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Circuit simulation of the process of film resistive elements laser trimming under the effect of a measuring current source

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Abstract. To solve the problems of film resistors laser trimming, finding the optimal cutting trajectory, and predicting the results, a discrete grid circuit model of the conductive medium is proposed. This circuit model is a grid with a sufficiently small step of elementary resistors, showing the distribution of currents and potential in a conducting medium. A mathematical model of a conducting medium is obtained in the form of algebraic equations system for the voltages at grid nodes, currents and voltages on elementary resistors. An algorithm for calculating the electrophysical parameters of a resistor is developed with a change in the structure of the grid model in the process of trimming and finding the optimal cutting trajectory.

1. Introduction

The resistance value of film resistive elements (RE) is largely determined by their shape. Alteration of the resistor shape by removing parts of the resisting material using laser radiation can bring the ohmage to its desired values and accuracy, i.e. resistor laser trimming. The main requirements for the trimming operation are: minimal resistor nominal deviation, minimal labour effort and time of operation, the impact of trimming trajectory on the resistance value after trimming, and the number of resistors failures during their functioning. Besides, depending on the product, the ratio of such requirements may vary. Thus, in case of general purpose silicone resistors mass production, adjustable cut selection becomes crucial for faster achievement of nominal values, while in case of high-precision solutions the cutting path becomes crucial in order to achieve the needed nominal value at given accuracy keeping in view minimal impact of trimming at further resistance alteration [1, p. 75-78]. This implies the necessity of developing a mechanism, which would not only suggest forecasting the resistance value at a given trimming trajectory, but would also become a flexible mechanism for the search of the best path depending on various criteria, and would allow a deep analysis of processes running in the resistor.

The resistors conductive medium can be broken into several models of different hierarchy. At a micro level, typical mathematical models of conductive medium are represented by differential equations with partial derivatives together with edge conditions. These models, named distributed models, refer
to many equations of mathematical physics. Fields of physical values become the objects of research here; this is required for analysing the distribution of current density and power in the resistor conductive field, and temperature distribution [2].

Thus, under the effect of the external measuring voltage source $U_c$, applied to bonding pads, electrical field $E$ is induced in the resisting film, and flat potential field $\varphi = \varphi(x, y)$ with respect to neutral grounded output. The potential scalar function $\varphi = \varphi(x, y)$ refers to the intensity $E$ by the ratio:

$$E = -\text{grad } \varphi.$$

In a general case, the potential field in a resistance film is governed by Laplace equation with partial derivatives:

$$\Delta \varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2},$$

where the resistance film is conductive medium with distributed parameters. [3, p. 56].

During its laser trimming, the configuration of conductive medium changes, and resistance calculation becomes a challenge. The number of mutually studied various media and material layers in practically used microlevel models cannot be large, due to the complexity of calculation. Computational efforts for distributed environments can be notably reduced only by applying another approach towards modelling, based on the admission of certain allowances.

In our opinion, the suitable approach here is circuit modelling of the trimming process for film resistance elements, which allows passing to macrolevel models, i.e. models with lumped parameters. [2, p. 66].

2. Building a circuit model for film resistor conductive medium

In order to build a circuit model for a rectangular film resistor with $L \times H$ dimensions ($L$ and $H$ are length and width respectively, see figure 1), let us approximate the conductive medium of the resistor film by a discrete mesh of elementary resistors ($ER$), which are equal and continuous for homogeneous isotopic medium. Selecting the mesh size $d$, the laser beam diameter and its movement discreteness shall be considered. The voltage values in the resistor mesh nodes are discrete approximation of the potential field in the resistive layer.

Figure 1. Resistor element model with discrete node mesh
The number of discrete steps and mesh nodes along the resistor element length shall be calculated based on the following expression:

\[ n_L = \frac{L}{d} \]

The number of nodes along the width of the resistor element shall be calculated in a similar way:

\[ n_H = \frac{H}{d} \]

The number of inner nodes in the scheme shall total:

\[ n = n_L \cdot n_H \]

where the first bonding pad forms the base zero node of the circuit model, respective to which voltages shall be considered in \( n \) inner nodes of the circuit model and in the node \((n + 1)\), the latter being represented by the outer node of the second bonding pad, to which the measuring current source is connected.

