Axisymmetric absorption of \textit{p} modes by an ensemble of thin, magnetic-flux tubes

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Abstract. The buffeting action of the solar acoustic waves (\textit{p} modes) on magnetic fibrils excites magnetohydrodynamic (MHD) tube waves. We model these fibrils as axisymmetric, untwisted, vertically oriented, thin, magnetic-flux tubes. The MHD tube waves propagate along the length of the tube and carry energy away from the \textit{p}-mode cavity creating a source of \textit{p}-mode absorption. We calculate the absorption arising from the excitation of \textit{sausage} MHD waves within a model plage composed of many flux tubes with differing plasma properties. We find that for a collection of tubes with normally distributed plasma parameters $\beta$, the macroscopic absorption coefficient of the collection effectively depends on only the mean value of $\beta$.

1. Introduction
It is well established that the solar \textit{f} and \textit{p} modes are influenced by the properties of magnetic structures such as sunspots and plages. It is also known that the effect of magnetic field is not just confined within the boundaries of the magnetised region but extends beyond it into an “acoustic shadow”. The work of Braun (1987, 1988), Bogdan and Braun (1995), and more recently Braun and Birch (2008) suggests that both sunspots and plages are ravenous absorbers of \textit{p}-mode power. Thus, the importance of sub-surface field structure in modifying the properties of \textit{f} and \textit{p} modes has been realised and many theoretical investigations have concentrated on understanding the physical mechanism responsible for this absorption (e.g., Spruit 1991; Bogdan and Cally 1995; Bogdan et al. 1996; Crouch and Cally 2005; Jain et al. 2009).

Jain et al. (2009) (hereafter referred to as JHBB) studied \textit{f}- and \textit{p}-mode absorption in a plage region by modelling the plage as an idealised forest of thin, untwisted, vertical, magnetic-flux tubes. By idealised, we mean each flux tube in the plage has the same properties, i.e., the same magnetic flux and plasma $\beta$ (the ratio of gas to magnetic pressure). However, the distribution of flux within a plage is generally quite heterogeneous and there is no reason to believe that a plage region is comprised of identical flux tubes. In this paper, we model the plage as consisting of many flux tubes each with the same magnetic flux but with a Gaussian distribution of $\beta$ values. We then investigate the effect of such an ensemble on the absorption of \textit{p} modes.

2. The Equilibrium
The fibril fields are embedded in a non-magnetic medium. We model the nonmagnetic medium as a plane-parallel gravitationally stratified polytropic atmosphere with gravity \textbf{g} = $-g\hat{z}$. The pressure, density and sound speed vary with depth $z$ as power laws with a polytropic index
a. Following Bogdan et al. (1996) and Hindman and Jain (2008) we truncate the polytrope at $z = -z_0$ which represents the model photosphere. Above the truncation depth $z > -z_0$ we assume the existence of a hot vacuum ($\rho_{\text{ext}} \rightarrow 0$ with $T_{\text{ext}} \rightarrow \infty$). For thin flux tubes, the lateral variation of the magnetic field across the tube is ignored and the plasma $\beta$ is constant with depth (see Bogdan et al. (1996) for details).

### 3. The Governing Equation

The incident acoustic wavefield in the non-magnetic medium can be expressed as a single partial differential equation for the displacement potential $\Phi$:

$$\frac{\partial^2 \Phi}{\partial t^2} = c^2 \nabla^2 \Phi - g \frac{\partial \Phi}{\partial z},$$

where $c^2$ is the square of the adiabatic sound speed. This equation can be transformed into an ordinary differential equation that supports plane wave solutions of the form:

$$\Phi(x,t) = A e^{-i \omega t} e^{i k_0 x} Q_n(z); \quad Q_n(z) = (-2 k_0 z)^{-(\mu+1)/2} W_{\kappa,\mu}(-2 k_0 z)$$

where $\mu = (a + 1)/2$, $\nu^2 = a \omega^2 z_0/g$ and $\kappa = \nu^2/(2k_0 z_0)$. We calculate the eigenvalues and eigenfunctions for the truncated polytrope by requiring that the Lagrangian pressure perturbation vanishes at the truncation depth, i.e., $\nabla \cdot \xi = 0$. Mathematically this takes the form $W_{\kappa,\mu+1}(2k_0 z_0) = 0$.

Thin flux tubes support both longitudinal (sausage) waves and transverse (kink) waves; here we only consider the sausage waves. These waves are driven by the external $f$ and $p$ modes. Using the formulation of Hindman and Jain (2008), the vertical fluid displacement due to the excitation of sausage waves within the tube can be described by

$$\left[ \frac{\partial^2}{\partial t^2} + \frac{2g_z}{2a + \beta(1 + a)} \frac{\partial^2}{\partial z^2} + \frac{g(1 + a)}{2a + \beta(1 + a)} \frac{\partial}{\partial z} \right] \xi_{\parallel} = \frac{(1 + a)(\beta + 1)}{2a + \beta(1 + a)} \frac{\partial^2 \Phi}{\partial z \partial t^2}$$

### 4. Absorption coefficient for an ensemble of flux tubes

Following the procedure in JHBB, we calculate the absorption coefficient, $\alpha_n$, for the $n$th-order $p$ mode for a single tube. We wish to apply our theoretical results to model the absorption coefficient measured for a solar plage region by examining the effect of a large number of thin flux tubes. We, thus assume that the tubes are sufficiently separated that we can ignore their acoustic jacket modes (Bogdan and Cally 1995) and the effects of multiple scattering (Hanasoge and Cally 2009).

