The $\Lambda$-$\Sigma$ coupling effect in the neutron-rich $\Lambda$-hypernucleus $^{10}_\Lambda$Li by microscopic shell model

A. Umeya* and T. Harada
Research Center for Physics and Mathematics, Osaka Electro-Communication University, Neyagawa, Osaka, 572-8530, Japan
(Dated: February 27, 2009)

We investigate the structure of the neutron-rich $\Lambda$-hypernucleus $^{10}_\Lambda$Li by using microscopic shell-model calculations considering a $\Lambda$-$\Sigma$ coupling effect. The calculated $\Sigma$-mixing probability in the $^{10}_\Lambda$Li ground state is found to be about 0.34% which is coherently enhanced by the $\Lambda$-$\Sigma$ coupling configurations, leading to the energy shift 0.28 MeV which is about 3 times larger than that in $^{10}_\Sigma$Li. The importance of the $\Sigma$ configuration obtained by the $\Sigma\gamma$ interaction and the potentiality of the neutron-rich environment are discussed.

PACS numbers: 21.60.Cs, 21.80.+a, 27.20.+n

I. INTRODUCTION

One of the most important subjects in strangeness nuclear physics is a study of neutron-rich $\Lambda$-hypernuclei [1]. It is expected that a $\Lambda$ hyperon plays a glue-like role in nuclei beyond the neutron-drip line, together with a strong $\Lambda N$-$\Sigma N$ coupling [2, 3], which might induce a $\Sigma$-mixing in nuclei. The knowledge of behavior of hyperons in a neutron-excess environment significantly affects our understanding of neutron stars, because it makes the Equation of State soften [4]. The purpose of our study is to theoretically clarify the structure of the neutron-rich $\Lambda$-hypernuclei by a nuclear shell model, which has successfully been applied for description of the neutron-excess nuclei [5–8].

Shell-model studies for $p$-shell $\Lambda$-hypernuclei have been performed by several authors [9–16]. A series of pioneering shell-model calculations was carried out by Gal, Soper and Dalitz [9] in the 1970s, using $\Lambda N$ effective interactions parametrized from the available data. The $\gamma$-ray transitions for $p$-shell $\Lambda$-hypernuclei [10, 11] and $\Lambda$ production cross sections of $(K^-,\pi^+)$ and $(\pi^+,K^+)$ reactions [12, 13] were successfully explained by shell-model descriptions. Recently, energy spacings of $p$-shell $\Lambda$-hypernuclei have been studied by Millener’s shell-model calculations including the $\Lambda$-$\Sigma$ coupling [14], so as to interpret the precise data of $p$-shell $N \approx Z$ $\Lambda$-hypernuclei from $\gamma$-ray measurements by Ge detector [17]. Moreover, properties of the $\Lambda N$ effective interactions derived form the recent Nijmegen potentials [18–20] in shell-model calculations have been discussed [15, 16].

Recently, Saha et al. have performed the first successful measurement of a neutron-rich $\Lambda$-hypernucleus $^{10}_\Lambda$Li by the double-charge exchange reaction $(\pi^-,K^+)$ on a $^{10}$B target [21]. However, the magnitude and incident-momentum dependence of the experimental production cross sections cannot be reproduced in a theoretical calculation by Tret'yakova and Lansky [22]. They predicted that the cross section for $^{10}_\Lambda$Li is mainly explained by a two-step process, $\pi^- p \rightarrow K^0\Lambda$ followed by $K^0 p \rightarrow K^- n$, or $\pi^+ p \rightarrow \pi^0 n$ followed by $\pi^0 p \rightarrow K^+\Lambda$ with the distorted-wave impulse approximation, rather than by a one-step process, $\pi^+ p \rightarrow K^+\Sigma^-$ via $\Sigma^-$ $p$ doorways due to the $\Sigma^- p \leftrightarrow \Lambda n$ coupling. This problem might suggest the importance of a $\Sigma$-mixing in the $\Lambda$-hypernucleus. The analysis of the $^{10}$B $(\pi^-,K^+)$ reaction provides a reason to carefully examine wave functions involving $\Sigma$ admixtures in $^{10}_\Lambda$Li, as well as a mechanism of this reaction [23].

In this paper, we investigate the structure of the neutron-rich hypernucleus $^{10}_\Lambda$Li, in microscopic shell-model calculations considering the $\Lambda$-$\Sigma$ coupling effect. We focus on the $\Sigma$-mixing probabilities and the energy shifts of the neutron-rich hypernucleus. Also, we investigate the effect of the $\Sigma N$ interaction to the core-nuclear state by the perturbation method. We discuss that the coupling strengths are enhanced in the neutron-rich $\Lambda$-hypernucleus and are related to the $\beta$-transition properties of the nuclear core state.

II. FORMALISM

A. Multi-configuration shell model for $\Lambda$-hypernuclei

We consider a $\Lambda$-nuclear state involving a $\Sigma$-mixing in a $\Lambda$-hypernucleus $^{A\Lambda}_Z$ with the mass number $A$ and the atomic number $Z$ in a multi-configuration nuclear shell model. The state of the $\Lambda$-hypernucleus is represented by $|\Lambda^Z\Sigma^T J M\rangle$, where $T$ and $T_J$ are the isospin and its $z$-component, respectively, and $J$ and $M$ for the angular momentum. The index $\nu$ is introduced to distinguish states with the same $T$ and $J$.

