Online monitoring for floating raft structure displacement based on optimal placement of measuring points

Guanghui Cheng¹,², Zhenhai Zhang³, Liang Shi¹,² and Yuanran Qiu¹,²

Abstract
The structure displacement of the raft of vibration isolation for ships is an important factor that affects shafting alignment accuracy and the safety of the equipment on the raft. In order to realize real-time monitoring of the raft structure displacement, which is the uncertain loadings and complex in shape, a measuring point placement strategy and an online monitoring method based on the displacement parameters to identify the displacement of the raft are proposed. FEM-based simulations for a prototyped raft have been conducted to obtain the displacement contour map. According to the displacement contour map based on displacement gradient and index evaluation, the measuring points have been combined and optimized. Displacement reconstruction principle based on a surface spline interpolation function method has been elaborated and derived. Structure displacement at any point can be obtained by a few measuring points. Finally, a case study of a certain type of floating raft is analyzed by the simulation analysis, and experimental research. The simulation and experimental results verify the validity of the measuring point placement strategy and the average of relative error with a value of less than 4% under different uncertain loadings. This method can effectively solve the problem of online monitoring of floating raft structure displacement under different changing loadings.

Keywords
Optimal layout of measuring points, floating raft, structural displacement, different loads, online monitoring

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Introduction
The raft vibration isolating device is widely used in the ship to achieve efficient vibration isolation of Marine power plant.¹ As the floating raft structure gets large-scale and light weight, not only the design load, but also the influence of external load changes on floating structure displacement are not ignorable. As shown in Figure 1, the large structural displacement could affect the safety of the equipment shafting and pipeline connection on the raft. Leads to excessive deformation of lower vibration isolator under the raft easily, which endangers the safety of equipment operation, as well as the performance of the vibration isolation system.²–⁵

Research and establishing a set of online monitoring methods for raft structure displacement, located in weak parts with large local displacement in time and take effective measures, which will improve the

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reliability of raft equipment and floating raft vibration isolation effect.

Some efforts have been made to monitor changes in the displacement of the raft structure, such as simplifying rigid bodies and making redundant sensor settings. Bu et al.\(^6\) simplified the raft frame to a rigid body and established a raft frame attitude monitoring model. Placing the four displacement measuring points between the raft frame and the base, the displacement deviation between the equipment on the raft and the external shafting and pipeline can be calculated online. However, this measurement means it is not suitable for large rafts with relatively low structural rigidity, nor does it consider the influence of raft structure deformation under variable different loadings. In terms of this issue, Wenzheng Qin et al.\(^7\) tried to install two redundant displacement sensors in the middle of the raft to obtain the rigid body displacement and elastic deformation of the raft frame. However, this method can only judge primarily the degree of deformation of the structure of the raft, and it cannot obtain the deformation of the raft quantitatively, nor can it monitor the part structural deformation of the raft. Therefore, on the basis of previous studies, developing an online measurement approach to comprehensively monitor the overall displacement of the raft which is a better choose to assess the displacement of the raft structure.

In recent years, in the field of structural health monitoring, foreign and domestic scholars did abundant research, but they pay more attention to structural dynamic response and module analyzation studies. Such as the effective independence method, Minimize modal assurance criterion (MinMAC) method, modal matrix summation and quadrature method, and other classical methods\(^8\) are widely used in structural monitoring. However, a little research on the optimal configuration of measuring points based on static deformation. Compared with the optimal configuration of dynamic measuring points, it can get accurate displacement with lower cost and plays an irreplaceable role in structural deformation monitoring.\(^9\)

Based on the static test, the measuring points are optimized by the responses, such as displacement, stress, strain, and other parameters. Theoretically, the more measuring points, the more comprehensive the structure information it can obtain, which is able to distinguish the structure change of the raft frame. However, fewer measuring points are applied in specific tests to obtain more information possibly.\(^10\) Zhang et al.\(^11\) optimized the displacement sensor and strain sensor integrated sensor system. By reducing the initial candidate sensor positions to minimize the structural response prediction error, and take a two-dimensional cantilever beam as an example to verify the rationality of the optimization. However, no experimental verification has been carried out, only theoretical analysis. Kim et al.\(^12\) in order to re-build plate-beam deformation, which is necessary to adopt the global or piecewise continuous basis function method to fit the measured surface strain into a strain field, and establish the strain-displacement algorithm to relate the strain data with the structural displacement. But it is necessary to obtain accurate strain mode and displacement mode, which puts forward high requirements for finite element analysis. Zhao et al.\(^13\) describes a finite element method for three-dimensional inverse deformation of the frame reconstruction. A sensor optimization layout model is proposed, which uses the particle swarm optimization algorithm to obtain the optimal sensor layout. It is verified by cantilever beam and frame structure, and the results show that it can be widely used in deformation reconstruction. Nevertheless, it is found that certain limits exist for the iFEM application. Li et al.\(^14\) proposed a two-dimensional structure measuring point

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**Figure 1.** (a) Schematic diagram of the floating raft vibration isolation device, (b) schematic diagram of the force of the raft frame.

