Probing light pseudoscalars with light propagation, resonance and spontaneous polarization

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Received 14 October 2004
Accepted 26 May 2005
Published 7 June 2005

Online at stacks.iop.org/JCAP/2005/i=06/a=002
doi:10.1088/1475-7516/2005/06/002

Abstract. Radiation propagating over cosmological distances can probe light weakly interacting pseudoscalar (or scalar) particles. The existence of a spin-0 field changes the dynamical symmetries of electrodynamics. It predicts spontaneous generation of polarization of electromagnetic waves due to mode mixing in the presence of background magnetic field. We illustrate this by calculations of propagation in a uniform medium, as well as in a slowly varying background medium, and finally with resonant mixing. Highly complicated correlations between different Stokes parameters are predicted, depending on the parameter regimes. The polarization of propagating waves shows interesting and complex dependence on frequency, the distance of propagation, coupling constants and parameters of the background medium such as the plasma density and the magnetic field strength. For the first time we study the resonant mixing of electromagnetic waves with the scalar field, which occurs when the background plasma frequency becomes equal to the mass of the scalar field at some point along the path. Dynamical effects are found to be considerably enhanced in this case. We also formulate the condition under which the adiabatic approximation can be used consistently, and find caveats about comparing different frequency regimes.

Keywords: dark energy theory, magnetic fields, axions, inter-galactic medium
1. Introduction

Light, weakly interacting spin-0 particles are predicted by many extensions of the Standard Model of particle physics. Such a field might arise as pseudo-Goldstone bosons of some spontaneously broken chiral symmetry (PQ) [1]–[3], in supergravity theories [4,5] and the low energy string action [6]–[8] as the Kalb–Ramond field. The mixing of light with pseudoscalar fields in propagation has a long history. Karl and Clark [9] looked for ‘cosmological birefringence’ more than 20 years ago. Pseudoscalar–electromagnetic field mixing has long been explored by several authors [10]–[16]. Our focus is on exploring the use of polarization observables that can separate models. Our calculations find that polarization evolution can sometimes be highly complicated and not easily estimated using simple dimensional arguments. Placing limits on the basis of rough arguments and dimensional analysis is far from reliable. Recently Csaki et al [17] raised the possibility that supernova dimming might be caused by photon–pseudoscalar ($\gamma-\phi$) mixing, re-igniting interest [18] in this problem. Whether or not supernovas dim, the overall physics of propagation turns out to be quite interesting, and the signals of polarization particularly sensitive to careful treatment. The variety of effects is huge. As a result, one must question...
the validity of placing limits based simply on dimensional analysis or naive generalities. Instead, the logic has to be reversed to seek particular effects contradicting conventional physics wherever they occur, and then investigate the causes.

There is another reason to consider polarization as a cosmological observable. Physical theories are not tested by fitting undetermined parameters: they are tested by testing their symmetries. The framework of general relativity (GR) has certain symmetries by construction. Among these symmetries is duality, which predicts that the polarization of light in free space does not change. By the principle of equivalence, GR then prescribes parallel transport of polarizations that are trivial in freely falling coordinates. It follows that observations inconsistent with duality might rule out perturbations of the metric as a false signal. Another symmetry of unitarity prevents the spontaneous appearance of polarization in an unpolarized beam of light. If such features contradicting GR are observed, then it rules out GR as an explanation of them, lifting a ‘degeneracy’ of interpretation, and putting responsibility uniquely on something new.

There is one spectacular signal. A spin-0 field generically induces polarization even in a completely unpolarized beam. Indeed such a field can spontaneously polarize the CMB itself. The effect is simple and quite distinctive. We are emboldened to suggest that the absence of polarization effects in propagation over cosmological distances, or in CMB derived quantities, would put stringent limits on the existence of a light spin-0 field. In this regard we have eagerly awaited the release of WMAP data on CMB polarization while preparing this paper. We feel that the time for discovery with polarization observables may be ripe with this and other sources of data coming in the future.

2. Symmetries of light in the presence of spin-0 field

The existence of a light scalar (or pseudoscalar) field changes the symmetries of the propagation of light. We consider a spin-0 field $\phi$ coupled to the electromagnetic field strength $F_{\mu\nu}$ by the action

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + g_\phi \phi \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} + g'_\phi \phi F^{\mu\nu} F_{\mu\nu} + j_\mu A^\mu + \frac{1}{2} \partial_\mu \phi \partial_\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 + V(\phi) \right]. \quad (1)$$

We include a coupling to a current $j_\mu$ for completeness, while in practice this term will lead to modified propagation such as plasma frequencies. For the purposes to be developed the potential $V(\phi)$ can be ignored as a small perturbation, and the metric $g$ be replaced by a given background form. Note that we have included coupling of $\phi$ to both $\epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}$ and $F^{\mu\nu} F_{\mu\nu}$, since both are allowed by the symmetries of the theory. If $\phi$ is a pseudoscalar, as we assume, then the coupling $g'_\phi$ breaks parity symmetry. However this approximate feature of the low energy world is not a symmetry of Nature, being broken by the weak interaction physics. Even if the coupling $g'_\phi$ is tuned to zero at some scale, a coupling will be induced by radiative corrections. In the rest of the paper, however, we ignore $g'_\phi$ since the limit on this coupling is much more stringent compared to the limit on $g_\phi$ [19].

Free space electrodynamics has an interesting symmetry called duality. In the Hamiltonian density $\mathcal{H}$ in natural units, the electric and magnetic fields $\vec{E}$ and $\vec{B}$ can
be rotated:

\[
\begin{pmatrix}
\vec{E}' \\
\vec{B}'
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\vec{E} \\
\vec{B}
\end{pmatrix};
\]

\[\mathcal{H} = \frac{1}{2} \vec{E}^2 + \frac{1}{2} \vec{B}^2 \rightarrow \mathcal{H}.\]  

(2)

Although a symmetry of the Hamiltonian density, duality is not a symmetry of the Lagrangian density, which changes by a pure divergence. Noether’s theorem represents this faithfully: the duality current \( K^\mu \) is not conserved, but its divergence is an identity:

\[
K^\mu = \epsilon_{\mu \nu \alpha \beta} A^\nu F^{\alpha \beta};
\]

\[
\partial^\mu K^\mu = \epsilon_{\mu \nu \alpha \beta} F^{\mu \nu} F^{\alpha \beta}.
\]

(3)

As a consequence of duality symmetry, \( \epsilon_{\mu \nu \alpha \beta} F^{\mu \nu} F^{\alpha \beta} \sim \vec{E} \cdot \vec{B} \), which is not invariant, must be zero in a free space plane wave. Otherwise symmetry would permit rotating it into a different magnitude, which is a contradiction. The duality current is deeply related to the helicity density:

\[
\frac{1}{2} K_0 = i \frac{k}{\omega} \vec{A}_k \times \vec{E}_k,
\]

where \( \vec{k} \) is the wavenumber, \( \omega \) the angular frequency and \( \vec{A} \times \vec{E} \) is the canonical spin density\(^3\). Thus duality symmetry implies separate conservation of the spin and orbital angular momentum of light in a \( \vec{k} \) eigenstate. Now the coupling of \( \phi \) to \( \epsilon_{\mu \nu \alpha \beta} F^{\mu \nu} F^{\alpha \beta} \) breaks duality symmetry, allowing the helicity of photons to mix, and the plane of polarization of light to change in propagation—at least in the general case. Remarkably, the coupling of equation (1) vanishes for a single plane wave: an isolated photon is ‘safe’. However light cannot be protected under all possible conditions, and below we explore the effects of a background magnetic field.

Let us contrast the \( \phi \)-coupling with propagation in the usual cosmological model. In that event there is a coupling of light and gravity: the metric enters as an ‘optical medium’ in the form of

\[
S = \int \mathrm{d}^4 x \sqrt{-g} \left[ -\frac{1}{4} F_{\mu \nu} g^{\nu \alpha}(x) g^{\mu \beta}(x) F_{\alpha \beta} \right].
\]

(5)

The energy of this system can be written as

\[
\mathcal{H} = \frac{1}{2} (\vec{D} \cdot \vec{E} + \vec{B} \cdot \vec{H});
\]

\[
D_k = g^{\nu 0}(x) g^{\mu k}(x) F_{\mu \nu};
\]

\[
H_k = \frac{1}{2} \epsilon_{ijk} g^{\nu j}(x) g^{\mu k}(x) F_{\mu \nu}.
\]

(6)

This system has a new duality of rotating \( \vec{D} \) and \( \vec{H} \) into \( \vec{E} \) and \( \vec{B} \). In a coordinate system where the metric is trivial, \( \vec{D} = \vec{E} \) and \( \vec{H} = \vec{B} \) and the state of polarization is preserved.

GR also maintains a symmetry under which the power transmitted scales with kinematic dependence on the scale factor, while the plane and magnitudes of polarizations are preserved, in the sense of parallel transport along geodesics. The prediction of GR

\(^3\) Strictly speaking, photon spin depends on the gauge choice. The interpretation comes from the gauge \( A^0 = 0 \), which is nearly unique in having valid canonical spin and orbital angular momenta.
effects then consists of a coordinate transformation. In the isotropic, homogeneous metrics assumed in cosmology, the observable result is a red-shift.

Free space symmetries map into GR symmetries due to the principle of equivalence. Thus conclusions about them do not hinge on particular solutions. With GR serving as a ‘null’ theory, the effects of $\gamma-\phi$ mixing can be separated and distinguished: simply observe the spontaneous polarization of light during propagation, for instance.

### 2.1. Mixing light with pseudoscalars

Astrophysics constraints limit the coupling $g_\phi$ to be very tiny, yet the enormous length scales of cosmology allow small cumulative effects to develop into large ones. Proceeding requires discussion of scales so that we can linearize equation (1) to discuss mixing of modes in propagation. There are two important classes: background $\phi$, where the electromagnetic field is solved with a fixed pseudoscalar field, and background $B$, where a combination of the electromagnetic field and $\phi$ is solved for a given magnetic field. Here we set the background $\phi$ field to zero and focus our effort on the second case.

