Strategic Bidding and Equilibria in Coupled Gas and Electricity Markets

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Abstract—The wide adoption of gas fired units and power-to-gas technology brings remarkable interdependency between natural gas and electricity infrastructures. This paper studies the equilibria of coupled gas and electricity energy markets driven by the strategic bidding behavior; each producer endeavours to maximizes its own profit subjecting to the market clearing outcome. The market equilibria is formulated as an equilibrium problem with equilibrium constraints (EPEC). A special diagonalization algorithm (DA) is devised, in which the unilateral equilibria of the rival’s strategy; the interaction of the two markets are tackled in the outer loop. Case studies on two test systems validates the proposed methodology.

Index Terms—Bilevel optimization, electricity market, natural gas market, interdependency, equilibrium problem with equilibrium constraints.

NOMENCLATURE

A. Sets and Indices

$s \in S$ Strategic electricity producer (SEP)

$o \in O$ Non-strategic electricity producer (Non-SEP)

$i \in I_s$ Units owned by SEP

$j \in I_o$ Units owned by non-SEP

$l_p \in I_p$ Power grid lines

$n_p \in N$ Power grid nodes

$b \in B$ Energy block of each generator

$v \in V$ Strategic gas producer (SGP)

$w \in W$ Non-strategic gas producer (Non-SGP)