Milligram is the gravity of the raft and the equipment on it, \(F(t)\) the resultant force of the gravity on the raft and the external disturbance, etc., is a changing force. The vertical displacement has the greatest influence on it. This article only discusses the vertical displacement of the raft. Therefore, it can be considered that the force \(F_{rad}(t)\) along the Z axis is the supporting force of each vibration isolator, and the number of vibration isolators is determined according to the actual structure.
placement method which is based on the principle of combination. The numerical optimization results show that the method has high accuracy and can be widely used in the test and actual measurement of the structure system under two-dimensional conditions. But its optimized combination method is complicated and inefficient. Hao Liu proposed the optimal configuration of measuring points for three-dimensional curved structures based on static test. And optimized the combination of measuring points through the principle of genetic algorithm, which has high precision, and solve the problem of optimal configuration of structural measuring points under three-dimensional conditions. Although many types of optimization problems and their applications have been proposed, few researchers have dealt with the deformation sensing of the raft. At present, more studies on structures with simple constrained boundaries such as bridges, cantilever beams, frame structure, and slabs at home and abroad, but fewer paper about complex constrained boundary conditions, such as plate-like structures with multi-flexible support on both sides.

To the best of our knowledge, there is still no an effective measurement technique to achieve online monitoring under different changing loadings of raft. To fill this study gap, this paper takes the floating raft as a topic, and according to the engineering background of the change of the force and load of the raft, the placement of the displacement sensor measuring points is optimized to meet the measurement requirements under different loaded conditions. According to the method of measured data and interpolation, to rebuild the raft displacement surface and achieve the purpose of accurately monitoring the displacement of the raft structure.

The remainder of this paper is organized as follows. Section 2 established the prototype of the raft, and has been its force and deformation characteristics. Optimal sensor placement strategy, the corresponding reconstruction algorithm and evaluation index function for the validity of the proposed approach are presented in Section 3. The FEM-based simulations for the validity of the proposed approach are presented in Section 4. The verification experiments in the raft experimental prototype are implemented in Section 5. Finally, the conclusion of the whole paper is presented in Sections 6.

**Structural modeling and analysis**

The raft structure shell is mainly composed of structural steel, and the bottom is braced by the elastic element of the vibration isolator. The main size of the raft frame of the floating raft vibration isolation is $5.54 \times 3.48 \times 0.191$ m, the plate thickness is 30 mm, and the rib thickness is 20 mm. After extracting the midplane, the shell element is used for modeling. The vibration isolator is represented by spring-damper elements with three-dimensional stiffness, the material is designed to be steel, and the density is 7850 kg/m$^3$.

Since the raft carries a larger fuel container, and the fuel consumption changes considerably as time goes by, this process can be regarded as a quasi-static process. It seemed like a variable load condition, and the total weight of the device and the raft is 160 T. In ANSYS Workbench, the system load is simplified as force, the direction is $-Z$, and the vibration isolators are all uniformly loaded. During the load change, the center of gravity of the raft frame does not change in the $X$ and $Y$ directions to ensure the vibration isolator to maintain a uniform load. Respectively, under full load, half load, and no load, three typical conditions of the load analysis of the raft. Loading force applied to the position shown in Figure 2, the forces are $1.469 \times 10^6$ N, $1.076 \times 10^6$ N, $0.684 \times 10^6$ N, when each isolator is under conditions 4, 6, 8 T load.

The force and deformation characteristics of the raft under different system loads can be obtained. The finite element simulation analysis results are shown in Figure 3. From Figure 3(a), it can be described that under the full load condition, the deformation characteristics of the raft frame are mainly the bulge in the middle part, which is recessed to both sides, respectively. Under no-load conditions, three typical conditions of the load analysis of the raft. Loading force applied to the position shown in Figure 2, the forces are $1.469 \times 10^6$ N, $1.076 \times 10^6$ N, $0.684 \times 10^6$ N, when each isolator is under conditions 4, 6, 8 T load.

