An Embedding of the BV Quantization into an N=1 Local Superfield Formalism

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Abstract

We propose an $N=1$ superfield formulation of Lagrangian quantization in general hypergauges by extending a reducible gauge theory to a superfield model with a local dependence on a Grassmann parameter $\theta$. By means of $\theta$-local functions of the quantum and gauge-fixing actions in terms of Darboux coordinates on the antisymplectic manifold, we construct superfield generating functionals of Green’s functions, including the effective action. We prove the gauge-independence of the S-matrix, obtain the Ward identities and establish a relation of the proposed local quantization with the BV method and the multilevel Batalin–Tyutin formalism.

1 Introduction

The quantization of gauge theories on the basis of BRST symmetry \cite{1} is usually carried out in the Hamiltonian \cite{2} or Lagrangian \cite{3} schemes, which were recently given a superfield description \cite{4,5} based on nontrivial \cite{4} and trivial \cite{5} relations between the even $t$ and odd $\theta$ components of superfields $\Phi$. These works realize a geometric interpretation of BRST transformations in terms of supertranslations, which originally provided a basis for the superspace formulation \cite{6} of quantum theories of Yang–Mills type \cite{7}, and, in a larger context, were applied to a classical and quantum description \cite{8,9,10} of generalized Poisson sigma-models \cite{11} and $D=1$ sigma-models with an arbitrary $N \geq 1$ number of Grassmann coordinates, as well as to a construction \cite{10} of the partition function with $N=2$ (for more details, see \cite{12}).

The Lagrangian formalism \cite{5} is a superfield modification of the BV method including non-Abelian hypergauges \cite{13}. The formalism \cite{5} provides a relatively complete insight into superfield quantization based on the properties of solutions to the generating equations; however, it does not indicate a detailed relation between these solutions and a gauge theory. It is therefore natural to complement the formalism by an explicit superfield description of the gauge algebra for a given model. This problem has so far remained open. Thus, the definition of a classical action of superfields, $A^i(\theta) = A^i + \lambda^i \theta$, on a superspace with coordinates $(x^\mu, \theta)$, $\mu = 0, \ldots, D-1$, as an integral of a nontrivial\textsuperscript{1} odd density $L(x, \theta)$ is a question for every given model. As a consequence, the vacuum functional $Z$ and generating functional of Green’s functions $Z[\Phi^\ast]$ of \cite{5} exhibit a peculiarity. Namely, these objects differ from their counterparts of the BV \cite{3} and Batalin–Tyutin \cite{13} methods, which is implied by a dependence of the gauge fermion $\Psi[\Phi]$ and quantum action $S[\Phi, \Phi^\ast]$ on the components $\lambda^A$ of superfields $\Phi^A(\theta)$ in the multiplet $(\Phi^A, \Phi^\ast_A(\theta)) = (\phi^A + \lambda^A \theta, \phi^\ast_A - \theta J_A)$, where $(\phi^A, \phi^\ast_A, \lambda^A, J_A)$ constitute the complete set of variables in the BV formalism.

The aim of this paper is to propose an $N=1$ local superfield Lagrangian quantization in which the quantities of an initial classical theory are realized through a $\theta$-local superfield model (LSM). Note that we adopt a terminology consistent with that of the papers \cite{14}, in which the quantization \cite{4} with a single fermion supercharge $Q(t, \theta)$ containing the BRST charge and unitarizing Hamiltonian was extended to $N=2$ (non-spacetime) supersymmetries, and then to an arbitrary number of supercharges, $Q^k(t, \theta^1, \ldots, \theta^N)$.

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\textsuperscript{1}A trivial density $L(x, \theta)$ is understood in the form $\int d^Dx d\theta L(x, \theta) = \int d\theta S_0(A(\theta)) = S_0(A)$, where $S_0(A)$ is a usual classical action.
whose range includes permits us to circumvent the mentioned peculiarity of the functions as following problems: We develop a superfield effective action we introduce a underlying the gauge-independence of the S-matrix. For the first time within superfield quantization, with first-class constraints of a higher stage of reducibility. An HS constructed from formulations of an arbitrary reducible LSM proposed in [15] for irreducible gauge theories (with bosonic classical fields and gauge parameters) in terms of a BRST charge related to a formal dynamical system with first-class constraints of a higher stage of reducibility. An HS constructed from \(\theta\)-local quantities (a quantum action, a gauge-fixing action, and an arbitrary bosonic function) encodes, through a \(\theta\)-local antibracket, both BRST and anticanonical-like transformations in terms of a universal set of equations underlying the gauge-independence of the S-matrix. For the first time within superfield quantization, we introduce a superfield effective action (also in the case of non-Abelian hypergauges). We establish a relation of the proposed local quantization with the BV and Batalin–Tyutin methods [3, 13], as well as with the superfield formalism [5].

We use DeWitt’s condensed notation and the conventions of [5]. As usual, the rank of an even \(\theta\)-local supermatrix \(M(\theta)\) with \(Z_2\)-grading \(\varepsilon\) is characterized by a pair of numbers \(m = (m_+, m_-)\), where \(m_+\) \((m_-)\) is the rank of the Bose–Bose (Fermi–Fermi) block of the \(\theta\)-independent part of the supermatrix \(M(\theta)\), \(\text{rank}[M(\theta)] = \text{rank}[M(0)]\). With respect to the same Grassmann parity \(\varepsilon\), we understand the dimension of a smooth supersurface, also supercharacterized by a pair of numbers in the sense of the definition [16] of a supermanifold, so that the pair \((m_+, m_-)\) coincides with the corresponding numbers of the Bose and Fermi components of \(z^i(0)\), being the \(\theta\)-independent parts of local coordinates \(z^i(\theta)\) parameterizing this supersurface.

2 Classical Description of a \(\theta\)-local Superfield Model

In this section, we propose odd Lagrangian and Hamiltonian descriptions of an LSM as extensions of a usual model of classical fields \(\lambda^I\), \(i = 1, \ldots, n = n_+ + n_-\), to \(\theta\)-local theories defined on the respective tangent \(\Pi T M_{\text{CL}} = \{A^I, \partial_\theta A^I\}\) \((\theta)\) and cotangent \(\Pi^* M_{\text{CL}} = \{\Gamma^I_{\text{CL}} = (A^I, A^*_I)\}\) \((\theta)\), \(I = 1, \ldots, N = N_+ + N_-\) odd bundles; \(^2(n_+, n_-) \leq (N_+, N_-)\). The superfields \((A^I, \partial_\theta A^I)(\theta)\) and superantifields \(A^*_I(\theta)\), \(A^*_I(\theta) = (A^I - \theta J_I)\) are defined in a superspace \(\mathcal{M} = \mathcal{M} \times P\) parametrized by \((z^M, \theta)\), where the coordinates \(z^M \in I \) include Lorentz vectors and spinors of the superspace \(\mathcal{M}\). The basic objects of the odd Lagrangian and Hamiltonian formulations of an LSM are Lagrangian and Hamiltonian actions, \([S_L, S_H]\): \([\Pi T M_{\text{CL}} \times \{\theta\}, \Pi^* M_{\text{CL}} \times \{\theta\}] \rightarrow \Lambda^1(\theta; \mathbb{R})\), being a respective \(C^\infty(\Pi T M_{\text{CL}}-\) and \(C^\infty(\Pi^* M_{\text{CL}}-)\)-functions taking values in a real Grassmann algebra \(\Lambda_1(\theta; \mathbb{R})\). The actions determine the respective functionals \(Z_L[A], Z_H[\Gamma_k]\), whose \(\theta\)-densities are defined with accuracy up to arbitrary functions \(g((A, \partial_\theta A)(\theta), \theta), g_1((A, \lambda A^I)(\theta), \theta)) \in \ker(\partial_\theta)\), \(\xi(g) = \xi(g_1) = 0\) \((d\theta = \partial_\phi \equiv \partial_\theta)\)

\[Z_L[A] = \partial_\theta S_L(\theta), \quad Z_H[\Gamma_k] = \partial_\theta \left[ V^0_k(\Gamma(\theta)) \partial_\theta \Gamma^P_k(\theta) - S_H(\Gamma_k(\theta), \theta) \right], \quad k = \text{CL}
\]

\[\xi(L) = \xi(S_H) = \xi(\theta) = (1, 0, 1), \quad \xi(S_L) = \xi(S_H) = 0\]

\(^2I\) denotes the operation that changes the coordinates of a (co)tanget fiber bundle \(T^{(*)} M_{\text{CL}}\) over a configuration \(A^I\) into the coordinates of the opposite Grassmann parity, whereas \(N_+, N_-\) are the numbers of bosonic and fermionic fields, among which there may be superfields corresponding to the ghosts of the minimal sector in the BV quantization scheme.
where $Z_H[\Gamma_k]$ is expressed in terms of an antisymplectic metric, $V_k^P(\Gamma(\theta)) = 1/2(\Gamma^Q \omega_k^{QP})(\theta)$, related to a flat antisymplectic metric $\omega_k^{PQ}(\theta)$ and an odd Poisson bivector $\omega_k^{PQ}(\theta)$: $\omega_k^{PQ}(\theta) = \delta^P_Q$, $\omega_k^{Q\theta}(\theta) = (\Gamma_k^\theta(\theta), \Gamma_k^\theta(\theta))_\theta$. The values $\varepsilon = (\varepsilon_P, \varepsilon_J, \varepsilon) = \varepsilon_P + \varepsilon_J$, of a $Z_2$-grading introduced in \cite{17}, with the auxiliary components $\varepsilon_J, \varepsilon_P$ related to the respective coordinates $(z^M, \theta)$, are defined by $\varepsilon(A^I) = (\varepsilon_P)I, (\varepsilon_J)I, (\varepsilon)I = \varepsilon(A_I^I) + (1, 0, 1)$. Note that $\mathcal{M}$ can be realized as the quotient of a symmetry supergroup $J = J \times U$ for the functional $Z_k[\mathcal{A}]$, where $\mu$ and $\mu_0$ are a nilpotent parameter and a generator of $\theta$-translations, whereas $J$ is chosen as a spacetime SUSY group. The quantities $\varepsilon_J, \varepsilon_P$ are the Grassmann parities of coordinates in some representation spaces of $J$, $P$. These objects are introduced for a correct spin-statistic relation in operator quantization.

