BOUNDS ON BESS MODEL PARAMETERS
FROM VECTOR-BOSON PRODUCTION
IN $e^+e^-$ COLLISIONS

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Abstract

The BESS model is the Higgs-less alternative to the standard model of electroweak interaction, based on non-linearly realized spontaneous symmetry breaking. Since it is non-renormalizable, new couplings (not existing in the SM) are induced at each loop order. On the basis of the one loop induced vector-boson self-couplings we study the two- and three-vector-boson-production processes in $e^+e^-$ collisions at $\sqrt{s} = 500\text{GeV}$, the expected energy of the next $e^+e^-$ linear collider (NLC). Assuming that NLC results will agree with the SM predictions within given accuracy we identify the bounds for the free parameters of the BESS model.
1 Introduction

One of the most important topics of future particle physics will be the investigation of the self-interactions of the electroweak gauge bosons $W^\pm$, $Z$, and $\gamma$. Since the fermionic part of the SM is well verified by experiments performed up to now, most alternative models are characterized by deviations of the gauge-boson self-couplings from the Yang–Mills type.

Presently we have only indirect empirical knowledge of these self-interactions due to loop effects, and $e^+e^-$-colliders with energies beyond the $W^+W^-$ threshold will be necessary for direct tests as well. The most powerful tests will come from measurements of the cross sections for two- and three-vector-boson-production processes in electron-positron annihilation. The first machine that is able to produce the process $e^+e^- \rightarrow W^+W^-$ will be LEP II, where the available energy of $\sqrt{s} \approx 190$ GeV is only slightly above the $W^+W^-$ threshold. However, LEP II will not be very sensitive to deviations of the self-couplings from the SM values, since these cause a violation of the gauge cancellations and lead to differences from the SM increasing with energy. The proposed $e^+e^-$-collider working at an energy of $\sqrt{s} \approx 500$ GeV and having a high luminosity of 20 fb (NLC) $^1$, however, will establish good possibilities to discriminate alternative models, like the BESS model, from the SM. Furthermore it will allow the measurement of three-gauge-boson-production processes, which are able to test the quartic self-couplings.

One principal way of testing the vector-boson self-interactions is to start from a Lagrangian containing the most general form of these self-couplings which can be constructed in agreement with Lorentz invariance. But due to the complicated structure of the general Lagrangian and the large number of free parameters this is a complicated procedure involving elaborate multi-parameter fits $^2$, especially if one investigates not only the cubic, but also the quartic self-couplings. In comparison it is more effective to test directly the sector of self-interactions in terms of physically meaningful deviations given by (reasonable) models.

In the present paper we investigate the implications of the structure of the vector-boson self-couplings in the BESS (Breaking Electroweak Symmetry Strongly) model $^3,^4$. This model is based upon a very different mechanism for gauge-boson-mass generation, since there are only as many scalar fields in this theory as needed to supply the additional degrees of freedom for massive gauge bosons. So all scalars are unphysical would-be-Goldstone bosons and the model contains no physical Higgs boson. However, avoiding the Higgs bosons necessarily implies nonlinear realization of the gauge symmetry in the scalar sector so that the model is nonrenormalizable. Consequently, it has to be considered as an effective theory, which describes physics only over a restricted energy range beyond which “new physics” arise. In difference to renormalizable models, at each loop-order new couplings are induced, which do not have the tree-level structure and cannot be removed by ordinary counterterms (that have the structure of the original Lagrangian). These quantum induced interactions have necessarily
to be taken into account if the model is taken seriously [3]. For gauge-boson self-interactions they have been completely calculated to one-loop level [5]. Using the resulting vertices, the cross sections for two- and three-gauge-boson production have been calculated for reasonable choices of the free parameters of the BESS model [7] with the result that the BESS model leads to measurable differences from the standard model at NLC energies (for these parameter choices).

