Embedding a very low scale inflation within a particle physics model is a challenging problem. It is not only difficult to obtain sufficient number of e-foldings, right amplitude for the scalar density perturbations, the right tilt in the power spectrum, but also generating baryon asymmetry and dark matter simultaneously.

Recently there has been a real progress in our understanding of embedding inflation within particle physics, particularly within the Minimal Supersymmetric Standard Model (MSSM), where the inflaton belongs to the MSSM instead of being an ad-hoc gauge singlet. Foremost the models [1, 2] not only predict the right amplitude of the scalar density perturbations and the tilted spectrum, see also [3], but are also testable at LHC [4].

In this paper we provide a simple example of a sub-eV Hubble scale inflation, $H_{\text{inf}} \sim 10^{-3} - 10^{-1}$ eV, embedded within MSSM. This is realizable provided supersymmetry (SUSY) breaking is communicated via gauge mediation, i.e. gauge mediated supersymmetry breaking (GMSB) [8], in contrast to Refs. [1, 2] where we assumed gravity mediation. We shall predict the reheat temperature around TeV, which strongly favors electroweak baryogenesis within MSSM and gravitino as a dark matter candidate. Thus all the ingredients for a successful cosmology are naturally contained within MSSM.

Let us first highlight relevant points of the model:

- The model is based on inflation in the vicinity of a saddle point, see [8, 9], which is predictive, radiatively stable, free from supergravity and Trans-Planckian corrections. The only scales which enters in the potential is the weak scale $\sim$ TeV, and the Grand Unified scale, $M_{\text{GUT}} \sim 10^{16.5}$ GeV.
- The reheat temperature of the model is just around TeV, which points towards thermal electroweak baryogenesis [10]. If the reheat temperature is slightly lower than this, we have the right conditions for a cold electroweak baryogenesis [10].
- Within GMSB the gravitino is the lightest SUSY particle (LSP). Therefore a reheat temperature of $O$(TeV) will lead to a sufficient relic abundance for the (stable) gravitino as a dark matter candidate [10].

Let us now consider an MSSM flat direction, $\phi$, lifted by a non-renormalizable ($n > 3$) superpotential term for a detailed dynamics on multiple flat direction, see [11]:

$$ W = (\lambda_n/n)(\Phi^n/M_{\text{GUT}}^{n-3}), $$

where $\Phi$ is the superfield which contains the flat direction $\phi$. Within MSSM all the flat directions are lifted by $n \leq 9$ operators [12, 13]. The cut-off scale is $M_{\text{GUT}}$, therefore the above superpotential is a reflection of integrating out the physics above the GUT scale, and we assume the non-renormalizable coupling to be $\lambda_n \sim O(1)$. Note that the new physics does not necessarily have to be tied to the GUT physics. For example a gauged $U(1)_{B-L}$ may appear at an intermediate scale $M_{B-L} \ll M_{\text{GUT}}$. This will amount to the same superpotential as in Eq. [14] parameterized by $M_{\text{GUT}}$, so long as $\lambda_n \ll 1$. This is conceivable as $\lambda_n$ is a product of the Yukawa couplings associated with the new interaction terms beyond the MSSM.

In GMSB the two-loop correction to the flat direction potential results in a logarithmic term above the messenger scale, i.e. $\phi > M_{S} [14]$. Together with the $A$-term this leads to the scalar potential

$$ V = M_{F}^{2}\ln\left(\frac{\phi^{2}}{M_{S}^{2}}\right) + A \cos(n\theta + \theta_{A}) \frac{\lambda_{n}\phi^{n}}{n^{n-3}M_{\text{GUT}}^{n-3}} + \lambda^{2}_{n}\phi^{2(n-1)}M_{\text{GUT}}^{2(n-3)} $$

where $M_{F} \sim (m_{\text{SUSY}} \times M_{S})^{1/2}$ and $m_{\text{SUSY}} \sim 1$ TeV is the soft SUSY breaking mass at the weak scale. For $\phi > M_{F}^{2}/m_{3/2}$, usually the gravity mediated contribution, $m_{3/2}^{2}\phi^{2}$, dominates the potential where $m_{3/2}$ is the gravitino mass. Here we will concentrate on the VEVs $M_{S} \ll \phi \lesssim M_{F}^{2}/m_{3/2}$.

