DYADOSPHERES DON’T DEVELOP

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Abstract

Pair production itself prevents the development of dyadospheres, hypothetical macroscopic regions where the electric field exceeds the critical Schwinger value. Pair production is a self-regulating process that would discharge a growing electric field, in the example of a hypothetical collapsing charged stellar core, before it reached 6% of the minimum dyadosphere value, keeping the pair production rate more than 26 orders of magnitude below the dyadosphere value.

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1 Introduction

Ruffini and his group [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14.15, 16] have proposed a model for gamma ray bursts that invokes a dyadosphere, a macroscopic region of spacetime with rapid Schwinger pair production [17], where the electric field exceeds the critical electric field value

\[ E_c = \frac{m^2}{q} = \frac{m^2 c^3}{\hbar q} \approx 1.32 \times 10^{16} \text{ V/cm}. \]  

(1)

(Here \( m \) and \(-q\) are the mass and charge of the electron, and I am using Planck units throughout.) The difficulty of producing these large electric fields is a problem with this model that has not been adequately addressed. Here I shall summarize calculations [18] showing that dyadospheres almost certainly don’t develop astrophysically.

The simplest reason for excluding dyadospheres is that if one had an astrophysical object of mass \( M \), radius \( R > 2M \), and excess positive charge \( Q \) in the form of protons of mass \( m_p \) and charge \( q \) at the surface, the electrostatic repulsion would overcome the gravitational attraction and eject the excess protons unless \( qQ \leq m_p M \) or

\[ \frac{E}{E_c} = \frac{qQ}{m^2 R^2} \leq \frac{m_p M}{m^2 R^2} < \frac{m_p}{4m^2 M} < 1.2 \times 10^{-13} \left( \frac{M_{\odot}}{M} \right), \]  

(2)

where \( M_{\odot} \) is the solar mass. (If the excess charge were negative and in the form of electrons, the upper limit would be smaller by \( m/m_p \).) Then the pair production would be totally negligible.

However, one might postulate the implausible scenario in which protons are bound to the object by nuclear forces, which in principle are strong enough to balance the electrostatic repulsion even for dyadosphere electric fields. Therefore, for the sake of argument, I did a calculation [18] of what would happen under the highly idealized scenario in which the surface of a positively charged stellar core with initial charge \( Q_0 = M \) (the maximum allowed before the electrostatic repulsion would exceed the gravitational attraction on the entire core, not just on the excess protons on its surface) freely fell from rest at radial infinity along radial geodesics in the external Schwarzschild metric of mass \( M \).

This idealization ignores the facts that a realistic charged surface would (a) not fall from infinity, (b) have one component of outward acceleration, relative to free fall, from the pressure gradient at the surface, (c) have another component of
outward acceleration from the electrostatic repulsion, and (d) fall in slower in the Reissner-Nordstrom geometry if the gravitational effects of the electric field with \( Q \sim M \) were included. Because of each one of these effects, the actual surface would fall in slower at each radius and hence have more time for greater discharge than in the idealized model. Hence the idealized model gives a conservative upper limit on the charge and electric field at each radius, even under the implausible assumption that the excess protons are somehow sufficiently strongly bound to the surface that they are not electrostatically ejected.

As we shall see, even in this highly idealized model, the self-regulation of the pair production process itself will discharge any growing electric field well before it reaches dyadosphere values. This occurs mainly because astrophysical length scales are much greater than the electron Compton wavelength, which is the scale at which the pair production becomes significant at the critical electric field value for a dyadosphere. Therefore, the electric field will discharge astrophysically even when the pair production rate is much lower than dyadosphere values.

These calculations lead to the conclusion that it is likely impossible astrophysically to achieve, over a macroscopic region, electric field values greater than a few percent of the minimum value for a dyadosphere, if that. The Schwinger pair production itself would then never exceed \( 10^{-26} \) times the minimum dyadosphere value.

## 2 Schwinger discharge of an electric field

In this section we shall analyze the pair production and discharge of an electric field produced by the collapse of the idealized hypothetical charged sphere or stellar core of mass \( M \) and initial positive charge \( Q_0 \), assuming that somehow the excess charge on its surface is not electrostatically ejected, and assuming that the surface falls in as rapidly as possible, which is free fall from rest at infinity in an assumed external Schwarzschild geometry.