$m \in M_v$ Gas wells owned by SGP

$x \in X_w$ Gas wells owned by non-SGP

$c \in C$ Active pipelines in gas network

$l_g \in L_g$ Passive pipelines in gas network

$n_g \in N_g$ Gas network nodes

$z \in Z$ Power-to-Gas facilities

$d_g \in D_p$ Electricity loads

$d_g \in D_g$ Gas loads

$\varphi \{\cdot \} (n_g)$ Components whose head node is $n_g$

$\varphi \{\cdot \} r_{\cdot} (n_g)$ Components whose tail node is $n_g$

$L_{l_p}(1, n_p)$ Power transmission lines whose head node is $n_p$

$L_{l_p}(2, n_p)$ Power transmission lines whose tail node is $n_p$

$\phi \{\cdot \} (n_p)$ Components connected to node $n_p$

$\Theta (n_p)$ Neighbouring nodes of node $n_p$

$\phi^{-1} (\cdot)$ Connection node of components in power system

$\phi^{-1} (\cdot)$ Connection node of components in gas system

$\varphi^{-1} (\cdot)$ Head node of components in gas system

$\varphi^{-1} (\cdot)$ Tail node of components in gas system

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B. Parameters

$\lambda_{ib}$ Marginal cost of non-gas-fired unit of SEP

$\lambda_{ib}$ Marginal cost of non-gas-fired unit of non-SEP

$P_d$ Electricity load demand

$P^{s, \text{max}}_b$ Upper limit of block $b$ of unit of SEP

$P^{o, \text{max}}_b$ Upper limit of block $b$ of unit of non-SEP

$E_l$ Electricity transmission line capacity

$B_{n_p}$ Admittance of transmission line

$q^{u}_{c}$ Upper limit of gas well of non-SGP

$q^{u}_{m}$ Upper limit of gas well of SGP

$q^{c, \text{max}}$ Capacity of active gas pipeline

$q^{m, \text{max}}$ Capacity of passive gas pipeline

$q_g$ Gas load demand

$\zeta$ Energy conversion constant

$\eta_{ib}, \eta_{ib}$ Efficiency of units

$\eta_{ib}$ Efficiency of power-to-gas facilities

$\alpha_{\text{max}}^o$ Maximal bidding price of SEP

$\delta_{\text{max}}^p$ Maximal bidding price of SGP

C. Variables

$\beta_{np}$ Local marginal electricity price

$\alpha_{ib}$ Bidding price of SEP

$P^{s}_{b}$ Cleared output of unit of SEP

$P^{o}_{b}$ Cleared output of unit of non-SEP

$\theta_{ib}$ Phase angle of power nodes

$\theta_{n_p}$ Local marginal gas price

$\delta_{m}$ Bidding price of SGP

$q^{w}_{c}$ Cleared output of gas well of SGP

$q^{w}_{m}$ Cleared output of gas well of non-SGP

$q_{c}$ Gas flow in active pipeline

$q_{m}$ Gas flow in passive pipeline

$P_z$ Demand of P2G facilities
I. INTRODUCTION

Over the past decade, there has been remarkable interest in the utilization of gas-fired units due to its high efficiency and low carbon emission, thanks to the breakthroughs in turbine technologies, say, the open-cycle gas turbine (OCGT) and the combined-cycle gas turbine (CCGT), as well as the dramatic decline in natural gas price, owing to the shale rock revolution \[1\]. Meanwhile, emerging P2G technology allows using excessive electricity produce gases in a vast form, such as hydrogen and natural gas \[2\], which can be stored in existing tanks in liquid or compressed format without intensive investments on upgrading the energy storage equipment. Such technologies greatly enhances the operating flexibility of power systems with volatile renewable energy integration. The potential bi-directional energy flows imposes stronger interdependency between gas and electricity systems, which calls for interdisciplinary research on the system operation and design.

Along this line, extensive researches can be found on the topic of coordinated operation of power and gas systems. For example, the optimal power and gas flow is analyzed in \[3\]; a stochastic unit commitment is proposed in \[4\], which aims to mitigate the variability of wind generation; a robust scheduling model considering uncertain wind power and demand response is investigated in \[5\]. To absorb excessive wind power, P2G technology is comprehensively discussed in \[6\], which allows a bi-directional interchange of energy. From the marketing perspective, the interdependency mentioned above impacts both gas and electricity markets: on one hand, the price of natural gas will influence the cost of gas-fired units as well as gas demand in the electricity market; on the other hand, the price of electricity will influence the cost of P2G facilities as well as electricity demand in the gas market. Some inspiring work have been done to address the correlation between electricity and gas markets. The day-ahead strategic bidding behavior of one CCGT owner is discussed in \[7\], taking operation constraints from both power and gas system into consideration, giving rise to a mathematical program with complementarity constraints (MPCC) formulation. The interaction of power and gas system models under a market environment in a medium-long scope is analyzed in \[8\], in which the single-level model includes power and gas operation constraints as well as the operational cost of either of the systems as its objective.

In the pool-based setting, the pool is cleared by a market operator (MO), an independent agency, giving the bidding strategies of all participants and necessary data. The market power of strategic electricity producers has been well explored in existing studies since the pioneer work \[9\]. The optimal bidding strategy of a single producer is analyzed in \[10\], resulting in an MPCC formulation. The optimal bidding strategies of multiple strategic producers are discussed in \[11\], leading to an EPEC formulation, and is solved by a stationary point method. The strategic behavior of wind power producers are studied in \[12\], the resulting EPEC is solved by diagonalization method. Similar to the pool-based power market, the gas pool can be cleared by a gas market operator as well. In this paper, we consider several energy producers who trade their resources strategically, in coupled electricity and gas markets. The strategic producers seek to maximize their own profits by bidding their offering prices, and the market will reach an equilibrium.

As one of the first few attempt, this paper proposes a market mechanism that allows di-directional energy trading between independently cleared power and natural gas markets. An EPEC model is established to study the optimal bidding strategies and market powers of energy producers. Moreover, a dedicated diagonalization algorithm is designed to compute the market equilibrium, in which the inner level provides the equilibria associating with either the gas or electricity market while fixing the exogenous variable from the other market; the outer level coordinate the bidding strategies of both markets.

The rest of this paper is organized as follows. The basic settings and market mechanism are elaborated in Section II. The EPEC formulation of the optimal bidding problem in coupled energy markets is presented in Section III. The diagonalization algorithm is introduced in Section IV. To validate the proposed model and algorithm, several numerical results on two testing systems are shown in Section V. Finally, Section VI draws the conclusion.

II. MARKET SETTINGS

A. Pool-based Market Mechanism

At the electricity side, SEPs such as generation companies receive local marginal gas price (LMGP) from the gas market operator (GMO), and then bid their offering price to the electricity market operator (EMO). The EMO clears the electricity market in terms of social welfare maximization, and the local marginal electricity price (LMEP) as well as the gas demand of each SEP become clear. At the gas side, strategic gas producers (SGPs) receive LMEP from the EMO, and then bid their offering price to the GMO. The GMO clears the gas market in terms of social welfare maximization, and the LMGP as well as electricity demand of each P2G facility become clear. It is apparent that the electricity market and the gas market impacts each other through bi-directional gas-electricity transitions. The schematic diagram of the coupled energy markets are shown in Fig. 1.

![Fig. 1. Electricity and natural gas market framework.](image)

B. Assumptions and Simplifications

The main assumptions made in the proposed model are summarized as follows.

