PARTICLE ACCELERATION IN SUPERLUMINAL STRONG WAVES

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Received 2015 February 10; accepted 2015 April 1; published 2015 May 28

ABSTRACT

We calculate the electron acceleration in random superluminal strong waves (SLSWs) and radiation from them using numerical methods in the context of the termination shocks of pulsar wind nebulae. We pursue the orbit of electrons by solving the equation of motion in the analytically expressed electromagnetic turbulences. These consist of a primary SLS and isotropically distributed secondary electromagnetic waves. Under the dominance of the secondary waves, all electrons gain nearly equal energy. On the other hand, when the primary wave is dominant, selective acceleration occurs. The phase of the primary wave for electrons moving nearly along the wavevector changes very slowly compared with the oscillation of the wave, which is “phase-locked,” and such electrons are continuously accelerated. This acceleration by SLSWs may play a crucial role in pre-shock acceleration. In general, the radiation from the phase-locked population is different from the synchro-Compton radiation. However, when the amplitude of the secondary waves is not extremely weaker than that of the primary wave, the typical frequency can be estimated from synchro-Compton theory using the secondary waves. The primary wave does not contribute to the radiation because the SLSW accelerates electrons almost linearly. This radiation can be observed as a radio knot at the upstream of the termination shocks of the pulsar wind nebulae without counterparts in higher frequency ranges.

Key words: acceleration of particles – radiation mechanisms: non-thermal – waves

1. INTRODUCTION

Particle acceleration is one of the most important physical processes in astrophysics and plasma physics. Acceleration by strong electromagnetic waves (EM waves) has been investigated for a long period in the field of laser physics (e.g., Jory & Trivelpiece 1968; Sarachik & Schappert 1970; Karimabadi et al. 1990; Kuznetsov 2014). In the field of astrophysics, such mechanisms have been investigated mainly in the context of pulsars. Magnetic dipole radiation was thought to be emitted from a pulsar that has a magnetic axis inclined toward the rotation axis (Pacini 1968). This EM wave was regarded as a “strong wave” in the vicinity of the pulsar. The strength of the EM wave is generally defined by the strength parameter $a \equiv eE/mc\omega$, where $E$ and $\omega$ are the amplitude and frequency of the EM wave, $m$ and $e$ are the mass and charge of the electron, and $c$ is the speed of light in a vacuum. The strength parameter is estimated to be much larger than unity for the EM wave around the pulsar, which means that this EM wave is capable of accelerating the electrons to relativistic energy. Electrons dropped in the strong EM wave at rest are strongly accelerated toward the direction of the wavevector. The phase of the wave for the electrons here changes very slowly (phase locking), since electron speed becomes close to the speed of light. The strong wave is nearly a stationary EM field for these electrons, and they are continuously accelerated. If we assume an infinite plane wave, the maximum Lorentz factor is $\gamma_{\text{max}} \sim a^2$, not $a$ (Gunn & Ostriker 1971). To achieve $\gamma_{\text{max}} \sim a^2$, the phase-locking effect plays a crucial role.

As is well known, the pulsar magnetosphere is not a vacuum as was assumed by the above papers, but is filled with dense plasma (Goldreich & Julian 1969). Kegel (1971) pointed out that when the refraction index $n_\perp$ is significantly smaller than unity because of the existence of the plasma, the phase locking becomes inefficient since the phase velocity $c/n_\perp$ of the EM wave becomes significantly larger than $c$. Such waves are called superluminal strong waves (SLSWs). Moreover, if the plasma is sufficiently dense, even the propagation near the light cylinder is prohibited. When we consider the pair creation in the magnetosphere, the plasma around the light cylinder is much larger than the Goldreich–Julian density. Low-frequency EM waves such as dipole radiation from the pulsar cannot propagate in such a high-density plasma. The magnetic energy is not carried by the EM wave but by the entropy wave in the magnetized plasma wind (i.e., striped wind in Coroniti 1990).

Recently, strong EM waves in pulsar environments have again attracted much interest. It has been pointed out that the entropy mode can be converted to a strong EM wave in the outer region of the pulsar wind (Arka & Kirk 2012). Moreover, it is numerically shown by using a relativistic two-fluid simulation that this conversion can occur through interaction with the termination shock (Amano & Kirk 2013). Such EM waves have a superluminal phase velocity and their strength parameter $a$ may be larger than 1. Around the termination shock of the pulsar wind, the electrons are accelerated by shock crossing and radiate synchrotron photons. If such SLSWs exist around the shock, they may affect the particle acceleration and radiation.

There are many unsolved problems for particle acceleration around the termination shocks of pulsar wind nebulae (PWNe). One is injection. In general, to be injected into the shock-crossing cycle, the particles have to be supra-thermal when they encounter the shock front. However, in Kennel & Coroniti (1984a), upstream plasma was assumed to be cold and all particles were accelerated. In other words, they assumed a very high injection rate. The lowest Lorentz factor $\gamma \sim 10^6$ at the immediate downstream of the shock corresponds to the bulk Lorentz factor of the upstream wind $\Gamma \sim 10^6$. These particles emit optical photons via synchrotron radiation. Their model can explain the observed spectrum at frequency regions higher than the optical range, but radio components were not discussed (Kennel & Coroniti 1984b). There are some models for the Crab Nebula that explain the radio components. For example,
we have time-dependent one-zone models (Tanaka & Takahara 2010, Bucciantini et al. 2011) and axisymmetric two-dimensional magnetohydrodynamic (MHD) models (Olmi et al. 2014, 2015). They reproduce the spectrum of the Crab Nebula by assuming the energy distribution of the electrons is a broken power law, but they do not specify the origin of the break Lorentz factor $\gamma \sim 10^6$. This may imply that a pre-acceleration mechanism exists in the upstream of the termination shock. That is to say, we can consider a model in which the pre-acceleration mechanism makes the energy distribution broader and only higher energy components are injected to the shock-crossing cycle. If SLSWs can cause a non-thermal energy distribution in the upstream, they may be good candidates for a pre-accelerator.

