Relaxed Recovery Conditions for OMP/OLS by Exploiting both Coherence and Decay

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Abstract—We propose new coherence-based sufficient conditions for the exact sparse recovery by orthogonal matching pursuit (OMP) and orthogonal least squares (OLS). Unlike standard uniform guarantees, we embed some information about the decay of the nonzero coefficients of the sparse vector in our conditions. As a result, we show that the standard conditions existing in the literature can be weakened as soon as the nonzero coefficients obey some decay (both in the noiseless and the bounded-noise scenarios). Some standard conditions of the literature are moreover shown to correspond to particular cases of our results when the non-zero coefficients are all equal, that is the sparse vector is “flat”.

1 Introduction

In this paper, we focus on two popular instances of greedy algorithms, namely orthogonal matching pursuit (OMP) [1] and orthogonal least squares (OLS) [2, 3]. These two iterative procedures build an estimate of the support of the sparse vector by adding one new element to it at each iteration. The algorithms exclusively differ in the way this new element is selected: OMP picks the atom leading to the maximum (absolute) correlation with the current residual while OLS selects the atom minimizing the norm of the new residual. We refer the reader to the reference papers [1, 2, 3] for a detailed description of the mechanics ruling these procedures.

The suboptimal nature of OMP and OLS has led many researchers to study conditions under which these procedures succeed in recovering the true sparse vector. This question has been widely addressed for OMP in the recent years, including worst-case uniform [7, 8] and probabilistic analyses [9]. The existing exact recovery analyses of OMP were also adapted to several extensions of OMP, namely regularized OMP [10], weak OMP [11], and stagewise OMP [12]. Although OLS has been known in the literature for a few decades, the proof of the success of OLS for a given sparsity level (or a given support) irrespective of the magnitude of the non-zero coefficients was provided by Tropp et al. in [14]. He showed that the relaxed condition

$$\mu < \frac{1}{2k-1},$$

ensures the success of OMP. The derivation of similar conditions for OLS is more recent and is due to Soussen et al. in [14].

Recently, condition (2) has been refined in [15] to analyze the “k-step” analysis encountered in many contributions of the literature [7, 8, 14]. More specifically, OMP/OLS with y defined in (1) as input will be said to succeed if and only if the atoms in $Q^*$ are selected during the first k iterations.

2 Context and Main Results

Let us assume that $y \in \mathbb{R}^m$ is a (noisy) linear combination of $k$ columns of $A \in \mathbb{R}^{m \times n}$ indexed by $Q^*$, that is

$$y = Ax + w \quad \text{with} \quad \{ \begin{array}{l} x_i \neq 0 \Leftrightarrow i \in Q^* \\ \text{Card}\{Q^*\} = k \end{array} \right.$$ (1)

where $w \in \mathbb{R}^m$ denotes some noise vector and Card{} stands for the cardinality operator. We assume that the columns $a_i$ of the dictionary are normalized: $\|a_i\|_2 = 1 \forall i$. In this section, we review some standard conditions ensuring the correct reconstruction of $x$ in $k$ steps and recast our contributions within these existing results. The noiseless case ($w = 0$) is considered in section 2.1 and the bounded-noise scenario ($\|w\|_2 \leq \epsilon$) in section 2.2.

Let us mention that the notion of success that will be used hereafter refers to the “k-step” analysis encountered in many contributions of the literature [7, 8, 14]. More specifically, OMP/OLS with $y$ defined in (1) as input will be said to succeed if and only if the atoms in $Q^*$ are selected during the first $k$ iterations.

