Network Simulation Model for Free Convective Flow from a Vertical Cone

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Abstract. An axisymmetric, mathematical models is obtainable for the laminar natural convective laminar flow with effects of heat as well as mass transmission on an incompressible viscid liquid past a cone by uniform wall concentration, variable surface temperature including with effect of chemical reaction. The solutions of non-dimensional leading boundary layer equation of flow which are coupled, unsteady and nonlinear PDE’s were obtained using NSM, a robust statistical method which demonstrate elevated effectiveness and accurateness and the computer code Pspice. The different profiles of temperature, velocity and concentration were analyzed for different parameter, namely \( \lambda \) (chemical reaction), \( \text{Sc} \) (Schmidt number), \( \text{Pr} \), \( \text{N} \) (buoyancy ratio constraint) and power law exponential of wall temperature \( n \) are discussed graphically. The current experiment result is compare with the existing outcome in the open literature and is found to be in outstanding conformity.

Keywords : Chemical reaction, natural convection, Thermal absorption/Generation, variable shell temperature, NSM, Unsteady, Vertical cone, PSPICE.

1. Introduction
The problem of natural convective boundary layer flow and mass transfer over a cone get a great deal of attention in various branches of science and engineering. The cooling of nuclear reactors is often used to study the structure of stars and planets through free convection studies. Intensive research effort, both theoretical and experimental, has been devoted to problems of free convection heat transfer in view not only of their own interest but also of the application to astro/geophysics and engineering during the last decades. Due to the influence of temperature variation and concentration variation or combination of both induces natural convection. Similarity or non-similarity solutions for 2D axisymmetric problems for natural laminar convective flow over vertical cone in steady state have developed by many authors [1-4] since 1953. Natural convective flow past over a vertical cone surface immersed in an infinite, compressible and viscous fluid combined with the effects of mass transfer is analyzed by Kafoussias [5]. Many authors [6-16] have analyzed the natural convective flow from a cone /rotating cone/ truncated cone with permeable/saturated permeable /non-Darcian porous regime for various boundary condition with Magnetohydrodynamic field, chemical reaction, radiation and absorption/generation etc.,

Network Simulation Solutions of present problem is well-tested, extremely compatible quantitative computational technique was applied. This computational procedure was invented by Nagel [17] in the application of semiconductors (1975) at University of California, Berkeley. Consequently it was
executed efficiently in engineering applications. NSM is based on the elegant thermoelectric equivalence between electrical and thermal variables. NSM can develop any type of nonlinear solutions affected by boundary conditions, thermal characteristics, temperature differences, etc. This procedure has been implemented quite currently to complicated highly nonlinear thermo fluid dynamical problems. The author’s solution to the differential equation in its current form [20-28] was developed using an estimation process discretization. NSM deals with the PDE which employ the mathematical representation of the physical system by applying spatial discretization which was earlier analyzed by Rektorys [18]. The continuous character of time factor is considered in the discretized boundary layer equations. The network model design is based on these equations. Current models developed by multiple networks are connected in series, and boundary conditions are added through special electrical equipment. Its main feature is that the network model consists of a few electrical components in series, and the surface conditions are taken into consideration to create the entire system model. The network model created here is a combination of current control generator, capacitor and resistor, and programming is very simple. PSpice computer code [17] is simulated complete network model to obtain numerical solutions. Gonzalez and Alhama [19] derived the correlation between network simulation method and heat transfer. Many authors [20-28] have solved different problems for various geometries of free convection using the Network simulation method. Convergence criteria does not require for this technique to obtain the solution of the finite difference equations formulated from the discretization of the partial differential equation corresponds to this model, as the robust software PSpice satisfies this work.

