I. INTRODUCTION

The rapidly emerging field of spin polarized transport is based on the ability of a ferromagnetic metal to conduct and accumulate spin-polarized current. Spin-polarized transport between ferromagnets (F) and superconductors (S) received considerable attention recently because of new physical phenomena and potential device applications. An introduction of the hybrid structures based on a combination of ferromagnetic and superconducting materials are not only interesting from a fundamental point of view but can bring further advantages for devices. In particular, spin accumulation effects in superconductors may play an important role because of a number of reasons. First, due to the gap in the excitation spectrum, the spin diffusion length in a superconductor can become quite long at low temperatures. Second, spin accumulation can take place at a FS interface since spin-polarized current in a ferromagnet has to be transformed into spinless supercurrent in a superconductor. An important step in the quantitative analysis of spin accumulation and spin injection in superconductors is the knowledge of the dependence of the resistance of a FS interface on the spin polarization in a ferromagnet. Furthermore, it was recently demonstrated that a combination of F and S metals can be used advantageously for measuring spin polarization in metallic ferromagnets either by measuring $T_c$ of FS multilayers, or by directly measuring the resistance of FS point contacts. So far, theoretical studies of the FS contact resistance were limited by calculations for ballistic FS contacts. It was argued in [1, 2] that the effects of impurity scattering are quite important in spin-polarized tunneling since the degree of spin polarization is defined differently in ballistic and diffusive contacts. However, no quantitative calculations on the effects of disorder have been published up to now. Moreover, the contribution of the contact resistance is most easily measured only in a point contact geometry, while for larger area contacts the contribution of an interface becomes rather small. The purpose of the present paper is twofold. First, we extend the theory in order to include the effects of impurity scattering in a contact. Second, we argue that an additional sensitive probe for spin polarization is the excess resistance $R_{ex}$ of a FS contact. This resistance is due to penetration of an electric field into a superconductor over macroscopically large charge-imbalance relaxation length $\lambda_Q$ and may exceed the direct interface resistance. We show that the magnitude of $R_{ex}$ is sensitive to spin polarization in a ferromagnet and provide an estimate for this effect.

II. BALLISTIC FS CONTACT

We start from the derivation of the general expression for the conductance of a FS contact in the absence of impurity scattering (ballistic case). We consider the atomically sharp interface barrier at $x = 0$ separating F metal ($x < 0$) and S metal ($x > 0$), modeled by a potential $U(r) = H\delta(x)$ and arbitrary relation between Fermi velocities in F and S, $v_{F_\uparrow}$, $v_{F_\downarrow}$ and $v_{Sx}$. Here $H$ is the barrier strength parameter, $v_{F_\uparrow,\downarrow} = \sqrt{2E_{F_\uparrow,\downarrow}/m} \equiv \sqrt{2E_F(1 \pm h)/m}$, $v_{Sx} = \sqrt{2E_{Sx}/m}$, where $E_{ex}$, $E_F$ and $E_{Sx}$ are respectively exchange energy in a ferromagnet and Fermi energies in F and S metals, the indices $\uparrow, \downarrow$ refer to the spin subbands and $h = E_{ex}/E_F$ denotes the dimensionless spin polarization in a ferromagnet. We assume that the effective electronic masses $m_F$, $m_S$ are equal to the free electron mass $m_e$, the mean free path is larger than the size of the contact and the pair potential is approximated by the step function $\Delta(x) = \Delta(T)\theta(x)$, $\Delta(T)$ being the bulk pair potential in a superconductor.

Charge and spin currents can be calculated within the framework of the BTK approach, i.e. considering explicitly Andreev and normal reflections at the FS interface and taking into account that an incoming electron and an Andreev reflected hole occupy opposite spin subbands. The electron- and hole-like excitations are represented by
two-component wave functions, which obey the Bogoliubov de-Gennes equations. An electron, incoming from the ferromagnet F into the superconductor S, is described by a plane wave with a wave vector $k^+_\uparrow$:

$$\psi_{\text{inc}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{i k^+_\uparrow x}.$$  \hfill (1)

Due to the four-fold degeneracy of an excitation in a superconducting state, the electron is partially reflected into F as an electron with the opposite wave vector $-k^+_\uparrow$ or as a hole $k^-\downarrow$:

$$\psi_{\text{refl}} = a \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i k^-\downarrow x} + b \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i k^+_\uparrow x}$$  \hfill (2)

and partially transmitted into S without branch crossing $k^+\downarrow$, or with branch crossing $k^-\uparrow$:

$$\psi_{\text{trans}} = c \begin{pmatrix} u \\ v \end{pmatrix} e^{i k^+\downarrow x} + d \begin{pmatrix} v \\ u \end{pmatrix} e^{-i k^-\uparrow x}.$$  \hfill (3)

Here $k^+\downarrow$, $k^+\uparrow$ and $k^-\uparrow$ are the projections of the Fermi wave vectors in two spin subbands and in a superconductor on the direction $x$ normal to the contact plane, index + (−) refers to electron- or hole-like quasiparticles.

