A numerical analysis method for identification of three-dimensional vortical structure of spiral vortex in wind turbine with two-dimensional velocity data in parallel planes at plural azimuthal angles

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Abstract
An identification method is presented to specify three-dimensional vortical flow topology of a spiral vortex (wing tip vortex) in a wind turbine, from two-dimensional velocity data in planes at different azimuthal angles. This method needs only the two-dimensional velocity field (data) in the parallel planes, and need not change the angle (horizontality) of the planes. The three-dimensional velocity structure is specified by physical properties associated with the velocity gradient tensor, and the formulation of the linear transformation between rotated coordinate systems associated with a spiral vortex derives the unknown components in the three-dimensional velocity gradient tensor. This method specifies the three-dimensional local vortical flow topology in detail including swirl plane, vortical axis and its orthogonality. Swirlity specifies the unidirectionality and intensity of the azimuthal flow, and sourcity does those of the radial flow. It also identifies the vortical flow symmetry that are associated with the important vortical features such as the pressure minimum.

Key words : Vortical flow, Topology, Spiral vortex, Wind turbine, Swirlity, Sourcity

1. Introduction

Vortices in turbulent flow influence the flow characteristics and also the design condition or performance of machines or facilities in fluid engineering. Flow around a wind turbine is not an exception. A wind turbine exhibits several types of vortices such as wing-tip and root vortices, and they influence the flow state of the wake and thus succeeding wind turbines in a wind farm. A spiral vortex generated at the tip of a rotating blade, that is associated with the wing-tip vortex is an important one for the characteristics of the wind turbine and environmental aspect.

An experimental approach is appropriate to simulate the spiral vortex with a high Reynolds number and investigate its vortical structure (Dobrev et al., 2008, Maalouf et al., 2009). For the purpose of clarifying the three-dimensional vortical structure of the spiral vortex, consequently it requires three-dimensional velocity field. However, unlike numerical analysis, the experiment may be difficult to obtain detail and sufficient flow information. Experimental condition often restricts the measurement of physical quantities or the range to be measured. Three-dimensional measurement is not always applicable and even two-dimensional measurement of velocity is often restricted to perform in a specified plane, which complicates clarifying the actual vortical structure.

On the other hand, as for the identification of the topological feature of a vortex, it is useful to specify the vortical flow structure independent of the inertia coordinate system, i.e., Galilei invariant flow topology (geometry). Then it
may not be appropriate to analyze the geometrical feature directly from the velocity field because it has an effect of the uniform velocity in general and then the streamline is not Galilei invariant. Identification of the vortical structure in terms of the Galilei invariant flow geometry needs the three-dimensional velocity gradient tensor $\nabla \mathbf{v}$ (Chong et al., 1990). The eigenvalues and eigenvectors of $\nabla \mathbf{v}$ specify the invariant local flow geometry, and have greatly contributed to clarify the vortical structure in turbulent flows (Soia et al., 1994, Blackburn et al., 1996). In addition, the physical interpretation of the eigenvalues has been clarified, and several physical quantities have been proposed to specify detail flow geometry (Nakayama, 2014), which not only identifies the precise geometry but also specifies relationships between the geometry and vortical features (Nakayama et al., 2014, Nakayama et al., 2015, Nakayama et al., 2016). Then a materialization of a numerical method to derive three-dimensional $\nabla \mathbf{v}$ from two-dimensional velocity data in experiment may bring about great contribution for clarifying a vortical structure and associated turbulent phenomena.

This paper presents a novel numerical method to estimate three-dimensional $\nabla \mathbf{v}$ from two-dimensional velocity field data and extracts the three-dimensional geometrical feature of a spiral vortex. This method supposes that velocity fields are measured in parallel planes near the trajectory of a rotating blade at different azimuthal angles of the blade. Then it can be assumed that a spiral vortex passes through a measured plane with a different angle associated with the azimuthal angle of the considered plane. That is, the velocity field in the measured planes is that in a cross section of the spiral vortex at an angle associated with the azimuthal angle. The components of $\nabla \mathbf{v}$ associated with a plane where the two-dimensional velocity is measured by experiment are estimated by a finite difference formula, and the relationship of a linear transformation of $\nabla \mathbf{v}$ between two different coordinate systems is applied to solve the missed components of $\nabla \mathbf{v}$ to complete the three-dimensional tensor. Incompressible condition is also applied in the present formulation.

