Faraday waves in strongly interacting superfluids

Diego Hernández-Rajkov¹, José Eduardo Padilla-Castillo¹, Alejandra del Río-Lima¹, Andrés Gutiérrez-Valdés¹, Freddy Jackson Poveda-Cuevas² and Jorge Amin Seman¹,*

¹ Instituto de Física, Universidad Nacional Autónoma de México, C.P. 04510 Ciudad de México, Mexico
² Cátedras CONACyT—Instituto de Física, Universidad Nacional Autónoma de México, C.P. 04510 Ciudad de México, Mexico
* Author to whom any correspondence should be addressed.
E-mail: seman@fisica.unam.mx

Keywords: Faraday waves, strongly interacting superfluids, collective excitations

Abstract

We report on the observation of Faraday waves in a cigar-shaped Fermi superfluid of $^6$Li parametrically excited by modulating the radial trap frequency. We characterize the phenomenon as a function of the interaction parameter employing a Feshbach resonance. Starting from the BEC side of the resonance, we observe a reduction of visibility of the Faraday pattern as we approach unitarity, possibly due to the increased incompressibility of the system. We probe the superfluid excitation spectrum by extracting an effective 1D speed of sound for different values of the interaction parameter, in good agreement with numerical simulations. We also discuss the effects introduced by the finite size of the sample. Finally, we introduce a novel model based on Floquet theory that we employ to perform a stability analysis in the parameter space, showing the emergence of the Faraday waves as unstable solutions to a Mathieu-like equation.

1. Introduction

Faraday waves are a non-linear parametric excitation in continuous media that manifest as a spatial and temporal periodic modulation of the density on a non-linear fluid [1]. Their study dates back to the 19th century when Michael Faraday first observed them on vertically shaken fluids [2]. Faraday waves are, in fact, a ubiquitous phenomenon in non-linear fluids and have been studied in a large variety of systems such as viscoelastic fluids [3], granular media [4], and even in living soft systems such as earthworms [5]. Naturally, superfluids are not an exception. Faraday waves were first observed and characterized in a weakly interacting BEC by Engels et al [6], demonstrating their presence in quantum fluids. Since then, they have been studied in these systems from very different perspectives. For instance, Faraday-like patterns were produced in a nearly ideal BEC as a starting point to explore more complex parametric excitations such as granulation [7]. Smits et al [8], in turn, identified that Faraday patterns in a BEC present the same behavior as a discrete space-time crystal [9], providing a new scenario for the research of nonequilibrium phase transitions. Indeed, Faraday-wave-like parametric instabilities represent an exceptional framework for the study of spontaneous temporal and spatial symmetry breaking phenomena [10–12]. Moreover, these excitations have also been explored in superfluid liquid $^4$He, where they are a precursor to the formation of classical (non-quantized) vortices [13].

From the theoretical point of view, Faraday patterns have also been intensively studied. Several works demonstrate that Faraday waves in BECs arise as solutions to the time-dependent Gross–Pitaevskii equation (GPE). Indeed, a Mathieu-like equation describing the phenomena can be deduced from this equation [14–17] and as a result from variational analysis [18]. They have also been investigated in Bose–Bose [19] and Fermi–Bose [20] mixtures in which the intra-species scattering length is modulated.

The theory of Faraday waves in Fermi superfluids has also been explored, particularly at the BEC-BCS crossover [21, 22]. However, these excitations have not been experimentally observed in such systems nor in...
any strongly interacting superfluid gas. Here, we explore these excitations in strongly interacting systems composed by a molecular Fermi superfluid. The advantage of using ultracold Fermi systems over Bose gases is the possibility to tune the interatomic interactions into strongly interacting regimes using a broad Feshbach resonance.

In this work, we parametrically excite Faraday waves in a cigar-shaped superfluid by modulating the radial trap frequency. We start our study on the deep BEC regime and observe that the associated wavevector decreases as the interaction strength is increased. As we approach unitarity, we observe an important decrease in the visibility of the patterns. We attribute this to the increasing incompressibility of the gas as the system becomes more interacting and also to heating effects due to the parametric nature of our excitation. To better understand our observations, we perform numerical simulations based on the mean-field extended Thomas–Fermi model (ETFM) and, independently, develop a new analytic stability analysis using Floquet theory; our data show very good agreement with the results from these theoretical studies. We also discuss the role of the finite size of the sample in our observations. An important result is that it is possible to extract an effective 1D speed of sound as a function of the interaction strength, indicating that Faraday waves can be used to probe the excitation spectrum of the system.

This article is organized in the following way. In section 2, we present the experimental setup and methods we use to create Faraday waves. Next, in section 3, we introduce our experimental results together with numerical simulations. In section 4, we explore the strongest interacting regimes discussing why we were not able to produce the Faraday waves along the full BEC-BCS crossover. Finally, in section 5, we introduce a new analytic model based on Floquet theory that we employ to perform a stability analysis of the phenomenon.

2. Experimental setup and methods

In our experiment, we can produce ultracold Fermi gases formed by a two-component spin mixture of $^6$Li atoms in the two lowest hyperfine states $|F = 1/2, m_F = \pm 1/2\rangle$. The interaction between these species is parameterized by the s-wave scattering length $a_s$, whose value determines the interaction strength and its sign defines if the interaction is effectively attractive ($a_s < 0$) or repulsive ($a_s > 0$). The value of $a_s$ can be precisely controlled using an external magnetic field employing a magnetic Feshbach resonance, allowing the creation of different interaction regimes, from weakly to strongly (repulsive or attractive) interacting gases [23]. In Fermi systems, the interaction regime is commonly described using the dimensionless interaction parameter $1/k_F a_s$, where $k_F$ is the Fermi wave vector which is related to the Fermi energy as $E_F = h^2 k_F^2 / 2m$, here $m$ is the mass of a single atom.

