Solution of the Boltzmann equation
in a random magnetic field

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Abstract

A general framework for solving the Boltzmann equation for a 2-dimensional electron gas (2DEG) in random magnetic fields is presented, when the random fields are included in the driving force. The formalism is applied to some recent experiments, and a possible extension to composite fermions at $\nu = 1/2$ is discussed.
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I. INTRODUCTION

Considerable interest both theoretically and experimentally has recently focussed on the response of a 2DEG in spatially inhomogeneous external fields. Examples include the so-called Weiss-oscillations when a periodic electric field modulation is imposed, similar effects are predicted for periodic magnetic fields, and random magnetic fields have been studied recently by several authors. In this paper we show how to set up a calculation of the transport coefficients of a 2DEG in a random magnetic field. The magnetic fields in the experiments cited above are all fairly low, well below the onset of Shubnikov-de Haas oscillations, leading us to expect a semiclassical treatment to be valid. We explicitly disregard any weak localization effects, which are well understood and can be subtracted from the experimental results.

The plan of the paper is as follows: In section II we describe the basic physical system and our model. In section III we discuss how to include the random magnetic field in the Boltzmann equation. Section IV and V are devoted to the calculation of the resistivity tensor. Finally, in section VI we give some applications of our theory to recent experiments, and discuss suggestions for future experiments. We also briefly consider the applicability of our theory to the Quantum Hall state at $\nu = 1/2$. The conclusion is in section VII.

II. PRELIMINARIES

Let us consider a 2DEG in a constant (external) electric field $E_0$, and a spatially varying magnetic field applied perpendicular to the plane of the electron gas: $B(r) = (0, 0, B(r))$ ($r$ is in the plane). There have been different realisations of the magnetic field: One, originally proposed by Rammer and Shelankov for studying weak localization effects in inhomogeneous magnetic fields, consists in a type II superconducting gate on top of the heterostructure containing the 2DEG. When above the lower critical field $B_{c1}$ for the superconductor, an applied field will penetrate in flux tubes, each containing an integral number of (superconductivity)
flux quanta $\Phi_0^{SC} = h/2e$. Because of disorder in the superconductor the flux tube distribution will not be the Abrikosov lattice, but a more or less random configuration, giving rise to a randomly modulated field in the 2DEG. Another approach was taken by Smith et al. who deposited small lead grains on the surface of a high mobility GaAs/AlGaAs heterostructure. For the grain sizes considered Pb is a type I superconductor. Below $B_c$ there will be (partial) flux expulsion from the grains, creating an inhomogenous magnetic field in the 2DEG. Still another possibility is to deposit small grains of a permanent magnetic material on top of the 2DEG. The different kinds of inhomogeneity will give rise to different field-dependences of the magnetoresistance.

We will model the random magnetic field by the following expression:

$$B(r) = \sum_i b(r - r_i),$$

where $r_i$ is the position of the $i$th impurity (flux tube, lead grain,...) and $b$ the field produced by the impurity (including any constant external field). We will assume that the positions are randomly distributed over the sample and denote an average over the positions $r_i$ by an overline. Defining $B_0 = \overline{B(r)}$ we can write

$$B(r) = B_0 + \delta B(r),$$

where $\delta B$ has zero average.

### III. THE BOLTZMANN EQUATION

The standard derivation of the Boltzmann equation from the full quantum mechanical transport equation relies on a separation of length scales: The external fields vary slowly in comparison with $1/k_F$, while the rapidly varying impurity potentials are included in a self-energy, giving rise to the collision term in the Boltzmann equation. A previous attempt to include random magnetic fields in the Boltzmann equation has viewed the magnetic ‘impurities’ as (asymmetric) scatterers, i.e. has included them in the scattering term on the
right hand side. This, of course, can always be done, even when the correlation length \( a \) of
the modulation (the London length or the size of the lead grains) is much greater than \( 1/k_F \).
However, when (as is the case in the experiments) \( a \gg 1/k_F \), the modulation \( \delta B \) can just
as well be treated as an ordinary external field in the driving force term of the Boltzmann
equation. (By contrast it would be inconsistent to put \( \delta B \) on the left hand side when
\( a \sim 1/k_F \).) In the present paper we will take the latter point of view, i.e. put the random
magnetic field on the left hand side of the Boltzmann equation. For the ordinary impurity
scattering we will assume the relaxation time approximation with a constant scattering time \( \tau \).