We will use symbol $\vec{B}$ for the magnetic background, and $\vec{B}$ for the field in propagation: the total magnetic field is $\vec{B} + \vec{B}$. Although galactic and intergalactic $\vec{B} \sim \mu G$ fields are small, their effects are likely to be large compared to the effects of $\phi$. A quick calculation shows that $\vec{E}/c$ and $\vec{B}$ for a typically weak but observable signal, such as light from a QSO or the CMB, are also small compared to $\mu G$. We seek to linearize consistently with $\vec{E}/c \ll \vec{B}$ and $\vec{B} \ll \vec{B}$. There is a well-defined three-state propagation problem of mixing two light polarizations with coupling to $\phi$. We first obtain the non-covariant form of Maxwell equations, as follows, and with no approximations:

\[
\nabla \cdot \vec{E} = g_\phi \nabla \phi \cdot (\vec{B} + \vec{B}) + \rho; \tag{7}
\]

\[
\nabla \times \vec{E} + \frac{\partial (\vec{B} + \vec{B})}{\partial t} = 0; \tag{8}
\]

\[
\nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = g_\phi \left( \vec{E} \times \nabla \phi - (\vec{B} + \vec{B}) \frac{\partial \phi}{\partial t} \right) + \vec{j}; \tag{9}
\]

\[
\nabla \cdot (\vec{B} + \vec{B}) = 0. \tag{10}
\]

Here $B_i + B_i = \frac{1}{2} \epsilon_{ijk} F^{jk}$ and $E_i = F^{0i}$ are the usual electric and magnetic fields. Besides this we have the equation for the pseudoscalar field

\[
\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi + m_\phi^2 \phi = -g_\phi \vec{E} \cdot (\vec{B} + \vec{B}). \tag{11}
\]

Gauge invariance is explicit and one can check current conservation directly,

\[
\nabla \cdot \vec{j} + \frac{\partial \rho}{\partial t} = 0.
\]

The dynamics of this coupled three-field system is complicated and has been visited by various approximations in the literature.
We assume $\vec{B}$ solves the zeroth-order Maxwell equations with no $\phi$ background. Our linearized equations for $\vec{E}/c \ll \vec{B}, \vec{B} \ll \vec{B}$ are

$$\nabla \cdot \vec{E} = g_\phi \nabla \phi \cdot \vec{B} + \rho; \quad (12)$$

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0; \quad (13)$$

$$\nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = -g_\phi \vec{B} \frac{\partial \phi}{\partial t} + \vec{j}; \quad (14)$$

$$\nabla \cdot \vec{B} = 0. \quad (15)$$

These equations remain exactly consistent with current conservation, $\vec{\nabla} \cdot \vec{j} + \partial \rho/\partial t = 0$.

Proceed to get a wave equation for $\vec{E}$ by taking the curl of Faraday’s law, and substituting into equations (7), (9). Replacing $\vec{B} \sim \vec{B}$ gives

$$-\vec{\nabla}^2 \vec{E} + \frac{\partial^2 \vec{E}}{\partial t^2} = g_\phi \vec{B} \frac{\partial^2 \phi}{\partial t^2} - g_\phi \vec{\nabla}(\vec{\nabla} \phi \cdot \vec{B}). \quad (16)$$

In this equation the longitudinal part of $\vec{E}$ mixes with $\vec{\nabla} \phi \cdot \vec{B}$. That is, $\vec{E}$ is not perfectly transverse in general when coupled to $\phi$.

Continuing along these lines towards a general solution greatly increases the complexity of the equations. We will be content here to show illustrative calculations restricted to the limit $\vec{\nabla} \phi \cdot \vec{B} = 0$. In this case $\vec{E}$ is transverse and the system readily collapses to two-state mixing. We have extensive calculations for the more general cases, but we feel that this paper would not be improved by introducing long and complex calculations. Our detailed calculations show that the numerical importance of $B_L \neq 0$ effects for the parameter regions considered in this paper are negligible. The only exception to this may possibly be in the case of resonance where the correction terms may be lumped together with other parameters in any event. The virtue of $\vec{\nabla} \phi \cdot \vec{B} = 0$, even if it is not the most general case in practice, is that one can get a feel for the rich interplay of several dimensionful scales including $g_\phi, \vec{B}, m_\phi^2, \omega_p$ and the propagation distance $z$. The amazing range of phenomena one should seek observationally is well illustrated.

**Requirement 0: existing physics.** Known physics of electromagnetic propagation in matter must be taken into account. It is well known that the plane of linearly polarized light rotates in a propagation through magnetized plasma: the Faraday effect. Our symmetry arguments are consistent, because duality is broken by the coupling to electric (as opposed to magnetic) charges and currents. Fortunately Faraday rotation is quite frequency dependent, and most important for low frequencies below the GHz regime.

We also take into account the practical ‘photon mass’ in the form of the plasma frequency. It is not an effect that can be ignored, and we do incorporate it in the momentum space propagation equations below.

**Requirement 1: phase accuracy.** Observables depend on the wavenumber differences $\Delta k$ accumulating phase $\Delta \theta$ by propagating over large distances $\Delta z$. An accurate approximation needs the absolute phase error (symbol $\delta$) to be small compared to $\pi$. 
namely $\delta(\Delta k_\perp \Delta z) \ll 1$. This requirement becomes more and more demanding as $\Delta z \to \infty$. Everyone recognizes this requirement, listed here for completeness. In the absence of degeneracies, the wavenumbers of propagating waves can often be obtained consistently in powers of $g_\phi$, provided the corrections are controlled: see below.

**Requirement 2: gauge invariance.** It is very important for all symmetries to be respected, and in particular, any violation of gauge invariance is unacceptable. Fortunately our equations set up in a gauge invariant manner coincide with the standard literature in the limit of $\vec{B} \cdot \vec{B} = 0$.

**Requirement 3: respect for limit interchange.** Shortly we will solve the propagation and present a common series expansion in $g_\phi^2$ to simplify the formulae. We will find that the series contains factors of the frequency $\omega$. The limit of $g_\phi^2$ fixed and small, and taking $\omega \to \infty$, does not commute with $\omega$ fixed and $g_\phi^2 \to 0$. Since the frequency of an eV photon in units of the cosmological length scale is huge, while the coupling constants contemplated for $\phi$ are tiny, one needs to examine series expansions carefully to be sure they apply to the problem being solved.

### 3. Pseudoscalar–photon mixing: uniform background

Choose the coordinate system with the $z$-axis along the direction of propagation of the wave and the $x$-axis parallel to the transverse component of the background magnetic field $\vec{B}_T$. Seek solutions with harmonic time dependence $e^{-i\omega t}$. Denote the component mixing by subscript $\parallel$ and its perpendicular complement by subscript $\perp$. Define symbol $\vec{A} = \vec{E}/\omega$, which simplifies equations much like the vector potential, except there is no ‘$i$’ and $\vec{A}$ is gauge invariant.

The perpendicular component $A_\perp$ does not mix with $\phi$. The wave equation for the mixing of $A_\parallel$ and $\phi$ can be written as

\[
(\omega^2 + \partial_z^2) \begin{pmatrix} A_\parallel(z) \\ \phi(z) \end{pmatrix} - M \begin{pmatrix} A_\parallel(z) \\ \phi(z) \end{pmatrix} = 0
\]

where the ‘mass matrix’ or ‘mixing matrix’ is

\[
M = \begin{pmatrix} \omega_\phi^2 & -g_\phi B_T \omega \\ -g_\phi B_T \omega & m_\phi^2 \end{pmatrix}
\]

and $B_T$ is the magnitude of the vector $\vec{B}_T$. At this point we took into account the plasma frequency $\omega_\phi^2$, which is a (non-local) gauge invariant mass for $\vec{E}$. Transform to a new basis

\[
\begin{pmatrix} A_\parallel(z) \\ \phi(z) \end{pmatrix} = O \begin{pmatrix} A_\parallel(z) \\ \phi(z) \end{pmatrix}
\]

where $O$ is the orthogonal matrix

\[
O = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}
\]
which diagonalizes the mixing matrix in equation (17). Diagonalization gives
\[
\tan 2\theta = \frac{2g_\omega B_\omega}{m_\phi^2 - \omega^2}
\]  
(21)
with mass eigenvalues
\[
\mu^2 \pm = \frac{\omega^2 + m_\phi^2}{2} \pm \frac{1}{2} \sqrt{\left(\omega^2 - m_\phi^2\right)^2 + (2g_\phi B_\omega \omega)^2}.
\]  
(22)
We list the leading order expansion in \(g_\phi\), by which
\[
\mu^2 = \omega^2 + \frac{g_\phi^2 B_\phi^2 \omega^2}{\omega^2 - m_\phi^2} \; \mu^2
\]  
(23)
\[
\mu^2 = m_\phi^2 - \frac{g_\phi^2 B_\phi^2 \omega^2}{\omega^2 - m_\phi^2} \; \mu^2
\]  
(24)
We noted earlier that examination of scales is needed to justify this step. Consistency requires
\[
\frac{g_\phi^2 B_\phi^2 \omega^2}{\left(\omega^2 - m_\phi^2\right)^2} \ll 1.
\]  
(25)
Continuing, the equations are written in the diagonal basis as
\[
(\omega^2 + \partial^2_0)A_\parallel - \mu^2 A_\parallel = 0
\]
\[
(\omega^2 + \partial^2_0)\phi - \mu^2 \phi = 0
\]  
(26)
which can be easily solved to give
\[
A_\parallel(z) = A_\parallel(0)e^{i(\omega + \Delta_\phi)z}
\]
\[
\phi(z) = \phi(0)e^{i(\omega + \Delta_\phi)z}
\]  
(27)
and
\[
\omega + \Delta_A = \sqrt{\omega^2 - \mu^2_+} \approx \omega - \frac{\mu^2_+}{2\omega}
\]
\[
\omega + \Delta_\phi = \sqrt{\omega^2 - \mu^2_-} \approx \omega - \frac{\mu^2_-}{2\omega}
\]  
(28)
We will always work in the limit \(\omega^2 \gg \mu^2\), justifying retention of just the leading terms. The phase difference \(\Delta_\phi - \Delta_A\) in this limit is found to be
\[
\Delta_\phi - \Delta_A \approx \frac{1}{2\omega} \sqrt{\left(\omega^2 - m_\phi^2\right)^2 + (2g_\phi B_\omega \omega)^2}.
\]  
(29)
The perpendicular component \(A_\perp\) does not mix with \(\phi\) and is given by
\[
A_\perp(z) = A_\perp(0)e^{ik_0z}
\]  
(30)
where \(k_0 = \sqrt{\omega^2 - \omega_p^2} \sim \omega - \omega_p^2/2\omega\).

