Early Fault Diagnosis of Rolling Bearing Based on Lyapunov Exponent

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Abstract. Lyapunov exponent is an important quantitative index to measure the dynamic characteristics of the system. It represents the average exponential rate of convergence or divergence between adjacent orbits in phase space. Whether there is dynamic chaos in the system can be judged intuitively from whether the maximum Lyapunov exponent is greater than zero. The Lyapunov exponent obtained by the small data method has the characteristics of simple calculation process and accurate calculation results, and it is applied to the fault diagnosis of rolling bearings. By calculating the Lyapunov exponents of bearing rollers, bearing outer rings, bearing inner rings and normal state bearings, and comparing the calculation results under different working conditions, it is concluded that Lyapunov exponents are of great significance in judging the early failure of rolling bearings.

1. CHAOS CHARACTERISTICS

Chaos theory plays an important role in the research of non-linear science. Up to now, non-linear science mainly includes non-linear systems, soliton theory, symbolic dynamics, chaotic dynamics, quantum chaos, fractal geometry and physics. So far, the concept of "chaos" has different understandings and expressions in different fields of science and technology, reflecting the universality of the existence of chaos phenomenon and its unique characteristics in their respective fields. Chaotic phenomena can be regarded as macro-nonlinear systems, which show uncertain or unpredictable characteristics under general conditions \(^1\). It is the unification of certainty and uncertainty, order and disorder, regularity and irregularity. Chaos has the following characteristics:

1. Similar randomness

Similar randomness is characterized by random disorder and chaotic trajectory at the macro level, but self-similar infinite nesting at the micro level, so it seems to occur randomly on the surface, but in fact it is determined by certain rules and can be deduced in a short period.\(^2\)

2. Sensitivity of initial conditions

The relative concept of initial-condition sensitivity and short-period computability is that the system can not be calculated in the long-term scale. Because the small change of initial conditions of the system will lead to large changes in subsequent motion, which reflects another characteristic of chaotic system, namely, sensitivity to initial conditions.

2. LYAPUNOV EXPONENT

Lyapunov exponent is a characteristic quantity describing attractor characteristics at macro level. It is an important parameter of chaotic system and represents the average index of convergence or
divergence of adjacent orbits in phase space dynamic system. In the one-dimensional dynamic system \( F(x_n) = x_{n+1} \), pay attention to \( \frac{dF}{dx} \). If \( \left| \frac{dF}{dx} \right| > 1 \), then the initial two-point iteration will diverge. If \( \left| \frac{dF}{dx} \right| < 1 \), after the initial two-point iteration, it will be aggregated. In subsequent iterations, the value of \( \frac{dF}{dx} \) is dynamic. In phase space, two orbits are separated and gathered. In order to reflect the degree of separation and gathering of two orbits as a whole, it is necessary to calculate the average number of iterations or time. [3].

Two points with a distance of \( \varepsilon \), after several iterations, the distance becomes:

\[
\varepsilon e^{\lambda(n)} = \left| F(x_0 + \varepsilon) - F(x_0) \right| \quad (1)
\]

If \( \varepsilon \rightarrow 0 \), \( n \rightarrow \infty \), the upper form becomes

\[
\lambda(n) = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left| \frac{F(x_0 + \varepsilon) - F(x_0)}{\varepsilon} \right| = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left| \frac{F(x) - F(x)}{\varepsilon} \right| \quad (2)
\]

The above formula is independent of the initial value and can be simplified to

\[
\lambda = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^{n} \ln \left| \frac{F(x)}{dx} \right|_{x=x_j} \quad (3)
\]

\( \lambda \) is the Lyapunov exponent in the prime mover system. It represents the average exponential separation rate caused by each iteration in several iterations. There are three main methods for calculating Lyapunov index, Wolf method, Jacobi method and small data quantity method. This paper mainly uses small data quantity method to calculate Lyapunov index.:

The small data quantity method is a widely used method to obtain Lyapunov exponent. It is an improvement of Wolf's method. It has the advantages of small data amount, high accuracy and simple calculation process. [4].

