2D DOA Estimation of Coherently Distributed Sources for Planar Arrays Based on Blind Signal Separation

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Abstract. A novel method is presented to estimate the direction-of-arrival (DOA) of coherently distributed (CD) sources for planar arrays. In the proposed method, the two-dimensional (2D) angular parameters are obtained by making use of the estimated generalized array manifold matrix, which is separated from the received observation data based on blind signal separation algorithm. The proposed method offers a number of advantages. First, the 2D central DOAs and angular spread variances can be estimated simultaneously. Second, the deterministic angular distributed functions (DADFs) of the CD sources do not need to know. Third, the special array geometries are not required, as the method is applicable to planar arrays. Computer simulations are shown to verify the efficacy of the proposed method.

Keywords: DOA estimation, coherently distributed source, blind signal separation

1. Introduction

DOA estimation, also known as ‘direction finding’, is an important technology in the field of array signal processing, and is widely used in military and civilian fields. From the perspective of application requirements, higher DOA estimation accuracy is its consistent pursuit in practical application. The traditional DOA estimation method has good performance under the assumption of point target model. However, in certain deterministic multipath environment, the DOA of the source will be extended. In this case, it is more appropriate to use a distributed source model to describe the multipath propagation characteristics of the signal component [1].

Distributed source can be categorized into CD source and incoherently distributed (ID) source [2]. The components of ID source in different incident directions are completely uncorrelated. However, for a CD source, the components of different incident directions differ only by a fixed phase delay and amplitude weighting. The 2D DOA estimation algorithms for CD sources have been widely studied in recent years. In the literature, the method in [3] combines the generalized ESPRIT algorithm to deal with the central DOA estimation of CD sources in the case of a uniform circular array (UCA). Subsequently, this method was extended to cento-symmetric planar arrays in [4] using noncircular signals. On the basis, by making use of a pair of UCAs, the method in [5] estimates the central DOAs based on the generalized ESPRIT algorithm and rank-reduction theory. The method in [6] decoherence of 2D CD sources by constructing double parallel uniform linear arrays (ULA). Based on the modified propagator method, the method in [7] obtains the angular parameters with the special geometry of L-
shape ULAs. These methods in [3]-[7], which are based on subspace theory, provide important improvements in the estimation performance. However, the ability of achieving the high performance is greatly influenced by the special array geometry used, and the angular spread parameters estimation of the CD sources is not considered.

In this paper, in order to overcome the shortcomings of subspace-based methods, we estimate the angular parameters of the CD sources based on blind signal separation. By making use of the joint approximation and diagonalization of eigenmatrices (JADE) algorithm, the construction of special array geometries is not required. Thus, a 2D CD source model for planar arrays is introduced and the angular signal distributed weight (ASDW) vectors of the CD source for planar arrays are derived. The property of the generalized array manifold matrix is then used to formulate the proposed method. The estimated generalized array manifold matrix cannot be directly used to estimate the central DOAs of the CD sources because of the estimation ambiguity. To solve the problem, we make some transformations of the estimated generalized array manifold matrix before the estimation of the central DOAs and angular spread variances.

2. Data Model
Consider a planar array composed of $M$ identical sensor elements as Fig.1 shows. The first sensor located at the coordinate origin is supposed to be the reference point. Suppose there are $D$ CD sources impinging on this planar array. The vector of the observation data can be expressed as

$$ x(t) = \sum_{i=1}^{D} \int \int a(\theta, \phi) s_i(\theta, \phi, t; \mu_i) d\theta d\phi + n(t) $$

where the signal $s_i(\theta, \phi, t; \mu_i)$ is written in the form of the angular signal density function. The central azimuth angle $\theta_i$, central elevation angle $\phi_i$, azimuth angular spread variance $\sigma_{\theta_i}^2$ and elevation angular spread variance $\sigma_{\phi_i}^2$ are indicated in the vector $\mu_i = (\theta_i, \sigma_{\theta_i}^2, \phi_i, \sigma_{\phi_i}^2)$. $n(t)$ is a vector of zero-mean white additive noise. $a(\theta, \phi)$ is the ideal array manifold vector at 2D direction $(\theta, \phi)$. Assume that the $m$-th sensor element is placed at $(x_m, y_m)$, we have

![Fig. 1. Geometry of a planar array and CD source.](image-url)
3. The Proposed Method

3.1. ASDW Vectors for Planar Arrays

Define \( \theta = \theta_0 + \hat{\theta} \) and \( \varphi = \varphi_0 + \hat{\varphi} \), for the small angular extension, using Taylor series approximation, the \( m \)-th element of \( \mathbf{a}(\theta, \varphi) \) in Eq. (5) can be given by

\[
\left[ \mathbf{a}(\theta, \varphi) \right]_m \approx \left[ \mathbf{a}(\theta_0, \varphi_0) \right]_m \cdot e^{j\xi_m} \tag{7}
\]

where \( \xi_m = (2\pi / \lambda) \left[ \hat{\theta} \left( -x_m \sin \theta + y_m \cos \theta \sin \varphi \right) + \hat{\varphi} \left( x_m \cos \theta \cos \varphi + y_m \sin \theta \cos \varphi \right) \right] \). Thus, the generalized array manifold vector can be simplified as

\[
\mathbf{b}(\mu) = \mathbf{a}(\theta_0, \varphi_0) \otimes \mathbf{g}(\mu) \tag{8}
\]

where the symbol \( \otimes \) stands for Hadamard product (element-wise product). Here, \( \mathbf{g}(\mu) \) is the ASDW vector which is given by

\[
\left[ \mathbf{g}(\mu) \right]_m = \int e^{j\xi_m} \rho(\theta, \varphi, \mu) d\theta d\varphi \tag{9}
\]

