New Approach for Measuring $|V_{ub}|$

at Future $B$–Factories

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Abstract

It is suggested that the measurements of hadronic invariant mass ($m_X$) distributions in the inclusive $B \to X_{c(u)} l \nu$ decays can be useful in extracting the CKM matrix element $|V_{ub}|$. We investigated hadronic invariant mass distributions within the various theoretical models of HQET, FAC and chiral lagrangian as well as ACCMM model. It is also emphasized that the $m_X$ distribution even at the region $m_X > m_D$ in the inclusive $b \to u$ are effective in selecting the events, experimentally viable at the future asymmetric $B$ factories, with better theoretical understandings.

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1. Introduction

The CKM matrix element $V_{ub}$ is important to the standard model description of CP violation. If it were zero, there would be no CP violation from the CKM matrix element (i.e. in the standard model), and we have to seek for other source of CP violation in $K_L \rightarrow \pi\pi$. Observations of semileptonic $b \rightarrow u$ transitions by the CLEO [1] and ARGUS [2] imply that $V_{ub}$ is indeed nonzero, and it is important to extract the modulus $|V_{ub}|$ from semileptonic decays of $B$ mesons as accurately as possible. Presently, the charged lepton energy is measured, and the $b \rightarrow u$ events are selected from the high end of the charged lepton energy spectrum. This method is applied to both inclusive and exclusive semileptonic $B$ decays.

However, this cut on $E_l$ is not very effective, since only below 10\% of $b \rightarrow u$ events survive this cut. As first discussed in [3], the measurements of hadronic invariant mass ($m_X$) distributions in $B \rightarrow X_{c,u}l\nu$ (inclusive decays) can be useful to extract a CKM matrix element $V_{ub}$. For $b \rightarrow c\nu$, one necessarily has $m_X \geq m_D = 1.86$ GeV. So, if we impose a condition $m_X < m_D$, the resulting events come from $b \rightarrow ul\nu$. According to the work of [3], $\sim 90\%$ of the $b \rightarrow u$ events survive this cut. This is in sharp contrast with the usual cut on $E_l$. Thus, one could get an order of magnitude improvement in selecting data.

Inclusive $m_X$ distributions for $b \rightarrow u$ transition can be obtained using the ACCMM model [4], or the work by BUSV [5]. In this work, we follow the result of Ref. [3] based on the ACCMM model, with a remark that theoretical uncertainties for the inclusive $b \rightarrow u$ decays are less compared to an exclusive mode.

Resonance contributions to the $m_X$ distributions in $B \rightarrow X_{c,u}l\nu$ can be estimated invoking various models. Once the decay rate for $B \rightarrow Rl\nu$ (where $R$ is a resonance) is known, the corresponding $m_X$ distribution can be written as

$$\frac{d\Gamma}{dm_X} \approx \frac{2m_X\Gamma(B \rightarrow Rl\nu)}{\pi} \frac{m_R\Gamma_R}{(m_X^2 - m_R^2)^2 + m_R^2\Gamma_R^2},$$

(1)

in the narrow width approximation. Here, $m_R$ and $\Gamma_R$ are the mass and the width of the resonance $R$. In the limit of $\Gamma_R \rightarrow 0$, we get

$$\frac{d\Gamma}{dm_X} = \Gamma(B \rightarrow R) \delta(m_X - m_R),$$

(2)

so that the correct decay rate for $B \rightarrow Rl\nu$ comes out upon integrating Eq. (2) over $dm_X$. 1
In Section 2, we discuss the result for $B \to D, D^*, (D^{**})$ in the heavy quark effective theory. In Section 3, $B \to (\pi, \rho)l\nu$ are considered in two different types of approaches: a nonrelativistic quark model and the chiral lagrangian with heavy mesons as well as light vector mesons. In Section 4, the $m_X$ distributions for $B \to X_{c(ud)}l\nu$ are shown, and it is emphasized that the $m_X$ distribution even at the region $m_X > m_D$ in the inclusive $b \to u$ are effective in selecting almost 100% of the events, experimentally viable at the future asymmetric $B$ factories. Since one can calculate the inclusive decay more reliably, one can achieve better determination of $V_{ub}$ both statistically and systematically.