A very important consequence of such control on interactions is the capability of creating different types of atomic bound states. For repulsive interactions, $1/k_F a_s > 0$, it is possible to associate two identical atoms into a molecule which, consequently, will exhibit bosonic statistics and the emergence of Bose–Einstein condensation of tightly bound molecules can occur [24–26]. On the other hand, attractive interactions, $1/k_F a_s < 0$, give rise to the formation of loosely bound Cooper-like pairs due to many-body correlations at the Fermi surface. The behavior of these pairs is described by the BCS theory [27–29]. In this way, using the Feshbach resonance it is possible to continuously transit from the BEC to the BCS regimes through the so-called BEC-BCS crossover [30, 31]. This crossover lies in the intermediate region, delimited by $-1 \leq 1/k_F a_s \leq 1$, and the limit where the scattering length diverges, $1/k_F a_s = 0$, is known as the unitary limit. These regimes are very interesting because, although their underlying pairing nature is very different, all of them exhibit superfluidity. The crossover and unitary regimes are particularly intriguing because the system is strongly interacting and strongly correlated. Physics across the BEC-BCS crossover is known to be related to other relevant phenomena such as high-$T_c$ superconductivity [32] and other strongly correlated superfluids such as neutron stars [33] and quark–gluon plasma [34].

We can access all the different superfluid regimes across the BEC-BCS crossover in our experiment. The setup and methods employed to produce such ultracold samples are described in detail in reference [35].

We produce the superfluid samples in a trap composed by the superposition of a single-beam far red-detuned optical dipole trap (ODT), which tightly confines the atoms along the radial direction, and a magnetic curvature generated by the Feshbach coils, which confines the atoms along the axial direction. In this trap, quantum degeneracy is achieved by runaway evaporation. We produce samples containing $5 \times 10^4$ pairs in a cylindrical symmetric harmonic trap with frequencies $\omega_x = 2\pi \times 163$ Hz and $\omega_z = 2\pi \times 11$ Hz. The cylindrical symmetry implies that $\omega_x = \omega_y \equiv \omega_z$, we guarantee this condition within a 1% by finely tuning $\omega_y$ using time-averaged potentials as described in reference [36].
Once the excitation is applied, we always observe the radial breathing mode independently of the value of $\alpha$. Observation and characterization of Faraday waves (FW) manifest as a periodic pattern along the axial direction of the cloud, as shown in figure 1(a). This pattern has specific values of the wavevector $k_{FW}$, as revealed by its spatial Fourier transform of the absorption image in (a), the dashed lines indicate the position of two visible components of the Faraday wavevector, $k_{FW}^{(1)}$ and $k_{FW}^{(2)}$. The presence of these components indicates the emergence of the density modulation associated to FW. Image taken at a magnetic field of 690 G, corresponding to $1/k_{FW} = 7.1$. Similar images are obtained for other values of $1/k_{FW}$.

In all experiments we reach a temperature of $T/T_F = 0.1$, where $T_F$ is the Fermi temperature, corresponding to a condensed fraction above 90% on the deep BEC regime. The interaction parameter ranges in the interval $0 \leq 1/k_{FW} \leq 11$, however, we only observe the Faraday patterns in the region $2.2 \leq 1/k_{FW} \leq 11$.

In our trap configuration, $\omega_r$ and $\omega_z$ are nearly decoupled: the radial frequency depends on the power $P$ of the ODT as $\omega_r \propto \sqrt{P}$, while the axial one depends on the curvature of the magnetic Feshbach field at the sample location $B_z(0)$ as $\omega_z \propto B_z(0)$. In this way, we can manipulate $\omega_r$ independently of $\omega_z$ by varying the ODT power. To excite the superfluid, we modulate the ODT intensity by means of an acousto-optic modulator [35], allowing us to control the radial frequency profile over time. All excitations studied in this paper are generated by periodically modulating the power of the ODT beam in the form $P(t) = P_0 [1 + \alpha \sin(\Omega t)]$, where $\Omega$ is the frequency of the excitation and $\alpha P_0$ its amplitude. Consequently, the radial trap frequency is $\omega_r(t) = \omega_{ro} \sqrt{1 + \alpha \sin(\Omega t)}$, where $\omega_{ro}$ is the unperturbed radial frequency. In our experiments $\alpha \ll 1$, meaning that the excitation can always be considered as a perturbation, hence, this modulated radial frequency is correctly approximated as $\omega_r(t) \simeq \omega_{ro} (1 + \frac{\alpha}{2} \sin(\Omega t))$. Therefore, $\alpha$ quantifies how much the modulation of the radial trap frequency varies from its unperturbed value.

After producing the ultracold sample, we wait an equilibration time of 50 ms, afterwards we apply the following excitation protocol. We modulate the ODT power during a fixed time of ten cycles, $\tau_e = 10/(\Omega/2\pi)$. Immediately after applying the excitation, we probe the sample in situ using an absorption imaging setup with a resolution of $\sim 2 \mu$m. We follow this protocol since we observe that the Faraday patterns exhibit maximum contrast after these ten cycles. To support this, we performed a continuous measurement of the cloud dynamics as a function of time while keeping the excitation on for all times (reported in figure 2). In all measurements we fix the excitation frequency to be equal to the radial breathing mode frequency to guarantee resonant wave formation [17]. In general, for Fermi superfluids this frequency depends on the interacting regime as well as on the aspect ratio of the trap [37–40]. However, in the interval where we observe the Faraday patterns ($2.2 \leq 1/k_{FW} \leq 11$) the breathing mode frequency is, to a very good approximation, that of an elongated molecular BEC, i.e. twice the static radial frequency, i.e. $\Omega = 2\omega_r^0 = 2\pi \times 326$ Hz, hence $\tau_e \simeq 30.7$ ms.

