Correlator of the quark scalar currents and 
$\Gamma_{\text{tot}}(H \rightarrow \text{hadrons})$ at $O(\alpha_s^3)$ in pQCD

K.G. Chetyrkin $^{a,b}$,

$^a$ Institute for Nuclear Research, Russian Academy of Sciences, 60th October Anniversary Prospect 7a Moscow 117312, Russia

$^b$ Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, Föhringer Ring 6, 80805 Munich, Germany

Abstract

We present the results of the analytical evaluation of the massless four-loop $O(\alpha_s^3)$ correction to the correlator of the quark scalar currents and the Higgs decay rate into hadrons. In numerical form we found (in $\overline{\text{MS}}$ scheme) that 

$$\Gamma(H \rightarrow \bar{b}b) = \frac{3G_F}{4\sqrt{2}\pi} M_H m_b^2(M_H) \left[ 1 + \frac{5.667}{\pi} \alpha_s + \left( 35.94 - 1.359 n_f \right) \left( \frac{\alpha_s}{\pi} \right)^2 
+ \left( 164.139 - 25.771 n_f + 0.259 n_f^2 \right) \left( \frac{\alpha_s}{\pi} \right)^3 \right]$$

where $\alpha_s = \alpha_s(M_H)$.

e-mail: chet@mppmu.mpg.de
1 Introduction

In the Standard Model (SM) one physical scalar Higgs boson is present as a remnant of the mechanism of mass generation. Particularly interesting for the observation of the Higgs boson with an intermediate mass $M_H < 2M_W$ is the dominant decay channel into a bottom pair $H \rightarrow b\bar{b}$. (Standard Model properties of the Higgs boson have been discussed in many reviews; see for example [1, 2]). The partial width $\Gamma(H \rightarrow b\bar{b})$ is significantly affected by QCD radiative corrections. First order $\alpha_s$ corrections including the full $m_b$ dependence were studied by several groups [3, 4, 5, 6, 7]. Second order corrections were calculated in the limit $m_b^2 \ll M_H^2$. Apart from the trivial overall factor $m_b^2$ due to the Yukawa coupling, corrections were obtained for otherwise massless quarks in Refs. [8] and for a nonvanishing mass of the virtual top quark in Ref. [9] (both results are confirmed in Ref. [10]). Subleading corrections in the $m_b^2/M_H^2$ expansion were found in Ref. [11] and also confirmed in Ref. [12]. In the latter work an additional quasi-massless (and numerically important) contributions of order $\alpha_s^2$ have been identified and elaborated. They come from so-called singlet diagrams with a non-decoupling top quark loop inside (similar effects for the decay of the Z-boson to quarks were first discovered in [12]). The results of Ref. [13] have been confirmed and extended by computing extra terms in the expansion in $M_H^2/m_t^2$ in Ref. [13].

In this work we compute the next-next-to-leading massless correction of order $\alpha_s^3$ to $\Gamma(H \rightarrow \text{hadrons})$.

2 Preliminaries

We start with considering the two-point correlators

$$\Pi_{ff'}^S(Q^2) = (4\pi)^2 i \int dx e^{ix} \langle 0 | T[J_S^f(x)J_S^{f'}(0)] | 0 \rangle. \quad (1)$$

Here $Q^2 = -q^2$, $J_S^f = \bar{\Psi}_f \Psi_f$ is the scalar current for quarks with flavour $f$ and mass $m_f$, which are coupled to the scalar Higgs bosons. The total hadronic decay rate of a scalar Higgs boson is naturally expressed in terms of the absorptive part of the correlator (1)

$$R_{ff'}^S(s) = \frac{1}{2\pi i} \text{Im} \Pi_{ff'}^S(-s - i\epsilon),$$

as follows [3, 4, 5]:

$$\Gamma(H \rightarrow \text{hadrons}) = \frac{G_F}{4\sqrt{2}\pi} M_H \sum_{ff'} m_f m_{f'} R_{ff'}^S(s = M_H^2). \quad (2)$$

