A proposal for the dynamic strain interpolation on hydroelectric turbine runner

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Abstract. The large operation range of hydroelectric turbine, caused by the increasing changes in the electrical network usage, leads to higher dynamic stress fluctuation on runner. However, the experimental data cannot cover all the possible operating conditions. The missing data in some turbine operation zones generates a challenge for the recovery of the highest dynamic strain. This paper proposes a methodology for the interpolation of such strains between operating conditions by using the kriging method. The case study focuses on the interpolation of the part load rope which is an important component in the dynamic strain of a Francis turbine runner. Two interpolation approaches, inspired by spatio-temporal kriging and cokriging, are applied and compared. Finally, some suggestions are proposed to improve the recovery of operating conditions using interpolation process.

1. Introduction

To adapt to the increasing changes in the electrical network usage, hydroelectric turbine operation range has been extended leading to greater dynamic stress fluctuation (e.g. at Part Load conditions, at Deep Part Load conditions…). This enlarged operation range increases the risk of fatigue failure caused by large dynamic mechanical stresses. For this reason, the dynamic behaviour prediction across the whole operation range is required. This leads to several difficulties such as high-cost measurements or time-consuming Finite Element Simulation (FES). Moreover, there are several unavoidable limitations of the experimental measurements (by gauges, sensors) which generate missing observations and limit prediction capability. In practice, it is not possible to have enough experimental data to cover the whole operation range of a turbine. Furthermore, the measurement length is short compared to the length required for long-term fatigue evaluation. Over the years, several researchers tried to improve the turbine runner fatigue prediction. Gagnon et al. (2013) [1] developed a fatigue reliability model by considering the High Cycle Fatigue (HCF) onset as the limit. Poirier et al. (2016) [2] extrapolated in time the short-term strain measurement of a turbine runner blade by using the cyclostationary decomposition. Pham et al. (2019) [3] interpolated between the periodic part of strain signals of a hydroelectric turbine.

The interpolation between different turbine conditions is an idea that can resolve many of the mentioned experimental limitations. The dynamic behavior of each turbine operation zone is composed of different physical phenomena hidden in the signals. Therefore, the interpolation quality depends on the distribution of the measured operating conditions for each of these phenomena. The kriging method was chosen as an appropriate method for the interpolation of strain dynamic between different operating conditions. The kriging is an interpolation method that proved to be suitable in many research fields [4] [7]. For the hydroelectric turbine, Salah et al. (2012) [4] and Pham et al. (2019) [3] already used the kriging for the interpolation of the runner strain value. This paper proposes an investigation of the aptitude and performance of kriging as a method of interpolating strain values over different operating conditions; and focusses on the recovery of dynamic strains generated by the
part load rope. The experimental strain measured from a hydro-Quebec medium head Francis turbine runner is used in the case study. The turbine operation zone of interest is related to the part load rope phenomenon (Section 2). Two kriging interpolation approaches, inspired by the Spatio-Temporal Kriging (STK) and the CoKriging (CK), are applied. Finally, the quality of the interpolation is evaluated via the comparison between these two approaches. From these results, some suggestions to improve measurement plan and for the further research are discussed.

The paper is organized as follows: different turbine operation zones are presented in Section 2. Then, the interpolation approaches are shortly explained in Section 3. Finally, some discussions based on a case study of a Francis turbine runner are presented in Section 4.

2. Steady Operation Zones
The steady-operating conditions are all regimes where the rotation speed is synchronized to the electrical network and the discharge is constant. When the steady-state operating range is expanded, this leads to more complicated dynamic behaviours in the low discharge conditions. Therefore, the interpolation between such conditions is more challenging. Indeed, to make understandable the complexity of this “interpolation space”, the range of operating conditions is separated into three zones given dynamic behaviour and its influence on the interpolation process. It must be noted that this separation is made for interpolation purpose, more information about the operation range of turbine can be found in [5].

- **Stable zone**: This zone contains the most “stable” conditions near the Best-Efficiency Point (BEP) where the flow is approximately between 85-110% of the BEP flow. The word “stable” means that the dynamic part is mainly periodic. Note that the fatigue damage in these conditions is usually lower than other one. Figure 1 shows an example of runner strain in this zone.

- **Stochastic zone**: The zone often contains the greatest dynamic ranges. It is composed of the Speed No Load, and the low par load where the flow is lower than 50% of BEP flow. In this zone, the stochastic parts often dominate the other components of the signal (e.g. periodic phenomena). The higher loading generates significant fatigue damage. Therefore, if we want to operate in this zone, it requires a good quality of interpolation to improve the fatigue evaluation. The behaviour diversity and the stochastic nature of this operation zone mean that if there are not enough input data in this zone, the interpolation might not be able to estimate the amplitude range properly. The Figure 2 shows an example of runner strain in this zone.

