The production of $\eta'$ mesons in the reactions $pp \rightarrow pp\eta'$ and $pn \rightarrow pn\eta'$ at threshold-near energies is analyzed within a covariant effective meson-nucleon theory. The description of cross section and angular distributions of the available data in this kinematical region in the $pp$ channel is accomplished by including meson currents and nucleon currents with the resonances $S_{11}(1650)$, $P_{11}(1710)$ and $P_{13}(1720)$. Predictions for the $pn$ channel are given. The di-electron production from subsequent $\eta'$ Dalitz decay $\eta' \rightarrow \gamma\gamma^* \rightarrow \gamma e^+e^-$ is also calculated and numerical results are presented for intermediate energy and kinematics of possible experiments with HADES, CLAS and KEK-PS.
I. INTRODUCTION

The pseudo-scalar mesons $\eta$ and $\eta'$ represent a subject of considerable interest since some time which has been addressed in many investigations (see, e.g. [1–5]). The physical states of $\eta$, $\eta'$ mesons can be constructed as a superposition of $\eta_0$, $\eta_8$ members of a SU(3) pseudoscalar nonet with one mixing angle. If the coupling constants (defined usually by the relation $\langle 0 | J_{\mu 5}^{\alpha} | P(p) \rangle = i f_\mu^{\alpha} p_{\mu}$, where $P$ is a member of the SU(3) nonet, and $\alpha = 0 - 8$) follow the same pattern as the state mixing then they must be related to each other as [6] $f_8^\eta = f_8 \cos \theta$, $f_0^\eta = -f_0 \sin \theta$, $f_8^\eta' = f_8 \sin \theta$, $f_0^\eta' = f_0 \cos \theta$, where $\theta$ is the state mixing angle. A combined phenomenological analysis [7] of data on two-photon decays of $\eta$ and $\eta'$ as well as $\gamma \eta$ transition form factors shows indeed that one can parameterize the relation among the coupling constants by such a relation, however, with an angle essentially differing from the state mixing angle. The interrelation between the two angles and different decay constants can be found by considering the divergences of axial vector currents including the $U(1)$ axial anomaly, which couples the $\eta'$ state with gluon fields [8, 9].

The $U(1)$ axial anomaly plays also a crucial role in understanding the physical masses of the $\eta$ and $\eta'$ mesons from the point of view of spontaneous breaking of chiral symmetry. It is known [3, 4] that the non-zero divergence of the axial current $J_{\mu 5}^0$ in the chiral limit in presence of interactions is due to couplings with gluons arising after renormalization. This anomaly is responsible for the generation of the heavy mass of the physical $\eta'$ meson. Yet, the anomaly is connected with a violation of the OZI rule for the nucleon and seems also to point to a small strangeness admixture in the proton wave function [10].

This also has a direct consequence in deep-inelastic scattering. For instance, the Goldberger-Treiman relation (see, e.g., [10] and references therein) relates the nucleon-nucleon-$\eta'$ coupling constant $g_{NN\eta'}$ (which is non-zero because of OZI rule violation) with the flavor-singlet axial constant $G_A(0)$ and to the coupling $g_{NNg}$ with gluons. If the gluon contribution is ignored in this relation then the EMC measurements lead to the so-called "spin crisis" [11]. A knowledge of all terms in the Goldberger-Treiman would allow to understand the origin of the spin crisis. However, neither $g_{NN\eta'}$ nor $g_{NNg}$ have been mea-
sured directly in experiments. From this point of view an investigation of the $pp \rightarrow ppp\eta'$ reactions can be considered a tool for supplying additional information on a wide range of fundamental theoretical issues.

The strong decay channels of $\eta$ and $\eta'$ are highly suppressed by various constraints (such as $G$ parity) so that their widths are extremely narrow implying a relatively long life-time and, consequently, a suitable means to investigate different theoretical aspects of the OZI rule, $U(1)$ axial anomaly, role of gluons in the $NN\eta'$ vertex etc. A further remarkable fact is that near the threshold the invariant mass of the $NN\eta'$ system in corresponding reactions is in the region of heavy nucleon resonances, i.e. resonances with isospin $\frac{1}{2}$ can be investigated via these processes. Furthermore, resonances weakly bound with nucleons (the so-called "missing resonances" [12, 13]) can be studied in principle. First SAPHIR data on $\eta'$ photo-production [14] has shown a strong rise of the cross section near the threshold which indicates a possibly large contribution from nucleon resonance excitation. Moreover, if one assumes that the production mechanism is solely governed by resonances then the new states $S_{11}(1897)$ and $P_{11}(1986)$ can be predicted [14]. However, further analysis of the SAPHIR and CLAS [15, 16] data has shown [17–19] that data are also compatible with a larger number of resonances, including higher spin states $\frac{3}{2}$ and $\frac{5}{2}$, i.e., data are still too scarce to firmly determine the properties of the relevant resonances.

Another aspect of $\eta'$ production in elementary hadron reactions is that the subsequent Dalitz decay may constitute a prominent source of di-electrons in intermediate-energy heavy-ion collisions. Indeed, the recent HADES data [20] exhibit a sizeable di-electron yield at large invariant masses which, besides the contribution from the vector mesons $\rho$ and $\omega$ [21], can be due to $\eta/\eta'$ mesons too. The $\eta$ Dalitz decay has been quantified [21] as prominent contribution to the invariant mass spectrum, indeed. One of the primary aims of the HADES experiments [22] is to seek for signal of chiral symmetry restoration in compressed nuclear matter. For such an endeavor one needs a good control of the background processes, including the $\eta'$ Dalitz decay, in particular at higher beam energies, as becoming accessible at SIS100 within the FAIR project [23].

The $\eta'$ Dalitz decays depend on the pseudo-scalar transition form factor, which encodes hadronic information accessible in first-principle QCD calculations or QCD sum rules [5].
The Dalitz decay process of a pseudo-scalar meson $P$ can be presented as $P \rightarrow \gamma + \gamma^* \rightarrow \gamma + e^- + e^+$. Obviously, the probability of emitting a virtual photon is governed by the dynamical electromagnetic structure of the "dressed" transition vertex $P \rightarrow \gamma^*$ which is encoded in the transition form factors [5, 24–27]. If the decaying particle were point like, then calculations of $\gamma^*$ mass distributions and decay widths would be straightforward along standard quantum electrodynamics (QED) techniques. Deviations of the measured quantities from the QED predictions directly reflect the effects of the form factors and, thus the internal hadron structure, and, consequently, can serve as experimental tests to discriminate the different theoretical models in the non-perturbative QCD regime. Yet, calculations of the transition form factor within perturbative QCD involve triangle and box diagrams, which are tightly connected to the $U(1)$ axial anomaly so that form factors provide additional information on the nature of anomalies in quantum field theories, here QCD.

