HERMITE-HADAMARD TYPE INEQUALITIES FOR MAPPINGS
WHOSE DERIVATIVES ARE $s$–CONVEX IN THE SECOND
SENSE VIA FRACTIONAL INTEGRALS

ERHAN SET$\star$, M. EMIN OZDEMIR$\star$, M. ZEKI SARIKAYA$\bullet$, AND FILIZ KARAKOC$\bullet$

Abstract. In this paper we establish Hermite-Hadamard type inequalities for
mappings whose derivatives are $s$–convex in the second sense and concave.

1. Introduction

Let $f: I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex function defined on the interval $I$ of real numbers
and $a, b \in I$ with $a < b$. Then

\begin{equation}
    f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_{a}^{b} f(x)dx \leq \frac{f(a) + f(b)}{2}
\end{equation}

is known that the Hermite-Hadamard inequality for convex function. Both in-
equalities hold in the reserved direction if $f$ is concave. We note that Hadamard’s
inequality may be regarded as a refinement of the concept of convexity and it follows
easily from Jensen’s inequality. Hadamard’s inequality for convex functions has re-
ceived renewed attention in recent years and a remarkable variety of refinements
and generalizations have been found; see, for example see ([5]-[17]).

Definition 1. ([4]) A function $f: [0, \infty) \to \mathbb{R}$ is said to be $s$–convex in the second
sense if

\[ f(\lambda x + (1 - \lambda)y) \leq \lambda^s f(x) + (1 - \lambda)^s f(y) \]

for all $x, y \in [0, \infty)$, $\lambda \in [0, 1]$ and for some fixed $s \in (0, 1]$. This class of $s$–convex
functions is usually denoted by $K_2^s$.

In ([3]) Dragomir and Fitzpatrick proved a variant of Hadamard’s inequality which
holds for $s$–convex functions in the second sense:

Theorem 1. Suppose that $f : [0, \infty) \to [0, \infty)$ is an $s$–convex function in the
second sense, where $s \in (0, 1)$ and let $a, b \in [0, \infty), a < b$. If $f' \in L^1 ([a, b])$, then
the following inequalities hold:

\begin{equation}
    2^{s-1} f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_{a}^{b} f(x)dx \leq \frac{f(a) + f(b)}{2}
\end{equation}

The constant $k = \frac{1}{s+1}$ is the best possible in the second inequality in (1.2).

2000 Mathematics Subject Classification. 26A33, 26A51, 26D07, 26D10, 26D15.
Key words and phrases. Hermite-Hadamard type inequality, $s$–convex function, Riemann-
Liouville fractional integral.
The following results are proved by M.I.Bhatti et al. (see [2]).

**Theorem 2.** Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a twice differentiable function on \( I^o \) such that \(|f''|\) is convex function on \( I \). Suppose that \( a, b \in I^o \) with \( a < b \) and \( f'' \in L[a,b] \), then the following inequality for fractional integrals with \( \alpha > 0 \) holds:

\[
\frac{|f(a) + f(b)|}{2} - \frac{\Gamma(\alpha + 1)}{2(b-a)^\alpha} \left| \int_{a}^{b} f(x) \, dx \right|
\]

\[
\leq \frac{(b-a)^2}{\alpha + 1} \beta(\alpha + 1) \frac{|f''(a)| + |f''(b)|}{2}
\]

where \( \beta \) is Euler Beta function.

**Theorem 3.** Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a twice differentiable function on \( I^o \). Assume that \( p \in \mathbb{R}, p > 1 \) such that \(|f''|^p \) is convex function on \( I \). Suppose that \( a, b \in I^o \) with \( a < b \) and \( f'' \in L[a,b] \), then the following inequality for fractional integrals holds:

\[
\frac{|f(a) + f(b)|}{2} - \frac{\Gamma(\alpha + 1)}{2(b-a)^\alpha} \left| \int_{a}^{b} f(x) \, dx \right|
\]

\[
\leq \frac{(b-a)^2}{\alpha + 1} \beta^\frac{1}{p}(\alpha + 1) \left( \frac{|f''(a)|^q + |f''(b)|^q}{2} \right)^\frac{1}{q}
\]

where \( \beta \) is Euler Beta function.

