Endophysical information transfer in quantum processes

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Abstract

We give a mathematical criterion for the concept of information flow within closed quantum systems described by quantum registers. We define the concepts of separations and entanglements over quantum registers and use them with the quantum zip properties of inner products over quantum registers to establish the concept of partition change, which is fundamental to our criterion of endophysical information exchange within such quantum systems.

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I. INTRODUCTION

This paper aims to give a mathematical definition of information exchange within quantum systems. This is motivated by the still unresolved questions of what an observer is and the meaning of measurement in quantum physics. Collectively, these and related issues will be called the measurement problem.

Before we discuss the definition of information exchange, we should say what we mean by information. According to Preskill [1], information is something that is encoded in a state of a physical system. According to Sippl [2], however, information is knowledge that was not previously known to its receiver. These two definitions do not seem equivalent to us. In fact, they are very different. The former presents a passive, classical perspective which suggests that information is a property of a system which is “there” waiting to be found, regardless of anything else in the universe. On the contrary, the latter definition is thoroughly dynamic, requiring both subject and observer and the passage of time for the definition to make any sense. We will discuss information according to the Sippl point of view, because it accords better with quantum principles. Quantum theory is not about systems. It is about interactions between systems and is therefore all about time. Data held in a system has no physical meaning in the absence of any measurement of that data. In this article, therefore, the terms information, information exchange, information acquisition and loss will be regarded as synonymous. The term “data” will be used to refer to mathematical properties encoded into state vectors representing physical systems.

Quantum mechanics is such an accurate and practical tool in the physical sciences that we should explain our interest in the measurement problem. In addition to a natural interest in constructing a completely consistent quantum theory, we have been motivated by two long-standing problems of modern theoretical physics; first, quantum field theory is littered with mathematical divergences and second, quantum gravity, the program attempting to rewrite Einstein’s classical theory of general relativity in quantum terms, has had limited success. It seems to us that the problems with quantum field theory and quantum gravity lie not with the general principles of quantum mechanics but with three historical legacies related to the measurement problem that these theories acquired at birth.

The first of these is the supposed continuity of space and time. Both quantum field theory and classical general relativity assume that space and time form a continuum in which various sorts of field can be embedded. Many attempts at quantum gravity also implicitly suppose the existence of some underlying manifold with a fixed dimension. The same is true of
string theory and developments of it such as brane physics. Our view is that classical spacetime is a throwback to the prequantum, classical mechanical view of the world sometimes referred to as the block universe \[3\]. There is no logical reason to suppose that any concept which originated before the advent of quantum mechanics should survive as an intrinsic one in a fully quantum theory of the universe. The many attempts to solve the problems associated with continuity rely on the introduction of ad hoc modifications to the continuum, such as spacetime lattice discretization, point splitting and dimensional regularization, all of which simply reinforce the view that spacetime continuity is a terminally sick concept.

The supposed continuity of space and time cannot in fact be empirically proved. Indeed, rather like pre-atomic continuum theories of liquids, it is an abstraction which arises from the particular way in which humans interact with their environment. Normally, this environment is so flooded with photons that the brain can maintain a consistent illusion that we exist in a three-dimensional continuum and evolve according to a continuous time. This gives a classical picture of a universe which advanced technology shows runs on quite different quantum principles.

The second historical legacy weighing down modern theory has been recognized and taken seriously by physicists only relatively recently. This is the issue of \textit{exophysics} versus \textit{endophysics}. In exophysics, the assumption is made that observers stand outside of the systems that they observe. This perspective is the basis of the standard Copenhagen School approach to quantum mechanics. In endophysics, on the other hand, observers and systems are all part of a greater whole. The ultimate expression of the endophysical perspective is the statement that there is only one system, the universe, which contains absolutely everything, including all forms of observers.

One essential difference between exophysics and endophysics lies in the meaning of information acquisition and storage. According to the exophysical perspective, whenever an observer measures something about a system, information is registered in some form of memory carried by that observer. The exophysical principles of classical mechanics generally assume that this registration process can be done without affecting the system being observed and effects the observer only via changes in the memory. This idea arises naturally, given the classical, three-dimensional model of external reality constructed by the human brain. The nature of this memory is rarely, if ever, discussed in exophysics, whereas it becomes crucial in endophysics to explain in what sense endophysical observers record “measurements”.

When applied to quantum physics, the exophysical perspective creates its own problems. First, quantum mechanics has been found to be valid at all scales looked at, so that no natural “Heisenberg cut” (the hypothetical
dividing line between classical observers and quantum systems) seems to exist. Second, quantum correlations seem oblivious to some of the properties associated with classical relativistic spacetime, such as the principle of local causes (i.e., Einstein locality \[4\]). For example, quantum correlation speeds vastly in excess of the speed of light have been reported recently \[5\].

Changing to the endophysical perspective appears to remove the problem with the Heisenberg cut, because observers are now regarded as quantum subsystems within a larger quantum system, so that quantum principles can cover everything without the need for any sort of cut. Unfortunately, changing perspective merely replaces one problem with another: now we have to explain what the difference between an observer and a system under observation is and what an act of measurement means.

Another consequence of this change is that there arises the extraordinarily difficult task of explaining how the classical world that we see on macroscopic scales could arise from a purely quantum theory. This program will be referred to as the problem of \textit{emergence}. If we went further and assumed no a priori Riemannian geometrical structures whatsoever, anticipating their appearance only in some emergent limit, then we would be dealing with what Wheeler has called \textit{pregeometry} \[6\].

The third historical legacy inherited by modern physics is its unwillingness to completely let go of all classical modes of thought, particularly concerning the concept of \textit{observer}. Relativistic covariance principles tend to be applied at all levels of modern theory, whereas more careful analysis reveals that these principles cannot be upheld everywhere at all costs. The problem occurs because the classical relativist believes that different observers can observe the same event, but the quantum theorist knows that this is physically incorrect. In spite of this, the language and thinking of relativity is predicated on the former point of view, which continues to infect modern theory at all levels. We shall discuss this issue in the context of the process time perspective in the next section.

In earlier work \[7\] we explored the idea discussed by Feynman \[8\] and others that the universe can be represented in terms of a vast \textit{quantum register}, i.e., a tensor product of a large number of quantum subregisters such as qubits. In our approach, the dynamics of the universe is postulated to be that of a self-referential quantum register with no external observers.

This approach has a number of important and useful properties. First, it is based on Hilbert space rather than on classical configuration or phase space. Therefore, rather than trying to quantize a classical theory in the traditional way, the quantum register approach starts off completely consistent with quantum principles. Second, the qubits making up the full quantum
register provide the ultimate source of the vast number of degrees of freedom which the physical universe is known to have (the reader is warned that the relationship between these concepts is not one-to-one and is considerably more subtle than expected). Third, we have shown that the properties of factorization and entanglement associated with quantum registers can be used to describe causal set dynamics in a natural way [7]. We found that the elements of the causal sets involved are not in fact the subregisters as might be expected. It is the subtle and intricate interplay between the patterns of factorization and entanglement in both the states of the quantum register and the Hermitian operators over that register which generates the causal set dynamics.

A particular feature of our approach is its use of state reduction rather than the unitary, no-collapse evolution favoured in the many-worlds and decoherence paradigms. There are several reasons for this. First, we wish to discuss physics as it appears to the experimentalist; state reduction corresponds to the registration of information in quantum experiments whereas Schrödinger evolution holds in the absence of such registration. Second, although the standard view is that quantum mechanics predicts only expectation values, it is an empirical fact that interference patterns can be built up over vast time scales from a succession of single outcomes which are well separated in time. This and the extraordinary difficulties encountered by hidden variables theories to account for all current experimental data suggests that single quantum outcomes do have an individual physical significance, albeit not a classical one. Probability is not just about averages and expectation values. It is too easily turned into a mathematical set-theoretic discussion of "measure", whereas in practice it is bound up with the counting of frequencies of quantum outcomes by observers.

A third reason is that the Schrödinger evolution of quantum systems can always be unitarily transformed away by moving to the Heisenberg picture, supporting the view that temporal evolution is not an intrinsic property of systems on their own, but of those systems and of the observers of those systems as well. Another way of saying this is that physical time is a marker of quantum information exchange.

The structure of this paper is as follows. In §2 we review the stages paradigm, which is the conceptual framework in which we work. In §3 we discuss its extension to quantum registers. In §4 we discuss the separation and entanglement properties of quantum registers, introducing the important concept of a lattice of partitions, on which we base our ideas of endophysical information exchange within quantum systems. In §5, 6 and 7 we discuss operators, eigenvalues, preferred bases and the separability properties of such
bases. In §8 we discuss the relationship between active and passive transformations, as this is a central issue in quantum dynamics, followed in §9 by the concept of local transformations, which are relevant to quantum registers. In §10 we discuss what is meant by state preparation, followed in §11 by a discussion of transition amplitude factors. In §12 and 13 we focus attention on the concept of an isolated quantum system. In §14 we state our principle of endophysical information exchange, based on the concept of partition change, followed in §15 by some concluding remarks.

In this paper we shall represent state vectors in two ways; when we discuss more abstract issues such as quantum cosmology we favour symbols such as \( \Psi \), but when we have more detailed statements to make we use the equivalent notation \( |\Psi\rangle \). This latter notation is more useful in the representation of operators. Inner products will also be represented in two ways, i.e., we will take

\[
(\Psi, \Phi) = \langle \Psi | \Phi \rangle.
\]

(1)

II. THE STAGES PARADIGM

Our account of time and information exchange rests on two observations: i) the laws of quantum mechanics appear to have universal application and ii) the universe contains a truly vast number \( N \) of degrees of freedom. Throughout our work we shall assume \( N \) is finite, principally because this ensures that we do not have any divergences in any of our equations and that all our Hilbert spaces are separable. In earlier work we used both of these ideas to develop further Feynman’s view of the universe as a gigantic form of quantum computation [8] which behaves as an autonomous quantum system with no external observers [9, 10]. We shall refer to this as the stages paradigm, the essential details of which are reviewed briefly as follows.

Because the laws governing observers of quantum systems are currently not understood [8], the conventional Copenhagen School approach to quantum mechanics assumes observers to be semiclassical objects with free will, standing outside of the quantum systems they are investigating. It is therefore an exophysical approach. In mathematical terms this translates to the use of differential equations governed by boundary conditions dictated by arbitrary factors, such as choice of initial state and experiment, external to the quantum systems under discussion.

Our approach is different. We take it as self evident that the universe itself organizes its own observations (or tests [4]), because by definition the universe contains everything and therefore there can be no semiclassical observers standing outside it. This forces us to make one (and only one) change in the
standard principles of quantum mechanics: we have to remove the rule that there are semiclassical observers external to quantum systems deciding how to measure them.

By this we do not mean that the concept of a semiclassical observer is wrong. Of course, it has proved to be enormously useful. What we do mean is that this notion is not an intrinsic or essential one. We believe that it is a derived or emergent aspect of quantum registers when looked at on certain scales, typically when extremely large numbers of subregisters are involved and when the complex factorization and entanglement properties of quantum registers permit it. In this paper we discuss what the notion of semiclassical observer should be replaced by.

In common with all theories, there are some aspects of our formalism which are unphysical, being auxiliary mathematical devices required to represent the physics. The quantum register itself and its states are examples of such devices. Another one is our concept of *exotime*; this is the underlying discrete time parameter, indexed by the integers, which labels successive states of the universe. This time is not an observable. How it relates to the time seen by endophysical observers (*endotime*) is a problem analogous to the question of how co-ordinate time relates to proper time in relativity. The former is integrable (i.e., path independent) and unphysical, whereas the latter is not and has direct physical significance.

At each instance \( n \) of exotime the universe is assumed to be in a well defined *stage* \( \Omega_n \). A given stage \( \Omega_n \equiv \Omega(\Psi_n, I_n, R_n) \) consists of three things:

1. a pure state \( \Psi_n \), known as the *state of the universe*, being a normalized element in a universal Hilbert space \( \mathcal{H} \) of some extremely large but finite dimension \( N \). The state of the universe \( \Psi_n \) is an eigenstate of some test \( \Sigma_n \), i.e., \( \Psi_n \) represents an outcome of some immediately previous test of the universe;

2. the *current information content* \( I_n \), representing dynamical information over and above that contained within \( \Psi_n \). For example, the dynamical laws governing future states of the universe may require a knowledge of which particular test \( \Sigma_n \) (or equivalently, which *preferred basis*) gave \( \Psi_n \) as an outcome;

3. the *current rules* \( R_n \) (or laws of physics), which determine how the current stage evolves.

