Gravitino Dark Matter and Light Gluino in an R-invariant Low Scale Gauge Mediation

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Abstract

We consider the simplest class of the R-invariant gauge mediation model with the gravitino mass in the one to ten keV range. We show that the entropy production from the supersymmetry breaking sector makes the gravitino into a warm dark matter candidate. We also discuss that the gluino mass can be lighter than the wino mass even when the messenger sector satisfies the GUT relations at the GUT scale.
1 Introduction

Dark matter is an important clue to the theory beyond the standard model (SM). There have been proposed a lot of models to explain dark matter. The light gravitino is a very interesting candidate for dark matter among them, since the gravitino itself is a unique and inevitable prediction of supergravity (SUGRA). If the gravitinos were in the thermal equilibrium in the early universe, the gravitino mass $m_{3/2}$ is required to be $m_{3/2} \simeq 100\,\text{eV}$ from the observed dark matter density, $\Omega_{\text{DM}} \simeq 0.1$. This prediction is very much interesting, since we can test the gravitino dark matter hypothesis at LHC. The gravitino mass, $m_{3/2} \simeq 100\,\text{eV}$, is, however, too small to be the cold dark matter and is disfavored for the successful galaxy formation [1].

The above argument is based on an unjustified assumption on the thermal history of the early universe. In fact, if we had late time entropy production after the decoupling time of the gravitino, the mass of the gravitino dark matter may be raised up to a few keV. Moreover, the gravitino dark matter with a mass in the one to ten keV range serves as the warm dark matter which has recently been invoked as possible solutions to the seeming discrepancies between the observation and the simulated results of the galaxy formation based on the cold dark matter scenario [2].

In this paper, we discuss the late-time entropy production from the SUSY breaking sector. As we will show, large entropy can be produced from the SUSY breaking sector when the sector has meta-stable particles whose lifetimes are long enough to dominate the energy density of the universe before they decay. As a result, the gravitino with a mass in the one to ten keV range can be a good candidate for the warm dark matter with the help of the entropy production.

The gravitino mass in a range of the one to ten keV also has an interesting implications on the phenomenological aspects of the supersymmetric standard model (SSM). For the gravitino mass in the keV range, we are led to consider the models with gauge mediation [3, 4] where the SUSY breaking effects are mediated to the SSM sector via the gauge interactions.

\footnote{The detailed analyses in Refs. [3, 4] have placed lower bounds on the warm dark matter mass around a few keV range. Thus, it is safer to assume that the dark matter is in the ten keV range.}
In this paper, as a particular example of the models with gauge mediation, we consider a class of the direct mediation models developed in Ref. [7], where the SUSY breaking vacuum is stable. The important feature of this class of models is that the models possess an R-symmetry. It should be noted that, independent of the SUSY-breaking mediation scheme, the (discrete) R-symmetry is considered to be a crucial symmetry for any low-energy SUSY extension of the Standard Model. This can be seen from the fact that SUSY should be broken at very high energy scale to obtain the nearly vanishing cosmological constant if the R-symmetry is largely broken by the constant term of the superpotential in supergravity. Therefore, it is quite tempting to consider a mediation mechanism which possesses an R-symmetry.

The notable feature of this class of gauge mediation models is a peculiar spectrum of the superparticles. Especially, the gaugino masses do not satisfy the so-called the Grand Unified Theory (GUT) relations even if the masses and the couplings of the messenger fields satisfying the GUT relations at the GUT scale [8]. For example, the light gluino of mass 300 GeV – 1 TeV is achieved with the heavier wino of mass 500 GeV – 2 TeV even for the boundary condition satisfying the GUT relations at the GUT scale. Such a light gluino will be easily produced at the LHC, and hence, almost all the parameter space is expected to be probed by the LHC experiment.

The organization of the paper is as follows. In section 2, we discuss the mass spectrum of the SSM particles in the simplest class of the R-invariant gauge mediation model for the gravitino with a mass in the one to ten keV range. There, we show that the typical gaugino mass spectrum is distinctive from the so-called minimal gauge mediation model. In section 3, we discuss an entropy production mechanism which makes the gravitino dark matter scenario with a mass in the one to ten keV range consistent with the observed dark matter density. The final section is devoted to our conclusions.

2 An R-invariant gauge mediation model

Let us discuss the minimal R-invariant gauge mediation model developed in Ref. [7]. We introduce two pairs of massive messengers, $\Psi_i, \tilde{\Psi}_i$ with $i = 1, 2$. Here, $\Psi_i$ and $\tilde{\Psi}_i$ transform as 5 and $5^*$ in terms of the minimal $SU(5)$ GUT representations, respectively. We further
introduce a SUSY-breaking gauge singlet field $S$ which has non-vanishing expectation values of the $F$ and $A$ terms,

$$\langle S(x, \theta) \rangle = \langle S \rangle + F \theta^2.$$  \hspace{1cm} (1)

(We abuse the notation for chiral fields and its lowest components.) We assume, throughout this paper, that the $F$ term is the dominant component of the SUSY breaking and hence the gravitino mass is given by

$$m_{3/2} \simeq \frac{|F|}{\sqrt{3} M_P}.$$  \hspace{1cm} (2)

Here, $M_P \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass.

Let us assume that the superpotential in the messenger sector is given by,

$$W = \begin{pmatrix} \bar{\Psi}_1 \Psi_2 \end{pmatrix} \begin{pmatrix} kS & m \\ m & 0 \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix},$$  \hspace{1cm} (3)

where $k$ and $m$ denote the coupling constant and the mass parameter, respectively. We see that the above superpotential is invariant under an R-symmetry with the charge assignment, $S(2)$, $\Psi_1(0)$, $\bar{\Psi}_1(0)$, $\Psi_2(2)$, $\bar{\Psi}_2(2)$. Notice that the vacuum expectation value of the scalar component of $S$ breaks the R-symmetry spontaneously.

So far, we have treated messenger fields in an $SU(5)_{GUT}$ symmetric way. Below the GUT scale, however, the $SU(5)_{GUT}$ messenger multiplets split into $\Psi_i \rightarrow (\Psi_i^{(d)}, \bar{\Psi}_i^{(d)})$ and $\bar{\Psi}_i \rightarrow (\tilde{\Psi}_i^{(d)}, \tilde{\Psi}_i^{(\ell)})$ which transform as $(3_{-1/3}, 2_{1/2})$ and $(\bar{3}_{1/3}, 2_{-1/2})$ under the SM gauge groups, $SU(3)_C \times SU(2)_L \times U(1)_Y$, respectively. In the followings, we name the messengers with the superscripts $d$ and $\ell$ “down-type” and “lepton-type”, respectively. In accordance with the above splitting, the coupling constants and the mass parameters in Eq. (3) may take different values for each type of messengers, i.e. $k^{(d,\ell)}$ and $m^{(d,\ell)}$. Especially, the renormalization group (RG) evolution makes them different at the lower energy scale, even if we impose $k^{(d)} = k^{(\ell)}$ and $m^{(d)} = m^{(\ell)}$ at the GUT scale.

In this model, we can take $k^{(x)}$, $m^{(x)}$, $\langle S \rangle$ and $F$ as real positive by the phase rotation of the fields without loss of generality. Therefore, we can avoid $CP$ violation in the present model. In the following of this paper, we take these parameters as real positive.

Let us discuss the condition for the messenger scalar not to be tachyonic. The mass parameters are required to satisfy $k^{(d)} F / m^{(d)2} < 1$ and $k^{(\ell)} F / m^{(\ell)2} < 1$ for the messenger
fields not to be tachyonic. By the RG effect, the condition for the lepton-type messenger

gives severer constraints than the one for the down-type messenger. This fact can be seen
as follows. The ratio of $k^{(d)} F / m^{(d)2}$ and $k^{(l)} F / m^{(l)2}$ evolves according the RG equations,

$$
\frac{d}{d \log \mu} \log \left( \frac{k^{(d)} F}{m^{(d)2}} / \frac{k^{(l)} F}{m^{(l)2}} \right) = -2 \left( \gamma_{\Psi_2^{(d)}} - \gamma_{\Psi_2^{(l)}} \right)
$$

$$
\simeq \frac{16 \alpha_3}{3} - \frac{3 \alpha_2}{4\pi} - \frac{1 \alpha_1}{3 \frac{4\pi}{4}},
\tag{4}
$$

where we have used the RG equations of $k$ and $m$ in terms of the anomalous dimensions

$$
\frac{\partial}{\partial \log \mu} k^{(\chi)} = (2 \gamma_{\Psi_1^{(\chi)}} + \gamma_S) k^{(\chi)},
\tag{5}
$$

$$
\frac{\partial}{\partial \log \mu} m^{(\chi)} = (\gamma_{\Psi_1^{(\chi)}} + \gamma_{\Psi_2^{(\chi)}}) m^{(\chi)}, \quad (\chi = d, \ell).
\tag{6}
$$

If we require $k^{(d)} = k^{(l)}$ and $m^{(d)} = m^{(l)}$ at the GUT scale, we can get

$$
\frac{k^{(d)} F}{m^{(d)2}} / \frac{k^{(l)} F}{m^{(l)2}} \simeq \exp \left[ -\int_{\text{med}}^{M_{\text{GUT}}} d \log \mu \left( \frac{16 \alpha_3}{3} - \frac{3 \alpha_2}{4 \pi} - \frac{1 \alpha_1}{3 \frac{4\pi}{4}} \right) \right]
$$

at the mediation scale. Due to the strong $SU(3)_C$ effect, $k^{(d)} F / m^{(d)2}$ becomes smaller than $k^{(l)} F / m^{(l)2}$ at the mediation scale. Therefore, the condition $k^{(l)} F / m^{(l)2} < 1$ guarantees

that all the messenger scalars have positive squared masses. The fact $k^{(d)} F / m^{(d)2} < k^{(l)} F / m^{(l)2}$ has an interesting consequence for the gaugino masses as will see later.

