γ-quanta propagation in single crystals

V.A.Maisheev *

Institute for High Energy Physics, 142284, Protvino, Russia

Abstract

Propagation of γ-quanta in the single crystals, oriented in a region of the coherent pair production is considered. The qualitative description of the process is also discussed. The theory of γ-quanta propagation in the anisotropic medium is illustrated by the help of the particular calculations of such a process in silicon single crystals.

It is shown that the single crystals are sensitive to the initial circular polarization of γ-beam despite the fact that the cross section of absorption is independent of it. The reason is that the normal electromagnetic waves (an eigenfunctions of the problem) are elliptically polarized. The speeds of absorption and motion of both the normal waves are different and as a result the process of γ-quanta propagation depend on the initial polarization state. The calculated value of asymmetry is about 20% for 25 GeV γ-quanta propagating in 100 cm silicon single crystal.

The obtained results are useful in creating of polarimeters for high energy electrons and γ-quanta.

1 Introduction

It is well known that the optical properties of a medium may be described with the use of a permittivity tensor \[ \mathbf{\varepsilon} \]. The anisotropic medium referred as such a medium in which the permittivity tensor is a symmetric tensor \[ \mathbf{\varepsilon} \]. In the general case the components of this tensor are complex values and, because of this, the complex permittivity tensor does not reduce to principal axes, except in specific cases. The simplest example of such an anisotropic medium of the general type is a combination of the two linearly polarized monochromatic laser waves with different frequencies (dichromatic wave) moving in the same direction. In general the directions of linear polarization of these monochromatic waves are different. Another more complicated example is a single crystal oriented in the region of coherent \( e^- e^+ \)-pair production \[ 3, 4, 5 \].

The permittivity tensor was found for both these examples in papers\[ 3, 4 \]. Besides, in \[ 3 \] the theory of γ-quanta propagation in the anisotropic medium of a general type is also considered. One of the main results of these papers is the prediction of sensitivity of the anisotropic medium to the initial circular polarization of propagating γ-quanta. It is important for creating of devices measuring the circular polarization of γ-quanta or longitudinal polarization of electrons. In the last case the electron beam is transformed into bremsstrahlung γ-beam by the use of amorphous target \[ 8 \]. The circular polarization of the end of bremsstrahlung spectrum is approximately equal to the longitudinal polarization of a primary electron beam. In this paper we consider the theoretical basis for creating of the γ-beam polarimeters.

*E-mail maisheev@mx.ihep.su
2 Qualitative consideration of process

As already noted, the theoretical description of γ-quanta propagation in the anisotropic medium is contained in paper [6]. The permittivity tensor in single crystals oriented in the region of coherent $e^-e^+$-pair production is found in [7]. In the case of high energy γ-quanta the process of propagation is determined primarily by the transverse part of the permittivity tensor, while the longitudinal components of the tensor are higher-order infinitesimals in the interaction constant $\alpha$ [9, 10]. In this way the permittivity tensor in anisotropic medium is a symmetric tensor of rank two.

Now we make an attempt of the qualitative consideration of the γ-beam propagation in the anisotropic medium. As is well known that the imaginary components of permittivity tensor in anisotropic medium describe the absorption of γ-quanta. The cross section $\sigma_{e^-e^+}$ depends on the linear polarization of γ-quanta

$$\sigma_{e^-e^+} = A + B(e_\tau)^2,$$

(1)

where the unit vectors $e$ and $\tau$ determine the polarization plane of the γ-quanta and some definite plane in single crystal (the wave vector of the γ-quanta lies in both planes). The formula (1) essentially determines a symmetric tensor of rank two, whose components we denote as $\sigma_{kl}$ ($k, l = 1, 2$). Then the imaginary parts of the permittivity tensor components are

$$\varepsilon_{kl} = \frac{N\sigma_{kl}c}{\omega},$$

(2)

where $N$ is the atomic density of single crystal, $c$ is the speed of light and $\omega$ is the γ-quanta frequency. The real components of the permittivity tensor we can find with the help of the dispersion relations

$$\varepsilon_{ij}' - \delta_{ij} = \frac{2}{\pi} \mathcal{P} \int_0^\infty \frac{x \varepsilon_{ij}''(x) \, dx}{x^2 - \omega^2},$$

