A Systematic Analysis of the Lepton Polarization Asymmetries in the Rare $B$ Decay, $B \to X_s \tau^+ \tau^-$

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Abstract

The most general model–independent analysis of the lepton polarization asymmetries in the rare $B$ decay, $B \to X_s \tau^+ \tau^-$, is presented. We present the longitudinal, normal and transverse polarization asymmetries for the $\tau^+$ and $\tau^-$, and combinations of them, as functions of the Wilson coefficients of twelve independent four–Fermi interactions, ten of them local and two nonlocal. These procedures will tell us which type of operators contributes to the process. And it will be very useful to pin down new physics systematically, once we have the experimental data with high statistics and a deviation from the Standard Model is found.
1 Introduction

Rare $B$-meson decays are very useful for constraining new physics beyond the Standard Model (SM). In particular, the processes $B \to X_s \gamma$ and $B \to X_s l^+ l^-$ are experimentally clean, and are possibly the most sensitive to the various extensions of the SM because these decays occur only through loops in the SM. Nonstandard model effects can manifest themselves in these rare decays through the Wilson coefficients, which can have values distinctly different from their Standard Model counterparts. (See for example, [1, 2, 3, 4, 5, 6, 7, 8, 9].) Compared to $B \to X_s \gamma$, the flavor changing leptonic decay $B \to X_s l^+ l^-$ is more sensitive to the actual form of the new interactions since we can measure experimentally various kinematical distributions as well as a total rate. While new physics will change only the systematically uncertain normalization for $B \to X_s \gamma$, the interplay of various operators will also change the spectra of the decay $B \to X_s l^+ l^-$. We can expect that the lepton polarization asymmetries in $B \to X_s \tau^+ \tau^-$ decay may also give useful information to fit parameters in the SM and constrain new physics [10, 11, 12, 13, 9, 14]. We note that the previous studies for lepton flavor asymmetries have been limited only to the subset of ten local four–Fermi interactions within specific extended models, such as the two-Higgs-doublet model, the minimal supersymmetric model, the left-right symmetric model, etc. (see for example [4, 15, 10].) In the SM and in many of its extensions, the decay $B \to X_s l^+ l^-$ is completely determined phenomenologically by the numerical values of Wilson coefficients of only three operators evaluated at the scale $\mu \sim m_b$. However, it would be most interesting if this three–parameter fit were found unsuccessful to explain the real experimental distributions, and if the new interactions even necessarily implied an extension of the ten local four–Fermi operator basis to new operators beyond the usual set [6]. And, therefore, the new physics scenario can be much richer than any of those models.

In our previous work [16], we studied the dependence on the four–Fermi interactions to the decay distribution and the forward-backward asymmetry of $B \to X_s l^+ l^-$, where $l$ is electron or muon [17]. We also studied the correlation between the branching ratio and the forward-backward asymmetry by changing each coefficient. From the study we also found that the dependence of the correlation on the coefficients of the vector–type interaction is large and we can use such information to identify the corresponding new physics contribution. However, we cannot get enough information for the scalar– and tensor–type interactions from such correlation studies. As the next step, we study here the case of $B \to X_s \tau^+ \tau^-$ and discuss the importance of measuring the $\tau$ polarization asymmetries to investigate the scalar– and tensor–type interactions. The contributions from the
scalar and tensor interactions will appear in the difference between the asymmetries for $\tau^-$ and $\tau^+$. The paper is organized as follows. In Section 2, we show the most general four–Fermi interactions and the decay distribution of $B \to X_s \tau^+ \tau^-$. In the previous work [16], we analyzed the decay $B \to X_s l^+ l^-$ based on ten local operators expansion. Here we expand our investigation by including the contribution from two nonlocal type operators. In Section 3, we present the longitudinal, normal and transverse polarization asymmetries of $\tau^+$ and $\tau^-$, and study their dependence on the four–Fermi interactions. We discuss the difference between the asymmetries for $\tau^-$ and $\tau^+$ in Section 4. The correlations between the branching ratio and the asymmetries are discussed in Section 5. Section 6 summarizes the results.

2 Dilepton Invariant Mass Distribution

We follow Refs. [11, 16, 18, 19, 20] for notations and for the choice of the parameters in the SM as well as for the incorporation of the long-distance effects of charmonium states. We start by defining the various kinematic variables. In this paper, the inclusive semileptonic $B$ decay is modeled by the partonic calculation, i.e., $b(p_b) \to s(p_s) + l^+(p_+) + l^-(p_-)$. This is regarded as the leading order calculation in the $1/m_b$ expansion [18, 21]. Then the decay distribution is described by the following two kinematic variables $s$ and $u$,

$$s = (p_b - p_s)^2 = (p_+ + p_-)^2 = m_b^2 + m_s^2 + m_+^2 + m_-^2 - t_+ - t_-,$$

$$u = t_+ - t_-,$$

with

$$t_+ = (p_s + p_+)^2 = (p_b - p_-)^2,$$

$$t_- = (p_s + p_-)^2 = (p_b - p_+)^2.$$

In the center of mass frame of the dileptons, $u$ is written in terms of $\theta$, i.e., the angle between the momentum of the $B$ meson and that of $l^+$,

$$u = -u(s) \cdot \cos \theta \equiv -u(s)z,$$

with

$$z = \cos \theta,$$

$$u(s) = \sqrt{(s - (m_b + m_s)^2)(s - (m_b - m_s)^2)(1 - \frac{4m_l^2}{s})}.$$

The phase space is defined in terms of $s$ and $z$,

$$4m_l^2 \leq s \leq (m_b - m_s)^2,$$

$$-1 \leq z \leq 1.$$
We want to consider the inclusive lepton polarization asymmetries as functions of the Wilson coefficients of the following twelve most general independent four–Fermi interactions. There are two nonlocal \((C_{SL}, C_{BR})\), and ten local four–Fermi interactions;

\[
\mathcal{M} = \frac{G_F}{\sqrt{2\pi}} V_{ts} V_{tb} \left[ C_{SL} \bar{s} \sigma_{\mu \nu} \frac{q^\nu}{q^2} (m_s L) b \bar{l} \gamma^\mu l \right. \\
+ C_{BR} \bar{s} \sigma_{\mu \nu} \frac{q^\nu}{q^2} (m_b R) b \bar{l} \gamma^\mu l \\
+ C_{LL} \bar{s} L \gamma^\mu b L \bar{l}_L \gamma^\mu l_L \\
+ C_{LR} \bar{s} L \gamma^\mu b L \bar{l}_R \gamma^\mu l_R \\
+ C_{RL} \bar{s} R \gamma^\mu b R \bar{l}_L \gamma^\mu l_L \\
+ C_{RR} \bar{s} R \gamma^\mu b R \bar{l}_R \gamma^\mu l_R \\
+ C_{LRLR} \bar{s} L \bar{b} L \bar{l}_L l_R \\
+ C_{RLLR} \bar{s} R \bar{b} L \bar{l}_L l_R \\
+ C_{LRRL} \bar{s} L \bar{b} R \bar{l}_R l_L \\
+ C_{RLRL} \bar{s} R \bar{b} R \bar{l}_R l_L \\
+ \left. iC_{TE} \bar{s} \sigma_{\mu \nu} b \bar{l} \sigma^\mu \nu \right] .
\] (4)