- **Part load rope zone**: The rope phenomenon generates a periodic pressure fluctuation in the turbine caused by the outflow vortex rope in the draft tube of turbine (see Figure 4). This outflow condition leads to a backflow in the centre of the draft tube cone and a vortex rope of helical shape [5]. This phenomenon can be observed in part load conditions via a high-pressure fluctuation at a low frequency. Generally, in fixed-blade turbines as Francis turbine, this phenomenon is found at the conditions where the flow is between 50% and 85% of the BEP flow [6]. The maximum amplitude is difficult to capture accurately during measurement campaign or with numerical simulation. This phenomenon tends to appear and then suddenly disappear in the neighbour conditions. Therefore, this operation zone is very sensitive for the interpolation. This zone is the main focus of this paper (see Section 4).
3. Interpolation Approaches

The interpolation methods can be separated into different types: the determinist methods (spline, barycentre, …), the probabilistic method like kriging or machine learning based methods. If the interpolation method provides only the estimated value (as in determinist methods), the uncertainty remains unknown. Methods based on machine learning require large amount of data (interpolation input). In our case, a probabilistic interpolation method like kriging seems a more suitable choice.

The kriging is a probabilistic interpolation method based on the spatial covariance between the target and the measured data. Salah (2012) used the kriging to study the uncertainty propagation of experimental strain measurement on a hydroelectric turbine blade [4]. Pham et al. (2019) applied using
for estimating the hidden periodic signal between different turbine conditions [3]. Therefore, the kriging is chosen for this study. In kriging the unknown value is estimated from the known values by finding the regression weights (see Eq 1 and Eq 4) that minimizes the variance of estimation error. The kriging interpolation requires the covariance map. With the intrinsic hypothesis, the kriging method uses a semivariogram for modeling the covariance.

3.1. First Approach (inspired by STK)
This approach is inspired by the idea of Spatio-Temporal Kriging (STK) where the spatial and temporal coordinates are treated as two independent coordinates. Precisely, the covariance map of datasets depends on two independent increments. For this approach, we consider two independent coordinates: operating conditions and the runner rotation runner.

The first step is to impose the regression linear equation:

\[ S^*(o,a) = \sum_{k=1}^{n} \sum_{i=1}^{m_k} \lambda_{ki}(o,a)S(o_k,a_i) \]  

where \( o \) is the operating condition, \( a \) is the rotation angle, \( S^* \) is the target strain value, \( S \) is the experimental strain values, \( n \) is the number of experimental turbine conditions, \( m_k \) is the measured rotation angle for the operating condition \( o_k \), \( \lambda_{ki} \) is the regression weight. The non-bias condition imposed for this approach is \( \sum_{k=1}^{n} \sum_{i=1}^{m_k} \lambda_{ki}(o,a) = 1 \).

The kriging requires the semivariogram for modelling the covariance. The semivariogram is defined by:

\[ \gamma(h,u) = \frac{1}{2} \times Var[S(o_k,a_i) - S(o_l,a_j)] \]  

where \( h = |o_k - o_l| \) is the operating condition increment, \( u = |a_i - a_j| \) is the rotation angle increment. This definition verifies the intrinsic hypothesis that the variance of two values does not depend on their locations but only on the increment \((h,u)\). The semivariogram is used in kriging by imposing a stationary and spatially isotropic assumption where the covariance \( \text{Cov}[S(o_k,a_i),S(o_l,a_j)] \) depends only on the increments, noted as \( C(h,u) \) [8]. Therefore, from the equation (2), the covariance can be expressed by the semivariogram via the relation \( \gamma(h,u) = C(0,0) - C(h,u) \).

In application, it is not evident to determine the semivariogram with the exact \( h = |o_k - o_l| \) and \( u = |a_i - a_j| \) because of the limited amount of the experimental data (the interpolation input). From the available experimental value, the semivariogram is first built using:

\[ \hat{\gamma}(h,u) = \frac{1}{2N(h,u)} \sum_{(k,l,i,j) \in \delta(h,u)} [S(o_k,a_i) - S(o_l,a_j)]^2 \]  

where \( \hat{\gamma}(h,u) \) is the experimental semivariogram, \( \delta(h,u) \) is a set of paired points separated such that \( |o_k - o_l| \in [h - dh, h + dh] \) and \( |a_i - a_j| \in [u - du, u + du] \) with \( dh, du > 0 \). \( N(h,u) \) is the cardinal of this set. To obtain a set of continuous values at every increment \((h,u)\). The experimental semivariogram is then modeled by a numeric model with the support of the least square method. When the semivariogram is modeled, the ordinary kriging is then applied for the interpolation (more details in [3, [7-8]]).