In the present paper we study the di-electron production from Dalitz decay of $\eta'$ mesons in $pp$ and $pn$ reactions at beam energies of a few GeV for kinematical conditions corresponding to the HADES setup [22]. Our focus is to elaborate a model which provides a reliable energy dependence of the total cross section of $\eta'$ meson production and its subsequent decay into a di-electron and a photon at threshold-near beam energies. To this end we calculate the dependence of the differential cross section for the reaction $pp \rightarrow pp\gamma e^+e^-$ upon the invariant mass of the subsystem $\gamma e^+e^-$ around the pole masses of $\eta'$. We calculate the invariant mass distribution of di-electrons in a suitable kinematical range as a function of the di-electron invariant mass and argue that such a quantity, normalized to the real photon point and supplemented by some specific kinematical factor, represents the transition form factor. In such way the extraction of the transition form factor becomes accessible.

$\eta'$ production processes have been analyzed in several papers. In Refs. [17–19, 28], photo-production of $\eta'$ has been studied to elucidate the role of resonances. An investigation of the $\eta'$ production in $NN$ reactions has been performed in Refs. [29, 30], and a consistent combined analysis of $\eta'$ production in the reactions $pp \rightarrow pp\eta'$ and $\gamma p \rightarrow p\eta'$ has been attempted in Ref. [31].
Our paper is organized as follows. Section II is devoted to the theoretical background for dealing with the reaction \( pp \to pp\eta' \to pp\gamma e^+ e^- \). It is essentially based on the effective model [26, 27, 32, 33] proposed to describe bremsstrahlung and vector meson production in \( NN \) reactions. The model is based on direct calculations of the relevant Feynman diagrams within a covariant phenomenological meson-nucleon theory. The model parameters have been fixed from independent experiments and adjusted to achieve a good description of the available experimental data. In the present paper, diagrams with excitation of nucleon resonances with masses close to the masses of a nucleon plus \( \eta' \) meson have been also included. These are \( S_{11}(1650) \), \( P_{11}(1710) \) and \( P_{13}(1720) \) resonances. The corresponding effective constants, whenever possible, have been obtained from the known decay widths of direct decay into the \( \eta' \) channel or radiative decay with subsequent use of vector meson dominance conjecture. Also we use effective constants commonly adopted in the literature and obtained from different considerations, e.g., SU(3) symmetry, fit of photo-absorption reaction etc. [29]. In Section III, total cross sections and angular distributions for \( \eta' \) production in \( NN \) reactions are presented. Comparison with available experimental data is also performed. In Section IV, the \( \eta' \) Dalitz decay is considered. Results of calculations of the invariant mass distribution as a function of the di-electron mass are presented for different initial energies. The role of the transition form factor in such processes is investigated as well. Our conclusions are summarized in Section V.

II. THE MODEL

We consider the reaction

\[
N_1 + N_2 \to N_1 + N_2 + \eta' \to N_1 + N_2 + \gamma + e^+ e^-.
\]

(2.1)

The invariant cross section is

\[
d^{11} \sigma = \frac{1}{2\sqrt{\lambda(s, m^2, m^2)}} \frac{1}{(2\pi)^{11}} \frac{1}{n!} \sum_{\text{spins}} | T(P'_1, P'_2, k_1, k_2, k_\gamma, \text{spins}) |^2 d^{11} \tau_f \frac{1}{n!},
\]

(2.2)

where the factor \( 1/n! \) accounts for \( n \) identical particles in the final state, \( |T|^2 \) denotes the invariant amplitude squared, \( m \) is the nucleon mass, \( s \) denotes the invariant mass of a
particle or a sub-system of particles with total four-momentum $p$, i.e., $s = p^2$, and $d\tau_f$ is the invariant phase volume. The kinematical factor $\lambda$, describing the incident flux, is defined as $\lambda(x^2, y^2, z^2) \equiv (x^2 - (y + z)^2)(x^2 - (y - z)^2)$. Eventually, $P'_1, P'_2, k_1, k_2, k_{\gamma}$ are the momenta of the final nucleons, leptons and photon, respectively.

\section{A. Interaction Lagrangians}

The invariant amplitude $T$ is evaluated here within a phenomenological meson-nucleon theory based on effective interaction Lagrangians (see \cite{26, 27, 29, 32, 33}) which include (i) isoscalar mesons: scalar ($\sigma$), pseudo-scalar ($\eta$), vector ($\omega$), (ii) isovector mesons: scalar ($a_0$), pseudo-scalar ($\pi$) and vector ($\rho$):

\begin{align}
\mathcal{L}_{\sigma NN} &= g_{\sigma NN} \bar{N}N \Phi_\sigma, \quad (2.3) \\
\mathcal{L}_{a_0 NN} &= g_{a_0 NN} \bar{N}(\tau \Phi_{a_0})N, \quad (2.4) \\
\mathcal{L}_{\pi NN} &= -\frac{f_{\pi NN}}{m_\pi} \bar{N}\gamma_5 \gamma^\mu \partial_\mu (\tau \Phi_\pi)N, \quad (2.5) \\
\mathcal{L}_{\eta NN} &= -\frac{f_{\eta NN}}{m_\eta} \bar{N}\gamma_5 \gamma^\mu \partial_\mu \Phi_\eta N, \quad (2.6) \\
\mathcal{L}_{\rho NN} &= -g_{\rho NN} \left( \bar{N}\gamma_\mu \tau N \Phi_\rho^\mu - \frac{K_\rho}{2m} \bar{N}\sigma_{\mu\nu} \tau N \partial^\nu \Phi_\rho^\mu \right), \quad (2.7) \\
\mathcal{L}_{\omega NN} &= -g_{\omega NN} \left( \bar{N}\gamma_\mu N \Phi_\omega^\mu - \frac{K_\omega}{2m} \bar{N}\sigma_{\mu\nu} N \partial^\nu \Phi_\omega^\mu \right), \quad (2.8)
\end{align}

where $N$ and $\Phi$ denote the nucleon and meson fields, respectively, and bold face letters stand for iso-vectors. All couplings with off-mass shell particles are dressed by monopole form factors $F_M = (\Lambda^2_M - \mu^2_M) / (\Lambda^2_M - k^2_M)$, where $k^2_M$ is the 4-momentum of a virtual particle with mass $\mu_M$.

To describe the Dalitz decay of the $\eta'$ meson Eqs. (2.3)-(2.8) must be supplemented with the corresponding Lagrangians describing the interaction of the electromagnetic field $A_\mu$ with di-electrons ($ll = e^+e^-$) and with the $\eta'$ meson,

\begin{align}
\mathcal{L}_{\gamma ll} &= -\bar{e}(\bar{\psi}_l \gamma_\mu \psi_l)A_\mu, \quad (2.9) \\
\mathcal{L}_{\eta' \gamma \gamma} &= f_{\eta' \gamma \gamma} \left( \varepsilon_{\alpha\beta\mu\nu} \partial^\alpha A^\beta A_\mu \right) \Phi_{\eta'}, \quad (2.10)
\end{align}

where $\varepsilon_{0123} = -1$. Lagrangians (2.3)-(2.10) determine the $S$ matrix from which one
generates the corresponding tree-level Feynman diagrams describing the "bremsstrahlung" of \( \eta' \) off nucleons and the subsequent Dalitz decay \( \eta' \rightarrow \gamma e^+ e^- \). Usually such diagrams are called in the literature as nucleonic currents in Dalitz decay of mesons.