2. The $B \to X_c l\nu$ decay in the heavy quark effective theory

Let us first consider $B \to X_cl\nu$, which is dominated by resonance contributions with $X_c = D, D^*, D^{**}$. Theoretical predictions based on the heavy quark effective field theory (HQET) \[6\] depend on one hadronic form factor $h_{A_1}(w)$. In order to calculate a decay rate, one has to know the shape of the form factor over the whole kinematic range of $w$. However, this form factor is not calculable from the first principle, except for the normalization : $h_{A_1}(1) = 0.99 \pm 0.04$, which is one of the predictions of HQET \[6\]. Thus, one necessarily resort to some model for the shape of the form factor. If one adopts the result of the QCD sum rule results, one may approximate the form factor as

$$h_{A_1}(w) \approx 0.99 \left( \frac{2}{w + 1} \right)^{2g_{A_1}^2},$$

with $g_{A_1}^2 \approx 0.8$. Since this result is based on the QCD sum rule, there exist some systematic uncertainties associated with the sum rule. This systematic uncertainty may be taken into account by allowing $g_{A_1}^2$ vary between 0.5 and 1.1, where the latter is on the border of the limit given by Voloshin’s sum rule \[7\]. For this range of $g_{A_1}^2$, one can predict the decay rates for $B \to D^{(*)}l\nu$ \[3\] :

$$\Gamma(B \to D l\nu) = (0.86 \sim 1.35) \times 10^{13} \ |V_{bc}|^2/\sec,$$

$$\Gamma(B \to D^* l\nu) = (2.59 \sim 3.48) \times 10^{13} \ |V_{bc}|^2/\sec,$$

where the smaller decay rates correspond to the larger $g_{A_1}^2$. For $B \to D^{**}l\nu$, we use
the observation by CLEO [8]:

\[ Br(B \to D^{**}l\nu) \simeq 0.5 \ Br(B \to Dl\nu). \] (6)

Our approach concerning the measurement of \( |V_{ub}| \) from the \( m_X \) distributions can be regarded independently of the uncertainties in Eqs. (4)–(6), because the \( m_X \) distributions from the three resonances \( D, D^* \) and \( D^{**} \) are essentially delta functions, Eq. (2), and we just require to exclude small regions around \( m_X = m_D, m_{D^*}, m_{D^{**}} \).

For more details, see Section 4 and Fig. 1.

3. The \( B \to X_u l\nu \) decay

Unlike the \( b \to c \) transition, the \( b \to u \) transition is largely nonresonant and multiple jet-like final states dominate [9]. The whole inclusive decay can be theoretically well understood in most of the kinematical region. The electron energy spectrum or the hadronic mass distribution for the inclusive semileptonic decay can be calculated rather reliably. In contrast, for the exclusive decays for \( b \to u \) such as \( B \to (\pi, \rho)^+ l\nu \), the model dependence becomes more pronounced, especially for the shape of the form factors. Here, we consider two classes of models, the FAC model (a nonrelativistic quark model) and the chiral lagrangian with heavy mesons. The results are compared with the \( m_X \) distribution obtained by the ACCMM model in Section 4.

A. The FAC model

The FAC model is based on the nonrelativistic quark model with assuming the form factors are factorized as [10]

\[ f_i^{\text{FAC}}(q^2) = f_i^{\text{FQM}}(q^2) \times F(q^2), \] (7)

where \( f_i^{\text{FQM}}(q^2) \) is the free quark model form factors arising from the overlap of the spin wavefunctions, and \( F(q^2) \) comes from the overlap of the spatial wave functions.

