Black hole as an Information Eraser

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ABSTRACT: We discuss some puzzles on the identity of black hole entropy by using the Landauer’s principle of information erasure. Especially, we discuss the information freezing on the black hole horizon in terms of information erasing process. We also calculate the quantized black hole mass spectra and entropy assuming that the black hole processes information in unit of bit. The black hole entropy gains a sub-leading contribution proportional to the logarithm of area in addition to the usual areal term without an artificial cutoff. We also argue that the minimum of a black hole mass is $\sqrt{\log 2/(8\pi)}M_P$.

KEYWORDS: black hole, Landauer’s principle, quantum black hole

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1. Introduction

Close connection between quantum information theory and general relativity have been discussed by Peres and Terno [1] and Hosoya [2]. One of their results is that generalized second law of black hole thermodynamics is satisfied even with quantum information [2], which was reexamined by Song [3]. Recently, roles of vacuum entanglement around black hole horizon has been taking much attentions [4, 5, 6, 7, 8, 9, 10, 11, 12]. Due to its area proportionality, the entanglement entropy [5, 6] appears to be a natural candidate for a black hole entropy [7, 8, 9]. Especially, Ryu and Takayanagi [10] and Fursaev [11] have studied entanglement entropy in the context of holographic anti-de Sitter/conformal-field-theory correspondence and Solodukhin [12] generalized it to include entanglement entropy of black holes living on the boundary of anti-de Sitter space. Role of information theory on cosmology is also discussed [13] in relation to dark energy from information erasure. Since entanglement entropy is related to non-locality of quantum theory, quantum information theory [14, 15] can plays an important role in black hole physics. The relation of the entanglement to Hawking radiation was studied [16, 17] too. In Ref. [11], it is conjectured, on the base of the relation between the entropy and the action, that in a fundamental theory the ground state entanglement entropy per unit area equals $k_Bc^3/(4G\hbar)$. Following their argument, the black hole entropy might be fully due to the vacuum entanglement. Horowitz and Maldacena [18] proposed a conjecture on final state of black hole evaporation as a completely entangled state, which makes the black hole evolution be unitary. It is also noted that information inside the horizon can be transferred to outside by means of quantum teleportation [19, 20].

In this paper we add another close connection between information theory and relativity, the Landauer’s principle of information erasure [14, 15] and the first law of black hole thermodynamics. A black hole satisfies the four laws of black hole mechanics [21], which are essentially the same as those of thermodynamics. Especially for a Schwarzschild black hole the first law

$$d(Mc^2) = \frac{\kappa c^2}{8\pi G} dA$$

(1.1)
relates its mass $M$ to horizon area $A$, where $\kappa$ is the surface gravity of the black hole. Hawking [22] found that a black hole really radiates as if it is a black body with temperature, $T_H = \frac{\hbar \kappa}{2\pi k_B c^2}$, by studying quantum field theory in classical background black hole spacetime. From the first law (1.1) and the analogy of $Mc^2$ with thermodynamic energy, the entropy of black hole [23] is given by

$$S_{bh} = \frac{k_B A}{4 l_p^2} \tag{1.2}$$

where $l_p = \sqrt{\frac{\hbar G}{c^3}}$ is the Planck length. The explicit presence of $\hbar$ in the Hawking temperature denotes the quantum mechanical nature of the black hole radiation. The first law relates the mass and entropy of black holes, however the origin of this link is not known. There have been appeared many different approaches to explain the origin of the black hole entropy including string theory [24] and brick wall method [25, 26]. The quantum mechanical nature and the geometric property of the black hole entropy give rise to many speculations on their quantum gravity origin, such as the entropy bound [27, 28], the holographic principle [24, 30], the holographic dark energy model [31], and the spacetime noncommutativity [32, 33]. These approaches try to find an appropriate models of the horizon or internal structures of the black hole which provide correct description of the black hole entropy.

In this paper, we suggest an alternative route to the problem. Rather than guessing the internal structure of the black hole, we ask “What happens when information is lost across the horizon?” employing the Landauer’s principle of information erasure. A merit of this approach is that we do not need to worry about the internal structure of the black hole, hence the results would be general, since we directly resort to our ignorance on missing information. We argue that the black hole mass and its relation to the entropy comes from the facts that “black hole has maximal entropy”, with which we postulate a principle of information erasure for black hole: black hole is a most efficient information eraser. We also ask black hole’s quantum nature from the viewpoint of information erasure since a black hole hides all incoming information except for global ones.