Radiation from electrons in a strong ($a > 1$) EM wave is usually called synchro-Compton radiation (Rees 1971) or nonlinear inverse Compton (NIC) radiation (Gunn & Ostriker 1971). The deflection angle in one cycle of the electron motion in the interaction with a strong wave tends to be larger than $1/\gamma$. As a result, the radiation signature resembles synchrotron radiation. However, we do not know the radiation spectra when there are many waves. Moreover, even for an interaction with one SLSW, there are ambiguous points in the consideration of the typical frequency. To estimate the typical frequency of the radiation, we have to know the photon formation time (Akhiezer & Shul’ga 1987, Reville & Kirk 2010), which is the inverse of the cyclotron frequency $mc^2/eB$ in the context of synchrotron radiation. Previous studies considered cases for which the phase-locking effect is weak, which means that the velocity is oblique to the wavevector direction. For this type of case, the photon formation time is $\sim mc^2/eB$. In general, the motion of the electrons in a strong wave can result in a different photon formation time. Therefore, the resultant radiation spectra can be different from the NIC or synchro-Compton theories.

In this paper, we study electron acceleration in the SLSWs. The radiation signature in such a situation also is studied. We use numerical methods to investigate such highly nonlinear motions. The contents of this paper are as follows. In Section 2, we discuss the physical parameters for the computational study and describe the methods we use. The results are shown in Section 3. Section 4 presents discussions including observational features. We summarize this paper in Section 5.

2. FORMULATION

2.1. Parameters

Here, we estimate the physical parameters around the termination shock of the PWN by using the Crab Pulsar and its nebula as a representative system. First, we estimate the strength parameter $a$ of the entropy mode in the immediate upstream of the termination shock by following the estimation by Kirk & Mochol (2011) in which they estimated $a$ in active galactic nuclei jets. We note that this strength parameter is not identical to the strength parameter of the EM wave. The strength parameter of the entropy mode is defined by the wavelength $\lambda_{sw}$ and magnetic field strength $B$ as $eB\lambda_{sw}/2\pi mc^2$ in the observer frame. There are two feasible assumptions. One is that the magnetic field strength in a pulsar wind is inversely proportional to the distance from the pulsar $r$ as $B \propto r^{-1}$, (here we implicitly assumed the magnetic field configuration to be pure toroidal). The other is that the spindown luminosity of the pulsar $L_{sd}$ is carried by the Poynting flux (high sigma) and is isotropic. Given the above assumptions, the entropy wave in the observer frame resembles an EM wave due to the fact that the electric field is perpendicular to the magnetic field, the strength ratio is nearly unity, and the wavevector is perpendicular to the electric field and magnetic field. We obtain $a$ from the observed quantities as

$$a = \left(\frac{n_c}{r}\right)\left(\frac{e^2L_{sd}}{m^2c^5}\right)^{1/2} \approx 3.4 \times 10^{16} \left(\frac{n_c}{r}\right)\left(\frac{L_{sd}}{10^{38} \text{ erg s}^{-1}}\right)^{1/2},$$

where $n_c = 1.6 \times 10^8 \text{ cm}$ is the light cylinder radius of the Crab Pulsar. The radius of the termination shock of the Crab Nebula is $\sim 10^9\,r_{LC}$ and the spindown luminosity of the Crab Pulsar is $\sim 6 \times 10^{38} \text{ erg s}^{-1}$. Thus, the strength parameter of the entropy mode is estimated as $a \approx 80$ at the termination shock. Since the EM wave is expected to be converted from this entropy wave, we can expect this wave to be a superluminal “strong” wave (SLSW).

Next, we compare the inertial length and the wavelength of the entropy mode in the upstream and the downstream by assuming that the typical length scale does not change even if some fraction of energy is converted from the EM energy to the kinetic energy such as magnetic reconnection (Lyubarsky & Kirk 2001). If the wavelength of the entropy mode is shorter than the inertial length, the MHD approximation breaks down and the entropy wave can be converted to the other waves. The ratio of these length scales in the upstream is

$$\frac{\Gamma\lambda_{sw}}{2\pi c/a_{p,up}}, \text{up} \equiv \eta_{ap} \sim 2.7 \times 10^2 \sqrt{n_{ap}\Gamma^2},$$

where $\Gamma$ is the bulk Lorentz factor of the pulsar wind, $\lambda_{sw} = 10^9 \text{ cm}$ is the wavelength of the striped wind in the observer frame, $a_{p,up} = \sqrt{4\pi n_{ap}e^2/m}$ is the upstream plasma frequency and $n_{ap}$ is the comoving number density. Here we assumed that the plasma does not have a relativistic temperature. On the other hand, a constraint on $n_{ap}$ and $\Gamma$ at the termination shock is obtained from the spindown luminosity. By expressing the spindown luminosity as a sum of isotropic kinetic and Poynting fluxes and substituting the radius of the termination shock of the Crab Nebula into it, we obtain

$$\Gamma^2(1 + \sigma)n_{ap} = 2 \times 10^{-2}.$$

Here $\sigma$ is the ratio of the Poynting flux to the kinetic flux. Using this constraint, Equation (2) gives

$$\eta_{ap} \sim 3.8 \times 10^4 \times (1 + \sigma)^{-1/2}.$$

We can estimate $\eta_{ap}$ at the immediate upstream of the termination shock by using $\sigma$. At the light cylinder, the pair cascade models predict $\sigma \sim 10^6$ (e.g., Hirota 2006). It should be reduced by the magnetic reconnection in the wind region. If this process is extremely inefficient and $\sigma$ is close to $10^3$ at the termination shock, $\eta_{ap} < 1$ is realized. In this case, the entropy wave may break down by a non-MHD effect. On the other hand, if we adopt a more conventional value of $\sigma < 10^3$, $\eta_{ap}$
should be larger than unity. In this case, the entropy wave can survive in the upstream. Hereafter we consider the latter case.

