The entrance pupil of the human eye: a three-dimensional model as a function of viewing angle

Cathleen Fedtke,1,2,3,* Fabrice Manns,2,4,5 and Arthur Ho1,2,3

1 The Brien Holden Vision Institute, Sydney, NSW 2052, Australia
2 The Vision Cooperative Research Centre, Sydney, NSW 2052, Australia
3 School of Optometry and Vision Sciences, The University of New South Wales, Sydney, NSW 2052, Australia
4 Department of Biomedical Engineering, University of Miami College of Engineering, Coral Gables, FL 33146, USA
5 Ophthalmic Biophysics Center, Bascom Palmer Eye Institute, University of Miami Miller School of Medicine, Miami, FL 33136, USA

* c.fedtke@brienholdenvision.org

Abstract: Precise peripheral ocular measurements have become important in vision research. These measurements are influenced by the shape and position of the peripherally observed entrance pupil. A long-held assumption is that its apparent shape is elliptical and is optically centered in its position. Our three-dimensional model shows that as viewing angle increases, the entrance pupil moves forward, tilts and curves towards the observer’s direction. Moreover, the tangential pupil size narrows and exhibits asymmetric distortions. Consequently, its shape is non-elliptical and its geometric mid-point departs from the optical center. These findings may have implications on the accuracy of peripheral ocular measurements.

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Nippon NVision-K5001 autorefractor [13]. Alignment or centration for measurement of centre is decreased for peripheral refraction than for central refraction when using the Shinmeasurements. It has been shown that tolerance to lateral misalignment of the entrance pupil precise alignment appears to be of greater importance when conducting peripheral can incorrectly introduce spurious Zernike coefficients to the results [12]. The requirement for example, a modeling study has demonstrated that incorrect alignment of a wavefront sensor shape of the entrance pupil with peripheral viewing. Thus, a more precise understanding of peripheral ocular optics is one aspect of the more general change in the three-dimensional optics of the eye suggest systematic peripheral refractive profiles for hyperopic, emmetropic and myopic eyes [3–5]. In contrast to hyperopic and emmetropic eyes, which exhibit more or less relative myopic shifts with increasing peripheral visual field angle, myopic eyes exhibited an increasingly hyperopic periphery. These findings led to one current hypothesis that peripheral hyperopic defocus stimulates eye growth and hence, myopia development.

Based on these findings, peripheral refractometry has become a frequently-performed procedure in myopia research, used as a means to gain a greater insight into the mechanisms related to refractive error changes and the periphery of the eye [6,7]. For this purpose, requisite methods had to be established to enable the measurement of the peripheral ocular optics. While contrast detection tasks, which are necessarily subjective, have been considered the most appropriate in determining the quality of peripheral vision [8,9], for characterization of peripheral refraction, objective methods, such as autorefractors and wavefront sensors are coming into prominence. Technical intricacies arise, however, when applying such instruments, which were originally designed to measure on-axis refraction [10]. In one aspect, the correct alignment of the instrument axis, which typically is centered to the entrance pupil for on-axis measurements (i.e. the line of sight), is critical to measurement accuracy [11]. For example, a modeling study has demonstrated that incorrect alignment of a wavefront sensor can incorrectly introduce spurious Zernike coefficients to the results [12]. The requirement for precise alignment appears to be of greater importance when conducting peripheral measurements. It has been shown that tolerance to lateral misalignment of the entrance pupil centre is decreased for peripheral refraction than for central refraction when using the Shin-Nippon NVision-K5001 autorefractor [13]. Alignment or centration for measurement of peripheral ocular optics is one aspect of the more general change in the three-dimensional shape of the entrance pupil with peripheral viewing. Thus, a more precise understanding of

1. Introduction

Much of the clinical emphasis on vision improvement has conventionally been in achieving best foveal correction for distance and near. Refractive correction for peripheral vision, in contrast, has been largely ignored, mainly due to the perceived marginal benefit given the poor peripheral retinal acuity. More recently, vision researchers have shown increased interest in peripheral vision due to its association with refractive error development. Primarily, this association is founded on animal studies demonstrating that peripheral visual feedback can influence the emmetropisation process [1,2]. In addition, measurements of the peripheral optics of the eye suggest systematic peripheral refractive profiles for hyperopic, emmetropic and myopic eyes [3–5]. In contrast to hyperopic and emmetropic eyes, which exhibit more or less relative myopic shifts with increasing peripheral visual field angle, myopic eyes exhibited an increasingly hyperopic periphery. These findings led to one current hypothesis that peripheral hyperopic defocus stimulates eye growth and hence, myopia development.
how the geometry of the entrance pupil behaves as a function of peripheral viewing angle would be of value in understanding the accuracy of techniques directed to measuring peripheral optics.

