Field-tuned spin excitation spectrum of $k\pi$ skyrmion

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Abstract

We study spin wave excitation modes of $k\pi$ skyrmion ($k = 1, 2, 3$) in a magnetic nanodot under an external magnetic field along the $z$ direction using micromagnetic simulations based on the Landau–Lifshitz–Gilbert equation. We find that a transition of $k\pi$ skyrmion to other skyrmion-like structures appears under some critical external fields, the corresponding spin wave excitations are simulated for each state under magnetic field. For skyrmion, the frequencies of excitation modes increases and then decreases with the low frequency mode splitting at a critical magnetic field. In addition to the well-known two in-plane rotation modes and an out-of-plane breathing mode of skyrmion, more excitation modes are found with a higher $k$ ($k = 2, 3$). The excitation modes vary as a function of magnetic field, and the excitation frequencies for different modes exhibit a rapid or slight change depending on the field-induced change of magnetization profile. Our study indicates the rich spin wave excitations for $k\pi$ skyrmion and opens up the possibility for theoretical or experimental investigation of magnonics application.

1. Introduction

As a particle-like magnetic chiral structure, magnetic skyrmion attracts a lot of attention due to its topology and small size since it has been observed in MnSi [1–3]. The direction of spins inside the skyrmion points up (down), while the surrounding spins of the ferromagnetic background are pointing in opposite direction which is down (up) [4–7]. Depending on the chirality of the circular domain wall determined by the Dzyaloshinsky–Moriya interaction (DMI) type [8, 9], which separates the spins inside of the skyrmion and the surroundings, there are two distinct types skyrmions with rotation symmetry defined as Bloch skyrmion and Néel skyrmion [10, 11]. They have been observed in chiral bulk helimagnets in the presence of bulk DMI [12–15] and in ultrathin magnetic films contacted with a heavy metal layer in the presence of interfacial DMI [16–19], respectively. Antiskyrmions with rotation symmetry breaking have been predicted in magnetic materials belonging to crystallographic classes $D_{2d}$ and $S_4$ by Bogdanov et al [12], which have been investigated theoretically and experimentally in recent years [16, 20–24]. While for skyrmions or antiskyrmions, the angle of progressive magnetization rotation is $1\pi$, while the angle is defined as the number of sign changes of the perpendicular magnetization when moving along the radial direction.

In addition to $1\pi$ skyrmion, $k\pi$ skyrmions ($k \geq 2$) were theoretically found and stabilized in uniaxial ferromagnets with DMI by Bogdanov et al [25], where the magnetization profiles and equilibrium parameters of individual $k\pi$ skyrmions, as well as their stability as a function of magnetic field are investigated. For $k\pi$ skyrmion, the direction of perpendicular magnetizations rotate multiple times with number $k$ along radial direction. The $k\pi$ skyrmion can be created and remain stable in magnetic nanodots with DMI [17, 23, 26]. In a larger DMI, the skyrmion with large $k$ is favorable with more magnetization rotation, and the nanodot confinement provides a topological barrier and limits the size of $k\pi$ skyrmion being centered in the nanodot. Meanwhile, due to the nanodot confinement, the small nanodot sizes cannot stabilize $k\pi$ skyrmion consisting of...
many rings. Compared to the rich investigations of skyrmion, skyrmion-like structures with higher $k$ attract much less attention both theoretically and experimentally [26–33]. The generation and the dynamics of $2\pi$ skyrmion under current are studied by Liu et al [34] and Zhang et al [35]. Moreover, the $2\pi$ skyrmion can be driven to propagate under a spin wave [36, 37], which exhibit large differences without skyrmion Hall effect compared to skyrmion. Recently, the controlled creation of $k\pi$ skyrmions on a discrete lattice and localized spin waves are investigated [27, 38].

