \frac{1}{2} \frac{\partial^2 G_{ij}(\vec{r}, \vec{r}_0)}{\partial z_0^2} N_j(\vec{r}_0) \right] d\vec{x}_0 d\vec{y}_0, \tag{20} \]

where \( T_j, M_j \) and \( N_j \) represent the moments of the force \( f_j \) and are given by the following integrals:
The second step aims at eliminating the derivatives of the Green’s functions over the thickness which appear in equation (20). Using the deformation tensor definition and the isotropic elasticity of the material, the local equilibrium law yields:

$$\sum_{j} K_{ij}(\vec{r}_0) u_j(\vec{r}_0) = f_i(\vec{r}_0) \ ,$$

where:

$$K_{ij}(\vec{r}_0) = -\rho \omega^2 - c_{11} \frac{\partial^2}{\partial x_i^2} - c_{44} \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y_i^2} - (c_{12} + c_{44}) \frac{\partial^2}{\partial x_i \partial y_j} - (c_{12} + c_{44}) \frac{\partial^2}{\partial x_j \partial y_i} \ ,$$

By multiplying equation (22) by $G_{ki}$, summing over $i$, integrating over the space and comparing the result with equation (18), we find:

$$u_k(\vec{r}) = \sum_{i,j} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_{ki}(\vec{r}, \vec{r}_0) K_{ij}(\vec{r}_0) u_j(\vec{r}_0) \, dx_i \, dy_j \, dz_0 \ .$$

Considering the free surface condition and the energy conservation which imply that both $\vec{G}$ and $\vec{u}$ tend to zero at infinity, the double integration by parts of the right-hand side of equation (24) gives:

$$u_k(\vec{r}) = \sum_{i,j} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{0} u_j(\vec{r}_0) K_{ij}(\vec{r}_0) G_{ki}(\vec{r}, \vec{r}_0) \, dx_0 \, dy_0 \, dz_0 \right. \left. + u_x(\vec{R}_0) C_{44} \left( \frac{\partial G_{kx}(\vec{r}, \vec{R}_0)}{\partial x_0} + \frac{\partial G_{kx}(\vec{r}, \vec{R}_0)}{\partial z_0} \right) \right] \, dx_0 \, dy_0 \ .$$

From this equation, the conditions of existence of the Green’s functions are written as:

$$\sum_{i} K_{ij}(\vec{r}_0) G_{ki}(\vec{r}, \vec{r}_0) = \delta^{ij}(\vec{r} - \vec{r}_0) \delta_{kj} \ ,$$

$$\frac{\partial G_{kx}(\vec{r}, \vec{R}_0)}{\partial x_0} + \frac{\partial G_{kx}(\vec{r}, \vec{R}_0)}{\partial z_0} = 0 \ ,$$

$$\frac{\partial G_{ky}(\vec{r}, \vec{R}_0)}{\partial z_0} + \frac{\partial G_{kx}(\vec{r}, \vec{R}_0)}{\partial y_0} = 0 \ ,$$

$$C_{11} \frac{\partial G_{kx}(\vec{r}, \vec{R}_0)}{\partial z_0} + C_{12} \left( \frac{\partial G_{kx}(\vec{r}, \vec{R}_0)}{\partial x_0} + \frac{\partial G_{ky}(\vec{r}, \vec{R}_0)}{\partial y_0} \right) = 0 \ .$$
where $\delta_{ij}$ denotes the Kronecker symbol. Using the development of equation (26) with the expression of the tensor $\bar{R}$, equation (23), and the derivatives of equations (27) to (29), the first order and the second order of the derivatives in $z_0$ of the Green’s functions are given by:

\[
\begin{align*}
\frac{\partial G_{kz}(\vec{r}, R_0)}{\partial z_0} &= -\frac{C_{12}}{C_{11}} \left( \frac{\partial G_{kx}(\vec{r}, R_0)}{\partial x_0} + \frac{\partial G_{ky}(\vec{r}, R_0)}{\partial y_0} \right), \\
\frac{\partial G_{kx}(\vec{r}, R_0)}{\partial z_0} &= -\frac{\partial G_{kz}(\vec{r}, R_0)}{\partial x_0}, \\
\frac{\partial G_{ky}(\vec{r}, R_0)}{\partial z_0} &= -\frac{\partial G_{kz}(\vec{r}, R_0)}{\partial y_0}, \\
\frac{\partial^2 G_{kx}(\vec{r}, R_0)}{\partial z_0^2} &= C_{12} \left( \frac{\rho \omega^2}{C_{44}} + \frac{\partial^2}{\partial x_0^2} + \frac{\partial^2}{\partial y_0^2} \right) G_{kx}(\vec{r}, R_0), \\
\frac{\partial^2 G_{ky}(\vec{r}, R_0)}{\partial z_0^2} &= - \left( \frac{\rho \omega^2}{C_{44}} + \frac{\partial^2}{\partial x_0^2} + \frac{\partial^2}{\partial y_0^2} \right) G_{ky}(\vec{r}, R_0) - \left( 1 + \frac{C_{12}}{C_{11}} \right) \frac{\partial^2}{\partial x_0 \partial y_0} G_{kx}(\vec{r}, R_0), \\
\frac{\partial^2 G_{kz}(\vec{r}, R_0)}{\partial z_0^2} &= - \left( \frac{\rho \omega^2}{C_{44}} + \frac{\partial^2}{\partial x_0^2} + \frac{\partial^2}{\partial y_0^2} \right) G_{kz}(\vec{r}, R_0).
\end{align*}
\]

Finally, substituting equations (30) to (35) into equation (21) yields:

\[
u_i(\vec{r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \frac{1}{2} \left( 1 + \frac{C_{12}}{C_{11}} \right) \frac{\partial^2}{\partial x_0^2} + N_x(\vec{R}_0) \frac{\partial^2}{\partial y_0^2} \right] \left( \frac{1}{2} \frac{\partial^2}{\partial x_0^2} + N_y(\vec{R}_0) \frac{\partial^2}{\partial y_0^2} \right) \left( T_x(\vec{R}_0) \frac{\partial}{\partial x_0} + M_x(\vec{R}_0) \frac{\partial}{\partial y_0} \right) + \left( \frac{1}{2} \frac{\partial^2}{\partial x_0^2} + N_y(\vec{R}_0) \frac{\partial^2}{\partial y_0^2} \right) \left( T_y(\vec{R}_0) \frac{\partial}{\partial y_0} + M_y(\vec{R}_0) \frac{\partial}{\partial x_0} \right) \right] d\vec{x}_0 d\vec{y}_0.
\]

The equivalent surface stresses appearing in equation (19) are readily identified as:

\[
\begin{align*}
\sigma_x^{eq} &= T_x - \frac{\rho \omega^2}{2 C_{44}} N_x - \frac{C_{12}}{C_{11}} M_x \frac{\partial}{\partial x_0} - \frac{1}{2} \left[ N_x \left( \frac{1}{2} \frac{\partial^2}{\partial x_0^2} + 2 \left( \frac{1}{2} + \frac{C_{12}}{C_{11}} \right) \frac{\partial^2}{\partial y_0^2} \right) + N_y \left( 1 + \frac{C_{12}}{C_{11}} \right) \frac{\partial^2}{\partial x_0 \partial y_0} \right], \\
\sigma_y^{eq} &= T_y - \frac{\rho \omega^2}{2 C_{44}} N_y - \frac{C_{12}}{C_{11}} M_y \frac{\partial}{\partial y_0} - \frac{1}{2} \left[ N_x \left( \frac{1}{2} \frac{\partial^2}{\partial x_0^2} + 2 \left( \frac{1}{2} + \frac{C_{12}}{C_{11}} \right) \frac{\partial^2}{\partial y_0^2} \right) + N_y \left( 1 + \frac{C_{12}}{C_{11}} \right) \frac{\partial^2}{\partial x_0 \partial y_0} \right], \\
\sigma_z^{eq} &= T_z + \frac{\rho \omega^2}{2 C_{44}} C_{12} N_z - M_z \frac{\partial}{\partial x_0} + M_y \frac{\partial}{\partial y_0} + \frac{1}{2} \left[ 1 + \frac{C_{12}}{C_{11}} \right] \frac{\partial}{\partial x_0 \partial y_0} + \frac{\partial}{\partial x_0} \left[ \frac{1}{2} \frac{\partial^2}{\partial x_0^2} + 2 \left( \frac{1}{2} + \frac{C_{12}}{C_{11}} \right) \frac{\partial^2}{\partial y_0^2} \right].
\end{align*}
\]

