The Development of Algorithm for Determining Optimal Route for Distribution of Goods Based on Distance, Time, And Road Quality Using Fuzzy Set and Clarke And Algorithm Wright Savings

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Abstract. The distribution of goods from distributors to several distribution agents is very important to be studied in order to minimize the total transportation costs and the availability of goods at each agent. The model generally developed in this problem is based on the shortest road distance. Basically, road distance is not the only factor considered but also the travel time and road quality that will be traversed. The travel time is uncertain, it really depends on the time and the density of the vehicles. Likewise, road quality is uncertain. Travel time and road quality are fuzzy. This research developed an algorithm to determine the route of distribution of goods by considering the distance, travel time, and road quality which are fuzzy. The optimal distribution route of the goods is a combination of three fuzzy criteria.

1. Introduction

1.1 Research Background

Competition in the business world today is very competitive so that every businessman tries to minimize operational costs including the cost of distributing goods to all agents. One effort to reduce the cost of distributing the goods is to determine the optimal distribution route. Generally, the basis for determining the route is time and distance. This can also be known by using the google-map or Waze application. It’s just that both of these applications do not consider the demand for goods at each agent and the transport capacity of the transport mode used for delivery. In addition, the travel time and road conditions are fuzzy which means that the value is relatively relative depending on various factors, including delivery hours, weather, and traffic density. Therefore, this blurring factor is resolved by using the fuzzy sets approach.

Meanwhile, to determine the optimal route based on one criterion can be solved using the Vehicle Routing Problem (VRP). VRP can be described as a problem in designing the optimal route from a distributor to a dealer. VRP has several classifications, one of which is Capacitated Vehicle Routing Problem (CVRP) where each transport mode has a limited capacity. There are several methods for completing CVRP, namely the exact method and the heuristic method. One of the heuristic methods that can be used to solve CVRP is the Clarke and Wright Savings algorithm. The Clarke and Wright Savings algorithm is a very popular heuristic method for solving VRP problems [10]. The algorithm is used to minimize the distance in running the route by combining two or more routes by taking into account the distance savings, the number of customer requests and the capacity of the vehicle used. This study applies a combination of two approaches in determining the optimal route with multiple objectives, namely: Fuzzy Sets and the Clarke and Wright Savings Algorithm.
1.2 Problems of the Study
The problems in this study is as follows:
What is the algorithm to distribute of goods that considers three criteria: (1) minimum total distance, (2) total travel time from the distributor warehouse to each minimum agent, and (3) total maximum the road quality, where there is a fuzzy environment, namely travel time and road quality, with a limited transport capacity?

1.3 Objective of the Study
The objective of this study is to develop an algorithm to determine the optimal route to distribute goods from the warehouse to agents which includes three criteria: (1) minimum total mileage, (2) total travel time from the distributor warehouse to each minimum agent, and (3) total maximum the road quality.

2. Literature Review

2.1 Vehicle Routing Problem (VRP)
Vehicle Routing Problem (VRP) was first introduced by Dantzig and Ramser in a study entitled "The Truck Dispatching Problem" in 1959. In their research, Dantzig and Ramser set the formulation of mathematical programs and approach algorithms to solve the problem of dispatching gasoline. Since then, interest in VRP has grown from a group of mathematicians to a broad range of researchers and practitioners from various disciplines. [13]

VRP is a matter of determining the route of a vehicle where there is a set of consumers who need a certain product and each consumer is known the location and number of requests. All consumer requests are met from a source using a number of vehicles. The purpose of VRP is to determine the most optimal delivery route for each vehicle, so that the total mileage of all vehicles can be minimized [13].

VRP can be defined as a search for a solution which involves determining a number of routes, where each route is passed by a transportation mode that starts and ends at its original depot, so that the needs/requests of all customers are met while still meeting existing operating constraints, also by minimizing costs global transportation [13]. Each customer may only be served by a vehicle. This is done to minimize the costs required by considering the capacity of a vehicle in one delivery.

