NLO QCD corrections to Drell-Yan processes in the SANC framework

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Abstract

NLO QCD corrections to charged and neutral current Drell–Yan processes and their implementation in the computer system SANC are considered. On the partonic level both quark-antiquark and quark-gluon scattering channels are taken into account. Subtractions of the collinear singularities in the massive case are compared with ones in the $\overline{\text{MS}}$ scheme. Results of SANC on the hadronic level are presented. Comparison with results of the MCFM package is shown.

1 Introduction

Charged and neutral current Drell–Yan (DY) processes \footnote{1} on the eve of the first proton collisions at the LHC become very important for precision tests of the Standard Model. They are easily detected and will provide standard candles for detector calibration during the first stage of LHC running. They will be also used for extraction of partonic density functions (PDF) in the kinematical region which has not been accessed by earlier experiments. Therefore it is crucial to control the theoretical predictions for production cross sections and kinematic distributions of these processes.

In the previous paper \cite{2} we presented a part of the QCD sector of our computer system SANC \footnote{2} (http://sanc.jinr.ru/ and http://pcphsanc.cern.ch/) where the NLO QCD processes are treated. There we considered the implementation into SANC the calculation of the charged (CC) and neutral (NC) current quark–antiquark Drell–Yan processes on the partonic level and briefly presented some numerical results for the hadronic level. The QCD corrections to DY processes are known in the literature for many years, see Refs. \cite{5} \cite{6} \cite{8}. Recently the corresponding NNLO corrections for differential distributions have been received \cite{9} \cite{10}.
In this paper with respect to Ref. [2] we add into consideration quark–gluon and gluon-antiquark Drell–Yan processes on the partonic level side by side with the quark–antiquark ones. We implemented into SANC the calculation of QCD corrections to DY processes on the hadronic level. Working with massive quarks, we regularized the collinear singularities by masses of quarks. But on the hadronic level we have to remove these collinear singularities to avoid double counting, because they are already included in PDF. Therefore we compared analytical results obtained in our treating of collinear singularities with analogous results obtained in the $\overline{\text{MS}}$ scheme calculated in the $n$-dimensional phase space. In this way we extracted the subtraction terms needed to remove the collinear singularities in our massive quarks case. We show here also comparison with the corresponding results of the MCFM [11] package.

One-loop electroweak radiative corrections were computed for the DY processes by SANC in Refs. [12, 13, 14] and extensively compared with results of other groups, see e.g. Refs. [15, 16, 17].

The paper is organized as follows. In the second section we calculated the hard gluon bremsstrahlung contributions both to charged and neutral current Drell-Yan processes on a quark-parton level in the massless $\overline{\text{MS}}$ scheme and compared with our massive quarks results. We used FORM3.1 [18] for analytical calculations. In the third section we calculated the quark–gluon and gluon-antiquark DY processes with charged and neutral currents in our massive quarks treating and draw parallel with calculation in the massless $\overline{\text{MS}}$ scheme. In Conclusion we discuss the results and give some illustrations of DY distributions at the hadronic level.

2 Quark–antiquark Drell–Yan processes

2.1 Massive quarks treatment of the hard gluon contribution

Working with massive quarks we showed in the previous paper [2] the NLO QCD corrections due to the hard gluon bremsstrahlung of charged current (CC) $\bar{d}(p_1) + u(p_2) \to W \to \nu_e(p_3) + \bar{\ell}(p_4) + g(p_5)$ and neutral current (NC) $\bar{q}(p_1) + q(p_2) \to [A, Z] \to \ell(p_3) + \bar{\ell}(p_4) + g(p_5)$ Drell–Yan processes were obtained in the form

$$\hat{\sigma}^{\text{CC}}_{\text{Hard}} = \frac{\alpha_s}{2\pi} \int_{z_{\text{min}}}^{z_{\text{max}}} d\hat{s} \hat{\sigma}_0^{\text{CC}}(z\hat{s}) P_{qq}(z) \left[ 2\ln\left(\frac{\hat{s}}{\mu^2}\right) + \ln\left(\frac{\mu^2}{m_u^2}\right) + \ln\left(\frac{\mu^2}{m_d^2}\right) - 2 \right],$$

(1)

$$\hat{\sigma}^{\text{NC}}_{\text{Hard}} = \frac{\alpha_s}{\pi} \int_{z_{\text{min}}}^{z_{\text{max}}} dz \hat{\sigma}_0^{\text{NC}}(z\hat{s}) P_{qq}(z) \left[ \ln\left(\frac{\hat{s}}{\mu^2}\right) + \ln\left(\frac{\mu^2}{m_d^2}\right) - 1 \right],$$

(2)

where $\mu$ is the factorization scale. The energy of the emitted gluon is

$$p_5^0 = \frac{\sqrt{\hat{s}}}{2} (1 - z),$$

(3)
where \( z = \frac{s'}{s} \), \( \hat{s} = -2(p_1, p_2) \), \( p_1 \) and \( p_2 \) are momenta of the incoming quarks and \( s' = -(p_3 + p_4)^2 \) is the invariant mass of the outgoing lepton.\(^1\)

\[
P_{qq}(z) = C_F \frac{1 + z^2}{1 - z} \tag{4}
\]

is the leading order (LO) quark-quark splitting function.

In the charged current case we have the limits of integration over variable \( z: z_{\text{max}} = 1 - \frac{2\overline{\omega}}{\sqrt{\hat{s}}} \), where \( \overline{\omega} \ll \sqrt{\hat{s}} \) and \( z_{\text{min}} = \frac{m_\ell^2}{\hat{s}} \), \( m_\ell \) being the mass of charged leptons. The auxiliary parameter \( \overline{\omega} \) is the maximal energy of a soft gluon in the c.m.s. of the incoming partons. In the neutral current case we have correspondingly the same \( z_{\text{max}} \) and \( z_{\text{min}} = \frac{4m_\ell^2}{\hat{s}} \).

