Transport of hard probes through glasma

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based on: arXiv:2202.00357, arXiv:2112.06812, arXiv:2001.05074

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EARLY-TIME DYNAMICS:
→ the least understood phase of the collision
→ lack of a direct experimental access to it
→ initial conditions for subsequent hydrodynamic evolution

• transition between early-time dynamics and hydrodynamics
• impact of pre-equilibrium phase on hard probes
  * expected hierarchy in energy loss: $\Delta E_g > \Delta E_q > \Delta E_c > \Delta E_b$
  (experimental data on $R_{AA}$ do not confirm it)
  * influence of initial dynamics on hard probes ignored for a long time

GLASMA:
→ initial phase of HIC at highest energies described by QCD
→ highly anisotropic system made mostly of gluon fields

METHOD:
→ expansion of glasma fields in the proper time (analytical, purely classical approach)

OBJECTIVES:
→ insight into macroscopic properties of QCD matter soon after the collision
  (see talk by M. Carrington, Thursday, 11:30am)
→ impact of the initial phase on energy losses of hard probes
→ limitations, consistency and reliability of the approach
**CGC - effective theory of QCD to describe a nucleus in terms of QCD quanta**

| before the collision (MV model): | after the collision (glasma): |
|----------------------------------|--------------------------------|
| - large-\(x\) partons:          | - valence quarks fly away     |
|  valence quarks, source partons \(\rho(x^-, \vec{x}_\perp)\) | - glasma fields \(\alpha(\tau, \vec{x}_\perp)\) and \(\alpha^i_\perp(\tau, \vec{x}_\perp)\) |
| - small-\(x\) partons:          | - glasma fields evolve in \(\tau\) according to    |
|  soft gluon fields \(\beta^\mu(x^-, \vec{x}_\perp)\) |  source-less classical YM equations   |
| - classical Yang-Mills equations: | - current dependence enters through   |
|  \([D_\mu, F^{\mu\nu}] = J^\nu\) |  boundary conditions at \(\tau = 0\):   |
| - solution:                      |  \(\alpha_\perp^i = \beta_1^i + \beta_2^i\), \(\alpha = -\frac{ig}{2}[\beta_1^i, \beta_2^i]\)   |
|  \(\beta^-(x^-, \vec{x}_\perp) = 0\) | - general solutions to CYM eqs. not known   |
|  \(\beta^i(x^-, \vec{x}_\perp) = \theta(x^-) \frac{i}{g} U(\vec{x}_\perp) \partial^i U^\dagger(\vec{x}_\perp)\) | - expansion of the glasma fields in \(\tau\):   |
| - saturation scale \(Q_s\) - UV regulator | \(\alpha_\perp^i(\tau, \vec{x}_\perp) = \sum_{n=0}^\infty \tau^n \alpha_\perp^i(n)(\vec{x}_\perp)\)   |
| - \(m \sim \Lambda_{QCD}\) - IR regulator | \(\alpha(\tau, \vec{x}_\perp) = \sum_{n=0}^\infty \tau^n \alpha(n)(\vec{x}_\perp)\)   |
| - Gaussian averaging over colour charges: | - solutions of CYM eqs. found recursively   |
|  \(\langle \rho_a(x^-, \vec{x}_\perp) \rho_b(y^-, \vec{y}_\perp) \rangle \) | - 0th order coefficients = boundary conditions   |
|  \(= g^2 \delta_{ab} \lambda(x^-, \vec{x}_\perp) \delta(x^- - y^-) \delta^2(\vec{x}_\perp - \vec{y}_\perp)\) | Chen, Fries, Kapusta, Li, Phys. Rev. C 92, 064912 (2015) |

**Basic building block: 2-point correlator (with Wick’s theorem)**

\[
\delta_{ab} B_n^{ij}(\vec{x}_\perp, \vec{y}_\perp) \equiv \lim_{w \to 0} \langle \beta_{n a}^i(x^\mp, \vec{x}_\perp) \beta_{n b}^j(y^\mp, \vec{y}_\perp) \rangle 
\]

\[
B_n^{ij}(\vec{x}_\perp, \vec{y}_\perp) = \frac{2}{g^2 N_c \tilde{\Gamma}_n(\vec{x}_\perp, \vec{y}_\perp)} \left[ \exp \left( \frac{g^4 N_c}{2} \tilde{\Gamma}_n(\vec{x}_\perp, \vec{y}_\perp) \right) - 1 \right] \partial_x^i \partial_y^j \tilde{\gamma}_n(\vec{x}_\perp, \vec{y}_\perp) 
\]

\(\tilde{\Gamma}_n(\vec{x}_\perp, \vec{y}_\perp)\) and \(\tilde{\gamma}_n(\vec{x}_\perp, \vec{y}_\perp)\) - determined by modified Bessel functions
Hard probes in glasma

**Fokker-Planck equation** - evolution equation on the distribution function $n(t, x, p)$ of hard probes interacting with a medium