Assuming that the Higgs boson mass $M_H$ is less then $2m_t$ and much larger than $m_b$, we shall treat correlators (1) within the massless five-quark QCD. (In fact, even in the limit of heavy top the Higgs boson coupling with the current $J_i^S$ still contributes to $\Gamma_H$; we shall ignore these effects in our work. For a recent discussion see e.g. Ref. [14].) Under such an assumption the nondiagonal correlators $\Pi_{ff'}^S$ with $f \neq f'$ vanish identically while the diagonal correlators get flavour independent, that is

$$\Pi_{ff}^S(Q^2) \equiv \Pi^S(Q^2) \quad \text{and} \quad R_{ff}^S(s) \equiv R^S(s).$$
The renormalization mode of the polarization operator $\Pi^S(Q^2)$ reads (see, e.g. Ref. [13])

$$\Pi^S(Q^2, a_s, \mu^2) = Z_{qs}^S Q^2 + Z_m^S(2, a_s^0),$$

(3)

where $a_s = \alpha_s/\pi = g^2/(4\pi^2)$, $g$ is the strong coupling constant; $Z_m$ is the quark mass renormalization constant. Within the $\overline{\text{MS}}$ scheme [10]

$$Z_{qs}^S = \sum_{1\leq j \leq i} \left( Z_{qs}^{S}\right)_{ij} \frac{\alpha_s^{i-j}}{\epsilon^j}$$

(4)

and the coefficients $(Z_{qs}^{S})_{ij}$ are just numbers, with $D = 4 - 2\epsilon$ standing for the space-time dimension. As a result we arrive at the following renormalization group (RG) equation for the polarization operator $\Pi^S(Q^2)$

$$\left( \mu^2 \frac{\partial}{\partial \mu^2} + a_s \beta(a_s) \frac{\partial}{\partial a_s} + 2\gamma_m(a_s) \right) \Pi^S = \gamma_q^{SS}(a_s)$$

(5)

or, equivalently, $(L_Q = \ln \frac{\mu^2}{Q^2})$

$$\frac{\partial}{\partial L_Q} \Pi^S = \gamma_q^{SS}(a_s) - \left( 2\gamma_m(a_s) + \beta(a_s) a_s \frac{\partial}{\partial a_s} \right) \Pi^S.$$  

(6)

Here the anomalous dimensions $\gamma_q^{SS}(a_s)$ and $\gamma_m(a_s)$, and the $\beta$-function $\beta(a_s)$ are defined in the usual way

$$\gamma_q^{SS} = \mu^2 \frac{d}{d\mu^2} (Z_{qs}^S) - \epsilon Z_{qs}^{SS} = - \sum_{i \geq 0} (i+1)(Z_{qs}^{SS})_{i0} a_s^i,$$

(7)

$$\mu^2 \frac{d}{d\mu^2} a_s = \alpha_s \beta(a_s) \equiv - \sum_{i \geq 0} \beta_i a_s^{i+1}, \quad \mu^2 \frac{d}{d\mu^2} m(\mu) = \bar{m}(\mu) \gamma_m(a_s) \equiv - \bar{m} \sum_{i \geq 0} \gamma_m^i (a_s)^{i+1}.$$  

(8)

The relation (1) demonstrates explicitly the main computational advantage of finding at first the polarization function $\Pi^S(Q^2)$ against a direct calculation of $R^S(s)$ in the case of massless pQCD. Indeed, in order $a_s^3$ the derivative $\frac{\partial}{\partial L_Q} \Pi^S$ and, consequently, $R^S(s)$ depends on the very function $\Pi^S$ which is multiplied by at least one factor of $a_s$. Thus, one needs only to know $\Pi^S$ to order $a_s^2$ and the anomalous dimension $\gamma_q^{SS}(a_s)$ to order $a_s^3$ to find all $Q$-dependent terms in $\Pi^S$ at $\mathcal{O}(a_s^3)$, since the beta function and the quark mass anomalous dimension $\gamma_m$ are reliably known to order $a_s^3$ from Refs. [17-20].

As we shall see later both problems are eventually reducible to calculation of a specific class of diagrams representing some massless Feynman integrals (FI) depending on only one external momentum (to be named $p$-integrals below) and with the number of loops not exceeding three. Note that this is in an obvious disagreement with a statement from Ref. [13] about the inevitable necessity to compute finite parts of four-loop $p$-integrals in order to find the $\mathcal{O}(a_s^3)$ contribution to $R^S(s)$. Yet, we agree with the author of Ref. [13] that if such a point were correct it would certainly preclude, at least at the present state of the art, any possibility of analytical calculation of $R^S(s)$ to order $a_s^3$. 

2
3 Calculation of $\Pi^S$

In order $\alpha_s^2$ the polarization operator $\Pi^S$ is contributed by twelve three-loop p-integrals. Such a calculation is now to be considered as almost trivial one because of three facts:

\textbf{a} There is an elaborated algorithm which provides a way to evaluate analytically divergent as well as finite parts of any three-loop dimensionally regulated p-integral [21, 22].

\textbf{b} The algorithm is reliably implemented in the language of FORM [23] as the package named MINCER in Ref. [24].

