Seismic Search for Strange Quark Nuggets

Eugene T. Herrin  
Geology Department, Southern Methodist University, Dallas, TX 75275

Doris C. Rosenbaum  
Physics Department, Southern Methodist University, Dallas, TX 75275

Vigdor L. Teplitz  
NASA Goddard Space Flight Center, Greenbelt, MD 20771

(Dated: Revised manuscript December, 2005)

Bounds on masses and abundances of Strange Quark Nuggets (SQNs) are inferred from a seismic search on Earth. Potential SQN bounds from a possible seismic search on the Moon are reviewed and compared with Earth capabilities. Bounds are derived from the data taken by seismometers implanted on the Moon by the Apollo astronauts. We show that the Apollo data implies that the abundance of SQNs in the region of 10 kg to one ton must be at least an order of magnitude less than would saturate the dark matter in the solar neighborhood.

PACS numbers: 93.85.+q, 95.35.+d, 96.20.Dt, 97.60.Jd

I. INTRODUCTION

It has now been more than two decades since 1984 when Witten raised the question of the existence of Strange Quark Matter (SQM) as the possible ground state of baryonic matter. In the interim, searches have been made in accelerators, stars and other exotic and non-exotic locales. They have been unsuccessful in discovering evidence for SQM as well as in the harder task of demonstrating its non-existence. This paper addresses limits on SQM that have been, and that might be, established by seismology – on Earth and on the Moon.

SQM might be bound at zero pressure. Nuclear matter made of up, down and strange quarks would have the same potential energy from the color force as nuclear matter made from just up and down, but would have three Fermi seas instead of two. With just up and down, nuclear matter is not bound, but rather condenses into protons and neutrons which, in turn, form finite sized nuclei at zero external pressure. With up, down and strange, on the other hand, it might well be that, at zero pressure, there is binding of large assemblies of quark matter. SQM binding is also aided by the fact that it tends to be electrically neutral except for effects from the color force. The strange quark mass is larger than those of the up and down. The nuclear physics of SQM was worked out in 1984 by Farhi and Jaffe and elaborated by others since; see, for example, for a recent review and references. The argument for SQM binding has recently become stronger from the realization that the color force should be expected to form “color-flavor locked” Cooper pairs thereby increasing the binding over that expected from the earlier work; see Alford for review and references.

De Rujula and Glashow outlined, in 1984, a variety of places in which one might search for nuggets of SQM (SQNs), including accelerators, mica, and among cosmic rays. For these, results have so far been all negative. The cosmic ray search would be significantly augmented were the AMS spectrometer deployed in orbit. Reference also pointed out that SQN passage through the Earth, or other body, would cause a seismic signal which, for large enough SQN mass, would be detectable. This paper addresses that phenomenology.

The plan of the paper is as follows: in the remainder of this section we review our past work on seismic detection of SQNs passing through the Earth which was based on seismic reports collected by the U.S. Geological Survey over the years 1990-1994. In Section II, we address the question of what limits can be placed on the distribution of SQNs from that work. Given the currency of the President’s Exploration Initiative (see, for example), in Section III we move on to the Moon, reviewing our work on the relative sensitivities of Earth and Moon for detecting SQNs and also presenting new limits on SQN abundance deduced from the data collected by the five seismometers implanted by the Apollo astronauts. Section IV gives a brief summary.

We turn now to our past work on SQNs. In 1996, two of us used a Monte Carlo calculation to investigate the sensitivity of the Earth in detecting SQN passage seismically. We took Cherenkov radiation as the model for generating seismic waves since the galactic virial velocity, about 250 km/s, is about 25 times the speed of sound (and seismic waves) in the Earth. Since it takes six variables to specify an SQN passage (time of entry, entry point, direction, and speed), we asked that seven or more seismic stations detect the passage and that each subset of six stations determine the same chord for the passage. Note from Fig. 1 in the difference in order of first arrival times between point (epicentral) events and epilinear ones. We considered essentially all real stations detectable. This paper addresses that phenomenology.
dom sets of values for the 6 parameters cited. We found that 97 percent of detections were from class 1 stations (the most sensitive ones). We asked for the lowest mass that would yield detection by 7 or more stations. The results are tabulated in Table I of the present work and also partially displayed graphically in Figure 1. Roughly, we found that ten percent of the minimum masses, \( m_{\text{min}} \), were below one metric ton, 30 percent were below ten and 90 were below 220. These results are sensitive to the assumption made for the fraction \( \epsilon \) of nugget energy loss converted to detectable seismic waves (\( m_{\text{min}} \sim \epsilon^{-3/2} \)).

