The $t\bar{t}$ Threshold and Machine Parameters at the NLC

D.Cinabro
Wayne State University, Detroit, MI 48202, USA

One of the problems in the design of a high energy $e^+e^-$ linear collider is the distribution of the luminosity as a function of real collision energy, $d\mathcal{L}/dE$, due to initial state radiation, beamstrahlung, and the energy spread of the collider. These effects smear a sharp feature in the cross section of $e^+e^-\rightarrow$ hadrons such as the $t\bar{t}$ threshold into a flatter structure. This study reviews the impact on $d\mathcal{L}/dE$ of these effects as a function of machine parameters, explores some methods of measuring $d\mathcal{L}/dE$ with Bhabha scattering, how the $d\mathcal{L}/dE$ flattens the $t\bar{t}$ cross section near threshold, and the effect the $d\mathcal{L}/dE$ measurement has on extracting parameters of the top quark, such as the mass and width, in a 2.5/fb scan of the $t\bar{t}$ threshold.

One of the prime goals of a high energy $e^+e^-$ linear collider is the study of sharp features in the $e^+e^-\rightarrow$ hadrons cross section. The $t\bar{t}$ threshold is an excellent example of such a structure. The cross section for $e^+e^-\rightarrow t\bar{t}$ is expected to rise by an order of magnitude with only a 10 GeV change in center-of-mass energy around 350 GeV. Careful study of this $t\bar{t}$ threshold structure can precisely measure many parameters of the top quark, including its mass and width.

One of the major problems in the design of a high energy $e^+e^-$ linear collider is the effects that cause the luminosity spectrum of the collisions, $d\mathcal{L}/dE$, to be smeared. One of these, initial state radiation, ISR, is simply the effect of QED on the initial electron and positron in $e^+e^-$ collision and cannot be controlled. On the other hand beamstrahlung, electromagnetic radiation by the particles in one bunch caused by their interaction with the electric field of the bunch they are colliding with, and the energy spread of the linac can be controlled by varying parameters of the collision. These parameters, such as the size of collision region and the bunch current, are also crucial in determining the luminosity of the machine. In general the parameters that lead to the highest luminosity also lead to largest smearing of $d\mathcal{L}/dE$.

This work studies the question of the impact of the machine parameters of a high energy $e^+e^-$ linear collider on the extraction of the parameters of a sharp feature in the $e^+e^-\rightarrow$ hadrons cross section. First I review the effects that smear $d\mathcal{L}/dE$. Next I explore various techniques of measuring $d\mathcal{L}/dE$ in Bhabha scattering. Then I show the effect of $d\mathcal{L}/dE$ on the $t\bar{t}$ threshold and the effect the measurement of $d\mathcal{L}/dE$ with Bhabha scattering has on the extraction of top quark parameters in a scan of the $t\bar{t}$ threshold.
There are three effects that smear the collision energy of any $e^+e^-$ collider. The first is a consequence of QED; the incoming electron and positron can radiate photons leaving them at an energy below the nominal beam energy. This is called initial state radiation, ISR. It is simulated using the Pandora Monte Carlo which uses the Skrzypek-Jadach approximation for the energy spectrum of the incoming beams. ISR depends only on the nominal beam energy and its effect on a 350 GeV center of mass collision is shown in Figure 1(a).

The second effect is the interaction of the particles in one beam with the electric field generated by the other beam as the two collide. This interaction causes the beams to radiate photons and is called beamstrahlung. Beamstrahlung is a large effect at a machine like the NLC where the beam sizes are very small and thus produce very intense fields. It is controlled by the number of particles in the colliding bunches, the relativistic factor gamma, and length and longitudinal sizes of the beam. These are used to calculate the disruption factor, $\Upsilon$, the size of which controls the amount of beamstrahlung experienced by the colliding beams. Table 1 shows some possible beam parameters for a 350 GeV nominal center of mass collision energy NLC design. Note that in general as $\Upsilon$ and the size of the beamstrahlung effect increases, the luminosity also increases. Figures 1(b-e) show the effect on the $d\mathcal{L}/dE$ spectrum of the
Table 1: Possible parameter choices for a 350 GeV NLC.

