Double Neutron Star Mergers from Hierarchical Triple-star Systems

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Received 2019 July 18; revised 2019 August 12; accepted 2019 August 12; published 2019 September 18

Abstract

The isolated binary evolution model for merging neutron stars (NSs) involves processes such as mass transfer, common-envelope evolution, and natal kicks, all of which are poorly understood. Also, the predicted NS–NS merger rates are typically lower than the rates inferred from the LIGO GW170817 event. Here, we investigate merger rates of NS and black hole–NS binaries in hierarchical triple-star systems. In such systems, the tertiary can induce Lidov–Kozai (LK) oscillations in the inner binary, accelerating its coalescence and potentially enhancing compact object merger rates. However, because compact objects originate from massive stars, the prior evolution should also be taken into account. Nataal kicks, in particular, could significantly reduce the rates by unbinding the tertiary before it can affect the inner binary through LK evolution. We carry out simulations of massive triples, taking into account stellar evolution starting from the main sequence, secular and tidal evolution, and the effects of supernovae. For large NS birth kicks ($\sigma_k = 265$ km s$^{-1}$), we find that the triple NS–NS merger rate is lower by a factor of $\sim2$–$3$ than the binary rate, but for no kicks ($\sigma_k = 0$ km s$^{-1}$), the triple rate is comparable to the binary rate. Our results indicate that a significant fraction of NS–NS mergers could originate from triples if a substantial portion of the NS population is born with low kick velocities, as indicated by other work. However, uncertainties and open questions remain because of our simplifying assumption of dynamical decoupling after inner binary interaction has been triggered.

Unified Astronomy Thesaurus concepts: Binary stars (154); Neutron stars (1108); Stellar dynamics (1596); Stellar evolution (1599)

1. Introduction

The detection of a double neutron star (NS) merger by LIGO’s O2 observing run (Abbott et al. 2017b) has definitively shown that double neutron stars (NSs) can merge in the universe, and the accompanying electromagnetic signals in gamma-rays (e.g., Abbott et al. 2017a; Goldstein et al. 2017; Savchenko et al. 2017), X-rays (e.g., Margutti et al. 2017), optical (e.g., Coulter et al. 2017; Nicholl et al. 2017; Shappee et al. 2017; Soares-Santos et al. 2017), near-infrared (e.g., Chornock et al. 2017; Cowperthwaite et al. 2017; Pian et al. 2017), and radio wavelengths (e.g., Alexander et al. 2017), have revealed a wealth of information on kilonova transients (e.g., Lattimer & Schramm 1974; Freiburghaus et al. 1999; Metzger et al. 2010; Just et al. 2015; Rosswog 2015), and short gamma-ray bursts (e.g., Eichler et al. 1989; Piran 1999; Berger 2014). The origin of compact object mergers is unclear because, in standard stellar evolution theory, the progenitor stars would merge before evolving to compact objects in an orbit that would cause them to merge due to gravitational wave (GW) emission within a Hubble time. Two main scenarios have been proposed for compact object mergers: isolated binary evolution (e.g., Tutukov & Yungelson 1973, 1993; Tutukov & 1992; Lipunov et al. 1997; Belczynski et al. 2002; Voss & Tauris 2003; Kalogera et al. 2007; Dominik et al. 2012, 2013; Belczynski et al. 2014, 2016a, 2017; Stevenson et al. 2017; Chruslinska et al. 2018; Giacobbo & Mapelli 2019) and dynamical interactions, such as those in triple-star systems (e.g., Thompson 2011; Hamers et al. 2013; Antonini et al. 2017; Liu & Lai 2017, 2018; Silsbee & Tremaine 2017; Toonen et al. 2018; Liu et al. 2019), globular clusters (e.g., Sigurdsson & Hernquist 1993; Portegies Zwart & McMillan 2000; O’Leary et al. 2006; Ziosi et al. 2014; Rodriguez et al. 2015, 2016; Kimpson et al. 2016; Mapelli 2016; Samsing & Ramirez-Ruiz 2017; Rodriguez et al. 2018; Samsing 2018; Samsing et al. 2018a, 2018b), and galactic nuclei (e.g., Antonini & Perets 2012; Antonini et al. 2014; Prodan et al. 2015; Antonini & Rasio 2016; Stephan et al. 2016; VanLandingham et al. 2016; Petrovich & Antonini 2017; Arca-Sedda & Gualandris 2018; Fragione et al. 2019; Gondán et al. 2018; Hamers et al. 2018; Hoang et al. 2018; Randall & Xianyu 2018a, 2018b; Arca-Sedda & Capuzzo-Dolcetta 2019).

The isolated binary channel involves binary interactions such as mass transfer, common-envelope (CE) evolution, and supernovae (SNe) kicks associated with the birth of NSs or black holes (BHs). The details of these processes are highly uncertain, yet, taking into account these uncertainties, it is challenging in the isolated binary model to predict rates (e.g., Kalogera et al. 2004a, 2004b, see also Abadie et al. 2010 for a review) that are on the same order of magnitude as derived from the LIGO observation of GW170817, $1540\pm1220$ Gpc$^{-3}$ yr$^{-1}$ (Abbott et al. 2017b). For example, Giacobbo & Mapelli (2018) find rates of up to $1 \times 10^3$ Gpc$^{-3}$ yr$^{-1}$, but only assuming low SNe kicks and a high CE efficiency.

Triple systems have been invoked in a large variety of contexts to explain an enhanced rate of interaction in binary systems; see, e.g., Naoz (2016) for a review. This occurs through oscillations of the inner binary eccentricity as a result of the torque of the tertiary, known as Lidov–Kozai (LK) oscillations (Kozai 1962; Lidov 1962). In particular, in the context of compact objects, it has been suggested that LK-driven eccentricity excitation can accelerate the mergers of compact objects (e.g., Blaes et al. 2002; Miller &
Hamilton 2002; Wen 2003; Thompson 2011; Antonini et al. 2017; Bonetti et al. 2018; Hoang et al. 2018; Fragione & Loeb 2019; Stephan et al. 2019). However, because compact objects originate from massive stars, the prior stellar evolution should also be taken into account, similarly to how it is in studies of binary evolution (e.g., Hamers et al. 2013; Naoz et al. 2016; Stephan et al. 2016, 2019; Hamers 2018a; Toonen et al. 2018). Natal kicks, in particular, could significantly reduce the rates by unbinding the tertiary before it can affect the inner binary through LK evolution (e.g., Hamers & Thompson 2019; Lu & Naoz 2019).

In this paper, we investigate the interplay between these processes in massive triple-star systems, taking into account stellar evolution starting from the main sequence (MS), secular and tidal evolution, and the effects of SNe. In contrast to previous studies, which focused on wide inner binaries that do not interact in the absence of a tertiary star (e.g., Hamers et al. 2013; Antonini et al. 2017; Hamers & Thompson 2019), we here consider triples with no restrictions on the inner binary orbital separation. In particular, this implies that in the “binary case” of our triples, i.e., when the effect of the tertiary on the inner binary is ignored, the system can in fact interact—and possibly produce a double NS merger via the standard binary evolution channel.

This approach requires the modeling of binary processes such as mass transfer and CE evolution, as well as the effects associated with the gravitational perturbations from the tertiary (i.e., LK oscillations). Combining these processes is challenging and complicated; see, e.g., Hamers & Dosopoulou (2019) for an exploratory study for the case of mass transfer taking into account orbital effects due to mass transfer and LK evolution. Here, we take a simpler approach in which we model the evolution of the tertiary initially using a secular code taking into account dynamical, stellar, and tidal evolution, but not mass transfer or CE evolution. We track the onset of mass transfer (i.e., Roche-Lobe overflow; RLOF), and in this case, we continue the evolution of the inner binary system using a dedicated binary population synthesis code that includes all the required binary physics—but not any effects associated with the tertiary star. Here, we make the assumption that, after the onset of RLOF in the inner binary, the latter is dynamically decoupled from the tertiary. This assumption is typically justified in the case of CE evolution, in which case the inner binary shrinks significantly (see Figure 8 of Hamers et al. 2013)). However, in the case of mass transfer, the inner orbit can also expand (if the donor has become less massive than the accretor). Here, we ignore that complication, instead taking the simpler, decoupled approach.

