Deformation of beam structures in an internal explosion

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Abstract. The paper considers the deformation of a hinged beam exposed to the action of an explosive load at the initial stage of an internal explosion, when the excess pressure increases according to the third degree of time. With this law of load change, the rate of its increases all the time, which leads to an increase in dynamism. Analytical expressions are obtained for the deformation value and its velocity, both at the growth stage and at the pressure drop. The case of transition of elastic deformation to plastic deformation both at the stage of load growth and decline is considered. The decline was approximated by a linear function of the start time of the decline. Calculations have shown that the deformation rate increases significantly towards the end of the lifting stage. If plastic deformation occurs at the stage of lifting close to its end, the coefficient of plasticity is close to 10 or more. If plastic deformation occurs at the stage of decline, the coefficient of plasticity varies between 1.4÷ 2.

1. Introduction

Internal explosions of gas-air mixtures occur constantly. Especially, recently, explosions of household gas in residential, most often, kitchen premises cause alarm. Premises of industrial explosive objects are categorized by fire and explosion hazard. The room is considered to be explosive if the pressure it reaches the value of 5kPa [1]. This pressure is often conditional and is not related to the load-bearing capacity of building structures, so the opinion that this pressure corresponds to the load-bearing capacity of building structures is erroneous. The carrying capacity should be determined by the calculation of the first group of limit States [2-3]. However, this calculation is not regulated in regulatory documents. Protection of explosive premises from the action of internal explosion is limited by the requirements of the use of safety structures. In [4] the area of apertures which have to be blocked by safety designs is regulated; for rooms of category "A" this area makes 0,05V0, and for category "B" 0,03 V0, V0-free volume of the room. The area of the openings of the overlapped safety design does not depend in [4] on the bearing capacity of the structures, which is undoubtedly a disadvantage. After all, it is obvious that a building with a high load-bearing capacity requires for its protection a smaller area of openings blocked by safety structures. The proportionality of the area occupied by the PC, the volume of the room contradicts the considerations of the theory of dimension and this relationship should be revised. In a large number of works [5-11] it is shown that the area of
openings of overlapped safety designs is proportional to $V_0^2$. The document [12] mainly uses the data of the monograph [13] uses the concept of permissible pressure, that is, the limit value of the explosion pressure, below which the load-bearing capacity of the building is provided. However, how to determine the allowable pressure is not shown. In works [2,3] the method of calculation on explosive loadings of buildings of different constructive execution is resulted, replacing conditional dynamic loading with equivalent static. The values of the dynamism coefficient are determined by analogy with the external explosion [13-15], that is, the load is approximated by a linear function of time. This assumption is too crude for a responsible calculation. Taking into account that most industrial and residential buildings have a bearing capacity corresponding to the pressure of a quasi-static explosion of not more than 20 kPa, it can be assumed that the pressure at the initial stage of the explosion increases $\Delta P \sim t^3$, as in a closed volume [16-22]. The calculation of building structures for such a load has never been carried out. This work assumes to eliminate this lack on the example of calculation of the hinged beam.

2. Methodology
The authors of rare works, which determined the load-bearing capacity of building structures in internal explosions, used either the concept of equivalent static load [2-3], or wave loading with a front shock front [15,23]. Emergency explosions indoors in the vast majority of cases are deflagration quasi-static explosions, and the pressure at which the loss of bearing capacity does not exceed 20kpa, and sometimes 3-5 kPa [24]. In the case of complete gas contamination of a closed volume with an explosive mixture, the final pressure of the explosion reaches $\Delta P_w \approx 800$ kPa. At the initial stage of the development of the explosion, when the pressure in the closed volume does not exceed 20kpa, its dependence on time is described by the expression:

$$\Delta P(t) = \frac{P_0}{\nu_0^2} \frac{\pi^2 (6-1) U^2 t^3}{\nu_0}$$

The use of the variation approach [25] to determine the deflection of the beam in the elastic stage of deformation in the sinusoidal form of oscillations

$$y_0(x,t) = y_0(t) \sin \frac{\pi x}{L}$$

leads to the equation for maximum deflection $y_0(t)$

$$\ddot{y}_0 + \ddot{y}_0 = \frac{3RABt^3}{\pi^2 t^4}$$

The solution of this equation under zero initial conditions is as follows:

$$\ddot{y}_0 = \frac{0.755Bt}{\ell^3} \left( \sin \ell - \ell + \frac{\ell^3}{6} \right)$$

$$\ddot{y}_0 = \frac{0.755Bt}{\ell^3} \left( \cos \ell - 1 + \frac{\ell^2}{2} \right)$$
Solution (4-5) is valid for elastic deformation \( y_0 \leq y_{el} \), in dimensionless form \( \bar{y}_0 \leq 1 \) and only at the stage of load growth \( \bar{t} \leq \bar{t}_+ \).

