On the heat current in the magnetic field: Nernst-Ettingshausen effect above the superconducting transition

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The observation of large thermomagnetic effects in cuprates turns out to be crucial for understanding high-
Tc superconductivity [12, 15]. In the Nernst effect, the electric current is induced by crossed temperature gradient and magnetic field, \( j = N \nabla T \times H \). In an open circuit, the Nernst voltage \( E_y \) is given by

\[
\frac{E_y}{(\nabla T)_x H_z} = \frac{N_{xy}}{\sigma_{xx}} - \frac{\eta_{xx} \sigma_{xy}}{\sigma_{xx}^2},
\]

where \( \eta_{xx} \) is the thermoelectric coefficient, \( \sigma_{xx} \) and \( \sigma_{xy} \) are the electrical and Hall conductivities.

In metals, the Nernst voltage is small, because it requires strong particle-hole asymmetry (PHA) in the electron systems. PHA of the thermoelectric coefficient \( \eta_{xx} \) is given by the factor \( T / \epsilon_F \), where \( \epsilon_F \) is the Fermi energy; PHA of the Hall conductivity \( \sigma_{xy} \) may be presented as \( \Omega_H T \), where \( \Omega_H = eH/mc \) and \( \tau \) is the electron momentum relaxation time. Coefficient \( N_{xy} \) contains both PHA factors of \( \eta_{xx} \) and \( \sigma_{xy} \). Thus, in Eq. (1) both terms are proportional to the square of PHA. Moreover, if \( \tau \) is independent on energy, they cancel each other.

The important result of Ref. [1] is that the large measured Nernst voltage originates from \( N_{xy} \), i.e. the first term in Eq. (1) strongly dominates over the second term in the temperature range extending from below \( T_c \) to a broad interval above. It now seems clear that the region above \( T_c \) is in fact the vortex fluid phase. Various phenomenological models are suggested to describe this phase and heat transfer by vortices [2, 3, 4, 5, 6]. It is also believed (see a review [7]), that, at least close to \( T_c \), the Nernst voltage is described by the microscopic Gaussian-fluctuation theory, which predicts the finite \( N_{xy} \) in the zero order in PHA [8, 9, 10, 11]. Below we will show that this statement is wrong. In the Gaussian model with ordinary particle-hole excitations, it is not possible to change the PHA of \( N_{xy} \). To obtain the large Nernst effect due to vortex-like excitations, it is necessary to go beyond fluctuations of the modulus of the order parameter.

The Gaussian-fluctuation theory allows one to calculate the Ettingshausen coefficient \( \Upsilon \), which describes the heat flux induced by crossed electric and magnetic fields, \( j^h = -\Upsilon [E \times H] \). According to the Onsager relation,

\[
N = \Upsilon / T
\]

Results of the previous theoretical works can be formulated by two statements. First, in the infinite sample the coefficient \( \Upsilon \) near \( T_c \) does not require PHA,

\[
\delta \Upsilon_{inf} = -\frac{e^2 T}{4\pi c} \times \left\{ \frac{2\alpha/\eta}{\sqrt{\alpha/\eta}} \propto \frac{(T - T_c)^{-1}}{(T - T_c)^{-1/2}} \right\} \text{ for 2D,}
\]

where \( \eta = (T - T_c)/T_c \) and \( \alpha = (T - T_c) |\xi(T)|^2, \xi(T) \) is the coherence length. For the first time, Eq. (3) has been obtained in Ref. 6 using the time-dependent Ginzburg-Landau (TDGL) theory. Later it has been reproduced in Ref. [10]. Finally, Eq. (3) has been confirmed by the diagrammatic calculations in the frame of the Aslamazov-Larkin (AL) approximation [11].

Second, according to Refs. [10, 11, 12] in the finite sample the Eq. (3) should be corrected due to surface currents related to thermal magnetization currents,

\[
\delta \Upsilon = \delta \Upsilon_{inf} - c \mu,
\]

where \( \mu = -\frac{e^2 T}{6\pi c^2} \times \left\{ \frac{2\alpha/\eta}{\sqrt{\alpha/\eta}} \right\} \text{ for 2D,}
\]

\[
\text{for 3D,}
\]

This correction would decrease the \( \Upsilon \) to one third of its value given by Eq. (3)
In the current paper we develop consistent microscopic description of the heat current in the magnetic field. Calculating $\delta Y$ by the Kubo method and $\delta N$ by the transport equation, we show that both statements expressed by Eqs. \[4\] and \[5\] are wrong. Our conclusion is that in the frame of Gaussian-fluctuation theory the coefficients $\delta Y = \delta N = 0$ in the the zero order in PHA.

