Solution of Inverse and Forward Kinematics Problems for Mobile Robot with Six Mecanum Wheels

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Abstract. The wheeled mobile robot has been considered a planner mechanism with linear and angular movement over a horizontal plane. In particular, the wheeled mobile robot's movement is assumed to result from pure rolling of wheels without slipping. The kinematics model is one of the most fundamental steps in studying any mobile robot. This research presents the kinematic equations of movement of a mobile robot with mecanum wheels. The resulting mathematical model of a wheeled mobile robot was produced using the MATLAB R2014a program. The kinematic modeling technique was investigated by simulating a robot's walk on a square path.

Keywords. Kinematics, Trajectory tracking, Wheeled mobile robot.

1. Introduction
In the last years, clever and adaptable assembling has spurred the improvement of autonomous mobile robots for workpiece and equipment handling and transportation, and specifically, AGV innovation is broadly considered and applied [1-5]. There is much research that has studied the kinematics of mobile robots of different designs. A kinematic model is worked to do a vigorous model prescient control system for the trajectory tracking control of a four-mecanum wheeled omnidirectional mobile robot under different constraints [6]. The dynamic and kinematic equations of a reconfigurable non-holonomic mobile robot used for space explorations and rescue operations are presented in reference [7]. To provide a theoretical kinematic basis for accurate motion control, a kinematic model with the combined mobile system's velocity compensation is created in references [8, 9]. A novel approach for simultaneous balancing and tracking tasks is presented for a Mecanum wheeled mobile robot with a cylinder [10]. In built-up an innovative technique of modeling and kinematics simulation in RecurDyn for an omnidirectional wheeled chair with four mecanum wheels [11-13]. An inverse kinematic model was applied to a mobile robot with four mecanum wheels [14]. Inverse and forward kinematics models for a holonomic mobile robot with three mecanum wheels have been obtained in reference [15]. The kinematic modeling technique has been examined by simulation examples. The four-mecanum wheeled drive mobile robot wheels arrangements were dissected that will give close to wanted execution with one fault and two faults for both set-point control and trajectory-tracking (circular profile) utilizing kinematic movement control scheme within the tolerance limit [16]. To maintain a specific location and behavior for a mobile robot, fuzzy logic and kinematic equations were used to
control the motor speed of wheels [17]. An approach was proposed to avoid obstacles without location and map information for mobile robots with mecanum wheels using a series of developed solutions that were provided using fuzzy logic and gap tracking [18]. This work aims to develop a kinematic model for a mobile robot with six mecanum wheels. Moreover, it aims to evaluate the resulting model by tracking a specified path (square shape). The rest of the research is organized as follows: Section 2 includes the core of this work, which is devoted to inferring the specific equations of the forward and inverse kinematic models of a mobile robot with six mecanum wheels. The motion control simulation is included in section 3, where the forward and inverse kinematic models were tested on a set of motions, and then the kinematic model was tested on a square path. The conclusion is clarified in section 4.

2. Description of the robot kinematics
In this work, six mecanum wheels have been used in the mobile robot. Mecanum wheel consists of a set of k congruent rolls placed symmetrically around the wheel body; a is the wheel's axis, and b is the roll axis, as shown in Figure 1. The angle δ is equal to ±45°. To study the mecanum wheel's kinematics, a robot was viewed as proceeding onward level ground and outfitted with mecanum wheels. The situation to be studied at a particular moment $t$ consists of 4 systems: the terrain, the platform, the wheels, and the rollers, which at that point C contact the ground.

Figure 1. Mecanum wheel.

Depending on reference [19], it was found that the formula that computes the speed of the wheel is given as follows:

$$\dot{\varphi} = -\frac{1}{r \sin(\delta)} \left[ \cos(\alpha + \delta)(v_x - \beta a_y) + \sin(\alpha + \delta)(v_y - \beta a_x) \right]$$

(1)

Work will be done on a wheeled mobile robot, as shown in Figure 2. In this case $\dot{\varphi}_i$ is the angular velocity of the wheel where $i$ is the wheel number ($i=1,2,3,4,5,6$). The distances between the two front wheels are $2\alpha$, and the distances between the midpoints of the front and back axle (points $S_1$, $S_2$) and the point O are in both cases equal to $\alpha$. The robot can be depicted in a coordinate system $(O, e_x, e_y, e_z)$. In this case $\alpha = \frac{\pi}{2}$. 

