Standard Model Confronting
New Results for $\varepsilon'/\varepsilon$

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Abstract
We analyze the CP violating ratio $\varepsilon'/\varepsilon$ in the Standard Model in view of the new KTeV results. We review the present status of the most important non-perturbative parameters $B_6^{(1/2)}$, $B_8^{(3/2)}$, $\hat{B}_K$ and of the strange quark mass $m_s$. We also briefly discuss the issues of final state interactions and renormalization scheme dependence. Updating the values of the CKM parameters, of $m_t$ and $\Lambda^{(4)}_{\overline{MS}}$ and using Gaussian errors for the experimental input and flat distributions for the theoretical parameters we find $\varepsilon'/\varepsilon$ substantially below the NA31 and KTeV data: $\varepsilon'/\varepsilon = (7.7^{+6.0}_{-3.5}) \cdot 10^{-4}$ and $\varepsilon'/\varepsilon = (5.2^{+1.6}_{-2.7}) \cdot 10^{-4}$ in the NDR and HV renormalization schemes respectively. A simple scanning of all input parameters gives on the other hand $1.05 \cdot 10^{-4} \leq \varepsilon'/\varepsilon \leq 28.8 \cdot 10^{-4}$ and $0.26 \cdot 10^{-4} \leq \varepsilon'/\varepsilon \leq 22.0 \cdot 10^{-4}$ respectively. Analyzing the dependence on various parameters we find that only for extreme values of $B_6^{(1/2)}$, $B_8^{(3/2)}$ and $m_s$ as well as suitable values of CKM parameters and $\Lambda^{(4)}_{\overline{MS}}$, the ratio $\varepsilon'/\varepsilon$ can be made consistent with data. We analyze the impact of these data on the lower bounds for $\text{Im} V_{td} V_{ts}^\ast$, $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$, $\text{Br}(K_L \rightarrow \pi^0 e^+ e^-)_{\text{dir}}$ and on $\tan \beta$ in the Two Higgs Doublet Model II.
1 Introduction

One of the most fascinating phenomena in particle physics is the violation of CP symmetry in weak interactions. In the Standard Model CP violation is supposed to originate in a single complex phase $\delta$ in the charged current interactions of quarks $[1]$. This picture is consistent, within theoretical hadronic uncertainties, with CP violation in $K^0 - \bar{K}^0$ mixing (indirect CP violation) discovered in $K_L \to \pi\pi$ decays already in 1964 $[2]$ and described by the parameter $\varepsilon$ $[3]$: 

$$\varepsilon = (2.280 \pm 0.013) \cdot 10^{-3} \exp(i\Phi_\varepsilon), \quad \Phi_\varepsilon \approx \frac{\pi}{4}. \quad (1.1)$$

It is also consistent with the recent measurement of $\sin 2\beta$ from $B \to \psi K_S$ at CDF $[4]$, although the large experimental error precludes any definite conclusion.

It should be emphasized that the agreement of the Standard Model with the experimental value of $\varepsilon$ is non-trivial as $|\sin \delta| \leq 1$. Indeed in the Standard Model 

$$\varepsilon = \hat{B}_K \text{Im} \lambda_t \cdot F_\varepsilon(m_t, \text{Re} \lambda_t) \exp(i\pi/4) \quad (1.2)$$

where $\hat{B}_K$ is a non-pertubative parameter $\mathcal{O}(1)$ and $\lambda_t = V_{td}V_{ts}^*$ with $V_{ij}$ being the elements of the CKM matrix $[4,5]$. The function $F_\varepsilon$ results from well known box diagrams with $W^\pm, t, c, u$ exchanges $[6]$ and includes NLO QCD corrections $[7,8]$. An explicit expression for $F_\varepsilon$ can be found in (10.42) of $[9]$. It is an increasing function of the top quark mass $m_t$ and of $\text{Re} \lambda_t$. The QCD scale ($\Lambda_{\overline{MS}}$) dependence of $F_\varepsilon$ is very weak.

Now, $\text{Im} \lambda_t$ is an important quantity as it plays a central role in the phenomenology of CP violation in $K$ decays and is furthermore closely related to the Jarlskog parameter $J_{CP}$ $[10]$, the invariant measure of CP violation in the Standard Model: $J_{CP} = \lambda \sqrt{1 - \lambda^2} \text{Im} \lambda_t$ with $\lambda = 0.221$ denoting one of the Wolfenstein parameters $[11]$. To an excellent approximation one has 

$$\text{Im} \lambda_t = |V_{ub}| |V_{cb}| \sin \delta. \quad (1.3)$$

As can be inferred from (1.2) and (1.3) only for sufficiently large values of $|V_{ub}|, |V_{cb}|, m_t$ and $\hat{B}_K$ can $\varepsilon$ in (1.2) and consequently the indirect CP violation in the Standard Model be consistent with the one observed experimentally. It turns out that using the known values of $|V_{ub}|, |V_{cb}|, m_t$ (see Section 3) and taking $\hat{B}_K = 0.80 \pm 0.15$ in accordance with lattice and large-N calculations (see Section 2), the experimental value of $\varepsilon$ can be reproduced in the Standard Model provided $\sin \delta \geq 0.69$. This determination of $\sin \delta$ includes constraints from $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixings. We also find 

$$1.04 \cdot 10^{-4} \leq \text{Im} \lambda_t \leq 1.63 \cdot 10^{-4}. \quad (1.4)$$
It should be noticed that $\sin \delta = \mathcal{O}(1)$ and that the extracted range for $\text{Im} \lambda_t$ is not far from the upper limit of $1.73 \cdot 10^{-4}$ following from the unitarity of the CKM matrix. It should also be emphasized that the large top quark mass plays an important role in obtaining the experimental value for $\varepsilon$. Had $m_t$ been substantially lower than it is, the theoretical value of $\varepsilon$ would be below the experimental one.

While indirect CP violation in $K_L \rightarrow \pi \pi$ reflects the fact that the mass eigenstates in the $K^0 - \bar{K}^0$ system are not CP eigenstates, the so-called direct CP violation is realized via direct transitions between states of different CP parities: CP violation in the decay amplitude. In $K_L \rightarrow \pi \pi$ decays this type of CP violation is characterized by the parameter $\varepsilon'$. In the Standard Model one has

$$\frac{\varepsilon'}{\varepsilon} = \text{Im} \lambda_t \cdot F_{\varepsilon'}(m_t, \Lambda_{\overline{\text{MS}}}^{(4)}, m_s, B_6^{(1/2)}, B_8^{(3/2)}, \Omega_{\eta+\eta'})$$

where the function $F_{\varepsilon'}$ results from the calculation of QCD penguin and electroweak penguin diagrams. Here $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are non-perturbative parameters related to the dominant QCD penguin and electroweak penguin contributions respectively, $\Lambda_{\overline{\text{MS}}}^{(4)}$ is the QCD scale and $\Omega_{\eta+\eta'}$ represents isospin breaking effects.

The expression (1.5) has been obtained by calculating $\varepsilon'$ and dividing it by the experimental value of $\varepsilon$ in (1.1) in order to be able to compare with the experimental value of $\varepsilon'/\varepsilon$. This procedure exhibits the nature of $\varepsilon'$ which representing direct CP violation is proportional to $\text{Im} \lambda_t$. However, one could also proceed differently and ignoring the constraint (1.1) calculate $\varepsilon'/\varepsilon$ fully in theory. In this case (1.5) is replaced by

$$\frac{\varepsilon'}{\varepsilon} = \frac{\bar{F}_{\varepsilon'}(m_t, \Lambda_{\overline{\text{MS}}}^{(4)}, m_s, B_6^{(1/2)}, B_8^{(3/2)}, \Omega_{\eta+\eta'})}{B_K F_\varepsilon(m_t, \text{Re} \lambda_t)}$$

where $\bar{F}_{\varepsilon'} = |\varepsilon_{\text{exp}}| F_{\varepsilon'}$ is independent of $\varepsilon$. One should notice that $\text{Im} \lambda_t$ cancelled out in $\varepsilon'/\varepsilon$ calculated in this manner and $\varepsilon'/\varepsilon$ is actually a function of $\text{Re} \lambda_t$ and not of $\text{Im} \lambda_t$. However, once the constraint (1.1) has been taken into account (1.6) reduces to (1.5). We will return to this point in Section 3.

There is a long history of calculations of $\varepsilon'/\varepsilon$ in the Standard Model. The first calculation of $\varepsilon'/\varepsilon$ for $m_t \ll M_W$ without the inclusion of renormalization group effects can be found in [12]. Renormalization group effects in the leading logarithmic approximation have been first presented in [13]. For $m_t \ll M_W$ only QCD penguins play a substantial role. First extensive phenomenological analyses in this approximation can be found in [14]. Over the eighties these calculations were refined through the inclusion of QED penguin
effects for \( m_t \ll M_W \) \cite{13,14,17}, the inclusion of isospin breaking in the quark masses \cite{16,17,18}, and through improved estimates of hadronic matrix elements in the framework of the \( 1/N_c \) approach \cite{19}. This era of \( \varepsilon'/\varepsilon \) culminated in the analyses in \cite{20,21}, where QCD penguins, electroweak penguins (\( \gamma \) and \( Z^0 \) penguins) and the relevant box diagrams were included for arbitrary top quark masses. The strong cancellation between QCD penguins and electroweak penguins for \( m_t > 150 \) GeV found in these papers was confirmed by other authors \cite{22}.

During the nineties considerable progress has been made by calculating complete NLO corrections to \( \varepsilon' \) \cite{23}-\cite{27}. Together with the NLO corrections to \( \varepsilon \) and \( B^0 - \bar{B}^0 \) mixing \cite{4,5,28}, this allowed a complete NLO analysis of \( \varepsilon'/\varepsilon \) including constraints from the observed indirect CP violation (\( \varepsilon \)) and \( B^0_{d,s} - \bar{B}^0_{d,s} \) mixings (\( \Delta M_{d,s} \)). The improved determination of the \( V_{ub} \) and \( V_{cb} \) elements of the CKM matrix, the improved estimates of hadronic matrix elements using the lattice approach as well as other non-perturbative approaches and in particular the determination of the top quark mass \( m_t \) had of course also an important impact on \( \varepsilon'/\varepsilon \).

In a crude approximation (not to be used for any serious analysis)

\[
F_{\varepsilon'} \approx 13 \cdot \left( \frac{110 \text{ MeV}}{m_s(2 \text{ GeV})} \right)^2 \left( B_6^{(1/2)}(1 - \Omega_{\eta+\eta'}) - 0.4 \cdot B_8^{(3/2)} \left( \frac{m_t}{165 \text{ GeV}} \right)^{2.5} \right) \left( \frac{\Lambda^{(4)}_{\text{MS}}}{340 \text{ MeV}} \right) 
\]

where \( \Omega_{\eta+\eta'} \approx 0.25 \). This formula exhibits very clearly the dominant uncertainties in \( F_{\varepsilon'} \) which reside in the values of \( m_s, B_6^{(1/2)}, B_8^{(3/2)}, \Lambda^{(4)}_{\text{MS}} \) and \( \Omega_{\eta+\eta'} \). Because of the accurate value \( m_t(m_t) = 165 \pm 5 \) GeV, the uncertainty in \( \varepsilon'/\varepsilon \) due to the top quark mass amounts only to a few percent. A more accurate formula for \( F_{\varepsilon'} \) will be given in Section 2.

A comparison of the formulae (1.2) and (1.3) reveals that the analysis of \( \varepsilon \) is theoretically cleaner. Indeed, \( \varepsilon \) depends on a single non-perturbative parameter \( \hat{B}_K \), whereas \( \varepsilon'/\varepsilon \) is a sensitive function of \( B_6^{(1/2)}, B_8^{(3/2)}, m_s, \Lambda^{(4)}_{\text{MS}} \) and \( \Omega_{\eta+\eta'} \). Moreover, the partial cancellation between QCD penguin (\( B_6^{(1/2)} \)) and electroweak penguin (\( B_8^{(3/2)} \)) contributions requires accurate values of \( B_6^{(1/2)} \) and \( B_8^{(3/2)} \) for an acceptable estimate of \( \varepsilon'/\varepsilon \).

Until recently the experimental situation on \( \varepsilon'/\varepsilon \) was rather unclear. While the result of the NA31 collaboration at CERN with \( \text{Re}(\varepsilon'/\varepsilon) = (23.0 \pm 6.5) \cdot 10^{-4} \) \cite{29} clearly indicated direct CP violation, the value of E731 at Fermilab, \( \text{Re}(\varepsilon'/\varepsilon) = (7.4 \pm 5.9) \cdot 10^{-4} \) \cite{30}, was compatible with superweak theories \cite{31} in which \( \varepsilon'/\varepsilon = 0 \). This controversy is
now settled with the very recent measurement by KTeV at Fermilab ~[32]: \[
\text{Re}(\frac{\varepsilon'}{\varepsilon}) = (28.0 \pm 4.1) \cdot 10^{-4} \tag{1.8}
\]
which together with the NA31 result confidently establishes direct CP violation in nature.

The grand average including NA31, E731 and KTeV results reads
\[
\text{Re}(\frac{\varepsilon'}{\varepsilon}) = (21.8 \pm 3.0) \cdot 10^{-4} \tag{1.9}
\]
very close to the NA31 result but with a smaller error. The error should be further reduced once the first data from NA48 collaboration at CERN are available and complete data from both collaborations have been analyzed. It is also of great interest to see what value for \(\varepsilon'/\varepsilon\) will be measured by KLOE at Frascati, which uses a different experimental technique than KTeV and NA48.

Does the direct CP violation observed in \(K_L \to \pi\pi\) decays agree with the Standard Model expectations? Before entering the details let us take a set of “central” values for the parameters entering \(F_{\varepsilon'}\). Together with \(B_K = 0.80\), \(m_t(m_t) = 165\) GeV, \(|V_{ub}| = 3.56 \cdot 10^{-3}\) and \(|V_{cb}| = 0.040\) needed for the \(\varepsilon\)-analysis we set
\[
B_{6}^{(1/2)} = 1.0, \quad B_{8}^{(3/2)} = 0.8, \quad m_s(2\text{ GeV}) = 110 \text{ MeV}, \quad \Omega_{\theta^+\eta'} = 0.25 \tag{1.10}
\]
and \(\Lambda_{\text{MS}}^{(4)} = 340\) MeV. Using the formula (2.38) for \(F_{\varepsilon'}\), we find \(F_{\varepsilon'} = 5.2\). On the other hand the \(\varepsilon\)-analysis gives \(\text{Im}\lambda_t = 1.34 \cdot 10^{-4}\). Consequently
\[
\left(\frac{\varepsilon'}{\varepsilon}\right)^{\text{central}} = 7.0 \cdot 10^{-4} \tag{1.11}
\]
well below the experimental findings in (1.9).