The cross sections in the Born approximation read

\[
\hat{\sigma}_{CC}^0(\hat{s}) = |V_{ud}|^2 \frac{G_F^2}{18\pi} \frac{M_W^4 \hat{s}}{|\hat{s} - \hat{M}_W^2|^2} \left( 1 - \frac{3m_\ell^2}{2\hat{s}} + \frac{m_\ell^6}{2\hat{s}^2} \right), \tag{5}
\]

\[
\hat{\sigma}_{NC}^0(\hat{s}) = \frac{4\pi \alpha^2}{3\hat{s}} \beta(\hat{s}, m_\ell^2) \left[ \frac{1}{3} \left( 1 - \frac{m_\ell^2}{\hat{s}} \right) V_0(\hat{s}) + \frac{m_\ell^2}{\hat{s}} V_a(\hat{s}) \right], \tag{6}
\]

where \( \hat{s} = -(p_1 + p_2)^2 \), \( p_1 \) and \( p_2 \) are 4-momenta of the initial quarks; \( \hat{M}_W^2 = M_W^2 - iM_W \Gamma_W \);

\[
\beta(\hat{s}, m_\ell^2) = \sqrt{1 - \frac{4m_\ell^2}{\hat{s}}}. \quad \text{Here we denoted}
\]

\[
V_0(\hat{s}) = Q_q^2 Q_\ell^2 + 2 Q_q Q_\ell |\chi_Z(\hat{s})| |v_q v_\ell| + |\chi_Z(\hat{s})|^2 \left( v_q^2 + I^{(3)}_q \right) \left( v_\ell^2 + I^{(3)}_\ell \right),
\]

\[
V_a(\hat{s}) = V_0(\hat{s}) - 2 |\chi_Z(\hat{s})|^2 \left( v_q^2 + I^{(3)}_q \right) \left( I^{(3)}_\ell \right)^2,
\]

\[
v_q = I^{(3)}_q - 2Q_q \sin^2 \theta_W, \quad v_\ell = I^{(3)}_\ell - 2Q_\ell \sin^2 \theta_W. \tag{7}
\]

The \( Z/\gamma \) propagator ratio \( \chi_Z(\hat{s}) \) with \( \hat{s} \)-dependent or constant \( Z \)-width is

\[
\chi_Z(\hat{s}) = \frac{\hat{s}}{s - M_Z^2 + i\hat{s} \frac{\Gamma_Z}{M_Z}} \frac{1}{4 \sin^2 \theta_W \cos^2 \theta_W}. \tag{8}
\]

We see in Eqs. (1) and (2) that the collinear singularities appear as quark mass singularities.

### 2.2 Hard gluon contribution in \( \overline{\text{MS}} \) scheme with massless quarks

Because the collinear singularities calculated in the \( \overline{\text{MS}} \) scheme with massless quarks are already included in PDF, we have to find which terms in our massive quarks treatment of the cross sections have to be subtracted to avoid the double counting. Therefore we calculate the

\(^1\)We use the \((-+, +, +)\) metrics, \( p = (p^0, \vec{p}) \).
same cross sections in the \( \overline{\text{MS}} \) scheme following the well known [19] manner of working with massless quarks and compare with our results.

Calculating the three particle phase space element of the hard gluon emission in the \( n \)-dimensional phase space we used a cascade in two steps:

\[
d\Phi^{(3)} = \frac{ds'}{2\pi} d\Phi^{(2)}_1 d\Phi^{(2)}_2. \tag{9}
\]

For the first step of the charged current process \( \bar{d}(p_1) + u(p_2) \to W^*(Q') + g(p_5) \) we obtained the same formula as in [19]:

\[
\Phi^{(2)}_1 = \frac{1}{8\pi} \left( \frac{4\pi \mu^2}{s'} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} z^\epsilon (1-z)^{1-2\epsilon} \int_0^1 dy \, y^{-\epsilon} (1-y)^{-\epsilon}, \tag{10}
\]

where we introduced in addition to \( z \) the variable \( y \)

\[
y = \frac{1 + \cos(\theta_g)}{2}. \tag{11}
\]

Here \( \theta_g \) is an angle between vectors \( \vec{p}_1 \) and \( \vec{p}_5 \), the angle of the emitted gluon.

The phase space element of the second step \( W^*(Q') \to \nu_\ell(p_3) + \ell^+(p_4) \) of the cascade is:

\[
\Phi^{(2)}_2 = \frac{1}{16\pi^2} \left( \frac{4\pi \mu^2}{s'} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left( 1 - \frac{m_\ell^2}{s'} \right)^{1-2\epsilon} \int_0^1 dR \, y_R^{-\epsilon} (1-y_R)^{-\epsilon} \int_0^{2\pi} d\varphi_R, \tag{12}
\]

where

\[
y_R = \frac{1 + \cos(\theta^R)}{2}, \tag{13}
\]

and \( \theta^R \) is an angle between the charged lepton and gluon in the rest frame of the outgoing leptons.

Having in mind that the cross section of the charged current process in Born approximation has the form

\[
\hat{\sigma}_0^{CC}(\hat{s}, \epsilon) = \left| V_{ud} \right|^2 \frac{G_F^2 M_W^2 \hat{s}}{6\pi} \left( \frac{4\pi \mu^2}{\hat{s}} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left( 1 - \frac{m_\ell^2}{\hat{s}} \right)^{2-2\epsilon} \int_0^1 dy_0 \, y_0^{-\epsilon} (1-y_0)^{-\epsilon} \left[ y_0 - \left( 1 - \frac{m_\ell^2}{\hat{s}} \right) y_0(1-y_0) - \frac{1}{2} \epsilon \right], \tag{14}
\]

we obtained a factorized expression of the hard gluon NLO correction to the charged current process.