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According to 0.1mm precision drawn contour diagram shown in Figure 3(b) to (d), the accuracy is set according to the actual accuracy requirements. Because the center of gravity of the raft on the XOY plane remains unchanged under different load conditions, the trend of the displacement contour map is consistent, and the difference is the degree of deformation. This result can better direct the combination and placement strategy of measuring points.

Surface reconstruction method based on optimal layout of measuring points

Optimization method of measuring point combination based on gradient theory

Appropriate measuring points were determined as the premise to achieve online monitoring of the displacement of the raft structure. The deformed raft surface can be regarded as curved. The raft is not a complete plate structure, as shown in Figure 2. Based on the actual slab rib structure of the raft, the orthogonal grid of the raft is constructed and can be seen in Figure 4. Each node represents the intersection of the ribs of each component of the raft. The nodes are named from the stern to the bow and are sorted from top to bottom. No nodes are set for incomplete areas of rafts.

The research of displacement sensor layout, which is necessary to consider not only the area with large structural displacement, but also the area with large displacement changes of the raft frame. From the finite element analysis in Chapter 2, it can be seen that the displacement contour diagrams under each load condition are similar. Choose one of the working conditions arbitrarily, and take the displacement contour diagram for research. Because the XOZ surface of the raft is longer, it can better reflect the weak rigidity effect of the large raft. Here we choose the X and Z directions for analysis, draw the raft frame contour map, and propose “displacement gradient” as the principle of...
selecting the combination of measuring points. The principle is as follows:

Suppose the displacement of point A at any point is $f(x, z) = f_x(x, z)i + f_z(x, z)j$. $f_x(x, z)$ and $f_z(x, z)$ are the displacement components in the $x$ and $z$ directions, respectively. Its “displacement gradient” at point A is:

$$\text{grad} f(x, z) = \frac{\partial f_x}{\partial x}i + \frac{\partial f_x}{\partial z}j + \frac{\partial f_z}{\partial x}i + \frac{\partial f_z}{\partial z}j$$

(1)

Where the contours are dense, the gradient change is large; where the contours are sparse, the gradient change is small. Based on this, to improve the accuracy and sensitivity of raft displacement monitoring, the displacement surface can be divided into the following regions according to the gradient theory: (1) the area where the displacement value changes greatly, (2) the area where the displacement gradient changes greatly, (3) the insensitive area, displacement measuring point is placed near the contour in different regions, can obtain the maximum degree of displacement information of the test structure. Take the selection of measuring point combinations in the displacement contour map under no load conditions as an example, as shown in Figure 5.

Above all, the placement principles should follow, trying to use three measuring points for static testing in one area. Then place the measuring points at both ends of the displacement contour and the middle part, and select the nodes near the maximum displacement gradient of the contour line for the middle part; if the area is relatively small or only two measuring points can be placed, the measuring points are placed at both ends of the displacement contour; At the same time, the measuring points between each area are distributed as equally as possible.

As shown in Figure 6, the structural displacement of the measuring point in this paper refers to the vertical from each node to the data plane of the initial position of the raft. Assuming that the initial data plane of the raft is parallel to the raft base, the initial height is $D$, and the current height of the measuring point B on the curve is $Z_m$, then the displacement $Z_i$ of point B can be obtained as:

$$Z_i = D - Z_m$$

(2)

**Surface spline interpolation function based on Green's function**

In this paper, the nodes are selected based on the actual structure of the raft frame, and the interval of nodes is irregular, so the biharmonic spline interpolation function is used to reconstruct the surface. Interpolation surfaces are linear combinations of Green functions centered at each data point. The amplitudes of the Green functions are found by solving a linear system of equations. The surface $s(x)$ is expressed as:

$$s(x) = T(x) + \sum_{j=1}^{n} \omega_j g(x, x_j)$$

(3)

Where $n$ is the number of data points $x_j = (x_j, y_j)$, $g(x, x_j)$ is Green’s function and $\omega_j$ is the weight of data point $j$. The weights $\omega_j$ are determined by requiring that the surface $s(x)$ passes exactly through the $n$ data points, that is,
The interpolated value of any point \( n_o \) is:
\[
Z_n = G_n W
\]  

(Evaluation index function)