The problem of natural heat flow on the changing vertical cone under the condition of irregular temperature and uniform and constant concentration of the wall, in the presence of heat generation/absorption, the combined effect of chemical reactions has not yet caused researchers to try. It also has many applications in various fields, such as the safety of nuclear reactors, solar power plants, dehydration and drying in food and chemical processes, steam generators, spacecraft design, etc. Hence in the present work investigated transient natural convective flow of non-isothermal vertical cone with effects as mentioned earlier. The governing boundary layer equations are solved by Network Simulation Method. The velocity, temperature and concentration profiles for various values of characters $\lambda$, $\Delta$, $\text{Sc}$, $n$, $\text{Pr}$ and $N$ are discussed. To justify the current numerical results are compared with the existing results of Chamkha[8] and has excellent compatibility.

2. Mathematical Formulation

After the following assumptions, the transient heat flux of a viscous incompressible liquid with an improved layered natural size through a vertical cone is obtained, which has uneven wall temperature and uniform surface concentration, and generates. The effects of viscous dissipation and pressure gradient along the boundary layer are neglected and there exists only chemical reaction of first order in combination of species concentration and liquid. Further chemical species concentration of the diffusing substance is treated to be very small which is far away from the wall of the cone. Hence the Dufour and Soret effects are ignored. Also cone surface and surrounding fluid is in equal temperature with concentration. Then at time $t$, the cone surface temperature is raised to $T_0$ where $\Delta t$ is the power law exponents variable in wall temperature. Concentration around surface the cone is raised to $C_0$ and both are retained in the consistent level. The physical model system is considered such that $x$ represents the length of the interval along cone surface from the apex ($x = 0$) and $y$ represents the length of the interval along exterior perpendicular. The properties of the fluid are considered as stable except the density difference which induces body force in boundary layer equation of momentum. And the governing boundary layer equation with respect to Boussinesq approximation is given by:

Continuity Equation:

$$\frac{\partial}{\partial x} (ru) + \frac{\partial}{\partial y} (rv) = 0$$

(1)
Momentum Equation:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g \beta (T' - T_{\infty}) \cos \phi + \nu \frac{\partial^2 u}{\partial y^2} + g \beta_c (C' - C_{\infty}) \cos \phi
\]  

(2)

Energy Equation:

\[
\frac{\partial T'}{\partial t} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \alpha \frac{\partial^2 T'}{\partial y^2} + \frac{Q_0}{\rho c} (T' - T_{\infty})
\]  

(3)

Concentration Equation:

\[
\frac{\partial C'}{\partial t} + u \frac{\partial C'}{\partial x} + v \frac{\partial C'}{\partial y} = D \frac{\partial^2 C'}{\partial y^2} - k_i (C' - C_{\infty})
\]  

(4)

The conditions imposed on boundaries at initially are

\[
t' \leq 0; \ u = 0, v = 0, \ T' = T_{\infty}', \ C' = C_{\infty}' \ \text{for all} \ x \ \text{and} \ y
\]

\[
t' > 0; \ u = 0, v = 0, T' = T_{\infty}' + a x'^n, C' = C_{\infty}' \ \text{at} \ y = 0
\]

\[
u = 0, \ \ T' = T_{\infty}', \ C' = C_{\infty} \ \text{at} \ x = 0
\]

\[
u = 0, \ \ T' \rightarrow T_{\infty}', \ C' \rightarrow C_{\infty} \ \text{as} \ y \rightarrow \infty
\]  

(5)

where \( u, v \) are velocities towards \( x \) and \( y \) direction, \( r \) is the local radius of the cone, \( x \) and \( y \) are the spatial coordinates, \( g \) is the acceleration due to gravity, \( \beta \) is the volumetric coefficient of thermal expansion, \( \beta_c \) is the volumetric coefficient of concentration expansion, \( T' \) is the temperature, \( C' \) is the concentration, \( \mu \) is the dynamic viscosity, \( \nu \) is the kinematic viscosity, \( \alpha \) is the thermal diffusivity, \( \rho \) is density, \( Q_0 \) is the dimensional heat generation/absorption coefficient, \( c_p \) is the specific heat in constant pressure, \( D \) is the mass diffusivity, \( k_1 \) is the dimensional parameter due to chemical reaction.