Several types of compact object mergers have been studied in triple systems, including WD–WD mergers (Thompson 2011; Katz & Dong 2012; Hamers et al. 2013; Toonen et al. 2018; Stephan et al. 2019), BH–BH mergers (Antonini et al. 2017; Liu & Lai 2017, 2018; Silsbee & Tremaine 2017; Hoang et al. 2018; Liu et al. 2019), and BH–NS mergers (Fragione & Loeb 2019; Stephan et al. 2019). To our knowledge, NS–NS mergers in triples with stellar-mass tertiaries (taking into account stellar evolution and dynamical evolution; see Stephan et al. 2019) for a comparable study, but with supermassive BH tertiaries) have not been studied. Our focus is therefore on mergers of double NSs.

The structure of this paper is as follows. We describe our methodology in Section 2, and the initial conditions of our simulations in Section 3. We present our results, most notably the merger rates, in Section 4. We discuss our findings in Section 5, and conclude in Section 6.

2. Numerical Method

We adopt a hybrid method in which we use SECULARMULTIPLE (Hamers & Portegies Zwart 2016a; Hamers 2018b) to model the secular dynamical, tidal, and stellar evolution of a binary or hierarchical triple star system, and the binary stellar evolution code BSE (Hurley et al. 2000, 2002) to model the evolution of systems in which we consider the tertiary to be unimportant. The latter case, which we will refer to as “isolated” binary evolution, includes interacting systems that undergo mass transfer in the inner binary, as well as systems in which the tertiary star becomes unbound from the inner binary due to an SNe event—but with the inner binary remaining bound and possibly merging at a later time. Both codes, SECULARMULTIPLE and BSE, are implemented within the AMUSE framework (Pelupessy et al. 2013; Portegies Zwart et al. 2013).

2.1. SECULARMULTIPLE

In SECULARMULTIPLE, we model the evolution of a binary or triple system, starting from MS stars, and taking into account stellar evolution, tidal evolution, and secular dynamical evolution (in the case of triples, and up to and including fifth order in the expansion of the separation ratio of the inner to the outer binary; see Hamers & Portegies Zwart (2016a) and Hamers (2018b)). The modeling in SECULARMULTIPLE is similar to that in previous works in which we coupled secular dynamical evolution to stellar and tidal evolution (Hamers & Portegies Zwart 2016b; Hamers 2018a; Hamers & Thompson 2019).

Stellar evolution is taken into account by using Single Stellar Evolution (SSE; Hurley et al. 2000), as implemented in AMUSE. SSE is based on analytical fits to detailed stellar evolution models, and uses the same stellar tracks as (i.e., is consistent with) BSE. We set the metallicity to solar, Z = 0.02. Quantities that are used from SSE include the stellar type and (convective envelope) mass and radius as a function of age. These quantities are used to take into account mass loss from the system, assumed to occur either adiabatically or because of an impulsive change due to SNe. In the former case, we assume isotropic and adiabatic mass loss, i.e., $a M_1$ and $e_1$ constant (Huang 1956, 1963) for an orbit $i$ in the system ($i = 1$ and $i = 2$, respectively, for the inner and outer orbits, if applicable). Here, $M_1 = m_1 + m_2$ for the inner orbit, and $M_2 = M_1 + m_3$ for the outer orbit.

In the case of SNe, we use the routines incorporated into SECULARMULTIPLE and described in Hamers (2018b) to compute the effects on the inner and outer orbits of the (assumed to be instantaneous) mass loss and (possible) kick to the newly formed NS or BH, assuming a random orbital phase of both orbits at the moment of SNe. We assume that the kick distribution is a Maxwellian distribution, i.e., the probability

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Footnote 4: Excluding the possibility of primordial compact objects.

Footnote 5: The reference frame in SECULARMULTIPLE is an arbitrary frame, rather than the invariable plane.
density function of the kick speed $V_k$ is given by
\[
dN = \frac{\sqrt{2}}{\pi \sigma_k^3} V_k \exp \left( - \frac{V_k^2}{2\sigma_k^2} \right). \tag{1}\]

Here, $\sigma_k$ characterizes the typical kick speed and is assumed to be given (see Section 3 below). For BHs, we use the prescription from Section 4.1 of Fryer et al. (2012) to rescale the sampled speed $V_k$ in Equation (1) according to $V_{k,BH} = V_k (1 - f_{fb})$, where the fallback factor $f_{fb}$ is given by
\[
f_{fb} = \begin{cases} 
0, & M_{CO} < 5 M_\odot; \\
0.378 M_{CO} - 1.889, & 5.0 M_\odot \leq M_{CO} < 7.6 M_\odot; \\
1, & M_{CO} \geq 7.6 M_\odot,
\end{cases} \tag{2}\]

where $M_{CO}$ is the mass of the carbon–oxygen core of the proto-BH, which is extracted from SSE.

Tidal evolution is taken into account with the assumption of the equilibrium tide model (Hut 1981; Eggleton et al. 1998). Specifically, we use Equations (81) and (82) of Eggleton et al. (1998), with the nondissipative terms $X$, $Y$, and $Z$ given explicitly by Equations (10)–(12) of Eggleton & Kiseleva-Eggleton (2001), and the dissipative terms $V$ given explicitly by Equations (A7)–(A11) of Barker & Ogilvie (2009). We apply these terms to both the inner and outer orbits (if applicable), in the latter case treating the inner binary as a point mass. The stellar spins are included in the tidal calculations; the initial spin–orbit angles are assumed to be zero (i.e., zero obliquities). The tidal dissipation efficiency is computed as a function of the stellar parameters as part of the set of ordinary differential equations using the prescription of Hurley et al. (2002). For all stars, we assume a fixed apsidal motion constant of $\kappa_{AM,i} = 0.014$, and a gyration radius of 0.08 (these parameters are not provided by SSE in AMUSE).

Many uncertainties still remain in tidal evolution; see, e.g., Ogilvie (2014) for a review. Especially for high eccentricities, which could be excited by LK oscillations, the equilibrium tide model could break down. A more sophisticated treatment of tides is beyond the scope of this work.

During the integration of the system in SECULARMULTIPLE, we check for a number of stopping conditions. These are listed and explained below.

1. For both binaries and triples, we check for RLOF in the inner binary using analytic fits (Sepinsky et al. 2007, Equations (47)–(52)). Specifically, we check for RLOF at periapsis as a function of the current radii, spins, and inner binary semimajor axis and eccentricity. If RLOF occurs, we stop the simulation within SECULARMULTIPLE, and continue the evolution of the inner binary using BSE; see Section 2.2 below for further details. We also check for RLOF by the tertiary star on the inner binary, treating the binary as a point mass and using the same fits of Sepinsky et al. (2007), although this scenario occurs much less commonly compared to RLOF in the inner binary (see Tables 2 and 3).

2. For both binaries and triples, after each SNe event, we check whether the system is still bound (i.e., if $a_i > 0$ for $i = 1$ and $i = 2$ if applicable). If not, we continue the evolution of the inner binary with BSE if the outer orbit became unbound but the inner binary is still bound (and could interact and merge at a later time); see Section 2.2 below.

3. For triples, we check for dynamical stability of the system using the criterion of Marling & Aarseth (2001), and stop the simulation in SECULARMULTIPLE if the system is unstable according to this criterion. We do not consider the subsequent evolution of the system.

