Gamma ray emission from a baryonic dark halo

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Abstract. A re-analysis of EGRET data by Dixon et al has led to the discovery of a statistically significant diffuse γ-ray emission from the galactic halo. We show that this emission can naturally be accounted for within a previously proposed model for baryonic dark matter, according to which dark clusters of brown dwarfs and cold self-gravitating H$_2$ clouds populate the outer galactic halo and can show up in microlensing observations. Basically, cosmic-ray protons in the galactic halo scatter on the clouds clumped into dark clusters, giving rise to the observed γ-ray flux. We derive maps for the corresponding intensity distribution, which turn out to be in remarkably good agreement with those obtained by Dixon et al. We also address future prospects to test our predictions.

1. Introduction

Observations of the diffuse γ-ray emission during the last 20 years [1] have been successfully interpreted in terms of a two-component structure:

(i) a highly anisotropic component strongly concentrated along the galactic disc and
(ii) an apparently isotropic component.

While the former is evidently galactic in nature—being actually accounted for by cosmic ray (CR) interactions in the interstellar medium [2]—the origin of the latter still remains an unsolved problem in high-energy astrophysics (see e.g. [3]–[6]). We will restrict our attention to the latter component throughout the present paper.
12.2

We begin by recalling that EGRET observations have detected a diffuse \( \gamma \)-ray flux \[7\]
\[
\Phi_{\gamma}(E_{\gamma} > 0.1 \text{ GeV}) = (1.45 \pm 0.05) \times 10^{-5} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}
\] (1)
with a spectral slope of \(-2.10 \pm 0.03\), which, for \( E_{\gamma} > 1 \text{ GeV} \), gives
\[
\Phi_{\gamma}(E_{\gamma} > 1 \text{ GeV}) = (1.14 \pm 0.04) \times 10^{-6} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}.
\] (2)

A question naturally arises. Where does the \( \gamma \)-ray emission in question come from? No doubt, its characteristic isotropy calls for an extragalactic origin—an option which is further supported by the fact that it fits remarkably well with the extragalactic hard x-ray background \[8\].

The next question to address is that of whether the considered \( \gamma \)-ray background arises from a truly diffuse process or rather from the contribution of very many unresolved point sources. Both options have received considerable attention. Among the theories of diffuse origin are a baryon-symmetrical Universe \[9\], primordial black hole evaporation \[10, 11\], early collapse of supermassive black holes \[12\], a new population of Geminga-like pulsars \[13\] and annihilation of weakly interacting massive particles (see e.g. \[14\]). Models based on discrete source contributions include a variety of possibilities. What has been clear for a long time is that normal galaxies fail to account for the observed isotropic background—at least as long as their disc emission is considered \[15–18\]—since the corresponding intensity falls short by a factor of about ten with respect to the detected flux. A more realistic option is provided by active galaxies \[19, 20\]. Indeed, blazars seem to yield a successful explanation of the isotropic \( \gamma \)-ray emission \[21–25\]. Finally, a somewhat hybrid model in which the isotropic \( \gamma \)-ray background is produced in clusters of galaxies through the interaction of CRs with the hot intracluster gas has recently been proposed \[26\]. However, this model has been criticized severely \[27, 28\]. In fact, it gives rise to a \( \gamma \)-ray spectral index in disagreement with the observed one and relies upon a value for the CR density in the intracluster space which is too high to be plausible. More generally, it has been shown that the contribution to the isotropic \( \gamma \)-ray emission from clusters of galaxies is negligible \[28\].

Dixon et al \[3, 4\] recently re-analysed the EGRET data concerning the diffuse \( \gamma \)-ray flux with a wavelet-based technique, using the expected (galactic plus isotropic) emission as a null hypothesis. Although the wavelet approach does not allow a good estimate of the errors, they find a statistically significant diffuse emission from an extended halo surrounding the Milky Way. This emission traces a somewhat flattened halo and its intensity at high galactic latitude is \[3, 4\]
\[
\Phi_{\gamma}(E_{\gamma} > 1 \text{ GeV}) \simeq 10^{-7} - 10^{-6} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}.
\] (3)

Clearly, the comparison of equations (2) and (3) entails that the newly discovered halo \( \gamma \)-ray flux is a relevant fraction of the standard isotropic diffuse emission (at least for \( E_{\gamma} > 1 \text{ GeV} \)).

Our aim is to show that the observed halo \( \gamma \)-ray emission naturally arises within a previously proposed model for baryonic dark matter, according to which dark clusters of brown dwarfs and cold self-gravitating \( \text{H}_2 \) clouds populate the outer galactic halo and can show up in microlensing observations \[29–33\]. Basically, CR protons in the galactic halo scatter on the clouds clumped into dark clusters, giving rise to the newly discovered \( \gamma \)-ray flux.

Although we have already pointed out that a signature of the model is a diffuse \( \gamma \)-ray emission from the galactic halo \[29, 30\], a more thorough study is required in order to compare the predicted intensity distribution with the observed one. A short account of these results has
been presented elsewhere [34]. In the present paper, we provide a more exhaustive analysis. In addition, we estimate the $\gamma$-ray emission from the nearby M31 galaxy.

The paper is organized as follows. In section 2 we recall the main features of our model for baryonic dark matter in the galactic halo. In section 3 we address the CR confinement in the galactic halo and we estimate the CR energy density. In section 4 we compute the halo $\gamma$-ray flux—produced by the clouds clumped into dark clusters through proton–proton scattering—that is detected on Earth. Section 5 is devoted to the study of the $\gamma$-ray flux due to inverse Compton (IC) scattering of electrons off background photons. In section 6 we present $\gamma$-ray intensity maps, pertaining both to proton–proton scattering and to IC scattering, and discuss their interplay. Finally, in section 7 we address future prospects to test our predictions.

2. Dark clusters in the galactic halo

Ever since the discovery that standard big-bang nucleosynthesis correctly accounts for the abundance of light elements, it has become clear that most of the baryons in the Universe happen to be in nonluminous form, thereby making a strong case for baryonic dark matter. In order to see how this comes about, we recall that the fraction of the critical density contributed by luminous matter is estimated to be $\Omega_L \sim 0.005$ [35]. Yet, agreement between the predicted and observed abundances of nucleosynthetic yields is achieved only provided that the similar contribution from baryons—in whatever form—lies in the range $0.01 \lesssim \Omega_B \lesssim 0.05$ [36]. Actually, this conclusion has recently been sharpened by deuterium measurements in absorption spectra of quasi-stellar objects, which probe regions of space much farther away than had previously been explored and give $\Omega_B \simeq 0.05$ [37]. So, about 90% of the baryonic matter in the Universe is expected to be dark.

Needless to say, one is naturally led to wonder about the distribution and form of baryonic dark matter. Several possibilities have been contemplated over the last few years. Although no logically compelling reason in favour of any particular option has emerged so far, it is intriguing that a naturalness argument strongly suggests that the galactic dark halos should be predominantly baryonic.