Equivalently, with \(F_{\varepsilon'} = 5.2\), the experimental value in (1.9) implies \(\text{Im}\lambda_t = (4.2 \pm 0.6) \cdot 10^{-4}\) which lies outside the range (1.14) extracted from the standard analysis of the unitarity triangle. Moreover it violates the upper bound \(\text{Im}\lambda_t = 1.73 \cdot 10^{-4}\) following from the unitarity of the CKM matrix.

The fact that for central values of the input parameters the size of \(\varepsilon'/\varepsilon\) in the Standard Model is well below the NA31 value of \((23.0 \pm 6.5) \cdot 10^{-4}\) has been known for some time. The extensive NLO analyses with lattice and large-N estimates of \(B_6^{(1/2)} \approx 1\) and \(B_8^{(3/2)} \approx 1\) performed first in [25, 26] and after the top discovery in [33]-[35] have found \(\varepsilon'/\varepsilon\) in the ball park of \((3 - 7) \cdot 10^{-4}\) for \(m_s(2\text{ GeV}) \approx 130\) MeV. On the other hand it has been stressed repeatedly in [3, 34] that for extreme values of \(B_6^{(1/2)}, B_8^{(3/2)}\) and \(m_s\) still
consistent with lattice, QCD sum rules and large-N estimates as well as sufficiently high values of $\text{Im}\lambda_t$ and $\Lambda_{\text{MS}}^{(4)}$, a ratio $\varepsilon'/\varepsilon$ as high as $(2 - 3) \cdot 10^{-3}$ could be obtained within the Standard Model. Yet, it has also been admitted that such simultaneously extreme values of all input parameters and consequently values of $\varepsilon'/\varepsilon$ close to the NA31 result are rather improbable in the Standard Model. Different conclusions have been reached in [33], where values $(1 - 2) \cdot 10^{-3}$ for $\varepsilon'/\varepsilon$ can be found. Also the Trieste group [38], which calculated the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ in the chiral quark model, found $\varepsilon'/\varepsilon = (1.7 \pm 1.4) \cdot 10^{-3}$. On the other hand using an effective chiral lagrangian approach, the authors in [39] found $\varepsilon'/\varepsilon$ consistent with zero.

The purpose of the present paper is to update the analyses in [9, 34] and to confront the Standard Model estimates of $\varepsilon'/\varepsilon$ with the experimental findings in (1.9). Other very recent discussions of $\varepsilon'/\varepsilon$ can be found in [40]-[42]. We will comment on them below. In the present paper we address in particular the following questions:

- What is the maximal value of $\varepsilon'/\varepsilon$ in the Standard Model consistent with the usual analysis of the unitarity triangle as a function of $B_6^{(1/2)}$, $B_8^{(3/2)}$, $m_s$ and $\Lambda_{\text{MS}}^{(4)}$?

- What is the lowest value of $B_6^{(1/2)}$ as a function of $B_8^{(3/2)}$ for fixed values of $m_s$ and $\Lambda_{\text{MS}}^{(4)}$ for which the Standard Model is simultaneously compatible with (1.9) and the analysis of the unitarity triangle?

- What is the sensitivity of the analysis of $\varepsilon'/\varepsilon$ to the values of $\Omega_{\eta+\eta'}$ and $\hat{B}_K$?

- What is the impact of the experimental value for $\varepsilon'/\varepsilon$ on $\text{Im}\lambda_t$, on the usual analysis of the unitarity triangle and in particular on Standard Model expectations for the rare decays $K_L \to \pi^0\nu\bar{\nu}$ and $K_L \to \pi^0e^+e^-$ in which direct CP violation plays an important role?

- What are the general implications of (1.3) for physics beyond the Standard Model? In particular, what is the impact on the allowed range in the space $(M_H, \tan\beta)$ in the so called two Higgs doublet model II (2HDMII) [43]?

While addressing these questions we would like to emphasize that it is by no means the purpose of our paper to fit $B_6^{(1/2)}$, $B_8^{(3/2)}$, $m_s$, $\Lambda_{\text{MS}}^{(4)}$, $\Omega_{\eta+\eta'}$ and $\hat{B}_K$ in order to make the Standard Model compatible simultaneously with experimental values on $\varepsilon'/\varepsilon$, $\varepsilon$ and the analysis of the unitarity triangle. Such an approach would be against the whole philosophy...
of searching for new physics with the help of loop induced transitions as represented by $\varepsilon'/\varepsilon$ and $\varepsilon$. Moreover it should be kept in mind that:

- $B_6^{(1/2)}$, $B_8^{(3/2)}$ and $\hat{B}_K$, in spite of carrying the names of non-perturbative parameters, are really not parameters of the Standard Model as they can be calculated by means of non-perturbative methods in QCD. The same applies to $\Omega_{\eta+\eta'}$.

- $m_s$, $\Lambda^{(4)}_{\text{MS}}$, $m_t$, $|V_{cb}|$ and $|V_{ub}|$ are parameters of the Standard Model but there are better places than $\varepsilon'/\varepsilon$ to determine them. In particular the usual determinations of these parameters can only marginally be affected by physics beyond the Standard Model, which is not necessarily the case for $\varepsilon$ and $\varepsilon'/\varepsilon$.

Consequently, the only parameter to be fitted by direct CP violation is $\sin\delta$ or $\text{Im}\lambda_t$. The numerical analysis of $\varepsilon'/\varepsilon$ as a function of $B_6^{(1/2)}$, $B_8^{(3/2)}$, $m_s$, $\Lambda^{(4)}_{\text{MS}}$, $\Omega_{\eta+\eta'}$ and $\hat{B}_K$ should only give a global picture for which ranges of parameters the presence of new physics in $\varepsilon'/\varepsilon$ and $\varepsilon$ should be expected.

Our paper is organized as follows. In Section 2 we recall briefly the basic formulae for $\varepsilon'/\varepsilon$ in the Standard Model. We also review the existing methods for estimating hadronic matrix elements of relevant local operators and we present a rather accurate analytic formula for $F_{\varepsilon'}$. In Section 3 we address several of the questions listed above. In Section 4 we discuss briefly general implications for physics beyond the Standard Model. In particular we investigate the lower bound on $\tan\beta$ as a function of the charged Higgs mass in the 2HDMII. Conclusions and outlook are given in Section 5.

2 Basic Formulae

2.1 Formulae for $\varepsilon'/\varepsilon$

The parameter $\varepsilon'$ is given in terms of the isospin amplitudes $A_I$ as follows

$$\varepsilon' = \frac{1}{\sqrt{2}} \text{Im} \left( \frac{A_2}{A_0} \right) \exp(i\Phi_{\varepsilon'}), \quad \Phi_{\varepsilon'} = \frac{\pi}{2} + \delta_2 - \delta_0,$$

(2.1)

where $\delta_I$ are final state interaction phases. Then, the basic formula for $\varepsilon'/\varepsilon$ is given by

$$\frac{\varepsilon'}{\varepsilon} = \text{Im}\lambda_t \cdot F_{\varepsilon'},$$

(2.2)

where

$$F_{\varepsilon'} = \left[ P^{(1/2)} - P^{(3/2)} \right] \exp(i\Phi),$$

(2.3)
with
\[ P^{(1/2)} = r \sum y_i \langle Q_i \rangle_0 (1 - \Omega_{\eta'\eta}) , \]  
\[ P^{(3/2)} = \frac{r}{\omega} \sum y_i \langle Q_i \rangle_2 . \]  
\[ (2.4) \]

Here
\[ r = \frac{G_F \omega}{2|\varepsilon| \text{Re} A_0} , \quad \langle Q_i \rangle_I^\prime \equiv \langle (\pi\pi)_I^\prime |Q_i| K \rangle , \quad \omega = \frac{\text{Re} A_2}{\text{Re} A_0} . \]  
\[ (2.6) \]

Since
\[ \Phi = \Phi_{\varepsilon'} - \Phi_{\varepsilon} \approx 0 , \]  
\[ (2.7) \]

\( F_{\varepsilon'} \) and \( \varepsilon'/\varepsilon \) are real to an excellent approximation.

The operators \( Q_i \) are given explicitly as follows:

**Current–Current :**
\[ Q_1 = (\bar{s}_\alpha u_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A} \quad Q_2 = (\bar{s}u)_{V-A} (\bar{u}d)_{V-A} \]  
\[ Q_3 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V-A} \quad Q_4 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A} \]  
\[ Q_5 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V+A} \quad Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A} \]  
\[ (2.8) \]

**QCD–Penguins :**
\[ Q_7 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V+A} \quad Q_8 = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A} \]  
\[ Q_9 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V-A} \quad Q_{10} = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V-A} . \]  
\[ (2.10) \]

**Electroweak–Penguins :**
\[ \begin{align*} 
Q_7 &= \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V+A} \\
Q_8 &= \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A} \\
Q_9 &= \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V-A} \\
Q_{10} &= \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V-A} . 
\end{align*} \]
\[ (2.12) \]

Here, \( \alpha, \beta \) are colour indices and \( e_q \) denotes the electric quark charges reflecting the electroweak origin of \( Q_7, \ldots, Q_{10} \).

The Wilson coefficient functions \( y_i(\mu) \) were calculated including the complete next-to-leading order (NLO) corrections in [23]-[27]. The details of these calculations can be found there and in the review [44]. Their numerical values for \( \Lambda^{(4)}_{\text{MS}}(M_Z) = 0.119 \pm 0.003 \) and two renormalization schemes (NDR and HV) are given in table [4]. There we also give the coefficients \( z_{1,2} \) relevant for the discussion of hadronic matrix elements.
Table 1: $\Delta S = 1$ Wilson coefficients at $\mu = m_c = 1.3$ GeV for $m_t = 165$ GeV and $f = 3$ effective flavours. $y_1 = y_2 \equiv 0$.

| Scheme | $\Lambda_{\overline{MS}}^{(4)} = 290$ MeV | $\Lambda_{\overline{MS}}^{(4)} = 340$ MeV | $\Lambda_{\overline{MS}}^{(4)} = 390$ MeV |
|--------|---------------------------------|---------------------------------|---------------------------------|
| $z_1$  | $0.393$                         | $0.477$                         | $0.521$                         |
| $z_2$  | $1.201$                         | $1.256$                         | $1.286$                         |
| $y_3$  | $0.270$                         | $0.303$                         | $0.341$                         |
| $y_4$  | $-0.054$                        | $-0.056$                        | $-0.061$                        |
| $y_5$  | $0.006$                         | $0.015$                         | $0.016$                         |
| $y_6$  | $-0.072$                        | $-0.074$                        | $-0.083$                        |
| $y_7/\alpha$ | $-0.038$               | $-0.037$                        | $-0.036$                        |
| $y_8/\alpha$ | $0.118$                     | $0.127$                         | $0.143$                         |
| $y_9/\alpha$ | $-1.410$                    | $-1.410$                        | $-1.437$                        |
| $y_{10}/\alpha$ | $0.496$                    | $0.502$                         | $0.546$                         |

It is customary in phenomenological applications to take $\text{Re}A_0$ and $\omega$ from experiment, i.e.

$$\text{Re}A_0 = 3.33 \cdot 10^{-7} \text{ GeV}, \quad \omega = 0.045,$$

where the last relation reflects the so-called $\Delta I = 1/2$ rule. This strategy avoids to a large extent the hadronic uncertainties in the real parts of the isospin amplitudes $A_I$. In order to be consistent the constraint (2.13) should also be incorporated in the matrix elements $\langle Q_i \rangle_I$ necessary for the evaluation of $\varepsilon'/\varepsilon$. This in fact has been done in [25] and we will return to this approach briefly below. Studies of the $\Delta I = 1/2$ rule can be found in [45, 46, 47].

The sum in (2.4) and (2.5) runs over all contributing operators. $P^{(3/2)}$ is fully dominated by electroweak penguin contributions. $P^{(1/2)}$ on the other hand is governed by QCD penguin contributions which are suppressed by isospin breaking in the quark masses ($m_u \neq m_d$). The latter effect is described by

$$\Omega_{\eta+\eta'} = \frac{1}{\omega} \frac{(\text{Im}A_2)_{1B}}{\text{Im}A_0}.$$  \hspace{1cm} (2.14)

For $\Omega_{\eta+\eta'}$ we will first set

$$\Omega_{\eta+\eta'} = 0.25,$$  \hspace{1cm} (2.15)
which is in the ball park of the values obtained in the $1/N$ approach \[17\] and in chiral perturbation theory \[16, 18\]. $\Omega_{\eta+\eta'}$ is independent of $m_t$. We will investigate the sensitivity of $\varepsilon'/\varepsilon$ to $\Omega_{\eta+\eta'}$ in Section 3.

### 2.2 Hadronic Matrix Elements

The main source of uncertainty in the calculation of $\varepsilon'/\varepsilon$ are the hadronic matrix elements $\langle Q_i \rangle_I$. They generally depend on the renormalization scale $\mu$ and on the scheme used to renormalize the operators $Q_i$. These two dependences are canceled by those present in the Wilson coefficients $y_i(\mu)$ so that the resulting physical $\varepsilon'/\varepsilon$ does not (in principle) depend on $\mu$ and on the renormalization scheme of the operators. Unfortunately, the accuracy of the present non-perturbative methods used to evaluate $\langle Q_i \rangle_I$ is not sufficient to have the $\mu$ and scheme dependences of $\langle Q_i \rangle_I$ fully under control. We believe that this situation will change once the lattice calculations and QCD sum rule calculations improve. A brief review of the existing methods including most recent developments will be given below.

In view of this situation it has been suggested in \[25\] to determine as many matrix elements $\langle Q_i \rangle_I$ as possible from the leading CP conserving $K \to \pi\pi$ decays, for which the experimental data is summarized in (2.13). To this end it turned out to be very convenient to determine $\langle Q_i \rangle_I$ in the three-flavour effective theory at a scale $\mu \approx m_c$. With this choice of $\mu$ the operators $Q^c_{1,2}$, being present only for $\mu > m_c$, are integrated out and the contribution of penguin operators to $\text{Re} A_I$ turns out to be very small. Unfortunately, since the charm mass is not much larger than the scale $M_K$ of the process we are studying, the matching procedure between the four- and three-flavour effective theories contains an ambiguity related to the choice of external momenta in the matching \[23, 25\]. Furthermore, as pointed out in \[33\], there is an ambiguity due to the contribution of higher dimensional operators which are unsuppressed for $\mu \approx m_c$. However, all these ambiguities are of $O(\alpha_s)$ and one can easily verify that their possible contribution to $\text{Re} A_I$ is at the level of a few percent at most. Consequently, they have only a minor impact on our determination of $\langle Q_i \rangle_I$ at $\mu = m_c$ from $\text{Re} A_I$. Using the renormalization group evolution one can then find $\langle Q_i \rangle_I$ at any other scale $\mu \neq m_c$. The details of this procedure can be found in \[25\].

