Supercontinuum Generation in Media with Sign-Alternated Dispersion

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When an ultrafast optical pulse with high intensity propagates through transparent material a supercontinuum can be coherently generated by self-phase modulation, which is essential to many photonic applications in fibers and integrated waveguides. However, the presence of dispersion causes stagnation of spectral broadening past a certain propagation length, requiring an increased input peak power for further broadening. Overcoming such spectral stagnation will be key to achieve practical integrated supercontinuum devices. Here, a concept is presented to drive supercontinuum generation with significantly lower input power by counteracting spectral stagnation via repeatedly alternating the sign of group velocity dispersion along in excellent agreement with modeling, revealing almost an order of magnitude reduced peak power compared to uniform dispersion. Calculations also reveal a similar power reduction with integrated optical waveguides, simultaneously with a significant increase in flat bandwidth, which is important for on-chip broadband photonics.

1. Introduction

Supercontinuum generation (SCG) is a nonlinear optical process where injecting intense, ultrashort pulses in transparent optical materials generates light with ultrawide spectral bandwidth.\[1–3\] Maximum coherence is achieved when spectral broadening is dominated by self-phase modulation based on the intensity-dependent Kerr index.\[4\] The achieved bandwidths, often spanning more than two octaves\[5–11\], have become instrumental in a plethora of fields.\[12\] Examples are metrology based on optical frequency combs.\[13–17\] optical coherence tomography.\[18–21\] ranging.\[22\] or subcycle pulse compression.\[23–25\]

The widest spectral bandwidths at lowest input power are typically achieved in waveguiding geometries, such as in fibers\[26–28\] and integrated optical waveguides on a chip\[10,11,29,30\] because transverse confinement of light enhances the intensity, which promotes the nonlinear generation of additional frequency components. However, although significant spectral broadening can be achieved with low input power using highly nonlinear materials,\[31\] generating bandwidth beyond one or two octaves still requires significant input peak powers in the order of several kilowatt,\[11,32,33\] hundreds of kilowatt,\[34\] or close to megawatt levels.\[3,35\] The reason is that dispersion terminates spectral broadening beyond a certain propagation length, which means that the peak power required for a desired bandwidth cannot be reduced by extending the propagation length.

Such lack of scalability towards lower input power constitutes a critical limitation for implementations of supercontinuum generation, specifically, in all-integrated optical systems, for the generation of high repetition rate supercontinuum pulses and for supercontinuum generation with long input pulse durations. In case of normal dispersion (ND) (positive acronym: group velocity dispersion (GVD)) the reason for termination of broadening is that initial pulse duration gets increased and the waveguide-encoded peak power is lowered versus propagation. This brings self-phase modulation (SPM), the main mechanism of coherent spectral broadening,\[36\] to stagnation past a certain propagation length.\[4,36,37\]

Lower input peak powers are sufficient with anomalous dispersion (AD) because negative group velocity dispersion leads to pulse compression by making use of the additionally generated bandwidth. However, spectral stagnation occurs again, here due to soliton formation, because self-phase modulation comes into balance with anomalous dispersion.\[1,38\] At that point, typically also soliton fission and Raman redshifting occurs, and dispersive waves are generated. However, fission does not increase the bandwidth,\[1,36\] dispersive waves generate only disconnected components of narrowband radiation, and Raman conversion redshifts the frequency bandwidth, without increasing it.
It was shown that the bandwidth of dispersive waves can be broadened in the anomalous dispersion regime by waveguide tapering.\(^{[28,39,40]}\) However, unavoidable and intrinsic bandwidth stagnation occurs by the taper geometry itself when the anomalous dispersion is no longer present and by the early onset of increasing pulse duration and decreasing peak power, introduced by these schemes.\(^{[19,40]}\) after an initial nonlinear pulse compression.

For avoiding both pulse stretching and soliton formation, it seems desirable to provide zero GVD, to keep self-phase modulation ongoing and to increase the bandwidth proportional to the interaction length. However, this would not yield the maximum possible broadening with length, because newly generated spectral bandwidth is not used for pulse compression during propagation.

Alternating the sign of dispersion was used previously to shorten pulses and increase their bandwidth.\(^{[41]}\) However, this had relied on a regime where spectral stagnation is absent by choice of parameters, i.e., segment length is chosen to let nonlinear generation dominate dispersive effects. While interesting for maintaining a certain pulse shape (soliton-type pulse linear generation dominate dispersive effects. While interest-


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We present a novel approach that increases the ratio of spectral bandwidth to input peak power, by overcoming spectral stagnation with alternating the sign of group velocity dispersion along propagation (see Figure 1). The alternation is done repeatedly in a segmented medium, to both counter temporal broadening and disrupting soliton formation, and thereby lowers the input peak power and pulse energy required for supercontinuum generation. In its simplest setting, self-phase modulation in normal dispersive segments generates spectral broadening and subsequent segments with anomalous dispersion make use of the newly generated bandwidth for pulse recompression and peak power increase, which lets spectral broadening continue. An approximate analysis reveals close-to-exponential growth of bandwidth versus propagation at weak dispersion and linear growth at strong dispersion. This scaling with interaction length enables to generate a given spectral bandwidth with substantially reduced pulse energy and peak power as compared to conventional supercontinuum generation based on uniform dispersion.

