Neutrino oscillation experiments have been successful in determination of the three mixing angles ($\theta_{12}, \theta_{13}$ and $\theta_{23}$) and the two mass squared differences ($\Delta m_{21}^2$ and $\Delta m_{31}^2$). What remains to be studied is the mass hierarchy of neutrinos (either normal (NH): $\Delta m_{31}^2 > 0$ or inverted (IH): $\Delta m_{31}^2 < 0$), the precise value of the mixing angle $\theta_{23}$ and the Dirac CP phase $\delta_{CP}$. There are many future experiments planned to determine these unknown quantities. In the mean time, a few tensions in neutrino experiments have been reported recently. They are: (i) the tension between the mass squared differences by the solar and KamLAND data [1] which gives a non-zero best-fit value of the non-standard interaction parameters $\epsilon_P$ and $\epsilon_N$. This rules out the standard oscillation scenario at 90% C.L., (ii) the tension between the T2K and NO$\nu$A experiments regarding the measurement of the mixing angle $\theta_{23}$ [2, 3] and (iii) the tension in the measurement of the mixing angle $\theta_{13}$ by the reactor and T2K experiments [4, 5].

In this work we present a scenario in which a nonstandard interaction in neutrino propagation can explain the three major tensions in the neutrino oscillation data at present. These tensions are: (i) a non-zero best-fit value of the non-standard oscillation parameters in the global analysis of the solar and KamLAND data which rules out the standard oscillation scenario at 90% C.L., (ii) the measurement of the non-maximal value of $\theta_{23}$ by NO$\nu$A which excludes the maximal mixing at 2.5$\sigma$ C.L. and (iii) a discrepancy in the $\theta_{13}$ measurement by T2K which has a tension with the reactor best-fit value of $\sin^2 \theta_{13} = 0.021$ at 90% C.L. Our results show that all these three above mentioned anomalies can be explained if one assumes the existence of the non-standard interactions in neutrino propagation with $\theta_{23} = 45^o$ and $\sin^2 \theta_{13} = 0.021$ in the case of normal hierarchy. In our scenario the phase of $\epsilon_{ee}$ is zero and the most favorable value of the Dirac CP phase is approximately 255$^o$.


\[ L_{\text{NSI}} = -2\sqrt{2} \epsilon_{\alpha\beta}^P G_F \left( \bar{\nu}_{\alpha} L \gamma_{\mu} \nu_{\beta L} \right) \left( \bar{f}_P \gamma^\mu f_P \right), \]

where $f_P$ stands for fermions with chirality $P$ and $\epsilon_{\alpha\beta}^P$ is a dimensionless constant which is normalized by the Fermi coupling constant $G_F$. In the presence of this NSI, the neutrino evolution is governed by the Dirac equation:

\[ i \frac{d}{dx} \begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \\ \nu_\tau(x) \end{pmatrix} = \left[ U \text{diag} \left( 0, \frac{\Delta m_{21}^2}{2E}, \frac{\Delta m_{31}^2}{2E} \right) U^{-1} + A \right] \begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \\ \nu_\tau(x) \end{pmatrix}, \]

where

\[ A \equiv A \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{\mu e} & 1 + \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{\tau e} & \epsilon_{\tau\mu} & 1 + \epsilon_{\tau\tau} \end{pmatrix}, \]

\[ A \equiv \sqrt{2} G_F N_e, U \text{ is the leptonic mixing matrix, } \Delta m_{jk}^2 \equiv m_j^2 - m_k^2, \epsilon_{\alpha\beta} \text{ is defined by } \]

\[ \epsilon_{\alpha\beta} = \sum_{f = e, u, d} \frac{N^f}{N_e} e^{f \epsilon_{\alpha\beta}}, \]

\[ 1 \text{ The latest measurement by Daya Bay [5] gives } \sin^2 \theta_{13} = 0.021 \text{ and this lies within } 90\% \text{CL of the T2K allowed region (See Fig. 31 of Ref. [4]). Although this may not be called a tension at present, if this trend persists as the statistics increases, the discrepancy between the mixing angles } \theta_{13} \text{ by the reactor and T2K experiments should be taken seriously in future.}

\[ 2 \text{ The recent update of the T2K data can be found in Ref. [6]. As the details of the fit are not available yet we take the latest published results for our analysis.}

\[ 3 \text{ For recent studies of NSI in long-baseline experiments see [12].} \]
TABLE I. Recent data of T2K and NOvA.