As we will see below this method allows to determine only the matrix elements of the $(V-A) \otimes (V-A)$ operators. For the central value of $\text{Im} \lambda_t$ these operators give a negative contribution to $\varepsilon'/\varepsilon$ of about $-2.5 \cdot 10^{-4}$. This shows that these operators are
only relevant if $\varepsilon'/\varepsilon$ is below $1 \cdot 10^{-3}$. Unfortunately the matrix elements of the dominant \((V - A) \otimes (V + A)\) operators cannot be determined by the CP conserving data and one has to use non-perturbative methods to estimate them.

Before giving the results for \(\langle Q_i \rangle_I\) in our approach we would like to emphasize why it is reasonable to extract hadronic parameters from Re\(A_I\), while this would not be the case for \(\text{Im}A_I\), which govern \(\varepsilon'/\varepsilon\). The point is that Re\(A_I\), in contrast to \(\text{Im}A_I\), are not expected to be affected by new physics contributions.

It is customary to express the matrix elements \(\langle Q_i \rangle_I\) in terms of non-perturbative parameters \(B_i^{(1/2)}\) and \(B_i^{(3/2)}\) as follows:

\[
\langle Q_i \rangle_0 \equiv B_i^{(1/2)} \langle Q_i \rangle_{(\text{vac})}^{(1/2)}, \quad \langle Q_i \rangle_2 \equiv B_i^{(3/2)} \langle Q_i \rangle_{(\text{vac})}^{(3/2)}.
\] (2.16)

The label “\(\text{vac}\)” stands for the vacuum insertion estimate of the hadronic matrix elements in question for which \(B_i^{(1/2)} = B_i^{(3/2)} = 1\).

Then the approach in \(\text{[25]}\) gives at \(\mu = m_c\):

\[
\langle Q_1( m_c) \rangle_0 = \frac{0.187 \text{ GeV}^3}{z_1( m_c)} - \frac{z_2( m_c)}{z_1( m_c)} \langle Q_2( m_c) \rangle_0, 
\] (2.17)

\[
\langle Q_2( m_c) \rangle_0 = \frac{5}{9} X B_2^{(1/2)}( m_c), 
\] (2.18)

\[
\langle Q_3( m_c) \rangle_0 = \frac{1}{3} X B_3^{(1/2)}( m_c), 
\] (2.19)

\[
\langle Q_4( m_c) \rangle_0 = \langle Q_3( m_c) \rangle_0 + \langle Q_2( m_c) \rangle_0 - \langle Q_1( m_c) \rangle_0, 
\] (2.20)

\[
\langle Q_5( m_c) \rangle_0 = \frac{1}{3} D_6^{(1/2)}( m_c) \langle Q_6( m_c) \rangle_0, 
\] (2.21)

\[
\langle Q_6( m_c) \rangle_0 = -4 \sqrt{\frac{3}{2}} \left[ \frac{m_K^2}{m_b( m_c) + m_d( m_c)} \right]^2 \frac{F_\pi}{\kappa} B_6^{(1/2)}( m_c), 
\] (2.22)

\[
\langle Q_7( m_c) \rangle_0 = -\left[ \frac{1}{6} \langle Q_6( m_c) \rangle_0(\kappa + 1) - \frac{X}{2} \right] B_7^{(1/2)}( m_c), 
\] (2.23)

\[
\langle Q_8( m_c) \rangle_0 = -\left[ \frac{1}{2} \langle Q_6( m_c) \rangle_0(\kappa + 1) - \frac{X}{6} \right] B_8^{(1/2)}( m_c), 
\] (2.24)

\[
\langle Q_9( m_c) \rangle_0 = \frac{3}{2} \langle Q_1( m_c) \rangle_0 - \frac{1}{2} \langle Q_3( m_c) \rangle_0, 
\] (2.25)

\[
\langle Q_{10}( m_c) \rangle_0 = \langle Q_2( m_c) \rangle_0 + \frac{1}{2} \langle Q_1( m_c) \rangle_0 - \frac{1}{2} \langle Q_3( m_c) \rangle_0, 
\] (2.26)

\[
\langle Q_1( m_c) \rangle_2 = \langle Q_2( m_c) \rangle_2 = \frac{8.44 \cdot 10^{-3} \text{ GeV}^3}{z_1( m_c)}, 
\] (2.27)

\[
\langle Q_i \rangle_2 = 0, \quad i = 3, \ldots, 6, 
\] (2.28)

\[
\langle Q_7( m_c) \rangle_2 = -\left[ \frac{\kappa}{6\sqrt{2}} \langle Q_6( m_c) \rangle_0 + \frac{X}{\sqrt{2}} \right] B_7^{(3/2)}( m_c), 
\] (2.29)
\[ \langle Q_8(m_c) \rangle_2 = -\left[ \frac{\kappa}{2\sqrt{2}} \langle Q_6(m_c) \rangle_0 + \frac{\sqrt{2}}{6} X \right] B_8^{(3/2)}(m_c), \quad (2.30) \]
\[ \langle Q_9(m_c) \rangle_2 = \langle Q_{10}(m_c) \rangle_2 = \frac{3}{2} \langle Q_1(m_c) \rangle_2, \quad (2.31) \]

where
\[ \kappa = \frac{F_\pi}{F_K - F_\pi}, \quad X = \sqrt{\frac{3}{2}} F_\pi \left( m_K^2 - m_\pi^2 \right), \quad (2.32) \]

and
\[ \langle Q_6(m_c) \rangle_0 = \frac{\langle Q_6(m_c) \rangle_0}{B_6^{(1/2)}(m_c)}, \quad z_+ = z_1 + z_2. \quad (2.33) \]

The equality of the matrix elements in (2.27) follows from isospin symmetry of strong interactions. Finally, by making the very plausible assumption, valid in known non-perturbative approaches, that \( \langle Q_-(m_c) \rangle_0 \geq \langle Q_+(m_c) \rangle_0 \geq 0 \), where \( Q_\pm = (Q_2 \pm Q_1)/2 \), \( B_2^{(1/2)}(m_c) \) can be determined as well. This gives for \( \Lambda^{(4)}_{\overline{MS}} = 340 \text{ MeV} \)
\[ B_{2,NDR}^{(1/2)}(m_c) = 6.5 \pm 1.0, \quad B_{2,\overline{MS}}^{(1/2)}(m_c) = 6.1 \pm 1.0. \quad (2.34) \]

The actual numerical values used for \( m_K, m_\pi, F_K, F_\pi \) are collected in the appendix of [43]. In particular \( F_\pi = 131 \text{ MeV} \).

It should be noted that this method allows to determine not only the size but also
the renormalization scheme dependence of those matrix elements which can be fixed in
this manner. This dependence enters through \( z_1, z_2(m_c) \) and the scheme dependence
of \( B_2^{(1/2)}(m_c) \). In obtaining the results above one also uses operator relations valid for \( \mu \leq m_c \)
which allow to express \( Q_4, Q_9 \) and \( Q_{10} \) in terms of \( Q_1, Q_2 \) and \( Q_3 \). Theoretical issues
related to these relations in the presence of NLO QCD corrections and the case of matrix

elements for \( \mu > m_c \) are discussed in detail in [43].

In order to proceed further one has to specify the remaining \( B_i \) parameters in the
formulae above. As the numerical analysis in [25] shows \( \varepsilon'/\varepsilon \) is only weakly sensitive
to the values of the parameters \( B_3^{(1/2)}, B_5^{(1/2)}, B_7^{(1/2)}, B_8^{(1/2)} \) and \( B_7^{(3/2)} \) as long as their
absolute values are not substantially larger than 1. As in [25] our strategy is to set
\[ B_{3,7,8}^{(1/2)}(m_c) = 1, \quad B_5^{(1/2)}(m_c) = B_6^{(1/2)}(m_c), \quad B_7^{(3/2)}(m_c) = B_8^{(3/2)}(m_c) \quad (2.35) \]
and to treat \( B_6^{(1/2)}(m_c) \) and \( B_8^{(3/2)}(m_c) \) as free parameters.

The approach in [25] allows then in a good approximation to express \( \varepsilon'/\varepsilon \) or equivalently \( F_{\varepsilon'} \) in terms of \( \Lambda^{(4)}_{\overline{MS}}, m_t, m_s \) and the two non-perturbative parameters \( B_6^{(1/2)} \equiv B_6^{(1/2)}(m_c) \) and \( B_8^{(3/2)} \equiv B_8^{(3/2)}(m_c) \) which cannot be fixed by the CP conserving data.
2.3 The Issue of Final State Interactions

In (2.1) and (2.7) the strong phases \( \delta_0 \approx 37^\circ \) and \( \delta_2 \approx -7^\circ \) are taken from experiment. They can also be calculated from NLO chiral perturbation for \( \pi\pi \) scattering \[37\]. However, generally non-perturbative approaches to hadronic matrix elements are unable to reproduce them at present. As \( \delta_I \) are factored out in (2.1), in non-perturbative calculations in which some final state interactions are present in \( \langle Q_i \rangle_I \) one should make the following replacements in (2.4) and (2.5):

\[
\langle Q_i \rangle_I \rightarrow \frac{\text{Re} \langle Q_i \rangle_I}{(\cos \delta_I)_{\text{th}}} \quad (2.36)
\]

in order to avoid double counting of final state interaction phases. Here \((\cos \delta_I)_{\text{th}}\) is obtained in a given non-perturbative calculation. In leading large-N calculations and in quenched lattice calculations the phases \( \delta_I \) vanish and this replacement is ineffective. When loop corrections in the large-N approach \[19, 47, 48\] and in the chiral quark model \[38\] are included an absorptive part and related non-vanishing phases are generated. Yet, in most calculations the phases are substantially smaller than found in experiment. For instance in the chiral quark model \((\cos \delta_0)_{\text{th}} \approx 0.94\) to be compared to the experimental value \((\cos \delta_0)_{\text{exp}} \approx 0.8\). Even smaller phases are found in \[19, 47, 48\].

The above point has been first discussed by the Trieste group \[38\] who suggested that in models in which at least the real part of \( \langle Q_i \rangle_I \) can be calculated reliably, one should make the following replacements in (2.4) and (2.5):

\[
\langle Q_i \rangle_I \rightarrow \frac{\text{Re} \langle Q_i \rangle_I}{(\cos \delta_I)_{\text{exp}}} \quad (2.37)
\]

where this time the experimental value of \( \delta_I \) enters the denominator. As \((\cos \delta_0)_{\text{exp}} \approx 0.8\) and \((\cos \delta_2)_{\text{exp}} \approx 1\) this modification enhances \( P^{(1/2)} \) by 25% leaving \( P^{(3/2)} \) unchanged. The same procedure has been adopted in \[17\]. To our knowledge there is no method for hadronic matrix elements which can provide \( \delta_0 \approx 37^\circ \) and consequently the replacement (2.37) may lead to an overestimate of the matrix elements.

As in our paper the matrix elements of \((V - A) \otimes (V - A)\) operators are extracted from the data, the replacements in (2.36) and (2.37) are ineffective for the determination of the corresponding contributions. They merely change the definition of the \( B_i \) parameters in the matrix elements of \((V - A) \otimes (V - A)\) operators. The situation is different with the matrix elements of \((V - A) \otimes (V + A)\) operators which are taken from theory. Yet in view of the remarks made above, in our analysis we will use exclusively (2.4) and (2.5) including possible effects of this sort in the uncertainties in \( B_6^{(1/2)} \) and \( B_8^{(3/2)} \).
2.4 An Analytic Formula for $\varepsilon'/\varepsilon$

As shown in [19], it is possible to cast the formal expressions for $\varepsilon'/\varepsilon$ in (2.2)–(2.5) into an analytic formula which exhibits the $m_t$ dependence together with the dependence on $m_s$, $\Lambda^{(4)}_{\overline{MS}}$, $B_6^{(1/2)}$ and $B_8^{(3/2)}$. To this end the approach for hadronic matrix elements presented above is used and $\Omega_{\eta+\eta'}$ is set to 0.25. The analytic formula given below, while being rather accurate, exhibits various features which are not transparent in a pure numerical analysis. It can be used in phenomenological applications if one is satisfied with a few percent accuracy. Needless to say, in our numerical analysis in Section 3 we have used exact expressions.

In this formulation the function $F_{\varepsilon'}$ is given simply as follows ($x_t = m_t^2/M_W^2$):

$$F_{\varepsilon'} = P_0 + P_X X_0(x_t) + P_Y Y_0(x_t) + P_Z Z_0(x_t) + P_E E_0(x_t).$$  \hspace{1cm} (2.38)

Exact expressions for the $m_t$-dependent functions in (2.38) can be found for instance in [9, 44]. In the range $150 \text{ GeV} \leq m_t \leq 180 \text{ GeV}$ one has to an accuracy much better than 1% $X_0(x_t) = 1.51 \left( \frac{m_t}{165 \text{ GeV}} \right)^{1.13}$, $Y_0(x_t) = 0.96 \left( \frac{m_t}{165 \text{ GeV}} \right)^{1.55}$, $Z_0(x_t) = 0.66 \left( \frac{m_t}{165 \text{ GeV}} \right)^{1.90}$, $E_0(x_t) = 0.27 \left( \frac{m_t}{165 \text{ GeV}} \right)^{-1.08}$.  \hspace{1cm} (2.39)

In our numerical analysis we use exact expressions.

The coefficients $P_i$ are given in terms of $B_6^{(1/2)} \equiv B_6^{(1/2)}(m_c)$, $B_8^{(3/2)} \equiv B_8^{(3/2)}(m_c)$ and $m_s(m_c)$ as follows:

$$P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8.$$  \hspace{1cm} (2.41)

where

$$R_6 \equiv B_6^{(1/2)} \left[ \frac{137 \text{ MeV}}{m_s(m_c) + m_d(m_c)} \right]^2, \quad R_8 \equiv B_8^{(3/2)} \left[ \frac{137 \text{ MeV}}{m_s(m_c) + m_d(m_c)} \right]^2.$$  \hspace{1cm} (2.42)

The $P_i$ are renormalization scale and scheme independent. They depend, however, on $\Lambda^{(4)}_{\overline{MS}}$. In table 2 we give the numerical values of $r_i^{(0)}$, $r_i^{(6)}$ and $r_i^{(8)}$ for different values of $\Lambda^{(4)}_{\overline{MS}}$ at $\mu = m_c$ in the NDR renormalization scheme. This table differs from the ones presented in [8, 34] in the values of $\Lambda^{(4)}_{\overline{MS}}$ and the central value of $m_s(m_c)$ in $R_8$ which has been lowered from 150 MeV to 130 MeV. The coefficients $r_i^{(0)}$, $r_i^{(6)}$ and $r_i^{(8)}$ depend only very weakly on $m_s(m_c)$ as the dominant $m_s$ dependence has been factored out. The numbers given in table 2 correspond exactly to $m_s(m_c) = 130$ MeV. However, even for $m_s(m_c) \approx 100$ MeV or $m_s(m_c) \approx 160$ MeV, the analytic expressions given here reproduce
the numerical calculations of $\varepsilon'/\varepsilon$ given in Section 3 to better than 4%. For different scales $\mu$ the numerical values in the tables change without modifying the values of the $P_i$'s as it should be. The values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ should also be modified, in principle, but as a detailed numerical analysis in [29] showed, it is a good approximation to keep them $\mu$-independent for $1\text{ GeV} \leq \mu \leq 2\text{ GeV}$. We will return to this point below.