We note that in the limit of linear optics, dispersive sign alternation is well known in optical fiber communications for stabilizing the spectral and temporal dynamics, to prevent pulse stretching and to prevent that the dynamics enters the nonlinear regime, which avoids, specifically, cross-talk of pulses.\(^{[43,44]}\) Similarly, periodic dispersion oscillation is also known to be very important in the nonlinear regime where it can provide quasiphase matching of dispersive waves with solitons, for enhancing spectrally narrowband modulation instability\(^{[45–47]}\) or to generate soliton trains.\(^{[48–50]}\)

The concept of alternating dispersion presented here goes well beyond, because it offers a novel path toward higher effective nonlinearity, i.e., a higher ratio of spectral broadening versus required power. The goal here is rather to maximize spectral bandwidth generation by the sign-alternation through self-phase modulation.

### 2. Results

#### 2.1. Basic Dynamics with Alternating Dispersion

Figure 1 shows calculated developments of the optical spectrum and pulse along propagation for a dispersion alternated waveguide (Figure 1A) made of normal dispersive (ND) segments (positive group velocity dispersion) and anomalous dispersive (AD) segments (negative group velocity dispersion). For unveiling the novel spectral dynamics caused by repeatedly alternating dispersion, the calculations retain only the two main physical processes, which are SPM causing coherent spectral broadening and second-order dispersion responsible for temporal stretching or compressing the pulse. For convenience, we take transform limited Gaussian pulses as input and consider spectral broadening only in the ND segments. Other situations, e.g., non-Gaussian input pulses, spectral broadening in both types of segments, higher-order dispersion, or propagation loss are taken into account as well, when comparing with experimental data as in Figures 2 and 3.

Figure 1B shows the increase in spectral bandwidth versus propagation coordinate as obtained by numerical integration of the nonlinear Schrödinger equation (NLSE) using parameters as in telecom ND and AD fiber (Section S1, Supporting Information). The NLSE is shown in the Experimental Section. It can be seen that there is an initial spectral broadening through SPM in the first ND segment, but broadening stagnates beyond approximately \(z = 15\) cm. The reason for stagnation in this ND segment is loss of peak intensity and increased duration by normal dispersive temporal broadening of the pulse (Figure 1C).

To reinitiate spectral broadening by re-establishing high peak intensity, the pulse is temporally recompressed in an AD segment back to its transform limit (between \(z_1\) and \(z_2 = 30\) cm).

The pulse at \(z_2\) is shorter and has a higher intensity than the incident pulse (\(z_0 = 0\) cm) because SPM in the first ND segment has increased the bandwidth above that of the input pulse. The consequence of higher intensity and shorter pulse duration is that SPM in the second ND segment yields more additional bandwidth (between \(z_2\) and \(z_3\)) than what was gained in the first ND segment (between \(z_0\) and \(z_1\)). Also, the rate of bandwidth growth and dispersion are higher, so that spectral stagnation is reached after a shorter propagation distance than in the first ND segment. Following this concept, further dispersion alternating segments keep the bandwidth growing and let the pulse become shorter as shown.

Figure 1D provides a bandwidth comparison with conventional supercontinuum generation (uniform AD, uniform ND) where the growth of bandwidth stagnates. In alternating dispersion (ND–AD), bandwidth generation remains ongoing with each next ND segment (AD segments omitted in for direct comparison with generation in uniform AD and ND media). With the chosen fiber and pulse parameters, the spectral bandwidth grows close-to-exponential (see blue-shaded exponential fit curve) and reaches about 200 THz via SPM in 50 cm of bandwidth generating ND segments. In uniformly dispersive fiber (ND or AD) there is spectral stagnation, limiting the bandwidth to 15 THz (after 20 cm) and 20 THz (after 10 cm), respectively.