Not only would information relating to the peripheral entrance pupil be useful in the current quest to understand the role peripheral refraction plays in myopia progression, but would be of value in many aspects of vision science, such as those requiring knowledge of retinal irradiation (e.g. calculations of Stiles-Crawford effect and ocular radiation safety).

The peripheral entrance pupil has been studied previously [14,15]. However, as these were in vivo human studies, the size and position of the anatomical pupil (aperture stop) of the eye is unknown. Hence, information such as pupil magnification and the relationship between entrance pupil and actual pupil centers could not be ascertained. In addition, in these studies, the axial position and the consequential three-dimensional shape of the entrance pupil were not investigated. Finally, only a limited number of pupil sizes were studied (limited to e.g. large versus normal or dilated versus natural).

Thus, the specific aim of this work was to extend the existing work on peripheral entrance pupil by modeling and assessing the three-dimensional entrance pupil position, shape and centration as a function of viewing angle and pupil size.

2. Methods

2.1 Model of the entrance pupil for different viewing angles

The entrance pupil of the eye was modeled as a function of viewing angle and pupil size by ray-tracing using Zemax EE (version 17 October 2008, Zemax Development Corporation, Washington, USA). Nine horizontal viewing angles ranging from 0° to 80° in 10° steps, and six pupil diameters ranging from 1.0 to 6.0 mm in 1.0 mm steps were analyzed.

The anterior segment (i.e. cornea and iris surfaces) of the Navarro schematic model for the human eye [16] was used for the optical modeling of the entrance pupil. The components of this model were assumed to be circular and co-axial, and the actual iris/pupil surface had zero thickness.

For the ray-tracing analysis of one pupil size at one viewing angle, the iris/pupil surface was set as the object. Viewing angles were modeled as directions in the horizontal plane (around a vertical axis). Thus within this model system, the tangential meridian lies in the horizontal plane while the sagittal meridian lies in a vertical plane. The iris/pupil diameter was assigned the pupil size to be modeled. Sixteen points on the iris/pupil surface and lying on the iris/pupil margin were defined, representing points on sixteen equally spaced semi-meridians at 22.5° increments. In addition, the point at the iris/pupil centre was also analyzed. Thus a total of seventeen object points were analyzed.

Using the robust, real ray-aiming options in Zemax, from each of these object points, 24 rays consisting of 3 rings by 8 arms of rays (defined using the Zemax default merit function whereby the rings represent ray-heights of 0.336, 0.707 and 0.942 times the pupil diameter, and the arms represent ray meridians 0.25π apart starting at 0.125π) were traced towards the ‘observer’, which is defined as a surface with a 10-mm clear diameter. The orientation of the observer is constrained so the chief ray has an angle equal to the viewing angle being analyzed and the observer surface is perpendicular to the chief ray. In addition, the position of the observer is constrained so it remains at 100 mm distance from the anterior corneal apex. This distance portrays a typical slit-lamp microscope configuration. Within the above constraints, the observer is translated along the horizontal plane until the chief ray from the object point passes through the centre of the observer surface.

For each object point, on emergence from the final surface (anterior cornea) following ray-tracing, the position of its virtual image point is determined using a merit function criteria of minimizing RMS radius of the 24 ray-intercept points from their centroid.
In this way, the sixteen image points defining the margin of the entrance pupil and the single point defining the image of the centre of aperture stop were computed. This procedure was repeated for the range of pupil sizes and viewing angles defined. Figure 1 illustrates the optical layout for modeling of the entrance pupil at a 40° viewing angle.