**Mass spectrum of the present model**

The gaugino masses are given by

$$
M_1 = \frac{\alpha_1}{2 \pi} \left( \frac{2}{5} \Lambda_1^{(d)} + \frac{3}{5} \Lambda_1^{(l)} \right),
\tag{8}
$$

$$
M_2 = \frac{\alpha_2}{2 \pi} \Lambda_1^{(l)},
\tag{9}
$$

$$
M_3 = \frac{\alpha_3}{2 \pi} \Lambda_1^{(d)},
\tag{10}
$$

and the squared masses of sfermion $\tilde{f}$ are given by

$$
m_\tilde{f}^2 = 2 \left( \frac{\alpha_1}{4 \pi} \right)^2 C_1 \left( \frac{2}{5} \Lambda_0^{(d)2} + \frac{3}{5} \Lambda_0^{(l)2} \right) + 2 \left( \frac{\alpha_2}{4 \pi} \right)^2 C_2 \Lambda_0^{(l)2} + 2 \left( \frac{\alpha_3}{4 \pi} \right)^2 C_3 \Lambda_0^{(d)2},
\tag{11}
$$
where $\alpha_a$ ($a = 1, 2, 3$) are gauge coupling fine structure constants of $U(1)_Y$, $SU(2)_L$, $SU(3)_C$, and $C_a$ ($a = 1, 2, 3$) are quadratic casimir invariants\(^2\) of the sfermion $\tilde{f}$ under the group $U(1)_Y$, $SU(2)_L$, $SU(3)_C$. Here, $\Lambda_{1/2}^{(\chi)}$ and $\Lambda_0^{(\chi)^2}$ ($\chi = d, \ell$) are functions of $m^{(\chi)}, k^{(\chi)} \langle S \rangle$ and $k^{(\chi)} F$ whose explicit forms can be read from Ref. [8, 9].

**Gaugino-sfermion mass ratio**

In the case of the so-called minimal gauge mediation (mGM) [6], with a messenger superpotential of the form $(m^{(\chi)} + k^{(\chi)} S) \tilde{\Psi} \Psi$, both the $\Lambda_{1/2}^{(\chi)}$ and $\Lambda_0^{(\chi)}$ are of order

$$\Lambda_{1/2}^{(\chi)} \big|_{\text{mGM}} \sim \Lambda_0^{(\chi)} \big|_{\text{mGM}} \sim \frac{k^{(\chi)} F}{m^{(\chi)}},$$

where we have neglected $\langle S \rangle$ (inclusion of it is straightforward). However, there is a significant difference in the gaugino masses in the present model. The rough behavior of the soft masses are as follows. For $k^{(\chi)} \langle S \rangle \lesssim m^{(\chi)}$, the soft masses $\Lambda_{1/2}^{(\chi)}$ and $\Lambda_0^{(\chi)^2}$ are approximately given by

$$\Lambda_{1/2}^{(\chi)} \sim \mathcal{O}(0.1) \times \frac{(k^{(\chi)} \langle S \rangle)(k^{(\chi)} F)^2}{m^{(\chi)}^6} \left(1 + \mathcal{O}\left(\frac{|k^{(\chi)} F|^2}{m^{(\chi)^2}}\right)\right),$$

$$\Lambda_0^{(\chi)^2} \sim \mathcal{O}(1) \times \left(\frac{k^{(\chi)} F}{m^{(\chi)}}\right)^2 \left(1 + \mathcal{O}\left(\frac{|k^{(\chi)} F|^2}{m^{(\chi)^2}}\right)\right),$$

and they decrease when $k^{(\chi)} S$ becomes much larger than $m^{(\chi)}$. Notice that there are no terms of order $k^{(\chi)} F/m^{(\chi)}$ in the gaugino masses, which would be present in the minimal gauge mediation. Because the mass parameters are required to satisfy $k^{(\chi)} F < m^{(\chi)^2}$ for the messenger fields not to be tachyonic, the above approximated expressions show that the gaugino masses are suppressed compared with the sfermion masses.

**Wino-gluino mass ratio**

In the case of the minimal gauge mediation, the ratio of gaugino mass contributions from the down-type and the lepton-type messengers, $\Lambda_{1/2}^{(d)} \big|_{\text{mGM}}$ and $\Lambda_{1/2}^{(\ell)} \big|_{\text{mGM}}$, is given by

$$\frac{\Lambda_{1/2}^{(d)}}{\Lambda_{1/2}^{(\ell)}} \bigg|_{\text{mGM}} \approx \left(\frac{k^{(d)}}{k^{(\ell)}}\right) \cdot \left(\frac{m^{(d)}}{m^{(\ell)}}\right)^{-1}$$

\(^2\)We use the GUT normalization for the $U(1)_Y$ gauge group. In particular, the quadratic casimir is given by $C_1 = \frac{2}{3} Y^2$ in terms of the hypercharge $Y$.  

6
where we have neglected the higher order terms in $k^{(x)} F/m^{(x)^2}$. One can easily check that this ratio is invariant under the RG flow. Thus, if we impose the GUT relations, $k^{(d)} = k^{(\ell)}$ and $m^{(d)} = m^{(\ell)}$, at the GUT scale, we have $\Lambda_{1/2}^{(d)} \simeq \Lambda_{1/2}^{(\ell)}$ at the messenger scale. This indicates that the gaugino masses obey the famous GUT relation $M_{\text{bino}} : M_{\text{wino}} : M_{\text{gluino}} \simeq \alpha_1 : \alpha_2 : \alpha_3 \simeq 1 : 2 : 6$.

In the present model, on the other hand, the ratio $\Lambda_{1/2}^{(d)}/\Lambda_{1/2}^{(\ell)}$ is given by

$$
\frac{\Lambda_{1/2}^{(d)}}{\Lambda_{1/2}^{(\ell)}} \simeq \left( \frac{k^{(d)}}{k^{(\ell)}} \right)^4 \cdot \left( \frac{m^{(d)}}{m^{(\ell)}} \right)^{-6}.
$$

By using similar argument to derive Eq. (17), the ratio Eq. (16) at the scale of the gauge mediation, $M_{\text{med}} = \mathcal{O}(m^{(d,\ell)})$, is roughly given by

$$
\frac{\Lambda_{1/2}^{(d)}}{\Lambda_{1/2}^{(\ell)}} \simeq \exp \left[ - \int_{M_{\text{med}}}^{M_{\text{GUT}}} d \log \mu \left( 2(\gamma_{\psi_1}^{(d)} - \gamma_{\psi_1}^{(\ell)}) - 6(\gamma_{\psi_2}^{(d)} - \gamma_{\psi_2}^{(\ell)}) \right) \right] 
\simeq \exp \left[ - \int_{M_{\text{med}}}^{M_{\text{GUT}}} d \log \mu \left( \frac{32 \alpha_3}{3\pi} - \frac{6 \alpha_2}{4\pi} - \frac{2 \alpha_1}{3\pi} + \frac{1}{2\pi} \frac{k^{(d)^2} - k^{(\ell)^2}}{4\pi} \right) \right].
$$

if we impose the GUT relations at the GUT scale, $M_{\text{GUT}}$, i.e. $m^{(d)} = m^{(\ell)}$ and $k^{(d)} = k^{(\ell)}$. The strong $SU(3)_C$ interaction makes $\Lambda_{1/2}^{(d)}$ smaller than $\Lambda_{1/2}^{(\ell)}$ at the mediation scale. Therefore, from Eqs. (8) - (10), we see that the gluino mass is rather suppressed in the present model compared with the one in the minimal gauge mediation. Furthermore, when $k^{(\ell)} F \simeq m^{(\ell)^2}$, the ratio $\Lambda_{1/2}^{(d)}/\Lambda_{1/2}^{(\ell)}$ is more suppressed than Eq. (17). This is because the higher order contributions of $k^{(\ell)} F/m^{(\ell)^2}$ in Eq. (13) make $\Lambda_{1/2}^{(\ell)}$ enhanced, while $\Lambda_{1/2}^{(d)}$ is not so enhanced since the RG equation indicates $k^{(d)} F/m^{(d)^2} < k^{(\ell)} F/m^{(\ell)^2}$ as we have discussed above.