In 1961, Rolf Landauer had the important insight that there is a fundamental asymmetry in the way that the nature allows us to process information [14]. There, he showed that copying classical information can be done reversibly and without wasting energy, but when information is erased there is always an energy cost larger than $k_B T \log 2$ per classical bit to be paid [15]. Explicitly, consider an 1-bit memory which consists of a cylinder and an atom in it as shown in Fig. 1. A thermal bath with temperature $T$ keeps in contact with the cylinder. The location of the atom, the left or the right partition, represents a memory. To erase this memory we use the piston to push the atom into the left partition regardless of its initial position. Then, the initial information of the atom is erased irreversibly because we will never find out where the atom was originally. According to the Landauer’s principle the entropy of the cylinder decreases by $k_B \log 2$ at the cost of at least $k_B T \log 2$ free energy consumption. This energy is eventually converted to thermal energy.
and increases the total energy of the whole system (the bath + the cylinder) by

$$ \delta E \geq k_B T \log 2. \quad (1.3) $$

In this work, we will describe Schwarzschild black hole using the information erasing procedure in Sec. II. Some puzzles about the black hole entropy are discussed and resolved using the Landauer’s principle of information erasure. In Sec. III, quantized black hole mass spectrum is obtained by using the fact that the information is quantized by bit. It is shown that there is a minimum value of black hole mass. In section IV, we summarize the results and discuss them. We also briefly mention the possibility of experiment on the minimum black hole mass.

2. Black hole as an information eraser

The quantum mechanical nature of black hole is represented by the Hawking radiation, which was explained as a quantum tunneling effect around horizon [34]. Even though it is physical, the Hawking radiation is dependent on the choice of coordinates in the sense that a free falling observer can not feel its presence since the system is in his vacuum state, Hartle Hawking vacuum. This confliction for different coordinates systems is explained due to the presence of event horizon and coordinates singularity there.

There are some puzzles related to the identity of black hole entropy. Consider a drop of particle of energy $m$ into a Schwarzschild black hole of mass $M \gg m$. For simplicity, we assume that the particle does not contain information except for the energy. However, once the particle falls into the black hole, the mass of the black hole is increased by $m$ and the entropy is also increased by the amount proportional to $Mm$ following the entropy formula (1.2). A natural question on the origin of the entropy aries “where does the entropy come from?” One may think that the entropy contained into the particle will be absorbed into the black hole since the information of the particle becomes invisible to the outside observer. However, this cannot explain the difference of the entropy increased by the mass $m$ from the information contents contained in the particle. Since no information except for the global energy go into the black hole, the origin of the increased entropy becomes

Figure 1: Information erasing process of an 1-bit memory consists of a cylinder and an atom.
obscure. We can also think the converse situation: two particles of the same energy but have different information contents (e.g. $I_1$ and $I_2$) fall into black hole. Once the two particles are engulfed by the black hole, the resulting change of the black hole entropy are exactly the same irrespective of its informational contents. Then, where does the information of the particle gone? What is the black hole entropy really represent? These are basic questions on the identity of black hole entropy.

Consider a black hole spacetime in which a radially freely falling rocket emits a light ray of a given frequency periodically to $r \to \infty$. A freely falling observer with the rocket may notice that the rocket enters simply into the black hole and may not appreciate even the presence of the horizon. On the other hand, from the point of view of outside stationary observer, there appears another coordinates dependent phenomenon: To the observer, the rocket takes infinite time to get to the horizon and permanently approaches to the horizon and the waves radiated by the rocket experience gravitational red-shift. In this sense, the “information freezing” happens around the horizon since the information contained in the rocket is freeze there. To emphasize this phenomenon, the black hole was once called as a frozen star.

The Hawking radiation is regarded as a physical phenomenon and it leads to the decrease of the black hole mass. In contrast, the “information freezing” at the horizon is regarded as a simple coordinates artifact and is short of physical implication. Even though there exist differences between the information freezing and the Hawking radiation, their origin is the same: The event horizon breaks the general covariance. Therefore, it is plausible to think that the information freezing may also represent certain physical phenomena. In this paper, we provide a possible explanation which may happen around the horizon to resolve the entropy puzzles and discuss physical meaning of the information freezing and erasing which happens at the black hole horizon.

Figure 2: The black hole is surrounded by a system, intermediate region of information. The energy going into the black hole is used to erase the information of the system.

