Does the wavefunction describe individual systems?

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We analyze the issue of the interpretation of the wavefunction, namely whether it should be interpreted as describing individual systems or ensembles of identically prepared systems. We propose an ideal experiment, leaning on the above results of either of them is measured along an axis $\vec{n}$. When the two particles are far apart, and the spin system composed of two spin 1/2 particles forming a singlet, let us consider the paradigmatic example of an entangled system: a wavefunction describes individual systems. Let us agree with their conclusions, however, one must accept their definition of objectivity, which is not susceptible of experimental resolution. In this note we propose an experiment which can decide the issue, based on the simultaneous measurement of the same observable with different detectors, and we discuss the theoretical implications of the possible experimental outcomes.

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity.

In this paper, we shall introduce a weaker definition of objectivity, which we hope may be widely accepted:

If, by simultaneously measuring the same physical quantity of a system by means of several detectors, the outcomes of the latter agree, then the value of that physical quantity is an objective property of the system.

The ensemble interpretation has been held by a minority of the physics community, a minority that is now growing in size, at least in our perception. The dichotomy between the wavefunction describing ensembles or individual systems is believed to be a metaphysical issue, not susceptible of experimental resolution. In this note we propose an ideal experiment, leaning on the above definition of objectivity, which may provide the answer.

First, we explain why the issue at hand is of great interest. Let us assume, for the moment being, that the wavefunction describes individual systems. Let us consider the paradigmatic example of an entangled system: a system composed of two spin 1/2 particles forming a singlet. When the two particles are far apart, and the spin of either of them is measured along an axis $\vec{n}$, giving an outcome $\pm \hbar/2$, then the wavefunction of the other particle instantaneously collapses into the state $|+\vec{n}\hbar/2\rangle$. Since we are assuming that the wavefunction describes individual systems, and thus possesses a physical reality, this is an instance of non-locality. From this point of view, Quantum Mechanics is a mysterious and surprising theory.

On the other hand, if one interprets the wavefunction as describing ensembles, the collapse of the second particles’ state means that, if one measures the spin of the second particle along a direction $\vec{n}'$, repeating the measurement many times, the frequency with which the outcomes of the two measurements agree tends to $P_{ag} = (1 + \vec{n} \cdot \vec{n}')/2$. Much of the surprise fades away, since the wavefunction appears now not as an objective property, but as a computational tool. The mystery, however, remains, since Bell has proved that the probability above cannot be reproduced by a local hidden variable theory (hidden variable theories attempt to describe individual systems by adding parameters having a stochastic distribution that is adjustable in order to match the predictions of Quantum Mechanics). Therefore, if one holds the view that Quantum Mechanics is incomplete, he must accept that a complete theory compatible with Quantum Mechanics must be nonlocal.

The usual discussion of measurement involves a single detector measuring at different times an ensemble of identically prepared systems. We consider instead an ensemble of detectors simultaneously measuring the same individual system. For illustrative purposes, we consider a two-level system. Let $|0\rangle$ and $|1\rangle$ be the eigenstates of the measured quantity. Let there be $N$ detectors coupled to the system, which is prepared in the state

$$c_0|0\rangle + c_1|1\rangle.$$ (1)

The question we ask is: What will be the output of the detectors in every single trial? We envision two possible scenarios compatible with Quantum Mechanics. In the first scenario, all the detectors give the same output, either 0 or 1. Upon repeating the measurement $M$ times on identically prepared systems, the detectors will all indicate 0 a number of times $M_0 \sim |c_0|^2 M$, and 1 a number of times $M_1 = M - M_0 \sim |c_1|^2 M$ (the approximate equality accounts for an expected spread of $\sqrt{M}$ of the binomial distribution). In the second scenario, in each individual trial a fraction $N_0 \sim |c_0|^2 N$ of the detectors will indicate 0, and the remaining $N_1 = N - N_0 \sim |c_1|^2 N$ will indicate 1. More precisely, the probability that $N_0$ detectors give 0 is

$$P(N_0) = \binom{N}{N_0} |c_0|^{2N_0} |c_1|^{2(N-N_0)}.$$ (2)
This probability can be recovered by repeating the measurement several times. There is also a third scenario that we shall not consider: in each trial, the detectors will give different outcomes, however with a probability $P_3(N_0)$ different from $P(N_0)$ of Eq. 2, having still the property that $\sum N_0 P_3(N_0) = |c_0|^2 N$. We could not figure out how this last scenario could be made compatible with Quantum Mechanics, thus we shall leave it out of our discussion.

