Neutron star heating constraints on wave-function collapse models

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Collapse models, like the Continuous Spontaneous Localization (CSL) model \cite{1, 2}, aim at solving the measurement problem of quantum mechanics through a stochastic non-linear modification of the Schrödinger equation \cite{3, 4}. Such modifications have sometimes been conjectured to be caused by gravity, the most famous example being the Diósi-Penrose (DP) model \cite{5, 6}. In general, collapse models posit an intrinsic (possibly gravitational) noise, which endogenously collapses superpositions of sufficiently macroscopic systems (in a particular basis), while preserving the predictions of quantum mechanics at small scales. One notable consequence of these models is spontaneous heating of massive objects. Neutron stars, which are extremely dense, macroscopic quantum-limited objects, offer a unique system on which to test this prediction. Here, we estimate the equilibrium temperature of a neutron star radiating heat generated from spontaneous collapse models. We find that neutron stars are competitive to constrain the parameter diagram of collapse models. Theoretically or experimentally improving upper bounds for neutron star equilibrium temperatures could in principle allow to eliminate historically proposed (and one day maybe all) CSL parameter values.

\textbf{Collapse models} – Continuous Markovian collapse models modify the Schrödinger equation with a non-linear noise term:

\[
\frac{\partial}{\partial t}\psi(t) = -\frac{i}{\hbar}H\psi(t) + F(\eta_t, \psi_t)
\]

where \(\eta_t\) is a white noise process and \(F\) some function which is partially constrained by consistency conditions \cite{7, 8}, and is chosen to yield a spontaneous collapse in the position basis.

Although this stochastic description (1) of the state vector is required to understand why collapse models actually achieve their purpose and solve the measurement problem, their empirical content is fully contained in the master equation obeyed by \(\rho_t = E[|\psi_t\rangle\langle\psi_t|]\). For most Markovian non-dissipative collapse models proposed so far \cite{4}, it takes the form

\[
\frac{\partial}{\partial t}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + D[M](\rho(t))
\]

where \(D[M]\rho = -\int dxdy f(x-y)[M_r(x), [M_r(y), \rho]]\) \textbf{(2)}

with \(f\) is a positive definite function and \(M_r(x)\) is a regularized mass density operator:

\[
M_r(x) = g_{r_c} \ast M(x) = g_{r_c} \ast m \rho f(x) a(x).
\]

In this expression, \(m\) in the mass of the particle considered (we will consider neutrons), \(a^*_k(x), a^*_k(x)\) denote the usual (here fermionic) creation and annihilation operators, \(g_{r_c}\) is a regulator which smooths the mass density over a length scale \(r_c\) and “\(\ast\)” denotes the convolution product. Typically, the regulator function is taken to be Gaussian:

\[
gr_{r_c}(x) = e^{-x^2/(2r^2_c)}/(\sqrt{2\pi r^2_c}).
\]

The regulator length scale has to be much larger than the Planck length and even the nucleon Compton wavelength, the usual choice being \(r_c \simeq 10^{-7}\) m \cite{9}.

The two most common continuous collapse models are the Continuous Spontaneous Localization (CSL) model and the Diósi-Penrose model (the latter having a heuristic link with gravity):

\textbf{1. The CSL model is obtained for:}

\[
f_{CSL}(x-y) = \frac{\gamma}{2m_{N}^2} \times \delta(x-y)
\]

where \(m_N\) is the mass of a nucleon and \(\gamma\) is the collapse “strength”. It is a rate \(\times\) distance\(^3\), the corresponding rate is \(\lambda_{CSL} \equiv \gamma/(4\pi r^2_c)^{3/2}\) historically fixed at \(\lambda_{CSL} \simeq 10^{-16}s^{-1}\) (the so called “GRW” value).

\textbf{2. The DP model is obtained for:}

\[
f_{DP}(x-y) = \frac{G}{4\hbar} \times \frac{1}{|x-y|}.
\]

Because the collapse strength is fixed by the gravitational constant, there is one parameter less \cite{10}. A modern motivation for eq. (6) is given by attempts at constructing models of fundamentally semiclassical gravity \cite{11, 12}.