Skyrmions are promised in rich application, such as racetrack memory [6, 18, 39–41], spin transfer nano-oscillator [42, 43], transistor [44, 45] or in other spintronic devices [44, 46, 47]. In addition to the application on spintronics, a possible application for skyrmions is in the field of magnonics. It is important in fundamental physics and manipulation to investigate the spin wave modes of skyrmions [48–51]. The spin wave modes of skyrmions were investigated in skyrmion lattice and isolated skyrmion in confined magnetic infrastructures [51, 52]. Typical spin wave excitation modes include a couple of gyrotropic modes (counterclockwise (CCW) and clockwise (CW)) for in-plane microwave magnetic field and a breathing mode for out-of-plane microwave magnetic field. The localized spin wave frequencies of $k\pi$ skyrmion are studied in [27]. A perpendicular magnetic field leads to a transition between different $k\pi$ skyrmion [35, 38], while the magnetic field-induced magnetization configuration transitions and mapping of corresponding spin excitations are still unexplored.

Here, we investigate the effect of external out-of-plane magnetic field on spin wave excitation modes of $k\pi$ skyrmion in a circular magnetic nanodot. The parameters of the system are taken to ensure that the $k\pi$ skyrmion ($k = 1, 2, 3$) can exist simultaneously. By applying magnetic field, the transformations between different $k\pi$ skyrmions are studied. We also demonstrated that the spin wave excitation modes for $k\pi$ skyrmion ($k = 2, 3$) are more complicated than that of skyrmion. Using a microwave magnetic field, different excitation modes are depicted. The frequency of excitation modes as a function of out-of-plane magnetic field are depicted as well. These results enable us to distinguish and classify skyrmions with different $k\pi$, and maybe promise in the application of magnonics by controlling the magnetization dynamics using spin wave.

2. Micromagnetic framework

The simulation system considered in our study is a circular ferromagnetic nanodot, where the radius and thickness are fixed as $R = 60$ nm and $t = 0.6$ nm, respectively. We use Mumax3 code to perform micromagnetic simulations [53]. The dynamics of $k\pi$ skyrmion and its spin excitation spectrum are governed by Landau–Lifshitz–Gilbert (LLG) equation of magnetization [6, 7]

$$\frac{\partial \mathbf{m}}{\partial t} = \gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + \mathbf{m} \times \frac{\partial \mathbf{H}}{\partial t},$$

where $\mathbf{m}$ is unit magnetization vector, which is defined as $\mathbf{M}/M_s$ and $M_s$ is the saturation magnetization. $\gamma$ is the gyromagnetic ratio, $\alpha$ is the Gilbert damping parameter. $\mathbf{H}_{\text{eff}} = \mathbf{H}_{\text{exch}} + \mathbf{H}_{\text{anis}} + \mathbf{H}_{\text{DMI}} + \mathbf{H}_{\text{ext}} + \mathbf{H}_{d}$ is the effective magnetic field of the system, which includes the Heisenberg exchange field $\mathbf{H}_{\text{exch}}$, the uniaxial perpendicular magnetic anisotropy field $\mathbf{H}_{\text{anis}}$, the Dzyaloshinsky–Moriya exchange field $\mathbf{H}_{\text{DMI}}$, the external magnetic field $\mathbf{H}_{\text{ext}}$ and the demagnetizing field $\mathbf{H}_d$.

The simulation code resolves the LLG equation using finite difference method and the unit cell size is set as $1$ nm $\times$ $1$ nm $\times$ $0.6$ nm. To obtain the initial equilibrium $k\pi$ skyrmion states ($k = 1, 2, 3$), we use the same magnetic parameters for Pt/Co multilayer, which has been found to exhibit magnetic skyrmions [54, 55]. The micromagnetic simulation parameters are chosen as [6]: exchange stiffness $A = 15 \times 10^{-12}$ J m$^{-1}$, uniaxial magnetocrystalline anisotropy $K_u = 0.8 \times 10^6$ J m$^{-3}$, saturation magnetization $M_s = 580 \times 10^3$ A m$^{-1}$. The interfacial DMI is considered and the DMI constant is set as $D = 4.0$ mJ m$^{-2}$, in which the energy density as a sum of Lifshitz invariant is [6, 56]