As the surface radius \( R \) collapses, the electric field \( E = Q/r \) outside \( (r > R) \) produces pairs, with the positrons escaping and the electrons propagating in to the surface to reduce its charge \( Q \). The pair production rate \( \mathcal{N} \) per 4-volume [17] is given by

\[
\mathcal{N} = \frac{q^2 E^2}{4 \pi^3} \exp \left( -\frac{\pi m^2}{qE} \right) \equiv \frac{m^4 e^{-w}}{4 \pi w^2},
\]

(3)
where

\[
\begin{align*}
  w & \equiv \frac{\pi m^2}{qE} \equiv \frac{\pi E_c}{E} = \frac{\pi m^2 r^2}{qQ} \\
  &= \frac{4\pi m^2 M_\odot}{q} \left( \frac{M}{M_\odot} \right) \left( \frac{Q}{M} \right)^{-1} \left( \frac{r}{2M} \right)^2 \\
  &\equiv \frac{1}{B} \left( \frac{M}{M_\odot} \right) \left( \frac{Q}{M} \right)^{-1} \left( \frac{r}{2M} \right)^2, \\
  B &\equiv \frac{q}{4\pi m^2 M_\odot} \approx 42475. 
\end{align*}
\]

(4)

A dyadosphere has \( E \geq E_c \equiv m^2/q \Rightarrow w \leq \pi \Rightarrow \)

\[
\left( \frac{M}{M_\odot} \right) \left( \frac{Q}{M} \right)^{-1} \left( \frac{r}{2M} \right)^2 \leq \pi B \equiv \frac{q}{4m^2 M_\odot} \approx 1.33 \times 10^5. 
\]

(5)

For astrophysical electric fields anywhere near dyadosphere values, the electrons and positrons produced will quickly be accelerated to very near the speed of light, so one will effectively get a null number flux 4-vector \( n_+ \) of highly relativistic positrons moving radially outward and another null number flux 4-vector \( n_- \) of highly relativistic electrons moving radially inward, with total current density 4-vector \( j = qn_+ - qn_- \).

It is most convenient to describe this current in terms of radial null coordinates, say \( U \) and \( V \), so that the approximately Schwarzschild metric outside the collapsing core may be written as

\[
ds^2 = -e^{2\sigma} dU dV + r^2(U, V) (d\theta^2 + \sin^2 \theta d\phi^2). 
\]

(7)

Then Maxwell’s equations (Gauss’s law) gives

\[
4\pi j \equiv 4\pi q(n_+^V \partial_V - n_-^V \partial_V) = \nabla \cdot F \equiv \frac{1}{r^2} \left( Q^V \partial_V - Q^U \partial_U \right). 
\]

(8)

The 4-divergence of each of the number flux 4-vectors \( n_+ \) and \( n_- \) is equal to the pair production rate \( N \), which leads to the following relativistic partial differential equation for the pair production and discharge process:

\[
Q_{,UV} = -2\pi qr^2 e^{2\sigma} N = -\frac{q^3 Q^2 e^{2\sigma}}{2\pi^2 r^2} \exp \left( -\frac{\pi m^2 r^2}{qQ} \right), 
\]

(9)

or

\[
8\pi qr^2 N = \frac{2q^3 r^2 E^2}{\pi^2} \exp \left( -\frac{\pi m^2}{qE} \right) = 2\Box (r^2 E) = r\Box (r E) - r^3 E \Box \left( \frac{1}{r} \right), 
\]

(10)
where $\Box$ is the covariant Laplacian in the 2-dimensional metric $2ds^2 = -e^{2\sigma} dU dV$
and where $\Box$ is the covariant Laplacian in the full 4-dimensional metric.

In my much more detailed paper [18], I have analyzed this partial differential equation for the charge distribution over the entire spacetime region exterior to the charged surface and have found an approximation that reduces it to a relativistic ordinary differential equation for the evolution of the charge at the surface itself. However, here I shall confine myself to a Newtonian approximation to this evolution equation, which turns out to give results very close to the relativistic approximation.

