Exclusive photoproduction of excited quarkonia in ultraperipheral collisions

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Abstract

In this paper, we discuss the exclusive photoproduction of ground and excited states of $\psi(1S, 2S)$ and $\Upsilon(1S, 2S)$ in ultraperipheral collisions (UPCs). Using the potential model in order to obtain the vector meson wave function, we find a good agreement of our calculations with data from the LHC and HERA colliders for $J/\psi(1S, 2S)$ and $\Upsilon(1S)$ in $\gamma p$ collisions. We extend the calculations to the nuclear target case applying them to $AA$ UPCs with the use of the shadowing and finite coherence length effects fitted to the data. Our results are compared to the recent LHC data, in both incoherent ($J/\Psi(1S)$ at 2.76 TeV) and coherent ($J/\Psi(1S)$ at 2.76 and 5.02 TeV) processes. We also provide the corresponding predictions for the excited states, in the hope that future measurements could provide a more detailed information about the vector meson wave functions and the nuclear effects.

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I. INTRODUCTION

Phenomenology of exclusive quarkonia photoproduction processes offers very sensitive and powerful probes for the associated soft and hard QCD phenomena. In the case of bottomonia $\Upsilon(1S, 2S)$ photoproduction, the heavy quark mass $m_Q$ provides a sufficiently hard scale for the perturbative QCD framework to be applicable for precision description of the underlined production mechanism [1–4]. However, charmonia $\psi(1S, 2S)$ photoproduction probes predominantly nonperturbative phenomena at a semihard scale. This means that a simultaneous description of the existing charmonia and bottomonia photoproduction data is required for validation of universality of the quarkonia production mechanism that is expected to incorporate both hard and soft QCD effects on the same footing. For a detailed review on quarkonia physics, see e.g. Refs. [5, 6] and references therein.

A notable progress in understanding of the mechanisms of heavy quarkonia elastic electro-(with large photon virtuality $Q^2 \gg 0$) and photo- (with quasireal photon $Q^2 = 0$) production has been done over the past two decades starting from $ep$ collisions at HERA [7–11]. More recently, the ultra-peripheral $pA$ and $AA$ collisions (UPCs) at the LHC have provided clean experimental means for probing the real photoproduction mechanisms of heavy quarkonia in photon-Pomeron fusion with intact colliding nucleons or nuclei, thanks to low QCD backgrounds. Over the past few years, a wealth of phenomenological information on elastic (exclusive) and quasi-elastic $J/\psi \equiv \psi(1S)$ and $\psi' \equiv \psi(2S)$ photoproduction in UPCs has become available from the LHC measurements, in particular, from LHCb [12–14], ALICE [15–19] and CMS [20, 21] experiments. Meanwhile, the existing theoretical approaches remain rather uncertain due to poorly known nonperturbative and a $D$-wave admixture in the corresponding $S$-wave quarkonia wave functions [22–24], as well as the coherence phenomena particularly relevant for photoproduction in $AA$ UPCs [1, 25–27].

A standard view on quarkonia production mechanism is encapsulated in the framework of non-relativistic QCD (NRQCD) where one assumes a small relative intrinsic motion of the heavy (non-relativistic) quark and antiquark, with the perfectly harmonic interaction potential (see e.g. Refs. [4, 28]). For charmonia photoproduction, nonperturbative and relativistic corrections may be significant since the mass of $c$-quark is not large enough in order to safely rely on the perturbative QCD approach. Besides, as of tradition, the lowest-order transition amplitudes for an $S$-wave quarkonium $Q\bar{Q} \rightarrow V (V = \psi(nS), \Upsilon(nS), n = 1, 2)$ is conventionally assumed to have the same form as the lowest Fock state of the photon in the LF (or infinite-momentum) frame, $\gamma \rightarrow Q\bar{Q}$. Such an assumption about a photon-like LF quarkonium wave function is unjustified as the corresponding rest-frame wave function necessarily has an admixture of a $D$-wave component whose weight cannot be established in the framework of a suitable interquark interaction potential [25]. Such an uncontrollable $D$-wave contribution has a considerable impact on quarkonia photoproduction observables as was recently advocated in Ref. [24].

In this work, we perform an analysis of the exclusive quarkonia photoproduction in $AA$ UPC collisions at the LHC in the phenomenologically successful color dipole picture [29, 30] (for an early analysis of diffractive charmonia photoproduction in the dipole framework, see e.g. Ref. [1–4, 31]). The main focus is the quarkonia photoproduction observables in $AA \rightarrow A + V + X$ in both the coherent ($X = A$) and incoherent ($X = A'$ with $A'$ being an excited state of the nucleus) scattering regimes. Here, one of the important ingredients of the production amplitude is the Light-Front (LF) quarkonium wave function found in the framework of potential approach going beyond the NRQCD approximation. In order
to avoid an unjustified $D$-wave effect one starts with a pure $S$-wave $Q\bar{Q} \to V$ transition in the $Q\bar{Q}$-pair rest frame as a product of spin-dependent and radial components. The radial component is found by the solution of the Schrödinger equation for a given model of the interquark interaction potential. A particularly relevant feature of the excited quarkonia states is the presence of one or several nodes in their radial wave functions [4]. These imply a possible cancellation of contributions to the photoproduction amplitude coming from the regions below and above the node position, in particular, in the $\psi' \equiv \psi(2S)$ wave function causing its relative suppression compared to the $J/\psi$ production amplitude [32]. In the spin-dependent component, a transformation of (anti)spinor of a heavy (anti)quark $Q$ ($\bar{Q}$) from the $Q\bar{Q}$ rest frame to the LF frame known as the Melosh transform is employed. The resulting LF quarkonium wave function has been recently validated in a detailed analysis of the $S$-wave quarkonia electro- and photoproduction observables at HERA in Refs. [22, 23] providing a consistent estimate of the underlined theoretical uncertainties.

