Advanced LIGO will be the first experiment to detect gravitational waves. Through superradiance of stellar black holes, it may also be the first experiment to discover the QCD axion with decay constant above the GUT scale. When an axion’s Compton wavelength is comparable to the size of a black hole, the axion binds to the black hole, forming a “gravitational atom.” Through the superradiance process, the number of axions occupying the bound levels grows exponentially, extracting energy and angular momentum from the black hole. Axions transitioning between levels of the gravitational atom and axions annihilating to gravitons produce observable gravitational wave signals. The signals are long-lasting, monochromatic, and can be distinguished from ordinary astrophysical sources. We estimate \( \mathcal{O}(1) \) transition events at aLIGO for an axion between \( 10^{-11} \) and \( 10^{-10} \) eV and up to 1000 annihilation events for an axion between \( 10^{-12} \) and \( 10^{-11} \) eV. Axion annihilations are particularly promising for much lighter masses at future lower-frequency gravitational wave observatories, where we expect as many as \( 10^5 \) events. Our results are robust against perturbations from the black hole environment and account for our updated exclusion on the QCD axion of \( 6 \times 10^{-13} eV < \mu_a < 1.5 \times 10^{-11} eV \) suggested by stellar black hole spin measurements.
tion of the medium, and radiation that scatters inside the cone ($\omega, \vec{v} \cdot \vec{k}_\gamma$) is amplified [5].

Similarly, superradiance occurs for a conducting axisymmetric body rotating at a constant angular velocity $\Omega_{cylinder}$ [7]. Here, superluminal motion is in the angular direction: a rotating conducting cylinder amplifies any light wave of the form $e^{im\phi-\omega t}$ when the rotational velocity of the cylinder is faster than the angular phase velocity of the light:

$$\frac{\omega_\gamma}{m} < \Omega_{cylinder}, \quad (1)$$

where $\omega_\gamma$ and $m$ are the photon energy and angular momentum with respect to the cylinder rotation axis, respectively. This is the same as the superradiance condition for rotating black holes, with $\Omega_{cylinder}$ substituted by the angular velocity of the black hole at the horizon. The only difference is in the interaction required for superradiance to occur: in electromagnetism, it is the ordinary gauge interaction between electron and photon, while for black holes, it is gravity.

Although the kinematic condition is easy to satisfy, the amplification rate is typically small, and rotational SR in particular is very hard to observe. The amplification rate is determined by the overlap of the scattered wave with the rotating object; for non-relativistic rotation, this overlap is proportional to $(\omega_\gamma R)^{2m}$ where $R$ is the size of the object. The superradiance condition in eq. (1) implies that this quantity is generically much less than 1. As Beackenstein notes, only superradiance for the $m = 1$ mode could potentially be observed in the lab [6]. For black holes, however, several modes with $m > 1$ can be superradiating within the evolution time scale of the black hole since their rotation is relativistic.

The smallness of the superradiance rate also highlights the importance of axisymmetry. For non-axisymmetric objects, superradiant modes mix with non-superradiant (decaying) ones, and hence the amplification rate is even smaller or non-existent. This is another complication for observing rotational superradiance in the lab as well as around astrophysical objects such as stars and planets.

To summarize, rotating black holes are just one type of system in which superradiance can occur. However, they have special properties that make them ideal for observing superradiance of massive bosonic particles:

- They are perfectly axisymmetric due to the no-hair theorem.
- Their rotation is relativistic so the superradiance rate can be significant.
- Gravity provides the interaction necessary for superradiance to occur, so the effect is universal for all particles.

In particular, the superradiance rate for black holes can be significantly faster than the dynamical black hole evolution rate. It is maximized when the Compton wavelength of the massive bosonic particle is comparable to the black hole size: astrophysical black holes are sensitive detectors of bosons with masses between $10^{-20}$ and $10^{-10}$ eV.

This mass range encompasses many theoretically motivated light bosons. In particular, the QCD axion, a pseudo-Goldstone boson proposed to solve the strong CP problem [8], falls in this mass range for high decay constant, $f_a$. Many light axions can also arise in the landscape of string vacua needed for the anthropic solution to the cosmological constant problem [9]. Other classes of particles probed by superradiance include light dilatons [9] and light gauge bosons of hidden $U(1)$s (see [10] and references therein). Black hole superradiance can probe parameter space that is inaccessible to laboratories or astrophysics since naturally light bosons have small or no couplings to the standard model.

As long as the self-interaction of the boson is sufficiently weak and its Compton wavelength comparable to the size of astrophysical black holes, superradiance will operate, regardless of the model or the abundance of the boson. We mostly refer to the QCD axion, but our result is directly applicable to general scalars via $\lambda \leftrightarrow (\mu_a/f_a)^2$ for a scalar with mass $\mu_a$ and quartic interaction $\mathcal{L} \supset \lambda \phi^4/4!$. The same results can also be approximately applied to light vector bosons.

When the superradiance effect is maximized, a macroscopic “cloud” of particles forms around the black hole, giving dramatic experimental signatures [4]. This is true even after taking into account bounds suggested by current black hole spin measurements: we exclude part of the mass range using astrophysical measurements of high spin black holes, since those black holes would have spun down quickly in the presence of light bosonic particles of appropriate masses.

The black hole superradiance is fast enough to allow multiple levels to superradiate within the dynamical evolution time scale of astrophysical black holes. Axions occupying these levels can annihilate to a single graviton in the presence of the black holes gravitational field. Levels with the same angular momentum quantum numbers but different energies can be simultaneously populated; axions that transition between them emit gravitational radiation. As we will see, both axion transitions and annihilations produce monochromatic gravitational wave radiation of appreciable intensity. The gravitational wave frequency and strain for GUT- to Planck-scale QCD axions fall in the optimal sensitivity band for Advanced LIGO (aLIGO) [11] and VIRGO [12].

The annihilations signature is also promising at future, low-frequency gravitational wave observatories. Another signal is relevant for bosons with self-coupling stronger than the QCD axion: the “bosonova” effect [4], where the bosonic cloud collapses under its self-interactions, produces periodic gravitational wave bursts.

In this paper, we focus on the prospects for detecting gravitational wave signals at aLIGO and discuss the reach for future gravitational wave detectors operating at lower frequencies. In section [11] we review the param-
eters for black hole superradiance and how it evolves for an astrophysical black hole. In section [III] we estimate expected event rates at aLIGO and at future lower frequency detectors. In section [IV] we revisit bounds from black hole spin measurements and include our results for both stellar and supermassive black holes. We examine the effects of black hole companion stars and accretion disks on superradiance in section [V] and conclude in section VI.

II. THEORETICAL BACKGROUND

A. The Gravitational Atom in the Sky

The bound states of a massive boson with the black hole (BH) are closely approximated by hydrogen wave functions: except in very close proximity to the black hole, the gravitational potential is \( \propto 1/r \). The “finestructure constant” \( \alpha \) of the gravitational atom is:

\[
\alpha = r_g \mu_a, \quad r_g \equiv G_N M, \tag{2}
\]

where \( r_g \) is the gravitational radius of the BH, \( M \) its mass, and \( \mu_a \) the boson mass. Like the hydrogen atom, the orbitals around the black hole are indexed by quantum numbers \( \{n, \ell, m\} \) with energies:

\[
\omega \simeq \mu_a \left( 1 - \frac{\alpha^2}{2n^2} \right). \tag{3}
\]

The orbital velocity is approximately \( v \sim \alpha/\ell \), and the axions form a “cloud” with average distance

\[
r_c \sim \frac{n^2}{\alpha^2 r_g} \tag{4}
\]

from the black hole.

A level with energy \( \omega \) and magnetic quantum number \( m \) can extract energy and angular momentum from the black hole if it satisfies the superradiance condition analogous to eq. [1]:

\[
\frac{\omega}{m} < \omega^+, \quad \omega^+ \equiv \frac{1}{2} \left( 1 - \frac{\alpha^2}{1 + \sqrt{1 - a_*^2}} \right) r_g^{-1}, \tag{5}
\]

where \( \omega^+ \) can be thought of as the angular velocity of the black hole and \( 0 \leq |a_*| < 1 \) is the black hole spin \( (a_* = \alpha/r_g \text{ in Boyer-Lindquist coordinates}) \). The superradiance condition requires

\[
\alpha/\ell \leq 1/2, \tag{6}
\]

with the upper bound saturated for \( m = \ell \) and extremal black holes \( (a_* = 1) \), so superradiating bound states are indeed well-approximated by solutions to a \( 1/r \) gravitational potential \( (r_c \gg r_g) \) with sub-leading relativistic corrections \( (v^2 \ll 1) \).

The occupation number of levels that satisfy the superradiance condition grows exponentially with a rate \( \Gamma_{sr} \),

\[
\left. \frac{dN}{dt} \right|_{sr} = \Gamma_{sr} N, \tag{7}
\]

\[
\Gamma_{sr}(a_*, \alpha, r_g) = \mathcal{O}(10^{-7} \cdots 10^{-14}) r_g^{-1}. \tag{8}
\]

The boson is not required to be dark matter or be physically present in the vicinity of the black hole: just like spontaneous emission, superradiance can start by a quantum mechanical fluctuation from the vacuum, and proceed to grow exponentially. If the SR condition is satisfied, the growth will occur as long as the rate is faster than the evolution timescales of the BH, the most relevant of which is the Eddington accretion time, \( \tau_{\text{Eddington}} = 4 \times 10^8 \text{ years} \). The growth stops when enough angular momentum has been extracted so that the superradiance condition is no longer satisfied. At that point the number of bosons occupying the level is

\[
N_{\text{max}} = \frac{G_N M^2}{\sqrt{\ell(\ell+1)}} \Delta a_* \sim 10^{76} \left( \frac{\Delta a_*}{0.1} \right) \left( \frac{M}{10 M_\odot} \right)^2, \tag{8}
\]

where \( \Delta a_* = \mathcal{O}(0.1) \) is the difference between the initial and final BH spin.