Our purpose is now to determine the empirical limits which can be assigned to the (yet free) BESS model parameters if the standard model cross section for vector-boson production will be confirmed in NLC experiments, i.e. to find out those values of the free parameters for which the predicted differences of the BESS model to the SM do not exceed the empirical error. In other words, if the BESS model is realized in nature and the parameters lie outside the abovementioned regions, the NLC experiments would exhibit measurable deviations from the SM predictions. The present analysis will also show which physical quantities are most sensitive to variations of the free parameters. Such parameter fits have already been performed for the BESS model on the basis of processes which get contributions from the induced fermionic couplings of the gauge bosons [8] while here we study the implications of the vector-boson self-couplings, thereby taking the quantum induced contributions fully into account. Our investigations are based on the total cross sections for the reactions $e^+e^- \rightarrow W^+W^-$, $e^+e^- \rightarrow W^+W^-Z$ and $e^+e^- \rightarrow W^+W^-\gamma$ (production of polarized and of unpolarized gauge bosons) and on the forward-backward asymmetry for the two-gauge-boson-production process.

In Section 2 we give a brief overview of the BESS model and sketch the features which are most important for our purposes. In Section 3 we explain the methods and the phenomenological input used to perform the parameter fits. In Section 4 we present and discuss the results of these parameter fits. Section 5 contains our conclusions.

2 The BESS Model

We will not present the full model here (for this see [3, 4]), instead we give a short overview and outline the aspects which are fundamental for the following analysis.

The BESS model is a spontaneously broken gauge theory of electroweak interaction. In difference to the SM, the scalars are realized by nonpolynomial expressions and transform nonlinarly under gauge transformations. By this way of constructing spontaneous symmetry breaking one can avoid the introduction of physical Higgs-bosons: there are as many scalars as massive gauge bosons. The simplest model of this type for electroweak interaction is the $SU(2)_L \times U(1)_Y$-gauged nonlinear $\sigma$-model [5], which is just the limit of the SM for infinite Higgs mass. This model contains additional local ("hidden") symmetries [3], which can be made apparent by increasing the number of unphysical scalar degrees of free-
dom and introducing the gauge bosons connected to the hidden symmetry groups \[3, 4\]. The BESS model is the simplest of these extensions of the nonlinear $\sigma$-model with one additional SU(2)$_V$ gauge symmetry and the corresponding gauge boson triplet.

The fundamental parameters of the BESS model are the following: $g, g', g''$ (the coupling constants of the gauge groups SU(2)$_L$, U(1)$_Y$ and SU(2)$_V$), the vacuum expectation value $f^2$ and the relative strength of the hidden symmetry $\lambda^2$. The (unphysical) gauge fields are $\tilde{W}, \tilde{Y}$ and $\tilde{V}$. They belong to the three gauge groups, that mix to the mass eigenstates $W^\pm, Z, \gamma, V^\pm$ and $V^0$, which are all except for the photon massive. Low energy phenomenology demands a large coupling constant $g''$ which yields heavy $V$ bosons.

To perform loop calculations within the nonrenormalizable BESS model one has to introduce a cut-off $\Lambda$ for making all divergent integrals finite. $\Lambda$ roughly represents the scale of new physics. In calculating loops one has to distinguish between

- Those cut-off dependent terms whose structure exists already in the starting Lagrangian and which can therefore be treated by the usual renormalization procedure (addition of counterterms) and
- The new “induced” couplings that reflect the nonrenormalizability of the BESS model.

For the results of these calculations see \[4, 6, 7\]. The main points are:

- All induced couplings have a logarithmic cut-off dependence.
- The induced fermionic couplings are suppressed by a factor of $(m_f/M_W)^2$ and therefore negligible for all fermions except for $b$- and $t$-quarks.
- The induced gauge-boson self-couplings are proportional to polynomials in the parameter $\lambda^2$, of the third power for cubic and of the fourth power for quartic self-interactions.
- The induced cubic self-couplings show Yang–Mills structure while the induced quartic couplings do not.

These results show, that the most genuine feature of the BESS model (and in fact of all theories with nonlinear symmetry realization), namely the existence of quantum induced interactions, shows up most clearly in processes which get contributions from gauge boson self-couplings. The most prominent examples are the various IVB-production processes in $e^+e^-$-collisions. Furthermore, since the size of the induced coupling strength grows with $\lambda^2$, which on the other hand also governs the size of $M_V$ (the mass of the heavy vector bosons), one expects that the influence of the induced interactions increases with increasing $M_V$. 

