COMPLEXITY IN COSMOLOGY

Statistical properties of galaxy large scale structures

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Abstract The question of the nature of galaxy clustering and the possible homogeneity of galaxy distribution is one of the fundamental problem of cosmology. It is well established that galaxy structures are characterized, up to a certain scale, by fractal properties. The possible crossover to homogeneity is instead still matter of debate. However, independently on the specific value of the homogeneity scale, the fractal nature of galaxy clustering requires new methods and theoretical concepts developed in the area of statistical physics and complexity. In this lecture we discuss a new perspective on the problem of cosmological structures formation, both from the experimental and the theoretical points of view. This new perspective leads to a very interesting and constructive interaction between the fields of cosmic structures, statistical physics and complexity with very challenging open problems which we also discuss.

Keywords: Galaxy: correlation Cosmology: Large Scale Structures

1. INTRODUCTION

The large amount of new data which is accumulating for galaxy distribution and for the cosmic microwave background radiation (CMBR) calls for a characterization of structures and correlations in terms of concepts developed in the area of statistical physics and complexity. The two observations, however, appear quite different. On one hand the CMBR is extremely isotropic and the small amplitude, possible Gaus-
sian, approach to characterize its fluctuations seems rather adequate. On the other hand galaxies show highly structured patterns, with fractal like properties and for which the definition of background average density is still a matter of debate. Up to now our activity has been focused mostly on galaxy distribution. We have shown the importance of fractal properties and their implications on the statistical methods and on the theoretical framework. For example one of the consequences of these studies is the meaning of the so-called correlation length $r_0$ defined by $\xi(r_0) = 1$, which is usually considered to be $r_0 \approx 5h^{-1}\text{Mpc}$. In our opinion, due to the fractal properties identified in this distribution, such a length scale is not a characteristic length (nor a “correlation length”, neither a length scale which corresponds to the transition from non-linear to linear structures), but simply a fraction of the sample’s size. Future larger samples, like 2dF or SDSS, will permit us to check these specific properties on larger scales. However, even beside the question of the possible crossover to homogeneity, all the structures we see in galaxy distribution have fractal properties, and hence require a new theoretical framework for their understanding.

In this lecture we describe our activity in the field which includes: (i) data analysis, (ii) N-body simulations and (iii) theoretical modeling. We refer the interested reader to [7, 16, 34, 30] for further material on the subject. We also refer to the web page http://pil.phys.uniroma1.it where most of the work we present has been collected.

2. COMPLEXITY

“More is different”. This epochal paper of 1972 by Phil Anderson [1] has set the paradigm for what has now evolved into the science of Complexity. The idea that “reality has a hierarchical structure in which at each stage entirely new laws, concepts, and generalizations are necessary, requiring inspiration and creativity to just as great a degree as in the previous one” has set a new perspective in our view of natural phenomena. The reductionist view focuses on the elementary bricks of which matter is made, but then these bricks are put together in marvelous structures with highly structured architectures. Complexity is the study of these architectures which depend only in part on the nature of the bricks, but also have their fundamental laws and properties which cannot be deduced from the knowledge of the elementary bricks. In physical sciences the geometric complexity of structures often corresponds to fractal or multi-fractal properties [2]. It is not clear whether this is an intrinsic unique property or it is due to the fact that we can only recognize what we know. May be in the future we shall see much more
but for the moment one of the elements we can identify in complexity is its fractal structure. Considering then the dynamical processes often associated with complex structures we have as basic concepts: chaos, fractals, avalanches \cite{3} and 1/f noise. Often complex structures arise from processes which are strongly out of equilibrium and dissipative. There is a broad field, however, which is in between equilibrium and non-equilibrium phenomena. This is the field of glasses and spin glasses which leads to highly complex landscapes and to the concept of frustration \cite{4}. Finally an important field in which these ideas can also be applied is that of adaptation via evolution which is characterized by a degree of self-organization and a critical balance between periods of smooth evolution and dramatic changes \cite{5}.

