Inducing topological superconductivity at the LaAlO$_3$/SrTiO$_3$ interface

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Abstract. The discovery of superconductivity in the two-dimensional electron gas at the interface between two band insulators SrTiO$_3$ and LaAlO$_3$ has opened up possibilities for novel physics. We show that due to the presence of Rashba spin-orbit interaction at the interface, a topological superconductivity can be induced by applying a magnetic field perpendicular to the interface plane even in the presence of the intrinsic in-plane magnetization. The in-plane magnetization favours a finite momentum pairing of electrons and therefore weakens the topological superconducting phase. We show that the tunable Rashba spin-orbit interaction can help in stabilizing the topological superconducting state against the deleterious in-plane magnetization.

1. Introduction

In the last few years, there has been a burst of activity to manipulate the fascinating properties of the highly conductive two-dimensional electron gas (2DEG) formed at the LaAlO$_3$/SrTiO$_3$ interface (001) [1]. Besides the low-temperature ($\leq 200$ mK) superconductivity [2], the 2DEG exhibits exciting properties such as coexistence of superconductivity and ferromagnetism [3, 4, 5], gate-voltage induced superconductor-insulator transition and metal-insulator transition [6]. There are proposals for phonon-mediated superconductivity [7, 8, 9, 10] as well as unconventional pairing [11, 12]. Also, there is suggestion that the superconducting transition belongs to a universal Berezinskii-Kosterlitz-Thouless (BKT) class signifying 2D nature of the superconductivity [6]. However, the temperature variation of the pairing gap has a good BCS fit with $2\Delta_0/k_B T_{gap} = 3.4$ [13].

Application of a magnetic field perpendicular to the interface plane provides a possible route to achieve a topological superconducting state [14, 15] which exhibits gapless excitations. The idea, as proposed earlier [16, 17], is that the Rashba spin-orbit interaction (SOI), in this interface 2DEG, can convert the existing $s$-wave superconductivity into a chiral $p$-wave superconducting state and the perpendicular Zeeman field lifts the spin-degeneracy, transforming the system into a $p+ip$-wave superconducting state which is a canonical example of topological superconductivity. We show that this topological superconductivity appears even in presence of intrinsic in-plane magnetization which results in a finite-momentum pairing thereby weakening the topological superconducting phase. We also show that by tuning Rashba SOI by external gate-voltage it is possible to overcome the dephasing effect of in-plane magnetization on the topological
superconductivity. The effect of intrinsic disorder such as the Oxygen vacancies [18, 19, 20] on the topological superconductivity is discussed.

2. Model for interface 2DEG
The 2DEG is formed at the TiO$_2$ layer in the SrTiO$_3$ side by an electronic transfer mechanism, known as polar catastrophe [21, 22], which removes the charge discontinuity at the interface between polar LaAlO$_3$ and non-polar SrTiO$_3$. The interface electrons occupy the $t_{2g}$ bands of the Ti atoms and participate in establishing ferromagnetism and superconducting order at low temperature. Since the $\Gamma$-point of the $d_{xy}$ band is situated well below the other two quasi-1D bands $d_{yz,zx}$ [23], electron occupying the $d_{xy}$ band get localized due to Coulomb correlation. These localized moments interact via an exchange interaction with the itinerant electrons in the conduction band, situated just below the interface, and give rise to an in-plane ferromagnetic order. On the other hand, at low temperatures the conduction electrons participate in superconducting pairing which, therefore, competes with the in-plane magnetization. The other indispensable part of the interface is the gate-tunable Rashba SOI, due to the broken inversion symmetry along $\hat{z}$-direction [24]. Although electron-pairing is possible, in principle, in all the itinerant bands, since superconductivity appears at low electron concentration, we are considering a single-band ($d_{xy}$) analysis for simplicity. The Hamiltonian describing the 2DEG at low temperature is given by