|        |               | MeV  |         |         | Expt $\sin^2 \theta_{13}$ NH (IH) | MeV  |         |         | NOvA $\sin^2 \theta_{23}$ NH (IH) | MeV  |         |         | $\delta_{CP}$ NH (IH) | MeV  |         |         |
|--------|---------------|------|---------|---------|----------------------------------|------|---------|---------|----------------------------------|------|---------|---------|-----------------------|------|---------|---------|
| T2K    | 0.0422 (0.0491) | 0.524 (0.523) | 1.91 (1.01) |          |                                  | 0.021 | 0.040   | 1.49 $\pi$ |                                  |      |         |         |

We defined the new NSI parameters as $\epsilon_{\alpha\beta}^f \equiv \epsilon_{\alpha\beta}^L + \epsilon_{\alpha\beta}^R$ since the matter effect is sensitive only to the coherent scattering and only to the vector part in the interaction, and $N_f (f = e, \mu, \tau)$ stands for the number densities of fermions $f$.

To discuss the effect of NSI on solar neutrinos, the $3 \times 3$ Hamiltonian in the Dirac equation Eq. (2) is reduced to an effective $2 \times 2$ Hamiltonian given by

$$H_{\text{eff}} = \frac{\Delta m_{21}^2}{4E} \left( -\cos 2\theta_{12} \, \sin 2\theta_{12} \right) + \left( c_{13}^2 A \begin{pmatrix} e^i \theta_{12} & 0 \\ 0 & -e^i \theta_{12} \end{pmatrix} + A \sum_{f=e,\mu,\tau} \frac{N_f}{N_e} \begin{pmatrix} -\epsilon_{\alpha\beta}^f & \epsilon_{\alpha\beta}^f \\ \epsilon_{\alpha\beta}^f & -\epsilon_{\alpha\beta}^f \end{pmatrix} \right),$$

(5)

where $\epsilon_{\alpha\beta}^f$ and $\epsilon_{\alpha\beta}^f$ are linear combinations of the standard NSI parameters:

$$\epsilon_{\alpha\beta}^f = -\frac{c_{13}^2}{2} (\epsilon_{ee} - \epsilon_{\mu\mu}) + \frac{s_{13}^2 - s_{13}^2 c_{23}}{2} (\epsilon_{\tau\tau} - \epsilon_{\mu\mu}) + c_{13} s_{13} \Re \left[ s_{23} \epsilon_{ee} + c_{23} \epsilon_{\mu\mu} \right] \sqrt{\epsilon_{\mu\tau}}$$

(6)

and $c_{13} \equiv \cos \theta_{13}$, $s_{13} \equiv \sin \theta_{13}$. In the analysis of Ref. [1], one particular choice of $f = u$ or $f = d$ was taken at a time because of the nontrivial composition profile of the Sun, and it was found that the best fit values are $\epsilon_{\alpha\beta}^f = (0.22, -0.30)$ (for solar neutrinos) and $\epsilon_{\alpha\beta}^f = (0.12, -0.16)$ (for atmospheric neutrinos) for the solar neutrino and KamLAND data only.

In this work we look for a scenario with NSI which gives a good fit to the solar and KamLAND data, the NOvA data and the T2K data. For our analysis we use the GLOBES [13] and MonteCUBES [14] softwares. For our fit we will assume that the mixing angle $\theta_{23}$ in vacuum is maximal, i.e., $\theta_{23} = 45^\circ$ and the mixing angle $\theta_{13}$ in vacuum is given by the reactor data, i.e., $\sin^2 \theta_{13} = 0.021$. We will do our analysis in the $(\epsilon_{D}^f, \epsilon_{N}^f)$ plane and for this we need to express $\epsilon_{\alpha\beta}$ as a function of $(\epsilon_{D}^f, \epsilon_{N}^f)$. So we proceed in the following way. As can be seen from the definition of $\epsilon_{\alpha\beta}$, the neutrino oscillation experiments on the Earth are sensitive only to the sum of $\epsilon_{\alpha\beta}^f$. However, since the analysis of solar neutrinos was done either for $f = u$ or $f = d$ only, we also analyze the long baseline experiments assuming the same condition. Since the arrival times of and that of electron is approximately equal in the Earth, if we turn on NSI for $f = u$ or $f = d$ only, then from Eq. (4) we get

$$\epsilon_{\alpha\beta} = 3 \epsilon_{\alpha\beta}^f.$$

(8)

