Target–superspace in 2d dilatonic supergravity

Thomas Strobl

Institut für Theoretische Physik, RWTH Aachen, D-52056 Aachen, Germany

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The $N = 1$ supersymmetric version of generalized 2d dilaton gravity can be cast into
the form of a Poisson $\sigma$–model, where the target space and its Poisson bracket are graded. The target space
consists of a 1+1 superspace and the dilaton, which is the generator of Lorentz boosts therein.
The Poisson bracket on the target space induces the invariance of the worldsheet theory against both
diffeomorphisms and local supersymmetry transformations (superdiffeomorphisms). The machinery of
Poisson $\sigma$–models is then used to find the general local solution to the field equations. As a
byproduct, classical equivalence between the bosonic theory and its supersymmetric extension is
found.

The most general 2d gravity action for a metric $g$ and a
dilaton field $\phi$ yielding second order differential equations
is of the form

$$L = \int_{\mathcal{M}} d^2x \sqrt{-g} \left[ U(\phi) R + V(\phi)(\nabla\phi)^2 + W(\phi) \right], \quad (1)$$

where $R$ denotes the Ricci scalar and $U$, $V$, and $W$ are
some arbitrary (reasonable) functions of the dilaton. Its
supersymmetrization, considered first in $[1]$, may be ob-
tained in a most straightforward manner by using the
superfield formalism of $[2]$. In this framework the action
takes the same form as above, where, however, each term
is replaced by an appropriately constrained supersym-
metric extension and, simultaneously, the volume form
d$^2x$ is replaced by its (worldsheet) superspace analog
$d^2x \, d^2\theta$. A term such as $U(\phi)$, e.g., is replaced by $U(\Phi)$,
where $\Phi$ is the superfield $\Phi = \phi + \theta \xi + \theta^2 f$ with $\phi$
being the bosonic dilaton field, $\xi$ a Majorana spinorial
superpartner, and $f$ an auxiliary bosonic scalar. (For fur-
ther details the reader is referred to the literature cited
above).

For many practical calculations it is necessary to re-
express the supersymmetric extension of $L$ in terms of
its component fields ($\phi$, $\xi$, $f$ etc.). The resulting action
and all the more its field equations become lengthy and
their analysis involved. In the present letter we propose
a different formulation, which greatly simplifies not only
the notation but also the analysis of the supersymmetric
theory.

This latter formulation is provided by a Poisson $\sigma$–
model $[1]$, the definition of which we will briefly recapit-
ulate now, generalizing it to the case of graded Poisson
manifolds, before we then come to its relation to $[1]$ (for
an introduction to bosonic Poisson $\sigma$–models cf. $[3]$). The action is a functional of $n$ scalar fields $X^i$, $i = 1, \ldots, n$, on
the one hand, which may be viewed as coordinates in
an $n$–dimensional (not necessarily linear) manifold $N$, as
well as of $n$ oneforms $A_i \equiv A_{i\mu} dx^\mu$, on the other hand,
which may be regarded as oneforms on the worldsheet
(coordinates $x^\mu$, $\mu = 0, 1$) taking values in $T^*N$ (more
precisely, $(A_i)$ is a oneform on the world sheet which is
simultaneously the pullback of a section of $T^*N$ by the
map $X(x)$). In the present context we allow $N$ to carry
also a $\mathbb{Z}_2$–grading, i.e. some of the fields $X^i$ and $A_{i\mu}$ may
be Grassmann valued, $\sigma_i$ denoting their respective parity
(so $X^i X^j = (-1)^{\sigma_i \sigma_j} X^i X^j$ etc.).

To define an action we require $N$ to be equipped with a (graded) Poisson bracket

$$\{X^i, X^j\} \equiv \mathcal{P}^{ij}. \quad (2)$$

Note that the Poisson bracket is anti–symmetric only
for the case that at least one of its entries is an even (commuting)
quantity; in general one has $\mathcal{P}^{ij} = (-1)^{\sigma_i \sigma_j + 1} \mathcal{P}^{ji}$
(while $\mathcal{P}^{ij}$ itself has grading $\sigma_i + \sigma_j$).