3.0.1. Typical scales and units. We use parameters typical of the Virgo supercluster for illustration. Generally the plasma frequency is given by
\[
\omega_p^2 = \frac{4\pi e n_e}{m_e} = \frac{n_e}{10^{-6} \text{ cm}^{-3}}(3.7 \times 10^{-14} \text{ eV})^2.
\]  
(31)
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For the supercluster, the plasma density $n_e \sim 10^{-6} \text{ cm}^{-3}$ and the magnetic field is of the order of $1 B \sim 0.1 \mu \text{G}$ coherent over a distance scale of order 10 Mpc [20]. As mentioned earlier, use of typical $B$ values in $B T$ formulae is made to take advantage of the transparent simplicity of that limit.

In the limit of $m_\phi \ll \omega_p$, there is a useful length scale $l$ defined by

$$l = \frac{2 \omega}{\omega_p^2 - m_\phi^2} \approx \frac{\nu}{10^6 \text{ GHz}} \times 0.04 \text{ Mpc} \quad (32)$$

where $\nu = \omega / 2 \pi$. We refer to this as the oscillation length in analogy with a similar variable in neutrino physics [21]. The value $10^6 \text{ GHz} \sim 4 \text{ eV}$ is a handy upper order of magnitude for optical frequencies.

A typical upper limit [22]–[25] on the coupling parameter $g_\phi$ is $6 \times 10^{-11} \text{ GeV}^{-1}$. In figure 1 below, we show results for $g_\phi \sim 10^{-12} \text{ GeV}^{-1}$, more than one order of magnitude smaller than the limit. For a magnetic field of 0.1 $\mu \text{G}$ we find that the product $g_\phi B$ can be expressed as $g_\phi B = 0.215 \text{ Mpc}^{-1}$. For convenience the scales are summarized in different units in table 1.

3.0.2. Small mixing limit: a simple example. For an example we examine the most innocuous case of mixing angle $\theta \to 0$. Specifically, this is the limit

$$\frac{g_\phi \omega B_T}{|m_\phi^2 - \omega_p^2|} \ll 1.$$ 

By equation (25), the limit of small mixing is just the same limit in which the naive Taylor expansion of equation (24) in small $g_\phi \sim 0$ applies. In this limit $E_\parallel, E_\perp$ and $\phi$ are the
Table 1. Typical values of dimensionful scales in different units. If not otherwise specified, we use $\hbar = c = 1$. The value of $g_\phi$ listed is far below published limits of $g_\phi < 6 \times 10^{-11}$ GeV$^{-1}$ [22].

| Quantity | Typical value | Alternative units |
|----------|---------------|-------------------|
| $B$      | $0.1 \, \mu$G | $16.8 \times 10^{49}$ Mpc$^{-2}$ |
| $g_\phi$ | $10^{-11}$ GeV$^{-1}$ | $6.4 \times 10^{-50}$ Mpc |
| $\omega_p$ | $3.7 \times 10^{-23}$ GeV $\sqrt{\frac{n_e}{10^{-6} \text{ cm}^{-3}}}$ | $5.7 \times 10^{15} \sqrt{\frac{n_e}{10^{-6} \text{ cm}^{-3}}} \text{ Mpc}^{-1}$ |
| $\omega$ | $10^{-5}$ eV | $1.6 \times 10^{24} - 1.6 \times 10^{29}$ Mpc$^{-1}$ |

approximate propagation eigenstates. One might think there are no observable effects, and in particular, that there would be no substantial ‘dimming’ of intensity.

However there is an important relative phase shift in propagation:

$$E_{\parallel}(z) = E_{\parallel}(0)e^{i(\omega + \Delta A)z};$$
$$E_{\perp}(z) = E_{\perp}(0)e^{i\alpha z}.$$

The physically observable density matrix $\rho$ is given by

$$\rho = \begin{pmatrix} \langle E_{\parallel}E_{\parallel}^* \rangle & \langle E_{\parallel}E_{\perp}^* \rangle \\ \langle E_{\perp}E_{\parallel}^* \rangle & \langle E_{\perp}E_{\perp}^* \rangle \end{pmatrix},$$

(33)

where $\langle \rangle$ denotes the statistical averages occurring in propagation. Under coherent conditions this result predicts a cumulative rotation of the plane of a linear polarization due to the off-diagonal term:

$$E_{\parallel}E_{\perp}^*(z) = E_{\parallel}E_{\perp}^*(0)e^{i(\omega_p^2/2\omega - \mu_\perp^2/2\omega)z}.$$  

(34)

As long as the mixing is small, we may insert the $g_\phi^2$ expansions of equation (24) and predict that the angle of rotation increases linearly with frequency. Yet there is always a limit in which the frequency is large enough to cause strong mixing. We explore this for $m_\phi^2 \ll \omega_p^2$.

Let the magnitude of the phase angle between $E_{\parallel}$ and $E_{\perp}$ be denoted $\chi$. From equation (34) we have

$$\chi = z\left(\frac{\omega_p^2}{2\omega} - \frac{\mu_\perp^2}{2\omega}\right) \sim \frac{g_\phi^2 B_T^2 \omega}{2\omega_p^2}, \quad \omega \ll \frac{\omega_p^2}{g_\phi B_T};$$

(35)

Using $g_\phi B_T \sim \text{Mpc}^{-1}$ as a typical value, the phenomenon of a rotating polarization might be readily observed to be linear in $\omega$ at radio frequencies for decades above the GHz regime.

Approaching the optical region, both the formula for small mixing and the expansion of equation (24) break down, leading to very interesting possibilities.
3.0.3. Comments on propagation. It is interesting to contrast the results above with propagation in a dispersionless and non-dissipative medium, such as ‘free space’ with gravitational fields. It is obvious that the intensity of a wave (Stokes $I$) is preserved up to the kinematic red-shift of propagation\(^4\). It is less obvious that the degree of polarization (Stokes $Q/I$) cannot be changed by any two-state purely electromagnetic propagation in a dissipationless medium. The origin of this is ‘unitarity’ of propagation which can be written as a unitary evolution operator, just as in quantum mechanics \[^2\text{6}\]. As a consequence of two-state electromagnetic unitarity, a particular circular polarization (say) can be converted into a linear one, but a linear polarization cannot be made to come from an unpolarized ensemble. In the weak mixing region, an unpolarized density matrix (a multiple of the identity) evolves with no change whatsoever. This means that basing observations or parameter limits on unpolarized quantities can easily miss effects that polarized observables would readily detect.

Spontaneous polarization is more than rare, and in pure electrodynamics any form of polarization is usually associated with a corresponding extinction. An ordinary polarizer plate is typical, with the degree of polarization scaling directly with the degree of extinction. The same goes for common sources of polarization such as Compton scattering invoked in astrophysics. Shortly we will discuss stronger mixing cases where mixing of light with pseudoscalars can lead to spontaneous polarization.

3.1. General density matrix

We can now determine the general density (coherency) matrix elements after propagation through distance $z$ in terms of the matrix elements at the source. Inasmuch as we can interchange $\vec{A}$ with $\vec{E}/\omega$, it is convenient to report density matrices of $\vec{A}$. Then,

\[
\langle A_\parallel^*(z)A_\parallel(z) \rangle = \frac{1}{2}\left[ \langle A_\parallel^*(0)A_\parallel(0) \rangle + \right. \\
+ \left. \frac{\langle \phi_\parallel^*(0)\phi_\parallel(0) \rangle}{2} \right] \left[ 1 + \cos^2 2\theta + \sin^2 2\theta \cos[z(\Delta_\phi - \Delta_A)] \right] \\
+ \left\{ \frac{\langle A_\parallel^*(0)\phi_\parallel(0) \rangle}{2} \right\} \left[ \sin 2\theta \cos 2\theta - \sin 2\theta \cos 2\theta \cos[z(\Delta_\phi - \Delta_A)] \right] \\
- i \sin 2\theta \sin[z(\Delta_\phi - \Delta_A)] + \text{c.c.} \right}. \tag{36}
\]

We gave the most general expression above keeping all the correlators at $z = 0$. One might think it reasonable to assume that $\langle A_\parallel^*(0)\phi(0) \rangle$ is zero. However in considering propagation through an intergalactic medium one expects a large number of magnetic domains uncorrelated with one another. The general expression is needed to describe sequentially the propagation through different domains. Given the randomness of such processes, the generic situation is one where all the correlators on the right-hand side will be non-zero.