As we can see from Eqs. (6) and (7), the mapping $\epsilon_{\alpha\beta} \rightarrow (\epsilon_{D}^f, \epsilon_{N}^f)$ is not one to one, and in general it is difficult to obtain the possible region for the $\epsilon_{\alpha\beta}$ parameters analytically. Here, instead of exhausting all the possible regions for $\epsilon_{\alpha\beta}$, we postulate the following:

$$\epsilon_{D}^f = -\frac{c_{13}^2}{2} (\epsilon_{ee} - \epsilon_{\mu\mu}) + \frac{s_{13}^2 - s_{13}^2 c_{23}}{2} (\epsilon_{\tau\tau} - \epsilon_{\mu\mu}) + c_{13} s_{13} \Re \left[ s_{23} \epsilon_{ee} + c_{23} \epsilon_{\mu\mu} \right] \sqrt{\epsilon_{\mu\tau}}$$

(9)

$$\epsilon_{N}^f = -\frac{c_{13}^2}{2} s_{23} \epsilon_{ee} + s_{13} c_{23} s_{23} \epsilon_{\mu\tau}$$

(10)

$$0 = c_{13} s_{13} \Re \left[ s_{23} \epsilon_{ee} + c_{23} \epsilon_{\mu\mu} \right] \sqrt{\epsilon_{\mu\tau}}$$

(11)

$$0 = c_{13}^2 s_{23} \epsilon_{ee} + s_{13} c_{23} s_{23} \Re \left[ \sqrt{\epsilon_{\mu\tau}} \right]$$

(12)

Furthermore, for simplicity, we postulate $\Im (s_{23} \epsilon_{\mu\tau} - c_{23} \epsilon_{ee}) = 0$, which implies that $\epsilon_{\mu\tau}$ is a real parameter in the case of $\theta_{23} = 45^\circ$, and following the bound from the high energy atmospheric neutrino data, we take $[16, 17]$

$$\epsilon_{\tau\tau} = \frac{|\epsilon_{ee}|^2}{1 + |\epsilon_{ee}|}.$$

(13)

Another constraint comes from the atmospheric neutrino data, and the following must be satisfied:[18]

$$\left| \frac{\epsilon_{ee}}{1 + |\epsilon_{ee}|} \right| \lesssim 0.8 \quad \text{at 2.5}\sigma \text{CL}.$$  

(14)

Finally, we put $\epsilon_{\mu\mu} = 0$ because we can always redefine $\epsilon_{ee} \rightarrow |\epsilon_{ee}|$ and $\epsilon_{\tau\tau} \rightarrow |\epsilon_{\tau\tau}|$. From these assumptions and Eqs. (8), (9), (10), (11), (12), (13) and (14), after putting $\theta_{23} = 45^\circ$, we get the following expressions:

$$\epsilon_{ee} = -\frac{3 \sqrt{2}}{c_{13}} \epsilon_{N}^f$$

(15)

$$\epsilon_{ee} = -\frac{3}{c_{13}} \epsilon_{D}^f - \frac{1}{2} + \left( \frac{\epsilon_{D}^f}{c_{13}} \right)^2 + \frac{1}{c_{13}^2} |\epsilon_{N}^f|^2$$

(16)

$$\epsilon_{\mu\mu} = -\frac{s_{13} \epsilon_{ee} - i \delta_{CP}}{\sqrt{2} c_{13}}$$

(17)

$$\epsilon_{\mu\tau} = \frac{\sqrt{2}}{1 + s_{13}^2} c_{13} \Re \left[ \epsilon_{\mu\tau} \left( \epsilon_{ee} + \epsilon_{\tau\tau} \right) \right]$$

(18)

