A New QCD Correction to Gauge Boson Decay into Heavy Flavor

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We find that, at order $\alpha_s$, the partial width of $Z^0$ to heavy flavors receives a power correction from a novel QCD mechanism, which is not suppressed by inverse powers of $M_Z$, but only by two unknown $O(\Lambda_{\text{QCD}}/m)$ constants. The hadronic $W$ width also receives a similar correction. These parameters may be fitted from the global electroweak analysis, and consequently the Standard Model predictions of various electroweak observables will be updated. This new mechanism is of no help to reconcile the discrepancy in $b$ forward-backward asymmetry. We also point out the implication of this mechanism to heavy flavor production in other collider experiments.

The precision electroweak measurements at LEP and SLC provide the most accurate knowledge of the fundamental parameters in the Standard Model (SM), and meanwhile place stringent constraints on the new physics beyond SM. Though the overall agreement between the measurements and the SM fits is acceptable, few acute discrepancies still persist. For the $Z$-pole observables, there are currently a 2.4 $\sigma$ discrepancy in the forward-backward asymmetry of $b$ quark ($A_{FB}^b$), and a 1.9 $\sigma$ deviation in the peak hadronic cross section ($\sigma_{\text{had}}^0$). While there was a significant deviation in $R_b \equiv \Gamma(\bar{b}b)/\Gamma_{\text{had}}$ in past years, the latest measurement is consistent with the SM prediction at 1 $\sigma$ level.

In order to search the possible hint of new physics, one needs first scrutinize all the uncertainties within SM. Ubiquitous (nonperturbative) QCD effects constitute a particularly important and difficult source of the theoretical uncertainties. This is best illustrated by muon $g-2$, where the current theory error is dominated by the low energy hadronic contribution.

In this Letter, we will report a novel QCD mechanism which renders a new correction to the partial width of $Z^0$ to heavy flavors at order $\alpha_s$. It can be calculated in perturbative QCD, up to two unknown constants of $O(\Lambda_{\text{QCD}}/m)$. Incorporating this new effect will generally influence the current SM predictions of many electroweak observables, especially $\Gamma_{\text{had}}, \sigma_{\text{had}}^0$, as well as $R_b, R_c$. It will also affect the hadronic $W$ width in a similar way. We also comment on the implication of this mechanism to the heavy flavor production in other collider experiments.

The hadronic $Z^0$ width has been measured to per mille accuracy. To match such a precision, radiative corrections from both QCD and electroweak sectors must be computed in comparable orders. For instance, perturbative QCD corrections have been included to 3-loop order for massless quark, and $b$ quark mass effects have been included up to order of $m_b^3/M_Z^0$.

Aside from the leading power contribution, the hadronic width of $Z^0$ are also affected by power corrections. For $Z^0$ decaying into light hadrons, the well-known power corrections are characterized by the quark and gluon condensates, whose effects are suppressed by a factor of $1/M_Z$. Therefore they can be neglected in accordance with present experimental precision.

The situation for heavy flavors is dramatically different. Because the heavy quark mass sets a new scale, the most significant power correction may start at order $\Lambda_{\text{QCD}}/m$, thus more important than those suppressed by powers of $\Lambda_{\text{QCD}}/M_Z$ and $m/M_Z$. Surprisingly, this possibility has been largely overlooked, perhaps due to the difficulty for the standard Operator Product Expansion (OPE) to tackle such a multi-scale problem.