By applying formula (1) to wheel 1, formula (1) takes the following form:

$$\dot{\phi}_1 = -\frac{1}{r \sin(\frac{\pi}{6})} \left[ \cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right) (v_x - \dot{\beta} a_y) + \sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right) (v_y + \dot{\beta} a_x) \right]$$

$$\Rightarrow \dot{\phi}_1 = -\frac{1}{r} \left[ -\sin\left(\frac{\pi}{6}\right) (v_x - \dot{\beta} a_y) + \cos\left(\frac{\pi}{6}\right) (v_y + \dot{\beta} a_x) \right]$$

$$\Rightarrow \dot{\phi}_1 = \frac{1}{r} \left[ v_x - \dot{\beta} a_y - v_y - \dot{\beta} a_x \right]$$

$$\Rightarrow \dot{\phi}_1 = \frac{1}{r} [v_x - v_y - \dot{\beta} (a_y + a_x)]$$  \hspace{1cm} (2)

Where:

$$a_y = R \cdot \sin\left(\frac{\pi}{6}\right) = \frac{R}{2}$$  \hspace{1cm} (3)

$$a_x = R \cdot \cos\left(\frac{\pi}{6}\right) = \frac{R \sqrt{3}}{2}$$  \hspace{1cm} (4)

Compensating formulas (3),(4) into formula (2) gives:

$$\dot{\phi}_1 = \frac{1}{r} [v_x - v_y - \dot{\beta} \left(\frac{R \sqrt{3} + R}{2}\right)]$$  \hspace{1cm} (5)

Applying formula (1) to wheel 2,3,4, taking into account formulas (3),(4) gives:

$$\dot{\phi}_2 = \frac{1}{r} [v_x + v_y + \dot{\beta} \left(\frac{R \sqrt{3} + R}{2}\right)]$$  \hspace{1cm} (6)

$$\dot{\phi}_3 = \frac{1}{r} [v_x + v_y - \dot{\beta} \left(\frac{R \sqrt{3} + R}{2}\right)]$$  \hspace{1cm} (7)

$$\dot{\phi}_4 = \frac{1}{r} [v_x - v_y + \dot{\beta} \left(\frac{R \sqrt{3} + R}{2}\right)]$$  \hspace{1cm} (8)

For wheels 5,6 $a_x = 0$, $a_y = R$. Applying formula (1) to wheel 5 gives:
\[
\dot{\varphi}_5 = -\frac{1}{r \sin(-\frac{\pi}{4})} \left[ \cos\left(\frac{\pi}{2} - \frac{3\pi}{4}\right) (v_x - \beta R) + \sin\left(\frac{\pi}{2} - \frac{3\pi}{4}\right) (v_y) \right] \\
\Rightarrow \dot{\varphi}_5 = \frac{1}{r} \left[ \sin\left(\frac{\pi}{4}\right) (v_x - \beta R) + \cos\left(\frac{\pi}{4}\right) (v_y) \right] \\
\Rightarrow \dot{\varphi}_5 = \frac{1}{r} [v_x + v_y - \beta R] \\
\] (9)

Likewise, it has been found that:

\[
\dot{\varphi}_6 = \frac{1}{r} [v_x + v_y + \beta R] \\
\] (10)

Formulas (5), (6), (7), (8), (9), (10) can be presented in the form of a relationship presented below:

\[
\dot{\varphi} = JV_o \\
\] (11)

Where:

\[
\dot{\varphi}^T = [\dot{\varphi}_1 \; \dot{\varphi}_2 \; \ldots \; \dot{\varphi}_6] \\
V_o = \begin{bmatrix} v_x \\ v_y \\ \beta \end{bmatrix} \\
J = \begin{bmatrix} 1 & -1 & -(R\sqrt{3} + R) \\ 1 & 1 & \frac{R\sqrt{3} + R}{2} \\ 1 & 1 & -(R\sqrt{3} + R) \\ 1 & -1 & \frac{R\sqrt{3} + R}{2} \\ 1 & 1 & -R \\ 1 & 1 & R \end{bmatrix} \\
\] (12) (13) (14)

