Deep recommender engine based on efficient product embeddings neural pipeline

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Abstract — Predictive analytics systems are currently one of the most important areas of research and development within the Artificial Intelligence domain and particularly in Machine Learning. One of the "holy grails" of predictive analytics is the research and development of the "perfect" recommendation system. In our paper we propose an advanced pipeline model for the multi-task objective of determining product complementarity, similarity and sales prediction using deep neural models applied to big-data sequential transaction systems. Our highly parallelized hybrid pipeline consists of both unsupervised and supervised models, used for the objectives of generating semantic product embeddings and predicting sales, respectively. Our experimentation and benchmarking have been done using very large pharma-industry retailer Big Data stream.

Keywords — recommender systems; efficient embeddings; machine learning; deep learning; big-data; high-performance computing, GPU computing.

I. INTRODUCTION

Recommender systems are by far one of the most important areas where machine learning in conjunction with Big Data are applied with proven success. The goal is to find what is likely to be of interest to a certain customer or group of customers and to provide personalized services to them. The interest in this area is very high because finding the "perfect" recommendation system is crucial and will allow retailers to structure their offer including sales strategy and marketing campaigns very well in order to optimize consumers choices.

This paper proposes a state-of-the-art deep neural model for the multi-task objective of determining product complementarity/similarity, which leads to diversified market baskets and sales prediction. The pharmaceutical industry has been proposed as an experimental environment in our research. As a result, we used the advantage of a Big-Data sequential transaction system provided by a successful pharma retailer.

Recommender systems in pharma industry. An important aspect related to the challenge of constructing pharma retail recommender systems resides in the transactional flow of this particular industry. In pharma retail we have both returning/traceable customers and unknown customers. Nevertheless, the recommender system should be able to predict various targets such as: best potential offer for a market basket starting from a product (or products), automatic inference of customer health/medical condition needs, product complementarity, similarity and actual product cannibalization. All of these mentioned use-cases within the pharma industry have to be tackled either if we have time-series data available for a particular customer (time-series based profiling) or time-series and customer-agnostic predictive analytics. To be more specific, in most other retail verticals, the focus is on customer loyalty based on historical data and collaborative filtering [1] with little focus on history-agnostic approaches. Even more, most retail verticals do not have meta-products or relevant meta-information similar to these examples from pharma retail vertical: target disease, target physiological need, target body part etc.

Our work presents a way to extend capabilities of recommender systems by learning low-dimensional vector space representation of products and users – embeddings – used in the following stage for sales prediction.

Semantically, our pipeline can be divided in two separate general models with end-to-end learning capabilities: the early stage of product semantic analysis and later stage of product sales regression. The key point within the initial stages of our pipeline system is learning product and customer feature vector embeddings. Our work is analogous to Word2Vec [2] and GloVe [3] approaches which are used to learn linguistic regularities and semantic information for natural language processing. The general approach within the initial stage is based on analyzing each sequence from the transactions database, choosing $c$ products around a target product $p$ in the same way a NLP system would analyze text semantics. The result will reveal that the embeddings generate clusters of complementary and similar products. As a result, through this approach we can identify a set of $k$ items that will be of interest to a certain customer. Moreover, besides the product similarity, the resulting embeddings will bring to light an important aspect: if a product is not available anymore, it can be replaced with other two products whose embeddings sum will be very close to the initial product feature vector. Our product embeddings can be directly used in order to determine pharmacy product complementarity assessment based on cosine similarity. A more advanced use, also derived from known neural linguistic models...
approaches, is that of generating “concept” products that capture specific needs and answer advanced queries such as “What natural remedies are good for back-pain as it is Vitamin C for simple cold?” that actually translates in a more domain-specific query such as “What products are complementary to product A as it is product Z for product X?”.

The main need of such a predictive analytics model that can easily recommend market baskets based on the latent semantic space of products comes from the increasing evolution of online advertisements that should target very well the users in order to reach their demands. This is an important aspect that make people to visit again the sites or to use the applications without being annoyed by not interesting advertisements. Consequently, the financial success of retailers is now strongly related to their users’ retention, which is an effect of tailoring for their tastes every single moment.

In order to finish the pipeline, our work includes also a deep neural model which uses the feature vectors and predicts the sales. As we mentioned, this is a crucial aspect for all companies if they want to set up an efficient marketing campaign.

