New Determination of the Hubble Parameter Using the Principle of Terrestrial Mediocrity

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1 Abstract

Measurements of the linear diameters of 12 nearby spiral galaxies with distances determined from primary indicators suggest that both the Milky Way Galaxy and M31 are in the middle of the range of sizes for such galaxies. By comparing the measured linear diameters of these nearby systems with the inferred diameters of a sample of more than 3000 spirals with known redshifts, we conclude that the most likely value of the Hubble Parameter lies in the range 50 - 55 km s\(^{-1}\) Mpc\(^{-1}\).

2 Introduction

It is widely accepted that our Milky Way Galaxy and M31 are larger than most spirals (see, for example, van der Kruit, 1990). This belief is largely a historical accident, resulting from early high estimates of the Hubble Parameter, \(H_0\). Until recently, there has been no direct way to test this assumption; analysis of volume limited surveys of galaxies in the Local Group certainly reveals that the Milky Way is among the largest galaxies in our immediate vicinity when we consider galaxies of all Hubble types, but since the Local Group is dominated in number by dwarf galaxies this is hardly surprising. We have found, on the other hand, from an analysis of nearby spirals of similar Hubble type to our galaxy, and with well-determined distances, that the Milky Way is very much average in size (Goodwin et al., 1997). This is strong evidence in support of the “principle of terrestrial mediocrity” (Vilenkin, 1995), that we live in an ordinary galaxy in an ordinary part of the Universe.

If the linear sizes of enough nearby galaxies can be determined, a comparison of these sizes with the inferred sizes of galaxies at higher redshift gives an indication of the value of \(H_0\).

Even without knowing whether the sizes of the Milky Way and M31 are typical for spirals, some researchers have attempted to use this kind of argument to find a value for \(H_0\). Notably, Sandage (1993a,b; 1996) chose M101 as a “typical” spiral, and argued that if the linear diameter of M101 is equal to the mean of field galaxies with similar appearance then the most probable value for \(H_0\) is 43 km s\(^{-1}\) Mpc\(^{-1}\). A key step in his chain of inference was the assumption “that M101 not be the largest in a distance-limited sample”. Unfortunately, we have found that M101 is indeed one of the largest galaxies in our part
of the Universe. A similar analysis in Sandage (1996) using M31 as the “typical” spiral yielded $H_0 = 43 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

In this paper we adopt a similar method to estimate $H_0$, based on the principle that a sample of nearby spirals of specific Hubble type represents a “fair” sample of the intrinsic population. We extend the range of Hubble types considered, however, in order to increase the size of both the calibrating sample of nearby objects and the distant population of several thousand field spiral galaxies. (Sandage’s analysis used 60 galaxies). We, therefore, aim to find in this way a more reliable value for $H_0$. In this report we summarise our preliminary results.

3 Galaxy diameters

The corrected 25 B-mag arcsec$^{-2}$ isophotal diameters for all of our galaxies were taken from the RC3 catalogue (de Vaucoulers et al. 1991). The observed major axis of this isophote was corrected for galactic extinction but not for inclination as spiral galaxies are found by those authors to be “substantially optically thick” at this isophotal level. This provides us with a uniform sample using a surface brightness diameter that is hence independent of distance (the redshifts involved are too low to require any $k$ corrections) allowing us to directly compare our local sample with the more distant sample.

3.1 Local calibrators

A sample of 11 spiral galaxies with independent Cepheid distances was chosen to act as the local calibrators. The Cepheid distances have been tabulated in Giovanelli (1996) and Freedman (1996); the figure for NGC3351 comes from Graham et al. (1997). Table 1 summarises the data, converting the corrected isophotal diameters in arcmin to a linear diameter in kpc using the Cepheid distances.

In our earlier paper (Goodwin et al. 1997) we also make a calculation for the Galactic diameter at this isophote using data from van der Kruit (1987, 1990) which yields a 25 B-mag arcsec$^{-2}$ diameter of

$$d_{25(\text{true})} = 26.8 \pm 1.1\text{kpc}$$

leading to the conclusion, from comparison with the data in Table 1, that the Milky Way is a very average-sized galaxy, especially for its Hubble type, $2 < T < 6$ (van der Kruit 1987). This small sample of galaxies also seems approximately to confirm the idea that linear diameter is relatively independent of Hubble type within this range (de Jong 1996).

