A generalization of IPMs for linear optimization to convex quadratic SDO based on a trigonometric kernel function

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Abstract. In this paper, we generalize primal-dual interior-point methods for linear optimization to convex quadratic semidefinite optimization, which is a wide class of optimization problems that contains linear optimization, convex quadratic optimization, second-order cone optimization and semidefinite optimization as special cases. Based on the Nesterov and Todd scaling scheme, we establish the currently best known complexity bounds of large- and small-update interior-point methods for convex quadratic semidefinite optimization, namely, \( O(\sqrt{n \log n \log(n/\epsilon)}) \) and \( O(\sqrt{n \log(n/\epsilon)}) \), respectively, which are as good as the linear optimization analogue.

1. Introduction

Kernel functions play an important role in the design and analysis of interior-point methods (IPMs). They are not only used for determining the search directions but also for measuring the distance between the given iterate and the \( \mu \) -center for the algorithms [1, 2]. Currently, Kernel function based on IPMs is one of the most effective methods for solving linear optimization (LO) [3-7], second-order cone optimization (SOCO) [8], semidefinite optimization (SDO) [9,10], and symmetric optimization (SO) [11] and a very active research areas in mathematical programming. For a comprehensive treatment of kernel-function based IPMs for SO and the Cartesian \( P_\kappa \)-linear complementarity problem over symmetric cones (the Cartesian \( P_\kappa \)-SCLCP) [12-14], we refer to the recent monographs on this subject [1, 2] and the references cited therein.

The so-called trigonometric kernel function was first studied by El Ghami et al. [3] for primal-dual IPMs in LO. They established the worst case complexity bounds for large- and small-update methods, namely, \( O(n^{3/4} \log(n/\epsilon)) \) and \( O(\sqrt{n \log(n/\epsilon)}) \), respectively. Subsequently, a class of trigonometric kernel functions was considered for various optimization problems and complementarity problems [4-7, 10, 13, 14]. Recently, Bouafia et al. [7] introduced a new kernel function with a trigonometric barrier term as follows...
They proposed a class of primal-dual IPMs for LO based on the parametric kernel function (1) and obtained the currently best known complexity bounds of large- and small-update methods. Later on, El Ghami [14] extended the kernel-function based IPMs for LO presented in [7] to the SDO analogue and the obtained complexity bounds coincide with the derived for LO.

The purpose of the paper is to generalize the primal-dual IPMs for LO based on the parametric kernel function presented in [7] to the extension of SDO, i.e., convex quadratic SDO (CQSDO), which is a wide class of optimization problems that contains LO, CQO, SOCO and SDO as special cases. The symmetricization of the search directions used in this paper is based on the Nesterov and Todd (NT) scaling scheme. Furthermore, we obtain the complexity bounds that match the currently best known complexity bounds for large- and small-update methods.

2. Properties of the kernel (barrier) function

For ease of reference, we list some useful properties of the parametric kernel function \( \psi(t) \) that are needed in this paper.

**Lemma 2.1** (Lemma 3.2 in [7]): Let \( t > 0 \). Then \( \psi(\sqrt{t_1t_2}) \leq \frac{1}{2}(\psi(t_1)+\psi(t_2)). \)

The property described above is exponential convexity, which has been proven to be very useful in the analysis of primal-dual IPMs based on the eligible kernel functions [2].

**Lemma 2.2** (Lemma 3.3 in [7]): Let \( t > 1 \). Then \( \psi(t) \leq \frac{p\pi + 8}{8}(t-1)^2. \)

Let \( V \in S^n \) and \( V = Q^T \text{diag}(\lambda_1(V), \ldots, \lambda_n(V))Q \), where \( Q \) is any orthonormal matrix that diagonalizes \( V \). The matrix valued function \( \psi(V) \) is given by

\[
\psi(V) = Q^T \text{diag}(\psi(\lambda_1(V)), \ldots, \psi(\lambda_n(V)))Q.
\]

Furthermore, we define the barrier function \( \Psi(V) \) and the norm-based proximity measure \( \delta(V) \) based on the parametric kernel function (1) as follows

\[
\Psi(V) := \text{Tr}(\psi(V)) = \sum_{i=1}^n \psi(\lambda_i(V)), \quad \delta(V) := \frac{1}{2} \| \nabla \Psi(V) \|,
\]

Respectively, where \( \nabla \Psi(V) \) is the gradient of the barrier function \( \Psi(V) \). It follows that \( \psi(1) = \psi'(1) = 0 \). Furthermore, the proposed parametric kernel function \( \psi(t) \) is strongly convex due to the fact that Lemma 3.2 in [7], i.e. \( \psi''(t) \geq 1 \). Then we can conclude that \( \Psi(V) \) is strictly convex with respect to \( V > 0 \) and vanishes at its global minimal point \( V = E \), i.e., \( \psi(E) = \psi'(E) = 0 \). Moreover, \( \Psi(E) = 0. \)

As a consequence of Lemma 2.1, we have the following lemma, which is crucial for the analysis of the algorithms.