Basically, the idea is as follows. As is well known, both optical and H I observations have shown that all galactic rotation curves exhibit a universal qualitative behaviour: after a steep rise corresponding to the bulge, they stay approximately constant out to the last measured point. This feature—namely the lack of a Keplerian fall-off—provides stark evidence in favour of there being a spheroidal dark halo surrounding the luminous part of any galaxy. This is however not the end of the story. For rotation curves trace the luminous—hence baryonic—matter within the optical disc, but are dominated by the halo dark matter at larger galactocentric distances. Yet both contributions invariably turn out to match smoothly and exactly, thereby signalling a striking visible–invisible conspiracy (also called disc–halo conspiracy). Before proceeding further, a point should be stressed. With only a rather limited sample of available rotation curves, that conspiracy was initially understood as a fine-tuning whereby the disc and the halo of spiral galaxies manage to produce a flat rotation curve [38, 39]. Further studies have shown that such a flatness is only approximate: brighter galaxies tend to have slightly falling rotation curves, whereas fainter ones possess slightly rising rotation curves [40]. Still, what really matters for the visible–invisible conspiracy (as stated above) is the lack of any jump in the rotation curve within the disc–halo transition region, besides the approximate flatness.

† We are using throughout the value of the Hubble constant favoured at present, $H_0 \simeq 70$ km s$^{-1}$ Mpc$^{-1}$.
A priori, only a mysterious fine-tuning could justify the conspiracy in question if the halo dark matter were different in nature from luminous matter, that is to say if it were nonbaryonic. So, baryonic dark matter looks like a natural constituent of galactic halos. Incidentally, this situation is very reminiscent of the case of grand unified theories in particle physics, in which supersymmetry has been invoked as a successful way out of a similar, mysterious fine-tuning needed to stabilize the gauge hierarchy against radiative corrections [41, 42]. Thus, we are led to the conclusion that—much in the same way as fundamental interactions ought to be supersymmetrical—galactic halos ought to be predominantly baryonic!

Remarkably enough, a specific model of baryonic dark halos emerges naturally from the present-day understanding of globular clusters. Indeed, a few years ago we realized [29, 30] that the Fall–Rees theory for the formation of globular clusters [43]–[45] automatically predicts—without any further physical assumption—that dark clusters made of brown dwarfs† and cold 
\[ \text{H}_2 \] clouds should lurk in the galactic halo at galactocentric distances larger than 10–20 kpc. Accordingly, the inner halo is populated by globular clusters, whereas the outer halo chiefly consists of dark clusters‡. We summarize the main features of our model below.

Although the mechanism of galaxy formation is not yet fully understood, the theory for the origin of globular clusters seems to be fairly well established—thanks to the pioneering work of Fall and Rees [43]—and can be summarized as follows. After its initial collapse, the proto-galaxy is expected to be shock heated up to its virial temperature \( \sim 10^6 \text{K} \). Because of thermal instability, density enhancements rapidly grow as the gas cools. Actually, overdense regions cool more rapidly than average, so proto-globular-cluster (PGC) clouds form in pressure equilibrium with the hot diffuse gas. When the PGC cloud temperature drops to \( \sim 10^4 \text{K} \), recombination of hydrogen atoms occurs: at this stage, the PGC cloud mass and size are \( \sim 10^5(R/kpc)^{1/2}M_\odot \) and \( \sim 10(R/kpc)^{1/2} \text{pc} \), respectively (\( R \) being the galactocentric distance). Below \( \sim 10^4 \text{K} \), efficient cooling can be brought about only by photon emission from roto-vibrational transitions in \text{H}_2. Whether this mechanism actually operates crucially depends on the intensity of the environmental ultraviolet (UV) radiation field, as we are now going to discuss.

In fact, in the central region of the proto-galaxy an active galactic nucleus and a first population of massive stars are expected to form, which act as strong sources of UV radiation that dissociates the \text{H}_2 molecules. It is not difficult to estimate that the destruction of \text{H}_2 should occur for galactocentric distances smaller than 10–20 kpc. As a consequence, cooling is heavily suppressed in the inner halo, so here the PGC clouds remain for a long time in quasi-hydrostatic equilibrium at a temperature of \( \sim 10^4 \text{K} \), resulting in the imprinting of a characteristic mass \( \sim 10^6 M_\odot \). Eventually, the UV flux decreases, thereby allowing the formation and survival of \text{H}_2. Accordingly, the PGC clouds can cool further, collapse and fragment, ultimately producing ordinary stars clumped into globular clusters.

What is most relevant for the present considerations is that, in the outer halo—namely for galactocentric distances larger than 10–20 kpc—no substantial destruction of \text{H}_2 should take place, owing to the suppression over distance of the UV flux. Therefore, here the PGC clouds monotonically cool, collapse and fragment. When their number density exceeds

† Although we concentrate our attention on brown dwarfs, it should be mentioned that red dwarfs as well can be accommodated within the considered setting.
‡ Similar ideas have been proposed by Ashman and Carr [46], Ashman [47], Fabian and Nulsen [48, 49], and Kerins [50, 51]. Moreover, a scenario almost identical to the one investigated here has been put forward by Gerhard and Silk [52]. Somewhat different baryonic pictures have been worked out by Pfenniger et al [53], Sciama [54] and Gibson and Schild [55] (see also [56]).

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\[ \sim 10^8 \text{ cm}^{-3} \], virtually all hydrogen gets converted into molecular form by three-body reactions \((H + H + H \rightarrow H_2 + H \text{ and } H + H + H_2 \rightarrow H_2 + H_2)\), which in turn increases the cooling efficiency dramatically [57]. As a result, no imprinting of a characteristic mass on the PGC clouds shows up and the fragment Jeans mass can drop to values considerably smaller than \(M_\odot\). The fragmentation process stops when the PGC clouds become optically thick to their own line emission—this happens for a fragment Jeans mass as low as \(\sim 10^{-2}M_\odot\) [57]. In this manner, dark clusters containing brown dwarfs in the mass range \((10^{-2}–10^{-1})M_\odot\) should form in the outer halo. Typical values of the dark cluster radius are \(\sim 10\) pc.

In spite of the fact that the dark clusters resemble globular clusters in many respects there is an important difference. Since practically no nuclear reactions occur in the brown dwarfs, strong stellar winds which are at present lacking. Therefore the leftover gas—which is ordinarily expected to exceed 60\% of the original amount—is not expelled from the dark clusters but remains confined inside them. Thus, also cold gas clouds are clumped into the dark clusters. Although these clouds are primarily made of \(H_2\), they should be surrounded by an atomic layer and a photo-ionized ‘skin’. Typical values of the cloud radius are \(\sim 10^{-5}\) pc.