The U.S. Geological Survey kindly made available seismic reports received from around the world from 1981 through 1993. We investigated in some detail the last 4 years of that data. These consisted of roughly a million reports, including first signal arrival times, that had not been associated into Earthquakes (and about twice that number that had). One of our collaborators calculated travel time tables for signals originating from deeper than 750 km (above that they have been tabulated for some time); see [18]. We eliminated reports within an hour of large Earthquakes. We tried all candidate lines, in meshes of increasing fineness near good fits, seeking ones that would minimize a figure of merit consisting of the sum of the squared differences between reported times of first signal arrival and calculated (on the basis of the assumed chord) times of first signal arrival. We eliminated all chords such that the wave travel path from the point of closest approach would involve passage through the Earth’s iron core where travel times are not so well understood.

The result of Reference [17] was one candidate event with a very good fit and waveforms that seemed to add additional evidence of SQN passage. The four most sensitive (and world-class) stations in Australia recorded strong signals from the event in question. The arrival times did not fit the spherical wave expected from a point source. They did, however, fit that expected from a line source. It was later discovered, however, after publication of Reference [18], that one of the four historically reliable stations had a large clock error (offset) for the entire month in which the candidate set of reports occurred [19]. After correcting for that offset, there was a good fit to an Earthquake. That is, when that station was deleted, the remaining three Australian stations, as well as stations in South America, had arrival times that did fit a point source. The final result, therefore, was that no SQN passages were detected in 4 years of seismic data. In Section II below, we use this fact combined with the results of the Monte Carlo calculation in Table I to determine limits on SQN abundance in the region of the galaxy near the sun.

II. LIMIT ON NUMBER REACHING THE EARTH

We estimate the bound on the number of strange quark nuggets (SQNs) in our region of the galaxy implied by the negative results of Anderson et al. [17]. That bound will naturally depend on the nugget mass distribution. If that distribution is skewed too much toward low mass nuggets there will be a shortage of nuggets capable of leaving detectable seismic signals and the abundance bound will be weak. If, on the other hand, it extends too far toward large masses only a relatively small number will be needed to saturate the abundance needed for galactic dark matter DM and the bound from the absence of seismic events will again be weak. This point is made in [16].

To estimate the bound on SQNs, we need convolve an assumed abundance function \( n(m) \), the number of nuggets of mass \( m \) per unit volume, with a probability \( p(m) \) for detecting a nugget of mass \( m \) incident from a random direction. We have for the number of nuggets that should be detected in time \( T \)

\[
N = 4\pi R_E^2 (v/4) T \int p(m) \frac{dn(m)}{dm} dm
\]  

where \( R_E \) is the radius of the Earth, \( v \) is the galactic virial velocity, and here \( T \) is the four year period over which Anderson et al. searched unassociated seismic reports. We assume nuggets are distributed between \( m_{\text{min}} \) and \( M_{\text{MAX}} \) with \( \frac{dn}{da} = K a^{-\gamma} \) where \( a \) is nugget radius. We take the normalization constant \( K \) in terms of the local density of dark matter, \( \rho_{\text{DM}} \), approximately \( 5 \times 10^{-25} \text{g cm}^{-3} \). The factor of \( 1/4 \) takes account of the fraction of SQNs per unit volume that will hit the nearby Earth. We work in the approximation of all nuggets having the galactic virial velocity (about 250 km/s).

\[
\frac{dn}{dm} = K m^{-(\gamma+2)/3}
\]  

\[
K = [(4 - \gamma)/3] \rho_{\text{DM}} / [M_{\text{MAX}}^{(4-\gamma)/3} - m_{\text{min}}^{(4-\gamma)/3}]
\]  

We also need to specify the probability of seismic detection \( p(m) \) as a function of nugget mass \( m \). We do this by means of the Monte Carlo results of [18]. These are given in Table I and Figure 1. With them, we can evaluate Eq. (1). The results are given in Tables II, III and IV for a few values of \( \gamma \) around 4. Note that \( \gamma \) is the exponent in the distribution in terms of nugget radius. The value 3.5 is special, as found by Dohnanyi [20]: if that is the distribution given by particle collisions it will be maintained under continuing collisions between the collision fragments. The value \( \gamma = 4.0 \) is also special in that the integral for \( K \) becomes a logarithm.

