The Less Intelligent the Elements, the More Intelligent the Whole. Or, Possibly Not?

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July 18, 2023

Abstract

We explore a Leviathan analogy between neurons in a brain and human beings in society, asking ourselves whether individual intelligence is necessary for collective intelligence to emerge and, most importantly, what sort of individual intelligence is conducive of greater collective intelligence. We first review disparate insights from connectionist cognitive science, agent-based modeling, group psychology, economics and physics. Subsequently, we apply these insights to the sort and degrees of intelligence that in the Lotka-Volterra model lead to either co-existence or global extinction of predators and preys.

We find several individual behaviors — particularly of predators — that are conducive to co-existence, eventually with oscillations around an equilibrium. However, we also find that if both preys and predators are sufficiently intelligent to extrapolate one other’s behavior, co-existence comes along with indefinite growth of both populations. Since the Lotka-Volterra model is also interpreted to represent the business cycle, we understand this finding as a condition for economic growth around oscillations. Specifically, we hypothesize that pre-modern societies may not have exhibited limitless growth also because capitalistic future-oriented thinking based on saving and investing concerned at most a fraction of the population.

Keywords: Collective Intelligence, Crowd Wisdom, Social Connectionism, Lotka-Volterra, Prey-Predator, Unlimited Economic Growth
1 Introduction

Since the 1980s, connectionism has been sweeping cognitive sciences with radically innovative concepts such as distributed memories and endogenous formation of mental categories, which made us understand intuition as information loops coming closer to one another and made it the root of intelligence, rather than logic \cite{47} \cite{16}. After conquering our understanding of brains, connectionism eventually ventured into the social realm conjecturing the possibility that some sort of collective mind could arise out of individual interaction \cite{87} \cite{18}. Although an explicit analogy between neurons in the brain and individuals in society has been seldom made, such a correspondence inevitably looms behind any attempt to transpose connectionist ideas onto the social sciences and indeed, a key concept of connectionism such as information loops flowing through neurons appears to be closely mirrored by the concept of organizational routines, namely sequences of actions that the members of an organization eventually repeat over and over within a loop \cite{39} \cite{65}.

According to the connectionist paradigm, information loops are the building blocks of distributed memories, and ultimately of intelligence itself. But if mind and consciousness arise out of huge networks of relatively simple neurons, does anything arise of networks of humans, such as societies and organizations are? Are we possibly the elements of a greater organism we are unaware of?

In spite of the seemingly intractable character of such a question, we submit that there exists a specific angle from which it can be approached in purely scientific terms. Science may be unable to ascertain whether anything like a Superorganism exist, but it can investigate what relations exist between the “intelligence,” or sophistication, of the elements that compose a connectionist system, and the capabilities of the system as a whole. What are the relations between individual intelligence and collective intelligence? What sort of individual abilities are necessary in order for collective abilities to arise? Conversely, what individual abilities are ineffective and useless from a collective point of view? And in its turn, how does collective intelligence impact on individual intelligence? To the extent that “intelligence” can be captured by algorithmic sophistication \cite{69} these are perfectly tractable research questions, that have already been posed in several disciplines indeed.

In particular, Agent-Based Models (ABMs) can be seen as a class of connectionist models that are particularly well-suited to pose and ask such questions because their component elements — the agents — can be quite sophisticated, and generally much more “intelligent” than artificial neurons. Indeed, in the community of agent-based modellers the above questions reflect into a debate between the advocates of the KISS principle (Keep It Simple, Stupid) \cite{23}, who suggest that complex social phenomena should be reproduced by means of minimalist agents, and the purporters of the KIDS approach (Keep It Detailed, Stupid) \cite{22}, who stress that sophisticated individual behavior is often essential in order to explain collective patterns. Some attempts have been made at resolving this debate by comparing specific ABMs, but results have been inconclusive hitherto \cite{21}.

We approached the above questions from a slightly different point of view. We focused on one single model, namely an ABM version of the Lotka-Volterra prey-predator model \cite{51} \cite{84}. However, within this model we explored forms of individual
intelligence that are suggested by empirical findings, theoretical statements and numerical simulations in a wide array of disciplines. Albeit to our knowledge no-one has ever explored the consequences of endowing the predators and preys of the Lotka-Volterra model with degrees of intelligence, the logic of our method mirrors the very common practice of exploring the collective consequences of individual behavior in the simulated and iterated multi-player Prisoner’s Dilemma [6] [54]. Just like researchers ask what individual behaviors generate cooperation in the Prisoner’s Dilemma, we ask what individual behaviors generate co-existence of predators and preys in the Lotka-Volterra model.

In the ensuing sections § 2 and § 3 we distill propositions from several disciplines concerning the relations between individual and collective intelligence in non-hierarchical and hierarchical systems, respectively. In § 4 we translate these propositions into behavioral algorithms that we ascribe to either predators or preys. Besides behaviors that generate co-existence of stable populations we also find behaviors that generate co-existence of exploding populations of both predators and preys, a circumstance that we discuss in the concluding section § 5 as representative of the unlimited growth of capitalistic economies. The mathematical properties of the Lotka-Volterra model, the parameters and outputs of our model and a sensitivity analysis of its results are expounded in appendices § A § B and § C, respectively. Our code is available at OpenABM.\(^1\)

### 2 Individual and Collective Intelligence

Connectionism [35] [85] is based on the idea that intelligence can arise out of a network of relatively simple elements. In a nutshell, we may subsume its Fundamental Idea as follows:

**Proposition 1** (The Fundamental Idea of Connectionism).

*Collective intelligence can arise out of a complex network of elements that are not expected to display individual intelligence.*

This idea echoes, and possibly inspired certain perspectives that in the social sciences assume individual intelligence to be irrelevant for the arousal of collective intelligence [60]. One extreme instance are the so-called “zero-intelligence” models to be found in economics, notably for double-auction markets [30] [31], financial markets [24] and, more in general, prediction markets [62]. Such models typically suggest that even futures markets can work perfectly well with decision makers who have no memory and are incapable of whatsoever form of cognition. At first sight, it might appear that at least in certain settings the Fundamental Idea of Connectionism can be straightforwardly transposed from networks of neurons onto networks of humans.

However, some of the above models have been found to hide constraints that artificially generate their most impressive results [29]. More in general, common sense suggests that the Fundamental Idea of Connectionism may require adaptations in order to be transposed from neurons in a brain to individuals in a society. In the rest of this

\(^1\)URL: www.comses.net/codebases/0eada5b3-3d18-4fc6-92af-841ff0971d28.
section we shall review findings and intuitions from different disciplines concerning
the relationship between individual and collective intelligence, which suggest qualifi-
cations to Proposition (1). Such qualifications may have different degrees validity, they
may may be relevant for different domains, and they may even contradict one another.
They should not be understood as pieces of Truth, but rather as directions to explore in
order to generalize Proposition (1).

The first qualification (2.1) states that some form of heterogeneity of the individual
elements is necessary for a connectionist system to work. The second qualification
(2.2) strengthens the Fundamental Idea of Connectionism by maintaining that individ-
ual intelligence can even impair the emergence of collective intelligence. By contrast,
the third qualification (2.3) suggests that at least the individual ability to make pre-
dictions is often essential in order to obtain some sort of collective intelligence. Thus,
while the first qualification is quite general the second and the third ones may contradict
with one another.

2.1 Simple, but Heterogeneous

The simplest instance of this qualification is possibly provided by Artificial Neural
Networks (ANNs), which are based on stylized neurons that yield an output signal y
by weighting input signals $x_i$ by means of appropriate coefficients $a_i$, $i = 1, \ldots, N$:

$$y = \sum_{i=1}^{N} a_i x_i$$  \hspace{1cm} (1)

Coefficients $a_i$ enable categorization of input signals $x_i$. ANNs arrange a large
number of such neurons in a network that collectively has the ability to classify inputs
into categories whose scope and classification criteria are coded by coefficients $a_i$.

