Gauge Invariant Overlaps for Identity-Based Marginal Solutions

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Abstract

We investigate identity-based solutions associated with marginal deformations in open string field theory. We find that the identity-based marginal solutions can be represented as a difference of wedge-based solutions plus an integration of a deformed BRST exact state. Using this expression, the gauge invariant overlap can be calculated analytically for the identity-based solutions. Moreover, we show that, by gauge transformation, the overlap is transformed into a disk correlation function with the integrations of currents at the boundary.
1 Introduction

Analytic classical solutions corresponding to the tachyon vacuum were constructed in bosonic cubic open string field theory in a marginally deformed background \[1\]. The theory in the background is derived from expanding a string field around the identity-based marginal solution \[2, 3, 4\], which is an analytic solution in the original theory characterized by the BRST operator \(Q_B\). A remarkable feature of the tachyon vacuum solution in the background is that the vacuum energy of the solution can be calculated exactly, although that of the identity-based marginal solution is given as an indefinite quantity. The resulting vacuum energy of the tachyon vacuum solution allows us to expect that the vacuum energy of the identity-based solution vanishes.

The gauge invariant overlap \[5, 6, 7\] can also be evaluated analytically for the tachyon vacuum solution in the marginally deformed background \[1\]. It is defined as, for a closed string vertex operator \(V(i)\) on the midpoint,

\[
O_V(\Psi) = \langle I| V(i) |\Psi \rangle, \tag{1.1}
\]

where \(\Psi\) is an open string field and \(I\) is the identity string field. The resulting overlap for the tachyon vacuum solutions reproduces correctly the effect of marginal deformations for open-closed string couplings. However, as in the case of the vacuum energy, the overlap for the identity-based marginal solution is apparently indefinite.

These observables for the identity-based solution become indeterminate because a string field is a state in a Hilbert space with an indefinite metric. For instance, if expanding the solution in a Fock space, the observables are indeterminate forms of type \(\infty - \infty\). In general, the indefiniteness occurs widely in string field theory and several possible ways are suggested to avoid divergence \[8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\]. To obtain a definite value, it is necessary that some sort of regularization technique makes the observables finite; this is probably controlled by symmetry.

The purpose of this paper is to calculate the gauge invariant overlaps analytically for the identity-based marginal solution. By the definition (1.1), the overlap is linear with respect to a string field, in contrast to the vacuum energy. In addition, it is invariant under BRST transformation in any background. In fact, we find that

\[
O_V(Q_{\phi} \Lambda) = 0, \tag{1.2}
\]

\[
Q_{\phi} \Lambda = Q_B \Lambda + \phi \ast \Lambda - (-1)^{|\Lambda|} \Lambda \ast \phi, \tag{1.3}
\]

where \(Q_{\phi}\) is a deformed BRST operator in the background generated by a classical solution \(\phi\) in the original theory. These properties have an essential role in the analytic calculation of the overlap for the identity-based solution. We will find that, owing to the linearity and the BRST invariance, a finite value of the overlap can be derived by a norm-cancellation mechanism.

The level truncation analysis works well in the theory expanded around identity-based solutions \[20, 21, 22\] and permits us to check exact results indirectly. Actually, numerical solutions have been constructed in a marginally deformed background \[23\]. We note that numerical overlaps for the solutions consistently support our analytic results given for the identity-based marginal solutions.
This paper is organized as follows. In section 2 we briefly review the identity-based marginal solution in the original theory and the tachyon vacuum solution in the deformed background. Next, we provide a new expression for the identity-based marginal solution in terms of the tachyon vacuum solutions. Using this expression, we calculate the overlap for the identity-based solution. Then, we discuss the idea that the resulting overlap includes an expression that has been given for wedge-based marginal solutions. In section 3, we give some concluding remarks. In Appendix A, we give an expression of the overlap for the tachyon vacuum solutions.