This method enables to estimate three-dimensional vortical structure, including the swirl plane, vortical axis and the angle (orthogonality) between the swirl plane and vortical axis. The vortical flow symmetry (skewness of vortical flow) is also given that is associated with the pressure minimum feature (Nakayama et al., 2014, Nakayama et al., 2016). Furthermore the flow symmetry is associated with the vortex stretching, as the radial flow feature specified by the symmetry influences the effective vortex stretching that strengthens the vorticity associated with the swirling. Swirlity and sourcity specify the unidirectionality and intensity of the respective azimuthal and radial flows in the swirl plane (Nakayama, 2014). This analysis specifies not only the detail flow geometry but also important feature corresponding to the development or stability of a vortex in terms of flow kinematics.

2. Formulation to derive 3-dim velocity gradient tensor

We derive the formulation that composes the three-dimensional $\nabla \mathbf{v}$ with two-dimensional velocity fields in plural planes. In this formulation, it is assumed that a spiral vortex is at the same state (vortical structure) in the subjected planes, and that the vortex passes through the plane with an angle associated with the azimuthal angle of the plane, as shown in Fig. 1. The horizontal azimuth (azimuthal) plane is defined as the plane at $\theta = 0$ in Fig. 1.

In the plane where $\theta = 0$, we consider a Cartesian coordinate system $x_i \ (i = 1, 2, 3)$ with orthonormal bases $e_i \ (i = 1, 2, 3)$ where $e_1$ is parallel to mean flow and the $x_1$-$x_2$ plane is the horizontal subjected plane and identical to the azimuth plane. The velocity $v_i \ (i = 1, 2, 3)$ is supposed to be measured in the $x_1$-$x_2$ plane in terms of $v_1$ and $v_2$ components. The matrix form of $\nabla \mathbf{v}$, say $A(0) = [a_{ij}] = [\partial v_j / \partial x_i] \ (i, j = 1, 2, 3)$, is expressed in the following form:

$$
A(0) = \begin{bmatrix}
\tilde{a}_{11} & \tilde{a}_{12} & a_{13} \\
\tilde{a}_{21} & \tilde{a}_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix},
$$

(1)

where $\tilde{a}_{ij} \ (i, j = 1, 2)$ denote the components that can be estimated by the measured velocity data using the finite differential formulas. Then the measurement of the velocity field in a plane gives four components of $A$, and five components are unknown.

Next we consider the representation of $\nabla \mathbf{v}$ associated with another parallel (horizontal) plane at a different azimuthal angle rotated by $\theta$ clockwise, denoted as $A(\theta) = [a'_{ij}] \ (i, j = 1, 2, 3)$. Then, because the state of the spiral vortex is assumed to be the same, the velocity field in terms of the coordinate system associated with this plane is similar to that in the original plane where $\theta = 0$, except that the bases $e_2$ and $e_3$ are rotated. That is, we rotate $e_2$ and $e_3$ along $e_1$ by $\theta$ clockwise (counterclockwise from the viewpoint of Fig. 1), and express as $e'_2(\theta) = (\cos \theta, - \sin \theta)$ and $e'_3(\theta) = (0, \sin \theta, \cos \theta)$, respectively. It is noted that the $x_1$ direction (the base $e_1$) does not change and that $A(\theta)$ is a similar matrix of $A(0)$. Then
A(0) and A(θ) have the following relation:

\[ A(θ) = P_θ^{-1} A(0) P_θ, \]  \tag{2}  

\[ P_θ = [e_1(θ), e_2(θ), e_3(θ)] \]
\[ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos θ & \sin θ \\ 0 & -\sin θ & \cos θ \end{bmatrix}. \]  \tag{3}  