Table 2: Coefficients in the formula (2.41) for various $\Lambda_{\overline{\text{MS}}}^{(4)}$ in the NDR scheme. The last row gives the $r_0$ coefficients in the HV scheme.

| $i$ | $\Lambda_{\overline{\text{MS}}}^{(4)} = 290\text{ MeV}$ | $\Lambda_{\overline{\text{MS}}}^{(4)} = 340\text{ MeV}$ | $\Lambda_{\overline{\text{MS}}}^{(4)} = 390\text{ MeV}$ |
|-----|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
|     | $r_i^{(0)}$ | $r_i^{(6)}$ | $r_i^{(8)}$ | $r_i^{(0)}$ | $r_i^{(6)}$ | $r_i^{(8)}$ | $r_i^{(0)}$ | $r_i^{(6)}$ | $r_i^{(8)}$ |
| 0   | -2.771     | 9.779     | 1.429     | -2.811     | 11.127     | 1.267     | -2.849     | 12.691     | 1.081     |
| $X_0$ | 0.532     | 0.017     | 0         | 0.518      | 0.021      | 0         | 0.506      | 0.024      | 0         |
| $Y_0$ | 0.396     | 0.072     | 0         | 0.381      | 0.079      | 0         | 0.367      | 0.087      | 0         |
| $Z_0$ | 0.354     | -0.013    | -9.404    | 0.409      | -0.015     | -10.230   | 0.470      | -0.017     | -11.164   |
| $E_0$ | 0.182     | -1.144    | 0.411     | 0.167      | -1.254     | 0.461     | 0.153      | -1.375     | 0.517     |
| 0   | -2.749     | 8.596     | 1.050     | -2.788     | 9.638      | 0.871     | -2.825     | 10.813     | 0.669     |

The inspection of table 2 shows that the terms involving $r_0^{(6)}$ and $r_Z^{(8)}$ dominate the ratio $\varepsilon'/\varepsilon$. Moreover, the function $Z_0(x_t)$ representing a gauge invariant combination of $Z^0$- and $\gamma$-penguins grows rapidly with $m_t$ and due to $r_Z^{(8)} < 0$ these contributions suppress $\varepsilon'/\varepsilon$ strongly for large $m_t$ [20, 21].

2.5 Renormalization Scheme Dependence

Concerning the renormalization scheme dependence only the coefficients $r_0^{(0)}$, $r_0^{(6)}$ and $r_0^{(8)}$ are scheme dependent at the NLO level. Their values in the HV scheme are given in the last row of table 2. We note that the parameter $r_0^{(0)}$ is essentially the same in both schemes as the dominant scheme independent contributions to $r_0^{(0)}$ have been determined by the data on Re$A_I$. Since $P_0$ must be scheme independent and $r_0^{(6)}$ and $r_0^{(8)}$ are scheme dependent, we conclude that $B_6^{(1/2)}$ and $B_8^{(3/2)}$ must be scheme dependent. Indeed the matrix elements in the NDR and HV schemes are related by a finite renormalization which can be found in equation (3.7) of [29]. Using this equation together with the approach to matrix elements presented above, we find approximate relations between the values of
\[(B_6^{(1/2)}, B_8^{(3/2)})\) in the NDR scheme and the corresponding values in the HV scheme:

\[
(B_6^{(1/2)})_{\text{HV}} \approx 1.2(B_6^{(1/2)})_{\text{NDR}}, \quad (B_8^{(3/2)})_{\text{HV}} \approx 1.2(B_8^{(3/2)})_{\text{NDR}}.
\] (2.43)

One can check that the scheme dependence of \((B_6^{(1/2)}, B_8^{(3/2)})\) cancels to a very good approximation the one of \(r_0^{(6)}\) and \(r_0^{(8)}\) so that \(P_0\) is scheme independent.

On the other hand the coefficients \(r_i, i = X, Y, Z, E\) are scheme independent at NLO. This is related to the fact that the \(m_t\) dependence in \(\varepsilon'/\varepsilon\) enters first at the NLO level and consequently all coefficients \(r_i\) in front of the \(m_t\) dependent functions must be scheme independent. Strictly speaking then the scheme dependence of \(B_6^{(1/2)}\) and \(B_8^{(3/2)}\) inserted into \(P_i\) with \(i \neq 0\) is really a part of higher order contributions to \(\varepsilon'/\varepsilon\) and should be dropped at the NLO level. Formally this can be done by not performing the finite renormalization when going from the NDR to the HV scheme. Then the coefficients \(P_i\) with \(i \neq 0\) are clearly scheme independent.

In practice the situation is more complicated. The present non-perturbative methods used to evaluate \(B_6^{(1/2)}\) and \(B_8^{(3/2)}\) like the large-N approach are not sensitive to the renormalization scheme dependence and we do not know which renormalization scheme the resulting values for these parameters correspond to. Lattice calculations, QCD sum rule calculations and the chiral quark model can in principle give us the scheme dependence of \(B_6^{(1/2)}\) and \(B_8^{(3/2)}\) but the accuracy of these methods must improve before they could be useful in this respect.

In view of this situation our strategy will be to use the same values for \(B_6^{(1/2)}\) and \(B_8^{(3/2)}\) in the NDR and HV schemes. This will introduce a scheme dependence in \(P_0\) and consequently in \(\varepsilon'/\varepsilon\) but will teach us something about the uncertainty in \(\varepsilon'/\varepsilon\) due to the poor sensitivity of present methods to renormalization scheme dependence.

It should also be noted that even if we knew the scheme dependence of \(B_6^{(1/2)}\) and \(B_8^{(3/2)}\) without the ability of separating a scheme independent part in these parameters, the resulting \(\varepsilon'/\varepsilon\) would be scheme dependent at the NLO level. This time the scheme dependence would enter through the scheme dependence of \(P_i\) with \(i \neq 0\). The latter scheme dependence could only be reduced by including the next order of perturbation theory in the Wilson coefficients: a formidable task. We should also stress that the scheme dependences discussed here apply not only to QCD corrections but also to QED corrections. That is QED corrections to the matrix elements of operators have to be also known.

For similar reasons the NLO analysis of \(\varepsilon'/\varepsilon\) is still insensitive to the precise definition
of $m_t$. In view of the fact that the NLO calculations needed to extract $\text{Im}\lambda_t$ have been performed with $m_t = m_t(m_t)$ we will also use this definition in calculating $F_{\epsilon'}$.

### 2.6 Status of the Strange Quark Mass

At this point it seems appropriate to summarize the present status of the value of the strange quark mass. Since different methods provide $m_s$ at different values of $\mu$ we give in table 3 a dictionary between the $m_s$ values at $\mu = 1\text{ GeV}$, $\mu = m_c = 1.3\text{ GeV}$ and $\mu = 2\text{ GeV}$.

In the case of quenched lattice QCD the present status has been summarized recently by Kenway [50]. Averaging the results presented by him at LATTICE 98, we obtain $m_s(2\text{ GeV}) = (120 \pm 20)\text{ MeV}$. It is expected that unquenching will lower this value but it is difficult to tell by how much. Strange quark masses as low as $m_s(2\text{ GeV}) = 80\text{ MeV}$ have been reported in the literature [51], although the errors on unquenched calculations are still large. Lacking more precise information on unquenched lattice calculations we take as the average lattice value

$$m_s(2\text{ GeV}) = (110 \pm 20)\text{ MeV}.$$  \hspace{1cm} (2.44)

which is very close to the one given by Gupta [52].

A large number of determinations of the strange quark mass from QCD sum rules exist in the literature. Historically, QCD sum rule results for $m_s$ are given at a scale 1 GeV. Taking an average over recent results [53]-[58] we find $m_s(1\text{ GeV}) = (170 \pm 30)\text{ MeV}$. This translates to $m_s(2\text{ GeV}) = (124 \pm 22)\text{ MeV}$, somewhat higher than the lattice result but compatible within the errors. QCD sum rules also allow to derive lower bounds on the strange quark mass. It was found that generally $m_s(2\text{ GeV}) \gtrsim 100\text{ MeV}$ [59]-[61]. If these bounds hold, they would rule out the very low strange mass values found in unquenched lattice QCD simulations.

Finally, one should also mention the very recent determination of the strange mass from the hadronic $\tau$-spectral function [62, 63] which proceeds similarly to the determination of $\alpha_s$ from $\tau$-decays. Normalized at the $\tau$ mass, the ALEPH collaboration obtains $m_s(m_\tau) = (176^{+46}_{-57})\text{ MeV}$ which translates to $m_s(2\text{ GeV}) = (170^{+44}_{-55})\text{ MeV}$. We observe that the central value is much larger than the corresponding results from lattice and sum rules although the error is still large. In the future, however, improved experimental statistics and a better understanding of perturbative QCD corrections should make the determination of $m_s$ from the $\tau$-spectral function competitive to the other methods.
Table 3: The dictionary between the values of \( m_s(\mu) \) in units of MeV. \( \Lambda_{\overline{MS}}^{(4)} = 340 \) MeV and \( m_c = 1.3 \) GeV have been used.

| \( m_s(m_c) \) | 105 | 130 | 155 | 180 |
|-----------------|-----|-----|-----|-----|
| \( m_s(2 \text{ GeV}) \) | 90  | 111 | 132 | 154 |
| \( m_s(1 \text{ GeV}) \) | 123 | 152 | 181 | 211 |

We conclude that the error on \( m_s \) is still rather large. In our numerical analysis of \( \varepsilon'/\varepsilon \), where \( m_s \) is evaluated at the scale \( m_c \), we will set

\[
m_s(m_c) = (130 \pm 25) \text{ MeV},
\]

roughly corresponding to \( m_s(2 \text{ GeV}) \) given in (2.44).

2.7 Review of \( \hat{B}_K \), \( B_6^{(1/2)} \) and \( B_8^{(3/2)} \)

2.7.1 \( \hat{B}_K \)

The renormalization group invariant parameter \( \hat{B}_K \) is defined through

\[
\hat{B}_K = B_K(\mu) \left[ \alpha_s^{(3)}(\mu) \right]^{-2/9} \left[ 1 + \frac{\alpha_s^{(3)}(\mu)}{4\pi} J_3 \right],
\]

(2.46)

\[
\langle \bar{K}^0| \bar{s}d \rangle_{V-A} \langle \bar{s}d \rangle_{V-A} |K^0 \rangle \equiv \frac{8}{3} B_K(\mu) F_K^2 m_K^2
\]

(2.47)

where \( J_3 = 1.895 \) and \( J_3 = 0.562 \) in the NDR and HV scheme respectively.

There is a long history of evaluating \( \hat{B}_K \) in various non-perturbative approaches. The status of quenched lattice calculations [64, 65, 66] as of 1998 has been reviewed by Gupta [52]. The most accurate result for \( B_K(2 \text{ GeV}) \) using lattice methods has been obtained by the JLQCD collaboration [64]: \( B_K(2 \text{ GeV}) = 0.628 \pm 0.042 \). A similar result has been published by Gupta, Kilcup and Sharpe [65] last year. The APE collaboration [66] found \( B_K(2 \text{ GeV}) = 0.66 \pm 0.11 \) which is consistent with [64, 65]. The final lattice value given by Gupta was then

\[
(\hat{B}_K)_{\text{Lattice}} = 0.86 \pm 0.06 \pm 0.06
\]

(2.48)

where the second error is attributed to quenching. The corresponding result from the APE collaboration [66] was \( \hat{B}_K = 0.93 \pm 0.16 \). The most recent global analysis of lattice
data including also the UKQCD results gives

\[ \hat{B}_K = 0.89 \pm 0.13 \]  

in good agreement with (2.48).

In the 1/N approach of [19] one finds \( \hat{B}_K = 0.70 \pm 0.10 \) [68, 69]. The most recent analysis in this approach with a modified matching procedure and inclusion of higher order terms in momenta gives a bigger range \( 0.4 < \hat{B}_K < 0.7 \) [17] which results from a stronger dependence on the matching scale between short and long distance contributions than found in previous calculations. It is hoped that inclusion of higher resonances in the effective low energy theory will make the dependence weaker.

QCD sum rules give results around \( \hat{B}_K = 0.5 - 0.6 \) with errors in the range 0.2–0.3 [70]. Still lower values are found using the QCD Hadronic Duality approach (\( \hat{B}_K = 0.39 \pm 0.10 \)) [71], the SU(3) symmetry and PCAC (\( \hat{B}_K = 1/3 \)) [72] or chiral perturbation theory at next-to-leading order (\( \hat{B}_K = 0.42 \pm 0.06 \)) [73]. However, as stressed in [69, 74], SU(3) breaking effects considerably increase these values. Finally, the analysis in the chiral quark model gives a value as high as \( \hat{B}_K = 1.1 \pm 0.2 \) [75].

In our numerical analysis presented below we will use

\[ \hat{B}_K = 0.80 \pm 0.15 \]  

which is in the ball park of various lattice and large-N estimates. We will, however, discuss what happens if values outside this range are used.

### 2.7.2 General Comments on \( B_6^{(1/2)} \) and \( B_8^{(3/2)} \)

As the different methods for the evaluation of these parameters use different values of \( \mu \), it is useful to say something about their \( \mu \)-dependence. As seen in (2.22) and (2.30) the \( \mu \)-dependences of \( \langle Q_6(\mu) \rangle_0 \) and \( \langle Q_8(\mu) \rangle_2 \) are governed by the known \( \mu \)-dependence of \( m_s \) and \( m_d \) and could also in principle be present in \( B_6^{(1/2)}(\mu) \) and \( B_8^{(3/2)}(\mu) \).