**Theorem 4.** Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a twice differentiable function on \( I^o \). Assume that \( q \geq 1 \) such that \(|f''|^q \) is convex function on \( I \). Suppose that \( a, b \in I^o \) with \( a < b \) and \( f'' \in L[a,b] \), then the following inequality for fractional integrals holds:

\[
\frac{|f(a) + f(b)|}{2} - \frac{\Gamma(\alpha + 1)}{2(b-a)^\alpha} \left| \int_{a}^{b} f(x) \, dx \right|
\]

\[
\leq \frac{\alpha(b-a)^2}{4(\alpha + 1)(\alpha + 2)} \left[ \left( \frac{3\alpha + 5}{3\alpha + 9} |f''(a)|^q + \frac{3\alpha + 5}{3\alpha + 9} |f''(b)|^q \right)^\frac{1}{q} \right]
\]

where \( \beta \) is Euler Beta function.

**Theorem 5.** Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a twice differentiable function on \( I^o \). Assume that \( p \in \mathbb{R}, p > 1 \) with \( q = \frac{p}{p-1} \) such that \(|f''|^q \) is concave function on \( I \). Suppose that \( a, b \in I^o \) with \( a < b \) and \( f'' \in L[a,b] \), then the following inequality for fractional integrals holds:

\[
\frac{|f(a) + f(b)|}{2} - \frac{\Gamma(\alpha + 1)}{2(b-a)^\alpha} \left| \int_{a}^{b} f(x) \, dx \right|
\]

\[
\leq \frac{(b-a)^2}{\alpha + 1} \beta^\frac{1}{p}(\alpha + 1) \left| f'' \left( \frac{a + b}{2} \right) \right|
\]

where \( \beta \) is Euler Beta function.

We will give some necessary definitions and mathematical preliminaries of fractional calculus theory which are used further this paper.

**Definition 2.** Let \( f \in L[a,b] \). The Reimann-Liouville integrals \( J_{a+}^\alpha f(x) \) and \( J_{b-}^\alpha f(x) \) of order \( \alpha > 0 \) with \( \alpha \geq 0 \) are defined by
HERMITE-HADAMARD TYPE INEQUALITIES VIA FRACTIONAL INTEGRALS

\[ J^\alpha_{a+} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t) \, dt, \ x > \alpha \]

and

\[ J^\alpha_{b-} f(x) = \frac{1}{\Gamma(\alpha)} \int_x^b (t-x)^{\alpha-1} f(t) \, dt, \ x < b \]

respectively. Where \( \Gamma(\alpha) = \int_0^\infty e^{-u} u^{\alpha-1} \, du \) is the Gamma function and \( J^0_{a+} f(x) = J^0_{b-} f(x) = f(x) \).

In the case of \( \alpha = 1 \) the fractional integral reduces to the classical integral.

For some recent results connected with fractional integral inequalities, (see [18]-[27]).

In this paper, we establish fractional integral inequalities of Hermite-Hadamard type for mappings whose derivatives are \( s \)-convex and concave.

2. Main Results

In order to prove our main theorems we need the following lemma (see [2]).

**Lemma 1.** Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a twice differentiable function on \( I^o \), the interior of \( I \). Assume that \( a, b \in I^o \) with \( a < b \) and \( f'' \in L[a,b] \), then the following identity for fractional integral with \( \alpha > 0 \) holds:

\[
\frac{f(a) + f(b)}{2} - \frac{\Gamma(\alpha + 1)}{2(b-a)^{\alpha}} \left[ J^\alpha_{a+} f(b) + J^\alpha_{b-} f(a) \right] = \frac{(b-a)^{2\alpha}}{2(\alpha+1)^2} \int_0^1 t (1-t^\alpha) \left[ f''(ta+(1-t)b) + f''((1-t)a+tb) \right] \, dt
\]

where \( \Gamma(\alpha) = \int_0^\infty e^{-u} u^{\alpha-1} \, du \).