We also find

\[
\langle A_\parallel^*(z)A_\perp(z) \rangle = \left( \cos^2 \theta e^{iFz} + \sin^2 \theta e^{iGz} \right) \langle A_\parallel^*(0)A_\perp(0) \rangle \\
+ \cos \theta \sin \theta (e^{iFz} - e^{iGz}) \langle \phi_\parallel^*(0)A_\perp(0) \rangle \tag{37}
\]

\(^4\) Reflections, namely the generation of backwards moving waves, are always possible in any varying medium. We work in the limit where they are negligible.
and \( A^*_\perp(z)A^\perp(z) \rangle = \langle A^*_\perp(0)A^\perp(0) \rangle \). The phase factors \( F \) and \( G \) are given by

\[
F = \omega \sqrt{1 - \frac{\omega_p^2}{\omega^2}} - \omega \sqrt{1 - \frac{\mu_+}{\omega^2}} \approx \frac{1}{2\omega}(\mu_+ - \omega_p^2),
\]

\[
G = \omega \sqrt{1 - \frac{\omega_p^2}{\omega^2}} - \omega \sqrt{1 - \frac{\mu_-}{\omega^2}} \approx \frac{1}{2\omega}(\mu_- - \omega_p^2).
\]

Here again it is reasonable to expand \( F \) and \( G \) and keep only the leading order terms in \( \omega_p^2/\omega^2 \) and \( \mu_+/\omega^2 \). Using these correlation functions we can compute reduced Stokes parameters \( I, Q, U, V \) (or \( S_0, S_1, S_2, S_3 \)):

\[
I = \langle A^*_{\parallel}(z)A_{\parallel}(z) \rangle + \langle A^*_{\perp}(z)A_{\perp}(z) \rangle
\]

\[
Q = \langle A^*_{\parallel}(z)A_{\parallel}(z) \rangle - \langle A^*_{\perp}(z)A_{\perp}(z) \rangle
\]

\[
U = \langle A^*_{\parallel}(z)A_{\perp}(z) \rangle + \langle A^*_{\perp}(z)A_{\parallel}(z) \rangle
\]

\[
V = i(-\langle A^*_{\parallel}(z)A_{\perp}(z) \rangle + \langle A^*_{\perp}(z)A_{\parallel}(z) \rangle)
\]

at any position \( z \). The standard Stokes parameters of the same name are simply \( \omega^2 \) times the above. We will often remove the scale of intensity, normalizing \( I = 2 \), as in an unpolarized wave \( |A\parallel|^2 = |A\perp|^2 = 1 \).

### 3.1.1. Special cases 1: spontaneous appearance of polarization.

Suppose the initial beam is unpolarized and the initial correlator \( \langle A^*_{\parallel}(0)\phi(0) \rangle \) is also zero. We scale out normalizations so that

\[ \langle A^*_{\parallel}(0)A_{\parallel}(0) \rangle = \langle A^*_{\perp}(0)A_{\perp}(0) \rangle = 1 \]

and assume that all other correlators vanish at \( z = 0 \). The expressions simplify to

\[ \langle A^*_{\parallel}(z)A_{\parallel}(z) \rangle = \frac{1}{2}(1 + \cos^2 2\theta + \cos[z(\Delta_\phi - \Delta_\lambda)] \sin^2 2\theta) + \frac{1}{2}(\phi^*(0)\phi(0))(1 - \cos[z(\Delta_\phi - \Delta_\lambda)]) \sin^2 2\theta \]

(39)

\[ \langle A^*_{\parallel}(z)A_{\perp}(z) \rangle = 0 \]

(40)

\[ \langle A^*_{\perp}(z)A_{\perp}(z) \rangle = 1. \]

(41)

This wave spontaneously acquires a linear polarization oriented along \( \vec{B} \) during propagation. No circular polarization is developed. We plot the degree of polarization in figure 1 for initial conditions \( \langle \phi^*(0)\phi(0) \rangle = 0 \). The degree of polarization \( p \) here is given by

\[ p = \frac{|Q|}{I} = \frac{|\langle A^*_{\parallel}(z)A_{\parallel}(z) \rangle - \langle A^*_{\perp}(z)A_{\perp}(z) \rangle|}{\langle A^*_{\parallel}(z)A_{\parallel}(z) \rangle + \langle A^*_{\perp}(z)A_{\perp}(z) \rangle}, \]

(42)

given Stokes \( U = V = 0 \). The degree of polarization accumulates to a sizable magnitude and could produce observable consequences even for exceedingly small couplings. In figure 1 the degree of polarization is shown for two different values of the coupling parameter \( g_\phi = 1 \times 10^{-12}, 2 \times 10^{-12} \text{ GeV}^{-1} \). The background medium parameters are
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\[ p = \sqrt{Q^2 + U^2 + V^2} \]

\[ I = \frac{2\omega}{\omega^2 - m^2} \]

**Figure 2.** Reduced Stokes parameter ratios \(Q/I, U/I\) and \(V/I\) as a function of the length parameter \(l\) in units of Mpc (equation (32)), for uniform background propagation with \(g_\phi = 2 \times 10^{-12}\) GeV\(^{-1}\). The degree of polarization \(p = \sqrt{Q^2 + U^2 + V^2}/I\) and the variable \(\sqrt{U^2 + V^2}/I\) are also shown. Other parameters are \(B = 0.1\) \(\mu\)G, \(n_e = 10^{-6}\) cm\(^{-3}\), Propagation distance = 10 Mpc, \(m_\phi/\omega_p = 0.1\). The initial state of the polarization has been chosen randomly such that \(Q/I = 0, U/I = 0.4\) and \(V/I = 0.1\).

chosen to be \(B = 0.1\) \(\mu\)G and \(n_e = 10^{-6}\) cm\(^{-3}\). The propagation distance is taken to be 10 Mpc. This is also the distance over which the background magnetic field is assumed to be coherent.

The experimental signature of this mixing for a uniform background would require fitting the observed degree of polarization to a number of parameters. As shown by the previous examples, the extrapolation between radio frequency and optical frequency observations needs care and attention to series expansions. Nevertheless the spontaneous polarization due to mixing is clearly distinguishable from extinction, which is expected to show a simple monotonic increase in the degree of polarization with frequency.

3.1.2. Special cases 2: initial polarization.

Consider now the case where the incident beam is partially polarized. A sample of results for randomly chosen initial conditions is shown in figure 2. The Stokes \(Q/I, U/I\) and \(V/I\) parameters are presented as a function of the length parameter \(l\), equivalent to the frequency of the incident wave. Initial conditions are \(\langle \phi^*(0)\phi(0) \rangle = 0, \langle A_\parallel^*(0)\phi(0) \rangle = 0\) and \(\langle A_\perp^*(0)\phi(0) \rangle = 0\). A more general result is discussed later. One interesting pattern that emerges from this figure is that \(U\) and \(V\) show a much larger variation with frequency compared to \(Q\). This is related to observations of [27] that at low frequencies the dominant physical effect arises due to the phase in \(A_\parallel\), an evolution of the polarization direction of the wave, which is permitted by two-state unitarity. The figure supports near conservation of \(\sqrt{U^2 + V^2}\) which is relatively independent of frequency, both in the low and high frequency regimes.
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Figure 3. A sample of results showing the correlation between the Stokes parameters $U$ and $V$ for randomly chosen parameters and initial state of polarization under propagation in a uniform background. The parameters (in arbitrary units) are (a) $g\phi B = 2$, $L = 10$, $0.04 < l < 20$, (b) $g\phi B = 10$, $L = 10$, $0.4 < l < 300$, (c) $g\phi B = 1$, $L = 50$, $0.2 < l < 800$ and (d) $g\phi B = 10$, $L = 10$, $0.04 < l < 100$; $m_{\phi}^2/\omega^2_{\pi} = 0.1$ for all the plots. A simple correlation is seen only for frequencies larger than a minimum frequency. At low frequencies the relationship becomes very complicated.

Indeed this variable shows spectral dependence only in a relatively narrow region. In the high frequency regime, of course, all the Stokes parameters show dependence on frequency.

A possible signal of the pseudoscalar mixing is the correlation between the different Stokes parameters, in particular between $U$ and $V$. In figure 3 we show a sample of results to illustrate the correlation between the Stokes parameters $U$ and $V$ for some randomly chosen parameters and initial state of polarization. We find these Stokes parameters do show an observable dependence on each other. The dependence of $U$ on $V$ is found to approximately follow an ellipse for large frequencies. If the frequency is small such that the oscillation length $l \ll L$, where $L$ is the propagation distance, then we find that the dependence of $U$ on $V$ is considerably more complicated. Remarkably, the Stokes parameter $Q$ (degree of polarization) also shows a simple dependence on $U$ and $V$ (polarization direction controllers) at large frequencies. This is shown for some randomly chosen parameters in figure 4.
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Figure 4. A sample of results showing the correlation between the Stokes parameters $Q$ and $U$. The parameters and initial states of polarization are taken to be same as used those in figure 3. The oscillation length $l$ is (a) $1.1 < l < 20$, (b) $0.4 < l < 300$, (c) $1.7 < l < 800$ and (d) $0.27 < l < 100$. The lower limits on $l$ are taken slightly larger than the values used in figure 3 in order to have a simple relationship emerge between $Q$ and $U$.

These illustrative calculations show that a fascinating range of physical phenomena are generated by the mixing of light and pseudoscalars.

4. Pseudoscalar–photon mixing: slowly varying background

For the rest of the paper we will consider cases in where the background is slowly varying. There exists a large literature regarding the integration of differential equations with slowly varying parameters. The reader interested in the mathematical problem can consult the classic reference of Olver [28]. It is an understatement to say that the variety of situations is huge. Rather than make sweeping statements, we will use a ‘workmanlike’ series of definitions and approximations that we check are suited to the limits that we study.

First we restrict attention to the simple case where the direction of $\vec{B}$ is uniform. Assume that the background varies sufficiently slowly to justify neglect of terms involving the derivatives of the background plasma density (or the magnitude of the magnetic field). We thus enter into the adiabatic approximation, a general branch of mechanics, which holds that the amplitude in an eigenstate is relatively constant when the parameters defining
the eigenstate change sufficiently slowly. We again obtain two decoupled equations (26) which can be solved by using the ansatz

\[ \overline{A}_\parallel(z) = \overline{A}_\parallel(0) \exp \left( i \omega z + i \int_0^z \Delta A \, dz' \right) \]

\[ \overline{\phi}(z) = \overline{\phi}(0) \exp \left( i \omega z + i \int_0^z \Delta \phi \, dz' \right). \]

(43)

The phase factors are given in equation (28).