Now, as can be demonstrated in the large-N limit, the \( \mu \)-dependence of \( 1/(m_s(\mu) + m_d(\mu))^2 \) in \( \langle Q_6 \rangle_0 \) and \( \langle Q_8 \rangle_2 \) is exactly cancelled in the decay amplitude by the diagonal evolution (no operator mixing) of the Wilson coefficients \( y_6(\mu) \) and \( y_8(\mu) \) taken in the large-N limit. An explicit demonstration of this feature is given in [3]. In the large-N limit one also finds

\[ B_6^{(1/2)} = B_8^{(3/2)} = 1, \quad \text{(Large - N Limit).} \]
The \( \mu \)-dependence of \( B^{(1/2)}_6 \) and \( B^{(3/2)}_8 \) for \( N = 3 \) and in the presence of mixing with other operators has been investigated in [24]. This analysis shows that \( B^{(1/2)}_6 \) and \( B^{(3/2)}_8 \) depend only very weakly on \( \mu \), when \( \mu \geq 1 \) GeV. In such a numerical renormalization study the factors \( B^{(1/2)}_6 \) and \( B^{(3/2)}_8 \) have been set to unity at \( \mu = m_c \). Subsequently the evolution of the matrix elements in the range \( 1 \text{ GeV} \leq \mu \leq 2 \) GeV has been calculated showing that for the NDR scheme \( B^{(1/2)}_6 \) and \( B^{(3/2)}_8 \) were \( \mu \)-independent within an accuracy of 2%. The \( \mu \) dependence in the HV scheme has been found to be stronger but still below 6%. Similar weak \( \mu \)-dependences have been found for \( B^{(1/2)}_5 \) and \( B^{(3/2)}_7 \).

These findings simplify the comparison of results for \( B^{(1/2)}_5 \) and \( B^{(3/2)}_7 \) obtained by different methods.

### 2.7.3 \( B^{(1/2)}_6 \) and \( B^{(3/2)}_8 \) from the Lattice

The lattice calculations of \( B^{(1/2)}_{5,6} \) and \( B^{(3/2)}_{7,8} \) have been reviewed by Gupta [52] and the APE collaboration [66]. They are all given at \( \mu = 2 \) GeV and in the NDR scheme. The most reliable results are found for \( B^{(3/2)}_{7,8} \). The “modern” quenched estimates for these parameters are collected in table 4 [52]. The errors given there are purely statistical. The first three calculations use perturbative matching between lattice and continuum, the last one uses non-perturbative matching. All three groups agree within perturbative matching that \( B^{(3/2)}_{7,8} \) are suppressed below unity: \( B^{(3/2)}_{7} \approx 0.6 \) and \( B^{(3/2)}_{8} \approx 0.8 \). The non-perturbative matching seems to increase these results by about 20%. It is important to see whether this feature will be confirmed by other groups.

Concerning the lattice results for \( B^{(1/2)}_{5,6} \) the situation is worse. The old results read

\[
B^{(1/2)}_{5,6}(2 \text{ GeV}) = 1.0 \pm 0.2 \quad [77, 78].
\]

More accurate estimates for \( B^{(1/2)}_6 \) have been given in [79]:

\[
B^{(1/2)}_6(2 \text{ GeV}) = 0.67 \pm 0.04 \pm 0.05 \quad \text{(quenched)} \quad \text{and} \quad B^{(1/2)}_6(2 \text{ GeV}) = 0.76 \pm 0.03 \pm 0.05 \quad \text{(f = 2)}.
\]

However, as stressed by Gupta [72], the systematic errors in this analysis are not really under control. A recent work of Pekurovsky and Kilcup [46], in which \( B^{(1/2)}_6 \) is even found to be negative, unfortunately supports this criticism. We have to conclude that there are no solid predictions for \( B^{(1/2)}_{5,6} \) from the lattice at present.

### 2.7.4 \( B^{(1/2)}_6 \) and \( B^{(3/2)}_8 \) from the 1/N Approach

The 1/N approach to weak hadronic matrix elements was introduced in [19]. In this approach the 1/N expansion becomes a loop expansion in an effective meson theory. In the strict large-N limit only the tree level matrix elements of \( Q_6 \) and \( Q_8 \) contribute and
one finds (2.51) while $B_{5}^{(1/2)} = B_{7}^{(3/2)} = 0$. The latter fact is not disturbing, however, as the operators $Q_5$ and $Q_7$ having small Wilson coefficients are unimportant for $\varepsilon'/\varepsilon$.

In view of the fact that for $B_{6}^{(1/2)} = B_{8}^{(3/2)} = 1$ and the known value of $m_t$ there is a strong cancellation between quark and electroweak penguin contributions to $\varepsilon'/\varepsilon$, it is important to investigate whether the $1/N$ corrections significantly affect this cancellation. This has been investigated in [48], where a calculation of $\langle Q_6 \rangle_0$ and $\langle Q_8 \rangle_2$ in the twofold expansion in powers of external momenta $p$, and in $1/N$ has been presented. The final results for $\langle Q_6 \rangle_0$ and $\langle Q_8 \rangle_2$ in [48] include the orders $p^2$ and $p^0/N$. For $\langle Q_8 \rangle_2$ also the term $p^0$ contributes. Of particular interest are the $O(p^0/N)$ contributions resulting from non-factorizable chiral loops which are important for the matching between long- and short-distance contributions. The cut-off scale $\Lambda_c$ in these non-factorizable diagrams is identified with the QCD renormalization scale $\mu$ which enters the Wilson coefficients.

Table 5: Results for $B_{6}^{(1/2)}$ and $B_{8}^{(3/2)}$ obtained in the $1/N$ approach.

| $\Lambda_c$ | $B_{6}^{(1/2)}$ | $B_{8}^{(3/2)}$ |
|------------|----------------|----------------|
| 0.6 GeV    | 1.10           | 0.96           |
| 0.7 GeV    | (1.30)         | (1.19)         |
| 0.8 GeV    | 0.64           | 0.56           |
| 0.9 GeV    | (0.71)         | (0.65)         |

In table 3, taken from [48, 47], we show the values of $B_{6}^{(1/2)}$ and $B_{8}^{(3/2)}$ as functions of the cut-off scale $\Lambda_c$. The results depend on whether $F_\pi$ or $F_K$ is used in the calculation, the difference being of higher order. The results using $F_K$ are shown in parentheses. The decrease of both B-factors with $\Lambda_c = \mu$ is qualitatively consistent with their $\mu$-dependence.
found for \( \mu \geq 1 \) in [23], but it is much stronger. Clearly one could also expect a stronger \( \mu \)-dependence in the analysis of [23] for \( \mu \leq 1 \) GeV, but in view of large perturbative corrections for such small scales a meaningful test of the dependence in table 5 cannot be made. We note that for \( \Lambda_c = 0.7 \) GeV the value of \( B_6^{(1/2)} \) is close to unity as in the large-N limit. However, \( B_8^{(3/2)} \) is considerably suppressed. An interesting feature of these results is the near \( \Lambda_c \) independence of the ratio \( B_6^{(1/2)}/B_8^{(3/2)} \). Consequently the results in [48, 47] can be summarized by

\[
\frac{B_6^{(1/2)}}{B_8^{(3/2)}} \approx 1.72 \ (1.84), \quad 0.72 \ (0.99) \leq B_6^{(1/2)} \leq 1.10 \ (1.30). \tag{2.52}
\]

It is difficult to decide which value should be used in the phenomenology of \( \varepsilon'/\varepsilon \). On the one hand, for \( \Lambda_c \geq 0.7 \) GeV neglected contributions from vector mesons in the loops should be included. On the other hand for \( \Lambda_c = \mu = 0.6 \) GeV the short distance calculations are questionable. Probably the best thing to do at present is to vary \( \Lambda_c = \mu \) in the full range shown in table 5. This has been done in a recent analysis [80] in which \( \varepsilon'/\varepsilon \) has been found to be a decreasing function of \( \Lambda_c \).

Finally, we would like to mention that the first non-trivial \( 1/N \) corrections to the matrix elements of \( Q_7 \) have been calculated in [81] using the methods developed in [82]. In particular it has been found that \( B_7^{(3/2)} \) is a rather strongly increasing function of \( \mu \) with negative values for \( \mu \leq m_c, B_7^{(3/2)}(m_c) = 0 \) and positive values for \( \mu > m_c \). This strong \( \mu \)-dependence of \( B_7^{(3/2)} \) is rather surprising as the numerical renormalization group analysis in [23] has shown a rather weak dependence of this parameter. We suspect that the inclusion of the full mixing between \( Q_7 \) and other operators in the analysis of [81] would weaken the \( \mu \)-dependence of \( B_7^{(3/2)} \) considerably. While this issue requires an additional investigation, the value of \( B_7^{(3/2)} \) has fortunately only a minor impact on \( \varepsilon'/\varepsilon \). Setting \( B_7^{(3/2)}(m_c) = 0 \) instead of \( B_7^{(3/2)}(m_c) = B_8^{(3/2)}(m_c) \) used here would change our results for \( \varepsilon'/\varepsilon \) only by a few percent.

### 2.7.5 \( B_6^{(1/2)} \) and \( B_8^{(3/2)} \) from the Chiral Quark Model

Effective Quark Models of QCD can be derived in the framework of the extended Nambu-Jona-Lasinio model of chiral symmetry breaking [83]. For kaon decays and in particular for \( \varepsilon'/\varepsilon \), an extensive analysis of this model including chiral loops, gluon and \( O(p^4) \) corrections has been performed over the last years by the Trieste group [84, 85]. The crucial parameters in this approach are a mass parameter \( M \) and the condensates \( \langle \bar{q}q \rangle \) and...
\(\langle \alpha_s GG \rangle\). They can be constrained by imposing the \(\Delta I = 1/2\) rule.

Since there exists a nice review \cite{Trieste} by the Trieste group, we will only quote here their estimates of the relevant \(B_i\) parameters. They are given in the HV scheme as follows

\[
B_6^{(1/2)} = 1.6 \pm 0.3, \quad B_8^{(3/2)} = 0.92 \pm 0.02, \quad \text{(Chiral QM).} \quad (2.53)
\]

Translating these values into the NDR scheme by means of (2.43) one finds

\[
B_6^{(1/2)} = 1.33 \pm 0.25, \quad B_8^{(3/2)} = 0.77 \pm 0.02, \quad \text{(NDR).} \quad (2.54)
\]

We observe a substantial enhancement of \(B_6^{(1/2)}\) in the chiral quark model, not found in other calculations, and a moderate suppression of \(B_8^{(3/2)}\). The errors given above arise from the variation of \(m_s\). We will return to this point in subsection 3.5.

It should be remarked that the definitions of the \(B_i\) parameters used in \cite{Trieste} agree with our definitions only if in the vacuum insertion formulae in \cite{Trieste} the \(\langle \bar qq \rangle\) condensate is given in terms of \(m_s\) as follows:

\[
\langle \bar qq \rangle^2 = \frac{F_\pi^4}{4} \left[ \frac{m_K^2}{m_s + m_d} \right]^2. \quad (2.55)
\]

This means that in the usual PCAC relation one has to set \(F_K = F_\pi\).

It is interesting to observe that in this method \(B_6^{(1/2)}/B_8^{(3/2)} = 1.74 \pm 0.33\) in the ballpark of (2.52). It will be of interest to see whether future lattice calculations will confirm this correlation between \(B_6^{(1/2)}\) and \(B_8^{(3/2)}\).

### 2.7.6 \(B_6^{(1/2)}\) and the \(\Delta I = 1/2\) Rule

In one of the first estimates of \(\varepsilon'/\varepsilon\), Gilman and Wise \cite{GW} used the suggestion of Vainshtein, Zakharov and Shifman \cite{VZS} that the amplitude \(\text{Re}A_0\) is dominated by the QCD-penguin operator \(Q_6\). Estimating \(\langle Q_6 \rangle_0\) in this manner they predicted a large value of \(\varepsilon'/\varepsilon\). Since then it has been understood \cite{Gasser,Leutwyler,Ekstein} that as long as the scale \(\mu\) is not much lower than 1 GeV the amplitude \(\text{Re}A_0\) is dominated by the operators \(Q_1\) and \(Q_2\), rather than by \(Q_6\). Indeed, at least in the HV scheme the operator \(Q_6\) does not contribute to \(\text{Re}A_0\) for \(\mu = m_c\) at all, as its coefficient \(z_6(m_c)\) relevant for this amplitude vanishes. Also in the NDR scheme \(z_6(m_c)\) is negligible.

For decreasing \(\mu\) the coefficient \(z_6(\mu)\) increases and the \(Q_6\) contribution to \(\text{Re}A_0\) is larger. However, if the analyses in \cite{Gasser,Leutwyler,Ekstein} are taken into account, the operators \(Q_1\) and \(Q_2\) are responsible for at least 90\% of \(\text{Re}A_0\) if the scale \(\mu = 1\) GeV is considered.
Therefore in our opinion there is no strict relation between the large value of $\varepsilon'/\varepsilon$ and the $\Delta I = 1/2$ rule as sometimes stated in the literature. Moreover, if the 90% contribution of the operators $Q_1$ and $Q_2$ to $\text{Re}A_0$ is taken into account and $z_6(1 \text{ GeV})$ is calculated in the NDR scheme, $B_6^{(1/2)}$ cannot exceed 1.5 if $m_s(1 \text{ GeV}) = 150 \text{ MeV}$. Consequently we do not think that values of $B_6^{(1/2)}$ in the NDR scheme as high as 4.0 suggested in [40] are plausible. Unfortunately, due to the very strong $\mu$ and renormalization scheme dependences of $z_6(\mu)$, general definite conclusions about $B_6^{(1/2)}$ cannot be reached in this manner at present. Similarly, we cannot exclude the possibility that $B_6^{(1/2)}$ is substantially higher than unity if it turned out that the present methods overestimate the role of $Q_1$ and $Q_2$ in $\text{Re}A_0$.

2.7.7 Summary

We have seen that most non-perturbative approaches discussed above found $B_8^{(3/2)}$ below unity. The suppression of $B_8^{(3/2)}$ below unity is rather modest (at most 20%) in the lattice approaches and in the chiral quark model. In the $1/N$ approach $B_8^{(3/2)}$ is rather strongly suppressed and can be as low as 0.5.

Concerning $B_6^{(1/2)}$ the situation is worse. As we stated above there is no solid prediction for this parameter in the lattice approach. On the other hand while the average value of $B_6^{(1/2)}$ in the $1/N$ approach is close to 1.0, the chiral quark model gives at $\mu = 0.8$ GeV and in the NDR scheme the value for $B_6^{(1/2)}$ as high as $1.33 \pm 0.25$. Interestingly both approaches give the ratio $B_6^{(1/2)}/B_8^{(3/2)}$ in the ball park of 1.7.

