

Dynamics of Dusty Radiation Pressure Driven Shells: Fast Outflows from Galaxies, Star Clusters, Massive Stars, & AGN

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23 June 2014

ABSTRACT

It is typically assumed that radiation pressure driven winds are accelerated to an asymptotic velocity of \(v_\infty \approx v_{\text{esc}}\), where \(v_{\text{esc}}\) is the escape velocity from the central source. We note that this is not the case for dusty shells. Instead, if the shell is initially optically-thick to the UV emission from the source of luminosity \(L\), then there is a significant boost in \(v_\infty\) that reflects the integral of the momentum absorbed by the shell as it is accelerated. For shells reaching a generalized Eddington limit, we show that \(v_\infty \approx (4R_{\text{UV}} L/M_{\text{sh}} c)^{1/2}\), in both point-mass and isothermal-sphere potentials, where \(R_{\text{UV}}\) is the radius where the shell becomes optically-thin to UV photons, and \(M_{\text{sh}}\) is the mass of the shell. The asymptotic velocity significantly exceeds \(v_{\text{esc}}\) for typical parameters, and can explain the \(\sim 1000 – 2000\) km s\(^{-1}\) outflows observed from rapidly star-forming galaxies and active galactic nuclei if their geometry is shell-like and if the surrounding halo has low gas density. Similarly fast shells from massive stars can be accelerated on \(\sim \text{few} – 10^3\) yr timescales. We further consider the dynamics of shells that sweep up a dense circumstellar or circumgalactic medium. We calculate the “momentum ratio” \(M_\text{v}/(L/c)\) in the shell limit and show that it can only significantly exceed \(\sim 2\) if the effective optical depth of the shell to re-radiated FIR photons is much larger than unity. We discuss simple prescriptions for the properties of galactic outflows for use in large-scale cosmological simulations. We also briefly discuss applications to the dusty ejection episodes of massive stars, the disruption of giant molecular clouds, and AGN.

Key words: galaxies: formation, evolution, starburst — galaxies: star clusters: general

1 INTRODUCTION

In the galactic context, radiation pressure on dust grains has been discussed as a mechanism for launching galactic-scale winds in starbursts and rapidly star-forming galaxies (Harwit 1962; Chiao & Wickramasinghe 1972; Ferrara et al. 1990; Murray et al. 2005, 2011; Hopkins et al. 2012; Krumholz & Thompson 2012; Davis et al. 2014), in disrupting the dusty gas in individual star clusters (Harwit 1962; O’dell et al. 1967; Scoville et al. 2001; Krumholz & Matzner 2009; Murray et al. 2014), in launching fast outflows from AGN (Scoville & Norman 1995; Roth et al. 2012), in setting the \(M – \sigma\) relation (Fabian 1999; Murray et al. 2005), and in supporting starbursts and AGN disks against their own self-gravity (Ferrara 1993; Scoville 2003; Thompson et al. 2005; Andrews & Thompson 2011; Krumholz & Thompson 2012). Radiation pressure and momentum injection by supernovae and stellar winds plays an important role in models of feedback in star-forming galaxies (Thompson et al. 2003; Hopkins et al. 2011; Ostriker & Shetty 2011; Hopkins et al. 2012; Faucher-Giguère et al. 2013).

In the stellar context, dusty shells are produced during the eruptions of supernova impostors and luminous blue variables, including \(\eta\)-Carinae (Davidson & Humphreys 1997), Smith & Gehrz 1998, Smith et al. 2003; Smith 2008, 2013), ultra-bright supernovae such as SN 2006gy (Miller et al. 2010), and SN 2008S-like transients (Kochanek 2011b; Kochanek et al. 2013; Prieto et al. 2008; Thompson et al. 2009; Prieto et al. 2009; Bond et al. 2009). Dusty shell formation and dynamics are also important to the phenomenology of R Coronae Borealis stars (e.g., Gillett et al. 1986), and continuous dusty winds are also produced generally by AGB stars (Ivezic & Elitzur 1993, 1997) and OH-IR stars and cool hypergiants like IRC+10420 (e.g., Ridgway et al. 1986; Humphreys et al. 1997).

The dynamics of radiation pressure-driven shells has been treated by a number of authors. Here, we provide a brief discussion that makes clearer the asymptotic velocity and momentum of an
initially optically-thick shell and connect with observations in several contexts, but with a focus on rapidly star-forming galaxies. In particular, we critically examine the assumption that the asymptotic velocity of a radiation pressure driven shell is of order the escape velocity from the central body. This expectation follows from consideration of the momentum equation for a continuous time-steady radiation pressure driven flow with constant opacity from a point mass $M$ and luminosity $L$ (e.g., eq. 9 of Salpeter 1974):

$$\frac{dv}{dt} = -\frac{GM}{r^2} + \frac{\kappa L}{4pr^2c} \Rightarrow v_{\text{esc}}^2 = v_{\text{esc}}^2(R_0) \left( \Gamma - 1 \right),$$

where $R_0$ is the initial radius, $\Gamma = L/cGMC/R_0$, and where the initial velocity of the medium has been neglected. For line-driven winds from hot sources (e.g., main sequence O stars, Wolf-Rayet stars, the central sources of planetary nebulae, AGN) the opacity is dominated by a forest of Doppler-shifted metal lines, and the shell becomes optically-thin to the UV radiation, and we have scaled for both the galaxy and stellar contexts, by detached blue-shifted absorption line profiles in the case of AGN winds and by Dijkstra & Loeb (2008, 2009) in the case of Ly$\alpha$ scattering.

As we discuss in more detail below, the dynamics of a single geometrically thin shell is different because as it expands it goes through an extended phase where it is optically-thin to the assumed incoming UV photons from the source, but optically-thin to the re-radiated IR emission from the grains. In this so-called single scattering limit (see eq. [19] below),

$$M_{\text{sh}} \frac{dv}{dr} = -\frac{GM_{\text{sh}}}{r^2} + \frac{L}{c} \Rightarrow v_{\text{esc}} \sim \frac{R_{\text{UV}}L}{M_{\text{sh}}c},$$

where $M_{\text{sh}}$ is the mass of the shell, $R_{\text{UV}}$ is the radius at which the shell becomes optically-thin to the UV radiation, and we have assumed $R_{\text{UV}} \gg R_0$ and that $L/c \gg GM_{\text{sh}}/R_0^2$. The lack of radial dependence to the radiation pressure driving term shifts the momentum deposition to large scales, $\sim R_{\text{UV}}$, instead of $R_0$ as in equation [1]. Because the shell sees the entire source luminosity $L$ during its entire evolution, it reaches high velocity.

Equations [1] and [2] are not as different as they first appear. Both expressions can be written as

$$v_{\text{esc}} \sim v_{\text{esc}} \Gamma^{1/2}$$

in the limit that $L \gg 4\pi GMc/\kappa$ and $L \gg GM_{\text{sh}}c/R_0^2$, respectively. But, whereas in the case of a continuous flow the right hand side of equation [3] is evaluated at $R_0$, yielding the result of equation [1], in the case of a shell the right hand side is evaluated at $R_{\text{UV}}$, yielding equation [2]. Another way to put the difference is that equation [1] implies the gas is accelerated in its first dynamical time at $t \sim R_0$, whereas equation [2] says that the “last” dynamical time at $R_{\text{UV}}$ dominates the shell’s acceleration and asymptotic velocity.

An analogous point, but without the associated dust physics, is made in King (2003, 2005) for the case of shells driven by an AGN wind and by Dijkstra & Loeb (2008, 2009) in the case of Ly$\alpha$ scattering.

In this paper, we develop this piece of physics in more detail and apply it to several physical systems. A shell geometry is motivated by observation of shells in the massive star and GMC contexts, by detached blue-shifted absorption line profiles in the case of some rapidly star-forming galaxies and AGN, and by theoretical arguments and modeling (e.g., Yeh & Matzner 2012). The key point is that shell-like outflows can attain significantly higher velocity than one might guess from an incorrect application of equation [1].

This issue is of particular importance in the extended gravitational potential wells of galaxies since the asymptotic velocity attained near the source is crucial in determining whether or not it will escape to the scale of the virial radius, or, if it falls back, on what timescale. We are particularly motivated by the recent discoveries of very fast outflows from post-starburst galaxies by Tremonti et al. (2007) and Diamond-Stanic et al. (2012) (see also Sell et al. 2014).

