Ergonomic, epistemological and existential challenges of integrating digital tools into school mathematics

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Abstract
Despite half a century of sustained advocacy and effort, the degree to which use of digital mathematical tools has become integral to the practice of school mathematics remains limited. The main sections of this paper identify three fundamental dimensions of challenge to such integration, illustrating the kinds of adaptation required. The ergonomic dimension relates to environmental and cognitive features which structure interaction between humans and digital tools in the everyday practice of school mathematics. The epistemological dimension relates to disciplinary and didactical knowledge available to guide use of digital tools in school mathematics. The existential dimension concerns conceptions of self and subject which shape use (and non-use) of such tools. In the light of this framework, a further section reviews the impact of the recent pandemic shock on use of digital tools in school mathematics.

Keywords
arithmetic calculator, classroom teaching, digital tools, dynamic geometry, mathematics education, online teaching, technology integration

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1. Introduction
The use of digital tools is now commonplace in mathematical practices outside school, but the degree to which use of such tools has become integral to school mathematics remains limited. The main sections of this paper focus on three fundamental dimensions of the challenge to such integration. To illustrate these, I focus on two particular types of digital mathematical tool – dynamic geometry systems and simple arithmetic calculators. The sources that I draw on come mainly from European countries – and from my experience in Britain in particular. Yet, it seems plausible that, given their fundamental character, these same dimensions are relevant to Asian countries.

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2. The Ergonomic Dimension

An ergonomic perspective focuses on interaction between humans, tools and other elements of a system, taking account of physical, cognitive and environmental factors (International Ergonomics Association, 2021). In particular, established practices take place in a smooth and efficient manner thanks to a highly evolved material/social organisation and the participant expertise which underpins it. The introduction of digital tools into the everyday practice of school mathematics brings perturbations to, and adaptations of, established relations between teacher, students, subject and tools which can be analysed in terms of a system of structuring features (Bozkurt & Ruthven, 2017; Ruthven, 2009).

Use of digital resources often involves changes in the working environment of lessons, requiring modification of the classroom routines which enable lessons to flow smoothly. While new technologies broaden the range of tools and resources available to support school mathematics, they present the challenge of building a coherent resource system (Ruthven, 2019) of compatible elements functioning in a complementary manner, which participants are capable of using effectively. Such innovation may call for adaptation of the established repertoire of activity formats that frame the action and interaction of participants during particular types of classroom episode, combining to create prototypical activity structures (Bozkurt & Ruthven, 2018) for particular styles of lesson. Incorporating new tools and resources into lessons requires teachers to develop their curriculum script for a mathematical topic, the cognitive structure which informs their planning of lesson agendas, and enables them to teach in a flexible and responsive way: it interweaves mathematical ideas to be developed, possible topic-related tasks to be undertaken, corresponding activity formats to be used, and potential student difficulties to be anticipated. Finally, teachers operate within a time economy in which they seek to optimise use of the available time to support desired educational outcomes.

Let me illustrate this framework through the case of a mathematics teacher developing his teaching practice to make use of dynamic geometry (Ruthven, 2010). The teacher’s developing craft knowledge conveys the extent of the professional learning involved in adapting everyday practice to the changing ergonomy created by the introduction of digital tools.

In terms of working environment, each session started in the teacher’s normal classroom and then moved to a nearby computer suite. Starting sessions in the classroom avoided disrupting established routines for launching lessons, providing an environment more conducive to maintaining student attention “without the distraction of computers in front of each of them.” In the new computer suite the teacher was establishing start-up routines with students for opening a workstation, logging on to the school network, using shortcuts to access resources, and maximising the document window. Similarly, near the end of each session, the teacher was developing shut-down routines, prompting students to save their files and print out their work, reminding them to give their file a name indicating its contents and to put their name on their document to make it findable amongst output from the shared printer. Compared to those other ergonomic aspects which follow, this aspect relates more to use of the digital platform hosting the dynamic geometry system than specifically to use of dynamic geometry itself.