If collapsing of matter into a black hole is a simple in-falling process even to the outside observer, these entropy problems cannot be resolved. Therefore, we suggest that the freezed information compose a “system” in Fig. 2 around the horizon. The “system” does the same role as the cylinder in the previous example. We regard the “system” is
in thermal contact with the black hole, which does the role of thermal bath, in Hawking temperature $T = T_H = \frac{\hbar c^3}{8\pi k_B G M}$. For example, in systems with entangled states, the use of the Hawking temperature as the system’s temperature was justified [35]. Once a particle falls into the horizon, its energy is included into the black hole mass, the thermal bath. In this sense, we regard the energy of in-falling particle is used to erase the information of the “system”. Therefore the energy of in-falling particle does the role of the work done by the piston in the previous example.

For simplicity, we assume the “system” consists of bits. We divide the in-falling procedure into three parts: First, the energy of the particle is used to erase system’s information. During one bit of information erasure, the entropy of the “system” decreases to zero and this entropy is transferred to the thermal bath through the thermal contact to increase the black hole entropy by $k_B \log 2$ per a bit. The energy provided to the “system” is also absorbed into the black hole. Second, the increased entropy of the black hole gives rise to the increase of the black hole horizon, which engulfs the system itself. Third, the information of in-falling particle are freeze and eventually contained into the “system”. This explains how the “system” is constructed and how the incoming information are erased consistently.

The energy used to erase the system’s information satisfies the Landauer’s principle. Here we postulate a principle of information erasure for black hole: “black hole is a most efficient information eraser.” This principle is in parallel with the fact that the black hole is a maximal entropy object in the sense that minimum energy is required to erase a given quantity of information. Physically, this implies that the information erasing process is optimal so that the energy provided to the system by in-falling matter during the erasing process saturates the Landauer’s bound, $\delta E = k_B T_H \log 2$. As a result, this saturating energy $\delta E$ is transferred into the black hole to increase its mass by

$$\delta (Mc^2) = \delta E = k_B T_H \log 2. \quad (2.1)$$

The right hand side of Eq. (2.1) is just the black hole temperature $T_H$ times the increased entropy, $\delta S = k_B \log 2$, of the black hole in unit of $k_B$ during the process of information erasure. Note that the relation (2.1) does not comes from the first law of black hole thermodynamics. It just says that the energy $k_B T_H \log 2$ used to erase the information is transferred from the “system” to the black hole through the thermal contact. The relation with the entropy is provided by the principle of information erasure of black hole. In this way, the increase of the black hole mass $\delta M$ is related with the information erasing of the “system” directly.

From now on in this letter, we use the natural unit and set $k_B = 1 = G$. For macroscopic black hole, we may assume $\delta S = k_B \log 2$ to be infinitesimal. Using $M(S = 0) = 0$ and $T_H = 1/(8\pi M)$ we integrate the equality in Eq. (2.1) to get the relation between the black hole mass and the entropy for erased information

$$M(S) = \sqrt{\frac{S}{4\pi}}. \quad (2.2)$$

Note that the black hole mass is directly obtained from its entropy contents through the principle of information erasure for black hole. If some mass $m_0$ can enter into the horizon without information erasure, then the black hole mass formula should be changed to
\[ M(S) = \sqrt{\frac{S}{\pi}} + m_0 \] due to the energy conservation law with \( m_0 \) be independent of the parameters of black hole. However, the known formula for black hole entropy fixes \( m_0 = 0 \), which implies no incoming energy go into the horizon without information erasing. This is why the contents of missing information provides the correct relation between the black hole entropy and mass.

### 3. Quantum black hole from information erasing

We now consider quantization of black hole spectrum by using a sequence of \((N - 1)\)-bits of information-erasing process bit by bit. Consider a small black hole with entropy \( S_1 = \log 2 \). We leave its mass \( M_1 \) as a free parameter since quantum gravitational natures will be important for a microscopic black hole. We determine \( M_1 \) later so that it maximizes the entropy of macroscopic black holes. To avoid conical singularity of Wick rotated black hole metric, it is natural to select the black hole temperature to be

\[ T_1 = \frac{1}{8\pi M_1} \]

Later in this paper, we discuss the possibility that the quantum gravity effect may change this temperature mass relation. After one bit of erasing, the black hole mass is increased by

\[ M_2 - M_1 = \frac{\log 2}{8\pi M_1} \]

and its temperature becomes

\[ T_2 = \frac{1}{8\pi} \left( M_1 + \frac{\log 2}{8\pi M_1} \right) - 1 \]

We may repeat this procedure which can be represented with the recurrence formula

\[ X_{n+1} = X_n + X^{-1}_n, \quad (3.1) \]

where the temperature and the mass of the black hole are

\[ M_n = \sqrt{\frac{\log 2}{8\pi}} X_n, \quad T_n = \sqrt{\frac{1}{8\pi \log 2}} \frac{1}{X_n}, \quad n \geq 1, \quad (3.2) \]

respectively. The informational entropy of final black hole of mass \( M = \sqrt{\frac{\log 2}{8\pi}} X_N \) is

\[ S_{bh} \equiv S_N = S_1 + (N - 1) \log 2 = N \log 2. \quad (3.3) \]

In the large \( N \) limit, we should have Eq. (2.2), which gives the limiting behavior, \( X_N \to \sqrt{2N} \).

