Update on Partial-Wave Analysis

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Abstract

Partial-wave analysis is one step in a process connecting experimental measurements to the N* states we are studying. Progress has been made in the area of ‘model-independent’ analysis. However, more model-dependent approaches are needed to cover broader energy ranges. An example of the problems faced by these more ambitious analyses is given.

I. OVERVIEW

The majority of recent partial-wave analyses (PWA), applied to the study of N* physics, have focused on photo- or electroproduction processes [1]. In the following, I will discuss only this topic and further restrict my comments to pseudoscalar-meson photoproduction. I will first mention fits that have focused on the near-threshold region, where a single resonance is dominant, and one can argue for a truncated multipole expansion. Here the goal is a ‘model-independent’ extraction of amplitudes. Other more ambitious fits have extended over wider energy ranges where model-dependence is inevitable. As a result, in such cases, it becomes difficult to distinguish between models, with fitted parameters, and data-fits (with theoretical constraints). The addition of Born contributions will be discussed as an example illustrating the problems one encounters in fits to the full resonance region.

Before moving on to examples, it is worth repeating that the requirements for an ‘amplitude’ analysis and a ‘multipole’ analysis are not equivalent. This has been pointed out in the past [2], but the distinction is rarely noted in the literature. Amplitude analyses have been performed for pion-nucleon and nucleon-nucleon scattering. However, in most [3] reactions of interest, arguments concerning the requirements for a ‘complete experiment’ are academic. We will be dealing with incomplete experimental information and should concentrate on finding methods which are as model-independent as possible.

In a workshop preceding this meeting, the Baryon Resonance Analysis Group (BRAG) compared the results of seven different fits to a benchmark dataset, with an agreed upon method for handling errors [4]. These fits will be refined and repeated over an expanded dataset (of pion photoproduction data), giving an objective measure of the model-dependence in multipole amplitudes extracted from this reaction.
II. NEAR-THRESHOLD STUDIES

Much of the recent work on pseudoscalar-meson photoproduction has focused on the ‘first bump’ region. In pion photoproduction, this structure is due to the $\Delta(1232)$ resonance. Analyses of $\pi^0$ production (both photo and electroproduction) have used truncated (S- and P-wave) multipole expansions with simplifications following from the dominance of the $M_{3/2}^{1+}$ contribution. In $\eta$ photoproduction, one again has a dominant $(E_{1/2}^{1+})$ multipole near threshold and higher waves are playing a minor role. A similar fitting strategy has been applied to $\eta'$ photoproduction, with less conclusive results. While the total cross section displays a near-threshold peak, there is little agreement on the underlying resonance components.

In $\pi$ and $\eta$ photoproduction, interesting results follow from fits including measurements sensitive to the interference of dominant and sub-dominant amplitudes. In both cases, the photon asymmetry ($\Sigma$) has played an important role, providing information on the E2/M1 ratio (for $\pi$) and the $D_{13}(1520)$ contribution (for $\eta$). This observable has also had a significant impact on pion photoproduction at higher energies and the ‘second bump’ in $K^+$ photoproduction.

These fits have suggested a few new states (in $K^+$ and $\eta'$ photoproduction), and have given new determinations of the N(1520) and N(1535) photo-decay amplitudes (in $\eta$ photoproduction). Unfortunately, the new determinations disagree with older ones obtained from fits to $\pi$ photoproduction data. As a result, the status of photo-couplings is probably less certain than we would have claimed prior to this renewed experimental effort.

More ambitious analyses, covering broader energy ranges, will be required to convincingly establish the existence (or absence) of the many ‘missing resonances’ predicted by quark models. Clearly, we must also have mutually consistent photo-couplings from fits to the different reactions. This important issue is being studied by a number of groups.

III. FITTING THE FULL RESONANCE REGION

Most photo-couplings have been determined from fits to $\pi$ photoproduction data. This reaction has been extensively studied, with measurements covering the full resonance region, and at least an order of magnitude more data than exists for the photoproduction of other pseudoscalar mesons. In $\pi$ photoproduction one also has the advantage of knowing where resonances should occur, given the equally extensive study of elastic $\pi^\pm p$ scattering.

Moving to other channels, we must rely either on quark model predictions or much less reliable hadronic information (for example, $\pi N \rightarrow \eta N$ and $\pi N \rightarrow KY$) for a guide to possible resonance content. The total cross section is of little help, generally having a rapid increase near threshold, and a relatively smooth fall-off at higher energies. As a result, photo-decay amplitudes will be much more difficult to determine (unambiguously) in these reactions.

One method gaining popularity is the use of a multi-channel formalism which incorporates information from many channels simultaneously. This approach requires a single formalism capable of describing all included channels. An ingredient common to each of these processes is the Born term. In the following, I will briefly describe how this contribution behaves and explain why its inclusion is problematic.
A. The Born Terms: General Features

The most obvious influence of Born terms in photoproduction is the forward peak seen in the $\pi^+n$ differential cross section [1]. For example, at a photon energy of 1 GeV, both the data and Born term contribution have a forward peak and a dip near $t = -\mu^2$. However, away from this forward peak, the Born contribution has entirely the wrong shape. In addition, the total cross section from this piece alone exceeds the experimental value just a few hundred MeV above threshold. This divergence of the Born contribution from the experimental total cross sections is also seen in $K^+\pi^0$ photoproduction [13,14].

