A Sequential Bayesian Updated Wiener Process Model for Remaining Useful Life Prediction

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ABSTRACT Wiener processes have been extensively used to model the degradation processes exhibiting a linear trend for predicting the remaining useful life (RUL) of degrading components. To incorporate the real-time degradation monitoring information into degradation modeling, the Bayesian method has been frequently utilized to update the model parameter, particularly for the drift parameter in Wiener process. However, due to the inherent independent increment and Markov properties of Wiener process, the Bayesian updated drift parameter only utilizes the current degradation measurement and cannot incorporate the whole degradation measurements up to now. As such, once the updated degradation model in this way is used to predict the RUL, the obtained result may be dominated by partial degradation observations or lower the prognosis accuracy. In this paper, we propose a sequential Bayesian updated Wiener process model for RUL prediction. First, a Wiener process model with random drift efficient is used to model the degradation process with the linear trend. To estimate the model parameters, the historical degradation measurements are used to determine the initial model parameters based on the maximum likelihood estimation (MLE) method. Then, for the degrading component in service, a sequential Bayesian method is proposed to update the random drift parameter in Wiener process model. Differing from existing studies using the Bayesian method, the proposed sequential method uses the Bayesian estimate for random drift parameter in the last time as the prior of the next time. As such, the Bayesian estimate for random drift parameter in the current time is dependent on the whole degradation measurements up to current time, and thus the problem of depending only on the current degradation measurement is solved. Finally, we derive the analytical expressions of the RUL distribution based on the concept of the first passage time (FPT). Two case studies associated with the gyroscope drift data and lithium-ion battery data are provided to show the effectiveness and superiority of the proposed method. The results indicate that the proposed method can improve the RUL prediction accuracy.

INDEX TERMS Remaining useful life, degradation, Wiener process, sequential Bayesian, first passage time.

I. INTRODUCTION
Over the last decades, the industrial equipment is becoming more and more complex and automated [1]–[4]. It is much more difficult to grasp the health status and predict the pending failure for such equipment. This difficulty leads to an emerging concept called prognostics and health management (PHM) and promotes its rapid development. PHM is generally regarded as an efficient tool for assessing the reliability of the equipment under the actual operating conditions, and reducing operating costs or failure risks through some management activities [5]–[11]. After the continuous exploration of scholars, the researches on PHM technique have obtained a huge amount of theoretical achievements and been extensively applied in various fields including electronics, batteries, bearings, motor drives, industrial production, aerospace and military application [1], [6], [8], [9], [12]–[16].

As we all know, PHM mainly consists of two parts: remaining useful life (RUL) prediction and health management. The former as the key components of PHM, aims at determining the length of the time from the present to the end of useful life and providing the significant information support.
for the condition-based maintenance in engineering practice. Predicting the RUL of the equipment accurately can not only extend the lifecycle, but also schedule the scientific health care actions. Owing to the fact that the degradation process exhibits the strongly stochastic nature, the issue on the uncertainty in RUL prediction may be frequently encountered [11], [17]–[19]. Therefore, a primary duty is to obtain the probability density function (PDF) of the RUL for the concerned equipment on the premise of making full use of condition monitoring (CM) information [1], [10].

The existing RUL prediction methods can be mainly divided into two categories: namely, physics of failure method and data driven methods [4], [20]. The key task for the former is to find the failure mechanisms for the equipment. In fact, this task is difficult and cost-expensive for modern complex equipment, which limits the development of such methods. With the advancement of the monitoring technology, various data for equipment can be collected in reality. The latter is independent of failure mechanism, and extracts the information related to the RUL from the condition monitoring data including degradation data, life data, censored data, and environmental data. Thus, data driven methods have gained more and more attention due to their universality and flexibility in RUL prediction in recent years.