To the best of our knowledge, this work represents the first study that offers an end-to-end recommendation solution in the pharmaceutical field.

A final important observation regarding our research and experimental development is that our focus has been exclusively on behavioral analytics with marginal direct impact on sales forecasting.

In Section II we will present the approaches that our work relates to. In Section III we present our proposed P2E (Product-to-Embeddings), U2E (User-to-Embeddings) and ProVe (Product Vectors) models, together with our proposed deep neural model for final stages of sales prediction. In Section IV, the results of the proposed recommender system using a pharmaceutical transactional database. Finally, Section V will emphasize the conclusions of this work and the future development directions.

II. RELATED WORK

Our work relates with several approaches either derived from Natural Language Processing or from classic methods that address the problem of recommendation systems.

A. Traditional Approaches

Existing methods for recommender systems can easily be categorized into collaborative filtering methods [1], [4], [5] and content-based methods [6], which make use of the user or product content profiles. Collaborative filtering is based on user-item interactions and predict which products a user will most likely be interested in by exploiting purchase behavior of users with similar interests or by using user’s interaction with other products. CF methods increased in popularity because they can discover interesting associations between products and do not require the heavy knowledge collection needed by content-based methods. To mitigate the cold-start problem, which CF methods suffer from, matrix factorization-based models have been developed and now they are very popular after their success in the Netflix competition.

Matrix factorization [7], [8] for collaborative filtering models can approximate a sparse user-item interaction matrix by learning latent representation of users and items using SVD or stochastic gradient descent which give the optimal factorization that globally minimizes the mean squared prediction error over all user-item pairs.

B. Neural NLP Models

In a number of Natural Language Processing (NLP) tasks, such as computing similarity between two documents, learning linguistic regularities and semantic information are essential. Therefore, a mathematical model has been developed by Mikolov et al. [2] (Word2Vec), which can be used for learning high-quality low-dimensional word embeddings from huge datasets and huge vocabularies, using two architectures of neural networks: continuous bag-of-words (CBOW) and skip-gram (SG).

This powerful and efficient model takes advantage of the word order in the text documents, explicitly modelling the assumption that closer words in a context window are statistically more dependent. In the SG architecture, the objective is to predict a word’s context given the word itself, whereas the objective in the CBOW architecture is to predict a word given its context.

GloVe [3] is a novel model for learning low-dimensional vector representations of words by combining the advantages of two major model families in the NLP literature: global matrix factorization and local context window methods (Word2Vec).

They consider the primary source of information available about a corpus of words being the word-word co-occurrence counts which is used to train the fine-grained word embeddings. Explicitly, the ratio of the co-occurrence probabilities of two words (rather than their co-occurrence probabilities themselves) is what contains the information encoded as word embeddings.

III. APPROACH

A. From NLP to Recommender Systems

Traditionally, in NLP applications, each word is represented as a feature vector using a one-hot representation where a word vector has the same length as the size of the vocabulary. Our first approach was to create a corpus of words from all pharmaceutical prospectuses and to encode each product, using hand engineered features, where feature i is 1 if the word i appears in the prospectus of a product and 0 otherwise. Then, we created a model based on XGBoost Regression Trees [9] which predicted sales for segments of users with a 74% r2-score. However, this approach suffers from high dimensionality and data sparsity and does not meet our first scope - predicting market baskets.

B. Products Embeddings

Considering the big improvement word embeddings brought to NLP domain, we were confident that from language models to product business analytics is a very slight difference.
To address the task of finding complementarity/similarity between products and diversified market baskets for a certain customer, we proposed to learn representations of products in low-dimensional space, using the Big-Data sequential transaction system provided by the pharma retailer, which was used also for computing the word-word co-occurrence counts.

More specifically, we developed two models (P2E and ProVe) inspired by the NLP models Word2Vec and GloVe. Our first model uses the sequential transactions. The transaction system can be represented as a set \( S \), where:

- \( S = (t_1, t_2, t_3, \ldots, t_M) \), \( M \) = total number of transactions/market baskets/receipts
- \( t_i = (p_{i_1}, p_{i_2}, \ldots, p_{i_{T_i}}) \), \( T_i \) = total number of products on \( i \)-th receipt.