![Optical layout for modeling the entrance pupil at a 40° viewing angle using ray-tracing of several pupil margin points. Each individual pupil point (object point) projects 24 rays which are traced to the observer. The corresponding virtual image point is identified by applying a minimum RMS radius criterion to the emergent rays. By joining the locus of image points, as exemplified here (for clarity, only 8 points for the lower pupil margin and the central pupil point are shown), the three-dimensional entrance pupil (dotted line) is determined.](image)

**Fig. 1.** Optical layout for modeling the entrance pupil at a 40° viewing angle using ray-tracing of several pupil margin points. Each individual pupil point (object point) projects 24 rays which are traced to the observer. The corresponding virtual image point is identified by applying a minimum RMS radius criterion to the emergent rays. By joining the locus of image points, as exemplified here (for clarity, only 8 points for the lower pupil margin and the central pupil point are shown), the three-dimensional entrance pupil (dotted line) is determined.

### 3. Results

#### 3.1 Entrance pupil relative to the actual pupil

Figure 2 shows the position of the entrance pupil relative to the eye’s actual pupil for front-on viewing (0°) and for selected peripheral viewing angles (for clarity, only select viewing angles are shown; i.e., 20°, 40°, 60° and 80°) and six pupil sizes. The observer is located in a positive tangential and axial distance from the actual pupil. For each viewing angle, the composite annuli of all six pupil diameters represent the entrance pupil surface showing its three-dimensional shape and its position relative to the actual pupil. The ‘sidewall’ provides the two-dimensional projections of the entrance pupils. An animated illustration of the entrance pupil’s position for all nine viewing angles is provided in Fig. 2, Media 1.

Overall, the three-dimensional position and the side projection of the entrance pupil reveal that the center and the distal marginal points of the peripheral entrance pupil move anteriorly as viewing angle increases. In contrast, the proximal entrance pupil margin moves posteriorly at low peripheral viewing angles and then anteriorly at higher viewing angles. Moreover, it can be seen that the peripheral entrance pupil tilts towards the direction of the viewing angle and curves (primarily concaves along the tangential meridian) towards the observer as peripheral viewing angle increase.
Fig. 2. The three-dimensional entrance pupil for six and nine (Media 1) actual pupil sizes (1 mm to 6 mm) at various viewing angles relative to the actual pupil position. The observer is located in the positive tangential and axial distance quadrant from the actual pupil. Each annulus represents the entrance pupil margin corresponding to one actual pupil diameter. The ‘sidewall’ of the graph shows the two-dimensional side-projection of the entrance pupils revealing their increasing tilt and curvature with viewing angle. Note axial distance scale has been exaggerated for clarity.

3.2 Entrance pupil relative to the viewing direction

As the entrance pupil exists only from the observer’s perspective, the position changes of the entrance pupil are best interpreted relative to the direction of the observer. To evaluate the peripheral entrance pupil as perceived by the observer, the pupil-referenced model (Fig. 2) requires correction via rotation according to the viewing angles. In this way, the observer-referenced entrance pupil can be constructed.

The entrance pupil shape and position relative to the viewing direction are shown for five selected viewing angles in the three-dimensional graph in Fig. 3 and for all viewing angles in succession in the animated Media 2. The observer’s viewing direction is indicated by the blue phantom line. From Fig. 3 and Media 2, the two-dimensional back-projection of the entrance pupil annuli shows the narrowing and distortion of the entrance pupil’s shape with increasing viewing angle. The three-dimensional entrance pupil shapes and positions, as well as their corresponding side-projection, highlight the entrance pupil tilt relative to the observer’s direction of view.

The tangential side-projection of the entrance pupil is individually plotted for all nine viewing angles in Fig. 4. It shows the effect of increasing viewing angle on the axial positions of points defining the margin of entrance pupils of different sizes. Relative to front-on viewing (at 0°) these changes in axial positions across the entrance pupil can become substantial for large peripheral angles. For example, at 60° viewing angle the difference in axial position relative to front-on viewing ranges from −1.66 mm for the proximal pupil margin to +3.20 mm for the distal pupil margin for an actual pupil diameter of 6 mm. For the viewing angle of 60°, the change in axial position of the entrance pupil centre may be as large as +0.90 mm.

