Bulk Reconstruction Beyond the Entanglement Wedge

Ning Bao, 1, 2, * Aidan Chatwin-Davies, 3, † Benjamin E. Niehoff, 3, ‡ and Mykhaylo Usatyuk 1, §

1 Center for Theoretical Physics and Department of Physics
University of California, Berkeley, CA 94720, USA
2 Computational Science Initiative
Brookhaven National Lab, Upton, NY 11973, USA
3 KU Leuven, Institute for Theoretical Physics
Celestijnenlaan 200D B-3001 Leuven, Belgium

We study the portion of an asymptotically Anti de Sitter geometry’s bulk where the metric can be reconstructed, given the areas of minimal 2-surfaces anchored to a fixed boundary subregion. We exhibit situations in which this region can reach parametrically far outside of the entanglement wedge. In a holographic setting, in which the bulk geometry is dual to a state in a conformal field theory (CFT), these minimal 2-surface areas can be deduced from the expectation values of Wilson loops in the boundary subregion. This suggests that the reduced CFT state encodes significant information about the bulk beyond the entanglement wedge, challenging conventional intuition about holographic subregion duality.

I. INTRODUCTION

Over the last decade, the introduction of quantum information theory into the Anti de Sitter/Conformal Field Theory (AdS/CFT) correspondence [1, 2] has had a profound impact on the field. The most basic result of this program is arguably the Ryu-Takayanagi formula [3], which says that the Von Neumann entropy of a reduced state on a subregion of the boundary CFT is proportional to the area of the smallest bulk extremal surface that is homologous to the subregion. More recently, investigations of the bulk region in between this extremal surface and the boundary subregion, called the entanglement wedge, have led to deep results on holographic quantum error correction [4, 5], the ability to reconstruct bulk perturbations of the vacuum within the entanglement wedge from boundary data [6, 7], and the equivalence of bulk and boundary relative entropy for perturbatively close states [8]. Altogether, this paints a picture of “subregion duality,” according to which information about the bulk gravitational state, reduced on the entanglement wedge, is encoded in the subtended boundary region’s reduced CFT state, and vice-versa. Earlier work offers hints, however, that the full story may be more subtle [9, 10].

To pursue this subtlety, we go back even earlier. It has been argued since the nascency of AdS/CFT that the expectation value of Wilson lines in the boundary CFT are given by the areas of 2-dimensional extremal surfaces anchored to the boundary [11–13], and this result has come to be well-established as a central and fundamental ingredient in the holographic dictionary.

In this letter, we will show that there exist situations in which these 2-dimensional extremal surfaces probe arbitrarily far outside of the entanglement wedge. Coupled with a recent result that shows that knowing the areas of these surfaces is sufficient to reconstruct the space-time metric in the region that they probe [14], we conclude that a boundary subregion reveals information about much more of the bulk than just the entanglement wedge.

II. EXTENT OF BULK MINIMAL SURFACES

Consider an asymptotically Anti de Sitter (aAdS) spacetime, $\mathcal{M}$, with $d + 1$ space-time dimensions, which we take to be static so that we may restrict to a spatial slice. Let $B_k$ be some simply-connected, $k$-dimensional submanifold in the boundary of the slice, where $1 \leq k \leq d - 1$, such that the boundary of $B_k$ is non-empty: $\partial B_k \neq \emptyset$. (One can take, e.g., $B_k$ to be a $k$-ball whose boundary is a topological $k$-sphere.) Given this submanifold, consider the set of $k$-dimensional surfaces that share a boundary with $B_k$, that can penetrate into the spatial bulk, that are homologous to $B_k$, and whose areas are stationary with respect to deformations. Let $m(B_k)$ denote the surface whose area is the smallest, or one such surface if there are many with the same minimum area. For concreteness, if $k = 1$, then $m(B_1)$ is a boundary-anchored geodesic; or, if $k = d - 1$ and the space-time has a holographic CFT dual, then $m(B_{d-1})$ is the Ryu-Takayanagi surface for the boundary subregion $A = B_{d-1}$ [3].