- usually applied to probes moving through QGP in equilibrium
- probes as Brownian particles: probe’s momentum $\gg$ momentum transfer

**Fokker-Planck equation for hard probes interacting with glasma:**

$$\left(D - \nabla_p^i X^{ij}(v) \nabla_p^j - \nabla_p^i Y^i(v)\right)n(t, x, p) = 0$$

**Collision terms:**

$$X^{ij}(v) = \frac{1}{2N_c} \int_0^t dt' \langle F^i_a(t, x) F^j_a(t', x - v(t - t')) \rangle, \quad Y^i(v) = X^{ij}(v) \frac{v^j}{T}$$

$T$ - temperature of a plasma that has the same energy density as in equilibrium

$F(t, r) = g(E(t, r) + v \times B(t, r))$ - color Lorentz force

$g$ - constant coupling, $E(t, r), B(t, r)$ - chromoelectric and chromomagnetic fields

$v = \frac{p}{E_p}$ - velocity of the probe:

$v \approx 1$ - light quarks and gluons  \quad v \leq 1$ - heavy quarks

Mrówczyński, Eur. Phys. J, A54 no 3, 43 (2018)

**Energy loss and momentum broadening determined through correlators of chromodynamic fields (or equivalently correlators of glasma potentials):**

$$- \frac{dE}{dx} = \frac{v}{T} \frac{v^i v^j}{v^2} X^{ij}(v)$$

$$\hat{q} = \frac{2}{v} \left(\delta^{ij} - \frac{v^i v^j}{v^2}\right) X^{ij}(v)$$

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Time dependence of $\hat{q}$ and $dE/dx$

- $dE/dx$ and $\hat{q}$ calculated up to $\tau^5$ order
- parameters $m = 0.2$ GeV, $Q_s = 2$ GeV, $N_c = 3$, $g = 1$
- in case of $dE/dx$ we need temperature $T$, which we compute from the relation to the energy density of the glasma

Consistency and reliability of the approach are fixed by convergence of the proper time expansion and saturation of the results.

- $\hat{q}$: saturation observed before the $\tau$ expansion breaks down, $\hat{q} \simeq 6$ GeV$^2$/fm - maximal value, similar result was found using real-time QCD calculations: Ipp, Müller, Schuh, Phys. Lett. B 810, 135810 (2020)
- $dE/dx$: reaches a maximal value 0.9 GeV/fm, no saturation $\to$ order of magnitude estimate only
purely transverse motion: saturation is evident, the higher $v = v_\perp$, the less time the probe spends in the region of correlated fields $\rightarrow$ reduction of $\hat{q}$

for larger values of $v_\parallel$ saturation is less evident

fixed $v_\perp$: the effect of the velocity dependence of the Lorentz force $\rightarrow$ the role of electric contribution decreases when $v_\parallel$ increases

fixed $v$: the effect of changing the amount of time that the probe spends in the region of correlation $\rightarrow$ probes with larger $v_\perp$ escape from the region of correlated fields fast, before the fields become large $\rightarrow$ probes with smaller $v_\perp$ remain longer in the region and eventually interact with very large glasma fields
dependence on spatial rapidity $\eta \rightarrow$ dependence on the initial position of the probe in the glasma

- $\hat{q}$ at orders $\tau^4$ and $\tau^5$ agree well up to $\tau \simeq 0.07$ fm
- $\hat{q}$ is weakly dependent on $\eta$ for small values of $\eta$ (CGC is expected to work best in the region of mid-spatial-rapidity region)
total accumulated transverse momentum: \( \Delta p^2_T = \int_0^L dt \, \hat{q}(t) \)

- non-equilibrium case: \( \Delta p^2_T \bigg|_{\text{non-eq}} = \int_0^{t_0} dt \, \hat{q}(t) = \frac{1}{2} \hat{q}_{\text{max}} t_0 + \frac{1}{2} \hat{q}_0 (t_0 - t_{\text{max}}) \)

- equilibrium case: \( \Delta p^2_T \bigg|_{\text{eq}} = \int_{t_0}^L dt \, \hat{q}(t) = 3 T_0^3 t_0 \ln \frac{L}{t_0} \)

where we used \( \hat{q}(t) = 3 T^3 \) and \( T = T_0 \left( \frac{t_0}{t} \right)^{1/3} \)

- parameters:
  - \( \hat{q}_{\text{max}} \approx 6 \text{ GeV}^2/\text{fm}, \ t_{\text{max}} \approx 0.06 \text{ fm} \)
  - \( L = 10 \text{ fm}, \ \hat{q}_0 \approx 1.4 \text{ GeV}^2/\text{fm}, \ t_0 \approx 0.6 \text{ fm}, \ T_0 = 0.45 \text{ GeV} \)

\( \frac{\Delta p^2_T \bigg|_{\text{non-eq}}}{\Delta p^2_T \bigg|_{\text{eq}}} = 0.93 \)

Non-equilibrium phase gives comparable contribution to the radiative energy loss as the equilibrium phase.

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