The situation drastically changes in the downstream. First we estimate the ratio in the downstream $\eta_{\text{down}}$ under the assumption that the entropy mode does not convert to the SLSWs. From this estimation, we can realize that this assumption is not appropriate. Then, we consider an alternative scenario.

According to observational fact, the bulk Lorentz factor of the downstream plasma is $\Gamma \sim 1$. The wavelength of the entropy wave changes due to shock compression, but their compression ratio measured in the downstream frame is at most $O(1)$. Therefore, this scale is $\sim \lambda_{\text{sw}}$ in the downstream frame. On the other hand, the inertial length in the downstream frame increases drastically because of the thermalization. The ratio becomes

$$\eta_{\text{down}} = \frac{\lambda_{\text{sw}}}{2\pi c/\omega_{p,\text{down}}} = \left(\gamma_{\text{th}}\Gamma\right)^{-1/2} \eta_{\text{up}},$$

where $\gamma_{\text{th}}$ is the Lorentz factor of the thermal motion of the downstream plasma, $\omega_{p,\text{down}} = \sqrt{4\pi n_{\text{down}} e^2/\gamma_{\text{th}} m}$ is the plasma frequency, which takes into account the relativistic correction, and $n_{\text{down}}$ is the downstream number density. We assume $\gamma_{\text{th}} \sim \Gamma$ since the downstream bulk velocity is nonrelativistic. As a result, $\eta_{\text{down}}$ is smaller than $\eta_{\text{up}}$ by a factor of $\Gamma$. This bulk Lorentz factor is estimated as $10^2 \lesssim \Gamma \lesssim 10^6$ by pair cascade models (e.g., Hirota et al. 2006), many MHD models of PWNe, and the induced Compton scattering constraint (Tanaka & Takahara 2013). Using the constraint, we estimate $\eta_{\text{down}}$ to be

$$10^{-6} \lesssim \eta_{\text{down}} \lesssim 1.$$  

Thus, the MHD approximation is not adequate for describing entropy waves in the downstream, and it should be dissipated (Petri & Lyubarsky 2007; Sironi & Spitkovsky 2011) or converted to some other waves (Amano & Kirk 2013).

Before proceeding further, let us summarize the above estimates. The strength parameter $a$ of the SLSWs is estimated as $O(10)$. The wavelength of the entropy mode is expected to be longer than the inertial length in the upstream, while the opposite is expected in the downstream. The expected values of their ratios are $\eta_{\text{up}} \gtrsim 1$ and $\eta_{\text{down}} \sim 10^{-3}$.

The entropy mode cannot be sustained by the downstream plasma. It should be converted to EM waves by the interaction with the shock front and the EM waves propagate back to the upstream region (Amano & Kirk 2013). As we will see below, the entropy mode can convert to SLSWs even in the upstream. Hereafter we consider the propagation condition of an SLSW, which is affected by the strength of the EM wave and the thermalization of the background plasma.

First, we review the effect of the strength of the wave. The frequency of the SLSW should be $\omega \sim \omega_{\text{sw}}$. In the shock rest frame. Here, we assume that this SLSW propagates toward the shock front in this frame. The frequency of this SLSW in the upstream frame is $2\pi c/\Gamma \lambda_{\text{sw}}$. For an ordinary ($a < 1$) EM wave in the plasma, the condition for the propagation is $\omega > \omega_{p,\text{up}}$. This condition coincides with $\eta_{\text{up}} < 1$. However, for SLSWs, the condition is modified by the strength parameter. The dispersion relation of the SLSW for pair plasma is described by Kaw & Dawson (1970) as

$$\omega^2 = \frac{2\omega_p^2}{\sqrt{1 + a^2}} + k^2c^2,$$

where $k$ is the wavevector. Hence, the condition for the propagation becomes

$$\frac{\omega^2}{\omega_p^2} > \frac{2}{\sqrt{1 + a^2}}.$$  

Therefore, the wave can propagate when $a$ is sufficiently large even if $\eta_{\text{up}} > 1$.

Next, we discuss the thermalization. The SLSWs in the overdense plasma (i.e., $\eta > 1$) are unstable (Max & Parkins 1971), and also it is shown by a two-fluid simulation that these waves generate EM waves and sound waves via stimulated Brillouin scattering (Amano & Kirk 2013). As a result, the SLSWs thermalize the plasma in the upstream region by dissipation of sound waves. If this mechanism works well, the upstream plasma gains a relativistic thermal energy. The plasma frequency becomes lower as the Lorentz factor of the thermal motion becomes large: $\omega_p \propto \gamma_{\text{th}}^{-1/2}$. Thanks to this nonlinear effect, the local plasma frequency in the upstream becomes lower than the SLSW frequency. The instabilities of the SLSWs are not fully understood. The growth rate is only known for limited conditions (c.f. Max & Parkins 1972; Asseo et al. 1978; Lee & Lerche 1978). It tends to be low when $a$ is not much larger than 1 and $\omega$ is sufficiently larger than $\omega_p$ (T. Amano 2004, private communication). This is natural because the wave becomes an ordinary EM wave in a vacuum in the limit of $a \ll 1$ and $\omega \gg \omega_p$.

As we have seen above, SLSWs can exist in the upstream region of the termination shock. In this paper, we study the electron acceleration in the upstream restframe. We assume that $a = O(10)$ and $\omega \gg \omega_p$. These assumptions are acceptable near the immediate upstream region.