1. The gas market and the electricity market are coupled through bi-directional energy trading (P2G and G2P).
2. The gas market and the electricity market are modeled as competitive markets.
3. The gas market operator (GMO) and the electricity market operator (EMO) are independent agencies.
4. The strategic producers are able to trade their resources strategically in both markets.
5. The exogenous variable in one market is fixed by the other market.
In the electricity market, all SEPs aim to maximize their individual profit, leading to the following formulation

\[
\forall s \in S, \min_{\alpha_{ib}^s} \left( \sum_{(i \in \phi_b^s)} \lambda_{ib}^s P_{ib}^s + \sum_{(i \in \phi_c(n))} \tau q_n \frac{P_{ib}^s}{\eta_{ib}} \right) - \sum_{(i \in \phi(b))} \beta_{ib} \frac{P_{ib}^s}{\eta_{ib}} \tag{1}
\]

\[
s.t. \quad 0 \leq \alpha_{ib}^s \leq \alpha_{i(b+1)}^s \leq \alpha_{i+B}^s, \forall i, \forall b < B \tag{2}
\]

\[
P_{ib}^s, \beta_{ib} \in \arg\min_{P_{ib}^s, \beta_{ib}} \left( \sum_{sib} \alpha_{ib}^s P_{ib}^s + \sum_{o \in \phi_b^o} \lambda_{jo} \frac{P_{jo}^o}{\eta_{jo}} + \sum_{(j \in \phi_b^o)} \tau q_n \frac{P_{ib}^s}{\eta_{ib}} \right) \tag{3}
\]

\[
s.t. \quad 0 \leq P_{ib}^s \leq P_{ib}^{s,\max} : \beta_{ib}^{s,\min} \leq \beta_{ib}^{s,\max}, \forall s, \forall i, \forall b \tag{4}
\]

\[
0 \leq P_{jb}^o \leq P_{jb}^{o,\max} : \beta_{jb}^{o,\min} \leq \beta_{jb}^{o,\max}, \forall o, \forall j, \forall b \tag{5}
\]

\[
\sum_{s \in \phi(n)} P_{ib} + \sum_{o \in \phi(n)} P_{jo} - \sum_{z \in \phi(z)} P_z = 0 \tag{6}
\]

\[
\sum_{d \in \phi(d)} B_{nb} \left( \theta_{nb} - \theta_k \right) = \beta_{nb}, \forall n, \forall p \tag{7}
\]

\[
-F_{lp} \leq B_{n_{p1}n_{p2}} (\theta_{n_{p1}} - \theta_{n_{p2}}) \leq F_{lp} : \beta_{lp}^{\min} \leq \beta_{lp}^{\max}, \forall n_{p1}, \forall n_{p2} \in Line_{lp}, \forall l \tag{8}
\]

\[
\theta_{n_0} = 0 : \varepsilon_1, n_0 = 1 \ \} \tag{9}
\]

In this formulation, the upper level bidding problem consists of (1)-(2) as well as the lower level electricity market clearing problem (3)-(9). Objective function (1) represents the opposite profit of strategic producer s, where the first two components are the generation cost of non-gas-fired units and gas-fired units, respectively, and the last component is the utility of all generating units. (2) imposes upper and lower bounds on the offering price, as well as the monotonicity with respect to the energy block. The LMEP \( \beta_{n} \) and the energy contract \( P_{ib}^s \) are determined by (3)-(9), which represents the electricity market clearing process. Objective (3) represents the opposite social welfare, where the first component is the generation expense of strategic producers and the last two component are the generation cost of non-gas-fired and gas-fired units owned by non-strategic producers, respectively. Constraints (4) and (5) are the energy block capacity of all type of units. Equality (6) depicts the nodal power balancing condition. Inequality (7) is the flow limit of transmission lines. Inequality (8) describes the upper and lower phase angle limits of the complex bus voltage. (9) sets the phase angle for the reference node. Dual variables are indicated at the corresponding equations following a colon.