4. For triples, we check whether the secular approximation made in the equations of motion breaks down at any point in the evolution. Such a breakdown can occur if the timescale for the inner binary angular momentum to change significantly becomes comparable to the inner or outer orbital periods (e.g., Ćuk & Burns 2004; Ivanov et al. 2005; Antonini & Perets 2012; Katz & Dong 2012; Seto 2013; Antognini et al. 2014; Antonini et al. 2014; Bode & Wegg 2014; Luo et al. 2016; Grishin et al. 2018; Lei et al. 2018). To evaluate whether or not this has occurred, we use the criterion of Antonini et al. (2014), i.e.,
\[
\sqrt{1 - e_1} < 5 \pi \frac{m_3}{m_1 + m_2} \left( \frac{a_1}{a_2(1 - e_2)} \right)^{3/2}.
\tag{3}\]

Although we check for this regime (also known as the semisecondary regime), it occurs very rarely in our simulations. This is because extremely high eccentricities are required, and in most cases, this implies that the stars in the inner binary would undergo mass transfer well before.

5. The inner binary components collide directly, i.e., $a_1(1 - e_1) < R_1 + R_2$, where $R_1$ and $R_2$ are the primary and secondary radius, respectively. This typically can only occur for compact objects, because the larger sizes of precompact objects generally imply that, when the eccentricity is high, usually RLOF occurs or possibly the semisecondary regime is triggered.

6. The system exceeds an age of 10 Gyr.

In contrast to previous work related to white dwarfs (Hamers 2018a; Hamers & Thompson 2019), here we do not include the effects of flybys on the binary or triple system. We find that most NS–NS mergers occur relatively early, i.e., within ~100 Myr (see Section 4 below). For systems in the field, flybys are typically unimportant on such timescales. Mergers occurring as a result of perturbations from passing stars are beyond the scope of this paper (e.g., Kaib & Raymond 2014; Michaely & Perets 2019).

2.2. BSE

As described in Section 2, we consider two situations in the simulations with SECULARMULTIPLE in which we assume for triples that the inner binary can subsequently be decoupled from the tertiary star. We then continue the evolution with the binary population synthesis code BSE. The latter code includes prescriptions for binary-star evolution, most notably mass transfer and CE evolution, which are both not modeled in SECULARMULTIPLE. In BSE, we assume a CE binding energy factor of $\lambda = 0.5$ and a CE efficiency of $\alpha = 1$. Furthermore, in BSE we assume the same NS kick speed distribution as in SECULARMULTIPLE (see Equation (1)). Due to limitations of BSE, we do not include the fallback correction to kicks applied to BHs, as described by Equation (2). Instead, in BSE, we assume zero kick speeds for BHs. This is strictly inconsistent with the prior simulations in SECULARMULTIPLE, but our main
focus in this work is on merging NSs, so we do not expect this
inconsistency to strongly affect our conclusions.

The two situations in which we hand off the evolution in
SECULARMULTIPLE to BSE correspond to stopping conditions
(1) and (2) described in Section 2.1. In situation (1), the inner
binary undergoes RLOF, which can be triggered by several
factors: expansion of the stars in the inner binary due to stellar
evolution, possibly combined with tidal evolution, and/or high
eccentricity due to secular evolution. Note that, contrary to
previous studies, ours is not restricted to wide systems.
Therefore, many systems will undergo RLOF even in the
“binary” case, i.e., in absence of a tertiary star.

The crucial assumption made in situation (1) for triples is
that the subsequent evolution of the inner binary is completely
decoupled from the tertiary star. This is a strong assumption,
and is certainly not correct for all systems. The motivation for
this approach is simplicity, as we currently do not have the
tools to self-consistently simulate the evolution of the triple
system taking into account the secular dynamical evolution
with mass transfer, particularly if the orbit is eccentric and
driven to varying eccentricity due to LK oscillations with
similar RLOF and LK timescales. Preliminary work (Hamers &
Dosopoulou 2019) indicates that the evolution in this regime
is complicated and merits future study. The decoupling assump-
tion likely gives rise to a significant systematic uncertainty
in our results. We further discuss this issue in Section 5.1.

3. Initial Conditions

Here, we describe the initial conditions assumed in our
simulations. We consider several sets of initial conditions; in
each set, we generate $N_{\text{MC}} = 10^3$ systems through Monte Carlo
sampling. Each set is characterized by: the type of system,
“Triple” or “Binary,” where “Binary” refers to the same system
but in absence of the tertiary star; the dispersion $q_i$ assumed in
the kick speed distribution (see Equation (1)); and the type of
assumption on the tertiary mass distribution (see Section 3.1
below).

Below, we describe in more detail the assumptions made
in the Monte Carlo sampling. An overview of our notation and
assumptions is given in Table 1.

3.1. Masses

We sample the mass of the primary star, $m_1$, from a Salpeter
distribution (Salpeter 1955), i.e., $dN/dm_1 \propto m_1^{-2.35}$. Because
our interest is in NS mergers, we sample with the range
$8 < m_1/M_\odot < 50$. The mass of the secondary star, $m_2$ is
sampled assuming a flat distribution in $q_2 \equiv m_2/m_1$, consistent
with observations of massive stars (e.g., Sana et al. 2012;
Duchène & Kraus 2013; Kobulnicky et al. 2014). Here, we set
the lower limit on $m_2$ to be $4 M_\odot$. The tertiary mass $m_3$ is
sampled according to two methods: sampling from (1) a flat
distribution in the mass ratio $q_3 \equiv m_3/(m_1 + m_2)$, and (2) a flat
distribution in the mass ratio $q_3^* \equiv m_3/m_2$. In either case, we set
the lower limit on $m_3$ to be $0.1 M_\odot$. These two choices give
different results in our simulations, as the tertiary star in case
(1) can be more massive than the primary star and therefore can
evolve first. Consequently, the tertiary star can become
unbound from the inner binary as it explodes in an SNe event.
In contrast, in method (2), the tertiary star is always less
massive than the primary star, such that the inner binary always
evolves first.

In Figure 1, we illustrate the difference between the two
assumptions on the tertiary mass ratio distribution. With
method (1), the median tertiary mass is $\approx 5 M_\odot$, whereas with
method (2) it is $\approx 12 M_\odot$.

3.2. Orbits

For both inner and outer orbits, we assume a flat distribution
in the logarithm of the orbital period, i.e., flat in $\log_{10}(P_{\text{orb},i})$
(also known as an Ópik distribution; see Ópik (1924)). The
limits for both inner and outer orbits are set to
$1 < \log_{10}(P_{\text{orb},i}/d) < 10$. The eccentricities $e_i$ of both inner
and outer orbits are sampled from flat distributions in $e_i$, with
$0 < e_i < 0.9$. The orbital period and eccentricity distributions

| Symbol | description | Initial value(s) and/or distribution in population synthesis |
|--------|-------------|----------------------------------------------------------|
| $m_1$  | Mass of the primary star. | 8–50 $M_\odot$ with a Salpeter initial mass function (Salpeter 1955, i.e., $dN/dm_1 \propto m_1^{-2.35}$). |
| $m_2$  | Mass of the secondary star. | $m_2q_1$, where $q_1$ has a flat distribution, and with $m_2 > 4 M_\odot$. |
| $m_3$  | Mass of the tertiary star. | Either (1) $m_3 = q_3(m_1 + m_2)$, or (2) $m_3 = q_3^*m_2$, where both $q_2$ and $q_3$ have a flat distribution, subject to $m_3 > 0.1 M_\odot$. |
| $Z_i$  | Metallicity of star $i$. | 0.02 |
| $R_i$  | Radius of star $i$. | From stellar evolution code. |
| $P_{i,\text{orb}}$ | Spin period of star $i$. | 10 day |
| $\theta_{i,\text{orb}}$ | Obliquity (spin–orbit angle) of star $i$. | $0^\circ$ |
| $\nu_{i,\text{vis}}$ | Viscous timescale of star $i$. | Computed from the stellar properties using the prescription of Hurley et al. (2002). |
| $k_{\text{AMS},i}$ | Apsidal motion constant of star $i$. | 0.014 |
| $\epsilon_{i,\text{g}}$ | Gyration radius of star $i$. | 0.08 |
| $V_{i,\text{SN}}$ | SN e kick speed. | Maxwellian distribution with $\alpha_i = 0, 40, \text{or} 265 \text{ km s}^{-1}$. |
| $P_{i,\text{orb},i}$ | Orbital period of orbit $i$ (inner orbit: $i = 1$; outer orbit: $i = 2$). | Flat distribution in $\log_{10}(P_{\text{orb},i})$, with $1 < \log_{10}(P_{\text{orb},i}/d) < 10$, and subject to dynamical stability constraints. |
| $a_i$ | Semimajor axis of orbit $i$. | Computed from $P_{\text{orb},i}$ and the $m_i$ using Kepler’s law. |
| $e_i$ | Eccentricity of orbit $i$. | Flat distribution between 0 and 0.9. |
| $i_i$ | Inclination of orbit $i$. | $0–180^\circ$ (flat distribution in $\cos i$) |
| $\omega_i$ | Argument of periastron of orbit $i$. | $0–360^\circ$ (flat distribution in $\omega$) |
| $\Omega_i$ | Longitude of the ascending node of orbit $i$. | $0–360^\circ$ (flat distribution in $\Omega$) |
are roughly consistent with observations of massive binary stars (Kobulnicky et al. 2014). Given the uncertainties in the observed orbital distributions of massive triples, we do not take into account a more sophisticated initial distribution, even though the observed distribution for massive MS triples reflects the initial distribution (Rose et al. 2019).