Besides accounting for the halo dark matter in a natural fashion—without demanding any new physical assumption—this model elegantly explains the visible–invisible conspiracy. For whether ordinary matter is luminous or dark ultimately depends on the intensity of the environmental UV radiation field during the proto-galactic epoch—no fine tuning is indeed involved! Moreover, the UV field in question is expected to be stronger for brighter galaxies. Accordingly, brighter galaxies should have the dark clusters lying farther away from the galactic centre than do fainter galaxies, thereby making the contribution of dark matter to the rotation curve of brighter galaxies less significant than that for fainter ones: this circumstance precisely agrees with the above-mentioned observed pattern of rotation curves [40].

Observationally, the present model makes a crucial prediction: very high-energy CR proton scattering on the clouds should give rise to a detectable diffuse \(\gamma\)-ray flux from the halo of our galaxy. This topic will be dealt with in great detail in the next sections.

Further support in favour of the baryonic scenario in question comes from the understanding of the extreme scattering events (ESEs): dramatic flux changes over several weeks during monitoring of compact radio quasars [58]. It is generally agreed that ESEs are not intrinsic variations, but rather apparent flux changes caused by refraction when a (partially) ionized cloud crosses the line of sight. Walker and Wardle [59] recently pointed out that the first consistent explanation of ESEs requires the refracting clouds to have precisely the same properties as those of the cold \(H_2\) clouds predicted by the present model (it is their photo-ionized ‘skin’ that causes the radio wave refraction).

Last but not least is the issue of massive astrophysical compact halo objects (MACHOs), which have been detected since 1993 in microlensing experiments towards the Magellanic Clouds. Regrettfully, their origin remains controversial. Although the events detected towards the Small Magellanic Cloud seem to be a self-lensing phenomenon [60, 61], a similar interpretation of all the events discovered towards the Large Magellanic Cloud looks unlikely [62]. Yet—even if most of the MACHOs are dark matter candidates lying in the galactic halo—their physical nature remains unknown, since their average mass strongly depends on the still uncertain galactic model, ranging from \(\sim 0.1M_\odot\) for a maximal disc up to \(\sim 0.5M_\odot\) for a standard isothermal sphere.

Superficially, white dwarfs look to be the best explanation, but the resulting excessive metallicity of the halo makes this option untenable, unless their contribution to halo dark matter

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is not substantial (see [63, 64]). So, some variations on the theme of brown dwarfs have been considered.

An option is that the galactic halo resembles a minimal halo (maximal disc) more closely than it does an isothermal sphere, in which case MACHOs can still be brown dwarfs†. In this connection, two points should be stressed. First, a large fraction (up to 50% in mass) can be binary systems—much like ordinary stars—thereby counting as objects of twice the mass [66]. Second, within our model brown dwarfs can actually be beige dwarfs—with masses substantially larger than $\simeq 0.1 M_\odot$—as suggested by Hansen [67], since slow accretion from cloud gas is likely to occur [68].

An alternative possibility has been pointed out by Kerins and Evans [69]. Since, in the present model, the initial mass function obviously changes with the galactocentric distance‡, it could well happen that brown dwarfs dominate the halo mass density without, however, dominating the optical depth for microlensing. What are MACHOs then? Quite recently, faint blue objects discovered by the Hubble Space Telescope have been understood to be old halo white dwarfs lying closer than $\sim 2$ kpc from the Sun [71]–[73]: they look to be good candidates for MACHOs within this context.

Finally, we remark that recent ISO observations [74] of the nearby NGC891 galaxy have detected a huge amount of molecular hydrogen, which might account for almost all dark matter, at least within its optical radius. Other observations suggest that similar clouds are also present farther away [75]. In addition, Sciama [54] has argued that a known excess in the far-infrared emissivity of our galaxy (over that expected from a standard warm interstellar dust model) would be naturally accounted for by a population of cold H$_2$ clouds building up a thick galactic disc.

3. Cosmic ray confinement in the galactic halo

Neither theory nor observation at present allows one to make definitive statements about the propagation of CRs in the galactic halo‡. Therefore, the only possibility of gaining some insight into this issue rests upon the extrapolation from the knowledge of CR propagation in the disc. Actually, this strategy looks sensible, since the main effect is CR scattering on inhomogeneities of the magnetic field over scales from $10^2$ pc down to less than $10^{-6}$ pc [76] and—according to our model—inhomogeneities of this kind are expected to be present in the halo as well, because of the existence of molecular clouds—with a photo-ionized ‘skin’—clumped into dark clusters. Indeed, typical values of the dark cluster radius are $\sim 10$ pc, whereas typical values of the cloud radius are $\sim 10^{-5}$ pc [33].

It is well known that CRs up to energies of $\sim 10^6$ GeV are confined in the galactic disc for $\sim 10^7$ years [76]. It can be shown that in the diffusion model for the propagation of CRs, the escape time $\tau_{esc}$ is given by [76]

$$\tau_{esc} \simeq \frac{R^2_h}{3D(E)} \left[ 1 - \frac{1}{2} \left( \frac{h_d}{R_h} \right)^2 + \frac{1}{8} \left( \frac{h_d}{R_h} \right)^3 \right]$$

† Notice that also the H$_2$ clouds can give rise to microlensing events [65].
‡ Evidence for a spatially varying initial mass function in the galactic disc has been reported [70].
‡ We stress that—contrary to the practice used in the CR community—by halo we mean the (almost) spherical galactic component which extends beyond $\sim 10$ kpc.

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where $D(E)$ is the diffusion coefficient, while $h_d$ and $R_h$ are the half-thickness of the disc and the radius of the confinement region, respectively. We remind the reader that—for CR propagation in the disc—the diffusion coefficient is $D(E) \simeq D_0 (E/7 \text{ GeV})^{0.3} \text{ cm}^2 \text{ s}^{-1}$ in the ultra-relativistic regime, whereas it reads $D(E) \simeq D_0 \simeq 3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$ in the nonrelativistic regime [76]. CRs escaping from the disc will further diffuse in the galactic halo, where they can be retained for a long time, owing to the scattering on the above-mentioned small inhomogeneities of the halo magnetic field†.

Indirect evidence that CRs are in fact trapped in a low-density halo has recently been reported. For example, Simpson and Connell [78] argue on the basis of the cosmic ratio of isotopic abundances $^{26}\text{Al}/^{27}\text{Al}$ that the CR lifetimes are perhaps a factor of four larger than had previously been thought, thereby implying that CRs traverse an average density smaller than that of the galactic disc.

A straightforward extension of the diffusion model implies that the time for escape of CRs from the halo (of size $R_H \equiv R_h \sim 100 \text{ kpc}$, much larger than the half-thickness of the disc) $\tau_{\text{esc}}^H$ is given by

$$\tau_{\text{esc}}^H \simeq \frac{R_H^2}{3D_H(E)}$$

where $D_H(E)$ is the diffusion coefficient in the galactic halo.