In Tables II, III and IV, we give the results for \( \gamma = 3.0, 4.0, \) and 5.0. The individual tables use the \( \gamma \) values just cited. In each, the rows and columns correspond to
$m_{\text{min}}, M_{\text{MAX}}$ values in the limit of the integral in Eq. (1). That is, SQNs are distributed in mass with index $\gamma = (\gamma + 2)/3$ from $m_{\text{min}}$ to $M_{\text{MAX}}$, not confined to just the mass values of Table I, i.e. to values such that there is chance of seismic detection on Earth. In the tables, the top row has $m_{\text{min}} = 10^{-1.2}$ tons (corresponding to the lowest value in Table I) and each succeeding row below has $m_{\text{min}}$ in Eq. (3) down by a factor 10. Similarly, column 1 has $M_{\text{MAX}} = 10^4$ tons, the highest value in Table I, and each succeeding column has $M_{\text{MAX}}$ up by a factor of 10. Tables II-IV are made under the assumption that the fraction $\epsilon = 0.05$ of SQN energy loss is turned into seismic waves. Mass values in the tables should be multiplied by $(0.05/\epsilon)^{3/2}$ for other assumptions [see Eq. (5) below].

Tables II-IV indicate that, over the four year period studied by [17], a value of $\gamma$ near 4 would have produced detectable nuggets for a fairly wide range in $M_{\text{MAX}}$ and $m_{\text{min}}$. For other $\gamma$ not too far from $\gamma = 4$, significant areas of the $[M_{\text{MAX}}, m_{\text{min}}]$ plane should similarly have produced detectable nuggets. A summary statement of the results recorded in Tables II-IV is that Reference [17] precludes distributions (of a total density $\rho_{\text{DM}}$ of SQNs) with $\gamma$ in a small range about 4.0 and places some restrictions on distributions with $\gamma$ near that range.

A second, in some sense opposite, way of presenting the results recorded in Tables II-IV is that Reference [17] is that of de Rujula and Glashow [8]. They take all SQN of one mass with an abundance that yields the dark matter density in the solar neighborhood ($\approx 5 \times 10^{-25}$ gm cm$^{-3}$). This is a useful tool for comparisons even though, in real life, we would expect a distribution in mass. It could correspond, in some approximation to primordial SQN dark matter production for which one might expect mass determined by the number of quarks within the horizon at the time of production.

We will make comparisons with seismic detection on the Moon in the next section. If $\rho_{\text{DM}}$ is all in SQNs of mass $m$, and if $p(m)$ is a theta function, we have, in 4 years, from Eq. (11)

### Table I: Distribution of minimum detectable masses for 120,000 random events.

| Mass  | Number of events | Fraction | Cumulative fraction |
|-------|------------------|----------|---------------------|
| 0.063 | 9                | 0.000    | 0.000               |
| 0.100 | 26               | 0.000    | 0.000               |
| 0.158 | 69               | 0.001    | 0.001               |
| 0.251 | 200              | 0.002    | 0.003               |
| 0.398 | 513              | 0.004    | 0.007               |
| 0.631 | 1043             | 0.009    | 0.016               |
| 1.000 | 2031             | 0.017    | 0.033               |
| 1.585 | 3449             | 0.029    | 0.061               |
| 2.512 | 5345             | 0.045    | 0.106               |
| 3.981 | 7462             | 0.062    | 0.169               |
| 6.310 | 9281             | 0.078    | 0.246               |
| 10.000 | 10678         | 0.089    | 0.335               |
| 15.849 | 11612         | 0.097    | 0.433               |
| 25.119 | 11939         | 0.100    | 0.532               |
| 39.811 | 11613         | 0.097    | 0.630               |
| 63.096 | 10180         | 0.085    | 0.715               |
| 100.000 | 8747          | 0.073    | 0.788               |
| 158.489 | 7027          | 0.059    | 0.847               |
| 251.189 | 5459          | 0.046    | 0.892               |
| 398.107 | 3988          | 0.033    | 0.926               |
| 630.958 | 2879          | 0.024    | 0.950               |
| 1000.001 | 2101         | 0.018    | 0.967               |
| 1584.894 | 1486         | 0.012    | 0.980               |
| 2511.888 | 1006         | 0.008    | 0.988               |
| 3981.075 | 659           | 0.006    | 0.994               |
| 6309.580 | 475           | 0.004    | 0.998               |
| 10000.011 | 267          | 0.002    | 1.000               |

### Table II: Number of events expected for $\gamma = 3$ as $M_{\text{MAX}}, m_{\text{min}}$ vary. The I-1 element has upper and lower masses $M_{\text{MAX}}, m_{\text{min}}$ in the distribution equal to the first and last masses in Table I. Succeeding rows (columns) decrease (increase) $m_{\text{min}}(M_{\text{MAX}})$ by 10.

