A new type of dark energy model

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Abstract. In this paper, we propose a general form of the equation of state (EoS) which is the function of the fractional dark energy density $\Omega_d$. At least, five related models, the cosmological constant model, the holographic dark energy model, the agegraphic dark energy model, the modified holographic dark energy model and the Ricci scalar holographic dark energy model are included in this form. Furthermore, if we consider proper interactions, the interactive variants of those models can be included as well. The phase-space analysis shows that the scaling solutions may exist both in the non-interacting and interacting cases. And the stability analysis of the system could give out the attractor solution which could alleviate the coincidence problem.

Keywords: gravity, cosmological phase transitions, dark energy theory

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1 Introduction

Nowadays there is a wide consensus among observational cosmologists that our universe is accelerating [1–4]. However, the essence of this acceleration is still an open question [5]. The cosmological constant is the simplest explanation, but it has the fine-tuning problem which requires the observed cosmological constant much smaller than the fundamental Planck scale, and the coincidence problem as well: why the cosmological constant and the matter have comparable energy density today even their evolution behavior is so different.

Dynamical dark energy (DE) models have been proposed as alternatives to the cosmological constant, such as quintessence and phantom [6, 7]. In principle, the dark energy problem may be one part of the puzzles of quantum gravity. However, we have not invented a complete theory of quantum gravity as yet. For all that, important progress in the study of the black hole theory and string theory is the holographic principle [8], which could be considered as a fundamental principle of quantum gravity and then shed some light on the DE problem. Its typical application in cosmology is holographic dark energy [9, 10], the agegraphic dark energy [11], modified holographic dark energy [12] and Ricci scalar dark energy [13] as well. They are dynamical models in which the dark energy equation of state (EoS) \( \omega_d \) can be written as a function of the fractional dark energy density \( \Omega_d \).

However, we emphasize that such flexibility and generality are particularly important to our research on \( \omega_d \), not only because they increase the range of possibility to be tested, but also because in principle they may reduce the possibility of misleading results an incorrect EoS parameterization can produce. For detailed research on \( \omega_d \), we assume a kind of holographic model in which the dark energy equation of state (EoS) \( \omega_d \) can be written as a function of the fractional dark energy density \( \Omega_d \). If we introduce a barotropic fluid to the system, the dynamics make the system an autonomous system.
In the previous work, many studies focus on the stability property of cosmological scaling solutions in an expanding universe in the scalar field dark energy model. A phase-space analysis of the spatially flat FRW (Friedmann-Robertson-Walker) models shows that there exist cosmological scaling solutions which are the unique late-time attractors [14–17]. With the assumption of $\omega_d$ being a function of $\Omega_d$, it may be a self-similar system that we can do some research on its phase space from the point of stability analysis. In addition, the cosmological evolution of the dark energy interacting with background perfect fluid could be investigated at the same time. We consider the cases of dark energy interacting with background perfect fluid, while the interaction terms are taken to be three different forms which are familiar in the literature [18–20]. And the physical consequence of these results should be given out as well. In this paper, we also do many observational constraint discussions on these interacting models [21–23].

This paper is organized as follows. In section 2, we propose our dark energy model and present the physical background that we refer to. In section 3, we extend to the interaction cases. In section 4, we give out the results of exact phase-analysis. In section 6, we try to analyze the four examples in detail. Finally, a short summary will be presented.

2 Physical background

The standard cosmology suggests that our universe went through the radiation dominated period, the matter dominated period, and now the dark energy dominated period. For consistence, any theoretical models should coincide with this history of the universe.

In this work, we assume that the geometry of space-time is described by the flat FRW metric which seems to be consistent with today’s cosmological observations

$$ds^2 = -dt^2 + a^2(t) \sum_{i=1}^{3} (dx^i)^2,$$

where $a$ is the scale factor. Based on the subject we are interested in, we assume that there are two main components in the universe: the background matter and the dark energy. The background matter is assumed to be described by a perfect fluid with barotropic equation of state

$$\rho_m' + 3\gamma \rho_m = 0,$$

where $\rho_m$ is the energy density of the background matter, a prime denotes the derivative with respect to the e-folding time $N \equiv \ln a$, and the barotropic index $\gamma$ is a constant and satisfies $0 < \gamma \leq 2$. In particular, $\gamma = 1$ and $\gamma = 4/3$ correspond to dust matter and radiation, respectively. And the dark energy component leads to

$$\rho_d' + 3(1 + \omega_d) \rho_d = 0,$$

where the $\rho_d$ is the energy density of dark energy, and the dark energy EoS parameter is $\omega_d = p_d/\rho_d$.

With respect to the whole system, the Friedmann and Raychaudhuri equations read

$$H^2 = \frac{\rho_{tot}}{3m_{pl}^2} = \frac{1}{3m_{pl}^2}(\rho_m + \rho_d),$$

$$\dot{H} = -\frac{1}{2m_{pl}^2}(\rho_{tot} + p_{tot}) = -\frac{(1 + \omega_{tot})\rho_{tot}}{2m_{pl}^2},$$

where $m_{pl}$ is the Planck mass and the index “tot” notes the variables for the whole system.
It is convenient to introduce the fractional energy densities
\[ \Omega_i \equiv \rho_i / (3m_{pl}^2 H^2), \]  
with \( i \) being \( m \) or \( d \). The Friedmann Equation can be rewritten as
\[ \Omega_m + \Omega_d = 1. \]  
(2.7)
Furthermore, the EoS parameter can be reexpressed as
\[ \omega_{\text{tot}} = \frac{\Omega_m \omega_m + \omega_d}{1 + \Omega_d}. \]  
(2.8)

As the universe evolves, combining eqs. (2.2), (2.3), (2.5) and (2.6), we can get the equations of motion for \( \Omega_i \) separately
\[ \Omega_m' = 3f_n \Omega_m \Omega_d, \]  
(2.9)
\[ \Omega_d' = -3f_n \Omega_d \Omega_m, \]  
(2.10)
with \( f_n = \omega_d - (\gamma - 1) \), where the index \( n \) means there is no interaction between the background matter and the dark energy.