$$\xi = D (\mathbf{m} \cdot \nabla m_z - m_z \nabla \cdot \mathbf{m}).$$

By giving different $k\pi$ skyrmion states ($k = 1, 2, 3$) as the initial magnetization configuration of the nanodot separately, $k\pi$ skyrmions are relaxed in the center of the nanodots. The parameters taken in our system ensuring the skyrmion, $2\pi$ skyrmion and $3\pi$ skyrmion can exist simultaneously. As shown in figure 1, the magnetization profiles of three $k\pi$ skyrmion magnetization states are depicted with the rotation angle (a) $1\pi$ (b) $2\pi$ and (c) $3\pi$. Color map characterizes the direction of out-of-plane magnetizations $\mathbf{m}_z$, where the red color represents the magnetization orientation along $z$ direction and the blue color represents that along $-z$ direction. For $k\pi$ skyrmions, the magnetizations in the center are along $z$ direction.

In the following simulation, the static magnetic field is applied along $z$ direction for the above three magnetization configurations in the nanodot, which varies from $0$ to $300$ mT. The corresponding magnetization states are stabilized.
under out-of-plane magnetic field. In order to investigate the spin excitation dynamics of $k\pi$ skyrmion, we consider another uniform magnetic field pulse along $x$ or $z$ direction as excitation field, where the direction depends on the spin excitation modes with different symmetry we are interested. The excitation field exhibits low amplitude and has a time dependence with sinc function type, $H_{exc}(t) = H_0 \text{sinc}(2\pi ft) = \sin(2\pi ft)/(2\pi ft)$, where the amplitude is represented by $H_0$ which is 10 mT and the cut-off frequency $f$ is set as 50 GHz. After applying the excitation field under different static magnetic field, the magnetization components as a function of time and space are transformed as a function of frequency using Fourier transform, thus we can acquire the corresponding frequencies of different spin excitation modes, as well as the spatial distribution of the fast Fourier transformation power at specific oscillation frequency of magnetization components. The damping constant $\alpha$ is set as 0.3 in the relaxed procedure and 0.01 in the process of excitation. To show a continuous transitions and mapping of the spin eigenmodes between different $k\pi$ skyrmion under out-of-plane bias magnetic field, we limit our simulation to one single sample with same magnetic parameters. The influence of other materials or parameters and size of sample on the magnetization configuration and spin wave excitations will be in further consideration [57–60].

Figure 1. Three different magnetization configurations of the circular magnetic nanodot, (a) skyrmion, (b) $2\pi$ skyrmion and (c) $3\pi$ skyrmion. The magnetization components in the center of the nanodot are all along $z$ direction.
3. Results and discussion

3.1. Field dependent $k\pi$ skyrmion size

Firstly, we discuss the effect of static magnetic field on the size of $k\pi$ skyrmion as well as the transitions between different magnetization states, where the static magnetic field is applied normal to the nanodot along $z$ direction. As shown in figure 2, the initialized magnetization profiles of skyrmion, $2\pi$ skyrmion and $3\pi$ skyrmion are marked with capitalized Roman numerals I, II, III, respectively. The magnetization states represented by IV, V and VI are the transformed states for skyrmion, $2\pi$ skyrmion and $3\pi$ skyrmion we are interested. The corresponding magnetization profiles of $\mathbf{m}$ are depicted under the magnetization configurations of different states, as well as the radius determined.

Figure 2. The radius of $k\pi$ skyrmion as a function of out-of-plane magnetic field for (a) skyrmion, (b) $2\pi$ skyrmion and (c) $3\pi$ skyrmion. The magnetization profiles in different magnetic field range are marked as I, II, III, IV, V and VI. I, II and III correspond to skyrmion, $2\pi$ skyrmion and $3\pi$ skyrmion.