The superradiance rates (or dumping rates for the levels that are not superradiating) are given by the small imaginary part of the energy of a free-field solution in the Kerr background. Unless otherwise specified, we use the semi-analytic approach for massive spin-0 fields presented in [1], which agrees well with analytical formulæ for \( \alpha/\ell \ll 1 \) [2] and the WKB approximation for \( \alpha/\ell \sim \mathcal{O}(1/2) \) [3, 4], as well as with partial numerical results in [5]. Rates for massive spin-1 fields are expected to be larger, and some numerical progress has been made toward calculating them [14]; we choose to focus on the spin-0 case (including the QCD axion) for the

![FIG. 1. Superradiance times of levels \( \ell = 1 \) to 4 (left to right) for spins \( a_* = 0.99 \) and 0.9, fixing \( m = \ell \) and \( n = \ell + 1 \). Time in years is shown for a 10 \( M_\odot \) black hole as a function of boson mass \( \mu_a \); on the right axis, we show the dimensionless superradiance rate \( \Gamma_{sr} \) as a function of the gravitational \( \alpha \) (top axis).](image-url)
remainder of this paper, but further studies with spin-1 fields are very worthwhile.

In fig. [1] we show representative values of the superradiance rates, \( \Gamma_{sr} \). The rate varies with the relevant parameters of the system as follows:

- \( r_g \) — The dimensionless quantity \( \Gamma_{sr} r_g \) depends on the coupling \( \alpha \), BH spin \( a_\star \) and the quantum numbers of the state; the physical superradiance time can be as short as 100 s for stellar black holes and is longer for heavier black holes.
- \( \alpha \) — For given level, \( \Gamma_{sr} \) is a steep function of the coupling, reaching its maximum close to the superradiance boundary. A single BH is sensitive to a range of boson masses: stellar BHs (2–30M\(_{\odot}\)) correspond to masses of 10\(^{-12}\)–10\(^{-10}\) eV, and supermassive BHs (10\(^6\)–10\(^8\)M\(_{\odot}\)) to masses of 10\(^{-19}\)–10\(^{-16}\) eV.
- \( m \) — \( \Gamma_{sr} \) decreases with increasing \( m \), and the dependence is strong: for \( \alpha/\ell \ll 1 \), \( \Gamma_{sr} \propto \alpha^4 \ell^2 \). For \( \alpha/\ell \approx 0.5 \), the WK approximation \( [2,4] \) gives \( \Gamma_{sr} \propto e^{-3.7\alpha} \approx (0.15)^\ell \).
- \( m \) — \( \Gamma_{sr} \) is largest for \( m = \ell \) and is much smaller for \( m < \ell \). Unless otherwise specified, we only consider levels with \( m = \ell \) below.
- \( n \) — For fixed \( \ell \) and \( m \), the dependence on \( n \) is mild and \( \Gamma_{sr} \) generally decreases with larger \( n \).

So far, we have considered a free bosonic field. Self-interactions between bosons will affect superradiance when the interaction energy becomes comparable with the binding energy of the boson in the cloud. Axions, for example, have attractive self-interactions which cause the cloud to collapse when it reaches a critical size; laboratory experiments have observed such collapse of bosonic states, known as “bosenova”, in the analogous system of trapped BECs \( [15] \). After a bosenova occurs, the superradiant growth restarts. Even weak self-interactions can have a significant effect: for example, for an axion with decay constant \( f_\alpha \), the critical bosenova occupation number is

\[
N_{bosenova} \approx 10^{78} \left( \frac{M}{10 M_\odot} \right)^2 \left( \frac{c_0}{3} \right)^{\frac{2}{n}} \left( \frac{2m^2}{\alpha^3} \right) \left( \frac{f_\alpha}{M_{pl}} \right)^2 , \tag{9}
\]

where \( M_{pl} = 2 \times 10^{18} \text{ GeV} \) and \( c_0 \approx 5 \) is determined by numerical simulation \( [16] \). Comparing eqs. \( (8) \) and \( (9) \), we see that the bosenova occurs before all the spin is extracted if

\[
f_\alpha \lesssim 2 \times 10^{16} \text{ GeV} \left( \frac{1}{n} \right) \left( \frac{\alpha}{0.4} \right)^{\frac{2}{5}} \left( \frac{\Delta m}{0.1} \right)^{\frac{1}{5}} \left( \frac{5}{c_0} \right)^{\frac{1}{2}} . \tag{10}
\]

For the QCD axion this gives \( \mu_\alpha > 3 \times 10^{-10} \text{ eV} \), too heavy to be relevant for astrophysical black holes (\( M \gtrsim 3 M_\odot \)). Nevertheless, the bosenova can lead to interesting gravitational wave signals for axion-like particles and other light bosons (section \( \text{III C} \)). For strongly interacting bosons, the superradiance instability can be slowed to a stand-still with the cloud collapsing before it can grow to macroscopic size.

B. A (Not So) Brief History of Superradiance

The superradiance condition can be satisfied for several levels of the black hole-axion “atom”, and for each level and boson mass, there is a region in the BH spin vs. mass plane that is affected (fig. [2]). As discussed previously, the superradiance condition is a kinematic one and SR can affect BHs with masses a factor of 10 to 100 around the optimal value. The affected region is set by the SR condition and is further limited by whether superradiance happens faster than the accretion rate of the BH.

In order to understand how superradiance affects astrophysical black holes, let us assume there exists a boson with mass \( \mu_\alpha = 10^{-11} \text{ eV} \) and self-interaction strength of the QCD axion (decay constant \( f_\alpha = 6 \times 10^{17} \text{ GeV} \)). The Compton wavelength of this particle is 20 km, the size of a typical stellar BH horizon.

Consider a BH that is born with spin \( a_\star = 0.95 \) and mass 6M\(_{\odot}\). Once the environment settles to a steady state after the supernova explosion, superradiant levels begin to grow exponentially with their respective SR rates. The fastest-growing level dominates — in this case, the \( \ell = 2 \) level, since the smallest \( \ell \) that satisfies the SR condition has the largest rate. It takes \( \log N_{\text{max}} \approx 200 \) e-folds — in this case, about 2 years — of growth to extract enough spin so that the SR condition is no longer satisfied for the \( \ell = 2 \) level. While losing 20% of the spin, the BH only loses about 5% of its mass, because the cloud is larger in extent than the black hole and so more efficient at carrying angular momentum. As the cloud grows, the gravitational wave signal from axion annihilation (section \( \text{III B} \)) increases until reaching a maximum when the SR condition is no longer satisfied.

At this point, we expect superradiance to start populating the \( \ell = 3 \) at a slower rate. If self-interactions are present, this does not happen right away: the \( \ell = 2 \) level perturbs the potential around the black hole such that the \( \ell = 3 \) level mixes with levels that do not satisfy the SR condition. Therefore the \( \ell = 3 \) level does not grow until the \( \ell = 2 \) level is depleted to the point when level mixing is a subdominant effect. The time scale for the \( \ell = 3 \) level to be depleted sufficiently is dominated by two-boson one graviton annihilations \( [4] \),

\[
\tau_{\text{regge}} = (N_{bosenova} \Gamma_\alpha)^{-1} |\Gamma_{sr}/\Gamma_{sr}^\ell|^{1/2} \tag{11}
\]

where \( \Gamma_\alpha \) is the annihilation rate (section \( \text{III B} \)). For interacting particles \( \tau_{\text{regge}} \) can be much longer than
the superradiance time since \( N_{\text{bosenova}} \Gamma_a < \Gamma_{sr} \) and \( |\Gamma_{sr}^{-1}/\Gamma_{sr}^{i+1}|^{1/2} \gg 1 \). The black hole can therefore spend a long time on the line where \( \omega = m_\omega a_+ \). This line thus defines for black holes the analog of Regge trajectories in particle physics. If black hole spin measurements become accurate enough, we could diagnose the presence of an axion by fitting the curve of BH spin vs. mass to the superradiance condition.

Once the \( \ell = 2 \) level is depleted through annihilations, the \( \ell = 3 \) level starts to grow and the BH makes another jump in the BH spin vs. mass plane. The previous process then repeats itself, but for the parameters chosen, the \( \ell = 4 \) superradiance rate is too slow and once the spin drops to \( a_* = 0.55 \), superradiance no longer affects the BH.

If the black hole is heavier such that the \( \ell = 4 \) superradiance rate is significant, the \( 5g \) and \( 6g \) levels grow with comparable superradiance rates to large occupation numbers. This sets the stage for a large gravitational wave signal from level transitions (section III A); \( \ell = 4 \) is the smallest \( \ell \) for which transitions occur.

The BH trajectory is more complicated if the bosenova is possible. In that case, the cloud reaches a maximum size of \( N_{\text{bosenova}} \) and collapses before saturating the superradiance condition. Then, the bosenova has to repeat many times before the superradiance condition is saturated, and the occupation number of the cloud at the Regge trajectory is smaller by a corresponding factor. As we discuss in section III C, the periodic repetition of bosenova can give rise to interesting signals.

For supermassive black holes the story changes slightly, since their spin and mass are acquired through accretion. As a supermassive BH grows, spin extraction by the cloud happens adiabatically with the black hole accretion, moving along the boundary of the region in the spin vs. mass plane affected by superradiance. Only a violent event such as a merger will perturb the system enough so the BH can jump between different levels. The long time spent on the trajectories can lead to exciting annihilation signals at low-frequency gravitational wave detectors (section III B 2).