In terms of the two–tensor $\mathcal{P}^{ij}$ the standard, graded Jacobi
identity (cf., e.g., $[3]$) may be brought into the form

$$(-1)^{\sigma_i \sigma_k} \left( \mathcal{P}^{ij} \frac{\partial}{\partial X^s} \right) \mathcal{P}^{sk} + \text{cycl}(ijk) = 0, \quad (3)$$

where a sum over the index $s$ is understood (but not over $i$ or $k$ in the first of the three cyclic terms). Using left
derivatives $\overleftarrow{\partial} = \partial$ (in contrast to the above right deriv-
atives $\overrightarrow{\partial}$), this equation may be written equivalently as

$$(-1)^{\sigma_i \sigma_k} \mathcal{P}^{is} \overleftarrow{\partial} \mathcal{P}^{jk} + \text{cycl}(ijk) = 0.$$

The action of the 2d theory is

$$S = \int A_i \wedge dX^i - \frac{1}{2} A_i \wedge A_j \mathcal{P}^{ij}, \quad (4)$$

where the order of the terms and indices has been chosen
so as to avoid unnecessary signs in the considerations to
follow while simultaneously $S$ coincides with its purely
bosonic counterpart in the previous literature.

Before further investigating the general model $[1]$, we
now discuss its relation to $N = 1$ supersymmetric
dilaton gravity. For this purpose we first restrict our at-
tention to the case where $U \equiv \phi$ and $V \equiv 0$ in $[1]$. This
is not a serious restriction since the general action can be
brought into this form always (at least locally) by a
dilaton–dependent conformal rescaling of the metric $g$
and a simultaneous change of the dilaton field $\phi \rightarrow F(\phi)$
$[1]$: the information about $U$ and $V$ is then “stored” in
the relation to the original variables. (In this process
global information may be lost, as happens, e.g., if $U$ has
critical points. In the following we thus restrict our at-
tention to such choices of the potentials $U,V,W$ where
the above transformation is sufficiently global; otherwise the considerations to follow are of local nature and the global information has to be restored in a subsequent step, cf. also \[\text{[3]}\] for details.) Following a recent paper by

\begin{equation}
L = \phi d\omega + X_a (de^a + e^a_\alpha \omega e^b + 2i\bar{\psi}\gamma^a \psi) - (2u u' - \frac{iu''}{16} \chi \varepsilon_{ab} e^a e^b + 4iu\bar{\psi}\gamma^3 \psi + iu' \bar{\chi} e^a \gamma_a \psi + i\bar{\chi} (d\psi + \frac{1}{2} \omega^3 \gamma_3 \psi). \tag{5}
\end{equation}

Here \(e^a\) is the zweibein of the metric \(g\) (or the conformally rescaled metric, respectively), \(\varepsilon^a_\beta \omega\) is the spin connection, and \(\psi\) is a oneform–valued Majorana spinor. \(\chi\) is a (zeroform–valued) Majorana fermion, which, such as \(\psi\), is of odd Grassmann parity, and \(X^a\) denotes a pair of scalar functions (the Lorentz index \(a\) running over two values in two spacetime dimensions). The fields \(X^a\) are introduced in the bosonic theory as Lagrange multipliers so as to determine \(\omega\) through the vanishing torsion constraint; as the field equations for \(\omega\) determine \(X^a\) uniquely in terms of the remaining variables, the fields \(X^a\) may be eliminated from (or introduced to) the action without changing the theory (at least on the classical level). \(u\) is a function of the dilaton \(\phi\), which relates to \(W\) in \(\text{[1]}\) through \(W = 4(u^2)\), the prime denoting differentiation with respect to the argument \(\phi\). For the conventions we conform to \(\text{[3]}\): the Lorentz metric has signature \((-\,+, +\,\,\,),\) Lorentz indices are underlined, \(\varepsilon^{\underline{\underline{ab}}} = +1\), and the gamma matrices are related to the usual Pauli matrices \(\sigma_i\) according to: \(\gamma^1 = -i\sigma_2, \quad \gamma^\alpha = \sigma_1, \quad \gamma^3 = \gamma^\underline{3}, \quad \gamma^\underline{3} = \gamma^3\).

