A Low-Complexity 2D-DOA Estimation Algorithm with Massive URA

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Abstract. Conventional Direction of Arrival (DOA) estimation algorithms, such as Multiple Signal Classification (MUSIC), Root-MUSIC, and Estimating Signal Parameters via Rotational Invariance Techniques (ESPRIT) algorithms, are difficult to apply to large-scale antenna arrays. Because these algorithms not only require a lot of snapshots of the received data, but also the hardware equipment cannot meet the algorithm complexity requirements. This paper presents a low-complexity DOA estimation algorithm for large-scale Uniform Rectangle Array (URA). We first obtain coarse initial DOA estimates via the two-dimensional fast Fourier transform and then search for the accurate estimates within a very small region. The proposed algorithm only needs one snapshot of the received data to obtain the estimation accuracy of the traditional algorithm. The algorithm also has good robustness, and it can estimate DOA accurately even under extremely low signal-to-noise ratio and many sources. Theoretical results and numerical simulations are provided to assess the superiority of the proposed algorithm.

1. Introduction
Massive multi-input multi-output (MIMO) technology is the key technology for the Space Division Multiplexing in 5G communications. It has high spectral efficiency, large antenna gain, high energy efficiency, strong reliability and strong anti-interference ability [1]. However, large-scale array antennas are difficult to meet the real-time processing requirements of signals with limited hardware costs. If the Direction of Arrival (DOA) information of the source signal can be obtained in advance, the difficulty of real-time signal processing in 5G communication can be greatly eased [2-3]. DOA estimation technology has been developed for decades. Subspace-based parameterization algorithms have been research hotspots for a long time. For example, MUSIC [4] and ESPRIT [5] have very high resolution. In recent years, some researchers have applied the ESPRIT algorithm and its improved algorithms to massive MIMO systems [6-7]. But the all algorithms need eigenvalue decomposition to construct the signal subspace. The complexity of this operation is $O(L^3)$, where $L$ is the number of array elements in the array. When the number of antennas is large, the hardware costs will be high. In addition, all subspace-based algorithms require many snapshots of received data to construct the subspace, which increases the hardware storage burden [8]. Although the use of spatial smoothing to improve the ESPRIT algorithm can reduce the number of snapshots required, it also reduces the effective aperture of the array and increases the computational complexity [9], which makes the hardware cost higher. Furthermore, the subspace-based algorithms often fail when faced with a low signal-to-noise ratio or a large number of sources. This paper proposes a low-complexity DOA
estimation algorithm based on a large-scale uniform rectangular array (URA). We first obtain coarse initial DOA estimates via the fast two-dimensional Fourier transform (2D-DFT) and then search for the accurate estimates within a very small region. This algorithm requires only a single snapshot of the data to accurately estimate the DOA. It also has good robustness. When the signal-to-noise ratio is low or the number of sources is large, the DOA can still be estimated accurately. Among them, 2D-DFT can use fast algorithms with low complexity, which is convenient for engineering implementation.

2. System Model

As shown in Figure 1, an \(M \times N\) dimensional uniform rectangular array is placed on the X-Y plane, where \(M\) and \(N\) represent the number of antenna elements arranged in the X-axis and Y-axis directions of the array, and the inter-element spacing is \(\frac{\lambda}{2}\), where \(\lambda\) is the wavelength of the source signals. Assume \(K\) far-field sources from \(\Theta = (\theta_1, \theta_2, \ldots, \theta_K)\), \(\Phi = (\phi_1, \phi_2, \ldots, \phi_K)\) impinge on the array.