Wiener process-based method, as one of the most classical data driven methods, have been extensively used to model the degradation processes exhibiting a linear trend for predicting the RUL of degrading equipment [7], [8], [17]. Doksum and Hoyland firstly adopted Wiener process to construct the accelerated degradation model and transformed the non-stationary Wiener process into the stationary Wiener process by time scale transformation [21]. Tseng et al. researched the RUL prediction for LED lamp of contact image scanners based on Wiener process only considering the current condition monitoring information [22]. To incorporate the historical degradation data into the model, Tseng et al. proposed an integrated Wiener process to model the cumulative degradation path of a product’s quality characteristics [23]. The random effect of a drift coefficient in Wiener process was considered to describe the individual difference and improve the prediction accuracy [24]. Elwany et al. utilized a Brownian motion with positive drift to predict the RUL online under the framework of Bayesian [25]. Si et al. derived an exact and closed-form RUL distribution on the basis of the Wiener process-based degradation model based on the concept of the first passage time (FPT) and applied the proposed method to the gyroscope [26]. When considering the measurement error in practical monitoring, Tang et al. proposed a new RUL prediction method for lithium-ion batteries based on the Wiener process with measurement error, which closed to the actual situation [27]. Wang et al. constructed the performance degradation model for the aviation axial piston pump by the Wiener process and utilized the expectation maximization (EM) algorithm and Kalman filter method to estimate the model parameters [28]. In order to provide more flexibility for degradation modeling under dynamic environments, a new Wiener process model with an adaptive drift for degradation-based RUL prediction was proposed [29].

Among the Wiener process-based methods, we can find that the Bayesian method has been frequently utilized to update the model parameter, which can incorporate the real-time degradation monitoring information into degradation modeling. There are plenty of studies associated to the Bayesian-based the drift parameter estimation, such as [25], [26], [30]. According to the current researches on this issue, the drift parameter estimation can achieve the online updating conditional on the available condition monitoring data. However, an obvious limitation exists for such processing: the Bayesian updated drift parameter only utilizes the current degradation measurement and cannot incorporate the whole degradation measurements up to now due to the inherent independent increment and Markov properties of Wiener process. Therefore, once the updated degradation model in this way is used to predict the RUL, the obtained result may be dominated by partial degradation observations or lower the prognosis accuracy.

Driven by the above inherent problem for Bayesian-based the drift parameter estimation over the related works, the purpose of this paper is to develop a sequential Bayesian updated Wiener process model for RUL prediction and fully utilize the whole degradation measurements up to current time. Firstly, we adopt a Wiener process model with random drift efficient to model the degradation process with the linear trend. The historical degradation measurements are used to determine the initial model parameters based on the maximum likelihood estimation (MLE) method. Then, for the degrading component in service, a sequential Bayesian method is proposed to update the random drift parameter in Wiener process model. Differing from existing studies using the Bayesian method, the proposed sequential method uses the Bayesian estimate for random drift parameter in the last time as the prior of the next time. As such, the Bayesian estimate for random drift parameter in the current time is dependent on the whole degradation measurements up to current time, and thus the problem of depending only on the current degradation measurement is solved. Finally, we derive the analytical expressions of the RUL distribution based on the concept of the FPT. Two case studies associated with the gyroscope drift data and lithium-ion battery data are provided to show the effectiveness and superiority of the proposed method. Overall, the main contribution of this work is to solve the problem that the Bayesian estimate for random drift parameter in the current time only depends on the current degradation measurement for the traditional Bayesian updated Wiener process model.

The remaining parts of this paper are organized as follows. Section II describes the problem. Section III estimates the model parameters and update hyper-parameters of the drift coefficient. The PDF and expectation of the RUL are derived in Section IV. Section V provides two experimental studies for model verification. We conclude this paper in Section VI.
II. PROBLEM DESCRIPTION

Owing to the characteristic of independent increment and Markov properties for Wiener processes, it is beneficial for simplifying the parameters estimation derivation. Another advantage for Wiener processes is to describe not only monotonic degradation processes but also non-monotonic degradation process. Thus, they have been extensively used to construct the degradation processes exhibiting a linear trend for predicting the RUL of degrading components [21]–[31]. In general, the degradation model based on a Wiener process can be represented by the following expression:

$$X(t) = x_0 + \theta t + \sigma_B B(t)$$

(1)

where $x_0$ represents the initial degradation level for equipment, $\theta$ and $\sigma_B$ are the drift and diffusion coefficients respectively. In order to characterize the unit-to-unit heterogeneity, the drift coefficient is generally regarded as a random variable. The commonly used distribution form is normal distribution, i.e., $\theta \sim N(\mu_\theta, \sigma_\theta^2)$. Note that when selecting other distribution, the analytical results for prognosis may be unavailable. $\sigma_B$ is frequently termed as deterministic parameter. $B(t)$ is the Standard Brownian Motion (SBM) to characterize the stochastic dynamics during the operation process.

