Interferences in the density of two initially independent Bose-Einstein condensates

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Abstract

It is shown that the density of two initially independent condensates which are allowed to expand and overlap can show interferences as a function of time due to interparticle interaction. Using many-body theory, explicit expressions for the density are given which are exact in the weak interaction limit. General working equations are discussed which reproduce exactly the density in this limit. Illustrative examples are presented.

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The interference of two Bose-Einstein condensates (BECs) has attracted much attention both from the theoretical and experimental sides, see, e.g., [1–7]. In a popular scenario studied, two BECs comprised of identical atoms are produced in separate traps which we may call the left and right traps. The energy barrier between the two traps is so high and broad that tunneling between them is completely negligible. By removing the traps and the barrier between them, the atoms expand freely and can overlap. In experiment, the photographs obtained show spectacular interference fringes [1,2], and there has been much debate concerning the origin of these fringes.

In the above scenario the initial state of the many-body system before removing the traps reads

\[ |\Psi\rangle = \left( N_L!N_R! \right)^{-1/2} (b_L^{\dagger})^{N_L} (b_R^{\dagger})^{N_R} |\text{vac}\rangle , \quad N_L + N_R = N , \]  

(1)

where the \( b_L^{\dagger} \) and \( b_R^{\dagger} \) are the usual creation operators for bosons in the left and right traps, respectively, which contain definite numbers \( N_L \) and \( N_R \) of atoms in them. After removing the traps, the state \( |\Psi\rangle \) is no longer an eigenstate of the system’s Hamiltonian \( H_0 \) and expands in space as a function of time. The time-dependent density, i.e., the expectation value of the density operator \( \hat{\rho}(x) \) as a function of time becomes

\[ \rho(x,t) \equiv \langle \Psi(t) | \hat{\rho}(x) | \Psi(t) \rangle = N_L |\Phi_L(x,t)|^2 + N_R |\Phi_R(x,t)|^2 , \]  

(2)

where the \( \Phi_{L,R}(x,t) \) are the single-atom states corresponding to \( b_{L,R}(t) = \exp(iH_0 t) b_{L,R} \exp(-iH_0 t) \). Obviously, the density is a sum of the individual densities of the two condensates and does not exhibit an interference term. So how can one explain the interference fringes observed in experiment? It has been widely argued that what we see in an individual experimental run (individual photograph) is described by higher-order correlation functions and that the density (2) is the result of the statistical average over many experimental runs [6,7]. Interestingly, higher-order correlation functions, for instance, the density-density correlation function, do show an interference pattern in contrast to the density \( \rho(x,t) \) [3].
We would like to draw attention to the fact that the literature result (2) has been obtained under the assumption that atoms belonging to the two different BECs do not interact with each other. In the following we demonstrate that in the presence of interaction, the density \( \rho(x, t) \) does show an interference term. This finding has many consequences. In particular, the corresponding interference structures remain after the statistical averaging over many experimental runs. Of course, as \( \rho(x, t) \) changes in time, the average must be carried out at the same value of \( t \).

The density operator \( \hat{\rho}(x) = \hat{\Psi}^\dagger(x) \hat{\Psi}(x) \), where \( \hat{\Psi}(x) \) is the usual field operator, can be expressed in any complete basis set of one-particle functions. Using (box normalized) plane waves, one has

\[
\hat{\rho}(x) = v^{-1} \sum_{k,k'} e^{i(k-k')x} a_k^\dagger a_k,
\]  

which is a suitable choice for our scenario. Here, \( v \) is the volume and \( a_k \) is the destruction operator of a free boson with momentum \( k \) having the usual commutation relations \([a_k, a_k^\dagger] = \delta_{kk'}\). In this representation, the Hamiltonian of interacting identical bosons of mass \( m \), after release of the traps, is given by

\[
H = H_0 + V, \quad H_0 = \sum_k \frac{k^2}{2m} a_k^\dagger a_k,
\]

\[
V = \frac{\lambda_a}{2v} \sum_{k_1,k_2,k_3,k_4} \delta_{k_1+k_2+k_3+k_4} a_{k_1}^\dagger a_{k_2}^\dagger a_{k_3} a_{k_4}.
\]

For the ease of presentation, the widely used contact interaction \( V(x, x') = \lambda_a \delta(x-x') \), where \( \lambda_a \) is proportional to the s-wave scattering length, is used [7,8]. Of course, any other interparticle interaction can be used as well. As usual, \( H_0 \) describes the motion of the free atoms.