Consider now the distribution of (natural) log linear diameters for this local sample of Table 1, supplemented by the Milky Way. We will use log linear diameters throughout this paper since these have been shown to be well-modelled by a Gaussian distribution (c.f. Paturel 1979; Lynden-Bell et al. 1988). Calculating the sample mean value, $< \log D >$, and dispersion, $\sigma$, of the (natural) log linear diameter distribution for this local sample, we obtain
Table 1: The Hubble type $T$, face-on angular 25 B-mag arcsec$^{-2}$ isophotal diameters $d_{25(\text{ang})}$, Cepheid distances, $R$, and actual linear diameters $D_{25(\text{true})}$ corresponding to this isophote, for 11 local spiral galaxies of Hubble types 2 to 6.

| NGC | M | T | $d_{25(\text{ang})}$ | $R$ | $D_{25(\text{true})}$ |
|-----|---|---|---------------------|-----|----------------------|
| 224 | 31| 3 | 204 | 0.77 | 45.7 |
| 598 | 33| 6 | 74.1 | 0.85 | 18.4 |
| 1365| 3 | 11.2 | 18.2 | 59.3 |
| 2403| 6 | 22.9 | 3.18 | 21.2 |
| 3031| 81| 2 | 27.5 | 3.63 | 29.0 |
| 3351| 3 | 7.59 | 10.1 | 22.3 |
| 3368| 96| 2 | 7.59 | 11.6 | 25.6 |
| 4321| 100| 4 | 7.59 | 16.1 | 36.2 |
| 4536| 4 | 7.59 | 16.7 | 36.8 |
| 4639| 4 | 2.82 | 25.1 | 20.6 |
| 5457| 101| 6 | 28.8 | 7.38 | 61.8 |

Fig. 1 shows the sample cumulative distribution of log linear diameter, compared with the cumulative distribution function of a Gaussian with mean and dispersion equal to the sample values. It can be seen that the local data give a good fit to a Gaussian distribution, as borne out by the significance level (or $p$ value) of the Kolmogorov Smirnov (KS) statistic.

3.2 Distant galaxy sample

If the nearby galaxy sample is typical of the universal intrinsic population of galaxies in the same range of Hubble types, we can use this information to obtain an estimate of the Hubble parameter. We have used angular diameters of more than 3827 spiral galaxies out to a redshift of over 20000 km s$^{-1}$ (although the vast majority of galaxies in the sample have redshifts of 1000 - 5000 km s$^{-1}$) taken from the RC3 catalogue.

All these galaxies have known redshifts, so the assignment of their true linear diameters depends only on the assumed value of the Hubble Parameter to give their distances, together with the assumption that the Hubble flow is uniform. The sample of galaxies is spread across the sky including both cluster and field galaxies and we assume no bulk local motion that may change distances. Unlike Sandage, we have included spirals of all Hubble types 2 through 6. The justification for this is that the linear diameters for a chosen value of $H_0$ are the same for all of these Hubble types (de Jong 1996).

3.2.1 Completeness

The sample of distant galaxies is biased strongly toward large galaxies as the RC3 catalogue has an angular diameter cut-off at $\approx$ 1 arcmin. At a redshift distance of 5000 km s$^{-1}$
this would correspond to a galaxy with diameter $14.5h^{-1}$ kpc. This result shows that galaxies even the size of the Milky Way will not appear in 1 arcmin angular diameter limited catalogues further away than $9200h$ km s$^{-1}$. At even larger distances only the most superlarge galaxies (similar to NGC 1365 or M 101) would appear in such catalogues. We address the question of completeness in two different ways, which we now summarise. A comprehensive discussion of our statistical analysis will follow shortly in a full report.

