Probable Approach to the Estimation of the Durability of Reinforced Concrete Structures

Mikhail Berlinov¹, Marina Berlinova¹

¹Moscow State University of Civil Engineering, National Research University, 26, Yaroslavskoye Shosse, Moscow, Russia
marina tvor@mail.ru

Abstract. A method for calculating the durability of reinforced concrete structures under conditions of rheological deformation, taking into account corrosion damage, is presented, reflecting their real work under the combined effect of the load and aggressive environment, and also the random nature of the effects. The possibility of considering the processes of long-term deformation of reinforced concrete from the standpoint of a probabilistic approach to the designation of the calculated reliability coefficients for the load and the material is shown. The regularity of the change in the maximum bending moment at different lifetimes is given, taking into account the rheological properties of materials by the example of the coating shell.

1. Introduction

The urgency of issues of durability of structures of civil buildings is currently justified by the popularity of the use of reinforced concrete as the main building material [1-4,11]. It is known that the nonlinear and non-equilibrium properties of materials lead to a redistribution of forces from more to less loaded areas and components of the cross-sections of reinforced concrete structures [1, 5], including multi-layer structures[7-10], as well as between concrete and reinforcement. Modelling of work and durability forecast can allow not only to save materials without reducing reliability [12], but also to determine more accurately the timing of the need to repair reinforced concrete structures.

2. Material and methods

During the operation of buildings and constructions, reinforced concrete structures are subject to loads that are not constant and change both during exposure and during operation. The pattern of change is random and is subject to statistical distribution curves. The values of the arising internal forces in the section and the law of redistribution depend on the accepted method of reinforcement of the concrete element.

Assuming that taking into account the above factors, it is possible to regulate the stress-strain state of reinforced concrete structures based on a probabilistic approach to building a method for calculating the durability assessment of various structures, including multilayer ones.

An integral method was used to calculate the durability of a reinforced concrete thin-walled coating shell. Then the resolving system of equations will have the following form:

\[
\frac{\partial^2}{\partial x^2} \left[ B_{xx}(t, \sigma) \frac{d^2 \omega}{\partial x^2} \right] + 2 \frac{d}{dx dy} \left[ B_{xy}(t, \tau) \frac{d^2 \omega}{\partial x \partial y} \right] + \frac{\partial^2}{\partial y^2} \left[ B_{yy}(t, \tau) \frac{d^2 \omega}{\partial y^2} \right] + K_x \frac{\partial^2 \varphi}{\partial y^2} + K_y \frac{\partial^2 \varphi}{\partial x^2} = q(t);
\]
\[
\frac{\partial^2}{\partial x^2} \left[ E_{xx}(t, \tau) \frac{d^2 \varphi}{dx^2} \right] + 2 \frac{\partial^2}{\partial x \partial y} \left[ E_{xy}(t, \tau) \frac{d^2 \varphi}{dx \partial y} \right] + \frac{\partial^2}{\partial y^2} \left[ E_{yy}(t, \tau) \frac{d^2 \varphi}{dy^2} \right] + K_x \frac{\partial^2 \omega}{\partial y^2} + K_y \frac{\partial^2 \omega}{\partial x^2} = 0,
\]

where: \( B \) and \( E \) - nonlinear integral operators depending on the level of the stress state, geometric characteristics, \( \omega \) and \( \varphi \) - respectively, vertical displacement and stress function; \( K_x, K_y \) - curvature of the shell.

3. Results and discussions

Closed integration of system (1) meets insurmountable mathematical difficulties, therefore, to obtain numerical results, one should resort to the linearization of the problem using the method of integral estimates. The essence of this technique consists in step-by-step viewing of solutions within each time interval. Due to the adoption of such assumptions, the system of integro-differential equations (1) turns into a nonlinear differential one that can be solved on the basis of the finite difference method using successive iterations. For a grid with a square cell of width \( b \), discrete equations in finite differences will take the following form:

\[
\begin{align*}
&\frac{B_{xy}(t, \tau)}{b^2} (\omega_{i+1,j} - 4\omega_{i,j+1} + 6\omega_{i-1,j} - 4\omega_{i-1,j-1} + \omega_{i-1,j+1}) + \frac{2B_{xy}(t, \tau)}{b^2} (\omega_{i+2,j+1} - 2\omega_{i,j+1} + \omega_{i+1,j+1} - 2\omega_{i,j} + 2\omega_{i-1,j} + \omega_{i-1,j-1} - 4\omega_{i,j-1} + 6\omega_{i,j-1} + \omega_{i,j+1}) + \\
&\quad + \frac{K_x}{b^2} (\varphi_{i+1,j} - 2\varphi_{i,j} + \varphi_{i-1,j}) + \frac{K_y}{b^2} (\varphi_{i+1,j} - 2\varphi_{i,j} + \varphi_{i+1,j}) - q_{ij}(t).
\end{align*}
\]

The second group of equations of system (2) will look similar, it suffices to replace \( \omega \) with \( \varphi \) in expression (2) and take \( q_{ij}(t) = 0 \).