Formula (11) represents the systematic equation for the inverse kinematics of the mobile robot. The inversion operation for matrix J must be computed for the forward kinematics issue. The Moore-Penrose theorem can be used to compute the inversion matrix because matrix J is a rectangular matrix. The following relationship is obtained from applying the mentioned theory [20]

\[
J_{od} = (J^T J)^{-1} J^T \\
\] (15)

The solution to the forward kinematics problem can be received from the following relationship

\[
V_o = J_{od} \dot{\varphi} \\
\] (16)

From formula (16), it can be found that:

\[
v_x = r \left( \frac{\dot{\varphi}_1}{4} + \frac{\dot{\varphi}_2}{8} + \frac{\dot{\varphi}_3}{8} + \frac{\dot{\varphi}_4}{8} + \frac{\dot{\varphi}_5}{8} + \frac{\dot{\varphi}_6}{8} \right) \\
\] (17)

\[
v_y = r \left( -\frac{\dot{\varphi}_1}{4} + \frac{\dot{\varphi}_2}{8} + \frac{\dot{\varphi}_3}{8} - \frac{\dot{\varphi}_4}{8} + \frac{\dot{\varphi}_5}{8} + \frac{\dot{\varphi}_6}{8} \right) \\
\] (18)

\[
\beta = \frac{r}{4R(\sqrt{3} + 3)} \left[ - (\sqrt{3} + 1) \dot{\varphi}_1 + (\sqrt{3} + 1) \dot{\varphi}_2 - (\sqrt{3} + 1) \dot{\varphi}_3 + (\sqrt{3} + 1) \dot{\varphi}_4 - 2 \dot{\varphi}_5 + 2 \dot{\varphi}_6 \right] \\
\] (19)
3. Motion control simulation

Inverse Kinematic Simulation Results:
For motion simulation of the robot, it has been assumed that \( R = 0.5m, \ r = 0.52m \). The robot is assumed to move in different cases, including four cases in straight lines along the x-axis, y-axis (front and rear), four cases in oblique directions each 45°, and two cases in rotation clockwise and counterclockwise. In each case, the mobile robot center's linear and angular velocities have been assumed to evaluate the mobile wheels' angular velocity according to the inverse kinematic model equation (11). Figure 3 shows the Simulink model of inverse kinematic. Table (1) shows the evaluated wheels' angular velocities resulting from the robot linear, angular velocities, and the variation of its direction.

![Figure 3. The Simulink model of inverse kinematic.](image)

| Line of the robot movement                  | Robot center Velocities | Wheels angular velocities rad/s |
|---------------------------------------------|-------------------------|--------------------------------|
|                                             |  \( v_x \) m/s  |  \( v_y \) m/s  |  \( \beta \) rad/s | \( \phi_1 \) | \( \phi_2 \) | \( \phi_3 \) | \( \phi_4 \) | \( \phi_5 \) | \( \phi_6 \) |
| Straight lines along x-axis (front)         | 2                      | 0                        | 0                          | 38.46 | 38.46 | 38.46 | 38.46 | 38.46 | 38.46 |
| Straight lines along x-axis (rear)          | -2                     | 0                        | 0                          | -38.46 | -38.46 | -38.46 | -38.46 | -38.46 | -38.46 |
| Straight lines along y-axis (front)         | 0                      | 2                        | 0                          | -38.46 | -38.46 | -38.46 | -38.46 | -38.46 | -38.46 |
| Straight lines along y-axis (rear)          | 0                      | -2                       | 0                          | 38.46 | 38.46 | 38.46 | 38.46 | 38.46 | 38.46 |
| Oblique directions +45°                     | 1                      | 1                        | 0                          | 0     | 38.46 | 38.46 | 38.46 | 38.46 | 38.46 |
| Oblique directions -45°                     | 1                      | -1                       | 0                          | 38.46 | 0     | 0     | 0     | 38.46 | 38.46 |
| Oblique directions +135°                    | -1                     | 1                        | 0                          | -38.46 | 0     | 0     | -38.46 | 0     | 38.46 |
| Oblique directions -135°                    | -1                     | -1                       | 0                          | 0     | -38.46 | -38.46 | 0     | -38.46 | -38.46 |
| Rotation clockwise                          | 0                      | 0                        | -2                         | 26.27 | -26.27 | 26.27 | -26.27 | 19.23 | -19.23 |
| Rotation counterclockwise                   | 0                      | 0                        | 2                           | -26.27 | 26.27 | -26.27 | 26.27 | -19.23 | 19.23 |
Forward Kinematic Simulation Results:
To examine the forward kinematics equation, the mobile robot is assumed to move in the same movements as inverse kinematics example, four cases in straight lines along x-axis, y-axis (front and rear), and four cases in oblique directions each 45° and two cases in rotation clockwise and counterclockwise. In each case, the wheels' angular velocities are known to evaluate the mobile robot center linear and angular velocities according to the forward kinematic model equation (16). Figure 4 shows the Simulink model of the forward kinematic. Table (2) shows the evaluated mobile robot linear and angular velocities resulting from the mobile robot's forward kinematics.