1. Under the assumption that the spatial distribution of galaxies is uniform and the intrinsic distribution of log linear diameters is Gaussian, we determine the probability density function of log linear diameter for observable galaxies subject to a selection function characterised by a sharp apparent angular diameter limit at 1 arcmin, and an upper and lower redshift cutoff. The primary function of the redshift cutoff was to exclude significant deviations from uniform Hubble flow due to galaxies at low redshift with substantial peculiar motions. As discussed by e.g. Gould (1993), the presence of a redshift (or equivalently distance) selection function also introduces a bias in the distribution of observable objects. Our analysis sets out to both compute and remove this bias in conjunction with the more familiar ‘Malmquist’ bias directly associated with the angular diameter limit. We thus estimate from the distribution of observable galaxies the parameters of the intrinsic, Gaussian, distribution of log linear diameters.

2. As an alternative to the above approach, instead of attempting to remove the selection bias to which the distant sample is subject, we subject the distant sample to a new selection limit, dependent only on the log linear diameter. Since the selection is now only a function of an intrinsic quantity we can apply the same selection criterion to the local sample.

We discuss quantitative aspects of these two treatments of selection effects in the Results section below.

4 Results

The extreme range of possible values for the Hubble parameter determined by different techniques and published recently is from about 40 to 80 km s$^{-1}$ Mpc$^{-1}$. If the value of $H_0$ were as low as 40 km s$^{-1}$ Mpc$^{-1}$ then the Galaxy (and most other galaxies from the local calibrating sample) would not appear above the angular diameter cut-off at a redshift distance of 5000 km s$^{-1}$ Mpc$^{-1}$ or greater. This would seriously undermine several other recent determinations of $H_0$, as our local sample comprises e.g. those galaxies used to calibrate the I band Tully-Fisher relation (c.f. Giovanelli, 1996). If, however, $H_0$ were higher than 80 km s$^{-1}$ Mpc$^{-1}$, then the Milky Way and M31 (which seem to be typical representatives of nearby galaxies over the considered range of Hubble types, in the centre of our calibrating sample) would be large spirals and NGC 1365 and M 101 would be among the largest spirals in the observable Universe.

We have used three methods in concert to examine the size distribution of the distant galaxies and estimate $H_0$, quantifying the above two general conclusions. First a subset of distant galaxies between 1500 and 5000 km s$^{-1}$ was selected. As mentioned in the preceding section, the lower limit of this sample was chosen to eliminate the worst of the contamination
of the sample by peculiar motions, whilst the upper limit was chosen so as to provide an approximately distance-limited sample. We found that the extension of the upper limit to 40000 km s$^{-1}$ (i.e. effectively no upper redshift limit!) yielded an observable distribution of log linear diameters which deviated significantly from the Gaussian distribution, with a shifted mean, predicted by our model of uniform spatial density and a sharp angular diameter limit. We considered this to be due to the breakdown of our model assumptions; in particular the selection function at high redshift was unlikely to be depend only on angular diameter, but also on inclination and would probably begin to display a sensitivity to morphological type. When we adopted the limit of 5000 km s$^{-1}$, however – while this introduced a new selection bias similar to that discussed by Gould (1993) and thus rendered the predicted distribution of log linear diameters formally non-Gaussian – we found that the computed observable distribution with this new selection function showed deviations from Gaussianity of only a few percent. We thus continued to model the predicted observable distribution as a Gaussian. Moreover, the distant sample now matched this predicted observable distribution very well. Figure 2 shows the sample cumulative distribution of log linear diameter for the selected galaxies and, for comparison, the best-fit Gaussian to the predicted distribution of observable galaxies with this new selection function. Note that application of the KS test suggests a good fit of the data to this approximately Gaussian model.

The observed mean of the best-fit Gaussian distribution lies at $< \log D > = 3.48 \pm 0.01$, if we assume that $H_0 = 60$ km s$^{-1}$ Mpc$^{-1}$. Correcting for the effects of bias introduced by the angular diameter and redshift selection limits, we deduce that the best-fit mean value of the intrinsic distribution of log linear diameter is $< \log D > = 3.25 \pm 0.02$. The mean of the calibrating sample, however, lies at $< \log D > = 3.39 \pm 0.12$, where the quoted error is the standard error on the mean derived from the sample dispersion. These two samples would, therefore, have the same mean if