However, we do not really have the exact form of \( \omega_d \), so we could not know the form of \( f_n \). There is a kind of holographic dark energy type model where \( \omega_d \) is a function of \( \Omega_d \), which can make the dynamic system an autonomous system. This kind of model has been widely researched. It was firstly motivated from the effective quantum field theory. Cohen, et al. suggested that the quantum zero-point of a system with the size \( L \) should not exceed the mass of a black hole with the same size, i.e., \( L^3 \rho_d < L m_{pl}^3 \), where \( \rho_d \) is the quantum zero-point energy density which we could use as dark energy. Thus, the ultraviolet (UV) cutoff scale of a system is connected to its infrared (IR) cut-off scale. Applying this idea to the whole universe, the vacuum energy can be considered as DE. Choosing the largest IR cutoff \( L \) which saturates the inequality, we obtain the holographic DE density
\[ \rho_d = 3c^2 m_{pl}^2 L^{-2}, \]  
(2.11)
where \( c \) is an unknown constant due to the theoretical uncertainties and can only be determined by observations.

We can see that, with \( L = \text{constant} \), it recovers the cosmological constant model. Interestingly, taking \( L \) to be the size of the current universe which is the Hubble radius \( H^{-1} \), it yields a wrong equation of the state for DE, but a correct DE density which is close to the observed values. Choosing different \( L \)’s, we can get different EoS which are listed in table 1.

In particular, holographic dark energy chooses the event horizon as the cutoff, agegraphic dark energy choose the age of the universe as the cutoff, modified holographic dark energy model relates the cutoff to the blackhole mass, and Ricci scalar holographic model choose the value of Ricci scalar as cutoff. In each of the models, the dark energy state of equation \( \omega_d \) is related to the dark energy fractional energy \( \Omega_d \), which could reduce a self-similar system. For a complete and exact examination, we could treat \( f_n \) as a general form of the \( \Omega_d \) which is
\[ f_n(\Omega_d) = \omega_d - w_m = \omega_d(\Omega_d) - (\gamma - 1), \]  
(2.12)
where the form of \( f_n \) or \( \omega_d \) is determined by the unknown underlying theory. Assuming \( f_n \) exists, we can deal with the system as an autonomous system. Then we can discuss the properties based on the existence and the stability of its critical points.
Dark energy models | $L$ | $\omega_d$
---|---|---
Cosmological constant | Constant | $\omega_d = -1$
Holographic | Event horizon | $\omega_d = -\frac{1}{3} \left( 1 + \frac{2\sqrt{\Omega_d}}{c} \right)$
Agegraphic | Age of the universe | $\omega_d = -\frac{1}{3} \left( 3 - \frac{2\sqrt{\Omega_d}}{n} \right)$
Modified holographic | Blackhole mass | $\omega_d = \frac{\alpha - 2}{2 - \alpha \Omega_d}$
Ricci | Ricci scalar | $\omega_d = -\frac{2}{3\gamma} + \frac{1}{3\gamma} - \frac{\Omega_m}{\Omega_d} (\gamma - 1)$

Table 1. The cutoff and the equation of state in cosmological constant model, holographic dark energy model, agegraphic dark energy model, modified holographic dark energy model and Ricci scalar dark energy model. $c, n, \alpha$ and $\beta$ are all related parameters in these models.

3 Interacting effects

Some interacting models are discussed in many works in order to understand or alleviate the coincidence problem. The basic idea is to consider the possible interaction between dark energy and dark matter owing to the unknown nature of dark energy and dark matter. In this section, by including the interaction between the dark energy and the background fluid, we will consider the change of the form $f_\gamma$. Although the interaction can significantly change the cosmological evolution, with proper interactions, the system is still an autonomous system.

Assuming the dark energy and the background matter exchange energy through interaction term $Q$, the continuity equations become

$$\rho_d + 3(1 + \omega_d)\rho_d = -Q, \quad (3.1)$$
$$\rho_m + 3\gamma \rho_m = Q, \quad (3.2)$$

which still preserve the total energy conservation equation $\dot{\rho}_{\text{tot}} + 3H(\rho_{\text{tot}} + p_{\text{tot}}) = 0$. When $Q = 0$, there is no interaction between matter and dark energy. When $Q > 0$, the dark energy will lose energy to the the background matter. When $Q < 0$, the dark energy will get energy from the background matter. The interaction term $Q$ can be assumed to be some special forms. For convenience, we consider the following specific interaction forms:

- Case (I) $Q_1 = 3\gamma_m \rho_m$, \quad (3.3)
- Case (II) $Q_2 = 3\gamma_d \rho_d$, \quad (3.4)
- Case (III) $Q_3 = 3\gamma_{\text{tot}} \rho_{\text{tot}}$. \quad (3.5)

We use the indices 1, 2, 3 notify the different interacting cases. We can write the effective EoS parameter for both dark energy and background matter

$$\omega_{d1} = \omega_d(\Omega_d) + \gamma_m \frac{1 - \Omega_d}{\Omega_d}, \quad \omega_{m1} = \gamma - 1 - \gamma_m \quad (3.6)$$
$$\omega_{d2} = \omega_d(\Omega_d) + \gamma_d, \quad \omega_{m2} = \gamma - 1 - \gamma_d \frac{\Omega_d}{1 - \Omega_d} \quad (3.7)$$
$$\omega_{d3} = \omega_d(\Omega_d) + \gamma_{\text{tot}} \frac{1}{\Omega_d}, \quad \omega_{m3} = \gamma - 1 - \gamma_{\text{tot}} \frac{1}{1 - \Omega_d} \quad (3.8)$$
Table 2. Four main critical points.

| Label | Physical meaning | (Ω, Ω) | f_j | (λ_1, λ_2) | ωtotal |
|-------|-----------------|--------|-----|------------|---------|
| M     | Matter dominated | (1, 0) | f_j | (0, -3f_j) | ωtotal = ω_m |
| F_M   | Matter dominated | (Ω, Ω) | f_j = 0 | (0, -3f_j, Ω_dΩ_m) | ωtotal = Ω_mω_mΩ_d + ω_d |
| D     | DE dominated     | (0, 1) | f_j | (0, 3f_j) | ωtotal = Ω_d |
| F_D   | DE dominated     | (Ω, Ω) | f_j = 0 | (0, -3f_j, Ω_dΩ_m) | ωtotal = Ω_mω_mΩ_d + ω_d |

where the index e means the EoS parameter of dark energy or matter is “effective”. Furthermore, we find

\[ f_j = \omega_{dej} - \omega_{mej} \]  

(3.9)

where \( j = 1, 2, 3 \), and

\[ f_1 = f_n + \frac{\gamma_m}{\Omega_d} = \omega_d(O_d) - (\gamma - 1) + \frac{\gamma_m}{\Omega_d} \]  

(3.10)

\[ f_2 = f_n + \frac{\gamma}{1 - \Omega_d} = \omega_d(O_d) - (\gamma - 1) + \frac{\gamma}{1 - \Omega_d} \]  

(3.11)

\[ f_3 = f_n + \frac{\gamma_{tot}}{\Omega_d(1 - \Omega_d)} = \omega_d(O_d) - (\gamma - 1) + \frac{\gamma_{tot}}{\Omega_d(1 - \Omega_d)} \]  

(3.12)

In conclusion, the fractional energies evolve as

\[ \Omega'_m = 3f_j\Omega_m\Omega_d, \]  

(3.13)

\[ \Omega'_d = -3f_j\Omega_d\Omega_m. \]  

(3.14)

When \( j \) takes \( n \), it is the non-interacting case, and when \( j \) is 1,2,3, it is the different interacting cases presented in eqs. (3.3), (3.4) and (3.5). This is an autonomous system with eq. (2.7) as its constraint.

