Two novel cluster states in $^{10}$Be

Fumiharu Kobayashi and Yoshiko Kanada-En’yo
Department of Physics, Kyoto University, Kyoto 606-8502, Japan
E-mail: f-kobayashi@ruby.scphys.kyoto-u.ac.jp

Abstract. We have investigated the cluster structure of $^{10}$Be by using $^6$He+$\alpha$ cluster wave functions and $2\alpha+2n$ dineutron condensate wave functions. We found two novel cluster $0^+$ states, one of which has an $\alpha+\alpha+dineutron$ gas-like structure and the other contains $^6$He and a well-separated $\alpha$ cluster. In this paper, we discuss their cluster structures.

1. Introduction
Cluster structure is a hot topic in nuclear physics, and it has been eagerly investigated both theoretically and experimentally. In Be isotopes, an $\alpha$ cluster, which is the most representative cluster, tends to be developed. As for the $0^+$ states of $^{10}$Be, the $0_{1,2}^+$ states have been confirmed experimentally, and their structures are explained well by a $2\alpha$ core and two valence neutrons in the molecular orbits around the core. In addition to these $0^+$ states, a new $0^+$ state was proposed experimentally [1], which may consist of a $^6$He and a well-developed $\alpha$.

In conjunction with the cluster structure of finite nuclei, we are also interested in the dineutron correlation. Two neutrons coupled to spin-singlet have strong spatial correlation and become compact to be considered as a kind of cluster. Its universal properties are not well-known and we constructed a model, which we call the “dineutron condensate (DC) wave function”, to investigate the dineutron correlation in various nuclei systematically [2].

In this work, we have investigated cluster states in $^{10}$Be and put forward the possibility of a state which contains a compact dineutron and another state which contains $^6$He and a well-separated $\alpha$. We discuss their cluster structures in this paper.

2. Theoretical framework
In the present work, we superpose two kinds of cluster wave functions, namely, $^6$He+$\alpha$ cluster wave functions used in Ref. [3] and $2\alpha+2n$ DC wave functions proposed in Ref. [2].

In the $^6$He+$\alpha$ cluster wave function, the system is composed of a $^6$He and an $\alpha$ cluster separated by a distance $d_c$. The single particle wave functions are described by the harmonic oscillator wave functions, and the configuration of an $\alpha$ is $(0s)^4$. The $^6$He cluster is composed of an $\alpha$ core and two valence neutrons in the $(0p)^2$ configuration represented by Gaussians shifted with respect to the $\alpha$ core. By using such $^6$He+$\alpha$ cluster wave functions, the $0_{1,2}^+$ states of $^{10}$Be are well-described as shown in Ref. [3].

In addition to the $^6$He+$\alpha$ wave functions, we superpose the $2\alpha+2n$ DC wave functions. In the $2\alpha+2n$ DC wave functions, we assume that the nuclear system is composed of a $2\alpha$ core and two valence neutrons whose spins are coupled to singlet and their relative motion is an s-wave.
The relative and centre of mass (c.o.m.) wave functions of two neutrons are denoted by $\psi_r$ and $\psi_G$, and the explicit form of the whole DC wave function is as follows.

$$\Phi_{DC} = \frac{1}{\sqrt{10!}} \text{det} \left[ \psi_{\alpha 1}(r_1) \cdots \psi_{\alpha 1}(r_4) \psi_{\alpha 2}(r_5) \cdots \psi_{\alpha 2}(r_8) \psi_r(r) \psi_G(r_G) \right],$$

$$\psi_r(r) \propto \exp \left[ -\frac{r^2}{4b_n^2} \right], \quad \psi_G(r_G) \propto \exp \left[ -\frac{r^2_G}{\beta^2} \right],$$

where $\psi_{\alpha i} (i = 1, 2)$ are the single particle wave functions of the $2\alpha$ core whose relative distance is $d_n$; $r$ and $r_G$ are the relative and c.o.m. coordinates of the two extra neutrons, respectively. The Gaussian widths in the relative and c.o.m. wave functions are $b_n$ and $\beta$, and these parameters set the size and the spatial expansion of the dineutron around the core, respectively. In the DC wave functions, we fix the parameters $\beta$ and $b_n$ to various values to describe the system as a superposition of dineutron subsystems with various sizes and distributions.

