The Hypernuclei $^3\Lambda$He and $^3\Lambda$H: Challenges for Modern Hyperon-Nucleon Forces.

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The hypernuclei $^3\Lambda$He and $^3\Lambda$H provide important information on the hyperon-nucleon interaction. We present accurate Faddeev-Yakubovsky calculations for the $\Lambda$ separation energies of the $0^+$ ground and the $1^+$ excited states based on the Nijmegen SC YN interactions. We explicitly take the $\Sigma$ admixture into account. Mass differences of the baryons and the charge-dependence of the interaction are considered. The results show that the Nijmegen models cannot predict all separation energies simultaneously hinting to failures of the current interaction models. It is pointed out that the differences of the $\Lambda$ separation energies of $^3\Lambda$He and $^3\Lambda$H are interesting observables to probe the YN interaction models.

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Several nucleon-nucleon (NN) interaction models have been adjusted successfully to the rich set of NN scattering data ($^4\Lambda$, $^3\Lambda$, $\Lambda$H). Deviations of the predictions of these models to the experimental data (like the underbinding of the 3N bound states) might be traced back to the action of 3N or higher order forces ($^4\Lambda$, $^3\Lambda$, $\Lambda$H) and are presumably no hints to failures of these NN interactions. Therefore the hyperon-nucleon (YN) system provides interesting and important new insights into the interaction mechanisms ($^4\Lambda$, $^3\Lambda$, $\Lambda$H), because the predictions of todays interactions are sensitive to the used model ($^4\Lambda$, $^3\Lambda$, $\Lambda$H). Unfortunately there are hardly scattering data for the YN system available. Therefore interaction models are generally constrained by flavor-SU(3) symmetry, which, however, is considerably broken. In this spirit one-meson exchange ($^4\Lambda$, $^3\Lambda$, $\Lambda$H) and quark-cluster ($^4\Lambda$, $^3\Lambda$, $\Lambda$H) forces have been developed, which are all consistent with the scarce YN data base. In this letter we will present results based on the Nijmegen soft core interactions SC89 ($^4\Lambda$, $^3\Lambda$, $\Lambda$H) and SC97a-f ($^4\Lambda$, $^3\Lambda$, $\Lambda$H) and study their predictions for the $\Lambda$ separation energies (SE’s) of the 4-body hypernuclei.

Both Nijmegen models are based on one-meson exchange and take pseudoscalar, vector and also a scalar meson nonets into account. The models are augmented by Pomeron and tensor meson exchange. The coupling constants within these nonets are related by flavor-SU(3) symmetry and are mostly determined by fits to NN scattering data. The physical masses of the mesons and mixing between mesons and between baryons introduce a sizable charge symmetry breaking (CSB). Additionally, the new models include more flavor-SU(3) breaking mechanisms (for details see ($^4\Lambda$, $^3\Lambda$, $\Lambda$H)). Tuning the magnetic $F/(F+D)$ ratio for the vector mesons, the Nijmegen group provided a series of models SC97a-f, which give a very different spin-spin interaction, to enable research on the strength of this part of the force.

As most of the YN models, the Nijmegen group incorporates the strong conversion process of the $\Lambda N$ to a $\Sigma N$ system explicitly. This is very important, because the conversion process is strongly affected by the nuclear medium. The YN interaction is generally weaker than the NN interaction leading to a core-hyperon structure of the hypernuclei. In some cases $\Lambda-\Sigma$ conversion is suppressed, if a change of the isospin of the core nucleus requires excitations ($^4\Lambda$, $^3\Lambda$, $\Lambda$H). The contribution of this process is much stronger than $\Delta-N$ conversion in ordinary nuclei, because it accounts for the long-range part of the interaction, it is not suppressed in s-waves and because the $\Lambda-S$ mass difference is much smaller. Because of this, effective $\Lambda N$ interactions require strong 3-baryon and even higher order forces, which are quite unknown. Predictions for few-baryon systems only based on 2-body effective $\Lambda N$ interactions are meaningless. On the other side an understanding of the medium dependence of the $\Lambda N$ force provides insights into the interaction mechanisms, which cannot be obtained studying the NN or even the YN system.