However, as we will see, the recently-developed heavy-quark recombination mechanism (HQR) can realize such an $O(\Lambda_{\text{QCD}}/m)$ correction to $Z^0$ hadronic width. In fact, when HQR was introduced and first applied to the $B$ hadroproduction, it was shown that HQR generates an $O(\Lambda_{\text{QCD}}/m_b)$ correction to the total $B$ cross section. With a more concrete hadronization picture, HQR was originally motivated as a “higher twist” mechanism, to supplement the usual heavy quark fragmentation. However, by studying $Z^0$ decay into $B$ at $O(\alpha_s^2)$, one recently realizes that under some circumstances, HQR can overlap with the fragmentation mechanism. In that work, the $O(\Lambda_{\text{QCD}}/m_b)$ contribution from HQR must be identified with the contribution to the fragmentation function. Had we been able to extract the finite “higher twist” contribution by removing the “leading twist” term, it would represent an $O(\alpha_s^2\Lambda_{\text{QCD}}m_b/M_Z^2) \sim 10^{-6}$ correction to the partial width of $Z^0$ to $b\bar{b}$, thus negligible with current experimental precision. In fact, the goal of this work is to show there is a new HQR process occurring at order $\alpha_s$ only, with a genuine “higher twist” contribution of order $\Lambda_{\text{QCD}}/m_b$. Therefore, it is mandatory to consider its impact on the heavy flavor observables.

The central picture of HQR is that, a heavy quark can bind with a light parton which emerges from the hard-scattering and carries an $O(\Lambda_{\text{QCD}})$ momentum in the rest frame of heavy quark. It is somewhat analogous to that a proton can capture an incident low energy electron to form a hydrogen atom. The word parton deserves some elaboration. Intuitively, one expects that $b$ and $q$ are more inclined to bind into a $\overline{B}$ meson, and $bq$ diquark tends to evolve into a $b$ baryon. Indeed, the $bq$ and $cq$ recombination have been developed and applied...
to a variety of fixed-target experiments to explain the observed charm meson and baryon asymmetries \[6, 7\].

The last recombination mechanism awaiting exploration then is the **bg recombination**, when the parton is a gluon. At first sight, it seems in contradiction with the picture of constituent quark model. Indeed, this objection is justified for the heavy quarkonium case, e.g., it is unlikely for **bg** to evolve into \(B_c\). However, the dynamics of heavy-light hadron is rather different from that of heavy quarkonium. Since the soft gluon in \(bg\) can easily split into \(q\bar{q}\) pair, and \(bg\) can easily pick up a light antiquark from the vacuum to hadronize, there is no dynamical reason for **bg** recombination to be suppressed relative to \(b\bar{b}\) and **bg** recombination. On the other hand, **bg** recombination does receive one parametric suppression from the large \(N_c\) consideration. There is a \(1/N_c\) suppression for **bg** to evolve into a \(\overline{B}\) meson relative to \(b\bar{b}\), since it requires an extra \(q\bar{q}\) pair. Similarly, it is even less probable for **bg** to evolve into a b baryon than \(\overline{B}\), due to a further penalty of \(1/N_c\).

At order \(\alpha_s\), bottoms are produced through \(Z^0 \rightarrow b\bar{b}g\). This represents an ordinary 3-jet event if each parton independently fragments. Nevertheless, in a small corner of phase space where \(g\) is soft in the rest frame of \(b\), they can form an intermediate \(bg\) state with definite color and angular momentum. This state then hadronizes into a \(b\) baryon plus additional soft hadrons. Therefore, we end up with a jet containing a \(b\) hadron from the recombination and a recoiling \(\overline{b}\). The corresponding Feynman diagrams are depicted in Fig. 1. We label the momenta of \(bg\) and \(\overline{b}\) by \(p\) and \(\overline{p}\).

The \(b\bar{b}\) recombination respects a simple multiplicative factorization \[13\]. It is suggestive that the inclusive \(\overline{B}\) production from \(bg\) recombination may also be written in a factorized form (The symbol \(\overline{B}\) schematically represents any ground state hadron containing a \(b\) quark):

\[
\Gamma[\overline{B}] = \sum_n \Gamma[Z^0 \rightarrow bg(n) + \overline{b}] \xi[bg(n) \rightarrow \overline{B}],
\]

where \(\Gamma_n\) are the perturbatively calculable partonic cross sections, and \(\xi_n\) are so-called recombination factors, which roughly amount to the probability for \(b\) and \(g\) to evolve into a state including \(\overline{B}\). The color and angular momentum quantum numbers of \(bg\) are collectively labeled by \(n\). The inclusive \(B\) production from \(bg\) recombination is identical to Eq. 11, because of \(CP\) invariance.

