Statistical game-theoretic model of the optimal labor resources distribution

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Abstract. In this paper, a competitive model of the static assignment problem is constructed and investigated. The paper considers the static theoretical and game model of the optimal purpose problem. The algorithm of finding the equilibrium situation is constructed. This algorithm based on the given utility matrixes for workers and enterprises shows what purpose will be equilibrium. Finding compromise gains and equilibrium situations are analyzed by examples.

1. Introduction

Many practical tasks of economic activity and a number of important questions of economic theory are connected with tasks of definition of an optimum variant of the decision. Studying of methods of mathematical programming become necessary for practical work of the economist [1-4].

In the mathematical economy, the task of the most efficient distribution of resources between n industries is of great importance [5-11]. The tasks of this type are relevant and constantly arise in various areas of our lives, such as economics, industry, etc.

At present, there are a lot of works devoted to the task of appointment, which is discussed in many books, for example: H. Papadimitritz, K. Stayglitz "Combine Optimization", D. Phillips, A. Garcia-Diaz "Methods of network analysis", etc. However, in contrast to the assignment task discussed in the books of these authors, in the constructed model, the goal of each participant, in addition to maximizing their income, is to achieve a compromise with the other participants. As a result, it is possible that not all participants in the process will receive their maximum possible income. In a static model, the role of compromise will be played by an appointment that satisfies all participants.

Definition 1. Position game \( \Gamma \) is an object consisting of the following elements: \( \Gamma = \{I, \Gamma_A, S, H\} \), where \( I = \{1, \ldots, n\} \) - array of players, graph \( \Gamma_A \) - finite tree (finite graph without cycles); the vertex (root) \( A \) is selected - the beginning of the game; \( S = S_T \cup \bar{S} \), where \( S_T \) - multiple terminal positions; \( S \) - many nonterminal positions; \( \bar{S} = \cup S_i \), where \( S_i \) - a lot of queuing for the \( i \) player, is a lot of positions in which the \( i \) player chooses his alternative, \( i = \{1, n\}; S_0 \) - multiple queues; \( S_{ij} = \cup S_{ij} \), where \( S_{ij} \) - player \( i \) information sets. If a player is in the information set \( S_{ij} \), it means he doesn't know exactly where in the information set he is.
The vector function of the winnings is set \( H = (H_1, \ldots, H_n) \), where \( H_i \) - the winning function of the \( i \) player, defined on all the terminal positions of the game [37].

**Definition 2.** The strategy of the player \( i \) in the positional game \( \Gamma \) is the display specified on the information sets of this player. This display takes on values on many alternatives. Informally speaking, the strategy indicates to the player his choice in any position of the game.

Let's denote the set of strategies of the player \( i \) in the positional game \( \Gamma \) through: \( \Phi_i = \{ \phi_i \} \). Then the set of strategies of the players \( (\phi_1, \ldots, \phi_n) = \varphi \) clearly defines the trajectory in the game tree and, thus, the value of the winning function in each situation. Thus, on this position game we have built its normal form in an unambiguous way

\[
\Gamma_H = \{ I = \{1, \ldots, n\}, \{\Phi_i\}_{i=1}^{n}, \{H_i\}_{i=1}^{n} \}.
\]

**Definition 3.** A game in its normal form \( \Gamma_n \) is called a set

\[
\Gamma_H = \{ I = \{1, \ldots, n\}, \{\Phi_i\}_{i=1}^{n}, \{H_i\}_{i=1}^{n} \}.
\]

Where \( I = \{1, \ldots, n\} \) - array of players; \( \Phi_i \) array of player strategies \( i \); \( H_i \) - the winning function of player \( i \), defined on the set \( \Phi = \Pi_i \Phi_i \to R_i \).

A set of strategies \( \varphi = (\phi_1, \ldots, \phi_n) \) is called a normal game situation \( \Gamma_n \). In this game, each player \( i \), regardless of the others chooses his own strategy \( \phi_i \), in an ongoing situation \( \varphi = (\phi_1, \ldots, \phi_n) \) each player \( i \) wins \( H_i(\varphi) \).

