Dependence of energy loss of jets on the initial thermodynamic state of deconfined matter at RHIC

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Abstract

The dependence of the radiative energy loss of fast partons on the initial thermodynamic parameters is studied for deconfined matter to be expected at RHIC. We demonstrate that the specific QCD radiation pattern with a quadratic dependence of the energy loss on the propagated distance leads to a strong increase of the energy loss with increasing initial entropy of deconfined matter supposed its life-time is less than the average time to pass through the medium. This is in contrast to a parameterization with constant energy loss per unit length of propagation. For a sufficiently high initial temperature a two-regime behavior of the energy loss as a function of the initial parton momentum occurs. The angular structure of the energy loss of hard jets with respect to the initial temperature is also discussed for RHIC conditions.

Key Words: heavy-ion collisions, deconfinement, jets
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I. INTRODUCTION

One of the primary goals of the current experiments at the Relativistic Heavy-Ion Collider (RHIC) in Brookhaven National Laboratory is the investigation of quark-gluon matter in a deconfined state. While this new state of strongly interacting matter is thought to be basically very different from a confined hadronic medium, its verification in heavy-ion collisions seems to demand a combination of various observables [1]. Among the presently disputed signals of the quark-gluon plasma are penetrating probes, like dileptons and photons, and hard probes, like jets and the $J/\psi$, and accumulative probes, like hadron multiplicity ratios. Since the hadron multiplicity measurements reflect mainly the chemical equilibration and hadronization processes in deconfined matter, they do not deliver direct information on the early stage with maximum density and/or temperature, where a quark-gluon plasma is expected to be formed. Dileptons and photons leave the medium without strong interactions from the very beginning of the matter's evolution and, therefore, can probe the initial stage of deconfined matter. At the same time, the dileptons are emitted in all stages so that the corresponding spectra appear as a convolution of the production rate and the whole space-time history of the matter. This leads to the possibility to describe the secondary (thermal) dilepton and photon spectra, at least for CERN-SPS energies, by one averaged
temperature [2]. While the value of such an effective temperature is found to be greater than the kinetic hadron freeze-out temperature, it still gives only indirect information on the initial or hottest stage of deconfined matter.

At RHIC, where one expects a much higher temperatures than at CERN-SPS, the measurements of jets seem to offer an access to a probe of the early stage of the deconfined matter. Being produced due to the hard initial collisions inside the volume which is a moment later occupied by the quark-gluon medium, a parent parton of a jet with high transverse momentum propagates then for some time through the strongly interacting matter and suffers an energy loss due to collisions and induced gluon radiation. As a result, the momentum of the jet parton is attenuated before hadronizing. This is the famous jet quenching anticipated in [3]. The energy loss in deconfined matter is estimated to be much larger than in confined hadronic matter. Therefore, the energy loss is expected to be most sensitive to both the size and the life-time of the deconfinement stage. One obvious way to study the system size dependence of jet quenching is a variation of the centrality of the heavy-ion collision [4,5]. Another opportunity is connected with variations of the beam energy in central collisions. This enables one to investigate the dependence of the energy loss on the life-time of the deconfined system and, consequently, on the initial conditions, such as temperature and parton density, expected to be achieved in RHIC experiments.

The dependence of the energy loss of a fast parton on the life-time and size of the deconfinement region is also of particular interest due to the non-trivial QCD behavior of the local radiative energy loss with respect to the passed-through distance. In QCD the radiative energy loss is mainly related to the coherence conditions (Landau-Pomeranchuk-Migdal effect) of the emitted gluons which suffer multiple non-Abelian interactions within the ambient medium. In so far, the radiative energy loss mechanism in QCD has some analogy with the Ter-Mikaelian effect in QED where the emitted photons suffer multiple Compton scatterings in the medium (cf. [6] for references and an explanation of these issues in the context of QCD). As a result, a specific quadratic distance dependence of the total radiative energy loss appears in deconfined matter. This quadratic dependence is confirmed in various detailed calculations [7,8] and persists also in an expanding quark-gluon plasma [9]. For further elaborate approaches to the energy loss we refer the interested reader to [10,12,13] and for a recent application on the $p_{\perp}$ dependence of pion and kaon ratios to [14].