We may now proceed to evaluate the time-dependent density \( \rho(x, t) \) of the interacting particles. First we transform to the interaction picture and write \( \hat{\rho}_I(x, t) = e^{iH_0t} \hat{\rho}(x)e^{-iH_0t} \) and all other quantities in this picture are extremely easy to evaluate. In particular, \( a_k(t) \equiv e^{iH_0t} a_k e^{-iH_0t} = e^{-i\frac{k^2}{2m} t} a_k \). To proceed we make use of the identity

\[
\rho(x, t) = \langle \Psi_I(t) | \hat{\rho}_I(x, t) | \Psi_I(t) \rangle
\]  

(5a)
where $|\Psi_I(t)\rangle = e^{iH_0t}e^{-iHt}|\Psi(t)\rangle$. This expression for the density can be systematically evaluated using the textbook expansion

$$|\Psi_I(t)\rangle = \left\{ 1 - i \int_0^t V_I(t_1)dt_1 + (-i)^2 \int_0^t V_I(t_1)dt_1 \int_0^{t_1} V_I(t_2)dt_2 + \ldots \right\} |\Psi(0)\rangle. \quad (5b)$$

Inserting this expression into the expression (5a) for $\rho(x,t)$ gives

$$\rho(x,t) = \rho^0(x,t) + i \int_0^t dt_1 \langle \Psi |[V_I(t_1), \hat{\rho}_I(x,t)]| \Psi \rangle + (i)^2 \int_0^t dt_1 \int_0^t dt_2 \langle \Psi |[V_I(t_2), [V_I(t_1), \hat{\rho}_I(x,t)]]| \Psi \rangle + \ldots \quad (6)$$

which is a systematic expansion of $\rho(x,t)$ in terms of the interparticle interaction. $|\Psi\rangle \equiv |\Psi(0)\rangle$ is the initial state in Eq. (1), which is an eigenstate of the system in the presence of the trap potentials, and $\rho^0(x,t) = \langle \Psi |\hat{\rho}_I(x,t)| \Psi \rangle$ is the free density evolving without the impact of this interaction. With (6) and the commutator relations for the boson operators $a_k$ and $a_{k'}^\dagger$, one obtains an explicit expansion of $\rho(x,t)$. With growing order, the terms of this expansion contain higher products of destruction and creation operators, $a_{k_1}^\dagger a_{k_2}^\dagger a_{k_3} a_{k_4}$, and so on.

How can one now evaluate the desired density $\rho(x,t)$ in view that the initial state $|\Psi\rangle$ in Eq. (1) is expressed in terms of other operators and not in terms of $a_k^\dagger$? To this end we remind that the field operator $\hat{\Psi}$ can be expressed in any complete set of functions, plane waves or others. Since $b_L$ and $b_R$ obviously commute, the respective functions $\Phi_L$ and $\Phi_R$ may be seen as the first two members in the expansion of the field operator: $\hat{\Psi}(x) = \Phi_L(x)b_L + \Phi_R(x)b_R + "Rest"$. It immediately follows that

$$a_k = \varphi_L(k)b_L + \varphi_R(k)b_R + "Rest(k)". \quad (7)$$

where $\varphi(k)$ is the Fourier transform of $\Phi(x)$ and the "Rest" contains all other destruction operators commuting with $b_L$ and $b_R$. Luckily, all the latter operators have a vanishing impact when applied to the initial state $|\Psi\rangle$. Consequently, if we order all the creation operators to the left and all the annihilation operators to the right when taking the expectation value of $\hat{\rho}(x,t)$ with the many-body state $|\Psi\rangle$ in Eq. (1), we can throw the "Rest" out in the
following. Inserting the result (7) into the above mentioned higher products of destruction and annihilation operators in the expansion of \( \hat{\rho}(x, t) \), one readily obtains

\[
\rho(x, t) = \rho_{LL}(x, t) + \rho_{RR}(x, t) + \rho_{LR}(x, t),
\]

(8)

where \( \rho_{LL} \) and \( \rho_{RR} \) are the densities of the expanding separated BECs as if the two BECs do not communicate, and \( \rho_{LR} \) is the change of the density due to the interaction between them. The terms contributing to \( \rho_{LL} \) (\( \rho_{RR} \)) contain only \( b_L b_R \) and \( b_L^\dagger b_R^\dagger \) operators, e.g., \( b_L^\dagger b_L^\dagger b_R b_L \), and those contributing to \( \rho_{LR} \) contain only mixed products, e.g., \( b_L^\dagger b_R^\dagger b_L b_R \).

Note that the calculation of \( \rho(x, t) \) for interacting bosons amounts to solving a full many-body problem. Wishing to arrive at an exact analytical result, we concentrate first on weak interparticle interactions.