$$
\mathcal{H} = \sum_{k,\sigma} (\epsilon_k - \mu) c^\dagger_{k\sigma} c_{k\sigma} + \alpha \sum_{k,\sigma,\sigma'} (g_k \cdot \sigma) c^\dagger_{k\sigma} c_{k\sigma} - \sum_{k,\sigma,\sigma'} (h_x \sigma_x + h_y \sigma_y + h_z \sigma_z) c^\dagger_{k\sigma} c_{k\sigma'} - \sum_{k,\sigma,\sigma'} (\Delta c^\dagger_{k\sigma} c_{-k\sigma} + h.c.)
$$

where $\epsilon_k = -2t(\cos k_x + \cos k_y)$ is the dispersion of the electrons with $t = 0.277$ eV [23], $\mu$ is the chemical potential, $g_k = (\sin k_y, -\sin k_x)$ describes the Rashba SOI of strength $\alpha$, $h_x$ represents the in-plane Zeeman field due to the ferromagnetic order, $\Delta = -U < c_{k\uparrow} c_{-k\downarrow} >$ is the pairing gap and $\sigma = [\sigma_x, \sigma_y, \sigma_z]$ are the Pauli matrices.

**Figure 1.** (a) The bands created by the Rashba SOI are shown with dashed lines. The solid curves represent the bands when a perpendicular magnetic field is applied (b) the assymmetry introduced in the bands in presence of the in-plane magnetization (c) colormap of the Berry curvature $\Omega$ which takes finite value only at the point $P (0, -h_x/\alpha)$ with $h_x = 0.1$, $h_z = 0.2$, $\alpha = 0.2$. 


3. In presence of a perpendicular magnetic field

The Rashba SOI lifts the spin-degeneracy and creates two new helical bands (as shown by the dotted curve in Fig. 1(a)) in which the spins are aligned perpendicular to the momentum direction. When a magnetic field, described by the Zeeman-splitting term \( \mathcal{H}_Z = -h_z \sum_k [c_{k\uparrow}^\dagger c_{k\downarrow} - c_{k\downarrow}^\dagger c_{k\uparrow}] \), is applied perpendicular to the interface plane, the two Rashba bands get separated as shown in Fig. 1(a). The in-plane field \( h_x \) shifts the Berry curvature, given by \( \Omega = \alpha^2 h_z/2(\alpha^2 k_x^2 + (\alpha k_y + h_x)^2 + h_z^2)^{3/2} \), from the \( \Gamma \)-point to a point \( P \) \((0, -h_x/\alpha)\) as described in Fig. 1(b), (c). Therefore, the electron pairing is energetically favourable at a finite momentum proportional to \( h_x \) \[25, 26\]. The resultant bands, in presence of both in-plane magnetization and the perpendicular magnetic field, are given by \( \epsilon_{\pm}(k) = \epsilon_k - \mu \pm \xi \) where \( \xi = (\alpha^2 |g_k|^2 + h_x^2 + h_z^2 - 2\alpha h_x \sin k_y)^{1/2} \).

The pairing symmetry of superconductivity changes dramatically when we consider pairing in these Rashba bands. The pairing amplitudes in the Rashba bands are given by

\[
\Delta_{\pm\pm} = -\frac{\alpha}{2\xi} \Delta (\sin k_y \pm i \sin k_x), \quad \Delta_{+-} = \frac{h_z \Delta}{\xi}
\]

where \( \Delta_{\pm\pm} \) and \( \Delta_{+-} \) are respectively intra-band \( p_x \pm ip_y \)-wave pairing amplitude and inter-band \( s \)-wave pairing amplitude. When the magnetic field is increased beyond a critical value \( h_{zc} \), there is only one band at the Fermi level and we have only \( p_x \pm ip_y \)-wave pairing which is a topological superconducting state \[27\]. Therefore, with increasing \( h_z \) there is a quantum phase transition from normal superconductivity to a topological superconductivity. In the Nambu basis \( \Psi(k) = [c_{k\uparrow}, c_{k\downarrow}, c_{-k\downarrow}^\dagger, -c_{-k\uparrow}^\dagger] \), the effective Hamiltonian \( \mathcal{H}_{eff} = \mathcal{H} + \mathcal{H}_Z \) of the system is written as