The bandwidth at stagnation in uniform dispersion may be increased as well, however, this requires significantly increased...
Figure 1. Supercontinuum generation (SCG) using alternating dispersion. A) Waveguide with segments of normal dispersion (ND, red) and anomalous dispersion (AD, yellow). The basic pulse and spectral propagation dynamics is modeled with Gaussian input pulses using parameters as in telecom fiber (Section S1, Supporting Information), accounting for second-order dispersion in ND and AD segments, and self-phase modulation in the ND segments. B) Normalized energy density spectrum versus propagation with alternating dispersion. C) Intensity and shape of the pulse versus propagation. Undesired pulse stretching in the bandwidth generating (ND) segments is repeatedly compensated by anomalous dispersive (AD) segments. D) Spectral bandwidth of supercontinuum generation in alternating dispersion (ND–AD) compared to zero uniform dispersion (zero GVD) and to uniform normal and anomalous dispersion (uniform AD, uniform ND). For direct comparison with the uniform waveguides the graph omits nongenerating AD segments. With the chosen fiber parameters and input pulse, alternating dispersion yields close-to-exponential growth (blue-shaded exponential fit curve under blue trace). Supercontinuum generation in uniform ND and AD exhibits spectral stagnation. E) Calculated spectral bandwidth increase factor, $F$, in single ND segments versus the relative strength of dispersion and self-phase modulation, $R$, in terms of the ratio of nonlinear length to dispersive length ($R = L_{nl} / L_d$).
Figure 2. A) Experimental setup for supercontinuum generation (SCG) in optical fiber with alternating normal dispersion (ND) and anomalous dispersion (AD). HWP: half-wave plate; PBS: polarization beam splitter; L1, L2: lenses; SM: switchable mirror; OSA: optical spectrum analyzer; AC: autocorrelator. B) Dispersion parameter, $D = \beta_2 \cdot \left(\frac{2\pi c}{\lambda^2}\right)$, versus wavelength of the AD and ND fiber segments. The wavelengths of zero second-order dispersion are indicated with dotted vertical lines. C) Relative spectral energy density of supercontinuum generation measured behind each ND segment in the alternating waveguide structure. The spectral bandwidth grows in steps with added ND segments. D) Measured bandwidth versus ND segment number retrieved from the spectra in (C). The measured pulse duration and peak power entering the segments, used to determine the $R$-value for each segment (not separately shown), suggest an approximately exponential growth via the first two segments (compare to exponential curve beginning at input bandwidth). Thereafter, weaker growth transiting to linear is expected. E) Measured normalized spectral energy density at the end of the dispersion alternating fiber (violet trace). For direct bandwidth comparison, measured supercontinuum spectra obtained with uniform ND fiber (brown trace) and AD fiber (green) of the same length are shown, generated with the same input pulse parameters. The dotted traces show theoretical spectra obtained with the generalized nonlinear Schrödinger equation, using the experimental input spectrum, containing the full fiber dispersion (B) and weak SPM also in the AD segments. F) Calculated normalized intensity (color coded) versus time (horizontal axis) and versus the propagation coordinate (vertical axis). Dotted white lines indicate the transitions between ND and AD segments. G) Measured width of autocorrelation trace (FWHM) versus propagation distance in the dispersion alternated fiber. The dotted trace connects experimental data. The recompressed pulse is becoming shorter after each further AD segment.
power, because the bandwidth grows approximately only with the square root of the input power.\cite{1, 4, 36} For instance, generating 200 THz in uniform dispersion with the fiber parameters used in Figure 1, would require about 180 times higher power (with ND), and 100 times higher power with uniform AD. The bandwidth comparison in Figure 1D is thus indicating that alternating dispersion allows to significantly reduce the power required for generating a given bandwidth.

### 2.2. Scaling of Bandwidth with Segment Number and Input Power

Many applications benefit from a broad spectral bandwidth particularly if there is significant power also at the edges of the specified bandwidth, i.e., if spectra are approximately flat in the range of interest.\cite{36, 37} The standard definition of bandwidth for supercontinuum generation leads to wider bandwidth values as it accepts low power at the edges, typically at the –30 dB level. To follow a conservative definition of bandwidth addressing approximately flat spectra, we chose as bandwidth, $\Delta \nu$, the full width frequency bandwidth at the 1/e level of the spectral energy density.

To provide spectra with minimum interference structure,\cite{4, 36, 37} we continue considering that spectral broadening is generated in the ND segments with Gaussian pulses, and that the AD segments just recompress the pulse to the transform limited duration, $\Delta t = 2/\pi \Delta \nu$, where $\Delta t$ is the 1/e half-width duration.

For calculating the bandwidth increase in a particular ND segment when a transform limited pulse enters the segment, it is important to realize that the achievable broadening is determined by only two parameters. The first is the nonlinear length, $L_{nl} = 1/(\gamma P)$, depending reciprocally on the peak power $P$, where $\gamma$ is the nonlinear coefficient of the ND segment. $L_{nl}$ denotes the propagation length where the bandwidth increases by a factor $\sqrt{3}$ if dispersion were absent.\cite{51} The second parameter is the dispersion length, $L_d = \Delta t^2/|\beta_2|$ depending quadratically on the pulse duration, where $|\beta_2|$ is the coefficient for second-order (group velocity) dispersion.\cite{51} $L_d$ denotes the propagation length after which the pulse duration increases by a factor of $\sqrt{2}$ if SPM were absent.

Given a certain length of the ND segment, it is the ratio of $L_{nl}$ and $L_d$ which determines the bandwidth gained in that segment. For instance, if dispersion is weak compared to spectral broadening, i.e., if the ratio $R = L_d/L_{nl}$ is small ($R \ll 1$), one expects the bandwidth to increase by a factor of $F = \sqrt{5}$ if the segment length is chosen equal to the nonlinear length. If dispersion is strong versus SPM, due to a short pulse duration and thus a wide spectrum ($R \gg 1$), there will still be broadening in the beginning of the segment, but the bandwidth increase factor will only be slightly above unity, $F \approx 1$.

To quantify the relation between the ratio $R$ and spectral broadening in an ND segment, we devise a lower-bound calculation of the bandwidth increase factor, $F(R)$, and display the result in Figure 1E. For the calculation, we chose the length of the ND segment to be its nonlinear length, to encompass all nonlinear generation up to the point where the bandwidth stops increasing.

