The hybrid grey wolf optimization-slime mould algorithm

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Abstract—In this paper, we hybridize the grey wolf optimization (GWO) algorithm with the newly proposed slime mould algorithm (SMA). Comparisons had been made and three kinds of benchmark functions were introduced to verify the capability. 100 Monte Carlo simulation experiments had been carried on to reduce the influence of randomness as less as possible. Results showed that the performance of hybrid GWO-SMA would base on the given characteristics of problems themselves because of the random threshold parameter p and the multiple branches in the updating equation. The hybridization of the GWO and SMA might be not recommended for steady applications and engineering problems.

1. INTRODUCTION
Lots of nature-inspired algorithms have been proposed by now, and most of the swarm-based algorithms focused on the behavior of swarms in nature. Because of the inherent nature of swarms, most of the algorithm were formulated with unique equations and different to each other. For instance, in the ant colony optimization algorithm[1], the current and averaged information would control the updating for every ants. The global best, historical best and the current positions would all be involved in updating for every birds in the particle swarm optimization algorithm[2]. In 2014, the grey wolf optimization algorithm[3] was raised and three best candidates were introduced during the updating, moreover, four best candidates including an averaged one would play their roles in updating in the equilibrium optimization algorithm[4]. Every algorithm had its unique and better performance might be gained in applications.

One kind of the improvements for the existed swarm-based algorithm is to hybridize two or more unique characteristics and construct a new algorithm, such operations are called hybridization and the improved algorithms are called hybrid algorithms literally.
In this paper, we would focus on the slime mould algorithm \[^5\] (SMA) which has just been proposed recently. Considering the global best candidates is involved, we hybridize the GWO algorithm with SMA and we would carry out some simulation experiments to verify the performance.

2. The SMA

The SMA is similar to other swarm-based algorithms, the individuals would spread all over the research domain and they would be guided towards the global optimum during iterations. The optimization procedure would be classified into several steps, such as initialization, searching and exploiting.

2.1 The initializing procedure

All of the individuals in swarms would be initialized randomly and uniformly all over the domain \([\text{LB}, \text{UB}]\):

\[
x_i = r_1 (\text{UB} - \text{LB}) + \text{LB}
\]

Where \(r_1\) is a random number in Gauss distribution. All of the values for every parameter should be initialized and setup, such as the population size, the maximum allowed iteration time \(\text{maxIter}\), and so on.

2.2 The iterations

During the exploring and exploiting procedure, iterations would be carried out and during the iterations, the positions for each individuals would be updated and guided to the global optimum:

\[
x_i(t + 1) = \begin{cases} 
    r_2 (\text{UB} - \text{LB}) + \text{LB} & r_2 < z \\
    x_b + r_2 \cdot (W \cdot x_c(t) - x_b(t)) & r_3 < p \\
    v_c \cdot x_i(t) & p \leq r_3 \leq 1
\end{cases}
\]

Where \(x_i(t)\) and \(x_i(t + 1)\) represent the position for \(i\)-th candidate in swarms in the current iteration \(t\) and next iteration \(t+1\); \(r_2\) is another random number in Gauss distribution; \(z\) is a proportional number to randomly select some candidates to re-begin the initialization, \(z=0.03\) as defaults. \(x_A(t)\) and \(x_B(t)\) are two randomly selected candidates in the current iteration. \(v_b\) and \(v_c\), are another two random numbers yet in uniform distribution with the interval of \([-a, a]\) and \([-b, b]\) respectively. Here \(a\) and \(b\) are two variables relevant to the iteration number and the maximum allowed iteration time:

\[
a = \tanh \left(1 - \frac{t}{\text{maxIter}}\right)
\]

\[
b = 1 - \frac{t}{\text{maxIter}}
\]

\(p\) is another proportional number to control the choice of branches, it is relevant to the global best fitness value \(DF\):

\[
p = \tanh |S_i - DF|
\]

