Investigation of Magnetostriction in Second Generation High Temperature Superconductors

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Abstract. Superconductors help to drastically improve the efficiency in every field by making the transmission losses of electrical energy to minimum. Superconductors show special properties only when their parameters like temperature, magnetic field, current density, mechanical strain are within their critical limits. Effect of magnetic field on second generation high temperature superconducting compounds have been studied in past few years. In this paper, the numerical simulation of the magnetostriction effect of second generation superconductor REBCO is discussed. The results obtained from numerical studies is compared with the experimental results reported in literature. The findings show that there is good agreement and all the three samples are showing rapid change in magnetostriction when the magnetic field is above $10^6$ A/m.

1. Introduction

Magnetostriction is one of the peculiar phenomena shown by superconductors. It describes the change in dimensions of a material due to a change in its magnetization. This phenomena was discovered by James Joule in 1840. Magnetostriction depends on the strength of the magnetic field and the material. Magnetostriction of a single crystal first generation superconductor BSCCO (Bi$_2$Sr$_2$CaCu$_2$O$_{8+}$) material is in the order of $10^{-4}$ [1-2]. This falls in the category of giant magnetostriction as the value is greater than the magnetostriction shown by other superconductors. Superconducting crystal of Magnesium Dibromide (Mg$_2$B) has the magnetostriction in the order of $10^{-5}$ [3]. Most of the high temperature superconductors (HTS) show this effect in the order of $10^{-6}$ [4]. But the type II superconductor Niobium Titanium (NbTi) shows in the order of $10^{-7}$ [5]. Strong ferro and antiferromagnetic materials attains magnetostriction value in the order of $10^{-4}$ to $10^{-2}$; while that for diamagnetic and paramagnetic materials they are in the order of $10^{-7}$ to $10^{-5}$ [6].

Compared to other superconducting materials, the rare earth Barium Cupric oxide (REBCO) is found to be a promising one. Maruyamaa et al. reported the development of REBCO HTS power cables and succeeded in reducing current loss to one-sixth the loss of that of a BSCCO cable [7]. REBCO coated conductors are more attractive because of their high critical temperature of 92 K. They also have high mechanical strength [8]. Considering the positives of REBCO, it may be expected that superconducting wires and cables will be the next generation ones. However, there is limited numerical studies on REBCO especially in bringing out the effects of magnetostriction on their performance while in application.

The effect of magnetostriction in REBCO materials has an influence when they are used to make superconducting cables. The type-II superconductors being brittle in nature and the changes in their dimensions due to magnetostriction will result in increased strain inside the cables made out of them.
Consequently, the performance of these superconducting cables will be degraded. Through experimental investigations it is difficult to identify the individual influence of each parameter on the performance due to the intertwined effects. Mathematical modelling has an important role in such situations. Studies using the mathematical model will not only help to save the money and time, but also in improving the theoretical knowledge of these effects through various parametric investigations.

This paper provides the details of the process by mathematically modelling the effects of magnetostriction in various superconducting materials. The approach will be to construct and analyse the mathematical model representing various physical situations. Being a multi-physics problem, modelling is a complex process here and it requires modification and customization of existing routines in the commercially available software. Literature shows that COMSOL Multiphysics was used to successfully model the Surface Acoustic Wave (SAW) sensors made out of magnetostrictive material; the model was generated using the AC/DC and the structural mechanics modules of COMSOL Multiphysics [9]. Numerical model to examine the magnetostriction of bulk high temperature superconductor under uniform and non-uniform magnetic field is reported by Li X et al. [10].

2. Basics Concepts for the calculation of Magnetostriction

The magnetostriction component along any direction can be calculated as a nonlinear function of the magnetization.

\[ \lambda_i = \frac{3}{2} \lambda_s \left( \frac{M_i}{M_s} \right)^2 \left( \frac{1}{3} \right) \]  

Where \( \lambda_s \) is the saturation of magnetostriction and direction cosine is the ratio of magnetization along the required direction \( M_i \) and the saturation magnetization \( M_s \) of the material. In this work, it is assumed that the material is sufficiently pre-stressed such that all magnetic moment are perpendicular to the direction of magnetization at the beginning of magnetization process.