We reject a sampled system if it is unstable according to the criterion of Mardling & Aarseth (2001). The orientations of the orbits are taken to be random, i.e., the inclinations $i_i$ and longitudes of the ascending node $\Omega_i$ are sampled from flat distributions. There are indications that lower-mass triples have more aligned inner and outer orbits if the system is compact ($a_2 < 100$ au), whereas the wider systems are more isotropically distributed (Tokovinin 2017). However, it is unclear if this trend also persists for higher-mass triples (primaries with masses $\gtrsim 8 M_\odot$). For simplicity, we here restrict to randomly oriented triples.

### 3.3. Kick Distributions

As discussed in Section 2, we sample the kick distribution for SNe from a Maxwellian distribution (e.g., Hansen & Phinney 1997). We take the dispersion $\sigma_k$ to be fixed for each set of Monte Carlo simulations, and adopt three values: 0, 40, and 265 km s$^{-1}$. The value $\sigma_k = 0$ km s$^{-1}$ is to evaluate the importance of mass loss associated with SNe only (i.e., only the Blaauw kick, Blaauw 1961; Boersma 1961). The value $\sigma_k = 265$ km s$^{-1}$ is a commonly adopted value inferred from the proper motions of pulsars (Hobbs et al. 2005). However, NS kicks are uncertain and their magnitude is still highly debated. For example, Arzoumanian et al. (2002) found a two-component distribution of kick speed distributions with characteristic velocities of 90 and 500 km s$^{-1}$ based on the velocities of isolated radio pulsars, and Beniamini & Piran (2016) also found evidence for a bimodal distribution based on observed binary NSs, with a low-kick population with $V_k < 30$ km s$^{-1}$, and a high-kick population with kicks up to 400 km s$^{-1}$. Rather than adopting a bimodal distribution, here we choose to carry out another set with $\sigma_k = 40$ km s$^{-1}$, to evaluate the importance of low but nonzero kicks.

### 4. Results

#### 4.1. Outcome Fractions

#### 4.1.1. Main Channels

In Tables 2 and 3, we show the fractions for the main channels in our simulations assuming a flat distribution in $q_2 = m_2/(m_1 + m_2)$ and $q_2' = m_2'/m_2$, respectively (henceforth, we refer to the latter two assumptions on the tertiary mass ratio as the “high-mass tertiary” and “low-mass tertiary” cases, respectively). These data are based on simulations with SECULARMULTIPLE only. The first three data columns correspond to triple systems, whereas the last three data columns correspond to the binary case, i.e., taking the same triple systems but in the absence of the tertiary star. For each case, we show results for the three different values of the kick velocity dispersion, $\sigma_k$.

The channels shown in these tables include “No interaction,” i.e., the triple or binary survived for 10 Gyr without triggering interaction such as mass transfer, or dynamical instability or instability as a result of SNe. In most cases, this “inert” channel is unlikely; instead, much more common are RLOF or the unbinding of the system due to SNe. The only notable exception is for the binary case with $\sigma_k = 0$ km s$^{-1}$, in which case the noninteracting fraction is $\sim 0.2$. In the equivalent triple case, the noninteracting fraction is only $\sim 0.02$, i.e., a factor 10 times smaller. This can be attributed largely due to the Blaauw kick in the inner binary, which can keep the inner binary bound but make the tertiary unbound (compare the “NS+MS” Unbound fractions between the triple and binary cases with $\sigma_k = 0$ km s$^{-1}$).

For RLOF, we distinguish between RLOF of the primary, secondary, or tertiary star (in the latter case, the inner binary is treated as a point mass in the fits of Sepinsky et al. (2007)). RLOF is predominantly triggered by the primary star, with a fraction of $\sim 0.6$ assuming $\sigma_k = 0$ km s$^{-1}$, and $\sim 0.4$ assuming $\sigma_k > 0$ km s$^{-1}$ for the triple systems with high-mass tertiaries. The decrease in the RLOF fraction with increasing $\sigma_k$ in the latter case can be ascribed to the higher fraction of unbound systems, which in turn is mostly due to kicks imparted on the tertiary star when it evolves first (see the unbound MS + MS fractions for triples in Table 2). This trend is much less pronounced in the case of lower-mass tertiaries (Table 3)—in this case, the tertiary, which always has mass equal to or less than that of the secondary star, does not evolve first, and therefore is less likely to unbind the triple system due to its SNe kick.

In contrast to triples, for binaries, the RLOF star 1 fraction is independent of $\sigma_k$. This can be explained by noting that, for binaries, the primary star is always the most massive and evolves the fastest; whether or not a star fills its Roche lobe is completely determined by the initial masses ($m_1$ and $m_2$), $a_i$, and $e_i$, and independent of $\sigma_k$. This is no longer the case for the secondary star, e.g., RLOF of the secondary star can be triggered by an SNe event of the primary star, whose properties in turn are set by $\sigma_k$.

It may be surprising that the RLOF fraction is typically $\sim 0.6$ for both triple and binary cases. It might be expected that the RLOF fraction would be much higher for triple systems, as RLOF can be triggered by LK evolution. To explain this, we note that most previous studies of the onset of RLOF in triples (e.g., Hamers et al. 2013; Antonini et al. 2017; Hamers & Thompson 2019) have assumed the inner binaries to be wide.
Table 2
Outcome Fractions of the Simulations with the Tertiary Mass Assuming a Flat Distribution in $q_t \equiv m_3/(m_1 + m_2)$