The article is organised as follows. In Sect. II, we provide a brief discussion of the LF potential approach for the proton target case and present the corresponding numerical results on the energy dependence of the $\gamma p \to V p$ integrated cross section compared against all the available data from the HERA, LHC and fixed-target experiments. In Sect. III, an analysis of coherent and incoherent quarkonia photoproduction off nuclear targets is performed and the numerical results are shown against the available LHC data on $AA$ UPCs. Finally, concluding remarks and a summary are given in Sect. IV.

II. EXCLUSIVE PHOTOPRODUCTION OFF THE PROTON TARGET

A. $\gamma p \to V p$ cross section

Consider first the case of exclusive quarkonia photoproduction in high-energy photon-proton $\gamma p \to V p$ scattering in the rest frame of the proton target. According to the dipole picture [29, 30], in the high-energy regime, both the photon and the heavy quarkonium $V = \psi(nS), \Upsilon(nS) \ (n = 1, 2)$ can be considered as color dipoles whose transverse separations are frozen during their interactions with a target nucleon. The lowest-order $Q\bar{Q}$ Fock fluctuation of the projectile quasi-real photon scatters off the target nucleon at a certain impact parameter and with a fixed interquark separation. This is an elementary QCD process described by means of the universal dipole cross section as an eigenstate of the elastic scattering operator. The same $Q\bar{Q}$ dipole is then projected $Q\bar{Q} \to V$ to a given quarkonium state $V$ by means of the corresponding LF wavefunction. In the non-relativistic limit, the $Q\bar{Q}$ separation is found to be $r_V \simeq 6/M_V$, in terms of the quarkonium mass $M_V$ [3, 22]. The perturbative domain then corresponds to $r_V \lesssim r_g$ where the gluon propagation length in the nucleon $r_g \sim 0.3$ fm represents the soft scale of the process [33, 34]. In the case of photoproduction, this condition is satisfied only for bottomonia, while charmonia are produced predominantly in the soft regime.

The exclusive photoproduction cross section integrated over the impact parameter of $\gamma p$ collisions [25]

$$\sigma_{\gamma p \to V p}(W) = \frac{1}{16\pi B} \left( \text{Im} A_{\gamma p \to V p}(W) \right)^2,$$

is found in terms of the forward exclusive photoproduction amplitude $A(W)$, the elastic slope parameter $B$ typically fitted to the exclusive quarkonia electroproduction data available from
the HERA collider, and \( W \) is the \( \gamma p \) center-of-mass energy. For the numerical values of the slope \( B \) we adopt the most recent energy-dependent parametrisation of the HERA data known from Ref. \[23\].

The forward \( Q\bar{Q} \) photoproduction amplitude reads

\[
\text{Im} \mathcal{A}^{\gamma p \rightarrow Vp}(W) = \int_0^1 d\beta \int d^2r_\perp \Psi_{\gamma}^\dagger(\beta, r_\perp) \Psi_{\gamma}(\beta, r_\perp) \sigma_{q\bar{q}}(x, r_\perp), \quad x = \frac{M_v^2}{W^2},
\]

where \( \Psi_{\gamma}(\beta, r_\perp) \) is the LF wave function of a transversely-polarized (quasi-real) photon fluctuating into a \( Q\bar{Q} \) dipole and \( \Psi_{\gamma}^\dagger(\beta, r_\perp) \) is the LF quarkonium wave function. The transverse \( Q\bar{Q} \) dipole size is \( r_\perp, \beta = p_Q^+/p_\gamma^+ \) is the longitudinal momentum fraction of the photon momentum \( p_\gamma^+ = E_\gamma + p_\gamma \) carried away by a heavy quark \( Q \). The universal dipole cross section describing the \( Q\bar{Q} \) dipole elastic scattering off the proton target is \( \sigma_{q\bar{q}}(x, r_\perp) \), where \( x \) is the standard Bjorken variable used e.g. in diffractive DIS \[35\] and associated with the proton energy loss in the dipole-proton scattering.

In the NRQCD limit, one typically assumes an equal energy sharing between \( Q \) and \( Q \), such that the LF quarkonium wave function is approximated as \( \Psi_{\gamma}(\beta, r_\perp) \propto \delta(\beta - 1/2) \) \[1\]. Another imposed approximation is a photon-like \( Q\bar{Q} \rightarrow V \) transition amplitude, with the radial wavefunction based upon a naive harmonic interquark potential \[1, 28\]. In our analysis below, we go beyond these approximations following the formalism of Ref. \[23, 25\] for the proton target, and then adopt it for the studies of quarkonia photoproduction observables in AA UPCs as done in Refs. \[25, 27\].

