Protocol for Counterfactually Transporting an Unknown Qubit

Hatim Salih\textsuperscript{1}\textsuperscript{∗}

\textsuperscript{1}Qubet Research, London NW6 1RE, UK

(Dated: April 15, 2014)

Quantum teleportation circumvents the uncertainty principle using dual channels: a quantum state, an effect often paraphrased as “a watched kettle never boils”. Crucially, the Zeno effect can dramatically boost the efficiency of interaction-free measurements.

We now turn to our protocol for the counterfactually transporting an unknown qubit. The protocol uses a dual version of the chained quantum Zeno effect (CQZE) to construct a fully counterfactual quantum CNOT gate. Counterfactual transport then follows straightforwardly.

Let’s first consider the Mach-Zehnder Zeno setup of FIG. 1(a)\textsuperscript{2}. (Throughout, we will adopt similar notation and style to Salih et al.\textsuperscript{2} for consistency.) The first concept we require here is Bob effecting a quantum superposition of blocking and not blocking the transmission channel\textsuperscript{11} \textsuperscript{12} \textsuperscript{14}. Although this is easier to imagine from a practical point of view for the Michelson version we will discuss shortly, we will for now stick to the Mach-Zehnder one, which is easier to explain. Here, BS stands for beamsplitter. The action of BS on Alice’s photon is the following, $|10\rangle \rightarrow \cos\theta|10\rangle + \sin\theta|01\rangle$ and $|01\rangle \rightarrow \cos\theta|01\rangle - \sin\theta|10\rangle$, where the state $|10\rangle$ corresponds to the photon being on the left of BS, the state $|01\rangle$ corresponds to the photon being on the right of BS, and $\cos\theta = \sqrt{R}$, with R being the reflectivity of BS. We set $\theta = \pi/2N$, where N is the number of beamsplitters. Let the initial combined state of Bob’s quantum object together with Alice’s photon, impinging on the first beamsplitter BS from the top left, be $(|\alpha\rangle + \beta|\text{block}\rangle) \otimes |10\rangle$. After n beamsplitters,

$$(\alpha|\text{pass}\rangle + \beta|\text{block}\rangle) \otimes |10\rangle \rightarrow \cos^{n-1}\theta(\alpha|\text{pass}\rangle \otimes (\cos n\theta|10\rangle + \sin n\theta|01\rangle)+\beta|\text{block}\rangle \otimes (\cos n\theta|10\rangle + \sin n\theta|01\rangle)),$$

And after N beamsplitters, with N very large, the combined state of Bob’s quantum object and Alice’s photon becomes $(\alpha|\text{pass}\rangle|01\rangle + \beta|\text{block}\rangle|10\rangle)$. The factor $\cos^{n-1}\theta$ squared is the probability that Alice’s photon is not lost due to measurement by Bob’s object, which brings about the Zeno effect. We have implemented a CNOT gate, with Bob’s the control bit, $|\text{block}\rangle \equiv |0\rangle$, and $|\text{pass}\rangle \equiv |1\rangle$, and Alice’s the target bit, $|10\rangle \equiv |0\rangle$, and $|01\rangle \equiv |1\rangle$, albeit for only one of Alice’s possible input states, namely $|0\rangle$. Moreover, the scheme is only counter-
factual for the part of the superposition corresponding to Bob blocking and is not counterfactual for the part of the superposition corresponding to Bob not blocking, where Alice’s photon gradually “leaks” into the channel.

We now show how to achieve complete CNOT counterfactuality, for Alice’s input state $|0\rangle$, using the chained quantum Zeno effect (CQZE) setup of FIG. 1b) [2]. Here, Alice’s photon goes through $M$ beamsplitters $BS_M$, with $\theta_M = \pi/2M$. Between successive $BS_M$, the photon goes through $N$ beamsplitters $BS_N$, with $\theta_N = \pi/2N$. The state $|100\rangle$ corresponds to Alice’s photon being on the left of $BS_M$, the state $|010\rangle$ corresponds to the photon being on the right of $BS_M$ and on the left of $BS_N$, and the state $|001\rangle$ corresponds to the photon being on the right of $BS_N$. For the $m$-th cycle,

$$(\alpha |\text{pass}\rangle + \beta |\text{block}\rangle) \otimes |100\rangle \rightarrow$$

$$\cos^{m-1}\theta_N [\alpha |\text{pass}\rangle \otimes (\cos n\theta_N |010\rangle + \sin n\theta_N |001\rangle) + \beta |\text{block}\rangle \otimes (\cos \theta_N |010\rangle + \sin \theta_N |001\rangle)].$$