Guided by the results presented above and biased to some extent by the results from the large-N approach and lattice calculations, we will use in our numerical analysis below $B_6^{(1/2)}$ and $B_8^{(3/2)}$ in the ranges:

$$B_6^{(1/2)} = 1.0 \pm 0.3, \quad B_8^{(3/2)} = 0.8 \pm 0.2$$ (2.56)

keeping always $B_6^{(1/2)} \geq B_8^{(3/2)}$. In our 1996 analysis [34] we have used $B_6^{(1/2)} = 1.0 \pm 0.2$ and $B_8^{(3/2)} = 1.0 \pm 0.2$ without the constraint $B_6^{(1/2)} \geq B_8^{(3/2)}$. The decrease of $B_8^{(3/2)}$ below unity is motivated by the recent results discussed above. The increase in the range of $B_6^{(1/2)}$ is supposed to take effectively into account the uncertainty in $\Omega_{\eta+\eta'}$ which we estimate to be at most $\pm 30\%$ i.e $\Omega_{\eta+\eta'} = 0.25 \pm 0.08$. We will return to this point in Section 3.
3 Numerical Results in the Standard Model

3.1 Input Parameters

In order to make predictions for $\varepsilon'/\varepsilon$ we need the value of $\text{Im}\lambda_t$. This can be obtained from the standard analysis of the unitarity triangle which uses the data for $|V_{cb}|$, $|V_{ub}|$, $\varepsilon$, $\Delta M_d$ and $\Delta M_s$, where the last two measure the size of $B^0_{d,s} - \bar{B}^0_{d,s}$ mixings. Since this analysis is very well known we do not list the relevant formulae here. They can be found for instance in [9, 88].

The input parameters needed to perform the standard analysis of the unitarity triangle are given in table 6, where $m_t$ refers to the running current top quark mass defined at $\mu = m_{\text{pole}}^t$. It corresponds to $m_t^\text{pole} = 174.3 \pm 5.1$ GeV measured by CDF and D0 [87].

We also recall that the lower bound on $\Delta M_s$ together with $\Delta M_d$ puts the following constraint on the ratio $|V_{td}|/|V_{ts}|$:

$$|V_{td}|/|V_{ts}| < \xi \sqrt{m_{Bs}/m_{Bd}} \sqrt{\Delta M_d/\Delta M_s^\text{min}}, \quad \xi = F_{Bs}/F_{Bd} \sqrt{B_{Bs}/B_{Bd}}. \quad (3.57)$$

The range for $\Lambda^{(4)}_{\text{MS}}$ in table 6 corresponds roughly to $\alpha_s(M_Z) = 0.119 \pm 0.003$.

3.2 Monte Carlo and Scanning Estimates of $\varepsilon'/\varepsilon$

In what follows we will present two types of numerical analyses of $\text{Im}\lambda_t$ and $\varepsilon'/\varepsilon$:

- Method 1: The experimentally measured numbers are used with Gaussian errors and for the theoretical input parameters we take a flat distribution in the ranges given in table 6.
- Method 2: Both the experimentally measured numbers and the theoretical input parameters are scanned independently within the ranges given in table 6.

Using the first method we find the probability density distributions for $\text{Im}\lambda_t$ and $\varepsilon'/\varepsilon$ in figs. 1 and 2 respectively. From the distributions in figs. 1 and 2 we deduce the following results:

$$\text{Im}\lambda_t = (1.33 \pm 0.14) \cdot 10^{-4} \quad (3.58)$$

$$\varepsilon'/\varepsilon = (7.7^{+6.0}_{-3.5}) \cdot 10^{-4} \quad \text{(NDR)} \quad (3.59)$$
Table 6: Collection of input parameters. We impose $B_b^{(1/2)} \geq B_s^{(3/2)}$.

| Quantity          | Central | Error       | Reference |
|-------------------|---------|-------------|-----------|
| $|V_{cb}|$         | 0.040   | ±0.002     | [3]       |
| $|V_{ub}|$         | $3.56 \cdot 10^{-3}$ | ±$0.56 \cdot 10^{-3}$ | [89]     |
| $\hat{B}_K$      | 0.80    | ±0.15      | See Text |
| $\sqrt{B_d F_{B_d}}$ | 200 MeV | ±40 MeV    | [90]     |
| $m_t$             | 165 GeV | ±5 GeV     | [87]     |
| $\Delta M_d$     | 0.471 ps$^{-1}$ | ±0.016 ps$^{-1}$ | [91]     |
| $\Delta M_s$     | $> 12.4$ ps$^{-1}$ | $95\%$C.L.  | [91]     |
| $\xi$            | 1.14    | ±0.08      | [90]     |
| $\Lambda_{MS}^{(4)}$ | 340 MeV | ±50 MeV    | [3, 92]  |
| $m_s(m_c)$        | 130 MeV | ±25 MeV    | See Text |
| $B_b^{(1/2)}$     | 1.0     | ±0.3       | See Text |
| $B_s^{(3/2)}$     | 0.8     | ±0.2       | See Text |

Since the probability density in fig. 1 is rather symmetric we give only the mean and the standard deviation for $\text{Im} \lambda_t$. On the other hand, the resulting probability density distribution for $\epsilon'/\epsilon$ is very asymmetric with a very long tail towards large values. Therefore we decided to quote the median and the $68\%(95\%)$ confidence level intervals. This means that 68\% of our data can be found inside the corresponding error interval and that 50\% of our data has smaller $\epsilon'/\epsilon$ than our median.

We observe that negative values of $\epsilon'/\epsilon$ can be excluded at 95\% C.L. For completeness we quote the mean and the standard deviation for $\epsilon'/\epsilon$:

$$\epsilon'/\epsilon = 9.1 \pm 6.2 \quad \text{(NDR)}$$  \hspace{1cm} (3.60)

Using the second method and the parameters in table 6 we find:

$$1.04 \cdot 10^{-4} \leq \text{Im} \lambda_t \leq 1.63 \cdot 10^{-4} \quad \text{(3.61)}$$

$$1.05 \cdot 10^{-4} \leq \epsilon'/\epsilon \leq 28.8 \cdot 10^{-4} \quad \text{(NDR).}$$ \hspace{1cm} (3.62)

The above results for $\epsilon'/\epsilon$ apply to the NDR scheme. $\epsilon'/\epsilon$ is generally lower in the HV scheme if the same values for $B_b^{(1/2)}$ and $B_s^{(3/2)}$ are used in both schemes. As discussed in
\[ \text{Im} \lambda_t(\epsilon) = (1.33 \pm 0.14) \cdot 10^{-4} \]

\[ \text{Im} \lambda_t(\epsilon') = (1.38 \pm 0.14) \cdot 10^{-4} \]

Figure 1: Probability density distributions for \( \text{Im} \lambda_t \) without (solid line) and with (dashed line) the \( \epsilon'/\epsilon \)-constraint.

subsection 2.5, such treatment of \( B_6^{(1/2)} \) and \( B_8^{(3/2)} \) is the proper way of estimating scheme dependences at present.

Using the two error analyses we find respectively:

\[ \frac{\epsilon'}{\epsilon} = (5.2^{+4.6}_{-2.7}) \cdot 10^{-4} \quad \text{(HV)} \]  

(3.63)

and

\[ 0.26 \cdot 10^{-4} \leq \frac{\epsilon'}{\epsilon} \leq 22.0 \cdot 10^{-4} \quad \text{(HV)}. \]  

(3.64)

Moreover, the mean and the standard deviation read

\[ \frac{\epsilon'}{\epsilon} = 6.3 \pm 4.8 \quad \text{(HV)}. \]  

(3.65)

The corresponding probability density distribution for \( \epsilon'/\epsilon \) is compared to the one obtained in the NDR scheme in fig. 2. Assuming, on the other hand, that the values in (2.56) correspond to the NDR scheme and using the relation (2.43), we find for the HV scheme the range \( 0.58 \cdot 10^{-4} \leq \frac{\epsilon'}{\epsilon} \leq 26.9 \cdot 10^{-4} \) which is much closer to the NDR result.
This exercise shows that it is very desirable to have the scheme dependence under control.

We observe that the most probable values for $\varepsilon'/\varepsilon$ in the NDR scheme are in the ballpark of $1 \cdot 10^{-3}$. They are lower by roughly 30% in the HV scheme if the same values for $(B_6^{(1/2)}, B_8^{(3/2)})$ are used. On the other hand the ranges in (3.62) and (3.64) show that for particular choices of the input parameters, values for $\varepsilon'/\varepsilon$ as high as $(2 - 3) \cdot 10^{-3}$ cannot be excluded at present. Let us study this in more detail.

### 3.3 Anatomy of $\varepsilon'/\varepsilon$

#### 3.3.1 Global Analysis

In table 7 we show the values of $\varepsilon'/\varepsilon$ in units of $10^{-4}$ for specific values of $B_6^{(1/2)}$, $B_8^{(3/2)}$ and $m_s(m_c)$ as calculated in the NDR scheme. The corresponding values in the HV scheme are lower as discussed above. The fourth column shows the results for central values of all remaining parameters. The comparison of the the fourth and the fifth column demonstrates how $\varepsilon'/\varepsilon$ is increased when $\Lambda^{(4)}_{\text{MS}}$ is raised from 340 MeV to 390 MeV. As
stated in (1.7) $\varepsilon'/\varepsilon$ is roughly proportional to $\Lambda_{\text{MS}}^{(4)}$. Finally, in the last column maximal values of $\varepsilon'/\varepsilon$ are given. To this end we have scanned all parameters relevant for the analysis of $\text{Im}\lambda_t$ within one standard deviation and have chosen the highest value of $\Lambda_{\text{MS}}^{(4)} = 390\text{ MeV}$. Comparison of the last two columns demonstrates the impact of the increase of $\text{Im}\lambda_t$ from its central to its maximal value and of the variation of $m_t$.

Table 7 gives a good insight in the dependence of $\varepsilon'/\varepsilon$ on various parameters which is roughly described by (1.7). We observe the following hierarchies:

- The largest uncertainties reside in $m_s$, $B_6^{(1/2)}$ and $B_8^{(3/2)}$. $\varepsilon'/\varepsilon$ increases universally by roughly a factor of 2.3 when $m_s(m_c)$ is changed from 155 MeV to 105 MeV. The increase of $B_6^{(1/2)}$ from 1.0 to 1.3 increases $\varepsilon'/\varepsilon$ by $(55 \pm 10)\%$, depending on $m_s$ and $B_8^{(3/2)}$. The corresponding changes due to $B_8^{(3/2)}$ are approximately $(40 \pm 15)\%$.
- The combined uncertainty due to $\text{Im}\lambda_t$ and $m_t$, present both in $\text{Im}\lambda_t$ and $F_{\varepsilon'}$, is approximately ±25%. The uncertainty due to $m_t$ alone is only ±5%.
- The uncertainty due to $\Lambda_{\text{MS}}^{(4)}$ is approximately ±16%.

The large sensitivity of $\varepsilon'/\varepsilon$ to $m_s$ has been known since the analyses in the eighties. In the context of the KTeV result this issue has been analyzed in [40]. It has been found that provided $2B_6^{(1/2)} - B_8^{(3/2)} \leq 2$ the consistency of the Standard Model with the KTeV result requires the $2\sigma$ bound $m_s(2\text{ GeV}) \leq 110\text{ MeV}$. Our analysis is compatible with these findings.

It is of interest to investigate the impact of the relation (2.52) on our results. Scanning all parameters in the ranges given in table 6 and imposing $B_6^{(1/2)} = 1.7 \cdot B_8^{(3/2)}$ we find

$$3.7 \cdot 10^{-4} \leq \varepsilon'/\varepsilon \leq 26.2 \cdot 10^{-4} \quad (3.66)$$

which is somewhat reduced with respect to (3.62).

Finally we would like to comment on formula (1.6) in which $\text{Re}\lambda_t$ appears instead of $\text{Im}\lambda_t$. Since $F_{\varepsilon}$ decreases with decreasing $\text{Re}\lambda_t$ one can come closer to the experimental data for $\varepsilon'/\varepsilon$ by choosing $\text{Re}\lambda_t$ sufficiently small. In the Wolfenstein parametrization $\text{Re}\lambda_t$ is proportional to $1 - \varrho$ and a small $\text{Re}\lambda_t$ corresponds to a sufficiently large positive value of the parameter $\varrho$. Yet it is known from analyses of the unitarity triangle that $\varrho$ is bounded from above by the ratio $|V_{ub}/V_{cb}|$ and even stronger by the value of $\varepsilon$. If these constraints are taken into account the analysis using (1.6) reduces to the one presented above.
Table 7: Values of $\varepsilon'/\varepsilon$ in units of $10^{-4}$ for specific values of $B_{6}^{(1/2)}$, $B_{8}^{(3/2)}$ and $m_s(m_c)$ and other parameters as explained in the text.

| $B_{6}^{(1/2)}$ | $B_{8}^{(3/2)}$ | $m_s(m_c)$ [MeV] | Central | $\Lambda_{\text{MS}}^{(4)} = 390$ MeV | Maximal |
|----------------|----------------|------------------|---------|-------------------------------|---------|
| 1.3            | 0.6            | 105              | 20.2    | 23.3                          | 28.8    |
|                |                | 130              | 12.8    | 14.8                          | 18.3    |
|                |                | 155              | 8.5     | 9.9                           | 12.3    |
| 1.3            | 0.8            | 105              | 18.1    | 20.8                          | 26.0    |
|                |                | 130              | 11.3    | 13.1                          | 16.4    |
|                |                | 155              | 7.5     | 8.7                           | 10.9    |
| 1.3            | 1.0            | 105              | 15.9    | 18.3                          | 23.2    |
|                |                | 130              | 9.9     | 11.5                          | 14.5    |
|                |                | 155              | 6.5     | 7.6                           | 9.6     |
| 1.0            | 0.6            | 105              | 13.7    | 15.8                          | 19.7    |
|                |                | 130              | 8.4     | 9.8                           | 12.2    |
|                |                | 155              | 5.4     | 6.4                           | 7.9     |
| 1.0            | 0.8            | 105              | 11.5    | 13.3                          | 16.9    |
|                |                | 130              | 7.0     | 8.1                           | 10.4    |
|                |                | 155              | 4.4     | 5.2                           | 6.6     |
| 1.0            | 1.0            | 105              | 9.4     | 10.9                          | 14.1    |
|                |                | 130              | 5.5     | 6.5                           | 8.5     |
|                |                | 155              | 3.3     | 4.0                           | 5.2     |

3.3.2 Parametric vs. Hadronic Uncertainties

One should distinguish between parametric and hadronic uncertainties. Parametric uncertainties are related to $m_t$, $|V_{ub}|$, $|V_{cb}|$ and $\Lambda_{\text{MS}}^{(4)}$. One should in principle include $m_s$ in this list. However, in order to extract $m_s$ from the kaon mass one encounters large non-perturbative uncertainties. Clearly such uncertainties are also present in the determination of $|V_{cb}|$ and in particular in the determination of $|V_{ub}|$, but they are substantially smaller. Hence the hadronic uncertainties discussed below are related to $\hat{B}_K$, $B_{6}^{(1/2)}$, $B_{8}^{(3/2)}$ and $m_s$.