\textbf{c} The package has been extensively tested and a chance of a bug in it seems to be very small.

We have computed the $\Pi^S_0$ to $\alpha_s^2$ using the general gauge with the gluon propagator $(g_{\mu\nu} - \xi q_{\mu} q_{\nu})/q^2$. On performing the renormalization according to eq. (3) we have obtained the following result:

$$\Pi^{SS}(Q^2) = d[R]Q^2 \left\{ -4 - 2 \ln \frac{\mu^2}{Q^2} + a_s C_F \left[ -\frac{131}{8} + 6 \zeta(3) - \frac{17}{2} \ln \frac{\mu^2}{Q^2} - \frac{3}{2} \ln \frac{\mu^2}{Q^2} \right] ight.$$ 

$$+ a_s^2 \left[ C_F^2 \left( -\frac{1613}{64} + 24 \zeta(3) - \frac{9}{4} \zeta(4) - 15 \zeta(5) - \frac{691}{32} \ln \frac{\mu^2}{Q^2} \right. 
\left. + \frac{9}{2} \zeta(3) \ln \frac{\mu^2}{Q^2} - \frac{105}{16} \ln \frac{\mu^2}{Q^2} - \frac{3}{4} \ln^3 \frac{\mu^2}{Q^2} \right) 
+ C_F C_A \left( -\frac{14419}{288} + \frac{75}{4} \zeta(3) + \frac{9}{8} \zeta(4) + \frac{5}{2} \zeta(5) - \frac{893}{32} \ln \frac{\mu^2}{Q^2} \right. 
\left. + \frac{31}{4} \zeta(3) \ln \frac{\mu^2}{Q^2} - \frac{71}{12} \ln \frac{\mu^2}{Q^2} - \frac{11}{24} \ln^3 \frac{\mu^2}{Q^2} \right) 
+ C_F T n_f \left( \frac{511}{36} - 4 \zeta(3) - \frac{65}{8} \ln \frac{\mu^2}{Q^2} - 2 \zeta(3) \ln \frac{\mu^2}{Q^2} + \frac{11}{6} \ln \frac{\mu^2}{Q^2} + \frac{1}{6} \ln^3 \frac{\mu^2}{Q^2} \right) \right\}.$$ 

Here $a_s = \alpha_s(\mu)/\pi$, $C_A$ and $C_F$ are the Casimir operators of the adjoint and quark (defining) representations of the colour group; $T$ is the normalization of the trace of generators of quark representation $Tr(t^a t^b) = T \delta^{ab}$; $n_f$ is the number of quark flavours; $d[R]$ is the dimension of the quark representation of the colour group. Note, that all the $Q$-dependent terms in (9) are in agreement with the results of [8]. The independence of (9) from the gauge parameter $\xi$ is of course expected on general grounds.

4 Calculation of the anomalous dimension $\gamma_{q}^{SS}$

There is about a hundred of diagrams contributing to $\Pi^S$ in order $\alpha_s^3$. According to (3), to compute $\gamma_{q}^{SS}$ one should determine the UV counterterms (that is essentially divergent parts) of all the corresponding four-loop p-integrals. Unfortunately, at present there exists no way to compute directly the divergent part of a generic four-loop p-integral.
The only available (unfortunately rather involved) approach to perform such calculations analytically is to use the method of Infrared Rearrangement (IRR) discovered by A. Vladimirov in Ref. \[23\] (see also Refs. \[20\]). The method simplifies calculation of UV counterterms by the effective use of the following important theorem: in a class of minimal renormalization schemes (including \(\overline{\text{MS}}\)- and MS-ones) any UV counterterm has to be polynomial in external momenta and masses \[27\]. It amounts to an appropriate transformation of the IR structure of FI’s by expanding the latter in a formal Taylor series with respect to some external momenta and masses, with resulting FI’s being much simpler to calculate.

The method of IRR was significantly extended with elaborating a so-called \(R^*\)-operation in Refs. \[28, 29, 30\]. By an explicit construction of the corresponding algorithm, it has been shown in Ref. \[29\] how to reduce calculation of the UV counterterm for an arbitrary (h+1)-loop FI to evaluation of divergent and finite parts of some appropriately chosen h-loop p-integrals.

It should be stressed that the \(R^*\)-operation is absolutely essential for the algorithm to work in general case, though in most (but not in all) practical cases one could proceed without it. However, such a practice forces the use of a diagram-wise renormalization procedure; the latter, being very difficult to perform by a computer, implies a huge amount of highly error-prone and time-consuming manipulations with hundreds of diagrams.

