Two photon decays of scalar mesons in the quark NJL model

Yu. L. Kalinovsky
Laboratory of Information Technologies, Joint Institute for Nuclear Research, 141980 Dubna, Russia

M. K. Volkov
Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia

Two photon decays of scalar mesons \( f_0(980), a_0(980), \sigma(600) \) in the quark Nambu - Jona - Lasinio (NJL) model are calculated. The contributions of the meson loops are taken into account along with the quark loops (Hartree - Fock approximation). This corresponds to the next order of the \( 1/N_c \) expansion, where \( N_c = 3 \) is the number of quark colors. It is shown that the meson and quark loops give comparable contributions to the amplitude. Moreover, in the process \( f_0(980) \to \gamma \gamma \) the kaon loop plays the dominant role. A similar situation takes place in the decay \( \phi \to f_0(980)\gamma \) [1]. Our results are in satisfactory agreement with the recent experimental data.

I. INTRODUCTION

Experimental studies of two photon decays of scalar mesons play an important role in testing different theoretical models. In the last few years a number of experimental investigations have been devoted to this problem [2, 3, 4, 5].

There is a number of models which attempt to explain the inner structure of these scalar mesons and their interactions. One of them considers the scalar mesons as four - quark states[6, 7]. Another model interprets these mesons as an admixture of quark - antiquark and diquark - antidiquark states[8]. There is a model which describes the scalar states as kaon molecules[9]-[12].

Here we use the quark NJL model[13, 14]. This model allows us to successfully describe the low - energy hadron physics using the chiral symmetry of the strong interaction [14, 15, 16]. As a rule, in this model the Hartree - Fock approximation is used for consideration of the quark loops only. However, there are some processes, for instance, the radiative decays with participation of scalar mesons, where the meson loops can also play a very important role[1, 17]. Consideration of the meson loops implies allowance for the next order to the \( 1/N_c \) approximation in the NJL model. Recently, it has been shown that in the process \( \phi \to f_0(980)\gamma \) the kaon loop plays even the dominant role as compared to the quark loop[1]. In this case the kaon loop completely defines the decay width of this process in agreement with the experimental data.

Our paper is devoted to study of the two photon decays of the scalar mesons \( f_0(980), a_0(980) \) and \( \sigma(600) \). It is a natural continuation of the investigations fulfilled in [1]. Here we also show that in the decay \( f_0(980) \to \gamma \gamma \) the kaon loop again plays the dominant role.

The obtained results are in satisfactory agreement with the recent experimental data.

In the next section, we present part of the meson - quark Langrangian obtained from the quark NJL model [14]. The electromagnetic interaction is introduced by the standard method. We describe two photon decays of the scalar mesons \( f_0(980), a_0(980), \sigma(600) \) with the help of this Lagrangian.

In section III, we calculate two photon decays of scalar mesons taking into account the contributions of the quark and meson loops.

In conclusion, we shortly discuss the obtained results.

II. THE NJL MODEL.

The part of the Lagrangian which we need for the description of the two photon decays of the scalar mesons has the form[14]

*Electronic address: kalinov@jinr.ru
†Electronic address: volkov@theor.jinr.ru
\[ \mathcal{L} = q \left\{ i\partial^0 - m + eQA \right. \\
+ g_{\sigma}\sigma_u \sigma_u + g_{\sigma_s}\lambda_s \sigma_s + g_{\sigma_u}\lambda_\sigma a_0 + \\
+ i\gamma_5 \left( \lambda_\pi \pi^+ + \lambda_{\pi^-} \right) \\
+ g_K \left( \lambda_K K^+ + \lambda_{K^-} K^- \right) \right\} q. \]

