Implementation of a medium-modified parton shower algorithm

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Abstract. We present a Monte Carlo implementation of medium-induced gluon radiation in the final-state branching process. Medium effects are introduced through an additive term in the splitting functions. We have implemented such modification within PYTHIA. We show the medium effects on the hump-backed plateau, and the transverse momentum and angular distributions with respect to the parent parton. As expected, with increasing medium densities there is an increase (decrease) of partons with small (large) momentum fraction, and angular broadening is observed. The effects on the transverse-momentum distributions are more involved, with an enhancement of low- and intermediate-\(p_T\) partons and a decrease at large \(p_T\), which is related to energy conservation, and to the lack of momentum exchange with the medium in our approach.

1 Introduction

Jet quenching - the suppression of particles with large transverse momentum produced in nucleus-nucleus collisions compared with the expectations from an incoherent superposition of nucleon-nucleon ones - is one of the most striking experimental observations at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory [1]. Its usual explanation is radiative energy loss (see the reviews [2]). Nevertheless, these measurements at RHIC suffer from a trigger bias - the requirement of a high transverse momentum particle in the event - which affects the production mechanism. This bias is very difficult to consider in analytical models as it comes strongly related to energy-momentum constraints, while most models have been developed within high-energy approximations. On the other hand, radiative energy loss is not the unique explanation for jet quenching, and several implementations of radiative energy loss exist [3].

To better characterize the produced medium and to distinguish among these different possibilities, both more differential observables at large transverse momentum and unbiased measurements like jets, are required. While RHIC is starting to look at some of these new opportunities [4], the Large Hadron Collider [5] will be the ideal place due both to the higher collision energy and to the characteristics of the detectors.

Radiative energy loss implies a modification of the standard QCD radiation pattern. The proper tool for considering the QCD branching process in the final state with full energy-momentum conservation, is a Monte Carlo simulator. In spite of the fact that a probabilistic interpretation of radiation in a medium requires phenomenological assumptions, the practical advantages of a Monte Carlo are numerous. First, it allows the access to other observables different from the limited single inclusive measurements, such as different jet shapes, jet multiplicities, multiparticle intrajet correlations,... Moreover, such an implementation makes it
possible to explore new physical mechanisms in jet development, such as the interplay of the multigluon radiation with the medium length, effects of the color flow and reconnections, effects of recoil with the medium, etc. Several implementations of radiative energy loss in Monte Carlo codes exist [6], but in them either the radiation process is superimposed on other effects like collisional energy loss, or it is treated in a simplified way as an enlargement of the QCD evolution or through a multiplicative increase of the collinear parts of the splitting functions [7].

In this paper we present a Monte Carlo with medium-modified final-state radiation, based on the ideas described in [8]. There, medium effects enter as an additive correction to the standard, vacuum splitting functions:

\[ P_{\text{tot}}(z) = P_{\text{vac}}(z) \rightarrow P_{\text{tot}}(z) = P_{\text{vac}}(z) + \Delta P(z, t, \hat{q}, L, E). \]  

This correction, \( \Delta P(z, t, \hat{q}, L, E) \) which we write as \( \Delta P(z, t) \) in the following, depends not only on the energy fraction \( z \) but also on the virtuality \( t \) of the radiating parton and its energy \( E \), and on the medium characteristics relevant for radiative energy loss: transport coefficient \( \hat{q} \) and medium length \( L \). This simple medium modification is implemented in the standard final-state showering routine PYSHOW in PYTHIA [9]. In our approach, the inelastic energy loss and the angular broadening of the shower are dynamically related through a single parameter, the transport coefficient \( \hat{q} \). The longitudinal evolution of the shower is implemented by considering the formation time of the radiated gluons. However, in its present form our implementation does not consider either the recoil of the scattering centers - and consequently elastic energy loss is not taken into account - or the exchange of color with the scattering centers in the medium.

