Monopoles and Vortices in the SU(2) Positive Plaquette Model

John D. Stack \textsuperscript{a} \textsuperscript{*} and William Tucker\textsuperscript{a}

\textsuperscript{a}Dept. of Physics, University of Illinois at Urbana–Champaign, 1110 W. Green Street, Urbana, IL 61801, USA.

We study the heavy quark potential in the SU(2) positive plaquette model using monopoles in the maximum abelian gauge, and vortices. Monopoles give a quantitative description of the string tension. Vortices approximately reproduce the entire heavy quark potential.

After many years of work, monopoles and vortices have emerged as the main possibilities for explaining confinement. Most research has been carried out for the Wilson action in SU(2) lattice gauge theory. Here we study the case of the positive plaquette model (PPM) which weights configurations in the same way as the Wilson action for positive plaquettes, but suppresses negative plaquettes entirely \cite{1}. A negative plaquette represents a huge field strength of order $1/a^2$, an obvious lattice artifact. Since negative plaquettes are suppressed in the PPM, the PPM should give a clearer view of continuum physics. Our goal here is to see how closely monopoles and vortices describe the long range confining physics in the PPM. We work on a $16^4$ lattice at couplings $\beta_{\text{PPM}} = 1.790, 1.840,$ and $1.886$. These couplings were chosen by using the accurately determined deconfining temperature $T_c$ for the PPM \cite{1}. In particular, $T_c = 1/8a$ corresponds to $\beta_{\text{PPM}} = 1.886(6)$. To find a value of $\beta_{\text{PPM}}$ appropriate for use on a $16^4$ lattice, we assumed the ratio $T_c/\sqrt{\sigma}$ (where $\sigma$ is the string tension) is universal. Using $T_c/\sqrt{\sigma} = 0.69(2)$, which is the known value for the Wilson action \cite{2}, we find that $\xi = 1/\sqrt{\sigma} \sim 5.5a$ for $\beta_{\text{PPM}} = 1.886(6)$. Thus $\beta_{\text{PPM}} = 1.886(6)$ has a similar value of correlation length $\xi$ to $\beta_W = 2.50$. We generated 500 well-separated PPM configurations at this $\beta_{\text{PPM}}$ and the two neighboring ones listed in Table 1.

For the case of monopoles, the configurations were put into the maximum abelian gauge and the magnetic current extracted. From the magnetic current, the magnetic vector potential, $A^m_\mu(x)$ was constructed,

$$A^m_\mu(x) = \sum_y v(x-y)\tilde{m}_\mu(y). \quad (1)$$

Then the monopole contribution to a Wilson loop, $W_{\text{mon}}$, was calculated,

$$W_{\text{mon}} = \left\langle \exp\left(\frac{i2\pi}{2} \sum_x D_{\mu\nu}(x)F^*_{\mu\nu}(x)\right)\right\rangle_m \quad (2)$$

where $F^*_{\mu\nu}(x)$ is the dual of the field strength constructed from $A^m_\mu(x)$. The resulting heavy quark potentials are shown in Figure \textsuperscript{2}, and the string tensions vs. those for full PPM SU(2) are shown in Table 1.

In our previous work on the PPM \cite{3}, the full SU(2) string tension was evaluated slightly incorrectly, since we used a multi-hit program to calculate Wilson loops which updated the links along the sides of the loop using the Wilson action instead of the PPM action. Correcting this made only a minor change at $\beta_{\text{PPM}} = 1.790$, shown in Table 1. The other two $\beta_{\text{PPM}}$ values are the old numbers. The important thing to note in Table 1

\begin{table}
\begin{tabular}{|c|c|c|}
\hline
$\beta_{\text{PPM}}$ & $\sigma_{\text{SU(2)}}$ & $\sigma_{\text{mon}}$ \\
\hline
1.790 & 0.0426(6) & 0.043(2) \\
1.840 & 0.036(1) & 0.036(1) \\
1.886 & 0.029(1) & 0.028(2) \\
\hline
\end{tabular}
\caption{Table 1}
\end{table}

\textsuperscript{*}Talk presented by J. Stack
is the excellent agreement between monopole and full SU(2) string tensions.