Coefficients $a_i$ change with time, either through a learning phase (supervised net-
works) or by means of feed-backs and -forwards that are built within each neuron
(unsupervised networks). Either case, coefficients $a_i$ change in response to signals $x_i$
that make the network learn which patterns it will recognize.

Learning requires random initialization of coefficients $a_i$ in order to work, for it is
their heterogeneity that makes neurons specialize into specific classes of input signals.
Thus, the initial heterogeneity of coefficients $a_i$ is essential for ANNs to operate.

There are clues that heterogeneity is just as important in many other distributed
systems. In particular, heterogeneity increases the range where critical, neither ordered
nor chaotic states can occur where the most complex computations are made [68]. One
intuitive example is provided by real human brains, whose neurons fire in unison under
epileptic seizures and other mental diseases but exhibit complex patterns otherwise [12]
[56].

One other clue, at a much more aggregate level, comes from the observation of
psychotic group behaviour. According to group psychoanalysis, irrational collective
behavior modes necessarily fall into one of the following categories [14]:

**Fight-Flight** occurs when a group identifies an enemy to fight or escape from. A
leader is selected in order to take control of the group until this task is accom-
plished. It may appear the rational thing to do, except that such mental states can easily degenerate into paranoia.

**Dependence** occurs when a group becomes dependent of an unaccountable leader who is the sole interpreter of some sort of sacred text, law, or moral legacy. Non-conforming individuals are sanctioned and eventually expelled from the group.

**Pairing/Utopian** takes place when a group dives into inaction out of expectation of some sort of messiah who will solve all the problems. At a deeper level, such dreams are bound to sexual fantasies related to giving birth.

Our point is that all three sorts of irrational behavior imply that decision-makers give away their intellectual capabilities in order to conform to a collective state of mind. Be it due to real or imagined threats, sectarian dynamics or daydreaming, all collective irrational behavior is characterized by homogeneity, which is not quintessential of human intelligence and creativity.

Finally, consider crowd wisdom, or the ability of large groups to provide solutions that would be difficult for isolated individuals to reach [41]. Heterogeneity is essential for crowd wisdom as well, for it is based on the fact that different individuals are able to conceive alternative interpretations even if they are exposed to the same information [37] [38].

On the whole, cross-disciplinary evidence suggests the following qualification to Proposition 1:

**Qualification 1.1.** *Heterogeneity of the elements is necessary to reach collective intelligence.*

It is appropriate to remark that a qualification of a general principle is neither a proof nor a matter of fact. However, this specific qualification is likely to be sufficiently basic to hold nearly everywhere.

**2.2 Lack of Individual Intelligence May Be Necessary for Collective Intelligence**

This qualification is meant to reinforce Proposition 1 by making lack of individual intelligence a necessary condition for collective intelligence to arise, rather than a mere possibility. In other words, it implies the existence of a trade-off between individual and collective intelligence.

Limitations of human intellectual capabilities are known in economics and psychology under the label of *bounded rationality.* This expression has been originally conceived with the understanding that bounded rationality is something one has to come to terms with, but certainly not strive to [73] [74].

However, this attitude has been subsequently reversed by the observation that, in quite many situations, simple heuristics can outperform complex rational deliberation in terms of time and efficiency [28] — one very early example being the *tit-for-tat* strategy in the iterated Prisoner’s Dilemma [6]. Such heuristics suggest that rather than rationality, at least in certain settings individual intellectual limitations seem to be
The Traveller’s Dilemma

Two travellers, returning home from a remote island where they have bought identical antiques, discover that those have been damaged during the flight. The airline offers refund according to the following scheme.

Each of the two travelers has to write down the cost of their antique. The two travelers can select any amount between 2$ and 100$. The airline refunds the smallest amount that has been written down. Furthermore, it prizes the traveler who wrote the smallest amount with a bonus of 2$ and charges the other with 2$.

Thus, each traveller has an incentive to write down 99$ in order to get 101$. But then each traveller, by figuring out what the other traveler is likely to do, has an incentive to write down 98$, and so on down to 2$ each if both are perfectly rational decision-makers [8] [9].

necessary for collective intelligence to obtain. Notably, the argument is not that heuristics are often sufficiently close to optimality while being less resource-consuming but, rather, that simple heuristics make us avoid mistakes that we would almost certainly make if we were to use a rational deliberation process. For instance, Box[22] illustrates The Traveller’s Dilemma, a game where rational players endowed with unlimited ability to read one other’s minds (“I think that she thinks that I think that ...”) receive lower payoffs than boundedly rational players who carry out mind reading for a couple of steps at most [8] [9]. Interestingly, when this game is played with real people they invariably behave the “stupid,” more efficient way.

Similar insights have been suggested by computational models of the stock market where both sophisticated and simple agents were assumed to operate. Quite unexpectedly, sophisticated agents made lower profits and, moreover, they were also globally less efficient [77] [33]. In this case, the problem is that employing sophisticated estimation techniques in a very noisy environment is likely to generate overfitting.

Out of the above examples we extract the following qualification to Proposition[1]

Notably, its formulation implies that it is not expected to hold everywhere:

Qualification 1.2. There exist settings where lack of individual intelligence is necessary to reach collective intelligence.

2.3 Prediction and Mind Reading

Striving for simple, albeit heterogeneous agents, is a pervasive principle and a powerful drive throughout connectionist approaches to collective behavior. However, there exists one exception. The one individual ability that — across disciplinary boundaries —
connectionist models suggest might improve collective behavior is the ability to make predictions.

One hint comes from brains, for which the Fundamental Idea of Connectionism was first conceived indeed [35] [85]. In brains, neurons are arranged in layers, and the ability of each layer of neurons to predict the behavior of downstream layers is believed to enable them to detect mistakes and learn by feed-back [17]. Furthermore, primates’ mirror neurons have been shown to allow figuring out what other individuals will likely do [27].

Several computational models broadly confirm the importance of the ability to make predictions. For instance, it has been found that sophisticated robots do not necessarily perform better than those that are based on simple heuristics, except for those that are endowed with the ability to map their environment and predict what other robots will eventually do [70]. Likewise, models of distributed resource allocation has highlighted that agents that are able to extrapolate general trends perform better than those who do not [42] [43].

Shifting to the social domain, one may remark that the above insights resonate fairly well with what we know about the ability of human groups to act as teams, which is essentially based on individuals making efforts to figure out what other group members have in mind [44]. More precisely, team-building rests on a two-sided effort in reading one other’s mind, and communicating intent instead of issuing detailed orders [20]. Experimental tests confirm that indicators of social attentiveness, such as eye movements or taking turns at speaking, make for better team performance [93]. Our point is that although the principles of team-building are expressed in a language that is quite different from those of brain anatomy or computational models of distributed decision-making, coordination necessarily ensues from team members predicting one other’s behavior [66].

Out of these examples we distill the notion that individual ability to make predictions can contribute to collective intelligence. We subsume this concept by means of the following qualification to Proposition 1:

**Qualification 1.3.** Individual intelligence, if employed in order to predict and coordinate behavior, can improve collective intelligence.

While Qualification 1.2 strengthens the Fundamental Idea of Connectionism making it a necessary condition for collective intelligence, Qualification 1.3 rather restrains its validity with the observation that at least one sort of individual intelligence — the ability to make predictions — can be conducive to greater collective intelligence. In a sense, these two Qualifications pull towards opposite directions.