Shear stress \( \tau_x \), Heat transfer rate \( Nu_x \) and local Sherwood number \( Sh_x \) are given by

\[
\tau_x = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, \ Nu_x = -\frac{x}{T_{\infty}' - T_{\infty}} \left( \frac{\partial T'}{\partial y} \right)_{y=0}, \ Sh_x = -\frac{x}{C_{\infty}' - C_{\infty}} \left( \frac{\partial C'}{\partial y} \right)_{y=0}
\]  

(6)

The following quantities are used dimensionlessly:

\[
X = \frac{x}{L}, \ Y = \frac{y}{L} (Gr_l)^{\frac{1}{2}}, \ R = \frac{r}{L}, \ \text{where} \ r = x \sin \phi, \ V = \frac{v L}{\nu} (Gr_l)^{-\frac{1}{2}}, \ U = \frac{u L}{\nu} (Gr_l)^{-\frac{1}{2}}, \ t = \frac{v l}{L} (Gr_l)^{\frac{1}{2}}.
\]

\[
U = \frac{(T' - T_{\infty})}{(T_{w}' - T_{\infty}')}, \ Gr = \frac{g \beta (T_{w}' - T_{\infty}') L^2 \cos \phi}{\nu^2}, \ Pr = \frac{\nu}{\alpha}, \ Sc = \frac{\nu}{D}, \ C = \frac{(C - C_{\infty})}{(C_{\infty}' - C_{\infty}')},
\]

\[
Gr^* = \frac{g \beta_c (C_{\infty}' - C_{\infty}) L^2 \cos \phi}{\nu^2}, \ N = Gr^* (Gr_l)^{-\frac{1}{2}}, \ \Delta = \frac{Q_0}{c_p \mu L} (Gr_l)^{-\frac{1}{2}}, \ \lambda = k_1 L^2 (Gr_l)^{-\frac{1}{2}}
\]  

(7)

Equations (1), (2), (3), (4) and (5) are transformed into the following dimensionless form:

\[
\frac{\partial (UR)}{\partial X} + \frac{\partial (VR)}{\partial Y} = 0, \ \frac{\partial (U)}{\partial X} + \frac{\partial (V)}{\partial Y} + U = 0
\]  

(8)

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = T + NC + \frac{\partial^2 U}{\partial Y^2}
\]

\[
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = NC + \frac{\partial^2 V}{\partial Y^2}
\]
Local Shear stress \( \tau_X \), heat transfer rate \( Nu_X \) and local Sherwood number \( Sh_X \) in dimensionless quantities are respectively

\[
\tau_X = Gr_X^{\frac{3}{2}} \left( \frac{\partial U}{\partial Y} \right)_{Y=0} \quad Nu_X = \frac{X}{T} \left( \frac{-\partial T}{\partial Y} \right)_{Y=0} \quad Gr_X^{\frac{1}{2}} \quad Sh_X = \frac{X}{C} \left( \frac{-\partial C}{\partial Y} \right)_{Y=0} \quad Gr_X^{\frac{1}{2}}
\]

Average Shear stress \( \overline{\tau} \), average Sherwood number \( \overline{Sh} \) and average heat transfer rate \( \overline{Nu} \) in non-dimensional quantities are respectively

\[
\overline{\tau} = 2Gr_X^{\frac{3}{2}} \int_0^1 X \left( \frac{\partial U}{\partial Y} \right)_{T=0} dX, \quad \overline{Nu} = 2Gr_X^{\frac{1}{2}} \int_0^1 X \left( \frac{-\partial T}{\partial Y} \right)_{Y=0} dX, \quad \overline{Sh} = 2Gr_X^{\frac{1}{2}} \int_0^1 X \left( \frac{-\partial C}{\partial Y} \right)_{T=0} dX
\]

Use the new method to solve nonlinear partially coupled differential equations (8) to (11) with initial and boundary conditions (12), Network Simulation Method. The estimation of the boundary layer equation is based on the finite difference discretization formula and the estimation of spatial coordinates An electrical network circuit design is formulated for each boundary layer equation. Electric analogy is applied in which the variable voltage (V) is similar to velocities (U, V), temperature (T) and concentration (C); the variable electric current (J) is equivalent to the velocity fluxes \((\partial U / \partial X, \partial U / \partial Y, \partial V / \partial Y)\), temperature fluxes \((\partial T / \partial X, \partial T / \partial Y)\) and the concentration fluxes \((\partial C / \partial X, \partial C / \partial Y)\).

