NEUTRINO EVENT RATES FROM GAMMA-RAY BURSTS

F. HALZEN AND D. W. HOOPER

Department of Physics, University of Wisconsin at Madison, 2531 Sterling Hall, Madison, WI 53706

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ABSTRACT

We recalculate the diffuse flux of high-energy neutrinos produced by gamma-ray bursts in the relativistic fireball model. Although we confirm that the average single burst produces only \( \sim 10^{-2} \) high-energy neutrino events in a detector with a 1 km\(^2\) effective area, i.e., about 10 events yr\(^{-1}\), we show that the observed rate is dominated by burst-to-burst fluctuations that are very large. We find event rates that are expected to be larger by 1 order of magnitude, likely more, which are dominated by a few very bright bursts. This greatly simplifies their detection.

Subject heading: gamma rays: bursts

1. HIGH-ENERGY NEUTRINOS FROM RELATIVISTIC FIREBALLS

The evidence has been steadily accumulating that gamma-ray burst (GRB) emission is the result of a relativistically expanding fireball energized by a process involving neutron stars or black holes (Piran 1999). In the early stage, the fireball cannot emit efficiently because the radiation is trapped as a result of the very large optical depth. The fireball’s energy is dissipated in kinetic energy until it becomes optically thin and produces the observed display. This scenario can accommodate the observed energy scales and timescales, provided that the bulk Lorentz factor \( \gamma \) of the expanding shock is \( \sim 300 \).

The production of high-energy neutrinos is anticipated: protons, accelerated in the kinetic phase of the shock, interact with photons, producing charged pions that are the parents of neutrinos (Waxman & Bahcall 1997). Standard particle physics and fireball phenomenology are sufficient to compute the neutrino flux (Waxman & Bahcall 1997; Halzen 1999) as well as the observed rates in high-energy neutrino telescopes.

The observation of GRB neutrinos over a cosmological baseline has scientific potential beyond testing the “best-buy” fireball model: the observations can help us test, with unmatched precision, special relativity and the equivalence principle and can help us study the oscillating neutrino flavors over the ultimate baseline of \( \gamma \sim 1 \).

The anticipated neutrino flux traces the broken power-law spectrum observed for the photons, which provide the target material for neutrino production:

\[
\phi = \frac{A}{E_B} \text{for } E < E_B, \tag{1}
\]

\[
\phi = \frac{A}{E^2} \text{for } E > E_B, \tag{2}
\]

where \( A \) is a normalization constant that is determined from energy considerations and \( E_B \approx 700 \text{ TeV} \) (Halzen 1999). The total energy in GRB neutrinos is given by

\[
F_{\text{tot}} = \frac{\gamma}{4\pi} \left( \frac{1}{2} f_r t_{\text{Hubble}} \rho_E \right), \tag{3}
\]

where \( f_r \) is the fraction of proton energy that goes into pion production, \( t_{\text{Hubble}} \) is 10 Gyr, and \( \rho_E \) is the injection rate into protons accelerated to high energies in the kinetic fireball. This is a critical parameter in the calculations; it can be fixed, for instance, by assuming that GRBs are the source of the highest energy cosmic rays beyond the ankle (Waxman 1995; Milgrom & Usov 1995; Vietri 1995) in the spectrum, or \( \rho_E = 4 \times 10^{44} \text{ ergs Mpc}^{-3} \text{ yr}^{-1} \) (Halzen 1999). The factor \( f_r \) represents the fraction of total energy going into pion production. It is calculated by known particle physics and is of order 15% (Waxman & Bahcall 1997; Halzen 1999). We remind the reader that these assumptions reproduce the observed average photon energy per burst if equal energy goes into the hadronic and electromagnetic component of the fireball (Waxman & Bahcall 1997).