### Numerical results

We show numerical results of the soft SUSY breaking masses of gauginos and sfermions in the minimal R-invariant gauge mediation model. We impose $m^{(d)} = m^{(\ell)}$ and $k^{(d)} = k^{(\ell)}$ at the GUT scale. The larger soft masses are obtained for the larger coupling constants,
Figure 1: The soft SUSY breaking masses of gauginos and sfermions in the minimal R-invariant model for $m_{3/2} = 1$ keV (left) and $m_{3/2} = 10$ keV (right). We set $k^{(ℓ)} F/m^{(ℓ)2} = 0.9$.

Figure 2: The soft SUSY breaking masses of gaugino and sfermion in the minimal R-invariant model for $m_{3/2} = 1$ keV (left) and $m_{3/2} = 10$ keV (right). We take $⟨S⟩$ as a value which maximizes the gluino mass.
$k^{(d)}$ and $k^{(ℓ)}$ at the mediation scale. The coupling constants are, however, not able to be arbitrary large, because the Yukawa-type interactions are not asymptotically free. Too large coupling constants result in the Landau-pole problem below the GUT scale. In order to avoid the Landau-pole problem, we put $k^{(d)} = k^{(ℓ)} = 4\pi$ at the GUT scale as the upper bound on the coupling constants. With this boundary condition, we obtain $k^{(d)} = 0.99$, $k^{(ℓ)} = 0.74$ and $m^{(d)}/m^{(ℓ)} = 0.71$ at the mediation scale in the minimal R-invariant gauge mediation model for $m_{3/2} = O(1) - O(10)$ keV. The explicit form of the RG equations are given in Ref. [9]. In this analysis, we have neglected the contribution of the SUSY breaking sector to the RG equations, which may make the values of $k^{(x)}$ a little smaller.

In Fig. 1 we show the SSM mass spectrum as a function of the parameter $⟨S⟩/m^{(ℓ)}$ for $m_{3/2} = 1$ keV and $m_{3/2} = 10$ keV. We fixed $k^{(ℓ)}F/m^{(ℓ)2} = 0.9$ in both figures. The figures show that the sfermion masses are much heavier than the gauginos. The sfermion-gaugino mass ratio is larger for the heavier gravitino mass if we fix the order of the gaugino masses. The figure also shows that the gaugino masses are maximized when $k^{(ℓ)}⟨S⟩ ∼ m^{(ℓ)}$. In Fig. 2 we also show the mass spectrum of gaugino and sfermion as a function of the parameter $k^{(ℓ)}F/m^{(ℓ)2}$. In the figure, we took $⟨S⟩$ as a value which maximizes the gluino mass.

The interesting observation here is that the gluino mass is predicted to be rather light and can be lighter than the wino even though we have assumed the GUT boundary conditions. Thus, the gaugino mass spectrum is distinguishable from the one in the minimal gauge mediation, $M_{bino} : M_{wino} : M_{gluino} ≃ 1 : 2 : 6$.

In Fig. 3 and 4 we also show the SSM mass spectrum for the model with two additional pairs of massive messengers $Ψ_i, ˜Ψ_i (i = 1, 2)$ which have the masses and the couplings to $S$ similar to those of $Ψ_i$ and $˜Ψ_i$ in Eq. (3). In the next section, we consider the SUSY breaking model where the R-symmetry is spontaneously broken in a perturbative way with the help of the $U(1)$ gauge interaction. In that model, the messenger sector is required

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4 The perturbative analysis is no more viable for $k = O(4\pi)$. Our result, however, does not strongly depend on the values of the coupling constants at the GUT scale as long as they are large.

5 As we see shortly, the mediation scale is required to be close to $\sqrt{F}$ to obtain heavy enough gaugino masses, which is determined for a given gravitino mass.

6 In Refs. [14, 15], the SSM spectrum in the R-invariant gauge mediation with the very light gravitino with a mass below $16$ eV has been considered. There, the sfermion-gaugino mass ratio is much smaller.
Figure 3: The soft SUSY breaking masses of gauginos and sfermions in the R-invariant model with double messenger for $m_{3/2} = 1\text{ keV}$ (left) and $m_{3/2} = 10\text{ keV}$ (right). We set $k^{(t)} F/m^{(t)} = 0.9$.

Figure 4: The soft SUSY breaking masses of gaugino and sfermion in the R-invariant model with double messenger for $m_{3/2} = 1\text{ keV}$ (left) and $m_{3/2} = 10\text{ keV}$ (right). We take $\langle S \rangle$ as a value which maximizes the gluino mass.
to be doubled (see also Eq. (7)). The sfermion-gaugino mass ratio is smaller than the minimal model, because the gaugino mass is proportional to the messenger flavor number but the sfermion mass is proportional to square root of the messenger flavor number.

The figure shows that the gluino mass is more close to the bino mass in the doubled messenger model. For such a peculiar spectrum, the collider phenomenology can be significantly different from the usual models with the gaugino masses which satisfies GUT relations, and hence, may require different search strategies than the usual SUSY scenarios (see for example Ref. [12]). The detailed collider study for the above gaugino spectrum will be given elsewhere [13].

3 Entropy production from SUSY breaking sector

In the previous section, we have discussed the mass spectrum in the minimal R-invariant gauge mediation model, and the peculiar mass spectrum are predicted for the gravitino mass in the one to ten keV range. As we have mentioned in the introduction, however, the relic density of the gravitino with a mass in this range is too high to be consistent with the observed dark matter density if the gravitinos were in the thermal equilibrium in the early universe. The thermally produced gravitino density is roughly given by,

$$\Omega_{3/2}h^2 \simeq 0.1 \times \left( \frac{100}{g^*(T_D)} \right) \left( \frac{m_{3/2}}{100 \text{ eV}} \right),$$

where $g^*(T_D) \simeq 100$ denotes the effective massless degree of freedom in the thermal bath at the decoupling temperature, $T_D$, of the gravitino from the thermal bath [16],

$$T_D \sim \max \left[ M_{\text{gluino}}, \frac{26 \text{ GeV} \left( g^*(T_D) \right)^{1/2} \left( \frac{m_{3/2}}{1 \text{ keV}} \right)^2 \left( \frac{500 \text{ GeV}}{M_{\text{gluino}}} \right)^2}{100} \right].$$

Notice that the effective interactions between the gravitino and the SM fermions after integrating the SUSY particles out are so suppressed that they cannot keep the gravitino in the thermal bath after the gauginos decouple. In the above expressions, we have neglected the contributions from the Winos and Binos which could give comparable or even a larger contributions. The following discussion is not affected as long as the order of the magnitude of $T_D$ is not changed.\footnote{One may obtain more precise expressions of $T_D$ by using more recent analysis on the gravitino production cross section given in Ref. [17].}
From the above discussion, for the gravitino with a mass in the one to ten keV range to be a consistent dark matter candidate, the above relic density should be diluted by

$$\Delta \simeq 100 \times \left( \frac{100}{g_* (T_D)} \right) \left( \frac{m_{3/2}}{10 \text{ keV}} \right),$$

(20)

after the decoupling of the gravitino. It should be noted that the late time entropy production dilutes also the primordial baryon asymmetry by the same factor, but it may not cause any serious problem [18] in the thermal leptogenesis [20] for $\Delta \simeq 10 - 100$.

In the previous section, we have also made a tacit but a crucial assumption; the spontaneous R-symmetry breaking, $\langle S \rangle \neq 0$. It is not trivially realized in many SUSY breaking models.

In this section, we propose a very ambitious solution to both the above problems, the dilution of the thermal gravitino and the R-symmetry breaking, at the same time, by considering the entropy production from the SUSY breaking sector, where the spontaneous R-symmetry breaking is realized.

### 3.1 Extended vector-like SUSY breaking sector

As an example of the SUSY breaking model where the spontaneous R-symmetry breaking is realized, we consider the extended model of the vector-like dynamical SUSY breaking model based on $SU(2)$ gauge theory in Ref. [21, 22]. In the extended model, one of the global $U(1)$ symmetry is upgraded to a gauge symmetry [23] which is spontaneously broken at the SUSY breaking vacuum. As discussed in Ref. [24], the spontaneous R-symmetry breaking is achieved in a perturbative way, which is not the case in the original SUSY breaking model.

The notable property of the extended model is that it possesses an accidental discrete symmetry even after the spontaneous $U(1)$ symmetry breaking [24]. Thus, the lightest particle which is charged under the unbroken discrete symmetry has a long lifetime. As we will show shortly, the energy density of such a long lived particle can dominate the universe and cause the entropy production when it decays, which dilutes the thermally produced gravitino.

