DREAM: a fluid-kinetic framework for tokamak disruption runaway electron simulations

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Runaway electrons

TCV tokamak

Aurora

Lightning

Solar flares
Can be generated during startup or shutdown

Disruptions can suddenly occur \(\Rightarrow\) significant runaway electron generation

- Runaways mostly benign in today’s devices
- …but avalanche multiplication gives

\[
n_{RE} \sim n_{RE,0} \exp \left(\frac{I_p}{170 \text{kA}}\right)
\]

- Big problem in reactor-scale devices!

Runaway impact in Alcator C-Mod.
(from R. A. Tinguely et al, *Nucl. Fusion* 58 076019 (2018))
Disruptions unfold in three main phases:

1. **Thermal quench (TQ): rapid temperature drop**
   - Due to turbulent transport and radiation
   - Resistivity increases significantly

![Schematic overview of disruption.](Image)

\[
\begin{align*}
I_{\text{re}} & \quad \text{Plasma current} \\
E_\parallel & \quad \text{Electric field} \\
T_e & \quad \text{Temperature}
\end{align*}
\]
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1. **Thermal quench (TQ):** rapid temperature drop
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2. **Current quench (CQ):** ohmic current drops
   - Electric field is induced
   - Electrons are accelerated
   - Runaway electrons multiply

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*Schematic overview of disruption.*
Evolution of a tokamak disruption

Disruptions unfold in three main phases:

1. **Thermal quench (TQ):** rapid temperature drop
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   - Resistivity increases significantly

2. **Current quench (CQ):** ohmic current drops
   - Electric field is induced
   - Electrons are accelerated
   - Runaway electrons multiply

3. **Runaway plateau**
   - Remaining current carried by runaway electrons

*Schematic overview of disruption.*
Runaway electron generation during disruptions has typically been treated using fluid models:

- $T_e$ (temperature): $P_\Omega - P_{\text{rad}} - P_{\text{transp}}$
- $n_i$ (ion density): $I - R$ (ionized – recombined)
- $E_\parallel$ (electric field): Ampère-Faraday equation
- $j$ (current density): $j_\Omega + j_{\text{re}}$
- $n_{\text{re}}$ (runaway density): $\gamma_{\text{Dreicer}} + \Gamma_{\text{ava}} n_{\text{re}} + \ldots$
Runaway electron generation during disruptions has typically been treated using fluid models:

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However, some aspects of RE are difficult to capture with a fluid model...
Hot-tail mechanism

Difficult to model because of its inherently transient nature

Requires kinetic equation to be solved
Hot-tail mechanism

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Distribution function

Temperature drop

"Hot tail"
Hot-tail mechanism

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Disruption Runaway Electron Analysis Model
Hoppe, Embreus and Fülöp, Comp. Phys. Comm. 268 108098 (2021)
Goal: Flexible and lightweight tokamak disruption simulator

Axisymmetric geometry
- 1D space
- 0D/1D/2D momentum space

Emphasis on advanced RE models
- Many possible hot-tail models
- Accounting for partial screening
- Fast electron impact ionization
- RE radial transport

https://github.com/chalmersplasmatheory/DREAM

1 Smith and Verwichte, Phys. Plasmas 15 (6) 072502 (2008) doi:10.1063/1.2949692
2 Aleynikov and Breizman, Nucl. Fusion 57 (4) 046009 (2017) doi:10.1088/1741-4326/aa5895
3 Hesslow et al., $(E_{c,\text{eff}})$ Plasma Phys. Control. Fusion 60 (7) 074010 (2018) doi:10.1088/1361-6587/aac33e
4 Hesslow et al., (Avalanche growth rate) Nucl. Fusion 59 (8) 084004 (2019) doi:10.1088/1741-4326/ab26c2
5 Hesslow et al., (Dreicer rate) J. Plasma Phys. 85 (4) 475850601 (2019) doi:10.1017/S0022377819000874
6 Garland et al., Phys. Plasmas 27 (4) 040702 (2020) doi:10.1063/5.0003638
7 Rechester and Rosenbluth, Phys. Rev. Lett. 40 (1) 38 (1978) doi:10.1103/PhysRevLett.40.38
8 Svensson et al., J. Plasma Phys. 87 (2) 905870207 (2021) doi:10.1017/S0022377820001592
Ampère’s & Faraday’s laws — $E_\parallel$, $\psi$, $j_{\text{tot}}$

