Can QFT on Moyal-Weyl spaces look as on commutative ones?

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Abstract

We sketch a natural affirmative answer to the question based on a joint work [11] with J. Wess. There we argue that a proper enforcement of the “twisted Poincaré” covariance makes any differences \((x-y)\) of coordinates of two copies of the Moyal-Weyl deformation of Minkowski space like undeformed. Then QFT in an operator approach becomes compatible with (minimally adapted) Wightman axioms and time-ordered perturbation theory, and physically equivalent to ordinary QFT, as observables involve only coordinate differences.

1 Introduction: twisting Poincaré group and Minkowski spacetime

In the last decade a broad attention has been devoted to the construction of QFT on Moyal-Weyl spaces, perhaps the simplest examples of noncommutative spaces. These are characterized by coordinates \(\hat{x}^\mu\) fulfilling the commutation relations

\[
[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \tag{1}
\]

*Talk given at the 21st Nishinomiya-Yukawa Memorial Symposium on Theoretical Physics “Noncommutative Geometry and Spacetime in Physics”, Nishinomiya-Kyoto, Nov. 2006. Preprint 07-16 Dip. Matematica e Applicazioni, Università di Napoli; DSF/12-2007.
where $\theta^{\mu\nu}$ is a constant real antisymmetric matrix. For present purposes $\mu = 0, 1, 2, 3$ and indices are raised or lowered through multiplication by the standard Minkowski metric $\eta_{\mu\nu}$, so as to obtain a deformation of Minkowski space. We shall denote by $\hat{A}$ the algebra “of functions on Moyal-Weyl space”, i.e. the algebra generated by $1, \hat{x}^\mu$ fulfilling \( [1] \). For $\theta^{\mu\nu} = 0$ one obtains the algebra $\mathcal{A}$ generated by commuting $x^\mu$.

Clearly \( [1] \) are translation invariant, but not Lorentz-covariant. As recognized in \([5, 18, 13, 14]\), they are however covariant under a deformed version of the Poincaré group, namely a triangular noncocommutative Hopf $*$-algebra $H$ obtained from the UEA $U\mathcal{P}$ of the Poincaré Lie algebra $\mathcal{P}$ by twisting \([9]\). This means that (up to isomorphisms) $H$ and $U\mathcal{P}$ (extended over the formal power series in $\theta^{\mu\nu}$) are the same $*$-algebras, have the same counit $\varepsilon$, but different coproducts $\Delta, \hat{\Delta}$ related by

$$
\Delta(g) \equiv \sum_I g_{(1)}^I \otimes g_{(2)}^I \quad \longrightarrow \quad \hat{\Delta}(g) = \mathcal{F}\Delta(g)\mathcal{F}^{-1} \equiv \sum_I g_{(1)}^I \otimes g_{(2)}^I
$$

for any $g \in H \equiv U\mathcal{P}$. The antipodes are also changed accordingly. The so-called twist $\mathcal{F}$ is not uniquely determined, but what follows does not depend on its choice. The simplest is

$$
\mathcal{F} \equiv \sum_I \mathcal{F}_I^{(1)} \otimes \mathcal{F}_I^{(2)} := \exp \left( i \theta^{\mu\nu} P_\mu \otimes P_\nu \right).
$$

$P_\mu$ denote the generators of translations, and in \( [2], [3]\), we have used Sweedler notation: $\sum_I$ may denote an infinite sum (series), e.g. $\sum_I \mathcal{F}_I^{(1)} \otimes \mathcal{F}_I^{(2)}$ comes out from the power expansion of the exponential. A straightforward computation gives

$$
\hat{\Delta}(P_\mu) = P_\mu \otimes 1 + 1 \otimes P_\mu = \Delta(P_\mu), \quad \hat{\Delta}(M_\omega) = M_\omega \otimes 1 + 1 \otimes M_\omega + P[\omega, \theta] \otimes P \neq \Delta(M_\omega),
$$

