Nuclear Structure Relevant to Neutrinoless Double $\beta$ Decay: the Valence Protons in $^{76}$Ge and $^{76}$Se.

B.P. Kay, J.P. Schiffer, S.J. Freeman, T. Adachi, J.A. Clark, C.M. Deibel, H. Fujita, Y. Fujita, P. Grabmayr, K. Hatanaka, D. Ishikawa, H. Matsubara, Y. Meada, H. Okamura, K.E. Rehm, Y. Sakemi, Y. Shimizu, H. Shimoda, K. Suda, Y. Tameshige, A. Tamii, and C. Wrede

\[ \text{Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA} \]
\[ \text{University of Manchester, Manchester M13 9PL, United Kingdom} \]
\[ \text{Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan} \]
\[ \text{Yale University, New Haven, CT 06520, USA} \]
\[ \text{Department of Physics, Osaka University, Toyonaka, Osaka 567-0043, Japan} \]
\[ \text{Physikalisches Institut, Universität Tübingen, D-72076 Tübingen, Germany} \]
\[ \text{Department of Applied Physics, Miyazaki University, Miyazaki 889-2192, Japan} \]
\[ \text{Cyclotron Radioisotope Center, Tohoku University, Sendai, Miyagi 980-8578, Japan} \]
\[ \text{Center for Nuclear Study, University of Tokyo, Bunkyo, Tokyo 113-0033, Japan} \]
\[ \text{Department of Physics, Kyushu University, Higashi, Fukuoka 812-8581, Japan} \]

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The possibility of observing neutrinoless double $\beta$ decay offers the opportunity of determining the effective neutrino mass if the nuclear matrix element were known. Theoretical calculations are uncertain and the occupations of valence orbits by nucleons active in the decay are likely to be important. The occupation of valence proton orbits in the ground states of $^{76}$Ge, a candidate for such decay, and $^{76}$Se, the corresponding daughter nucleus, were determined by precisely measuring cross sections for proton-removing transfer reactions. As in previous work on neutron occupations, we find that the Fermi surface for protons is much more diffuse than previously thought, and the occupations of at least three orbits change significantly between the two $0^+$ ground states.

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Major experimental efforts are under way to observe neutrinoless double $\beta$ decay, an essential step in determining the nature of the neutrino. If this process were to be observed, it would not only demonstrate that neutrinos are their own antiparticles, but the rate may well give the first direct measure of the neutrino mass if the corresponding nuclear matrix element can be reliably calculated. For one of the likely candidates, $^{76}$Ge, theoretical calculations have yielded answers that vary widely and this uncertainty will be a major obstacle to deducing the neutrino mass. It is important to explore experimental data that could help constrain theoretical models. The difference in the configuration of nucleons between the initial and final states (the $0^+$ ground states of $^{76}$Ge and $^{76}$Se) is a major ingredient in the matrix element. We have undertaken several experiments to better define the knowledge of the two ground-state wave functions and the difference between them. In previous experiments we have focused on the difference in neutron configurations, determining the difference in neutron valence-orbit occupancies and in correlations. As was done for neutrons, here we utilize transfer reactions and the Macfarlane-French sum rules to extract the proton occupancy of states from proton-removing reactions.

These reactions had been studied before, however the two previous experiments were done at different times and apparently with slightly different techniques, so that the relative cross sections may have additional uncertainties. In addition, the publications do not report cross sections, just figures showing selected angular distributions and tables of spectroscopic factors.

There are very few facilities remaining that are capable of carrying out precision reaction measurements of the type required here, with high-quality beams, the necessary beam energies, high-resolution spectrometers and detector systems. We used the RCNP facility at Osaka University to determine accurate cross sections for the $(d,^3He)$ reaction using the Grand Raiden (GR) spectrograph. Evaporated targets of $^{76}$Ge and $^{76}$Se were used, as well as targets of $^{74}$Ge and $^{76}$Se as a check on systematic errors. The experiment was performed at a beam energy of 80 MeV, the lowest energy feasible, since the focal-plane detector of the GR requires passage through two foils between the spectrograph vacuum and the detectors. The angular distributions were found to be consistent with Distorted Wave Born Approximation (DWBA) calculations, and are shown in Figure 3(a). Polarization measurements were also made to distinguish the spins $j$ of $\ell = 3$ transitions, removing uncertainties in attributing strength to $f_{5/2}$ or $f_{7/2}$ orbitals.

In order to obtain accurate cross sections, the product of target thickness and spectrometer aperture was obtained from 10-MeV $\alpha$ particles at an angle of $30^\circ$ on the assumption of Rutherford scattering. The AVF Cyclotron, usually the injector to the main Ring Cyclotron, was used to deliver a singly-charged $^4$He beam directly through a bypass beam line, to the target position. The spectrometer aperture, $60\times40$ mrad in the vertical and horizontal directions, was the same as for all the $(d,^3He)$
measurements with the same current integrator to measure the total beam charge.