In expressions (3-5) the notation is used

\[
\bar{t} = \frac{t}{t_c}, \quad t_c = \left( \frac{Me}{k_e} \right)^{\frac{1}{2}}
\]

, where \( t_c \)- characteristic oscillation time of the structure (beam);

\( Me = \frac{M}{2}, \) where \( Me \)- equivalent mass; \( M \)- mass of the beam taking into account the attached mass

\( k_e = \frac{\pi^4 E I}{2L^4} \) – the coefficient of stiffness of the hinged beam at elastic deformation, where

\( E \)- beam modulus of elasticity; \( I \)- moment of inertia of the beam section

\( \bar{t}_+ = \frac{t_+}{t_c} \)- the dimensionless magnitude of the explosion pressure build-up to \( \Delta P_1 \).

\( B_\ell = \frac{\Delta P_1 L^2 b}{Z_p \sigma_{\ell,\ell}} \)- force factor, \( \Delta P_1 \)- the power factor for maximum burst pressure at the moment \( \ell = t_+ \).

\( L \)- the calculated span length, \( b \)- width of load collection area, \( Z_p \)- plastic moment of resistance to bending section, \( \sigma_{\ell,\ell} \)- the dynamic yield strength of the beam material.

\[
\bar{y}_0 = \frac{y(t)}{y_{el}}, \quad \bar{y}_{el} = \frac{8 M_p}{3 L^2 E I} = \frac{5 M_p L^2}{48 E I},
\]

where \( M_p \)- plastic moment of bending resistance \( M_p = Z_p \sigma_{\ell,\ell} \).

When the deflection value \( \bar{y}_0 = 1 \) occurs plastic deformation with a plastic hinge in the center of the span. The shape of the bent beam is described by the ratio:

\[
y_{(xt)} = 2 y_0 \frac{x}{L}; \quad 0 \leq x \leq \frac{1}{2} L \quad (6)
y_{(xt)} = 2 y_0 \left[ 1 - \frac{x}{L} \right]; \quad \frac{1}{2} L \leq x \leq L_0
\]

The equation describing the motion of the beam in the plastic stage has the form:

\[
\ddot{y}_1 + \frac{115.2}{\pi^4} \left( \frac{t_0}{t_1} \right) \left(1 + \frac{\bar{t}_1}{t_0} \right)^3 = \frac{14.4B_l}{\pi^4} \left( \frac{t_0}{t_1} \right)^3 \left(1 + \frac{\bar{t}_1}{t_0} \right)^3
\]

(7)

The solution of this equation is as follows:

\[
\bar{y}_1 = y_{1(0)} + \frac{3.6B_l}{\pi^4} \left( \frac{t_0}{t_1} \right)^3 \dot{t}_0 \left[ \left(1 + \frac{\bar{t}_1}{t_0} \right)^4 - 1 \right] - \frac{115.2}{\pi^4} \bar{t}_1
\]

(8)
\[
\overline{y_1} = \overline{y_1(0)} t_+ + \frac{3.6B_t}{\pi^4} \left( t_{+10} \right)^3 \frac{2}{t_+} \left( \frac{1 + \overline{t_1}}{t_{+10}} \right)^{\frac{5}{5}} - \left( 1 + \overline{t_1} \right)^{\frac{4}{5}} + \frac{57.6}{\pi^4} \frac{t_{+10}^2}{t_+^2}
\]  

(9)

Here \( \overline{y_1} = \overline{y_1, y_1} \) - counted from \( y_{el} \) and so when \( t_1 = 0, y_1 = 0 \);

\( t_1 \) - it is counted from the beginning of plastic deformation \( \overline{t_1} = \frac{\bar{t}_1}{t_+} \);

The constant \( \overline{y_{1(0)}} \) is determined from the conditions of preserving the maximum deflection in the center of the beam during the transition from elastic to plastic deformation. Simultaneous equality of velocities is impossible, as the shape of the bend changes. It is assumed that the amount of movement of the beam is preserved when changing the deformation mode. In this case, the initial velocity of the plastic motion \( \overline{y_{10}} \) is correlated with the final velocity of the elastic stage \( \overline{y_{el}} \) as: \( \overline{y_{10}} = \frac{4}{\pi} \overline{y_{el}} \)

Where \( \overline{y_{el}} \) is determined from (5) provided \( t = t_{+10} = t_{el} \) is the start time of plastic deformation. Under the assumptions made

\[
\overline{y_{1(0)}} = \frac{3.02 B_t}{\pi t_+} \left[ \cos \overline{t_1} - 1 + \frac{\overline{t_1}^2}{2} \right]
\]

(10)

By substituting the value \( t = t_+ = t_{+10} \) in expressions (8-9), the deformation rate and its value are obtained at the moment when the explosion pressure reaches its maximum value, and then the pressure drop begins.