We start our calculations with discussion of the energy and heat current operators. The energy current of interacting electrons may be obtained from the equation of motion for the electron field operators or from the energy-momentum tensor \[14\]. For electrons interacting in the Cooper channel, the energy and heat current operators have been derived in our work \[12\]. In the external fields $H = i[k_H \times A_H]$ and $E = -i k_E \phi$, the electron energy has a gauge-invariant form

$$
\tilde{E} = (p + eA_H/c)^2/2m + e\phi.
$$

The energy current operator is easily generalized to include the external fields,

$$
J^e = \sum_p \sum_c \sum_{\gamma} \frac{c}{2}(vA_H)_{ap} a^+_p a_p + \sum_p \sum_c \sum_{\gamma} v c \phi a^+_p a_p + \sum_p \sum_{\gamma} v p \mu a^+_p a_p - \lambda/2 \sum_{p, p', p''} (v + v') a^+_p p'' a^+_p a_p a_p \\
+ \sum_{p, p', R_i} (v + v') U_{imp}(R_i) \exp[i(p + p') R_i] a^+_p a_{p'}.
$$

where $a^+_p$ and $a_p$ are the electron creation and annihilation operators, $\xi_p = p^2/2m - \mu$ (is the chemical potential), $U_{imp}$ is the impurity potential. The first and the forth terms in the Eq. \[6\] describes the energy flux of non-interacting electrons without fields. The second and third terms are due to electron interaction with magnetic and electric fields. Two last terms are many-body corrections due to the electron-electron and electron-impurity interactions. According to Ref. \[13\], these two terms generate diagram blocks (AL-blocks) proportional to PHA, but contributions of all blocks with PHA cancel each other. Anyway, these many-body corrections can be neglected, if $\tilde{Y}$ is calculated in the zero order in PHA.

The thermal energy can be defined as the electron energy counted from the electro-chemical potential $e\phi + \mu$. Then the heat current may be presented in terms of the energy and electric currents as $J^h = J^e - (\mu/e + \phi)J_e$. Obviously, the third and the forth terms in Eq. \[7\] do not contribute to the heat current. Thus, calculating the heat current we should take into account only two heat current vertices $\gamma^h_1$ and $\gamma^h_2$ corresponding to the first (kinetic) and second (magnetic) terms in Eq. \[6\].

In the linear response method, the thermoelectric coefficient is given by the correlator of the heat and charge currents, \[12\]. Calculating response to $E \times H$, it is convenient to use the gauge conditions $k_H \cdot A_H = k_E \cdot A_H = 0$, in this case $E \times H = -iA_H(k_H \cdot E)$.

Two leading terms in the heat current operator generate two diagrams, which describe $Y$ in the Aslamazov-Larkin (AL) approximation. The left blocks $B^e_1$ and $B^e_2$ in Figs. 1.a and 1.b are blocks of electron Green functions connected with heat current vertices $\gamma^h_1$ (kinetic) and $\gamma^h_2$ (magnetic). General for both diagrams, the right block $B^e$ includes the electric current vertex $\gamma = ev \cdot e_E$, $e_E = E/E$. Block $B^H$ includes the magnetic vertex $\gamma^h = (e/c)v \cdot A_H$. The wavy lines correspond to the fluctuation propagator, which is given by \[13\] and \[14\]

$$
L^{R,A}(q, \omega) = (\lambda^{-1} - P^{R,A}(q, \omega))^{-1},
$$

where $\lambda$ is a constant of the interaction in the Cooper channel and $P(q, \omega)$ is the polarization operator,

$$
p^{R,A}(q, \omega) = \frac{\nu}{2} \left( \ln \frac{2 c \omega P}{\pi T} - a q^2 + \frac{4 \pi \omega}{\gamma q} \right).
$$

$\nu$ is the electron density of states, $\omega_P$ is the Debye frequency, and $C$ is the Euler constant. The last term in Eq. \[9\] is proportional to PHA and can be ignored in zero order in PHA calculations.

For an arbitrary electron momentum relaxation time $\tau$, the AL blocks $B^{e,h,H}$ of electron Green functions $G^{R,A}(\omega)$ with vertices $\gamma^e$, $\gamma^h$, and $\gamma^H$ are given by \[13\] and \[15\]

$$
B^{e,h,H}(q) = 2c \nabla_q P^{R,A}(q, 0) \cdot e_E = 2e \nu \alpha q \cdot e_E.
$$

