Simultaneous optimisation of support structure regions and part topology for additive manufacturing

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Abstract
Support structures are required to enable the build of additively manufactured parts. The supports reinforce overhanging regions on the part and/or counteract the thermally-induced residual stresses generated during printing. However, the optimal design of the part for its intended use case is decoupled from the design of the support structures in a conventional design for additive manufacturing (DfAM) workflow. In this work, a novel methodology is presented that simultaneously optimises the part topology and its support structure regions. A two-model topology optimisation approach is considered. One model describes the combined part and support structure regions subject to a pseudo-gravity load and a second model describes the part subject to its intended application load cases. A novel load-aligned trunk and branch support structure is generated from the topology optimisation results. Generating the fine support features in a post-processing step avoids the computational expense of topology optimising the intricate supports directly. Thermo-mechanical simulations of a selective laser melting process confirms that this new approach to optimising support structures can reduce manufacturing process-induced deformation when benchmarked against a conventional DfAM workflow.

Keywords Topology optimisation · Additive manufacturing · Support structures · Lattice

1 Introduction

Topology optimisation (TO) is a powerful design tool for the realisation of complex, lightweight, high performance, structures (Eschenauer and Olhoff 2001; Jihong, et al. 2021). There are numerous industry applications for TO structures including aerospace (Zhu et al. 2016; Berrocal, et al. 2019), automotive (Wang et al. 2004) and medical (Al-Tamimi et al. 2020). Additive manufacturing (AM) can realise such geometrically complex designs with ease in comparison to traditional manufacturing processes such as casting (Wang et al. 2019). Lightweight design is critical for such applications, not just for product performance, but also to reduce fabrication costs, which are a function of the quantity of material used (Gisario, et al. 2019). These reasons make TO and AM well suited technologies to one another in Design for Additive Manufacturing (DfAM) (Panesar, et al. 2018). Even though AM enables great design freedom, there are still manufacturing process constraints that need to be observed. For example, several AM processes, such as Selective Laser Melting (SLM), usually require additional supporting structures to be added to the part. Manufacturing constraints can impact on the manufacturing feasibility and final performance of the design, so it is critical consider such constraints in the design of both the part and support structures (Yang and Zhao 2015).

An important consideration in the design of support structures is the build orientation of the part (Allaire et al. 2020). There are various strategies to determine the optimal orientation. These include minimising the support structure volume to reduce material costs or to avoid the need for supports entirely (Wang and Qian 2020). Finding the orientation that minimises the height of the build is also another valid approach as this impacts on the time taken to print the part (Matos et al. 2020). Another consideration is how to orientate the part to reduce tolerance errors and the perception
of build quality (Zhang et al. 2015). Reducing the cross-sectional area of any given layer can also be an important parameter in avoiding print job failures in manufacturing processes such as SLM that work by melting successive layers of metal powder. In this process, the upper surface tends to warp upwards due to the thermal residual stresses, which can result in collisions between the part and the powder recoat blade (Mugwagwa et al. 2018; Song et al. 2020).

Since support structures need to be removed after printing, it is necessary to make their attachment points comparatively weaker than the part that they support. The ease of removal of support structures will influence the total manufacturing cost as they frequently need to be removed manually. This poses a difficult design trade-off between strength during printing and ease of removal afterwards (Jiang et al. 2018). Perhaps the simplest support design, and the easiest to automate, is a repeated infill approach using columns (Mirzendehdel and Suresh 2016). Tree-like supports are more complex, but they are reported to reduce the required support volume in comparison to column supports (Vanek et al. 2014; Zhang et al. 2020). Thin wall/shell and lattice designs, based on a tessellated infill approach, have also been used as supports (Cheng et al. 2019). More structurally efficient lattice designs include those that have lattice members aligned with principal load bearing directions (Wu et al. 2019; Daynes and Feih 2022).

There are several AM process design rules that should be observed and they will depend on the specifics of a given printer. A common rule to observe with SLM is to limit overhanging features to angles greater than approximately 45° (Barroqueiro et al. 2019; Mass and Amir 2017; Wang et al. 2020). Overhangs can result in manufacturing imperfections or print job failure (Luo et al. 2020). In addition, every AM process will have a minimum print resolution, which should be kept in mind during TO to avoid generating unfeasibly fine features. A common approach to avoid minimum feature size issues with the Solid Isotropic Material with Penalization (SIMP) TO method is through a selection of an appropriate finite element (FE) mesh size and filtering (Sigmund 2001).