A convenient time parameter for describing the collapse of the surface is the velocity $v$ the surface has in the frame of a static observer at fixed $r$ when the surface radius $R$ crosses that value of $r$. For free fall from rest at infinity in the external Schwarzschild metric of mass $M$, with $\tau$ being the proper time along the surface worldlines, one gets
\[ v = \frac{-dR}{d\tau} = \sqrt{\frac{2M}{R}}, \]
so that the proper time remaining until the proper time $\tau_c$ at which the surface reaches the curvature singularity at $R = 0$ is $\tau_c - \tau = (4/3) M/v^3$ and $R = 2M/v^2 = (4.5M)^{1/3}(\tau_c - \tau)^{2/3}$. The Newtonian limit of this is when $R \gg 2M$, which gives $v \ll 1$ and $\tau \approx t$, the Schwarzschild coordinate time. In this limit, the surface moves negligibly during the time it takes for electrons to move inward from where they are created to the surface to reduce $Q(t)$.

If $w \equiv \pi E_c/E$ is defined at each point outside the surface, let
\[ z(t) \equiv w(t, R(t)) \equiv \frac{\pi E_c}{E(R(t))} = \frac{\pi m^2 R(t)^2}{qQ(t)} = \frac{\pi m^2 M^2}{qQv^4} \]
be the value of $w$ at the surface itself. Since we shall find that the electric field $E$ always stays far below the critical dyadosphere value $E_c$, we have $z \gg 1$, which will be used for various approximations below.

Now the pair production rate (and assumed effectively instantaneous propagation of the electrons produced to the surface of the collapsing stellar core) gives
\[ \frac{dQ}{dt} \approx -q\int_R^\infty N(r)4\pi r^2 dr \approx -q \frac{m^4 R^4}{z^2} \int_R^\infty \frac{dr}{r^2} e^{-z^2 r^2} \approx -q m^4 R^5 \frac{z^2}{2z^2 e^z}. \]
Then using $R = 2M/v^2$ and $dv/dt \approx dr/d\tau = v^4/(4M)$ leads to the following ordinary nonlinear first-order differential equation for $z(v)$ in the Newtonian limit:
\[ \frac{v}{z} \frac{dz}{dv} \approx -4 \left[ 1 - \frac{(Mmq)^2}{\pi v^5 z^2 e^z} \right] = -4 \left[ 1 - \frac{A \mu^2}{v^5 z^2 e^z} \right] = -4 \left[ 1 - e^{-U} \right], \]
where
\[
A \equiv \frac{(M_\odot m_q)^2}{\pi} \approx 3.39643251 \times 10^{28},
\]
\[
\mu \equiv \frac{M}{M_\odot},
\]
\[
U \equiv z + \ln z - \ln A - 2 \ln \mu + 5 \ln v.
\]

From Eq. (12), one can see that the boundary conditions for Eq. (14) are that initially (\(\tau \approx t = -\infty \iff R = \infty \iff v = 0\)) \(v^4 z = \pi m^2 M^2/(qQ)\) with the surface charge \(Q\) having its asymptotically constant initial value \(Q_0\), which will be taken to be its maximum allowed value, \(M\), unless otherwise specified. One can see from this that both \(z\) and \(U\) start off initially at infinite values. The final value will be when the surface enters the event horizon of the black hole at \(R = 2M\) or \(v = 1\). This is beyond the applicability of the Newtonian approximation being used here, but it turned out that the relativistic analysis [18] gave very nearly the same answers.

One can now differentiate Eq. (17) for \(U(v)\), using Eq. (14), to obtain
\[
\frac{v}{v} \frac{dU}{dv} \approx -4(z + 2) \left(1 - e^{-U}\right) + 5.
\]

Since \(z \gg 1\), this equation implies that \(U\) decreases to near zero (though it cannot reach zero, for if it could, the right hand side would be positive, contradicting the assumption that it dropped to zero and hence had a negative derivative on the left hand side). Then when \(U \ll 1\), Eq. (17) may be solved approximately for \(z(v)\) to give
\[
z(v) \approx \ln A + 2 \ln \mu - 5 \ln v - 2 \ln \left(\ln A + 2 \ln \mu - 5 \ln v\right)
\]
\[
\approx \ln A - 2 \ln \ln A + \left(1 - \frac{2}{\ln A}\right) (2 \ln \mu - 5 \ln v)
\]
\[
\approx 57.33 + 1.94 \ln \mu + 4.85 \ln \frac{1}{v}.
\]

This then gives the ratio of the electric field at the surface to the critical electric field of a dyadosphere as being
\[
\frac{E}{E_c} = \frac{\pi}{z} \approx 0.0548 - 0.00185 \ln \mu - 0.00463 \ln \frac{1}{v} \ll 1.
\]