In this note we focus on the description of the implementation of medium-induced gluon radiation in the \( t \)-ordered final-state-radiation routine in PYTHIA, and present some results. For a discussion of the basis of the modification of the splitting functions [11], we refer the reader to [8]. Extensive discussions together with a comparison between medium effects on branching processes for different ordering variables, will be published elsewhere [10]. A final version of our routine, called Q-PYTHIA, will be soon made publicly available [10].

2 The Monte Carlo

2.1 Basics steps

Basically, a branching algorithm must solve the following problem: given a parton coming from a branching (or production) point with coordinates \((t_1, x_1)\), with \( t_1 \) the virtuality and \( x_1 \) its energy fraction, which are the coordinates \((t_2, x_2)\) for the next branching?

Ignoring, for simplicity, parton labels, the Sudakov form factor

\[ \Delta(t_1) = \exp \left[ - \int_{t_0}^{t_1} \frac{d\tau'}{\tau'} \int_{z_1}^{z_1^+} dz' \frac{\alpha_s(t_1)}{2\pi} P(z) \right] \]  

(2)

gives the probability for a parton not to branch while evolving from an initial scale \( t_0 \) to another scale \( t_1 \). Consequently, \( \Delta(t_2)/\Delta(t_1) \) stands for the probability of evolving from \( t_1 \) to \( t_2 \) without branching. Thus \( t_2 \) can be generated by solving the equation

\[ \frac{\Delta(t_2)}{\Delta(t_1)} = R, \]  

(3)

\( R \) being a random number.

The energy fraction kept by the parton in the next branching \( z_2 \) can be diced down by solving the equation:

\[ \int_{z_2}^{z_2^+} dz \frac{\alpha_s}{2\pi} P(z) = R' \int_{z_2}^{z_2^+} dz' \frac{\alpha_s}{2\pi} P(z'), \]  

(4)
with $R'$ another random number. (3) and (4) are the two basic steps of a branching Monte Carlo algorithm. The only difference between the standard case and the one considering medium effects comes through the substitution (1).

In all equations above, the value of the maximum initial virtuality, $t_{\text{max}} = 4E_{\text{jet}}^2$, of the lower virtuality limit, $t_0 = 1 \text{ GeV}^2$, the lower and upper limits of the $z$-integrals, $z_{\pm} \equiv z_{\pm}(t)$, and the scale at which $\alpha_s$ runs, as well as all other aspects of the $t$-ordered evolution, are the PYTHIA defaults, see [9]. These aspects will be described and discussed more extensively elsewhere [10].

PYTHIA approximates the splitting functions by their $z \to 1$ forms, and later this approximation is corrected via a rejection method. On the other hand, our splitting functions also include small $z$ corrections - we use for the vacuum the exact splitting functions. This is the main source of the very small differences between our vacuum implementation ($\hat{q} = 0$) and PYTHIA default shown in Fig. 1. We take this comparison as a check of the quality of our procedure. Note also that our medium modifications are for $g \to g g$ and $q(\bar{q}) \to q(\bar{q}) g$ splittings \[8\]. The $g \to q\bar{q}$ branching is the same as in vacuum because its splitting probability is not singular at $z \to 1$.

![Fig. 1. Intrajet parton distributions in $\xi = \ln (E_{\text{jet}}/p)$ (left), $p_T$ (middle) and $\theta = \cos(p_z/p)$ (right) for a gluon of initial energy $E_{\text{jet}} = 100 \text{ GeV}$. PYTHIA default (black lines) and our results with $\hat{q} = 0$ (blue lines) are compared.](image)

### 2.2 Length and energy evolution

The length traveled by a parton before a gluon decoheres from its wave function and is radiated, can be estimated \[2\] by the gluon formation length $l_{\text{coh}} = 2\omega/k_T^2$, where $\omega$ and $k_T$ are the energy and transverse momentum (with respect to the parent parton) of the emitted gluon, respectively.

The shower begins with a parton that faces the full length of the medium $L$, so the medium effects on the probability of the first branching are evaluated at $L$. The coherence length of the emitted gluon is then computed being its next branching evaluated at $L - l_{\text{coh}}$. The process is iterated.
Also the energy degradation is considered. For a process \( a(E_b + E_c) \rightarrow b(E_b) + c(E_c) \), the medium effects in the branching process of \( a \) is considered at energy \( E_b + E_c \), while the subsequent branchings of \( b \) and \( c \), if any, are considered at \( E_b \) and \( E_c \) respectively. In our default results for the medium, both the evolution in length and the energy degradation are considered. The separate effect of these aspects of evolution will be discussed elsewhere [10].