Rather than adopting a tessellated infill design approach to generating supports, advanced concepts are also emerging from the research community for the support structures to be generated via TO (Mezzadri et al. 2018). Some of these methods enable TO concurrently with part orientation, accounting for both structural performance and manufacturing constraints (Fritz and Kim 2020). Supporting structures can also be optimised with respect to thermal compliance to minimise mean temperature (Giraldo-Londoño et al. 2020) or for maximising heat dissipation (Miki and Nishiwaki 2022). Porosity constraints can be introduced between the part and topology optimised supports to promote ease of removal (Zhou et al. 2020). Simultaneous optimisation of the shape and the supports based on an objective function that considers both part and support compliance has also been investigated (Allaire and Boge 2018). Support compliance can be described using a pseudo-gravity vector applied on the supported part and/or at various stages of layered construction (Amir and Amir 2021). Since the part and the support structures often have different relative material properties, either through direct use of two materials or one material of varying porosity, many of these part-support TO strategies adopt a multi-material optimisation approach (Wang and Wang 2004; Bruggi and Talerico 2020).

In this work a new TO methodology to simultaneously optimise the part and its supports is presented by considering two model configurations, representing the build process and the parts final use case. During TO, support regions are modelled as continuum solid volumes and optimised based on minimising the compliance of the part subject to pseudo-gravity, implicitly reducing overhanging regions. A novel method for generating trunk and branch support structures aligned with principal load paths is introduced after TO. It is shown using thermomechanical simulations of the SLM build process that thermally induced deformations can be reduced using the pseudo-gravity approach. A key advantage of this work is that the intricate support structure features (~0.5 mm) are generated after the bulk support structure volumes have been determined through TO. Therefore, the TO mesh only needs to be fine enough for the minimum feature size of the part itself (~3 mm), reducing computational effort.

The structure of the paper is as follows. The novel simultaneous optimisation approach is presented in Sect. 2, followed by details of the support structure generation process in Sect. 3. Section 4 details how this new optimisation approach is benchmarked against a conventional DFAM workflow and subsequent results and discussion are provided in Sect. 5. All sections are presented in terms of an industry application of an aircraft engine bracket, which has a complex 3D geometry and multiple load cases. Finally, conclusions are given in Sect. 6.

## 2 Simultaneous optimisation approach

To highlight the advantage of simultaneously optimising the support structures and part topology, the optimisation approach described in the paper is presented in the context of an industry application. The part selected is an additively manufactured bracket intended to be mounted onto an aircraft engine. The bracket needs to be stiff, lightweight and feasible to additively manufacture (GrabCAD 2022).
2.1 Multiple models with common design variables

Consider a final part shape $\alpha$ for the bracket to be printed with its supporting structures $\beta$. Both $\alpha$ and $\beta$ are open sets of $\mathbb{R}^3$ and they can potentially be composed of different materials and/or lattice structures. Both the shape $\alpha$ and the supports $\beta$ are optimised simultaneously with respect to at least two states of static equilibrium:

1) Equilibrium state A: counteracting pseudo-gravity loads on $\alpha \cup \beta$ at the end of the build process.
2) Equilibrium state(s) B: the final (in-service) use case(s) for the shape $\alpha$.

Two FE models are required (Fig. 1) for the simultaneous TO of the two structural configurations, in a single optimisation run, with common design variables $x$ in $\mathbb{R}^3$. Configuration A represents the structure’s self-weight load case at the end of the printing process while support structures are still attached and the part is not yet removed from the build platform. The base plate has a clamped boundary condition $\Gamma_A$ assigned on a plane and the acceleration due to gravity vector $g$ is applied normal to this plane. $\Gamma_A$ is selected to be on one of the flat outer surfaces of the design space. Configuration B represents the part subject to its designed (in-service) load cases after the support structures are removed. The bracket is bolted to a rigid surface using four Ø5 mm bolts. The surfaces where the bolts would contact the part are assigned clamped boundary conditions $\Gamma_B$. There are three static load cases, each non-dimensional load is applied to model B as a distributed load over the inner surface of a Ø8 mm loading pin:

1) Horizontal load in $+X$ ($f = 1$)
2) Load inclined at 45° in $+X$, $-Y$ ($f = 1.3$)
3) Vertical load in $-Y$ ($f = 1.8$)

The non-design space, $\Omega_1$, is assigned around the loading points of Model B to ensure that the bolted and pinned connections remain circular during TO. The part’s shape design space, $\Omega_2$, is determined by the space constraints of the part’s operating environment, see Fig. 2. The bounding box external dimensions of $\Omega_2$ are $125 \times 85 \times 38$ mm. The final part shape is contained within domains 1 and 2: $\alpha \subseteq (\Omega_1 \cup \Omega_2)$. The support structures are contained within domains 2 and 3: $\beta \subseteq (\Omega_2 \cup \Omega_3)$. The design space for domain $\Omega_3$, which attaches to the printer’s base plate, is externally a quadrilateral prism encapsulating $\Omega_1 \cup \Omega_2$. An additional 1 mm thickness offset is added to domain $\Omega_3$ to ensure that the volume $\Omega_1 \cup \Omega_2$ does not directly contact the base plate of the printer. This is an important consideration when removing a printed part from the base plate in manufacturing processes such as SLM where a wire cutting operation if often required (Daynes et al. 2021).