To incorporate the real-time degradation monitoring information into degradation modeling, the Bayesian method, as a typical method, has been frequently utilized to update the drift parameter in Wiener process. In Bayesian framework, the posterior distribution of $\theta$ can combine its prior distribution with the conditional likelihood of real-time degradation data. Let $X_{1:k} = \{x_1, x_2, \cdots, x_k\}$ represents the sets of degradation monitoring information up to time $t_k$. The prior distribution of the $\theta$ can be denoted by $p(\theta)$ with parameters $(\mu_\theta, \sigma_\theta^2)$, i.e., $\theta \sim N(\mu_\theta, \sigma_\theta^2)$. Thus the posterior distribution of $\theta$ at time $t_k$ can be formulated as:

$$p(\theta | X_{1:k}) = \frac{p(X_{1:k} | \theta) \cdot p(\theta)}{p(X_{1:k})} \propto p(X_{1:k} | \theta) \cdot p(\theta)$$

(2)

where $p(X_{1:k} | \theta)$ represents the conditional likelihood of real-time degradation data up to time $t_k$. According to the derivation in [26], the posterior estimation of $\theta$ conditional on $X_{1:k}$ still follows the normal distribution with mean $\mu_{\theta,k}$ and variance $\sigma_\theta^2,k$, i.e., $\theta | X_{1:k} \sim N(\mu_{\theta,k}, \sigma_\theta^2,k)$. Accordingly, the posterior estimation results of $\theta$ at time $t_k$ can be written as [25], [26]:

$$\mu_{\theta,k} = \frac{\mu_\theta \sigma_\theta^2 + x_k \sigma_{\theta,0}^2}{\sigma_\theta^2 + \sigma_{\theta,0}^2} + \frac{\mu_{\theta,0} \sigma_{\theta,0}^2}{\sigma_\theta^2 + \sigma_{\theta,0}^2}$$

$$\sigma_{\theta,k}^2 = \frac{\sigma^2 + \sigma_{\theta,0}^2}{\sigma_\theta^2 + \sigma_{\theta,0}^2}$$

(3)

$$\sigma_{\theta,k}^2$$

(4)

It can be observed that the posterior estimation can be updated once the new monitoring data $x_k$ is obtained. The core of Bayesian method lies in fully utilizing the new observation at time $t_k$, which can guarantee the accuracy of parameters estimation and RUL prediction at some extent.

However, an obvious drawback exists for the posterior estimation results of $\theta$ in Eqs. (3)-(4). That is, the Bayesian updated drift parameter only utilizes the current degradation measurement $x_k$ and cannot incorporate the whole degradation measurements up to now, i.e., $X_{1:k} = \{x_1, x_2, \cdots, x_k\}$. Especially, an abnormal observation at the current time will lead to inaccurate estimation results. As such, once the updated degradation model in this way is used to predict the RUL, the obtained result may be dominated by partial degradation observations or lower the prognosis accuracy.

From the above description, the following issues will be researched in this paper:

1) How to estimate the model parameters and update the drift coefficient based on the whole degradation measurements up to now?

2) How to determine the distribution and expectation of the RUL for the concerned equipment according to the estimated parameters and updated drift coefficient?

III. PARAMETERS ESTIMATION

A. THE OFFLINE PARAMETERS ESTIMATION VIA MLE METHOD

The unknown parameters of the Wiener process-based degradation model in Eq. (1) contains the drift coefficient $\theta$ and diffusion coefficient $\sigma_B$. Owing to the stochastic characteristic of the drift coefficient, the hyper-parameters in drift coefficient include $\mu_\theta$ and $\sigma_\theta$. Hence, the parameters to be estimated can be recorded as $\Theta = [\mu_\theta, \sigma_\theta, \sigma_B]$. The MLE method in [31] is adopted to determine the online estimation results on the basis of the historical degradation measurements $\{X_i(t_i) = x_{i,j}, i = 1 \cdots N, j = 1 \cdots, m_i\}$, where $N$ represents the number of independently tested equipment, and $m_i$ denotes the available number of degradation observations for $i$th group of the degraded equipment. Consequently, the degradation measurement at $j$th monitoring time for the $i$th group of equipment can be formulated as:

$$X_i(t_j) = x_{0,i} + \theta_i t_j + \sigma_B B(t_j)$$

(5)

where $x_{0,i}$ is the initial degradation level for $i$th group of equipment. $\theta_i$ are $i$-independent and identically distributed following $N(\mu_\theta, \sigma_\theta^2)$. Let $T_i = [t_1, \cdots, t_{m_i}]^T$, $I = [1, 1, \cdots, 1]^T$, and $x_i = [x_{i,1}, \cdots, x_{i,m_i}]^T$, where, $I \in \mathbb{R}^{m_i \times 1}$, and $[\cdot]^T$ represents the vector transposition. Due to the independent increments property of Wiener process, it can be found that $x_i$ follows the multivariate normal distribution, i.e., $x_i \sim MVN(\mu_i, \Sigma_i)$. Its mean and variance can be represented as respectively:

$$\mu_i = x_{0,i} I + \mu_\theta T_i$$

$$\Sigma_i = \Omega_i + \sigma_B^2 T_i T_i^T$$

(6)