VRP is often referred to as Multi Traveling Salesman Problem (MTSP) where VRP is a combinatorial problem of two problems, namely Traveling Salesman Problem (TSP) and Bin Packing Problem (BPP) [13]. TSP and BPP are categorized as NP-hard problems, so that VRP can also be categorized as NP-hard problems, in combinatorial optimization where exact methods have not been found to find the optimal value. To solve small-scale VRP with multiple customers and all vehicles have the same capacity, the Branch and Bound Algorithm is proven to be the best method for finding optimal solutions. To solve large-scale VRP can be solved heuristically. Although the heuristic approach does not guarantee its optimization, it is able to produce the best solution.

Many factors arise in the use of VRP in the real world. These factors affect the appearance of variations in VRP, such as:

1. Capacitated VRP (CVRP), i.e. each vehicle has a limited capacity
2. VRP with Time Windows (VRPTW), i.e. every customer must be supplied within a certain period
3. Multiple Depot VRP (MDVRP), i.e. distributors have many depots to supply customers
4. VRP with Pick-up and Delivering (VRPPD), i.e. the customer might return the goods to the original depot
5. Split Delivery VRP (SDVRP), i.e. customers are served with different vehicles
6. Stochastic VRP (SVRP), which is the emergence of 'random values' such as number of customers, number of requests, service time or travel time
7. Periodic VRP, that is, delivery is only done on certain days
2.2 Capacitated Vehicle Routing Problem
Capacitated Vehicle Routing Problem (CVRP) is one of the variations of the VRP problem, where there are additional constraints of identical vehicle capacity to visit a number of consumers according to their respective requests. The problem of CVRP is that the total number of consumer requests on a route does not exceed the capacity of the vehicle serving the route and each customer is visited only once by one vehicle. The problem of CVRP aims to minimize the total distance travelled by vehicle travel routes and minimize the number of vehicles used in distributing goods from the delivery place (depot) to a number of consumers.

2.3 Clarke and Wright Savings Algorithm
The Clarke and Wright Savings algorithm is an algorithm developed for CVRP problems and is often used. The purpose of the savings method is to minimize the total travel distance of all vehicles and indirectly to minimize the number of vehicles needed to service all stops [2].

In 1964, Clarke and Wright published an algorithm as a solution to problems of various vehicle routes, which are often referred to as the classical problems of vehicle routes (the classical vehicle routing problem). This algorithm is based on a concept called the concept of savings. This algorithm is designed to solve vehicle route problems with the following characteristics. From a depot the goods must be delivered to customers who have ordered. A number of vehicles have been provided for the transportation of these goods, where each vehicle has a certain capacity in accordance with the goods transported. Each vehicle used to solve this problem must take a predetermined route, starting and ending at the depot, where goods are delivered to one or more customers.

2.4 Fuzzy Multi Objective Decision Making (FMODM)
Fuzzy Multi Criteria Decision Making (FMCDM) can be grouped into two models, namely: Fuzzy Multi Objective Decision Making (FMODM) and Fuzzy Multi Attribute Decision Making (FMADM). The application of FMADM to multi-attribute decision making in fuzzy environments has been found in the literature. That applies FMADM to select high achieving student graduates. [14] describe the taxonomy of Fuzzy Multi-Attribute Decision Making Systems in terms of models, inventors, and data types.

3. Algorithm Development
3.1 Clarke and Wright Savings Algorithm for CVRP Problem
The Capacitated Vehicle Routing Problem (CVRP) is to satisfy the demand of a set of customers using a fleet of vehicles with minimum cost. The problem is described as follows [4]:
Let:
- \( C = \{1, 2, ..., n\} \): the set of customer location.
- \( 0 \): depot location.
- \( G = (N, E) \): the graph representing the vehicle routing network with \( N = \{0, 1, ..., n\} \) and \( E = \{(i, j) : i, j \in N, i \neq j\} \)
- \( q \): demand of customer j.
- \( Q \): common vehicle capacity.
- \( m \): number of delivery vehicles.
- \( c_{ij} \): distance or associated cost between locations \( i \) and \( j \).
- \( L \): maximum distance a vehicle can travel.
- \( P_j \): a lower bound on the cost of travelling from the depot to customer j.
- \( \ell(S) \): lower bound on the number of vehicles required to visit all locations of \( S \) in an optimal solution. Note that \( S \subseteq C \) and \( \ell(S) \geq 1 \).
- \( S \): the complement of \( S \) in \( C \)
- \( x_{ij} \): 1,2, or 0