We already emphasized that the YN data are not sufficient to probe the interaction models. Therefore the few-body hypernuclear bound states are very important benchmarks for these interactions ($^4\Lambda$, $^3\Lambda$, $\Lambda$H) because exact solutions based on the full interaction models can now be obtained and because the SE’s are experimentally known. There is no YN 2-body bound state. The lightest bound system, the $^3\Lambda$H, has already been solved by Miyagawa in ($^4\Lambda$, $^3\Lambda$, $\Lambda$H). We confirmed that from the set of Nijmegen YN forces only SC89, SC97e and SC97f bind $^3\Lambda$H. For those models we summarize our results in Table ($^4\Lambda$, $^3\Lambda$, $\Lambda$H). While SC97e underbinds $^3\Lambda$H considerably, SC89 and SC97f are in agreement with the experiment. The results are converged within 2 keV. We also confirmed the independence of the results from the used NN force. The results shown are based on the Nijm 93 ($^4\Lambda$, $^3\Lambda$, $\Lambda$H) or Bonn B ($^4\Lambda$, $^3\Lambda$, $\Lambda$H) NN interaction. We emphasize that our values are based on the full Nijmegen interaction models. We also applied the Gaussian approximation SC97-sim, which recently has
been developed to simulate the SC97f interaction [2]. We found that this interaction overestimates the original result by 109 keV and that conclusions on the interaction based on approximated potentials should be taken with caution.

In the Faddeev equations, the interaction enters via their t-matrices [7]. Therefore one simulates an effective AN interaction, which predicts the same AN phase shifts as the original interaction, if one takes Λ-Σ conversion for the evaluation of the t-matrix into account, but keeps only the AN-AN elements for the 3-body calculation. We found that one loses about 116 keV binding in this way (in case of SC89). We consider this result as an important additional hint that effective ΛN interactions fail in the AN version is important. From our wave functions (WF’s) we extracted that the Λ particle predominantly stays far apart from the 2 nucleons, with a r.m.s. distance from

| YNF  | NNF  | $E^\Lambda_{sep}$ |
|------|------|-----------------|
| SC97e Nijm 93 | -0.023 |
| SC97f Nijm 93 | -0.080 |
| SC89 Nijm 93 | -0.143 |
| SC89 Bonn B | -0.155 |
| Expt. | -0.130(50) |

TABLE I: A SE’s of $\Lambda$H for different YN and NN force combinations (YNF and NNF) compared to the experimental value. All energies are in MeV.

Using the permutation $P = P_{12}P_{23} + P_{13}P_{23}$, we refer the reader to [21, 23] for a detailed exposition how to solve that set in a numerically precise manner. The $1^+$ states are more demanding, therefore we had to put a more rigid limit on the partial wave decomposition for this state. However, we checked that the SE’s for both states are converged within 50 keV. The difference $\Delta$ of the SE’s of both states is based on equivalent truncations for $0^+$ and $1^+$ states. Note that therefore the results for the SE’s and $\Delta$ do not exactly match in the following tables, but that we could obtain a more accurate result for $\Delta$ in this way.

To gain insight into CSB effects we take the mass differences for the nucleons and the $\Sigma$’s into account, also CSB of the YN interactions and the Coulomb forces in the pp, $\Sigma^+p$ and $\Sigma^-p$ pairs. Our calculations are, however, restricted to the total 4-body isospin $T = \frac{3}{2}$, which we expect to be a very good approximation.

In Table II we document that the SE’s are only moderately dependent on the used NN interaction and on a 3N force (we used Tucson-Melbourne (TM) [22]). The NN interactions Nijm 93 and Bonn B have got a very different functional form. Therefore the results can serve as an estimation of the systematic uncertainty of the SE’s due to the choice of nucleon interactions. We find 120 keV uncertainty for the SE’s and 50 keV for the energy splitting $\Delta$ between the $0^+$ and $1^+$ states. The following results are based on the Nijm 93 NN potential and the 3N force will be neglected.

\[ +(1 + P)\psi_{2A} + (1 + P)\psi_{2B} \]

\[ \text{(6)} \]

\begin{itemize}
  \item SC97e Bonn B — 1.66 0.80 0.84
  \item SC97e Nijm 93 — 1.54 0.72 0.79
  \item SC97e Nijm 93 TM 1.56 0.70 0.82
  \item SC89 Bonn B — 2.25 — —
  \item SC89 Nijm 93 — 2.14 0.02 2.06
  \item SC89 Nijm 93 TM 2.19 — —
  \item Expt. 2.39(3) 1.24(5) —
\end{itemize}