(7)

where $\Omega_i = \sigma_{\theta,0}^2 \Omega$, and $Q_i = \left[ \begin{array}{cccc} t_{i,1} & t_{i,1} & \cdots & t_{i,1} \\ t_{i,1} & t_{i,2} & \cdots & t_{i,2} \\ \vdots & \vdots & \ddots & \vdots \\ t_{i,1} & t_{i,2} & \cdots & t_{i,m_i} \end{array} \right]$. 

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The historical degradation measurements for all the equipment can be recorded as $X = \{x_1, x_2, \cdots, x_N\}^T$. Since the degradation measurements of different equipment are assumed to be independent, the log-likelihood function of all historical degradation measurements $X$ over parameters vector $\Theta$ can be expressed as [31]:

$$
\ell (\Theta | X) = - \frac{\ln (2\pi)}{2} \sum_{i=1}^{N} m_i - \frac{1}{2} \sum_{i=1}^{N} \ln | \Sigma_i | - \frac{1}{2} \sum_{i=1}^{N} \left( (x_i - \mu_i)^T \Sigma_i^{-1} (x_i - \mu_i) \right)
$$  \hspace{1cm} (8)

where $| \Sigma_i | = | \Omega_i | \left( 1 + \sigma_B^2 T_i^T \Omega_i^{-1} T_i \right)$, $\Sigma_i^{-1} = \Omega_i^{-1} - \frac{\sigma^2}{1 + \sigma_B^2 T_i^T \Omega_i^{-1} T_i} \Omega_i^{-1}$. To maximize the log-likelihood function in Eq.(8), taking the first partial derivative of the log-likelihood function with respect to $\mu \theta$ yields:

$$
\frac{\partial \ell (\Theta | X)}{\partial \mu \theta} = \sum_{i=1}^{N} \left[ T_i^T \Sigma_i^{-1} (x_i - x_0, I - \mu \theta T_i) \right]
$$  \hspace{1cm} (9)

For specified values of $\sigma \theta$ and $\sigma_B$, we can obtain the result of MLE for $\mu \theta$ by setting the derivative in Eq.(9) to zero, which can be expressed as:

$$
\hat{\mu} \theta = \frac{\sum_{i=1}^{N} \left[ T_i^T \Sigma_i^{-1} (x_i - x_0, I) \right]}{\sum_{i=1}^{N} \left[ T_i^T \Sigma_i^{-1} T_i \right]}
$$  \hspace{1cm} (10)

Then, by substituting Eq.(10) into Eq.(8), the profile likelihood function for $(\sigma \theta, \sigma_B)$ can be represented as:

$$
\ell (\Theta | X) = - \frac{\ln (2\pi)}{2} \sum_{i=1}^{N} m_i - \frac{1}{2} \sum_{i=1}^{N} \ln | \Sigma_i | - \frac{1}{2} \sum_{i=1}^{N} \left( (x_i - x_0, I - \hat{\mu} \theta T_i) \right)
$$

As such, the results of MLE for $(\sigma \theta, \sigma_B)$ can be determined by maximizing the profile likelihood function in Eq.(11) through the two-dimensional search. Furthermore, the result of MLE for $\mu \theta$ can be obtained by substituting the MLE for $(\sigma \theta, \sigma_B)$ into Eq.(10). In order to achieve the two-dimensional search in this section, the “fminsearch” function in MATLAB is frequently adopted in practice engineering. In additional, diffusion coefficient $\sigma_B$ is identical for different equipment, while the hyper-parameters $(\mu \theta, \sigma_B)$ in drift coefficient exhibit the difference to some extent. As a result, these two hyper-parameters should be updated by utilizing the real-time degradation monitoring information.