Let's say that \( \varphi \parallel \varphi' = (\phi_1, \ldots, \phi_{i-1}, \phi'_i, \phi_{i+1}, \ldots, \phi_n) \).

**Definition 4.** Let the game be played in its normal form \( \Gamma_n \). The situation \( \varphi \) in this game is called equilibrium if inequality is fulfilled for \( i = 1, \ldots, n \) and \( \varphi'_i \in \Phi_i \) the inequality is fulfilled:

\[
H_i(\varphi) \geq H_i(\varphi \parallel \varphi'_i).
\]

That is, the situation \( \varphi \) is balanced, if an individual deviation from this situation is disadvantageous to any of the players.

**Definition 5.** Position game \( \Gamma \) is called a game with full information if all information sets in this game are single-point. (This means that each player in any position is aware of this position and the choices made by all players in this game.)

**Theorem 1 (Zermelo-Neumann).** In the final position game with complete information, there is a situation of equilibrium on Nash in net strategies.

**Definition 6.** A two-player game with zero sum is called antagonistic if \( H_1 = -H_2 \).

The final game of two faces with any sum can be described by a pair of dimensional matrices \( m \times n : A = (a_{ij}), B = (b_{ij}) \). The elements \( a_{ij} \) and \( b_{ij} \) are the winnings (in utility units) of players \( I \) and \( II \) respectively, assuming that they will choose their \( i \) and \( j \) net strategies. It's called a bimatrix game.

**Definition 7.** Antagonistic games in which both players have finite sets of strategies are called matrix games.

The normal form of the final antagonistic game is the matrix \( A, A = (a_{ij}) \). The number of rows equals the number of strategies of player \( I \), the number of columns equals the number of strategies of player \( II \). The first player gets a win \( a_{ij} \), if \( a_{ij} > 0 \) and pays a loss \( a_{ij} \), if \( a_{ij} > 0 \); matrix \( A \) has a dimension \( m \times n \).
The situation (a pair of strategies) \((i, j)\) will be balanced if and only if the corresponding element \(a_{ij}\) is both the largest in column \(j\) and the smallest in row \(i\).

The first player tries to maximize his winnings, therefore, he must choose the line in which the winnings are maximal. His winnings look like this: \(\max_i \max_j a_{ij} = V_A\). This value is called the lower value of the matrix game \(A\).

Since the second player tries to minimize his loss, he chooses: \(\max_j \min_i a_{ij} = \bar{V}_A\). The value \(\bar{V}_A\) is called the upper value of the matrix \(A\).

**Theorem 2 (Neiman) (o minmax).** In any matrix game, there is a \(V_A\) equilibrium situation in \(A \Rightarrow V_A = \bar{V}_A = V_A\).

**Definition 8.** A player’s mixed strategy is a probability distribution on a set of his pure strategies.

**Definition 9.** It is said that the situation (a pair of mixed strategies \((x_0, y_0)\)) in a bimatrix game is a balance situation if for any other mixed strategies \(x \in X^m\) and \(y \in Y^n\)

\[
xAy_0 \leq x_0Ay_0
\]

\[
x_0By \leq x_0By_0
\]

where many mixed strategies look like:

\[
X^m = \left\{ x \in R^m \mid \sum_{i=1}^{m} x_i = 1, x_i \geq 0 \right\}
\]

\[
Y^n = \left\{ y \in R^n \mid \sum_{j=1}^{n} y_j = 1, y_j \geq 0 \right\}
\]

**Theorem 3 (Nash).** In any final noncoalistic game of \(n\) faces there is a situation of equilibrium in mixed strategies.

