$SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ model for three families

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Abstract. An extension of the Standard Model to the local gauge group $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ as a three-family model is presented. The model does not contain exotic electric charges and we obtain a consistent mass spectrum by introducing an anomaly-free discrete $Z_2$ symmetry. The neutral currents coupled to all neutral vector bosons in the model are studied. By using experimental results from the CERN LEP, SLAC Linear Collider and atomic parity violation we constrain the mixing angle between two of the neutral currents in the model and the mass of the additional neutral gauge bosons to be $-0.0032 \leq \sin \theta \leq 0.0031$ and 0.67 TeV $\leq M_Z^X \leq 6.1$ TeV at 95% C.L., respectively.

PACS. 12.60.Cn Extensions of the electroweak gauge sector – 12.15.Mm Neutral currents – 12.15.Ff Quark and lepton masses and mixings

1 Introduction

The Standard Model (SM), based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, can be extended in several different ways: first, by adding new fermion fields (adding a right-handed neutrino field constitute its simplest extension and has profound consequences, as the implementation of the see-saw mechanism, and the enlarging of the possible number of local abelian symmetries that can be gauged simultaneously); second, by augmenting the scalar sector to more than one Higgs representation, and third by enlarging the local gauge group. In this last direction $SU(4)_L \otimes U(1)_X$ as a flavor group has been considered before in the literature [2,3,4], which, among its best features, provides with an alternative to the problem of the number $N_f$ of families, in the sense that anomaly cancellation is achieved when $N_f = N_c = 3$, $N_c$ being the number of colors of $SU(3)_c$ (also known as QCD). Moreover, this gauge structure has been used recently in order to implement the so-called little Higgs mechanism [4].

The analysis of the local gauge structure $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ (hereafter the 3-4-1 group) presented in Sect. 4 shows that we may write the most general electric charge operator for this group as

$$Q = a T_{3L} + b \frac{2}{3} T_{8L} + c \frac{1}{6} T_{15L} + X I_4,$$

where $a, b$ and $c$ are free parameters, $T_{ijL} = \lambda_{ijL}/2$, with $\lambda_{ijL}$ the Gell-Mann matrices for $SU(4)_L$ normalized as $\text{Tr}(\lambda_{ijL}) = 2 \delta_{ij}$, and $I_4 = Dg(1,1,1,1)$ is the diagonal $4 \times 4$ unit matrix. The $X$ values are fixed by anomaly cancellation of the fermion content in the possible models and an eventual coefficient for $XI_4$ can be absorbed in the $X$ hypercharge definition. The free parameters $a, b$ and $c$ fix the gauge boson structure of the electroweak sector $[SU(4)_L \otimes U(1)_X]$, and also the electroweak charges of the scalar representations which are fully determined by the symmetry breaking pattern implemented. In particular $a = 1$ gives the usual isospin of the known electroweak interactions, with $b$ and $c$ remaining as free parameters, producing an infinite plethora of possible models.

Restricting the particle content of the model to particles without exotic electric charges and by paying due attention to anomaly cancellation, a few different models are generated [3]. In particular, the restriction to ordinary electric charges, in the fermion, scalar and gauge boson sectors, allows only for two different cases for the simultaneous values of the parameters $b$ and $c$, namely: $b = c = 1$ and $b = 1, c = -2$, which become a convenient classification scheme for these type of models. Models in the first class differ from those in the second one not only in their fermion content but also in their gauge and scalar boson sectors. Four of the identified models without exotic electric charges are three-family models in the sense that anomalies cancel among the three families of quarks and leptons in a nontrivial fashion. Two of them are models for which $b = c = 1$, and one of them has been analyzed in Ref. [3]. The other two models belong to the class for which $b = 1, c = -2$ and one of them, the so-called “Model E” in the appendix of Ref. [3], will be studied in this paper. It is worth noticing that in the four different models at least one of the three families is treated differently.

This paper is organized as follows. In the next section we describe the fermion content of the particular model we are going to study. In Sect. 3 we introduce the scalar sector. In Sect. 4 we study the gauge boson sector pay-
ing special attention to the neutral currents present in the model and their mixing. In Sect. we analyze the fermion mass spectrum. In Sect. we use experimental results in order to constrain the mixing angle between two of the neutral currents in the model and the mass scale of the new neutral gauge bosons. In the last section we summarize the model and state our conclusions.