Equations (37-39) constitute a general result though they were derived to deal with typical forces generated by EMAT. Each term depends on the various components of the force considered as it depends of the force moments.
It must be noticed that the basic assumptions and approximations made in deriving them depend only on elastodynamic quantities. Therefore, these formulae can be used to transform a distribution of body forces of any physical origin into an equivalent distribution of surface stresses; the main condition of applicability is that body forces must be concentrated in the vicinity of the free surface of an elastic medium. The notion of vicinity is understood as follows: whatever the direction of the vector forces, the extent of the distribution of forces in the direction normal to the free surface must be small compared to the elastic wavelength.

Note that a result based on similar assumptions and approximations was proposed in [4] in the more restrictive case of a 2D distribution of horizontal forces assumed to be normal to the 2D plane of elastic wave propagation, therefore limiting the applicability to sources of shear horizontal waves.

3. Example of application in 2D: radiation of Lamb waves by a meander-coil EMAT
The coordinate system is again that of Figure 1a but in 2D, *i.e.*, the ̂y axis is the infinite dimension along which derivatives vanish.

3.1. Green’s function for Lamb waves generated in a plate by a surface stress
The Green’s functions appearing in equation (36) are not explicitly given. Here, 2D Green’s functions for a line source at a finite distance are not explicitly given. However, they can be obtained using the reciprocity relation given by Auld [11].

Let ̃A_m^σ_xz and ̃A_m^σ_{xz} denote the amplitudes associated with the Lamb mode m due to a tangential stress σ_{xz} and to a normal stress σ_{zz}, respectively. Their calculations are based on the integration over the plate thickness of the complex reciprocity relation [2]. Equivalent results were obtained for the shear horizontal case in [4] and a similar derivation is made here to get the modal amplitudes for the Lamb case. Amplitudes are:

\[
A_m^{σ_{xz}}(x, x_0) = \frac{e^{-jβ_m x}}{4P_{mm}} \bar{v}_{m}(z = 0)e^{jβ_m x_0},
\]

\[
A_m^{σ_{xz}}(x, x_0) = \frac{e^{-jβ_m x}}{4P_{mm}} \bar{v}_{m}(z = 0)e^{jβ_m x_0},
\]

where x represents the observation point, x_0 the position of the line source, β_m the wavenumber of the m-th mode; ̃v_m and ̃v_m are the components of the particle velocity of the m-th mode taken at the plate surface. The factor P_{mm} is given by:

\[
P_{mm} = -\frac{1}{4} \int_{-h}^{0} \left( \bar{\sigma}_m \cdot \bar{\sigma}_m \right) . \bar{v} dz,
\]

where ̃σ_m stands for the amplitude of the stress tensor of the m-th mode and h the plate thickness. The notation * denotes the complex-conjugate value of the designated quantity. The wavenumbers β_m are analytically known for shear horizontal (SH) waves but must be numerically evaluated in the case of Lamb waves. For this, our implementation of the semi analytical finite element method is used.