We now collect the one– and zeroforms into two multiplets:

\begin{equation}
(X^i) := (X^a, \chi^\alpha, \phi), \quad (A_i) := (e_a, i\bar{\psi}_\alpha, \omega). \tag{6}
\end{equation}

After a simple partial integration, the action \(\text{[3]}\) is seen to take the form \(\text{[4]}\) and the coefficient matrix \(P^{ij}\) may be read off by straightforward comparison:

\begin{equation}
P^{ab} = -\varepsilon^{ab}(4uu' + \frac{1}{16} u'' \varepsilon_{\lambda\alpha} \chi^\alpha), \quad P^{\alpha\alpha} = u' (\gamma^\alpha \chi)^\alpha, \quad P^{\alpha\beta} = -8iu (\gamma^3)^{\alpha\beta} - 4iX_a (\gamma^a)^{\alpha\beta}, \quad \{X^a, \phi\} = \varepsilon^a_b X^b, \quad \{\chi^\alpha, \phi\} = -\frac{1}{2} (\gamma^3 \chi)^\alpha
\end{equation}

where \((\gamma^\alpha \chi)^\alpha \equiv (\gamma^\alpha)^\alpha \chi^\beta\), spinor indices have been raised and lowered by means of \(\varepsilon^{\alpha\beta} (\chi_\alpha = \varepsilon_{\alpha\beta} \chi^\beta\) with \(\varepsilon^{01} := 1\), and, in the last line, the identification \(\text{[3]}\) was used.

Up to now it may seem that we have not gained much and in particular it is by no means clear that the matrix \(P\) obtained above indeed satisfies the (graded) Jacobi identity \(\text{[3]}\), which, however, is at the heart of Poisson \(\sigma\)–models.

In \(\text{[3]}\) it was shown that the field equations of \(\text{[3]}\) form a free differential algebra (FDA) \(\text{[1]}\). In the present formulation \(\text{[3]}\) the field equations take the compact form

\begin{equation}
dX^i - A_j P^{ji} = 0, \quad dA_i - \frac{1}{2} A_k A_i (P^{lj} \partial_j P^{ik}) = 0. \tag{8}
\end{equation}

Applying an exterior derivative to the first set of these equations, we obtain \(dA_i P^{ji} = (P^{lj} \partial_j P^{ik}) dX^k = 0\). Eliminating \(dA_j\) and \(dX^k\) by means of \(\text{[3]}\), we end up with an expression bilinear in the \(A_i\)’s. By definition of an FDA, the resulting equations have to be fulfilled identically, without any restriction to the oneforms \(A_i\). It is a simple exercise to show that this requirement is fulfilled if and only if \(P^{ij}\) satisfies Eqs. \(\text{[3]}\). Thus, using the result of \(\text{[3]}\), the validity of the graded Jacobi identities is proven.

Certainly the Jacobi identities may be verified also by a direct calculation using the specific form \(\text{[3]}\) of the bracket. This is in fact simpler than proving the FDA property of the field equations of the specific action \(\text{[3]}\) (cf. also first sentence of the following paragraph). Seeking brackets fulfilling the graded Jacobi identities with restrictions specified below will automatically provide 2d supergravity theories and to us this route seems to be the technically simplest for the construction of such models. This idea will be made clearer in what follows.

Applying an exterior derivative to the second set of equations, the requirement for an FDA does not lead to any further relations beside \(\text{[3]}\). Thus we may conclude that the field equations of a general graded Poisson \(\sigma\)–model \(\text{[3]}\) form an FDA, iff the tensor \(P\) is a Poisson tensor (i.e., by definition, iff \(P\) satisfies \(\text{[3]}\)). Alternatively, the Jacobi identity may be verified to be the necessary and sufficient condition for the constraints in a

\[1\] After completion of this letter, I became aware that the action below was found already in \(\text{[4]}\) where the latter of the two works was cited in a different context only. \(\text{[4]}\) contains also parts of the considerations to follow, but, such as in the case of bosonic Poisson \(\sigma\)–models, the hidden structure of a (graded) Poisson manifold is not noted (and, consequently, no statement about, e.g., the classical solutions could be obtained there).
Hamiltonian formulation of \( (3) \) to be of first class. This is tantamount to requiring the model to have maximal local symmetries.