2 The Fermion Content of the Model

In what follows we assume that the electroweak gauge group is $SU(4)_L \otimes U(1)_X$ which contains $SU(2)_L \otimes U(1)_Y$ as a subgroup. We will consider the case of a non-universal cellation is concerned.

Here we are interested in studying the phenomenology of three-family models without exotic electric charges and with values $b = 1, c = -2$ for the parameters in the electric charge generator in Eq. (3). As an example we take Model E of Ref. for which the electric charge operator is given by $Q = T_{3L} + T_{3L}/\sqrt{3} - 2T_{15L}/\sqrt{6} + X I_L$. The model has the following anomaly free fermion structure:

$$Q_{1L} = \begin{pmatrix} d_1 & u_1 & d_1^c & u_1^c \end{pmatrix}_L$$
$$\begin{array}{c|c|c|c}
3, 4^* & 3^* & 3^* & 3^* \\
1, 1 & 1 & 1 & 1
\end{array}$$

$$Q_{2L} = \begin{pmatrix} d_2 & u_2 & d_2^c & u_2^c \end{pmatrix}_L$$
$$\begin{array}{c|c|c|c}
3, 4^* & 3^* & 3^* & 3^* \\
1, 1 & 1 & 1 & 1
\end{array}$$

$$L_{\alpha L} = \begin{pmatrix} e_\alpha & \nu_\alpha & \nu_\alpha & e_\alpha \end{pmatrix}_L$$
$$\begin{array}{c|c|c|c}
1, 4^* & 1, 1 & 1, 1 & 1, 1
\end{array}$$

where $j = 2, 3$ and $\alpha = 1, 2, 3$ are two and three family indexes, respectively. The numbers in parenthesis refer to the $[SU(3)_C, SU(4)_L, U(1)_X]$ quantum numbers, respectively. Notice that if needed, the lepton structure of the model can be augmented with an undetermined number of neutral Weyl singlet states $N^0_{L,n} \sim [1, 1, 0], n = 1, 2, \ldots$, without violating our assumptions, neither the anomaly constraint relations, because singlets with no X-charges are as good as not being present as far as anomaly cancellation is concerned.

3 The Scalar Sector

Our aim is to break the symmetry following the pattern

$$SU(3)_C \otimes SU(4)_L \otimes U(1)_X$$

$$\begin{array}{c}
\rightarrow SU(3)_C \otimes SU(3)_L \otimes U(1)_X \\
\rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\
\rightarrow SU(3)_C \otimes U(1)_Q
\end{array}$$

where $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ refers to the so-called 3-3-1 structure introduced in Ref. [5]. At the same time we want to give masses to the fermion fields in the model. With this in mind we introduce the following four Higgs scalars: $\phi_1[1, 4^*, -1/2]$ with a Vacuum Expectation Value (VEV) aligned in the direction $\langle \phi_1 \rangle = (0, v', 0, 0)^T$, $\phi_2[1, 4^*, -1/2]$ with a VEV aligned as $\langle \phi_2 \rangle = (0, 0, V, 0)^T$, $\phi_3[1, 4, -1/2]$ with a VEV aligned in the direction $\langle \phi_3 \rangle = (v', 0, 0, 0)^T$, and $\phi_4[1, 4, -1/2]$ with a VEV aligned with $\langle \phi_4 \rangle = (0, 0, V')^T$, with the hierarchy $V \sim V' \gg \sqrt{v^2 + v'^2} \approx 174$ GeV (the electroweak breaking scale).

4 The Gauge Boson Sector

In the model there are a total of 24 gauge bosons: One gauge field $B^\mu$ associated with $U(1)_X$, the 8 gluon fields associated with $SU(3)_C$ which remain massless after breaking the symmetry, and another 15 gauge fields associated with $SU(4)_L$ which, for $b = 1$ and $c = -2$, can be written as

$$\frac{1}{2} \lambda_{\alpha \beta} A_{\alpha \beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} D^\mu_1 W^{+\mu} K^{+\mu} X^{0\mu} \\ W^{-\mu} D^\mu_2 K^{0\mu} X^{-\mu} \\ K^{-\mu} K^{0\mu} X^{+\mu} \\ X^{0\mu} X^{+\mu} Y^{+\mu} \\ D^\mu_4 \end{pmatrix} ,$$

where $D^\mu_1 = A^\mu_1/\sqrt{2} + A^\mu_2/\sqrt{6} + A_3^0/\sqrt{12}$, $D^\mu_2 = -A^\mu_3/\sqrt{2} + A_3^0/\sqrt{6} + A_3^0/\sqrt{12}$, $D^\mu_3 = -A^\mu_3/\sqrt{2} + A_3^0/\sqrt{12}$, and $D^\mu_4 = -3A_3^0/\sqrt{12}$.