At higher energies (10 GeV), a simple Born approximation continues to give a qualitative description of the forward peak in $\pi^+n$ photoproduction. This rather unexpected behavior has been explained in terms of finite energy sum rules [15]. Thus we have a Born contribution which is ’correct’ for the forward peak, but clearly is problematic for the total cross section. These features, together with the constraints of gauge invariance and proper analytic structure, can be used to assess the model input adopted in fitting data.

B. Taming the Born Contribution

Born term plus Breit-Wigner fits are adequate to describe cross section data. However, extrapolation beyond the fitted energy range tends to be very unstable. This problem is linked to the behavior of the Born component described above. One simple way to compensate for the Born contribution, is to multiply by an overall form factor. This approach clearly violates crossing symmetry and reduces the full angular distribution uniformly, including the forward peak which requires no suppression. It can even result in a ‘resonancelike’ Born contribution to the total cross section [13], as demonstrated in a fit to $\eta'$ photoproduction.

Some of these problems can be avoided if each strong vertex, in the Born terms, is modified by a form factor depending on the off-shell 4-momentum. In this case a contact term is required to restore gauge invariance. This can be done in a way which preserves crossing symmetry. The effect of these form factor recipes, at the multipole level, is very different. An overall form factor, $F(s)$, having no angular dependence, reduces each multipole by the same amount as $s$ increases. If the recipe of Ref. [16] is used instead, the effect is most important for low partial waves, and becomes negligible as the angular momentum increases. However, using simple and common (real) form factors, we have found that the recipe of Ref. [16] also tends to kill the forward peak seen in $\pi^+n$. As a result, it is difficult to see how the use of real form factors can be justified, based both on formal arguments and in comparison with general features seen in the data.

C. Results from Dispersion Relations

All of the above approaches to the Born contribution implicitly separate the ‘resonance’ and ‘background’ contributions. If one examines this problem within the context of dispersion relations, an entirely different picture emerges. In a fixed-\(t\) dispersion relation, the real part is given by the Born term plus a weighted integral over imaginary parts. In $\pi$ photoproduction, it has been shown that the Born term is damped primarily through the
inclusion of $M_{1+}^{3/2}$ in the integral [17]. Adding this contribution cancels the non-forward Born contribution and enhances the forward peak [18]. Other resonance contributions appear to play a much smaller role, and this holds far above the energy of the $\Delta(1232)$ resonance.

It is possible to write a Born term with form factors in terms of an unmodified Born term plus a contact term. This contact term must grow with energy to cancel the unmodified Born contribution, and therefore represents a significant contribution to the real part of the invariant amplitude. However, in fits such as the one reported by Crawford at this meeting, the dispersion integral appears to be well approximated by resonance contributions alone (over the resonance region).

D. SAID versus MAID

How then are the Born terms handled in the SAID [1] and MAID [19] fits over the resonance region? In both analyses, the Born terms are added (without form factors). In MAID, the full Born contribution is added to resonance contributions. In SAID, the low partial waves are parametrized and the high partial waves are given by a (real) Born contribution.

In Figure 13 of Ref. [19], a cancellation of the unitarized Born and $P_{33}$ multipoles was noted in $\pi^+n$ photoproduction. We have checked this behavior in SAID and find a very similar result. In fact, this simple model is remarkably successful in reproducing the forward $\pi^+n$ peak up to the limit of the MAID analysis (1 GeV). We have also found that this model results in a qualitative description of very forward $(4^\circ) \pi^0p$ differential cross sections over the same energy region. We did not anticipate this result and haven’t found any specific mention of it in the literature.

In SAID and MAID the Born contribution is damped both by the (K-matrix) unitarization and a cancellation with the $P_{33}(1232)$ multipoles. Both contributions are required and important in the observed cancellation.

IV. CONCLUSIONS AND FURTHER QUESTIONS

Many recent studies have focused on reactions and kinematic regions with simplifying features. These have generally been near-threshold fits where a large signal is present, the final meson is neutral, and only a few partial-waves are important. If our goal is to find the many missing states predicted by quark models, these exploratory fits must be followed by more complete treatments of the full resonance region.

We have seen that the SAID and MAID fits have features similar to those mentioned long ago in the context of fixed-t dispersion relations. Aspects of the forward peaking and suppression of Born contributions at non-forward angles are largely due to the $\Delta(1232)$, with smaller contributions from other states. These results weigh against approaches which attempt to separate Born (or generic background) and resonance contributions, suppressing the Born contribution through the use of phenomenological form factors.

Whether the photoproduction of other pseudoscalar mesons can be treated in an analogous manner is less clear. High energy $K^+$ photoproduction has a forward dip, rather than the peak seen in $\pi^+n$. [Aside: This forward region was, however, used by Dombey to estimate
the $K\Lambda N$ coupling constant $[13]$. Some cancellation of the nucleon pole term and resonance contributions has been mentioned in the work of Zhao $[8]$ applied to $\eta'$ photoproduction.

It would be interesting to see how well the various models, used in fitting data (or partial-wave amplitudes), reproduce the forward peak seen in $\pi^+n$ photoproduction. A second test would involve determining the consistency of each approach with the constraints imposed by fixed-$t$ dispersion relations. Checks of this kind may help to select the most promising approach to pursue.

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