The most complicated parameter is the weights \(W\), it is a matrix and would be calculated as follows:

\[
W_{si(i)} = \begin{cases} 
    1 + r_4 \cdot \log \left(1 + \frac{bF - S_i}{bF - wF}\right) & \text{condition} \\
    1 - r_4 \cdot \log \left(1 + \frac{bF - S_i}{bF - wF}\right) & \text{others}
\end{cases}
\]

Where \(bF, wF\) are the best and worst fitness values for among all the fitness values \(S_i\) \((i = 1, 2, \cdots, n)\). And \(si\) is the sort of all of the fitness values for all of the individuals.

\[
si = \text{sort}(S)
\]

3. The GWO algorithm and hybrid GWO-SMA

3.1 The GWO algorithm

The updating equation for individuals in grey wolf swarms is shown as follows discarding the detailed calculations:

\[
x_i = \frac{x_a + x_b + x_c}{3}
\]

In order to simulate the real social hierarchy of grey wolf swarms, we had proposed the variable weights for the standard GWO algorithm\[^6\].
\[ \varphi = \frac{1}{2} \tan(t) \]  
\[ \theta = \frac{2}{\pi} \cos \left( \frac{1}{3} \tan(t) \right) \]  
\[ \omega_1 = \cos \theta, \omega_2 = \frac{1}{2} \sin \theta \cdot \cos \varphi, \omega_3 = 1 - \omega_1 - \omega_2 \]  

3.2 The hybrid GWO-SMA
Hybridizing the GWO and SMA, we replace the best candidate \( x_b \) with the average of alpha, beta and delta candidates, and consequently, the updating equation for hybrid GWO-SMA algorithm would be:

\[
\begin{align*}
    x_i(t + 1) &= r_1 \cdot (UB - LB) + LB \\
    &+ \omega_1 x_b + \omega_2 x_{\beta} + \omega_3 x_{\delta} + v_b \cdot \left[ W \cdot x_A(t) - x_B(t) \right] \\
    &+ v_c \cdot x_i(t) \\
    &\quad \text{if } r_2 < z \\
    &+ v_e \cdot x_i(t) \\
    &\quad \text{if } r_3 < p \\
    &+ v_f \cdot x_i(t) \\
    &\quad \text{if } p \leq r_3 \leq 1
\end{align*}
\]  

4. SIMULATION EXPERIMENTS
In this section, we would carry out some simulation experiments and verify the capability of the hybrid GWO-SMA. We would introduce some benchmark functions for simplicity. More efforts for real engineering problems could be done if the results are promising.

Furthermore, we would carry on 100 Monte Carlo simulation experiments and make an average for all of the outputs, such operations are believed to reduce the influence of randomness as less as possible and the averaged results would be more convinced too.

Efforts have proved and verified that some of the benchmark functions are easy to optimize, some of them not. The main difference between them might be based on their modality, dimensionality, scalability, continuity, or basins/valleys in their profiles\(^7\). Therefore, we would carry on three kinds of simulation experiments.

4.1 Experiments on unimodal benchmark functions
This kind of benchmark functions have only one local optima, which are also the global optima. Most of these benchmark functions are easy to optimize and the best solutions might be obtained after several iterations. A representative called Chung Reynolds function would be used here:

\[
f(x) = \sum_{i=1}^{d} x_i^2\]  

Chung Reynolds function is continuous, differentiable, partially-separable, scalable, and unimodal function, it has no constraints for every parameter and the global optimum is located at the Origin \( x^* = (0, 0, \cdots, 0) \) and \( f(x^*) = 0 \). This function looks like a basket seen from the three-dimensional profile in Figure 1. And the best fitness values in each iteration are shown in Figure 2.

![Figure 1 Three-dimensional profile for Chung Reynolds function](image)
We can see from Figure 2 that the performance of SMA is indeed better than that of GWO, however, the performance of hybrid GWO-SMA might remain in the same level with that of SMA.