TABLE II: NN and 3N interaction dependence of the $\Lambda$He SE’s $E^\Lambda_{sep}$ and the $0^+-1^+$ splitting $\Delta$. We show results for different combinations of YN, NN and 3N force (YNF,NNF and 3NF). All energies are given in MeV.

\begin{itemize}
  \item YNF  NNF  3NF  $E^\Lambda_{sep}(0^+)  E^\Lambda_{sep}(1^+)  \Delta$
  \item SC97e Bonn B — 1.66 0.80 0.84
  \item SC97e Nijm 93 — 1.54 0.72 0.79
  \item SC97e Nijm 93 TM 1.56 0.70 0.82
  \item SC89 Bonn B — 2.25 — —
  \item SC89 Nijm 93 — 2.14 0.02 2.06
  \item SC89 Nijm 93 TM 2.19 — —
  \item Expt. 2.39(3) 1.24(5) —
\end{itemize}

TABLE III: Predictions for the $\alpha$He $E^\Lambda_{sep}$ and the $0^+-1^+$ splitting $\Delta$ of the different YN potential models (YNF) in combination with Nijm 93 in comparison to the experimental values. The 3NF has been neglected. All energies are in MeV.
We show in Table IV the SE’s of \( ^3\text{He} \) for the \( 0^+ \) and \( 1^+ \) states and their difference choosing the YN forces, which bind \( ^3\text{H} \) and in addition SC97d, since it predicts a larger triplet than singlet \( \Lambda N \) scattering length. We see that SC89 comes closest to the experimental value for the \( 0^+ \) state, but fails totally for the excited state. In case of the SC97 potentials the SE’s drop from \( t \) to \( d \) in case of the \( 0^+ \) state, but increase for the excited state. In no case one comes close to the experimental values. The experimental \( 0^+\)\(-1^+\) splitting, however, is reached by SC97f. For all interactions the ordering of the spin states agrees with the experimental result, independent on the size of the \( \Lambda N \) scattering length predictions of the models. This clearly shows that direct conclusions from the 4-body binding energies on the singlet and triplet scattering lengths are not possible. We would like to add that SC97f-sim leads to an increased \( V_{\Lambda N,\text{nucl.}} \) for the \( 0^+ \) state SE of 600 keV in relation to SC97f and thus comes very close to the experimental value. This does, however, not reflect the properties of the full interaction. Conclusions on the Nijmegen YN interactions based on simulating forces are misleading.

Truncating again the YN \( t \)-matrix to the \( \Lambda N \)-\( \Lambda N \) channel, we find for SC97e, as an example, a 400 keV reduction for the \( 0^+ \) state, but a slight increase of 7 keV for the \( 1^+ \) state, showing that the inclusion of the \( \Sigma \)-channel is strongly spin dependent and can be repulsive or attractive. Again, it is mandatory to take the \( \Lambda \)-\( \Sigma \) conversion fully into account.

In recent years many studies focused on the splitting of the \( 0^+ \) and \( 1^+ \) states \( \Delta \) and its connection to the spin-spin interaction \([25]\) in the YN system. Unfortunately, \( \Delta \) is affected strongly by other parts of the interaction, namely the \( \Lambda \)-\( \Sigma \) conversion \([26, 27, 28, 29, 30]\). Additionally, we argue that unknown \( 3\text{BF’s} \) in the \( \text{YN} \) system probably affect the \( 0^+ \) and \( 1^+ \) state differently and have strong impact on \( \Delta \). The effect of those forces are known to be visible from the ordinary nuclei \([12, 22]\). Fortunately, one can expect approximate isospin invariance for those forces. Therefore their contribution to CSB is presumably small. This makes the SE’s differences \( \Delta_{\text{CSB}} \) of \( ^3\text{He} \) and \( ^3\text{H} \) a very interesting observable to pin down properties of the YN 2-body interaction.

