Cosmology with coalescing massive black holes

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Abstract. The gravitational waves generated in the coalescence of massive binary black holes will be measurable by LISA to enormous distances. Redshifts \( z \sim 10 \) or larger (depending somewhat on the mass of the binary) can potentially be probed by such measurements, suggesting that binary coalescences can be made into cosmological tools. We discuss two particularly interesting types of probes. First, by combining gravitational-wave measurements with information about the universe’s cosmography, we can study the evolution of black hole masses and merger rates as a function of redshift, providing information about the growth of structures at high redshift and possibly constraining hierarchical merger scenarios. Second, if it is possible to associate an “electromagnetic” counterpart with a coalescence, it may be possible to measure both redshift and luminosity distance to an event with less than \( \sim 1\% \) error. Such a measurement would constitute an amazingly precise cosmological standard candle. Unfortunately, gravitational lensing uncertainties will reduce the quality of this candle significantly. Though not as amazing as might have been hoped, such a candle would nonetheless very usefully complement other distance-redshift probes, in particular providing a valuable check on systematic effects in such measurements.

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As other contributions to these Proceedings will make clear, there are currently major uncertainties in our understanding of the astrophysics of massive binary black hole coalescences. We are quite unsure how often black holes merge, or what mass spectrum describes mergers, or how these quantities are likely to evolve as the universe evolves. What is certain is that if such mergers occur for binaries whose total masses \( M \) are roughly in the range \( 10^4 \ M_\odot < (1+z)M < 10^7 \ M_\odot \), LISA will measure these waves out to redshifts of order 10. Even if these events are rare, LISA will measure them; and, because the source of these waves arises at such large distances, these measurements have the potential to provide detailed information about the large-scale structure of the universe. In this contribution, we discuss the potential of such measurements as cosmological probes.

Coalescing binaries can be considered standard candles because general relativity predicts a unique form for the two polarizations of the binary waveform. For LISA measurements, the strongest \( l = 2 \), \( m = 2 \) harmonic of the emitted waves has the form

\[
h_+ = \frac{2M^{5/3}}{r} \left[ \pi f(t) \right]^{2/3} \left[ 1 + (\mathbf{L} \cdot \hat{n})^2 \right] \cos \left[ \Phi(t) \right],
\]

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\[ h_\times = \frac{4M^{5/3}}{r} \left[ \pi f(t)^{2/3} \left[ \hat{L} \cdot \hat{n} \right] \sin \left[ \Phi(t) \right] \right]. \]  

(1)

In this equation, \( M = m_1^{3/5} m_2^{3/5} / (m_1 + m_2)^{1/5} \) is the so-called “chirp mass”. The vector \( \hat{L} \) gives the orientation of the binary (it is the direction of the orbital angular momentum); \( \hat{n} \) is the direction to the source according to an observer that rides along with the LISA antenna. The phase function \( \Phi(t) \) depends on intrinsic parameters such as masses and spins, and so should be written \( \Phi(t; m_1, m_2, \vec{S}_1, \vec{S}_2) \). The frequency \( f(t) = df/dt \), and \( r \) is the distance to the source. Note that the waveform \( \Pi \) does not include certain effects such as Lense-Thirring precession of the binary’s orbital plane and multipoles other than \( l = m = 2 \); this will likely impact the quantitative details of our results somewhat.

By accurately measuring the evolution of the phase function \( \Phi(t) \), we measure the binary’s intrinsic parameters. In particular, the chirp mass \( M \) can be measured with exquisite precision: \( \delta M / M \sim 10^{-4} \) is a reasonable expectation. This is because \( M \) largely determines the total accumulated phase in the measurement, and hence is very sensitive to that number; cf. Refs. [1, 2, 3]. The LISA antenna pattern and modulations that arise from the antenna’s orbital motion constrain \( \hat{L} \cdot \hat{n} \) fairly well. The only remaining parameter in the waveform is the source distance \( r \). Most error in determining \( r \) comes from correlations with orientation and position errors [4]; in practice, \( r \) is likely to be measured to a precision \( \delta r / r \sim 1 - 25\% \) [5].

