State of Charge Modeling of Arbitrary Cell Connection

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I. NOMENCLATURE

| Symbol | Description                                |
|--------|--------------------------------------------|
| $V_o^B$ | Open-circuit voltage of the $i^{th}$ single-cell battery |
| $V_C^B$ | Output voltage of the $i^{th}$ cell |
| $V_B$  | Output voltage of the multi-cell battery |
| $V_F$  | Cutoff voltage of the single-cell battery |
| $R_F$  | Self discharge resistance of the $i^{th}$ cell |
| $R_i$  | Serial resistance of the $i^{th}$ cell |
| $R_S$  | Short-transient resistance of the $i^{th}$ cell |
| $R_L$  | Long-transient resistance of the $i^{th}$ cell |
| $C_S$  | Short-transient capacitance of the $i^{th}$ cell |
| $C_L$  | Long-transient capacitance of the $i^{th}$ cell |
| $\varphi_i^B$ | SOC of the single battery the $i^{th}$ cell |
| $\varphi_i^P$ | SOC of the multi-cell battery |
| $\varphi_i^S$ | SOC of the cell string in parallel connection |
| $\alpha_i^C$ | Capacity of the $i^{th}$ cell |
| $\alpha_i^T$ | Full capacity of a single cell |
| $\alpha_i^A$ | Consumed capacity of a single-cell battery |
| $\alpha_i^B$ | Remaining capacity of multi-cell battery |
| $\alpha_i^P$ | Remaining capacity of cell string in parallel |
| $\alpha_i^S$ | Remaining capacity of cell string in serial |
| $I_i^C$  | Discharge current rate of the $i^{th}$ cell |
| $I_i^B$  | Discharge current rate of the multi-cell battery |
| $\omega$ | Frequency of discharge current |

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circuit-based multi-cell battery model to understand and quantify the multi-cell battery discharge behavior under various cell connections, discharge current rates, and cell status variations. The proposed multi-cell battery models provide a guide for dynamic reconfiguration of multi-cell battery. Furthermore, both simulation and experiment are conducted to validate the multi-cell battery models.

III. MULTI-CELL BATTERY MODEL

The multi-cell battery can be recursively constructed by applying the connection of series and parallel, starting with a single battery cell. The last step is either a series connection or a parallel connection. If the last step is the series connection, the multi-cell battery could be looked as an equivalent cell string in series connection as shown in Figure 1(a). If the last step is the parallel connection, then the multi-cell battery could be modeled as an equivalent cell string in parallel connection as shown in Figure 1(b). In other words, any given cell connection in multi-cell battery can be composed by two basic cell connections: cell strings in series connection and cell strings in parallel connection. In the following sections, we will develop performance models for cell strings in both series and parallel connections in details.

IV. MODEL OF CELL STRING IN PARALLEL CONNECTION AND SERIES CONNECTION

A. Related works-Single Cell Battery Model

A circuit-based model as shown in Figure 2 has been adopted in this paper. Assuming that the single-cell battery is healthy. The single-cell battery model can capture both simulation and experiment are conducted to validate the multi-cell battery models.

\[
\begin{align*}
\alpha^A (I^C, \beta, L, t_s, t_e) &= I^C [(t_s - t_e) + F(L, t_s, t_e, \beta)] \\
F(L, t_s, t_e, \beta) &= 2 \sum_{m=1}^{\infty} \frac{e^{\beta m^2 L(t_s-t_e)^2}}{m^2} \frac{e^\beta}{\beta m^2} \\
\varphi^C &= \frac{1 - \alpha^A}{\alpha^A} \\
V^o (\varphi^C) &= a_1 e^{a_2 \varphi^C} + a_3 \varphi^C + a_4 \varphi^2 + a_5 \varphi^3 + a_6 \\
R^o (\varphi^C) &= b_1 e^{b_2 \varphi^C} + b_3 \varphi^C + b_4 \varphi^2 + b_5 \varphi^3 + b_6 \\
R^S (\varphi^C) &= d_1 e^{-d_2 \varphi^C} + d_3 \\
C^S (\varphi^C) &= f_1 e^{f_2 \varphi^C} + f_3 \\
R^L (\varphi^C) &= g_1 e^{g_2 \varphi^C} + g_3 \\
C^L (\varphi^C) &= l_1 e^{l_2 \varphi^C} + l_3 \\
V^C (\varphi^C) &= V^o (\varphi^C) - R^o (\varphi^C) I^C \\
&- \frac{R^o (\varphi^C) f^o (\varphi^C)}{R^o (\varphi^C) f^o (\varphi^C) + I^C} I^C \\
&- \frac{R^o (\varphi^C) f^o (\varphi^C)}{R^o (\varphi^C) f^o (\varphi^C) + I^C} I^C
\end{align*}
\]