With this, the total number of resistor branches \( b \) in the circuit model shall be calculated based on the total sum of resistors, included between the nodes along the length and the width; it exceeds the number of nodes greatly:

\[ b = (n_L + 1) \cdot n_H + (n_H - 1) \cdot n_L = 2 \cdot n_L \cdot n_H + (n_H - n_L) \]

The voltage \( u_{n+1} \) in the node \((n + 1)\) compared to the basic one represents the potentials difference between bonding pads; this means loss of measuring voltage on film resistor \( U_C \). With rated measuring current \( I_C = \text{const} \) it is proportional to RE resistance:

\[ u_{n+1} = U_C = R \cdot I_C \]

With this, the measuring current source \( I_C \) acts as the additional element \((b + 1)\) of the circuit model with zero conductivity.

The electrical analogy of the finite difference method and of the finite element method allows, during modelling, the use of complex electrical systems calculation methods, where potential functions at the mesh nodes can be homologated with the voltage at the model nodes relative to the zero one. As the basic elements of energy conversion are ERs, connecting and disconnecting them in certain ways will give an electric circuit reflecting the electric process that happens in film RE during trimming, as well as the distribution of current, voltage, and power values.

Figure 2 shows a resisting element made of a discrete mesh of resistors, in which \( n_L = n_H = 4 \). It should be noted that such a low resolution of the discrete mesh has been selected for illustrative purposes showing the circuit model composition; it is obvious that in the case of real objects, an adequate model will contain a much greater number of nodes.
During the laser adjustment modelling process, ERs located on the laser cutting path are removed from the circuit model. With this, changes occur in the model configuration, its equivalent resistance, and distribution of RE current and power in various areas. In order to calculate such parameters as voltage in nodes, and current, voltage and power values at the mesh branches, mathematic modelling of electrical processes in a complex multi-branched circuit is needed, based on the circuit theory, and solving large-scale equation systems referring to current and voltage values at the circuit elements.

3. Mathematical formulation of resistive conductive medium circuit models

The initial data for building mathematical models of circuits are component equations, which describe the attributes of separate elements, and continuous equations, which describe the correlation of components within the circuit being modelled [2, p. 66].

In order to build the mathematical model of the circuit and to obtain results concerning node voltage values, and voltage and current values for branches depending on external measuring current, each branch of the circuit model will be treated in the aggregate, which consists of paralleled resistor \( R_k \) and current source \( J_k \) \[4, p. 55\]. With this, all the circuit branches shall be equitype. Depending on which parameter equals zero: \( R_k \) or \( J_k \), the generalized branch shall become either a source of current or a resistor. The correlation of current and voltage for generalized branches shall be defined not only by their conductivity but also by the current sources of branches:

\[
i_k = i_R - J_k = \frac{u_k}{R_k} - J_k = u_k - J_k = u_k - y_k - J_k.
\]

In its matrix notation, the correlation of current and voltage of the generalized branches with current sources shall be defined as:

\[
i_B = Y_B \cdot u_B - J_B, \tag{1}
\]

where \( i_B = [i_{R_1}, i_{R_2}, ..., i_{R_{b+1}}]^T \) is the vector of branch currents, \( u_B = [u_{R_1}, u_{R_2}, ..., u_{R_{b+1}}]^T \) is the vector of the generalized branches voltages, \( Y_B \) is the matrix of branches conductivity, and \( J_B \) is the vector of branches current sources.