3. Observation and characterization of Faraday waves

Once the excitation is applied, we always observe the radial breathing mode independently of the value of $\alpha$. Above a certain critical value of $\alpha$, we observe the periodic pattern characteristic of Faraday waves (FW), which arises following a time of the order of 20 ms after the excitation is applied, consistent with previous observations [6–8]. FW manifest as a periodic pattern along the axial direction of the cloud, as shown in figure 1(a). This pattern has specific values of the wavevector $k_{FW}$, as revealed by its spatial Fourier
different values of the interaction parameter $1$ transform shown in figure 1(b). Similar images were obtained for all the explored values of $1$ where we can clearly observe the appearance of the Faraday pattern as separate figure 2(a) we highlight with a red rectangle. Figure 2(b) shows the spatial Fourier transform of the series function of time. In these series, the Faraday patterns can be seen as a periodic structure which in and for every value of the evolution time $t$ select a time in which the Faraday pattern is fully formed in the Fourier space (for instance, at figure 2(d) we also observe that these four peaks are located in the frequency domain at $(indicated by the solid lines), showing that the FW undergo a death-revival sequence with a frequency equal to $\Omega_k$ as the interaction strength increases (see further discussion in section 4).

We observe these dynamics for different values of the excitation amplitude $m$ and the excitation period $1$ for a given value of the interaction parameter $1$ is not turned off. We observe these dynamics for different values of the excitation amplitude $\alpha$ and for different values of the interaction parameter $1/k_\alpha a$. To precisely track the dynamics of the excited cloud, for a given value of the interaction parameter $1/k_\alpha a$, we acquire images with a time step of $\Delta t = 0.5$ ms and for every value of the evolution time $t$, we take the average of five different images under same conditions.

To easily visualize all the data, we integrate the optical density and its Fourier transform along the radial direction, and create the time series shown in figures 2(a) and (b), where we can see the axial profile as a function of time. In these series, the Faraday patterns can be seen as a periodic structure which in figure 2(a) we highlight with a red rectangle. Figure 2(b) shows the spatial Fourier transform of the series where we can clearly observe the appearance of the Faraday pattern as separate $k$-components. The patterns show maximum contrast at $\sim 30$ ms (after ten cycles of excitation) and start to fade away after a time of $\sim 40$ ms due to heating associated to the driving.

The space-time Fourier transform of figure 2(a) is shown in figures 2(c) and (d), providing insightful information about the dynamics of the system. Figure 2(c) shows the space-time Fourier transform of the system before the onset of the Faraday waves ($t < 20$ ms). Two peaks can be observed, each one at $k_z = 0$ and $\omega = \pm \Omega$ which correspond to the breathing/driving mode (indicated by the dashed lines). However, for a later time, the emergence of the Faraday patterns can be seen in figure 2(d) by the appearance of four new peaks at approximately $k_z \approx \pm 0.25 \mu m^{-1}$ this is a clear signal of the onset of the Faraday instability. In figure 2(d) we also observe that these four peaks are located in the frequency domain at $\omega = \pm \Omega/2$ (indicated by the solid lines), showing that the FW undergo a death-revival sequence with a frequency equal to $\Omega/2$, which is characteristic of this phenomenon.

By increasing the value of the excitation amplitude $\alpha$, we observe two things: (i) the Faraday pattern appears at a shorter time and (ii) higher $k$-components appear. We are able to clearly observe a second $k$-component (see figure 2(b)) and, in some cases, even a third one. For this reason, we denote the Faraday wavevector as $k_{FW}^{(n)}$, where the superscript $(n)$ indicates the $n$th $k$-peak. To characterize the effect of $\alpha$, we select a time in which the Faraday pattern is fully formed in the Fourier space (for instance, at $t = 31$ ms in figure 2(b)) and plot that profile for different values of $\alpha$, generating a $k-\alpha$ diagram shown in figure 3(a) where three different $k$-components ($k_{FW}^{(1)}$, $k_{FW}^{(2)}$, and $k_{FW}^{(3)}$) are clearly distinguished. Similar diagrams are obtained for other values of $1/k_\alpha a$, however, the contrast and the number of Faraday components decrease as the interaction strength increases (see further discussion in section 4).

In the diagram of figure 3(a) the peak corresponding to $k_{FW}^{(1)}$ appears broader than the other two peaks, this is an effect associated to the difference between their relative heights. To clarify this point we present figure 3(b) where we plot the intensity of the Faraday peaks (orange curve) for $\alpha = 0.06$. Hence, in

**Figure 2.** Time evolution of the measured (a) integrated optical density and (b) its spatial Fourier transform. In (a) the red rectangle shows the region in which the periodic structure associated to the Faraday waves is clearly observed. In (b) the red horizontal arrows indicate the position of two visible $k$-components, $k_{FW}^{(1)}$ and $k_{FW}^{(2)}$, the presence of these components indicates the emergence of the density modulation associated to FW. Panels (c) and (d) show the space-time Fourier transform of (a) before ($t < 20$ ms) and after the onset of the Faraday pattern ($20 < t < 40$ ms), respectively. The dashed lines indicate the position of the breathing mode frequency, while the solid lines correspond to the Faraday temporal frequency. In these measurements the excitation is never turned off, $1/k_\alpha a = 7.1$ (690 G) and $\alpha = 0.17$. 

transform shown in figure 1(b). Similar images were obtained for all the explored values of $1/k_\alpha a$. The Faraday patterns exhibit the best visibility when the trap is cylindrically symmetric because the excitation process is resonant with both radial frequencies. For this reason, we employ time-averaged potentials [36] to produce a perfectly cylindrical trap.
order to make visible all three \( k \)-components in figure 3(a), we need to include \( k_{FW}^{(i)} \) from the base, where it is broader. For comparison, the blue curve in figure 3(b) corresponds to a superfluid with no excitations.