### III. Gravitational Wave Signals

Processes that have forbiddingly small rates for a single particle can be enhanced in the bosonic cloud, since the occupation number of a single level in the BH-boson gravitational atom can be exponentially large. Transitions of bosons between two different levels are enhanced by \( N_1 N_2 \) where \( N_i \) is the occupation number for each level; two-boson annihilations to a single graviton are also enhanced by \( N_2^2 \). Because of this “lasing” effect, the peak strain of the resulting gravitational waves (GW) can be within reach of GW detectors. Superradiance for stellar black holes can lead upcoming observatories, Advanced LIGO [11] and Advanced VIRGO [12] — beginning science runs in 2015-2016 [17] — into the realm of discovery. Superradiance for supermassive black holes has exciting prospects for future, low-frequency observatories.

There are three types of GW signals from the bosonic cloud:

- graviton emission from level transitions
- axion annihilations into gravitons
- bosenova collapse of the axion cloud

The axions involved in transitions and annihilations are in exact energy eigenstates of the black hole potential

### TABLE I. Characteristic superradiance timescales.

| Process | (see also) | Stellar BHs | Supermassive BHs |
|---------|------------|-------------|------------------|
| Superradiance \((2p, \alpha = 0.3, a_* = 0.9)\) | \(\Gamma_{sr}^{-1}\) fig. 1 | \(10^{-4}\) yr | \(100\) yr |
| Superradiance \((5g, \alpha = 1.2, a_* = 0.9)\) | \(\Gamma_{sr}^{-1}\) fig. 1 | \(10\) yr | \(10^2\) yr |
| Regge trajectory \((2p, \alpha = 0.3, f_a = 10^{17}\text{ GeV})\) | \(\tau_{\text{regge}}\) eq. (11) | \(10^6\) yr | \(10^{12}\) yr |
| Eddington Accretion, \((M/M_\odot)^{-1}\) | \(\tau_{\text{Eddington}}\) | \(4 \times 10^8\) yr | \(4 \times 10^8\) yr |

FIG. 2. Effect of superradiance for a QCD axion with mass \(\mu_a = 10^{-11}\) eV and decay constant \(f_a = 6 \times 10^{17}\) GeV. Shaded regions correspond to BH parameters which would result in spin down within a binary lifetime (\(10^8\) years), for \(\ell = 1\) (dark blue) to \(\ell = 5\) (light blue) levels. We also show an example evolution of a \(6M_\odot\) black hole with initial spin \(a_* = 0.95\).
and thus emit monochromatic GWs. \footnote{This disagrees with what was originally stated in \cite{Dubovsky2014} regarding the monochromaticity of GWs from annihilations. We thank S. Dubovsky and S. Dimopoulos for discussions clarifying this issue.}

In this section, we compute the experimental reach of gravitational wave observatories and estimate expected event rates. Table \ref{tab:timescales} summarizes the timescales typical for these processes. We focus on light axions as a prime example of bosons relevant for superradiance, since their mass and self-interaction are naturally small due to shift symmetry. When relevant, we assume the self-coupling is that of the QCD axion.

To calculate the gravitational wave strain we use

$$ h = \left( \frac{4G_N P}{r^2 \omega^2} \right)^{1/2} $$

(12)

for a source emitting power $P$ at angular frequency $\omega$ at a distance $r$ away from the Earth. We do not include effects from the angular dependence and orientation of the gravitational wave detectors.

### A. Level Transitions

Analogously to atomic transitions emitting photons, level transitions of axions around black holes emit gravitons. The GW angular frequency for transitions between an “excited” and a “ground” level with principal quantum numbers $n_e$ and $n_g$, respectively, is

$$ \omega_{tr} = \frac{1}{2} \mu_a c^2 \left( \frac{1}{n_g^2} - \frac{1}{n_e^2} \right). $$

(13)

If the superradiance rates for the two levels dominate, their occupation numbers $N_{e,g}$ evolve with their SR rates, modified by axions transitioning from the excited to the ground state via graviton emission,

$$ \frac{dN_g}{dt} = \Gamma_{e} N_e + \Gamma_{tr} N_g N_e, \quad \frac{dN_e}{dt} = \Gamma_{tr} N_e N_g - \Gamma_{e} N_e N_g, $$

(14)

where $\Gamma_{e,g}$ are the superradiance rates for the two levels and $\Gamma_{e}$ the transition rate for a single axion. A quadrupole formula estimate gives

$$ \Gamma_{e} \sim \frac{2G_N \omega^5}{5} \mu_a^2 c^4 \mathcal{O}(10^{-6} - 10^{-8}) \frac{G_N \alpha^9}{r_g^3}; $$

(15)

for our numerical results, we compute more precise rates (see app. \ref{app:rates}).

Although the single axion transition rate is tiny ($\Gamma_{e} \ll \Gamma_{e,g}^{sr}$), the emission of gravitational waves is enhanced by the occupation numbers of each level:

$$ h_{tr}(t) = \sqrt{\frac{4G_N}{r^2 \omega_{tr}}} \Gamma_{e} N_g(t) N_e(t). $$

(16)

When the axion clouds are small, transitions are negligible, and both levels grow exponentially with their respective SR rates. The transition terms in eq. \ref{eq:level_transitions} become important when the transition rate starts to compete with the growth rate. The occupation number of the excited level is maximized when

$$ N_e = \Gamma_{e,g}^{sr}/\Gamma_{e}, $$

(17)

after which the excited state depopulates rapidly. The size of the signal depends on whether $\Gamma_{e,g}^{sr} > \Gamma_{e}^{sr}$ or vice-versa. An example of the $N_{e,g}$ and $h$ time evolution for the two cases is shown in fig. \ref{fig:transitions}.

If $\Gamma_{e,g}^{sr} > \Gamma_{e}^{sr}$ (fig. \ref{fig:transitions} left), $N_e \gg N_g$ at the time when the transition rates become relevant. The transition GW strain keeps growing as the excited level gets depleted, until both levels are populated with an equal number of axions. After that, the signal drops precipitously as the excited level empties out and the ground state returns to growing with rate $\Gamma_{e,g}^{sr}$.

If $\Gamma_{e,g}^{sr} > \Gamma_{e}^{sr}$ (fig. \ref{fig:transitions} right), the excited level depopulates very quickly once it reaches the maximum, so the transition term for the ground state is never important: $N_g$ continues to grow at its superradiance rate. The smaller occupation number of the excited level suppresses the overall GW strain: the peak transition strain is smaller compared to the previous case by an additional factor of

$$ \sqrt{\Gamma_{e}^{sr}/\Gamma_{e}/\Gamma_{e,g}^{sr}} \sim \mathcal{O}(10^{-35}) \Gamma_{e}^{sr}/\Gamma_{e,g}^{sr}. $$

In both cases, the transition process has a characteristic timescale of $\Gamma_{e,g}^{sr}^{-1}$, typically decades for stellar BHs.
The maximal occupation numbers are controlled by the ratio $\Gamma^e_s/\Gamma_t$, and the peak strain is proportional to the superradiance rate:

$$h_{\text{peak}} \propto \frac{\Gamma^e_s}{\sqrt{\omega_s \Gamma_t}}.$$  (18)

The maximum signal occurs before the occupation numbers reach $N_{\text{max}}$: the ground state will continue to grow exponentially until all available angular momentum is extracted and the SR condition is no longer satisfied. At this point, annihilations become the dominant process. For general light bosons, the transition signal strain is unaffected as long as $N_{e,q}^{\text{peak}} < N_{\text{bosenova}}$, or $f_q \gtrsim 10^{14}$ GeV for sufficiently quickly spinning BHs.

We assume above that the initial number of axions occupying each level is $N_{e,0} = N_{20} = 1$, as expected when superradiance starts from scratch. If it restarts after being disturbed e.g. by a bosenova which leaves one level partially occupied, the other level does not have time to grow to the optimal occupation number and the transition rate is not significant. This implies that the transition signal will most likely appear only once in the lifetime of a stellar black hole.

Figure 3 shows the relative GW strains of various transitions for a BH of mass $10M_\odot$. The analysis above shows that transitions are relevant for the evolution of superradiance only when the two levels can be simultaneously populated. The most promising cases for transition signals are the ones with the smallest difference between superradiance rates: $\Delta n \neq 0$ and $\Delta \ell = \Delta m = 0$. The $\ell = 4$ (‘$g$’) levels are ones with lowest $\ell$ that satisfy $\Gamma^e_s > \Gamma^g_s$ to avoid the suppression factor discussed above; levels with higher $\ell$ have lower superradiance rates and correspondingly lower peak strains. When three or more levels have similar superradiance rates, the transitions between them may be inhibited; such situations require further analysis. For instance, the $7\ell \rightarrow 6\ell$ transition suppresses the $8\ell \rightarrow 6\ell$ transition power (suppression not shown in the figure).

We see that the $6g \rightarrow 5g$ transition is the most likely to be seen by GW detectors, followed by $7\ell \rightarrow 6\ell$; these are the levels that we use in our signal estimates below.

1. **Advanced LIGO/VIRGO Prospects**

Transitions between superradiant levels around stellar black holes fall in the sensitivity band of Advanced LIGO and VIRGO: they have frequency $f \sim 15$ Hz $\times (\mu_{\alpha}/10^{-11}$ eV) and peak strains as high as $h \sim 10^{-22}/\sqrt{\text{Hz}}$ for a BH at 10 kpc away. A search for such a signal with GW observatories is very promising, especially since the GW emission is monochromatic and the signal can last decades or longer.

