Generalized string compactifications with spontaneously broken supersymmetry

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Abstract
The Narain lattice construction of string compactifications is generalized to include spontaneously broken supersymmetry. Consistency conditions from modular invariance and Lorentz symmetry are solved in full generality. This framework incorporates models where supersymmetry breaking is inversely proportional to the radii of compact dimensions. The enhanced lattice description, however, might allow for models with a different geometrical or even non-geometrical interpretation.
1 Introduction

The problem of supersymmetry breaking in string theory is one of the basic problems the theory faces if it is to be relevant for elementary particle physics. One possibility, which has to some extent been explored in the past \cite{1, 2, 3}, is that supersymmetry may be broken spontaneously at tree level in string theory. This possibility is actually rather special, since it is known \cite{4} that vacua with broken supersymmetry cannot be continuously connected to ones where supersymmetry is unbroken. The only possibility which the theorem leaves open is that the broken and unbroken vacua are connected, but one cannot be reached from the other by a sequence of infinitesimal transformations. This is realized, for example, when the supersymmetry breaking order parameter vanishes as one of the compactification radii goes to infinity: only in this decompactification limit is supersymmetry restored.

This paper presents a systematic method of constructing a large class of N=4 models with spontaneously broken supersymmetry, which can also be twisted to reduce the number of supersymmetries. The approach followed here is a direct generalization of the work of Narain et al. \cite{5, 6} in the spirit of \cite{7, 8, 9}. In this approach, all toroidal compactifications are described by specifying a family of Narain lattices parameterized by a set of moduli. These can either appear in the basis vectors of the lattice, or the basis can be constant over moduli space while the moduli enter as components of various background fields. We will make use of both these pictures. The virtue of the Narain formulation of toroidal string vacua is that the moduli dependence of the theory is explicit. A given model is not locked into some special point in moduli space, as is the case for covariant lattice constructions \cite{10}, or the free fermionic construction \cite{11, 12}.

The Narain lattice (for the case of a 4 dimensional compactification) has dimension (6,22). The worldsheet NSR fermions play no role in the compactification process – that sector is the same in the compact theory as in the non-compact case. The basic idea put forward here is to bosonize the NSR fermions and treat them on the same footing as the other compact coordinates. These additional bosonic degrees of freedom will be referred to as “NSR bosons” in the sequel. In particular, the construction outlined below allows backgrounds which mix these additional directions in the lattice with the ones present already in the work of Narain, Sarmadi and Witten. It is this new class of backgrounds which makes spontaneous supersymmetry
breaking possible.

The key issues are whether modular invariance and Lorentz covariance can be maintained in the presence of the new backgrounds. It is straightforward to check that one-loop modular invariance holds\(^1\) for a nontrivial class of backgrounds. These new backgrounds bear a formal analogy to the Wilson lines; however rather than correlating the space compactification lattice with the \(E_8 \times E_8\) lattice, they connect it with the part corresponding to the NSR bosons. In what follows, the new backgrounds will be referred to as generalized Wilson lines. In accordance with general theorems these extra background parameters will be seen to be quantized in order to preserve Lorentz covariance.

The construction presented here gives theories with \(N=4\) supersymmetry. Some of these supersymmetries are spontaneously broken for an appropriate choice of the backgrounds (or equivalently, of the Narain lattice). It is natural to look at orbifolds of these models which would give spontaneously broken \(N=1\) supersymmetric theories.\(^2\) This should be straightforward to achieve using, for example, the approach of [8, 9, 13]. The formulation of the problem presented here is set up so as to carry out this step. The results of that study will be presented in a more detailed communication in the future.

The constructions of string compactifications with spontaneously broken supersymmetry which exist in the literature were carried out via so-called coordinate dependent compactifications [2] from 5 to 4 dimensions, first in the context of fermionic string models, and more recently also for Narain models [14]. These constructions involve specially chosen \(U(1)\) currents – these currents correspond to particular directions in the extended Narain lattices introduced in this paper.

The construction given here works for compactifications down to any dimension \(2 < d < 10\). The discussion is presented for compactification down to \(d = 4\) to be specific. Apart from generalizing known examples and giving a systematic means of studying models with spontaneously broken symmetry in a familiar framework, the construction presented here could play a role in finding pairs of dual string theories with a reduced number of supersymmetries [15, 16]. Indeed, it is natural to consider strings compactified to low dimensions with reduced supersymmetry: such compactifications would then

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\(^1\)In this communication we do not address the issue of higher loop modular invariance.

