Multiplexing schemes for an achromatic programmable diffractive lens

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Abstract. A multiplexed programmable diffractive lens, displayed on a pixelated liquid crystal device under broadband illumination, is proposed to compensate for the severe chromatic aberration that affects diffractive elements. The proposed lens is based on multiplexing a set of sublenses with a common focal length for different wavelengths. We consider different types of integration of the optical information (spatial only, temporal only and hybrid spatial-temporal) combined with a proper selection of the spectral bandwidth. The properties and limits of the achromatic programmable multiplexed lens are described. Experimental results are presented and discussed.

1. Introduction

Although diffractive lenses show good properties of large collecting aperture, light weight, ease of replication, flexible design to correct aberrations in comparison with conventional refractive lenses, their usefulness in broadband imaging systems is very little so far because of their severe chromatic aberration. The focal length of a phase diffractive lens is given by $f(\lambda) = \frac{\lambda_0}{\lambda} f_0$, where $\lambda$ is the illumination wavelength and $f_0$ is the focal length for the design wavelength $\lambda_0$. For broadband optical processors with high demands of compactness or for applications where there is no possibility to use a combination of spaced elements, diffractive and refractive, it is interesting to investigate how to obtain an achromatic phase Fresnel lens.

Electronically addressed spatial light modulators (SLM), such as liquid-crystal pixelated displays, have been effectively used to generate programmable diffractive optical components. Concerning phase Fresnel lenses, they are capable of dynamically changing their focal length by addressing the proper phase function onto a well characterized display. Under polychromatic illumination, as obtained from a white-light source, the performance of a diffractive lens encoded on SLM shows the expected severe chromatic aberration [1] that deteriorates quickly the image. The advantages of being programmable permit to compensate for this effect as well as to encode a variety of non-uniform amplitude transmission filters jointly with a lens onto a single SLM [2-4].

This work deals with a multiplexed programmable phase Fresnel lens displayed on a pixelated liquid crystal device working under broadband or white light illumination. The proposed lens is based on multiplexing a set of sublenses with a common focal length for different wavelengths. We consider different types of integration of the optical information (spatial only, hybrid spatial-temporal and temporal only) combined with a proper selection of the spectral bandwidth. We explore three possibilities to design the multichannel phase Fresnel lens [3,4]:

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- Spatial integration only: it uses a mosaic color filter against a multilens with mosaic aperture;
- Spatial and time integration: it uses a color filter and a multilens aperture, both divided into multiple circular sectors. Both the color filter and the multilens rotate synchronized.
- Time integration only: it uses a tunable spectral filter that transmits a temporal sequence of narrow bands centred at the selected wavelengths with respect to which the sublenses have been programmed on the SLM aperture.

2. Design of multiplexed Fresnel lens

Our method is based on designing a multichannel phase Fresnel lens that works nearly monochromatically in each channel and has a common focal point where the different focusing wavefronts add with temporally incoherent superposition. We design a set of \( N \) different phase Fresnel lenses \( L_i \), with \( i=1...N \), centered on a common optical axis, with their apertures placed at the same plane, and with the same focal length \( f_0 \) for the respective design wavelengths \( \lambda_i \). The lenses are then combined, or multiplexed, by carrying out some spatial integration, or time integration, or hybrid spatial-time integration, as we present in the following subsections. The mathematics of the procedure is described in Refs. 3-4.

Let us consider a converging lens \( L_i \), placed at the plane of rectangular coordinates \((x, y)\), with a quadratic phase function given by

\[
L_i(x,y) = \exp \left( -j \frac{\pi}{\lambda_i f_0} (x^2 + y^2) \right),
\]

where \( f_0 \) is the focal length and \( \lambda_i \) the design wavelength. Let us consider that this lens function is sampled with a sampling period given by the pixel space (or pixel pitch). The lens is displayed on a \( M \times M \) pixel array SLM, with square pixel pitch \( \Delta \) and fill factor less than unity. We assume that the lens pattern reaches, at most, the Nyquist frequency at the circular contour of the aperture and, consequently, no secondary lenses appear. This implies that the focal length \( f_0 \) has to be longer than or equal to the reference focal \( f_0 \) (called critical distance in Ref. 5) that depends on the sampling period \( \Delta \), the number of samples \( M \) and the wavelength \( \lambda_i \).