After breaking the symmetry with $\langle \phi_1 \rangle + \langle \phi_2 \rangle + \langle \phi_3 \rangle$ and using for the covariant derivative for 4-pelts $iD^\mu = iD^\mu - g\lambda_{\alpha \beta} A^\mu_\alpha / 2 - g' X B^\mu$, where $g$ and $g'$ are the $SU(4)_L$ and $U(1)_X$ gauge coupling constants respectively, we get the following mass terms for the charged gauge bosons:

$$M^2_{\pm} = g^2 (v^2 + v'^2)/2, M^2_{\pm} = g^2 (v^2 + V^2)/2, M^2_{\pm} = g^2 (v^2 + V'^2)/2, M^2_{\pm} = g^2 (v^2 + V + V'^2)/2, M^2_{\pm} = g^2 (v^2 + V + V'^2)/2. Since w \pm does not mix with the other charged bosons we have that $v^2 + v'^2 \approx 174$ GeV as mentioned in the previous section.

For the four neutral gauge bosons we get mass terms of the form

$$M = \frac{g^2}{2} \left\{ V^2 \left( g' B^\mu \frac{2 A_3^0}{\sqrt{3}} + A_3^0 \frac{A_3^0}{\sqrt{6}} \right)^2 + V'^2 \left( g' B^\mu \frac{3 A_3^0}{\sqrt{6}} \right)^2 + v^2 \left( A_3^0 \frac{A_3^0}{\sqrt{3}} + A_3^0 \frac{A_3^0}{\sqrt{6}} - g' B^\mu \frac{A_3^0}{\sqrt{6}} \right)^2 \right\}$$
gauge bosons $Z$, $Z'$, and $Z''$. Since we are interested in the low energy phenomenology of our model, we can choose $V = V'$ in order to simplify matters. Also, the mixing between the three neutral gauge bosons can be further simplified by choosing $v' = v$. For this particular case the field $Z'' = 2A'_C\sqrt{2} + A''_C/\sqrt{3}$ decouples from the other two and acquires a squared mass $(g^2/2)(V^2 + v^2)$. By diagonalizing the remaining $2 \times 2$ mass matrix we get other two physical neutral gauge bosons which are defined through the mixing angle $\theta$ between $Z_{\mu}$, $Z'_{\mu}$, $Z''_{\mu}$,

$$
Z_{\mu} = Z_{\mu} \cos \theta + Z'_{\mu} \sin \theta ,
Z''_{\mu} = -Z_{\mu} \sin \theta + Z'_{\mu} \cos \theta ,
$$

where

$$
\tan(2\theta) = \frac{S_W^2 C_{2W}}{(1 + S_W^2)^2 + \frac{4}{3} C_W^4 S_W^2 - 2} .
$$

The photon field $A^{\mu}$ and the fields $Z_{\mu}$ and $Z'_{\mu}$ are given by

$$
A^{\mu} = S_W A'_C + C_W \left[ \frac{T_W}{\sqrt{3}} \left( A^{\mu}_C - 2\frac{A''_C}{\sqrt{2}} \right) + (1 - T_W^2)^{1/2} B^{\mu} \right] ,
$$

$$
Z^{\mu} = C_W A'_C + S_W \left[ \frac{T_W}{\sqrt{3}} \left( A^{\mu}_C - 2\frac{A''_C}{\sqrt{2}} \right) + (1 - T_W^2)^{1/2} B^{\mu} \right] ,
$$

$$
Z''^{\mu} = \frac{1}{\sqrt{3}} (1 - T_W^2)^{1/2} \left( A^{\mu}_C - 2\frac{A''_C}{\sqrt{2}} \right) - T_W B^{\mu} .
$$

We can also identify the $Y$ hypercharge associated with the SM abelian gauge boson as

$$
Y^{\mu} = \frac{T_W}{\sqrt{3}} \left( A^{\mu}_C - 2\frac{A''_C}{\sqrt{2}} \right) + (1 - T_W^2)^{1/2} B^{\mu} .
$$