### B. THE ONLINE UPDATION OF DRIFT PARAMETER BY THE SEQUENTIAL BAYESIAN METHOD

Although the Bayesian method can update the drift parameter for a degrading equipment in service based on Eqs.(3)-(4), the current degradation measurement is only utilized and the previous degradation measurements are ignored in the parameters updating. We propose a sequential Bayesian method to update the random drift parameter in Wiener process model. Specifically, the posterior distribution of $\theta$ at time $t_k$ can be expressed as:

$$
p(\theta | X_{1:k}) = \frac{p(X_{1:k} | \theta) \cdot p(\theta)}{p(X_{1:k})} = \frac{p(x_k | X_{1:k-1}, \theta) \cdot p(\theta | X_{1:k-1}) \cdot p(X_{1:k-1})}{p(x_k | X_{1:k-1}) \cdot p(\theta | X_{1:k-1})}
$$  \hspace{1cm} (12)

where $p(x_k | X_{1:k-1}, \theta)$ represents the probability distribution of $x_k$ conditional on $X_{1:k-1}$ and $\theta$, and $p(\theta | X_{1:k-1})$ denotes the posterior distribution of $\theta$ at time $t_{k-1}$ with parameters $(\mu \theta_{k-1}, \sigma \theta_{k-1})$. The above equation constructs the recursive relation between the posterior distribution of $\theta$ at time $t_{k-1}$ and the posterior distribution of $\theta$ at time $t_k$. That is, the Bayesian estimate for random drift parameter in the last time is regarded as the prior of the next time.

According to the basic characteristics of Wiener process, $x_k | X_{1:k-1}, \theta$ follows a normal distribution, and its probability distribution holds

$$
p(x_k | X_{1:k-1}, \theta) = \frac{1}{\sqrt{2\pi \sigma_B^2 (t_k - t_{k-1})}} \cdot \exp \left[ - \frac{(x_k - x_{k-1} - \theta (t_k - t_{k-1}))^2}{2\sigma_B^2 (t_k - t_{k-1})} \right]
$$  \hspace{1cm} (13)

Thus, by substituting Eq.(13) and the posterior estimation results of $\theta$ at time $t_{k-1}$ into Eq.(12), the posterior distribution of $\theta$ at time $t_k$ can be further written as:

$$
p(\theta | X_{1:k}) \propto \exp \left[ - \frac{(x_k - x_{k-1} - \theta (t_k - t_{k-1}))^2}{2\sigma_B^2 (t_k - t_{k-1})} \right] \cdot \exp \left[ - \frac{(\theta - \mu \theta_{k-1})^2}{2\sigma_{\theta,k-1}^2} \right]
$$
\[ \propto \exp\left( \frac{1}{2\sigma_B^2} \frac{1}{(t_k - t_{k-1})} \right) \cdot \left[ \left( \frac{1}{2\sigma_B^2} \frac{(t_k - t_{k-1})}{(t_k - t_{k-1} - \sigma_{\theta,k-1}^2)} + \frac{1}{2\sigma_B^2} \frac{(t_k - t_{k-1})}{(t_k - t_{k-1} - \sigma_{\theta,k-1}^2)} \right) \theta^2 \\
+ \left. \left( \frac{1}{2\sigma_B^2} \frac{(x_k - x_{k-1})}{(t_k - t_{k-1} - \sigma_{\theta,k-1}^2)} + \frac{1}{2\sigma_B^2} \frac{(x_k - x_{k-1})}{(t_k - t_{k-1} - \sigma_{\theta,k-1}^2)} \right) \theta \right] \]
\[ \propto \exp\left( \frac{-(\theta - \mu_{\theta,k})^2}{2\sigma_{\theta,k}^2} \right) \] (14)

According to Eq.(14), the posterior estimation results of \( \theta \) at time \( t_k \) can be calculated as:

\[ \mu_{\theta,k} = \frac{\sigma_B^2}{\sigma_{\theta,k-1}^2} (t_k - t_{k-1} - \sigma_{\theta,k-1}^2) + \frac{1}{\sigma_B^2} \frac{(x_k - x_{k-1})}{(t_k - t_{k-1} - \sigma_{\theta,k-1}^2)} \]
\[ \sigma_{\theta,k} = \left[ \frac{\sigma_B^2 \sigma_{\theta,k-1}^2}{\sqrt{\sigma_{\theta,k-1}^2 (t_k - t_{k-1} - \sigma_{\theta,k-1}^2) + \sigma_B^2}} \right] \] (16)

From Eqs.(15)-(16), it can be observed that the recursive relations indicate \( \mu_{\theta,k} \) and \( \sigma_{\theta,k} \) are not only associated with the current degradation measurement, but also associated with \( \mu_{\theta,k-1} - \sigma_{\theta,k-1} \). Thus, the Bayesian estimate for random drift parameter in the current time is dependent on the whole degradation measurements up to current time, and thus the problem of depending only on the current degradation measurement is solved. It is noted that the offline estimated results of the hyper-parameters \( (\mu_{\theta}, \sigma_{\theta}) \) can be regarded as the prior distribution parameters of \( \theta \) at the initial moment.