2 Analytic evaluation of the gauge invariant overlap for identity-based marginal solutions

2.1 Identity-based marginal solutions and tachyon vacuum solutions

We briefly summarize identity-based marginal solutions in the original theory characterized by the BRST charge $Q_B$ and a tachyon vacuum solution in the theory expanded around the marginal solutions.

The action in bosonic open cubic string field theory is given by

$$S[\Psi; Q_B] = \int \left( \frac{1}{2} \Psi \ast Q_B \Psi + \frac{1}{3} \Psi \ast \Psi \ast \Psi \right),$$

where $Q_B$ is the original BRST charge. The equation of motion is $Q_B \Psi + \Psi \ast \Psi = 0$. Using holomorphic current operators $j^a(z)$ associated with a Lie algebra, and the identity string field $I$, we have an identity-based marginal solution [4]:

$$\Psi_0 = -V^a_L(F_a)I - \frac{1}{4} g^{ab} C_L(F_a F_b)I.$$ (2.2)

The half-string operators are defined by

$$V^a_L(f) = \int_{C_{left}} dz \frac{1}{2 \sqrt{2}} f(z) c j^a(z), \quad C_L(f) = \int_{C_{left}} dz \frac{2 \pi i}{f(z)} c(z),$$ (2.3)

where $c(z)$ is the ghost operator and $f(z)$ is a function on the unit circle $|z| = 1$. $C_{left}$ is the path along a unit half circle: $\text{Re} \, z \geq 0$. The function $F_a(z)$ in (2.2) satisfies $F_a(-1/z) = z^2 F_a(z)$. The parameters describing marginal deformations are given by

$$f_a = \int_{C_{left}} dz \frac{F_a(z)}{2 \pi i}.$$ (2.4)

Expanding the string field around the solution (2.2) as $\Psi = \Psi_0 + \Phi$, we obtain the action in a marginally deformed background: $S[\Phi; Q_{\Psi_0}]$. Here, the deformed BRST operator $Q_{\Psi_0}$ is
given by the definition \( (1.3) \). As a feature of the identity-based solution, we have a simpler expression for the deformed BRST operator:

\[
Q_{\Psi_0} = Q_B - V^a(F_a) - \frac{1}{4} g^{ab} C(F_a F_b), \tag{2.5}
\]

\[
V^a(f) = \oint \frac{dz}{2\pi i} \frac{1}{\sqrt{2}} f(z) c_j^a(z), \quad C(f) = \oint \frac{dz}{2\pi i} f(z) c(z), \tag{2.6}
\]

where the integration path is given by a whole unit circle.

The equation of motion in the deformed background is given by \( Q_{\Psi_0} \Phi + \Phi^* \Phi = 0 \). Then, we can construct a tachyon vacuum solution in the marginally deformed background \([1]\):

\[
\Phi_T = \frac{1}{\sqrt{1 + K'}} (c + cK'Bc) \frac{1}{\sqrt{1 + K'}}, \tag{2.7}
\]

Here, \( K' \) is defined by \( K' = Q_{\Psi_0} B \) and therefore it is given as a state deformed from \( K (= Q_B B) \) by the currents and a numerical term:

\[
k' = K + J, \tag{2.8}
\]

\[
J = -\frac{\pi}{2} \int_{C_{\text{left}}} \frac{dz}{2\pi i} \frac{1}{\sqrt{2}} (1 + z^2) F_a(z) j^a(z) I - \frac{\pi}{8} \int_{C_{\text{left}}} \frac{dz}{2\pi i} (1 + z^2) g^{ab} F_a(z) F_b(z) I. \tag{2.9}
\]

\( K', B, c \) and \( Q_{\Psi_0} \) have the same algebraic structure as that of the \( KBc \) algebra with \( Q_B \) and, in analogy with the Erler-Schnabl solution \([24]\), we can easily find that \( \Phi_T \) is a classical solution in the marginally deformed background.