The apparent (i.e. as seen by the observer) tilt of the tangential entrance pupil meridian plotted as a function of viewing angle (Fig. 5) shows that the amount of tilt becomes...
progressively smaller than the actual viewing angle as the latter increases. For example, when the eye is observed from a 60° viewing angle, the entrance pupil tilt is approximately 15° smaller than the viewing angle.

![Graph showing three-dimensional entrance pupil for six and nine actual pupil sizes at various viewing angles.](image)

Fig. 3. The three-dimensional entrance pupil for six and nine (Media 2) actual pupil sizes at various viewing angles as seen by the observer. The observer’s viewing direction is indicated by the blue dotted line. The apparent rotation of the actual pupil axis relative to viewing direction is towards the positive axial and negative tangential distance quadrant. Each annulus represents the entrance pupil margin corresponding to one actual pupil diameter. The ‘back-wall’ of the graph shows the two-dimensional back-projection of the entrance pupils, which represent the entrance pupil shapes as seen by the observer. The ‘floor’ of the graph gives the side-projection showing the tilt of the entrance pupils relative to the direction of the observer.

This increasing difference between the entrance pupil tilt and the viewing angle, together with the increasing curvature along the tangential meridian, are the predominant factors that produce the asymmetric distortion of the peripheral entrance pupil shape, which becomes more noticeable with increasing viewing angle. Indeed, the shape of the peripheral entrance pupil does not correspond to an ellipse as often assumed. Instead, although mathematically different, it resembles the shape of a convex limaçon of Pascal. Figure 6 provides a comparison of the shapes of the actual circular pupil with the peripheral entrance pupil as viewed from a 60° angle. One consequence of this asymmetric distortion of the peripheral entrance pupil is that the bisected (geometric) centre of the peripheral entrance pupil does not correspond to the center of the actual pupil.
Fig. 4. The tangential profile (side-projection) of the peripheral entrance pupil from the point-of-view of the observer for nine viewing angles.

Fig. 5. Apparent tilt of the tangential entrance pupil meridian as a function of viewing angle and pupil size. The broken line of negative 1:1 slope represents the expected apparent tilt. Apparent tilt has negative values as it is opposite in direction to viewing angle.

When pupil decentration is considered with respect to viewing angle and actual pupil size, the overall general trend indicates is it can be seen that with increasing viewing angle or increasing pupil size the mid-point (geometrical centre) of the peripheral entrance pupil increasingly departs from the optical centre of the actual pupil (Fig. 7). That is, the geometrical centre of the peripheral entrance pupil does not map to the geometrical centre of the actual pupil. As a consequence, the light ray corresponding to the line-of-sight passes through different points in the actual pupil at different peripheral viewing angles. Our model predicts that the systematic error when alignment is made to the apparent pupil centre can reach around 0.2 mm for a 6 mm pupil diameter at approximately 60° viewing angle.
Fig. 6. Two-dimensional (frontal) projection of (a) the actual pupil and (b) the entrance pupil at 60° observation angle showing the shape as seen by the observer. Each annulus represents one pupil diameter from 1 mm to 6 mm in 1 mm step. The blue dotted line indicates the geometrical mid-point of the peripheral entrance pupil for the 6 mm actual pupil diameter. Actual pupil center is located at the origin (0, 0).

Fig. 7. Entrance pupil decentration as a function of viewing angle and actual pupil diameter.

With increasing pupil size or viewing angle, the peripheral entrance pupil is gradually not only more decentered as described above, but also more asymmetric in its shape (Fig. 6b). To evaluate these pupil size dependent shape changes as a function of viewing angle, the size and associated magnification changes of the peripheral entrance pupil along the two orthogonal (tangential and sagittal) meridians, were plotted with respect to actual pupil diameter (Fig. 8).

As expected and from Fig. 8, along the tangent meridian the entrance pupil size decreases with viewing angle. This decreasing effect appears to be more pronounced for smaller pupils. From the geometry standpoint, if there is no optical component between the pupil and the observer, the tangential entrance pupil magnification would be expected to follow a cosine function of viewing angle as shown in Fig. 8b. However, due to the entrance pupil tilt and anterior movement towards the observer, the tangential entrance pupil magnification decreases more slowly than the cosine function as viewing angle increase.