In terms of resource system, this teacher saw work with dynamic software as complementing established construction work with classical manual tools by strengthening attention to geometric properties. Nevertheless, he felt that old and new tools lacked congruence because certain manual techniques appeared to lack computer counterparts. Accordingly, he saw dynamic software as involving different methods and having a distinct function: “I don’t think there’s a great deal of connection. I don’t think it’s a way of teaching constructions, it’s a way of exploring the geometry.” The teacher was also concerned that students were spending too much time on cosmetic aspects of presentation. Through a new lesson segment showing students an example illustrating to what degree, and for what purpose, it was legitimate to “slightly adjust the font and change the colours a little bit, to emphasise
the maths, not to make it just look pretty”, he was establishing socio-mathematical norms for using the new tool. Equally, the teacher reported that he was learning about “unusual” and “awkward” aspects of software operation liable to “cause a bit of confusion” amongst students, as well as of how to turn such difficulties to advantage in helping students.

In terms of **activity structure**, the teacher’s account of his lesson pointed to a combination of formats, “a bit of whole class, a bit of individual work and some exploration”, a structure that the teacher wanted “to pursue because it was the first time [he]’d done something that involved all those different aspects.” The teacher also highlighted how arrangements had not worked as well as he would have liked in fostering discussion during student activity. He would be giving more thought to how best to organise this. The teacher noted ways in which use of the software helped to structure and support his exchanges with students, creating three-way formats of interaction between student, computer and teacher. Such opportunities arose from helping students to identify and resolve bugs in their dynamic geometry constructions. The use of text-boxes created conditions under which students could be more easily persuaded to revise their written comments.

In terms of **curriculum script**, one episode will serve as illustration. In the first session of the lesson, after an opening demonstration by the teacher, students used the software to construct a dynamic figure consisting of a triangle and the perpendicular bisectors of its edges, and then investigated this figure. The teacher was developing strategies for helping students appreciate that concurrency of perpendicular bisectors was geometrically significant, by getting them to drag the dynamic figure: “I don’t think anybody got that without some sort of prompting. It’s not that they didn’t notice it, but they didn’t see it as a significant thing to look for.” In the second session, using the dynamic figure that they had constructed the previous day, students investigated how the position of the point of concurrency of the perpendicular bisectors was affected by dragging vertices to change the shape of the triangle (see Figure 1). During this session the teacher asked the class about the position of this ‘centre’ when the triangle was dragged to become right angled. Afterwards, he commented that he “was just expecting them to say it was on the line” and that he had not anticipated what a student pointed out: “I don’t know why it hadn’t occurred to me, but it wasn’t something I’d focused on in terms of the learning idea, but the point would actually be on the mid point.” One can reasonably infer that the teacher’s curriculum script absorbed this new variant as a direct result of this episode.

![Figure 1. Example of a dynamic figure under investigation in the lesson.](image-url)
In terms of time economy, this teacher linked his overall management of time to key stages of investigation – “the process of exploring something, then discussing it in a quite focused way as a group, and then writing it up” – in which students moved from being “vaguely aware of different properties” to being able to “actually write down what they think they’ve learned.” Because he viewed the software as a way of engaging students in disciplined interaction with a geometric system, he was willing to spend time to make them aware of the construction process underlying dynamic figures by “actually putting it together in front of the students so they can see where it’s coming from.” Equally, he was willing to invest time in having students learn to use the software.

3. The Epistemological Dimension

An epistemological perspective focuses on the development, evaluation, and organisation of knowledge, taking account of logical, psychological and social factors (Goldman, 1986). For mathematics teaching, the ways in which digital tools mediate mathematical concepts and processes may be sufficiently distinctive to pose epistemological challenges. This calls for the development of disciplinary and didactical knowledge to guide such use of tools, through mathematical didactical analysis aiming to establish a coherent intellectual framework covering the digital and the traditional, and to generate appropriate curricular sequences. Equally, many types of digital mathematical tool are still at a relatively early stage in their evolution, with significant differences of design between alternative tools of similar type, and between successive generations of a particular tool. This variability and ephemerality increases the demands made of users, and adds to the complexity of establishing stable mathematical didactical analyses.

For example, Mackrell (2011) found considerable diversity in basic features of the most commonly used dynamic geometry packages:

- Different repertoires of tools and organisation of them.
- Different styles of interface and modes of interaction.
- Different and inconsistent order of selecting action and object.
- Differing modes of behaviour of figures under dragging.

