Multi-Scale Anisotropic Gaussian Kernels for Image Edge Detection

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ABSTRACT In this paper, a new edge detection method is proposed where multi-scale anisotropic Gaussian kernels (AGKs) are used to obtain an edge map from an input image. The main advantage of the proposed method is that high edge detection accuracy and edge resolution are attained while maintaining good noise robustness. The proposed method consists of three aspects: First, anisotropic Gaussian directional derivatives (AGDDs) are derived from the AGKs which are used to acquire local intensity variation from an input image with multiple scales. Second, multi-scale AGDD based edge strength maps (ESMs) are fused into a new ESM with high edge resolution and little edge stretch effect which has the ability to solve the contradiction issue between noise robustness and accurate edge extraction. Third, the fused ESM is embedded into the framework of Canny detection for obtaining edge contours. Finally, the criteria on precision-recall curve, detection accuracy, and noise robustness are used to evaluate the proposed detector against four state-of-the-art methods. The experimental results show that our proposed detector outperforms all the other tested edge detection methods.

INDEX TERMS Edge detection, anisotropic Gaussian kernel, anisotropic Gaussian directional derivative, multi-scale.

I. INTRODUCTION

EDGE detection is a very important basic operation in image understanding and image processing. Edge is the basic visual feature of displaying image pixel information in computer vision research, such as image segmentation [1], image retrieval [2], and corner detection [3], [4]. Therefore, an efficient and reliable edge detection algorithm is very important for many computer vision tasks.

The existing edge detection methods can be roughly divided into two groups [5]: local analysis method and global analysis method. The former is mainly based on differential [6]–[8], statistical [9]–[11], and multi-scale [12] methods. The latter is represented by active contour [13] methods. This paper focuses on the differential-based edge detection methods.

The differential-based methods find edges [14] by looking for the maximum of the absolute values of the first-order directional derivatives or the zero crossings of the second-order directional derivatives. Early edge detection methods such as Prewitt [6], Rosenfeld and Thurston [15], and Lyvers and Mitchell [16] methods used local gradient operators to extract edge contours from an input image. It is indicated in [5], [8] that the aforementioned methods [6], [15], [16] are sensitive to noise and cannot accurately obtain intensity variation information from an input image. To solve the aforementioned problems, Marr and Hildreth [17] used a Laplacian of Gaussian (LoG) operator for detecting edges. Later, Lindeberg [18] proposed the nearest-neighbor operation using approximate discrete derivative. The regular Laplacian zero-crossing is used as the optimal edge integrator [19] and the improved LoG operator [20] is used for detecting edges.

In view of image acquisition, the digital image is always corrupted by noise. Under this situation, edge detection can be regarded as the problem of optimal filter design. Canny [7] firstly derived an optimal filter for the extraction of a one-dimensional ideal step edge under the condition of white Gaussian noise. It is indicated [7] that the optimal filter
can be approximated by the first derivative of isotropic Gaussian kernel. Canny edge detector cascades Gaussian smoothing, gradient calculation, non-maxima suppression, and bi-threshold decision which was considered as the standard edge detection algorithm. It is worth to note that it is still difficult to choose parameters \[8\], \[21\] for Canny edge detector between scale selection and noise suppression. A Gaussian kernel with a small scale can extract fine detail of intensity variation information from an input image, but it tends to be sensitive to noise. On the other hand, A Gaussian kernel with a large scale achieves good noise suppression capability but degrades edge localization precision.

In [22]–[24], multi-scale techniques were adopted to form multi-scale edge detection methods which intend to avoid the difficulty of finding the appropriate scale and alleviate the difficulty of noise suppression and edge localization. Lindeberg [22] proposed an edge detection method with automatic scale-selection based on the regularized isotropic Gaussian kernel. Sumengen and Manjunath [23] presented a statistical approach using data driven probability distributions for edge cues at multi-scales. Another commonly used multi-scale technique is the fusion of multiple edge strength maps which are obtained at different scales. Bao et al. [25] extended Canny edge detector by using multi-scale multiplication for detecting edges. Shui and Zhang [8] proposed an edge detection method based on the multi-scale multiplication technique. Then the anisotropic Gaussian kernel and bi-threshold decision which was considered as the standard edge detection algorithm. It is worth to note that it is still difficult to choose parameters \[8\], \[21\] for Canny edge detection accuracy, high edge resolution, and noise robustness. Finally, a multi-scale multiplication technique which has high edge resolution, edge localization, and noise robustness while it has serious edge stretch effects. An AGDD is introduced first. Then, the anisotropic Gaussian kernel and its corresponding anisotropic directional derivative are presented.