It is a nice and simple exercise to show that due to \( (3) \) the variations

\[
\delta X^i = \epsilon_j P^{ji}, \quad \delta A_i = d\epsilon_i - A_j \epsilon_k (P^{kj} \delta_{ji})
\]

change the action only by a total divergence: \( \delta S = \int d(x \delta X^i) \). Thus there is a local symmetry for any pair of fields \((X^i, A_i)\). Since the action is of first order in these fields, this implies that there are at most a finite number of physical (gauge invariant) degrees of freedom. (More precisely, being a oneform, \( A_i \) has two components for each value of \( i \); however, the “time component” \( A_0 \) of the oneform \( A_i \) enters a Hamiltonian formulation as Lagrange multiplier for the constraints only and therefore it must not be included in the above naive counting.)

In the context of \( (2) \) resp. \( (3) \) there are five independent local symmetries contained in \( (3) \): \( \epsilon_0 \) generates local Lorentz symmetries (cf. last line in Eq. \( (3) \)). The Grassmann spinor \( \epsilon_0 \), on the other hand, generates precisely the local supersymmetry transformations of \( (3) \). The remaining two symmetries correspond to the obvious diffeomorphism invariance of the action. Indeed, a simple calculation shows that the Lie derivative of \( X^i \) and \( A_i \) along a spacetime vector field \( v \) differs from \( (3) \) with the specific choice \( (\epsilon_0) := (v^m A_{mi}) \) only by an additive term proportional to the field equations \( (3) \). For invertible zweibein \( A_\mu \) the first two entries establish a bijection between any possible choice of \( v \) and \( \epsilon_0 \), moreover. As this will be of some importance later on, we note, however, that while diffeomorphisms cannot change an invertible matrix \( A_\mu \) into a noninvertible one and vice versa, the symmetries \( (3) \) can.\(^2\)

We return to the structure found in the target space \( N \) of the theory. The target space is spanned by a Lorentz vector \( X^\alpha \), a Majorana spinor \( \chi^\alpha \), and the dilaton \( \phi \). \( X^\alpha \) and \( \chi^\alpha \) may be combined into a \((1 + 1)\)-dimensional superspace. \( \phi \), on the other hand, generates Lorentz boosts in this superspace by means of the Poisson brackets \( (3) \), last line. Indeed, \( \varepsilon^\alpha_b \) is the Lie algebra element of the (one-dimensional) Lorentz group in the fundamental representation and \( \gamma^3 \) is easily identified with the generator of Lorentz boosts in a two-dimensional spinor space: \( \gamma^3 \equiv \gamma_2 \gamma_\perp = \frac{1}{2} [\gamma_2, \gamma_\perp] \) (irrespective of the choice of presentation for the generators \( \gamma_2 \) and \( \gamma_\perp \) in the Clifford algebra).

This structure in the target space will remain also within generalizations to more general 2d supergravity theories including the supersymmetrization of theories with nontrivial torsion, such as the Katanaev–Volovich (KV) model \( (4) \), for which a supersymmetrization has not been provided in the literature yet. Using the same fields \( (3) \) as before, such theories may be found by searching for other solutions to the Jacobi identity \( (3) \). However, the then yet unknown Poisson tensor should be restricted to agree with the last line in \( (3) \). This is due to the relation of the Poisson bracket on the target space and the local symmetries \( (3) \) and thus implicitly required by local Lorentz invariance, present in any gravity theory when formulated in Einstein–Cartan variables.