![Uniform rectangular array](image)

Figure 1. Uniform rectangular array

Let the element of the coordinate origin be the reference element, and the single snapshot data received by the array can be expressed as

\[
X = AS + N
\]

Where \(A = [a(\theta_1, \phi_1), \ldots, a(\theta_K, \phi_K)] \in \mathbb{C}^{MN \times K}\) is the steering matrix with \(a(\theta, \phi) = a_x(\theta, \phi) \otimes a_y(\theta, \phi)\), where

\[
a_x(\theta, \phi) = \begin{bmatrix}
e^{-j 2 \pi \sin \theta \cos \phi / \lambda}, & \ldots, & e^{-j 2 \pi (M-1) \sin \theta \cos \phi / \lambda}
\end{bmatrix}^T \in \mathbb{C}^{M \times K}
\]

and

\[
a_y(\theta, \phi) = \begin{bmatrix}
e^{-j 2 \pi \sin \phi / \lambda}, & \ldots, & e^{-j 2 \pi (N-1) \sin \phi / \lambda}
\end{bmatrix}^T \in \mathbb{C}^{N \times K}\]

\(S = [s_1, \ldots, s_K]^T \in \mathbb{C}^{K \times 1}\) is the narrow-band noncircular signal vector, \(N = [n_1, \ldots, n_{MN}]^T \in \mathbb{C}^{MN \times 1}\) denotes the additive white Gaussian noise and \(E[NN^H] = \sigma^2 I_{MN}\).

3. DOA Estimation Algorithm

Discrete Fourier transform (DFT) has been applied to DOA estimation for a long time, and the DOA estimation algorithm based on DFT is considered as a non-parametric spectrum analysis algorithm. Since the optimal resolution of this algorithm is inversely proportional to the number of antennas \(M\), it performs worse than some parameterized subspace algorithms for traditional small array and is gradually substituted by the latter algorithms. Correspondingly, the DFT-based DOA estimation algorithms were rarely studied in the literature of the last decade. However, DFT algorithms have their advantages in large-scale arrays since the resolution of DFT algorithms can be significantly improved with the very large number of antennas.

3.1. The Initial Estimation Based on 2D-DFT

The normalized 2D-DFT of the received data matrix can be given by \(\hat{F}_s = F_s X_{MN}^H F_N\), where \(X_{MN}\) is the \(M \times N\) received data matrix. The \((p,q)\)-th element of \(X_{MN}\) is represented by \(X_{(p-1)N+q}(t)\). \(F_s\) and \(F_N\) are normalized DFT matrices, and the \((p,q)\)-th element can be given by \([F_s]_{pq} = e^{-j2\pi pq / \sqrt{M}}\) and \([F_N]_{pq} = e^{-j2\pi pq / \sqrt{N}}\), respectively. Considering no noise interference and the number of antennas tends to infinity, i.e., \(M, N \to \infty\), we can obtain that \(\frac{M}{2} \sin \theta \cos \phi\) and \(\frac{N}{2} \sin \theta \sin \phi\) are both integers, and
all energy is concentrated at the point \( \left( \frac{M}{2}\sin\theta_k\cos\phi_k, \frac{N}{2}\sin\theta_k\sin\phi_k \right) \), as shown in Figure 2. The DOA estimation angle can be uniquely determined by searching this non-zero point.

However, the number of antennas in practical scenarios is limited, then \( \frac{M}{2}\sin\theta_k\cos\phi_k \) and \( \frac{N}{2}\sin\theta_k\sin\phi_k \) are not integers. As a result, the energy of \( \hat{F}_k \) will leak to the adjacent points of \( \left( \left[ \frac{M}{2}\sin\theta_k\cos\phi_k \right], \left[ \frac{N}{2}\sin\theta_k\sin\phi_k \right] \right) \), as shown in Figure 3. In fact, \( \hat{F}_k \) consists of sinc functions, and the amount of leaked energy is inversely proportional to \( M \) while proportional to \( \frac{1}{2}\sin\theta_k\cos\phi_k - \left[ \frac{M}{2}\sin\theta_k\cos\phi_k \right] \) and \( \frac{1}{2}\sin\theta_k\sin\phi_k - \left[ \frac{N}{2}\sin\theta_k\sin\phi_k \right] \). For large-scale URA arrays, \( M, N \gg 1 \), most energy of \( \hat{F}_k \) is still concentrated at the point \( \left( \left[ \frac{M}{2}\sin\theta_k\cos\phi_k \right], \left[ \frac{N}{2}\sin\theta_k\sin\phi_k \right] \right) \) and its around, and the initial DOA value can be estimated using the peak position of \( \hat{F}_k \).

\[ \theta_k = \arcsin(u_k^2 + v_k^2) \]
\[ \phi_k = \arctan(v_k/u_k) \]