2.1 The Noiseless Case

The first thoughtful “k-step” analysis of OMP is due to Tropp, see [7, Th. 3.1 and Th. 3.10]. He provided a sufficient and worst-case necessary condition for the exact recovery of any sparse vector with a given support $Q^*$. Moreover, he showed that the relaxed condition

$$\mu < \frac{1}{2k-1},$$

where $\mu \triangleq \max_{i \neq j} |a_i^T a_j|$ is the mutual coherence of $A$, ensures the success of OMP. The derivation of similar conditions for OLS is more recent and is due to Soussen et al. in [14]. Recently, condition (2) has been refined in [15] to analyze the success of OLS from one given iteration. More specifically, if one assumes that OLS has selected $g$ atoms in $Q^*$ during the first $g$ iterations, the following condition

$$\mu < \frac{1}{2k-g-1},$$

ensures that the other atoms in $Q^*$ are selected during the remaining $k - g$ iterations.
Let us mention that conditions (2) and (3) are uniform, that is Oxx can recover any $k$-sparse vector irrespctive of the amplitude of the non-zero coefficients if they are satisfied. On the other hand, it was shown in [15, 16] that (2) and (3) are tight in some sense. In particular, the authors provided counter-examples showing that one cannot further improve these conditions. Nevertheless, it is useful to note that the sparse vectors involved in the latter counter-examples are “flat”, that is such that

$$x_i = \text{constant} \quad \forall i \in \mathbb{Q}^*.$$  

(4)

Theorem 1 extends this result one step further by showing that weaker sufficient conditions than (2)-(3) can be obtained as soon as the non-zero coefficients in $x$ obey some decay.

In our statement below, we assume without loss of generality\(^2\) that

$$\mathbb{Q}^* = \{1, 2, \ldots, k\},$$  

(5)

and

$$|x_1| \geq |x_2| \geq \ldots \geq |x_k| > 0.$$  

(6)

**Theorem 1.** If

$$\mu < \frac{1}{k},$$  

(7)

and

$$|x_i| > \frac{2\mu(k-i)}{1 - i\mu} |x_{i+1}| \quad \forall i \in \{1, \ldots, k\},$$  

(8)

then Oxx selects atoms in $\mathbb{Q}^*$ from noiseless data during the first $k$ iterations.

We note that Theorem 1 encompasses the standard conditions (2)-(3) as particular cases. Indeed, the decay factor appearing in the right-hand side of condition (8) is such that

$$\frac{2\mu(k-i)}{1 - i\mu} < 1,$$  

(9)

as soon as

$$\mu < \frac{1}{2k-i}.$$  

(10)

Thus, by virtue of our initial assumption (6), (9) implies that condition (8) trivially holds for any $i \in \{g+1, \ldots, k\}$ as soon as $\mu < 1/(2k-g-1)$. In other words, if (3) is satisfied and atoms in $\mathbb{Q}^*$ have been selected during the first $g$ iterations, then Oxx selects atoms in $\mathbb{Q}^*$ during the last $k-g$ iterations irrespective of the amplitudes of the non-zero coefficients.

As a matter of fact, it also turns out that the worst case for the strongest conditions on the mutual coherence, namely the standard sufficient conditions (2)-(3), occur when considering “flat” vectors only. Indeed, reexpressing the conditions in (8) for $i \in \{1, \ldots k-1\}$ in terms of constraints on the mutual coherence, we have

$$\mu < \frac{|x_i|}{|x_{i+1}|}.$$  

(11)

Clearly, the right-hand side of (11) is an non-decreasing function of $\frac{|x_i|}{|x_{i+1}|}$ and the strongest conditions on $\mu$ (namely (2)-(3)) are therefore obtained for $\frac{|x_i|}{|x_{i+1}|} = 1$.

Finally, let us mention that the condition $\mu < 1/k$ appearing in Theorem 1 is the best bound which can be achieved by any coherence-based result. Indeed, the linear independence of any group of $k+1$ columns is clearly a necessary condition for the success of Oxx. On the one hand, this is ensured by the condition $\mu < 1/k$ (see for example [7, Lemma 2.3]). On the other hand, one can find dictionaries with $\mu \geq 1/k$ for which a group of $k+1$ columns are linearly dependent (see for example [15, Example 2]).

2.2 The Bounded-Noise Case

We now assume that the noise affecting $y$ has a bounded $\ell_2$ norm, that is $\|w\|_2 \leq \epsilon$. Although many researchers have emphasized that the noiseless conditions can be generalized to the case where the noise level is low in comparison to the smallest nonzero amplitudes [17, 18, 19], we stress that no tight sufficient conditions on the mutual coherence, we have

$$\mu < \frac{1}{2k-1},$$  

(12)

and

$$|x_i| > \frac{2\epsilon}{1 - (2k-1)\mu} \quad \forall i \in \{1, \ldots, k\}.$$  

(13)

To the best of our knowledge, the extension of this result to the success of OLS has never been made in the literature.