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8 The gauge mediation mechanism often involves natural mechanisms of late time entropy production from, for example, the messenger sector [18] or the intermediate SUSY breaking sector [19].
Table 1: The symmetries of the model. $SU(2) \times U(1)$ are gauge symmetries and $Z_4 \times U(1)_R$ are global symmetries. Notice that both the global symmetries are anomaly free.

|        | $SU(2)$ | $U(1)$ | $Z_4$ | $U(1)_R$ |
|--------|---------|--------|-------|----------|
| $S_{12}$ | 1       | -1     | $e^{i\pi}$ | 2        |
| $S_{34}$ | 1       | 1      | $e^{i\pi}$ | 2        |
| $S_{13,14,23,24}$ | 1 | 0      | $e^{i\pi}$ | 2        |
| $Q_{1,2}$ | 2       | 1/2    | $e^{i\pi/2}$ | 0        |
| $Q_{3,4}$ | 2       | -1/2   | $e^{i\pi/2}$ | 0        |

The vector-like SUSY breaking model consists of four fundamental representations $Q_k (k = 1, \cdots, 4)$ and six singlets $S_{ij} = -S_{ji} (i, j = 1, \cdots, 4)$ which interact with each other in the superpotential,

$$W = \sum \lambda_{ij}^k S_{ij} Q_k Q_l ,$$

where $\lambda$'s denote the coupling constants.

In the extended model, we gauge one of the $U(1)$ subgroup of the maximal subgroup of $SU(4)$ of the model by assigning the gauge charges, $Q_{1,2}(1/2)$, $Q_{3,4}(-1/2)$, $S_{12}(-1)$, $S_{34}(+1)$, and $S_{13,14,23,24}(0)$. With this charge assignment, the above superpotential reduces to

$$W = \lambda^{(+)} S_{12} Q_1 Q_2 + \lambda^{(-)} S_{34} Q_3 Q_4 + \sum \lambda_{ij}^{kl} S_{ij} Q_k Q_l ,$$

where $\lambda_{ij}^{kl} = 0$ for $ij = 12, 34$ or $kl = 12, 34$. The global symmetries of the model are $U(1)_R \times Z_4$. The charge assignment of the R-symmetry is $S(2)$ and $Q(0)$. The fields are transformed to $iQ$ and $-S$ under the $Z_4$ symmetry. We list the symmetries of the model in Table 1.

Below the dynamical scale $\Lambda$, the model is well described by using the composite fields, $M_{ij} \sim Q_i Q_j / \Lambda$, whose superpotential terms are approximated by,

$$W_{\text{eff}} = \lambda^{(+)} \Lambda S_{12} M_{12} + \lambda^{(-)} \Lambda S_{34} M_{34} + \sum \lambda_{ij}^{kl} \Lambda S_{ij} M_{kl} + \lambda'(\text{Pf}(M_{ij}) - \Lambda^2) ,$$

9Classically, the $Z_4$ symmetry can be realized as a continuous $U(1)$ symmetry with the charge assignment $S(2)$ and $Q(-1)$. The anomaly against the $SU(2)$ gauge symmetry breaks the $U(1)$ symmetry down to the discrete $Z_4$ subgroup.
where $\mathcal{X}$ denotes the Lagrange multiplier which expresses the quantum deformed moduli constraint, \(\text{Pf}(M) = \Lambda^2\). Here, the ambiguity of the normalizations of the meson fields are implicitly absorbed by $\lambda$’s. Thus, strictly speaking, the parameters $\lambda$’s appearing in Eqs. (23) and (24) are different from the ones in Eq. (22). By using appropriate linear combinations of $S$’s and $M$’s we may rewrite the above effective field theory by,

$$
W = \lambda^{(+)} \Lambda S_+ M_+ + \lambda^{(-)} \Lambda S_- M_+ + \sum_{a=1}^{4} \lambda_a \Lambda S_a M_a + \mathcal{X} \left( M_+ M_- + \sum_{a,b=1}^{4} \frac{y_{ab}}{2} M_a M_b - \Lambda^2 \right) .
$$

(24)

Here, the newly introduced matrix $y_{ab}$ is generically given by,

$$
y_{ab} = (U^T U)_{ab}, \quad U \in SU(4) .
$$

(25)

By assuming that $\lambda$’s are perturbative, and $\lambda^{\pm}$ are smaller than $\lambda$’s, we may parametrize the deformed moduli space, by\[^{10}\]

$$
M_+ = e^{\phi/\sqrt{2} \Lambda} \sqrt{\Lambda^2 - \sum_{a,b=1}^{4} \frac{y_{ab}}{2} M_a M_b} , \quad M_- = e^{-\phi/\sqrt{2} \Lambda} \sqrt{\Lambda^2 - \sum_{a,b=1}^{4} \frac{y_{ab}}{2} M_a M_b} .
$$

(26)

Notice that the $U(1)$ gauge symmetry is spontaneously broken on the deformed moduli space. Then, the above effective theory can be reduced to

$$
W = \left( \lambda^{(+)} \Lambda S_+ e^{-\phi/\sqrt{2} \Lambda} + \lambda^{(-)} \Lambda S_- e^{\phi/\sqrt{2} \Lambda} \right) \sqrt{\Lambda^2 - \sum_{a,b=1}^{4} \frac{y_{ab}}{2} M_a M_b} + \sum_{a=1}^{4} \lambda_a \Lambda S_a M_a .
$$

(27)

In the followings, we simplify the model by taking $\lambda^{(\pm)} = \lambda$ and $y_{ab} = \delta_{ab}$, although we can easily generalize the results for more generic coupling constants.

By expanding the above superpotential around $\phi = 0$ and $M_a = 0$, we obtain,

$$
W = \lambda \Lambda^2 (S_+ + S_-) - \frac{\lambda}{\sqrt{2}} \Lambda (S_+ - S_-) \phi + \frac{\lambda}{4} (S_+ + S_-) \left( \phi^2 - \sum_{a=1}^{4} M_a^2 \right) + \sum_{a=1}^{4} \lambda_a \Lambda S_a M_a ,
$$

$$
= \sqrt{2} \lambda \Lambda^2 S - \lambda \Lambda T \phi + \frac{\lambda}{2 \sqrt{2}} S \left( \phi^2 - \sum_{a=1}^{4} M_a^2 \right) + \sum_{a=1}^{4} \lambda_a \Lambda S_a M_a ,
$$

(28)

where we have defined $S$ and $T$ by,

$$
S = \frac{1}{\sqrt{2}} (S_+ + S_-) , \quad T = \frac{1}{\sqrt{2}} (S_+ - S_-) .
$$

(29)

\[^{10}\] The “radial” component of the $M_\pm$ becomes a “mass partner” of $\mathcal{X}$, and hence, we can integrate the radial component out.
Table 2: The symmetries of the model in terms of the low energy fields. By the condensation, \( \langle M_+ M_- \rangle = \Lambda^2 \), the symmetries \( U(1) \times Z_4 \) break down to a global \( Z_2 \) symmetry under which \( S_a \) and \( M_a \) are odd.

|       | \( U(1) \) | \( Z_4 \) | \( U(1)_R \) |
|-------|------------|------|----------|
| \( S_- \) | \(-1\)     | \( e^{i\pi} \) | 2         |
| \( S_+ \) | 1          | \( e^{i\pi} \) | 2         |
| \( S_a \) | 0          | \( e^{i\pi} \) | 2         |
| \( M_- \) | \(-1\)     | \( e^{i\pi} \) | 0         |
| \( M_+ \) | 1          | \( e^{i\pi} \) | 0         |
| \( M_a \) | 0          | \( e^{i\pi} \) | 0         |

The newly defined \( S \) corresponds to the pseudo-flat direction which \( F \)-term breaks SUSY.

The above effective theory possesses an discrete symmetry, \( Z_2 \), after the SUSY and the \( U(1) \) gauge symmetry breaking, which is a diagonal subgroup of the discrete subgroup of the \( U(1) \) gauge symmetry and the global \( Z_4 \) symmetry. Under the \( Z_2 \) symmetry, \( M_a \) and \( S_a \) are odd while the other \( S, T, \) and \( \phi \) are even (see Table 2). Therefore, the lightest particle which is odd under \( Z_2 \) symmetry is stable. In the appendix, we give the mass spectrum of the SUSY breaking sector which shows that the lightest \( Z_2 \) odd particle is an appropriate linear combination of the scalar components of \( M_a \) and \( S_a \). In the followings, we name the lightest \( Z_2 \) odd particle, \( x_L \).

