Where are the Uranus Trojans?

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Abstract The area of stable motion for fictitious Trojan asteroids around Uranus’ equilateral equilibrium points is investigated with respect to the inclination of the asteroid’s orbit to determine the size of the regions and their shape. For this task we used the results of extensive numerical integrations of orbits for a grid of initial conditions around the points $L_4$ and $L_5$, and analyzed the stability of the individual orbits. Our basic dynamical model was the Outer Solar System (Jupiter, Saturn, Uranus and Neptune). We integrated the equations of motion of fictitious Trojans in the vicinity of the stable equilibrium points for selected orbits up to the age of the Solar system of $5 \times 10^9$ years. One experiment has been undertaken for cuts through the Lagrange points for fixed values of the inclinations, while the semimajor axes were varied. The extension of the stable region with respect to the initial semimajor axis lies between $19.05 \leq a \leq 19.3$ AU but depends on the initial inclination. In another run the inclination of the asteroids’ orbit was varied in the range $0^\circ < i < 60^\circ$ and the semimajor axes were fixed. It turned out that only four ’windows’ of stable orbits survive: these are the orbits for the initial inclinations $0^\circ < i < 7^\circ$, $9^\circ < i < 13^\circ$, $31^\circ < i < 36^\circ$ and $38^\circ < i < 50^\circ$. We postulate the existence of at least some Trojans around the Uranus Lagrange points for the stability window at small and also high inclinations.

Keywords Uranus · Trojan asteroids · stability area

1 Introduction

The first discovery of a Jupiter Trojan in 1906 (Achilles by Max Wolf in Heidelberg) proved that the equilateral equilibrium points in a simplified dynamical model Sun – planet – massless body are not only of hypothetical interest. Ever since many of such Trojan asteroids of Jupiter have been found and now we have knowledge of several thousands of objects in the 1:1 mean motion resonance with Jupiter. Several investigations show the symmetry of these
two stable equilibrium points not only in the restricted three body problem (Sándor et al. (2002); Erdi (1988)) but also in the realistic dynamical model of the Outer Solar System (OSS) consisting of Jupiter, Saturn, Uranus and Neptune, like in e.g. Dvorak and Schwarz (2005); Freistetter (2006); Schwarz et al. (2004); Nesvorný (2000); Robutel et al. (2005); Tsiganis et al. (2005a).

But we have also evidence for the existence of asteroids in orbit around the equilateral equilibrium points of Neptune and also of Mars. Since the discovery of the first Neptune Trojan (Chiang et al. (2003)) – the minor planet 2001 QR322 around its equilibrium point \( L_5 \) – a lot of work has been dedicated to understand the dynamics of Neptune Trojans. On one hand the stability of hypothetical bodies was studied in the model of the Outer Solar System, on the other hand long term investigations tried to understand how Trojans can survive, or even get captured in these kind of orbits when one allows the large planets to migrate. An extensive stability study of the Trojans for the four gas giants has been undertaken by Nesvorný and Dones (2002) in different dynamical models. The results of the realistic n-body simulations for clouds of hypothetical Trojans around the Lagrange point \( L_4 \) showed how fast the depletion of asteroids is acting for Saturn, Uranus and Neptune. For several hundreds of bodies, different eccentricities and inclinations as well as semimajor axes the integrations were carried out over 4 Gyrs. Saturn Trojans turned out to be unstable already after several \( 10^5 \) years, Uranus Trojans were also found to be unstable but only after several million years, whereas Neptune’s Trojans mostly survived for the whole integration time of 4 Gyrs. In a similar study for Uranus and Neptune Trojans Marzari et al. (2003) determined the diffusion speed of Trojans and derived even expressions for the secular frequencies using their numerical results. Both studies were undertaken for initial inclinations of the Trojans up to \( i = 30^\circ \) and showed quite similar results for the dependence on the inclination.