This paper proposes a fitness function as an evaluation index for the layout of measuring points. Selecting the method of minimum average error of the sum of the differences to establish the fitness function and analyzing the pros and cons of different measuring point layout methods. The response value of the unplaced measuring points is obtained through spline interpolation, and the ability of sensors in each optimized layout to sense structural deformation is evaluated. The smaller the minimum average error of the sum of the differences, the more similar the integration calculation node displacement and the finite element calculation displacement, and the stronger the perception ability. The larger the value, the lower the degree of similarity between the interpolation calculation node displacement and the finite element calculation displacement, and the weaker the perception ability. The mathematical expression is:
\[
f_p = \frac{1}{n} \sum_{i=1}^{n} |y_i - y_{ij}| < \varepsilon
\]  

Where \( f_p \) in the formula represents the node displacement value of combination \( p \) obtained by interpolation calculation; \( y_i \) is the \( i \)-th node actual displacement value. \( n \) is the total number of nodes. The value of the fitness function reflects the degree of similarity between the interpolation calculated nodal displacement and the actual displacement. The closer the \( f_p \) is to 0, the more similar the displacement value calculated by the interpolation is to the actual displacement value. \( \varepsilon \) is selected according to the actual accuracy requirements.

Simulation experiment and analysis based on finite element

Analysis of the force and deformation of the raft

As shown in Table 1, the raft deformation surface is divided into 88 parts according to the node placement method, summing up 98 nodes. Use ANSYS finite element software for static analysis. Due to space, only the full load conditions’ displacement value of each node is listed. The displacement contour diagram and the displacement deformation surface are shown in Figure 7.

Optimized configuration of raft surface displacement measurement points

Applying the measuring point combination method based on the gradient theory to configure the measuring
Table 1. Raft FEM vertical displacement of the measuring point value.

| Coordinates | y/x | 2 | 2.246 | 2.62 |
|-------------|-----|---|-------|------|
| 1.351       | 0.816 | 1.32 | 1.75 | 2.17 |
| 1.871       | 2.245 | 2.717 | 2.798 | 3.040 |
| 0.875       | 1.75  | 2.403 | 2.675 | 3.030 |
| 0.435       | 1.32  | 2.498 | 2.626 | 2.900 |
| 0.0435      | 0.875 | 2.382 | 2.498 | 2.900 |
| -0.435      | 0     | 2.293 | 2.382 | 2.900 |
| -1.351      | -0.816| 2.197 | 2.293 | 2.900 |
| -1.871      | -2.245| 2.187 | 2.293 | 2.900 |
| -2.245      | -2.62 | 2.113 | 2.293 | 2.900 |
| -2.62       | -2.177| 2.09 | 2.293 | 2.900 |
| -2.177      | -1.75 | 2.038 | 2.293 | 2.900 |
| -1.75       | -1.32 | 2.006 | 2.293 | 2.900 |
| -1.32       | -0.875| 1.983 | 2.293 | 2.900 |
| -0.875      | -0.435| 1.960 | 2.293 | 2.900 |
| -0.435      | -0   | 1.937 | 2.293 | 2.900 |
| 0           | 0.435 | 1.914 | 2.293 | 2.900 |
| 0.435       | 0.875 | 1.891 | 2.293 | 2.900 |
| 0.875       | 1.32  | 1.868 | 2.293 | 2.900 |
| 1.32        | 1.75  | 1.846 | 2.293 | 2.900 |
| 1.75        | 2.245 | 1.824 | 2.293 | 2.900 |
| 2.245       | 2.62  | 1.802 | 2.293 | 2.900 |
| 2.62        | 2.177 | 1.780 | 2.293 | 2.900 |
| 2.177       | 1.351 | 1.758 | 2.293 | 2.900 |
| 1.351       | 0.816 | 1.725 | 2.293 | 2.900 |
| 0.816       | 0.435 | 1.692 | 2.293 | 2.900 |
| 0.435       | 0     | 1.660 | 2.293 | 2.900 |
| 0           | 0.435 | 1.627 | 2.293 | 2.900 |
| 0.435       | 0.875 | 1.594 | 2.293 | 2.900 |
| 0.875       | 1.32  | 1.562 | 2.293 | 2.900 |
| 1.32        | 1.75  | 1.530 | 2.293 | 2.900 |
| 1.75        | 2.245 | 1.497 | 2.293 | 2.900 |
| 2.245       | 2.62  | 1.465 | 2.293 | 2.900 |
| 2.62        | 2.177 | 1.433 | 2.293 | 2.900 |
| 2.177       | 1.351 | 1.401 | 2.293 | 2.900 |
| 1.351       | 0.816 | 1.369 | 2.293 | 2.900 |
| 0.816       | 0.435 | 1.336 | 2.293 | 2.900 |
| 0.435       | 0     | 1.304 | 2.293 | 2.900 |
| 0           | 0.435 | 1.272 | 2.293 | 2.900 |
| 0.435       | 0.875 | 1.239 | 2.293 | 2.900 |
| 0.875       | 1.32  | 1.207 | 2.293 | 2.900 |
| 1.32        | 1.75  | 1.175 | 2.293 | 2.900 |
| 1.75        | 2.245 | 1.143 | 2.293 | 2.900 |
| 2.245       | 2.62  | 1.111 | 2.293 | 2.900 |
| 2.62        | 2.177 | 1.079 | 2.293 | 2.900 |
| 2.177       | 1.351 | 1.047 | 2.293 | 2.900 |
| 1.351       | 0.816 | 1.015 | 2.293 | 2.900 |
| 0.816       | 0.435 | 0.983 | 2.293 | 2.900 |
| 0.435       | 0     | 0.951 | 2.293 | 2.900 |
| 0           | 0.435 | 0.919 | 2.293 | 2.900 |

