Proving the Pythagorean Theorem to Junior High School students through the Asian method

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Abstract. The development of right triangle concept has been known since ancient Egypt and is still be used in daily life until now. One of the most popular theorems that relate to the right triangle is Pythagorean Theorem. In its development, the Pythagorean Theorem has been proven by mathematicians with different methods. Based on historical developments there are approximately 200 proofs of the Pythagorean Theorem that has been found. Nevertheless, history still has interesting mysteries to study especially for the development of the Pythagorean Theorem in Asia. In student textbooks, the proof of Pythagorean Theorem introduction often comes from the European continent such as proof by Pythagoras, Euclid, and Leonardo Da Vinci. This caused Pythagorean Theorem’s development in Asia as if it were excluded, the proof of Pythagorean Theorem in Asia also developed by mathematicians such as Bhaskara and Zhao Shang with different names. Even so, the discussion of those Pythagorean Theorem proof can be introduced to students as an initial concept in Pythagorean Theorem chapter and presented pleasantly in the form of mathematics historical comic to make it more attractive. The discussion result in this journal is also in line with the 2013 curriculum which emphasizes student activity and fun learning model.

1. Introduction

Since the enactment of government regulation number 24 of 2016 by the Ministry of Education and Culture of Republic Indonesia about curriculum, the curriculum that applies in Indonesia is the Curriculum 2013 [1]. In this curriculum, students are required to be more active in learning and find information through discovery-based learning. To facilitate this, the student textbook has been given various activities that invite students to actively think and find their own concepts. These activities are often inspired by historical developments in order that the concepts taught will be more meaningful.

Correspondingly, Ernst Haeckel [2] suggests the theory that individual development (ontogeny) is a reflection of the history of its evolution (phylogeny). Then if it is associated with mathematics and the history of mathematics, the development of mathematics in a person is in line with the development of the history of mathematics itself so that it will be easier for students to understand mathematical concepts if they are related to the history. One of the mathematical concepts at the junior high school level that is close to the history of mathematics is the concept of the Pythagorean Theorem. The history of the Pythagorean Theorem begins with the development of the history of right triangles which have been known by ancient Egyptian civilization a long time ago.
In its development, the Pythagorean Theorem has been proven by mathematicians with different methods. Based on historical developments there are approximately 200 proofs of the Pythagorean Theorem that has been found. However, the history of the Pythagorean Theorem still has an interesting mystery to study especially the development of the Pythagorean Theorem in Asia. In textbooks, proof of the introduction of the Pythagorean Theorem often comes from the European continent, for example, proof by Pythagoras, Euclid, and Leonardo Da Vinci. This resulted in the development of the Pythagorean Theorem in Asia as if it were excluded, the proof of Pythagorean Theorem in Asia also developed by mathematicians such as Bhaskara and Zhao Shang with different names. Although with different terms, both of the Pythagorean Theorems in Europe and in Asia both examine the same thing, the hypotenuse Theorem in the right triangle. Therefore, the history of mathematics regarding the Pythagorean Theorem in Asia can be used in introducing and proving the Pythagorean Theorem to students at the Junior High school level as one of the activities in mathematics learning.

2. Method
This study uses a literature review method of various historical resources of the Pythagorean Theorem, especially in Asia. Before searching for information about the history of the Pythagorean Theorem, the researchers conducted a study of student textbook on the Pythagorean Theorem taught at the Junior High School level in the Curriculum 2013. Then the researchers focused on looking for information on the history of Pythagorean Theorem in Asia proposed by several mathematicians such as Bhaskara and Zhao Shang. The study was conducted through various books, journals, proceedings, and the internet to find out the historical development of the Pythagorean Theorem. Then the information obtained is re-explained so that they can be understood and used to be one of the choices in introducing and proving the Pythagorean Theorem to grade 8 of Junior High School students.

3. Result
Through the study of student books, it was found that the student book only introduced proof of the Pythagorean Theorem from Europe and America only (Figure 1).