Interpreting these formulas becomes somewhat more complicated when the source generating \( h_+ \) and \( h_\times \) is at a distance where cosmological effects are important. Without going into the details (see [6] for further discussion), Eq. (1) works provided we redshift all frequencies and timescales, and replace the naive distance measure \( r \) with the transverse comoving distance \( D_M \) (see Ref. [8] for a detailed description of cosmological distance measures). Putting these two replacements into Eq. (1) and using the fact that the luminosity distance \( D_L = (1 + z)D_M \), we find

\[ h_+ = \frac{2[(1 + z)M]^{5/3}}{D_M} \left[ \pi f(t)^{2/3} \left[ 1 + \left( \hat{L} \cdot \hat{n} \right)^2 \right] \cos \left[ \Phi(t) \right] \right], \]

\[ h_\times = \frac{4[(1 + z)M]^{5/3}}{D_L} \left[ \pi f(t)^{2/3} \left[ \hat{L} \cdot \hat{n} \right] \sin \left[ \Phi(t) \right] \right]. \]  

(2)

Now \( \Phi(t) \) is strictly the measured gravitational-wave phase function (and \( f(t) \) is likewise the measured instantaneous frequency). It depends on redshifted values of the intrinsic parameters: \( \Phi(t) = \Phi(t; (1 + z)m_1, (1 + z)m_2, (1 + z)^2 S_1, (1 + z)^2 S_2) \). The reason for redshifting these parameters can be simply explained using dimensional analysis. In general relativity, a mass \( m \) can only impact the evolution of the system as a timescale \( \tau_m = Gm/c^3 \). (General relativity has no intrinsic scale, so the scales seen in any particular problem must follow from that problem’s specific parameters, such as masses.) When the system is placed at redshift \( z \), the timescale is redshifted. Thus, the apparent mass likewise picks up the factor \( 1 + z \). Similarly, a spin \( S \) impacts the system as a squared timescale \( \tau_S^2 = GS/c^4 \) and picks up a factor \( (1 + z)^2 \). As a consequence of this, the phase evolution of a cosmologically distant binary is indistinguishable from a “local” binary with redshifted masses and spins.

Measuring the waves from a distant binary black hole coalescence thus provides us with two particularly interesting types of information — redshifted masses of the form \((1 + z)m\), and the luminosity distance \( D_L \) (as well as information about the source’s location on the sky and the spins of the binary’s members). There are two direct ways
to exploit this information for cosmological studies. First, we can assume that we know the universe’s cosmography. This allows us to build a map between $z$ and $D_L$. From our inferred $z(D_L)$, we can break the mass-redshift degeneracy and learn about black hole masses as a function of redshift. Second, if it is somehow possible to obtain the redshift independently, one can use the simultaneous measurement of $z$ and $D_L$ to improve the cosmography. (In principle, there is also a third track: if one knows the mass spectrum of the binaries, or has enough events to statistically sample the range of the spectrum, then it should be possible to infer the cosmological properties of this distribution; see Ref. [9] for details. We thank Sam Finn for pointing this out to us. Given the large uncertainties in our understanding of the massive black hole merger rate and the likely breadth of the merger mass spectrum, we will not discuss this third direction here.)

Let us begin by considering the first possibility. We will describe the universe using the currently popular “concordance cosmology”, with matter density given by $\Omega_m \simeq 0.35$, dark energy in the form of a cosmological constant (that is, with equation of state $p = w \rho$ and $w = -1$) with $\Omega_\Lambda \simeq 0.65$, and a Hubble constant whose present value is $H_0 = 65 \text{ km/sec Mpc}^{-1}$. We will assume the universe is precisely flat, and that the relative errors in $\Omega_\Lambda$ and $H_0$ are $\sim 10\%$ [10]. These assumptions allow us to build a map from redshift to luminosity distance [8] which is simple to invert (at least numerically). We then ask: How well can LISA measure the parameters characterizing massive binary black hole systems, particularly the luminosity distance, redshift, and masses? To do this calculation, we use the restricted second post-Newtonian waveform described in Ref. [3] and a description of the LISA antenna as described in Ref. [4]; see Ref. [5] for complete details.

An example of a binary that is measured particularly well is shown in Fig. 1. The histograms describe parameter measurement accuracies found by randomly distributing 100 binaries over the sky at $z = 1$ with random orientations, and with the merger time randomly distributed within an assumed 3 year LISA mission. Each binary is taken to have masses $m_1 = m_2 = 10^5 M_\odot$. In many cases the redshifted masses and luminosity distance are measured with exquisite precision. The distribution for the distance peaks at $\delta D_L/D_L \simeq 2\%$, and most events have $\delta D_L/D_L < 20\%$; the peak for the redshifted chirp mass is at $\delta M_\chi/M_\chi \simeq 10^{-4}$, with most events having $\delta M_\chi/M_\chi < 10^{-3}$; and the peak for the redshifted reduced mass is at $\delta \mu_z/\mu_z \simeq 4\%$, with most events at $\delta \mu_z/\mu_z < 10\%$. The redshift does not appear to be determined as well, but this is entirely due to our assumed errors in the cosmography. If the parameters determining the mapping between $z$ and $D_L$ were known precisely, we would find $\delta z/z \simeq \delta D_L/D_L$. Again, we emphasize that these distributions are computed with the restricted waveform given by Eq. (1), are likely to change when effects we have neglected are taken into account.

