INTRODUCTION

Let me start by an introductory warning: one of our contributors to the meeting [1], has discussed the existence of an unbiased estimator for dynamical fluctuations (we will return more seriously to this important topics). To be clear, this summary is not made by an unbiased estimator! There are at least two reasons for that. First, being a theoretician I have no competence to evaluate the validity of experimental results. The best thing I can do is to propose a suitable observable and ask for advice from my experimentalist friends. Second, theory is a ground for subjectivity. One way towards objectivity is to take care of everybody’s work. I will thus try my best! In fact, I will only use the contributions to the workshop, and their remaining tracks in the proceedings, for the discussion and references.

During the meeting there were nice contributions to the so-called ”Soft Particle Physics” which do not concern fluctuations/correlations in multiparticle production. Section 1 of this talk is devoted to these aspects. Experimental aspects of the search for soft photons, theoretical contributions to the search of a non-perturbative ap-
approach to Quantum ChromoDynamics (QCD) enabling to describe "Soft Physics", the use of studies based on nuclei, were all important features of our meeting, and will be shortly reviewed.

The correlation/fluctuation question and related studies occupied a large fraction of the meeting. The main reason for this are the stimulating discussions that this field of research has been leading to during the last few years, especially in relation with the concept of "Intermittency". Indeed, many of the unclear issues have now been clarified, but, all in all, not solved. That is the reason why the next sections of this summary talk are devoted to the most prominent results and open questions which appeared during the meeting.

In section 2, I will refer to the rapid evolution of the experimental techniques involved in the study of correlations/fluctuations in multiparticle physics. The comparison of the tools available nowadays with the ones originally proposed shows the technical advances made since then. In section 3, we will focus on three main collective results which were obtained thanks to these techniques: i) the emergence of Bose-Einstein interference effects in the study of fluctuations, ii) the comparison with perturbative QCD calculations obtained by different groups for the first time, and iii) the intermittent behaviour in connection with phase transitions.

Section 4 is devoted to a series of questions left for future work which I noticed open during the meeting. May be, some of them will find their answer by mail return,...or before the next multi-particle meeting.

1. Problems in Soft Particle Physics

"Soft Physics", at least when concerning elementary particles, appears really as the "Hard Problem". Indeed, the processes involving hadrons with small transverse momentum between each other and/or with respect to the incident ones remain a theoretical mystery. While such processes have been experimentally studied since a long time, it seems that their theoretical understanding remains at a standing point, except perhaps for the "static" QCD calculations of the low-lying states of the hadronic spectrum. We remain thus far from a deep understanding of "soft" hadronic reactions at high energy which implies a knowledge of the non-perturbative behaviour of Quantum Field Theory. More precisely the confinement problem in QCD, which requires a non-perturbative vacuum shift from the elementary quark and gluon fields to hadrons, is for the moment beyond reach. So, what to do?

In this general context, one could think that multi-particle processes is an even tougher problem in Soft Physics. They involve a lot of particles, and thus a lot of
variables for the description of the final states, which seems to forbid any reasonable treatment of the scattering amplitudes in terms, say, of Feynman diagrams in the perturbative expansion. However, less superficially, one may realize that there are two hidden advantages which could help the understanding on a basic level. First, particles are to be considered as elementary quanta of a relativistic quantum field, and thus operating with many particles could be a better revelator of the quantum field structure than low energy and/or low multiplicity events. Second, the number of particles involved in modern experiments at high energy is quite high (from dozens at LEP to hundreds at tevatron and future accelerators). This casts a bridge between particle physics and systems with many degrees of freedom. For instance, some powerful Statistical-Mechanics tools may become relevant, and their connection with Field Theory may be of great help, as it has already been the case for lattice calculations. To my mind, the goal and spirit of the workshop is to make progress towards these directions.

On the theoretical point of view, questions related to the field theoretical vacuum and more specifically the QCD vacuum at large distance have been discussed during the meeting. In analogy with QED, some hypotheses on the behaviour of quarks in such a vacuum have been modeled, see[2]. It is an ambitious approach, though apparently very difficult. Another approach is to consider an effective field theory of hadrons, for instance in the framework of a $\sigma$-model of pions and sigma resonances. The interesting suggestion discussed in [3] is that the large number of pions to be produced in future accelerators would justify a quasi-classical field-theoretical approach. This leads to dramatic predictions, like the formation of domains of disordered chiral condensates (DCC) following a very high energy collision. In more practical terms, one could observe bunches of particles with definite charge, e.g. neutrals. This picture may remain a theoretician dream, but it is worth investigating its consequences. It has the merit to show that imagination and theoretical rigor can live together in this domain of research. Several studies are now in action about DCC, and one is waiting for their results with curiosity.