These \(\xi_n\) factors are analogous to those \(\rho_n, \eta_n\) associated with \(b\bar{b}\) and \(bg\) recombination. Thus far, a rigorous definition in terms of nonperturbative matrix elements is only available for \(\rho_n\) \[13\], but generalization to \(\eta_n\) and \(\xi_n\) should be straightforward. We also assume \(\xi_n^b = \Lambda_{QCD}/m_b\), the same as \(\rho_n\) \[13\].

The \(bg\) that emerges from the hard-scattering can be in either of three irreducible color representations: 3, \(\overline{3}\) or 15. Fortunately, due to very simple color structure, only the color-triplet channel survives in this process. The color-triplet state \(bg\) can be labeled by a fundamental \(SU(3)\) index \(i\) and has the normalized color wave function

\[
|bg(3)\rangle = \frac{\sqrt{3}}{2} T^3 a \langle b_j | g_a \rangle ,
\]

where \(SU(3)\) generators \(T^a\) \((a = 1 \cdots 8)\) satisfy the normalization \(\text{tr}(T^a T^b) = \delta_{ab}/2\). In more complicated \(bg\) recombination processes, the other two color channels will also contribute.

We also need project the \(bg\) onto the states of definite angular momentum. We will consider only the S-wave states, because the higher orbital angular momentum states will result in towers of \(\Lambda_{QCD}/m_b\) suppression. Therefore, the \(bg\) state can be either \(2S_1/2\) or \(4S_3/2\), corresponding to a spin-\(1/2\) or spin-\(3/2\) \((\text{Rarita-Schwinger})\) fermion. The momentum eigenstates of these fermions are conveniently represented by a vector-spinor \(u^a_\mu(p)\) \[13\], which carries an extra Lorentz index relative to the Dirac spinor. Both of \(u^{i = 1}_\mu(p)\) and \(u^{i = 2}_\mu(p)\) satisfy the on-shell condition \(p \cdot u^a_\mu(p) = m_b\) and transversality constraint \(p \cdot u^a_\mu(p) = 0\). The spin-\(1/2\) vector-spinor is subject to a further constraint \(p^\mu u^a_\mu = 0\).

In addition to projecting the amplitude onto states of definite color and spin, we need isolate those most singular terms when the gluon has small momentum in the rest frame of the \(b\) quark \[13\]. This can be accomplished by setting \(p_g \rightarrow x_g p\), where \(x_g\) is a momentum fraction that is presumably of order \(\Lambda_{QCD}/m_b\), and then taking the limit \(x_g \rightarrow 0\). The factor \(1/x_g\) appearing in the amplitude must be absorbed into the nonperturbative factor \(\xi_n\). Pragmatically, this can be described by a simple prescription. First we make the following substitution in the \(bbg\) amplitude:

\[
\pi^a_j(p_b) \epsilon^{a\mu}(p_g) \rightarrow x_g \xi[bg(3, J)] \frac{\sqrt{3}}{2} T_a \langle j | m_b \pi^a_j(p) \rangle ,
\]

then set \(p_g = x_g p\) in the rest of the amplitude and take the limit \(x_g \rightarrow 0\).