In Section 2 we first consider the dynamics of a shell surrounding a point mass, and then treat extended mass distributions, as is more appropriate for the dynamics in a galactic gravitational potential. In Section 3 we provide a discussion of our results, including a discussion of fast outflows from galaxies and AGN, the total asymptotic momentum of radiation pressure accelerated shells and clouds, including the momentum ratio $Mv_{\text{esc}}/(L/c)$ in the shell limit, and we provide simple prescriptions for cosmological simulations that captures the expulsion of gas from rapidly star-forming galaxies if it is shell-like.

## 2 DYNAMICS OF SHELLS DRIVEN BY RADIATION PRESSURE

### 2.1 Point Mass

Assume a point source with UV luminosity $L$ and total mass $M$, surrounded by a dusty gas shell of mass $M_{\text{sh}}$ an initial distance $R_0$ from the central source. We define two characteristic radii:

$$R_{\text{IR}} = (\kappa_{\text{IR}} M_{\text{sh}}/4\pi)^{1/2} \approx 0.3 \text{kpc} \kappa_{\text{IR},0.7} M_{\text{sh}}^{1/2},$$

$$R_{\text{UV}} = (\kappa_{\text{UV}} M_{\text{sh}}/4\pi)^{1/2} \approx 1.9 \times 10^3 \text{AU} \kappa_{\text{UV},0.7} M_{\text{sh}}^{1/2}$$

where the shell becomes optically-thin to the re-radiated IR, and

$$R_{\text{UV}} = (\kappa_{\text{UV}} M_{\text{sh}}/4\pi)^{1/2} \approx 4 \text{kpc} \kappa_{\text{UV},3} M_{\text{sh}}^{1/2},$$

$$R_{\text{UV}} = (\kappa_{\text{UV}} M_{\text{sh}}/4\pi)^{1/2} \approx 4.6 \times 10^4 \text{AU} \kappa_{\text{UV},9} M_{\text{sh}}^{1/2}$$

is where the shell becomes optically-thin to the UV radiation from the source. In the above, $M_{\text{sh},x} = M_{\text{sh}}/10^x M_\odot$, we have scaled for both the galaxy and stellar contexts, $\kappa_{\text{UV},3} = \kappa_{\text{UV}}/10^3 f_{\text{d,g, MW}} \text{cm}^2 \text{g}^{-1}$ of gas, where $f_{\text{d,g, MW}}$ is the dust-to-gas ratio scaled to the Milky Way value, and $\kappa_{\text{IR},0.7} = \kappa_{\text{IR}}/10^{0.5} f_{\text{d,g, MW}} \text{cm}^2 \text{g}^{-1}$ roughly approximates the Rosseland-mean dust opacity over a range of temperatures from $\sim 100 - 1000$ K.

The general expression for momentum conservation for a thin shell of mass $M_{\text{sh}}$ approximately valid in the limits of both small and large UV and IR optical depth is

$$\frac{d}{dt}(M_{\text{sh}} v) = -\frac{GM_{\text{sh}}}{r^2} + \left( 1 + \tau_{\text{IR}} - e^{-\tau_{\text{UV}}/4\pi r^2} \right) \frac{L}{c},$$

where

$$\tau_{\text{IR}, \text{UV}} = \kappa_{\text{IR}, \text{UV}} M_{\text{sh}}/(4\pi r^2).$$
are the IR and UV optical depths of the shell and where we have assumed that the dust and gas are dynamically coupled. Note the three terms multiplying $L/c$ in equation (3). The first term (“1”) is due to the direct radiation field. It represents the radiation pressure force exerted if each photon interacts just once with the dusty medium, is converted into an IR photon, and then escapes the system. The second term accounts for reprocessed radiation if the shell is optically-thick to the re-radiated FIR emission. The third term goes to the appropriate limit when $\tau_{\text{UV}} \ll 1$ ($\tau_{\text{IR}} \ll 1$ also), canceling the “1” and yielding the familiar optically-thin radiation pressure force for a source dominated by UV emission ($k_{\text{UV}} L / 4\pi r^2 c$).

Setting the acceleration in equation (6) equal to zero, we obtain the generalized Eddington limit for a shell starting at $R_0$:

$$L_{\text{Edd}} = \frac{GM M_{\text{sh}} c}{R_0^2} \left( 1 + \tau_{\text{IR}} - e^{-\tau_{\text{IR}}} \right)^{-1},$$

where $M$ and $M_{\text{sh}}$ are understood to be the total mass interior to $R_0$ and the shell mass at $R_0$, respectively. The Eddington ratio is then

$$\Gamma_{\text{tot}} = L/L_{\text{Edd}}.$$  

(9)

There are three characteristic Eddington ratios, depending on the optical depth of the shell at $R_0$. If the shell is optically-thick to the IR ($\tau_{\text{IR}}(R_0) > 1$), then the relevant Eddington ratio is

$$\Gamma_{\text{IR}} = L/(4\pi GMc/k_{\text{IR}}).$$

(10)

If the shell is optically-thin to the IR, but optically-thick to the UV ($\tau_{\text{IR}} < 1$, $\tau_{\text{UV}} > 1$) at $R_0$, then the single-scattering Eddington ratio (the “1” in eqs. 8 and 9)

$$\Gamma_{\text{SS}} = L/(GM M_{\text{sh}} c/R_0^2),$$

(11)

determines the dynamics. Finally, if the shell is optically-thin to the incident UV radiation ($\tau_{\text{UV}} < 1$ at $R_0$), then the relevant Eddington ratio is

$$\Gamma_{\text{UV}} = L/(4\pi GMc/k_{\text{UV}}).$$

(12)

Before solving equation (6) for specific example systems, we consider several simple analytic limits.

We first assume that the mass of the shell is constant as a function of radius (i.e., the shell expands into vacuum). Then, dropping the $e^{-\tau_{\text{UV}}}$ term in equation (6) in the regime $R_0 \lesssim r \lesssim R_{\text{UV}}$ ($\tau_{\text{UV}} > 1$) and solving for the velocity at $v_{\text{UV}} = v(R_{\text{UV}})$, one finds that

$$v_{\text{UV}}^2 = v_0^2 + \frac{2GM}{R_0} \left( \Gamma_{\text{SS}} \frac{R_{\text{UV}}}{R_0} + \Gamma_{\text{IR}} - 1 \right) \left( 1 - \frac{R_0}{R_{\text{UV}}} \right),$$

(13)

where $v_0 = v(R_0)$. Once the shell reaches $R_{\text{UV}}$ it becomes optically-thin to the incident UV photons, and the momentum equation for the shell in the regime $R_{\text{UV}} \leq r \leq \infty$ is just

$$v \frac{dv}{dr} = -\frac{GM}{r^2} + \frac{k_{\text{UV}} L}{4\pi r^2 c}.$$  

(14)

Solving, and substituting from equation (13) one finds that

$$v_{\infty}^2 = v_0^2 + \frac{2GM}{R_0} \left[ \left( \Gamma_{\text{SS}} \frac{R_{\text{UV}}}{R_0} + \Gamma_{\text{IR}} - 1 \right) \left( 1 - \frac{R_0}{R_{\text{UV}}} \right) \right.$$  

$$\left. + \frac{GM}{R_{\text{UV}}} \left( \Gamma_{\text{UV}} - 1 \right) \right],$$

(15)

which can be rewritten as

$$v_{\infty}^2 = v_0^2 + \frac{2GM}{R_0} \left[ 2\Gamma_{\text{SS}} \frac{R_{\text{UV}}}{R_0} \left( 1 - \frac{R_0}{2R_{\text{UV}}} \right) \right.$$  

$$\left. + \Gamma_{\text{IR}} \left( 1 - \frac{R_0}{R_{\text{UV}}} \right) - 1 \right].$$

(16)

These expressions are key. The ratio $R_{\text{UV}}/R_0$ can be much larger than unity and thus, even if the initial Eddington ratio of the flow is equal to unity $\Gamma_{\text{tot}} \simeq 1$ at $R_0$, the shell can still be accelerated to
As discussed in Section 3.6, equation (16), and the simple results given in equations (13) and (19) for the asymptotic velocity of a radiation pressure driven shell are qualitatively and quantitatively different from the expectation that \( v_\infty \approx v_{\text{esc}}(R_0)(\Gamma_{\text{IR}} - 1)^{1/2} \) (eq. 1). In particular, from equation (18) one sees that \( v_\infty \) can significantly exceed \( v_{\text{esc}}(R_0) \), even for an Eddington ratio near unity at \( R_0 \); high Eddington ratios are not required for high velocities with respect to the escape velocity. Instead, the initial value of the UV optical depth through the shell determines its dynamical evolution. The typical “boost” in the asymptotic velocity compared to \( v_{\text{esc}}(R_0) \) is

\[
\Gamma_{\text{SS}}^{1/2} [4 \tau_{\text{UV}_0}]^{1/4} \approx \frac{9}{4} \frac{\Gamma_{\text{SS}}^{1/2} R_{0,0.1\text{kpc}}^{1/2}}{\kappa_3^{1/2}} M_9^{1/4} \sim 73 \frac{\Gamma_{\text{SS}}^{1/2} R_{0,10\text{kpc}}^{1/2}}{\kappa_3^{1/2}} M_9^{1/4}.
\]