4 The critical points for the general form

In the following, we firstly obtain the critical points of the autonomous system by imposing the conditions \( \Omega'_m = \Omega'_d = 0 \). Of course, the critical points should satisfy the Friedmann constraint, namely \( \Omega_m + \Omega_d = 1 \). Then, we will discuss the existence and stability of these critical points. An attractor is one of the stable critical points of the autonomous system.

Based on the history of the universe and eqs. (3.13), (3.14), we can get that the critical points in table 2. Critical point \( M \) is the matter dominated phase with \( \Omega_m = 1 \). And Critical point \( D \) is the dark energy dominated phase with \( \Omega_d = 1 \). If \( f_j \propto \frac{1}{\Omega_m} \) or \( f_j \propto \frac{1}{\Omega_d} \), these two fixed point may not exist. Besides the above two fixed points, there are other solutions caused by \( f_j = 0 \). Because the universe went through the matter dominated phase and the dark energy dominated phase, we consider the critical points caused by \( f_j = 0 \), with \( \omega_{tot} = 0 \) or \( \omega_{tot} = -1 \), which is matter dominated phase \( F_M \) or dark energy dominated phase \( F_D \).

If we substitute linear perturbations about the critical point \( (\Omega_m, \Omega_d) \) into dynamical system eqs. (3.13) and (3.14) and linearize them, we can get that

\[ \delta\Omega'_m = 3f(\Omega_d)\delta\Omega_m + 3 \left( f(\Omega_d)\delta\Omega_m + \frac{df(\Omega_d)}{d\Omega_d}\Omega_m\Omega_d \right) \delta\Omega_d, \]  

(4.1)

\[ \delta\Omega'_d = -3f(\Omega_d)\delta\Omega_m - 3 \left( f(\Omega_d)\delta\Omega_m + \frac{df(\Omega_d)}{d\Omega_d}\Omega_m\Omega_d \right) \delta\Omega_d. \]  

(4.2)
The two eigenvalues of the coefficient matrix of the above equations determine the stability of the corresponding critical point, which yield two eigenvalues

\[ \lambda_1 = 0, \]
\[ \lambda_2 = 3f(2\Omega_d - 1) - 3f_d\Omega_d(1 - \Omega_d). \]  

(4.3)

(4.4)

Here \( f'_d = df/d\Omega_d \). When \( \lambda_2 \) is positive, the corresponding critical point is an unstable node. “Unstable” means that the phase will not stay in the phase for long, eventually it will evolve to other phases. When \( \lambda_0 \) is negative, the corresponding critical point is a stable node and the phase will last long. We list these critical points in table 2 as well.

With regard to the history of the universe, we expect that the matter dominated phase should be unstable; otherwise, the universe will not enter the dark energy dominated phase. The stability of the matter dominated phase will determine whether the model coincides with the history of the universe or not. If the matter dominated phase (\( M \) or \( F_M \)) is stable, we will see that there is no way for the universe to get into a dark energy dominated phase. The stability of the dark energy dominated phase (\( D \) or \( F_D \)) will determine the fate of the universe. If it is stable, the dark energy phase is an attractor that the universe will be dark energy dominated in the future. With a preceding unstable matter-dominated phase, the universe will enter the phase at last. If the critical points are unstable, then, even through the EoS parameter might be smaller than \(-1\), the universe would escape the big rip singularity.

5 The interacting terms

As we discussed, the four critical points in table 2 may not all exist, especially in the interaction cases.

With interacting term one, we will not have the critical point \( M \). With the interacting term two, we will not have the critical point \( D \). With the interacting term three, we will not have both the critical points \( M \) and \( D \). When \( f_j = 0 \), from eqs. (2.8) and (3.9), we can get that

\[ \omega_{tot} = \omega_{de} = \omega_{me}. \]  

(5.1)

There are two scaling solutions. That the solutions may appear in the non-interacting or interacting cases or not depends on the exact form of \( \omega_d \).

At the critical points \( F_M \), we assume \( \omega_{tot} = 0 \) for a matter dominated phase, then we can get that

\[ \omega_{tot} = \gamma - 1 - \gamma_m = 0, \]  

(5.2)

\[ \omega_{tot} = \gamma - 1 - \gamma_d\frac{\Omega_d}{1 - \Omega_d} = 0, \]  

(5.3)

\[ \omega_{tot} = \gamma - 1 - \frac{\gamma_{tot}}{1 - \Omega_d} = 0. \]  

(5.4)

In the interacting term one case, \( \gamma - 1 = \gamma_m \). In the interacting term two case, \( \Omega_d = (\gamma - 1)/(\gamma_d + \gamma - 1) \) and in the interacting term three case, \( \Omega_d = 1 - \gamma_{tot}/(\gamma - 1) \).

And, at the critical points \( F_D \), as \( \omega_{tot} = -1 \), we can get

\[ \omega_{tot} = \gamma - 1 - \gamma_m = -1, \]  

(5.5)

\[ \omega_{tot} = \gamma - 1 - \gamma_d\frac{\Omega_d}{1 - \Omega_d} = -1, \]  

(5.6)

\[ \omega_{tot} = \gamma - 1 - \frac{\gamma_{tot}}{1 - \Omega_d} = -1. \]  

(5.7)
In the interacting term one case, $\gamma = \gamma_m$. In the interacting term two case, $\Omega_d = \gamma/(\gamma_d + \gamma)$) and in the interacting term three case, $\Omega_d = 1 - \gamma_{\text{tot}}/\gamma$. Because $0 \leq \Omega_d \leq 1$, the interaction parameter should satisfy $\gamma_d > 0$ and $\gamma_{\text{tot}} > 0$.