### 3.2 Decay of the \( Z_2 \) charged particle

So far, we have assumed that the \( Z_2 \) accidental symmetry is an exact symmetry. Generically, such an accidental symmetry could be broken by higher dimensional operators which are suppressed by the reduced Planck scale. For example, the lowest dimensional operator which breaks the \( Z_2 \) symmetry is

\[
W \sim \frac{c}{M_P} QQH_u H_d \sim \frac{c \Lambda}{M_P} M H_u H_d,
\]

\( \text{(30)} \)

\(11\) In addition to the \( Z_2 \) symmetry, the model apparently possesses another discrete symmetry under which \( \phi \) and \( T \) are odd. This symmetry stems from the charge conjugation symmetry of the gauged \( U(1) \) symmetry. The charge conjugation symmetry is, however, expected to be broken in generic models with \( \lambda^+ \neq \lambda^- \). In the followings, we do not consider the charge conjugation symmetry as a good symmetry.

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\( \text{Page 15} \)
where \( c \) is a coupling constant and \( H_u \) and \( H_d \) denote the Higgs doublets in the SSM. Here, we have assumed that the \( R \)-charge of \( H_uH_d \) is 2, assuming that the \( R \)-symmetry (or its discrete subgroup) is better symmetry than the accidental \( Z_2 \) symmetry.

Through this operator, the lightest \( Z_2 \) odd particle decays into Higgs (Higgsino) with the decay rate

\[
\Gamma_{x_L} \sim \frac{c^2 \Lambda^2}{8\pi M_*^2} m_{x_L}. \tag{31}
\]

Here, we have neglected the masses of the Higgs and Higgsinos in the final state. As a result, the decay temperature is roughly given by

\[
T_{\text{decay}} \sim \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \sqrt{\Gamma_{x_L} M_P} \sim 1 \text{ GeV} \times c \left(\frac{10}{g_*}\right)^{1/4} \left(\frac{\Lambda}{10^7 \text{ GeV}}\right) \left(\frac{m_{x_L}}{10^6 \text{ GeV}}\right)^{1/2}. \tag{32}
\]

Therefore, the decay temperature of the lightest \( Z_2 \) odd particle is expected to be very low even if the \( Z_2 \) symmetry is explicitly broken by the higher dimensional operators.

### 3.3 The entropy production and dilution of the gravitino

Since the decay temperature of the lightest \( Z_2 \) odd particle is very low, the energy density of the lightest \( Z_2 \) odd particle is expected to dominate the universe, where the domination temperature is estimated by,

\[
T_{\text{dom}} \simeq \frac{4}{3} m_{x_L} Y_{x_L}, \tag{33}
\]

\(^{12}\)The detailed analysis is given in the appendix.
Figure 6: The dilution factor for a given SUSY breaking scale. In the figure, we have fixed $T_{\text{decay}} \simeq 100 \text{MeV}$ by choosing an appropriate coefficient $c$ in Eq. (32).

where $Y_{x_L}$ is the yield (number density divided by entropy density) of $x_L$ after its freeze-out. The yield is roughly given by,

$$Y_{x_L} \simeq \min \left[ \frac{0.278}{g_*}, \frac{0.76}{g_*^{1/2} M_P T_f \langle \sigma v_{\text{rel}} \rangle} \right], \quad (34)$$

where $g_*$ is the effective massless degree of freedom at the freeze-out temperature $T_f$, and the $\langle \sigma v_{\text{rel}} \rangle$ is the thermal averaged annihilation cross section,

$$\langle \sigma v_{\text{rel}} \rangle \simeq \frac{1}{8\pi} \frac{\lambda^4}{m_{x_L}^2}. \quad (35)$$

As we show in the appendix, the dominant mode of the annihilation process of $x_L$ is the one into a pair of the gravitinos.\textsuperscript{13}

In Fig.5 we show the yield and the domination temperature. The figure shows that the domination temperature is typically below the decoupling temperature of the gravitino given in Eq. (19). Therefore, after the decay of $x_L$, the yield of the gravitino is diluted by the factor $\Delta^{-1}$,

$$\Delta \equiv \frac{s|_{\text{after decay}}}{s|_{\text{before decay}}} \simeq \frac{4}{3T_{\text{decay}}} \frac{\rho|_{\text{after decay}}}{s|_{\text{before decay}}} \simeq \frac{4}{3T_{\text{decay}}} \frac{m_{x_L} n_{x_L}|_{\text{before decay}}}{s|_{\text{before decay}}} \simeq \frac{4}{3T_{\text{decay}}} m_{x_L} Y_{x_L}$$

where $s$ is the entropy density, $\rho$ is the radiation energy density, and $n_{x_L}$ is the number

\textsuperscript{13}The gravitinos produced at the annihilation of $x_L$ interact with the thermal bath and are thermalized immediately since $T_f \gg T_D$. 

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density of \(x_L\). Thus we obtain (using Eq. \((33)\))

\[\Delta \sim \frac{T_{\text{dom}}}{T_{\text{decay}}} . \tag{36}\]

In Fig. 6, we show the resultant dilution factor for a given SUSY breaking scale. In the figure, we assumed \(T_{\text{decay}} \simeq 100\) MeV which roughly corresponds to the lower bound on the decay temperature not to affect the Big-Bang Nucleosynthesis\(^{14}\). The figure shows that the dilution factor of the order of \(10^{-100}\) is achieved for \(\lambda \sim O(10^{-1})\), which is required to achieve the consistent gravitino dark matter scenario with a mass in the one to ten keV range.

### 3.4 Gravitino from the process of entropy production

The thermal relic gravitinos are diluted as we have discussed above. In this subsection, we discuss other sources of gravitinos. The decay of \(x_L\) produces higgsinos with a branching ratio of order 1, and they eventually decay into gravitinos after some cascade decay. Besides, the particles from the \(x_L\) decay have very large energies of order \(m_{x_L}\), and they interact with the thermal bath and may produce more SUSY particles.

Suppose that \(N_{x_L}\) gravitinos are produced in average from a decay of single \(x_L\). Then the yield of the gravitinos coming from the decay is,

\[Y_{3/2}^{\text{decay}} \sim N_{x_L} \frac{n_{x_L}}{s|_{\text{after decay}}} \sim \frac{N_{x_L}}{m_{x_L}} \rho|_{\text{after decay}} \sim \frac{3}{4} N_{x_L} \frac{T_{\text{decay}}}{m_{x_L}} . \tag{37}\]

Thus, the contribution of these gravitinos to \(\Omega h^2\) is,

\[\Omega_{3/2}^{\text{decay}} h^2 \sim 2 \times 10^{-4} \times N_{x_L} \left( \frac{m_{3/2}}{10\text{ keV}} \right) \left( \frac{T_{\text{decay}}}{100\text{ MeV}} \right) \left( \frac{10^6\text{ GeV}}{m_{x_L}} \right) . \tag{38}\]

The decay remnants of \(x_L\) whose initial energies are of the order of \(E \simeq m_{x_L}\) lose most of their energies by interacting with the particles in the thermal bath. Then, the SUSY particle productions take place when the energies of the remnants decrease down to the threshold energy, \(E \simeq M_{\text{gaugino}}/T_{\text{decay}}\). As a result, the average number of the gravitinos from the decay of \(x_L\) is at most of the order of unity\(^{18}\).

Here, instead of analyzing the detail of the above process, we give a very rough upper bound on \(N_{x_L}\) by neglecting the energy loss of the remnants, which is good enough for

\(^{14}\)The decay temperature, \(T_{\text{decay}} \simeq 100\) MeV, can be achieved by choosing appropriate coefficient \(c\).
our discussion. That is, the maximum number of the SUSY particles from a decay of \( x_L \) is achieved if the initial energy \( E \simeq m_{x_L} \) is distributed to \( \tilde{N} \simeq m_{x_L}/(M_{\text{gaugino}}/T_{\text{decay}}) \) particles with energy \( E \simeq M_{\text{gaugino}}^2/T_{\text{decay}} \), and each particle with this energy produces a gaugino. Thus, the absolute upper bound on the number \( N_{x_L} \) is given by,

\[
N_{x_L} \lesssim \tilde{N} \simeq 10 \cdot \left( \frac{100 \text{ GeV}}{M_{\text{gaugino}}} \right)^2 \left( \frac{T_{\text{decay}}}{100 \text{ MeV}} \right) \left( \frac{m_{x_L}}{10^6 \text{ GeV}} \right) + 2\text{Br}(x_L \rightarrow \tilde{H}_u \tilde{H}_d), \tag{39}
\]

where the second term comes from the direct higgsino production by the decay of \( x_L \). Therefore, we find that the gravitino coming from the \( x_L \) decay does not become a dominant component of the dark matter.  

3.5 Spontaneous R-symmetry breaking

Finally, we discuss the R-symmetry breaking in the extended vector-like SUSY breaking model. As we see from the superpotential in Eq. (28), the SUSY breaking field \( S \) has the flat potential at the tree level, and hence, \( S \) corresponds to the pseudo-flat direction. The potential of \( S \) is, however, deformed by the radiative correction, and especially, the origin of \( S \) can be destabilized [23].