It is noted that \( P_θ \) is a unitary matrix and that \( P_θ^{-1} = \dag P_θ \), where the superscript \( \dag \) denotes the transpose of the matrix. Equations (2) and (3) derive the following equations:

\[ A(θ) = [a_{ij}'] \]
\[ = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} \cos θ - a_{13} \sin θ & \bar{a}_{12} \sin θ + a_{13} \cos θ \\ \bar{a}_{21} \cos θ - a_{23} \sin θ & \bar{a}_{21} \sin θ + a_{23} \cos θ & c_1 \cos θ - c_3 \sin θ + c_3 \sin θ + c_4 \cos θ \\ \bar{a}_{31} \cos θ - a_{33} \sin θ & \bar{a}_{31} \sin θ + a_{33} \cos θ & c_1 \cos θ + c_2 \sin θ + c_3 \sin θ + c_4 \cos θ \end{bmatrix}. \]  \tag{4}  

\[ c_1 = \bar{a}_{22} \cos θ - a_{23} \sin θ, \]  \tag{5}  

\[ c_2 = a_{32} \cos θ - a_{33} \sin θ, \]  \tag{6}  

\[ c_3 = \bar{a}_{22} \sin θ + a_{23} \cos θ, \]  \tag{7}  

\[ c_4 = a_{32} \sin θ + a_{33} \cos θ. \]  \tag{8}  

As \( a_{ij}' (i, j = 1, 2) \) can be estimated by the measured velocity data (denoted as \( \bar{a}_{ij} \) hereafter), equations for \( a_{12}', a_{21}' \) and \( a_{23}' \) in Eq. (4) give the three simultaneous equations for unknown components.

If the fluid can be considered incompressible, then the equation of continuity derives

\[ a_{33} = -(\bar{a}_{11} + \bar{a}_{22}). \]  \tag{9}  

In this case, both \( a_{33} \) and \( a_{33}' \) are given by estimated \( \bar{a}_{ii} \) and \( \bar{a}_{ii}' \) (\( i = 1, 2 \)). Then the following equations can be derived from Eq. (4) with respect to the unknowns \( a_{13}, a_{23}, a_{31} \) and \( a_{32} \):

\[ a_{13} = \frac{1}{\sin θ} (\bar{a}_{12} \cos θ - \bar{a}_{12}'), \]  \tag{10}  

\[ a_{31} = \frac{1}{\sin θ} (\bar{a}_{21} \cos θ - \bar{a}_{21}'), \]  \tag{11}  

\[ a_{32} = \frac{1}{\cos θ \sin θ} (\bar{a}_{22} \cos^2 θ - a_{23} \cos θ \sin θ + a_{33} \sin^2 θ - \bar{a}_{22}'). \]  \tag{12}  

\( a_{13} \) and \( a_{31} \) are solved by Eqs. (10) and (11), respectively. However, Eq. (12) includes two unknowns \( a_{23} \) and \( a_{32} \). It is noted that the equation associated with \( a_{33}' \) in Eq. (4) is equivalent to Eq. (12). Then we consider \( \nabla u \) associated with the other plane at a different azimuthal angle rotated by \( θ' \) clockwise, denoted as \( A(θ') = [a_{ij}]' (i, j = 1, 2, 3) \). Similarly the following equation is derived:

\[ a_{32} = \frac{1}{\cos θ' \sin θ'} (\bar{a}_{22} \cos^2 θ' - a_{23} \cos θ' \sin θ' + a_{33} \sin^2 θ' - \bar{a}_{22}'). \]  \tag{13}  