By least-squares fitting to the results from the ray-tracing model, a parametric model involving pupil diameter and viewing angle for the tangential pupil magnification can be derived:

\[
M_{\text{tan}} = (1.133 \times 10^4 p^3) \cos \left[ (-0.8798 + 4.8 \times 10^3 p) \theta + 3.7 \times 10^{-4} \theta^2 \right]
\] (1a)
where \( p \) corresponds to the pupil diameter (in mm) and \( \theta \) corresponds to the viewing angle (in °). Equation (1a) yielded an RMS Error of 0.0015.

It should be noted that the choice of the form of Eq. (1a) was ‘semi-arbitrary’. Conventionally, pupil magnification has been assumed to be related to viewing angle by the cosine function. Since our model showed that the departure from the elliptical shape and cosine relationship is due at least to the field curvature, we decided to employ a modification to the cosine function in which the variable (viewing angle) has been ‘rescaled’ according to a second-order function. It is possible that other forms of function can provide a better fit. But given the excellent resultant RMS, we did not search for a more precise form.

While Eq. (1) quite precisely predicts tangential pupil magnification, it is somewhat complex in structure. A simplified equation involving only viewing angle (\( \theta \) in °), still with good precision (RMS Error = 0.0066), may be obtained:

\[
M_{\text{tan}} = 1.121 \cos(0.8359\theta)
\]  

(1b)

Fig. 8. Entrance pupil diameter (a) and (c) and magnification (b) and (d) along the tangential (a) and (b) and sagittal (c) and (d) meridians as a function of viewing angle and actual pupil size. The broken line in (b) represents the cosine function with viewing angle.

Figure 8 also shows that the sagittal entrance pupil size and magnification increases slightly with increasing viewing angle. This small increase in sagittal pupil size is also slightly greater for smaller pupils. In a similar manner as for tangential pupil magnification, a parametric equation can be derived to predict sagittal pupil magnification with good precision (RMS Error = 0.0083 mm) from the viewing angle \( \theta \) (in °):

\[
M_{\text{sag}} = 4.4 \times 10^{-6} (\theta^{2.299}) + 1.125
\]  

(2)

4. Discussion

Much of what is known about the peripheral entrance pupil was established many decades ago. Spring & Stiles (1948) [15] then later Jay (1962) [14] measured the peripheral entrance
pupil shape for in vivo human subjects. They established the change in diameters along the horizontal and vertical meridians with viewing angle and identified the departure of the peripheral entrance pupil size from a cosine function with viewing angle. Jay also noted the departure of the peripheral pupil shape from an ellipse [14]. However, being in vivo studies, those early works had some limitations. For example, without knowledge of the actual pupil size, the pupil magnification could not be estimated. Also, no attempt was made to evaluate the changes in the entrance pupil in the axial dimension.

In the present study, we attempted to extend the knowledge on the peripheral entrance pupil. In particular, the model predicted the pupil magnification for a range of pupil sizes and viewing angles. The changing shape and its departure from an ellipse of the peripheral entrance pupil were evaluated with the consequence that the peripheral entrance pupil centre does not correspond to the actual pupil centre. In addition, by considering the axial dimension, the forward movement, compensatory tilt and increasing curvature of the peripheral entrance pupil with viewing angle was revealed.

4.1 Comparison of the entrance pupil model with in vivo pupils

There are a number of differences between the peripheral entrance pupil modeled in the present study and the direct measurement studies mentioned [14,15]. For simplicity, our model portrays a thin (zero thickness), circular pupil that is co-axial with the corneal surfaces. In addition to the temporal viewing angles, Spring & Stiles measured a nasal viewing angle and were thus able to identify the presence of nasal-temporal asymmetry in the peripheral entrance pupil with respect to viewing angle that, presumably, is due to the decentration and tilt of the corneal surfaces relative to the iris [15]. Jay suggested that iris thickness may become relevant and would have the effect of reducing tangential entrance pupil size at high viewing angles [14]. Despite these differences, the model appears to produce predictions of acceptable precision with respect to the range of in vivo measurement errors. The ratio of tangential (horizontal) to sagittal (vertical) entrance pupil diameters as a function of viewing angle for the entrance pupil model of this study, together with in vivo measurements previously published [14,15] are shown in Fig. 9. In lieu of Spring & Stiles who refer to large and small pupil diameters and Jay who refers to dilated and natural pupil diameters, Fig. 9 plots the values for the actual pupil diameters of 3 and 6 mm. It can be seen that the predictions lie well within the spread of measured results, particularly of Jay [14]. In addition, as the actual pupil size is known in our model, the pupil magnification was also calculable.