4.2 Neutral currents

The neutral currents $J_{\mu}(EM)$, $J_{\mu}(Z)$, $J_{\mu}(Z')$, and $J_{\mu}(Z'')$ associated with the Hamiltonian

$$
H^0 = e A^{\mu} J_{\mu}(EM) + (g/C_W) Z^{\mu} J_{\mu}(Z) + (g'/2\sqrt{2}) Z''^{\mu} J_{\mu}(Z'') ,
$$

are:

$$
J_{\mu}(EM) = \frac{2}{3} \sum_{j=2}^{3} \left( \bar{u}_{a} \gamma_{\mu} u + \bar{t}_{a} \gamma_{\mu} t + \bar{\nu}_{a} \gamma_{\mu} \nu \right) + \bar{u}_{1} \gamma_{\mu} u_{1} + \bar{t}_{1} \gamma_{\mu} t_{1} + \bar{\nu}_{1} \gamma_{\mu} \nu_{1} + \bar{u}_{1} \gamma_{\mu} u_{1} + \bar{t}_{1} \gamma_{\mu} t_{1} + \bar{\nu}_{1} \gamma_{\mu} \nu_{1} ,
$$

$$
J_{\mu}(Z) = J_{\mu,L}(Z) = S_W^2 J_{\mu}(EM) ,
$$

4.1 Charged currents

The Hamiltonian for the charged currents in the model is given by

$$
H^{CC} = \frac{g}{\sqrt{2}} \left\{ W^+_\mu \left[ \sum_{j=2}^{3} \bar{u}_{a} L_{\gamma^{\mu} d_{a} L} - \bar{u}_{1} L_{\gamma^{\mu} d_{1} L} \right] - \frac{3}{2} \sum_{\alpha=1}^{3} \bar{\nu}_{a} \gamma_{\mu} \nu_{a} \right\} .
$$
\[ J_\mu(Z') = J_{\mu,L}(Z') - T_W J_\mu(EM), \]
\[ J_\mu(Z'') = \sum_{\alpha=2}^{3} (\bar{u}_{\alpha L} \gamma_\mu u_{\alpha L} + \bar{d}_{\alpha L} \gamma_\mu d_{\alpha L} - \bar{D}_{\alpha L} \gamma_\mu D_{\alpha L}) 
- \bar{U}_1 \gamma_\mu U_1 - \bar{U}_2 \gamma_\mu U_2 
+ \sum_{\alpha=1}^{3} (-\bar{e}_{\alpha L} \gamma_\mu e_{\alpha L} - \bar{\nu}_{e\alpha L} \gamma_\mu \nu_{e\alpha L}) 
+ N_{\alpha L}^0 \gamma_\mu N_{\alpha L}^0 + \bar{E}_{\alpha L} \gamma_\mu E_{\alpha L}), \quad (5) \]

where \( e = g S_W = g' C_W \sqrt{1 - T_W^2} > 0 \) is the electric charge, \( q_f \) is the electric charge of the fermion \( f \) in units of \( e \), and \( J_\mu(EM) \) is the electromagnetic current. Note from \( J_\mu(Z'') \) that, notwithstanding the extra neutral gauge boson \( Z''_\mu \) does not mix with \( Z_\mu \) or \( Z''_\mu \) (for the particular case \( V = V' \) and \( v = v' \)), it still couples to ordinary fermions. The left-handed currents are

\[ J_{\mu,L}(Z) = \frac{1}{2} \sum_{j=2}^{3} (\bar{u}_{\alpha L} \gamma_\mu u_{\alpha L} - \bar{d}_{\alpha L} \gamma_\mu d_{\alpha L}) 
- \bar{D}_1 \gamma_\mu D_1 - \bar{U}_1 \gamma_\mu U_1 
+ \sum_{\alpha=1}^{3} (-\bar{e}_{\alpha L} \gamma_\mu e_{\alpha L} - \bar{\nu}_{e\alpha L} \gamma_\mu \nu_{e\alpha L}) 
+ N_{\alpha L}^0 \gamma_\mu N_{\alpha L}^0 + \bar{E}_{\alpha L} \gamma_\mu E_{\alpha L}), \quad (6) \]

The values of \( g_{i V}, g_{i A} \) with \( i = 1, 2 \) are listed in Tables \[ ] and \[ ]

As we can see, in the limit \( \theta = 0 \) the couplings of \( Z_1^\mu \) to the ordinary leptons and quarks are the same as in the SM; due to this we can test the new physics beyond the SM predicted by this particular model.

