Twin Higgs Asymmetric Dark Matter

Isabel García García,1 Robert Lasenby,1 and John March-Russell1,2

1Rudolf Peierls Centre for Theoretical Physics, University of Oxford, 1 Keble Rd., Oxford OX1 3NP, UK
2Department of Physics, Stanford University, Stanford, CA 94305, USA

We study Asymmetric Dark Matter (ADM) in the context of the minimal (Fraternal) Twin Higgs solution to the little hierarchy problem, with a twin sector with gauged SU(3)′ × SU(2)′, a twin Higgs, and only third generation twin fermions. Naturalness requires the QCD′ scale λ′ QCD ~ 0.5 – 20 GeV, and t′ to be heavy. We focus on the light b′ quark regime, m_{b'} \lesssim \lambda'_{QCD}, where QCD′ is characterised by a single scale \lambda'_{QCD} with no light pions. A twin baryon number asymmetry leads to a successful DM candidate: the spin-3/2 twin baryon, \Delta′ ∝ b′b′', with a dynamically determined mass (∼ 5\lambda'_{QCD}) in the preferred range for the DM-to-baryon ratio \Omega_{DM}/\Omega_{baryon} ∼ 5. Gauging the U(1)′ group leads to twin atoms (\Delta′ - \overline{\Delta′} bound states) that are successful ADM candidates in significant regions of parameter space, sometimes with observable changes to DM halo properties. Direct detection signatures satisfy current bounds, at times modified by dark form factors.

PACS numbers:

I. INTRODUCTION

Despite overwhelming evidence for the existence of Dark Matter (DM), its precise nature remains a mystery. Moreover, the closeness of DM and baryon energy densities, \Omega_{DM} / \Omega_{baryon} ~ 5, is fundamentally puzzling: there seems to be no reason for these two quantities, a priori unrelated, to be so close to each other. This puzzle motivates the idea of Asymmetric Dark Matter (ADM) [1]–[12], based on the assumption that the present DM density is set by an asymmetry \eta_{DM} in the DM sector, analogous to the baryon asymmetry \eta_{baryon}. Then

\[ \frac{\Omega_{DM}}{\Omega_{baryon}} = \frac{m_{DM}}{m_N} \frac{\eta_{DM}}{\eta_{baryon}} \]

where \text{m}_N is the nucleon mass and \text{m}_{DM} the mass of the DM particle. A linked asymmetry of the same order in both sectors, |\eta_{DM}| / |\eta_{baryon}|, is relatively easy to achieve, but a successful explanation of \Omega_{DM}/\Omega_{baryon} requires a reason for \text{m}_{DM} / \text{m}_N, and this has been the Achilles heel of ADM model building.

Another pressing worry is the LHC naturalness problem: why have the new particles and/or dynamics that stabilise the weak scale not been observed? The Twin Higgs (TH) solution to this little hierarchy problem is based on the realisation of the Higgs boson as a pseudo-Nambu-Goldstone of an approximate global SU(4) symmetry [13]–[15]. The TH mechanism introduces a Standard Model (SM) neutral sector, the twin sector, that is an approximate copy of the SM, and the Higgs sector of the theory must respect, at tree-level, an SU(4) global symmetry that acts on the two (visible and twin sector) Higgs doublets. A Z_2 between sectors imposes all couplings to be equal and ensures that radiative corrections to the Higgs soft mass squared are SU(4) symmetric. The global SU(4) is only broken at 1-loop and the Z_2 must be broken, explicitly or otherwise, for the vacuum expectation value (vev) of the twin and visible sector Higgs (denoted f and v ≈ 246 GeV respectively) to be different. As the possibility of \text{f} = \text{v} is ruled out by Higgs coupling measurements, the minimal fine-tuning in the electroweak sector is then given by \simeq \frac{2\pi}{7} \approx 20\% (only \sim 20\% for \text{f}/\text{v} ≃ 3, the minimum experimentally allowed ratio).