The unit of node coordinate in the table is meter, and the unit of displacement value is millimeter.
structural displacement changes greatly, the node displacement curve fits the reference curve more closely.

At the same time, the error between the interpolated calculated value of the unconfigured node and the true value obtained by the combined configuration scheme of each measuring point under the three working conditions can be calculated, as shown in Table 2. As the number of measuring points increases, the maximum error, and average error of each combination plan will decrease. For the same conditions, compared to a combination regimen 1, combination regimen 4 maximum error is reduced by more than 61.8%, the average error is reduced by more than 83%, it shows that the number of measuring points has a greater influence on the interpolation calculation error, and the error value of the third and fourth combinations are small, indicating that after reaching a certain number of measuring points, the spline interpolation function can better estimate the response value of the unconfigured node.

Under the same combination scheme, as the load weight decreases, both the maximum error and the average error decrease. Combination regimen 1 compared to a combination regimen 4 the maximum error is reduced by more than 27.8%, the average error is reduced by more than 21.3%. Compared with the number of measuring points, the load change has less influence on its error. This result further verifies the feasibility of the scheme in this paper to accurately measure the displacement of the raft under complex loads and unknown physical parameters.

**Comparative experimental verification and result analysis**

The experimental device is mainly composed of air spring vibration isolation unit, raft posture, and deformation monitoring device, hydraulic loading system, intelligent control system and other equipment as shown in Figure 10. Raft carried by the 20 air springs, each air spring ranges 0–8 T, hydraulic loading point shown in Figure 2, can simulate the deformed state structure raft actual operation. Selects the second combination scheme for verification, and disposes displacement sensors at nodes 1, 3, 8, 10, 29, 45, 54, 69, 72, 89, 91, 96, 98 as configuration measuring points.

The hydraulic loading device applies three different load conditions. By adjusting the air pressure of the air
spring, the load of each vibration isolator under each operating condition is 4, 6, and 8 T, which simulates three different operating conditions. Using the method in this paper, select other nodes except the measuring points as comparison points to verify whether the optimized layout method in this paper can meet the needs of actual experiments. As shown in Figure 11, select the four points of nodes 2, 7, 92, and 97, and use the dial indicator to collect the true displacement.

The Spline function interpolation method is used for interpolation fitting, and the displacement values of the 4 comparison nodes obtained from the experiment are compared with the analysis values of the comparison nodes obtained by interpolation of 13 configuration

Table 2. Error analysis table under different working conditions.

| Conditions | Scheme 1 | Scheme 2 | Scheme 3 | Scheme 4 |
|------------|----------|----------|----------|----------|
| 1          | 0.4168   | 0.2248   | 0.2785   | 0.0973   | 0.2469   | 0.0552   | 0.1534   | 0.0342   |
| 2          | 0.3573   | 0.1919   | 0.2384   | 0.0835   | 0.2106   | 0.0471   | 0.1321   | 0.0323   |
| 3          | 0.2905   | 0.1590   | 0.1985   | 0.0700   | 0.1742   | 0.0395   | 0.1108   | 0.0269   |

The error data unit in the table is mm. The right data is the max error and the left data is mean error in each scheme.

![Figure 9](image-url)
measuring points. Introduce the absolute error and relative error for evaluation to verify the effectiveness and performance of the method in this paper, as shown in Tables 3 to 5.

The error is mainly affected by the number of sensors and the size of the interpolation step. Tables 3–6 show that with the increase of the applied load, the average absolute error and average relative error of the deformation value obtained from the test increase compared with the analysis value. However, the relative error of each point is kept within 5%, and the average relative error is less than 4%.