### 5 Fermion Masses

The Higgs scalars introduced in Sect. \[ ] break the symmetry in an appropriate way. Now, in order to generate both a simple mass splitting between ordinary and exotic fermions and a consistent mass spectrum, we introduce an anomaly-free discrete \( Z_2 \) symmetry \[ ], with the following assignments of \( Z_2 \) charge

\[ q(Q_{\alpha L}, u_{\alpha L}^c, d_{\alpha L}^c, L_{\alpha L}, \epsilon_{\alpha L}^c, \phi_1, \phi_3) = 0, \]
\[ q(U_{\alpha L}^c, D_{\alpha L}^c, E_{\alpha L}^c, \phi_2, \phi_4) = 1. \quad (9) \]

Notice that ordinary fermions are not affected by this discrete symmetry.

The gauge invariance and the \( Z_2 \) symmetry allow for the following Yukawa lagrangians:
Table 1. The $Z'_1 \rightarrow \bar{f}f$ couplings.

| $f$ | $g(f)_{1W}$ | $g(f)_{1A}$ |
|-----|-------------|-------------|
| $u_{1,2,3}$ | $\cos \theta (\frac{1}{2} - \frac{4 S_W^2}{3}) - \frac{5 \sin \theta}{6 C_W} S_W^2$ | $\frac{1}{2} \cos \theta + \frac{\sin \theta}{2 C_W} S_W^2$ |
| $d_{1,2,3}$ | $-\frac{1}{2} - \frac{2 S_W^2}{3} \cos \theta + \frac{5 \sin \theta}{6 C_W} S_W^2$ | $-\frac{1}{2} \cos \theta + \frac{\sin \theta}{2 C_W} S_W^2$ |
| $D_{1,2,3}$ | $\frac{2 S_W^2}{3} \cos \theta + \frac{\sin \theta}{2 C_W} S_W^2$ | $-\frac{1}{2} \cos \theta + \frac{\sin \theta}{2 C_W} S_W^2$ |
| $U_{1,2,3}$ | $-\frac{4 S_W^2}{3} \cos \theta - \frac{5 \sin \theta}{6 C_W} S_W^2$ | $-\frac{1}{2} \cos \theta + \frac{\sin \theta}{2 C_W} S_W^2$ |
| $e_{1,2,3}$ | $\cos \theta (\frac{1}{2} - \frac{2 S_W^2}{3}) + \frac{5 \sin \theta}{6 C_W} S_W^2$ | $\frac{1}{2} \cos \theta + \frac{\sin \theta}{2 C_W} S_W^2$ |
| $\nu_{1,2,3}$ | $\frac{1}{2} \cos \theta + \frac{\sin \theta}{2 C_W} S_W^2$ | $\frac{1}{2} \cos \theta + \frac{\sin \theta}{2 C_W} S_W^2$ |
| $N_{1,2,3}$ | $\frac{2 S_W^2}{3} \cos \theta + \frac{\sin \theta}{2 C_W} S_W^2$ | $\frac{1}{2} \cos \theta + \frac{\sin \theta}{2 C_W} S_W^2$ |
| $E_{1,2,3}$ | $2 S_W^2 \cos \theta + \frac{\sin \theta}{2 C_W} S_W^2$ | $\frac{1}{2} \cos \theta + \frac{\sin \theta}{2 C_W} S_W^2$ |

Table 2. The $Z'_2 \rightarrow \bar{f}f$ couplings.