Due to a $J$-scalar nature of $Z_k[\mathcal{A}], Z_k[\Gamma_k]$ it is only $[S_L, S_H]$, among $[S_L, S_H], [Z_k[\mathcal{A}], Z_k[\Gamma_k]]$, invariant under a $J$-superfield representation $T$ restricted to $J$, $T_J$, that transform nontrivially with respect to the total representation $T$ under $\mathcal{A}^I(\theta) \to \mathcal{A}^I(\theta)(\theta, \mu)$, for instance,

$$\delta S_L(\theta) = S_L(\mathcal{A}(\theta), \partial_\theta \mathcal{A}(\theta), \theta) - S_L(\theta) = -\mu [\partial/\partial \theta + P_0(\theta)(\partial U(\theta))] S_L(\theta).$$

Here, we have introduced the nilpotent operator $(\partial U(\theta)) = \partial_\theta \mathcal{A}(\theta) \partial_\theta \mathcal{A}(\theta) = [\partial_\theta, U(\theta)]$, $U(\theta) = P_1(\theta) \mathcal{A}(\theta) \partial_\theta / \partial_\theta \mathcal{A}(\theta)$, and a set of projectors onto $C^\infty(\Pi^{(\varepsilon)} \mathcal{M}_{CL}) \times \{\theta\}$, $\{P \theta(\theta) = \delta_{\theta \theta}(1 - \theta \theta) + \delta_{\theta \theta} \theta \theta$, $a = 0, 1\}$.

Assuming the existence of critical configurations for $Z_H[\Gamma_k], Z_k[\mathcal{A}]$, we present the HS dynamics through a $\theta$-local antibracket, and the LS dynamics in terms of superfield Euler–Lagrange equations,

$$\partial_\theta \Gamma^\theta_{CL}(\theta) = \{\Gamma^\theta_{CL}(\theta), S_H(\theta)\}_\theta;$$
$$\delta Z_k[\mathcal{A}] / \delta \mathcal{A}^I(\theta) = \left[ \partial_\theta / \partial_\theta \mathcal{A}(\theta) - (-1)^{\varepsilon_I} \partial_\theta / \partial_\theta \mathcal{A}(\theta) \right] S_L(\theta) \equiv \mathcal{L}^I(\theta) S_L(\theta) = 0,$$

where the latter system is equivalent (since $\partial_\theta \mathcal{A}(\theta) \equiv 0$ to an LS characterized by $2N$ formally second-order differential equations in $\theta$:

$$\partial_\theta \mathcal{A}(\theta)^I / \partial \theta \mathcal{A}(\theta)^I \equiv \partial_\theta \mathcal{A}(\theta)^I(S^I_{\mathcal{L}}(\theta) = 0),$$

$$\Theta_I(\theta) = \partial_\theta \mathcal{L}^I(\theta) / \partial \theta \mathcal{A}(\theta)^I = (-1)^{\varepsilon_I} \left[ \partial_\theta \partial_\theta \mathcal{A}(\theta)^I / \partial_\theta \mathcal{A}(\theta)^I \right] = 0.$$ An equivalence of the two descriptions is implied by the nondegeneracy of the supermatrix $||S^I_{\mathcal{L}}(\theta)\|_\theta$ in \cite{12}, under a Legendre transformation of $S_L(\theta)$ with respect to $\partial_\theta \mathcal{A}(\theta)^I$, $S_H(\Gamma_{CL}(\theta), \theta) = \mathcal{A}^I(\theta) \partial_\theta \mathcal{A}(\theta)^I - S_L(\theta)$, $\mathcal{A}^I(\theta) = \partial / \partial \theta \mathcal{A}(\theta)^I S_L(\theta)$.

In this case, the equivalence of an LS and HS is guaranteed by the respective settings $(\theta = 0, k = \text{CL})$ of the Cauchy problem for integral curves $\mathcal{A}(\theta)$ and $\Gamma_k^\theta(\theta)$, modulo the continuous part of $I$,

$$\left( \mathcal{A}(\theta), \partial_\theta \mathcal{A}(\theta)^I \right)(0) = \left( \mathcal{A}, \mathcal{A}^2_{\theta} \mathcal{A}^2 \right), \Gamma_k^\theta(0) = \left( \mathcal{A}, \mathcal{A} \right): \mathcal{A} = P_0 \left[ \partial_\theta \mathcal{L}(\theta) / \partial_\theta \mathcal{A}(\theta)^I \right] (\mathcal{A}, \mathcal{A}^2_{\theta}).$$

The Lagrangian constraints $\Theta_I(\theta) = \Theta_I(\mathcal{A}(\theta), \partial_\theta \mathcal{A}(\theta), \theta)$ identical to half the HS equations, $\Theta_I(\theta) = -\left( \partial_\theta \mathcal{A}(\theta)^I + S_H(\theta) \right) (-1)^{\varepsilon_I}$, in view of transformations \cite{12}, rest the setting of the Cauchy problem for an LS and HS, and may be functionally dependent as first-order equations in $\theta$.

On condition that there exists (at least locally) a supersurface $\Sigma \subset \mathcal{M}_{CL}$ such that

$$\Theta_I(\theta)|_\Sigma = 0, \dim \Sigma = \overline{\mathcal{M}}, \text{rank } \left[ \mathcal{L}_I(\theta) \right] \left[ \mathcal{L}_I(\theta) S_L(\theta)(-1)^{\varepsilon_I} \right] \|_\Sigma = \overline{\mathcal{M}} - \mathcal{M},$$

there exist $M = M_+ + M_-$ independent identities:

$$\partial_\theta \left[ \delta Z_k[\mathcal{A}] / \delta \mathcal{A}(\theta) \right] \mathcal{A}^I_{\theta_0}(\theta; \theta_0) = 0, \mathcal{A}^I_{\theta_0}(\theta; \theta_0) = \sum_{k \geq 0} \left( \partial_\theta \right)^k \left( \theta - \theta_0 \right) \mathcal{R}^I_{\theta_0}(\mathcal{A}(\theta), \partial_\theta \mathcal{A}(\theta), \theta).$$

The generators $\mathcal{R}^I_{\theta_0}(\theta; \theta_0)$ of general gauge transformations,

$$\delta_{\theta} \mathcal{A}^I(\theta) = \partial_\theta \left[ \mathcal{R}^I_{\theta_0}(\theta; \theta_0) \chi_{\theta_0}(\theta) \right], \varepsilon(\chi_{\theta_0}) = \varepsilon_{\theta_0}, \chi_{\theta_0} = 1, ..., M = M_{\theta_0} + M_{\theta_0}$$. 
that leave $Z_L[A]$ invariant are functionally dependent on the assumption of locality and $\bar{J}$-covariance, provided that rank $\left\|\sum_{k=0}^{\infty} \bar{R}_k^{J\alpha_0}(\theta)(\partial^k)\right\|_\Sigma = N - M$. The dependence of $\bar{R}_k^{J\alpha_0}(\theta;\theta_0)$ implies the existence (on solutions of an LS) of proper zero-eigenvalue eigenvectors, $\hat{Z}_{A_0}^{\theta}(A(\theta_0), \partial_0 A(\theta_0), \theta_0; \theta_1)$, with a structure analogous to $\hat{R}_k^{J\alpha_0}(\theta;\theta_0)$ in (9), which exhaust the zero-modes of the generators and are dependent in case rank $\left\|\sum_k \hat{Z}_{A_0}^{\theta}(\theta_0)(\partial^k)\right\|_\Sigma = N - M < N_1$. As a result, the dependence relations for eigenvectors that define a general $L_g$-stage reducible LSM are given by

$$\int d\theta \hat{Z}_{A_{s-1}}^{\alpha_{s-2}}(\theta_{s-2}; \theta') \hat{Z}_{A_s}^{\alpha_1}(\theta'; \theta_s) = \int d\theta' \Theta_f(\theta') L_{\bar{A}_s}^{\alpha_{s-2}J}(\bar{\theta} A(\theta_{s-2}), \theta_{s-2}; \theta_s),$$