For instance, the only QCD four-loop problem performed by now is the evaluation of the ratio \(R(s) = \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}\) to order \(\alpha_s^3\). It was done in Refs. \[31, 32\] within the diagram-wise renormalization approach. The core of the problem is the calculation of the four-loop contribution to the photon anomalous dimension entering into the RG equation for the photon polarization operator. The initial 98 four-loop diagrams contributing to the photon polarization operator proliferate to about 250 after the IRR procedure is applied. In addition these diagrams contain about 600 various subdiagrams which should be computed separately in order to subtract UV subdivergences.

Technically, the calculation of \(\gamma_q^{SS}\) is obviously very similar to that of the photon anomalous dimension. Consequently, any attempt of straightforward repetition of those calculations for the case of the scalar correlator would mean a few man-years of routine and boring work and, thus, would be not acceptable for the present author.

Below we will use, instead, the power of the \(R^*\)-operation and simplify the application of IRR to the problem so far that both UV and IR renormalizations can be done in a global form and, consequently, can be simply performed by computer.

We begin with from the Dayson–Schwinger equation for the correlator (\[1\]) written in the bare form\[4\]

\[
\Pi^S(q, a_s^0) = - \int dp \frac{(4\pi)^2}{(2\pi)^D} Tr[G^0(p + q, a_s^0)\Gamma^0_S(p, q, a_s^0)G^0(p, a_s^0)].
\]

Here \(G^0\) and \(\Gamma^0_S\) are the full quark propagator and the scalar current vertex function respectively; below the integration with respect to the loop momentum \(p\) with the weight function \(\frac{(4\pi)^2}{(2\pi)^D}\) will be only understood but not explicitly displayed.

\[4\]For simplicity we set the 't Hooft-Veltman unit of mass \(\mu\) equal to 1 below.
The renormalized version of (10) reads
\[ \Pi^S(q, a_s) = Z^S_q Q^2 - \frac{Z^S_q}{Z^2} Tr[G^0(p + q, a_s^0) \Gamma^0_S(p, q, a_s^0) G^0(p, a_s^0)]. \] (11)

Here \( Z_2 \) is the quark wave function renormalization constant; \( Z_S \) is the renormalization constant of the scalar quark current defined as
\[ \left[ \bar{\psi}\psi \right] = \frac{Z_S}{Z_2} \bar{\psi}_0 \psi_0, \]
where the current inside squared brackets is the renormalized one. A well-known equality \( Z_S = Z_m Z_2 \) implies the equivalence of (3) and (11).

From the finiteness of the renormalized correlator one gets
\[ Z^S_{SS} q = -K_\epsilon \left\{ \frac{1}{2D} \left( \frac{Z_S}{Z^2_2} \Box_q Tr[G^0(p + q, a_s^0) \Gamma^0_S(p, q, a_s^0) \tilde{G}^0(p, a_s^0)] \right) \right\} \] (12)
where \( K_\epsilon f(\epsilon) \) stands for the singular part of the Laurent expansion of \( f(\epsilon) \) in \( \epsilon \to 0 \) and \( \delta Z_S = Z_S - 1 \). In Eq. (12) we have let a Dalambertian with respect to the external momentum \( q \) act on quadratically divergent diagrams to transform them to the logarithmically divergent ones. We also have introduced an auxiliary mass dependence to a quark propagator — the one entering into the “left” current \( J_S \) — by making the following replacement
\[ G^0(p, a_s^0) \to \tilde{G}^0(p, a_s^0) \frac{p^2}{(p^2 - m_0^2)}. \] (13)

Note, please, that the auxiliary mass dependence has caused somewhat more complicated structure of UV renormalizations in the right hand side of Eq. (12).

The idea of the method of IRR is quite simple: since the renormalization constant \( Z^S_q \) does not depend on anything dimensionful one could significantly simplify its calculation by nullifying the momentum \( q \) in Eq. (12). The only requirement which must be respected is the absence of any IR singularities in the resulting integrals. Unfortunately, a mass introduced to a propagator distinguished by some topological property like the one we created above is not always sufficient to suppress all IR divergences in all diagrams. For instance, if \( q = 0 \) then there appear completely massless tadpoles in the second term on the rhs of Eq. (12). Thus, the eq. (12) with \( q = 0 \) is not valid unless the unwanted IR poles are all identified and subtracted away.