| $f$ | $g(f)_{2W}$ | $g(f)_{2A}$ |
|-----|-------------|-------------|
| $u_{1,2,3}$ | $-\sin \theta (\frac{1}{2} - \frac{4 S_W^2}{3}) - \frac{5 \cos \theta}{6 C_W} S_W^2$ | $-\frac{1}{2} \sin \theta + \frac{\cos \theta}{2 C_W} S_W^2$ |
| $d_{1,2,3}$ | $\frac{1}{2} - \frac{2 S_W^2}{3} \sin \theta + \frac{5 \cos \theta}{6 C_W} S_W^2$ | $\frac{1}{2} \sin \theta - \frac{\cos \theta}{2 C_W} S_W^2$ |
| $D_{1,2,3}$ | $-\frac{2 S_W^2}{3} \sin \theta + \frac{5 \cos \theta}{6 C_W} S_W^2$ | $-\frac{1}{2} \sin \theta + \frac{\cos \theta}{2 C_W} S_W^2$ |
| $U_{1,2,3}$ | $\frac{4 S_W^2}{3} \sin \theta - \frac{5 \cos \theta}{6 C_W} S_W^2$ | $\frac{1}{2} \sin \theta - \frac{\cos \theta}{2 C_W} S_W^2$ |
| $e_{1,2,3}$ | $\sin \theta (\frac{1}{2} - \frac{2 S_W^2}{3}) + \frac{5 \cos \theta}{6 C_W} S_W^2$ | $-\frac{1}{2} \sin \theta + \frac{\cos \theta}{2 C_W} S_W^2$ |
| $\nu_{1,2,3}$ | $\frac{1}{2} \sin \theta + \frac{\cos \theta}{2 C_W} S_W^2$ | $\frac{1}{2} \sin \theta + \frac{\cos \theta}{2 C_W} S_W^2$ |
| $N_{1,2,3}$ | $\cos \theta \frac{\sin \theta}{2 C_W} S_W^2$ | $\frac{1}{2} \sin \theta + \frac{\cos \theta}{2 C_W} S_W^2$ |
| $E_{1,2,3}$ | $-2 S_W^2 \sin \theta + \frac{\cos \theta}{2 C_W} S_W^2 (2 - \frac{5}{2} C_W)$ | $-\frac{1}{2} \sin \theta + \frac{\cos \theta}{2 C_W} S_W^2$ |

For quarks:

$$L_Y^Q = \sum_{j=2}^{3} Q^T_{cL} C \left\{ \phi_3^3 \sum_{\alpha=1}^{3} h_{\alpha j}^u u_{\alpha L}^c + \phi_4^3 \sum_{\alpha=1}^{3} h_{\alpha j}^U U_{\alpha L}^c \right\}$$

where

$$M_{uU} = \begin{pmatrix} M_{u(3 \times 3)} & 0 \\ 0 & M_{U(3 \times 3)} \end{pmatrix},$$

$$M_u = \begin{pmatrix} h_{11}^U & h_{21}^U v' & h_{31}^U v' \\ h_{12}^U v' & h_{22}^U & h_{32}^U v' \\ h_{13}^U v' & h_{23}^U v' & h_{33}^U v' \end{pmatrix},$$

$$M_U = \begin{pmatrix} h_{11}^U V & h_{21}^U V' & h_{31}^U V' \\ h_{12}^U V' & h_{22}^U & h_{32}^U V' \\ h_{13}^U V' & h_{23}^U V' & h_{33}^U V' \end{pmatrix},$$

$$M_{dD} = \begin{pmatrix} M_{d(3 \times 3)} & 0 \\ 0 & M_{D(3 \times 3)} \end{pmatrix},$$

$$M_d = \begin{pmatrix} h_{11}^D v' & h_{21}^D v' & h_{31}^D v' \\ h_{12}^D v' & h_{22}^D & h_{32}^D v' \\ h_{13}^D v' & h_{23}^D v' & h_{33}^D v' \end{pmatrix},$$

$$M_D = \begin{pmatrix} h_{11}^D V' & h_{21}^D V' & h_{31}^D V' \\ h_{12}^D V' & h_{22}^D & h_{32}^D V' \\ h_{13}^D V' & h_{23}^D V' & h_{33}^D V' \end{pmatrix}.$$
For the charged leptons the Lagrangian \( L_{L} \), in the basis \((e_{1}, e_{2}, e_{3}, E_{1}, E_{2}, E_{3})\), also produces a block diagonal mass matrix

\[
M_{L} = \begin{pmatrix}
M_{e(3 \times 3)} & 0 \\
0 & M_{E(3 \times 3)} 
\end{pmatrix},
\]

where the entries in the submatrices are given by

\[
M_{e,\alpha \beta} = h_{e,\beta}^{\alpha} v' \quad \text{and} \quad M_{E,\alpha \beta} = h_{E,\beta}^{\alpha} V'.
\]

The former mass matrices exhibit the mass splitting between ordinary and exotic charged fermions and show that all the charged fermions in the model acquire masses at the three level. Clearly, by a judicious tuning of the Yukawa couplings and of the mass scales \( v \) and \( v' \), a consistent mass spectrum in the ordinary charged sector can be obtained. In the exotic charged sector all the particles acquire masses at the scale \( V \sim V' \gg 174 \, \text{GeV} \). Note that in the low energy limit our model corresponds to a Type III two Higgs doublet model \( \texttt{[1]} \) in which both doublets couple to the same type of fermions, with the quark and lepton couplings treated asymmetrically.