For \( B \to D^{(*)} \) transitions, one can impose the normalization condition, \( F(q_{\text{max}}^2) = 1 \), by heavy quark flavor symmetry. For \( B \to \pi (\text{or } \rho) \) transitions, this normalization is not valid and it may be reasonable to assume that the form factor suppression for
$B \rightarrow \pi$ begins at the $B \rightarrow \rho$ threshold. With these assumptions, one gets

$$\Gamma(B \rightarrow D l \nu) = (0.71 \sim 0.85) \times 10^{13} \vert V_{bc} \vert^2 \text{sec},$$  \hspace{1cm} (8)

$$\Gamma(B \rightarrow D^* l \nu) = (2.17 \sim 2.44) \times 10^{13} \vert V_{bc} \vert^2 \text{sec},$$  \hspace{1cm} (9)

$$\Gamma(B^0 \rightarrow \pi^+ l \nu) = (0.24 \sim 0.86) \times 10^{13} \vert V_{ub} \vert^2 \text{sec},$$  \hspace{1cm} (10)

$$\Gamma(B^0 \rightarrow \rho^+ l \nu) = (0.77 \sim 2.10) \times 10^{13} \vert V_{ub} \vert^2 \text{sec},$$  \hspace{1cm} (11)

for certain ranges of pole masses (see Ref. [10] for more details.) Note that the FAC model predictions for $B \rightarrow D^*$ are consistent with (although they are systematically lower than) those by the HQET discussed in the previous section. Hence, the FAC model for the $B \rightarrow D^{(*)}$ transitions are rather reliable. For $B \rightarrow \pi$(or $\rho$) transitions, the predictions are very sensitive to the shape of the form factors because of the large phase space available. Therefore, we simply regard the above numbers for $B \rightarrow \pi$(or $\rho$) transitions as exemplary values in a nonrelativistic quark model, without giving much meanings to the specific values.

B. The chiral lagrangian with heavy mesons

Recently, the chiral lagrangian with heavy mesons and baryons has been developed [11]–[13]. This lagrangian was originally invented in order to describes interactions among heavy mesons and light mesons such as $\pi$ and $K$ in the soft pion limit. Then, heavy baryons [14] as well as $\rho$ [15]–[16] have been incorporated in the leading order in $1/m_Q$ and chiral expansions. The weak current can be represented in terms of physical fields like heavy hadrons and light mesons, allowing us to calculate the matrix element of the weak current between hadrons and thus the semileptonic decays of heavy hadrons.

However, this approach has its own limitations. First of all, the hadronic form factors given by this chiral lagrangian is valid only in very limited regions of the whole kinematic region. Therefore, one often assumes certain shape of form factors and normalize them at a point to a value given by the chiral lagrangian with heavy hadrons. Furthermore, if one considers the next–to–leading order corrections in $1/m_Q$ and chiral expansions, there come in a lot of unknown parameters and one essentially loose predictability. Although the reparametrization invariance of the heavy quark field leads to some constraints to the parameters in the next–to–leading order terms, it still leaves many other parameters unconstrained. Therefore, results based on the
chiral lagrangian with heavy baryons should be understood, keeping in mind the uncertainties just mentioned above.

One of the extensive studies of semileptonic decays of heavy mesons in the framework of the chiral lagrangian with heavy hadrons is the work by R. Casalbuoni and his collaborators [15]. Their results are

\[
\Gamma(B^0 \rightarrow \pi^- l\nu) = 38.8 \left( \frac{f_B(\text{MeV})}{200} \right)^2 \times 10^{13} |V_{ub}|^2 / \text{sec}.
\]

\[
\Gamma(B^0 \rightarrow \rho^- l\nu) = 22.7 \left( \frac{f_B(\text{MeV})}{200} \right)^2 \times 10^{13} |V_{ub}|^2 / \text{sec}.
\]

At this point, a remark on the $B-$meson decay constant $f_B$ in Eqs. (12) and (13) is in order. In the lowest order in the $1/m_Q$ expansion,

\[
\frac{f_B}{f_D} = \sqrt{\frac{M_D}{M_B}}.
\]

On the other hand, the lattice QCD and the QCD sum rule [17] suggest that

\[
f_B \approx f_D \approx 200 \text{ MeV},
\]

which violates the scaling relation, Eq. (14). Thus, the results in Ref. [15] are expressed as above, although it is not systematic in $1/m_Q$ expansion to use Eq. (15).