B. EPEC model for gas market

In the gas market, all SGPs aim to maximize their individual profit, leading to the following formulation

\[
\forall v \in V, \min_{\delta_m} \left( \sum_{m \in \phi_m} \zeta_m^v q_m^v - \sum_{(m \in \phi_m(n))} \theta_{nq}^m q_m^w \right) \tag{10}
\]

\[
s.t. \quad 0 \leq \delta_m^v \leq \delta_{m,\max}, \forall m \tag{11}
\]

\[
q_m^v, \theta_{nq} \in \arg\min_{q_m^v, \theta_{nq}} \left( \sum_{n \in V} \sum_{q_m^w} \zeta_m^v q_m^w + \sum_{w \in \Omega} \xi_w q_w^+ + \sum_{z \in \Omega} \zeta_z q_z^w + \sum_{(z \in \phi_z(n))} \beta_{nq} \right) \tag{12}
\]

\[
s.t. \quad 0 \leq q_m^v \leq \alpha_m^v : \rho_m^{\min}, \rho_m^{\max} \forall m, \forall v \tag{13}
\]

\[
0 \leq q_w^+ \leq \alpha_w^+ : \rho_w^{\min}, \rho_w^{\max} \forall w \tag{14}
\]

\[
-\delta_{lq}^{\max} \leq \delta_{lq} \leq \delta_{lq}^{\max} : \delta_{lq}^{\min}, \delta_{lq}^{\max} \forall g \tag{15}
\]

\[
0 \leq q_c^{\max} : \rho_c^{\min}, \rho_c^{\max} \forall c \tag{16}
\]

\[
0 \leq \delta_l^{\max} : \rho_l^{\min}, \rho_l^{\max} \forall l \tag{17}
\]

\[
\sum_{n \in \phi(n)} q_m^v + \sum_{w \in \phi(w)} q_w^+ + \sum_{z \in \phi(z)} \tau q_z^w = 0 \tag{18}
\]
In this formulation, the upper level bidding problem consists of (10)-(11) as well as the lower level gas market clearing problem (12)-(18). The objective function (10) represents the utility of (10)-(11) as well as the lower level gas market clearing process. Objective function (12) defines the opposite social welfare, where the first two components are the generation expenses of strategic and non-strategic gas producers, respectively, and the last component is the electricity purchase cost of P2G facilities. Constraints (13) and (14) give the production limits of gas wells of strategic and non-strategic producers, respectively. In the natural gas network, pipelines without/with compressors are determined by (12)-(18), which represents the gas market clearing process. Objective function (11) restricts the gas demand from gas-fired units as well as the bidding strategies from other SEPs. Constraints (15) and (16) represent the gas flow capacity of pipelines with/without compressors. Constraints (17) indicate a non-negative constraint on the electricity demand of P2G facilities. (18) is the nodal gas balancing equations following a colon.

IV. Solution Methodology

In this section, we first present equivalent MPCC formulations for the bilevel bidding problems associated with each player in the upper level. Then a special diagonalization algorithm (DA) is introduced to solve the proposed EPEC models by iteratively solving a sequence of MPCCs, until a certain convergence criteria is met.

A. Solution Methodology for MPCCs

Take the electricity market as an example. For SEP $s^*$, if the LMGP $q_m$ and the gas contract $q_n$, are determined by (12)-(18), which represents the gas market clearing process. Objective function (11) restricts the gas demand from gas-fired units as well as the bidding strategies from other SEPs. Constraints (13) and (14) give the production limits of gas wells of strategic and non-strategic producers, respectively.

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\[ 0 \leq P_z \perp \rho_z \geq 0, \forall z \] (46)

where, \((43, 446)\) are the KKT optimality conditions of the lower level gas market clearing problem.

There are two sorts of nonlinearities in the proposed MPCC: one is the summation of bilinear terms \( \sum_{(i \in \phi_1(n_p))} \beta_{np} P_{ib}^s \) in \((19)\) and \( \sum_{(m \in \phi_m(n_p))} \gamma_{nm} q_{im}^w \) in \((32)\), consisting of products of primal variable and dual variable; the other is the complementarity constraints, including \((24)-(31)\) and \((38)-(46)\). The linearization methods are introduced below.