Now, normalizing the flux for this total energy,

\[
F_{\text{tot}} = \frac{A}{E_B} \int_{E_{\text{min}}}^{E_B} dE + A \int_{E_B}^{E_{\text{max}}} \frac{dE}{E}. \tag{4}
\]

Approximating \( E_B - E_{\text{min}} \approx E_B \), the integration constant is found to be \( 1.20 \times 10^{-12} \text{ TeV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \). The quantities \( f_r \) and \( E_B \) are calculated using equations (4) and (5) in Waxman & Bahcall (1997). The GRB parameters entering these equations are chosen following Halzen (1999) and are assumed to be independent of \( \gamma \). We assumed \( E_{\text{max}} = 10^7 \text{ TeV} \).

Given the neutrino flux \( \phi \), the event rates in a neutrino detector are obtained by a straightforward method (Halzen 1999):

\[
N = \int \phi(E) P_{\gamma \mu} dE, \tag{5}
\]

where \( P_{\gamma \mu} \) is the detection probability for neutrinos of muon flavor. It is determined, as a function of energy, by the neutrino cross sections and the range of the secondary muon (Gaisser, Halzen, & Stanev 1995; Gandhi et al. 1996). We obtain an observed neutrino flux of order 10 yr\(^{-1}\) km\(^{-2}\) cm\(^{-2}\) s\(^{-1}\) assuming 10\(^5\) bursts yr\(^{-1}\). This result is somewhat lower than but not inconsistent with those obtained previously. One should keep in mind that neutrinos can also be produced, possibly with higher fluxes, in other stages of the fireball, e.g., when it expands through the interstellar medium (Katz 1994; Halzen & Jaczko 1996; Vietri 1998; Böttcher & Dermer 1998).

2. BURST-TO-BURST FLUCTUATIONS

We here want to draw attention to the fact that this result should not be used without consideration of the burst-to-burst fluctuations, which are very large indeed (see also Dermer,
Fig. 1.—Rate of high-energy neutrino events in a detector with a 1 km$^2$ effective area as a function of the range of the bulk Lorentz factor $\gamma$. The range is assumed to be Gaussian with width $\sigma$ below the average of 300 and $\sigma'$ above. The latter is taken to be either 0 or 300.

Fig. 2.—Yearly rate of individual neutrino bursts with more than seven events in a detector with a 1 km$^2$ effective area as a function of the range of the bulk Lorentz factor $\gamma$. The range is assumed to be Gaussian with width $\sigma$ below the average of 300 and $\sigma'$ above. The latter is taken to be either 0 or 300.

Fig. 3.—Neutrino multiplicity distribution in individual GRBs in a detector with a 1 km$^2$ effective area. The range of the bulk Lorentz factor $\gamma$ is assumed to be Gaussian with widths $\sigma$ = 30, 60, and 75 below the average of 300 and with $\sigma'$ taken to be 0.

Böttcher, & Chiang 1999). We will, in fact, conclude that from an observational point of view, the relevant rates are determined by the fluctuations and not by the average burst. Our calculations are performed by fluctuating individual bursts according to the model described above (1) with the square of the distance that is assumed to follow a Euclidean or cosmological distribution; (2) with the energy, assuming that the neutrino rate depends linearly on energy and follows a simple step function with 10% of GRBs producing more energy than average by a factor of 10 and 1% by a factor of 100; and, most importantly, (3) with fluctuations in the $\gamma$-factor around its average value of 300. The fluctuations in $\gamma$ affect the value of $f_\nu$, which varies approximately as $\gamma^{-4}$ (Waxman & Bahcall 1997), as well as the position of the break energy, which varies as $\gamma^2$. For a detailed discussion, see Halzen (1999). Both effects are taken into account. Clearly, a factor of 10 variation of $\gamma$ leads to a change in flux by roughly 4 orders of magnitude.

The origin of the large fluctuations with $\gamma$ should be a general property of boosted accelerators. With high luminosities emitted over short times, the large photon density renders the GRB opaque unless $\gamma$ is very large. Only transparent sources with large boost factors emit photons. They are, however, relatively weak neutrino sources because the actual photon target density in the fireball is diluted by the large Lorentz factor.