We note that, at least at the master equation level, the regulator applied on the mass density operator can equivalently be applied on the kernel \(f\):

\[
D[M]\rho = -\int dxdy f_{r_c}(x-y)[M(x), [M(y), \rho]],
\]

with \(f_{r_c} = g_{r_c} \ast f \ast g_{r_c}\).
Spontaneous heating – The additional decoherence term eq. (2) in the master equation does not commute with the kinetic part of the Hamiltonian, hence the expectation of the energy $\langle H \rangle_t = \text{tr}[H \rho_t]$ is no longer conserved. This spontaneous heating provides a natural test of collapse models [13–15].

Recent proposals to test these models have e.g. been built around ultra cold atoms [16], which may provide good platforms to obtain bounds on the parameters in the theory as the heating effect should be significant in relative terms. An alternative, which has been overlooked so far, is to consider instead maximally dense systems, exploiting the mass density dependence of the heating for all collapse models. In this respect, neutron stars are ideal candidates.

Neutron star cooling has been studied theoretically and observationally. At early stages when $T_{\text{star}} \sim 10^9$ K, they cool by various baryonic emission processes, but at later stages, when $T_{\text{star}} \sim 10^6$ K or colder, the cooling is radiation dominated [17–19]. Thus, the equilibrium temperature is attained by the balance of the spontaneous collapse induced heating with Stefan-Boltzmann radiation, so is determined by the heat balance condition $P_{\text{heat}} = P_{\text{rad}}$, where

$$P_{\text{heat}} = \partial_t \langle H \rangle_t = \text{tr}[H \mathcal{D} [\hat{M}] \rho_t] \quad (8)$$

and

$$P_{\text{rad}} = S \sigma T^4 \quad (9)$$

where $S$ is the neutron star surface area and $\sigma = 5.6 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$ is Stefan’s constant. It follows that at equilibrium $T_{\text{star}} = (P_{\text{heat}}/(S\sigma))^{1/4}$.

For a system of $N$ fermions with non-relativistic Hamiltonian, one can show that the spontaneous collapse induced heating $P_{\text{heat}}$ is independent of the potential (which commutes with the mass density) and more surprisingly does not even depend on the quantum state. For the CSL model it reads:

$$P_{\text{heat}} = \text{tr}[H \mathcal{D} [\hat{M}] \rho_t] = \frac{3\lambda h^2}{4r_c^2 m} N, \quad (10)$$

where $N$ is the number of neutrons in the star. Similarly, for the DP model it reads:

$$P_{\text{DP,heat}} = \frac{Gh m}{\sqrt{\pi r_c^3}} N. \quad (11)$$

The CSL model – We take the typical neutron star radius $L \sim 10$ km and mass $M_{\text{star}} \sim M_\odot \simeq 2.0 \times 10^{30}$ kg, hence $N = M_{\text{star}}/m_N \simeq 10^{57}$ neutrons. For the values historically proposed for the CSL model, $\lambda = 10^{-15}$s and $r_c = 10^{-1}m$, one finds $P_{\text{heat}} \sim 10^{14}$W. On the other hand the lowest observed temperature of an astronomical neutron star is $T_{\text{obs}} \sim 0.28$ MK for the object PSR J 840-1419 [20]. This observed temperature corresponds to a radiative dissipation rate of $P_{\text{rad}}^{\text{obs}} \sim 10^{26}$ W, well above the power that would be radiated by the CSL model. Hence, the neutron stars we can currently observe are not cold enough to straightforwardly falsify the CSL model.

Naturally, neutron stars are expected to cool down to much lower temperatures than the ones we currently manage to see directly [19] and the bound from PSR J 840-1419 is thus an excessively conservative one. Indeed, we can only observe the radiation from young and hot neutron stars, in their first few million years of existence, before they cool down into irrelevance. While it is expected that there are more than 100 million neutron stars in our galaxy, only 2000, among the brightest, have so far been observed. However, if all neutron stars ultimately reached an equilibrium temperature far above that of the interstellar medium, we would have presumably observed many more of them already.