Take the most commonly used DADFs for example, such as Gaussian distribution and Uniform distribution functions. Insert them into Eq. (9), the ASDW vectors \( \mathbf{g}(\mu) \) for planar arrays can be presented as follows,

\[
\text{Gaussian}: \left[ \mathbf{g}(\mu) \right]_m = e^{\frac{-\alpha_m^2 \xi_m^2}{2}} e^{\frac{-\beta_m^2 \xi_m^2}{2}} \\
\text{Uniform}: \left[ \mathbf{g}(\mu) \right]_m = \frac{\sin(\alpha_m \sigma_\theta)}{\alpha_m \sigma_\theta} \cdot \frac{\sin(\beta_m \sigma_\varphi)}{\beta_m \sigma_\varphi} \tag{10}
\]

where \( \alpha_m \) and \( \beta_m \) are defined as

\[
\alpha_m \@ (2\pi / \lambda) \left( -x_m \sin \theta \sin \varphi + y_m \cos \theta \sin \varphi \right) \\
\beta_m \@ (2\pi / \lambda) \left( x_m \cos \theta \cos \varphi + y_m \sin \theta \cos \varphi \right) \tag{11}
\]

In this paper, the signals of the CD sources are supposed to be uncorrelated with each other. The distance between the adjacent array elements is not longer than half a wavelength.
3.2. Generalized Array Manifold Matrix Estimation

The JADE algorithm, which can separate the incident signal and the array manifold matrix with high estimation accuracy and fast convergence, is a conventional algorithm. In this letter, we separate the vector and the generalized array manifold matrix $B$ by making use of the JADE algorithm. The detail algorithm implementation can refer to [9]. The estimated matrix is expressed as

$$
\hat{B}_{JADE} = \begin{bmatrix} b_j(\mu_1), b_j(\mu_2), \ldots, b_j(\mu_K) \end{bmatrix}
$$

Since the first array element located at the coordinate origin is the reference array element. By normalizing the matrix $\hat{B}_{JADE}$, the estimate of the generalized array manifold matrix can be obtained as

$$
\hat{B} = \begin{bmatrix} b(\mu_1), b(\mu_2), \ldots, b(\mu_K) \end{bmatrix}
$$

3.3. DOA estimation

From Eq. (10) and (11), it can be found that the elements in the ASDW vector are real values. Thus, the phase of $a(i, \theta, \varphi)_{\mu_i}$ is equal to the phase of $\hat{b}(\mu_i)_{\mu_i}$. The estimate of $a(i, \theta, \varphi)_{\mu_i}$, denoted as $\hat{a}(i, \theta, \varphi)_{\mu_i}$, can be estimated from

$$
\hat{a}(i, \theta, \varphi)_{\mu_i} = \frac{|b(\mu_i)_{\mu_i}|}{|\hat{b}(\mu_i)_{\mu_i}|}, \quad i = 1, 2, \ldots, D
$$

where $||$ indicates the module for complex numbers. According to Eq. (2) and (14), we have

$$
(2\pi / \lambda) (x_n \cos \theta \sin \varphi + y_n \sin \theta \sin \varphi)
= 2\pi n + \text{arg}(b(\mu_i)_{\mu_i})
$$

where $\text{arg}(\cdot)$ denotes the argument of complex numbers. Thus, without the unknown number $n$, the argument of $b(\mu_i)_{\mu_i}$ cannot be directly used to calculate the central DOAs of the CD sources, which may lead to estimation ambiguity. In order to avoid the ambiguity, it is necessary to make some transformations on both sides of Equation (14). Define

Fig. 2. (a) 2D-RMSEs of central DOAs for different azimuth angular spread. (b) 2D-RMSEs of central DOAs for different elevation angular spread.
Here, \([\cdot]^T\) denotes the transpose operation. As the distance between adjacent array elements is less than half a wavelength, we have

\[
(2\pi / \lambda)[(x_{n+1} - x_n)\cos \theta \sin \phi + (y_{n+1} - y_n)\sin \theta \sin \phi] = \arg\left(\begin{bmatrix} \Delta \hat{b}(\mu) \end{bmatrix}_m\right)
\]