To the best of our knowledge \( \Delta_{\text{CSB}} \) has never been completely estimated before. It has been suggested that the \( \Sigma^+, \Sigma^0 \) and \( \Sigma^- \)-states are not equally populated in the 2 hypernuclei. Because of the mass difference within the \( \Sigma \)-multiplet this leads to a shift in the kinetic energy, which is different in \( ^3\text{He} \) and \( ^3\text{H} \).\([31]\). Further the \( \Lambda \)-\( \Sigma \) conversion creates charged \( \Sigma \) pairs and pp pairs. This leads to Coulomb force effects \([11]\). In addition a “core compression effect” has been studied \([8, 31]\): the \( 3\text{He} \) core in \( ^3\text{He} \) should be slightly compressed because of the increased binding and this should lead to an increased Coulomb repulsion for the pp pairs. All these effects were separately estimated based on simplified models. They showed that the kinetic energy effect and the direct Coulomb effect might cause \( \Delta_{\text{CSB}} \). The “core compression effect” was found to be less important \([31]\).

We performed complete calculations for \( \Delta_{\text{CSB}} \) in the \( 0^+ \) and \( 1^+ \) states for SC97e. For SC89 we restricted ourselves to the \( 0^+ \) state, because of its unrealistic SE for the \( 1^+ \) state. The results are given in Table V. Using SC97e \( \Delta_{\text{CSB}} \) for the \( 0^+ \) state is visibly underestimated. To our surprise it agrees with the experimental value in case of SC89. For SC97e and the \( 1^+ \) state \( \Delta_{\text{CSB}} \) has even the wrong sign. One has to conclude that none of the present day meson-theoretical Nijmegen YN forces describes the \( 0^+ \) state energies, the \( 0^+\)-\( 1^+ \) spin splittings and the differences in the SE’s for \( ^3\text{He} \) and \( ^3\text{H} \) correctly.

Nevertheless, it is interesting to shed light on the origin of the CSB in the SE’s. To this aim we performed a perturbative study of the origin of \( \Delta_{\text{CSB}} \) and estimated the contributions from the expectation values for the kinetic energy \( \Delta V^\text{CSB}_{\text{kin}} \), the strong YN interaction \( \Delta V^\text{CSB}_{\text{YN,nucl.}} \), the Coulomb interaction in pp pairs \( \Delta V^\text{CSB}_{\text{YN,pp}} \), and in YN pairs \( \Delta V^\text{CSB}_{\text{YN,NN}} \), which we extracted from our realistic WF’s for \( ^3\text{He} \) and \( ^3\text{H} \) as described in \([21]\). We study the YN force models SC89 and SC97e in the \( 0^+ \)-state only and neglect the contribution of the strong NN interaction \( \Delta V^\text{CSB}_{\text{NN,nucl.}} \). This appears justified, since the CSB of NN forces should be very similar for the pairs \( ^3\text{He} \) and \( ^3\text{H} \). Our perturbative results are presented in Table V. They agree within 10 keV with the nonperturbative ones from Table IV which shows that the perturbative estimates are sufficiently accurate. We see that the Coulomb force plays a minor role, in contrast to the estimation given in \([11]\). We confirm a significant contribution of the \( \Sigma \)-mass differences showing up in \( \Delta V^\text{CSB}_{\text{YN,nucl.}} \). There are 2 effects contributing to \( \Delta V^\text{CSB} \), a mass shift and a change in the momentum dependent part of the kinetic energy \( T \). The first one can be shown to be \( \Delta V^\text{CSB}_{\text{\Sigma mass}} = (P_{\Sigma^+} - P_{\Sigma^-}) \left( m_{\Sigma^0} - m_{\Sigma^-} \right) \). Here \( P_{\Sigma^\pm} \) are the \( \Sigma \)-probabilities in \( ^3\text{He} \). The \( \Sigma \) probabilities extracted from our wave functions lead to 80(210) keV for SC97e(SC89) for this quantity. The mass shift contribution is reduced by a contribution from the momentum

| \( J^\pi \) | YNF | \( E_{\text{sep}}^\Lambda (^3\text{He}) \) | \( E_{\text{sep}}^\Lambda (^3\text{H}) \) | \( \Delta_{\text{CSB}} \) |
|---|---|---|---|---|
| \( 0^+ \) SC97e | 1.54 | 1.47 | 0.07 |
| SC89 | 2.14 | 1.80 | 0.34 |
| Expt. | 2.39(3) | 2.04(4) | 0.35 |
| \( 1^+ \) SC97e | 0.72 | 0.73 | -0.01 |
| Expt. | 1.24(5) | 1.00(6) | 0.24 |