If one of the critical points $F_M$ or $F_D$ exists, it can be treated as a scaling solution which is helpful to solve the coincidence problem. And the above discussions show that the phase-analysis method is effective to describe the dark energy models even with interactions included. In the following, we will give exact examples.

### 6.1 Cosmological constant model

First, we investigate the cosmological constant case with $L = \text{constant}$. In the non-interacting case, we can see that

$$\omega^c_d = -1, \quad \omega_m = \gamma - 1, \quad f^c_n = -\gamma,$$

where the superscript $c$ means the cosmological constant model. If we put the above equations into the evolution of the energy density we can get that

$$\Omega'_m = -3\gamma \Omega_m \Omega_d,$$

$$\Omega'_d = 3\gamma \Omega_d \Omega_m.$$  

In table 3, we list the main results related to the critical points. The non-interacting cosmological constant model gives out a unstable matter dominated phase, and a dark energy attractor as well.

In the interacting term cases, we need replace the EoS parameter $\omega_m$ and $\omega_d$ with the effective EoS parameter $\omega_{me}$ and $\omega_{de}$. In the interaction one case,

$$f^c_{de1} = \omega_{de1} - \omega_{me1} = -\gamma + \frac{\gamma_m}{\Omega_d},$$

$$f^c_{d1} = \frac{\gamma_m}{\Omega_d^2}.$$
Table 4. The properties of the critical points in the cosmological constant model with interacting term one.

|        | \((\Omega_m, \Omega_d)\) | \((\lambda_1, \lambda_2)\) | existence       | stability       | \(\omega_{\text{tot}}\) |
|--------|---------------------------|-----------------------------|-----------------|-----------------|-------------------|
| \(F_M\) | \((1 - \frac{\gamma_m}{2}, \frac{\gamma_d}{2})\) | \((0, \frac{\gamma_d}{2})\) | \(0 \leq \gamma_m \leq \gamma\) | unstable         | 0                 |
| \(D\)  | \((0,1)\)                | \((-3(\gamma - \gamma_m))\) | always          | \(\gamma_m < \gamma\), stable | -1                |

Table 5. The properties of the critical points in the cosmological constant model with interacting term two.

|        | \((\Omega_m, \Omega_d)\) | \((\lambda_1, \lambda_2)\) | existence       | stability       | \(\omega_{\text{tot}}\) |
|--------|---------------------------|-----------------------------|-----------------|-----------------|-------------------|
| \(M\)  | \((1,0)\)                 | \((0,3\gamma)\)            | always          | unstable        | \(\gamma - 1\)  |
| \(F_D\) | \((\frac{\gamma_d}{\gamma}, 1 - \frac{\gamma_d}{\gamma})\) | \((-3\frac{\gamma_d\Omega_m}{\Omega_d})\) | \(0 \leq \gamma_d \leq \gamma\) | stable          | -1                |

Table 6. The properties of the critical points in the cosmological constant model with interacting term three. For conciseness, we define a new variable \(k = \sqrt{-\frac{\gamma_{\text{tot}}}{\frac{\gamma}{2}} + \frac{1}{2}}\).

In the interacting term two,

\[
f_{c2}' = -\gamma + \frac{\gamma_d}{(1 - \Omega_d)}, \tag{6.7}
\]

\[
f_{d2}' = \frac{\gamma_d}{(1 - \Omega_d)^2}. \tag{6.8}
\]

In the interacting term three,

\[
f_{c3}' = -\gamma + \frac{\gamma_{\text{tot}}}{\Omega_d(1 - \Omega_d)}, \tag{6.9}
\]

\[
f_{d3}' = \gamma_{\text{tot}} \left( -\frac{1}{\Omega_d^2} + \frac{1}{(1 - \Omega_d)^2} \right). \tag{6.10}
\]

As are listed in tables 4, 5 and 6, we can all get the unstable matter dominated phase and the stable dark energy dominated phase. We can see that the critical points require \(\gamma_m > 0\), \(\gamma_d > 0\) and \(\gamma_{\text{tot}} > 0\). The interactions bring further constraint to the parameter \(\gamma\).

### 6.2 Holographic dark energy

The successive holographic dark energy model uses the future event horizon of the universe as the IR cutoff, which leads to

\[
L = a \int_{a}^{\infty} \frac{dt'}{a(t')} \tag{6.11}
\]

\[
\rho_d^h = 3c^2m_{\text{pl}}^2/L^2, \tag{6.12}
\]

where the superscript \(h\) means the holographic dark energy model.
Table 7. The properties of the critical points in holographic dark energy model.

| Label | \((\Omega_m, \Omega_d)\) | \((\lambda_1, \lambda_2)\) | existence | stability | \(\omega_{\text{tot}}\) |
|-------|-----------------|-----------------|----------|----------|-----------------|
| M     | \((1,0)\)       | \((0, \gamma - \frac{2}{3})\) | always   | \(\gamma > \frac{2}{3}\) unstable | \(\gamma - 1\) |
| D     | \((0,1)\)       | \((0, 2(1 - \frac{1}{c}) - 3\gamma)\) | always   | \(\gamma > \frac{2}{3}\) unstable | \(-\frac{1}{3}(1 + \frac{1}{c})\) |

Table 8. The properties of the critical points in holographic dark energy model with interacting term one. For conciseness, \(\Omega_d = \frac{\gamma_m}{2\gamma + 3\gamma_c}\) for the \(F_M\) phase is presented in the caption.

In the non-interacting term,

\[
\omega^b_d = -\frac{1}{3} \left(1 + \frac{2}{c} \sqrt{\Omega_d}\right), \quad \omega_m = \gamma - 1, \quad (6.13)
\]

\[
f^b_{h1} = \omega_{de1} - \omega_{me1} = -\frac{1}{3} \left(1 + \frac{2}{c} \sqrt{\Omega_d}\right) - (\gamma - 1), \quad (6.14)
\]

If we put the above equations into the evolution of the energy density, we can list the main results related to the critical points in table 7. In the non-interaction case, if \(\gamma > 2/3\), the matter-dominated phase is unstable, in the meanwhile the dark energy dominated phase is also unstable. For example, assuming \(\gamma = 1\), \(c > -1\) will make the unstable case, which means we can get the big-rip fate in holographic dark energy and the realization of the dark energy dominated phase hardly depends on the initial conditions.