Further expanding the string field around the solution \( (2.7) \), we find the kinetic operator \( Q'_{\Phi_T} \) at the tachyon vacuum. For an arbitrary string field \( \Xi \), \( Q'_{\Phi_T} \) is defined as

\[
Q'_{\Phi_T} \Xi = Q_{\Psi_0} \Xi + \Phi_T^* \Xi - (-1)^{|\Xi|} \Xi^* \Phi_T. \tag{2.10}
\]

Thanks to the \( K'Bc \) algebra, we can construct a homotopy operator \( \hat{A} \) such that \( \{ Q'_{\Phi_T}, \hat{A} \} = 1 \) and \( \hat{A}^2 = 0 \):

\[
\hat{A} \Xi = \frac{1}{2} (A \, \Xi + (-1)^{|\Xi|} \Xi^* A), \quad A = \frac{1}{\sqrt{1 + K'}} B \frac{1}{\sqrt{1 + K'}}. \tag{2.11}
\]

Therefore, we can formally prove that \( Q'_{\Phi_T} \) has vanishing cohomology.

### 2.2 The gauge invariant overlaps for identity-based marginal solutions

Let us introduce a parameter \( t \) into the weighting functions of the solution \( (2.2) \) as \( F^i_a(z) = t F_a(z) \). We denote the solution given by \( F^i_a(z) \) as \( \Psi^l_0 \):

\[
\Psi^l_0 = -t V^a(L_a) I - \frac{t^2}{4} g^{ab} C_L(F_a F_b) I. \tag{2.12}
\]
\( \Psi_t^i \) provides a one-parameter family of identity-based solutions connecting a trivial configuration and \( \Psi_0 \).

Correspondingly, we can consider the tachyon vacuum solution \( \Phi_t^i \), which is constructed by using \( K_t' = Q_{\Psi_t^i} B \) instead of \( K' \) in \( \Phi_T \) (2.7). \( \Phi_T^i \) becomes \( \Phi_T \) for \( t = 1 \) and the Erler-Schnabl solution \( \Phi_{T}^{ES} \) for \( t = 0 \). Similarly, we can find that a homotopy operator exists for \( Q_{\Phi_t^i} \), which is given by replacing \( K_t' \) in (2.11) with \( K_t' \).

Now, let us consider the sum of the identity-based marginal solution \( \Psi_t^i \) and the tachyon vacuum solution \( \Phi_t^i \) in the deformed background: \( \Psi_T^i = \Psi_0^i + \Phi_T^i \). It connects \( \Psi_T^{i=0} = \Phi_{T}^{ES} \) and \( \Psi_T^{i=1} = \Psi_0 + \Phi_T \). We find that \( \Psi_T^i \) is a classical solution in the original background. In fact, by adding the two equations, \( Q_B \Psi_0^i + \Psi_0^i \star \Psi_0^i = 0 \) and \( Q_{\Psi_0^i} \Phi_T^i + \Phi_T^i \star \Phi_T^i = 0 \), we have the equation,

\[
Q_B \Psi_T^i + \Psi_T^i \star \Psi_T^i = 0. \tag{2.13}
\]

Expanding the string field around \( \Psi_T^i \) in the original theory, we have the deformed BRST operator \( Q_{\Psi_T^i} \) by following the definition (1.3). Substituting \( \Psi_0^i + \Phi_T^i \) for \( \Psi_T^i \) in \( Q_{\Psi_T^i} \), we can easily find that \( Q_{\Psi_T^i} \) is the kinetic operator at the tachyon vacuum in the deformed background; i.e., \( Q_{\Psi_T^i} = Q_{\Phi_T^i} \). We remember that \( Q_{\Phi_T^i} \) has no cohomology because of the existence of a homotopy operator as mentioned above. Consequently, we conclude that \( Q_{\Psi_T^i} \)-closed states are \( Q_{\Psi_T^i} \)-exact states.