It is worth noting that restricting the Poisson tensor only by the last line of \( (3) \), the Jacobi identities \( (3) \) with one of the indices corresponding to \( \phi \) requires the Poisson tensor components \( P^{ab} \), \( P^{a\alpha} \), and \( P^{a\beta} \) to transform covariantly under Lorentz transformations! E.g., for \( P^{a\alpha} \) the Jacobi identities require \( \{ P^{a\alpha}, \phi \} = \varepsilon^b_{\alpha} P^{b\alpha} - \frac{1}{2} (\gamma^3)^a_{\beta} P^{a\beta} \). Thus to obtain the most general supergravity theory that fits into the present framework, we can proceed as follows: It must be possible to build the unknown tensor components \( P \) by means of the Lorentz covariant quantities \( X^\alpha \), \( \chi^\alpha \), \( \varepsilon^{ab} \), \( (\gamma^3)^a_{\alpha} \beta \), and \( (\gamma^3)^a_{\beta} (\varepsilon^a_{\beta}) \) is incorporated automatically by raising and lowering spinor indices) with coefficients that are Lorentz invariant functions, i.e. functions of \( X^\alpha \), \( \chi^\alpha \), and \( \phi \). Thus, e.g., the antisymmetric tensor \( P^{ab} \) must of the form \( P^{ab} = \varepsilon^{ab} (F_1 + \chi^\alpha \chi^\alpha F_2) \), where \( F_{1,2} \) are functions of the two arguments \( X^\alpha \), \( \chi^\alpha \) and \( \phi \). The remaining Jacobi identities then reduce to \( a \) (a comparatively simple) set of differential equations for these coefficient functions.

Proceeding in this way, e.g., by replacing all (five) coefficients in its first two lines of the bracket \( (3) \) by yet undetermined coefficient functions of \( X^\alpha \), \( \chi^\alpha \), \( \phi \), one can show that the remaining Jacobi identities \( (3) \) force the coefficients to agree with those provided already in \( (3) \) (except for a simultaneous global prefactor). More general theories can thus be obtained only by using further covariant entities to build \( P^{a\alpha} \) and (possibly also) \( P^{a\beta} \). Indeed, Lagrangians quadratic in torsion require an extra additive term \( F(X^\alpha X^\alpha, \phi) X^\alpha (\gamma^3)^{\alpha}_{\beta} \chi^\beta \) in \( P^{a\alpha} \), also perfectly compatible with Lorentz covariance. In more general supergravity theories will be constructed by the above method. By construction, the resulting theories will be invariant against superdiffeomorphisms incorporated within \( (3) \), thus allowing for an interpretation as supergravity theory. The supersymmetrization of the KV–model will be contained as a particular example in the class of models constructed in this way.
We finally turn to the solution of the field equations. Beside its notational compactness the main advantage of the formulation \( \text{I} \) as opposed to \( \text{II} \) is its inherent target space covariance. Thus we may change coordinates in the target space of the theory so as to simplify the tensor \( \mathcal{P} \), while the field equations in the new variables still will take the form \( \text{III} \) (but then with the transformed, simplified Poisson matrix). We will first use this method to show that locally the space of solutions to the field equations of \( \text{III} \) modulo gauge symmetries is just one–dimensional. Thereafter we will provide a representative of this one–parameter family in terms of the original variables used in \( \text{III} \). As a byproduct we will find that locally the space of solutions is identical to the one of the bosonic theory; all the fermionic fields may be put to zero by gauge transformations. Since any global solution can be obtained by patching together local solutions, we conclude that the local equivalence between the bosonic theory; all the fermionic fields may be put to zero by gauge transformations. Since any global solution can be obtained by patching together local solutions, we conclude that the local equivalence between the bosonic \( \text{II} \) and its supersymmetrization holds also on a global level. It would be interesting, however, to confirm this result in a more direct way.

Locally (more precisely, in the neighborhood of a generic point) any (bosonic) Poisson manifold allows for Darboux (CD) coordinates \( (C^A, Q^I, P_J) \) \( \text{III} \); constant values of the Casimir functions \( C^A \) label symplectic leaves in \( N \), on which the remaining coordinates are standard Darboux coordinates: \( \{ Q^I, P_J \} = \delta^I_J \) (all other brackets vanishing). According to \( \text{III} \), Darboux coordinates exist also for supersymplectic manifolds (manifolds with a graded, nondegenerate Poisson bracket). We thus assume that CD coordinates exist in the case of general, graded Poisson manifolds. However, at least in the case of the bracket \( \text{III} \), they definitely do: Although we did not succeed to find such coordinates explicitly in the present paper, we will provide a Casimir function below. On its level surfaces (defined by the appropriate quotient algebra of superfunctions) the Poisson bracket is (almost everywhere) nondegenerate and the result on supersymplectic manifolds may be applied.