| Mass  | Number of events | Fraction | Cumulative fraction |
|-------|------------------|----------|---------------------|
| 3.0   | 1.4              | 0.6      | 0.3 0.1 0.1 0.0 0.0 0.0 0.0 0.0 0.0 |
| 3.0   | 1.4              | 0.6      | 0.3 0.1 0.1 0.0 0.0 0.0 0.0 0.0 0.0 |
| 3.0   | 1.4              | 0.6      | 0.3 0.1 0.1 0.0 0.0 0.0 0.0 0.0 0.0 |
| 3.0   | 1.4              | 0.6      | 0.3 0.1 0.1 0.0 0.0 0.0 0.0 0.0 0.0 |
| 2.9   | 1.4              | 0.6      | 0.3 0.1 0.1 0.0 0.0 0.0 0.0 0.0 0.0 |
| 2.9   | 1.4              | 0.6      | 0.3 0.1 0.1 0.0 0.0 0.0 0.0 0.0 0.0 |
| 2.9   | 1.4              | 0.6      | 0.3 0.1 0.1 0.0 0.0 0.0 0.0 0.0 0.0 |
| 2.9   | 1.4              | 0.6      | 0.3 0.1 0.1 0.0 0.0 0.0 0.0 0.0 0.0 |

### Table III: Number of events expected for $\gamma = 4$ as $M_{\text{MAX}}, m_{\text{min}}$ vary. The I-1 element has upper and lower masses $M_{\text{MAX}}, m_{\text{min}}$ in the distribution equal to the first and last masses in Table I. Succeeding rows (columns) decrease (increase) $m_{\text{min}}(M_{\text{MAX}})$ by 10.

| Mass  | Number of events | Fraction | Cumulative fraction |
|-------|------------------|----------|---------------------|
| 10.6  | 8.7              | 7.3      | 6.3 5.5 4.8 4.3 3.9 3.5 3.2 |
| 9.0   | 7.6              | 6.5      | 5.7 5.0 4.5 4.0 3.6 3.3 3.0 |
| 7.9   | 6.8              | 5.9      | 5.2 4.7 4.2 3.8 3.4 3.1 2.9 |
| 7.1   | 6.2              | 5.4      | 4.8 4.4 3.9 3.6 3.3 3.0 2.8 |
| 6.4   | 5.7              | 5.1      | 4.5 4.1 3.7 3.4 3.1 2.9 2.7 |
| 5.9   | 5.3              | 4.7      | 4.3 3.9 3.5 3.2 3.0 2.8 2.5 |
| 5.5   | 4.9              | 4.4      | 4.0 3.7 3.4 3.1 2.9 2.6 2.5 |
| 5.2   | 4.6              | 4.2      | 3.8 3.5 3.2 3.0 2.8 2.6 2.4 |
| 4.9   | 4.4              | 4.0      | 3.7 3.4 3.1 2.9 2.7 2.5 2.3 |
| 4.6   | 4.2              | 3.8      | 3.5 3.2 3.0 2.8 2.6 2.4 2.2 |
TABLE IV: Number of events expected for $\gamma = 5$ as $M_{\text{MAX}}, m_{\text{min}}$ vary. The 1-1 element has upper and lower masses $M_{\text{MAX}}, m_{\text{min}}$, in the distribution equal to the first and last masses in Table I. Succeeding rows (columns) decrease (increase) $m_{\text{min}}$, $M_{\text{MAX}}$ by 10.

| $M_{\text{MAX}}$ | $m_{\text{min}}$ |
|-----------------|-----------------|
| 15.8            | 15.6            |
| 15.5            | 15.5            |
| 15.5            | 15.5            |
| 15.5            | 15.5            |
| 15.5            | 15.5            |
| 7.3             | 7.3             |
| 7.3             | 7.3             |
| 7.2             | 7.2             |
| 7.2             | 7.2             |
| 7.2             | 7.2             |
| 3.4             | 3.4             |
| 3.4             | 3.4             |
| 3.4             | 3.4             |
| 3.4             | 3.4             |
| 3.4             | 3.4             |
| 1.6             | 1.6             |
| 1.6             | 1.6             |
| 1.6             | 1.6             |
| 1.6             | 1.6             |
| 0.7             | 0.7             |
| 0.7             | 0.7             |
| 0.7             | 0.7             |
| 0.7             | 0.7             |
| 0.3             | 0.3             |
| 0.3             | 0.3             |
| 0.3             | 0.3             |
| 0.3             | 0.3             |
| 0.2             | 0.2             |
| 0.2             | 0.2             |
| 0.2             | 0.2             |
| 0.2             | 0.2             |
| 0.1             | 0.1             |
| 0.1             | 0.1             |
| 0.1             | 0.1             |
| 0.1             | 0.1             |
| 0.0             | 0.0             |
| 0.0             | 0.0             |
| 0.0             | 0.0             |
| 0.0             | 0.0             |

\[
dN_{\text{hits}}/dt = \left(\rho_D m / m\right) (v/4) 4\pi R_E^2 p(m) \rightarrow (5 \times 10^8 / m) / yr
\]

where $m$ is in grams. The limit in Eq. 11 is for large $m$ for which the probability of detection goes to one. Using the cumulative fraction in the results of the Monte Carlo of Table I gives the results of Figure 2.