Then \( a_{32} \) and \( a_{23} \) are solved by Eqs. (12) and (13). As an another method to derive \( a_{13} \) and \( a_{23} \), the measurement of velocity field in the planes with a minute interval enables to estimate the derivative of \( v_1 \) and \( v_2 \) with respect to the \( x_3 \) direction, i.e., \( \partial v_1/\partial x_3 \) and \( \partial v_2/\partial x_3 \), by a finite differential formula.
coordinate system, the radial and azimuthal directions, respectively. Note that \( e \) can be decomposed into the azimuthal and radial flows, to the sign of \( \varepsilon \) and \( \eta \). \( \eta \) shows that \( \varepsilon \) and \( \eta \) are expressed as specific quadratic forms:

\[
Q_\varepsilon = \begin{bmatrix}
\varepsilon R & \varepsilon \eta \\
-\varepsilon / \varepsilon & \varepsilon \eta \\
0 & 0
\end{bmatrix},
\]

\( Q_\eta \) has a pair of complex conjugate eigenvalues \( \psi \) and \( \eta \), and proceeds (or approaches) along a vortical axis \( \varepsilon \). It is noted that \( \varepsilon \) and \( \eta \) can be orthogonal, i.e., \( \varepsilon \) and \( \eta \), and their relative lengths, i.e., ratio \( e = |\varepsilon|/|\eta| \) is specified by the eigenequations of \( \nabla \psi \).

We define an orthonormal coordinate system \( \hat{\mathbf{e}}_i \) \( (i = 1, 2, 3) \) with the orthonormal bases \( \hat{\mathbf{e}}_i \) \( (i = 1, 2, 3) \) where \( \hat{\mathbf{e}}_1 \) and \( \hat{\mathbf{e}}_2 \) are parallel to \( \varepsilon \) and \( \eta \), respectively, so that the \( \hat{x}_1 \)-\( \hat{x}_2 \) plane is \( \mathcal{P} \) (the invariant subspace of \( \nabla \psi \)). Then \( \nabla \psi \) in this coordinate system, \( \hat{\alpha} = [\hat{\alpha}_i] (i, j = 1, 2, 3) \) can be expressed in the form

\[
\hat{\alpha} = \begin{bmatrix}
\hat{\alpha}_1 \\
\hat{\alpha}_2 \\
\hat{\alpha}_3
\end{bmatrix} = \begin{bmatrix}
\hat{\alpha}_{13} \\
\hat{\alpha}_{23} \\
\hat{\alpha}_{12}
\end{bmatrix}.
\]

Here we focus on the local flow \( \hat{b} = (\hat{b}_1, \hat{b}_2) \) in the \( \hat{x}_1 \)-\( \hat{x}_2 \) plane, which is expressed as \( \hat{b}_i = (\partial \psi / \partial \hat{x}_i) (i, j = 1, 2) \). \( \hat{b} \) can be decomposed into the azimuthal and radial flows, \( v_\theta \) and \( v_r \), so that \( \hat{b} = v_r \hat{e}_r + v_\theta \hat{e}_\theta \), where \( \hat{e}_r \) and \( \hat{e}_\theta \) are unit vectors in the radial and azimuthal directions, respectively. Note that \( \hat{e}_r = 1/|\hat{r}|(\hat{x}_1, \hat{x}_2) \) and \( \hat{e}_\theta = 1/|\hat{r}|(-\hat{x}_2, \hat{x}_1) \), where \( \hat{r} = (\hat{x}_1, \hat{x}_2) \) and \( |\hat{r}| = \sqrt{\hat{x}_1^2 + \hat{x}_2^2} (i = 1, 2) \). \( v_\theta \) is given by the inner product of \( \hat{b} \) and \( \hat{e}_\theta \), i.e., \( (\hat{b} \cdot \hat{e}_\theta) \), and similarly \( v_r \) is \( (\hat{b} \cdot \hat{e}_r) \). Then \( v_\theta \) and \( v_r \) are expressed as specific quadratic forms:

\[
v_\theta = \frac{1}{|\hat{r}|} \hat{Q}_\theta \hat{r}, \quad v_r = \frac{1}{|\hat{r}|} \hat{Q}_r \hat{r},
\]

where \( Q_\theta \) and \( Q_r \) denote the associated \( 2 \times 2 \) matrices. The eigenvalues of \( Q_\theta \) and \( Q_r \), say \( \lambda_\theta \) and \( \lambda_r \) \( (i = 1, 2) \), specify the direction and intensity of these flows. Then the swirlity \( \phi \) and sourcity \( \sigma \) indicate the unidirectionality and intensity of \( v_\theta \) and \( v_r \) in terms of their geometrical averages, respectively. \( \phi \) is defined as \( \phi = \text{sgn} (\lambda_\theta \lambda_r) \sqrt{|\lambda_\theta \lambda_r|} \) where \( \text{sgn}(\alpha) \) denotes