We have been working for some years \cite{7} on the characterization of galaxy large scale structures and we have found that these are fractal in a certain range of scales. A possible crossover towards homogeneity is not yet identified and it is a matter of a wide debate \cite{8,9}. However, no matter the possible value of the crossover, the structures observed in three dimensional samples are fractal and they require new methods both for the techniques of analysis and for the theoretical interpretation and understanding.

3. SCALE INVARIANT STRUCTURES

In order to better define what is an “irregular structure” let us briefly discuss the properties of a regular one (see Fig.1) An analytical distribution of points is characterized by a small scale granularity which turns, at larger scales, into a well defined background density with specific structures corresponding to local over-densities (or under-densities). Let us consider a simple example of a single structure (over-density) super-imposed on a uniform background. Such a simple structure can be characterized by its position, size and intensity. One can also define a density profile along a line: This profile can be well approximated by a smooth (analytical) function, which for example can be a constant plus a Gaussian function. If we consider the dynamical evolution of our structure including the specific interactions between its constituent points, we can write a differential equation for the smooth function of the density profile. In this perspective the structure is essentially represented by the three elements: position, size and intensity (amplitude). The typical result of this study is to understand whether the structure moves, if it becomes more or less extended or more or less intense. This is the traditional approach to the study of structures based on the implicit assumption of regularity or analyticity which has been the one adopted
Figure 1. Example of analytical and non-analytic structures. Top panels: (Left) A cluster in a homogeneous distribution. (Right) Density profile. In this case the fluctuation corresponds to an enhancement of a factor 3 with respect to the average density. Bottom panels: (Left) Fractal distribution in the two dimensional Euclidean space. (Right) Density profile. In this case the fluctuations are non-analytical and there is no reference value, i.e. the average density. The average density scales as a power law from any occupied point of the structure. (From Sylos Labini et al., 1998 Phys.Rep. 293, 66)

in Statistical Physics before the advent of Critical Phenomena in the seventies.

Instead for the case of a strongly irregular structure, like for example a simple fractal distribution, all the concepts used to characterize the previous picture loose their meaning. There is no background density, there are structures in many zones and at various scales but it is not possible to assign them a specific size or intensity. The density profile is highly irregular at any scale. In order to give a proper characterization of the properties of this structure, one has to look at it from a new perspective. A structure which consists, for example, of a simple stochastic fractal has its regularity in the *scale transformation*. This naturally leads to power law correlations characterized by an exponent, the fractal dimension. Also from a theoretical point of view, the understanding of the origin of the irregular or fractal properties cannot arise
from the traditional differential equation approach but it requires new methods of the type of the renormalization group \[2, 6, 10\].

4. PHYSICS OF SCALE-INARIANT AND COMPLEX SYSTEMS

The physics of scale-invariant and complex systems is a novel field which is including topics from several disciplines ranging from condensed matter physics to geology, biology, astrophysics and economics. This widespread inter-disciplinarity corresponds to the fact that these new ideas allow us to look at natural phenomena in a radically new and original way, eventually leading to unifying concepts independently of the detailed structure of the systems. The objective is the study of complex, scale-invariant structures, that appear both in space and time in a vast variety of natural phenomena. New types of collective behaviors arise and their understanding represents one of the most challenging areas in modern statistical physics.

The activity in this field (see e.g the web page of the EC Network of "Fractal structures and self-organization" [11]) results in a cooperative effort of numerical simulations, analytical and experimental work, and it can be characterized by to the following three levels:

- \((i)\) Mathematical or geometrical level.

This consists in applying the methods of fractal geometry into new areas to get new insights into important unresolved problems and contribute to a better overall understanding. Such an approach permits to include into the scientific areas many phenomena characterized by intrinsic irregularities which have been previously neglected because of the lack of an appropriate framework for their mathematical description. The main examples of this type can be found in the geophysical and astrophysical data.