**IV. RUL PREDICTION**

After estimating the model parameters and updating the drift parameter, we will implement the derivations of the PDF and expectation of RUL for the concerned equipment. Using the FPT of degradation process \( X(t) \) crossing the preset failure threshold \( w \), the RUL at time \( t_k \) can be defined as:

\[ L_k = \inf \{ l_k : X(t_k + l_k) \geq w \mid X_{1:k} \} \] (17)

Based on the work in [26], the conditional PDF of the RUL at time \( t_k \) on condition that \( \theta \) is given can be represented as:

\[ f_{L_k \mid \theta, X_{1:k}} (l_k \mid \theta, X_{1:k}) = \frac{w - x_k}{\sqrt{2\pi l_k^2 \sigma_{\theta,k}^2}} \exp\left( -\frac{(w - x_k - \theta l_k)^2}{2\sigma_{\theta,k}^2} \right) \] (18)

When the stochastic nature of \( \theta \) is considered, it is generally to adopt the law of total probability to calculate the PDF of the RUL, which can be formulated as:

\[ f_{L_k \mid \theta, X_{1:k}} (l_k \mid \theta, X_{1:k}) = \int_{-\infty}^{\infty} f_{L_k \mid \theta, X_{1:k}} (l_k \mid \theta, X_{1:k}) \cdot \mu (\theta \mid X_{1:k}) \, d\theta \] (19)

where \( \mu (\theta \mid X_{1:k}) \) is the posterior distribution of \( \theta \) at time \( t_k \) derived from Eq.(14). Since Eq.(19) contains the complex integrand function, the lemma in [26] can be introduced to calculate the PDF of the RUL for equipment. Hence, Eq.(19) can be further expressed as:

\[ f_{L_k \mid X_{1:k}} (l_k \mid X_{1:k}) = \frac{w - x_k}{\sqrt{2\pi l_k^2 \sigma_{\theta,k}^2}} \exp\left( -\frac{(w - x_k - \mu_{\theta,k} l_k)^2}{2l_k (\sigma_{\theta,k}^2 + \sigma_{\theta,k}^2 l_k^2)} \right) \] (20)

Moreover, the expectation of the RUL can be calculated as:

\[ E [L_k \mid X_{1:k}] = \int_{-\infty}^{\infty} l_k f_{L_k \mid X_{1:k}} (l_k \mid X_{1:k}) \, dl_k \]
\[ = \frac{w - x_k}{\sqrt{2\pi l_k^2 \sigma_{\theta,k}^2}} \exp\left( -\frac{(w - x_k - \mu_{\theta,k} l_k)^2}{2l_k (\sigma_{\theta,k}^2 + \sigma_{\theta,k}^2 l_k^2)} \right) \]
\[ = \sqrt{2} \frac{(w - x_k)}{\mu_{\theta,k}} D \left( \frac{\mu_{\theta,k}}{\sqrt{2\sigma_{\theta,k}^2}} \right) \] (21)

where \( D(y) = \exp\left( -\frac{y^2}{2} \right) \int_{0}^{\infty} \exp\left( \frac{u^2}{2} \right) \, du \) represents the Dawson integral over \( y \). The Dawson integral has the following properties, that is, when \( y \) is an extremely large number, \( D(y) \approx 1/2y \).

**Remark 1:** If all the realizations of \( \theta \) are greater than zero, i.e., \( \mu_{\theta,k} \gg \sigma_{\theta,k} \), the expectation of RUL can be approximated as:

\[ E [L_k \mid X_{1:k}] = \frac{w - x_k}{\mu_{\theta,k}} \] (22)

In order to better understand the whole process, a flowchart towards the proposed sequential Bayesian method for RUL prediction is provided in Fig.1, which can be classified as two part: offline stage and online stage.