\[
\hat{\sigma}_\text{Hard}^{CC}(\epsilon) = \frac{\alpha_s}{2\pi} C_F \int_{z_{\text{min}}}^{z_{\text{max}}} dz \, \hat{\sigma}_0^{CC}(z\hat{s}, \epsilon) \left( \frac{4\pi \mu^2}{z\hat{s}} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} z^\epsilon (1-z)^{-2\epsilon} \int_0^1 dy \, y^{-\epsilon} (1-y)^{-\epsilon} \left[ \frac{1}{y(1-y)} \left( \frac{1}{1-z} - 1 + \frac{1}{2} (1-z) \right) + (1-z) \left( -1 - \frac{\epsilon}{2y(1-y)} \right) \right], \tag{15}
\]
Integration over $y$ gives

$$
\hat{\sigma}_{\text{Hard}}^{\text{CC}}(\varepsilon) = \frac{\alpha_s}{2\pi} \int_{z_{\min}}^{z_{\max}} dz \, \hat{\sigma}_0^{\text{CC}}(z \hat{s}, \varepsilon) \left( \frac{4\pi \mu^2}{z \hat{s}} \right)^\varepsilon \frac{\Gamma(1 - \varepsilon)}{\Gamma(1 - 2\varepsilon)} z^\varepsilon (1 - z)^{-2\varepsilon} P_{\gamma \gamma}(z) \left( -\frac{2}{\varepsilon} \right). \tag{16}
$$

One can see that the collinear divergence appears here as a pole $\frac{1}{\varepsilon}$. To compare this expression with the analogous expression (1), where the collinear divergence manifests itself in the form of logarithms $\ln \left( \frac{\hat{s}}{m^2} \right)$, $\ln \left( \frac{\hat{s}}{m^2} \right)$ one has to take the limit $\varepsilon \to 0$. Then one obtains an expression

$$
\hat{\sigma}_{\text{Hard}}^{\text{CC}} = \frac{\alpha_s}{2\pi} \int_{z_{\min}}^{z_{\max}} dz \, \hat{\sigma}_0^{\text{CC}}(z \hat{s}) \, P_{\gamma \gamma}(z) \left( -\frac{2}{\varepsilon} + 2 \ln \left( \frac{\hat{s}}{\mu^2} \right) + 4 \ln(1 - z) \right) \tag{17}
$$
to be compared with the corresponding expression (11). We see which terms in the expression (11) correspond to the collinear divergent term $-\frac{1}{\varepsilon}$ which in $\overline{\text{MS}}$ scheme has to be subtracted from the hard gluon contribution to the considered process because it is already included into PDF.

So, on the quark-parton level we have subtract from $\hat{\sigma}_{\text{Hard}}^{\text{CC}}$ (11), the following expression:

$$
\hat{\sigma}_{\text{Subtr}}^{\text{CC}}(\mu^2) = \frac{\alpha_s}{2\pi} \int_{z_{\min}}^{z_{\max}} dz \, \hat{\sigma}_0^{\text{CC}}(z \hat{s}) \, P_{\gamma \gamma}(z) \left[ \ln \left( \frac{\mu^2}{m^2} \right) + \ln \left( \frac{\mu^2}{m^4} \right) - 2 - 4 \ln(1 - z) \right]. \tag{18}
$$

Factorization properties and general relations between amplitudes with massive and massless partons can be found in Ref. [20].

For the neutral current process calculating the three particle phase space element (9) of the hard gluon emission in the $n$-dimensional phase space we obtain the same result (11) for the first step $\bar{q}(p_1) + q(p_2) \to \{\gamma, Z\}^*(Q') + g(p_3)$. The phase space element of the second step $\{\gamma, Z\}^*(Q') \to \ell^- (p_3) + \ell^+ (p_4)$ of the cascade is similar to (12):

$$
\Phi_2^{(2)} = \frac{1}{16\pi^2} \left( \frac{4\pi \alpha^2}{s'} \right)^\varepsilon \frac{1}{\Gamma(1 - \varepsilon)} \beta^{1 - 2\varepsilon} \int_{0}^{1} dy_R y_R^{-\varepsilon} (1 - y_R)^{-\varepsilon} \int_{0}^{2\pi} d\varphi_R. \tag{19}
$$

Analogously, we obtained a factorized expression for the hard gluon NLO correction to the neutral current process. It has exactly the same structure as Eq. (15), but the cross section in Born approximation $\hat{\sigma}_0^{\text{NC}}(z \hat{s}, \varepsilon)$ of the neutral current process has a different form:

$$
\hat{\sigma}_0^{\text{NC}}(\hat{s}, \varepsilon) = \frac{4 \pi \alpha^2}{3 \hat{s}} \beta^{1 - 2\varepsilon} \left( \frac{4\pi \mu^2}{\hat{s}} \right)^\varepsilon \frac{1}{\Gamma(1 - \varepsilon)} \int_{0}^{1} dy_0 y_0^{-\varepsilon} (1 - y_0)^{-\varepsilon} \left[ V_0(\hat{s}) \left( -\beta^2 \left( \frac{m^2}{\hat{s}} \right) y_0 (1 - y_0) + (1 - \varepsilon) \left( \frac{1}{2} - \frac{m^2}{\hat{s}} \right) \right) \right]
$$

$$
+ V_a(\hat{s}) (1 - \varepsilon) \frac{m^2}{\hat{s}} + A_0(\hat{s})(1 - \varepsilon)(1 - 2\varepsilon) \beta(\hat{s}, m^2) \left( \frac{1}{2} - y_0 \right), \tag{20}
$$

where

$$
A_0(\hat{s}) = 2 Q_q Q_\ell \left| \chi_\ell(\hat{s}) \right| I_q^{(3)} I_\ell^{(3)} + \left| \chi_\ell(\hat{s}) \right|^2 4 v_q v_\ell I_q^{(3)} I_\ell^{(3)}. \tag{21}
$$
Integration over $y$ gives for $\hat{\sigma}_{\text{Hard}}^{\text{NC}}(\hat{s}, \varepsilon)$ the same result as (16). So, in the limit $\varepsilon \to 0$ we come to the expression almost the same as (17),

$$\hat{\sigma}_{\text{Hard}}^{\text{NC}} = \frac{\alpha_s}{2\pi} \int_{z_{\text{min}}}^{z_{\text{max}}} dz \, \hat{\sigma}_0^{\text{NC}}(z\hat{s}) \, P_{qq}(z) \left( -\frac{2}{\varepsilon} + 2 \ln \left( \frac{\hat{s}}{\mu^2} \right) + 4 \ln(1 - z) \right). \quad (22)$$