The Boltzmann equation for the distribution function \( f(r, v) \) is then

\[
\mathbf{v} \cdot \frac{\partial f}{\partial r} + \mathbf{a}(r) \cdot \frac{\partial f}{\partial \mathbf{v}} = \left( \frac{\partial f}{\partial t} \right)_{\text{coll}},
\]

where the Lorentz force \( m\mathbf{a}(r) \) is

\[
a(r) = -\frac{e}{m}(E_0 + \mathbf{v} \times \mathbf{B}(r)),
\]

and

\[
\left( \frac{\partial f}{\partial t} \right)_{\text{coll}} = -\frac{f - f^0}{\tau}.
\]

We calculate to linear order in the external electric field \( E \), i.e. replace
\[-(e/m)E \cdot \frac{\partial f^0}{\partial \mathbf{v}} \rightarrow -eE \cdot v^0 \frac{\partial f^0}{\partial E}, \]

where \( f^0 \) is the Fermi-Dirac distribution. Introducing polar co-ordinates \( v, \phi \) for the
velocity, at \( T = 0 \) (the only case considered here) \( v \) only enters through
\[
\frac{\partial f^0}{\partial E} = -\delta(E - E_F) = -(mv_F)^{-1}\delta(v - v_F)
\]
and can be put equal to \( v_F \). Writing \( f(r, v, \phi) = f^0 + g(r, \phi)\delta(v - v_F) \)
we get the following equation for \( g \):

\[
\left\{ v_F \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix} \cdot \frac{\partial}{\partial r} + (\omega_0 + \delta \omega(r)) \frac{\partial}{\partial \phi} + \frac{1}{\tau} \right\} g(r, \phi) = -\frac{e}{m} E \cdot \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix}.
\]

Here \( \omega_0 = eB_0/m \) and \( \delta \omega(r) = e\delta B/m \). With the following definitions (the \( i \)'s are for
convenience):
\[ D_0 = i \left\{ v_F \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix} \cdot \frac{\partial}{\partial r} + \omega_0 \frac{\partial}{\partial \phi} + \frac{1}{\tau} \right\} \]  

(7)

\[ W = i \delta E(r) \frac{\partial}{\partial \phi} \]  

(8)

\[ \chi(\phi) = -i \frac{e}{m} E \cdot \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix}, \]  

(9)

we can write Eq. (6) as an operator equation

\[ Dg \equiv (D_0 + W)g = \chi. \]  

(10)

The strategy is now to find the Green's function \( G \) for \( D \). Then we get

\[ g(r, \phi) = \int dr' d\phi' G(r, \phi; r', \phi') \chi(\phi'); \]  

(11)

and the current density

\[
\begin{align*}
\mathbf{j} &= -2e \int \frac{d\mathbf{p}}{(2\pi \hbar)^2} v_g(r, \phi) \delta(v - v_F) \\
&= -\frac{1}{\pi} ne \int_0^{2\pi} d\phi \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix} g(r, \phi),
\end{align*}
\]  

(12)

where \( n \) is the electron density.

\textbf{IV. EIGENFUNCTIONS AND GREEN'S FUNCTION FOR } \( D_0 \)

It is straightforward to see that \( D_0 \) has the following complete set of eigenfunctions and corresponding eigenvalues:

\[ \psi_{nk}(r, \phi) = \frac{1}{\sqrt{2\pi A}} e^{i[k(r - R(\phi)) - n\phi]} \]  

(13)

\[ \lambda_n = n\omega_0 + \frac{i}{\tau}. \]  

(14)

Here \( A \) is the area of the sample and we have defined

\[ \mathbf{R}(\phi) = r_c \begin{pmatrix} \sin \phi \\ -\cos \phi \end{pmatrix}, \]  

(15)
where \( r_c = \frac{v_F}{\omega_0} \) is the average cyclotron radius.

The Green’s function is then given by

\[
G_0(\mathbf{r}, \phi; \mathbf{r}', \phi') = \sum_{nk} \frac{\psi_{nk}(\mathbf{r}, \phi) \psi_{nk}^*(\mathbf{r}', \phi')}{n\omega_0 + i/\tau} = -i\omega_0 \frac{1}{e^{2\pi/\omega_0 \tau} - 1} e^{i(\phi' - \phi)/\omega_0 \tau} \delta(\mathbf{r} - \mathbf{r}' - \mathbf{R}(\phi) + \mathbf{R}(\phi'))
\]  

(16)

(17)

(see appendix \[A\] for details). We use the notation \([\theta]\) to denote \(\theta \mod 2\pi\). Note that \(G_0\) is only a function of \(\mathbf{r} - \mathbf{r}'\) because of the translational invariance of \(D_0\). Inserting Eq. (17) in Eqs. (11) and (12) for the current, the ordinary Drude formula is recovered.