These ingredients represent the minimum we believe is needed to model any self-referential quantum universe. Indeed, the equivalent of these ingredients are implicit in the alternative many-worlds and decoherence approaches;
they assume states of the universe, Hamiltonian operators (equivalent to dynamical information about the system), and the Schrödinger equation, which gives the rules for dynamical evolution.

A. Stage dynamics

An essential feature of the stages paradigm is the dynamical relationship between successive stages, i.e., the jump from $\Psi_n$ to $\Psi_{n+1}$. Generalizing the standard rules of quantum mechanics [4], we assume that each given stage $\Omega_n$ is replaced by another one, $\Omega_{n+1}$, such that the latter’s state of the universe $\Psi_{n+1}$ is a quantum outcome (an eigenstate) of some quantum test $\Sigma_{n+1}$, which itself was determined in some way by $\Omega_n$ and by no other factor. One way of seeing stage dynamics is as a perpetual sequence of ideal measurements [4] interlaced with a sequence of self-determined tests.

In the real world of the human observer, the physical experience of time appears very different to its representation in classical theories such as Newtonian mechanics and relativity. Real time appears to us not as a continuum but as a single point, the enigmatic "moment of the now" also known as the present, albeit it is one which appears to change constantly. Only our memories and our ability to anticipate the future allow us to think of the past and the future, all of which thinking takes place in the present. This view of time has been labelled process time, in contrast to the manifold time or block universe perspective used widely throughout relativistic physics, which is a geometric view of space and time including past, present and future.

It is here that a conceptual clash occurs between the principles of quantum mechanics and the principles of relativity. The Kochen-Specker theorem [11] states that quantum systems do not have classical properties per se waiting to be discovered. Taking this to its logical conclusion, quantum mechanics has to deny the reality of the future and therefore should strictly avoid using block universe concepts. The logic behind this is based on the following assertions: i) it is self-evident that the future cannot be more physically real than the present and ii) according to the Kochen-Specker theorem, the present is not there in any classical sense. Therefore, the future is not there in any classical sense and so the block universe model is fundamentally incorrect. This argument also applies to the concept of closed timelike curves (CTCs) in general relativity. These cannot be reconciled with the principles of quantum mechanics, which is probably why they have never been observed.

There are two reasons why the block universe model is used throughout theoretical physics. First, it allows us to order data in a way consistent with the classical principles of Newtonian mechanics. These were formulated from
an exophysical perspective long before the discovery of quantum mechanics and make use of an exophysical time concept known to Newton as absolute time. Second, the concept of process time is not a Lorentz covariant one, because simultaneity is frame dependent.

Relativity poses perhaps the most serious problem for the stages paradigm from a number of perspectives. Apart from the problem with simultaneity, the dimension of spacetime and the Lorentzian signature of the metric would have to be explicable in terms of our pregeometric framework.

Actually, the mathematical structures associated with the stages paradigm have particular properties which we believe can explain the emergence of relativity. First of all, the issue of simultaneity is not the intractable problem it appears to be. It arises because relativity itself arose from purely classical ways of thinking about systems, observers, and what is meant by observation. Consequently, some of the assertions frequently made in the subject are either incorrect, incomplete, or incompatible with the principles of quantum mechanics. For instance, there are actually no infinitely extended inertial frames, yet many discussions in relativity assume that they exist (we have in mind here the standard discussion of elementary particle scattering in relativistic quantum field theory).

Another example is the widespread use of covariance arguments in relativity, one of the most powerful principles employed in relativistic physics being that the theory should be Lorentz covariant. This is misleading and incorrect from a quantum measurement point of view. No single experiment is Lorentz covariant. It sits in its own rest frame. Moreover, no single outcome of any quantum experiment (i.e., a single run) can be “observed” by different observers sitting in different inertial frames. Otherwise we could get around the Heisenberg uncertainty principle by having different observers test a state for different incompatible observables. As emphasized by Peres [12], Lorentz covariance relates only to the transformation properties of ensemble averages, i.e., expectation values, which is quite a different matter to what actually happens in any single run of an experiment. Expectation values are statistical summaries of information either already taken or planned to be taken from very many runs of a given experiment. The stages paradigm on the other hand is designed to discuss the process physics description of single runs, which has to take place before ensemble averaging can be considered.

As for the other issues with relativity, we have shown [7] that a causal set structure arises naturally within the stages paradigm once we extend it to large rank quantum registers. This then opens the door to discussions of how concepts of space and Lorentz signature metric can arise in emergent limits [13]. Moreover, the possibility of null tests [7] permits the emergence of a non-integrable endophysical time local to observers, thereby providing
the basis for the proper time concept in relativity. From the stages paradigm point of view, therefore, relativity is but an emergent view of the quantum universe and will not be discussed further here.

In the stages paradigm, the concept of process time cannot be modelled directly; it is taken account of by the following rule: in any discussion, only one stage (referred to as the present) can ever be regarded as certain. All other stages can be discussed only in conditional probability terms relative to that stage. This reflects the essential feature of process time, that only the “present” exists; the past is gone and the future has no direct physical significance.

Each test $\Sigma_n$ is represented mathematically by some element $\hat{\Sigma}_n$ in $\mathbb{H}(\mathcal{H})$, the set of all nondegenerate Hermitian operators on $\mathcal{H}$. Nondegeneracy of outcomes is as necessary here as in standard quantum mechanics, because otherwise the interpretational power of quantum mechanics (such as the Born probability rule) collapses. The eigenstates of $\hat{\Sigma}_n$ collectively constitute a unique (up to inessential phase factors of its elements) basis $B_n \equiv B_{\Sigma_n}(\mathcal{H})$ for $\mathcal{H}$, which gives a “preferred basis” at each instant of exotime.

In standard quantum mechanics, attention is generally focused on observables, their eigenstates and the corresponding eigenvalues, the theory being particularly good at predicting the latter. The really important problem physically, however, is how preferred basis sets should arise in the first place. Eigenvalues in themselves have only a relative value, because classical information, when it is expressed solely in the form of eigenvalues, is not in general absolute. For instance, energy, charge and momentum are all expressed relative to arbitrary levels and scales of definition, which can only be done on emergent scales anyway. Moreover, mathematically it is possible for different tests to correspond to the same preferred basis set.

The stages paradigm incorporates these comments directly. There are no Hamiltonians, no phase space, no differential equations of motion, and eigenvalues are not taken to be important per se. Instead, it is the uniqueness of the elements of the preferred basis set which really matters.

The problem of how a preferred basis arises at each instant of exotime is an unsolved problem, common to our stages paradigm, the many-worlds paradigm and decoherence theory generally. If we knew the answer we would have a more complete understanding of time and the universe. Whilst there is currently no general understanding of how such bases arise in any theory, we shall rule out free will in any shape or form and follow Feynman [8] in asserting that observers are part of the universe and are therefore governed by its laws, whatever they are. The stages paradigm asserts that $B_{n+1}$ is
determined solely by $\Omega_n$, i.e.,
\[ B_{n+1} = B(\Omega_n) = B(\Psi_n, I_n, R_n). \] (2)

We do not exclude here the possibility that $B_{n+1}$ is itself a random outcome of some higher form of quantum process, involving the selection of one out of various potential elements of $\mathfrak{B}(\mathcal{H})$, the set of all orthonormal bases for $\mathcal{H}$. If there were some sort of random process governed by Born-type rules, this would most appropriately be referred to as “second quantization”. We cannot rule out the possibility that the information content $I_n$ includes a knowledge of $B_n$ and possibly of earlier preferred bases, all of which could be used in the determination of $B_{n+1}$.

Whatever the actuality, the stages paradigm is designed to describe our existence in a well defined branch of reality and not in any superpositions (as per many-worlds paradigm). We will assume that, given $\Omega_n$, there is always some subsequent selection process which picks out a definite preferred basis $B_{n+1}$ from $\mathfrak{B}(\mathcal{H})$.

### B. Probabilities

The stages paradigm accepts quantum randomness as an intrinsic property of the universe which is quantified by the Born probability interpretation of state vectors. Relative to a given stage $\Omega_n$ and to a given preferred basis $B_{n+1}$, the conditional probability (or propensity) $P(\Psi_{n+1} = \theta^\alpha \in B_{n+1}|\Omega_n, B_{n+1})$ of $\Psi_{n+1}$ being an element $\theta^\alpha$ of $B_{n+1}$ is given by the Born rule
\[ P(\Psi_{n+1} = \theta^\alpha \in B_{n+1}|\Omega_n, B_{n+1}) = |(\theta^\alpha, \Psi_n)|^2. \] (3)

All basis states are taken to be normalized to unity, so we may write
\[ \sum_{\alpha=1}^{\dim \mathcal{H}} P(\Psi_{n+1} = \theta^\alpha \in B_{n+1}|\Omega_n, B_{n+1}) = 1. \] (4)

Note that the probability (3) is not in general the same thing as the answer to the question: what is the probability of jumping to an arbitrary element $\Theta$ in $\mathcal{H}$, given $\Psi_n$? For most elements in $\mathcal{H}$ the answer is zero, even for those states $\Theta$ such that $|(\Theta, \Psi_n)| > 0$. The reason is that $\Theta$ has to be an element of the preferred basis $\Sigma_{n+1}$ before there is any possibility of such a jump occurring. In any such discussion, it is important to keep in mind that tests are as important as states in determining how the universe evolves. For instance, the microscopic reversibility implied by the mathematical symmetry
\[ |(\theta^\alpha, \Psi_n)|^2 = |(\Psi_n, \theta^\alpha)|^2 \] (5)
The meaning of the probability rule (3) when applied to the universe requires careful interpretation, because it is here that criticisms of the quantum universe concept have been raised \cite{14}. By definition the universe is not in an ensemble, so there is no direct exophysical meaning to the concept of the probability of the outcome $\Psi_{n+1}$.

We make two comments about this issue. First, in probability theory, particularly in the Bayesian approach to statistics, there are two forms of probability: epistemic uncertainty is uncertainty arising from a lack of knowledge whereas aleatory uncertainty is uncertainty due to inherent randomness. The former is reducible and may even be eliminated by the acquisition of sufficient data. In quantum mechanics, this form of uncertainty is encoded into the classical probabilities associated with mixed states and is clearly predicated on the concept of some observer external to a system having a lack of information about that system. Aleatory uncertainty is irreducible and intrinsic, on the other hand. The stages paradigm is based on the belief that quantum uncertainties are inherently aleatory in nature. The problem with many-worlds and decoherence is that they purport to derive quantum aleatory uncertainties from epistemic foundations, which is inherently impossible. This accounts for their general failure to explain the Born probability rule without the introduction of extra assumptions which in the long run will amount to the state reduction concept. It is for this reason that we have chosen to take state reduction as a fundamental physical phenomenon in the first place.

The other point is that although cosmologists and quantum theorists are themselves embedded in the universe and are therefore endophysical objects in their own right, this does not prevent them from discussing possible future states of the universe, including their own futures. This is somewhat surprising given that another indirect criticism of quantum cosmology has been the suggestion that Gödel-type incompleteness rules out the possibility of endophysical observers determining the exact state of the universe they find themselves in \cite{15}. While this may be true in detail, this does not rule out cosmologists discussing conditional or relative probabilities, or making counterfactual statements about the universe that they are part of.

**III. QUANTUM REGISTERS**

Although the stages paradigm provides the basic framework for our quantum dynamics, it is not specific enough as it stands to permit a discussion
of the measurement problem. We need to add to it the specific assumption that $\mathcal{H}$ is a quantum register consisting of a vast number $N$ of subregisters.

By definition, a quantum register is a Hilbert space which is the tensor product of a finite number of quantum subregisters, each of which is a distinct Hilbert space of finite dimension. A simple calculation based on Planck scales and the expansion of the observed universe suggests that $N$ has to be at least of the order $10^{180}$ if we want some chance of describing the universe that we can see [10]. It is almost certainly much greater than that estimate, given that physical space and all the currently known quantum fields describing matter should emerge from such a pregeometric foundation. The dimension of $\mathcal{H}$ is at least $2^N$, this lower bound occurring in the case that each subregister is a qubit. This means that we are faced with the prospect of dealing with mathematical structures capable of very great complexity indeed, which is a double-edged sword. Whilst the mathematical structures associated with quantum registers should in principle be capable of describing any physical situation, their great complexity makes it near impossible to make detailed calculation in any but the simplest situations.