1) Linearizing the primal-dual product: linear expressions can be obtained to replace the bilinear terms in the original objective function by strong duality theorem and KKT complementarity constraints. The linearized objective functions are given below. Details can be found in Appendix.

\[
\begin{align*}
\min_{\alpha_{ib}, \beta_{ib}} & \left( \sum_{(i \in \phi_1(n_p))} \tau s_{ib}^* P_{ib}^s / \eta_{ib} + \sum_{(i \in \phi_1(n_p))} \lambda_{ib}^s P_{ib}^* \right) \\
& - \sum_{np} \pi \left( e_{np}^s + e_{np}^m \right) + \sum_{l_p} F_{lp} (\beta_{lp}^m + \beta_{lp}^p) \\
& + \sum_{ib} \left( \sum_{s} \beta_{ib}^s \sum_{\alpha_{ib}^s} P_{ib}^s + \sum_{o} \beta_{ib}^o P_{ib}^o \right) + \sum_{\beta_{ib}} P_{db} + \sum_{o(j \in \phi_2(n_p))} \lambda_{ij}^s P_{ij}^m \bar{a}_b + \sum_{s, n, b} P_{db} + \sum_{o(j \in \phi_2(n_p))} \lambda_{ij}^s P_{ij}^m \bar{a}_b - \sum_{s, n, b} \beta_{ib} P_{ib}^s \\
& + \sum_{s, n, b} \beta_{ib} P_{ib}^s + \sum_{s, n, b} \alpha_{ib}^s P_{ib}^s + \sum_{s, n, b} \beta_{ib} P_{db} + \sum_{s, n, b} \beta_{ib} \sum_{o} \beta_{ib}^o P_{db} + \sum_{s, n, b} \beta_{ib} \sum_{o} \beta_{ib}^o P_{db} + \sum_{s, n, b} \beta_{ib} \sum_{o} \beta_{ib}^o P_{db} + \sum_{s, n, b} \beta_{ib} \sum_{o} \beta_{ib}^o P_{db} \end{align*}
\]

(47)

2) Complementarity Constraint: All complementarity constraints in KKT condition share the similar form of

\[ 0 \leq P_z - \rho_z \leq \rho_z \geq 0 \] (49)

Where \(a, f, g\) represent the primal variable, constant and dual variable, respectively. By introducing a binary variable \(h\) and the following constraints, \((49)\) can be fully linearized.

\[
\begin{align*}
a - f & \leq \text{BigM} \cdot h \\
g & \leq \text{BigM} (1 - h)
\end{align*}
\]

(50)

Where \(\text{BigM}\) is a sufficient large positive number.

In view of the linearized objective functions and constraints, the MPCCs of SEP and SGP have been converted into mixed integer linear problems (MILPs), and are readily solvable by using commercial solvers, such as Cplex and Gurobi.

\[ B. \text{The Nested Diagonalization Algorithm} \]

In this subsection, a nested diagonalization algorithm is proposed to find the equilibria in the coupled electricity-gas market. For notation brevity, we use \(P_z(s \in S)\) and \(G_v(v \in V)\) to represent the strategic bidding problem for SEPs and SGPs in MILP form, respectively. The aggregated problems \(\{P_z\}, \forall s\) and \(\{G_v\}, \forall v\) represent denote the EPECs of the electricity market and gas market, respectively.

1) Inner Diagonalization Algorithms: Two inner DAs are introduced to solve the EPEC models for electricity and gas market, respectively, the flow charts are given below.

Algorithm 1 DA for EPEC of electricity market

1. Get current \(P_z, g_{na}\) as input. Set \(\alpha_{ib}^s = \alpha_{ib}^{max}\) for all \(s, i, b\). Set the maximum iteration number \(r_{max}\) and convergence criterion \(\epsilon, r = 1, \text{Flag}_{pp} = 0\).
2. Do for \(s^* = 1, \ldots, S\), regard \(\alpha_{ib}^s (s \neq s^*)\) as the bidding price from others. Solve \(P_{s^*}\) and obtain \(\alpha_{ib}^{s^*, r}\). Let \(\alpha_{ib} = \alpha_{ib}^{s^*, r}\)
3. If \(|\alpha_{ib}^{s^*, r} - \alpha_{ib}^{s, r-1}| \leq \epsilon \cdot \max\{\alpha_{ib}^{s, r}, \alpha_{ib}^{s, r-1}\}\) for all \(s, r\), then terminate, and report \(\alpha_{ib}^{s, r}\). Else, set \(r = r + 1\), quit and return the inner DA for electricity market fails to converge. Else, \(r = r + 1\) and return to Step 2.