The optimal search strategy is similar to an existing search for periodic gravitational waves from e.g. asymmetric neutron stars [18]. Unlike the case of neutron stars which spin down due to GW emission, the transition signal’s frequency is set by the level splitting and is constant up to corrections from the nonlinearities of the cloud itself. For the QCD axion, the maximum signal occurs when the occupation number of the cloud is much smaller than the nonlinear regime, resulting in a
tiny frequency drift,
\[
\frac{df}{dt} \simeq 10^{-11} \text{Hz s}^{-1} \left( \frac{f}{35 \text{ Hz}} \right) \left( \frac{M}{10 M_\odot} \right) \left( \frac{M_{\text{GUT}}}{f_a} \right)^2 \left( \frac{10^{11} \text{ yr}}{T} \right)^2
\]
where \( T \) is the characteristic signal length, set by \( \Gamma^{-1}_r \) and \( M_{\text{GUT}} = 2 \times 10^{16} \text{ GeV} \). This is compared to \( df/dt \lesssim 10^{-9} \text{ Hz s}^{-1} \) in the periodic GW search.

The signal strain varies with the superradiance timescale. The amplitude and sign of the frequency drift are correlated with the amplitude of the signal. For bosons with attractive self-interaction such as the axion, if \( \Gamma_{\text{g}} > \Gamma_r \), the energy of the ground state decreases as the number of axions in the cloud grows while the energy of the excited state is less affected, so the frequency of the emitted graviton increases with time. For the more promising case of \( \Gamma_{\text{g}} < \Gamma_r \), the frequency decreases as both levels grow, stays constant when the signal peaks, and finally increases as the excited level empties. For particles with repulsive interactions, the frequency drift is in the opposite direction. While the absolute value of the change in frequency is small, observing such a drift and correlating it with the signal amplitude would provide additional handles on the magnitude and sign of the particle’s self-interaction.

In fig. [5] we estimate the maximum reach radius of aLIGO for the promising \( 6g \rightarrow 5g \) transition. We base our calculation on the design aLIGO noise level [17] and 121 segments of 25 hr. integration time motivated by the periodic gravitational wave search [18]; we expect similar reach for Advanced VIRGO. For each mass \( \mu_a \), we calculate the reach to a black hole of mass \( M \) that optimizes the signal strength. Because the transition signal scales with the superradiance rate, a black hole with larger spin will be visible from further away: the signal from a BH with \( a_0 = 0.99 \) has a reach of up to 40 Mpc and one with \( a_0 = 0.9 \) has reach of up to 1 Mpc. For fixed \( \alpha \), the sensitivity grows with decreasing axion mass since \( h_{\text{peak}} \propto \mu_a^{-1} \), but drops off as the aLIGO noise floor comes up sharply at 10 Hz; the best reach is for masses around \( 3 \times 10^{-11} \text{ eV} \).

Given the promising reach in fig. [5] we attempt to quantify the number of events aLIGO could observe. To do so, we consider the probability to find a black hole with high spin and mass in the appropriate range to lead to transitions, as well as the number and spatial distributions of black holes in our neighborhood young enough to be undergoing transitions today. We focus on the transitions with the largest strains, \( 6g \rightarrow 5g \) and \( 7h \rightarrow 6h \). The length of the signal for most of the observable parameter space is 10 years or longer, so if a signal is detectable on Earth it will persist for longer than an observatory’s science run.

We provide optimistic, realistic, and pessimistic estimates for the astrophysical distributions following the event rate estimates of the LIGO collaboration [19]. The relevant astrophysical distributions of stellar BHs are:

- Mass distribution: we use a fit to data with a minimum BH mass and exponential drop toward high
masses [20]. In fig. 6 we use a minimum mass of 4\(M_{\odot}\); the events shift proportionally to higher axion masses if the minimum BH mass is smaller. The width of the event distribution is set by the exponential tail (choosing three values gives the different shapes in fig. [4]).

- Spin distribution: we take the measured distribution, 30% of \(a_s > 0.8\), as a realistic estimate for natal spins. We consider 90% above 0.9 optimistic and a flat distribution pessimistic.

- Formation rate: barring rare violent events, the maximal transition signal occurs once per stellar BH lifetime; therefore the event rate is directly proportional to the BH formation rate. We estimate the BH birth rate to be \(0.4^{+0.5}_{-0.3}\) per century based on supernova rates [21, 22] and BH fraction of supernova remnants [23].

- Distance distribution: we assume the spatial distribution of BHs is proportional to the stellar distribution inside [24] and outside [25] the Milky Way. These mass, spin, distance, and formation rate distributions are folded into our transition event rate estimates at aLIGO (see app. [B] for details).

The resulting rates in fig. 6 compare favorably to realistic estimates of coalescences of compact object binaries [19]. The event rates are dominated by black holes near the center of our galaxy; an increase in detector sensitivity by a factor of 10 (such as the Einstein Telescope [18]) would increase the number of events by the same factor because signals with smaller strain last for a proportionally longer time. If the detector sensitivity is increased by a factor close to 100, the event rate will scale cubically with the sensitivity, since the reach will be beyond 30 Mpc away, at which point the density of galaxies scales in proportion to the volume.

The astrophysical uncertainties are large, but the event rates are promising: if there is an axion or light boson close to \(10^{16} - 10^{15} \text{eV}\) in mass, aLIGO could see a signal lasting for many years. Once identified, further study of the signal amplitude and frequency as a function of time, correlated with astrophysical measurements of the black hole source, could determine the mass and coupling of the superradiating particle.

2. Future Gravitational Wave Observatories

Upcoming observatories such as Advanced LIGO and VIRGO are perfectly suited to search for superradiance signals from stellar black holes; to detect SR signals from supermassive black holes (SMBHs), we look to future, lower-frequency proposals: AGIS [26] and eLISA [27].

In fig. 7 we show the reach radius of AGIS and eLISA to transition signals around SMBHs; similar to the previous section, we assume that there exists a BH with the optimal mass for each axion mass. The reach is as far as a few hundred Mpc, but the best detector sensitivity falls in the range of intermediate-mass BHs, \(M \lesssim 10^5 M_{\odot}\). Lack of estimates for mass distribution of intermediate BHs makes even an approximate estimate of event rates difficult. In the limit where all supermassive black holes have mass \(M = 10^6 M_{\odot}\) and most have maximal spin (see app. [B]), low-frequency detectors can observe between 1 and 500 transition signals.

B. Annihilations

Another source of gravitational wave emission is axions annihilating to gravitons in the black hole background, \(a + a \rightarrow g + g_{bg}\); this process is analogous to electron-positron annihilation to a photon in the background of a nucleus [28]. The GW frequency is

\[
\omega_{\text{ann}} = 2\omega_a \approx 2\mu_a \left(1 - \frac{\alpha^2}{2n^2}\right) .
\]  

(20)

When a single level dominates the evolution of the axion-BH system, its occupation number \(N(t)\) grows with its SR rate while being depleted by axions pair annihilating into gravitons,

\[
\frac{dN}{dt} = \Gamma_{sr} N - \Gamma_a N^2.
\]  

(21)
Here, $\Gamma_a$ is the annihilation rate for one pair of axions, of order

$$\Gamma_a \simeq 10^{-10} \left( \frac{\alpha/\ell}{0.5} \right)^p + \mathcal{O} \left( \frac{\alpha/\ell}{0.5} \right)^{p+1} \frac{G_N}{r_s^3},$$

(22)

where $p = 17$ for $\ell = 1$ and $p = 4\ell + 11$ for $\ell \geq 2$ (see app. A for full expressions). The annihilation rates close to the superradiance boundary ($\alpha/\ell = 1/2$) are similar for all $\ell$-levels. At smaller $\alpha/\ell$, the annihilation rate is velocity suppressed, with the suppression more pronounced at higher $\ell$.

Comparing eqs. 15 and 22, we see that annihilation is the slowest process, $\Gamma_a \ll \Gamma_t \ll \Gamma_{\text{sr}}$. Nevertheless, annihilations are important when the occupation number of a single level is far larger than the others, as is the case when a single superradiance rate dominates.

When $N < \Gamma_{\text{sr}}/\Gamma_a$, the axion population grows exponentially with the superradiance rate. Once the axion cloud extracts the maximum possible spin from the BH, $N(t) = N_{\text{max}}$, superradiance shuts off and the occupation number evolves as:

$$N(t) = \frac{N_{\text{max}}}{1 + \Gamma_a N_{\text{max}}^2 t}.$$  

(23)

The corresponding gravitational wave strain is

$$h(t) = N(t) \sqrt{\frac{4G_N}{r^2 \omega_{\text{ann}}^2} \Gamma_a}.$$  

(24)

Both the peak strain,

$$h_{\text{peak}} \simeq 10^{-22} \left( \frac{1 \text{ kpc}}{r} \right) \left( \frac{\alpha/\ell}{0.5} \right)^2 \frac{\alpha^{1/2}}{\ell} \left( \frac{M}{10M_\odot} \right),$$

(25)

and peak duration, $(N_{\text{max}} \Gamma_a)^{-1}$, are independent of the superradiance rate. For stellar BHs, the signal at its peak value can last for thousands of years.

For supermassive black holes, $(N_{\text{max}} \Gamma_a)^{-1}$ is of order $10^8$ years, comparable to the Eddington accretion time. In the absence of superradiance, accretion spins up black holes to maximal spin in a few Eddington times 29. If superradiance can occur, after the axion cloud reaches size $N_{\text{max}}$, the black hole continues to superradiate as it accretes, while the axion cloud grows slowly with the characteristic timescale of accretion to a steady state value of

$$N_{\text{steady-state}} \simeq \left( \frac{\Gamma_{\text{accretion}} N_{\text{max}}}{\Gamma_a} \right)^{1/2}. \tag{26}$$

For accretion at the Eddington rate, $N_{\text{steady-state}}$ — and hence $h_{\text{peak}}$ — can be enhanced by a factor of up to $10^3$ compared to $N_{\text{max}}$ and eq. 25.

So far, we have assumed that the self-interaction is of QCD axion strength. If the interaction is stronger such that bosonovae are relevant, the axion cloud only grows to $N_{\text{boosenova}} < N_{\text{max}}$, and the maximum annihilation signal is proportionally smaller and lasts for a shorter time.