At the one-loop level, the effective potential of the pseudo-flat direction is given by the so-called Coleman-Weinberg potential,

\[
V_{\text{CW}}(S) = \frac{1}{64\pi^2} \text{tr}(-)^F M^4(S) \log \frac{M^2(S)}{\mu_R^2}, \tag{40}
\]

where \( \mu_R \) denotes the renormalization scale, and \((-)^F = 1 \) for bosons and \((-)^F = -1 \) for fermions. The mass spectrum in the extended model is given in the appendix.

In Fig. 7, we show the parameter space where the spontaneous R-symmetry breaking is achieved for given values of \( \lambda \). In the figure, \( g_X \) denotes the gauge coupling constant of the \( U(1) \) gauge interaction. The figure shows that spontaneous R-symmetry breaking is realized for \( g_X \gtrsim \lambda, k^{(i)} \), which corresponds to the large contribution from the \( U(1) \) gauge interaction. Notice that for \( g_X \gtrsim 0.5 \) at the mediation scale, the \( U(1) \) gauge interaction has the Landau pole below the GUT scale. Thus, if we require that the extended model

\[\text{[15] Although the energy density of the gravitino component from the decay of } x_L \text{ is subdominant, such gravitinos may have much larger velocity than the one of the thermally produced gravitino. Thus, the gravitino component from the decay of } x_L \text{ could have some impacts on the structure formation.}\]
is perturbative up to the GUT scale, the gauge coupling constant should satisfy $g_X \lesssim 0.5$ at the mediation scale.

For the larger field value of $S \gg M_{\text{med}}$, the radiatively generated potential is further approximated by \textit{[25]},

\begin{align*}
V_{\text{CW}}(S) &= \frac{|F|^2}{Z_S}, \\
Z(S) &= \exp \left[ -\int_{\ln M_{\text{med}}}^{\ln S} 2\gamma_S d\ln \mu \right].
\end{align*}

(41)

Here, $\gamma_S$ is the anomalous dimension of $S$ which is, at the one-loop level, given by,

\begin{align*}
2\gamma_S &= \frac{4}{2\pi} \frac{k^{(l)2}}{4\pi} + \frac{6}{2\pi} \frac{k^{(d)2}}{4\pi} - \frac{1}{\pi} \frac{g_X^2}{4\pi} + O(\lambda^2),
\end{align*}

(42)

where $O(\lambda^2)$ contribution comes from mesons $M$ in the region $\lambda S \lesssim \Lambda$ or quarks $Q$ in the region $\lambda S \gtrsim \Lambda$, and this contribution is always positive in the region where perturbative calculation is valid (i.e. $\lambda S$ is not near $\Lambda$). In order for the potential not to show the runaway behavior, we need to have $\gamma_S > 0$ at the larger value of $S$. Because $O(\lambda^2)$ contribution is positive, we can conservatively neglect this contribution to constrain the viable parameter space. So we neglect this contribution in $\gamma_S$. In Fig.\textit{7} we have shown the region where $\gamma_S > 0$ at least around the mediation scale. The figure shows that spontaneous R-breaking is realized for $g_X \gtrsim \lambda, k^{(l)}$, while the runaway behavior is avoided for not too large $g_X$ compared with $k^{(l)}$\textit{[16]}.

Put the above discussions together, we find that spontaneous R-symmetry breaking and the right amount of the entropy production are achieved at the same time for $\lambda = O(10^{-1})$ in the extended vector-like SUSY breaking model.

4 Conclusion

In this paper, we discussed the simplest class of the R-invariant gauge mediation model for the gravitino with a mass in the one to ten keV range. The gravitino dark matter

\textit{[16]} In the figure, we have fixed $\sqrt{k^{(l)} F/m^{(l)}} = 0.8$. For the heavier messenger masses, the hatched regions in the figure are shifted to right while the light-shaded region is not shifted. This is because the contribution to the Coleman-Weinberg potential in Eq. \textit{[40]} from the messenger is suppressed by the heavier masses. Thus, for the heavier messengers, the allowed parameter space is larger.
Figure 7: Spontaneous R-symmetry breaking is realized by the radiatively generated potential of $S$ in the hatched region for given values of $\lambda$. The $U(1)$ gauge coupling $g_X$ and the messenger-SUSY breaking field coupling $k(\ell)$ are defined at the mediation scale. In the figure, we have fixed $\lambda' = 2\lambda$, $k(d) \simeq 1.8k(\ell)$, $m(d) = 2.1m(\ell)$, and $m(\ell) = \sqrt{k(\ell)F/0.8}$. We have fixed the value of $F$. In the dark shaded region, the $U(1)$ gauge coupling constant at the mediation scale is too strong and has a Landau pole problem below the GUT scale, i.e. $g_X \gtrsim 0.5$. In the light-shaded region, the radiatively generated potential does not curl up for the large value of $S$.

scenario with the mass in this range is drawing attention as an interesting interpretation of seeming discrepancies between the observation and the simulation of the structure formation based on the cold dark matter model.

For a consistent gravitino dark matter scenario with a mass in the one to ten keV range, the relic density of the gravitino is needed to be diluted by a factor of $\Delta = 10 - 100$. In this paper, we discussed entropy production from the vector-like SUSY breaking model, which is extended so that spontaneous R-symmetry breaking is achieved. Spontaneous R-symmetry breaking is necessary ingredient for the successful R-invariant gauge mediation mechanism. As a result, we find that R-symmetry breaking and right amount of entropy production are achieved at the same time for a certain parameter space.

The interesting prediction of the R-invariant gauge mediation model is the peculiar gaugino mass spectrum with much heavier sfermions. Especially, the gluino can be lighter.
than the wino even if the messenger masses and coupling constants satisfy the GUT relation at the GUT scale. The light gluino \( m_{\text{gluino}} \approx 300\,\text{GeV} - 1\,\text{TeV} \) is quite advantageous to be produced at the LHC. Therefore, it is expected that this model can be probed by the LHC.

Another notable point is that the solution to the \( \mu \)-problem proposed in Ref. [26] works for the gravitino mass of order \( O(1) - O(10) \,\text{keV} \). It is very interesting that the gravitino mass of this order is favored from several things, i.e. the warm dark matter, the interesting region of the gaugino masses at the LHC, and the solution to the \( \mu \)-problem.

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### A More on the extended SUSY breaking sector

#### A.1 Mass spectrum

**Scalar spectrum**

In order to analyze the spectrum of the scalar particles, we decompose the scalar components as:

\[
S = \left( \langle S \rangle + \frac{1}{\sqrt{2}} \sigma \right) e^{i a / \sqrt{2}}, \quad T = \frac{1}{\sqrt{2}} \left( x_T + i y_T \right) e^{i a / \sqrt{2}},
\]

\[
S_a = \frac{1}{\sqrt{2}} \left( x_s + i y_s \right) e^{i a / \sqrt{2}}, \quad M_a = \frac{1}{\sqrt{2}} \left( x_m + i y_m \right),
\]

\[
\phi = \frac{1}{\sqrt{2}} \left( x_\phi + i y_\phi \right).
\]

In the followings, we suppress the index \( a \) of \( x_{s,m} \) and \( y_{s,m} \).

\footnote{In this paper, we assume that all the parameters in the SUSY breaking sector are real valued, so that the \( CP \)-symmetry is not broken. However, notice that the \( CP \) violation in the hidden sector is insignificant for the \( CP \) violation of the SSM, because the phases of the gaugino masses are the same as \( \langle S \rangle^* F \), which can always be rotated away.}
The squared mass matrices of $x$'s are given by
\[ M^{(x)^2}_{T\phi} = \begin{pmatrix} \lambda^2 \Lambda^2 + 2g_X^2 \langle S \rangle^2 & -\lambda^2 \langle S \rangle / \sqrt{2} + 2\sqrt{2}g_X^2 \Lambda \langle S \rangle \\ -\lambda^2 \langle S \rangle / \sqrt{2} + 2\sqrt{2}g_X^2 \Lambda \langle S \rangle & 2\lambda^2 \Lambda^2 + 4g_X^2 \Lambda^2 \end{pmatrix}, \]
for $(x_T, x_\phi)$ and
\[ M^{(x)^2}_{sm} = \begin{pmatrix} \lambda^2 \Lambda^2 & -\lambda \langle S \rangle / \sqrt{2} \\ -\lambda \langle S \rangle / \sqrt{2} & (\lambda^2 - \lambda^2) \Lambda^2 + \frac{1}{2} \lambda^2 \langle S \rangle^2 \end{pmatrix}, \]
for $(x_s, x_m)$. On the other hand, the squared mass matrices of $y$'s are given by,
\[ M^{(y)^2}_{T\phi} = \begin{pmatrix} \lambda^2 \Lambda^2 & -\lambda \langle S \rangle / \sqrt{2} \\ -\lambda \langle S \rangle / \sqrt{2} & (\lambda^2 + \lambda^2) \Lambda^2 + \frac{1}{2} \lambda^2 \langle S \rangle^2 \end{pmatrix}, \]
for $(y_T, y_\phi)$ and
\[ M^{(y)^2}_{sm} = \begin{pmatrix} \lambda^2 \Lambda^2 & -\lambda \langle S \rangle / \sqrt{2} \\ -\lambda \langle S \rangle / \sqrt{2} & (\lambda^2 + \lambda^2) \Lambda^2 + \frac{1}{2} \lambda^2 \langle S \rangle^2 \end{pmatrix}, \]
for $(y_s, y_m)$.