With \( V \) in Eq. (4) and \( \hat{\rho}(x) \) in Eq. (3) and the very simple appearance of \( a_k(t) \) given above, we evaluated explicitly the commutator \([V_I(t_1), \hat{\rho}_I(x, t)]\) and with it the leading term in Eq. (6). The calculation is somewhat lengthy, but very straightforward. Using the simple “trick” (7) we obtain the exact results up to first order

\[
\rho_{LR} = -4\lambda a N_L N_R \text{Im}\{\Phi_L(x, t)A_{LR}(x, t) + \Phi_R(x, t)A_{RL}(x, t)\},
\]

\[
\rho_{LL} = N_L |\Phi_L(x, t)|^2 - 2\lambda a N_L (N_L - 1) \text{Im}\{\Phi_L(x, t)A_{LL}(x, t)\}.
\]

(9a)

Here \( \Phi_{L,R}(x, t) \) is the freely expanding \( \Phi_{L,R}(x) \) and the amplitude \( A_{LR} \) reads

\[
A_{LR} = \frac{1}{v^{3/2}} \sum_{k_1, k_2, k_3} e^{i(k_3-k_1-k_2)x} \varphi_L(k_1)\varphi_R^*(k_2)\varphi_R(k_3) \int_0^t dt_1 e^{i[(k_1^2+k_2^2-k_3^2)t_1+(k_1+k_2-k_3)^2(t-t_1)]/2m}.
\]

(9b)

To obtain \( \rho_{RR} \) just interchange \( L \) and \( R \) in (9a). The quantities \( A_{RL}, A_{LL} \) and \( A_{RR} \) are obtained analogously from (9b). The integration over \( t_1 \) in Eq. (9b) can, of course, be performed explicitly, but then the summations over the three momenta are more cumbersome to carry out. Let us briefly discuss the result (9). Clearly, the interference term \( \rho_{LR} \) vanishes for \( t \to 0 \). Furthermore, \( \rho_{LR}(x, t) \) vanishes as expected if the atoms do not interact with each other (\( \lambda a \to 0 \)). The interference term \( \rho_{LR}(x, t) \) is enhanced by the product \( N_L N_R \) of the numbers of atoms in the two initial BECs.
The above results make clear that the interaction between the particles gives rise to an interference term in the density of two initially independent BECs of identical bosons. Before presenting a numerical example we go one step further and pose the question whether we can formulate a mean-field theory which reproduce exactly the exact small $\lambda_a$ result (9a). Such a theory would open the door for real applications. The standard mean-field leads to the well-known and widely used Gross-Pitaevskii equation [7,8]. For coherent states this equation gives exact results for small $\lambda_a$. Clearly, it is inapplicable to fragmented states (1). For fragmented states a more general multi-orbital mean-field theory has been recently derived [9]. In the present scenario two orbitals are involved and the respective time-dependent mean-field (TDMF(2)) takes on the appearance (for the general derivation of TDMF, see [10]):

$$
\begin{align*}
\dot{\psi}_L &= \mathcal{P} \left[ \hat{h} + \lambda_a(N_L - 1) |\psi_L|^2 + 2\lambda_a N_R |\psi_R|^2 \right] \psi_L, \\
\dot{\psi}_R &= \mathcal{P} \left[ \hat{h} + \lambda_a(N_R - 1) |\psi_R|^2 + 2\lambda_a N_L |\psi_L|^2 \right] \psi_R
\end{align*}
$$

where the initial conditions are $\psi_{L,R}(x, t = 0) = \Phi_{L,R}(x)$. $\hat{h}$ is the usual one-particle Hamiltonian (in our scenario just the kinetic energy operator) and $\mathcal{P} = 1 - |\psi_L\rangle \langle \psi_L| - |\psi_R\rangle \langle \psi_R|$. $\mathcal{P}$ is a projector which ensures orthonormalization of the orbitals $\psi_L$ and $\psi_R$ [10].

We now prove that the TDMF(2) in (10) exactly reproduces the exact many-body small $\lambda_a$ result (9a). In TDMF(2) the density can be expresses by $\rho(x, t) = N_L |\psi_L(x, t)|^2 + N_R |\psi_R(x, t)|^2$. Since $\lambda_a$ in (9a) is taken to be small, we may write $\psi_L = \Phi_L(x, t) + \delta\psi_L(x, t)$ and analogously for $\psi_R$. Inserting into the latter expression for the density and comparing with (9a), we immediately identify $\delta\psi_L$:

$$
\delta\psi_L = -i\lambda_a(N_L - 1) A_{LL}^* - i2\lambda_a N_R A_{LR}^*.
$$

To obtain $\delta\psi_R$ just interchange $L$ and $R$. Of course, we still have to show that $\psi_L = \Phi_L + \delta\psi_L$ with $\delta\psi_L$ from Eq. (11) indeed fulfills the TDMF equations, i.e., is the solution of (10). Taking the derivative of $\Phi_L + \delta\psi_L$ with respect to time and using $i\dot{\Phi}_L = \hat{h}\Phi_L$, leads to

$$
\dot{\psi}_L = \hat{h}\Phi_L + \lambda_a(N_L - 1) A_{LL}^* + 2\lambda_a N_R A_{LR}^*.
$$
$\hat{A}_{LL}^*$ and $\hat{A}_{LR}^*$ can be deduced from Eq. (9b). The time $t$ appears there twice. The derivative with respect to the upper limit of the integral leaves us with a separable triple sum of terms like $v^{-1/2} \sum e^{ik_1 x} e^{ik_1^2/(2m)} \varphi_L(k_1) = \Phi_L(x,t)$, which leads to $|\Phi_L|^2 \Phi_L$ and $|\Phi_R|^2 \Phi_L$, respectively. The derivative with respect to $t$ in the exponential function gives an expression which is identical to $-\hat{h} A_{LL}^*$ and $-\hat{h} A_{LR}^*$, respectively (remember that $\hat{h} = -\hbar \frac{\partial^2}{\partial x^2}$).

Collecting all terms yields

$$i \dot{\psi}_L = \hat{h} \psi_L + \left[ \lambda_a (N_L - 1) |\Phi_L|^2 + 2 \lambda_a N_R |\Phi_R|^2 \right] \Phi_L$$

which is, as the orthonormalization is of second order in $\lambda_a$, identical with the TDMF(2) in Eq. (10) to first order in the interaction strength $\lambda_a$.

In the following we present two illustrative numerical examples. In the first the interaction is very weak and the initial single-atom functions $\Phi_{L,R}(x)$ are normalized Gaussians located at $\pm x_0$, i.e., $\Phi_{L,R}(x) = (\frac{2}{\pi a^2})^{1/4} \exp\left\{-(x \mp x_0)^2/a^2\right\}$. This is a realistic choice for harmonic traps. The Fourier transforms of these functions simply are $\varphi_{L,R}(k) = a^{1/2}/(2\pi)^{3/4} \exp(-k^2 a^2/4) \exp(\mp ikx_0)$. They evolve in time as $\varphi_{L,R}(k,t) = e^{-\frac{i}{\hbar} \frac{k^2}{2m} t} \varphi_{L,R}(k)$ and their back-transforms give $\Phi_{L,R}(x,t)$ which are normalized Gaussians expanding in space as a function of time and can be found in many elementary text books. For the ease of presentation our example is in one dimension. After transferring the summations in Eq. (9b) to integrals over the momenta, all three integrals over $k_3$, $k_2$ and $k_1$ can be carried out analytically using that $\int_{-\infty}^{+\infty} e^{-q^2(k+p)^2} dk = \sqrt{\pi}/q$ for any complex quantities $q, p$ as long as $\Re q^2 > 0$. We have checked that this condition is fulfilled for all three integrals. The final result is of the form $A_{LR}(x,t) = \int_0^t dt_1 f(t_1) e^{g(x,x_0,t_1,t)}$, where $f(t_1)$ and $g(x,x_0,t_1,t)$ are simple but lengthy algebraic expressions.

Fig. 1 shows the interference term of the density $\rho_{LR}(x,t)$ for very weak atom-atom interaction. Without interaction $\rho_{LR}(x,t) = 0$ and the density is simply $N_L |\Phi_L(x,t)|^2 + N_R |\Phi_R(x,t)|^2$. The interaction leads to $\rho_{LR}$ for which we use expression (9). To simplify the discussion, we put $m = a/2 = \hbar = 1$ and express $x$ in units of $a/2$, $t$ in units of $(a/2)^2m$, and $\lambda_a$ in units of $\frac{1}{(a/2)^3m}$. Furthermore, $N_L = N_R = 500$, $x_0 = 6$ and $\lambda_a = 2.5 \cdot 10^{-7}$. At
$t = 0$ one has $\rho_{LR}(x,0) = 0$ and then it starts to grow as $\sim t^2$ at very short times and much faster later on. $\rho_{LR}(x,t)$ exhibits an oscillatory behavior which changes as time proceeds.