In the case where there is a conservation law—the conservation of transmitted energy flux, similar to the conservation of probability in quantum mechanics—the WKB approximation would indicate certain pre-factors of order \(1/\sqrt{k(z)}\), where \(k(z)\) are the exponential integrands. We review this with an iterative WKB procedure developed in section 5 which controls coefficients in a certain expansion. However extensive calculations support wide applicability of the simpler formulae, inasmuch as \(k(z)\) tends to cancel out of polarization ratios and not to develop cumulative run-outs in many circumstances.

Using these results we can now evaluate the electromagnetic field at any position. For this purpose we express the fields \(\overline{A}_\parallel(0)\) and \(\overline{\phi}(0)\) in terms of \(\overline{A}_\parallel(0)\) and \(\overline{\phi}(0)\) and the mixing angle \(\theta(0)\). The final expressions for the fields \(\overline{A}_\parallel(z)\) and \(\overline{\phi}(z)\) are given by

\[ \overline{A}_\parallel(z) = e^{i \omega z} \overline{A}_\parallel(0) \left[ \cos \theta \cos \theta_0 \exp \left( i \int_0^z \Delta A \, dz' \right) + \sin \theta \sin \theta_0 \exp \left( i \int_0^z \Delta \phi \, dz' \right) \right] \]

\[ + e^{i \omega z} \phi(0) \left[ \cos \theta \sin \theta_0 \exp \left( i \int_0^z \Delta A \, dz' \right) - \sin \theta \cos \theta_0 \exp \left( i \int_0^z \Delta \phi \, dz' \right) \right] \]

\[ - \sin \theta \cos \theta_0 \exp \left( i \int_0^z \Delta \phi \, dz' \right) \]

\[ \overline{\phi}(z) = e^{i \omega z} \overline{A}_\parallel(0) \left[ \sin \theta \cos \theta_0 \exp \left( i \int_0^z \Delta A \, dz' \right) - \cos \theta \sin \theta_0 \exp \left( i \int_0^z \Delta \phi \, dz' \right) \right] \]

\[ + e^{i \omega z} \phi(0) \left[ \sin \theta \sin \theta_0 \exp \left( i \int_0^z \Delta A \, dz' \right) + \cos \theta \cos \theta_0 \exp \left( i \int_0^z \Delta \phi \, dz' \right) \right] \]

\[ + \cos \theta \cos \theta_0 \exp \left( i \int_0^z \Delta \phi \, dz' \right) \]

(44)

where on the right-hand side \(\theta = \theta(z)\) and \(\theta_0 = \theta(0)\). The resulting expression for the coherency matrix elements is given by

\[ \langle \overline{A}_\parallel^*(z) \overline{A}_\parallel(z) \rangle = \left\langle \frac{A_\parallel^*(0) A_\parallel(0)}{2} \right\rangle \left[ 1 + \cos 2\theta \cos 2\theta_0 + \sin 2\theta \sin 2\theta_0 \cos \Phi \right] \]

\[ + \left\langle \frac{\phi^*(0) \phi(0)}{2} \right\rangle \left[ 1 - \cos 2\theta \cos 2\theta_0 - \sin 2\theta \sin 2\theta_0 \cos \Phi \right] \]

\[ + \left( \langle A_\parallel^*(0) \phi(0) \rangle \right) \left[ \cos 2\theta \sin 2\theta_0 - \sin 2\theta \cos 2\theta_0 \cos \Phi \right] \]

\[ + i \sin 2\theta \sin \Phi + \text{c.c.} \]

(45)
where
\[ \Phi = \int_0^z (\Delta_A - \Delta_\phi) \, dz' \]  
(46)

\[ \langle A_\parallel(z) A_\perp(z) \rangle = \langle A_\parallel(0) A_\perp(0) \rangle \left[ \cos \theta \cos \theta_0 \exp \left( \frac{i}{2\omega} \int_0^z dz' \left[ \mu_+^2 - \omega_p^2 \right] \right) \right. \]
+ \sin \theta \sin \theta_0 \exp \left( \frac{i}{2\omega} \int_0^z dz' \left[ \mu_-^2 - \omega_p^2 \right] \right) \]
+ \langle \phi^*(0) A_\perp(0) \rangle \left[ \cos \theta \sin \theta_0 \exp \left( \frac{i}{2\omega} \int_0^z dz' \left[ \mu_+^2 - \omega_p^2 \right] \right) \right. \]
- \sin \theta \cos \theta_0 \exp \left( \frac{i}{2\omega} \int_0^z dz' \left[ \mu_-^2 - \omega_p^2 \right] \right) \right]. \]
(47)

The above result is valid as long as the medium changes slowly so that the term
\[ 2O_T(\partial_z O) \partial_z \left( \begin{array}{c} A \\ \phi \end{array} \right) \right) = 2i\omega \theta' \left( \begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right) \left( \begin{array}{c} A \\ \phi \end{array} \right) \]
(48)

is negligible compared to mass term
\[ \left( \begin{array}{cc} \mu_+^2 & 0 \\ 0 & \mu_-^2 \end{array} \right) \left( \begin{array}{c} A \\ \phi \end{array} \right). \]

We are justified in ignoring the derivative of the mixing angle \( \theta \) if
\[ |\theta'| \ll \frac{\mu_+^2}{2\omega}. \]
(49)

The condition for adiabaticity will be formulated in greater detail in section 5. There we estimate the probability of transition from one local eigenstate to another. As we shall see, the condition given above, equation (49), is sufficient provided that the difference between \( \mu_+ \) and \( \mu_- \) is of the order of these eigenvalues. If there is a delicate cancellation between these two eigenvalues then on the right-hand side in equation (49) more care and specialized techniques may be needed.

### 4.1. Resonance

We next discuss the interesting case where the plasma frequency \( \omega_p \) becomes equal to the pseudoscalar mass \( m_\phi \) somewhere along the path. Given the wide range of plasma frequencies observed in the astrophysics there may be many situations where such resonance might occur. Examples include propagation of light or pseudoscalars from a dense medium, such as an AGN, GRB or cluster of galaxies into the intergalactic medium. Another example includes propagation of light from pulsar magnetosphere into the interstellar medium. Let the radiation initially propagate from the region \( z = 0 \) containing high plasma density \( \omega_p > m_\phi \) towards increasing \( z \) where a resonance occurs. Assume that initially the mixing angle \( \theta \ll 1 \), and recall that
\[ \tan 2\theta = \frac{2g_0^2 \omega B_T}{m_\phi^2 - \omega_p^2} \ll 1. \]
(50)
Let the plasma density decrease slowly along the path such that at the observation point \( m_\phi > \omega_p \). Somewhere along the path at \( z = z^* \) the right-hand side of equation (50) becomes infinite, which we will implement with angle \( 2\theta(z) \rightarrow \pi/2 \). To finally fix the initial conditions, let \( \langle A^\parallel_0(0)A^\parallel(0) \rangle \neq 0, \langle A^\parallel_0(0)A^\perp(0) \rangle \neq 0 \) and \( \langle A^\perp_0(0)A^\perp(0) \rangle \neq 0 \), that is, a generically mixed wave. The remaining correlators, which involve \( \phi \), are taken to be zero at \( z = 0 \). We compare the conditions after travelling a distance \( \Delta z = L \) to the observation point. From equations (45), (47) we find
\[
\langle A^\parallel(L)A^\parallel(L) \rangle \approx 0, \\
\langle A^\parallel(L)A^\perp(L) \rangle \approx 0, \\
\langle A^\perp(L)A^\perp(L) \rangle = \langle A^\perp_0(0)A^\perp(0) \rangle
\]
if \( |\tan 2\theta| \ll 1 \) at \( z = L \). These are predicted independently of frequency and the distance travelled, provided that the wave crosses the region where \( m_\phi = \omega_p \). Almost independently of the state of polarization at the origin, the wave becomes completely linearly polarized at the observation point. The orientation of the electric field vector is perpendicular to the background magnetic field\(^5\).

Studying resonance using a slowly varying approximation requires some care. The consistent requirement is that the assumption of slowly varying background be satisfied. We now make this assumption more precise in order to obtain a constraint on the derivative of the background plasma frequency. For this purpose we assume that the background plasma density varies linearly with position such that
\[
\frac{m_\phi^2}{\omega_0} - \frac{\omega_p^2}{\omega_0} = C - \alpha z
\]
where \( C \) and \( \alpha \) are constants, \( z \) is the distance of propagation and \( \omega_0 \) is some chosen value of the frequency of the wave. It is convenient to work with the rescaled variables,
\[
\tilde{\omega}_p^2 = \frac{\omega_p^2}{\omega_0}, \quad \tilde{m}_\phi^2 = \frac{m_\phi^2}{\omega_0^2}, \quad \tilde{\omega} = \frac{\omega}{\omega_0},
\]
since for cosmological applications \( \tilde{\omega}_p^2 \) has a value of order unity in units of \( \text{Mpc}^{-1} \). The origin of the wave corresponds to \( z = 0 \) and the observation point \( z = L \).

If at \( z = 0 \) the plasma frequency is larger than pseudoscalar mass then the constants \( C \) and \( \alpha \) are taken to be negative. We assume that the wave crosses the resonance region where \( m_\phi = \omega_p \). As discussed earlier, in this case our approximation is valid if
\[
|\theta'| \ll \frac{|\mu_+^2 - \mu_-^2|}{2\omega}.
\]
This is discussed in greater detail in the next section. This constraint reduces to
\[
|\alpha| \ll 4(g_\phi B)^2 \tilde{\omega}
\]
at the resonance point \( m_\phi = \omega_p(z) \). Now we come to the point of these numbers: for the coupling \( g_\phi = 6 \times 10^{-11} \text{ GeV}^{-1} \) the constraint is satisfied for the magnetic field, plasma density and the distance scale corresponding to the Virgo supercluster, provided

\(^5\) Generally all resonant systems have a special phase at resonance. We acknowledge R Buniy for discussions and work long ago on similar resonant phase evolution in classical mechanics.
the frequency of the wave $\omega \gg 1.7 \times 10^{-4}$ eV. Since this is well below optical frequencies and well above the regime of GHz radio frequency studies, one should be prepared to see different phenomena in the optical and radio regimes. One can both use a slowly varying approximation and observe resonant mixing consistently.