$$M_{s-1} > \sum_{k=0}^{s-1} (-1)^k M_{s-2} = \text{rank} \left\|\sum_{k=0}^{\infty} \hat{Z}_{A_{s-1}}^{\alpha_{s-2}}(\theta_{s-2})(\partial_{\theta_{s-2}}^k)\right\|_\Sigma,$$

$$M_{L_g} = \sum_{k=0}^{L_g} (-1)^k M_{L_g-k-1} = \text{rank} \left\|\sum_{k=0}^{\infty} \hat{Z}_{A_{L_g-k}}^{\alpha_{L_g-k}}(\theta_{L_g-k-1})(\partial_{\theta_{L_g-k-1}}^k)\right\|_\Sigma,$$

$$\varepsilon(\hat{Z}_{A_{s-1}}) = \varepsilon_{A_s} + \varepsilon_{A_{s-1}} + (1, 0, 1), \hat{Z}_{A_0}^{\theta} = \bar{R}_k^{J\alpha_0}(\theta; \theta_0),$$

$$L_{\bar{A}_s}^{\alpha_{s-1}J}(\theta_{1}, \theta'; \theta_1) \equiv K_{\bar{A}_s}^{\alpha_{s-1}J}(\theta_{1}, \theta'; \theta_1) = (-1)^{\varepsilon_{s-1}(\varepsilon_{s-1})} K_{\bar{A}_s}^{\alpha_{s-1}J}(\theta'; \theta_1).$$

for $s = 1, \ldots, L_g$, $A_s = 1, \ldots, M_s = M_s + M_{s-1}, M \equiv \overline{M}_1 - 1$. For $L_g = 0$, an LSM is an irreducible general gauge theory.

For an LSM of the form $S_L(\theta) = T(\partial_0 A(\theta)) - S(A(\theta), \theta)$, the functions $\Theta_f(\theta), \Theta_I(\theta) \in \mathcal{M}_{CL} \times \{\theta\}$, take the form of the usual extremals $\Theta_f(\theta) = -S_f(A(\theta), \theta) - \bar{M}(\theta) = 0$ for the functional $S_f(A(\theta)) = S(A(0), 0)$ corresponding to $\theta = 0$. Condition (E) and identities (D) have a form usual for $\theta = 0$, with linearly-dependent (for $M_0 > M$) generators of special gauge transformations, $\delta A^I(\theta) = R_{A_0}^{J\alpha_0}(A(\theta), \theta) \xi_{A_0}^{\alpha_0}(\theta)$, which leave invariant only $S(\theta)$, in contrast to $T(\theta)$. The dependence of $R_{A_0}^{J\alpha_0}(\theta)$, as well as of their zero-eigenvalue eigenvectors $\hat{Z}_{A_0}^{\theta}(A(\theta), \theta)$, and so on, can also be expressed by special relations of reducibility for $s = 1, \ldots, L_g$, namely,

$$Z_{A_{s-1}}^{\alpha_{s-2}}(\theta_0) Z_{A_s}^{\alpha_1}(\theta_0, \theta) = S_{IJ}(\theta_0) L_{\bar{A}_s}^{\alpha_{s-2}J}(\bar{\theta}_A(\theta_0), \theta), \varepsilon(Z_{A_{s-1}}) = \varepsilon_{A_{s-1}} + \varepsilon_{A_s},$$

$$Z_{A_0}^{\alpha} = R_k^{J\alpha_0}(\theta), \hat{L}_{\bar{A}_s}^{\alpha_{s-1}J}(\theta_1, \theta'; \theta_1) \equiv K_{\bar{A}_s}^{\alpha_{s-1}J}(\theta_1, \theta'; \theta_1) = (-1)^{\varepsilon_{s-1}(\varepsilon_{s-1})} K_{\bar{A}_s}^{\alpha_{s-1}J}(\theta'; \theta_1).$$

In case $M_{L_g} = \sum_{k=0}^{L_g} (-1)^k M_{L_g-k-1} = \text{rank} \left\|Z_{A_{L_g-k}}^{\alpha_{L_g-k}}\right\|_\Sigma$, we shall refer to (11) and (12) as a special gauge theory of $L_g$-stage reducibility. The gauge algebra of such a theory is $\theta$-locally embedded into the gauge algebra of a general gauge theory with the functional $Z[A] = \partial_0(T(\theta) - S(\theta))$, which leads to a relation between the eigenvectors,

$$\hat{Z}_{A_0}^{\alpha_0}(\theta_{s-1}, \theta_{s-1}; \theta_s) = -\delta(\theta_{s-1} - \theta_s) \hat{Z}_{A_s}^{\alpha_{s-1}}(A(\theta_{s-1}), \theta_{s-1}),$$

and to a possible parametric dependence of structure functions on $\partial_0 A^I(\theta)$. For special gauge theories in the Hamiltonian formulation, definitions (11) and (12) retain their form; while for general gauge theories of $L_g$-stage reducibility definitions (8) and (9) are transformed:

$$\hat{Z}_{A_s}^{\alpha_{s-1}}(\Gamma_k(\theta_{s-1}), \theta_{s-1}; \theta_s) = \hat{Z}_{A_s}^{\alpha_{s-1}}(A(\theta_{s-1}), \partial_0 \Gamma_k(\theta_{s-1}), \theta_{s-1}, \theta_{s-1}; \theta_s), s = 0, \ldots, L_g.$$
Provided that $S_{H}(\theta)$ or $S_{L}(\theta)$ do not depend on $\theta$ explicitly, \[(16)\] yields the equations $(S_{H}(\theta), S_{H}(\theta))_{\theta} = 0$ or $(\partial_{\theta}U)(\theta)S_{L}(\theta)|_{L(\theta)} = 0$, having no analogy in a $t$-local field theory and implying the condition \[(3)\] that $S_{H}(\theta)$ or $S_{L}(\theta)$ be proper, however for an LSM on the classical level. Then the $\theta$-superfield integrability of the HS in \[(3)\] is implied by the properties of the antibracket, in particular, the Jacobi identity,

\[
(\partial_{\theta})^{2}T_{\theta}^{\mu}(\theta) = \frac{1}{2} \left( \Gamma_{\theta}^{\mu}(\theta), \left( S_{H}(\Gamma_{\theta}(\theta)), S_{H}(\Gamma_{\theta}(\theta)) \right)_{\theta} = 0. \tag{17}\]

This yields a $\theta$-translation formula and the nilpotency of the BRST-like generator $\tilde{s}_{l}(\theta)$ of $\theta$-shifts along the $(\varepsilon_{\mu}, \varepsilon)$-odd vector field $Q(\theta) = (S_{H}(\theta), \cdot)_{\theta}$

\[
\delta_{\mu}F(\theta)|_{\Gamma_{\theta}(\theta)} = \mu [\partial/\partial \theta - Q(\theta)]F(\theta) \equiv \mu \tilde{s}_{l}(\theta)F(\theta). \tag{18}\]

Depending on additional properties (see Section 3) of a gauge theory, we shall suppose

\[
\Delta^{k}(\theta)S_{H}(\theta) = 0, \quad \Delta^{k}(\theta) \equiv \frac{1}{2} (-1)^{\varepsilon} \omega_{Q_{P}}^{k}(\theta) \left( \Gamma_{\theta}^{\mu}(\theta), \left( \Gamma_{\theta}^{Q}(\theta), \cdot \right)_{\theta} \right)_{\theta}. \tag{19}\]

which is equivalent to a vanishing antisymplectic divergence of $Q(\theta)$, $(\partial_{\theta}/\partial \theta)Q(\theta) = 2\Delta(\theta)S_{H}(\theta) = 0$, which holds trivially for its symplectic counterpart. The Hamiltonian master equation $(S_{H}(\theta), S_{H}(\theta))_{\theta} = 0$ for $(\partial/\partial \theta)S_{H}(\theta) = 0$ explains the interpretation of the equivalent equation in \[(16)\], for $(\partial/\partial \theta)S_{L}(\theta) = 0$, $(\partial_{\theta}U)(\theta)S_{L}(\theta)|_{L_{1}S_{1}=0} = 0$, as a Lagrangian master equation.