Where \( u_k = \frac{\lambda m_k}{Md} \) and \( v_k = \frac{\lambda n_k}{Nd} \)

### 3.2. The Precise Estimation Based on Phase Rotation

The phase rotation factors with respect to \( \alpha \) and \( \beta \) are respectively defined as

\[
\Phi_M(\Delta\alpha_k) = \text{diag}(1, e^{j\alpha_k}, \ldots, e^{j(M-1)\Delta\alpha_k})
\]
\[
\Phi_N(\Delta\beta_k) = \text{diag}(1, e^{j\beta_k}, \ldots, e^{j(N-1)\Delta\beta_k})
\]

Where \( \Delta\alpha_k \in \left[ -\frac{\pi}{M}, \frac{\pi}{M} \right] \) and \( \Delta\beta_k \in \left[ -\frac{\pi}{N}, \frac{\pi}{N} \right] \). The phase rotation operation can be given by

\[
\hat{F}_k^{\alpha} = F_k \Phi_M(\Delta\alpha_k)X_{MN}\Phi_N(\Delta\beta_k)F^\ast_k
\]
When $\Delta \alpha_k = (\frac{2m_k}{M} - \frac{2n_k}{N} \sin \theta_k \cos \phi_k) \in \left[ -\frac{\pi}{M}, \frac{\pi}{M} \right]$ and $\Delta \beta_k = (\frac{2m_k}{N} - \frac{2n_k}{M} \sin \phi_k \sin \theta_k) \in \left[ -\frac{\pi}{N}, \frac{\pi}{N} \right]$, $\tilde{F}_k^{\alpha}$ only has one non-zero point, and the sum of $\Delta \alpha_k$ and $\Delta \beta_k$ is called the optimal phase shift amount. $u'_k = u_k - \frac{\Delta \alpha_k}{2\pi d}$, $v'_k = v_k - \frac{\Delta \beta_k}{2\pi d}$. At this time, the precise DOA estimate of the first source is given by

$$
\theta_i = \arcsin(u'^2_i + v'^2_i) = \arcsin\left(\frac{m_i^2}{M^2} - \frac{n_i^2}{N^2}\right)^2 + \frac{\Delta \alpha_k}{2\pi d}^2)
\phi_i = \arctan\left(v'_i/u'_i\right) = \arctan\left(\frac{\Delta \beta_k}{2\pi d} \right)
$$

(5)

The compression the spectral peaks with phase rotation for the incidences of two far-field narrowband independent sources are shown in figure 4 and figure 6, where noise is not considered for better demonstration. The angles of the two incident sources are assumed to be $(30^\circ, 50^\circ)$ and $(35^\circ, 55^\circ)$, respectively. The optimal phase shift of sources is usually different from each other, as shown in figure 4 and figure 6. Hence, it is necessary to find the optimal phase shifts separately.

![Figure 4. The case without phase rotation](image)

![Figure 5. The case with phase rotation for the source($\theta_1 = -30^\circ, \phi_1 = 50^\circ$)](image)

![Figure 6. The case with phase rotation for the source ($\theta_2 = -35^\circ, \phi_2 = -55^\circ$)](image)

When searching for the best phase shift amount, we can search $\Delta \alpha_k$ in a small area $\left[ -\frac{\pi}{M}, \frac{\pi}{M} \right]$ and similarly search $\Delta \beta_k$ in $\left[ -\frac{\pi}{N}, \frac{\pi}{N} \right]$. The $\Delta \alpha_k$ and $\Delta \beta_k$ which shrink the $k$-th peak to a point are the optimal phase shift of the $k$-th source, which can be calculated by

$$
\Delta \alpha_k = \arg \max_{\Delta \alpha_k \in \left[ -\frac{\pi}{M}, \frac{\pi}{M} \right]} \left\| F_m(m_k) \Phi_m(\Delta \alpha_k) X_M F_N(n_k) \right\|^2
\Delta \beta_k = \arg \max_{\Delta \beta_k \in \left[ -\frac{\pi}{N}, \frac{\pi}{N} \right]} \left\| F_m(m_k) X_M \Phi_N(\Delta \beta_k) F_N(n_k) \right\|^2
$$

(6)
Where, the $m$-th row of $F_M$ is given by $F_M(m, \cdot)$ and the $n$-th column of $F_N$ is given by $F_N(\cdot, n)$.

