Spontaneously broken, local $Z_2$ symmetry and the anatomy of a lepton

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In the framework of the gauge symmetry $SU(2)_{\text{CMB}} \times SU(2)_e \times SU(2)_{\mu} \times SU(2)_\tau$, we provide a microscopic argument why a spontaneously broken, local $Z_2$ symmetry surviving the 1st order phase transition to the confining phase of one of the SU(2) factors generates two stable spin-1/2 excitations of vastly different mass. A mixture of Fermi-Dirac and Bose-Einstein distributions with the former favored by a factor two over the latter is obtained by applying inequivalent, local-in-Euclidean-time $Z_2$ transformations to one and the same field configuration contained in the description of the center-vortex condensate.

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Introduction. In [1] we have constructed and applied a thermal theory for charged leptons. The underlying gauge symmetry for this theory is:

$$SU(2)_{\text{CMB}} \times SU(2)_e \times SU(2)_{\mu} \times SU(2)_\tau. \quad (1)$$

At the present temperature of the Universe $T_{\text{CMB}} \sim 2.728 \times 10^{-4}$ eV the dynamics due to the latter three gauge factors is confining. Moreover, the factor $SU(2)_{\tau}$ implies gauge dynamics which is contaminated by strong interactions. As was noticed in [1], the experimental observation that the CMB photon is very close to massless [4] implies that the theory associated with the gauge factor $SU(2)_{\text{CMB}}$ is at the magnetic side of the phase boundary to the electric phase. The gauge dynamics due to SU(2)$_{\text{CMB}}$ apparently is the only non-confining dynamics in our present universe. The purpose of this Letter is to show that the spontaneously broken, local $Z_2$ symmetries associated with each of the factors in SU(2)$_e \times SU(2)_{\mu} \times SU(2)_\tau$ in their confining phases render the respective two distinct stable excitations effectively spin-1/2 fermions. We provide statistical and microscopic arguments in favor for this claim.

$t'$ Hooft loop. The confining phase of a pure SU(N) gauge theory is related to a vanishing expectation of the Polyakov loop. As was explained in [8, 9] and stressed in [8, 10] there are severe problems associated with this operator: it creates a static and fundamental color source which is not in the physical Hilbert space, it is not defined at zero temperature, and it is plagued by ultraviolet divergences in the continuum limit. Another, much healthier order parameter for the deconfinement-confinement transition is the vacuum expectation of the ’t Hooft-loop operator $B[C]$ [8]. This operator is associated with a closed curve $C$ which is linked to another curve $C''$ $n$ times. Both curves $C$ and $C''$ are purely spatial when working in Minkowski signature. In 4D Euclidean space, $C$ and $C''$ can also extend into the time direction. The action of the operator $B[C]$ results in a multi-valued gauge transformation $\Omega[C]$ of the field $A_i$ along the curve $C''$. Namely, parametrizing $C''$ by an angle $0 \leq \theta \leq 2\pi$ we have

$$\Omega[C](2\pi) = \Omega[C](0)e^{\pm 2\pi i n/N}, \quad (2)$$

where the $+$ sign refers to the clockwise (anticlockwise) sense of running through the curve $C''$ [9]. In the absence of any preferred direction in the system (for the case in Eq. [1] this could be a homogeneous, external magnetic field with respect to the unbroken U(1)$_{\text{EM}}$ gauge theory in the Standard Model [11]) the clockwise and anticlockwise sense are physically equivalent. $B[C]$ creates an elementary magnetic flux along the curve $C$ out of the ground state. In contrast to the Polyakov loop a nonvanishing expectation value of the ’t Hooft-loop operator $B[C]$ indicates a spontaneous breakdown of a the (local) center symmetry $Z_0$ of SU(N) [8]. Such a phase is identified with the confining phase of the SU(N) Yang-Mills theory [8]. It is striking that the author of [8] stresses a similarity with fermionic Green functions in his discussion of correlation functions of ’t Hooft-loop operators in Euclidean spacetime. This similarity is related to an ambiguity in the definition of the phase factor.

The fields $\Phi_n(x)$ introduced in [2] are defined by the action of the ’t Hooft loop. Explicitly, we have

$$\Phi_n(x) = \left\langle \text{tr} \mathcal{P} \exp \left[ \frac{ie}{n} \oint_{C_n(x)} dz A_i(z) \right] \right\rangle, \quad (3)$$

where $e$ denotes the Yang-Mills coupling constant, and $\mathcal{P}$ is the path-ordering symbol. In Eq. [3] $C_n(x)$ denotes a spatial curve centered at $x$ which is linked $n$ times to a curve $C$. The ‘radius’ of $C_n(x)$ is not much larger than the core-size of the $n$th center vortex ($n = 1, \cdots, N - 1$) piercing it along the curve $C$ [10].