(2)

And after $N$ beamsplitters $BS_N$, with $N$ very large, the combined state of Bob’s quantum object and Alice’s photon becomes $(\alpha |\text{pass}\rangle |001\rangle + \beta |\text{block}\rangle |010\rangle)$. The factor $\cos^{n-1}\theta_N$ squared is the probability that the photon is not lost due to measurement by Bob’s object, which brings about the Zeno effect. But Alice’s single photon is initially in the state $|100\rangle$, as shown in FIG. 1b), with all unused ports in the vacuum state. After the $m$-th $BS_M$,

$$(\alpha |\text{pass}\rangle + \beta |\text{block}\rangle) \otimes |100\rangle \rightarrow$$

$$\cos^{m-1}\theta_M [\alpha |\text{pass}\rangle \otimes (\cos m\theta_M |100\rangle + \sin m\theta_M |010\rangle) + \beta |\text{block}\rangle \otimes (\cos m\theta_M |100\rangle + \sin m\theta_M |010\rangle)].$$

(3)

And after the $M$-th $BS_M$, with $M$ very large, the combined state of Bob’s quantum object and Alice’s photon $\propto (|\text{pass}\rangle |100\rangle + |\text{block}\rangle |010\rangle)$. The factor $\cos^{m-1}\theta_M$ squared is the probability that Alice’s photon is not lost through detection by one of her $D_3$s, which bring about the Zeno effect for the part of the superposition corresponding to Bob not blocking. We have thus implemented a fully counterfactual CNOT gate, with Bob’s the control bit $|\text{pass}\rangle \equiv |0\rangle$, and $|\text{block}\rangle \equiv |1\rangle$, and Alice’s the target bit $|100\rangle \equiv |0\rangle$, and $|010\rangle \equiv |1\rangle$, again for only one of Alice’s possible input states; $|0\rangle$.

The probability that the photon successfully avoids detection by all $D_3$s for the $|\text{pass}\rangle$ part of the superposition is $\cos^{2(M-1)}\theta_M$. This part has a probability amplitude $\alpha$. While for the $|\text{block}\rangle$ part of the superposition, the probability that the photon avoids being lost due to measurement by Bob’s object (or $D_3$) is $$\prod_{m=1}^{M-1} (1 - \sin^2 m\theta_M \sin^2 \theta_N)^N.$$ This part has a probability amplitude $\beta$. Thus the maximum efficiency of this counterfactual CNOT gate is $|\alpha|^2 \cos^{2(M-1)}\theta_M + |\beta|^2 \prod_{m=1}^{M-1} (1 - \sin^2 m\theta_M \sin^2 \theta_N)^N$. FIG. 2 plots this success probability for different $M$s and $N$s, for $\alpha = \beta = 1/\sqrt{2}$. We can see that efficiency approaches unity for large $M$ and $N$, given ideal implementation. For instance for $M = 50$ and $N = 1250$, efficiency is already 95%.

(Unlike Michelson-Zehnder, where for Bob passing, the last $BS_M$ produces an undesired rotation—tiny for large $M$—in the Michelson implementation we will discuss next, a small number of cycles does not lead to output errors from the counterfactual CNOT gate, but would instead lead to more instances of the gate failing through photon loss. Imperfect implementation, however, would [2].)

Complete counterfactuality is ensured: Any photon going into the channel would either be lost due to measurement by Bob’s object or else end up at one of the detectors $D_4$. Moreover, the probability amplitude of the photonic state $|001\rangle$ corresponding to the photon being in the channel is virtually zero for large enough $M$ and $N$. Nevertheless, the scheme is not practical, and more fundamentally, it only works for one of Alice’s input states.