|                    | Fraction of All Systems |                  |
|--------------------|-------------------------|------------------|
|                    | Triple                  | Binary           |
|                    | $\sigma_t$/km s$^{-1}$  | $\sigma_t$/km s$^{-1}$ |
| No interaction     | 0.024 ± 0.005           | 0.004 ± 0.002    |
|                    | 0.044 ± 0.021           | 0.004 ± 0.002    |
|                    | 0.442 ± 0.021           | 0.253 ± 0.016    |
|                    | 0.011 ± 0.003           | 0.008 ± 0.003    |
| RLOF +1            | 0.607 ± 0.025           | 0.445 ± 0.021    |
|                    | 0.066 ± 0.008           | 0.063 ± 0.008    |
|                    | 0.061 ± 0.008           | 0.061 ± 0.008    |
|                    | 0.460 ± 0.021           | 0.460 ± 0.021    |
|                    | 0.460 ± 0.021           | 0.460 ± 0.021    |
|                    | 0.460 ± 0.021           | 0.460 ± 0.021    |
| MS                 | 0.002 ± 0.001           | 0.000 ± 0.000    |
|                    | 0.000 ± 0.000           | 0.000 ± 0.000    |
|                    | 0.000 ± 0.000           | 0.000 ± 0.000    |
|                    | 0.000 ± 0.000           | 0.000 ± 0.000    |
|                    | 0.000 ± 0.000           | 0.000 ± 0.000    |
| RLOF +2            | 0.009 ± 0.003           | 0.009 ± 0.003    |
|                    | 0.009 ± 0.003           | 0.009 ± 0.003    |
|                    | 0.009 ± 0.003           | 0.009 ± 0.003    |
|                    | 0.009 ± 0.003           | 0.009 ± 0.003    |
|                    | 0.009 ± 0.003           | 0.009 ± 0.003    |
| Dynamical inst.    | 0.007 ± 0.002           | 0.007 ± 0.002    |
|                    | 0.007 ± 0.002           | 0.007 ± 0.002    |
|                    | 0.007 ± 0.002           | 0.007 ± 0.002    |
|                    | 0.007 ± 0.002           | 0.007 ± 0.002    |
|                    | 0.007 ± 0.002           | 0.007 ± 0.002    |
| NS + G             | 0.003 ± 0.002           | 0.003 ± 0.002    |
|                    | 0.003 ± 0.002           | 0.003 ± 0.002    |
|                    | 0.003 ± 0.002           | 0.003 ± 0.002    |
|                    | 0.003 ± 0.002           | 0.003 ± 0.002    |
|                    | 0.003 ± 0.002           | 0.003 ± 0.002    |
| Secular collision  | 0.004 ± 0.002           | 0.004 ± 0.002    |
|                    | 0.004 ± 0.002           | 0.004 ± 0.002    |
|                    | 0.004 ± 0.002           | 0.004 ± 0.002    |
|                    | 0.004 ± 0.002           | 0.004 ± 0.002    |
|                    | 0.004 ± 0.002           | 0.004 ± 0.002    |
| Unbound (SNe)      | 0.001 ± 0.000           | 0.001 ± 0.000    |
|                    | 0.001 ± 0.000           | 0.001 ± 0.000    |
|                    | 0.001 ± 0.000           | 0.001 ± 0.000    |
|                    | 0.001 ± 0.000           | 0.001 ± 0.000    |
|                    | 0.001 ± 0.000           | 0.001 ± 0.000    |
| Inner bound        | 0.286 ± 0.017           | 0.304 ± 0.017    |
|                    | 0.295 ± 0.017           |                  |

Notes. The first three data columns correspond to triple systems, whereas the last three data columns correspond to the binary case, i.e., taking the same triple systems but in the absence of the tertiary star. For each case, we show results for the three different values of the kick velocity dispersion, $\sigma_k$. Fractions are based on $N_{\text{tot}} = 10^5$ simulations, and quoted errors are based on Poisson statistics. Outcomes that do not apply (e.g., dynamically unstable systems in the binary case) are marked with "--." Some of the stellar evolutionary states are G—giants star (including red giants and asymptotic giants) and core helium burning star (CHeB). Refer to the text in Section 4.1.1 for a description of the different channels.

enough to avoid interaction in the absence of a tertiary star. In contrast, here we include tight systems as well. Therefore, a significant fraction of systems already interact in the "binary case." In addition, as shown by the higher unbound fractions for the triple cases compared to the binary cases, orbital changes due to SNe associated with the tertiary star play an important role in triples with NSs.
The fraction of dynamically unstable systems for triples is relatively small, with the fraction being at most \( \sim 0.03 \) if \( \sigma_k = 0 \text{ km s}^{-1} \). Dynamical instability is typically triggered by mass loss from the inner binary, i.e., the traditional triple dynamical instability scenario (Perets & Kratter 2012). Instability can also be triggered by SNe events, but this is a rare event. In fact, the dynamical instability fraction decreases with increasing \( \sigma_k \), which can be understood from the larger fraction of systems becoming unbound due to SNe before a dynamical instability can be triggered.

As discussed in Section 2, we also check for the semisecular regime in our simulations. This regime is triggered only very rarely, and predominantly with the inner binary consisting of two MS stars. The reason for the rarity of this channel is that very high eccentricities are required to trigger it, which usually instead lead to RLOF in eccentric orbits.
We also check for direct collisions (indicated with “Secular collision” in the tables). These do not occur for precompact objects, because RLOF is expected to ensue before direct collision. However, direct collisions could occur during later stages, when compact objects have formed and the inner orbit is excited in eccentricity due to secular evolution (e.g., Thompson 2011; Katz & Dong 2012). Nevertheless, we find no such direct collisions of compact objects. This can be attributed to the high fractions of systems that undergo RLOF or become unbound. In other words, there is a low probability that the system survives without interacting or becoming unstable due to SNe and a collision is triggered at later stages. We emphasize that our simulations are limited in terms of the number of systems. We therefore cannot exclude the possibility that direct collisions would occur if \( N_{\text{MC}} \) were increased. However, we focus here on the largest contributions to NS–NS mergers, which we find originate from interacting or unbound systems (i.e., RLOF-induced and “unbound” mergers). The latter are discussed in further detail below.

### 4.1.2. Mergers from RLOF Systems

As described in detail in Section 2.2, we continue the evolution of the inner binary after RLOF occurs in the inner binary using the binary population synthesis code BSE. In Tables 4 and 5 for the high- and low-mass tertiary simulations, respectively, we show the fractions for merger outcomes of the subsequent “isolated binary” simulations. These fractions are with respect to the systems undergoing RLOF triggered by the primary or secondary star; the fractions of the latter cases corresponding to all systems were given in Table 2. Here, “He” refers to a helium-burning star. Fractions are based on the systems in which RLOF was triggered in the inner binary system.
mass-loss induced eccentric LK mechanism (Shappee & Thompson 2013; Naoz et al. 2016).

The channel of most interest here is the merger of two NSs, which occurs for a relatively large fraction of \( \sim 0.1 \) of RLOF systems if \( \sigma_k = 0 \) km s\(^{-1} \), but drops quickly to a tenth of this fraction, to \( \sim 0.01 \), if \( \sigma_k = 265 \) km s\(^{-1} \). As expected, the kick dispersion has a large impact on the fraction of NS–NS mergers. Also, these fraction are mostly sensitive to \( \sigma_k \) and do not depend very strongly on the assumption of the tertiary mass ratio distribution (it should be taken into account, however, that the latter does affect the overall fraction of RLOF systems).

### 4.1.3. Mergers from Systems with Unbound Tertiaries

In addition to considering the subsequent “isolated binary” evolution of systems that undergo RLOF, we also consider the “isolated binary” evolution of systems in which the tertiary becomes unbound from the system, due to an SNe event, but the inner binary remains bound. In the latter case, the inner binary can subsequently merge due to “isolated binary” evolution, which we take into account by evolving these systems with BSE (see Section 2.2). Similarly to Tables 4 and 5, we show in Tables 6 and 7 the outcome fractions of several merger channels for these “unbound tertiary” systems. As is evident, there are no equivalent “binary” systems in this case (because this channel originates exclusively from triples). The fractions in these tables are given with respect to the systems in which the tertiary becomes unbound, but with a bound inner binary; the latter fractions with respect to all systems are given in the bottom rows in Tables 2 and 3 for the high-mass and low-mass tertiaries, respectively. Note that the latter fraction is very small for the low-mass tertiary simulations and non-zero \( \sigma_k \).

Similarly to RLOF-induced mergers, mergers originating from unbound tertiary systems are dominated by NS–NS mergers, particularly for nonzero \( \sigma_k \). The fraction of NS–NS mergers is relatively high at \( \sim 0.25 \) for \( \sigma_k = 0 \) km s\(^{-1} \), but rapidly decreases with increasing \( \sigma_k \), dropping to \( \sim 0.01 \) for \( \sigma_k = 265 \) km s\(^{-1} \) in the high-mass tertiary case, and to zero (within our statistical certainty) for \( \sigma_k = 265 \) km s\(^{-1} \) in the low-mass tertiary case.