FIG. 8. aLIGO reach as a function of the axion mass for an axion cloud currently undergoing annihilations at the maximum rate, assuming that there exists a BH with optimal mass for each axion mass such that $\alpha$ is optimal for each level ($\ell = 1$ and 5), and $a_s = 0.9$. The top axis shows the corresponding GW frequency. We indicate with vertical lines when the optimal BH masses are heavier than $100M_\odot$ and lighter than $3M_\odot$. The shaded region is disfavored by black hole spin measurements assuming QCD axion coupling strength.

1. Advanced LIGO/VIRGO Prospects

The annihilation signal is quite distinctive: it is monochromatic with frequency of order twice the axion mass, $f = 10\text{ kHz} \times (\mu_a/10^{-13}\text{ eV})$, possibly lasting for thousands of years. The signal grows to a maximum at the superradiance rate, and the cloud then slowly depletes, resulting in a small positive drift in frequency due to attractive self-interactions,

$$\frac{df}{dt} \simeq 10^{-12} \text{ Hz/s} \left( \frac{f}{\text{300 Hz}} \right) \left( \frac{M_{\odot}}{f_a} \right)^2 \left( \frac{10^3 \text{ yr}}{T} \right)^2,$$

(27)

where $T = (\Gamma_a N_{\text{max}})^{-1}$ is the typical length of the signal. We plot the reach of aLIGO based on their monochromatic search sensitivities to a maximally filled axion cloud undergoing annihilations in fig. 8 using 121 segments of 25 hr. integration time as described in section III A 1.

The optimal reach is for axion masses around $5 \times 10^{-13}\text{ eV}$ which correspond to intermediate mass black holes, $M > 50M_\odot$; these are poorly understood and have only been recently observed [20]. To estimate the event rate, we use the mass distribution of stellar BHs, which includes an exponential tail extending to heavier BHs. In fig. 9 we estimate the event rate expected in aLIGO from axion annihilations using the same BH astrophysical distributions as in section III A 1. We folded the BH mass distribution width into the optimistic and pessimistic estimates: at the low axion mass edge, the difference between the optimistic and pessimistic estimate is dominated by the shape of the exponential tail of the
BH mass distribution. In fig. 9 we indicate the expected event rate for different maximum stellar BH mass cutoffs (see app. B). Especially at light axion masses, the signal is subject to large astrophysical uncertainties of heavier (30–100 $M_\odot$) BH mass distributions.

The event rates range from $\mathcal{O}(0.1)$ to $\mathcal{O}(1000)$; unfortunately, much of the parameter space with appreciable event rates is disfavored by BH spin measurements (section IV). The event rates for axion masses lighter than the excluded range are very promising, with possibly dozens of monochromatic signals due to a Planck-scale QCD axion or another boson in the same mass range.

2. Future Gravitational Wave Observatories

The frequency sensitivities of AGIS and eLISA are ideal for detection of axion annihilations around supermassive black holes. In fig. 10 we plot the reach of these proposed GW detectors to axions annihilating in a maximally filled 2p level around SMBHs, with a promising reach of up to 1000 Mpc in distance for axion masses between $10^{-18} - 10^{-17}$ eV. We use a maximal integration time of 121 segments of 25 hours as in the current LIGO monochromatic search.

The masses and spins of supermassive black holes are determined by long periods of accretion and so possible to model theoretically. We use the following distributions for the event rates (more details in app. B):

- Mass and distance distribution: we use the distributions of [31] [32], with most SMBHs between $10^6$ and $10^7 M_\odot$ in mass and about one $10^7 M_\odot$ BH per Milky-Way type galaxy.

- Spin distribution: given the conclusions of [33], we use that 70% of SMBHs have spins $a_\star \geq 0.998$ (optimistic), 70% have spins $a_\star \geq 0.93$ (realistic) or 80% have spins $a_\star \geq 0.8$ (pessimistic).

- Signal length: much of the evolution of a SMBH is at the regime where accretion, superradiance, and axion annihilation happen adiabatically. In this case BH spin and axion cloud size remain in a steady-state with annihilations at the maximum rate.

- Accretion rate: When the superradiance rate is much faster than the time between violent events (in-falling compact objects), the size of the annihilation signal is limited only by the rate of black hole spin-up due to accretion. We estimate that 10% or more of SMBHs accrete at 1%, 10%, and $\mathcal{O}(1)$ of the Eddington rate for the pessimistic, realistic, and optimistic values, respectively.

We estimate the event rate expected by AGIS and eLISA in fig. 11, giving the realistic as well as the optimistic and pessimistic estimates based on the astrophysical uncertainties above. The differences in sensitivity of the two detectors are small compared to the astrophysical uncertainty. If there is a light boson with mass of $10^{-19} - 5 \times 10^{-17}$ eV, the annihilation signal is dramatic, with thousands of continuous events visible.
C. Bosenovae

A very different signature is the periodic collapses of the axion cloud, known as “bosenovae” in analogous condensed matter systems. If the axion self-interaction is sufficiently strong ($f_a < M_{pl}$), the axion cloud will collapse at the critical size $N_{bosenova}$ before extracting all the BH’s spin as allowed by the superradiance condition. During the bosenova, a fraction of the cloud falls into the black hole and the rest escapes to infinity, emitting a gravitational wave burst.

The collapse lasts for approximately an infall time, $t_{bn} = (c_{bn} r_*)$, and has primary frequency component

$$f_{bn} = 30 \text{ Hz} \left( \frac{16}{c_{bn}} \right) \left( \frac{\alpha/\ell}{0.4} \right)^2 \left( \frac{10 M_{\odot}}{M} \right),$$

(28)

where $c_{bn}$ parametrizes the collapse time ($c_{bn} \sim 16$ for the $2p$ level [16]).

Once the size of the cloud is reduced such that the system is no longer nonlinear, the level grows again with its superradiance rate until the next bosenova, and the growth-collapse cycle repeats until the superradiance condition is no longer satisfied. The separation between bursts depends on the fraction of the cloud which remains bound to the BH after the bosenova event.

For example, the collapse of a typical axion cloud around a $10M_{\odot}$ black hole with spin $a_* = 0.99$ will emit a burst lasting $10^{-3}$ s. If the axion coupling is e.g. $f_a = 6 \times 10^{16}$ GeV, and each bosenova depletes the cloud to a size of $10^{-4} N_{bosenova}$, the signal will be in the distinct pattern of 10 spikes separated by quiet periods of 300 s.

The strain of these periodic bursts can be large enough to be observed by aLIGO: at a distance of a kpc, the quadrupole estimate gives a signal strain of

$$h \simeq 10^{-21} \left( \frac{\sqrt{\epsilon/ c_{bn}}}{10^{-2}} \right)^2 \left( \frac{\alpha/\ell}{0.4} \right) \left( \frac{M}{10 M_{\odot}} \right) \left( \frac{f_a}{f_{a,\text{max}}} \right)^2,$$

(29)

where $\epsilon$ is the fraction of the cloud falling into the BH ([19] gives $\epsilon \sim 5\%$) and $f_{a,\text{max}}$ is the largest $f_a$ for which bosenovae occur, eq. (10). For a $10M_{\odot}$ BH, $f_{a,\text{max}}$ corresponds to a small quartic coupling of $\lambda \sim 10^{-77}$ for a generic scalar boson.

As we saw earlier, the QCD axion is too weakly coupled to cause bosenovae around astrophysical black holes; light bosons with larger self-couplings can lead to bosenovae that occur with a tell-tale regularity and with signal frequency and strain that fall promisingly in the range of aLIGO. Calculation of detector reach and event rates are beyond the scope of this work; dedicated numerical study is necessary to determine the precise shape, timing, and amplitude of the bosenovae emission.

IV. BOUNDS FROM BLACK HOLE SPIN MEASUREMENTS

Several techniques for spin measurements of black holes have been developed and are constantly improving. The leading methods are continuum fitting and the X-ray relativistic reflection (for recent reviews, see [34] and [35]). Both are based on the measurement of the innermost stable circular orbit of the accretion disk: the radius ($R_{\text{ISCO}}$) at which matter in the disk stops smoothly orbiting and rapidly falls into the black hole. $R_{\text{ISCO}}$ is a monotonically decreasing function of $a_*$ that becomes steeper for $a_* \sim 1$, reducing error on high-spin measurements [36].

Continuum fitting measures the $R_{\text{ISCO}}$ through the temperature and luminosity of the accretion disc. As the innermost stable circular orbit gets closer to the BH horizon, the matter becomes denser and hotter, increasing the luminosity of emitted radiation. The luminosity does not depend in detail on the disk model: as matter orbits toward the BH, its gravitational potential energy is converted to orbital kinetic energy, with the amount radiated away determined only by assumptions of steady state rotation, axisymmetry, and conservation laws [34]. Just like the measurement of a star’s radius from its temperature and luminosity, the $R_{\text{ISCO}}$ measurement relies on the absolute distance to the BH and the disk inclination with respect to line-of-sight. These, along with the BH mass needed to convert $R_{\text{ISCO}}$ to the dimensionless quantity $a_*$, lead to the dominant sources of error in the spin measurement [34,37,38]. Sub-leading uncertainties from disk modeling result in less than 10% error in $R_{\text{ISCO}}$ and < 3% errors in $a_*$ at high spins. The limitation is that the peak emission must be clearly visible, excluding SMBHs for which emission is in the unobserved far-UV or soft X-ray frequencies.