We now switch to a much more practical and versatile Michelson implementation, with a massive saving of physical resources, where the function of $BS$ in the CQZE setup of FIG. 1b) is achieved by the combined action of switchable polarisation rotator $SPR$ and polarising beamsplitter $PBS$, FIG. 3. The action of $SPR_i^{H(V)}$ on Alice’s $H(V)$ photon is the following, $|H(V)\rangle \rightarrow \cos \theta_i |H(V)\rangle + \sin \theta_i |V(H)\rangle$ and $|V(H)\rangle \rightarrow \cos \theta_i |V(H)\rangle - \sin \theta_i |H(V)\rangle$, with $i = 1, 2$ corresponding to $SPRs$ with different rotation angles. We set the rotation angle $\theta_1(2) = \pi/2M(N)$ with $SPR_1(2)$ switched on once per cycle, when the photon, or part of it rather, is moving in the direction from $SM_1(2)$ towards $PBS_1(2)$. Initially, switchable mirror $SM_1(2)$ is turned off allowing the photon in, but is then turned on for $M(N)$ cycles before it is turned off again, allowing the photon out.

Note that the Michelson CQZE setup of FIG. 3 takes $H(V)$ polarsed photons as input with $PBS_i^{H(V)}$ passing $H(V)$ photons and reflecting $V(H)$ as shown in FIG. 3. Alice sends a $H(V)$ photon into the $H(V)$-input CQZE setup. By a similar evolution to Eqs. 2 and 3, if Bob does not block, then Alice’s existing photon will be $H(V)$ polarised, and if Bob blocks then Alice’s exiting photon will be $V(H)$ polarised.

This means that Alice can encode her bit using polarisation. She encodes a “0” (“1”) by sending a $H(V)$ photon into the corresponding $H(V)$-input CQZE setup. But can Alice encode a quantum superposition of “0” and “1”? Crucially, the answer is yes. She first passes her photon through $PBS_1$ in order to separate it into $H$ and $V$ components as shown in FIG. 4a). The $H$ component is then fed into the corresponding $H(V)$-input CQZE setup. Bob can block or not block the transmission channel—or a quantum superposition of blocking and not blocking—for both $H$ and $V$ components which are first recombined using $PBS_2$, FIG. 4a). The polarisation of Alice’s exiting photon is determined by Bob’s bit choice. This is our dual chained quantum Zeno effect.
Consider the most general case where Alice sends a photon in the superposition $\lambda |H\rangle + \mu |V\rangle$, with Bob’s object in the superposition $\alpha |\text{pass}\rangle + \beta |\text{block}\rangle$. We get the following superposition for Alice’s exiting photon, from the upper path $H$-input CQZE module and the lower path $V$-input CQZE module, FIG. 4(b),

$$\lambda(\alpha |\text{pass}\rangle |H\rangle + \beta |\text{block}\rangle |V\rangle) \otimes |\text{upper-path}\rangle + \mu(\alpha |\text{pass}\rangle |V\rangle + \beta |\text{block}\rangle |H\rangle) \otimes |\text{lower-path}\rangle.$$

(4)

All we need now is to combine the two photonic states from the upper and lower paths. This is done by replacing $PB_{SL}$ in FIG. 4(a) by a 50:50 beamsplitter $BS$, as shown in FIG. 4(b). We define the upper-path as above or to the right of $BS$, and the lower-path as below or to the left of $BS$. Let’s rename the states $(\alpha |\text{pass}\rangle |H\rangle + \beta |\text{block}\rangle |V\rangle)$ and $(\alpha |\text{pass}\rangle |V\rangle + \beta |\text{block}\rangle |H\rangle)$ as $\langle \lambda |H\rangle$ and $\langle \mu |V\rangle$ respectively. We can rewrite the exiting state, Eq. (4) as $(\lambda |\langle \lambda \rangle |\text{upper-path}\rangle + \mu |\langle \mu \rangle |\text{lower-path}\rangle)$. Feeding this state into $BS$, which applies a $\pi/2$-rotation to the path qubit, gives,

$$1/\sqrt{2}(\lambda |\langle \lambda \rangle + \mu |\langle \mu \rangle \rangle \otimes |\text{lower-path}\rangle +$$

$$1/\sqrt{2}(\lambda |\langle \lambda \rangle - \mu |\langle \mu \rangle \rangle \otimes |\text{upper-path}\rangle).$$