This diagram is very important since it can be interpreted as a stability diagram showing the conditions under which Faraday waves can be formed. Indeed, this is reminiscent of the stability diagram shown in reference [14] for a 2D BEC.

To support and better interpret our data, we perform numerical calculations to simulate the dynamics of the superfluid trapped in a time-dependent harmonic potential in which the radial frequency is modulated as in our experiment.

We employ a zero-temperature mean-field approximation known as ETFM which is useful to describe Fermi superfluids composed by atomic pairs at any interaction regime. This model is described in detail in references [41–43]. According to this model, the wavefunction of the system

\[
\Psi(\vec{r}, t) = \alpha \Psi_0 + n(\vec{r}, t) \Psi_0 \]

is a mean-field non-linear term that describes the interactions in the system. \( \gamma_d \) is a dimensionless phenomenological parameter that accounts for dissipation. It is used to model the presence of any dissipative mechanism such as the effect of the finite temperature of the sample since the existence of a thermal component surrounding the superfluid is one of the main sources of damping [44, 45]. We provide more details about this parameter in the supplementary materials (https://stacks.iop.org/NJP/23/103038/mmedia) [46].

The ETFM can be employed to describe the dynamics of the system in all superfluid regimes as long as the equation of state \( \mu(n) \) is known. In our experiments we observe Faraday waves for interactions in the interval \( 2.2 \lesssim 1/k_{FW} \alpha_s \lesssim 11 \), which corresponds to a very good approximation to the molecular BEC regime, as explained at section 2. Hence, we use in equation (1) the expression for the chemical potential [47]

\[
\mu(n) = g n = \frac{4\pi \hbar^2 a_M}{M} |\Psi|^2 \]

where \( a_M = 0.6a_s \) is the inter-molecular scattering length [48]. We explain how other superfluid regimes can be described within the ETFM in section 4.

In this way, equation (1) reduces to the well-known GPE,

\[
(i - \gamma_d)\hbar \frac{\partial \Psi}{\partial t} = \left( -\frac{\hbar^2}{2M} \nabla^2 + \frac{M}{2} \left( \omega_r^2(t)r^2 + \omega_z^2 z^2 \right) + \mu |n(\vec{r}, t)| \right) \Psi, \tag{1}
\]

which has been widely used to describe the dynamics of Bose–Einstein condensates [47, 49].

To solve equation (2) we use a spectral split–step method to evolve the equation as a function of time and solve it under similar conditions as in the experiments (see for instance [50]). For the study of Faraday waves, considering dissipation is important to ensure the stability of the solutions, as has been previously shown (see for instance [14, 15, 51]). In our simulations, for \( \gamma_d = 0 \), FW are still formed, but the system
becomes unstable due to the parametric nature of the excitation. Therefore, in our simulations we set $\gamma_d = 0.01$ to stabilize the system, this value also reproduces our observations.

Figures 4(a) and (b) show the evolution of the radially integrated sample, as well as its Fourier transform, respectively. Figures 4(c) and (d) show the space-time Fourier transform before ($t < 25$ ms) and after ($25$ ms $< t < 43$ ms) the onset of the Faraday pattern, respectively. Comparing this results with figure 2 we can see that the agreement is remarkable. It is important to notice that in figure 2(a) there is a dephasing on the Faraday patterns that is not captured by our simulations in figure 4(a). This dephasing arises from the fact that our imaging technique is destructive and, hence, figure 2(a) is constructed by numerous independent measurements at different evolution times. As investigated in reference [10], a phase difference between independent realizations is expected.

It is important to mention that although our simulations also exhibit the non-linear regime reported in references [7, 51], experimentally we observe the destruction of the superfluid state when using longer evolution times or larger excitation amplitudes, as indicated by a significant reduction of the condensed fraction. This result is to be expected due to parametric heating associated with the excitation. Moreover, to explore the non-linear regime observed in [51] it would be better to acquire a momentum resolved image rather than a resolution limited in situ image. We are indeed interested in developing such imaging techniques in the near future. Additionally, we do not observe the granulation state observed in [7] for which a key ingredient is the use of lower frequencies, $\Omega < \omega_r$, and large excitation amplitudes.

From these measurements and simulations, we can determine the wave vector $k_{FW}^{(n)}$ associated to the Faraday pattern as a function of the interaction strength. Inspired by figure 7 from reference [21], in figure 5(a) we plot the product, $k_{FW}^{(n)}a_0/\omega_r/(n\Omega)$, where $a_0 = \sqrt{\hbar/m\omega_r}$ is the harmonic oscillator length along the radial direction. Our measured values of $k_{FW}^{(n)}$ and $k_{FW}^{(2)}$ correspond to the orange circles and green squares, respectively. The Faraday wavevector decreases as the interaction strength increases, meaning that the pattern spacing increases with the scattering length. This is consistent with the $k_{FW}^{(n)}$ extracted from our GPE simulations (purple points, the purple shaded region corresponds to the error associated to the width of the first Faraday peak), as well as with the Floquet analysis (red triangles) detailed in section 5. The solid blue curve in figure 5(a) corresponds to the solution of the superfluid hydrodynamic equations presented by Capuzzi and Vignolo in reference [21], showing good agreement with our experimental data. The superfluid hydrodynamic equations are a model equivalent to the ETFM in which rather than obtaining the wavefunction of the system $\Psi(\vec{r}, t)$, the density profile $n(\vec{r}, t)$ and the velocity field $\vec{v}(\vec{r}, t)$ of the superfluid are calculated.