As a matter of fact, radio observations in clusters of galaxies yield for the corresponding diffusion constant $D_0$ a value similar to that found in the galactic disc [79]‡. So, it looks plausible that a similar value for $D_0$ also holds on intermediate length scales, namely within the galactic halo. Given the lack of any further information on the energy dependence of $D_H(E)$, we assume the same dependence as that established for the disc. Hence, from equation (5) we find that, for energies $E \lesssim 10^3 \text{ GeV}$, the time taken for escape of CRs from the halo is greater than the age of the Galaxy $t_0 \sim 10^{10} \text{ years}$ (notice that below $\tau_{\text{esc}}^H$ gets even longer below the ultra-relativistic regime). As a consequence—since the CR flux scales like $E^{-2.7}$ (see the next section)—protons with $E \lesssim 10^3 \text{ GeV}$ turn out to give the main contribution to the CR flux.

We are now in a position to evaluate the CR energy density in the galactic halo, getting

$$\rho_{CR}^H \simeq \frac{3t_0 L_G}{4\pi R_H^2} \simeq 0.12 \text{ eV cm}^{-3}$$

where $L_G \simeq 10^{41} \text{ erg s}^{-1}$ is the galactic CR luminosity [81]. Notice, for comparison, that $\rho_{CR}^H$ turns out to be about one tenth of the disc value [82]. In fact, this value is consistent with the EGRET upper bound on the CR density in the halo near the Small Magellanic Cloud [83].

We remark that we have taken specific realistic values for the various parameters entering the above equations in order to make a quantitative estimate. However, somewhat different values can be used. For instance, $R_H$ may range up to $\sim 200 \text{ kpc}$ [35], whereas $D_0$ might be slightly larger than the above value, e.g. $\sim 10^{29} \text{ cm}^2 \text{ s}^{-1}$, consistently with our assumptions. Moreover, $L_G$ can be as large as $3 \times 10^{41} \text{ erg s}^{-1}$ [84]. It is easy to see that these variations do not substantially affect our previous conclusions.

† A similar idea has been proposed with a somewhat different motivation in [77].
‡ Moreover, we note that average magnetic field values in galactic halos are expected to be close to those of galaxy clusters, i.e. $0.1$–$1 \mu\text{G}$ [80].

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4. Proton–proton scattering in the galactic halo

We proceed to estimate the halo $\gamma$-ray flux produced by the clouds clumped into dark clusters through the interaction with high-energy CR protons. CR protons scatter on cloud protons giving rise (in particular) to neutral pions, which subsequently decay into photons. A highly nontrivial question concerns the opacity effects in the clouds. Quite recently, Kalberla et al [85] addressed precisely this issue, showing that optical-depth effects both for protons and for photons are negligible within our model. Finally, we expect an irrelevant high-energy ($\geq 100$ MeV) $\gamma$-ray photon absorption outside the clouds, since the mean free path is orders of magnitudes larger than the halo size.

Insofar as the energy dependence of the halo CRs is concerned, we adopt the same power law as that applied in the galactic disc (see below) [82]

$$\Phi^{H}_{CR}(E) \simeq \frac{A}{\text{GeV}} \left( \frac{E}{\text{GeV}} \right)^{-\alpha} \text{ particles cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}. \quad (7)$$

The constant $A$ is fixed by the requirement that the integrated energy flux agrees with the above value of $\rho_{H}^{0}$. Explicitly

$$\int d\Omega \, dE \, E \, \Phi^{H}_{CR}(E) \simeq 5.7 \times 10^{-3} \text{ erg cm}^{-2} \text{ s}^{-1}$$

where for definiteness we take the integration range to be $1 \text{ GeV} \leq E \leq 10^{5} \text{ GeV}$. A nontrivial point concerns the choice of $\alpha$. As an orientation, the observed spectrum of primary CRs on Earth would yield $\alpha \simeq 2.7$. However, this conclusion cannot be extrapolated to an arbitrary region in the halo (and in the disc), since $\alpha$ crucially depends on the diffusion processes undergone by CRs. For instance, the best fit to EGRET data in the disc towards the galactic centre yields $\alpha \simeq 2.45$ [86], thereby showing that $\alpha$ is increased by diffusion. Given the lack of any direct information, we conservatively take $\alpha \simeq 2.7$ even in the halo, but in table 1 we report some results for different values of $\alpha$ for comparison. At any rate, the flux does not vary substantially.

Let us next turn our attention to the evaluation of the $\gamma$-ray flux produced in halo clouds through the reactions $pp \rightarrow \pi^{0} \rightarrow \gamma\gamma$. Accordingly, the source function $q_{\gamma}(> E_{\gamma}, \rho, l, b)$—yielding the photon number density at distance $\rho$ from the Earth with energy greater than $E_{\gamma}$—is [82]

$$q_{\gamma}(> E_{\gamma}, \rho, l, b) = \frac{4\pi}{m_{p}} \rho_{H_{2}}(\rho, l, b) \sum_{n} \int_{E_{\gamma}(E_{\gamma})}^{\infty} dE_{p} \, dE_{\gamma} \, \Phi^{H}_{CR}(E_{p}) \frac{d\sigma_{p+\gamma}^{n}(E_{\pi})}{dE_{\pi}} \, n_{\gamma}(E_{p}) \quad (9)$$

where the lower integration limit $E_{p}(E_{\gamma})$ is the minimal proton energy necessary to produce a photon with energy $> E_{\gamma}$, $\sigma_{p+\gamma}^{n}(E_{\pi})$ is the cross-section for the reaction $pp \rightarrow n\pi^{0}$ ($n$ is the $\pi^{0}$ multiplicity), $\rho_{H_{2}}(\rho, l, b)$ is the halo gas density profile and $n_{\gamma}(E_{p})$ is the photon multiplicity.