The example shown in Fig. 1 is particularly good, but is not far off what can be achieved for a broad range of system masses in the rough band $10^4 M_\odot < (1+z)M_{\text{total}} < (\text{several}) \times 10^6 M_\odot$. In this band, the redshifted masses and the distance are typically measured with a relative error of a few tens of percent. These numbers are also approximately independent of mass ratio, at least for $m_1/m_2 > 1/10$ or so — the loss in signal-to-noise ratio that comes from the reduced mass ratio is mostly compensated by an increase in the number of measured cycles, so that measurement precision remains roughly constant.

We thus conclude that, at the very least and using gravitational-wave information alone, LISA will be able to provide useful and interesting data on black hole masses.
Figure 1. Parameter measurement accuracy distributions for binaries with \( m_1 = m_2 = 10^{5} \, M_{\odot} \) at \( z = 1 \). In this case, the parameters are measured particularly well: the peak determination of luminosity distance is at \( \delta D_L / D_L \simeq 2\% \), the peak in the redshifted chirp mass is at \( \delta M_z / M_z \simeq 10^{-4} \), and the peak in the redshifted reduced mass is at \( \delta \mu_z / \mu_z \simeq 4\% \). The relatively large error in redshift determination (\( \delta z / z \simeq 15\% \)) is because our cosmological model assumes that the cosmological parameters are themselves only accurate to about 10%. If those parameters were known precisely, we would find \( \delta z / z \simeq \delta D_L / D_L \).

and redshifts, making it possible to study the mass and merger history of black holes in the universe. Particularly at moderate to high redshift, this could provide a wealth of data on the formation and evolution of the universe’s structures [5, 11, 12].

Before moving on to the next track in our study — redshift determined independently — we would like to comment on the accuracy with which distances are determined. Distance error is strongly correlated with the errors with which the orientation and sky position of the source are determined: the measured waveform (which is a weighted sum of the two polarizations \( h_{\perp} \) and \( h_{\times} \)) takes the form

\[
h_{\text{meas}} = \frac{[ (1 + z) M ]^{5/2}}{D_L} [\pi f]^{2/3} \mathcal{F}(\text{angles}) \cos [\Phi(t) + \phi(\text{angles})].
\]

(3)

The functions \( \mathcal{F}(\text{angles}) \) and \( \phi(\text{angles}) \) schematically indicate the dependence of the measured amplitude and phase on a source’s position and orientation angles; see [4] for further discussion and details. To measure the distance accurately, we must nail down these angles as precisely as possible.

As has already been mentioned, the various source angles are determined by exploiting LISA’s orbital motion. Roughly speaking, the “angles” indicated schematically in Eq. (3) are effectively time dependent from the viewpoint of an observer who rides along with the LISA antenna (though of course they are constant with respect to the solar system’s barycentre). This time dependence modulates \( h_{\text{meas}} \), with the exact modulation encoding the values of the source angles. As a rough rule
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Figure 2. Comparison of distance determination for binaries at $z = 1$ with $m_1 = m_2 = 10^6 M_\odot$. In the top panel, we assume that LISA’s noise gets extremely bad below $10^{-4}$ Hz; the bottom panel assumes the cutoff is at $3 \times 10^{-5}$ Hz. Distance determination improves by about a factor of ten in the lower panel. This is because the wider band allows LISA to follow the binaries’ phase evolution for a much longer time: in the top panel, the binaries radiate in band for about 15 days; in the bottom, the time in band is about 10 months. Controlling the low frequency performance will have an important impact on LISA’s science on high mass binaries.

of thumb, measuring the angles well requires that LISA move through at least one radian of its orbit, translating to a rough minimum of 2 months of observation to adequately pin down the source angles. Sources that don’t radiate in band for long enough (typically high mass systems) tend to determine these angles badly and hence have poor distance determinations. A cure for these systems is to open the LISA band by reducing noise at the low frequency end. Figure 2 compares how well the luminosity distance is determined under the assumption LISA’s noise becomes extremely large below $f = 10^{-4}$ Hz (top panel) and below $f = 3 \times 10^{-5}$ Hz (bottom panel). Both cases look at measurements of binaries that have $m_1 = m_2 = 10^6 M_\odot$ and $z = 1$; as in Fig. 1, we randomly distribute the binaries’ orientations, sky positions, and merger times (within an assumed 3 year mission). In the first case ($10^{-4}$ Hz cutoff), the binaries only radiate in band for about 15 days. Distance determination is concomitantly poor, with a peak in the distribution at $\delta D_L/D_L \approx 25\%$. (This actually isn’t too bad, largely because these binaries generate a very strong signal.) When the cutoff is lowered to $3 \times 10^{-5}$ Hz, the binaries radiate in band for 10 months, and the distance determination is dramatically improved — the peak is now at $\delta D_L/D_L \approx 2\%$. Keeping the low frequency behavior under control is clearly very desirable in order to study high mass binaries.