On the phenomenological ground of soft physics (excluding correlations /fluctuations) soft photon physics and the interaction of particles with nuclei, (or of nuclei with nuclei) were the topics represented in the meeting. Soft photons can be considered as the Loch Ness Monster (or the legendary Krakow dragon!) of particle physics, since they are claimed to appear from time to time. More precisely, the question remains to know whether a significant excess of /it direct photons of low energy is produced with respect to the known background, mainly due to
bremsstrahlung. The results shown to us by the WA83 collaboration at CERN, see [4], give evidence for a strong excess, but amazingly similar in shape and properties with the bremsstrahlung. It is clear that this interesting experimental finding has to be confronted with other ones which give negative results. Let this research contribute to solve a long-standing controversy on this important question. A theoretical discussion can be found in [5].

Nuclei, beside their own interest as quantum systems of hadrons provide valuable tools for particle production: they may represent the finest existing microdetectors of sizes below 10 fermis. In particular they may give some information on the first stages of hadron production, at least if one is able to distinguish nuclear from particle effects in a suitable way. In [6] a series of results and models are discussed, which can pave the way towards the use of nuclei as detectors. Nuclei properties by themselves are evoked in [7] together with the interesting experimental detection of a possible critical phenomenon during the multifragmentation of heavy nuclei.

2. Tools for correlations/fluctuations: Past/Present

In order to figure out the decisive progress made in the detection of dynamical fluctuations in multi-particle data, it is useful to compare the tools used and discussed at the present workshop with the original method based on the factorial multiplicity moments, their binsize dependence and the \( \alpha \)-model of intermittency.

Factorial moments have been designed in order to remove from the measurements of multiplicity fluctuations the statistical fluctuations associated to a Poisson (or at fixed multiplicity, Bernouilli) noise. This simple assumption had the merit to exhibit unknown features of dynamical particle correlations, which are related to these moments by construction. Let us recall the conventional definition of factorial moments of rank \( q \):

\[
F_q(\delta) \equiv \frac{\langle n(n-1)\ldots(n-q+1) \rangle}{\langle n^q \rangle} \simeq \delta^{-f_q},
\]

where \( n \) is the multiplicity of particles observed in a phase-space interval \( \delta \). The third term of the equality represents by definition the intermittent behaviour characterized by a set of indices \( f_q \). Note that the factorial moments can also be expressed in terms of integrals of the \( q \)–correlation functions integrated in \( \delta \).

When experimentalists became interested in the game, it was recognized that the method could and should be improved. In particular, the factorial moments
suffered from one important defect: they were rather unstable at small binsize and high rank. At our meeting, these problems have been discussed, see [1,8], and one method has been found providing a good improvement: the correlation integrals’ method. The correlation integrals, which appear under different forms, have in common the following recipe: contrary to factorial moments, where the q-uple groups of particles are counted in phase-space boxes defined \textit{a priori}, the correlation integrals count all q-uple groups within a given distance \(\delta\). Removing the arbitrariness of the phase-space division gives a much stronger stability to the results. My conclusion, which is perhaps the main positive conclusion about the meeting, is that, after a few years of hesitation, one disposes of good tools to evaluate dynamical fluctuations/correlations in multi-particle physics.

As noticed since their proposal, factorial moments do not give a dynamical information on multi-particle fluctuations unless one varies the bin-size. However, different \textit{technical} difficulties appeared when this binning has to be done in the fully-dimensional phase-space: dependence of the moments on a non-homogeneous average multiplicity, instabilities of different kinds, etc... As a result of the improvements on the measure of fluctuations, some important empirical results have been found on the 2− or 3− dimensional cases. Let us quote in particular the universality of moments’ behaviour in this last case, which has been confirmed at the meeting[11].

The \(\alpha\)--model of intermittent fluctuations has been useful to model out genuine intermittency properties in multi-particle physics. As a mathematical tool, it allows to confront experimental data or phenomenological models to typical sets of fluctuations/correlations without scale (self-similar). However, it is a very crude type of modelization when compared to the sophistication of data. At least in two cases, it has been modified or improved. First, intermittent fragmentation models have been worked out[12], which possess both \textit{local} intermittent structures and \textit{global} features of the multiplicity distributions as seen in high-energy reactions. Second, Monte-Carlo simulations of great technicity have incorporated intermittent correlations[13]. Let us however remark that the level of accuracy obtained in Monte-Carlo simulations for \(e^+ - e^-\)−reactions[9] do not exist in other cases. Indeed, the problem here is due to the mismatch between intermittency and the Bose-Einstein correlations for hadron-induced reactions, which is discussed later on.

