Operational Semantics of Process Monitors

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Abstract. CSP$_E$ is a specification language for runtime monitors that can directly express concurrency in a bottom-up manner that composes the system from simpler, interacting components. It includes constructs to explicitly flag failures to the monitor, which unlike deadlocks and livelocks in conventional process algebras, propagate globally and aborts the whole system’s execution. Although CSP$_E$ has a trace semantics along with an implementation demonstrating acceptable performance, it lacks an operational semantics. An operational semantics is not only more accessible than trace semantics but also indispensable for ensuring the correctness of the implementation. Furthermore, a process algebra like CSP$_E$ admits multiple denotational semantics appropriate for different purposes, and an operational semantics is the basis for justifying such semantics’ integrity and relevance. In this paper, we develop an SOS-style operational semantics for CSP$_E$, which properly accounts for explicit failures and will serve as a basis for further study of its properties, its optimization, and its use in runtime verification.

Keywords: Operational Semantics, Concurrency, Runtime Monitoring, Communicating Sequential Processes

1 Introduction

Specification-based runtime monitoring \cite{5} checks a program’s execution trace against a formal specification. Often more rigorous than testing due to the presence of a formal specification, this technique is also computationally much cheaper than formal verification methods like model checking, as it only needs to look at concrete program runs with instrumentation. CSP$_E$ \cite{10} is a language based on Hoare’s Communicating Sequential Processes \cite{6} for developing the formal specification. Unlike many other languages in this niche, CSP$_E$ can directly express concurrency. Moreover, it builds up the specification in a bottom-up manner by composing smaller, interacting components, helping to model complex behavior.

CSP$_E$’s main appeal as a specification language, compared to plain CSP, is a \texttt{FAIL} construct that signals a global failure, aborting all processes in the model at once. This construct can be used like \texttt{assert(false)} in C or Java, allowing to code invariants that mark some states as (should-be) unreachable. By contrast,
**Event** \( e \in \Sigma \)  
**Event Variable** \( x \in X \)  
**Term** \( P, Q \in \text{Terms} ::= \text{STOP} \mid \text{FAIL} \mid ?x : E \rightarrow P \mid P \underline{\square} Q \mid P \parallel E \)  
**Event Set** \( E ::= f(y_1, \ldots, y_n) \) where \( f : \Sigma^n \rightarrow 2^{\Sigma} \) is computable  
**Event Set Param** \( y ::= x \mid e \)  

![Fig. 1. Syntax of CSP\(_E\).](image)

deadlocks and livelocks, the conventional notions of failure in CSP, affect only the deadlocked or livelocked process(es). These failures are thus very difficult to propagate into a failure for the entire system, as desired for assertion failures. However, the semantics of how \text{FAIL} propagates throughout the model requires special treatment. Because the propagation preempts all other activities, normal execution rules must apply only when \text{FAIL} is not currently propagating. This is a negative constraint, which is generally problematic [4].

While earlier work [10] demonstrated a trace semantics and a reasonably efficient implementation for CSP\(_E\), an operational semantics has been lacking. Developing an operational semantics is highly desirable for several reasons. Firstly, though a trace semantics more naturally defines the set of behaviors (i.e. traces) that comply with a CSP\(_E\) specification, an operational semantics more directly defines the implementation. Secondly, process algebras admit multiple denotational semantics capturing different aspects of operationally defined behavior [3]. Investigating the full spectrum of such semantics requires an operational semantics. Finally, an operational semantics provides a more accessible presentation of the semantics than denotational semantics.

### 1.1 Contributions

In this paper, after reviewing the syntax and trace semantics of CSP\(_E\) (Section 2), we present the following contributions.

- We define an operational semantics in SOS format [8], which properly captures the propagation of \text{FAIL} while avoiding the complexities of rules with negative premises (Section 3).
- We prove that the operational semantics induces the previously published trace semantics (Section 4).

### 2 Syntax and Trace Semantics of CSP\(_E\)

This section reviews the syntax and trace semantics of CSP\(_E\). Figure 1 presents the syntax. A CSP\(_E\) term represents a process, which is an entity that successively emits events drawn from an alphabet \( \Sigma \). Terms are built from the following constructs, with the indicated meanings. For a thorough explanation, see [10].