where \(C_{XX}\)'s are the coefficients of the four–Fermi interactions. Among them, there are two nonlocal four–Fermi interactions denoted by \(C_{SL}\) and \(C_{BR}\), which correspond to \(-2C_7\) in the SM, and which are constrained by the experimental data of \(b \to s \gamma\). There are four vector–type interactions denoted by \(C_{LL}, C_{LR}, C_{RL},\) and \(C_{RR}\). Two of them \((C_{LL}, C_{LR})\) are already present in the SM as the combinations of \((C_9 - C_{10}, C_9 + C_{10})\). Therefore, they are regarded as the sum of the contribution from the SM and the new physics deviations \((C_{LL}^{new}, C_{LR}^{new})\). The other vector interactions denoted by \(C_{RL}\) and \(C_{RR}\) are obtained by interchanging the chirality projections \(L \leftrightarrow R\). There are four scalar–type interactions, \(C_{LRLR}, C_{RLLR}, C_{RLRL}\) and \(C_{RLRL}\). The remaining two denoted by \(C_T\) and \(C_{TE}\) correspond to tensor–type. The subindices, \(L\) and \(R\), are chiral projections, \(L = \frac{1}{2}(1 - \gamma_5)\) and \(R = \frac{1}{2}(1 + \gamma_5)\).

The differential decay rate of \(B \to X_s l^+ l^-\) as a function of the dilepton invariant mass is shown as follows:

\[
\frac{d\mathcal{B}}{ds} = \frac{1}{2m_b^8} \mathcal{B}_0 \Re \left[ S_1(s) \left\{ m_s^2 |C_{SL}|^2 + m_b^2 |C_{BR}|^2 \right\} \\
+ S_2(s) \left\{ 2m_b m_s C_{SL}^* C_{BR} \right\} \\
+ S_3(s) \left\{ 2m_s^2 C_{SL} (C_{LL}^* + C_{LR}^*) + 2m_b m_s C_{BR} (C_{RL}^* + C_{RR}^*) \right\} \right] .
\]
\[\begin{align*}
&+ S_4(s) \{2m_b^2C_{BR}(C_{LL}^* + C_{LR}^*) + 2m_b m_s C_{SL}(C_{RL}^* + C_{RR}^*)\} \\
&+ S_5(s) \{2(m_s C_{SL} + m_b C_{BR})C_T^*\} \\
&+ S_6(s) \{4 (m_bC_{BR} - m_s C_{SL})C_{TE}\} \\
&+ M_2(s) \{ |C_{LL}|^2 + |C_{LR}|^2 + |C_{RL}|^2 + |C_{RR}|^2 \} \\
&+ M_6(s) \{ -2(C_{LL}C_{RL}^* + C_{LR}C_{RR}^*) \\
&\quad + (C_{LRLR}C_{RLRL}^* + C_{LRLL}C_{RLRL}^*)\} \\
&+ M_8(s) \{ |C_{LRLR}|^2 + |C_{RLRL}|^2 + |C_{LRLL}|^2 + |C_{RLRL}|^2 \} \\
&+ M_9(s) \{ 16|C_T|^2 + 64|C_{TE}|^2 \} \\
&+ N_1(s) \{ 2(C_{LL}C_{LR}^* + C_{RL}C_{RR}^*) \\
&\quad - (C_{LRLR}C_{RLRL}^* + C_{LRLL}C_{RLRL}^*)\} \\
&+ N_5(s) \{ -2(C_{LL}C_{RL}^* + C_{LR}C_{RR}^*) \\
&\quad + (C_{LRLR}C_{RLRL}^* + C_{LRLL}C_{RLRL}^*)\} \\
&+ N_5(s) \{ 2(C_{LL}C_{RR}^* + C_{LR}C_{RL}^*) \\
&\quad + \frac{1}{2}(C_{LRLR}C_{RLRL}^* + C_{LRLL}C_{RLRL}^*)\} \\
&+ N_6(s) \{ 2(C_{LL} - C_{LR})(C_{RLRL}^* - C_{LRLR}^*) \\
&\quad + 2(C_{RL} - C_{RR})(C_{RLRL}^* - C_{LRLR}^*)\} \\
&+ N_7(s) \{ 2(C_{LL} - C_{LR})(C_{RLRL}^* - C_{LRLR}^*) \\
&\quad + 2(C_{RL} - C_{RR})(C_{RLRL}^* - C_{LRLR}^*)\} \\
&+ N_8(s) \{ 2(C_{LL} + C_{LR})(C_{T}^* + 2(C_{RL} + C_{RR})(C_{T}^* )) \} \\
&+ N_9(s) \{ 2(C_{LL} + C_{LR})(C_{TE}^* + 2(C_{RL} + C_{RR})(C_{TE}^* )) \} \\
&+ N_5(s) \{ -192|C_{TE}|^2 \}],
\end{align*}\]

where \(B_0\) is a normalization factor normalized to the semileptonic decay

\[B_0 = B_{sl} \frac{3 \alpha_s^2}{16 \pi} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{1}{f(m_c) \kappa(m_c)},\]

and the phase space factor, \(f(m_c = \frac{m_c}{m_b})\), and the \(O(\alpha_s)\) QCD correction factor \(\kappa(m_c)\), of \(b \to c \ell \nu\) are given by

\[\begin{align*}
f(m_c) &= 1 - 8m_c^2 + 8m_c^6 - m_c^8 - 24m_c^4 \ln m_c, \\
\kappa(m_c) &= 1 - \frac{2 \alpha_s(m_b)}{3 \pi} \left( [\pi^2 - \frac{31}{4}](1 - m_c)^2 + \frac{3}{2} \right).
\end{align*}\]

For the numerical calculations, we set \(|V_{ts}^* V_{tb}|^2/|V_{cb}|^2 = 1\) and use the experimental value of the
semileptonic branching fraction $B_{st} = 10.4\%$. $S_n(s), M_n(s)$ and $N_n(s)$ are functions of the dilepton invariant mass $s$,

$$
S_1(s) = \frac{1}{s^2} u(s) [-16m_1^2(m_b^2 + m_s^2)s - (m_b^2 - m_s^2)^2] - 4s\{s^2 - \frac{1}{3}u(s)^2 - (m_b^2 - m_s^2)^2\}], \\
S_2(s) = \frac{1}{s} u(s)m_bm_s(-32m_1^2 - 16s), \\
S_3(s) = \frac{1}{s} u(s)\{8m_1^2(s + m_b^2 - m_s^2) + 4s(s + m_b^2 - m_s^2)\}, \\
S_4(s) = \frac{1}{s} u(s)\{8m_1^2(s - m_b^2 + m_s^2) + 4s(s - m_b^2 + m_s^2)\}, \\
S_5(s) = \frac{1}{s} u(s)m_l\{96m_bm_s s + 16s(m_b^2 + m_s^2 - s) + 32(s^2 - (m_b^2 - m_s^2)^2)\}, \\
S_6(s) = \frac{1}{s} u(s)m_l\{96m_bm_s s - 16s(m_b^2 + m_s^2 - s) - 32(s^2 - (m_b^2 - m_s^2)^2)\}, \\
M_2(s) = 2u(s)(-\frac{1}{3}u(s)^2 - s^2 + (m_b^2 - m_s^2)^2), \\
M_6(s) = 8u(s)m_bm_s(2m_1^2 + s), \\
M_8(s) = -2u(s)(m_b^2 + m_s^2 - s)(2m_1^2 - s), \\
M_9(s) = 2u(s)\{4m_1^2(m_b^2 - 6m_bm_s + m_s^2 - s) - \frac{2}{3}u(s)^2 - 2(m_b^2 + m_s^2)s + 2(m_b^2 - m_s^2)^2\}, \\
N_1(s) = 8u(s)m_1^2(m_b^2 + m_s^2 - s), \\
N_5(s) = -32u(s)m_bm_s m_1^2, \\
N_6(s) = 2u(s)m_1m_b(s - m_b^2 + m_s^2), \\
N_7(s) = 2u(s)m_1m_b(s + m_b^2 - m_s^2), \\
N_8(s) = 24u(s)m_l(-(m_b + m_s)s + (m_b - m_s)(m_b^2 - m_s^2)), \\
N_9(s) = 48u(s)m_l((m_b - m_s)s - (m_b + m_s)(m_b^2 - m_s^2)).
$$