The amplitudes $a, b, c$ and $d$ have to be determined from the matching conditions for $\Psi_F = \psi_{\text{inc}} + \psi_{\text{refl}}$ and $\Psi_S = \psi_{\text{trans}}$ at the interface, $x = 0$:

$$d\Psi_F(0)/dx - d\Psi_S(0)/dx = 2m_eH\Psi_F(0)/\hbar.$$  \hfill (4)

Using these conditions we find the Andreev and normal reflection coefficients, $A = |a|^2$ and $B = |b|^2$, and the transmission coefficients with or without branch crossing, $C = |c|^2$ and $D = |d|^2$, respectively, which determine charge and spin currents in a ballistic FS contact.

Charge current in a FS contact is given by

$$I = \frac{e}{2\pi\hbar} \sum_{\uparrow, \downarrow} P_{\uparrow, \downarrow} \int \frac{d^2k_F}{(2\pi)^2} \int_{-\infty}^{+\infty} \left[ 1 + \frac{k_{F\uparrow, \downarrow}}{k_{F\uparrow, \downarrow}} A_{\uparrow, \downarrow}(E) - B_{\uparrow, \downarrow}(E) \right] [f(E + eV) - f(E)] dE,$$  \hfill (5)

where $P_{\uparrow, \downarrow} = (E_F \pm E_{x\pm})/2E_F = (1 \pm \hbar)/2$, $v_{F\uparrow, \downarrow}$ is the projection of the Fermi velocity on the direction $x$, $f(E)$ is the Fermi distribution function, and $k_F$ is the component of the Fermi momentum parallel to the junction plane, which is conserved for each individual scattering process. The ratio $k_{F\uparrow, \downarrow}/k_{F\uparrow, \downarrow}$ provides the normalization of the total probability current, taking into account that Andreev scattering involves different spin subbands.

In the limit of low temperatures $T \ll T_c$, we arrive the following expression for the charge conductance of a ballistic FS contact at the subgap bias voltage $eV < \Delta(T)$:

$$G_{FS} = 2G_0 T_\uparrow T_\downarrow \frac{(1 + \alpha^2) P_{\text{trans}}(v_{F\uparrow} + v_{F\downarrow})/v_{FS}}{(1 - r^{-1}_\uparrow + \alpha^2)/(1 + r^{-1}_\uparrow + \alpha^2)^2}.$$  \hfill (6)

Here $G_0 = e^2k_pF/4\pi^2\hbar$ is the normal state (Sharvin) conductance of the contact, $S$ is the contact area, $T_\uparrow$ and $T_\downarrow$ are the transmission probabilities for scattering from the spin up(down) subband into a superconductor

$$T_\uparrow = \frac{4v_{F\uparrow}v_{FS}}{4Z^2 + (v_{F\uparrow} + v_{FS})^2}, \quad T_\downarrow = \frac{4v_{F\downarrow}v_{FS}}{4Z^2 + (v_{F\downarrow} + v_{FS})^2},$$  \hfill (7)

$$r_{\uparrow, \downarrow} = \sqrt{1 - T_{\uparrow, \downarrow}}, \quad Z = H/\hbar v_{FS}, \quad \alpha = \sqrt{\Delta^2(T) - (eV)^2}/eV.$$  \hfill (8)

In relevant limits the expressions [5]-[11] agree with the results derived in [12]-[18], while the advantage of the representation [5]-[8] is, that the charge conductance is directly expressed in terms of the individual probabilities $T_{\uparrow, \downarrow}$ and therefore is particularly suitable for consideration of the impurity scattering, as explained in the next section. It follows from eq. (8) that in a NS contact ($T_\uparrow = T_\downarrow = T$) at zero bias the BTK result $G_{NS} = 2G_0T^2/(2 - T)^2$ is
recovered, which yields the conductance doubling $G_{NS} = 2G_0$ for a fully transmissive contact, $T = 1$. This conductance enhancement is suppressed in a FS contact when $T_\uparrow \neq T_\downarrow$ due to spin polarization in a ferromagnet.

It is straightforward to extend the expressions (10) to (12) to the regime of high bias voltage, $eV > \Delta(T)$. The results of numerical calculations of the dependence of charge current and zero-bias conductance are presented in Figs.1-3 for various values of spin polarization. It is seen that the zero-bias charge conductance is quite sensitive to the spin polarization $h = E_{ex}/E_F$. Fig.3 shows that for small values of $h$ this dependence is linear, which reflects the simple fact that the number of transmitted electronic modes scales like $1/h$. Fig.1 shows the results of numerical calculations of the dependences of the low temperature spin conductance on $h$. In the previous section the case of a ballistic FS contact was considered, when the contact size is smaller than the electronic mean free path. However the latter condition is not always fulfilled in experiments, and it is therefore of interest to evaluate the effect of impurity scattering in a contact. As a model for a FS contact we consider two bulk reservoirs (S and F), which in addition to the interface potential $U(r) = H\delta(x)$ are separated by the scattering region (a diffusive conductor) with a size smaller than the electronic mean free path.

