Optimized Reduction of Unsteady Radial Forces in a Single-channel Pump for Wastewater Treatment

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Abstract. A single-channel pump for wastewater treatment was optimized to reduce unsteady radial force sources caused by impeller-volute interactions. The steady and unsteady Reynolds-averaged Navier–Stokes equations using the shear-stress transport turbulence model were discretized by finite volume approximations and solved on tetrahedral grids to analyze the flow in the single-channel pump. The sweep area of radial force during one revolution and the distance of the sweep-area center of mass from the origin were selected as the objective functions; the two design variables were related to the internal flow cross-section al area of the volute. These objective functions were integrated into one objective function by applying the weighting factor for optimization. Latin hypercube sampling was employed to generate twelve design points within the design space. A response-surface approximation model was constructed as a surrogate model for the objectives, based on the objective function values at the generated design points. The optimized results showed considerable reduction in the unsteady radial force sources in the optimum design, relative to those of the reference design.

1. Introduction

Commonly, single-channel impellers are used to pass solid matter smoothly in wastewater treatment pumps. Since such an impeller has only one free annulus passage and no blade, the pressure distribution is not rotationally symmetric; furthermore, the pressure rise is not smooth, compared to that in typical bladed impellers. Consequently, radial forces are created in the impeller, which rotate at a frequency given by the rotor speed [1]. Thus, unsteady radial force sources are generated by the interaction of the rotating impeller and the volute. These unsteady sources can adversely affect the overall performance of the single-channel pump.

Interest has grown in analyzing the unsteady dynamic radial force effect generated by impeller-volute interactions in centrifugal pumps. Gonzalez et al. [2] performed both unsteady numerical analysis and experimental tests to demonstrate the dynamic interactions between the flow at the impeller exit and the volute tongue in a centrifugal pump. Gonzalez and colleagues [3] also investigated the static and dynamic effects of fluid flow in a vanless-volute centrifugal pump with two different types of impellers. Kurokawa et al. [4] experimentally investigated the flow characteristics in a double volute in order to balance the unsteady radial thrust in centrifugal pumps. Wei et al. [5] performed numerical and experimental studies on the dynamic radial forces in a single-
stage pump with diffuser vanes having different outlet diameters. However, no systematic studies on single-channel pumps have yet been attempted, and only a few studies have investigated such concepts and patents. To the best of the authors’ knowledge, such studies are hindered by difficulties in establishing a methodical design, manufacturing, and solving the balancing problem related to the impeller-volute interactions of single-channel pumps, rather than those for general centrifugal pumps.

This work presents an optimization procedure for reducing unsteady radial forces in a single-channel pump, based on three-dimensional steady and unsteady Reynolds-averaged Navier–Stokes (RANS) analyses. The aims of this work were to optimize the volute geometry in order to reduce the unsteady radial force sources in a single-channel pump via the proposed design method and to examine the differences in the internal flow fields of the reference and the optimum designs, clarifying the flow physics associated with the reduced radial force sources.

2. Single-channel pump design
The initial single-channel pump impeller for wastewater treatment investigated in this work was designed based on the Stepanoff theory from a previous study [6]. Hence, the initial model has an area distribution that constantly increases with increases in the angle theta of the impeller from the inlet to outlet, as shown in Fig. 1 [7].

The reference volute used in this study was also designed according to the Stepanoff theory through a previous work [7]. Since the Stepanoff theory generally minimizes the flow loss resulting from differences in flow speed by increasing the cross-sectional area of the internal flow at a fixed rate, according to the theta angle position, the theory is especially useful in designing the stationary volute. Therefore, the cross-sectional area distribution of the internal flow changed proportionally along the theta angle position, to maintain a constant flow velocity in the volute. Moreover, in the previous work [7], a novel design method for the cross-section of high-efficiency single-channel pump volutes was invented, as shown in Fig. 2. The total head and hydraulic efficiency for the reference single-channel pump model are 9.41 m and 81.06%, respectively, at the selected volumetric flow rate of 85 m$^3$/h.

3. Steady and unsteady analyses
The flow of the single-channel pump was analyzed by solving the three-dimensional steady and unsteady incompressible RANS equations in the shear-stress transport (SST) turbulence model, using a finite volume solver provided by the commercial code ANSYS CFX-14.5 [8].