### 2.2. Setup

We describe the EM turbulences using the superposition of EM waves. In this paper, we ignore wave–wave interactions for simplicity. Such interactions may not be negligible and will be treated in future works. This description of the turbulence is based on Giacalone & Jokipii (1999) who calculated the transport of the cosmic rays in a static magnetic field. Recently, they studied particle accelerations using this scheme (e.g., Giacalone & Jokipii 2009; Guo & Giacalone 2014). Here, we note the difference between our assumption about the waves and theirs. We assume propagating (superluminal) EM waves, whereas their magnetic field is static in the fluid restframe (entropy waves), and the motional electric field $E = -(v \times B)/c$ is used to calculate the particle acceleration in the shock restframe, where $v$ and $B$ are the fluid velocity and magnetic field, respectively. This difference has a big impact on particle acceleration mechanisms such as phase locking.

The electric field and magnetic field are expressed by the superposition of elliptically polarized waves. They are decomposed to linearly polarized sinusoidal waves. Strictly speaking, the waveform of an SLSW does not have a pure sinusoidal shape, but rather a sawtoothlike shape (Max & Parkins 1971). Our approximation of a sinusoidal wave is adequate for $\omega \gg \omega_p$. We assume that there is a primary wave which is...
generated from the entropy mode and isotropically distributed daughter EM waves. It is assumed that the entropy mode is completely converted into the primary wave for simplicity. The primary wave is assumed to be linearly polarized and propagates in the $z$ direction as

$$E_0 = A_0 \cos(\omega_0 t - k_0 z) \hat{e}_z,$$

$$B_0 = (A_0/\beta_{ph,0}) \cos(\omega_0 t - k_0 z) \hat{e}_y,$$

where $A_0$, $\omega_0$, $k_0$, and $\beta_{ph,0}$ are the amplitude, frequency, wavenumber, and phase velocity, respectively. Here $\hat{e}_z$ and $\hat{e}_x$ are the unit vector in the $x$ direction and $y$ direction, respectively. The secondary components are described as

$$E_{sec}(x, t) = \sum_{n=1}^{N} A_n \exp\left\{i\left(k_n \cdot x - \omega_n t + \xi_n\right)\right\} \hat{e}_{E,n},$$

$$B_{sec}(x, t) = \sum_{n=1}^{N} A_n \exp\left\{i\left(k_n \cdot x - \omega_n t + \xi_n\right)\right\} \hat{e}_{B,n},$$

where $A_n$, $k_n$, $\omega_n$, and $\xi_n$ are the amplitude, wavevector, frequency, and phase of each mode, respectively. Here $\beta_{ph,n} = v_{ph,n} / c$ is the phase velocity of each mode, which is calculated from the dispersion relation (Equation (7)), as we will show later. Since the phase velocities are larger than unity, the amplitude of the electric field is larger than the magnetic field. We note that it can be understood by considering Faraday’s law $\nabla \times E = -\frac{\partial B}{\partial t}$. Each wave propagates toward $\hat{e}_{x,n} = k_n / |k_n|$, where $\hat{e}_{x,n}$ and $\hat{e}_{y,n}$ form the orthogonal coordinate system. The polarization vectors are written as $\hat{e}_{E,n} = \cos \psi_n \hat{e}_{x,n} + i \sin \psi_n \hat{e}_{y,n}$ and $\hat{e}_{B,n} = -i \sin \psi_n \hat{e}_{x,n} + \cos \psi_n \hat{e}_{y,n}$. Since the distribution of the secondary waves is uncertain, we assume it to be isotropic. In this paper, $\psi_n$ is chosen randomly to make the distribution of the secondary waves isotropic. The polarization $(\hat{e}_{E,n})$ and phase $\xi_n$ are also randomly distributed. The amplitude of each mode is given by

$$A_n^2 = \sigma_{sec}^2 G_n \left[ \sum_{n=1}^{N} G_n \right]^{-1},$$

where $\sigma_{sec}^2$ represents the mean intensity of the secondary waves. The number of Fourier components $N$ is $10^2$ in this paper. We use the following form for the power spectrum

$$G_n = \frac{4\pi\omega_n^2 \Delta \omega_n}{1 + (\omega_n T_c)^2},$$

where $T_c$ is the coherence time which is set as $\omega_0 T_c = 1$, and $\alpha - 2$ is the power-law index of the energy spectrum of the secondary waves. We set $\alpha = 11/3$, which makes Kolmogorov-like turbulences. Here, $\Delta \omega_n$ is chosen such that there is an equal spacing in logarithmic $\omega$-space, over the finite interval $\omega_{min} \leq \omega \leq \omega_{max}$. We set the minimum frequency and the maximum frequency as $\omega_{min} = \omega_0$ and $\omega_{max} = 10^5 \omega_0$ in all calculations. The sum of the EM energy density of the primary and secondary waves is set to be constant in each run as

$$\varepsilon^2 = A_0^2 + \sigma_{sec}^2 \varepsilon_{sec}^2.$$

Using $\varepsilon$, the unit of time in this paper is defined as

$$\varepsilon \equiv \frac{mc}{e \omega},$$

The strength parameter is defined by using $\varepsilon$ and $\omega_0$ as

$$a = \frac{e \omega_0}{mc \varepsilon}.$$

When there is no primary wave, the strength parameter is defined by replacing $\omega_0 \rightarrow \omega_{min}$. The phase velocities $\beta_{ph,n} = \omega_n / (k_n c)$ are calculated from the dispersion relation

$$\omega_n^2 = \frac{2\omega_p^2}{1 + a^2} + k_n^2 c^2.$$

Here we note that the superposition of the modes is an approximation, because this dispersion relation is nonlinear. The appropriateness of this approximation for the obtained results should be checked in future works. We assume that the electrons initially have a relativistic energy $\gamma_0 mc^2 = 10mc^2$ and have an isotropic velocity distribution that mimics thermalized particles in the precursor region of the termination shock. In this paper we use $10^4$ electrons for the calculation. Since we neglect the back reaction from particles to EM fields, the electron number is important only for the statistics and does not change the physics. The injection points of these particles are chosen to be homogeneous. We fix the plasma frequency as $\omega_p = \sqrt{5} \times 10^{-2} \omega_0$ and it does not change in time. This fulfills the stable propagation condition. It is lower than the plasma frequency, which is estimated from the initial thermal Lorentz factor. However, as the Lorentz factor of the electrons becomes higher, the plasma frequency becomes lower. To avoid complexity, we approximate the plasma frequency by a fixed value, which corresponds to $\gamma_{th} \sim 100$.