In this article the separation and entanglement properties of states and operators over quantum registers are crucial for the formulation of our concept of endophysical information exchange. We shall focus our attention on the relation between pure quantum states of a quantum register and strong operators, which are a particular class of Hermitian operator acting on those states and are discussed below.

It is frequently asserted that one of the crucial features of quantum mechanics distinguishing it from classical mechanics is the occurrence of entangled states. Whilst this is an important point, we have found that the separable states are equally important, being used to represent classically distinct observers and subsystems under observation. They represent the nearest thing to classicality in quantum mechanics, because factors in a separable state have an identifiable physical identity, something which components in an entanglement do not have in the absence of measurement. Separable states, therefore, are an essential component of our account of endophysical measurement theory.

In the next section we introduce a notation designed to represent the concepts of separations and entanglements of quantum registers.

**IV. SPLITS, PARTITIONS, SEPARATIONS AND ENTANGLEMENTS**

We shall use the notation $F[x] \in \mathbb{N}$ to denote the number of factors in
the object $x$, when evaluated on the required contextual level. For example, in the case of real numbers, we have $F[4] = 1$ but $F[2 \times 2] = 2$.

The notation $\mathcal{H}_{[12...N]}$ denotes a quantum register of rank $N$, consisting of the tensor product

$$
\mathcal{H}_{[12...N]} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N
$$

(6)
of a finite number $N \geq 1$ of factor Hilbert spaces $\mathcal{H}_i$, $1 \leq i \leq N$, each known as a (quantum) subregister. The dimension $d_i$ of the $i^{th}$ subregister $\mathcal{H}_i$ will generally be assumed to be finite. When this dimension is two, such a subregister is called a quantum bit, or qubit. Note that $F[\mathcal{H}_{[12...N]}] = 1$ but $F[\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N] = N$. This is an example where an equality is contextual, that is, does not hold under all circumstances.

The left-right ordering of the tensor product in (6) is not significant in our approach. Left-right ordering turns out to be an inadequate way of labelling products in the case of three or more subregisters, because entanglements can occur between elements of any of the subregisters. Instead, we shall use subscript labels as in (6) to identify specific subregisters, which are therefore regarded as having their own physical identities. For example,

$$
\mathcal{H}_{[12]} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2 = \mathcal{H}_2 \otimes \mathcal{H}_1 \equiv \mathcal{H}_{[21]}.
$$

(7)

This invariance to left-right re-ordering applies to states of subregisters as well as the subregisters themselves. For example, if $\psi_1 \in \mathcal{H}_1$ and $\phi_2 \in \mathcal{H}_2$, then

$$
\phi_2 \otimes \psi_1 = \psi_1 \otimes \phi_2.
$$

(8)

A. Splits

A quantum register consisting of two or more subregisters can be split in a number of ways. A split is just an arrangement of the subregisters into a convenient number of nonintersecting groupings of tensor products. For example, a rank-3 quantum register can be split in five different ways:

$$
\mathcal{H}_{[123]} \equiv \mathcal{H}_{[12]} \otimes \mathcal{H}_3 = \mathcal{H}_{[13]} \otimes \mathcal{H}_2 = \mathcal{H}_{[23]} \otimes \mathcal{H}_1 = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3.
$$

(9)
The equality here refers to the fact that each of these splits is the same as a vector space, but they are not equivalent in terms of split structure. For example, $F[\mathcal{H}_{[123]}] = 1$ but $F[\mathcal{H}_{[12]} \otimes \mathcal{H}_3] = 2$ and $F[\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3] = 3$. The significance of such splits is that states which are factorizable relative to one split need not be factorizable relative to another [16]. This underlines
the fact that entanglement and factorization are context dependent, that is, depend on the physical interpretation of the subregisters concerned.

Although they are crucial to the development of quantum causal set theory [7], splits by themselves do not go far enough to describe physics, however, and we need to develop the notion of a partition, which is based on the concepts of separations and entanglement, discussed next.

B. Separations

We define the separations first because entanglements can only be defined in terms of them. Given a rank-\(N\) quantum register \(\mathcal{H}_{[12...N]}\), each of its component subregisters \(\mathcal{H}_i\), \(1 \leq i \leq N\) will be called a rank-1 subregister. Rank-1 subregisters will be assumed to be elementary, in that the concepts of entanglements and separations (defined below) do not apply to them. Qubits are examples of such elementary subregisters. An arbitrary element in a subregister \(\mathcal{H}_i\) will be denoted by \(\psi_i\), except when the index is the letter \(n\), in which case it refers to exotime.

Assuming \(N > 1\), for any choice of two rank-1 subregisters \(\mathcal{H}_i, \mathcal{H}_j\) in \([6]\) such that \(1 \leq i < j \leq N\), we define the rank-2 subregister \(\mathcal{H}_{[ij]} = \mathcal{H}_{[ji]}\) of \(\mathcal{H}_{[12...N]}\) as the tensor product

\[
\mathcal{H}_{[ij]} \equiv \mathcal{H}_i \otimes \mathcal{H}_j,
\]

there being a total of \(\binom{N}{2}(N-1)\) distinct rank-2 subregisters. Rank-2 subregisters can contain both entangled and separable states and are vector spaces in their own right. Consistent with our notation, an element in \(\mathcal{H}_{[ij]}\) will be denoted by \(\psi_{[ij]}\).

For a given rank-2 subregister \(\mathcal{H}_{[ij]}\), we define the rank-2 separation \(\mathcal{H}_{ij}\) to be the proper subset of \(\mathcal{H}_{[ij]}\) consisting of all separable elements in it, i.e.,

\[
\mathcal{H}_{ij} \equiv \{ \psi_i \otimes \phi_j : (\psi_i \in \mathcal{H}_i) \ & \ (\phi_j \in \mathcal{H}_j) \}.
\] (11)

In this notation, the subscripts are not basis set indices. By definition we include in \(\mathcal{H}_{ij}\) the zero vector \(0_{[ij]}\) of \(\mathcal{H}_{[ij]}\).

We shall use lower indices without square brackets to denote separations, reserving the use of lower indices within square brackets to represent tensor products of subregisters. The concept of rank-2 separation generalizes readily to higher rank separations [7]. For example, the rank-3 separation \(\mathcal{H}_{ijk}\) is the subset of \(\mathcal{H}_{[ijk]}\) defined by

\[
\mathcal{H}_{ijk} \equiv \{ \phi_i \otimes \psi_j \otimes \eta_k : (\phi_i \in \mathcal{H}_i) \ & \ (\psi_j \in \mathcal{H}_j) \ & \ (\eta_k \in \mathcal{H}_k) \}.
\] (12)
C. Entanglements

Entanglements may be constructed once the separations have been defined. Starting with the lowest rank possible, we define the rank-2 entanglement $H^{ij}$ to be the complement of $H_{ij}$ in $H_{[ij]}$, i.e.,

$$H^{ij} \equiv H_{[ij]} - H_{ij} = (H_{ij} \cap H_{ij})^c. \quad (13)$$

Hence $H_{[ij]} = H_{ij} \cup H^{ij}$. $H_{ij}$ and $H^{ij}$ are disjoint and $H^{ij}$ does not contain the zero vector. An important aspect of this decomposition is that neither $H_{ij}$ nor $H^{ij}$ is a vector space.

The generalization of the entanglements to higher rank subregisters is straightforward but requires the concept of separation product. If $A_i$ and $B_j$ are arbitrary, non-empty subsets of $H_i$ and $H_j$ respectively, where $i \neq j$, then we define the separation product $A_i \bullet B_j$ to be the subset of $H_{[ij]}$ given by

$$A_i \bullet B_j \equiv \{ \psi_i \otimes \phi_j : \psi \in A_i, \phi \in B_j \}. \quad (14)$$

This generalizes immediately to any sort of product. For example, $H_{ij} = H_i \bullet H_j$. Separation products are associative, commutative and cumulative, i.e.,

$$ (H_i \bullet H_j) \bullet H_k = H_i \bullet (H_j \bullet H_k) \equiv H_{ijk}, \\ H_{ij} \bullet H_k = H_{ijk}, \quad (15)$$

and so on. Separation products can also be defined for the entanglements. For example,

$$H^{ij} \bullet H_k = \{ \phi^{ij} \otimes \psi_k : \phi^{ij} \in H^{ij}, \psi_k \in H_k \}, \\ H^{ij} \bullet H^{rs} = \{ \phi^{ij} \otimes \psi^{rs} : \phi^{ij} \in H^{ij}, \psi^{rs} \in H^{rs} \}. \quad (16)$$

A further notational simplification is to use a single $H$ symbol, using the vertical position of indices to indicate separations and entanglements and incorporating the separation product symbol $\bullet$ with indices directly. For example,

$$H^{15} \bullet H^{97} \bullet H_{28} \bullet H_{4} \bullet H_{36} \equiv H_{28}^{15\bullet97} \bullet H_{4} \bullet H_{36} = H_{23468}^{15\bullet97}. \quad (17)$$

Associativity of the separation product applies to both separations and entanglements.

Rank-3 and higher entanglements such as $H^{ijkl}$ are defined in terms of complements. For example, we define

$$H^{abc} \equiv H_{[abc]} - \{ H_{abc} \cup H^{bc}_{a} \cup H^{ac}_{b} \cup H^{ab}_{c} \}. \quad (18)$$

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D. Partitions

Every quantum register \( \mathcal{H} \) can be represented uniquely as a union of disjoint elements, such as separations, entanglements, and separation products of these two types (such as \( \mathcal{H}_{a}^{bcd} \equiv \mathcal{H}_{a} \bullet \mathcal{H}_{bcd} \)). These elements will be called partitions and together they form the natural lattice \( \mathcal{L}(\mathcal{H}) \) of \( \mathcal{H} \). The number of elements in each natural lattice is given by the Bell numbers \([7]\). For example, a rank-3 quantum register has five partitions:

\[
\mathcal{H}_{[abc]} = \mathcal{H}_{abc} \cup \mathcal{H}_{a}^{bc} \cup \mathcal{H}_{b}^{ac} \cup \mathcal{H}_{c}^{ab} \cup \mathcal{H}_{abc}.
\] (19)

Each partition itself may be the separation product of a number of blocks, each block being an individual separation or entanglement. For example, the partition \( \mathcal{H}_{cd}^{def \bullet fgh} \) has four blocks: two separations, \( \mathcal{H}_{c}, \mathcal{H}_{d} \) and two entanglements, \( \mathcal{H}_{cd}^{ab}, \mathcal{H}_{efg}^{e} \).

We may also use the above index notation to label the various elements of the entanglements and separations. For example, \( \psi_{abc}^{def \bullet g} \) is interpreted to be some element in the partition \( \mathcal{H}_{abc}^{def \bullet g} \) and so on. With this notation we may write for example

\[
\psi_{abc}^{def \bullet g} = \psi_{a} \otimes \psi_{b} \otimes \psi_{c} \otimes \psi_{def} \otimes \psi_{g},
\] (20)

where \( \psi_{a} \in \mathcal{H}_{a}, \psi_{b} \in \mathcal{H}_{b}, \psi_{c} \in \mathcal{H}_{c}, \psi_{def} \in \mathcal{H}_{def} \) and \( \psi_{g} \in \mathcal{H}_{g} \). Each factor such as \( \psi_{a}, \psi_{def}, etc. \) lies in a particular block in the partition to which \( \psi_{abc}^{def \bullet g} \) belongs.

An important feature of the concepts of splits, separations, entanglements and partitions is that they are all independent of basis, the only requirement for their definition being that the enclosing Hilbert space is a tensor product of identifiable subregisters.