**Definition 10.** Let \(X\) - be a compact metric space , \(H_i : X \rightarrow R, i \in I = \{1, \ldots, n\}\) - the essence of continuous functions, \(M_i = \max\{H_i(x) \mid x \in X\}\). The compromise set of \(C_H\) is defined as follows:

\[
C_H = \left\{ x \in X \mid \max_i (M_i - H_i(x)) \leq \max_i (M_i - H_i(x')) \forall x' \in X \right\}
\]

**Definition 11.** Let \(N = \{1, \ldots, n\}\) - a lot of all the players. Any non-empty subset \(S \subseteq N\) is called a coalition.

**Definition 12.** The characteristic function of the cooperative game of \(n\) the faces will be called the real function \(v\), defined on coalitions \(S \subseteq N\), while for any disjoint coalitions \(T, S (T \subseteq N, S \subseteq N)\) inequality holds:

\[
v(T) + v(S) \leq v(T \cup S), v(\emptyset) = 0.
\]

2. **Game-theoretic model of optimal resource allocation**

2.1. **Formulation of the problem**

It considers a lot of workers \(S = \{s_1, \ldots, s_m\}\), who want to get a job, and many enterprises \(H = \{h_1, \ldots, h_n\}\) that offer jobs. We assume that each enterprise \(h_j\) has one vacant position for which it wishes to accept an employee, and employee \(s_i\) can be accepted for only one enterprise. It is required to make the appointment of workers in an optimal way [21].
Formalized the game in normal form \( \Gamma = \{ I, X, [H]_{i=1}^{m+n} \} \), in which the players are workers and enterprises, and the situation is a substitution \( p_k \). Each substitution is one of the possible assignments of workers to the enterprise from among the many \( P = \{ p_k, k = 1,2,\ldots,n! \} \).

Each employee \( s_i \) evaluates for himself the work of a particular enterprise \( h_j \). Evaluation criteria may include, for example, wages, compliance with the specialty received, the amount of time required for the journey from home to work, benefits provided, etc. Taking into account all the advantages and disadvantages, the employee makes his assessment \( \alpha_{ih} \in Z^+ \), (where \( Z^+ \) is the set of positive integers, \( i = 1,2,\ldots,m; j = 1,2,\ldots,n \)) to the enterprise [12-15]. The number \( \alpha_{ih} \) will be called the utility for the employee \( s_i \) from the appointment to the enterprise \( h_j \) and we will understand by it the degree of satisfaction of the player’s interests \( s_i \). Similarly, the company \( h_j \) evaluates each employee \( s_i \) by a certain number \( \beta_{h,i} \in Z^+ \). This assessment is influenced by a combination of factors such as the professional qualifications of the employee, the salary assigned, the ability to work in a team, creative initiative, etc. We will call the number \( \beta_{h,i} \) utility for the enterprise \( h_j \) from appointing an employee \( s_i \) to it and we will mean by it the degree of player \( h_j \) satisfaction.

The estimates of both sides are written in the form of utility matrices \( A_{mxn} = (\alpha_{ih}) \) and \( B_{mxn} = (\beta_{h,i}) \) and from which the player’s payoff matrix \( W_{nh(n+m)} \) is constructed in the game \( \Gamma \). Rows of the payoff matrix \( W_{nh(n+m)} \) correspond to situations \( p_k \) from a variety of situations \( X = P = \{ p_k, k = 1,2,\ldots,n! \} \), columns correspond to players from a set \( I \). The matrix element \( W_{nh(n+m)} \) is the player’s payoff function in a specific situation.

A compromise set \( C_H = \{ p_k \in X \} \) is constructed from the matrix \( W_{nh(n+m)} \) in the game \( \Gamma \), and a compromise win is found, which is the guaranteed win of the least satisfied player. The algorithm for constructing a compromise set \( C_H \) is described in steps [16-18].

2.2. A formal description of a mathematical model in the form of a game in normal form \( \Gamma \)

Consider the game in normal form.

\[ \Gamma = \{ I, X, [H]_{i=1}^{m+n} \} \]

where \( I = \{ 1,2,3,\ldots,m+n \} \) - is a set of players, \( X \) - a set of situations in the game, \( H_i : X \rightarrow R_i \) - player \( i \) win function.