The eigen-modes of each mass matrices are given by,
\[ m_1^2 = \frac{1}{2} \left( \text{tr} M^2 - \sqrt{\text{tr} M^2 - 4 \det M^2} \right), \]
\[ m_2^2 = \frac{1}{2} \left( \text{tr} M^2 + \sqrt{\text{tr} M^2 - 4 \det M^2} \right). \]

Notice that the lighter mode of $(y_T, y_\phi)$ is massless, which corresponds to the would-be Nambu-Goldstone mode of the gauged $U(1)$ symmetry.

**Fermion mass spectrum**

The mass matrix of the fermion components of $S_a$ and $M_a$ is given by
\[ M^{(f)}_{sm} = \begin{pmatrix} 0 & \lambda \Lambda \\ \lambda \Lambda & -\lambda \langle S \rangle / \sqrt{2} \end{pmatrix}. \]

The mass matrix of the fermion components of $T$, $\phi$ and the $U(1)$ gaugino is given by,
\[ M^{(f)}_{T\phi\bar{g}} = \begin{pmatrix} 0 & -\lambda \Lambda & -i\sqrt{2}g_X \langle S \rangle \\ -\lambda \Lambda & \lambda \langle S \rangle / \sqrt{2} & -i2g_X \Lambda \\ -i\sqrt{2}g_X \langle S \rangle & -i2g_X \Lambda & 0 \end{pmatrix}. \]

The fermion component of $S$ corresponds to the Goldstino.

---

18. Note that the low-energy effective Kähler potential of $\phi$ is given by $K \simeq \Lambda^2[(\phi + \phi^*)/\sqrt{2} - 2g_X V_X]$, where $V_X$ is the gauge supermultiplet of $U(1)_X$.

19. There is no mixing between the R-axion and the would-be Nambu-Goldstone boson.
Vector boson mass

The mass of the vector boson is given by,

\[ M_V^2 = 2g_X^2 (\langle S \rangle^2 + 2\Lambda^2) . \tag{51} \]

The lightest particle

As we have discussed, the model possesses a discrete \( Z_4 \) symmetry which is effectively a \( Z_2 \) symmetry under which \( x_{m,s} \) and \( y_{m,s} \) are odd. From the above mass matrices, we find that the lightest, and hence stable, \( Z_2 \) odd particle resides in \( x_{m,s} \). In the followings, we name the lightest particle \( x_L \).

In addition to the lightest \( Z_2 \) odd particle, the model possesses two massless scalars which correspond to the pseudo-flat direction, \( \sigma \), and the R-axion, \( a \), respectively. Besides, the model also has a massless fermion, the fermion component of \( S \), which corresponds to the Goldstino. As we will discuss, the mass of the pseudo-flat direction is generated by the radiative corrections. The R-axion mass is also generated by the effects of the explicit R-symmetry breaking terms, e.g. the constant term in the superpotential generates the axion mass in supergravity. The Goldstino becomes massive by the super-Higgs mechanism of supergravity.

Relevant interactions

The relevant interaction terms to analyze the relic density of the lightest \( Z_2 \) charged particle is summarized below.

First, the R-axion interactions only appear in the kinetic terms. In the basis we have defined above, the R-axion interactions come from the kinetic terms of \( S, T \) and \( S_a \),

\[ \mathcal{L} = \frac{1}{2} (\partial a)^2 \left( 1 + \frac{\sigma}{\sqrt{2} \langle S \rangle} \right)^2 + \sum_{i=s,T} \frac{(\partial a)^2}{4 \langle S \rangle^2} (x_i^2 + y_i^2) + \frac{\partial \mu a}{\sqrt{2} \langle S \rangle} (x_i \partial^\mu y_i - y_i \partial^\mu x_i). \tag{52} \]

The other important interaction term is

\[ \mathcal{L} = \frac{\lambda}{2} (x_m + iy_m) \tilde{G} \psi_m + \frac{\lambda}{2} (x_m - iy_m) \tilde{G}^\dagger \psi_m^\dagger , \tag{53} \]

where \( \tilde{G} \) denotes the Goldstino and \( \psi_m \) the fermion component of \( M_a \).


A.2 Annihilation of the lightest $Z_4$ charged ($Z_2$ odd) field

The annihilation modes of $x_L$ are $x_Lx_L \rightarrow \tilde{G}\tilde{G}$, $x_Lx_L \rightarrow \sigma\sigma$, and $x_Lx_L \rightarrow aa$. In terms of $x_L$, the relevant interaction terms can be rewritten by,

$$
\mathcal{L} = \frac{\lambda^2 \langle S \rangle}{2\sqrt{2}} \frac{m_L^2}{m_H^2 - m_L^2} \sigma x_L^2 - \frac{\lambda^2 \langle S \rangle}{4} \frac{\lambda^2 \langle S \rangle}{(m_H^2 - m_L^2)} \sigma x_Lx_H - \frac{\lambda^4 \langle S \rangle}{16 (m_H^2 - m_L^2)(m_H^2 - \lambda^2 \langle S \rangle)} \sigma x_L^2
$$

$$
+ \frac{\cos^2 \theta_x}{4 \langle S \rangle^2} (\partial \mathcal{L})^2 \sigma x_L^2 + \frac{\cos \theta_x}{\sqrt{2} \langle S \rangle} \partial \mathcal{L} (\cos \theta_y (x_L \partial y_L - y_L \partial x_L) - \sin \theta_y (x_L \partial y_H - y_H \partial x_L))
$$

$$
+ \frac{1}{(\partial \mathcal{L})^2} \left( 1 + \frac{\sigma}{\sqrt{2} \langle S \rangle} \right) + \frac{\lambda}{2} \sin \theta_x x_L \tilde{G} (\sin \theta_f \psi_L + \cos \theta_f \psi_H) + h.c.
$$

Here, $\theta_x,y$ are mixing angles of $x$ and $y$ components of $S_a$ and $M_a$, and $\theta_f$ is the angle of the fermion components. The mixing angles are given by,

$$
\tan \theta_x = \frac{\sqrt{2}(m_L^2 - \lambda^2 \langle S \rangle)}{-\lambda^2 \langle S \rangle}, \quad \tan \theta_y = \frac{\sqrt{2}(m_L^2 - \lambda^2 \langle S \rangle)}{-\lambda^2 \langle S \rangle}, \quad \tan \theta_f = \frac{\sqrt{2}(m_L^2 - \lambda^2 \langle S \rangle)}{-\lambda^2 \langle S \rangle}
$$

A.2.1 $x_Lx_L \rightarrow \sigma\sigma$

The process, $x_Lx_L \rightarrow \sigma\sigma$, proceeds via the $t$ and $u$-channel exchanges of $x_L$ and $x_H$ as well as via the contact interaction. The amplitude of the $t$ and $u$-channel $x_L$ exchange is given by,

$$
\mathcal{M}_{2x_L \rightarrow 2\sigma} = \frac{1}{2} \lambda^4 \langle S \rangle^2 \left( \frac{m_L^2}{m_H^2 - m_L^2} \right)^2 \left( \frac{1}{m_H^2 - t} + \frac{1}{m_H^2 - u} \right),
$$

$$
\simeq \frac{1}{2} \lambda^2 \langle S \rangle^2 \left( \frac{m_L^2}{m_H^2 - m_L^2} \right)^2
$$

where we have neglected the mass of the flaton and used $t \simeq u \simeq -m_L^2$ in the non-relativistic limit. The amplitude of the $t$ and $u$-channel $x_H$ exchanges is given by,

$$
\mathcal{M}_{2x_L \rightarrow 2\sigma} = \frac{\lambda^6 \langle S \rangle^2}{16} \left( \frac{m_L^2}{m_H^2 - m_L^2} \right)^2 \left( \frac{1}{m_H^2 - t} + \frac{1}{m_H^2 - u} \right)
$$

$$
\simeq \frac{\lambda^2}{8} \left( \frac{2\lambda^2 \langle S \rangle^2}{m_L^2 - m_H^2} \right)^2 \left( \frac{\lambda^2 \langle S \rangle^2}{m_L^2 + m_H^2} \right).
$$

The amplitude of the contact term interaction is given by,

$$
\mathcal{M}_{2x_L \rightarrow 2\sigma} = -\frac{\lambda^2}{4} \left( \frac{\lambda^2 \langle S \rangle^2}{m_L^2 - m_H^2} \right) \left( \frac{\lambda^2 \langle S \rangle^2}{m_L^2 - \lambda^2 \langle S \rangle^2} \right).
$$

Notice that the sum of all the above matrix elements vanishes for $\lambda = \lambda'$. 