Comparison of this expression where the collinear divergence appears as a pole $1/\varepsilon$, with (2) where the collinear divergence manifests itself in the form of logarithms permits us to find what expression one has to subtract from $\hat{\sigma}_{\text{Hard}}^{\text{NC}}(\hat{s})$, namely

$$\hat{\sigma}_{\text{Subtr}}^{\text{NC}}(\mu^2) = \frac{\alpha_s}{\pi} \int_{z_{\text{min}}}^{z_{\text{max}}} dz \, \hat{\sigma}_0^{\text{NC}}(z\hat{s}) \, P_{qq}(z) \left[ \ln \left( \frac{\mu^2}{m_q^2} \right) - 1 - 2 \ln(1 - z) \right]. \quad (23)$$

where $m_q$ is the mass of the pair quark and antiquark coming from the both protons.

### 2.3 Virtual and soft gluon contribution

Working in our massive quark treatment we obtained for the sum of virtual and soft gluon contributions an expression free from infrared divergences. In the case of charged current processes we have

$$\hat{\sigma}_{\text{Virt}}^{\text{CC}} + \hat{\sigma}_{\text{Soft}}^{\text{CC}} = \frac{\alpha_s}{2\pi} C_F \left( \frac{3}{2} + \ln \left( \frac{4\tilde{\omega}^2}{\hat{s}} \right) \right) \left[ \ln \left( \frac{\hat{s}}{m_u^2} \right) + \ln \left( \frac{\hat{s}}{m_d^2} \right) - 2 \right] - 1 - \frac{\pi^2}{3}. \quad (24)$$

Correspondingly, in the case of neutral current processes we have

$$\hat{\sigma}_{\text{Virt}}^{\text{NC}} + \hat{\sigma}_{\text{Soft}}^{\text{NC}} = \frac{\alpha_s}{\pi} C_F \left( \frac{3}{2} + \ln \left( \frac{4\tilde{\omega}^2}{\hat{s}} \right) \right) \left[ \ln \left( \frac{\hat{s}}{m_q^2} \right) - 1 \right] - \frac{1}{2} - \frac{\pi^2}{6}. \quad (25)$$

One can find the collinear divergent expressions to be subtracted from these virtual and soft gluon contributions because they are already included into PDF, taking the corresponding expressions which one has subtract from the hard gluon contributions to the considered processes. But one has to take them with opposite sign, to substitute the argument of Born cross sections taken $z = 1$, namely

$$\hat{\sigma}_0^{\text{CC}}(z\hat{s}) \quad \text{and} \quad \hat{\sigma}_0^{\text{NC}}(z\hat{s}) \implies \hat{\sigma}_0^{\text{CC}}(\hat{s}) \quad \text{and} \quad \hat{\sigma}_0^{\text{NC}}(\hat{s}) \quad (26)$$

and to integrate over $z$ from 0 to $z_{\text{max}}$. In this way we obtained the expressions to be subtracted from virtual and soft gluon contributions.

For charged current processes collinear divergent subtraction is:

$$\hat{\sigma}_{\text{SVSubtr}}^{\text{CC}}(\mu^2) = -\frac{\alpha_s}{2\pi} \hat{\sigma}_0^{\text{CC}}(\hat{s}) \int_0^{z_{\text{max}}} dz \, P_{qq}(z) \left[ \ln \left( \frac{\mu^2}{m_u^2} \right) + \ln \left( \frac{\mu^2}{m_d^2} \right) - 2 - 4 \ln(1 - z) \right]. \quad (27)$$

And for neutral current processes it is:

$$\hat{\sigma}_{\text{SVSubtr}}^{\text{NC}}(\mu^2) = -\frac{\alpha_s}{\pi} \hat{\sigma}_0^{\text{NC}}(\hat{s}) \int_0^{z_{\text{max}}} dz \, P_{qq}(z) \left[ \ln \left( \frac{\mu^2}{m_q^2} \right) - 1 - 2 \ln(1 - z) \right], \quad (28)$$
Integration over $z$ gives the expressions to be found, namely, first - a subtraction for the charged current contribution:

$$
\hat{\sigma}_{SV_{\text{Subtr}}}^{CC}(\mu^2) = \frac{\alpha_s}{2\pi} C_F \hat{\sigma}_0^{CC}(s) \left\{ \left( \frac{3}{2} + \ln \left( \frac{4\omega^2}{\hat{s}} \right) \right) \left[ \ln \left( \frac{\mu^2}{m_u^2} \right) + \ln \left( \frac{\mu^2}{m_d^2} \right) - 2 \right] 
+ 7 - \ln^2 \left( \frac{4\omega^2}{\hat{s}} \right) \right\} \right. 
$$

(29)

and second - a subtraction for the neutral current contribution:

$$
\hat{\sigma}_{SV_{\text{Subtr}}}^{NC}(\mu^2) = \frac{\alpha_s}{\pi} C_F \hat{\sigma}_0^{NC}(\hat{s}) \left\{ \left( \frac{3}{2} + \ln \left( \frac{4\omega^2}{\hat{s}} \right) \right) \left[ \ln \left( \frac{\mu^2}{m_u^2} \right) - \frac{1}{2} \right] 
+ 7 - \frac{1}{2} \ln^2 \left( \frac{4\omega^2}{\hat{s}} \right) \right\} \right. 
$$

(30)

One can see that subtractions (29) and (30) really subtract the collinear divergent terms in the expressions of virtual and soft gluon contributions (24) and (25), correspondingly.