Considering that the conductivity of the current source branch equals zero, we obtain the following matrix structure for branches conductivity and external current sources vector:
Kirchhoff’s equation for currents for the whole scheme with rected towards the\n\n\[
\begin{bmatrix}
1/r & & & & \\
& 1/r & & & \\
& & 1/r & & \\
& & & & 1/r \\
& & & & 0
\end{bmatrix}
= \frac{1}{r}
\begin{bmatrix}
1 & & & & \\
& 1 & & & \\
& & 1 & & \\
& & & & 1 \\
& & & & 0
\end{bmatrix}
= y
\begin{bmatrix}
1 & & & & \\
& 1 & & & \\
& & 1 & & \\
& & & & 1 \\
& & & & 0
\end{bmatrix},
\]

In order to solve the topology problem in the linear circuit analysis, i.e. defining conformity in the distribution of voltage and current at different elements connection, graph theory is used. Figure 3 represents an oriented graph of the circuit model, the latter corresponding to RE in figure 2.

The information about the connected oriented graph of the circuit is contained in the incidence matrix [5, p. 126]. For a circuit with \((n+1)\) inner nodes and \((b+1)\) branches, elements of the reduced incidence matrix \(A\), size \((n+1)\times(b+1)\), not including the zero basic node, shall be defined as follows:
- \(a_{ij} = 1\), if the branch \(j\) exits from the node \(i\);
- \(a_{ij} = -1\), if the branch \(j\) enters the node \(i\);
- \(a_{ij} = 0\), if the branch \(j\) is not connected with the node \(i\).

With this, the measuring current source connected between the zero and the \((n+1)\) nodes and directed towards the \((n+1)\) node, is reflected in the incidence matrix \(A\) as the only element always having the value \(-1\): \(a_{(n+1),(b+1)} = -1\).

![Figure 3. Oriented graph of the RE circuit model](image)

As each \(i\)-row of matrix \(A\) shows which branches (considering their direction) are connected with \(i\)-node, this row can be considered as the record of indices of Kirchhoff’s equation for currents of \(i\)-node, while the whole matrix \(A\) can be considered as the matrix of indices of the whole system of Kirchhoff’s equation for currents for the whole scheme. In this case, if the currents of branches of the

\[y_i = \frac{1}{r} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} = y \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, \]

\[\mathbf{j}_B = [0, ..., 0, J_{b+1}]^T, J_{b+1} = -I_c. \]
circuit model are represented as column-vector \( \mathbf{i}_B = \begin{bmatrix} i_{R_1}, i_{R_2}, \ldots, i_{R_m} \end{bmatrix}^T \), size \( (b+1) \times 1 \), and matrix \( \mathbf{A} \) columns and vector \( \mathbf{i}_B \) rows refer to the same branches, then, using a reduced incidence matrix \( \mathbf{A} \), the Kirchhoff’s currents law is formed for the inner \( (n+1) \) nodes of the circuit (for all the nodes except the zero basic one) [6, p.88]

\[
\mathbf{A} \cdot \mathbf{i}_B = 0 .
\]

The current values of the circuit model branches \( \mathbf{i}_B = \begin{bmatrix} i_{R_1}, i_{R_2}, \ldots, i_{R_m} \end{bmatrix}^T \) are unknown, their number exceeds the number of equations of the system (4) – \( b > n \), and it cannot be solved in a single-valued way referring to branch currents. In order to define the voltages of the branches, voltages of the circuit model nodes are explored referring to the zero grounded node, i.e. the vector \( \mathbf{u}_N = \begin{bmatrix} u_{1_0}, u_{2_0}, \ldots, u_{n_0} \end{bmatrix}^T \), where \( u_{k_0} \) are node voltages (node potentials), \( k = 1, \ldots, n + 1 \). The vector of node voltages has a much smaller number of dimensions compared to the vector of branch currents and voltages. After this, using the incidence matrix \( \mathbf{A} \), we get the expression of branch voltages vector by means of the vector of node voltages (node transformation):

\[
\mathbf{u}_B = \mathbf{A}^T \cdot \mathbf{u}_N .
\]