Hereafter, we provide new (weaker) conditions of success for Oxx in the noisy setting (Theorems 2 and 3). The result stated in Theorem 3 encompasses, as a corollary, (12)-(13) as sufficient conditions for both OMP and OLS.

In order to state Theorem 2, we first need to define the following quantity:

$$\gamma_i \triangleq \left\{ \begin{array}{ll}
\frac{1-(i+1)\mu}{1+(i+1)\mu} & \text{for OMP,} \\
\frac{1-(i+1)\mu}{1+(i+1)\mu} & \text{for OLS.}
\end{array} \right.$$  

(14)

Following the same conventions as in (5)-(6), our result then writes as follows:

**Theorem 2.** If

$$\mu < \frac{1}{k},$$  

(15)

and

$$|x_i| > \frac{2\mu(k-i)}{1 - i\mu} |x_{i+1}| + 2\gamma_{k-1}\epsilon \quad \forall i \in \{1, \ldots, k\}$$  

(16)

then Oxx selects atoms in $\mathbb{Q}^*$ from noisy data during the first $k$ iterations.

**Theorem 3.** If

$$\mu < \frac{1}{2k-1},$$  

(17)

and

$$|x_i| > \frac{2\epsilon}{1 - (2k-i)\mu} \quad \forall i \in \{1, \ldots, k\},$$  

(18)

then Oxx selects atoms in $\mathbb{Q}^*$ during the first $k$ iterations.
Theorem 3 makes a direct connection with the result in [17], [19]. More specifically, we see that the condition by Donoho et al. in (13) is sufficient for (18) to be satisfied and is therefore stronger than the conditions mentioned in Theorem 3. On the other hand, if one deals with “flat” vectors, (18) obviously reduces to (13) since \( \frac{(\mu+1)(1-\mu)}{1-(g-1)\mu} \) \( \leq \frac{2e}{(2k-1)g} \) \( \forall i \in \{1, \ldots, k\} \). In such a case, Theorem 3 does not improve over the standard conditions by Donoho et al.

3 Technical Details

In this section, we provide a proof of the theorems stated in section 2. We first recall the main principles ruling OMP and OLS in section 3.1. We then introduce some technical lemmas in section 3.2. Finally the proof of the main results is reported in section 3.3.

3.1 OMP and OLS

In order to precisely describe the update rules characterizing Oxx, let us first introduce some notations: given a set of indices \( Q \), \( A_Q \) represents the submatrix of \( A \) specified by the columns indexed in \( Q \); the projector onto the orthogonal complement of the span of \( A_Q \) is defined as \( P_A^Q \triangleq I - A_QA_Q^\dagger \), where \( A_Q^\dagger \) is the pseudo inverse of \( A_Q \); in particular, \( r_Q \triangleq P_A^Qy \) is the residual error when projecting \( y \) onto the span of \( A_Q \). Finally, \( \langle \cdot, \cdot \rangle \) represents the vector inner product and \( 0_m \) is the null vector of size \( m \times 1 \).

Oxx can be understood as an iterative procedure generating an estimate of \( Q^* \) by sequentially adding one new element to the current support estimate, say \( Q \). OMP and OLS differ in the way this new element is selected. At each iteration, OLS selects the atom \( a_\ell \) yielding the minimum residual error \( \|r^{Q_\ell(l)}\|_2 \):

\[
\ell \in \arg\min_{i \notin Q} \|r^{Q_\ell(l)}\|_2,
\]

and \( n - \text{Card}\{Q\} \) least-square problems are being solved to compute \( \|r^{Q_\ell(l)}\|_2 \) for all \( i \notin Q \) [2]. On the contrary, OMP adopts the simpler rule

\[
\ell \in \arg\max_{i \notin Q} \|\tilde{a}_i, r^Q_i\|,
\]

to select the new atom \( a_\ell \) and then solves only one least-square problem to update the new residual \( r^{Q_\ell(l)} \).