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1 The inflaton candidates are, $LLe$, $udd$ [1] and $NH_{4}L$ [2] flat directions. Here $L$ denotes left-handed sleptons, $u, d, e$ denote right-handed squarks and sleptons, respectively, $N$ denotes right-handed sneutrinos and $H_{u}$ is the Higgs which gives masses to the up type quarks. The $LLe, udd$ mass should be $\geq 340$ GeV [8], which is within the reach of LHC. For other attempts of inflation with flat directions, see [8, 9]. For a review on MSSM flat directions, see [5].
In Eq. (2), $\phi$ and $\theta$ denote the radial and the angular coordinates of the complex scalar field $\Phi = \phi \exp[i\theta]$ respectively, while $\theta_A$ is the phase of $A$-term (thus $A$ is a positive quantity with a dimension of mass). Note that the first and third terms in Eq. (2) are positive definite, while the $A$-term leads to a negative contribution along the directions where $\cos(n\theta + \theta_A) < 0$. The cosmological importance of an $A$-term can be found in Refs. [1, 2, 3, 4].

Although individual terms are unable to support a sub-Planckian VEV inflation, but as shown in Refs. [1, 2, 3, 4], a successful inflation can be obtained near the saddle point, which we find by solving, $V'(\phi_0) = V''(\phi_0) = 0$ (where derivative is w.r.t $\phi$).

$$\phi_0 = \left( \frac{M_{GUT}^n - M_{F}^n}{\lambda_n} \right) \sqrt{n(n-1)(n-2)}^{1/(n-1)},$$

$$A = \frac{4(n-1)^2 \lambda_n}{nM_{GUT}^n} \phi_0^{-2}.$$  

In the vicinity of the saddle point, we obtain the total energy density and the third derivative of the potential to be:

$$V(\phi_0) = M_F^2 \left[ \log \left( \frac{\phi_0^2}{M_S^2} \right) - \frac{3n-2}{n(n-1)} \right],$$

$$V''(\phi_0) = 4n(n-1)M_F^2 \phi_0^{-3}.$$  

There are couple of interesting points, first of all note that the scale of inflation is extremely low in our case, barring some small coefficients of order one, the Hubble scale during inflation is given by:

$$H_{inf} \sim M_F^2/M_P \sim 10^{-3} - 10^{-1} \text{ eV},$$

for $M_F \sim 1 - 10 \text{ TeV}$. For such a low scale inflation usually it is extremely hard to obtain the right phenomenology. But there are obvious advantages of having a low scale inflation, $M_F \gg H_{inf}$. The supergravity corrections and the Trans-Planckian corrections are all negligible [4], therefore the model predictions are trustworthy.

Perturbations which are relevant for the COBE normalization are generated a number $N_{COBE}$ e-foldings before the end of inflation. The value of $N_{COBE}$ depends on thermal history of the universe and the total energy density stored in the inflaton, which in our case is bounded by, $V_0 \leq 10^{16} \text{ (GeV)}^4$. The required number of e-foldings yields in our case, $N_{COBE} \sim 40$ [15], provided the universe thermalizes within one Hubble time. Although within SUSY thermalization time scale is typically very long [11], however, in this particular case it is possible to obtain a rapid thermalization.