A problem remained regarding the measurements at 4.5°. It is suspected that the Faraday cup used for the beam measurement at 0° was partially obstructed by components of the spectrometer system. To appropriately normalize the data, a second spectograph, the Large-Acceptance Spectrometer (LAS) placed at a fixed angle of 60°, was used at the same time as the GR. The ratio of the scattering yield in LAS to a beam current integrator was determined when the GR was at larger angles. This ratio was constant for a given target to the statistical accuracy of ~±3%.

FIG. 1: (color online) (a) Measured angular distributions for ℓ=1, 3 and 4 on 76Ge (points) compared to DWBA calculations (lines). The interval shown (blue) indicates the angular range covered in the high-statistics measurements. (b) Red points are asymmetries with the vector-polarized deuteron beam, between spin up and spin down for 76Ge, with the data summed over the peaks. A histogram representing the spectrum on the focal plane (in arbitrary units) is inserted at the bottom, with spin assignments shown. The circled spins were determined in this experiment.

High-statistics measurements of the unpolarized yield were carried out at the three angles, near the peaks of the angular distributions for ℓ=1, 3, and 4, at 4.5, 8, and 12°, with the aperture subtending ± 1°. The choice of the smallest angle was about half a degree larger than optimal, because of limitations imposed by the counting rates from slit scattering at the very forward angles. The operation of the focal-plane detectors and the method of monitoring efficiencies and dead times have been described elsewhere [8].

The polarization measurements were carried out separately. A vector-polarized deuteron beam (Ap=−0.520±0.010) was used and the measurement was made with the GR spectograph at 8°, close to optimal for separating the j values for the ℓ=3 transitions. Spin assignments were made to five ℓ=3 transitions whose j values had not been previously assigned, and five definite 5/2− assignments were made, including spins of the 198-keV level in 73Ga and the 229-keV level in 76Ge, both with dominant spectroscopic factors. The results for one of the targets, 76Ge, are shown in Figure 1(b).

In order to extract spectroscopic factors, the finite-range DWBA program PTOLEMY [9] was used with several sets of optical-model parameters from the literature [10]. The observed cross sections for ℓ=1, 3, and 4, were divided by the DWBA cross sections. A single overall normalization was established for the (d, 3He) data by requiring that the total occupancy of the valence proton orbits add up to four for Ge and six for Se.

The DWBA normalizations obtained in this fashion had a mean value of 0.63 with an rms variation of 23% between the different parameter sets. However, the summed spectroscopic factors were very nearly the same and varied by less than 0.1 units. The values given in Table I are the average of results from analyses with the various potential sets.

We note that ‘absolute’ spectroscopic factors for ‘good’ single-particle states in doubly-magic nuclei are usually around ~0.6 because of short-range correlations [12]. Such correlations are a uniform property of nuclei, formally they move a given fraction of spectroscopic strength to very high excitation energy, and this fraction does not change appreciably between nearby nuclei or configurations. Thus the normalization obtained by our procedure is meant as a way of estimating the relative populations of valence orbits—and not as a measurement of absolute spectroscopic factors.

The summed spectroscopic factors of Table I are the proton occupancies and are shown graphically in Figure 2 for the A = 76 nuclei, along with the values of these occupancies from various theoretical calculations. The first QRPA calculations [13] were the only ones available at the time the neutron occupancies were published [3] and were also shown there. The more recent QRPA

| Target | ℓ=1 | ℓ=3 | ℓ=4 | Sum |
|--------|-----|-----|-----|-----|
| 74Ge   | 1.88| 1.52| 0.37| 3.76|
| 76Ge   | 1.77| 2.04| 0.23| 4.04|
| 76Se   | 2.08| 3.16| 0.84| 6.08|
| 78Se   | 2.26| 1.81| 2.20| 6.27|
| Uncertainty | ±0.15 | ±0.25 | ±0.25 | ±0.38 |
calculation [14] and a shell-model calculation [15] were published subsequently.

The uncertainties in the experimental values are difficult to estimate. Statistical errors in the summed strengths are less than 1% and relative systematic errors between targets are believed to be less than 3%. The biggest uncertainties stem from possible missing states and, to a lesser extent, from the validity of the DWBA method in extracting spectroscopic factors. We estimate that the occupancy is determined to about 0.15 nucleons and, to a lesser extent, from the validity of the DWBA method in extracting spectroscopic factors. We estimate that the occupancy is determined to about 0.15 nucleons

We also made measurements of the ($^3$He,d) reaction on the same targets and at the same scattering angles, at a corresponding $^3$He beam energy of 72 MeV—determined by the average difference in $Q$-values between the proton-adding and proton-removing reactions on these targets. Even though Ge has only 32 protons, and thus the major shell between $Z=28$ and 50 is barely started, we intended to attempt to apply the sum rule to the proton-adding reaction as well as to the proton-removing one.