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the sign of $\sigma \in \mathbb{R} (\sigma \neq 0)$. In a vortical region where $\nabla \bm{v}$ has the complex conjugate eigenvalues, then $\lambda_{0i}, \lambda_{0j} = -\psi/c, -c\psi$ thus $\phi$ is equal to $\psi$. On the other hand, $\lambda_{ri}, \lambda_{rj} = e_R \pm |c - 1/c|\psi/2$ and $\sigma$ is expressed as:

$$\sigma = \text{sgn}(\epsilon) \sqrt{\left|\epsilon\right|},$$

$$\epsilon = \lambda_{ri} \lambda_{rj} = e_R^2 - \frac{1}{4} \left(\frac{1}{c} \right)^2 \psi^2.$$  (17)

In a vortical flow where $0 < \phi$, $\sigma$ can be positive or negative, and it specifies the whole inflow/outflow in all directions ($0 < \sigma$) or mixed inflow and outflow ($\sigma < 0$). It is noted that $(\lambda_{ri} + \lambda_{rj})/2 = e_R$, and that $e_R$ does not specify the precise feature of the radial flow. Figure 2 show the local flow $\tilde{\phi}$ in the $\tilde{x}_1-\tilde{x}_2$ plane, i.e., $\mathcal{P}$, with decomposed azimuthal and radial flows (denoted as $\dot{v}_\theta \theta$ and $\dot{v}_r r$). The eigenvectors $\zeta_{\theta i}$ and $\zeta_{r i}$ ($i = 1, 2$) of respective $\lambda_{0i}$ and $\lambda_{ri}$, and typical contours of $\tilde{\phi}_Q \tilde{x}$ and $\tilde{\phi}_Q \tilde{x}$ are also shown. It indicates that, although the complex eigenvalues are the same ($e_R = -1, \psi = 2$), the vortical flow (symmetry) depends on $c$. If $c$ is nearly equal to 1, then both radial and azimuthal flows have symmetrical features. However, If $c$ is lower (or higher than 1), these flows and the whole vortical flow become asymmetric, and the radial flow is difficult to be unidirectional.

Thus, when the three-dimensional $\nabla \bm{v}$ is given, these eigenvalues and the above physical quantities specify the detail vortical flow geometry.

4. Numerical examples with experiment

We apply PIV (Particle Image Velocimetry) data of the velocity field of a spiral vortex in an experiment. The PIV is applied in the wake near the blade, and the measurement is synchronized with the rotating blade (at approximately 900 [rpm]). Then the velocity data of the PIV in each plane is synchronized with the blade at the same azimuthal angle (phaselocked). We apply the velocity data in PIV planes at $\theta = 0$ and $\theta = \pi/4$, and the planes at a minute interval to the $\theta = 0$ plane are also used to derive $a_{13}$ and $a_{23}$. The mesh size (node interval) in the planes of the PIV in the present example is less than 2 [mm], and the interval between the parallel planes at $\theta = 0$ for estimating $a_{13}$ and $a_{23}$ is 2 [mm]. Thus the interval between these two respective planes and the $\theta = 0$ plane is almost the same as the mesh size of the velocity data. Then the computational accuracy of the finite difference for $a_{13}$ and $a_{23}$ is similar to that for the other components $a_{ij}$ ($i, j = 1, 2$) using the finite difference approximation in a plane. The composition of the three-dimensional $\nabla \bm{v}$ applies the velocity data in the respective planes where several instantaneous data is averaged and adjusted. We show the contours of quantities associated with the vortical structure (geometry) derived from the three-dimensional $\nabla \bm{v}$ hereafter.
Figure 3 shows the contour of φ in the vortical region of the spiral vortex where 0 < φ. In Fig. 3 (and figures hereafter), the contour is shown in the area of 31 × 31 nodes where the node interval (mesh size) is roughly smaller than 0.1 of diameter of the spiral vortex, and non vortical region where φ < 0 is colorless. Figure 3 exhibits a feature of the spiral vortex that φ has a symmetric distribution and a local maximum in the core region. The contour of c is shown in Fig. 4, which indicates that the flow symmetry is high in the core region and nearly symmetric in the center of the spiral vortex.