### 3 Superfield Quantization

#### 3.1 Superfield Construction of a Local Quantum Action

Here, the reducibility relations of a restricted special LSM are transformed into new gauge transformations for ghost superfields. Along with the gauge transformations of $A'(\theta)$ extracted from $A'(\theta)$, the new gauge transformations imply a Hamiltonian system related to an initial restricted HS and leading to quantum and gauge-fixing actions subject to respective $\theta$-local master equations. With the standard distribution of ghost number \[(3)\] $gh(A_{1}) = -1$ and $gh(A_{1}) = -1$ and the choice $gh(\theta, \partial_{\theta}) = (-1, 1)$ implying the absence of ghosts among $A_{1}$, $(\varepsilon_{\mu})_{I} = 0$, the quantization is firstly given by

\[
(gh, \partial/\partial \theta)S_{H(L)}(\theta) = (0, 0). \tag{20}\]

Assuming the existence in $S_{H(L)}(\theta)$ of a potential term, $S(A(\theta), 0) = S(A(\theta))$, a solution of \[(21)\] extracts a usual gauge theory with a classical action $S_{0}(A)$, where $A'$ are extended to $A'(\theta)$. The generalized HS in \[(3)\] then transforms into a $\theta$-integrable system on $\text{III}^{*}M_{c_{1}} = \{ \Gamma_{c_{1}}^{p}(\theta) \} = \{ (A', \xi^{A}(\theta)) \}$ with $\Theta(A(\theta)) \in M_{c_{1}},$

\[
\partial_{\mu} \Gamma^{p}_{c_{1}}(\theta) = (\Gamma_{c_{1}}^{p}(\theta), S_{0}(A(\theta)))_{\theta}, \Theta(A(\theta)) = -(1)^{\varepsilon}S_{0; i_{1}}(A(\theta)). \tag{21}\]

The restricted special gauge transformations $\delta A'(\theta) = R_{\alpha_{0}}^{i} (\varepsilon_{\alpha_{0}}(\theta), \xi^{\alpha_{0}}(\theta)) = \varepsilon_{\alpha_{0}}$, $(\varepsilon_{\mu})_{\alpha_{0}} = 0$, are embedded by $\xi^{\alpha_{0}}(\theta) = d\xi^{\alpha_{0}}(\theta) = C^{\alpha_{0}}(\theta)d\theta$, $\alpha_{0} = 1, ..., m_{0} = m_{0} + m_{0} + m_{0}$, into an HS of 2$n$ equations for $\Gamma_{c_{1}}^{p}(\theta)$ with the Hamiltonian $S_{0}^{i_{1}}(\Gamma_{c_{1}}, C_{0}(\theta)) = (A', \xi_{0}^{A}(\theta))(A^{\alpha_{0}}(\theta))$. A union of this system with the HS in \[(21)\], extended to $2(n + m_{0})$ equations, has the form

\[
\partial_{\mu} \Gamma^{p}_{[0]}(\theta) = \left( \Gamma^{p}_{[0]}(\theta), S_{0}^{i_{1}}(\theta) \right)_{\theta}, \Gamma^{p}_{[0]}(\theta) = \left( S_{0}^{i_{1}}(\theta), S_{0}^{i_{1}}(\theta) \right)_{\theta}, \Gamma^{p}_{[0]}(\theta) \equiv (\Gamma^{c_{1}}_{c_{1}}, \Gamma^{c_{1}}_{c_{1}}), \Gamma^{c_{1}}_{c_{1}} = (C^{\alpha_{0}}, C^{\alpha_{0}}). \tag{22}\]

Due to \[(22)\], \(S_{0}^{i_{1}}(\theta)\) is invariant, modulo $S_{0; i_{1}}(\theta)$, under special gauge transformations for the ghost superfields $C^{\alpha_{0}}(\theta)$ with arbitrary functions $\xi^{\alpha_{1}}(\theta)$, $(\varepsilon_{\mu})_{\alpha_{1}} = 0$, defined in $M$:

\[
\delta C^{\alpha_{0}}(\theta) = Z^{\alpha_{0}}_{\alpha_{1}}(A(\theta))\xi^{\alpha_{1}}(\theta), (\varepsilon, gh)\xi^{\alpha_{1}}(\theta) = (\varepsilon_{\alpha_{1}}, (1, 0, 1, 1)). \tag{23}\]

In \[(23)\], we now choose $\xi^{\alpha_{1}}(\theta) = d\xi^{\alpha_{1}}(\theta) = C^{\alpha_{1}}(\theta)d\theta$, $\alpha_{1} = 1, ..., m_{1}$, and extend the $m_{0}$ equations for $C^{\alpha_{0}}(\theta)$ to an HS of $2m_{0}$ equations with the Hamiltonian $S^{i_{1}}(A, C_{0}, C_{1})(\theta) = (C_{\alpha_{0}}^{\alpha_{1}}Z_{\alpha_{1}}^{\alpha_{0}}(A)C^{\alpha_{1}}(\theta))$. We then

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\[\text{3} \text{We use the notion of } \theta \text{-superfield integrability by analogy with } \text{[13].}\]
obtain a system for $\partial_\theta^{(p)} \Gamma_0^\alpha(\theta)$. The extension of a union of the latter HS with eqs. is formally identical to 22 with the replacement

$$(\Gamma_0^{(p)} [0]_1, S_0^1) \to (\Gamma_0^{(p)} [1]_1, S_1^1) : \left\{ \Gamma_0^{p[1]} = \Gamma_0^{p[0]}, \Gamma_1^{p[1]} = (C^{\alpha_1}, C^{\alpha_2}_*) \right\}.$$ 

For an $L$-stage-reducible restricted LSM at the $s$-th step, $0 < s < L$, $\Gamma_0^{p[1]} = \Gamma_0^{p[1]}$, an iteration corresponding to reformed special gauge transformations for $C^{\alpha_0}, \ldots, C^{\alpha_{s-2}}$ implied by (possibly) enhanced 4 formulae yields invariance transformations for $S_1^{s-1}(\theta)$, modulo $S_0, (\theta)$,

$$\delta \xi^{\alpha_0} \Gamma_0^{p[1]}(\theta) = Z^{\alpha_0} \delta(\theta), (\xi \partial_\theta) \Gamma_0^{p[1]}(\theta) = (\xi \partial_\theta) \Gamma_0^{p[1]}(\theta), \quad (\xi \partial_\theta) \Gamma_0^{p[1]}(\theta) = (\xi \partial_\theta) \Gamma_0^{p[1]}(\theta) = (0, 0).$$

The replacement $\xi^{\alpha_0} = d\xi^{\alpha_0} = (C^{\alpha_0}, C^{\alpha_1})(\theta), \partial_\theta \xi^{\alpha_0} = 0$, transforms $\xi^{\alpha_0}$ into $S_0$ equations for $C^{\alpha_1}(\theta)$, enlarged by an introduction of $C^{\alpha_1}_*$ to the following HS:

$$\partial_\theta \Gamma_0^{p[1]}(\theta) = (\Gamma_0^{p[1]}(\theta), S_0^1(\theta)) = (\xi \partial_\theta) \Gamma_0^{p[1]}(\theta) = (C^{\alpha_0}, C^{\alpha_1})(\theta), \Gamma_0^{p[1]}(\theta) = (C^{\alpha_0}, C^{\alpha_1})(\theta).$$

Making a combination of 24 with an HS of the same form, however with $\partial_\theta \Gamma_0^{p[1]}(\theta)$ and with the Hamiltonian $S_0^1(\theta) = (S_0 + \sum_{s=0}^{s-1} S_1^s(\theta)$, and presenting the result for $2 (n + \sum_{s=0}^{s-1} m_r)$ equations with $S_0^1(\theta) = (S_1^{s-1} + s(\theta))$, we obtain, by induction,

$$\partial_\theta \Gamma_0^{p[1]}(\theta) = (\Gamma_0^{p[1]}(\theta), S_0^1(\theta)), S_0^1(\theta) = (C^{\alpha_0}, C^{\alpha_1})(\theta), \Gamma_0^{p[1]}(\theta) = (C^{\alpha_0}, C^{\alpha_1})(\theta).$$

With the antibracket extended to $\Pi^* \mathcal{M}_k$, the function $S_0^1(\theta)$ is a proper solution of the classical master equation with accuracy up to $O(C^{\alpha_0}$, modulo $S_0, (\theta)$. An integrability of the HS in 20 is implied by a deformation of $S_0^1(\theta)$ in powers of both $\Phi^A_k(\theta)$ and $C^{\alpha_0}(\theta)$, by virtue of a superfield counterpart of the existence theorem 3 for the classical master equation in the minimal sector:

$$(S_{H;i,k}(\Gamma_k(\theta)), S_{H;k}(\Gamma_k(\theta)))) = (0, 0), \quad (\xi \partial_\theta) S_{H;i,k}(\Gamma_k(\theta)) = (0, 0).$$

The construction of $S_{H;i,k}(\theta)$ is a superfield analogue of the Koszul–Tate complex resolution. Assuming now $k = 0$, we remind that the enlargement of $S_{H;i,k}(\theta)$ to $S_{H;k}(\Gamma_k(\theta))$, $S_{H;k}(\theta) = S_{H;i,k}(\theta) + \sum_{s=0}^{s=0} \sum_{s=0}^{s=0} (C^{\alpha_0}, E^{\alpha_0}_s)(\theta)$, being a proper solution in $\Pi^* \mathcal{M}_k$, with a deformation in the Planck constant $\hbar$, determines a quantum action $S_{H;i,k}(\Gamma(\theta), \hbar)$, e.g., in case of an Abelian hypergauge,

$$S_{H;i,k}(\Gamma(\theta), \hbar)$$

The functions $S_{H;i,k}(\theta, \hbar)$ obey equations provided that the $\theta$-deformation of $S_{H;i,k}(\theta)$ is their solution. Such equations are known to ensure an integrability of the non-equivalent HS constructed from $S_{H;i,k}(\theta)$, as well as the anticanonical [respecting the volume $dV_k(\theta) = \prod_{\alpha} dT_k^\alpha(\theta)$] nature of transformations related to a $\theta$-shift by a constant $\mu$ along the corresponding HS solutions. At the same time, the quantum master equation

$$\Delta_k(\theta) \exp[(i/\hbar)E(\theta, \hbar)] = 0, \quad E \in \{ S_{H;i,k}, S_{H;k} \}$$

introduces a non-integrable HS, with the corresponding anticanonical transformation preserving $dV_k(\theta) = \exp[(i/\hbar)E(\theta, \hbar)]dV_k(\theta)$. The vector field related to the latter HS determines, for $\partial_\theta \Phi^A_k(\theta) = 0$, a $\theta$-local generator $\Phi^A_k(\theta)$ of BRST transformations crucial for the BV formalism (in case $\theta = 0)$.