### 3 Quark–gluon Drell–Yan processes

#### 3.1 Charged current quark–gluon processes

In the framework of the Drell–Yan process $pp \rightarrow W \rightarrow \ell \nu_\ell$ we have to take into account the presence of gluons in the protons. Therefore we consider on the quark-parton level the attendant processes with incoming gluon $u(p_2) + g(p_5) \rightarrow d(p_1) + \nu_\ell(p_3) + \ell^+(p_4)$, see the Feynman diagrams on the Fig.1 and $d(p_1) + g(p_5) \rightarrow \bar{u}(p_2) + \nu_\ell(p_3) + \ell^+(p_4)$ (similar diagrams).

![Charged current diagrams with coming gluon.](image)

The contributions of these processes do not contain infrared divergences but have quark mass singularities.

The three particle phase space element of the process $u g \rightarrow d \ell^+ \nu_\ell$ can be treated as a cascade analogously to (9) but here gluon $g$ is incoming and $d$ - quark is outgoing. The phase space element of the first step $g(p_5) + u(p_2) \rightarrow W^+(Q') + d(p_1)$ of the cascade is the same as in Eq. (10). Variable $z$ has the same meaning, $z = \frac{s'}{\hat{s}}$ and $s' = -(p_3 + p_4)^2$ is the invariant mass of outgoing leptons, but here $\hat{s} = -2(p_2.p_5)$. Variable $y$ has the same form (11) and $\theta_g$
is the angle between vectors $\vec{p}_1$ and $\vec{p}_5$, but now $\vec{p}_5$ is the momentum of the incoming gluon and $\vec{p}_1$ is the momentum of the outgoing quark.

The phase space of the second step $W^*(Q') \to \nu_\ell(p_3) + \ell^+(p_4)$ of the cascade, obviously, has the same form \[12\].

In our massive quarks treatment in the 4-dimensional phase space we obtain the cross section of the process $u g \to d \nu_\ell \ell^+$ in the form factorized to the Born cross section of the corresponding quark-antiquark process $u d \to \nu_\ell \ell^+$:

$$\sigma_{ug}^{CC} = \frac{\alpha_s}{2\pi} \int_{z_{min}}^{z_{max}} dz \, \sigma_{0}^{CC}(z\hat{s}) \left\{ P_{qg}(z) \left[ \ln \left( \frac{\hat{s}}{m_d^2} \right) + 2 \ln(1-z) - \frac{7}{4} \right] + \frac{9}{8} - \frac{1}{4} z \right\}, \quad (31)$$

where

$$P_{qg}(z) = T_f \left[ z^2 + (1-z)^2 \right] \quad (32)$$

is the quark-gluon splitting function and $T_f = \frac{1}{2}$. We neglected the quark masses except of in the logarithm, where we have a mass of the outgoing quark, namely we face a mass singularity. Integration over $z$ here is up to $z_{max} = \left( 1 - \frac{m_d}{\sqrt{s}} \right)^2 \approx 1$.

In the \textit{MS} scheme with massless quarks working in the $n$-dimensional phase space we obtain a factorized expression:

$$\sigma_{ug}^{CC}(\varepsilon) = \frac{3\alpha_s}{8\pi} \frac{C_F}{(1-\varepsilon)} \int_{z_{min}}^{z_{max}} dz \, \sigma_{0}^{CC}(z\hat{s}, \varepsilon) \left( \frac{4\pi\mu^2}{z\hat{s}} \right)^{\varepsilon} \frac{z^{\varepsilon}(1-z)^{-2\varepsilon}}{\Gamma(1-\varepsilon)} \int_0^1 dy \, y^{-\varepsilon} (1-y)^{-\varepsilon} \times \left\{ \frac{1}{y} \left( z^2 - z + \frac{1}{2}(1-\varepsilon) \right) + z - z^2 + (1-z)\varepsilon + \frac{1}{2} y (1-z)^2 (1-\varepsilon) \right\}. \quad (33)$$

We divided here by $1 - \varepsilon$ because of the number of the gluon spin projections in the $n$-dimensional phase space is $n - 2 = 2(1 - \varepsilon)$. After integration over $y$ we have

$$\sigma_{ug}^{CC}(\varepsilon) = \frac{\alpha_s}{2\pi} \int_{z_{min}}^{z_{max}} dz \, \sigma_{0}^{CC}(z\hat{s}, \varepsilon) \left( \frac{4\pi\mu^2}{z\hat{s}} \right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \frac{z^\varepsilon(1-z)^{-2\varepsilon}}{1-\varepsilon} \times \left\{ P_{qg}(z) \left[ -\frac{1}{\varepsilon} - \frac{3}{4} - \frac{1}{4} \varepsilon \right] + \frac{9}{8} - \frac{1}{4} z + \varepsilon \left( \frac{7}{8} - \frac{3}{4} \varepsilon \right) \right\}. \quad (34)$$

In this expression the collinear divergence appears as a pole $\frac{1}{\varepsilon}$. It has to be compared with the analogous expression \[31\] where the collinear divergence appears as a mass singularity. In the limit $\varepsilon \to 0$ we have

$$-\frac{1}{\varepsilon} \left( \frac{4\pi\mu^2}{z\hat{s}} \right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \frac{z^\varepsilon(1-z)^{-2\varepsilon}}{1-\varepsilon} = -\frac{1}{\varepsilon} - 1 + \ln \left( \frac{\hat{s}}{\mu^2} \right) + 2 \ln(1-z) + O(\varepsilon). \quad (35)$$

In this way we obtain

$$\sigma_{ug}^{CC}(\varepsilon) = \frac{\alpha_s}{2\pi} \int_{z_{min}}^{z_{max}} dz \, \sigma_{0}^{CC}(z\hat{s}) \left\{ P_{qg}(z) \left[ -\frac{1}{\varepsilon} + \ln \left( \frac{\hat{s}}{\mu^2} \right) + 2 \ln(1-z) - \frac{7}{4} \right] + \frac{9}{8} - \frac{1}{4} z \right\}. \quad (36)$$
Comparing with (31) we see that the collinear divergent term which in \( \overline{\text{MS}} \) scheme has to be subtracted from the cross section of the considered process (because it is already included into PDF of gluons) in our treatment has the form

\[
\hat{\sigma}_{\text{CC}ug_{\text{Subtr}}} = \frac{\alpha_s}{2\pi} \int_{z_{\text{min}}}^{1} dz \, \hat{\sigma}_{0}^{\text{CC}}(z\hat{s}) \, P_{qq}(z) \left[ \ln \left( \frac{\mu^2}{m_q^2} \right) \right].
\]

(37)

Here \( m_q \) is the mass of the outgoing quark.