B. Quarkonium wave function

A consistent computation of the LF quarkonium wave function \( \Psi_{\gamma}(\beta, r_\perp) \) in the infinite-momentum frame remains a challenging problem even for the lowest Fock \( V \rightarrow |Q\bar{Q} \rangle \) state \[25\]. In this work, we follow the potential approach of Ref. \[23, 25\] starting from the factorised wave function for a pure \( S \)-wave state in the light-cone (LC) momentum representation defined as

\[
\Psi_{\gamma}(\mu, \bar{p}_T) = U^{(\mu, \bar{p}_T)}(\beta, \bar{p}_T) \psi_{\gamma}(\beta, p_T), \quad U^{(\mu, \bar{p}_T)}(\beta, \bar{p}_T) = \frac{1}{\sqrt{2}} \xi^\mu_Q \bar{\sigma}_{\bar{e}} \xi_Q^\mu, \quad (2.3)
\]

\[
\xi_Q^\mu = i\sigma_\gamma S_Q^\mu, \quad \xi_Q^\mu = R(\beta, \bar{p}_T) \chi_Q^\mu, \quad \xi_Q^\mu = R(1 - \beta, -\bar{p}_T) \chi_Q^\mu,
\]

in terms of the spin-dependent and spatial (radial) parts of the vector meson wave function denoted as \( U^{(\mu, \bar{p}_T)}(\beta, \bar{p}_T) \) and \( \psi_{\gamma}(\beta, p_T) \), respectively. Here, \( \bar{e}_V \) is the vector meson polarisation vector, \( \xi_Q^\mu \) and \( \xi_Q^\mu \) are the heavy quark and antiquark spinors in the \( Q\bar{Q} \) rest frame, respectively, related to their counterparts in the infinite momentum frame, \( \chi_Q^\mu \) and \( \chi_Q^\mu \), by means of the Melosh spin transformation matrix given by \[25, 36\]

\[
R(\beta, \bar{p}_T) = \frac{-i\bar{p}_T(\bar{\sigma} \times \bar{n}) + m_Q + \beta M_V}{\sqrt{p_T^2 + (m_Q + \beta M_V)^2}}.
\]

One may assume that the corrections due to Melosh spin transformation are numerically not very relevant for a non-relativistic \( Q\bar{Q} \) system. In the case of the ground-state quarkonium states such as \( J/\psi \), indeed, the photoproduction cross section increases by roughly 30% only.
However, in the case of excited states the spin rotation effect is much larger and enhances \( \psi(2S) \) photoproduction by about a factor 2–3 [22, 23, 25].

The spatial wave function \( \psi_V(\beta, p_T) \) is typically found by using a simple Lorentz boost prescription [37] based upon the conservation of probability. Starting from the corresponding spatial wave function \( \psi(p) \) in the \( QQ \) rest frame one writes

\[
\psi_V(\beta, p_T) = \left( \frac{p_T^2 + m_Q^2}{16(\beta(1-\beta))^3} \right)^{\frac{1}{4}} \psi(p),
\]

(2.5)

\[
\int |\psi_V(p)|^2 d^3p = 1, \quad \int |\psi_V(\beta, p_T)|^2 d^2p_T d\beta = 1,
\]

in terms of the heavy-quark 3-momentum \( p \equiv |\vec{p}| \). This prescription, also known as the Terent’ev recipe, has been found to be quite successful in describing the HERA data on exclusive electro- and photoproduction of charmonia states in Refs. [22, 23, 25]. A justification of this prescription has been discussed in Ref. [38] and no significant deviation in predictions has been found between the exact calculation and the Terent’ev recipe in the phenomenologically relevant domains of the phase space.

The spatial momentum-space wave function \( \psi_V(p) \) is found by a Fourier-transform from the coordinate-space radial wave function \( \psi(R) \) where \( R \equiv |\vec{R}| \) is the interquark separation. The latter is found as a solution of the Schrödinger equation for a given interquark \( Q - \bar{Q} \) interaction potential. In numerical calculations we employ five distinct models for the interquark potential: harmonic oscillator (osc), power-like model [39, 40] (pow), Buchmuller-Tye parametrisation [41] (but), Cornell potential [42, 43] (cor) and logarithmic potential [44] (log). In each such parametrisation of the long-distance interaction potential, the heavy quark mass \( m_Q (Q = c, b) \) was considered as an adjustable parameter fitted to describe the charmonia and bottomonia spectra. In the short-distance production amplitude, however, we use in numerical analysis the universal values for the charm \( m_c = 1.4 \text{ GeV} \) and bottom \( m_b = 4.75 \text{ GeV} \) quark masses.

C. Dipole formula for photoproduction amplitude

Using the LF quarkonia wave function in Eq. (2.3), the resulting photoproduction amplitude (2.2) can be represented in the following form [23]

\[
\text{Im}A^{p \rightarrow Vp}(W) = \int_0^1 d\beta \int d^2r_\perp \left[ \Sigma^{(1)}(\beta, r_\perp)\sigma_{qq}(x, r_\perp) + \Sigma^{(2)}(\beta, r_\perp) \frac{d\sigma_{qq}(x, r_\perp)}{dr_\perp} \right],
\]

(2.6)

where the coefficient functions read

\[
\Sigma^{(1)} = \frac{Z_Q \sqrt{N_c \alpha_{em}}}{2\pi \sqrt{2}} \frac{2K_0(m_Qr_\perp)}{2K_0(m_Qr_\perp)} \int dp_T J_0(p_T r_\perp) \psi_V(\beta, p_T) \frac{m_T m_L + m_T^2 - 2\beta(1-\beta)p_T^2}{m_L + m_T},
\]

(2.7)

and

\[
\Sigma^{(2)} = \frac{Z_Q \sqrt{N_c \alpha_{em}}}{2\pi \sqrt{2}} \frac{2K_0(m_Qr_\perp)}{2K_0(m_Qr_\perp)} \int dp_T J_1(p_T r_\perp) \psi_V(\beta, p_T) \frac{p_T^2}{2} \frac{m_L + m_T + (1 - 2\beta)^2 m_T}{m_T(m_L + m_T)}. \]
Here, $\alpha_{\text{em}}$ is the fine structure constant, $N_c = 3$ is the number of colors in QCD, $Z_Q$ is the electric charge of the heavy quark, $J_{0,1} (K_0)$ are the (modified) Bessel functions of the first (second) kind, respectively, $p_T$ is the transverse momentum of the produced quarkonium state, and

$$m_T = \sqrt{m_Q^2 + p_T^2}, \quad m_L = 2m_Q \sqrt{\beta(1 - \beta)}. \quad (2.8)$$