V. CALCULATION OF THE RESISTIVITY

We first have to consider how to perform the average over the random magnetic field. Since the distribution function \(g\) is the physically relevant quantity, it is \(g\) we have to average. From Eq. (11) we see that this means we have to find the averaged Green’s function \((\chi, \text{ of course, only depends on the external electric field})\) for \(D\). To do this we consider the expansion of \(D^{-1}\):

\[
D^{-1} = (D_0 + W)^{-1} = D_0^{-1} - D_0^{-1}WD_0^{-1} + D_0^{-1}WD_0^{-1}WD_0^{-1} - \cdots
\]

(18)

We now average Eq. (18) term by term. This is formally similar to the quantum mechanical treatment of ordinary impurity scattering. Like in quantum mechanics we can organize the terms into diagrams as shown in figure 1. Summing the appropriate geometrical series we can rewrite the equation for the averaged Green’s function in terms of a self-energy:

\[
\overline{D^{-1}} = (D_0 + \Sigma)^{-1},
\]

(19)

where \(\Sigma\) is the sum of all irreducible diagrams, i.e., diagrams that do not fall apart when an internal line is cut, as shown in figure 2.

We shall only go to second order in \(W\), i.e., only keep the first diagram in figure 2. The higher order diagrams (including the maximally crossed ones, of which the first is shown in
the figure) can in principle be included; they are all finite and we do not expect them to change our conclusions (as we have explicitly verified for the second diagram in figure 2). There are no divergences corresponding to ‘weak localization’ in this purely semiclassical calculation.

We thus truncate the self-energy and write

$$D^{-1} = (D_0 - WD_0^{-1}W)^{-1},$$

The ‘self-energy’ $WD_0^{-1}W$ is calculated as follows. Using that $W$ is Hermitian we get:

$$\langle nk | WD_0^{-1}W | n'k' \rangle = \int dr \ d\phi \ dr' \ d\phi' (W \psi_{nk}(r, \phi))^* G_0(r, \phi; r', \phi') W \psi_{n'k'}(r', \phi')$$

$$\times [n + k \cdot r(\phi)] [n' + k' \cdot r'(\phi')] \psi_{nk}(r, \phi) \psi_{n'k'}(r', \phi') \delta \omega(r) \delta \omega(r') \langle n \rangle \langle k \rangle \langle n' \rangle \langle k' \rangle.$$  \hspace{1cm} (21)

Here $r(\phi) = r_c (\cos \phi, \sin \phi)$. Now the averaging gives (see appendix B) $\delta \omega(r) \delta \omega(r') = f(r - r')$, where the correlation function $f$ (which actually only depends on $|r - r'|$) depends on the nature of the random magnetic field modulation. The averaging restores translational invariance, meaning that the self-energy (21) (and therefore $D^{-1}$) is diagonal in $k$. This is also seen explicitly by changing integration variables to $r_1 = r - r'$ and $r_2 = r + r'$. The only $r_2$-dependence of the integrand is then through the factor $\exp(-i(k - k') \cdot r_2/2)$, giving a $\delta_{kk'}$ upon integration. Therefore $G$ only depends on $r - r'$. From Eq. (11) we see that only $(A \times)$ the $k = 0$ component is needed for the current. We can therefore put $k = 0$ in the self-energy (21). Inserting the expression (17) for $G_0$ we can use the delta function to do the $r_1$-integration, giving

$$\langle n0 | WD_0^{-1}W | n'0 \rangle = -\frac{i}{\omega_0} \frac{nn'}{e^{2\pi/\omega_0} - 1} \int_0^{2\pi} d\phi \int_0^{2\pi} d\phi' f(R(\phi) - R(\phi')) e^{i\phi' - \phi} e^{i\phi} e^{-i\phi'}. \hspace{1cm} (22)$$

