Anomalous proximity effect in a superconductor under a strong electric field

B.Ya. Shapiro, I. Shapiro and Y. Atzmon

Abstract. We considered a superconductor subjected to a strong dc electric field. We show that even a short distance penetrating electric field can strongly affect a superconducting-normal interface. Using the Ginzburg-Landau equation complemented by the boundary conditions modified by the electric field we calculated the order parameter, distribution at the surface, the surface energy, the critical Josephson current and the voltage current characteristic of a superconductor, separated by a thin layer where a strong electric field is applied.

Department of Physics, Institute of Superconductivity, Bar-Ilan University, Ramat-Gan 52900, Israel
E-mail: shapib@mail.biu.ac.il

1. Introduction
It is widely accepted that a DC electric field applied to a metallic surface can only slightly affect the superconducting properties in the bulk. This is so since in a metallic superconductor the electric field penetrates through a short (Thomas-Fermi) distance \( l_{TF} \approx 1\, \text{A} \) as in a normal metal and cannot affect the superconductor properties due to the proximity effect [1]. Recently, renewed interest in this problem has arisen again due to Tao experiment where it was discovered that if the applied electric field exceeds a critical value, typically of order \( 0.5 - 1\, \text{kV/mm} \), then millions of metallic superconducting microparticles each of size \( 5\, \mu \text{m} \) separated by the liquid nitrogen inside the cavity, spontaneously aggregate into spherical clusters of millimeter size [2]. This phenomenon still remains mysterious. Here we show, however, that even a short distance penetrating electric field can strongly affect some of superconducting properties. In fact the boundary condition at the superconducting-normal interface are strongly modified by the applied electric field. Using the Ginzburg-Landau equation complemented by the boundary conditions modified by the electric field we calculated the order parameter distribution at the surface, the surface energy which is found to be positive and the critical Josephson current. In particular we concluded that the electric field can suppress superconductivity on a distance much longer than the electric field penetration length \( \xi_{GL} >> l_{TF} \).

2. Basic Equations.
In order to calculate the modified in the electric field boundary conditions we employ the model of a superconducting metal with the electron-electron constant \( g = g_0 \) for \( x > 0 \) and \( g = 0 \) for negative \( x \) and with a narrow dielectric layer \( (d \sim l_{TF} << \xi_0 \) here \( \xi_0 \) is the coherence length) at \( x = 0 \) with the dc electric field inside. In this case the spatial distribution of the order parameter \( \Psi \) in a superconducting semi-space is described by the Ginzburg-Landau Hamiltonian...
\[ \Omega = N \int dx \left\{ -t |\Psi(r)|^2 + \xi^2 |D\Psi(r)|^2 + \frac{\beta}{2} |\Psi(r)|^4 + \frac{B^2}{8\pi} \right\} \] (1)

\[ \partial = \frac{\partial}{\partial r} - \frac{2ei}{c} A; \ B = \nabla \times A; \ t = 1 - \frac{T}{T_c} \]

(here \(N\) is the density of states at the Fermi level) which is complemented by the boundary condition at \(x = 0\) [3]

\[ (\partial \Psi)_{x=0} = \frac{\Psi(0)}{\xi q} \] (2)

where \(\xi\) is the coherence length and

\[ q = \frac{\pi^4}{56\zeta(3)} \int_0^1 y (2 - D(y)) dy + \frac{21\zeta(3)}{\pi^2} \left( \int_0^1 y (2 - D(y)) dy \right)^2 \] (3)

Here \(D(y)\) is the transparency of the superconducting-normal boundary for a single normal electron, (here \(y = \cos \theta\), \(\theta\) is the incidence angle between the electron momentum and the surface). In particular, for completely nontransparent surface when \(D = 0, q \to \infty\).

In this consideration the transparency of the superconducting-normal border is treated as a transparency of the barrier for a normal electron. The electric field inside the barrier diminishes its magnitude \(U(U_{\text{eff}} = U - Eex)\) and consequently the interface transparency is increased. As a result, the boundary condition for initially nontransparent barrier in reads

\[ (\partial \Psi)_{x=0} = \varepsilon \frac{\Psi(0)}{\xi} \] (4)

where \(\varepsilon < 1\) is the dimensionless electric field inside the barrier (in the atomic field units).

The superconducting free energy reads

\[ \Omega = -N\beta \int_0^\infty |\Psi|^4 dx \] (5)

where \(\Psi(x)\) is the solution of the Ginzburg-Landau equation for the GL Hamiltonian 1.

### 3. Interface Energy

In the absence of the magnetic field the order parameter is described by the GL equation of the form

\[ -\xi^2 \frac{d^2 \Psi}{dx^2} - t \Psi + \beta \Psi^3 = 0 \] (6)

with the boundary condition

\[ \left( \frac{d\Psi}{dx} \right)_{x=0} = \varepsilon \frac{\Psi(0)}{\xi} \] (7)
Considering a nontransparent interface in the first order ($\Delta << 1$) we obtain for the order parameter

$$\Psi = \Psi_0 \left( 1 - \frac{\varepsilon}{\sqrt{2}} \exp \left( -\sqrt{2}tx/\xi \right) \right); \Psi_0^2 = t/\beta$$

and for additional positive interface energy due to the electric field at the surface.

$$\Omega_E - \Omega_0 = N\varepsilon S \frac{\xi^{3/2}}{\sqrt{2}\beta}$$

Here $S$ is the area of the contact.

It should be noted that in principle, it may be favorable energetically to create one big superconducting ball instead of large number of small superconducting particles (Tao effect).

### 4. The Josephson critical current

The expression for the Josephson critical current can be easily obtained from the boundary conditions (2). It reads

$$I_c = \varepsilon \xi N \left| \Psi_0^2 \right| = \frac{e\xi N t}{\beta} \frac{V}{V^*}$$

This dependence on the electric field results in crucial modification of the Josephson equations for the contact. In particular, the basic Josephson relation for the phases difference $\Phi$ is transformed into the form:

$$\alpha = \frac{I}{I_c} = \frac{\hbar}{2eV^*} \frac{\partial \Phi}{\partial \tau} \sin \Phi$$

here $\tau$ is the time.

This effect can change dramatically all of the predictions for the Josephson electrodynamics, making the electric field in the contact an effective tool in driving the Josephson effect devices. In particular this term can significantly modify the voltage-current characteristic of the Josephson contact resulting in additional Ohmic resistance below the critical current. In a more realistic models where the transparency of the interface is not zero, the boundary condition (4) is

$$(\partial \Psi)_{x=0} = (a + \varepsilon) \frac{\Psi (0)}{\xi}$$

where $a \sim 1$ and is the electric field independent. In this case the basic equations of the Josephson circuit has the following form (for the overdamped contact):

$$\alpha = \left( 1 + \sigma_J \frac{\partial \Phi}{\partial \tau} \right) \sin \Phi + \beta_J \frac{\partial \Phi}{\partial \tau}$$

where $\beta_J, \sigma_J$ are the dimensionless Josephson parameters [4].

The voltage-current characteristic of the Josephson contact is significantly modified and takes the form

$$\left\langle \frac{\partial \Phi}{\partial \tau} \right\rangle = \frac{\sqrt{\alpha^2 - 1}}{\sigma_J \left[ \alpha - \sqrt{\alpha^2 - 1} \right] + \beta}$$
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