### 3 All Elements are Equal, but Some Elements are More Equal than Others

Connectionism was born with the idea that relatively simple neurons would connect into an extremely complex network where information could circulate in loops, generating distributed associative memories and, somehow, all the higher-order functions
 However, progress in neurosciences has highlighted that neurons in the brain are actually organized in dynamic hierarchies whose layers process increasingly abstract information while exerting influence upon one another. These developments meet a fundamental concern of any attempt at transposing connectionist principles into the social realm, namely the fact that Proposition 1 ignores the existence of hierarchy, which is pervasive of human organizations in fact.

Human organizations are spontaneously capable of collective adaptation to the vagaries of their environments by using alternative routines, and even changing the available routines by means of locally conceived and locally implemented improvements. This generally unproblematic adaptation of collective behavior has been variously labelled as “single-loop” [2], “lower-level” [25], “adaptive” [72], “incremental” [58] or “first-order” [4] organizational learning because it does not require specific individuals to reason and design collective behavior. Basic connectionism as expressed by Proposition 1 is sufficient to reproduce collective adaptation.

However, the possibility that one or a few individuals are capable to grasp organizational dynamics may open up opportunities for steering organizations. In other words, if one or a few individuals become aware of the overall consequences of certain organizational routines, they may conceive alternative arrangements. Borrowing the expression employed in psychology [44], let us call this capability meta-cognition and condense it in the following Proposition 2:

**Proposition 2 (Meta-Cognition).**

*Single individuals may be able to envision the collective intelligence of their organization.*

Proposition 2 opens up possibilities for certain individuals to change the collective intelligence of their organization. Such possibilities evidently depend on hierarchical position [61] [48], a circumstance that suggests qualifications that will be discussed in the ensuing § 3.1. The subsequent § 3.2 and § 3.3 concern the importance of collective agreements and the possibility of unplanned, unwanted changes of the collective dynamics, respectively.

### 3.1 Double-Loop and Deutero-Learning

Double-loop learning [2], also called “higher-level” [25], “adaptive” [72], “radical” [58], “second-order” [4] or “meta” [1] learning, has the action-oriented flavor of management studies where one or a few leaders understand causal relations and conceive innovative solutions that other organization members could not figure out. For instance, in the following episode [44] a few members of a large organization realize that they are pursuing a failing strategy and steer the whole organization into a new direction:

The fire, an order of magnitude greater than anyone had ever seen, is entirely out of control. The commanders pull in all kinds of fire crews, outfitting them and sending them to different parts of the state. Yet the news keeps coming back that they aren’t making much progress.

(...) They decide to stop fighting every fire in the state. They list all the fires, and select the one that will be easiest to put out with available resources. Then they move to the next easiest fire, and so on. (...)
Their shift in strategy isn’t easy. The hardest part is to let some fires go. The crews have been working hard to keep these fires checked. Now they are told that the Forest Service is going to let those fires rage uncontrolled, with the crews transferred elsewhere. It feels like a betrayal. Friendships are broken, some permanently.

It is apparent that this story could only happen because the commanders had the authority to impose the change that they had conceived. Thus, we add the following qualification to Proposition 2:

Qualification 2.1. Meta-Cognition generates double-loop learning if the individuals who envision the collective intelligence of their organization have the authority to steer it towards alternative dynamics.

By contrast, if those individuals who envision the collective intelligence of their organization do not have the authority to steer it, Deutero-learning ensues. Deutero-learning originated in anthropology and it is rather focused on the case of individuals who must passively accept a collective intelligence of which they become aware, whose aims they do not share. For instance, employees experiencing work conditions that are opposite to official company policy are deutero-learning the unwritten rules of the game. Contradiction between official declarations and real life generates a double bind between employer and employees, which negatively impacts on employee behavior.

Let us summarize the possibility of deutero-learning with the following qualification to Proposition 2:

Qualification 2.2. Meta-Cognition generates deutero-learning if the individuals who envision the collective intelligence of their organization are frustrated in their desire to steer it towards alternative dynamics.

Both double-loop and deutero-learning have been inspired by the early cybernetics of W. Ross Ashby. In particular, Ashby stressed a distinction between variables taking different values with given parameters, and changing the parameters themselves. Parameter regulation was eventually seen as a metaphor for going beyond normal operations by means of some sort of double-loop, or deutero-learning.

In system-theoretic terms, single-loop learning occurs when a system moves within its current basin of attraction, whereas double-loop or deutero learning implies that the system’s Lyapunov function changes its shape, eventually enabling jumps towards different basins of attraction. Figure 1 illustrates the difference.

The coefficients that eventually modify the shape of a Lyapunov function are its order parameters. Technically, the difference between first-loop and second-loop, or deutero-learning translates into different time scales, with order parameters regulating slow, unstable dynamics that allow fast and stable dynamics to unfold within a given basins of attraction. One simple example is the anharmonic oscillator (see Box 2). In §4 we shall interpret the consequences of certain individual choices in terms of their impact on order parameters.
The Anharmonic Oscillator

The anharmonic oscillator swings with force $F(x) = -ax - bx^3$. Its state variable $x$ moves along the $x$-axis projection of the Lyapunov function $V(x) = 1/2ax^2 + 1/4bx^4$. With $b > 0$ this Lyapunov function has only one minimum if $a > 0$, but two minima appear if $a < 0$. Thus, $a$ is an order parameter for this system [34].
3.2 Collective Agreement

A serious difficulty arises when different individuals, or coalitions of individuals who are capable of meta-cognition, have competing ideas on how an organization should be steered. This does not merely entail the rather simple case when alternative proposals are openly made, but also the more subtle, more dangerous cases when policies are formally welcome but covertly and effectively opposed. Deutero-Learning may even take the form of sabotage, with the consequence of nullifying any attempt at double-loop learning.

While Game Theory focuses on the intricacies of strategic behavior, we limit ourselves to remark that one obvious solution consists of openly discussing problems without making any decision until the vast majority of stakeholders has agreed upon a course of action \[45\] \[92\]. Albeit difficult to practice, we believe it is worth a qualification to Proposition 2:

Qualification 2.3. Agreement between individuals capable of meta-cognition may be necessary for individual intelligence to influence collective intelligence.

Agreement between individuals makes a certain state of affairs a stable equilibrium, to which the collective intelligence will converge. However, individual intelligence can still make a difference insofar it concerns the speed of convergence.

Macroeconomics has a good point in case. In the 1970s, many Central Banks were trying to stimulate the economy by increasing the money supply in the belief that economic actors would use it to increase GDP, but those actors were eventually able to anticipate that the consequence of more money being made available would be inflation in the long run, which expectation generated inflation already in the short run \[52\]. In a nutshell, greater individual intelligence — eventually labelled as rational expectations — made for faster convergence to a high-inflation, high-unemployment equilibrium. Note that in this case “agreement” exists among economic actors other than the Central Bank, which is treated as an exogenous disturbance in fact.

Let us capture this insight by means of the following qualification to Proposition 2:

Qualification 2.4. If an agreement between individuals who are capable of meta-cognition exists, then individual intelligence accelerates convergence towards the equilibrium generated by this agreement.

3.3 The Extended Butterfly Effect

The butterfly effect epitomizes situations where a tiny small cause, such as a butterfly flapping its wings somewhere on Earth, triggers a chain of events that ultimately generates a substantial consequence such as a tornado some tens of thousands kilometers away. Originally, it was meant to illustrate that non-linearities, through chains of bifurcations, are capable of generating chaotic dynamics which may remain confined around a "strange attractor" nevertheless \[50\]. Such chains of events can happen in connectionist systems, too, making collective intelligence depend on individual decisions that may not imply any substantial degree of “intelligence” however defined.