4From $gh(A^1) = 0$ in eqs. with $(\varepsilon \partial_\theta) A^1 = (\varepsilon \partial_\theta) A^1 = 0, s = 0, \ldots, L_g$, it follows that $\eta, \eta_\alpha, \eta_{\alpha}^- \eta_{\alpha}^+$ may be both larger and smaller than $M, M_\alpha$, in contrast to $\eta, \eta_\alpha$. In fact, a restricted LSM may possess additional gauge symmetries. Thus, we suppose that (possibly) enhanced sets of restricted functions $R_\alpha^{(l)}(\theta), Z_{\alpha_{s-1}}^\alpha(\theta)$ exhaust, on the surface $S_0, (\theta) = 0$, the zero-modes of both $S_0, (\theta)$ and $Z_{\alpha_{s-1}}^\alpha(\theta)$, respectively. At the final stage of reducibility for a restricted model, the above implies that $L \neq L_g$. 

6
3.2 BV–BFV Duality

We now propose a dual description for an LSM. Namely, an embedding of a restricted LSM gauge algebra with $S_{H_{\min}}(\theta)$ and equations (27) into the gauge algebra of a general gauge theory in Lagrangian formalism (8–12) can be realized by dual counterparts, with the opposite $(\varepsilon, \bar{\varepsilon})$-parity, of the action and antibracket, following, in part, Refs. (8, 15). For this purpose, consider the functional

$$Z_k[\Gamma_k] = -\partial_0 S_{H,k}(\theta), \quad (\varepsilon, \bar{\varepsilon}) Z_k = ((1, 0, 1), 1)$$

in $\Pi T(\Pi T^*M_k) = \{ (\Gamma^0_k, \partial_0 \Gamma^0_k) | \theta, k = \text{min} \}$, with a symplectic and odd Poisson structures which define an even functional $\{ \cdot, \cdot \}$ with canonical pairs $\{ (\Phi^s_A, \partial_0 \Phi^s_A), (\partial_0 \Phi^s_A, \Phi^s_A) \}$ and a 0-local odd Poisson bracket $(\cdot, \cdot)_{\theta}$. The latter bracket acts on $C^\infty(\Pi T(\Pi T^*M_k) \times \theta)$ and provides a lifting of $(\cdot, \cdot)_{\theta}$ defined in $\Pi T^*M_k$. For any $F_t[\Gamma_k] = \partial_0 F_t ((\Gamma_k, \partial_0 \Gamma_k))(\theta), t = 1, 2$, there holds a correspondence between the Poisson brackets of the opposite Grassmann gradings:

$$\{ F_1, F_2 \} = \int d\theta \left[ \frac{\delta F_1}{\delta \Phi^s_A(\theta)} \frac{\delta F_2}{\delta \Phi^s_A(\theta)} - \frac{\delta F_1}{\delta \Phi^s_A(\theta)} \frac{\delta F_2}{\delta \Phi^s_A(\theta)} \right] = \int d\theta (F_1(\theta), F_2(\theta))_{\theta}^{(\Gamma_k, \partial_0 \Gamma_k)}.$$

(30)

Here, for instance, the Euler–Lagrange superfield derivative with respect to $\Phi^s_A(\theta)$, for a fixed $\theta$, is given by $\mathcal{L}^{\cdot A_1} = \partial / \partial \Phi^s_A(\theta) - (-1)^{s+1} \partial_0 / \partial (\partial_0 \Phi^s_A(\theta))$.

The functional $Z_k$, for $k = \text{min}$, is nilpotent by construction: $(Z_k, Z_k) = \partial_0 (S_{H,k}(\theta), S_{H,k}(\theta))_{\theta} = 0$. The absence of a time coordinate implies that $Z_k$ formally corresponds to a BRST charge for a dynamical system with first-class constraints [2]. In fact, after identifying ($\Gamma_k, \partial_0 \Gamma_k)(0)$ with phase-space coordinates (of the minimal sector) canonical with respect to the $(\varepsilon, \bar{\varepsilon})$-even BFV bracket for a first-class constrained system of $(L + 1)$-stage reducibility,

$$(q^i, p_i) = (A^i, \partial_0 A^i)(0), \quad (C^{A^i}, P_{A^i}) = \left( (\partial_0^0 C^{\alpha_{s-1}}, C^{\alpha_{s}}), (C^{\alpha_{s-1}}, \partial_0 C^{\alpha_{s}}) \right)(0), \quad A_s = (\alpha_{s-1}, \alpha_s), \quad s = 0, \ldots, L, \quad (C^{A^s}, P_{A^s}) = \left( (\partial_0^0 C^{\alpha_s}, 0), (C^{\alpha_s}, 0) \right)(0),$$

(31)

the functional $Z_k$ acquires the form

$$Z_k[\Gamma_k] = T_{A_0}(q, p) C^{A^0} + \sum_{s=1}^{L+1} P_{A^s-1} Z^{A^s-1}_{A^s}(q) C^{A^s} + O(C^2).$$

(32)

The structure functions of the initial $L$-stage reducible restricted LSM in the enhanced eqs. (49) determine $T_{A_0}(q, p)$ and a set of $(L + 1)$-stage reducible eigenvectors $Z^{A^s-1}_{A^s}(q)$:

$$T_{A_0}(q, p) = (S_{H_0}(q), -p_i R_{A_0}(q)), \quad Z^{A^s-1}_{A^s}(q) = \text{diag} \left( Z^{A^s-2}_{A^s}, \left[ 1 - \delta^{A^s}_{A^s+1} \right] Z^{A^s-1}_{A^s} \right)(q),$$

(33)

$$Z^{A^s-2}_{A^s} Z^{A^s-1}_{A^s} = T_{A_0} L_{A^s-2}^{A^s}(q), \quad Z^{A^s-1}_{A^s} = T_{A_0} L_{A^s-2}^{A^s}(q) = 0,$$

$$L_{A^s-2}^{A^s} = \text{diag} \left( L^{A^s-3}_{A^s-1}, L^{A^s-2}_{A^s-1} \right), \quad L^{A^s-2}_{A^s-2} = 0, \quad L^{A^s-2}_{A^s-1} = 0 = (A^s-1)(q, p) = (-1)^{s+1} p_i K_{A^s-1}^{ij}(q).$$

(34)

Formulas (30–34) generalize to any reducible theories the dual description (proposed for $\varepsilon = \varepsilon_{\text{ext}} = L = 0$) of a quantum action in the minimal sector via a nilpotent BRST charge in the minimal sector [15]. One can show that the corresponding dual description in terms of the extended variables of the BV and BFV methods yields the only possible embedding of $Z_k[\Gamma_k], k = \text{ext}$, and $\Pi T(\Pi T^*M_k)$ into the BRST charge and total phase space of the BFV method.

A characteristic property of the duality problem is an equivalent definition of the systems with the Hamiltonians $S^\Phi_M(\Gamma(\theta), \bar{h}), \quad S_{H,k}(\theta), \quad k = \text{min, ext}$, by means of dual fermionic functionals, $Z_k[\Gamma_k], Z^\Phi[\Gamma] = -\partial_0 S^\Phi_M(\Gamma(\theta), \bar{h})$, in terms of even Poisson brackets:

$$\partial_0^0 \Gamma^p(\theta) = (\Gamma^p(\theta), S^\Phi_M(\Gamma(\theta), \bar{h}))_{\theta} = - \left\{ \Gamma^p(\theta), Z^\Phi[\Gamma] \right\}.$$  

(35)

Thus, BRST transformations in a Lagrangian formalism with Abelian hypergauces can be expressed in terms of a formal BRST charge $Z^\Phi[\Gamma]$ related to $Z_k[\Gamma_k], k = \text{ext}$, by a canonical transformation with an even phase, $F^\Phi[\Phi] = \partial_0 \Phi(\Phi(\theta)), \quad Z^\Phi[\Gamma] = \exp \left\{ F^\Phi, \cdot \right\} Z_k[\Gamma_k]$. 