A Gaussian kernel with a large scale achieves good noise suppression capability but degrades edge localization precision. Konishi et al. proposed a novel multi-scale edge detection by choosing finer scales for edge localization. Sumengen and Manjunath [23] proposed a novel multi-scale edge detection by choosing isotropic Gaussian kernel. Sumengen and Manjunath [23] proposed a novel multi-scale edge detection by choosing finer scales for edge localization. Konishi et al. [24] presented a statistical approach using data driven probability distributions for edge cues at multi-scales. Another commonly used multi-scale technique is the fusion of multiple edge strength maps which are obtained at different scales. Bao et al. [25] extended Canny edge detector by using multi-scale multiplication for detecting edges. Shui and Zhang [8] proposed an edge detection method based on the multi-scale multiplication technique. Then the anisotropic Gaussian kernel and bi-threshold decision which was considered as the standard edge detection algorithm. It is worth to note that it is still difficult to choose parameters \[8\], \[21\] for Canny edge detection accuracy, high edge resolution, and noise robustness. Finally, a multi-scale multiplication technique which has high edge resolution, edge localization, and noise robustness while it has serious edge stretch effects. An AGDD is introduced first. Then, the anisotropic Gaussian kernel and its corresponding anisotropic directional derivative are presented.

II. PROPERTIES OF THE ANISOTROPIC GAUSSIAN DIRECTIONAL DERIVATIVE

In this section, the multi-scale multiplication technique is introduced first. Then, the anisotropic Gaussian kernel and its corresponding anisotropic directional derivative are presented.

A. MULTI-SCALE MULTIPLICATION TECHNIQUE

Providing that a noisy edge \(D(x)\) consists of true edge \(S(x)\) and Gaussian white noise \(w(x)\). The filter is denoted by \(f(x)\), which is dilated by scale \(\sigma\) as \(f_{\sigma}(x) = f(x/\sigma)/\sigma\). The support of \(f_{\sigma}(x)\) is \([-\sigma \Gamma, \sigma \Gamma]\), where \(\Gamma\) is the interval value. Thus, the response of \(D(x)\) to \(f_{\sigma}(x)\) can be denoted as \(H_{D}^{\sigma}(x) = H_{S}^{\sigma}(x) + H_{w}^{\sigma}(x)\), where \(H_{S}^{\sigma}(x)\) and \(H_{w}^{\sigma}(x)\) are the response of \(S(x)\) and \(w(x)\), respectively. In this paper, the anisotropic directional derivative is adopted as an edge detection filter, which is derived from the function of an anisotropic Gaussian kernel. Three scales \(\sigma_{1}, \sigma_{2}\), and \(\sigma_{3}\) with increasing values are fitted to detect the edge. The responses at the three scales are \(H_{D}^{\sigma_{1}}(x)\), \(H_{D}^{\sigma_{2}}(x)\), and \(H_{D}^{\sigma_{3}}(x)\). The scale multiplication [25] is defined as the product of the \(H_{D}^{\sigma_{1}}(x)\), \(H_{D}^{\sigma_{2}}(x)\), and \(H_{D}^{\sigma_{3}}(x)\)

\[
R_{D}(x) = H_{D}^{\sigma_{1}}(x) \cdot H_{D}^{\sigma_{2}}(x) \cdot H_{D}^{\sigma_{3}}(x). \tag{1}
\]