The TH mechanism does not require the twin sector to be an exact copy of the SM. A minimal realisation, the Fraternal Twin Higgs (FTH) [16], only requires in the twin sector SU(3)′ × SU(2)′ interactions, top and bottom quarks (Q′, t_R′, b_R′), and lepton (L′) and Higgs (H′) doublets. Twin right-handed leptons are not required but may be added, and a \text{U}(1)′ gauge group is not required by naturalness, although it remains an accidental global symmetry. Masses of twin fermions are set by their Yukawa couplings and the ratio f/v. Naturalness requires a twin top Yukawa y_{t'} ∝ y_t, but only imposes y_{t'} ≪ 1 (t′ ≠ t'). Most important for us, values of the \text{g}_3 gauge coupling consistent with naturalness imply a QCD′ scale \lambda'_{QCD} ~ 0.5 – 20 GeV for a 5 TeV cutoff [17]. (The theory needs UV-completion at some scale \text{M}_{UV} \lesssim 4\pi \text{f}.)

The purpose of this letter is to explore the possibility of ADM in the FTH context [18]. We work in the regime, m_{b'} \lesssim \lambda'_{QCD}, where the twin QCD′ theory is determined by a single scale, arguing that the baryon \Delta′ ∝ b′b′', either on its own, or in an atomic bound state with a \overline{\Delta′} in the gauged \text{U}(1)′ case, is a successful ADM candidate.

II. STABLE & RELATIVISTIC TWINS

Within the FTH scenario, the twin sector respects three accidental global symmetries: twin baryon number B’, lepton number L’ and ‘charge’ Q’. If these are not too badly broken by higher dimensional operators (HDOs), as we will assume, then the lightest twin particles carrying these quantum numbers will be cosmologically stable states. Twin \text{CP} could be a good discrete symmetry of the twin sector, although both P and C are violated by
SU(2)′ interactions.

We consider massive τ′ but allow for heavy or massless ν′, usually with m_{τ′} < m_{ν′} such that W^±′ gauge bosons decay, although a possibly interesting scenario arises if m_{τ′} + m_{ν′} > m_{ν′} and W^±′ are stable. For m_{ν′} ≤ Λ_{QCD}′, the lowest QCD′ states are W^±′ mesons, the lightest being a pseudoscalar ˆν and a scalar ˆχ with masses m_{ν′} ≈ (2 − 3)Λ_{QCD}′ and m_{χ′} ≈ 1.5m_{ν′} [20]. (A distinctive feature is the absence of pseudo-Nambu-Goldstone bosons due to the chiral anomaly.) The glueball spectrum is heavier and only weakly mixed with the mesons, with the lightest being a 0^{++} state of mass m_{ν′} ∼ 7Λ_{QCD}′ [21, 22]. Meson/glueball states decay quickly via SU(2)′ interactions to W^±′ pairs if m_{ν′} ≈ 0 (and multi-γ′ states if U(1)′ is gauged) and lighter mesons/glueballs, or to SM states via twin-scalar–Higgs mixing [19, 23]. Independently of m_{ν′}, the lightest twin meson ˆν may decay very fast via dimension-six HDOs that preserve total CP, of the form ∼ (qγ5q Bγ5b′)/M^2, where q denotes SM quarks (for M ∼ 10 TeV, this gives a lifetime τ_{ν′} ≈ 10^{-14} s).

The spin-3/2 twin Δ′ baryon with mass m_{Δ′} ≈ 5Λ_{QCD}′ [20] and U(1)′ global charge +1, is naturally extremely long-lived since it is the lightest B′ ≠ 0 object. Moreover, the leading HDO violating SM and twin baryon numbers but preserving a linear combination is dimension-12, resulting in a lifetime τ_{Δ′} ∼ 10^{26} s for m_{Δ′} ∼ 10 GeV and M ∼ 10 TeV. Thus even in the presence of HDOs, Δ′ can be stable on cosmological timescales. For the purposes of this paper we assume that the Δ′ is the only twin baryon number carrying state with a cosmologically relevant lifetime. (The presence of heavier stable twin baryon states would not qualitatively change our conclusions.)