3.2. Modal amplitude of Lamb guided modes generated by an EMAT
By assuming the previous modal decomposition, the displacement ̃u_{m} generated by a line source of normal or tangential stresses over a plate is written, respectively:

\[
̃u_{m}^{σ_{xz}}(x, x_0), z = \sum_m A_m^{σ_{xz}}(x, x_0) \bar{v}_{m}(z) = \sum_m e^{-jβ_m x} \frac{1}{4P_{mm}} \bar{v}_{m}(z = 0)e^{jβ_m x_0} \bar{v}_{m}(z),
\]

\[
̃u_{m}^{σ_{xz}}(x, x_0), z = \sum_m A_m^{σ_{xz}}(x, x_0) \bar{v}_{m}(z) = \sum_m e^{-jβ_m x} \frac{1}{4P_{mm}} \bar{v}_{m}(z = 0)e^{jβ_m x_0} \bar{v}_{m}(z),
\]
The expression (18) of the particle displacement uses Green’s functions for a half space which verifies the free surface conditions. The Green’s functions developed in the previous paragraph verify similar conditions at both plate surfaces. However, at the surface opposite to that where the source applies, the generated body forces are negligible as soon as the condition “plate thickness much larger than the skin depth” is respected. This is generally true at typical frequencies and for the materials considered in NDT applications. Therefore, the half-space Green’s functions may be substituted with the elastic plate Green’s functions.

For a transducer of length \( L \), a derivation similar to that made in the simpler case of SH waves in [4] gives for a force \( f_x \) or \( f_z \), respectively:

\[
A_{m}^{f_{x}}(x) = \frac{e^{-j\beta_{m}x}}{4\rho_{mm}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \left[ T_{x} - j \beta_{m}M_{x} - \frac{1}{2} \frac{\rho\omega^{2}}{C_{44}}N_{x} + \frac{1}{2} \left( 2 + \frac{C_{12}}{C_{11}} \right) \beta_{m}^{2}N_{x} \right] e^{j\beta_{m}x_{0}} dx_{0} ,
\]

\[
A_{m}^{f_{z}}(x) = \frac{e^{-j\beta_{m}x}}{4\rho_{mm}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \left[ T_{z} - j \frac{C_{12}}{C_{11}} \beta_{m}M_{z} + \frac{1}{2} \frac{\rho\omega^{2}}{C_{44}} C_{12} N_{z} - \frac{1}{2} \frac{C_{12}}{C_{11}} \beta_{m}^{2} N_{z} \right] e^{j\beta_{m}x_{0}} dx_{0} .
\]

These expressions derive from equations (37-39) when considering the simplifications resulting from the 2D configuration considered in this part.

3.3. Meander-coil EMAT configuration

The theory described in previous paragraphs is now applied to model a meander-coil EMAT operating on a 3-mm-thick iron plate. The configuration and the parameters used in the computations are first presented. All the data are given at room temperature and the temperature dependence is not considered.

Iron is modelled as elastic and isotropic with mass density of 7700 kg.m\(^{-3}\), longitudinal velocity of 5690 m.s\(^{-1}\) and shear velocity of 3145 m.s\(^{-1}\). The static magnetic permeability and the dynamic are set to 200 and 1000 respectively, and the magnetic susceptibility is 200. The electric conductivity is 9.93 MS.m\(^{-1}\). The magnetization at saturation is 1720 kA.m\(^{-1}\). The magnetostriction curve is analytically approximated by Hirao and Ogi [8]. The EMAT used is schematized on Figure 2. The meander-coil is made of 10 wires of diameter \( a = 1 \) mm, and the inter-space between two wires is \( c = 0.5 \) mm, so that its total length \( L \) equals 15 mm. The lift-off is 0.1 mm and the induction current sent to the coil is 1 mA with a frequency varying in the [0.1; 5] MHz range by 10 kHz steps (491 calculation frequencies).