This is certainly the job the \( R^* \)-operation was created for! The rules of Ref. [29] spell how to do it on the diagram-wise level. The only remaining problem is to disentangle the relevant combinatorics and put down the IR subtractions in a global form. The task is facilitated by the fact that, as shown in Ref. [30] the IR counterterms for an arbitrary diagram can be determined in terms of some properly chosen combination of the UV ones. The final formula incorporating all necessary UV and IR subtractions reads
\[ Z^S_q = -K_\epsilon \left\{ \frac{1}{2D} \left( \frac{Z_S}{Z^2_2} \Box_q Tr[G^0(p + q, a_s^0) \Gamma^0_S(p, q, a_s^0) \tilde{G}^0(p, a_s^0)] \right) \right\} q = 0 \]
\[ - \frac{Z_S}{Z^2_2} \frac{1}{4} Tr[\delta \Gamma^0_S(0, 0, a_s^0)] \frac{Z^S_{SS}}{Z^2_m} - \frac{\delta Z_S Z_S Z^S_{SS}}{Z^2_2 Z^2_m} \right\}. \] (14)
Here, by $\delta \Gamma_0^S(p,q,a_0^q)$ we denote the vertex function of the scalar current with the tree contribution removed. The “tilde” atop $S$ again means that in every diagram the quark propagator entering to the vertex $J^S$ is softened at small momenta by means of the auxiliary mass $m_0$ according to Eq. (13). The bare coupling constant $a_0^q$ is to be understood as $a_s = Z_a a_s$, with $Z_a$ being the coupling constant renormalization constant. To our accuracy in $a_s$

$$Z_a = 1 + a_s \left( -\beta_0/\epsilon \right) + a_s^2 \left( \beta_0^2/\epsilon^2 - \beta_1/(2\epsilon) \right).$$

Finally, an inspection of (14) immediately shows that, in order to find the $(n+1)$-loop correction to $Z_{qSS}$, one needs only to know the renormalization constants $Z_{SS}$, $Z_2$ and $Z_m$ to order $a_s^n$ as well as the bare Green functions

$$G^0(p,a_0^q), \quad \left. \frac{\partial}{\partial q} [\Gamma_0^S(p,q,a_0^q)] \right|_{q=0} = 0, \quad \left. \Box_q [\Gamma_0^S(p,q,a_0^q)] \right|_{q=0} = 0, \quad \delta \Gamma_0^S(0,0,a_0^q) \quad (15)$$

up to (and including) $n$-loops. Thus, we have obtained a general formula for $Z_{qSS}$ in terms of bare $p$-integrals with explicitly resolved UV and IR subtractions.

5 Results and discussion

We have computed with the program MINCER [24] the unrenormalized three-loop Green functions (14) as well as the renormalization constants $Z_m$ and $Z_2$ to order $a_3^q$. The calculations have been performed in the general covariant gauge. The total calculational time with an IBM workstation is about 40 hours for the general gauge; for the Feynman one it is reduced to about 4 hours.

Then we have used Eqs. (14) and (7) to find $\gamma_{qSS}$ to order $a_3^q$. Our results for $\gamma_{qSS}$ and $R^S(s)$ read

$$\gamma_{qSS} = d[R] \left\{ -2 + a_s C_F \left[ -\frac{5}{2} \right] + a_s^2 \left[ C_F^2 \left( \frac{119}{32} - \frac{9}{2} \zeta(3) \right) + C_F C_A \left( -\frac{77}{16} + \frac{9}{4} \zeta(3) \right) + C_F T n_f \right] + a_s^3 \left[ C_F^3 \left( \frac{4651}{384} - \frac{29}{4} \zeta(3) + \frac{27}{8} \zeta(4) + \frac{45}{4} \zeta(5) \right) + C_F^2 C_A \left( \frac{641}{48} - \frac{259}{16} \zeta(3) + \frac{39}{16} \zeta(4) + \frac{45}{8} \zeta(5) \right) + C_F C_A^2 \left( \frac{267889}{31104} + \frac{475}{48} \zeta(3) - \frac{33}{16} \zeta(4) - \frac{45}{8} \zeta(5) \right) + C_F^2 T n_f \left( \frac{125}{32} - \frac{1}{2} \zeta(3) - 3 \zeta(4) \right) + C_F C_A T n_f \left( \frac{631}{1776} + \frac{5}{3} \zeta(3) + \frac{9}{4} \zeta(4) \right) + C_F T^2 n_f^2 \left( \frac{1625}{1944} - \frac{2}{3} \zeta(3) \right) \right] \right\} \quad (16).$$
and

\[ R^8(s, \mu) = d[R] \left\{ 1 + a_s(\mu) \left[ s_1 + 2 \gamma_m^0 \ln \frac{\mu^2}{\mu^2} \right] + a_s^2(\mu) \left[ 6 \beta_0 + 2 \gamma_m^0 + 2 \gamma_m^1 + \ln \frac{\mu^2}{\mu^2} (\beta_0^0 + 2(\gamma_m^0)^2) \right] + a_s^3(\mu) \left[ 2 s_3 + \ln \frac{\mu^2}{\mu^2} (s_3 (\beta_0^0 + 2 s_1 \gamma_m^0 + 2 s_1 \gamma_m^1) + \ln \frac{\mu^2}{\mu^2} (\beta_0 \gamma_m^0 + 2(\gamma_m^0)^2)) \right] \right\} \]