where we have set $M_\omega := \omega^{\mu\nu} M_{\mu\nu}$ and used a row-by-column matrix product on the right. The left identity shows that the Hopf $P$-subalgebra remains undeformed and equivalent to the abelian translation group $\mathbb{R}^4$. Therefore, denoting by $\triangleright, \hat{\triangleright}$ the actions of $U\mathcal{P}, H$ (on $\mathcal{A}$ $\triangleright$ amounts to the action of the corresponding algebra of differential operators, e.g. $P_\mu$ can be identified with $i \partial_\mu := i \partial/\partial x^\mu$), they coincide on first degree polynomials in $x^\mu, \hat{x}^\mu$,

$$
P_\mu \triangleright x^\rho = i \delta^\rho_\mu = P_\mu \hat{\triangleright} \hat{x}^\rho, \quad M_\omega \triangleright x^\rho = 2i (x\omega)^\rho, \quad M_\omega \hat{\triangleright} \hat{x}^\rho = 2i (\hat{x}\omega)^\rho, \quad (4)
$$

and more generally on irreps (irreducible representations); this yields the same classification of elementary particles as unitary irreps of $\mathcal{P}$. But $\triangleright, \hat{\triangleright}$ differ on products of coordinates, and more generally on tensor products of representations, as $\triangleright$ is extended by the rule $g \triangleright (ab) = (g_{(1)} \triangleright a)(g_{(2)} \triangleright b)$ involving $\Delta(g)$ (the rule reduces to the usual Leibniz rule for $g = P_\mu, M_{\mu\nu}$), whereas $\hat{\triangleright}$ is extended as at the lhs of

$$
g \hat{\triangleright} (\hat{a} \hat{b}) = \sum_I (g_{(1)}^I \hat{\triangleright} \hat{a})(g_{(2)}^I \hat{\triangleright} \hat{b}) \iff g \hat{\triangleright}_* (a \ast b) = \sum_I (g_{(1)}^I \hat{\triangleright}_* a)(g_{(2)}^I \hat{\triangleright}_* b), \quad (5)
$$

1In section 4.4.1 of \([14]\) this was formulated in terms of the dual Hopf algebra
involving \( \hat{A}(g) \) and a deformed Leibniz rule for \( M_\alpha \hat{b} \). Summarizing, the \( H \)-module unital \(*\)-algebra \( \hat{A} \) is obtained by twisting the \( UP\)-module unital \(*\)-algebra \( A \).

**Several spacetime variables.** The proper noncommutative generalization of the algebra of functions generated by \( n \) sets of Minkowski coordinates \( x^\mu_i, i = 1, 2, \ldots, n, \) is the noncommutative unital \(*\)-algebra \( \hat{A}^n \) generated by real variables \( \hat{x}^\mu_i \) fulfilling the commutation relations at the lhs of

\[
[i^\mu_\mu, j^\nu_j] = 1i^{\theta\mu\nu} \quad \Leftrightarrow \quad [x^\mu_i, x^\nu_j] = 1i^{\theta\mu\nu}; \quad (6)
\]

note that the commutators are not zero for \( i \neq j \). The latter are compatible with the Leibniz rule \([5]\), so as to make \( \hat{A} \) a \( H \)-module \(*\)-algebra, and dictated by the braiding associated to the quasitriangular structure \( \mathcal{R} = \mathcal{F}_{21}\mathcal{F}^{-1} \) of \( H \).

As \( H \) is even triangular, an essentially equivalent formulation of these \( H \)-module algebras is in terms of \(*\)-products derived from \( F \). For \( n \geq 1 \) denote by \( A^n \) the \( n \)-fold tensor product algebra of \( A \) and \( x^\nu_1 \otimes 1 \otimes \ldots \otimes 1 \otimes x^\mu_2 \otimes \ldots \) respectively by \( x^\mu_1, x^\mu_2, \ldots \). Denote by \( A^n_0 \) the algebra obtained by endowing the vector space underlying \( A^n \) with a new product, the \(*\)-product, related to the product in \( A^n \) by

\[
a \ast b := \sum_i (\mathcal{F}_i^0 \triangleright a)(\mathcal{F}_i^1 \triangleright b), \quad (7)
\]