Thus, Proposition 2 should be complemented by some qualification stating that qualitative transformations of the Lyapunov function can also happen out of chains
of unlikely events that do not necessarily imply meta-cognition. Such a qualification would be loosely inspired by the butterfly effect, but it should extend beyond its original meaning. In particular, the following aspects should be covered:

- Individuals may trigger chains of events that have a substantial impact on collective intelligence without awareness of what they are doing, without meta-cognition, and even without remarking that they are doing anything other than routine.
- Individuals at any hierarchical level may trigger one such chain of events.
- The impact of such disruptive chains of events includes but it is not limited to deterministic chaos. Generation of novel but standard basins of attraction in a Lyapunov function may be generated as well, including limit cycles and stable equilibrium points.

Let us expressed this extended butterfly effect by means of the following qualification to Proposition 2:

**Qualification 2.5.** Meta-cognition is not required for individual intelligence, however limited and however constrained by hierarchical structures, to impact very heavily on collective intelligence.

### 4 Individual and Collective Intelligence of Predators and Preys

In this section we apply the aforementioned insights to the Lotka-Volterra model of predators and preys [51] [84]. Specifically, we explore whether any sort of collective intelligence appears by endowing predators and preys with sorts of individual intelligence inspired by the propositions and qualifications outlined in § 2 and § 3.

In general, co-existence of predators and preys is the preferred outcome in any application of the Lotka-Volterra model, either because one desires some sort of ecological equilibrium, or because the model performs more sophisticated, more interesting computations if species co-exist, or both. Henceforth, we shall take co-existence as an indication of collective intelligence, looking for sorts of individual intelligence that make it appear.

There are several reasons for focusing on the Lotka-Volterra model. The first one is that it is relevant to a number of settings other than animal species, including as diverse applications as the business cycle [32] [63] [75], technological substitution [13] [59], ideological struggles [83] and market-share dynamics [56] [86]. Thus, it arguably reflects important aspects of human relations and interactions. Endowing predators and preys with sophisticated intelligence rather reflects these sorts of applications.

The second reason is that although the basic Lotka-Volterra is a mathematically well-known object, recent research has highlighted that its dynamics change substantially if realistic features are added to it, such as random noise [55] [96] [49] [80], time
delays and discrete dynamics. Thus, it is a good candidate for ABMs that aim at reconstructing collective dynamics out of individual decisions.

However, existing ABMs achieve consistency with the aggregate Lotka-Volterra by adding features that are specific to animal populations, such as availability of food and shelters for preys, or development stages for predators. By contrast, we are interested in collective dynamics that can be influenced by sophisticated individual decisions. Thus, a final reason for investigating the interplay of individual and collective decision-making on the Lotka-Volterra model is that this endeavour has not been attempted hitherto.

Let us begin with the basic Lotka-Volterra model. Let $x \in \mathbb{R}^+$ and $y \in \mathbb{R}^+$ denote the environmental density of preys and predators, respectively. In its simplest version, the Lotka-Volterra model captures their dynamics by means of the following pair of differential equations:

$$
\begin{aligned}
\dot{x} &= ax - bxy \\
\dot{y} &= -cy + dxy
\end{aligned}
$$

where $a, b, c, d \in \mathbb{R}^+$ are suitable constants.

The Lotka-Volterra model has three outcomes:

$E_1 = (0, 0)$ No preys, no predators. If $x(0) = 0$ and $y(0) = 0$, then $x(t) = y(t) = 0 \forall t > 0$.

However, this equilibrium is also approached if $x(0) = 0$ and $y(0) > 0$, in which case $x(t) = 0 \forall t > 0$ and $y(t) = y(0)e^{-ct}$, which implies that $\lim_{t \to +\infty} y(t) = 0$.

$E_2 = (+\infty, 0)$ Only preys, no predators. If $x(0) > 0$ and $y(0) = 0$, $x(t) = x(0)e^{at}$ and $y(t) = 0 \forall t > 0$. The population of preys grows indefinitely because in the limit $\lim_{t \to +\infty} x(t) = +\infty$.

$E_3 = (c/d, a/b)$ Preys and predators coexist. In the basic Lotka-Volterra, the two species co-exist in a limit cycle where both populations oscillate.

where $E_* = (x^*, y^*)$ are the equilibrium coordinates.

Let us look for individual behavior that makes $E_3$ sustain itself. One possible Lyapunov function for the Lotka-Volterra model is:

$$
V(x, y) = d(x - c/d \ln x) + b(y - a/b \ln y) + \text{const}
$$

with $a, b, c$ and $b$ as in eq. 2.

It is possible to show that $E_3$ realizes if coefficients $a$ and $d$ are sufficiently small and coefficients $b$ and $c$ are sufficiently large (see § A). In other words, preys and predators are more likely to co-exist if preys do not grow too quickly and, furthermore, their growth is effectively checked by predators. Predators in their turn should grow with preys, but they must quickly slow down their growth as soon as their population becomes too large.

Let us turn to the discrete-time, agent-based version of the Lotka-Volterra model in order to analyze what behavioural algorithms take the system into either $E_1$, $E_2$ or $E_3$.
Figure 2: Three Lyapunov functions. Left (a), a saddle with equilibria at its extremes. Centre (b), a stable limit cycle along which the two populations oscillate. Right (c), a stable equilibrium point to which the two populations converge. By courtesy of ©Rong Ge.

respectively. Notably, moving onto computational models based on heterogeneous interacting agents implies that Qualification 1.1 applies.

Henceforth, we shall use Wilensky and Reisman’s Wolf Sheep Predation model [88][89] in the NetLogo environment. In order to experiment with behavioral hypotheses we added additional features to the basic model. Our extended version is available at OpenABM.

In Wilensky and Reisman’s Lotka-Volterra, $E_3$ almost never sustains itself. We observed $E_1$ (both preys and predators disappear) 41% of the times, $E_2$ (only preys survive) 58.7% of the times whereas only 0.3% of the times the two species managed to co-exist (see Appendix B for details). Wilensky and Reisman obtained stable cycles by adding a renewable source of energy for preys [89].

We took on a different route. We started from the basic, unstable model that almost invariably ends into either $E_1$ or $E_2$, investigating whether either predators’ or preys’ intelligence, or both, can modify its Lyapunov function to make $E_3$ the centre of a basin of attraction.

Figure 2 qualitatively illustrates this concept by means Lyapunov functions. On the left (a), a saddle from which the system sooner or later ends either in $E_1$ or in $E_2$. This is the sort of Lyapunov function that likely describes the Wilensky-Reisman model as it is. We would like that preys and predators behave in ways such that the Lyapunov function is either as in (b), in which case both species coexist with oscillating populations, or (c), in which case constant populations of predators and preys obtain. Our research question is whether any sort of individual intelligence exists, that helps them reach one of these configurations.

Henceforth we explore what happens to Wilensky and Reisman’s model by endowing preys and predators with alternative sorts of intelligence inspired by the Propositions and Qualifications expounded in § 2 and 3. Details about the model are in Appendix B. All reported results have been averaged over 1,000 runs.

3NetLogo is freely available at https://ccl.northwestern.edu/netlogo. The Wolf Sheep Predation model is in the emphNetLogo standard library.

4Our extended model is available at www.comses.net/codebases/0ead5b3-3d18-4fc6-92af-841ff0971d28/
4.1 Failing to Escape the Fundamental Idea

Suppose that predators and prey are endowed with a degree of intelligence that enables them to devise a reproductive strategy which, in their intentions, should be able to keep the system at \( E_3 \). By exercising single-loop learning they might devise some flexible and adaptive strategy based on a negative feedback (see § 3) which they may expect to generate co-existence. For instance, in order not to be either too many when the other species shrinks, or too few when the other species thrives, either predators, or prey, or both of them may decide to bind their reproduction strategies to the success of the other species:

1. Predators reproduce proportionally to the fraction of prey.
2. Prey reproduce proportionally to the fraction of predator.
3. Predators reproduce proportionally to the fraction of prey and prey reproduce proportionally to the fraction of predator.

