The nonlinear deflection response of CNT/nanoclay reinforced polymer hybrid composite plate under different loading conditions

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Abstract. The carbon nanotubes (CNTs) are used as significant reinforcement materials in the advanced composites due to their improved mechanical properties. The carbon nanotubes reinforced nanocomposites have improved mechanical strength, and stiffness properties in addition to their reduced weight. Only a few percentages of CNTs (2–5% by weight) could be added to these nanocomposites as more volume fraction will deteriorate their mechanical properties. A computational approach is used in the present investigation to find the mechanical properties of the hybrid nanocomposite which is modeled by adding nanoclay particles into the conventional CNTs reinforced epoxy system. Using Halpin-Tsai approach (which is based on the micromechanical modeling) the mechanical properties of hybrid nanocomposites are evaluated. The effect of the CNTs and the nanoclay particles on the nonlinear transverse central deflection response of nanocomposite plate under different loading conditions is studied in detail. The simulation results of present investigation have been validated with the data sets of the deflection responses available in the references.

1. Introduction

The investigations about the material properties of CNTs are making way to variation of the elasticity modulus within 200 GPa to 5.6 TPa due to the number of the CNT walls, length, diameter, etc. Nowadays, the carbon nanotubes (CNTs) are in limelight due to their improved physical and chemical features [1]. The increase in CNTs in the matrix could enhance the required mechanical properties of the smart CNTs reinforced composites. It is important to note that increase in CNT volume fractions will lead to improvement in stiffness of CNTs reinforced polymer composites. On the contrary, the experimental data however, advocates about the reduction in stiffness after addition of CNTs volume fractions beyond a certain limit in the matrix [2]. It is due to the agglomeration of CNTs within the matrix material. In the present investigation, we aim to investigate the elastic nonlinear deflection response of the hybrid nanocomposite plate under different loading conditions using the shear deformation theory based on the secant function (SFSDT) [1, 2].
2. Modeling of the hybrid nanocomposite plate
The modeling of the CNTs/nanoclay based nanocomposite plate is proposed in two stages such as micromechanical modelling and numerical modelling using finite element method (FEM) [1]. The elastic properties of effective CNT fibers and Clay particles including CNTs, and Clay interphases are computed using modified Halpin–Tsai model as shown in the set of equations (1).

\[
\begin{align*}
\frac{P}{P_m} &= \frac{1 + (\phi_1 \eta_{CP} V_{CP} + \phi_2 \eta_{CF} V_{CF})}{1 - (\eta_{CP} V_{CP} + \eta_{CF} V_{CF})}, \\
\eta_{CP} &= \frac{(P_{CP} / P_m) - 1}{(P_{CP} / P_m) + \epsilon_1}, \\
\eta_{CF} &= \frac{(P_{CF} / P_m) - 1}{(P_{CF} / P_m) + \epsilon_2}, \\
V_m + V_{CP} + V_{CF} &= 1
\end{align*}
\]

where, \(\epsilon_1 = \frac{2l}{d_p}\), \(\phi_2 = \frac{2l}{d}\) and \(\phi_1 = \sqrt{3} \log\left(\frac{l}{d_p}\right)\). \(\epsilon_1 = 1\) are used for the longitudinal and transverse moduli of elasticity, respectively. The symbols “l” and “d” are used to represent the length and diameter of the CNTs. In the above equation (1), \(V_m, V_{CP}, \) and \(V_{CF}\) are the volume fractions of matrix, Clay, and CNT, respectively, and \(\epsilon_1\) and \(\epsilon_2\) are parameters used for the Clay and CNT, respectively. Further, thicknesses of the effective Clay particle and CNT can be calculated as \(d_p = d_c + 2d_l\), and \(d = 2r\), respectively [1].

![Figure 1](image1.png)  **Figure 1.** The layout for evaluating elastic properties of the SWCNTs reinforced hybrid nanocomposite plate [1] (Under CC Attribution 3.0 License. Copyright 2016 IOP Publishing Ltd)

![Figure 2](image2.png)  **Figure 2.** The schematic representation of unit volume element of the CNTs/nanoclay reinforced polymer [1] (Under CC Attribution 3.0 License. Copyright 2016 IOP Publishing Ltd)

In Figure 1 the layout for evaluating the elastic properties of single walled carbon nanotubes (SWCNTs)/nanoclay reinforced hybrid nanocomposite plate using Halpin–Tsai approach is illustrated. In Figure 2 the schematic representation of unit volume element of the CNTs/nanoclay reinforced polymer nanocomposite plate is demonstrated.

