Millicharged Atomic Dark Matter

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We present a simplified version of the atomic dark matter scenario, in which charged dark constituents are bound into atoms analogous to hydrogen by a massless hidden sector U(1) gauge interaction. Previous studies have assumed that interactions between the dark sector and the standard model are mediated by a second, massive Z' gauge boson, but here we consider the case where only a massless $\gamma'$ kinetically mixes with the standard model hypercharge and thereby mediates direct detection. This is therefore the simplest atomic dark matter model that has direct interactions with the standard model, arising from the small electric charge for the dark constituents induced by the kinetic mixing. We map out the parameter space that is consistent with cosmological constraints and direct searches, assuming that some unspecified mechanism creates the asymmetry that gives the right abundance, since the dark matter cannot be a thermal relic in this scenario. In the special case where the dark “electron” and “proton” are degenerate in mass, inelastic hyperfine transitions can explain the CoGeNT excess events. In the more general case, elastic transitions dominate, and can be close to current direct detection limits over a wide range of masses.

1. INTRODUCTION

In recent years there has been increased interest in dark matter models in which the dark sector has some of the richness of the visible sector, such as hidden gauge interactions [1]–[5] and flavor. A natural possibility to consider is an unbroken U(1) gauge symmetry that would give rise to bound states, i.e., atomic dark matter [8]–[11]. In this case there should be a further resemblance to the visible sector in that the dark matter must be asymmetric [12] in order to have the right abundance; otherwise the U(1) coupling must be so weak that recombination in the dark sector does not occur efficiently and the would-be atoms remain predominantly ionized [12].

Atomic dark matter can have interesting properties with respect to direct detection, because of the possibility of inelastic scattering to excited states of the atom, notably through hyperfine transitions. In previous studies it has been shown that inelastic transitions can help to reconcile the DM interpretation of CoGeNT events [14] with null results from Xenon10 [15]. In refs. [10, 11] it was assumed that the hyperfine transitions were mediated by the kinetic mixing of the photon with a massive dark vector boson that couples to the axial vector current of the DM. In the present work we explore a simpler possibility [8]: one can rely upon mixing of the photon with the massless $\gamma'$ that is already present due to the unbroken U(1) gauge symmetry. This is a very economical model, while still endowed with the rich phenomenology of the atomic DM scenario.

Because of the kinetic mixing, the constituents of the dark atoms acquire small electric charges $\epsilon e$; thus the interactions that give rise to direct detection are electromagnetic. We show that direct searches in fact give the strongest bounds on $\epsilon$ in this model; thus such detections could be imminent. In fact in the special case where the two constituents have equal mass, we offer an interpretation for the excess events reported by CoGeNT, relying upon inelastic interactions.

2. DIRECT DETECTION

We follow the notation of ref. [10] by denoting the dark analogues of the proton, electron, and hydrogen atom by \( \mathbf{p}, \mathbf{e}, \mathbf{H} \). The Lagrangian is

\[
L = \bar{\epsilon} (i \gamma^\mu p_\mu - m_\epsilon) \epsilon + \bar{\mathbf{p}} (i \gamma^\mu p_\mu - m_p) \mathbf{p} - \frac{1}{4} F_{\mu\nu}^\prime F^\prime_{\mu\nu} - \frac{1}{4} \tilde{F}^{\prime*}_{\mu\nu} \tilde{F}^{\prime}_{\mu\nu} + \frac{1}{2} \epsilon F_{\mu\nu} \tilde{F}^{\prime*}_{\mu\nu} \tag{1}
\]

where \( F \) is the electromagnetic field strength and \( \tilde{F}^{\prime} \) is that of the massless \( \gamma' \). \( D' = \partial \pm i q A' \) where \( q \) is the U(1)$_d$ dark coupling constant. We ignore the small mixing of the \( \gamma' \) with the Z boson, and hence refer to \( F \) as the electromagnetic rather than the hypercharge field strength. \( \tilde{\epsilon} \) is the gauge kinetic mixing parameter. The gauge boson kinetic terms can be diagonalized to first order in \( \tilde{\epsilon} \) by letting \( \tilde{F}' = F' + \tilde{\epsilon} F \). Then \( A'_\mu \) couples only to the dark current \( q J^\mu_d \) while the photon \( A_\mu \) couples to \( \epsilon J^\mu_e + \tilde{\epsilon} q J^\mu_d \) [16, 17]. The DM particles thus acquire millicharges \( \tilde{\epsilon} q \equiv \epsilon e \) under the electromagnetic U(1). We will refer to \( \epsilon \) rather than \( \tilde{\epsilon} \) in the remainder of the paper.

