Determination of the Deformed Shape of a Masonry Wall Exposed to Fire Loading by a Homogenization Method

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Abstract. The present contribution shows how it is possible to determine the homogenized thermo-elastic characteristics of a natural stone masonry wall, taking into account the material properties of stone and mortar as functions of temperature increase, as well as the geometrical characteristics of their assembly. Joints are incorporated in the analysis through a numerical homogenization procedure. As a result, membrane and bending stiffness coefficients, as well as thermal-induced efforts, of an equivalent plate are obtained. Such homogenized thermo-mechanical characteristics make it possible to determine the deformed shape of the wall after a certain time of fire exposure. As an example, the calculation procedure is performed on a particular configuration of infinitely wide wall, illustrating the influence of the joints on its thermal deformed shape. To assess the practical validity of this homogenization-based calculation procedure, results of the numerical homogenized model (incorporating joints) are compared to those of a homogeneous model (without joints), and to available experimental results obtained on a 3 m-high, 3 m-wide wall exposed to fire loading.

1. Introduction
In engineering practice, the reference text for the design of masonry structures in Europe is Eurocode 6 [1]. However, recommendations for the analysis and design of natural stone masonry walls in fire conditions are still missing. In this context, a preliminary experimental campaign has been carried out on two natural stone masonry walls, showing geometry changes of such structures in fire conditions. Indeed, when exposed to fire, a natural stone masonry wall is subjected to high thermal gradients through its thickness. Due to the thermal-induced deformations, such a structure exhibits out-of-plane displacements, which lead to an eccentricity of the vertical load. Consequently, bending moments are generated in the wall in addition to the pre-existing compressive axial force. These bending moments caused by the wall deflection, combined with the fire-induced degradation of material properties, may lead to the collapse of the structure.
Based on a simplified 1D modelling of the problem, a calculation procedure for evaluating the deformed configuration of masonry walls exposed to fire, has recently been proposed [2]. However, the main limitation of the latter approach lies in the fact that the thermo-mechanical characteristics of the mortar joints are assumed to be close to those of the natural stone blocks, so that the wall can be considered as being homogeneous. Consequently, such an approach may not provide reliable predictions for poorer quality mortars where the influence of the joints on the thermal induced change of geometry of the wall may be no longer negligible.

In this context, the purpose of this contribution is to show how it is possible to take into account the joints when determining the deflection of the wall using a numerical homogenization method. More precisely, a 1D heat transfer analysis is firstly carried out in order to obtain the temperature distribution across the thickness of the wall. Then, a representative elementary cell of the heterogeneous masonry wall is subjected to this temperature distribution, as well as to periodic in-plane and out-of-plane displacement boundary conditions. As a result, membrane and bending stiffness coefficients, as well as thermal-induced efforts, of an equivalent plate are obtained for any temperature profile. Such homogenized thermo-mechanical characteristics make it possible to determine the deformed shape of the wall after a certain time of fire exposure. By way of example, the calculation procedure is performed on the particular configuration of an infinitely wide wall, illustrating the influence of the joints on its thermal deformed shape. To assess the practical validity of this homogenization-based calculation procedure, results of the numerical homogenized model (incorporating joints) are compared to those of a homogeneous model (without joints), and to available experimental results obtained on a 3 m-high, 3 m-wide wall exposed to fire loading.

2. Statement of the problem and general equations

2.1. Definition of the representative unit cell of the masonry wall

The masonry wall is initially represented by a three-dimensional volume composed of a periodic arrangement of blocks and joints in the orthonormal frame $O_{x_1,x_2,x_3}$ (Figure 1). From this periodic arrangement, it is possible to extract a representative unit cell which contains all the information about the geometry and the material characteristics of the wall. It follows that the entire wall can then be reconstructed from the assembly of many cells obtained from translating such a representative unit cell in its $(O_{x_1,x_2})$ plane.

Figure 1. 3D masonry wall and corresponding representative unit cell.

Figure 2 represents the representative unit cell, composed of blocks of height $H$, width $L$, and thickness $h$, bonded by horizontal joints of thickness $e_h$ and vertical joints of thickness $e_v$.