She comments that “This diversity is an indication that creating [a] program is not simply a matter of representing the conventions of static Euclidean geometry on a screen, but is dependent on the epistemology of the designer and is influenced by both cultural conventions and pedagogical considerations” (p. 384). Mackrell comments on the limited volume of research on the impact of such design decisions, even for well-known ones such as selection order and the draggability of objects.

Moreover, the representations and actions provided by digital mathematical tools may diverge in important respects from those associated with traditional written inscription. This calls for mathematical didactical analysis to establish a coherent intellectual framework covering the digital and the traditional, and to establish appropriate curricular sequences. For example, the idea of dragging has developed in ways quite unanticipated when the first dynamic geometry software was created. Arzarello et al. (2002) have identified a wide variety of ways in which dragging may be used with dynamic figures, including:

- Wandering dragging: of points without a plan in order to discover configurations or regularities in the drawing.
- Bound dragging: of a point already linked to an object.
- Guided dragging: of the basic points of a drawing in order to give it a particular shape.
- Dummy locus dragging: of a basic point so that the drawing keeps a property.
- Line dragging: drawing new points along a line in order to keep the regularity of the figure.
- Linked dragging: of a point attached to an object.
We still await development of a full mathematical theorisation of dragging: Baccaglini–Frank (2019) comments that “the discussion is still open on how to link phenomena experienced in a DGE with their interpretations in the Euclidean world” (p. 779). Equally, little of the didactical analysis necessary to incorporate dragging into the curriculum and to underpin a systematic development of curricular sequences has yet been undertaken. Such curricular sequences must acknowledge the expansion of mathematical concepts and techniques which digital tools make available. While it is relatively straightforward to bring such tools to bear on familiar tasks, ultimately a renewed curriculum must incorporate tasks which would be inconceivable without the mediation of digital mathematical tools, and this, of course, calls for a corresponding shift in mathematical thinking. Laborde (2001), reflecting on a multi-year project working with teachers, identified a progression in types of curricular scenario employing dynamic software (which I exemplify here in relation to the geometrical topic already discussed):

- Facilitates material aspects of a familiar task: e.g. construction of a diagram consisting of a triangle and its perpendicular bisectors.
- Assists mathematical analysis of a familiar task: e.g. through dragging the triangle to identify the concurrence of perpendicular bisectors as an invariant property.
- Substantively modifies a familiar task: e.g. dragging the triangle to identify a variable characteristic which correlates with the internal or external positioning of the circumcentre.
- Creates a task which could not be posed without dynamic software: e.g. a task in which three circles have been constructed with a common free centre, each circle passing through a different vertex of the triangle; dragging of the ‘free’ centre is then used to identify the conditions under which two or all three of the circles coincide (see Figure 2).

But dynamic geometry systems are relatively complex tools which radically augment available mathematical representations and actions, and which have not yet achieved stability in design. By contrast, the arithmetic calculator is much less complex, employs broadly familiar mathematical representations and actions, and has achieved relative stability in design. Its use in primary mathematics was the subject of extensive developmental work through the ‘calculator aware’ number project which influenced the original English National Curriculum (Ruthven, 2001; Shuard et al., 1991). In particular, this led to the inclusion in that curriculum of a detailed section on calculator methods – alongside more extensive sections on mental and written methods. At that time, official guidance recognised that, despite its congruence with established mathematical representations and actions, the introduction of the arithmetical calculator to primary school mathematics had a

![Figure 2](image-url)  
*Figure 2.* Construction for investigating where to position a common centre so that circles, each passing through a different triangle vertex, coincide.
number of implications for curricular sequences. In particular, the availability of a calculator made it possible for children to tackle problems using real data relating to familiar situations from an early stage; use of, and experimentation with, the calculator led to children encountering negative numbers and decimal fractions much earlier than in the traditional curriculum; the ease of computation with a calculator made experimental methods of problem solving based on trial and improvement much more feasible.