We inject relativistic electrons as test particles in this prescribed EM field. We solve the equation of motion

$$\frac{d}{dt} (\gamma m_e v) = -e\left(\frac{E}{c} + \frac{v}{c} \times B\right)$$

by using the Buneman–Borris method. We neglect the radiation back reaction. This is an adequate approximation for the electrons that have not been injected in the shock-crossing process because the cooling timescale is much longer than the dynamical timescale (Amano & Kirk 2013). The radiation spectra are calculated from the information of the motion by directly using the Lienard–Wiechert potential

$$\frac{dW}{do d\Omega} = \frac{\omega^2}{4\pi c^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} n \times \left\{ \frac{(n - \beta) \times \hat{\beta}}{(1 - \beta \cdot n)^2} \right\} \exp\left\{ i \omega \left( t' - \frac{n \cdot r(t')}{c} \right) \right\}^2,$$

where $\hat{\beta} = v/c$ is the velocity of the electron, $n$ is the observer direction, and $t'$ is the retarded time (Hededal 2005). This formula can be applied for the frequency range of $\omega > \gamma \omega_p$ for
the radiation in the plasma. In the next section, we will calculate the radiation spectra. We will see that the above condition satisfied almost all frequency ranges. When an exception is encountered, we will note it.

3. RESULTS

3.1. Particle Acceleration

First, we demonstrate one of the notable features of particle acceleration by SLSWs, namely the strong acceleration in the wavevector direction. For this calculation, we set $\omega_0 = 0.1$. Since the unit of frequency is $e\varepsilon/mc$ as noted earlier, the strength parameter is $a = e\varepsilon/mc\omega_0 = 10$. We show the distribution of the $x$ and $z$ components of a four-velocity at $t = 3 \times 10^4\omega_0^{-1}$ in Figure 1. The red crosses are the four-velocities for $e\varepsilon_{\text{sec}}/mc = 1$, which means that the turbulence consists of secondary waves without a primary wave. On the other hand, green dots are the four-velocities for $e\varepsilon_{\text{sec}}/mc = 0.1$. The corresponding amplitude of the primary wave satisfies $eE_0/mc = \sqrt{\varepsilon^2 - \varepsilon_{\text{sec}}^2}/mc \approx 0.995$, and therefore the primary wave is dominant in this case. The distribution of the red crosses is nearly isotropic, while the distribution of green dots is quite anisotropic. The anisotropy is due to the acceleration of the electrons near the wavevector of the primary wave. Even though there are other waves, the primary wave can dominate the electron motion and phase locking occurs for $e\varepsilon_{\text{sec}}/mc = 0.1$. The distribution for the $x$ direction is symmetric and the width is around 10 times smaller than that for the case with $e\varepsilon_{\text{sec}}/mc = 1$. This is because the amplitude of the secondary components $\varepsilon_{\text{sec}}$ is 10 times smaller.

In Figure 2, we show the energy spectra for different amplitudes of secondary components ($e\varepsilon_{\text{sec}}/mc = 1, 0.5, 0.1, \text{and } 10^{-2}$) at $t = 3 \times 10^4\omega_0^{-1}$. For $e\varepsilon_{\text{sec}}/mc = 10^{-3}$, the energy distribution has a pure power-law distribution. In this case, the secondary components are quite weak compared to the primary component. As a result, electrons that are initially moving nearly parallel to the wavevector of the primary wave are in a state of “phase locking” and selectively accelerated. The cutoff energy reaches an expected value of $a^2\varepsilon_0 = 10^3$ in the timescale of $a^2\varepsilon_0^{-1} = 10^4\omega_0^{-1}$, which is the typical oscillation timescale of a “phase-locked” particle. The other electrons also tend to accelerate toward the wavevector direction of the primary wave, but this acceleration is weaker because they are not in the phase-locking state. To clarify what is meant by selective acceleration, we show the time series of the energy spectra for $e\varepsilon_{\text{sec}}/mc = 10^{-3}$ in Figure 3. The high-energy cutoff evolves in time due to selective acceleration. On the other hand, the peak Lorentz factor does not vary and remains at $\gamma \sim 10$ since the acceleration by secondary waves is quite weak in this case. This distribution shows a power-law shape of $dN/d\gamma \propto \gamma^{-2}$. We do not intend to claim that this power-law index is universal because it can be altered by initial conditions. For example, the initial velocity (direction) distribution, assumed to be isotropic, apparently changes the resultant energy distribution, since the phase-locking effect is strongly dependent on the angle between the velocity and wavevector of the SLSW. Here we stress that we obtain high-energy electrons that have an energy much higher than the peak energy, and the phase-locking effect from the strong wave is an important factor for realizing this higher energy.

For $e\varepsilon_{\text{sec}}/mc = 1$, many EM waves accelerate the electrons without long-term phase locking, which is prevented by the disturbance of the velocity direction by the other EM waves. As
a result, electrons diffuse in the momentum space, and the distribution function \( f(p) \) is well described by the Gaussian function. We can see that the energy distribution \( dN/d\gamma \propto p^2 f(p) \) is consistent with a power-law distribution in the low energy side with an index of +2 and an exponential cutoff in the high-energy side.

