1. Introduction

The Preisach model [1] is very suitable for a complete description of the hysteresis. Because of the hysteresis, the model must be two-dimensional (2D). The Preisach model uses fictive hysterons, exhibiting the rectangular hysteresis loop, and arranges them systematically in the triangle grid; see the last chapter of the paper for some details. The simulated magnetic material is characterized by its weighting function. Although its analytical description is possible, in practice, the digital form is used. If the weighting function is well-known, the material can be analyzed by numerical methods efficiently and correctly. There are two basic ways for finding this function: using an estimated analytical form, or determining it from an experiment [2]. The analytical estimation has several serious limitations. Therefore, theoretically, the best choice is to use the experiment. The experiment requires a measurement of a lot of elementary hysteresis loops, starting from the negative saturation. In practice, the experimentally accessible negative level of excitation is used. The excitation magnetic field has a harmonic waveform with negative bias and its amplitude gradually increases. The decreasing branches of the loops are used to form the 2D Everett surface [1]. Usually, they are termed the first order reverse curves (FORC). The weighting function is given by partial derivations of the Everett surface both horizontally and vertically. Due to the experimental inaccuracy the derivation exhibits serious errors [3]. The accuracy of the weighting function and the Preisach model application is a subject of the paper.

2. Measurement in the time domain

The current source was used to ensure the harmonic field strength excitation. The standard apparatus, described in [4] was used. The measured material was a grain oriented steel with 4% of silicon that was used to create the toroid transformer. The core of the transformer is 24.3 mm high, inner diameter was 125 mm, outer diameter was 135 mm and was used 500 turns for both the primary and the secondary winding. The well-defined starting level was the negative saturation. The time varying field strength for partial loop is \( H_d \), while its systematically increasing amplitude (up to the positive saturation) is \( H_u \). Only decreasing part of the loops (FORC) is used to form the Everett surface. Since \( H_d \leq H_u \), the Everett surface is defined on the Preisach triangle. Its horizontal axis is \( H_d \), while the vertical one is \( H_u \). The amplitude \( H_d \) should increase by the smallest possible steps because of precise determination of values, especially in the neighborhood of the weighting function main peak, typical for magnetically soft materials. Basic experimental limitation of the procedure is demonstrated in Fig. 1. The total of 1800 loops were measured. The illustrative choice of the loops in the time domain is given in Fig. 1. The increment of four was used to improve the curve visibility. The excitation is in the upper graphs, in the lower one is the response. In the left hand graphs of Fig. 1, there are wave parts near the maximum excitation, which is the time \( t = T/2 \). In the right hand graphs the situation near the excitation zero crossing at time \( t = (3/4)T \) is shown. Symbol \( T \) is used for the period of excitation. For each curve set (both excitation and response) the points in its central part are taken and presented in the summarizing graphs on its right hand side. The curve number is on the horizontal axis of the graph. These graphs allow to judge the measurement quality of both the excitation and the response. For the excitation at FORC starting point (the maximum of excitation), the excitation curves are well defined, the amplitude increases from the expected linear shape (upper right hand graph of Fig. 1). The excitation, therefore, exhibits a high quality.
Fig. 1. Excitation and response in the time domain for the maximum excitation and points where the excitation crosses the time axis.

The response is measured with a lower accuracy. The curve numbers are preserved at FORC start and the deviation from the linear course is acceptable. Although, at the response in the FORC centre, three quarters of period, the monotonic increase does not exist. It means that the impossible negative derivation with respect to $H_u$ takes a place. The selected excitation and response curves originate at the area of rapidly increasing flux density. Therefore, the distance between response curves is relatively high. Near the saturation, the curve disorder is much higher and appears also in the start of FORC. Several cuts of the Everett surface are in Fig. 2. The selected area is close to the central part of the hysteresis loop. The change speed of flux density reaches maximum in this area. The cut curves along the rows in Fig. 2a are flat and increasing. Therefore, the partial derivation with respect to $H_d$ will be positive. The decrease followed by constant value at the end part of several curves in Fig. 2a was explained in [4]. The vertical cuts, in Fig. 2b, behave quite differently. In this case, the magnetic field strength $H_u$ is a variable and $H_d$ is a parameter. In this case noise is superposed to the expected flat and increasing shapes. Although the noise is small, the derivation with respect to $H_u$ will change the sign, giving incorrect values of the weighting function. It should be also stressed that there is different number of points forming the curves. In Fig. 2a each curve contains million of points, because the sample rate was 500 kHz and the sample time was 2 s. While curves in Fig. 2b contain only 1800 points, that is restricted by the minimum step of the power supply current magnitude. Other reasons are current accuracy, stability, noise from the current source, random effects in a material, and total time of measurement. Therefore, we achieved almost the practical limit.