For each dimensionless boundary layer equation, three circles are developed. The entire network is converted into a suitable program, which is solved by the Pspice circuit simulator [17]. The time interval required for convergence is not necessary, because the Pspice code is used with the calculation method of a highly advanced mathematical algorithm, which is common to most numerical methods currently used.

3. Design of Network Model

Set the inclination height of the semi-infinite vertical cone to \( L = 1 \), which can be considered as a rectangular field, where \( X \) ranges from 0 to 1, and \( Y \) ranges from 0 to \( Y_{\text{max}} = 20 \), where \( X = L \) corresponds to the inclination of the vertical cone Height, and is considered \( Y_{\text{max}} \) as \( \infty \), where \( Y_{\text{max}} \) is located in the external thermal category, momentum and species boundary. The integration area is regarded as a rectangle with grid size \( \Delta X = 0.25 \) and \( \Delta Y = 0.25 \).

The network model is designed as follows. The finite difference differential equations formulated from non-dimensional continuity, momentum balance, energy balance and mass balance Equations (8) – (11) by implementing electrical equivalence together with Kirchhoff’s law is
Each term of the finite difference Equations (22)-(24) is considered as an electrical current, and is written as the combinations of Resistors, Capacitors and Generators as derived in Zueco[20]. More rigorous The numerical analysis carried out by excellent procedure described by Gonzalez-Fernandez and Alhama[19], Zueco[20]. Figure 1a, 1b, 1c describe the network model equivalent to the equations (16)-(18).

\[
\begin{align*}
\left( U_{i-\Delta X,j} - U_{i+\Delta X,j} \right) / \Delta X + ( V_{i,j-\Delta Y} - V_{i,j} ) / (\Delta Y / 2) \\
+ U_{i,j} / (i \Delta X / 2) &= 0 \\
\Delta Y dU_{i,j} / dt + \Delta Y U_{i,j} \left( U_{i-\Delta X,j} - U_{i+\Delta X,j} \right) / \Delta X \\
+ V_{i,j} \left( U_{i,j-\Delta Y} - U_{i,j+\Delta Y} \right) = \left( U_{i,j-\Delta Y} - U_{i,j+\Delta Y} \right) / (\Delta Y / 2) \\
- \left( U_{i,j} - U_{i+\Delta X,j} \right) / (\Delta Y / 2) + \Delta Y T_{i,j} + \Delta Y N C_{i,j}
\end{align*}
\]

\[
\begin{align*}
\Delta Y Pr dT_{i,j} / dt + \Delta Y Pr U_{i,j} \left( T_{i,j+\Delta X,j} - T_{i,j-\Delta X,j} \right) / \Delta X \\
+ Pr V_{i,j} \left( T_{i,j+\Delta Y} - T_{i,j-\Delta Y} \right) = \left( T_{i,j+\Delta Y} - T_{i,j-\Delta Y} \right) / (\Delta Y / 2) \\
- \left( T_{i,j} - T_{i+\Delta X,j} \right) / (\Delta Y / 2) + \Delta Y Pr \Delta T_{i,j}
\end{align*}
\]

\[
\begin{align*}
\Delta Y dC_{i,j} / dt + \Delta Y U_{i,j} \left( C_{i,j+\Delta X,j} - C_{i,j-\Delta X,j} \right) / \Delta X \\
+ V_{i,j} \left( C_{i,j+\Delta Y} - C_{i,j-\Delta Y} \right) = \\
\left( C_{i,j+\Delta Y} - C_{i,j-\Delta Y} \right) / (Sc \Delta Y / 2) \\
- \left( C_{i,j} - C_{i,j+\Delta Y} \right) / (Sc \Delta Y / 2) - \lambda C_{i,j}
\end{align*}
\]