A noisy step edge \(D(x)\) is shown in Fig. 1(a). The position of the step edge is marked by a vertical line. Three filters, \(f_{\sigma_{1}}, f_{\sigma_{2}},\) and \(f_{\sigma_{3}}\) (\(\sigma_{1}=1.5, \sigma_{2}=2.5, \) and \(\sigma_{3}=3.5\)) as shown in Fig. 1(b), (d), and (f) respectively, are used to smooth the step edge \(D(x)\). Filter responses \(f_{\sigma_{1}}, f_{\sigma_{2}},\) and \(f_{\sigma_{3}}\) to \(D(x)\) are shown in Fig. 1(c), (e), and (g) respectively. The filter response of multi-scale multiplication to \(D(x)\) is shown in Fig. 1(h). It can be observed from Fig. 1(c) that the filter response with small scale has accurate edge localization while it has some obvious false local maxima. When a step edge is smoothed by a filter with a scale \(\sigma_{2}\), it can be observed that the edge localization accuracy decreases and there still exists some false local maxima. When step edge is smoothed by a filter with scale \(\sigma_{3}\), it can be observed that the false local maxima on step edge decreases while the edge location greatly deviates from the real edge position. On the contrary, the filter response using the multi-scale multiplication...
corrupted by zero-mean white noise $\varepsilon$ the technique to $D(x)$ has the ability to obtain good edge localization and noise suppression as shown in Fig. 1(h).

B. ANISOTROPIC GAUSSIAN KERNELS AND ANISOTROPIC GAUSSIAN DIRECTIONAL DERIVATIVE FILTER

In Canny edge detector [7], an image $I(x)$, $x = (x, y)^T$ is smoothed by an isotropic Gaussian kernel with a scale $\sigma$

$$I_\sigma(x) = I * g_\sigma(x) = \int \int I(x-u) g_\sigma(u) \, du,$$  

where superscript $T$ represents matrix transpose and symbol "$*$" represents a convolution operation. The Canny detector extracts an edge map from an input image by seeking the maxima of the pixels’ gradient magnitudes $|\nabla I_\sigma(x)|$.

$$\nabla I_\sigma(x) = \left[ \frac{\partial}{\partial x} I_\sigma(x), \frac{\partial}{\partial y} I_\sigma(x) \right]^T = I * \nabla g_\sigma(x),$$

$$\nabla g_\sigma(x) = -\frac{x}{\sigma^2} \nabla g_\sigma(x).$$

In the spatial domain, an anisotropic Gaussian kernel (AGK) [5], [8], [26], [28]–[30] $g_{\sigma, \rho, \theta}(x)$ can be represented as

$$g_{\sigma, \rho, \theta}(x) = \frac{1}{2\pi \sigma^2} \exp\left(-\frac{x^T R_\theta^{-1} x}{2\sigma^2}\right),$$

with

$$R_\theta = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix},$$

where $\rho$ is the anisotropic factor ($\rho > 1$), and $R_\theta$ is the rotation matrix with angle $\theta$. Assuming that an image is corrupted by zero-mean white noise $\varepsilon(x)$ with a variance $\varepsilon_w^2$, the $\varepsilon_w^2$ of the noise $\varepsilon(x)$ smoothed by the AGKs with a scale $\sigma$ is [8]

$$\varepsilon_w^2 = E \left( (w * g_{\sigma, \rho, \theta}(x))^2 \right) = \frac{\varepsilon_w^2}{4\pi \sigma^2}. \quad (7)$$

The capability of an AGK to suppress noise is uniquely determined by its scale and is independent of the anisotropic factor $\rho$ and orientation angle $\theta$. From an AGK with orientation angle $\theta$, the anisotropic directional derivative (AGDD) along the direction $\theta$ of an image is defined as

$$\frac{\partial I(x)}{\partial \theta} = \frac{\partial}{\partial \theta} (I * g_{\sigma, \rho, \theta}(x)) = I * g'_{\sigma, \rho, \theta}(x), \quad (8)$$

$$g'_{\sigma, \rho, \theta}(x) = -\frac{\rho^2 \cos \theta, \sin \theta \, |x|}{\sigma^2} g_{\sigma, \rho, \theta}(x), \quad (9)$$

where $g'_{\sigma, \rho, \theta}$ is referred to as the AGDD filter, which reflects the gray-scale variation of the image along the orientation $\theta$. Since the absolute value of $g'_{\sigma, \rho, \theta}(x)$ at and $\theta + \pi$ is identical, it means that only the AGDDs for the orientation range $\theta \in [0, \pi]$ need to be calculated for an image. The capability of the AGDD filters to suppress noise can be measured by the variance of its response to zero-mean white noise $\varepsilon(x)$ with a variance $\varepsilon_w^2$. The variance of the noise response is [8]

$$\varepsilon_w^2 = E \left( (w * g'_{\sigma, \rho, \theta}(x))^2 \right) = \frac{\varepsilon_w^2}{8\pi (\alpha(\rho)^2)^2}. \quad (10)$$

It is directly proportional to the noise variance and inversely proportional to the square of the scale and the square of ratio of the scale to the anisotropic factor.