2.1. Governing equations
The displacement field is expressed using the shear deformation theory (SFSDT) based on the secant function [3, 4], which involves the nonlinear shear stress distribution. The displacement fields for laminated Single walled carbon nanotube (SWCNT) plate are shown in equations (2), and (3).
\[ \bar{u}(x, y, z) = u(x, y) - \frac{\partial w}{\partial x} + \left( g(z) + z \Omega \right) \theta_x (x, y); \]
\[ \bar{v}(x, y, z) = v(x, y) - \frac{\partial w}{\partial y} + \left( g(z) + z \Omega \right) \theta_y (x, y); \]
\[ \bar{w}(x, y, z) = w(x, y); \]

where,
\[ g(z) = z \sec \left( \frac{rz}{h} \right); \quad \Omega = \frac{\sec \left( \frac{r}{2} \right)}{1 + \frac{r}{2} \tan \left( \frac{r}{2} \right)} \]  

In the above equation (3), the symbol \( 'r' \) is used for the transverse shear stress and its value is ascertained by the inverse method in post processing step, the symbol \( '\Omega' \) is used for a constant and its value is evaluated by including transverse shear stress boundary conditions to nullify values of transverse shear stresses at the boundary, and the symbol \( 'g(z)' \) is used for the stress strain functions.

The complexity and difficulty involved while making a choice of \( C^1 \) continuity are well known, and hence a \( C^0 \) continuous element would be sufficient for the finite element analysis. Using independent variables such as \( \varphi_x \) and \( \varphi_y \), \( C^1 \) continuity is modified to \( C^0 \) continuity as shown in equation (4).

\[ \varphi_x = \frac{\partial w}{\partial x}; \quad \varphi_y = \frac{\partial w}{\partial y} \]  

The displacement field vector \( \{ \Lambda \} \) can be expressed as shown using equation (5).

\[ \{ \Lambda \} = \left[ u \quad v \quad w \quad \theta_x \quad \theta_y \quad \varphi_x \quad \varphi_y \right]^T \]  

2.1.1 Strain-displacement relationships. The final strain is shown using equation (6).

\[ \varepsilon = \varepsilon_i + \varepsilon_n; \]  

The Von-Karman theory is used to express the relationships between strains and displacements as shown in equation (7).

\[ \varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2; \]
\[ \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2; \]
\[ \varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \left( \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right); \]
\[ \varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial x}; \]
\[ \varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{\partial w}{\partial y}; \]

The linear strain vectors are shown using equations (8), and (9).

\[ \{ \varepsilon \} = \left[ \varepsilon_1 \varepsilon_2 \varepsilon_6 \kappa_1^0 \kappa_2^0 \kappa_4^0 \kappa_5^2 \kappa_6 \varepsilon_4^0 \varepsilon_5^0 \kappa_1^2 \kappa_2^2 \right]^T \]  

where,
\[ \varepsilon_i = \varepsilon_i^* + z \left( k_i^{*} + z^2 k_i^{**} \right); \]
\[ \varepsilon_2 = \varepsilon_2^* + z \left( k_2^{*} + z^2 k_2^{**} \right); \]
\[ \varepsilon_3 = 0; \]
\[ \varepsilon_4 = \varepsilon_4^* + z^2 k_4^{*}; \]
\[ \varepsilon_5 = \varepsilon_5^* + z^2 k_5^{*}; \]
\[ \varepsilon_6 = \varepsilon_6^* + z \left( k_6^{*} + z^2 k_6^{**} \right) \] (9)

The nonlinear strain vector \( \varepsilon_{nl} \) corresponding to displacement fields are shown using equation (10).

\[
\begin{bmatrix}
\frac{\partial w}{\partial x} \\
0
\end{bmatrix} + \theta = \begin{bmatrix}
\frac{\partial w}{\partial x} \\
\frac{\partial w}{\partial y}
\end{bmatrix}
\]

A = \frac{1}{2} \begin{bmatrix}
0 & \frac{\partial w}{\partial x} & 0 \\
0 & 0 & \frac{\partial w}{\partial y} \\
\frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & 0
\end{bmatrix}

(10)

2.1.2 Constitutive relation. Stress–strain relationship can be expressed as shown using equation (11).

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & Q_{16} \\
Q_{12} & Q_{22} & Q_{26} \\
Q_{16} & Q_{26} & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

where, the symbols ‘\( Q_{ij} \)’ and ‘\( \theta_k \)’ are used for the elastic constants and fiber orientation of the lamina.

3. Results and discussions
Using MATLAB code, the nonlinear transverse central deflection of single walled carbon nanotubes reinforced composite plate, is evaluated. In Table 1 [1], the material properties of all phases in 5-phase composite are listed and the results have been validated in Table 2 for different CNTs and Clays conditions.

**Table 1. Proposed material properties [1]**

| Material Specification | Modulus of Elasticity (GPa) | Poisson’s Ratio (v) | Details |
|------------------------|----------------------------|--------------------|---------|
| Epoxy                  | 2.026                      | 0.4                | 25×25mm |
| CNT                    | 1054                       | 0.25               | \( r_l = 0.315 \text{mm}; r_o = 0.650 \text{mm} \) |
| Clay                   | 178                        | 0.28               | \( t = 4 \text{mm}; d_c = 1 \text{mm} \) |
| Interphase (CNT/Polymer)| 16.10                      | 0.4                | \( r_{int} = 1.404 \text{nm} \) |
| Interphase (Clay/Polymer)| 16.10                    | 0.4                | \( d_i = 3 \text{nm} \) |

In Table 2, the effects of nanoclay particles and CNTs in nanocomposite are discussed through the variations in Clays and CNTs. The results show that with increase in Clay and CNT particles, the material properties will be improved as shown in Table 2.
The convergence of the transverse central deflection was observed at \((4 \times 4)\) mesh size as shown in Table 3. Therefore, a minimum of \((4 \times 4)\) mesh size is considered in the present investigation to reduce the computational effort.