V. OPERATORS

In the stages paradigm, operators representing tests are assumed to be Hermitian because this guarantees that their eigenvalues are real and that nondegenerate eigenstates are orthogonal. In general, for a finite dimensional Hilbert space \( \mathcal{H} \) of dimension \( d \) we can find \( d^{2} \) linearly independent Hermitian operators out of which we can build all the other Hermitian operators on \( \mathcal{H} \) \([4]\). These independent operators can then be used as a basis for the real vector space \( \mathbb{H}(\mathcal{H}) \) of all Hermitian operators on \( \mathcal{H} \). Furthermore, multiplication of elements in \( \mathbb{H}(\mathcal{H}) \) by other elements in \( \mathbb{H}(\mathcal{H}) \) is well defined and closed, so that \( \mathbb{H}(\mathcal{H}) \) is also an algebra \([17]\) over the real number field.
Given a quantum register consisting of $N$ subregisters, we can go further and define skeleton sets of operators. A skeleton set is a basis for $\mathbb{H}(\mathcal{H}_{[1\ldots N]})$ such that every element of the set is factorizable into a product of $N$ factors, the $n^{th}$ factor being associated with the $n^{th}$ subregister. To construct such a skeleton set, we first construct an operator basis for each algebra $\mathbb{H}(\mathcal{H}_i)$, $1 \leq i \leq N$, and then take tensor products of elements of bases from all the different subregisters. For example, for a two qubit register $\mathcal{H}_{[12]}$, a skeleton set for $\mathbb{H}(\mathcal{H}_{[12]})$ is given by the sixteen elements $\{\hat{\sigma}_i^\mu \otimes \hat{\sigma}_j^\nu : 0 \leq \mu, \nu \leq 3\}$ where $\hat{\sigma}_i^0$ is the identity operator in $\mathcal{H}_i$ and $\{\hat{\sigma}_i^1, \hat{\sigma}_i^2, \hat{\sigma}_i^3\}$ are equivalent to Pauli spin matrices.

Skeleton sets of operators permit us to define separations and entanglements for $\mathbb{H}(\mathcal{H}_{[1\ldots N]})$ in much the same way as for the quantum register itself and we may use the same index notation to represent separations and entanglements for operators as for the states. For example, $\hat{A}_1^{23}$ will be understood to be an operator of the form $\hat{A}_1 \otimes \hat{B}_{12}$, where $\hat{A}_1$ is an element of $\mathbb{H}_1 \equiv \mathbb{H}(\mathcal{H}_1)$ and $\hat{B}_{12}$ is an element of $\mathbb{H}_{12}$, which is the set of all entangled elements of $\mathbb{H}(\mathcal{H}_{[12]})$, i.e., those not of the form $\hat{C}_2 \otimes \hat{D}_3$ where $\hat{C}_2 \in \mathbb{H}_2$ and $\hat{D} \in \mathbb{H}_3$.

At first sight it may seem incorrect to apply the concept of entanglement to Hermitian operators, given that in standard quantum mechanics they usually represent physical observables corresponding to real physical laboratory equipment. Such equipment appears to us to be rather classical and quite correctly we would not normally think of applying the superposition principle to it per se. In the context we are discussing here, however, superposition refers to vector addition in the abstract space $\mathbb{H}(\mathcal{H})$ of operators representing physical systems, and this will not in general translate directly to anything like the “addition” of physical pieces of equipment together. If $\hat{\Sigma}_1$ and $\hat{\Sigma}_2$ are legitimate tests representing real physical experiments $E_1$ and $E_2$ respectively, then $\hat{\Sigma}_1 + \hat{\Sigma}_2$ might also represent some other real physical experiment $E_3$, but $E_3$ need not have anything to do with either $E_1$ or $E_2$ separately.

The study of the separation and entanglement properties of operators representing tests has been relatively neglected in quantum theory at the expense of the study of the separation and entanglement properties of states, but it is clearly an important part of quantum dynamics nevertheless. For instance, in the EPR thought experiment discussion of entanglement, the spatially separated tests used to observe the components of an entangled state are implicitly assumed to be represented by factorizable operators (otherwise the spatially separated observers could not make independent choices of what to measure), whilst Theorem 3 (discussed below) says that the test which had created the initial entangled state had to be entangled.
Not only are the separation and entanglement properties of operators important in their own right, but their relationship with the separation and entanglement properties of states is an important and subtle one, leading to a complex pattern of causal relationships which generates quantum causal set structure [7]. Fortunately, there are some simple yet powerful theorems controlling the sort of outcomes we should expect from various quantum tests. To understand these results, we should keep in mind that the important structures in our paradigm are not the operators as such but their associated basis sets. This was something recognized by Everett [19].

VI. EIGENVALUES AND PREFERRED BASES

In the following we shall assume all Hilbert spaces are finite dimensional and make frequent references to the following terms:

i) a degenerate operator is a Hermitian operator with at least two linearly independent eigenstates having identical eigenvalues of that operator;

ii) a weak operator is a Hermitian operator which is either degenerate or at least one of its eigenvalues is zero;

iii) a strong operator is a Hermitian operator which is not weak; that is, none of its eigenvalues are zero and all its eigenvalues are distinct.

The subset of $\mathbb{H}(\mathcal{H})$ consisting of all weak elements of $\mathbb{H}(\mathcal{H})$ will be denoted by $\mathbb{W}(\mathcal{H})$ whilst the subset of $\mathbb{H}(\mathcal{H})$ consisting of all strong elements of $\mathbb{H}(\mathcal{H})$ will be denoted by $\mathbb{S}(\mathcal{H})$. Then clearly

$$\mathbb{S}(\mathcal{H}) \cup \mathbb{W}(\mathcal{H}) = \mathbb{H}(\mathcal{H}), \quad \mathbb{S}(\mathcal{H}) \cap \mathbb{W}(\mathcal{H}) = \emptyset. \tag{21}$$

Neither $\mathbb{S}(\mathcal{H})$ nor $\mathbb{W}(\mathcal{H})$ are vector spaces.

We shall now discuss some important theorems involving strong and weak operators which have significant implications for the physics associated with quantum registers. Proofs of most of these are elementary and are discussed in [7].

Theorem 1

The normalized eigenstates of any strong operator $\hat{A} \in \mathbb{S}(\mathcal{H})$ form a unique, orthonormal basis set for $\mathcal{H}$, referred to as the preferred basis of $\mathcal{H}$ relative to $\hat{A}$, denoted by $\mathcal{B}_A \equiv \mathcal{B}_A(\mathcal{H})$. 

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We can use this theorem to relate the concepts of preferred basis sets and strong operators as follows. First we note that associated with our universal quantum register $\mathcal{H}$ is the set $\mathfrak{B}(\mathcal{H})$ of distinct orthonormal basis sets for $\mathcal{H}$. Second, there is a many-to-one mapping from $\mathcal{S}(\mathcal{H})$ onto $\mathfrak{B}(\mathcal{H})$, defined by virtue of Theorem 1. Third, this mapping defines an equivalence relationship for elements in $\mathcal{S}(\mathcal{H})$ which we call basis equivalence: $\hat{A}, \hat{B} \in \mathcal{S}(\mathcal{H})$ are basis equivalent if and only if

$$B_A = B_B.$$  \hspace{1cm} (22)

Basis equivalence is symmetric, transitive and reflexive and therefore defines an equivalence relationship which divides $\mathcal{S}(\mathcal{H})$ into disjoint equivalence classes. All the elements of a given equivalence class have the same preferred basis.

Suppose now we have a rank-2 quantum register $\mathcal{H}_{[12]} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2$ and suppose $\hat{O}_1 \in \mathbb{H}(\mathcal{H}_1)$ and $\hat{O}_2 \in \mathbb{H}(\mathcal{H}_2)$. Then the tensor product operator $\hat{O}_{12} \equiv \hat{O}_1 \otimes \hat{O}_2$ is a separable element of $\mathbb{H}(\mathcal{H}_{[12]})$ and the following theorem holds:

**Theorem 2**

i) If $\hat{O}_1$ or $\hat{O}_2$ is weak then $\hat{O}_{12} \equiv \hat{O}_1 \otimes \hat{O}_2$ is necessarily weak;

ii) equivalently, a tensor product operator is strong only if each of its factors is strong;

iii) if both $\hat{O}_1$ and $\hat{O}_2$ are strong, $\hat{O}_{12}$ need not be strong.

Theorem 2 leads to the following theorem which has important implications for the outcomes of certain kinds of physics experiments:

**Theorem 3 (the fundamental theorem)**

i) All the eigenstates of a separable strong operator are separable;

ii) equivalently, entangled states can be the outcomes of entangled operators only.

These results generalize to higher rank quantum registers. The importance of Theorem 3 is that it forces the factorization properties of the tests
to drive the factorization properties of the states. It is this mechanism which underpins our analysis of quantum causal set theory \cite{7}. To see how this works, consider an initial state of the universe $\Psi_n$ which has precisely $a$ factors, i.e.,

$$\Psi_n = \psi_1 \otimes \psi_2 \otimes \ldots \otimes \psi_a,$$

(23)

where $\psi_i$ is a completely entangled element in some rank-$r_i$ factor of a particular split $S_1$ of the universal register $\mathcal{H}_{[1...N]}$. By relabelling the sub-registers appropriately, we may always write this split in the form

$$S_1 = \mathcal{H}_{[1...r_1]} \otimes \mathcal{H}_{[(r_1+1)...(r_1+r_2)]} \otimes \ldots \otimes \mathcal{H}_{[(N+1-r_a)...N]}.$$  

(24)

Now suppose that the next test of the universe $\hat{\Sigma}_{n+1}$ factorizes into precisely $b$ factors, i.e.,

$$\hat{\Sigma}_{n+1} = \hat{\sigma}_1 \otimes \hat{\sigma}_2 \otimes \ldots \otimes \hat{\sigma}_b,$$

(25)

where each of the factors $\hat{\sigma}_i$ is a completely entangled strong operator acting over some rank-$s_i$ factor of another split $S_2$ ($\mathcal{H}_{[1...N]}$) of the universal register. In general, it will not be possible to easily relate the two splits $S_1$, $S_2$ sub-register by sub-register, because for any two randomly chosen splits of the same register, the likelihood is that none of their factors coincide. It is this which creates the rich structure of quantum causal set theory \cite{7}.

Now according to Theorem 3, the outcome $\Psi_{n+1}$ of test $\Sigma_{n+1}$ has to factorize into $c$ factors, where $c \geq b$. These factors can be arranged into precisely $b$ groups of factors, i.e.

$$\Psi_{n+1} = \phi_1 \otimes \phi_2 \otimes \ldots \otimes \phi_b,$$

(26)

where each group of factors $\phi_i$ defines a factor state of $\Psi_{n+1}$ which is an eigenstate of $\hat{\sigma}_i$ and therefore lies in the corresponding factor of the split $S_2$. It is possible for each of these $b$ groups of factors to consist of one or more factors, because this is not ruled out by the theorem (entangled strong operators can have separable outcomes, but not the other way around).

The relevance to quantum cosmology of these observations is that, if successive tests of the universe contain progressively more factors, then successive states of the universe must factorize at least to the same extent. If now we identify factorization of states with classicality (the appearance of distinct physical identity), then an explanation for the expansion of the universe could be that it is driven by the increasing factorization properties of successive tests of the universe.

A. Quantum zipping

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A fundamental feature of quantum register dynamics which is central to all of this discussion is that inner products between successive states have to follow the rule that, whatever the details of these states and regardless of which split each is in, individual subregister component states always have to “zip” together in inner products. For example, for a rank-2 quantum register, if \( B \equiv \{|i\rangle_1 \otimes |j\rangle_2\} \) and \( B' \equiv \{|a\rangle_1 \otimes |b\rangle_2\} \) are two factorizable basis sets for the register, the inner product \((\Phi, \Psi)\) of states \( \Psi \equiv \sum_{i,j} \psi_{ij} |i\rangle_1 \otimes |j\rangle_2 \) and \( \Phi \equiv \sum_{a,b} \phi_{ab} |a\rangle_1 \otimes |b\rangle_2 \) is of the form

\[
(\Phi, \Psi) = \sum_{i,a} \sum_{j,b} \phi^*_{ab} \psi_{ij} \langle a|i\rangle_1 \langle b|j\rangle_2.
\]  (27)

Quantum zipping is discussed in more detail in §XI.

**VII. NATURAL BASES**

Given two finite dimensional Hilbert spaces \( \mathcal{H}_1, \mathcal{H}_2 \) with dimensions \( d_1, d_2 \) respectively, then their tensor product \( \mathcal{H}_{[12]} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2 \) is also a finite dimensional Hilbert space with dimension \( d_{[12]} = d_1 d_2 \). However, \( \mathcal{H}_{[12]} \) is more than just a Hilbert space with dimension \( d_{[12]} \). Because it is a tensor product, it is possible to discuss separations and entanglements as explained above, which cannot be done with a Hilbert space with the same dimension but not known to be a tensor product. This is an elementary fact which is fundamental for the development of our view of the quantum universe. It arises precisely because a tensor product has component spaces (in this example \( \mathcal{H}_1 \) and \( \mathcal{H}_2 \)) which have their own identities. In other words, a quantum register exists as such simply because it has identifiably distinct subregisters.

This identifiability of subregisters is unrelated to the concepts of distinguishability and indistinguishability of particles in quantum mechanics. Jordan and Wigner showed a long time ago that quantum registers based on the principles we employ here can be used to construct quantum fields with bosonic or fermionic symmetry [20, 21].