- \((ii)\) Development of physical models: The Active principles for the generation of Fractal Structures.

Computer simulations represent an essential method in the physics of complex and scale-invariant systems. A large number of models have been introduced to focus on specific physical mechanisms which can lead spontaneously to fractal structures. Here we list some of them, which, in our opinion, represent the active principles for processes which generate scale invariant properties based on physical processes. In Ref. [6] one can find many papers on models like Diffusion Limited Aggregation and the Dielectric Breakdown Model. These models are the prototype of the so-called
fractals in which an iteration process based on Laplace equation leads spontaneously to very complex structures. The concept of self-organization is common to all the models discussed here but it has been especially emphasized in relation to the sandpile model. In addition, to these simplified models we know that fractal structures are naturally generated in fluid turbulence as described by Naiver-Stokes Equations as the fractional portion of space in which dissipation actually occurs. Also the studies of gravitational instabilities suggest that gravity with random initial conditions may be enough to generate fractal clustering (see below). Up to now, however, the connections between the two important problems of turbulence and gravitational clustering and the above simplified models are only indirect. Each phenomenon and model mentioned seems to belong to a different universality class.

(iii) Development of theoretical understanding

At a phenomenological level scaling theory, inspired by usual critical phenomena, has been successfully used. This is essential for the rationalization of the results of the computer simulations and experiments. This method allows us to identify the relations between different properties and to focus on the essential ones. From the point of view of the formulation of microscopic fundamental theories the situation is still in evolution. With respect to usual equilibrium statistical mechanics these systems are far from equilibrium and their dynamics is intrinsically irreversible. This situation does not seem to lead to any sort of ergodic theorem and the temporal dynamics has to be explicitly considered in the theory [10]. This, together with the concept of self-organization, as compared to criticality, represent the main new elements for the formulation of microscopic theories.

5. GALAXY STRUCTURES AND CORRELATIONS

The existence of large scale structures (LSS) and voids in the distribution of galaxies up to several hundreds Megaparsecs is well known for twenty years [12, 13]. The relationship among these structures on the statistics of galaxy distribution is usually inferred by applying the standard statistical analysis as introduced and developed by Peebles and coworkers [14]. Such an analysis assumes implicitly that the distribution is homogeneous at very small scale ($\lambda_0 \approx 5 \div 10 h^{-1} \text{Mpc}$). Therefore the system is characterized as having small fluctuations about a finite average density. If the galaxy distribution had a fractal nature the sit-
uation would be completely different. In this case the average density in finite samples is not a well defined quantity: it is strongly sample-dependent going to zero in the limit of an infinite volume. In such a situation it is not meaningful to study fluctuations around the average density extracted from sample data. The statistical properties of the distribution should then be studied in a completely different framework than the standard one. We have been working on this problem since some time [7] by following the original ideas of Pietronero [15]. The result is that galaxy structures are indeed fractal up to tens of Megaparsecs [16]. Whether a crossover to homogeneity at a certain scale $\lambda_0$, occurs or not (corresponding to the absence of voids of typical scale larger than $\lambda_0$) is still a matter of debate [8]. At present, the problem is basically that the available red-shift surveys do not sample scales larger than $50 \div 100 h^{-1} Mpc$ in a wide portion of the sky and in a complete way.

Note that Gerard de Vaucouleurs [17] has been the first who has considered a possible hierarchical structure of galaxy clustering, which also implies a different interpretation of galaxy counts: “When inhomogeneities are considered (if at all) they are treated as unimportant fluctuations amenable to first order variational treatment. Mathematical complexity is certainly an understandable justification, and economy or simplicity of hypotheses is a valid principle of scientific methodology: but submission of all assumptions to the test of empirical evidence is an even more compelling law of science”. He has related the presence of large scale structures to the power-law behavior of the (conditional) average density and then to a non-Euclidean exponent in the number counts as a function of magnitude.