Dark radiation (DR) contributions to the number of effective neutrino species, ΔN_{eff}, can arise from light twin neutrinos, and twin photons when U(1)′ is gauged. Due to the extremely fast decay of the lightest twin meson ˆν into SM states naturally present via HDOs, we expect the ν′ and γ′ sectors to remain in equilibrium with the SM after the QCD′ phase transition, even for values of Λ_{QCD}′ as small as ∼ 0.5 GeV. As a result, in the case of m_{ν′} ≈ 0 and no gauge U(1)′ we expect a contribution to ΔN_{eff} of ±0.075 (as argued in Section VIII of [24]) and of ±0.16 when twin photons are also present. Notice these are the minimum possible contributions to ΔN_{eff} and are compatible with the current measured value ΔN_{eff, SM} ≈ 0.1 ± 0.2 [25], although future experiments may achieve an accuracy of ∼ 0.05 [26, 27] and therefore probe these two scenarios.

III. TWIN BARYON & W’ DARK MATTER

The ADM scenario necessarily has a linked asymmetry in SM- and twin-sector quantum numbers. The generation of such an asymmetry is a UV issue — here we simply assume that it is present. In addition, ADM requires efficient annihilation of the symmetric component of stable DM states, so that the final DM abundance is dominantly set by the asymmetry. In our case, annihilation of the symmetric component of the twin baryon states happens efficiently via twin strong interactions. Sufficiently heavy τ′ and ν′ states also annihilate efficiently, mainly to ˆτ′/ ˆν states (see Figure 2 in [24]). The QCD′ phase transition for m_{ν′} ≤ Λ_{QCD}′ is a smooth crossover [28–30], so we expect neither significant non-equilibrium dynamics nor entropy production affecting relic densities.

A twin baryon number asymmetry implies an asymmetric relic population of Δ′ baryons. If η_{QCD}′ = 0, then the (ungauged) charge density of the Δ′ population must be balanced by a population of twin charged states. So, if the Δ′ baryons are to be the only significant DM component, either m_{ν′} ∼ 0 so that an asymmetric abundance of these can exist as DR, or we must have a compensating asymmetry in (global) twin charge, η_{ν} ∼ −η_{ν′}. Depending on UV dynamics there may be a non-zero twin lepton asymmetry setting an asymmetric ν′ DR relic density (the τ′ density is fixed by η_{ν} and η_{QCD}′).

As anticipated in Section II, m_{Δ′} ∼ 5Λ_{QCD}′ [20], which translates into η_{ν′}/η_{baryon} ∝ m_{Δ′}/Λ_{QCD}′, with Λ_{QCD}′ = 0.5 − 20 GeV [19]. Thus this framework allows for a successful realisation of ADM in which the mass of the DM particle is not tuned to be O(10 GeV), but rather is set by the confinement scale of the DM sector, whose range is restricted directly by naturalness arguments. The value of η_{ν′} is irrelevant for the DM mass as long as m_{ν′} ≤ Λ_{QCD}′ is realised. DM in this framework is then made of individual Δ′ baryons. Bound states, if they exist in the spectrum, will not form in the early universe, since the only states parametrically lighter that could be emitted in the binding processes are ν′ or light SM states, but these both only interact via tiny sub-weak interactions. Moreover, we find that even in the presence of twin photons, radiative capture is not fast enough to give a significant population of Δ′ − Δ′ bound states as the electric and magnetic dipole radiative capture rates vanish. (This situation can be significantly different when lighter generations are present, in which case bound states may form allowing for a scenario of nuclear DM [31, 32].)

Regarding Δ′ self-interaction bounds we have, parametrically, σ_{Δ′}/m_{Δ′} ∼ (Λ_{QCD})^{-3} ∼ 10^{-3} − 10^{-8} cm^2 g^{-1} for Λ_{QCD}′ = 0.5 − 20 GeV, well below the current experimental upper bound of ∼ 0.5 cm^2 g^{-1} [33].

Finally, in the case where m_{τ′} + m_{ν′} > m_{W′}, W′± are also a stable states, and even if η_{W} = −η_{QCD}, an asymmetric population of ν′ (t) states could survive, whose charge is balanced by an equal number of asymmetric W^±′ (W^′−) states. Notice that for small values f/v ≈ 3 − 5 (see Figure 4 in [24]), annihilation of the symmetric populations of τ′, ν′ and W′± occurs very efficiently. For this latter possibility to be realised without introducing significant extra tuning, one would need m_{τ′}, m_{ν′} ∼ 10^2 GeV (since m_{W′} ∼ (f/v)m_{W}), above the
mass range where ADM scenarios work most naturally. (Scattering cross sections of such states off SM nucleons via the Higgs portal are \( \lesssim 10^{-45} \text{ cm}^2 \) for \( f/\nu \gtrsim 4 \), close to current bounds.)