(9)

The terms with the functions $S_5(s), S_6(s)$ and $N_n(s)$ appear only in the case of massive leptonic decay, $B \rightarrow X_s\tau^+\tau^-$. For the massless cases, $B \rightarrow X_s e^+e^-$ and $B \rightarrow X_s \mu^+\mu^-$, we already discussed the influence of new interactions on the differential decay rate in Ref. [16]. Now we want to expand our discussion for the massive case, $B \rightarrow X_s\tau^+\tau^-$. However, in this case it is not easy to determine the dependence of each coefficient of the twelve new interactions from the given differential decay rate only, because of some terms with $N_n(s)$ which show the cross term dependence between the different type operators, as shown in Eq. (5). In the next Section we investigate instead the lepton polarization asymmetries. The dilepton’s invariant mass distribution of $B \rightarrow X_s\tau^+\tau^-$ within the SM is shown in Fig. [13]
3 Lepton Polarization Asymmetries

We now compute the lepton polarization asymmetries from the four–Fermi interactions defined in Eq. (4). We define the following orthogonal unit vectors, $S$ in the rest frame of $l^-$ and $W$ in the rest frame of $l^+$, for the polarization of the leptons [10, 11] to the longitudinal direction ($L$), the normal direction ($N$) and the transverse direction ($T$)

\begin{align*}
S^\mu_L &\equiv (0, e_L) = (0, \frac{p_-}{|p_-|}), \\
S^\mu_N &\equiv (0, e_N) = (0, \frac{p_s \times p_-}{|p_s \times p_-|}), \\
S^\mu_T &\equiv (0, e_T) = (0, e_N \times e_L), \\
W^\mu_L &\equiv (0, w_L) = (0, \frac{p_+}{|p_+|}), \\
W^\mu_N &\equiv (0, w_N) = (0, \frac{p_s \times p_+}{|p_s \times p_+|}), \\
W^\mu_T &\equiv (0, w_T) = (0, w_N \times w_L),
\end{align*}

where $p_\pm$ and $p_s$ are the three momenta of the $l^\pm$ and the final strange ($s$) quark in the center–of–mass (CM) frame of the $l^+l^-$ system. The longitudinal unit vectors, $S_L$ and $W_L$, are boosted by Lorentz transformation to CM frame of $l^+l^-$,

\begin{align*}
S^\mu_{LCM} &= \left( \frac{|p_-|}{m_l}, \frac{E_l p_-}{m_l |p_-|} \right), \\
W^\mu_{LCM} &= \left( \frac{|p_-|}{m_l}, -\frac{E_l p_-}{m_l |p_-|} \right),
\end{align*}
while the vectors of perpendicular direction are not changed by the boost.

The differential decay rate of $B \to X_d l^+ l^-$ for any spin direction of leptons can be computed from the following spinor expression of the matrix elements:

\[
\mathcal{M} = C_{SL} \bar{u}(p_s) i\sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_s L) u(p_b) \bar{u}(p_-) P^{\gamma\mu} Q v(p_+) \\
+ C_{BR} \bar{u}(p_s) i\sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_b R) u(p_b) \bar{u}(p_-) P^{\gamma\mu} Q v(p_+) \\
+ C_{LL} \bar{u}(p_s) \gamma_\mu L u(p_b) \bar{u}(p_-) P^{\gamma\mu} L Q v(p_+) \\
+ C_{LR} \bar{u}(p_s) \gamma_\mu L u(p_b) \bar{u}(p_-) P^{\gamma\mu} R Q v(p_+) \\
+ C_{RL} \bar{u}(p_s) \gamma_\mu R u(p_b) \bar{u}(p_-) P^{\gamma\mu} L Q v(p_+) \\
+ C_{RR} \bar{u}(p_s) \gamma_\mu R u(p_b) \bar{u}(p_-) P^{\gamma\mu} R Q v(p_+) \\
+ C_{LRLR} \bar{u}(p_s) R u(p_b) \bar{u}(p_-) P R Q v(p_+) \\
+ C_{LRLR} \bar{u}(p_s) R u(p_b) \bar{u}(p_-) P L Q v(p_+) \\
+ C_{RRLR} \bar{u}(p_s) R u(p_b) \bar{u}(p_-) P R Q v(p_+) \\
+ C_{RLRL} \bar{u}(p_s) R u(p_b) \bar{u}(p_-) P L Q v(p_+) \\
+ C_T \bar{u}(p_s) \sigma_{\mu\nu} u(p_b) \bar{u}(p_-) P^{\sigma\mu\nu} Q v(p_+) \\
+ i C_{TE} \bar{u}(p_s) \sigma_{\mu\nu} u(p_b) \bar{u}(p_-) P^{\sigma\mu\nu} Q v(p_+) \epsilon^{\nu\alpha\beta}, \tag{10}
\]

where $P = \frac{1}{2}(1 + \gamma_5 \gamma_\mu S^\mu)$ and $Q = \frac{1}{2}(1 + \gamma_5 \gamma_\mu W^\mu)$ are spin projection operators. Then the lepton polarization asymmetries are defined as

\[
P_x^- \equiv \frac{\frac{d\Gamma(S_x, W_x)}{ds} + \frac{d\Gamma(S_x, -W_x)}{ds}}{\frac{d\Gamma(S_x, W_x)}{ds} + \frac{d\Gamma(S_x, -W_x)}{ds}} - \frac{\frac{d\Gamma(-S_x, W_x)}{ds} + \frac{d\Gamma(-S_x, -W_x)}{ds}}{\frac{d\Gamma(-S_x, W_x)}{ds} + \frac{d\Gamma(-S_x, -W_x)}{ds}}, \tag{11}
\]

\[
P_x^+ \equiv \frac{\frac{d\Gamma(S_x, W_x)}{ds} + \frac{d\Gamma(S_x, -W_x)}{ds}}{\frac{d\Gamma(S_x, W_x)}{ds} + \frac{d\Gamma(S_x, -W_x)}{ds}} - \frac{\frac{d\Gamma(-S_x, W_x)}{ds} + \frac{d\Gamma(-S_x, -W_x)}{ds}}{\frac{d\Gamma(-S_x, W_x)}{ds} + \frac{d\Gamma(-S_x, -W_x)}{ds}}, \tag{12}
\]

where the subindex $x$ is $L, T$ or $N$. $P^\pm$ denotes lepton polarization asymmetry of charged lepton $l^\pm$. $P_L$ denotes the longitudinal polarization, $P_T$ is the polarization asymmetry in the decay plane and $P_N$ is the normal component to both of them. As we can see from the direction of the lepton polarization, $P_L$ and $P_T$ are P-odd, T-even and CP-even observables while $P_N$ is P-even, T-odd and CP-odd observable.