Another important aspect of this discussion is that the concepts of splits, separations, entanglements and partitions are defined without reference to any basis sets, either for the subregisters or for quantum register itself. For example, given that \( \Psi \in \mathcal{H}_{[12]} \) is separable, then we can be sure that there exist elements \( \psi_1 \in \mathcal{H}_1 \) and \( \psi_2 \in \mathcal{H}_2 \) such that \( \Psi = \psi_1 \otimes \psi_2 \), this statement being independent of any choice of basis for any of the spaces concerned.

This point is more subtle than its first appears, because it is possible to find basis sets for the tensor product space \( \mathcal{H}_{[12]} \) such that all their elements are entangled. Somewhat surprisingly, such a basis set can then be used to
describe separable states. A discussion of this and related concepts is given in [16]. There we discussed the possibility of finding basis sets for tensor product spaces which have mixed factorization properties. For example, in the case of $\mathcal{H}_{[12]}$, a basis of type $(p,q)$ has $p$ elements which are entangled (i.e., are elements of $\mathcal{H}^{[2]}$) and $q = d_1d_2 - p$ which are separable (i.e., are elements of $\mathcal{H}_{12}$).

In [16] we defined a completely entangled basis for $\mathcal{H}_{[12]}$ to be one of type $(d_1d_2,0)$, whereas a completely separable basis is of type $(0,d_1d_2)$. A completely separable basis will also be referred to as a natural basis, because such a basis displays the full separability properties underlying a tensor product space, whereas the other types either partially or completely hide these properties. In particular, the use of a completely entangled basis is equivalent (on a formal level) to replacing the tensor product space $\mathcal{H}_{[12]}$ with a featureless Hilbert space which happens to have dimension $d_1d_2$. As we have stated, however, matters are more subtle than they appear. The reason we can use a completely entangled basis to discuss separable states is that we still need to have an underlying knowledge of the component spaces of $\mathcal{H}_{[12]}$ in order to define entanglement in the first place and it is this knowledge allows us to say what we mean by “separable” state.

We shall use the symbol $\mathfrak{B}(p,q)$ to denote the set of all type $(p,q)$ bases for a rank-2 quantum register.

VIII. ACTIVE VERSUS PASSIVE TRANSFORMATIONS

Mathematicians are generally concerned with functions, transformations and mappings in a precise way and it is somewhat surprising therefore that the mathematician’s concept of a transformation needs to be qualified when it comes to physics. In physics, it is most important to distinguish between the concepts of passive and active transformations.

Passive transformations are purely formal changes in the descriptions of mathematical structures representing physical systems, one of the main characteristics of these changes being that they are applied to all elements of a set. By definition, passive transformations have no physical or observable consequences.

An example of a passive transformation is a change from one spatial coordinate frame of reference to another. Normally, such a change is regarded by physicists has having no intrinsic effect on the measurable physical relationships between material objects embedded in the space. This idea becomes of the greatest importance in theories such as general relativity, where
a main objective is to identify those aspects of the theory which are generally covariant, i.e., are invariant to arbitrary co-ordinate transformations.

Another example of a passive change is a unitary transformation which acts on all the elements in a Hilbert space. Such a transformation leaves all inner products invariant and on account of this is regarded as physically undetectable.

In contrast to the mathematician’s passive transformation, a real physics experiment always involves a physical change in some parts of the universe and not in others. Locality is one of the characteristics of an active transformation. An active transformation never acts on the universe as a whole, because this would also include the physicists performing the experiment. Clearly, the essential difference between active and passive transformations involves the issue of exophysics versus endophysics.

Active and passive transformations are readily confused when the non-global nature of an active transformation is overlooked. In such cases, sign changes may be the only way of seeing the difference, as in the case of spatial rotations. An analogous situation arises in quantum cosmology when the role of time is considered. In the many-universe scenario, the state of the universe \( \Psi \) is assumed to satisfy the Schrödinger equation

\[
i \hbar \partial_t \Psi = \hat{H} \Psi,
\]

(28)

which gives rise to the unitary evolution conventionally regarded as an essential ingredient of quantum cosmology. Now this equation is given in the Schrödinger picture, in which operators are usually time independent. It is well known however that alternative pictures can be used. In particular, the Heisenberg picture may be used to freeze the quantum state, locating the intrinsic time dependence within the operators representing the observables. The interpretation of this is that the semiclassical observers assumed to be present in the standard view of quantum mechanics carry an intrinsic (i.e. physical) time dependence which is not removed by the passive unitary transformation from Schrödinger picture to Heisenberg picture. When these observers decide to perform a measurement at a given moment \( T \) of their time, the corresponding Heisenberg picture operator representing the said measurement carries a memory of the observer’s time \( T \).

From this we arrive at the standard equality of expectation values between each of these two pictures:

\[
\langle \Psi, T | \hat{A}_s | \Psi, T \rangle_s = \langle \Psi | \hat{A}_H (T) | \Psi \rangle_H,
\]

(29)

where \( | \Psi, T \rangle_s = \hat{U} (T) | \Psi \rangle_H \), \( \hat{A}_H (T) \equiv U^+ (T) \hat{A}_s \hat{U} (T) \) and \( \hat{U} (T) \) is the unitary temporal evolution operator associated with (28). The big problem
for quantum cosmology is that it is generally assumed that there are no external observers in the first place. Therefore, there is no form of external memory of when any measurement is taken, which is why time itself seems to have been transformed away by the change from the Schrödinger picture to the Heisenberg picture in quantum cosmology. A similar line of argument, associated with general covariance, is behind the lack of a global time in the Wheeler-de Witt equation

\[ \hat{H}|\Psi\rangle = 0. \]  

(30)

We are at risk of a similar problem arising in the stages paradigm, because we too do not have any external observers. To avoid it, we must ensure that our endophysical definition of information exchange is one which cannot be undermined by any passive transformation of any sort. Such a definition can be found based on the concept of partition change. The next two sections set the scene for our statement of this definition.

IX. LOCAL TRANSFORMATIONS

Given a Hilbert space \( \mathcal{H} \), we define \( \mathbb{U}(\mathcal{H}) \) to be the set of all unitary transformations on \( \mathcal{H} \). Similarly, if \( \mathcal{H}_1 \) and \( \mathcal{H}_2 \) are two Hilbert spaces, then we denote the set of all unitary transformations on their tensor product \( \mathcal{H}_{[12]} \) by the symbol \( \mathbb{U}_{[12]} \equiv \mathbb{U}(\mathcal{H}_{[12]}) \).

Before we discuss local unitary operators in more detail, we need to clarify one small point; \( \mathbb{U}_1 \equiv \mathbb{U}(\mathcal{H}_1) \) and \( \mathbb{U}_2 \equiv \mathbb{U}(\mathcal{H}_1) \) act on different spaces and are therefore unrelated. Technically, neither is a subset of \( \mathbb{U}_{[12]} \), for example. However, whenever it suits our purposes, we shall assume without further comment that when we write \( \mathbb{U}_1 \), for example, we may also mean \( \mathbb{U}_1 \otimes \hat{I}_2 \), where \( \hat{I}_2 \) is the identity operator on \( \mathcal{H}_2 \), and so on, depending on context.

An important subset of \( \mathbb{U}_{[12]} \) is the set \( \mathbb{U}_{12} \equiv \mathbb{U}_1 \bullet \mathbb{U}_2 \) of local unitary transformations, all the elements of which are of the form

\[ \hat{U}_{12} \equiv \hat{U}_1 \otimes \hat{U}_2, \quad \hat{U}_i \in \mathbb{U}_i, \quad i = 1, 2. \]  

(31)

The local unitary transformations form a nonabelian group under operator product multiplication.

The significance of local unitary transformations is that they transform natural bases into other natural bases, as can be readily proved. For example, a natural basis \( \mathcal{B}_{(0,d_1,d_2)} \subset \mathcal{B}_{(0,d_4,d_2)} \) has elements of the form \( \phi_{12} \equiv a_1 \otimes b_2 \in \mathcal{H}_{12} \), where \( a_1 \in \mathcal{H}_1 \) and \( b_2 \in \mathcal{H}_2 \). Then a local unitary transformation \( \hat{U}_{12} \)
of each such element $\phi_{12}$ gives
\[
\hat{U}_{12}\phi_{12} = (\hat{U}_1 \otimes \hat{U}_2)(a_1 \otimes b_2) = (\hat{U}_1 a_1) \otimes (\hat{U}_2 b_2) = \phi'_{12} \in \mathcal{H}_{12}. \quad (32)
\]
This, and the fact that unitary transformations preserve inner products, proves the assertion. Formally, we shall write
\[
\mathcal{U}_{12}\mathcal{B}_{(0,d_1d_2)} = \mathcal{B}_{(0,d_1d_2)}. \quad (33)
\]
This generalizes in an obvious way to higher rank tensor product spaces.
This leads to the following theorem which is important in our concept of information exchange:

**Theorem 4**

Local unitary transformations are invariances of separations and entanglements.

**Proof**

We prove this first for separations and then for entanglements:

i) **Separations**: let $\hat{U}_{1...n} \equiv \hat{U}_1 \otimes \ldots \otimes \hat{U}_n$ be any element of $\mathcal{U}_{1...n}$ and let $\phi_{1...n} \equiv \phi_1 \otimes \ldots \otimes \phi_n$ be any element of $\mathcal{H}_{1...n}$. Then
\[
\hat{U}_{1...n}\phi_{1...n} = \left(\hat{U}_1 \otimes \ldots \otimes \hat{U}_n\right)\left(\phi_1 \otimes \ldots \otimes \phi_n\right) = \left(\hat{U}_1 \phi_1\right) \otimes \ldots \otimes \left(\hat{U}_n \phi_n\right), \quad \in \mathcal{H}_{1...n}. \quad (34)
\]
Hence
\[
\mathcal{U}_{1...n}\mathcal{H}_{1...n} = \mathcal{H}_{1...n} \quad (35)
\]
as required.

ii) **Entanglements**: Let $\hat{U}_{1...m} \equiv \hat{U}_1 \otimes \ldots \otimes \hat{U}_m$ be any element of $\mathcal{U}_{1...m}$ and let $\phi^{1...m}$ be any element of the entanglement $\mathcal{H}^{1...m}$ ($m > 1$). Now suppose that $\hat{U}_{1...m}$ takes $\phi^{1...m}$ out of $\mathcal{H}^{1...m}$ into some state $\phi'$ not in $\mathcal{H}^{1...m}$. Because $\phi'$ is given as not in $\mathcal{H}^{1...m}$, it is necessarily in the complement $\mathcal{H}_{[1...m]} - \mathcal{H}^{1...m}$ of $\mathcal{H}^{1...m}$ in the register $\mathcal{H}_{[1...m]}$. Then
because $m > 1$ and by definition and construction of entanglements, the result $\phi'$ must necessarily be a separable state, i.e., we may write

$$\hat{U}_{1...m} \phi^{1...m} = \Phi \otimes \Psi \in \mathcal{H}_{[1...m]} - \mathcal{H}^{1...m}, \quad (36)$$

for some factor states $\Phi$, $\Psi$. Now without loss of generality we may always relabel the subregisters so that we may write

$$\Phi \in \mathcal{H}_{[1...k]}, \quad \Psi \in \mathcal{H}_{[(k+1)...m]}, \quad (37)$$

for some $k$ satisfying the condition $1 \leq k < m$.

Next, because $\hat{U}_{1...m}$ is a unitary operator, it has an inverse, $\hat{U}_{1...m}^{-1}$, given by

$$\hat{U}_{1...m}^{-1} = \hat{U}_{1}^{-1} \otimes \ldots \otimes \hat{U}_{m}^{-1}. \quad (38)$$

This inverse is also a local unitary transformation. Applying this inverse to equation (36) gives

$$\phi^{1...m} = \hat{U}_{1...m}^{-1} (\Phi \otimes \Psi) = \left(\hat{U}_{1...k}^{-1} \Phi\right) \otimes \left(\hat{U}_{(k+1)...m}^{-1} \Psi\right), \quad (39)$$

which contradicts the given condition that $\phi^{1...m}$ is fully entangled. Hence the result is proven for entanglements as well as separations.

Because all partitions can be written as separation products of separations and entanglements, we readily deduce that the result holds for all partitions generally.

**X. STATE PREPARATION**

In the stages paradigm, each outcome of a jump serves also as a preparation or initial state for the next jump. This means that the dynamics of the universe involves a sequence of *ideal measurements* [4], but this does not mean that information is being extracted by any external observer.