Formally, the appointment of workers to work can be represented by substituting \( p_k \) of the form:

\[
\begin{pmatrix}
1 & 2 & 3 \\
h_k & h_j & h_m
\end{pmatrix}
\]

where the first line is unchanged and corresponds to the numbers of workers from the set \( S \), and the second to the players from the set \( H \). The number of such substitutions is \( |P| = n! \). The situation in the game will be considered a substitution. In this way \( |X| \leq |P| = n! \).

Let each player evaluate his appointment with a certain positive number. We call this number a utility for a given player from the received destination. We assume that the greater the utility, the more
the player is satisfied with the received appointment. Thus, utility shows the degree of satisfaction of the interests of the player.

We write utilities for players from the sets $S$ and $H$ into matrices $A$ and $B$, which we call utility matrices.

Matrices and $A_{m,n}=(\alpha_{i,h})$ и $B_{m,n}=(\beta_{h,l})(l=1,\ldots,m;k=1,\ldots,n)$ (index $l$ corresponds to numbers of players from the set $S$, index $k$ corresponds to numbers of players from the set $H$) have the form:

$$A = \begin{pmatrix}
\alpha_{1h} & \alpha_{2h} & \ldots & \alpha_{1h} \\
\alpha_{2h} & \alpha_{2h} & \ldots & \alpha_{2h} \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_{mh} & \alpha_{mh} & \ldots & \alpha_{mh}
\end{pmatrix}, \quad B = \begin{pmatrix}
\beta_{h1} & \beta_{h2} & \ldots & \beta_{h1} \\
\beta_{h2} & \beta_{h2} & \ldots & \beta_{h2} \\
\vdots & \vdots & \ddots & \vdots \\
\beta_{hm} & \beta_{hm} & \ldots & \beta_{hm}
\end{pmatrix}.$$

We define the payoff functions of the players on the set of permutations $P$ as follows:

$$H_1(p_k) = \alpha_{1h_1}, H_2(p_k) = \alpha_{2h_2}, \ldots, H_m(p_k) = \alpha_{mh_m},$$

$$H_{m+1}(p_k) = \beta_{h_1}, H_{m+2}(p_k) = \beta_{h_2}, \ldots, H_{m+n}(p_k) = \beta_{h_m}, k=1,2,\ldots,n!.$$

We form the payoff matrix $W_{n!(m+n)}$ (the rows correspond to the permutations forming the set of situations $X$, the columns correspond to the numbers of players from the set$I$):

$$W = \begin{pmatrix}
H_1(p_1) & H_2(p_1) & \ldots & H_{m+n}(p_1) \\
\vdots & \vdots & \ddots & \vdots \\
H_1(p_k) & H_2(p_k) & \ldots & H_{m+n}(p_k) \\
\vdots & \vdots & \ddots & \vdots \\
H_1(p_n) & H_2(p_n) & \ldots & H_{m+n}(p_n)
\end{pmatrix}.$$

A compromise set is proposed as a solution to the problem. An algorithm for finding a compromise point is given.

2.3. Compromise Set Algorithm

In our notation $|I|=m+n$, $|X|=n!$, $H_i(p_k), W_{n!(m+n)}$.

1 step: The payoff functions for each player from each game situation $p_k \in X, H_i(p_k)$ are calculated, and we form the payoff matrix $W_{n!(m+n)}$, where $S$ – is the set of game situations $\Gamma$, $k=(1,2,\ldots,n!)$ - is the situation number, $i=(1,2,\ldots,n+m)$ - is the player number.

2 step: For each player $i=1,n+m$ the values are calculated:

$$M_i = \max_{p_k \in X} H_i(p_k), \forall k = 1,2,\ldots,n!,$$

where $n!$ – the number of situations in game $\Gamma$, and the "ideal vector" $M = (M_1,\ldots,M_{n+m})$ is formed.