5.1. SELF-ORGANIZED CRITICALITY IN SELF-GRAVITATING SYSTEMS

The clustering of matter in the universe is hence an important example of the fields in which scale invariance has been observed as a common and basic feature. However, the fact that certain structures exhibit fractal and complex properties does not tell us why this happens. A crucial point to understand is therefore the origin of the general scale-invariance of in the gravitational clustering phenomenon. This would correspond to the understanding of the origin of self-gravitating fractal structures and of the properties of Self-Organized Criticality (SOC) from the knowledge of the microscopic physical processes at the basis of this phenomenon: Most of the scale free phenomena observed in nature are self-organized, in the sense that they spontaneously develop from the
generating dynamical process. Such a project requires a close interaction between three different lines, (i) Data Analysis, (ii) N-body simulations, (iii) Formulations of simple physical models, the first steps towards a real theoretical understanding. Let us see these three points in more detail.

5.2. DATA ANALYSIS

Nowadays there is a general agreement about the fact that galactic structures are fractal up to a distance scale of $\sim 30 \div 50 h^{-1} Mpc$ [7, 16] and the increasing interest about the fractal versus homogeneous distribution of galaxy in the last year [18, 19, 8, 20, 21, 23, 9, 22] has mainly focused on the determination of the homogeneity scale $\lambda_0$ (See the web page http://pil.phys.uniroma1.it/debate.html where all these materials have been collected). The main point in this discussion is that galaxy structures are fractal no matter what is the crossover scale, and this fact has never been properly appreciated. Clearly, qualitatively different implications are related to different values of $\lambda_0$, which could be possibly found in the new galaxy three-dimensional samples which will be completed in the next few years. From the point of view of data analysis we may identify different problems which must be addressed for a correct understanding of galaxy structures.

5.2.1 Characterization of scaling properties. Given a distribution of points, the first main question concerns the possibility of defining a physically meaningful average density. In fractal-like systems such a quantity depends on the size of the sample, and it does not represent a reference value, as in the case of an homogeneous distribution. Basically a system cannot be homogeneous below the scale of the maximum void present in a given sample. However the complete statistical characterization of highly irregular structures is the objective of Fractal Geometry [2].

The major problem from the point of view of data analysis is to use statistical methods which are able to properly characterize scale invariant distributions, and hence which are also suitable to characterize an eventual crossover to homogeneity. Our main contribution [7], in this respect, has been to clarify that the usual statistical methods, like correlation function, power spectrum, etc. [14], are based on the assumption of homogeneity and hence are not appropriate to test it. Instead, we have introduced and developed various statistical tools which are able to test whether a distribution is homogeneous or fractal, and to correctly characterize the scale-invariant properties. Such a discussion is clearly relevant also for the interpretation of the properties of artificial simula-
The agreement about the methods to be used for the analysis of future surveys such as the Sloan Digital Sky Survey (SDSS) and the two degrees Fields (2dF) is clearly a fundamental issue [7].

Then, if and only if the average density is found to be not sample-size dependent, one may study the statistical properties of the fluctuations with respect to the average density itself. In this second case one can study basically two different length scales. The first one is the homogeneity scale ($\lambda_0$), which defines the scale beyond which the density fluctuations become to have a small amplitude with respect to the average density ($\delta \rho < \rho$). The second scale is related to the typical length scale of the structures of the density fluctuations, and, according to the terminology used in statistical mechanics [24], it is called correlation length $r_c$. Such a scale has nothing to do with the so-called "correlation length" used in cosmology and corresponding to the scale $\xi(r_0) = 1$ [14], which is instead related to $\lambda_0$ if such a scale exists. Such a confusion being at the origin of the misinterpretation of the concept of clustering in modern cosmology [7].