3 Results

At the parton level, the general expectations of medium-induced gluon radiation formalisms are:

- A softening of the spectra.
- An increase of the parton multiplicity.
- An angular broadening of the jet.

To see the results, shown in Figs. 1-2, we run PYTHIA final-state-radiation routine PY-SHOW with our medium modifications, on a gluon of energy \( E_{\text{jet}} = 100 \) GeV moving along the positive z-axis, and study the intrajet distribution of final partons in energy fraction (actually in \( \xi = \ln (E_{\text{jet}}/p) \) with \( p = |p| \) the modulus of the momentum of the final particle - the hump-backed plateau plot), transverse momentum \( p_T = \sqrt{p_x^2 + p_y^2} \) of the final particle, and polar angle \( \theta = \cos(p_z/p) \) of the final particle. The medium length is fixed to \( L = 2 \) fm. For this note, we focus on representative results which illustrate the effects of medium-induced gluon radiation in the branching process. The statistics we have used is \( 10^5 \) generated events. Further results will be published elsewhere [10].

**Fig. 2.** Intrajet parton distributions in \( \xi \) (left), \( p_T \) (middle) and \( \theta \) (right) for a gluon of initial energy \( E_{\text{jet}} = 100 \) GeV in a medium of length \( L = 2 \) fm and for different transport coefficients \( \tilde{q} = 0 \) (black), 5 (red) and 50 (blue lines) GeV\(^2\)/fm.

In Fig. 2, we show the results at parton level, for two values of the transport coefficient \( \tilde{q} = 5 \) and 50 GeV\(^2\)/fm. We observe a suppression of high-\( z \) particles and a large enhancement
of particles with low-intermediate $z$-values, as expected. We also observe a suppression of high-$p_T$ particles and the corresponding enhancement of intermediate-$p_T$ particles. The $p_T$-spectrum should be softer than vacuum at low transverse momentum since low-$p_T$ particles should be kicked towards higher values of the transverse momentum. However we find a clear enhancement of low-$p_T$ particles. Here, the lack of exchange of energy and momentum with the medium plus energy conservation in PYTHIA, is making a large effect. Finally, we see that the angular distribution broadens with increasing transport coefficient, as expected.

4 Conclusions and outlook

In this note we have presented an implementation of medium-induced gluon radiation in the final-state branching process. Medium effects are introduced through an additive term in the splitting functions. We have implemented such modification within PYTHIA [9]. In this note, we focus on a case study. Extensive discussions of the proposed formalism, together with a comparison between medium effects on branching processes for different ordering variables, will be published elsewhere [10]. The corresponding implementation in HERWIG will be also presented there and computer codes will be released for public use.

In this brief discussion, we have observed the appearance of the different medium effects on the hump-backed plateau, and the transverse momentum and angular distributions with respect to the parent parton. As expected, with increasing medium densities there is an increase (decrease) of partons with small (large) momentum fraction, and angular broadening is observed. The effects on the transverse-momentum distributions are more involved, with an enhancement of low- and intermediate-$p_T$ partons and a decrease at large $p_T$, which is related to energy conservation and to the lack of momentum exchange with the medium in our approach.

Let us mention several caveats of our approach:

- A Monte Carlo implementation of medium-induced gluon radiation assumes that there is an ordering variable in medium, this not having been theoretically proved yet.
- As mentioned, there is no energy and momentum exchange with the medium in our present implementation. This could be of importance in an energy conserving Monte Carlo such as PYTHIA.
- Elastic energy loss is not considered.
- The effects on the color flow between the jet and the medium have to be considered.

Together with the possibility of studying the medium modifications in a realistic environment using state-of-the-art reconstruction and pileup subtraction techniques, these caveats constitute our to-do list.

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