Our objective is to find \( D \)-dimensional real-valued representations \( \mathbf{w}_{p_j} \in \mathbb{R}^D \) of each product \( p_i \) such that they lie in a latent vector space. The general approach within this initial stage is based on analyzing each transaction, predicting each target product \( p_i \) using all context products (all products that are on the same receipt) (Figure 1). If there is only one product in a market basket, it is skipped due to the fact that the transaction does not contain complementarity information.

Therefore, the objective function of CBOW architecture is defined as follows

\[
J = \frac{1}{M} \sum_{i=1}^{M} \log P(p_i | p_{i-c}, \ldots, p_{i-1}, p_{i+1}, \ldots, p_{i+c})
\]

where probability \( P(p_i | p_{i-c}, \ldots, p_{i-1}, p_{i+1}, \ldots, p_{i+c}) \) of predicting the current product \( p_i \), given a product context is defined using the softmax function (2):

\[
P(p_i | p_j) = \frac{e^{w_j^T w_{p_i}}}{\sum_{k=1}^{P} e^{w_j^T w_{p_k}}}
\]

The original Word2Vec model proposes a sampled-softmax loss \([2]\) which is a computationally efficient approximation of the full softmax. Given the fact that our models are trained in a high-performance computing environment, we used the non-approximated version which resulted in better results. Simultaneously, the projection layer computes a concatenation of all \( c * D \) embeddings, instead of summation (Figure 1). This aspect leads to more computation for the softmax layer, but also to a better accuracy.

The second model (ProVe) seeks to learn low-dimensional product embeddings using the ratio of co-occurrence probabilities of two products. Therefore, the model generates two sets of product vectors \( \mathbf{W} \) and \( \mathbf{\tilde{W}} \) and it should minimize the weighted least squares objective \( J \) defined as follows:

\[
J = \sum_{i,j=1}^{P} f(X_{ij}) (w_{p_i}^T \tilde{w}_{p_j} + b_i + \tilde{b}_j - \log X_{ij})^2
\]

The product-product co-occurrence score measures how often two products are bought together in the same market basket (4).

\[
X_{ij} = X_{ij} + \frac{1}{d(p_i, p_j)}
\]

For both P2E and ProVe, we applied KMeans algorithm [10] on the resulting embeddings which is our first approach for the objective of determining the concept vectors which should capture all the needs of a certain product. For the particular case of pharmaceutical products, the concept vectors may capture, for example, information about the position of the ‘flu’ concept in the semantic latent space, which leads to obtaining full complementarity in the generation of market basket.

The market baskets are efficiently created starting from a certain product by using the cosine similarity (5) between it and all product vectors. Algorithm 1 defines the methodology used for market baskets definition.

\[
similarity(p_i, p_j) = \frac{w_{p_i} \cdot w_{p_j}}{||w_{p_i}|| \cdot ||w_{p_j}||}
\]

**Algorithm 1**

```
MarketBasket(Product, EmbeddingSpace, k)
    e <- get embedding of Product from EmbeddingSpace
    cos_dist <- compute_cos_sim(e, EmbeddingSpace)
    top_k <- choose top k closest products based on cos_dist
    Eliminate products that have the same concept vector as Product
    market_basket <- set of all complementary products
    Return market_basket
```

**Figure 1** – P2E CBOW architecture
Our approach is similar to current state-of-the-art recommender systems (Grbovic et al. [11], Vasile et al. [12]) that use multi-dimensional representation of an ecosystem’s entities (products, services etc.).

C. Users Embeddings

Motivated by the doc2vec algorithm proposed by Le et al. [13], which jointly optimizes both word embeddings and the global context of the entire document, we also employed this methodology in order to create a latent multi-dimensional semantic space of the users along with the product embeddings.

Considering that a receipt (market basket) is defined by the products that are bought and by the “global context” (user/customer), the P2E architecture is modified (Figure 2) for the objective of jointly learning users and products embeddings. Therefore, the cost function that should be minimized is defined as follows:

\[
J = \frac{1}{M} \sum_{i=1}^{M} \log \mathbb{P}(p_i | u_j, p_{i-c}, ..., p_{i-1}, p_{i+1}, ..., p_{i+c}) \quad (6)
\]

where probability \( \mathbb{P}(p_i | u_j, p_{i-c}, ..., p_{i-1}, p_{i+1}, ..., p_{i+c}) \) is defined using the softmax function (2).