This method is applied in actual raft structural displacement monitoring, which is simple and quick, and clearly, is not affected by raft material and force. Each combination plan is placed according to the displacement contour map, and it is not necessary to perform a large number of combination calculations on the measuring points. Practically, it can be selected according to the accuracy requirements combined with each plan. The actual number of measuring points may be less than the designed number.

Figure 10. (a) Schematic diagram of experimental verification system and (b) Experimental device of floating raft air spring vibration isolation device.

Figure 11. Schematic diagram of configuration node and comparison node.
than the optimal number of measuring points, which is more versatile and meets normal needs. It is suitable for large-scale floating raft structure displacement monitoring whose accuracy requirements are not particularly high, and can also be further extended to other structural deformation monitoring fields. In view of aiming at the space constraints of the actual ship's installation location and the accuracy of the measuring device, the next step is to consider the use of FBG components to monitor the deformation of the raft structure, to be more suitable for the complex, and harsh environments on the ship.

**Conclusion**

This paper proposes an online measure method for the displacement of a variable-load raft structure based on the optimized placement of displacement sensors. First, based on the finite element analysis, the combined placement method of its measuring points is investigated according to the gradient theory. Obtaining the structural displacement value of the node through the displacement sensor, and utilizing the interpolation function to fit the surface. Finally, the evaluation function is calculated and the error of different combination configuration methods. According to the actual demand analysis, the optimal layout of measuring points is selected. To verify the rationality of the method described in this article in practical application through experiments. The results show that the measurement system in this paper has simple and strong adaptability, can sense the displacement of the raft structure very well, and can be further extended to the field of large-scale floating raft deformation monitoring. It is better to assist the design and verification of the attitude control algorithm of the floating raft.

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**Table 3.** Error analysis table for full load conditions.

| Node label | Measured value/mm | Calculated value/mm | Absolute error Δ/mm | Relative error δ/% |
|------------|-------------------|---------------------|---------------------|--------------------|
| 2          | 2.749             | 2.681               | 0.068               | 2.47               |
| 7          | 1.551             | 1.604               | 0.053               | 3.42               |
| 92         | 3.560             | 3.387               | 0.173               | 4.86               |
| 97         | 2.160             | 2.101               | 0.059               | 2.73               |

**Table 4.** Error analysis table of half load condition.

| Node label | Measured value/mm | Calculated value/mm | Absolute error Δ/mm | Relative error δ/% |
|------------|-------------------|---------------------|---------------------|--------------------|
| 2          | 1.351             | 1.312               | 0.039               | 2.96               |
| 7          | 1.520             | 1.476               | 0.044               | 3.29               |
| 92         | 1.712             | 1.783               | 0.071               | 4.09               |
| 97         | 1.690             | 1.650               | 0.04                | 2.36               |

**Table 5.** Error analysis table under no-load conditions.

| Node label | Measured value/mm | Calculated value/mm | Absolute error Δ/mm | Relative error δ/% |
|------------|-------------------|---------------------|---------------------|--------------------|
| 2          | 0.605             | 0.594               | 0.011               | 1.82               |
| 7          | 0.990             | 0.998               | 0.008               | 0.81               |
| 92         | 0.872             | 0.850               | 0.022               | 2.52               |
| 97         | 0.909             | 0.889               | 0.020               | 2.20               |

**Table 6.** Average error analysis table under each working condition.

| Condition  | Average absolute error | The mean relative error δ/% |
|------------|------------------------|-----------------------------|
| Full load  | 0.0733                 | 3.440                       |
| Half load  | 0.0485                 | 3.175                       |
| No load    | 0.0153                 | 1.838                       |
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Appendix

Notation

| Symbol | Description |
|--------|-------------|
| FEM    | Finite Element Method |
| iFEM   | Inverse Finite Element Method |
| gradf(x, z) | XOZ plane displacement gradient |
| D      | The initial height of the raft |
| Zi     | The current height of a measuring point |
| zm     | The displacement of point |
| s(x)   | Fitting Surfaces |
| n      | The number of data points |
| Zn     | The interpolated value of any point |
| f(x)   | A fitness function |
| f p(i) | The i-th node displacement value of combination obtained by interpolation calculation |
| y i    | The i-th node actual displacement value |
| e      | The actual accuracy requirements |
| Δ      | Absolute error |
| δ      | Relative error |
| | Average absolute error |
| | The mean relative error |