An AGK with $\sigma = 1$, $\rho^2 = 1.5$, and $\theta = \frac{3\pi}{4}$ and an AGDD filter with $\sigma = 1$, $\rho^2 = 1.5$, and $\theta = \frac{3\pi}{4}$ are shown in Fig. 2(a) and (b) respectively. It is proved in [26] that the designed ANDD filter $g'_{\sigma, \rho, \theta}$ has the ability to effectively suppress noise and accurately extract intensity variation information from an input image. The aforementioned AGDD properties will help us to present a new edge detection measure in the following section.

III. A NEW EDGE DETECTION MEASURE

In this section, the signal-to-noise ratio (SNR) of the anisotropic Gaussian directional derivative is analyzed. Then, multi-scale edge strength maps of the AGDD are presented. Finally, a new edge detection measure is derived which has good edge detection accuracy and noise robustness.
A. SIGNAL-TO-NOISE RATIO (SNR) OF THE AGGD

In [5], a step edge is expressed as

\[ E_{\alpha}(x) = TH([\sin \alpha, \cos \alpha]x), \quad (11) \]

where \( T \) is the edge strength and \( H(x) \) is a Heaviside function \((H(x) = 0, x < 0; H(x) = 1, x \geq 0)\). The anisotropic Gaussian directional derivation representation of the step edge is

\[ h_{\alpha,T}(\theta) = \frac{T}{\sqrt{2\pi}\sigma} \frac{\cos(\theta - \alpha)}{\sqrt{\rho^2 \sin^2(\theta - \alpha) + \rho^{-2} \cos^2(\theta - \alpha)}}. \quad (12) \]

When \( \alpha = \theta \), the AGDDs achieve the maximal magnitude \( T\rho/(\sqrt{2\pi}\sigma) \). The SNR is defined as the ratio of the maxima magnitude to the standard deviation of the noise \( \varepsilon_{\hat{\omega}} \)

\[ \text{SNR} = \frac{1}{\varepsilon_{\hat{\omega}}} \max_{\theta \in [0, \pi]} \{|h_{\alpha,T}(\theta)|\} = \frac{T\rho}{\sqrt{2\pi}\sigma} \frac{\sqrt{\pi}\sigma^2}{\rho\varepsilon_{\hat{\omega}}} = \frac{2T\sigma}{\varepsilon_{\hat{\omega}}}. \quad (13) \]

It can be derived from Equation (13) that the SNR of the AGDD is directly proportional to the product of the edge strength and the scale and inversely proportional to the noise standard deviation [5], [8]. In terms of the SNR representation, the AGDD-based ESM is noise-robust and with high edge resolution in the case of a large scale and a small ratio of the scale to the anisotropic factor. The \( \max \) operator performs a simple selective smoothing of an image [31] and the best smoothing result is selected for each pixel according to the predefined number of directions \( K \) of AGKs. Though smoothing necessarily blurs edges, image smoothing is necessary for edge detection of noisy images. Actually, smoothing also causes edge stretching that severely affects the quality of an edge map extracted from an AGDD-based ESM.

When an image is smoothed by an AGGD with large scale and anisotropic factor, the detected edge will be widened along the direction perpendicular to itself. Meanwhile, the edge will be also elongated along its direction. The two phenomena are referred to as edge blurring and edge stretching respectively. As a result, the edge detection using an ANDD-based ESM tends to generate pseudo-edge pixels around the actual edges. Considering the superiority of a small scale AGDD-based ESM with little edge stretch, the new ESM is defined as

\[ \eta_f(x) = \frac{1}{3} \prod_{i=1}^{3} \max_{k=1,2,\ldots,K} \{ f \ast g_{\alpha,\rho,\theta_k}(x) \}. \quad (14) \]

The fused ESM based on Equation (14) has the ability to solve the impact of edge stretch effect and maintain high edge resolution, and noise robustness. Under this approach, the proposed edge detection measure is defined as the fused ESM of the multi-scale AGGDDs.