with \( \mathcal{F} = \mathcal{F}^{-1} \). This encodes both the usual \(*\)-product within each copy of \( A \), and the \("*\-tensor product" \) algebra \([2][3]\). As a result one finds the isomorphic \(*\)-commutation relations at the rhs of \([3]\) (this follows from computing \( x^\mu_1 \ast x^\nu_2 \), which e.g. for the specific choice \([3]\) gives \( x^\mu_1 x^\nu_2 + i^{\theta\mu\nu}/2 \) and that \( \hat{A}^n, A^n_0 \) are isomorphic \( H \)-module unital \(*\)-algebras, in the sense of the equivalence \([5]\). More explicitly, on analytic functions \( f, g \) \([7]\) reads \( f(x_i) \ast g(x_j) = \exp[\frac{i2}{\theta} \partial x_i \theta \partial x_j] f(x_i)g(x_j) \), and must be followed by the indentification \( x_i = x_j \) after the action of the bi-pseudodifferential operator \( \exp[\frac{i2}{\theta} \partial x_i \theta \partial x_j] \) if \( i = j \). It should be extended to functions in \( L^1 \cap \mathbb{F}L^1 \) in the obvious way using their Fourier transforms \( \mathbb{F} \). In the sequel we shall formulate the noncommutative spacetime only in terms of \(*\)-products and construct QFT on it replacing all products of functions and/or fields with \(*\)-products.

Let \( a_i \in \mathbb{R} \) with \( \sum_i a_i = 1 \). An alternative set of real generators of \( A^n_0 \) is:

\[
\xi^\mu_i := x^\mu_{i+1} - x^\mu_i, \quad i = 1, \ldots, n-1, \quad X^\mu := \sum_{i=1}^n a_i \xi^\mu_i \quad (8)
\]

It is immediate to check that \( [X^\mu, X^\nu] = 1i^{\theta\mu\nu} \), so \( X^\mu \) generate a copy \( A_{\theta, X} \) of \( A_\theta \), whereas \( \forall b \in A^n_0 \)

\[
\xi^\mu_i \ast b = \xi^\mu_i b = b \ast \xi^\mu_i \quad \Rightarrow \quad [\xi^\mu_i, b] = 0, \quad (9)
\]

so \( \xi^\mu_i \) generate a \(*\)-central subalgebra \( A_{\xi}^{n-1} \), and \( A^n_0 \sim A^{n-1}_\xi \otimes A_{\theta, X} \). The \(*\)-multiplication operators \( \xi^\mu \ast \) have the same spectral decomposition on all \( \mathbb{R} \) (including 0) as multiplication opertaors \( \xi^\mu \). by classical coordinates, which make up a space-like, or a null, or
a time-like 4-vector, in the usual sense. Moreover, \( A^{n-1}_{\xi}, A_{\theta,X} \) are actually \( H \)-module subalgebras, with

\[
\begin{align*}
g \hat{\circ} a &= g \triangleright a \quad a \in A^{n-1}_{\xi}, \quad g \in H \\
g \hat{\circ}(a \star b) &= (g(1) \triangleright a) \star (g(2) \hat{\circ} b), \quad b \in A^n_{\theta},
\end{align*}
\]

i.e. on \( A^{n-1}_{\xi} \) the \( H \)-action is undeformed, including the related part of the Leibniz rule. [By \( (10) \) \( \star \) can be also dropped]. All \( \xi^\mu_i \) are translation invariant, \( X^\mu \) is not.

2 Revisiting Wightman axioms for QFT and their consequences

As in Ref. [17] we divide the Wightman axioms [16] into a subset (labelled by \( \text{QM} \)) encoding the quantum mechanical interpretation of the theory, its symmetry under space-time translations and stability, and a subset (labelled by \( \text{R} \)) encoding the relativistic properties. Since they provide minimal, basic requirements for the field-operator framework to quantization we try to apply them to the above noncommutative space keeping the QM conditions, “fully” twisting Poincaré-covariance R1 and being ready to weaken locality R2 if necessary.