Here \( q = (u, d, s) \) are the quark fields with the masses \( m = \text{diag}(m_u, m_d, m_s) \); \( m_u, m_d, m_s \) are the constituent quark masses \( (m_u = m_d) \); \( A_\mu \) are the photon fields, \( e \) is the electric charge, \( Q = (\lambda_3 + \lambda_8/\sqrt{3})/2 \). \( \pi^-, \pi^+, K^-, K^+ \) are the pseudoscalar mesons; \( a_0(980) \) is the isovector scalar meson with the mass 980 MeV; \( \sigma_u, \sigma_s \) are components of isoscalar scalar mesons \( \sigma(600), f_0(980) \). The state \( \sigma_u \) consists of light \( u \) and \( d \) quarks, and the state \( \sigma_s \) consists of \( s \) quarks only. The physical scalar mesons \( \sigma(600), f_0(980) \) are expressed in terms of \( \sigma_u, \sigma_s \) with the help of the angle \( \alpha = \theta_0 - \theta \). Here \( \theta_0 = 35.3^\circ \) and \( \theta = 24^\circ \) are the ideal and the real singlet - octet mixing angles \( u, d, s \).

\[ \sigma = \sigma_u \cos \alpha - \sigma_s \sin \alpha, \]
\[ f_0 = \sigma_u \sin \alpha + \sigma_s \cos \alpha. \]

The matrices \( \lambda_i \) are expressed in terms of the Gell - Mann matrices: \( \lambda_u = (\lambda_8 + \sqrt{2}\lambda_8)/\sqrt{3}, \lambda_s = (\lambda_3 + \sqrt{2}\lambda_8)/\sqrt{3}, \lambda_{\pi^\pm} = (\lambda_1 \mp i\lambda_2)/\sqrt{2}, \lambda_{K^\pm} = (\lambda_4 \mp i\lambda_5)/\sqrt{2}, \lambda_\sigma = \sqrt{2}/3 \). The coupling constants can be written in terms of the logarithmic divergent integral as \( (1/4) \)

\[ g_{\sigma_u} = (4\lambda^u(m_u, m_u))^{-1/2}, \]
\[ g_{\sigma_s} = (4\lambda^s(m_s, m_s))^{-1/2}, \]
\[ g_\pi = Z_\pi g_{\sigma_u}, \]
\[ g_K = Z_K (4\lambda^K(m_u, m_s))^{-1/2}, \]

where

\[ \lambda^p(m_1, m_2) = \frac{N_c}{(2\pi)^4} \int \frac{d^4k}{(k^2 + m_1^2)(k^2 + m_2^2)} \left[ \frac{\theta(\Lambda^2 - k^2)}{(k^2 + m_1^2)(k^2 + m_2^2)} \right] \]
\[ -m_1^2 \ln \left( \frac{\Lambda^2}{m_1^2} + 1 \right) / (m_2^2 - m_1^2). \]

All these integrals are presented in the Euclidean space. The constituent quark masses are \( m_u = m_d = 260 \) MeV, \( m_s = 410 \) MeV and the cutoff parameter \( \Lambda = 1.27 \) GeV \( [1, 14, 19] \). As a result, we obtain \( g_{\sigma_u} \approx 2.42, g_{\sigma_s} \approx 3.0 \); \( Z_\pi \) and \( Z_K \) are the factors which take into account the transitions of pseudoscalar mesons to axial - vector mesons. \( Z_\pi = (1 - \frac{6m_s^2}{M_{\pi^i}^2})^{-1/2}, Z_K = (1 - \frac{3m_u^2 + m_s^2}{2M_{K^i}^2})^{-1/2} \) \[ [14] \), where \( M_{\pi^i} = 1260 \) MeV, \( M_{K^i} = 1403 \) MeV are the masses of the axial - vector mesons \( [2] \). In our calculations we suppose \( Z_\pi \approx Z_K = Z = 1.2 \). As a result, we have \( g_\pi \approx 2.9, g_K \approx 3.3 \).

### III. TWO PHOTON DECAYS OF SCALAR MESONS.

Two photon decays of scalar mesons are described by the diagrams in Fig. 1 where the first diagram 1a. determines the contribution of the quark loops (Hartree - Fock approximation), and the other two (1b.,1c.) describe the contribution of the meson loops (next order of the 1/Nc expansion). The strong vertices in the first diagram are given in the Lagrangian \( \mathcal{L} \). The strong vertices in the meson loops are defined by the divergent parts of the quark loops (see eqs. 3 and 4). It is a standard method of the local NJL model \( [1, 14, 17] \).