With the interacting term one,

\[
f^{h1}_{d1} = -\frac{1}{3c\sqrt{\Omega_d}} - \frac{\gamma_m}{\Omega_d}, \quad (6.15)
\]

\[
f^{h1}_{d2} = -\frac{1}{3c\sqrt{\Omega_d}} - \frac{\gamma_m}{\Omega_d}. \quad (6.16)
\]

We can get the critical point \(D\), and the existence of \(\gamma_m\) extends the parameter regime of \(c\). There is no \(M\) point because of the interaction. However, there is a new matter dominated phase \(F_M\). To get such a phase, we require \(\omega_{\text{tot}} = 0\) and \(f^{h1}_{d1} = 0\), so we can find when the total universe behaves like matter-dominated. As the matter dominated, we need \(\Omega_d\) to be much smaller than \(\Omega_m\). Specifically speaking, we can assume \(\Omega_d \ll c^2/2\), \(\Omega \approx \frac{\gamma_m}{2\gamma + 3\gamma_c}\). \(\gamma_m > 0\) could lead to the unstable matter dominated phase. And we set \(\gamma_m = \gamma - 1\), which means \(\omega_{\text{tot}} = 0\).

In the interacting term two case,

\[
f^{h2}_{d1} = -\frac{1}{3c\sqrt{\Omega_d}} - \frac{\gamma_d}{(1 - \Omega_d)}, \quad (6.17)
\]

\[
f^{h2}_{d2} = -\frac{1}{3c\sqrt{\Omega_d}} + \frac{\gamma_d}{(1 - \Omega_d)^2}. \quad (6.18)
\]
Table 9. The properties of the critical points in holographic dark energy model with interacting term two.

| Label | \((\Omega_m, \Omega_d)\) | \((\lambda_1, \lambda_2)\) | existence | stability | \(\omega_{\text{tot}}\) |
|-------|----------------|----------------|-----------|-----------|----------------|
| M     | \((1,0)\)    | \((0, -3(\frac{2}{3} - \gamma - \gamma_d))\) | always    | \(\gamma > \frac{2}{3} + \gamma_d\), unstable | \(\gamma_m + \gamma - 1\) |
| \(F_D\) | \((\frac{\gamma_{\text{tot}}}{\gamma_d+\gamma}, \frac{\gamma_{\text{tot}}}{\gamma_d+\gamma})\) | \((0, -3(\frac{2}{3} \Omega_d - \frac{\Omega_m \sqrt{\Omega_d}}{3c}))\) | \(\gamma_d > 0\) | \(\frac{\gamma_d^2}{9(\gamma_d + \gamma)} < c^2\), stable | \(-1\) |

Table 10. The properties of the critical points in holographic dark energy model with interacting term three.

| Label | \((\Omega_m, \Omega_d)\) | \((\lambda_1, \lambda_2)\) | existence | stability | \(\omega_{\text{tot}}\) |
|-------|----------------|----------------|-----------|-----------|----------------|
| \(F_M\) | \((\frac{\gamma_{\text{tot}}}{\gamma - 1}, 1 - \frac{\gamma_{\text{tot}}}{\gamma - 1})\) | \((0, -3f_{d3}^H \Omega_m \Omega_d)\) | \(0 \leq \frac{\gamma_{\text{tot}}}{\gamma - 1} \leq 1\) | \(f_{d3}^H < 0\), unstable | 0 |
| \(F_D\) | \((\frac{\gamma_{\text{tot}}}{\gamma - 1}, 1 - \frac{\gamma_{\text{tot}}}{\gamma})\) | \((0, -3f_{d3}^H \Omega_m \Omega_d)\) | \(0 \leq \frac{\gamma_{\text{tot}}}{\gamma} \leq 1\) | \(f_{d3}^H > 0\), stable | \(-1\) |

The critical points \(M\) show that if \(\gamma > 2/3 + \gamma_d\), the matter dominated phase is unstable. And the \(F_D\) critical point shows that we can get a dark energy attractor in holographic model when the first term in the eq. \((6.18)\) is much smaller than the second term.

With the interacting term three,

\[
f_3^h = -\frac{1}{3} \left( 1 + \frac{2}{c} \sqrt{\Omega_d} \right) - (\gamma - 1) + \frac{\gamma_{\text{tot}}}{\Omega_d(1 - \Omega_d)},
\]

\[
f_{d3}^{h'} = -\frac{1}{3c\sqrt{\Omega_d}} - \frac{\gamma_{\text{tot}}}{\Omega_d^2} + \frac{\gamma_{\text{tot}}}{(1 - \Omega_d)^2}.
\]

Firstly, as is noted in table 10, the existence of a critical point \(F_D\) needs \(\gamma_{\text{tot}} > 0\). When \(\Omega_d\) is large, the absolute value of the third term in eq. \((6.20)\) will be larger than the first two terms, \(f_{d3}^H > 0\), and we get the stable dark energy dominated phase. Instead, if \(\Omega_d\) is small, \(f_{d3}^H < 0\), we will get the unstable matter dominated phase.

We can see that in holographic dark energy model, as we predicted, we can get the matter dominated and the dark energy dominated phase. The interacting term could take an important part to make the dark energy dominated phase stable.

### 6.3 Agegraphic dark energy

Agegraphic dark energy model, whose cutoff is the age the universe by the age of the universe, is proved to be consistent with the evolution of universe,

\[
L = T = \int_0^a \frac{da}{Ha},
\]

\[
\rho_a^d = \frac{3n^2m_{pl}^2}{T^2},
\]

where the superscript \(a\) means the agegraphic dark energy model.

In the non-interaction case,

\[
\omega_a^d = -1 + \frac{2\sqrt{\Omega_d}}{3n},
\]

\[
f_n^a = \frac{2}{3n}\sqrt{\Omega_d} - \gamma,
\]

\[
f_{dn}^a = \frac{1}{3n\sqrt{\Omega_d}}.
\]
| Label | $(\Omega_m, \Omega_d)$ | $(\lambda_1, \lambda_2)$ | existence | stability | $\omega_{\text{tot}}$ |
|-------|------------------|------------------|---------|---------|-----------------|
| M     | $(1,0)$          | $(0,3\gamma)$    | always  | unstable | $\gamma - 1$    |
| $F_M$ | $(1 - \frac{9n^2\gamma^2}{4}, 9n^2\gamma^2)$ | $(0, -\frac{3\gamma^3}{2}(1 - \frac{9n^2\gamma^2}{4}))$ | $n < \frac{2}{3\gamma}$ | stable | $\gamma - 1$ |
| D     | $(0,1)$          | $(0, \frac{2}{n} - 3\gamma)$ | always  | $n > \frac{2}{3\gamma}$, stable | $-1 + \frac{2}{3\gamma}$ |