The block $B^e$ with the electric current vertex, $\gamma^e = ev \cdot e_E$, may be presented as \[13\] and \[15\]

$$
B^e(q) = 2c \nabla_q P^{R,A}(q, 0) \cdot e_E = 2e \nu \alpha q \cdot e_E.
$$

The block $B^H$ with the vertex $\gamma^H = (e/c)v \cdot A_H$ is

$$
B^H(q) = 2(e/c) \nu \alpha q \cdot A_H.
$$

The block $B^h$ with the kinetic heat current vertex, $\gamma^h = \xi v \cdot e_{jh} (e_{jh} = \gamma^h_jh/\gamma^h_H || A)$, is given by \[12\] (see also \[10\])

$$
B^h(q, \omega) = \omega \nabla_q P^{R,A}(q, 0) \cdot e_{jh} = \omega \nu \alpha q \cdot e_{jh}.
$$

Next, we calculate the block $B^h_2$ with the magnetic heat current vertex $\gamma^h_2 = (v \cdot A_H)(v \cdot e_{jh})$. The integral over angles of the electron momentum involves only the vertex $\gamma^h_2$, because the heat current is in the direction of $A_H$. To obtain an imaginary part in Eq. \[10\] the integral

$$
\int d\xi (G^{\alpha A})^2 G^{R} = \frac{2\pi i}{(2\xi - \omega - q \cdot v - i/\gamma)^2},
$$

should be expanded in $\omega$ (in calculations of $B^e$ it is expanded in $q \cdot v$). Finally, we get

$$
B^h_2(q, \omega) = 2A_H \frac{\omega}{q} \nabla_q P^{R,A}(q, 0) = 2(e/c) \nu \alpha \omega A_H.
$$
The first AL diagram (Fig. 1.a) for the Ettingshausen coefficient is based on the blocks $B^A_i$ and $B^c$, the vector potential $A$ enters the fluctuation propagator. This diagram was calculated in Ref. [11] and its contribution to $\Upsilon_{inr}$ is given by Eq. [3]. The same result has been obtained in the TDGL formalism in Refs. [8,10].

The second AL diagram (Fig. 1.b) is based on the blocks $B^L_3$ and $B^c$. The analytical expression corresponding to this diagram is

$$\Upsilon_{inr}^{(2)}H = \Im \int \frac{dq}{(2\pi)^n} \frac{dw}{2\pi} \frac{B^{L}_3 B^{c}}{2\Omega}(L^C L^A + L^R L^C), \quad (18)$$

where $L^C = \coth(\omega/2T)(L^R - L^A)$, $L_\pm$ is used for $L(q \pm k/2, \omega \pm \Omega/2)$, and $n$ is the system dimensionality with respect to the coherence length $\xi(T)$. Expanding the integrant to the linear order in $\Omega$ and $k$ and calculating the integrals over $\omega$ and $q$, we find that the contribution of the second diagram, $\Upsilon_{inr}^{(2)}$ cancels completely the contribution of the first one (Eq. [3]). The analogous cancellation of two diagrams with the heat current vertices $\gamma_1^1$ and $\gamma_2^1$ takes place in the case of noninteracting electrons (see Appendix in Ref. [10]).

Thus, without PHA the Ettingshausen effect is absent, $\Upsilon = 0$. To get nonzero result, we should expand the fluctuation propagator (Eq. [3]) up to the second order in PHA (in the first order in PHA both diagrams give zero results), i.e. we expand the polarization operator (Eq. [3]) to the second order in $\gamma_\omega$. In the second order in PHA, the thermomagnetic coefficients are

$$\frac{\delta \Upsilon}{T} = \delta N = -\frac{5e^2}{4\pi c} \left( \frac{8T \gamma}{\pi} \right)^2 \left\{ \frac{2\alpha/\eta}{\sqrt{\alpha/\eta}} \right\} \quad \text{for 2D},$$

$$\frac{\delta \Upsilon}{T} = \delta N = -\frac{5e^2}{4\pi c} \left( \frac{8T \gamma}{\pi} \right)^2 \left\{ \frac{2\alpha/\eta}{\sqrt{\alpha/\eta}} \right\} \quad \text{for 3D}; \quad (19)$$

where $\gamma = \frac{1}{2T} \frac{\partial \ln \epsilon_F}{\partial \ln c} \ln \frac{2C_e \omega_D}{\sqrt{2Tc}}$. Thus, the Ettingshausen and Nernst coefficients in the fluctuation region are proportional to $(T/\epsilon_F)^2$. Taking into account that in this region the thermoelectric coefficient $\eta_{xx}$ and the Hall conductivity $\sigma_{xy}$ are proportional to $(T/\epsilon_F)^2$, we see that in Eq. [1] for the Nernst voltage both terms are of the same order, $(T/\epsilon_F)^2$.