(3)

As is seen from (1) the cross sections for γ-quanta with direction of linear polarization parallel and perpendicular with respect to unit vector $\tau$ are different. As is well known that the ellipse is a geometric interpretation of the symmetric tensor. These ellipses are represented on Fig.1 for three energies of γ-quanta. Let take the above-mentioned dichromatic laser wave as an example of the anisotropic medium. Fig.1a illustrates the behavior of the absorption ellipses at different energies in case when linear polarizations of the both waves are parallel or perpendicular with respect to unit vector $\tau$. Fig.1b illustrates the similar behavior, but the angle between polarizations of the two waves is not equal to 0 or $\pi$/2. In both cases the cross sections are described by the equation (1). However, in the case on Fig.1a the common symmetric plane exists for all energies of γ-quanta. This is true because of the natural symmetry of the problem. As Fig.1.b illustrates, in this case the symmetric plane also exists but the position of the plane in space is different for every energy of γ-quanta. In particular the position of this plane depends on the ratio of intensities of the dichromatic wave components. It is obviously that the symmetry plane is determined in main by the strong wave when the intensity one of two waves is very weak in the comparison with another. In such a manner, in this case the symmetry have the dynamic character, which defined by the particular mechanism of the interaction. The numerical quantities of this interaction are different for every energy of γ-quanta. Hence it follows that dynamic symmetry plane rotate at changing of the γ-quanta energy.

We can see from Eq(3) that real components of permittivity tensor is determined by the integration over all energies of γ-quanta. It is means that in general the position
of the symmetry plane for real permittivity tensor differ from the similar position for imaginary part of this tensor. In other words, the axes of both tensors are not parallel or perpendicular in between. Hence we get that in general the preferred two planes exist in a space for propagating γ-quanta of a fixed energy.

As is well known that γ-beam in a medium present the superposition of two states. These states referred as normal electromagnetic waves [2]. The normal waves are the eigenfunctions of the problem and they have determinate polarization characteristics, which depend on the optical properties of a medium. So, the γ-quanta, having the polarization characteristics identical to one of normal waves, are conserve their at the propagation in a medium. In general the speeds of absorption of normal waves are different.

It is evidently that both normal waves are linearly polarized in case illustrated in Fig.1a. These polarizations are perpendicular in between and their directions coincide with the principal axes of ellipses. In other case (see Fig.1b), in general the principal axes of both ellipses are not parallel, and it is believed that normal wave are elliptically polarized. It is also follows from paper [7]. In the case, when the absorption in medium is absent, the all imaginary components of permittivity tensor are equal to zero and because of this the normal waves are always linearly polarized.

In the general case the initial polarization state of γ-quanta changes at propagation in a medium. The intensity of γ-beam on thickness x in the anisotropic medium can calculate by the help of follows relation [6]

\[ J_γ(x) = A(x) + B(x)ξ_1 + C(x)ξ_2 + D(x)ξ_3 \]  

where \( ξ_i \), \( i = 1, 3 \) are the initial Stokes parameters of γ-quanta, A,B,C,D are simple functions of x including also some parameters as, for example, refractive indices of normal waves. When the normal waves have only linear polarization the function C(x) is equal to zero on any thickness x. In the case, when the normal waves have nonzero component of circular polarization the function C(x) is equal to zero at \( x = 0 \) and in general it is nonzero at \( x > 0 \). Besides, the first derivative of the C(x)-function is equal to zero at \( x = 0 \). It means that the cross section of the absorption process independent of the circular polarization of the γ-quanta. Here we employ the well-known formula \( dJ = -J(0)Nσdx \) for thin targets.

In this way the relation (4) show that intensity of γ-quanta propagating in the anisotropic medium depends on the initial polarization despite the fact that the cross section of absorption is independent of it. The reason is that the normal waves are elliptically polarized. The speeds of absorption and motion of both the normal waves are different due to the process γ-quanta propagation depends on the initial polarization state. On the other hand the initially unpolarized γ-beam became elliptically polarized [3]. This is an essential prerequisite for creating of the γ- polarimeter. To do this requires computations of the γ-quanta propagation in single crystals.