We have the same situation with the process \( g \, \bar{d} \rightarrow \bar{u} \, \nu_\ell \ell^+ \). Analogously we obtain that from the cross section of this process we have to subtract the corresponding expression:

\[
\hat{\sigma}_{\text{CC}dg_{\text{Subtr}}} = \frac{\alpha_s}{2\pi} \int_{z_{\text{min}}}^{1} dz \, \hat{\sigma}_{0}^{\text{CC}}(z\hat{s}) \, P_{qg}(z) \left[ \ln \left( \frac{\mu^2}{m_u^2} \right) \right].
\]

(38)

In this process anti-up quark is outgoing and it can move collinearly with the gluon, so the logarithm is from the mass \( m_u \) of the \( u \)-quark.

### 3.2 Neutral current quark–gluon processes

In the case of the DY processes \( pp \rightarrow [A, Z] \rightarrow \ell^- \ell^+ \) we consider on the quark-parton level the additional process \( q(p_2) + g(p_5) \rightarrow q(p_1) + \ell^- (p_3) + \ell^+ (p_4) \) (corresponding Feynman diagrams are similar to the Fig.1) and process \( \bar{q}(p_1) + g(p_5) \rightarrow \bar{q}(p_2) + \ell^- (p_3) + \ell^+ (p_4) \) with an incoming gluon. The contributions of these processes also do not contain infrared divergences but have quark mass singularities. We considered these processes analogously as the charged current quark–gluon processes. Difference is only in the Born cross section to which they are factorized.

In our massive quarks treatment in the 4-dimensional phase space the cross section of both kind of neutral current quark–gluon processes have equal form:

\[
\hat{\sigma}_{qq}^{\text{NC}} = \frac{\alpha_s}{2\pi} \int_{z_{\text{min}}}^{z_{\text{max}}} dz \, \hat{\sigma}_{0}^{\text{NC}}(z\hat{s}) \left\{ P_{qq}(z) \left[ \ln \left( \frac{\hat{s}}{m_q^2} \right) - 1 + 2 \ln(1 - z) - 3 \right] + \frac{9}{8} - \frac{1}{4} z \right\},
\]

(39)

where \( m_q \) is the mass of incoming as well as outgoing quark (or anti-quark). The upper limit of integration over \( z \) is the same, namely \( z_{\text{max}} = \left( 1 - \frac{m_q^2}{\sqrt{\hat{s}}} \right) \approx 1.0 \). Here we also neglected the quark masses except of under the logarithm.

In the \( \overline{\text{MS}} \) scheme with massless quarks working in the \( n \)-dimensional phase space we obtain the same expressions (33), (34), (36). as those of charged current quark–gluon processes.

Analogously comparing gave us that the collinear divergent term which in \( \overline{\text{MS}} \) scheme has to be subtracted from the cross section of the both considered processes, because is already included into PDF of gluons, in our treatment has a form

\[
\hat{\sigma}_{qq_{\text{Subtr}}}^{\text{NC}} = \frac{\alpha_s}{2\pi} \int_{z_{\text{min}}}^{1} dz \, \hat{\sigma}_{0}^{\text{NC}}(z\hat{s}) \, P_{qq}(z) \left[ \ln \left( \frac{\mu^2}{m_q^2} \right) \right].
\]

(40)
4 Numerical calculations on hadronic level

4.1 Hadronic level kinematics

In the c.m.s. of the quark-quark or quark-gluon pair (see Fig. 2a) we have (neglecting masses of quarks):

\[
p_1^0 = p_2^0 = |\vec{p}_1| = |\vec{p}_2| = \frac{\sqrt{s}}{2}, \quad \vec{p}_2 = -\vec{p}_1, \quad \hat{s} = -2p_1 \cdot p_2. \tag{41}
\]

\[
\theta_q \quad \vec{p}_1 \quad \vec{p}_3 \quad \text{a)}
\]

\[
\theta_q \quad \vec{p}_2 \quad \vec{p}_4
\]

Figure 2: a) Quark c.m.s. b) Proton c.m.s. (Born approximation).

In the c.m.s. of protons \(p, p\) (see Fig. 2b) we have

\[
p_1^0_N = p_2^0_N = |\vec{p}_1^N| = |\vec{p}_2^N| = \frac{\sqrt{s}}{2}, \quad \vec{p}_2^N = -\vec{p}_1^N, \quad s = -2p_1 \cdot p_2
\]

\[
p_1 = x_1 p_1^N, \quad p_2 = x_2 p_2^N, \quad \hat{s} = -2p_1 \cdot p_2 = -2p_1^N \cdot p_2^N x_1 x_2 = s x_1 x_2. \tag{42}
\]

Equations of transition from quarks c.m.s to protons c.m.s. for any momentum have the form

\[
\hat{Q}^0 = \gamma \left(\hat{Q}_N^0 - \beta Q_N^z\right), \quad \hat{Q}^z = \gamma \left(\hat{Q}_N^z - \beta Q_N^0\right), \tag{43}
\]

where

\[
\gamma = \frac{x_1 + x_2}{2\sqrt{x_1 x_2}}, \quad \gamma \beta = \frac{x_1 - x_2}{2\sqrt{x_1 x_2}}. \tag{44}
\]

Replacing the scalar products

\[
p_1 \cdot p_3 = -p_1^0 p_3^0 \left(1 - \cos \theta_q\right), \quad p_2 \cdot p_3 = -p_2^0 p_3^0 \left(1 + \cos \theta_q\right), \tag{45}
\]

where \(\theta_q\) is an angle between the 3-momenta of the quark \(\vec{p}_1^N\) and neutrino \(\vec{p}_3\), from quarks c.m.s. to protons c.m.s.