The X-ray relativistic reflection method (also known as Fe-K or broad iron line method) measures the prop-
FIG. 12. The shaded regions are affected by superradiance and bounded by Regge trajectories in the presence of a QCD axion with mass $\mu_a = 10^{-11}$ eV (see section 11B). The points are stellar black holes measurements with 1 $\sigma$ error bars (for the two fastest spinning BHs, the 3$\sigma$ lower bound for spin is shown).

properties of the Fe-K$\alpha$ spectral line, which is excited in the accretion disk by an external X-ray source (e.g. the disk corona or the base of a jet); it can be used to measure spins of both supermassive and stellar BHs. The X-rays are partially absorbed, leading to an emission line from de-excitation with a particularly distinctive shape. No emission occurs inside $R_{\text{ISCO}}$ since there the density of matter drops sharply. For higher spin, matter can be closer to the horizon, resulting in an iron line with a longer gravitationally red-shifted tail [39]. If the disk is tilted, the rotation of the disk Doppler-shifts the line to the blue and the red, resulting in two peaks. Since iron has high abundance and high fluorescence, and is isolated in the spectrum, the broadened and distorted line can be fit to find $R_{\text{ISCO}}/r_g$ and the inclination of the disk [10]. The Doppler and gravitational shifts both depend on the dimensionless quantity $R_{\text{ISCO}}/r_g$, so knowledge of the BH mass is not needed to find $a_*$.

Spin Limits

Dozens of stellar and supermassive black hole spins have been measured to date with the techniques described above. Since a black hole that satisfies the superradiance condition loses its spin quickly on astrophysical timescales, these measurements place limits on previously unexplored light boson parameter space.

In fig. 12, we show example regions in the BH spin-mass plane affected by the superradiance of a QCD axion with $\mu_a = 10^{-11}$ eV. The shaded areas correspond to the $\ell = 1, \ldots, 5$ levels that satisfy the SR condition, separated by Regge trajectories. A black hole excludes the axion mass if the SR condition is satisfied for at least one $\ell$-level and, within experimental error, the corresponding superradiance rate is fast enough to grow a maximally filled cloud, $\Gamma_{sr} \tau_{\text{bh}} \geq \log N_{\text{max}}$. The relevant timescale $\tau_{\text{bh}}$ is the shortest on which SR can be disturbed; for stellar BHs, we use the shorter of the age and the Eddington accretion time; for SMBHs, we use the compact-object infall time ($\tau_{\text{bh}}$ varies between systems so the regions shown in fig. 12 are approximate). In the following section, we show that the environment of BHs does not further affect these results.

FIG. 13. Limits on mass and self-coupling of light axions derived from (a) quickly rotating stellar black holes (at 2$\sigma$) and (b) quickly rotating supermassive black holes (at 1$\sigma$). The limits disappear for lower $f_a$ because the axion cloud can collapse due to self-interactions before extracting a significant fraction of the BH’s spin. The lighter regions are where the BH may be on a Regge trajectory and are therefore not excluded. We also translate the $f_a$ dependence to the quartic coupling $\lambda$ (right axis).

FIG. 13. Limits on mass and self-coupling of light axions derived from (a) quickly rotating stellar black holes (at 2$\sigma$) and (b) quickly rotating supermassive black holes (at 1$\sigma$). The limits disappear for lower $f_a$ because the axion cloud can collapse due to self-interactions before extracting a significant fraction of the BH’s spin. The lighter regions are where the BH may be on a Regge trajectory and are therefore not excluded. We also translate the $f_a$ dependence to the quartic coupling $\lambda$ (right axis).
| # | Object   | Mass ($M_\odot$) | Spin | Age (yrs) | Period (days) | $M_{\text{comp, star}}$ ($M_\odot$) | $M / M_\odot$ |
|---|----------|------------------|------|-----------|--------------|------------------------------------|---------------|
| 1 | M33 X-7  | 15.65 ± 1.45     | 0.84$^{+0.05}_{-0.07}$ | $3 \times 10^6$ | 3.4530$^{+12}_{-12}$ | > 20                                | > 0.12        |
| 2 | LMC X-1  | 10.91 ± 1.4      | 0.92$^{+0.05}_{-0.07}$ | $5 \times 10^6$ | 3.9092$^{+13}_{-13}$ | 31.79 ± 3.48                       | 0.16          |
| 3 | GRO J1655-40 | 6.3 ± 0.5      | 0.7$^{+0.2}_{-0.2}$ | $3.4 \times 10^8$ | 2.622$^{+14}_{-14}$ | 2.3 ± 4                            | < 0.25         |
| 4 | Cyg X-1  | 14.8 ± 1.0       | > 0.95          | $4.8 \times 10^9$ | 5.599829$^{+11}_{-11}$ | 17.8                               | 0.02$^{+11}_{-11}$ |
| 5 | GRS1915+105 | 10.1 ± 0.6      | > 0.95          | $4 \times 10^9$ | 33.85$^{+13}_{-13}$ | 0.47 ± 0.27                        | 1$^{+18}_{-18}$ |

TABLE III. Stellar black holes that set limits on light bosons (data compiled in [34]). Errors are quoted at 1σ, lower spin limits at 3σ. GRO J1655-40 has a 2σ-discrepancy between continuum fitting and X-ray reflection [35]; we include this in the error. GRS1915+105 has periods of unusually high luminosity; we conservatively use $\tau_{\text{bh}} = \tau_{\text{eddington}} / 10$ to set the limit.

| # | Object   | Mass ($10^6 M_\odot$) | Spin |
|---|----------|------------------------|------|
| 1 | NGC 3783 | 20.8 ± 5.4             | > 0.88 |
| 2 | Mrk 110  | 25.1 ± 6.1             | > 0.89 |
| 3 | MCG-6-30-15 | 2.9$^{+0.18}_{-0.16}$ | > 0.98 |
| 4 | NGC 4051 | 1.91 ± 0.78            | > 0.99 |

TABLE IV. Supersymmetric black holes with reliable mass and spin measurements (compiled in [35, 40]) used to set limits on light bosons in Fig. 13(b). The mass errors are quoted at 1σ and the spin measurements at 90% confidence. While many more spin measurements are available, our analysis excludes BHs which do not have an error estimate on the mass.

of growing to maximum size all at once, each time the cloud reaches the critical size $N_{\text{bosenova}}$ it collapses, so the SR rate has to be larger to extract the spin in the same period, \( \Gamma_{\text{sr}} \tau_{\text{bh}} (N_{\text{bosenova}} / N_{\text{max}}) \geq \log N_{\text{bosenova}} \). In addition, the BH can be trapped on the Regge trajectories; if the spin and mass of a black hole indicate that it may be on a Regge trajectory, we only use it to place bounds if it stays there for a short time, $\tau_{\text{regge}} \ll \tau_{\text{bh}}$.

We show the resulting bounds in fig. 13. Each black hole places a limit on a range of axion masses and each $\ell$-level leads to the distinct lobes of the exclusion region; higher levels have longer superradiance times and give increasingly weaker constraints. For large axion masses, there is no measured BH that satisfies the SR condition; for axion masses too small, the “atomic coupling” $\alpha$ is too weak, resulting in a too-slow spin extraction rate. For strong self-interactions ($N_{\text{bosenova}} \ll N_{\text{max}}$), the bounds no longer apply; this is in contrast with typical experimental limits which are relaxed if interactions are sufficiently weak.

We present more details about the stellar black holes we use to set limits in Table 11. These have spins determined by both methods, as well as precise mass measurements and an estimated age for the BH system. Stellar BH limits are quite robust: the binary systems are well mined by both methods, as well as precise mass measurements.

V. EFFECT OF THE BLACK HOLE ENVIRONMENT

So far, we have considered an isolated black-hole-axion system; in this section, we show that this is a good approximation. The companion star orbits far from the black hole and the accretion disk contains a small fraction of the black hole mass spread over a large region; we will see that the perturbation due to the environment is irrelevant for GW signal parameter space, although it can slightly affect the bound derived from high-spin BHs for very large clouds, $\alpha / \ell \ll 1$.

A non-uniform, asymmetric perturbation near the black hole can mix the superradiating and dumping (infalling) levels of the cloud and cause part of the axion cloud to collapse. We consider a static perturbing potential $\delta V(\vec{r})$: orbital frequencies of the companion star and accretion disk are much smaller than the axion energy so...
the perturbation is adiabatic. The condition that at the horizon the energy flux is negative, i.e. more axions are extracted from the ergo-region than fall back in, is

$$\frac{\Gamma_{\text{dump}}}{\Gamma_{\text{sr}}} \left| \left\langle \psi_{\text{sr, dump}}^* \left| \begin{array}{c} \psi_{\text{sr}}^n \end{array} \right| \delta V(\vec{r}) \right\rangle \right| \Delta E < 1,$$

(30)

where $\psi_{\text{sr, dump}}$ are the wave functions of the superradiating and dumping levels, $\Delta E$ is the energy difference between them, and $\Gamma_{\text{dump}}$ and $\Gamma_{\text{sr}}$ are the analytical dumping and superradiant rates [2] (an excellent approximation at small $\alpha/\ell$). The ratio of the rates comes from relating the wave functions at the cloud to those at the horizon [41] and scales as a high power of $\alpha$,

$$\frac{\Gamma_{\text{dump}}}{\Gamma_{\text{sr}}} \propto \alpha^{4(\ell'-\ell)} \left( \frac{\alpha}{\ell} \ll 1 \right),$$

(31)

with a weaker dependence on $m$.

The energy differences $\Delta E$ are $\mu_a(\alpha/\ell)^2$ between levels with $\Delta n \neq 0$, with $\sim \mu_a(\alpha/\ell)^4$ “fine” level-splitting from relativistic corrections (if $\Delta \ell \neq 0$) and $\sim \mu_a(\alpha/\ell)^5$ “hyperfine” splitting from spin-orbit coupling (if $\Delta m \neq 0$),

$$\Delta H_{\text{s.o.}} = \mu_{\text{BH}} \cdot B_{\text{axion-orbit}} \sim \alpha_a r_g \left( \frac{\alpha}{\ell} \right)^6 L_z,$$

(32)

and higher order corrections to the Newtonian potential from the black hole.