In the configuration space for the $\Lambda$-hypernucleus involving a $\Lambda$-$\Sigma$ coupling, the Hamiltonian is given as

$$H = H_\Lambda + H_\Sigma + V_{\Lambda\Sigma} + V_{\Sigma\Lambda},$$

where $H_\Lambda$ is the Hamiltonian in the $\Lambda$-configuration space and $H_\Sigma$ is that in the $\Sigma$ configuration space. $V_{\Lambda\Sigma}$ and its Hermitian conjugate $V_{\Sigma\Lambda}$ denote the two-body $\Lambda$-$\Sigma$ coupling interaction, $\Lambda N \leftrightarrow \Sigma N$. Then, we can write the $\Lambda$-nuclear state with $T, J$ as

$$|\Lambda^Z\Sigma^T J M\rangle = \sum_\mu C_{\mu \nu} |\Lambda^\mu \Sigma^T J M\rangle + \sum_\mu' D_{\mu' \nu} |\Sigma^\mu' \Sigma^T J M\rangle,$$

where $|\Lambda^\mu \Sigma^T J M\rangle$ and $|\Sigma^\mu' \Sigma^T J M\rangle$ are eigenstates for the $\Lambda$ and $\Sigma$ configurations, respectively, which are given by

$$H_\Lambda |\Lambda^\mu \Sigma^T J M\rangle = E_\mu^\Lambda |\Lambda^\mu \Sigma^T J M\rangle,$$

$$H_\Sigma |\Sigma^\mu' \Sigma^T J M\rangle = E_{\mu'}^\Sigma |\Sigma^\mu' \Sigma^T J M\rangle.$$
Although the coefficients $C_{\nu,\mu}$ and $D_{\nu,\mu}$ are determined by diagonalization of the full Hamiltonian $H$, we treat $V_{\Lambda\Sigma}$ and $V_{\Sigma\Sigma}$ as perturbation because a $\Sigma$ hyperon has a larger mass than a $\Lambda$ hyperon by about 80 MeV. When taking into account up to the first-order terms, the coefficients can be written as
\begin{equation}
C_{\nu,\mu} = \delta_{\nu,\mu},
\end{equation}
\begin{equation}
D_{\nu,\mu} = \frac{\langle \psi^\Lambda_{\nu}; T J | V_{\Lambda\Sigma} | \psi^\Sigma_{\mu}; T J \rangle}{E^\Sigma_{\mu} - E^\Sigma_{\nu}}.
\end{equation}
Since a $\Lambda\Sigma$ coupling strength for each $\Sigma$ eigenstate $|\psi^\Sigma_{\nu}; T J\rangle$ is obtained as $|D_{\nu,\mu}|^2$, the $\Sigma$-mixing probability in the $\Lambda$-nuclear state $|\psi^\Lambda_{\nu};\nu T J\rangle$ is given as
\begin{equation}
P_\Sigma = \sum_{\mu} |D_{\nu,\mu}|^2
\end{equation}
or
\begin{equation}
P_\Sigma^{(R)} = \left( \sum_{\mu} |D_{\nu,\mu}|^2 \right) \left( 1 + \sum_{\mu} |D_{\nu,\mu}|^2 \right),
\end{equation}
where the latter probability is a renormalized value. The binding energy $E^\Sigma_{\nu} - \Delta E$ is calculated by the energy shift which is given as
\begin{equation}
\Delta E = \sum_{\mu} \left( E^\Sigma_{\mu} - E^\Sigma_{\nu} \right) |D_{\nu,\mu}|^2
\end{equation}
in the perturbation theory.

It is well known that a $\Lambda$ hyperon in a $\Lambda$-hypernucleus is described by the single-particle picture very well because the $\Lambda N$ interaction is weak. On the other hand, in terms of a $\Sigma$ hyperon, the nuclear configuration would change due to the strong spin-isospin dependence in the $\Sigma N$ interaction [24]. In order to evaluate the single-particle picture for a hyperon, we consider a spectroscopic factor for a hyperon-pickup from $|\psi^\Sigma_{\nu};T J\rangle$,
\begin{equation}
S_\nu(v_N T N J_N, j_N) = \frac{|\langle \psi^\Lambda_{\nu}; T J | a^\dagger_{j_N} | (\Lambda-1)Z N T N J_N \rangle|^2}{(2T + 1)(2J + 1)},
\end{equation}
where $|(\Lambda-1)Z N T N J_N \rangle$ is an eigenstate of the core nucleus, which is obtained by diagonalizing in the nucleon configurations, and $a^\dagger_{j_N}$ is a creation operator of a single-particle state of the hyperon in an orbit $j_N$. The matrix element $\langle \psi^\Lambda_{\nu}; T J | a^\dagger_{j_N} | (\Lambda-1)Z N T N J_N \rangle$ with the Edmonds' convention [25] is reduced with respect to both the isospin and the angular momentum. The spectroscopic factor satisfies the sum rule
\begin{equation}
\sum_{v_N T N J_N} S_\nu(v_N T N J_N, j_N) = n_{j_N},
\end{equation}
where $n_{j_N}$ is the number of hyperons in the orbit $j_N$. If a hyperon in the hypernucleus provides the single-particle nature, the state $|\psi^\Lambda_{\nu}; T J\rangle$ is represented as a tensor product of a core-nuclear state $|(\Lambda-1)Z N T N J_N \rangle$ and a hyperon state $|j_N\rangle$: we obtain $S_\nu(v_N T N J_N, j_N) = \delta_{v_{\psi,\text{core}}}$, where $v_{\psi} = v_{\text{core}}$ means the core state is equivalent to the $(\Lambda-1)Z$ state in the weak-coupling limit.