The exact equilibrium temperature we may exclude from current lack of observation of warm remnants is not straightforward to rigorously obtain. We may however contemplate several increasingly speculative bounds. For example, if all 100 million neutron stars in our galaxy ultimately reached an equilibrium temperature above $10\%$ that of PSR J 840-1419, $T_{\text{obs}}^{10\%} \sim 3.10^4$K, we would have likely observed several nearby ones. Accepting this reasonable bound already makes neutron stars a competitive platform to test the CSL model, roughly comparable with spontaneous X-ray emission studies [21]. More speculatively, we might hope to one day be able to eliminate the possibility of an equilibrium temperature similar to that of our own sun $T_{\odot}^{\text{sun}} \sim 6.10^3$K, which would make the 100 million neutron stars in our galaxy tiny white objects. Even more speculatively, we might hope to one day be able to eliminate the possibility of an equilibrium temperature like that of our own planet, $T_{\text{earth}}^{\text{earth}} \sim 3.10^2$K, which would falsify the historical GRW values by two orders of magnitude (see Fig. 1).

What would be the ultimate limit? Let us dream one were able to get a probe next to an isolated infinitely old neutron star in thermal equilibrium only with the cosmic microwave background (CMB). Then we might be able to disprove equilibrium temperatures of the order of a few $K$, say $T_{\text{ultimate}}^{\text{ultimate}} \sim 5K$ to fix the ideas. In that case, no plausible value of parameters for the CSL model, and not only the historically proposed ones, would survive. Indeed, to be of any use, collapse models need to provide sufficient decoherence to suppress macroscopic superpositions (e.g. the smallest visible object, $\sim 10\mu$m, should become localized faster than the biological perception time of the eye $\sim 10$ms [24]). This typically requires $\lambda \gtrsim 10^{-13}$s$^{-1}$ (see Fig. 1 for more detail), although this theoretical lower bound is obviously not sharp and weaker conditions on $\lambda$ have been put forward [26].

The DP model – Following the same reasoning as for the CSL model, we can constrain the only free parameter, the regularization length $r_c$, of the DP model using
The analysis presented in this letter makes ‘lumped-element’ approximations that provide robust bounds on the radiated power. For example, we have assumed that the emissivity of a neutron star is unity and that the thermal conductivity throughout the core is large enough that the star temperature is approximately uniform. If these assumptions are relaxed, then the core temperature may be substantially higher than the observed surface temperature. Neutron superfluidity [18] has been hypothesized in the core of neutron stars. This phase will have a corresponding critical temperature $T_c$, which may provide a sensitive thermometric bound on tolerable heat generation rates in the star core: superfluidity will be suppressed if the internal temperature is too high. More generally, heat transfer models that include realistic constitutive models for the neutron star body may thus be able to provide even more stringent bounds on collapse model parameters than the lumped-element approximations we have adopted here.

On the other hand, refinements and extensions of the CSL model with colored noise (cCSL) [28, 29], dissipation (dCSL) [30], or both [31], containing additional parameters (such as a high frequency cutoff or a temperature) are known to yield weaker heating effects. Consequently, the constraints we put forward here would be weaker for these models.

In summary, with a conservative estimate of neutron star cooling based on the currently observed coldest neutron stars, one obtains constraints on the CSL model (albeit weaker than from spontaneous X-ray emission studies) and on the DP model ($r_c \gtrsim 10^{-13}$m, competitive with state of the art gravitational wave interferometer data). Improving the experimental upper bound on neutron star equilibrium temperatures would yield substantial improvements. In the most optimistic scenario in which one could get close to an old isolated neutron star and measure its temperature to a few K, one could test the whole CSL parameter diagram and severely narrow down possible DP regulator values. This prospect yields an additional motivation to study cold neutron stars.

Discussion – Currently observed neutron stars provide competitive bounds and collapse models, with substantial perspectives for improvement if colder neutron stars are observed, or if a large population of warm remnants can be excluded from lack of observation.

The Newtonian force is well measured for distances $r < 10 \mu m$ [27], which provides a conservative upper bound for $r_c$. Hence the range of values allowed for the DP model could not (even in principle) be closed by the temperature of neutron stars alone, and gravitational upper bounds would need to be improved in parallel.

For the DP model, upper bounds on $r_c$ can be obtained if the model is required to provide a consistent theory of fundamental semiclassical gravity [12]. In this context, the regulator $g_c$, affects the Newtonian potential and the $1/r^2$ law of the gravitational force breaks down for $r \sim r_c$. The Newtonian force is well measured for distances as short as $100 \mu m$ [27], which provides a conservative upper bound for $r_c$. Hence the range of values allowed for the DP model could not (even in principle) be closed by the temperature of neutron stars alone, and gravitational upper bounds would need to be improved in parallel.