However, the English National Curriculum has never raised the issue of standard calculator methods of computation to mirror its references to standard written methods. In the cases of addition, subtraction and multiplication, of course, the direct way in which such operations are performed on the calculator means that there can hardly be said to be a calculator method as such. However, the case of division is less straightforward, because the calculator carries out a particular form of division, meaning that it can be necessary to interpret the calculator result and translate it into an alternative form. Were the curriculum to make explicit the need for a calculator-based method of quotient and remainder division (Figure 3), this would provide a publicly visible capstone for a ‘calculator aware’ curriculum. Such a capstone could stand alongside – if not replace – the long division algorithm, the culmination of the traditional written arithmetic curriculum of the primary school, which, for many members of the public, is a totemic mathematical achievement.

4. The Existential Dimension
An existential perspective focuses on the representations, valorisations and identities through which people orient themselves and act within their material and social world (Moscovici, 1984). Here the focus is on conceptions of self and subject which shape the uptake and use (or non-use) of digital tools within school mathematics. In particular, the introduction of digital mathematical tools has the potential to modify or even question established features of school mathematics (often perceived as ‘natural’). To the extent that such features are valued, particularly by powerful and influential groups, the introduction of these tools, or development of their use beyond a certain point, is likely to encounter reluctance or more active resistance.

The calculator has become a popular archetype around which prevalent social representations of digital mathematical tools are formed. In England, at least, this is reflected in a shift in policy that has taken place at all levels of schooling towards a curriculum that is more ‘calculator beware’ than ‘calculator aware’. On the occasion of the most recent review of the National Curriculum, discussion on

Figure 3. A calculator method of quotient and remainder division.
an online comment board – from The Guardian, a newspaper with a broadly liberal readership – pro-
vided an opportunity to gauge popular opinion on this matter and to analyse the associated social
representations (Ruthven, 2014).

Many of the comments depict the use of calculators by pupils as antagonistic to thought and sub-
versive of intelligence:

One of the most important things that a child learns is the ability to think. If you give them a tool
that discourages that at such a young age, that aspect of their thinking will be stunted.

Some comments portray use of calculators not only as developmentally debilitating but as a
morally iniquitous avoidance of effort:

Using a calculator… rots the brain, not to mention the poor ethic it instils… if they don’t work out
the answers with hard graft.

Where contributions concede that using a calculator does involve a degree of expertise, this tends
to be presented as distinct from mathematics itself:

Learning to work a calculator is only learning to work a calculator, not learning how to do maths.

A common suggestion is that access to calculators should be granted pupils only once they have
become confident with number and proficient in mental or written calculation:

They should learn how basic arithmetic works first, which means doing it either in their head or on
paper.

Some comments are salutary in showing how opposition to calculators is embedded in contribu-
tors’ sense of personal worth, grounded in their own educational experiences. One such identity nar-
rative from a contributor conveys a sense of personal accomplishment associated with mastery of
mental and written calculation, expressed in a continuing proud refusal of the calculator:

I learned arithmetic the old-fashioned way, using a sums book, following the methods demon-
strated by the teacher on the blackboard. By six, I could add and subtract up to a hundred, by
eight I had long division and multiplication, and all the tables to ten… My mathematical skills
took me all the way through A level into degree-level statistics, and then a (boring) first job in
Health Service data analysis. I have never owned a stand-alone calculator, and I don’t use the
one in MacOs X.

Another (atypical) identity narrative conjures up a very different type of personal history, offering
a sense of how, for some pupils at least, calculators serve as a catalyst for developing interest and
capability with numbers:

I am really good at mental arithmetic, but as a child abhorred rote learning of times tables, couldn’t
see the point as I could work them out in an instant. It almost alienated me completely from maths,
luckily playing with calculators… rekindled my interest in number games. So when I was older
and scientific calculators starting coming in… I used to play with it, especially the functions that
worked out means and standard deviations. That set me up well for the types of maths I used in
later life, inferential statistics.
This last narrative indicates that there are some threads of popular opinion which are more positive about calculator use in school mathematics. Nevertheless, it seems that the currently dominant strands of popular thought tend to devalorise the use of calculators in particular, and of digital mathematical tools more generally, in terms of the following polarities:

- **Cognitive self-sufficiency**: thinking ‘independent’ of digital tools versus unthinking ‘dependence’ or ‘over-reliance’ on such tools.
- **Mathematical essence**: ‘purely’ mathematical mental/written methods versus (wholly/partially) ‘non-mathematical’ use of digital tools.
- **Moral virtue**: ‘effortful’ use of ‘rigorous’ mental/written methods versus ‘lazy’ recourse to ‘slipshod’ use of digital tools.
- **Epistemic value**: use of mental/written methods taken as exercising intelligence and developing understanding versus use of digital tools taken as doing neither.