(5)

which means we can obtain the desired state $\lambda |\langle \lambda \rangle + \mu |\langle \mu \rangle \rangle$ with 50% probability upon measuring the path qubit. This measurement is carried out at $D_{0}$, FIG. 4(b), without destroying the photon when ideal $\text{t trom B to A? The problem with the circuit is that the CNOT gates are on opposite sides. But there is a way around it. By means of four Hadamard gates, the circuit of FIG. 5(b) interchanges the control and target qubits of a CNOT gate. Applying this to the circuit of FIG. 5(a) we get the circuit of FIG. 5(c) which forms the basis of our protocol.

What about the case of Alice’s exiting photon ending up on path $D_{0}$? From Eq. (3) the combined state of Bob’s object and Alice’s photon in this path is, $\lambda(\alpha |\text{pass}\rangle |H\rangle + \beta |\text{block}\rangle |V\rangle) - \mu(\alpha |\text{pass}\rangle |V\rangle + \beta |\text{block}\rangle |H\rangle)$, which in binary is, $\lambda(\alpha |0\rangle |0\rangle + \beta |1\rangle |1\rangle) - \mu(\alpha |0\rangle |1\rangle + \beta |1\rangle |0\rangle)$. This is equivalent to the output of a CNOT gate but with Alice applying a $Z$-gate to her input qubit. Incorporating this in the circuit of FIG. 5(c) we get the circuit of FIG. 5(d). It turns out that an $X$-gate is needed at the end for the state $\alpha |0\rangle + \beta |1\rangle$ to be transferred from one side (top) to the other (bottom). We finally arrive at our protocol for counterfactually transporting an unknown qubit.

Protocol for counterfactual quantum transportation—Alice starts by sending a $H$ photon into the dual CQZE setup (FIG. 4) with Bob’s quantum object, his qubit to be counterfactually transported, placed in a superposition of blocking and not blocking the channel: $\alpha |\text{pass}\rangle + \beta |\text{block}\rangle$. Alice then applies a Hadamard transformation to (the polarisation of) her exiting photon, as does Bob to his qubit. Alice sends her photon back into the dual CQZE setup. If her exiting photon is not found in path $D_{0}$, she knows it is in the other path travelling towards the left. She applies a Hadamard transformation to (the polarisation of) her photon, as does Bob to his qubit. The photon is now in the state $\alpha |H\rangle + \beta |V\rangle$. If Alice’s exiting photon is found instead in path $D_{0}$, she first applies a Hadamard transformation to (the polarisation of) her photon, as does Bob to his qubit. She finally applies an $X$-transformation to her qubit. The photon is now in the state $\alpha |H\rangle + \beta |V\rangle$. Bob’s qubit has been counterfactually transported to her. His original qubit ends up in the state $|0\rangle$ or $|1\rangle$ randomly; in other words destroyed.

We have so far not said anything about how Bob may practically implement his qubit. Tremendous recent advances mean that there are several candidate technologies. Perhaps most promising for our purpose here are trapped-ion techniques, whereby a carefully shielded and controlled ion can be placed in a quantum superposition of two spatially separated states—one of which in our case blocks the channel. Trapped ions offer relatively long decay times, needed for a large-number-of-cycles implementation of the protocol. Moreover, the Hadamard transformation, key to this protocol, can be directly applied. What this means is that all elements of our protocol are implementable using current technology. The protocol could well be useful for transferring atomic qubits to photonic ones inside a future quantum computer—with the key advantage that no previously shared entanglement is required. (If, for practicality, counterfactuality is relaxed, only the inner cycles of the counterfactual CNOT gate would be required, quadratically reducing the number of cycles.) On the flip side, the counterfactual CNOT gate at the heart of this protocol can itself be used to entangle atomic and photonic qubits from scratch.
We have proposed a protocol for the counterfactual, disembodied transport of an unknown qubit—much like in quantum teleportation except that Alice and Bob do not require previously-shared entanglement nor a classical channel. No physical particles travel between them either. Here, Bob’s qubit is gradually “beamed up” to Alice. In the ideal asymptotic limit, efficiency approaches unity as the probability amplitude of the photon being in the channel approaches zero. This brings into sharp focus both the promise and mystery of quantum information.

