Comparison of tabular and numerical methods of designing regulators for control of objects with delay

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Abstract. Controlling an object with delay remains an actual task, since each object has a delay. If it is not introduced into the model, it means not its absence, but only that the delay is not the main factor that forms the transition process. However, if it is not taken into account, this can lead to a discrepancy between the results obtained in theory or in simulation with practical results. In work [1] it was suggested to use the same model of objects to compare the design methods of regulators, which will allow ensuring the objectivity of comparing different design methods. In particular, in [2] the same problem is solved in three ways, and by the result conclusions are drawn about the advantages of some methods over others. Unfortunately, in this article [2], not all methods are applied. In particular, the method of numerical optimization is not investigated [3–5]. This paper fills this gap.

1. Introduction
The paper [1] compares the results of applying various methods for calculating the regulator for an object with a transfer function of the following form:

\[ W(s) = \frac{\exp(-55s)}{4.7s + 1}. \]  \hspace{1cm} (1)

For the design of the regulator, the following methods are used: a) Ziegler-Nichols method; b) the Chen-Hrones-Resvik method; c) the Cohen-Kuhn method. Although the paper does not contain results in the form of the desired PID-regulator coefficients, it gives graphs of transient processes in the form of a response to a stepwise input action. These graphics are reproduced below in Fig. 1-3, since without their consideration further conclusions will not be clear. If the authors would give the results in the form of specific numerical values, we could construct these graphs by mathematical modeling in single axes, which would facilitate their comparison, but in the absence of these data, we are forced to use a graphical citation.

Consideration of these transient processes allows us to draw the following conclusions:
1. In the best case, the transient process is characterized by overshooting of more than 10%, in which case the duration of the transient process is about 270 s.
2. In the worst case, overshooting exceeds 40%, the process duration is more than 1000 s.
3. The intermediate variant is characterized by over-regulation of more than 10%, followed by a deviation in the opposite direction by 20%; the duration of the transient is about 600 s.

Thus, the obtained results demonstrate the impossibility of controlling the object (1) without overshooting; the minimum overshooting is 10%. The minimum control time is 270 s. The robustness of the received solution is not investigated in the paper [1]. If the problem solved in [1] has an applied meaning, then the results that allow reducing the overshooting, or shortening the duration of the transient process, and even less to reduce both, should be useful.

In this paper, the goal is to improve these results for both indicators simultaneously.
2. The Method of numerical optimization

The method of numerical optimization is described in [3–8]. To optimize the regulator, it is required to apply a structure containing the system model and a number of auxiliary modules. The model of the system includes a regulator and an object, as well as other specific blocks.

In addition to the system model, the structure should contain:

1. Means of formation of test signals.
2. Means of indicating the results of optimization.
3. Means of calculating the value function.
4. Means of formation of initial values of parameters.
5. Optimization tool.

The possible structure for optimization is shown in Fig. 4.
Figure 5 shows this structure for the specific case of optimization of the PID-regulator. The cost function is calculated by the cost function estimator. This device performs, firstly, the calculation of the integral of the error module multiplied by the time, and secondly, by taking the integral also from the output of the error growth detector. The error growth detector multiplies the error by its derivative with the following restriction from below (the level of the constraint is zero) and then integrating the result of this multiplication.

The optimization result is shown as the values of the PID-regulator coefficients. The resulting transient process is shown in Fig. 6.

Consideration of this process shows that overshooting can be reduced to zero, and the duration of the transient process can be reduced to 200 s, both these results being obtained simultaneously, that is, with the same controller settings.

![Figure 5. Structure for optimization of the regulator for a system with an object (1) using the numerical optimization method.](image)

![Figure 6. The graph of the transient process in the system with the object (1) when using a regulator calculated using the numerical optimization method using the structure of Fig. 4.](image)

3. Check for the System Robustness

The result obtained with the help of the structure shown in Fig. 5, can be investigated for robustness. To do this, a developer can change one of the time constants of the object model, or both, by a small amount. The change in these values by 1% does not significantly change the transient processes, so the result can be considered robust.

It can be assumed that the model of object (1) is not complete, that is, identification did not take into account some inertial links of higher order with a smaller time constant. In particular, we can assume that the true transfer function of the object has the following form:

$$W_1(s) = \frac{\exp(-55s)}{4.7s + 1} \cdot \frac{1}{k \cdot 0.47s + 1}. \quad (2)$$
Here we can put \( k = 1 \) or more. Fig. 7 shows the transient process for the cases \( k = 1, k = 2 \) and so on for \( n = 1 \). Also, for \( k = 0 \), we can assume that \( n = 1.1, n = 1.2 \) and so on. The results are shown in Fig. 8. It can also be assumed that \( k > 0, n > 1 \). The results are shown in Fig. 9.

Thus, it can be argued that the calculated regulator forms a robust system, and even if any of the time constants deviates by 20% (or less), the overshoot does not exceed 10%, and the duration of the transient process does not exceed 300 s. If both time constants are rejected by 10% simultaneously, the overshooting also does not exceed 10%, the duration of the transient process does not exceed 420 s. The system remains stable even if both time constants are increased by 20%.

4. Research of the System Behavior with an Unaccounted Oscillation Link

We can investigate the behavior of the system in the event that the object model contains an unaccounted oscillation link. Consider the transfer function of the following form:

\[
W_c(s) = \frac{\exp(-55s)}{4.7s+1} \cdot \frac{1}{s^2 + 0.05s + 1}. \tag{3}
\]
Fig. 10 shows a transient process in the system with such object under the condition of using of the regulator calculated from a model of the form (1). It can be seen that stability is destroyed in the system due to the occurrence of high-frequency oscillations at the resonant frequency of the oscillation link. These oscillations increase in amplitude as the transient process develops.

To eliminate the oscillation, it is sufficient to perform numerical optimization of the regulator using the refined model (3). The transient process in the system with the refined regulator is shown in Fig. 11. It can be seen that although residual oscillations are present, they are limited in magnitude and amount to about 1% of the output value.

Thus, it was demonstrated that the regulator design method, which is not taken into account by the authors of [1], is much more effective than the best method considered in this paper.

5. Forced Limitation of the Frequency Area of Model Used in the Optimization of the Regulator

The tools of methods of numerical optimization are expanded by publications [3–5]. Let's turn to the patent [5]. Its essence lies in the fact that it is proposed to artificially decrease the speed of an object by numerical optimization by introducing a pure delay element into it, if it is not available in the model. If there is a delay link in the initial model, then the time constant of this link increases by some values, for example, by 10%. In particular, optimization was performed with an increase in the time constant of the delay link in the initial model of the object (1) by 10%.

This allows forcefully reducing of the frequency range obtained as a result of optimization of the system with increasing of the margin of its stability. Fig. 12 shows the corresponding modified structure for this purpose to optimize the regulator. Its difference from the structure shown in Fig. 1 is that an additional delay link is introduced into the object model. The obtained regulator gives a system with transient processes shown in Fig. 13 with a line 2. A process with an object in which the time constant has an initial value is shown on the same Fig. 13 by line 1, and the process is shown by a line 3 is when this constant is changed by 20%. It can be seen that the system remains stable and of good enough quality (overshooting does not exceed 5%) for all specified values of the time constant of the delay link.
Conclusions
An analysis of all the transient processes, which were obtained even with a significant deviation of the actual model of the object from the one used in numerical optimization, has shown that the numerical optimization method is a highly efficient method of regulator design. Comparison of the graphs in Fig. 6–9, 11 and 13 with the graphs in Fig. 1–3 convincingly proves this.

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