The customer embedding space is created exclusively using transactional information (no personal information). Therefore, this latent space encodes the similarity between customers according to their buying behavior. The latent vectors corresponding to each individual are already used in pharmaceutical commercial applications for various objectives:

- Segmentation of customers according to their tastes (which products they have bought);
- Individual marketing campaigns;
- Products propensity-to-buy prediction.

The training data used for the user embeddings model jointly trained with the product embeddings has over 4 million user data ranging from customers with a transaction up to customers with 2000 transactions. The actual user embeddings calculation can be evaluated based on the number of transactions per each user (the more the better). On average, our model optimizes the user embeddings (starting from an initial random uniform allocation) on more than 60% of the population.

In order to evaluate customer (user) embeddings we have to first select the ones that have passed the minimal number of transactions in order to ensure that the backpropagation optimization process has managed to assign a viable position to the customer in the latent space. Following this first step, we can either use the user embeddings in order to train shallow or deep models for customer-product propensity to buy inference or use simple analytics approaches in order to compare customer average product baskets for customers that reside in the same area of the embedding space.

D. Sales Regression Model

The last model in the proposed deep neural pipeline predicts customer buying propensities for any product in the ecosystem based on their optimized embedding spaces, in which every entity has a well determined position "extracted" from the sequential transactional information.

This model’s output depends on the input. In our case, the input, as described in 0, represents the total sales registered during a year. Therefore, the sales regression model will generate the customer-product propensity-to-buy in the next year. This model is involved in pharmaceutical marketing campaigns which offer discounts for customers based on their propensities-to-buy.

For this particular model, we researched and developed a fully-connected deep neural network [14] with 3 layers. This neural network has two inputs: one for user embeddings and one for product embeddings and the readout layer has a single neuron which is a regressor (7) and predicts totally supervised how much will spend the customer for a certain product.

\[
J = \frac{1}{M} \sum_{i=1}^{M} (f(u_i, p_i) - y_i)^2 \quad (7)
\]

E. End-to-end Pipeline Formalization

Our approach manages to build embeddings for products and users from transactional pharmaceutical data. The resulting embeddings are used to predict the propensity-to-buy of each customer to any product. The obtained prediction is used for recommendation, together with information extracted the market basket prediction. An end-to-end pipeline formalization is presented below:

- \( \{w_{p_1}, w_{p_2}, ..., w_{p_n}\} = f(p_1, p_2, ..., p_n) \), where \( n \) is the total number of products and \( f \) is the product embedding function \( f \) is the product embedding function optimized using (7) via backpropagation,
which returns the \( D_1 \)-dimensional real-valued representations \( w_{p_1} \in \mathbb{R}^{D_1} \);

- \( \{w_{u_1}, w_{u_2}, \ldots, w_{u_m}\} = g(u_1, u_2, \ldots, u_m) \), where \( m \) is the total number of users and \( g \) is the user embedding function optimized using (6) via backpropagation, which returns the \( D_2 \)-dimensional real-valued representations \( w_{u_j} \in \mathbb{R}^{D_2} \);

- \( \{p_{\text{compl}_1}, p_{\text{compl}_2}, \ldots, p_{\text{compl}_k}\} = MB(p_i, f, k) \), where \( p_i \) is the product for which is needed to find complementary products, \( f \) is the optimized product embedding function, \( k \) is the dimension of the market basket and \( MB \) is the function that computes the set of all complementary products based on similar embeddings;

- \( \text{total\_amount} = h(u_j, p_i) \), where \( h \) is the function optimized using (7) via backpropagation which returns the propensity-to-buy of the customer \( u_j \) to the product \( p_i \).

IV. EXPERIMENTS AND RESULTS

The pipeline was trained and tested using a Big-Data sequential transaction system comprising more that 200 million purchases made by 4.3 million users, involving about 27,000 unique pharmaceutic products. All the models were trained on a high-performance computing (HPC) environment using CUDA [15] kernels that are deployed on a NVIDIA QUADRO P5000 GPU card.

A. Market Basket Experiment

For the first objective of creating market baskets, we trained our product and user embeddings models - P2E, ProVe and U2E - during 50 epochs and using Adagrad [16] optimizer with learning rate = 1 and initial accumulator value = 0.1. Each epoch lasted 1 hour and 35 minutes on the mentioned HPC.