ACKNOWLEDGMENTS

The author thanks Sam Braunstein, M. Suhail Zubairy, M. Al-Amri, Zheng-Hong Li, Gilles Puetz, and Nichola Gisin for useful comments. This work is partially supported by Qubet Research, a start-up in quantum information.

[1] Bennett, C. H. et al. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. 70, 1895 (1993).
[2] Salih, H., Li, Z. H., Al-Amri, M. & Zubairy, M. S. Protocol for direct counterfactual quantum communication. Phys. Rev. Lett. 110, 170502 (2013).
[3] Hosten, O., Rakher, M. T., Barreiro, J. T., Peters, N. A. & Kwiat, P. G. Counterfactual quantum computation through quantum interrogation. Nature (London) 439, 949-952 (2006).
[4] Wootters, W. K. & Zurek, W. H. A single quantum cannot be cloned. Nature (London) 299, 802-803 (1982).
[5] Gisin, N. Quantum cloning without signaling. Phys. Lett. A 242, 1 (1998).
[6] Bouwmeester, D. et al. Experimental quantum teleportation. Nature 390, 575-579 (1997).
[7] Furusawa, A. et al. Unconditional quantum teleportation. Science 282, 706-709 (1998).
[8] Riebe, M. et al. Deterministic quantum teleportation with atoms. Nature 429, 734-737 (2004).
[9] Barrett, M. D. et al. Deterministic quantum teleportation of atomic qubits. Nature 429, 737-739 (2004).
[10] Sherson, J. F. et al. Quantum teleportation between light and matter. Nature 443, 557-560 (2006).
[11] Dicke, R. H. Interaction-free quantum measurement: A paradox? Am. J. Phys. 49, 925-930 (1981).
[12] Elitzur A. C. & Vaidman, L. Quantum mechanical interaction-free measurements. Found. Phys. 23, 987-997 (1993).
[13] Kwiat, P. G., Weinfurter, H., Herzog, T., Zeilinger, A. & Kasevich, M. A. Interaction-free measurement. Phys. Rev. Lett. 74, 4763-4766 (1995).
[14] Kwiat, P. G. et al. High-efficiency quantum interrogation measurements via the quantum Zeno effect. Phys. Rev. Lett. 83, 4725-4728 (1999).
[15] Misra, B., Sudarshan, E. C. G. The Zeno’s paradox in quantum theory. J. Math. Phys. (N.Y.) 18, 756-763 (1977).
[16] Asher Peres, Zeno paradox in quantum theory. Am. J. Phys. 48, 931-932 (1980).
[17] Agarwal G. S. & Tewari, S. P. An all-optical realization of quantum zeno effect. Phys. Lett. A 185, 139-142 (1994).
[18] Noh, T.-G. Counterfactual quantum cryptography. Phys. Rev. Lett. 103, 230501-230504 (2009).
[19] Nogues, G. et al. Seeing a single photon without destroying it. Nature 400, 239-242 (1999).
[20] Haroche, S. Nobel Lecture: Controlling photons in a box and exploring the quantum to classical boundary. Rev. Mod. Phys. 85, 1083 (2013).
[21] Mermin, N. D. From classical state swapping to quantum teleportation. Phys. Rev. A 65, 012320 (2001).
[22] Mermin, N. D. Copenhagen computation: How I learned to stop worrying and love Bohr. IBM Journal of Research and Development 48, 53-62 (2004).
[23] Monroe, C., Meekhof, D. M., King, B. E. & Wineland, D. J. A “Schrodinger cat” superposition state of an atom. Science 272, 1131-1136 (1996).
[24] Wineland, D. Nobel Lecture: Superposition, entanglement, and raising Schrodinger’s cat. Rev. Mod. Phys. 85, 1103-1114 (2013).
FIG. 1. **Counterfactual Mach-Zehnder CNOT.**

(a) Partially counterfactual CNOT gate. Beamsplitters $BS$ are highly reflective, with reflectivity $R = \cos^2 \pi/2N$, where $N$ is the total number of $BS$s. Bob effects a quantum superposition of blocking and not blocking the channel (not practical for this Mach-Zehnder implementation) which acts as the CNOT’s control qubit. The gate, however, only works for one of Alice’s inputs (“0”). Moreover, it is not counterfactual for the part of the superposition where Bob does not block, in which case the photon passes through the channel.