The \( \eta' \) meson can be produced also by an internal conversion of the exchanged mesons, the so-called conversion or meson current. The dominant exchange mesons in this case are \( \omega \) and \( \rho \) mesons with the interaction Lagrangians

\[
\mathcal{L}_{\eta'\omega\omega} = -\frac{g_{\eta'\omega\omega}}{2m_\omega} \varepsilon_{\mu\nu\alpha\beta} \left( \partial^\mu \Phi^\nu_\omega \partial^\alpha \Phi^\beta_\omega \right) \Phi_{\eta'},
\]
\[
\mathcal{L}_{\eta'\rho\rho} = -\frac{g_{\eta'\rho\rho}}{2m_\rho} \varepsilon_{\mu\nu\alpha\beta} \left( \partial^\mu \Phi^\nu_\rho \partial^\alpha \Phi^\beta_\rho \right) \Phi'_{\eta'}.
\]

The corresponding diagrams are the same as in \( \eta \) production [27], see Fig. 1.

In the threshold-near kinematics for the \( \eta' \) production in \( NN \) reactions there are several firmly determined nucleon resonances (four stars, according to the Particle Data Group classification [34]) and a large number of the so-called "missing resonances" [12, 13], which, in principle, can be excited in these processes. Consequently, \( \eta' \) production can serve as additional tool to search for and to investigate the missing resonances in the mass interval \( 1.5 - 2.0 \text{ GeV}/c^2 \). As mentioned in the Introduction, in Refs. [18, 28, 30] a preliminary analysis of SAPHIR data [14] has been performed assuming that only resonance currents contribute in this kinematical region. Then, excitations of missing \( S_{11} \) and \( P_{11} \) resonances have been considered and their pole masses and widths are estimated. A further combined analysis [19], including also the CLAS data [15, 16], has demonstrated that excitations of only two resonances are not sufficient to describe the data, and at least two more missing resonances, \( P_{13} \) and \( D_{13} \), ought to be involved into the calculations. Eventually, a systematic analysis of the \( \eta' \) production in photo and \( NN \) reactions [31] has shown that an equally well description of data could be achieved with several diverse sets of diagrams, which include different numbers of known and missing resonances. This means that up to now the available data is too scarce to determine unambiguously the kind and characteristics of resonances contributing in this kinematical region. Moreover, since in photo and \( NN \) reactions one can observe excitations of different resonances, it is not mandatory to analyze simultaneously photo and \( NN \) data within the same set of diagrams. We are interested in finding reliable parameterizations of the energy dependence of the
total cross section and angular distributions near the threshold. Consequently, in order to reduce the number of free parameters and to avoid additional ambiguities, in the present analysis we consider only the known (four stars) lowest spin resonances $S_{11}(1650)$ with odd parity, and $P_{11}(1710)$ and $P_{13}(1720)$ with even parity. The corresponding nucleon-meson-resonance interaction Lagrangians can be found in Ref. [29]. It should be mentioned that the inclusion of higher spin resonances leads to additional uncertainties. It is known that the Lagrangian for particles with spins $s \geq \frac{3}{2}$ possesses an additional symmetry, i.e., additional free parameters (see Ref. [35] and references therein quoted). Thus, the invariance of the Lagrangian with respect to the point transformation causes an additional transformation parameter $A$ and the off-mass shell parameter $z$. In the present calculation we adopt $A = -1$ and $z = -1/2$. Also, the choice of the form of higher-spin propagators has been a subject of discussion in the literature [33, 36–39] with respect to the choice of the spin projector operator $P_{\frac{3}{2}}$ (off mass shell or on mass shell) and the order to write the product of the energy projection operator $\hat{P}_{N^*}$ with the spin projection operator $P_{\frac{3}{2}}$ (only for on-mass shell particles these two operators commute). In the present paper we take the spin-$\frac{3}{2}$ propagator in the form [33]

$$S_{\frac{3}{2}}(P) = -i\frac{\hat{P}_{N^*} + m_{N^*}}{P^2 - m_{N^*}^2}P_{\frac{3}{2}}(P),$$

(2.13)

where the spin projection operator is of the commonly adopted form in the Rarita-Schwinger formalism [40]

$$P_{\frac{3}{2}}(P) = g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{2P^\mu P^\nu}{3m_{N^*}^2} - \frac{1}{3m_{N^*}^2}(\gamma^\mu P^\nu - \gamma^\nu P^\mu).$$

(2.14)

B. Fixing effective parameters

The coupling constants, masses and cut-offs for the dipole form factors for the nucleon currents are taken the same as in the Bonn potential C [41], except for the $\omega$ meson where the coupling is chosen as $g_{\omega NN} = 11$ (see also comments in Refs. [29, 30]). The choice of the magnitude of the coupling constant $g_{\eta' NN}$ is still matter of debate. The OZI rule would imply a small value of $g_{\eta' NN}$, while the above mentioned $\eta-\eta'$ mixing conjecture can relate the corresponding coupling constants and express $g_{\eta' NN}$ via $g_{\eta NN}$ providing in such
a way a relatively large $g_{\eta'NN}$ [18, 30]. An analysis based on $SU(3)$ symmetry and $\eta$-$\eta'$ mixing angle, as performed in Ref. [30], has established an upper limit of $g_{\eta'NN} = 6.1$. Note that in the analysis [30] some 50% of the pseudoscalar-pseudovector admixture in the $NN\eta'$ Lagrangian has been adopted. Subsequent investigations have shown that such a constant is too large, and a new upper limit $g_{\eta'NN} < 2$ has been proposed [17] for the pseudo-vector coupling. Moreover, even values consistent with zero are also admitted in attempts to estimate the couplings and masses of possible excitations of the missing resonances [18, 31]. Evidently, the choice of a small value $g_{\eta'NN}$ implies a negligible contribution of the nucleonic currents, therefore increasing the role of meson conversion and nucleon-resonance currents in the $\eta'$ production.

In our calculations we take $g_{\eta'NN} \simeq 1.42$ as reported in Ref. [2] and recently confirmed by the CLAS data [15]. The effective coupling constants for the resonance currents, whenever possible, have been estimated from the known decay widths of direct decay into $\eta'$ channels or radiative decay with subsequent use of the vector meson dominance (VMD) conjecture. The few remaining unknown cut-off parameters are taken either as constant or are adjusted to the experimental data. These parameters are listed in Tab. I.

| TABLE I: Resonance Parameters. For the spin-$\frac{3}{2}$ resonance $P_{13}$ the off-shell parameter is taken as $z = -1/2$ and the second coupling constant with vector mesons [29] is given in parenthesis. |
|------------|------------|------------|------------|
| $S_{11}(1650)$ | $P_{11}(1710)$ | $P_{13}(1720)$ |
| $g_{MNN^*}$ | $g_{MNN^*}$ | $g_{MNN^*}$ |
| $\Lambda[GeV]$ | $\Lambda[GeV]$ | $\Lambda[GeV]$ |
| $\pi$ | 1.47 | 1.2 | 1.47 | 1.2 | 0.2 | 1.2 |
| $\eta$ | 0.7 | 1.2 | 1.9 | 1.2 | 0.6 | 1.2 |
| $\omega$ | 2.8 | 1.2 | 6.28 | 1.2 | 10(2) | 1.2 |
| $\rho$ | 1.1 | 1.2 | 2.1 | 1.2 | -10(-6) | 1.2 |
| $\eta'$ | 1.18 | 0.9 | 1.4 | 1.2 | 1.0 | 1.2 |