In order to compare our observations with previous experiments where FW were parametrically excited, namely $^7$Li [7] and $^{87}$Rb [6], we normalized the interaction parameter as $1/a_0^2 n(0)$ where $n(0)$ is the peak density of the BEC at the center of the trap [49] and compare them with our data ($k_{FW}^{(n)}$, orange circles, and $k_{FW}^{(2)}$, green squares) in figure 5(b). We have, in fact, accessed a regime two orders of magnitude more interacting than what has been previously studied. One can also observe that the dimensionless parameter $k_{FW}^{(n)}a_0/(n\Omega)$ decreases logarithmically as the interaction parameter increases.

We can now use the obtained Faraday wavevector to extract the associated phase velocity of the Faraday waves as a function of the interaction parameter. A relevant quantity is the phase velocity of the $n$th
Figure 5. (a) Measurement of the Faraday wavevector. Here we plot the product $k_{\text{FW}}^2 \omega_r / (n\Omega)$ (orange circles and green squares) so it can be compared with the model from [21] (solid blue curve). The red triangles correspond to the Floquet model from section 5. (b) Comparison of our data with other parametrically excited BEC systems composed by $^7\text{Li}$ [7] (brown star), and $^{87}\text{Rb}$ [6] (red triangle), as a function of the interacting parameter $1/\tilde{a}_3 n(0)$. In both panels the purple dots correspond to the results from the GPE simulation while the shaded region is associated to the width of the first Faraday peak. The error bars of our data correspond to the width of the $n$th Faraday peak.

Figure 6. Measured phase velocity of FW as a function of the interaction parameter (orange circles and green squares). Different models (see text) indicate that this velocity corresponds to the effective 1D superfluid speed of sound. The error bars of our data correspond to the width of the $n$th Faraday peak.

excitation given by

$$v_{\text{ph}}^{(n)} = \frac{\omega_{\text{FW}}}{k_{\text{FW}}^2} = \frac{m\Omega/2}{k_{\text{FW}}^2}.$$  

We claim that this phase velocity corresponds to the effective 1D speed of sound of the superfluid. To support this claim, we plot our data in figure 6 (orange circles and green squares) and compare them with the expected speed of sound according to different models (we scale them with the Fermi velocity, $v_F$). The red curve represents the speed of sound of a Bose–Einstein condensate, that is $c_{\text{BEC}} = \sqrt{\frac{g n(0)}{M}}$, where $n(0)$ is the peak density of the molecular gas at the center of the trap [47]. The results from the GPE simulations are shown by the purple points and shaded region. We also compare our data with a simple model based on Floquet theory (gray triangles) which we will discuss with more detail in section 5. We finally compare our data with the results reported by Joseph et al in reference [52] where the authors measure the speed of sound of a Fermi superfluid in the BEC-BCS crossover using a different procedure. In said work, the authors also present a quantum Monte Carlo calculation in excellent agreement with their data. This calculation is shown in the blue solid curve in figure 6, showing that our data is in good agreement with the results reported in [52].

This result is important since it provides a new and simple way to measure the speed of sound of the superfluid, complementing other available methods [52–54].

We see that, in general, the agreement between our data and the different models is very good considering that no fitting parameters were employed. Nonetheless, it must be noticed that in figures 5(a) and 6 there is a jump in both, our data and the GPE simulations, at approximately $1/k_F a_s \approx 5$. We attribute this behavior to the finite size of the confined sample. As the scattering length varies, the size of the cloud
also changes, becoming bigger as the interaction strength increases. In consequence, the number of fringes \( N \) composing the Faraday pattern also changes [21]. However, while the size of cloud varies continuously, \( N \) does it discretely. Hence, a change in \( N \) has as a consequence an abrupt change in \( k_{FW} \). We do observe this change in \( N \) in both, our data and GPE simulations. This is also observed for different values of \( 1/k_\text{a} \) in the much broader domain considered in figure 5(b). We discuss this matter with more detail in the supplementary materials [46].

The specific values of \( 1/k_\text{a} \) where the jumps occur depend on the aspect ratio of the trap, becoming less abrupt for more elongated traps and completely disappearing for an infinitely long system (\( \omega_z = 0 \)). This is the reason why no jumps are present in the blue solid curve in figure 5(a) [21] and in figure 6 [52] where an infinite system was considered. In consequence, the precision of FW to probe the superfluid sound velocity improves for more elongated traps.

It should be also noticed that in figure 6 our data points lay below the expected speed of sound. Besides the finite size effect, as pointed out by Mossman et al [55], highly excited superfluid gases may exhibit an effective lower speed of sound. In this sense, a more detailed analysis of the physics involving this process needs to be carried out.

**4. Unitary regime**

Being a strongly correlated regime, observing FW at unitarity is certainly very interesting. We applied the same protocol that we employed on the molecular condensate to generate FW at or near unitarity. Although we do excite a breathing mode, we could not observe a Faraday pattern in the absorption images. The numerical simulations from references [21, 22] indicate that Faraday waves should be formed throughout the crossover.

We have performed numerical simulations using the ETFM presented in equation (1). At unitarity, the scattering length diverges \( (1/k_\text{a} = 0) \) so the equation of state does not depend on \( a_\text{a} \) anymore and takes a rather simple form, \( \mu(n) = \xi E_F = \xi \hbar^2/(3\pi^2 n)^{2/3} = \xi \hbar^2/(3\pi^2)^{2/3} |\Psi|^{4/3} \), where \( E_F \) is the Fermi energy and \( \xi = 0.370 \) is the Bertsch parameter [56, 57]. This results in the following equation

\[
(i - \gamma_d) \hbar \frac{\partial \Psi}{\partial t} = \left[ -\frac{\hbar^2}{2M} \nabla^2 + \frac{\omega_r^2(t)r^2 + \omega_z^2z^2}{2} + \frac{\hbar^2}{M}(3\pi^2)^{2/3} |\Psi|^{4/3} \right] \Psi. \tag{4}
\]

For other superfluids regimes, the ETFM can be employed using different approaches to obtain the equation of state. A commonly used model is known as the polytropic approximation (see for instance [37–41, 59], the Faraday pattern emerges very clear, as can be seen in figures 7(c), (d) and (f). Indeed, our numerical simulations suggest that FW at unitarity are very sensitive to the value of the excitation frequency. This is not a surprise since the unitary gas is much more incompressible and, therefore, more rigid than a BEC [37], making the system response narrower and FW harder to observe. As pointed out in reference [60], density perturbations have lower amplitude near unitarity due to the increase of the superfluid speed of sound in the stronger interacting regimes (see figure 6).