Figure 4. The Simulink model of the forward kinematic.
Table 2. Robot center velocities resulting by solving forward kinematics.

| Line of the robot movement | Wheels angular velocities rad/s | Robot center Velocities |
|---------------------------|---------------------------------|------------------------|
|                           | $\dot{\psi}_1$ | $\dot{\psi}_2$ | $\dot{\psi}_3$ | $\dot{\psi}_4$ | $\dot{\psi}_5$ | $\dot{\psi}_6$ | $v_x$ | $v_y$ | $\beta$ |
| straight lines along the x-axis (front) | 38.46 | 38.46 | 38.46 | 38.46 | 38.46 | 38.46 | 2 | 0 | 0 |
| straight lines along x-axis (rear) | -38.46 | -38.46 | -38.46 | -38.46 | -38.46 | -38.46 | 0 | 2 | 0 |
| straight lines along the y-axis (front) | 38.46 | 38.46 | 38.46 | 38.46 | 38.46 | 38.46 | 0 | -2 | 0 |
| straight lines along y-axis (rear) | 38.46 | 38.46 | 38.46 | 38.46 | 38.46 | 38.46 | 1 | 1 | 0 |
| oblique directions +45º | 0 | 38.46 | 38.46 | 0 | 38.46 | 38.46 | 1 | 1 | 0 |
| oblique directions -45º | 38.46 | 0 | 0 | 38.46 | 0 | 0 | 1 | -1 | 0 |
| oblique directions +135º | -38.46 | 0 | 0 | -38.46 | 0 | 0 | -1 | 1 | 0 |
| oblique directions -135º | 0 | -38.46 | -38.46 | 0 | -38.46 | -38.46 | -1 | -1 | 0 |
| rotation clockwise | 26.27 | 26.27 | 26.27 | 26.27 | 19.23 | 19.23 | 0 | 0 | -2 |
| rotation counterclockwise | -26.27 | 26.27 | 26.27 | -26.27 | 19.23 | 19.23 | 0 | 0 | 2 |

To test the forward kinematic model, the robot will be made to walk on a square trajectory using the Simulink model in Figure 5. To do this, it was assumed that the robot's initial position is (1,1,0) and the direction of the robot is parallel to the x-axis. The reference trajectory is illustrated in Figure 6. Here the trajectory needs to be passed in 40 sec, so each side of the square will be passed in 10 sec. The robot will move in a straight line along the x-axis (front), then move in a straight line along the y-axis (front), then move in a straight line along the x-axis (rear) and then move in a straight line along the y-axis (rear). To know the required wheels angular velocities, the inverse kinematic will be used, assuming that $V_o = 1$ in the desired direction. Table (3) shows the desired wheels' angular velocities resulting from the inverse kinematic. In Simulink, there is a need to calculate the distance from the velocity; to do this, the integrator (1/s) will be used. In this model, the block labeled “Subsystem” is a 2-dimensional rotation matrix shown in Figure 7.