Thus, in the context of galactic winds, even if \( \Gamma_{\text{SS}} \sim 1 \), one expects the asymptotic velocity of the shell to exceed the escape velocity from the launch region by nearly an order of magnitude (see Sections 2.2 and 2.3 for a discussion of extended galactic potentials). In the stellar context, the boost may be larger. The physics of this enhancement in the asymptotic velocity comes simply from the radial dependence of the single-scattering radiation pressure term; in particular, aside from acting only until \( r \approx R_{\text{UV}} \) it has no radial fall-off, whereas both the gravitational acceleration and the flux drop with radius as \( r^{-2} \). The result that \( v_\infty^2 / R_{\text{UV}} \sim L / M_{\text{sh}c} \) is precisely what one would then get from dimensional analysis of equation (6), which is equivalent to the result \( v_\infty \sim v_{\text{esc}} \Gamma_{\text{SS}}^{1/2} \), but evaluated at \( R_{\text{UV}} \) (see discussion after eq. 2).

As an aside, note that the dynamical stability of a dusty shell to radial perturbations is different depending on whether or not the radiation pressure force is \( f_{\text{rad}} = \kappa L / 4 \pi r^3 c^2 \) or \( L_{\text{IR}} / M_{\text{sh}c} \). In the latter, single-scattering limit, shells are unstable to radial perturbations since \( f_{\text{rad}} \propto r^5 \), whereas the gravitational force is \( f_{\text{grav}} \propto r^{-2} \). Thus, if an equilibrium is established with \( \Gamma_{\text{SS}} = 1 \) small perturbations would drive the shell to smaller \( r \) causing collapse, or larger \( r \), causing dynamical escape with \( v_\infty \) given by equation (19), different from the behavior if \( f_{\text{rad}} \propto r^{-2} \).

Based on the fact that the initial IR optical depth is larger than unity at \( R_0 = R_{\text{sub}}(\approx 100 \text{ AU}) \) and that \( \Gamma_{\text{IR}} \gg \Gamma_{\text{SS}} \), one might have expected that \( v_\infty \approx v_{\text{esc}}(R_{\text{sub}})(\Gamma_{\text{IR}} - 1)^{1/2} \approx 250 \text{ km s}^{-1} \), but this is not the case. Instead, the single-scattering term in equation (6) dominates the dynamics, accelerating the shells to velocities much larger than \( v_{\text{esc}}(R_{\text{sub}}) \sim 40 \text{ km s}^{-1} \) since \( R_{\text{UV}} / R_0 \gg 1 \). One might have also have expected that the shell with the highest mass and highest initial optical depth to have the highest asymptotic velocity, but because these results are shown for fixed luminosity, and because \( \Gamma_{\text{SS}} \ll M_{\text{sh}}/\kappa \), one has that \( v_\infty \propto M_{\text{sh}}^{-1/4} \) (eq. 19).

Thus, low mass shells are driven to higher asymptotic velocity than higher mass shells at fixed \( L \), for \( M_{\text{sh}} = 0.01 \text{ M}_\odot \) (not shown), \( v_\infty \approx 970 \text{ km s}^{-1} \). More discussion of these types of eruptions are included in Section 3.6. Since such outbursts have high initial IR optical depths, their dynamics may be important in assessing multi-dimensional instabilities that may limit the momentum coupling between the radiation field and the shell as it is accelerated (see Section 3.6).

Finally, note that the characteristic acceleration timescale \( t_{\text{acc}} \sim v_\infty / (dv/\text{dt}) \) is long on the scale of observations of a single massive star outburst:

\[
v_{\text{acc}} \sim \left( \frac{2c}{L} \frac{k_{\text{UV}}}{4\pi} \right)^{1/4} M_{\text{sh}}^{3/4} \sim 1600 \text{ yr } L_{7}^{-1/2} M_{\text{sh},0.01M_\odot}^{3/4} \sim 9 \times 10^6 \text{ yr } L_{7}^{-1/2} M_{\text{sh},0.01M_\odot}^{3/4} \text{.}
\]

The scaling for the massive star outburst agrees with the calculations shown in Figure 4 and implies that high-velocity shells driven by this physics will be associated with many old outbursts.

\footnote{Note that throughout this work we assume that \( L_{7}/(M_{\text{sh}}v^2/2) \gg 1 \). For high enough IR optical depths it is possible for the radiation field to do sufficient work on the matter to enter the “photon tiring” regime discussed by Dowski & Gavrylov (1997) and Dowski et al. (2001) in the context of line-driven winds. Examining \( L_{7}/(M_{\text{sh}}v^2/2) \) as a function of radius for the models in Figure 4 we find that it is always larger than unity, although it becomes as low as \( \sim 2 \) on scales smaller than the \( \Gamma_{\text{IR}} \) point (red circle) for the model with \( M_{\text{sh}} = 10 \text{ M}_\odot \). An estimate of the critical shell mass such that \( L_{7}/(M_{\text{sh}}v^2/2) = 1 \) and photon tiring becomes important is \( M_{\text{sh,\,tiring}} = L_{7}^{3/4} \kappa_{3/2} M_{\odot}^{2/3} / (2/3 \kappa_{3/2}^{2/3} T_{4\,\text{kpc}}^{2/3} \kappa_{3/2}^{2/3}) \), obtained by taking \( \Gamma_{\text{IR}}(R_{\text{sub}}) = 2c/v_\infty \) in the limit that \( \Gamma_{\text{IR}} \gg \Gamma_{\text{SS}} \). For the parameters of Figure 4 this is \( M_{\text{sh,\,tiring}} \approx 8 \text{ M}_\odot L_{7}^{-3/4} \kappa_{3/2}^{2/3} T_{4\,\text{kpc}}^{2/3} \). Note the strong dependencies.}
2.2 Extended Mass Distributions with Fixed Shell Mass

In the context of outflows driven from galaxies, it is important to consider the extended stellar and dark matter potentials. For illustration we assume that the total mass distribution is an isothermal sphere: \( M(r) = 2\sigma^2 r/G \), where \( \sigma \) is the velocity dispersion. Assuming momentarily that the mass of the shell is constant, we then have that

\[
M_{\text{sh}} \frac{dv}{dr} = -\frac{2\sigma^2}{r} + \frac{1 + \tau_{\text{IR}}}{r} - \frac{e^{-\tau_{\text{UV}}}}{r^2} \frac{L}{c}.
\]

and then integrating to an outer radius \( R_{\text{out}} \) where the approximation of an isothermal potential breaks down,

\[
v_{\text{out}}^2 = v_0^2 + 4\sigma^2 \left[ \frac{2 \Gamma_{\text{SS}} R_{\text{UV}}}{R_0} \left( 1 - \frac{R_0}{R_{\text{UV}}} - \frac{R_{\text{UV}}}{2R_{\text{out}}} \right) + \Gamma_{\text{IR}} \left( 1 - \frac{R_0}{R_{\text{UV}}} - \ln \left( \frac{R_{\text{out}}}{R_0} \right) \right) \right].
\]

The same factor that appears in the point-mass limit — \( 2 \Gamma_{\text{SS}} R_{\text{UV}}/R_0 \) — which can boost the asymptotic velocity far above \( v_{\text{out}}(R_0) \) (eq. 19) also appears in the limit of an extended mass distribution. Assuming that \( \Gamma_{\text{tot}} \gg 1 \), \( R_0 \ll R_{\text{UV}} \), \( R_{\text{UV}} \ll R_{\text{out}} \), and that \( \Gamma_{\text{IR}} \ll 2 \Gamma_{\text{SS}} R_{\text{UV}}/R_0 \), one finds that

\[
v_{\text{out}} \approx 2\sigma \Gamma_{\text{SS}}^{1/2} \left( \frac{2 \kappa_{\text{UV}} \sigma^2 f_{\text{sh}}}{\pi R_0 G} \right)^{1/4}
\]

and

\[
v_{\text{out}} \approx 4 \times 10^3 \text{ km s}^{-1} R_{\text{out}}^{1/4} R_{0, 0.1 \text{kpc}}^{1/4} R_{\text{UV}}^{1/4} R_{0, 0.1 \text{kpc}}^{-1/4}.
\]