If the experiment reveals that the first case occurs, we will have evidence that the wavefunction describes ensembles and not individual system. We would conclude that initially the detectors are at equilibrium position corresponding to the outcome $\sigma = 1$. If we include the influence of the environment in the time evolution, assuming that the measurement process is much larger than the relaxation time, we have that

\[ \rho_{\text{det}}(\tau) = |c_0|^2 \rho_{\text{det}}(0) + |c_1|^2 U_A U_B \rho_{\text{det}}(0) U_B^\dagger U_A^\dagger, \]  

where $U_a$ is the time evolution operator due to the Hamiltonian

\[ H^{(1)}_a = H_a + \lambda_a \hat{x}_a = \frac{\hat{p}_a^2}{2m_a} + \frac{1}{2} m_a \omega_a^2 (\hat{x}_a - X_a)^2 + \text{const}, \]

with $X_a = \lambda_a / m_a \omega_a^2$ the equilibrium position corresponding to the outcome $\sigma = 1$. We include the influence of the environment in the time evolution, assuming that the measurement time is much larger than the relaxation time, we have that

\[ \rho_{\text{det}}(\tau) = |c_0|^2 \rho_{A|0} \otimes \rho_{B|0} + |c_1|^2 \rho_{A|1} \otimes \rho_{B|1} + O(e^{-\gamma \tau}), \]

where $\gamma$ is the relaxation rate, and $\rho_{A|\sigma} = \exp (-\beta H^{(\sigma)}_a) / \text{Tr} \exp (-\beta H^{(\sigma)}_a)$ is the density matrix of detector $a$ indicating the outcome $\sigma$. In order for the outcomes to be distinguishable from thermal noise we have to require that $X_a \gg \Delta x_a$, which ensures that the outcomes are also macroscopically distinguishable.

Thus, when two detectors are simultaneously measuring the same two-valued observable, both will indicate 0 with probability $|c_0|^2$ or 1 with probability $|c_1|^2$. The extension to observables having more than two values is trivial. We point out that the result is independent of the detection model. In order to have the detectors’ outcomes disagree with probability $2 |c_0 c_1|^2$, the reduced detector density matrix should read:

\[ \rho_{\text{det}} = |c_0|^4 \rho_{A|0} \otimes \rho_{B|0} + |c_1|^4 \rho_{A|1} \otimes \rho_{B|1} + |c_0 c_1|^2 \left( \rho_{A|0} \otimes \rho_{B|1} + \rho_{A|1} \otimes \rho_{B|0} \right). \]

Eq. 10 violates the linearity of the amplitude evolution, which implies that only second powers of $c_0, c_1$ appear in the evolution of the density matrix.

Next, we consider a weak measurement, as the one used in 3: two quantum point contacts, biased with potentials $V_A, V_B$, are coupled to a double quantum dot, having one excess electron. The electron being in the left (right) dot corresponds to the observable $\sigma$ having value 0 (1). When the electron is in the left dot, the transmission probabilities of one electron through the spin degenerate channel of the left or right QPC are

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\[ |c_0| |c_0|^\dagger + |c_1| |c_1|^\dagger. \]
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$T_{A|0}$ and $T_{B|0}$, respectively, while, when the electron is in the right dot, they are $T_{A|1} := T_{A|0} + \delta T_A > T_{A|0}$ and $T_{B|1} := T_{B|0} + \delta T_B < T_{B|0}$. The larger the coupling of the dot to the QPC’s, the larger the differences $T_{A|1} - T_{A|0}$ and $T_{B|0} - T_{B|1}$. The different values of the transmission probabilities are due to the coupling to the system

$$\hat{H}_{\text{int}} = \hat{\sigma} \left( \hat{V}_A + \hat{V}_B \right),$$

where $\hat{V}_A$ is a one-body operator on the detector’s degrees of freedom. As a consequence, for $\sigma = 0$ the transmission through the QPC is governed by a scattering matrix $S^{(0)}_\sigma$ corresponding to the Hamiltonian $\hat{H}^{(0)}_\sigma$, and, for $\sigma = 1$, by $S^{(1)}_\sigma$ corresponding to $\hat{H}^{(1)}_\sigma = \hat{H}^{(0)}_\sigma + \hat{V}_A$. If we write the scattering matrix as

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

we have that the transmission probabilities are $T = |t|^2$.