Even in the best case, the position determination that LISA is likely to achieve
is not great by the usual standards of astronomy — the best angular resolution is on the order of several to several tens of arcminutes. If it is possible to associate the gravitational waves from a binary black hole merger with some kind of “electromagnetic” counterpart, the situation improves dramatically. By getting an independent pointing solution for the binary, many of the degeneracies that impact distance determination are broken. We illustrate this in Fig. 3. The top panel shows the distribution of $\delta D_L/D_L$ that can be expected when only gravitational waves are used to analyze the binary. The lower panel shows how the distribution changes if a counterpart exists and provides an independent pointing solution. The distance precision is improved by about an order of magnitude in this case.

Associating a counterpart with a merger is not going to be easy. The number of galaxies in each LISA “pixel” can reasonably be expected to number in the hundreds. Also, it is far from clear what kind of electromagnetic signature will characterize a counterpart. Some work discussing the kinds of counterparts one can imagine has appeared [13, 14, 15]; we hope that the promise of coordinating electromagnetic observations with gravitational-wave measurements will motivate additional work in this vein.

Associating a counterpart with a gravitational-wave merger measurement offers another exciting possibility: from such an association, it may be possible to directly measure the event’s redshift, rather than inferring it by combining the distance with cosmographic information [16]. In principle, this could provide simultaneous...
Figure 4. Likelihood contours (1 sigma) for the matter density $\Omega_m$ and the equation of state parameter $w$ (relating the pressure and density of dark energy, $p = w \rho$). We assume that the universe is flat, and that the underlying cosmology has $\Omega_m = 0.3$, $w = -1$. We compare parameter determination following measurement of two gravitational-wave candles (at $z = 1$ and $z = 3$), and 3000 Type-Ia supernovae with the SNAP [21] (evenly distributed from $z = 0.7$ to $z = 1.7$). Gravitational lensing has a dramatic impact on the gravitational-wave candle, puffing the likelihood contour out by a large amount. The event rate of supernovae is high enough that lensing effects can be averaged out.

measurements of $D_L$ and $z$, each with precision $< 1\%$. Such a measurement, though possibly rare and difficult to make, would provide invaluable information about our cosmography (complementing other well developed probes such as Type-Ia supernovae [17, 18]). At the very least, because the systematics are so different from that of other candles, a merger would increase confidence in all candles (assuming that their measurements are in accord!). At best, because the merger candle could be exquisitely precise, it could impact cosmological parameter determination with high weight.

In fact, as has recently been shown [19], all candles have a fundamental limit to their precision set by gravitational lensing, arising from the propagation of radiation through the lumpy, inhomogeneous universe in which we live. Lensing impacts gravitational waves exactly as it impacts electromagnetic radiation. A lens with magnification $\mu$ will cause an event whose true luminosity distance is $D_L$ to appear to be at distance $D_L/\sqrt{\mu}$. (Note that $\mu$ can be less than 1 — “demagnification” is in fact quite likely.) By convolving this error with the expected magnification distribution $P(\mu)$ [19], we find that $\delta D_L/D_L \approx 5 - 10\%$ due to lensing is probable (with some dependence on the redshift of the merger event) [20]. Although intrinsically of high quality, the actual effectiveness of the merger plus counterpart standard candle will
be significantly reduced by lensing.

Figure 4 illustrates cosmological parameter determination using standard candles. We assume that the universe is flat, and that the total density is given by matter ($\Omega_m$) plus “dark energy” with equation of state $p = w \rho$. The figure shows the 1-$\sigma$ likelihood contours in the $\Omega_m$-$w$ plane for several cases. The heavy black line shows the contour that would be obtained if two merger events with counterparts are measured, neglecting gravitational lensing. This contour is nearly identical to (indeed, slightly tighter than) that expected for 3000 Type-Ia supernovae measured by the proposed SNAP satellite (dotted line) [21]. The dashed line illustrates what happens to the heavy black line when the systematic uncertainty induced by gravitational lensing is taken into account. It’s rather sobering to note how much of an effect the lensing has on the gravitational-wave measurements. Lensing does not impact supernovae nearly as much: the supernova event rate is high enough that measurements can average away lensing effects, essentially sampling the full range of the lensing probability distribution.

Combining these lensed gravitational wave events with the supernovae changes the supernova contour very little. We believe that, in the end, the most important contribution of a gravitational-wave standard candle will be as a check on systematic and evolutionary effects in the candle dataset. As a candle with drastically different properties, coordinated gravitational wave/electromagnetic measurements would improve confidence in all standard candles.

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