Here the coefficients of the beta-function \( \beta_0, \beta_1 \) and the quark mass anomalous dimension \( \gamma_m^i \), \( i = 0, 1, 2 \) are defined according to (17) and read

\[ \gamma_m^0 = \frac{1}{4} [3 C_F], \quad \gamma_m^1 = \frac{1}{16} \left[ \frac{3}{2} C_F - \frac{97}{6} C_F C_A - \frac{10}{3} C_F T_{nf} \right], \]

\[ \gamma_m^2 = \frac{1}{64} \left[ \frac{129}{2} C_F^2 - \frac{129}{4} C_F C_A + \frac{11413}{108} C_F C_A^2 \right], \]

\[ \quad + C_F^2 T_{nf} (48 \zeta(3) - 46) + C_F C_A T_{nf} \left( -48 \zeta(3) - \frac{556}{27} \right) - \frac{140}{27} C_F T^2 n_f^2 \]

\[ \beta_0 = \frac{1}{4} \left[ \frac{11}{3} C_A - \frac{4}{3} T_{nf} \right], \quad \beta_1 = \frac{1}{16} \left[ \frac{34}{3} C_A - 4 C_F T_{nf} - \frac{20}{3} C_A T_{nf} \right]. \]

At last, the coefficients \( s_1, s_2 \) and \( s_3 \) are found to be

\[ s_1 = C_F \left[ \frac{17}{4} \right], \quad s_2 = C_F^2 \left[ \frac{691}{64} - \frac{3}{8} \pi^2 - \frac{9}{4} \zeta(3) \right] + C_F C_A \left[ \frac{893}{64} - \frac{11}{48} \pi^2 - \frac{31}{8} \zeta(3) \right] + C_F T_{nf} \left[ -\frac{65}{16} + \frac{1}{12} \pi^2 + \zeta(3) \right], \]

\[ s_3 = \left[ \frac{23443}{768} - \frac{27}{16} \pi^2 - \frac{239}{16} \zeta(3) + \frac{45}{8} \zeta(5) \right] \]

\[ + C_F \left[ \frac{13153}{192} - \frac{383}{96} \pi^2 - \frac{1089}{32} \zeta(3) + \frac{145}{16} \zeta(5) \right] \]

\[ + C_F C_A \left[ \frac{3894493}{62208} - \frac{1715}{96} \pi^2 - \frac{2329}{96} \zeta(3) + \frac{25}{48} \zeta(5) \right] \]

\[ + C_F^2 T_{nf} \left[ -\frac{88}{3} + \frac{65}{48} \pi^2 + \frac{65}{4} \zeta(3) + \frac{3}{4} \zeta(4) - 5 \zeta(5) \right] \]

\[ + C_F C_A T_{nf} \left[ -\frac{33475}{972} + \frac{571}{432} \pi^2 + \frac{22}{3} \zeta(3) - \frac{3}{4} \zeta(4) + \frac{5}{6} \zeta(5) \right] \]

\[ + C_F T^2 n_f^2 \left[ \frac{15511}{3888} - \frac{11}{54} \pi^2 - \zeta(3) \right]. \]

We observe that neither \( \gamma_q^S \) nor \( R^8(s) \) depend on the gauge fixing parameter \( \xi \) as it must be. For the standard QCD colour group values \( d[R] = 3, C_F = 4/3, C_A = 3 \) and \( T = 1/2 \).
we get for $R_S(s)$ with $\mu^2$ set to $s$:

$$
R_S(s) = 3 \left\{ 1 + a_s \left[ \frac{17}{3} \right] + a_s^2 \left[ \frac{10801}{144} - \frac{19}{12} \pi^2 - \frac{39}{2} \zeta(3) - \frac{65}{24} n_f + \frac{1}{18} \pi^2 n_f + \frac{2}{3} \zeta(3) n_f \right] \\
+ a_s^3 \left[ \frac{6163613}{5184} - \frac{3535}{72} \pi^2 - \frac{109735}{216} \zeta(3) + \frac{815}{12} \zeta(5) - \frac{46147}{486} n_f + \frac{277}{72} \pi^2 n_f \right] \\
+ \frac{262}{9} \zeta(3) n_f - \frac{5}{6} \zeta(4) n_f - \frac{25}{9} \zeta(5) n_f + \frac{15511}{11664} n_f^2 - \frac{11}{162} \pi^2 n_f^2 - \frac{1}{3} \zeta(3) n_f^2 \right\}, 
$$

