The research of statistical properties of colorimetric features of screens with a three-component color formation principle

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Abstract. The problem of the research is concerned with quantitative analysis of influence of technological variation of the screen color profile parameters on chromaticity coordinates of the displayed image. Some mathematical expressions which approximate the two-dimensional distribution of chromaticity coordinates of an image, which is displayed on the screen with a three-component color formation principle were proposed. Proposed mathematical expressions show the way to development of correction techniques to improve reproducibility of the colorimetric features of displays.

1. Introduction

Developers of software for a color multi-function display units (MFDU) in avionics indication equipment have to solve the problem of the image color palette selection, which is displayed on the LCD (liquid crystal display), or LED (light emitting diode) screen panel [1-3]. The color palette selection involves the necessity to comply the requirements of the aviation industry technical documentation and the necessity to show to the observer the image with the optimal visual characteristics of perception. Quality of avionics indication equipment is evaluated using a lighting test with the measuring instruments. Final result of these tests is the color palette — an array of records encoded with the RGB code (R – Red, G – Green, B – Blue) for each involved color [2, 4-6].

The practice of the color MFDU development reveals that once formed color palette, which is approved with the results, and the lighting test protocol of a particular color MFDU type is used in the software of the following developments of this type of equipment. This software engineering approach based on the application of the maximally uniform and standardized successful solutions has some significant disadvantages [7-10]. Mainly problem is caused by unavoidable technological variation of the screen color profile parameters. As the result, random shifts of chromaticity coordinates appear for the same once approved color palette. Because of this reason, the research of statistical properties of colorimetric features of screens with a three-component color formation principle is an actual problem.

2. Color space transformation rules used in avionics
Results of tests of the color MFDU manufactured on the basis of the LCD panels revealed that the same RGB code which determines the image displayed color leads to different XYZ-tristimulus values and (x,y)-chromaticity coordinates in the different samples of color multi-function displays.

For each pair of the color MFDU samples that use the software with the same color palette according to the transformation rules [2] for the color spaces XY, XYZ and the RGB color space the following expressions are correct:

\[
\begin{bmatrix}
X_2 - X_1 \\
Y_2 - Y_1 \\
Z_2 - Z_1
\end{bmatrix} =
\begin{bmatrix}
X_{x_2} X_{x_1} X_{x_0} \\
Y_{y_2} Y_{y_1} Y_{y_0} \\
Z_{z_2} Z_{z_1} Z_{z_0}
\end{bmatrix}^{-1}
\begin{bmatrix}
X_1 \\
Y_1 \\
Z_1
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
X_2 - X_1 \\
Y_2 - Y_1 \\
Z_2 - Z_1
\end{bmatrix} =
\begin{bmatrix}
X_{x_2} X_{x_1} X_{x_0} \\
Y_{y_2} Y_{y_1} Y_{y_0} \\
Z_{z_2} Z_{z_1} Z_{z_0}
\end{bmatrix}^{-1}
\begin{bmatrix}
X_1 \\
Y_1 \\
Z_1
\end{bmatrix}
\]

Where \(X_1, Y_1, Z_1\) are the displayed image color coordinates of the first color MFDU sample; \(X_2, Y_2, Z_2\) - the displayed image color coordinates of the second color MFDU sample; \(x_{x_2}, y_{y_2}, z_{z_2}\) - the chromaticity coordinates and the image relative brightness component on the first color MFDU screen; \(x_{x_1}, y_{y_1}, z_{z_1}\) - the chromaticity coordinates and the image relative brightness component on the second color MFDU screen; \(X_{x_1}, X_{x_0}, X_{y_1}, X_{y_0}, X_{z_1}, X_{z_0}, Y_{y_1}, Y_{y_0}, Z_{z_1}, Z_{z_0}\) - the LCD screen profile matrix coefficients used in the color MFDU; RGB - the sample color code.

Analysis of the LCD panels technical documentation reveals that the color difference of the (x,y)-coordinates of an image displayed in the different color MFDU samples with the same screen type and the same program RGB code may reach up to 0.1 unit of chromaticity value, which is visible for an observer and measurable with instruments.