TABLE IV: CSB splitting of the \( ^3\text{He},^3\text{H} \) mirror nuclei. We show SE’s \( E_{\text{sep}}^\Lambda \) and the CSB splitting \( \Delta_{\text{CSB}} \) of the SC89 and SC97e potential models (YNF). The first 3 rows compare \( 0^+ \) state results to the experimental values, the last 2 \( 1^+ \) states results. The calculations are based on Nijm 93, the 3NF has been neglected. All energies are given in MeV.
dependent part of $T$. However, the sign of $\Delta T^{CSB}$ is driven by the mass shift. Therefore an increase of mass of the $\Sigma$’s leads to a decrease in binding, just opposite to the effect of increasing nucleon masses in ordinary nuclei.

The mass shift contribution also increases for higher $\Sigma$-probabilities, which is directly governed by the $\Lambda$-$\Sigma$ conversion. Thus we find again that the study of the conversion process is crucial for hypernuclear physics and that the hypernuclei provide important information on this process. We suggest to focus more on the CSB because of larger uncertainties in the predictions of the spin splitting.

The second very important part of $\Delta_{CSB}$ comes from $\Delta V_{YN,nucl}^{CSB}$. In the Nijmegen interaction models the $V_{YN,nucl}$ are generated by the mass differences of the baryons and mesons and by the $\Lambda - \Sigma^0$ mixing. If we just switch off that mixing the $\Delta V_{YN,nucl}^{CSB}$ value reduces drastically to 52(-1) keV for SC89(SC97e). Thus the mixing plays in these interaction models an important role.

Finally we present some WF properties of the hypernuclei $^3_\Lambda$He and $^3_\Lambda$H. The $0^+\rightarrow 1^+$ states differ predominantly in their spin structure and they can be seen as spin flip states. This is supported by the probability to find the total spins $S$=0,1, and 2 for the 2 states. For instance for SC89 and $^3_\Lambda$He we find 90.32%, 0.13% and 9.55% for $0^+$ and 0.02%, 95.71% and 4.25% for $1^+$. It is also interesting to see the probabilities in ground states of $^3_\Lambda$He for the hyperon to be a $\Sigma$. The SC97d-f models predict $P_S=1.49\%$ to 1.76\%, whereas SC89 leads to $P_S=4.08\%$. Therefore our $\Delta_{CSB}$ results seem to support a large $\Sigma$ component in the WF. Recently a similar conclusion has been drawn from the observed $\pi^+$ decay width of $^3_\Lambda$He [22]. Note that the $\Sigma$-probabilities are also much larger than the ones for $^3_\Lambda$H.

Again we calculated the r.m.s. distances $r$ of the hyperons to the center of mass of the nucleons. For $\Lambda$ they depend strongly on the SE ranging from 20.1 fm for the $1^+$ state for SC89 to 3.5 fm for the $0^+$ state for SC89. The $\Sigma$ particle is again forced to stay close to the nucleons (1.6 fm to 2.0 fm for all considered states, similar to $^3_\Lambda$H).

In summary we presented the first SE results based on complete meson-theoretical YN interactions for 4-body hypernuclei. The models of the SC89 and SC97 series, when taken alone, fail. Additional, though still unknown, YNN 3BF’s might influence the $0^+\rightarrow 1^+$ splitting. Therefore we suggest to focus on the CSB of the SE’s and showed a strong connection to the interesting $\Lambda$-$\Sigma$ conversion process.

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\begin{table}[h]
\begin{tabular}{|c|c|c|c|c|}
\hline
YNF & $\Delta T^{CSB}$ & $\Delta V_{YN,nucl}^{CSB}$ & $\Delta V_{YN,nucl}^{CSB}$ & $\Delta V_{YN,nucl}^{CSB}$ \\
\hline
SC89 & 132 & -9 & 255 & -27 \\
SC97e & 47 & -9 & 44 & -7 \\
\hline
\end{tabular}
\caption{Perturbative calculation of the CSB splitting of $^3_\Lambda$He and $^3_\Lambda$H SE’s in the $0^+$ states. See text for explanations of the various parts. The results are based on the SC89 or SC97e YN force (YNF) and on Nijm 93. The 3NF has been neglected. All energies are given in keV.}
\end{table}