Considering the mechanical characteristics of materials as random variables that obey the normal distribution curves, we can assume:

\[
B(\lambda) = \frac{1}{\sqrt{2\pi}} \int e^{-\frac{x^2}{2}} dx,
\]

where \( \lambda \) is a safety characteristic corresponding to a given reliability.

So, for example, damage to a concrete sample unloaded by external forces, caused by unilateral exposure to corrosion in conditions of stable humidity of the environment and concrete, is considered to obey the non-linearly generalized Goldberg-Waag law, which was originally formulated in a linear formulation and used in the theory of reinforced concrete:

\[
\frac{d\Delta \delta_0 (t, t_0)}{dt} = -\alpha [\Delta \delta_0 (t, t_0)]^m \quad \text{at} \quad m \geq 0,
\]

where \( \Delta \delta_0 \) - the damage depth by the time \( t \), \( \Delta \) - the deficit \( \delta \) compared to the limit, \( t \) - the time count, \( \delta(t_{st}) \) - the damage depth \( \delta(t_{st}, t_0) \) by the time of stabilization \( t_{st} \),

Solution (4) can be obtained in the form:

\[
\delta(t, t_0) = f_m(\alpha, m, t) \delta_{st}(t_0),
\]

\( \alpha \) and \( m \) - some empirical values, depending on the type of corrosion (\( \alpha \) - characterizes the decay rate of the process, \( m \) - reflects the type of corrosion type).
With known distributions of load and material properties, the minimum allowable level of safety that guarantees the basic safety of the structure from mechanical effects (load), taking into account the degradation of the material during operation under the influence of the environment:

\[
R(t) - Q(t) = \lambda_{\text{min}}
\]  
(6)

where: \( \lambda_{\text{min}} \) - is the minimum allowable safety margin.

That is, the estimate of the probability of destruction is associated with reliability, subject to the possible reduction of the safety margin to \( \lambda_{\text{min}} \).

The probability of destruction with the simultaneous action of power and non-force factors is determined using the safety characteristic in the following form:

\[
\lambda(t) = \frac{R(t) - Q(t) - tv_R}{\sqrt{s^2_R(t) + s^2_Q(t) + t^2 v_R}}
\]

(7)

\( v_R \) - resistance reduction rate, determined by the expression (5); \( t \) - duration of exposure to aggressive environment; \( s_R \) and \( s_Q \) are the corresponding standard deviations.

The value of the temporary external load in the general case is also a random value, which can be taken in the form of repeated loads:

\[
q_{ij}(t) = \alpha(t) - \beta(t) \ln \left[ -\ln \frac{P_q}{n(t)} \right],
\]

(8)

where \( \alpha(t) \) and \( \beta(t) \) are numerical factors depending on the type of load; \( P_q \) - security of necessary reliability; \( n(t) \) is the number of loadings during a certain time \( t \).

Considering the value of the constant load unchanged, the value of the full load included in the expression (8), we can accept

\[
p_{ij}(t) = g_{ij}(t_0) - \alpha(t) - \beta(t) \ln \left[ -\ln \frac{P_q}{n(t)} \right],
\]

(9)

here \( g_{ij}(t_0) \) - the constant load applied at time \( t_0 \).

In accordance with expression (9), the value of the external load will change over time according to a logarithmic law.

For practical calculations under the action of loads, continuously changing over time according to random law(Fig.1.), we recommend an approximate method of partitioning time into separate intervals \( \Delta t \), the value of which should satisfy the condition when the correlation function of arguments \( t_i \) and \( t_i + l \) separated by a time interval \( \tau \) greater than \( \Delta t \), disappears is small.
Figure 1. The probabilistic nature of changes in external load. $P_0(t)$ is the constant load; $\Delta(t)$ is the time interval; $\alpha(t)$ is a constant coefficient.

Then, on the basis of the experimental data, the distribution of the load maxima over time $t$ is found, after which the solution of the problem is reduced to a similar one with repeated loadings. The reliability of the design can be interpreted using the safety characteristic $\lambda_{\text{min}} = 3$, which corresponds to the security $P_q = 0.9987$.

In this case, their distribution centres are taken as the standard values of the reliability coefficients for the material and load; for the calculated values, the same security is assigned, i.e. the probability of not exceeding them, then:

$$\bar{y}_m = \frac{(R - \gamma R)}{R}, \quad \gamma_f = (\bar{Q} - \gamma Q)/\bar{Q},$$

$\bar{R}$ and $\bar{Q}$ - expectation, respectively, strength and design loads.