\(^2\)The class of theories obtained in this way includes the examples of references [2, 3].
lead to higher dimensional theories with higher numbers of supersymmetries in various “decompactification” limits. It is also interesting to see how the models described here fit in with the general considerations of string spectra [17]. In particular it would be relevant for phenomenology to check whether the supertrace of the mass square matrix indeed vanishes in these models [18].

The paper is structured as follows: section 2 describes a fully bosonized incarnation of the heterotic string compactified on a Narain lattice in a form suitable for further developments. In section 3 the generalized Wilson lines are introduced and consistency with one-loop modular invariance and Lorentz symmetry is discussed. Section 4 presents some general considerations of the spectrum and shows how supersymmetry breaking can take place in these models.

2 Bosonization

This section describes how the Narain compactified heterotic string can be formulated in purely bosonic terms. The fact that this can be done is well known – the reason for reviewing the subject here is to extend the framework of [8] to include bosonized fermions, so that the generalization discussed in the following section can be easily stated.

The heterotic string in the light front gauge can be described purely in terms of the following bosonic fields:

1. Non-compact (transverse) spacetime coordinates $X^a$, $a = 1, 2$.
2. Left and right–moving bosons corresponding to “internal” degrees of freedom $X^i, \bar{X}^i$, $i = 1...6$.
3. Left–moving bosons $Y^I$, $I = 1...16$, compactified on the $E_8 \times E_8$ lattice.
4. Right–moving bosons $H^A$, $A = 1..4$, which bosonize the world-sheet NSR fermions (the “NSR bosons”).

The zero-modes of the compact fields lie on a generalized Narain lattice of dimension $(10, 22)$. The possibility of writing the partition function for the heterotic string in the light–front gauge in terms of these degrees of freedom has been noted in [11, 10].
For the moment only the metric moduli of the (6,6) compactification lattice will be considered. The remaining moduli of the Narain lattice – the antisymmetric tensor field and the Wilson lines will be incorporated later.

The spectrum of the theory follows from the Virasoro generators, which can be written as\footnotemark
\begin{align}
    L_0 & = \frac{1}{4} p^2 + L'_0 + N - \frac{1}{2}, \tag{1} \\
    \bar{L}_0 & = \frac{1}{4} p^2 + \bar{L}'_0 + \bar{N} - 1, \tag{2}
\end{align}

where \( p \) is the spacetime 4-momentum, \( N, \bar{N} \) are integer valued oscillator number operators, and the (compact) zero-mode contributions read (in matrix notation)
\begin{align}
    L'_0 & = \frac{1}{4} p_R^T g^{-1} p_R + \frac{1}{2} \tilde{p}_R h^{-1} \tilde{p}_R, \tag{3} \\
    \bar{L}'_0 & = \frac{1}{4} p_L^T g^{-1} p_L + \frac{1}{2} \tilde{p}_L c^{-1} \tilde{p}_L. \tag{4}
\end{align}

Here
\begin{align}
    p_R & = \hat{m} - g \hat{n}, \tag{5} \\
    p_L & = \hat{m} + g \hat{n}, \tag{6}
\end{align}

and
\begin{align}
    \tilde{p}_R & = \hat{r} + \hat{t}, \tag{7} \\
    \tilde{p}_L & = \hat{t}, \tag{8}
\end{align}

are the momenta in the lattice basis. The hatted quantities have integer entries apart from the vector \( \hat{t} \), which is given by
\begin{equation}
    \hat{t} = (-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, 1). \tag{9}
\end{equation}

In the following it will sometimes be convenient to use the notation
\begin{equation}
    \hat{w} \equiv \hat{r} + \hat{t}, \tag{10}
\end{equation}

\footnotemark[3] All formulae assume \( \alpha' = 1 \).
which is also not integer.

The shift vector \( \hat{t} \) appears because of the spacetime fermions present in the spectrum of the heterotic string. In a canonical basis, described later on, the shift by \( \hat{t} \) is responsible for transforming the \( o \) and \( c \) cosets of the \( D_4 \) lattice to the \( v \) and \( s \) cosets actually appearing in the physical partition function. The vectors \( \hat{m}, \hat{n} \) are the momenta and windings of the 6 compactified coordinates, \( \hat{l} \) are the momenta of the \( Y \) bosons, while \( \hat{r} \) denote the momenta of the chiral bosons \( H \) corresponding to the NSR fermions. These are all (column) vectors of dimensions 6, 6, 16, 4 respectively.