**Lemma 2.3** (Theorem 4 in [15]): Let \( V_1, V_2 \in S^+ \). Then \( \Psi\left(\left[V_1^{1/2}V_2V_1^{1/2}\right]^{1/2}\right) \leq (\Psi(V_1) + \Psi(V_2))/2. \)

In what follows, we need to derive a lower bound on \( \delta(V) \) in terms of \( \Psi(V) \).

**Theorem 2.4** (Theorem 5 in [15]): Let \( \varphi: [0, +\infty) \to [1, +\infty) \) be the inverse function of the parametric kernel function \( \psi(t) \) for \( t \geq 1 \), and \( V \in S^+ \). Then \( \delta(V) \geq \psi'(\varphi(\Psi(V)))/2. \)

As a consequence of Theorem 2.4, we have the following corollary, which gives a lower bound on \( \delta(V) \) in terms of \( \Psi(V) \).

**Corollary 2.5**: Let \( V \in S^+ \) and \( \Psi(V) \geq 1. \) Then \( \delta \geq (\Psi(V))^{1/2} / 6. \)
It is well known that the largest values of $\Psi(V)$ occur just after the update of $\mu$ during the course of the algorithm [15, 16]. The analysis of the algorithms enables us to derive an estimate for the effect of a $\mu$-update on the value of $\Psi(V)$.

Theorem 2.6 (Theorem 8 in [15]): Let $\beta \geq 1$ and $V \in S^n_{++}$. Then $\Psi(\beta V) \leq n\Psi\left(\beta\rho\left(\Psi(V) / n\right)\right)$.

As a consequence of Theorem 2.6, we have the following corollary, which yields an upper bound on $\Psi(V)$.

Corollary 2.7: Let $\Psi(V) \leq \tau$ and $V_\tau = V / \sqrt{1 - \theta}$ with $0 \leq \theta < 1$. Then $\Psi(V_\tau) \leq n\Psi\left(\rho(\tau) / \sqrt{1 - \theta}\right)$.

3. Kernel Function Based IPMs for CQSDO

3.1. The CQSDO problem

Consider the CQSDO problem in standard form

$$(P) \quad \min \{ C \cdot X + 0.5 X \cdot \Omega(X) : A_i \cdot X = b_i, i = 1,2,\ldots,m, X \succeq 0 \}$$

and its dual

$$(D) \quad \max \{ b^T y - 0.5 X \cdot \Omega(X) : \sum_{i=1}^{m} y_i A_j - \Omega(X) + S = C, S \succeq 0 \}$$

where $\Omega(X) : S^n \rightarrow S^n$ is a given self-adjoint positive semidefinite linear operation on $S^n$, i.e., for any $A, B \in S^n$, then $\Omega(A) \cdot B = A \cdot \Omega(B)$ and $\Omega(A) \cdot A \succeq 0$. To simplify matters, we will restrict ourselves to the following special case $\Omega(X) = \sum_{i=1}^{q} H_i^T X H_i$, where $H_i, i = 1,2,\ldots,q$ are the matrices in $\mathbb{R}^{n \times n}$ and $q$ is an integer not greater than $n^2$. Throughout the paper, we assumption that the matrices $A_i, i = 1,2,\ldots,m$ are linearly independent.

3.2. The central path for CQSDO

Without loss of generality, we assume that $(P)$ and $(D)$ satisfy the interior-point condition (IPC), i.e.,

$$A_i \cdot X^0 = b_i, X^0 > 0, i = 1,2,\ldots,m, \sum_{i=1}^{m} y_i^0 A_j - \Omega(X^0) + S^0 = C, S^0 > 0.$$  

Under the assumption of IPC, the optimality condition for $(P)$ and $(D)$ is equivalent to solve the following system

$$A_i \cdot X = b_i, i = 1,2,\ldots,m, X \succeq 0, \sum_{i=1}^{m} y_i A_i - \Omega(X) + S = C, S \succeq 0, XS = 0.$$  

The third equation in the system (6) is the so-called complementarity condition for $(P)$ and $(D)$ of CQSDO. The core idea of primal-dual IPMs is to replace $XS = 0$ by the parameterized equation $XS = \mu E$ with $\mu > 0$. This yields the following system

$$A_i \cdot X = b_i, i = 1,2,\ldots,m, X \succeq 0, \sum_{i=1}^{m} y_i A_i - \Omega(X) + S = C, S \succeq 0, XS = \mu E.$$  