### TABLE I: Radial integrals for $\Sigma N$ effective interactions in unit of MeV. The values are listed in Ref. [14] for the $\Lambda N$ interaction $V_\Lambda$ and the $\Lambda\Sigma$ coupling interaction $V_{\Lambda\Sigma}$, and Ref. [27] for the $\Sigma N$ interaction $V_\Sigma$.  

| Isospin | $V$ | $\Delta$ | $S_N$ | $S_\Sigma$ | $T$ |
|---------|-----|-------|------|------|-----|
| $V_\Lambda$ | $T = \frac{1}{2}$ | -1.2200 | 0.4300 | -0.2025 | 0.1875 | 0.0300 |
| $V_\Sigma$ | $T = \frac{1}{2}$ | 1.0100 | -7.2150 | -0.0010 | 0.0000 | -0.3640 |
| $V_\bar{\Sigma}$ | $T = \frac{1}{2}$ | -1.1070 | 2.2750 | -0.2680 | 0.0000 | 0.1870 |
| $V_{\Lambda\Sigma}$ | $V_{\Sigma\Sigma}$ | $T = \frac{1}{2}$ | 1.4500 | 3.0400 | -0.0850 | 0.0000 | 0.1570 |

### B. Shell-model setup and effective interactions

In the present shell-model calculations, we construct wave functions of $\Lambda Z$ as follows: Four nucleons are in the $^3$He core and $(A - 5)$ valence nucleons move in the $p$-shell orbits. The $\Lambda$ or $\Sigma$ hyperon is assumed to be in the lowest $0s_{1/2}$ orbit. For the $NN$ effective interaction, we adopt the Cohen-Kurath (8–16) $2BME$ [26], which is a traditional and empirical interaction for ordinary $p$-shell nuclei, and is one of the reliable effective interactions for stable and semi-stable nuclei. The $YN$ effective interaction is written [9–11] as
\begin{equation}
V_Y = V_0(r) + V_{1\sigma}(r)s_N \cdot s_Y + V_{1T}(r)\ell \cdot (s_N + s_Y) + V_{1S}(r)\ell \cdot (s_N + s_Y) + V_{1T}(r)S_{12},
\end{equation}
where $V(r)$’s are radial functions of the relative coordinate $r = |r_N - r_Y|$ between the nucleon and the hyperon. $s_N$ and $s_Y$ are spin operators for the nucleon and the hyperon, respectively, $\ell$ is the angular momentum operator of the relative motion. The tensor operator $S_{12}$ is defined by
\begin{equation}
S_{12} = 3 (\hat{r} \cdot \sigma_N)(\hat{r} \cdot \sigma_Y) - (\sigma_N \cdot \sigma_Y)
\end{equation}
with $\sigma = 2s$ and $\hat{r} = (r_N - r_Y)/r$. In Table I, we list the parameters of radial integrals $V, \Delta, S_N, S_\Sigma$ and $T$, which correspond to $V_0, V_{1\sigma}, V_{1T}, V_{1S}$ and $V_{1T}$, respectively. The parameters $S_N$ and $S_\Sigma$ are defined as the coefficients of $\ell_N \cdot (s_N + s_Y)$ and $\ell_N \cdot (s_N + s_Y)$, respectively, where $\ell_N$ is the angular momentum operator of the nucleon and is proportional to the relative $\ell$ for the hyperon in the $0s_{1/2}$ orbit. We adopt the values of $V_\Lambda$, which is given in Ref. [14]. For $V_{\Lambda\Sigma}$ and $V_{\Sigma\Sigma}$, we used the $\Lambda N, \Sigma N$ and $\Sigma \Sigma$ effective interactions [14, 27] based on the NSC97e,f potentials [19].

### III. NUMERICAL RESULTS

#### A. $\Sigma$-mixing probabilities and energy shifts

We perform numerical calculations of the neutron-rich $\Lambda$-hypernucleus $^{10}\text{Li}$ in order to evaluate the $\Sigma$-mixing probabilities and the energy shifts which are obtained by Eqs. (7)–(9). In order to check our shell-model calculation, we compare our numerical results for the $Z \approx N$ $\Lambda$-hypernuclei to other work [14]. We obtain the energy shifts, e.g., $\Delta E = 0.085$ and 0.073 MeV for the ground states of $^7\text{Li}$ and $^{11}\text{B}$, which are
taken from Ref. [28]. All states in the figure have the isospin and comparable to the Millener’s results of 0.078 and 0.066 MeV, respectively. Therefore, we confirm that our calculations fully reproduce the Millener’s results.