2.1. Spatial multiplexing by using a mosaic aperture

Let us define a set of \( N \) phase Fresnel lenses \( L_i, i=1...N \) that have the same focal length \( f_0 \) for the set of wavelengths \( \lambda_i \). We combine these \( L_i \) lenses in the same aperture by using a mosaic structure and analyze the polychromatic PSF of the resulting diffractive multichannel lens in the common focal plane. If the phase Fresnel lens function of Eq. (1) is displayed on the SLM (phase quantization effects are not considered in this work) and a uniform plane wave of \( \lambda_i \) impinges the aperture, the amplitude distribution behind the lens is

\[
T_i(x,y) = \left( L_i(x,y) M_i(x,y) \right) \otimes \text{rect} \left( \frac{x}{\Delta x'}, \frac{y}{\Delta y'} \right),
\]

where symbol \( \otimes \) indicates convolution and the mosaic sampling function for the wavelength \( \lambda_i \) is

\[
M \left( \lambda_i, x,y \right) = M_i \left( x,y \right) = \tau_i \left( \Delta \lambda \right) \text{circ} \left( \frac{1}{R_i} \left( x^2 + y^2 \right)^{1/2} \right) \sum_{n,m} \delta \left( x - \left[ n \Delta l + a \right], y - \left[ m \Delta s + b \right] \right).
\]

In Eq. (3) \( \tau_i \left( \Delta \lambda \right) \) is the amplitude transmittance of the quasimonochromatic filter that selects a narrow bandwidth centred in \( \lambda_i \) (consequently, \( \tau_i \left( \Delta \lambda \right) \approx 0 \) except for \( \Delta \lambda = \left| \lambda - \lambda_i \right| \approx 0 \)); the circ function...
corresponds to a circular pupil of radius $R_0$ (with $R_0 \leq R = M\Delta/2$) and the summation corresponds to a 2D-comb function that establishes the positions of the sampling points.

The mosaic filter of Eq. (3) and Fig. 1(a) originates from a basic pattern that replicates throughout the filter aperture. We assume that this basic pattern consists of $N$ similar elements of size $\Delta \times \Delta$ that coincides with the SLM pixel size. The grid of the mosaic color filter is assumed to perfectly match the SLM grid. The basic pattern has rectangular dimensions $\Delta l \times \Delta s$ with $\Delta l/\Delta s = N\Delta^2$. The point of coordinates $(a_i, b_i)$ gives the position of the $i$-cell containing the $\lambda_i$-quasimonochromatic filter inside the basic pattern.

We are interested in the maximum of the central order of the amplitude distribution of light in the focal plane [6]. Calculating the Fresnel propagation by following an analysis analogous to that carried out in Refs. [3,7], and neglecting the slow varying phase terms, we obtain

$$U_{00}(u,v) = \frac{\tau_{i}(\Delta \lambda ) \pi d_i R_i}{\Delta l \Delta s} \left[ \frac{J_1 \left( 2\pi d_i \left( u^2 + v^2 \right)^{1/2} \right)}{2\pi d_i \left( u^2 + v^2 \right)^{1/2}} \right] \otimes \text{rect} \left( \frac{u}{\Delta u' \Delta v'}, \frac{v}{\Delta u' \Delta v'} \right),$$

where $d_i = R_i/\lambda_i f_0$. Eq. (4) is the convolution of a wavelength dependent term by a wavelength independent rectangle function. The variation of the central lobe of $U_{00}$ with $\lambda_i$ can therefore be analyzed through the variation of the first term of Eq. (4) with $\lambda_i$. The width of the central lobe of the Bessel $J_1$ function in Eq. (4) is $1.22 \lambda_i f_0 / R_i$ and its height is weighted by the precedent factor. Since the focal plane is the same for all $\lambda_i$, the solution given by Eq. 4 shows compensation for longitudinal chromatic aberration. The central lobe of $U_{00}$ shows different sizes and, as a result, transversal chromatism is produced unless the condition $d_i = \text{constant}$ or, equivalently, $R_i/\lambda_i = \text{constant}$ (PSF$_S$-condition), is fulfilled. On the other hand, the central order focalization of Eq.(4) shows different maximum intensity with wavelength unless the multiplicative factor meets the condition $\tau_{i}(\Delta \lambda ) d_i R_i = \text{constant}$ or, equivalently, $\tau_{i}(\Delta \lambda ) R_i^2 / \lambda_i = \text{constant}$ (PSF$_T$-condition) (Fig. 2).