This raises the unanswered question of whether there are bursts with lower $\gamma$-factors. Because of absorption, such sources would emit fewer photons of lower energy and could have been missed by selection effects; they would be spectacular neutrino sources. Some have argued (Stern 1999) on the basis that the unusual fluctuations in the morphology of GRBs can only be produced in a relatively dense, turbulent medium.

3. MONTE CARLO SIMULATION RESULTS

We will illustrate the effect of fluctuations by Monte Carlo simulation of GRBs. For a Euclidean distribution, the calculation can be performed analytically; in other cases, the Monte Carlo method evaluates the integrals. The overwhelming effect of fluctuations is demonstrated in sample results shown in Figures 1, 2, and 3 and in Table 1. Not knowing the distribution of $\gamma$-factors around its average, we have parameterized it as a Gaussian with widths $\sigma$ and $\sigma'$ below and above the average.
value of 300. We chose \( \sigma' \) to be either 0 or 300, to illustrate the effect of allowing GRBs with Lorentz factors up to \( 10^3 \) with a Gaussian probability. The critical, and unknown, parameter is \( \sigma \). It may, in fact, be more important than any other parameter entering the calculation. As far as we know, neither theory nor experiment provide compelling information at this point. Note that we require \( \sigma > 70 \) in order to allow a significant part of the bursts to have \( \gamma \)-factors less than \( 10^2 \), or one-third the average value (see Table 1). The value of \( \sigma' \) is less critical. A value of 300 allows for Lorentz factors as large as \( 10^3 \). The dominant effect of this is a renormalization of the neutrino rates because a fraction of the bursts now has a large \( \gamma \)-factor and, as a consequence, a low neutrino flux. We discuss some quantitative examples next.

First, even in the absence of the dominant fluctuations in \( \gamma \), the rate of nine detected neutrinos per year over \( 10^5 \) bursts becomes 30–90 in the presence of fluctuations in energy and distance. The range covers a variety of assumptions for GRB distributions, which range from Euclidean to cosmological; the issue is not important here because other factors dominate the fluctuations. Every 2 years, there will be an event with seven neutrinos in a single burst. For \( \sigma = 70 \), the rate is \( \sim 600 \) yr\(^{-1} \) in a kilometer square detector, with 23 individual bursts yielding more than four neutrinos and more than 10 in a single year! The results for other values of \( \sigma \) are tabulated in Table 1. The number of events per year for a range of values of \( \sigma \) is shown in Figure 1. The frequency of GRBs producing seven or more neutrinos is shown in Figure 2. The neutrino multiplicity of bursts for \( \sigma = 30, 60, \) and 75 is shown in Figure 3. Note that absorption in the source eventually reduces the overall rates when \( \gamma \) is much smaller than average or when \( \sigma \) is larger than 60. This factor is included in our calculations (Halzen 1999).

With a Gaussian probability, the critical, and unknown, parameter of 300. We chose \( \sigma' \) to be either 0 or 300, to illustrate the effect of allowing GRBs with Lorentz factors up to \( 10^3 \) with a Gaussian probability. The critical, and unknown, parameter is \( \sigma \). It may, in fact, be more important than any other parameter entering the calculation. As far as we know, neither theory nor experiment provide compelling information at this point. Note that we require \( \sigma > 70 \) in order to allow a significant part of the bursts to have \( \gamma \)-factors less than \( 10^2 \), or one-third the average value (see Table 1). The value of \( \sigma' \) is less critical. A value of 300 allows for Lorentz factors as large as \( 10^3 \). The dominant effect of this is a renormalization of the neutrino rates because a fraction of the bursts now has a large \( \gamma \)-factor and, as a consequence, a low neutrino flux. We discuss some quantitative examples next.

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Up to this point, we have assumed that all quantities fluctuate independently. Interestingly, the value of \( \sigma = 70 \) is obtained in the scenario where, instead, fluctuations in \( \gamma \) are a consequence of fluctuations in energy. In order not to double count, fluctuations in energy itself, which only contribute a factor of 3 to the neutrino rate anyway, should be omitted.