While a full derivation of the lower bound calculation of $F(R)$ is shown in Section S2 in the Supporting Information, we give here only the final expressions with a brief description of how they are obtained. To obtain the lower bound expression of $F(R)$ we subdivide the ND segment length (i.e., its nonlinear length), into two subintervals of propagation distance spanning half of the nonlinear length. The splitting of the overall ND segment length into subintervals ensures that the analytic lower bound estimate is a close fit to the bandwidth increase factor found from numerically solving the NLSE. Then we find analytic expressions for the spectral bandwidth and temporal duration of Gaussian-like pulses that are functions solely of the parameter $R$ for the propagation in each of these subintervals. These expressions use assumptions on the pulse’s temporal propagation that guarantee a lower bound calculation of the bandwidth and, thus, also of $F(R)$. The spectral bandwidth (i.e., Equation (1) multiplied by the input bandwidth) and the temporal duration found at the end of the first subinterval (called region 1) are the initial conditions for the corresponding analytic expressions in the second subinterval (region 2), to determine the lower bound increase factor at the end of the ND segment.

**Figure 3.** A) Normalized spectral energy density (dB) versus wavelength compared for the same length of uniform normal dispersive waveguide (blue trace), and the spectrum obtained with alternating dispersion (red trace). B) Dispersion parameter curves for the AD (red) and ND (blue) segments used in the integrated optical waveguide (for parameters see Section S5 in the Supporting Information).
For the first region, \( z \in [0, \frac{t_i}{2}] \), we find the ratio of bandwidth at the end of this region (i.e., at \( z = \frac{t_i}{2} \)) versus the initial bandwidth, labeled as \( \text{LBR}_{R1} \) (lower bound ratio, region 1). This expression is given as

\[
\text{LBR}_{R1} = \left[ 1 + \left( \frac{\sqrt{2}}{1 + (1 + R)^2} + \sqrt{R(1 + R)} \right)^{\frac{1}{2}} \right]^2 \times \frac{\sqrt{2}}{C(R)^{\frac{1}{2}}} \tag{1}
\]

Once this is obtained, we proceed to find the expression for the second region \( (z \in (\frac{t_i}{2}, L_{in})] \) in analogy with the first. Using the initial conditions for region 2, the total lower bound calculation of the bandwidth increase factor \( F(R) \) at the end of the ND segment (i.e., at \( z = L_{in} \)) is found as

\[
F(R) = \left[ 1 + \left( \frac{\sqrt{5}}{C(R)^{\frac{1}{2}}} + \sqrt{\frac{C(R)}{5} \left( \text{LBR}_{R1} \right)^{\frac{1}{2}} - 1} \right)^{\frac{1}{2}} \right] \times \frac{\sqrt{5}}{C(R)^{\frac{1}{2}}} \tag{2}
\]

where \( C(R) = 1 + (\frac{1}{2} R + \sqrt{\frac{3}{2} R(R+2)+4})^2 \).

While we only give the lower bound expression of \( F(R) \) for the specific case that the ND segment length is equal to its nonlinear length, the expressions can be extended for an ND segment of arbitrary length. The procedure to this is described in Section S2 in the Supporting Information (i.e., Equation (S25b), Supporting Information).

The comparison between our analytic expression of \( F(R) \) and the bandwidth increase corresponding to the NLSE results of Figure 1D is found in Figure S1 in Section S2 (Supporting Information). Figure S1 (Supporting Information) indeed shows that \( F(R) \) follows the NLSE results closely but remains a lower bound.

From the above expression of \( F(R) \), i.e., Equation (2) and displayed in Figure 1E it can be seen that \( F \) varies with \( R \) as expected. \( F \) has its values close to \( \sqrt{5} \) at small \( R \) and reduces asymptotically to unity toward increasing \( R \).

We note that \( F(R) \) does not only determine the broadening in a single ND segment, but it determines also the broadening in all following ND segments. To illustrate this, we recall Figure 1 where, due to recompression in AD segments, the peak power increases and pulse duration shortens before entering a next ND segment. This causes \( R \) to increase with each ND segment since \( R \propto 1/\Delta t \). More specifically, because the \( R \)-value of a previous ND segment determines the bandwidth increase factor \( F \) in the according segment (see Figure 1E), it also determines the bandwidth, transform limited pulse duration and peak power in front of the next ND segment. The peak power and pulse duration then set the \( R \)-value of the next ND segment and, via \( F(R) \), the next bandwidth increase. Labeling the ND segment number with \( p \), the recursive dependence of \( R_p \) on \( R_{p-1} \) is why the first segment, \( R_1 \), predetermines the bandwidth growth along the entire structure.

The bandwidth at the end of the structure with a number of \( n \) ND segments, if \( \Delta \nu_0 \) is the frequency bandwidth of the incident pulse, can then be expressed as a product of broadening factors

\[
\Delta \nu_n = \Delta \nu_0 \prod_{p=1}^{n} F_p \tag{3}
\]

Here \( F_p \equiv \Delta \nu_p/\Delta \nu_{p-1} \) is the bandwidth increase factor of the \( p \)th ND segment, where \( \Delta \nu_{p-1} \) is the bandwidth in front of the segment and \( \Delta \nu_p \) is the bandwidth behind the segment.