Figure 2, conservatively, shows that Earth should be a reasonable detector of DM SQNs if they are peaked in mass about a value in the range 0.05. It could be considerably higher as discussed in Reference 17 thereby decreasing the minimum detectable mass by $(\epsilon/0.05)^{-3/2}$. It could also, of course, be lower. A reliable calculation of $\epsilon$ would appear to be an important goal.

We believe that a continuation of this search effort with terrestrial seismology should make use of real time seismic data now available from most seismic stations, rather than the old data used by 17. However, as will be discussed in the following two sections, a better approach to seismology might be to apply it to other solar system bodies with lower seismic backgrounds and hence the capability to detect nuggets of smaller mass. These nuggets are likely to be more abundant in any distribution, if SQM is indeed bound at zero pressure.

III. SEISMIC DETECTION ON THE MOON AND BEYOND

The Apollo astronauts implanted five seismometers at various locations on (the near side of) the Moon. These functioned for several years and give some picture of lunar seismic activity. In brief, there are weak, deep quakes caused by the tides as the Moon’s position relative to the Earth and Sun varies; there are (relatively strong and infrequent) shallow quakes caused by unknown geologic processes (it is believed there is no tectonic activity), and there are impacts. There is no background from winds and waves. This feature means that seismometers used on Earth should be sensitive to seismic waves of amplitude about one third as great if used on the Moon. In this section, we first review the implications of this fact for the seismic search for strange quark nuggets drawing on our discussion in Banerdt et al. 18. A second important lunar seismic feature is the Moon’s tendency to “ring” for some time after seismic excitation. We leave this feature for later study. Seismology on the Moon is reviewed on the NASA Johnson Space Flight Center web site, in the Apollo summary of the Moon 19, as well as in standard text books such as Carroll and Ostlie 20.

In this section we focus primarily on the implications of the Apollo seismic bounds, numbers of seismic events (around 2500/yr) and total lunar yearly seismic energy. The measured amount of the latter, or, more precisely, the amount directly inferred from the measurements made, is $10^{17}$ ergs per year, which can be compared to $2 \times 10^{24}$ ergs for the Earth. It should be noted, however, that, as pointed out by Nakamura 21, the actual figure could be, on the average, several orders larger if one extrapolates the curve of numbers of relatively strong, shallow quakes as a function of shallow quake magnitude. For present purposes, however, we just address the question as to the extent to which the observed limit gives information on the abundance of SQNs in our part of the Galaxy. We note that this question was raised in discussion at the Caltech Jet Propulsion Laboratory April, 2004, Physics in Solar System Exploration conference held in Solvang, California. We begin by briefly reviewing, from Reference 15 the major factors that enter in the relative sensitivities of the Earth and the Moon when used as seismic SQN detectors. These include:

1. Relative cross sectional areas of Earth and Moon;
2. Likely numbers of seismic stations and station placement;
3. Earthquake backgrounds;
4. Ocean and atmospheric backgrounds;
5. Attenuation with distance;
6. Effective blackout of signals by Earth’s iron core; and
7. Lunar ringing.

We address each of these items in turn below.

1. Areas. The ratio of the cross sectional area of Moon to Earth is $a_M/a_E \sim 0.075$.

2. Numbers and placement of stations. Assume about 10 seismic stations for good coverage of the Moon. The number of stations needs to be considered both per unit area (Moon wins if stations can be affordably placed optimally since Earth has no sensitive stations in or near oceans) and in the context of 7 or more station reports
needed both to fix and to confirm the 6 nugget trajectory parameters (Earth wins). We take the rough approximation that these two factors cancel. We believe that this approximation is conservative in the sense that it likely favors the Earth and penalizes the Moon.

3. Earthquake backgrounds. Anderson et al. 17 found it desirable to remove all station reports within one hour of a quake of magnitude 4.0 or more because of the difficulty of reliably identifying reverberations. The result was to remove signals from 1/3 of the minutes in the year. Low seismic activity frees the Moon of such a penalty but see item 7 below.