Near the vicinity of the saddle point, $\phi_0$, the potential is extremely flat and one enters a regime of self-reproduction [21]. The self-reproduction regime lasts as long as the quantum diffusion is stronger than the classical drag: $H_{inf}/2\pi > \phi_0/\dot{\phi}$ for $\phi_0 \leq \phi \leq \phi_s$, where $\phi_0 - \phi_s \approx M_F(\phi_0/M_P)^{3/2}$. From then on, the evolution is governed by the classical slow roll. Inflation ends when $|\eta| \sim 1$, which happens at $\phi \approx \phi_s$, where

$$\phi - \phi_0 = -\sqrt{\frac{\sqrt{3}V_0\phi_0^3}{2n(n-1)M_F^2 M_P}}.$$  

Assuming that the classical motion is due to the third derivative of the potential, $V''(\phi) \approx (1/2)V'''(\phi)(\phi - \phi_0)^2$, the total number of e-foldings during the slow roll period is found to be:

$$N_{tot} = \int_{\phi_s}^{\phi_0} \frac{H_{inf}d\phi}{\phi} \approx \frac{2V_0\phi_0^3}{4n(n-1)M_F^2 M_P^2} \left( \frac{1}{\phi_0 - \phi_s} \right).$$

This simplifies to

$$N_{tot} \approx \phi_0^{3/2}/(M_F^2/2M_P).$$  

Let us now consider the adiabatic density perturbations. Despite $H_{inf} \ll 1 \text{ eV}$, the flat direction can generate adequate density perturbations as required to explain to match the observations. Recall that inflation is driven by $V''' \neq 0$, we obtain

$$\delta H \approx (1/5\pi)(H_{inf}/\dot{\phi}) \sim M_F^2/2M_P N_{tot}^{2/3} \sim 10^{-5}.$$  

Note that for $M_F \sim 10 \text{ TeV}$, and $N_{COBE} \sim 40$, we match the current observations [21], when $\phi_0 \sim 10^{11} \text{ GeV}$. The validity of Eq. (2) for such a large VEV requires that $M_F^2 \gg (10^{11} \text{ GeV}) \times m_{3/2}$. For $M_F \sim 10 \text{ TeV}$ this yields the bound on the gravitino mass, $m_{3/2} < 1 \text{ MeV}$, which is compatible with the dark matter constraints as we will see.

We can naturally satisfy Eq. (11) provided, $n = 6$. The non-renormalizable operator, $n = 6$, points towards two MSSM flat directions out of many,

$$LLe \text{ and } udd.$$  

As we discussed before in [1], these are the only directions which are suitable for inflation as they give rise to a non-vanishing $A$-term. Note that the inflatons are now the gauge invariant objects. The total number of e-foldings, during the slow roll inflation, after using Eq. (11) yields,

$$N_{tot} \sim 10^4.$$  

While the spectral tilt and the running of the power spectrum are determined by $N_{COBE} \sim 40 \ll N_{tot}$.

$$n_s = 1 + 2\eta - 6\epsilon \approx 1 - \frac{4}{N_{COBE}} \sim 0.90,$$

$$\frac{dn_s}{d\ln k} = 16\epsilon \eta - 24\epsilon^2 - 2\xi^2 \approx -\frac{4}{N_{COBE}} \sim -10^{-3},$$

where $\xi^2 = M_F^2 V'/V''.\text{ Note that the spectral tilt is slightly away from the 2r result of the current WMAP3}$.  

years data, on the other hand running of the spectrum is well inside the current bounds \[21\].

At first instance one would discard the model just from the slight mismatch in the spectral tilt from the current observations. However note that our analysis strictly assumes that the slow roll inflation is driven by \(V''(\phi_0)\). This is particularly correct if \(V'(\phi_0) = 0\) and \(V''(\phi_0) = 0\). Let us then study the case when \(V'(\phi_0) \neq 0\), as discussed in \[3\].