The expressions derived above are particularly suitable for the introduction of impurity scattering, since they allow straightforward application of the scattering theory. According to this theory (see and references therein) any diffusive conductor having size larger than the electronic mean free path is characterized by universal distribution of transmission eigenvalues $t$ over different channels. An average conductance of a diffusive metal is then given as a sum of contributions of those channels, each having the conductance $G_0 = e^2/2\pi\hbar$

$$G_\sigma = \frac{G_{NS}}{G_0} \int_0^1 g_\sigma(t)\rho(t)dt,$$

(12)

where $\sigma = \uparrow, \downarrow$ is the spin index, $g_\sigma$ is the conductance of a channel with transmission coefficient $t$, $G_{NS} = e^2N_\sigma(0)D_\sigma$ is the normal state conductance per spin direction, $N_\sigma$ is the density of states at the Fermi level. The expression (12) is valid when the impurity scattering does not mix different spin directions.

Function $\rho(t)$ is the universal distribution function of transmission eigenvalues for different channels given by

$$\rho(t) = \frac{1}{2t\sqrt{1-t}}$$

(13)

and does not depend on microscopic parameters of a diffusive conductor. Eq.(13) shows that the transmission eigenvalues have a bimodal distribution with a peak at unit transmission and a peak at exponentially small transmission.

As a model for the diffusive SF contact we consider two scattering regions in series: an incoming electron is first transmitted through the diffusive region with probability $T_\uparrow$, then crosses the FS interface with probability $T_{\uparrow\downarrow}$.
turn, an Andreev-reflected hole is first scattered by the interface (probability $T_{1,2}$), then by the diffusive region. The probabilities of these two-step processes $T_{1,2}$ are given by the expression

$$T_{1,2} = \frac{tT_{\uparrow\downarrow}}{t + T_{\uparrow\downarrow} - tT_{\uparrow\downarrow}}, \quad (14)$$

which follows from averaging over transmission resonances between two scattering regions, assuming that all relevant distances exceed the electronic wave-length.

The charge conductance in a diffusive FS contact is given by the expression eq. (6) in which the probabilities $T_{\uparrow\downarrow}$ should be substituted by the probabilities $T_{1,2}$ of the two-step scattering processes described above. Here we present the result for low temperatures and $eV < \Delta(T)$

$$G_{FS} = G_N \int_0^1 T_1T_2(1 + \alpha^2)P_\uparrow(\nu_\uparrow^2/v_{\uparrow s}^2 + 1) \frac{dt}{(1 - r_1r_2)^2 + \alpha^2(1 + r_1r_2)^2} \frac{1}{t\sqrt{1 - t}} \quad (15)$$

where the probabilities $T_{1,2}$ are given by eq. (14), $r_{1,2} = \sqrt{1 - T_{1,2}}$, $\alpha = \sqrt{\Delta^2(T) - (eV)^2/eV}$ and $G_N = 2\varepsilon^2N(F_D)D(E_F)$ is the conductance of a contact in the unpolarized state.

In the NS case with a transparent interface ($Z=0$) and $v_{F\uparrow} = v_{F\downarrow} = v_{Fs}$ the above expression at $V = 0$ yields

$$G_{NS}(V = 0) = G_N \int_0^1 \frac{tdt}{(2 - t)^2\sqrt{1 - t}} \equiv G_N, \quad (16)$$

i.e. we reproduce the well known result that the zero-bias conductance of the diffusive contact $G_{NS} = G_N$, in contrast to the ballistic case when $G_{NS} = 2G_N$, first obtained by Artemenko, Volkov and Zaitsev by a different method.

Fig. 4 shows the results of numerical calculations of the dependence of the zero-bias conductance of a disordered FS contact vs spin polarization. It is seen by comparison of Figs. 3 and 4, that assuming ballistic transport in a FS contact one can overestimate the spin polarization in a ferromagnet. The results presented here correspond however to the strong scattering regime. For a more quantitative comparison with experiments the model should be further extended to the regime of arbitrary scattering strength.

**IV. EXCESS RESISTANCE**

So far we have taken into account both the interface and impurity scattering in the contact, but neglected the contribution of an electric field penetrating a superconductor. The latter can be indeed neglected in a point contact geometry, while it becomes important in planar contacts, in particular close to $T_c$, when an electric field penetrates into a superconductor over the macroscopically large charge-imbalance relaxation length $\lambda_{Q}^{\uparrow\downarrow}$.