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3 Fit of Free Parameters

The BESS model contains the five abovementioned free parameters, two more than the nonlinear $\sigma$-model. These five parameters are related to the physical coupling constants and masses as described in [3, 4, 7]. In particular, these parameters can be reconstructed from the values of $M_W$, $M_Z$ (gauge boson masses), $\alpha$ (electromagnetic fine structure constant), $g/g''$ and $\lambda^2$. While the first three of these are given by experiments at present energies, the latter two are yet unspecified. Future high energy experiments at NLC will allow a specification of these two free parameters within experimental errors.

Our question is now which numerical values of the free parameters $g/g''$ and $\lambda^2$ will yield measurable differences to the SM predictions at NLC for various physical quantities (cross sections and asymmetries for two- and three-gauge-boson-production processes) and which will not. So on the one hand we derive bounds on the two free parameters for the case that NLC confirms the SM predictions and on the other hand we determine the regions in the $(\lambda^2, g/g'')$ plane where the BESS model will be empirically distinguishable from the SM and which physical quantities to be measured at NLC are especially sensitive to the BESS model. In addition, we will demonstrate which quantities are more sensitive to $g/g''$ and which are more sensitive to $\lambda^2$.

As an input for our calculations we need, as mentioned above, the gauge boson masses $M_W$ and $M_Z$ and the electromagnetic fine structure constant $\alpha$. The latter is taken as

$$\alpha = \frac{1}{127},$$

which is the value of the running coupling constant in the NLC region. A further needed parameter is the cut-off $\Lambda$ that is used to calculate the induced couplings. We choose

$$\Lambda = 5 \text{ TeV}^1.$$ 

Based upon these inputs and using freely chosen values for $g/g''$ and $\lambda^2$, it is possible to determine the cross sections, asymmetries, etc. and to compare them with the SM results\(^2\). The cross sections for two-gauge-boson production are calculated completely analytically using the usual trace techniques, while for the calculation of three-gauge-boson-production processes we use the same numerical techniques as in ref. [10]. In agreement with [10] we impose the following cuts on transverse-momentum $P_{T,\gamma}$ and pseudorapidity $\eta_\gamma$ of the photon produced in

\(^1\)Because of the logarithmic cut-off dependence a slightly different choice of $\Lambda$ does not significantly influence the results.

\(^2\)For the SM reference values we neglect all Higgs effects (which contribute to $e^+e^- \rightarrow W^+W^-Z$), since for a Higgs mass $M_H > \sqrt{s} - M_Z$ these contributions are negligible. Otherwise the Higgs boson would be found at NLC in the reaction $e^+e^- \rightarrow ZH$ and the BESS model would be ruled out anyway.
\( e^+e^- \rightarrow W^+W^-\gamma: \)
\[
P_{T,\gamma} > 20\text{ GeV}, \quad |\eta_\gamma| < 2.
\] (3)

All calculations are performed at the NLC energy of
\[
\sqrt{s} = 500\text{ GeV}.
\] (4)

To perform the parameter fit one has to know the expected experimental accuracy (sum of the statistical and the systematical error). The statistical error is predicted from the event rate, assuming an integrated luminosity of 20 fb\(^{-1}\) per year and taking into account a reduction factor (branching fraction and acceptance correction) of 0.3 for \( e^+e^- \rightarrow W^+W^-\) \(^[11]\) and \( e^+e^- \rightarrow W^+W^-\gamma \) and of 0.2 for \( e^+e^- \rightarrow W^+W^-Z \) \(^[10]\). We determine the statistical error to 90\% confidence level. We consider both a “conservative” accuracy, meaning one year data collecting time plus a systematical error of 1.5 \% and an “ideal” one, assuming three years of running plus a systematical error of 1.2 \% for \( e^+e^- \rightarrow W^+W^- \). For three gauge-boson production we assume a systematical error of 2\% and three years of collecting data.