Figure 2 shows the velocity evolution of massive dusty shells launched in an isothermal potential. We assume this potential extends to 100 kpc for simplicity, even though this approximation breaks down for real galaxies on the scale of \( \sim 10^5 \) of kpc. For massive shells (e.g., \( f_{\text{sh}} = 1 \)), we include the self-gravity of the shell.

\[\text{Figure 2.} \text{ Velocity as a function of radius (left panel) and time (right panel) for dusty shells in an isothermal potential with } \sigma = 200 \text{ km s}^{-1}, \text{ starting from a launch radius of } R_0 = 0.2 \text{ kpc and } \Gamma_{\text{tot}} = 2 \text{ (eq. 8 & 9). The two solid lines show the evolution for a freely expanding shell with } f_{\text{sh}} = 0.1, \text{ and 1, where } M_{\text{sh}} = f_{\text{sh}} 2\sigma^2 R_0/G, \text{ such that } M_{\text{sh}} \approx 3.7 \times 10^8 M_\odot \text{ and } \approx 3.7 \times 10^9 M_\odot, \text{ respectively. Given } \Gamma_{\text{tot}}, \text{ the total luminosities are } \approx 8.9 \times 10^{12} L_\odot \text{ and } \approx 2.6 \times 10^{13} L_\odot. \text{ The short dashed lines show the evolution of both shells if they interact with a constant density halo that has } n_{\text{ext}} = 10^{-4} \text{ cm}^{-3}. \text{ The dotted and long dashed lines show the evolution of the } f_{\text{sh}} = 1 \text{ shell if the external gas density distribution follows equation (11) with } f_{\text{sh}} = 0.01 \text{ and 0.1, respectively. The metallicity of the swept up gas was assumed to have a dust content equivalent to 0.1 times the Milky Way value. The red, and blue circles denote } R_{1\text{IR}} \text{ and } R_{\text{UV}} \text{ (eqs. 30 & 31), respectively. See Fig. 3.} \]
itself in the total mass $M$ in our solution to the momentum equation (6) using $M = M(< r) + M_{sh}/2$. The two solids lines show velocity as a function of radius (left panel) and time (right panel) for $f_{sh} = 0.1$ and $f_{sh} = 1$, corresponding to $M_{sh} = 3.7 \times 10^8$ and $3.7 \times 10^9 M_\odot$, respectively, launched from a galaxy with $\sigma = 200$ km s$^{-1}$, and from a radius $R_0 = 0.2$ kpc. For each shell, we assume $\Gamma_{tot} = 2$, so that $L \approx 8.9 \times 10^{12}$, $2.6 \times 10^{13} L_\odot$ as might be provided by a central starburst and/or active galactic nucleus. Again, the naive expectation in many models of galactic winds would have been that $v_{\infty} \simeq 2\sigma (\Gamma_{tot} - 1)^{1/2} \simeq 400$ km s$^{-1}$, for the parameters of Figure 2. However, because of the factor $(2\Gamma_{SS} R_{UV}/R_0)^{1/2}$ in equation (28) the actual velocities are $\sim 4$ times this value, reaching $\sim 1600$ km s$^{-1}$ on $\sim 1 - 10$ kpc scales.

### 2.3 Evolving Shells

In the general case of a massive shell driven into the circumgalactic medium of highly star-forming galaxies, we expect the shell to sweep up mass and the assumptions of the previous section break down. In the limiting case that the shell sweeps up less than its initial mass by the time it reaches $R_{UV}$, we expect the dynamics to be qualitatively similar. However, if the mass of the swept-up material approaches the initial mass of the shell on the scale of $R_{UV}$, we expect the shell dynamics to be altered. If we assume that the circumgalactic gas takes the form of a static isothermal sphere with gas density

$$\rho = \frac{f_{sh} \sigma^2}{2\pi G r^2}, \quad (31)$$

the critical value for $f_{sh}$ such that the swept up gas mass $M_{sh}(R_{UV})$ is equal to the initial shell mass $M_{sh}(R_0)$ is

$$f_{sh, crit} = \left(\frac{\rho R_{UV}}{\kappa_{UV}}\right)^{1/2} \simeq 0.01 \sigma_{200}^{-2} M_{sh, 9}^{1/2} \kappa_{UV, 3}^{-1/2}. \quad (32)$$

Thus, for $f_{sh} \geq f_{sh, crit}$, we expect the shell dynamics to be different from the solid lines shown in Figure 2. In particular, we expect the shell to decelerate, in accord with momentum conservation.

An analogous estimate can be made in the stellar case, where the shell from the eruption sweeps up the matter in a steady preceding stellar wind of mass loss rate $\dot{M}$. In this case, the density profile is an isothermal sphere with $\rho = \dot{M} / (4\pi r^2 v_w)$, where $v_w$ is the wind velocity. Setting the total swept up mass $\dot{M}(r-R_0)/v_w$ equal to the initial shell mass, one derives a critical mass loss rate such that the shell sweeps up its own mass on a scale $R_{UV}$:

$$M_{crit} = \frac{4\pi \dot{M}_{sh} v_w^2}{\kappa_{UV}} \left(\frac{R_{UV}}{v_w}\right)^{1/2} \simeq 8 \times 10^{-4} M_\odot \text{ yr}^{-1} M_{sh, 9}^{1/2} \kappa_{UV, 3}^{-1/2} v_{w, 100}^{1/2}, \quad (33)$$

where $v_{w, 100} = v_w / 100$ km s$^{-1}$. \footnote{For the purposes of this estimate we include $v_w$ in the wind mass profile, but neglect the bulk flow of the wind matter in the momentum equation in calculating $M_{crit}$.}

If we take the surrounding external medium to have constant density, we can estimate the critical density such that the swept up mass is equal to the initial shell mass at $R_{UV}$. This limit is applicable to massive stars in constant density circumstellar envelopes, or shells interacting with the surrounding ISM, and to shells driven from galaxies that sweep up matter from the hot near-constant density halo (e.g., \cite{2004MNRAS.349..559M}). The critical external den-
sity required to slow the shell on a scale \( R_{UV} \) is

\[
\eta_{\text{ext}, \text{crit}} = 4.5 \times 10^3 \text{ cm}^{-3} \left( \frac{\kappa_{\text{UV}, 3}}{M_{\text{sh}, 0}} \right)^{-3/2} \times 0.1 \text{ cm}^{-3} \left( \frac{\kappa_{\text{UV}, 3}}{M_{\text{sh}, 0}} \right)^{-3/2}.
\]

(34)

Note that both normalizations are large on the scale of what might be expected in the stellar and galactic environments, respectively. Because the swept up mass scales with \( r^3 \) in the constant density case we expect the shell to slow significantly on scales larger than \( R_{UV} \) even for \( n_{\text{ext}} \ll \eta_{\text{ext}, \text{crit}} \), but that the maximum velocity of the shell at \( R_{UV} \) will not be much smaller than the estimates above as long as \( n_{\text{ext}} \) is not greater than \( \eta_{\text{ext}, \text{crit}} \). An example is shown in the stellar case by the dashed line in Figure 1, which shows the evolution of a shell driven into a constant density medium with \( n_{\text{ext}} = 5 \text{ cm}^{-3} \). Of course, the circumstellar medium around a massive star in outburst is likely to have a complex density structure with a wind-blown bubble, but equations (33) and (34) show that the density must be very high to slow the shell on scales much smaller than \( R_{UV} \).

Similar examples of a shell interacting with a medium, but in the galactic case, are shown by the dashed and dotted lines in Figure 2. The short dashed lines show the \( f_{\text{sh}} = 0.1 \) and \( f_{\text{sh}} = 1.0 \) models, but including a surrounding constant density medium of \( n_{\text{ext}} = 10^{-4} \text{ cm}^{-3} \) as motivated by the hot halo models of [Maller & Bullock 2004]. Both models attain high velocities, but then decelerate as they accumulate more mass, effectively stopping after 108 yr of evolution. In an isothermal potential, these shells would fall back again, particularly since the radiation pressure driving is not likely to be strong for timescales much larger than \( 10^7 \text{–} 10^8 \) yr. For these models, \( L \) is held constant in time.

The dotted line in Figure 2 shows the evolution of the \( f_{\text{sh}} = 1 \) model with a constant density external medium, but also with an isothermal sphere gas reservoir of the form in equation (31). For \( f_g = 0.01 \), the maximum velocity of the shell is significantly decreased, and for \( f_g = 0.1 \) the velocity of the shell only reaches \( \sim 300 \text{ km s}^{-1} \).