All vertices with off-mass shell nucleons and nucleon resonances are dressed with (i.e.
are to be multiplied by a form factor

\[ F(p^2, \Lambda) = \frac{\Lambda^4}{\Lambda^4 + (p^2 - m^2)^2}. \]  \hspace{1cm} (2.15)

The coupling constants \( g_{\omega \omega' \eta} \) and \( g_{\rho \rho' \eta} \) for the meson conversion current have been derived from a combined analysis of radiative decays within the vector meson dominance (VMD) model and within effective Lagrangians with SU(3) symmetry providing

\[ g_{\omega \omega' \eta} \simeq 4.90, \quad g_{\rho \rho' \eta} \simeq 5.84. \]  \hspace{1cm} (2.16)

Note that naive direct calculations of these constants within the VMD model alone can provide slightly larger values, e.g. \( g_{VV'\eta'} \sim 6.5 - 7.0 \). The corresponding cut-offs, \( \Lambda_{\omega \omega' \eta} = \Lambda_{\rho \rho' \eta} = 1.2 \text{GeV} \), determine the form factor in the conversion vertex \( \eta'VV \) (where \( V \) denotes \( \omega \) or \( \rho \)) which is chosen as (see Refs. [27, 30])

\[ F_{VV\eta'}(\Lambda, k_1^2, k_2^2) = \frac{\Lambda^2 - m_V^2}{\Lambda^2 - k_1^2} \frac{\Lambda^2}{\Lambda^2 - k_2^2}. \]  \hspace{1cm} (2.17)

In accordance with the procedure of determining the coupling constant, the form factor (2.17) is normalized to unity if one vector meson is on-mass shell, while the other one becomes massless, e.g., \( F_{VV\eta}(\Lambda, k_1^2 = m_V^2, k_2^2 = 0) = 1 \). In calculations performed in Ref. [31] the form factor (2.17) has been changed from the monopole to dipole form. As a result, the conversion contribution becomes essentially suppressed in comparison to previous results [30].

III. THE REACTION \( pp \rightarrow pp\gamma e^+ e^- \)

For the Feynman diagrams in Fig. 1 generated by the Lagrangians (2.3)-(2.10), the invariant amplitude \( T \) can be cast in a factorized form

\[ T = T^{(1)}_{NN \rightarrow NN\eta'} \frac{i}{q^2 - \left( m_{\eta'} - \frac{i}{2} \Gamma_{\eta'} \right)} 2 T^{(2)}_{\eta' \rightarrow \gamma e^+ e^-}. \]  \hspace{1cm} (3.1)

The amplitude \( T^{(1)}_{NN \rightarrow NN\eta'} \) describes the production process of a pseudoscalar meson (an off-mass shell \( \eta' \) meson in the \( NN \) collision), while the second amplitude \( T^{(2)}_{\eta' \rightarrow \gamma e^+ e^-} \) describes the Dalitz decay of the produced meson into a real photon and a di-electron. In
the propagator of the $\eta'$ meson the mass $m_{\eta'}$ is replaced by $m_{\eta'} - i\Gamma_{\eta'}/2$ to take into account the finite life time of the $\eta'$ meson.

For such factorized Feynman diagrams one can separate, in the cross section, the dependence on the variables connected with the Dalitz decay vertex and perform the integration analytically [26, 27]. Then, formally, Eq. (3.1) allows one to write the differential cross section (2.2) in a factorized form as well,

$$\frac{d\sigma}{ds_{\eta'}ds_{\gamma^*}} = \frac{d\Gamma_{\eta'\rightarrow\gamma^+e^-}}{ds_{\gamma^*}} \frac{1}{4\pi\sqrt{s_{\eta'}}} \frac{1}{\left(\sqrt{s_{\eta'}} - m_{\eta'}\right)^2 + \frac{1}{4}\Gamma_{\eta'}^2} d^5\sigma_{NN\rightarrow NN\eta'}^{tot},$$

where the integrals over the final di-electron and photon variables have been carried out analytically (see for details [26]). The decay rate $d\Gamma/ds_{\gamma^*}$ for the $\eta'$ meson is defined as

$$\frac{d\Gamma_{\eta'\rightarrow\gamma^+e^-}}{ds_{\gamma^*}} = \frac{2\alpha_{em}}{3\pi s_{\gamma^*}} \left(1 - \frac{m_{\eta'}^2}{s_{\gamma^*}}\right)^3 \Gamma_{\eta'\rightarrow\gamma\gamma^*} |F_{\eta'\gamma\gamma^*}(s_{\gamma^*})|^2.$$  

(3.3)

The electromagnetic fine-structure constant is denoted as $\alpha_{em}$, and $F_{\eta'\gamma\gamma^*}(s_{\gamma^*})$ is the transition $\eta' \rightarrow \gamma\gamma^*$ form factor. The cross section for the production of a pseudoscalar meson with $\eta'$ quantum numbers but with $s_{\eta'} \neq m_{\eta'}^2$ is

$$d^5\sigma_{NN\rightarrow NN\eta'}^{tot} = \frac{1}{2(2\pi)^5 \sqrt{\lambda(s, m^2, m^2)}} \times \frac{1}{4\pi} \sum_{\text{spins}}|T_{NN\rightarrow NN\eta'}^{(1)}|^2 ds_{12}dR_2(N_1 N_2 \rightarrow s_{\eta'} s_{12})dR_2(s_{12} \rightarrow N_1' N_2'),$$

(3.4)

where $s_{12}$ is the invariant mass of the two nucleons in the final state, and the two-particle invariant phase-space volume $R_2$ reads

$$R_2(ab \rightarrow cd) = \frac{\sqrt{\lambda(s_{ab}, m_{\gamma}^2, m_{\gamma}^2)}}{8s_{ab}} d\Omega_c.$$  

(3.5)

It can be seen from Eq. (3.2) that all the peculiarities of the cross section for the full reaction $NN \rightarrow NN\gamma e^+e^-$ are basically determined by the sub-reaction $NN \rightarrow NN\eta'$ with creation of a virtual $\eta'$ meson. Hence, before analyzing the full reaction, we shall proceed with a detailed study of the sub-reaction $NN \rightarrow NN\eta'$, i.e. the production of an on-mass-shell $\eta'$ meson.
A. $\eta'$ production in NN $\rightarrow$ NN$\eta'$ reactions: initial and final state interaction

It ought to be mentioned that generally the Feynman diagrams deal with asymptotically free particles, i.e. account for reactions with non-interacting particles in initial and final states. However, in the real process (2.1) the two nucleons can interact in the initial state (ISI) before the $\eta'$ creation, and in the final state (FSI) as well, provoking distortions of the initial and final $NN$ waves.