\[
2\pi\mu_0 \langle \mathbf{B} \cdot \nabla \phi \rangle \frac{j_{\text{tot}}}{B} = \frac{1}{V'} \frac{\partial}{\partial r} \left[ V' \left\langle \frac{|\nabla r|^2}{R^2} \right\rangle \frac{\partial \psi}{\partial r} \right], \quad (1a)
\]

\[
\frac{\partial \psi}{\partial t} = -V_{\text{loop}} + \frac{\partial}{\partial \psi_t} \left( \psi_t \mu_0 \Lambda \frac{\partial}{\partial \psi_t} \frac{j_{\text{tot}}}{B} \right). \quad (1b)
\]

Boundary conditions accounting for conducting structures.
Ion charge state densities — $n^{(Z_0)}_i$, $Z_0 = 0, 1, \ldots , Z$

\[
\frac{\partial n^{(Z_0)}_i}{\partial t} = \left( I^{(Z_0-1)}_i \langle n_e \rangle + \langle \sigma^{(Z_0-1)}_{\text{ion},i} v \rangle \right) n^{(Z_0-1)}_i - \left( I^{(Z_0)}_i \langle n_e \rangle + \langle \sigma_{\text{ion},i} v \rangle \right) n^{(Z_0)}_i \\
+ R^{(Z_0+1)}_i \langle n_e \rangle n^{(Z_0+1)}_i - R^{(Z_0)}_i \langle n_e \rangle n^{(Z_0)}_i ,
\]

with

\[
\langle \sigma^{Z_0}_{\text{ion},i} v \rangle = \int dp \int_{-1}^1 d\xi_0 \frac{v'}{V} v \sigma^{(Z_0)}_{\text{ion},i} f_e(t, r, p, \xi_0). \tag{3}
\]

Rate coefficients $I^{(Z_0)}_i, R^{(Z_0)}_i$ taken from ADAS$^9$.

Ionization cross-section $\sigma^{(Z_0)}_{\text{ion},i}$ taken as in Ref. 6.

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$^6$Garland et al., Phys. Plasmas 27 (4) 040702 (2020) doi:10.1063/5.0003638

$^9$Summers, The ADAS user manual, version 2.6 http://www.adas.ac.uk (2004)
Energy balance — $T_e, T_i$

$W_k = 3n_kT_k/2 =$ thermal energy of species $k$.

Electron balance:

$$\frac{\partial W_e}{\partial t} = \frac{j\Omega}{B} \langle E \cdot B \rangle$$

(Ohmic heating)

$$- \langle n_e \rangle \sum_i \sum_{Z_0=0}^{Z_i-1} n_i(Z_0) L_i(Z_0)$$

(Ionization/radiation)

$$- \langle Q_c \rangle$$

(Collisions)

$$+ \frac{1}{V'} \frac{\partial}{\partial r} \left[ V' \frac{3}{2} \langle n_e \rangle \left( A_W T_e + D_W \frac{\partial T_e}{\partial r} \right) \right],$$

(Transport)

and ion balance

$$\frac{\partial W_i}{\partial t} = \sum_j Q_{ij} + Q_{ie}$$

(Collisions)
Runaways treated as a fluid in “conventional” simulation mode:

\[
\frac{\partial n_{re}}{\partial t} = \gamma_{Dreicer} + \gamma_{hottail} + \Gamma_{ava}n_{re} + \gamma_{tritium} + \gamma_{Compton} + \frac{1}{V'} \frac{\partial}{\partial r} \left[ V' \left( A \langle n_{re} \rangle + D \frac{\partial \langle n_{re} \rangle}{\partial r} \right) \right]
\]

(can also be evolved fully kinetically)
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\]

(can also be evolved fully kinetically)

\[
F_{\text{kinetic}} = \int V' \left( \left\{ A^p \right\} f_e + \left\{ D^{pp} \right\} \frac{\partial f_e}{\partial p} \right) d\xi_0 \bigg|_{p=p_{re}}
\]
Injection of cold impurities