25
A.2.2 $x_Lx_L \rightarrow aa$

The process $x_Lx_L \rightarrow aa$ has four contributions, from the contact term interaction, the $t$ and $u$-channel exchange of $y_L$ and the $s$-channel flaton exchange. The matrix element of the contact term interaction is given by,

$$ M_{x_Lx_L \rightarrow aa} = -\cos^2 \theta_x \frac{p_3 \cdot p_4}{\langle S \rangle^2} \sim -2 \cos^2 \theta_x \frac{m_L^2}{\langle S \rangle^2}, \quad (59) $$

where we have neglected the mass of the R-axions in the final state. The matrix element of the $t$-channel exchange of $y$’s is given by,

$$ M_{x_Lx_L \rightarrow aa} = \cos^2 \theta_x \frac{2 m_L^4}{\langle S \rangle^2} \left( \frac{p_3 \cdot (-2p_1 + p_3) p_4 \cdot (-2p_2 + p_4)}{m_L^{(y)2} - t} + p_4 \cdot (-2p_1 + p_4) p_3 \cdot (-2p_2 + p_3) \right) \frac{m_L^{(y)2}}{m_L^{(y)2} - u}, \quad (60) $$

where we again neglected the mass of the R-axions. The contributions from the $y_H$ exchanges are given by,

$$ M_{x_Lx_L \rightarrow aa} \sim \frac{4 \cos^2 \theta_x \sin^2 \theta_y}{\langle S \rangle^2} \frac{m_H^4}{m_L^2 + m_H^{(y)2}}. \quad (61) $$

Finally, the flaton exchange contribution is given by,

$$ M_{x_Lx_L \rightarrow aa} = \lambda^2 \frac{m_L^2}{m_H^2 - m_L^2} \frac{p_3 \cdot p_4}{s} \sim \frac{\lambda^2}{2} \frac{m_L^2}{m_H^2 - m_L^2}. \quad (62) $$

A.2.3 $x_Lx_L \rightarrow \tilde{G}\tilde{G}$

The annihilation cross section into the gravitino is given by $t$ and $u$-channel exchange of $M_a$ and $S_a$ fermions \footnote{Here, we neglect the $s$-channel flaton decay which utilizes the higher dimensional operators.}. The matrix element is given by,

$$ M_{x_Lx_L \rightarrow \tilde{G}\tilde{G}} = \frac{\lambda^2}{4} \sin^2 \theta_x \sin^2 \theta_f \left( \bar{u}(p_3) \frac{\hat{q}_t + m_L^{(f)}}{m_L^{(f)2} - t} v(p_4) + \bar{u}(p_3) \frac{\hat{q}_u + m_L^{(f)}}{m_L^{(f)2} - u} v(p_4) \right), \quad (63) $$

where $q_t = p_3 - p_1$ and $q_u = p_3 - p_2$. The heavier field exchange is then given by,

$$ M_{x_Lx_L \rightarrow \tilde{G}\tilde{G}} = \frac{\lambda^2}{4} \sin^2 \theta_x \cos^2 \theta_f \left( \bar{u}(p_3) \frac{\hat{q}_t + m_H^{(f)}}{m_H^{(f)2} - t} v(p_4) + \bar{u}(p_3) \frac{\hat{q}_u + m_H^{(f)}}{m_H^{(f)2} - u} v(p_4) \right), \quad (64) $$
Thus, the unpolarized squared amplitude is given by,

\[ |M_{xLxL \to G\bar{G}}|^2 \simeq 2\lambda^4 \sin^4 \theta_x \sin^4 \theta_f \frac{m_L^2 m_f^2}{(m_L^2 + m_f^2)^2} + 2\lambda^4 \sin^4 \theta_x \cos^4 \theta_f \frac{m_H^2 m_f^2}{(m_L^2 + m_H^2)^2} \]

\[ -4\lambda^4 \sin^2 \theta_x \sin^2 \theta_f \frac{m_L^2 m_f^2}{(m_L^2 + m_f^2)^2}. \]  

(65)

Here, we have used the fact that the product of the two fermion masses is the negative valued since the determinant of the mass matrix in Eq. (49) is negative.

A.2.4 Total cross section

By adding up all the above modes, we obtain the total \( S \)-wave cross section,

\[ \sigma_{\text{rel}} \simeq \frac{1}{2} \frac{1}{32\pi} \frac{|M|^2}{m_L^2}, \]  

(66)

where a factor 1/2 represents the statistical factor of the final state particles. The thermal average can be trivially taken and the resultant cross section appearing in the Boltzmann equation is given by,

\[ \langle \sigma_{\text{rel}} \rangle \simeq \frac{1}{2} \frac{1}{32\pi} \frac{|M|^2}{m_L^2}. \]  

(67)

A.3 Decay of the \( Z_2 \) charged particle

As we have discussed in the paper, the \( Z_2 \) symmetry is expected to be broken by the reduced Planck suppressed operators in Eq. (30). This operator generates the interaction terms of the \( Z_2 \) odd particle such as,

\[ \mathcal{L} = \left( \frac{\lambda \langle S \rangle}{\sqrt{2}} M_a - \lambda' A S_a \right) \frac{c\Lambda}{M_P} M_a (H_u H_d)^* - \frac{c\Lambda}{M_P} M_a \psi_{H_u} \psi_{H_d} + h.c. \]

\[ \rightarrow \left( \frac{1}{2} \sin \theta_x \lambda \langle S \rangle - \frac{1}{\sqrt{2}} \cos \theta_x \lambda' A \right) \frac{c\Lambda}{M_P} x_L (H_u H_d)^* - \sin \theta_x \frac{c\Lambda}{\sqrt{2} M_P} x_L \psi_{H_u} \psi_{H_d} + h.c. \]  

(68)

Thus, the decay rate of the lightest \( Z_2 \) odd particle is given by,

\[ \Gamma_{xL} \simeq \frac{1}{4\pi} \left( \frac{1}{2} \sin \theta_x \lambda \langle S \rangle \rho_{xL}^2 - \frac{1}{\sqrt{2}} \cos \theta_x \lambda' A \rho_{xL}^2 \right)^2 \frac{c^2 \Lambda^2}{M_P^2} m_{xL} + \frac{1}{8\pi} \sin^2 \theta_x \frac{c^2 \Lambda^2}{M_P^2} m_{xL}. \]  

(69)
A.4 The messenger interaction

For the extended vector-like SUSY breaking model, we need at least two sets of the messengers, which couple to $S_\pm$,

$$W = \left(\tilde{\Psi}_1, \tilde{\Psi}_{1+}\right) \left(\begin{array}{cc} k^{(-)} S_+ & m \\ m & 0 \end{array}\right) \left(\begin{array}{c} \Psi_1^- \\ \tilde{\Psi}_1 \end{array}\right) + \left(\tilde{\Psi}_2, \tilde{\Psi}_{2-}\right) \left(\begin{array}{cc} k^{(+)} S_- & m \\ m & 0 \end{array}\right) \left(\begin{array}{c} \Psi_2^+ \\ \tilde{\Psi}_2 \end{array}\right), \quad (70)$$

where we have rewritten $S_\pm$ in terms of the pseudo-flat direction $S$, and we have taken $k^{(+)} = k^{(-)} = \sqrt{2}k$.

Thus, the Dirac-type fermion mass matrix is given by,

$$M(f) = \left(\begin{array}{cc} k \langle S \rangle & m \\ m & 0 \end{array}\right). \quad (71)$$

for each $\Psi_1$’s and $\Psi_2$’s. The squared mass matrix of the complex scalars is given by,

$$M(s) = \left(\begin{array}{cccc} k^2 \langle S \rangle^2 + m^2 & km \langle S \rangle & kF & 0 \\ km \langle S \rangle & m^2 & 0 & 0 \\ kF & 0 & k^2 \langle S \rangle^2 + m^2 & km \langle S \rangle \\ 0 & 0 & km \langle S \rangle & m^2 \end{array}\right). \quad (72)$$

The messenger sector has the $U(1)_{d1} \times U(1)_{\ell1} \times U(1)_{d2} \times U(1)_{\ell2}$ global symmetries, which act on the down type and lepton type $\Psi_1$’s and $\Psi_2$’s, respectively. We have to break these symmetries explicitly to avoid the messenger dark matter overclosing the universe. For example, by introducing a small mixing such as $\delta m \tilde{\Psi}_1 \Psi_2$, we can break the symmetry down to $U(1)_d \times U(1)_\ell$. Furthermore, we can completely break the symmetry by introducing interactions such as $\epsilon \tilde{\Psi}_1 10_{SSM} 5^*_{SSM}$, where $10_{SSM}$ and $5^*_{SSM}$ are the SSM matter fields written in the $SU(5)_{GUT}$ representations and $\epsilon$ is a very small Yukawa coupling. What type of interactions are allowed depends on the precise R-charge assignment to the SSM matter fields, which we do not explicitly specify in this paper.

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