Inter-particle collision in particle-laden isotropic turbulence

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Abstract. Direct numerical simulations of heavy particles in isotropic homogeneous turbulence are performed to study inter-particle collisions in a wide range of Stokes numbers. Dynamics of the particles are described by Stokes drag including particle-particle interactions via hard-sphere collisions while fluid turbulence is solved using a pseudo-spectral method. The statistics related to rotation and dissipation rates as well as collision frequency are investigated. A particular emphasis is placed on the statistics of the fluid experienced by heavy particles, which provide essential information on the collision process. A plausible explanation of the collision mechanism for various Stokes numbers is provided.

1. Introduction

Heavy particles suspended in turbulence are commonly found in nature and industrial flows, such as droplets in cloud, dust transportation in the atmosphere, etc. Particle motion is usually considered as a result of local aerodynamic forces, but inter-particle collision cannot be neglected when particle volumetric fraction becomes larger (Crowe, 1982). Saffman & Turner (1956) found that a collision frequency for particles that are smaller than Kolmogorov turbulence scale is independent of particle properties. Abrahamson (1975) extended kinetic theory of gases to incorporate the effect of large/dense particle inertia on collision rates. Sundaram & Collins (1997) systematically investigated the influence of particle parameters such as response time, diameter and number density on collision frequency. They found a strong dependency of collision frequency on Stokes number and explained the collision mechanism with accumulation of heavy particles towards low vorticity and high strain rate regions and de-correlation of particle motion from the fluid. However, it is still unclear whether the inter-particle collision affects on the preferential concentration and how background turbulence affects on the de-correlation. The objective of this paper is to study a complex interplay between the both effects for finite-inertia particles on collisions and to suggest a plausible explanation of the collision process for various Stokes numbers.

2. Numerical methods

Direct numerical simulations of isotropic turbulence at $Re_\lambda = 46$ with various Stokes ($St$) numbers ranged from 0.1 to 40 were performed to investigate the inter-collision of heavy particles by using a pseudo-spectral method. Note that $Re_\lambda$ denotes the Taylor-scale Reynolds number.

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and $St$ is defined as the ratio of the particle response time ($\tau_p = \rho_p d_p^2 / 18 \mu$) to the Kolmogorov time scale ($\tau_\eta$) of turbulence, where $d_p$ and $\mu$ are the diameter of a particle and dynamic viscosity of fluid, respectively. To maintain stationary statistics in turbulence, we used the forcing scheme utilizing Uhlenbeck-Ornstein random processes (Eswaran & Pope, 1988). The fluid domain (a cube of size, $L = 2\pi$) is uniformly discretized with $64^3$ grid points. Details for the numerical methods can be found in Jung et al. (2008). Under the assumption for heavy particles, all the transient drags are negligible in the BBO (Basset-Boussinesq-Oseen) equation of motion (Maxey & Riley, 1983). Thus, simplified equations of particle motion including particle-particle collisions can be written as:

$$\frac{dv_p}{dt} = \frac{1}{\tau_p}(u - v_p) + a_c \quad \text{and} \quad \frac{d\omega_p}{dt} = \frac{10}{3\tau_p} (\Omega - \omega_p).$$

(1)

where $u$ and $\Omega$ are instantaneous fluid velocity and rotation rate, while $v_p$ and $\omega_p$ are instantaneous particle velocity and rotation rate, respectively. Initially, $2 \times 10^6$ particles were released. To obtain flow quantities at the locations of the particles, the four-point Hermite interpolation scheme was used, while particles were tracked using the third-order Runge-Kutta time advancement scheme (Choi et al., 2004). The particle collision model in Gui et al. (2008) was used in determining the acceleration ($a_c$) caused by collision forces based on the conservation law of momentum and angular momentum for hard-sphere collision without energy loss (perfect elastic collision). We also used the detection procedure of the inter-particle collision in Gui et al. (2008), which is based on a geometric solution of relative displacements of any pairs of colliding particles. For the inter-particle collision, we updated the particle velocities at the post-collision state by the pre-collision velocities.
Figure 2. Effect of inter-particle collision on particle distribution: (a) probability density function (PDF) and (b) flatness of particle number density. Note that solid and dashed lines represent the PDFs of number density without and with collision, respectively.