The matrices \( g, h, c \) are the metrics of various parts of the full lattice. The matrix \( g \) is a real symmetric matrix containing the metric moduli of the \( (6, 6) \) compactification lattice of the internal compact coordinates. The matrices \( c, h \) are fixed, integer matrices whose form is required by modular invariance: \( c \) is the \( E_8 \times E_8 \) lattice Cartan matrix, while \( h^{-1} \) is given by

\[
h^{-1} = \begin{pmatrix}
2 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 \\
1 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 \\
\end{pmatrix}.
\]

The form of this metric can be found, for example, by studying the Poisson resummed partition function in the standard formulation.

It is often convenient \[8\] to use the quantities \( H \) and \( P \) defined by

\[
H = L'_0 + \bar{L}'_0,
\]

\[
P = L'_0 - \bar{L}'_0.
\]

Note that only the zero-mode contributions enter here – the oscillators and normal ordering constants do not appear in this definition. One finds

\[
H = \frac{1}{2}(u + s)^T \chi(u + s),
\]

\[
P = -\frac{1}{2}(u + s)^T \eta(u + s),
\]

where

\[
u = \begin{pmatrix}
\hat{m} \\
\hat{n} \\
\hat{l} \\
\hat{r} \\
\end{pmatrix},
\]

5
is a vector of 32 integers grouping all the zero-mode quantum numbers of the various string states, while

\[ s = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hat{t} \end{pmatrix}. \]  

(17)

The matrix \( \eta \) is the constant Narain lattice metric:\n
\[ \eta = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & c^{-1} & 0 \\ 0 & 0 & 0 & -h^{-1} \end{pmatrix}, \]  

(18)

while \( \chi \) contains all the modulus dependence:

\[ \chi = \begin{pmatrix} g^{-1} & 0 & 0 & 0 \\ 0 & g & 0 & 0 \\ 0 & 0 & c^{-1} & 0 \\ 0 & 0 & 0 & h^{-1} \end{pmatrix}. \]  

(19)

When other moduli are given non-vanishing values, the matrix \( \eta \) retains its (constant) form – the moduli always appear in \( \chi \) only. Thus the issue of introducing new backgrounds can be formulated as finding forms of the background matrix \( \chi \) which satisfy the requirements of modular invariance.

To check modular invariance one has to study the behaviour of the partition function of the theory under transformations of the worldsheet modular parameter \( \tau \). Using the objects given above, the moduli dependent zero-mode contribution to the one-loop partition function can be written in the form

\[ P(\tau, \bar{\tau}) = \sum_u \exp \{ -\pi (u + s)^T M(\tau, \bar{\tau})(u + s) + 2\pi i u^T \eta s \}, \]  

(20)

where

\[ M(\tau, \bar{\tau}) = iRe(\tau)\eta + Im(\tau)\chi. \]  

(21)

The term linear in \( u \) which appears in the exponential in (20) is responsible for the GSO projection in the spectrum. It is straightforward to check that this leads to the known result for the toroidally compactified heterotic string.

\[ ^4 \text{Note that the extended Narain lattice is no longer even.} \]
one-loop partition function, expressed in terms of Jacobi theta functions, the
\( E_8 \times E_8 \) lattice sum and a soliton sum over the (6, 6) compactification lattice.

Modular invariance of these theories can be demonstrated using the fol-
lowing remarkable properties of the matrix \( M \):

\[
M(\tau + 1, \bar{\tau} + 1) = M(\tau, \bar{\tau}) + i\eta, \tag{22}
\]

\[
M\left(-\frac{1}{\tau}, -\frac{1}{\bar{\tau}}\right) = \eta M^{-1}(\tau, \bar{\tau})\eta. \tag{23}
\]

The first of these properties is true for any background, as is clear from the
definition (21). The second property, however, places stringent restrictions
on the possible background matrices \( \chi \).

When nontrivial antisymmetric tensor and Wilson line backgrounds are
introduced in the correct way, these transformation properties of \( M \) remain
unaltered.