| Parameter Set | A (×10^{-10}) | B | C | D |
|---------------|---------------|---|---|---|
| N (×10^{10})  | 0.95          | 0.75 | 0.95 | 1.1 |
| $\sigma_x$ (nm) | 402          | 246 | 327 | 365 |
| $\sigma_y$ (nm) | 6.40       | 3.39 | 4.88 | 7.57 |
| $\sigma_z$ (µm) | 120         | 90 | 120 | 145 |
| $\Upsilon$    | 0.061        | 0.094 | 0.075 | 0.065 |
| $\mathcal{L} \times 10^{33}$/cm$^2$/sec | 4.00 | 6.51 | 5.84 | 5.21 |

beamstrahlung effect on top of the ISR effect described above respectively for the four possible NLC parameters given in Table 1 using the Pandora Monte Carlo.

The third effect at a high energy $e^+e^-$ linear collider is the intrinsic energy spread of the linac. An expectation for the NLC is summarized in 4. In general the energy spread is not Gaussian, but it can be accurately measured using synchrotron radiation, and it is an effect is in the range of 0.1-0.5% of the nominal beam energy. For this study I simply smeared the incoming beam energy by a Gaussian with an illustrative width of 0.3% of the nominal beam energy. This effect on $\mathcal{L}/dE$ is shown in Figures 1(f-i) convoluted with the effects of ISR and beamstrahlung respectively for the four 350 GeV NLC parameter sets of Table 1.

Many methods have been proposed to extract the $d\mathcal{L}/dE$ spectrum at a high energy $e^+e^-$ linear collider. The method which has been very effective at LEP is small angle Bhabha scattering, $e^+e^- \rightarrow e^+e^-$, and I have explored using this process at the NLC. My method of exploration was to generate about 1000 single electrons of 200 GeV and fully simulate their interactions in the American S and L detectors in various polar angle slices of the detector. These events were then reconstructed using JAS based reconstruction algorithms and selections of the single electron were made to choose well reconstructed energies and momenta. These selections resulted in a selection efficiency and a resolution on the energy of a single electron which depended on the polar angle and the analysis technique. These were then used to smear the legs of $e^+e^- \rightarrow e^+e^-$ events generated with the Pandora Monte Carlo. Cross sections are also from Pandora and are for both scattered particles to be in the given polar angle range. Four polar regions and three techniques are considered in a 2.5/fb run at 350 GeV.

The simplest analysis to consider is the barrel region. For example with
|cos θ| < 0.6 the Bhabha cross section is 3.5 pb and the event energy is best measured with the tracking system. Good single electrons are selected by requiring one and only one track pointing to only a single calorimeter cluster, no other calorimeter clusters, and requiring that the track precisely point back to the collision point. Such a selection is 60% efficient and has a resolution of 0.60% on the momentum of the track in the American L detector. In the smeared events the resolution on the event energy is extracted by looking beyond the end point of collision and in this scan is measured to be 0.803±0.080 GeV. With this resolution the dL/dE spectrum and the total luminosity can be extracted with an error limited by the statistics of the barrel Bhabha sample and a size essentially given by the statistics of the Bhabha’s.

The second simplest region is to consider is the forward region, for example 0.99824 < |cos θ| < 0.99987 with a cross section of 33200 pb. Here forward calorimetry is the only option for measuring the event energy. For single electrons making only one calorimeter cluster the efficiency is 90% and the energy resolution is 1%. In smeared events the event energy resolution is again extracted with the beyond the end point technique at is 2.702 ± 0.018 GeV. Again this resolution can be used to extract the dL/dE spectrum and total luminosity to an accuracy given by the statistics of the Bhabha sample. In this case the error on the luminosity in this scan is 0.05%, which is below the theoretical uncertainty claimed for the current generation of predictions.