Following Reference \[4\], we consider the Full Counting Statistics of the two detectors. We have that the probability of getting a number $n_A$ of electrons transmitted in the left QPC and $n_B$ in the right one within a time $\tau$ is

$$P(n_A, n_B) = \sum_{\sigma} |c_\sigma|^2 P(n_A, n_B|\sigma),$$

where

$$P(n, n|\sigma) = P_A(n|\sigma)P_B(n|\sigma).$$

In the shot noise limit $k_B T \ll eV_A, eV_B$, the probability distribution for each QPC is simply the binomial distribution:

$$P_A(n|\sigma) = \binom{N_{\sigma}}{n} R_{a|\sigma}^{-n} T_{a|\sigma}^n.$$  

We put $R := 1 - T$ and $N_{\sigma} := 2eV_{\sigma}\tau/h$, then, for large observation time, the probability density $\Pi_a(I|\sigma)$ for the average current measured within $\tau$ is

$$\Pi_a(I|\sigma) \simeq \frac{1}{\sqrt{2\pi S_{a|\sigma}}} \exp \left\{ \frac{(I - I_{a|\sigma})^2}{2S_{a|\sigma}} \right\},$$

$\Pi_a(I|\sigma)$ has a peak at $I_{a|\sigma} = 2G_Q eV_a T_{a|\sigma}$ ($G_Q = e^2/h$ is the quantum of conductance and the factor of two accounts for spin degeneracy), taking the value $\Pi_a(I_{a|\sigma}|\sigma) \simeq \sqrt{\tau/2\pi S_{a|\sigma}}$ and a width $\Delta I_{a|\sigma} = \sqrt{S_{a|\sigma}/\tau}$, with the current noise $S_{a|\sigma} = 2G_Q eV_a R_{a|\sigma} T_{a|\sigma}$. If the observation time and the coupling with the double dot are such that $(S_{a|0} + S_{a|1})/\tau \ll (I_{a|0} - I_{a|1})^2$ the observation of current is sufficient to discriminate in which dot the electron is. The joint probability distribution for the two detectors has two peaks in the $I_A - I_B$ plane, at points $(I_{A|0}, I_{B|0})$ and $(I_{A|1}, I_{B|1})$, while it is negligible at $(I_{A|0}, I_{B|1})$ and $(I_{A|1}, I_{B|0})$. This means that it is practically impossible that the two detectors will give discarding outcomes. Furthermore, the negligible probability of this disagreement depends weakly on the state of the system, and it is to be attributed to the imperfection of the detection.

Thus, we come to the conclusion that Quantum Mechanics, consistently applied to system and detectors, supports the ensemble interpretation of the wavefunction. Only experiment, however, can settle the issue. We discuss the theoretical implications of the possible experimental outcomes.

**The experiments support the single-system interpretation of the wavefunction.**

This would contradict Quantum Mechanics. Since the latter has been confirmed in many experiments, and provides reliable predictions, it could be retained by restraining its domain of validity, i.e. we would be forced to conclude that Quantum Mechanics is non-universal, in the sense that it is unable to describe the measurement process. In order to account for the latter, rules external to Quantum Mechanics have to be invoked, consisting in a generalized projection postulate for simultaneous measurements.

**The experiments support the ensemble interpretation of the wavefunction.**

Then we could conclude that individual systems do possess objective properties, which are ascertained by measurement. We could also conclude that Quantum Mechanics is incomplete, in the sense that it does not describe these objective properties.

In conclusion, we have proved that Quantum Mechanics is either complete or universal, but not both, and we have provided a simple experimental proposal which may settle the issue.

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