Fatigue Life of Welded Structures under Random Load Study on Frequency Domain Method

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Abstract. In this paper, based on the finite element software, the residual stress of the structure weld is obtained through simulation analysis. The influence of stress concentration is considered by using the mean square stress concentration factor; and the S-N curve equation of the structure is derived. Then, the stress power spectrum density of dangerous parts is obtained by random response analysis; Then, the vibration mode of the welded structure is obtained by random response analysis, and the acceleration power spectral density of the structure is input, and then the stress power spectrum density of the dangerous part is obtained. Finally, the appropriate stress probability density distribution model is selected to predict the fatigue life with S-N curve equation and Miner damage accumulation theory. The results show that the fatigue life predicted by this method is very close to that of rain flow counting method.

Keywords. Welded structure, random load, fatigue life, frequency domain method.

1. Introduction

In the engineering field, welding is a widely used component connection mode, and fatigue fracture is also a main form of engineering structure failure [1]. Vibration fatigue failure occurs in a large number of welded structures of various engineering structures [2]. When the structure is subjected to complex random excitation, with the constant change of load frequency, when approaching the natural frequency of the structure, the structure will produce resonance, and then fatigue fracture will occur in the weld.

At present, the frequency domain analysis methods for fatigue life of welded structures can be divided into two categories: probability based peak stress distribution method and stress amplitude distribution method. Among them, the amplitude distribution method is the most widely studied and widely used. According to different welding forms and load conditions, Li Xiang wei et al. [3] proposed a new principal S-N curve method to calculate the fatigue life of welded structures. In order to consider the influence of residual stress in welded structure, MI et al. [4] established the algorithm model of structure based on response surface method, and used genetic algorithm to find the optimal solution in global variables, which improved the accuracy of fatigue life prediction. Zheng et al. [5] proposed a correlation integral method to separate mixed vibration signals to solve the problem of multi-axis random excitation. For engineering structures under complex random loads, Li et al. [6] proposed a new fatigue damage modeling method, which decomposed the evolutionary PSD response of the structure and predicted the fatigue damage through each discrete PSD function. Gao et al. [7] proposed a new multi-axial random vibration fatigue damage parameter for multi-axial high cycle fatigue problem. Based on the equivalent stress PSD under this parameter, the fatigue life under multi-axial random load can be effectively predicted.
The traditional random fatigue life prediction method usually counts the rain flow of the stress or strain time history of the dangerous parts in time domain, then calculates the damage caused by each rain flow cycle, and finally estimates the fatigue life combined with the damage accumulation theory. The fatigue analysis method based on frequency domain uses a probability density function to describe the stress amplitude distribution, and then uses some statistics of the function to estimate the life. In this paper, the welded T-joint is taken as the research object. Firstly, the fatigue life of the model is predicted by frequency domain method, and then the fatigue life in time domain is obtained by rain flow counting method. The effectiveness of the method is verified by comparing the results obtained by the two methods.

2. Random Vibration Fatigue Analysis of Welded Structure

2.1. Random Vibration Characteristics
Power spectral density (PSD) is a probabilistic statistical method. The spectral moment is introduced into the frequency domain characteristic parameters to describe the statistical characteristic parameters of the random process power spectral density. The spectral moment \( m_i \) of the stationary random process is defined by the unilateral power spectral density function:

\[
m_i = \int_0^\infty f^i G_x(f)df
\]  

(1)

The stress power spectral density (PSD) of dangerous parts is expressed as follows:

\[
G_x(f) = H^2(f)w(f)
\]  

(2)

where: \( H(f) \) is the transfer function; \( w(f) \) is the input excitation power spectrum. In the stationary random process, the irregular factor is used to identify the bandwidth characteristics of the random process, and its expression is:

\[
\gamma = E[0^+] / E[P]
\]  

(3)

where: \( E[0^+] \) and \( E[P] \) are spectral width parameters, as the irregular factor approaches zero, the signal can be regarded as a wideband stochastic process. As the irregular factor tends to 1, it is regarded as a narrow-band random process.