### 3 γ-quanta propagation in single crystals

In this section we present some results of the calculations of γ-beam propagation in silicon single crystals. The investigated process is determined with the help of such parameters as refractive indices and polarization states of normal electromagnetic waves. The imaginary parts of refractive indices are responsible for the speed of normal wave absorption. The real parts and polarization states of normal waves are also in term C(x) (see equation (4)). Let us define the polarization state of one normal wave with the help of Stokes parameters \( X_1, X_2, X_3(X_1^2 + X_2^2 + X_3^2 = 1) \). Then the polarization state of another
wave is correspondingly equal to $-X_1$, $X_2$, $-X_3$. Intuition suggest that the parameter $X_2$ should be significant for circular polarization detection. As it is follows from [4] parameter $X_2$ is significant when $\gamma$-beam moving under no large angle with respect to any strong crystallographic axis. We select the <001> axis in silicon for calculations. We also select the Cartesian coordinate system with one axis along <001> and two another axes along <110> and <1−10> axes. Direction of $\gamma$-quanta motion we can determine with the use of angle $\theta$ with respect to <001> axis and azimuth angle $\alpha$ around of this axis ($\alpha = 0$, when the $\gamma$-quanta momentum lies in the (110)-plane). However another angles are more convenient to use $\varphi_H = \theta \cos \alpha$, $\varphi_V = \theta \sin \alpha$.

The characteristics of process have the same numerical values at different $\gamma$-quanta energies $E_\gamma$ when the following invariant parameters are used: $W_H = E_\gamma G_2 \varphi_H/(2mc^2)$, $W_V = E_\gamma G_3 \varphi_V/(2mc^2)$. Here $m$ is the electron mass, $G_2, G_3$ are the reciprocal lattice constants (in our case $G_2 = G_3 = 0.01264$ in units of the inverse electron Compton length).

In the case being considered we can write these parameters as $W_H = 12.366 E_\gamma \gamma H$, $W_V = 12.366 E_\gamma \gamma V[GeV radian]$. The calculations are carried out for $E_\gamma = 25 GeV$. The Figs.2-6 illustrate these results of calculations. In the calculations the Moliere form factor was employed [10].

As Fig.3 illustrates, the regions with high circular polarization $X_2$ exist in single crystals. The absolute value of $X_2$ is near 1 at $\varphi_H = 3.008$ mrad and $\varphi_V = 3.558$ mrad. We can see that the high quantity of $X_2$ is observed in region $\approx \pm 0.1$ with respect to central $\varphi_H$, $\varphi_V$-values. The direction, in which the circular polarization $X_2$ is equal to 1, is the direction of the so-called singular axis, described in the crystal optic of the visible light [2]. We can see in Fig.2 that refractive indices are approximately equal to one another near this direction. Notice, that the value $Im(n) - \varepsilon_A/2$, as a function of invariant $W_H, W_V$-parameters, is independent of the $\gamma$-quanta energy. The $\varepsilon_A$-value is determined in [7], and for the silicon single crystal is numerically equal to $1.32 \times 10^{-15}/E_\gamma$. Fig.5 illustrates the variation of polarization state of the propagating $\gamma$-quanta. We can see that the initially unpolarized beam obtain in general the linear and circular polarization. Notice, that equations of paper [8] break down for $X_2 = 1$ (see also [4]). However they are true in the neighborhood of this point and our calculations are made for $X_2 = 0.995$.

Figs.4 and 6 illustrate the absolute and relative losses of $\gamma$-quanta intensity. One can see that these losses depend on the initial polarization of the $\gamma$-quanta. We select the value $A_s = |I_p - I_0|/I_0$ as the degree of relative losses of intensity, where $I_0$ is the losses of initially unpolarized beam and $I_p$ is the losses of completely circularly polarized beam.