Taking into account the early evolution of the Solar System the migration processes need to be included. This has been done in a paper by Kortenkamp et al. (2004) for the Neptune Trojans where it has been shown that only 5% of an initial Trojan population could survive this early stage when Jupiter migrated inward (5.4 to 5.2 AU), and Saturn (8.7 to 9.5 AU), Uranus (16.2 to 19.2 AU) and Neptune (23 to 29 AU) migrated outwards. Three different theories for the origin of Neptune Trojans are developed and tested by Chiang and Lithwick (2005), they use the results to constrain the circumstances of the formation of planets and the accretion of Trojans. In a more recent study Nesvorný and Vokrouhlický (2009) in the NICE model (e.g. Tsiganis et al. (2005b)) – where Uranus and Neptune could interchange their location in the Solar System and Jupiter and Saturn go through the 2:1 Mean Motion Resonance – it was shown that the Neptune Trojans were captured by a process quite similar to the chaotic capture of Jupiter Trojans (Morbidelli et al. (2005)). A similar process can be assumed for the Uranus Trojans, but up to now no Uranus Trojans have been detected which can be explained by the results of the pure dynamical studies mentioned before (Nesvorný and Dones (2002); Marzari et al. (2003)). These investigations stopped at an inclination of \( i = 30^\circ \); the discovery of a highly-inclined Neptune Trojan (Sheppard and Trujillo (2006)) raised the question of the general stability of Trojans at high inclinations. In a recent study concerning the Neptune Trojans (Zhou et al. (2009)) it was shown that especially for larger inclinations Trojans may be in stable orbits. We have therefore undertaken an extensive numerical study of the – hypothetical – Uranus Trojan cloud for the whole range of inclinations \( 0^\circ \leq i \leq 60^\circ \).
2 The method of investigation

2.1 The model and the grid of initial conditions

As dynamical model for the determination of the stable regions around Uranus’ $L_4$ and $L_5$ points the Outer Solar System was considered, consisting of the Sun and the four gas giants. The fictitious asteroids were assumed to be massless. No test computations had to be undertaken, because former studies for the Jupiter and the Neptune Trojans provided realistic results for that model (Dvorak et al. (2008); Zhou et al. (2009); Erdi et al. (2009)). Thus the equations of motion (only in the Newtonian framework) were integrated using the Lie-series method, a code which uses an adaptive step size and which has already been extensively tested and used in many numerical studies (e.g. Hanslmeier and Dvorak (1984); Lichtenegger (1984); Delva (1984)).

Different time scales were tested: First estimations have been undertaken for only $10^6$ years of integration time to separate the regions in phase space which are more or less immediately unstable. Then, for the remaining stable orbits, the integrations were extended up to $10^7$ and $10^8$ years, and finally for three selected orbits the time scale was set to the age of the Solar system ($5 \times 10^9$ years).

The initial conditions in the vicinity of the triangular equilibrium points were chosen such that the orbital elements mean anomaly, eccentricity and longitude of the node for the Trojans were set to the corresponding values of Uranus and only the argument of the perihelion was set to $\omega = \omega_{\text{Uranus}} \pm 60^\circ$.

For one run different values of the semimajor axes close to the one of Uranus were taken: $a_{\text{Trojan}} = a_{\text{Uranus}} \pm m \times 0.007$ AU, for $1 \leq m \leq 50$. The inclination was set to be equal to the one of Uranus. For a second run the semimajor axis was set to the one of Uranus, but now the inclinations of the fictitious Trojans were changed in the range $i_{\text{Uranus}} \leq i_{\text{Trojan}} \leq 60^\circ$ for steps of $\Delta i = 1^\circ$. Both studies were undertaken separately for the leading and the trailing Lagrange-points (see chapter 4). For the whole grid of initial conditions around $L_4$ we integrated some thousand orbits to get a first picture of the stable region (see chapter 3, Fig. 1).

2.2 The methods of analysis

Different tools of analysis were used to determine on one hand the stable regions in phase space and on the other hand to find out the reason for the instability of orbits:

- the maximum eccentricity within the integration time,
- the libration amplitude around the Lagrange point,
- the escape time for the unstable orbits,
- the involved frequencies.

