Virtual Modules in Discrete-Event Systems: Achieving Modular Diagnosability

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Abstract—This paper deals with the problem of enforcing modular diagnosability for discrete-event systems that don’t satisfy this property by their natural modularity. We introduce an approach to achieve this property combining existing modules into new virtual modules. An underlying mathematical problem is to find a partition of a set, such that the partition satisfies the required property. The time complexity of such problem is very high. To overcome it, the paper introduces a structural analysis of the system’s modules. In the analysis we focus on the case when the modules participate in diagnosis with their observations, rather than the case when indistinguishable observations are blocked due to concurrency.

Index Terms—Discrete Event Systems, Modular Structure, Distributed Diagnosability

I. INTRODUCTION

Discrete-Event Systems (DES) has successfully concurred significant area in the systems engineering discipline due to their enormous capabilities of designing and managing complex systems. While “real-world” systems are growing in scale, the solutions provided by DES have to evolve to tackle the increasing complexity issues. For that reason the development of solutions relying on the fact that the most of complex systems have naturally modular structure has been under focus for the last two decades. Particularly, the task of design verification and diagnosis with respect to undesired behaviour, commonly refereed as to faulty behaviour of discrete systems, has a fairly developed theory nowadays. In this paper we consider the automata framework for diagnosability analysis, where a behaviour of DES is modeled by regular languages and represented by automata.

Diagnosability analysis requires to verify if one can detect if the system executes a faulty behaviour, i.e. a fault occurred, and to verify if one can isolate a certain type of fault from other faults. This analysis implies that the system’s behaviour can be observed only partially. In DES built from more then one modules it may be necessary to verify if the faults originated from one module can be detected by observing only the same module, or by observing only other modules, or under other implications with respect to the possible flow of observations. Moreover, the verification of a modular system is preferred to be done without composing its entire model from the system’s components since such composition may be not even feasible to perform due to correspondent high computational burden.

The approaches aimed to solve the problem of diagnosability verification consider different architectures of DES and differ with respect to some implications they take into account, i.e. either they require an entire model of the system or not, what information is presented to each observation spot if any, and etc. In this paper we use a classification as in [1] where a diagnosability approach can be centralized, decentralized or distributed. The centralized approach is presented in [2] and, with improved complexity, in [3] and [4]. The decentralized approach can be found in [5], [6], [7], [8] and others. The distributed approach is presented in [9], and a related notion of modular diagnosability is introduced in [10]. We briefly review all the approaches in the next section.

The original contribution of this paper can be summarized as follows. We consider the distributed approach for a DES with modular structure, i.e. no entire model of the system is presented and the fault diagnosis procedure assumes that observation spots can not communicate. As the starting point we consider the definition of modular diagnosability property and the correspondent verification algorithm from [10]. We assume that the systems’ modules are given by its designer. The design may reflect a physical or functional structure of the system, or may follow other underlining design principals, which make the modules natural for the designer. Thus, we assume that preserving the systems’ modularity as close as possible is required. We investigate the case when the system is not modular diagnosable initially, but the modules of the system can be composed into new virtual modules in order to force the modular diagnosability property. We study how to choose the system’s modules for the composition. For this goal a structural analysis of the system is introduced.

We refer to [11] as to one of the recent works addressing diagnosability problem for distributed approach. The work introduces a notion of regional diagnosability exploiting a subset of the modules. The authors do not address modular diagnosability property, however, we may consider this work as a similar due to the correlated notion, and optimisation techniques, which can be applied in our approach.

This paper is organized as follows. Section II covers the necessary notation and describes the diagnosability problem. Section III reviews diagnosability verification of a modular system. In Section IV we focus on diagnosability by virtual
modules, and analysis of system’s module structure with respect to the faulty behaviour. The Section [V] shows an example. The last Section [VII] concludes the current results and discusses possible directions for the further research.