III.2. Direct Detection

Scattering of \( \Delta' \) baryons off SM nucleons happens via Higgs exchange or by exchanging a twin scalar state (\( \hat{\chi} \) meson or \( 0^{++} \) glueball) that mixes with the Higgs. Couplings between scalar mesons/glueballs and a pair of twin baryons are unknown and require dedicated lattice computation. We find that within a reasonable range for the couplings and mixing angles either Higgs exchange or meson/glueball exchange can dominate the scattering. We therefore separately consider the processes (ignoring interference effects) to give an idea of the possible scattering cross sections.

In the case where Higgs exchange dominates, the spin independent scattering cross section is given by

\[
\sigma_{\text{SI}}^\chi \approx \frac{1}{\pi} \frac{\mu_{\Delta'N}^2}{m_{\chi}^4 \nu^2} \frac{(m_{\Delta}f_{\Delta})^2}{(f/\nu)^4} \tag{2}
\]

where \( \mu_{\Delta'N} = m_{\Delta}m_{\Delta'}/(m_{\Delta} + m_{\Delta'}) \) is the reduced mass of the \( \Delta' \)-nucleon system. \( f_{\Delta} \approx 0.32 \) \cite{34,39} and \( f_{\Delta'} = (2 + 87f_{\Delta})/31 \) (following \cite{67}) are the effective Higgs couplings to nucleons and \( \Delta' \) baryons, respectively, where \( f_{\Delta} \) is the dimensionless part of the matrix element of \( b' \) in \( \Delta' \). In the light \( b' \) case, one expects \( f_{\Delta} \ll 1 \) albeit its exact value requires dedicated lattice study. In the case where the dominant process is meson exchange, the cross section can be written as

\[
\sigma_{\text{SI}}^\chi \approx \frac{1}{\pi} \frac{\mu_{\Delta'N}^2}{m_{\chi}^4 \nu^2} \frac{(m_{\Delta}f_{\Delta})^2}{(f/\nu)^4} \tag{3}
\]

where \( \lambda' \) is the coupling between \( \hat{\chi} \) and a pair of \( \Delta' \) baryons and \( \theta' \) is the Higgs-\( \hat{\chi} \) mixing angle

\[
\theta' = \frac{f_{\chi}m_{\chi}}{2f(f/\nu)m_\chi^2 - m_\chi^2} \tag{4}
\]

with \( F_{\chi} \) the \( 0^{++} \) meson decay constant that we define as \( F_{\chi} \equiv a'm_{\chi}^2 \) (with \( a' \) an unknown dimensionless constant) and \( f_{\chi} = (2 + 58f_{\Delta})/31 \) accounts for the effective coupling between meson and Higgs. Numerical evaluation shows that for \( \lambda' \lesssim 1 \) Higgs exchange dominates, whereas for \( \lambda' \gtrsim 4\pi \) (the NDA value) meson exchange provides the leading interaction. In the event of glueball exchange being the dominant process, the scattering cross section is given by Eq.\( \text{(3)} \) after performing the appropriate substitutions.

Figure 1 shows these spin independent scattering cross sections for particular choices of the unknown parameters. To illustrate the range possible we have chosen the minimum Higgs-exchange cross section (i.e. \( f_{\nu} = 0 \)), while for meson exchange we have selected reasonably large values of the parameters. Note that different choices allow Higgs or glueball exchange to dominate. A significant portion of parameter space is covered by the neutrino floor, in particular the region \( m_{\Delta'} \approx 5 \text{ GeV} \) that would allow for \( \eta_{\Delta'} \approx \eta_{\text{baryon}} \). For values \( m_{\Delta'} \approx 10 - 50 \text{ GeV} \), that correspond to \( \eta_{\Delta'}/\eta_{\text{baryon}} \approx 0.5 - 0.1 \), predicted cross sections escape the neutrino background and sit close to (or within) the region that will be probed by next-generation experiments such as LZ \cite{35}.