3.2. Second Approach (inspired by CK)
In this second approach, an exogenous signal is incorporated in the interpolation phase to add more information for the strain value interpolation (Eq 4). This secondary signal correlated to the strain signal.
This approach is inspired by CoKriging (CK) where we treat \( o \) and \( a \) as two spatial coordinates. That means \( (o, a) \) represents a spatial location in the interpolation space. Therefore, to simplify the equation, \( (o, a) \) can be replaced by a vector \( z \) with coordinates \( o \) and \( a \) (e.g. \( S(o, a) = S(z) \)). The linear regression equation for this approach is thus expressed as:

\[
S^*(z) = \sum_{i=1}^{n_S} \lambda_i(z)S_1(z_i) + \sum_{j=1}^{n_Z} \alpha_j(z)S_2(z_j)
\]  

(4)

where \( S^* \) is the target strain value, \( S_1 \) is the experimental strain value (also called as the primary signal or the signal of interest), \( S_2 \) is the experimental values come from a secondary signal that has the similar dynamic behaviour with \( S_1 \) (see details in Section 4), \( \lambda_i \) and \( \alpha_j \) correspond to regression weights, \( n_S \) and \( n_Z \) are the numbers of experimental operation conditions for the primary signal \( S_1 \) and the secondary signal \( S_2 \). There are two non-bias constraints corresponding to two signal:\( \sum_{i=1}^{n_S} \lambda_i(z) = 1 \) and \( \sum_{j=1}^{n_Z} \alpha_j(z) = 0 \). The semivariogram used in this approach is called pseudo cross semivariogram [11]:

\[
\gamma_{12}(h_{CK}) = \frac{1}{2} \times Var [S_1(z_i) - S_2(z_k)]
\]  

(5)

where \( \gamma_{12} \) is the pseudo cross semivariogram of two signals \( S_1 \) and \( S_2 \). \( h_{CK} = \|z_i - z_k\| \) is spatial increment. To create the semivariogram from the experimental dataset, the experimental pseudo cross semivariograms can be built by:

\[
\hat{\gamma}_{12}(h_{CK}) = \frac{1}{2N(h_{CK})} \sum_{(i,k) \in u(h_{CK})} \{[S_1(z_i) - \bar{S}_1] - [S_2(z_k) - \bar{S}_2]\}^2
\]  

(6)

where \( \bar{S}_1 \) and \( \bar{S}_2 \) are the means of the experimental observations of the signals \( S_1 \) and \( S_2 \), \( \|z_i - z_k\| \in [h_{CK} - dh_{CK}, h_{CK} + dh_{CK}] \) with \( dh_{CK} > 0 \). The next step is to find the fitted semivariogram model in order to obtain a set of continuous semivariogram values at every increment \( z \). The most common approach for the cross semivariogram in the CK is to fit Linear Model of CoRegionalization (noted as LMC). This fitted LMC ensures that all the fitted semivariogram models are negative semidefinite leading to ensures the covariance values are positive semidefinite. The detail of LMC approach is available in [12]. After having the semivariogram map, the ordinary kriging formulations (see [7],[10]) will be applied for the interpolation.

4. Case Study

The study case uses the experimental measurements from a Francis hydroelectric turbine runner in a power plant of Quebec, Canada. The strain gauges were located close to the critical locations (hotspot) of the runner blade where the stress levels are the highest. All the values are normalized for confidentiality purposes. This study focusses on the part load rope operation zone as mentioned in the Section 2. This operation zone poses challenges for the interpolation process because of the low number of experimental data and the shift of strain amplitude across the operation zones 50-85% of the BEP.

To study exclusively the rope and remove the influence of other dynamic components, the rope dynamic components were extracted from the strain signal. The extraction methodology is based on the cyclostationary second order extraction method proposed by Poirier et al. (2016) [2]. This method allows a generation of a new angular domain (here that links to the rope frequency) around a given frequency (corresponding to the runner rotation frequency). Now, the signal of interest corresponds to the extracted rope value. The two interpolation approaches are applied to interpolate the extracted rope signal. The target turbine condition in this study is the highest measured amplitude condition which allows comparisons for quality assessment.
4.1. Part Load Rope Observations

![Frequency Spectrum Diagram]