The parameterized system (7) has a unique solution $(X(\mu), y(\mu), S(\mu))$ for each $\mu > 0$ due to the fact that the matrices $A_i$ are linearly independent and the IPC holds. Let $X(\mu)$ and $(y(\mu), S(\mu))$ be the $\mu$-center of $(P)$ and the $\mu$-center of $(D)$, respectively. The set of $\mu$-centers (with $\mu$ running through all the positive real numbers) gives a homotopy path, which is called the central path of $(P)$.
and \((D)\). If \(\mu \to 0\), then the limit of the central path exists, and since the limit points satisfy the complementarity condition, the limit yields an optimal solution for \((P)\) and \((D)\).

3.3. The new search directions for CQSDO

IPMs follow the central path approximately and approach the optimal set of CQSDO by letting \(\mu\) go to zero. Applying Newton's method to the system (3), we have

\[
A_i \cdot \Delta X = 0, i = 1, 2, \ldots, m, \sum_{i=1}^{m} \Delta y_i A_i - \Omega(\Delta X) + \Delta S = 0, \Delta X + X \Delta S^{-1} = \mu S^{-1} - X. \tag{8}
\]

Similar to the SDO analogue, the above system unfortunately does not have a unique symmetric solution due to the third equation in the system (8). Many researchers have proposed methods for symmetrizing the third equation in the above Newton system such that the resulting new system has a unique symmetric solution [1, 9, 15].

In this paper, we use the symmetrization scheme from which the NT search direction is derived. The most motivation for this choice is that the NT scaling technique transfers the primal variable \(X\) and the dual \(S\) into the same space: the so-called \(V\)-space. Let \(P := X^{1/2} (X^{1/2} S X^{1/2})^{-1/2} X^{1/2}\), and also define \(D = P^{1/2}\). Then

\[
V := 1/ \sqrt{\mu D^{-1} X^{-1}} - 1/ \sqrt{\mu D S D}. \tag{9}
\]

By replacing the term \(X \Delta S^{-1}\) in the third equation of the system (8) by \(P \Delta S P\), we can compute the scaled NT-search directions from the following system

\[
\overline{A}_i \cdot D_{\chi} = 0, i = 1, 2, \ldots, m, \sum_{i=1}^{m} \Delta y_i \overline{A}_i - \overline{\Phi}(D_{\chi}) + D_{\chi} = 0, \quad D_{\chi} + D_{\delta} = V^{-1} - \nabla V, \tag{10}
\]

where

\[
\overline{A}_i := 1/ \sqrt{\mu} \Delta A \overline{D}, i = 1, 2, \ldots, m; \quad D_{\chi} := 1/ \sqrt{\mu D^{-1} X^{-1}}; \quad D_{\delta} := 1/ \sqrt{\mu D S D}, \tag{11}
\]

and

\[
\overline{\Phi}(D_{\chi}) := \sum_{i=1}^{m} D H^T D_{\chi} D H D. \tag{12}
\]

Now, following [15, 16] we turn to the new approach of this paper. We replace the right-hand side \(V^{-1} - \nabla V\) in the third equation in the system (10) by the negative gradient, i.e., \(-\nabla \Psi(V)\), of the barrier function \(\Psi(V) := \Psi(X, S; \mu)\). This yields the following system, which defines the scaled NT search directions.

\[
\overline{A}_i \cdot D_{\chi} = 0, i = 1, 2, \ldots, m, \sum_{i=1}^{m} \Delta y_i \overline{A}_i - \overline{\Phi}(D_{\chi}) + D_{\chi} = 0, \quad D_{\chi} + D_{\delta} = -\nabla \Psi(V). \tag{13}
\]

The new search directions \((D_{\chi}, D_{\delta})\) are obtained by solving the system (13) so that \((\Delta X, \Delta S)\) are computed via (11). If \((X, y, S) \neq (X(\mu), y(\mu), S(\mu))\), then \((\Delta X, \Delta y, \Delta S)\) is nonzero. The new triple \((X_+, y_+, S_+)\) is given by \(X_+ := X + \alpha \Delta X, \quad y_+ := y + \alpha \Delta y, \quad S_+ := S + \alpha \Delta S\), where \(\alpha\) denotes the default step size, \(\alpha \in (0, 1]\), which has to be chosen appropriately.

The generic primal-dual IPMs for CQSDO as follows.