Behaviors 1, 2, and 3 appear sensible, but surprisingly, none of them works very well. With 1, the model ends up in \( E_1 \) 65.3% of the times, in \( E_2 \) 12.8% of the times and only 21.9% of the times in \( E_3 \). With 2 and 3 it is even worse, making the model reach \( E_2 \) 100% of the times (predators go extinct, the population of prey grows indefinitely).

One may claim that reproducing proportionately to the fraction of predator is a great strategy for prey because they end up as the sole surviving species independently of what predators do, except for the fact that unlimited growth of one species is not sustainable in the long run. Indeed, collective intelligence is rather understood as co-existence of predators and prey.

Thus, this experiment apparently confirms Proposition 1 (the Fundamental Idea of Connectionism), or even its Qualification 1.2, namely that individual intelligence destroys collective intelligence. However, these results were obtained with mere adaptation, single-loop learning of individuals who were not even attempting to figure out the global consequences of their behavior. Perhaps, meta-cognition can help to devise individual behavior that generates co-existence.

4.2 Agreeing to Coordinate

Let us now explore the possibility that, as expressed by Proposition 2, at least some individuals can envision collective dynamics and therefore are able to influence it by selecting some appropriate behavior. Specifically, let us suppose that either predators, or prey, or both of them take on a collaborative attitude that may favor a collective agreement as prescribed by Qualification 2.3.

Let us suppose that the predators realize that, if there are too many of them, prey will disappear so in the end they will have no food and go extinct. Therefore, they collectively decide to stop reproducing if their population becomes larger than the population of prey.

Likewise, prey may collectively decide to stop reproducing if their population becomes larger than the population of predators. Notably, such a behavior would
be purely altruistic whereas predators have the incentive of not endangering their own species’ long-run survival.

Finally, these behaviors can be combined by assuming that both predators and preys stop reproducing if their own population becomes larger than the other one. While the previous decisions were the outcome of agreements between either predators or preys, the combined behavior may be the outcome of a general agreement involving both populations.

4. Predators stop reproducing if their population becomes larger than the population of preys.

5. Preys stop reproducing if their population becomes larger than the population of predators.

6. Predators stop reproducing if their population becomes larger than the population of preys and preys stop reproducing if their population becomes larger than the population of predators.

Note that 4, 5 and 6 are very different from 1, 2 and 3. Since they have been elaborated out of a shared understanding of global dynamics, they non-linearly depend on global thresholds rather than relying on continuous adaptation.

These reproductive strategies are also remarkably similar to those we found for the mathematical continuous-time model described by eq. 2. Indeed, assuming that predators are willing to stop reproducing if preys are too few (behavior 4) corresponds to a small coefficient $a$ in eq. 2, whereas preys willing to stop reproducing if predators are too few corresponds to a large coefficient $b$. Eventually, small $a$ and large $c$ reinforce the effect of a large $b$ and small $d$.

If predators behave as in 4, $E_1$ is reached only 2.8% of the times, $E_2$ just 0.5% of the times, whereas $E_3$ is reached 96.7% of the times. Thus, this behavior is successful to make predators and preys co-exist with one another. To a closer scrutiny, this 96.7% arises out of a 43.7% of outcomes where the two populations are roughly constant, a 13.2% where the population of preys oscillates whereas predators do not, a 21.0% where the population of predators oscillates whereas preys do not and a 18.8% where both populations oscillate.

However, if preys behave as in 5, the outcome is radically different. In this case, $E_1$ (extinction of both predators and preys) occurs 0.1% of the times, $E_2$ (exclusive survival and indefinite growth of preys) happens overwhelmingly 99.8% of the times whereas with a mere 0.1% $E_3$ (co-existence) is just as unlikely as generalized extinction. Once again, preys’ altruistic behavior turns to their own advantage in the short run but an ever-increasing population of preys is not sustainable in the long run. Once again, collective intelligence is not there.

If both predators and preys take on the collaborative attitude 6, the outcome is somewhat mixed, with $E_1$ inexistent at 0.0% but both $E_2$ and $E_3$ quite substantial at 22.9% and 77.1%, respectively. Co-existence is the most likely outcome but happens less often than in 4. Unlimited growth of preys is substantial but far from inevitable as in case 5.
Here we can notice that, in the Lotka-Volterra model, predators and preys do not enjoy the same status. Predators capable of meta-cognition are in position to steer the ecology towards co-existence by exerting double-loop learning (Qualification 2.1). By contrast, preys capable of meta-cognition understand the system but cannot steer it, as it is typical of deutero-learning (Qualification 2.2). Preys can at most obtain dominance in the short run, but at the cost of unsustainable explosive dynamics for themselves.

One may remark that case 4 is remindful of the sort of relations that humans (the predators) are entertaining with respect to the world ecosystem (the preys). Humans are predators who are capable of meta-cognition, and therefore in a position that enables them to steer the ecosystem towards sustainable co-existence. By contrast, cases 5 and 6 do not appear to have an immediate counterpart in the real world.

4.3 Prediction with Perfect Foresight

The problem with behavioral rules 4, 5 and 6 is that either preys or predators, or both, are supposed to be capable of sacrificing their immediate interests. In practice, this may be difficult to attain.

Is it possible to obtain stable coexistence of species without such a high degree of public spirit? We still want individuals to use their intelligence to understand the system’s global dynamics, but we also want them to pursue their own interests.

Qualification 1.3 is most compatible with Proposition 2, so let us endow our agents with the ability to make predictions based on global understanding of the Lotka-Volterra model that they inhabit. Specifically, predators predict their own extinction if they either see the population of preys or their own population growing very quickly, whereas preys predict their own extinction if they see predators growing very quickly.

If they are rational, neither predators nor preys wait for their Lotka-Volterra world to unfold their inevitable destiny. As Qualification 2.4 suggests, they collectively agree to commit suicide if they see their inevitable end forthcoming. Thus, let us explore the following behavioral rules:

7. Predators commit suicide if either the population of preys is growing much faster than their own population, or if their own population is growing much faster than the population of preys.

8. Preys commit suicide if the population of predators is growing much faster than their own population.

9. Predators commit suicide if either the population of preys is growing much faster than their own population, or if their own population is growing much faster than the population of preys, and preys commit suicide if the population of predators is growing much faster than their own population.

These behaviors require an additional parameter to specify what “much faster” means. We introduced this parameter with the default assumption that “much faster” means ten times faster, carrying out a sensitivity analysis reported in Appendix C.

The results are quite discomforting. With 7, E₂ is reached 100% of the times. With either 8 or 9, E₁ (generalized extinction) is reached 100% of the times.
Notably, by assuming that either predators or preys, or both of them are capable to predict the correct and inevitable outcome of the Lotka-Volterra model, co-existence $E_3$ can never be reached. We may sensibly ask whether Qualification 1.2 is relevant to this case, which implies that some less-than-perfect individual intelligence is necessary in order to reach co-existence. Some limited ability to make predictions, suggested by some understanding of global dynamics but without knowledge of the true model.

4.4 Prediction based on Extrapolation

Let us consider the possibility that predators and preys make predictions by making use of some simple heuristics, combining Proposition 2 with Qualifications 1.3 and 1.2. In particular, let us assume that either predators, or preys, or both of them reproduce out of extrapolation of the other species’ reproductive trends:

10. Predators reproduce with probability proportional to the variation of the number of preys in the last simulation steps.

11. Preys reproduce with probability proportional to the variation of the number of predators in the last simulation steps.

12. Predators reproduce with probability proportional to the variation of the number of preys in the last simulation steps and preys reproduce proportionally to the variation of the number of predators in the last simulation steps.

Similarly to § 4.3, also in this case it is necessary to add a parameter in order to specify how many past steps the above variations are computed on. We introduced this parameter with the default value of five steps while carrying out a sensitivity analysis reported in Appendix C.