Lastly let us focus on the cases for \( \varepsilon_{\text{sec}}/m_{\text{c}} = 0.1 \) and 0.5. In these cases, both the primary and secondary waves affect the particle distribution (green and light blue lines in Figure 2). While most particles are diffusively accelerated by the turbulence (secondary waves), a small fraction of particles are selectively accelerated by the primary wave. This selection can clearly be seen in the four-velocity distribution for \( \varepsilon_{\text{sec}}/m_{\text{c}} = 0.1 \) in Figure 1. The peak energy difference between two distributions comes from the energy density of the secondary waves. The electrons in this peak energy range are accelerated diffusively by the secondary waves. Therefore, the energy density of the secondary waves is smaller and the peak energy is lower. If the \( A_0 \) is slightly larger than \( \varepsilon_{\text{sec}} \) in the upstream of the termination shock, this acceleration mechanism can produce a broad energy distribution that contains a high-energy power-law tail.

3.2. Radiation

The radiation spectra of the electrons moving in the SLSWs for \( \varepsilon_{\text{sec}}/m_{\text{c}} = 0.1 \) are shown in Figures 4 and 5. We calculate the radiation spectra in the restricted time from \( t = 3 \times 10^5 \omega_0^{-1} \) to \( t = 3 \times 10^{4} + 2 \times 10^{5} \omega_0^{-1} \). The starting time \( t = 3 \times 10^{4} \omega_0^{-1} \) corresponds to the time of the energy distribution of Figure 2. This integration timescale is longer than the photon formation time of the typical frequency of the synchro-Compton radiation by a factor of \( 2 \times 10^5 \). This ensures that we can resolve the radiation spectrum down to \( \sim 10^3 \) times lower frequencies than the peak one. On the other hand, the time step for pursuing the electron motion is \( 10^{-2} \). This ensures that we can resolve the radiation spectrum up to \( \sim 10^2 \) times higher frequencies than the peak one. Strictly speaking, the time step should be smaller than the inverse of the radiation frequency, and it is much shorter than \( 10^{-2} \). However, it is shown that the radiation spectrum is well described by using the large time step which is about only one order shorter than the photon formation time (Reville & Kirk 2010). The horizontal axis is the frequency normalized by \( \varepsilon_{\text{sec}}/m_{\text{c}} \). The vertical axis is the flux in an arbitrary unit. The jagged lines are the calculated radiation spectra, and the smooth lines are the analytical synchrotron curve, which are shown for comparison.

In Figure 4, the observer direction is nearly along the \( z \) direction. To be precise, we set the observer direction as \( \mathbf{n} = (n_x, n_y, n_z) = (10^{-2}, 0, \sqrt{1 - 10^{-4}}) \). The reason for not choosing \( \mathbf{n} = (0, 0, 1) \) will be explained later. The peak frequency of the radiation spectrum in Figure 4 is \( \omega_{\text{peak}} \sim 10^3 \). This can be understood by using the synchro-Compton theory (Gunn & Ostriker 1971; Rees 1971). The cutoff Lorentz factor \( \gamma_{\text{cut}} \) is around \( 10^3 \) for \( \varepsilon_{\text{sec}}/m_{\text{c}} = 0.1 \) at \( t = 3 \times 10^5 \omega_0^{-1} \) as is seen in Figure 2. The peak frequency can be estimated as \( \omega_{\text{peak}} \sim \gamma^2 \varepsilon_{\text{sec}}/m_{\text{c}} = 10^5 \) in the same manner as the synchrotron radiation. This can be justified as follows. First, we consider an electron that is in the phase-locking state in a strong wave and tentatively ignore the other waves. In this case, the Lorentz force is negligible except for the direction parallel to the velocity. The resultant trajectory is nearly straight, and the curvature radius of the orbit is very long compared to \( y_{\text{mc}}/\varepsilon_{\text{sec}} \). Next we add the other waves which are isotropically distributed. The electron trajectory obtains a wiggling shape with a curvature radius for each case of \( \sim y_{\text{mc}}/\varepsilon_{\text{sec}} \). The deflection angle during a typical deflection is \( \sim 1/\gamma^2 \) since the strength parameter defined by using only secondary components such as \( \varepsilon_{\text{sec}}/m_{\text{c}} \) is unity. As a result, electrons emit “synchro-Compton” radiation around the peak frequency. The large part of this radiation power comes from the electrons moving nearly along the \( z \) direction since the selectively accelerated electrons are moving in this way. Thus, the radiation spectrum has a clear peak at \( \omega \sim \gamma_{\text{max}}^2 \varepsilon_{\text{sec}}/m_{\text{c}} \sim 10^5 \). We note that the deviation from the exponential cutoff in the highest frequency region comes from the jitter radiation contribution since there are secondary waves that have higher frequencies than \( \omega_{\text{min}} = \varepsilon_{\text{sec}}/m_{\text{c}} \) (compare Teraki & Takahara 2011). The spectrum in the frequency
region lower than the peak is harder than the “isotropic” synchrotron theoretical one. In our case, the velocity distribution is quite anisotropic. The spectral index of the synchrotron radiation toward some direction, due to an electron, is 2/3 (compare Jackson 1999). The calculated spectrum seems to be slightly harder than $F_\nu \propto \nu^{2/3}$, so that additional mechanisms may contribute to this spectrum, but we do not discuss this topic further. More detailed analyses will be done in our future work. The important thing that we should stress here is that the primary wave does not contribute to the radiation directly. It works only for the energy gain of the electrons. The radiation power and typical frequency are determined by the energy of the electrons and EM energy density of the secondary waves.