Unfortunately, it would be exceedingly difficult to keep track of the clumpiness of the actual gas distribution in the halo, so we assume that its density is smooth and goes like the dark matter density—anyhow, the very low angular resolution of $\gamma$-ray detectors would not permit one to distinguish between the two situations (evidently this strategy would be meaningless if optical-depth effects were not negligible). Accordingly, the halo gas density profile reads

$$\rho_{H_{2}}(x, y, z) = f \rho_{0}(q) \frac{\hat{a}^{2} + R_{0}^{2}}{\hat{a}^{2} + x^{2} + y^{2} + (z/q)^{2}} \quad (10)$$

for $(x^{2} + y^{2} + z^{2}/q^{2})^{1/2} > R_{min}$ ($R_{min} \simeq 10$ kpc is the minimal galactocentric distance of the dark clusters in the galactic halo). We recall that $f$ denotes the fraction of halo dark matter in the

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Table 1. The halo $\gamma$-ray intensity at high galactic latitude for a spherical halo evaluated for $R_{\text{min}} = 10$ and 15 kpc at energies above 0.1 and 1 GeV, for various values of the CR spectral index $\alpha$, is given in units of $10^{-7} \gamma \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$.

| $R_{\text{min}}$ (kpc) | $E_{\gamma}$ (GeV) | $\alpha$ | $\Phi_{\text{DM}}^{\gamma}(b = 90^\circ)$ |
|------------------------|---------------------|---------|-----------------------------------|
| 10                     | $> 0.1$             | 2.45    | 62                               |
|                        | 2.70                | 59      | 49                                |
|                        | 3.00                | 3.3     |                                   |
| 10                     | $> 1.0$             | 2.45    | 37                               |
|                        | 2.70                | 35      |                                   |
|                        | 3.00                | 29      |                                   |
| 15                     | $> 0.1$             | 2.45    | 37                               |
|                        | 2.70                | 35      |                                   |
|                        | 3.00                | 29      |                                   |
| 15                     | $> 1.0$             | 2.45    | 6.5                              |
|                        | 2.70                | 4.0     |                                   |
|                        | 3.00                | 1.9     |                                   |

Form of gas, $\rho_0(q)$ is the local dark matter density, $\tilde{a} = 5.6$ kpc is the core radius and $q$ measures the halo flattening. For the standard spherical halo model $\rho_0(q = 1) \simeq 0.3$ GeV cm$^{-3}$, whereas it turns out that e.g. $\rho_0(q = 0.5) \simeq 0.6$ GeV cm$^{-3}$.

In order to proceed further, it is convenient to re-express $q_{\gamma}(> E_{\gamma}, \rho, l, b)$ in terms of the inelastic pion production cross-section $\sigma_{in}(p_{lab})$. Since

$$\sigma_{in}(p_{lab}) \langle n_{\gamma}(E_p) \rangle = \sum_n \int dE_\pi \frac{\sigma_{n-p-\pi}^n(\pi_n)}{dE_\pi} n_{\gamma}(E_p)$$  \(11\)

equation (9) becomes

$$q_{\gamma}(> E_{\gamma}, \rho, l, b) = \frac{4\pi}{m_p} \rho_H^2(\rho, l, b) \int_{E_\rho(E_{\gamma})}^{\infty} dE_p \Phi_{CR}(E_p) \sigma_{in}(p_{lab}) \langle n_{\gamma}(E_p) \rangle$$  \(12\)

where $\rho_H(\rho, l, b)$ is given by equation (10) with $x = -\rho \cos b \cos l + R_0$, $y = -\rho \cos b \sin l$ and $z = \rho \sin b$.

For the inclusive cross-section of the reaction $pp \rightarrow \pi^0 \rightarrow \gamma\gamma$ we adopt the Dermer [87] parametrization

$$\sigma_{in}(p) \langle n_{\gamma}(E_p) \rangle = 2 \times 1.45 \times 10^{-27} \times \begin{cases} 0.032\eta^2 + 0.040\eta^6 + 0.047\eta^8 & 0.78 \leq p \leq 0.96 \\ 32.6(p - 0.8)^{3.21} & 0.96 \leq p \leq 1.27 \\ 5.40(p - 0.8)^{0.81} & 1.27 \leq p \leq 8.0 \\ 32 \ln p + 48.5p^{-1/2} - 59.5 & p \geq 8.0 \end{cases}$$  \(13\)

where $p$ is the proton laboratory momentum in GeV/c, the factor 2 comes from the fact that each pion decays into two photons, whereas 1.45 accounts for the CR composition [87], which
12.10

includes also heavy nuclei. The quantity

\[ \eta \equiv \frac{[(s - m_\pi^2 - m_p^2)^2 - 4m_\pi^2m_p^2]^{1/2}}{2m_\pi s^{1/2}} \]  

(14)
is expressed in terms of the Mandelstam variable \( s \), while \( m_\pi \) and \( m_p \) are the pion mass and the proton mass, respectively.

Because \( dV = \rho^2 \phi \Omega \), it follows that the observed \( \gamma \)-ray flux per unit solid angle is

\[ \Phi_{DM}^\gamma(> E_\gamma, l, b) = \frac{1}{4\pi} \int_{\rho_1(l,b)}^{\rho_2(l,b)} d\rho \quad q_\gamma(> E_\gamma, \rho, l, b). \]  

(15)

So, we find

\[ \Phi_{DM}^\gamma(> E_\gamma, l, b) = f_{\rho_0(q)} \frac{m_p}{I_1(l,b)} I_2(> E_\gamma) \]  

(16)

where \( I_1(l,b) \) and \( I_2(> E_\gamma) \) are defined as

\[ I_1(l,b) \equiv \int_{\rho_1(l,b)}^{\rho_2(l,b)} d\rho \left( \frac{a^2 + R_0^2}{(a^2 + x^2 + y^2 + z/q)^2} \right) \]  

(17)

\[ I_2(> E_\gamma) \equiv \int_{E_p(E_\gamma)}^{\infty} dE_p \Phi_C^{H\gamma}(E_p) \sigma_{in}(p_{lab}) \langle n_\gamma(E_p) \rangle \]  

(18)

and \( m_p \) is the proton mass.

According to the discussion in sections 2 and 3, typical values of \( \rho_1(l,b) \) and \( \rho_2(l,b) \) in equations (15) and (17) are 10 kpc and 100 kpc, respectively. Numerical values for \( \Phi_{DM}^\gamma \) in the cases \( \alpha = 2.45, 2.7 \) and 3.0 are reported in table 1.

5. Inverse Compton scattering

Another mechanism whereby \( \gamma \)-ray photons are produced is inverse Compton (IC) scattering of high-energy CR electrons off galactic background photons. Here we estimate the resulting flux, while the interplay between proton–proton scattering and IC scattering will be discussed in the next section.

The electron injection spectrum which best fits the locally observed electron spectrum is given by the following power law valid for \( E_e \gtrsim 10 \) GeV (see e.g. [88])

\[ I_e(E_e; \rho, l, b) = K(\rho, l, b) E_e^{-a} \ e^{-cm^{-2} \ s^{-1} \ sr^{-1} \ GeV^{-1}} \]  

(19)

with \( a \simeq 2.4 \) and \( K_0 \equiv K(0) \simeq 6.3 \times 10^{-3} \ e^{-cm^{-2} \ s^{-1} \ sr^{-1} GeV^{-1}} \) (the value of \( K_0 \) is obtained by normalizing equation (19) with respect to the observed local CR electron spectrum at 10 GeV). Since \( a \) is somewhat model dependent (in particular it depends on the diffusion processes), its actual value is not well determined and indeed it could be as low as \( a \simeq 2 \) [5] or even \( a \simeq 1.8 \) [6]. However, what is relevant is the electron spectrum where the \( \gamma \)-ray production occurs and—due to diffusion processes—the value of \( a \) is expected to increase with the distance from the galactic plane where the electrons are mostly produced.