A test image is shown in Fig. 3(a). The test image smoothed with Gaussian standard deviation \( \varepsilon_{\hat{\omega}} = 15 \) is shown in Fig. 3(b). The AGGD-based ESM with a small scale (\( \sigma = 1.5 \)) is shown in Fig. 3(c). It can be observed from Fig. 3(c) that it has a little edge stretching on the ESM and the background of the ESM is dirty. The reason is that the ESM with a small scale is sensitive to noise. With the scale increasing, it can be observed from Fig. 3(d) and (e) that the edge stretch effect of the ESM increases and the effect of noise on ESM is weakened. The fused ESM as shown in Fig. 3(f) inherits the merits of AGDD-based ESM at three different scales which has little edge stretch effect and it is noise robust.

IV. EDGE DETECTION USING MULTI-SCALE SPACE ANISOTROPIC GAUSSIAN KERNEL

In this section, a novel multi-scale edge detection method based on fused ESM is proposed. The new edge detection includes three main steps: smoothing the image using AGGD filters at three different scales, calculating ESM fusion, and deciding the hysteresis threshold.

A. DISCRETE AGGD FILTERS AND FUSED ESM

Given a scale \( \sigma \) and an anisotropic factor \( \rho \), sampling in the 2D integer lattice \( Z^2 \) yields the discrete AGKs and AGGD filters with the expressions as follows:

\[ g_{\alpha,\rho,\theta}(n) = \exp \left( -\frac{1}{2\sigma^2} n^T R_k^T \right) \frac{\rho^2}{\rho^2 - \rho^{-2}} R_k n, \]

\[ g'_{\alpha,\rho,\theta}(n) = -\frac{[\cos \theta_k, \sin \theta_k] n}{\theta_k^2 \rho^{-2}} g_{\alpha,\rho,\theta}(n), \]

\[ R_k = \begin{bmatrix} \cos \theta_k & \sin \theta_k \\ -\sin \theta_k & \cos \theta_k \end{bmatrix}, \quad n = [n_x, n_y] \in Z^2, \]

\[ \theta_k = k\pi/K, k = 1, 2, \ldots, K. \quad (15) \]
For an input image $I(n)$, the AGDD-based ESM of the input image is calculated by

$$h_{\sigma}(n) = \max_{k=1,2,...,K} \left[ l \ast g_{\sigma,\rho,k}(n) \right].$$  \hspace{1cm} (16)

Then, the fused ESM is calculated by

$$\eta(n) = \sqrt{\frac{3}{\sum_{i=1}^{3} h_{\sigma}(n)}}.$$  \hspace{1cm} (17)

The new edge detection measure is defined as the fused ESM.

**B. CONTRAST EQUALIZATION AND NOISE STATISTIC**

The fused ESM provides the global gradient changes of an input image, while the edge judgment in the visual system is usually based on the local relative changes. For instance, in a rough region with texture or fine periodic oscillations, the ESM values at some pixels are small, but these pixels do correspond to visual edges, such as the boundary of two constant areas with a small gray difference. To solve this problem, this paper applied a contrast equalization technique [8] which was based on local image intensity variation to reduce spurious edges in rough areas and find weak edges in small intensity variation areas.

The overall average change of an image can be calculated as

$$\bar{s} = \frac{1}{MN} \sum_{n} \eta(n),$$  \hspace{1cm} (18)

where $MN$ is the size of the input image, $\eta(n)$ is the value in Equation (17). The grayscale variation of the image is often uneven, and the texture area has more changes than the smooth area. The local average variation of pixel $n$ is defined as

$$\bar{s}_{Local}(n) = \frac{1}{W^2} \sum_{\tau \in Q} \eta(n + \tau),$$  \hspace{1cm} (19)

where $Q$ is a $W \times W$ squared window and $\tau$ is the changeable distance in the window. Simulating the visual system, the contrast equalization is used for the modified fused ESM, which is defined as

$$\tilde{\eta}(n) = \frac{\eta(n)}{\bar{s} + 0.5 \bar{s}_{Local}(n)}.$$  \hspace{1cm} (20)

Without loss of generality, assume that noise $w(n)$ is zero-mean white Gaussian noise with a variance $\epsilon_w^2$. Then the noise threshold can be expressed as

$$T_{noise} = \begin{cases} 0, & \tilde{\eta}(w(n)) = 0 \\ \epsilon_w^2, & \epsilon_w^2 \neq 0 \end{cases}$$  \hspace{1cm} (21)

where $p_f$ is the probability for spurious pixels, the symbol “[]” is zero-rounding operation, $n_{[0.05MN]}$ represents the $[0.05MN]$th pixel arranged by the gray value from small to large.