As for the radial flow, Fig. 5 shows the contours of ε_R and σ. ε_R indicates an intensity of the inflow/ourflow in the arithmetic mean, i.e., average radial flow, and σ specifies the unidirectionality of its flow in all directions around the point, as described above. The distribution of ε_R shows that this spiral vortex includes both (average) inflow and outflow. Although σ is negative in most of the vortical region, ε_R itself indicates that both inflow and outflow exist irrespective of the sign of σ, and the spiral vortex may seem to have complex radial flow. If we compare |ε_R| to φ, ε_R is enough small in the core region. Then the reason of these features may be due to the small (absolute) intensity of the radial flow, and it shows that the present spiral vortex is mainly composed of the azimuthal flow.

Contours of the vorticity vector are shown in Fig. 6 with respect to ω_i (i = 1, 2, 3) components. It shows that ω_3 is the main component and then the normal vector of P may be close to e_3 (the x_3 direction). On the other hand, Fig. 7 shows contours of the vorticity vector with respect to components parallel and normal to the swirl plane, where ˆω_1 and ˆω_2 are components parallel to P, and ˆω_3 denotes the component normal to P. We note that the directions of ˆe_1 and ˆe_2, i.e., ˆξ_pl and ˆη_pl, are defined in each point in the vortex region. It shows that ˆω_1 and ˆω_2 have non-zero values to some extent, which exhibit a feature that the vortical axis is not normal to P. But these intensities are less than ˆω_3. Figure 8 shows the angle between the vortical axis ˆξ_axi and e_1 (the x_3 direction), and that between the normal vector of P and e_3, denoted as θ_axi and θ_e, respectively. θ_axi is almost parallel to e_3 in the core region, whereas the swirl plane is inclined and not parallel to the x_1-x_2 plane. The angle between ˆξ_axi and the normal vector of P, denoted as ˆθ_axi, is shown in Fig. 9. It shows the feature that ˆξ_axi is not orthogonal to P, but that the orthogonality increases in the center of the vortex.

5. Discussion

The present example shows that the composition of the three-dimensional $\nabla \boldsymbol{u}$ derives the detail vortical flow geometry and associated vortical features. Once φ, ε_R, σ, c, and the eigenvectors of $\nabla \boldsymbol{u}$ are estimated, the flow geometry or respective azimuthal and radial flow characteristics are clarified. Figure 10 shows the local flow geometry in the core of the vortex derived from their quantities, and the decomposed azimuthal and radial flows.

These quantities enable to investigate the important characteristics of a vortex associated with the stability in terms
of the flow kinematics, that are the pressure minimum and vortex stretching. And the above quantities or $\nabla \mathbf{u}$ specify the pressure minimum features defined by the $Q$- (Hunt et al., 1988), $\Lambda_2$- (Jeong and Hussain, 1995) and their integrated definition (Nakayama et al., 2014) that specifies the pressure minimum in the swirl plane. The vortex stretching (Tennekes and Lumley, 1972) can be estimated with respect to the vorticity components associated with swirling and $\mathcal{P}$, which directly clarifies the effect of the stretching in the point of the vortical flow geometry. The stretching terms (effects) associated with the vorticity components parallel to $\mathcal{P}$ classify the vortex stretching as developing or breaking the orthogonality of the vortical axis (Nakayama et al., 2016), and these terms can be specified by $\lambda_i$ because they are associated with the eigenvalues of the rate of strain in $\mathcal{P}$ (Nakayama, 2014). Specifically it is noted that the vortical flow symmetry is important not only in stability of a vortex (Lundgren, 1982), but also in pressure minimum feature and vortex stretching. Thus this analysis derives important information of a vortex in the three-dimensional feature, from several two-dimensional data.