In order to estimate the galactic radiation field, we adopt the model of Mazzei, Xu and De Zotti [89] for the photometric evolution of disc galaxies. This model reproduces well the present broad-band spectrum of the Galaxy over about four decades in frequency, from the UV to the far IR. Accordingly, the two main contributions to the galactic radiation field come from stars
at wavelength $\lambda \sim 1 \mu\text{m}$ and diffuse dust at $\lambda \sim 100 \mu\text{m}$. The total stellar luminosity of the Galaxy is $L_\ast \sim 3.5 \times 10^{10} L_\odot$ and the amount of starlight absorbed and re-emitted by dust is $L_d \sim 1.2 \times 10^{10} L_\odot$ (see e.g. [89, 90]). Regarding the photon energy distribution, we can roughly approximate the emission spectrum (see figure 4 in [89]) with the sum of two Planck functions with temperatures $T_\ast \sim 2900 \text{ K}$ and $T_d \sim 29 \text{ K}$, respectively.

According to the previous assumptions, the source function $q_{ph}(E_\gamma)$ for $\gamma$-ray production through IC scattering is given by [76]

$$q_{ph}(E_\gamma) = \frac{1}{2} \sigma_T \left( \frac{4}{3} \langle \epsilon_{ph}(T_{\ast,d}) \rangle \right)^{(a-1)/2} \left( mc^2 \right)^{1-a} K_0 E_\gamma^{-(a+1)/2} \gamma \text{ s}^{-1} \text{ sr}^{-1} \text{ GeV}^{-1}. \tag{20}$$

Here $\langle \epsilon_{ph}(T_{\ast,d}) \rangle \sim 8kT_{\ast,d}/3$ is the average energy of background photons emitted by stars or dust and $\sigma_T$ is the Thompson cross-section. In deriving equation (20), we use the fact that the $\gamma$-ray energy is related to the electron and background photon energies according to

$$\langle E_\gamma \rangle = \frac{4}{3} \langle \epsilon_{ph}(T_{\ast,d}) \rangle \left( \frac{E_e}{mc^2} \right)^2 \tag{21}$$

so that very high-energy electrons are needed in order to produce $\gamma$-rays. For example, a $\gamma$-ray with $E_\gamma \simeq 1 \text{ GeV}$ produced by this mechanism requires $E_e \simeq 170 \text{ GeV}$ for a target photon emitted by dust, while $E_e \simeq 17 \text{ GeV}$ is demanded for starlight.

The intensity of diffuse galactic $\gamma$-rays of energy $E_\gamma$ produced in this way and coming to Earth along the line of sight $(l, b)$ turns out to be

$$\Phi_\gamma(>E_\gamma, l, b) = \int_0^\infty d\rho \langle n_{ph}(\rho, l, b) \rangle f_e(\rho, l, b) \int_{E_\gamma}^\infty q_{ph}(E_\gamma) dE_\gamma \gamma \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \tag{22}$$

where we have introduced the function $f_e(\rho, l, b) \equiv K(\rho, l, b)/K_0$ as the ratio of the electron CR intensity to the local intensity, while $\langle n_{ph}(\rho, l, b) \rangle$ is the average density of background photons. Let us next focus our attention on the functions $f_e(\rho, l, b)$ and $\langle n_{ph}(\rho, l, b) \rangle$. The electron component of CRs is galactic in origin, mainly produced by supernovae and pulsars located inside the disc. Electrons diffuse through the Galaxy and their distribution is energy dependent and not uniform, namely, the characteristic diffusion length scale decreases for higher electron energy. This feature cannot be described in the framework of the widely used leaky box model, so in order to obtain the electron density at an arbitrary point in the Galaxy, one has to resort to the transport equation (see e.g. [5, 76]). Unfortunately, several fairly unknown parameters enter this equation, such as the electron diffusion coefficient, the rate at which electrons lose energy, the density of sources and the electron spectrum.

An alternative approach relies upon the experimental evidence of the thick disc†, in which high-energy electrons may be retained for a long time before escaping into the galactic halo. Indeed, the observed characteristics of the radio emission spectra of our and other galaxies lead to a relative density distribution of electrons $f_e(R_0, z) \equiv n_e(z)/n_e(0)$ extending up to 5–12 kpc perpendicularly to the galactic plane, as shown in figure 5.29 of [76]. These numerical results can be approximated by $f_e(R_0, z) = \exp[-(z/z_e)^{3/2}]$, with the parameter $z_e$ depending on the electron energy. From equation (21) and the ensuing discussion, it turns out that $z_e \simeq 2.5 \text{ kpc}$ for $E_e \simeq 170 \text{ GeV}$ while $z_e \simeq 3.5 \text{ kpc}$ for $E_e \simeq 17 \text{ GeV}$. Insofar as the radial dependence of the electron distribution is concerned, we assume that $f_e(R, 0)$ follows the same $R$ dependence as

† Often defined as the ‘halo’ by the CR community.
where the total disc luminosity as

\[ f_e(R, 0) = \exp \left[ 0.48 - 0.36(R/R_0) - 0.12(R/R_0)^2 \right]. \quad (23) \]

However, following Bloemen [91]—who suggested a stronger radial gradient for the electron component of the CRs—we also tested the effect of using a steeper radial electron distribution on the IC γ-ray flux. As expected, the corresponding results show that the IC γ-ray flux does not change significantly for galactic longitudes |l| ≤ 90° (irrespective of the latitude values), whereas it increases by up to a factor of two at l = 180° for |b| ≤ 30°.

The last quantity to be specified in equation (22) is the average background photon density \( \langle n_{ph}(\rho, l, b) \rangle \) or, equivalently, the background photon flux \( \Phi_{ph}(\rho, l, b) \) emitted by stars and dust

\[ \langle n_{ph}(\rho, l, b) \rangle = \frac{\Phi_{ph}(\rho, l, b)}{c} \gamma \text{ cm}^{-3}. \quad (24) \]

Note that the photon flux \( d\Phi_{ph}(\rho, l, b) \) at a point \( P(\rho, l, b) \) from the solid angle \( d\Omega \) subtended by an infinitesimal area \( dl' \) centred at \( P'(R', \phi', z' = 0) \) on the galactic plane is given by

\[ d\Phi_{ph}(\rho, l, b) = I_{*,d}(R') \left( \frac{d\Omega}{4\pi} \right) \cos \alpha \gamma \text{ cm}^{-2} \text{ s}^{-1} \quad (25) \]

where \( \alpha \) is the angle between the normal to the area \( dl' \) and the direction PP'. We can trace the surface brightness \( I_{*,d}(R') \) to the stellar/dust distribution. Assuming that visible matter makes up an exponential disc, we set

\[ I_{*,d}(R') = A_{*,d} e^{-(R'-R_0)/h_{*,d}} \gamma \text{ cm}^{-2} \text{ s}^{-1} \quad (26) \]

where \( h_{*,d} \simeq 3.5 \text{ kpc} \) is the scale length for the visible matter and the constant \( A_{*,d} \) is fixed by the total disc luminosity as

\[ \int_0^{R_d} I_{*,d}(R') 2\pi R' dR' = \frac{L_{*,d}}{2 \epsilon_{ph}(T_{*,d})} \gamma \text{ s}^{-1}. \quad (27) \]