2.1. \( m_\epsilon \ll m_p \) case

Dark atom interactions. The low-energy interactions of \( \mathbf{H} \) are screened due to its net charge neutrality. Let us first consider the generic regime where \( m_\epsilon \ll m_p \). In this limit, the Fourier transform of the \( \mathbf{H} \) electric charge density is given by \( \tilde{\rho}_\mathbf{H} \propto \epsilon e a_0^2 q'^2/2 \) at low wavenumber \( q \ll 1/a'_0 \), where \( a'_0 \approx 1/(\alpha' m_\epsilon) \) is the Bohr radius of \( \mathbf{H} \), and \( \alpha' = g^2/4\pi \). We assume that \( m_\epsilon \ll m_p \) in our approximation for \( a'_0 \). The factor of \( q'^2 \) in \( \tilde{\rho} \) cancels the factor of \( 1/q'^2 \) coming from the gauge boson propagator in Coulomb gauge, so that the scattering of \( \mathbf{H} \) on a proton in a DM detector will not be long-range, but will instead appear as a contact interaction, with

\[
\sigma_p = 4\pi \alpha^2 e^2 \mu^2 a_0^4 \tag{2}
\]
where $\mu$ is the reduced mass of the \( p-H \) system. This expression relies upon the Born approximation, whose validity depends upon the properties of the central potential experienced by \( p \) due to \( H \):

$$V = \frac{\alpha}{a_0'} e^{-2r/a_0'} \left( 1 + \frac{a_0'}{r} \right)$$  \hspace{1cm} (3)

The Born approximation is justified if \(|V(a_0')| \ll 1/(\mu a_0'^2)\), which implies

$$\epsilon \frac{\alpha}{a'} \ll \frac{m_e}{\mu} = \frac{m_e (m_p + m_H)}{m_p m_H}$$  \hspace{1cm} (4)

We will see that this condition is satisfied for parameters of interest for direct detection. In comparing $\sigma_p$ to direct detection bounds, we must take into account that \( H \) interacts only with protons and not all nucleons. This weakens the experimental limit on $\sigma_p$ by a factor of \((Z/A)^2\) for a target with atomic number and weight \( Z, A \). For Xenon, \((Z/A)^2 = 0.17\). We define $\sigma_{p, \text{eff}} = (Z/A)^2 \sigma_p$ to facilitate comparison with the Xenon excluded region.

The cross section for direct detection is proportional to the combination

$$\beta \equiv \frac{\epsilon^2}{\alpha^4} \left( 1 + x_e \right)^4 \frac{1}{x_e^2}$$  \hspace{1cm} (5)

when we reexpress $\mu$ and $a_0'$ in terms of $m_e$ and $m_H$, where \( x_e \equiv m_e/m_p \equiv m_e/m_H \). The values of $\beta$ that are interesting for direct detection can be read from fig. 1, where contours of constant $\beta$ (labeled by the value of $\log_{10} \beta$), are plotted in the $m_H$-$\sigma_{p, \text{eff}}$ plane, on top of constraints from the Xenon100 experiment. To give more concrete examples, let us take $a' = 0.1$ and $x_e = 0.1$, which tend to give a small ionization fraction and therefore are more robust with respect to structure formation constraints. (The ionization fraction $f$ is computed in analogy to that of visible hydrogen in ref. 10, and goes roughly as $f \sim 10^{-10 \alpha^4 - 4 m_e m_p \text{ GeV}^{-2}}$.) The values of $\epsilon$ needed to saturate the Xenon100 bound 18 for a range of $m_H$ are shown in figure 2. These scale as $x_e^2 \alpha^2$ for different values of $x_e$ and $a'$. The values shown in fig. 2 easily satisfy condition (4).

