High Linearity Voltage Response Parallel-Array Cell

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Abstract. We studied in detail a cell consisting of two parallel SQUID arrays or two parallel superconducting interference filters (SQIFs) connected differentially with the goal of achieving highly linear voltage response to magnetic signal. In these different cell designs, we accounted for realistic values of coupling inductances in contrast to limiting case of vanishing inductances considered earlier. We found that a cell based on regular parallel SQUID arrays produces higher linearity as compared to the cell based on SQIFs. This high-linearity cell can be used for realizing Superconducting Quantum Arrays (SQA) capable of providing a broadband, highly-linear magnetic field-to-voltage transfer function and high dynamic range.

Introduction

A parallel array of Josephson junctions (a parallel SQUID array) is one of the frequently used elements of superconductive electronics [1-6]. Among many applications, the parallel array was suggested to design quantum cells for Superconducting Quantum Arrays (SQA) capable of providing both highly linear magnetic signal to voltage transformation and high dynamic range [7-9]. Such an implementation exploits the close-to-parabolic form of the main peak sides of the parallel array voltage response. As long as subtraction of two identical parabolic functions with the mutually shifted vertex positions results in linear dependence, differential connection of two parallel arrays oppositely biased by some magnetic flux \(\delta\Phi\) results in the cell providing a linear voltage response (quantum cell) [9]. Figure 1 shows schematically the differential cell and a realistic schematic of the parallel array.

An accurate analysis of the parallel array is a complex numerical problem. Therefore in many cases, one has to consider the array in the limit of zero coupling inductances between Josephson junctions in order to use an analytic formula to calculate a voltage response to magnetic flux. However, the realizable in practice arrays have inevitably non-zero coupling inductances which lead to characteristics quite different from the ones obtained for the idealized array with zero coupling inductances.

This paper presents an analysis of achievable linearity of the differential cell voltage response. The

\[ I = I_b/N \]

where \(I_b\) is a total bias current of the array, and \(N\) is total number of Josephson junctions.

Figure 1. Schematics of a practicable parallel array of Josephson junctions (a) and a differential cell (b). Here \(\Phi\) is an input signal, \(\delta\Phi\) is a frustration magnetic flux, \(I = I_b/N\) is a Josephson-junction bias current, where \(I_b\) is a total bias current of the array, and \(N\) is total number of Josephson junctions.
study is based on the detailed numerical simulation of the parallel array with realistic parameters and an accurate analysis of the array characteristics.

**Approximation Approach**

To analyze a linearity of the differential cell response, the terms of the next order of infinitesimality in the parallel array voltage response approximation should be taken into account. The terms can be expressed as a sum of the 4-th and 6-th order parabolas, and therefore the shape of the array peak sides can be described by the following relation:

\[ V_{R,I}(\Phi) = C_0 - k(\Phi - \Phi^*)^2 + a_4(\Phi - \Phi_4)^4 + a_6(\Phi - \Phi_6)^6, \]  

where \( C_0, k, a_4, a_6 \) are constants, \( \Phi^*, \Phi_4, \Phi_6 \) - the vertex coordinates, which are positive for the right side relation \( V_R(\Phi) \) and negative for the left side relation \( V_L(\Phi) \). This allows us to write the differential voltage response as follows:

\[ V(\Phi) \equiv V_R(\Phi + \delta\Phi) - V_L(\Phi - \delta\Phi) = \\
[4k(\Phi^* - \delta\Phi) - 2a_4(\Phi_4 - \delta\Phi)^4 - 3a_6(\Phi_6 - \delta\Phi)^6] \Phi - 8[a_4(\Phi_4 - \delta\Phi) + 5a_6(\Phi_6 - \delta\Phi)^4]\Phi^4 - \\
-12a_6(\Phi_6 - \delta\Phi)\Phi^6. \]  

At \( \delta\Phi = \Phi_4 \), the 4-th order parabola does not generate nonlinear terms in (4). Similarly, at \( \delta\Phi = \Phi_6 \), the 6-th order parabola does not generate any nonlinear terms. If one applies sinusoidal signal \( \Phi = A \sin(\omega t) \) to the differential cell, the output signal \( V(\Phi) \) consists of the fundamental tone with amplitude \( B_1 \approx 4k(\Phi - \Phi^*)A \) and two harmonics with the following amplitudes:

\[ B_3 = [10a_6(\Phi_6 - \delta\Phi)^4 + 2a_4(\Phi_4 - \delta\Phi)]A^4 + (15/4)a_6(\Phi_6 - \delta\Phi)A^5, \]  

\[ B_5 = -(3/4)a_6(\Phi_6 - \delta\Phi)A^5. \]  

In accordance with the one-tone analysis technique, the linearity of the magnetic flux to voltage transformation can be expressed by formula

\[ Lin = B_1/\max\{B_3, B_5\}. \]  

To realize the highest linearity, one has to apply and hold the needed magnetic frustration \( \delta\Phi \) with the utmost precision.

**Array Response Fitting**

As it was found earlier [10,11], at high enough number \( N \) of Josephson junctions (\( N > 10 \)), voltage response of the idealized parallel array with critical current biasing \( I_b = I \), approaches the ultimate shape of its main peak and the side peak shape becomes close to parabolic starting right from the extreme point at \( \Phi = 0 \). Moreover, the range of the magnetic frustration \( \delta\Phi \) needed to realize the high linearity of the response can be increased by implementation of a parallel SQIF to provide an inhomogeneous flux distribution along the array with minimum in the array center [10]:

\[ \Phi_m \propto [1 - \chi \sin^2(\pi(m-1)/(N-1))]. \]  

As the factor \( \chi \) increases up to 1, the range of \( \delta\Phi \) becomes 1.5 to 2 times larger.