3. Results

Dependency of the steady-state collision frequency on Stokes numbers is shown in Figure 1. We have also plotted two theoretical limits: Saffman and Turner theory for non-inertial particles (Saffman & Turner, 1956) and kinetic theory for large/dense particles (Abrahamson, 1975) as well as other collision frequency models described in Ayala et al. (2008). Similar to findings in Sundaram & Collins (1997), the collision frequencies for finite-inertia particles from the present simulations are higher than for the Saffman-Turner limit while lower than for kinetic theory but asymptotically approach both theoretical limits. We also found that maximum of the collision frequency is at about \( St = 4 \). This can be explained with two competing phenomenon: 1) preferential concentrations due to high strain and low vorticity and 2) crossing trajectory effects (See Figure 5 in Jung et al. (2008)). Thus, an intermediate Stokes number (\( St \approx O(1) \)) regime shows high collision frequency when particles are accumulated near vortical structures due to interaction between suspended particles and background turbulence but the particles are sufficiently de-correlated with background turbulence to cross the vortical structures.

The probability density functions (PDFs) of the number density are plotted in Figure 2 (a) for the different Stokes numbers in order to see the tendency of the preferential concentrations. Note that the number density \( n_p \) is defined as the ratio of the number of particles in a computational cell and its volume. There is no obvious difference of the PDFs of the number density between without and with inter-particle collision. In both cases, long tail of the PDF is observed for cases \( St = 1 \), while the PDFs for \( St = 0.1 \) and 10 are similar to the PDF of fluid particle. Such a long tail of the PDF for \( St = 1 \) reflects localized high concentrations of the particles. It is also observed in Figure 2 (b) that the flatness of the number density \( \langle n_p^4 \rangle / \langle n_p^2 \rangle^2 \) in both cases is maximized at \( St = 1 \), while the flatness for \( St \ll 1 \) or \( St \gg 1 \) seems to converge asymptotically at 3, which is the value for Gaussian distribution. Consequently, high collision rate is expected at the intermediate Stokes numbers (\( St \approx O(1) \)) due to the preferential concentrations. However, it is worthy to note that the maximum collision rate and flatness of the number density occur at different Stokes numbers. This implies that an interplay between particles and background turbulence influences the increase of the collision rate at \( St > 1 \).

For a statistically stationary turbulence, the Lagrangian autocorrelation function \( \rho_p(t) \) is
defined as
\[ \rho_p(t) = \frac{\langle v_p(t_0)v_p(t+t_0) \rangle}{\langle v_p^2(t_0) \rangle}, \] (2)

where \( t_0 \) is the reference time and its integral time scale is
\[ T_p = \int_{0}^{\infty} \rho_p(t) dt. \] (3)

Figure 3 shows the effect of inter-particle collision on \( \rho_p \) of the particle velocity and normalized integral time scale \( (T_p/T_L) \) by Lagrangian time scale of fluid particle \( (T_L) \). General trends show that \( \rho_p \) extends further as the Stokes number increases due to the fact that a heavier particle has more inertia (Jung et al., 2008). In the collision model, \( \rho_p \) shows an apparently quicker decaying for \( St > 7 \) than in the non-collision model. As results, \( T_p \) is reduced due to the collision for higher Stokes number particles. This indicates that temporal memory effects of particle motion are weakened and de-correlated after inter-particle collisions. Considering the correlation between the integral time scale and dispersion coefficient (Jung et al., 2008), less particle dispersions are expected due to the collision.