The initial state distortion due to the $NN$ interaction before the $\eta'$ creation is to be evaluated at relatively high energies, i.e., larger than the threshold of the $\eta'$ meson production, $T_{kin} \sim 2.41\, GeV$. Therefore, one can expect that the variation with the kinetic energy of ISI effects is small. As shown in Ref. [42], the effect of ISI can be factorized in the total cross section and henceforth it plays a role of a reduction factor in each partial wave in the cross section. This reduction factor depends on the inelasticity and phase shifts of the partial waves at the considered energies. At the threshold, the number of initial partial wave is strongly limited by the partial waves of the final states and, in principle, one can restrict oneself to $^3P_0$ and $^1P_1$ waves. Experimentally it is found [43] that at kinetic energies of the order of few $GeV$ the phase shifts $^3P_0$ and $^1P_1$ are indeed almost energy independent and the reduction factor for each partial wave can be taken constant. In our calculations we adopt, for the reduction factor $\zeta$, the expression from Ref. [42], which leads to $\zeta = 0.277$ for the $^1P_1$ wave and $\zeta = 0.243$ for the $^3P_0$ waves (cf. Ref. [29]).

Final state interaction effects in the $NN$ system have been calculated within the Jost function formalism [44] which reproduces the singlet and triplet phase shifts at low energies. Details of calculations of FSI with the Jost function can be found in Ref. [45].

B. Results for $\eta'$ meson production in NN collisions

The amplitude $T_{NN\rightarrow NN\eta'}^{(1)}$, besides the above listed meson conversion contribution, includes the nucleonic and resonance currents, each of them being a coherent sum over all the considered exchanged mesons. The corresponding parameters are described above
in Section II B. It is worth mentioning that, in spite of the large number of considered diagrams and the large number of effective parameters, there is not too much freedom in fitting the cross section as long as the known resonances are considered. However, if one fits data with additional inclusion of missing resonances, a bulk of parameters remain practically free and can be fine-tuned to optimize the description of data. We performed some investigation of such a model with unknown resonances by considering few missing resonances with masses about $2\, GeV/c^2$ and found that, since the energy dependence of the cross section is rather smooth, various sets of parameters fit equally well the data near the threshold. However, for each of the considered set it turns out that the cross section rises too rapidly at large values of the excess energy. Therefore, to reconcile this dependence with a smoother behavior at large excess energies one can, in principle, add more and more resonances and adjust appropriately the relative (unknown) signs of the couplings to compensate contributions from different diagrams (the interested reader can find a detailed analysis of such a situation in Ref. [31]).

In our present calculations we included only the known four-stars resonances $S_{11}(1650)$, $P_{11}(1710)$ and $P_{13}(1720)$. Results of numerical calculations for the total cross section $\sigma^{NN\to NN\eta'}$ are presented in Figs. 2 and 3, for proton-proton and proton-neutron reactions, respectively. Experimental data are taken from [46]. The total contribution of nucleonic and resonance currents (dot-dashed lines) is found to be much smaller than the mesonic diagrams alone providing a constructive interference in $pp$ and a destructive one in $pn$ reactions. As already discussed, the small contribution of the nucleonic and resonance currents is due to the smallness of the coupling constants $g_{\eta'NN}$ and $g_{\eta'N^*N}$. The magnitude of other parameters entering in our calculations are dictated by the known experimental data. A reasonable agreement with data is achieved almost by the contribution of the meson conversion currents. This does not contradict to the results reported in Ref. [30], however, if one adopts a dipole form factor [31] for the meson conversion vertex, then additional resonances with freely adjustable parameters have to be accommodated in the model.

To further investigate the role of different diagrams, one needs to consider quantities being more sensible to the production mechanism. For instance, a detailed investigation of
the angular distribution (see Figs. 4 and 5) shows that the mesonic current provides a form of the differential cross section which evolves from a flat to a convex curve with increasing excess energy. Contrarily, the nucleonic and resonance currents give a concave shape of the corresponding contributions. The interference effects (which are maximum in forward-backward directions) lead to a slightly convex form of the resulting curves. Experimental data are still too scarce to determine more precisely the relative contributions of different diagrams. Note again that, taking into account more resonances, the description of the experimental data on angular distribution at larger excess energies cannot be improved [31]. New data will essentially enlighten the problem.

Note also that, even achieving a good fit of the cross section in proton-proton reactions, it is not 

\textit{a priori} clear whether the obtained set of parameters equally well describes also the proton-neutron reactions. The isospin dependence of the amplitude is determined by a subtle interplay of different diagrams with different exchange mesons (scalar, vector, iso-scalar, iso-vector etc.). Once the parameters for $pp$ reactions are fixed, the $pn$ amplitude follows directly from this set of parameters without any further readjustment. Of course, the ISI and FSI factors are different for $pp$ and $pn$ systems, but they are fixed by independent information. Our results displayed in Fig. 3-5 may serve as predictions for the $pn$ channel.

Figure 6 exhibits the isospin dependence of the total cross section. Similar to $\omega$, $\phi$ and $\eta$ production [26, 32] the ratio of the $pn$ channel to the $pp$ channel depends on the excess energy. This means that a simple (i.e. constant) isospin factor (see discussions in Ref. [47]) cannot relate these channels. In addition, our calculations point also to the possibility of rather different angular distributions at the same excess energy, as evidenced in Figs. 4 and 5. The angular distribution in the $pn$ channel is fairly flat. The rise of the isospin dependence at low excess energies is entirely determined by the FSI effects which are rather different in the $pp$ and $pn$ channels. At larger excess energy the FSI effects vanish and the isospin ratio is determined entirely by the corresponding Feynman diagrams. Figure 6 exposes the importance of the $pn$ channel for heavy-ion collisions. This seems to hold for the production of light vector mesons and virtual ($e^+e^-$) bremsstrahlung too [33].
IV. DALTZ DECAY

A. Preliminaries

Consider first the general process of Dalitz decay of an on-mass shell pseudo-scalar meson $P$ into a real and a virtual photon (di-electron). Within the present approach this reaction can be considered as a two-stage process, when the meson decays firstly into two photons, and secondly, one of the virtual photon decays into a di-electron pair. The decay width of production of two real photons is calculated from Eq. (2.10) as

$$\Gamma_{P \rightarrow \gamma\gamma} = \frac{s^{3/2}_P}{64\pi} f_{P\gamma\gamma}(0)$$

and serves for a determination of the coupling constant $f_{P\gamma\gamma}$ from experimental data. The square of the $\gamma\gamma$ invariant mass is denoted by $s_P$. Since in what follows we are interested in production and decay of $\eta'$ mesons, being generally off-mass shell, we use the notation $s_{\eta'}$ instead of $s_P$, bearing in mind that $s_{\eta'} \neq m_{\eta'}^2$. Contrarily to the vector meson case [26], instead of the factor $1/3$ (due to averaging over three projections of the spin of the vector particle) in Eq. (4.1) a factor of $1/2$ appears due to two photons in the final state. Equation (4.1) yields $|f_{\eta'\gamma\gamma}| \simeq 0.0314$ GeV$^{-1}$ for the known width $\Gamma_{\eta' \rightarrow \gamma\gamma} = (4.30 \pm 0.15)$ keV [34]. In our calculations the sign of the coupling constant has been taken positively.