Figure 1. Proof of the Pythagorean Theorem snippet in the student textbook.
In Figure 1, it can be observed that the proof of the Pythagorean Theorem presented in the student book has not shown any of the names of mathematicians from Asia or their proof of the Pythagorean Theorem. This resulted in the assumption that Pythagorean Theorem did not develop in Asia because it did not introduce the proof of Pythagorean Theorem from Asia. However, in fact, the theorem regarding the hypotenuse of a right triangle develops in the Asian Continent even though in different terms. To get to know the brief journey of Pythagorean Theorem in Asia, this study will explain the development of the Pythagorean Theorem in Asia through proof which made by Asian mathematicians.

At first, the Pythagorean Theorem was known as the Theorem regarding the length of the hypotenuse of a right triangle, usually known as Pythagorean Theorem. However, it is not certain who the first originator of the Theorem was, whether Pythagoras discovered the Theorem himself or not. Nevertheless, Pythagoras was the person who first put forward and proved the Theorem formally in ancient times [3]. Based on historical records, the form of the Pythagorean Theorem appeared in ancient Egyptian civilization (2000 BC) which can be found in the use of worker tools. At that time, the workers used a rope 12 tied nodes which formed into a 3-4-5 right triangle. Some sources claim that knowledge of right triangles began to enter Asia through the silk trade route which was later developed and proven by mathematicians in Asia.

In Asia, the knowledge of right triangles spread and developed among mathematicians and astronomers, so that the development of the Pythagorean Theorem developed which can be observed through the following simple timeline [4][5][6][7].

![Figure 2. The Timeline of Pythagorean Theorem](image-url)

The timeline began in 1100 BC after the concept of a right triangle entered Asia, Tschou Gun from China wrote down the characteristics of right triangle. In the year 100 AD in the Zhou dynasty there was a discussion about how to measure heaven and earth with gnomon. At that discussion, Shang Rong answered questions from Duke Zhou by explaining the use of gnomon in estimating the distance of the earth and sky using size 3-4-5 ratio. When explaining this, Rong also mentioned the assumption that the earth was flat, Gai Tian model. Gai Tian model has been trusted by mathematicians and astronomers in China since 83 AD.

Furthermore, the use of gnomon in astronomy was increasingly developed which later inspired Liu Hiu and Zhao Shang in 300 AD to prove the relationship between the sides of a right triangle (Gougu Theorem) which known as the current formula of the Pythagorean Theorem. Besides in China, in other
Asian countries, mathematicians began to prove the theorem in their own way. In Turkey, a mathematician named Thabit Ibn Kurrah (836 - 901 AD) proved the theorem with a method named Bride's Chair. Then in India, Bhaskara (1115 - 1185 AD) also tried to prove the Theorem using diagrams. The diagram used by Bhaskara is similar to the diagram used by Zhao Shang, but with a different point of view.

The development of the history of the Pythagorean Theorem in Asia can be used in learning as an activity of students in proving the Pythagorean Theorem. In order to be used and easier to understand, here are some proving steps that can be used in learning that are in accordance with the methods used by the mathematicians.

3.1. Proof by Zhao Shang
Zhao Shang's proof of the Gougu Theorem was written in a notebook entitled Zhou Bi Suan Jiang (The Arithmetic Classic of the Gnomon and the Circular Paths of Heaven) or Zhou Bi in the Zhou dynasty in 300 BC [8]. Shang explained his proof through a diagram, following the diagram used by Shang.

Figure 3. Diagram of Zhao Shang proof.

From the picture above, it can be arranged steps to prove the Pythagorean Theorem as follows:

i. Make four right triangles of the same size, where the length of the triangular legs is made different. The goal is to be easily differentiated.

ii. Arrange the four triangles so that they form a new plane with the side length represents the hypotenuse of the triangle. From this step, the square shape will be formed from the hypotenuse of the right triangle.

iii. Turn the four triangles towards the square that is formed so that the four triangles appear to cover the square as shown in the following figure.

Figure 4. Representation for step iii.

iv. From Figure 4 above, it can be seen that in the middle the square is formed a new square which is represented by a red square. Make a square which resembles the red square to fill the empty space on the red square so that it is obtained that

\[
\text{Area of } c - \text{square} = 4 \times \text{Area of triangles} + \text{Area of small square}
\]
v. Arrange two new square shapes from the four right triangles and the previous red square as Figure 5.