**C. PROPOSED EDGE DETECTION METHOD**

For an input image $I(n)$, the proposed new edge detection method applied the multi-scale AGDD filters to obtain the fused edge maps from an input image.

The proposed corner detection method is described as follows:

(i) Computation of ESMs: calculate ESMs at three different scales: $h_{\sigma_i}$, $i = 1, 2, 3$, and fuse them into a new ESM: $\eta(n)$ in terms of Equation (17).

(ii) Contrast equalization: calculate the average variation of image $\bar{s}$ and local average variation $\bar{s}_{Local}(n)$, using Equation (20) for contrast equalization to obtain $\tilde{\eta}(n)$.

(iii) Non-maxima suppression: for each pixel, the numerical value $\tilde{\eta}(n)$ and the gradient orientation $\theta(k)$ are used to decide whether it is a maximum of $\tilde{\eta}(n)$. All the maxima form the candidate set of edge pixels, which are labelled by $\Omega(n)$.

(iv) Upper and lower thresholds: the upper and lower thresholds are required for hysteresis operations. The upper threshold is determined by the percentage of the histogram of $\tilde{\eta}(n)$. In the noiseless image, $T_{upper}$ and $T_{lower}$ are given by the following formula

$$T_{upper} = \Omega(I(n_{[\beta_{upper}MN]})), \hspace{1cm} (22)$$

$$T_{lower} = \Omega(I(n_{[0.5MN]})). \hspace{1cm} (23)$$

where the symbol “[]” is zero-rounding operation, $n_{[\beta_{upper}MN]}$ and $n_{[0.5MN]}$ represent the $[\beta_{upper}MN]$th pixel and the $[0.5MN]$th pixel arranged by the gray value from small to large, and $\beta_{upper}$ is in the interval $[0.6, 0.95]$. For noisy images, the lower threshold also depends on the noise level and the expected of the false alarm probability $p_f$. According to Equation (21), the lower threshold is given by the following equation

$$T_{lower} = \max \left\{ \tilde{T}_{lower}, T_{noise} \right\}.$$  \hspace{1cm} (24)

The lower threshold is spatially dependent because contrast equalization changes the stationarity of noise $\tilde{\eta}(w(n))$.

(iv) Hysteresis determination: the determination of an edge pixel is implemented in two steps. $\Omega(n)$, the maxima set, and all pixels whose intensity exceeds $T_{upper}$ are first identified as edge pixels. These pixels form a strong edge set

$$J_{edge} \equiv \{ n \in \Omega(n): \Omega(n) \geq T_{upper} \}. \hspace{1cm} (25)$$

In addition to strong edge pixels, pixels in set $C \equiv \{ n \in \Omega(n): T_{lower} < \Omega(n) < T_{upper} \}$ are recognized as edge pixels if the pixels are connected to strong edge pixels according to eight adjacent fields. These edge pixels form a weak edge pixel group, represented by $V_{edge}$. The collections $J_{edge}$ and $V_{edge}$ together form the edge image.

An example of the flow chart of the proposed algorithm is shown in Fig. 4.

**V. EXPERIMENTAL RESULTS AND PERFORMANCE EVALUATION**

This section reports the full performance evaluation results of the proposed edge detection method. The proposed method
is compared with four state-of-the-art detectors [7], [8], [25], [27]. The criteria on precision-recall curve based on the Berkeley segmentation data set and benchmark 500 (BSDS500) [32], detection accuracy, and noise robustness are used to compare the performance of the five edge detection methods.

A. EVALUATION INDEX

Edge detection can be considered as a binary classification process where pixels of an input image are divided into edge pixels and non-edge pixels. Under this way, the edge detection results and its corresponding ground truth edge maps should be given. Meanwhile, an evaluation criteria is given to judge the similarity between the detection results and the ground-truth edge maps. Finally, the edge detection method can be evaluated whether it is successful or not.