II. Preliminaries

A. Notation

The notation used in this document is the one in [12]. Let \( \Sigma \) be a finite set of events. A sequence of events is a string. \( \Sigma^* \) denotes a set of all finite strings over \( \Sigma \). \( L \subseteq \Sigma^* \) is a language over \( \Sigma \). Given strings \( s \) and \( t \), \( st \) is their concatenation. Given strings \( s \) and \( w \), \( w \) is a prefix of \( s \) if there exists \( t \) such that \( wt = s \). Prefixed closure of \( L \), denoted by \( \bar{L} \) is a set of all prefixes of all the strings in \( L \). If \( \bar{L} = L \) then \( L \) is prefix-closed. The post language of \( L \) after a string \( s \) is denoted as \( L/s \), i.e. \( L/s := \{ t \mid st \in L \} \). We write \( \sigma \in s \) if the event \( \sigma \in \Sigma \) appears in the string \( s \in \Sigma^* \). If \( \{ s \} \) is a singleton, we write \( s \) for operations on languages.

An automaton \( G \) is a tuple

\[
G := \langle X, \Sigma, \delta, x_0, X_m \rangle,
\]

where \( X \) is a set of states, \( x_0 \in X \) is an initial state, \( X_m \subseteq X \) is the set of marked states, and \( \delta : X \times \Sigma \to X \) is the transition function. We say a language \( L := \mathcal{L}(G) \) is generated or recognized by the automaton \( G \). In this paper we assume that for each language there is always a correspondent automaton, and vice versa. The marked language \( L_m \subseteq L \) is intended to make a part of the automaton’s behaviour distinguishable in a certain context.

Some events of DES can not be observed. To reflect that the set of events \( \Sigma \) is partitioned into disjoint sets of observable events \( \Sigma_o \) and not observable events \( \Sigma_{oo} \), i.e. \( \Sigma = \Sigma_o \cup \Sigma_{oo} \). The \( M : \Sigma^* \to \Sigma_o^* \) denotes the natural projection that erases unobservable events. The correspondent inverse projection is \( M^{-1} : \Sigma_o^* \to 2^{\Sigma^*} \). If a set of events is partitioned into subsets, \( \Sigma := \bigcup_{i \in \mathbb{N}} \Sigma_i \), the natural projection over the partition members is denoted as \( M_i : \Sigma_i^* \to \Sigma_{io}^* \).

Let \( I := \{1, 2, \ldots, n\} \subset \mathbb{N} \) be an index set. A system is defined by a set of automata \( \{G_{i \in I}\} \) and a correspondent set of languages \( \{L_{i \in I}\} \). We use the term local in context of the automata and languages from these sets. The global language of the system is defined by the parallel composition [12] of its local languages: \( L := \|_{i \in I} L_i \). The natural projection is commonly defined over Kleene closure on event sets. We restrict it, for simplicity of notation, to the system’s languages as follows: \( P_i(L) := \{ s \mid s \in L_i \} \), and \( P_i^{-1}(L_i) := \{ s \mid s \in L \} \), \( i \in I \).

B. Architectures for on-line diagnosis

Architectures for on-line diagnosis can be categorized as follows: centralized, decentralized and distributed.

1) Centralized approach: This architecture refers to a global model (language). If the system is modular, then the global language is built by the parallel composition of the local languages. All the observations are performed at one site. In this architecture only one diagnoser \( D \) [2] is constructed. Upon the current state of the diagnoser a decision on the fault occurrence is made. The structure is depicted in Figure 1.

2) Decentralized approach: This approach also exploits the entire model built from its modules, but several local sites perform observations using only local diagnosers. The diagnosers do not communicate to each other, but they provide necessary information (via a protocol) to a central decision node. This architecture is depicted in Figure 2.

3) Distributed approach: The architecture is depicted in Figure 3. The distributed approach does not require to built the entire model of the system. The architecture implies that the system has a set of observation spots, and each spot observes only one module of the system. A communication among observation spots is possible in order to make a decision about a fault occurrence.