Note that in integrating the evolution of these shells we have had to make an assumption about the dust content per unit mass of the swept up material, adjusting the UV and IR opacities accordingly. We have adopted a simple parameterization by assuming that the dust-to-gas ratio of the swept up material is a constant, normalized to the Milky Way value: \( \xi = f_{\text{DG, swept}} / f_{\text{DG, MW}} \). All the models in black in Figure 2 assume \( \xi = 0.1 \), but the results are not qualitatively different if we assume the swept up medium is completely dust-less, \( \xi = 0 \). As an example, for \( f_{\text{sh}} = 1, f_g = 0.1 \), and \( n_{\text{ext}} = 10^{-4} \text{ cm}^{-3} \) (long dashed lines) we show models for \( \xi = 0 \) and \( \xi = 1 \). The other models shown are not as strongly affected by this change in \( \xi \).

In addition to the calculations presented in Figure 2, we have done a number of tests with a more realistic NFW dark matter potential. Because the density profile is steeper on large scales, the shells launched in the NFW potential generally attain higher asymptotic velocity than those launched in a singular isothermal sphere, all else being equal. However, it is clear from Figure 2 that the dynamics of shells is dominated by the large-scale gas distribution, and not the large scale potential. The maximum velocity of a shell, its velocity profile, and its long-term evolution depend sensitively on both the radial dependence of the ambient density profile and its normalization. For this reason, we have opted to focus on the simpler isothermal case, for which some analytic estimates are easily made.

3 DISCUSSION

3.1 The Asymptotic Momentum of Shells

A key diagnostic of observed outflows in galaxies and AGN is the momentum ratio

\[
\zeta = \frac{\dot{M}v/(L/c)}{M_{\text{sh}}v/(L/c)/(r/v)}.
\]

(35)

where the first equality is applicable to a continuous wind with mass loss rate \( \dot{M} \), and the second equality is applicable to a shell. These two definitions are equivalent since, typically, one measures the column density in blue-shifted absorption features (such as the resonance lines of Fe, Mg, and Na) to infer

\[
\dot{M} = 4\pi r N m_p v \quad \text{or} \quad M_{\text{sh}} = 4\pi r^2 N m_p,
\]

(36)

where \( N \) is the column density of gas, so that

\[
\zeta = 4\pi r N m_p v/(L/c).
\]

(37)

in either case. In general, one must assume a relative abundance of the tracer (e.g., Na or Mg) with respect to total gas, which usually involves an uncertain ionization correction (e.g., Murray et al. 2007).

A primary observational difference between a continuous wind and a shell would of course be in the absorption line profile, which for a perfect geometrically-thin single shell would be a delta-function in velocity along the line of sight toward a point source of radiation. For an extended source, such as a galaxy, this is not true, and the observed absorption line would be broadened geometrically by the projection of the moving shell onto the source.

The momentum ratio \( \zeta \) is important because for \( \zeta \gg 1 \), either (1) the shell or wind had very high effective \( \tau_L \) (e.g., Murray et al. 2010), or (2) the shell was initially energy-driven, as in the early evolution of a supernova remnant. Since the radial scale of the absorbing material is in general not known, absorption line studies determine \( \zeta \) with significant uncertainties. Emission-line studies with molecular emission, [CII], or in the optical/UV (e.g., H\textalpha, [NI]) provide a complementary view of winds, and in some cases find evidence for \( \zeta > 1 \) (e.g., Cicone et al. 2014; Genzel et al. 2011, Section 3.2).

The momentum ratio in the case of a single shell of fixed mass, observed at a radius \( r \gg R_{UV} \), can be approximated by

\[
\zeta = \frac{M_{\text{sh}}v^2_{\text{UV}}}{rL/c} \approx 2f(\tau_0) \left( \frac{\Gamma_{\text{SS}}}{\Gamma_{\text{tot}}} \right) \left( \frac{R_{UV}}{r} \right),
\]

(38)

where \( f(\tau_0) = (1 + \tau_R - e^{-\tau_{UV}}) \) evaluated at the launch radius \( R_0 \). For \( \tau_{UV} > 1 \), but \( \tau_R < 1 \), \( \Gamma_{\text{SS}} \approx \Gamma_{\text{tot}} \) and \( f(\tau_0) \approx 1 \), implying that \( \zeta \) should always be of order unity or smaller, since for \( r > R_{UV} \), \( \zeta \) rapidly decreases. This behavior can be seen in both the \( f_{\text{sh}} = 0.1 \) and the \( f_{\text{sh}} = 1 \) models shown by the solid lines in Figure 2(left), which shows \( \zeta(r) \). All models have \( \zeta \approx 0.1 \) on 10–100 kpc scales. Note that even though the shells accelerate beyond \( R_{UV} \), the measured momentum decreases, because
in this regime $v^2(r)/r$ decreases. Models that sweep up mass decelerate, and $\zeta(r)$ decreases more rapidly. In the single-scattering limit of $\tau_{\text{UV}}>1$, but $\tau_{\text{IR}}<1$ at $R_0$, one would simply estimate $v_{\text{UV}}^2 \sim R_{\text{UV}}L/\dot{M}_{\text{sh}}c$ by dimensional analysis of the momentum equation, and then substituting into equation (38), one again finds that $\zeta(R_{\text{UV}}) \approx 1$.

Since the two key observables for characterizing shell-like outflows are velocity and column density, and since these enter the calculation of $\zeta$ directly, in the right panel of Figure 2 we show $M_{\text{sh}}/4\pi r^2 n_p$ versus $r$ for the same models as in the left panel, and in both panels of Figure 2 More discussion is provided in Section 3.2.

If the effective IR optical depth of the shell is much larger than unity at the launch point (see Section 3.7 below for caveats), then $f(\tau_0) \sim \tau_{\text{IR}}(R_0)$, $\Gamma_{\text{tot}} \sim \Gamma_{\text{IR}}$ and $\zeta(r)$ can be significantly increased. For the $f_{\text{sh}} = 1$ example in Figures 2 and 3, $\tau_{\text{IR}}(R_0) \approx 8$ and $\zeta(r)$ peaks at $\approx 2$ on the scale of a few times $R_0$. For higher $\tau_{\text{IR}}(R_0)$ one finds higher values of $\zeta$ on the scale of $R_{\text{IR}}$. In particular, when $\tau_{\text{IR}}(R_0) \gg 1$ and $\Gamma_{\text{IR}} \gg 1$, one finds that at $R_{\text{IR}}$

$$
\zeta(R_{\text{IR}}) = \frac{M_{\text{sh}}v_{\text{sh}}^2}{R_{\text{IR}}L/c} \approx \frac{2R_{\text{IR}}}{\dot{M}_{\text{sh}}} \approx 5.8^{1/2} M_{\text{sh}}^{1/2} R_{\odot}^{-1} R_{\text{uc}}. \text{ } (39)
$$

Note that the above is not proportional to $\tau_{\text{IR}}(R_0)$. Instead, the maximum value of $\zeta$ occurs on scales smaller than $R_{\text{IR}}$ and is proportional to $\tau_{\text{IR}}(R_0)$ (for fixed $\Gamma_{\text{IR}} \gg 1$). The red solid lines in Figure 3 show $\zeta(r)$ and $N(v)$ for a model with $f_{\text{sh}} = 5$ so that $\tau_{\text{IR}}(R_0) \approx 40$. The peak in $\zeta(r)$ occurs at a few times $R_0$ and is about 4 times lower than $\tau_{\text{IR}}(R_0)$. See Section 3.7 for a discussion of some of the uncertainties associated with high-$\tau_{\text{IR}}$ solutions in the context of radiation pressure driven shells.

3.2 Fast Outflows from Rapidly Star-Forming Galaxies, Starbursts, & Post-Starbursts

Fast Outflows in Emission: Cicone et al. (2014) have recently presented data on a collection of AGN and star-formation dominated systems with outflows seen in molecular emission. We show $\zeta$ as a function of $L_{\text{bol}}$ and a histogram of $\zeta$ for their sample in the panels of Figure 4. Red, blue, and green denote AGN fractions of $L_{\text{AGN}}/L_{\text{bol}} > 0.5$, $0.2 \leq L_{\text{AGN}}/L_{\text{bol}} < 0.5$, and $L_{\text{AGN}}/L_{\text{bol}} < 0.2$, respectively, for the systems with outflows detected at high significance.