As shown in figure 2 for (a) skyrmion, (b) $2\pi$ skyrmion and (c) $3\pi$ skyrmion as a function of out-of-plane magnetic fields. The direction of the magnetic field is parallel to the direction of magnetization in the center of the nanodot, which varies in the range of $0\sim300$ mT. For skyrmion, the radius $r_s$ increases rapidly with increasing the magnetic field until $160$ mT, and then it increases slowly when the magnetic field reaches $280$ mT due to the competition of the field-induced skyrmion expanding and boundary induced reduction. When the field increases larger than $280$ mT, the magnetization state in the nanodot collapses to a uniform state (IV) with magnetization components along $z$ direction. While for $2\pi$ skyrmion, $r_{2s}$ and $r_{2s}'$ decreases as a function of magnetic field. Compared to the energy of the inner ring, the energy of the outer ring is more higher. It is worth noting that the $2\pi$ skyrmion collapses to another skyrmion state (VI) when $B_{\text{ext}} = 180$ mT, in which the polarity is opposite to the skyrmion state I. For $3\pi$ skyrmion, $r_{3s}$ and $r_{3s}'$ decrease with increasing field until $60$ mT, while $r_{3s}$ increases at the same time. When the magnetic field is larger than $60$ mT, $3\pi$ skyrmion transforms to a $2\pi$ skyrmion state (V) with the inner ring vanishing. The polarity of $2\pi$ skyrmion V is opposite to that of the $2\pi$ skyrmion state II. The radius corresponding to the second ring $(r_{2s}')$ and the outer ring $(r_{3s}')$ decreases and increases as a function of field with increasing to $280$ mT, respectively. Moreover, it collapses to a skyrmion state (VI) when the field exceeds $280$ mT.

3.2. Field dependent spin excitations of $k\pi$ skyrmion

After investigating that the size of $k\pi$ skyrmion depends on the out-of-plane magnetic field, the spin eigenmodes for $k\pi$ skyrmion are performed in this section, as well as the spin excitations of different transformed states at
each out-of-plane magnetic field. The spin wave excitation modes for skyrmion include the eigenmode with radial symmetry, also known as breathing mode, and the eigenmode with broken radial symmetry known as gyrotropic modes containing CW and CCW rotations, which can be described by azimuthal index $m = -1$ and $m = 1$, respectively [50]. CCW and CW modes are a precession of the topological center around the initial state in the nanodot center. To excite the breathing mode, a field pulse normal to the nanodot was applied. The gyrotropic modes including rotation around CW and CCW directions can be excited by an in-plane magnetic field pulse.

Using the same excitation method, the calculated frequency of spin excitation modes as a function of magnetic field for skyrmion, $2\pi$ skyrmion and $3\pi$ skyrmion are presented in figure 3. Figure 3(a) shows the gyrotropic modes for in-plane excitation magnetic field and figure 3(b) depicts the breathing mode for out-of-plane excitation magnetic field. The regions with different $k\pi$ skyrmion are separated with perpendicular yellow dashed lines. In each region, the magnetic field increases from 0 to 300 mT with a step of 10 mT. As we have mentioned, $k\pi$ skyrmion states collapse to another magnetization configurations at some critical magnetic fields. In figure 3, we use several perpendicular gray dashed lines to mark critical magnetic fields, and the corresponding magnitudes are depicted. These transformed states can be differentiated using topological number $S_n$ and magnetization components along $z$ direction $m_z$, as shown in figures 3(c) and (d), respectively. The capitalized Roman numerals I–VI represent relaxed magnetization states formed in specific magnetic fields, which have been clarified in figure 2. The simulated magnetization configurations in the nanodot are characterized by the topological number, named skyrmion number [5]

\[
S_n = \frac{1}{4\pi} \int \int \mathbf{m} \cdot \left( \frac{\partial \mathbf{m}}{\partial x} \times \frac{\partial \mathbf{m}}{\partial y} \right) \, dx \, dy.
\]

In ideal situation, the skyrmion number is 1 for skyrmion, 0 for $2\pi$ skyrmion and 1 for $3\pi$ skyrmion in our simulation system. However, it is not exactly equal to them due to the DMI induced magnetization rotation at the boundary of the restricted nanodot. It should be mentioned that the skyrmion numbers are equal for state II and state V, which are 0, while the magnetization states are different. The magnetizations in the center of the nanodot are opposite for them. Thus, we use another static property that is average magnetization component
along z direction \( \mathbf{m}_z \) to characterize different magnetization configurations, as depicted in figure 3(d). We can see that \( \mathbf{m}_z > 0.5 \) for 2\( \pi \) skyrmion (II) and \( \mathbf{m}_z < 0.5 \) for state V, which increase with increasing magnetic field.