Transforming such representations, many of which are shared, at least in part, by educators themselves, represents a considerable challenge for the field.

5. *Initial Discussion*

I have used the examples of arithmetic calculator and dynamic geometry to illustrate ergonomic, epistemological and existential dimensions of challenge to the integration of digital mathematical tools into school mathematics. Similar issues arise in relation to other digital mathematical tools – digital tools devised for purposes of mathematical representation and computation, such as spreadsheet and computer algebra. In particular, to the extent that the representational conventions and computational operations associated with digital mathematical tools differ from classical ones, the epistemological dimension becomes prominent. Equally, in response to associated shifts in the system of mathematical tools and techniques available – notably towards mathematical interdependence of tool and user – the existential dimension is activated.

Such explanations imply that the epistemological and existential dimensions will be less significant in relation to the integration of digital tools which do not involve the types of representational, computational and operational shift that have been outlined above. Certainly, such shifts are less pronounced for the types of digital tool reported as being more widely used in school mathematics, notably digital presentational tools – such as interactive whiteboard and presentation software – and digital curricular resources – such as prepared presentations and interactive worksheets (Bretscher, 2021). The wider uptake of such tools may, at least in part, be explicable in terms of a relative absence of epistemological and existential challenges associated with their use. Equally, while integrating them still faces ergonomic challenges, the potential of such tools to support and enhance already well established ‘dominant practices’ in school mathematics provides a strong incentive to resolve such challenges (Bretscher, 2021).

6. *The Pandemic Shock*

Circumstances recommend me to pursue this a little further. The preceding sections are based on a talk that I gave to the International Symposium on Mathematics Education held at the Asian Centre for Mathematics Education at East China Normal University in November 2019. At that time, little did any of us anticipate the unprecedented disruption which the Covid-19 pandemic was soon to unleash. Only months later, in countries around the world, many schools shifted from on-site classroom education to some form of remote online or distance education. While this has often been presented, in rather general terms, as a great leap forward for the use of digital technology in education,
the preceding discussion suggests the need for a more discriminating analysis. In writing up the talk for publication as a commentary paper, then, it seemed remiss not to add a new section examining the influence on mathematics teaching of this extraordinary pandemic shock. Although it is, of course, too early to make any definitive assessment, I have repurposed relevant findings from two available large-scale survey studies of how mathematics teachers adapted to the shift online.

One of these studies allows comparison of teacher-reported use of ‘digital delivery tools’ in three European systems (Flanders, Germany and the Netherlands) before and after the shift (Drijvers et al., 2021). The most striking change was in the reported use of video conferencing software, rising from no greater than 4% in any system to 56% (Germany), 87% (Flanders) and 97% (Netherlands). High proportions of teachers also reported inducting students to this new working environment in terms of providing instructions on how to use the platform and establishing rules for behaviour. Ergonomically, then, this highlights the significant change in working environment faced by many teachers in shifting to online education and hints at some of the adaptation required. Another development for many teachers in this study was embracing use of video-clips, either sourced online or home-made. For these teachers, then, this represented a further change not just in working environment (extended to include use of video tools) but in resource system (modified to incorporate video resources).

The other study fills out this ergonomic dimension, reporting on the types of difficulty encountered by teachers in a system in which “face-to-face teaching has been such an entrenched practice that it seemed challenging for teachers and students to switch to online instruction” (Cao et al., 2021, p. 148). Many of the themes identified by this Chinese study can be related to particular ergonomic aspects of adapting established classroom practices to the online medium: ‘Difficulties in using the different functions of the online teaching platform’ and ‘Inability to monitor student participation during lessons’ to aspects of working environment; ‘Difficulties in combining digital pedagogical resources in teaching’ to resource system; ‘Difficulties following mathematics teaching procedures as in normal classroom instruction’ to activity structure; ‘Problems with teacher–student interactions during online instruction’ to working environment, activity structure and time economy. For example, the study illustrates this last theme with the teacher comment: “Sometimes when you ask questions, you can’t get a response instantly, since you need to ask a student to turn on their video or microphone first, and then maybe repeat the questions and so on…[t]his slows the teaching pace” (p. 162).