In matrix form

\[
P \begin{bmatrix} \cos \theta \sin \phi \\ \sin \theta \sin \phi \end{bmatrix} = \frac{\arg\left(\Delta \hat{b}(\mu)\right)}{2\pi} \frac{\lambda}{\alpha_k}
\]

where \(P\) is a \((M-1)\times 2\) matrix which is defined as

\[
P = \begin{bmatrix} x_2 - x_1, y_2 - y_1 \\ x_3 - x_2, y_3 - y_2 \\ \vdots \\ x_M - x_{M-1}, y_M - y_{M-1} \end{bmatrix}
\]

According to Eq. (18), we have

\[
\begin{bmatrix} \cos \theta \sin \phi \\ \sin \theta \sin \phi \end{bmatrix} = P^\dagger \frac{\arg\left(\Delta \hat{b}(\mu)\right)}{2\pi} \frac{\lambda}{\alpha_k} \begin{bmatrix} l_k \\ \alpha_k \end{bmatrix}
\]

Here, \((\cdot)^\dagger\) denotes the pseudo-inverse. Thus, the central DOA of the \(i\)-th source can be estimated numerically from

\[
\hat{\theta}_i = \arctan\left(\frac{l_k}{\alpha_k}\right)
\]

\[
\hat{\phi}_i = \arcsin\left(\frac{k_i}{\cos \hat{\theta}_i}\right)
\]

After we find the central DOA \((\hat{\theta}_i, \hat{\phi}_i)\), respecting Eq. (8) and (9), the ASDW vector can be obtained as

\[
\hat{g}(\mu) = |\hat{b}(\mu)|
\]

which means the azimuth and elevation angular spread variances can be estimated from

\[
\int e^{i\rho^2} \rho \cos \phi \cdot \mu d\phi d\theta = |\hat{b}(\mu)|^2, \quad m = 1, 2, K, M
\]

For example, when the DADF is Gaussian distributed, the azimuth and elevation angular spread variances can be obtained numerically from

\[
\begin{bmatrix} \hat{\sigma}_{\hat{\phi}}^2 \\ \hat{\sigma}_{\hat{\phi}}^2 \end{bmatrix} = Q^\dagger -2\ln|\hat{b}(\mu)|
\]

where \(Q\) is defined as

\[
Q = \begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \\ \vdots & \vdots \\ \alpha_M & \beta_M \end{bmatrix}
\]

Finally, we obtain the estimation vector \(\hat{\mu} = (\hat{\theta}, \hat{\phi}, \hat{\sigma}_{\hat{\phi}}, \hat{\sigma}_{\hat{\phi}})\) of the \(i\)-th CD source.
4. Numerical Simulations

Some simulations are presented to investigate the performance of the proposed method. Each simulation was performed 500 times of Monte Carlo experiments to obtain the RMSE curve. The 2D-RMSE used in the following simulations is given by

$$\text{2D-RMSE} \propto \sqrt{\left(\text{RMSE}(\theta)\right)^2 + \left(\text{RMSE}(\phi)\right)^2}$$

(26)

In the first numerical simulation, the influence of the DADFs is examined in the presence of two CD sources with $$(\theta_1, \phi_1) = [35^\circ, 40^\circ]$$ and $$(\theta_2, \phi_2) = [55^\circ, 60^\circ]$$. Consider a UCA consisting of 12 array elements, and the spacing between the adjacent sensor elements is $$\lambda/2$$. The number of snapshots is 1000 and the signal-to-noise ratio (SNR) is 10dB. We perform the method in three different cases: case 1 is that the two DADFs are both Gaussian distribution functions, case 2 is that they are both Uniform distribution functions, case 3 is that the two DADFs are Gaussian distribution function and Uniform distribution function respectively. Fig. 2a and Fig. 2b illustrate the 2D-RMSE curves for fixed angular spread of $$\sigma_\theta = 2^\circ$$ and $$\sigma_\phi = 2^\circ$$, respectively. Fig. 2 shows that the proposed method performs well in all these cases.

In the second experiment, the performance of the proposed method is compared with the Nam algorithm in [3] with respect to the SNR from 0dB to 20dB. Consider a UCA consisting of 8 sensor elements, and the spacing between the adjacent elements is $$\lambda/2$$. The angular parameters of the two Uniform shaped CD sources are assumed as $$\mu_1 = (15^\circ, 3^\circ, 20^\circ, 5^\circ)$$ and $$\mu_2 = (35^\circ, 5^\circ, 40^\circ, 4^\circ)$$, respectively. The 2D-RMSE curves of central DOA estimation are illustrated in Fig. 3a. It can be found that the estimation performance of the proposed method is higher than that of the Nam algorithm over a wide range of SNRs. Since the Nam algorithm cannot estimate the angular spread variances, we only present the angular spread RMSE curves of the proposed method in Fig. 3b.

Fig. 3. (a) 2D-RMSEs of central DOAs versus the SNR. (b) RMSEs of angular spread versus the SNR.

5. Conclusion

In this paper, we propose a central DOA and angular spread variances estimation method of CD sources for planar arrays. A 2D CD sources model for a planar array is presented. Based on blind signal separation, the central DOA and angular spread variances are obtained by making use of the estimated generalized array manifold matrix, without multi-dimensional spectrum searching. In addition, the information about the DADFs of multiple CD sources is not required to know, even if the DADF of each CD source is different.
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