Statistical situation. In [2] we have constructed the effective potential for the local field $\Phi_n$ from the requirements that (a) a matching to the magnetic phase takes place in thermal equilibrium and (b) the (spontaneously broken) $Z_0$ symmetry is implemented in the potential in a nontrivial way. Requirement (a) implies, that the Euclidean time dependence of the center-vortex condensate $\Phi_n$ is Bogomol’nyi-Prasad-Sommerfield (BPS) saturated along the compactified Euclidean time coordinate. Requirement (b) implies that the vacuum pressure vanishes identically in the confining phase. According to the proposed potential [2]

$$V^{(n)}_C \equiv \psi^{(n)}_C \bar{\psi}^{(n)}_C, \quad (4)$$

where $\psi^{(n)}_C$ and $\psi^{(n)}_C$ are defined in [2] and [3] respectively.
where
\[
\psi^{(n)}_C \equiv i(\Lambda_C^3/\Phi_n - \Phi^{N-1}_n/\Lambda_C^{N-3}) ,
\]
requirements (a) and (b) are simultaneously only then satisfied if \( N \to \infty \). For finite \( N \) and at the phase boundary the form of the BPS saturated solution along the compactified Euclidean time coordinate subject to the potential in Eq. (5) indicates the breakdown of thermal equilibrium by itself [9].

Let us from now on only consider the case \( SU(2) \). This case is very likely relevant for an understanding of the statistics and the anatomy of charged leptons and neutrinos, see below and [10]. For our statistical consideration we assume that reheating process during the phase transition from the magnetic to the confining phase of the \( SU(2) \) gauge theory is efficient enough so that the system can be described thermodynamically not too long after the onset of vortex condensation. A check of this assumption could be done by using methods of nonequilibrium field theory subject to the potential of Eq. (4). It is mandatory for our argument that such a check will be performed in the future. For now we assume a Euclidean field theory with compactified time coordinate \( 0 \leq \tau \leq 1/T \) and the following action [2]:
\[
S_C = \int_0^{1/T} d\tau \int d^3x \left\{ (\partial_\tau \Phi)^2 + \Lambda_C^2 \left( \frac{\Lambda_C^3}{\Phi} - \Phi \right)^2 \right\} .
\]
(6)
At the phase boundary the field \( \Phi \) still satisfies periodic boundary conditions [11]. Due to the breakdown of thermal equilibrium the expectation of \( \Phi \) for a short period (probably a fraction of the inverse Yang-Mills scale \( \Lambda_C^{-1} \)) develops a violent time dependence. Thermal equilibrium is restored as this expectation relaxes towards the minimum of the potential in [6]. Since the spontaneously broken \( Z_2 \) symmetry is discrete the action of a genuinely local \( Z_2 \) transformation leads to a physical effect. Let us consider the following, local \( Z_2 \) transformations
\[
T_\pm = \tau 2\Theta(\frac{1}{2T} - \tau) \pm 1
\]
(7)
where \( \Theta \) denotes the usual Heaviside function (\( \Theta(0) = 1/2 \)). The transformations \( T_\pm \) are depicted in Fig. 1(a) and Fig. 1(b). Applying \( T_\pm \) to a field configuration \( \Phi \) subject to \( \Phi(0, \vec{x}) = \Phi(1/T, \vec{x}) \) generates boundary conditions \( \Phi(0, \vec{x}) = -\Phi(1/T, \vec{x}) \) in two inequivalent ways [12]. Both transformations change the action formally by a piece
\[
\Delta S_C = 4 \delta(0) \int d^3x (\Phi(1/(2T), \vec{x})^2).
\]
(8)
\( T_+ \) creates and destroys an explicit center vortex of infinite spatial extension, respectively. At \( \tau = 1/(2T) \) a 't Hooft-loop operator, defined by a circular loop say, in the \( \tau - x_1 \) plane and centered at \( (\tau, \vec{x}) \), creates or destroys one unit of magnetic flux, say, along the \( x_2 \) direction. The transformations \( T_\pm \) are the basic building blocks for all nontrivial, local \( Z_2 \) transformations along the Euclidean time interval which change periodic into antiperiodic boundary conditions. Global \( Z_2 \) transformations neither change the action [6] nor the nature of the boundary conditions of the given configuration \( \Phi \). All global \( Z_2 \) transformations must thus be considered equivalent [13]. We conclude that it is two times more likely to generate antiperiodic boundary conditions out of a given periodic field configuration \( \Phi \) than it is to maintain periodic boundary conditions if no a priori bias on possible \( Z_2 \) transformations along the \( \tau \) direction exists. If the system is autonomous, which safely can be presumed not too far away from the phase boundary, there is no reason for such a bias to exist. At this point it is interesting to look back at Einstein’s 1925 paper [10]. In that paper an expression for the mean squared deviation of the energy of an electron gas was obtained. This expression separates into two parts. The first part was interpreted as a fluctuation term for distinguishable, Poisson distributed particles while the second term is due to indistinguishable particles or a wavefield a la de Broglie. Let us relate this our field theoretical result: The electron (and neutrino) ensemble is generated by 't Hooft-loop actions generating particles whose existence is based on the explicit occurrence of center-vortex loops or magnetic fluxes (see below). This ensemble is described by antiperiodic boundary conditions (fermion). Whenever a particle is being created the condensate of center vortices it is immersed in proliferates the propagation of a longitudinal soundwave in the condensate. The associated degree of freedom is described by periodic boundary conditions (boson). Since a longitudinal soundwave is a scalar and fermions come in two spin orientations the statistical weighting is 1:2, respectively.