7
3.3 Local Quantization

Let us define a generating functional of Green’s functions $\mathcal{Z}(\theta)$ and an effective action $\Gamma(\theta)$, using an invariant description of super(anti)fields on a general anti-symplectic manifold. For this purpose, we use a choice of Darboux coordinates $(\varphi, \varphi^*)$ consistent with the properties of a quantum action. We suppose that a model is described by a quantum action $W(\theta) = W(\theta, \hbar)$ defined on an arbitrary (without connection) anti-symplectic manifold $\mathcal{N} = \{\Gamma^p(\theta)\}$, dim $\mathcal{N} = \text{dim} \text{PIT}^*\text{M}_{\text{ext}}$, with a density function $\rho(\Gamma(\theta))$ determining an invariant volume element $d\mu(\Gamma(\theta))$. A local antibracket and a nilpotent second-order operator $\Delta_\mathcal{N}(\theta)$ are defined with the help of an odd Poisson bivector, $\omega^{po}(\Gamma(\theta)) = (\Gamma^p(\theta), \Gamma^q(\theta))_{\theta}^{\mathcal{N}}$,

$$\Delta_\mathcal{N}(\theta) = \frac{1}{2}(-1)^{\ell(\Gamma^p)}\rho^{-1}\omega_{qp}(\theta) \left(\Gamma^p(\theta), \rho(\Gamma^q(\theta), \cdot)\right)_{\theta}^{\mathcal{N}}. \quad (36)$$

In perturbation theory, a generating functional of Green’s functions $Z((\partial_\theta \varphi^*, \varphi^*, \partial_\theta \varphi, \mathcal{I}) (\theta)) = \mathcal{Z}(\theta)$ can be defined as a path integral (for a fixed $\theta$) by introducing on $\mathcal{N}$ some Darboux coordinates $\Gamma^p(\theta) = (\varphi^*, \varphi_\theta^*) (\theta)$ in a vicinity of the stationary points of $W(\theta)$ such that $\rho = 1$ and $\omega^{po}(\theta) = \text{antidiag}(-\delta^b_\theta, \delta^b_\theta)$. The function

$$Z(\theta) = \int d\mu \left(\tilde{\Gamma}(\theta)\right) d\Lambda(\theta) \exp \left\{ (i/\hbar) \left[ W \left(\tilde{\Gamma}(\theta), \hbar\right) + X \left(\left(\varphi, \varphi^*, \Lambda, \Lambda^*\right)(\theta), \hbar\right) \right] \right\}, \quad \mu(\Gamma(\theta)) = \rho(\Gamma(\theta))d\Gamma(\theta), \quad (37)$$

depends on extended sources $(\partial_\theta \varphi^*, \partial^b_\theta \varphi^*, \mathcal{I}) (\theta) = (-J_\theta^a, \lambda^a, I_{\theta 0}, I_{\theta a} \theta)$ for $(\varphi^*, \varphi_\theta^*, \Lambda^a(\theta))$ with the properties $(\varepsilon, g) \partial_\theta \varphi^*_a = (\varepsilon, g) \mathcal{I}_a + ((1, 0, 1, 1) = (\varepsilon, g) \varphi^a$, where $\Lambda^a(\theta) = (\lambda^a_\theta + \lambda^a \theta)$ are Lagrangian multipliers to independent non-Abelian hypergences, $G_a(\Gamma(\theta)), a = 1, ..., k = \text{dim}_+ \mathcal{N}$; see [13]. The functions $G_a(\Gamma(\theta)), (\varepsilon, g) G_a = (\varepsilon, g) \mathcal{I}_a$, determine a boundary condition (when $\Lambda^a_\theta = \hbar = 0$) for the gauge-fixing action $X(\theta) = X((\Lambda, \Lambda^a(\theta), \hbar)$ defined on the direct sum $\mathcal{N}_{\text{tot}} = \mathcal{N} \oplus \text{PIT}$, where $\text{PIT}^* = \{(\Lambda^a, \Lambda^a_\theta)\}$. Hypergences in involution $(G_a(\theta), G_b(\theta))_{\theta}^{\mathcal{N}} = G_c(\theta) \mathcal{U}_{ab}(\Gamma(\theta))$ obey various unimodularity relations [13] depending on equations with a solution $X(\theta)$, in terms of the antibracket $(\cdot, \cdot)_{\theta} = (\cdot, \cdot)_{\theta}^\mathcal{N} + (\cdot, \cdot)_{\theta}^\mathcal{I}$ and the operator $\Delta(\theta) = (\Delta^\mathcal{N} + \Delta^\mathcal{I})(\theta)$, trivially lifted from $\mathcal{N}$ to $\mathcal{N}_{\text{tot}}$.

$$1) \ (E(\theta), E(\theta))_{\theta} = 0, \ \Delta(\theta) E(\theta) = 0; \ 2) \ (\Delta(\theta) \exp [(i/\hbar) E(\theta)] = 0, \ E \in \{W, X\}.$$

(38)

The functions $G_a(\theta)$ solvable with respect to $\varphi^*_a$ determine a Lagrangian surface, $Q_\theta = \{(\varphi^*, \Lambda(\theta))\} \subset \mathcal{N}_{\text{tot}}$, on which $X(\theta)|_{Q_\theta}$ is non-degenerate. Then integration over $(\varphi^*, \Lambda)$ in (37) yields (when $\partial_\theta \varphi^* = \mathcal{I}_a = 0$) a function whose restriction to the Lagrangian surface $Q = \{\varphi(\theta)\}$ is also non-degenerate.

Using the properties of $(W, X)(\theta)|_{Q_\theta}$, one can introduce an effective action $\Gamma(\theta) = \Gamma(\varphi, \varphi^*, \partial_\theta \varphi, \mathcal{I})(\theta)$ by a Legendre transformation of $\ln Z(\theta)$ with respect to $\partial_\theta \varphi^*_a(\theta)$,

$$\Gamma(\theta) = (h/i) \ln Z(\theta) + (\partial_\theta \varphi^*_a(\theta) \varphi^a(\theta)) = i \hbar \delta \ln Z(\theta) \frac{\partial}{\partial \varphi^*_a(\theta)}.$$

(39)

The properties of $(Z, \Gamma)(\theta)$ are implied by a $\theta$-nonintegratable Hamiltonian-like system with an arbitrary $(\varepsilon, \varepsilon)$-even $C^\infty(\mathcal{N}_{\text{tot}})$-function $R(\theta) = R \left( (\tilde{\Gamma}, \Lambda, \Lambda^*)(\theta), \hbar \right)$, $g R(\theta) = 0$, for $T(\theta) = \exp [(i/\hbar) (W - X)(\theta)]$,

$$\partial^\theta_\theta \left( \tilde{\Gamma}, \varphi^*, \varphi_\theta^*, \Lambda^* \right) = -i \hbar T^{-1}(\theta) \left( \cdot, \cdot, T(\theta) \right) (\theta) \left|_{\Lambda^* = 0} = 0. \quad (40)$$

The integrand in (37) is invariant (for $\partial_\theta \varphi^* = \partial_\theta \varphi = \mathcal{I} = 0$) under the superfield BRST transformations

$$\tilde{\Gamma}_{\text{tot}}(\theta) = (\tilde{\Gamma}, \Lambda, \Lambda^*)(\theta) \to \left( \tilde{\Gamma}_{\text{tot}} + \delta_{\mu} \tilde{\Gamma}_{\text{tot}} \right)(\theta), \ \delta_{\mu} \tilde{\Gamma}_{\text{tot}}(\theta) = \left( \partial_\theta^\theta \tilde{\Gamma}_{\text{tot}} \right)|_{\tilde{\Gamma}_{\text{tot}}} \mu, \quad (41)$$

related to a $\theta$-shift by a constant $\mu$ along an arbitrary solution $\tilde{\Gamma}_{\text{tot}}(\theta)$ of (40), for $R(\theta) = 1$, with the arguments of $(W, X)(\theta)$ being those in (37).

The function $Z_X(\theta) = Z(0, \varphi^*, 0, 0)(\theta)$ is gauge-independent: it does not change if $X(\theta)$ is replaced by $(X + \Delta X)(\theta)$ that obeys equations (38) for $X(\theta)$ and respects nondegeneracy on $Q_\theta$. This means that $\Delta X(\theta)$ obeys a set of linearized equations with a nilpotent operator $Q_j(X)${

$$Q_j(X) \Delta X(\theta) = 0, \delta_{j1} \Delta(\theta) \Delta X(\theta) = 0; \ Q_j(X) = \text{ad} X(\theta) - \delta_{j2} (i \hbar \Delta(\theta)), \ j = 1, 2,$$

(42) where $j$ labels those equations in (38) which are met by $X(\theta)$. By analogy with the theorems [20], the fact that solutions $X(\theta)$ of each system in (38) are proper implies that the cohomologies of $Q_j(X)$ on functions
\[f(\Gamma_{\text{tot}}(\theta)) \in C^\infty(N_{\text{tot}})\] vanishing for \(\Gamma_{\text{tot}}(\theta) = 0\) are trivial. Thus, the general solution of Eq. (12) is given by a certain function \(\Delta Y(\theta), \Delta Y(\theta)|_{\Gamma_{\text{tot}}=0} = 0\),

\[\Delta X(\theta) = Q_j(X)\Delta Y(\theta), \delta_1\Delta(\theta)\Delta Y(\theta) = 0, \quad (\varepsilon, gh, \partial/\partial\theta) \Delta Y(\theta) = ((1, 0, 1), -1, 0). \tag{43}\]

Making in \(Z_X + \Delta X(\theta)\) a change of variables induced by a \(\theta\)-shift by a constant \(\mu\), related to \(10\), and choosing \(2R(\theta)\mu = \Delta Y(\theta)\), we have \(Z_X + \Delta X(\theta) = Z_X(\theta)\), which implies the gauge-independence of the S-matrix, in view of the equivalence theorem \(21\).