The neutral leptons remain massless as far as we use only the original fields introduced in Sect. \( \texttt{[2]} \). But as mentioned earlier, we may introduce one or more Weyl singlet states \( N_{L,b}^{b} \), \( b = 1, 2, \ldots \), which may implement the appropriate neutrino oscillations \( \texttt{[8]} \).

### 6 Constrains on the \((Z^{\mu} - Z'^{\mu})\) Mixing Angle and the \(Z_{SM}^{\mu}\) Mass

To bound \( \sin \theta \) and \( M_{Z_{SM}} \) we use parameters measured at the \( Z \) pole from CERN \( e^{+}e^{-} \) collider (LEP), SLAC Linear Collider (SLC), and atomic parity violation constraints which are given in Table \( \texttt{[4]} \).

The expression for the partial decay width for \( Z_{1}^{\mu} \rightarrow ff \) is

\[
\Gamma(\bar{Z}_{1}^{\mu} \rightarrow \bar{f}f) = \frac{N_{C}G_{F}M_{Z_{1}}^{3}}{6\pi\sqrt{2}}\rho\left\{ \frac{3\beta - \beta^{3}}{2} |g(f)_{1V}|^{2} + \beta^{3}|g(f)_{1A}|^{2}\right\}(1 + \delta_{f})R_{EW}R_{QCD},
\]

where \( f \) is an ordinary SM fermion, \( Z_{1}^{\mu} \) is the physical gauge boson observed at LEP, \( N_{C} = 1 \) for leptons while for quarks \( N_{C} = 3(1 + \alpha_{s}/\pi + 1.405\alpha_{s}^{2}/\pi^{2} - 12.77\alpha_{s}^{3}/\pi^{3}) \), where the 3 is due to color and the factor in parenthesis represents the universal part of the QCD corrections for massless quarks (for fermion mass effects and further QCD corrections which are different for vector and axial-vector partial widths see Ref. \( \texttt{[9]} \)); \( R_{EW} \) are the electroweak corrections which include the leading order QED corrections given by \( R_{QED} = 1 + 3a/(4\pi) \); \( R_{QCD} \) are further QCD corrections (for a comprehensive review see Ref. \( \texttt{[10]} \) and references therein), and \( \beta = \sqrt{1 - 4m_{f}^{2}/M_{Z_{1}}^{2}} \) is a kinematic factor which can be taken equal to 1 for all the SM fermions except for the bottom quark. The factor \( \delta_{f} \) contains the one loop vertex contribution which is negligible for all fermion fields except for the bottom quark for which the contribution coming from the top quark at the one loop vertex radiative correction is parametrized as \( \delta_{b} \approx 10^{-2}[m_{t}^{2}/(2M_{Z_{1}}^{2}) + 1/5] \). The \( \rho \) parameter can be expanded as \( \rho = 1 + \delta_{\rho_{0}} + \delta_{\rho_{V}} \) where the oblique correction \( \delta_{\rho_{0}} \) is given by \( \delta_{\rho_{0}} \approx 3G_{F}m_{t}^{2}/(8\pi^{2}\sqrt{2}) \), and \( \delta_{\rho_{V}} \) is the tree level contribution due to the \((Z_{\mu} - Z'_{\mu})\) mixing which can be parametrized as \( \delta_{\rho_{V}} \approx (M_{Z_{SM}}^{2}/M_{Z_{1}}^{2} - 1)\sin^{2}\theta \).

Finally, \( g(f)_{1V} \) and \( g(f)_{1A} \) are the coupling constants of the physical \( Z_{SM}^{\mu} \) field with ordinary fermions which are listed in Table \( \texttt{[1]} \).

In what follows we are going to use the experimental values \( \texttt{[12]} \): \( M_{Z_{1}} = 91.188 \, \text{GeV}, m_{t} = 174.3 \, \text{GeV}, \alpha_{s}(m_{Z}) = 0.1192, \alpha(m_{Z})^{-1} = 127.938, \) and \( \sin^{2}\theta_{W} = 0.2333 \). The experimental values are introduced using the definitions \( R_{\eta} \equiv \Gamma(\eta \eta) / \Gamma(\text{hadrons}) \) for \( \eta = e, \mu, \tau, b, c \).