![Figure 5. The square result formed in step v.](image)

vi. In Figure 5 above it can be can be concluded that

\[
c - \text{side square area} = a - \text{side square area} + b - \text{side square area}
\]

\[
c^2 = a^2 + b^2
\]

\[
c = \sqrt{a^2 + b^2}
\]

The Pythagorean Theorem that “In any right triangle, the sum of the areas of the squares on the legs equals the area of the square on the hypotenuse” [9] which can be represented algebraically as \(c^2 = a^2 + b^2\).

3.2. Bride’s Chair concept of Thabit Ibn Kurrah

The steps of the proof are:

i. Bride Chair consists of two square pieces that have different sizes. Thus, first make two squares that have different sizes, for example, sizes a and b.

ii. Then, cut the bottom edge of the two square into two right triangles with the two triangular legs representing the size of the two squares (Figure 6).

![Figure 6. Representation of steps i and ii.](image)

iii. Turn the two triangles to the top so they form a new large square with the sides being the hypotenuse of a right triangle (Figure 7).

![Figure 7. Representation of step iii.](image)

iv. If the hypotenuse of a right triangle is c, it is obtained that \(c^2 = a^2 + b^2\).
3.3. **Proof by Bhaskara**

In his proof, Bhaskara used a diagram which is similar to Zhao Shang diagram. The difference is in the algebraic step used, where Zhao Shang only concludes his findings that the area of a large square with sides $c$ equals the number of two squares with sides $a$ and $b$ so that $c^2 = a^2 + b^2$ proved. Whereas in Bhaskara, he uses the quadratic equation where side $c$ equals the difference between sides $a$ and $b$, then it gets:

$$A_{big\ square} = A_{little\ square} + 4.A_{triangle}$$

$$c^2 = (a - b)^2 + 4\left(\frac{1}{2}ab\right)$$

$$c^2 = a^2 - 2ab + b^2 + 2ab$$

$$c^2 = a^2 + b^2$$

Those methods above can be inserted into learning through worksheet so students can practice and prove the Pythagorean Theorem as the mathematicians did.

4. **Discussion**

Based on the development of the history of mathematics regarding the Pythagorean Theorem in the results above, it can be seen that the development of the Pythagorean Theorem also developed in Asia even though with a different name. These developments can be introduced to students so that they will not only understand the concept of Pythagorean Theorem but also know that in Asia the theorem also develops. In order to make the presentation more attractive to students, the development of Pythagorean Theorem’s proof can be presented in educational comic form.

The use of comic as an information delivering tool to students has recently been highlighted. It is due to the interest in reading for comics is quite high among the public, especially students. Based on the survey, the reading appeal of comic types in Indonesia gets the highest ranking in Asia according to the Line Webtoon version in 2018 with 17 million active readers per month [10]. The use of comics as a media for delivering information has been used for a long time. Thorndike stated that comics have advantages in terms of improving vocabulary and children's reading skills which are comparable to reading textbooks every year [11]. In addition, through images in the comic, students can develop their imagination in imagining the information provided through the fun method. Therefore, it stimulates an idea to combine the history of mathematics and comic into one to convey mathematical concepts to students. By combining comic with the development of the history of mathematics, it is expected that students can better understand the concepts given through the course of the history of mathematics inserted in the story.

5. **Conclusion**

Based on the results and discussions that have been discussed before, it can be concluded that:

- The development of the Pythagorean Theorem in Asia also developed and spread even with different names. The development of the concept in Asia can be proven by the appearance of mathematicians who tried to prove the theorem such as Bhaskara, Zhao Shang, Thabit Ibn Kurrah, and Liu Hui.
- The history of mathematics can be inserted into learning activities through worksheet so students are able to find their own concepts to be built. Insertion of the history of mathematics can add information which is obtained by students so that the concepts can be more meaningful.
- In order to make the presentation of the history of mathematics into learning more enjoyable, the media for educational comics can become an option. By combining comic with the development of the history of mathematics, it is expected that students can better understand the concepts given through the course of the history of mathematics inserted in the story.
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