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1 Let us note that in \[ [14] \) some other values of these parameters were used which corresponded to the weak pion decay constant \( F_\pi = 93 \) MeV and the width \( \Gamma_{\rho \rightarrow \pi \pi} = 155 \) MeV. Here we use modern experimental data \( [2] \) for fixing our model parameters: \( F_\pi = 92.4 \) MeV and \( \Gamma_{\rho \rightarrow \pi \pi} = 149.4 \) MeV.
Here we have used the Goldberger -Treiman relation for the constants of radiative meson decays with participation of pseudoscalar and vector mesons such as \((\pi, \eta, \eta') \rightarrow \gamma \gamma, (\rho, \omega) \rightarrow (\pi, \eta) \gamma, K^+ \rightarrow K \gamma, \eta' \rightarrow (p, \omega) \gamma, \phi \rightarrow (\eta, \eta') \gamma\) \((14,16)\).

The strong meson vertices describing the two photon decays of the scalar mesons (see Fig. 1) have the form

\[
T_S^{\mu \nu} = \frac{\alpha}{\pi F_\pi} (A_S^u + A_S^M) (g^{\mu \nu}(q_1 q_2) - q_1^\nu q_2^\mu),
\]  

where \(A_S^u\) is the quark loop contribution and \(A_S^M\) is the meson loop contribution, \(q_1, q_2\) are the photon momenta, \(\alpha = e^2/4\pi = 1/137\).

The contributions to the amplitude \(T_S^{\mu \nu}\) from the quark loops are calculated in detail in \((14,16)\). In the \(q^2\)-expansion over photon momenta they have the following form:

\[
A^u_{a_0(980)} = \frac{2}{3Z} = 0.48,
\]

\[
A^u_{a_0(600)} = \frac{10}{9Z} \cos \alpha = 0.8, \quad A^u_{a_0(600)} = \frac{2\sqrt{2} F_\pi}{9Z F_s} \sin \alpha = 0.24
\]

\[
A^u_{f_0(980)} = \frac{10}{9Z} \sin \alpha = 0.6, \quad A^u_{f_0(980)} = -\frac{2\sqrt{2} F_\pi}{9Z F_s} \cos \alpha = -0.35.
\]

Here we have used the Goldberger -Treiman relation for the constants \(g_{\sigma_u}\) and \(g_{\sigma_s}\): \(g_{\sigma_u} = g_{\sigma_s} = g_{\sigma}/Z = m_u/\langle ZF_\pi \rangle\), \(g_{\sigma_s} = m_s/\langle ZF_s \rangle\), where \(F_s = 1.28 F_\pi\) \((14,16)\).

This approximation allows us to obtain, in the framework of the local NJL model, the Wess - Zumino terms in the phenomenological chiral Lagrangian \((20)\). With help of these terms it is possible to succesfully describe main radiative meson decays with participation of pseudoscalar and vector mesons such as \((\pi^0, \eta, \eta') \rightarrow \gamma \gamma, (\rho, \omega) \rightarrow (\pi, \eta) \gamma, K^+ \rightarrow K \gamma, \eta' \rightarrow (p, \omega) \gamma, \phi \rightarrow (\eta, \eta') \gamma\) \((14,16)\).

The strong meson vertices describing the two photon decays of the scalar mesons (see Fig. 1) have the form

\[
G_{a_0(980)\pi^+ \pi^-} = 0, \quad G_{a_0(980)K^+ K^-} = 2(2m_u - m_s) \frac{q_K^2}{g_{\sigma_u}},
\]

\[
G_{f_0(980)\pi^+ \pi^-} = 4m_u \frac{g_2^2}{g_{\sigma_u}} \sin \alpha
\]

\[
G_{f_0(980)K^+ K^-} = 2 \left[ \sqrt{2}(2m_u - m_s) \frac{g_K^2}{g_{\sigma_u}} \cos \alpha - (2m_u - m_s) \frac{g_K^2}{g_{\sigma_u}} \sin \alpha \right]
\]

\[
G_{\sigma(600)\pi^+ \pi^-} = 4m_u \frac{g_2^2}{g_{\sigma_u}} \cos \alpha,
\]

\[
G_{\sigma(600)K^+ K^-} = 2 \left[ \sqrt{2}(2m_u - m_s) \frac{g_K^2}{g_{\sigma_u}} \sin \alpha + (2m_u - m_s) \frac{g_K^2}{g_{\sigma_u}} \cos \alpha \right]
\]

The sum of diagrams 1b. and 1c. leads to the converged integral having the gauge invariant form.