Instead of solving the recurrence formula (3.1) exactly, we try to find an approximate solution. Summing over \( n \) after multiplying \( X_n \) to Eq. (3.1), we get

\[ N - 1 = \sum_{n=1}^{N-1} X_n \delta X_n = \int_{X_1}^{X_N} X \, dX - \frac{1}{2} \sum_{n=1}^{N-1} \delta X_n^2 \]

\[ = \frac{X_N^2 - X_1^2}{2} - \frac{1}{2} \sum_{n=1}^{N-1} \frac{1}{X_n} \delta X_n, \quad (3.4) \]

where \( \delta X_n = X_{n+1} - X_n \). To find an approximate expression for Eq. (3.4), we introduce a large value \( H \gg N \) in Eq. (3.4) and divide the summation into two parts: 1 to \( H \) and \( H \) to \( N \). Then, we approximately change the second summation into integral form \( \int_{X_H}^{X_N} X^{-1} \, dX \) for large \( N \) to get,

\[ N - 1 = \frac{1}{2} \left( X_N^2 - \log X_N - 1 - \tilde{\gamma}(X_1) \right), \quad (3.5) \]
where the function $\tilde{\gamma}(X_1)$ has a well defined value,

$$\tilde{\gamma}(X_1) \equiv \lim_{H \to \infty} \left( \sum_{n=1}^{H-1} \frac{1}{X_n^2} - \log X_H \right) + X_1^2 - 1. \quad (3.6)$$

Note that $\tilde{\gamma}(X_1)$ is guaranteed to converge since the limiting procedure is the same as that of the Euler gamma because $X_n \sim \sqrt{2n}$ for $n \gg 1$. $\tilde{\gamma}(X_1)$ is a function of $X_1$ only and is minimized at $X_1 = 1$ as seen in Fig. 3. This $X_1 = 1$ is not numerical but exact, since $X_n$ with $n \geq 2$ is a function of $X_1 + X_1^{-1}$, of which derivative vanishes at $X_1 = 1$. As seen in Fig. 3: $X_N(X_1)$ (black curve) and $\tilde{\gamma}(X_1)$ (gray curve) as a function of $X_1$. In this figure we use $N = 10^4$. The minima of $X_N$ and $\tilde{\gamma}(X_1)$ are placed at $X_1 = 1$ and its value is $X_N \simeq 141.43659$ and $\tilde{\gamma}(1) \simeq 0.376569$.

Fig. 1, for fixed $N$, $X_N$ is minimized at $X_1 = 1$ too.

The entropy of a black hole is given in terms of $X_1$ and $X_N$ by

$$S_{bh} = \frac{\log 2}{2} \left[ X_N^2 - \log X_N + 1 - \tilde{\gamma}(X_1) \right].$$

For a given mass $M$, this entropy will be maximized for $X_1 = 1$, which determines the initial mass

$$M_1 = \sqrt{\frac{\log 2}{8\pi}}. \quad (3.7)$$

Therefore, the informational entropy of a black hole with mass $M = M_1 X_N$ is

$$S_{bh} = \frac{\log 2}{2} \left( \frac{M^2}{M_1^2} - \log \left( \frac{M}{M_1} \right) \right) + 1 - \tilde{\gamma}(1), \quad (3.8)$$

where the mass is measured in the Planck unit and $\tilde{\gamma}(1) \approx 0.37657$.

As seen in Fig. 4, the approximate formula (3.8) provides the correct black hole entropy for $n \geq 2$. Because of the negative sign in front of $\log M$ in Eq. (3.8), the informational black hole entropy is slightly smaller than the classical one given by the area law.

In addition, the formula (3.8) also gives the quantized black hole mass because the informational entropy (3.3) is $N \log 2$ with integer $N$. The mass spectrum of the spherical black hole is given by solving,

$$\frac{M^2}{M_1^2} - \log \left( \frac{M}{M_1} \right) = 2N - (1 - \tilde{\gamma}(1)), \quad (3.9)$$
Figure 4: Entropy $S_{bh}/\log 2$ as a function of black hole mass squared $X^2 = \frac{8\pi}{\log 2} M^2$ in log-log plot. The gray curve and the dashed line denote the logarithm of Eq. (3.7) and the logarithm of the classical black hole entropy formula $4\pi M^2$, respectively. The dots denotes the entropy for each quantized mass of black hole for $n = 0, 1, 2 \cdots$ obtained from the recurrence relation (3.1). The informational black hole entropy (dots) is slightly smaller than the classical expectation (dashed line).

where $N = 1, 2 \cdots$ is an integer.