In Fig. 1, we show the schematic energy levels for the Σ and S ground states of $^9\mathrm{Li}$. Here, we assume that the difference between Λ and Σ threshold energies is $E(9\mathrm{Li}_g.s. + \Sigma) - E(9\mathrm{Li}_g.s. + \Lambda) = 80$ MeV. Thus the energy of the Σ ground state $|\phi_{g.s.}\rangle$ is calculated to be $E^\Sigma_{g.s.} = \Delta M + B_\Sigma - B_\Lambda = 69.3$ MeV, measured from that of the Λ ground state $|\phi_{g.s.}\rangle$.

The calculated energy levels for the ground and low-lying excited states in $^{10}_\Lambda\mathrm{Li}$ are shown in Fig. 2. Here, the experimental $^9\mathrm{Li}$ binding energies [28] are used for the core-nucleus energies, instead of the calculated ones. The energy shift $\Delta E$ for $^{10}_\Lambda\mathrm{Li}$ is obtained to be 0.280 MeV which is about 3 times larger than that for $^7\mathrm{Li}$ or $^{11}_\Lambda\mathrm{Be}$. As shown in Fig. 2, the values of energy shifts for the low-lying excited states also account for about a few hundred keV. In particular, the second 2$^-$ state with the 4.604 MeV excitation energy has the large energy shift of 2.822 MeV. The calculated Σ-mixing probabilities are shown in Table II. It is found that the Σ-mixing probability is $P_\Sigma = 0.345$ % in the $^{10}_\Lambda\mathrm{Li}$ ground state, where the effect of renormalization is very small, $\mu_{\Sigma g.s.} = 0.344$ %. The $\Sigma^+$ and $\Sigma^0$ admixtures for the $^{10}_\Lambda\mathrm{Li}$ ground state are $P_{\Sigma^+} = 0.183$ % and $P_{\Sigma^0} = 0.159$ %, respectively. The $\Sigma^+$ admixture is negligible. This is the first result that the Σ-mixing probabilities and the energy shifts in the neutron-rich hypernucleus are coherently enhanced by the configuration mixing in the nuclear shell model.

### B. Λ and Σ eigenstates

We examine an enhancement of Σ-mixing probabilities and energy shifts of $^{10}_\Lambda\mathrm{Li}$ eigenstates by the configuration mixing in the Σ states, which couple to the Λ states by the Λ-Σ coupling. In order to investigate a change of the core-nuclear configuration due to the $YN$ interaction, we estimate the energy spectra and the hyperon-pickup spectroscopic factors in the Λ and Σ eigenstates with $T = \frac{3}{2}$. The $J^\pi = 1^−$ including the ground and excited states.

The calculated energy spectra of Λ eigenstates $|\psi_{g.s.}^\Lambda; JT\rangle$ are shown in the left panel of Fig. 3, measured from the ground state. We find that the energy spacings between the levels of $^{10}_\Lambda\mathrm{Li}$ are very similar to those of $^9\mathrm{Li}$ with $T = \frac{1}{2}$. The $J^\pi = 0^+$, as seen in Fig. 3. We confirm that the $^9\mathrm{Li}$ core state is slightly changed by the addition of the Λ hyperon, and that the Λ hyperon behaves as a single-particle motion in the nucleus [24] because the ΛN interaction is rather weak. The results are supported by the calculated Λ-pickup spectroscopic factors of Eq. (10). In Fig. 3, we also show the calculated spectroscopic factors $S_\Lambda$ for the Λ ground and two excited states; (a) $T^J = (\frac{1}{2}, 0^-)_{g.s.}$ at 0.0 MeV, (b) $\approx (\frac{1}{2}, 1^-)_{g.s.}$ at 9.5 MeV, and (c) $\approx (\frac{1}{2}, 0^-)_{g.s.}$ at 16.0 MeV. In the case of (a), we obtain $S_\Lambda \approx 1$ for the $^9\mathrm{Li}$ ground state and $S_\Lambda \approx 0$ for other eigenstates. Similarly, in the case of (b), $S_\Lambda \approx 1$ for the $^9\mathrm{Li}$ eigenstate. In the case of (c), $S_\Lambda$ has a large value for the two eigenstates, because these states are almost degenerate.

In Fig. 4, we display the calculated energy spectra of Σ eigenstates in $^{10}_\Lambda\mathrm{Li}$ and of $^9\mathrm{Be}$ eigenstates with $T = \frac{1}{2}, \frac{3}{2}$. The $J^\pi = \frac{1}{2}^+, \frac{3}{2}^+$, together with the Σ-pickup spectroscopic factors $S_\Sigma$ for three states; (a) $\approx (\frac{1}{2}, 1^-)_{g.s.}$ at 0.0 MeV, (b) $\approx (\frac{1}{2}, 1^-)_{g.s.}$ at 9.4 MeV, and (c) $\approx (\frac{1}{2}, 1^-)_{g.s.}$ at 19.7 MeV. The excited states of (b) and (c) are strongly coupled to the Λ ground state in the energy regions of $E^\Sigma_{g.s.} - E^\Lambda_{g.s.} \approx 80$ and 90 MeV, respectively, as we will mention in Fig. 5. The distributions of $S_\Sigma$ for (b) and (c) widely spread with the multi-configuration of $^9\mathrm{Be}^*$, as seen in Fig. 4. This implies that the Σ hyperon has the ability of largely changing the core-nuclear configuration.