**QM1.** The states are described by vectors of a (separable) Hilbert space \( \mathcal{H} \).

**QM2.** The group of space-time translations \( \mathbb{R}^4 \) is represented on \( \mathcal{H} \) by strongly continuous unitary operators \( U(a) \). The spectrum of the generators \( P^\mu \) is contained in \( \mathcal{V}^+ = \{ p_\mu : p^2 \geq 0, p_0 \geq 0 \} \). There is a unique Poincaré invariant state \( \Psi_0 \), the vacuum state.

**QM3.** The fields (in the Heisenberg representation) \( \varphi^\alpha(x) \) [\( \alpha \) enumerates field species and/or \( SL(2,\mathbb{C}) \)-tensor components] are operator (on \( \mathcal{H} \)) valued tempered distributions on Minkowski space, with \( \Psi_0 \) a cyclic vector for the fields, i.e. \( \star \)-polynomials of the (smeared) fields applied to \( \Psi_0 \) give a set \( \mathcal{D}_0 \) dense in \( \mathcal{H} \).

We shall keep QM1-3. Taking v.e.v.’s we define the *Wightman functions*

\[
W^{\alpha_1,\ldots,\alpha_n}(x_1,\ldots,x_n) := (\Psi_0, \varphi^{\alpha_1}(x_1) \star \ldots \star \varphi^{\alpha_n}(x_n) \Psi_0),
\]

which are in fact distributions, and (their combinations) the *Green’s functions*

\[
G^{\alpha_1,\ldots,\alpha_n}(x_1,\ldots,x_n) := (\Psi_0, T[\varphi^{\alpha_1}(x_1) \star \ldots \star \varphi^{\alpha_n}(x_n)] \Psi_0)
\]

where also *time-ordering* \( T \) is defined as on commutative space (even if \( \vartheta^{0i} \neq 0 \)),

\[
T[\varphi^{\alpha_1}(x) \star \varphi^{\alpha_2}(y)] = \varphi^{\alpha_1}(x) \star \varphi^{\alpha_2}(y) \star \vartheta(x^0-y^0) + \varphi^{\alpha_2}(y) \star \varphi^{\alpha_1}(x) \star \vartheta(y^0-x^0)
\]
(ϑ denotes the Heavyside function). This is well-defined as \( \vartheta(x^0 - y^0) \) is \( \star \)-central.

QM1-3 (alone) imply exactly the same properties as on commutative space:

**W1.** Wightman and Green’s functions are translation-invariant tempered distributions and therefore may depend only on the \( \xi^\mu \):

\[
W^{\alpha_1, \ldots, \alpha_n}(x_1, \ldots, x_n) = W^{\alpha_1, \ldots, \alpha_n} (\xi_1, \ldots, \xi_n),
\]

\[
G^{\alpha_1, \ldots, \alpha_n}(x_1, \ldots, x_n) = G^{\alpha_1, \ldots, \alpha_n} (\xi_1, \ldots, \xi_n).
\] (13)

**W2.** (Spectral condition) The support of the Fourier transform \( \hat{W} \) of \( W \) is contained in the product of forward cones, i.e.

\[
\hat{W}^{(\alpha)}(q_1, \ldots q_{n-1}) = 0, \quad \text{if } \exists j: q_j \notin \mathbb{V}_+.
\] (14)

**W3.** \( W^{(\alpha)} \) fulfill the Hermiticity and Positivity properties following from those of the scalar product in \( \mathcal{H} \).

**R1.** (Untwisted Lorentz Covariance) \( SL(2, \mathbb{C}) \) is represented on \( \mathcal{H} \) by strongly continuous unitary operators \( U(A) \), and under extended Poincaré transformations \( U(a, A) = U(a) U(A) \)

\[
U(a, A) \varphi^\alpha(x) U(a, A)^{-1} = S^\alpha_\beta(A^{-1}) \varphi^\beta(\Lambda(A)x + a),
\] (15)

with \( S \) a finite dimensional representation of \( SL(2, \mathbb{C}) \).

In ordinary QFT as a consequence of QM2,R1 one finds

**W4.** (Lorentz Covariance of Wightman functions)

\[
W^{\alpha_1, \ldots, \alpha_n}(\Lambda(A)x_1, \ldots, \Lambda(A)x_n) = S^\alpha_{\beta_1}(A) \ldots S^\alpha_{\beta_n}(A) W^{\beta_1, \ldots, \beta_n}(x_1, \ldots, x_n).
\] (16)

In particular, Wightman (and Green) functions of scalar fields are Lorentz invariant.