Equations (4) and (5) are the record of generalized Kirchhoff’s laws for currents and voltages of the circuit model. Based on expression (1) and using node transformation (5), the correlation of generalized branches currents vector with node voltages vector is formed, taking into account the vector of the external current sources:

\[
\mathbf{i}_B = \mathbf{Y}_B \cdot \mathbf{A}^T \cdot \mathbf{u}_N - \mathbf{j}_B .
\]

Then, expression (4) of the generalized Kirchhoff’s equation for currents will be written as:

\[
\mathbf{A} \mathbf{Y}_B \mathbf{A}^T \cdot \mathbf{u}_N - \mathbf{A} \cdot \mathbf{j}_B = 0 .
\]

From here, we obtain a linear algebraic equation system relative to the vector of the mesh node voltages \( \mathbf{u}_N \):

\[
\mathbf{A} \mathbf{Y}_B \mathbf{A}^T \cdot \mathbf{u}_N = \mathbf{A} \cdot \mathbf{j}_B .
\]

The matrix of the system \( \mathbf{A} \mathbf{Y}_B \mathbf{A}^T \) is the matrix of the overall node conductivity forming a square non-generate matrix, size \( (n+1) \times (n+1) \):

\[
\mathbf{A} \mathbf{Y}_B \mathbf{A}^T = \mathbf{Y}_N, \quad \det \mathbf{Y}_N \neq 0 .
\]

The additional column \( (n+1) \) and row \( (n+1) \) refer to the external node \( (n+1) \) of measuring current source connection.

The right member of the system (6) is defined by the vector of equivalent node current sources, size \( (n+1) \times 1 \):

\[
\mathbf{j}_N = \mathbf{A} \cdot \mathbf{j}_B .
\]

The obtained system of node equations

\[
\mathbf{Y}_N \cdot \mathbf{u}_N = \mathbf{j}_N
\]

has the only solution referring to the vector of node voltages (potentials) in the mesh \( \mathbf{u}_N \):

\[
\mathbf{u}_N = \mathbf{Y}_N^{-1} \mathbf{j}_N .
\]

The components of the vector of node voltages \( \mathbf{u}_N \), as a result of node analysis, i.e. the solution of the system (9), are explicitly defined by the value of the external measuring current \( I_c \), as the vector of the right member (8), being the vector of equivalent node current sources (7), taking into account the incidence matrix structure, has only one last non-zero element equalling \( I_c \):

\[
\mathbf{Y}_N^{-1} \mathbf{j}_N = \begin{bmatrix} I_1, I_2, \ldots, I_n, I_c \end{bmatrix}^T
\]

...
Based on expression (2), the system (6) relatively to node potentials will be simplified after multiplication by resistance \( r \) due to conductivity cancel out in the left member and is written as:

\[
A_0 A_0^T \cdot u_N = A_0 j_N \cdot r ,
\]

\[
Y_N = A_0 A_0^T ,
\]

\[
j_N = A j_N \cdot r ,
\]

where the matrix \( A_0 \) is the incidence matrix, size \((n+1) \times (b+1)\), obtained from the matrix \( A \) by zeroing column \((b+1)\). This takes place due to zero conductivity of the current source in the matrix of resistor conductivity values (2).

We shall call the obtained system (11) the reduced system of node equations; we shall call the matrix \( Y_N = A_0 A_0^T \) the reduced matrix of node conductivity values. The elements of this matrix do not depend on the mesh resistors values, they are integer numbers, which reflect the structure (the topology) of the circuit model, and are defined by the number of mesh elements, connected to a certain node.