3. Evaluation of the chromaticity coordinates shift

The system (1) binds the chromaticity coordinates and the image relative brightness components with the same color palette RGB. The difference in the (x,y)-chromaticity coordinate values can be explained by the different values of the LCD panel profile coefficients and can be calculated by solving the system (1) as follows:

\[
x_2 = \frac{X_{x_2} - X_{x_1}}{Y_{y_1}} + \frac{X_{x_1} Y_{y_0}}{Y_{y_1}} + \frac{X_{x_0} Y_{y_1}}{Y_{y_1}} (1 - x_1 - y_1) \frac{1}{Y_{y_1}}
\]

(2)
The expressions (2) and (3) define the chromaticity coordinates conversion rule on the XY-plane for an image programmed with the same RGB code but displayed on two color MFDU with different LCD panel matrix profiles.

4. LCD panel color reproduction stochastic model

Stochastic model of the LCD panel color reproduction includes their colorimetric characteristics technological dispersion in mass production, and it may be represented as:

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
= \begin{bmatrix}
X_r + \Delta \cdot \xi_{x_r} & X_g + \Delta \cdot \xi_{x_g} & X_b + \Delta \cdot \xi_{x_b} \\
Y_r + \Delta \cdot \xi_{y_r} & Y_g + \Delta \cdot \xi_{y_g} & Y_b + \Delta \cdot \xi_{y_b} \\
Z_r + \Delta \cdot \xi_{z_r} & Z_g + \Delta \cdot \xi_{z_g} & Z_b + \Delta \cdot \xi_{z_b}
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix},
\]

where \(\xi_{x_r} \in [-X_r/2;+X_r/2], \xi_{x_g} \in [-X_g/2;+X_g/2], \xi_{x_b} \in [-X_b/2;+X_b/2], \xi_{y_r} \in [-Y_r/2;+Y_r/2], \xi_{y_g} \in [-Y_g/2;+Y_g/2], \xi_{y_b} \in [-Y_b/2;+Y_b/2], \xi_{z_r} \in [-Z_r/2;+Z_r/2], \xi_{z_g} \in [-Z_g/2;+Z_g/2], \xi_{z_b} \in [-Z_b/2;+Z_b/2]\), \(-\) uniformly distributed random values, \(\Delta\) – the technological dispersion parameter explained with the screens production quality of the same manufacturer.

The geometric representation of the color reproduction stochastic model is shown in figure 1.

5. Simulation results

There are two-dimensional histogram sections (shaped as distorted ellipses) shown in figure 2, which describe the chromaticity coordinates distribution \(p(x,y)\) for the different technological dispersion.

\[\begin{array}{ccc}
y, & \text{un.} & \text{\(\Delta = 0.1\)} \\
0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 \\
\hline
x, & \text{un.} & \\
0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 \\
\end{array}\]

Figure 1. The color gamut variability of the mass production of screens.
values: $\Delta=0.2$; $\Delta=0.1$; $\Delta=0.05$; $\Delta=0.01$. The point (marked in figure 2 with a mark $«+»$) which has the $(x,y)$-coordinates calculated with the formula (4) when $\Delta=0$ corresponds to the ellipse central point which is the same for all the values of $\Delta$.

The threshold level $L$ for the horizontal histogram section of the $p(x,y)$ chromaticity coordinates distribution is calculated using the following equation:

$$L = \arg \min_{L} \left( \sum_{k_{1}=0}^{N-1} \sum_{k_{2}=0}^{N-1} I\left(H_{k_{1},k_{2}} > L\right) \sum_{k_{1}=0}^{N-1} H_{k_{1},k_{2}} - P_{D} \right), \quad I(q) = \begin{cases} 0, q < 0 \\ 1, q > 0 \end{cases},$$

(5)

where $H_{k_{1},k_{2}}$ - intersection relative frequency of the $(x,y)$-chromaticity coordinates in the histogram subinterval $p(x,y)$; $I(q)$ - indicator function; $N$ - the number of the $p(x,y)$ histogram subintervals for each coordinate; $P_{D}$ - the confidence probability level.

The equation (5) is solved for the $L$ with the use of a numerical computing methods in software MathCad 15.0. The subinterval dimensions are 0.0003 x 0.0003 units of chromaticity values on the $XY$-plane. In figure 2 the horizontal section of the histogram $p(x,y)$ is shown for the confidence probability level $P_{D}=0.999$ in the different parts of the color gamut triangle for five different RGB code values which are non-zero. In particular, if $\Delta=0.2$ the ellipse area includes almost $5 \cdot 10^{5}$ of the $(x,y)$-chromaticity coordinates values.