The question that we begin with is the following: how far into the bulk does $m(B_k)$ reach? The reach, or extent, of a boundary-anchored minimal surface was studied by Hubeny in aAdS space-times having planar symmetry in Ref. [15]. Concretely, working in the Poincaré patch, these are metrics of the form

$$\text{ds}^2 = {1 \over \tilde{z}^2} \left( - f(\tilde{z}) \text{d}t^2 + \text{d}x_i \text{d}x^i + h(\tilde{z}) \text{d}\tilde{z}^2 \right),$$

where $i = 1, \ldots, d - 1$, the AdS boundary is at $\tilde{z} = 0$, and $f(\tilde{z}) > 0$ is a smooth function 

* ningbao75@gmail.com
† aidan.chatwindavies@kuleuven.be
‡ ben.niehoff@kuleuven.be
§ musatyuk@berkeley.edu
and \(f(\tilde{z}), h(\tilde{z}) \to 1\) as \(\tilde{z} \to 0\). Here, we follow Hubeny and use coordinates that are better-adapted to finding the bulk reach of minimal surfaces instead of the usual Fefferman-Graham coordinates \([16]\), the relation to which is discussed in Ref. \([15]\). The coordinate \(\tilde{z}\) labels the hyperbolic bulk direction, and so its largest value on \(m(B_k)\), which we denote by \(\tilde{z}_*\), characterizes the bulk depth to which \(m(B_k)\) reaches.

We will be interested specifically in two cases: either an infinite strip with \(k = d - 1\), or a round \(k\)-ball for any \(1 \leq k \leq d - 1\). The infinite strip is the region bounded by two (spatial) co-dimension-1 planes; from the coordinates \(x^i\), label one coordinate by \(x\) and the remaining \(d-2\) coordinates by \(y^i\), where \(i = 2, \ldots, d-1\). Then, with a proper choice for the origin and orientation of the coordinate system, an infinite strip of width \(L\) is the boundary region described by the ranges \(x \in [-L/2, L/2]\) and \(y^i \in \mathbb{R}\). Similarly, a \(k\)-ball is given by \(\sum_{i=1}^k (x^i)^2 \leq R\) for some radius \(R\), with the remaining \(x^i = 0\) for \(k+1 \leq i \leq d-1\).

In the case of pure \(\text{AdS}_{d+1}\), for which \(f(\tilde{z}) = h(\tilde{z}) = 1\), the deepest reach of the minimal surface anchored to an infinite strip is given by \([15]\)

\[
z_{\text{strip}} = L (d - 1) \frac{\Gamma(\frac{2d-1}{2d-2})}{\sqrt{\pi} \Gamma(\frac{d}{2d-2})}.
\] (2)

For a \(k\)-ball, the minimal surface is a spherical cap for all values of \(k\) whose reach is given by \([15]\)

\[
z_{\text{disk}} = R.
\] (3)

Away from pure \(\text{AdS}\), one can expect more general behavior; however, what is important are these scalings of \(\tilde{z}_*\) with respect to \(L\) and \(R\) which hold whenever the bulk metric is sufficiently close to pure \(\text{AdS}\).

Next, we consider the following question: given a \((d-1)\)-dimensional boundary subregion \(A\), what part of the bulk can we reach with \(k\)-dimensional minimal surfaces that are anchored to submanifolds \(B_k\) in the interior of \(A\), where \(k \leq d - 1\)? Let us call such bulk regions \(k\)-wedges:

**Definition II.1** Let \(A\) be a simply-connected, \((d-1)\)-dimensional subregion in the boundary of a slice of a static \(\text{AdS}_{d+1}\) space-time. The \(k\)-wedge of \(A\), denoted \(W_k(A)\), is the set of all points that lie on a \(k\)-dimensional minimal surface \(m(B_k)\) for at least one simply-connected submanifold \(B_k \subseteq A\), where \(1 \leq k \leq d - 1\).

Again for illustration, in the case of a holographic space-time, \(W_{d-1}(A)\) coincides with the entanglement wedge of \(A\) when the latter contains no entanglement shadows \([9, 17]\).

Different \(k\)-wedges for the same fixed subregion \(A\) differ from each other. This is not very surprising, and indeed, it is a straightforward consequence of Hubeny’s investigations. It is perhaps a bit more surprising, however, that in certain situations, different \(k\)-wedges can differ by an arbitrarily large bulk region.