The noise is caused by the incorrect order of the response curves as it is explained in Fig. 1. The main problem is that the curves cannot be shifted to the correct position. Their orders change along the curves.

The result of numeric derivation of the Everett surface, weighting function, is given in Fig. 3. It was obtained using several corrections of data. The only cut at the vertical plane in $H_u = -5 \text{ A/m}$ is shown for clarity.
Experimental Determination of the Preisach Model for Grain Oriented Steel

3. Reduction of weighting function

Response curve disorder causes errors in the weighting function determination. The best solution should be to increase the experiment accuracy. It is a very difficult task. Furthermore, the disorder may be due to the material itself, at least partially. Therefore, another way of experiment and data processing should be used.

The simplest way is a change of the experiment. The step of the magnetic excitation field increase is constant. Due to the strong nonlinearity, the increase step of flux density will differ strongly. In the central part the step will be large, while near the saturation it will be small. The reason is that the response curve order is not kept. The better experiment should use almost constant steps of the flux density. In this case the grid for the Everett surface should be also non-uniform and the weighting function will be reduced. We simulated the effect of reduction of the weighting function. In the central part, near the maximum, the grid is not changed, but at the distant parts, every second row and column are withdrawn. The grid is sketched in Fig. 4. Of course, in practice, the number of rows and columns is significantly higher.

Hysteresis loops reconstructed from the reduced weighting function are in Fig. 5. The loop shape is not considerably changed by the grid reduction, except for quantization. The inset shows that the reduction causes major deviations of steps in the central part. According to the second inset of Fig. 5, in the area of saturation, the reduction effect is negligible. The legend contains the number of nodes in the Preisach triangle. The steps are made predominantly by the low number of nodes and not by the row and column reduction.
4. Proposal of an ideal non-uniform excitation

Usually a constant step in the excitation field is used as the simplest way. But the corresponding flux density that responds the excitation steps vary in a large extent. Therefore an approximately constant step in response is the best solution [6]. In the design of variable steps in excitation, we use the approximation of the major hysteresis loop.

There are a lot of approximations of the hysteresis loop branches: functions arctan, arctanh, erf, step response, etc. We used the simplest function arctan with several variable parameters for the approximation of the flux density in one branch of the hysteresis loop

$$B_a(H) = A \arctan(k(H - H_0)) + cH,$$

where $A$ is the amplitude of approximation that defines the limit values, $k$ is the coefficient of argument that determines the slope of approximation especially in the zero area, and $H_0$ is the coercitive magnetic field strength of the selected branch that is either positive or negative. The linear term with constant $c$ is added, since the saturation does not appear practically. Since the excitation is symmetric, the response given by (1) is symmetric, too. Therefore, absolute term in (1) is zero and omitted. All the four parameters were found by the use of fminsearch function in MATLAB. This function is called many times with different initial values of parameters. As the optimum, the set of parameters with minimum deviation from experiment is selected.

The approximation is presented in Fig. 6. The loop in Fig. 6a is well approximated. For a better evaluation of the approximation, the increasing part of the loop together with experimental points is in Fig. 6b. The accordance is good with an exception of the initial part. Only the positive part is shown in graph for clarity. The deviation of experimental data and the approximation is less than about 20 mT with an exception of the area near the zero excitation. Therefore, in practice, the approximation is quite acceptable. The current, necessary for excitation of the hysteresis loop with the constant step in the flux density magnitude of 38 mT, is given by the inverse function to (1), practically by tangent function. The total scale is in Fig. 7a. The very large step of more than 2 A is in the initial part, at the half time of period, and at the end of period. Otherwise, the current change is small, usually several mA or less. The current difference of flat areas is the effect of hysteresis. Figure 7b shows the detail of the current near the half of period, where the abrupt current change takes place. In this critical part the current change is several mA only. In practice, a big current change will be impossible experimentally and could lead to other negative effects. In the saturation area, a smaller flux density step should be projected.

![Image](image_url)

Fig. 5. Hysteresis loops by the Preisach model with a strong reduction of number of hysterons (rows and columns).