However, when we apply the same normalization to the ($^3$He,d) data, unlike the neutron case, the summed strengths for proton vacancies fall short of the expected values, indicating considerable missed strength. For $\ell=1$ an average of 16% of the total strength (12, 14, 21 and 18 % respectively for $^{74}$Ge, $^{76}$Ge, $^{76}$Se, and $^{78}$Se), for $\ell=3$ 24% (25, 30, 4, and 35), and for $\ell=4$ 48% (56, 41, 57, 37) are missing. A small correction, for the unobserved

The shortfall may be a consequence of the fact that these nuclei are near the beginning of a major shell for protons, the centroids of the expected single-particle strength are at higher excitation energy and the strength becomes appreciably fragmented in a high level-density region such that the many small individual components cannot be resolved. Indeed, the discrepancy is worst for the $9/2^+$ orbit, which is at the highest excitation energy. Previous work on the ($^3$He,d) reaction at lower bombarding energies [11], and some of it at higher energy resolution, has not identified significant components with the requisite $\ell$ values. We have also attempted to identify this missing strength by carrying out measurements at the Yale ESTU tandem of the (a,t) reaction which is particularly selective for $\ell=4$ transitions, where the problem is the most serious. No candidates for missing $\ell=4$ strength were found with a strength greater than $\sim 4\%$. This is unlike the neutron-transfer case, where excellent

FIG. 2: (color online) Experimentally determined proton occupancies from Table I for the three valence orbits in $^{76}$Ge and $^{76}$Se. These are compared to (A) QRPA calculations of Ref. [13], (B) of Ref. [14] and (C) the shell-model calculations of Ref. [15]. The experimental uncertainties are indicated by error bars on the left.

FIG. 3: (color online) The difference between proton and neutron occupancies of the ground states of $^{76}$Ge and $^{76}$Se deduced from the present measurement and compared to theoretical calculations with the notation as in Fig. 2.
consistency was obtained between the neutron-adding and removing reactions with $\ell=4$ using the $(\alpha,^{3}\text{He})$ reaction and its inverse. We therefore conclude that we cannot utilize the data from proton-adding reactions in the present case and must rely on the consistency of the proton-removing reactions.

Indeed, the strength in the proton-removing reactions so close to the $Z=28$ closed shell, is restricted to relatively few states at low excitation energies and, as is shown in Table I, gives self-consistent results among the four nuclei studied. Even though the summed strength was used for the normalization, the four determinations are independent and the rms deviation from the expected value is 0.19 protons – well within the estimated uncertainty. We believe that, together with the neutron occupation study [3], the internal consistency of these results perhaps constitutes the most quantitative test of the validity of the sum rules in one-nucleon transfer reactions.

Utilizing the information in Table I we can state the differences in proton occupations between $^{76}\text{Se}$ and $^{76}\text{Ge}$ as 0.31±0.15 in $\ell=1$ (it appears that this difference is mostly in the $0f_{1/2}$ occupation), 1.12±0.35 in $0f_{5/2}$, and 0.61±0.35 in $0g_{9/2}$. The uncertainties were estimated taking possible correlations into account. This comparison is shown in Figure 3, displaying both the neutron differences from [3] and the proton differences from the present work, along with the same set of calculations as in Figure 2. As is evident from both Figures 2 and 3, the $0g_{9/2}$ proton orbit is considerably more involved in the changes in the Fermi surface than the original QRPA [13] calculations suggested. The more recent calculations are consistent with the proton as well as the neutron data on occupancies. Some weak $\ell=2$ transitions are also seen in the proton removal from the two Se isotopes. The spectroscopic factors indicate 1$d_{5/2}$ strength of no more than 0.1 protons. Low-lying $7/2^-$ states seen in the $(^{3}\text{He},d)$ reaction indicate similarly low strengths for these transitions. Though this evidence is not conclusive – we see no appreciable admixture from beyond $Z=50$ or below $Z=28$.

We have thus characterized the microscopic changes in the valence occupations for neutrinoless double beta decay. Measurements of neutron pair-transfer on these nuclei suggest that correlations between neutrons are very similar in these two ground states [4]. We hope to determine proton correlations in a future experiment. Although the most recent theoretical calculations by [11, 12] and [10] appear to be closer to each other in their predictions for the $^{76}\text{Ge}$ matrix element, firm anchor points to the measured properties of these states are likely to be essential. At present, the theoretical calculations do not yet seem to be able to specify how the experimental observables that characterize the wave functions of these two ground states influence the matrix element for neutrinoless double $\beta$ decay.

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Our data are available on-line in the XUNDL database [17].

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