In table 8 we show ranges for $\varepsilon'/\varepsilon$ related to various uncertainties. The parametric
uncertainties have been obtained for central values of \( \hat{B}_K \) and \( m_s \) and two choices of \((B_6^{(1/2)}, B_8^{(3/2)})\). The hadronic uncertainties due to \( \hat{B}_K \), \( B_6^{(1/2)} \) and \( B_8^{(3/2)} \) have been found by setting all the remaining parameters at their central values. The uncertainty due to \( m_s \) has been shown for two choices of \((B_6^{(1/2)}, B_8^{(3/2)})\) and all other parameters set at their central values. The last row in table 8 shows the total hadronic uncertainty. It is evident from this table that hadronic uncertainties dominate, although the reduction of parametric uncertainties is very desirable.

Table 8: Uncertainties in \( \varepsilon'/\varepsilon \) in units of \( 10^{-4} \) as explained in the text.

| Uncertainties | \( B_6^{(1/2)} \) | \( B_8^{(3/2)} \) | \( (\varepsilon'/\varepsilon)_{\text{min}} \) | \( (\varepsilon'/\varepsilon)_{\text{max}} \) |
|---------------|----------------|----------------|----------------|----------------|
| Parametric    | 1.0            | 0.8            | 5.0            | 9.5            |
| Parametric    | 1.3            | 0.8            | 8.4            | 15.1           |
| Hadronic (\( B_i \)) | –              | –              | 3.0            | 13.6           |
| Hadronic (\( m_s \)) | 1.0            | 0.8            | 4.5            | 11.3           |
| Hadronic (\( m_s \)) | 1.3            | 0.8            | 7.6            | 17.9           |
| Hadronic (full ) | –              | –              | 1.7            | 21.3           |

3.4 \( B_6^{(1/2)}-B_8^{(3/2)} \) Plot

In fig. 3 we show the minimal value of \( B_6^{(1/2)} \) for two choices of \( m_s(m_c) \) and \( \Lambda^{(4)}_{\text{MS}} \) as a function of \( B_8^{(3/2)} \) for which the theoretical value of \( \varepsilon'/\varepsilon \) is higher than \( 2.0 \cdot 10^{-3} \). To obtain this plot we have varied all other parameters in the ranges given in table 6. We show also the line corresponding to the relation \( (2.52) \). We observe that as long as \( B_8^{(3/2)} \geq 0.6 \), the parameter \( B_6^{(1/2)} \) is required to be larger than unity. This plot should be useful when our knowledge of \( B_6^{(1/2)}, B_8^{(3/2)}, m_s \) and \( \Lambda^{(4)}_{\text{MS}} \) improves.

3.5 Approximate Scaling Laws for \( \varepsilon'/\varepsilon \)

3.5.1 Preliminaries

Table 7 contains a lot of information on \( \varepsilon'/\varepsilon \). This information can be further extended by noting that \( \varepsilon'/\varepsilon \) depends to a very good approximation on certain combinations of the input parameters. This is seen in \( (1.7) \) and \( (2.41) \). Here we want to provide scaling laws
Table 7 values for \( \varepsilon' / \varepsilon \) for different sets of input parameters.

### 3.5.2 \( B_{6}^{(1/2)} \), \( B_{8}^{(3/2)} \) and \( m_s \)

As seen in (2.41), \( \varepsilon' / \varepsilon \) depends on these important three parameters only through \( R_6 \) and \( R_8 \) defined in (2.42). Using this property one can for instance immediately find that the values for \( \varepsilon' / \varepsilon \) in the tenth row of table 7 can also be obtained for the set

\[
B_{6}^{(1/2)} = 1.50, \quad B_{8}^{(3/2)} = 0.90, \quad m_s(m_c) = 130 \text{ MeV}.
\]  

This set of parameters is similar to the input parameters used by the Trieste group [38].

At this point we would like to remark that in principle the determination of \( B_{6}^{(1/2)} \) and \( B_{8}^{(3/2)} \) in a given non-perturbative framework could depend on the value of \( m_s \). This turns out not to be the case in the large-N approach [19, 48, 47]. In the lattice approach this question has still to be investigated. On the other hand there are results in the literature showing a strong \( m_s \)-dependence of the \( B_i \) parameters. This is the case for \( B_{6}^{(1/2)} \) in the
chiral quark model where \( B_6^{(1/2)} \) scales like \( m_s \) \[38\]. Similarly values for \( B_7^{(3/2)} \) calculated in \[31\] show a strong \( m_s \)-dependence.

In the present paper we have varied \((B_6^{(1/2)}, B_8^{(3/2)})\) and \( m_s \) independently which is in accordance with large-N calculations. This resulted in the following ranges for \( R_6 \) and \( R_8 \)

\[ 0.5 \leq R_6 \leq 1.95, \quad 0.4 \leq R_8 \leq 1.5 \]  (3.68)

which are correlated through their common dependence on \( m_s \).

If \((B_6^{(1/2)}, B_8^{(3/2)})\) depend on \( m_s \) these ranges could change. In this context one should remark that in the chiral quark model \[38\] the highest value of \( R_6 \) corresponds to the minimal value of \( B_6^{(1/2)} \) and consequently the comparison of the results from the chiral quark model and the large-N approach has to be made with care.

Finally, it should be remarked that the decomposition of the relevant hadronic matrix elements of penguin operators into a product of \( B_i \) factors times \( 1/m_s^2 \) although useful in the \( 1/N \) approach will become unnecessary in the lattice approach, once matrix elements of dimension three will be calculable with improved accuracy.

### 3.5.3 \( \Lambda_{\text{MS}}^{(4)} \) and \( \text{Im} \lambda_t \)

For \( \alpha_s(M_Z) = 0.119 \pm 0.003 \), the ratio \( \varepsilon' / \varepsilon \) is within a few percent proportional to \( \Lambda_{\text{MS}}^{(4)} \). On the other hand \( \varepsilon' / \varepsilon \) is exactly proportional to \( \text{Im} \lambda_t \) at fixed \( m_t \). However, if \( m_t \) is varied the correlation in \( m_t \) between \( \text{Im} \lambda_t \) extracted from \( \varepsilon \) and \( F_{\varepsilon'} \) has to be taken into account. Consequently the simple rescaling of \( \varepsilon' / \varepsilon \) with the values of \( \text{Im} \lambda_t \) is only true within a few percent.

### 3.5.4 Sensitivity to \( \Omega_{\eta+\eta'} \)

The dependence of \( \varepsilon' / \varepsilon \) on \( \Omega_{\eta+\eta'} \) can be studied numerically by using the formula (2.4) or incorporated approximately into the analytic formula (2.38) by simply replacing \( B_6^{(1/2)} \) with an effective parameter

\[
(B_6^{(1/2)})_{\text{eff}} = B_6^{(1/2)} \frac{1 - 0.9 \Omega_{\eta+\eta'}}{0.775}
\]  (3.69)

A numerical analysis shows that using \((1 - \Omega_{\eta+\eta'})\) overestimates the role of \( \Omega_{\eta+\eta'} \). In our numerical analysis we have incorporated the uncertainty in \( \Omega_{\eta+\eta'} \) by increasing the error in \( B_6^{(1/2)} \) from \( \pm 0.2 \) to \( \pm 0.3 \).
The last estimates of $\Omega_{\eta+\eta'}$ have been done more than ten years ago \cite{16-18} and it is desirable to update these analyses which can be summarized by

\[ \Omega_{\eta+\eta'} = 0.25 \pm 0.08 \, . \]  

(3.70)

The uncertainty in $\varepsilon'/\varepsilon$ due to $\Omega_{\eta+\eta'}$ alone is approximately $\pm 12\%$ and is slightly lower than the one originating from $\Lambda_{\text{MS}}^{(4)}$.

3.5.5 Sensitivity to $\hat{B}_K$

As $\Im \lambda_t$ extracted from $\varepsilon$ increases with decreasing $\hat{B}_K$, there is a possibility of increasing $\varepsilon'/\varepsilon$ by decreasing $\hat{B}_K$ below the range considered in table 3. It should be remarked that $\varepsilon'/\varepsilon$ is not simply proportional to $1/\hat{B}_K$ as the extraction of $\Im \lambda_t$ from $\varepsilon$ involves also $\Re \lambda_t$ (see (1.2)). For the phase $\delta$ in the first quadrant as favoured by the analyses of the unitarity triangle \cite{88}, the dependence of $\varepsilon'/\varepsilon$ on $\hat{B}_K$ is weaker than $1/\hat{B}_K$ \cite{93}.

Now, the highest value of $\Im \lambda_t$ consistent with the unitarity of the CKM matrix is $1.73 \cdot 10^{-4}$. It is obtained from $\varepsilon$ for $\hat{B}_K = 0.52$. This increase of $\Im \lambda_t$ beyond the range in (3.61) would increase the maximal values in table 6 by approximately 6%. On the other hand it should be emphasized that for $\hat{B}_K = 0.8 - 0.9$, as indicated by lattice calculations, $\varepsilon'/\varepsilon$ is generally smaller than found in our paper unless $B_6^{(1/2)}$ is substantially increased. This is what happens in the chiral quark model \cite{38} where on the one hand $\hat{B}_K = 1.1 \pm 0.2$ and on the other hand $B_6^{(1/2)} = 1.6 \pm 0.3$.

3.6 Impact on $\Im \lambda_t$ and the Unitarity Triangle

As we stressed at the beginning of this paper the main new parameter to be fitted by means of $\varepsilon'/\varepsilon$ is $\Im \lambda_t$. Our analysis indicates that the Standard Model estimates of $\varepsilon'/\varepsilon$ are generally below the data. If the parameters $m_s$, $B_6^{(1/2)}$, $B_8^{(3/2)}$, $\Lambda_{\text{MS}}^{(4)}$ and $\Omega_{\eta+\eta'}$ are such that $\Im \lambda_t$ consistent with $\varepsilon$ (see (3.61)) cannot accomodate the experimental value of $\varepsilon'/\varepsilon$, one has to conclude that new contributions from new physics are required. On the other hand if the data on $\varepsilon'/\varepsilon$ can be reproduced within the Standard Model, then generally a lower bound on $\Im \lambda_t$ excluding a large fraction of the range (3.61) can be obtained. Unfortunately, the strong dependence of the lower bound on the parameters involved precludes any firm conclusions. Similar comments apply to the possible impact of $\varepsilon'/\varepsilon$ on the analysis of the unitarity triangle: the presently allowed area in the $(\bar{\rho}, \bar{\eta})$ plane \cite{88} can be totally removed or an improved lower limit on $\bar{\eta}$ from $\varepsilon'/\varepsilon$ will decrease the allowed
region considerably. As an illustration we show in table 9 the lower bound on \( \text{Im}\lambda_t \) from \( \varepsilon'/\varepsilon \) as a function of \( B_{8}^{(3/2)} \) for \( m_s(m_c) = 105 \text{ MeV}, B_6^{(1/2)} = 1.3 \) and \( \Lambda_{\text{MS}}^{(4)} = 390 \text{ MeV} \).

To this end we have used the formula (2.2) with \( \varepsilon'/\varepsilon \geq 2.0 \cdot 10^{-3} \). Comparing with (3.61) we indeed observe that the lower bound on \( \text{Im}\lambda_t \) has been improved.

The impact of \( \varepsilon'/\varepsilon \)-data as given in (1.9) on \( \text{Im}\lambda_t \) can also be investigated by the method 1 which was used to obtain (3.58) and (3.59). We find

\[
\text{Im}\lambda_t = (1.38 \pm 0.14) \cdot 10^{-4}.
\]  

(3.71)

The corresponding distribution is compared with the one without \( \varepsilon'/\varepsilon \)-constraint in fig. 1.

We observe a very modest but visible shift towards higher values for \( \text{Im}\lambda_t \).

Table 9: Minimal values of \( \text{Im}\lambda_t \), \( \text{Br}(K_L \to \pi^0 \nu \bar{\nu}) \) and \( \text{Br}(K_L \to \pi^0 e^+ e^-)_{\text{dir}} \) for \( m_s(m_c) = 105 \text{ MeV}, B_6^{(1/2)} = 1.3 \) and \( \Lambda_{\text{MS}}^{(4)} = 390 \text{ MeV} \) and specific values of \( B_{8}^{(3/2)} \) assuming \( \varepsilon'/\varepsilon \geq 2.0 \cdot 10^{-3} \).

| \( B_{8}^{(3/2)} \) | \( (\text{Im}\lambda_t)_{\text{min}} \) | \( \text{Br}(K_L \to \pi^0 \nu \bar{\nu})_{\text{min}} \) | \( \text{Br}(K_L \to \pi^0 e^+ e^-)_{\text{min}} \) |
|---------|-----------------|-------------------|-------------------|
| 0.6     | 1.14 \cdot 10^{-4} | 1.8 \cdot 10^{-11} | 3.0 \cdot 10^{-12} |
| 0.8     | 1.27 \cdot 10^{-4} | 2.2 \cdot 10^{-11} | 3.7 \cdot 10^{-12} |
| 1.0     | 1.42 \cdot 10^{-4} | 2.7 \cdot 10^{-11} | 4.7 \cdot 10^{-12} |

3.7 Impact on \( K_L \to \pi^0 \nu \bar{\nu} \) and \( K_L \to \pi^0 e^+ e^- \)

The rare decay \( K_L \to \pi^0 \nu \bar{\nu} \) is the cleanest decay in the field of K-decays. It proceeds almost entirely through direct CP violation [93] and after the inclusion of NLO QCD corrections [95] the theoretical uncertainties in the branching ratio are at the level of 1 – 2%. Similarly the contribution of direct CP-violation to the decay \( K_L \to \pi^0 e^+ e^- \) is very clean. Using the known formulae for these decays [9, 95] and scanning the parameters given in table 8 we find:

\[
1.6 \cdot 10^{-11} \leq \text{Br}(K_L \to \pi^0 \nu \bar{\nu}) \leq 3.9 \cdot 10^{-11}
\]  

(3.72)

\[
2.8 \cdot 10^{-12} \leq \text{Br}(K_L \to \pi^0 e^+ e^-)_{\text{dir}} \leq 6.5 \cdot 10^{-12}
\]  

(3.73)

Since these branching ratios are proportional to \( (\text{Im}\lambda_t)^2 \) any impact of \( \varepsilon'/\varepsilon \) on the latter CKM factor will also modify these estimates. We illustrate this in table 9 where an
improved lower bound on Im$\lambda_t$ implies improved lower bounds on the branching ratios in question. With decreasing $B'_6(1/2)$ and increasing $m_s$ these lower bounds continue to improve excluding a large fraction of the ranges in (3.72) and (3.73). In obtaining the results in table 9 correlations in $m_t$ and the CKM parameters between $\epsilon'/\epsilon$, $\epsilon$ and the branching ratios for the decays considered have been taken into account. Unfortunately, due to large hadronic uncertainties in $\epsilon'/\epsilon$, no strong conclusions can be reached at present.