The transition form factor $F_{\eta'\gamma\gamma^*}(s_{\gamma\gamma})$ requires separate consideration. As stressed above, the functional dependence of form factors upon the momentum transfer is a source of information on general characteristics of hadrons, such as charge and magnetic distributions, size etc. Also, form factors are known as important quantities characterizing bound states within non-perturbative QCD. The main theoretical tools in studying exclusive processes within the non-perturbative QCD are approaches based on light cone sum rules and factorization theorem (see, e.g., [48–52] and references therein quoted). At high $Q^2$, i.e., at high virtualities of quarks the hard gluon exchange is predicted to be dominant. However, since the virtuality of a quark is determined by $x_i p$ ($x_i$ is fraction part of the total momentum $p$ carried by quark) the QCD asymptotic regime for low values of $x_i$ can be postponed until extremely large values of the momentum transfer and
non-perturbative (soft) contributions play an important role in exclusive processes. In this connection an investigation of form factors in large intervals of momentum transfer, including the time-like region, serves as an important tool to provide additional information about the QCD regimes and the interplay between soft and hard contributions. Besides, there is a number of more phenomenological models, e.g., based on the dispersion relation technique [53, 54], or on use of vector meson dominance models (for details see Refs. [55–58]), or on effective SU(3) chiral Lagrangians with inclusion of the $U(1)$ non-Abelian anomaly [58–60]. It is also known that the asymptotic behavior of the transition form factors can be determined from the corresponding axial anomaly [61]. Traditionally, the electromagnetic form factors are studied by electron scattering from stable particles which provides information in the space-like region of momenta where, as well known, the experimental data can be peerless parameterized by dipole formulae. This in turn means that in the unphysical region, i.e., for kinematics outside the reach of experiments with on-mass shell particles, the analytically continued form factors display a pole structure. Intensively studied form factors are the ones of the light pseudo-scalar mesons, in particular the pion. Heavier pseudo-scalar meson form factors have received less attention since their experimental determination is more difficult. However, there are already experimental data on form factors of the heavier pseudoscalar mesons, such as $\eta$ and $\eta'$, most of them in the space-like region [25, 62]. With the spectrometer HADES [22] which can investigate with a high efficiency di-electrons production in $pp$ collisions in a wide kinematical range of invariant masses, an additional study of the transition form factor for $\eta' \rightarrow \gamma e^+e^-$ process becomes feasible.

The simplest and quite successful theoretical description of form factors can be performed [55, 57, 58] with the VMD conjecture, within which a reasonably good description of elastic form factors in the time-like region has been accomplished. By using the current-field identity [55, 58]

$$J^\mu = -e \frac{m_\rho^2}{f_{\gamma\rho}} \Phi^\rho_\mu - e \frac{m_\omega^2}{f_{\gamma\omega}} \Phi^\omega_\mu - e \frac{m_\phi^2}{f_{\gamma\phi}} \Phi^\phi_\mu + \cdots \quad (4.2)$$

with the coupling constants $f_{\gamma\rho}$, $f_{\gamma\omega}$ and $f_{\gamma\phi}$ known [2, 55, 63, 64] from experimentally measured electromagnetic decay widths, one can also compute the transition form factor...
$F^{\gamma\gamma^*}_{\eta'}(s_{\gamma'})$ by evaluating the corresponding Feynman diagrams. The result is

$$F^{\gamma\gamma^*}_{\eta'}(s_{\gamma'}) = \sqrt{4\pi\alpha_{em}} \sum_{V=\rho,\omega,\phi...} \frac{f_{\gamma'V\gamma}}{f_{\gamma'\gamma}} \frac{m_V^2}{m_V^2 - s_{\gamma'}} \equiv \sum_{V=\rho,\omega,\phi...} C_V \frac{m_V^2}{m_V^2 - s_{\gamma'}}, \quad (4.3)$$

where the summation over vector mesons $V$, besides $\rho$, $\omega$ and $\phi$ mesons, contains also heavier vector particles [55]. The normalization of the form factor at the origin, $F^{\gamma\gamma^*}_{\eta'}(s_{\gamma'} = 0) = 1$ implies that $\sum_V C_V = 1$. Actually, in real calculations one takes into account only a restricted number of vector mesons, for which $\sum_V C_V \approx 1$, i.e. in order to obtain correct normalization, the coefficients $C_V$ must be rescaled. Here, we take into account $\rho$, $\omega$ and $\phi$ mesons. The absolute values of $C_V$ in Eq. (4.3) have been calculated from the rates Eq. (4.1) as

$$\Gamma(V \to e^+e^-) = \frac{4\pi\alpha_{em}^2 m_V}{3 f_{\gamma'}^2}. \quad (4.4)$$

The relative signs have to be fixed from some more general considerations. For instance, SU(3) symmetry implies an opposite sign for $f_{\gamma'\phi}$ relative to the corresponding couplings for $\rho$ and $\omega$, viz. $f_{\gamma\rho} : f_{\gamma\omega} : f_{\gamma'\phi} = 1 : 3 : \left(-\frac{3}{\sqrt{2}}\right)$. We suppose that the sign of the coupling constant $f_{\eta'V\gamma}$ does not depend upon the mass of the vector particle, i.e., the ratio $f_{\eta'V\gamma}/f_{\eta'\gamma}$ in the present paper is taken positive for all mesons. Numerical calculations of $C_V$ with experimental decay rates [34] result in $C_\rho = 0.946$, $C_\omega = 0.095$ and $C_\phi = -0.041$, with a rescaling factor $\approx 0.83$. The accordingly computed transition form factors (see also Refs. [5, 24, 65]) are in a reasonable agreement with the available experimental data [62], which usually are parameterized in a monopole form

$$F^{\gamma\gamma^*}_{\eta'}(Q^2) = \frac{1}{1 - Q^2/\Lambda_{\eta'}^2}, \quad (4.5)$$

where the effective pole-mass parameter $\Lambda_{\eta'} \simeq 0.81 GeV$ is a combination of the masses of the vector mesons. It is worth emphasizing that the experimental parameter $\Lambda_{\eta'}$ has been obtained by fitting data mostly in the space-like region ($Q^2 < 0$). In case of time-like arguments of the form factors, i.e., when $Q^2 \equiv s_{\gamma'} > 0$ the pole mass parameters $m_V$ in Eq. (4.3) receive a small imaginary part, proportional to the total $V$ meson widths, $m_V \rightarrow m_V - i\Gamma_V/2$. For the monopole parameter $\Lambda_{\eta'}$ in Eq. (4.5), we adopt the imaginary part as $\Gamma_\rho/3$. 

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In Fig. 7 we compare the transition form factor computed by the VMD formula, Eq. (4.3), with the corresponding monopole experimental fit (4.5). It is seen that the form factors obtained within VMD model and fitted to data are in a good agreement in a wide interval of di-electron masses. Note that in the Dalitz decay $\eta' \rightarrow \gamma e^+e^-$ the $\rho$ and $\omega$ poles occur in the physical region allowed for the invariant mass of the di-electrons. This implies that an experimental investigation of the $e^+e^-$ mass spectrum in the vicinity of the $\rho$ and $\omega$ poles will provide an excellent test of the validity of the VMD conjecture at large virtualities in the time-like region.