2.2. S-N Curve Equation
The existence of residual stress has different influence on the fatigue life of welded structure. In addition, the change of geometry near the weld will cause stress concentration in these parts. In order to consider this effect, the mean square stress concentration coefficient \( K_{\sigma} \) is used to describe the stress concentration degree of the notch under random vibration load by referring to the theoretical stress concentration coefficient \( K_c \). Guan [9] established the S-N curve equation of welding structure under symmetrical circulation by using the S-N curve prediction theory proposed by him, and verified the correctness of the method by comparing with the experimental results. The relevant formula is as follows

\[
\log_{10} N = C - m \log_{10} S_a
\]  

(4)

\[
m = (-b + 0.2776 \log_{10} \beta_{-1})^{-1}
\]  

(5)

\[
C = 6.301 + m \log_{10} \left[ \sigma_f' \left( 4 \times 10^6 \right)^b / \beta_{-1} \right]
\]  

(6)

\[
\beta_{-1} = K_f \sigma_f' / (\sigma_f' - \sigma_r)
\]  

(7)

\[
K_{\sigma} = \frac{\text{maximum local elastic stress RMS} \sigma_{\text{max}}}{\text{nominal stress} \sigma_0}
\]  

(8)
where, $C$ and $m$ are curve parameters; $N$ is the number of cycles; $S_a$ is the stress amplitude; $\beta_{-1}$ is the effective stress concentration coefficient under symmetrical circulation; $\sigma'_f$ is the fatigue strength coefficient; $b$ is the fatigue strength index; $K_f$ is fatigue notch coefficient; $a$ is the material constant; $r$ is the root radius of welding toe.

2.3. Mean Stress Correction in Frequency Domain
Niesłony and Böhm [10] put forward a simple and reliable method to modify the average stress directly acting on the power spectral density function, as shown in Figure 1. The relevant calculation formula is as follows:

![Figure 1. Mean stress correction model in frequency domain.](image)

\[
\sigma_m = \lim_{T \to \infty} \frac{1}{T} \int_0^T \sigma(t) \, dt
\]

\[
k(\sigma_m) = \frac{1}{1 - \frac{\sigma_m}{R_m}}
\]

\[
G_{\sigma_T}(f) = [K(\sigma_m)]^2 G_{\sigma}(f)
\]

where: $\sigma_m$ is the average stress; $R_m$ is the yield strength of the material; $K(\sigma_m)$ mean stress correction factor; $G_{\sigma}(f)$ and $G_{\sigma_T}(f)$ are the inputs and outputs of the PSD.

2.4. Frequency Domain Life Prediction Method
According to the Miner linear fatigue damage accumulation theory [11], fatigue damage can be calculated by the following formula:

\[
D = \sum_i D_i = \sum_i \frac{n_i}{N_i}
\]

where: $n_i$ is the number of the $i$th stress amplitude; $N_i$ is the number of failure cycles under the $i$-th stress amplitude. When within the cumulative total damage time $T$, the number of cycles of the stress amplitude at $(S_a, S_a + \Delta S_a)$ is

\[
n(S_a) = \nu T P(S_a) \Delta S_a
\]

where, for random broadband process $\nu = E[P]$; $P(S_a)$ is the probability density function of the stress amplitude.

For random broadband processes, Dirlik approximates the rain flow amplitude probability density function with an exponential distribution and two Rayleigh distributions:

\[
P(S_a) = \frac{D_1 e^{-\frac{Z^2}{2}} + D_2 e^{-\frac{Z^2}{2 \lambda^2}} + D_3 e^{-\frac{Z^2}{2 \lambda_0^2}}}{2 \sqrt{\pi \lambda_0}}
\]

where: $D_1, D_2, D_3, Q, Z$ and $R$ can all be obtained from spectrum moment $m_i$ and the specific expressions are shown in reference [12], combined with equations (12) ~ (14), and the expressions of vibration fatigue damage at dangerous parts of welded structures can be obtained:

\[
D = \nu T 10^{-C} \int_0^\infty (S_a)^m P(S_a) dS_a
\]

when $D=1$, fatigue failure is considered to occur, and the fatigue life is:
The calculation example model is the welded T-shaped bearing joint of Q345 steel. The welding foot size is 6mm, and the geometric size is shown in Figure 2. The model is established by finite element software, as shown in Figure 3. In this paper, the life and death unit technology is used to simulate the welding process, and the direct coupling method is used to solve the temperature field and stress field.

3. Examples and Analysis

3.1. Joint Model and Welding Residual Stress Solution

Because of the modal analysis is the basis of random response analysis, modal analysis was carried out on the model first, and then according to the analysis results to extract a sufficient number of modal order, then according to the extraction of modal order to determine the frequency sweep range. Finally, the acceleration power spectral density as shown in Figure 4 was applied to the joint for random vibration analysis to obtain the stress power spectral density at the welded toe, as shown in Figure 5.

Inverse Fourier transform was applied to the PSD without mean stress correction as shown in Figure 5 to obtain the stress-time history of the danger point, as shown in Figure 7. Then the stress-time history was counted by rain flow counting method and the stress amplitudes and mean values were obtained for each determined number of cycles. In order to consider the influence of stress mean value, Goodman equation was introduced to obtain the stress amplitude under equivalent symmetric cycle, and then the time-domain fatigue life was obtained by combining damage accumulation theory and S-N curve equation.
Figure 6 shows the stress distribution cloud map of t-joint. It can be seen from the cloud map that the stress at the toe is the maximum, which indicates that fatigue failure is most likely to occur at the toe.

The spectral moments of each order were calculated by the stress power spectral density, as shown in Table 1. The probability density function of the stress amplitude is shown in Figure 8.

![Stress time history](image)

**Figure 6. Root mean square stress nephogram.**

**Figure 7. Stress time history.**

![Probability density function image of stress amplitude](image)

**Figure 8. Probability density function image of stress amplitude.**

**Table 1.** Spectral moments of each order

|                | Before the average stress is corrected | Before the average stress is corrected |
|----------------|----------------------------------------|----------------------------------------|
| $m_0$          | $5.3608 \times 10^{13}$                | $3.9855 \times 10^{14}$                |
| $m_1$          | $1.8745 \times 10^{17}$                | $1.3959 \times 10^{18}$                |
| $m_2$          | $7.0056 \times 10^{20}$                | $5.2256 \times 10^{21}$                |
| $m_4$          | $1.3692 \times 10^{28}$                | $9.9988 \times 10^{28}$                |

$K_p = 5.83$ was obtained by finite element analysis, and the S-N curve was obtained as equation (17) by combining equations (5) ~ (8).

$$\log_{10} N = 11.43 - 3.25 \log_{10} S_o$$

(17)

The fatigue life of the structure was obtained by combining (14), (16) and (17), and the comparison with the results of rain-flow counting method was shown in Table 2.

**Table 2.** Comparison of fatigue life among estimation results

| Dirlik life time /s | Life of rain flow counting method /s | Relative error /% |
|---------------------|-------------------------------------|-------------------|
| 15106               | 16351                               | 7.61              |
4. Conclusion
In this paper, a method for predicting vibration fatigue life of welded structures with residual stress is presented. Based on finite element software, the residual stress was obtained through simulation analysis, the mean square stress concentration coefficient was introduced, and the S-N curve equation at the welding toe was obtained by combining the fatigue notch coefficient. Then the random response analysis is carried out to obtain the stress power spectral density and modify the average stress. Finally, combined with S-N curve equation and Miner damage accumulation theory, fatigue life was predicted by stress probability distribution model. The results show that the fatigue life predicted by this method is less error than that of the rain flow algorithm, and the method has clear thinking and simple calculation, which can be used for reference in engineering application.

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