\[
\left( \begin{array}{cc} 3\mathcal{H}_0(k) & \Delta \\ \Delta & -\sigma_y \mathcal{H}_0(k) \sigma_y^* \end{array} \right) \Psi(k) = E_{\pm}(k) \Psi(k)
\]

where \( \mathcal{H}_0(k) = \epsilon_k^\prime + \alpha g_k \sigma_y - h_z \sigma_z - h_x \sigma_x \), \( \epsilon_k^\prime = \epsilon_k - \mu \). The quasi-particle spectrum is given by \( E_{\pm}^2(k) = (\epsilon_k^2 + \Delta^2 + \xi^2) \pm \zeta \) where \( \zeta = [\Delta^2 h_z^2 + \epsilon_k^2 \xi^2]^{1/2} \). When \( \zeta = \epsilon_k^2 + \Delta^2 + \xi^2 \), the gap in the spectrum closes signifying a transition to the topological superconducting state. This condition

![Figure 2](image.png)

**Figure 2.** (a) Variation of the excitation gap \( E_y \) with the perpendicular field strength \( h_z \) (with \( h_x = 0, \mu = 0, \Delta = 0.1 \)) describing the quantum phase transition from normal superconductivity to topological superconductivity. Beyond \( h_{zc} = 0.1 \), the new excitation gap is generated by the Rashba SOI. (b) The in-plane magnetization \( h_x \) increases the critical field \( h_{zc} \) but the Rashba SOI \( \alpha \) competes against \( h_x \) in stabilizing the topological superconductivity. Parameters: \( \mu = 0, \Delta = 0.1 \) (c) Variation of \( h_{zc} \) with the chemical potential \( \mu \) with \( h_x = 0.05, \Delta = 0.1 \). All energies are in eV.
is satisfied (with $\Delta \neq 0$) when either $\xi^2 = h_{\text{zc}}^2$ or $\epsilon_k^2 + \Delta^2 = h_{\text{zc}}^2$ which essentially reduces to the well-known relation $h_{\text{zc}} = \sqrt{\Delta^2 + \mu^2}$ when $h_x = 0$ [28]. Beyond $h_{\text{zc}}$, a topologically protected excitation gap $E_g$ (given by the minimum of $E_-(k)$) is induced. This gap $E_g$ is proportional to the Rashba SOI strength $\alpha$ in the limit of small $h_z$ as shown in Fig. 2(a). Consequently, $E_g$ keeps track of the quantum phase transition from ordinary superconductivity to the topological one. There are two competing energy gaps, the minimum of which tries to destroy the topological state; one is the induced bulk excitation gap at the Fermi level ($\Delta_{FS} = 2\Delta - \xi$), the other is the Zeeman gap at the point $P$, given by $\Delta_z = E_-(0, -h_x/\alpha)$. Additionally, the in-plane magnetization, by introducing a finite-momentum pairing, weakens the topological superconductivity. However, the Rashba SOI competes with the in-plane magnetization to restore the topological state as shown in Fig. 2(b). Fig. 2(c) shows the variation of $h_{\text{zc}}$ with the chemical potential which controls the electron concentration. Since the gate-voltage controls both the Rashba SOI and the electron concentration [6, 24], it has be tuned properly to stabilize the topological state.

4. Discussion and conclusion

In summary, we have shown that a topological superconducting state can be realized at the interface 2DEG by applying a magnetic field perpendicular to the interface due to the presence of the Rashba SOI. The Rashba SOI converts the $s$-wave superconductivity into a $p_x + ip_y$-wave topological superconductivity beyond a critical magnetic field. The intrinsic in-plane magnetization introduces asymmetry in the bands created by the Rashba SOI and favours a finite momentum pairing of electrons. We have shown that the in-plane magnetization tries to destroy the induced topological superconductivity but the Rashba SOI stabilizes the topological superconducting state against the inimical in-plane magnetization. We finally show that the gate-voltage can be used as an extra knob to ensure the topological superconducting state. However, intrinsic disorder such as the Oxygen vacancies at the interface, developed during the deposition process of the film, may damage the topological superconducting state [29]. Therefore, to realize this topological superconductivity experimentally, considerable attention has to be given to remove the Oxygen vacancies as much as possible.

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