Algorithm 2 DA for EPEC of gas market

1. Get current \(P_{db}, P_{oj}^0, \beta_{nj}\) as input. Set \(\delta_{m}^{v, 0} = \delta_{m}^{max}\) for all \(v, m\). Set the maximum iteration number \(r_{max}\) and convergence criterion \(\epsilon, r = 1, \text{Flag}_{gp} = 0\).
2. Do for \(v^* = 1, \ldots, V\), regard \(\delta_{m}^{v, 0} (v \neq v^*)\) as the bidding price from others. Solve \(G_{v^*}\) and obtain \(\delta_{m}^{v^*, r}\). Let \(\delta_{m}^{v, r} = \delta_{m}^{v^*, r}\)
3. If \(|\delta_{m}^{v, r} - \delta_{m}^{v, r-1}| \leq \epsilon \cdot \max\{\delta_{m}^{v^*, r}, \delta_{m}^{v^*, r-1}\}\) for all \(v, r\), then terminate, and report \(\delta_{m}^{v, r}\). Else, set \(r = r + 1\), quit and return the inner DA for gas market fails to converge. Else, \(r = r + 1\) and return to Step 2.

2) Outer Diagonalization Algorithm: Similar to \((17)\), we adopt an iterative-based algorithm to find the equilibria of the coupled energy markets, which is referred to as outer DA. The flow chart is given below.

Other than the proposed nested DA, traditional DA is also a straightforward method to find the equilibria in the gas-electric market. However, the computational cost of traditional DA is much higher than the nested DA, sometimes even fails to solve the EPEC model. The reason is that the updating variables in the outer DA are already equilibriums, say equilibriums in electricity market and gas market, respectively, while the updating variables in traditional DA are not.

V. ILLUSTRATIVE EXAMPLE

In this section, we present numerical experiments on two testing systems to show the effectiveness of the proposed model and algorithm. The experiments are performed on a
Algorithm 3 DA for equilibria of the coupled markets

1: Set $\delta_{n_q}^0 = \delta_{z}^{max}$, $\delta_{i_b}^0 = 0$, $P_{i_b}^{0,z} = 0$, $P_{j_b}^{0,z} = 0$, $\alpha_{n_q}^0 = \alpha_{z}^{max}$. Set the maximum iteration number $r^{max}$ and convergence criterion $\epsilon$, $r = 1.$
2: Call Algorithm 1 and obtain $\alpha_{i_b}^r$. Solve problem (3)-(9), and obtain $P_{i_b}^{r,s}, P_{j_b}^{r,s}, \beta_{n_q}^r$.
3: Call Algorithm 2 and obtain $\delta_{m}^r$. Solve problem (12)-(18), and obtain $P_{s}^{r}, \delta_{m}^r$.
4: If $|P_{i_b}^{r,s} - P_{i_b}^{r,s-1}| \leq \epsilon \cdot \max \{ |P_{i_b}^{r,s} - P_{i_b}^{r,s-1}| \}$ for all $z$, $|P_{j_b}^{r,s} - P_{j_b}^{r,s-1}| \leq \epsilon \cdot \max \{ |P_{j_b}^{r,s} - P_{j_b}^{r,s-1}| \}$ for all $s$, $|P_{j_b}^{r,s} - P_{j_b}^{r,s-1}| \leq \epsilon \cdot \max \{ |P_{i_b}^{r,s} - P_{i_b}^{r,s-1}| \}$ for all $o$, all holds, then quit, and report $\alpha_{i_b}^m, \delta_{m}^r, P_{s}^r, P_{i_b}^r, P_{j_b}^r$. Else,
5: If $r = r^{max}$, or $\Flag_{dp} = 1$ or $\Flag_{dq} = 1$, then quit and report the algorithm fails to converge. Else, $r = r + 1$ and return to Step 2.

A. 6-Bus Power System with 7-Node Gas System

Fig. 2 depicts the topology of the interconnected infrastructure. It has 4 gas-fired units ($P_2, P_3, P_6, P_7$), 4 traditional coal-fired units ($P_1, P_3, P_5, P_6$), 6 gas wells ($G_1 - G_6$), 1 compressor, 2 P2G facilities ($P2G_1, P2G_2$), 4 power loads and 3 gas loads. Specifically, the head and tail node of the compressor is 4 and 2, respectively. The parameters of the system can be found in [18]. The total electricity and gas demand are 525MW and 5520Sm3, respectively. In the following cases, we have 4 strategic producers in the electricity market, who own generator $P_1$ to $P_4$, respectively, and the rest units are owned by a non-strategic producer. Also, we have 3 strategic producers in the gas market, who own gas well $G_1$ to $G_3$, respectively, and the rest gas wells are owned by a non-strategic producer. $\epsilon$ in the proposed diagonalization algorithm is 1% and the maximum iteration number is 20.