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FIG. 1. CSL parameter diagram – Top: Zones formerly excluded by gravitational wave detectors [22, 23] (red), spontaneous X-ray emission [21] (blue), and insufficient macroscopic localization [24]. The value historically proposed by GRW [9] and the range put forward by Adler [25] are shown with black dots. The green line delineate the upper left regions that are excluded by currently observed neutron stars (continuous line). More speculative delineate the upper left regions that are excluded by currently observed neutron stars (continuous line). More speculative bounds, obtained assuming various equilibrium temperatures for neutron stars, are showed in hashed green.

eq (11). The most conservative bound, given by PSR J 840-1419, yields $r_c \gtrsim 10^{-13}$m, which excludes a regulator of the order of the neutron radius which was historically conjectured to be a possible cutoff. This lower bound is of the same order of magnitude as the current upper bound for $r_c \gtrsim 10^{-13}$m, which provides a conservative estimate of neutron star cooling based on the currently observed coldest neutron stars, one obtains constraints on the CSL model (albeit weaker than from spontaneous X-ray emission studies) and on the DP model ($r_c \gtrsim 10^{-13}$m, competitive with state of the art gravitational wave interferometer data). Improving the experimental upper bound on neutron star equilibrium temperatures would yield substantial improvements. In the most optimistic scenario in which one could get close to an old isolated neutron star and measure its temperature to a few K, one could test the whole CSL parameter diagram and severely narrow down possible DP regulator values. This prospect yields an additional motivation to study cold neutron stars.

Discussion – Currently observed neutron stars provide competitive bounds and collapse models, with substantial perspectives for improvement if colder neutron stars are observed, or if a large population of warm remnants can be excluded from lack of observation.
We compute the power \( P_{\text{heat}} = \text{tr} [H D [\hat{M}] \rho_t] \) generated by spontaneous collapse for a generic continuous Markovian non-dissipative collapse model and then evaluate the result for the CSL and DP models.

We consider a rather generic non-relativistic Hamiltonian \( \hat{H} = \hat{H}_0 + \hat{V} \) for fermions with:

\[
\hat{H}_0 = -\frac{\hbar^2}{2m} \int \text{d}x a^\dagger(x) \nabla_x^2 a(x) \\
\hat{V} = \int \text{d}x \text{d}y a^\dagger(x) a(x) V(x - y) a^\dagger(y) a(y),
\]

where \( a^\dagger(x) \) and \( a(x) \) are the local anti-commuting creation / annihilation operators \( \{a^\dagger(x), a(y)\} = \delta(x - y) \) and we neglect spin. We note that \( \hat{V} \) commutes with the local mass density and thus does not contribute to \( P_{\text{heat}} \).

Introducing \( \hat{P}_{\text{heat}} \) such that \( P_{\text{heat}} = \text{tr} \left[ \hat{P}_{\text{heat}} \rho_t \right] \) we have:

\[
\hat{P}_{\text{heat}} = \frac{\hbar^2}{2m} \int \text{d}x \text{d}y \text{d}z f_r(x, y) \times \left[ a^\dagger(x) a(x), [a^\dagger(y) a(y), a^\dagger(z) \nabla_x^2 a(z)] \right]
\]

We shall prove that:

\[
\hat{P}_{\text{heat}} = \frac{\hbar^2}{2m} \left( -2 \nabla_x^2 f_r(x) \right) N
\]

where \( N = \int \text{d}x a^\dagger(x) a(x) \) is the total number of neutrons. Let us compute the first commutator using the canonical anti-commutation relations and evaluate the \( y \) integral:

\[
\int \text{d}y f_r(x - y) \left[ a^\dagger(x) a(x), a^\dagger(z) \nabla_x^2 a(z) \right] = \int \text{d}y f_r(x - y) \left[ \delta(y - z) a^\dagger(x) a^\dagger(y) \nabla_x^2 a(z) - \nabla_x^2 \delta(y - z) a^\dagger(x) a(y) \right] = a^\dagger(z) \left[ f_r(x - z) \nabla_x^2 a(z) - \nabla_x^2 [f_r(x - z) a(z)] \right]
\]

\[
= -a^\dagger(z) a(z) \nabla_x^2 f_r(x - z) - 2 a^\dagger(z) \nabla_x f_r(x - z) \nabla_x a(z)
\]