One can improve this result by using a slightly improved formula (giving a roughly 2.5% correction for \(z \sim 57\)) for the radial integral in Eq. (13) for the pair production rate, by using a better explicit approximation for what \(U\) should
approach at \( v = 1 \), and by using a numerical solution of the thus-corrected form of Eq. (17) for \( z \) with this expression for \( U \), to estimate that when \( \mu = 1 \) and \( v = 1 \), \( z \approx 57.5843 \). Including the improved formula for the right hand side of Eq. (13) into the Newtonian differential equation (14) and then integrating it numerically from \( v = 0 \) to \( v = 1 \) for \( \mu = 1 \) gave the result at the horizon of the solar black hole of \( z \approx 57.5845 \), so only the 6th digit changed from the algebraic estimate obtained without numerically solving the Newtonian differential equation.

In [18] I used a relativistic ordinary differential equation approximation to the partial differential equation (9) and was able to deduce an explicit approximate relativistic result for \( \mu = 1 \) and \( v = 1 \) of \( z \approx 57.60483 \), whereas the numerical solution of the relativistic ordinary differential equation gave \( z \approx 57.60480 \), differing from the explicit formula (using as input the values of \( m, q \), and \( M_\odot \)) by only about one part in two million. However, I would estimate that the relativistic ordinary differential equation approximation to the partial differential equation would itself introduce absolute errors of the order of \( 10^{-4} \)–\( 10^{-3} \) in the value of \( z \), so the numerical solution of that ordinary differential equation is not necessarily any better than the completely explicit approximate solution I also obtained.

The difference between the Newtonian and the relativistic approximations for \( z \) on the horizon (\( v = 1 \)) of a solar mass collapsing core (\( \mu = M/M_\odot = 1 \)) is about 0.02, which I would guess is considerably larger than the error of my relativistic approximation (not given here, but in [18]), but it is still a relative difference of only about one part in three thousand for the idealized upper limit of the value of the electric field of a hypothetical charged stellar core collapsing into a black hole after falling in freely (no nongravitational forces on the surface) from starting at rest at radial infinity with the external metric being Schwarzschild.

We can also give a heuristic derivation of the Newtonian result in the following way: We expect the self-regulation of the electric field to make \( z = \pi E_c/E = \pi m^2 R^2/(qQ) \) change slowly, so

\[
\frac{1}{Q} \frac{dQ}{dt} \sim \frac{2}{R} \frac{dR}{dt} \approx -\frac{1}{M} \left( \frac{2M}{R} \right)^{\frac{3}{2}} = -\frac{v^3}{M}.
\]

Then the pair production rate per 4-volume, \( \mathcal{N} = (m^4/4\pi) e^{-w} w^2 \approx (m^4/4\pi z^2) e^{-zr^2/R^2} \) for \( z \gg 1 \) decreases roughly exponentially with \( r \) with e-folding length \( \Delta r \approx R/(2z) \). Thus

\[
\frac{1}{Q} \frac{dQ}{dt} \approx -\frac{q}{Q} \int_R^\infty \mathcal{N}(r) 4\pi r^2 dr \approx -\frac{q}{Q} \mathcal{N}(r = R) 4\pi R^2 \Delta r
\]
≈ -q \frac{q^2}{\pi m^2 R^2} \frac{m^4}{4\pi z^2} e^{-z} 4\pi R^2 \frac{R}{2z} = -\frac{1}{M} \frac{M^2 m^2 q^2 e^{-z}}{\pi z^2} \left( \frac{R}{2M} \right). \quad (22)

Equating this to \(-(1/M)(2M/R)^{3/2}\) gives

\[ 1 \approx \frac{M^2 m^2 q^2 e^{-z}}{z^2} \left( \frac{R}{2M} \right)^{5/2} = \frac{A\mu^2 e^{-z}}{v^5 z^2}, \quad (23) \]

which implies \( z + 2 \ln z \approx \ln A + 2 \ln \mu - 5 \ln v \), just as we got from the approximated solution of the ordinary differential equation in the Newtonian approximation.