In principle, inelastic scattering can also occur, in which the internal atomic state changes. The lowest energy excitation available is the hyperfine transition, with energy gap $\Delta E \approx \frac{4}{3} \alpha^2 m_e^2 m_p$. For our fiducial parameters $a' = x_e = 0.1$, this would correspond to $\sim 30 \text{ keV}$ if $m_p \approx 10 \text{ GeV}$. Although this may be an interesting value for direct detection, the rate of such transitions is suppressed compared to the elastic ones by a factor of $\alpha^4 x_e^2 (m_H/m_p)^2$ so they make a subdominant contribution if $m_H \lesssim \text{TeV}$. Transitions that excite the $\epsilon$ orbital state are less suppressed by powers of $a'$, but have a larger energy gap, and so are also irrelevant.

**Dark ion interactions.** An interesting feature of atomic dark matter is that a small fraction remains in the ionized state, which has a larger cross section on nucleons than does the atomic state, because its charge is unscreened. Therefore one might question whether the scattering of ionic DM could dominate the direct detection signal, even though it is a subdominant component. Ref. 19 has argued that the ionized component would necessarily have been blown out of the galactic disk by supernova shock waves if $\epsilon$ was in the range required for direct detection. The galactic magnetic field subsequently shields the disk from being repopulated by the millicharged ions, unless the strong scatterings between the ions themselves sufficiently randomize their directions contrary to the Lorentz force from the magnetic field. Adapting the estimate (8) of 21 for the dis-
tance scale $l$, over which randomization occurs, to our models with $\alpha' = 0.1$, $m_{\alpha}/m_p = 0.1$ and using the $m_H$-dependent ionization fraction from fig. 1 of ref. [11], we find that $l \approx 300$ pc, which is larger than the height of the galactic disk and thus ineffective for overcoming magnetic shielding.

Nevertheless, in case there may be some other way of evading this argument, we indicate by the dashed line of fig. 2 the value of $\epsilon$ at which dark ionic scattering would saturate the CoGeNT signal, which is adapted from ref. [11] by taking into account the ionization fraction $f$ mentioned above. Thus even if the ions penetrate to the earth, their signal is much weaker than that of the atoms unless $m_H > 100$ GeV. For consistency one must also check that dark ions can penetrate $\sim 1$ km of rock, which we have done along the lines of refs. [11, 12]. Fig. 2 shows that much larger values of $\epsilon$ are needed to stop the ions in the earth to detect them.

2.2. Special case $m_\alpha = m_p$

If for some reason (e.g., a discrete symmetry) $m_\alpha = m_p$, the matrix element for elastic scattering vanishes in the Born approximation since the average charge density in the $H$ atom vanishes. This is an interesting situation since then inelastic scattering can be the dominant effect for direct detection. The hyperfine splitting is given by

$$E_{\text{hf}} = \frac{\tilde{g} e g_p}{m_p [m_\alpha + m_p]^2} \cdot \frac{m_p^2}{(m_e + m_p)^2} \cdot \frac{1}{m_p} \cdot \frac{1}{m_H}$$

assuming gyromagnetic ratios $g_e = g_p = 2$ and $m_H \approx 2m_p$. The transitions from the spin singlet to triplet atomic DM states are dominated by the spin-orbit coupling between the proton and the constituents of $H$,

$$H_{\text{int}} = \frac{\tilde{e} e}{4\pi m_p r^3} \tilde{p} \cdot \tilde{p} + \{ e \rightarrow p \}$$

where $\tilde{p} = g_{\text{em}}/m_e$ is the dark magnetic moment of $e$ (hence $\tilde{p}_e = e\tilde{e}/m_e$ is its normal magnetic moment) and $r$ is the distance between $p$ and $e$. (Notice there is no reduction by 1/2 for Thomas precession since the electron rest frame is effectively inertial.) We neglect the spin-spin couplings because these give rise to spin-dependent interactions with the nucleus that are suppressed due to the lack of coherence.

The squared matrix element, summed over the final spin states of the triplet, and taking into account the equal contributions from $e$ and $p$, is given by

$$\sum_s |\langle \tilde{p}', s | H_{\text{int}} | \tilde{p}, 0 \rangle|^2 = \frac{C^2}{2\pi^2} \sum_s |\langle \tilde{p}' \bigg| \frac{\tilde{p} - m_p r^3}{m_p r^3} \tilde{p} \bigg| s | \tilde{p}_e | 0 \rangle|^2$$

$$= \frac{C^2 \mu_p}{p q^2} \frac{|\tilde{v} \times \tilde{q}|^2}{(1 + q^2 \alpha_0^2 / 4)^4}$$