According to these mean field models [21, 22], it should be possible to excite and observe FW across the BEC-BCS crossover. Nonetheless, it is important to consider the fact that these models assume that the condensed fraction at unitarity is 100%. However, this is not the case in a real unitary Fermi gas, where we know that even at \( T = 0 \) the maximum condensed fraction that can be obtained is 50% due to beyond mean-field effects associated to strong interactions [57, 59, 61]. Hence, one possibility is that Faraday excitations are being produced but they are not visible due to the presence of a very large non-condensed fraction that hides the pattern, just as it happens when quantized vortices are nucleated at strongly interacting regimes [62]. To support this hypothesis we studied the contrast of the Faraday pattern as a function of temperature for different values of the interacting parameter.
Numerical simulations of the extended Thomas–Fermi model at unitarity. Panels (a) and (b) show, respectively, the time evolution of the integrated density and spatial Fourier transform for an excitation frequency of $\Omega = 2\omega_0$. Panels (c) and (d) show the same quantities for an excitation frequency that matches the breathing mode at this regime, $\Omega = \sqrt{10/3} \omega_0$. Panels (e) and (f) show the space-time Fourier transform of (c) before ($t < 40$ ms) and after ($40$ ms $< t < 60$ ms) the onset of the Faraday pattern, respectively. Dashed lines indicate the position of the breathing mode frequency, while the solid lines correspond to the Faraday waves temporal frequency. In these simulations the excitation is never turned off, $1/k_F a_s = 0$, $\gamma_d = 0.01$ and $\alpha = 0.1$.

We estimate this contrast by calculating the integrated Fourier signal associated to the first Faraday peak $k_{FW}^{(1)}$ using the formula

$$ C = \int \left( \tilde{n}_{z}(k_z) - \tilde{n}_{0z}(k_z) \right) dk_z, $$

(5)

where $\tilde{n}_{0z}(k_z)$ and $\tilde{n}_{z}(k_z)$ are the radially integrated spatial Fourier transform of the density profile before and after the Faraday pattern is formed, respectively. In equation (5), the integration is performed in the region around the first Faraday peak. The results are shown in figure 8(a), where we can see how the contrast of the pattern rapidly drops as the condensed fraction decreases. Below the dashed line the Faraday pattern is not visible anymore, however $C$ is not zero due to the residual effect of the change in the cloud’s width in the Fourier transform. Moreover, in figure 8(b) we can see that the maximum observed contrast (at lowest temperature) decreases as the system becomes more interacting. This supports the idea that FW are not observed at unitarity due to the presence of a large non-condensed cloud.
In these numerical simulations we observe that, even exciting at resonance with the breathing mode of the unitary superfluid, the growth rate of the FW excitation is lower than in the BEC, comparing figures 4(b) and 7(d) we see that the patterns appear at a later time at unitarity. This was also noted by Tang et al [22]. This result suggests that the duration of the excitation should be longer. Moreover, being a parametric process, the excitation will tend to heat up the sample [63, 64]. This effect is stronger in a less compressible system, where this kind of excitations are harder to produce and hence the pumped energy will be more easily employed to heat up the gas, which will result in a decrease in the condensed fraction. Indeed, we have measured the temperature of the sample after the excitation is applied, we do observe an important increase of the temperature after 20 cycles of excitation. Therefore, there is a possibility that we are not generating the Faraday patterns at unitarity, and a different excitation scheme is required. Perhaps, a quench in the trapping potential rather than a parametric process, like the one described in [8], could generate the pattern without significantly increasing the temperature of the system. To unambiguously detect the Faraday pattern at unitarity we will require momentum resolving techniques, such as Bragg spectroscopy [65, 66]. We expect that this technique would show a signal located at the corresponding Faraday wavevector \( k_{FW} \), indicating the presence of the excitation. This remains to be investigated in a future study.

5. Floquet analysis and stability diagram

From the discussion in section 3, we see that the Faraday patterns on the BEC side of the resonance are well described by the GPE. In this section, we develop a different analysis which allows us to investigate the phenomenon in terms of stability.

In classical systems, Faraday waves can be explained as unstable solutions of the Mathieu equation [1]. This is also the case in superfluid systems as has been previously shown [14–17]. Here we show the relation between FW and the Mathieu equation by introducing a new analytic Floquet model similar to the one presented in reference [67]. Floquet theory has been successfully used to describe BECs in driven optical lattices [67–70] or employed to design time-dependent potentials to dynamically control the state of different many-body quantum systems (see for instance the review article [71] and references therein).