where, \( \alpha^A \) is the accumulated capacity during time period \([t_s, t_e] \) at rate of \( I^C \). The first term \( I^C (t_s - t_e) \) is the consumed capacity by the load \( I^C \) during the load period \([t_s, t_e] \). The second term \( I^C F(L, t_s, t_e, \beta) \) is the amount of discharging loss due to the current effect, which is the maximum recoverable battery capacity at \( t_s, \beta \) is a constant related to the diffusion rate within battery. The larger the \( \beta \), the faster the battery diffusion rate is, thus the less the discharging loss. \( L \) is the total operating time of the battery, \( m \) determines the computational complexity and accuracy of the model. \( R^o, V^o, \alpha^A, \) and \( \varphi^o \) are battery internal resistance, open-circuit voltage, the full capacity, and SOC, respectively. \( R^S, R^L, C^S, \) and \( C^L \) are resistances and capacitors to capture the transient response of battery voltage. \( a_1 \sim a_6, b_1 \sim b_6, d_1 \sim d_3, f_1 \sim f_3, g_1 \sim g_3, l_1 \sim l_3 \) are coefficients of the model, which can be achieved by using the standard least-square estimator [14], [15].

B. Model of Cell String in Parallel Connection

A circuit-based model of cell string in parallel connection can be described in Fig. 3, which can be described as:

\[
\begin{align*}
V^C_1 (\varphi^C) &= V^C_2 (\varphi^C) = \cdots = V^C_N (\varphi^C) = V^B \\
V^o (\varphi^C) &= V^B \\
\varphi^B &= \frac{1}{N} \sum_{i=1}^{N} \varphi^C_i \\
I^B &= \frac{1}{N} \sum_{i=1}^{N} I^C_i
\end{align*}
\]

where \( \alpha^A \) is the full cell capacity. \( \alpha^C_i (I^C) \) is the available capacity of the \( i \)th cell. \( \varphi^B \) is the SOC of the whole multi-cell battery. \( V^B (\varphi^C) \) is the output voltage of the \( i \)th cell.

1) Internal Resistance: Internal resistance of the \( i \)th battery cell can be denoted as:

\[
Z^B_i (\varphi^C_i) = R^o (\varphi^C_i) + \frac{R^e (\varphi^C_i)}{R^o (\varphi^C_i) + \omega^C (\varphi^C_i)} + \frac{R^e (\varphi^C_i)}{R^o (\varphi^C_i) + \omega^C (\varphi^C_i) + 1}
\]

where \( \omega^C \) is the available capacity of the \( i \)th cell in parallel connection. \( \omega^C \) can be defined as the sum of the consumed capacities on all cells until the weakest cell of a multi-cell battery reaches its cutoff voltage, which can be
expressed:

$$\alpha^P(I^B, \beta, L, t_s, t_e) = \sum_{i=1}^{N} \alpha^C(I^C_i, \beta, L, t_s, t_e), \quad I^C_i \geq 0$$  \hspace{1cm} (5)

3) Cell Output Voltage and Current Distribution in a Cell String in Parallel Connection: The discharge current rate of the cell string in parallel connection is a decreasing function of the output voltage, which can be written as

$$I^B = \sum_{i=1}^{N} I^C_i = \sum_{i=1}^{N} \frac{V_o^0 - V^B}{r_i}$$  \hspace{1cm} (6)

where, $V^B$ and $I^B$ are the output voltage and the discharge current rate of cell string in parallel connection, respectively. $V_o^0$ and $r_i$ are the open circuit voltage and the internal resistance of the $i^{th}$ cell.