(21)

or, in the numerical form,

$$
R_S(s) = 3 \left\{ 1 + 5.66667 \frac{\alpha_s(s)}{\pi} + (35.93996 - 1.35865 n_f) \left( \frac{\alpha_s(s)}{\pi} \right)^2 \\
+ \left( 164.13921 - 25.77119 n_f + 0.258974 n_f^2 \right) \left( \frac{\alpha_s(s)}{\pi} \right)^3 \right\}. 
$$

(22)

At last, for the phenomenologically relevant case of $n_f = 5$ we obtain

$$
R_S(s) = 3 \left\{ 1 + 5.66667 \frac{\alpha_s(s)}{\pi} + 29.1467 \left( \frac{\alpha_s(s)}{\pi} \right)^2 + 41.7576 \left( \frac{\alpha_s(s)}{\pi} \right)^3 \right\}. 
$$

(23)

Due to eq. (2) the combination $m_b^2 R_S(s = M_H^2)$ is directly related to the Higgs decay rate to the $b\bar{b}$ pairs plus gluons. The corresponding expression including power suppressed corrections reads

$$
\Gamma(H \rightarrow b\bar{b}) = \frac{3 G_F}{4 \sqrt{2} \pi} M_H m_b^2(M_H) \left[ 1 + 5.67 \alpha_s(M_H) + 29.15 a_s^2(M_H) + 41.76 a_s^3(M_H) \right] \\
+ \frac{m_b^2(M_H)}{M_H^2} \left[ -6 - 40 \alpha_s(M_H) - 87.72 a_s^2(M_H) \right]. 
$$

(24)

Let us take as input parameters a $\Lambda_{QCD}^{(5)} = 233$ MeV and a bottom pole mass of $m_b^{\text{pole}} = 4.7$ GeV. The latter translates into the running mass $m_b(M_H^2) = 2.84/2.75/2.69$ GeV for Higgs masses of $M_H = 70/100/130$ GeV. All other light quarks are assumed to be massless. One arrives at the following values for the strong coupling constant: $\alpha_s(M_H^2) = 0.125/0.118/0.114$ corresponding to the three different values of $M_H$. The corresponding numerical values for different contributions to $R_S(M_H^2)$ are given in Table 1.

To summarize: we have suggested a new convenient way to compute the UV renormalization constant of the correlator of the scalar quark currents. Our final formula (14) directly
expresses the constant in terms of unrenormalized p-integrals, with all UV and IR subtractions being implemented in a global form. The formula is ideally suited for carrying out completely automatic calculations and can easily be extended for the case of general bilinear quark currents. Detailed derivation of the formula will be presented elsewhere. Using the formula and the FORM version of the MINCER we have computed the \( \mathcal{O}(\alpha_s^3) \) correction to the anomalous dimension \( \gamma_{SS}^{q} \), to the absorptive part of the scalar correlator \( R_S(s) \) and to \( \Gamma(H \rightarrow \text{hadrons}) \) in pQCD. All the calculations have been performed with the use of the general covariant gauge. The gauge independence of the results constitutes a strong check of the correctness of our approach.

Numerically, the correction of order \( \alpha_s^3 \) to \( \Gamma(H \rightarrow \text{hadrons}) \) proves to be relatively small. However, for the Higgs boson with intermediate mass it is more important than the power suppressed \( \mathcal{O}(\frac{m_b^2}{M_H^2} \alpha_s^2) \) contribution. (see Table 1).

### Acknowledgments

I would like to thank J. Kühn, B. Kniehl, S. Larin and M. Steinhauser for stimulating and useful discussions. I am specially grateful to S. Larin and J. Vermassen for providing me with the FORM package MINCER as well useful advice about its features. I would like to thank S. Jadach for the occasion to present the results of this paper at the Cracow International Symposium on Radiative Corrections, 1-5 August 1996.

I am deeply thankful to the Institute of Theoretical Particle Physics of the Karlsruhe University and the theoretical group of the Max-Plank-Institute of Physics and Astrophysics for the warm hospitality. Financial support by Deutsche Forschungsgemeinschaft (grants No. Ku 502/3-1 and Ku 502/6-1) and by INTAS under contract No. INTAS-93-0744 is gratefully acknowledged.