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\(^2\) Let us note that in \((17)\) some other method for calculation of the quark loops was used. Here we do not use this method, because it does not allow us to obtain the agreement with the experimental data for the above-mentioned radiative decays with pseudoscalar and vector mesons. The method in \((17)\) also has the problem with the quark confinement. Let us note that we use a different value for the singlet - octet mixing angle \(\theta\) obtained with the help of taking into account the t’Hooft interaction \((18)\).
where

\[ a = \sqrt{\frac{x}{1 + \sqrt{x - 1}}}, \quad x \leq 1 , \]

and \( x_1 = 4M_K^2/M_a^2 \). This leads to the numerical value \( A_{a^0}^{K^+} \approx -0.1 \).

For the total amplitude we have

\[ T_{a^0}^{\mu\nu} = \frac{\alpha}{\pi F_\pi} (0.48 - 0.1)(g_{\mu\nu}(q_1 q_2) - q_1^\nu q_2^\mu) = \frac{\alpha}{\pi F_\pi} 0.38(g_{\mu\nu}(q_1 q_2) - q_1^\nu q_2^\mu) \]

For the decay width we obtain

\[ \Gamma_{a^0 \to \gamma\gamma} = \frac{M_a^3}{64\pi} |T_{a^0 \to \gamma\gamma}|^2 = 0.39 \text{ keV}. \]

The experimental value is \( \Gamma(a_0(980) \to \gamma\gamma) = 0.3^{+0.11}_{-0.10} \text{ keV} \). For the decays \( f_0(980) \to \gamma\gamma \) and \( \sigma(600) \to \gamma\gamma \) the meson contributions to the amplitudes have the form

\[ A_{f_0(980)}^\pi = \frac{4 m_f \pi}{M_{f_0}} g_\pi^2 (x_2 \varphi(x_2) - 1) \sin \alpha = 0.07 + i0.18 \] \[ A_{f_0(980)}^K = 2\sqrt{2} \frac{(2m_g - m_h) F_\pi g_K^2}{M_{f_0}^2} (x_3 \varphi(x_3) - 1) \cos \alpha \]

\[ -2\frac{(2m_g - m_h) F_\pi g_K^2}{M_{f_0}^2} (x_3 \varphi(x_3) - 1) \sin \alpha = 0.536 \]

where \( x_2 = 4M_\pi^2/M_{f_0}^2, \quad x_3 = 4M_K^2/M_{f_0}^2 \). The total amplitude is

\[ T_{f_0}^{\mu\nu} = \frac{\alpha}{\pi F_\pi} (0.5 - 0.35 - (0.07 + i0.18) - 0.536)(g_{\mu\nu}(q_1 q_2) - q_1^\nu q_2^\mu) = \frac{\alpha}{\pi F_\pi} (-0.23 - i0.18)(g_{\mu\nu}(q_1 q_2) - q_1^\nu q_2^\mu) \]