V. THE COMPLETE $\text{NN} \rightarrow \text{NN} \gamma e^+e^-$ REACTION

In order to emphasize the dependence upon the di-electron invariant mass we rewrite the cross section (3.2) in the form

$$\frac{d\sigma}{ds_{\gamma^*}} = \frac{2\alpha_{em}}{3\pi^2} |F_{\eta'\gamma\gamma^*}(s_{\gamma^*})|^2 \frac{Br(\eta' \rightarrow 2\gamma)}{m_{\eta'}^2} I(s_{\gamma^*}),$$

$$I(s_{\gamma^*}) \equiv \int_1^{\xi_{max}} d\xi \frac{\xi(\xi - 1)^3}{\xi} \frac{b/a}{(\xi - a)^2 + b^2} \sigma_{\text{tot} \text{NN} \rightarrow \text{NN} \eta'}(\xi),$$

where we introduced dimensionless variables $a, b$ and $\xi$ as follows

$$a \equiv \frac{m_{\eta'}^2}{s_{\gamma^*}}; \quad b \equiv \frac{m_{\eta'} \Gamma_{\eta'}}{s_{\gamma^*}}; \quad \xi \equiv \frac{s_{\eta'}}{s_{\gamma^*}}; \quad \xi_{max} = \left(\frac{\sqrt{s} - 2m}{s_{\gamma^*}}\right)^2$$

and $Br(\eta' \rightarrow 2\gamma)$ is the branching ratio of the $\eta'$ meson decay into two real photons, $Br(\eta' \rightarrow 2\gamma) = (2.12 \pm 0.14)\%$ [34]. Since the total width of the $\eta'$ meson is fairly small, the parameter $b$ in (5.1) provides a sharp maximum of the integrand function at $\xi = a$ as far as the parameter $a > 1$ (or, equivalently, the di-electron invariant mass $s_{\gamma^*} > m_{\eta'}^2$).

This allows to take out from the integral the smooth function $\sigma_{\text{tot} \text{NN} \rightarrow \text{NN} \eta'}(\xi = a)$ and calculate it at $s_{\eta'} = m_{\eta'}^2$, i.e. at the $\eta'$ meson pole mass. The remaining integral can then be carried out analytically. However, at $a < 1$ when the di-electron mass is larger than the $\eta'$ pole mass, i.e., the integrand does not exhibit anymore a resonant shape, and the integral needs to be calculated numerically.

Equation (5.1) shows that the di-electron invariant mass distribution $d\sigma/ds_{\gamma^*}$ is proportional to the transition form factor $F_{\eta'\gamma\gamma^*}(s_{\gamma^*})$ so that measurements of this distribution
provide a direct experimental information about this quantity. At values $s_{\gamma^*} < m_{\eta'}^2$, the smooth part of the integrand, $\frac{b(\xi - 1)^3}{a\xi} \sigma^{NN\rightarrow NN\eta'}(\xi)$, can be pulled out the integral at $\xi = a$ obtaining

$$I(s_{\gamma^*}) = \frac{\pi(a - 1)^3}{a^2} \sigma^{NN\rightarrow NN\eta'}(s_{\eta'} = m_{\eta'}^2).$$

Equations (5.1) and (5.3) allow to define an experimentally measurable ratio which is directly proportional to the form factor

$$R(s_{\gamma^*}) = \frac{d\sigma/ds_{\gamma^*}}{(d\sigma/ds_{\gamma^*})|_{s_{\gamma^*} = s_{\gamma^* min}}} \left(\frac{s_{\gamma^* min}}{s_{\gamma^* min}} - \frac{1 - s_{\gamma^* min}/m_{\eta'}^2}{1 - s_{\gamma^*}/m_{\eta'}^2}\right)^3 = \frac{|F_{\eta'\gamma\gamma^*}(s_{\gamma^*})|^2}{|F_{\eta'\gamma\gamma^*}(s_{\gamma^* min})|^2},$$

where $s_{\gamma^* min}$ is the minimum value accessible experimentally (in the ideal case this is the kinematical limit $s_{\gamma^* min} = 4\mu_{\eta'}^2$). At low enough values of $s_{\gamma^* min}$, the transition form factor is close to its normalization point $F_{\eta'\gamma\gamma^*}(0) = 1$ and the ratio (5.4) is just the transition form factor as a function of $s_{\gamma^*}$.

As $a$ approaches unity keeping the maximum position still within the integration range, one can again take out the integral the smooth function $\sigma^{NN\rightarrow NN\eta'}$ at $s_{\eta'} = m_{\eta'}^2$. However, now the function $(\xi - 1)^3/\xi$ cannot be considered smooth enough and must be kept under the integration. Nevertheless, even in this case the remaining integral can be computed analytically [27] and one can still define a ratio analogously to Eq. (5.4) which allows for an experimental investigation of the form factor near the free $\eta'$ threshold ($s_{\gamma^*} \rightarrow m_{\eta'}^2$).

At $s_{\gamma^*} > m_{\eta'}^2$ the integral $I(s_{\gamma^*})$ does not have anymore a sharp maximum and it ought to be calculated numerically.

In Figs. 8 and 9 the di-electron invariant mass distribution $d\sigma/ds_{\gamma^*}^{1/2}$ is exhibited as a function of the invariant mass $\sqrt{s_{\gamma^*}}$ calculated by (i) Eq. (5.1) with the transition form factor $F_{\eta'\gamma\gamma^*}(s_{\gamma^*})$ from the VMD model (solid line), (ii) with the monopole experimental fit (4.5) (dashed line), and (iii) without accounting for the form factor, i.e. with $F_{\eta'\gamma\gamma^*}(s_{\gamma^*}) = 1$, known also as QED calculations (dotted line). It is clearly seen that without form factors the calculated cross section rapidly vanishes as $\sqrt{s_{\gamma^*}} \rightarrow m_{\eta'}$, while the inclusion of the form factors results in a resonant shape of the distribution.

It should be noted that the considered final state (two nucleons, one real photon and one di-electron) can be produced by various competing channels too. For instance, prominent
channels are Dalitz decays of pions and $\eta$ mesons as well. However, the contribution of different pseudo-scalar mesons to the di-electron mass distribution is well separated kinematically. Thus, at low values of $s_{\gamma^*}$ the pion contribution is predominant [21]. At intermediate $s_{\gamma^*}$, close to the $\eta$ meson pole mass, the invariant contribution of $\eta$ Dalitz decay is much larger than the corresponding contribution from $\eta'$ mesons. This can be understood from an inspection of Eq. (5.1) which is valid also for $\eta$ meson production. The main difference in mass distributions occurs from different values of the total cross section $\sigma_{NN\rightarrow NN\pi}^{\text{tot}}$ for $\eta$ and $\eta'$ production. For an illustration, in Fig. 10 we present a comparison of the total $\eta$ and $\eta'$ cross sections as a function of the excess energy $\Delta s^{1/2}$. It is seen that the shape of the energy dependence is basically the same, however, the $\eta$ production cross section, at equal excess energy, is larger by more than one order of magnitude. Moreover, at equal initial kinetic beam energy the $\eta$ curve is essentially shifted towards higher excess energies. For instance, at $T = 2.5 \text{ GeV}$ the excess energy for $\eta'$ is $\Delta s^{1/2} \sim 32 \text{ MeV}$ while for $\eta$ meson $\Delta s^{1/2} \sim 442 \text{ MeV}$. In this case, the $\eta$ cross section becomes larger than the $\eta'$ cross section by more than three orders of magnitude. The remaining terms in Eq. (5.1) are of the same order for both $\eta$ and $\eta'$ mesons. Hence, at $s_{\gamma^*}$ close to the $\eta$ pole mass the invariant mass distribution, for the specified final state with one real photon and one di-electron, is entirely governed by the $\eta$ Dalitz decay. However, at larger di-electron masses, where the (virtual) $\eta$ meson is highly off-mass shell, this contribution rapidly vanishes [27] and, consequently, beyond the $\eta$ pole mass the distribution is governed by the $\eta'$ Dalitz decay.