For the first three tools there is not much to say about, because they are self explaining, except that we define the libration width (or libration amplitude) as the difference of the maximum and minimum value of the mean longitude-difference of the Trojan with respect to Uranus. The program package SIGSPEC used for the frequency analysis was provided by Reegen (2007). This is a code which is very efficient when used for data with equidistant time-domain sampling interval but also for non-equally spaced data. The computer pro-

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1 This does NOT mean that we used a constant step size for the numerical integration of the equations of motion.
gram is a novel method in time series analysis. It incorporates the frequency and phase of the signal appropriately and takes also into account the properties of the time-domain sampling. Thus it provides a very accurate peak detection (Kallinger et al. (2008)). The benefit of frequency- and phase-resolved statistics and the capability of full automation are important also in this application with equidistant data.

3 Stability of $L_4$ depending on the inclination: a first overview

To get an idea of the stability we used the results of numerically integrated orbits of several thousand fictitious Trojans for an integration time of $10^6$ years. For an increment of $1^\circ$ for inclinations between $0^\circ \leq i_{\text{Trojan}} \leq 60^\circ$ we used 100 orbits with different semimajor axes defined before. As mentioned we used three different methods for the analysis, which provided all more or less the same results concerning the stability of an orbit. In Fig. 1 we mark the amplitudes $A$ of libration around the $L_4$ point in different greytones and with contour-lines. The innermost lines on the upper part of the graph ($i \geq 20^\circ$) enclose orbits in the stable region with $A \leq 15^\circ$ (black region); the fuzzy edges show the coexistence of stable and unstable orbits in the transition regions. Furthermore it is interesting to note that for small inclinations there are no orbits with amplitudes of libration smaller than $A \leq 30^\circ$ present, which was already mentioned in the paper by Marzari et al. (2003). On the contrary quite small libration amplitudes occur for large inclination ($42^\circ \leq i \leq 52^\circ$). The non existence of orbits from moderate inclinations on ($15^\circ \leq i \leq 30^\circ$) was already found in the papers by Nesvorny (2000); Nesvorny and Dones (2002) and Marzari et al. (2003); in our study we can confirm it, but then, for larger eccentricities, new regions of stable motion appear.

4 The Results of the Cuts

The integration of thousands of orbits for longer time would need a huge amount of computer time – even with a fast program and several CPUs for running the code. We decided to limit the integration time to $10^7$ years (or $10^8$ years where appropriate) in the dynamical model of the OSS and specified the following initial conditions for the fictitious Trojans: the initial orbital elements for the massless asteroids were set to the one of Uranus, but for the $L_4$ Trojans we set $\omega = \omega + 60^\circ$ and for the $L_5$ Trojans $\omega = \omega - 60^\circ$. To be able to determine the sizes of stable regions with respect to the semimajor axes and the inclination of the Trojans we chose the following two different experiments:

- **a-cut** where we varied the initial semimajor axes of 100 fictitious Trojans and set the initial inclination – as an illustrative example – to $i = 3^\circ$; the integration time was limited to $10^7$ years.
- **i-cut** where we fixed the semimajor axis to the one of Uranus and set the inclination of 60 fictitious Trojans to $15^\circ < i < 60^\circ$; the integration time was set to $10^8$ years.

In addition two different runs – independently for $L_4$ and $L_5$ – have been undertaken to check any possible differences.

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2 It is the first technique relying on an analytic solution for the probability distribution produced by white noise in an amplitude spectrum (Discrete Fourier Transform). Returning the probability that an amplitude level is due to a random process, it provides an objective and unbiased estimator for the significance of a detected signal.

3 compare section 4.2, Fig[4]
Fig. 1 The initial semi-major axes of the fictitious object (x-axis) are plotted versus the initial inclination of the Trojan (y-axis). The grey-tone visualizes the libration amplitude around the Lagrange point \( L_4 \); the contour-lines show the limits of the libration amplitudes with a step of 15° (for detail see in the text).