Figure 1a: Grid model of volume control-(momentum equation)
The finite difference scheme corresponding to equation (10) is

\[ V_{i,j} = \left( U_{i-\Delta X,j} - U_{i+\Delta X,j} \right) \Delta Y / (2 \Delta X) \]

\[ + U_{i,j} \Delta Y / (i\Delta X) - V_{i,j-\Delta Y} \]  \hspace{1cm} (19)

By implementing Kirchhoff’s law, the above Equations (22) – (24) can be expressed as

\[ j_{U,i,j+\Delta Y} - j_{U,i,j-\Delta Y} - j_{UT,i,j} - j_{UC,i,j} \]

\[ + j_{UM,i,j} + j_{UX,i,j} + j_{UY,i,j} + j_{UT,i,j} = 0 \]  \hspace{1cm} (20)

\[ j_{T,i,j+\Delta Y} - j_{T,i,j-\Delta Y} + j_{T_{i,j}} + j_{YT,i,j} \]

\[ + j_{TY,i,j} + j_{T_{i,j}} = 0 \]  \hspace{1cm} (21)

\[ j_{C,i,j+\Delta Y} - j_{C,i,j-\Delta Y} + j_{C_{i,j}} + j_{CY,i,j} \]

\[ + j_{CY,i,j} + j_{C_{i,j}} = 0 \]  \hspace{1cm} (22)

In order to achieve temperature, speed and concentration, the terminal conditions (when X = 0 and Y→) are used for the grounded unit. The constant voltage and constant current at Y = 0 are used to reproduce uniform wall temperature and uneven wall concentration, respectively.
Finally, in the three capacitors, $C_{ij}$, $T_{ij}$, $C_{ij}$ for $t$, the initial condition of the voltage parameter $U = T = C = 0$.

4. Result and Discussion

In order to prove the validity of the current results, the obtained experimental values are compared with the current results of Chamkha [8], and the mutual compatibility is very good. In particular, the numerical solution of local skin friction $\tau_X$ with different Nusselt values $Nu_X$ and Prandtl $Pr$ values in Table 1 was compared with the results of Chamkha [8].

Table 1: Comparison of locally stable skin friction conditions and local Nusselt value with Chamkha [8] value (for full cone) when $X = 1.0$, for $Pr$ when $n = 0$, $M = 0$, $N = 0$ and $R_d = 0$ for different values.

| Pr  | Chamkha (2001) $f'(\infty,0)$ | Present values $\frac{\tau_X}{Gr^3}$ | Chamkha (2001) $-\theta'(\infty,0)$ | Present values $\frac{Nu_X}{Gr^3}$ |
|-----|-------------------------------|---------------------------------------|-----------------------------------|----------------------------------|
| 0.001 | 1.5135 | 1.4149 | 0.0245 | 0.0294 |
| 0.01 | 1.3549 | 1.3356 | 0.0751 | 0.0797 |
| 0.1 | 1.0962 | 1.0911 | 0.2116 | 0.2115 |
| 1 | 0.7697 | 0.7688 | 0.5111 | 0.5125 |
| 10 | 0.4877 | 0.4856 | 1.0342 | 1.0356 |
| 100 | 0.2895 | 0.2879 | 1.9320 | 1.9316 |
| 1000 | 0.1661 | 0.1637 | 3.4700 | 3.5186 |