We now construct one such situation, which we illustrate in Fig. 1. An important tool in our construction is the notion of the *bounding width* of a subregion, given as follows:

**Definition II.2** Let \(A\) be a simply-connected, \((d-1)\)-dimensional subregion in the boundary of a slice of a static \(\text{AdS}_{d+1}\) space-time. The bounding width of \(A\), denoted \(L(A)\), is the width \(L\) of the smallest \((d-1)\)-dimensional infinite strip which contains \(A\).

For example, if \(d = 3\) and \(A\) is the interior of an ellipse with semimajor axis \(a\) and semiminor axis \(b\), then the bounding width of \(A\) is \(L(A) = 2b\), the smaller of the two diameters.

Once again suppose that \(M\) is simply pure \(\text{AdS}_{d+1}\), and suppose that \(d \geq 3\). Let \(A\) be a simply-connected, \((d-1)\)-dimensional boundary subregion such that \(\partial A \neq \emptyset\), and suppose that it has a bounding width \(L(A) < \infty\).

Because a strip of width \(L(A)\) contains \(A\), it follows that the \((d-1)\)-wedge of the strip contains the \((d-1)\)-wedge of \(A\). This follows from entanglement wedge nesting \([18]\), since in this case, the \((d-1)\)-wedges of both \(A\) and the strip coincide with these subregions’ entanglement wedges. It therefore follows that the deepest reach of \(m(A)\) is bounded by Eq. (2).

Now consider inscribing a round \(k\)-dimensional ball inside \(A\), with \(k \leq d - 2\). (A 1-dimensional ball is just a line segment.) In particular, if \(A\) is sufficiently larger than it is wide in at least \(k\) directions, then we can inscribe a \(k\)-ball of radius \(R > z_{\text{strip}}\). Then according to Eq. (3), the bulk reach of such a ball will exceed the bulk reach of the strip, and hence also that of \(m(A)\).

However, nothing prevents us from choosing a boundary subregion that is arbitrarily longer than it is wide. It will remain possible to inscribe such a region in an infinite strip of width \(L\), and so the extent of its \((d-1)\)-wedge will remain bounded by \(z_{\text{strip}}\). At the same time, based on being able to inscribe \(k\)-balls of arbitrarily large radius transversely to \(A\)’s bounding width, we can arrange for the subregion’s \(k\)-wedges to probe arbitrarily deeply into the bulk.

To summarize, if a \((d-1)\)-dimensional boundary subregion \(A\) in \(\text{AdS}_{d+1}\) can be inscribed with lower-dimensional balls whose radii are sufficiently larger than the bounding width of \(A\), then the \(k\)-wedges of \(A\) will generically reach deeper into the bulk than \(W_{d-k}(A)\) for \(1 \leq k \leq d - 2\). Furthermore, we can immediately construct situations to which this observation applies in general static \(\text{AdS}\) space-times as well. Since \(A\) can be chosen such that its bounding width is arbitrarily small, the minimal surface of the infinite strip which bounds \(m(A)\) can be made arbitrarily close to the pure \(\text{AdS}\) case, due to the asymptotic \(\text{AdS}\) boundary conditions. Therefore, the bulk depth to which \(W_{d-k}(A)\) reaches remains bounded by \(z_{\text{strip}}\) (plus subleading corrections that are calculated in Ref. \([15]\)), while \(W_k(A)\) can be arranged to reach as deep into the bulk as one wishes, being obstructed only by topological barriers such as horizons.
Reduced CFT state $\rho$, meaning, since data about certain minimalARY Wilson loops contained in surfaces whose anchors lie in and large-moment wedge for $k$ is not possible to reconstruct the full bulk metric in $W_2(A)$ for a large class of boundary subregions. This is essentially the content of the BCFK theorem [14]: Let $d \geq 3$, and let $A$ be a $(d-1)$-dimensional boundary subregion that is topologically a ball. Suppose that every point in $W_2(A)$ lies on a minimal 2-surface $m(B)$, which is itself part of a smooth family of minimal surfaces which begins at a single point in $A$. In other words, for each $p \in W_2(A)$, suppose that there exists a smooth family of minimal 2-surfaces $m(S(\lambda)), \lambda \in [0,1]$, such that $S(\lambda) \subset A$, $S(1) = B$, $p \in m(B)$, and $S(0)$ is a single point in the closure, $A$ [20]. Then, knowledge of the areas (or, in fact, just the area variations) of all minimal 2-surfaces whose anchors lie in $A$ guarantees a unique reconstruction of the bulk metric in $W_2(A)$. Therefore, knowing the expectation values of Wilson loops localized within $A$ ultimately makes it possible to reconstruct the bulk metric in the region $W_2(A)$, provided that $W_2(A)$ satisfies the foliation condition in the BCFK theorem. When $d = 3$, the bulk region foliated by 2-surfaces will in general be equal to or contained by the entanglement wedge. When $d \geq 4$, however, according to our earlier geometric observation, we can generically find subregions $A$ such that $W_2(A)$ is both reconstructible and extends parametrically far outside of the entanglement wedge.