Two gyrotrropic modes are excited after applying the excitation field along x direction, which can be found in the spin excitation spectrum of skyrmion (figure 3(a)). The lower frequency mode is the CCW mode \( (m = 1) \), and the higher frequency mode corresponds to the CW mode \( (m = -1) \). The frequencies of two gyrotrropic modes increase and then decrease when static magnetic field increases until 280 mT. With the skyrmion expanding as a function of magnetic field, the interaction between the skyrmion and nanodot boundary increases, and the excitation frequencies are affected. We will discuss this phenomenon later. When the magnetic field is larger than 280 mT, the skyrmion state collapses to a uniform state (IV in figure 2), the spin wave excitations of this single domain states are not indicated in figure 3(a). For 2\( \pi \) skyrmion, there are four gyrotrropic spin wave excitation modes, two low frequency modes and two high frequency modes. The frequencies of lower frequency modes decrease as a function of magnetic field, while the frequencies of higher frequency modes increase. The frequency is not shown when 2\( \pi \) skyrmion transforms to an another skyrmion state with \( S_n = -1 \) (VI in figure 2) when the magnetic field increases to 160 mT, for that the frequency of spin wave excitation mode exceeds the cut-off frequency of excitation field. Compared to skyrmion and 2\( \pi \) skyrmion, the in-plane excitation modes for 3\( \pi \) skyrmion are more complicated, where five spin wave excitation modes exist. The frequencies of two high frequency modes decrease with the magnetic field increase, and the frequencies for three low frequency modes exhibit a weak increase or decrease as a function of magnetic field. When the static magnetic field exceeds 60 mT, 3\( \pi \) skyrmion transforms to 2\( \pi \) skyrmion (V in figure 2), the corresponding spin excitation modes are different from that of 2\( \pi \) skyrmion (II in figure 2), for that the polarities of them are in opposite direction, and the field-induced magnetization variations are quite different, as depicted in figure 2. Three excitation modes are indicated, where the frequency of the highest frequency mode increases, while for the other two modes, the frequencies variation are similar to that of skyrmion (I in figure 2). Then, the excitation modes of skyrmion (VI in figure 2) appear with \( B_{\text{ext}} \) increase to 280 mT.

In figure 3(b), a particular mode can be found in the excitation spectrum for skyrmion, which is the breathing mode with skyrmion expansion and contraction periodically. Similar to the two spin excitations depicted in figure 3(a), the frequency of breathing mode increases and then decreases as a function of magnetic field. In a critical field, the skyrmion edge is localized near the nanodot edge and related edge effects appears, which influences the excitation frequency. We will talk the edge effects later. For 2\( \pi \) skyrmion, there are two main modes in the excitation spectrum. The applied magnetic field induces the increasing of excitation frequency for high frequency mode and decreasing for low frequency mode. At the critical field, only one breathing mode depicted for skyrmion state (VI in figure 2). Three breathing modes are excited for 3\( \pi \) skyrmion when \( B_{\text{ext}} < 60 \) mT, where the frequencies of highest and lowest frequency modes increase, and it decreases for the other mode. When 3\( \pi \) skyrmion transforms to a 2\( \pi \) skyrmion state (V in figure 2), the frequency decreases and then increases for high frequency mode, while the frequency variation as a function of field is the opposite for low frequency mode. The change trend is quite different with that of 2\( \pi \) skyrmion (II in figure 2). Moreover, only one excitation mode is depicted with the magnetic field exceeding 280 mT. The results show that the frequency variation for each region of out-of-plane magnetic field depends on the field-induced magnetization profile change and the confinement of nanodot.