It is of interest to ask what is the difference between the monopole description of the heavy quark potential for the PPM and the Wilson action. At $\beta_{PPM} = 1.840$, we have $\sigma_{mon} = 0.036(1)$, very close to $\sigma_{mon} = 0.034(1)$ at $\beta_W = 2.50$ for the Wilson action $\mathbb{I}$. However, for the number of links with magnetic current, we find 2936(25) for the PPM, whereas the Wilson action has 3565(22), so there are $\sim 600$ more links carrying magnetic current for the Wilson action. The origin of this difference becomes clear when the magnetic current is resolved into loops. It is known that only large loops contribute to the string tension. When loops with less than 50 links of magnetic current are dropped, the string tension retains the same value for both actions. Comparing the number of magnetic current links for loops larger than 50 links, we now find quite comparable results, 1520(20) for the PPM, and 1573(20) for the Wilson action. The excess of $\sim 600$ links found for the Wilson action is then mainly in small loops which do not affect the string tension. Suppression of negative plaquettes suppresses these small loops, or in other words, suppression of an obvious lattice artifact gives a magnetic current more concentrated in the large loops known to be relevant for continuum physics.

Turning now to vortices, there is a long history of work initiated by Mack and collaborators, $\mathbb{I}$, and Tomboulis and collaborators $\mathbb{II}$, which seeks to explain confinement via vortices associated with the center of the gauge group, $Z(2)$, for the present case of an SU(2) gauge group. In SU(2), vortices can be classified as ‘thin’ (composed of negative plaquettes), ‘hybrid’ (composed partly of negative plaquettes), and ‘thick’ (composed entirely of positive plaquettes) $\mathbb{III}$. In the PPM, there can only be thick vortices. In all cases the Wilson loop is a vortex counter, the vortex contribution to a Wilson loop being

$$W_{vort} = \langle (-1)^n \rangle,$$

(3)

where $n$ is the number of vortices piercing the Wilson loop. Vortices are supposed to control the long range physics, and therefore $W_{vort}$ should produce the same string tension as full SU(2).

This has been checked for the Wilson action $\mathbb{III}$, but the PPM provides a more stringent test, since it only allows thick vortices. Calculation of $W_{vort}$ is particularly simple; one just replaces the loop by its sign. To enhance statistics, we smeared the ends of the loops. The overall appearance of the potential derived from $W_{vort}$ is rather similar to the full SU(2) potential as seen in Figure 2.

The noise level is comparable, and the potential has the same shape even in the Coulomb region at small R. This is in contrast to the monopole potential. Monopoles are quiet; no smearing or multi-hitting is needed to see the potential out to R=8a, and the monopole potential clearly differs from full SU(2) in the Coulomb region, being basically linear at all values of R. We have extracted PPM results for the vortex string tension $\sigma_{vort}$ at only one coupling so far, which we
show in Table 2. As can be seen there, the vortex string tension is somewhat low compared to monopoles and full SU(2). A possible explanation is that thin and hybrid vortices are still alive and contributing to the string tension for the Wilson action, where it was found that $\sigma_{\text{vort}} = \sigma_{\text{SU(2)}}$. Since the PPM has only thick vortices, this would naturally give it a lower string tension. Of course, as the value of $\beta_{\text{PPM}}$ is increased, thin and hybrid vortices are heavily suppressed, and only thick vortices remain. In this limit presumably, we will have $\sigma_{\text{vort}} = \sigma_{\text{SU(2)}}$ for the PPM as well as the Wilson action.

Assuming the small discrepancy for $\sigma_{\text{vort}}$ goes away with increasing $\beta_{\text{PPM}}$, one then has two viable descriptions of the long range confining physics, one from monopoles and the other from vortices. In the future, we plan to work in two directions. The first is to make the vortex picture more concrete by starting a program of vortex location. This we plan to do without gauge-fixing by

| $\beta_{\text{PPM}}$ | $\sigma_{\text{SU(2)}}$ | $\sigma_{\text{mon}}$ | $\sigma_{\text{vort}}$ |
|----------------------|------------------------|----------------------|----------------------|
| 1.790               | 0.0426(6)              | 0.043(2)             | 0.036(2)             |

Table 2

using small Wilson loops e.g. $2 \times 2$, $3 \times 3$, $4 \times 4$ etc. as vortex detectors. By this means, the physical picture of a large Wilson loop as pierced by many vortices can be tested. Our second line of investigation is to try to see what connection there is between the monopole and vortex descriptions, in particular to investigate the magnetic current distribution near vortices.

This work was supported in part by the National Science Foundation under Grant No. NSF PHY 94-12556.

REFERENCES

1. J. Fingberg, U. M. Heller, and V. Mitryishkin, Nucl. Phys. 435 (1995) 311.
2. J. Fingberg, U. Heller, and F. Karsch, Nucl. Phys. 392 (1993) 493.
3. J. D. Stack and S. D. Neiman, Physics Letters B 377 (1996) 113.
4. J. D. Stack, S. D. Neiman, and R. J. Wensley, Phys. Rev. D 50 (1994) 3399.
5. G. Mack and V. B. Petkova Ann. Phys. 123 (1979) 442.
6. E. T. Tomboulis and T. G. Kovács, Phys. Rev. D 57 (1998) 4054.