(b) Fully counterfactual CNOT gate—based on the chained quantum Zeno effect (CQZE). Between successive $BS_M$, of which there are $M$, there are $N$ beamsplitters $BS_N$. While the scheme only works for Alice’s “0” input state, complete counterfactual property is ensured as any photon going into the channel would be lost due to measurement by Bob’s object or else end up at one of the detectors $D_3$: the chained quantum Zeno effect. For large enough $M$ and $N$, the probability amplitude of the photon being in the channel is virtually zero.
FIG. 2. Efficiency of counterfactual CNOT. The success probability of an ideally implemented counterfactual CNOT gate, FIG. 1(b), with Alice encoding “0”, plotted against the number of outer and inner cycles M and N, with M ranging from 2 to 75, and N from 2 to 1500. (Bob’s control qubit is assumed an equal superposition here.) Efficiency approaches 100% for large M and N. This equally applies to the more versatile Michelson-type counterfactual CNOT gate, FIG. 3, with Alice encoding either “0” or “1”.
FIG. 3. Fully counterfactual CNOT gate based on a Michelson CQZE. By using polarising beamsplitter $PBS^H(V)$ that passes $H(V)$ photons and reflects $V(H)$, this setup can take $H(V)$ input from Alice, i.e. “0” (“1”), but not yet a superposition. Bob implements his qubit as a superposition of blocking and not blocking the channel using his quantum object $QO_B$. Initially, switchable mirror $SM_{1(2)}$ is turned off allowing the photon in, but is then turned on for $M(N)$ outer(inner) cycles before it is turned off again, allowing the photon out. The combined action of switchable polarisation rotators $SPR$ and polarising beamsplitters $PBS$ achieves the function of beamsplitters $BS$ in the Mach-Zehnder version of FIG. 1(b). $MR$ stands for mirror, and $OD$ for optical delay. Again, complete counterfactuality is ensured as any photon going into the channel would either be lost due to measurement by Bob’s object $QO_B$ or else end up at detector $D_3$: the chained quantum Zeno effect. For large enough $M$ and $N$, the probability amplitude of the photon being in the channel is virtually zero.
FIG. 4. **Fully counterfactual general input CNOT gate based on a dual CQZE.**

**a**, Alice sends a photon in the qubit state $\alpha |H\rangle + \beta |V\rangle$, while Bob’s quantum object $QO_B$ is in the qubit state $\lambda |\text{pass}\rangle + \mu |\text{block}\rangle$. Alice’s incoming photon is first separated into $H$ and $V$ components using polarising beamsplitter $PBS_L$, which are then respectively fed into the $H$-input and $V$-input CQZE modules from FIG. 3.

**b**, For Alice’s exiting photon, $PBS_L$ is replaced by a 50:50 beamsplitter $BS$. If it is not detected at $D_0$, then the photon exits towards the left in the correct state. For the case of Alice initially sending a $H$ photon, as in the first step of our protocol, there is no need for BS, the photon exits towards the left in the correct state. $S$ stands for single photon source.
FIG. 5. Circuit for quantum state transfer. a, By means of two CNOT gates a qubit $\alpha |0\rangle + \beta |1\rangle$ can be transferred from one side (top) to the other (bottom). Our counterfactual CNOT gate has Bob's as the control qubit, which is a problem since in this circuit the control qubits of the two CNOT gates are on opposite sides. b, By means of four Hadamard gates the control and target qubits of any CNOT gate can be interchanged. c, Applying b to a, the control qubits of both CNOT gates are now on the same side, Bob's. This circuit forms the basis of our protocol for counterfactually transporting an unknown qubit. d, In case of a Z-gate before the right hand side CNOT, we need an X-gate at the end in order for the state $\alpha |0\rangle + \beta |1\rangle$ to be correctly transferred. This circuit is relevant to one of the two possible paths for Alice's exiting photon from the Dual CQZE setup of FIG. 4(b).