**Figure 5.** Observation of the runner strain frequency spectrum at a “rope” condition.

The experimental strains are measured in several steady operations identified by the different guide vane openings. The guide vanes control the flow rate through the runner. For this studied Francis turbine, the rope phenomenon is found in the conditions where the turbine guide vanes open around 40-50% of the maximum opening. **Figure 5** shows the frequency spectrum of runner strains during the rope phenomenon at 40% vane opening condition (noted as 40%VO). Beside the rotation frequency $f_0$ and its harmonics, there are two other dominant frequencies, $0.26f_0$ and $0.74f_0$ which correspond to the rope phenomenon. The $0.26f_0$ is the axial component the vortex rope in the draft tube. The latter $0.74f_0$ is the radial component related to the shift between the rotation of turbine runner and the rotation of vortex rope in the draft tube ($0.74f_0 = f_0 - 0.26f_0$). The subtraction means that the runner and the vortex rope rotate in the same direction.

The rope phenomenon occurs in a limited number of steady conditions and mostly disappears in the neighbour conditions. The maximum amplitude peak-to-peak ranges are plotted in **Figure 6** for operating conditions related to the studied phenomenon. The highest value is around 40%VO, the amplitudes in other neighbour conditions are mostly similar and much smaller than the 40%VO. As observed in **Figure 5**, the rope frequencies dominate other structures (stochasticity or periodicity) in this operating zone. Therefore, the lack of data renders difficult the identification of the condition with the highest expected strain range using the interpolation process.

4.2. Interpolation Results

Based on the experimental condition shown in **Figure 6**, the recovery of the highest rope (at 40%VO) is challenging because of the huge amplitude difference between the target condition and the input conditions. The interpolation input is the measured operating conditions excluding 40%VO which we try to estimate. In this study, the interpolation is applied on the extracted rope at 0.74$f_0$.

The CK interpolation requires a secondary signal with physical behaviour correlated to the signal of interest (extracted rope component of the runner strain). The part load rope can be observed in the signals measured in other structural components like the draft tube or the turbine shaft line. In this study, we choose shaft bending measurement as the secondary information for the CK (**Figure 7**). Thus, beside the extracted rope signal of runner strain coming from the experimental conditions (excluding 40%VO), the interpolation input for the based approach contains additionally information of the rope seen from the shaft bending measurements at all the experimental conditions (including at the 40%VO). The interpolation output is the part load rope component of runner strain at 40%VO.
Figure 6. Evolution of the maximum amplitude range of the rope phenomenon (at \(0.74f_0\)) at different turbine conditions.

Note that amplitudes of extracted rope component of shaft bending are normalized to obtain the similar relative amplitudes to the extracted rope component of runner strain. This normalization avoids an unusual pseudo cross semivariogram.

The Figures 8 and 9 show the comparison between the interpolated rope and the experimental one for the two interpolation approaches. For the STK based approach, the interpolated result is smaller than the experimental one. This lower performance was expected because of large difference between the amplitude range at 40%VO and the other operating conditions. This means that there is not enough information to predict accurately the high amplitude at 40%VO. However, note that Pham et al. [3] found that STK based approach gives a good performance for the periodic part interpolated over the entire range of OV which change more slowly over a larger number of data points. On the other hand, the CK based approach gave much better result by using a secondary signal. The predicted maximum amplitude is close to the experimental one (Figure 10).

Figure 7. Observation of the shaft bending frequency spectrum at a part load rope condition.
4.3. Discussion

The interpolation results confirm that the lack of experimental data at specific turbine operations can highly influence on the dynamic amplitude prediction. The case study shows that the CK based approach provides better interpolation than the STK based approach for the part load rope zone. However, it is not enough to confirm that the CK is the best interpolator but the use of supplementary signal for the interpolation is a suitable candidate to resolve the dynamic prediction challenge.

It is not obvious to foresee which conditions are needed to recover the highest amplitude. In this studied case, the measurement plan used a regular condition increment between conditions (±5 %VO). The highest measured strain amplitude of the rope is at 40%VO while it mostly disappeared at 35% and 50%VO. Therefore, decreasing the condition increments to 1 or 2 % VO in this operation zone could provide valuable data points. Nonetheless, such a small measurement increment might not be practical on the whole operating range as the time to perform the measurements would lead to much higher cost. It is therefore important to restrict the use of such a small measurement increments to only the relevant zones.