The calculated Λ-Σ coupling strengths $|D_\mu|^2$ between the

**TABLE II**: The calculated Σ-mixing probabilities of $^{10}_\Lambda\mathrm{Li}$. All states in the table have the isospin $T = 3/2$.

| $J^\pi$ | $P_\Sigma$ [$\%$] | $P_\Sigma^-$ [$\%$] | $P_\Sigma^+$ [$\%$] | $P_\Sigma^0$ [$\%$] |
|--------|----------------|----------------|----------------|----------------|
| $^1^\Lambda_{g.s.}$ | 0.345 | 0.183 | 0.159 | 0.002 |
| $^2^\Lambda$ | 0.166 | 0.096 | 0.070 | 0.000 |
| $^0^\Lambda$ | 0.185 | 0.098 | 0.086 | 0.001 |
| $^1^-_{\Lambda}$ | 0.227 | 0.128 | 0.098 | 0.001 |
| $^2^+_{\Lambda}$ | 0.350 | 0.187 | 0.162 | 0.001 |
| $^3^-_{\Lambda}$ | 0.175 | 0.104 | 0.071 | — |

FIG. 1: Schematic energy levels for Λ and Σ ground states of $^{10}_\Lambda\mathrm{Li}$.

FIG. 2: The energy spectra of $^9\mathrm{Li}$ by the shell-model calculation with the Λ-Σ coupling. The experimental $^9\mathrm{Li}$ binding energies are taken from Ref. [28]. All states in the figure have the isospin $T = \frac{1}{2}$. 

| $E^\Lambda_{g.s.}$ [MeV] | $J^\pi$ | $^9\Lambda_{g.s.}$ | $E^\Lambda_{g.s.}$ [MeV] | $J^\pi$ | $^9\Lambda_{g.s.}$ |
|-----------------|--------|-----------------|-----------------|--------|-----------------|
| 4.296 | 5/2$^-$ | 3$^-$ | 4.944 | 0.139 |
| 2.691 | 1/2$^-$ | 2$^-$ | 4.590 | 0.282 |
| 0.000 | 3/2$^-$ | 1$^-$ | 3.566 | 0.181 |
| 0.000 | 3/2$^-$ | 0$^-$ | 3.551 | 0.150 |
| 0.000 | 3/2$^-$ | 0$^-$ | 0.395 | 0.134 |
| 0.000 | 3/2$^-$ | 0$^-$ | 0.000 | 0.280 |
MeV in NSC97e,f, lead to the attractive because of recent suggestions of repulsive labeled by (a), (b), and (c), are shown in each panel.

Further theoretical investigation are needed.

FIG. 3: Calculated energy spectra for Λ eigenstates of $^{10}_{\Lambda}\text{Li}$ and eigenstates of $^9\text{Li}$. Λ-pickup spectroscopic factors for three eigenstates, labeled by (a), (b), and (c), are shown in each panel.

Σ eigenstates $|\psi^{\Sigma}_g\rangle$ and the Λ ground state $|\psi^{\Lambda}_{g.s.}\rangle$ are shown in Fig. 5. It should be noticed that a contribution of the Σ ground state $|\psi^{\Sigma}_{g.s.}\rangle$ to the Σ-mixing of the ground state of $^{10}_{\Lambda}\text{Li}$ is reduced to $|D_{\mu}^{\Lambda}|^2 = 0.002\%$, whereas the several Σ excited states in the $E^{\Sigma}_{g.s.} - E^{\Lambda}_{g.s.} \approx 80$ MeV region considerably contribute to the Σ-mixing. These contributions are coherently enhanced by the configuration mixing which is caused by the $\Sigma N$ interaction. It is shown that the nature of the Σ-nuclear states plays an important role in the Λ-Σ coupling.

However, it should be noticed that the ΣN interaction has still ambiguities. The values of radial integrals in the ΣN $T = \frac{3}{2}$ effective interaction, $\bar{V} = -1.107$ and $\Lambda = 2.275$ MeV in NSC97e,f, lead to the attractive $^{3}\Sigma_1$ interaction of $^{33}V = \bar{V} + \frac{3}{2}\Lambda = -0.538$ MeV, as pointed out in Ref. [29], whereas the $T = \frac{5}{2}$, $^{3}\Sigma_1$ interaction may be repulsive [30, 31] because of recent suggestions of repulsive Σ-nuclear potentials [29, 32–34]. Further theoretical investigation are needed.

FIG. 5: Σ-Λ coupling strengths $|D_{\mu}^{\Lambda}|^2$ of the Σ eigenstates in the ground state of $^{10}_{\Lambda}\text{Li}$.