In the present work, we include $^6\text{He}+\alpha$-type basis states with $d_c = 1, 2, \cdots, 8$ fm as well as $2\alpha+2n$ DC-type basis states with $d_n = 1, 2, \cdots, 6$ fm, $\beta = 2, 3, \cdots, 9$ fm and five $b_n$ values for each $\beta$.

3. Results

By superposing the wave functions explained above, we obtain two kinds of excited states having distinct cluster structures above the $0^+_2$ state. We explain the structures of two new $0^+$ states below.

3.1. The $\alpha+\alpha+$dineutron gas-like state

We obtained a state, which we label $0^+_3$ here, containing a rather compact dineutron separated from the core. To see its dineutron component, we calculated the overlaps with the DC wave functions whose parameters $d_n$, $\beta$ and $b_n$ are fixed to various values. A component with a small $b_n$ corresponds to a state containing a compact dineutron, while a component with a large $\beta$ corresponds to a state containing two neutrons far from the $2\alpha$ core. We show the overlap of the $0^+_3$ state obtained with DC wave functions whose $d_n$ is fixed to 4.0 fm in the $\beta$-$b_n$ plane in Figure 1 (a), and that whose $b_n$ is fixed to 2.2 fm on the $\beta$-$d_n$ plane in Figure 1 (b). It can be seen in Figure 1 (a) that the peak is located at $(\beta, b_n) \sim (4.0, 2.2)$ with a broad width in both directions of $\beta$ and $b_n$, and in Figure 1 (b) that the peak is at $(\beta, d_n) \sim (4.0, 4.0)$ also.

![Figure 1](image_url)

**Figure 1.** (a) The overlap of the $0^+_3$ state with the DC wave functions with $d_n = 4$ fm. The horizontal axis is $\beta$ and the vertical one is $b_n$. (b) The overlap of the $0^+_3$ state with the DC wave functions with $b_n = 2.2$ fm. The horizontal axis is $\beta$ and the vertical one is $d_n$. These figures are from Ref. [4]
with broad widths in both horizontal and vertical directions. This means that the dineutron is rather compact but its size is changeable to some extent, and such a soft dineutron and the two \( \alpha \) clusters are weakly interacting with each other and spread broadly. We propose that this \( 0^+ \) state is a gas-like cluster state since it contains well-separated clusters loosely bound.

3.2. The state with a \( ^6\text{He} \) plus a well-separated \( \alpha \)

![Image](image_url)

**Figure 2.** The overlaps of the \( 0^+_3 \) and \( 0^+_4 \) states with the \( ^6\text{He}(0^+)+\alpha \) wave functions. The horizontal axis is \( d_{\alpha} \). This figure is from Ref. [4].

We also obtained a \( ^6\text{He}+\alpha \) state. We label it by \( 0^+_4 \). It lies energetically above the \( \alpha+\alpha+2\text{n} \) state \( 0^+_3 \). In order to see its components consisting of a \( ^6\text{He} \) and an \( \alpha \) cluster, as a function of their distance, in Figure 2 we show the overlap of the \( 0^+_4 \) state with \( ^6\text{He}(0^+)+\alpha \) wave functions (the subsystem \( ^6\text{He} \) is projected to \( J^\pi = 0^+ \)) whose relative distance is \( d_{\alpha} \). The \( 0^+_4 \) state has a large amplitude in the region of \( d_{\alpha} \sim 7 \text{ fm} \). This indicates that in the \( 0^+_4 \) state there is a well-developed \( \alpha \) cluster displaced far from \( ^6\text{He} \). On the other hand, the component of \( ^6\text{He}+\alpha \) is minor in the \( 0^+_3 \) state because the two \( \alpha \) clusters and the one dineutron in this state are weakly correlated, so that a \( ^6\text{He} \) cluster cannot be formed. Thus these two \( 0^+ \) states have cluster structures distinct from each other.

4. Summary

In the present work, we investigated excited cluster states in \(^{10}\text{Be}\) and indicated that there may exist two novel \( 0^+ \) states with significantly different cluster structures. One of them has a gas-like structure of \( \alpha+\alpha+\text{dineutron} \) interacting weakly and extended spatially. Another consists of a \( ^6\text{He} \) cluster and a well-separated \( \alpha \) cluster.

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