2.2 Multi-material interpolation

With density-based TO approaches such as SIMP, the design variables determine the material properties. For single material TO, the SIMP method uses the following power-law to relate stiffness and relative density (Sigmund 2001):

Fig. 1 Simultaneous TO of two models with common design variables but different boundary conditions. Model A represent the printing configuration whereas model B represent the part’s in-service load cases. Stepped section views of the XY plane

Fig. 2 Model domains; $\Omega_1$ is the non-design space, $\Omega_2$ is the design space for the part shape and the support structures, whereas $\Omega_3$ is the design space for support structures exclusively
where \( x_1 \in [0,1] \) is the relative density bounded between a fully void and a fully solid state. The penalty factor is given by \( n \), \( E_1 \) is the Young’s modulus of the solid material and \( E \) is the interpolated Young’s modulus. Equation (1) can be extended to the optimisation of two materials (Li and Kim 2018; Ge and Kou 2021):

\[
\begin{align*}
E(x_1, x_2) &= (x_1 x_2)^{n} E_2 + x_1^{n}(1-x_2^{n}) E_1
\end{align*}
\]

where \( x_2 \in [0,1] \) represents which material the element belongs to, bounded between material 1 (\( x_2 = 0 \)) and material 2 (\( x_2 = 1 \)). The Young’s modulus of the second solid material is given by \( E_2 \). In this work a value of \( n = 4 \) is used to penalise the formation of intermediate design variable values. Intermediate values would be undesirable as this would be representative of a gradual transition between part and supports and make support structure removal challenging.

The non-dimensional material properties used in this study are provided in Table 1. Non-dimensional values are used since the optimisation objective function, presented in Sect. 2.5, is based on weighted compliance. Different material properties are used to represent the shape \( \alpha \) and the supports \( \beta \). For the supported configuration A the support stiffness \( E_{\beta} \) is \( 1/10^{th} \) of the solid material \( E_{\alpha} \) representing a support in-fill of 10% relative density. Direct generation of lattice-like fine features as part of larger structures using TO is computationally expensive (Aage, et al. 2017). In addition, the minimum feature size for supports (typically \( \approx 0.5 \) mm) (Daynes et al. 2021) is necessarily smaller than the 3 mm minimum feature size of the final part. Instead, the lattice is represented in \( \beta \) by a homogenised material with reduced stiffness compared to the build material \( E_{\alpha} \). The homogenised density of the support volume \( \rho_{\beta} \) is not included for the pseudo-gravity loading case since the support will be shown later in Sect. 3 to be primarily composed of self-supporting columns. Due to the column-like design of the support structures, the Poisson’s ratio of the supports, \( v_\beta \), is also set to 0. Although a column-dominated support structure in-fill will clearly not have bulk isotropic material properties, in this work it is argued that the loading in the supports due to gravity is predominantly unidirectional so it is not necessary to characterise the orthotropic stiffness matrix. In configuration B, the support structures will be removed, in practice, before the part is utilised. However, a non-zero value is still required for the supports \( E_{\beta} \) in configuration B to ensure the robustness of the optimisation. To reflect this in model B, the relative stiffness of the supports is set to a negligible value of \( 10^{-10} \) so that any supports generated within domain \( \Omega_2 \) have a negligible influence on the performance of the final part.

### 2.3 Configuration A: supported structure under gravity loading

During the SLM printing process, a laser melts the powder and the liquid metal pool will seep into the powder below, creating an irregular under surface in overhanging regions (Hong, et al. 2021). As the metal solidifies and cools, the thermally induced residual stresses tend to lift the part upwards, which can cause permanent deformation of the part and/or a collision with the recoat blade (Song, et al. 2020). Therefore, supports are required to keep such regions in their intended position. From an optimisation point of view, minimising the compliance of the supported part subject to self-weight is a computationally inexpensive analogy to describing such complexities in the build process (Song, et al. 2020). Inclusion of a gravity-like load will result in large deformations on structural regions that are not aligned with the build direction. In this way, reducing the self-weight compliance of the supported structure \( \alpha \cup \beta \) will implicitly suppresses the formation of overhang regions and reduce support structure requirements (Amir and Amir 2021). Using this approach there is no need to explicitly include an overhang angle constraint in the optimisation.

For configuration A, a build platform plane is selected and the gravity vector is aligned to be normal with this plane. The optimisation can be run multiple times to investigate different build orientations. It will be seen with the example in this paper that the number of intuitive build orientations to investigate is limited to four, to correspond to the four largest external surfaces on \( \Omega_3 \). The structure is built layer-by-layer in the opposite sense to the direction of the gravity vector. The build chamber containing \( \Omega_1 \cup \Omega_2 \cup \Omega_3 \) is typically a rectangular box for most AM processes. The baseplate \( \Gamma \) is situated on the lower surface of the build chamber. Gravity forces (self-weight) are applied to \( \alpha \) and the deformation of this supported structure, \( \mathbf{u}_A \), is determined by the equations of linear elasticity. The mechanical performance of the supported structure \( \alpha \cup \beta \) under self-weight can therefore be evaluated in terms of its structural compliance:

\[
\begin{align*}
c_A &= \sum_e \mathbf{u}_A^T \mathbf{K}_A(x_1, x_2) \mathbf{u}_A = \sum_e \mathbf{u}_A^T \mathbf{K}_A(x_1, x_2) v_A g
\end{align*}
\]