Above we have demonstrated the frequency spectrum of gyrotrropic and breathing modes of \( k\pi \) skyrmion under different out-of-plane magnetic fields, the green dashed circles in figures 3(a) and (b) show the \( k\pi \) skyrmion spin wave excitation modes in the absence of magnetic field. Next, we perform a detailed discussion about these modes, which are shown in figure 4. According the rotation directions, two gyrotrropic modes are classified with CCW mode \( (m = 1) \) and CW mode \( (m = -1) \). First, we discuss the gyrotrropic modes. For skyrmion, the spatial profiles of the amplitude \( \text{Abs}(m_i) \) \( (i = x, y, z) \) and phase \( \text{Phase}(m_i) \) for dynamic magnetization components of skyrmion are plotted in the second row and middle column of figure 4. It is characterized by the out-of-plane magnetization component \( \mathbf{m}_z \) localized around the ring of skyrmion composed by in-plane magnetization. The amplitude distributions of CCW or CW modes form a ring around the skyrmion edge, and the phase changes continuously from \( -\pi \) to \( \pi \), which means the dynamic magnetization oscillations are in different phase in the nanodot. The spatial profiles of \( \text{Abs}(m_i) \) and \( \text{Phase}(m_i) \) for dynamic magnetization of 2\( \pi \) skyrmion are indicated in the third row and middle column in figure 4, which shows that in low frequency \( f = 1.98 \) GHz, the amplitude characterized by \( \mathbf{m}_z \) formed a bright ring in the center and a slightly bright ring at the outer part in the nanodot. Two rings formed in the nanodot at \( f = 6.59 \) GHz and only one formed at the outer part \( (f = 20.76 \) GHz) or at the inner part \( (f = 45.42 \) GHz) in the nanodot. The dynamic magnetization oscillations are in different phase, where the phase changes continuously in each magnetization parts separated by circular domain wall in 2\( \pi \) skyrmion (figure 1(b)). While for 3\( \pi \) skyrmion, the spatial profiles of \( \text{Abs}(m_i) \) and \( \text{Phase}(m_i) \) for dynamic magnetization at 0 mT are indicated in the fourth row and middle column. Depending on the frequency excited, different rings are formed characterized by \( \mathbf{m}_z \). In addition, the
profi les of Phase($m_n$) for $3\pi$ skyrmion are still in different phase and the phase changes continuously in each magnetization part separated by domain walls in $3\pi$ skyrmion (figure 1(c)).

The breathing modes for $k\pi$ skyrmion are depicted in the right column of figure 4. For skyrmion, it shows that the amplitude of out-of-plane magnetization components are localized in the circular domain wall of skyrmion which formed a ring, and the phase of breathing is in phase. Whereas the areas of the mode localization are different for $2\pi$ skyrmion (third row and right column in figure 4), as well as the phase distribution. The amplitude of low frequency mode is localized at the two domain walls of $2\pi$ in the nanodot, while the high frequency mode is mainly localized in the inner domain of $2\pi$ skyrmion. The oscillation mode for low frequency is in different phase and it is in phase for high frequency mode. Moreover, the excitations are localized in three domain walls region of $3\pi$ skyrmion at $f = 5.06$ GHz and the phase distribution shows that the oscillations of magnetization are in different phase. When $f = 16.39$ GHz, the spatial profile of amplitude is similar to at $f = 5.06$ GHz, while the spatial profiles of phase are quite different. The phase distribution at $f = 5.06$ GHz shows that the inner part and the outer part separated by circular domain walls are in phase, while the middle part is opposite. Whereas the phase of inner part is opposite to that of the middle part and the outer part at $f = 16.39$ GHz. For a high frequency breathing mode at $f = 43.10$ GHz, the spatial profile of amplitude is mainly localized in the inner part, and the spatial profiles of the phase are same. It is worth noting that the numbers of modes increase with increasing circular domain walls quantity for $k\pi$ skyrmion. For each circular domain wall, two gyrotropic modes and a breathing mode are excited, and the numbers of gyrotropic modes and breathing modes for $k\pi$ skyrmion equal to $2k$ and $k$, respectively. The phase distributions exhibit circular symmetry whether the breathing is in phase or not.