We start by imposing a condition in the GPE (2) in which the potential and the mean-field interaction energies are balanced, that is,

\[
V(\vec{r}, t) + g |\Psi(\vec{r}, t)|^2 = E_F,
\]

where \( g = 4\pi\hbar^2 a_M / M \) is the interaction parameter and the potential energy is given by

\[
V(\vec{r}, t) = \frac{M}{2} \left( \omega_r^2(t) r^2 + \omega_z^2 z^2 \right),
\]

\[
\omega_r(t) = \frac{\omega_0^2}{\sqrt{1 + \alpha \sin(2\Omega t)}}.
\]

In the balance condition (6), \( E_F \) is a constant that we will refer to as the Floquet energy. This condition can be interpreted as the driving being an adiabatic-like process in which the variation of the potential is compensated by the density change during the whole dynamics of the system. Notice that said condition implies that at any time the density at \( r = 0 \) is kept constant, which is not the case in the experiment, nonetheless, this simplified model gives a qualitative description of the process.

Using this balance condition, we find solutions to be

\[
\Psi_F = e^{iS(\vec{r}, t)} \left( \frac{E_F - V(\vec{r}, t)}{g} \right)^{1/2},
\]

where the phase \( S(\vec{r}, t) \) is chosen such that the equation

\[
\hbar \frac{\partial \Psi_F}{\partial t} = -\frac{\hbar^2}{2M} \nabla^2 \Psi_F + E_F \Psi_F
\]

is satisfied. We assume that this equation has a non trivial solution. It is important to mention that the balance condition imposes a non-dissipative equation in which \( \gamma_d = 0 \). From equations (7) and (8) we can see that \( \Psi_F \) corresponds to the breathing/driving mode of the system.

We investigate the stability of the Floquet state (8) using linear stability analysis [72, 73]. To do so, we introduce a small perturbation onto the Floquet states of the form \( \Psi = \Psi_F \Psi_p \), where \( \Psi_p = 1 + w(t) \cos(kz) \) is the perturbation. Here, \( w(t) \) is a complex function given by \( w(t) = u(t) + iv(t) \).
Now we substitute \( \Psi \) into equation (2) and consider the following approximations. First, we perform a linear stability analysis, meaning that we will neglect the second and higher order powers in \( u(t) \). Next, we consider our trap to be harmonically confining along the radial direction, but to be homogeneous along the axial direction, i.e. \( \partial \Psi / \partial z \approx 0 \). This is equivalent to consider the axial frequency of the trap to be zero, \( \omega_z = 0 \). Since we want to describe the Faraday pattern along the axial direction, only the time-varying part of the potential is important, so we spatially average the radial part, this is

\[
V(t) = \frac{M \omega^2 R^2}{4} (1 + \alpha \sin \Omega t),
\]

(10)

where \( R \) is the radial Thomas–Fermi radius. After this analysis we obtain the following equation for the real part of \( u(t) \):

\[
\ddot{u}(t) + \omega^2 \left( \frac{E_k + 2E_F}{E_k} - 2 \frac{\ddot{V}(t)}{E_k} \right) u(t) = 0,
\]

(11)

where we have defined \( E_k \equiv \hbar^2 k^2 / 2M \) and \( E_k \equiv \hbar \omega_k \). In this procedure we also found a similar equation for the imaginary part \( v(t) \), however, since we are neglecting higher orders of \( u(t) \), the amplitude of the perturbation is given by \( |\Psi(t)|^2 \approx 1 + 2u(t)\cos(kz) \) which does not depend on \( v(t) \) and, consequently, we can ignore the imaginary part of \( u(t) \). In this way, the time-dependent function \( u(t) \) can be interpreted as the actual amplitude of the Faraday excitation.

Notice that from the balance condition (6), we can obtain the Floquet energy \( E_F \) in different ways. By evaluating equation (6) at \( z = 0, r = R \) and, \( t = 0 \) we obtain an expression for the Floquet energy:

\[
E_F = M \omega^2 R^2 / 2. \text{ Simultaneously, by evaluating the balance condition at the center of the trap at } t = 0 \text{ we obtain } E_F = g|\Psi(0)|^2.
\]

We now define the following parameters

\[
\tau = \frac{1}{2} \left( \Omega t - \frac{\pi}{2} \right),
\]

(12)

\[
a = \frac{4}{\hbar^2 \Omega^2} E_k (E_k + E_F),
\]

(13)

\[
q = \frac{4}{\Omega^2} \frac{E_k E_F}{\hbar^2 \alpha}.
\]

(14)

By substituting equations (12)–(14) into equation (11) we finally obtain the following Mathieu equation for \( u(t) \)

\[
\frac{\partial^2 u(\tau)}{\partial \tau^2} + (a - 2q \cos 2\tau) u(\tau) = 0.
\]

(15)

The solutions to the Mathieu equation are well known for all values of \( a \) and \( q \), in fact, they can be categorized in stable and unstable solutions forming well defined regions in the \( a - q \) diagram \[72–74\]. Faraday waves can be identified as unstable solutions of the Mathieu equation, as pointed out in previous works \[14–17\].

We know that in the limit of small \( q \) the instability 'tongues' of the Mathieu equation are located at \( a = n^2 \) with \( n \) an integer. Substituting this result in equation (13), we find an expression for the Faraday wavevector \( k_{FW}^{(n)} \),

\[
k_{FW}^{(n)} = \pm \sqrt{\frac{ME_F}{\hbar^2}} \sqrt{-1 + \sqrt{1 + \left( \frac{nh\Omega}{E_F} \right)^2}}.
\]

(16)

It is important to mention that although other approaches do arrive to the Mathieu equation (15) \[14–17\] our approximation allows us to obtain an analytic expression for the Faraday wavevector (16) under the assumption of the balance condition of equation (6).