The notion of modular diagnosability meets the same architectural implications, and we refer to it as to the distributed approach when the amount of information the observation spots communicate to each other is equal to zero.

III. Diagnosability of a Modular System

Diagnosability analysis uses a notion of a faulty language to describe the faulty behaviour of a discrete-event system.
This section discusses design issues related to representations of the faulty language and focuses on a definition of modular diagnosability.

The faulty behavior is usually modeled by introducing fault events or by faulty specifications. We refer to this approaches as to event-based and specification-based correspondingly. All the aforementioned works exploit the event-based approach, whereas the works [13] and [14] are examples of the specification-based one.

In the event-based approach fault events are a special type of event such that $\Sigma_{af}$ can be disjointed into the sets of faults $\Sigma_f$ and non-faults $\Sigma_{af} \setminus \Sigma_f$. A string containing a fault event is called faulty string. A set of faulty strings is called faulty language, i.e. formally

$$ L_f := \{ s \in L \mid \sigma \in s, \sigma \in \Sigma_f \}. $$

By definition, the faulty language is not necessarily prefix-closed, $L_f \subseteq \mathcal{T}_f$. Thus, in the event-based approach the language of the system can be partitioned into faulty and non-faulty languages, where the non-faulty language is defined as $L_{n,f} := L \setminus L_f$.

In the case of the specification-based approach the faulty specification allows us to define undesired behavior when the fault events are not necessarily introduced. In this case this behaviour can be represented by a marked language $L_f := L_m \subseteq L$. Labeling automata’s states for the same purpose can be considered as an equal technique.

Different types of undesired behaviors (or types of faults) are defined by partitioning $\Sigma_f$ into subsets (not necessarily disjoint) or by several faulty specifications for the same language.

A faulty language defined in the event-based approach can be simply converted into a faulty specification by marking faulty strings, and erasing fault events. Then, we can assume that if fault events are defined, then faulty specifications can also be defined. Consequently, a set of different types of faults requires a correspondent set of specifications. Thus, a method suitable for the specification-based approach implies that it can be adopted for the event-based approach. In this paper, for the sake of unification, we use specification-based approach. For this reason the definitions of diagnosability originally developed by their authors for the event-based approach are slightly modified with no loss of meaning.

For the sake of simplicity, in the following we assume that there is only one type of fault, and that the language of the system is live.

We define diagnosability of a fault as follows:

**Definition 1:** Given a system’s language $L$ with a fault defined by the sublanguage $L_f$. The fault is diagnosable if there is no two strings in the language $L$ with the same observation such that one string is faulty and of arbitrary cardinality, and another is non-faulty, i.e. if the following holds:

$$ \forall(s \in L_f, t \in L_f/s) \quad \exists n \in \mathbb{N} (|t| \geq n) \quad [M(st) \cap M(L_{n,f}) = \emptyset]. $$

(1)

We define diagnosability property of a language as follows:

**Definition 2:** The language is diagnosable if all its faults are diagnosable.

The two above definitions altogether are similar to the Definition 1 in [2]. We recall the statement in [10] proved by Theorem 2, that the global language of the system is not diagnosable only if exists at least one non-diagnosable local language. If all the local languages are diagnosable then the global language is diagnosable. We refer to this property as to a local diagnosability property:

**Definition 3 (Local diagnosability):** Given the set of languages $\{L_i\}_{i \in I}$. The global language $L := \| L_i$ is diagnosable locally if each local language $L_i$ is diagnosable, i.e. if the following holds:

$$ \forall(i \in I, s \in L_i,f, t \in L_i,f/s) \quad \exists n \in \mathbb{N} (|t| \geq n) \quad [M_i(st) \cap M_i(L_{n,f}) = \emptyset]. $$

(2)

The definition of modular diagnosability extends the definition of local diagnosability as it takes into account the case when a faulty string locally indistinguishable in one module becomes distinguishable due to the composition with another module:

**Definition 4 (Modular diagnosability):** Given the set of local languages $\{L_i\}_{i \in I}$ and its correspondent sets $\{L_i,f\}$ and $\{L_i,n,f\}$. The global language $L := \| L_i$ is modularly diagnosable with respect to $M_i$ : $\Sigma^r \rightarrow \Sigma_{i,o}$ if the following holds:

$$ \forall(i \in I, s \in L_i,f, t \in L_i,f/s) \quad \exists n \in \mathbb{N} (|t| \geq n) \quad [M_i(P_i^{-1}(st)) \cap M_i(P_i^{-1}(L_{i,n,f})) = \emptyset]. $$

(3)

It was proved in [10] by Theorem 2, Part 2 that the local diagnosability implies the modular diagnosability\(^4\), i.e.

$$ \forall(i \in I, s \in L_i,f, t \in L_i,f/s) \quad \exists n \in \mathbb{N} (|t| \geq n) \quad [(M_i(st) \cap M_i(L_{i,n,f}) = \emptyset) \Rightarrow (M_i(P_i^{-1}(st)) \cap M_i(P_i^{-1}(L_{i,n,f}) = \emptyset)]. $$

(4)

Recall the Definition of the diagnosable fault. If a module is not diagnosable locally then exist at least two strings in its language, one is faulty and the other one is not, with the same observation of arbitrary length, i.e. the strings are not distinguishable. The indistinguishability can disappear if and only if: $a)$ at least one string is not in its language due to concurrency with other module, and then the strings would be distinguishable locally - the verification of the modular diagnosability property is devoted to find if this is the case; $b)$ indistinguishability is broken globally by interleaving sequences of the module’s events with observable events of other modules. The later case is expressed in the following conjecture:

**Conjecture 1:** Given a system of two modules with languages $L_1$ and $L_2$, and the global language $L := L_1 \| L_2$.\(^4\)

\(^4\)In [13] the authors show that the local diagnosability and modular diagnosability are not comparable but they have a different setup for the problem.
Suppose there is only one faulty string $s \in L_1$ such that it is not distinguishable from at least one string of $L_1 \setminus s$. Thus, $L_1$ is not locally diagnosable. Suppose the system is not modular diagnosable. Then, the global language $L$ is diagnosable only if all the strings $t \in P_1^{-1}(s)$ change their observation due to the composition with the language $L_2$.

The above conjecture gives the insight into the underlining idea of our approach. If we find a module which makes the faulty string distinguishable then the composition of that module with a faulty one would result in a new module satisfying the property of local diagnosability, thus improving the modular diagnosability property of the system. In the following section we provide a formal description of the problem.

IV. Virtual Modules and Structural Analysis

Our goal is to have the system modularly diagnosable. If the initial modularity does not satisfy the property of modular diagnosability then we assume that the set of modules can be partitioned such that all the modules in each element of the partition can be considered as a virtual module, and the system with the new modularity satisfies the property of modular diagnosability.

**Definition 5 (Diagnosability of virtual modules):** Let $I := \{1, 2, \ldots, n\} \subset \mathbb{N}$ be an index set, and $J$ be a partition of $I$. Given the set of local languages $\{L_i\}_{i \in J}$ and its corresponding subsets $\{L_{i,j}\}$ and $\{L_{i,n,f}\}$. The global language $L := \bigl\| L_i \bigr\|$ is modularly diagnosable with respect to $M_j: \Sigma^* \rightarrow \Sigma_{j,o}^{*} | j \in J$, $\Sigma_{j,o} := \bigcup_{i \in J} \Sigma_{i,o}$ if the following holds:

\[
\forall(i \in I, s \in L_{i,j}, t \in L_{i,j}/s) \\
(\exists n \in \mathbb{N})(|t| \geq n) \\
\left[ M_j(P_i^{-1}(st)) \cap M_j(P_i^{-1}(L_{i,n,f})) = \emptyset \right].
\]

If $\forall j \in J, L_{i,j} := \bigl\| L_{i,j} \bigr\|$ then, by definition, $L := \bigl\| L_{i,j} \bigr\|$ and all the statements related to the modular diagnosability property can be applied for the diagnosability by virtual modules.