\[ \text{FIG. 1: Illustrative range of possible spin independent scattering cross sections of } \Delta' \text{ baryons off SM nucleons when either Higgs or } \hat{\chi} \text{ meson exchange dominates (dashed and thick lines respectively). We take } m_{\chi} = 3\Lambda_{\text{QCD}}, \lambda' = 4\pi, a' = 1, f_{\nu} = 0 \text{ and } f_{\nu} = 0.1 \text{ for illustration. Blue: LUX excluded } \cite{39}; \text{ blue line: LUX projected sensitivity (300 live-days) } \cite{40}; \text{ orange: neutrino background } \cite{35}; \text{ pink dotted line: LZ sensitivity } \cite{38}; \text{ pink: values of } m_{\Delta'} \text{ (equivalently, of } \Lambda_{\text{QCD}}' \text{) that imply extra tuning } \cite{19}. \]

IV. TWIN ATOMS

Once the \( U(1)' \) group is gauged, the physics becomes substantially richer. Twin-charge neutrality of the Universe requires \( \eta_{Q'} = 0 \), which means that a \( B' \) asymmetry resulting in a non-zero asymmetric population of \( \Delta' \) baryons must be balanced by an \( L' \) asymmetry, such that an equal asymmetric population of \( \vec{\tau} \) is present (we here assume that \( W^{\pm}_{\tau} \) are unstable). Due to twin electromagnetic interactions, the asymmetric populations of \( \Delta' \) and \( \vec{\tau} \) states may form bound states. In fact, the late-time DM population must consist of overall-neutral ‘twin atoms’, rather than a plasma of charged states, for
values of the twin electromagnetic coupling $\alpha'$ that are not extremely small; otherwise, the long-range interactions between DM particles result in plasma instabilities that strongly affect Bullet Cluster-like collisions [41–43]. Requiring that efficient twin recombination takes place imposes non-trivial constraints on the sizes of $\alpha'$ and the mass of the twin atom $\hat{H}$ [44]. Further constraints are present due to DM self-interactions: Low energy atom-atom scattering processes have cross sections $\sigma \approx 10^2 (a_0')^2$ where $a_0' = (\alpha' \mu_{\hat{H}})^{-1}$ is the atomic Bohr radius and $\mu_{\hat{H}}$ the reduced mass of the atomic system, although the exact value of $\sigma$ depends strongly on the ratio $R \equiv m_{\Delta'}/m_{\tau'}$ for values $R \gtrsim 15$ [45]. We impose the constraint $\sigma/m_{\hat{H}} \lesssim 0.5 \text{ cm}^2 \text{ g}^{-1}$ applicable to contact-like DM scattering, since the effect of hard scatterings generally dominates over soft or dissipative processes for atom-atom scattering in the regimes we consider. Figures 2 and 3 show constraints from recombination [44] and DM self-interactions, for ratios $R \equiv m_{\Delta'}/m_{\tau'} = 1$ and 10 respectively.

For values of $\alpha'$ and $m_{\hat{H}}$ satisfying recombination and self-interaction constraints, and for the parameter ranges we consider, annihilation of the symmetric populations of $\Delta'$ and $\tau'$ happens very efficiently. As can be seen from Figures 2 and 3, the minimum value of $\alpha'$ consistent with all constraints is $\alpha' \approx 10^{-2}$, in which case the twin atom mass is constrained to be $m_{\hat{H}} \approx 20, 40 \text{ GeV}$ for $R = 1, 10$ respectively. This results in binding energies of order $O(10^2) \text{ keV}$, and a hyperfine splitting of the first atomic energy level of order $\Delta E \sim 10 \text{ eV}$.