Figure 3 shows the computational domain, which consists of the single-channel pump impeller and volute, for the numerical analysis. A tetrahedral grid system is constructed in the computational domain with a prismatic mesh near the walls, as shown in Fig. 3. The single-channel pump impeller and the volute domains are each constructed using approximately 1,300,000 and 1,200,000 grid points, respectively. Here, the grids near the walls are densely arranged and thus the y+ value does not exceed
2. Therefore, the total optimum grid system, selected using a grid independence test, has approximately 2,500,000 grid points. The total pressure and designed mass flow rate are set at the inlet and outlet of the computational domain, respectively. Water is chosen as the working fluid and the solid surfaces in the computational domain are considered hydraulically smooth with adiabatic and no-slip conditions, as shown in Fig. 3. The stage average and transient rotor-stator methods are employed to connect the interface between the rotating impeller and the volute domains in the steady and unsteady analyses, respectively [8].

The root-mean-square values of the governing equation residuals were set to at least $10^{-5}$ for all equations as the convergence criteria. The physical timescale was set to $1/\omega$, where $\omega$ is the angular velocity of the single-channel impeller. The converged solutions were obtained after approximately 1,000 iterations. The computations were performed using an Intel Xeon CPU with a clock speed of 2.70 GHz. The computational time was ~4 h, depending upon the geometry being considered and the rate of convergence.

The results of the steady RANS analysis were used in the unsteady RANS analysis to obtain the characteristics of the radial force sources in the region near the exit surface of the impeller, according to the impeller-volute interactions in the single-channel pump. In the unsteady computations, the time step and the coefficient loop for the time scale control were set to 0.000947 s and three times, respectively. The solutions were obtained after 180 iterations with an unsteady total time duration of 0.1704775 s (five revolutions), and the computational time for the unsteady calculation was ~8 h.

4. Objective functions and design variables

The aim of the current optimization problem was to reduce the unsteady radial force sources caused by impeller-volute interactions in the single-channel pump. The objective functions relating to the sources of unsteady radial forces were defined as the sweep area of the radial force during one revolution and the distance of the center of mass of the sweep area from the origin, as follows:

$$A_s = \frac{1}{2} \sum_{i=0}^{n-1} (x_i y_{i+1} - x_{i+1} y_i)$$

where $A_s$ is the signed area of the polygon as the sweep area of the radial force during one revolution of the single-channel pump impeller. The centroid of a non-self-intersecting closed polygon defined by the $n$ vertices $(x_0, y_0), (x_1, y_1), ..., (x_{n-1}, y_{n-1})$ is defined as the point $(C_x, C_y)$ as follows:

![Figure 3. Computational domain and grids](image1)

![Figure 4. Definition of the design variables](image2)
\[ C_x = \frac{1}{6A_s} \sum_{i=0}^{n-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \]  

(2)

\[ C_y = \frac{1}{6A_s} \sum_{i=0}^{n-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \]  

(3)

In these formulas, the vertices are assumed to be numbered in the order of occurrence along the perimeter of the polygon. Hence, the distance, \( D_s \), of the center of mass of the sweep area from the origin is finally defined as follows.

\[ D_s = \sqrt{C_x^2 + C_y^2} \]  

(4)

These functions (1) and (4) were integrated into one objective function by applying the weighted-sum-of-objective-functions method for the optimization. \( A_s \) and \( D_s \) are linearly combined with a weighting factor \( \beta \) to constitute a mono-objective function, \( f_{\text{radial}} \):

\[ f_{\text{radial}} = A_s + \beta D_s \]  

(5)

Therefore, the current optimization problem is the minimization of \( f_{\text{radial}} \) with the weighting factor \( \beta \), which is selected by design requirements. However, since the two objective functions are identically important in reducing unsteady radial forces, the weighting factor is equal to 1.0.

In this work, geometric parameters related to the internal flow through the cross-sectional area of the volute were selected as design variables to reduce the unsteady radial force sources in the single-channel pump for wastewater treatment. The distribution of the cross-sectional area of the volute with the angle theta can be changed smoothly by adjusting the control points, represented by a third-order Bezier-curve, as shown in Fig. 4. Thus, the variations in the y values for the two control points CP1 and CP2 were selected as design variables, to obtain the most sensitive results for the curve variation among the control points [9]. Figure 4 shows these defined design variables and ranges of variation. When the volute optimization is processed with these variables, the optimized single-channel pump impeller maximizing the total efficiency by using a radial-basis neural-network surrogate model from the previous work [9] is employed.