In what follows, the constitutive law of this wall, modelled as a plate, is established by an upscaling procedure based upon the solution to an auxiliary thermo-elastic problem defined on this representative unit cell.
2.2. Loading mode of the representative unit cell

The thermo-mechanical loading applied on the representative unit cell is assumed to be uncoupled in order to simplify the analysis procedure:

- Firstly, a transient thermal transfer analysis is performed to determine the temperature distribution in the unit cell;
- Then, a thermo-mechanical analysis, taking into account the thermal strains, is conducted in order to determine membrane and bending stiffness coefficients, as well as thermal-induced efforts.

2.2.1. Thermal loading. The first step of the homogenization method consists in determining the temperature distribution within the unit cell through a preliminary heat transfer analysis. The thermal characteristics of the joints are assumed to be close enough to those of the blocks so that the wall may be considered as being homogeneous, and heat propagation across the wall may be considered as one-dimensional along the Ox3 axis. In such a thermal analysis, the conductivity, the specific heat, and the bulk unit of the stone material are dependent on the temperature.

The thermal loading consists in applying an ISO 834 fire [3] on one face of the wall during a given time. The resulting increase in temperature across the wall thickness induces thermal strains, defined as:

\[ \varepsilon_{th} = \int_{T_i}^{T_f} \alpha(\xi, \theta) d\theta \]

where \( \alpha \) is the coefficient of isotropic thermal expansion and \( T_0 \) the initial ambient temperature. These thermal strains are then taken into account in the mechanical analysis. The corresponding secant coefficient of thermal expansion, defined as:

\[ \alpha_s = \frac{1}{T - T_0} \int_{T_i}^{T_f} \alpha(\theta) d\theta \]

will also be used.

2.2.2. Mechanical loading. In addition to the thermal loading and resulting thermal strains, the representative unit cell is subjected to the following mechanical loading conditions (similar to those presented in [4] at ambient temperature for instance):

- (a) No volume forces;
- (b) The lower and upper faces of the cell (\( S^a \) - Figure 2) are stress free;
- (c) The stress vector is antiperiodic on the lateral face of the cell (\( S^l \) - Figure 2);
- (d) The displacement on the lateral face of the cell \( S^l \) is prescribed of the following form:

\[ u_{F,x} \left( \xi \in S^l \right) = F_x \xi - \xi_x + \frac{1}{2} \left( \xi_x \cdot \xi_x \right) \xi + u_{\text{per}} \left( \xi \right) \]

where the displacement \( u_{\text{per}} \) is periodic on the lateral face, and \( F_x \) and \( \chi \) are two second-order plane tensors, with \( \chi \) being symmetric.
The linearized strain at each point of the unit cell is then defined as:
\[
\varepsilon = \frac{1}{2} \left( \nabla F_{,x} + \nabla F_{,y} \right)
\]  
(4)

Introducing the tensor \( \varepsilon = \frac{1}{2} \left( F + F^T \right) \), the principle of virtual work shows that, for any statically admissible stress field \( \sigma \) (i.e. complying with the equilibrium equation along with conditions (a-c)) and any velocity field kinematically admissible with \( (\varepsilon, \chi) \) (i.e. complying with condition (d)), the work of internal forces writes [5]:
\[
\int_{V} \sigma : \varepsilon dV = \int_{S} \left( N : \varepsilon + M : \chi \right) dS
\]  
(5)
which means that the representative unit cell is subjected to a six parameters loading mode.

Given that \( V \) is the volume of the unit cell, and \( |S| = 2 \left( L + e_r \right) \left( H + 2e_q \right) \) the area of its mid-surface, the loading parameters are the generalized membrane forces \( N \) and bending moments \( M \), defined as :
\[
N = \frac{1}{|S|} \int_{S} \sigma dS \quad \text{and} \quad M = \frac{1}{|S|} \int_{S} \chi dS
\]  
(6)
These loading parameters are in duality with the membrane strains \( \varepsilon \) and curvature strains \( \chi \).