Even though bursts with somewhat lower \( \gamma \) produce neutrinos with reduced energy (compared with 700 TeV), over a longer timescale than 1 s, the signatures of these models are spectacular. Existing detectors with effective areas at the 1%–10% of a kilometer squared should produce results relevant to the open question on the distribution of bulk Lorentz factors in the fireball model.

Independent of the numerics, the fact that a single GRB with high energy, close proximity, and a relatively low Lorentz factor can reasonably produce more detectable neutrino events than all other GRBs over several years time renders the result of the straightforward diffuse flux calculation observationally misleading. Our calculations suggest that it is far more likely that neutrino detectors detect one GRB with favorable characteristics rather than hundreds with average values. Clearly, our observations are relevant for other GRB models as well as for blazars and any other boosted sources. They are also applicable to photons and may represent the underlying mechanism because of the fact that the TeV extragalactic sky consists

| \( \sigma \) | \( 0-10 \) | \( 10-50 \) | \( 50-100 \) | \( 100-200 \) | \( 200-300 \) | \( \sigma = 0 \) |
|---|---|---|---|---|---|---|
| 80 | 0.04 | 0.15 | 1.90 | 19.83 | 78.9 | 1276 |
| 75 | 0.01 | 0.08 | 0.68 | 17.4 | 81.8 | 814 |
| 70 | 0 | 0.04 | 0.39 | 14.8 | 84.8 | 611 |
| 65 | 0 | 0.02 | 0.20 | 12.1 | 87.7 | 483 |
| 60 | 0 | 0 | 0.09 | 9.46 | 90.5 | 394 |
| 55 | 0 | 0 | 0.04 | 6.86 | 93.1 | 326 |
| 50 | 0 | 0 | 0.01 | 4.52 | 95.5 | 271 |
| 45 | 0 | 0 | 0 | 2.62 | 97.4 | 227 |
| 40 | 0 | 0 | 0 | 1.28 | 98.7 | 193 |
| 35 | 0 | 0 | 0 | 0.44 | 99.6 | 167 |
| 30 | 0 | 0 | 0 | 0 | 99.9 | 146 |
| 0 | 0 | 0 | 0 | 0 | 100 | 86 |

Table 1: High-Energy Neutrino Events in Detector with 1 km\(^2\) Effective Area as Function of Range of Bulk Lorentz Factor \( \gamma \)

| \( \sigma \) | \( \sigma' = 0 \) | \( \sigma' = 300 \) |
|---|---|---|
| 80 | 0.01 | 0.05 |
| 75 | 0 | 0.03 |
| 70 | 0 | 0.01 |
| 65 | 0 | 0.05 |
| 60 | 0 | 0.02 |
| 55 | 0 | 0.01 |
| 50 | 0 | 0.04 |
| 45 | 0 | 0.03 |
| 40 | 0 | 0.17 |

\( ^* \) The range is assumed to be Gaussian with width \( \sigma \) below the average of 300 and \( \sigma' \) above. The latter is taken to be either 0 or 300. The total number of neutrinos, as well as the yearly number of individual bursts with more than four, seven, and 11 high-energy neutrinos, is shown as a function of \( \sigma \). For comparison, we also show the result when all fluctuations are removed.
of a few bursting sources. We expect no direct correlation between neutrino and high-energy gamma sources because cosmic events with abundant neutrino production are almost certainly opaque to high-energy photons.

4. CONCLUSIONS

Although the average event rates predicted with typical GRB parameters appear somewhat discouraging to present and future Cerenkov neutrino detector experiments, the fluctuations in these calculations are more significant and affect the prospects for detection. We speculated on the distribution of the parameters entering the fireball model calculation and used a Monte Carlo simulation to estimate the actual event rates. The result of these simulations shows that a kilometer-scale detector could be expected to observe tens or hundreds of events per year. To improve on the reliability of this estimate, a well-defined distribution for the Lorentz factor must be determined. Contrariwise, not observing neutrino GRBs after years of observation will result in a strong limit on the number of accelerated protons in the kinetic phase of the burst or in fine-tuned, high values of the Lorentz boost factor.

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