The diagnosability problem with virtual modules can be solved in two ways. In the first, in order to find a partition of system’s modules satisfying the modular diagnosability property one may take a faulty module, enumerate all possible sets of other modules, compose all the languages from each set with the faulty language, and check the resulting language for modular diagnosability. Since, in general, there may be many partitions such that the system is diagnosable with virtual modules, only one partition should be chosen taking some heuristic guiding criteria. The entire process is computationally expensive, since the number of possible partitions $J$ is double exponential with respect to the cardinality of $I$. However, not each module can change diagnosability. Consequently, we propose the second approach that can significantly decrease the complexity by selecting only the modules which can probably change diagnosability and, thus, check only the partitions made of such modules. For this purpose a procedure to check if an arbitrary module potentially can change diagnosability is required. Then, we can have a heuristic procedure to choose which module to pick to verify diagnosability.

In the sequel, for the sake of simplicity, we suppose that the system consists only of two modules with the correspondent languages $L_1$ and $L_2$. The language of the system is $L := L_1 \parallel L_2$. Suppose that only one module has a faulty behaviour: $L_1 := L_{1,f} \cup L_{1,n,f}$. Suppose that $L_1$ is not diagnosable locally, but $L$ is diagnosable.

Firstly, we define the notion of observation changing of a string in a global language.

**Definition 6:** Given two languages $L_1$ and $L_2$, a string $s \in L_1$ changes its observation $M_1(s)$ in the language $L$ if there is no the same observation in $P_1^{-1}(s)$, i.e. if the following holds:

\[
M(L) \cap M_1(s) = \emptyset. \tag{6}
\]

**Lemma 1:** Given two languages $L_1$ and $L_2$, and a string $s \in L_1$. Assume that $s \in P_1(L)$. The string $s$ changes its observation in the language $L$ if and only if:

\[
(\exists \sigma \in s | \sigma \in \Sigma_1 \cap \Sigma_2) \land \tag{7a}
(\forall t \sigma \in L_2) [M_2(t) \neq \emptyset], \tag{7b}
\]

where $M_2 : \Sigma^* \rightarrow (\Sigma_2,o \setminus \Sigma_1)^*$. \tag{7c}

**Proof:** In order to prove sufficiency of $(7)$ we use its converse relation and prove by contradiction that the change of observation $(6)$ is necessary. Assume $\exists w \in L$ and $\exists s \in L_1$ such that $M(w) = M_1(s)$ and, therefor, $(6)$ is false. Let $\exists \sigma \in \Sigma_1 \cap \Sigma_2$ such that $\sigma \in w$ and also $(7a)$ holds. Then may $\exists u \sigma \in w$ such that $M_2(u) = \emptyset$, and then $M_2(P_2(u)) = \emptyset$ which contradicts $(7b)$. Now, let $(7b)$ be true for all $t \sigma \in P_2(w)$. Then the assumption $M(w) = M_1(s)$ holds only if $\sigma \notin s$, which contradicts $(7a)$.

We prove necessity of $(7)$ by contradiction. Let $(7a)$ holds, and $\exists \sigma \in L_2$ such that $M_2(t) = \emptyset$. Then may $\exists t \sigma \in L \subseteq P_2^{-1}(t \sigma)$ such that $M_2(t \sigma) = \emptyset$ and $M_1(t \sigma) = M_1(s) \neq \emptyset$, which contradicts $(6)$. Now, let $(7b)$ holds and $\exists \sigma \in s \cdot w \in L_1 \cap \Sigma_2$. Then may $\exists u \sigma \in w \in L \subseteq P_2^{-1}(w)$ such that $M(w \sigma) = M_1(s)$, which contradicts $(6)$.