The latter case can be studied by parameterizing a small deviation from the exact saddle point condition by solving near the point of inflection, where we wish to solve \(V''(\phi_0) = 0\) and we get up-to 1st order in the deviation, \(\delta \ll 1\),

\[
\tilde{A} = A(1 - \delta), \quad \tilde{\phi}_0 = \phi_0 \left(1 - \frac{n-1}{n(n-2)} \delta\right), \quad (16)
\]

with \(A\) and \(\phi_0\) are the saddle point solutions. Then the 1st derivative is given by

\[
V'(\phi_0) = 4\frac{n-1}{n-2} M_F \phi_0^{-1} \delta. \quad (17)
\]

Therefore the slope of the potential is determined by,

\[
V'(\phi) \simeq V'(\phi_0) + (1/2)V''(\phi_0)(\phi - \phi_0)^2.
\]

Note that both the terms on the right-hand side are positive. The fact that \(V'(\phi_0) \neq 0\) can lead to an interesting changes from the saddle point behavior, for instance the total number of e-foldings is now given by

\[
N_{\text{tot}} = \frac{V(\phi_0)}{M_F^2} \int_{\phi_{\text{end}}}^{\phi_0} \frac{d\phi}{V'(\phi_0) + \frac{1}{2}V''(\phi_0)(\phi - \phi_0)^2}. \quad (18)
\]

First of all note that by including \(V'\), we are slightly away from the saddle point and rather close to the point of inflection. This affects the total number of e-foldings during the slow roll. It is now much less than that of \(N_{\text{tot}}\), i.e. \(N_{\text{tot}} \ll 10^3\), see Eq. \[18\].

When both the terms in the denominator of the integrand contributes equally then there exists an interesting window.

\[
\frac{\kappa}{8} \leq \delta \leq \frac{\kappa}{2}, \quad (19)
\]

where

\[
\kappa \equiv \frac{n-2}{n(n-1)} \left[\ln \left(\frac{\phi^2_0}{M_F^2}\right) - \frac{3n-2}{n(n-1)}\right]^2 \frac{\phi^4_0}{M_F^4 N_{\text{COBE}}^2}. \quad (20)
\]

The lower limit in Eq. \[19\] is saturated when \(V'(\phi_0) = 0\), while the upper limit is saturated when \(N_{\text{tot}} \simeq N_{\text{COBE}} \approx 40\). It is also easy to check that there will be no self-reproduction regime for the field values determined by \(\delta\).

It is a straightforward but a tedious exercise to demonstrate that when the upper limit of Eq. \[19\] is saturated the spectral tilt becomes \(n_s \approx 1\), when the lower limit is satisfied we recover the previous result with \(n_s = 0.90\). This value, \(n_s \to 1\), can be easily understood as \(\phi_{\text{COBE}} \to \phi_0\) (where \(\phi_{\text{COBE}}\) corresponds to the VeV where the end of inflation corresponds to \(N_{\text{COBE}} \sim 40\)), in which case, \(\eta \to 0\). Therefore the spectral tilt becomes nearly scale invariant. We therefore find a range \[2\],

\[
0.90 \leq n_s \leq 1, \quad (21)
\]

whose width is within the 2\(\sigma\) error of the central limit \[21\]. Similarly the running of the spectral tilt gets modified too but remains within the observable limit \[2\], while the amplitude of the power spectrum is least affected \[3\].

Let us now discuss the issue of reheating and thermalization. Important point is to realize that the inflaton belongs to the MSSM, i.e. \(LLe\) and \(udd\), both carry MSSM charges and both have gauge couplings to gauge bosons and gauginos. After inflation the condensate starts oscillating. The effective frequency of the inflaton oscillations in the Logarithmic potential, Eq. \[2\], is of the order of \(M_F^2/\phi_0\), while the expansion rate is given by \(H_{\text{inf}} \sim M_F^2/\Delta F\). This means that within one Hubble time the inflaton oscillates nearly \(M_F/\phi_0 \sim 10^7\) times. The motion of the inflaton is strictly one dimensional from the very beginning. During inflation, the imaginary direction is very heavy and settles down in the minimum of the potential.

