Transverse Flow of Gluon Fields in Heavy Ion Collision

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Abstract. We describe the dynamics of initial gluon fields in heavy ion collision using a formal recursive solution of the Yang Mills equations and solving for the energy momentum tensor analytically in a boost-invariant setup. We generalize the original McLerran-Venugopalan (MV) model in order to allow for realistic nuclear profiles. This leads to a transverse flow of gluon fields. This flow pattern is inherited by the quark gluon plasma fluid after thermalization. Its most interesting aspect is a rapidity-odd flow component. We show that this rapidity-odd flow does not break boost invariance and that it emerges naturally from the Yang Mills equations. It leads to directed flow of particles and introduces angular momentum to the system.

1. Introduction
Experiments at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) provide opportunities to study a deconfined phase of QCD matter, Quark Gluon Plasma (QGP) [1, 2, 3]. In high energy nuclear collisions, the wave functions of incident nuclei are dominated by wee partons. High occupation numbers permit a quasi-classical description of the gluon fields in the nuclei before collisions called Color Glass Condensate (CGC), which should be valid up to a time scale inversely proportional to the saturation scale $Q_s$. Predictions from CGC can serve as initial conditions for a further hydrodynamic evolution of the bulk matter created in high energy nuclear collision [4, 5, 6].

There have been many efforts devoted to solving the Yang Mills equations for the gluon field in the forward light cone after the collision, both numerically and analytically [7, 8, 9]. Here we use a formal recursive solution of the Yang-Mills equations found by Fries et al. [7] to calculate the energy momentum tensor. In a generalized version of the McLerran-Venugopalan (MV) model [10, 11], which allows for transverse dynamics (MVTD), we calculate the flow of energy in transverse direction and find a rapidity-odd flow term which will eventually lead to directed flow of particles. These results had first been reported in [12, 13].

This article is organized as follows. First we briefly review the MV model in section 2. Then, in section 3 we show that the rapidity-odd flow term is a natural consequence of Gauss' Law in an electrodynamic analogue. We compute the Poynting vector of the gluon field in the forward light cone in the MVTD framework in section 4. Finally we discuss the phenomenological consequence of flowing gluon fields in heavy ion collision in section 5.
2. Gluon Fields in the MV Model

In the McLerran-Venugopalan model, the simplest incarnation of CGC, partons with large Bjorken-$x$ moving along the light cone can be treated as a $SU(3)$-current $J^\mu = \delta^{\mu+} \delta(x^-) \rho_\Sigma \delta^2(\bf{r})$ in light cone gauge, which generates soft gluons through the Yang Mills equations $D_\mu F^{\mu\nu} = J^\nu$.

The local charge density $\rho_\Sigma(\bf{r})$ is stochastic with vanishing mean because of local color neutrality, but the variance is finite. A Gaussian distribution with variance $\mu$ is used to describe $\rho_\Sigma(\bf{r})$ in the MV model,

$$\langle \rho_\Sigma(\bf{r}_1) \rho_\Sigma(\bf{r}_2) \rangle = \frac{g^2}{N_c^2 - 1} \delta_{\bf{r}_1} \delta_{\bf{r}_2} \delta^2(\bf{r}_1 - \bf{r}_2).$$

In our notation, $\mu$ is the average, color-summed, squared density of charges in the transverse plane.

A recursive solution [7] has been found for the Yang-Mills equations [14] of the gluon field after collisions of two charges. The chromo-electric and chromo-magnetic fields $\vec{E}$ and $\vec{B}$ immediately after the collision of nuclei can then be calculated.