![FIG. 2: Constraints in the $\alpha', m_{\hat{H}}$ plane, for a ratio $R = m_{\Delta'}/m_{\tau'} = 1$. Blue: twin recombination is inefficient, an ionised fraction $\gtrsim 0.1$ remaining; pink: self-interaction cross section is $\gtrsim 0.5 \text{ cm}^2 \text{ g}^{-1}$; green: twin atom masses small enough that significant extra tuning is present.](image)

Before twin sector recombination occurs, the $\Delta'$ and $\tau'$ are coupled to the twin photon bath, constituting a dark plasma that can undergo 'dark acoustic oscillations' [44]. If twin sector recombination is late enough, these oscillations can leave an imprint in the power spectrum of baryonic matter. However, since $\alpha' \gtrsim 10^{-2}$ in our allowed regions, the binding energy of our twin atoms is sufficiently high ($\gg 10 \text{ keV}$) that twin recombination is always too early to realise this possibility.

Another possibility is that, after dark recombination, molecular bound states may form at lower temperatures. However, radiative capture of two neutral atoms to a 'dark hydrogen molecule' is very suppressed [46], with molecule formation requiring that there is an abundance of charged particles to catalyse the reactions. Given the constraints that must already be satisfied, our estimates indicate that a significant proportion of molecules will not be formed, either in the early universe, or in halos.

We remark that most of the physics discussed in this section is not specific to FTH models, relying only on asymmetric DM charged under a dark $U(1)$. There is a large body of literature on the physics of such ‘dark atoms’, e.g. [47–50], which in particular can arise in many ‘mirror world’ models [51, 52].

### IV.2. Direct Detection

We first neglect the impact of kinetic mixing between the twin and SM photons on direct detection (DD) signatures and concentrate on the process of scattering purely via Higgs exchange or by exchange of a twin scalar that mixes with the Higgs. An interesting situation arises for $R \approx 1$. In this case, $m_{\Delta'} \approx m_{\tau'}$, and therefore the
Higgs couples to both states with equal strength. On the other hand, the typical size of the atom is set by \( a_0' = (\alpha' \hat{R})^{-1} \), which is \( \approx 4 \text{ fm} \) for \( \alpha' \approx 10^{-2} \) and \( m_{\hat{R}} \approx 20 \text{ GeV} \), values consistent with all constraints (see Figure 2). The size of the atomic system is thus comparable to that of SM nuclei relevant for DD experiments, and the possibility of a detectable ‘dark form factor’ arises (with the form factor approximately given by the Fourier transform of the ground state atomic wavefunction squared). While such a signal would be degenerate with modifications to the DM halo velocity distribution for data from a single DD experiment \([53]\), multiple experiments with different SM target nuclei could allow the dark form factor contribution to be disentangled \([54]\).

Alternatively, if \( R \gg 1 \) then the atom’s coupling to the Higgs is dominantly through the \( \Delta' \), whose structure is on smaller scales than SM nuclei, since \( \Lambda_{QCD}' > \Lambda_{QCD} \). Thus, in this case, we would have a basically momentum-independent dark form factor, and spin independent cross sections would be like those shown in Figure 1.

Finally, kinetic mixing between the two sectors can arise via the operator \( (\epsilon/2) F_{\mu\nu} F^{\mu\nu} \). This results in twin sector particles acquiring SM-sector electric charges of size \( \sim \epsilon' \), with \( \epsilon' = \sqrt{4\pi\alpha} \). Low-energy radiative contributions to the kinetic mixing parameter appear to be absent up to three-loop order \([14, 19]\) and therefore one can expect \( \epsilon \sim (16\pi^2)^{-4} \sim 10^{-9} \) if a non-vanishing four-loop contribution to \( \epsilon \) indeed exists (UV contributions to kinetic mixing can be present depending on the completion). Notice that our DM atoms are neutral under both visible and twin sector electromagnetism and have vanishing permanent electric dipole moments, due to their spherical distribution of charge. Nevertheless, twin atoms have magnetic dipole moments under both sectors, with the visible sector moment suppressed by a factor of \( \epsilon \). Experimental constraints on the size of \( \epsilon \) arise from a combination of astrophysical, accelerator, and direct detection considerations \([55-59]\). The nature of the dominant constraint depends strongly on the values of \( \alpha' \), \( m_{\hat{R}} \) and \( R \) chosen, but for the range of parameters considered in this work, values of \( \epsilon \lesssim 10^{-9} \) are likely to satisfy all current bounds.

\[ \text{V. CONCLUSIONS} \]