The free strain due to magnetostriction is incorporated using an extension of the Hooke’s Law. COMSOL uses a generalized Hooke’s Law to describe the constitutive relationship between the stress and strain tensors in a solid mechanics problem. This general form of the linear constitutive relation allows the incorporation of initial strain in a material which does not contribute to mechanical stress unless the material is mechanically constrained. Because, by definition, the magnetostriction or magnetization-induced free strain in the material is equivalent to an initial strain, this idea is used to incorporate the magnetostriction calculated from equation 1.

3. Modelling

The size of superconducting crystal was taken as 1.05 x 3.3 mm as shown in the figure 1. An air domain is created around the superconductors to realistically model the magnetic flux path. The boundaries of this air domain are magnetically insulated which ensure that flux does not diverge out of the modelling domain.

![Figure 1: Modelling arrangement](image)
The current is applied externally by creating a coil around the superconductors. The external current density is the total current through the coil divided by the longitudinal cross sectional area. The steel housing used in the model is designed to create a closed magnetic flux path thereby minimizing flux leakage. The non-linear magnetic behaviour of the steel is modelled by using a hysteresis curve to specify the magnetic constitutive relation in the steel housing. Bottom side of the superconductor is fixed to the steel housing. Due to the external current around the superconductors create a magnetic field and magnetize the superconductor. The current density is varied from 0 to $10^8$ A/m$^2$.

Table 1: Properties of YBa$_2$Cu$_3$O$_{7-x}$, DyBa$_2$Cu$_3$O$_{7-x}$ and HoBa$_2$Cu$_3$O$_{7-x}$ [4, 13, 15]

| Compound          | Young’s Modulus (GPa) | $\lambda_x \times 10^6$ (Anisotropic magnetostriction Constant) | Poisson’s ratio |
|-------------------|------------------------|---------------------------------------------------------------|-----------------|
| YBa$_2$Cu$_3$O$_{7-x}$ | 148                    | 5                                                             | 0.3             |
| DyBa$_2$Cu$_3$O$_{7-x}$ | 95                     | -84                                                           | 0.3             |
| HoBa$_2$Cu$_3$O$_{7-x}$ | 95                     | -66                                                           | 0.3             |

Incorporation of a nonlinear hysteresis curve help in modelling magnetic saturation at sufficiently high magnetic field. COMSOL Multiphysics have good collection of magnetic field vs magnetisation (HB) properties of different materials. In the same manner as mentioned above, a non-linear HB relation is also used for superconducting material [11-12].Physics controlled mesh is used to solve the model. It may be noted that in this work, the magnetostriction effect parallel to the magnetic field is only considered as the Poisson’s ratio is 0.3 and the effect will be more significant in the longitudinal direction. This has also been experimentally shown by Lu J et al. [11], where the magnetization and hysteresis loss of a 1x1 mm$^2$ REBCO sample was measured in different field and field orientation. They have found that hysteresis losses are dependent on the magnetic field and its field angle for a Super Power SCS 4050 REBCO conductor.

The various issues faced during the process of numerically simulating is discussed. The HB properties of superconducting compound is not available at higher magnetic fields and therefore, had to be extrapolated and assumed. Another challenge is that COMSOL Multiphysics can’t take magnetization and demagnetization values corresponding to the single magnetic field. So magnetostriction effect during the magnetization is considered in this study. The demagnetisation studies need to be conducted separately. At the end, for validation, the results obtained through numerical investigation are compared with the experimental data available in the literature.

4. Results and Discussion
Numerical studies have been conducted on the 3 type of samples selected is given separately. The dimensional variation in the longitudinal direction is captured with the increase in magnetic field and is plotted accordingly. Figure 2 shows the effect of magnetic field on magnetostriction for Yttrium Barium Cupric Oxide (YBCO) YBa$_2$Cu$_3$O$_{7-x}$ specimen. The results obtained from experimental studies are also shown in the graph for comparison. It may be observed that the numerical results are in good agreement wherever the experimental results are available. It may be also inferred that when the magnetic field reaches the range of $10^6$ A/m, there is a sudden increase in the effect of magnetostriction on the material.