Differentiating (2.13) with respect to \( t \), we have

\[
Q_{\Psi_T^i} \frac{d}{dt} \Psi_T^i = 0. \tag{2.14}
\]

Then, we find that, for some state \( \Lambda_t \),

\[
\frac{d}{dt} \Psi_T^i = Q_{\Psi_T^i} \Lambda_t. \tag{2.15}
\]

Integrating (2.15) from 0 to 1, we get

\[
\Psi_0 = \Phi_{T}^{ES} - \Phi_T + \int_0^1 Q_{\Psi_T^i} \Lambda_t dt. \tag{2.16}
\]

Thus, the identity-based marginal solution \( \Psi_0 \) is expressed by the difference of the wedge-based solutions plus the integration of a deformed BRST exact state.

Equation (2.16) is a useful expression for evaluating the gauge invariant overlaps for the identity-based marginal solution. Substituting (2.16) into (1.1), it can be written as

\[
O_V(\Psi_0) = O_V(\Phi_{T}^{ES}) - O_V(\Phi_T), \tag{2.17}
\]

where the contribution of the last term in (2.16) vanishes due to the BRST invariance (1.2). Although the left hand side of (2.17) seems to be indefinite due to singular property of the

\footnote{The Erler-Schnabl solution is given by \( \Phi_{T}^{ES} = \frac{1}{\sqrt{1 + K}}(c + cK B c)\frac{1}{\sqrt{1 + K}} \).}

\footnote{We can take \( \Lambda_t = \hat{A}^i \frac{d}{dt} \Psi_T^i \) using the homotopy operator \( \hat{A}^i \).}
identity string field, the right-hand side can be calculated analytically. In fact, $O_{V}(\Phi_{ES})$ and $O_{V}(\Phi_{T})$ have been already computed in Refs. [24] and [1], respectively. Thus, the overlap for the identity-based marginal solution is reduced to the difference of the overlaps for wedge-based solutions.

As a first concrete example, let us consider the overlap with the graviton vertex operator, $V \sim c \bar{c} \partial X^0 \partial X^0$ and the identity-based solution generated by spatial currents. Since $V$ does not correlate with spatial currents, we find the relation $O_{V}(\Phi_{T}) = O_{V}(\Phi_{ES})$ as discussed in Ref. [1]. From the relation (2.17), it follows that the overlap for the identity-based marginal solution is zero: $O_{V}(\Psi_{0}) = 0$.

Secondly, let us consider the identity-based marginal solution generated by the current

$$j(z) = \frac{i}{\sqrt{2\alpha'}} \partial X^{25}(z).$$

(2.18)

Then, we consider the case that the 25th direction is compactified on a circle of radius $R$ and the vertex operator $V$ is given as

$$V = \tilde{V} e^{ik_{L}X^{25}(i) + ik_{R}X^{25}(-i)},$$

(2.19)

where $\tilde{V}$ is an operator containing no $X^{25}$. In Ref. [1], it is shown that a phase shift occurs in the overlap when the background is deformed by the current (2.18):

$$O_{V}(\Phi_{T}) = \exp \left( i \frac{\pi w R}{\sqrt{\alpha'}} f \right) O_{V}(\Phi_{ES}),$$

(2.20)

where $w$ is the winding number in the 25th direction\(^3\) and $f$ is a marginal parameter given by Eq. (2.4)\(^4\) for the current (2.18).

Consequently, from (2.17), we obtain

$$O_{V}(\Psi_{0}) = \left\{ 1 - \exp \left( i \frac{\pi w R}{\sqrt{\alpha'}} f \right) \right\} O_{V}(\Phi_{ES}).$$

(2.21)

It should be noted that this is a non-zero analytic result for physical observables of the identity-based solutions.