We have shown that for the values of \( \Lambda_{QCD}' \) allowed by naturalness, and in the ungauged \( U(1)' \) case, the twin hadron \( \Delta' \sim b'b'b' \) is a successful ADM candidate, with a mass, \( \mathcal{O}(10 \text{ GeV}) \), automatically in the most attractive regime for ADM theories to explain the \( \mathcal{O}(1) \) ratio of DM-to-baryon energy densities. If \( U(1)' \) is gauged, an asymmetric population of \( \Delta' \) baryons is balanced by an equal number of asymmetric \( \bar{\Delta}' \). In significant regions of parameter space, twin atoms are formed and are successful DM candidates consistent with all current constraints, although modified halo dynamics and direct detection signals are possible.

\[ \text{Acknowledgments:} \] We wish to thank N. Craig, R. Harnik, F. Kahlhoefer, M. McCullough & the participants of the CERN Neutral Naturalness workshop where this work was first presented for useful discussions. IGG & JMR are especially grateful to K. Howe for many discussions of Twin Higgs theories. RL and JMR thank, respectively, the Berkeley Center for Theoretical Physics, and the CERN Theory Group, for hospitality during this work. IGG & RL acknowledge financial support from STFC studentships, and IGG from a University of Oxford Scatcherd European Scholarship.

\[ \]
[24] I. Garcia Garcia, R. Lasenby, and J. March-Russell (2015), 1505.07109.
[25] P. Ade et al. (Planck) (2015), 1502.01589.
[26] S. Hannestad, J. Hamann, and Y. Y. Wong, J. Phys. Conf. Ser. 485, 012008 (2014).
[27] S. Bashinsky and U. Seljak, Phys. Rev. D 69, 083002 (2004), astro-ph/0310198.
[28] R. D. Pisarski and F. Wilczek, Phys. Rev. D 29, 338 (1984).
[29] C. Alexandrou, A. Borici, A. Feo, P. de Forcrand, A. Galli, et al., Phys. Rev. D 60, 034504 (1999), hep-lat/9811028.
[30] M. Fromm, J. Langelage, S. Lottini, and O. Philipsen, JHEP 1201, 042 (2012), 1111.4953.
[31] E. Hardy, R. Lasenby, J. March-Russell, and S. M. West, JHEP 1506, 011 (2015), 1411.3739.
[32] G. Krnjaic and K. Sigurdson (2014), 1406.1171.
[33] D. Harvey, R. Massey, T. Kitching, A. Taylor, and E. Tittley, Science 347, 1462 (2015), 1503.07675.
[34] I. Low, P. Schwaller, G. Shanghnessy, and C. E. Wagner, Phys. Rev. D 85, 015009 (2012), 1110.4405.
[35] J. Giedt, A. W. Thomas, and R. D. Young, Phys. Rev. Lett. 103, 201802 (2009), 0907.4177.
[36] A. Crivellin, M. Hoferichter, and M. Procura, Phys. Rev. D 89, 054021 (2014), 1312.4951.
[37] M. A. Shifman, A. Vainshtein, and V. I. Zakharov, Phys. Lett. B 78, 443 (1978).
[38] P. Cushman, C. Galbiati, D. McKinsey, H. Robertson, T. Tait, et al. (2013), 1310.8327.
[39] D. Akerib et al. (LUX), Phys. Rev. Lett. 112, 091303 (2014), 1310.8214.
[40] M. Horn et al. (LUX), Nucl. Instrum. Meth. A 784, 504 (2015).
[41] H. Vogel and J. Redondo, JCAP 1402, 022 (2014), 1405.1420.
[42] S. Davidson, S. Hannestad, and G. Raffelt, JHEP 0005, 003 (2000), hep-ph/0001179.
[43] A. Prinz, R. Baggs, J. Ballam, S. Ecklund, C. Fertig, et al., Phys. Rev. Lett. 81, 1175 (1998), hep-ex/9804008.
[44] J. Kopp, L. Michaels, and J. Smirnov, JCAP 1404, 022 (2014), 1401.6547.
[45] M. Farina (2015), 1502.01589.
[46] Related investigations of DM in Twin Higgs models have been carried out by other groups [30] [31].