We have mentioned that the skyrmion size increases as a function of magnetic field, as well as the interaction between skyrmion and nanodot edge. In figure 5, we present a detailed microwave absorption spectra (a) $\text{Im} \chi(x)$ and (b) $\text{Im} \chi(z)$ of skyrmion when the excitation field is applied along x and z direction, respectively, as a function of microwave frequency for different perpendicular magnetic field. Two gyrotropic modes marked as M1 and M2 are corresponding to CCW mode and CW mode, and the breathing mode is marked as M3. Note that the excitation frequency of these three modes increase with increasing magnetic field until a critical field $B_{\text{ext}} = 240$ mT. Then, they decrease as a function of field. The reason is that, increasing out-of-plane magnetic field will expand the skyrmion, when the edge of skyrmion is localized near the nanodot edge and related edge effects appears. In a particular magnetic field, $\text{Im} \chi(x)$ and $\text{Im} \chi(z)$ split a low frequency mode when $B_{\text{ext}} = 270$ mT. The corresponding spatial profile of amplitude and phase of dynamic magnetization are shown in figure 5(c), which shows that the

| $m = 1$ | $m = -1$ | Breathing mode ($m = 0$) |
|---------|---------|------------------------|
| ![Image](https://example.com/image1.png) | ![Image](https://example.com/image2.png) | ![Image](https://example.com/image3.png) |

Figure 4. The static magnetization components $m_x$, $m_y$, and $m_z$, (left column) and the profiles of the FFT power and phase of magnetization components in different frequency for skyrmion, $2\pi$ skyrmion and $3\pi$ skyrmion, including in-plane CW ($m = -1$) and CCW ($m = 1$) gyrotropic modes (middle column) and breathing mode (right column).
skyrmion edge almost reaches to the boundary of nanodot. The left column of figure 5(c) shows the spatial distribution of magnetization components \( m_x, m_y, m_z \) and spatial profiles of amplitude and phase distribution for the three excitation modes with \( B_{ext} = 270 \) mT.

**Figure 5.** Imaginary parts of the dynamical magnetic susceptibilities (a) \( \text{Im} \chi(x) \) and (b) \( \text{Im} \chi(z) \) as a function of frequency for several perpendicular magnetic fields for a skyrmion confined in the nanodot, the activation magnetic field is applied along x and z direction, respectively. These modes are marked as M1 (CCW mode), M2 (CW mode) and M3 (Breathing mode). (c) Static magnetization components \( m_x, m_y, m_z \) and spatial profiles of amplitude and phase distribution for the three excitation modes with \( B_{ext} = 270 \) mT.

For 2\( \pi \) skyrmion, the imaginary parts of the dynamical magnetic susceptibilities (a) \( \text{Im} \chi(x) \) and (b) \( \text{Im} \chi(z) \) for 2\( \pi \) skyrmion as a function of frequency, several magnetic fields are applying. Four in-plane gyrotropic modes are defined (M1-4) and two breathing modes are defined (M5, M6). The horizontal dotted line separates 2\( \pi \) skyrmion states II and VI shown in figure 2.

**Figure 6.** Imaginary parts of the dynamical magnetic susceptibilities (a) \( \text{Im} \chi(x) \) and (b) \( \text{Im} \chi(z) \) for 2\( \pi \) skyrmion as a function of frequency, several magnetic fields are applying. Four in-plane gyrotropic modes are defined (M1-4) and two breathing modes are defined (M5, M6). The horizontal dotted line separates 2\( \pi \) skyrmion states II and VI shown in figure 2.
microwave field (M5-6) when $B_{\text{ext}} = 20$ mT. They are activated via application of a microwave magnetic field
\[ B_i(t) = B_i \sin(2\pi ft) \quad (i = x, z) \]
with a corresponding excitation frequency. For the purpose of visualization, we depicted the profiles of net magnetization in $z$ direction $\delta m_z(t)$ defined as
\[ m_z(t) = m_z(0) + \delta m_z(t). \]

We use the initialized $2\pi$ skyrmion state as the equilibrium state $m_z(0)$. The results show that M1 and M3 are CCW mode, and the rotation of inner and outer rings of $2\pi$ skyrmion are in opposite phase. For modes M2 and M4, the $2\pi$ skyrmion rotates with CW mode. Two breathing modes (M5, M6) depict that the $2\pi$ skyrmion expands and shrinks periodically, where the inner skyrmion and outer skyrmion are breathing in different phase for M5 and in phase for M6.