In this expectation, the performance of the five methods are assessed through the precision-recall framework [33] which has been widely adopted in the literature [11], [34], [35]. Precision is the ratio of the number of detected real edge pixels and the number of detected edge pixels. Recall is the ratio of the the number of detected real edge pixels and the number of missing edge pixels in the ground truth map.

Providing that $\varphi_d$ are the results of edge detection from input images, and $GT = \{G_1, G_2, \ldots, G_{N_G}\}$ are ground truth edge maps which correspond to the detection results. $N_G \in N_+$ represents the total number of ground truth edge maps. The cost scaling assignment (CSA) [36] algorithm is used to perform pixel-to-pixel matching between $\varphi_d$ and the ground-truth edge map in $G$. For each detected edge pixel in $\varphi_d$, the detected edge pixel will be marked as a true positive detected pixel if it matches the edge pixel in the ground truth edge map within a spatial tolerance distance. Otherwise, it will be marked as a false positive detection pixel. On the other hand, for a true edge pixel in each ground truth edge map, if it is matched by the detected edge pixel within the spatial tolerance distance, the edge pixel of the ground truth is marked as a matched ground-truth pixel. Otherwise, it is treated as an unmatched benchmark pixel. These matched ground truth pixels and unmatched ground truth pixels in all ground-truth maps constitute the aggregated matched ground truth pixels and aggregated unmatched ground truth pixels respectively.

In this way, the rates of precision and recall are calculated by

$$
\zeta_{\text{precision}} = \frac{\nu_{TP}}{\nu_{TP} + \nu_{FP}},
$$

$$
\zeta_{\text{recall}} = \frac{\nu_{MT}}{\nu_{MT} + \nu_{UM}},
$$

where $\nu_{TP}$, $\nu_{FP}$, $\nu_{MT}$, and $\nu_{UM}$ are true positive detection pixels, false positive detection pixels, aggregated matched ground truth pixels, and aggregated unmatched ground truth pixels respectively. It is worth to note that different thresholds will lead to different detection results. The performance of the edge detection methods are evaluated by different thresholds and multi scales. Then all performance results are interpolated subsequently to form a precision-recall (PR) curve [37]. For PR curve, whether an algorithm is good or nor depend on the two factors: PR curve with a distance from the origin or area under the PR curve. It is worth to note that the area enclosed by the curve can also be evaluated by the parameter of average precision (AP) [37].
The F-measure [33] is used to calculate the harmonic mean of precision and recall.

\[ F = \frac{2 \cdot \text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}. \]  

(28)

The threshold is needed to produce the edge map when the ESM is given. There are two ways to choose the optimal threshold. The first way is to apply a fixed threshold to all images in the dataset, which named the optimal dataset scale (ODS) threshold. The second way is to select an optimal threshold for each image, which named the optimal image scale (OIS) threshold. The F-measure of ODS \((F_{ODS})\) and OIS \((F_{OIS})\) are calculated in this experiment.

The R50 is used to evaluate the precision when recall is at 50\%, which indicates the detection accuracy in high recall regime [11].

In this paper, Pratt’s figure of merit (FOM) [38] is used to test the performance of the noise robustness of the five edge detection methods. In FOM, loss of true edge pixels, spurious edge pixels, and edge localization errors are considered. The FOM is defined as

\[ FOM = \frac{1}{\max(\gamma_1, \gamma_0)} \sum_{i=1}^{\gamma_0} \frac{1}{1 + 0.25d^2(i)}, \]  

(29)

where \(\gamma_1\) and \(\gamma_0\) are sets of the numbers of the edge pixels in the ideal edge map and the detected edge map respectively, \(d(i)\) represents the distance from the \(i\)th detected edge pixel to the ideal edge map. Ideally, the value of FOM is 1. In the experiment, the same edge pixels in at least three detected edge maps from the five methods are selected to form the ideal edge maps.

B. EXPERIMENTAL CONFIGURATION

The proposed method is compared with four state-of-the-art detectors (Canny [7], IAGK [8], AAGK [5], MCanny [27]). All the competing methods are based on the gray difference. Meanwhile, all the five methods are based on the Gaussian filtering and adopt different strategies to measure edge strength. In order to highlight the influence of different strategies on edge detection, the binarization process remains the same for all methods.