**Table 2.** Validation of the material elastic property with variation in CNT and Clay particle

| Particle variation | \(E_1\) | \(E_2\) | \(G_{23}\) | \(\nu_{12}\) | \(\nu_{21}\) |
|-------------------|------|------|--------|--------|--------|
| Present \([1]\)    | 5.941 | 5.3245 | 2.563  | 2.420  | 0.7868 |
| \(1\text{CNT} + 1\text{Clay}\) | 7.667 | 10.2846 | 2.948  | 3.0908 | 0.9929 |
| \(1\text{CNT} + 4\text{Clay}\) | 10.039 | 10.3201 | 2.820  | 2.6521 | 0.8381 |
| \(1\text{Clay} + 4\text{CNT}\) | 10.3201 | 10.3201 | 2.6521 | 2.556  | 0.8381 |

The boundary conditions are employed for nonlinear responses in the nanocomposites: (a) for clamped four edges (CCCC) \((u_0 = v_0 = w_0 = \theta_x = \theta_y = \varphi_x = \varphi_y = 0\); at \(x = 0\), \(a\), and \(y = 0\), \(b\)), (b) for simply supported four edges (SSSS) \((u_0 = w_0 = \theta_x = \psi_y = 0\); at \(x = 0\), \(a\); and \(u_0 = w_0 = \theta_x = \psi_y = 0\); at \(y = 0\), \(b\)).

In Figures 3 and 4, the values of the transverse central deflection using different load parameters for various practical variations, and loading conditions, respectively as per boundary condition such as \(\text{CSCS}; a/h=10\); and stacking sequence as \((0^\circ / 45^\circ / -45^\circ / 90^\circ)\), are shown. The different combinations of CNTs and Clays under uniformly distributed loading conditions have been tested in Figure 3 to investigate the maximum transverse central deflection in case of 1Clay layout. It is observed about the plate (having 1Clay) that it is less stiff than other combinations considered for analyses under the plot (Figure 3) so far.

**Figure 3.** Values of transverse central deflection using different load parameters for various practical variations as per boundary condition

**Figure 4.** Values of transverse central deflection using different load parameters for different loading conditions as per boundary condition such
such as CSCS; $a/h = 10$; and stacking sequence as $CSCS; a/h = 10$; and stacking sequence as $(0^\circ / 45^\circ / -45^\circ / 90^\circ) 
(0^\circ / 45^\circ / -45^\circ / 90^\circ)$

In Figure 5, variation in the transverse central deflection using different load parameters for different boundary conditions while $a/h = 20$; and stacking sequence as $(90^\circ / 0^\circ / 0^\circ / 90^\circ)$, is shown. The transverse central deflection response is however maximum when edges of the nanocomposite plate are simply-supported. In that way the nonlinear transverse central deflection response is minimum when edges of the nanocomposite plate are clamped.

![Figure 5. Variation in transverse central deflection using different load parameters for different boundary conditions while $a/h = 20$; and stacking sequence as $(90^\circ / 0^\circ / 0^\circ / 90^\circ)$](image)

![Figure 6. Variation in transverse central deflection using different load parameters for linear and nonlinear analyses as per boundary condition such as CSCS; $a/h = 20$; and stacking sequence as $(45^\circ / 90^\circ / 90^\circ / -45^\circ)$.](image)

In Figure 6, variation in the transverse central deflection using different load parameters for linear and nonlinear analyses as per boundary condition such as $CSCS; a/h = 20$; and stacking sequence as $(45^\circ / 90^\circ / 90^\circ / -45^\circ)$, is shown.

4. Conclusions
In the present investigation, the nonlinear deflection response of CNTs reinforced composite plate (modeled using Halpin–Tsai Model in MATLAB environment based on the present mathematical model) is successfully carried out so far. The transverse central deflection is more in case of the uniformly distributed loading conditions; however, it is less in case of hydrostatic loading conditions under different load parameters (plotted along X-axis). It is observed that transverse central deflection is reduced while increasing number of CNTs to four. The response of nonlinear transverse central deflection is less in case of CCCC boundary condition among the other support conditions considered in the present investigation. The plate, whose edges are simply supported, has shown highest transverse central deflection during analyses.

References
[1] Arvind Kumar Thakur et al. 2016 IOP conf. ser., Mater. sci. 115 012007
[2] Kulmani Mehar and Subrata Kumar Panda 2017 Adv Polymer Technol. 37(6) 1–15
[3] Achchhe Lal et al. 2016 Comp Mater Sci. 5(4) 1650020
[4] Grover Neeraj et al. 2014 Latin Am. J. Solids Struct. 11(7) 1275–1297
[5] N. S. Putcha and J. N. Reddy 1986 Compos. Struct. 22(4) 529-538