Finally, in order to take into account corrections due to the real part of the forward photoproduction amplitude we adopt the formalism of Ref. [25] by applying a multiplicative factor

$$\mathcal{A}^{\gamma p \to V p} = i \text{Im} \mathcal{A}^{\gamma p \to V p} \left(1 - i \frac{\pi}{2} \frac{\partial \ln \text{Im} \mathcal{A}^{\gamma p \to V p}}{\partial \ln W^2}\right). \quad (2.9)$$

containing the derivative of the imaginary part of the amplitude which is, thus, sensitive to the $x$-dependence of the dipole cross section.

### D. Saturated dipole cross section

An essential ingredient of the photoproduction amplitude [2.6] at high energies is the universal dipole cross section $\sigma_{q\bar{q}}(x, r_\perp)$ related to the gluon distribution in the proton target. At very low-$x$, one expects to enter a non-linear QCD evolution regime known as saturation that constrains the maximum of the gluon density that can be achieved in the hadronic wavefunction (see e.g. Refs. [45–47] and references therein). This effect is typically accounted for in phenomenological parametrisations for the dipole cross section whose saturated shape is characterised by the $x$-dependent saturation scale, $Q_s(x)$.

One of such simplest and most phenomenologically successful saturated models is known as the Golec-Biernat–Wustoff (GBW) [48] parametrisation,

$$\sigma_{q\bar{q}}(x, r_\perp) = \sigma_0 \left(1 - e^{-\frac{r_\perp^2 Q_s^2(x)}{4}}\right), \quad (2.10)$$

satisfying the renown color transparency property of the dipole scattering, $\sigma_{q\bar{q}}(r_\perp) \propto r_\perp^2$ as $r_\perp \to 0$. Besides, we stick to the standard assumption about the quark flavor invariance of the dipole cross section in this study. In Eq. (2.10), the parametrisation of the saturation scale squared in the proton target case

$$Q_s^2(x) \equiv R_0^{-2}(x) = Q_0^2 \left(\frac{x_0}{x}\right)^\lambda, \quad (2.11)$$

is valid at very small $x \lesssim 0.01$ only. The saturated ansatz (2.10) have given rise to a whole family of dipole models attempting in particular to incorporate the hard scale dependence of the dipole cross section via e.g. QCD DGLAP-like evolution. In the case of photoproduction, however, an effect of such a scale dependence remains minor for not very large masses of the produced states and will be safely ignored in what follows. An early fit of the HERA data for the GBW model parameters in Eq. (2.10)

$$Q_0^2 = 1 \text{GeV}^2, \quad x_0 = 3.04 \times 10^{-4}, \quad \lambda = 0.288, \quad \sigma_0 = 23.03 \text{mb}. \quad (2.12)$$

has been performed in Ref. [49] yielding a good description of a big variety of various observables in both $ep$ and $pp$ collisions at high energies.
Motivated by an analysis of theoretical uncertainties performed in Ref. [23], in what follows we use another parametrisation from Ref. [50] known as the KST model providing a reasonably good description of the real photoproduction data available from the HERA collider. When $Q^2 \rightarrow 0$ the Bjorken variable $x$ becomes inappropriate, such that $\sigma_0$ and $R_0$ in Eq. (2.10) should be replaced by functions $\bar{\sigma}_0(\hat{s})$ and $\bar{R}_0(\hat{s})$ of the dipole-target collision center-of-frame energy squared $\hat{s} \equiv W^2$ found as follows (for more details, see Ref. [51])

$$\bar{R}_0(\hat{s}) = 0.88 \text{ fm } (s_0/\hat{s})^{0.14}, \quad \bar{\sigma}_0(\hat{s}) = \sigma_{\text{tot}}^{\pi p}(\hat{s}) \left(1 + \frac{3R_0^2(\hat{s})}{8\langle r_{\text{ch}}^2/\pi \rangle} \right),$$

respectively, where

$$\sigma_{\text{tot}}^{\pi p}(\hat{s}) = 23.6(\hat{s}/s_0)^{0.08} \text{ mb}, \quad \langle r_{\text{ch}}^2/\pi \rangle = 0.44 \text{ fm}^2,$$

are the total pion-proton scattering cross section and the mean pion radius squared [52], respectively. Here, $s_0 = 1000 \text{ GeV}^2$. The KST model is generally considered to be applicable for soft and semi-hard processes up to scales of $Q^2 \sim 20 \text{ GeV}^2$ or so and, thus, particularly suitable for the predominantly soft charmonia photoproduction observables.

In our analysis of quarkonia photoproduction in UPCs, following Refs. [25, 27] we employ only GBW and KST models described above. This is also motivated by an observation of Ref. [23] that these two models provide similar results at low-$x$ and both lead to a reasonably good description of the charmonia photoproduction data at center-of-mass energies $W \lesssim 200 \text{ GeV}$, while most of other well-known dipole parametrisations noticeably underestimate the HERA data and hence are less favored.

In the large-$x$ limit (i.e. when $x \rightarrow 1$), however, it is necessary to take into account the contribution from the quark distributions. To achieve that, following the Ref. [53], we multiply the structure functions by a factor $(1-x)^{2n_s-1}$, from the constituent quarks counting rule, where $n_s$ denotes the number of spectator quarks, taken in this work to be equal to four.