**V. EXPERIMENTAL STUDIES**

In this section, two experimental studies associated with the gyroscope drift data and lithium-ion battery data are provided to compare the performance of the sequential Bayesian updated Wiener process model with the traditional Bayesian updated Wiener process model existing in the current literatures for RUL prediction. We adopt a loss function to appraise the prediction accuracy and implement a direct comparison with the alternative modeling method. It is noted that the mean-squared error (MSE) is generally selected as the loss function, which can be formulated as [32]:

\[ \text{MSE}_k = \int_{-\infty}^{\infty} (\tilde{l}_k - \tilde{l}_k) f_{L_k \mid X_{1:k}} (l_k \mid X_{1:k}) \, dl_k \] (23)

where \( \tilde{l}_k \) represents the actual RUL at the observation time \( t_k \) and \( f_{L_k \mid X_{1:k}} (l_k \mid X_{1:k}) \) is the according PDF of the RUL obtained by Eq.(20). Here, the total mean-squared error
where $S$ represents the total number of prediction points. Owing to the advantage of incorporating the prediction uncertainty into the evaluation criteria, we utilize the MSE between the predicted RUL and the real RUL as the comparing indicator in the following numerical example for further comparisons. To facilitate better illustration, the proposed model can be recorded as Model 1, while the traditional Bayesian updated Wiener process model in [25] can be recorded as Model 2.

### A. THE RUL PREDICTION FOR GYROSCOPE

It is acknowledged that the most critical component of the inertial navigation system (INS) is the inertial platform, which directly determines the navigation precision. In order to guarantee the navigation precision, the health condition of the inertial platform should be mastered by the precision testing during the service period. The engineering practice indicates the observations of the gyro, i.e., the drift data can be served as the characteristic parameter for reflecting the health condition of the inertial platform. In general, the larger the drift data are, the worse the health performance is. As such, we can implement the RUL prediction by modeling the drift data monitored periodically or irregularly. Due to the various practical uncertainties, the irregular monitoring is more accordance with the actual circumstance.

In this part, six groups of historical drift data of gyroscopes are collected by the condition monitoring technique, as shown in Fig.2. It can be observed that the drift data of all the gyroscopes shows an upward trend as a whole. This is because the rotating part of inertial platform with a high speed can result in the rotation axis wear, and such wear will accumulate over time. Once the accumulation exceeds a certain level, the inertial platform cannot meet the requirements of normal operation. Moreover, the time interval between the two successive monitoring is not identical, which implies the monitoring is irregular.

The drift data of these six gyroscopes are utilized to estimate the model parameters offline by the MLE method. The estimated results are listed in Table 1. Gyroscope #3 is selected as the research object for RUL prediction. Thus, it is natural to consider that utilizing the drift data of this gyroscope can adjust the model parameters. As described previously, the diffusion coefficient in Wiener process model is a deterministic parameter and the drift coefficient is a stochastic parameter. Subsequently, the hyper-parameters of drift coefficient ($\mu_\theta$ and $\sigma_\theta$) should be updated by the proposed sequential Bayesian method in Eqs.(15)-(16). In addition, for the purpose of RUL prediction, the failure threshold of Gyroscope #3 can be defined as the last monitoring data, i.e., $w = 0.2516$.

After updating $\mu_\theta$ and $\sigma_\theta$ at each inspection point, we can substitute the updated results for drift coefficient and the estimated results for diffusion coefficient into Eq.(20), and then the PDFs and corresponding expectations of the RUL at 1th-8th inspection points can be obtained, as shown in Fig.3. Meanwhile, Fig.4 depicts the PDFs and corresponding expectations of the RUL at the same inspection points by Model 2.
FIGURE 3. The PDF of RUL for gyroscope #3 by Model 1.

FIGURE 4. The PDF of RUL for gyroscope #3 by Model 2.

From Fig. 3-4, it can be intuitively found that the PDF curves of the RULs are getting more and more higher with the time passes, and the uncertainty and accuracy of RUL prediction at the initial stage by these two models are similar. However, the PDF curves of the RULs by Model 1 are higher than those by Model 2 at the later stage, especially after the 5th inspection point, which implies the uncertainty of RUL prediction by Model 1 is relatively smaller. As for the accuracy of RUL prediction, the predicted RULs by the proposed model are more approaching the real RULs. Because the Model 1 can fully utilize the whole degradation measurements up to current time to guarantee the accuracy of random drift parameter estimation and overcome the problem of depending only on the current degradation measurement for Method 2. For quantitative comparison, Fig. 5 provides the MSEs of RULs at 1th-8th inspection points by Model 1 and Model 2 respectively.

As seen from Fig. 5, two MSE curves of RUL show the downward trend overall during the life cycle. Nevertheless, the MSE curve of RUL by Model 1 falls faster than that by Model 2. Furthermore, the TMSEs of the presented model and Models 2 are 150.94, 189.89 respectively. Obviously, the presented model has the less TMSE and improves the RUL prediction accuracy compared with the traditional Bayesian updated Wiener process model, which verifies the effectiveness and superiority of the proposed model in RUL prediction.