Let us turn to the phases that appear in equation (47) for the case of linearly varying background. We find

$$\int_0^z dz' \frac{\mu_+^2 - \omega_p^2}{\omega_0} = \frac{1}{4} [-\alpha z^2 + 2zC] \pm \frac{C}{4\alpha} \sqrt{4g^2B^2\omega^2 + C^2}$$

$$\pm \frac{\alpha z - C}{4\alpha} \sqrt{4g^2B^2\omega^2 + (\alpha z - C)^2}$$

$$\pm \frac{(g_\phi B\omega)^2}{\alpha} \left[ \log \left( \alpha z - C + \sqrt{(2g_\phi B\omega)^2 + (\alpha z - C)^2} \right) \right]$$

$$- \log \left( -C + \sqrt{(2g_\phi B\omega)^2 + C^2} \right).$$

(56)

Since these phases are equal to $\int_0^z dz' [\mu_+^2 - \omega_p^2]/2\omega$ we find that for a wide range of parameter space these phases vary roughly as $1/\omega$. The corresponding polarization observables change roughly inversely proportionally to $\omega$.

We can understand this behaviour of the phases appearing in the correlator $\langle A_\parallel^* (z) A_\perp (z) \rangle$ as follows. Consider the phase $\int_0^z dz' (\mu_+^2 - \omega_p^2)/2\omega$, which is basically the integral of the difference of the squared masses of one of the eigenmodes and the perpendicular component divided by $2\omega$, as expected for relativistic particles. In the limit of small mixing the difference of the squared masses is proportional to the mixing angle squared and hence is proportional to $\omega^2$. In this limit the phase increases linearly with $\omega$. However if $\omega_p^2 - m_\phi^2 \rightarrow 0$ along the path, there is a limit interchange. We need a different series expansion, for example

$$\mu_+^2 \sim g_\phi B_T \omega + \frac{1}{2} (\omega_p^2 + m_\phi^2) + \frac{(\omega_p^2 - m_\phi^2)^2}{8g_\phi B_T \omega}.$$  

(57)

This contradicts perturbation theory in $g_\phi$, because resonance is always a non-perturbative phenomenon, as signalled by the appearance of $1/g_\phi$ in the expansion. In any event, given that $\mu_+^2/\omega$ occurs, the linear increase of phase flattens out near $\omega_p^2 - m_\phi^2 \sim 0$, and it can even decrease for large $\omega$.

So far we made the resonance occur. In the limit of small frequencies the phase factors become very large. For example in equation (56) the first factor contributes the term $m_\phi^2 z/2\omega$ to the phase. For large propagation distance $z$ and small $\omega$ this factor is clearly much greater than unity. Hence one expects that in general the Stokes parameters will show rapid fluctuations as a function of $\omega$ in this limit. However since $g_\phi B$ occur in a product, the small $g_\phi$ limit can again be reversed if $B$ becomes very large!

Continuing, we evaluate the phase $\Phi$ appearing in equation (45) for the case of a linearly varying background plasma frequency, equation (52). Results can be extracted from the integrals given in equation (56) by using

$$\Phi = \int_0^z (\Delta_A - \Delta_\phi) dz' = \int_0^z \left( \frac{\mu_+^2}{2\omega} - \frac{\mu_+^2}{2\omega} \right) dz'.$$

(58)

In figures 5 and 6 we show two examples of the result expected in the case of resonant mixing. Here the initial state of polarization has been chosen randomly. The
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\[
-\frac{m^2}{2} \phi \frac{2\omega}{\omega_p^2(L) - m^2_\phi} = \omega^2_p(L)
\]

Figure 5. The Stokes parameters \(Q/I, U/I\) and \(V/I\) as a function of the length parameter \(2\omega/(\omega^2_p(z) - m^2_\phi)\) (in arbitrary units) for the case of resonant mixing. Solid and dashed lines represent the results obtained by direct numerical integration and by using the analytic expressions (45) and (47) in the adiabatic approximation. The analytic result is in very good agreement with numerical result as long as equation (55) is satisfied. Here \(\tilde{\omega}_p^2(z) = \tilde{m}^2_\phi - C + \alpha z; g_B = 0.1; \tilde{m}^2_\phi = 1.\) The initial conditions at \(z = 0\) are \(p = 1.0, Q = 0.0, U = 1.0\) and \(V = 0.0.\)

Figure 6. Another example of Stokes parameters for resonant mixing. The parameters \(g_B B\) and \(L\) are approximately the same as for figure 2 and \(\tilde{m}^2_\phi = 1.\) The initial conditions are \(p = 1.0, Q = 0.0, U = 0.894\) and \(V = -0.447.\) Solid and dashed lines represent the results obtained by direct numerical integration and by using the analytic expressions equations (45) and (47) in the adiabatic approximation. The analytic result is in very good agreement with the numerical result as long as equation (55) is satisfied.
Degree of Polarization

Figure 7. Degree of polarization as a function of distance for the case of resonant mixing. The wave is taken to be unpolarized at source. The parameters are $g_\phi B = 1.0$, $\tilde{m}_\phi^2 = 1$, $C = -0.5$, $\alpha = -0.1$, $l = 2\omega/(\omega_0^2(L) - m_\phi^2) = 0.2$. The curve for uniform background (non-resonant mixing) is shown for comparison. All the parameters in this case are same as above, with the (uniform) mixing length $l = 2\omega/(\omega_0^2 - m_\phi^2) = 0.2$.

parameter values (in arbitrary units) given in the figure may be translated to those relevant for cosmological propagation by taking the units in the appropriate powers of Mpc. The pseudoscalar mass is chosen such that $\tilde{m}_\phi^2 = m_\phi^2/\omega_0 = 1$, where $\omega_0$ is defined in equation (52). As expected, observable effects are large at low frequencies compared to the corresponding results of a uniform background. We also find rapid fluctuations in this region. The analytic result, obtained in the adiabatic limit, is found to be in good agreement with the numerical result as long as equation (55) is satisfied. At smaller frequencies the two results start to disagree. As expected, we find numerically that the Stokes parameters approach their initial values in the limit $\omega \to 0$.

In figure 7 we show the dependence of the degree of polarization on the distance of propagation for the case of resonant mixing. Here the wave is taken to be unpolarized at source. The remaining parameters are taken to be $g_\phi B = 1.0$, $m_\phi = 1$, $C = -0.5$, $\alpha = -0.1$ and the oscillation length at the observation point $l = 2\omega/(\omega_0^2(L) - m_\phi^2) = 0.2$. The figure clearly shows the large change in the degree of polarization as the wave crosses the resonant point. For comparison we also show the case of uniform background with essentially the same parameters as for the resonant case. The only difference is that the (uniform) oscillation length parameter $l = 2\omega/(\omega_0^2 - m_\phi^2) = 0.2$. In this case the wave acquires a very small degree of polarization in comparison to the resonant case.

5. Adiabatic approximation and corrections

In this section we analyse the pseudoscalar–photon mixing for slowly varying background more systematically. This will give us an estimate of corrections to our results above. We assume that the adiabatic limit applies: for any changes of parameters, we take derivatives such as $\partial B/\partial z \to 0$, with the propagation distance $L \to \infty$, and the cumulative change $\Delta B$ fixed.
Borrowing standard notation from quantum mechanics, we rewrite the basic equation, equation (17), as follows:
\[(\omega^2 + \partial_z^2)\ket{\psi} - M\ket{\psi} = 0.\] (59)

We denote the eigenvectors of the mixing matrix as \(\ket{n}\), where in the case of resonant mixing of \(A_\parallel\) with \(\phi\), \(n = 1, 2\). The eigenvalue equation can be written as
\[M\ket{n} = \mu_n^2\ket{n}.\] (60)

Here the eigenvectors also vary slowly with \(z\). The general solution to equation (59) can be written as
\[
\ket{\psi} = \sum_n a_n(z) \exp\left(\int_0^z dz' \omega_n(z)\right)\ket{n}
\] (61)

where \(\omega_n \approx \omega - \mu_n^2/2\omega\). Substituting this into equation (59), and taking the overlap of the resulting equation with \(\ket{m}\) we find
\[
\partial_z a_m = -\frac{1}{2} \frac{\partial_z \omega_m(z)}{\omega_m(z)} a_m - \frac{1}{\omega_m(z)} \sum_n a_n \omega_n \exp\left(\int_0^z dz' (\omega_n(z) - \omega_m(z))\right) \langle m|\partial_z|n\rangle.
\] (62)

We have dropped all terms involving two derivatives of the slowly varying quantities such as \(a_n(z)\) and the exponent. Using the eigenvalue equation, equation (60), we find
\[
\langle m|\partial_z|n\rangle = \frac{\langle m|\partial_z M|n\rangle}{\mu_n^2 - \mu_m^2} \quad m \neq n.
\] (63)

The resulting solution for \(a_m\) can be written as
\[
a_m(z) = \exp\left(-\frac{1}{2} \int_0^z dz' (\partial_z \omega_m(z))/\omega_m(z)\right) b_m(z)
\] (64)

where \(b_m(z)\) satisfies
\[
\partial_z b_m = \frac{1}{\omega_m(z)} \sum_{n,n \neq m} b_n \omega_n \frac{\langle m|\partial_z M|n\rangle}{\mu_n^2 - \mu_m^2} \sqrt{\frac{\omega_n(0)\omega_m(z)}{\omega_m(0)\omega_n(0)}} \exp\left(\int_0^z dz' (\omega_n(z) - \omega_m(z))\right).
\] (65)

We pause to assess what the result would be if we simply assumed \(b_m(z)\) to be constant. The pre-factor in equation (64) is integrated via
\[
\exp\left(-\frac{1}{2} \int_0^z dz' (\partial_z \omega_m(z))/\omega_m(z)\right) = \sqrt{\frac{\omega_m(0)}{\omega_m(z)}},
\] (66)

which is the well-known WKB \(1/\sqrt{k(z)}\) factor cited earlier. In effect we have derived this factor and pushed the remaining derivatives into the equation for \(b_m(z)\), equation (65). It is clear that this exponential term is approximately equal to one.