**E. Numerical results for $\gamma p \rightarrow V p$ cross sections**

Let us now turn to a discussion of numerical results for the integrated diffractive $\gamma p \rightarrow V p$ photoproduction cross sections (i.e. with the proton target), for $V = \psi(nS)$, $\Upsilon(nS)$, $n = 1, 2$. In Fig. 1, we present the dipole model results for $\psi(1S)$ (left panel) and $\psi(2S)$ (right panel) cross sections as functions of $\gamma p$ center-of-mass energy, $W$. In this analysis, we have used five different models for the interquark potential available from the literature and mentioned earlier. We notice that for charmonia photoproduction both dipole parametrisations, GBW and KST, discussed above in Sect. [11] give very similar results so we have chosen the GBW parametrisation for the presentation purposes here. Our results are compared to the data available from H1 [8], ZEUS [10], ALICE [15] and LHCb [14] measurements as well as from the fixed-target measurements at Fermilab [54–56]. One observes that all five potentials used in our calculations give a relatively good description of the data for diffractive photoproduction of both $\psi(1S)$ and $\psi(2S)$ states in the considered energy range.

In Fig. 2, we show the numerical results for the integrated cross sections of diffractive $\Upsilon(1S)$ (left panel) and $\Upsilon(2S)$ (right panel) photoproduction as functions of $W$. In analogy to the previous figure, in our calculations of the radial wavefunction of the bottomonia states, we employed five different models for the $b \bar{b}$ interaction potential. The results for the
ground state are confronted against the available ψ(1S) photoproduction data from CMS [21], H1 [7], ZEUS [9, 11] and LHCb [12] collaborations. In this figure, we have shown the results with the KST parametrisation of the dipole cross section (2.13) since it provides the best description of ψ photoproduction data. It is worth noticing that all five potentials provide a comparatively good description of the available data on ψ(1S) in the considered energy range. This is the reason why we have used the same dipole model parametrisation and the interquark potentials for making predictions for the photoproduction cross section of the excited ψ(2S) state shown in the right panel. For reliable and thorough estimates of underlined theoretical uncertainties in our calculations, we refer the reader to Ref. [23].
where such uncertainties have been discussed in detail.

III. EXCLUSIVE PHOTOPRODUCTION OFF THE NUCLEUS TARGET

Let us now turn to analysis of exclusive $\psi(nS), \Upsilon(nS)$, $n = 1, 2$ quarkonia photoproduction off the heavy nucleus target $A$ relevant for the corresponding recent measurements in $AA$ UPCs at the LHC. Here, we briefly overview an extension of the dipole model framework and the formalism of LF quarkonia wavefunctions to this case, as well as study the corresponding observables and confront them with all currently available data.

A. Differential $AA \rightarrow AVX$ cross section and the photon flux

In exclusive quarkonia photoproduction off the proton target considered above, the experimental data are typically provided for $\sigma_{\gamma p \rightarrow Vp}$, since the photon flux, in this case, is factorised from the elastic $\gamma p \rightarrow Vp$ cross section. However, such factorisation does not hold in the case of a nucleus target. Indeed, a nontrivial impact parameter dependence of the photon flux becomes relevant in the $AA$ UPCs. In the considering kinematics, the photons can only interact with the target when there is no overlap between the projectile and the target in impact parameter $b$-space such that $b > 2R_A$, in terms of the nucleus radius $R_A$ (for more details and review, see Ref. [57] and references therein). We also define $b'$ as the position of the interaction point with respect to the target nucleus center, and $\vec{b}_\gamma = \vec{b}' - \vec{b}$ is the same position but with respect to the projectile nucleus.

In the center-of-mass frame of the colliding particles, the produced quarkonium state $V$ has mass $M_V$ and rapidity $y$. Also, the photon energy is labelled as $\omega$ and

$$\omega \approx \frac{M_V}{2} e^y = \frac{W^2}{2\sqrt{s}},$$

(3.1)

where $\sqrt{s}$ is the projectile-target (with the projectile being a nucleon inside of an incoming nucleus and the target – a nucleon inside the target nucleus) center-of-mass energy. Of course, in numerical computations of the differential cross section one has to take into account that the photon can be originated from each of the nuclei, which is done by incorporating $y \rightarrow -y$.

The differential cross section of quarkonia photoproduction in $AA$ UPCs when the projectile photon is taken from one of the colliding nuclei is given in the following standard form [58]

$$\frac{d\sigma^{AA \rightarrow AVX}}{dy} = \int d^2b \int d^2b' \omega \frac{dN_\gamma(\omega, \vec{b}_\gamma)}{d\omega d^2b_\gamma} \frac{d\sigma^{A \rightarrow VX}(\omega, \vec{b}')}{{d^2b}'}, \quad \vec{b}_\gamma = \vec{b}' - \vec{b},$$

(3.2)

where $X = A$ or $A^*$ for the coherent and incoherent production, respectively. Provided that the photons are quasi-real in the considering UPCs, i.e. $Q^2 \approx 0$, only transversely polarised photons are relevant. The photon number density in the projectile nucleus is conventionally described by the differential Weiszäcker-Williams (WW) photon flux [59, 60], with a broad spectrum as given by

$$\frac{d^3N_\gamma(\omega, \vec{b}_\gamma)}{d\omega d^2b_\gamma} = \frac{Z^2\alpha_{em} k^2}{\pi^2 \omega b_\gamma^2} \left[K_1^2(k) + \frac{1}{2}, K_0^2(k)\right], \quad k = \frac{b_\gamma \omega}{\gamma},$$

(3.3)
where \( \gamma = \sqrt{s}/2m_p \) is the Lorentz factor, the proton mass is given by \( m_p = 0.938 \text{ GeV} \), and \( Z \) is the charge of the projectile nucleus. Since at the LHC energies the Lorentz factor is large \( \gamma \gg 1 \) (e.g. in 2016 \( p\Pb \) run with \( \sqrt{s} = 8.16 \text{ TeV}, \gamma_{\Pb} \approx 4350 \)), the second term in Eq. (3.3) can be safely omitted in practical calculations.