Informally, the above lemma says that the string of the local language $L_1$ changes its observation in the global language $L$ if and only if the string has an event in common with the language $L_2$, and all the strings of $L_2$ which have this common event have observable events in the prefixes, and some of the observable events in the prefixes are not common with $L_1$.

We call the subset of stings $\{t \in L_2\}$ satisfying condition $(7b)$ as the adjacent observable support for the given string $s \in L_1$.

**Definition 7:** Given two languages $L_1$ and $L_2$. We say that a string $s \in L_1$ is distinguished from all the other local strings $L_1 \setminus s$ in the language $L$ if the following holds:

\[
(\forall w \in L_1 \setminus s) \\
\left[ M_{P_1^{-1}(w)}(L_2) \cap M_{P_1^{-1}(s)}(L_2) = \emptyset \right]. \tag{8}
\]

**Lemma 2:** Given two languages $L_1$ and $L_2$. Assume that $L_1 = P_1(L)$. The string $s \in L_1$ is distinguished from $L_1 \setminus s$...
in the language $L$ if $s$ has an adjacent observable support $L_{2,s} \subseteq L_2$ which satisfies the following condition:

$$(\forall t \in L_{2,s}) [(\exists \sigma \in t \mid \sigma \in \Sigma_1 \cap \Sigma_2) \land (\forall w \in L_1 \setminus s \mid w \not\in w') \land (\forall t' \in T) [M_2(t/t') \not\in \emptyset],$$

where $M_2$ is defined as in (7c).

Proof: Assume $[4]$ is false, i.e., $\exists w' \in P^{-1}_1(w)$ and $\exists t' \in P^{-1}_2(s)$ such that $M(w') = M(s')$.

Assume $[2a]$ and $[9b]$ hold. Then may $\exists t' \in T$ such that $M(t') = M_{2}(w)$, and $t \in P_2(t')$ such that $M_2(t) = M_2(w)$. And may $\exists t' \in T$ such that $M_2(t) = M_2(w)$. Since $\sigma \in t$ and $\sigma \not\in w$, then $M(t \sigma') = \emptyset$ for any $t' \sigma' \in T$, which contradicts $[9c]$.

Assume $[2a]$ and $[9c]$ hold. Let $M_1(L_1) = \emptyset$ and $M_2(L_2) = M_2(s) = M_2(s')$. Then $\forall s' \in M(P^{-1}_1(s))$ there exists $s' \in T$, which contradicts $[9b]$.

Assume $[2b]$ and $[9c]$ hold. If $[8]$ is false, then $[2b]$ is false. However, $[9c]$ is sufficient for $[2b]$, which contradicts the former statement.

Informally, the above lemma says that a string $s$ becomes distinguishable from the other strings $L_1 \setminus s$ in the global language, when the occurrence of events from the observable support happens only in $P^{-1}(s)$ due to common events. Thus, whenever we observe events of the observable support of $L_2$, we are sure the string $s$ is being executed.

As it was discussed in the Section 3 indistinguishability can be changed either by blocking the string in the local language due to concurrency, or by interleaving with observable events from other languages. Under assumption that all the strings are not affected by concurrency, i.e., $L_1 = P_1(L)$ we can deduce, that the conditions of Lemma 2 are also necessary for changing distinguishability.

The Figure 4 depicts an automaton which accepts the sublanguage of $L_2$ satisfying conditions (7a), (5a) and (9c) of the above lemmas. The Figure 5 depicts an automaton which marks a sublanguage of $L_1$ satisfying conditions (7a) and (2a).

A procedure verifying if a string $s \in L_1$ is distinguishable in the global language $L$ consists of two steps. First, the string $s$ should be marked by the automaton depicted in the Figure 5. Then the set of common events $\Sigma_1 \cap \Sigma_2$ is reduced to the set of events causing transitions in the automaton. Second, all the continuations of the strings of the language $L_2$ which have these common events should be accepted by the automaton depicted in the Figure 4.