We now use expression (16) and compare it with our experimental data and the GPE calculations shown in figure 5. The agreement is very good, indicating that our simple analytic model based on Floquet states provides a good qualitative description of this phenomenon. Nevertheless, it should be noted that this model slightly underestimates the value of \( k_{FW}^{(n)} \) since it is consistently below the predictions from the hydrodynamic and GPE models, as can be seen in figure 5. This is due to the fact that the balance condition from equation (6) considers that the density of the cloud at the center is unperturbed by the excitation.
Our model also provides an insightful interpretation of the parameters of Mathieu equation. We should notice that in the expression for $a$, the term $E_k(E_k + E_F)$ corresponds to the Bogoliubov phononic spectrum [47], so equation (13) can be rewritten as

$$a = \frac{4\hbar^2 \Omega^2}{\hbar^2} E_k(E_k + E_F) = \left( \frac{2\hbar \omega_{\text{Bog}}}{\hbar \Omega} \right)^2,$$

where $\omega_{\text{Bog}}$ is the frequency of a Bogoliubov phonon. Faraday waves are formed whenever $a = n^2$, that is, when the condition $2\hbar \omega_{\text{Bog}} = n\hbar \Omega$ is met. The physical interpretation is that FW are formed whenever the energy associated to two Bogoliubov phonons is an integer multiple of the excitation energy $\hbar \Omega$. This is in agreement with the interpretation presented in [21, 75], where Faraday waves are seen as two counter propagating phonons.

In fact, said interpretation reinforces the claim presented in the previous section in which the phase velocity of the Faraday waves corresponds to the superfluid speed of sound. Using equations (3) and (16), we obtain an expression for the phase velocity of the $n$th Faraday excitation as

$$v_{\text{ph}}(n) = \sqrt{\frac{2}{M} \sqrt{\frac{(\hbar k_F^n)^2}{2M} + E_F}}.$$

This expression fits our data and is in agreement with other theoretical models, as shown by the gray triangles in figure 6, indicating that our simple model offers a good description of the system. Notice that in this case the finite size effects explained at the end of section 3 are not present because we are considering an infinite elongated system through the use of the potential of equation (10) where $\omega_z = 0$ is assumed.

Finally, we further analyze the stability properties of the system. Using the expressions for $a$ and $q$ we can map figure 3 into an equivalent stability-like diagram and obtain figure 9, where we indicate the position of the instability tongues of the Mathieu equation with vertical lines ($a = n^2$ with $n = 1, 2$ and 3). We see that the regions where FW are observed (darker regions) correspond to the unstable solutions of the Mathieu equation.

This analysis introduces a novel and simple way to visualize the competition between the driving and the breathing frequencies in the formation of FW, and provides a universal tool to determine the conditions under which Faraday waves can be formed regardless of the interaction strength.

6. Conclusions

In this work we investigated the phenomenon of Faraday waves in Fermi superfluids composed by atomic pairs of $^6\text{Li}$ with tunable interactions. We explore the interaction regimes within the interval $0 \leq 1/k_F a_s \leq 11$. As the interaction strength increases, we observe an important decrease in the visibility of the patterns, in such a way that the Faraday patterns are only observable in the interval $2.3 \leq 1/k_F a_s \leq 11$, which corresponds to the molecular BEC regime. We attribute this to the fact that the incompressibility of the system considerably increases in the more interacting regimes and, also, to heating processes associated to the parametric nature of the employed excitation.
To better understand our results, we perform numerical simulations employing the ETFM, obtaining good agreement between theory and experiment. From our experimental data and numerical analysis we are able to characterize the Faraday patterns by measuring the associated Faraday wavevector. A particularly important result is that the Faraday excitation can be employed to measure the superfluid speed of sound. However, the precision of this measurement is limited by finite size effects associated to the axial confinement of the cloud, improving for more elongated systems. In this way, our work provides insights on how the Faraday waves can be employed to probe the excitation spectrum of these systems.

Finally, we develop a simple analytic model based on Floquet theory that allows to study the Faraday waves in terms of a stability analysis, enabling the exploration of the parameter space and indicating under which conditions the Faraday instability can be observed independently of the interaction regime. This also provides new ideas on the use of Floquet theory in the description of strongly interacting systems.

In the near future we plan to explore alternative excitation schemes to produce Faraday patterns in the stronger interacting regimes in the BEC-BCS crossover. We also plan to implement momentum-resolved imaging techniques to address additional features in the study of these collective excitations. Finally, we would like to explore these excitations in different trap geometries.

Acknowledgments

We would like to thank Giacomo Roati, Rocío Jáuregui Renaud, Patrizia Vignolo, Pablo Capuzzi, Víctor Romero-Rochín, Rosario Paredes Gutiérrez, and Santiago Caballero Benítez for fruitful discussions. In particular, we thank Víctor Romero for providing useful ideas in the development of the theory presented in section 5. We also thank the anonymous referees for their useful suggestions. We acknowledge the technical support from Carlos Alberto Gardea Flores, Rodrigo Alejandro Gutiérrez Arenas and Maira Pérez Vielma from the electronics workshop at IFUNAM for their contribution in the development of instrumentation, as well as Roberto Gleason Villagrán for his support in the building of our research infrastructure. We acknowledge the following grants: Instituto de Física UNAM (PIIF-8, PIIF-9 and LANMAC-2021); DGAPA-UNAM (PAPIIT projects IA101716, IN103818, IN109021, and IN109619); CONACyT (Ciencia Básica 255573, 254942, and A1-S-39242; Laboratorio Nacional 299057, 314850 and 315838), and Coordinación de la Investigación Científica UNAM (Grant Number LANMAC-2019, and support during 2021). DHR, JEPC, ARL and AGV acknowledge their scholarships from CONACyT (programs 000306 and 000328, and Ciencia Básica 254942), and DGAPA-UNAM (PAPIIT projects IA101716, and IN103818). Finally, we want to thank the company Seman Baker S.A. de C.V. for their generous donation of numerous machined pieces.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

ORCID iDs

Freddy Jackson Poveda-Cuevas https://orcid.org/0000-0001-9808-3593
Jorge Amin Seman https://orcid.org/0000-0003-1473-8689

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