For the process \( e^+e^- \rightarrow W^+W^- \) (unpolarized gauge bosons) the total cross section is 8 pb, and so there will be a large number of events and the systematical error dominates, but if one studies cross sections for the production of polarized gauge bosons, partial cross sections or three-gauge-boson-production processes, the event rate is considerably smaller (for \( e^+e^- \rightarrow W^+W^-Z \) the total cross section is 47 fb) and the statistical error becomes more important. So larger deviations to the SM in these cases do not necessarily imply stricter bounds on the free parameters. That is why we did not consider bounds obtained after only one year of run for three-gauge-boson production; the error would be too large to get useful results.

4 Discussion of the Results

Out of the observables that were calculated in ref. \(^[7]\) we find the following ones to be most interesting for deriving bounds to the BESS model:

- The total cross section for production of two unpolarized Ws, i.e. \( e^+e^- \rightarrow W^+W^- \).
- The total cross section for production of two longitudinally polarized Ws, i.e. \( e^+e^- \rightarrow W^+_LW^-_L \).
- The forward-backward asymmetry of \( W^+W^- \) production.
- The total cross sections for the three-gauge-boson-production processes \( e^+e^- \rightarrow W^+W^-Z \) and \( e^+e^- \rightarrow W^+W^-\gamma \) (production of unpolarized gauge bosons).
The total cross section for production of three longitudinally polarized gauge bosons $e^+e^- \rightarrow W^+_L W^-_L Z_L$.

Fig. 1 shows the bounds (in the $(\lambda^2, g/g'')$ plane) stemming from production of an unpolarized W pair. In all figures the shaded areas below the curves are the allowed regions, the dashed line corresponds to the conservative, the solid one to ideal precision, as characterized in the last section. From these data $\lambda^2$ is restricted to be lower than 10 and $g/g''$ to be lower than 0.12 for the conservative measurement. The bounds obtained from the ideal measurement are only slightly stricter because the statistical error is small anyhow. Production of two longitudinally polarized gauge bosons yields stronger limits (Fig. 2), since the greatest deviations of the BESS results from the SM predictions occur in the production of purely longitudinal gauge bosons due to the violation of the gauge cancellations caused by the induced couplings. Here, accuracy increases much for the ideal measurement, i.e. for a longer time of collecting data since this reduces the statistical error arising as a consequence of the small cross-section for this channel. Fig. 3 shows the bounds stemming from the forward-backward asymmetry. The conservative measurement does not yield any interesting bounds, especially with respect to $g/g''$. The ideal measurement gives bounds on $\lambda^2$ comparable to those from $\sigma_{\text{tot}}$ but worse limits on $g/g''$.

Figs. 4 and 5 exhibit the bounds that result from the total cross sections for $e^+e^- \rightarrow W^+W^-Z$ and $e^+e^- \rightarrow W^+W^-\gamma$ (unpolarized gauge bosons), respectively. These processes are of big importance, since they allow a direct test of four-gauge-boson self-interactions (together with three-boson ones). Because of the stronger dependence of the induced quartic self-couplings on $\lambda^2$ there are larger deviations from the SM (at least for higher $\lambda^2$-values) within these processes. Unfortunately however, the corresponding cross sections are much smaller than the two gauge-boson-production cross sections. Therefore, the statistical accuracy to be obtained at NLC is worse and we get bounds which are not much better than those stemming from two-gauge-boson production. Fig. 4 shows that $g/g''$ can be restricted to be lower than 0.1 and $\lambda^2$ to be lower than 10, if the SM is confirmed at NLC. $e^+e^- \rightarrow W^+W^-\gamma$ yields slightly stricter bounds on both parameters (Fig. 5) due to the larger cross section of 153 fb, i.e., the smaller statistical error.

We also have analyzed the bounds arising from the reaction $e^+e^- \rightarrow W^+_L W^-_L Z_L$ (Fig. 6) with longitudinal final gauge bosons. Unfortunately the cross section is so small (0.5 fb) and the statistical error becomes so large that the effect of larger deviations is again compensated. This reaction therefore does not yield stricter bounds than production of unpolarized gauge bosons at the NLC (in difference to W-pair production). However, comparing Figs. 4 and 6, one can see that production of unpolarized gauge bosons is more sensitive to $g/g''$, while

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3The results should be compared to those of the ideal measurement in Fig. 1, which are based on three years’ data, too.
production of longitudinally polarized gauge bosons is more sensitive to $\lambda^2$, i.e. to the strength of the induced couplings.