In the MV model, most of the physical observables are related to the gluon distribution function of a nucleus in light cone gauge [15]$(i, j = 1, 2)$

$$\langle A_i^\mu(\bf{r}) A_j^\nu(\bf{r}) \rangle = \frac{g^2}{8\pi(N_c^2 - 1)} \delta_{\bf{r}_1} \delta_{\bf{r}_2} \mu(\bf{r}) \ln \frac{Q^2}{m^2},$$

where $Q$ is a UV cutoff and $m$ is an infrared scale that can be introduced as an effective gluon mass. In the original calculation [15], $\mu$ was taken to be a constant area density, which is not a realistic approximation for nuclei. We have checked that the QCD dynamics at larger distances can be separated from color glass physics if variations of $\mu(\bf{r})$ are small on the IR length scale $\sim 1/m$, i.e.

$$m^{-1}|\nabla \mu(\bf{r})| \ll \mu(\bf{r}).$$

Many well-known results of the MV model, in particular Eq. (2), hold with corrections appearing as an expansion in gradients $m^{-1}\nabla^2 \mu(\bf{r})$. Details of this calculation will be reported elsewhere [16].

The longitudinal chromo-electric and -magnetic fields after the collision for small times $\tau$ can be written in terms of purely transverse gauge fields $A_1^i = A_1^i(\bf{r}) t^2$ and $A_2^i = A_2^i(\bf{r}) t^2$ of the nuclei before the collision, in their respective light cone gauge [7, 16, 17],

$$E^3 = \left(1 + \frac{\tau^2}{4} D^2\right) E_0 + O(\tau^4)$$

$$B^3 = \left(1 + \frac{\tau^2}{4} D^2\right) B_0 + O(\tau^4),$$

where $D^2 = D^i D_i$ is the square of the covariant derivative with respect to gauge field $A_1^i + A_2^i$. $E_0 = iq \left[ A_1^1, A_2^3 \right]$, $B_0 = ig\epsilon^{ijk} \left[ A_1^j, A_2^k \right]$ are the longitudinal fields at $\tau = 0$, which are consistent with boundary conditions. Transverse fields do not receive contribution from zeroth order in $\tau$, i.e. they vanish at $\tau = 0$. At order $\tau$, the transverse fields can be expressed in terms of the initial longitudinal fields as [7, 16, 17]

$$E^i = -\frac{\tau}{2} \left( \sinh \eta D^i E_0 + \cosh \eta \epsilon^{ij} D^j B_0 \right)$$

$$B^i = -\frac{\tau}{2} \left( \sinh \eta D^i B_0 - \cosh \eta \epsilon^{ij} D^j E_0 \right).$$

The transverse fields will receive corrections at order $\tau^3$ and higher, but here we focus on the early time behavior. Higher order (in $\tau$) contributions are discussed in [16, 18].
Figure 1. Rapidity-even transverse fields from Ampère’s and Faraday’s Laws and rapidity-odd transverse fields from Gauss’ Law illustrated by transverse fields at \( z = z_0 \) and \( z = -z_0 \). Initial longitudinal fields are indicated by solid grey arrows, thickness reflects field strength. Refer to [13] for details.

Now the energy momentum tensor can be calculated from chromo-electric and -magnetic fields straightforwardly, e.g. the initial energy density is \( \epsilon_0 = T^{00}(\tau = 0) = (E_0^2 + B_0^2)/2 \). We are particularly interested in the Poynting vector \( T^{0i} \) which represents the initial transverse energy flux. The leading behavior of \( T^{0i} \) at small \( \tau \) can be decomposed into two terms

\[
S^i_+ = \frac{\tau}{2} \cosh \eta \left( E_0 D^j E_0 + B_0 D^j B_0 \right)
\]

is even in \( \eta \), and,

\[
S^i_- = \frac{\tau}{2} \sinh \eta \epsilon^{ij} \left( E_0 D^j B_0 - B_0 D^j E_0 \right)
\]

is odd in \( \eta \).