Particularly strong growth is obtained from Equation (3) if generation remains in the range of relatively weak dispersion \( (R \leq 1) \). In this case \( F_p \) remains close to its maximum value \( (F_p(0) = \sqrt{5} \approx 2.23) \) or decreases only slowly with segment number. The bandwidth growth then remains approximately exponential with the number of ND segments, resulting in

\[
\Delta \nu_n = \Delta \nu_0 F^n \tag{4}
\]

as is found also numerically for the example in Figure 1D. In the other limit of strong dispersion \( (R \gg 1) \), we find (more details in Section S3 in the Supporting Information) that the decrease in \( F_p \) with each additional ND segment reduces Equation (3) to a linear growth of bandwidth with the number of ND segments

\[
\Delta \nu_n \approx n \left[ g_c F_n \frac{E}{4 \beta_1} \right] + \Delta \nu_0 \tag{5}
\]

with \( g_c \approx 0.81 \) being a constant related to the Gaussian pulse shape, and with \( E = \sqrt{2} P \cdot \Delta t \) being the pulse energy.

Equations (4) and (5) display that the basic dynamics of supercontinuum generation in alternating dispersion is a growth of spectral bandwidth versus propagation length, while with uniform dispersion spectral stagnation occurs. The growth may remain close to exponential if \( R \) remains small throughout all segments, show transition from exponential to linear growth at around a certain segment number \( n \gamma \) where \( R \approx 1 \), or is close to linear if dispersion is strong in the first and thus also in all segments \( (R \gg 1) \).

It is interesting to evaluate to what extent higher-order dispersion would let the analytical model deviate from a full simulation based on the generalized NLSE. To explore this question, we have compared the spectral broadening predicted by Equations (2) and (5) to that obtained with the generalized NLSE for the ND fiber used in our experiments, where significant third-order dispersion is present (see details in Figures S2 and S3 in Section S3 in the Supporting Information). We note that for generated spectra with frequency ranges past the normal dispersion of the ND fiber a comparison is not possible with our simplified model, since the sign of dispersion must stay constant for it to be valid.

In the regime of exponential growth, where \( R < 1 \), we find that the results of Equation (2) remain close to the numerical results from the GNLSE (differing by at most 17% for the example investigated here; see Figure S2 in the Supporting Information). This indicates that approximately exponential spectral growth is maintained even when higher order dispersion is significant. In the strongly dispersive regime \( (R \gg 1) \) in spite of strong higher-order dispersion, the GNLSE predicts linear growth of the
spectral bandwidth with the number of segments (see Figure S3 in the Supporting Information). The spectral increase per segment becomes constant for $R \gg 1$ even if the pulse duration decreases further. Compared to the simplified model, the effect of higher-order dispersion is a somewhat lower slope of growth. Here the GNLS reveals 25 THz broadening per ND segment as compared to 35 THz per segment predicted by Equation (5).

Using these properties, we find approximate bandwidth-to-input peak power scaling laws (more details in Section S3 in the Supporting Information) shown in Table 1. In uniform dispersion there is only a square-root growth of bandwidth versus input peak power. With alternating dispersion, the bandwidth increases in a mixture of exponential and linear growth versus input peak power, where the slope and exponent can be increased monotonically with the total number of ND segments. Correspondingly, the generated bandwidth changes much more strongly with input peak power in the alternating dispersion structure.

These relations show that the power of the method lies not explicitly in increasing the SCG bandwidth. The method rather increases the effective nonlinearity of the medium which is the spectral broadening generated with a given power. Therefore, the power required to achieve a given bandwidth can be much reduced by increasing the number of ND segments.

Summarizing the analytical results, repeatedly sign-alternating dispersion offers new spectral bandwidth dynamics that are fundamentally different than conventional supercontinuum generation schemes in uniform dispersion or tapered waveguides. These nonconventional dynamics increase the ratio of bandwidth to input peak power while the dynamics follow different regimes across the propagation coordinate or when changing the input peak power.

### 2.3. Experimental Results

For demonstrating increased spectral broadening with repeated sign-alternating dispersion, we use supercontinuum generation with ultrashort optical pulses injected into a repeatedly dispersion sign-alternated optical fiber structure shown in Figure 2A. The incident pulses have 74 fs FWHM pulse duration at 50 mW average power and 79.9 MHz repetition rate (central wavelength of 1560 nm). The pulses are provided by a mode-locked Erbium doped fiber laser, amplifier and a temporal compressor system. The power coupled into the fiber is 35.6 mW (446 pJ pulse energy) and is held constant throughout the main experiments.

The dispersion alternated fiber structure is selected to comprise alternating segments of standard single-mode ND and AD telecom fiber with well-characterized linear and nonlinear properties (see Figure 2B and parameters in Section S4 in the Supporting Information) because this enables numerical modeling with high reliability.

We note that the wavelength range across which the sign of the dispersion can be alternated and the concept can be applied is restricted by the ND and AD zero-dispersion wavelengths indicated with dashed lines in Figure 2B. On the other hand, the according proof of concept experiment was designed to demonstrate that our concept works in real cases where there is uncompensated higher order dispersion in the segments, limited sign-inverted bandwidth, losses and low nonlinearity.