2.3 The relation to one-point functions

The gauge invariant overlap for $\Phi_{T}$ (2.7) is represented by using a correlation function on a cylinder of circumference $\pi$ (or on the sliver frame obtained by the conformal mapping arctan $z$ from the canonical upper half plane) [1, 25]:

$$O_{V}(\Phi_{T}) = \frac{e^{-\pi c}}{\pi} \left\langle V(i\infty) c \left( \frac{\pi}{2} \right) \exp \left( \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dx J(x) \right) \right\rangle_{C_{\pi}}$$

(2.22)

\(^3\) $w$ is related to $k_L$ and $k_R$ as $w = (k_L - k_R)\alpha'/R$.

\(^4\)Since we consider an Abelian case, $f$ is given by using a function $F(z)$: $f = \int_{C_{\text{left}}} \frac{dz}{2\pi i} F(z)$. 
where $J$ is given in terms of the current $j^a$ in the marginal solution (2.2), and $C$ comes from their operator product expansion (OPE):

$$J(x) = \int_{-\infty}^{\infty} dy f_a(y) j^a(x + iy),$$  \hspace{1cm} (2.23)

$$C = \int_{-\infty}^{\infty} dy (-\pi) g^{ab} f_a(y) f_b(y).$$  \hspace{1cm} (2.24)

In the above integrations, the weighting function $f_a(y)$ is given by

$$f_a(y) = \frac{F_a(\tan(\frac{iy}{4}))}{2\pi\sqrt{2}\cos^2(\frac{iy}{4})}.$$  \hspace{1cm} (2.25)

We give a derivation of (2.22) in Appendix A. In the correlation function (2.22), the current operator is integrated inside the cylinder, which is parametrized by $x + iy$. In contrast, the gauge invariant overlap for the wedge-based solutions [26, 27, 28, 29] is expressed by a correlation function with the $x$-integrations of currents inserted at the open string boundary, namely $y = 0$ [30, 31]. The boundary insertion is a natural consequence from the viewpoint of boundary marginal deformations in conformal field theory. However, this is not a necessity in string field theory.

Here, we will show that the expression (2.22) can be related to a one-point function with the boundary insertion. First, let us take $F_a(z)$ in $\Psi_0$ as

$$F_a(z; s) = 2\lambda_a \frac{s(1 - s^2)}{\arctan(\frac{2s}{1 - s^2})} \frac{1}{1 - s^2 (z + \frac{1}{z})^2 + s^4},$$  \hspace{1cm} (2.26)

where $s$ is a parameter from 0 to 1 and $\lambda_a$ corresponds to a marginal parameter by following (2.4):

$$\int_{C_{\text{left}}} \frac{dz}{2\pi i} F_a(z; s) = \frac{2\lambda_a}{\pi}.$$  \hspace{1cm} (2.27)

This indicates that the marginal parameters are independent of the parameter $s$ in (2.26). Since the form of $F_a$ except the half integration mode (2.4) can be changed by gauge transformations of $\Psi_0$ [4], the parameter $s$ is redundant in the gauge invariant overlap (2.17) and therefore in (2.22).

For the limit $s \to 0$, the function becomes

$$F_a(z; s) \to \lambda_a \left( z + \frac{1}{z} \right) \frac{1}{z}.$$  \hspace{1cm} (2.28)

This limiting form is a function discussed as a simple example in Ref. [1].

Taking the limit $s \to 1$, the function approaches the delta function. In fact, (2.26) can be expanded into a Laurent series:

$$F_a(z; s) = 2\lambda_a \frac{1}{\arctan(\frac{2s}{1 - s^2})} \sum_{n=0}^{\infty} s^{2n+1} \left( z^{2n+1} + \frac{1}{z^{2n+1}} \right) \frac{1}{z}.$$  \hspace{1cm} (2.29)

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Then, putting \( z = e^{i\theta} \), we find that, for \( s \to 1 \),
\[
F_a(z; s) \to 4\lambda_a \frac{1}{\pi} \sum_{n=0}^{\infty} \left( z^{2n+1} + \frac{1}{z^{2n+1}} \right) \frac{1}{z} = 4\lambda_a \left\{ \delta(\theta) + \delta(\pi - \theta) \right\}. \tag{2.30}
\]

On the unit disk \(|z| = 1\), \( \theta = 0 \) and \( \theta = \pi \) correspond to open string boundaries. As a result, the support of the function \( F_a(z; s) \) localizes at the open string boundaries for the limit.