Two electron populations:
- Injected, **cold** electrons: \( n_{\text{cold}} \) (fluid)
- Original, **hot** electrons: \( f_{\text{hot}} \) (kinetic)

Only account for hot-cold collisions

Superthermal limit of FP equation
- \( v \gg v_{\text{th}} \)
- Sink in \( p = 0 \)

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\(^2\)Aleynikov and Breizman, Nucl. Fusion **57** (4) 046009 (2017) doi:10.1088/1741-4326/aa5895
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Aleynikov and Breizman, Nucl. Fusion 57 (4) 046009 (2017) doi:10.1088/1741-4326/aa5895
RE seed transport in ITER

from ‘Hot-Tail Runaway Seed Landscape during the Thermal Quench in Tokamaks’
Svenningsson et al, Phys. Rev. Lett. 127 (3) 035001 (2021)

doi:10.1103/PhysRevLett.127.035001
Simulation setup

- MMI-triggered ITER disruption: **Deuterium + Neon**
- Evolve hot-tail with **two-temperature** model
- Include **radial transport** of hot electrons\(^7\) and heat
- Scan in:
  - Amount of injected Ne
  - Magnetic field perturbation amplitude \(\delta B/B\)
- Consider mainly TQ (no avalanche generation)

We optimize for
- Minimal seed RE current \(I_{RE}^{\text{seed}}\)
- Maximal cooling via radiation

\(^7\)Rechester and Rosenbluth, Phys. Rev. Lett. **40** (1) 38 (1978) doi:10.1103/PhysRevLett.40.38
Thermal quench dynamics

- TQ evolves edge $\rightarrow$ core
  - Cold front propagates inwards
- Rapid TQ near edge $\rightarrow$
  RE generation near edge $\Rightarrow$
  - More easily deconfined via external perturbations

$I_p = 15$ MA, $T_0 = 15$ keV

$n_D = 1 \times 10^{21}$ m$^{-3}$, $n_{Ne} = 9 \times 10^{19}$ m$^{-3}$ (uniform)

$\delta B/B = 0.16\%$ (except (c) and (e))
 suppression with transport is possible!

- Care must be taken to avoid
  - Too large runaway seed (solid white)
  - Too high heat flux to PFCs (dashed white)

**Flat profiles:** impurities uniformly distributed

![Graph showing suppression of runaway seed with different profiles and scenarios.](image-url)
(b) Lower $I_p$

- Higher $I_{RE}^{\text{seed}}$ allowed
- Weaker electric fields induced

![Graph showing Runaway seed suppression comparison of scenarios](image-url)
(b) Lower $I_p$
- Higher $I_{RE}^{seed}$ allowed
- Weaker electric fields induced

(c) Quadratic impurity profile
- Radiated power $\propto n_i^2$
- Transported heat $\propto n_i$
- More heat \textbf{radiated away}
- Low core density
- Lower collisionality, \textit{survival} of fast electrons
(b) Lower $I_p$
   - Higher $I_{RE}^{seed}$ allowed
   - Weaker electric fields induced

(c) Quadratic impurity profile
   - Radiated power $\propto n_i^2$
   - Transported heat $\propto n_i$
   $\Rightarrow$ More heat radiated away
   - Low core density
   $\Rightarrow$ Lower collisionality, survival of fast electrons

(d) Lower temperature
   - TQ time decreases faster than the collision time with $T_e$
   $\Rightarrow$ Hot-tail generation more efficient
Some completed features:

- Everything shown in this presentation
- Fluid runaway transport model\textsuperscript{8}
- Shattered Pellet Injection (SPI) module\textsuperscript{10}

Currently under development:

- Ion transport
- Kinetic + synthetic diagnostic simulations: synchrotron radiation in JET\textsuperscript{11} and TCV
- Prevention of RE formation in SPARC with passive coils\textsuperscript{12}
- Tokamak burn-through model