We note that the equation for \(b_m\) is very similar to equation (62) for \(a_n\). This sets up an iterative scheme. Since we have slowly varying \(\omega_n \sim \omega\), we can approximate equation (65) as
\[
\partial_z b_m \approx \sum_{n,n \neq m} b_n \frac{\langle m|\partial_z M|n\rangle}{\mu_n^2 - \mu_m^2} \exp\left(\int_0^z dz' (\omega_n(z) - \omega_m(z))\right),
\] (67)
plus small corrections in a series expansion of \(|\omega - \omega_n|/\omega \ll 1\). For the cosmological applications we consider here, the parameter ranges are such that the correction terms are extremely small.

The value of \(b_m(L) - b_m(0)\) gives an estimate of the amplitude of transition between the two local eigenstates after the wave has propagated a distance \(L\). Let us now estimate this using dimensional arguments. In the adiabatic limit, \(b_m\), as well as the term \(\langle m | (\partial_z M)| n \rangle / (\mu_n^2 - \mu_p^2)\), varies slowly compared to the exponential. Hence we may take these to be approximately constant along the path. We can now integrate equation (67) to obtain the transition amplitude. We find that

\[
\langle b \rangle \approx \frac{2g_\phi B \omega}{\mu_\phi^2 - \mu_p^2} \exp \left( -i \int_0^L \frac{dz'}{2\omega} \left( \mu_\phi^2 - \mu_p^2 \right) \right). \tag{68}
\]

Now if we assume that the exponential is the most rapidly varying factor on the right-hand side of equation (68) as long as we ignore the case of resonant mixing, where the two eigenmodes come very close to one another at some point along the path. Hence we find that in the limit \(L \gg \omega/\Delta \mu^2\) the transition probability is negligible: this is the usual justification for the adiabatic approximation when the internal dynamical timescale is very short compared to the timescale for changes of parameters.

### 5.0.1. Estimating the transition probability for resonant mixing.

We turn to estimating the transition probability in the case of resonant mixing. We assume that the plasma frequency changes linearly along the path according to the relation equation (52). We again take the direction of the background magnetic field fixed so that we consider the two-state mixing problem. In this case we find, for example,

\[
\partial_z b_2 = \frac{\alpha}{2} \frac{2g_\phi B \omega}{(C - \alpha z)^2 + (2g_\phi B \omega)^2} \exp \left( -i \int_0^z \frac{dz'}{2\omega} \left( \frac{\mu_\phi^2}{(C - \alpha z)^2 + (2g_\phi B \omega)^2} - \frac{\mu_p^2}{(C - \alpha z)^2 + (2g_\phi B \omega)^2} \right) \right). \tag{69}
\]

Now if we assume that the exponential is the most rapidly varying factor on the right-hand side of the above equation, then we can replace the coefficient by its average value along the path. We then find

\[
b_2(L) \approx b_2(0) + ib_1(0) \frac{\omega}{L} \left[ \tan^{-1} \frac{m_\phi^2 - \omega_p^2(0)}{2g_\phi B \omega} - \tan^{-1} \frac{m_\phi^2 - \omega_p^2(L)}{2g_\phi B \omega} \right] \times \left( \frac{\exp \left( -i \int_0^L \frac{dz'}{(C - \alpha z)^2 + (2g_\phi B \omega)^2} \frac{\mu_\phi^2}{(C - \alpha z)^2 + (2g_\phi B \omega)^2} - \frac{\mu_p^2}{(C - \alpha z)^2 + (2g_\phi B \omega)^2} \right)}{\sqrt{(m_\phi^2 - \omega_p^2(L))^2 + (2g_\phi B \omega)^2}} - \frac{1}{\sqrt{(m_\phi^2 - \omega_p^2(0))^2 + (2g_\phi B \omega)^2}} \right) \tag{69}
\]

where we have used

\[
\frac{2g_\phi B \omega}{(m_\phi^2 - \omega_p^2(z))^2 + (2g_\phi B \omega)^2} \bigg|_{\alpha z} = \frac{1}{L \alpha} \left[ \tan^{-1} \frac{m_\phi^2 - \omega_p^2(0)}{2g_\phi B \omega} - \tan^{-1} \frac{m_\phi^2 - \omega_p^2(L)}{2g_\phi B \omega} \right].
\]

It is clear that the second term on the right-hand side of equation (69) is small and hence the probability of transition from one state to another is negligible as long as \(L\) is sufficiently large that

\[
L \gg \frac{\omega}{|\mu_\phi^2 - \mu_p^2|}. \tag{70}
\]

However this condition is not general and is applicable only in the limit of large \(\omega\). The basic problem is that close to the region where \(\omega_p = m_\phi\), our original assumption that the coefficient of the exponential in the right-hand side of equation (68) varies slowly compared
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to the exponential term is not correct. We see this by comparing the magnitudes of the logarithmic derivatives of these two factors. We find
\[ D_1 = \frac{d}{dz} \log \left[ (C - \alpha z)^2 + (2g_\phi B \omega)^2 \right] = \frac{-2\alpha(C - \alpha z)}{(C - \alpha z)^2 + (2g_\phi B \omega)^2} \]
\[ D_2 = \frac{d}{dz} \log \exp \left( -i \int_0^z \frac{dz'}{2\omega} \left( \mu_+^2 - \mu_-^2 \right) \right) = \frac{1}{2\omega} \sqrt{(C - \alpha z)^2 + (2g_\phi B \omega)^2}. \]

The assumption \(|D_1| \ll |D_2|\) implies
\[ \frac{|2\alpha(C - \alpha z)|}{(C - \alpha z)^2 + (2g_\phi B \omega)^2} \ll \frac{1}{2\omega} \sqrt{(C - \alpha z)^2 + (2g_\phi B \omega)^2}. \] (71)

In regions where \(|C - \alpha z| \gg 2g_\phi B \omega\) this condition demands
\[ |\alpha| \ll \frac{1}{4\omega}(C - \alpha z)^2. \]

Since \((C - \alpha z)\) is of order \(m_\phi^2\) this condition implies
\[ |\alpha| \ll \frac{1}{4\omega}m_\phi^2. \]

We next consider the limit where \(|C - \alpha z|\) is small or comparable to \(2g_\phi B \omega\). We test the condition, equation (71), at the point \(|C - \alpha z| = \beta(2g_\phi B \omega)\). In this case we find
\[ |\alpha| \ll \frac{(\beta^2 + 1)^{3/2}}{\beta}(g_\phi B)^2 \omega. \]

This condition is most stringent for very small values of \(\beta\). However in this region, as we show below, the transition probability between instantaneous eigenstates continues to be very small provided the condition of equation (70) is obeyed. If we take \(\beta\) of order unity then this condition implies \(|\alpha| \ll (g_\phi B)^2 \omega\), which can be expressed as
\[ L \gg \frac{|\omega_0^2(L) - \omega_0^2(0)|}{(g_\phi B)^2 \omega}, \] (72)

which is rather stringent for small frequencies and magnetic fields. In regions where this condition is violated we can no longer ignore the transition between different instantaneous eigenstates.

We can obtain an estimate of the transition probability in regions where equation (71) is violated. Assume that in these regions the exponential term in equation (68) varies much more slowly compared to the coefficient. We then replace the exponential with its average over the path. We perform the integration over the region where \(C - \alpha z\) ranges from \(\beta(2g_\phi B \omega)\) to \(-\beta(2g_\phi B \omega)\). In this region we find
\[ \Delta b_2 = \tan^{-1} \beta \left[ b_2 \exp \left( -i \int_0^z \frac{dz'}{2\omega} \left( \mu_+^2 - \mu_-^2 \right) \right) \right]_{av}. \] (73)

The exponential term gives a contribution of order unity. It follows that the probability of transition between different eigenstates is non-negligible, unless \(\beta \ll 1\).
Summary. The above results can be summarized by stating that in the case of resonant mixing the adiabatic approximation is valid only if both the equations (70) and (72) are satisfied. If either of these equations is not respected, then the probability of transition between instantaneous eigenstates cannot be ignored.

6. Applications

The basic aim of the present paper is to make detailed predictions for polarization observables due to pseudoscalar–photon mixing, which can be tested in future astrophysical and cosmological observations. Tests may involve CMBR, polarization observations from distant sources and propagation of electromagnetic waves through regions of strong magnetic fields such as the pulsar magnetosphere and active galactic nuclei. The resonance phenomenon may play an important role in many of these situations since the waves propagate through regions with large variations in plasma density. A detailed study of all these phenomena is beyond the scope of the present paper. Here we shall confine ourselves to simple estimates.

We first determine the range of frequencies for which the adiabaticity condition is satisfied in the case of resonance in the supercluster magnetic fields. Using equation (72), with $B = 0.1 \mu G$ [20], $n_e = 10^{-6} \text{ cm}^{-3}$, $L = 10 \text{ Mpc}$ and the current limit on the coupling $g_\phi = 6 \times 10^{-11} \text{ GeV}^{-1}$, we find that adiabaticity is satisfied if $\omega \gg 5 \times 10^{-3} \text{ eV}$. Hence we expect a significant effect at optical frequencies but negligible at radio ones. We should point out that if resonance occurs it gives a considerable enhancement of the mixing phenomenon in comparison to the case of uniform background. For the latter the mixing probability is limited by the oscillation length $l$ in the medium and is proportional to $(gBl)^2$ [15]. For low frequencies this is very small and gives a significant effect only if $\omega > 5 \text{ eV}$. On the other hand for a spatially varying medium, if $\omega_p = m_\phi$ somewhere along the path, the mixing probability is of order unity as long as the adiabaticity constraint equation (72) is satisfied. This constraint is satisfied at much smaller frequencies in comparison to what is required to obtain a significant effect if $\omega_p \neq m_\phi$ all along the path.