4.1 The a-cuts

As mentioned before these computation are very cumbersome from the point of view of the CPU times. Therefore we made the following decisions for the initial conditions of the a-cuts: the inclinations of the Trojans were – as mentioned before – \( i = 3° \) and the semimajor axes were set to 100 values between \( 18.9 \leq a \leq 19.6 \) AU, such that the Lagrange points should be in the middle of this interval. This choice showed the apparent ‘asymmetry’ between \( L_4 \) and \( L_5 \), but displayed well the actual location of the Lagrange points. This kind of asymmetry also appeared in the investigations of the Jupiter Trojans already in the first studies (e.g. Holman and Wisdom (1993); Nesvorný and Dones (2002)) and was discussed in more detail in recent papers (Robutel and Gagner (2006); Lhotka et al. (2008)), also for Neptune (Zhou et al. (2009); Dvorak et al. (2007, 2008)). The apparent ‘asymmetry’ is due to the differently chosen initial conditions for the hypothetical Trojans when one investigates the area around \( L_4 \) and \( L_5 \) because of the presence of Jupiter which changes the location of the equilibrium points. Strictly speaking they don’t even exist in the n-body problem, but they can be defined as the minimum in the libration angle. Nevertheless the stable regions are symmetric and it is sufficient to explore only one (e.g. \( L_4 \)). In fact comparing the stability diagram of the fictitious Trojans with different semimajor axes for \( L_4 \) and \( L_5 \) first of all the shift in the stable region is eye-catching: the Lagrange point itself is lying at \( a = 19.235 \) AU, whereas the middle of the stable area for \( L_4 \) is at \( a = 19.18 \) AU, and for \( L_5 \) it is at \( a = 19.3 \).
AU (cf. Figs. 2, 3). The respective values for position in \( a \) for the Lagrange points were then chosen as the initial semimajor axis for the \textbf{i-cuts} described in the next section.

The time interval to show the extent of the stable region with respect to the semimajor axis was set to \( 10^7 \) years; we are aware that this is not enough to determine the effective largeness, but gives a good second estimate after the results for the \( 10^6 \) years integration for the Lagrange point \( L_4 \) of Fig. 1. In fact one can see that in Fig. 2 the outer parts, which still show stable orbits in the \( 10^6 \) year integration, are unstable, but the central area is still very stable with librations with amplitudes \( A \leq 80^\circ \). It has been mentioned in the former section that on the edge stable and unstable orbits are close to each other which explains the unstable gap around \( a = 19.07 \) AU especially visible in the libration plot (Fig. 2, bottom).

This unstable gap is not visible for orbits around the Lagrange point \( L_5 \); it shows a smooth stable region with increasing libration amplitudes on both sides of the equilibrium point itself (Fig. 3, bottom). The different escape times and larger eccentricity values in the unstable region are not surprising, because these orbits are – after leaving the region around the Lagrange point – in a chaotic state suffering from multiple encounters with the planets.

\subsection*{4.2 The \textbf{i-cuts}}

In Fig. 1 there are two main regions with stable orbits separated by a curved strip of instability for an inclination around \( i = 16^\circ \). Although we expected that the stable regions will shrink for an integration time 100 times longer (\( 10^8 \) years), the opening of an unstable gap around \( i = 8^\circ \) was a surprise. In the next section, where we carefully discuss the dynamics of three fictitious Trojans, a possible explanation will be given.

According to our results we can distinguish four different connected region in Fig. 4 where the orbits are stable with \( e_{\text{max}} \leq 0.2 \) (upper graph), no escapes occur up to 100 million years (middle graph) and only with small libration angles (bottom graph):
Fig. 3 a-cut for $L_5$: semimajor axis versus the maximum eccentricity (top), versus the escape time (middle) and versus the libration width around the Lagrange point (bottom).