The aim of the present work is to study the dependence of the energy loss on the initial thermodynamic variables of deconfined matter formed in central heavy-ion collisions at RHIC. In particular, in section II we search for an evidence of the specific QCD pattern of the parton energy loss which reflects the appearance of a high-temperature non-Abelian quark-gluon plasma. These considerations are supplemented by estimates of the dependence of the energy loss signal on the jet opening cone (section III). The discussion and conclusions can be found in section IV.

II. ENERGY LOSS EFFECTS OF FAST PARTONS

According to [9] the total radiative energy loss of a parton propagating through an expanding quark-gluon plasma is given by $\Delta E = \xi \Delta E|_{T_f}^{\text{static}}$, where $\xi = 2$ (6) for a parton created inside (outside) the medium. The medium is assumed to expand and obeys the
temperature evolution $T \propto \tau^{-\beta}$ with $\tau$ as proper time in the piece of matter undergoing boost-invariant longitudinal expansion, and $\beta \leq 1$. $T_f$ denotes the plasma temperature at which the parton leaves the deconfined region either through a time like or a space like boundary. $\Delta E_{T}^{\text{static}}$ corresponds to the energy loss in a static system with temperature $T$ and is given by

$$\Delta E_{T}^{\text{static}} = -\frac{1}{4} \alpha_s N_c \times \begin{cases} \hat{q} L^2 / \sqrt{q E L}, & L < L_{\text{crit}} \\ \sqrt{q E L}, & L > L_{\text{crit}} \end{cases},$$

(1)

where $L$ is the distance in transverse direction the fast jet quark propagates till escaping the region with deconfined matter; $\hat{q}$ denotes a transport coefficient, and $N_c = 3$. The critical length $L_{\text{crit}}$ depends on the initial quark energy as

$$L_{\text{crit}} = \sqrt{\frac{E}{\hat{q}}},$$

(2)

The transport coefficient is approximated in the subsequent calculations by $\hat{q} = \mu^2 / \lambda$ with $\lambda$ as mean free path of the partons and $\mu$ as the average transverse momentum kick which the fast quark suffers per scattering while propagating through the deconfined medium. We use the screening mass as scale for $\mu$. The temperature dependence of the parameters describing the medium is as follows: $\lambda^{-1} = 2.2 \alpha_s T$ [15], $\mu^2 = 6 \pi \alpha_s T^2$ [16], and $\alpha_s = 0.3$. We use here $\xi = 2$, i.e., the lower limit of the QCD energy loss from [4].

One should keep in mind that the estimates of the energy loss based on the above described equations are semi-quantitative due to the following reasons. (i) The parameter $\hat{q}$ is model dependent. Accordingly the numerical value of the energy loss according to Eq. (1) can change by a constant factor of about 2. (ii) One can also expect an additional suppression factor in Eq. (1) if the number of parton scatterings inside deconfined matter is not large and one has to use an opacity expansion [10] (cf. also [11]). However, the most important $L^2$ dependence of the energy loss and the specific QCD pattern governed by the parameter $L_{\text{crit}}$ are expected to be stable with respect to variations of the medium dependent parameters. (For a recent discussion of the $L^2$ and $L_{\text{crit}}$ dependences cf. [12].) (iii) Equations (1, 2) are based on one-gluon emission. A reliable estimate of multi-gluon emission is not yet available. Nevertheless, we use Eqs. (1, 2) as benchmark and explore in the following their consequences.