![Figure 2](image_url)

**Figure 2.** Representation of the meander-coil EMAT used in simulations.

The magnetic fields, and thus, the three electromagnetic forces, are computed at each point of a grid in the plate. The length of the transducer (in the \( \hat{x} \) direction) is discretized by 150 regularly-spaced points (i.e. 10 points per wire and 5 points between two wires). The thickness of the material (in the \( \hat{z} \) direction) is divided into two regions: the first region from the surface to a depth equals to five times the skin depth is discretized by 50 points; the remaining thickness is meshed by only 5 points, because the fields, thus, the forces, are confined close to the surface.

Before calculating the modal amplitudes, it is important to check if the assumptions made in deriving the model are satisfied. At first, the static field set at 0.5 T is much larger than the dynamic one which order of magnitude is \( 10^{4} \) T for the induction current considered. Then, discretizations in space and time are chosen so that the assumptions made are fulfilled.
3.4. Results

The modal amplitudes of SH and Lamb waves calculated are presented for different static magnetization directions, \( \hat{x} \), \( \hat{y} \) and \( \hat{z} \). The static field is considered uniform under the material and is set to 0.5 T in the three system directions, successively.

Results shown in figure 3 are presented to illustrate the capability of our model to predict elastic wave fields radiated by EMAT of various designs. Here, the three possible orientations of the static field relatively to the wire direction of a meander-coil EMAT are considered. Several observations can be made from these results.

First, a modulation is visible in the amplitudes displayed, whatever the static field orientation. It is related to the coil design as the peak values arise when the wire spacing of the coil coincides with the wavelength of the mode. Then, shear horizontal (SH) modes are generated only when the static field direction coincides with the electric current direction: SH waves result from forces acting along the \( \hat{y} \) axis which only exist when the static field and the electric current directions are aligned (in the 2D simulations considered here). Interestingly, in this case, the amplitude of SH waves can reach higher values than that of symmetric (S) and antisymmetric (A) Lamb modes. However, the relative amplitudes of the various modal contributions depend on the excitation frequency.

![Figure 3](image-url)
Numerous results were obtained but it is not the purpose of the present paper to discuss them. Many parameters can be varied in our modelling approach, even in the simple 2D configurations that are treated. Depending on the material characteristics, one of the three forces may dominate the others. Note that 2D configurations correspond to actual cases of EMAT use in NDT applications. Experimental studies will be performed in order to validate the theoretical model derived here.

4. Conclusions
The magnetic fields and the electromagnetic forces generated by an EMAT in a ferromagnetic material have been described. The double frequency effect was highlighted. The three generated body forces were then transformed into equivalent surface stresses and the role of the Green’s functions involved in this transformation has been described. Under this transformation, the modal amplitudes of Lamb waves generated by an EMAT in a ferromagnetic plate have been expressed as quasi-closed form solutions; these solutions complement our earlier closed form solutions restricted to SH waves. Examples of application have been given to illustrate the capabilities of our model.

It must be noticed that a similar transformation could be applied to the case of the radiation of bulk or surface waves by simply adopting the appropriate Green’s functions to describe their specific propagation characteristics.

To apply the presented theory, some assumptions must be verified. However, they are not restrictive in regards to typical EMAT designs and to the characteristics of ferromagnetic materials of interest in NDT applications.

These analytical (for SH waves) and quasi-analytical (for Lamb waves) calculations allow fast parametric studies for optimizing EMAT design for a given material.

Various stages in the model derivation could be adapted to get an overall model which could apply in more general cases, such adaptations being compatible with the overall approach proposed herein. For examples, the magnetization field can be calculated from the Weiss molecular field theory and the magnetic permeability can be expressed as a function of the applied field amplitude and the induction frequency. The magnetostrictive model can also be improved, especially in relating the magnetostrictive deformation calculated by Bozorth [5] and the magnetostrictive strain $\xi$, by considering for example a micro-macro homogenization model [12].

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