The corresponding contribution to the boundary resistance of a FS contact can be calculated by the generalization of the approach of [12] valid for a clean superconductor. Excess resistance $R_{ex}$ is given

$$R_{ex} = F \lambda_Q \rho_s / S. \quad (17)$$

Here $\rho_s$ is the normal state resistivity of a superconductor and $F = Y^*/Y$, where $Y^*$ represents the charge current in FS contact

$$Y^* = \sum_{\uparrow\downarrow} P_{\uparrow\downarrow} \int \frac{d^2k}{(2\pi)^2} \int_{-\infty}^{+\infty} \left(-\frac{\partial f}{\partial E}\right)^2 [1 - C_{\uparrow\downarrow}(E) - D_{\uparrow\downarrow}(E)] N_s^{-1}(E)dE \quad (18)$$

and $Y$ represents the total current

$$Y = \sum_{\uparrow\downarrow} P_{\uparrow\downarrow} \int \frac{d^2k}{(2\pi)^2} \int_{-\infty}^{+\infty} \left(-\frac{\partial f}{\partial E}\right) \left[1 + \frac{k_{F\uparrow\downarrow}}{k_{F\uparrow\downarrow}} A_{\uparrow\downarrow}(E) - B_{\uparrow\downarrow}(E)\right] dE. \quad (19)$$

Here $N_s(E) = E/\sqrt{E^2 - \Delta^2(T)}$ is the density of states in a superconductor.

At $E < \Delta(T)$ the coefficients $C, D$ vanish, while $A, B$ are given by

$$\frac{k_{F\uparrow\downarrow}}{k_{F\uparrow\downarrow}} A_{\uparrow\downarrow}(E) = 1 - B_{\uparrow\downarrow}(E) = \frac{T_1T_2(1 + \alpha^2)}{(1 - r_1r_2)^2 + \alpha^2(1 + r_1r_2)^2}, \quad (20)$$
where $\alpha = \sqrt{\Delta^2(T) - E^2}/E$. At $E > \Delta(T)$

$$\frac{k_{F\uparrow} A_{\uparrow\downarrow}(E)}{k_{F\downarrow}} = \frac{T_\uparrow T_\downarrow (1 - \beta^2)}{[1 + \beta - r_\uparrow r_\downarrow (1 - \beta)]^2}; \quad 1 - B_{\uparrow\downarrow}(E) = \frac{T_\uparrow T_\downarrow (1 - \beta)^2 + 4\beta(T_\uparrow + T_\downarrow)}{[1 + \beta - r_\uparrow r_\downarrow (1 - \beta)]^2}; \quad (21)$$

$$C_{\uparrow\downarrow}(E) = \frac{2(1 + \beta)(T_\uparrow + T_\downarrow)}{[1 + \beta - r_\uparrow r_\downarrow (1 - \beta)]^2}; \quad \frac{k_{F\uparrow\downarrow} A_{\uparrow\downarrow}(E) + B_{\uparrow\downarrow}(E) + C_{\uparrow\downarrow}(E) + D_{\uparrow\downarrow}(E)}{k_{F\uparrow\downarrow}} = 1; \quad (22)$$

where $\beta = \sqrt{E^2 - \Delta^2(T)}/E$.

The results of the calculations of the excess resistance factor $F = Y^*/Y$ for a FS contact are shown in Fig.5. It is seen that $F$ increases strongly at temperatures close to $T_c$. Given the fact that the charge-imbalance relaxation length $\lambda_Q$ becomes macroscopically large near $T_c$, we conclude that measuring the excess resistance in a FS contact can provide a sensitive probe for measuring spin polarization.

In conclusion, we have presented the results of a theoretical study of interface resistance in ferromagnet/superconductor junctions. The Andreev reflection theory is extended in order to take into account the impurity scattering within the contact in the regime of strong disorder. The model is applied to the calculation of the excess resistance of a FS contact caused by penetration of an electric field into a superconductor. The latter contribution could be important in contacts with planar geometry and provides an additional method for measuring spin polarization in ferromagnets.

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FIG. 1. Low temperature spin and charge currents in a ballistic FS contact for different spin polarizations in a ferromagnet for the barrier strength parameter $Z=0$.
FIG. 2. Low temperature spin and charge currents in a ballistic FS contact for different spin polarizations in a ferromagnet for the barrier strength parameter $Z=1$. 

$G_{FS}/G_0$ vs $eV/\Delta$ for $Z=1$.
FIG. 3. Zero-bias conductance of a ballistic FS contact as a function of the spin polarization at various barrier strengths.
FIG. 4. Zero-bias conductance of a disordered FS contact as a function of the spin polarization at various barrier strength.
FIG. 5. Temperature dependence of the excess resistance in a FS contact.