- The stuck term \text{STOP} does not emit anything.
- The failing term \text{FAIL} aborts all processes.
Trace Set
\( s, t \in \Sigma^* \)
Trace Set Operations
\[
e T := \{ \varepsilon \} \cup \{ et \mid t \in T \}
\]
\[
T(e) := \{ t \mid et \in T \}
\]
\[
\emptyset \parallel e T := e T \parallel \emptyset := \emptyset
\]
\[
T_1 \parallel e T_2 := \bigcup_{e \in E} e(T_1(e) \parallel e T_2(e)) \cup \bigcup_{e \in \Sigma \setminus E} (e(T_1(e) \parallel e T_2) \cup e(T_1 \parallel e T_2(e)))
\]
Trace Semantics
\[
[STOP] := \{ \varepsilon \}
\]
\[
[P \Box Q] := [P] \cup [Q]
\]
\[
[\square x : E \rightarrow P] := \{ \varepsilon \} \cup \bigcup_{e \in E} e[e/x]P
\]

Fig. 2. Trace semantics of CSP. Equation (1) takes precedence over eq. (2), so the latter applies only if the former does not.

– Prefix \(? x : E \rightarrow P\) chooses and emits an event \(e \in E\), then executes \([e/x]P\).
– Choice \(P \Box Q\) executes \(P\) or \(Q\), whichever manages to emit something first.
– Parallel composition \(P \parallel E Q\) executes \(P\) and \(Q\) in parallel. Their events are interleaved arbitrarily, except events in \(E\) are synchronized.

An event set \(E\) can be specified by any computable function parametrized by the \(x\)'s bound by surrounding prefix operators.

In this short paper, we omit recursion and the terminating action \(\checkmark\) in the interest of conciseness. This paper’s focus is on analyzing \(\text{FAIL}\), and \(\checkmark\) complicates the presentation substantially without adding anything of conceptual significance. Recursion seems to be similar, though it is still under investigation.

Figure 2 presents the trace semantics. A trace is a (possibly empty) sequence of events, and \(\Sigma^*\) is the set of all traces. The concatenation of traces \(s\) and \(t\) is written \(st\). A trace set is any prefix-closed set of traces, which can be empty, unlike in conventional process algebras. The trace semantics of CSP\(_E\) assigns to each term \(P\) a trace set \([P]\), which is intuitively the set of traces \(P\) can emit.

The semantic map uses some operations on trace sets. If \(T\) is a trace set, \(e T\) prepends \(e\) to all members of \(T\) and adjoins \(\varepsilon\), while \(T(e)\) discards all traces in \(T\) that do not start with \(e\) and drops the leading \(e\) from all remaining traces. The \(\parallel e\) operator is defined by eqs. (1) and (2). Though significantly simplified, these equations are equivalent to the ones found in [10] modulo the absence of \(\checkmark\). In [10], this operator was defined “coinductively”, which was correct but misleading. Formally, by the Knaster-Tarski Theorem, the defining equations (1) and (2) have a greatest solution in the complete lattice of total binary functions on \(\text{TraceSets}\) ordered by point-wise inclusion, which was taken to be \(\parallel e\). However, if \(\parallel e'\) and \(\parallel e''\) are any two solutions of these equations, then for any \(T_1\) and \(T_2\), every trace in \(T_1 \parallel e' T_2\) is also in \(T_1 \parallel e'' T_2\), by straightforward induction on the trace’s length. Thus, the solution is unique, and \(\parallel e\) is this unique solution.
Action

\[ a ::= e \mid \tau \]

Doomed Term

\[ D \in \text{Doomed} ::= \text{FAIL} \mid D \parallel D \mid D \parallel E \parallel P \parallel P \parallel E \parallel D \]

Viable Term

\[ \hat{P}, \hat{Q} \in \text{Terms} - \text{Doomed} \]

Operational Semantics

\[
\begin{align*}
\varepsilon &\in E \\
(P \mapsto P') \mapsto [e/x]P &\quad Q \mapsto Q' \\
P \parallel Q \mapsto P' &\quad P \parallel Q \mapsto Q' \\
P \parallel E \parallel Q \mapsto P' \parallel E \parallel Q &\quad a \notin E \\
\hat{P} \mapsto P' &\quad \hat{Q} \mapsto Q' \\
\hat{P} \parallel E \hat{Q} \mapsto P' \parallel E \hat{Q} &\quad \hat{D}_1 \mapsto P_1 \\
\hat{D}_2 \mapsto P_2 &\quad \hat{P} \parallel E \hat{D}_2 \mapsto \hat{P} \parallel E \hat{D}_2 \mapsto P_1 \parallel E \hat{D}_2 \\
\text{FAIL} \parallel \text{FAIL} \mapsto \text{FAIL} &\quad \text{FAIL} \parallel E \parallel P \mapsto \text{FAIL} \\
P \mapsto P' &\quad P \mapsto P'' \\
\end{align*}
\]

Fig. 3. Operational semantics of CSP\(_E\).