4. Ocean and atmospheric background. This very important factor means that seismic detection on the Moon is only limited by instrument noise and ringing (below). The relative contributions of atmospheric noise and instrument noise (ocean noise is less than atmospheric in land) is unknown. We estimate that atmospheric (amplitude) noise is the greater effect by about an order of magnitude in energy.

5. Attenuation with distance. Since seismic energy falls, as with other forms of energy, with distance as \( r^{-2} \), seismic amplitude falls as \( 1/r \) making Earth seismic signals received at a station weaker, on average, than those received at lunar stations by the ratio of the radii (0.273).

6. Iron core blackout. We compute for the Earth the volume of the cone segment \( z \sim [r_E - r_C/2, 2r_E] \) (where the ratio of the core radius, \( r_C \) to the Earth radius \( r_E \) is about 0.5) from which seismic signals will not reach a station at \( z = 0 \) in reliably predictable times (because we have found no reliable way of following propagation of SQN signals through the Earth’s iron core). The result, weighting with the attenuation factor, is that about one third of signals are eliminated for the Earth. There is controversy with regard to a possible, relatively small iron core for the Moon. We assume/approximate that none is present.

7. Lunar ringing. Seismic signals on the Moon exhibit codas. These persisted for some time with Apollo instruments. We do not know the rate of decrease of these signals for small values. The codas could set a lower limit on achievable sensitivities. Additionally, at greater sensitivities they could require a subtraction procedure as in item 3 above. We do not attempt here to quantify these issues.

We include these considerations as needed below. Our aim is to be relatively conservative in our estimates of lunar capability for seismic SQN detection. We assume that seismic signals on the Moon can be detected to a factor of \( \sqrt{10} \) in amplitude of ground motion below those on Earth. The factor of \( \sqrt{10} \) in amplitude implies a factor of 10 below in energy, making modern seismometers on the Moon sensitive to signals on the order of 0.013 erg/cm²s. Note that significant further sensitivity improvement would be possible with, for example, superconducting technology. In addition to the sensitivity improvement, for identifying epilinear signal generation we assume only minimal seismometer emplacements, say 6 or 7 widely separated. Thus we require signal strength sufficient to be detected at distances of \( 2R_M \), the lunar diameter. As above, we set \( \epsilon \), the fraction of SQN energy loss converted to detectable seismic waves, at \( \epsilon \sim 0.05 \) except as noted.

With these assumptions, the minimum detectable mass \( m_d \) for all transit trajectories for an SQN with galactic virial speed \( v_v \) can be found from equating its signal strength to our assumed instrument noise

\[
P_m = 1.3 \times 10^{-2} \text{erg cm}^{-2} \text{s}^{-1} = \epsilon \rho_M \pi (3m_d/4\pi \rho_N)^{2/3}v_v^2/4\pi R_M^2 \sim 10^{-6}m_d^{2/3} \tag{5}
\]

This gives (with nugget density \( \rho_N = 2 \times 10^{14} g/cm^3 \))

\[
m_d \sim 125 kg \tag{6}
\]

\( \epsilon = 0.1 \) would imply \( m_d \sim 50 \) kg and \( \epsilon = 0.5 \) 3 kg. Nuggets of mass below \( m_d \) might also be detected depending on the location of their transit trajectories. For \( m > m_d \), detection probability would be 100%.

We assume below that, for nuggets of mass \( m_d \), the detection probability on the Moon is one for \( m > m_d \) and zero for \( m < m_d \). This is consistent with our requirement for signal detection at distances \( 2R_M \). More detailed modeling and more nuanced discussion would need mass and velocity distributions. We consider first the lunar companion to Figure 2, the number of SQN detections, for given single SQN mass \( m \) and for sufficient abundance to constitute the local DM density.

\[
dN/dt = (\rho_{DM}/m)(v/4)\pi R_M^2 \text{yr}^{-1} \sim (3 \times 10^7/m) / \text{yr} \tag{7}
\]

At the lower mass sensitivity limit, we would expect about 600 events each year if \( \rho_{DM} \) were in the form of 50 kg SQNs with \( \epsilon = 0.1 \). The mass, \( m_d \) decreases as \( P_m^{3/2} \) so another factor of 10 or so decrease would bring the minimum detectable signal down to kilograms. Equations (6) and (7) say that the Moon and the Earth are somewhat complementary as SQN detectors. The Earth with its larger area gets eight times the events for the same abundance assumption while the Moon has the sensitivity to detect significantly lower nugget mass. Together they span a range of roughly \( 10^4 \) at least in mass detection.