### 4.2. Orbital Properties

We further discuss the RLOF-induced (Section 4.1.2) and tertiary unbound (Section 4.1.3) channels by showing (in Figures 2 and 3) the distributions of the inner binary semimajor axis \( a_1 \) and eccentricity \( e_1 \) at the moment of the onset of RLOF, and the unbinding of the tertiary star. Here, Figures 2 and 3 correspond to the high- and low-mass tertiary cases, respectively.

As expected, RLOF-induced systems tend to have significantly smaller inner binary semimajor axes compared to all systems (compare the red and black lines in the figures), with RLOF systems having a median of \( a_1 \sim 1 \) au, compared to \( a_1 \sim 10 \) au for all systems overall. Also, SNe kicks have very little impact on the orbital properties RLOF systems, as expected (because in our simulations, RLOF typically occurs before stars evolve to compact objects). In addition, for the RLOF systems, there are only slight differences in terms of \( a_1 \).
between the triple and binary cases. There is some difference in terms of the eccentricity—in the triple case, RLOF-induced systems tend to have slightly higher eccentricities compared to the binary case. This can be explained by eccentricity excitation by LK oscillations. There are no noticeable differences in the orbital properties of RLOF systems between the high- and low-mass tertiary simulations.

The “tertiary unbound” systems show typically larger semimajor axes compared to all systems (compare the green and black lines in the figures). With higher kicks, the semimajor axes tend to be larger compared to without ($\sigma_k = 0 \text{ km s}^{-1}$); this can be understood by noting that kicks tend to unbind the inner binary if it is wide—so, for the inner binary to remain bound, the inner binary semimajor axis should be smaller. Note that the number of unbound systems in the low-mass tertiary simulations for $\sigma_k > 0 \text{ km s}^{-1}$ are very small (see the bottom row in Table 3), causing the jagged behavior in Figure 3.

### 4.3. Delay-time Distributions

In Figures 4 and 5, we show delay-time distributions (DTDs) for various merger channels for the triple and binary cases, respectively. We include the RLOF-induced (red lines) and tertiary unbound (dashed green lines) cases. Each panel corresponds to a different channel, labeled in the top left, and we show results only for channels with a significant number of systems ($>5$). DTDs apply to the simulations with $\sigma_k = 40 \text{ km s}^{-1}$, as well as the high-mass tertiary assumption.

Generally, RLOF-induced mergers occur earlier than tertiary unbound mergers. This can be attributed to the generally tighter orbits (see Section 4.2). There are generally no large differences between the triple (Figure 4) and binary (Figure 5) RLOF-induced DTDs, which can be explained by the similarities in the initial orbits.

Of most interest here are the NS–NS mergers. Most of these mergers occur within $\sim 100 \text{ Myr}$, although there is a small tail with delay times of several Gyr.

The DTDs for simulations with different parameters (low-mass tertiary and different $\sigma_k$; not shown here) are qualitatively similar to those in Figures 4 and 5.

### 4.4. Merger Rates

Here, we compute the NS–NS merger rates based on the merger fractions from our simulations. To convert merger fractions to absolute rates, we start with the local star formation rate (SFR) density, which we take to be...
$R_{\text{SFR}} = 0.025 \times 10^9 M_\odot \text{Gpc}^{-3} \text{yr}^{-1}$ (Bothwell et al. 2011). This is the mass of all stars formed per unit volume and time. We are agnostic about the type of galaxy in which the stars are formed and consider the local universe only; this is justified by the typically short delay times (see Section 4.3). We assume that all stellar systems consist of either single, binary, or triple stars (and ignore high-order systems). Consider a population of stars with $N_{\text{sys}}$ stellar systems. The number of single, binary, and triple systems is then $\alpha_s N_{\text{sys}}, \alpha_{\text{bin}} N_{\text{sys}},$ and $\alpha_{\text{tr}} N_{\text{sys}}$, respectively, where $\alpha_s + \alpha_{\text{bin}} + \alpha_{\text{tr}} = 1$. We assume $\alpha_s = 0.19, \alpha_{\text{bin}} = 0.56,$ and $\alpha_{\text{tr}} = 0.25$ (Sana et al. 2014), independent of mass. It must be remarked that other recent observational studies (e.g., Moe & Di Stefano 2017) indicate that these fractions could be more heavily biased toward more high-multiplicity systems for higher primary masses.

$M_\ast = \int_{m_{\text{low}}}^{m_{\text{upp}}} \frac{dN}{dm} dm = N_{\text{sys}} \bar{M}$,  
where $N_{\text{sys}}$ is the number of (single) stars, $m_{\text{low}} = 0.01 M_\odot$ and $m_{\text{upp}} = 150 M_\odot$, and we calculate $\bar{M}$ to be $\bar{M} \simeq 0.38 M_\odot$ for a Kroupa (2001) IMF. In our mixed population with single, binary, and triple stars, the mass contribution from single stars is therefore $\alpha_s N_{\text{sys}} \bar{M}$. Assuming a flat mass ratio distribution (i.e., flat in $q_1 \equiv m_2/m_1$), the binaries have a mass contribution that is approximately $(1 + \frac{1}{2}) \bar{M} \alpha_{\text{bin}} N_{\text{sys}} = \frac{1}{2} \alpha_{\text{bin}} N_{\text{sys}} \bar{M}$. For triples, the mass contribution depends on our assumption regarding the tertiary mass ratio; we assumed distributions that

$\epsilon$(Bothwell et al. 2011). This is the mass of all stars formed per unit volume and time. We are agnostic about the type of galaxy in which the stars are formed and consider the local universe only; this is justified by the typically short delay times (see Section 4.3). We assume that all stellar systems consist of either single, binary, or triple stars (and ignore high-order systems). Consider a population of stars with $N_{\text{sys}}$ stellar systems. The number of single, binary, and triple systems is then $\alpha_s N_{\text{sys}}, \alpha_{\text{bin}} N_{\text{sys}},$ and $\alpha_{\text{tr}} N_{\text{sys}}$, respectively, where $\alpha_s + \alpha_{\text{bin}} + \alpha_{\text{tr}} = 1$. We assume $\alpha_s = 0.19, \alpha_{\text{bin}} = 0.56,$ and $\alpha_{\text{tr}} = 0.25$ (Sana et al. 2014), independent of mass. It must be remarked that other recent observational studies (e.g., Moe & Di Stefano 2017) indicate that these fractions could be more heavily biased toward more high-multiplicity systems for higher primary masses.

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$\epsilon$ here is inconsistent with the assumed slope for the massive stars in our simulations (primary masses $8 < m_1/M_\odot < 50$; slope $-2.55$). However, the difference in slope is very small.
from all hierarchies, the total mass of our population is (in the high-mass tertiary case)

\[ M_{\text{tot}} \approx \left( \alpha_1 + \frac{3}{2} \alpha_{\text{bin}} + \frac{9}{4} \alpha_{\text{tr}} \right) N_{\text{sys}} \dot{M} \]

\[ = \left( 1 + \frac{1}{2} \alpha_{\text{bin}} + \frac{5}{4} \alpha_{\text{tr}} \right) N_{\text{sys}} \dot{M} . \]

The number of binaries/triples is therefore given by

\[ N = \alpha N_{\text{sys}} = \frac{\alpha M_{\text{tot}}}{\left( 1 + \frac{1}{2} \alpha_{\text{bin}} + \frac{5}{4} \alpha_{\text{tr}} \right) \dot{M}} . \]  

where \( \alpha = \alpha_{\text{bin}} \) for binaries and \( \alpha = \alpha_{\text{tr}} \) for triples. The corresponding occurrence rates (per unit volume and time) are obtained by replacing \( M_{\text{tot}} \) in Equation (6) by \( \dot{R}_{\text{SFR}} \).