3 step: For each player $i=1,n+m$ and for each situation $p_k \in X, k=1,\ldots,n!$ of the game $\Gamma$ the following deviations of the payoff function $H_i(x)$ from the component of the ideal vector $M_i; \Delta_i(p_k) = M_i - H_i(p_k)$ are calculated [52].
4 step: The maximum deviations are found for all players \( i = 1, n + m \) at each permutation (game situation) \( p_k \in X \):

\[
\alpha(p_k) = \max_i \Delta_i(p_k),
\]

where \( k = 1, \ldots, n! \) - the number of situations in game \( \Gamma \), \( i \) – set of players of the game \( \Gamma \).

5 step: The minimum of these maximum deviations is selected, i.e., it is: \( \min_{p_k \in X} \alpha(p_k) \), where \( k = (1, \ldots, n!) \).

3. **An example of finding a compromise situation**

Let \( m = n = 3, S = \{s_1, s_2, s_3\}, H = \{h_1, h_2, h_3\} \).

\( I = \{1, 2, 3, 4, 5, 6\} \) - the set of players, and with numbers 1, 2, 3 correspond to players \( s_1, s_2, s_3 \) from the set \( S \), and players 4, 5, 6 correspond to players \( h_1, h_2, h_3 \) from the set \( H \).

Utility matrices \( A, B \) for players from the set \( S \) and \( H \) respectively:

\[
\text{Matrix } A: \quad A = \begin{pmatrix} 76 & 22 & 94 \\ 33 & 41 & 86 \\ 45 & 13 & 54 \end{pmatrix}, \quad \text{Matrix } B: \quad B = \begin{pmatrix} 94 & 71 & 17 \\ 30 & 32 & 18 \\ 59 & 85 & 38 \end{pmatrix}.
\]

We list all the appointments of workers to work (many situations \( P = \{p_1, \ldots, p_6\} \)):

\[
p_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \quad p_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad p_3 = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad p_4 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad p_5 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \quad p_6 = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}.
\]

Functions for winning players from the substitution \( p_i \):

\[
H_1(p_1) = \alpha_{th_1} = 76, \quad H_2(p_1) = \alpha_{th_2} = 41, \quad H_3(p_1) = \alpha_{th_3} = 54, \quad H_4(p_1) = \beta_{h_1} = 94, \quad H_5(p_1) = \beta_{h_2} = 32, \quad H_6(p_1) = \beta_{h_3} = 38, \ldots
\]

We set the payoff functions of players from substitutions \( p_2, p_3, p_4, p_5, p_6 \) in a similar way.

Then:

- The payoff matrix has the form (rows correspond to permutations \( p_1, \ldots, p_6 \), forming a set of situations \( X \), columns correspond to numbers of players from the set \( I \)):

\[
\begin{pmatrix}
76 & 41 & 54 & 94 & 32 & 38 \\
22 & 33 & 54 & 71 & 30 & 38 \\
94 & 41 & 45 & 17 & 32 & 59 \\
94 & 33 & 13 & 71 & 18 & 59 \\
76 & 86 & 13 & 94 & 18 & 85 \\
22 & 86 & 45 & 17 & 30 & 85
\end{pmatrix}
\]

- The ideal vector \( M \) is: \( M = (94, 86, 54, 94, 32, 85) \).

- After calculation \( \{M_i - H_i(x)\} \) we get:
77, 41, 53, 77, 72, 47 (max = x_{HM}^{ii}) = (47, 72, 77, 53, 41, 77).

we found min_{x \in X} \max_{i} \{ M_i - H_j(x) \} .

A compromise is substitution \( p_5 \). Each player’s win is:

\( H_1(p_5) = 76, H_2(p_5) = 86, H_3(p_5) = 94, H_4(p_5) = 18, H_5(p_5) = 18 \).