2.3. Deflection of the homogenized plate

2.3.1. Constitutive law of the wall components. Both stone and mortar are supposed to be isotropic materials. As a result, the constitutive equation at each point of the unit cell writes:
\[
\sigma(\varepsilon) = \lambda(\varepsilon) \text{tr}(\varepsilon(\varepsilon) - \varepsilon^0(\varepsilon))\frac{1}{1+2\mu(\varepsilon)} + 2\mu(\varepsilon)(\varepsilon(\varepsilon) - \varepsilon^0(\varepsilon))
\]  
(7)
In this expression, \( \lambda \) and \( \mu \) are the two Lamé coefficients, defined as \( \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \) and \( \mu = \frac{E}{2(1+\nu)} \), where \( E \) is the Young modulus and \( \nu \) is the Poisson coefficient.

2.3.2. General case. From the loading mode used above, the constitutive equations of the homogenized plate may be written as:
\[
N = A : \varepsilon - B : \chi + N^T
\]  
\[
M = -B : \varepsilon + D : \chi + M^T
\]  
(8)
with:
\[
N^T = \frac{1}{|S|} \int_{S} \sigma^T dS \quad \text{and} \quad M^T = \frac{1}{|S|} \int_{S} \chi \sigma^T dS
\]  
(9)
where \( \sigma^T \) is the stress field due to the thermal loading when membrane strains and curvature strains are prescribed to zero.

Because of the local isotropy of the constitutive material at any point of the unit cell, the homogenized constitutive law of the plate writes:
In the specific case when the constitutive material of the plate is initially homogeneous (which is equivalent to have the same material in the blocks and in the joints), the constitutive law of the plate reduces to [5]:

\[
\begin{bmatrix}
N_{11} \\
N_{22} \\
N_{12}
\end{bmatrix} =
\begin{bmatrix}
A_{1111} & A_{1122} & 0 \\
A_{1211} & A_{2222} & 0 \\
0 & 0 & A_{1212}
\end{bmatrix}
\begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
\epsilon_{12}
\end{bmatrix} -
\begin{bmatrix}
B_{1111} & B_{1122} & 0 \\
B_{1211} & B_{2222} & 0 \\
0 & 0 & B_{1212}
\end{bmatrix}
\begin{bmatrix}
\chi_{11} \\
\chi_{22} \\
\chi_{12}
\end{bmatrix} +
\begin{bmatrix}
N_{11}^T \\
N_{22}^T \\
N_{12}^T
\end{bmatrix}
\] (10)

\[\begin{bmatrix}
M_{11} \\
M_{22} \\
M_{12}
\end{bmatrix} =
\begin{bmatrix}
B_{1111} & B_{1122} & 0 \\
B_{1211} & B_{2222} & 0 \\
0 & 0 & B_{1212}
\end{bmatrix}
\begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
\epsilon_{12}
\end{bmatrix} -
\begin{bmatrix}
D_{1111} & D_{1122} & 0 \\
D_{1211} & D_{2222} & 0 \\
0 & 0 & D_{1212}
\end{bmatrix}
\begin{bmatrix}
\chi_{11} \\
\chi_{22} \\
\chi_{12}
\end{bmatrix} +
\begin{bmatrix}
M_{11}^T \\
M_{22}^T \\
M_{12}^T
\end{bmatrix}
\]

In the specific case when the constitutive material of the plate is initially homogeneous (which is equivalent to have the same material in the blocks and in the joints), the constitutive law of the plate reduces to [5]:

\[
\begin{bmatrix}
N_{11} \\
N_{22} \\
N_{12}
\end{bmatrix} =
\begin{bmatrix}
1 & v & 0 \\
v & 1 & 0 \\
0 & 0 & 1-v
\end{bmatrix}
\begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
\epsilon_{12}
\end{bmatrix} -
\begin{bmatrix}
A_{1111} & A_{1122} & 0 \\
A_{1211} & A_{2222} & 0 \\
0 & 0 & A_{1212}
\end{bmatrix}
\begin{bmatrix}
\chi_{11} \\
\chi_{22} \\
\chi_{12}
\end{bmatrix} +
\begin{bmatrix}
N_{11}^T \\
N_{22}^T \\
N_{12}^T
\end{bmatrix}
\] (11)

with:

\[
A = \frac{1}{1-v^2} \int_{-h/2}^{h/2} E(x_3) d\xi_3; \quad B = \frac{1}{1-v^2} \int_{-h/2}^{h/2} \frac{\partial E(x_3)}{\partial x_3} d\xi_3; \quad D = \frac{1}{1-v^2} \int_{-h/2}^{h/2} \frac{\partial^2 E(x_3)}{\partial x_3^2} d\xi_3
\]

\[
N^T = -\frac{1}{1-v^2} \int_{-h/2}^{h/2} E(x_3) \alpha(x_3) T(x_3) d\xi_3; \quad M^T = \frac{1}{1-v^2} \int_{-h/2}^{h/2} \frac{\partial E(x_3) \alpha(x_3) T(x_3)}{\partial x_3} d\xi_3
\] (12)

In addition to the constitutive law, the solution must comply with the boundary conditions of the problem and with the following equilibrium equations:

\[
\text{div}N = 0 \quad \text{div}M = 0
\] (13)

In general, it would be necessary to perform a finite element analysis in order to determine the deflection of a wall obeying the constitutive law given by equation (10). However, in some simple cases, it is possible to determine this deflection in an analytical way.

2.3.3. Case of an infinitely wide wall. For illustrative purpose, the proposed calculation procedure is now performed on an infinitely wide wall of height H, hinged at its bottom and simply supported at its top, as shown in figure 3.

\[
\begin{align*}
N_{22} &= 0 \\
M_{22} &= 0 \\
M_{12} &= 0
\end{align*}
\]

Figure 3. Simplified 1D model for an infinitely wide wall subjected to fire loading.
As this wall is infinitely wide in the \( x_1 \) direction, it can be modelled in plane strain conditions in the \((Ox_2x_3)\) plane. In this case, the constitutive law (10) simplifies to:

\[
\begin{align*}
N_{22} &= A_{2222} \varepsilon_{22} + B_{2222} \chi_{22} + N_{22}^T \\
M_{22} &= B_{2222} \varepsilon_{22} + D_{2222} \chi_{22} + M_{22}^T
\end{align*}
\]

(14)

Combining the equilibrium equations (13) and the boundary conditions in figure 3, it appears that \( N_{22} = 0 \) and \( M_{22} = 0 \) in every section of the wall. The curvature component \( \chi_{22} \), written \( \chi_{22}^T \) as it is related to the thermal loading only, is then:

\[
\chi_{22}^T = -\frac{A_{2222} M_{22}^T + B_{2222} N_{22}^T}{A_{2222} D_{2222} - B_{2222}^2}
\]

(15)

Finally, the deflection of the wall writes:

\[
u(x_2) = \frac{\chi_{22}^T x_2}{2} (x_2 - H)
\]

(16)

3. Numerical example

The homogenization method presented in the previous section is now applied to the determination of the deformed shape of a natural stone masonry wall. This wall is composed of 72 cm-wide, 36 cm-high, 20 cm-thick blocks of Saint-Maximin limestone bonded together by 1 cm-thick mortar joints made of natural hydraulic lime NHL 3.5. It is exposed to an ISO 834 fire on one of its faces during 120 min.

3.1. Thermo-mechanical properties of the constituent materials

3.1.1. Blocks. As stated previously, blocks are made of Saint-Maximin limestone. The evolution of the mechanical properties of this limestone as a function of temperature is described in [6]. The Poisson coefficient is \( \nu_b = 0.28 \) and is assumed to remain unaffected by the temperature increase. The evolution of the elastic modulus and of the coefficient of thermal expansion of Saint-Maximin limestone are recalled in figure 4.

3.1.2. Joints. Joints are made of a natural hydraulic lime mortar NHL 3.5. As the evolution of the thermo-elastic properties of this material as a function of temperature is not known, the following hypotheses are adopted:

- At ambient temperature, the elastic modulus of the mortar is \( E_J = 4.3\) GPa and its Poisson ratio is \( \nu_J = 0.2 \) [7];
- The reduction coefficient of the elastic modulus as a function of temperature is the same as the concrete’s one given by Eurocode 2 [8], while the Poisson coefficient is assumed to remain unaffected by the temperature increase;
- The coefficient of thermal expansion is considered equal to the one of calcareous aggregate concrete [8].