$$H_0 = 52 \pm 6 \text{km s}^{-1}\text{Mpc}^{-1}$$

As a further means of placing limits on the likely value of $H_0$ it is useful to consider the order statistics of the modelled intrinsic distribution of log linear diameters (c.f. Hendry, O’Dell & Collier-Cameron 1993). For a sample of 12 calibrating galaxies, which we are assuming to be drawn from the same intrinsic population as the distant sample, one can therefore pose the question of how likely it is, for a given value of $H_0$, that one would obtain e.g. as small a galaxy as the galaxy with smallest linear diameter, or conversely as large a galaxy as that with largest linear diameter. There are many different ways to combine the order statistics and apply statistical tests to their values, and we will describe such an analysis in detail in our follow-up paper. Here we present one plot, which succinctly illustrates the same principal features as a more sophisticated analysis. Figure 3 shows the lower tail of the third order statistic (i.e. the probability that the third smallest galaxy in a sample of size 12 drawn from the distant sample would be as small as, or smaller than, the smallest observed member of the local calibrating sample) and the upper tail of the tenth order statistic (i.e. the probability that the third largest galaxy in a sample of size 12 drawn from the distant sample would be as large as, or larger than, the largest observed member of the local calibrating sample) for different assumed values of $H_0$ in the range 30 - 100 km s$^{-1}$ Mpc$^{-1}$. We could, of course, choose any of the order statistics for this illustration but
the third and tenth are suitable examples. As can be seen from this figure, the value of $H_0$ where these two probability curves cross is found at $H_0 \approx 48 \text{ km s}^{-1} \text{ Mpc}^{-1}$. In addition it can be seen that values of $H_0 > 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ are not favoured (as the largest galaxies in the local sample would then be unreasonably large compared with the distant galaxies) nor are values of $H_0 < 35 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (as then the smallest galaxies in the local sample would then be unreasonably small compared with the distant galaxies).

The above analysis assumes that we have correctly accounted for, and corrected for, the observational selection effects within the distant sample, and that the local sample is a “fair” sample of the intrinsic distribution of log linear diameters. A further statistical analysis of the data was made by, instead, imposing a sharp linear diameter limit on both the distant and nearby samples. We adopt a limit of $14.5 h^{-1} \text{ kpc}$ since at the redshift limit of $5000 \text{ km s}^{-1}$ this corresponds to an angular size of 1 arcmin – above which we assume that the entire sample is reasonably complete. Provided again that the local galaxies are assumed to be a fair sample of the intrinsic population, the distribution of log linear diameter in the truncated local and distant samples should be identical. This analysis is less powerful than that using the entire sample (especially at low $H_0$ where the number of calibrators above the cut-off diameter becomes small), but has the advantage of applying the same selection function to both samples instead of essentially reconstructing the ‘missing’ galaxies from the observed distribution. Thus, what is lost in statistical power is partially gained in robustness, and we consider this approach a useful consistency check.

Fig. 4 shows a comparison of the means of the truncated distant and local samples, together with $1 - \sigma$ errors on the means, as a function of $H_0$ (which, of course, determines the truncation limit). As can be seen from this figure, good agreement between the truncated means of the local and distant data are found at $H_0 \approx 53$ and $H_0 \approx 65 - 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$. This weaker analysis therefore suggests that $50 < H_0 < 75$ gives a reasonable bound. As above, however, one can place some further constraints on $H_0$ by considering the order statistics of the largest galaxies in the local sample, and on this basis we find that the lower end of this range is favoured by the data. Assuming the local calibrators to be a fair sample of the intrinsic distribution, we carried out a series of Monte Carlo simulations in order to determine the probability of a truncated random sample of 12 galaxies larger than $14.5 h^{-1} \text{ kpc}$ in diameter containing two galaxies larger than $59 \text{ kpc}$ in diameter, for a range of different values of $H_0$. This probability is less than 10% for all $H_0 > 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and drops below 5% for $H_0 > 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