In the sliver frame, the line \( u = \pi/4 + iy \) corresponds to the left half of an open string, \(|\theta| \leq \pi/2\). Following the relation \( e^{i\theta} = \arctan(\pi/4 + iy) \), we find that \( \theta \) is given by a function of \( y \):
\[
\theta(y) = \arctan(\sinh(2y)). \tag{2.31}
\]

Then, for the limit \( s \to 1 \), the function \( f_a(y) \) \textbf{[2.25]} approaches \( \sqrt{2} \pi \lambda_a \delta(y) \) and then the integration \( J(x) \) \textbf{[2.23]} becomes a local operator:
\[
J(x) \to \sqrt{2} \pi \lambda_a j^a(x). \tag{2.32}
\]

Consequently, in the limit \( s \to 1 \), \textbf{[2.22]} can be expressed as the correlation function deformed by the boundary operator:
\[
\frac{e^{-\pi c}}{\pi} \left\langle V(i\infty)c(\frac{\pi}{2}) \exp\left( \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dx j^a(x) \right) \right\rangle_{C_{\pi}} \to \frac{e^{-\pi c}}{\pi} \left\langle V(i\infty)c(\frac{\pi}{2}) \exp\left( \sqrt{2} \pi \lambda_a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dx j^a(x) \right) \right\rangle_{C_{\pi}}. \tag{2.33}
\]

In general, the boundary integration in \textbf{[2.33]} diverges whenever the OPE among the currents contains poles. However, the correlation function \textbf{[2.22]} does not suffer from divergence \textbf{[1]} and then, as far as \( s \neq 1 \), \textbf{[2.33]} can be calculated finitely. Accordingly, it turns out that the divergence in the limit should be canceled by the factor depending on \( C \) \textbf{[2.24]}, which diverges in the limit \( s \to 1 \). In other words, the function \( F_a(z; s) \) can be applied to regularize the boundary integration of the currents and then \( C \) takes a role as a subtraction term to cancel the divergence.

Finally, from the above result of \( O_V(\Phi_T) \) and the relation \textbf{[2.17]}, we conclude that the overlap of the identity-based marginal solution is represented by the difference between the two disk amplitudes:
\[
O_V(\Psi_0) = \frac{1}{\pi} \left\langle V(i\infty)c(\frac{\pi}{2}) \left\{ 1 - e^{-\pi c} \exp\left( \sqrt{2} \pi \lambda_a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dx j^a(x) \right) \right\} \right\rangle_{C_{\pi}}. \tag{2.34}
\]

This expression just corresponds to the result in Ref. \textbf{[30]} for wedge-based marginal solutions.  

\textbf{5} It should be noted that this is essentially the same result as that found by T. Erler and C. Maccaferri, which was presented at the SFT2012 conference in Jerusalem. According to their discussion, the gauge invariant overlap can be computed by using the “phantom term” \textbf{[32, 33]}, which emerges when connecting the two solutions, the solution \( \Psi_0 \) and the tachyon vacuum solution \( c(1 - K) \). Taking the limit in the phantom term, the bulk integration of the current in the overlap is localized at the open string boundary. Like our results, \( C \) serves as a subtraction term to cancel the divergence. Consequently, the correlation function can be regarded as the disk amplitude with the integration of the currents at the boundary.
3 Concluding remarks

We have shown that the gauge invariant overlaps for identity-based marginal solutions can be calculated exactly in terms of the overlaps for tachyon vacuum solutions. We emphasize that the relation (2.16) enables us to derive non-zero values of the overlaps for identity-based solutions. Then, we have found that, by gauge transformations, the resulting overlap is transformed to the disk amplitude with the boundary deformation. The result is identical to the gauge invariant overlap for wedge-based marginal solutions.