To justify a perturbative treatment, we need show the parton process is really governed by the short distance
The subamplitude of Fig. 1a) involves a term
$$\mathcal{M} \left[ bg(3,2S_{1/2}) \right] \propto \frac{m_b}{p \cdot \bar{p}} \mathcal{M}_b \left( p \cdot \bar{p} \right) \cdot \pi \Gamma^\alpha v(\bar{p}) \epsilon_a(Z),$$
where $\Gamma^\alpha$ is the $Zb\bar{b}$ coupling, and $\epsilon_a(Z)$ is the polarization vector of $Z^0$. Squaring the amplitude and summing over the color are standard. Less familiar is the sum over two polarizations of $u_{1/2}^b$, yet can be accomplished with the aid of the following formula:

$$\sum_{ \frac{1}{2} } u_{ \frac{1}{2} }^b(p) \mathcal{M}_b \left( p \cdot \bar{p} \right) \cdot \pi \Gamma^\alpha v(\bar{p}) \epsilon_a(Z) = \frac{1}{3} \left( \frac{p + m}{\bar{p}} \right)^2 - \frac{1}{\bar{p}^2}. \quad (6)$$

Note this resembles summing over polarizations of a Dirac spinor, up to a normalization. Finally we arrive at a succinct expression:

$$\hat{\Gamma}[bg(3,2S_{1/2})] = \frac{16 \pi \alpha_s(m_Z)}{9} \frac{1 - 4 \gamma}{(1 - 2 \gamma)^2} \Gamma_0[bb], \quad (7)$$
where $\gamma = m_b^2/M_Z^2$, and $\Gamma_0$ is the lowest order $Z^0$ partial decay width into $bb$.

$$\Gamma_0[bb] = \frac{G_F M_Z^3}{2 \sqrt{\pi}} \beta \left[ g_V^b (1 + 2 \gamma) + g_A^b (1 - 4 \gamma) \right], \quad (8)$$
where $\beta = \sqrt{1 - 4 \gamma}$, the vector coupling $g_V^b = -\frac{1}{2} + \frac{3}{4} \sin^2 \theta_W$ and the axial-vector coupling $g_A^b = -\frac{1}{2}$.

The HQR contribution appearing in Eq. (7) suppresses the HQR contribution when the center-of-mass energy is near the $bb$ threshold. Nevertheless, on the top of $Z$ pole, setting this kinetic factor to 1 is an excellent approximation for $b$ and $c$. Since the HQR parton cross sections are finite when $\gamma = 0$, it might be tempting to apply Eq. (7) also to the light hadron production. However, the validity of HQR factorization in Eq. (1) is crucially based on the heavy quark dynamics, thus one should not erroneously use Eq. (7) to describe $Z^0$ decay into light hadrons, by literally taking $\gamma \to 0$.

It may look surprising that the HQR parton cross sections are $O(1)$ relative to $\Gamma_0$, not suppressed by $m_b^2/M_Z^2$ as indicated in Eq. (6). In fact, the explicit suppression from $m_b$ in Eq. (6) is compensated by $\bar{p} \cdot v$ in Eq. (6). This non-suppression has important implication to the heavy flavor production in the hadron collision. By examining the $bg$ recombination process $gg \to bg(3) + b$, one finds that the differential cross section $d\sigma(bg(3))/dp^2 \sim 1/p^4$ at $p_t \gg m_b$, a priori of the same order as fragmentation.

In fact, it is equal to the differential cross section for $gg$ annihilating into massless $qq$ pair, up to a numerical constant. This finding diametrically challenges the general belief, that any non-fragmentation process at large $p_t$ will be suppressed by powers of $1/p^2$.

To expedite the analysis, we choose the following linear combination of recombination factors: $\xi = \xi_{\text{eff}}[bg(3,2S_{1/2})] = 2 \xi[bg(3,4S_{3/2})]$. We further define $\xi_{\text{tot}} = \sum_b \xi_{\text{eff}}[bg \to Z]$, where the sum is over all the lowest-lying $b$ mesons and baryons. So the net contribution of this mechanism to the partial width of $Z^0$ to $bb$ is

$$\Delta \Gamma[bb] = \frac{32 \pi \alpha_s \xi_{\text{tot}}}{9} \Gamma_0[bb], \quad (9)$$
where we have doubled Eq. (7) to include the contribution from $CP$ conjugate of Eq. (7). This is the crux of this work. From now on, we will use $\xi_3$ as a shorthand for $\xi_{\text{tot}}$ for simplicity.