Following Subsection 3.2, the stated properties of \(Z_X(\theta)\) can be independently derived from a Hamiltonian-like system in terms of an even superfield Poisson bracket in general coordinates (see footnote 4),

\[\partial^\mu \left( \Gamma^p, A^a, \Phi^*_a, \Lambda^*_a \right)_{\theta} = \left\{ \cdot, (Z_X + \ii \hbar Z_R)[\Gamma_{\text{tot}}] - Z^W(\Gamma) \right\} \left( \Gamma^p, 2 \Lambda^a, 0, 0 \right)_{\theta}, \quad \Lambda^*_a = 0, \tag{44}\]

where \(Z^F[\Gamma_{\text{tot}}] = -\partial_\theta E(\Gamma_{\text{tot}}(\theta), \hbar), \quad E \in \{ W, X, R \}\). If \((W, X)(\theta)\), obey the first system in \(13\), then \(Z^W, Z^X\), playing the role of a usual and gauge-fixing BRST charge, are nilpotent with respect to the Poisson bracket \(\{ \cdot, \cdot \} = \{ \cdot, \cdot \}^{\Gamma^W} + \{ \cdot, \cdot \}^{\Gamma^X}\). Here, the first bracket is defined on any functionals over \(\Pi T N \times \{ \cdot \}\) in terms of a \(\theta\)-local extension \(\{ \cdot, \cdot \}^{\Pi T N} = ( (L^\nu_{\theta}), \omega_{\theta}^{\Pi T N}(\Gamma(\theta)) L_{\theta} \} \) of antibracket \(30\).

\[\{ F_1, F_2 \}^{\Pi T N} \equiv \int d\theta \frac{\delta F_1}{\delta \Gamma^p(\theta)} \omega_{\theta}^{\Pi T N}(\Gamma(\theta)) \frac{\delta F_2}{\delta \Gamma^p(\theta)} = \partial_\theta \left( F_1(\theta), F_2(\theta) \right)^{\Pi T N}_{\theta}, \quad F_1[\theta] = \partial_\theta F_1((\Gamma, \partial_\theta \Gamma(\theta), \theta), \tag{45}\]

where \(L^\nu_{\theta}(\Gamma)\) is the left-hand Euler–Lagrange superfield derivative with respect to \(\Gamma^p(\theta)\).

The functions \((\Gamma, \Gamma \theta)\) obey the Ward identities

\[\left\{ \partial_\theta \Phi^a(\theta) \theta, \partial^a \Phi^a(\theta) \right\} + \ii \hbar \left[ \partial_\theta \Phi^a(\theta) \theta, \partial^a \Phi^a(\theta) \right] = \partial_\theta X(\theta, \partial_\theta \Phi^a(\theta)) \left( i \hbar \partial_\theta \partial^a \Phi^a(\theta) - \partial^a \Phi^a(\theta) \partial_\theta \right), \tag{46}\]

\[\frac{\delta \Gamma^{\nu}_{\theta}(\theta)}{\delta \Phi^a(\theta)} X \left( \phi^b + \ii \hbar (\Gamma^\nu-1)^{bc} \partial^c \phi^a ; i \hbar \partial^c \phi^a \right) \left( \partial^a \phi^a X = \frac{\delta \Gamma^{\nu}_{\theta}(\theta)}{\delta \Phi^a(\theta)} \left( i \hbar \partial^a \phi^a \right) - \partial^a \phi^a = 0 \right); \tag{47}\]

which follow from the functional averaging of the respective system for \(W(\theta)\) and \(X(\theta)\) in \(58\), as well as from integration by parts in the path integral, with allowance for \((\partial/\partial \phi^a + \partial/\partial \phi^a) X(\theta) = 0\).

For Abelian hypergagues \(G_A(\Phi, \Phi(\theta)) = \Phi_A(\theta) - \partial \Phi(\Phi(\theta))/\partial \Phi_A(\theta) \equiv 0 \) related to eq. \(28\), with \((\phi, \phi^a, W) = (\Phi, \Phi^*, \Theta_{\text{ext}})\), \(\Phi^* A = \Lambda = 0\), the functional \(Z(\partial_\theta \Phi^*, \Phi^*)(\theta)\) acquires the form

\[Z(\partial_\theta \Phi^*, \Phi^*)(\theta) = \int d\Phi(\theta) \exp \left\{ \ii \hbar \left[ S^\Phi_{\text{H}}(\Gamma(\theta), \hbar) - (\partial_\theta \Phi^*_A(\theta)) \right] \right\}, \tag{48}\]

For an HS with a Hamiltonian \(S^\Phi_{\text{H}}(\theta, \hbar)\) and a solution \(\Gamma(\theta)\) defined in \(\Pi T^* \mathcal{M}_{\text{ext}}\), the BRST transformations are given by an anticanonical (for a constant \(\mu\) transformation:

\[\Gamma(\theta) \rightarrow \Gamma^{(1)}(\theta) = \exp \left[ \mu s(\Phi(\theta)) \right] \Gamma^p(\theta), s(\Phi(\theta)) \equiv \partial/\partial \theta - \left( S^\Phi_{\text{H}}(\theta, \hbar), \cdot \right)_\theta. \tag{49}\]

From the permutation rule for the functional integral, \(\varepsilon(\Phi(\theta)) = 0, \)

\[\partial_\theta \int d\Phi(\theta) \mathcal{F}(\Phi, \Phi^*(\theta), \theta) = \int d\Phi(\theta) \left[ \partial \Phi(\theta) + (\partial_\theta V(\theta)) \right] \mathcal{F}(\theta), \quad \partial_\theta V(\theta) = (\partial_\theta \Phi^*_A(\theta)) \partial \theta \Phi^*_A(\theta), \]

with \(i \hbar \partial_\theta \ln Z(\theta) = (\partial_\theta \Phi^*_A \partial_\theta \Phi^*_A(\theta) - \partial^c \Gamma(\theta), \) follow the relations

\[\partial_\theta Z(\Gamma(\theta)) \Gamma(\theta) = (\partial_\theta V(\theta)) Z(\theta) = 0, \quad (\partial_\theta \Gamma(\theta)) \Gamma(\theta) = (\Gamma(\Gamma(\theta)), \Gamma(\Gamma(\theta)))\theta = 0, \tag{50}\]
which are implied by functional averaging with respect to \( Z(\theta) \) and \( \Gamma(\theta) \),

\[
(\partial_0^\mu \Gamma^\nu)_{|Z} = \left( \frac{\hbar}{i} Z^{-1} \frac{\partial Z(\theta)}{\partial \Phi_0(\theta)} - \partial_0 \Phi_0(\theta) \right), \quad (\partial_0^\mu \Gamma^\nu) = (\Gamma^\nu(\theta)), \Gamma(\theta)_{|\theta = \theta} = \partial_0^\mu \Gamma^\nu(\theta),
\]

without the sign of averaging in (50) for \( \Gamma^\nu(\theta) \) and \( \Gamma^\nu(\theta) \). Formulæ (50) relate the Ward identities in a theory with Abelian hypergauces to the invariance of the generating functional of Green’s functions under superfield BRST transformations.

4 Relation between Lagrangian Quantizations

A relation between the conventional and \( \theta \)-local quantizations can be established through the component form of \( \Gamma^\nu = \Gamma^\nu_0 + \Gamma^\nu_1 \), \( k = \text{tot} \), for \( \theta = 0 \): \( (M, N_0, \Lambda^a, I_0) \rightarrow (M, N_0, \Lambda^a, I_0) = (\Gamma^\nu_0, \lambda_0, I_{00}) \).