The Narain models of [6] are expressed in this language by the following
form of the background field matrix [8, 9] :

\[
\chi = \begin{pmatrix}
g^{-1} & -g^{-1}b' & -g^{-1}a^Tc^{-1} & 0 \\
-b^Tg^{-1} & (g + b^T)g^{-1}(g + b') & (g + b^T)g^{-1}a^Tc^{-1} & 0 \\
-c^{-1}ag^{-1} & (g + b^T)c^{-1} & (g + b^T)g^{-1}a^Tc^{-1} & 0 \\
0 & 0 & 0 & h^{-1}
\end{pmatrix}, \tag{24}
\]

where

\[
b' \equiv b + \frac{1}{2}a^2, \tag{25}
\]

\[
a^2 \equiv a^Tc^{-1}a. \tag{26}
\]

Here \( b \) is an antisymmetric matrix (6 by 6) containing the “axionic” moduli
and \( a \) is the matrix (16 by 6) containing the Wilson line moduli. One may
check directly that (23) is fulfilled.

3 Generalized Narain Models

This section presents the main technical development of this paper. We
consider background field matrices \( \chi \) which involve lattice directions corre-
sponding to the zero-modes of the NSR bosons. It is clear that the appear-
ance of such backgrounds will generically lead to the breaking of worldsheet
superconformal invariance and consequently to the spontaneous breaking of spacetime supersymmetry. It will later be shown that in order that these backgrounds do not lead to a model with broken Lorentz symmetry, the new background parameters have to be quantized.

The new moduli are grouped into a 4 by 6 matrix denoted by $x$. To simplify the formulae somewhat, the ordinary Wilson lines have been set to naught, since in any case they will not be discussed further here. The modified background matrix, including the generalized Wilson lines $x_T$, reads:

$$
\chi = \begin{pmatrix}
g^{-1} & -g^{-1}b' & 0 & g^{-1}x_T h^{-1} \\
-b'^T g^{-1} & (g - b'^T)g^{-1}(g - b') & 0 & (g - b'^T)g^{-1}x_T h^{-1} \\
0 & 0 & c^{-1} & 0 \\
h^{-1}xg^{-1} & h^{-1}xg^{-1}(g - b') & 0 & h^{-1} + h^{-1}xg^{-1}x_T h^{-1}
\end{pmatrix}.
$$

(27)

Here

$$
b' \equiv b - \frac{1}{2}x^2,
$$

(28)

and

$$
x^2 \equiv x^T h^{-1} x.
$$

(29)

It is straightforward to verify that the new background fields $x$ enter in such a way that the crucial property (23) remains intact. This way one ensures that the resulting partition function is modular invariant.

To make the formulae more readable it is convenient at this stage to introduce a (moduli dependent) canonical basis defined by the condition that in that basis the metrics $g, h, c$ should be unit matrices. When referring to this basis, the vectors $\hat{m}, \hat{n}, \hat{l}, \hat{w}$ will be written without the hats, while the 32 dimensional objects like $u$ or $\chi$ will be written with a tilde over them.

The background fields, and the left and right momenta hitherto written in lowercase letters, will be written in capitals when expressed in the canonical basis. The basis transformation matrices satisfy

$$
g = e_g^T e_g, \quad h = e_h^T e_h, \quad c = e_c^T e_c,
$$

(30)

so that

$$
m = e_g^* \hat{m},
$$

$$
n = e_g \hat{n},
$$

$$
w = e_g^* \hat{w},
$$

$$
l = e_c^* \hat{l}
$$

(31)
where $A^*$ denotes $(A^T)^{-1}$.

For future reference let us note, that the basis transformation matrix relating the lattice basis and the canonical basis in the NSR block is given by

$$e_h^* = \begin{pmatrix}
-1 & -1 & -1 & -1/2 \\
1 & 0 & 0 & 1/2 \\
0 & 1 & 0 & 1/2 \\
0 & 0 & 1 & 1/2 \\
\end{pmatrix}.$$  \hspace{1cm} (32)

The generalized Wilson line is transformed as

$$X = e_h^*xe^{-1},$$

so in the canonical basis this depends on the metric moduli.