We emphasize that the above discussion is a statistical one.

Microscopic situation. In Fig. 2 we indicate the particle content in the confining phase of one of the \( SU(2) \) gauge factors in Eq. (4). These particles are generated.
by the action of ‘t Hooft loops during the relaxation of the Φ expectation towards the minimum of the potential in Eq. (6). At the minimum of the potential the action of ‘t Hooft loops during the relaxation to the confining phase \cite{2}. Distinguishing situations (a1), (a2) and (b1), (b2) is only possible if a direction is singled out in 3-space (a homogeneous magnetic field). The creation of the solitons (b) is only possible if a direction is singled out in 3-space (a homogeneous magnetic field). The creation of the solitons (a) and (b) by ‘t Hooft loop action (indicated by the dashed curves C′ and C′′) takes place as each of the factor theories SU(2) × SU(2) × SU(2), undergoes the 1st order transition to the confining phase \cite{2}.

Since the two different flux orientations in each soliton are interpreted as intrinsic flux momenta 1/2 (see below), which is the same for soliton (a) and (b), we conclude that the g factor for a charged lepton is twice as high as that for a neutrino.

The reader may easily convince himself that a soliton of the form (b) but with one of the center vortex loops rotated out of the common plane is instable: The local interaction energy-density \( \rho_{\text{int}} \) between the magnetic fields (w.r.t. U(1)_{SM}) \( \vec{B}_1 \) and \( \vec{B}_2 \) generated by the electric fluxes on either side of the isolated electric charge is given as

\[
\rho_{\text{int}} = -\vec{B}_1 \cdot \vec{B}_2.
\] (10)

This is minimal for \( \vec{B}_1 \parallel \vec{B}_2 \).

Applying an external, static, and homogeneous magnetic field \( \vec{B} \) (w.r.t. U(1)_{SM}) along an axis perpendicular to the center vortex loops in soliton (b) (Fig. 2) and keeping the soliton fixed by means of a static electric field sensitive to the isolated electric charge discriminates between the situations (b1) and (b2). The level splitting \( \Delta E \) originating from the effective interaction of the magnetic moment \( \vec{\mu}_b \) with the magnetic field \( \vec{B} \)

\[
H^B_{\text{int}} = -\vec{\mu}_b \cdot \vec{B}
\] (11)

is

\[
\Delta E = 2|\vec{\mu}_b||\vec{B}|.
\] (12)

In \cite{3} we have identified soliton (a) with the (Majorana) neutrino and soliton (b) with the charged lepton associated with one of the gauge-group factors in SU(2) × SU(2) × SU(2). The above situation is realized for a valence electron in an atom (soliton (b)), the level splitting of Eq. (12) is known as the anomalous Zeeman effect. This effect was interpreted as an intrinsic angular momentum \( \vec{J} \) of the electron in \cite{12,13}.

Conclusion. We have shown that the only stable solitons arising in the confining phases of each factor in SU(2) × SU(2) can be interpreted as spin-1/2 fermions. Statistically, a fermions are inseparably linked to bosons. The latter are interpreted as longitudinal soundwaves propagating in the center-vortex condensate. So far we have not investigated how the weak interactions of the Standard Model can be understood in the solitonic framework presented here.

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[14] The mass bound from the measurement of the Coulomb potential is $\sim 10^{-14}$ eV.
[15] The availability of such an $n$th center vortex is guaranteed in a condensate of center vortices existing in a phase of spontaneously broken $Z_N$ symmetry. This condensate is created by a diverging magnetic gauge coupling $g$ in the magnetic phase of the theory.
[16] Referring to the field $\Phi$ in our statistical discussion we do not mean the expectation as in Eq. but a field configuration as it contributes to the partition function.
[17] This can be seen by adding to the action density in a small perturbation $B\Phi$ which breaks the $Z_2$ symmetry explicitly.
[18] In contrast to the transformations $T_{\pm}$ they are continuous gauge transformations and thus do not change the physics.
[19] Imagine a dynamical chain of magnetic monopoles and antimonopoles with the monopoles moving to the right and the antimonopoles to the left.