Because of the insufficient statistics caused by the small cross sections an analysis of partial cross sections would not yield stricter bounds. The same is true for the reaction $e^+e^- \rightarrow ZZZ$ with a tiny total cross section of only 1 fb. So we did not apply our analysis to these quantities.

The resulting bounds on BESS model parameters should be compared to those which are obtained from fermionic processes measured at LEP I [8] (Figs. 8 - 12). We see that gauge boson production will yield stricter bounds in most cases. In particular, higher values of $\lambda^2$, which are allowed by most data from fermionic processes, can be excluded. This is not surprising, since $\lambda^2$ governs the strength of the induced couplings and these do not contribute very much to the fermionic sector of the theory but to the bosonic self-interactions. Only the fermionic process $e^+e^- \rightarrow b\bar{b}$ at LEP I energies yields bounds comparable to the ones obtained here or even better.

5 Conclusion

We summarize our main results:

- The strictest bounds to the free BESS-Model parameters arise from the total cross sections while the forward-backward asymmetry for $e^+e^- \rightarrow W^+W^-$ is less sensitive to $g/g''$.

- The processes $e^+e^- \rightarrow W^+W^-$ and $e^+e^- \rightarrow W^+W^−γ$ yield better bounds than $e^+e^- \rightarrow W^+W^−Z$ due to better statistics.

- Production of longitudinally polarized gauge bosons yield better limits for two-gauge-boson production. For three-gauge-boson production the results are comparable to those from production of unpolarized gauge bosons but more sensitive to $\lambda^2$.

- Measurements of gauge-boson-production at NLC will improve the information on the evidence of the model with respect to fermionic processes measured at LEP I [8]. This is due to the induced self-couplings of the vector bosons, whereas the induced fermionic interactions are very small.

\footnote{However the small errors for $A_{FB}$ and $\sin^2 \theta_W$ assumed in [8] are not reached by LEP I data so that the bounds from $e^+e^- \rightarrow b\bar{b}$ are not so strict as given there.}

\footnote{See, in comparison, the treatment of [12], which represents a principally different theoretical acces to nonstandard vector-boson self-couplings. There gauge invariant extra terms with \textit{linearly} realized symmetry are added to the SM Lagrangian which yield anomalous couplings on \textit{tree-level}. In the BESS model gauge symmetry is realized \textit{nonlinearly} and anomalous couplings are \textit{loop implied}. However, both acceses yield the result that anomalous self-interactions (within the two different formalisms) are not restricted very well by LEP I measurements.}
The above discussion has shown that, if the SM will be confirmed at NLC, the free BESS model parameters will be restricted to a quite narrow region. For a wide parameter range the structures of the BESS model (heavy vector bosons, mixing between light and heavy bosons, new induced couplings due to nonrenormalizability) will yield measurable effects on two- and three-gauge-boson-production processes at NLC. So there is hope that indications of the BESS model (if this is realized in nature) may be found at NLC. However one has to collect data over a longer period of time to get useful results due to limited statistics.

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**Figure Captions**

**Figure 1:** Bounds from the total cross section for $e^+e^- \rightarrow W^+W^-$ (unpolarized)

**Figure 2:** Bounds from the total cross section for $e^+e^- \rightarrow W_L^+W_L^-$ (longitudinally polarized)

**Figure 3:** Bounds from the forward-backward asymmetry for $e^+e^- \rightarrow W^+W^-$ (unpolarized)

**Figure 4:** Bounds from the total cross section for $e^+e^- \rightarrow W^+W^-Z$ (unpolarized)

**Figure 5:** Bounds from the total cross section for $e^+e^- \rightarrow W^+W^-\gamma$ (unpolarized)

**Figure 6:** Bounds from the total cross section for $e^+e^- \rightarrow W_L^+W_L^-Z_L$ (longitudinally polarized)