For the present purpose it will suffice to keep just the generalized Wilson lines, setting the antisymmetric background to naught. Using the notation

$$X^2 \equiv X^TX,$$

the background matrix in this basis is

$$\tilde{\chi} = \begin{pmatrix}
1 & \frac{1}{2}X^2 & 0 & X^T \\
\frac{1}{2}X^2 & (1 + \frac{1}{2}X^2)^2 & 0 & (1 + \frac{1}{2}X^2)X^T \\
0 & 0 & 1 & 0 \\
X & X(1 + \frac{1}{2}X^2) & 0 & 1 + XX^T \\
\end{pmatrix}.$$  \hspace{1cm} (35)

The various lattice metrics do not appear explicitly here, but the consequence is that $X$ is now moduli dependent.

While one-loop modular invariance is guaranteed by construction for all backgrounds of this kind, the requirement of Lorentz covariance of the theory introduces further restrictions. The preservation of Lorentz symmetry in the light-front gauge requires that the supercurrent be well defined\footnote{This means that under $\sigma \to \sigma + 2\pi$ the supercurrent should either be periodic (in the Neveu–Schwarz sector) or anti-periodic (in the Ramond sector).}. This means that for the generalized Wilson lines, note that the supercurrent has the form

$$T_F = \sum_{A=1}^{4} \exp(iH^A)\partial X^A.$$  \hspace{1cm} (36)
The $X^A$ are complex linear combinations of the original $X$’s. This follows from the usual form of the supercurrent in the fermionic language via the bosonization relations

$$\psi^A = \exp(iH^A),$$

where $\psi^A$ are the (complex) NSR fermions. The supercurrent boundary condition puts constraints on the allowed spectrum of zero-modes of the NSR bosons, that is, on the allowed eigenvalues of $\tilde{p}_R$. From the explicit form of the background matrix $\chi$ it follows that

$$\tilde{p}_R = \hat{w} + \hat{x}\hat{n}.$$ (38)

The supercurrent condition requires that in the canonical basis these vectors should have either all integer components, or all half-integer components. Explicitly,

$$\tilde{P}_R = w + e^*_h x\hat{n}.$$ (39)

The states in the Hilbert space for which $\tilde{P}_R$ is integral belong to the Neveu-Schwarz sector, and the ones for which it is half integral belong to the Ramond sector. This quantization condition on the allowed spectrum of $\tilde{P}_R$ translates immediately to a quantization condition on the generalized Wilson lines $x$.

Therefore the generalized Wilson lines do not describe continuous deformations of the extended Narain lattice, but rather parameterize a discrete family of extended Narain lattices.

4 The spectrum

To see the impact of the new background on the spectrum of the model it is useful first to recall what the spectrum looks like in the case of a purely metric background.

In full generality the spectrum is given by

$$\frac{1}{2}M^2 = H + N + \bar{N} - \frac{3}{2},$$

$$0 = P + N - \bar{N} + \frac{1}{2},$$ (40)
where $H$ contains all the dependence on the backgrounds via $\chi$. Thus we have
\[
\frac{1}{2} M^2 = \frac{1}{2} (u + s)^T \chi (u + s) + N + \tilde{N} - \frac{3}{2},
\]
\[
0 = -\frac{1}{2} (u + s)^T \eta (u + s) + N - \tilde{N} + \frac{1}{2}.
\] (41)

Inserting the purely metric background given by (19) and going to the canonical basis one finds
\[
\frac{1}{2} M^2 = \frac{1}{2} (m^2 + n^2 + l^2 + w^2) + N + \tilde{N} - \frac{3}{2},
\]
\[
0 = -mn + \frac{1}{2} (w^2 - l^2 + 1) + N - \tilde{N}.
\] (42)

Recall here that
\[
w = r + t
\] (43)
is a vector belonging to the $v$ or $s$ coset of the $D_4$ lattice. This gives the known massless spectrum of toroidal compactifications, with an $N = 4$ supersymmetric structure.

Indeed, for massless states with $m, n = 0$ the formulae (42) are equivalent to
\[
N = \frac{1}{2} (1 - w^2),
\]
\[
\tilde{N} = 1 - \frac{1}{2} l^2.
\] (44) (45)

On the right, massless states have $N = 0$ and $w^2 = 1$. The latter condition has 16 possible solutions:
\[
w = (\pm 1, 0, 0, 0)
\] (46)
and permutations (8 vectors in all), and
\[
w = (\pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2})
\] (47)
with an even number of minus signs (another 8 vectors). These solutions express the supersymmetry of the spectrum: the states come in multiples of 16 (8 bosonic degrees of freedom, 8 fermionic).
On the left one has either $N = 0$ or $\bar{N} = 1$, with $l^2 = 2$ or $l^2 = 0$ respectively.