Figure 2a shows the velocity curves of different values of Prandtl $Pr$ and heat generation/absorption parameters $\Delta$. The negative values correspond to heat absorption and the presence of heat generation represented by positive values of $\Delta$. It can be seen from Figure 2a that when heat is generated, the buoyancy increases, which leads to an increase in the flow velocity to the velocity distribution, and its volume increases, but the momentum boundary layer becomes thinner due to the rise in $Pr$. It can be seen from Figure 2b that the boundary layer thickness and velocity are reduced to a higher $Sc$ and $\lambda$. Figure 2c depicts that velocity raises steadily with time and attain a temporal maximum and consequently it attains the steady state. However, time taken to attain the steady state based on buoyancy ratio parameter $N$. An raise in $N$ leads to an raise in the velocity near the cone surface. It can be seen from Figure 2d that the velocity of the entire boundary layer has increased to a lower value of $n$.  


**Figure 2a**: Different values of Pr and $\Delta$, the transient velocity curve when $X = 1.0$.

**Figure 2b**: Different values of Sc and $\lambda$, the transient velocity curve when $X = 1.0$.

**Figure 2c**: Different values of $N$, the transient velocity profile at $X = 1.0$.

**Figure 2d**: Different values of $n$, the transient velocity profile at $X = 1.0$. 

---

*steady state value

- $Pr = 0.71$
- $\Delta = 1.0$
- $N = 1.0$
- $n = 0.25$
- $Sc = 0.6$
- $\lambda = 1.0$

| $Pr$ | $\Delta$ | $N$ | $Sc$ | $\lambda$ | $n$ |
|------|----------|-----|------|-----------|-----|
| 0.71 | 1.0      | 1.0 | 0.6  | 1.0       | 0.25|

---

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---

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---

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---

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---

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| $Pr$ | $\Delta$ | $N$ | $Sc$ | $\lambda$ | $n$ |
|------|----------|-----|------|-----------|-----|
| 0.71 | 1.0      | 1.0 | 0.6  | 1.0       | 0.25|
**Figure 3a**: For different values of Pr & Δ, the transient temperature curve

**Figure 3b**: For different values of Sc and λ, the transient temperature curve

**Figure 3c**: For different values of N, the transient temperature curve at X = 1.0

**Figure 3d**: For different values of n, the transient temperature curve at X = 1.0
Figure 4a: For different Pr and ∆ values, the concentration distribution graph at X = 1.0.

Figure 4b: For different values of Sc and λ, the transient concentration curve at X = 1.0.

Figure 4c: Transient concentration profiles at X = 1.0 for different values for N.

Figure 4d: For different values of n, the transient concentration curve at X = 1.0.
Figure 5a: Local skin friction with different Pr and $\Delta$ values during the transition period

Figure 5b: Local Skin Friction with different values of Sc and $\lambda$ during transient period

Figure 5c: Local Skin Friction with different values of $N$ during transient period

Figure 5d: Local Skin Friction with different values of $n$ during transient period
Figure 6a: Local Nusselt Number with various values of \( Pr \) & \( \Delta \) during transient period

Figure 6b: Local Nusselt Number with various values of \( Sc \) & \( \lambda \) in transient period

Figure 6c: Local Nusselt Number with various values of \( N \) during transient period

Figure 6d: Local Nusselt Number with various values of \( n \) during transient period
Figure 7a: Local Sherwood Number for various values of Pr & Δ in transient period

Figure 7b: Local Sherwood Number for various values of Sc & λ in transient period

Figure 7c: Local Sherwood Number for various values of N in transient period

Figure 7d: Local Sherwood Number for various values of $n$ in transient period
It's noted from Figure 3a that the value of the temperature increase is greater or less relative to the higher value of Pr and $\Delta$, the thermal boundary layer becomes thinner. Figure 3b reveals that temperature raises for higher values of Sc and lower values of $\lambda$ and thermal boundary layer thickness reduced for lower values of Sc and $\lambda$. From Figure 3c noticed that as we move far away from the cone surface, the temperature reduces for all the values of $N$, thus for larger value of $N$ the fluid cools rapidly. From Figure 3d one can observe that the temperature is maximised for lower values of $n$.