Lemma 1. \( \| E \) is continuous, i.e. \( (\bigcup S_1) \| E (\bigcup S_2) = \bigcup_{T_1 \in S_1, T_2 \in S_2} T_1 \| E T_2 \).

Proof. The defining equations \( \mathbf{(1)} \) and \( \mathbf{(2)} \) preserve continuity, so in fact the Knaster-Tarski construction can be carried out in the space of continuous binary operators, which is also a complete lattice under point-wise inclusion.

3 Operational Semantics

This section presents the operational semantics. The semantics is given in Figure 3, which defines internal transitions \( P \mapsto^a Q \) between terms. Some transitions do not emit events but instead emit the silent action \( \tau \). A visible transition \( P \Rightarrow Q \) happens when \( P \) internally transitions to \( Q \) in zero or more steps, and the non-\( \tau \) actions it emits along the way forms \( s \).

The main challenge in this semantics is capturing the propagation of \text{FAIL}. For example, in \( P \parallel \text{FAIL} \), the \( P \) must not be allowed to keep emitting events, for then \( P \) could do so indefinitely, withholding the propagation of \text{FAIL}. Instead, \text{FAIL} should kill all processes including \( P \), transitioning the whole term to \text{FAIL}.

To achieve this effect, the usual rule that allows the left operand to transition must apply only when the right operand is not failing. This constraint is tricky to capture because it is a negative constraint.

In our semantics, the constraint is captured by the viability annotation \( \hat{P} \). This annotation restricts the range of the metavariable \( \hat{P} \) to exclude doomed terms, i.e. terms for which transitioning to \text{FAIL} has become inevitable and are now propagating \text{FAIL} within themselves. These annotations are placed so that
Then, a lemma characterizing those $Q$ the inductive hypothesis applies to give $\llbracket h \rrbracket$. Lemma 5. Proof. Induction on the size of $P$. Example 4. Proposition 4. Take for $s$ induction on $P$. Proof. Induction on $P$. Furthermore, trace sets faithfully follow non-silent transitions, in that the traces which follow an event $e$ in $\llbracket P \rrbracket$ are precisely the traces of terms $Q$ that follow $P$ after a sequence of transitions that emit $e$.

Lemma 5. $\llbracket P \rrbracket(e) = \bigcup_{P \xrightarrow{e} Q} \llbracket Q \rrbracket$.

Proof. Induction on the size of $P$, where event sets do not count toward size, e.g. $|?x : E \rightarrow P'| := |P'| + 1$. This way, $|\llbracket e/x \rrbracket P'| = |P'|$; so when $P = (?x : E \rightarrow P')$, the inductive hypothesis applies to $\llbracket e/x \rrbracket P'$, despite it not being a subterm. Several lemmas are needed along the way, two of which are of particular note. Take for example $P = P_1 \llbracket E P_2$ with $\llbracket P_1 \rrbracket, \llbracket P_2 \rrbracket \neq \emptyset$ and $e \in E$. Inductive hypotheses give $\llbracket P \rrbracket(e) = (\bigcup_{P_1 \xrightarrow{e} Q_1, Q_2} \llbracket Q_1 \rrbracket) \llbracket E (\bigcup_{P_2 \xrightarrow{e} Q_2} \llbracket Q_2 \rrbracket)\rrbracket$. Then, continuity (Lemma 7) lets us commute the $\bigcup$ and $\llbracket E$, equating this to $\bigcup_{P_1 \xrightarrow{e} Q_1, P_2 \xrightarrow{e} Q_2} (\llbracket Q_1 \rrbracket \llbracket E \llbracket Q_2 \rrbracket\rrbracket)$.

Then, a lemma characterizing those $Q$ with $P \xrightarrow{e} Q$ equates this to $\bigcup_{P \xrightarrow{e} Q} \llbracket Q \rrbracket$.

Theorem 3 is a straightforward consequence of these facts.