In the future the situation will be reversed. As pointed out in [96] the cleanest measurement of Im$\lambda_t$ is offered by $Br(K_L \to \pi^0\nu\bar{\nu})$:

$$\text{Im} \lambda_t = 1.41 \cdot 10^{-4} \left[ \frac{165 \text{ GeV}}{m_t(m_t)} \right]^{1.15} \left[ \frac{Br(K_L \to \pi^0\nu\bar{\nu})}{3 \cdot 10^{-11}} \right]^{1/2}. \quad (3.74)$$

Once Im$\lambda_t$ is extracted in this manner it can be used in $\epsilon'/\epsilon$ thereby somewhat reducing the uncertainties in the estimate of this ratio.

4 Implications for Physics Beyond the Standard Model

4.1 General Comments

We have seen that the Standard Model estimates of $\epsilon'/\epsilon$ are generally below the experimental results from NA31 and KTeV. In view of the large theoretical uncertainties it is, however, impossible at present to conclude that new physics is signaled by the $\epsilon'/\epsilon$-data. Still, we can make a few general comments on the extensions of the Standard Model with respect to $\epsilon'/\epsilon$:

- In models where the phase of the CKM matrix is the only source of CP violation, the modifications with respect to the Standard Model come through new loop contributions to $\epsilon$ and $\epsilon'/\epsilon$. If the new contributions to $\epsilon$ are positive and the contributions to $F_{\epsilon'}$ are negative, then Im$\lambda_t$, $F_{\epsilon'}$ and consequently $\epsilon'/\epsilon$ are smaller than in the Standard Model putting these models into difficulties. An example of this disfavoured situation is the two-Higgs doublet model II in which $\epsilon'/\epsilon$ has been analysed a long time ago [97]. We will update this analysis below.

- In the Minimal Supersymmetric Standard Model, the last analysis of $\epsilon'/\epsilon$ after the top quark discovery has been performed in [98]. Here in addition to charged Higgs exchanges in loop diagrams, also charginos contribute. The chargino contribution to $\epsilon$ has always the effect of decreasing Im$\lambda_t$. However, depending on the choice
of the supersymmetric parameters, the chargino contribution to $F_{\epsilon'}$ can have either sign. Consequently, $\epsilon'/\epsilon$ in the MSSM can be enhanced with respect to the Standard Model expectations for a suitable choice of parameters, low values of chargino (stop) masses and high charged Higgs masses. Yet, as stressed in [98], generally $F_{\epsilon'}$ is further suppressed by chargino contributions and the most conspicuous effect of minimal supersymmetry is a depletion of $\epsilon'/\epsilon$.

- The situation can be different in more general models in which there are more parameters than in the two Higgs doublet model II and in the MSSM, in particular new CP violating phases. As an example, in more general supersymmetric models $\epsilon'/\epsilon$ can be made consistent with experimental findings [42, 99]. Unfortunately, in view of the large number of free parameters such models are not very predictive. Similar comments apply to models with anomalous gauge couplings [41] and models with additional fermions and gauge bosons [100] in which new positive contributions to $\epsilon'/\epsilon$ are in principle possible. A recent discussion of new physics effects in $\epsilon'/\epsilon$ can also be found in [10]. In the past, there have of course been several other analyses of $\epsilon'/\epsilon$ in the extensions of the Standard Model but a review of these analyses is clearly beyond the scope of this paper.

- Finally, models with an enhanced $\bar{s}dZ$ vertex, considered in [101], can give rise to large contributions to $\epsilon'/\epsilon$ as pointed out in [102]. As analyzed in the latter paper, in these models there exist interesting connections between $\epsilon'/\epsilon$ and rare K decays.

4.2 An Update on $\epsilon'/\epsilon$ in the Two-Higgs Doublet Model II

A detailed renormalization group analysis of $\epsilon'/\epsilon$ in the Two-Higgs Doublet Model II [13] has been presented in [97]. It has been found that due to additional positive charged Higgs contributions to $\epsilon$ and corresponding negative contributions to $F_{\epsilon'}$ through the increase of the importance of $Z^0$-penguin diagrams, the ratio $\epsilon'/\epsilon$ is suppressed with respect to the Standard Model expectations. Since this analysis goes back to 1990 and several input parameters, in particular $m_t$, have been modified we would like to update this analysis.

We recall that the two new parameters relevant for our analysis are the charged Higgs mass ($M_H$) and tan $\beta$, the ratio of the two vacuum expectation values. The expressions for the new contributions with charged Higgs exchanges to $\epsilon$ are rather complicated and will not be repeated here. They can be found in Section 3 of [77]. The QCD corrections to
these contributions are given there in the leading logarithmic approximation. As of 1999 only NLO corrections to box diagram contributions with internal top-quark exchanges to $B^0 - \bar{B}^0$ mixing are known [28]. Unfortunately the NLO QCD analysis for $\varepsilon$ in the 2HDMII is still lacking. For this reason we have used the leading order expressions for the Higgs contributions to $\varepsilon$ [97] except for the box diagram contributions with internal top-quark exchanges where we took the NLO QCD factor obtained in the Standard Model. While such a treatment is clearly an approximation, it is sufficient for our purposes.

The analysis of $F_{\varepsilon'}$ on the other hand can be done fully at the NLO level. We only have to add to the functions $X_0(x_t)$, $Y_0(x_t)$, $Z_0(x_t)$ and $E_0(x_t)$ the contributions from charged Higgs exchanges. They are given as follows:

$$\Delta X_0 = \Delta Y_0 = \frac{x_t}{\tan^2 \beta} \left[ \frac{y}{8(y-1)} - \frac{y}{8(x-1)^2} \log y \right]$$  (4.75)

$$\Delta Z_0 = \Delta X_0 + \frac{1}{4\tan^2 \beta} D_H(y)$$  (4.76)

$$\Delta E_0 = \frac{1}{\tan^2 \beta} \left[ \frac{y(7y^2 - 29y + 16)}{36(y-1)^3} + \frac{y(3y - 2)}{6(y-1)^4} \log y \right]$$  (4.77)

where

$$D_H(y) = \frac{y(47y^2 - 79y + 38)}{108(y-1)^3} + \frac{y(-3y^2 + 6y - 4)}{18(y-1)^4} \log y$$  (4.78)

with $y = m_t^2/M_{H^2}$.

We observe that all new contributions to $F_{\varepsilon'}$ are inversely proportional to $\tan^2 \beta$. In $\varepsilon$ they are inversely proportional to $\tan^2 \beta$ and $\tan^4 \beta$. This should be contrasted with the case of $B \to X_s \gamma$ where there are new contributions with charged Higgs exchanges, which do not involve $\tan \beta$. Thus $\varepsilon'/\varepsilon$ is more sensitive to $\tan \beta$ than $B \to X_s \gamma$. This implies that in principle a better constraint for $\tan \beta$ could be obtained from $\varepsilon'/\varepsilon$ than from the latter decay.

It is obvious from this discussion that $\varepsilon'/\varepsilon$ in the 2HDMII is lower than in the Standard Model for any choice of input parameters. Consequently, for low $M_H$ and $\tan \beta$, the ratio $\varepsilon'/\varepsilon$ is generally well below the experimental data. On the other hand if the Standard Model is consistent with the experimental value of $\varepsilon'/\varepsilon$, it is possible to put a lower bound on $\tan \beta$ as a function of $M_H$. In fig. 4 we show the result of such an analysis for $B_8^{(3/2)} = 0.8$ and selected values of $B_6^{(1/2)}$ and $m_s$. The remaining parameters have been scanned in the ranges given in table 6. We require $\varepsilon'/\varepsilon \geq 2.0 \cdot 10^{-3}$. We observe that for the lowest values of $M_H \approx 200$ GeV allowed by the $B \to X_s \gamma$ decay [103] - [107].
Figure 4: Lower bound on $\tan \beta$ as a function of $M_{H}$ consistent with $\varepsilon'/\varepsilon \geq 2.0 \cdot 10^{-3}$.

For $m_s(m_c) = 105\text{MeV}$ and $B_6^{(1/2)} = 1.3$ the lower bound on $\tan \beta$ is similar to the one obtained from $B \to X_s \gamma$. For higher values of $m_s(m_c)$ and lower values of $B_6^{(1/2)}$ the bound on $\tan \beta$ becomes stronger than from $B \to X_s \gamma$.

5 Conclusions and Outlook

We have presented a new analysis of $\varepsilon'/\varepsilon$ in the Standard Model in view of the recent KTeV measurement of this ratio, which together with the previous NA31 result firmly establishes direct CP violation in nature. Compared with our 1996 analysis \cite{34}, the present analysis uses improved values of $|V_{ub}|$, $|V_{cb}|$, $m_t$, $\Lambda_{\text{MS}}^{(4)}$, and $m_s$ as well as new insights in the hadronic parameters $B_6^{(1/2)}$, $B_8^{(3/2)}$ and $B_K$. Our findings are as follows:

- The estimates of $\varepsilon'/\varepsilon$ in the Standard Model are typically below the experimental data. Our Monte Carlo analysis gives

$$
\varepsilon'/\varepsilon = \begin{cases} 
(7.7 \pm 0.9) \cdot 10^{-4} & \text{(NDR)} \\
(5.2 \pm 1.6) \cdot 10^{-4} & \text{(HV)}
\end{cases}
$$

(5.79)
The difference between these two results indicates the leftover renormalization scheme dependence.

- On the other hand, a simple scanning of all input parameters gives

\[ 1.05 \cdot 10^{-4} \leq \frac{\epsilon'}{\epsilon} \leq 28.8 \cdot 10^{-4} \quad \text{(NDR)} \]  

and

\[ 0.26 \cdot 10^{-4} \leq \frac{\epsilon'}{\epsilon} \leq 22.0 \cdot 10^{-4} \quad \text{(HV)} \]  

This means that for suitably chosen parameters, \( \frac{\epsilon'}{\epsilon} \) in the Standard Model can be made consistent with data. However, this happens only if all relevant parameters are simultaneously close to their extreme values. This is clearly seen in table 7 and fig. 3. Moreover, the probability density distributions for \( \frac{\epsilon'}{\epsilon} \) in fig. 2 indicates that values of \( \frac{\epsilon'}{\epsilon} \) in the ballpark of NA31 and KTeV results are rather improbable.

- Unfortunately, in view of very large hadronic and substantial parametric uncertainties, it is impossible to conclude at present whether new physics contributions are indeed required to fit the data. Similarly it is difficult to conclude what is precisely the impact of the \( \frac{\epsilon'}{\epsilon} \)-data on the CKM matrix. However, there are indications as seen in table 8 that the lower limit on \( \text{Im}_\lambda_t \) is improved. The same applies to the lower limits for the branching ratios for \( K_L \rightarrow \pi^0 \nu \bar{\nu} \) and \( K_L \rightarrow \pi^0 e^+ e^- \) decays.

- Finally, we have pointed out that the \( \frac{\epsilon'}{\epsilon} \) data puts models in which there are new positive contributions to \( \epsilon \) and negative contributions to \( \epsilon' \) in serious difficulties. In particular we have analyzed \( \frac{\epsilon'}{\epsilon} \) in the 2HDMII demonstrating that with improved hadronic matrix elements this model can either be ruled out or a powerful lower bound on \( \tan \beta \) can be obtained from \( \frac{\epsilon'}{\epsilon} \).

The fact that one cannot firmly conclude at present that the data for \( \frac{\epsilon'}{\epsilon} \) requires new physics is rather unfortunate. In an analogous situation in the very clean rare decays \( K \rightarrow \pi \nu \bar{\nu} \) a departure of the experimental result from the Standard Model expectations by only 30% would give a clear signal for new physics. This will indeed be the case if the improved measurements of the \( Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \) from BNL787 collaboration at Brookhaven [106] find this branching ratio above \( 1.5 \cdot 10^{-10} \). All efforts should be made to measure this branching ratio and the branching ratio for \( K_L \rightarrow \pi^0 \nu \bar{\nu} \), which while being directly CP violating is almost free of theoretical uncertainties.
The future of $\varepsilon'/\varepsilon$ in the Standard Model and in its extensions depends on the progress in the reduction of parametric and hadronic uncertainties. We have analyzed these uncertainties in detail in Section 3 with the results given in table 8.

Concerning parametric uncertainties related to $|V_{ub}|$, $|V_{cb}|$, $m_t$ and $\Lambda^{(4)}_{\text{MS}}$, we expect that they should be reduced considerably in the coming years. This will, however, result only in a modest reduction of the total uncertainty in $\varepsilon'/\varepsilon$. In this respect a measurement of $\text{Im}\lambda_t$ in a very clean decay like $K_L \to \pi^0\nu\bar{\nu}$ would be very useful.

A real progress in estimating $\varepsilon'/\varepsilon$ will only be made if the non-perturbative parameters $\hat{B}_K$, $B_6^{(1/2)}$, $B_8^{(3/2)}$ and $\Omega_{\eta+\eta'}$ as well as the strange quark mass $m_s$ will be brought under control. In particular the sensitivity of non-perturbative methods to $\mu$ and renormalization scheme dependences of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ is clearly desirable. We expect that considerable progress on $\hat{B}_K$ and $B_8^{(3/2)}$ should be made in the coming years through improved lattice calculations. Progress on $\Omega_{\eta+\eta'}$ should also be possible in the near future. Moreover, as various estimates of $\hat{B}_K$, $B_8^{(3/2)}$ and $\Omega_{\eta+\eta'}$ by means of several non-perturbative methods are compatible with each other we do not expect big surprises here. Similar comments apply to $|V_{ub}|$, $|V_{cb}|$, $m_t$ and $\Lambda^{(4)}_{\text{MS}}$.

On the other hand, it appears that it will take longer to obtain acceptable values for $m_s$ and $B_6^{(1/2)}$. In view of the bounds [54]-[51], it is difficult to imagine that $m_s(m_c) \leq 105$ MeV. Consequently we expect that future improved estimates of $m_s$ will most probably exclude the lowest values of $m_s$ considered in this paper. This would simultaneously exclude the highest values for $\varepsilon'/\varepsilon$ obtained by us unless $B_6^{(1/2)}$ is found to be higher than used here. In this respect improved estimates of $B_6^{(1/2)}$, if found substantially higher than unity, could have considerable impact on our analysis. Finally, it should be stressed that future lattice calculations will give the full matrix elements without the necessity to use separately $(B_6^{(1/2)},B_8^{(3/2)})$ and $m_s$.

In any case $\varepsilon'/\varepsilon$ already played a decisive role in establishing direct CP violation in nature and its rather large value gives additional strong motivation for searching for this phenomenon in cleaner K decays like $K_L \to \pi^0\nu\bar{\nu}$ and $K_L \to \pi^0e^+e^-$, in B decays, in D decays and elsewhere.

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