The above results obtained for the infinite sample (or in the interior of the sample) do not show any contribution of the magnetization currents and, therefore, in the finite sample no additional corrections are required. Because the role of magnetization currents is widely misinterpreted in literature, below we briefly analyze the energy current induced by the magnetization current and show that this term is erroneously attributed to the heat flux (Eq. [4]). In the potential $\phi(r)$, the electric magnetization current $j^e_{mag}$ transfers the energy flux $j^e_{mag} = \delta \phi_{mag}$ (Eq. 37 in [18]). Taking into account that $j^e_{mag} = e\mu c \mathbf{A}_H$, we get

$$j^e_{mag} = c\mu |\mathbf{H} \times \mathbf{E}|. \quad (20)$$

In the linear response method, this term is given by the diagram with the energy current vertex $\gamma^e = e\phi \mathbf{v}$ (the third term in the energy current operator, Eq. [7] and the magnetic vertex $\gamma^A = (e/\epsilon)(\mathbf{A}_H \cdot \mathbf{v})$. Without $\phi$ this diagram gives the $j^A_{mag}$ and, therefore, we get $j^A_{mag} = \phi \mathbf{J}_{mag}$. As we discussed, the vertex $\gamma^e = e\phi \mathbf{v}$ does not contribute to the heat current, because the heat energy is counted from the electro-chemical potential.

While the magnetization currents do not contribute to the heat transfer, they play an important role in the charge transfer. In the interior of the sample the electric current consists of the transport and magnetization components, $j^e_{inr} = j^r_{inr} + j^e_{mag}$. According to Ref. [18],

$$j^e_{mag} = c\frac{\partial \mu}{\partial T} (\nabla T \times \mathbf{H}). \quad (21)$$

The magnetization currents are divergence-free. Therefore, the total magnetization current through the sample cross-section must be zero, i.e. the bulk magnetization currents are canceled by the surface currents. Therefore, in the finite sample, the measured Nernst coefficient $N$ is determined by the transport currents, $N = j^r_{inr}/(\nabla T \times \mathbf{H})$. Calculated from the theory, the Nernst coefficient in the infinite sample may be associated with the current in the interior of the finite sample, $N_{inf} = j^r_{inr}/(\nabla T \times \mathbf{H})$. Using Eq. [21] we get

$$\delta N = \frac{j^r_{inr} - j^e_{mag}}{\nabla T \times \mathbf{H}} = \delta N_{inf} - c\frac{\partial \mu}{\partial T}. \quad (22)$$

The coefficient $\delta N_{inf}$ may be calculated by the quantum transport equation, where $\nabla T$ can be easy incorporated (as a nonmechanical perturbation, $\nabla T$ cannot be incorporated into the Kubo formalism). In the AL approximation, calculation of the Nernst coefficient is analogous to calculation of the Hall effect with the only difference that the derivatives of the fluctuation propagator $L^{R,A}(q,\omega)$ with respect to $\omega$ in the Hall effect should be replaced by the temperature derivative in calculations of $\delta N_{inf}$. Taking into account temperature dependencies of all coefficients in the fluctuation propagator (Eqs. [8] and [9]), for the 2D-superconductor we get

$$\delta N_{inf} = \delta N + c^2 \frac{\epsilon^2}{3\pi e} \left( \frac{\alpha}{\eta^2} - \frac{\alpha}{\eta} - \frac{\alpha}{\eta} \frac{\partial \alpha}{\partial T} \right). \quad (23)$$

Thus, in the infinite sample or in the interior of the finite sample, the Nernst coefficient consists of a large term (the second term in Eq. [23], which is nonzero to zero order in PHA. However, in the finite sample this term is canceled by the contribution of the magnetization currents, $c\partial \mu/\partial T$. According to Eq. [22] the rest is equal to $\delta N$, which satisfies to the Onsager relation $\delta N = 5\Upsilon/T$.

Thus, in the finite sample both Nernst and Ettingshausen coefficients in the fluctuation region (Eq. [18]), as well as the coefficients in the normal state ($N_0$ and
The corrections are of the same order as the relative corrections to the conductivity, thermoelectric and Hall coefficients. Our consideration was limited by the one-band model. However, Eq. 24 is also valid for the two-band model, where the electron and hole contributions are additive and thermomagnetic effects are large.

These corrections are of the same order as the relative corrections to the conductivity, thermoelectric and Hall coefficients. The previous works gave $\delta N/N_0$ with an additional large factor of $(\epsilon_f/T)^2$. However, none of experiments has shown such huge effect in ordinary superconductors. While claiming opposite, the very recent work on SiN requires for the thermomagnetic coefficients. The large Nernst effect observed in high-$T_c$ cuprates requires vortex-like excitations related to the phase fluctuations.

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Fluctuation AL diagrams describing the heat current in crossed electric and magnetic fields. Wavy lines stand for the fluctuation propagators and straight lines stand for the electron Green functions, which form the AL blocks.