Fig. 4 i-cut for $L_4$: inclination versus the maximum eccentricity (top), versus the escape time (middle) and versus the libration width around the Lagrange point (bottom).
Uranus Trojans $L_5$, $a = 19.30$ AU

![Graphs showing inclination versus maximum eccentricity, escape time, and libration width]

**Fig. 5** i-cut for $L_5$: inclination versus the maximum eccentricity (top), versus the escape time (middle) and versus the libration width around the Lagrange point (bottom).

**A** for $0^\circ \leq i_{Trojan} \leq 7^\circ$

**B** for $9^\circ \leq i_{Trojan} \leq 13^\circ$

**C** for $31^\circ \leq i_{Trojan} \leq 36^\circ$

**D** for $38^\circ \leq i_{Trojan} \leq 50^\circ$

This description and the finding of the regions A to D has been done for the $L_4$ environment. The picture for $L_5$ is different in the following features: the unstable strip at around $i = 8^\circ$ is slightly larger, also the unstable gap close to $i = 37^\circ$ is broader which makes region D significantly smaller. What we expect is that for even longer integration time the stable region C will disappear and we will be left with three ‘islands’ of stability with respect to the inclinations around both Lagrangian equilibrium points. We could not identify the small instability strip between A and B with a special secular frequency (visible also in the signal of the Ura3 orbit). The instability between B, respectively C and D is caused by $g_5$ (Jupiter) and $g_7$ (Uranus) according to our analysis of the main frequencies which corresponds quite well with the results of Marzari et al. (2003) (their Fig. 10). No stable orbits are present for $i \geq 50^\circ$ which is due to the presence of $g_8$ (Neptune). A detailed investigation of all frequencies involved in these regions is in preparation, where not only cuts will be used but also the extensions in semimajor axes and inclination.

### 4.3 Escapes during 100 Million years

Another plot was drawn concerning the number of escapers from the region around $L_4$ of Uranus during 100 million years. Here we show the number of escapers and compare the results with the ones derived in Nesvorny and Dones (2002) (Fig. 6). In our investigations,
Fig. 6 Escape times of Uranus Trojans close to \( L_4 \); the full line summarizes the results.

where we restricted the initial semimajor axes to \( 19.1 \leq a \leq 19.3 \) AU, \( \sim 3100 \) orbits were integrated. Here it is possible to distinguish for different inclinations; it is evident that the escape behaviour is quite different for \( i = 1, 20, 40 \) and \( 60^\circ \). For all the inclinations together the plot shows that at the beginning – the first million years – almost no objects escape, but then a constant depletion up to 100 million years occurs. Finally after the total integration time some 20\% of the original population survive, which is comparable to the results derived by Nesvorný and Dones (2002). The different escaping rate for different initial inclinations reveals that there are different mechanisms taking effect on different timescales, which will be analyzed in detail in our future work.

5 Long term integrations

To assure that the determined stable regions will survive for times up to the age of the solar system three orbits close to the Lagrange point \( L_4 \) were integrated for \( 5 \times 10^9 \) years for low inclined orbits. The initial conditions were the ones of Uranus except for the inclinations:

- \( \text{Ura1} \) with \( i_{\text{Trojan}} = i_{\text{Uranus}} \),
- \( \text{Ura2} \) with \( i_{\text{Trojan}} = i_{\text{Uranus}} + 2^\circ \),
- \( \text{Ura3} \) with \( i_{\text{Trojan}} = i_{\text{Uranus}} + 4^\circ \).

In the respective plot where we compare the eccentricities of fictitious Trojans directly with Uranus (Fig. 7), it is evident that there is not a big difference in the time development of the eccentricities; only the most inclined Trojan \( \text{Ura3} \) suffers from larger amplitudes than \( \text{Ura1} \) and \( \text{Ura2} \) in \( e \). The comparison of the inclination plots unveils why there is an instability gap at about \( i = 6^\circ \); Whereas on the first graph (Fig. 5) no difference is visible at all between the Trojan \( \text{Ura1} \) and Uranus itself, already in the graph for \( \text{Ura2} \) (Fig. 9) a significant feature can be observed. Large periodic changes occur with \( i_{\text{max}} \geq 4^\circ \). From the
Fig. 7 The eccentricities of 3 fictitious Uranus Trojans after 5 billion years compared to the one of Uranus: Ura1 (top), Ura2 (middle) and Ura3 (bottom).