FIG. 6: (a) Fermi-type coupling strengths $|D_{\mu}^{\Lambda}|^2$ and (b) Gamow-Teller-type coupling strengths $|D_{\mu}^{\Gamma T}|^2$ of the Σ eigenstates in the ground state of $^{10}_{\Lambda}\text{Li}$.

IV. DISCUSSION

It is important to understand a mechanism of the Λ-Σ coupling in neutron-rich nuclei microscopically. When a Λ state in $^{10}_{\Lambda}\text{Li}$ converts to a $^\Sigma^-$ state by the Λ-Σ coupling interaction, the $^9\text{Li}$ core state changes into the $^9\text{Be}$ core state. In other words, the $^\beta^-$-transition, $^9\text{Li} \rightarrow ^9\text{Be}$, occurs in the core-nuclear state. Thus it is interesting to consider the $^\beta^-$-transitions between the core-nuclear components of Λ and Σ eigenstates in order to investigate the strength distribution of the Λ-Σ coupling. The two-body Λ-Σ coupling interaction $V_{\Sigma \Lambda}$ is approximately rewritten as

$$V_{\Sigma \Lambda} \approx V_{\Sigma \Lambda}^\beta + V_{\Sigma \Lambda}^{GT}$$  \hspace{1cm} (14)
with
\[ V^F_{\Sigma \Lambda} = V_F(r) \cdot \phi_{\Sigma \Lambda}, \]
\[ V^{GT}_{\Sigma \Lambda} = V_{\Gamma T}(r) \cdot (\sigma_{\Sigma} \cdot \sigma_{\Lambda}) \cdot t_X \cdot \phi_{\Sigma \Lambda}, \]
where \( t_X \) and \( \sigma_{\Sigma} t_X \) denote the Fermi and Gamow-Teller \( \beta \)-transition operators for nucleons, respectively, and the operator \( \phi_{\Sigma \Lambda} \) changes the \( \Lambda \) hyperon into the \( \Sigma \) hyperon,
\[ |j_\Sigma \rangle = \phi_{\Sigma \Lambda} |j_\Lambda \rangle, \]
and has tensorial rank-0 in the spin space and rank-1 in the isospin space; \( \sigma_{\Sigma} \) is the spin operator for a hyperon. Therefore, \( V^F_{\Sigma \Lambda} \) and \( V^{GT}_{\Sigma \Lambda} \) are regarded as the Fermi-type and Gamow-Teller-type coupling interactions, respectively. We stress that the \( \Lambda-\Sigma \) coupling strengths \( |D_{\mu}|^2 \) are extremely affected by strengths of these \( \beta \)-transitions between the core-nuclear states. We discuss which type interaction in \( V^F_{\Sigma \Lambda} \) contributes to the coupling strengths in the regions of \( E^F_{\mu} - E^A_{\mu,s} \approx 80 \text{ MeV} \), as shown in Fig. 5. We can evaluate the strengths,
\[ |D^F_{\mu}|^2 = \frac{|\langle \psi^A_{\mu,s}; T_j | (V^F_{\Sigma \Lambda} + V^{GT}_{\Sigma \Lambda}) | \psi_{\mu}^T \rangle|^2}{E^F_{\mu} - E^A_{\mu,s}}, \]
\[ |D^{GT}_{\mu}|^2 = \frac{|\langle \psi^A_{\mu,s}; T_j | V^{GT}_{\Sigma \Lambda} | \psi_{\mu}^T \rangle|^2}{E^F_{\mu} - E^A_{\mu,s}}, \]
using \( V_{\Sigma \Lambda} = 1.45 \text{ MeV} \) and \( \Delta_{\Sigma \Lambda} = 3.04 \text{ MeV} \) in Table I. The calculated strength distributions of the Fermi- and Gamow-Teller-type couplings are shown in Figs. 6 (a) and (b), respectively. Comparing Fig. 6 with Fig. 5, we recognize the Fermi and Gamow-Teller components coherently contribute to the \( \Lambda-\Sigma \) coupling strengths in the energy region of \( E^F_{\mu} - E^A_{\mu,s} \approx 80 \text{ MeV} \). The calculated \( \Sigma \)-mixing probability involving both couplings is
\[ \sum_{\mu} \frac{|\langle \psi^A_{\mu,s}; T_j | (V^F_{\Sigma \Lambda} + V^{GT}_{\Sigma \Lambda}) | \psi_{\mu}^T \rangle|^2}{E^F_{\mu} - E^A_{\mu,s}} = 0.350 \%, \]
which is close to the full calculated probability \( P_\Sigma = 0.345 \% \), while \( \sum_{\mu} |D^F_{\mu}|^2 = 0.144 \% \) is obtained for the Fermi type and \( \sum_{\mu} |D^{GT}_{\mu}|^2 = 0.098 \% \) for the Gamow-Teller type.