![Figure 8. First approach (STK approach) result. The interpolated value is compared with the experimental extracted rope at 40%VO.](image1)

![Figure 9. Second approach (CK approach) result. The interpolated value is compared with the experimental extracted rope at 40%VO.](image2)

![Figure 10. Maximum amplitude peak-to-peak ranges between the interpolated results and the experimental one.](image3)
The choice of the secondary signal (or covariable) is an important parameter influencing on the interpolation quality of CK. The more covariable is correlated to the signal of interest, the better interpolation quality is. Thus, the strain measurement in other locations of the runner blade could be a suitable secondary signal if we would like to recover the data from a lost sensor. Notice that some other signals could be used like the pressure in the draft tube, the other signal from the turbine shaft line (bending, vibration, twisting…). Many of these signals can be measured in the stationary domain and are therefore easier to obtain than the runner blade deformations. The difficulty when using such signals is the low level of the correlations with the signal of interest. For further research, we propose trying other signals for covariables in the CK interpolation. The CK based approach proposed in this paper used only one covariable. Theoretically, the CK method does not have a limitation on the number of the covariables used in the interpolation.

5. Conclusions

The missing of experimental data requires an interpolation process for recovering the dynamic information in all the hydroelectric turbine conditions. In several turbine operating conditions, the part load rope phenomenon is the dominating part in the dynamic behaviour. The lack of data points and the huge amplitude difference in the rope operation zone create a challenge to recover the highest rope amplitude. Via a case study using measurements from a Francis turbine runner, this paper proposed two interpolation approaches and the CoKriging based approach which uses covariable for interpolation performed significantly better. This method is considered as a first step to resolve the challenge of the dynamic behaviour prediction. This method could be improved by increasing the number of measured conditions using smaller increments between operating conditions and by exploring the use of other suitable covariables.

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Nomenclature

| S, S₁ | Rope value extracted from runner strain |
| S₂ | Rope value extracted from shaft bending |
| λ | Regression weight |
| α | Operating condition [%VO] |
| a | Runner rotation angle |
| h | Operating condition increment |
| u | Runner rotation angles |
| CK | CoKriging |
| %VO | Percentage of Vane Opening |
| BEP | Best Efficiency Point |
| γ | Experimental semivariogram |
| α∗ | Regression weight for covariable |
| hacam | Spatial increment in the cokriging |
| γ | Semivariogram value |
| STK | Spatio-Temporal Kriging |
| f₀ | Rotation frequency |

References

[1] Gagnon, M., Tahan, A., Bocher, P., & Thibault, D. (2013). A probabilistic model for the onset of High Cycle Fatigue (HCF) crack propagation: Application to hydroelectric turbine runner. International Journal of Fatigue, 47, 300-307

[2] Poirier, M., Gagnon, M., Tahan, A., Coutu, A., & Chamberland-lauzon, J. (2017). Extrapolation of dynamic load behaviour on hydroelectric turbine blades with cyclostationary modelling. Mechanical Systems and Signal Processing, 82, 193-205.

[3] Pham, Q. H., Gagnon, M., Antoni, J., Tahan, A. S., & Monette, C. (2019, July). Interpolation of periodic hidden signal measured at steady-operating conditions on hydroelectric turbine runners. In SURVISHNO Conference Lyon, France.

[4] Ben Salah, F. (2014). Modélisation de la propagation des incertitudes des mesures sur l’aube
d’une turbine hydraulique par Krigeage et simulations stochastiques (Master dissertation, École de technologie supérieure, QC, Canada).

[5] Seidel, U., Mende, C., Hübner, B., Weber, W., & Otto, A. (2014). Dynamic loads in Francis runners and their impact on fatigue life. In *IOP conference series: earth and environmental science* (Vol. 22, No. 3, p. 032054). IOP Publishing.

[6] Dörfler, P., Sick, M., & Coutu, A. (2012). *Flow-induced pulsation and vibration in hydroelectric machinery: engineer’s guidebook for planning, design and troubleshooting*. Springer Science & Business Media.

[7] Goovaerts, P. (1997). *Geostatistics for natural resources evaluation*. Oxford University Press on Demand.

[8] Pebesma, E., & Heuvelink, G. (2016). Spatio-temporal interpolation using gstat. *RFID Journal*, 8(1), 204-218.

[9] Presas, A., Luo, Y., Wang, Z., & Guo, B. (2019). Fatigue life estimation of Francis turbines based on experimental strain measurements: Review of the actual data and future trends. *Renewable and Sustainable Energy Reviews*, 102, 96-110.

[10] Myers, D. E. (1982). Matrix formulation of co-kriging. *Journal of the International Association for Mathematical Geology*, 14(3), 249-257.

[11] Papritz, A., Künsch, H. R., & Webster, R. (1993). On the pseudo cross-variogram. *Mathematical Geology*, 25(8), 1015-1026.

[12] Goulard, M., & Voltz, M. (1992). Linear coregionalization model: tools for estimation and choice of cross-variogram matrix. *Mathematical Geology*, 24(3), 269-286.