The longitudinal polarization asymmetries for each lepton are

\[
P_L^\pm = \frac{B_0}{m_b} \frac{u(s)}{s} \frac{dL_1(s)}{ds} \Re\left\{ 2 L_1(s) \left( m_b^2 C_{BR}(C_{LL}^* - C_{LR}^*) + m_b m_s C_{SL}(C_{RL}^* - C_{RR}^*) \right) \right\} \tag{4}
\]

This is because the time reversal operation changes the signs of momentum and spin, and because the parity transformation changes only the sign of momentum.
\[ + 2L_2(s) \{ -m_s^2 C_{SL}(C_{LL}^* - C_{LR}^*) - m_b m_s C_{BR}(C_{RL}^* - C_{RR}^*) \} \]
\[ + L_3(s) \{ |C_{LL}|^2 - |C_{LR}|^2 + |C_{RL}|^2 - |C_{RR}|^2 - 128C_T C_{TE}^* \} \]
\[ + L_4(s) \{ 2C_{LL} C_{RL}^* - 2C_{LR} C_{RR}^* - C_{LRLR} C_{RLLR}^* + C_{LRLR} C_{RLRL} \} \]
\[ + 2L_5(s) \{ m_b C_{BR}(-C_{T}^* + 2C_{TE}^*) + m_s C_{SL}(C_{T}^* + 2C_{TE}^*) \} \]
\[ + 2L_6(s) m_b \{(C_{LL} - C_{LR})(C_{LRLR}^* + C_{RLRL}^*) \}
\[ + (C_{RL} - C_{RR})(C_{RLLR}^* + C_{RLRL}) \]
\[ + 4[(-3C_{LL} + C_{LR})(C_{T}^* - 2C_{TE}^*) \]
\[ + (-C_{RL} + 3C_{RR})(C_{T}^* + 2C_{TE}^*)] \}
\[ + 2L_7(s) m_s \{(C_{LL} - C_{LR})(C_{RLLR}^* + C_{RLRL}^*) \}
\[ + (C_{RL} - C_{RR})(C_{LRLR}^* + C_{RLRL}) \]
\[ + 4[(C_{LL} - 3C_{LR})(C_{T}^* + 2C_{TE}^*) \]
\[ + (3C_{RL} - C_{RR})(C_{T}^* - 2C_{TE}^*)] \}
\[ + L_8(s) \{ -|C_{LRLR}|^2 + |C_{LRLR}|^2 - |C_{LRLR}|^2 + |C_{RLRL}|^2 - 128C_T C_{TE}^* \} \]

and

\[
P_L^+ = \frac{R_0 u(s)}{m_b^8 \frac{d\beta}{ds}} \text{Re} \left[ + 2L_1(s) \{ -m_b^2 C_{BR}(C_{LL}^* - C_{LR}^*) - m_b m_s C_{SL}(C_{RL}^* - C_{RR}^*) \} \right]
\[ + 2L_2(s) \{ m_b^2 C_{SL}(C_{LL}^* - C_{LR}^*) + m_b m_s C_{BR}(C_{RL}^* - C_{RR}^*) \} \]
\[ + L_3(s) \{ -|C_{LL}|^2 + |C_{LR}|^2 - |C_{RL}|^2 + |C_{RR}|^2 - 128C_T C_{TE}^* \} \]
\[ + L_4(s) \{ -2C_{LL} C_{RL}^* + 2C_{LR} C_{RR}^* - C_{LRLR} C_{RLLR}^* - C_{LRLR} C_{RLRL} \} \]
\[ + 2L_5(s) \{ m_b C_{BR}(-C_{T}^* + 2C_{TE}^*) + m_s C_{SL}(C_{T}^* + 2C_{TE}^*) \} \]
\[ + 2L_6(s) m_b \{(C_{LL} - C_{LR})(C_{LRLR}^* + C_{RLRL}^*) \}
\[ + (C_{RL} - C_{RR})(C_{RLLR}^* + C_{RLRL}) \]
\[ + 4[(-3C_{LL} + C_{LR})(C_{T}^* - 2C_{TE}^*) \]
\[ + (-C_{RL} + 3C_{RR})(C_{T}^* + 2C_{TE}^*)] \}
\[ + 2L_7(s) m_s \{(C_{LL} - C_{LR})(C_{RLLR}^* + C_{RLRL}^*) \}
\[ + (C_{RL} - C_{RR})(C_{LRLR}^* + C_{RLRL}) \]
\[ + 4[(C_{LL} - 3C_{LR})(C_{T}^* + 2C_{TE}^*) \]
\[ + (3C_{RL} - C_{RR})(C_{T}^* - 2C_{TE}^*)] \}
\[ + L_8(s) \{ -|C_{LRLR}|^2 + |C_{LRLR}|^2 - |C_{LRLR}|^2 + |C_{RLRL}|^2 - 128C_T C_{TE}^* \} \]
where \( L_n(s) \) are the functions of kinetic variables \( s \)

\[
\begin{align*}
L_1(s) &= 2(-s + m_b^2 - m_s^2)v(s), \\
L_2(s) &= 2(s + m_b^2 - m_s^2)v(s), \\
L_3(s) &= \{(s^2 - (m_b^2 - m_s^2)^2)v(s) + u(s)^2 \frac{1}{3v(s)}\}, \\
L_4(s) &= 4m_bm_s v(s), \\
L_5(s) &= \frac{8}{s} m_t \{(m_b^2 - m_s^2)^2 v(s) + s(m_b - m_s)^2 v(s) + u(s)^2 \frac{1}{3v(s)}\}, \\
L_6(s) &= m_t(-s + m_b^2 - m_s^2)v(s), \\
L_7(s) &= m_t(s + m_b^2 - m_s^2)v(s), \\
L_8(s) &= s(m_b^2 + m_s^2 - s)v(s),
\end{align*}
\]

with \( v(s) = \sqrt{1 - \frac{4m_t^2}{s}}. \)

We note that the contributions from the SM (i.e. from \( C_{BR}, C_{SL}, C_{LL} \) and \( C_{LR} \)) to \( P^-_L \) are exactly the same as those to \( P^+_L \), but with the opposite sign. However, the contributions from new interactions are totally different. This difference is very interesting and useful for the search for the new physics effects later.