**Algorithm1**

**Step0** Input a threshold parameter \(0 < \tau < 1\), an accuracy parameter \(\varepsilon > 0\), a fixed barrier update parameter \(0 < \theta < 1\), a strictly feasible \((X^0, y^0, S^0)\) and \(\mu^0 = 1\) such that \(\Psi(X^0, S^0; \mu^0) < \tau\). Set \(X := X^0; \quad y := y^0; \quad S := S^0; \quad \mu := \mu^0\).
Step 1 If \( n \mu < \varepsilon \), stop, \((X, y, S)\) is an optimal solution; otherwise, update \( \mu := (1 - \theta)\mu \), go to Step 2.

Step 2 If \( \Psi(X, S; \mu) \leq \tau \), go back to Step 1; otherwise, go to Step 3.

Step 3 Solve the system (13) and use (11) to obtain \((\Delta X, \Delta y, \Delta S)\), choose a default step size \( \alpha \), update \( X := X + \alpha \Delta X; y := y + \alpha \Delta y; S := S + \alpha \Delta S \), go back to Step 2.

The parameters \( \tau, \theta \) and the step size \( \alpha \) should be chosen in such a way that the algorithm is "optimized" in the sense that the number of iterations required by the algorithm is as small as possible [9, 15].

4. The Analysis and Complexity of the Algorithms
By taking a default step size \( \alpha \), we have
\[
X' = X + \alpha \Delta X = \sqrt{\mu D(V + \alpha D_X)D}, \quad S' = S + \alpha \Delta S = \sqrt{\mu D^{-1}(V + \alpha D_x)D^{-1}}. \tag{14}
\]

It follows from (5) that \( V' = 1/\sqrt{\mu (D^{-1}X_S D)^{1/2}} \). We can verify that \( V' \) is unitarily similar to the matrix \( X'^{1/2}S'X'^{1/2} \) and thus to \( (V + \alpha D_x)^{1/2}(V + \alpha D_s)(V + \alpha D_x)^{1/2} \). This implies that the eigenvalues of \( V' \) are precisely the same as those of the matrix
\[
\overline{V}' := (V + \alpha D_x)^{1/2}(V + \alpha D_s)(V + \alpha D_x)^{1/2})^{1/2}. \tag{15}
\]

Then \( \Psi(V') = \Psi(\overline{V}') \). Furthermore, we have, by Lemma 2.3,
\[
\Psi(V') = \Psi(\overline{V}') \leq \frac{1}{2}(\Psi(V + \alpha D_x) + \Psi(V + \alpha D_s)). \tag{16}
\]

Let \( f(\alpha) := \Psi(V') - \Psi(V) = \Psi(\overline{V}') - \Psi(V) \), \( f_1(\alpha) := \frac{1}{2}(\Psi(V + \alpha D_x) + \Psi(V + \alpha D_s)) - \Psi(V) \).

Then \( f(0) = f_1(0) \) and \( f(\alpha) \leq f_1(\alpha) \). This implies that \( f_1(\alpha) \) gives an upper bound for the decrease of the barrier function \( \Psi(V) \). It is worth pointing out that \( f_1(\alpha) \) is convex and in general \( f(\alpha) \) is not convex. Furthermore, we have
\[
\begin{align*}
f_1'(\alpha) &= \frac{1}{2}(\psi'(V + \alpha D_x)D_x + \psi'(V + \alpha D_s)D_s), \tag{17} \\
f_1''(\alpha) &= \frac{1}{2}Tr(\psi''(V + \alpha D_x)D_x^2 + \psi''(V + \alpha D_s)D_s^2). \tag{18}
\end{align*}
\]

Below we use the following notation \( \delta := \delta(V) \). The following lemma provides an upper bound of \( f_1''(\alpha) \), which plays an important role in the analysis of the algorithms.

Lemma 4.1: One has \( f_1''(\alpha) \leq 2\delta^2\psi''(\lambda_{\min}(V) - 2\alpha \delta) \).

The idea underlying our approach is that the default step size should be chosen such that \( X' \) and \( S' \) are feasible and \( f(\alpha) = \Psi(V') - \Psi(V) \) decreases sufficiently. Let \( \alpha^* := \max\{\alpha : f_1'(\alpha) \leq 0\} \). The default step size that we are going to use will satisfy \( f_1'(\alpha) \leq 0 \), and as a consequence also \( \alpha \leq \alpha^* \). Following the strategy considered in [15, 16], we briefly recall how to choose the default step size. Suppose that the step size \( \alpha \) satisfies
\[
-\psi'(\lambda_{\min}(V) - 2\alpha \delta) + \psi'(\lambda_{\min}(V)) \leq 2\delta, \tag{19}
\]

Then \( f_1'(\alpha) \leq 0 \). The largest possible value of the step size of \( \alpha \) satisfying (19) is given by
where $\rho(s):[0, +\infty) \to (0,1]$ is the inverse function of $-\psi(t)/2$ for $t \in (0,1]$. Furthermore, we have $\bar{\alpha} := 1/\psi^n(\rho(2\delta))$. Similar to the LO analogue [7], we use $\bar{\alpha} := 1/1320p\delta^{1/(p+1)} \leq \bar{\alpha}$ as the default step size.