**Theorem 6.** Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a twice differentiable function on \( I^o \) and let \( a, b \in I^o \) with \( a < b \) and \( f'' \in L[a,b] \). If \( |f''| \) is \( s \)-convex in the second sense on \( I \) for some fixed \( s \in (0,1] \), then the following inequality for fractional integrals with \( \alpha > 0 \) holds:

\[
\left| \frac{f(a) + f(b)}{2} - \frac{\Gamma(\alpha + 1)}{2(b-a)^{\alpha}} \left[ J^\alpha_{a+} f(b) + J^\alpha_{b-} f(a) \right] \right| \leq \frac{(b-a)^{2\alpha}}{2(\alpha+1)^2} \left\{ \alpha \left[ \frac{\beta(2,s+1)}{(s+2)(\alpha+s+2)} + \beta(\alpha+2,s+1) \right] \left[ |f''(a)| + |f''(b)| \right] \right\}
\]

where \( \beta \) is Euler Beta function.

**Proof.** From Lemma 1 since \( |f''| \) is \( s \)-convex in the second sense on \( I \), we have
\[
\left| \frac{f(a) + f(b)}{2} - \frac{\Gamma(\alpha + 1)}{2(b-a)^\alpha} \left[ J_{a^+}^\alpha f(b) + J_{b^-}^\alpha f(a) \right] \right| \\
\leq \frac{(b-a)^2}{2(\alpha + 1)} \int_0^1 |t(1-t^\alpha)| \left[ |f''(ta + (1-t)b)| + |f''((1-t)a + tb)| \right] \, dt \\
\leq \frac{(b-a)^2}{2(\alpha + 1)} \left[ \int_0^1 t(1-t^\alpha) \left[ t^s |f''(a)| + (1-t)^s |f''(b)| \right] \, dt + \int_0^1 (t(1-t^\alpha)(1-t)^s \left[ |f''(a)| + |f''(b)| \right] \right] \\
= \frac{(b-a)^2}{2(\alpha + 1)} \left[ \frac{\alpha}{(s+2)(\alpha + s + 2)} + \beta(2, s + 1) - \beta(\alpha + 2, s + 1) \right] \left[ |f''(a)| + |f''(b)| \right]
\]

where we used the fact that

\[
\int_0^1 t^{s+1}(1-t^\alpha) \, dt = \frac{\alpha}{(s+2)(\alpha + s + 2)}
\]

and

\[
\int_0^1 t(1-t^\alpha)(1-t)^s \, dt = \beta(2, s + 1) - \beta(\alpha + 2, s + 1)
\]

which completes the proof. \(\square\)

**Remark 1.** In Theorem 6 if we choose \(s = 1\) then \((2.2)\) reduces the inequality \((1.3)\) of Theorem 4.

**Theorem 7.** Let \(f : I \subseteq \mathbb{R} \to \mathbb{R}\) be a twice differentiable function on \(I^o\). Suppose that \(a, b \in I^o\) with \(a < b\) and \(f'' \in L[a,b]\). If \(|f''|^q\) is \(s\)-convex in the second sense on \(I\) for some fixed \(s \in (0,1]\), \(p,q > 1\) then the following inequality for fractional integrals holds:

\[
(2.3) \quad \left| \frac{f(a) + f(b)}{2} - \frac{\Gamma(\alpha + 1)}{2(b-a)^\alpha} \left[ J_{a^+}^\alpha f(b) + J_{b^-}^\alpha f(a) \right] \right| \\
\leq \frac{(b-a)^2}{\alpha + 1} \beta \left( p + 1, \alpha p + 1 \right) \left[ \frac{|f''(a)|^q + |f''(b)|^q}{s + 1} \right]^{\frac{1}{q}}
\]