To simplify our considerations we model the space-time evolution of the quark-gluon plasma by the Bjorken scenario [17] with boost-invariant longitudinal expansion and conserved entropy per rapidity unit. This results in the life-time of deconfined matter $\tau_c = (T_i / T_c)^3 \tau_i$, where $T_i = T(\tau_i)$ is the ”initial” temperature at ”initial” time $\tau_i$. (Actually, $T_i$ and $\tau_i$ refer to the earliest stage where the partonic system is sufficiently near to equilibrium.) $T_c \approx 170 \text{ MeV}$ is the temperature where in a baryon-poor plasma the confinement sets in according to QCD lattice calculations [18]. We also assume that transverse expansion is negligible for the early (deconfined) state of matter.

Taking Eq. (2) as a dynamical ingredient we calculate with Monte Carlo simulations the average energy loss of a fast quark created randomly, but with uniform distribution over the central slice of matter at mid-rapidity, with random transverse direction. Due to the fast longitudinal expansion of the matter and the not too long life-time of the plasma expected
for RHIC conditions, the propagated distance within deconfined matter is determined not only by the geometrical size but mainly by the life-time $\tau_c$. Therefore, $L$ depends sensitively on the initial thermodynamic state of the deconfined matter.

To see some evidence of the specific QCD behavior of the energy loss that includes the two regimes, i.e., the linear and quadratic $L$ dependences in Eq. (1) governed in turn by the parameter $L_{\text{crit}} \propto \sqrt{E}$ in Eq. (2), we plot in Fig. 1 the dependence of the average energy loss as a function of the initial transverse momentum of the parent quark $p_{\perp}^0$. For the central rapidity region we are considering one can approximate $p_{\perp}^0 \approx E$. The value of $p_{\perp}^0$ is most easily measured for photon tagged jets since the high-energy photon does not suffer any noticeable influence by the ambient medium. (For estimates of the count rates and so on cf. [19]. By using the corresponding parts of PYTHIA [20] we have checked that intrinsic parton transverse momenta before the hard scattering process $g+q \rightarrow \gamma+q$ with momentum spread $\sqrt{\langle k_{\perp}^2 \rangle} = 0.8$ GeV does not spoil the results. In doing so we identified $p_{\perp}^0$ with $p_{\perp}^\gamma$.)

The results displayed in Fig. 1 refer to a transverse radius $R_A = 7$ fm and an initial time of $\tau_i = 0.2$ fm/c. In contrast to a simple energy loss scenario with $\Delta E = \eta L$ with the widely used constant $\eta = -1$ GeV/fm, one can observe indeed the two-regime behavior for the QCD based energy loss according to Eqs. (1, 2). Due to the above mentioned dependence of $L$ on the life-time of the quark-gluon phase, only in restricted ranges of initial temperatures and initial jet momentum the inequality $L > L_{\text{crit}}$ is satisfied, i.e., a high enough initial temperature but not too large jet momentum, so that the regime $\Delta E \propto L$ with increasing energy loss $\Delta E \propto \sqrt{E}$ for increasing $p_{\perp}^0$ becomes visible. For large values of $p_{\perp}^0$, one enters ultimately the regime $\Delta E \propto L^2$ which does not display any dependence on $p_{\perp}^0$. Also for smaller initial temperatures and shorter life-times $\tau_c$ and consequently smaller values of $L$, one enters again the $p_{\perp}^0$ independent regime with $\Delta E \propto L^2$. Notice that the energy loss, at larger values of $p_{\perp}^0$, for the QCD behavior according to Eqs. (1, 2) and the linear model with $\Delta E = \eta L$ deliver similar results for smaller values of $T_i$, while with increasing $T_i$ the energy loss becomes larger in the former approach due to the $L^2$ dependence.