5. Response surface approximation model

As one design-of-experiment (DOE) method, Latin hypercube sampling (LHS) was employed to generate twelve design points, which are the initial data for constructing the response surface in the optimization. The objective function values at these design points were evaluated by the unsteady RANS analysis.

The response surface approximation (RSA) model was applied as a surrogate model to predict the objective function values, based on the twelve design points generated in the design space using LHS. The RSA model, as a methodology of fitting a polynomial function to discrete responses obtained from numerical calculations, represents the association between the design variables and response functions [10]. As the RSA model is highly effective for optimization problems, it has been used for both single- and multidisciplinary turbomachinery optimizations [11, 12, 13]. The construction function for a second-order polynomial RSA can be expressed as follows:

\[ f(x) = D_0 + \sum_{j=1}^{N} D_j x_j + \sum_{j=1}^{N} D_{jj} x_j^2 + \sum_{i<j}^{N} D_{ij} x_i x_j \]  

(6)
where $D$, $N$, and $x$ denote the regression analysis coefficients, number of design variables, and a set of design variables, respectively, and the number of regression analysis coefficients ($D_0, D_1, \text{etc.}$) is \[(N + 1) \times (N + 2)/2.\]

After constructing the RSA surrogate model, a sequential quadratic programming (SQP) [14] algorithm was applied to find the optimum point on the RSA model. The SQP algorithm depended on the initial guess of the optimum point; thus, multiple trials were performed before the final optimum point was obtained from any surrogate.

### 6. Results and discussion

In a previous study [15], the accuracy of the numerical analysis was evaluated by validating the results of the steady-state flow analysis.

In the optimization, the RSA model was obtained by using the numerical results at twelve design points sampled by LHS. Here, an analysis of variance (ANOVA) and a regression analysis with t-statistics [15] were conducted to measure the uncertainty in the set of coefficients of the polynomial. Here, the values of $R^2$ and $R^2_{\text{adj}}$ for second-order curve fitting and the root mean square error (RMSE) are 0.998 and 0.994, respectively, representing the correlation coefficient in the least-squares surface fitting and the adjusted correlation coefficient, respectively. These values are reliable, according to the $0.9 < R^2_{\text{adj}} < 1.0$ range, for the accuracy of predictions by the RSA model [16]. The constructed RSA model for the objective function can be expressed in terms of the normalized design variables as follows.

$$f_{\text{radial}} = 1.310175 - 1.07203 x_1 - 2.23478 x_2 + 0.692144 x_1 x_2 + 1.567291 x_1^2 + 1.913658 x_2^2 \quad (7)$$

Table 1 lists the results of the optimization of the design variables and objective function. Here, the values of $f_{\text{radial}}$ for both the reference and optimum designs are normalized by the value of the reference design. In the design variables of the optimum design, CP1 changes significantly while CP2 varies slightly, compared to the reference design. It is thought that the distribution of the cross-sectional area of the optimum volute design has a convex shape by the effect of the varied design variables. In Table 1, the normalized value of the objective function of the reference design, obtained through the unsteady RANS analysis, is 1.000. The optimized value of the objective function, related to the unsteady radial force sources of the optimum design, is predicted as 0.584 using the RSA model and calculated as 0.526 by unsteady RANS analysis. Hence, the RSA model produces a good prediction with a slight error. The resulting value of the objective function related to the unsteady radial force sources of the optimum design represents a decrement of 0.474, compared to that of the reference design. This is nearly half of that of the reference design.

A three-dimensional mesh plot of the constructed final RSA model is shown in Fig. 5. The optimum point, located at (807, 4,175) on the surface, is clearly plotted in this figure. The sensitivity of the objective function to changes in each design variable is tested at the optimum point, as shown in Fig. 6. The change in each design variable is restricted to $\pm 10\%$ of the optimum values. $F_{\text{opt}}$ indicates the value of the objective function at the optimum point. The results show that the objective function is much more sensitive to changes in CP2 than to those in CP1 near the optimum point. Notably, the

| Designs            | Design variables | Objective function (Normalized $f_{\text{radial}}$) | Decrement |
|--------------------|------------------|----------------------------------------------------|-----------|
|                    |                  | Surrogate Unsteady RANS                            |           |
| Reference design  | 2,000            | 4,000                                              | -         |
|                    |                  | 1.000                                              | -         |
| Optimum design     | 807              | 4,175                                              | 0.584     |
|                    |                  |                                                    | 0.526     |
|                    |                  |                                                    | 0.474     |
cross-sectional area near the volute exit has greater influence on the unsteady radial force sources generated by the impeller-volute interactions in a single-channel pump.