\textsuperscript{8}Svensson \textit{et al.}, J. Plasma Phys. \textbf{87} (2) 905870207 (2021) doi:10.1017/S0022377820001592
\textsuperscript{10}Vallhagen, MSc Thesis (2021) https://hdl.handle.net/20.500.12380/302296
\textsuperscript{11}Brandström, MSc Thesis (2021) https://hdl.handle.net/20.500.12380/302636
\textsuperscript{12}Tinguely \textit{et al}, TSDW Princeton (2021) https://tsdw.pppl.gov/Talks/2021/Tinguely.pdf
DREAM: a new framework for tokamak disruption simulations, with focus on the runaway generation

Many options for simulation runaway electrons:
- As a fluid (density $n_{re}$)
- Kinetic hot-tail (e.g. two-temperature model)
- Kinetic hot-tail and runaway

Magnetic perturbations (naturally induced or externally applied) may provide safe suppression of seed electrons in ITER

Hoppe, Embreus and Fülöp, Comp. Phys. Comm. 268 108098 (2021)
Svenningsson et al, Phys. Rev. Lett. 127 (3) 035001 (2021)
Benchmark
Comparison with figure 4 of ‘Vallhagen et al, J. Plasma Phys. 86 (4) 475860401 (2020)’

- Good quantitative agreement
- Differences attributed to improved model for $E_{c, \text{eff}}$ used in DREAM
Benchmark: DREAM and CODE

Conductivity

![Conductivity Graph]

- CODE = lines
- DREAM = crosses

(agreement to within 0.3%)

Dreicer runaway rate

![Dreicer Runaway Rate Graph]

\[ \bar{E} = \frac{(E - 2E_c)}{0.04E_D - 2E_c} \]

(agreement to within 3.5%)
Energy balance — $T_e, T_i$

$W_e, W_i = \text{thermal energy of electrons/ions}$

\[
\frac{\partial W_e}{\partial t} = \frac{j \Omega}{B} \langle E \cdot B \rangle - \langle n_e \rangle \sum_i \sum_{Z_0=0}^{Z_i-1} n_i(Z_0) L_i(Z_0) - \langle Q_c \rangle + \frac{1}{V'} \frac{\partial}{\partial r} \left[ V' \frac{3 \langle n_e \rangle}{2} \left( A_W T_e + D_W \frac{\partial T_e}{\partial r} \right) \right]
\]

(5a)

\[
\frac{\partial W_i}{\partial t} = \sum_j Q_{ij} + Q_{ie},
\]

(5b)

with

\[
\langle Q_c \rangle = \int dp \int_{-1}^1 d\xi_0 \frac{V'}{V} \Delta \dot{E}_{ee}(t, r, p, \xi_0) + \sum_i Q_{ei}
\]

\[
Q_{kl} = \frac{\langle n Z^2 \rangle_k \langle n Z^2 \rangle_l}{(2\pi)^{3/2} e_0^2 m_k m_l} e^4 \ln \Lambda_{kl} \frac{T_k - T_l}{\left( \frac{T_k}{m_k} + \frac{T_l}{m_l} \right)^{3/2}},
\]

\[
L_{i}(Z_0) = L_{\text{line}} + L_{\text{free}} + \Delta W_i(Z_0) \left( I_{i}(Z_0) - R_{i}(Z_0) \right).
\]
\[ \frac{\partial f}{\partial t} = \sum_{m,n} \frac{1}{\mathcal{V}'} \frac{\partial}{\partial z^m} \left[ \mathcal{V}' \left( -\{A^m\} f + \{D^{mn}\} \frac{\partial f}{\partial z^n} \right) \right] + \{S_A\} \]

\[ A = A_E + A_C + A_B + A_S + A_T \]
\[ D = D_C + D_T \]

with

\[ A_E = \text{Electric field acceleration}, \]
\[ A_C = \text{Collisional friction}, \]
\[ A_B = \text{Bremsstrahlung radiation reaction}, \]
\[ A_S = \text{Synchrotron radiation reaction}, \]
\[ A_T = \text{Radial transport}, \]
\[ D_C = \text{Collisional momentum-space diffusion}, \]
\[ D_T = \text{Radial diffusion}, \]
\[ S_A = \text{Avalanche source}. \]