5.2.2 Fluctuations. In the characterization of scaling properties, one would like to determine other statistical quantities beyond the fractal dimension. Such a global parameter is in fact the first one to be determined, but then fractals with the same dimension can have completely different morphological properties (higher order correlations). One point to be studied is the identification and characterization of some relevant global quantities such as porosity, lacunarity and three point correlation function, which are poorly studied in the general case of mathematical fractal, and never considered in the studies of large scale structures. Another possibility [25] concerns the study of fluctuations around the average counts as a function of scale and we have developed tests to study of galaxy distribution both in red-shift and magnitude space. Briefly, fluctuations in the counts of galaxies, in a fractal distribution, are of the same order of the average number at all scales as a function of red-shift and magnitude. For the case of an homogeneous distribution fluctuations are instead exponentially or power-law damped. We point out that the study of these kind of fluctuations can be a powerful test to understand the nature of galaxy clustering at very large scales as these analysis can be performed on both photometric and redshift galaxy catalogs. It is worth to notice that one of the anomalous statistical properties of critical systems, characterized by power-law long-range correlations systems is that, whatever their size, they can never be divided into mesoscopic regions that are statistically independent. As a result they do not satisfy the basic criterion of the central
limit theorem and one should not necessarily expect global, or spatially averaged quantities to have Gaussian fluctuations about the mean value [26]. The probability density function (PDF) of a global measure in a large class of highly correlated systems is then strongly non-Gaussian. The measurement of such a quantity in galaxy data is then very interesting to understand the PDF of galaxies and its possible relation to other critical systems.

5.2.3 Implication of the fractal structure up to scale $\lambda_0$.
The fact that galactic structures are fractal, no matter what is the homogeneity scale $\lambda_0$, has deep implication on the interpretation of several phenomena such as the luminosity bias, the mismatch galaxy-cluster, the determination of the average density, the separation of linear and non-linear scales, etc., and on the theoretical concepts used to study such properties [7]. An important point is then to consider the main consequences of the power law behavior of the galaxy number density, by relating various cosmological parameters (e.g. $r_0$, $\sigma_8$, $\Omega$, etc.) to the length scale $\lambda_0$ [27]. This has been partially done, but a more complete picture is still lacking. We also note that the properties of dark matter are inferred from the ones of visible matter, and hence they are closely related. If now one observes different statistical properties for galaxies and clusters, this necessarily implies a change of perspective on the properties of dark matter. For example in most direct estimates of the mass density (visible or dark) of the Universe, a central input parameter is the luminosity density of the Universe. We have considered [27] the measurement of this luminosity density from red-shift surveys, as a function of the yet undetermined characteristic scale $\lambda_0$ at which the spatial distribution of visible matter tends to a well defined homogeneity. Making the canonical assumption that the cluster mass to luminosity ratio $M/L$ is the universal one, we can estimate the total mass density as a function $\Omega_m(R_H,M/L)$. Taking the highest estimated cluster value $M/L \approx 300 h M_\odot/L_\odot$ and a conservative lower limit $R_H \gtrsim 20 h^{-1} \text{Mpc}$, we obtain the upper bound $\Omega_m \lesssim 0.1$. Note that for values of the homogeneity scale $\lambda_0$ in the range $\lambda_0 \approx (90 \pm 45) h \text{Mpc}$, the value of $\Omega_m$ may be compatible with the nucleosynthesis inferred density in baryons [27]. From this perspective one of the main arguments used as an indirect evidence of non-baryonic dark matter fails, and one has no need to invoke an unknown kind of matter to reconcile the observed amount of matter in galaxy clusters with the limits of primordial nucleosynthesis (e.g. [28]).
5.2.4 Determination of the homogeneity scale $\lambda_0$. This is, clearly, a very important point at the basis of the understanding of galaxy structures and more generally of the cosmological problem. We distinguish here two different approaches: direct tests and indirect tests. By direct tests, we mean the determination of the conditional average density in three dimensional surveys, while with indirect tests we refer to other possible analysis, such as the interpretation of angular surveys, the number counts as a function of magnitude or of distance or, in general, the study of non-average quantities, i.e. when the fractal dimension is estimated without making an average over different observes (or volumes). While in the first case one is able to have a clear and unambiguous answer from the data, in the second one is only able to make some weaker claims about the compatibility of the data with a fractal or a homogeneous distribution. For example the paper of Wu et al. [8] mainly concerns with compatibility arguments, rather than with direct tests. However, also in this second case, it is possible to understand some important properties of the data, and to clarify the role and the limits of some underlying assumptions which are often used without a critical perspective. Clearly the availability of new three dimensional galaxy samples in the next few years would allow one to study larger volume of space with a better statistics, and, possibly, to determine the homogeneity scale.