To provide statistical evidences for high collision frequency at the intermediate Stokes number regime, we investigated Lagrangian statistics of rotation and dissipation rates of fluid seen by particles with or without the inter-particle collision. We also plotted conditionally averaged rotation and dissipation rates of fluid seen by colliding particles in Figure 4(a) and (b), respectively. The statistics are normalized by their values for the non-inertial particles (fluid particles). There are no obvious differences in the statistics for the simulation results with or without the collision. The particles with finite-inertia still tend to accumulate in a preferential region of high dissipation rate and low rotation rate before or after the inter-particle collision. The conditional statistics indicate that colliding particles in the intermediate Stokes number regime have higher vorticity and dissipation rate than the non-colliding particles. This may imply that particles having highly intermittent accelerations in the core of vortical structure escape from the vortical structures due to centrifugal forces and are then collided with particles around the vortical structures.

Next, we investigate collision statistics such as collision angle \( (\theta_C) \) and collision time interval \( (t_C) \) for providing an evidence of collision mechanism. For a pair of colliding particles, a colliding
angle ($\theta_C$) is defined as the angle between the velocity vectors of the particles. The PDF of the collision angle shown in Figure 5 (a) indicates that the most probable collision angles are closer to zero for small Stokes numbers ($St \ll 1$) while the PDF becomes flat and symmetric as Stokes number increases. This implies that two particles are contacting rather than colliding with each other for the small Stokes numbers but the probability of head-on collision is increased as the Stokes number increases. Since the particle motion for large Stokes numbers ($St \gg 1$) are mainly governed by the particle inertia, collisions with an arbitrary angle are expected due to the de-correlated particle motions from the background turbulence. The collision time interval ($t_C$) is defined as an interval between two consecutive collisions of each particle. Figure 5 (b) shows the PDF of the collision intervals and the averaged collision interval. For the small Stokes numbers, most of inter-particle collisions are observed within a short time interval ($t_C/\tau_\eta < 50$), which means that particles tend to collide consecutively and to be aligned with each other. However, the PDF shows an exponential decay at long collision time intervals for the large Stokes numbers, which is observed for primary collisions where the time interval between two random collisions in Cate et al. (2004). As expected, the inset of Figure 5 (b) shows that the
mean of the collision interval time is increased with the increase of Stokes number while the local maximum of the collision interval is observed at $St = 1$.

4. Conclusions

Characteristics of the inter-particle collision in particle-laden forced isotropic turbulence at $Re_\lambda = 46$ was investigated using direct numerical simulation. Maximum collision frequency was observed at an intermediate Stokes number ($St = 4$) while the collision rate varied with the Stokes numbers. We also found that the number density distribution of the particles and Lagrangian statistics of the fluid seen by particles are not significantly influenced by the inter-particle collisions, except for the auto-correlation of the particle velocity for large Stokes numbers. Conditional statistics for the rotation rates and dissipation rates of the fluid seen by colliding particles provided an evidence of collision mechanism at the intermediate Stokes number regime where particles with highly intermittent accelerations in the core of vortical structure were collided with other particles around the vortical structures. The collision angle and interval showed that most of collision process for small Stokes number is a contact between a pair of aligned particles while for large Stokes number is a random collision with non-preferential collision angle due to the de-correlated particle motion from the background turbulence. Finally, mechanisms of inter-particle collision for three different Stokes number regimes can be illustrated in Figure 6: 1) For small Stokes numbers ($St \ll 1$), inter-particle collisions seem to occur when suspended particles near vortical structures follow streamlines around the vortices. Moreover, a particle tend to collide repeatedly with other particle moving towards same direction; 2) For intermediate Stokes numbers ($St \approx O(1)$), due to the preferential concentrations, particles are accumulated near vortical structures. Collisions seem to occur when particles having relatively high acceleration in vortical structures are moving outward from the structures due to centrifugal forces. Collision rates are enhanced by the preferential concentrations towards strong vortical and strain flow structures, and a moderate (or efficient) de-correlation of the particle motion from the fluid; 3) For large Stokes numbers ($St \gg 1$), particle motions become de-correlated from the fluid structures and collision process tends to be random. Collision rates are decreased with longer particle response time, approaching the value predicted by the kinetic theory.

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