We now are in the position to show that locally there is only a oneparameter family of gauge inequivalent solutions to the field equations of \( \text{III} \). For this purpose we only need to know about the local \textit{existence} of CD coordinates \( \tilde{X}^I \). In these coordinates \( \text{IV} \) the second set of field equations \( \text{V} \) reduce to \( dA_i = 0 \). Thus locally \( A_i = df_i \) for some functions \( f_i(x) \). However, the local symmetries \( \text{IV} \) also simplify dramatically in these new field variables: \( \delta A_i = d\tilde{e}_i \). This infinitesimal formula may be integrated easily showing that all the functions \( f_i \) may be put to zero identically. But then we learn from the first set of the field equations \( \text{VI} \) that all the functions \( \tilde{X}^i \) are constant. All of the constant values of \( Q^I \) and \( P_J \) may be put to an arbitrary value by means of the residual gauge freedom in \( \text{III} \) (constant \( \epsilon_i \)). What remains as gauge invariant information is only the constant values of the Casimir functions \( C^A \). Since the Poisson tensor \( \text{IV} \) has rank four (almost everywhere),\(^4\) the model defined by Eq. \( \text{VI} \) has just one Casimir function and its space of local solutions is thus indeed one–dimensional only.

The local solution obtained above in terms of CD-coordinates may be transformed back easily to any choice of target space coordinates. We find that also in the original variables: \( A_i \equiv 0 \) and \( X^i = \text{const} \), where the latter constants may be chosen at will as long as they are compatible with the constant values of the Casimir(s) \( C^A \), which characterize the (local) solution. As it stands, this solution corresponds to a solution with vanishing zweibein and metric. In a gravitational theory, the metric (and zweibein) is required to be a nondegenerate matrix, however. The vanishing zweibein is a result of using the symmetries \( \text{III} \), which, in contrast to diffeomorphisms, connect the degenerate with the nondegenerate sector of the theory. A similar phenomenon occurs, e.g., also within the Chern–Simons formulation of \((2 + 1)\)-gravity. The problem may be cured by applying a gauge transformation \( \text{IV} \) to the local solution \( A_i \equiv 0 \) so as to obtain a solution with nondegenerate zweibein. However, in contrast to Chern–Simons theory, where the behavior of \( A \) under \textit{finite} (nonabelian) gauge transformations is known, the infinitesimal gauge symmetries \( \text{IV} \) cannot be integrated in general (except for the case where the Poisson tensor is (at most) linear in the fields \( X^i \) and the theory reduces to a (non)abelian gauge theory). We thus need to introduce one further step.

For the Poisson brackets \( \text{III} \) a possible choice for a Casimir function \( C, \{ C, \cdot \} \equiv 0 \), is

\[
C = \frac{1}{2} X_\alpha X^\alpha + 2u^2 - \frac{i}{8} u' \chi_\alpha \chi^\alpha. \tag{10}
\]

\(^3\)To be sure: These are coordinates on the target space, not on the worldsheet spacetime. From the point of view of the field theory a change of coordinates \( X^i \rightarrow \tilde{X}^i \), which induces the change \( A_i \rightarrow \tilde{A}_i \equiv A_i (\partial X^i/\partial \tilde{X}^i) \), corresponds to a (local) change of field variables.