### 4.2 Experiments on multi-modal benchmark functions

The multimodal benchmark functions usually have quite a lot of local optima. The global optima might buried among them. Consequently, they are difficult to optimize and more iterations might be needed, furthermore, sometimes the optimization would fail. In this simulation experiment, we would introduce a representative with lots of local optima, seen from Figure 3, its equation is shown as follows:

\[
\begin{align*}
    f(x) &= 0.5 + \frac{\sin^2(x_1^2 + x_2^2)^2 - 0.5}{1 + 0.001(x_1^2 + x_2^2)^2} \\
    &= 14
\end{align*}
\]

Scahffer 1 function is a continuous, differentiable, non-separable, non-scalable, and multimodal function. The results were shown in Figure 4.

![Figure 2](image1.png) Optimized results for Chung Reynolds function

![Figure 3](image2.png) Three-dimensional profile for Scahffer 1 function

We can see from Figure 4 and draw some different conclusions this time. The averaged best fitness values along with iterations might cause a little confusions. The overall curves showed that the SMA perform worst and the GWO perform better, the hybrid GWO-SMA perform the best.
4.3 Experiments on benchmark functions with valleys

Another kind of benchmark functions difficult to optimize might be those who have valleys or basins in their profiles. In other words, the global optimum might locate in the valleys or basins. In such circumstances, the individuals could not gain enough information towards the global optimum when they are in the valleys or basins. Consequently, they are difficult to optimize. We would use Freudenstein Roth function this time:

\begin{equation}
\hat{f}(x) = (x_1^3 - 1 \frac{3}{5} + (5 - x_2) x_2^2 x_2^2) + (x_1^3 - 2 \frac{9}{5} + (x_1 + 1) x_2 - 14 x_2^2)
\end{equation}

Freudenstein Roth function is a continuous, differentiable, non-separable, non-scalable, unimodal benchmark function. Freudenstein Roth function has no constraints on every parameter. It is a symmetric function. The global optimum is located at point \( x^* = (5, 4) \) and \( f(x^*) = 0 \). Its three-dimensional profile is shown in Figure 5. There is a valley existed in its three-dimensional profile and the global optimum is located in the valley.

The simulation results are shown in Figure 6. The best fitness values along with iterations would result in a third kind of conclusions: The GWO algorithm would perform better during optimization whether the SMA perform the worst. And the hybrid GWO-SMA perform a litter better than the SMA and worse than the GWO algorithm.
5. DISCUSSIONS AND CONCLUSIONS

We might be confused about the conclusions after three types of experiments. Every simulation experiments would result in a different conclusion.

Having another glance on the updating equation (2), there are three branches and three ways for individuals to choose. Two threshold parameters are involved in this equation. The proportional value \( z=0.03 \) is independent which would result in a proportional number of individuals re-initializing again during each iteration. Another threshold parameter \( p \) is relevant to the fitness values and the global best fitness value according to equation (5). Consequently, the more the fitness values steer away from the best \( DF \), the larger values for \( p \), and the larger selected ratio for the second branch. Apparently, the unimodal are easy to optimize, thus most of the individuals are comparatively near to \( DF \), thus the second branch would be seldom chosen for individuals in swarms, consequently, the hybrid GWO-SMA might be the same with the standard SMA. For the multimodal benchmark functions and those who have basins or valleys in their profiles, the individuals might be comparatively far away from the best candidates, thus the second branch might be chosen for individuals with a large ratio, and consequently, the hybrid GWO-SMA will prevail and perform with large proportions. Another phenomenon that the GWO algorithm sometimes perform better than the SMA and sometimes not, might be relevant to the various ways for individuals, and such conclusions might need further study.

We can henceforth conclude that: (1) it would be unnecessary for us to hybridize the GWO algorithm with SMA in applications. (2) The SMA is unique and it has three ways for individuals to update their positions, therefore, hybridize one branch with other methods might cause some confusing problems. And (3) more attention should be paid to the parameters or other relevant governing equations, the updating equation for the SMA might be unique and outstanding and hard to improve.

![Figure 6 Optimized results for Freudenstein Roth function](image)

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