In JHBB, the absorption coefficient for an individual tube was linearly proportional to the magnetic flux $\phi$ contained by that tube,

$$\alpha_n = C_n(\beta, \omega) \phi.$$  \hspace{1cm} (2)

The function $C_n(\beta, \omega)$ is a smooth but rather complicated function of the frequency $\omega$ and the $\beta$ value of the tube. The absorption coefficient at position $x$ measured by local helioseismic techniques such as ridge-filtered holography (Braun and Birch 2008), can be related to the distribution of tubes through a spatial weighting function or kernel, $K_n(x, \omega)$,

$$\alpha_n(x, \omega) = \sum_i C_n(\beta_i, \omega) \phi_i K_n(x_i - x, \omega),$$
where each flux tube in the plage is labelled by an index \( i \) and \( \mathbf{x}_i, \phi_i \) and \( \beta_i \) are the position, magnetic flux and plasma parameter for tube \( i \).

In JHBB all the flux tubes were identical; thus, the quantity \( C_n(\beta_i, \omega) \) could be taken outside the summation and the remaining sum simply became the kernel weighted magnetic flux, \( \Theta_n(\mathbf{x}, \omega) \), which is a measurable quantity. Here, however, we will be considering a random distribution of flux tubes for a range of \( \beta \) values between 0 and 1. Thus, we compute an ensemble average of the absorption coefficient,

\[
\langle \alpha_n(\mathbf{x}, \omega) \rangle = \left\langle \sum_i C_n(\beta_i, \omega) \sum_i \phi K_n(\mathbf{x}_i - \mathbf{x}, \omega) \right\rangle.
\]

We will assume that the locations \( \mathbf{x}_i \) and \( \beta \) values of the tubes are not correlated. Thus, if all the tubes have the same flux,

\[
\langle \alpha_n(\mathbf{x}, \omega) \rangle = \langle C_n(\beta_i, \omega) \rangle \Theta_n(\mathbf{x}, \omega) ; \quad \langle C_n(\beta_i, \omega) \rangle = C \int_0^1 C_n(\beta, \omega) e^{-\frac{(\beta-\beta_0)^2}{2\sigma^2}} \, d\beta,
\]

where \( \sigma \) is the variance, \( C \) is a distribution normalization, and

\[
\Theta_n(\mathbf{x}, \omega) = \left\langle \sum_i \phi K_n(\mathbf{x}_i - \mathbf{x}, \omega) \right\rangle \approx \int d\mathbf{x}' \left| B(\mathbf{x}') \right| K_n(\mathbf{x}' - \mathbf{x}, \omega).
\]

We show the results for maximal flux boundary condition. (Hindman & Jain 2008).

5. Results and Discussion

We compute equation (3) for different distributions of \( \beta \) values. The magnetic flux \( \Theta_n \) and the kernel functions \( K_n \) are identical to that used by JHBB. In Figure 1, we plot the absorption coefficient for a modelled plage as a function of frequency. The curves in this figure are for a modelled plage that is composed of a host of magnetic flux tubes whose \( \beta \) values have been drawn from a gaussian distribution limited to values between 0 and 1. The three horizontal panels correspond to different peak values of the distribution \( \beta_0 \), while the different line styles for the curves denote different distribution widths \( \sigma \). While the curves for various \( \beta_0 \) and \( \sigma \) all differ, the primary dependence of the absorption coefficient is on the mean of the distribution \( \beta \). Note, \( \beta \) is not equal to the most probable value \( \beta_0 \) because of the fact that \( \beta \) is restricted to values within 0 and 1.

The Gaussian distribution used in Figure 1 is shown as curves, whereas the symbols correspond to a plage comprised of a multitude of identical flux tubes, all with the same value of \( \beta \) equal to the mean of the gaussian distribution \( \beta = \bar{\beta} \). One can clearly see that two very different tube distributions produce essentially the same absorption coefficient as long as their mean \( \beta \) values are the same.

Also note from Figure 1 that the absorption reduces significantly above \( \nu = 2.5-3 \) mHz for low radial orders. This is due to the fact that at low mode orders and high frequencies, waves have larger horizontal wavenumbers and thus the kernel function \( K_n \) (see JHBB) whose width depends on the horizontal wavelength samples relatively smaller spatial region, resulting in reduced total magnetic flux. Thus, from equation (3) the absorption coefficient \( \alpha \) reduces for low radial order and higher frequencies.

6. Conclusions

We compute absorption coefficients for collections of vertical, axisymmetric, thin, magnetic-flux tubes representing a solar plage region. We have shown that the macroscopic absorption coefficient of the collection effectively depends only on the mean value of \( \beta \) for the distribution.
Figure 1. Absorption coefficient as a function of frequency for a modelled plage constructed from many thin, magnetic-flux tube with plasma $\beta$ varying between 0 and 1. The distribution function is gaussian in form with a peak value of $\beta_0$ and a width of $\sigma$. The solid and dashed lines are for $\sigma = 0.1$ and $0.8$, respectively. Various symbols represent the corresponding absorption coefficients for identical tubes, the $\beta$ value for each of these tubes was selected such that it matched the mean value of the gaussian distributions. Each mode order is denoted by a different colour: black (p1), red (p2), turquoise (p3) etc. One can easily see that the only relevant parameter appears to be the mean of the distribution.

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