Now we are ready to apply Lemma 2 with respect to diagnosability property, but make some notes before. Intuitively, one would apply the conditions of the lemma for faulty and non-faulty languages. Recall, that faulty and non-faulty languages are disjoint, but they may have common prefixes. This common sublanguage is defined as $\overline{L_f} \cap (L \setminus L_f)$. Changing observability of this sublanguage has no effect for diagnosability, and we can exclude it from a verification procedure. Thus, the non-faulty sublanguage disjoint to all the prefixes of the faulty language is defined as $L \setminus L_f$, and the set of all prefixes of the faulty language disjoint to the above non-faulty sublanguage and to the common prefixes is defined as $\overline{L_f} \setminus (L \setminus \overline{L_f})$.

Lemma 3: Given $L_1, L_2, L_1 \cap L_2 \subseteq L_1$ and $L_1, L_2 \subseteq L_1$. A language $L := L_1 \parallel L_2$ is diagnosable if the sublanguages $L_f \setminus (L \setminus L_f)$ and $L_1 \setminus L_1$ have distinguished observable supports in $L_2$.

The proof can be deduced from the Lemma 2.

The automata depicted in Figures 4 and 5 can not be simply used in a procedure verifying diagnosability, since we should avoid the verification of $\overline{L_1,f \setminus L_1,f}$ and $L_1,nf \setminus L_1,nf$. We leave development of such procedure for future work. However, the automata can be used to demonstrate the approach in a trivial case, as it is shown in the next section.

V. Example

Consider the system of two automata $G_1$ and $G_2$ depicted in Figure 6 and Figure 7. The set of events for the system is $\Sigma = \{a, b, c, e, f\}$. Suppose the observable events are $\Sigma_a = \{c, e\}$, and the set of fault events is $\{f\}$. Thus, only the language $L_1$ has a fault, and $L_2$ has not. We use the verifier [4] to check if a language is diagnosable. The verifier for the language $L_1$ is depicted in the Figure 8. One can check that it has an indeterminate cycle. The strings $fbc^*$ and $ac^*$ are not distinguishable in the local language $L_1$. Hence, $L_1$ is not locally diagnosable.
We now use the verification procedure described in this paper to check if the language $L_2$ can change observation of either strings $fbc^*$ or $ac^*$ in the language $L_1 \parallel L_2$ such that the strings become distinguishable. The set of events common for $L_1$ and $L_2$ is $\{b, c\}$. It can be verified that only the strings $fbc^*$ are marked by the automaton depicted in the Figure 5. Next, it can be verified that all the strings of $L_2$ which have events common with the strings $fbc^*$ are accepted by the automaton depicted in the Figure 4. Thus, we conclude that $L_2$ changes observation of the strings $fbc^*$ in the virtual module $G$ built of modules $G_1$ and $G_2$, such that $G$ becomes diagnosable. Indeed, if we make a parallel composition of the modules and build a verifier for the result as it is depicted in the Figure 2, it can be checked that the verifier has no indeterminate cycles.

VI. CONCLUSION

In this paper we introduced a notion of virtual modules for DES, and proposed a new definition of modular diagnosability by virtual modules. The approach suggests to combine the existing modules of the system into virtual modules in a way that the system with the new modularity is modular diagnosable.

We introduced a structural analysis of the system’s modules, which allows to verify if a module may change its observation by composition with others, and if a module can change the observation of other modules. We defined correspondent sufficient and necessary conditions for the modules’ languages. If the languages satisfy those conditions then one can state that the system can be made modular diagnosable by creating virtual modules. The suggested verification procedure has linear complexity with respect to the number of states of a module.

We are actually working on the problem of defining criteria of how to select the best candidates for creating the virtual modules, what the optimal partition of the set of the modules can be, and generalization of the problem.

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