In Figure 5, we set $n = (1, 0, 0).$ The spectrum is well described by the isotropic synchro-Compton radiation. The spectral index at the frequency region lower than the peak coincides with one-third, and the peak frequency $\nu_{\text{peak}} \sim 10^7$ is the expected value. The Lorentz factor of the electrons moving around the x direction is $\gamma \sim 30$ (see Figures 1 and 2). The magnetic field (y direction) of the primary wave mainly contributes to the radiation power for this case. Thus, the peak frequency is described as $\gamma^2 eB_0/mc \sim (30)^2 \times 1 \sim 10^7$. The spectrum at the frequency region higher than the peak shows slower decline than the exponential cutoff. This is due to the superposition of the contributions from electrons with higher energies. Jitter radiation components do not stand out in this spectrum because of $eB_0/mc \gg 1$ (compare Teraki & Takahara 2011).

To confirm the fact that the scatterers needed for the radiation of Figure 4 are the secondary components, we show the radiation spectra for $\epsilon_{\text{sec}}/mc = 10^{-3}$ in Figure 6. The secondary waves are extremely weak compared to the primary wave, so that the primary wave causes the radiation. The integration time is identical to that used for depicting Figures 4 and 5. The lower (blue) line is the radiation spectrum for the observer located in the x direction. The peak frequency at $\sim 100$ can be explained in the same manner as above. The $B_y$ component of the primary wave $B_0y$ strongly deflects the electron and produces the peak frequency at $\sim \gamma^2 eB_0/mc \approx 100$ since the typical Lorentz factor for the unselected electrons (which is not moving toward the z direction) is $\approx 10$ and $eB_0/mc \approx 1$. The spectral shape roughly coincides with the theoretical synchrotron curve as seen in Figure 6. On the other hand, different features are found in the spectrum for the observer located in $n = (n_x, n_y, n_z) = (10^{-2}, 0, \sqrt{1 - 10^{-4}})$, which is shown by the red line. It is noted that the spectrum is shifted vertically by a factor of 200 to see the shape clearly. The spectral shape is clearly different from the synchro-Compton radiation because this radiation signature directly reflects the nonlinear orbit. This large-scale orbit is determined by the strong primary wave, and the small-scale deflection angle is much smaller than $1/\gamma$ since the strength parameter for the secondary waves $\epsilon_{\text{sec}}/mc\omega_{\text{min}} = 10^{-2}$ is very small. The sweeping behavior (of the beaming cone) is completely different from sweeping with a curvature radius $\gamma mc^2/eB$, which is assumed for the synchrotron radiation. The particle trajectory is nearly straight, and the sweeping time is much longer than $mc/e\gamma$ and $1/\omega_0$. In this case, the sweeping timescale is roughly 10 times shorter than the maximum phase-locked oscillation timescale $2\pi^{3/2} \sqrt{\gamma^2 - 1} / a_{0-1}$. In our calculation, the initial velocity directions do not coincide with the wavevector of the primary wave, and $|E| \neq |B|$. Moreover, the phase velocity of this wave is larger than $c$. Such effects shorten the sweeping timescale by a few times compared to the case for the vacuum. As a result, the typical frequency is $\gamma_{\text{max}}^2 \omega_0 / (\sqrt{3} a_{0-1}^2) \approx 0(10) \geq 10^2$. We note that the second harmonics can be seen around $\omega \sim 1000$. We note that this radiation spectra in the lowest frequency region $\omega \approx 10$ is not precise because $\omega_0 \sim 10^3 \times \sqrt{\gamma} \times 10^{-3}$ is $O(1)$. However, this frequency range is not important for the current discussion. Here, we showed that the scatterer for the radiation for $\epsilon_{\text{sec}}/mc = 10^{-2}$ is the primary wave. Furthermore, we confirmed that the radiation signature is not always described by the synchro-Compton theory. From this fact, we can understand that the scatterers that realize the spectra in Figure 4 are the secondary waves.

Lastly let us explain the reason for choosing the observer direction of $n = (n_x, n_y, n_z) = (10^{-2}, 0, \sqrt{1 - 10^{-4}})$ and not $n = (0, 0, 1)$. As mentioned above, the selectively accelerated electrons tend to move toward the z direction, but there are very few electrons moving “very close” to the z axis. The reason is as follows. When the phase locking occurs, the perpendicular forces (on the velocity) from the electric magnetic fields nearly cancel each other out. From the balance of perpendicular forces, we can estimate the angle $\theta$ between the wavevector and the velocity. By using the assumed parameter $a_{0-2} = \sqrt{3} \times 10^{-2}$, $\gamma \sim 1/\theta$ is obtained for $\gamma < 10^2$. On the other hand, the angle is constant as $\theta \approx 10^{-2}$ for $\gamma > 10^2$. The electrons moving along the z axis are slightly deflected and tend to have angles $\geq 10^2$ from the z axis. Since we want to see the radiation from the strongly accelerated particles, we set the observer in the direction $n = (n_x, n_y, n_z) = (10^{-2}, 0, \sqrt{1 - 10^{-4}})$.

4. DISCUSSION

SLSWs should exist around the termination shock of the PWNe. They may play an important role in the particle acceleration. Particularly, the particle acceleration by the SLSWs may work as a pre-acceleration mechanism for diffusive shock acceleration (DSA). We note that for electrons
undergoing DSA, the contribution to the “energy change” is small since it is much slower than the Bohm limit of DSA. To make the injection rate higher, the energy distribution should be broader. The amplitude ratio between the primary and secondary waves is a key point for achieving this. If the upstream EM field mainly consists of a primary SLSW, the resultant electron energy distribution tends to show a power-law shape, and the four-velocity distribution is anisotropic. On the other hand, if the SLSW distribution is isotropic and no primary wave exists, the obtained energy distribution is narrower than the former case and isotropic. The SLSW acceleration in the upstream may determine the injection rate of the DSA. The radiation from the electrons accelerated by the SLSW can be understood as synchro-Compton radiation in the secondary waves. In this section, we discuss the applicability of this acceleration to PWNe and the observational prospects.