Fig. 8 Inclination of Uranus and Ura1 for the first 50 and the last 50 million years of integration (upper, respectively lower graph). Note that the signal for Ura1 and Uranus are undistinguishable.
**Fig. 9** Inclination of Uranus and Ura2 for the first 50 and the last 50 million years of integration.

**Fig. 10** Inclination of Uranus and Ura3 for the first 50 and the last 50 million years of integration.
Fig. 11 The two main periods in the element \( k = e \cos(\Omega + \omega) \) for the 5 billion years integration of Ura3 namely the \( 4.4 \times 10^5 \) years period and the period which changes between \( \sim 10^7 \) and \( 2 \times 10^7 \) years compared to the 'same' period of \( \sim 10^7 \) of Ura2 which is shown as a straight wall in the middle of the graph: periods (y-axis) are plotted versus the time (x-axis) and the amplitudes (x-axis).

graph of Ura3 (Fig. 10) one can see that the period of the variation of the inclination at the end of the 5 billion years is only half of the period at the beginning.

A carefully undertaken frequency analysis showed that in contrast to Ura1 and Ura2 for Ura3 a dramatic change in the main frequencies occurred during the dynamical evolution. This is a sign of chaotic behaviour as was already pointed out in many papers (e.g. Laskar (1990, 1994, 1997)). To visualize this change we used the technique of running windows, each section spanning 50 million years with an overlap of 25 million years. The changes are well visible for the larger periods of some 12 to 18 million years which also show significantly larger amplitudes than the second largest period of about \( 4.4 \times 10^5 \) years (Fig. 11).

In addition, at the lower end of the high inclination window, we made numerical integrations up to 1Gyr which showed that even orbits close to the instability window have the typical behaviour of changing the periodicity. This can be seen already after several hundred million years in Fig. 12 for two orbits, one is well inside the stability region (\( i = 42^\circ \)) and the other one already close to the edge (\( i = 40^\circ \)). In fact the last one becomes unstable after another 100 Myrs (at 946 Myrs); this behaviour gives notice already when one compares the dynamics for the first 200 Myrs and the last 200 Myrs of this hypothetical Trojan (compare the 2nd and 4th panel in Fig. 12).

### 6 Conclusions

In our study we explored the regions of possible Trojan asteroids of Uranus for a large range of their inclinations up to \( i = 60^\circ \), which has never been done before. The method
of determining these regions was the analysis of the results of straightforward numerical integrations of the full equations of motion in the model of the OSS. Besides the escape from the region we also checked the amplitudes of libration of these fictitious asteroids around the Lagrangian equilibrium points. To assure the survival of the ‘stable’ Trojans we did also integrations of three specially chosen Trojans on low inclined orbits for the age of the Solar system of $4.5 \times 10^9$ years, where the accurate analysis of the frequencies involved was undertaken. Our results showed that close to the first instability window $6^\circ \leq i_{\text{Trojan}} \leq 8^\circ$ the strong variation of the frequency of a special secular resonance indicates chaotic motion. Two other unstable windows with respect to the inclination are $14^\circ \leq i_{\text{Trojan}} \leq 31^\circ$ and around $i_{\text{Trojan}} = 37^\circ$. From $i_{\text{Trojan}} \geq 50^\circ$ (for $L_4$) and $i_{\text{Trojan}} \geq 45^\circ$ ($L_5$) no stable Trojan orbits exist for Uranus. These stable islands in inclination are surrounded by more or less small stable regions in semimajor axis.

According to the results of Morbidelli et al. (2005) Jupiter Trojans could be trapped during the phase of early migration of the planets in our Solar system. If that was also the case for Neptune and Uranus Trojans (and why not) we should be able to detect them. So the question is: why don’t we observe them?

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Footnote:

5 up to now 6 Neptune Trojans were observed
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