5.2.5 Crossover towards homogeneity and Finite size effects. A related and important point under consideration concerns the correct modeling of the possible crossover towards homogeneity. If the average density will be ultimately defined one would like to properly describe the transition from a system with large fluctuations (fractal) to a distribution with small fluctuations (homogeneous with small amplitude and correlated fluctuations). A number of statistical tools (correlation function, power spectrum, etc.) can be useful in this respect, but one has to correctly understand some subtle properties due to finite size effects. For example, in a finite sample, the power spectrum will always show a maximum followed by a decay (for $k \to 0$): such a break is due to a finite size effect related to the determination of the average density inside the sample itself. This has been often and incorrectly associated with a real change of the correlation properties of the distribution. Even in the case of a smooth distribution the standard methods used for the characterization of correlation must be carefully revised. This also particularly interesting for the analysis of cosmological N-body simulations which indeed show a smooth transition from small scale fractality to large scale homogeneity.
6. **N-BODY SIMULATIONS**

We have started to study the problem of the self-gravitating gas in a periodic volume. We have used high resolution N-body simulations to study the dynamical evolution of a gas of particles, initially distributed according to Poisson statistics, with periodic boundary conditions, and (for the moment) without the effect of space expansion. The aim of this project is to understand first a simple case of clustering process to then study more sophisticated simulations which involve space expansion and a particular choice of initial conditions. The results, which must be tested with large simulations as the number of particles used is in the range \(10^3 \div 10^4\), is that the system spontaneously develops self-similar fluctuations, characterized by a fractal dimension \(D \approx 2\). There is a list of new type of question which we would like to address: Can gravity develop a critical equilibrium? Is the fractal dimension \(D \approx 2\) a characteristic exponent of gravity? How long in time and how deep in space does the critical behavior extend? Basically, in the standard picture described by the continuous equations one has a linear or non-linear amplification of smooth fluctuations. In the new perspective there is a transfer of non-analytic clustering (granulosity) from small to large scales. The continuous fluid description simple neglects the effect due to small scale granulosity.

There are large N-body cosmological simulations which are publicly available. The clustering in these simulations is due to a combination of effects (space expansion, initial conditions, properties of dark matter and gravitation) and hence it is more difficult to understand the influence of each of these effects at a time. However it is interesting to consider, as a first step, the statistical properties of both initial and final conditions in these simulations. We have studied the statistical properties of cosmological N-body simulations based on CDM-like models, showing that they develop fractal structures almost independently on a wide choice of initial conditions and cosmological parameters. In such a case, however, the fractal extends in a relatively small range of scales (i.e. \(0.1 \div 20h^{-1} Mpc\)) and a crucial point in this respect is the fact, that self-similar fluctuations require a long time to develop over a large range of scales (up to \(\sim 100h^{-1} Mpc\) or more) from Gaussian initial conditions.

A related question concern the implementation of the initial conditions of N-body simulations. In standard models (like CDM’s) the initial conditions are due to a combination of properties of the quantum fluctuations of the early universe and the specific properties of the considered dark matter. However one has predictions on the initial continuous density field and its correlation properties. How to discretize the initial
continuous density field? The answer to this basic question is clearly fundamental in order to relate the properties of the initial continuous density field to the properties of a discrete set of points, with which the initial conditions of N-body simulations are usually set up.