The principle of the present method is based on the linear transformation in different coordinate systems, in order to compose the three-dimensional $\nabla \mathbf{v}$ and derive the physical quantities associated with the vortical characteristics. Then this numerical method itself has enough accuracy for the tensor and quantities, and the accuracy depends on the finite difference approximation of the tensor and resolution of the velocity data, which is prerequisite for the present analysis. On the other hand, in the combining the $\nabla \mathbf{u}$ components in several planes, the velocity field in a plane ($R'$) at a different azimuthal angle is assumed to be equivalent to that in the inclined plane (rotated by $\theta$) at the standard azimuthal angle, e.g., $\theta = 0$. If the different azimuthal angle is large, it may be careful on the relationship between two points in $R'$ and standard plane ($R$ at $\theta = 0$). Even though these planes have the same node distribution and location where the velocity data is measured, the projected point from a node in $R'$ to $R$ may be different from the same node in $R$. They are the same for nodes in the $x_1$-axis, and shift along the $x_2$ direction for nodes where the $x_2$ components are not zero. Its effect is larger in the outer region from the $x_1$-axis. Although this correction of the node projection is not given in the present example, this case extracts the primary feature of a spiral vortex.

The size of the mesh for the velocity data depends on the scale of a subjected vortex. The spacial resolution of the velocity field to be measured may depend on the finite difference formula to estimate the $\nabla \mathbf{u}$. Moreover, the feature (complexity) of the velocity structure of the vortex is important and should be considered in the numerical and experimental condition.

Because the three-dimensional vortical structure is composed of the two-dimensional $\nabla \mathbf{v}$ in several planes, the state of a spiral vortex is assumed to be the same or in the similar condition. Then the distance between the planes with respect to the wake direction should be close. In addition, if the phase of the blade is locked in the measurement so that relative location between the blade and a measured plane is the same for all considered planes, the state of the spiral vortex can

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Fig. 7 Contours of vorticity components parallel and normal to $\mathcal{P}$. (a) $\hat{\omega}_1$, (b) $\hat{\omega}_2$, and (c) $\hat{\omega}_3$.

Fig. 8 Contours of angles (degree) between $e_1$ and vortical axis or normal vector of $\mathcal{P}$. (a) $\theta_{axis}$, and (b) $\theta_n$. 

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Fig. 9  Contour of angle (degree) between vortical axis and normal vector of $P$.

Fig. 10  Flow geometry in $P$ at the center of a spiral vortex, and decomposed azimuthal and radial flows denoted as $v_\theta e_\theta$ and $v_r e_r$ with typical contours of $\delta Q \hat{x}$ and $\delta Q \hat{r}$, where $\epsilon_R = -0.022, \psi = 1.7, c = 0.87$ and $\sigma = -0.23$. (Note that the vector lengths are adjusted in respective figures.)

be considered further similar.

6. Conclusion

A numerical method for identification of three-dimensional vortical structure with plural two-dimensional velocity data in parallel planes was investigated for a spiral vortex in wind turbine. This method enables to analyze the detail vortical flow geometry in terms of the local approach and three-dimensional characteristics of a vortex. The swirlity and sourcity give the intensity and unidirectionality of the azimuthal and radial flows in the swirl plane, respectively, and the flow symmetry is identified. The vorticity can be decomposed into parallel and normal components to the swirl plane, which categorizes the component associated with swirling motion and ones that influence the orthogonality of the vortical axis.

Since this identification method derives the three-dimensional velocity gradient field, it also enable to specify the important characteristics of a vortex associated with the stability or development of the vortex, such as the pressure minimum and vortex stretching, including vortical flow symmetry as described above. The present method facilitates the measurement of the velocity field in experiment for the analysis of the vortical structure, nevertheless it gives further useful flow information of a spiral vortex.

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