When we are discussing an actual physics experiment, the term *state preparation* will be reserved here to mean a particular class of stage jump, from a state of the universe $\Psi_{n-1}$ to a state of the universe $\Psi_n$ such that $\Psi_n$ is of the separable form

$$\Psi_n \equiv \psi_{[12..P]} \otimes \Theta_{[(P+1)...N]}, \quad (40)$$

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for some \( P \) such that \( 1 \leq P < N \), where \( \psi_{[12...P]} \) is in \( \mathcal{H}_{[12...P]} \), \( \Theta_{[(P+1)...N]} \) is in \( \mathcal{H}_{[(P+1)...N]} \), and \( \mathcal{H}_{[12...P]} \otimes \mathcal{H}_{[(P+1)...N]} \) is a particular split of the total Hilbert space \( \mathcal{H}_{[12...N]} \). Without loss of generality, we may always relabel the subregisters to give the above convenient representation. More specifically, \( \Psi_n \) will be in the particular partition

\[
\Psi_n \in \mathcal{H}_{[12...P]} \otimes \mathcal{H}_{[(P+1)...N]} \equiv \mathcal{H}_{[12...P] \otimes [(P+1)...N]} \tag{41}
\]

of the total Hilbert space.

Depending on the split, \( \psi_{[12...P]} \) may be thought of as the state of the subject of the experiment whilst \( \Theta_{[(P+1)...N]} \) is the state of the observer plus environment plus wider universe. It is clear from our paradigm, however, that this is not an intrinsic description; which is observer and which is subject will be irrelevant except on emergent scales. In actual physics experiments, \( P \) could be relatively small, such as 1, or enormous, such as of the order \( 10^{100} \), but this would pale into relative insignificance given an \( N \) of the order \( 10^{182} \) or more. Under these circumstances, i.e. \( 1 \leq P \ll N \), it is reasonable to call \( \psi_{[12...P]} \) the subject (system under observation) and \( \Theta_{[(P+1)...N]} \) the observer (which includes the laboratory, the local environment and the rest of the universe).

**XI. TRANSITION AMPLITUDE FACTORS**

Before we can give a definition of what we mean by information exchange in quantum systems, we need to make some observations concerning transition amplitudes. In the following, it is the “zipping” properties of quantum register inner products, discussed in §VI, which combine with the partition structure of the states concerned to produce the factorization properties of transition amplitudes.

Let \( \Psi_n \) and \( \Psi_{n+1} \) be two successive states of the universe. Each of these is a vector in \( \mathcal{H}_{[1...N]} \), the total quantum register. Now from our discussion of entanglements and separations, we know that \( \mathcal{H}_{[1...N]} \) is the union of a large number of partitions, the full set of which we call the natural lattice of partitions and denoted by \( \mathcal{L} \left( \mathcal{H}_{[1...N]} \right) \). Each element \( \Psi \) in \( \mathcal{H}_{[1...N]} \) lies in a unique partition \( P_\Psi \) in \( \mathcal{L} \left( \mathcal{H}_{[1...N]} \right) \) and has \( F \left[ P_\Psi \right] \) factors, each factor lying in a different block associated with \( P_\Psi \). For each state \( \Psi_n \), the pattern of the associated blocks defines a unique split \( S_n \) of the quantum register.

There are two cases to consider: \( i \) \( \Psi_n, \Psi_{n+1} \) are in the same partition and \( ii \) \( \Psi_n \) and \( \Psi_{n+1} \) are in different partitions. These need to be discussed separately.
i) Suppose that $\Psi_1, \Psi_2 \in \mathcal{H}_{[1...N]}$ lie in the same partition, i.e., $P_{\Psi_1} = P_{\Psi_2}$, and are not in the full entanglement $\mathcal{H}_{1...N}$. Then each state has at least two factors (assuming $N > 2$). Because they lie in the same partition, they have the same number of factors, each of which is a separation or an entanglement. Suppose there are $F$ such factors, such that

$$\Psi_i = \psi_i^{(1)} \otimes \psi_i^{(2)} \otimes \ldots \otimes \psi_i^{(F)}, \quad i = n, n + 1,$$

where each factor $\psi_i^{(k)}$ belongs to the $k^{th}$ block in the partition $P_{\Psi_i}$. Then by virtue of the “zipping” properties of the subregisters, the inner product $(\Psi_n, \Psi_n)$ necessarily factorizes into the same number $F \equiv F[P_{\Psi_n}] = F[P_{\Psi_{n+1}}]$ of factors formally, i.e.,

$$\left(\Psi_{n+1}, \Psi_n\right) = \left(\psi_{n+1}^{(1)}, \psi_n^{(1)}\right) \left(\psi_{n+1}^{(2)}, \psi_n^{(2)}\right) \ldots \left(\psi_{n+1}^{(F)}, \psi_n^{(F)}\right).$$

Each factor in this product is associated with a given factor in the split $\mathcal{S}_n$ of $\mathcal{H}_{[1...N]}$.

ii) Suppose on the other hand that $P_{\Psi_n} \neq P_{\Psi_{n+1}}$, i.e., $\Psi_n$ and $\Psi_{n+1}$ belong to different partitions. Then by inspection we find that for any transition involving such a change of partition, the number of factors $F[(\Psi_{n+1}, \Psi_n)]$ in the amplitude $(\Psi_{n+1}, \Psi_n)$ satisfies the relation

$$F[(\Psi_{n+1}, \Psi_n)] \leq \min \{ F(\Psi_n), F(\Psi_{n+1}) \} < \max \{ F(\Psi_n), F(\Psi_{n+1}) \}. \quad (44)$$

The upper bound $\max \{ F(\Psi_n), F(\Psi_{n+1}) \}$ is attained only if there is no partition change. If either $\Psi_n$ or $\Psi_{n+1}$ is in the full entanglement $\mathcal{H}_{1...N}$, then $F[(\Psi_{n+1}, \Psi_n)]$ attains its lowest bound, unity.
The set of all possible patterns of factorization of transition amplitudes becomes increasingly more complex as the rank of the quantum register increases. In Figure 1, we show all topologically inequivalent quantum zipper diagrams for rank-1, 2 and 3 quantum registers, disregarding the direction of time. Vertical lines represent inner products between subregisters whilst horizontal lines represent entanglements.

Table 1 shows the number of formal factors $F$ in the amplitudes between all the various possible initial and final state types for a rank-3 quantum register:

$$
\begin{array}{|c|ccccc|}
\hline
F & \psi_{123} & \psi_{12}^{23} & \psi_{13}^{32} & \psi_{23}^{12} & \psi_{123}^{123} \\
\hline
\phi_{123} & 3 & 2 & 2 & 2 & 1 \\
\phi_{1}^{23} & 2 & 2 & 1 & 1 & 1 \\
\phi_{2}^{13} & 2 & 1 & 2 & 1 & 1 \\
\phi_{3}^{12} & 2 & 1 & 1 & 2 & 1 \\
\phi_{123} & 1 & 1 & 1 & 1 & 1 \\
\hline
\end{array}
$$

Table 1. The number of amplitude factors $F$ for given initial and final states.

XII. THE ISOLATED QUANTUM SYSTEM

In this section we discuss what can be said about an isolated quantum system $S$, called the subject, when it is described by a rank-1 quantum register of dimension $d$. States of such a system will be assumed never to factorize. There are three scenarios which we shall consider in turn. The first two invoke the usual exophysical principles of standard quantum mechanics whilst the third gives an endophysical account.

A. Complete isolation from exophysical observers

In this scenario, exophysical observers stand outside of the subject $S$ with no information exchange whatsoever between it and them after it has been prepared by them in a given state $\psi_0$ at initial exophysical time $n = 0$. The external observers have only two roles in this scenario: first they prepare the subject state and then they arrange to keep it isolated from the rest of the universe. They retain a knowledge of how the subject state was prepared and are motivated to keep it isolated. We shall argue that after preparation, the subject will behave in effect as if it were an isolated subuniverse and, because
it is not observed, it may be considered by the observers to be “frozen” in
time, even if it evolves autonomically.

The issue here rests on what “complete isolation” means. Taken literally,
it can only mean that no test organized by the external observers is performed
on the subject after state preparation. No test means no outcome, which
means the state remains unchanged.

This is of course a rather trivial conclusion. The discussion is not quite
complete however, because we have to examine the possibility that the sub-
ject might test itself and jump into a new state without reference to the
external observers. The question is then, what could the external observers
say under those circumstances?

The problem faced by the external observers in discussing this possibility
is analogous to the problem of parallel transport in general relativity. This
arises because tangent vector spaces at different points in a manifold are
distinct spaces. In principle, there is no a priori or natural way of ensuring
that bases in different tangent vector spaces coincide and in a sense, such a
concept has no direct physical meaning anyway.

It was because of this problem that the concepts of Lie differentiation
and covariant differentiation had to be devised. These provide a way of
relating bases in different spaces to each other. For example, in the case of a
Riemannian manifold, the metric over the manifold can be used to construct
a metric connection, which can then be used to determine how basis vectors
change as we move over the manifold. In our case, we need to define carefully
what we mean by the idea that the state of the system “changes” from \( \psi_n \)
to state \( \psi_{n+1} \) at exotime \( n + 1 \) in the absence of any measurement of \( \psi_{n+1} \)
by the external observers.

To help us in the formal analysis and by analogy with the parallel trans-
port problem, we shall assume that states of the subject \( S \) at different times
lie in different Hilbert spaces, i.e., we shall suppose that for \( n \geq 0 \), \( \psi_n \in \mathcal{S}_n \),
where the \( \mathcal{S}_n \) are copies of \( \mathcal{S} \).

Now assuming that the stages paradigm holds, the observers know some
things for certain but have only a partial knowledge of other things. What
they do know is this: because they prepared the subject state and retained
a knowledge of it, the observers know all about the initial preferred basis set
\( \mathcal{B}_0 \equiv \{|i, 0\} : 1 \leq i \leq d\} \in \mathcal{B}(\mathcal{S}_0) \). Also, they know which particular element
of this basis set the prepared initial state \( \psi_0 \) happened to be.

Believing that the subject may have evolved via its own internal dynam-
ics by some arbitrary number of jumps, the observers are entitled to assume
that at any moment \( n > 0 \) of exotime after preparation, \( \psi_n \) is some element
of another preferred basis \( \mathcal{B}_n \equiv \{|i, n\} : 1 \leq i \leq d\} \in \mathcal{B}(\mathcal{S}_n) \). However, although they may believe that these things “exist”, the observers have no
knowledge of either the $B_0$ or the actual outcomes $\psi_n$.

Even with such limited information, the observers may always relabel the elements in $B_0$ so that $\psi_0 = |1, 0\rangle$. They can also assume a formal relabelling for the unknown bases $B_n$ such that for each $n$, $\psi_n = |1,n\rangle$, because this does not actually invoke any new knowledge. The question now is, what grounds are there for relating the elements of the basis sets $B_0$ and $B_n$? They are bases for different copies of the same Hilbert space and therefore something analogous to a connection would be required in order to allow us to compare vectors in one copy with vectors in another.

To formally define a process of “parallel transport of basis”, we introduce a linear map $\hat{U}(B_{n+1}, B_n)$ from $\mathcal{S}_n$ to $\mathcal{S}_{n+1}$ given by the expression

$$\hat{U}(B_{n+1}, B_n) \equiv \sum_{i=1}^{d} |i, n+1\rangle \langle i, n|.$$  \hfill (45)

This map transports ket states in $\mathcal{S}_n$ into ket states in $\mathcal{S}_{n+1}$ and has properties associated with Rota incidence algebras \[22\]. Such algebras encode some of the properties of causal sets. For instance, we have the product rule

$$\hat{U}(B_{n+2}, B_{n+1}) \hat{U}(B_{n+1}, B_n) = \hat{U}(B_{n+2}, B_n),$$  \hfill (46)

but the “product” $\hat{U}(B_{n+1}, B_n) \hat{U}(B_{n+2}, B_{n+1})$ is not defined. There has to be some chain of causality for such products to be meaningful.