C. Model of Cell String in Series Connection

Model of cell string in series connection is described in Fig. 4. For a $N$-cell string, the output voltage of the cell string can be expressed as

$$\begin{align*}
I^C_i(\varphi^C_i) &= I^C_2(\varphi^C_2) = \cdots = I^C_N(\varphi^C_N) = I^B \\
V_o^0(\varphi^C_i) &\geq V^F \quad i \in \{1, 2, \cdots, N\} \\
\varphi^C_i &= \frac{\alpha^C_i(\varphi^C_i)}{\alpha^C_1(\varphi^C_1)} \\
V^B &= \sum_{i=1}^{N} V^C_i  \quad V^C_i \geq 0, \quad i \in \{1, 2, \cdots, N\}
\end{align*}$$  \hspace{1cm} (7)

1) Internal Resistance: The equivalent resistance of cell string in series connection is the sum of internal resistance of each cell $Z^B$.

$$Z^B = \sum_{i=1}^{N} R(\varphi^C_i) + \sum_{i=1}^{N} \frac{R^B_i(\varphi^C_i)}{R^C_i(\varphi^C_i) + \varphi^C_i} + 1 + \cdots + 1$$  \hspace{1cm} (8)

2) The Available Capacity: For cell string in series connection, all battery cells are equally treated and discharged at the same discharge current rate and time. Each cell consumes the same capacity. Therefore, the available battery capacity of a cell string in series connection is determined by the weakest cell, which is:

$$\alpha^S(I^B, \beta, L, t_s, t_e) = \min \{\alpha^S_1(I^B, \beta, L_1, t_s, t_e), \alpha^S_2(I^B, \beta, L_2, t_s, t_e), \cdots, \alpha^S_N(I^B, \beta, L_N, t_s, t_e)\}$$  \hspace{1cm} (9)

3) The Output Voltage of Cell String: The output voltage of a cell string in series connection is the sum of the voltage of all cells, which can be denoted as follow:

$$V^S = \sum_{i=1}^{N} V^C_i(\varphi^C_i)$$  \hspace{1cm} (10)

The cutoff voltage of the cell string is determined by the weakest cell. As a result, the cell string is discharged to cutoff voltage before all cells in the cell string in series connection reach their cutoff voltages. The actual cutoff voltage of the multi-cell battery is much higher than nominal cutoff voltage.

V. MODEL VALIDATION

All simulations and experiments are conducted on LFP18650P battery cells whose full capacity, nominal voltage, and cutoff voltage are 1250mAh, 3.2V and 2.5V, respectively [16]. Battery cells are first charged to its full capacity through the method of Constant Current Constant Voltage (CCCV) [17]. Then, the multi-cell battery is discharged at desired configurations at the discharge current rate of 1A and 5A, respectively. The simulation results are derived by using the proposed mathematical model through MATLAB, while the experimental results are collected through the ARBIN battery testing instrument BT2000 [18].

In this work, without losing generality, a four-cell string in series connection is discharged at both discharge rates of 1A and 5A. As shown in Figure 5, the cutoff voltage of the cell string in series connection is 11.9V, which is much higher than its theoretical cutoff voltage 10V. We also studied the battery performance at a high discharge current rate of 5A as shown in Figure 6. High discharge current rate results in the incensement of battery temperature, and thus the battery output voltage will be increased gradually. There is a discrepancy between experimental and simulation results due to the fact that the adopted single-cell battery model in [9] has slightly different with experiment data due to the inadequate data for data fitting of single-cell battery model parameters. The close agreement between simulation and experiment results as shown in Figure
5 and 6 indicates the accuracy of the proposed model of cell-string in series connection.

Cell current distribution of multi-cell battery with parallel connection as shown in Figure 7. Without losing generality, the experiment and simulation are conducted on two-cell string in parallel connection, where each cell has different capacity. Even though the cell string in parallel connection is discharged at a constant discharge current rate 2.5A (2C), the current flow through each battery cell will not be proportionally divided by the capacity ratio due to the time varying nature of internal resistance raised by electrolyte and electrode, solid-electrolyte interface, charge transfer, and mass. Due to the cell status variations, each cell has different discharge current rate. When one of the battery cells reaches the cutoff voltage, the cell string in series connection will reach its full discharged states. Therefore, the actual available capacity of multi-cell battery is much less than its theoretical capacity.

The discharge voltage curve of multi-cell battery is shown in Figure 8 at discharge current rate of 1A. Failure of the single-cell battery results in a high cutoff voltage of the multi-cell battery. The close agreement between simulation and experimental results for multi-cell battery indicates the correctness of the proposed circuit-based battery model.

VI. CONCLUSION

A circuit-based multi-cell battery model has been proposed to characterize the multi-cell battery performance in terms of available capacity, output voltage, discharging current distribution, and internal resistance. The close agreement between simulations and experiments shows that the proposed model can accurately capture and derive the performance of multi-cell battery. The proposed model also offers the capability to design engineers to accurately derive the battery operating time and optimize the performance of battery power management. The proposed model is battery chemistry independent, so it can be used in design and optimization of various electrochemical battery energy storage systems.
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