B. Uncongested Network

The coupled energy market and its equilibria is firstly studied without considering congestions by neglecting capacity constraints, including those of power transmission lines and gas pipelines. The optimal bidding strategies and energy contracts are listed in Table I where both the superscript and the subscript are omitted for the price variables. The output of generators and gas wells which do not appear in Table I are 0. In the power network, all buses share the same LMEP, because the network is uncongested. The bidding price are equal to the corresponding LMEPs. The committed power from each block of generator can be obtained according to their capacity parameters. In the gas market, due to the existence of active pipeline as well as its radial topology, the LGMPs could be different at the head node and the tail nodes of an active pipeline. In this particular case, all the LGMPs are the same, and the strategic gas producers bid the same price equal to the LGMP. The energy exchange between electricity market and gas market are also listed in Table I. Intuitively, the power consumption of P2G facilities should be zero in an uncongested network as the equivalent gas price should be higher than the LGMPs in general, due to the dissipativity of energy conversion.

Then we change the electricity and gas load by their load ratios, respectively. The corresponding nodal prices are shown in Fig. 3 and Fig. 5. From Fig. 3, it can be observed that the LMEP is always nondecreasing with the gas load ratio (GLR) when electricity load ratio is fixed. However, the relationship between LMEP and electricity load ratio (ELR) is quite complicated, whose general trend is increasing yet sometimes decreases, say there is a drop when ELR is 0.8, which is mainly caused by competition in the electricity pool as the LGMPs remain the same when ELR varies from 0.75 to 0.9. Further, from Fig. 4 and Fig. 5, the LGMPs of two sides of active pipeline are different in most cases. Also, the general variation trend of Fig. 3 and Fig. 4 are the same.

C. Congested Network

The analysis in the previous subsection is performed again with transmission capacity limits becoming mandatory, including those of power transmission lines and gas pipelines. The transmission capacity parameter can be found in [18]. The results are listed in Table II. In the gas market, SGPs seek to
In this subsection, we apply the proposed model and algorithm to a larger test system, which consists of a modified IEEE 118-bus system and the Belgian high-calorific 20-node gas network. It possesses 30 gas-fired units, 24 traditional units, 10 gas wells, 3 compressors, 4 P2G facilities, 91 power loads and 9 gas loads. Refer to [18] for the network topology and system data set. Table III summarizes the computational times with different numbers of strategic producers indicated by a tuple \((a, b)\), where \(a\) is the number of SGPs and \(b\) is the number of SEPs. The details of ownership of units and gas wells can be found in [18]. From Table III, we see that the computational time is always acceptable for the coupled energy market in this moderately sized system.

**VI. Conclusion**

With the physical and economic interdependency between the natural gas and electricity systems becoming stronger, this paper proposes an EPEC model that allows bi-directional gas-power trading, and a nested diagonalization algorithm to compute the market equilibria. Simulation results corroborate the effectiveness of the proposed method. The proposed

**TABLE II**

| Power | Gas |
|-------|-----|
| \(\beta\) | \(\phi\) | \(\alpha\) |
| \(\beta\) ($/MWh) | \(\phi\) ($/Sm\(^3\)) | \(\alpha\) ($/MWh) |
| \(P_1\) | \(P_2\) | \(P_3\) |
| 54.25 | 0.6 | 7 |
| 55.15 | 0.9 | 2 |
| 82.75 | 3 | 3 |

**TABLE III**

| Case Setting | NSP | Time (s) | NSP | Time (s) | NSP | Time (s) |
|-------------|-----|----------|-----|----------|-----|----------|
| Uncongested | (3,3) | 99 | (3,4) | 167 | (3,5) | 259 |
| | Time (s) | 117 | 195 | 209 | NSP | Time (s) |
| | (3,3) | 48 | (3,4) | 203 | (3,5) | 271 |

| Congested | NSP | Time (s) | NSP | Time (s) | NSP | Time (s) |
|-----------|-----|----------|-----|----------|-----|----------|
| (3,3) | 40 | (3,4) | 296 | (3,5) | 327 |
| | Time (s) | 59 | (3,4) | 203 | (3,5) | 335 |