### Table 1 Non-dimensional material properties used for TO

| Model | \( E_\alpha \) | \( E_\beta \) | \( \rho_\alpha \) | \( \rho_\beta \) | \( v_\alpha \) | \( v_\beta \) |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|
| A     | 1           | \( 10^{-1} \) | 1           | 0           | 0.3         | 0           |
| B     | 1           | \( 10^{-10} \) | N/A         | N/A         | 0.3         | 0           |
where $K_A$, $\rho_A$ and $v_A$ are the global stiffness matrix, element densities and element volumes for model A, respectively. The element number is given by $e$. In Eq. (3) $g$ is the acceleration due to gravity vector that is aligned normal to the plane $\Gamma_A$.

### 2.4 Configuration B: structural use case

For the second state B, the shape $\alpha$ is considered for its final use case, which can be assumed independent of its support structures $\beta$ due to the low $10^{-10}$ value for $E_\beta$. The change in stiffness for $E_\beta$ and the omission of the domain $\Omega_3$ results in the global stiffness matrix for model A not being equal to model B; $K_A \neq K_B$. The deflections during the final load cases are given by $u_B$. The shape $\alpha$ is clamped on the boundary $\Gamma_\beta$ and load vector $f$ is applied for a given load case $i$. The remaining boundary is traction free. The mechanical performance of the final part $\alpha$ can similarly be evaluated in terms of its structural compliance. All load cases are assigned an equal compliance weighting.

$$c_B = \sum_i \sum_e u^T_B K_B(x_1, x_2) u_B = \sum_i \sum_e u^T_B f(x_1, x_2) \quad \text{(4)}$$

### 2.5 Multiple model optimisation

The global objective function to be minimised is the sum of the weighted compliance contributions from Eqs. (3) and (4)

$$\min_{x_1, x_2} c_{AB}(x_1, x_2) = \frac{1}{2} (w_A c_A + w_B c_B)$$

s.t. :

$$\begin{align*}
g_1(x_1, x_2) &= \sum_{e \in (\Omega_1 \cup \Omega_2)} v = 0 \\
g_2(x_1, x_2) &= \sum_{e \in \Omega_1} v - V_\alpha \leq 0 \\
g_3(x_1, x_2) &= \sum_{e \in \Omega_2} v - V_\beta \leq 0 \\
x_1 &= x_2 = 1, e \in \Omega_1 \\
0 &\leq x_1 \leq 1, e \in (\Omega_2 \cap \Omega_1) \\
0 &\leq x_2 \leq 1, e \in (\Omega_2 \cap \Omega_3) \\
\end{align*} \quad \text{(5)}$$

with :

$$\begin{align*}
K_A(x_1, x_2) u_A &= \rho_A(x_1, x_2) V_A g \\
K_B(x_1, x_2) u_B &= f(x_1, x_2)
\end{align*}$$

It should be noted that more general objective functions to replace (5) could also be implemented with this method. The constraint $g_1$ is required to prevent the final shape $\alpha$ from occupying the $\Omega_2$ domain and therefore exceeding its allowable final design space once the support structures are removed. For constraints $g_2$ and $g_3$, the allowable volumes $V_\alpha$ and $V_\beta$ are placed on the domains $\alpha$ and $\beta$, respectively. In this paper, both of these volume constraints are made equal to 40% of the initial shape volume $\Omega_1 \cup \Omega_2$:

$$V_\alpha = V_\beta = 0.4 \sum_{e \in (\Omega_1 \cup \Omega_2)} v \quad \text{(6)}$$

Of course, different values for $V_\alpha$ and $V_\beta$ could be considered, but it is found that keeping the two volumes similar works well for the bracket example investigated in this paper. Similar values for $V_\alpha$ and $V_\beta$ ensures that there is a similar amount of support structure volume $\beta$ to support the shape $\alpha$.

The terms $w_A$ and $w_B$ in Eq. (5) are the compliance weighting coefficients for Models A and B, respectively. Different compliance weighting coefficients could be considered to investigate how the design changes with the relative compliance weightings between $c_A$ and $c_B$. However, the magnitudes of the two terms $w_A$ and $w_B$ are selected in this paper so that the compliance contributions from models A and B are approximately equal. This is to ensure that the compliance contribution from one of the models does not dominate the optimised topology. The compliance weighting for model A is determined prior to TO by running a preliminary linear-static analysis on model A with $x_1 = 1$ and $x_2 = 0$:

$$w_A = V_\alpha / \left( V_\alpha \sum_e u^T_A K_A(1, 0) u_A \right) \quad \text{(7)}$$

The term $K_A(1, 0)$ represents the stiffness of a fully dense Model A comprising of only material 1 prior to optimisation. The volume of model A is represented by $V_A$ and, similarly, the volume of model B is represented by $V_B$.