The correction to the partial width of $Z^0$ into charm can be obtained by simply replacing $\xi_3^b$ and $\Gamma_0[bb]$ with $\xi_3^c$ and $\Gamma_0[cc]$ in Eq. (9). The heavy quark symmetry suggests that $\xi_3^c = \xi_3^b m_b/m_c \approx 3 \xi_3^b$. However, a caveat is that, the symmetry-breaking effect may be large in the real world and practically we better treat them as two independent unknown parameters.

The $bg$ recombination can also make a correction to the hadronic width of $W$ boson, through the decay channels $W^+ \to bg + c(u)$ and $W^+ \to cg + s(d)$. (Same reasoning can be also applied to $t \to bg + W^+$). Note it is not necessary for the recoiling quark to be heavy. The calculations are essentially the same as described above for $Z^0$ decay. However, unlike $\Gamma_Z$ which is measured to per mille accuracy, $\Gamma_W$ is measured with a 3% error. Therefore the $Z$-pole observables are more sensitive probes to ascertain the effects of this mechanism.
TABLE I: Some selected electroweak variables, where $R_c \equiv \Gamma_{\text{had}}/\Gamma[e^+e^-]$ and $\sigma^0_{\text{had}} \equiv 12\pi |e^+e^-|\Gamma_{\text{had}}/(M_Z^2 \Gamma_Z)$. Listed are the latest measurements $^1$ and the SM predictions $^2$. Pull is defined as $(\sigma_{\text{meas}} - \sigma_{\text{fit}})/\sigma_{\text{meas}}$.

| Meas | Predictions | Pull |
|------|-------------|------|
| $\Gamma_{\text{had}}$ [GeV] | $1.7444 \pm 0.0020$ | $1.7429 \pm 0.0015$ | - |
| $\Gamma_Z$ [GeV] | $2.4952 \pm 0.0023$ | $2.4972 \pm 0.0011$ | -0.9 |
| $\sigma^0_{\text{had}}$ [nb] | $41.541 \pm 0.037$ | $41.470 \pm 0.010$ | 1.9 |
| $R_c$ | $20.804 \pm 0.050$ | $20.753 \pm 0.012$ | 1.0 |
| $R_b$ | $0.21644 \pm 0.00065$ | $0.21572 \pm 0.00015$ | 1.1 |
| $R_c$ | $0.1718 \pm 0.0031$ | $0.17231 \pm 0.00006$ | -0.2 |
| $\Gamma_W$ [GeV] | $2.130 \pm 0.069$ | $2.0921 \pm 0.0025$ | 0.7 |

The discrepancy in $A_{FB}^{Z\ell}$ is believed to either be an experimental problem, or originate from some tree-level new physics effects on $Zbb$ vertex $^2$. Our $bg$ recombination doesn’t have much impact on the forward-backward asymmetries of $b$ and $c$, since the role of this mechanism is simply rescaling both forward and backward cross sections by a common factor, which cancels out in the ratio.

From Table I, we can see the general agreement between the measurements and SM fits is rather good, therefore demanding $\xi_3$ must be small. Naively, if we assume including the corrections in Eq. (2) doesn’t affect the values of SM fit parameters, then the relative variations of the SM predictions listed in Table I are (For simplicity, we take $\xi_5 = 3 \xi_3$ temporarily):

$$\delta \Gamma_{\text{had}}/\Gamma_{\text{had}} \approx \tau (R_b + 3R_c) \approx \xi_b^0,$$

$$\delta \Gamma_Z/\Gamma_Z = (\Gamma_{\text{had}}/\Gamma_Z) - \Gamma_{\text{had}}/\Gamma_{\text{had}} \approx 0.7 \xi_b^3,$$