7. INTERPRETATION AND MODELING

7.1. EXPONENTS VERSUS AMPLITUDES

From the theoretical point of view, the only relevant and meaningful quantity is the exponent of the power law correlation function (or of the space density), while the amplitude of the correlation function, or of the space density, is just related to the sample size and to the lower cut-offs of the distribution. The geometric self-similarity has deep implications for the non-analyticity of these structures. Indeed, analyticity or regularity would imply that at some small scale the profile becomes smooth and one can define a unique tangent. Clearly this is impossible in a self-similar structure because at any small scale a new structure appears and the distribution is never smooth. Self-similar structures are therefore intrinsically irregular at all scales and correspondingly one has to change the theoretical framework into one which is capable of dealing with non-analytical fluctuations. This means going from differential equations to something like the Renormalization Group to study the exponents. For example the so-called “Biased theory of galaxy formation” [29] is implemented considering the evolution of density fluctuations within an analytic Gaussian framework, while the non-analyticity of fractal fluctuations implies a breakdown of the central limit theorem which is the cornerstone of Gaussian processes [15, 10, 7].

7.2. FRACTAL COSMOLOGY IN AN OPEN UNIVERSE

The clustering of galaxies is well characterized by fractal properties, with the presence of an eventual cross-over to homogeneity still a matter of considerable debate. We have discussed and considered the cosmological implications of a fractal distribution of matter, extending to an arbitrarily large scale [30]. Such an open model of universe can be treated consistently within the framework of the expanding universe solutions of Friedmann, with the fractal being a perturbation to an open cosmology in which the leading homogeneous component is the cosmic microwave background radiation (CMBR). This new type of cosmology, inspired by the observed galaxy distributions, provides a simple explanation for the recent data which indicate the absence of deceleration in the ex-
pansion \((q_0 \approx 0)\). Moreover the ‘age problem’ is essentially eliminated. The model leads to a new scenario for the explanation of the observed isotropy of the CMBR. The radiation originates mostly in the annihilation processes which leave behind the large voids, with the residual fractal matter leading to small perturbations. Nucleosynthesis and the formation of structure can also be addressed in this new framework.

7.3. FORCE DISTRIBUTION

One of the main properties it is possible to calculate in the analysis of the gravitational characteristics of a poissonian distribution of points, is the probability distribution of the (Newtonian) gravitational force. Such a distribution is known as the Holtzmark distribution \([31]\). We have considered the generalization of the Holtzmark to the case of a fractal set of sources \([32]\). We have shown that, in the case of real structures in finite samples, an important role is played by morphological properties and finite size effects. For dimensions smaller than \(d - 1\) (being \(d\) the space dimension) the convergence of the net gravitational force is assured by the fast decaying of the density, while for fractal dimension \(D > d - 1\) the morphological properties of the structure determine the possible convergence of the force as a function of distance. The relationship between peculiar velocity and gravitational fields has been considered in \([30]\).

8. CONCLUSIONS

The clustering of matter in the universe is hence an important example of the fields in which scale invariance has been observed as a common and basic feature. However, the fact that certain structures exhibit fractal and complex properties does not tell us why this happens. A crucial point to understand is therefore the origin of the general scale-invariance of in the gravitational clustering phenomenon. This would correspond to the understanding of the origin of self-gravitating fractal structures and of the properties of Self-Organized Criticality (SOC) from the knowledge of the microscopic physical processes at the basis of this phenomenon: Most of the scale free phenomena observed in nature are self-organized, in the sense that they spontaneously develop from the generating dynamical process. For example some interesting attempts to understand why gravitational clustering generates scale-invariant structures have been recently proposed by de Vega et al. \([35, 36, 37]\). Basically, the Physics should shift from the study of "amplitudes" towards "exponents" and the methods of modern Statistical Physics should be adopted. This
requires the development of constructive interactions between the two fields.