For the galactic magnetic fields $B \approx 3 \mu G$ and plasma density $n_e \approx 0.03 \text{ cm}^{-3}$ and distance scale 50 kpc, we find that adiabaticity condition is satisfied only for $\omega \gg 50 \text{ eV}$. Hence here it is satisfied only for ultraviolet frequencies and we can expect significant effects due to pseudoscalar–photon mixing only at such high frequencies.

The effects of resonance are most dramatic if $(gBl)^2 \ll 1$, where $l$ is the oscillation length evaluated at some point far away from resonant region. In this case the resonant effects dominate. For supercluster magnetic fields this is found to be the case for a wide range of frequencies $5 \times 10^{-3} < \omega < 1 \text{ eV}$ (here the lower limit is determined by the adiabaticity condition). In this case the results of equation (51) apply and the wave becomes almost entirely linearly polarized after crossing the resonant region, independently of its state of polarization at origin. If the direction of the magnetic field remains fixed during propagation then its state of polarization is well described by equation (51). In general, however, the magnetic field will also twist along the path. In this case the wave will also acquire circular polarization during propagation. This phenomenon is discussed in a separate paper [29]. If the frequency is large, such that $(gBl)^2 \gg 1$, then the effect of pseudoscalar–photon mixing is large irrespective of whether resonance occurs or not.
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The pseudoscalar–photon mixing has been proposed [27] as a possible explanation of the observed large scale alignment of optical polarizations from distant quasars [30, 31]. The possibility of resonance in supercluster magnetic fields further enhances the parameter space over which this explanation may be applicable. This explanation can be further tested by observing the spectral dependence of all the Stokes parameters. Pseudoscalar–photon mixing also predicts correlations between these parameters, which can be tested in future observations.

The resonance phenomenon may also considerably enhance the production of pseudoscalars due to their mixing with photons as the electromagnetic wave propagates through magnetic fields in astrophysical objects such as the active galactic nuclei, pulsars, magnetars and galactic clusters. If we assume resonant production of pseudoscalars from active galactic nuclei at optical frequencies, then these pseudoscalars can convert back into photons during propagation in the local supercluster. It is reasonable to assume that the direct photon flux, integrated over the entire source, would not be strongly polarized. Hence the observed polarization may be determined dominantly due to the conversion of pseudoscalars into photons in the local magnetic field. This may explain the observed alignment of optical polarizations from large red-shift \( z > 1 \) quasars over the entire sky [31] as well as their correlation with the supercluster equatorial plane [30].

6.1. Cosmic microwave background radiation

We next consider the effect of the supercluster magnetic field on the CMBR due to its mixing with pseudoscalars. Here even a small effect may be observable due to the high precision with which the CMBR has already been measured.

**Intensity effects.** We first consider the intensity on the CMBR due to decay into pseudoscalars. Let \( P(\theta, \phi, E) \) be the probability of decay into pseudoscalars, where \( E \) is the energy of the photon and \( (\theta, \phi) \) are the angular coordinates. Let the original spectrum be denoted by \( f(E; T) \) and the distorted spectrum due to mixing with pseudoscalars by \( f'(E, T') \). These are related by

\[
 f'(E; T') = f(E; T)(1 - P(\theta, \phi, E)).
\]

Assuming that \( P(\theta, \phi, E) \ll 1 \) we find

\[
 T'(\theta, \phi, E) \approx T(\theta, \phi) - \frac{P(\theta, \phi, E)T_0^2}{E}(1 - \exp(-E/T_0))
\]

where \( T_0 \) is the mean CMBR temperature. As expected, mixing with pseudoscalars will produce both a frequency and angular dependence of the temperature. To be consistent with observations we expect that \( P(\theta, \phi, E) \) can have a maximum value of order \( 10^{-5} \).

**Angular distributions.** It is interesting to speculate that the mixing with pseudoscalars in the local supercluster might give rise to preferred orientation of the CMBR quadrupole and octupole [32, 33] due to a possible angular dependence of the mixing probability. This may also explain why higher multipoles are aligned with the CMBR dipole [34]. ‘Cosmic variance’, invoked to explain away magnitude puzzles, cannot credibly explain the multiple coincidence of multipole directions. Hence an explanation in terms of local foreground...
effects is attractive. For consistency, the mixing probability has to be of order $10^{-5}$, which is roughly the strength of these multipole moments. Current limits on the pseudoscalar photon coupling do not rule out this possibility. However if local effects are the source, why should they be of the same order, multipole by multipole, as cosmological CMBR fluctuations? Interaction with a coherent pseudoscalar field might be the explanation with the fewest number of arbitrary assumptions, although some fundamental revision of assumptions in cosmology might be needed to reconcile everything observed. Nevertheless it is interesting to investigate pseudoscalar–photon mixing in supercluster magnetic fields further since it may provide more stringent limits on the coupling $g_\phi$. Pseudoscalar–photon mixing will also give rise to a spectral dependence to the CMBR temperature. Hence the effect has to be sufficiently small that it does not conflict with the observed agreement [35] of CMBR with the black body radiation formula.

**Polarization.** Pseudoscalar–photon mixing can also generate polarization of the CMBR radiation. If the mixing probability is of order $10^{-5}$, which is required to explain the alignment of the quadrupole and octupole, then it will generate polarization of the same order of magnitude. Hence if the polarization is observed to be much smaller than this, it might be possible to impose further limits on the pseudoscalar–photon coupling. The pseudoscalar–photon mixing can also change the CMBR polarization parameters without changing the overall degree of polarization. This effect is in general much larger. However if the degree of polarization of the CMBR is small, as expected, any change in the polarization will be observable by the current detectors only if it is of order unity. Hence here we require a relatively large contribution in order for new effects to be observable. The shift in polarization is of the order of the phase acquired by the wave after propagating a distance $L$ [27]. For a uniform medium this is of order $(g_\phi B)^2 L$, where $l$ is the oscillation length. For the Virgo supercluster parameters and microwave frequencies of order 100 GHz we find that the phase is of order 0.1 for the coupling $g_\phi = 6 \times 10^{-11}$ GeV$^{-1}$. This phase will generate all the Stokes parameters with relative strength of order 0.1, assuming the current limit on the coupling $g_\phi$. Hence this contribution is small but it may be observable with future detectors. Similar results are found if we take into account the fluctuations in the plasma density [27] or propagation through the intergalactic medium [17].

**Condensate.** A pseudoscalar condensate can also affect the CMB polarization [12]. In this case the rotation of polarization is equal to $g_\phi \Delta \phi$, where $\Delta \phi$ is the total change of the pseudoscalar field along the trajectory of the electromagnetic wave. This effect, however, cannot generate circular polarization. This effect is independent of frequency and will also affect radio wave polarizations from distant sources [27,34]. If this pseudoscalar field distribution is anisotropic, it will lead to an anisotropy in both the CMB and radio polarizations [36]. Future observations could provide stringent limits on this effect.

7. Dark energy

**Dark energy** has come to denote a cosmic energy–momentum tensor of the vacuum. The source of dark energy is unknown. Unfortunately, the traditionally minimal option of employing one universal cosmological constant appears less and less credible. It appears
more likely that dark energy is the gravitational trace of a field, somehow chaperoning the evolution of Big Bang pressures, densities and phase transitions predicted by particle physics [37]. The need for a causal inflaton field is rather clearly manifested in the high uniformity of the cosmic microwave background (CMB) radiation.

By now, cosmology as a whole perhaps cannot do without a dark energy-associated field. Yet of all interactions, exploring the Universe with gravity has a problem, in that gravity is the finest example where the coupling to fields is unknown! Unconventional as this remark may seem, in quantum theory there exists no way to find independently that part of the energy–momentum tensor which couples to gravity. The rules of coupling are unknown, and if at all knowable, appear to hinge on ultrahigh energy physics not experimentally testable. The method of using gravity as a probe, when it is not really known what gravity couples to, flies in the face of the long successful tradition of using perturbatively stable interactions to study new situations. It seems ironic that entire fields are built using gravity to explore the Universe’s evolution when gravity is the least understood interaction.

If we assume that dark energy is associated with a scalar field $\phi$, such a field will also have a coupling to electromagnetism given in equation (1). Hence this field might produce all the physical effects that we discuss in this paper. Couplings of dark energy to ordinary matter fields as well as their limits are discussed by Carroll [19]. Yet the mass of such a conventional dark energy field is expected to be of order $10^{-33}$ eV, and thus much smaller than the intergalactic plasma frequency. Assuming such values, we do not expect the conditions for resonance to be applicable. One may, however, consider generalized models of dark energy. Indeed it turns out that by invoking a false vacuum it is possible to explain both dark matter and dark energy in terms of the invisible axion [38, 39]. In this case the mass of the background field can be much larger than currently assumed. Hence we cannot rule out the possibility that conditions for resonance may be applicable in some cases.

8. Summary

If one were to approach light–pseudoscalar field mixing ‘lightly’, then there would be a sequence of facile, dimensionally based arguments that could be used. First, since $g_{\phi}$ has dimensions of inverse mass, and electrodynamics is a theory with no scale, one might claim that all effects are relatively proportional to $g_{\phi}\omega$, and must vanish for $g_{\phi}\omega \ll 1$. This is false: the effects are cumulative, and observable for exceedingly small $g_{\phi} \ll 10^{-12}$ GeV$^{-1}$.

Next, one might observe that with a background field $\vec{B}$, the effects must be of relative size $g_{\phi}B/\omega$, and vanish for large $\omega$. This is again false, as there are several other scales, sometimes $\omega$ occurs in the numerator and we find that for realistic parameters there are observable effects persisting all the way from radio to optical frequencies.

Then we return to our original goal of probing pseudoscalar fields with light. It is certainly very interesting that light can be dimmed by interactions with pseudoscalars. Yet in comparison, the variety and variability of polarization-based observables seems almost unlimited. We believe that as polarization observations accumulate, more and more anomalies will appear. It would be gratifying to have data so that a new and systematic study of pseudoscalars via a coupling that is stable under perturbation theory could commence.
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Acknowledgment

This work was supported in part under Department of Energy grant number DE-FG02-04ER41308.

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