With the noticed improvement of tools for exploring the correlations /fluctuations, It is now time to go to the basic question we want to adress, namely the physical origin of the observed dynamical fluctuations, which are compatible with
the intermittency behaviour (1).

3. What is the origin of intermittency?

Good or bad, the fact is that no convincing explanation of the intermittent phenomenological structure of dynamical fluctuations in multi-particle reactions exists. Partly, it is due to the lack of precision data for some time. It is only recently that the emergence of the Bose-Einstein correlations in intermittency studies have been confirmed. Partly, it is due to the lack of theoretical understanding of long-distance effects in field theory, which is a subject treated during the workshop. There was also discussed an interesting connection between intermittent fluctuations and phase transitions. Let us review these subjects in turn.

i) Bose-Einstein correlations

The main finding concerning intermittency in the past year is that it is dominated by the same charge correlations between same-sign particles in very small bins of the whole 3-dimensional phase space at least for hadron or nuclei-induced reactions. This year, the evidence for the contribution of same-sign particle correlations have been confirmed and strengthened in various cases, namely in hadron-hadron reactions examined by the NA22 experiments at $22\text{GEV}/c^{[10]}$ and UA1 at $640\text{GEV}/c^{[14]}$. However, the situation is different in $e^{+} - e^{-}$ annihilation into hadrons as seen, e.g. by DELPHI at LEP$^{[9]}$. Indeed, the correlation integral method clearly shows that for hadron-induced reactions, same-sign particles contribute mainly at very small $q^{2}$, the Lorentz-invariant momentum distance between near-by particles. Note that a check should be done to know whether this variable gives the same results as the 3-dimensional phase space in terms of rapidity, azimuth and transfer momentum. For lepton-induced reactions, the opposite is true, namely correlations seem to be due to opposite-charge particles$^{[14]}$. The situation in the "mixed" case, that is lepton-hadron interactions is also "mixed"! Indeed, same-sign particles give the main contribution $^{[15]}$ at CERN energies, but are dominated by other effects in Monte-Carlo simulations at HERA energies, probably due to gluon cascading, see further on.

Does it mean that the conventional Bose-Einstein interference effect between identical particles can explain the intermittency phenomenon? Or what is the influence of Bose-Einstein correlations on the intermittency problem (as it is asked in ref.$^{[14]}$)? The investigations described during the meeting bring some interesting precisions on the problem. First, the form of Bose-Einstein correlations have been studied in detail, starting from the conventional gaussian or di-gaussian fits$^{[14,16]}$. 

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Quite unexpectedly, smaller is the interval in $q^2$ available, more peaked is the form of the correlation curve. One goes from gaussian to exponential [14,16], or to an edgeworth expansion (using Hermite polynomials) [17], or finally a power-law form without intrinsic scale for the source radius[10,14]. This last case, though required only by the very forward region of correlation, is the result obtained via the factorial or correlation integral methods. If confirmed by a detailed analysis of these very near-by correlations, this would mean a scale-invariant behaviour of the Bose-Einstein mechanism itself.

On the theoretical point of view, while waiting for a development of the experimental situation on this subject, it is stimulating to examine the motivations for having such a scale-invariant structure of the Bose-Einstein correlations. This has been analyzed in [18], with the following conclusions: in any case there must be an (effective) singularity in the space-time structure of the source of correlations (effective: it acts as a singularity in some region near-by, with a cut-off to avoid a true, unphysical, singularity in the cross-sections). Now, two cases, at least, are possible: the source is event-by-event smooth (not fractal), but it fluctuates with a singular distribution from event to event; Or, a source is not characterized by a smooth curve, but is itself a fractal object in space-time, and the scale-invariant behaviour is then a consequence of the scale-invariant structure of the dynamics (as in classical intermittency). One problem on previous studies raised by this approach is the lack of solid derivation of the Bose-Einstein correlations when there exist correlated sources, since the Hanbury-Twiss phenomenon is for uncorrelated sources. A key phenomenological conclusion of this work is the necessity to look for higher-order Bose-Einstein correlations (with more than 2 particles involved) as a way of distinguishing the various mechanisms.