The model formulation of CVRP is shown below:
Algorithm proposed by Clarke and Wright Savings to solve Capacitated Vehicle Routing Problem, is as follows:

1. Make \( n \) routes: \( v_0 \rightarrow v_i \rightarrow v_0 \), for each \( i \geq 1 \);
2. Compute the savings for merging delivery locations \( i \) and \( j \), which is given by \( s_{ij} = d_{i0} + d_{0j} - d_{ij} \), for all \( i, j \geq 1 \) and \( i \neq j \);
3. Sort the savings in descending order;
4. Starting at the top of the (remaining) list of savings, merge the two routes associated with the largest (remaining) savings, provided that:
   (a) The two delivery locations are not already on the same route;
   (b) Neither delivery location is interior to its route, meaning that both notes are still directly connected to the depot on their respective routes;
   (c) The demand \( G \) and distance constraints \( D \) are not violated by the merged route.
5. Repeat step (3) until no additional savings can be achieved.

3.2 An Example
Consider the nodes described below, and note that the depot is located at node 0. Suppose we would like to solve this capacitated vehicle routing problem (VRP) using the savings algorithm. The number of depots is 25 with the following demand in units.
The number of vehicles to distribute the product to each depot is 3 units with capacity 400 unit per vehicle. The distance, traveling time, and road quality between depots are provided. By implementing Clarke and Wright Savings, the following results are obtained:

a. Based on the distance

Table 1. Ordered Saving based on Distance

| Iteration | Savings Value | (i,j) | Iteration | Savings Value | (i,j) |
|-----------|---------------|------|-----------|---------------|------|
| 1         | 17,40         | (16,3) | 11        | 14,97         | (22,19) |
| 2         | 16,80         | (8,5)  | 12        | 14,96         | (20,4)  |
| 3         | 16,73         | (24,4) | 12        | 14,70         | (9,8)   |
| 4         | 15,70         | (4,2)  | 14        | 14,68         | (15,13) |
| 5         | 15,67         | (12,11)| 15        | 14,61         | (21,10) |
| 6         | 15,64         | (11,9) | 16        | 14,50         | (7,6)   |
| 7         | 15,45         | (25,16)| 17        | 14,42         | (23,17) |
| 8         | 15,43         | (14,12)| 18        | 14,10         | (17,5)  |
| 9         | 15,35         | (19,7) | 19        | 13,35         | (18,10) |
| 10        | 15,00         | (5,1)  |           |               |       |

Table 2. The optimal route for each vehicle and the total number of products are carried are as follows:

| Vehicle No and Total Product | Depot No | Demand (units) | Demand (units) | Demand (units) | Demand (units) |
|------------------------------|----------|----------------|----------------|----------------|----------------|
| I (399 Units)                | 16       | 15             | 8              | 78             | 15             |
|                              | 3        | 2              |               | 11             | 125            |
|                              | 8        | 78             | 12             | 125            | 11             |
|                              | 5        | 125            | 11             | 82             | 9              |
|                              | 24       | 12             | 14             | 110            | 18             |
|                              | 4        | 5              |               |                |                |
|                              | 2        | 5              |               |                |                |
|                              | 25       | 100            |               |                |                |
|                              | 1        | 15             |               |                |                |
|                              | 10       | 42             |               |                |                |

II (383 Units)                | 12       | 125            | 9              | 58             | 14             |
|                              | 11       | 82             | 14             | 110            | 18             |
|                              | 15       | 57             |               |                |                |
|                              | 19       | 5              |               |                |                |
|                              | 13       | 20             |               |                |                |
|                              | 14       | 57             |               |                |                |
|                              | 6        | 5              |               |                |                |
|                              | 23       | 20             |               |                |                |

III (392 Units)               | 12       | 125            | 9              | 58             | 14             |
|                              | 11       | 82             | 14             | 110            | 18             |
|                              | 15       | 57             |               |                |                |
|                              | 13       | 20             |               |                |                |
|                              | 14       | 57             |               |                |                |
|                              | 6        | 5              |               |                |                |
|                              | 23       | 20             |               |                |                |
|                              | 17       | 20             |               |                |                |
b. Based on traveling time