In order to evaluate the product embeddings, we transformed them in a two-dimensional “meta-map” (Figure 3) employing t-Distributed Stochastic Neighbor Embedding (t-SNE) technique [17]. In the figure are highlighted several different regions that actually contain strong semantic meanings: cardiovascular products (which can be used for high blood pressure, thrombosis, angina or stroke prevention), products used for woman care, products used against flu etc. The colors represent the concept vectors discovered using KMeans algorithm.

B. Sales Regression Experiments

For the sales regression objective using the fully-connected deep neural network (160-80-1) presented as a graph in Table 1, we structured a 1-year transactional information such that each observation was defined by a pair customer-product and the total amount of money spent by the customer on that product during a year. Formally, we can define the dataset as a pair \( (\{X_1, X_2\}, y) \), where \( X_1, X_2 \) represents the user and product IDs and \( y \) represents the target – the sales generated by the customer on that product.

The fully-connected deep neural network (160-80-1) was trained during 35 epochs using Adam [18] optimizer with learning rate = 0.0025 and batch size = 512. Each epoch lasted almost 90 seconds on the mentioned HPC.

| Layer (type) | Output Shape | Connected to |
|--------------|--------------|--------------|
| input_user (InputLayer) | (None, 1) | |
| input_prod (InputLayer) | (None, 1) | |
| emb_user (Embedding) | (None, 32) | input_user |
| emb_prod (Embedding) | (None, 128) | input_prod |
| dense_input (Concatenate) | (None, 160) | emb_user emb_prod |
| fc_layer_1 (Dense) | (None, 160) | dense_input |
| fc_layer_2 (Dense) | (None, 80) | fc_layer_1 |
| readout_layer (Dense) | (None, 1) | fc_layer_2 |

We have also experimented early in our research with hand-engineered features based on bag-of-words approach on product descriptions. Nevertheless, the product embeddings proved much more efficient and scalable both in terms of resource and performance.

The baseline for our sales regression model was our model based on XGBoost Regression Trees and hand-engineered features (III.A) which obtained a 74% r2-score. This model cannot be used in production-grade systems due to very high data sparsity. Another main drawback of this model is its lack of knowledge about the powerful embedding spaces created using our P2E and U2E models.

We defined four different experiments (Table 2) whose purpose was to determine the effect of continuing/not continuing the optimization of the embeddings based on the sales registered in the dataset used for the training process. Our insight was that
the optimization of the latent spaces in the sales prediction process will also encode the other meta information relating to how much a customer spend, who are the customers that have the propensity to buy more and which are the cheap/expensive products. Besides the optimization of the embeddings accordingly to how much each customer buy, the deep neural model acts like a prediction function $f(u, p)$ between the latent spaces and the total sales amounts.

| Experiment ID | Continue opt. user embeddings | Continue opt. prod embeddings | R² score |
|---------------|--------------------------------|-------------------------------|---------|
| 1             | ✗                              | ✗                             | 84%     |
| 2             | ✗                              | ✔                             | 76%     |
| 3             | ✔                              | ✗                             | 91%     |
| 4             | ✔                              | ✔                             | 88%     |

V. CONCLUSIONS AND FURTHER WORK

A. Conclusions

The presented work describes the research and development steps for an end-to-end recommender system which is part of a commercial application already used by a pharmaceutical retail company in their sales strategy and marketing campaigns. We can conclude that our work manages to innovate the area of predictive analytics through the following main approaches:

- An approach that generates completely unsupervised products’ and users’ semantical information (embeddings) using only sequential transactional data;
- A technique that use the products’ embeddings to compute the concept vectors of the ecosystem (in our particular case, the pharmaceutical ecosystem) in order to generate complementary market baskets;
- An approach that use the optimized embeddings and generates year-to-year sales prediction. Simultaneously, this approach enables further optimization of the embeddings in order to capture, besides the transactional patterns, also the amount information.

B. Underway and Future Improvements

It remains to explore which are the benefits of the current state-of-the-art neural networks in sequence processing (recurrent neural networks – particularly, long short-term memory [19]) for recommender systems. More precisely, we are currently researching a recurrent model that is able to process all the transaction of a customer and to predict which is the next basket that is most likely to be bought by that certain customer.

This recurrent model will be a powerful tool, because it will encode, besides the latent spaces of embeddings, also the seasonality and temporality.