To convert the total occurrence rate of binaries/triples to (NS–NS) merger rates, we need to take into account the fraction of systems that were taken into account in the simulations compared to all astrophysically occurring systems (which we denote as \( f_{\text{calc}} \)), and the actual merger fractions in the simulations (which we denote as \( f_{\text{merge}} \)). Because we did not restrict the orbital separation distributions in the Monte Carlo simulations (see Section 3), the calculated fraction \( f_{\text{calc}} \) is determined solely by the mass cutoffs. Specifically, we restricted the primary mass to the range \( 8 < m_1/M_\odot < 50 \) and the secondary mass to \( 4 < m_2/M_\odot < 50 \), whereas there was no restriction on \( m_3 \) (within the mass ranges of the IMF of Kroupa 2001). For the adopted Kroupa (2001) IMF, this implies that \( f_{\text{calc}} \approx 2.6 \times 10^{-3} \), which applies to both triples and binaries. The merger fractions \( f_{\text{merge}} \) can be inferred from Tables 2–7.

The merger rate for binaries/triples (number per unit volume and time) is then given by

\[ R_{\text{merge}} \approx \frac{\alpha R_{\text{SFR}}}{\left( 1 + \frac{1}{2} \alpha_{\text{bin}} + \frac{5}{4} \alpha_{\text{tr}} \right) \dot{M}} f_{\text{calc}} f_{\text{merge}} . \]

Note that this equation applies to the high-mass tertiary case; in the low-mass tertiary case, the factor \( \frac{5}{4} \) in Equation (7) should be replaced by \( \frac{3}{2} \). The resulting NS–NS merger rates for all our simulations are given in Table 8. We also give the BH–NS merger rates in the same table. We do not include BH–BH rates, as the associated fractions \( f_{\text{merge}} \) in the simulations are at the noise level. The NS–NS DTD distributions normalized to the rates are shown in Figure 6, and the total rates are plotted as a function of \( \sigma_1 \) in Figure 7. We discuss our rates further in Section 5.2.

5. Discussion

5.1. The Decoupling Assumption

As mentioned in Section 2.2, a (strong) assumption that we made in our modeling is that, after RLOF is triggered in the inner binary system, the subsequent evolution of the inner binary is completely decoupled from the tertiary star. This approach was taken for simplicity, as well as due to the current lack of an available framework to model this phase self-consistently. Here, we briefly investigate this assumption by considering the systems that, after RLOF is triggered in the
Table 8
Compact Object Merger Rates According to Our Simulations

|                  | Rate (Gpc⁻³ yr⁻¹) |                  | Rate (Gpc⁻³ yr⁻¹) |
|------------------|--------------------|------------------|--------------------|
|                  | αₗ/s km s⁻¹        | 0 40 265         | αₗ/s km s⁻¹        | 0 40 265         |
| Triple           | 1741              | 551 109          | 3874              | 2119 542         |
| Unbound          | 1893              | 104 54           | ...               | ...              |
| Total            | 3634              | 655 164          | 3874              | 2119 542         |
| Low-mass tertiary| 1937              | 909 309          | 4149              | 1731 627         |
| Unbound          | 1856              | 0 0              | ...               | ...              |
| Total            | 3793              | 909 309          | 4149              | 1731 627         |
| BH–NS             |                    |                  |                    |                  |
| High-mass tertiary| 166               | 257 268          | 186               | 840 759          |
| Unbound          | 421               | 423 263          | ...               | ...              |
| Total            | 586               | 680 531          | 186               | 840 759          |
| Low-mass tertiary| 109               | 346 345          | 274               | 747 823          |
| Unbound          | 514               | 21 0             | ...               | ...              |
| Total            | 622               | 367 345          | 274               | 747 823          |
| NS–NS             |                    |                  |                    |                  |
| High-mass tertiary| 3464              | 5178 7006        | 7710              | 15238 20883      |
| Unbound          | 1232              | 4097 4650        | ...               | ...              |
| Total            | 4697              | 9275 11656       | 7710              | 15238 20883      |
| Low-mass tertiary| 3928              | 7475 10351       | 8299              | 16088 23368      |
| Unbound          | 1564              | 28 0             | ...               | ...              |
| Total            | 5493              | 7504 10351       | 8299              | 16088 23368      |

Notes. Top part: NS–NS mergers; middle part: BH–NS mergers. We also include the rate of NS–MS mergers (Thorne–Zytkow objects) in the bottom part. We include mergers from triples and binaries, and for different kick dispersions qₗ. The assumed channels are mergers following RLOF, and after the tertiary becomes unbound from the inner binary (only applies to triples). The “Total” row gives the sum of the two channels (note: quoted numbers have been rounded to integers). Results are shown for sets of simulations with the tertiary mass sampled according to m₁ = qₗ(m₁ + m₂) (“high-mass tertiary” case), and according to m₁ = qₐm₂ (“low-mass tertiary” case), where both q₂ and qₗ have flat distributions.

inner binary, survive the binary evolution (i.e., do not merge within 10 Gyr).

To evaluate whether or not the inner system can be decoupled from the tertiary, we compute for these systems the ratio of the timescale t₁PN for the lowest-order post-Newtonian (PN) precession (e.g., Weinberg 1972), to the LK timescale t₁LK (e.g., Kinoshita & Nakai 1999; Antognini 2015). If t₁PN ≪ t₁LK, we expect 1PN precession to dominate and the decoupling assumption to be justified, whereas if t₁PN ≫ t₁LK, this is no longer the case (e.g., Blaes et al. 2002; Wu & Murray 2003; Fabrycky & Tremaine 2007; Thompson 2011; Naoz et al. 2013; Liu et al. 2015).

In our simulations, we do not model the subsequent evolution of the tertiary star and orbit after the onset of RLOF.

Here, we assume, for simplicity, that the tertiary mass does not change between the onset of RLOF and 10 Gyr, and we make two assumptions regarding the outer orbit semimajor axis a₂ after 10 Gyr: we either take it to be the value at the onset of RLOF, or the value if all mass in the inner binary between the time of the onset of RLOF and 10 Gyr were lost adiabatically, i.e., with a₂(m₁ + m₂) constant.

In Figure 8, we show the resulting distributions of the ratio t₁PN/t₁LK, for two values of σₖ, as well as the high-mass tertiary simulations. As shown, there is a non-negligible fraction of systems with t₁PN/t₁LK ≫ 1, in which case the decoupling assumption is not well-justified. The fraction of these systems depends on σₖ: the distribution of t₁PN/t₁LK shifts to larger values with increasing σₖ.

This indicates that not all systems are well-decoupled from the tertiary, which implies there is a systematic error in our results, particularly the NS–NS merger rates. Furthermore, here...
we only considered the surviving systems; the systems undergoing RLOF may also not be truly decoupled. At this point, it is difficult to estimate whether fully self-consistent modeling would lead to lower or higher merger rates. Such an endeavor is left for future work.

5.2. Rates

Here, we comment on our NS–NS merger rates (see Section 4.4). First, as also discussed above, it should be noted that there are likely signiﬁcant systematic errors in our rates as a consequence of simpliﬁcations made in the modeling. Therefore, our results should be interpreted as estimates.

Interestingly, and most importantly, the triple rates, although always lower, are typically comparable to the binary rates (see Table 8). Both the triple and binary rates are highly sensitive to the SNe kick dispersion $\sigma_k$, as expected. However, the triple rates are even more sensitive to $\sigma_k$ than the binary rates. This can be explained by the fact that a third star adds more possibilities for the system to become unbound, and this effect increases in importance with increasing $\sigma_k$. It should also be mentioned that we assumed a conservative triple fraction ($\alpha_tr = 0.25$) compared to the binary fraction ($\alpha_{bin} = 0.56$). If a higher triple fraction were assumed relative to the binary fraction, for which there is observational evidence in massive multiple systems (Moe & Di Stefano 2017), then the importance of triple systems to the NS–NS merger rate is even larger. In particular, the ratio of the triple to the binary merger rate is $\propto \alpha_{tr}/\alpha_{bin}$ (see Equation (7)); therefore, if we assumed, e.g., $\alpha_{tr} = 0.5$ and $\alpha_{bin} = 0.4$ instead of $\alpha_{tr} = 0.25$ and $\alpha_{bin} = 0.56$, this ratio would increase by a factor of $\approx 3$.