Behavior 10 is the second most effective in generating co-existence after 4 (altruistic predators seeking a collective agreement, § 4.2). In particular, with 10 generalized extinction $E_1$ never occurs. Preys-dominated equilibrium $E_2$ is reached 8.4% of the times and, most importantly, $E_3$ is reached 91.6% of the times. Out of this 91.6%, in 7.7% both populations oscillate, 78.1% only the population of predators oscillates, in a 0.4% of cases the opposite happens and, finally, in the remaining 5.4% neither population oscillates. In a nutshell, the most likely outcome is that the population of predators oscillates while the population of preys remains roughly constant.

By contrast, in the opposite case 11 generalized extinction $E_1$ is reached 99.5% of the times, whereas in the remaining 0.5% $E_2$ is reached. Most notably, $E_3$ is never reached. Once again, in the Lotka-Volterra model there exists a clear asymmetry between predators and preys insofar it concerns what they can achieve with a given level of individual intelligence. Specifically, intelligent predators can attain co-existence by making predictions (double-loop learning as in Qualification 2.1), whereas equally intelligent preys cannot (deutero-learning as in Qualification 2.2).

The mixed arrangement 12 is not some sort of average between 10 and 11 but rather an interesting case on its own. With 12, $E_1$ is reached 17.9% of the times, $E_2$ a mere 2.4% whereas $E_3$ is reached 79.7% of the times. Out of this 79.7%, 61.7% of the times both populations oscillate, 1.4% only the population of predators oscillates, with 14.2%
the opposite happens and, finally, in the remaining 2.4% neither population oscillates. Thus, co-existence is reached quite often, and most of the times it is reached with both populations oscillating.

However, this sort of co-existence is quite remarkable because differently from all previous cases, both populations explode. A large herd of preys moves in our artificial space, hunted by an even larger herd of predators. Preys reproduce while running away from predators, predators reproduce while chasing preys generating the two moving columns illustrate in Figure 3.

Figure 4 illustrates the average of the 797 time series that reach co-existence ($E_3$), plotted against one of them in order to show individual variability. Both populations oscillate, with the population of predators growing faster than preys. Thus, while in all previous cases co-existence was associated with sustainability, in case 12 co-existence of predators and preys does not imply a sustainable future for either species.

One may argue that humans are the only species able to make predictions based on variations of the populations of other species, and that since humans are predators, only the case 10 occurs in the real world. Specifically, humans can achieve co-existence with other species by observing their variations and behaving accordingly. This is interesting
in itself, but we already mentioned at the beginning of this section that there exist many interpretations of the Lotka-Volterra model beyond the ecological one.

In particular, let us consider its application to the rather irregular oscillations of economic activities that make economies swing with a highly irregular period of about 8-10 years [32] [53] [75]. The reason for interpreting these oscillations by means of the Lotka-Volterra model goes back to Marx, who was first to remark that capitalists cannot make profits if after many years of a booming economy unemployment has become so low that workers are able to obtain very high salaries, a circumstance which may induce capitalists to stop operating their companies plunging the economy into recession until unemployment becomes sufficiently high to induce workers to accept lower salaries again [57]. By mapping capitalists into predators and workers into preys the Lotka-Volterra model has been used to describe the Marxian theory of the business cycle [52], and by further disaggregating it into firms and households it is possible to recover several empirical stylized facts [53] [75]. However, these models say nothing about economic growth. In order to understand the psychological engine of economic growth, decision-making must be considered.

Capitalism is also a way of thinking, based on postponing consumption in order to save and invest. It requires making extrapolations about the future, a circumstance which, according to our computational experiments, turns oscillations around a fixed point into oscillations around a growing average. Notably, our model suggests that economic growth sets in once all actors have acquired an extrapolation-based way of thinking, a circumstance which in pre-modern societies may have never realized. Even if kings ordered the construction of infrastructures that would benefit society at large in the long run, insofar most people reasoned in terms of enjoying life day-by-day (carpe
diem), indefinite economic growth would not set in.

One interesting question is whether the explosive oscillations originated by behavior\[12\] are an instance of the extended butterfly effect (Qualification\[2.5\]). Do economic actors want indefinite growth, or do they want equilibrium with the rest of the ecosystem? How much aware are they of collective intelligence of the system as a whole? It is clear that insofar Qualification\[2.5\] holds, Proposition\[2\] does not.

5 Conclusions

Applying connectionist principles to the Social Sciences is a long-standing dream, feeding on even older concepts such as “social organism” and “collective mind” that intuitively appeal to our understanding of human organizations. There are many good reasons for pursuing this dream, including a clear analogy between organizational routines and information loops, a less clear but equally fascinating mapping between organizational culture and distributed memory, as well as organizations’ reactions to novel events depending on past experiences pretty much as individuals do.

One difficulty in pursuing this analogy is the appalling relevance of intention and strategic behaviour in human affairs. Although connectionism has moved on from basic neural networks to hierarchical structures of neurons, the gap with the intricacies of human interactions is still wide.

A specular difficulty is defining what “intention” or “strategic behavior” mean at the collective level. We started our exploration of the relationships between individual and collective intelligence within the limited world of the Lotka-Volterra model with the apparently obvious idea of equating collective intelligence to co-existence of predators and preys, whereas indefinite growth would implicitly mark collective stupidity.

Our first results appeared to fit into this framework. Locally adaptive individual behavior yields no collective intelligence whereas globally-thought self-restraint of predators does the miracle of co-existence. Yet the hierarchical nature of the Lotka-Volterra model does not assign preys the same sort of positive impact, for intelligent predators could generate collective intelligence whereas individually intelligent preys could not.

Finally, we explored the impact of individual prediction on global intelligence. Awareness of the Lotka-Volterra model and perfect foresight either brought about generalized extinction or indefinite growth of preys, apparently confirming the initial intuition of connectionism that intelligent individuals would destroy collective intelligence. However, since there exist hints that this may not be the case with less-than-perfect prediction, we tried extrapolation. Here the results have been appalling for our very definition of collective intelligence, for the Lotka-Volterra model achieves at the same time co-existence of the two species, which we had associated with collective intelligence, and indefinite growth of both predators and preys, which we had associated with collective stupidity. Notably, we could not dismiss this case as one that lacks practical significance because the Lotka-Volterra model is also employed to understand the business cycle and decision-making by extrapolation fits very neatly the capitalistic way of thinking based on postponing consumption in order to save and invest.

Out of metaphor, mankind has invented an economic system that is able to generate
an unrelenting flow of innovations out of checked competition between companies and social classes and which, from the point of view of its material achievements, can only be admired as an outstanding example of collective intelligence. Co-existence of predators and preys is being pursued and favored by institutions ranging from anti-trust laws to venture capitalists, generating an unprecedented level of collective intelligence. At the same time, this unrelenting flow of goods and innovations is jeopardizing the very existence of humans on Earth, which can be hardly named a form of intelligence.

Our difficulty in labeling such a collective behavior mirrors opposing values in different branches of Science, with economists pursuing GDP growth while ecologists plea for some sort of stationary state. Can collective intelligence and collective stupidity happen at the same time?

Or, perhaps, can collective intelligence be careless about the individual intelligence of its components? With cancer and parasites we observe the individual intelligence of relentlessly reproducing cells (or insects) can be perfectly careless of the collective intelligence of the organism that is hosting them, preparing their own demise as a consequence of the death of their host. Possibly, collective intelligence has a potential for being equally careless of the intelligence of its individuals, nurturing its own demise as a consequence of the death of its components.