where \(\beta\) is Euler Beta function and \(\frac{1}{p} + \frac{1}{q} = 1\).
Proof. From Lemma\[1\] using the well known Hölder inequality and $|f''|^q$ is $s-$convex in the second sense on $I,$ we have

\[
\begin{align*}
&\left| \frac{f(a) + f(b)}{2} - \frac{\Gamma (\alpha + 1)}{2 (b-a)^\alpha} [J_b^{\alpha} f(b) + J_a^{\alpha} f(a)] \right| \\
&\leq \frac{(b-a)^2}{2 (\alpha + 1)} \int_0^1 |t (1 - t^\alpha)| |f''(ta + (1-t)b)| dt \\
&\leq \frac{(b-a)^2}{2 (\alpha + 1)} \left( \int_0^1 t^p (1 - t^\alpha)^p dt \right)^{1-\frac{1}{q}} \left[ \left( \int_0^1 |f''(ta + (1-t)b)|^q dt \right)^{\frac{1}{q}} + \left( \int_0^1 |f''((1-t)a + tb)|^q dt \right)^{\frac{1}{q}} \right] \\
&\leq \frac{(b-a)^2}{2 (\alpha + 1)} \left( \int_0^1 t^p (1 - t^\alpha)^p dt \right)^{1-\frac{1}{q}} \left[ \left( \int_0^1 (t^s |f''(a)|^q + (1-t)^s |f''(b)|^q) dt \right)^{\frac{1}{q}} + \left( \int_0^1 (1-t)^s |f''(a)|^q + t^s |f''(b)|^q) dt \right)^{\frac{1}{q}} \right] \\
&\leq \frac{(b-a)^2}{\alpha + 1} \beta^+ (p + 1, \alpha p + 1) \left[ \frac{|f''(a)|^q + |f''(b)|^q}{s + 1} \right]^{\frac{1}{q}}
\end{align*}
\]

where we used the fact that

\[
\int_0^1 t^s dt = \int_0^1 (1-t)^s dt = \frac{1}{s + 1}
\]

and

\[
\int_0^1 t^p (1 - t^\alpha)^p dt \leq \int_0^1 t^p (1 - t)^p dt = \beta^+ (p + 1, \alpha p + 1)
\]

which completes the proof. \[\Box\]

Remark 2. In Theorem\[3\] if we choose $s = 1$ then \[(2.3)\] reduces the inequality \[(1.4)\] of Theorem\[5\].

Theorem 8. Let $f : I \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a twice differentiable function on $I$. Suppose that $a, b \in I$ with $a < b$ and $f'' \in L[a, b]$. If $|f''|^q$ is $s-$convex in the second sense on $I$ for some fixed $s \in (0, 1]$ and $q \geq 1$ then the following inequality for fractional integrals holds:

\[
(2.4) \quad \left| \frac{f(a) + f(b)}{2} - \frac{\Gamma (\alpha + 1)}{2 (b-a)^\alpha} [J_b^{\alpha} f(b) + J_a^{\alpha} f(a)] \right|
\]

\[
\leq \frac{\alpha (b-a)^2}{4 (\alpha + 1) (\alpha + 2)} \times \left[ \left( |f''(a)|^q \left( \frac{2s+4}{(s+2)(\alpha+2+s+2)} \right) + |f''(b)|^q \left( \frac{2s+4}{(s+2)(\alpha+2+s+2)} \right) \right)^{\frac{1}{q}} \right]
\]

\[
\left[ \left( |f''(a)|^q \left( \frac{2s+4}{(s+2)(\alpha+2+s+2)} \right) + |f''(b)|^q \left( \frac{2s+4}{(s+2)(\alpha+2+s+2)} \right) \right)^{\frac{1}{q}} \right]
\]