Fig. 6. Approximation of a maximum hysteresis loop by arctan function in the whole positive range (a), and the detail (b) with experimental points.
5. Model calculation

The important part of the model processing is to calculate the response from the model for any excitation. The geometrical presentation of the Preisach model is used in this case. Basics are shown in Fig. 8. The hysteron in Fig. 8a has a rectangular loop with unit magnetic momentum and switching levels $H_u$ and $H_d$. In the model, the hysterons are systematically arranged according to switching levels. The principle of the model is sketched in Fig. 8b. It contains a lot of hysterons, hundreds in the row. The effect of external field consists of switching the hysterons into a positive or a negative magnetization $M$. When the excitation field increases, its level in the model moves from the bottom to the top. All the hysterons under the excitation level are switched up to the positive magnetization $+M$, all other remain unchanged, as Fig. 9a explains. In the case of the excitation decrease, the corresponding level is vertical and moves from the right to the left, see Fig. 9b. In the figure the previous increasing excitation remains. All the hysterons on the right hand side of excitation level are switched down, and the magnetic momentum of all other remains unchanged. Now it is clear that 2D model provides the hysteresis, while the 1D model can be used only for modeling nonlinearity.
The setting of the hysteron momentum and their summing can be made by a very simple function in MATLAB. But the simplicity is connected with a higher computation time, since the function is used for each excitation level. However, only the level that meets the row or column with hysterons makes the change of the total momentum. Since the constant time increment is used for excitation, the inactive levels exist for both the directions of excitation, see Fig. 10. The case in Fig. 10b corresponds to the harmonic excitation near its maximum or minimum. Ignoring the inactive levels increases the computation speed several times.

6. Discussion

The weighting function is the complete characterization of a material when using the Preisach model. Therefore, it has a key meaning in the model application and should be found with a maximum accuracy. There are two basic ways of its determination: analytical approximation and experimental determination. The analytical approximation uses the fact that the weighting function exhibits sharp maximum for soft magnetic materials. The 2D analytical functions with their features are used, usually in combination. The unknown coefficients are found numerically by the best fit of approximation and experiment. The advantage of the method is that only one hysteresis loop for the maximum excitation is sufficient. That is why an experiment is simpler and faster. However, for lower excitations the accordance of approximation and experiment can decrease.

The experimental method needs to form the Everett surface by a complex experiment. Therefore, all the material information is included in the Everett surface. The disadvantage is that the numeric derivation must be used to get the weighting function from the Everett surface. Unfortunately, the derivation enhances experimental errors that were shown for the case of cuts of the Everett surface by several values of \( H_u \) in Fig. 2b, for instance. There are several sources of the curve imperfection. The integration is used to get the flux density from the secondary voltage. In principle, this method reduces the errors. On the other hand, a small positive or negative constant exists that shifts the resulting curve monotonically up or down. It is well-known as the drift in practice. The sign and value of the constant can change during the experiment that leads to a curve disorder. The improvement of integration is a very complicated technical task. A simple way of drift reduction is to decrease the period. However, the frequency increase leads to the increase of eddy currents. The elimination of the eddy current effect in the loop is almost impossible. The frequency of 1 Hz or little less was used as the limit. Probably, the material feature also can take place in errors. Existing thermal effects can slowly change magnetic momentum of the magnetic domain. Also a small change of the flux density for a constant magnetic field strength is reported and it is known as the after-effect [7, 8]. Unfortunately, it is a very difficult task to find whether the complication is of experimental nature or a material feature, or both of them with different contribution.

Since the calculation is numeric, the steps of response corresponding to the steps of excitation appear at the output in every case. The simulation showed that the grid reduction leads only to the increase of the response steps. The average curve shape is preserved. The steps are highest in the area of the abrupt change of the flux density.

An ideal case is the use of an excitation that leads to approximately constant steps of the flux density. Such theoretical excitation was found, however, there is a question if it can be realized experimentally for all its points.

7. Conclusion

Experimental inaccuracy limits the number of points in the Preisach triangle. The distance between rows and columns should be greater than the estimated experimental error. Fortunately, the simulation confirms that this limitation does not considerably affect the Preisach model prediction.

The excitation for the constant step of the flux density magnitude was found. The speed of repeated calculations can increase by two orders at the expense of lower algorithm robustness.
Acknowledgments

The result of the work was obtained thanks to the financial support the European Union within the scope of the project “Modular platform for autonomous chassis of specialized electric vehicles for freight and equipment transportation”, Reg. No. CZ.02.1.01/0.0/0.0/16_025/0007293.

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