The evolution of the elastic modulus and of the coefficient of thermal expansion of the mortar are represented in figure 4.

3.2. Preliminary thermal transfer analysis

Given that the thermal properties of the stone are rather close to those of the mortar, the masonry is assumed to be homogeneous for the thermal calculation and its thermal properties are those of Saint-Maximin limestone determined by Vigroux et al. [6]. The preliminary one dimensional heat transfer calculation can be performed using a FE software such as Safir [9]. Figure 5 represents the temperature profiles for 30 min and 120 min of fire exposure.

![Temperature Profiles](image)

**Figure 5.** Evolution of the temperature across the thickness of the wall for different fire exposures.

3.3. Numerical determination of the stiffness and thermal effort tensors components

3.3.1. Numerical model. The components of the stiffness tensors and of the thermal induced membrane efforts and bending moments have been determined using the finite element software Cast3M [10]. The complete determination of the homogenized constitutive law requires to perform seven elastic calculations (one for each deformation or curvature component and one for the thermal induced efforts).

![Mesh](image)

**Figure 6.** Mesh (a) of the blocks and (b) of the joints.

Figure 6 represents the mesh used to perform these calculations. It is refined around the joints in order to avoid inaccuracies due to distorted finite elements. In this case, the joints are rather thin compared to the bricks. For that reason, the mesh needs to be quite fine, which increases the size of the numerical model. Here, it is composed of 67602 nodes and 47967 10-noded tetrahedral elements. Each of the seven calculations takes around 2 min of computational time.
3.3.2. **Value of the components.** The components of the stiffness tensors, of the membrane thermal forces and thermal bending moments derived from these elastic finite elements calculations are given in table 1.

**Table 1.** Components of the stiffness tensor and thermal efforts for a wall with joints and for a homogeneous wall.

|                     | Wall with joints (N/m) | Homogeneous wall (N/m) | Relative gap | Wall with joints (Nm) | Homogeneous wall (Nm) | Relative gap |
|---------------------|------------------------|------------------------|--------------|------------------------|------------------------|--------------|
| \( A_{111} \)       | 1.84E+09               | 1.99E+09               | 7.8%         | \( D_{111} \)         | 5.17E+06               | 9.9%         |
| \( A_{122} \)       | 4.75E+08               | 5.57E+08               | 15.9%        | \( D_{122} \)         | 1.38E+06               | 14.6%        |
| \( A_{222} \)       | 1.78E+09               | 1.99E+09               | 11.4%        | \( D_{222} \)         | 5.01E+06               | 13.1%        |
| \( A_{122} \)       | 1.26E+09               | 1.43E+09               | 12.5%        | \( D_{122} \)         | 3.63E+06               | 12.3%        |
| \( B_{111} \)       | -3.63E+07              | -3.70E+07              | 1.7%         | \( N_{11}^T \)        | -3.96E+06              | 14.2%        |
| \( B_{122} \)       | -1.02E+07              | -1.03E+07              | 1.1%         | \( N_{22}^T \)        | -3.76E+06              | 19.4%        |
| \( B_{222} \)       | -3.68E+07              | -3.70E+07              | 0.6%         | \( M_{11}^T \)        | -1.58E+05              | 18.5%        |
| \( B_{122} \)       | -2.62E+07              | -2.66E+07              | 1.6%         | \( M_{22}^T \)        | -1.48E+05              | 25.5%        |

It should be noted that:
- The difference between the values of \( A_{111} \) and \( A_{222} \), or \( N_{11}^T \) and \( N_{22}^T \) for instance, reveals the orthotropic properties of the homogenized material;
- As could be expected from the presence of softer joints between the blocks, the homogenized stiffness coefficients are lower than those of the homogeneous wall, the relative gap being of the order of 10-15% for membrane and flexural stiffness parameters;
- Thermal generalized stresses are between 14 and 26% lower than in the homogeneous case (absence of joints). This may have a significant impact on the stresses within the wall, notably when zero displacement boundary conditions are prescribed along the wall edges.