An efficient bout of particle creation occurs when the inflaton crosses the origin, which happens twice in every oscillation. The reason is that the fields which are coupled to the inflaton are massless near the point of enhanced symmetry. Mainly electroweak gauge fields and gauginos are then created as they have the largest coupling to the flat direction. The production takes place in a short interval. Once the inflaton has passed by the origin, the gauge bosons/gauginos become heavy by virtue of VeV dependent masses and they eventually decay into particles sparticles, which creates the relativistic thermal bath. This is so-called instant preheating mechanism \[22\]. In a favorable condition, the flat direction VeV coupled very weakly to the flat direction inflaton could also enhance the perturbative decay rate of the inflaton \[23\]. In any case there is no non-thermal gravitino production \[24\] as the energy density stored in the inflaton oscillations is too low.

A full thermal equilibrium is reached when \(a)\) kinetic and \(b)\) chemical equilibrium are established \[14\]. The maximum temperature of the plasma is given by

\[
T_R \sim [V(\phi_0)]^{1/4} \sim M_F \lesssim 10 \text{TeV}, \quad (22)
\]

when the flat direction, either \(LLe\) or \(udd\) evaporates completely. This naturally happens at the weak scale.

\[2\] A similar exercise can be done for the running of the spectral tilt and the running lies between \(-16/N_{\text{COBE}}^2 \lesssim dn_s/d\ln k \lesssim -4/N_{\text{COBE}}^2\).
There are two very important consequences which we summarize below.

**Hot or cold electroweak Baryogenesis**: The model strongly favors electroweak baryogenesis within MSSM. Note that the reheat temperature is sufficient enough for a thermal electroweak baryogenesis.

However, if the thermal electroweak baryogenesis is not triggered, then cold electroweak baryogenesis is still an option. During the cold electroweak baryogenesis, the large gauge field fluctuations give rise to a non-thermal sphaleron transition. In our case it is possible to excite the gauge fields of $SU(2)_L \times U(1)_Y$ during instant preheating provided the inflaton is $L\bar{L}e$. The $L\bar{L}e$ as an inflaton carries the same quantum number which has a $B-L$ anomaly and large gauge field excitations can lead to non-thermal sphaleron transition to facilitate baryogenesis within MSSM.

**Gravitino dark matter**: Within GMSB gravitinos are the LSP and if the $R$-parity is conserved then they are an excellent candidate for the dark matter. There are various sources of gravitino production in the early universe. However, in our case the thermal production is the dominant one and mainly helicity $\pm 1/2$ gravitinos are created. Gravitinos thus produced have the correct dark matter abundance for $\frac{m_{3/2}}{100 \text{ keV}} \simeq \left(\frac{T_R}{1 \text{ TeV}}\right)\left(\frac{M_{\tilde{g}}}{1 \text{ TeV}}\right)^2$, (23)

where $M_{\tilde{g}}$ is the gluino mass. For $m_{3/2} \gtrsim 100 \text{ keV}$, Eq. (23) is easily satisfied for $M_{\tilde{g}} \sim 1 \text{ TeV}$ and $T_R \lesssim 10 \text{ TeV}$. We remind that for $\mathcal{O}(\text{keV}) \lesssim m_{3/2} < 100 \text{ keV}$ gravitinos produced from the sfermion decays overlap the universe.

Before concluding we should also highlight that the existence of a saddle point does not get spoiled through radiative corrections, see Ref. [1]. To summarize, we provided a truly low scale inflation model with $H_{\text{inf}} \sim 10^{-3} - 10^{-1} \text{ eV}$, embedded within MSSM, provided GMSB is the correct paradigm. Although inflation occurs at such low scales, the model predictions match the current WMAP data and the reheat temperature of $T_R \lesssim 10 \text{ TeV}$ is sufficient enough to trigger either hot or cold electroweak baryogenesis. The model also produces sufficient abundance of gravitinos to be the dark matter candidate. Thus inflation within GMSB connects the physics of microwave background radiation to a successful dark matter and a baryogenesis scenario whose ingredients are testable at the LHC.

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