It seems to be impractical to realize a parallel array of Josephson junctions with normalized coupling inductances less \( l = 2\pi I/L\Phi_0 \) than about 0.3, where \( I \) is Josephson-junction critical current (~ 100 \( \mu \)A for niobium process with critical current density 4.5 kA/cm\(^2\)). Our study shows that at low (but realizable) normalized coupling inductances \( l \sim 0.3 \ldots 0.7 \), the use of the aforementioned SQIF structure instead of regular array does not give any pronounced change in the array voltage response and therefore gives no improvement in linearity of the differential cell response. The insensitivity to the flux distribution along the array results from the weakening of the collective interaction between the junctions at Josephson oscillation frequency. Further increase in coupling inductances leads to a gradual degradation of the response shape due to a progressive reduction in the number of junctions which can interact with each other and, therefore, contribute to the array response.

Voltage response of the realistic parallel array has significantly wider main peak than the one at zero coupling inductances and its side has two domains which can be well approximated by parabolas (1) with different parameters. Figures 2(a) and 2(b) show two parabolic approximations of the main peak side of the voltage response of the array of 10 Josephson junctions coupled by inductances of value \( l = 0.5 \). Some increase in bias current \( I_b \), above \( I \) (up to ~ 1.06 \( I \)) allows widening this domain to \( \Phi = 0 \). The main fitting parabola has vertex position \( \Phi^* \approx 2.47\Phi_0 \) and factor \( k \approx 0.132 V_J/(\Phi_0)^2 \).
Positions of the 4-th and 6-th parabola vertexes are localized within a range from 0.79 $\Phi_0$ to $\Phi_0$. The vertex positions are very sensitive to the current biasing and at $I_b = 1.06 I_c$ they well approach each other.

For the second domain, the fit parameters are practically insensitive to a change in the bias current. The main fitting parabola has the practically fixed factor $k \approx 0.15 V_c/(\Phi_0)^2$ and vertex position $\Phi^* \approx 2.2 \Phi_0$. Vertex positions for the 4-th and 6-th parabolas are about fixed and distinct: $\Phi_4 \approx 1.36 \Phi_0$ and $\Phi_6 \approx 1.57 \Phi_0$.

Response Linearity

Figure 3 shows the differential cell response linearity vs magnetic frustration $\delta \Phi$ at bias current $I_b = 1.06 I_c$ and different amplitudes of the applied magnetic signal. The data reveal three domains of parabolic approximation on the voltage responses of the parallel arrays forming the differential cell. The additional domain with center at $\Phi_{add} \approx 0.4$ is superimposed on some part of the first domain shown in figure 2(a). When magnetic frustration $\delta \Phi$ coincides with the centres of the additional domain (the first one and the second one), the response linearity reaches the maxima.

Similarly to the second domain discussed above, the additional domain demonstrates the lack of convergence of the high-order parabola vertex positions and therefore both the side maxima in figure 3 show a shift with the input signal amplitude in accordance with the formulae (3), (4). In contrast to these peaks, the middle maximum does not shift with the signal amplitude due to the fact that the 4-th and 6-th parabola vertexes approach each other at $I_b = 1.06 I_c$. Thus regardless of the signal amplitude, the highest linearity is achievable at the same frustration $\delta \Phi \approx 0.71 \Phi_0$ and can be as high as ~95 dB and ~80 dB for the signals over 30% and 60% of the total response swing, correspondingly. Moreover, the requirement for an accuracy of the magnetic frustration $\delta \Phi$ seems not too stringent.

Load Impact Balancing

If a load $R_c$ is connected to the differential cell, some part of the bias currents $I_b$ applied to the cell shoulders (parallel arrays) flow through the load and this causes essential changes in the cell response. The effective currents $I_R, I_L$ flowing through the cell shoulders depend on output voltage $V = V_R - V_L$ and change in antiphase as follows:

$$I_R = I_b - V/R_c, \quad I_L = I_b + V/R_c. \quad (7)$$

Maximum distortion of the response takes place when signal $\Phi$ comes close to $-\delta \Phi$ or $+\delta \Phi$. In fact, at $\Phi \approx -\delta \Phi$ (output voltage $V < 0$) total magnetic fluxes $\Phi_R$ and $\Phi_L$ applied to the right and left shoulders approach correspondingly 0 (peak point of the shoulder response) and $2 \delta \Phi$ (far from peak point), and vice versa at $\Phi \approx +\delta \Phi$. Reduction in the currents $I_R$ and $I_L$ occurs at total fluxes $\Phi_R$ and $\Phi_L$ close to $2 \delta \Phi$ where the shoulder responses are not sensitive to the current reduction. At the same time, the increase in these currents takes place when the total fluxes $\Phi_R$ and $\Phi_L$ approach 0 (peak point) and the
shoulder responses are extremely sensitive to the current change. This fact points at the possible solution of the problem through the proper decrease in the bias currents. Our numerical simulations show that the increase in the current biasing of the cell shoulders allows us to balance the load impact within the load impedance $R_e$ down to about $10R_N$, where $R_N$ is the shoulder normal resistance. This means that output current can reach one tenth of the bias current.

Conclusion
Our detailed numerical analysis of the realistic differential cell based on two parallel arrays with non-zero coupling inductances confirms its capability of providing highly linear voltage response at low, but practical normalized coupling inductances between Josephson junctions $l \sim 0.3 \ldots 0.7$. The high linearity is achievable within significantly wider range of magnetic frustrations than the one following from analysis of the idealized cell with zero inductances. Moreover, the cell characteristics including response linearity show good tolerance to output load. The results have been used in designing of SQA reported in [8,9].

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