Compared to skyrmion or $2\pi$ skyrmion, the spin wave excitation modes are more richer for $3\pi$ skyrmion. The imaginary parts of the dynamical magnetic susceptibilities (a) $\text{Im}(\chi(x))$ and (b) $\text{Im}(\chi(z))$ for $3\pi$ skyrmion are shown in figure 8, as a function of frequency in different magnetic fields. When the out-of-plane magnetic field is under 60 mT, the $3\pi$ skyrmion is stabilized. From low frequency to high frequency, five in-plane gyrotrropic modes (M1-5) and three breathing modes (M6-8) are defined. We find that the frequency of M1 and M2 decrease slightly with magnetic field, while it increases slightly for M3. For M4 and M5, the frequency decreases as a function of field. Figure 8(b) depicts that the frequency of M6 and M8 increases opposite to the decrease of M7 with increasing magnetic field. Increasing magnetic field induces the collapse of $3\pi$ skyrmion to $2\pi$ skyrmion state V, where the excitation modes are similar to the results shown in figure 6. It is worth noting that state V and state II in figure 2 are all composed of two skyrmion nested together, where only the polarities are opposite. Depending on the magnetic field, the size change of these two states are different, thus the excitation spectrum shown in figure 3 are different. While it is same that there are two domain wall rings for state V and state II, and we have discussed the spin wave excitation modes for $2\pi$ skyrmion state II. Thus, we only focus the excitation modes for $3\pi$ skyrmion here. The corresponding snapshots for these eight excitation modes are shown in figure 9, for in-plane microwave magnetic field (M1-5) and out-of-plane microwave magnetic field (M6-8) when $B_{\text{ext}} = 20$ mT. Under an in-plane microwave with eigenfrequency of M1-5, the CCW mode and CW mode appear alternately. M6-8 are all corresponding to breathing modes under perpendicular microwave magnetic field. Depending on the excitation area in $3\pi$ skyrmion, the net magnetization of $m_z$ are lighted in different rings. For M6, the breathing phases of inner and outer rings are opposite to the middle ring, and for M7, the inner ring breathes in opposite phase compared to the middle and outer rings. While for M8, three rings are in same phase. These results show that the spin wave excitations depend on the magnetization configurations and excitation areas.

Figure 7. Snapshots of the six excitation modes (M1–6) at different times marked in figure 6 of $2\pi$ skyrmion, which are activated by in-plane microwave magnetic field (M1–4) and out-of-plane magnetic field (M5–6) under an external perpendicular magnetic field $B_{\text{ext}} = 20$ mT. Blue, red and green arrows represent CCW, CW and breathing modes, respectively.
4. Conclusion

In summary, we have determined spin excitation spectrum of $k\pi$ skyrmion ($k = 1, 2, 3$) in nanodot under static out-of-plane magnetic field, and shown the field-induced transformations of different skyrmion-like topological magnetization structures. We have investigated the transition states and related size as a function of perpendicular magnetic field. Moreover, we find that with a larger $k$, the number of spin excitation modes increases, for both in-plane rotation modes and the out-of-plane breathing modes. Under a static external
magnetic along z direction, the magnetization for different parts of $k\pi$ skyrmion expands or reduces depending on the magnetization orientation. Field-induced magnetization profile modulates the frequency of excitation modes for $k\pi$ skyrmion, which decreases or increases as a function of magnetic field. Therefore, using spin wave excitation modes of $k\pi$ skyrmion, the theoretical and experimental investigation and classification of these skyrmion-like structures are possible. These findings may open a promising application in spintronic devices or using in the magnonics.

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