The non-elliptical shape of the entrance pupil at high peripheral angles has been noted by Jay. Our model has been able to predict and describe this shape, which appears to be caused by the increasing curvature (primarily concave towards the observer along the tangential meridian) with viewing angle. As illustrated by Fig. 3, this curvature, when combined with the tilt of the entrance pupil, introduces an increasing ‘fore-shortening’ effect towards the proximal margin (i.e. the side of the pupil nearer the observer due to pupil tilt) from the point of view of the observer. This effect is present for all finite pupil sizes and viewing angles although it is most readily observable at higher viewing angles and pupil sizes.
4.2 Implications of our entrance pupil model

One of the major consequences of the asymmetric distortion of the peripheral entrance pupil is the loss of correspondence between its geometrical centre and the ‘true’ optical centre of the actual pupil. The light ray that passes through the centre of the peripheral entrance pupil is not the ray that passes through the centre of the actual pupil. This loss of correspondence may become relevant to measurements of the peripheral optics of the eye; especially where measurements rely on alignment to the peripheral entrance pupil’s centre, or its analysis requires knowledge of the centre of the actual pupil (aperture stop).

As shown in Fig. 6b, the systematic error of aligning the geometrical centre of the entrance pupil compared to the centre of the actual pupil, can exceed 0.2 mm for a 6 mm pupil diameter. Despite this systematic error being small in absolute terms and lying within normal measurement variability for some refraction instruments, the impact of this error could affect measurement accuracy; especially at large peripheral angles. Fedtke et al. [13] have shown that tolerance to pupil misalignment is much smaller for peripheral refraction than for central refraction. For example, when measuring peripheral refraction at the 30° nasal visual field using an open-view autorefractor, the refractive power vector components $M$ and $J_{180}$ reached clinical significance for pupil misalignment as small as 0.2 mm.

In addition, Applegate et al. have shown that misalignment from the pupil center during on-axis wavefront measurements can introduce spurious coefficients into Zernike polynomial descriptions of the wavefront. It was shown that this systematic error was not only Zernike mode-dependent but also that the larger the misalignment the more profound its effect [12]. Assuming these findings may be extrapolated to peripheral wavefront measurements, it may be reasonable to suggest that a combination of even small pupil misalignment errors, as well as the systematic error caused by the loss of correspondence of the peripheral entrance pupil mid-point with the actual pupil centre could, adversely affect the accuracy of peripheral ocular measurements.

In many studies of peripheral optics, comparisons are made to the central optics of the eye. Consideration of the errors in peripheral refractometry or wavefront measurements due to systematic misalignment, as well as the loss of correspondence between peripheral and actual pupil centers suggest that caution needs to be exercised when making comparisons of peripheral and central measurements.
The axial alignment of most autorefractor and aberrometer instruments requires that either the cornea or the pupil plane has to be in focus. This is of no difficulty for on-axis measurements, where both the cornea mire and the pupil margin appear completely and symmetrically in focus. According to our model, the axial position shifts forward for the peripheral entrance pupil but more significantly, the pupil tilt produces different axial positions for different points on the entrance pupil. From Fig. 4, for a 60° viewing angle the axial position range from the distal to the proximal pupil margin is around 4 mm for a 6 mm pupil. This is the axial focus range of the aberrometer with the peripheral entrance pupil, when focusing from the proximal to the distal pupil margin. However, it has been shown that the entrance pupil’s depth of focus range for on-axis measurements for the COAS aberrometer lies within ±2 mm [17]. Hence, dependent on the specific instrument’s depth of focus for peripheral measurements, the precise location of the axial position of the peripheral entrance pupil may be difficult to locate.