4. Equilibrium Algorithm

In this part of the thesis, we consider many enterprises \( H = \{ h_1, \ldots, h_m \} \), that offer jobs, and many workers \( S = \{ s_1, \ldots, s_n \} \), who want to get a job at a particular enterprise. Let each player evaluate his appointment with a certain positive number. We call this number a utility for a given player from the received destination. We assume that the greater the utility, the more the player is satisfied with the received appointment. Thus, utility shows the degree of satisfaction of the interests of the player. Utilities for players from the set \( S \) are presented in the form of a utility matrix \( A^{mn} = (a_{mn}) \). The utility for the set \( H \) can be represented as a utility matrix \( B^{mn} = (b_{h,l}) \). Finding equilibrium in this model is carried out according to the following algorithm:

- from the utility matrix for workers we find which of the enterprises delivers the greatest utility to each of the employees;
- from the utility matrix for enterprises we find which of the workers delivers the most utility to each of the enterprises;
- comparing the data, we can see that with the coincidence of the greatest utility for the employee and the enterprise, this appointment is the best for this pair. From the utility matrices for workers and enterprises, we discard this pair;
- next, we consider the utility matrix without a matched pair consisting of a student and an enterprise. We repeat the algorithm until we determine all the assignments with maximum utilities.

In the case when the number of enterprises is more than the number of employees, we analyze only the utility matrix for enterprises and find such an appointment of workers to enterprises, as a result of which it will not be profitable for one of the employees to deviate individually from this appointment (i.e., get a job at another enterprise) [19-20].

In the case when the number of employees is more than the number of enterprises, we analyze only the utility matrix for employees and find such an appointment of workers to enterprises, as a result of which not one of the enterprises individually deviates from this appointment (i.e., hire another employee).

5. Numerical example

There are three students who want to get a job at the enterprise upon graduation, and three enterprises offering jobs. Each student appreciates for himself the work at each enterprise with usefulness.
Usefulness depends on the time a student needs to get to the enterprise, and the salary in the enterprise, etc. Also, each enterprise evaluates the student's usefulness. The usefulness of assigning a student to an enterprise depends on the diploma and student characteristics [21-23].

Utility matrices for students and enterprises look like:

- for students:

\[
A = \begin{pmatrix} 76 & 94 & 22 \\ 45 & 41 & 86 \\ 54 & 38 & 33 \end{pmatrix}
\]

- for enterprise:

\[
B = \begin{pmatrix} 32 & 71 & 94 \\ 30 & 18 & 17 \\ 59 & 85 & 38 \end{pmatrix}
\]

Analyzing the utility matrix for students, we can see that for the first student the maximum utility when appointing to the second enterprise.

Analyzing the utility matrix for enterprises, we can see that the maximum utility for the first enterprise when appointing a third student, for the second enterprise from the appointment of the first student, for the third enterprise from the appointment of the second student.

Now, comparing the data, we can see that the pair coincided with the maximum utilities - this is the pair of the first student and the second enterprise.

Now we cross out from the matrices of utilities for students and enterprises the first student and the third enterprise. Utility matrices will take the form:

- for students:

\[
A' = \begin{pmatrix} 45 & 86 \\ 54 & 33 \end{pmatrix}
\]

- for enterprise:

\[
B' = \begin{pmatrix} 71 & 94 \\ 85 & 38 \end{pmatrix}
\]

Analyzing the utility matrix \(A'\) and \(B'\) we can see that from the utility matrix \(A'\) the maximum utility for the second student is assigned to the third enterprise.

Analyzing the matrix \(B'\), we see that the third student delivers the greatest utility to the third student, the second student to the third enterprise. Comparing the data, we see that the pair coincided with the maximum utilities, that is, the second student and the third enterprise, and the third student will get a job at the first enterprise. The equilibrium in this example will be the following assignment: the first student to the second enterprise, the second student to the third enterprise, the third student to the first enterprise.

6. Conclusion
A game-theoretic version of the problem of the optimal distribution of labor resources is proposed in the work, a statistical game-theoretic model of the optimal purpose is considered. Formalized the game in normal form. The algorithms for finding a compromise appointment of workers at the enterprise and finding equilibrium are described in detail. The examples are given.

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