The following lemma provides an upper bound for the decreasing value of the barrier function $\Psi(V)$ in each inner iteration.

Lemma 4.2 (Lemma 4.7 in [15]): Let $\alpha \leq \bar{\alpha}$. Then $f(\alpha) \leq -\alpha\delta^2$.

It follows from Corollary 2.5 that $\delta \geq 1/6$. We have the following theorem, by Lemma 4.2 in [15].

Theorem 4.3: One has $f(\bar{\alpha}) \leq -\Psi(V)^{p/(2p+2)} \times 7920p$.

Let $\Psi_0$ be the value of $\Psi(V)$ after the $\mu$-update and $\Psi_k$, $k=1, 2, \ldots, K$ be the subsequent values in the same outer iteration, where $K$ denotes the total number of inner iterations in the outer iteration. It follows from the decrease of $f(\bar{\alpha})$ obtained in Theorem 4.3 that

$$\Psi_{k+1} \leq \Psi_k - \beta(\Psi_k)^{-\gamma}, \quad k = 0, 1, \ldots, K-1,$$

where $\beta = 1/7920p$, and $\gamma = p + 2/(p+1)$.

From Lemma 9 in [7], we have the following lemma, which provides an estimate for the number of inner iterations between two successive barrier parameter updates, in terms of $\Psi_0$.

Theorem 4.5: One has $K \leq 7920p\Psi_0^{(p+2)/(2p+2)}$.

By multiplying the number of outer iterations, i.e., $1/\Theta \log n/\varepsilon$, and the number of inner iterations we get an upper bound for the total number of iterations, namely,

$$\frac{7920p\Psi_0^{(p+2)/(2p+2)}}{\Theta} \log n/\varepsilon.$$

(22)

Recall that $\psi'(t) \geq 1$. Let $t \geq 1$. Then $s = \psi(t) = \int_1^t \psi'(\zeta) d\zeta \geq \int_1^t d\zeta = (t-1)/2$. This implies that $t = g(s) \leq 1 + \sqrt{2s}$. It follows from Corollary 2.7, and $\psi(t) \leq (t^2 - 1)/2$ when $t \geq 1$, after some elementary reductions, we have $\Psi_0 \leq \Theta n + 2\sqrt{2\tau n} + 2\tau$. From (22), we have the following theorem, which gives the complexity bound for large-update methods.

Theorem 4.5: For large-update method, one takes $\Theta = \Theta(1)$, and $\tau = O(n)$. The iteration bound then becomes $O(\tau n^{(p+2)/(2p+2)} \log (n/\varepsilon))$.

Corollary 4.7: Let $p = O(\log n)$. Then the iteration bound reduces to $O(\sqrt{n} \log n \log (n/\varepsilon))$, which matches the currently best known complexity bound for large-update methods.

For the analysis of the complexity bound of small-update methods, we need to estimate the upper bound of $\Psi_0$ more accurately. From Corollary 2.7, Lemma 2.2 and $1 - \sqrt{1-\theta} \leq \theta$, after some elementary reductions, we have $\Psi_0 \leq \frac{p\pi + 8}{8(1-\theta)} (\theta \sqrt{n} + \sqrt{2\tau})^2$. From (22), we have the following theorem, which yields the complexity bound for small-update methods.
Theorem 4.8: For small-update method, one takes $\theta = \Theta(1/\sqrt{n})$ and $\tau = O(1)$. The iteration bound then becomes $O(\sqrt{n} \log(n/\epsilon))$, which matches the currently best known complexity bound for small-update methods.

5. Conclusions
In this paper, we have shown that a class of primal-dual IPMs for LO based on the parametric kernel function presented in [7] can be extended to the context of CQSDO. The currently best known complexity bounds for large- and small-update IPMs are established, namely, $O(\sqrt{n} \log n \log(n/\epsilon))$ and $O(\sqrt{n} \log(n/\epsilon))$, respectively. Moreover, this unifies the analysis for the LO, CQO, SOCO, and SDO analogues.

The generalization of SO and the Cartesian $P_\kappa(k)$-SCLCP deserves to be investigated.

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