Table 3. Ordered Saving based on Traveling Time

| Iteration | Savings Value | (i,j) | Iteration | Savings Value | (i,j) |
|-----------|---------------|------|-----------|---------------|------|
| 1         | 12,20         | (11,3) | 11        | 9,07          | (21,19) |
| 2         | 11,80         | (7,4) | 12        | 8,96          | (20,4) |
| 3         | 11,73         | (22,5) | 12        | 8,70          | (11,8) |
| 4         | 10,20         | (14,2) | 14        | 8,68          | (15,13) |
| 5         | 10,12         | (13,8) | 15        | 8,61          | (21,20) |
| 6         | 9,94          | (10,9) | 16        | 8,50          | (7,6) |
| 7         | 9,45          | (25,16) | 17        | 8,42          | (23,21) |
| 8         | 9,41          | (14,12) | 18        | 8,10          | (17,15) |
| 9         | 9,35          | (10,7) | 19        | 8,05          | (18,20) |
| 10        | 9,10          | (4,1) |           |               |      |

Table 4. The optimal route for each vehicle and the total number of products are carried are as follows:

| Vehicle No and Total Product | Depot No | Demand (units) | Vehicle No and Total Product | Depot No | Demand (units) | Vehicle No and Total Product | Depot No | Demand (units) |
|------------------------------|----------|----------------|------------------------------|----------|----------------|------------------------------|----------|----------------|
| I (392 Units)                | 16       | 15             | II (395 units)               | 12       | 125            | III (387 Units)              | 19       | 5              |
|                              | 25       | 100            |                              | 11       | 82             |                              | 7        | 60             |
|                              | 8        | 78             |                              | 9        | 58             |                              | 22       | 30             |
|                              | 5        | 125            |                              | 14       | 110            |                              | 20       | 125            |
|                              | 24       | 12             |                              | 18       | 8              |                              | 15       | 57             |
|                              | 6        | 5              |                              | 3        | 2              |                              | 13       | 20             |
|                              | 1        | 15             |                              | 4        | 5              |                              | 21       | 50             |
|                              | 10       | 42             |                              | 2        | 5              |                              | 23       | 20             |
|                              |          |                |                              |          |                |                              | 17       | 20             |

c. Based on road quality

Table 5. Ordered Saving based on Road Quality

| Iteration | Savings Value | (i,j) | Iteration | Savings Value | (i,j) |
|-----------|---------------|------|-----------|---------------|------|
| 1         | 9,20          | (10,2) | 11        | 8,60          | (20,19) |
| 2         | 9,13          | (17,4) | 12        | 8,56          | (18,14) |
| 3         | 9,03          | (21,15) | 12        | 8,50          | (11,8) |
| 4         | 8,98          | (11,13) | 14        | 8,48          | (14,13) |
| 5         | 8,90          | (12,18) | 15        | 8,40          | (20,16) |
| 6         | 8,80          | (9,16) | 16        | 8,35          | (17,15) |
| 7         | 8,75          | (22,6) | 17        | 8,32          | (22,24) |
| 8         | 8,71          | (14,12) | 18        | 8,30          | (17,15) |
| 9         | 8,65          | (10,17) | 19        | 8,25          | (16,13) |
| 10        | 8,62          | (4,1) |           |               |      |
Table 6. The optimal route for each vehicle and the total number of products are carried are as follows:

| Vehicle No and Total Product | Depot No 1 | Demand (units) 1 | Depot No 2 | Demand (units) 2 | Depot No 3 | Demand (units) 3 |
|-----------------------------|-----------|------------------|-----------|------------------|-----------|------------------|
| I                           | 2         | 5                | II        | 12               | 125       |
| (390 Units)                 |           |                  |           |                  |           |
| II                          | 16        | 15               | III       | 7                | 60        |
|                             | 25        | 100              |           | 22               | 30        |
|                             | 8         | 78               |           | 20               | 125       |
|                             | 5         | 125              |           | 15               | 57        |
|                             | 6         | 5                |           | 13               | 20        |
|                             | 1         | 15               |           | 21               | 50        |
|                             | 10        | 42               |           | 23               | 20        |
|                             | 19        | 5                |           | 17               | 20        |