\[
p_1^N \cdot p_3 = x_1 p_1^N \cdot p_3 = -x_1 p_1^0 p_3^0 \left(1 - \cos \theta_N\right), \quad p_2^N \cdot p_3 = x_2 p_2^N \cdot p_3 = -x_2 p_2^0 p_3^0 \left(1 + \cos \theta_N\right), \tag{46}
\]

and the angle \(\theta_N\) is between 3-momenta of the proton \(\vec{p}_1^N\) and neutrino \(\vec{p}_3\), after some algebra we come to the relation between angles \(\theta_q\) and \(\theta_N\)

\[
\cos \theta_q = \frac{x_2 - x_1 + (x_1 + x_2) \cos \theta_N}{x_1 + x_2 + (x_2 - x_1) \cos \theta_N}. \tag{47}
\]
The same connection we have for the angle $\theta^{14}_q$ between the quark momentum $\vec{p}_1$ and the charged lepton momentum $\vec{p}_4$ and for the angle $\theta^{14}_N$ correspondingly between the proton momentum $\vec{p}_{1N}$ and the charged lepton momentum $\vec{p}_4$. We have only to take into account that for the massive charged lepton we have multiply $\cos \theta_q$ and correspondingly $\cos \theta_N$ by $|\vec{p}_4|/p_4^0$, what is not equal to 1 for a massive charged lepton. The same is valid for the neutral current case when both final leptons are massive.

Working with particle momenta components in the protons c.m.s. we can determine the parameters needed for the presentation of results on a hadronic level:

- the transverse momenta of the charged lepton and the missing transverse momenta of the neutrino.

$$p_{4N}^\perp = \sqrt{(p_{4x}^N)^2 + (p_{4y}^N)^2} \quad \text{and} \quad p_{3N}^\perp = \sqrt{(p_{3x}^N)^2 + (p_{3y}^N)^2}. \quad (48)$$

- the rapidity of the charged lepton

$$\eta = \frac{1}{2} \ln \frac{p_{4N}^0 + p_{4z}^N}{p_{4N}^0 - p_{4z}^N} \quad (49)$$

- the transverse invariant mass of the final leptons

$$M^\perp = \sqrt{2p_{4N}^\perp p_{3N}^\perp (1 - \cos \varphi^3_4)}, \quad (50)$$

where $\varphi^3_4$ is an angle between the final leptons in a plane transversal to $z$ axis.

### 4.2 Integration over variables $x_1$ and $x_2$

We have to integrate over variables $x_1$ and $x_2$ the whole contribution to the cross section of the DY charged current as well as neutral current processes. The leading order contribution is

$$\sigma^{CC}_{LO} = \sum_{q_1 q_2} \int_0^1 dx_1 f(x_1, \mu^2) \int_0^1 dx_2 f(x_2, \mu^2) \hat{\sigma}^{CC}_0(x_1 x_2 s),$$

$$\sigma^{NC}_{LO} = \sum_q \int_0^1 dx_1 f(x_1, \mu^2) \int_0^1 dx_2 f(x_2, \mu^2) \hat{\sigma}^{NC}_0(x_1 x_2 s). \quad (51)$$

In the next to leading order (NLO) we have to add one-loop corrections: hard gluon contribution, virtual and soft gluon contribution and corresponding subtractions of quark-antiquark processes and also the contribution of quark-gluon processes with their subtractions.

$$\sigma^{CC}_{NLO} = \sum_{q_1 q_2} \int_0^1 dx_1 f(x_1, \mu^2) \int_0^1 dx_2 f(x_2, \mu^2) \left[ \hat{\sigma}^{CC}_0(x_1 x_2 s) \right.$$

$$+ \hat{\sigma}^{CC}_{\text{Hard}}(x_1 x_2 s) - \hat{\sigma}^{CC}_{H_{\text{Subtr}}}(\mu^2, x_1 x_2 s) + \hat{\sigma}^{CC}_{\text{SoftVirt}}(x_1 x_2 s) - \hat{\sigma}^{CC}_{S_{\text{Subtr}}}(\mu^2, x_1 x_2 s) \left.$$
The subtractions are needed because the $\overline{MS}$ scheme collinear divergent terms are already included into the quark distribution functions $f(x_1, \mu^2)$ and $f(x_2, \mu^2)$ and into the gluon distribution function $g(x_5, \mu^2)$. For neutral current processes we have the same formula.

Let us take into consideration the following expression with quark-antiquark hard and soft-virtual subtraction terms:

\[
\int_0^1 dx_1 \int_0^1 dx_2 f(x_1, \mu^2) f(x_2, \mu^2) \left[ \hat{\sigma}^{CC}(x_1 x_2 s) - \hat{\sigma}^{CC}_{H_{\text{subtr}}}(\mu^2, x_1 x_2 s) - \hat{\sigma}^{CC}_{SV_{\text{subtr}}}(\mu^2, x_1 x_2 s) \right]
\]

\[
= \int_0^1 dx_1 \int_0^1 dx_2 \hat{\sigma}^{CC}(x_1 x_2 s) \left\{ f(x_1, \mu^2) f(x_2, \mu^2) - \frac{\alpha_s}{2\pi} f(x_1, \mu^2) \int_{x_1}^1 dz f \left( \frac{x_1}{z}, \mu^2 \right) \right\}
\]

\[
\times \left[ P_{qq}(z) \left( \ln \left( \frac{\mu^2}{m_f^2} \right) - 1 - 2 \ln(1 - z) \right) \right]_+ - \frac{\alpha_s}{2\pi} f(x_2, \mu^2) \int_{x_2}^1 dz f \left( \frac{x_2}{z}, \mu^2 \right) \right\}. \tag{53}
\]

We used the “+” prescription because we have here difference $\hat{\sigma}^{CC}_0(z, x_1, x_2, s) - \hat{\sigma}^{CC}_0(1, x_1, x_2, s)$.