\(^4\)To determine the rank of the matrix \( \mathcal{P}_{ij} \), we may concentrate on the rank of the the two by two matrix \( \mathcal{P}^{\alpha\beta} \) and the three by three matrix in the purely bosonic sector; \( \mathcal{P}^{\alpha\beta} \), being linear in Grassmann variables, cannot contribute to the rank of the matrix, cf. \( \text{IV} \).
with respect to $\phi, \chi^\alpha$ Poisson commuting with all bosonic variables. We now choose new coordinates on (patches of) $N$ (where $X^a \neq 0$) according to
\[
\bar{X}^i := (C, \ln |X^+|, \chi^\alpha, \phi),
\]
with the null coordinates $X^\pm := (X^1 \mp X^2)/\sqrt{2}$. (An argumentation similar to the following one may be applied also if $\ln |X^+|$ is replaced by $X^+$ in (11)). $(C, \ln |X^+|, \phi)$ provides a CD coordinate system in the purely bosonic sector (cf. also [3]); however, in the five-dimensional target space $N$, these coordinates are far from being CD, several brackets still containing the potential $u(\phi)$. It thus seems rather difficult to solve the field equations for field variables (11). However, from our considerations above, we know that up to Poisson–σ gauge transformations the local solution always takes the from $A_\alpha \equiv 0$, $C = const$, $\ln |X^+| = \chi^\alpha = \phi = 0$. In these field variables, induced by the coordinates (11), it is now possible to gauge transform this explicitly to a solution with nondegenerate zweibein and, simultaneously, the resulting solution may be transformed back to the original variables used in (11) — this is possible since in contrast to the CD coordinates $\bar{X}^i$ used above, the coordinates $\hat{X}^i$ are known explicitly in terms of the original variables.

It is straightforward to verify that on the above solutions the infinitesimal gauge transformations (11) with $(\epsilon_i) := (\epsilon_C, \epsilon_+, 0, 0, 0)$ are simply:
\[
\delta A_C = \delta \epsilon_C, \quad \delta A_+ = d\epsilon_+, \quad \delta \phi = \epsilon_+,
\]
with all other variations vanishing. Note that e.g., in the second relation we dropped terms proportional to $A_\alpha$, since $A_\alpha$ can be kept zero consistently by the above transformations due to $\delta A_\alpha = 0$. It is thus possible to integrate the gauge symmetries (11): $A_C \rightarrow A_C + df_1$, $\phi \rightarrow \phi + f_2$, $\bar{A}_+ \rightarrow \bar{A}_+ + df_2$ (all other fields remaining unaltered) where $f_{1,2}$ is an arbitrary pair of functions on the 2d spacetime. The degenerate solution is then transformed into $\bar{A}_C = df_1$, $\bar{A}_+ = df_2$, $A_\alpha = A_\phi = 0$, $C = const$, $\ln |X^+| = \chi^\alpha = 0$, and $\phi = f_2$. Using $f_1$ and $f_2$ as coordinates $x^1$ and $x^0$ on the worldsheet, respectively, and transforming these solutions back to the original variables (11) (using $A_i = \bar{A}_j(\partial X^j/\partial X^i)$), we obtain:
\[
(e^+, e^-, \omega) = (dx^+, dx^0 + \frac{1}{2}h(x^0)dx^1, -h'(x^0)dx^1),
\]
\[
(X^+ = X^- = \phi = (1, \frac{1}{2}h(x^0), x^0),
\]
where $h(x^0) \equiv C - 2u^2(x^0)$, $C$ being the constant value of the Casimir (11). All the fermionic variables vanish identically.

Thus, up to gauge transformations, the local solution agrees completely with the one found in the purely bosonic dilaton theory (11). This applies at least to those patches where the above coordinate systems are applicable. Since, e.g., all the fixed points of the supersymmetric bracket (11) lie entirely within the bosonic sector of the target space, we expect that there are also no exceptional solutions, containing (necessarily) nonvanishing fermionic fields. Moreover, the subsequent global analysis of (11) may be applied to the solutions (13) without change.

So the characterization of the dynamics of the general supersymmetric extension of (11) turns out to be less difficult than it appeared at the time when (11) was written (cf the remarks following equation (50) of that paper). Rather, it appears that the supersymmetric extension of (11) is trivial (on–shell), at least at the classical level.

It would be interesting to check this result by some other method and to possibly establish it in a less indirect way. It is to be expected, moreover, that a similar result holds also on the quantum level.

Let us finally remark that the supersymmetric extension may still be of some value even on the purely classical level: In (11) it was used, e.g., to establish the positivity of (some notion of) the “mass” (presumably closely related to the Casimir $C$ above, cf. (11)) in a large class of (nonsupersymmetric) models (11) coupled to matter fields. Further investigations of 2d dilatonic supergravity theories, including generalizations to theories with nontrivial torsion and a comparison to the existing literature (11) is in preparation (14).

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