\[
\left[ \left( |f''(a)|^q \left( \frac{2s+4}{(s+2)(\alpha+2+s+2)} \right) + |f''(b)|^q \left( \frac{2s+4}{(s+2)(\alpha+2+s+2)} \right) \right)^{\frac{1}{q}} \right]
\]
Proof. From Lemma 4 using power mean inequality and \( f'' \) is \( s \)-convex in the second sense on \( I \) we have
\[
\begin{align*}
\left| \frac{f(a) + f(b)}{2} - \frac{\Gamma(\alpha + 1)}{2(b - a)^\alpha} [J_a^\alpha f(b) + J_a^\alpha f(a)] \right| & \leq \frac{(b - a)^2}{2(\alpha + 1)} \int_0^1 |t(1 - t^\alpha)| \left[ |f''(ta + (1 - t)b)| + |f''((1 - t)a + tb)| \right] dt \\
& \leq \frac{(b - a)^2}{2(\alpha + 1)} \left( \int_0^1 t(1 - t^\alpha) dt \right)^{1 - \frac{1}{q}} \left[ \left( \int_0^1 |f''(ta + (1 - t)b)|^q dt \right)^{\frac{1}{q}} + \left( \int_0^1 |f''((1 - t)a + tb)|^q dt \right)^{\frac{1}{q}} \right] \\
& \leq \frac{(b - a)^2}{2(\alpha + 1)} \left( \int_0^1 t(1 - t^\alpha) dt \right)^{1 - \frac{1}{q}} \left[ \left( \int_0^1 |f''(a)|^q |\beta(2s + 1) - \beta(\alpha + 2, s + 1)| + |f''(b)|^q |\beta(2s + 1) - \beta(\alpha + 2, s + 1)| \right)^{\frac{1}{q}} \right] \\
& \leq \frac{\alpha(b - a)^2}{4(\alpha + 1)(\alpha + 2)} \left( \int_0^1 t(1 - t^\alpha) dt \right)^{1 - \frac{1}{q}} \left[ \left( \int_0^1 |f''(a)|^q |\beta(2s + 1) - \beta(\alpha + 2, s + 1)|^{\frac{2\alpha + 4}{\alpha}} + |f''(b)|^q |\beta(2s + 1) - \beta(\alpha + 2, s + 1)|^{\frac{2\alpha + 4}{\alpha}} \right)^{\frac{1}{q}} \right]
\end{align*}
\]
where we used the fact that
\[
\int_0^1 t^{s + 1}(1 - t^\alpha) dt = \frac{\alpha}{(s + 2)(\alpha + s + 2)}
\]
and
\[
\int_0^1 t(1 - t^\alpha)(1 - t)^s dt = \beta(2s + 1) - \beta(\alpha + 2, s + 1)
\]
which completes the proof. \(\square\)

Remark 3. In Theorem 8 if we choose \( s = 1 \) then (2.4) reduces the inequality (1.5) of Theorem (4).

The following result holds for \( s \)-concavity.

Theorem 9. Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a twice differentiable function on \( I^\circ \). Suppose that \( a, b \in I^\circ \) with \( a < b \) and \( f'' \in L[a, b] \). If \( f'' \) is \( s \)-concave in the second sense on \( I \) for some fixed \( s \in (0, 1] \) and \( p, q > 1 \) then the following inequality for fractional integrals holds:
\[
(2.5) \quad \left| \frac{f(a) + f(b)}{2} - \frac{\Gamma(\alpha + 1)}{2(b - a)^\alpha} [J_a^\alpha f(b) + J_a^\alpha f(a)] \right| \leq \frac{(b - a)^2}{\alpha + 1} \frac{1}{\beta^p (p + 1, \alpha p + 1)} 2^{\frac{s - 1}{s}} \left| f'' \left( \frac{a + b}{2} \right) \right|
\]
where $\frac{1}{p} + \frac{1}{q} = 1$ and $\beta$ is Euler Beta function