The section of the histogram $p(x,y)$ corresponds to the section resulted from the theoretical two-dimensional probability density function for the normally distributed mutually dependent random values $(x,y)$:

$$f(x,y) = \frac{1}{2\pi \sqrt{D_{x} D_{y}} \sqrt{1-\rho^2}} \exp \left( -\frac{1}{2(1-\rho^2)} \left( \frac{\left(x-M_{x}\right)^{2}}{D_{x}} - \frac{2\rho(x-M_{x})(y-M_{y})}{\sqrt{D_{x} D_{y}}} + \frac{\left(y-M_{y}\right)^{2}}{D_{y}} \right) \right),$$

$$\rho = \frac{1}{N} \sum_{i=0}^{N-1} (x_{i}-M_{x})(y_{i}-M_{y}) \sqrt{D_{x} D_{y}},$$

(6)

where $\rho$ - correlation coefficient of the random values $(x,y)$; $M_{x}, M_{y}$ - expected value of the random values $(x,y)$; $D_{x}, D_{y}$ - dispersion of the random values $(x,y)$. In the formula (6) the following parameters $M_{x}, M_{y}, D_{x}, D_{y}, \rho$ can be substituted with the correspondent sample estimates after some colorimetric measurements of LCD panel samples are taken.

The equations of the equal probability density ellipse family formed with horizontal section of the function (6) on the level $\lambda$ must be represented as:

$$Q(x,y) = \frac{(x-M_{x})^{2}}{D_{x}} - \frac{2\rho(x-M_{x})(y-M_{y})}{\sqrt{D_{x} D_{y}}} + \frac{(y-M_{y})^{2}}{D_{y}} = \lambda^{2}.$$  

Different but constant probabilities for all ellipse points of the probability density function (6) correspond to the various values of the parameter $\lambda$, called threshold. The probability that random point $(x,y)$ with density function (6) can be found inside the equal probabilities ellipse must be represented as:

$$P_{D}(\lambda) = \frac{1}{2\pi \sqrt{D_{x} D_{y}} \sqrt{1-\rho^2}} \int_{\mathbb{R}^{2}} \exp \left( -\frac{1}{2(1-\rho^2)} Q(x,y) \right) dx dy = 1 - \exp \left( -\frac{\lambda^{2}}{2(1-\rho^2)} \right),$$

where $\lambda^{2} = -2(1-\rho^2)\ln\left(1-P_{D}\right)$, function $g(\lambda)$ specifies points $(x,y)$ located inside the ellipse section area.
Figure 2. The horizontal section of the histogram $p(x,y)$, projected on the XY-plane, and the two-dimensional distribution histogram $p(x,y)$ for the chromaticity coordinates on the XY-plane.

The ellipse sections of the theoretical probability density function $f(x,y)$ on the XY-plane are shown in figure 3. Horizontal sections of the histogram $p(x,y)$ are also presented in figure 3.

6. Conclusion
The analysis of the figure 3 shows that the $(x,y)$-chromaticity coordinates random shift caused by the technological variation of screen color profile parameters is an essential matter to be kept in mind. For example, if $\Delta=0.01$ or higher: the $x$-chromaticity coordinates dispersion is within 0.005 units, the $y$-chromaticity coordinates dispersion is within 0.005 units. The dispersion of 0.005 chromaticity units when $\Delta=0.01$ can be perceived by an observer and can be measured with the modern direct measurement instruments (colorimeters, spectroradiometers). This level of dispersion is acceptable for the quality of the household equipment (monitors, TV-sets, cell phones, i-pad etc.) when the observer looks at only one sample of the display unit. But the value $\Delta=0.01$ is insufficient for manufacturing the aviation display screens when the observer receives simultaneously one the same image from several (up to 6 units) supposed to be identical displays. Today the screen manufacturers provide the screen samples reproducibility with the dispersion level of $0.1 \leq \Delta \leq 0.2$. Proposed mathematical expressions show the
way to development of correction techniques to improve reproducibility of the colorimetric features of displays.

**Figure 3.** The ellipses of histogram section $p(x,y)$ (dotted line) and the theoretical $f(x,y)$ function of probability density (solid line).

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