The integrand is periodic in $\phi'$, so we can shift the limits on the $\phi'$-integration to $\phi$ and $\phi + 2\pi$. Changing variables to $\theta = \phi' - \phi$ and using

$$|R(\phi) - R(\phi')| = 2r_c |\sin \frac{\phi' - \phi}{2}|,$$

(23)
the $\phi$-integration gives a $\delta_{n,n'}$ (which was obvious from the start: when $k = 0$ the matrix element is also rotationally invariant.) The final result for the self-energy is

$$\Sigma_n \equiv \langle n|WD_0^{-1}W|0\rangle = -\frac{i}{\omega_0}e^{2\pi/\omega_0\tau} - 1 n^2 \int_0^{2\pi} f(2r_c\sin{\frac{\theta}{2}})e^{\theta/\omega_0\tau}e^{-in\theta}d\theta. \quad (24)$$

Now the left hand side of Eq. (20) is diagonal and we get

$$\langle n0|D_0^{-1}|n'0\rangle = \delta_{n,n'} n\omega_0 + i/\tau - \Sigma_n. \quad (25)$$

For the Green’s function we get

$$\int dr G(r, \phi; r', \phi') = \frac{1}{2\pi} \sum_n \frac{e^{-in(\phi-\phi')}}{n\omega_0 + i/\tau - \Sigma_n}. \quad (26)$$

It is straightforward to do the remaining angular integrals in Eq. (11) and (12) to get the current density. The result for the resistivity tensor is

$$\rho = \frac{m}{ne^2\tilde{\tau}} \begin{pmatrix} 1 & \tilde{\omega}\tilde{\tau} \\ -\tilde{\omega}\tilde{\tau} & 1 \end{pmatrix}, \quad (27)$$

where we have used $\Sigma_{-n} = -\Sigma_{n}^*$ and defined the renormalised quantities

$$\tilde{\omega} = \omega_0 - \text{Re} \Sigma_1 \quad (28)$$

$$\frac{1}{\tilde{\tau}} = \frac{1}{\tau} - \text{Im} \Sigma_1. \quad (29)$$

We see that the change in $\rho$ is directly related to $\Sigma_1$:

$$\frac{\Delta \rho_{xx}}{\rho_{xx0}} = -\tau \text{Im} \Sigma_1 \quad (30)$$

$$\frac{\Delta \rho_{xy}}{\rho_{xy0}} = -\omega_0^{-1} \text{Re} \Sigma_1. \quad (31)$$

VI. APPLICATIONS

In ref. [3] we have applied the above theory to the case where the field modulation is caused by the Meissner flux expulsion from deposited grains of Pb. Furthermore we also showed that the theory could be applied to the experiments by Geim et al. [4] where the field
modulation was caused by flux tubes. This establishes that the increase in the resistivity even in this case has an essentially classical origin.

The general analysis of the magnetoresistance can be quite complex owing to the three characteristic length scales in the problem: the mean free path $l_\tau = v_F \tau$, the magnetic field correlation length $l$, and the average cyclotron radius $r_c = v_F/\omega_0$ (when the magnetic field is such that quantum effects become important, a fourth length scale, $1/k_F$, enters). In this section we shall confine ourselves to considering the following correlation function for different parameter values:

$$f(r) = n_i \delta \omega \frac{\pi l^2}{2} e^{-r^2/2l^2}, \quad (32)$$

which arises from an assumed gaussian field from a single impurity:

$$\delta \omega(r) = \delta \omega e^{-r^2/l^2}. \quad (33)$$

This correlation function is applicable to the experiment in ref. 3, when the modulation was dominated by frozen flux in the grains (in this case $\delta \omega$ would depend on the external field and go to zero as $B_0 \to B_c$), or the case when the modulation is caused by permanent magnets.

We can find an explicit result for the change in the longitudinal resistance for $B_0 = 0$ by expanding the integral in the self-energy (24) (only $\theta \sim 2\pi$ contributes). We get

$$\frac{\Delta \rho_{xx}(B_0 = 0)}{\rho_{xx0}} = (\delta \omega \tau)^2 n_i \frac{\pi l^2}{2} \int_0^\infty e^{-(\frac{4}{l^2})s^2} ds. \quad (34)$$

The overall shape of the magnetoresistance is determined by the ratio $x = l_\tau/l$. When $x \lesssim 2$ there is a maximum in $\rho_{xx}$ at $B_0 = 0$, i.e., a negative magnetoresistance. As $x$ is increased $B_0 = 0$ is still a local maximum, but the magnetoresistance develops an intermediate minimum and has a maximum for $B_0 > 0$. The location of the maximum is approximately given by the condition that the average cyclotron radius should equal $l$. When $x \gtrsim 4$, $B_0 = 0$ becomes a local minimum. In fig. 3 we show a plot of the relative change of the longitudinal resistance for $x$ varying between 1 and 100. Fig. 4 shows the relative
change of the Hall resistance for the same range of $x$'es. By varying the electron density, the mobility, and the size $l$ of the impurities, the different types of behaviour might be seen experimentally. We note furthermore that it is possible to have a very large enhancement of the zero-field resistance in a random magnetic field (for reasonable parameters we can easily get a 20-fold increase).