A feature of the American S detector is forward silicon tracking disks. They occupy the 0.9074 < |cos θ| < 0.9903 region of the S detector with a cross section of 422 pb, and can be added to any of the detector designs considered for a high energy e⁺e⁻ linear collider. The analysis technique here is to measure the acollinearity between the two scattered particles and with the nominal beam energy extract the event energy. The single electron analysis requires there to be only one calorimeter cluster and a track which hits all four planes in the forward silicon tracker. This gives a very good resolution on the polar angle of the track of 0.02 milliradians with an efficiency of 85%. Then noting that the resolution on the event energy is related to the resolution on the acollinearity by $\sigma_{\sqrt{s}} = \sigma_{AP}/\sin \theta$, which in this case gives 18 MeV, it is easy to see that this technique measures the dL/dE and the total luminosity only limited by the statistics of the Bhabha sample.

Finally in the endcap region of the detector, 0.829 < |cos θ| < 0.996 with a Bhabha cross section of 1130 pb, any and all of the analysis techniques described above can be used. The tracking resolution on the momentum degrades from the barrel as the legs of the Bhabha become more forward but this is usually easy to measure and understand being dominantly an effect of the decreasing number of hits on the tracks. The calorimeter energy resolution
can easily be held to 1% as in the forward region. The acollinearity resolution worsens to 0.03-0.05 milliradians, but this still gives a very good event energy resolution. Track based techniques are 60% efficient and calorimeter based techniques 90%. Here the measurement of dL/dE and total luminosity is again limited by the statistics of Bhabha sample and in this scan gets into the range of the theoretical error on the cross section.

All these techniques of extracting the dL/dE spectrum and the total luminosity from Bhabha scattering at a high energy e⁺e⁻ linear collider were tried on the four different NLC machine parameter sets given in Table 1 and dependence on them is negligible. Since the extraction is limited by Bhabha statistics whatever machine gives the highest luminosity is the preferred machine from this point of view. This also implies that whatever technique samples the largest Bhabha cross section is preferred.

A caveat to both of these conclusions is that the simulations used here do not contain any treatment of beam related backgrounds. These have tremendous dependence on the details of the machine, especially in regards to the amount of beamstrahlung which besides smearing dL/dE also produces large numbers of low energy pairs in the forward direction which can degrade the resolution in the forward regions of the detector. To conclude that the best machine is the highest luminosity machine and that a forward calorimeter or forward tracker is the best place to measure dL/dE and the total luminosity this study needs to be repeated with the effects of beam related background considered. Nevertheless it should also be pointed out that a measurement based on the possible techniques in the endcaps of the proposed detectors are likely to be insensitive to the details of the machine induced backgrounds and achieve an accuracy similar to the current theoretical uncertainty.

The effects of the the dL/dE spectrum on the t\bar{t} threshold shape are quite large. Figure 2(a) shows the bare e⁺e⁻ → t\bar{t} cross section and successively the effects that smear the dL/dE spectrum have on that shape. By the end after ISR, beamstrahlung, and linac energy spread have been applied there is no sharp peak, but simply a smooth increase in the cross section. Figure 2(b) shows the smeared t\bar{t} cross section for different choices of the parameters of the top quark. This shows that the mass of the top quark effects where the rise in the cross section will take place and the width will control the steepness of the rise.

I can estimate how the measurement of the dL/dE will effect the extraction of the parameters of the top quark in a scan of the t\bar{t} threshold region. The assumptions of this study are simplistic, but seek to isolate the effects of luminosity spectrum measurement on the extraction of the top parameters. A brief scan is assumed with a total of 2.5/fb equally divided into five beam en-
energies: two below the rise; one at its center; and two at the top. Unrealistically
the number of observed $t\bar{t}$ events is assumed to be measured with an efficiency
of one and no background. Sources of error considered are the statistics of $t\bar{t}$
events and an error on the total luminosity of 0.5%, conservatively based on
the studies described in above. Such a scan is repeated 100 times and the re-
sulting cross sections are fit to shapes with the mass and width as parameters.
The width of the distribution of best fit top masses and widths are taken as
the errors.