Upon insertion in the second commutator, the first term in (18) will vanish. Using again the canonical anti-commutation relations we get:

\[
\begin{align*}
\{a^\dagger(x) a(x), -2 a^\dagger(z) \nabla_x f_r(x - z) \nabla_x a(z)\} &= -2 \delta(x - z) a^\dagger(x) \nabla_x f_r(x - z) \cdot \nabla_x a(z) \\
&\quad + 2 a^\dagger(z) \nabla_x f_r(x - z) \cdot \nabla_x \delta(x - z) a(x)
\end{align*}
\]

Once integrated over \( x \), the first term will be proportional to \( \nabla_x f_r(x - z) \big|_{x = z} = 0 \) by symmetry, and thus will not contribute. Using \( \nabla_x \delta(x - z) = -\nabla_x \delta(x - z) \) on the second term we get:

\[
\int \text{d}x \{a^\dagger \} = -2 \int \text{d}x a^\dagger(z) \nabla_x f_r(x - z) \cdot \nabla_x \delta(x - z) a(x)
\]

\[
= 2 \int \text{d}x \delta(x - z) a^\dagger(z) \left[ \nabla_x \cdot \nabla_x f_r(x - z) a(x) \right]
\]

\[
= 2 \left[ a^\dagger(z) \nabla_x \cdot \nabla_x f_r(x - z) a(x) \right]_{x = x}
\]

**SUPPLEMENTARY MATERIAL**

We compute the power \( P_{\text{heat}} = \text{tr} [H D [\hat{M}] \rho_t] \) generated by spontaneous collapse for a generic continuous Markovian non-dissipative collapse model and then evaluate the result for the CSL and DP models.
In this integration by part, we have neglected boundary terms which would vanish once applied to a density matrix $\rho_t$ sufficiently well behaved at infinity (which is reasonable for a compact object). As before, the gradient of $f_{rc}$ evaluated in 0 vanishes and we are left with:

$$\int dx \mathbf{\hat{a}} = -2 \nabla^2 x f_{rc}(x)|_{x=0} a^\dagger(z) a(z). \quad (21)$$

Carrying the final integration over $z$ yields as advertised:

$$\tilde{P}_{\text{heat}} = \frac{\hbar^2}{2m} \left(-2 \nabla^2 x f_{rc}(x)|_{x=0}\right) \int dz a^\dagger(z) a(z). \quad (22)$$

For the CSL model, we have:

$$f^{\text{CSL}}_{rc}(x) = \frac{\gamma}{2m^2} g_{rc} \ast g_{rc}(x)$$

$$= \frac{\gamma}{2m^2(\sqrt{4\pi r_c^2})^3} e^{-x^2/(4r_c^2)} \quad (23)$$

$$= \frac{3\lambda}{2m^2 r_c^2} \quad (24)$$

Hence:

$$-2 \nabla^2 x f^{\text{CSL}}_{rc}(x)|_{x=0} = \frac{\gamma}{m^2(\sqrt{4\pi r_c^2})^3} \frac{3}{2r_c^2} = \frac{3\lambda}{2m^2 r_c^2} \quad (25)$$

Finally, for the CSL model, this gives:

$$P_{\text{heat}} = \frac{3\lambda \hbar^2}{4r_c^2m} N, \quad (26)$$

which depends on the quantum state only through the total number of particles.

For the DP model, the regularized kernel $f^{\text{DP}}_{rc}$ can easily be evaluated in Fourier space:

$$f^{\text{DP}}_{rc}(x) = g_{rc} \ast f^{\text{DP}} \ast g_{rc}(x)$$

$$= 4\pi \frac{G}{4\hbar} \int \frac{dk}{(2\pi)^3} e^{-k^2 r_c^2} e^{i k \cdot x} \quad (27)$$

$$= \frac{G}{8\sqrt{\pi r_c^2}}, \quad (28)$$

hence:

$$-\nabla^2 x f^{\text{DP}}_{rc}(x)|_{x=0} = 4\pi \frac{G}{4\hbar} \int \frac{dk}{(2\pi)^3} e^{-k^2 r_c^2}$$

$$= \frac{G}{8\sqrt{\pi r_c^2}}, \quad (29)$$

and

$$P^{\text{DP}}_{\text{heat}} = \frac{G \hbar m}{8\sqrt{\pi r_c^3}} N. \quad (31)$$