The problem of the (quantum mechanical) localization of a particle in a random magnetic field has received a lot of attention. As we have seen we can get a negative magnetoresistance around $B_0 = 0$, reminiscent of weak localization. This enhancement of the resistance could be interpreted as a classical sign of ‘localization’ in the random magnetic field. However, varying the parameters it is also possible to get a local minimum around $B_0 = 0$, so there is probably no simple connection to the quantum mechanical theory of localization (although we always get an enhancement of the resistance).

We end this section by some speculations regarding the possibility of applying our theory to the Quantum Hall $\nu = 1/2$-state. As shown in the seminal paper by Halperin et al. it is possible to make a Chern-Simons gauge transformation and transform the $\nu = 1/2$-state to a system of fermions (so-called composite fermions, which can be viewed as an electron with two flux-tubes attached to it) in zero average magnetic field. The composite fermions interact through a fictitious Chern-Simons magnetic field given by

$$\delta b_{\text{CS}} = 2\Phi_0 \delta \rho,$$

where $\Phi_0 = h/e$ is the flux quantum and $\delta \rho$ is the deviation of the electron density from the average density $n$. In a static approximation $\delta \rho$ is nonzero simply because of the electrostatic coupling to the impurities. We then have a system of fermions moving in a static, random magnetic field, i.e. the system considered above. If the impurities are fully ionized we would have $n_i = n$, i.e. a much higher impurity density than in the grain experiments, and $l$ is typically much smaller than before (of the order of 500 Å). In our model this will give rise to a broad minimum in $\rho_{xx}$ around $B_0 = 0$ (that is around $B_{1/2}$) for our simple model correlation function (see fig. 6).
In fact, we are here dealing with the limit $x \gg 1$, in which case the self-energy (24) can be evaluated analytically for moderate fields. We get

$$\frac{\Delta \rho_{xx}}{\rho_{xx0}} = \left( \frac{\pi}{2} \right)^{\frac{3}{2}} n_i l^2 \frac{(\delta\omega\tau)^2}{x} \coth \left( \frac{\pi}{\omega_0\tau} \right)$$

(36)

This is consistent with the behaviour seen experimentally.\cite{14} We also note that typical values of $\rho_{\nu=1/2}/\rho_0$ are in the range 25–500. To do quantitative comparisons with experiment there is an additional complication: Since the impurities give rise both to the magnetic field and the impurity scattering it is inconsistent to put them on different sides of the Boltzmann equation, when in reality they are strongly correlated. Instead the potential scattering should be brought to the LHS of the Boltzmann equation as a random electric field. This does not contradict the discussion in the beginning of section \cite{13}, since $k_F d \sim 15 \gg 1$ in a typical QHE sample.\cite{14} ($d \sim$ the distance from the 2DEG to the impurity layer, sets the distance over which the impurity potential varies appreciably). Calculations are currently underway, addressing this issue.

\section*{VII. CONCLUSION}

We have shown how to calculate the magnetoresistance of a 2DEG in a random magnetic field in the semiclassical approximation. The magnetic field was included in the driving force term in the Boltzmann equation. Furthermore we have applied our results to different experiments and shown how the different types of random magnetic fields gives rise to the differences in the magnetoresistance. Finally we speculated on the possible relevance of our theory to the $\nu = 1/2$ Quantum Hall state.

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\section*{APPENDIX A: GREEN’S FUNCTION FOR $D_0$}

Inserting the expression (13) for the eigenfunctions into Eq. (16) we get
\[ G_0(\mathbf{r}, \phi; \mathbf{r}', \phi') = \frac{1}{2\pi} \sum_n \frac{e^{in(\phi' - \phi)}}{n\omega_0 + i/\tau} \frac{1}{\mathcal{A}} \sum_k e^{ik(\mathbf{r} - \mathbf{r}' - \mathbf{R}(\phi) + \mathbf{R}(\phi'))}. \]  

(A1)