3.4. **Application to the deformed shape prediction of an infinitely wide wall**

The deflection of an infinitely wide wall is now determined analytically from the coefficients determined in table 1. Figure 7 shows the deflection of an infinitely wide, 3 m-high, 20 cm-thick limestone wall after being exposed to fire during 120 min.

![Figure 7. Deformed shape of a masonry wall.](image-url)
It clearly appears from this figure that the thermal deflection of the wall with joints is lower than the one of the homogeneous wall. This result, which could at first glance be somewhat counterintuitive, was already reported by Yang [5] for precast concrete panels connected by horizontal joints. A possible explanation for this might be that the joints allow for a discontinuity of the thermal curvature, hence limiting the total thermal deflection.

Although the thermal deformed shape of the wall with joints is slightly lower than that without joints, the former is more flexible than the latter. Indeed, the pure flexural stiffness of the wall, defined by: \( D'_{222} = D_{222} - B_{222}^2 / A_{222} \), is lower for the wall with joints (\( D'_{222} = 4.2 \text{ MNm} \)) than for the homogeneous one (\( D_{222}^* = 4.5 \text{ MNm} \)). As a consequence, the deformed shape due to an additional bending moment distribution generated by a surcharge would then be greater for the wall with joints than for the homogeneous one.

3.5. Comparison with experimental results

The results of the present model are now compared to those of an experiment performed by CSTB and CTMNC on a 3 m-high, 3 m-wide, 20 cm-thick limestone wall. The thermomechanical properties of the Saint-Vaast limestone composing this wall are close to the ones of Saint-Maximin limestone described in Vigroux et al. [6]. On this wall, blocks are bonded together by a natural hydraulic lime mortar NHL 3.5, the properties of which have been described in paragraph 3.1.2.

The wall was exposed to an ISO 834 fire during 120 min. Figure 8 shows the experimental and the numerical deflections of this wall after 30 min (Figure 8 (a)) and 120 min (Figure 8 (b)) of fire exposure.

**Figure 8.** Experimental and numerical deflection after (a) 30 min and (b) 120 min of fire exposure.

From these figures, it can be seen that the experimental deformed shape is rather close to the model’s one after 30 min of fire exposure: the homogenized model underestimates the maximal deflection by 20 \%. Conversely, after 120 min: the homogenized model overestimates the experimental maximal deflection by 53 \%. This might originate from the heat transfer analysis for which material properties had to be estimated for temperatures greater than 600 °C. Another source of uncertainties comes from the boundary conditions used in the model. Indeed, the experimental boundary condition at the bottom of the wall lies just in between a hinge and a full rotational restrain. The deflection calculated with the present model, with a hinge, would then represent an upper bound estimate for the real deflection. Finally, these calculations rely on the hypothesis that the constituent materials remain thermo-elastic. Actually, plastic strains are likely to appear and affect the deformed shape; however, for preliminary engineering design, the present method may represent a good compromise between short computational time and accuracy of the results.
4. Concluding remarks
The present contribution has developed a procedure for evaluating the deformed configuration of masonry wall exposed to fire, taking into account the geometrical and material characteristics of the blocks and of the joints through a homogenization method. It is notably shown that the calculated deflection of masonry wall may be lower when the effect of the joints is taken into account than when the wall is considered homogeneous. Thanks to the smaller size of the finite element model of the representative unit cell, the proposed homogenization method allows for performing parametric studies in a much quicker way than when using a fully 3D finite element model for the entire wall.

Comparisons with first available experimental data have shown that the model is able to predict the deformed shape of a masonry wall exposed to fire in a rather accurate way, even though owing to the pretty similar mechanical characteristics of stone and mortar of the wall tested, the small difference between homogenized and homogeneous models does not allow to fully justify the need to account for the joints.

The present homogenization method has been applied to the example of a natural stone masonry wall. Such a method seems relatively simple to be expandable to every masonry composed of plain blocks, and could also be adapted for hollow blocks masonry structures. However, its main limitation lies in its inability to determine the deflection of walls of finite width under arbitrary boundary conditions. Developments are currently under way to extend this method to walls for which the infinite width assumption is no more relevant, by means of the finite element method.

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