Nevertheless, some comments about this kinematical region are in order. Since the $\eta'$ mass is above the $\rho$ and $\omega$ pole masses, the transition form factor and, correspondingly the mass distribution, display a sharp peak due to smallness of the $\omega$ meson width. To get a feeling on the required detector resolution to identify experimentally such a peak, we simulate it by

$$
\frac{d\sigma}{ds_{\gamma^*}^{1/2}} = N \int_{\sqrt{s_{\gamma^*} - 4d}}^{\sqrt{s_{\gamma^*} + 4d}} \frac{d\sigma}{d\tilde{m}} \exp\left(-\frac{(\tilde{m} - s_{\gamma^*}^{1/2})^2}{2d^2}\right) d\tilde{m}.
$$

(5.5)

In Fig. 11 we present the corresponding invariant mass distributions computed by taking into account this schematic detector resolution. It is seen that already for a resolution
characterized by $d = 15\, MeV$ the $\omega$ peak essentially decreases and practically disappears at lower resolutions, $d \sim 30\, MeV$.

VI. SUMMARY

In summary we have analyzed the di-electron invariant-mass distribution from Dalitz decays of $\eta'$ mesons produced in $pp$ and $pn$ collisions at intermediate energies. The corresponding cross section has been calculated within an effective meson-nucleon approach with parameters adjusted to describe the free vector and pseudo-scalar meson production [26, 32] in $NN$ reactions near the threshold. In particular, we found a suitable parametrization of the cross section for $pp \rightarrow pp\eta'$, which can be used for a prediction of the channel $pn \rightarrow pn\eta'$, which is important in heavy-ion collisions. We argue that by studying the invariant mass distribution of the final $e^+e^-$ system from $\eta'$ Dalitz decays in a large kinematical interval one can directly measure the transition form factor $F_{\eta'\gamma\gamma^*}$, e.g., in $pp$ collisions. Such experiments are possible, for instance at HADES, CLAS, KEK-PS, and our results may serve as predictions for these forthcoming experiments. Experimental information on form factors is useful for testing predictions of hadronic quantities in the non-perturbative QCD domain. In particular, as mentioned in Introduction, the use of the quark flavor basis in a combined analysis of $\eta$ and $\eta'$ mesons with one mixing angle, but with topological effects connected with the $U(1)$ axial anomaly kept, provides an analytical form of the transition form factors with two terms, one related to non-strange quarks another one being entirely expressed via contributions from strange components [24]. This is in full analogy with the vector meson dominance with the $\phi$ meson incorporated. Also, a precise determination of the nucleon-nucleon-$\eta'$ coupling constant $g_{NN\eta'}$ together with an investigation of the polarized deep inelastic lepton scattering, in connection with the Goldberger-Treiman relation [10], can supply additional information on the flavor-singlet axial constant $G_A(0)$ and on the gluon coupling $g_{NNg}$, hence understanding of the origin of the "spin crisis".

We predict the transition form factor and the invariant mass distribution to exhibit maxima at di-electron masses close to the $\rho$ and $\omega$ poles, in contrast to a pure QED
calculation without strong interaction effects which provides a smooth behavior of the distribution in the whole kinematical range.

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FIG. 1: Tree-level diagrams for the process $NN \to NN\gamma e^+e^-$ within an effective meson-nucleon theory. a) Dalitz Decay of $\eta'$-mesons from nucleonic and resonance currents. b) Dalitz decay of $\eta'$ mesons from internal meson conversion.
FIG. 2: Total cross section for the $pp \rightarrow pp\eta'$ reaction as a function of the excess energy $\Delta s^{1/2} = (\sqrt{s} - 2m - m_{\eta'})$. The dot-dashed line depicts the mesonic conversion contribution (MEC only), while the dashed curve represents the total contribution of nucleonic and resonances currents (Nucl.+Res.). The solid curve is the total contribution. Data are from Ref. [46].
FIG. 3: The same as in Fig. 3 but for the $pn \rightarrow p\eta'$ reaction.
FIG. 4: Angular distribution of $\eta'$ mesons in the center-of-mass system in $pp \rightarrow pp\eta'$ reactions at three different excess energies. Line codes as in Fig. 2. Data are from Ref. [46].
FIG. 5: Angular distribution of $\eta'$ meson in the center-of-mass system in $pn \rightarrow pnn'$ reactions.

Line codes as in Fig. 2
FIG. 6: Ratio of the cross sections $pn \rightarrow pnn'$ to $pp \rightarrow ppn'$ as a function of the excess energy.

Line codes as in Fig. 2.
FIG. 7: Transition form factor $\eta' \rightarrow \gamma\gamma^*$ computed within the VMD model (solid line) in comparison with the monopole fit (dashed line) $F_{\eta'\gamma\gamma^*} = 1/(1-s^{*}_{\gamma}/\Lambda_{\eta'}^2)$ with $\Lambda_{\eta'} = (0.81-i\Gamma_{\rho}/3)$ [25, 62].
FIG. 8: Invariant mass distribution of di-electrons from \( \eta' \) Dalitz decay in the reaction \( pp = pp\eta' \rightarrow pp\gamma e^+ e^- \) at proton kinetic energy \( T_p = 2.5 \text{GeV} \) (corresponding to the excess energy \( \Delta s^{1/2} \simeq 31.5 \text{MeV} \)). Solid line - calculations with the transition form factor within the vector meson dominance model, dashed line - transition form factor fit from Fig. 7, dotted line - pure QED form factor, meaning \( F_{\eta'\gamma\gamma^*} = 1 \).
FIG. 9: Same as in Fig. 8, but at $T_{\text{kin}} = 3.5 \text{ GeV}$ corresponding to $\Delta s^{1/2} \simeq 342 \text{ MeV}$. 
FIG. 10: Comparison of total cross sections for $\eta$ and $\eta'$ production in $pp$ collisions as a function of the excess energy. Details of calculations and references for data of the $\eta$ meson cross section can be found in Ref. [27].
FIG. 11: Invariant mass distribution weighted with a Gaussian corresponding to a finite mass resolution of the detector. Solid (dashed) line corresponds to the dispersion $d = 30 \, \text{MeV}$ ($d = 15 \, \text{MeV}$), while dotted line is without smearing. For $T_{\text{kin}} = 3.5 \, \text{GeV}$. 