The transverse asymmetries, \( P^-_T \) and \( P^+_T \), are:

\[
P^-_T = \frac{B_0 u(s)w(s)}{m_b^8 \frac{ds}{ds}} \text{Re}\left[ T_1(s) \{-m_b^2|C_{BR}|^2 + m_s^2|C_{SL}|^2\} \right. \\
+ 4m_t \{m_b^2 C_{BR}(3C_{LL}^* + C_{LR}^*) + m_s^2 C_{SL}(C_{LL}^* + 3C_{LR}^*) \\
- m_b m_s C_{BR}(3C_{RL}^* + C_{RR}^*) - m_b m_s C_{SL}(C_{RL}^* + 3C_{RR}^*)\} \right] \\
+ T_2(s) \{-|C_{LL}|^2 + |C_{RR}|^2\} \\
+ 2T_3(s) \{ -C_{LL} C_{LR}^* + C_{RR} C_{RR}^* \\\n+ (2T_T - 4C_{TE})(C_{LRLR}^* - C_{LRRL}^*) \\\n+ (2C_T + 4C_{TE})(C_{RLRL}^* - C_{RRLR}^*)\} \right] \\
+ T_4(s) \{|C_{LR}|^2 - |C_{RL}|^2\} \\
+ 2T_5(s) \{ m_b C_{BR}(C_{LRLR}^* - C_{LRRL}^*) + m_s C_{SL}(C_{LRLR}^* + C_{LRRL}^*)\} \\
+ 2T_6(s) \{ m_b C_{BR}(C_T^* - 2C_{TE}^*) - m_s C_{SL}(C_T^* + 2C_{TE}^*)\} \\
+ 4m_t^2 \{ m_b(C_{LL} C_{LRLR}^* - C_{LR} C_{LRRL}^*) + m_s(C_{LL} C_{RLRL}^* - C_{LR} C_{RLRL}^*) \\
+ m_s(C_{RL} C_{LRLR}^* - C_{RR} C_{LRRL}^*) + m_b(C_{RL} C_{RLRL}^* - C_{RR} C_{RLRL}^*) \\
- 12m_b(T_T - 2C_{TE})(C_{LL}^* - C_{RR}^*)\}.
\]
\[-12m_s(C_T + 2C_{TE})(C_{LR}^* - C_{RL}^*)\]
\[ + 2T_7(s)\{m_b(C_{LL}C_{LRRL}^* - C_{LR}C_{RLRL}^*) + m_s(C_{LL}C_{RLRL}^* - C_{LR}C_{RLLL}^*)\]
\[ + m_s(C_{RL}C_{LRRL}^* - C_{RR}C_{LRLR}^*) + m_b(C_{RL}C_{RLRL}^* - C_{RR}C_{RLLL}^*)\]
\[ + 4m_s(C_T + 2C_{TE})(C_{LL}^* - C_{RR}^*) + 4m_b(C_T - 2C_{TE})(C_{LR}^* - C_{RL}^*)\]
\[ + 256m_l(m_b^2 - m_s^2)C_T C_{TE}^*\],
\[ (16) \]

and

\[ P_T^+ = \frac{B_0 u(s) w(s)}{m_b^8 \frac{ds}{ds}} \text{Re} \left[ T_1(s) \{-m_b^2|C_{BR}|^2 + m_s^2|C_{SL}|^2\} \right. \]
\[ + 4m_l \{m_b^2C_{BR}(C_{LL}^* + 3C_{LR}^*) + m_s^2C_{SL}(3C_{LL}^* + C_{LR}^*)\}
\[ - m_b m_s C_{BR}(C_{RL}^* + 3C_{RR}^*) - m_b m_s C_{SL}(3C_{RL}^* + C_{RR}^*)\}\]
\[ + T_2(s) \{-|C_{LL}|^2 + |C_{RR}|^2\} \]
\[ + 2T_3(s) \{-C_{LL}C_{LR}^* + C_{RL}C_{RR}^*\}
\[ - (2C_T - 4C_{TE})(C_{LRLR}^* - C_{LLRL}^*)\]
\[ - (2C_T + 4C_{TE})(C_{RLRL}^* - C_{RLLR}^*)\}\]
\[ + T_4(s) \{|C_{LR}|^2 - |C_{RL}|^2\} \]
\[ + 2T_5(s) \{-m_b C_{BR}(-C_{LRLR}^* + C_{LLRL}^*) - m_s C_{SL}(-C_{LRLR}^* + C_{LLRL}^*)\}
\[ + 2T_6(s) \{m_b C_{BR}(C_T^* - 2C_{TE}^*) - m_s C_{SL}(C_T^* + 2C_{TE}^*)\}\]
\[ + 4m_l^2 \{m_b(C_{LL}C_{LRRL}^* - C_{LR}C_{RLRL}^*) + m_s(C_{LL}C_{RLRL}^* - C_{LR}C_{RLLL}^*)\}
\[ + m_s(C_{RL}C_{LRRL}^* - C_{RR}C_{LRLR}^*) + m_b(C_{RL}C_{RLRL}^* - C_{RR}C_{RLLL}^*)\]
\[ - 12m_s(C_T + 2C_{TE})(C_{LL}^* - C_{RR}^*)\]
\[ - 12m_b(C_T - 2C_{TE})(C_{LR}^* - C_{RL}^*)\}\]
\[ + 2T_7(s)\{m_b(C_{LL}C_{LRRL}^* - C_{LR}C_{RLRL}^*) + m_s(C_{LL}C_{LRRL}^* - C_{LR}C_{RLRL}^*)\}
\[ + m_s(C_{RL}C_{LRRL}^* - C_{RR}C_{LRLR}^*) + m_b(C_{RL}C_{RLRL}^* - C_{RR}C_{RLLL}^*)\]
\[ + 4m_b(C_T - 2C_{TE})(C_{LL}^* - C_{RR}^*) + 4m_s(C_T + 2C_{TE})(C_{LR}^* - C_{RL}^*)\]
\[ + 256m_l(m_b^2 - m_s^2)C_T C_{TE}^*\],
\[ (17) \]

where \( T_n(s), w(s) \) are the functions of kinetic variables \( s \)

\[ T_1(s) = \frac{8}{s} m_l(m_b^2 - m_s^2), \]
\[ T_2(s) = 2m_l(s + m_b^2 - m_s^2), \]
\[ T_3(s) = 2m_l s, \]

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Finally the normal asymmetries, \( P_{N}^{-} \) and \( P_{N}^{+} \), are:

\[
P_{N}^{-} = -\frac{B_0 u(s) z(s)}{m_b s^{2} \frac{dB}{ds}} \left[ \frac{8m_l}{s} \left\{ m_b^2 C_{BR}(C_{RL}^* - C_{LR}^*) + m_s^2 C_{SL}(C_{LL}^* - C_{LR}^*) \\
+ m_b m_s (C_{BR} + C_{SL})(-C_{RL}^* + C_{RR}^*) \right\} + 8m_l \left\{ C_{LL} C_{LR}^* - C_{RL} C_{RR}^* \right\} + 2(C_T - 2C_{TE})(C_{LRLR}^* + C_{LRRL}^*) + 2(C_T + 2C_{TE})(C_{RLRL}^* + C_{RRLR}^*) \right]
\]

\[
P_{N}^{+} = -\frac{B_0 u(s) z(s)}{m_b s^{2} \frac{dB}{ds}} \left[ \frac{8m_l}{s} \left\{ -m_b^2 C_{BR}(C_{LL}^* - C_{LR}^*) - m_s^2 C_{SL}(C_{LL}^* - C_{LR}^*) - m_b m_s (C_{BR} + C_{SL})(-C_{RL}^* + C_{RR}^*) \right\} + 8m_l \left\{ -C_{LL} C_{LR}^* + C_{RL} C_{RR}^* \right\} - 2(C_T - 2C_{TE})(C_{LRLR}^* + C_{LRRL}^*) - 2(C_T + 2C_{TE})(C_{RLRL}^* + C_{RRLR}^*) \right]
\]

\[
\]