The \( \mathbf{k} \)-sum just gives a delta function. The sum over \( n \) involves calculating

\[
\text{where we have used the Poisson summation formula. The integrand has a simple pole in } x = -i\alpha. \text{ If } 2\pi m + \phi > 0, \text{ we close the contour in the upper half plane and get 0. If } 2\pi m + \phi < 0, \text{ we close in the lower and pick up a contribution } -2\pi i e^{(2\pi m + \phi)\alpha} \text{ from the pole:}
\]

\[ h(\phi) = -i \sum_{2\pi m < \phi} e^{(2\pi m + \phi)\alpha}. \]

(A3)

Since \( h \) is periodic in \( \phi \) we can choose the argument to lie between 0 and \( 2\pi \) and get

\[
\text{Since } h \text{ is periodic in } \phi \text{ we can choose the argument to lie between 0 and } 2\pi \text{ and get}
\]

\[ h(\phi) = -ie^{\alpha[\phi]} \sum_{m=-\infty}^{-1} e^{2\pi \alpha m} = -ie^{\alpha[\phi]} \frac{e^{-2\pi \alpha}}{1 - e^{-2\pi \alpha}}. \]

(A4)

With \( \alpha = 1/\omega_0\tau \) we then get Eq. (17).

**APPENDIX B: CALCULATION OF THE CORRELATION FUNCTION**

We have to calculate \( \overline{\delta \omega(\mathbf{r})\delta \omega(\mathbf{r}') = (e^2/m^2)\delta B(\mathbf{r})\delta B(\mathbf{r}')} \). The averaging consists of integrating over the \( N \) positions (assumed independent) of the impurities. We get

\[
\overline{\delta B(\mathbf{r})\delta B(\mathbf{r}')} = \overline{B(\mathbf{r})B(\mathbf{r}')} - B_0^2
\]

\[
= \sum_{i,j} \overline{b(\mathbf{r} - \mathbf{r}_i)b(\mathbf{r}' - \mathbf{r}_j)} - B_0^2
\]

\[
= \frac{1}{\mathcal{A}^N} \sum_{i,j} \int d\mathbf{r}_k \overline{b(\mathbf{r} - \mathbf{r}_i)b(\mathbf{r}' - \mathbf{r}_j)} - B_0^2. \]

(B1)

The \( N(N - 1) \) terms with \( i \neq j \) are equal to \( A^{N-2}(\int d\mathbf{R} b(\mathbf{R}))^2 = A^N B_0^2/N^2 \) (since \( B_0 = \overline{B} = N\overline{b} \)), and the \( N \) terms with \( i = j \) are equal to \( A^{N-1} \int d\mathbf{R} b(\mathbf{r} - \mathbf{R})b(\mathbf{r}' - \mathbf{R}) \). Dropping the term of order \( 1/N \) we get
\[ \delta B(r) \delta B(r') = n_i \int dR \, b(r - R) b(r' - R), \] (B2)

where \( n_i = N/A \) is the density of impurities. The correlation function thus becomes

\[ f(r) = n_i \int dR \, b(r + R) b(R). \] (B3)

It is only a function of \( r \). Note that it does not matter if we write \( \delta b \) for \( b \) in (B3), as \( b \) and \( \delta b \) only differ by a constant of order \( 1/N \).
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FIGURES

FIG. 1. The perturbation series for the full propagator (Green’s function) $G$. The unperturbed propagator $G_0$ is denoted by a thin arrow. A cross denotes $W$, and the dashed lines an impurity average.

FIG. 2. The (irreducible) self-energy $\Sigma$. The calculations in the text approximates $\Sigma$ by the first diagram in the series.

FIG. 3. Relative change in longitudinal resistance of a 2DEG in a random magnetic field with the correlation function $^{(32)}$. Parameters as follows: $n = 4 \times 10^{15}$ m$^{-2}$, $n_i = 10^9$ m$^{-2}$, $db = 0.03$ T. The curves show (from the top) $x = 1, 2, 3, 5, 10, 20, 100$. We are varying $l$ and $l_\tau$ such that $ll_\tau = l_0^2$, where $l_0 = 8.1$ $\mu$m is constant, to keep the curves on the same scale.

FIG. 4. Relative change in Hall resistance for $x$ varying between 1 and 100, with parameters as in fig. 3. $x = 1$ corresponds to the lowest curve.

FIG. 5. The relative change in the longitudinal resistance with parameters as follows: $n = 8 \times 10^{15}$ m$^{-2}$, $n_i = 8 \times 10^{15}$ m$^{-2}$, $l = 500$ Å, $\mu = 100$ m$^2$/Vs, $db = 0.03$ T. The change in the Hall resistance (not shown) is very small, of the order of $10^{-3}$. 

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