Figure 7 shows the distribution of the unsteady radial force sources, averaged at the boundary surface near the impeller outlet, during one revolution for both the reference and optimum designs. Here, both values were normalized by the maximum radial force value in the reference design. The sweep area constructed from the unsteady radial force sources of the reference design leans toward the four quadrant directions from the origin, whereas the optimum design is formed near the origin. Furthermore, the sweep area of the optimum design is remarkably decreased compared to that of the reference design. Consequentially, the sweep area and the distance for the optimum design are decreased by 0.629 and 0.844, respectively, compared to those of the reference design.

Figure 8 shows the unsteady fluctuation of the net radial forces for the reference and optimum designs during one revolution, normalized by the maximum radial force value in the reference design. As shown, the amplitude values of the fluctuation of the net radial forces of the optimum design are significantly reduced for all theta angular positions. These results highlight the considerable decrease in the radial force sources in the optimized single-channel pump, as a result of the optimization.

Figure 9 shows the time history of the instantaneous unsteady pressure contours at the boundary surface near the impeller outlet for both the reference and optimum designs. Both of the instantaneous unsteady pressure contours are compared in one rotation $\tau$ of the single-channel impeller. This rotation is divided into four steps to clarify the changes in the flow structure with time during one revolution.
In the reference design, high-pressure zones occur widely on the boundary surface near the impeller outlet as shown in Fig. 9(a), and a large high-pressure zone caused by impeller-volute interactions becomes gradually larger. Consequently, unbalancing phenomena with the vibrations caused by unsteady radial forces result throughout the annulus passage area of the pump. In the optimum design, the pressure distribution is generally uniform; the large high-pressure zone caused by impeller-volute interactions is obviously suppressed, as shown in Fig. 9(b). The optimum design produces mostly stable flows throughout the annulus passage area of the pump.

Meanwhile, because the optimization was performed to reduce the unsteady radial force sources, the efficiency of the optimum design with the reduced radial force was improved remarkably to 82.48%, in comparison with the 81.06% efficiency of the reference design. To understand the improved pump efficiency, the internal flow fields of the reference and optimum designs are compared and analyzed as shown in Fig. 10. Figure 10 shows the distribution of the isosurfaces with the low velocity of 1.5 m/s in both pump models. As shown, an extensive low-velocity region forms along the internal wall in the impeller flow path of the reference design, whereas the low-velocity isosurface is considerably reduced in the optimum design (Fig. 10(b)). Moreover, the broad low-velocity region...
formed in the corner of the reference volute is suppressed in the optimum design. These results illustrate the enhancement of the single-channel pump impeller’s efficiency as a result of the optimization.

7. Conclusions
A single-channel pump volute for wastewater treatment was optimized using a RSA surrogate model with three-dimensional steady and unsteady RANS analyses. Optimization was performed to reduce the unsteady radial force sources caused by impeller-volute interactions by using two design variables related to the cross-sectional area of the internal flow of the volute. The sweep area of the radial force during one revolution and the distance of the mass center of the sweep area from the origin were selected as the two objective functions. The results of the optimum design showed a decrease of 0.474 in the radial force sources, normalized to the reference value, and an increase by 1.42% in the efficiency over that of the reference design. The sweep area of radial force during one revolution and the distance of the mass center of the sweep area from the origin were decreased considerably as 0.629 and 0.844, respectively, normalized to these values in the reference design. Through analyses of the unsteady pressure contours at the boundary surface near the pump impeller outlet, in the reference design, a high-pressure zone was found to form across a wide region; this large high-pressure zone caused by impeller-volute interactions grew with time. From these results, it was found that unbalancing phenomena caused by the unsteady radial force occurred throughout the annulus passage area of the pump. However, the pressure distribution in the optimum design was generally uniform, and the large high-pressure zone caused by impeller-volute interactions was obviously suppressed, compared to those in the reference design. This contributed to the decrease in unbalancing phenomena caused by the decreased unsteady radial force sources, as well as the increase in the efficiency.

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