Our rates vary from typically several hundred to several thousand Gpc$^{-3}$ yr$^{-1}$, strongly depending on $\sigma_k$. The rate inferred by GW170817 is $1540^{+3200}_{-1220}$ Gpc$^{-3}$ yr$^{-1}$ (Abbott et al. 2017b), which falls well within our ranges (for both triples and binaries). Based on observations of short gamma-ray bursts, Coward et al. (2012) find a rate of $\sim 8$–1800 Gpc$^{-3}$ yr$^{-1}$, Petrillo et al. (2013) of $\sim 500$–1500 Gpc$^{-3}$ yr$^{-1}$, Siellez et al. (2014) of $\sim 92$–1154 Gpc$^{-3}$ yr$^{-1}$, and Fong et al. (2015) of $\sim 90$–1850 Gpc$^{-3}$ yr$^{-1}$, also consistent with our rates.

Theoretical studies based on isolated binary evolution typically find NS–NS merger rates of several tens to hundreds Gpc$^{-3}$ yr$^{-1}$, up to the order of thousands in extreme cases. For example, Belczynski et al. (2016b) found rates of $\sim 50$–150 Gpc$^{-3}$ yr$^{-1}$, and Kruckow et al. (2018) found rates of $\sim 10$–400 Gpc$^{-3}$ yr$^{-1}$ (the upper limit being “rather optimistic”). The rates are highly sensitive to the CE $\alpha$ parameter, as shown by Giacobbo & Mapelli (2018), who found rates up to several hundred Gpc$^{-3}$ yr$^{-1}$ and up to $\sim 10^3$ Gpc$^{-3}$ yr$^{-1}$ if a high CE $\alpha$ value is assumed, as well as Chruslinska et al. (2018), who found similar results. Another crucial ingredient is the assumption on natal kicks (e.g., Giacobbo & Mapelli 2019).

Generally, isolated binary evolution studies find rates that are lower than our “binary” rates, as listed in Table 8. This can be understood by noting that our “binary” population consists of the inner binaries of a triple population. The requirement of dynamical stability implies that the inner binary separation distribution becomes skewed toward smaller values. As is evident in Figure 2 (see the black dotted lines), the inner binary semimajor axis distribution of the binaries in our simulations has a median value of $\approx 10$ au. In contrast, for a distribution flat in $\log_{10}(a_1)$ without the restriction of dynamical stability imposed by the tertiary, the median semimajor axis is $a_{low}(\alpha_{tr}/\alpha_{bin})^{1/2} \approx 32$ au (setting $\alpha_{low} = 0.02$ au, and $\alpha_{tr} = 5 \times 10^4$ au, see Figure 2). A more compact distribution of semimajor axes implies a higher formation rate of double NS binaries, explaining the higher base initial binary NS–NS merger rate in our simulations. Another way of understanding this trend is by noting that a more compact base initial semimajor axis distribution is somewhat analogous to assuming a higher CE efficiency. The latter indeed leads to higher merger rates, of several hundred and up to $\sim 10^3$ Gpc$^{-3}$ yr$^{-1}$ (Giacobbo & Mapelli 2018).

6. Conclusions

In this paper, we have estimated the rates of mergers of double neutron stars (NSs) in triple systems. We have accounted for secular, stellar, tidal, and binary evolution, and
the effects of supernovae (SNe) on the orbits, starting with main-sequence (MS) stars, until the merger of two NSs. We made different assumptions regarding the properties of the massive triple progenitors, including different kick distributions, and different assumptions on the tertiary mass distribution. Our main conclusions are given below.

1. Contrary to previous studies of the secular and stellar evolution of triples, which focused on wide inner binaries that do not interact in the absence of a tertiary star (e.g., Hamers et al. 2013; Antonini et al. 2017; Hamers & Thompson 2019), we found that the tertiary does not significantly affect the probability of Roche-lobe overflow (RLOF) in the inner binary system. This can be attributed to the fact that we did not restrict to wide, noninteracting systems, but instead considered the entire range of orbital separations. Consequently, the inner binary interacts also in the absence of the tertiary star. Typically, about 60% of systems undergo RLOF in our simulations, for both triple and binary cases (in the latter case, the same inner binary parameters are adopted, but without taking into account the effects of the tertiary star). We modeled the subsequent evolution of RLOF systems using a binary population synthesis code, neglecting the effect of the tertiary. Subsequently, the stars in the inner binary can merge as two NSs after periods of mass transfer and/or common-envelope evolution. Future work should not make the simplifying decoupling assumption, but rather model the system self-consistently (see Section 5.1).

2. For ∼10 up to ∼50% of systems, the inner and/or outer binary becomes unbound due to the effects of SNe (instantaneous mass loss and/or the effects of SNe kicks). Kicks are typically more important for triples, because a third star adds more possibilities for the system to become unbound, and this effect increases in importance with increasing kick dispersion. In up to ∼30% of simulated triples, the inner binary remains bound after an SNe event whereas the tertiary star becomes unbound. The inner binaries of these systems can still potentially merge at a later time due to “isolated binary” evolution. We continued the evolution of these isolated binaries using a binary population synthesis code (and considered these systems to be part of the original “triple” population). The fraction of triple systems without strong interactions is small, typically a few percent. This can be attributed to the large RLOF fraction (due to the inclusion of tighter systems), and the importance of SNe kicks, especially for NSs.

3. We considered two pathways for NS–NS mergers in triples: following binary interactions after RLOF (possibly induced by the tertiary star through Lidov–Kozai oscillations), or following binary interactions after the inner binary became unbound from the tertiary due to an SNe event. For the equivalent binaries (with the same properties as the inner binaries of the triples that we modeled), we considered only the channel of mergers following binary interactions after RLOF. Our rates (see Figure 7 for a visual summary) vary from typically several hundred to several thousand Gpc−3 yr−1, and are within the rate estimates of LIGO based on GW170817, 1540.2±228 Gpc−3 yr−1 (Abbott et al. 2017b). We find that the rates decrease strongly with increasing SNe kick speed, σk. Also, the ratio of the triple to binary NS–NS merger rate decreases with increasing σk. Our “binary” rates are higher than those of dedicated isolated binary evolution studies. This can be understood as due to the more compact inner binaries in our simulations, which are the result of the requirement of dynamical stability of the corresponding triple system.

4. Most of the NS–NS mergers in our models occur relatively early, with a delay-time distribution (DTD) peaked around several tens of Myr. Some mergers can occur at late times, i.e., several Gyr.

5. We also considered mergers of other types of stars (see Figures 4 and 5). In particular, we found a large fraction of NS–MS mergers in both triple and binary cases. Such mergers result in Thorne–Zytkow objects (Thorne & Zytkow 1977), and are also found in dedicated isolated binary evolution studies (e.g., Brandt & Podsiałowski 1995). Our results show that this channel is also possible (and relatively likely) in triples; in fact, the fractions of Thorne–Zytkow objects in our simulations are very similar in the triple and binary cases, giving formation rates on the order of several thousand Gpc−3 yr−1 (see Table 8). Taking into account the occurrence rates of triples and binaries, the relative formation rate of these objects formed in our simulations ranges between τRZ, triple/τRZ, binary ∼ 0.4 to ∼0.6 (depending on the assumed kick speed and tertiary mass ratio distributions). In addition, we found BH–NS mergers at a rate of typically several hundred Gpc−3 yr−1 (see Table 8).

We thank the anonymous referee for a helpful report. A.S.H. gratefully acknowledges support from the Institute for Advanced Study, and the Martin A. and Helen Chooljian Membership. T.A.T. is supported in part by a Simons Foundation Fellowship, an IBM Einstein Fellowship from the Institute for Advanced Study, NSF grant 1313252, and Scaligio Scholar grant 24216 from the Research Corporation.

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