In the experiment, ND and AD segments are added sequentially, with the choice of segment length guided by intermittent recordings of the energy spectrum and the pulse duration (see the Experimental Section). Due to the almost five times higher non-linear coefficient in the ND fiber compared to the AD fiber, spectral broadening comes almost entirely from the ND segments, similar to the setting used for illustrating the basic dynamics in Figure 1.

Figure 2C shows supercontinuum spectra measured behind each added ND segment. Starting with the injected light, the spectral bandwidth increases with each ND segment. For better analysis, Figure 2D displays the measured bandwidth growth versus segment number (red dots). To identify the regime of growth, we determine the experimental ratio $R = L_{\text{NL}}/L_o$ using the measured peak power and pulse duration. We find $R \leq 1$ for the first two segments, suggesting that the initial growth would be closer to exponential growth, while the third and fourth segments would introduce a transition toward linear growth. This is confirmed by finding a slightly upward curved growth along the first three data points, followed by a transit toward more linear growth with the remaining segments.

The maximum number of ND segments before no further spectral broadening occurs is seen to be four. This can be attributed to the finite width of the spectral interval across which dispersion parameters show opposite signs (Figure 2B), i.e., the range in which alternating dispersion can be applied with the used fibers.

Figure 2E shows supercontinuum spectra generated with dispersion alternated fiber with four ND segments (trace 1). For direct bandwidth comparison, we generate supercontinuum also in uniform fiber (traces 2 and 3 for uniform ND and AD fiber, respectively), using the same input pulse parameters and fiber length. The comparison shows that alternating dispersion increases the full 1/e bandwidth by a factor of 1.4 and 2.5 with regard to uniform ND and AD fiber, respectively. The full-width at −30 dB increases by a factor of 2.5 and 3.1, respectively. These numbers show, that the effective nonlinearity (as described in section 2.2) is indeed increased by sign-alternating dispersion.

An additional observation is that the spectral profile associated with the dispersion alternated fiber is free of large spectral

| Relative strength of dispersion over self-phase modulation in first segment | Scaling of bandwidth with input peak power using alternating dispersion | Scaling of bandwidth with input peak power in uniform dispersion |
|---|---|---|
| $R_l > 1$ for $R_l \leq 1$ | $\Delta \nu_1 \propto (c_t \sqrt{P_o})^{n_l} + (n_l - n_l + 1)P_o$ | $\Delta \nu_1 \propto P_o^{n_l}$ |
| $R_l < 1$ | $\Delta \nu_1 \propto [c_t(n_l)^{n_l} + (n_l - n_l + 1)]P_o$ | $\Delta \nu_1 \propto P_o^{n_l}$ |

$^a$Alternating dispersion yields a mixture of linear and exponential growth versus the input peak power, depending on the relative strength of dispersion over self-phase modulation in the first ND section (expressed as the ratio of nonlinear length to dispersive length, $R_l = L_{\text{NL}}^d/L_o^d$), $n_l$ is the segment number where exponential growth transits into linear growth (where $R \approx 1$) and $c_t$ is a constant (see Section S3 in the Supporting Information regarding the definition of $n_l$ and $c_t$).
modulations, which is due to spectral generation occurring predominantly in the ND segments. The dashed traces are numerical solutions of the generalized nonlinear Schrödinger equation (see the Experimental Section and Equation (6)), accounting for all orders of dispersion, the experimental pulse shape, loss between segments, and also weak SPM in the AD segments (see parameters in Section S4 in the Supporting Information). It can be seen that there is very good agreement with the experimental data in all cases, i.e., for supercontinuum generation in uniform ND and AD fiber, and also for the dispersion alternated fiber.

Figure 2F compares the development of the pulse duration with according measurements of the intensity autocorrelation (Figure 2G). We find good agreement between the modeled oscillation of the pulse duration with that of the autocorrelation measurements.

Motivated by the agreement and given the spectral bandwidth generated with dispersion alternated fiber, we use the model to extrapolate the pulse energy needed to generate the same bandwidth with uniform ND fiber. The model predicts that approximately one order of magnitude higher pulse energy is required with uniform dispersion. We note that this factor is a conservative number due to splice loss between segments (about 8% per splice) which lowers the intensity available for SPM toward the later segments. Lower required powers are expected with postprocessing the splices. However, the presence of high splice losses shows that our concept is robust to this influence.

The compression to ultrashort pulse durations shows that the spectral phase coherence is maintained in the alternated fiber. This is expected since the dominant nonlinear effect is SPM. The primary mechanism for degraded pulse-to-pulse coherence is modulation instability in the AD fiber segments. However, we find that this effect is negligible since the characteristic length for modulation instability is approximately an order of magnitude larger than the length of the AD segments. This favorable situation is due to the combination of a relatively long pulse duration entering the AD segments and the relatively low nonlinear coefficient of the AD segments. Closer investigation of shot-to-shot coherence will be the subject of future work. The proper working of the scheme also in the presence of substantial noncompensated third and higher-order dispersion (see Figure 2B) indicates a certain robustness of SCG with sign-alternating dispersion with respect to higher-order dispersion.