Given the immediate demands on teachers to develop a functional pedagogy through the medium of digital delivery tools, it is not surprising that the ergonomic dimension emerges prominently from these two studies. The Chinese study (Cao et al., 2021) reports that “[t]he restrictions brought by the shift to online instruction made many teachers feel helpless in terms of shaping their classes” (p. 166), but that “teacher participants… eventually got used to online teaching, and their anxiety about it declined” (p. 167). Likewise, the European study (Drijvers et al., 2021) reports that “teachers’ confidence in using digital technologies increased remarkably during the lockdown” (p. 35). This study also reports teacher ratings of how well remote education provided opportunities for specific aspects of school mathematics. Here the average rating was clearly positive “for teaching algorithms and procedures” but negative “for mathematical argument and reasoning” and strongly so “for authentical, complex mathematical tasks” (pp. 47, 58). These observations suggest that, while teachers may have become more confident in using digital delivery tools, once it becomes possible to return to on-site classroom education they are likely to use them at best sparingly and selectively.

These studies provide no support for the idea that growth in teachers’ expertise and confidence with digital delivery tools fosters uptake of digital mathematical tools. Indeed, the limited explicit evidence available is from the European study which found “a decrease in the use of mathematical tools embedded in online exercise platforms and learning environments” (p. 46).
A further study of a different type offers insights into the existential and epistemological dimensions. Based on qualitative thematic analysis of essays in which Italian mathematics teachers volunteered their experience of the shift from face-to-face to distance education during the pandemic, this study highlights a potential for longer-term and more far-reaching impact (Albano et al., 2021, pp. 32–33):

Generally speaking, the traumatic change in the educational setting plunged teachers into an unexpected and unthinkable world where the teachers become aware that the didactic system has to be reconstructed. Therefore, totally different educational worlds are imagined: a school where the summative assessment disappears; a school where mathematics is not a set of formula and procedure; a school where the teachers design activities to promote reasoning and, more generally, competency-oriented activities; and a school where technology is really integrated in teachers’ and students’ usual practice… We think that it is relevant that teachers take into consideration different possible worlds… that can assume an important role in the development and in the diffusion of a culture of mathematics education, even if these possible worlds will not (fully) become actual worlds.

It will be fascinating to see to what extent the various changes introduced and aspirations imagined during the period of remote education thrive after the possibility arrives of a return to classroom education!

7. Concluding Discussion
The main sections of this paper have outlined ergonomic, epistemological and existential dimensions of the integration of digital mathematical tools into school mathematics. In the light of the challenges involved, we should not be surprised at limited progress to date. At the same time, research has started to give us a better understanding of these dimensions, with the potential to help tackle the associated challenges. A further section has highlighted the primarily ergonomic issues that teachers encountered during the uptake of digital delivery tools in response to the recent Covid-19 shock, while suggesting that many teachers harbour some reservations about the use of these tools. Equally, the evidence available suggests that the uptake of digital delivery tools bears little relation to that of digital mathematical tools. Arguably it would take a very different kind of crisis to provoke a response to the epistemological and existential challenges posed by digital mathematical tools – one produced by wider recognition of the serious effects of the increasing gap between the mathematics taught in school and the techno-mathematical literacies (Bakker et al., 2006) required in everyday life and employment (Ruthven, 2016).

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Notes
1. An earlier version of these sections has appeared in conference proceedings (Ruthven, 2017).
2. I have wavered between ‘ecological’ and ‘ergonomic’, using the former in earlier versions. On balance, however, I now think that the latter is preferable as, in its strict definition, it more clearly demarcates the perspective.
3. In particular, since 2013, the English National Curriculum has had an appendix setting out “Examples of formal written methods for addition, subtraction, multiplication and division” (Department for Education, 2013).
4. Both studies involve volunteer samples of mathematics teachers which may not be representative. Nevertheless, the results which emerge from them can be taken as indicative at least of the experience of the many mathematics teachers participating.

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