ii) Perturbative QCD predictions

We have seen that in $e^+e^-$-annihilation into hadrons, the Bose-Einstein effect is likely not to be the dominant mechanism for scale-invariant fluctuations, even in full dimensionality. In this case, one can be confident that, at least in a first stage of the reaction, perturbative QCD calculations can give a hint on to the problem. At a deeper level, it is a basic problem to see whether the intermittency effect has some connection with the fundamental theory at all! Interesting developments have been reported at the conference [12,19], showing from three different calculations that perturbative QCD, within some approximation framework, is intermittent in the strict sense for emitted gluons (in the axial gauge). However, hadronization effects may be important, since the predicted behaviour for gluons is similar in form but
different in strength from that observed for hadrons[19].

Some conceptual and computational problems had to be solved before arriving at the results, which may explain why this well-defined problem in perturbative field theory took a long time and many efforts before yielding a solution. Conceptually, it was difficult to understand how a perturbative theory at short distance can say something for a typically long-distance problem. However, handling all orders of the perturbative expansion with the leading logarithm approximation, and the choice of a well-adapted gauge, allows one to make predictions for the multiplicity of gluons (quasi-real in such a gauge-fixing) in a small phase-space interval. On the computational point-of-view, it was rather striking that, within the same approximation scheme, analytical results on the behaviour of factorial moments could be obtained.

The overall result, besides some differences probably due to different assumptions on the initial conditions and on the observables, is that fractal dimensions can be predicted from these calculations. Let us first, for the sake of simplicity, fix the coupling constant of QCD. Then the behaviour of fluctuations is exactly fractal, that is effective singularities of the $q-$correlation functions exist and are concentrated into fractal regions of phase-space, which are random but with fixed non-integer dimension $d \equiv \frac{d_{\alpha}}{q-1}$, see equation (1). When the running of $\alpha_S$ is restored a more complex behaviour appears with multifractality and saturation at small bins. Multifractality means that now the $q-$correlation functions have $q-$dependent dimensions. Saturation corresponds to the breakdown of the intermittent behaviour since the amount of gluon radiation becomes so large that the fractality disappears and the full phase-space is filled by the gluons. Note that this breakdown appears already at the perturbative level, quite unexpectedly. However, for various reasons, the small-bin region becomes ambiguous and in fact forbidden to perturbative calculations. We are entering the no-man’s land of field theory (for the moment, I hope,) the non-perturbative regime. A proposal is made for damping hadronization effects by looking for ratios of factorial moments with varying resolution [12]. Concluding with that subject, there is now a theoretically motivated route to intermittency in QCD which deserves more study. The problem of non-perturbative methods remain completely open. Note, however, the interesting attempt of [20], introducing a new mechanism within the popular string fragmentation picture of hadronization.

**iii) Phase Transitions**

As is well established by the recent developments in field theory, when you
cannot solve a non-perturbative problem, you discretize your problem on a lattice and you study phase-transitions. This is precisely what people also did for the intermittency problem, connecting it to the study of fluctuations (both dynamical and geometrical) at a phase transition. There has been a significant activity in this domain using typical Statistical Mechanics methods. On a lattice, you may easily create the conditions of a critical behaviour leading to intermittent fluctuations, without the technical constraint of a weak coupling. For instance the effective Ginzburg-Landau theory of phase transitions have been advocated[21]. The main problem of this kind of studies is that one is not sure to meet the requirements for a "pure" phase transition at equilibrium for multi-particle production. Even in the case of heavy-ion reactions a thermal transition from the quark-gluon plasma is not an evidence. On the contrary, the observed fluctuations in this case seem to be quite different from those expected from the formation and decay of a plasma. I am nevertheless confident that the richness and the variety of physical situations which one meets in Statistical Mechanics systems will give powerful tools to particle physicist in the near future, for instance considering non-equilibrium systems. This is a guess.

4. Concluding by questions

From the preceding discussion, it is clear that the phenomenons, registered under the name of intermittency in multi-particle reactions, have not yet found a physical interpretation. While in $e^+ - e^-$ reactions into jets, the hadronization contribution is not understood and remains ambiguous in strength, see e.g.[22], the interplay of Bose-Einstein correlations with a possible scale-invariant structure of interactions remain a mystery in the other cases. It is thus too soon to draw any definite conclusion. Better is to propose a series of questions for further investigations. The interest of the situation is that, thanks to the sizeable improvements of the experimental and theoretical tools, some of the answers to the open questions could be within reach in the near future.