(18)
where
\[ z(s) = \frac{\pi \sqrt{su(s)}}{8}. \]

Note that several terms survive even if we take \( m_l \to 0 \) limit. In that limit, the terms \( T_5, T_6 \) and \( T_7 \) of Eq. (18) in \( P^\pm_T \), related to the coefficients of the scalar– and tensor–type operators, still remain. Similarly, the terms \( L_1, L_2, L_3, L_4 \) and \( L_8 \) of Eq. (15) survive in the massless lepton limit. Therefore, if we can measure the experimental values of \( P^\pm_T, L \) for \( B \to X_s \mu^+ \mu^- \), they may be very useful for searching for new interactions. Concerning the asymmetries \( P^\pm_N \), while \( P^-_N = -P^+_N \) in the SM, the difference between \( P^-_N \) and \( -P^+_N \) comes only from new interactions of the scalar- and tensor–type. By using this difference, we may be able to study the information about the imaginary part of these interactions. Of course, for the massive lepton case we can consider the difference between the asymmetries \( \tau^+ \) and \( \tau^- \) to find the contribution of new interactions. We are going to discuss this in the next Section.

4 Combined Analysis of the Asymmetries for \( \tau^- \) and \( \tau^+ \)

We can get very useful information to constrain the parameters and to find the evidence of new physics by measuring the asymmetries for each lepton, \( \tau^+ \) and \( \tau^- \), and combining those asymmetries. We show \( P^+_L + P^+_L, P^-_T - P^+_T \), and \( P^-_N + P^+_N \), because within the SM, \( P^+_L + P^+_L = 0, P^-_T - P^+_T \approx 0 \) and \( P^-_N + P^+_N = 0 \).

(A) For \( P^-_L + P^+_L \), the result is;

\[ P^+_L + P^-_L = \frac{B_0 \ u(s)}{m_b^2 \ \frac{d\sigma}{ds}} \ \text{Re}\{ 2L_4(s) \ \{-C^*_{LRLR}C^*_{RLRR} + C^*_{LRRL}C^*_{RLRL}\} \]
\[ + 4L_5(s) \ \{m_b C_{BR}(-C^*_T + 2C^*_TE) + m_s C_{SL}(C^*_T + 2C^*_TE)\}\]
\[ + 2L_6(s) \ m_b(2(C_{LL} - C_{LR})(C^*_{LRLR} + C^*_{LRRL}) \]
\[ + 2(C_{RL} - C_{RR})(C^*_LRLR + C^*_RLRL) \]
\[ - 8(C_{LL} + C_{LR})(C^*_T - 2C^*_TE) \]
\[ + 8(C_{RL} + C_{RR})(C^*_T + 2C^*_TE)\}\]
\[ + 2L_7(s) \ m_s \{2(C_{LL} - C_{LR})(C^*_{LRLR} + C^*_{RLRL}) \]
\[ + 2(C_{RL} - C_{RR})(C^*_LRLR + C^*_RLRL) \]
\[ - 8(C_{LL} + C_{LR})(C^*_T + 2C^*_TE) \]
\[ + 8(C_{RL} + C_{RR})(C^*_T + 2C^*_TE)\}\]
\[ + 2L_8(s) \ \{-|C_{LRLR}|^2 + |C_{LRLR}|^2 - |C_{RLRL}|^2 + |C_{RLRL}|^2\} \]

\[ (21) \]
As can be seen from Eq. (21), the contribution from the SM to $P_L^+ + P_L^-$ completely disappears. There must be new interactions and new physics in the decay $B \to X_s \tau^+ \tau^-$, if the value of $P_L^+ + P_L^-$ is nonzero. We can then determine the parameters of the scalar– and tensor–type interactions, as shown in Eq. (21). If we neglect the small strange quark mass $m_s$ (i.e. $L_4$ and $L_7$) and retain only terms with $m_b$, then we find the following:

- There is no contribution from the combination of vector–type and nonlocal operators because such terms are all canceled.

- The contribution from $C_{LRLR} + C_{LRRL}$ is much larger than from the other scalar–type coefficients, and it is

\[
- 8m_b L_6(s) \left[ C_{10}(C_{LRLR}^* + C_{LRRL}^*) \right] \\
- 2L_8(s) \left[ (C_{LRLR} + C_{LRRL})(C_{LRLR}^* - C_{LRRL}^*) \right].
\]  

(22)

- The tensor–type coefficients give the contribution

\[
- 256(L_3(s) + L_8(s)) \left[ C_T^* C_{TE} \right] \\
- 8L_5(s) m_b C_T \left[ -C_T^* + 2C_{TE}^* \right] - 32L_6(s) m_b \left[ C_{9}^{\text{eff}}(C_T^* - 2C_{TE}^*) \right].
\]  

(23)

(B) We now consider $P_T^- - P_T^+$:

\[
P_T^- - P_T^+ = \frac{\beta_0 u(s)w(s)}{m_b^8 \frac{ds}{d\tau}} \left[ 4m_l \left\{ 2m_b^2 C_{BR}(C_{LL}^* - C_{LR}^*) - 2m_s^2 C_{SL}(C_{LL}^* - C_{LR}^*) \right. \right.
\]

\[
- 2m_b m_s C_{BR}(C_{RL}^* - C_{RR}^*) + \left. 2m_b m_s C_{SL}(C_{RL}^* - C_{RR}^*) \right\} \\
+ 4T_3(s) \left\{ (2C_T - 2C_{TE})(C_{LRLR}^* - C_{LRRL}^*) \right. \\
\]

\[
+ 2(C_T + 2C_{TE})(C_{RLRL}^* - C_{RLRR}^*) \\
+ 2T_5(s) \left\{ 2m_b C_{BR}(-C_{LRLR}^* + C_{LRRL}^*) \right. \\
\]

\[
+ 2m_s C_{SL}(-C_{RLRR}^* + C_{RLRL}^*) \\
+ 4m_l^2 \left\{ m_b(C_{LL} + C_{LR})(C_{LRLR}^* - C_{LRRL}^*) \right. \right.
\]

\[
+ m_s(C_{LL} + C_{LR})(C_{RLRR}^* - C_{RLRL}^*) \\
+ m_s(C_{RL} + C_{RR})(C_{LRLR}^* - C_{LRRL}^*) \\
+ m_b(C_{RL} + C_{RR})(C_{LRRL}^* - C_{RLRR}^*) \\
- 12m_b(C_T - 2C_{TE})(C_{LL}^* - C_{RR}^* - C_{LR}^* + C_{RL}^*) \right\].
\]
\[ -12m_s(C_T + 2C_{TE})(-C^*_{LL} + C^*_{RR} + C^*_{LR} - C^*_{RL}) \]
\[ + 2T_7(s) \{ -m_b(C_{LL} + C_{LR})(C^*_{LRLR} - C^*_{LRLL}) \]
\[ -m_s(C_{LL} + C_{LR})(C^*_{RLLR} - C^*_{RLRL}) \]
\[ -m_s(C_{RL} + C_{RR})(C^*_{LRLR} - C^*_{LRRL}) \]
\[ -m_b(C_{RL} + C_{RR})(C^*_{RLLR} - C^*_{RLRL}) \]
\[ + 4m_s(C_T + 2C_{TE})(C^*_{LL} - C^*_{RR} - C^*_{LR} + C^*_{RL}) \]
\[ + 4m_b(C_T - 2C_{TE})(-C^*_{LL} + C^*_{RR} + C^*_{LR} - C^*_{RL}) \} \]

The SM model contribution to \( P_T^- - P_T^+ \) is only

\[ 4m_t \{ 2m_b^2 \text{Re}[C_{BR}(C^*_{LL} - C^*_{LR})] - 2m_s^2 \text{Re}[C_{SL}(C^*_{LL} - C^*_{LR})] \} = 32m_t \left[ m_b^2 C_7C_{10} - m_s^2 C_7C_{10} \right] \]

We show in Fig. 2 the values of \( P_T^- - P_T^+ \) in the SM. If we observe the experimental results different from Fig. 2, we will have a strong evidence of the existence of physics beyond the SM.