The selection rules described above can also be expressed in terms of the (normalized) projected atoms of the dictionary [14]. This formulation will turn out to be convenient in our proofs below. More specifically, let

\[
\tilde{a}_i \triangleq P_A^Qa_i,
\]

\[
\tilde{b}_i \triangleq \begin{cases} \frac{\tilde{a}_i}{\|\tilde{a}_i\|_2} & \text{if } \tilde{a}_i \neq 0_m \\ 0_m & \text{otherwise} \end{cases}
\]

With these notations, the selection rule of Oxx can be re-expressed as (see e.g., [6])

\[
\ell \in \arg\max_i \|\tilde{c}_i, r^Q_i\|, 
\]

(19)

where

\[
\tilde{c}_i \triangleq \begin{cases} \tilde{a}_i & \text{for OMP} \\ \tilde{b}_i & \text{for OLS} \end{cases}
\]

We note that, for simplicity, the dependence of \( \tilde{a}_i \), \( \tilde{b}_i \) and \( \tilde{c}_i \) on \( Q \) does not appear explicitly in our notations. The reader should however keep this dependence in mind in our subsequent derivations.

3.2 Some Useful Lemmas

As emphasized in the previous section, the projected atoms play an important role in the characterization of Oxx. In this section, we state two useful lemmas, connecting different functions of the projected atoms to the mutual coherence of the dictionary. These results will be the building blocks of the proof in the next section.

**Lemma 1.** Let \( |Q| = g \). If \( \mu < \frac{1}{g-1} \), then

\[
\|\tilde{a}_i\|^2 \geq \frac{(\mu+1)(1-\mu)}{1-(g-1)\mu} \quad \forall i \notin Q, \\
\|\langle \tilde{a}_i, \tilde{a}_j \rangle\| \leq \frac{\mu+1}{1-(g-1)\mu} \quad \forall j \neq i,
\]

(20)

**Proof:** The result is a direct consequence of Lemmas 4 and 10 in [15]. \( \square \)

**Lemma 2.** Let \( |Q| = g \). If \( \mu < \frac{1}{g} \), we have

\[
\min_{i \notin Q} \langle \tilde{c}_i, \tilde{a}_i \rangle \geq \alpha_g > 0, \\
\max_{i \neq j} |\langle \tilde{c}_i, \tilde{a}_j \rangle| \leq \mu_g,
\]

(21, 22)

where

\[
\alpha_g = \begin{cases} \frac{(\mu+1)(1-g\mu)}{1-(g-1)\mu} & \text{for OMP} \\ \sqrt{\frac{\mu+1}{1-(g-1)\mu}} & \text{for OLS} \end{cases}
\]

\[
\mu_g = \begin{cases} \frac{(\mu+1)(1-g\mu)}{1-(g-1)\mu} & \text{for OMP} \\ \sqrt{\frac{\mu}{1-\mu}} & \text{for OLS} \end{cases}
\]

(23, 24)

**Proof:** The result immediately follows from Lemma 1. Note that \( \mu < \frac{1}{g} \) implies that \( \tilde{a}_j \neq 0_m \) (see (20)). Thus, \( \tilde{b}_i \) reads \( \frac{\tilde{a}_i}{\|\tilde{a}_i\|_2} \). \( \square \)

3.3 Proofs of the Main Results

The proofs of Theorems 1, 2 and 3 are based on the following technical lemma:

**Lemma 3.** Assume that Oxx, with \( y \) defined as in (1) as input, has selected atoms in \( Q \) during the first \( g \) iterations and \( Q \subseteq Q^* \). Let

\[
j \in \arg\max_{i \in Q^* \setminus Q} |x_i|,
\]

and \( \alpha_g, \mu_g \) be defined as in Lemma 2. If

\[
\mu < \frac{1}{g+1},
\]

(25)

\[
|\langle \tilde{c}_j - \mu_g \rangle x_j - 2\mu_g \langle x_{Q \cup \{j\}} \rangle_1 | > 2 \epsilon,
\]

(26)

then Oxx selects an atom in \( Q^* \) at the next iteration.