Table 1 shows the values of the force parameter \( B_f \) and the deformation rate \( \frac{4}{\pi} \overline{y_{el}} \) at the beginning of the plastic stage, depending on the ratio of the time of the beginning of the plastic deformation to the duration of the pressure build-up phase \( \left( \frac{t_{+10}}{t_+} \right) \).

| \( \frac{t_{+10}}{t_+} \) | \( t_+ = 40 \) | \( t_+ = 20 \) | \( t_+ = 10 \) | \( t_+ = 5 \) | \( t_+ = 2 \) | \( t_+ = 1 \) |
|-----------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1               | 7.98         | 0.996        | 8.06         | 0.193        | 8.48         | 0.393        | 11.13        | 1.009        | 43.7         | 3.06         | 162.8        | 6.31         |
| 0.8             | 15.61        | 0.12         | 15.9         | 0.241        | 16.9         | 0.502        | 28           | 1.37         | 128.8        | 3.88         | 429.5        | 7.91         |
| 0.5             | 64.5         | 0.193        | 67.9         | 0.393        | 89           | 1.009        | 235.6        | 2.4          | 1302         | 6.31         | 5115.4       | 12.7         |

Table 2 shows the values of the force factor \( B_f \) and the speed of movement of the structure at the end of the pressure build-up phase \( \left( \overline{t_+} \right) \), if the maximum deflection at this time was \( 1) \overline{y_{+}} = 0.8 \) and \( 2) \overline{y_{+}} = 0.5 \).
Table 2. The values of the force factor and the speed of movement of the structure at the end of the pressure build-up phase

| $\bar{t}_+$ | 20   | 10   | 5    | 2    |
|-------------|------|------|------|------|
| $\bar{y}_{+} = 0,8; B_+$ | 6,44 | 6,78 | 8,9  | 34,93 |
| $\bar{y}_{+} = 0,8; \bar{y}_+$ | 0,152 | 0,308 | 0,792 | 2,406 |
| $\bar{y}_{+} = 0,5; B_+$ | 4,03 | 4,24 | 5,56 | 21,8 |
| $\bar{y}_{+} = 0,5; \bar{y}_+$ | 0,152 | 0,308 | 0,792 | 2,406 |

Data of table 1 provide an opportunity to determine the further deformation of the beam during the plastic work of its material (8-9), if $t_{10} < \bar{t}_+$. As soon as the build-up of pressure stops, the nature of the load changes-its decline occurs. The decay time depends on the process of opening the pressure relief openings. It is several times less than the pressure rise time and is approximated by the linear dependence.

In the case of elastic deformation, the equation describing the movement of the maximum deflection point with a linear pressure drop is as follows:

$$\bar{y}_2 + \bar{y}_y = \frac{38.4}{\pi^5} B_+ \left(1 - \frac{\bar{t}_2}{\bar{t}_c}\right)$$

(11)

And the solution of this equation will be written as

$$\bar{y}_2 = \bar{y}_{2(0)} \cos \bar{t}_2 + \frac{38.4 B_+}{\pi^5} \left[\cos \frac{\bar{t}_2}{\bar{t}_c} + \sin \frac{\bar{t}_2}{\bar{t}_c} - \frac{1}{\bar{t}_c}\right]$$

(12)

$$\bar{y}_y = \bar{y}_{y(0)} \sin \bar{t}_2 + \frac{38.4 B_+}{\pi^5} \left[\sin \frac{\bar{t}_2}{\bar{t}_c} - \cos \frac{\bar{t}_2}{\bar{t}_c} + 1 - \frac{\bar{t}_2}{\bar{t}_c}\right]$$

(13)

Here $\bar{y}_2$ is the dimensionless offset of the maximum deflection point measured from the start of the load drop. This means that $\bar{y}_{2(0)} = 0$. Time $\bar{t}_2$ is counted from the beginning of the load decay. As before $\bar{t}_2 = \bar{t}_2/\bar{t}_c$, where $\bar{t}_c$ as before the characteristic oscillation time of the structure $\bar{t}_c = \sqrt{\frac{1}{2} \frac{M}{Re}}$.