where $\tilde{p}$, $\tilde{p}'$ are the initial and final momenta of the proton (in the rest frame of $H$), $\tilde{q} = \tilde{p} - \tilde{p}'$ is the momentum transfer, $C = e^2 / m_e$, and $\mu_p / p = m_p m_H / [(m_p + m_H)p]$ is from normalizing the incoming plane wave to unit current density $22$. From this we obtain the inelastic differential cross section for protons on dark atoms, which can be rescaled to represent the nucleus-atom cross section by including the factor of $Z^2$ to sum over the individual proton contributions, letting $\mu_p \rightarrow \mu_N$, and inserting the Helm form factor $F_H^2(q)$ to account for the nuclear structure:

$$\frac{d\sigma}{d\Omega} \approx \frac{4\pi \alpha^2 Z^2 \mu_N^2}{m_H^2} \frac{q^2}{p^2} |\tilde{v} \times \tilde{q}|^2 F_H^2$$

where now $p$, $p'$ stand for the initial and final momenta of $H$ (in the lab frame), $\tilde{q} = \tilde{p} - \tilde{p}'$ as before, and we used $m_e = \frac{1}{2} m_H$.

Let us compare to the corresponding result (40, 42) of [10] in the $m_e = m_p$ limit:

$$\frac{d\sigma}{d\Omega} \approx \frac{4\pi \alpha}{\pi} Z^2 \mu_N^2 \frac{q^2}{p^2} F_H^2$$

By equating (9) with (10) using the preferred values for fitting to the CoGeNT data, $A = 73$, $q = \sqrt{2m_N E_R} \sim 26$ MeV for recoil energy $E_R = 5$ keV, $v \sim 2 \times 10^{-3}c$, and $f_{\text{eff}}^2 \sim 10^{-20}$ cm$^2$ [11], we find that

$$\epsilon \sim 10^{-2}$$

to explain CoGeNT. The splitting $E_{\text{hf}} = 15$ keV and atomic mass $m_H = 6$ GeV imply $\alpha' = 0.062$. We confirm the estimate [11] by comparing $\sigma_p \approx \frac{\pi^2 \alpha \mu_N^2}{(q m_H)^2}$ (c.f. eq. (9)) to the determination $\sigma_p \approx 10^{-38.3} \text{cm}^2$ from fig. 12 of ref. [23], which was the first to propose inelastic scattering (also with a mass difference of 15 keV) as an explanation for the CoGeNT observations. We note that dark ions will not penetrate 1 km of rock for such large $\epsilon$, considering fig. 2.

3. OTHER CONSTRAINTS

Laboratory and supernova bounds. Unlike models in which the $\gamma'$ has a mass, in ours no coupling of the $\gamma'$ to visible sector matter is induced by the kinetic mixing. Therefore a variety of bounds that would pertain to massive $\gamma'$s, from beam-dump experiments, contributions to the anomalous magnetic dipole moments of the electron and muon, and supernova emission of $\gamma'$, do not apply here. Note that the dark matter is too heavy to be in equilibrium in supernovae. Accelerator constraints for millicharged particles have a large open window for $\epsilon < 0.1$ and masses $\gtrsim 1$ GeV of interest here [24].

Exotic isotopes. Millicharged dark ions with ionization fraction $f = 10^{-4}$ would ostensibly have a flux of $2 \times 10^{20}$/s on the earth, and they would bind to normal nuclei unless they are unstable against thermal fluctuations, requiring $\epsilon \lesssim 10^{-3}$ [8, 25]. With $\epsilon \sim 10^{-2}$, they would be stopped in $\sim 1$ m of the atmosphere (10$^{24}$ atoms) and produce a relative abundance $10^{-7}$ of exotic
isotopes over 10 Gyr. In the mass range covering $m_e = 3$ GeV, heavy isotope searches have excluded abundances of $10^{-18.5}$ for deuterium from D$_2$O [28] and $10^{-14}$ for helium [27].