In this experimental validation section, each method is configured as follows:

- Canny [7]: The Gaussian scale is \(\sqrt{2}\) which is widely adopted in [8], [5], [27].
- IAGK [8]: The scale is 4, the anisotropic factor is \(2\sqrt{2}\), and the number of kernel orientation is 16.
- AAGK [5]: The number of kernel orientation is 8 [5], the scale is \(\sqrt{10}\), and the anisotropic factor is \(\sqrt{5}\).
- MCanny [25]: The values of \(\sigma_1\) and \(\sigma_2\) are 1 and 4 respectively.
- Proposed: The selection range of the three scales is \(\sigma \in [1, 6]\), the number of orientation is 8, and the anisotropic factor is \(\sqrt{15}\). The default scale factors \((\sigma_1, \sigma_2, \text{ and } \sigma_3)\) are \(1, \sqrt{2}, \text{ and } \sqrt{3}\) respectively.

As in [32], [39], [40], the matching tolerance distance in the evaluation process is set as 0.75\% of the diagonal line of the image.

C. EXPERIMENT RESULTS BASED ON THE BSDS500 DATASET

The BSDS500 is a widely used dataset in performance comparison on edge detection methods. It includes 200 training, 100 validation, and 200 test images. Each image is labeled by 4 to 9 annotators.

The PR curves of the five edge detection methods are shown in Fig. 5. It can be observed that the proposed method achieves the maximum area under the PR curve. The reason is that the proposed method utilized the anisotropic Gaussian directional derivative filters which have the ability to accurately obtain the local structure information from the input image. Furthermore, the proposed method applied the multi-scale multiplication technique which made the proposed method to have the better performance in noise robustness. Meanwhile, it can be found from Tab. 1 that the proposed method achieves better performance in terms of \(F_{ODS}, F_{OIS}, \text{ and } R50\) indexes.

In this experiment, six test images are shown in the first row of Fig. 6. The ground truths of the six test images are shown in the second row of Fig. 6. The edge detection results
FIGURE 6. Detection results of five methods on six test images. The first row is the test images. The second row is the ground truths of the six test images. The detection results of Canny [7], AAGK [5], IAGK [8], MCanny [25], and proposed method are shown in the third, fourth, fifth, sixth, and seventh rows respectively.

It can be observed that the proposed method can accurately detect edges from input images. Furthermore, the performance of the five edge detection methods are evaluated in noisy images with standard variation $\varepsilon_w^2 = 10$. The noisy test images are shown in the first row of Fig. 7. The ideal edge maps of the noisy test images are derived in terms of the FOM criteria [38] which are shown in the second row of Fig. 7. The detection results of Canny [7], AAGK [5], IAGK [8], MCanny [25], and the

| $\varepsilon_w^2 = 10$ | Elephant | Elk | Boat | Manor | Fish | House |
|----------------------|----------|-----|------|-------|------|-------|
| Y.                   | 69078    | 69273| 69165| 69067 | 68821| 69069 |
| Canny [7]            | 0.9796   | 0.9763| 0.9743| 0.9793 | 0.9794| 0.9782 |
| AAGK [5]             | 0.9583   | 0.9718| 0.9665| 0.9685 | 0.9619| 0.9638 |
| IAGK [8]             | 0.9587   | 0.9687| 0.9705| 0.9745 | 0.9566| 0.9775 |
| MCanny [25]          | 0.9808   | 0.9837| 0.9842| 0.9835 | 0.9795| 0.9837 |
| Proposed             | 0.9824   | 0.9852| 0.9869| 0.9835 | 0.9805| 0.9870 |
FIGURE 7. Detection results of five methods on six noisy images. The first row is the noisy images. The second row is the ideal edge map of the six test images. The detection results of Canny [7], AAGK [5], IAGK [8], MCanny [25], and proposed method are shown in the third, fourth, fifth, sixth, and seventh rows respectively.

It can be observed that the proposed method has better noise robustness.

VI. CONCLUSION
This paper proposed an edge detector with high edge detection accuracy and good noise robustness. It used multi directional anisotropic Gaussian directional derivative filters with multiple scales to smooth the input image. A Multi-scale multiplication technique is used to fuse an edge map from the multi ESMs. The proposed edge detection method is compared with four state-of-the-art edge detectors. Precision-recall curve based on the BSD500 dataset, edge detection accuracy, and noise robustness are used to assess the performance of the proposed edge detection method. The experimental results show that the proposed method is of very high quality.

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