The solution is valid until the elastic deformation at $\bar{y}_2 = 1 - \bar{y}_{y(\bar{t}_c)}$ ends, or the external load $\bar{t}_2 = \bar{t}_c$ ceases to act. After the termination of the load, the deformation will continue at the expense of the kinetic energy reserve and will move to oscillations relative to the equilibrium point. This motion will be described by the equation for harmonic oscillations:

At transition to plastic deformation at the moment of time $\bar{t}_2 = t_2(y_{el})$ at a stage of decline of loading movement of a point of the maximum deflection is described by expressions:
\[
\ddot{y}_3 = \ddot{y}_{3(0)} + \frac{14A}{\pi^4} B f t_3 \left( 1 - \frac{t_{30}}{t} - \frac{t_3}{3t} \right) - \frac{1152}{\pi^4} \frac{t_3}{t^4}
\]

(15)

\[
\ddot{y}_3 = \ddot{y}_{3(0)} + \frac{72B t_3^2}{\pi^4} \left[ 1 - \frac{t_{30}}{t} - \frac{t_3}{3t} \right] - \frac{576}{\pi^4} \frac{t_3^2}{t^4}
\]

(16)

Velocity $\ddot{y}_{3(0)}$ is determined from the expression (12) at $t_2 = t_{el}$, that is, during the transition of elastic deformation into plastic. The change of the deformation mode is taken into account by a factor of $\frac{4}{\pi}$. The time $t_{el}$ is determined from the expression (13) when $\ddot{y}_2 = 1 - \ddot{y}_3$, where $y_+ = \frac{1}{t}$ is the deflection at the end of the load rise or at the beginning of the pressure drop, and the time $t_3$ from $t_{30}$.

3. Results and discussion

Deformation and time when moving the structure are dimensioned by the same characteristic values, so that the total displacement of the point with the maximum deflection is obtained by summing the deflections on all successive solutions, the total time is also obtained by summing the duration of the successive stages of the process. The rate of deformation is obtained always corresponding to this moment, since the amount of motion is preserved during the transition from one stage of deformation to another.

We analyze the further behavior of the structure, if the plastic deformation occurs at the stage of load growth (table 1) at times $t_{10} = 0.8t_+$ and $t_{10} = 0.5t_+$. The maximum deflection in this case is determined by the formula (9). Calculations show that the structure by the end of the pressure rise receives significant plastic deformation and fails. The most real and favorable case at $\frac{t_{10}}{t_R} = 0.8$ and $t_+ = 20$, the deformation at the beginning of the pressure drop corresponds to the coefficient of plasticity $\mu = 14.4$. At the stage of pressure decline, the deformation can still increase, since the external load is still acting and the deformation rate increases by 10 times, which indicates a high, 100 times increased, kinetic energy.

Table 2 presents data on the magnitude of the velocity and the force factor for cases where the deflection at the time of maximum load is 0.8 and 0.5 of the maximum deflection at elastic deformation.

In both cases, the further deformation is calculated by the ratios (12-13) for the elastic deformation during load decay. The calculation is carried out until the onset of plastic deformation, that is, when $\ddot{y}_2 = 0.2$ for $y_+ = 0.8$ and $\ddot{y}_2 = 0.5$ for $y_+ = 0.5$.

The calculation in the plastic stage is carried out by (15-16) until the equilibrium or the end of the load decay time. In operation, the load decay time $t_-$ was assumed to be 0.2 of the rise time $t_+$. It should be noted that the load decay time has not been studied, and the value adopted in the work is purely intuitive. In the case of plastic deformation at the pressure drop, the deformation increases by another value $\ddot{y}_3 = 0.4$ for the case of $y_+ = 0.5$, and the structure stops until the end of the full load drop time.

4. Conclusions

In this paper, analytical dependences for the deformation of a hinged beam for a load varying according to the law $t \sim t^3$ at the growth stage and with a linear decline are obtained.
The problem is solved by using the concepts of equivalent mass, stiffness and load. The change of deformation mode was controlled by maintaining the amount of motion of the system. The obtained results indicate a large contribution to the overall deformation of the site of the fastest increase in load.

It is concluded that it is necessary to investigate not only the maximum pressure of the explosion and the rate of its growth, but also the patterns of decline. In the case of its most dangerous case, when plastic deformation occurs at the stage of pressure growth.

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