If interpreted as shells accelerated by radiation pressure on dust, as described in this paper, we find that the systems with $\zeta \lesssim 2$–3 are readily explained, given their bolometric luminosities. For example, Mrk 273 and 231, have $L_{\text{AGN}}/L_{\text{bol}} \approx 0.08$ and $\approx 0.3$, $L_{\text{bol}} \approx 1.7 \times 10^{12}$ and $4 \times 10^{12} L_{\odot}$, and total outflow mass of $\approx 10^{8.2}$ and $\approx 10^{8.4} M_{\odot}$, respectively, on scales of $\approx 0.6$ kpc, with average velocities of $\approx 600–700$ km s$^{-1}$. These parameters are all in the range expected for relatively low-mass shells accelerated by radiation pressure, with dynamics similar to the $f_{\text{sh}} = 0.1$ model shown in Figures 2 and 4.

Systems with high values of $\zeta$ in the Cicone et al. (2014) compilation likely require large initial values of $\tau_{\text{IR}}(R_0)$ (see red line in Figure 3 Sections 3.1 and 3.7). Another possibility is that a fairly rapid decrease in the AGN or starburst luminosity could imply large $\zeta$ even though the dynamics is consistent with radiation pressure acceleration in the single-scattering limit. Future explorations could employ the shell models described here and/or continuous wind

5 By assuming a uniform medium, Cicone et al. (2014) overestimate $\zeta$ by a factor of 3, which we have corrected in Figure 4. We also adopt the lowest values of the total inferred outflowing gas mass in their tables. In some cases $\zeta$ could be larger by a factor of $\sim 3$. 

Figure 4. (Left); Observed momentum ratio from a sample of AGN and starburst galaxies as a function of bolometric luminosity (Cicone et al. 2014). Red, blue, and green points show systems where the central AGN luminosity $L_{\text{AGN}}/L_{\text{bol}} > 0.5$, $0.2 \leq L_{\text{AGN}}/L_{\text{bol}} < 0.5$, and $L_{\text{AGN}}/L_{\text{bol}} < 0.2$, respectively. (Right): Histogram of momentum ratios from the same sample. A CO-to-H$_2$ conversion factor of 0.8 has been used for calculating $M_{\text{sh}}$. 

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models to constrain \( R_0 \) and the source of driving with the data on \( \zeta \). A careful comparison with the observed dynamics could be used to quantitatively test the radiation pressure driven shell picture discussed here.

Fast Outflows in Absorption: Fremonti et al. (2007) and Diamond-Stanic et al. (2012) report the discovery of fast \( \gtrsim 1000 \text{ km s}^{-1} \) outflows in starburst and post-starburst galaxies. In particular, Diamond-Stanic et al. (2012) show that the system J0905+5759 has strongly blue-shifted Mg II absorption with a shell-like velocity profile centered at \(-2470 \text{ km s}^{-1} \) (range from \(-3000 \) to \(-2200 \text{ km s}^{-1} \)), covering the entire galaxy. The effective radius of the galaxy is \( \approx 100 \) pc, with stellar mass of \( 10^{10.7} \text{ M}_\odot \), velocity dispersion of \( \approx 250 \text{ km s}^{-1} \), and total IR luminosity of \( 10^{12.6} \text{ L}_\odot \) (Diamond-Stanic, private communication).

Repeating the calculation shown in the left panel of Figure 2, but for \( \sigma = 250 \text{ km s}^{-1} \), \( R_0 = 100 \text{ pc} \), and \( \Gamma_{\text{tot}} = 2.5 \) \( (L_{\text{bol}} \approx 1.1 \times 10^{11} \text{ L}_\odot) \) we find \( v_{\text{UV}} \approx 1900 \text{ km s}^{-1} \) and \( v_{\infty} \approx 2500 \text{ km s}^{-1} \) for \( f_{\text{sh}} = 0.05 \) \( (M_{\text{sh}} \approx 1.5 \times 10^8 \text{ M}_\odot) \). If interpreted as a shell accelerated by the continuum radiation pressure on dust grains, the shell would have a radial scale of \( \gtrsim 1.6 \text{ kpc} \), which is consistent with the observed blue continuum.

We note that although the velocity and shell mass calculated are roughly consistent with the observational constraints, the required bolometric luminosity of the system is somewhat high compared to the observations. However, there are many factors that complicate straightforward modeling of this system. In particular, a realistic model for the luminosity of the system as a function of time and the large-scale gravitational potential change the velocity profile quantitatively. More importantly, though, as highlighted in Figure 2, the surrounding medium swept up by the shell can qualitatively affect its dynamics. In this context, it is again worth noting that the Eddington ratio for dusty shells is linearly dependent on the gas-to-dust-ratio: larger dust content per gram of gas lowers the critical luminosity for shell expulsion.

The Local Starburst M82: M82 has an extensively studied outflow, with evidence for \( \approx 200 - 600 \text{ km s}^{-1} \) line-emitting gas and dust on kpc scales, a molecular outflow on small scales, and hot nuclear X-ray emission (e.g., Shopbell & Bland-Hawthorn 1998, Walter et al. 2002, Strickland & Heckman 2009).

Using the dust-scattered UV emission from the central starburst (Hoopes et al. 2005, Coker et al. 2013) calculated the Eddington ratio for dusty gas on \( 0.5 - 5 \text{ kpc} \) scales. Using the large-scale rotation curve from Greco et al. (2013), Coker et al. (2013) found that although significantly more UV escapes the starburst along the minor axis than along our line of sight, the Eddington ratio is still much less than unity on large scales. These results imply that a shell-like radiation pressure-driven outflow cannot account for the dusty gas currently seen on kpc scales. Moreover, this result highlights the fact that the additional large-scale acceleration that produces the high velocities discussed here may not be generic, but may only occur in special circumstances or geometries. One mitigating factor is that Förster Schreiber et al. (2003) find that the bolometric luminosity of the M82 starburst was a factor of \( \approx 4 \) larger 6 Myr ago, indicating that the system may have potentially approached the single-scattering Eddington limit on scales within the starburst\(^6\) and would have certainly exceeded \( \Gamma_{\text{UV}} = 1 \) \( (\text{eq} \, 12) \).

3.3 Broad Absorption Line Quasars & Ultra-Fast Outflows

Some BAL quasars have detached potentially shell-like absorption profiles with blueshifted velocities of \( \approx 4000 - 5000 \text{ km s}^{-1} \). Some have velocities over \( 20,000 \text{ km s}^{-1} \) (e.g., Pounds et al. 2003, Tombesi et al. 2010, Gupta et al. 2013).

Taking \( L_{\text{AGN}} = 10^{47} \text{ erg s}^{-1} \) and \( R_0 = R_{\text{gal}} \approx 2 \text{ pc} \) and \( M_{\text{sh}} = 10^5, 10^6, \text{ and } 10^7 \text{ M}_\odot \), we find \( v_{\text{sh}} \approx 2.8 \times 10^4, 1.7 \times 10^4 \text{ and } 1.1 \times 10^4 \text{ km s}^{-1} \), respectively, for Milky Way gas-to-dust ratio. Such high velocity outflows might emerge along the line of sight to Type-I quasars in a very short timescale \( < 10^5 \text{ yr} \) (see eq. 11) and become optically-thin to the UV emission from the quasar on scales \(< 0.1 - 1 \text{ kpc} \) (see eq. 5). These types of dusty shells are related to the suggestion by Scoville & Norman (1995) that dusty material near quasars could be accelerated to \( \approx 0.1 \text{ c} \).

3.4 Previous Work & Prescriptions for Cosmological Simulations

Murray et al. (2005) discussed galactic winds driven by the combined momentum input of radiation pressure on dust and supernovae and wrote down the momentum equation for a shell in an isothermal potential, as in equation 32. The primary difference between their results and our work here is in the assumed distribution of mass swept up by the outflowing shell. In particular, they assumed an isothermal mass distribution for the gas so that \( M_{\text{sh}} \propto r \) (as in eq. 31). In this case, in the single-scattering limit,

\[
v_{\text{sh}} \frac{dv}{dr} = -\frac{\sigma^2}{r} + \frac{L}{c^2} \frac{G}{2\sigma^2 f_{\text{sh}} r} \tag{40}
\]

where \( f_{\text{sh}} = M_{\text{sh}}/M \) is a constant at each radius. Note that for constant \( f_{\text{sh}} \), both the gravitational acceleration and the radiation pressure acceleration have the same radial dependence. This is why the fairly large enhancement in the asymptotic velocity of a shell launched by radiation pressure we emphasize here was not noted in that work. The long dashed line in the panels of Figure 2 assumes an isothermal sphere for the surrounding gas distribution and shows a shell model that closely tracks the expectation from equation 40 and Murray et al. (2005).