1. **Wavelets and other methods.** Can we develop new experimental tools, such as wavelets[1], in order to even more improve the data on correlations /fluctuations? The key question seems to be the possibility of mixing these probabilistic methods with the "factorial trick” in order to avoid analyzing only the statistical "noise” by wavelets.

2. **The "Wall and Tower" problem.** As is clear, e.g. in the analysis
of ref.[14], the factorial analysis in different dimension can reveal different types of dynamical excitations. A ”wall” of particles represented in a Lego-plot could be seen by perpendicular projection in phase-space as a strong fluctuation, while a ”tower” is likely to dominate only the full dimensional phase-space. If then, Bose-Einstein correlations dominate in most cases the 3-dimensional studies, what about lower dimensionalities? In particular how to interpret the Ochs-Wosiek scaling (moments over moments) in 1- and 2-dimensional NA22 studies[10]?

3. The ”multiplicity and $P_T$” problem. In hadron-induced reactions, it has been noticed[14] that, in contradiction with lepton-induced reactions, simulations are far from reproducing the low multiplicity and $P_T$ behaviour of fluctuations. As such this remains an unexplained feature which, if a solution is found, can open some doors for the ”soft” part of the intermittency problem. One should notice in this respect the new Monte-Carlo simulations, with intermittent fluctuations, applied to soft physics[19].

4. The ”Universality” problem. It has been noticed that 3-dimensional factorial moments follow a quite striking general behaviour in various reactions, such as lepton-hadron, hadron-hadron and even heavy-ion ones[11]. More precisely, the second factorial cumulant $K_2 \equiv F_2 - 1$ has roughly the same scale-invariant behaviour in all cases, up to a constant which can be attributed to the different long-range correlations. Is this feature due to the common Bose-Einstein origin of the expected fluctuations? Could we expect such a large extension of the scale-invariant range (from 1 to $10^4$ subdivisions) in all cases?

5. ”Angular intermittency”. One property of intermittent fluctuations in particle physics suggested by perturbative QCD calculations is that an effective singularity may show up in the fluctuations/correlations for angular variables[12]. It would be interesting to look for such a behaviour, either in lepton-induced reactions by depressing as much as possible the effect of hadronisation[12], or by extension to all other cases, just for curiosity’s sake.

6. Intermittency at HERA? An interesting remark made during the conference[15], is that one expects in lepton-hadron reactions (deep-inelastic scattering) a competition between the Bose-Einstein type of correlations and the perturbative QCD-induced mechanism. More precisely Monte-Carlo simulations indicate a levelling off of the perturbative component with the energy. This gives hope that the properties of the underlying perturbative theory can be caught from the fluctuations/correlations observed in the final hadron radiation. Such a study may be of some help in discussing such matters as the improved perturbative expansions(cf.
the Lipatov regime), coherence of gluons in the space-like region or saturation effects of partons beyond the perturbative regime? This is a challenge for next future.

7. Intermittency and the space-time structure of Strong Interactions. On the theoretical side of the problem, a much better understanding of the space-time development of the processes is required[18]. The extension of the interactions to large distances, which is probably necessary to generate scale-invariant fluctuations, is the origin of most (if not all) open questions: what is the interplay between parton fragmentation, resonance production and decay, quantum interference effects, dynamical quark-gluon phase transitions, in the overall phenomenon? Is there a simple answer or do we face the physics of a complex system? Those are some of the basic questions one has to face in that game.

8. The ”Micro-Universe story”. Let us end by the a speculative touch; Why not dream sometimes? The problematics of intermittency leads to an analogy between the history of the macro-world and that of the micro-world. In the macro-world, e.g. the Universe, after the Big-Bang, there was a succession of self-organizing structurations, compensated (as the entropy increases) by an increasing disorder whose signature is the famous Background Radiation. In the Micro-Universe represented by a production of particles from the vacuum, it is tempting to find the origin of dynamical fluctuations in the structuration of partons during the scattering process. They tend to form colorless clusters, but they do not necessarily find their partners within the small range of their individual interaction. Fluctuationg structures are formed, rearrangements occur, till they succeed to form new objects, the hadrons, which are required by the new vacuum structure. These hadrons seem to be rather complicated and structured objects at low energy, which may explain the long duration of the structuration compared with the length of the fundamental interaction. Is this picture right or wrong?

5. Many Thanks To The Organizers!

It is a pleasure to warmly thank all the organizers of our meeting. The atmosphere of discussions, exchanges and friendship show the evidence that the goals of the conference have been realized much beyond the normal level. Thanks to all of them.

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