### 3.3. The Implementation Steps of the Proposed Algorithm

Based on the analysis above, the steps of the proposed DOA estimation algorithm in this paper are listed as follows:

1) Perform 2D-DFT transformation on the received data matrix $X_{MN}$ to obtain $\hat{F}_k$;
2) Calculate the modulus of each element of $\hat{F}_k$ and search the peak coordinate $(m_k, n_k)$ of $\hat{F}_k$;
3) calculate $u_k$ and $v_k$ according to $u_k = \frac{m_k \lambda}{2 \pi d}$ and $v_k = \frac{n_k \lambda}{2 \pi d}$;
4) search the optimal phase shift $\Delta \alpha_k$ and $\Delta \beta_k$ of the $k$-th source by utilizing equation (6);
5) calculate $u'_k$ and $v'_k$ according to $u'_k = u_k - \frac{\Delta \alpha_k}{2 \pi d}$ and $v'_k = v_k - \frac{\Delta \beta_k}{2 \pi d}$;
6) calculate the estimated DOA by utilizing equation (5).

### 4. Performance Analysis and Simulation Results

The use of sections to divide the text of the paper is optional and left as a decision for the author. Where the author wishes to divide the paper into sections the formatting shown in table 2 should be used.

#### 4.1. Performance Analysis

**4.1.1. Computational complexity analysis:** In order to reduce the computational complexity of the proposed algorithm, 2 dimension fast Fourier transform (2D-FFT) is usually used to achieve the 2D-DFT in the initial estimation, the computational complexity of the 2D-FFT operation of the $M \times N$ received data matrix is $O(MN \log(MN))$. Finding the peak of $\hat{F}$ requires calculating the modulus of elements of the $M \times N$ complex matrix with a computational complexity of $O(MN)$. For the precise estimation of the $k$-th source, the computational complexity corresponding to the decision of the optimal phase shift $\Delta \alpha_k$ is $O(2G_M M + MN)$, where $G_M$ is the amount of total quantization grids of $[-\frac{\pi}{d}, \frac{\pi}{d}]$. The computational complexity corresponding to the decision of the optimal phase shift $\Delta \beta_k$ of $\beta_k$ is $O(2G_N N + MN)$, where $G_N$ is the amount of total quantization grids of $[-\frac{\pi}{d}, \frac{\pi}{d}]$. The computational complexity of the precise estimation of K sources is $O(2K(G_M M + MN + G_N N))$. The values of $G_M$, $G_N$ needs to be adjusted according to $M$ and $N$. As $M$ and $N$ increases, the values of $G_M$ and $G_N$ can be appropriately reduced. It can be seen that the overall computational complexity of the proposed algorithm is very low, and the largest term is only $O(MN \log(MN))$.

**4.1.2. Algorithm advantage analysis:** The proposed algorithm in this paper has considerable advantages for large-scale uniform rectangular arrays, which can be listed as follows:

1) Low computational complexity (the largest term is only $O(MN \log(MN))$);
2) High estimation accuracy. The proposed algorithm obviously outperforms subspace-based estimation algorithms especially in low signal to noise ratio;
3) The total hardware cost can be reduced with no needs of additional source estimates;
4) The proposed algorithm can solve the estimation problem for more sources than subspace-based estimation algorithms;
5) Robustness. The DOA can still be estimated accurately by the proposed algorithm with very low SNR and large number of sources.
6) Simplicity. The main operation 2D-DFT of in the proposed algorithm can be achieved with the existing 2D-FFT hardware structure. Hence, the proposed algorithm is easy to implement.
4.2. Simulation Results

This section uses Monte Carlo simulations to assess the DOA estimation performance of the proposed method. We define the root mean square error (RMSE) of DOA as:

\[
RMSE = \frac{1}{K} \sum_{k=1}^{K} \sqrt{\frac{1}{Q} \sum_{n=1}^{Q} \left( \hat{\theta}_{k,n} - \theta_k \right)^2 + \left( \hat{\phi}_{k,n} - \phi_k \right)^2}
\]  