Several studies of the enrichment of the intergalactic medium have used the so-called "momentum scalings" (radiation pressure or supernovae) based on the work of Murray et al. (2005) (e.g., Oppenheimer & Davé 2006, 2008). Typically, these are that

\[
v_{\infty} \approx \text{few } \times \sigma \tag{41}
\]

and that

\[
M/\text{SFR} \propto \sigma^{-1} \tag{42}
\]

These same scalings and variants have also been tested in models of the mass-metallicity relation by Peeples & Shankar (2011).

\(^6\) \( \Gamma_{\text{SS}} \approx 1 \) for \( M(r < 300 \text{ pc}) = 10^9 \text{ M}_\odot \), \( M_{\text{sh}} = 10^8 \text{ M}_\odot \), \( R = 300 \text{ pc} \) and \( L = 2 \times 10^{11} \text{ L}_\odot \) (see eq. 11).

\(^7\) This follows from equating \( L/c \approx \epsilon \text{ SFR } c \sim \dot{M}_{\text{sh}} \) for a star-forming galaxy in the single-scattering limit (Murray et al. 2005).
Our work suggests prescriptions that would more accurately capture the physics of radiation pressure driven shells, as opposed to continuous winds. The simplest is that the “few” in equation 41 could in some cases be as large as \(~\sim\) 10, depending on the geometry of the outflow and the surrounding medium, a potentially important factor that should be taken into account and explored in more detail. Second, the Eddington luminosity for a shell is given simply by equation (23), which can be thought of as three different Eddington luminosities in three different regimes. If the shell is (1) optically-thick to the IR, (2) optically-thin to the IR, but optically-thick to the UV, or (3) optically-thin to the UV, the Eddington luminosity is given by equations (10), (11), and (12), respectively, depending on the surface density of the shell. If \(L \geq L_{\text{Edd}}\) in the appropriate limit, then the ISM is ejected. Note that in general \(L_{\text{Edd}}\) is dust-to-gas ratio dependent and thus metallicity dependent.

The dynamics of the shell and its interaction with the surrounding circumgalactic medium could be calculated from equations (6), either via a subgrid model in large-scale cosmological simulations, or explicitly in high-resolution simulations of individual galaxies. Such a prescription for large-scale simulations would differ qualitatively from what is currently done in that a large fraction of the ISM would be ejected in single events, fallback would be determined predominantly by the circumgalactic medium density profile, and the velocity of the material could approach \(\sim 10 \times \sigma\) along lines of sight with little gas (see eq. 23). In this picture, the ratio \(M_{\text{f}}/\text{SFR}\) should instead be thought of as the ratio of the total mass ejected to the total mass formed between each star formation and ejection event, where the timescale between ejections would be determined by the gas accretion rate from the IGM and from re-accreted (formerly ejected) gas. Ejection episodes and fallback might precipitate radiative cooling of the hot circumgalactic medium, as in the work of Fraternali & Binney 2008 (see also Marasco et al. 2012; Fraternali et al. 2013).

### 3.5 Star Cluster Disruption

The estimates made here can also effect giant molecular cloud (GMC) disruption. Murray et al. 2010 discussed the acceleration of GMC gas by radiation pressure and other forces (see also Krumholz & Matzner 2009, and Murray et al. 2011) provide a general description of launching these shells and clouds from star clusters to velocities high enough to escape the host galaxy and generate supershells. In these works, analytic estimates for the critical star cluster stellar mass required to generate extra-planar gas was estimated, based on the assumption that \(v_{\infty}\) for such a shell would be of order a few times the cluster escape velocity. If the self-gravity of the shell dominates the total gravitational force, and if the central star cluster reaches the single-scattering Eddington limit \((\Gamma_{\text{SS}} \simeq 1)\), then we have shown here that \(v_{\infty} \simeq (2GM_{\text{sh}}/R_0)^{1/2}(R_{\text{UV}}/R_0)^{1/2} \simeq 200 \text{ km/s} M_{\text{sh}}^{3/4} R_0^{1/4} (5 \text{ pc}/R_0)^{-1}\), implying that large-scale super-shells from GMC disruption could be driven high above the plane of a large galaxy by massive star clusters with total stellar mass significantly less than \(10^6 \text{ M}_\odot\). Faster asymptotic velocities are more easily obtained in a shell-like geometry.

Because shell formation during GMC disruption by radiation pressure may be generic (Yeh & Matzner 2012), our results are important for diagnosing the dynamics of observed systems like 30 Doradus (Lopez et al. 2011; Pellegrini et al. 2011).

### 3.6 Massive Star Eruptions

Figure 1 shows that shells from massive star outbursts can be accelerated to velocities much larger than the escape velocity at the dust formation radius. Here, we briefly discuss applications to the outbursts of Luminous Blue Variables and the shells observed around the yellow hypergiants VY CMa and IRC 10420.

Eta Carinae’s homunculus shows a number of different kinematic components. The primary mass reservoir is \(~\sim\) 10 M\(_\odot\) with a velocity of \(~\sim 500 \text{ km s}^{-1}\) (Smith et al. 2003; Smith 2006). There is also a faster component at \(~\sim 1000 - 2000 \text{ km s}^{-1}\) and a much faster, but much less massive component moving at \(~\sim 3000 - 6000 \text{ km s}^{-1}\) (Smith 2008). All are associated with the Great Eruption approximately 170 years ago in which Eta Car reached an estimated bolometric luminosity of \(~\sim 2 \times 10^7 \text{ L}_\odot\) (Davidson & Humphreys 1997).

In this context it is worth asking if radiation pressure on dust grains could have dominated the acceleration of any of the kinematic components. Examining Figure 1 and equations (19) and (21), this possibility appears unlikely for the more massive component because (1) the asymptotic velocity would only be \(~\sim 250 - 350 \text{ km s}^{-1}\) (allowing for some uncertainty in \(L\)) and (2) the timescale to reach this velocity is too long, \(~\sim 10^4 \text{ yr}\). For the less massive high velocity components radiation pressure on dust might have had more of a role, depending on their mass. Although not shown in Figure 1, calculations with eq. 19. Allowing for a factor of 2 higher luminosity and dust shell formation radius, as in the work of Fraternali & Binney 2008 (see also Owocki et al. 2004). It is worth noting that the maximum possible velocity for a dusty shell accelerated from the dust formation/destruction radius is

\[
v_{\infty} \approx \left( \frac{4L \kappa_{\text{UV}}^4 \sigma_{\text{SB}} T_{1500}^3}{\pi c^2} \right)^{1/4}
\]

\[
\sim 3500 \text{ km s}^{-1} L_7^{1/4} \kappa_{\text{UV}}^{1/2} T_{1500}^3
\]

(43)

for a shell that starts optically-thin to the incident UV radiation from the star and a nominal Milky Way gas-to-dust ratio (compare with eq 19). Allowing for a factor of 2 higher luminosity and dust-to-gas ratio boosts \(v_{\infty}\) to \(~\sim 4200 \text{ km s}^{-1}\).

The much lower velocity dusty outflows of the yellow hypergiants VY CMa and IRC 10420 are also of interest. The latter has \(L \sim 5 \times 10^5 \text{ L}_\odot\) and a mass of \(~\sim 10 - 20 \text{ M}_\odot\), with dusty shells observed at velocity \(~\sim 40 \text{ km s}^{-1}\) on \(~\sim 10^2 \text{ AU}\) scales, with equivalent mass loss rates of \(~\sim 10^{-3} - 10^{-4} \text{ M}_\odot \text{ yr}^{-1}\) (Oudmaijer et al. 1996; Humphreys et al. 1997; 2002; Dinh-V-Trung et al. 2009). The observed velocity is large compared to the escape velocity at the dust formation radius, \(~\sim 20 \text{ km s}^{-1}\) and the typical velocities of dusty AGB star winds. The observed parameters for VY CMa

---

8 The photospheric temperature of Eta Carinae and the yellow hypergiants discussed here is lower than needed for significant UV emission. For this reason, \(\kappa_{\text{UV}}\) used throughout this work should be replaced by the flux-mean dust opacity for a \(~\sim 5000 - 7000 \text{ K}\) blackbody: e.g., \(\kappa \approx 100 - 300 \text{ cm}^2 \text{ g}^{-1}\). This lowers the expected asymptotic velocity according to the scalings in equations (19) and (43).
are similar. It has a prominent dusty arc with velocity \( \sim 50 \text{ km s}^{-1} \) on \( 10^3 \text{ AU} \) scales, but with both slower \( (\sim 10 \text{ km s}^{-1}) \) and faster \( (\sim 100 - 200 \text{ km s}^{-1}) \) material observed smaller and larger scales, respectively (Monnier et al. 1992; Humphreys et al. 2005; Muller et al. 2007).