$$\delta \sigma^0_{\text{had}}/\sigma^0_{\text{had}} = (1 - 2\Gamma_{\text{had}}/\Gamma_Z) \delta \Gamma_{\text{had}}/\Gamma_{\text{had}} \approx -0.4 \xi_b^3,$$

$$\delta R_c/R_c = \delta \Gamma_{\text{had}}/\Gamma_{\text{had}} \approx \xi_b^3,$$

$$\delta R_b/R_b \approx (1 - R_b - 3R_c) \approx 0.4 \xi_b^3,$$

$$\delta R_c/R_c \approx \tau (3 - R_b - 3R_c) \approx 3 \xi_b^3,$$

$$\delta \Gamma_W/\Gamma_W \approx 3 \tau/2 (0.707 \text{GeV}/\Gamma_W) \approx 0.7 \xi_b^3,$$

where $\tau \equiv 32\pi\alpha_e \xi_b^3/9$, and $\alpha_e(M_Z) = 0.12$ is used. In the last row, we use $\Gamma[W^+ \to u_i d_j] \approx 0.707 |V_{ij}|^2 \text{GeV}^2$ $^2$, and only include the Cabbibo-favored channels $W^+ \to c \bar{g} + s \bar{d}$. To be compatible with most variables, $\xi_3^3 \approx 10^{-3}$ looks reasonable. It will drive $R_c$, $R_b$ and $\Gamma_W$ towards the correct direction, but deteriorate $\Gamma_Z$, $\sigma^0_{\text{had}}$ and $R_c$. Note the correction to $R_c$ is much more significant than to $R_b$. A $\xi_b^3$ as large as 0.01 becomes apparently unacceptable, since it would bring up the deviations in $\sigma^0_{\text{had}}$ and $R_c$ to 6.4 $\sigma$ and 1.8 $\sigma$, respectively.

Since many quantities are correlated in a complicated way, an unbiased strategy is to incorporate this mechanism into the global electroweak analysis. Consequently, the recombination factors $\xi_b^3$ and $\xi_3^3$, can be fitted together with the SM parameters $M_Z$, $M_H$, $m_t$, $\alpha_s(M_Z)$ and $\Delta \sigma^{(5)}_{\text{had}}$. It will be very interesting to see how these parameters vary, and how much the quality of global electroweak fit may improve.

To summarize, we have studied a new QCD mechanism that generates power corrections to partial widths of $Z\gamma$ and $W$ at $O(\alpha_s)$, which have previously eluded the OPE analysis. The present precision of electroweak measurements requires these terms must be included.

The phenomenological consequences of this mechanism to other collider experiments should be investigated. For example, we have noticed one unusual feature of $bg$ recombination in hadron collision – it is formally of the same order as fragmentation contribution at large $p_t$. This is in sharp contrast to $b\bar{q}$ and $bq$ recombination, whose effects are suppressed by $\Delta QCDm/p_t^2$ $^3$. In addition, because the dominant contribution to heavy flavor hadroproduction arises from the gluon-initiated subprocess, we expect $bg$ recombination is more important than $b\bar{q}$ and $bq$ recombination.

Unfortunately, a rather (unnaturally) small $\xi_b$ inferred from this work makes the color-triplet contribution virtually invisible in hadron and $ep$ collider experiments, due to much cruder consistency between data and the NLO QCD predictions. Since $\xi_b$ and $\xi_3$ remain unconstrained, we hope they perhaps are much larger than $\xi_3$, so that the color-$\bar{b}$ and 15 channels may result in noticeable effects.

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[12] Perhaps the $\sigma^0_{\text{had}}$ discrepancy is not so glaring. It was recently suggested $^1$ that, the luminosity in four LEP experiments may have been underestimated, and correcting that will decrease the measured $\sigma^0_{\text{had}}$ value.