B. Coherent and incoherent processes

In the \( \gamma A \to VX \) subprocess cross section entering in Eq. (3.2) one separates two distinct vector meson production modes. The first one, called coherent production, occurs when the nucleus target remains intact after the interaction, i.e. \( X = A \). In this case, the production cross section is computed in the framework of the Good-Walker formalism \[61\] by averaging the color dipole interactions over all possible configurations of the projectile nucleus, thus, probing the average distribution of low-\( x \) gluons in the target \[62, 63\]. The second process called incoherent production occurs when the outgoing target nucleus does not retain the same quantum state as the incoming nucleus, i.e. it becomes an excited state \( A^* \), which contains nucleons and nuclear fragments but no other hadrons. This process features a large gap in rapidity between the produced quarkonium and \( A^* \) system, and measures how much the scattering amplitude fluctuates between the different possible initial-state configurations \[64\] (for a recent detailed discussion, see e.g. Ref. \[65\]).

Compared to the proton target case considered above, two additional effects are known to play a critical role in quarkonia photoproduction off a heavy nucleus \[1, 27, 58\]. The first one is called the color filtering i.e. inelastic scatterings of the \( Q\bar{Q} \) pair in the course of its propagation through the nucleus. The second effect is associated with the nuclear shadowing of the gluon density due to a reduction of the dipole scattering cross section off the nucleus target compared to that off the proton due to interferences. Both effects cause a reduction of quarkonia photoproduction off the nuclear target, \( \gamma A \to VX \), compared to \( A\sigma^{\gamma p\to Vp} \), and are effectively accounted for in our analysis below.

At high energies, the coherence (or production) length \( l_c \) defined as the lifetime of the \( q\bar{q} \) fluctuation \[1, 66\],

\[
l_c = \frac{2\omega}{M_V^2},
\]

(3.4)
is often considered to be much larger than the nuclear radius, \( l_c \gg R_A \). This guaranties a small variation of the transverse size of the dipole system and no fluctuations during the propagation process through the nucleus by means of Lorentz time dilation. In this case, a \( Q\bar{Q} \) Fock fluctuation of the photon builds up long before it interacts with the nucleus target. This is called the “frozen” approximation, and the calculations are particularly simple and well-known in this case.

Note, there is yet another scale called the formation length of the heavy quarkonia defined as \( l_f = 2\omega/(M^2_{1S} - M^2_{2S}) \) \[1, 66\], which is larger than the coherence length scale. In the considered case of exclusive photoproduction the uncertainty principle enables to resolve between \( J/\psi \) and \( \psi' \) as long as the formation length \( l_f \) is smaller than the mean inter-nucleon separation in the target nucleus. Hence, at high energies, when \( l_f \gtrsim R_A \), the use of a different approach than a simple eikonalization of the photoproduction off the nucleon target is necessary.

In the “frozen” \( l_c \to \infty \) limit, the incoherent and coherent production cross sections are
found as $[1, 27, 58]$

$$\sigma_{\gamma A \rightarrow VA^*} = \int d^2b \frac{T_A(b)}{16\pi B} \left| \int d\beta d^2r_\perp \Psi_V^\dagger \sigma qq(x, r_\perp) \exp \left( -\frac{1}{2} \sigma qq(x, r_\perp) T_A(b) \right) \right|^2, \quad (3.5)$$

$$\sigma_{\gamma A \rightarrow VA} = \int d^2b \left| \int d\beta d^2r_\perp \Psi_V^\dagger \Psi_\gamma \left[ 1 - \exp \left( -\frac{1}{2} \sigma qq(x, r_\perp) T_A(b) \right) \right] \right|^2, \quad (3.6)$$

respectively, where $\Psi_V = \Psi_V(\beta, r_\perp)$ and $\Psi_\gamma = \Psi_\gamma(\beta, r_\perp)$ are the LF vector meson and real photon wave functions discussed above, and $T(b)$ is the thickness function of the nucleus

$$T(b) = \int_{-\infty}^{+\infty} dz \rho_A(b, z), \quad \int d^2b T(b) = 1, \quad (3.7)$$

defined as an integral of the LC nuclear density, $\rho_A(b, z)$, over the longitudinal coordinate $z$ at a fixed impact parameter $b$. In this paper, for the latter we employ the Wood-Saxon parametrisation $[67]$

$$\rho_A(b, z) = \frac{N_A}{1 + \exp \left[ \frac{r(b, \delta) - R_A}{\delta} \right]}, \quad r(b, z) = \sqrt{b^2 + z^2}, \quad (3.8)$$

where $r \equiv |\vec{r}|$ is the distance from the center of the nucleus, $N_A$ is an overall normalization factor, and the parameters $R_A = 6.62$ fm and $\delta = 0.546$ fm are taken from Ref. $[68]$.

### C. Finite coherence length

Effects due to a finite coherence length, i.e. when $l_c < R_A$, become particularly relevant at low energies $W$. In this case, the real photon propagates inside the nucleus target not experiencing any attenuation until it develops a $Q\bar{Q}$ fluctuation (at a short time scale $t_c$) which then instantly interacts with the nuclear medium. In the analysis of this dynamics, one should take into account that the produced $Q\bar{Q}$ dipole attenuates along its propagation path in the nuclear absorptive medium, whose typical mean length is roughly a half of the nuclear thickness $[27]$.