A. Companion Star

The observed stellar BHs are in binaries with companion stars, the masses and especially orbital periods of which are known to great precision (Table III). For the systems used in setting limits, the companions have masses $M_*$ of order the BH mass and orbit at a distance $R \sim 10^n r_g$ from the BH. To compute the effect of the companion, we decompose its gravitational potential into multipoles; schematically,

$$\frac{\delta V}{\mu_a} = \frac{M_* r_g}{M} \left( 1 + \frac{r_c}{R} Y_{1,0} + \frac{r_c^2}{R^2} Y_{2,0} + \ldots \right).$$

(33)

The leading contribution comes from the dipole ($\ell' = \ell - 1$) term of order $r_c^2/R^2 \ll 1$: the resulting mixing does not affect superradiant growth if

$$\left( \frac{\alpha}{\ell} \right) > (0.05) \left( \frac{M_*}{M} \right)^{1/8} \left( \frac{M}{10^6 M_\odot} \right)^{1/6} \left( \frac{\text{day}}{T} \right)^{1/6},$$

(34)

where $T$ is the orbital period of the companion star. Higher multipoles mix with other $\ell' \neq \ell$ modes and give a comparable bound, while mixing with $\Delta \ell = 0, m' \neq m$ modes gives a weaker bound since the ratio of dumping to superradiant rates for these modes is smaller. Thus, a typical companion star may disrupt superradiance only if the axion-black hole coupling is small, $\alpha/\ell < 0.05$; the bound has very weak dependence on the black hole binary parameters. We take this constraint into account in setting limits on axion parameter space. For our GW signal estimates, the bound on $\alpha/\ell$ is irrelevant: weak coupling regime produces signals too small to observe, and signals are just as likely to come from the over 50% of BHs which are not in binaries [52].

B. Accretion Disk

Stellar BHs in binaries and supermassive BHs are surrounded by accretion disks which extend from the innermost stable orbit to the distance at which stars begin to be tidally disrupted and become part of the disk. For stellar black holes, the accretion disk extends to the companion star at $10^6 r_g$, while for a $M = 10^6 M_\odot$ black hole, the disk ends at $\sim 10^3 r_g$ [39]. Although the disk can spatially overlap with the axion cloud, it does not source a large perturbing potential since the mass in the disk is very small compared to the BH mass. To compute the effect on the axion cloud, we use the thin-disk model for the surface mass-density (for $r \gg r_g$) [52]:

$$\sigma(r) \simeq \frac{10^{-17} M}{r_g^2} \left( \frac{0.01}{\alpha_\text{D}} \right)^{1/2} \left( \frac{L}{L_{\text{edd}}} \right)^{1/2} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{r}{r_g} \right)^{-3/2},$$

(35)

where $L \lesssim L_{\text{edd}}$ is the disk luminosity and $\alpha_\text{D} = 0.01$–0.1 characterizes the disk properties; we conservatively use $\alpha_\text{disk} = 0.01$ and $L = L_{\text{edd}}$ in our estimates. Even maximally accreting disks contain a tiny amount of mass, and the effect on the axion cloud is further suppressed by the small height of the disk, $z(r) \simeq 10^{-2} r_g$ [52].

Of course only non-uniformities in the disk contribute to the mixing of different levels and the fractional mass in a given mode is even smaller, but to avoid model-dependence of the disk substructure, we conservatively take the entire mass of the disk to be concentrated in a spherical harmonics mode that induces mixing of a given SR mode with the fastest-dumping mode. The scenario with the biggest ratios of $\Gamma_{\text{dump}}/\Gamma_{\text{sr}}$ (thus giving the most stringent bounds) is mixing with the rapidly decaying $\ell' = m' = 0$ mode, followed by the $m' = -m, \ell' = \ell$ mode.

In fig. 14 we show the resulting bound on $\alpha/\ell$ for the axion clouds of different levels. We see that for stellar BHs, the disk constraint on $\alpha/\ell$ is weaker than that from the companion star. The disks of SMBHs are fractionally smaller in extent, but disk density grows quickly with the BH mass, so the effect of accretion disks around SMBHs is relatively larger. The higher-$\ell$ modes are more affected by the accretion disk since their superradiance rates are increasingly smaller than the dumping rate of the $\ell' = m' = 0$ level. However, the superradiance time for these levels becomes long and limited by other factors, so they are not relevant for the limits or signals we consider. The above discussion agrees with numerical results in [54]. In sum, even with very conservative...
massive) black holes have mass $10^6 M_{\odot}$, and the axion, creating a macroscopic “cloud” of axions with large occupation number. Advanced LIGO will be sensitive to gravitational wave radiation from axion annihilations (for Planck-scale QCD axions) and axion transitions between levels (for GUT-scale QCD axions). Advance VIRGO have similar sensitivities.

Both signals are monochromatic and last dozens of years or more, with time-dependent intensity and a positive frequency drift of less than $10^{-11}$ Hz/s. These signals are distinct from other sources, including monochromatic radiation from rapidly spinning asymmetric neutron stars, since GWs from neutron stars appear at higher frequencies and the signal frequency decreases following the spin-down rate.

We extrapolate the current LIGO run’s monochromatic search [18] to aLIGO sensitivities, and we estimate that aLIGO should expect $\mathcal{O}(1)$ persistent events from axion transitions using expected astrophysical distributions of stellar black holes. Since the signal lasts longer than the duration of the experiment, this event estimate for transitions could be interpreted as the probability of observing the axion at aLIGO.

For axion annihilations, the optimal reach of aLIGO corresponds to BHs near $50 M_{\odot}$ in mass for which very little is known. Much of the parameter space where annihilation gives appreciable event rates is disfavored by BH spin measurements; if we extrapolate measured stellar BH distributions to higher masses, we expect hundreds of events at aLIGO just outside the disfavored region, for axions around $6 \times 10^{-13}$ eV in mass.

Both axion transitions and annihilations provide the opportunity to discover the QCD axion within a few years’ time through its gravitational coupling. Future GW observatories such as the Einstein Telescope may reach a factor of 100 further in sensitivity than current observatories [55] and detect a factor of 10 or more events than our aLIGO estimates.

As we can see from table [V] and fig. [11], the prospect for discovery of much lighter axions (between $10^{-19}$ and $10^{-16}$ eV) is particularly promising through the annihilation signal around supermassive BHs at future lower-frequency GW observatories such as eLISA and AGIS. We expect to see up to $10^9$ events. Signals from level transitions around supermassive black holes would mostly come from BHs with masses below $10^6 M_{\odot}$, and we know little about BHs in this mass range, but our naive estimate in table [V] shows that the expected event rate can also be high.

One signature we have not explored in detail is GW emission from “bosenovae”, the collapse of the cloud under its self-interaction, relevant for bosons with self-interaction stronger than that of the QCD axion. Rates for bosenova events are difficult to estimate because the shape and frequency of the signal are sensitive to the dynamics of cloud collapse. This is a particularly interesting avenue for future numerical studies, since the signal has promising size and a distinctive time profile.

The range of axion masses probed by GW detectors is already constrained by black hole spin measurements. For supermassive BHs, these measurements are less reliable given the uncertainties in the spin measurement method as well as the infall rates of compact objects. For
stellar BHs, spin measurements are confirmed by two independent techniques, and the environment of the BH is very well known in the relevant cases. This makes the exclusion of $4 \times 10^{17}$ GeV $< f_a < 10^{19}$ GeV on the axion decay constant quite robust. This bound is taken into account in our conclusions for the discovery potential of aLIGO.

 Astrophysical black hole superradiance diagnoses the presence of light axions in the theory independently of their cosmic evolution and abundance. We focus on the QCD axion in this paper, but the above discussion can be generalized to all spin-0 bosons with weak self-coupling since gravity is the only interaction required. Effects we discussed can also be extended to light spin-1 particles, although further study, in particular of their superradiance rate, is needed.

 Finally, we use approximations of the waveforms expected from superradiance which are sufficient for our estimates and for preliminary searches at GW detectors; full numerical studies that take into account all the non-linear effects would be required for further analysis in the event that a candidate signal is discovered. The work presented above may only be scratching the surface of the rich phenomena of black hole superradiance.

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 Appendix A: Gravitational Wave Power Calculation

 To calculate the transition rate $\Gamma$, and annihilation rate $\Gamma_a$ (sections III A and III B) we make two approximations here as in [4]: we use the flat space formula for the gravitational wave flux and the hydrogen wave functions for the axion wavefunctions, both of which are valid since the cloud is localized far away from the BH horizon. Further numerical study would be of interest.

 For source stress-energy tensor decomposed as

$$T_{\mu\nu} = \sum_{\omega} e^{-i\omega t} T_{\mu\nu}(\omega, \vec{x}) + \text{c.c.},$$  \hspace{1cm} (A1)

the power per solid angle emitted in a direction $\hat{k}$ is [50],

$$\frac{dP}{dt} = \sum_{\omega} G N \omega^2 \Lambda_{ijlm} T^{ij}(\omega, \vec{k}) T^{lm*}(\omega, \vec{k})$$  \hspace{1cm} (A2)

where $\vec{k}^2 = \omega^2$,

$$\Lambda_{ijlm} = P_{ml} P_{jl} - \frac{1}{2} P_{ij} P_{ml}, \quad P_{ij} = \delta_{ij} - \frac{k_i k_j}{k^2},$$  \hspace{1cm} (A3)

and $T_{ij}$ is the cartesian spatial component of the Fourier transform

$$T_{\mu\nu}(\omega, \vec{k}) = \int d^3x e^{-i\vec{k}\cdot\vec{x}} T_{\mu\nu}(\omega, \vec{x}).$$  \hspace{1cm} (A4)

We use indices $i,j,\ldots$ to denote cartesian spatial coordinates for the rest of this section.