Notice, that the initial polarization of $\gamma$-quanta is not needed to change for measuring of the asymmetry $A_s$. So, let $\gamma$-beam is completely circularly polarized at the entry of the single crystal which have the orientation angles are equal to $\varphi_{H1}$ and $\varphi_{V1}$. After measuring the intensity losses we change the single crystal orientation so, that new angles are equal to $\varphi_{H2} = \varphi_{V1}$ and $\varphi_{V2} = \varphi_{H1}$. Then we get that $|I_1 - I_2|/0.5(I_1 + I_2) = |I_1 - I_2|/I_0 = 2A_s$.

As already noted, the symmetry of problem have in general the dynamic character. It is means that our knowledge of such characteristics as refractive indices and $X_i$-values depends on the possible of uncertainty in the initial data for calculations. We think that main uncertainty is connected with the electric fields of single crystals. We carry out calculations of $X_2$-value with the use of experimental silicon form factors [11, 12], instead Moliere ones. The results of these calculations agree closely with each other. However, a little displacement of the singular axis direction (when $X_2 = 1$) take a place (in this case $W_H = 0.975, W_V = 1.075$, instead $W_H = 0.93, W_V = 1.10$ for Moliere potential).
4 Discussion and Summary

The results of calculations in the silicon single crystal oriented near <001> axis show detectable sensitivity to the circular polarization of propagating $\gamma$-quanta. Below we discuss some questions of the limitations and optimization.

Our consideration of the process are based on the traditional theory of the coherent $e^+e^-$-pair production in single crystals. However, this process is transformed into the analogous process in a "strong field" of the crystallographic axis and the mathematical description of the both processes is different. The process in the "strong field" take a place at angles $\phi_B \leq V_a/mc^2$ with respect to axis, where $V_a$ is the potential of this axis. In the case of <001> silicon axis this angle is equal to $\approx 0.15$ mrad. It is means that the upper bound for our consideration (<001>,Si) is $\sim 500 GeV$.

We believe that the asymmetry of process $A_s$ on the same thickness is growing with the $\gamma$-quanta energy increasing. It is true because of the increasing of the cross section of the $\gamma$-beam absorption. As Fig.6 illustrates, the detectable value $A_s$ take a place for orientations with not high quantity of the $X_2$-value. This orientation region is more wide and flat then region near singular axis. It is important for measuring polarization of the $\gamma$-beam with a large angle divergence.

It is well-known that the <011> and <111> axes in silicon single crystals have more strong electric fields, then <100> axis. Because of this, it is expected more high $A_s$-values for these axes.

We believe that the theoretical prerequisites exist for creating $\gamma$, $e$-polarimeters on the base of the considered here phenomenon. However, a lot of optimization calculations need to be done before for it.

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Figure 1: The geometrical interpretation of permittivity tensor in anisotropic medium. Further explanations are given in the text.
Figure 2: Real minus unit (1', 2') and imaginary (1, 2) parts of the refractive indices near of <001>-axis in silicon single crystal as a function of the parameter $W_H$. $E_\gamma = 25 GeV$, $W_V = 1.10$. Temperature of single crystal $T_{Si} = 300^\circ K$.

Figure 3: Absolute quantity of normal wave circular polarization $X_2$ near of <001>-axis in silicon single crystal as a function of the parameter $W_H$. $W_V = 1.10$, 1.13, 1.07 for 1,2,3-curves. $T_{Si} = 300^\circ K$. 
Figure 4: Intensity of 25 GeV γ-quanta as a function of the single crystal thickness. Central curve is the intensity of initially unpolarized beam, upper and bottom curves are the intensities of beam with the corresponding initial circular polarization equal to -1 and +1. $W_H = 0.93$, $W_V = 1.10$, $T_{Si} = 300^\circ$ K.
Figure 5: Stokes parameter variations of 25 GeV $\gamma$-quanta as a function of the single crystal thickness. The first number in the parenthesis is initial circular polarization $(1,0,-1)$, the second number is number of Stokes parameter $i=1-3$. $W_H = 0.93$, $W_V = 1.10$, $T_{Si} = 300^\circ$ K.
Figure 6: Asymmetry of the process as a function of the silicon single crystal thickness. Curve 1 is for $W_H = 0.93$, $W_V = 1.10$ and curve 2 is for $W_H = 2.0$, $W_V = 1.10. E_\gamma = 25GeV$, $T_{Si} = 300^\circ$ K.