5 Conclusions

The results presented in this report strongly suggest that values of $H_0$ higher than $75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ or below $40 \text{ km s}^{-1} \text{ Mpc}^{-1}$ are ruled out. Clearly this is not a dramatically new conclusion, but should serve to underline that the simple idea of comparing the linear size of local and distant galaxies is certainly a plausible method for estimating $H_0$. A value of $H_0$ in the range $50 - 55 \text{ km s}^{-1} \text{ Mpc}^{-1}$ gives a good fit between the mean log diameter of the local and distant samples – both with and without a lower diameter limit – and renders the statistical properties of the largest and smallest local galaxies easily compatible with those predicted from the distant sample. This result is also consistent with a number of independent recent estimates from other distance indicators.
Our analysis has, however, used distance moduli to the local galaxies derived via the traditional Cepheid calibration of Madore & Freedman (1991), adopting a true distance modulus of 18.5 for the Large Magellanic Clouds. Recently the LMC distance modulus has been revised, using Hipparcos data (Reid 1997; Gratton et al. 1997) to calibrate the Cepheid period-luminosity relation in galactic open clusters. The simplest interpretation of the new results is that our estimate for $H_0$ should be reduced by about 10 per cent. Taking this into account would imply that the true value of the Hubble parameter lies in the range $45 - 50 \text{ km s}^{-1}\text{Mpc}^{-1}$.

On the other hand, Kochanek (1997) has embarked on a comprehensive re-analysis of the Cepheid distance scale and has concluded that relative distance moduli between the LMC and a number of local galaxies, as determined from HST Cepheid observations, have been significantly underestimated. The effect of the Kochanek re-calibration of the Cepheid distance scale may well largely cancel out any reduction in the Hubble parameter resulting from the Hipparcos results. For the moment, then, we will consider the range $H_0 = 50 - 55 \text{ km s}^{-1}\text{Mpc}^{-1}$ as the range most likely from this analysis. In a simple Einstein-de Sitter universe with $\Omega = 1$ and $\Lambda = 0$ this range corresponds to an age of approximately 12 - 13 Gyr since the Big Bang.

For the value of the Hubble parameter to lie significantly outside of this range, not only the Milky Way but most of the spirals in our local sample must be collectively either significantly larger or significantly smaller than the population of spirals in the visible Universe. If that were the case, it would mean that we live in an atypical part of the Universe, and that would remove the justification for any extrapolation from the nearby region to the Universe at large. Given that, on the contrary, we find the Milky Way to be a very average-sized spiral galaxy compared with other local galaxies of similar Hubble type, it seems unlikely to us that the ‘principle of terrestrial mediocrity’ should hold on the scale of the local supercluster and yet break down on larger scales.

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Figure 1: the fit of a gaussian of \( < \log D > = 3.39 \) and \( \sigma = 0.41 \) to the sample of local calibrating galaxies.

Figure 2: Comparison of the sample cumulative distribution function of log linear diameter, and the cumulative distribution of the predicted best-fit Gaussian model, for 1388 galaxies between 1500 and 5000 km s\(^{-1}\). The Gaussian has a mean of 3.48 and \( \sigma = 0.42 \), assuming \( H_0 = 60 \) km s\(^{-1}\) Mpc\(^{-1}\).

Figure 3: The change in the lower tail probability of the third order statistic (dashed line) and the upper tail probability of the tenth order statistic (full line) with \( H_0 \).

Figure 4: The means, enclosed by 1 \( \sigma \) error bands, of the distant (full line) and local (dashed line) galaxies when the sample is truncated at a linear diameter of 14.5\( h^{-1} \) kpc, for 30 < \( H_0 \) < 100.
CDF, $\Phi(L)$, of calibrators and Gaussian with same mean and variance

$\text{mean} = 3.39$

$\text{disp} = 0.41$

$n_{\text{gal}} = 12$

$\text{K.S. statistic} = 0.121$

$p$ value $= 0.991$
CDF of data and Gaussian with same mean and variance ($H_0 = 60$)

$V_{\text{min}} = 0 \, \text{kms}^{-1}$

$V_{\text{max}} = 40000 \, \text{kms}^{-1}$

mean $= 3.61$

disp $= 0.50$

$n_{\text{gal}} = 2924$

K.S. statistic $= 0.049$

$p$ value $< 0.001$
Mean log linear diameter, with 1σ limits, in distant and local data as a function of $H_0$. 

- local data