The area where experimental results are available is zoom and plotted as inset in Figure 2. It may be seen that experimentally also there is a trend for a sudden increase in magnetostriction effect when increasing the field beyond certain limit.
Figure 2: a) Magnetostriction vs Magnetic Field (YBCO) b) Magnified View of specified area

Figure 3 illustrates the magnetostriction of DyBa$_2$Cu$_3$O$_{7-x}$ compound under varying magnetic field. DyBa$_2$Cu$_3$O$_{7-x}$ shows the magnetostriction in the negative direction in the order greater than that of YBa$_2$Cu$_3$O$_{7-x}$ compound shown in Figure 2. Negative value indicates that there is contraction in the material in the longitudinal direction parallel to the magnetic field. Comparison of experimental and numerical results depicts good agreement. In this sample also it may be observed that when the magnetic field is in the range of $10^6$ A/m, there is a sudden increase in magnetostriction effect. It shown that experimentally also there is a trend for a sudden increase in magnetostriction effect when increasing the field beyond certain limit.

Figure 4 shows the effect of magnetic field on magnetostriction of HoBa$_2$Cu$_3$O$_{7-x}$ compound. This compound shows a similar characteristics (negative values) as that YBCO compound along the direction parallel to the magnetic field. It may be observed that when the magnetic field is increased beyond the order of $10^6$ A/m, there is a sudden increase in magnetostriction as similarly observed for other samples. Magnetostriction at the field greater than $10^7$ A/m is not reported in the open literature due to the practical difficulty in applying such high magnetic fields.
Figure 4: a) Magnetostriction vs Magnetic Field (HoBa$_2$Cu$_3$O$_{7-x}$)

DyBa$_2$Cu$_3$O$_{7-x}$ and HoBa$_2$Cu$_3$O$_{7-x}$ shows similar results of negative variation and have good agreement with the experimental results. They have magnetostriction effect in the order of $10^{-5}$, when the magnetic field is in the range of $10^6$ A/m. It is also seen that the order of magnetostriction is higher in these compounds compared with YBCO superconducting compound. In YBCO, the expansion is observed as the effect of magnetostriction and is in the order of $10^{-6}$ and the sudden increase is at $10^6$ A/m.

The dimension of superconductor vary due to magnetostriction effect because, when the magnetic field increases above the lower critical magnetic field, magnetic lines penetrate through the superconductor and decreases the condensation energy. This effect is associated with change in volume and leading to magnetostrictive strain. When the magnetic field is loaded to the expansive (positive) magnetostrictive materials, the tensile stress occurs parallel to applied magnetic field. On the other hand, the opposite phenomena will occur for compressive (negative) magnetostriction [16].

Out of the 3 samples, 2 of them had a negative effect (contraction) due to magnetostriction. It may be due to the anisotropic magnetostriction properties of materials. The anisotropic magnetostriction constant is given in the Table 1. It indicates that Dy and Ho compounds have negative values and the 2 samples with their presence, showed negative magnetostriction in the direction parallel to the applied field.

All the 3 compounds showed rapid rise in magnetostriction when the magnetic field crosses a particular value. It may be due to the fact that the magnetostriction is directly proportional to the square of magnetization as understood from Equation 1.

5. Conclusion
The numerical studies have been conducted to analyse the effect of magnetic field on different high temperature superconducting samples and the following conclusions are obtained. When the magnetic field attain a value of $10^6$ A/m, it is found that magnetostriction effect in YBCO compound is in the order of $10^{-6}$ and other two compounds have the order of $10^{-5}$. In comparison with the experimental results available in the literature, it may be concluded that numerical simulation will be very helpful in calculating magnetostriction effect. It is also understood that the magnetostriction effect shown by DyBa$_2$Cu$_3$O$_{7-x}$, HoBa$_2$Cu$_3$O$_{7-x}$ are negative and the order of magnetostriction greater than the YBCO compound.

6. References
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