However, there are a number of reasons why these limits would not apply to our model. We evade the first one because $e$ binds much more strongly (400 eV) to oxygen than to deuterium (3 eV) for $\epsilon = 0.01$, making D[DeO] highly unstable to decay into D$_2$O [28]. The second is ameliorated by realizing that He has a lifetime of $\tau = 10^9$ y in the atmosphere [28], reducing the estimated abundance by a factor of $\tau/(10 \text{ Gyr})$ to $10^{-11}$. This does not yet take into account the magnetosphere which very effectively shields the earth from slow charged particles, including 3 GeV dark ions with $\epsilon \sim 10^{-2}$, whose gyroradius at the top of the atmosphere is $\sim 0.01$ earth radii. Solar x-rays are sufficiently energetic to break up the He $e$ bound state (binding energy 5 eV) and allow $e$ to rebind much more strongly to N or O in the atmosphere. Even though ref. [28] conservatively limits $\epsilon < 0.005$ for supernovae to be able to efficiently expel 3 GeV ions from the galaxy, which is marginally smaller than our preferred value, the galactic B field does prevent new ions from entering the galaxy, and this could decrease the expected flux. Moreover our determination of $\epsilon$ could decrease by a factor of $10^{10.3}$ in light of CoGeNT’s recent reanalysis of their background events [28]. Thus it is far from clear that exotic isotope searches rule out our $\epsilon \sim 10^{-2}$, $m_e = m_p = 3$ GeV model.

**Big bang nucleosynthesis (BBN).** The current bound of no more than one additional neutrino [30] in the plasma at BBN gives weak constraints on $\epsilon$ since the extra $\gamma'$ counts as only 8/7 of a neutrino. Thus even if $\gamma'$ remains completely in equilibrium, it only exceeds the 95% c.l. bound by 0.14 of an additional neutrino species. As expected, the constraints on $\epsilon$ are quite weak from demanding decoupling of processes keeping $\gamma'$ in equilibrium early enough to dilute this small fraction of a species.

**Neutron stars.** It has been pointed out that asymmetric bosonic dark matter must interact extremely weakly with baryons in order to avoid the destruction of neutron stars due to accumulation leading to gravitational collapse [31,33]. This restricts the cross section to values below that needed for direct detection, but it relies upon the absence of Fermi pressure for bosonic DM. Once the atoms become so dense that they are overlapping, their constituents start to behave as a Fermi gas, and thus avoid collapse until much higher densities, as long as the overlap of atoms occurs before they are within their Schwarzschild radius, $R_s$. For $N$ dark atoms of mass $m_H$, $R_s = 2N m_H/m_p^2$ and the average separation between atoms is $\Delta r = R_s/N^{1/3}$. The criterion to avoid collapse is that $\Delta r \ll a'_0$ at the bosonic Chandrasekhar limit $N \sim (m_p/m_H)^2$ [31]. We find that $\Delta r/a'_0 \sim \alpha' m_H/(m_H^{1/3} m_p^{2/3}) \ll 1$, so the system is no longer atomic, but a plasma of dark ions. In this case the much weaker fermionic Chandrasekhar limit $N \sim M_p^3/(m_e^{3/4} m_p^{9/4})$ [34] applies, and there is no constraint from neutron stars. ($\Delta r/a'_0 \sim \alpha' \sqrt{m_e/m_p}$ is still self-consistently $< 1$ in this case.)

**Halo shape constraints.** The most serious constraints on atomic dark matter self-interactions come from their distortions of the shapes of elliptical DM cores of galactic clusters. To avoid relaxation to more spherical shapes, ref. [35] finds the constraint $\sigma/m_H < 0.02$ cm$^2$/g on the cross section for $H\cdot H$ collisions. Ref. [10] argues that $\sigma = 4\pi (\kappa a'_0)^2$ with $3 \lesssim \kappa \lesssim 10$. Together with the bound on $\sigma$, this would rule out our inelastic model with $m_H = 6$ GeV. However, assuming that the elastic cross section for $H\cdot H$ scattering computed in [36] is the same as for $H\cdot H$, we find that $\kappa \approx 0.16$ at the relevant energy $E \sim (v/a'\gamma)^2 \sim 10^{-3}$ in atomic units. The ellipticity constraint is then satisfied for the inelastic model, with $m_H \gtrsim 2$ GeV. For the elastic model with $\alpha' = m_{e}/m_{p} = 0.1$, it implies $m_H \gtrsim 4$ GeV. Bounds on $\sigma$ from the Bullet Cluster give weaker constraints [37].