4.1. Length Scale of the Acceleration Region

The length scale over which the SLSWs can exist also remains unsolved, but is an important problem. Here we estimate the length scale needed for the acceleration. We consider the observer frame ($K'$ frame) and the upstream restframe ($K$ frame). First, we consider the case for which these electrons are diffusively accelerated. We assume the velocity direction is nearly parallel to the boost direction of the $K'$ frame in the $K$ frame. The length scale in the $K'$ frame for a typical deflection is $\sim c/\omega_{\text{min}}$, where $\omega_{\text{min}}$ is the typical frequency of the waves. This scale is written in the $K$ frame as $L \sim \Gamma (\gamma' + V)/\omega'_{\text{min}}$. In the $K$ frame, we have

$$L \sim 4\Gamma^2 c/\omega_{\text{min}}, \quad (21)$$

where $\omega_{\text{min}} = \omega_0$ is the frequency in the $K$ frame, which can be regarded as the inverse of the spin period of the pulsar. It is estimated by using the Crab parameters as

$$L \sim 6 \times 10^{12} \left( \frac{\Gamma}{10^2} \right)^2 \text{cm}. \quad (22)$$

Next we consider electrons in the phase-locking state. The length scale in the $K'$ frame is $\sim a_0^2 c/\omega_0$, where $\omega_0$ is the frequency of the primary wave. In the $K$ frame, it is

$$L \sim 6 \times 10^{16} \left( \frac{\Gamma}{10^2} \right)^2 \left( \frac{\gamma'_0}{10} \right)^2 \text{cm}. \quad (23)$$

This is shorter than the radius of the termination shock of the Crab Nebula only by a factor of five. From this estimation, it can be realized that the maximum Lorentz factor reached in our simulation is an upper limit since the electrons can get into the shock before achieving the maximum energy. On the other hand, the planar wave approximation becomes inadequate if the length scale is comparable to the termination shock radius. Therefore, the acceleration by secondary waves without phase locking can play a role in the upstream, but we should be cautious about interpreting the phase-locking acceleration. In other words, the maximum energy of the strong wave acceleration can be determined by the scale length for which the SLSWs exist.

4.2. Observational Prospect

Lastly we discuss the possibility of observing the signature of pre-acceleration by the primary wave. The typical radiation in this situation is synchro-Compton radiation from selectively accelerated electrons. The maximum frequency of the synchro-Compton radiation in the $K'$ frame is written as

$$\omega_{\text{max}} \sim \Gamma_m^2 eB/mc. \quad (24)$$

The maximum Lorentz factor may be determined by the scale limit, as we have discussed above. However, here we suppose the acceleration region is sufficiently large to reach the maximum energy in one cycle of the phase-locking electron $\gamma'_m \sim a^2\gamma_0$. We note that the photon formation length of the typical synchro-Compton photon (compare Teraki & Takahara 2014) is much shorter than the length scale of the whole orbit. Therefore, we can estimate the typical radiation frequency without information about the whole orbit. Here we suppose that the energy density of the SLSWs and electrons are highest near the region of the termination shock. The typical frequency of this radiation in the observer frame is estimated as

$$\omega \sim \Gamma_m^2 eB/mc \approx 2 \times 10^{11} \left( \frac{\gamma'_m}{10^3} \right)^2 \left( \frac{B}{10^{-3} \text{G}} \right) \text{s}^{-1}. \quad (25)$$

Thus, it can be observed as a radio knot. The observable area is restricted as $\sim (r_{\text{TS}}/\Gamma)^2$ by the beaming effect of the upstream bulk motion. Interestingly, small radio knots with a scale $\lesssim 10^{15}\text{cm}$ are observed near the termination shock by VLBI imaging (Lobanov et al. 2011). If we adjust the observable area to this observation, the bulk Lorentz factor is constrained to $\Gamma \gtrsim 10^2$. This value is consistent with our scenario and acceptable for PWNe models. If these radio photons are emitted as in our model, unfortunately, no high-energy counterparts emitted by inverse Compton scattering will be observed. The luminosity should be much smaller than the gamma-ray luminosity from the whole nebula, and current gamma-ray observations cannot resolve the spatial structure of PWNe.

5. SUMMARY

We have investigated electron acceleration in SLSWs and the radiation from them. We considered two classes of waves. One is the primary wave, and the other is the isotropically distributed secondary waves. We took the amplitude ratio of them as a parameter. When the primary wave is dominant, the electrons moving nearly along the wavevector direction are selectively accelerated and form a power-law distribution in the high-energy region. On the other hand, when the secondary waves are dominant, the energy distribution is narrow, shows an exponential cutoff, and does not show any power-law tails beyond the peak. We can expect both cases in the upstream of the termination shock of the PWNe. If the former case is realized, this acceleration mechanism may play a significant role for the injection to the shock acceleration. The radiation features can be described by synchro-Compton theory when the amplitudes of the secondary waves are similar to those of the primary wave. The radiation from the phase-locked electrons can show a different spectrum when the primary wave is extremely dominant and the strength parameter of the...
secondary waves is much smaller than unity. However, such a situation may not be realized around the termination shock of the PWNe since it is a very coherent situation. Radio knots that are illuminated by a synchro-Compton mechanism without counterparts in the higher frequency range can be observed in the immediate upstream of the termination shocks if SLSWs can propagate much longer than their wavelengths.

We are grateful to the referee for his/her constructive and helpful comments. We also thank Jin Matsumoto, Maria Dainotti, Maxim Barkov, Annop Wongwathanarat, Akira Mizuta, Tomoya Takiwaki, and Nodoka Yamanaka for fruitful discussions. Y.T. thanks Fumio Takahara, Takanobu Amano, Shimone Giacche, and John Kirk for stimulating suggestions. H.I. acknowledges support by a Grant-in Aid for Young Scientists (B:26800159) from the Ministry of Education, Culture Sports, Science, and Technology, Japan. This work is supported by RIKEN through the Special Postdoctoral Researcher Program.

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