**Figure 1.** (a) Spatial multiplexing (mosaic). (b) Hybrid spatial-time multiplexing.
2.2. Hybrid spatial-temporal multiplexing by using rotating multisector aperture

A rotating multisector color filter is placed against the SLM. We multiplexe the \( N \) phase Fresnel lenses \( L_i \) by considering circular sectors in the aperture plane. The focal length of design \( f_i \) is common for all the wavelengths. Similarly to the mosaic case, the amplitude distribution behind the lens is

\[
T_i(x, y) = \left[ Q_i(x, y) \sum_{n,m} \delta(x - n\Delta, y - m\Delta) \right] \circ \text{rect} \left( \frac{x}{\Delta x'}, \frac{y}{\Delta y'} \right). \tag{5}
\]

Function \( Q_i(x, y) \) represents a multicolor filter consisting of four sectors with equal angular amplitude \( A = \pi/2 \) and radius of at least \( R = M\Delta/2 \) to cover the SLM size (Fig. 1(b)). The lens \( L_i \) is centered in the optical axis, limited by a quadrant shaped pupil, and has a radial extension of \( R_i \leq R \).

The sector defined by \( R_i \leq r \leq R \) is left blank, or equivalently, it introduces a constant phase \( \phi \). The whole element could be mathematically described by

\[
Q_i(x, y) = \tau_i(\Delta\lambda) \left\{ L_i(x, y) \circ \text{circ} \left( \frac{r}{R_i} \right) + \text{circ} \left( \frac{r}{R} \right) - \text{circ} \left( \frac{r}{R_i} \right) \right\} \exp \{ j\phi \} H \left( -1 \right)^{1-x} x, (-1)^{\left( \frac{i-1}{2} \right)} y, \tag{6}
\]

where \( H(x, y) \) is the 2D Heaviside step function and \( 1(\cdot) \) is the integer part of the argument. All the SLM pixels belonging to a circular sector display a single lens function \( L_i \), which is now sampled with period \( \Delta \) (pixel pitch) inside the sector.

Using Fresnel propagation, we calculate the PSF corresponding to sublenses \( L_i \) in the focal plane (Fig. 3a). Their lack of circular symmetry is compensated when the filter and the multilens rotate around the optical axis with synchronism (Fig. 3b). For a better comparison with the results obtained for the multilenses with mosaic aperture (Fig. 2) we also show the time-average cross-section intensity profiles of the central order focalization for the lens with the same focal length \( f_o \) but with rotating aperture (Fig. 3c).

The multichannel phase Fresnel lens based on a mosaic aperture scheme produces smaller PSF sizes than the rotating aperture scheme, contributing to a better resolution in an imaging system. On the other hand, the first diffraction orders are closer to the central order, thus limiting the extension of the image field more than the rotating scheme [3]. The rotating aperture has higher range of programmable focal lengths. But, from the point of view of optomechanical requirements, the rotating aperture scheme could be more complex to implement than the static mosaic aperture.
2.3. Temporal multiplexing

Time multiplexing of sublenses can be achieved by using a white light source and a tunable spectral filter that transmits a sequence of narrowbands centred at the selected wavelengths with respect to which the sublenses have been programmed. A liquid crystal tunable filter is like an interference filter, but the wavelengths of the light it transmits are electronically controllable. The required synchronism of both the tunable filter and the phase distribution displayed by the SLM can be achieved by computer. All the channels or sublenses need to be displayed within the integration time of the sensor, otherwise chromatic distortions could be noticed and the whole system would not be completely effective. Experimental results of time multiplexing scheme are shown in Fig. 4. Longitudinal as well as transversal chromatic aberrations are compensated.

**Figure 3.** (a) Intensity of the PSF of $T_r(x,y)$, in the top right sector (Fig. 1b). The lack of circular symmetry is compensated when $W_{01}(u,v)$ rotates around the optical axis (b). (c) PSF comparison.

**Figure 4.** Time multiplexing scheme. Experimental compensation of transversal (bottom left) and longitudinal (bottom right) chromatic aberration for $\lambda_i=650, 600, 550, 500\text{nm}$. 
3. Conclusions
We have proposed three different types of lens multiplexing for a set of phase Fresnel sublenses and have outlined their practical advantages and disadvantages. In all the three multiplexing schemes, each sublens works almost monochromatically and, consequently, the system benefits from a common focal plane. For both the mosaic and the rotating aperture multiplexing schemes, the simulated results show a significant improvement in achieving compensation of both transversal and longitudinal chromatic aberrations. Both schemes present technical difficulties that can be overcome using time integration.

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