The Fermi-type operator \( t_X \cdot \phi_{\Sigma \Lambda} \) does not change the core-nuclear states in \|\psi^A_{\mu,s}\rangle\), which is equivalent to the Fermi \( \beta \)-transition, \(|(\Lambda Z)_{T \pi} \rightarrow \Lambda (\Lambda Z \pm 1)_{T \mp 1} \rangle\). Note that this transition can change only \( T_\pi \). By the Fermi \( \beta \)-transition from the \( ^9\text{Li} T_\pi = \frac{1}{2} \) ground state, the \( ^9\text{Be}^+ T_\pi = \frac{1}{2} \) excited state is populated near \( E^F_{\mu} \approx 40 \text{ MeV} \), measured from the \( ^9\text{Be}^+ T_\pi = \frac{1}{2} \) ground state. If the weak-coupling limit in the \( \Sigma N \) interaction works well like the \( \Lambda N \) interaction, the coupling is concentrated on the only one state. As a result, one peak arises at \( E^F_{\mu} - E^A_{\mu,s} \approx 80 \text{ MeV} \) in the coupling strength. However, the calculated strength distribution of \( |D^F_{\mu}|^2 \) widely spreads to the energy regions of \( E^F_{\mu} - E^A_{\mu,s} = 75-80 \) and about \( 90 \text{ MeV} \), as shown in Fig. 6 (a). This means that the \( \Sigma \) hyperon changes a nuclear configuration mixing due to the strong spin-isospin dependence in the \( \Sigma N \) interaction, leading to the coherence of the \( \Lambda-\Sigma \) coupling.

In conclusion, we have found that the Fermi-type coupling strength \( |D^F_{\mu}|^2 \) is proportional to \( T(T+1) \). Therefore, the Fermi-type coupling strengths play the important role in the \( \Sigma \)-mixing probability of neutron-rich \(^{10}_{\Lambda A}\) eigenstates because of \( T = \frac{3}{2} \). On the other hand, in \(^{11}_{\Lambda Li} T = 0 \) states, the Fermi-type coupling strengths vanish whereas the \( \Sigma \)-mixing probability is mainly caused by the Gamow-Teller-type coupling strengths.

In the Gamow-Teller transitions for ordinary nuclei, the sum rule [35]
\[ \sum B(\Gamma T) - \sum B(\Gamma T^+) = 3(N-Z) \]
is well known as a model independent formula, where \( B(\Gamma T^\mp) \) is a strength of the Gamow-Teller \( \beta^\mp \)-transition, \(|\langle \Lambda Z^\pm 1 \rangle_{T \mp 1} |^2 \). In general, \( \sum B(\Gamma T^+) \) becomes smaller as neutron-excess grows larger, leading to \( \sum B(\Gamma T) \approx 3(N-Z) \). Therefore, the Gamow-Teller-type coupling is very important in \( \Lambda \)-hypernuclei with large neutron excess.

\section{SUMMARY AND CONCLUSION}
We have investigated the structure of the neutron-rich \(^{10}_{\Lambda A}\) hypernucleus, in shell-model calculations considering the \( \Lambda-\Sigma \) coupling in the perturbation theory. We have found that the \( \Sigma \)-mixing probabilities and the energy shifts of \(^{10}_{\Lambda A}\) eigenstates are coherently enhanced by the \( \Lambda-\Sigma \) coupling configurations in the neutron-rich nucleus. We have argued the effects of the \( \Lambda-\Sigma \) coupling interaction in terms of the \( \beta \)-transitions for the core-nuclear states. The reasons why the \( \Sigma \)-mixing probabilities are enhanced are summarized as follows: (i) The multi-configuration \( \Sigma \) excited states can be strongly coupled with the \( \Lambda \) ground state with the help of the \( \Sigma N \) interaction. (ii) These strong \( \Lambda-\Sigma \) couplings are coherently enhanced by the Fermi- and Gamow-Teller-type coupling components. (iii) The Fermi-type coupling becomes more effective in the neutron-rich environment increasing as \( T(T+1) \).

In conclusion, we have found that the \( \Sigma \)-mixing probability is about 0.34 \% and the energy shift is about 0.28 MeV for
the neutron-rich $^{10}_{\Lambda}$Li ground state, which is about 3 times larger than that for $^{7}_{\Lambda}$Li. This is the first estimation of the $\Sigma$-mixing in the neutron-rich hypernuclei by microscopic shell-model calculations.

Acknowledgments

The authors are obliged to K. Muto for valuable discussion and useful comments. They are pleased to acknowledge D. J. Millener for providing the $\Sigma N$ effective interaction in $p$ shell based on NSC97e,f and for valuable discussion and comments. This work is supported by Grant-in-Aid for Scientific Research on Priority Areas (Nos. 17070002 and 20028010).