In accord with equations \((19)\) and \((21)\), low-mass shells with \( M_{sh} = 0.1 M_\odot \) from a star with \( L = 5 \times 10^3 L_\odot \), and \( M = 15 M_\odot \) can be accelerated to \( \sim 50 - 75 \text{ km s}^{-1} \) on scales of \( \sim 10 - 100 \text{ AU} \), but reach an asymptotic velocity of \( \sim 150 \text{ km s}^{-1} \) at \( \sim 10^4 \text{ AU} \). Higher mass shells of \( \sim 1 - 3 M_\odot \) reach only \( v_\infty \sim 100 \text{ km s}^{-1} \). Here, as throughout this paper, we assume that the shell subsumes 4\( \pi r \) and that it sees the central source throughout its acceleration to \( v_\infty \). Both assumptions should be cast into question for these mass ejection episodes since the dusty nebulae are observed to be asymmetric and since multiple shells exist for both stars. Even so, the large-scale acceleration that is the focus of this paper might be required to explain the \( \sim 100 - 200 \text{ km s}^{-1} \) material seen around VY CMa by Humphreys et al. (2005) (their Fig. 13).

3.7 Uncertainties & Assumptions

Geometry & Emergent Radiation Field: We assume a simple geometry throughout this paper: a point source with a surrounding spherical shell. In most contexts, the shell is unlikely to subsume 4\( \pi r \) and may break up into discrete clouds. The dynamics of an ensemble of clouds differs qualitatively and quantitatively from the shell geometry discussed here, and is left for a future investigation (see Murray et al. 2011). Perhaps more importantly, in the case of galactic-scale winds, the set of sources is not point-like, although the galaxy may be represented by a distribution of actively star-forming and disrupting star clusters.

In addition, we have assumed that the UV continuum escapes from the source to large scales and that it is time-steady. In the galaxy context, the central source may be obscured by the ISM of the galaxy, or by intervening shells of material. Although we have shown that the acceleration time is short with respect to the characteristic time for a stellar population to change luminosity (eq. \(21\)), the time evolution could be important effect for the long-term dynamics in galactic potentials \( (> 10^7 - 10^8 \text{ yr}) \). Similar to the discussion presented in Zhang & Thompson (2012) it is worth noting that strong blue-shifted absorption can be observed in post-starburst galaxies because the timescale for material driven to \( \sim 100 \text{ kpc} \) is longer than \( \sim 10^8 \text{ yr} \). It is worth emphasizing that the momentum ratio \( \zeta \) (eq. \(35\)) would then be underestimated. The time-dependence of a quasar might produce analogous effects; a fast outflow could be seen on large scales around a (now) less luminous AGN and a correspondingly low value of \( \zeta \) would be inferred.

Large \( \tau_{IR} \): Using 2D planar gray flux-limited diffusion (FLD) with a realistic dust opacity Krumholz & Thompson (2013) showed that the asymptotic momentum of shells driven with initially very large \( \tau_{IR} \) is not proportional to \( \tau_{IR} \). In particular, for an initial midplane optical depth \( \tau_{IR} \approx 100 - 1000 \) and IR Eddington ratio of \( \infty \) (gravitational acceleration of zero), they find that the asymptotic momentum taken up by the ejected material is only \( \zeta \approx 1 - 10 \) times that expected from the single-scattering limit. This result follows from the strong density-flux anti-correlation that develops as a result of channels which open in the 2D flow due in part to the radiation-driven Rayleigh-Taylor instability (RRTI).

Recent results from Davis et al. (2014) using a more sophisticated and accurate radiation transport algorithm (the Variable Eddington Tensor [VET] method; Davis et al. 2012; Jiang et al. 2012) supersede these earlier calculations, and produce different results than FLD (see Krumholz & Thompson 2012). In particular, Davis et al. (2014) find a less extreme flux-density anticorrelation that produces more net momentum coupling between the radiation and the dusty gas relative to FLD. This leads to a qualitatively different outcome in some simulations: Krumholz & Thompson (2012) find steady radiation pressure-driven convection whereas Davis et al. (2014) find an unbound outflow for the same initial conditions.

These results are important and should be more fully studied. There is yet no systematic study of the momentum coupling in super-Eddington dusty outflows with a large range of initial IR optical depths using multi-dimensional VET calculations. Such a study will be crucial in understanding the viability of radiation pressure driving in generating outflows with \( \zeta \gg 1 \) in a range of contexts. As implied by Figure 1 and the discussion in Section 3.6, outbursts from massive stars will in general have large initial \( \tau_{IR} \) if dust forms. In addition, we expect large average IR optical depths for massive star clusters and ULIRGs, and in the dusty pc-scale environments around AGN. If radiation pressure is a viable mechanism for the dynamics of these outflows then large effective IR optical depths for momentum coupling may be required by the data in some systems (e.g., Fig. 4). These results from observational and numerical works may ultimately point either to other sources of wind driving, such as energy-driven flows powered by supernovae (e.g., Chevalier & Clegg 1985; Strickland & Stevens 2000; Strickland & Heckman 2005 but see Zhang et al. 2014), or additional momentum input by supernovae (e.g., Murray et al. 2005; Thompson et al. 2005; Faucher-Giguère et al. 2013), cosmic-rays (e.g., Jelbelas et al. 2008; Socrates et al. 2008; Hanasz et al. 2013), magneto-centrifugal acceleration, or other processes.

Grain Physics: We have simplified a number of issues associated with grain physics. First, we have neglected the temperature dependence of the Rosseland-mean opacity of dust grains in the optically-thick limit, relevant when \( \tau_{IR} > 1 \) (Pollack et al. 1994; Semenov et al. 2003). In particular, \( \kappa_R(T) \approx 2.4(T/100 \text{ K})^2 \text{ cm}^2/\text{g} \) for \( T \leq 150 \text{ K} \) and \( \kappa_R(T) \approx \text{ const} \) for \( 150 \leq T \leq 1500 \text{ K} \) for Milky Way gas-to-dust ratio. Second, we have assumed that the gas and dust grains are completely dynamically coupled. In reality, the momentum coupling between
dust and gas is grain size dependent, and will depend both on the charge distribution and magnetic field strengths in the medium being accelerated [Draine & Salpeter 1979b]. Third, in the models presented throughout this paper, we assume that the grains in a central AGN or starburst, by grain-gas collisions, or sputtering (see, e.g., Draine & Salpeter 1979a; Draine & Salpeter 1979b; Voit 1991; Draine 1981; 1995). All of these issues deserve further investigation in the context of dusty radiation pressure accelerated shells.

4 SUMMARY

We have shown that the typical expectation for radiation pressure driven flows that $v_\infty \sim v_{\text{esc}}$ at the launch radius $R_0$ is not correct for dusty shells. In this case, the single-scattering phase of acceleration dominates the dynamics and the asymptotic velocity of the shells scales as $v_\infty \sim (R_{\text{UV}} L/M_\text{sh})^{1/4} \propto \kappa_{\text{UV}}^{-1/4} (L/M_\text{sh})^{1/2}$. As discussed in Section 1 and 2 this result is equivalent to $v_\infty \sim v_{\text{esc}} \Gamma^{1/2} R_0$ evaluated at the point where the shell becomes optically thin to the UV, $R_{\text{UV}}$. This result for $v_\infty$ has implications for the dynamics of dusty shells in a number of contexts, including giant molecular cloud disruption around forming star clusters, outbursts from massive stars, galactic winds driven by star formation, and fast dusty outflows driven by AGN. In particular, it appears possible to accommodate the surprising result from Diamond-Stanic et al. (2012) that post-starburst galaxies with velocity dispersions of order $\sim 200-250$ km s$^{-1}$ can drive shell-like outflows with velocity of $\sim 2000$ km s$^{-1}$ even though the Eddington ratio at the launch radius was only $\Gamma_{\text{tot}} \sim 1$.

ACKNOWLEDGMENTS

TAT is supported in part by NASA Grant NNX10AD01G. TAT thanks Aleks Diamond-Stanic, Nathan Smith, David Weinberg, Smita Mathur, and Dong Zhang for useful conversations and Chris Kochanek for a critical reading of the text. EQ is supported in part by NASA ATP Grant 12-ATP12-0183.

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