From Figure 4a it is seen that the concentration reduces and raise in $\Delta$ and Pr increased the time required to attain the steady state. Further concentration boundary layer thickness reduced for larger values of Pr and $\Delta$. Figure 4b depicts that concentration reduces for small values of $\lambda$ and larger values Schmidt number Sc. This reduces the effect of focal buoyancy. Figure 4c one can observe that concentration field reduces with higher values of Figure 4d shows that concentration field of the species raises for lower values of $n$.

Figure 5a indicates local Skin friction raises for higher values of Pr. Figure 5b shows the local skin friction behaviour for different values $\lambda$ of and Sc. Figure 5c shows that an increase in N is accompanied by an increase in auxiliary buoyancy. It greatly accelerates the flow and enhances the shear stress. As N increases, the time required to reach steady state decreases.

Figure 6a shows that when the value of Pr is lower and higher, the local Nusselt number increases; that is, the magnitude of the local Nusselt number increases with the heating/absorption coefficient $\Delta$. Evident from from Fig. 6b that as Sc increases, the local Nusselt number keeps decreasing. Inspection of Figure 6c shows that an increase in $N$ strongly boosts Nusselt number $Nu_x$, that is, it enhances the heat transfer gradient at the cone surface. Figure 6d illustrate that when the value of $n$ is low, the local Nusselt number.

It can be seen from Figure 7a above that the local Sherwood number is increased to a value higher $\Delta$ and a value lower than Pr. Figure 7b indicates that as $\lambda$ and Sc increase, the gradient of surface species (ie, the mass transfer rate on the surface of the cone) is very high. Figure 7c shows that the increase of N greatly enhances the Sherwood number. In other words, it enhances the gradient of the quality of the cone surface. Moreover, the number of Sherwood increases as N increases. The physical reason is that for liquids with a lower Prandtl number (Pr = 0.71), the positive force produces obvious overflow near the surface in the boundary layer, but for liquids with a higher Prandtl number (Pr = 6.7), the increase Speed is not obvious. At the same time, the time required to reach the steady state decreases as the value of $N$ increases. Figure 7d shows that for higher values of $n$, the local Sherwood number increases. For brevity, the graphical results of average skin contact, Nusselt number and Sherwood number are not included.

5. Conclusion

From the uneven wall temperature, the vertical cone with uniform wall concentration, combined with the chemical reaction coefficient and the comprehensive effect of heat generation/absorption, a mathematical model of natural convection is established. Parametric studies were carried out to clarify the influence of thermo physical factors on temperature, velocity and concentration. Have been observed

1. The time require to attain steady state raises with higher values of $\Delta$, Pr, $\lambda$, Sc, $N$ and $n$. 
2. The fluid velocity boosts for larger values of $\Delta$, $N$ and smaller values of $Pr$, $\lambda$, $Sc$ and $n$.

3. Temperature raises for higher values of $\Delta$, $Sc$ and lower values of $Pr$, $\lambda$, $N$.

4. Concentration of species decreases for lower values of $\lambda$, $n$ and higher values of $\Delta$, $Pr$, $Sc$, $N$.

5. The local skin friction raises for larger values of $Pr$, $\Delta$, $N$ and for smaller values of $\lambda$, $Sc$, $m$.

6. The local Nusselt number increases for higher values of $Pr$, $N$ and lower values of $\lambda$, $Sc$, $\Delta$.

7. The local Sherwood number increases for higher values of $\lambda$, $Sc$, $\Delta$, $n$, $N$ and lower values of $Pr$.

8. Momentum boundary layers become thick for larger values of $Sc$, $\Delta$, $\lambda$, $N$ and smaller values of $Pr$.

9. Thermal boundary layer becomes thick for larger values of $Sc$, $\Delta$ and smaller values of $Pr$, $\lambda$, $N$.

10. Concentration boundary layer becomes thick for higher values of $\lambda$, $n$ and smaller values of $\Delta$, $Pr$, $Sc$, $N$.

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