 We start with the classical axion field,

$$\phi(r, \theta, \varphi) = \sum_{n,l,m} e^{-i\omega_n t} \sqrt{\frac{N_{nlm}}{2\mu_a}} \Psi_{nlm}(r, \theta, \varphi) + \text{c.c.}$$  \hspace{1cm} (A5)

where $N_{nlm}$ is the occupation number and $\Psi_{nlm}$ is the normalized hydrogen wave function for the $(n, l, m)$ level with energy $\omega_n$ (eq. (3)), with stress energy tensor

$$T_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left( \frac{1}{2} \nabla_\rho \phi \nabla^\rho \phi + V(\phi) \right).$$  \hspace{1cm} (A6)

In the Minkowski metric the spatial part of the second term is proportional to $\delta_{ij}$ and does not contribute to eq. (A2); therefore the cartesian spatial component is

$$T_{ij}(r, \theta, \varphi) = \nabla_i \phi \nabla_j \phi,$$  \hspace{1cm} (A7)

where

$$\nabla_i = \frac{1}{r} \begin{pmatrix} r \cos \varphi \sin \theta & \cos \theta \cos \varphi & -\csc \theta \sin \varphi \\ r \sin \theta \sin \varphi & \cos \theta \sin \varphi & \cos \varphi \csc \theta \\ r \cos \theta & -\sin \theta & 0 \end{pmatrix} \begin{pmatrix} \partial_r \\ \partial_\theta \\ \partial_\varphi \end{pmatrix}$$  \hspace{1cm} (A8)

Decomposing (A7), we identify terms in the expansions

$$T_{ij}(r, \theta, \varphi)_{\text{ann}} \equiv T_{ij}(\omega_n + \omega_n', r, \theta, \varphi)$$  \hspace{1cm} (A9)

$$= e^{i(\omega_n + \omega_n') t} \left( \frac{N_{nlm} N_{nl'm'}}{2\mu_a} \right)^{\frac{1}{2}} (\nabla_i \Psi_{nlm}) (\nabla_j \Psi_{nl'm'})$$

as the GW source from annihilation of two axions from levels $(n, l, m)$ and $(n', l', m')$, and

$$T_{ij}(r, \theta, \varphi)_{\text{trans}} \equiv T_{ij}(\omega_n - \omega_n', r, \theta, \varphi)$$  \hspace{1cm} (A10)

$$= e^{i(\omega_n - \omega_n') t} \left( \frac{N_{nlm} N_{nl'm'}}{2\mu_a} \right)^{\frac{1}{2}} (\nabla_i \Psi_{nlm}) (\nabla_j \Psi_{nl'm'}) + \text{c.c.}$$
higher transition rates, we use the differential power. To calculate the annihilation and the spherical harmonics.

TABLE VI. Analytic expressions for GW power from axion annihilations, with \( m = l \). For brevity, we expand in \( \alpha \) for higher \( l \) levels, though the first few higher-order terms are comparable in magnitude.

| Level | \( \frac{dP}{d\Omega} N^{-2} \) |
|-------|-----------------|
| 2p    | \( \alpha^{18} G_N (6\alpha^3 + 40\alpha - 3(2\alpha^3 + 3)\tan^{-1}(2/\alpha)^3 (28\cos 2\theta_y + \cos 4\theta_y + 35)/2^{24}\pi^4 r_y^4) + \ldots \) |
| 3d    | \( \alpha^{20} G_N \sin^4 \theta_y (28\cos 2\theta_y + \cos 4\theta_y + 35)/2^{31}\pi^4 r_y^4) + \ldots \) |
| 4f    | \( \alpha^{24} G_N \sin^6 \theta_y (28\cos 2\theta_y + \cos 4\theta_y + 35)/5^{41} \pi^4 r_y^4) + \ldots \) |

TABLE VII. Analytic expressions for GW power from transitions. Higher order terms in \( \alpha \) are smaller by a factor of 10 or more.

as the GW source from transition from levels \( (n, l, m) \) to \( (n', l', m') \). To calculate the GW power, we use eqs. (A9) & (A10) and Fourier transform to momentum space.

\[
T_{ij}(\omega, \theta_k, \varphi_k) = \int d\theta d\varphi \, r^2 \sin \theta \times \sum_{l,m} 4\pi (-i)^l j_l(\omega r) Y_{lm}^*(\theta, \varphi) Y_{lm}(\theta_k, \varphi_k) T_{ij}(\omega, r, \theta, \varphi), \tag{A11}
\]

where \( j_l \) are the spherical Bessel functions and \( Y_{lm} \) are the spherical harmonics.

We then plug eq. (A11) into eq. (A2) to calculate the differential power. To calculate the annihilation and transition rates, we use

\[
\Gamma_{\alpha} = \frac{\int d\Omega dP/d\Omega}_{\text{ann}}, \quad \Gamma_{\tau} = \frac{\int d\Omega dP/d\Omega}_{\text{tr}}, \tag{A12}
\]

respectively. For the 2p level we find a cancellation in the leading \( \alpha \) term, which disagrees with results in [57]. We list the differential power for the most relevant annihilations and transitions in tables VI and VII.

### Appendix B: Event Rate Calculation

We calculate the event rates by incorporating the various astrophysical distributions:

\[
\# \text{ of Events} = \int_0^1 da_* P(a_*) \int_0^\infty Dr P(r) \int_0^\infty dMP(M) \times \frac{\tau_{\text{sig}}(M, a_*, r)}{\text{BHFR}}^{-1} \tag{B1}
\]

where BHFR is the black hole formation rate, \( P(a_*, M, P(r) \) are the normalized probability distributions to find a black hole with spin \( a_* \), mass \( M \), and distance \( r \) away, and \( \tau_{\text{sig}}(M, a_*, r) \) is the duration for which the GW signal from a BH with mass \( M \), spin \( a_* \), and distance \( r \) away is above the noise threshold of the detector.

### 1. Stellar Black Hole Distributions

**Mass distribution:** We quantify the probability of finding a black hole of a certain mass using a fit to current data [20].

\[
P(M) = M_0^{-1} e^{M - M_{\text{min}}/M_0}, \tag{B2}
\]

where \( M_{\text{min}} = 5.3^{+0.9}_{-1.2} M_\odot \) is the minimum BH mass which can be formed from stellar collapse and \( M_0 = 4.7^{+3.2}_{-1.8} M_\odot \) sets the width of the distribution. In figs. [9] and [10] we use \( M_{\text{min}} = 4.1 M_\odot \), within the experimental fit and above theoretical maximum of neutron star mass, \( < 2.9 M_\odot \) [59]; lowering \( M_{\text{min}} \) shifts the event distribution to higher axion masses. Larger values of \( M_0 \) give wider distributions. We expect the narrower distribution is more accurate for young BHs because some larger masses are attained by accretion. In fig. [9], we use the smaller and larger \( M_0 \) as part of our pessimistic and optimistic estimates, respectively, since the aLIGO event rate is very sensitive to the exponential tail of massive BHs. Not much is known about the heavier stellar BHs: the heaviest known stellar BH has mass \( 32.7 \pm 2.6 M_\odot \) [60], while theoretical modeling of BH formation from a single star can accommodate maximum stellar BH mass of \( 30 – 80 M_\odot \) from depending on the metalicity of the environment [61].

**Spin distribution:** The measured distribution is peaked at high spins (30% above 0.8) which we take as a realistic estimate. We consider 90% above 0.8 as optimistic – a high birth spin is likely since the progenitor star has to lose a lot of angular momentum to collapse to the small BH, and in the case a light boson is present, some observed BHs would have spin down since their birth. We take a flat initial spin distribution as pessimistic.

**Formation rate:** Barring rare violent events (e.g. neutron star-BH or BH-BH mergers, < 10\(^{-2}\) per century in the Milky Way [13]), event probability is proportional to the BH birth rate. Core collapse supernovae rates are estimated to be 1.9 ± 1.1 per century [21, 22]. The fraction of supernovae that form black holes is estimated to be 15% in metal-rich stars like the Sun [23] with a large uncertainty. Based on average metalicities today, 20 ± 10% [24] of supernovae form BHs.

Violent BH formation may impede superradiant growth and delay the signal until a more uniform accretion disk forms; our conclusions are unaffected by delays less than a Gyr, and the unlikely case of delays > 5 Gyr would increase the signal by a factor of 3 due to higher star formation rates [62].
Distance distribution: We assume the distance distribution of BHs in the Milky Way is proportional to the stellar distribution. Outside of our galaxy we scale the number of BHs in our galaxy $N_{MW}$ by the blue luminosity distribution in $N_{BH} = 0.0042 N_{MW} (r/\text{Mpc})^3$ at distances $r > 30 \text{Mpc}$.

2. Supermassive Black Hole Distributions

Mass and distance distribution: Supermassive BHs are generally understood to reach their mass through accretion, with most BHs with mass $10^6 - 10^7 M_\odot$ and a tail extending to $10^{10} M_\odot$. We use the distributions of [31, 32], which give a total amount of mass in SMBHs as $\rho_{BH} = (3.2 - 5.4) \times 10^5 M_\odot \text{ Mpc}^{-3}$, or about one $10^7 M_\odot$ BH per MW-type galaxy. We scale this distribution according to [25] at distances closer than 30 Mpc.

Spin distribution: Black holes quickly spin up to maximal spin, where maximal is less than 1 due to counteracting torques from either radiation emitted from the disk and absorbed by the BH ($a_{\text{max}}^* = 0.998$) or magnetic fields transporting angular momentum away from the BH ($a_{\text{max}}^* = 0.93$) in simulations of thick disk models [63]. Simulations of thick disk accretion find that 70% of SMBHs are maximally rotating. In the conservative case of thick disk accretion, 80% of SMBHs have $a_r > 0.8$ [33].

Given these conclusions, we use 70% with spins above $a_r = 0.998$ as optimistic, 70% above 0.93 as realistic and 80% above 0.8 as pessimistic.
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