Firstly, the phase space is reconstructed, and then the nearest neighbors of the orbits in the phase space are found:

\[
dt(0) = \min_{i(t)} \| X(t) - X(\hat{t}) \|, \quad \| t - \hat{t} \| > p \quad (4)
\]

In the formula, \( P \) is the average period of time series. Its value can be calculated by calculating the reciprocal of the average frequency of energy spectrum. From this, the maximum Lyapunov exponent can be estimated according to the average divergence rate of the nearest neighbor point of each orbit.

According to Sato et al:

\[
\Delta t = \frac{1}{i} \frac{1}{M-i} \sum_{j=1}^{M-i} \ln \left| \frac{d_j(i)}{d_j(0)} \right| \quad (5)
\]

In the formula, \( \Delta t \) is the sample period and \( d_j(i) \) is the distance of the \( j \) pair of adjacent points in orbit after \( i \) discrete time steps.

According to the above formula:

\[
d_j(i) = C_j e^{\lambda_j(i\Delta t)}, \quad C_j = d_j(0) \quad (6)
\]

Logarithms on both sides of the upper formula

\[
\ln d_j(i) = \ln C_j + \lambda_j(i\Delta t) \quad (7)
\]

In the formula \( j = 1, 2, \ldots, M \). The maximum Lyapunov exponent is the slope of the upper curve, which can be obtained by the least square method.
3. Characteristic analysis of test data
Next, the validity of Lyapunov exponent in early fault diagnosis of rolling bearings is verified by experiments. The bearing fault is simulated by manual faults and the whole life test platform of rolling bearing is used in the test. (Figure 1), Crack faults are manually manufactured on the inner ring, outer ring and rolling body of bearings. The crack size is 0.4 mm wide and 0.2 mm deep.

By using the vibration signal data acquisition system, the time-domain vibration signals of rolling bearings under different working conditions are collected, and a number of groups of digital sequences bearing fault information of rolling bearings are obtained. Since the motion of rotating machinery system inevitably shows chaotic state, we can take Lyapunov exponent as a characteristic quantity of the system and calculate different fault types. The maximum Lyapunov exponent of signal sequence under the same working condition is used as the feature of each fault type to identify the fault type.

The Lyapunov exponent is calculated by the small data method. The calculation steps are as follows:

1. Estimate the average period of the sequence by FFT transform.
2. Phase space reconstruction of time series. The parameters of phase space reconstruction refer to the previous calculation method.
3. Find the nearest distance point according to formula (4) to limit the short separation.
4. Calculate the average distance of adjacent points.
5. The maximum Lyapunov exponent is obtained by fitting the slope of the straight line according to equation (7), as shown in Fig. 2.

Through the above steps, the Lyapunov exponents of bearing vibration signal sequence under inner ring fault, outer ring fault, rolling element fault and normal state are calculated. Due to space limitation, Table 1 only gives the characteristic parameters of six sets of vibration signal sequences under four working conditions. The first two data names represent the fault type, and the last two represent the fault data groups, as shown in the table.
Table 1 Maximum Lyapunov exponent of sample data (six groups for each failure)

| Inner Ring Fault | Outer Ring Fault | Roller Fault | Normal Conditions |
|------------------|------------------|--------------|-------------------|
| D0101            | 0.0963           | D0301        | D0401             |
|                  | 0.0472           | D0302        | D0402             |
|                  | 0.0413           | D0303        | D0403             |
|                  | 0.0168           | D0304        | D0404             |
|                  | 0.0225           | D0305        | D0405             |
|                  | 0.0862           | D0306        | D0406             |

In order to intuitively express the difference and regularity of Lyapunov exponents among different groups of data, the data of each group under various working conditions in Table 1 are made into a three-dimensional histogram (Figure 3). From the graph, we can see the numerical difference and distribution regularity of the largest Lyapunov exponents among different types of faults.

Figure 3. Comparison chart of maximum Lyapunov exponent for each fault type

4. Conclusion
The maximum Lyapunov exponent of the vibration signal sequence of rolling element faults is high; the maximum Lyapunov exponent of the vibration signal sequence of outer ring faults is slightly larger than that of inner ring faults; the maximum Lyapunov exponent of the vibration signal sequence under normal working conditions is relatively small. The results show that because of the existence of rolling bearing faults and the different fault location, the chaotic characteristics of each group of data systems are obviously different. So the maximum Lyapunov exponent can be used as a feature for pattern recognition of early fault types of rolling bearings.

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