We started our investigation with a research question that we thought would be extremely tough, namely whether and what sorts of individual intelligence could give birth to some sort of collective intelligence, but we ended up finding out that our question was even more intricate than we had suspected. We thought that the causal relation from individuals to collective would be hard enough, but we have to acknowledge that a causal relation from the collective to individuals might exist as well, making the problem even harder to frame.

Extracting general conclusions from our investigation is difficult because of its many limitations. One obvious limitation is that it is based on the basic Lotka-Volterra model only. Extension to the generalized, \( n \)-species Lotka-Volterra would be in order, as well as the exploration of the relationships between individual and collective intelligence in other canonical models. One less obvious limitation is its unwarranted identification of “intelligence” with algorithmic complexity or computational ability, a quality that may overlap with “sophistication” but does not appear to include creativity. Finally, we did not even mention the possibility that collective intelligence may be boosted by repositories of individual intelligence, with examples ranging from social insects’ stigmergy to human libraries progressing through hand-writing to the printed press, to computer memories and the cloud.

We may have opened more questions than we actually solved. We humbly ask our few readers to take it as a stimulus to do better than us, rather than condemning us for not having made enough.
A Lyapunov’s Stability Theorem and its Application to the Lotka-Volterra Model

Henceforth, Lyapunov’s Stability Theorem will be illustrated for two-dimensional systems. Subsequently, it will be applied to the Lotka-Volterra prey-predator model by developing a Lyapunov function specifically designed to highlight the influence of the coefficients of eqs. (2) on the stability of the limit cycle.

Consider a nonlinear two-dimensional dynamical system:

\[
\begin{cases}
  \dot{x} = f(x, y) \\
  \dot{y} = g(x, y)
\end{cases}
\]

where \(x, y \in \mathbb{R}\) and \(f, g \in \mathbb{R}^2 \to \mathbb{R}\).

Without any loss of generality, let us assume that this system has an equilibrium point at \(E = (x^*, y^*)\). We want to know whether this equilibrium is stable.

The Lyapunov Theorem states that if a function \(V : \mathbb{R}^2 \to \mathbb{R}\) exists, such that:

1. \(V(x, y) = 0\) if and only if \((x, y) = (x^*, y^*)\);
2. \(V(x, y) > 0\) if and only if \((x, y) \neq (x^*, y^*)\);
3. \(\dot{V}(x, y) = \frac{\partial V}{\partial x} f(x, y) + \frac{\partial V}{\partial y} g(x, y) \leq 0\) for \((x, y) \neq (x^*, y^*)\);

then \(x = 0\) is a stable equilibrium point. Function \(V(x)\) is called a Lyapunov function.

The Lotka-Volterra model (2) has an equilibrium point at \((x^*, y^*) = (c/d, a/b)\). Let us consider the following function:

\[
V(x(t), y(t)) = d(x - x^* \ln x) + b(y - y^* \ln y) + [d(x^* - x^* \ln x^*) + b(y^* - y^* \ln y^*)]
\]

which we may also write as \(V(x(t), y(t)) = d(x - c/d \ln x) + b(y - a/b \ln y) + \text{const}\).

It is obviously \(V(x^*, y^*) = 0\). Thus, condition (1) is satisfied.

Let us check whether \(V(x, y) > 0\). Let us consider the first term, namely, \(d(x - x^* \ln x)\). It is \(d(x - x^* \ln x) > 0\) if \(dx - c \ln x > 0 \to dx > c \ln x \to x > c/d \ln x \to e^x > e^{c/d \ln x} \to e^x > e^{c/d}\) which is always true because the exponential function yields greater values than the power function. The third term, \(d(x^* - x^* \ln x^*)\), makes sure that the combination of the first and the third term starts to yield positive values just as soon as \(V\) leaves equilibrium \((x^*, y^*)\). Likewise, \(e^x > e^{c/d}\) is always true because the exponential function yields greater values than the power function and the fourth term \(b(y^* - y^* \ln y^*)\) ensures that this only happens outside \((x^*, y^*)\). Thus, condition (2) is satisfied.

Let us turn to the third condition. It is \(\dot{V} = (d - d x^*) (ax - bxy) + (b - b y^*) (-cy + dxy) = ad(x - x^*) - bc(y - y^*) + bd(x^*y - y^*x)\). Obviously, \(\dot{V}(x^*, y^*) = 0\). Let us investigate which parameters make for \(\dot{V}(x, y) < 0\) at \((x, y) \neq (x^*, y^*)\).

Let us explore the sign of \(\partial V/\partial x = ad - bdy^*\) and \(\partial V/\partial y = -bc + bdx^*\). \(V\) is negative if:

\[
\begin{cases}
  ad - bdy^* < 0 \\
  -bc + bdx^* < 0
\end{cases}
\]
We obtain that $\dot{V}$ is negative if:

$$\begin{cases} a < by^* \\ c > dx^* \end{cases}$$

This means that:

- The smaller $a$, the more likely that $\dot{V} < 0$;
- The greater $b$, the more likely that $\dot{V} < 0$.
- The greater $c$, the more likely that $\dot{V} < 0$.
- The smaller $d$, the more likely that $\dot{V} < 0$.

Thus, we can conclude that with small $a$, large $b$, large $c$ and small $d$ the equilibrium $(c/d, a/b)$ is more likely to be stable.

**B Our Agent-Based Lotka-Volterra**

Since we built our model out of Wilensky and Reisman’s model [88] [89], our code keeps naming predators as “Wolves” and preys as “Sheep,” respectively. Specifically, we derived our model from the sheep-wolves version which lacks the agent “Grass.” Correspondingly, we eliminated the parameters grass-regrowth-time, sheep-gain-from-food. By contrast, parameters initial-sheep, initial-wolves, sheep-reproduce, wolf-reproduce and wolves-gain-from-food carry on to our model.

We kept the five parameters that we mutated from Wilensky and Reisman’s model at their original default values, whereas we used switches to select among behavioral configurations in § 4. We introduced two new parameters in order to explore the behavior configurations of § 4.3 and § 4.4 which we named how-much-faster and Timespan, respectively. We kept these two additional parameters at their base values throughout our simulations, carrying out a sensitivity analysis whose results are reported in Appendix C. The ensuing Table summarizes the parameter values of our model for each behavior configuration, as well as the switches that we used to select among them.

By means of the parameters Simulation Length, max-wolves and max-sheep our model runs until either the maximum number of steps is reached, or both wolves and sheep have died out, or either population has reached the maximum allowed. In most behavioral configurations we ran the model for a maximum of 500 steps and allowed a maximum of 500,000 wolves and 500,000 sheep, respectively. These values were chosen because 500 steps were more than enough to observe results, and because the 500,000 threshold was reached only when either wolves or sheep were the only surviving species. However, case 12 was quite different because both populations were growing very fast and, specifically, the population of wolves was growing too large for available computational resources. In order to minimize the number of times when the population threshold would be reached we heightened it from 500,000 to 1,000,000 individuals and, since with this behavioral configuration the pattern became very clear.
| Parameter Name                  | 0     | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    |
|--------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| initial-sheep                  | 100   | 100   | 100   | 100   | 100   | 100   | 100   | 100   | 100   | 100   | 100   | 100   | 100   |
| initial-wolves                 | 50    | 50    | 50    | 50    | 50    | 50    | 50    | 50    | 50    | 50    | 50    | 50    | 50    |
| sheep-reproduce                | 4     | 4     | 4     | 4     | 4     | 4     | 4     | 4     | 4     | 4     | 4     | 4     | 4     |
| wolf-reproduce                 | 5     | 5     | 5     | 5     | 5     | 5     | 5     | 5     | 5     | 5     | 5     | 5     | 5     |
| wolves-gain-from-food          | 20    | 20    | 20    | 20    | 20    | 20    | 20    | 20    | 20    | 20    | 20    | 20    | 20    |
| Wolves-repr-by-frac-sheep      | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   |
| Sheep-repr-by-frac-wolves      | Off   | Off   | On    | On    | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   |
| Stop-rep-if-wolves->sheep      | Off   | Off   | Off   | Off   | Off   | On    | Off   | Off   | Off   | Off   | Off   | Off   | Off   |
| Stop-rep-if-sheep->wolves      | Off   | Off   | Off   | Off   | Off   | Off   | On    | On    | Off   | Off   | Off   | Off   | Off   |
| Wolves-perfect-foresight       | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   |
| Sheep-perfect-foresight        | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | On    | On    | Off   | Off   | Off   |
| how-much-faster                | 10    | 10    | 10    | 10    | 10    | 10    | 10    | 10    | 10    | 10    | 10    | 10    | 10    |
| Wolves-extrapolate             | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | On    | Off   |
| Sheep-extrapolate              | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | Off   | On    | On    |
| Timespan                       | 5     | 5     | 5     | 5     | 5     | 5     | 5     | 5     | 5     | 5     | 5     | 5     | 5     |
| Simulation Length              | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  |
| max-wolves                     | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  |
| max-sheep                      | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  | 500k  |
| Transitory                     | 50    | 50    | 50    | 50    | 50    | 50    | 50    | 50    | 50    | 50    | 50    | 50    | 50    |
| Benchmarking                   | 100   | 100   | 100   | 100   | 100   | 100   | 100   | 100   | 100   | 100   | 100   | 100   | 100   |