R1 needs a “twisted” reformulation **R1\( \star \)**, which we defer. Now, however R1\( \star \) will look like, it should imply that \( W^{(\alpha)} \) are \( SL_\theta(2, \mathbb{C}) \) tensors (in particular invariant if all involved fields are scalar). But, as the \( W^{(\alpha)} \) are to be built only in terms of \( \xi^\mu \) and other \( SL(2, \mathbb{C}) \) tensors (like \( \partial_{x^\mu}, \eta_{\mu\nu}, \gamma^\mu \), etc.), which are all annihilated by \( P_{\mu\nu}, \mathcal{F} \) will act as the identity and \( W^{(\alpha)} \) will transform under \( SL(2, \mathbb{C}) \) as for \( \theta = 0 \). Therefore we shall require **W4** also if \( \theta \neq 0 \) as a temporary substitute of R1\( \star \).

The simplest sensible way to formulate the \( \star \)-analog of locality is

**R2.** (Microcausality or locality) The fields either \( \star \)-commute or \( \star \)-anticommute at spacelike separated points

\[
[\varphi^\alpha(x) \star \varphi^\beta(y)]_\mp = 0, \quad \text{for } (x - y)^2 < 0.
\] (17)
This makes sense, as space-like separation is sharply defined, and reduces to the usual locality when $\theta = 0$. Whether there exist reasonable weakenings of $R_{2*}$ is an open question also on commutative space, and the same restrictions will apply.

Arguing as in [16] one proves that $QM1-3$, $W4$, $R_{2*}$ are independent and compatible, as they are fulfilled by free fields (see below): the noncommutativity of a Moyal-Weyl space is compatible with $R_{2*}$. As consequences of $R_{2*}$ one again finds $W5$. (Locality) if $(x_j - x_{j+1})^2 < 0$

$$W(x_1, ... x_j, x_{j+1}, ... x_n) = \pm W(x_1, ... x_{j+1}, x_j, ... x_n). \quad (18)$$

$W6$. (Cluster property) For any spacelike $a$ and for $\lambda \to \infty$

$$W(x_1, ... x_j, x_{j+1} + \lambda a, ..., x_n + \lambda a) \to W(x_1, ..., x_j) W(x_{j+1}, ..., x_n), \quad (19)$$

(convergence in the distribution sense); this is true also with permuted $x_i$’s.

Summarizing: our QFT framework is based on $QM1-3$, $W4$, $R_{2*}$, or alternatively on the constraints $W1-6$ for $W^{(\alpha)}$, exactly as in QFT on Minkowski space. We stress that this applies for all $\theta^{\mu\nu}$, even if $\theta^{0i} \neq 0$, contrary to other approaches.

3 Free and interacting scalar field

As the differential calculus remains undeformed, so remain the equation of motions of free fields. Sticking for simplicity to the case of a scalar field of mass $m$, the solution of the Klein-Gordon equation reads as usual

$$\varphi_0(x) = \int \mu(p) [e^{-ip\cdot x} a_p + a^\dagger_p e^{ip\cdot x}] \quad (20)$$

where $d\mu(p) = \delta(p^2 - m^2)\vartheta(p^0) d^4p = dp^0 \delta(p^0 - \omega_p) d^3p / 2\omega_p$ is the invariant measure ($\omega_p := \sqrt{p^2 + m^2}$). Postulating all the axioms of the preceding section (including $R_{2*}$), one can prove up to a positive factor the free field commutation relation

$$[\varphi_0(x) \; \hat{,} \; \varphi_0(y)] = 2 \int \frac{d\mu(p)}{(2\pi)^3} \sin [p \cdot (x-y)], \quad (21)$$

coinciding with the undeformed one. Applying $\partial_{y^0}$ to (21) and setting $y^0 = x^0$ [this is compatible with (6)] one finds the canonical commutation relation

$$[\varphi_0(x^0, x) \; \hat{,} \; \varphi_0(x^0, y)] = i \delta^3(x - y). \quad (22)$$

As a consequence of (21), also the $n$-point Wightman functions coincide with the undeformed ones, i.e. vanish if $n$ is odd and are sum of products of 2-point functions (factorization) if $n$ is even. This of course agrees with the cluster property $W6$. 