**Proof:** We want to show that (25)-(26) implies

\[
\max_{i \in Q^* \setminus Q} |\langle \tilde{c}_i, r^Q_i \rangle| > |\langle \tilde{c}_i, r^Q_i \rangle| \quad \forall i \notin Q^*.
\]

(27)
First, using the definitions of the residual \( r^Q = P^Q_y \) and the projected atoms \( \tilde{a}_i \), we have
\[
r^Q = s^Q + P^Q_w,
\]
where
\[
s^Q = \sum_{i \in \mathcal{Q}^c} \tilde{a}_i x_i.
\]
Noticing that \( \| \tilde{c}_i \|_2 \leq 1 \) and \( \| P^Q_w \|_2 \leq \| w \|_2 \leq \epsilon \), a sufficient condition for (27) is then as follows:
\[
\max_{i \in \mathcal{Q}^c} \| \tilde{c}_i s^Q \| - \| \tilde{c}_i, s^Q \| > 2 \epsilon, \quad \forall \ell \notin \mathcal{Q}^c.
\]
Since \( \mu < \frac{1}{g^2 k} \), we can apply Lemma 2 and bound the terms in the left-hand side of (30) as follows:
\[
\max_{i \in \mathcal{Q}^c} \| \tilde{c}_i, s^Q \| \geq \| \tilde{c}_j, \tilde{a}_i \| | x_j | - \sum_{i \in \mathcal{Q}^c \setminus (\mathcal{Q} \cup \{ j \})} | \tilde{c}_j, \tilde{a}_i | | x_i | \geq \alpha_g | x_j | - \mu_g \| s^Q \| \| x \|_1,
\]
and
\[
\| \tilde{c}_i, s^Q \| \leq \sum_{i \in \mathcal{Q}^c} \| \tilde{c}_i, \tilde{a}_i \| | x_i | \leq \mu_g \| s^Q \| \| x \|_1.
\]
Combining these two inequalities, we see that (25)-(26) is a sufficient condition for (27).

We are now ready to proceed to the proof of Theorems 1 and 2. The proof of these two results follow exactly the same lines of thought (obviously, Theorem 1 is a special case of the Theorem 2 when the noise \( \epsilon = 0 \)). Hence, we mainly focus on the proof of Theorem 2 hereafter.

**Proof of Theorem 2:** Assume that Oxx has selected atoms in \( \mathcal{Q} \subset \mathcal{Q}^c \) when \( g \) iterations have been completed; we apply Lemma 3 to show that, under the hypotheses of Theorem 2, the next atom selected by Oxx belongs to \( \mathcal{Q}^c \setminus \mathcal{Q} \).

First, note that the first condition of Lemma 3, \( \mu < \frac{1}{g^2 k} \), is always verified since \( \mu < 1/k \) by hypothesis and \( g \leq k - 1 \).

Let \( j \) be the lowest index such that:
\[
| x_j | = \max_{i \in \mathcal{Q}^c} | x_i |.
\]
From the definition of \( x_j \) and assumption (6), the following inequalities hold:
\[
\| x_{\mathcal{Q} \setminus (\mathcal{Q} \cup \{ j \})} \|_1 \leq | (k - g - 1) \sum_{i \in \mathcal{Q}^c \setminus (\mathcal{Q} \cup \{ j \})} | x_i |, \\
\leq | k - g - 1 | | x_{j+1} |.
\]
Hence,
\[
(\alpha_g - \mu_g) | x_j | - 2 \mu_g | x_{j+1} | | k - g - 1 | > 2 \epsilon,
\]
is a sufficient condition for (26). Since \( \mu < \frac{1}{g^2 k} \), we can exploit the expressions of \( \alpha_g \) and \( \mu_g \) in Lemma 2 to rewrite (35) as
\[
| x_j | > \frac{2 \mu_g | k - g - 1 |}{1 - (g + 1) \mu} | x_{j+1} | + 2 \gamma_g \epsilon,
\]
where \( \gamma_g \) is defined in (14). We finally obtain condition (16) by noticing that: i) \( j \leq g + 1 \) and the function \( \frac{2 \mu_g (k - g)}{1 - (g + 1) \mu} \) is decreasing on \([0, k] \) for \( \mu < \frac{1}{3} \); ii) the function \( \gamma_g \) is increasing on \([0, k - 1] \) if \( \mu < \frac{1}{3} \).