Besides the condition \( \theta = 0 \), a standard field model can be extracted by eliminating the quantities \( \partial_0 \Phi(\theta), \partial_0 \Phi(\theta) \) and the superfields \( \Phi(\theta) \) with a wrong spin-statistics relation, \( \varepsilon \rho(\Phi(\theta)) \neq 0 \). Such an elimination can be realized by eqs. (29) and the conditions \( \varepsilon \rho(\Phi(\theta)) = -1 - \varepsilon \rho(\Phi(\theta)) = 0 \), \( (\varepsilon \rho)_{I} = 0 \). For a restricted LSM of Section 3, a reduction to a model of the multilevel formalism [13] is achieved for vanishing \( \partial_0 \Phi, \partial_0 \Phi, \Phi, \Phi, \Lambda, I_0 \). Then the first-level functional integral \( (F^1) \) and its symmetry transformations [13], with \( \lambda_0^{\mu} \) instead of \( \pi^\mu \) for Lagrangian multipliers of [13],

\[
Z(1) = \int d\lambda_0 d\Gamma_0 M(\Gamma_0) \exp \left\{ \frac{i}{\hbar} \left[ (W(\Gamma_0) + G_0(\Gamma_0)\lambda_0^{\mu} \right] \right\},
\]

\[
[\delta \Gamma_0^{\mu}, \delta \lambda_0^{\nu}] = \left[ (\Gamma_0^{\mu}, -W + G_0^{\mu}, -U_{cb}^{\mu} \lambda_0^{\nu} \Lambda^c (1)^{-\varepsilon} + 2i\hbar V_0^{\mu} \lambda_0^{\nu} + (2i\hbar^2 G_0^{\mu}) \right] \mu,
\]

coincide with \( Z_X(0)_{|\varepsilon = 0} \) and its BRST transformations generated by 40 for \( R(\theta) = 1 \), under the identification \( \rho(\Phi(\theta)) = (M, E^{\rho}) \) \( (\Gamma_0) \) implying the coincidence of \( (\cdot, \cdot)_{\theta = 0} \) and \( \Delta(0) \) with their counterparts of [13]. The coincidence is implied by the choice of \( X(\theta) \) as

\[
X(\theta) = \left\{ G_0(\theta) \Lambda^a - \Lambda^a_{|\theta = 0} \left[ \frac{1}{2} T_{cb}^{\mu} (\Lambda^b \Lambda^c (1)^{-\varepsilon} - i\hbar V_0^{\mu} \Lambda^b - (2i\hbar^2 G_0^{\mu}) \right] \right\} \theta + o(\Lambda^a),
\]

where \( (V_0^{\mu}, G_0^{\mu}) \) and \( (U_{cb}^{\mu}, G_0^{\mu}) \) determine the unimodularity relations [13]. A connection between the local quantization and the generating functional of Green’s function \( Z[\Gamma, \phi^\dagger] \) of the BV method is evident after identifying \( Z(\partial_0 \Phi, \Phi^\dagger) = 0 \) in \( Z[\Gamma, \phi^\dagger] \) in [13], with the action \( S_{10} \) in \( (\Gamma_0, \hbar) \) in [25, 29].

An arbitrary function \( F(\theta) = F(\Gamma, \partial_0 \Gamma) \) \( \theta \in C^\infty(\Pi \Omega \times \{ \theta \}) \) can be represented (in case \( \Gamma^\nu = (\Phi^A, \Phi^a) \) see footnote 1) by a functional \( F[\Gamma] \) of the superfield scheme [5],

\[
F[\Gamma] = \partial_0 [\theta \partial (F\theta)] = F(\Gamma(0), \partial_0 \Gamma) = F(\partial_0 \Gamma), \Gamma = F(\Gamma, \Gamma) = F(\Gamma, \Gamma) = (53).
\]

This implies the independence of \( F[\Gamma] \) from \( \partial_0 \Gamma^\nu(\theta) \) in case \( F(\theta) = F(\Gamma(\theta), \theta) \). Formulæ (53) allows one to establish a relation between the objects \( (\cdot, \cdot), \Delta_N(\theta) \) and \( \Delta_N(\theta) \) in \( C^\infty(\Pi \Omega \times \{ \theta \}) \), with an extension to any \( (\Gamma, \omega^{\rho}, \rho(\theta)) \) of the flat operations \( (\cdot, \cdot), \Delta \) in [5], which coincide with their analogues of the BV method in case \( \Gamma^\nu = (\Phi^A, \Phi^a), \omega^{\rho}(\Gamma(\theta)) = \text{antidiag}(\delta^{\rho}_0, \delta^{\rho}_0), \rho(\theta) = 1 \), and in the case of a different odd Poisson bivector, \( \omega^{\rho,\mu}(\Gamma(\theta), \theta) = (1 + \theta^\rho \delta_0^{\rho}) \omega^{\rho,\mu}(\theta) \). This correspondence is implied by

\[
(F(\theta), G(\theta))^{N}_{\theta = 0} = (F[\Gamma], G[\Gamma])^N = \partial_0 \left[ \frac{\delta F[\Gamma]}{\delta \Gamma^\rho(\theta)} \partial_0 \left[ \frac{\delta G[\Gamma]}{\delta \Gamma^\rho(\theta)} \right] (1)^{\varepsilon(\Gamma^\rho) + 1}, \right]
\]

\[
\Delta_N(\theta) F(\theta)_{\theta = 0} = \Delta_N F[\Gamma] = \frac{1}{2} (1)^{\varepsilon(\Gamma^\rho)} \partial_0 \partial_0 \rho(\Gamma) \left[ \rho(\Gamma) \Gamma^\rho(\theta) \right] \left( \Gamma^\rho(\theta), \rho(\Gamma) \Gamma^\rho(\theta), F[\Gamma] \right)^N, \right]
\]

where \( (\rho(\Gamma), \omega^{\rho}) \) and \( (\partial_0 \Gamma^\rho, \omega^{\rho}) \) are related by \( \theta \partial_0^\rho \omega^{\rho}(\theta) \). To establish the correspondence with \( (\cdot, \cdot) \) and \( \Delta \) in [5], one needs a relation between superfield and component derivatives: \( \delta_0^\rho \Gamma^\nu(\theta) = (\Lambda^a, (1)^{-\varepsilon} J_\Lambda^a), \delta_0^\rho \delta_0^\rho \Gamma^\nu(\theta) = (1)^{\varepsilon(\Gamma^\rho)} \left( \delta_0^\rho / \partial_0^\rho - \delta_0^\rho / \partial_0^\rho \right) \left( \Gamma^\rho(\theta), \rho(\Gamma) \Gamma^\rho(\theta), F[\Gamma] \right)^N, \right)

In general coordinates, the operators \( \delta_0^\rho (V \pm U)^N(0) \) in \( N = \Pi T^* M_{\text{ext}} \theta = 0 \) reduced to

\[
\partial_0 (V \pm U)^N(0) = \partial_0 \Phi^A(0) \partial / \partial \Phi^A(0) + \partial_0 \Phi^A(0) \partial / \partial \Phi^A(0),
\]
coincide with the generalized sum and difference of $V$, $U$ in \[ \text{(44)} \], for $t = 1, 2$:

$$
\partial_\theta (V - (-1)^t U)^N \theta) F(\theta) \big|_{\theta = 0} = (S^t(\theta), F(\theta))^N \big|_{\theta = 0} = (V - (-1)^t U)^N F[\Gamma] = (S^t[\Gamma], F[\Gamma])^N,
$$

where the $\varepsilon$-bosonic quantities $S^t(\theta)$ or $S^t[\Gamma]$ must obey certain differential equations providing the anticommutativity of the operators $\{\Delta^N, \partial_\theta V^N, \partial_\theta U^N\}$, while $\omega^t_{pq}(\theta)$ and $\tilde{\omega}^t_{pq}(\theta, \theta')$, identical to $\omega_{pq}(\theta)$ and $\tilde{\omega}_{pq}(\theta, \theta')$ in case $t = 1$, are given by

$$
\tilde{\omega}^t_{pq}(\theta, \theta') = \theta' \omega^t_{pq}(\theta') = (-1)^{t+\varepsilon(\Gamma^p)\varepsilon(\Gamma^q)} \tilde{\omega}^t_{qp}(\theta', \theta), \quad \omega^t_{pq}(\theta) = (-1)^{\varepsilon(\Gamma^p)\varepsilon(\Gamma^q)+t} \omega^t_{qp}(\theta).
$$

## 5 Summary

We have proposed a $\theta$-local description of an arbitrary reducible superfield theory as a natural extension of a standard gauge theory with classical fields $A^I$ to a superfield model defined on extended cotangent $\{A^I, A^J\}(\theta)$ and tangent $\{A^I, \partial_\theta A^J\}(\theta)$ odd bundles in respective Hamiltonian and Lagrangian formulations. It is shown that the conservation, under the $\theta$-evolution, of a Hamiltonian action $S_H((A, A^*)((\theta), \theta))$, or, equivalently, of an odd analogue of the energy, $S_E((A, \partial_\theta A)((\theta), \theta))$, is equivalent to a respective Hamiltonian or Lagrangian master equation. We have proposed a $\theta$-local description of Lagrangian quantization in non-Abelian supergravities for a reducible gauge model extracted from a general superfield model by conditions of the $\theta$-independence of the classical action and the vanishing of ghost number for $A^I(\theta)$ and the action. To investigate the BRST invariance and gauge-independence of the generating functionals of Green’s functions, we have used two equivalent Hamiltonian-like systems, defined in terms of a $\theta$-local antibracket and an even Poisson bracket, respectively. These systems permit a simultaneous description of the $\theta$-independence of the classical action and the vanishing of ghost number for $A^I(\theta)$ and the action.

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