**Proof.** From Lemma 1 and using the Hölder inequality we have

\[
\begin{align*}
(2.6) & \quad \frac{1}{2} \left| f(a) + f(b) \right| - \frac{\Gamma(\alpha + 1)}{2(\beta - \alpha)} \left| J_{a+}^{\alpha} f(b) + J_{b-}^{\alpha} f(a) \right| \\
& \leq \frac{(b-a)^2}{2(\alpha + 1)} \int_0^1 |t(1-t^\alpha)| |f''(ta + (1-t)b)| + |f''((1-t)a + tb)| \, dt \\
& \leq \frac{(b-a)^2}{2(\alpha + 1)} \left( \int_0^1 t^\alpha (1-t^\alpha)^p \, dt \right)^{\frac{1}{p}} \left[ \left( \int_0^1 |f''(ta + (1-t)b)|^q \, dt \right)^{\frac{1}{q}} + \left( \int_0^1 |f''((1-t)a + tb)|^q \, dt \right)^{\frac{1}{q}} \right]
\end{align*}
\]

Since $|f''|^q$ is $s$-concave using inequality (1.2) we get (see [1])

\[
(2.7) \quad \int_0^1 |f''(ta + (1-t)b)|^q \, dt \leq 2^{s-1} \left| f'' \left( \frac{a+b}{2} \right) \right|^q
\]

and

\[
(2.8) \quad \int_0^1 |f''((1-t)a + tb)|^q \, dt \leq 2^{s-1} \left| f'' \left( \frac{b+a}{2} \right) \right|^q
\]

Using (2.7) and (2.8) in (2.6), we have

\[
\begin{align*}
& \frac{1}{2} \left| f(a) + f(b) \right| - \frac{\Gamma(\alpha + 1)}{2(\beta - \alpha)} \left| J_{a+}^{\alpha} f(b) + J_{b-}^{\alpha} f(a) \right| \\
& \leq \frac{(b-a)^2}{\alpha + 1} \beta^\frac{1}{\alpha} (p+1, \alpha p+1) 2^{\frac{\alpha+1}{\alpha+1}} \left| f'' \left( \frac{a+b}{2} \right) \right|
\end{align*}
\]

which completes the proof. \(\square\)

**Remark 4.** In theorem 4 if we choose $s = 1$ then (2.5) reduces inequality (1.6) of theorem 3.

**REFERENCES**

[1] M. Alomari, M. Darus, S.S. Dragomir, P. Cerone, Ostrowski type inequalities for functions whose derivatives are s-convex in the second sense, Appl. Math. Lett. 23 (2010) 1071-1076.

[2] S.S.Dragomir, M.I.Bhatti and M.Iqbal Some new fractional integral Hermite-Hadamard type inequalities, RGMIA Res. Rep. Coll., 16 (2013), Article 2.

[3] S.S.Dragomir, S.Fitzpatrick, The Hadamard’s inequality for s-convex functions in the second sense, Demonstratio Math 32 (4) (1999) 687-696.

[4] H. Hudzik, L. Maligranda, Some remarks on s-convex functions, Aequationes Math. 48 (1994) 100–111.

[5] M. Alomari, M. Darus, On the Hadamard’s inequality for log-convex functions on the co-ordinates, J. Inequal. Appl. 2009 (2009) 13. Article ID 283147.

[6] A.G. Azpeitia, Convex functions and the Hadamard inequality, Revista Colombiana Mat. 28 (1994) 7–12.
[7] M.K. Bakula, M.E. Özdemir, J. Pečarić, Hadamard type inequalities for m-convex and \((\alpha, m)\)-convex functions, J. Ineq. Pure Appl. Math. 9 (4) (2008) Art. 96.

[8] M.K. Bakula, J. Pečarić, Note on some Hadamard-type inequalities, J. Ineq. Pure Appl. Math. 5 (3) (2004) Article 74.

[9] S.S. Dragomir, C.E.M. Pearce, Selected topics on Hermite–Hadamard inequalities and applications, RGMIA Monographs, Victoria University, 2000.

[10] S.S. Dragomir, R.P. Agarwal, Two inequalities for differentiable mappings and applications to special means of real numbers and to trapezoidal formula, Appl. Math. lett. 11 (5) (1998) 91–95.

[11] S.S. Dragomir, On some new inequalities of Hermite–Hadamard type for m-convex functions, Tamkang J. Math. 3 (1) (2002).