Table 11. The properties of the critical points in agegraphic dark energy model.

| Label | $(\Omega_m, \Omega_d)$ | $(\lambda_1, \lambda_2)$ | existence | stability | $\omega_{\text{tot}}$ |
|-------|------------------|------------------|---------|---------|-----------------|
| $F_M$ | $(1 - \frac{2\Omega_m}{3\gamma}, \frac{2\Omega_m}{3\gamma})$ | $(0, \Omega_m(3\gamma - \sqrt{\frac{2\Omega_m}{3\gamma}}))$ | $\gamma_m = \gamma - 1$ | $\gamma_m < 9\gamma^3n^2$, unstable | 0 |
| D     | $(0,1)$          | $(0, \frac{2}{n} - 3\gamma + 3\gamma_m)$ | always  | $n > \frac{2}{3\gamma - 2\gamma_m}$, stable | $-1 + \frac{2}{3\gamma}$ |

Table 12. The properties of the critical points in agegraphic dark energy model with interacting case one.

Putting these forms in the eq. (3.13) and (3.14), we could get the stability of the critical points which is listed in table 11.

In table 11, we can see that, there is an interesting critical point $F_M$ which is an attractor, so it is impossible to have the dark energy dominated phase. There are three points. With $n$ larger than $2/3\gamma$, it could get the dark energy dominated phase; otherwise, there will always be matter dominated. And to make the universe accelerate, in the dark energy dominated phase $\omega_{\text{tot}} = -1 + 2/3n < -1/3$, it requires $n > 1$.

With the interacting case one,

$$f_1^a = \frac{2}{3n} \sqrt{\Omega_d - \gamma + \frac{\gamma_m}{\Omega_d}},$$  \hspace{1cm} (6.26)

$$f_d^a = \frac{\Omega_d^{3/2} - 3n\gamma_m}{3n\Omega_d^2},$$  \hspace{1cm} (6.27)

we can get the critical point $D$, and the existence of $\gamma_m$ extends the parameter regime of $n$. There is no $M$ point because of the interaction. However, there is another matter dominated phase $F_M$. To get such a phase, we require $\omega_{\text{tot}} = 0$ and $f_1^a = 0$. As the matter dominated, we could assume $\Omega_d$ is much smaller than $\Omega_m$. Then we can get the unstable matter dominated phase and the dark energy dominated phase.

With the interacting term two,

$$f_2^a = -1 + \frac{2}{3n} \sqrt{\Omega_d - (\gamma - 1) + \frac{\gamma_d}{(1 - \Omega_d)}} = 0,$$  \hspace{1cm} (6.28)

$$f_d'^a = \frac{(1 - \Omega_d)^2 + 3n\gamma_d\sqrt{\Omega_d}}{3n\sqrt{\Omega_d}(1 - \Omega_d)^2},$$  \hspace{1cm} (6.29)

In table 12, it only needs that $0 < \gamma_d < \gamma$, and we can get the unstable matter dominated phase and the stable dark energy dominated phase.

With the interacting term three,

$$f_3^a = \frac{2}{3n} \sqrt{\Omega_d - \gamma + \frac{\gamma_{\text{tot}}}{\Omega_d(1 - \Omega_d)}},$$  \hspace{1cm} (6.30)

$$f_d'^a = \frac{1}{3n\sqrt{\Omega_d}} + \frac{\gamma_{\text{tot}}}{\Omega_d\Omega_m} - \frac{\gamma_{\text{tot}}}{\Omega_d\Omega_m^2}.$$  \hspace{1cm} (6.31)
Table 13. The properties of the critical points in agegraphic dark energy model in interaction two.

| Label | $(\Omega_m, \Omega_d)$ | $(\lambda_1, \lambda_2)$ | existence | stability | $\omega_{\text{tot}}$ |
|-------|------------------------|--------------------------|-----------|-----------|------------------|
| $F_M$ | $(\frac{\gamma_d}{\gamma + \gamma_d}, 1 - \frac{2m}{\gamma + \gamma_d})$ | $(0, -3\frac{\gamma_d}{3m} \Omega_m \Omega_d)$ | always | $\gamma_d < \gamma$, unstable | $\gamma - 1$ |
| $F_D$ | $(\frac{\gamma_d}{\gamma + \gamma_d}, 1 - \frac{2m}{\gamma + \gamma_d})$ | $(0, -3f_{d3}' \Omega_m \Omega_d)$ | $0 \leq \frac{2m}{\gamma} \leq 1$ | $f_{d3}' < 0$, unstable | 0 |

Table 14. The properties of the critical points in agegraphic dark energy model in interaction three.

| Label | $(\Omega_m, \Omega_d)$ | $(\lambda_1, \lambda_2)$ | existence | stability | $\omega_{\text{tot}}$ |
|-------|------------------------|--------------------------|-----------|-----------|------------------|
| M     | (1,0)                  | $(0, 3 - \frac{3}{2} \alpha \gamma)$ | always | $\alpha < \frac{2}{\gamma}$, unstable | $\gamma - 1$ |
| D     | (0,1)                  | $(0, -3)$ | always | stable | $\gamma - 1$ |

Table 15. The properties of the critical points in Modified Holographic dark energy model.

| Label | $(\Omega_m, \Omega_d)$ | $(\lambda_1, \lambda_2)$ | existence | stability | $\omega_{\text{tot}}$ |
|-------|------------------------|--------------------------|-----------|-----------|------------------|
| $F_M$ | $(1 - \Omega_d, \Omega_d)$ | $(0, -3f_{d1}'' \Omega_d \Omega_m)$ | $0 \leq \Omega_d \leq 1$ | $f_{d1}'' < 0$, unstable | $-\gamma_m + \gamma - 1$ |
| D     | (0,1)                  | $(0, 3(\gamma_m - \gamma))$ | always | $\gamma_m < \gamma$, stable | $-1$ |

Table 16. The properties of the critical points in Modified Holographic dark energy model in interacting case one. For conciseness, $\Omega_d = \frac{2\gamma_m}{2\gamma - \alpha \gamma + \alpha \gamma_m}$ for the $F_M$ phase is presented in the caption.