Note that in all the best fit solutions from the solar+KamLAND analysis, both $\epsilon_{D}^f$ and $\epsilon_{N}^f$ have a phase ($-1$), so in the present case $\theta_{23}$ is close to $90^\circ$.
we see the following. For NH the two curves for both the hier-
archies. In Eq. (21) we approximated $\chi^2_{\text{sol}+\text{KL}}$ as $\chi^2_{\text{sol}+\text{KL}} \approx \chi^2(\epsilon_{\delta}) + \chi^2(\epsilon_N)$, where $\chi^2(\epsilon_{\delta})$ and $\chi^2(\epsilon_N)$ are $\chi^2$ obtained from the solar+KamLAND data in Ref. [1]. Obviously for the solar+KamLAND best-fit points, $\chi^2_{\text{sol}+\text{KL}} = 0$, while for the global best-fit points, $\chi^2_{\text{sol}+\text{KL}} = 0.1$. The latter was estimated from the Figure 2 of Ref. [1]. To estimate the goodness of fit, we compare our $\chi^2$ with $\chi^2_{\text{std}}$, i.e., the standard case. By $\chi^2_{\text{std}}$ we mean the value of $\chi^2$ at $(\theta^{\text{fit}}_{23}, \theta^{\text{fit}}_{13})$ without NSI. Here the $\chi^2_{\text{std}}$ for solar+KamLAND is 3.8 (4.4) for $f = u$ ($f = d$), which is estimated by the approximation mentioned earlier. In our analysis we take the value $\chi^2_{\text{sol}+\text{KL}} = 3.8$ for conservative estimation. On the other hand, the $\chi^2_{\text{std}}$ of T2K and NOνA depend on $\delta_{\text{CP}}$. For the standard case, therefore, we have $\chi^2_{\text{std}} \approx 3.8$. From Fig. 1 we see the following. For NH the two curves (solid-purple and dashed-blue) which correspond to the best-fit points of the global analysis of the solar data lie below the standard curve (solid-black) for all the values of $\delta_{\text{CP}}$. This
we give the allowed region in the scenario, the three tensions give a constraint on the phase of others. For IH the NSI does not give a better fit. In this sce-

Table II. Values of $\epsilon_{\alpha\beta}$ corresponding to global-$\nu$ best fit point of the solar data ($\epsilon_D = -0.14$, $\epsilon_N = -0.03$) at $\delta_{CP} = 255^\circ$. 

| $\epsilon_{ee}$ | $\epsilon_{e\tau}$ | $\epsilon_{\tau\tau}$ | $|\epsilon_{e\mu}|$ | $\epsilon_{\mu\tau}$ |
|---------------|------------------|------------------|------------------|------------------|
| 0.84885 | 0.12863 | 0.008950 | 0.00092689 | -0.0067963 |

implies that a nonstandard interaction at the solar best-fit point gives a better fit as compared to the standard case. Thus we found a new solution (the best-fit point of the solar+kamLAND data) with NSI which solves all the three neutrino tensions. Whereas in IH, a scenario with NSI in any region of $\delta_{CP}$ does not give $\chi^2$ which is smaller than the minimum $\chi^2$ in the standard case. From the plot we also see that in the case of NH, $\delta_{CP} \approx 255^\circ$ is the most preferred value of $\delta_{CP} \Delta$ which gives the best fit with NSI. In Table II we give the values for $\epsilon_{\alpha\beta}$ corresponding to the global-$\nu$ best-fit point of the solar data at $\delta_{CP} = 255^\circ$. For our information, in Fig. 2 we give the allowed region in the $\epsilon_D$, $\epsilon_N$ plane for NO+\A and T2K at $\delta_{CP} = 255^\circ$ in the case of NH. As mentioned earlier, since $\epsilon_N$ ($\epsilon_{\mu\tau}$) has a phase ($-1$) ($0$) in all the best-fit solutions from the solar+kamLAND analysis, we performed our analysis only for $\epsilon_N < 0$. For the solar+kamLAND we give just the best-fit points. From these plots we identify the allowed region which is consistent with all the three anomalies under discussion. For NH we see that the global best-fit of the solar data is consistent with the NO+\A and T2K data within 2$\sigma$ confidence regions.

In summary we found a scenario which explains the tension of the mass squared differences of the solar and KamLAND data, the one of mixing angles $\theta_{23}$ of the T2K and NO+\A data, and the discrepancy of $\theta_{13}$ of the reactor and T2K data. In our analysis we found that the goodness of fit for the NSI sce-

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|---------------|------------------|------------------|------------------|------------------|
| 0.84885 | 0.12863 | 0.008950 | 0.00092689 | -0.0067963 |

A different point of view.

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