### III. BULK MINIMAL SURFACES AND HOLOGRAPHIC RECONSTRUCTION

Now let us suppose that the $\mathcal{M}$ is holographic, meaning that it is dual to a state in some $d$-dimensional CFT, which we can think of as living on $\partial \mathcal{M}$, in the large-$N$ and large-$\lambda$ limit. Now we can ask: what are the holographic consequences of the geometric observation that $k$-wedges can probe arbitrarily deeper than the entanglement wedge for $k \leq d - 2$?

First, we note that the observation has an operational meaning, since data about certain minimal $k$-dimensional surfaces whose anchors lie in $A$ is accessible from the reduced CFT state $\rho_A$. As an example, consider boundary Wilson loops contained in $A$. Since $A$ is simply-connected, a Wilson loop can be specified as the boundary $\partial B$ of some disk $B \subseteq A$, and its expectation value can be computed by a bulk path integral of the form [11–13]

$$\langle W(\partial B) \rangle = \int_{\sigma \sim B} D\sigma \; e^{-\sqrt{\lambda}S[\sigma]}.$$  

The integral is over all 2-surfaces $\sigma$ that end on the Wilson loop. In the limit where $\lambda = g^2N$ is large, the above integral is approximated to leading order by its saddle point to obtain

$$\langle W(\partial B) \rangle \approx e^{-\sqrt{\lambda}A[m(B)]},$$

where $A[m(B)]$ is the area of the minimal 2-surface anchored to $B$, suitably regularized. Therefore the areas of minimal 2-surfaces anchored within $A$ can be deduced from the reduced density matrix $\rho_A$ by extracting the expectation values of Wilson loops. Similar expressions hold which relate boundary two-point functions to the length of boundary-anchored geodesics [19]. In principle there may also be similar relations between higher-dimensional surface operators in the boundary and the areas of higher-dimensional boundary-anchored minimal surfaces; however, we are not aware of results that suit our purposes.

We focused on boundary Wilson loops in particular because, if one knows the areas of all minimal 2-surfaces whose anchors lie on closed curves within $A$, then it is possible to reconstruct the full bulk metric in $W_2(A)$ for a large class of boundary subregions. This is essentially the content of the BCFK theorem [14]: Let $d \geq 3$, and let $A$ be a $(d-1)$-dimensional boundary subregion that is topologically a ball. Suppose that every point in $W_2(A)$ lies on a minimal 2-surface $m(B)$, which is itself part of a smooth family of minimal surfaces which begins at a single point in $A$. In other words, for each $p \in W_2(A)$, suppose that there exists a smooth family of minimal 2-surfaces $m(S(\lambda)), \lambda \in [0,1]$, such that $S(\lambda) \subset A$, $S(1) = B$, $p \in m(B)$, and $S(0)$ is a single point in the closure, $A$ [20]. Then, knowledge of the areas (or, in fact, just the area variations) of all minimal 2-surfaces whose anchors lie in $A$ guarantees a unique reconstruction of the bulk metric in $W_2(A)$.

The finding above encourages us to revisit the notion of subregion duality in holography. According to the conventional lore, a boundary subregion is dual to its entanglement wedge, and vice-versa. In contrast, our analysis suggests that the reduced state on a boundary subregion of a holographic CFT has information about much more than just its entanglement wedge. Moreover, since minimal 2-surfaces that are anchored to a boundary subregion can be made to reach arbitrarily far outside of the entanglement wedge, the effect cannot be attributed to a subleading quantum correction. It is leading order geometric information.