**Fig. 3.** LMEPs under different load ratios.

**Fig. 4.** LMGPs under different load ratios. (Node:1-3, 5-6)

**Fig. 5.** LMGPs under different load ratios. (Node:4, 7)

bid higher prices due to congestion, which would give rise to the marginal cost increment of gas-fired units. In the electricity market, SEPs also seek to bid higher prices due to congestion, moreover, some of them are suffering from a higher marginal cost. Therefore, the bidding prices of SEP 4 as well as SGP 1,3 reach their maximum value, respectively, in the equilibria. In this regard, the MO of electricity market may try to dispatch more traditional generation assets to maximize social welfare. Thus, the total gas consumption from gas-fired units has a 2385 $Sm^3/h$ reduction compared with the uncongested case. Meanwhile, traditional units, say \(P_1\) and \(P_2\), are put into operation in this case while they are out of service in the uncongested case. It can be observed that the P2G facilities are not working as they are still not cost-effective according to the gas market clearing model.

From the maximum social welfare perspective, the production cost of the power network in the congested case is increased by $221 compared with the uncongested case, from $7,539 to $7,760, and the production cost of the gas network in the congested case is increased by $439 compared with the uncongested case, from $9,537 to $9,976.

**D. 118-Bus Power System with 20-Node Gas System**

In this subsection, we apply the proposed model and algorithm to a larger test system, which consists of a modified IEEE 118-bus system and the Belgian high-calorific 20-node gas network. It possesses 30 gas-fired units, 24 traditional units, 10 gas wells, 3 compressors, 4 P2G facilities, 91 power loads and 9 gas loads. Refer to [18] for the network topology and system data set. Table III summarizes the computational times with different numbers of strategic producers indicated by a tuple \((a, b)\), where \(a\) is the number of SGPs and \(b\) is the number of SEPs. The details of ownership of units and gas wells can be found in [18]. From Table III, we see that the computational time is always acceptable for the coupled energy market in this moderately sized system.
methodology will be desired by policy makers and market designers.

APPENDIX

For strategic producer $s^*$, multiplying both sides of (20) by $P_{ib}^s$ and adding them together, yields following equation:
\[ \sum_{ib} (\alpha_{ib} + \beta_{ib}^{s,max} - \beta_{ib}^{s,min}) P_{ib}^s - \sum_{(i \in \phi(n_p))b} \beta_{np} P_{ib}^s = 0 \]  
(51)

From (24)-(25), we have
\[ \beta_{ib}^{s,min} P_{ib}^s = 0 \Rightarrow \sum_{ib} \beta_{ib}^{s,min} P_{ib}^s = 0 \]  
(52)

\[ \sum_{ib} \beta_{ib}^{s,max} P_{ib}^s = \beta_{ib}^{s,max} P_{ib}^s \Rightarrow \sum_{ib} \beta_{ib}^{s,max} P_{ib}^s = \sum_{ib} \beta_{ib}^{s,max} P_{ib}^s \]  
(53)

Substitute some terms in (51) with (52), we have
\[ \sum_{(i \in \phi(n_p))b} \beta_{np} P_{ib}^s = \sum_{(i \in \phi(n_p))b} (\alpha_{ib}^{s,max} P_{ib}^s + \beta_{ib}^{s,max} P_{ib}^s) \]  
(54)

From strong duality theorem, we have
\[ \sum_{ib} \alpha_{ib}^{s,max} P_{ib}^s + \sum_{(s \neq b)} \alpha_{ib}^{s,min} P_{ib}^s + \sum_{o(j \in \phi(n_b))b} \lambda_{oj}^{o} P_{oj} + \sum_{o(j \in \phi(n_b))b} \tau q_{n_g} P_{oj} + \sum_{o(j \in \phi(n_b))b} \pi \varepsilon_{n_p}^{max} + \pi \varepsilon_{n_p}^{min} \]  
(55)

\[ \sum_{ib} \beta_{ib}^{s,max} P_{ib}^s \} \Rightarrow \sum_{ib} \beta_{ib}^{s,max} P_{ib}^s \]  
(56)

By substituting (54) into (55), we arrive at the linear expression of $\sum_{(i \in \phi(n_p))b} \beta_{np} P_{ib}^s$ as shown in (47).

For strategic producer $s^*$, multiplying both sides of (33) by $q_m^s$ and adding them together with respect to $m$, yielding the following equation:
\[ \sum_{m} (q_m - m \rho_m - \rho_m) q_m^s - \sum_{(n_g = \varphi^{-1}(m))} \gamma_n q_m^s = 0 \]  
(57)

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