![Figure 2: \( P_T^- - P_T^+ \) in the SM.](image)

After we neglect the terms with \( m_s \), we get

\[ P_T^- - P_T^+ = \frac{B_0 u(s)w(s)}{m_b^8 \frac{ds}{ds}} \text{Re} \left[ 4m_t \{ 2m_b^2 C_{BR}(C^*_{LL} - C^*_{LR}) \} \right. \]
\[ + 4T_3(s) \{ 2(C_T - 2C_{TE})(C^*_{LRLR} - C^*_{LRLL}) \]
\[ + 2(C_T + 2C_{TE})(C^*_{RLLR} - C^*_{RLRL}) \} \]
We find the following from the above expression:

- If there are only new vector–type interactions in addition to the SM operators, the extra contributions come from

\[ -16m_l m_b^2 C_7 \text{ Re}(C_{L}^{\text{new}} - C_{L}^{\text{new}}). \] (26)

Since \( dB/ds \) depends on \( C_{L} \) much more strongly than on any other Wilson coefficient (this was found out in the previous work [16]), \( P_T^- - P_T^+ \) is most sensitive to \( C_{L}^{\text{new}} \).

- The dependence on the scalar–type interactions is

\[
4m_b \text{ Re}\{[2T_5(s)C_7 + (2m_l^2 - T_7(s))C_9]\} (C_{LRLR}^* - C_{LRLR}^*)
= 4m_b s \text{ Re}\{[2C_7 + C_9]\} (C_{LRLR}^* - C_{LRLR}^*). \]

(27)

We find that here the contribution comes only from \( (C_{LRLR}^* - C_{LRLR}^*) \), in contrast to the case of \( P_L + P_L^+ \).

- The dependence on the tensor–type interactions is

\[
(96m_l^2 + 16T_7(s))m_b C_{10}(C_T^* - 2C_{TE}^*). \]

(28)

(C) For \( P_N^- + P_N^+ \), we get

\[
P_N^- + P_N^+ = -\frac{\mathcal{B}_0 u(s)z(s)}{m_b^2 4\pi} \text{ Im} \left[ 4 \{ -m_b(C_{LL} - C_{LR})(C_{LRLR}^* - C_{LRLR}^*) \\
- m_s(C_{LL} - C_{LR})(C_{LRLR}^* - C_{LRLR}^*) \\
- m_s(C_{RL} - C_{RR})(C_{LRLR}^* - C_{LRLR}^*) \\
- m_b(C_{RL} - C_{RR})(C_{LRLR}^* - C_{LRLR}^*) \\
+ 4(C_T - 2C_{TE})(m_b(C_{LL}^* + C_{LR}^*) - m_s(C_{RL}^* + C_{RR}^*)) \\
+ 4(C_T + 2C_{TE})(-m_s(C_{LL}^* + C_{LR}^*) + m_b(C_{RL}^* + C_{RR}^*)) \\
+ \frac{32}{m_b^2 - m_s^2} \{ m_b C_{BR}(C_T^* - 2C_{TE}^*) + m_s C_{SL}(C_T^* + 2C_{TE}^*) \} \right]. \]

(29)
It appears that the experimental observation of the above quantity is much more difficult than the other asymmetries because of the small numerical value. However, if we measure it, we will be able to get information about the imaginary parts of the coefficients for the scalar– and tensor–type operators from Eq. (29). After we neglect small strange quark mass $m_s \to 0$, we find:

- The dependence on the scalar–type interactions is

$$-\text{Im}[4m_b (C_{LL} - C_{LR})(C_{LRLR}^* - C_{LRRL}^*)] = 8m_b C_{10} \text{ Im}(C_{LRLR}^* - C_{LRRL}^*).$$

Therefore, we can get the imaginary part of $C_{LRLR} - C_{LRRL}$ from the well known SM value of $C_{10}$.

- The dependence on the tensor–type interactions is

$$\text{Im}[16m_b (C_T - 2C_{TE})(C_{LL}^* + C_{LR}^*) - \frac{32}{s} m_b^2 (m_b C_{BR}^* (C_T - 2C_{TE})) = 32m_b \text{ Im}[(2m_b^2 C_T + C_{9eff}^*) (C_T - 2C_{TE})].$$

(31)

It would be quite difficult to determine (ascertain) the imaginary parts of $C_T - 2C_{TE}$, because $C_{9eff}^*$ is also a complex number. However, we could still get some hints of the new physics.

## 5 Averaged Values of the Asymmetries

It may be experimentally difficult to measure the polarization asymmetries of each lepton for all $l^+l^-$ center–of–mass energies $s$. However, we may get easily the averaged polarization asymmetries. So we define the following averaged polarization asymmetries,

$$< P_x > \equiv \frac{\int_{14 \text{ GeV}^2}^{(m_b-m_s)^2} P_x(s) \frac{dR}{ds} ds}{\int_{14 \text{ GeV}^2}^{(m_b-m_s)^2} \frac{dR}{ds} ds},$$

(32)

where we integrated from 14 GeV$^2$ to avoid the resonance regions below $\psi'$. Within the SM, the results are

$$< P_L^\mp >= \mp 0.43, \quad < P_T^- >= -0.56, \quad \text{and} \quad < P_T^\mp >= \pm 0.05.$$  

$$< P_T^+ >= -0.73.$$  

Note that here we neglected the final state Coulomb interaction effect of the leptons with the other charged particles, since the effect has been estimated to be much smaller than the averaged values of the SM [11, 13]. The final state interaction effect in the lepton polarization of $K_L \to \pi^+\mu^-\bar{\nu}$ or $K^+ \to \pi^+\mu^+\mu^-$ is estimated as the order of $10^{-3}$ [23]. In this paper, we follow Ref. [11, 13] and
neglect the final state interaction effect, because it will be difficult to measure such small effect in experiments and decide whether the measurement includes the effects from some new physics when \(< P_x^- > + < P_x^+ >\) is non zero but very small value, where the subindex \(x\) is \(L\) or \(N\).

We summarize the findings from the averaged asymmetries as follows:

- Within the SM, \(< P_L^- > + < P_L^+ > = 0\). As we find from Eq. (21), if there is any contribution from new interactions other than nonlocal–type and vector–type operators, \(< P_L^- > + < P_L^+ >\) can have a nonzero value. The contributions from the vector–type operators cancel each other. The main contributions come from the terms with \(C_{LRLR} + C_{LRRNL}\), \(C_T\) and \(C_{TE}\).