An additional issue relevant to observation of the peripheral entrance pupil as well as peripheral optical measurements should be considered. In general, the image points from which the entrance pupil is composed, degrades in quality as viewing angle increases. This can be seen in Fig. 10, where the tangential and sagittal transverse ray aberrations were plotted as a function of viewing angle for the vertical superior as well as the proximal and distal horizontal pupil margins. It shows that, the image points at the region of the horizontal pupil that is distal to the viewing direction suffers greater degradation than those proximal to the viewing direction. The vertical superior (and by symmetry, vertical inferior) pupil margin was least affected by viewing direction. From this it can be concluded that with increase in viewing angle the peripheral entrance pupil remains better defined along the sagittal than the tangential meridian.

Fig. 10. Tangential and sagittal spot sizes (in mm) for the horizontal proximal and distal pupil margins as well as the vertical superior pupil margin as a function of viewing angle.

4.3 The wide-field eye

The eye is exquisitely suited for extreme wide-field light-collection. The optical layout of the human eye resembles the design of a retrofocal, ‘fisheye’ camera lens [18], capable of collecting light at field angles well beyond the retinal-neural and facial anatomical limits of the eye. Our entrance pupil model demonstrated that this extreme wide-field capability is made possible by the forward movement towards the observer and the compensatory tilt of the entrance pupil with increasing viewing angle. As a result, the tangential entrance pupil magnification decreases more slowly than the cosine function as viewing angle increases. Extrapolation of the results in Fig. 8b shows that, even at 90° field angle, the tangential magnification relative to the front-on magnification is around 0.3. Since the sagittal
magnification increases with field angle, the outcome is that even at right-angle illumination, the entrance pupil is collecting greater than 30% of incident light. This capability represents an obvious advantage in defensive sensing of the environment but may also present a disadvantage in terms of radiation safety of the eye. For example, the phenomenon of “peripheral light focusing” [19,20], particularly the focusing of scattered light from the peripheral field, has been demonstrated to be a plausible explanation for the occurrence of radiation-related cataracts and other light-related ocular pathologies (coined the “ophthalmohelioses” [20]) at post-iris locations that are not involved in front-on light focusing.

4.4 Recommendations for future models

The present model has a number of limitations. The simplification of co-axial corneal and iris surfaces has already been mentioned, as have the assumptions of a circular, concentric pupil and a thin iris. In reality, the iris boundary is not perfectly circular and varies with age, illumination and pupil size [21]. Also, the position of the pupil can shift during constriction/dilation, which can be as much as 0.6 mm [22] and it has been found that the pupil constricts significantly more during oblique viewing when compared to the straight ahead gaze [23]. An improved model, incorporating non-circular, eccentric pupils may provide additional insight into the peripheral entrance pupil.

The measurements of Spring & Stiles suggested that the sagittal pupil magnification increases to a peak at 80°, decreasing again above that angle. Due to a limitation in the optical layout, our model was not able to compute results for viewing angles at or above 90°. It would be interesting to model and verify this reversal of sagittal pupil magnification at very high viewing angles.

Finally, for the study of peripheral retinal image quality or other retinal responses, such as the Stiles-Crawford Effect, it might be of more value to analyze the exit pupil shape and size with viewing angle. Such an analysis would need to take into the account the complex shape [24] and gradient refractive index of the crystalline lens [25]. Hopefully, future work can address this aspect of the peripheral optics of the eye.

5. Conclusion

As the viewing angle increases, the entrance pupil moves forward, exhibits compensatory tilt and increases in concavity towards the observer. In consequence the tangential pupil size does not follow a cosine relationship with viewing angle, the shape of the entrance pupil undergoes asymmetric (non-elliptical) distortion, and the geometrical centre of the entrance pupil does not represent precisely the centre of the actual pupil. Thus, peripheral ocular measurements may be affected adversely, particularly where alignment to the pupil center is required. Given the potential adverse impact misalignment may have on such ocular measurements as wavefront sensing, caution is warranted when comparing such results between peripheral and central viewing angles. Overall however, we conclude that typically, these departures are small and may only be of significance for large peripheral viewing angles.

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