In the following we apply the TDMF theory. As a first step we compute $\rho_{LR}(x,t)$ for the case of weak interaction discussed above. The results are also depicted in Fig. 1 and seen to coincide with those obtained using the analytic expression (9). Next, we enlarge the interaction strength $\lambda_a$. For coherent states the time-dependent Gross-Pitaevskii equation, which is exact in the weak interaction limit, has been demonstrated in many cases to be applicable for intermediate and stronger interactions [7,8]. Similarly, there is reason to expect that for fragmented states the TDMF theory, which has been proven above to be exact in the weak interaction limit, is applicable well beyond this limit. We mention that TDMF(1) is nothing but the time-dependent Gross-Pitaevskii equation.

For the very weak interaction discussed above we have chosen as initial conditions Gaussians located at $\pm x_0$. Now, as the interaction is increased to $\lambda_a = 0.1$, we choose the respective solutions of the stationary Gross-Pitaevskii equation at this $\lambda_a$ as initial conditions to account for the interaction when the harmonic traps centered at $\pm x_0$ are released. In Fig. 2 the density $\rho(x,t)$ computed using the TDMF(2) equations (10) is shown as a function of time. As seen in the figure, at $t = 0$ the density consists of two separated distributions centered at $\pm x_0$. The traps are removed at this time and the distributions start to broaden and to overlap. At about $t = 2.4$ one begins to see impact of the interference term in the density which becomes strongly pronounced as time proceeds.

We conclude that the density of two initially independent condensates which are allowed to overlap can show interference effects in the presence of interparticle interaction. The physics of so called fragmented states, like the state in Eq. (1), is generally very different from that of coherent states [11,12]. Coherent states of condensates have been extensively studied, mostly in the framework of the Gross-Pitaevskii equation [7,8]. A BEC in a coherent state can exhibit interference fringes even in the absence of interaction [7,13–16]. Take, for instance, the coherent state $|\Psi^{coh}\rangle = (N!)^{-1/2}(b^\dagger)^N |vac\rangle$ with $b^\dagger = (b^\dagger_L + b^\dagger_R)/\sqrt{2}$. 8
This immediately leads to $\rho^{coh}(x, t) = \frac{N}{2} |\Phi_L(x, t) + \Phi_R(x, t)|^2$ in the absence of interaction between the atoms. In analogy to Eq. (7) we can determine the interference term $\rho^{coh}_{LR}(x, t) = N \text{Re}(\Phi^*_L \Phi_R)$. For the expanding Gaussians discussed above, the oscillatory part of $\rho^{coh}_{LR}$ is simply given by $\cos[K(t)x]$ with $K(t) = 8x_0(t/m)/(a^4 + 4t^2/m^2)$. This interference term is qualitatively different from that arising due to the interaction between the particles. Another important difference between $\rho^{coh}_{LR}(x, t)$ and $\rho_{LR}(x, t)$ worth mentioning is that the former depends on the relative phase between $\Phi_L(x)$ and $\Phi_R(x)$, while the latter does not depend on this phase.

Whether in an experiment the initial state is coherent or fragmented depends on the experimental conditions. It is beyond the scope of this work to argue whether or not the initial state in the currently available experiments is fragmented. It is also not our intention to take side in the ongoing debate on whether these experiments detect the density or higher-order correlation functions, although we tend to share the opinion of some researchers, see, e.g., [4,13–15], that the density in measured. What we can state, is that if one measures the density of two freely expanding initially independent BECs, it will only show interferences in the presence of interaction. This leads to the following proposal for an experiment which makes use of the fact that nowadays one can vary the strength of the interaction between the atoms [17,18]. Two measurements are necessary. If the measurement with interaction shows interferences which disappear upon measuring with the interaction turned off, then (a) the initial state was a fragmented state and (b) the interaction is responsible for the interferences.

The theory presented here is easily extendable to any kind of interparticle interaction. It is also easily extendable to the case where one does not let the two BECs expand freely by removing the traps completely. One may, e.g., remove only the barrier and let the BECs expand in the new global trap. Since the interference structures depend on the interaction, a wealth of effects can be expected by varying the interaction, the form of the individual traps and of the numbers $N_L$ and $N_R$ of the particles.
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FIG. 1. (Color online) The interference term $\rho_{LR}$ for very weak interaction strength ($\lambda_a = 2.5 \cdot 10^{-7}$) at two values of $t$. The solid curves are computed using the analytic result (9). The dots are computed using the TDMF(2) equations (10). The quantities shown are dimensionless.
FIG. 2. (Color online) The density $\rho(x,t)$ of two condensates of 500 atoms each for $\lambda_a = 0.1$ as a function of time computed with TDMF(2) (black) compared to the density $\rho_{LL} + \rho_{RR}$ of two BECs which do not interact with each other, each computed with the Gross-Pitaevskii equation (red). The quantities shown are dimensionless. For more details see text.