A dyadosphere would have \( E \geq E_c \), which implies \( z = \pi E_c / E \leq z_c = \pi \) and \( N = (m^4/4\pi)e^{-z}/z^2 \geq N_c = m^4 e^{-\pi} / (4\pi^3) \). But

\[ N \approx \frac{m^4}{4\pi} \frac{v^5}{A\mu^2} = \frac{\pi^2 e^\pi v^5}{A\mu^2} N_c = \frac{\pi^3 e^\pi v^5}{q^2 m^2 M} N_c \]

\[ \approx 0.672 \times 10^{-26} \frac{v^5}{\mu^2} N_c < 10^{-26} N_c, \quad (24) \]

so the heuristic estimate gives the maximum pair production rate more than 26 orders of magnitude below that of a dyadosphere. By comparison, the numerical solution of the approximate relativistic ordinary differential equation \([18]\) gave, at \( v = 1 \),

\[ \frac{N}{N_c} \approx \frac{0.661168 \times 10^{-26}}{\mu^2} (1 + 0.0005545 \ln \mu - 0.00001759 \ln^2 \mu), \quad (25) \]

about 1.6% less than the heuristic estimate gives.

One can also calculate the maximum efficiency for converting the collapsing stellar core mass \( M \) into outgoing positron energy,

\[ \epsilon \approx -\int \frac{QdQ}{MR} \approx \frac{2\mu^2}{B^2} \int_0^1 e^{-U} \frac{dv}{v^2} \]

\[ \approx \frac{1}{3} B^{-1/2} \left[ \ln A + \frac{5}{4} \ln B - \frac{3}{4} \ln \ln (AB^2) + \frac{3}{4} \ln \mu \right]^{-1/2} \mu^{1/2} (Q_0/M)^{3/2} \]

\[ = \frac{m}{3M} \sqrt{\frac{4\pi Q_0^3}{q}} \left[ \frac{1}{4} \ln \left( \frac{q^{13} M^3}{2^{10} \pi^9 m^3} \right) - \frac{3}{4} \ln \ln \left( \frac{q^4}{16\pi^3 m^2} \right) \right]^{1/2} \]

\[ \approx 1.9 \times 10^{-4} \left[ 1 + \ln \left( \frac{M/M_\odot}{300} \right) \right]^{-1/2} \left( \frac{M}{M_\odot} \right)^{1/2} (Q_0/M)^{3/2} \]

\[ \approx 2 \times 10^{-4} \left( \frac{Q_0}{M} \right)^{3/2} \sqrt{\frac{M}{M_\odot}}. \quad (26) \]
By comparison, the numerical solution of the approximate relativistic ordinary
differential equation [18] gave the more precise result
\[ \epsilon \approx 0.0001855 \left( \frac{M}{M_\odot} \right)^{0.495} \left( \frac{Q_0}{M} \right)^{0.742}. \] (27)

The dominant factor in an estimate of the coefficient 0.0001855 (the upper limit on
the efficiency if \( Q_0 = M = M_\odot \)) is one-third the ratio of the electron mass to the
proton mass, which is 0.0001815. This efficiency is too low for the pair production
from these idealized collapsing charged cores to explain gamma ray bursts, even
if it is admitted that this very implausible scenario (of the excess charge \( Q \sim M \)
not getting expelled from the collapsing core by the huge electrostatic forces on it)
comes nowhere near being able to form a dyadosphere.

One can also calculate [18] that the probability of one of the particles annihilating
with an antiparticle is less than \( 10^{-26} \), so the direct interactions of individual
particles is negligible, consistent with what was assumed above.

### 3 Conclusions

If protons are bound to a collapsing stellar core purely gravitationally, the maximum
electric field is more than 13 orders of magnitude below dyadosphere values:
\[ E_{\text{max}} \leq 1.2 \times 10^{-13} \frac{M_\odot}{M} E_c. \]

If protons are much more strongly bound, \( E_{\text{max}} \leq 0.055 E_c \) and \( N_{\text{max}} \leq 10^{-26} N_c \),
where \( N_c \) is the minimal dyadosphere production rate.

The energy efficiency of the process for \( M \sim M_\odot \) is very low, \( \epsilon \approx 1.86 \times 10^{-4} (M/M_\odot)^{1/2} (Q_0/M)^{3/2} \approx (Q_0/M)^{3/2} \sqrt{M/(2.9 \times 10^7 M_\odot)}. \)

If one relaxed the assumptions of this model, such as the spherical symmetry,
one would expect to get similar results, perhaps changing the pair production rates
by factors of order unity that depend upon the precise geometry. However, it seems
very unlikely that any modification could increase the maximum possible pair pro-
duction rate by any significant fraction of the 26 orders of magnitude that the model
above fails to achieve dyadosphere values. Therefore, the example analyzed strongly
suggests that dyadospheres do not form astrophysically.
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