The existence of the critical point $F_D$ needs $\gamma_{\text{tot}} > 0$. When $\Omega_{\text{tot}}$ is large, the absolute value of the third term in eq. (6.31) will be larger than the first two terms, $f_{d3}' > 0$, then we get the stable dark energy dominated phase. Instead, if $\Omega_d$ is small, $f_{d3}' < 0$, then we will get the unstable matter dominated phase.

In conclusion, the interaction is very helpful to extend the parameter phase and then alleviate the coincidence problem.

### 6.4 Modified holographic dark energy

Furthermore, to realize a workable dark energy model, Gong use the Hubble scale as the IR cut-off $L$, and the UV and IR connection is modified by using the black hole mass $M$ in higher dimensions,

$$L^3 \rho_{d}^m \sim M = \frac{(N - 1) \Omega_{N-1}}{16\pi G_N} L^{N-2},$$  

$$\rho_{d}^m = \frac{d(N - 1) \Omega_{N-1}}{16\pi G_N} L^{N-5},$$

where the superscript $m$ means the modified holographic dark energy model, $N$ is the number of spatial dimensions, $d$ is the unknown constant related to the theoretical uncertainties, $G_N$ is the Newton constant and $\Omega_{N-1}$ is the volume of the black hole.
In the non-interaction case,

$$\omega_d^m = -1 + \frac{\alpha \gamma (1 - \Omega_d)}{2 - \alpha \Omega_d}, \omega_m = \gamma - 1, \quad (6.34)$$

$$f_1^m = \frac{\alpha \gamma (1 - \Omega_d)}{2 - \alpha \Omega_d} - \gamma, \quad (6.35)$$

where $\alpha = 5 - N < 2$.

In the non-interaction case, the critical point $M$ which presents the matter dominated phase would be unstable only when $\alpha < \frac{3}{2}$. If $\gamma = 1$, it means that, only for the spatial dimension $N > 3$, the universe could escape from the matter dominated phase. When the spatial dimension is $N = 3$, there exists a matter-dominated phase $F_M$ again. We should calculate to the second order perturbation to justify its stability. Nevertheless, the dark energy dominated phase $D$ is an attractor.

With the interaction term one,

$$f_1^m = \frac{\alpha \gamma (1 - \Omega_d)}{2 - \alpha \Omega_d} - \gamma + \frac{\gamma_m}{\Omega_d} = \frac{(\alpha - 2) \gamma}{2 - \alpha \Omega_d} + \frac{\gamma_m}{\Omega_d}, \quad (6.36)$$

$$f_{d1}^m = -\frac{\gamma_m}{\Omega_d} + \frac{\alpha \gamma (2 - \alpha)}{2 - \alpha \Omega_d} \frac{\gamma_m}{\Omega_d^2}, \quad (6.37)$$

By adding the interacting one, we will still get two phases. The matter dominated phase $F_M$ is unstable when $f_{d1}^m < 0$. We need that $0 \leq \alpha \leq 2$ and $\gamma_m > 0$. Furthermore, with $\gamma_m < \gamma$, the dark energy dominated phase is an attractor.

With the interaction term two,

$$f_2^m = \frac{(\alpha - 2) \gamma}{2 - \alpha \Omega_d} + \frac{\gamma_d}{1 - \Omega_d}, \quad (6.38)$$

$$f_{d2}^m = \frac{\gamma_d}{(1 - \Omega_d)^2} + \frac{\alpha \gamma (\alpha - 2)}{2 - \alpha \Omega_d} \frac{\gamma_d}{(2 - \alpha \Omega_d)^2}. \quad (6.39)$$

From table $17$, the instability of the matter dominated phase and the existence of the dark energy dominated phase $F_D$ require that $0 < \gamma_d < (2 - \alpha) \gamma / 2$, and it means $\alpha < 2$. Then for a stable dark energy phase, we need that the first term in eq. (6.39) is larger than the second term.
Table 19. The properties of the critical points in non-interaction Ricci dark energy case. For conciseness, $\Omega_d = \frac{\beta^2(4-3\gamma)}{2}$ for the $F_M$ phase is presented in the caption.

With the interacting case three,

\begin{align}
  f_{3m}^m &= \frac{(\alpha - 2)\gamma}{2 - \alpha \Omega_d} + \frac{\gamma_{tot}}{\Omega_d(1 - \Omega_d)}, \\
  f_{3t}^{m'} &= \frac{\gamma_{tot}}{\Omega_d^2 \Omega_m} + \frac{\alpha \gamma (\alpha - 2)}{(2 - \alpha \Omega_d)^2} - \frac{\gamma_{tot}}{\Omega_d \Omega_m^2},
\end{align}

(6.40)

(6.41)

The existence of the critical point $F_D$ needs $\gamma_d > 0$. If $\alpha < 2$, when $\Omega_d$ is large, the absolute value of the third term in eq. (6.41) will be larger than the first two terms, and $f_{3t}^{m'} > 0$, we get the stable dark energy dominated phase. Instead, if $\Omega_d$ is small, $f_{3t}^{m'} < 0$, we will get the unstable matter dominated phase. If $\alpha > 2$, when $\Omega_d$ is large, the absolute value of the last two terms in eq. (6.41) will be larger than the first term, $f_{3t}^{m'} > 0$, and we get the stable dark energy dominated phase. Instead, if $\Omega_d$ is small, $f_{3t}^{m'} < 0$, we will get the unstable matter dominated phase.

In conclusion, the interaction one and three cases alleviate the constraint on the spatial dimensions.

6.5 Ricci dark energy

Recently, the average radius of the Ricci scalar curvature has been chosen as the IR cutoff in Ricci DE model:

\begin{equation}
  L = R = 6(\dot{H} + H^2)\rho_d^m = 3\beta^2 m^2_{pl}(\dot{H} + H^2),
\end{equation}

(6.42)

where the superscript $m$ means the modified holographic dark energy model, $\beta$ is the model parameter. We can get the EoS parameter as:

\begin{align}
  \omega_d &= \frac{2}{3\beta^2} + \frac{1}{3\Omega_d} - \frac{\Omega_m}{\Omega_d}(\gamma - 1), \quad \omega_m = \gamma - 1, \\
  f_n' &= \frac{2}{3\beta^2} + \frac{1}{3\Omega_d} - \frac{1}{\Omega_d}(\gamma - 1). \\
  f_{dn}' &= \left(\gamma - \frac{4}{3}\right) \frac{1}{\Omega_d^2}.
\end{align}

(6.43)

(6.44)

(6.45)

In the non-interaction case, for the form of the $\omega_d'$, we could not get the $M$ critical point, but an $F_M$ critical point instead. From table 19, we can see the two critical points which could lead us to the stable dark energy dominated phase and the unstable matter dominated phase.