REFERENCES

[1] G. Linden, B. Smith and J. York, "Amazon.com recommendations: Item-to-item collaborative filtering," IEEE Internet Computing, vol. 1, pp. 76-80, 2003.
[2] T. Mikolov, J. Sutskever, K. Chen, G. Corrado and J. Dean, "Distributed representations of words and phrases and their compositionality," in proceedings of the 26th International Conference on Neural Information Processing Systems - Volume 2 (NIPS’13), C. J. C. Burges, L. Bottou, M. Welling, Z. Ghahramani, and K. Q. Weinberger (Eds.), Vol. 2. Curran Associates Inc., USA, 3111-3119, Lake Tahoe, Nevada, 2013.
[3] J. Pennington, R. Socher and C. Manning, "Glove: Global Vectors for Word Representation," in EMNLP. 14. 1532-1543. 10.3115/v1/D14-1162, 2014.
[4] A. Karatzoglou, X. Amatriain, L. Baltrunas and N. Oliver, "Multivariate recommendation: N-dimensional tensor factorization for context-aware collaborative filtering," Proceedings of the Fourth ACM Conference on Recommender Systems, RecSys ’10, pp. 79-86, 2010.
[5] X. Ning and G. Karypis, "SLIM: sparse linear methods for top-n recommender systems," in 11th IEEE International Conference on Data Mining, ICDM, Vancouver, Canada, December 11-14, 2011.
[6] M. J. Pazzani and D. Billsus, Content-Based Recommendation Systems, Brusilovsky P., Kobsa A., Nejdl W. (eds) The Adaptive Web. Lecture Notes in Computer Science, vol 4321, 2007.
[7] P. O. Hoyer, "Non-negative Matrix Factorization with Sparseness Constraints," Journal of Machine Learning Research, vol. 5, pp. 1457-1469, 2004.
[8] Y. Koren, R. Bell and C. Volinsky, "Matrix factorization techniques for recommender systems," in Computer, 42(8):30-37, Aug. 2009.
[9] T. Chen and C. Guestrin, XBBoost: A scalable tree boosting system, San Francisco, CA: Proceedings of the 22nd ACM SIGKDD Conference on Knowledge Discovery and Data Mining, 2016.
[10] R. Xu and D. Wunsch, "Survey of clustering algorithms," in IEEE Transactions on Neural Networks DOI: 10.1109/TNN.2005.845141 , 2005.
[11] M. Grbovic, V. Radosavljevic, N. Djuric, N. Bhamidipati, J. Savla, V. Bhagwan and D. Sharp, "E-commerce in Your Inbox: Product Recommendations at Scale," in proceedings of the 21th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD ’15). ACM, New York, NY, USA, 1809-1818. DOI: https://doi.org/10.1145/2783258.2788627 , Sydney, NSW, Australia, 2015.
[12] F. Vasile, E. Smirnova and A. Conneau, "Meta-Prod2Vec: Product Embeddings Using Side-Information for Recommendation," in proceedings of the 10th ACM Conference on Recommender Systems (RecSys ’16). ACM, New York, NY, USA, 225-232. DOI: https://doi.org/10.1145/2959100.2959160 , 2016.
[13] Q. Le and T. Mikolov, "Distributed representations of sentences and documents," in proceedings of the 31st International Conference on International Conference on Machine Learning - Volume 32 (ICML'14), Eric P. Xing and Tony Jebara (Eds.), Vol. 32. JMLR.org II-1188-II-1196, Beijing, China, 2014.
[14] J. Schmidhuber, "Deep Learning in Neural Networks: An Overview," Neural Networks DOI: 10.1016/j.neunet.2014.09.003, vol. 61, pp. 85-117, 2015.
[15] J. Ghorpade, J. Paradie and M. Kulkarni, "GP-PROCESSING IN CUDA ARCHITECTURE," Advanced Computing: An International Journal (ACIJ), vol. 3, 2012.
[16] J. Duchi, E. Hazan and Y. Singer, " Adaptive Subgradient Methods for Online Learning and Stochastic Optimization," Journal of Machine Learning Research 12, pp. 2121-2159, 2011.
[17] L. van der Maaten and G. Hinton, "Visualizing Data using t-SNE," Journal of Machine Learning Research 9, pp. 2579-2605, 2008.
[18] D. P. Kingma and J. Ba, "Adam: A Method for Stochastic Optimization," arXiv preprint arXiv:1412.6980, 2017.
[19] S. Hochreiter and J. Schmidhuber, "Long short-term memory," *Neural Comput.* 9, p. 1735–1780, 1997.