The map $\hat{U}(B_{n+1}, B_n)$ is invertible, with inverse map

$$\hat{U}^{-1}(B_{n+1}, B_n) \equiv \hat{U}(B_n, B_{n+1}) = \sum_{i=1}^{d} |i, n\rangle \langle i, n+1|.$$  \hfill (47)

Then we have the results

$$\hat{U}^{-1}(B_{n+1}, B_n) \hat{U}(B_{n+1}, B_n) = \hat{I}_n, \quad \hat{U}(B_{n+1}, B_n) \hat{U}^{-1}(B_{n+1}, B_n) = \hat{I}_{n+1},$$  \hfill (48)

where $\hat{I}_n$ and $\hat{I}_{n+1}$ are the identity operators in $\mathcal{S}_n$ and $\mathcal{S}_{n+1}$ respectively. The inverse map is formally equivalent to the “adjoint” map, i.e.,

$$\hat{U}^{-1}(B_{n+1}, B_n) = \hat{U}^+(B_n, B_{n+1}) \equiv \sum_{i=1}^{d} |i, n\rangle \langle i, n+1|,$$  \hfill (49)

which takes states in the dual (bra) space $\mathcal{S}_n^*$ into states in the dual space $\mathcal{S}_{n+1}^*$.

If now we apply the map $\hat{U}(B_n, B_0)$ to the state $\psi_0$ we find

$$\hat{U}(B_n, B_0) \psi_0 = \psi_n.$$  \hfill (50)
The effect of this is to make the internal jump of the subject state look formally like the result of some unitary evolution, such as that given by the integration of Schrödinger evolution in continuous time. We may now formally “undo” this evolution by transforming to the Heisenberg picture, effectively rotating the basis \( B_n \) into \( B_0 \). Now precisely because the external observers have no interaction with \( S \) after state preparation, this transformation has no physical consequences for them; there is no measurement undertaken by the external observers subsequent to state preparation which could register the effect of the change of picture.

Our conclusion is, therefore, that as long as the subject remains isolated, the external observers are entitled to regard it as frozen in time, even if they had grounds for believing it had some autonomous evolution.

### B. Standard exophysical quantum description

The above discussion is close to triviality because it ignores the physical presence of the external observers after state preparation. It is, in effect, a solipsist view of a subuniverse in which only the subject exists. In this subsection we discuss the conventional quantum description of an experiment on a nominally isolated quantum subject system in the active presence of external observers. Now the external observers have three roles: i) they prepare the subject state at time \( n = 0 \), ii) they isolate the subject during some interval \((0, M)\), of their endotime, where typically \( M \gg 0 \), and finally, iii) they test the subject state at time \( M \).

The analysis proceeds as for the first scenario, except for the final phase, state testing, which occurs at time \( M \). Now the external observers play an active role, because they create the testing equipment. This defines a final preferred basis \( B_M \), which is therefore now known to the observers. The observers also retain a knowledge of the initial preferred basis \( B_0 \) and indeed of the initial state \( \psi_0 \in B_0 \). It is the simultaneous knowledge of both bases which cannot be eliminated by any passive rotation of basis. A rotation of one basis must also be applied to the other basis, so that transition amplitudes between elements of the two bases remain invariant and cannot be eliminated.

Another important difference between this scenario and the previous one is that, before the final test, the principles of quantum mechanics do not now permit the observers to assume that the final state is any particular element of the final preferred basis \( B_M \), such as \( \psi_M = |1, M\rangle \). Therefore, the Rota incidence algebra discussion can only be undertaken after the whole experiment is over and outcome \( \psi_M \) has been observed. By that time, however, it will no longer be reasonable for the observers to argue that the subject state
has not changed, because a real active change in it will have been registered for certain by their equipment.

In standard quantum mechanics, external observers are a crucial component of information exchange. It is their knowledge of both the initial and final bases which cannot be transformed away by any unitary transformation of basis. This is why either the Schrödinger picture or Heisenberg picture can be used in standard quantum mechanics. This is not the case in any paradigm which has no external observers and no state reduction.

C. The endophysical description

The second scenario above is exophysical and is therefore one we wish to replace by an endophysical discussion. We now discuss what happens from the endophysical perspective, when the “observers” are contained within a greater quantum system $U$ (the universe), of which the original subject $S$ is but a part. The discussion in this situation requires more care concerning the tests involved.

Now regardless of whether an exophysical or endophysical description is being used, a quantum experiment generally requires a sequence of three things to happen. First the subject has to be prepared in some initial state (state preparation). Then the initial state then has to be given time to evolve in isolation. Finally, at the end of each run of the experiment, the state has to be tested. In the real world of experimental physics, each of these steps will be very complex, involving extremely sophisticated equipment and experimental protocols, even if the standard quantum description appears straightforward. Let us consider each of these steps separately for a given run of the experiment.

i) State preparation

Assuming that the greater quantum system $U$ containing the observer and the subject can be described via the stages paradigm, state preparation means that the state $\Psi_n$ of the universe at initial time $n = 0$ is of the form

$$\Psi_0 \equiv \Theta_0 \otimes \psi_0,$$

where the pure state $\Theta_0$ (the complement) represents the observers, apparatus and environment external to the subject, which is represented by $\psi_0$ as before. Note that $\Theta_0$ will necessarily be different for each run of the experiment, because the universe as a whole is not reversible. On the other hand, the subject state $\psi_0$ can be assumed to be identical at the start of
each run. If this last condition is relaxed, then a density matrix discussion of the experiment will be required.

The complement $\Theta_0$ is an element of some large rank quantum register $A_0$ whilst $\psi_0$ is an element of $S_0$, as before. The initial state of the universe $\Psi_0$ is an element of the quantum register $H_0 \equiv A_0 \otimes S_0$. Although $H_0$ contains both entangled and separable states, state preparation means that $\Psi_0$ is of the separable form (51) and we are entitled to write

$$\Psi_0 \in A_0 \cdot S_0 \subset A_0 \otimes S_0.$$  \hspace{1cm} (52)

In other words, state preparation in a physics experiment is equivalent to ensuring that an initial state of the universe is an element of a partition with two or more factors (blocks).

**ii) Isolation**

Now in a real experiment, the observers generally contrive in some way to ensure that the system under investigation remains isolated for a reasonable length of their time. Therefore, in our endophysical description of such a process, we may suppose that there is a sequence of jumps of the state of the universe,

$$\Psi_0 \rightarrow \Psi_1 \rightarrow \ldots \rightarrow \Psi_n \rightarrow \ldots \rightarrow \Psi_{M-1}, \quad M \gg 0.$$  \hspace{1cm} (53)

during which the subject state remains isolated, i.e., we have

$$\Psi_n \equiv \Theta_n \otimes \psi_n \in A_n \cdot S_n \subset A_n \otimes S_n, \quad 0 \leq n < M,$$  \hspace{1cm} (54)

where $A_n$ is a copy of $A_0$, provided we have not entered the partition change regime (which is where we state that real information exchange occurs).

In real physics experiments, the observers do not in general have absolute control of the universe and total isolation of a subject system cannot be guaranteed in general. For example, in particle scattering experiments, there is always the possibility of background effects, such as high energy cosmic rays passing through the subject system, interfering with any particular run of the experiment. It is only after the scattering information has been acquired that the observers can deduce what level of isolation had in fact been achieved.

From the stages paradigm point of view, complete isolation can be guaranteed by virtue of Theorem 3, if each test $\hat{\Sigma}_n$ of the universe between state preparation and outcome definitely factorizes into the form

$$\hat{\Sigma}_n = \hat{A}_n \otimes \hat{S}_n.$$  \hspace{1cm} (55)
where $\hat{A}_n \in \mathbb{S}(\mathcal{A}_n)$ and $\hat{S}_n \in \mathbb{H}(\mathcal{S}_n)$. Background effects cannot occur if this factorization condition holds. Another way of understanding background effects is that they involve entanglement between observer and subject.

Given perfect isolation, Theorem 3 tells us that because $\hat{\Sigma}_n$ is strong (according to our principles), then its preferred basis $\mathcal{B}_n$ is completely factorizable, i.e., is of type $(0, \dim \mathcal{A} \times \dim \mathcal{S})$. Indeed, we may write

$$\mathcal{B}_n = \mathcal{B}_{A_n} \cdot \mathcal{B}_{S_n}, \quad (56)$$

where $\mathcal{B}_{A_n}$ is the preferred basis for $\hat{A}_n$ and $\mathcal{B}_{S_n}$ is the preferred basis for $\hat{S}_n$. We can now apply the Rota incidence algebra discussion to each subject state basis $\mathcal{B}_{S_n}$ so that it is rotated back into the initial preferred basis $\mathcal{B}_{S_0}$. The incidence algebra operators are extended to the universal register in a straightforward way, i.e., for each $n$ between 0 and $M$ we define

$$\hat{U}(\mathcal{B}_n, \mathcal{B}_0) \equiv \hat{I}_{A_n} \bigotimes \sum_{i=1}^{d} |i, n\rangle \langle i, 0|, \quad 0 \leq n < M, \quad (57)$$

where $\hat{I}_{A_n}$ is the identity over $\mathcal{A}_n$. Then

$$\Psi_n \rightarrow \Psi'_n \equiv \hat{U}^{-1}(\mathcal{B}_n, \mathcal{B}_0) \Psi_n = \Theta_n \otimes |1, 0\rangle = \Theta_n \otimes \psi_0. \quad (58)$$

The result is that during the period of isolation, the state of the subject system may be regarded as subject to what we call a null test [9], the principal characteristic of which being that it tests one of its eigenstates, which therefore passes through unchanged.

These Rota incidence algebra transformations are equivalent to transforming to a partial Heisenberg picture wherein the subject state is frozen but the rest of the universe is not. Because in principle there is no formal difference in the stages paradigm between subject and complement, it should be possible to exchange roles and freeze the observers but not the subject. However, we should keep in mind that in real situations, the register $\hat{\mathcal{A}}$ associated with the observers will have vastly greater rank than the register $\mathcal{S}$ associated with the subject. It is the vastly greater complexity associated with $\hat{\mathcal{A}}$ which permits us to use anthropomorphic terminology occasionally, as if the observers had some sort of choice in what they were doing. In fact, the entire system $\hat{U}$ is behaving simply as a quantum automaton.

From now on, we shall drop the formal distinction between different temporal copies of the same vector space. We introduced this notion in order to facilitate the Rota algebra discussion. In fact, the concept of different copies of spaces corresponding to different times is a block universe idea which has
no place in process time thinking. Therefore, both \( \Psi_n \) and \( \Psi'_n \) may be considered to be in the same quantum register. An important point about the Rota analysis is that the observers believe that only one of these vectors represents the true state of the subject at time \( n \).

The sequence of states of the universe during isolation is now given by

\[
\Theta_0 \otimes \psi_0 \rightarrow \Theta_1 \otimes \psi_0 \rightarrow \ldots \rightarrow \Theta_n \otimes \psi_0 \rightarrow \ldots \rightarrow \Theta_{M-1} \otimes \psi_0. \tag{59}
\]

Essentially, the subject state can be regarded as frozen during the period of isolation, simply because it not being observed, whilst the rest of the universe undergoes dynamical change.

iii) Test and outcome

The most important aspect of the discussion concerns the nature of the test \( \hat{\Sigma}_M \) at the end of the period of isolation and the nature of the tests which follow it. Test \( \Sigma_M \) will still be separable and of the specific form (55), but the Rota incidence algebra transformation is not applied for the final time \( M \). This is because the act of measurement means that the observers believe that the hitherto isolated subject state has made an active jump into some eigenstate of their measuring equipment.

The final test \( \Sigma_M \) will have a preferred basis \( B_M = B_{AM} \bullet B_{SM} \), where

\[
B_{AM} \equiv \{ \Theta^\alpha_M : 1 \leq \alpha \leq \dim A \} \quad \text{and} \quad B_{SM} \equiv \{ \phi^\beta_M : 1 \leq \beta \leq \dim S \},
\]

and so the state of the universe at time \( M \) is of the form

\[
\Psi_M = \Theta^\alpha_M \otimes \phi^\beta_M.
\]

Tests after time \( M \) cannot take the form (55). Otherwise Theorem 3 tells us that isolation was continuing beyond time \( n = M \). Therefore, in order for information to be exchanged between the subject and the complement after time \( M \), \( \hat{\Sigma}_{M+1} \) must be basis inequivalent to \( \hat{\Sigma}_M \). In other words, information exchange requires a change of partition associated with the test of the universe. There is no way around this conclusion.