To support this quantitatively, we determine the spectral phase profile of the pulse after the last ND segment, through the autocorrelated pulse duration in front of the last AD segment and directly behind it, in combination with the measured spectral bandwidth. From this we estimate a maximum deviation from a purely parabolic phase of the pulse by at most 26% at the 1/e points of the bandwidth. If we compare this to the total fiber dispersion in all preceding segments (assuming SPM is turned off), we obtain a much higher deviation from a parabolic phase spectrum of approx. 90% at the 1/e end points. This means that SPM in sign-alternating media can bring the phase spectrum of the generated pulses closer to a parabolic shape. This is in analogy to optical wave breaking, where undesired higher-order contributions from SPM are reduced by dominating second-order dispersion.

The experiment verifies the theory of repeated sign-alternating dispersion, since there is a large bandwidth enhancement relative to uniform dispersion cases. This in turn indicates that dispersion alternating provides a considerably larger bandwidth to input peak power ratio. Additionally, the dispersion alternating fiber keeps advantages of normal dispersion supercontinuum generation[4,36,37] seen by a spectrum free of large modulations and the high temporal compressibility of the pulses.

While the experiment verifies the functioning of our concept, it is still a proof-of-concept experiment and further improvements (minimize splice losses, increase nonlinearities, etc.) promise to further increase the bandwidth generation. However, the experiment provides the insight that repeatedly sign-alternating dispersion can remove the need to use maximally nonlinear waveguides, since the alternation substantially increases spectral broadening in a waveguide system with a low nonlinear coefficient relative to nonlinear fiber (e.g., photonic crystal fibers). A wider range of waveguide platforms and materials is thus made available for supercontinuum generation.

The experiment also shows that our concept removes the need to use ultrashort or high peak power pulses, thus enabling supercontinuum generation with a wider range of lasers. This should enable, for example, implementation of supercontinuum generation with chip integrated lasers that have lower peak power, e.g., due to a high repetition rate.

2.4. Integrated Optical Waveguides with Alternating Dispersion

Particularly promising is supercontinuum generation in integrated optical waveguides because tight guiding reduces the required input power to provide high intensities. Further advantages are that lithographic fabrication methods provide increased freedom and precision in dispersion engineering.[52] For instance, different from fiber splicing, transitions between segments can be shaped as desired, and also the spectral shape of ND and AD dispersion may be designed to widen the wavelength range between ND and AD zero dispersion wavelengths.

To investigate the options for additional spectral broadening and reduced peak power in such systems, we use the generalized nonlinear Schrödinger equation[11,51,53] to model supercontinuum generation in silicon nitride integrated optical waveguides. Figure 3A compares generation in uniform anomalous dispersion with that in sign-alternating dispersion (see parameters in Section S5 in the Supporting Information). With uniform dispersion (see Figure 3B, red curve), the assumed input pulse energy and duration (450 pJ and 120 fs), the shape of the calculated spectrum (Figure 3A, blue trace) and the 1/e full bandwidth (21 mm) are comparable with previous modeling that matched according experiments.[10]

For modeling with alternating dispersion, we select four ND segments (see dispersion in Figure 3B, blue curve) and three AD segments, beginning with an ND segment (see parameters in Section S5 in the Supporting Information). We note that in this structure, different than with the investigated fibers, both types of segments contribute to spectral generation and high-order dispersion is included.

The second trace in Figure 3A (red) shows the spectrum obtained with the alternating structure and 80 pJ input pulse energy. Both approaches yield approximately the same ~30 dB bandwidth, however, the input pulse energy is about 5.6 times lower.
with alternating dispersion. A second difference is the much larger $1/e$ bandwidth (550 nm), which is a factor of about 26 wider. While the generated bandwidth is both larger and flatter in the alternating waveguide than in the uniform dispersion case, effects such as modulation instability, coherence degradation and complex spectral phase profiles may still be present due to a notable fraction of spectral generation in the anomalous dispersion segments. The complex dynamics introduced here will be subject of future work.

Furthermore, a potential drawback in integrated waveguides is that the lengths of the ND or AD segments cannot be varied in the experiment, to maximize the spectral bandwidth. However, we found that alternating dispersion is a robust approach because, even with nonoptimum segment lengths, there is still a repeated temporal compression through the AD segments, which induces a broader spectral output than with uniform waveguides. However, to optimize the segment lengths and thus the broadening factor also in integrated waveguides, one can apply two experimental variations. The first is based on the option that a large number of differently designed waveguides can be adjacently fabricated on one chip. For instance, using the SiN platform, more than several hundred design variations are typically possible per cm transverse width of a chip. A second experiment variation is to adjust the input peak power and pulse duration to maximize spectral broadening within these alternated waveguides.

3. Conclusions

Overcoming spectral stagnation by repeatedly alternating the sign of dispersion in supercontinuum generation is a novel and highly promising concept for increasing the ratio of generated spectral bandwidth to the required input peak power or pulse energy. Even in a situation where the $-30$ dB bandwidth is already extremely wide, the flat part of the spectrum specified via the $1/e$ bandwidth can be much increased and, simultaneously, the required power can be much reduced.