Table 1: The switches and parameters of our model, for each behavior configuration. Switches that are “On” for a specific configuration are highlighted in yellow. The basic Wilensky model is denoted as the zero-configuration, the remaining ones are numbered as in § 4. In the top five lines, the five parameters inherited from Wilensky’s model. Subsequently, the two parameters relative to configurations 1, 2 and the two parameters relative to configurations 3, 4, respectively. Then the three parameters that are relative to configurations 5, 6, 7 and 8, respectively. In the bottom lines, the five parameters that have been introduced in order to run and observe the model.
very early, we shortened the simulation length from 500 to 300 steps. Even with these values, the population of wolves hit the threshold in a 7.9% of our 1,000 runs.

In order to observe outputs we also introduced a parameter Transitory and a parameter Benchmarking that we set at 50 and 100 steps, respectively. Between Transitory and Benchmarking the model determines which dynamics count as oscillations. At each step within this interval, the minimum and maximum number of sheep and wolves that have been ever attained are updated. Let us call them $MIN_s, MAX_s, MIN_w$ and $MAX_w$, respectively. Sheep oscillations are counted if $(MAX_s - MIN_s)/2 \geq 0.2(MAX_s + MIN_s)/2$. If this condition is satisfied, one oscillation occurs if the number of sheep is greater or equal to $(MAX_s + MIN_s)/2 + 0.5(MAX_s - MIN_s)/2$ and, subsequently, the number of wolves is smaller or equal to $(MAX_w + MIN_w)/2 - 0.5(MAX_w - MIN_w)/2$. Wolves oscillations are counted in a similar way. Also the possibility of perfect foresight is only considered after Benchmarking steps have elapsed.

The parameters that have been introduced in order to run the model and obtain outputs are listed at the bottom of Table 1. Since we are dealing with a simulation model that must be necessarily stopped after a finite number of steps, we eventually obtain outcomes that exhibit richer details than the mathematically defined $E_1$, $E_2$ and $E_3$. We mapped simulation outcomes onto $E_1$, $E_2$ and $E_3$ as follows:

0. When the simulation is stopped, neither sheep nor wolves exist. This case corresponds to $E_1$.

1. When the simulation is stopped, only wolves exist. Since this is untenable in the long run, we assumed that wolves would go extinct as well. Thus, also this case corresponds to $E_1$.

2. When the simulation is stopped, only sheep exist. This case corresponds to $E_2$.

3. Both sheep and wolves exist when the simulation is stopped, and no oscillation has been observed. We classified this case as $E_3$.

4. Both sheep and wolves exist when the simulation is stopped. The number of sheep has been oscillating, but no oscillation of the number of wolves has been observed. We classified this case as $E_3$.

5. Both sheep and wolves exist when the simulation is stopped. The number of wolves has been oscillating, but no oscillation of the number of sheep has been observed. We classified this case as $E_3$.

6. Both sheep and wolves exist when the simulation is stopped. Both the number of sheep and the number of wolves have been oscillating. We classified this case as $E_3$.

7. The simulation is stopped because the number of sheep reached a threshold imposed by available computational power. When the simulation was stopped the population of sheep was growing faster than the population of wolves. By extrapolating a future where sheep continue to grow whereas wolves become negligible and finally die out we classified this case as $E_2$. 

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8. The simulation is stopped because the number of sheep reached a threshold imposed by available computational power. Contrary to the previous case, when the simulation was stopped the population of wolves was growing faster than the population of sheep. By extrapolating a future where wolves dominate the scene, eat all the sheep and finally go extinct we classified this case as $E_1$.

9. The simulation is stopped because the number of wolves reached a threshold imposed by available computational power. When the simulation was stopped the population of wolves was growing faster than the population of sheep. By extrapolating a future where wolves dominate the scene, eat all the sheep and finally go extinct we classified this case as $E_1$.

10. The simulation is stopped because the number of wolves reached a threshold imposed by available computational power. Contrary to the previous case, when the simulation was stopped the population of sheep was growing faster than the population of wolves. By extrapolating a future where sheep continue to grow whereas wolves become negligible and finally die out we classified this case as $E_2$.

C Sensitivity Analysis

We carried out a sensitivity analysis on the two parameters that we added to Wilensky’s model \cite{88, 89}, namely how-much-faster and Timespan. Since how-much-faster regulates the difference of growth rates between wolves and sheep that activates their perfect foresight, we analyzed its impact when behavioral rules (7), (8) and (9) are adopted. Likewise, since Timespan regulates the time interval over which extrapolations are made, we analyzed its impact when behavioral rules 10, 11 and 12 are adopted.

In both cases we explored the consequences on $E_1$, $E_2$ and $E_3$ of 40% increments and decrements of these parameters with respect to their base values. For how-much-faster this meant exploring the consequences of decreasing it to 6 and increasing it to 14 from its base value 10. For Timespan this meant exploring the consequences of decreasing it to 3 and increasing it to 7 from its base value 5. In both cases we measured the occurrence of $E_1$, $E_2$ and $E_3$ over 1,000 runs.

Table 2 reports the outcomes of the sensitivity analysis for parameter how-much-faster. The outcomes with this parameter at its base value are reported in the three central columns, whereas the outcomes when this parameter is decreased by 40% or increased by 40% are reported in the three columns on their left and their right, respectively. It appears that 40% variations of this parameter have no impact whatsoever.

Table 3 reports the outcomes of the sensitivity analysis for parameter Timespan. The three central columns report outcomes with this parameter at its base value, whereas the three columns on their left and their right report the outcomes when this parameter is decreased by 40% and increased by 40%, respectively.

It appears that behavioral rule 10 — predators extrapolate, preys do not — is the most sensitive to this parameter. In particular, with Timespan = 7 the prevalence of $E_3$ disappears, with $E_2$ and $E_3$ becoming almost equally likely.
By contrast, behavioral rule 11 — preys extrapolate, predators do not — is only marginally affected by Timespan. In spite of $E_2$ and $E_3$ appearing or disappearing across the three values of Timespan, $E_1$ remains the most likely outcome by far.

Finally, behavioral rule 12 — both predators and preys extrapolate — is somehow in between. Similarly to behavior 10, also in this case Timespan = 7 has an impact, not strong enough to destroy the overall pattern but sufficient to make $E_1$ substantially more likely, $E_3$ substantially less likely with respect to the base case.

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