$$V_A = \sum_{e \in (\Omega_1 \cup \Omega_2 \cup \Omega_3)} v \quad \text{(8)}$$

$$V_B = \sum_{e \in \Omega_2} v \quad \text{(9)}$$

The fraction $V_A'/V_A$ in Eq. (7) results in the compliance being scaled by the target volume fraction so that the final compliance will be approximately equal to unity after TO, see Fig. 3. Similarly, the compliance weighting for model B is found prior to TO by solving model B independently with $x_1 = 1$ and $x_2 = 0$. The fraction $V_B'/V_B = 0.4$ scales the compliance contribution of model B to also be approximately equal to unity after TO.

$$w_B = V_B / \left( V_B \sum_e u^T_B K_B(1, 0) u_B \right) \quad \text{(10)}$$

The solution of the optimisation problem in Eq. (5) is found using the Method of Feasible Directions (Chen and Kostreva 2000). This gradient-based approach requires sensitivity analysis using a first-order derivative calculation of the objective function. The derivatives of the objective function are given by (Li and Kim 2018):
where \( j = \{1, 2\} \) represents the design variable. Altair OptiStruct 2021 is used as the TO solver. The optimisation terminates when the change in the objective function between two successive iterations is less than 0.2%. All domains are meshed with first-order tetrahedral elements and a target edge length of 1 mm. To ensure that the finer details of the optimised designs are feasible to print, a minimum allowable feature size of 3 mm is implemented during TO, which has built-in checkerboard control (Zhou et al., 2001). Each of the two models consisted of approximately 594,000 nodes and 3,436,000 elements, with 178,200 degrees of freedom. Typically, the multi-model TO procedure converged after 52–56 iterations, which equated to a run time of approximately 5 h using an Intel Xeon CPU E5-2650 v4 @ 2.20 GHz and 7 cores.

### 2.6 Post-processing of the optimised shape

At the post-processing stage after TO, the iso-surfaces corresponding to \( x_1 = x_2 = 0.5 \) are extracted to determine the topology of the final shape \((\alpha)\) and support volume \((\beta)\) using the postprocessor Altair HyperView 2021. At this stage, the geometries can be extracted as triangulated surfaces in .stl file format. However, the triangulated surfaces remain rough and the geometry is imprecise around the dimensioned bolt holes in the non-design space, see Fig. 4a. For these reasons, a triangulated mesh of the final shape \((\alpha)\) with the threshold of \( x_1 = x_2 = 0.5 \) applied is imported into Altair Inspire 2021, where a Non-Uniform Rational B-Spline (NURBS) surface (Piegl 1989) can be automatically fit to the mesh, see Fig. 4b. The final step in post-processing of final shape \((\alpha)\) is to re-import the original non-design space CAD geometry, merging with the NURBS solid, to ensure that the bolted areas are accurately dimensioned, see Fig. 4c. An added advantage of this post-processing strategy is that the final shape can be exported as a modifiable CAD file format, such as .iges or .step. The resultant TO shapes and support structure volumes are shown in Fig. 5 for four different orientations. The support structure volume \((\beta)\) remains as a triangulated mesh at this stage; the detailed generation of support structures is addressed in Sect. 3.

### 3 Support structure generation

For multi-material AM processes, such as PolyJet, a secondary support material is used, which can be melted away from the final shape \((\alpha)\) after printing (Liu et al. 2020). In such cases, the supporting volumes \((\beta)\) shown in Fig. 5 could be directly interpreted as this secondary, lower stiffness, material. However, for AM processes such as SLM, there is no secondary material available so the supporting ‘volumes’ need to be comprised of lattice-like structures with reduced relative density (Gan and Wong 2016). This section describes the methodology whereby the homogenous supporting volume \((\beta)\) is re-interpreted as a structurally efficient lattice structure suited to the SLM process.
3.1 Maximum absolute principal stress field

The post-processor Altair HyperView 2021 is used to extract the stress tensor $\sigma$ and coordinates for each element in Model A after TO. This stress field data is imported into Matlab where the three principal stress orientations at each node in the domain $\beta$ are determined from the eigenvectors of $\sigma$. Of the three eigenvectors calculated at each node, the only vector of interest is the one that corresponds to the maximum absolute principal stress; $\max(|\sigma_1|,|\sigma_2|,|\sigma_3|)$. This is because Model A is subject to pseudo-gravity loading so this eigenvector of interest $k$ will be aligned to counteract this loading. A typical alignment of the maximum absolute principal stress vectors is shown in Fig. 6.