We turn now to the “Apollo limits,” limits on SQNs that can be inferred from the Apollo results that:

—As noted above, about 2500 seismic events per year were detected in the three categories: deep, weak, tidal Moonquakes; shallow, relatively strong Moonquakes; and impacts. We assume that a population of tens of SQNs over 50kg or even somewhat less would have been identified as an additional class of seismic events.

—The total lunar seismic energy in a year inferred from the data taken was about \( 2 \times 10^{17} \text{erg} \) (compared with \( 10^{24} \text{erg for Earth} \)).

We consider here only these two gross characteristics, ignoring more subtle arguments that might be exploited.
The first, the limit on numbers of events in an unrecognized class, implies roughly that the abundance of SQNs with masses in the range $10^{-3}$ kg must be at least an order of magnitude less than would be required to saturate the local DM density. However there were too few seismometers to be certain that some of the events identified as deep quakes were not actually SQN passages.

Moving to the second Apollo limit, we have that the seismic energy from each lunar SQN passage is, on the average, given by

$$E_S = \epsilon(\nu_T^2/2)\rho_M R M \pi (3m/4\pi \rho_N)^{2/3} \sim 5 \times 10^{12} m^{2/3}$$ (8)

where we have assumed an average SQN travel distance in the Moon of $R_M$. Putting this together with Eq. 7, we see that the seismic energy in one year, from SQNs of mass $m$ would be given by

$$E_T = (3 \times 10^7/m)(5 \times 10^{12} m^{2/3}) \sim 10^{20} / m^{1/3}$$ (9)

Equation 9 implies that the abundance of SQNs in the range 10 kg to a ton must be at least an order of magnitude less than that that the latter depends on identifying a class of SQN events from about ten sets of seismic reports, while the former just says that their contribution to total seismic energy would be noticed. These points are given graphically in Figure 3 where events expected if all local DM is in mass $m$ nuggets are compared with the Apollo limits on maximum number of nuggets found from the bound on total amount of seismic energy in a year. Note the desirability of determining instrument noise reliably and on decreasing it so as to enlarge the mass region in which DM can be bounded. Recall from Section II we expect that, if SQNs are dark matter, they should be relatively narrowly peaked in mass distribution around the mass of baryons $M_B$ within the horizon at formation time and temperature. $M_B$ is given by

$$M_B \sim 2 \times 10^{21} / T(GmV)^3$$ (see Appendix A in [23]).

Thus, in principle, the range of SQM dark matter formation temperatures covered by the Apollo data is roughly $10^5 - 10^6$ GeV and by the Earth data $10^4 - 10^5$ GeV. These temperature values would vary as $(\epsilon/0.05)^{3/2}$ for $\epsilon \neq 0.05$.

However, it is important to note that current understanding of quantum chromodynamics implies that the strong coupling constant decreases with increasing energy so that we would not expect nugget formation for temperatures over a few hundred MeV by which time the horizon contains about $10^{24}$ grams of baryons. This would imply dark matter of nuggets with masses about one percent the mass of the Earth and sizes around a meter. Thus the range of SQN masses that can be investigated by terrestrial and lunar seismology does not, in all likelihood, include that appropriate for SQM as DM. Nor, of course do any of the other methods now available such as those discussed in [8]. All of course do, however, include detecting fragments from colliding neutron stars if these are made from SQM, an eventuality considered more likely than SQM DM. In spite of the fact that DM is unlikely to be SQNs in the mass ranges to which the Earth and moon are sensitive, it is convenient to express limits on SQM abundance in terms of $\rho_{DM}$.

Could, however, $10^{24}$ gm DM ever be detected (in the spirit of exploring the moon, the solar system and beyond)? Equation (7) implies that, if the number density of DM particles is given by $\rho_{DM}/m$, the mass of DM that can be detected (say 10 events per year) by a system of size $R$ is

$$m \sim 10^7 \rho_{DM} v_R R^2 \sim 10^{-10} R^2$$ (10)

Helioseismology ($R_\odot \sim 10^{11}$ cm) could reach to a million metric tons providing nuggets of the corresponding size (millimeters) would leave behind a sufficiently energetic, detectable, identifiable signal. To reach to $10^{24}$ gm would appear to call for raising $R$ by a factor of $10^6$, i.e. to $10^4$ astronomical units (the size of the inner radius of the Oort cloud) not a near term project.

IV. SUMMARY

We briefly recapitulate our results.

— The work of Anderson et al. [17] precludes SQN distributions in our neck of the galaxy given by Eq. 2 with $\gamma$ in a small range about 4 and places some restrictions for $\gamma$ near that range, e.g. 3 and 5.