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A \( \varphi_0 \) fulfilling (24) can be obtained from (22) plugging \( a^p, a^\dagger_p \) satisfying
\[
\begin{align*}
 a^\dagger_q a^q &= e^{i \theta q} a^\dagger_q a^q, \\
 a^p a^q &= e^{ip \theta q} a^q a^p, \\
 a^p a^\dagger_q &= e^{-i p \theta q} a^\dagger_q a^p + 2 \omega p \delta^3 (p - q),
\end{align*}
\]
(with \( \theta' = \theta \)), and \([a^p, f(x)] = [a^\dagger_p, f(x)] = 0\). \hfill (23)
(here \( p \theta q := p \theta \mu^{\mu \nu} q_{\nu} \)), as adopted e.g. in [4 12 1]. We briefly consider the consequences of choosing \( \theta' \neq \theta \) \( \theta' = 0 \) gives CCR among the \( a^p, a^\dagger_p \), assumed in most of the literature, explicitly [8] or implicitly, in operator [6 7] or in path-integral approach to quantization. One finds the non-local \( \ast \)-commutation relation
\[
\varphi_0(x) \ast \varphi_0(y) = e^{i \partial_x (\theta - \theta') \partial_y} \varphi_0(x) \ast \varphi_0(y) + i F(x - y),
\]
and the corresponding (free field) Wightman functions violate W4, W6, unless \( \theta' = 0 \). One can obtain (23) also by assuming nontrivial transformation laws \( P_\mu \triangleright a^\dagger_p = p_\mu a^\dagger_p \), \( P_\mu \triangleright a^p = -p_\mu a^p \) and extending the \( \ast \)-product law (7) also to \( a^p, a^\dagger_p \). It amounts to choosing \( \theta' = \theta \) in (22), see [11] for details; the relations define examples of deformed Heisenberg algebras covariant under a (quasi)triangular Hopf algebra \( H \) [15 10].

**Normal ordering** is consistently defined as a map which on any monomial in \( a^p, a^\dagger_q \) reorders all \( a^p \) to the right of all \( a^\dagger_q \) adding a factor \( e^{-i \theta' q} \) for each flip \( a^p \leftrightarrow a^\dagger_q \), e.g.
\[
:a^p a^q: = a^p a^q, \quad :a^\dagger_p a^\dagger_q: = a^\dagger_p a^\dagger_q, \quad :a^\dagger_p a^q: = a^\dagger_p a^q, \quad :a^p a^\dagger_q: = a^p a^\dagger_q e^{-i \theta' q}.
\]
(for \( \theta' = 0 \) one finds the undeformed definition), and is extended to fields requiring \( A^0_\ast \)-bilinearity. As a result, one finds that the v.e.v. of any normal-ordered \( \ast \)-polynomial of fields is zero, that normal-ordered products of fields can be obtained from products by the same subtractions, and the **same Wick theorem** as in the undeformed case. Applying **time-ordered perturbation theory** to an interacting field again one can heuristically derive the Gell-Mann–Low formula
\[
G(x_1, ..., x_n) = \frac{\langle \Psi_0, T \{ \varphi_0(x_1) \ast ... \ast \varphi_0(x_n) \ast \exp \left[ -i \lambda \int dy_0 H_I(y_0) \right] \} \Psi_0 \rangle}{\langle \Psi_0, T \exp \left[ -i \int dy_0 H_I(y_0) \right] \Psi_0 \rangle}.
\]
(24)

Here \( \varphi_0 \) denotes the free “in” field, i.e. the incoming field in the interaction representation, and \( H_I(x^0) \) is the interaction Hamiltonian in the interaction representation. By inspection one finds that the **Green functions (24)** **coincide with the undeformed ones** (at least perturbatively). They can be computed by Feynman diagrams with the undeformed Feynman rules. See [11] for some conclusions on these results, in striking contrast with the ones found in most of the literature.

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