3.2 Find overhanging regions

Before generating the lattice supports it is necessary to identify the overhang regions where the supports will be attached to. The overhang regions are described using the points $p$, which form a subset of the vertices of the triangulated surface definition for $\beta$. These vertices on the surface of $\beta$ must satisfy three conditions to be included in the subset of points $p$. First, the point must lie on the surface of both domain $\alpha$ and $\beta$; $p \subset (\alpha \cap \beta)$. Second, the domain directly above point $p$ in the build direction must be in domain $\alpha$; $(p - \Delta g) \subset \alpha$. Third, the domain directly below point $p$ in the build direction must be in domain $\beta$; $(p + \Delta g) \subset \beta$. The points that satisfy these criteria for one of the optimised designs are shown in Fig. 7.
3.3 Determine solid-lattice overhang connection points

To ensure that the complex 3D geometries are accurately described, the triangulated surface definitions for the \( \alpha \) and \( \beta \) domains have a fine 1 mm mesh size. However, it would be impractical to generate a lattice structure whose trusses are spaced apart by 1 mm, since this is typically approaching the minimum pore size for removing loose powder after build for commercial SLM processes (Daynes et al. 2021). Instead, the points \( p \) are used to generate an agglomerative hierarchical cluster tree of points that are less densely distributed but still describe the overhang regions. The algorithm for computing the Euclidean distance between clusters is based on the Unweighted Pair Grouping Method with Arithmetic-mean (UPGMA) (Gronau and Moran 2007). The cluster tree reflects the structure present in a pairwise similarity matrix. The dissimilarity between clusters is defined as their average dissimilarly using the following reduction formula:

\[
D(C_k, C_i \cup C_j) \leftarrow \frac{|C_i|}{|C_i| + |C_j|} D(C_k, C_i) + \frac{|C_j|}{|C_i| + |C_j|} D(C_k, C_j)
\]

(12)

where \( D \) is the dissimilarity (i.e. distance) matrix and \( C_i \) and \( C_j \) are existing cluster sets, each of size (i.e. cardinality) \(|C_i|\) and \(|C_j|\), respectively. The new cluster set is given by \( C_k \). An example of the distribution on points \( p \) on a semi-ordered triangulated surface mesh, with an average nodal spacing of 1 mm, is shown in Fig. 8a. The UPGMA algorithm is applied to these points \( p \) with a cut-off distance of 2.5 mm. The resulting points \( q \) represent the mean value of these clusters, as shown in Fig. 8b. The newly formed points \( q \) form the ‘branches’ for the supporting lattice structure in Sect. 3.4. The UPGMA algorithm is implemented for a second time on the points \( q \) and with a cut-off distance of 5.0 mm. The resulting average points of these secondary clusters are the points \( r \), as shown in Fig. 8c. The points \( r \) form the ‘trunks’ for the supporting lattice structure in Sect. 3.4. Applying these cut-offs to a semi-ordered triangulated mesh, on average, results in 10 mesh nodes \( p \) for each branch \( q \) and 4 branches \( q \) for each trunk \( r \).

3.4 Generate stress trajectory-aligned trunks and branches

The next step is to trace stress trajectories emanating from the overhanging points \( r \) to form the ‘trunks’ of the lattice structure. This is done using a fourth-order Runge–Kutta approach (Kelly, et al. 2011).
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\[
\begin{align*}
\Delta k_{i+1} &= k_i + \frac{s}{6}(\Delta k_1 + 2\Delta k_2 + 2\Delta k_3 + \Delta k_4) \\
t_{i+1} &= t_i + s \\
\Delta k_1 &= f(t_i, k_i) \\
\Delta k_2 &= f(t_i + \frac{s}{2}, k_i + \frac{s}{2} \Delta k_1) \\
\Delta k_3 &= f(t_i + \frac{s}{2}, k_i + \frac{s}{2} \Delta k_2) \\
\Delta k_4 &= f(t_i + s, k_i + s \Delta k_3)
\end{align*}
\] (13)

In Eq. (13), \(k\) is an unknown function of time \(t\) travelled along the stress trajectory. At the initial time \(t_0\) the corresponding value of \(k_0 = r\) is provided by the clustering algorithm (Fig. 8c). A visual representation of the gradients used by the Runge–Kutta method is shown in Fig. 9. In Eq. (13) the step size \(s\) corresponds to the support structure element size, the determination of which will be discussed in Sect. 3.5. The trunks are traced for each overhang point \(r\) and terminate when the stress trajectory exceeds the boundary of the domain \(\beta\).

The next step is to add the ‘branches’, see Fig. 10. There are two situations that influence how the branches are generated: For trunks greater than or equal to 10 mm in length (i.e. twice the secondary cluster size), the top 10 mm of the trunk is removed and replaced with branches connecting the corresponding cluster points \(q\) to the corresponding trunk, see Fig. 10b–c. Otherwise, for trunks shorter than 10 mm, these trunks are completely removed and Eq. (13) is re-run but replacing the trunk starting point \(r\) with the branch starting points \(q\) in the corresponding cluster, see Fig. 10d–e. To ensure contact is achieved between the supports structures and the part and baseplate, all trunks and branches are then extended by a distance of \(s/2\).

The motivation for this trunk and branch approach is to ensure that the gravity loads are efficiently transferred into the load-bearing aligned trunks. However, these trunks are comparatively sparsely distributed and so the branches serve to better support the overhanging surfaces with more densely distributed contact points. A secondary motivation with this approach is to have mainly slender branches contacting with the part, which will be easier to remove after printing.

