Forced vibrations of a box-like structure of a multi-storey building under dynamic effect

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Abstract. The article is devoted to improving the model of a box-like structure of a building, taking into account the forces and moments in the zones of contact interaction of beam and plate elements. The equations of motion of the box-like elements, the boundary conditions in the base of the box and the contact conditions between the elements of the box are given; the graphs of panels and beams displacements are constructed.

Keywords: buildings, box-like structure, dynamic calculation, displacement, stress, strain, equation of motion, boundary conditions, contact conditions.

1. Introduction
Various dynamic problems in a plane and spatial statements are considered in [1–4]; the studies are devoted to estimating and predicting the dynamic behaviour of various structures, taking into account physical and geometric nonlinearity, inelastic properties of the material, and inhomogeneous structural features under multicomponent kinematic effects.

In [5,6], vibrations of structural elements of an isotropic viscoelastic plate of variable thickness under uniformly distributed vibration load applied on one of the parallel sides, leading (at certain combinations of natural vibration frequencies and disturbing force) to parametric resonance, are considered.

The studies in [7, 8] are devoted to the development of the methods for dynamic spatial calculation of a structure based on the finite difference method in the framework of the bimoment theory, which takes into account the spatial stress-strain state. Solutions were given to the problem of transverse and longitudinal vibrations of buildings and structures using a plate model developed in the framework of the bimoment theory of plates [9-11].
In the theory of vibrations of thin-walled structural elements, a special place is occupied by transverse vibrations of cantilever plates and beams. Such processes are described by high order differential equations. Meeting the boundary conditions, the choice of coordinate functions when applying variation methods or approximating the finite difference formulas of the third and fourth order derivatives are very difficult tasks. The free and forced vibrations of the boxes of a large-panel building and the cells of frame buildings are studied; a spatial box with a fixed lower end is considered as a design diagram [9-11].

This article poses the problem of forced vibrations of a spatial box of a building, consisting of interacting beams and rectangular panels under dynamic effect. As the design diagram of the building, consider a spatial box with a fixed lower end, consisting of beams and rectangular panels, see figure 1. All elements of the spatial frame are square beams of δ size made of the same material, with the same elastic and shear moduli E and G, Poisson's ratio ν and density ρ. J and Iₖᵣ are the moments of inertia of the beam section under bending and torsion.

![Figure 1. The spatial box of the building](image)

To study the movement of the box elements, introduce a Cartesian coordinate system, figure 1. Introduce the following notation for the panels of the spatial box-like structure of the building: Eₖ, νₖ, ρₖ and hₖ - the elastic modulus, Poisson's ratio, density and thickness of the k-th panel, respectively.

2. Statement of the problem

Dynamic problem of building box vibrations is considered, its base oscillates according to the law:

\[ U_0 = A_0 \sin \omega_0 t, \]  

where \( A_0 \) and \( \omega_0 \) are the amplitude and frequency of forced oscillations.

An analytical-numerical method is proposed for solving the box-like building oscillations problem taking into account spatial strains with overall contact conditions in the areas of butt joints of panels and beam elements of the building box.

The deflection of the flexural panels is presented in the form:

\[ W = W(x, y)\sin(\omega_0 t), \]
and the displacements of panels working on a shear, are written in the form:

\[ u = u(x, z) \sin(\omega_0 t), \quad \nu = \nu(x, z) \sin(\omega_0 t). \]  

(3)

where \( \omega_0 \) is the fundamental frequency of forced oscillations.

The deflection and the angle of twist of the beams can be represented as:

\[ W^{(i)} = W^{(i)}(x, y) \sin(\omega_0 t), \quad \alpha^{(i)} = \alpha^{(i)}(x, y) \sin(\omega_0 t) \]  

(4)

where \( \omega_0 \) is the principal frequency of external influence, \( i = I, II, III, IV \) (number of beams).

Consider the theoretical calculation of a box of a large-panel building under dynamic effect, taking into account the spatial work of transverse and longitudinal walls.

Based on the representation (1), we rewrite the kinematic laws of motion of the panel points.

Normal displacements of the points of bending panels are

\[ u_i = A_i \sin \omega_0 t + W(x, y, t), \]  

(5.a)

where \( W(x, y, t) \) is the deflection of bending panels, \( A_i \) and \( \omega_0 \) are the amplitude and frequency of forced oscillations.

The displacement field of the panels working on shear is described by the functions

\[ u_t = A_t \sin \omega_0 t + u(x, y, t), \quad u_z = \nu(x, y, t). \]  

(5.b)

Here \( u, \nu \) are the displacements of the panels working on shear.