We would like to emphasize that the energy momentum tensor with such a Poynting vector is boost invariant despite the rapidity-odd flow term. Here boost invariance is defined for the full tensor \( T^{\mu\nu} \) as

\[
\Lambda^{\nu'}(y) T_{\mu'\nu'}(\eta) \Lambda^{\nu'}(y) = T_{\mu\nu}(\eta + y),
\]

where \( \Lambda(y) \) is the Lorentz boost tensor with rapidity \( y \) along the \( z \)-axis. We will show in the next section that the rapidity-odd field is a natural consequence of the field equations.

3. An Electrodynamic Analogue

An electrodynamics problem with equivalent boundary conditions can provide an intuitive interpretation of the origin of the \( \eta \)-odd flow term. In the forward light cone the free electromagnetic fields obey the homogenous Maxwell Equations \( \partial_{\mu} F^{\mu\nu} = 0 \). We require equivalent boundary conditions on the light cone \( \tau = 0 \), i.e., \( \vec{E}(\tau = 0, \vec{r}) = E_0(\vec{r})\hat{e}_z \), \( \vec{B}(\tau = 0, \vec{r}) = B_0(\vec{r})\hat{e}_z \). In other words, the initial fields are again purely longitudinal. In principle this abelian problem can be solved analytically [14, 16], but we explore the solution order by order in powers of \( \tau \) as we did in the case of QCD for comparison.
The longitudinal fields in the abelian case can be obtained immediately from the QCD solutions,

\[ E^3 = \left(1 + \frac{t^2 - z^2}{4} \nabla^2 \right) E_0 \]  
\[ B^3 = \left(1 + \frac{t^2 - z^2}{4} \nabla^2 \right) B_0 , \]  
(11) (12)

while the transverse fields are

\[ E^i = \frac{z}{2} \nabla^i E_0 + \frac{t}{2} \epsilon^{ij} \nabla^j B_0 \]  
\[ B^i = \frac{z}{2} \nabla^i B_0 - \frac{t}{2} \epsilon^{ij} \nabla^j E_0 , \]  
(13) (14)

for small times \( \tau \), i.e. \( t^2 \approx z^2 \). Now we can check these solutions against Gauss’, Ampère’s and Faraday’s Laws.

It is easy to see that longitudinal fields \( E^3 \) and \( B^3 \) are decreasing away from the light cone \( t^2 = z^2 \). At the same transverse point, the electric (magnetic) flux through a transverse square area of size \( a^2 \) at fixed points \( z = z_0 > 0 \) and \( z = -z_0 \) have the same initial values \( E_0 a^2 \) and diminish at the same rates \( \nabla^2 E_0 a^2 t/2 \) \((\nabla^2 B_0 a^2 t/2)\). This reduction induces magnetic (electric) fields curling with the same chirality around the longitudinal fields at these two points, see Fig. 1.

On the other hand we consider two cubes of volume \( a^3 \). One cube has two sides located at \( z_0 \) and \( z = z_0 - a \) along the \( z \) axis, and the total electric (magnetic) flux out of the box due to the longitudinal field is \(-z_0 a^2 \nabla^2 E_0 /2 > 0 \) \((-z_0 a^2 \nabla^2 B_0 /2 > 0\). The other cube has two sides located at \(-z_0 \) and \( z = -z_0 + a \), and the net flux of longitudinal field of this cube has the opposite sign. From Gauss’ Law we will have transverse electric (magnetic) fields with opposite signs induced at \( z_0 \) and \( z = -z_0 \).

To summarize, the transverse fields contain \( \eta \)-odd terms due to Gauss’ Law and \( \eta \)-even terms due to Ampère’s and Faraday’s Laws. It should not be surprising that such transverse fields naturally give rise to an \( \eta \)-odd flow of energy as well.