3.5 Generate solid element support structures

In Sect. 3.4 the trunk and branch support structures are described as one-dimensional trajectories. It is necessary to describe the cross-sections of these supports, the orientations of these cross-sections and the geometry details connecting the ends of the supports to the part and baseplate. All support structures are assigned rectangular cross-sections with a thickness of 0.5 mm. A thickness of 0.5 mm is selected since it is approaching the print resolution of many commercially available SLM processes (Daynes et al. 2021). The cross-sectional width, \(s\), is determined by noting that there are, on average, four support structures per cluster, \(r\), in Fig. 8 and the target cluster distribution, \(l\), is 5 mm. By making the approximation that the support structures form a square grid, see Fig. 11, and that the target volume fraction of the support structures \((E_r/E_a)\) is 0.1 for model A, the support structure width \(s\) is calculated to be 2.75 mm.

\[s = \frac{E_r F + r^2}{4l} \] (14)

Note that the width \(s\) is intentionally the same quantity as the Runge–Kutta step size in Eq. (13) so that the generated finite elements do not have severely distorted aspect ratios. The uniform support structure infill shown in Fig. 11 is also used to benchmark the optimisation methodology presented in this work against a conventional DfAM workflow with a conventional support structure design.

To correctly align the cross-section to a given one-dimensional element, a normal vector is required, see Fig. 12a. Finding the normal vector can be determined as the cross-product of the line segment orientation and the relative positioning of the respective cluster points \((r - q)\). The trunk and branch line segments can then be exported from Matlab in either 2D shell or 3D solid FE meshes or triangulated .stl file format. The .stl mesh version of the trunk and branch segments is shown in Fig. 12b. Note
that triangular ends of length $s$ are placed on the final segment, see Fig. 12c. This reduction in cross-sectional area is intended to facilitate ease of removal of the support structures from the part and base plate after printing. When exporting the supports from Matlab for further FE analysis, a node-to-surface frozen contact is imposed between the surface of the optimised part and the nodes of the supports. Only nodes within a search distance of 1 mm from the part surface are included in the contact.

4 Benchmarking against a conventional DfAM workflow

The optimised supports for the four orientations under investigation are shown in Fig. 13a-d. For benchmarking purposes, the optimisation methodology is compared with a conventional DfAM approach, see Fig. 13e. For benchmarking, Model B is run independently in Altair...
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OptiStruct 2021 with the single objective to minimise the compliance $c_B$ given by Eq. (4). Therefore, no consideration is given to support structures or part orientation during TO of the benchmarking model. Parameters such as volume constraints, material properties, mesh size and minimum feature size are kept constant for all models during TO. In a second, decoupled, step the optimised benchmark geometry from TO is imported into Altair Inspire Print3D 2021 (Altair 2022). This software can optimise part orientation and generate support structures intended for the SLM printing process. The part is orientated to minimise part deformation and support structures are assigned to overhanging regions. Large variations in the surface areas of horizontal slices can induce large thermally-induced stresses (Allaire et al. 2020). It is the objective of Print3D to find the orientation for which the horizontal slices have the smallest variation. A column-type support structure infill is generated whose unit cell cross-sectional design and dimensions are identical to those in Fig. 11. The column-type supports were selected as the benchmark since both the cross-sectional geometry of the supports and their relative spacing were consistent with the Matlab parameters used to generate the optimised supports.

Print3D is then used for time-dependent thermo-mechanical simulation of the SLM printing process so that the benchmark approach can be compared with the optimised supports and part topologies generated in this work. The build process is modelled as a layer-by-layer simulation, followed by a cool-down step after the building process is completed. This enables deformations induced by the printing process to be evaluated. The models are voxel-based with a voxel size of 0.5 mm, set to match the minimum feature size of the lattice structures. For all simulations, the target printer was the EOS M290 SLM using Ti6Al4V powder (Daynes et al. 2021). This titanium alloy has a Young’s modulus of 117 GPa, Poisson’s ratio of 0.3, yield strength of 827 MPa, thermal expansion coefficient of $8.8 \times 10^{-6} / \text{K}$ and a thermal conductivity of 6.7 W/m/K. The printing process parameters are provided in Table 2.

5 Results and discussion

5.1 Mechanical performance

The resulting compliance values for the various optimisation approaches are shown in Fig. 14. The compliance components are normalised with respect to the benchmark DfAM workflow. There are two compliance components, $\tilde{c}_A$ and $\tilde{c}_B$, related to the self-weight compliance and the operational compliance, respectively. Note that the compliance components $c_A$ and $c_B$ from Eq. (5) are taken from TO and before post-processing whereas the compliance components $\tilde{c}_A$ and $\tilde{c}_B$ are extracted from the FE models shown in Fig. 13. By this stage, the various post-processing steps, such as NURBS fitting and lattice generation, presented in Sects. 2 and 3 will have some influence on the final compliance values.