[12] P.M. Gill, C.E.M. Pearce, J. Pečarić, Hadamard’s inequality for r-convex functions, J. Math. Anal. Appl. 215 (2) (1997) 461–470.

[13] U.S. Kırmacı, M.K. Bakula, M.E. Özdemir, J. Pečarić, Hadamard-type inequalities for s-convex functions, Appl. Math. Comput. 193 (2007) 26–35.

[14] M.E. Özdemir, M. Avci, E. Set, On some inequalities of Hermite–Hadamard type via m-convexity, Appl. Math. Lett. 23 (9) (2010) 1065–1070.

[15] E. Set, M.E. Özdemir, S.S. Dragomir, On the Hermite–Hadamard inequality and other integral inequalities involving two functions, J. Inequal. Appl. (2010) 9. Article ID 148102.

[16] E. Set, M.E. Özdemir, S.S. Dragomir, On Hadamard-type inequalities involving several kinds of convexity, J. Inequal. Appl. (2010) 12. Article ID 286845.

[17] J.E. Pečarić, F. Proschan, Y.L. Tong, Convex Functions, Partial Orderings and Statistical Applications, Academic Press, Boston, 1992.

[18] G. Anastassiou, M.R. Hooshmandasl, A. Ghasemi, F. Moftakhazadeh, Montgomery identities for fractional integrals and related fractional inequalities J. Ineq. Pure Appl. Math. 10 (4) (2009) Art 97.

[19] S. Belarbi, Z. Dahmani, On some new fractional integral inequalities, J. Ineq. Pure Appl. Math. 10 (3) (2009) Art. 86.

[20] Z. Dahmani, New inequalities in fractional integrals, Int. J. Nonlinear Sci. 9 (4) (2010) 493–497.

[21] Z. Dahmani, On Minkowski and Hermite–Hadamard integral inequalities via fractional integration, Ann. Funct. Anal. 1 (1) (2010) 51–58.

[22] Z. Dahmani, L. Tabharit, S. Taf, Some fractional integral inequalities, Nonlinear. Sci. Lett. A 1 (2) (2010) 155–160.

[23] Z. Dahmani, L. Tabharit, S. Taf, New generalizations of Gruss inequality using Riemann–Liouville fractional integrals, Bull. Math. Anal. Appl. 2 (3) (2010) 93–99.

[24] M.Z. Sarikaya, H. Ogunmez, On new inequalities via Riemann–Liouville fractional integration, Abstract and Applied Analysis, Volume 2012, Article ID 428983, 10 pages, doi:10.1155/2012/428983.

[25] M.Z. Sarikaya and H. Yaldız, On weighted Montgomery identities for Riemann-Liouville fractional integrals, Konuralp Journal of Mathematics, 1 (1) (2013) 48–53.

[26] M.Z. Sarikaya, E. Set, H. Yaldız and N. Basak, Hermite-Hadamard’s inequalities for fractional integrals and related fractional inequalities, Mathematical and Computer Modelling, doi:10.1016/j.mcm.2011.12.048, in press.

[27] E. Set, New inequalities of Ostrowski type for mappings whose derivatives are s-convex in the second sense via fractional integrals, Comp. Math. Appl., 63(7) (2012), 1147-1154.
HERMITE-HADAMARD TYPE INEQUALITIES VIA FRACTIONAL INTEGRALS

*Department of Mathematics, Faculty of Science and Arts, Düzce University, Düzce-TURKEY
E-mail address: erhanset@yahoo.com

*Atatürk University, K.K. Education Faculty, Department of Mathematics, 25240, Campus, Erzurum, Turkey
E-mail address: emos@atauni.edu.tr

■Department of Mathematics, Faculty of Science and Arts, Düzce University, Düzce-TURKEY
E-mail address: sarikayamz@gmail.com

♣Department of Mathematics, Faculty of Science and Arts, Düzce University, Düzce-TURKEY
E-mail address: filinz41@hotmail.com