As the equation of motion of the bending panel we take [9, 10], with (1) it can be written as

\[ D(\frac{\partial^2 W}{\partial x^2} + 2 \frac{\partial^2 W}{\partial x \partial y} + \frac{\partial^2 W}{\partial y^2}) + \rho h W = \rho h A_0 \omega_0^2 \sin \omega_0 t \]  

(6)

where \( D \) is the cylindrical rigidity of panels under lateral bending.

\[ D = \frac{E h^3}{12(1-\nu^2)}, \quad h - \text{the beam thickness}, \quad W - \text{the panel deflection (working on bending)}. \]

Two-dimensional equations of motion of the panel working on shear are taken [9,10] and written in the form

\[ B(\frac{\partial^2 u}{\partial z^2} + \frac{1+\nu}{2} \frac{\partial^2 u}{\partial x \partial z} + \frac{1-\nu}{2} \frac{\partial^2 u}{\partial x^2}) = \rho h \ddot{u} - \rho h A_0 \omega_0^2 \sin \omega_0 t, \]  

(7)

\[ B(\frac{\partial^2 \nu}{\partial x^2} + \frac{1+\nu}{2} \frac{\partial^2 \nu}{\partial x \partial z} + \frac{1-\nu}{2} \frac{\partial^2 \nu}{\partial z^2}) = \rho h \ddot{\nu}, \]

where \( B \) is the cylindrical rigidity of panels under extension and compression.

\[ B = \frac{E h}{1-\nu^2}, \quad u, \nu - \text{the displacements along the axes } OX \text{ and } OY. \]

The boundary conditions on the building base (\( x = 0 \)) are written for rigid fixation. The lower part of the building moves with the base and there is no turning.
\[ u_i = u_j = U_o(t), \quad u_z = 0, \quad \frac{\partial W}{\partial x} = 0. \]  \hspace{1cm} (8,a)

The boundary conditions (7) and (8) at \( x = 0 \) with (2) are rewritten as:

\[ W = 0, \quad \frac{\partial W}{\partial x} = 0, \quad u = 0, \quad v = 0. \] \hspace{1cm} (8,b)

The boundary conditions at the upper end \( x = H \) are:

The contact conditions at the joints of the floor and the wall working on bending have the form

\[- R_b^b + \eta_0 \rho h_b h_1 \frac{\partial^2 x}{\partial y} = h_1 h_1 \frac{\partial^2 x}{\partial y} - \eta_0 \rho h_b h_1 \frac{\partial^2 x}{\partial y}. \] \hspace{1cm} (9)

The contact conditions at the joints of the floor and the wall working on shear relative to the contact tangent stress are written as

\[- c h \tau_x + m_s \hat{u}_{n,s} = c h \frac{\partial x}{\partial z} - m_s \hat{U}_o. \] \hspace{1cm} (10)

The contact conditions at the joints of the floor and the wall working on shear relative to the contact normal stress are written as

\[- c h \sigma_{n} + m_s \hat{i}_{n,s} = c h \frac{\partial x}{\partial z}. \] \hspace{1cm} (11)

The initial conditions of the problem are assumed to be zero.

3. Method of the solution

The General solutions to the problem for a flexible panel is

\[ W(x, y, t) = A_0 (\sin \theta_0 t - A_1 \frac{\omega_0}{p_1} \sin p_1 t) W_s(x, y). \] \hspace{1cm} (12)

The expressions for the shear panel displacements have the form

\[ u(x, z, t) = A_0 (\sin \theta_0 t - A_1 \frac{\omega_0}{p_1} \sin p_1 t) u_s(x, z), \] \hspace{1cm} (13)

\[ v(x, z, t) = A_0 (\sin \theta_0 t - A_1 \frac{\omega_0}{p_1} \sin p_1 t) v_s(x, z). \]

The kinematic functions of the beams can be written as:

\[ W^{(1)}(x, t) = A_0 (\sin \theta_0 t - A_1 \frac{\omega_0}{p_1} \sin p_1 t) W^{(1)}_s(x), \] \hspace{1cm} (14)

\[ \alpha^{(1)}(x, t) = A_0 (\sin \theta_0 t - A_1 \frac{\omega_0}{p_1} \sin p_1 t) \alpha^{(1)}_s(x). \]