Nevertheless, duality between a boundary subregion and its entanglement wedge remains an accurate notion when transitioning from lore to more precise accounts of subregion duality, such as bulk reconstruction as formulated by Dong, Harlow, and Wall [6]. According to this
formulation, a boundary subregion and its entanglement wedge are dual in the sense that a sufficiently small algebra of bulk operators in an entanglement wedge, whose geometry is fixed to leading order in $N$ (the “code subspace”), can be represented by a corresponding set of CFT operators in the boundary subregion. There is thus no conflict between this precise notion of subregion duality and our observations about minimal 2-surfaces.

There is an apparent tension with the complementary error correction aspect of the story—that operators in the boundary subregion $A$ should commute with any operator derived solely from the interior of the complement of $A$’s entanglement wedge. It appears that this is explicitly untrue for Wilson loop operators, unless their proposed dual object in the holographic dictionary is not a minimal 2-surface. Here, the resolution lies in the fine print: the condition for complementary error recovery is a condition that is true for states within a fixed code subspace. In particular, if one were to consider two dramatically different completions of the entanglement wedge to a complete aAdS slice, as would need to be the case to have parametric differences in the reconstructed metrics for two states, then these states for the full boundary theory would not be in the same code subspace, and thus the complementary error correction theorem would not apply. In other words, Wilson loop operators have the same expectation value for all states in a code subspace to leading order in $N$.

One question that our result immediately motivates is how to think of boundary reconstruction. Given access to only the entanglement wedge, what information is learned about the reduced CFT state? Alternatively, what portion of the bulk is necessary to reconstruct the full reduced density matrix for a boundary subregion? There have been hints in earlier works, such as Refs. [9, 21], that one can affect the reduced density matrix of the boundary subregion by acting outside of the entanglement wedge, and our result only reinforces these hints. It would therefore be an interesting challenge to construct a systematic method of reconstructing the boundary reduced density matrix, in a sort of inverse problem to Ref. [6].

In this letter, we restricted ourselves to two dimensional surfaces to exploit the BCFK metric reconstruction results [14]. Metric reconstruction may also be possible knowing the lengths of all geodesics connecting pairs of boundary points. In the mathematics community, this problem is known as boundary rigidity [22–24]. While no such reconstruction theorem is currently known for aAdS space-times, reconstruction using geodesic data deduced from boundary correlators would be an obvious extension of our work [25].

Similarly, if the BCFK theorem could be extended to disjoint boundary subregions, then 2-wedge reconstruction could potentially be thought of as a way of smoothing out the discontinuous jump that the entanglement wedge experiences across entanglement entropy phase transitions. While the number of connected components in the entanglement wedge changes across such a transition, the number of connected components in the 2-wedge remains the same.

Another potential direction for further work is to study the problem in a time-dependent setting. Some of the pieces are already in place, as the BCFK theorem holds covariantly. However, it would still be necessary to extend our construction in Sec. II. This may follow from careful application of the maximin proposal [26], but the precise details remain to be seen.

V. CONCLUSION

In three or more bulk spatial dimensions, we observed that the bulk region swept out by (spatial) co-dimension 1 minimal surfaces anchored to a given boundary subregion can be significantly shallower than the regions swept out by the higher co-dimension minimal surfaces. In four or more dimensions, holographically, this translates into knowledge about the bulk metric far outside of the entanglement wedge, given access only to the reduced CFT state on the wedge’s boundary subregion. We therefore obtain new insights into holographic subregion duality, and, more broadly, are invited to wonder what further intuition changes when working in higher dimensions.

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[20] In general, one must consider extremal 2-surfaces, but here we work with minimal 2-surfaces since we have restricted to a static spatial slice. The condition that every point in $W_2(A)$ be reachable by a smooth family of minimal surfaces which begins at a boundary point is necessary for the theorem. In particular, it rules out, e.g., 2-wedges that contain a compact horizon, where the minimal surfaces in any family which reaches a point on the other side of the horizon with respect to the boundary subregion will experience a discontinuous jump when the minimal surface wraps the horizon.

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