The initial temperature of the plasma, $T_i$ remains as unknown parameter which is under intensive dispute now. In particular within the saturation model [21], where the mechanism of the transverse energy production is determined by gluon mini-jets [22–25], the value of $T_i$ scales with $\sqrt{s}$, and $T_i(\sqrt{s} = 200 \text{GeV}) \approx 0.6$ GeV is estimated. As well known, the value of the initial temperature depends strongly on the kinetic framework used to model the thermalization process in the partonic system. Within the Boltzmann equation in relaxation time approximation and imposed boost invariance [20], a lower initial temperature of $T_i \approx 340$ MeV is found in [27]. Under the reasonable assumption that the entropy is mainly produced in the very early stage of the heavy-ion collision, where $\tau \ll \tau_c$, and the partonic matter undergoes boost-invariant longitudinal expansion, the combination $\tau_i T_i^3$ describes the entropy per unit rapidity interval during both the pre-equilibrium and equilibrium stages and therefore can be related to the multiplicity of secondary particles measured experimentally. Keeping this in mind we quantify the sensitivity of the QCD governed energy loss of a fast quark on the initial thermodynamic state of matter via its dependence on the parameter $N_i = \tau_i T_i^3$.

In Fig. 2 we plot the dependence of scaled energy loss $(p_{\perp}^0 - \langle p_{\perp}^0 \rangle)/N_i$ on the value $N_i$ for various values of the initial time $\tau_i$. For the initial energy of the fast quark we take...
$E = p_0^2 = 50$ GeV as appropriate for RHIC energies. Due to the specific $L^2$ dependence (cf. Fig. 1) the scaled QCD energy loss increases almost linearly with $N_i$. This is in contrast to the linear energy loss model with $\Delta E = \eta L$ which is approximately independent of the value of $N_i$. The above behavior of the QCD energy loss has actually a qualitative character and is stable with respect to a variation of the medium parameters in a wide region. As already mentioned, to enter the $\Delta E \propto L^2$ regime one has to satisfy first the inequality $\tau_c < R_A$, where $R_A$ is the transverse radius of the deconfined region, and second $\tau_c < L_{\text{crit}}$, since the average propagated distance to escape the deconfined region is $\langle L \rangle \approx \tau_c$. Both these inequalities reflect the not too long life-time of the deconfined stage and a sufficiently high energy of the jet quark as realistic for RHIC conditions.

To check the robustness of our results we also calculate the energy loss by generating events with a probability $\propto (1 - (r/R_A)^2)$, where $r$ is the radial coordinate of the initial position of the hard quark. As seen in Fig. 3 the increase of the scaled QCD energy loss becomes stronger in this case while the linear energy loss model is still almost independent of $N_i$.

III. ANGULAR DEPENDENCE

Since the bremsstrahlung gluons are mainly radiated in nearly the same direction as the parent parton propagates and the energy loss is strongly dominated by the gluon radiation, the experimental measurement of the energy loss of the jet quark in QCD faces the problem of selecting the proper angular size of the jet cone [28,29]. This problem is studied in detail for LHC conditions in [4]. In our consideration we use the results of [30] where the complete calculations of the angular dependence of the energy loss are presented (cf. also [12] for a recent discussion of this topic).

The effect of the finite size of the jet cone for the parent parton’s energy loss can be characterized by a "suppression" factor $R(\theta) = \Delta E(\theta)/\Delta E$, where $\Delta E(\theta)$ is the energy radiated away outside of an angular cone with opening angle $\theta$, and $\Delta E$ is the total radiated energy. As shown in [30], the factor $R$ actually depends on the combination $c(L)\theta$ with $c(L) = \sqrt{2C_f \hat{q}(L)}^3$ and $C_f = N_c^2 - 1/2N_c$. While the explicit equations for $R(\theta)$ are somewhat involved, the simple parameterization $R(\theta) \approx \exp \left(-a\sqrt{c(L)\theta}\right)$ with $a = 0.432$ results as appropriate fit displaying also the $L$ dependence. As pointed out in the previous section, the averaged $L$ dependence can be translated into a dependence on the initial thermodynamic state parameters of the plasma. To estimate the initial temperature dependence of the factor $R$, which actually becomes $R(\theta,T_i)$, we approximate here the propagation distance within the Bjorken scenario by $L \approx \tau_i(T_i/T)^3$. Taking the initial time $\tau_i = 0.2$ fm/c and using for the parameters related to transport properties of the deconfined medium the same values as described in the previous section we get

$$R(\theta,T_i) \approx \exp \left[-5.306\sqrt{\theta} \left(\frac{T_i}{\text{GeV}}\right)^{9/4}\right].$$