In the interacting case one,

\begin{align}
  f_1' &= \frac{2}{3\beta^2} + \frac{1}{3\Omega_d} - \frac{1}{\Omega_d}(\gamma - 1) + \frac{\gamma_m}{\Omega_d}, \\
  f_{dn}' &= \left(-\gamma_m + \gamma - \frac{4}{3}\right) \frac{1}{\Omega_d^2}.
\end{align}

(6.46)

(6.47)
$\beta^2 < 1/3$, the universe could accelerating. And with certain constraint on the parameter $\beta$, we can get the expected unstable matter dominated phase and a stable dark energy dominated phase.

In the interacting case two, we can see that

$$f_2 = -\frac{2}{3\beta^2} + \frac{1}{3\Omega_d} - \frac{1}{\Omega_d} (\gamma - 1) + \frac{\gamma_d}{(1 - \Omega_d)} = 0,$$

$$f_{d2} = \left( \gamma - \frac{4}{3} \right) \frac{1}{\Omega_d^2} + \frac{\gamma_d}{(1 - \Omega_d)^2}.$$  \hfill (6.49)

The existence of the critical point $F_D$ needs $\gamma_d > 0$. Assuming $\gamma < 4/3$, when $\Omega_d$ is large, the absolute value of the second term in eq. (6.49) will be larger than the first term, $f_{d2} > 0$, and we get the stable dark energy dominated phase. Instead, if $1 - \Omega_d$ is large, $f_{d2} < 0$, we will get the unstable matter dominated phase.

With the interaction term three,

$$f_3 = -\frac{2}{3\beta^2} + \frac{1}{3\Omega_d} - \frac{1}{\Omega_d} (\gamma - 1) + \frac{\gamma_{tot}}{\Omega_d(1 - \Omega_d)},$$

$$f_{d3} = \left( \gamma - \frac{4}{3} \right) \frac{1}{\Omega_d^2} + \frac{\gamma_{tot}}{\Omega_d^2 m}.$$  \hfill (6.50)

The existence of the critical point $F_D$ needs $\gamma_{tot} > 0$. Assuming $\gamma - \gamma_{tot} < 4/3$, when $\Omega_d$ is large, the absolute value of the second term in eq. (6.49) will be larger than the first term, $f_{d3} > 0$, and we get the stable dark energy dominated phase. Instead, if $1 - \Omega_d$ is large, $f_{d3} < 0$, we will get the unstable matter dominated phase.

In conclusion, in the Ricci dark energy model, we successfully get the desired matter dominated and dark energy dominated phases. The interactions extend the regime of the parameter and help to get some scaling solutions.

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Label & $(\Omega_m, \Omega_d)$ & $(\lambda_1, \lambda_2)$ & existence & stability & $\omega_{tot}$ \\
\hline
$F_M$ & $1 - \Omega_d, \Omega_d$ & $(0, -3f_{d1}' \Omega_d \Omega_m)$ & $0 \leq \Omega_d \leq 1$ & $\gamma_m > \frac{r}{3}, \text{unstable}$ & $\gamma - 1$ \\
$D$ & $(0, 1)$ & $(0, -2+\beta^2(4-3\gamma+3\gamma_m))$ & always & $\frac{1}{\beta^2} > \frac{4-3(\gamma - \gamma_m)}{2}$, stable & $\beta^2 - 2$ \\
\hline
\end{tabular}
\caption{The properties of the critical points in Ricci dark energy model with interacting case one.}
\end{table}

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Label & $(\Omega_m, \Omega_d)$ & $(\lambda_1, \lambda_2)$ & existence & stability & $\omega_{tot}$ \\
\hline
$F_M$ & $(\frac{\gamma_d + \gamma}{2}, \frac{\gamma_d - 1}{2})$ & $(0, -3f_{d2}' \Omega_d \Omega_m)$ & $0 \leq \frac{\gamma_d}{\gamma_d + \gamma - 1} \leq 1$ & $f_{d2} < 0$, unstable & $0$ \\
$F_D$ & $(\frac{\gamma_d + \gamma}{2}, \frac{\gamma_d - 1}{2})$ & $(0, -3f_{d2}' \Omega_d \Omega_m)$ & $\gamma_d > 0$ & $f_{d2} > 0$, stable & $-1$ \\
\hline
\end{tabular}
\caption{The properties of the critical points in Ricci dark energy model in interacting case two.}
\end{table}

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Label & $(\Omega_m, \Omega_d)$ & $(\lambda_1, \lambda_2)$ & existence & stability & $\omega_{tot}$ \\
\hline
$F_M$ & $(\frac{\gamma_{tot}}{2}, 1 - \frac{\gamma_{tot}}{2})$ & $(0, -3f_{d3}' \Omega_m \Omega_d)$ & $0 \leq \frac{\gamma_{tot}}{\gamma_d - 1} \leq 1$ & $f_{d3} < 0$, unstable & $0$ \\
$F_D$ & $(\frac{\gamma_{tot}}{2}, 1 - \frac{\gamma_{tot}}{2})$ & $(0, -3f_{d3}' \Omega_m \Omega_d)$ & $0 \leq \frac{\gamma_{tot}}{\gamma_d - 1} \leq 1$ & $f_{d3} > 0$, stable & $-1$ \\
\hline
\end{tabular}
\caption{The properties of the critical points in Ricci dark energy model in interacting case three.}
\end{table}
6.6 Discussions

However, in the above discussions, we conclude that the interactions may have three-fold action: to make the unstable dark energy dominated phase to an attractor, to extend the regime of the parameter, and the last is to make out scaling solutions to alleviate the coincidence problem.

7 Conclusion

In the paper, we suggest that we could construct the models in which the equation of state is related to the fractional dark energy. Then, considering the evolution of the universe, we analyze the dynamical behavior in the holographic-like models. Since the universe experiences the matter dominated phase and the dark energy dominated phase, we focus on this two phases.

We have presented a phase-space analysis of the evolution for a spatially flat FRW universe containing dark energy and matter. Based on the established history of the universe, we want to get an unstable matter dominated phase and a stable dark energy dominated phase. The unstable matter dominated phase is necessary. If the dark energy dominated phase is unstable, it may depend on the initial conditions that the universe first experiences a matter dominated phase, and then a dark energy dominated phase. But if it is an attractor, we do not need to consider the initial conditions.

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