Hence, (15)-(16) are sufficient conditions for (25)-(26) and the next atom selected by Oxx thus belongs to \( \mathcal{Q}^c \setminus \mathcal{Q} \) by virtue of Lemma 3.

**Proof of Theorem 3:** We first note that \( \mu < \frac{1}{g^2 k} \) \( \forall g \in \{ 1, \ldots, k - 1 \} \) since \( \mu < \frac{1}{g^2 k} \) by hypothesis and \( 2k - 1 \geq k \geq g + 1 \). The beginning of the proof follows the same lines as the proof of Theorem 2 but exploits the fact that, by assumption (6),
\[
| x_j | > | x_{j+1} |,
\]
and therefore
\[
(\alpha_g - \mu_g) | x_j | - 2 \mu_g | x_{j+1} | (k - g - 1) > 2 \epsilon,
\]
is a sufficient condition for (35). Since \( \mu < \frac{1}{g^2 k} \), we can use Lemma 2 to rewrite the latter condition as
\[
\eta \frac{(\mu + 1)(1 - (2k - g - 1) \mu)}{1 - (g - 1) \mu} | x_j | > 2 \epsilon,
\]
where
\[
\eta = \begin{cases} 1 & \text{for OMP,} \\ \frac{1 - (g - 1) \mu}{(\mu + 1)(1 - (g - 1) \mu)} & \text{for OLS.} \end{cases}
\]
On the one hand, it can be seen that \( \eta \geq 1 \) for \( \mu < \frac{1}{g^2 k} \). On the other hand, \( \frac{1 - (g - 1) \mu}{(\mu + 1)(1 - (g - 1) \mu)} \geq 1 \) because \( \mu < \frac{1}{g^2 k} \leq \frac{1}{g - 1} \). As a consequence, (39) can be relaxed as
\[
1 - (2k - g - 1) \mu | x_j | > 2 \epsilon.
\]
Finally, since \( \mu < \frac{1}{g^2 k} \leq \frac{1}{g - 1} \) and \( | x_j | \geq | x_{j+1} | \), this condition can be relaxed as (18). Hence, the atom selected by Oxx at iteration \( g + 1 \) belongs to \( \mathcal{Q}^c \setminus \mathcal{Q} \) by virtue of Lemma 3.

**4 Conclusions**

In this paper, we derived new guarantees of success for OMP and OLS. In the noiseless setting, we showed that a sufficient condition of success for OMP and OLS reads \( \mu < \mu^* \) with \( \mu^* \in \left[ \frac{1}{2k - 1}, 1 \right] \) when the amplitude of the nonzero coefficients obey some decay, see (11). The condition thus weakens the traditional condition \( \mu < 1/(2k - 1) \) which applies to OMP and OLS in the uniform case, i.e., for any possible amplitudes.

The specific value of \( \mu^* \) is shown to be related to the rate of decay: the faster the decay, the larger is \( \mu^* \). In particular, our result emphasizes that \( \mu^* > \frac{1}{2k - 1} \) as soon as the sparse vector is not flat.

In the noisy case (for bounded-noise), we extended this result by showing that the constraint on the mutual coherence is a function of both the noise amplitude and the coefficient decay.

One of our results improve over the conditions proposed by Donoho et al. for OMP [17] when the sparse vector obey some decay.

Proving the (worst-case) necessity of the proposed conditions (namely, for a given decay and noise amplitude, is the proposed value of \( \mu^* \) the largest bound on the coherence ensuring the success of OMP and OLS?) is part of our perspectives.

Finally, we expect to generalize our results in the framework of sparse decaying signals to the case of compressible vectors, i.e., decaying representations that are not exactly sparse.
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