— Anderson et al. [17] should have found 5 or more epilinear events for $10^2$ gm $< m < 3 \times 10^8$ gm if all the local dark matter density were in SQNs in that range and 0.05 of SQN energy loss is into seismic waves.

— As pointed out in Ref. [13], deployment of seismometers on the Moon with instrument noise in the region $10^{-2}$ erg/cm$^2$ s would mean certain detection of SQNs down to the 125 kg level while ones with $10^{-4}$ erg/cm$^2$ s would permit detection to the 125 gm level providing persistence of lunar codas (ringing)did not create too great a background.

— The total yearly lunar seismic energy being $10^7$ less than that of Earth implies that the abundance of SQNs in the range 10 kg to a ton must be at least an order of magnitude less than would saturate the dark matter density in the solar neighborhood.

Acknowledgments

We very much appreciate discussions with D. Anderson, W. Banerdt, T. Chui, H. Paik, K. Penanen, J. Sandweiss, and D. Strayer. The work of Section III was greatly aided by J. Ormes and N. White suggesting we review helioseismology and lunar seismic data generally, by
the participant at the JPL Solvang Conference who raised the issue of the bound on total lunar seismic energy, and by helpful suggestions from the anonymous Phys. Rev. referee.

[1] E. Witten, Phys. Rev D 30, 279 (1984).
[2] See T. Siegfried, Strange Matters: Undiscovered Ideas at the Frontiers of Space and Time (Joseph Henry Press, Washington, D.C., 2002), for very readable, non-technical but informative review.
[3] E. Farhi, and R. Jaffe, Phys. Rev D 30, 2379 (1984).
[4] F. Weber, Prog. Part. Nuc. Phys 54, 193 (2005).
[5] M. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. 422B, 247 (1998).
[6] R. Rapp, T. Schäfer, E.V. Shuryak, and M. Velkovsky, Phys. Rev. Lett. 81, 53 (1998); Ann. Phys. 280, 35 (2000).
[7] M. Alford, Ann. Rev. Nuc. Sci (2002).
[8] A. de Rujula, and S. Glashow, Nature (London) 312, 734 (1984).
[9] J. Sandweiss, J. Phys. G 30, S51 (2004).
[10] P.B. Price, E.K. Shirk, W.Z. Osborne, and L.S. Pinsky., Further measurements and reassessment of the magnetic-monopole candidate. Phys. Rev. D38, 3813 (1988).
[11] J. Madsen, Phys. Rev. D71, 014026 (2005).
[12] www.nasa.gov/missions/solarsystem/explore_main.html.
[13] W.B. Banerdt, T. Chui, E.T. Herrin, D. Rosenbaum, V.L. Teplitz, in Proceedings of the 5th International Workshop on the Identification of Dark Matter, Edinburgh (2004), edited by V. Kudryavtsev (World Scientific, in press).
[14] G. Heiken, D. Vanimum, and B. French, Lunar Source Book, Cambridge university Press (Cambridge, 1991)
[15] Johnson Space Center Web Pages, [16] E.T. Herrin, and V.L. Teplitz, Phys. Rev. D53, 6762 (1996).
[17] Anderson,D, E. Herrin, V. Teplitz, and I. Tibuleac, Bull. Seis. Soc. of Am. 93, 2363 (2003).
[18] I. Tibuleac, (unpublished).
[19] N. Selby, J. Yang and A. Douglas Bull. Seis. Soc. of Am. 94, 2414 (2004).
[20] J. Dohnanyi, J. Geophys. Res. 74, 2531 (1969).
[21] B.W. Carroll, and D.A. Ostlie, An Introduction to Modern Astrophysics, Addison-Wesley (New York, 1996).
[22] Y. Nakamura, in Shallow moonquakes, proceedings of the Lunar Planet. Sci. Conf. 11th, (1980).
[23] E.W. Kolb and M.S. Turner, The Early Universe, Addison-Wesley (New York,1990).

FIG. 1: Curves give the fraction and cumulative fraction of SQNs of mass $m$ SQNs that would have been detected by terrestrial seismic stations existing in early 1990s.

FIG. 2: Curve uses Eq. (9) with probability of detection $p(m)$ from the Monte Carlo results (Table I and Fig. 1) to give SQN detections that should have been seen, in 1990-1993, if local DM were mass $m$ SQNs.

FIG. 3 Solid curve: (the log of) the number of lunar SQN passages expected if local DM were mass $m$ SQNs. Dashed curve: number of mass $m$ passages permitted by the Apollo bound on total lunar seismic energy. For $m > 1000$, no limit is implied.