The relation (2.16) is useful for evaluating the gauge invariant overlaps for the identity-based solutions. In addition, the linearity and the BRST invariance of the overlaps are important in the calculation. However, it remains difficult to calculate the vacuum energy of the identity-based solutions directly even if (2.16) is used, since the vacuum energy is non-linear with respect to the string field.

In §2.2 by using (2.16), the gauge invariant overlap with the graviton vertex $V \sim \bar{c} \partial X^0 \bar{\partial} X^0$ is found to be zero for the identity-based marginal solutions. Therefore, we can expect that the vacuum energy vanishes if we use the proportionality between the energy and the overlap with the vertex $[34]$. This is consistent with the result that was previously evaluated by the differentiation of the vacuum energy with respect to the marginal parameters $[2, 3, 4]$.

Our non-trivial analytic results are seen as a significant step in understanding identity-based solutions. The results seem to be easily extended to the case of identity-based marginal solutions in superstring field theories $[1, 35]$. One of the most important applications is the evaluation of the overlap and the vacuum energy for the identity-based tachyon vacuum solution $[3, 21, 36]$. Although it is not straightforward to apply our results to the tachyon vacuum case, we expect that they offer a new perspective on identity-based solutions.

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A A derivation of (2.22)

In the marginally deformed background $[1]$, the overlaps for the tachyon vacuum solution are given by

$$O_V(\Phi_T) = \text{Tr} \left( Vc \frac{1}{1 + K'} \right) = \int_0^\infty dt \, e^{-t} \langle I| V(i)| c e^{-iK'} \rangle. \tag{A.1}$$

To extract the dependence of $t$, let us consider the operator

$$\mathcal{L}'_0 = \{ Q_{\Psi_0}, B_0 \}, \tag{A.2}$$
where $Q_{\Psi_0}$ is the deformed BRST operator (2.5) and $B_0$ is an anti-ghost operator defined in Ref. [37]. As in the case of superstring field theories [25, 38], we can derive the equations

$$\frac{1}{2}(\mathcal{L}'_0 - \mathcal{L}'_0^\dagger)c = -c, \quad \frac{1}{2}(\mathcal{L}'_0 - \mathcal{L}'_0^\dagger)K' = K'. \quad (A.3)$$

Then, we find

$$c e^{-tK'} = \left( \frac{t}{\kappa} \right)^{1 + \frac{1}{2}(\mathcal{L}'_0 - \mathcal{L}'_0^\dagger)} c e^{-\kappa K'}, \quad (A.4)$$

where $\kappa$ is a constant.

Noting that we have [25, 31]

$$\langle I|V(i)(B_0 - B_0^\dagger) = 0, \quad \langle I|V(i)(\mathcal{L}'_0 - \mathcal{L}'_0^\dagger) = 0, \quad (A.5)$$

we can rewrite the overlap by using (A.4) and perform the integration in the overlap:

$$O_V(\Phi_T) = \int_0^\infty dt \; e^{-\frac{t}{\kappa}} \langle I|V(i)|c \; e^{-\kappa K'} \rangle = \frac{1}{\kappa} \langle I|V(i)|c \; e^{-\kappa K'} \rangle. \quad (A.6)$$

Then, by following Refs. [1, 25], we can express the overlap in terms of a correlation function on a cylinder of circumference $\pi \kappa / 2$:

$$O_V(\Phi_T) = \frac{2e^{-\frac{\pi c}{\kappa}}}{\pi \kappa} \left\langle V(i\infty) \; c(0) \; \exp \left( \int_0^{\frac{\pi}{2}} dx \; J(x) \right) \right\rangle e^{\frac{\pi c}{\kappa}}, \quad (A.7)$$

where $J(x)$ and $C$ are defined by (2.23) and (2.24), respectively. Setting $\kappa = 2$ and using the periodicity along the $x$ direction, we can derive the final expression of the overlap as in (2.22).

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