This study yields a 50 MeV error on the top mass and a 35 MeV on the top
width with the errors being equally contributed from the statistics of $t\bar{t}$ events
and the luminosity. The study is repeated for the four NLC parameter sets of
Table 1 and I note that the error on the width shows significant dependence
on the machine parameters. For the B parameter set the error on the width
is 10% larger than the A set. This is mainly an effect of the $dL/dE$ spectrum
which is the most smeared for the B parameters. This smearing results in fewer
$t\bar{t}$ events at the scan points on the top of the rise and thus an increased error
on the steepness of the rise and the top width. Thus I do conclude that the
parameters of the machine do matter from this point of view. While higher
luminosity is generally preferred it is worth sacrificing $\sim 10\%$ of the luminosity
for a sharper $dL/dE$ distribution.

While at first glance the effect of ISR, beamstrahlung, and the linac energy
spread seem daunting on the possibility of extracting top quark parameters in
a scan of the $t\bar{t}$ threshold at a high energy $e^+e^-$ linear collider, this detailed
study shows that such extraction is not limited by the $dL/dE$ spectrum. The
$dL/dE$ spectrum can be measured to an accuracy limited by the statistics of
Bhabha scattering in various techniques using various features of the proposed
d Detectors. The many possibilities suggests that the systematic errors of such a
measurement can be limited by extensive cross checks. Many other ideas for measuring the $d\mathcal{L}/d\mathcal{E}$ spectrum still remain to be explored, and such explorations are likely to improve the picture. A key caveat is the effect of beam induced backgrounds which could severely limit the use of the forward regions in measuring the $d\mathcal{L}/d\mathcal{E}$ spectrum. This is a critical area for further study.

As much as the $t\bar{t}$ threshold can be used as the prototype of a sharp feature in the $e^+e^- \to $ hadrons cross section my studies do reveal that the optimal high energy $e^+e^-$ linear collider is the one with the highest luminosity. There is some gain on the accuracy of the extraction the parameters of a sharp feature, the width of top quark is an example, by sharpening the $d\mathcal{L}/d\mathcal{E}$ spectrum at the expense of 10% of the luminosity, but since the statistics in both the total luminosity and the events produced by the sharp feature are a major factor in determining the errors on the feature’s parameters a further reduction in the luminosity to sharpen the $d\mathcal{L}/d\mathcal{E}$ spectrum would not be useful.

Acknowledgments

My research efforts are supported by the US NSF. I would like to thank Dave Strom, Andreas Kronfeld, David Gerdes, Charley Baltay, and Dave Burke for useful discussions.

References

1. R. Frey et al. in Summary Report for Snowmass 1996, eds. D.G. Cassel, L.T. Gennari, and R.H. Siemann (1997) (hep-ph/9704243).
2. K. Thompson and T. Raubenheimer, LCC-0014; private communications K.A. Thompson and T.L. Barklow.
3. M. Peskin these proceedings; [http://www.slac.stanford.edu/~mpeskin/LC/ pandora.html](http://www.slac.stanford.edu/~mpeskin/LC/pandora.html).
4. K. Thompson, NLC-Note-10.
5. David J. Miller in Proceedings of RADCOR 98 (1999) (hep-ex/9901039).
6. J. Brau these proceedings; R. Dubois these proceedings; [http://www-sldnt.slac.stanford.edu/nld/](http://www-sldnt.slac.stanford.edu/nld/).
7. M. Ronan these proceedings; [http://www-sldnt.slac.stanford.edu/jasweb/](http://www-sldnt.slac.stanford.edu/jasweb/).
8. B.F.L. Ward et al. in Proceedings of the 1998 Rochester Conference (1999) (hep-ph/9811247v3).
9. Y. Kurihara, contribution on behalf of the KEK JLC group to First EDFA/DESY workshop, Munich September 1996.
10. M. Peskin and M. Strassler, Phys. Rev. D 43, 1500 (1991).