Step 1: Transform all values into normal values as follows:

\[
D_{ij} = \frac{d_{ij}}{\max (\forall d_{ij})} \quad \text{for the distance from Depot } i \text{ to Depot } j.
\]

\[
T_{ij} = \frac{t_{ij}}{\max (\forall t_{ij})} \quad \text{for the traveling time from Depot } i \text{ to Depot } j.
\]

\[
Q_{ij} = \frac{q_{ij}}{\max (\forall q_{ij})} \quad \text{for the road quality average from Depot } i \text{ to Depot } j.
\]

Step 2: Calculate the average value between depots as follows:

\[
C_{ij} = \frac{(D_{ij} + T_{ij} + Q_{ij})}{3}
\]

Step 3: Calculate Ordered Saving using Clarke and Wright Savings Algorithm

Table 7. Ordered Saving for Combined Values

| Iteration | Savings Value | (i,j)   | Iteration | Savings Value | (i,j)   |
|-----------|---------------|---------|-----------|---------------|---------|
| 1         | 0.912         | (11,4)  | 11        | 0.840         | (20,19) |
| 2         | 0.913         | (16,8)  | 12        | 0.865         | (18,14) |
| 3         | 0.903         | (10,12) | 13        | 0.850         | (3,8)   |
| 4         | 0.898         | (15,18) | 14        | 0.838         | (14,13) |
| 5         | 0.890         | (17,22) | 15        | 0.830         | (20,11) |
| 6         | 0.880         | (16,19) | 16        | 0.825         | (17,15) |
| 7         | 0.878         | (22,14) | 17        | 0.822         | (20,14) |
| 8         | 0.870         | (14,12) | 18        | 0.810         | (17,15) |
| 9         | 0.862         | (8,9)   | 19        | 0.805         | (4,13)  |
| 10        | 0.852         | (3,10)  |           |               |         |
Table 8. The optimal route for each vehicle and the total number of products are carried are as follows:

| Vehicle No and Total Product | Depot No | Demand (units) | Vehicle No and Total Product | Depot No | Demand (units) | Vehicle No and Total Product | Depot No | Demand (units) |
|------------------------------|---------|----------------|------------------------------|---------|----------------|------------------------------|---------|----------------|
| I (392 Units)                | 2       | 5              | II (392 units)               | 12      | 125             | III                          | 18      | 8              |
|                              | 16      | 15             |                              | 11      | 82              |                              | 7       | 60             |
|                              | 25      | 100            |                              | 9       | 58              |                              | 22      | 30             |
|                              | 8       | 78             |                              | 14      | 110             |                              | 20      | 125            |
|                              | 5       | 125            |                              | 24      | 12              |                              | 15      | 57             |
|                              | 6       | 5              |                              | 4       | 5               |                              | 13      | 20             |
|                              | 1       | 15             |                              |         |                 |                              | 21      | 50             |
|                              | 10      | 42             |                              |         |                 |                              | 23      | 20             |
|                              | 19      | 5              |                              |         |                 |                              | 17      | 20             |
|                              | 3       | 2              |                              |         |                 |                              |         |                |

e. Result Comparison

Table 9. The result calculation can be summarized as follows:

| Criteria                | Total Distance | Total Traveling Time | Total Road Quality | Total Product carried by |
|-------------------------|----------------|----------------------|--------------------|--------------------------|
|                         |                |                      |                    | Vehicle 1 | Vehicle 2 | Vehicle 3 |                |
| Distance                | 56.15          | 530.4                | 8.12               | 399        | 383      | 392       |                |
| Traveling Time          | 62.54          | 510.3                | 8.20               | 392        | 395      | 387       |                |
| Road Quality            | 64.3           | 540.6                | 9.25               | 390        | 390      | 394       |                |
| Combined                | 60.2           | 505.24               | 9.12               | 392        | 392      | 390       |                |

4. Conclusion

The distribution problem is mainly focus on to find the best route such that minimize traveling distance, traveling time, and road quality. Based on traveling distance criteria, it can be determined the route that minimize the total distance but it is not best solution for traveling time and road quality. Similarly, the calculation based on traveling time will yields the best route for total traveling time but not for total distance and road quality. By using Fuzzy Sets which combines all criteria to be considered will provide the best-compromised solution for all criteria.

5. References

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