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References

[1] P.W. Anderson, Science 177 393 (1972).
[2] B.B. Mandelbrot, The Fractal geometry of Nature", (Freeman, San Francisco, 1982).
[3] P.Bak in the Proceedings of the Aspen winter conference in condensed matter 'fifty years of condensed matter physics' (2001)
[4] M. Mezard, in the Proceedings of the Aspen winter conference in condensed matter 'fifty years of condensed matter physics' (2001)
[5] S. Kauffman, in the Proceedings of the Aspen winter conference in condensed matter 'fifty years of condensed matter physics' (2001)
[6] C.J.G. Evertsz et al Eds, Fractal geometry and Analysis", (World Scientific, singapore 1996).
[7] Sylos Labini F., Montuori M., Pietronero L., Phys.Rep. 293, 66 (1998)
[8] Wu K.K., Lahav O.and Rees M. Nature, 225 230 (1999)
[9] Chown M. New Scientist, 2200, 22-26 (1999)
[10] Erzan A., Pietronero L. and Vespignani L., Rev Mod. Phys 67 545 (1995).
[11] EC TMR Network on “Fractal structures and self-organization” ERBFMRXCT980183 http://pil.phys.uniroma1.it/eece1.html
[12] De Lapparent V., Geller M. & Huchra J., Astrophys.J., 343, (1989) 1
[13] Tully B. R. Astrophys.J. 303, (1986) 25
[14] Peebles, P.J.E. "Large Scale Structure of the Universe", Princeton Univ. Press (1980)
[15] Pietronero L., Physica A, 144, (1987) 257
[16] Joyce M., Montuori M., Sylos Labini F., Astrophys. Journal 514,(1999) L5
[17] de Vaucouleurs, G., (1970) Science, 167, 1203-1213
[18] Coles P. Nature 391, 120-121 (1998)
[19] Scaramella R., et al. Astron.Astrophys 334, 404 (1998)
[20] Cappi A. et al., Astron.Astrophys 335, 779 (1998)
[21] Martinez V.J. Science 284, 445-446 (1999).
[22] Landy S.D. Scientific American 456, 30-37, (1999)
[23] Hatton S.J. Mon.Not.R. 310, 1128-1136, (1999)
[24] Gaite J., Dominuguez A. and Perez-Mercader J. Astrophys.J.Lett. 522 L5 (1999)
[25] Gabrielli A. & Sylos Labini F. Europhys.Lett. (Submitted) astro-ph/0012097
[26] S.T. Bramwell, K. Christensen, J.-Y. Fortin, P.C.W. Holdsworth, H.J. Jensen, S. Lise, J. Lopez, M. Nicodemi, J.-F. Pinton, M. Sellitto Phys. Rev. Lett. 84, 3744 (2000)
[27] M. Joyce and F. Sylos Labini Astrophys.J. Letters in the press (2001)
[28] Bahcall N., 1999 In the Proc. of the Conference “Particle Physics and the Universe (astro-ph/9901076)
[29] N. Kaiser, Astrophys. J. Lett. 284, L9 (1984).
[30] Joyce M., Anderson P.W., Montuori M., Pietronero L. and Sylos Labini F. Europhys.Letters 50, 416-422 (2000)
[31] Chandrasekhar S., Rev. Mod. Phys. 15, 1 (1943)
[32] A. Gabrielli, F. Sylos Labini and S. Pellegrini Europhys.Lett. 46, 127-133 (1999)
[33] Gabrielli A., Sylos Labini F. & Durrer R. Astrophys.J. Letters 531, L1-L4 (2000)
[34] M.Joyce, F. Sylos Labini, M. Montuori and L. Pietronero Astron.Astrophys. 344, 387-392, (1999)
[35] De Vega H.J., Sanchez N. and Combes F. Astrophys.J. 500, 8 (1998)
[36] De Vega H.J., Sanchez N. and Combes F. Nature 383, 56 (1996)
[37] De Vega H.J., Sanchez N. and Combes F. Phys.Rev.D. 54, 6008 (1996)