[1] L. Majling, Nucl. Phys. A585, 211c (1995).
[2] Khin Swe Myint, T. Harada, S. Shinmura, and Y. Akaishi, Few-Body Syst. Suppl. 12, 383 (2000).
[3] Y. Akaishi, T. Harada, S. Shinmura, and Khin Swe Myint, Phys. Rev. Lett. 84, 3539 (2000).
[4] M. Baldo, G. F. Burgio, and H.-J. Schulze, Phys. Rev. C 61, 055801 (2000).
[5] O. Sorlin and M.-G. Porquet, Prog. Part. Nucl. Phys. 61, 602 (2008), and references therein.
[6] A. Umeya and K. Muto, Phys. Rev. C 69, 024306 (2004).
[7] A. Umeya and K. Muto, Phys. Rev. C 74, 034330 (2006).
[8] A. Umeya, G. Kaneko, T. Haneda, and K. Muto, Phys. Rev. C 77, 044301 (2008).
[9] A. Gal, J. M. Soper, and R. H. Dalitz, Ann. Phys. (NY) 63, 53 (1971); A. Gal, J. M. Soper, R. H. Dalitz, Ann. Phys. (NY) 72, 445 (1972); A. Gal, J. M. Soper, R. H. Dalitz, Ann. Phys. (NY) 113, 79 (1978).
[10] R. H. Dalitz and A. Gal, Ann. Phys. (NY) 116, 167 (1978).
[11] D. J. Millener, A. Gal, C. B. Dover, and R. H. Dalitz, Phys. Rev. C 31, 499 (1985).
[12] E. H. Auerbach, A. J. Baltz, C. B. Dover, A. Gal, S. H. Kahana, L. Ludeking, and D. J. Millener, Ann. Phys. (NY) 148, 381 (1983).
[13] K. Itonaga, T. Motoba, and H. Bandō, Prog. Theor. Phys. 84, 291 (1990).
[14] D. J. Millener, Springer Lecture Notes in Physics 724, 31 (2007); D. J. Millener, Nucl. Phys. A804, 84 (2008).
[15] S. Fujii, R. Okamoto, and K. Suzuki, Phys. Rev. C 66, 054301 (2002).
[16] D. Halderson, Phys. Rev. C 77, 034304 (2008).
[17] H. Tamura, K. Hosomi, T. Koike, Y. Ma, M. Mimori, K. Miwa, K. Shiratori, and M. Ukai, Nucl. Phys. A804, 73 (2008), and references therein.
[18] P. M. M. Maessen, Th. A. Rijken, and J. J. de Swart, Phys. Rev. C 40, 2226 (1989).
[19] Th. A. Rijken, V. G. J. Stoks, and Y. Yamamoto, Phys. Rev. C 59, 21 (1999).
[20] Th. A. Rijken and Y. Yamamoto, Phys. Rev. C 73, 044008 (2006).
[21] P. K. Saha, T. Fukuda, W. Imoto, J. K. Ahn, S. Ajimura, K. Aoki, H. C. Bhang, H. Fujioka, H. Hotchi, J. I. Hwang, T. Itabashi, B. H. Kang, H. D. Kim, M. J. Kim, T. Kishimoto, A. Krutenkova, T. Maruta, Y. Miura, K. Miwa, T. Nagae, H. Nouni, H. Outa, T. Ohtaki, A. Sakaguchi, Y. Sato, M. Sekimoto, Y. Shimizu, H. Tamura, K. Tanida, A. Toyoda, M. Ukai, and H. J. Yim, Phys. Rev. Lett. 94, 052502 (2005).
[22] T. Yu. Tretyakova and D. E. Lanskoy, Phys. At. Nucl. 66, 1651 (2003).
[23] T. Harada, A. Umeya, and Y. Hirabayashi, Phys. Rev. C 79, 014603 (2009).
[24] C. B. Dover, D. J. Millener, and A. Gal, Phys. Rep. 184, 1 (1989).
[25] A. R. Edmonds, Angular Momentum in Quantum Mechanics, 2nd ed. (Princeton University Press, New Jersey, 1960).
[26] S. Cohen and D. Kurath, Nucl. Phys. 73, 1 (1965).
[27] D. J. Millener, private communication.
[28] D. R. Tilley, J. H. Kelley, J. L. Godwin, D. J. Millener, J. E. Purcell, C. G. Sheu, and H. R. Weller, Nucl. Phys. A745, 155 (2004).
[29] J. Dąbrowski, Phys. Rev. C 60, 025205 (1999).
[30] Y. Fujiwara, Y. Suzuki, and C. Nakamoto, Prog. Part. Nucl. Phys. 58, 439 (2007).
[31] Th. A. Rijken and Y. Yamamoto, Nucl. Phys. A804, 51 (2008).
[32] T. Harada and Y. Akaishi, Phys. Lett. B262, 205 (1991).
[33] H. Nouni, P. K. Saha, D. Abe, S. Ajimura, K. Aoki, H. C. Bhang, T. Endo, Y. Fujii, T. Fukuda, H. C. Guo, K. Imai, O. Hashimoto, H. Hotchi, E. H. Kim, J. H. Kim, T. Kishimoto, A. Krutenkova, K. Maeda, T. Nagae, M. Nakamura, H. Outa, M. Sekimoto, T. Saito, A. Sakaguchi, Y. Sato, R. Sawafita, Y. Shimizu, T. Takahashi, L. Tang, H. Tamura, K. Tanida, T. Watanabe, H. H. Xia, S. H. Zhou, L. H. Zhu, and X. F. Zhu, Phys. Rev. Lett. 89, 072301 (2002).
[34] E. Friedman and A. Gal, Phys. Rep. 452, 89 (2007).
[35] K. Ikeda, S. Fujii, and J. I. Fujita, Phys. Lett. 3, 271 (1963).