One can note that we can apply subtractions to the PDF instead of the cross section:

\[
f(x_1, \mu^2) \rightarrow f(x_1, \mu^2) - \frac{\alpha_s}{2\pi} \int_{x_1}^1 dz f \left( \frac{x_1}{z}, \mu^2 \right) \left[ P_{qq}(z) \left( \ln \left( \frac{\mu^2}{m_f^2} \right) - 1 - 2 \ln(1 - z) \right) \right]_+. \tag{54}
\]

The same manipulation can be done for the neutral current processes. In practical applications, if the subtraction is applied to PDFs, one has to take care on spurious $\mathcal{O}(\alpha_s^2)$ contribution. The can be done by means of linearization procedure described in Ref. [12].

We have a possibility to integrate numerically over three angles and over the independent variables $z, x_1$ and $x_2$ to obtain $\hat{\sigma}^{CC}_{\text{Hard}}(x_1 x_2 s), \hat{\sigma}^{CC}_{qg}(x_1 x_5 s)$ and $\hat{\sigma}^{CC}_{qg}(x_2 x_5 s)$.

We introduced a new variable $W_x = x_1 x_2$ when integrated the hard gluon contribution of the quark-antiquark process to obtain $\hat{\sigma}^{CC}_{\text{Hard}}(x_1 x_2 s)$. If we take $x_2, Wx$ for the set of independent variables, the Jacobian of the transition is equal to $\frac{1}{x_2}$. So we have the following transition of the integrals:

\[
\int_0^1 dx_1 \int_0^1 dx_2 f(x_1) f(x_2) \quad \Rightarrow \quad \int_0^1 dW_x \int_0^1 dx_2 \frac{1}{x_2} f \left( \frac{W_x}{x_2} \right) f(x_2). \tag{55}
\]

When we integrated the hard gluon contribution of the quark-gluon process to obtain $\hat{\sigma}^{CC}_{qg}(x_1 x_5 s)$ or $\hat{\sigma}^{CC}_{qg}(x_2 x_5 s)$ we introduced a new variable $W_y = x_1 x_5 z$ or $W_y = x_2 x_5 z$. In this case we have the following transition of the integrals:

\[
\int_0^1 dx_1 \int_0^1 dx_5 f(x_1) g(x_5) \int_{x_5}^{2\text{max}} dz \quad \Rightarrow \quad \int_0^1 dW_y \int_0^1 dx_5 \int_{x_5}^{2\text{max}} dz \frac{1}{x_5 z} f \left( \frac{W_y}{x_5 z} \right) g(x_5). \tag{56}
\]
5 Conclusions and Numerical Results

For the sake of comparison with MCFM [11] for numerical evaluations we used the following set of input parameters:

\[
\begin{align*}
G_F &= 1.16639 \times 10^{-5} \text{ GeV}^{-2}, & \alpha(0) &= 1/137.03599911, \\
\alpha_s(M_Z) &= 0.130, & \alpha_s(M_W) &= 0.1326, \\
M_W &= 80.419 \text{ GeV}, & \Gamma_W &= 2.06 \text{ GeV}, \\
M_Z &= 91.188 \text{ GeV}, & \Gamma_Z &= 2.49 \text{ GeV}, \\
M_H &= 115 \text{ GeV}, & m_t &= 170.9 \text{ GeV}, \\
m_u &= m_d = 66 \text{ MeV}, & m_c &= 1.5 \text{ GeV}, \\
m_s &= 150 \text{ MeV}, & m_b &= 4.62 \text{ GeV}, \\
|V_{ud}| &= |V_{cs}| = 0.975, & |V_{us}| &= |V_{cd}| = 0.222.
\end{align*}
\]

(57)

The CTEQ6L1 [21] set of PDF was used with the factorization scales being equal to $M_Z$ and $M_W$ for the NC and CC cases, respectively. The following cuts on the final state kinematics were applied:

\[
P_t > 25 \text{ GeV}, \quad M_{ll} > 20 \text{ GeV}, \quad \eta < 1.2,
\]

(58)

where $P_t$ is the transverse momentum of a lepton, $M_{ll}$ is the invariant mass of a charged lepton pair (only for NC), and $\eta$ is the pseudo-rapidity of a charged lepton.

In numerical evaluations we used an adaptive Monte Carlo integrator based on the VEGAS algorithm [22]. For the partonic sub-process cross sections we used the standard SANC FORTRAN modules which can either produced interactively by the system or just downloaded from the SANC webpage [4]. These modules are described in Ref. [23].

In Fig. 3 we show comparison of the SANC results with the MCFM ones for the transverse momentum distribution of $\mu^+$ in the charged current DY process at LHC. Fig. 4 shows the corresponding comparison of results for the $\mu^+\mu^-$ invariant mass distribution in the NC case. Note that the deeps in the first bins of both the distributions are not physical, they appeared due to kinematical cuts imposed just at the left borders. Application of the PDF factorization in SANC and MCFM are performed in different schemes: the scheme with massive quarks in SANC (as described above) and the \(\overline{\text{MS}}\) scheme with massless partons in MCFM. We see that for the given distributions the difference between these schemes is not numerically important.

For a realistic application, one has to take into account also QCD showers. It can be done with help of the standard packages like PYTHIA [24] and HERWIG [25]. Note that the showers will wash out the negatively weighted events, which can be seen in the resonance region in Fig. 3.

In this way we presented in detail the evaluation of NLO QCD corrections to Drell–Yan like processes. It is important that we performed it in the environment of the SANC system, so that now we have a self-consistent simultaneous treatment of QCD and electroweak radiative corrections the the DY processes. It is required for the forthcoming experiments at the LHC. Simultaneous implementation of the electroweak and NLO QCD corrections to

\[\text{The LO PDF were used just for the comparison. For practical applications of the described results NLO PDF should be chosen.}\]
Drell-Yan processes received with help of the SANC into a Monte Carlo event generator will be described elsewhere \cite{26}.

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