The quantization of black hole and its effects on black hole entropy were discussed by several authors [36, 37, 38, 39]. Based on the adiabatic invariance of the horizon area, Bekenstein [36] argued that the horizon area is quantized in unit of $4 \log k$. In addition, Hod [38] determined $k = 3$ using the Bohr’s correspondence principle and the asymptotic behavior of the ringing frequencies [40] of black hole’s quasi-normal modes. Our result is slightly different from Hod’s by the factor of $\log 2$. If for some reason, the black hole cannot absorb the information of bit, or the information of the “system” are not arranged as bit but arranged as trit with unit of 3, then the present result should be rewritten with $\log 3$. This results approach to that of the Hod [38] and Corichi [39] for large $N$. This is directly related to the quantum gravitational structure of the event horizon.

In the presence of quantum effect, the black hole horizon area may fluctuate, which makes the surface gravity and the temperature also fluctuate. In addition, for small black holes, the change of geometry with one bit of information erasure is non-negligible. In this case, the assumption of the black hole as a heat bath with constant temperature fails. We present a rough estimation which can take care of the effects. The consumption of one bit decreases the temperature of the black hole to $T_{n+1} = \sqrt{\frac{1}{8\pi \log 2} \frac{1}{X_n + X_{n+1}}}$ from $T_n$. Therefore, to calculate the increased mass effectively, we may use an intermediate temperature $T'_n = \sqrt{\frac{1}{8\pi \log 2} \frac{1}{X_n + \alpha_n X_n}}$ between $T_n$ and $T_{n+1}$, where $\alpha_n$ is a $n$ dependent constant smaller than one. This modification changes the recurrence relation to $X_{n+1} = X_n + (X_n + \alpha_n X_n^{-1})^{-1}$ and alters the sub-leading logarithmic contribution of the entropy from $-\frac{\log 2}{2} \log M$ to $\frac{\log 2}{2}(\alpha - 1) \log M$ if $\alpha_n = \alpha$ is independent of $n$.

4. Summary and discussions

The Landauer’s principle, the information erasure accompanies the energy cost, provides
a direct explanation of black hole mass from erased information by the black hole horizon. We have interpreted the information freezing and information erasing around the black hole horizon using the Landauer’s principle of information erasure. With this and assuming the black hole to be a most efficient information eraser, we reproduce successfully the first law of black hole thermodynamics by using microscopic information erasing process. We also discussed some puzzles on the identity of black hole entropy and argued clearly that the black hole entropy counts the contents of missing information rather than the internal degrees of freedom.

In conclusion, all incoming energy are used to erase the information around the horizon. The details of the erasing process are an interesting subject of future research. The present result illuminates the true nature of the black hole entropy. The contents of missing information provides the correct relation of the black hole entropy and mass relation. Therefore, the black hole entropy counts not the internal degrees of freedom but the missing information behind the horizon. This conceptual fact has not yet been argued clearly in the previous literatures.

We calculated the quantized the black hole mass using the fact that the information is quantized by bit. We have shown that there exists a minimum mass of a black hole, which has entropy corresponding to an 1-bit of information erasure. The presence of minimum of black hole mass \( M_{\text{min}} = M_1 = \sqrt{\log 2/(8\pi)} \) also implies the presence of maximum of black hole temperature, \( T_{\text{max}} = (8\pi \log 2)^{-1/2} \simeq 0.24 \). Since Planck scale vacuum fluctuation may compose the Planck scale black hole, the existence of maximal temperature of black hole implies that the absence of black hole with temperature larger than the critical temperature \( T > T_{\text{max}} \), however many energy is confined in a small region. Another insight one get from this result is that the black hole mass spectrum is discrete. Since the mass gap is big for small black hole, a particle of small energy may have difficulties in entering the black hole horizon \[1\]. This mass gap may prevent a small black hole from becoming a larger one and we need a mechanism to circumvent this situation. If the Planck scale is about TeV as in brane world scenarios \[2\], the discreetness of the mini-black hole might be observed at the Large Hadron Collider (LHC) in the near future. If the center of mass energy of two colliding particles is high enough, a microscopic black hole can be produced. The discreetness of the black hole mass spectrum could be observed from the peaks of the cross section graphs.

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