To summarize, the spectrum for $x = 0$ is $N = 4$ supersymmetric and consists of the following states:

- The graviton multiplet ($2 \times 16$ states):
  \[
  \bar{X}^a_i e^{i\omega H} |0 >.
  \] (48)

- The gauge sector ($(480 + 16) \times 16$ states)
  \[
  e^{i\bar{Y}^I} e^{i\omega H} |0 >, \quad \bar{Y}^I_{-1} e^{i\omega H} |0 >.
  \] (49) (50)

- The matter sector ($6 \times 16$ states):
  \[
  \bar{X}^i e^{i\omega H} |0 >.
  \] (51)

($\bar{X}_n$ and $\bar{Y}_n$ denote the mode operators of the string coordinates and the vectors $w$ appearing here span the 16 possibilities given earlier).

The full spectrum for non-vanishing background $x$ can be obtained by substituting the form of the background matrix given by (27) into the general formula (41). Setting the antisymmetric background to naught for simplicity, the resulting equations can be written in the form

\[
\frac{1}{2} M^2 = \frac{1}{2} (m + X^T w + (1 + \frac{1}{2} X^2)n)^2 + l^2 + 2 \bar{N} - 2, \quad \text{(52)}
\]

\[
0 = -mn - \frac{1}{2}(l^2 - w^2) + N - \bar{N} + \frac{1}{2}. \quad \text{(53)}
\]

To show explicitly where the metric $g$ enters it is better to rewrite this in the lattice basis:

\[
\frac{1}{2} M^2 = \frac{1}{2} (\hat{m} + x^T h^{-1} \hat{w} + (g + \frac{1}{2} x^2) \hat{n})^T g^{-1} (\hat{m} + x^T h^{-1} \hat{w} + (g + \frac{1}{2} x^2) \hat{n})
+ \hat{l}^T c^{-1} \hat{l} + 2 \bar{N} - 2, \quad \text{(54)}
\]

\[
0 = -\hat{m}^T \hat{n} - \frac{1}{2}(\hat{l}^T c^{-1} \hat{l} - \hat{w}^T h^{-1} \hat{w}) + N - \bar{N} + \frac{1}{2}. \quad \text{(55)}
\]
This makes it possible to study various decompactification limits where supersymmetries are restored.

The above formulae demonstrate that depending on $x$, some of the states which are massless at $x = 0$ become massive, with masses proportional to the inverse radius of the compact manifold. Thus, for an appropriate choice of $x$, one gets a behaviour reminiscent of Scherk–Schwarz type symmetry breaking in field theory. In particular, one sees that the gravitinos can acquire masses. Thus, the framework put forward here is geared toward analyzing various patterns of spontaneous supersymmetry breaking. This becomes particularly interesting in the case when orbifold twists are introduced.

5 Conclusions

We have formulated a class of models with spontaneously broken supersymmetry. This was done in the framework of Narain–like models on a Lorentzian $(14 - d, 26 - d)$ lattice, where $d$ is the number of non–compact dimensions. For compactifications down to $d = 4$ this leads to a $(10, 22)$ lattice instead of the usual $(6, 22)$. Such an extended lattice has additional “moduli”, the generalized Wilson lines. For consistency of the theory these have to be quantized, so the actual number of continuous moduli is the same as in the Narain case. One has therefore an infinite, discrete family of extended Narain lattices, each deformed in terms of the standard moduli.

The new backgrounds lead to (symmetry breaking) shifts in the spectrum which are proportional to $g^{-1}$. Thus, these effects will disappear in various decompactification limits (or strong coupling limits, in contexts where the role of the coupling is taken on by one of the elements of $g$). This should lead to new insights regarding the interconnections between string vacua.

This framework, due to its simplicity and the possibility to treat the general situation in a unified way, offers good prospects for nonperturbative extensions of models with spontaneously broken supersymmetries. It may also have interesting applications of purely mathematical nature, related to rather nontrivial lattice identities. These topics, as well as the obvious orbifoldization of this class of theories, are the subject of current research.
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