Then for the decay width we obtain \( \Gamma(f_0 \to \gamma\gamma) = 0.19 \text{ keV} \). There are several experimental data for the decay \( \Gamma(f_0(980) \to \gamma\gamma) = 0.29^{+0.07}_{-0.09} \text{ keV} \), \( \Gamma(f_0(980) \to \gamma\gamma) = 0.205^{+0.083}_{-0.117} \text{ keV} \), \( \Gamma(f_0(980) \to \gamma\gamma) = 0.31 \pm 0.14 \pm 0.09 \text{ keV} \), \( \Gamma(f_0(980) \to \gamma\gamma) = 0.29 \pm 0.07 \pm 0.12 \text{ keV} \). In other papers the following results were obtained: \( \Gamma(f_0(980) \to \gamma\gamma) = 0.29^{+0.07}_{-0.09} \text{ keV} \), \( \Gamma(f_0(980) \to \gamma\gamma) = 0.24 \text{ keV} \), \( \Gamma(f_0(980) \to \gamma\gamma) = 0.28^{+0.09}_{-0.13} \text{ keV} \), \( \Gamma(f_0(980) \to \gamma\gamma) = 0.31 \text{ keV} \), \( \Gamma(f_0(980) \to \gamma\gamma) = 0.33 \text{ keV} \), \( \Gamma(f_0(980) \to \gamma\gamma) = 0.27 \text{ keV} \), \( \Gamma(f_0(980) \to \gamma\gamma) = 0.20 \text{ keV} \), \( \Gamma(f_0(980) \to \gamma\gamma) = 0.22 \pm 0.07 \text{ keV} \), \( \Gamma(f_0(980) \to \gamma\gamma) = 0.21 - 0.26 \text{ keV} \).

For the decay amplitude \( \sigma(600) \to \gamma\gamma \) we have

\[ A_{\pi(600)}^\pi = \frac{4 m_{\pi} F_{\pi}}{M_\pi^2} g_{\pi}^2 (x_3 \varphi(x) - 1) \cos \alpha = -0.83 + i0.83 \]

where \( x = 4M_\pi^2/M_\pi \).

\[ A_{\sigma(600)}^K = 2\sqrt{2} \frac{(2m_g - m_h) F_\pi g_K^2}{M_{\sigma}^2} (x_4 \varphi(x_4) - 1) \sin \alpha \]

\[ +2\frac{(2m_g - m_h) F_\pi g_K^2}{M_{\sigma}^2} (x_4 \varphi(x_4) - 1) \cos \alpha \]
Keeping together all contributions to the amplitudes we have the following results: $A_{f_0}^{u,s,\pi^+}K^+ = 0.22$, $A_{f_0}^{s,\pi^+}K^+ = 1.6 \pm 0.8$ and the corresponding widths are $\Gamma(\sigma \to \gamma\gamma) = 1.03$ keV.

Unfortunately, for the decay $\Gamma(\sigma \to \gamma\gamma)$ there are no reliable experimental data. In other theoretical models the following estimations were obtained: $\Gamma(\sigma \to \gamma\gamma) = 0.45$ keV [7], $\Gamma(\sigma \to \gamma\gamma) < 1$ keV if $M_\sigma < 0.7 - 0.8$ MeV [28].

We see that our results do not contradict the recent experimental data. Let us note that if we use a smaller mass of $\sigma$ meson, we will obtain the result close to the prediction [7, 28].

IV. CONCLUSION

As a result, we have shown that in the two-photon decays of $f_0(980), a_0(980), \sigma(600)$ the quark and meson loops give comparable contributions to the amplitude of these processes.

In the decay $f_0(980) \to \gamma\gamma$ the kaon loops play even the dominant role. Indeed, in this process the contributions from the $u(d)$ and $s$ quark loops noticeably cancel each other. Therefore, the contributions of meson loops became dominant. Some other reasons lead to a similar result in the process $\phi \to f_0(980)\gamma$ [1].

On the other hand, in the processes $(a_0(980), \sigma(600)) \to \gamma\gamma$ the contributions from the quark and meson loops have the same order. However, the main contribution to these amplitudes is connected with the quark loop. Our results concerning the processes $f_0(980) \to \gamma\gamma$ and $\phi \to f_0(980)\gamma$ do not contradict the theoretical predictions obtained in other models where the full contributions were defined by the kaon loops.

Let us note that the amplitudes describing the radiative decays of scalar mesons contain two different parts: the first is connected with the quark intermediate state (quark loop); and the second, with the four-quark intermediate state (meson loop). Therefore, these amplitudes can be considered as a mixing of the two quark and four quark intermediate hidden states.

On the other hand, the radiative decays with pseudoscalar mesons (see section 3) are totally defined by the quark loops only. This fact confirms the $(\bar{q}q)$ structure of the pseudoscalar mesons.

In conclusion, we would like to emphasize that in the framework of our model we did not use any additional parameters for the description of radiative decays with participation of scalar mesons.

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