Once the universe has jumped into stage \( \Omega_M \), the stages paradigm dynamical principles take over. The information content \( I_M \) now contains information about the final state \( \phi^\beta_M \) of the subject. The rules \( \mathcal{R}_M \) now take over and dictate that the subsequent test \( \hat{\Sigma}_{M+1} \) will depend on this particular outcome. This particular outcome essentially influences the subsequent history of the universe in such a way that the information that the subject state jumped into state \( \phi^\beta_M \) is encoded into all subsequent stages (at least for as long as the observers retain a memory of the outcomes of the experiment).
After many runs of the same experiment, a frequency distribution for the various outcomes \( \phi^\alpha_M, \phi^\beta_M, \ldots \) will be encoded (registered) into the state of the universe and eventually the observers can compare this with the theoretical probability \( P(\phi^\alpha_M | \psi_0) \) of the transition \( \psi_0 \rightarrow \phi^\alpha_M \). This is given in self-explanatory notation by the rule

\[
P(\phi^\alpha_M | \psi_0) = \sum_{\Theta_1} \sum_{\Theta_2} \cdots \sum_{\Theta_M} P(\Theta_M \otimes \phi^\alpha_M | \Theta_{M-1} \otimes \psi_0) \times P(\Theta_{M-1} \otimes \psi_0 | \Theta_{M-2} \otimes \psi_0) \cdots P(\Theta_1 \otimes \psi_0 | \Theta_0 \otimes \psi_0) \]

\[
= \sum_{\Theta_1} \sum_{\Theta_2} \cdots \sum_{\Theta_M} P(\Theta_M | \Theta_{M-1}) |\langle \phi^\alpha_M | \psi_0 \rangle|^2 \times P(\Theta_{M-1} | \Theta_{M-2}) \cdots P(\Theta_1 | \Theta_0) \\
= |\langle \phi^\alpha_M | \psi_0 \rangle|^2 \sum_{\Theta_1} \sum_{\Theta_2} \cdots \sum_{\Theta_M} \prod_{n=1}^{M} |\langle \Theta_n | \Theta_{n-1} \rangle|^2 \\
= |\langle \phi^\alpha_M | \psi_0 \rangle|^2,
\]

which is the standard quantum result. In this calculation, summation is over all the basis elements of the bases for intermediate stages. We note that there are no interference terms.

This calculation does not take into account the fact that the state \( \Theta_0 \) of the complement at the start of each run would be different each time (necessarily so, because after each run, the universe has registered new information). Neither does it take into account the probability distribution of the tests \( \hat{A}_n \) of the complement during isolation. It is not hard to see that these effects would not alter the conclusion in any way. In other words, quantum experiments on isolated systems can be undertaken and the rules of quantum mechanics can be applied to those isolated systems, even when the observers are themselves evolving according to the rules of quantum mechanics.

**XIII. HIGHER RANK SUBJECT SYSTEMS**

For completeness, we discuss now what may happen when the subject system consists of two or more subregisters. Now we are able to discuss separations and entanglements of the subject state, which gives some important constraints on our ability to "parallel-transport away" changes in state during isolation. For simplicity we shall restrict our attention to the case when the subject register \( S \) consists of two subregisters, i.e., \( S = \mathcal{H}_{[12]} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2 \).

There are three cases to consider during the period of isolation; in each case we restrict our attention to the subject register.
i) separable to separable:

Consider a jump of the subject state of the form $\psi_{12} \rightarrow \phi_{12}$, where each of these states is in the separation $\mathcal{H}_{12}$. With our conventions for separations and entanglements, we can rewrite this process in a way reminiscent of consistent histories:

$$
\psi_{12} \rightarrow \phi_{12} \Rightarrow \psi_1 \otimes \psi_2 \rightarrow \phi_1 \otimes \phi_2 = \left( \psi_1 \rightarrow \phi_1 \right) \otimes \left( \psi_2 \rightarrow \phi_2 \right),
$$

(61)

where the tensor product in the last term carries a somewhat different meaning to that hitherto. It is a sort of product of “histories”. Essentially, the dynamical evolution here suggests that there are two completely distinct, noninteracting subject systems, each of which appears to be evolving in its own subuniverse. Moreover, each of these subuniverses is no different in its properties to the rank-1 subject register discussed extensively in the previous section. We can see then, that in this particular case, a passive local unitary transformation applied to the quantum register can transform away the change in the state, exactly as in the Rota algebra discussion applied above.

ii) separable to entangled (and vice-versa)

With a jump of the form $\psi_{12} \rightarrow \phi_{12}$ there is a change of partition, from a state in the separation $\mathcal{H}_{12}$ to a state in the entanglement $\mathcal{H}^{12}$. It will be seen upon inspection that there is no way of performing any local unitary transformation which can transform away such a change in the state. The reason of course is directly associated with Theorem 4, which states that partitions are invariant to local unitary transformations. This sort of jump therefore represents a nontrivial internal change of the subject state involving both subregisters.

If the observers external to the subject wish to use the Rota algebra method to transform away the change of the subject, then they can only do so if they perform a non-local unitary transformation. In other words, they have to ignore the fact that $\mathcal{S}$ is a tensor product space.

iii) entangled to entangled

A jump of the form $\psi^{12} \rightarrow \phi^{12}$ raises the interesting mathematical question of whether we can always find some local unitary transformation which can transform one entangled state $\psi^{12}$ into any other entangled state $\phi^{12}$.
If the answer were yes, then taking into account our previous analysis, we would conclude that without a change of partition of the subject state, we could always maintain the fiction that there is no intrinsic quantum dynamics (i.e., we could always regard fixed-partition change as due to a passive transformation of basis).

To answer this question we shall consider a two qubit quantum register. Suppose $\psi_{12}$ and $\phi_{12}$ are two states in the entanglement $H_{12}$ of the register $H_1 \otimes H_2$. According to Schmidt decomposition, we can always find a decomposition of each state in the following form:

$$\psi_{12} = \sqrt{p}|a\rangle_1 \otimes |b\rangle_2 + \sqrt{1 - p}|\bar{a}\rangle_1 \otimes |\bar{b}\rangle_2,$$

$$\phi_{12} = \sqrt{q}|u\rangle_1 \otimes |v\rangle_2 + \sqrt{1 - q}|\bar{u}\rangle_1 \otimes |\bar{v}\rangle_2,$$  \hspace{1cm} (62)

where $0 \leq p, q \leq \frac{1}{2}$, $B^a_1 \equiv \{|a\rangle_1, |\bar{a}\rangle_1\}$ and $B^u_1 \equiv \{|u\rangle_1, |\bar{u}\rangle_1\}$ are orthonormal bases for $H_1$ and $B^b_2 \equiv \{|b\rangle_2, |\bar{b}\rangle_2\}$ and $B^v_2 \equiv \{|v\rangle_2, |\bar{v}\rangle_2\}$ are orthonormal bases for $H_2$. Here, the real numbers $p$ and $q$ can be interpreted as conditional probabilities.

Now it is always possible to construct a unitary transformation $\hat{U}_1$ which transforms $B^a_1$ into $B^u_1$, such that

$$\hat{U}_1|a\rangle_1 = |u\rangle_1, \quad \hat{U}_1|\bar{a}\rangle_1 = \hat{U}_1|\bar{u}\rangle_1.$$  \hspace{1cm} (63)

In fact, $\hat{U}_1$ is unique and given by

$$\hat{U}_1 = |u\rangle_1\langle a| + |\bar{u}\rangle_1\langle \bar{a}|.$$  \hspace{1cm} (64)

Likewise there exists a unique unitary transformation $\hat{V}_2$ which transforms $B^b_2$ into $B^v_2$ such that

$$\hat{V}_2|b\rangle_2 = |v\rangle_2, \quad \hat{V}_2|\bar{b}\rangle_2 = \hat{V}_2|\bar{v}\rangle_2.$$  \hspace{1cm} (65)

The tensor product $\hat{W}_{12} \equiv \hat{U}_1 \otimes \hat{V}_2$ is a local unitary transformation on $H_{12} \equiv H_1 \otimes H_2$ which has the specific effect of transforming $\psi_{12}$ into a near clone of $\phi_{12}$:

$$\psi_{12} \rightarrow \hat{W}_{12} \psi_{12} = \sqrt{p}|u\rangle_1 \otimes |v\rangle_2 + \sqrt{1 - p}|\bar{u}\rangle_1 \otimes |\bar{v}\rangle_2.$$  \hspace{1cm} (66)

However, for $p \neq q$ it is clear that there is no way that we could transform $\psi_{12}$ into $\phi_{12}$ exactly via any local unitary transformation.

The conclusion from this is that if an entangled factor of a state of the subject jumps into another entangled factor within the same entanglement, the change in that factor cannot always be interpreted in terms of a passive,
local unitary transformation of basis. In other words, real dynamical changes can occur within an entanglement, not involving any change of partition.

This is an important result. It means that a universal quantum register can be discussed in terms of isolated subsystems evolving within greater subsystems. We can imagine a Schrödinger’s cat experiment locked away within a box, such that even though we have no contact with the contents of the box, we can legitimately imagine it evolving dynamically, with state reduction taking place out of sight inside the box. Note however, on the basis of our analysis in the previous section, that even though real dynamical changes may occur within a subject state, no consequences of those changes can be communicated to the observers unless there is a change in partition involving the observers and the subject, i.e., effectively entangling their states. In the case of the Schrödinger’s cat experiment, this corresponds to opening the box. Essentially, states evolving within fixed partitions behave as if they were in separate subuniverses, rather like regions of spacetime divided by event horizons, for as long as those partitions persist.

**XIV. THE BASIC PRINCIPLE OF ENDOPHYSICAL INFORMATION EXCHANGE**

By considering these basic examples we can see the appearance of a criteria for defining real (i.e., intrinsic) dynamical changes in quantum states, as opposed to those which could be removed by passive transformations. This leads us to state what we mean by endophysical information transfer:

“any quantum process in which meaningful information is exchanged within a quantum system is always accompanied by a change in partition”.

We shall call this the principle of endophysical information exchange, because it does not rely on any notion of observer or system per se. It says what information exchange means for closed systems.

Whilst this criterion does not at first sight look much like the conventional picture of products of states representing observers and systems changing in time due to interactions, it has a number of features which are physically appealing. First, it relies on the existence of an underlying quantum register. We have already demonstrated that the stage paradigm based on these leads to causal set structures [2]. Our concept of information exchange is a natural one in this paradigm. Second, there is no need to introduce the concepts of systems or observers to define information exchange. Changes of partition involve mathematical
relationships which apply equally well to either concept. This therefore gives a truly endophysical picture of quantum system dynamics. Certainly, if we needed to, we could always associate large numbers of subregisters in a given block (factor of a partition) with an “observer”, but this need not have any intrinsic meaning. It would be whatever happened to the subsequent tests and outcomes which would determine the viability of any defined “observer” or system concept. One criterion for this would be the relative persistence in exotime of various patterns of factorization. This is motivated by the observation that, on typical macroscopic scales associated with humans, we do not normally see macroscopic objects suddenly disappearing. All structures do disappear eventually, however, given enough jumps.

Third, when there is no change in partition over a number of jumps, then essentially the various blocks in the partition behave much like isolated subjects and observers between which no real information exchange occurs. It is only when partitions change that we can be sure that real dynamical exchange between systems and observers have occurred. When these occur in real experiments, changes of partition which may start off involving a relatively small number of subregisters may be amplified enormously over time, so that the effect in the emergent limit will be equivalent to the conventional picture of semiclassical observers appearing to record changes in quantum subsystems. With large rank quantum registers, it should be possible to represent memory and data storage within the complex of partition structure available. That cannot be the entire story however; it is the tests on states which will determine in what sense partition structure represents memory and data. In other words, the whole process of quantum measurement involves the entire dynamics, which must include the states and the tests. Indeed, how tests are determined is as fundamental to the running of the universe as what the outcomes of those tests are.

A fourth point about our definition of information exchange is that partition change is inherently consistent with the fundamental principle in quantum mechanics that the acquisition of real information from a state necessarily destroys that state. In fact, a partition change necessarily destroys all factor states involved, so that in essence, we can say that a quantum measurement changes not only the system being observed but the observer as well.

**XV. CONCLUDING REMARKS**

The basic idea we have put forward is remarkably simple but has many implications. Many aspects of the idea remain to be developed. We have had
only a limited opportunity here to discuss the details of the sort of experiments actually performed in physics laboratories. Although the principles are universal, such experiments are quite atypical of the tests of the universe occurring naturally. A physics experiment generally involves vast numbers of subregisters, most of which will be associated with the apparatus and the immediate environment, not to mention the physicists running the experiment. Such experiments need many stages before completion, but from our point of view are no more than enormous irreversible amplifications of elementary partition changes.

The implications of these concepts will be reported in due course.

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