(3)

The corresponding temperature dependence for various angles is displayed in fig. 4. The value of the characteristic angle where $R \approx 1$, and consequently the parton energy loss
can be associated directly with the jet energy, decreases for increasing initial temperature of the medium. At the same time, for the initial temperatures expected at RHIC, i.e., $T_i = 300 \cdots 550$ MeV, the characteristic angle is about $0.01 \cdots 0.1$ rad which looks acceptable for an experimental feasibility. That means one must measure the jet energy (or transverse momentum at mid-rapidity) in a sufficiently small cone around the jet axis to filter out the radiated energy.

The results of our calculations of the energy loss as a function of the initial plasma temperature for various values of the angular cone are displayed in fig. 5. In accordance with the above analysis for the temperature range expected for RHIC one can notice only a tiny modification of the total radiative energy loss if the jet cone is restricted to $\theta \sim 0.01 \cdots 0.1$ rad. Basing on results of [4] one can also conclude that within such a jet cone the radiative energy loss is much greater than the collisional one and can be related directly to the final jet energy.

**IV. CONCLUSIONS AND DISCUSSION**

In summary we have considered the dependence of the QCD radiative energy loss of a fast parton on the initial thermodynamic state of deconfined matter for conditions expected to be achieved in central heavy-ion collisions at RHIC. One of the appropriate ways to pin down the initial state is to vary the beam energies in central collisions which in turn should change the life-time of the deconfined matter. If the life-time of the deconfined matter is less than the time a fast parton needs to pass the typical geometrical size of matter, the jet energy loss depends strongly on this life-time due to the specific QCD governed $L^2$ dependence. We show that such a QCD behavior causes an increase of the jet energy loss with increasing entropy density of the system which is encoded in the combination $\tau_i T_i^3$. For the widely used value of $\tau_i = 0.2$ fm/c the proper range of initial temperatures is $300 \cdots 550$ MeV in agreement with usual predictions for RHIC. For a sufficiently high initial temperature of deconfined matter one can also expect a remarkable dependence of the parton energy loss on the initial parton energy which can be tagged by a hard photon: Two different regimes, related to two different QCD radiation patterns should become visible.

Our conclusions on the final jet energy loss are obviously valid if the QCD energy loss of the initial parton is much greater than the corresponding loss in the hadronic medium and if the collisional energy loss is negligible. In practice one needs to select a small enough angular cone around the jet axis to measure only the energy of the fast parent parton. For initial temperatures achieved at RHIC the opening angle should be not larger than 0.1 rad.

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FIG. 1. The measure of the energy loss $p_T^0 - \langle p_T \rangle$ as a function of the initial parton momentum $p_T^0$ for the QCD energy loss according to Eqs. (1, 2) (solid curves) and for the linear energy loss model $\Delta E = \eta L$ with $\eta = -1\text{GeV/fm}$ (dotted lines). The black (gray) curves are for $T_i = 550$ (450) MeV.

FIG. 2. The scaled energy loss as a function of $N_i = \tau_i T_i^3$ for various initial times $\tau_i$. Black (gray) curves are for the QCD energy loss according to Eqs. (1, 2) (linear energy loss model $\Delta E = \eta L$ with $\eta = -1\text{GeV/fm}$).
FIG. 3. The same as Fig. 2 but for another radial distribution of initial positions as described in text.

FIG. 4. The suppression factor $R(T_i, \theta)$ as a function of $T_i$ for various values of the cone angle $\theta$ (in units of rad).
FIG. 5. The energy loss as a function of $T_i$ for various opening angles of the jet cone (in units of rad).