In this way, we get \( A_* \simeq 4.71 \times 10^{20} \gamma \text{ cm}^{-2} \text{ s}^{-1} \) and \( A_d = 1.64 \times 10^{22} \gamma \text{ cm}^{-2} \text{ s}^{-1} \), with \( R_d \simeq 15 \text{ kpc} \). By integrating equation (25) over the galactic disc, we find

\[ \Phi_{ph}(\rho, l, b) = \int_0^{R_d} \int_0^{2\pi} I_{*,d}(R') R' dR' d\phi' \left( \frac{\cos \alpha}{4\pi|PP'|^2} \right) \gamma \text{ cm}^{-2} \text{ s}^{-1}. \quad (28) \]

Finally, by using equations (24), (26) and (28)—and recalling equation (20)—equation (22) can be rewritten in the form

\[ \Phi_{\gamma}^{IC}(>E_{\gamma}, l, b) = J_1(l, b) J_2(>E_{\gamma}) \gamma \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (29) \]

where we have set

\[ J_1(l, b) \equiv \int_0^{\infty} f_e(\rho, l, b) d\rho \int_0^{R_d} \int_0^{2\pi} \left( \frac{\cos \alpha}{4\pi|PP'|^2} \right) R' dR' d\phi' e^{-(R'-R_0)/h_{*,d}} \text{ cm} \quad (30) \]

\[ J_2(>E_{\gamma}) \equiv \frac{A_{*,d}}{2c \tau T} \left[ \frac{4}{3} \left( \epsilon_{ph}(T_{*,d}) \right) \right]^{(a-1)/2} \left( mc^2 \right)^{1-a} K_0 \int_{E_{\gamma}}^{\infty} E_{\gamma}^{-(a+1)/2} dE_{\gamma} \gamma \text{ cm}^{-3} \text{ s}^{-1} \text{ sr}^{-1}. \quad (31) \]

Numerical values of \( \Phi_{\gamma}^{IC}(>E_{\gamma}, l, b) \) at high galactic latitude are exhibited in table 2 and plotted in figure 2.
Table 2. The galactic diffuse $\gamma$-ray intensity due to IC scattering of high-energy electrons on background photons from stars and dust is given (in units of $10^{-7} \gamma \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$) for $a = 2.0, 2.4$ and $2.8$. The results for $a = 2, 2.8$ are reported for illustrative purposes. We adopt the following values: $T_{\star} = 2900$ K, $L_{\star} = 3.5 \times 10^{10} L_{\odot}$ and $T_{d} = 29$ K, $L_{d} = 1.5 \times 10^{10} L_{\odot}$.

| $z_e$ (kpc) | $E_\gamma$ (GeV) | $\Phi_{\gamma}^{IC}(90^\circ)$ | $\Phi_{\gamma}^{IC}(90^\circ)$ | $\Phi_{\gamma}^{IC}(90^\circ)$ |
|-------------|-----------------|-----------------|-----------------|-----------------|
| Stars       | 3.5             | $>0.1$          | 3.8             | 3.5             | 3.4             |
|             | $>1.0$          | 1.2             | 0.7             | 0.4             |
| Dust        | 2.5             | $>0.1$          | 12              | 4.4             | 1.7             |
|             | $>1.0$          | 3.8             | 0.9             | 0.2             |

Table 3. Rough values of the measured residual $\gamma$-ray flux at $E_\gamma \geq 1$ GeV (after subtraction both of the isotropic background and of the standard galactic diffuse component) for various galactic latitudes and longitudes (interpolated from figure 3 in [3]) are given. Fluxes are given in units of $10^{-6} \gamma \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$.

| $b$ (deg) | $l = 0^\circ$ | $l = 60^\circ$ |
|-----------|--------------|--------------|
| 45$^\circ$| 1            | 1            |
| 30$^\circ$| 2            | 1.5          |
| 15$^\circ$| 5.5          | 2            |

6. Discussion

Our main results are maps for the intensity distribution of the $\gamma$-ray emission from baryonic dark matter (DM) in the galactic halo and from IC processes in the galactic disc. In order to make the discussion definite, we take the fraction of halo dark matter in the form of molecular clouds $f \simeq 0.5$. Insofar as the IC emission is concerned, the standard electron spectral index $a = 2.4$ is used. We stress that the shape of the IC maps does not depend on the value of $a$.

In figure 1 we exhibit the contour plots in the first quadrant of the sky ($0^\circ \leq l \leq 180^\circ, 0^\circ \leq b \leq 90^\circ$) for the halo $\gamma$-ray flux $\Phi_{\gamma}^{DM}(E_\gamma > 1 \text{ GeV})$. Corresponding contour plots for $E_\gamma > 0.1$ GeV are identical, up to an overall constant factor equal to 8.74, as follows from equation (16).

Figure 1(a) refers to a spherical halo, whereas figure 1(b) pertains to a $q = 0.5$ flattened halo. Regardless of the value adopted for $q$, $\Phi_{\gamma}^{DM}(E_\gamma > 1 \text{ GeV})$ lies in the range $(6-8) \times 10^{-7} \gamma \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ at high galactic latitude. However, the shape of the contour lines strongly depends on the flatness parameter. Indeed, for $q \gtrsim 0.9$ there are two contour lines (for each flux value) that are approximately symmetrical with respect to $l = 90^\circ$ (see figure 1(a)). On the other hand, for $q \lesssim 0.9$ there is a single contour line (for each value of the flux) which varies much less with the longitude (see figure 1(b)).
Figure 1. Contour values for the $\gamma$-ray flux due to the DM at $E_\gamma > 1$ GeV are given for the indicated values in units of $10^{-7} \gamma$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$, in the cases of (a) a spherical halo and (b) a flattened halo with $q = 0.5$.

As we can see from table 1 and figure 1, the predicted value for the halo $\gamma$-ray flux at high-galactic latitude is close to that found by Dixon et al [3, 4] (see also table 3). This conclusion holds almost irrespectively of the flatness parameter.

Moreover, the comparison of the overall shape of the contour lines in our figures 1(a) and 1(b) with the corresponding ones in figure 3 of [3] entails that models with flatness parameter $q \lesssim 0.8$ are in better agreement with the data, thereby implying that it is most likely that the halo dark matter is not spherically distributed. This result has also been confirmed recently in the analysis in [85].