$$R_x(\beta) = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$V_o = R_x(\beta) \cdot V_o$$

$$P_o = \int_0^t V_o(t) \, dt$$

Table 3. The desired wheels angular velocities resulting from the inverse kinematic.

| Line of the robot movement | Robot center Velocities | Wheels angular velocities rad/s |
|----------------------------|------------------------|--------------------------------|
|                           | $v_x$ | $v_y$ | $\dot{\beta}$ | $\dot{\psi}_1$ | $\dot{\psi}_2$ | $\dot{\psi}_3$ | $\dot{\psi}_4$ | $\dot{\psi}_5$ | $\dot{\psi}_6$ |
| straight lines along the x-axis (front) | 1 | 0 | 0 | 19.23 | 19.23 | 19.23 | 19.23 | 19.23 | 19.23 |
| straight lines along x-axis (rear) | -1 | 0 | 0 | -19.23 | -19.23 | -19.23 | -19.23 | -19.23 | -19.23 |
| straight lines along y-axis (front) | 0 | 1 | 0 | -19.23 | 19.23 | 19.23 | -19.23 | 19.23 | 19.23 |
| straight lines along y-axis (rear) | 0 | -1 | 0 | 19.23 | -19.23 | -19.23 | 19.23 | -19.23 | -19.23 |
Figure 5. Simulink model for square trajectory.

Figure 6. The reference trajectory.

Figure 7. 2-Dimensional rotation matrix in “Subsystem”.
The robot trajectory will be shown in the output of the XY Graph block, as shown in Figure 8. To calculate the error of simulation, the result was sent to the workspace in MATLAB and compared to the reference trajectory matrixes. As a result, it was found that the error rate is \( E = 1.9950 \times 04 \text{m} \).

![Figure 8. The robot trajectory.](image)

4. Conclusion

This research presents a wheeled mobile robot with six mecanum wheels design. Inverse and forward kinematic models have been effectively acquired. A Simulink model has been built up to investigate the kinematics and path tracking for this type of robot. The inverse kinematics simulation results showed that the robot movement directly influenced the variation of wheels' angular velocities values and directions. Also, the inverse kinematics simulation results showed that the usage of mecanum wheels is like the conventional wheels' usage when moving the robot forward along a straight line because the angular velocities of all mecanum wheels were equal in values and directions. The forward kinematics simulation results show the affirmation of the models. MATLAB has been utilized to simulate the mobile robot velocities controlling. Results showed a good matching between expected inverse, forward kinematics models mobile robot wheels angular velocities and its translation and orientation velocities, where the transition from the forward model to the inverse model and vice versa gives very accurate values, and for the tracking trajectory, the error value was minimal \((E < 1\text{mm})\) between the reference trajectory and the resulting trajectory for that all results were in acceptable values.

### Nomenclatures

| Symbol | Description |
|--------|-------------|
| AGV    | Automated Guided Vehicle |
| a      | The axis of the wheel |
| b      | The roll axis |
| \( \delta \) | The angle between a and b [rad] |
| \( \hat{\phi} \) | Angular velocity for mecanum wheel [rad/s] |
| \( a \) | The angle between the \( e_x \)-axis and the wheel axis a [rad] |
| \( r \) | The distance between axis a and the ground [m] |
| \( v_x \) | Velocity of robot center (O) in x direction [m/s] |
| \( v_y \) | Velocity of robot center (O) in y direction [m/s] |
| \( \hat{\beta} \) | Angular velocity of the robot [rad/s] |
| \( a_x, a_y \) | The coordinates of the wheel center A |
| \( O, e_x, e_y, e_z \) | A co-ordinate system |
| \( O \) | The mass center of the robot |
| \( R \) | The radius of the platform of the robot [m] |
| \( J \) | Jacobian matrix |
| \( J_{od} \) | Generalized inverse (pseudoinverse) of \( J \) |
| \( P_o \) | Robot center positioning vector |
| \( E \) | Trajectory error[m] |
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