### References

[1] J.F. Gunion, H.E. Haber, G. Kane, S. Dawson, The Higgs Hunter’s Guide, Addison Wesley 1990.

[2] B. Kniehl, Phys. Rep. 240 (1994) 211.

[3] E. Braaten, J.P. Leveille, Phys. Rev. D 22 (1980) 715.
[4] N. Sakai, Phys. Rev. D 22 (1980) 2220.
[5] T. Inami, T. Kubota, Nucl. Phys. B 179 (1981) 171.
[6] M. Drees, K. Hikasa, Phys. Rev. D 41 (1990) 1547; Phys. Lett. B 240 (1990) 455; erratum ibid. B 262 (1991) 497.
[7] A.L. Kataev, V.T. Kim, Mod. Phys. Lett. A Vol. 9, No. 14 (1994) 1309.
[8] S.G. Gorishny, A.L. Kataev, S.A. Larin, L.R. Surguladze, Mod. Phys. Lett. A 5 (1990) 2703; Phys. Rev. D 43 (1991) 1633.
[9] B.A. Kniehl, Phys. Lett. B 343 (1995) 299.
[10] K.G. Chetyrkin and A. Kwiatkowski, Nucl. Phys. B461 (1966) 3.
[11] L.R. Surguladze, Phys. Lett. B 341 (1994) 60.
[12] B.A. Kniehl, J.H. Kühn, Nucl. Phys. B 329 (1990) 547.
[13] S.A. Larin, T. van Ritbergen, J.A.M. Vermaseren, Phys. Lett. B362 (1995) 134.
[14] A. Djouadi, M. Spira and P.M. Zerwas Z. Phys. C70 (1966) 427.
[15] K.G. Chetyrkin, J.H. Kuehn and A. Kwiatkowski, QCD Corrections to the $e^+e^-$ Cross-Section and the Z Boson Decay Rate, In the Report of the Working Group on Precision Calculations for the $Z_0$ Resonance, CERN Yellow Report 95-03, eds. D. Yu. Bardin, W. Hollik and G. Passarino, page 175.
[16] W.A. Bardeen, A.J. Buras, D.W. Duke and T. Muta, Phys. Rev. D18 (1978) 3998.
[17] O.V. Tarasov, A.A. Vladimirov, A.Yu. Zharkov, Phys. Lett. B 93 (1980) 429.
[18] O.V. Tarasov, preprint JINR P2-82-900 (1982).
[19] S.A. Larin and J.A.M. Vermaseren, Phys. Lett. B 303 (1993) 334.
[20] S.A. Larin, Preprint NIKHEF-H/92-18, hep-ph/9302240 (1992); In Proc. of the Int. Baksan School "Particles and Cosmology" (April 22-27, 1993, Kabardino-Balkaria, Russia) eds. E.N. Alexeev, V.A. Matveev, Kh.S. Nirov, V.A. Rubakov (World Scientific, Singapore, 1994).
[21] F.V. Tkachov, Phys. Lett. B 100 (1981) 65.
[22] K.G. Chetyrkin and F. V. Tkachov, Nucl. Phys. B 192 (1981) 159.
[23] J.A. M. Vermaseren, Symbolic Manipulation with FORM, Version 2, CAN, Amsterdam, 1991.
[24] S.A. Larin, F.V. Tkachov, J.A.M. Vermaseren, Preprint NIKHEF-H/91-18 (1991).
[25] A.A. Vladimirov, Teor. Mat. Fiz. 43 (1980) 210.
[26] K.G. Chetyrkin, A.L. Kataev, F.V. Tkachov, Nucl. Phys. B 174 (1980) 345.
[27] J.C. Collins, Nucl. Phys. B 92 (1975) 477.
[28] K.G. Chetyrkin and F. V. Tkachov, Phys. Lett. B114 (1982) 340.
[29] K.G. Chetyrkin and V. A. Smirnov, Phys. Lett. B 144 (1984) 419.
[30] K.G. Chetyrkin, preprint MPI-PAE/PTh 13/91 (Munich, 1991).
[31] S.G. Gorishny, A.L. Kataev and S.A. Larin, Phys. Lett. B 259 (1991) 144.
[32] L.R. Surguladze and M.A. Samuel, Phys. Rev. Lett. 66 (1991) 560; erratum ibid, 2416.
[33] N. Gray, D.J. Broadhurst, W. Grafe and K. Schilcher, Z. Phys. C 48 (1990) 673.