A manifest quantum-mechanical mechanism for propagation of such a fluctuating $Q\bar{Q}$ dipole through the nuclear medium is consistently implemented in the framework of the LC Green function approach $[24]$. An analytical solution for dipole Green function is known only for the simplest quadratic dependence of the dipole cross section, $\sigma qq \propto r^2$, while a generic case is treatable only numerically and is rather challenging.

Here we follow the approximation of Ref. $[69]$ (with the explicit formulae given in Ref. $[27]$) in which the effect of the finite coherence length is effectively taken into account by multiplying the infinite coherence length results $[3.5]$ and $[3.6]$ by a corresponding form factor as

$$\sigma_{\gamma A \rightarrow VA^*}(W^2) \Rightarrow \sigma_{\gamma A \rightarrow VA^*}(W^2) F^{\text{inc}}(W^2, l_c), \quad (3.9)$$

$$\sigma_{\gamma A \rightarrow VA}(W^2) \Rightarrow \sigma_{\gamma A \rightarrow VA}(W^2) F^{\text{coh}}(W^2, l_c), \quad (3.10)$$

for the incoherent and coherent production cross sections, respectively. In the first, incoherent case, the form factor is represented in the form explicitly normalized to the $l_c = \infty$ case as follows

$$F^{\text{inc}}(W^2, l_c) = \int d^2b \int_{-\infty}^{\infty} dz \rho_A(b, z) \left| F_1(W^2, b, z) - F_2(W^2, b, z, l_c) \right|^2 \left/ \left( \ldots \right) \right|_{l_c \rightarrow \infty}. \quad (3.11)$$
Here,
\[ F_1(W^2, b, z) = \exp \left( -\frac{1}{2} \sigma_{VN}(W^2) \int_z^\infty dz' \rho_A(b, z') \right) \] (3.12)
takes into account that the incident photon fluctuates into the $\bar{Q}Q$ pair and interacts with nucleus at some point with longitudinal coordinate $z$, which then propagates through the nucleus and forms a vector meson. The latter then leaves without experiencing inelastic interactions. The above relation is written in terms of the energy-dependent meson-nucleon total cross section, $\sigma_{VN}(W^2)$, since the result is dominated by the $\bar{Q}Q$ dipole propagation with typical sizes relevant for $\bar{Q}Q$ projection to a particular meson state $V$. The second contribution in Eq. (3.11) reads
\[ F_2(W^2, b, z, l_c) = \frac{1}{2} \sigma_{VN}(W^2) \int_{-\infty}^z dz' \rho_A(b, z') F_1(W^2, b, z') e^{i(z'-z)/l_c}. \] (3.13)

It accounts for a possibility that the photon first elastically produces a quarkonium state at a point $z'$, $\gamma A \rightarrow VA$, which then propagates through the nucleus target without interacting till another point $z > z'$, where the last (quasi-elastic) scattering occurs giving rise to the final-state quarkonium, $VA \rightarrow VA^*$. In the coherent case, the fact that the mesons produced at different longitudinal coordinates and impact parameters add up coherently simplifies the expression for the form factor compared to the previous case yielding
\[ F^{\text{coh}}(W^2, l_c) = \int d^2b \left| \int_{-\infty}^\infty dz \rho_A(b, z) F_1(W^2, b, z) e^{iz/l_c} \right|^2 \left/ \left( \ldots \right) \right|_{l_c \rightarrow \infty}. \] (3.14)

A more sophisticated Green functions analysis of the coherence length effects against the recent data on quarkonia photoproduction in UPCs will be done elsewhere.

D. Gluon shadowing

At small $x$, the gluon density of a nucleon inside the target nucleus is suppressed compared to that of a free nucleon – the phenomenon known as the gluon (or nuclear) shadowing. In the target rest frame, this is understood as a result of the interference among incoming dipoles due to the presence of the higher Fock states of the projectile photon. The leading-order gluon shadowing correction emerges via eikonalization of the next Fock component of the photon containing the $\bar{Q}Q$ dipole plus a gluon. This effect can be accounted for by renormalizing the dipole cross section as follows \[51, 58\]
\[ \sigma_{\bar{q}q}(r, x) \rightarrow \sigma_{\bar{q}q}(r, x) R_g(x, \mu^2), \] (3.15)
where the factor $R_g$ represents the ratio of the gluon density inside a nucleon in the nucleus compared to the one inside the free proton, i.e.
\[ R_g(x, \mu^2) = \frac{x g_A(x, \mu^2)}{Ax g_p(x, \mu^2)}. \] (3.16)

In this work, we compute this factor phenomenologically using the EPPS16 nuclear parton distributions fitted to the LHC data \[70\], with $\mu = M_V/2$ as the factorization scale. At this point, our calculations differ from most of previous analyses in the literature which instead have used theoretical models to predict the amount of the gluon shadowing.
E. Numerical results for $AA \rightarrow AVX$ cross sections

FIG. 3. Differential (in rapidity) cross section for coherent $\psi$ photoproduction in PbPb UPCs at 2.76 TeV (left) and 5.02 TeV (right) using the GBW dipole model. The results with 2.76 TeV are compared with data from CMS \cite{20} and ALICE \cite{15,16} for $J/\psi$ and from ALICE \cite{17} for $\psi'$, while the ones with 5.02 TeV are compared with data from ALICE \cite{19} and preliminary results of LHCb \cite{13}.