(7)

Where \( \theta_k \) and \( \phi_k \) are the accurate azimuth and elevation angles of the \( k \)th signal, respectively. \( \hat{\theta}_{k,n} \) is the estimate of \( \theta_k \) of the \( n \)th Monte Carlo trial and \( \hat{\phi}_{k,n} \) is the estimate of \( \phi_k \) of the \( n \)th Monte Carlo trial, where \( n = 1,2,\cdots,Q \). All the numerical results were obtained from \( Q = 1000 \) independent trials.

Figure 7. DOA estimated scatter plot

Because traditional subspace algorithms need to estimate the number of sources, and they are not robust. DOA cannot be estimated when the signal-to-noise ratio is very low and the number of sources is large. The algorithm proposed in this paper only needs a snapshot to receive data to estimate DOA more accurately. Fig. 6 demonstrates the effectiveness of the proposed algorithm through simulation experiments. Assume that \( M = 32 \) and \( N = 32 \), the distance between adjacent array elements \( d = \lambda/2 \), there are 13 ( \( K = 13 \) ) source signals impinging on the array with the number of source signal \( K = 13 \) with azimuth angles \( \theta = [15^\circ,15^\circ,15^\circ,30^\circ,45^\circ,45^\circ,45^\circ,60^\circ,60^\circ,75^\circ,75^\circ,75^\circ] \) and elevation angles \( \phi = [15^\circ,45^\circ,75^\circ,30^\circ,60^\circ,15^\circ,45^\circ,75^\circ,30^\circ,60^\circ,15^\circ,45^\circ,75^\circ] \), \( SNR = -5dB \). Figure 7 shows that even in the case of \( SNR = -5dB \) and the number of sources is 13, the angle estimation results are still close to the precise angle. The absence of excessive estimation errors or meaningless results is sufficient to show that the algorithm proposed in this paper has good robustness and has advantages in multi-source estimation with low signal-to-noise ratio.

Figure 8: Performance comparison between different algorithms

Figure 7 presents the DOA estimation performance of the proposed method with different SNR. In this simulation, \( M = 32 \), \( N = 32 \), \( K = 3 \), \( \theta = [15^\circ,30^\circ,45^\circ] \), \( \phi = [10^\circ,25^\circ,45^\circ] \). Figure 7 shows the RMSE comparison of the proposed algorithm, spatial smoothing ESPRIT algorithm and spatial smoothing PM algorithm when the SNR is from -5dB to 20dB. It can be seen from Fig.7 that not only the computational complexity of the spatial smoothing PM algorithm is higher than the algorithm proposed in this paper, but also the estimation performance is poor. Especially at extremely low signal-to-noise ratio ( \( SNR < 0dB \) ), the RMSE of the algorithm is already greater than 10°, which makes it difficult to achieve DOA estimation. Although the spatial smoothing ESPRIT algorithm can estimate DOA more accurately, its performance is worse than the algorithm proposed in this paper at
lower signal-to-noise ratio ($ SNR < 15 dB$). Moreover, the spatial smoothing ESPRIT algorithm requires eigenvalue decomposition with complexity $O(\lambda MN^3)$, which is difficult to achieve in large-scale array antennas. The algorithm proposed in this paper can estimate DOA using single snapshot data without any additional operations. The other two algorithms require spatial smoothing. The spatial smoothing of the two-dimensional rectangular array itself also requires more hardware resources. In large-scale uniform rectangular arrays, the proposed algorithm has obvious advantages over the spatially smoothed PM and spatially smoothed ESPRIT algorithms.

5. Conclusion
Based on large-scale URA, this paper proposes a low-complexity DOA estimation algorithm that only requires a single snapshot to receive data. This algorithm first uses 2D-DFT to implement the initial estimation, and then searches for the precise estimation value in a small area around the initial estimation. This algorithm has the advantages of high accuracy, low computational complexity, good robustness, simple algorithm, and easy engineering implementation. It can estimate DOA more accurately even when the signal-to-noise ratio is extremely low and the number of sources is large.

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7. References
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