B. THE RUL PREDICTION FOR LITHIUM-ION BATTERY

The lithium-ion batteries have plenty of advantages in practice compared with lead–acid, nickel–cadmium, and nickel–metal–hydride cells, such as the higher power density, longer cycle life, higher galvanic potential, lower weight. Hence, it is generally to regard the lithium-ion batteries as the energy solution in many fields including consumer electronics, electric vehicles, aircrafts, satellites. However, lithium-ion battery performance will undergo the degradation process inevitably with cycling and aging, which can result in the reduced performance and catastrophic failure. Such degradation can be characterized by the decrease in capacity over repeated charge cycles. That is to say, battery capacity can be served as an indicator of health status. Currently, scholars and engineers usually evaluate the health status of lithium-ion batteries by the capacity analysis.

A series of degradation testing for lithium-ion batteries are conducted by the Center for Advanced Life Cycle Engineering (CALCE) of Maryland University [34], [35]. Four groups of prismatic cells in CS2 are selected to verify the proposed model, as depicted in Fig. 6. The corresponding cathode is LiCoO₂, and the trace elements of manganese can be found from EDS results. The charging process is as follows, that is, the constant current rate is 0.5C until the voltage reaches...
4.2V and then 4.2V is sustained until the charging current drops to below 0.05A. The failure threshold can be set as the 80% of the rated capacity, i.e., 0.88Ah. It is noted that the proposed model is derived based on the case that the degradation process exhibits the increasing trend. In order to process the decreasing capacity data, we can transform the original data by making the initial capacity minus all the capacity data of battery.

Similarly, these transformed capacity data of four batteries are utilized to estimate the model parameters online by MLE method. After that, battery CS32-35 is chosen for updating drift parameter and predicting RUL. To illustrate the effect of the RUL prediction, we implement the RUL prediction every 50 cycles from 100th cycle to 550th cycle. Fig.7 and Fig.8 provide the PDF of the RUL at corresponding predicted points for CS32-35 by Model 1 and Model 2, respectively. It can be observed from these two figures that all the PDF curves can cover the real RULs, which implies these two models can be effectively utilized to predict the RUL for lithium-ion battery. Nevertheless, the predicted RULs obtained by Model 1 are closer to the corresponding real RULs than those obtained by Model 2. In addition, it is especially important to concentrate on the prediction performance in the late stage. Specifically, the RUL curves in the late stage by Model 1 are higher and tighter than those by Model 2 after 300th cycle, indicating the prediction uncertainty of the proposed model is smaller than that of Model 2. This improvement in the aspects of prediction accuracy and uncertainty is due to the fact the proposed model can fully use the whole degradation measurements up to current time when updating the drift parameter. According to the definition of MSE in Eq.(23), we can further obtain the MSEs of RULs at the above prediction points by these two models, as depicted in Fig.9.

From Fig.9, we can find that the whole MSE curve of Model 1 is below that of Model 2, which indicates the prediction effect of the proposed model is superior to that of the traditional Bayesian updated Wiener process model.
In particular, the MSEs of the predicted RUL at 550th cycle by these two models are 143.14 and 682.42. Moreover, the TMSEs for all the prediction points by Model 1 and Model 2 are 16459 and 31462 respectively. As a result, it can be concluded that the proposed model can dramatically improve the prediction accuracy and uncertainty compared with traditional Bayesian updated Wiener process model.

VI. CONCLUSION
A sequential Bayesian updated Wiener process model for RUL prediction is proposed in this paper. Firstly, we adopt a Wiener process model with random drift efficient to model the degradation process with the linear trend. The historical degradation measurements are used to determine the initial model parameters based on the MLE method. Then, for the degrading component in service, a sequential Bayesian method is proposed to update the random drift parameter in Wiener process model. Finally, we derive the analytical expressions of the RUL distribution based on the concept of the FPT. Two case studies associated with the gyroscope drift data and Lithium-ion battery data are provided to show the effectiveness and superiority of the proposed method. It is noted that the proposed model has an obvious advantage.

That is, the Bayesian estimate for random drift parameter in the current time is dependent on the whole degradation measurements up to current time, and thus the problem of depending only on the current degradation measurement is solved. The future direction is to apply sequential Bayesian techniques to non-linear degradation models.

There are some issues untouched in this paper but deserving future research. Firstly, the diffusion coefficient should also be updated utilizing the online degradation measurements to guarantee the accuracy of RUL prediction. Secondly, the research on hidden observed degradation cases is worth further exploration.

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