With all the formalism presented previously, we obtained some of the most relevant numerical results for the differential photoproduction cross sections in lead-lead collisions at the LHC energies. In Fig. 3, it is shown the differential cross sections for the coherent photoproduction of $\psi(1S)$ and $\psi(2S)$ at 2.76 TeV (left panel) and $\psi(1S)$ at 5.02 TeV (right panel) as functions of the vector meson rapidity $y$. In these plots, we used again the five different models for the interquark potentials and the GBW dipole model. In the left panel, we compared our results with the data from CMS \cite{20} and ALICE \cite{15,16} collaborations for $J/\psi$, and from ALICE collaboration \cite{17} for $\psi'$. It can be noticed that our theoretical calculations describe very well the ground state production data with any of the considered interquark potentials. However, in the case of the excited state production, only the result obtained with the oscillator $c\bar{c}$ potential reaches the corresponding data within error bars. In the right panel, the results are compared with the data from the ALICE collaboration \cite{19} and with the preliminary data from the LHCb collaboration \cite{13}. In this case, our results provide a good description of the LHCb data, but fail to describe the ALICE data. We would like to point out that there is, in fact, a significant tension between these data sets themselves.

In Fig. 4, we present our results for coherent photoproduction of $\Upsilon(1S)$ (left panel) and $\Upsilon(2S)$ (right panel) at 5.02 TeV. The observable analyzed here is the differential cross section as a function of the meson rapidity calculated with the KST dipole cross section for the five distinct models of the $b\bar{b}$ potentials, in analogy to Fig. 3. As the reader can notice in the left panel, there is a significant spread between the results obtained with different potentials in the $\Upsilon(1S)$ production. This spread is especially pronounced at small values of the rapidity, since at large values of $y$ the effect of the finite coherence length dominates. This is a direct consequence of the shape of the LF $\Upsilon(1S)$ wave functions that differs more from one potential to another than the other vector meson wavefunctions. This does not
FIG. 4. Differential (in rapidity) cross section for coherent $\Upsilon(1S)$ (left) and $\Upsilon(2S)$ (right) production at 5.02 TeV in PbPb collision, using the KST dipole model.

occur in the production of the excited state, however, as can be seen in the right panel. In this case, there is a good agreement between the results obtained for each potential, except for the oscillator one which gives a higher prediction.

FIG. 5. Differential (in rapidity) cross section for incoherent $\psi(1S)$ (left) and $\psi(2S)$ (right) photoproduction at 2.76 TeV in PbPb UPCs. The ground state results are compared with the data from ALICE [16].

Fig. 5 shows the differential cross section for the incoherent photoproduction of $\psi(1S)$ (left panel) and $\psi(2S)$ (right panel) as a function of the meson rapidity at 2.76 TeV. These results are obtained using the GBW model and the wave functions in the potential approach and are compared, in the case of ground state production, with the single data point from ALICE collaboration [16]. As can be seen, the results obtained with the formalism described above do not describe the data. This means that the approach has to be improved in the case of incoherent production, and a theoretical further analysis is necessary. Nevertheless, for completeness, in Fig. 6 we show the predictions for incoherent photoproduction of $\Upsilon(1S)$ (left) and $\Upsilon(2S)$ (right) at 5.02 TeV obtained with the same formalism. As in Fig. 4, there is a spread in the predictions obtained with different interquark potentials in the case of
ground state photoproduction. Such a strong sensitivity to the potential models would, in principle, enable one to set more stringent constraints on the heavy quark interaction potential once the corresponding precision data become available.

**FIG. 6.** Differential (in rapidity) cross section for incoherent $\Upsilon(1S)$ (left) and $\Upsilon(2S)$ (right) photoproduction at 5.02 TeV in PbPb UPCs.

### IV. CONCLUSIONS

With the idea of reproducing what is already well known, we have described all the existing data on photoproduction of $J/\psi(1S)$, $\psi(2S)$, and $\Upsilon(1S)$ off the proton target with a good accuracy using the GWB and KST dipole models and different interquark interaction potentials for the light-front vector meson wavefunctions. We have also made predictions for the $\Upsilon(2S)$ photoproduction in this case, for the sake of completeness.

After establishing that the proton target case was well understood in our case, we moved to nucleus target. In order to do so, we included the important effects of gluon shadowing and finite coherence length of the quark-antiquark dipole. So far in the literature, the combined use of the gluon shadowing fitted to data and the interquark potential models (with Melosh spin rotation) for the quarkonia wavefunction has not been employed. Furthermore, no predictions for the bottomonia using the potential model were available as far as we know.

To start with, we considered the case of coherent photoproduction when the whole nucleus interacts with the photon. We achieved a good description of the $J/\psi(1S)$ and $\psi(2S)$ ALICE and CMS data sets from $AA$ UPCs with center-of-mass energy of 2.76 TeV. At a higher energy of 5.02 TeV, our description is consistent with the LHCb data, while there is some tension with the ALICE data. We have also made predictions for the $\Upsilon(1S)$ and $\Upsilon(2S)$ photoproduction in $AA$ UPCs that can potentially help in determining the best interquark potential for the $b\bar{b}$ vector mesons.

In the incoherent case, we make predictions for the same four vector meson states. The only available datapoint (from ALICE) does not agree with our calculation. Nevertheless, we chose to show these predictions because there is in the literature a well-known difficulty of describing the incoherent production data of any type. Therefore, we have demonstrated the best description that can be achieved with the (approximate) models described in this paper and in the literature.
As a final conclusion, we would like to mention that the coherent production case can be described very well if the complete treatment of nuclear effects and vector-meson light-front wavefunctions discussed here is employed. Regarding the incoherent case, we see a strong need of further precise measurements and a better theoretical description of the underlined physics.

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