In figure 2 we present contour plots for the $\gamma$-ray flux due to the IC scattering for $E_\gamma > 1$ GeV. The corresponding contour plots for $E_\gamma > 0.1$ GeV are identical, up to an overall constant factor equal to 5 (this follows from equation (31)). The contour lines decrease with increasing longitude.

We remark that equation (16) yields $\Phi_{\gamma,DM}(E_\gamma > 0.1 \text{ GeV}) \simeq 5.9 \times 10^{-6} \gamma$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$ at high galactic latitude (for a spherical halo). This value is roughly 40% of the diffuse $\gamma$-ray emission of $(1.45 \pm 0.05) \times 10^{-5} \gamma$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$ found by the EGRET team [7] and in
agreement with the conclusion of Dixon et al. [3, 4] that the halo $\gamma$-ray emission is a relevant fraction of the standard isotropic diffuse flux also for $E_\gamma > 0.1$ GeV.

Nevertheless, given the large uncertainties both in the data and in the model parameters (such as for instance the electron scale height and the electron spectral index $a$), one might also explain the observations with a nonstandard IC mechanism [6]. Our calculation, however, seems to indicate that the IC contour lines in figure 2 decrease much more rapidly than do the observed ones for the halo $\gamma$-ray emission (see figure 3 in [3]). More precise measurements with the next generation of satellites are certainly required in order to settle the issue.

7. Gamma rays from the halo of M31

Since M31 resembles our galaxy, the discovery of Dixon et al. [3, 4] naturally leads to the expectation that the halo of M31 should give rise to a $\gamma$-ray emission as well. Below, we will try to address this issue in a quantitative manner, assuming that the halo of M31 is structurally similar to that of our galaxy and that our model for baryonic dark matter is correct.

We suppose that the various parameters entering the calculations in sections 3 and 4 take similar values for M31 and for the Galaxy, apart from the central dark matter density of M31 being $\rho(0) \simeq 2.5 \times 10^{-24}$ g cm$^{-3}$ and the core radius of M31 being $\tilde{a} \simeq 5$ kpc. Accordingly, the evaluation of the corresponding flux $\Phi_{\gamma \text{halo}}^{M31}$ proceeds as before, with only minor modifications. Specifically, we can use again equation (16)—with $I_2$ still given by equation (18)—but now $I_1$ is to be replaced by $L_1$ (see below), in order to account for the different geometry. Notice that $f$ in equation (16) now denotes the fraction of halo dark matter of M31 that is in the form of H$_2$ clouds.

Consider a generic point P in the halo of M31 and let $R$ and $r$ denote its distance from the centre O of M31 and from Earth, respectively. Since the distance of O from the Earth is $D \simeq 650$ kpc, we have $R(r) = (r^2 + D^2 - 2rD \cos \theta)^{1/2}$, where $\theta$ is the angular separation between P and O as seen from the Earth. For simplicity, we suppose that the M31 halo is described by an
isothermal sphere with radius $R_H$ and density profile
\[ \rho(R) = \frac{\rho(0)}{1 + (R/a)^2}. \]  \hfill (32)

Note that the ensuing amount of dark matter in M31 turns out to be about twice as large as that of the Galaxy.

According to the discussion in section 2, the dark clusters should populate only the outer halo of M31. So, we compute $\Phi_{\gamma, \text{halo}}^{M31}$ from regions of the M31 halo with $R_{\text{min}} < R < R_H$, with $R_{\text{min}} \simeq 10$ kpc and $R_H \simeq 100$ kpc, for definiteness. It is easily seen that the values of $\theta$ corresponding to $R_{\text{min}}$ and $R_H$ are $\theta_{\text{min}} \simeq 1^\circ$ and $\theta_H \simeq 9^\circ$, respectively.

We are now in a position to compute $L_1$, which reads
\[ L_1 = 2\pi \int_{\theta_{\text{min}}}^{\theta_H} \sin \theta \, d\theta \int_{r_{\text{min}}}^{r_{\text{max}}} dr \left( \frac{\tilde{a}^2}{\tilde{a}^2 + R^2(r)} \right) \simeq 1.9 \times 10^{20} \text{ cm sr} \]  \hfill (33)
with $r_{\text{max}}(\theta) \equiv D \cos \theta + (-)(R_H^2 - D^2 \sin^2 \theta)^{1/2}$. Recalling equations (16) and (18), we get
\[ \Phi_{\gamma, \text{halo}}^{M31}(E_\gamma > E_\gamma) = 1.9 \times 10^{20} f \rho(0) m_p I_2(E_\gamma) \gamma \text{ cm}^{-2} \text{ s}^{-1}. \]  \hfill (34)

Observe that regions of M31 halo with angular separation less than $\theta_{\text{min}}$ from O do not contribute in equations (33) and (34), so $\Phi_{\gamma, \text{halo}}^{M31}$ should be regarded as a lower bound on the total $\gamma$-ray flux from the M31 halo. Specifically, equation (34) yields
\[ \Phi_{\gamma, \text{halo}}^{M31}(E_\gamma > 0.1 \text{ GeV}) \simeq 3.5 \times 10^{-7} f \gamma \text{ cm}^{-2} \text{ s}^{-1}. \]  \hfill (35)

This value has to be compared both with the $\gamma$-ray flux from the M31 disc and with the $\gamma$-ray emission from the halo of the Galaxy. The former quantity has been estimated to be $\simeq 0.2 \times 10^{-7} \gamma \text{ cm}^{-2} \text{ s}^{-1}$ for $E_\gamma > 0.1 \text{ GeV}$ [92, 93] within a field of view of $1.5^\circ \times 6^\circ$, whereas the latter quantity, integrated over the entire field of view of the M31 halo, is $\simeq 4.3 \times 10^{-7} \gamma \text{ cm}^{-2} \text{ s}^{-1}$ for $E_\gamma > 0.1 \text{ GeV}$, according to our results in sections 4 and 6.†

Insofar as observation is concerned, no $\gamma$-ray flux from M31 has been detected by EGRET. Accordingly, the EGRET team has derived the upper bound [94]
\[ \Phi_{\gamma, M31}(E_\gamma > 0.1 \text{ GeV}) \lesssim 0.8 \times 10^{-7} \gamma \text{ cm}^{-2} \text{ s}^{-1}. \]  \hfill (36)

Unfortunately, a direct comparison between equations (35) and (36) is hindered by the fact that equation (36) is derived under the assumption of a point-like source.

Clearly, a good angular resolution of about $1^\circ$ or less is necessary in order to discriminate between the halo and disc emission from M31. So, the next generation of $\gamma$-ray satellites such as AGILE and GLAST can test our predictions.

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† For simplicity, we suppose here that the halo of the Galaxy is spherical and we employ equation (16) with $f = \frac{1}{2}$. 

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