**Cosmic microwave background (CMB).** Bounds on $\epsilon$ were obtained on millicharged DM in ref. [19] considering a variety of physical processes; the most stringent limits were obtained from demanding that dark matter has decoupled from the photon-baryon plasma before recombination, under the assumption that the DM was fully ionized. These are weakened in our model due to the screened electromagnetic interactions of the atoms or the ionization fraction $f$ being small. By extrapolating the results of fig. 1 of ref. [19] to $m_e = m_p = 3$ GeV we find that $f = 10^{-4}$ at $\alpha' = 0.002$. This is well below the limit $f = \Omega_{\text{ion}}/\Omega_{\text{atom}} < 0.007/0.11$ from distortions of the cosmic microwave background [38]. One also requires that the DM be out of kinetic equilibrium with the baryon-photon plasma before recombination. For the atoms, the dominant interaction is $H \rightarrow H$ through the Compton cross section $\sigma_c = 32\pi^2 a'\nu^2/3m_e^2$. We find the weak limit $\epsilon < 0.02$, which is marginally consistent with [11]. Thus we evade the strongest constraints on $\epsilon$ in fig. 1 of [19] in our inelastic scattering model with $m_e = m_p$.

In the case $m_e < m_p$ where elastic scattering dominates in direct detection, we need not consider how much the bounds of ref. [19] are softened by having dark atoms rather than ions. In this case, the Xenon100 limit on $\epsilon$, at least for the model $\alpha' = m_{e}/m_{p} = 0.1$, is well below the most stringent limit of [19] even if the dark matter was fully ionized. This is shown in fig. 2.

**Virialization of dark matter.** The next-strongest bounds in [19] arise from heating of the DM by scattering from baryons, which would interfere with DM collapse and virialization in galaxy formation. However in the $m_e = m_p$ case, the tree-level exchange of photons between $H$ and normal H atoms vanishes because of the perfect mutual screening of $e$ and $p$. Thus there is no efficient way of heating up the dark atoms; only the small ionized fraction suffices this fate, and it should have a negligible effect on galaxy formation. For the $m_e < m_p$ case, our Xenon100 bound on $\epsilon$ is more restrictive than the virialization bound.
4. CONCLUSIONS

We have reconsidered a minimal alternative for atomic dark matter, namely that at the Lagrangian level its ionic constituents couple only to one massless $\gamma'$ gauge boson, responsible for their binding, but due to kinetic mixing of the $\gamma'$ with the photon, they acquire small charges $\pm e$ giving the dark atoms a weak coupling to the ordinary photon. We have shown that $\epsilon$ is sufficiently unconstrained so that the first evidence of this interaction might come through direct detection of the dark atom.

There are two interesting regions of parameter space for the model with respect to direct detection. If the dark "electron" is much lighter than the dark "proton," $m_e \ll m_p$, then dark atoms scatter primarily elastically on nuclei, and the cross section can be close to the limits from direct detection over a wide range of masses $m_H$. However if $m_e = m_p$, elastic scattering is suppressed and inelastic hyperfine transitions dominate. We find that if the $\gamma'$ gauge coupling is $\alpha' = 0.06$ and $m_e = m_p \gtrapprox 3$ GeV, the hyperfine splitting is $\sim 15$ keV, and $\epsilon \gtrapprox 10^{-2}$ gives the right cross section for explaining candidate DM events reported by CoGeNT.

This model has further testable implications. The detailed effects of atomic DM on the CMB, depending upon exactly how and when it recombines, have yet to be studied. An important unexplored aspect is the fraction of atoms that combine into molecular $H_2$. This could result in multiple signals for direct detection, where dark atoms or molecules interact with nuclei, producing recoils at energies corresponding to several different DM masses. Searches for exotic isotopes consisting of bound states of e and normal atoms appear to be tantalizingly close to ruling out the $\epsilon = 0.01$ model, but more work must be done to quantify the predicted abundances. The framework appears to be consistent with strong DM halo ellipticity constraints, but an independent determination of the $H$-$H$ scattering cross section at the relevant energies should be done to confirm this. The origin of the DM asymmetry [11] should also be addressed. We hope to return to these issues in the near future.

Acknowledgements. We thank S. Davidson, R. Foot, D.E. Kaplan, G. Krnjaic, A. Kurkela, S. McDermott, G. Moore, M. Pospelov, P. Scott, N. Toro, and K. Zurek for helpful interactions. We are supported by NSERC.

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