The problem of determining the unknown coordinate functions in expressions (12), (13) and (14) is solved by the finite difference method. To determine the value of the function derivatives, we use the
well-known formulas of the finite difference method. Introduce formulas for determining the value of the function derivatives \( f(x, y, t) \) with respect to spatial coordinates \( y \) and denote by primes

\[
\begin{align*}
    f'_i &= \left. \frac{\partial f}{\partial y} \right|_{y=y_i}, \\
    f''_i &= \left. \frac{\partial^2 f}{\partial y^2} \right|_{y=y_i}, \\
    \ldots \\
    f''''_i &= \left. \frac{\partial^4 f}{\partial y^4} \right|_{y=y_i}.
\end{align*}
\]

The first and second derivatives are determined by formulas

\[
\begin{align*}
    f'_i &= \frac{f_{i+1} - f_{i-1}}{2\Delta y} ; \\
    f''_i &= \frac{f_{i+1} - 2f_i + f_{i-1}}{\Delta y^2}.
\end{align*}
\]

Now write the difference formula for the third and fourth derivatives

\[
\begin{align*}
    f''''_i &= \frac{1}{\Delta y^3} \left( f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2} \right) , \\
    f''''''_i &= \frac{1}{\Delta y^4} \left( f_{i+2} - 4f_{i+1} + 6f_i - 4f_{i-1} + f_{i-2} \right).
\end{align*}
\]

4. **Analysis of numerical results**

The following parameters are set as initial data. The ratio of the height to the width of the bending panel is \( \frac{H}{b} = \frac{3.25}{6} \), the ratio of the height to the width of the panel working on shear is \( \frac{H}{c} = \frac{3.25}{6} \).

The ratio of the thickness to the width of the bending panel is \( \frac{h_b}{b} = \frac{0.5}{6} \), and the ratio of the thickness of the bending panel to the thickness of the shear panel is \( \frac{h}{h_b} = \frac{0.25}{0.5} \).

The ratio of the elastic moduli of bending and shear panels is \( \frac{E_1}{E_2} = \frac{3}{8} \). Poisson's ratio of panel materials is \( \nu = 0.3 \).
Figure 2. Changes in the deflection of panels working on bending.
   a) at the middle b) at the edges of the opening c) at the edges of the panel

Figure 2 shows the graphs that characterize vertical changes of the dimensionless maximum deflection $\frac{W}{A_0}$ on three characteristic sections of panels working on bending, from the middle of the panel to one of its edges. As seen, the panel deflections increase approaching the middle of the panels.

The maximum deflection value (see figure 2, a), reached at the top of the vertical section located in the middle of the bending panels, is

$$W = 0.369 A_0 = 0.369 \cdot 2 cm = 0.738 cm$$

The maximum value of the overall horizontal displacement of the upper point of the bending panel is determined by the formula (4.1a) and is

$$u_z = A_0 + 0.369 A_0 = 2 cm + 0.369 \cdot 2 cm = 2.738 cm$$

As expected, maximum displacements were found at the upper points of the panels and beams.
Figure 3 shows the graphs of vertical changes of the dimensionless maximum deflection \( \frac{W}{A_0} \) of the beams.

Calculations show (see figure 3) that the beams I and IV, located on the side of loading, bend more than the beams II and III.

The maximum deflection value (see figure 3,a) is reached at the upper point of the bending beams:

\[
W^{(f)} = 0.22A_0 = 0.22 \cdot 2\,cm = 0.44\,cm
\]

Then the maximum value of the overall horizontal displacement of the upper point of the bending panel is determined by the formula (5.1a)

\[
u_3 = A_0 + 0.22A_0 = 2\,cm + 0.22 \cdot 2\,cm = 2.44\,cm
\]

**Figure 3.** Changes in deflection along the height of the beam. a) beams I and IV b) beams II and III

From figure 3 it can be seen that the deflection of the vertical edge of the bending panel is several times less than the deflection in the middle part, since the panel edges are held by transverse panels that work on shear; therefore, the bending shapes of the beam elements and panels are not identical. As expected, maximum displacements were found at the upper points of the panels and beams.

Figure 4 shows the graphs of changes in dimensionless displacement \( \eta(t) = \frac{u_1(t)}{A_0} \) and deflection \( \xi(t) = \frac{u_3(t)}{A_0} \) on the panels depending on time.
8

Figure 4. Graph of changes in dimensionless displacement and deflection:
a) in panels working on shear; b) in panels working on bending

5. Conclusion

The equations of motion of the panels and beams points of the building box, the boundary, contact and initial conditions of the problem of forced oscillations are given. Within the framework of finite difference method, the methods for dynamic calculation of displacements in beam and panel elements of box-like structures of buildings have been developed. The laws of changes in the maximum values of deflections and stresses at characteristic points of panels and beam elements are graphically presented depending on time.

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