Because any design is subjected to a system of actions, the calculated combination of correlative unrelated loads is found based on a probabilistic approach in the form:

$$\psi = 1 + \sqrt{a_i^2(y_{f_1} - 1)^2 + a_i^2(y_{f_2} - 1)^2 + \cdots + a_i^2(y_{f_n} - 1)^2},$$

here: $a_i - F_i / F$ is the share of the i-th load in the total loading.

The value of the reliability coefficient for the intended purpose of the structure is taken in the form:

$$\gamma_n = \left[1 + \frac{\gamma_y(y_{m} - \gamma_m)\gamma_{y_f}(2 - \gamma_{y_f})}{\gamma_m(y_{m} - \gamma_{y_f})}\right].$$

As an illustration, we present the results of the calculation of a shallow thin-walled reinforced concrete shell loaded with its own weight and snow load, taking into account the reliability and nonlinearity of deformation. The shell is made of concrete of class B30 with an initial modulus of...
elasticity \( E_o(t) = 14.5 \times 10^{-3} \) MPa, prismatic strength \( R_b = 17 \) MPa. Armature class A400, \( E_s = 20-10^{-4} \) MPa, \( R_s = 365 \) MPa. The functions of nonlinearity, creep, as well as other characteristics of the physicomechanical properties of materials were determined by the method described in detail above. The results of the numerical calculation are shown in Fig.1.

![Figure 2. Plots of bending moments (Mnm) in the shell.](image)

- _____ excluding corrosion and deformation nonlinearity;
- - - - - - the same with a 30 year service life without corrosion;
- _____ the same with a 30 year period - taking into account the corrosion damages.

4. Conclusions
Thus, a probabilistic approach to the calculation of reinforced concrete structures requires assignment of calculated coefficients individually in each case, depending on the operating conditions of the structures, including for multilayer elements, in contrast to the calculation of the limiting states, when the calculated coefficients are fixed.

Compared to the traditional method, calculations using the reinforced concrete thin-walled shell coating method proposed above showed that the maximum bending moment perceived by the cross section decreased by 15% when taking into account nonlinear rheological deformation ensuring the required level of reliability with a 30-year service life.

A more complete account of the real properties of materials under conditions of both force and corrosion effects will lead to a decrease in the value of the maximum bending moment by 31%.

A further increase in service life leads to an even greater decrease in the maximum bending moment. Full exhaustion of the carrying capacity will occur, respectively, when the design ceases to conform to the specified level of reliability during operation without carrying out repair and restoration work.

Considering the above, it can be assumed that the safety of the structure, also taking into account corrosion damage, including multilayer ones, must be ensured in two ways, either by increasing the bearing capacity using higher classes of concrete and reinforcement (having high strength characteristics), or increasing cross-sectional dimensions of elements to ensure normal conditions during long-term operation.

References
[1] V.M. Bondarenko, V.I. Kolchunov. Calculation models of the power resistance of reinforced concrete, 112 (2004).
[2] M.V. Berlinov. Strength resistance of reinforced concrete elements of high-rise buildings under dynamic loads. E3S Web of Conferences. 2018. Volum 33. C.02049.
[3] M.V. Berlinov, M.N. Berlinova Durability of reinforced concrete constructions in conditions of prolonged operation//Bulletin of construction equipment. 2019. No. 1 (1013). C. 60-61.
[4] M.V. Berlinov, M.N. Berlinova, A.G. Gregorian Operational durability of reinforced concrete structures // E3S Web of Conferences. 2019. Volum 91. C.02012.
[5] V.M. Bondarenko, V.I. Rimshin The theory of dissipative power resistance of reinforced concrete, 287 (2015).
[6] E.M. Zverjaev, M.V. Berlinov, M.N. Berlinova. International Journal of Applied Engineering Research, 11(8), 5811 (2016).
[7] E.A. Korol. Development of methods of calculation of multilayer enclosing structures with a monolithic link layers // Korol E.A., Berlinova M.N. / [electronic resource]: scientific publication/monograph. Moscow. Publisher MISI-MGSU. 2019/-78 c.
[8] E.A. Korol, M.N. Berlinova Features calculate wall panels with solid communication layers on the stages of installation, transportation and exploitation//Vestnik MGSU/2019. Vol. 14, no. 3 (125). Pp. 367-375.
[9] E.A. Korol, M.V. Berlinov, M.N. Berlinova, Procedia Engineering, 165, 292 (2016)
[10] E.A. Korol, Yu.A. Kharkin, Book of Reports of XX Russian-Polish-Slovak Workshop “Theoretical Fundamentals of Construction” 401 (2011).
[11] V.I. Puhonto. Durability of reinforced concrete structures engineering structures//Moscow ACB. 2004.424 p.
[12] A.G. Tamrazyan. Calculation of structural elements with the specified normal distribution and reliability and load bearing capacity//Vestnik MGSU. 2012. No. 10. Pp. 109-115.