As a first result notice from Table \( \texttt{[1]} \) that our model predicts \( R_{c} = R'_{\mu} = R_{\tau}, \) in agreement with the experimental results in Table \( \texttt{[3]} \).

The effective weak charge in atomic parity violation, \( Q_{W} \), can be expressed as a function of the number of protons \((Z)\) and the number of neutrons \((N)\) in the atomic nucleus in the form

\[
Q_{W} = -2[(2Z + N)c_{1u} + (Z + 2N)c_{1d}],
\]

where \( c_{1u} = 2g(e)_{1A}g(q)_{1V} \). The theoretical value for \( Q_{W} \) for the Cesium atom is given by \( \texttt{[14]} \)

\[
Q_{W}(85^{3} \text{Cs}) = -73.09 \pm 0.04 + \Delta Q_{W},
\]

where the contribution of new physics is included in \( \Delta Q_{W} \) which can be written as

\[
\Delta Q_{W} = \left[1 + \frac{4S_{W}^{4}}{1 - 2S_{W}^{2}}\right]Z - N\delta_{\rho_{V}} + \Delta Q_{W}' (12).
\]

The term \( \Delta Q_{W}' \) is model dependent and it can be obtained for our model by using \( g(e)_{1A} \) and \( g(q)_{1V}, i = 1, 2 \), from Tables \( \texttt{[1]} \) and \( \texttt{[2]} \).

The value we obtain is

\[
\Delta Q_{W}' = (3.75Z + 2.56N)\sin \theta + (1.22Z + 0.41N)\frac{M_{Z_{1}}^{2}}{M_{Z_{SM}}^{2}}.
\]

The discrepancy between the SM and the experimental data for \( \Delta Q_{W} \) is given by \( \texttt{[15]} \)

\[
\Delta Q_{W} = Q_{W}^{exp} - Q_{W}^{SM} = 1.03 \pm 0.44,
\]

which is 2.3 \( \sigma \) away from the SM predictions.

Introducing the expressions for \( Z \) pole observables in Eq. \( \texttt{[10]} \), with \( \Delta Q_{W} \) in terms of new physics in Eq. \( \texttt{[12]} \) and using experimental data from LEP, SLC and atomic parity violation (see Table \( \texttt{[5]} \), we do a \( \chi^{2} \) fit and we find the best allowed region in the \((\theta - M_{Z_{1}})\) plane at 95\% confidence level (C.L.). In Fig. \( \texttt{[4]} \) we display this region which gives us the constraints

\[
\begin{align*}
-0.0032 \leq \theta \leq 0.0031, \quad 0.67 \, \text{TeV} \leq M_{Z_{1}} \leq 6.1 \, \text{TeV}.
\end{align*}
\]
Table 3. Experimental data and SM values for the parameters.

| Parameter | Experimental results | SM |
|-----------|----------------------|----|
| $\Gamma_Z$(GeV) | $2.4952 \pm 0.0023$ | $2.4966 \pm 0.0016$ |
| $\Gamma$(had) (GeV) | $1.7444 \pm 0.0020$ | $1.7429 \pm 0.0015$ |
| $\Gamma(l^+l^-)$ (MeV) | $83.984 \pm 0.086$ | $84.019 \pm 0.027$ |
| $R_e$ | $20.804 \pm 0.150$ | $20.744 \pm 0.018$ |
| $R_\mu$ | $20.785 \pm 0.033$ | $20.944 \pm 0.018$ |
| $R_\tau$ | $20.764 \pm 0.045$ | $20.790 \pm 0.018$ |
| $R_b$ | $0.21664 \pm 0.00068$ | $0.21569 \pm 0.00016$ |
| $R_c$ | $0.1729 \pm 0.0152$ | $0.17230 \pm 0.00007$ |
| $Q^2_W$ | $-72.65 \pm 0.28 \pm 0.34$ | $-73.10 \pm 0.03$ |
| $M_{Z_1}$(GeV) | $9.1872 \pm 0.0021$ | $9.1870 \pm 0.0021$ |

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Fig. 1. Contour plot displaying the allowed region for $\theta$ vs. $M_{Z^2}(\text{TeV})$ at 95% C.L.