As expected, the benchmark design has the lowest value for part compliance $\tilde{c}_B$ since it has the single objective function to minimise $c_B$. In terms of self-weight compliance, $\tilde{c}_A$, it is Design (d) that that performs the best and the benchmark approach performs the worst. The smallest values of $\tilde{c}_A$ for Designs (b) and (d) are expected since both of these designs have large flat surfaces in close proximity to the base plate. However, a potential disadvantage
of Design (d) is the asymmetry of the design about the $Z = 0$ plane, which results in some out-of-plane deformation when the loads $f$ are applied.

The last set of columns in Fig. 14 take an average of $\tilde{c}_A$ and $\tilde{c}_B$, as means to evaluate the multi-objective performance of the various designs on an equal weighting basis. Based on this performance metric, it is Designs (a) and (b) that have the lowest average compliance of 0.87 and 0.84, respectively. Whereas the benchmark design and Design (c) have the highest average compliances of 1.00 and 1.01, respectively.

5.2 Manufacturing simulation

Here the printing induced distortions between the designs generated using the simultaneous optimisation method are compared with the benchmark DfAM approach. The printing process simulation results are shown in Fig. 15, in terms of
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It was found that the maximum absolute distortions after cool-down occurred in the parts themselves, rather than in the supports, so only the displacement results for the parts are shown for clarity in Fig. 15. Comparing the displacements at the end of the printing process shows that maximum absolute displacements are greatest on overhanging features, for all five models. This highlights the value of the pseudo-gravity approach, which implicitly suppresses the formation of unsupported overhanging features.

However, measuring maximum displacements only characterises the deformation of the structure based on one point. A more holistic measure is to calculate the mean value of all nodal deformations in the models, see Fig. 16. Considering all nodal displacements is also a fairer means of comparing against the self-weight compliance results, in which the deformations of all elements contribute to the total elastic strain energy.

It can be seen in Fig. 16 that the benchmark DfAM workflow results in the highest mean nodal displacement and Model (d) performs the best based on this metric. There is a clear correlation between decreasing the mean nodal displacement in Fig. 16 and reducing the self-weight compliance $\tilde{c}_A$ in Fig. 14. For brevity, one aerospace bracket example is presented in this work. However, by investigating five different part orientations, combined with the correlation between the pseudo-gravity compliance in Fig. 14 and the mean nodal displacements in Fig. 16, strengthens the case for the extensibility of the approach to other problems.

Typically, one thermo-mechanical simulation takes 23 h to complete using an Intel Xeon CPU E5-2650 v4 @ 2.20 GHz and four cores. Note that this manufacturing process simulation is significantly more computationally expensive compared to the simultaneous TO approach described in Sect. 2. The computational time is an important consideration from an industrial point of view (Allaire et al. 2020). This highlights the value of using the pseudo-gravity approach as a computationally efficient analogy for optimising support structure placement.

### 6 Conclusions

TO is increasingly being used as a part of DfAM workflows to realise complex and structurally efficient designs. However, the design of support structures, which also impact on manufacturing cost and feasibility, are usually only considered once the part’s shape has been fixed. In addition, support structure design often requires separate post processing steps. This paper explores the potential for simultaneously optimising the support structures along with the part topology using an automated approach. The novel approach combines both multi-model and multi-material TO along with the generation of structurally efficient load-aligned lattices in complex 3D design spaces.

If a single objective exists to optimise part compliance then a conventional DfAM workflow that decouples the part optimisation and the design of support structures is appropriate. However, quantifying the compliance of the supported structure under pseudo-gravity loads enables the support structures to be simultaneously optimised. In addition, investigating different part orientations and weightings between the two compliance contributions enables multi-objective designs to be generated.

Thermo-mechanical simulations of the SLM printing process indicate a correlation between minimising the compliance of the supported structure under
pseudo-gravity loads and the resultant thermally induced deformations. Thus, the pseudo-gravity approach is a useful analogy for the computationally efficient exploration and optimisation of multiple design configurations for this case-study.

Other design parameters to consider in future works include reducing the structure’s build height (i.e. build time) and reducing its planform area (required space on build platform). These parameters typically contradict one another but one or both could be implemented as additional design constraints in future. Minimisation of the maximum planform area to reduce re-coater blade impact loads could also be considered.

Although this paper considers a specific part, load cases and an AM process with specific parameters, the findings should be generally applicable. This is because the TO methodology is applicable to other optimisation problems and other AM process that require support structures. It is believed that the multi-objective optimisation methodology will inform engineers and designers about the trade-off between part performance and manufacturability. It is envisaged that such an optimisation approach can enable greater automation in the DfAM workflow and the realisation of increasingly complex and structurally efficient AM parts.

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**Declarations**

**Conflict of interest** The corresponding author states that there is no conflict of interest.
Simultaneous optimisation of support structure regions and part topology for additive fabrication is presented in this manuscript. The OptiStruct input files for topology optimisation, the Matlab scripts for generating support structures and the .stl files of the optimised geometries are available on request from the corresponding author.

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