Proof (of Theorem 3). We show $s \in \llbracket P \rrbracket \iff \exists Q. P \xrightarrow{s} Q \notin \text{Doomed}$ by induction on $s$. For the base case, $e \in \llbracket P \rrbracket \iff P \notin \text{Doomed}$ by Proposition 4. If $P \notin \text{Doomed}$, then $P \xrightarrow{e} P \notin \text{Doomed}$, and if $P \in \text{Doomed}$, then $P \xrightarrow{e}$ only finitely many times, while staying doomed. Another induction shows $\forall D \neq \text{FAIL}. \exists D'$. $P \xrightarrow{e} D'$, so a doomed term keeps transitioning until it reaches \text{FAIL}.

4 Correspondence Between the Semantics

This section establishes a correspondence between the two semantics: a process’ denotation is precisely the set of traces it can emit, up to but not including any transitions that doom the process. This means that the monitor comparing a system to $P$ can declare a failure as soon as the system’s trace strays out of $\llbracket P \rrbracket$.

Theorem 3. $\llbracket P \rrbracket = \{s \mid \exists M. P \xrightarrow{e} M \notin \text{Doomed}\}$.

A special case of this theorem is particularly illuminating: the doomed set is precisely the set of terms with empty trace sets, corresponding to the fact that doomed terms silently transition to \text{FAIL}.

Proposition 4. $P \in \text{Doomed} \iff \llbracket P \rrbracket = \emptyset$.

Proof. Induction on $P$.

Doomed terms silently transition to \text{FAIL} when a term is doomed, rules that propagate \text{FAIL} become the only applicable ones, thus forcing the propagation to take place.

Proposition 2. A doomed process always transitions to \text{FAIL} while emitting nothing but $\tau$.

Proof. $D \xrightarrow{P}$ implies $a = \tau \land P \in \text{Doomed} \land |D| > |P|$, where $|P|$ denotes term size, by induction on $D$. Thus, a doomed term can only $\tau$-transition, and only finitely many times, while staying doomed. Another induction shows $\forall D \neq \text{FAIL}. \exists D'$. $D \xrightarrow{\tau} D'$, so a doomed term keeps transitioning until it reaches \text{FAIL}.
5 Related Works

The main issue with CSP\textsubscript{E} semantics is the propagation of \textit{FAIL}, which entails the negative constraint that normal computation rules apply only if \textit{FAIL}-propagation rules do not. Negative premises of the form \( P \not\rightarrow \) come quite naturally as a means for codifying such constraints, but negative premises are generally quite problematic. A transition relation satisfying negative rules may be not-existent, or non-unique, with no obvious guiding principle (such as minimality) in choosing the “right” one. Some formats do guarantee well-definedness, such as GSOS with the witnessing constraint \cite{2} and \textit{ntyft/ntyxt} \cite{4}. But even then, negative rules tend to betray desirable properties such as compositionality of some forms of bisimulation \cite{4}.

Our approach exploits the fact that we only have a very specific negative constraint – the absence of doomed subprocesses – and encodes it with a restriction on the range of metavariables in transition rules. With trick, we manage to avoid negative premises altogether, essentially turning the system into a positive one. This approach is very commonly employed, e.g. in reduction rules for the call-by-value \( \lambda \) calculus \cite{7}, where the argument in a function application should be evaluated only if the function expression cannot be evaluated any further.

We identify \textit{FAIL}-induced failures by transitions into \textit{FAIL}, but an alternative approach would be to have \textit{FAIL} emit a special event \( \mathcal{F} \), just as termination is signalled by \( \checkmark \). Though we have not pursued this idea in detail, the central concern there will be to give \( \mathcal{F} \) higher priority than all other events. Prioritized transition also involves a negative constraint but is known to be quite well-behaved, being translatable to plain CSP \cite{9}. At the moment, it is not clear if \textit{FAIL} propagation can be translated to the prioritized-transition primitive in \cite{9}.

6 Conclusion

We gave an operational semantics for CSP\textsubscript{E} that adequately captures the behavior of \textit{FAIL}, the global failure operator, with positive operational rules. This semantics induces the previously defined trace semantics. As noted in the introduction, this development enables studies of other types of denotational semantics, while informing the implementation. An interesting direction of future work is to see if \textit{FAIL} can be specified by priorities, and if that approach yields better-behaved semantics.

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