- Within the SM, \(< P_T^- > - < P_T^+ > \approx 0.17\). The change of the value for \(< P_T^- > - < P_T^+ >\) from the SM prediction would come from \(C_{LRLR} - C_{LRRNL}\) and \(C_T - 2C_{TE}\), as we discussed earlier. Therefore, for the scalar–type operators we can find rather easily which type of operators gives contributions, by using the measured values of both \(< P_L^- > + < P_L^+ >\) and \(< P_T^- > - < P_T^+ >\).

- \(< P_N^\pm >\) could give us interesting information on the phases of new physics parameters. Within the SM \(< P_N^- > + < P_N^+ > = 0\), and the experimentally measured value, if it is nonzero, will give us some hints of the new physics.

- In Figs. 3–8, we show correlations between the integrated branching ratio \(\mathcal{B}\) and the averaged lepton polarized asymmetries of \(\tau^-\) and \(\tau^+\), by varying the interaction coefficients. In drawing Figs. 3–8 we assume for simplicity that all the new interaction coefficients in Eq. (10) are real. Since \(< P_N^- > + < P_N^+ >\) comes only from the imaginary parts of the old (SM) coefficients, due to our temporary assumption of real new coefficients, the possible correlation between this combined asymmetry and the integrated branching ratio comes primarily \((m_s \approx 0)\) from varying \(C_T - 2C_{TE}\), as seen from Eq. (31).

- In Figs. 3–8 we show the flows in \((\mathcal{B}, < P_L^- > + < P_L^+ >)\) plane, where

\[
\mathcal{B} = \int_{14 \text{ GeV}^2}^{(m_b - m_s)^2} d\mathcal{B} \int ds,
\]

by varying the values of scalar–type coefficients (Fig. 3), and those of tensor–type coefficients (Fig. 4). In Figs. 5–7 we show the flows in \((\mathcal{B}, < P_T^- > - < P_T^+ >)\) plane by varying the values of vector–type (Fig. 5), scalar–type (Fig. 6) and tensor–type coefficients (Fig. 7). In Fig. 8 we show the flows in \((\mathcal{B}, < P_N^- > + < P_N^+ >)\) plane by changing the values of tensor–type coefficients.
• We note that the influence of varying of the coefficients in the Figures is confirming our previous findings. In Figs.3-4, we find that the influence of varying $C_{LRLR} + C_{LRRL}$ and $C_T - 2C_{TE}$ is quite large. In Fig.5, the flow by varying $C_{LL}$ is large. In Figs.6-7, we find that the variation of $C_{LRLR} - C_{LRRL}$ and $C_T - 2C_T$ will make large flow in the $(B, <P_T^L> - <P_T^R>)$ plane. In Fig.8, we find that varying $C_T - 2C_{TE}$ will lead to a flow in the $(B, <P_N^- > + <P_N^+>)$ plane, but it will not be so large one.

![Figure 3: The flows in $(B, <P_T^L> + <P_T^R>)$ plane. In each flow, $C_{LRLR} + C_{LRRL}$ (thick solid line), $C_{RLLR} + C_{RLRL}$ (thin line) are varied respectively.](image-url)
Figure 4: The flows in \((B, \langle P^-_L \rangle + \langle P^+_L \rangle)\) plane. In each flow, \(C_T + 2C_{TE}\) (thick solid line), \(C_T - 2C_{TE}\) (thin line) are varied respectively.

Figure 5: The flows in \((B, \langle P^-_T \rangle - \langle P^+_T \rangle)\) plane. In each flow, \(C_{LL}\) (thick solid line), \(C_{LR}\) (thick dashed line), \(C_{RL}, C_{RR}\) (thin solid line) are varied respectively.
Figure 6: The flows in \((B, <P_T^- > - <P_T^+ >)\) plane. In each flow, \(C_{LRLR} + C_{LRRL}\) (thick solid line), \(C_{LRLR} - C_{LRRL}\) (thick dashed line), \(C_{RLLR} + C_{RLRL}\) (thick solid line), \(C_{RLLR} - C_{RLRL}\) (thick dashed line) are varied respectively.

Figure 7: The flows in \((B, <P_T^- > - <P_T^+ >)\) plane. In each flow, \(C_T + 2C_{TE}\) (thick solid line), \(C_T - 2C_{TE}\) (thin solid line) are varied respectively.
Figure 8: The flows in \((B, <P_N^-> + <P_N^+>)\) plane. In each flow, \(C_T + 2C_{TE}\) (thick solid line), \(C_T - 2C_{TE}\) (thin solid line) are varied respectively.

6 Summary and Conclusions

The most general model-independent analysis of lepton polarization asymmetries in the rare \(B\) decay \(B \to X_s \tau^+ \tau^-\) is presented. We have presented the longitudinal, normal and transverse polarization asymmetries of \(\tau^+\) and \(\tau^-\) as functions of the coefficients of the twelve four–Fermi operators, ten local and two nonlocal ones. Even though the experimental observations of all those asymmetries may be very challenging, we found that such observations will be very useful to pin down new physics beyond the Standard Model (SM) systematically, telling us which type of operators contributes to the process.

We also investigated various combinations of the polarization asymmetries: For the longitudinal polarization \(P_L\), the contribution from the SM to \(P_L^-\) of \(\tau^-\) is exactly the same as that to \(P_L^+\) of \(\tau^+\), with just the opposite sign. Therefore, if there is any difference in absolute values between the longitudinal asymmetries of \(\tau^+\) and \(\tau^-\), it must come from interactions beyond the SM – in this case from the terms with the scalar– and tensor–type interactions, because the contributions from the combination of vector types and nonlocal types are all canceled. We also found that the contribution from \(C_{LRLR} + C_{LRRL}\) is much larger than from the other scalar–type interactions. We also showed the usefulness of the transverse asymmetry \(P_T\). From the difference between \(P_T^-\) and \(P_T^+\), we can find the dependence on \(C_7 \ast C_{10}\), if no contributions beyond those of the SM exist. However, if there exist new physics interactions, the contribution from the scalar–type operators,
$C_{LRLR} - C_{LRRL}$, will dominate the difference $P_T^- - P_T^+$. Therefore, if there are the scalar–type interactions from new physics, we can find the interaction strength by using the results of both $P_L^- + P_L^+$ and $P_T^- - P_T^+$. As is well known, many new physics models (for example, multi Higgs doublet models \[12\], SUSY \[9, 14\], R-parity violation model \[13\] etc.) include such scalar–type interactions. Concerning $< P_N^+ >$, experimental observation may be much more difficult than in the case of other asymmetries because of small numerical value. However, if we can measure it, we will be able to get very useful information on the imaginary part of $C_T - 2C_{TE}$. Of course, within the SM, $< P_N^- > + < P_N^+ > = 0$.

To summarize, we presented the most general model-independent analysis of the lepton polarization asymmetries in the rare $B$ decay, $B \to X_s \tau^+ \tau^-$. The longitudinal, normal and transverse polarization asymmetries for the $\tau^+$ and $\tau^-$, and the combinations of them as the functions of the Wilson coefficients of the twelve independent four–Fermi interactions are also presented. It will be very useful to pin down new physics systematically, once we have the experimental data with high statistics and the deviation from the Standard Model is found.

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