### 4. Averaging Over Field Configurations

The expectation value of the initial energy density can be calculated from gluon distribution functions after averaging over source configurations \[19\],

\[ \varepsilon_0(\vec{r}) = \frac{g^6}{32\pi^2 N_c} \frac{N_c}{N_c^2 - 1} \ln^2 \frac{Q^2}{m^2} \mu_1(\vec{r}) \mu_2(\vec{r}) . \]  
(15)

The \( \eta \)-even flow term of the Poynting vector becomes,

\[ S_{+}^i = -\frac{\tau}{2} \cosh \eta \frac{g^6}{32\pi^2 N_c} \frac{N_c}{N_c^2 - 1} \ln^2 \frac{Q^2}{m^2} \nabla^i \(\mu_1 \mu_2\) = -\frac{\tau}{2} \cosh \eta \nabla^i \varepsilon_0 . \]  
(16)

It mimics hydrodynamic flow by following the gradient of the energy density. The rapidity-odd flow is

\[ S_{\perp}^i = -\frac{\tau}{2} \sinh \eta \frac{g^6}{32\pi^2 N_c} \frac{N_c}{N_c^2 - 1} \ln^2 \frac{Q^2}{m^2} \(\mu_2 \nabla^j \mu_1 - \mu_1 \nabla^j \mu_2\) , \]  
(17)

where the expectation value of two gluon fields in light cone gauge with one derivative has been used \[16\],

\[ \left\langle \left( \partial^k A^l_{1/2}(\vec{r}_\perp) A^l_{1/2}(\vec{r}_\perp) \right) \right\rangle = \frac{g^2}{16 \pi (N_c^2 - 1)} \delta_{ab} \ln \frac{Q^2}{m^2} \nabla^l \mu(\vec{r}_\perp) \left( \delta^{jl} \delta^{ik} - \delta^{il} \delta^{jk} - \delta^{kl} \delta^{ij} \right) . \]  
(18)
5. Discussion of Results

It is useful to normalize the initial average transverse energy flow by the average initial energy density,

\[ V^i = \frac{T^{0i}}{\epsilon_0} = -\frac{\tau}{2} \left( \cosh \eta \frac{\nabla^i (\mu_1 \mu_2)}{\mu_1 \mu_2} + \sinh \eta \frac{\mu_2 \nabla^i \mu_1 - \mu_1 \nabla^i \mu_2}{\mu_1 \mu_2} \right) \] (19)

which is independent of the UV cutoff and the IR regulator. It is a proxy for the velocity field if the system were to decay rapidly into thermalized particles. Using Woods-Saxon profiles for incident nuclei, \( V^i \) has been calculated in several situations [13].

Fig. 2 shows the average flow field \( V^i \) for the collision of two gold nuclei at impact parameter \( b = 6 \) fm in \( \eta - x \)-plane. The nucleus centered at \( x = 3 \) fm travels in the positive \( \eta \)-direction. The fireball shows a rotation pattern as if the initial gluon flux tubes preferred to expand in the wake of spectator nucleons.

Fig. 3 shows the average flow fields \( V^i \) for Au+Cu collisions in the \( \eta - x \)-plane at impact two parameters: \( b = 0 \) fm and \( b = 2 \) fm. The gold nucleus is traveling in the positive \( \eta \) direction. The
flow field leads to a more prominent expansion on the copper side. The flow pattern becomes even more involved for Au+Cu collisions at finite impact parameter [13].

The flow of energy implies a finite angular momentum with magnitude comparable to that found in other calculations [20, 21]. Even without detailed knowledge of the thermalization mechanism, it is still reasonable to expect such a flow pattern to survive in a hydrodynamic phase after (partial) thermalization because of energy and momentum conservation. Eventually it should contribute to directed flow of particles observed at RHIC and LHC [22, 23, 24]. The matching to ideal hydrodynamic initial conditions has been worked out in [25]. A matching to 3+1D viscous hydrodynamics initial conditions will provide a tilted fireball similar to the fire streak model [26]. Our calculation suggests that color glass could be the microscopic mechanism to transfer angular momentum to the fireball. A detailed comparison to experimental data will require hydrodynamic simulations which are ongoing.

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