Due to the generality of the concept presented here, we expect that it can have important impact in a large variety of waveguiding systems that aim at coherent generation of broadband optical light fields. Examples may include photonic crystal fibers,[58] low-power broadband generation for dual-comb applications in the mid-infrared,[54] on-chip generation of frequency combs,[56] and coherent spectral broadening in liquid[57] and gaseous media.[53]

The basic concept as presented, encourages further investigations into various directions. For instance, spectrally shaping normal and anomalous dispersion via dispersion engineering in photonics waveguides can be explored to extend the range between zero-dispersion wavelengths. Emphasizing self-phase modulation in normal dispersive segments, rather than in anomalous dispersion, might be used to reduce undesired spectral structure, nonlinear phase spectra, and noise from modulation instability, as is found in supercontinuum generation with anomalous dispersion.[4,16,37]

For optimum results, i.e., maximally wide spectra at lowest possible power, alternating dispersion requires an optimum choice of the segment lengths. Sets of adjacent waveguide structures on chip may be used to cope with wider variations of the input power or pulse shape. Nevertheless, because alternating dispersion always counteracts the detrimental effects of uniform dispersion, we observe that noticeably increased bandwidth and lowered power requirements can be obtained even with nonoptimal matched segment lengths, for instance with simple periodic sign-alternation.

4. Experimental Section

To assemble the dispersion alternated fiber, segments of standard single-mode doped-silica step-index optical fiber were used (Corning Hi1060flex for ND, Corning SMF28 for AD). The structure was assembled using fiber splicing and cut back, starting with a 25 cm piece of ND fiber. While measuring the power spectrum at the fiber output with an optical spectrum analyzer, the length was cut back until a slight reduction of the spectral width becomes noticeable at the $-30$ dB level. Terminating the first segment at this length ensures that the spectral generation is ongoing throughout the entire segment but stagnates at the end of the segment. The cutback also removes further pulse defocusing by second and higher-order dispersion while no further broadening takes place. Removing higher-order phase contributions improves pulse compression in the following AD segment because the AD fiber compensates for second-order dispersion but, in demonstration of this experiment, cannot compensate for the high-order dispersion of the ND fiber.

To assemble the next segment, a 20 cm piece of AD fiber was spliced, the splice loss was measured with a power meter and the pulse duration was measured with an intensity autocorrelator (APE, Pulse Check). The AD fiber segment was cut down until the autocorrelation trace indicates a minimum pulse duration, i.e., that the pulse is closest to its transform limit at the end of the AD segment. The procedure was then repeated with the next pieces of ND and AD fibers. The lengths of the fiber segments obtained with this method and the measured splice loss are summarized in Section 54 in the Supporting Information.

The experiment was modeled using a generalized nonlinear Schroedinger equation (GNLSE) under the slowly varying envelope approximation[11]

\[
\frac{\partial u}{\partial z} = \sum_{k=1}^{k+1} \frac{\rho_k}{k!} \frac{\partial^k u(z, \tau)}{\partial \tau^k} + i \gamma \left( 1 + i \tau_s \frac{\partial}{\partial \tau} \right) |u(z, \tau)|^2 u(z, \tau),
\]

where $u$ is the complex field envelope and $\rho_k$ are the Taylor series coefficients of the expansion of the frequency-dependent wave number about the central frequency $\omega_0$. $\tau = 1 - V g \tau_s$ is the time coordinate, comoving in the frame of reference of the group velocity dispersion ($V g \approx \beta_2^{-1}$). $\tau_s$ is the characteristic timescale of self-steepening (shock time). For the input light field, the complex field envelope of the experimental laser pulse obtained from FROG measurements was used.

The equation was evaluated using the split-step exponential Fourier method[59] iteratively along the propagation coordinate $z$. Splice losses between fiber segments were accounted for by being applied at the boundary between fiber segments. The dispersive contributions were evaluated in the frequency domain using as input the frequency-dependent group velocity dispersion of the ND or AD fiber segment under consideration, obtained from a waveguide mode solver. The nonlinear coefficient $\gamma$ also changes depending on the ND or AD segments.

The simulations showed good agreement to experimental findings (e.g., as shown in Figure 2E) by setting $\tau_s = 0$ (typically $\tau_s \approx \frac{1}{\omega_0}$ for single-mode waveguides) and by omitting any Raman contributions, denoting that higher-order SPM effects and the Raman effect are not significant in the fiber experiment. In the integrated SiN waveguide calculations, Raman contributions were also omitted, motivated by earlier observations and estimates that in that material the effect can be neglected.[60]
Supporting Information
Supporting Information is available from the Wiley Online Library or from the author.

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Author Contributions
H.Z. conceived and developed the concept, the theory and numerical GNLSE model and designed the experimental methodology. N.L. and C.F. designed and built the experimental setup with the help of H.Z. N.L. and H.Z. conducted the experiment. H.Z. and N.L. analyzed the experimental data. N.L. calculated the fiber and waveguide dispersion. K.-J. B., H.Z., C.F., and N.L. wrote the manuscript. H.Z. wrote the Supporting Information section. K.-J. B. and C.F. supervised the work. All authors contributed to the scientific discussion.

Conflict of Interest
Haider Zia and Klaus Boller are inventors of a pending filed patent owned by University of Twente of which a part covers the main idea of this work. The patent application no. is 16/204614.

Keywords
coherent broadband generation, integrated photonics, nonlinear optics, nonlinear pulse compression, supercontinuum generation, ultrafast optical pulses

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