The solution of the reduced system (11) is in linear dependence on mesh ERs resistance values at their equal size. According to (3) we get:

\[
j_N = \begin{bmatrix} 0, ..., 0, I_c \end{bmatrix}^T ,
\]

\[
Y_N \cdot u_N = \begin{bmatrix} 0, ..., 0, I_c \end{bmatrix}^T .
\]

Solving the system (13) relatively to the vector of node voltages of the mesh \( u_N \)

\[
\begin{align*}
    u_N &= Y_N^{-1} j_N \\
    &= \left[A_0 A_0^T \right]^{-1} : j_N
\end{align*}
\]

we shall separate out element \( U_{n+1} \), which equals the voltage drop at film resistor between bonding pads \( U_{n+1} = U_c \), and knowing the measuring current \( I_c \) we shall calculate the resulting resistance of the film RE:

\[
R = U_c / I_c .
\]

Further on, we shall calculate voltages on the mesh resistors basing on nodes transformation (5):

\[
u_B = A_0^T \cdot u_N ,
\]

resistors currents

\[
i_B = Y_B \cdot u_B ,
\]

and, consequently, power on mesh resistors

\[
P_k = U_k \cdot I_k , \quad k = 1, ..., n .
\]

4. Calculation of resistor parameters at restructuring circuit model during RE trimming

During laser trimming of film resistor, parts of film are removed, as laser radiation evaporates the resisting layer. With this, the resulting resistance of the film resistor increases. During the trimming process modelling via a circuit model, mesh resistors are removed in it on the laser beam trajectory. With this, the circuit structure will change, as well as the elements of node conductivity matrix elements will, and node voltages and currents through branch elements.

After removal of another ER in the mesh on laser trimming trajectory, the decision on new current and voltage values can be made during formation of a new node conductivity matrix. However, a better approach suggests forming an initial system matrix and the right member vector for a fully filled circuit model ER mesh before the trimming starts. After this, when another ER is removed from the circuit at a trimming stage, the only correction of the matrix elements of the system nodes conductivity is to be performed [5, p. 171].
Changes shall be made according to system properties registered earlier (6), (11) and to numbers of nodes to which an ER to be removed was connected. In branch number $k$ is removed, which is directed from node $i$ to node $j$ with connectivity $y_{k}$, then in $Y_N$ matrix $y_{k}$ is removed in four places: $y_{k}$ is twice subtracted from elements $y_{ii}$ and $y_{jj}$ of $Y_N$ matrix in its diagonal, and twice added to non-diagonal elements $y_{ij}$ and $y_{ji}$. This procedure of nodes conductivity matrix renewal at each stage requires less computational efforts.

During trimming with circuit restructuring due to resistors removal, only the nodes conductivity matrix changes, while the vector of equivalent current sources (12) remains unchanged. After each trimming stage, the system (13) shall be solved anew relative to node voltages, and after this, the value of film resistor is re-calculated. The calculation of voltage, current and power distribution can be performed only at the last stage, after the trimming process is finished.

Let us explore the cut trajectory showed at the graph of the simplified circuit model, figure 4. At the first stage, ER $R_1$ with conductivity $y_{11}$, connected to nodes 4 and 8 is removed. As a result, in matrix $Y_N$ of system (13) diagonal elements $y_{4,4}$ and $y_{8,8}$ are reduced by $y_{11}$ (by 1 in a reduced matrix), and elements $y_{4,k}$ and $y_{k,4}$ are increased by $y_{11}$ (by 1 in a reduced matrix) and become zero values. Stages 2, 3, and 4 are performed in the same way, with removal of branches with numbers 10, 13, and 20 respectively.

![Figure 4. Oriented graph of the simplified circuit model with trimming trajectory](image)

**5. Conclusion**

Now it can be seen that it is possible to model the whole process of film resisting elements trimming. The circuit model allows us to define the actual resistance of the resisting layer, the actual distribution of current and power in the circuit elements, and the actual voltage in the mesh nodes and elements at every trimming stage by solving large-scale algebraic equation systems.

In general, operations for removing ER and calculating the obtained film resistor ohmage during trimming modelling should be repeated until such a trajectory is found at which the resistor will meet all the requirements of the design documentation, i.e. resistance value and its deviation from nominal, geometric layout related to film edges, and the allowed power dissipation of the product [7].
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