We note that the results of Ref. [15] are substantially larger than those based on the FAC model. Especially, relative ratios between $B \rightarrow \pi$ and $B \rightarrow \rho$ are opposite in two models, and may be checked in the near future. For the isospin–related decay $B^- \rightarrow \rho^0 l^- \bar{\nu}_l$, the predicted decay rate is the half of Eq. (13), with the corresponding branching ratio

\[
Br(B^- \rightarrow \rho^0 l^- \bar{\nu}_l) = 0.44 \times 10^{-3} \left( \frac{f_B(\text{MeV})}{200} \right)^2 \left| \frac{V_{ub}}{0.0045} \right|^2,
\]

assuming $\tau_B = 1.29 \text{ ps}$. The data from ARGUS and CLEO seem contradictory with each other:

\[
Br(B^- \rightarrow \rho^0 l^- \bar{\nu}_l) = (1.13 \pm 0.36 \pm 0.26) \times 10^{-3} \quad \text{(ARGUS)}
\]

\[
< 0.3 \times 10^{-3} \quad \text{(CLEO)}.
\]

Note that two data are incompatible with each other. The ARGUS result [18] is consistent with the prediction by R. Casalbuoni et al., but is inconsistent with the
FAC model prediction. On the other hand, if the result by CLEO [19] is confirmed, the prediction based on the chiral lagrangian with heavy mesons would be excluded. In this case, there can be many possible reasons for it. First of all, interactions between $\rho$ and heavy mesons may not be well described by the chiral lagrangian in the lowest order because of relatively heaviness of $\rho$. This would be contrary to the better known case, the chiral lagrangian with vector mesons ($\rho$), where dynamics of $\pi, \rho$ are rather well described. Secondly, and most likely, the usual simple assumption on the shape of the form factor may not be right. In most cases including Ref. [15], it is assumed that a form factor $f(q^2)$ satisfies a monopole form:

$$f(q^2) = \frac{f(0)}{1 - q^2/M_{\text{pole}}^2}, \quad (19)$$

where $M_{\text{pole}}$ is a pole mass. This extrapolation of the form factor through the whole kinematic range does not have justifications from the first principle, and is a source of uncertainties in any models.

4. Discussions and conclusions

The resulting $m_X$ distributions for $B \to Rl\nu$ for $R = \pi, \rho, D, D^*, D^{**}$ are shown in Fig. 1, along with the inclusive $m_X$ distribution for the $b \to u$ transition, with $|V_{ub}/V_{cb}| = 1$. The $b \to c$ transition is dominated by the $X_c = D, D^*, D^{**}$, and can be reliably calculated in the HQET as described in the previous section. The regions between the triangles are the range of the predicted rate when the $dm_X$ integration over the delta function is performed. On the other hand, the $b \to u$ transition is largely nonresonant. The cases with $X_u = \pi, \rho$ are shown explicitly for two different models discussed in Section 3. For $X_u = \pi$, the region between the open triangles are predictions by Hagiwara et al. [10], and the region between the closed triangles are predictions by Casalbuoni et al. [15]. For $X_u = \rho$, the regions between the lower two curves are predictions by Hagiwara et al., whereas the region between the upper two curves are predictions by Casalbuoni et al. The inclusive $m_X$ distribution for $b \to u$ was obtained from the ACCMM model with hadronic mass constraint of $m_X \gtrsim 2m_\pi$. The exclusive decay of $B \to \pi l\nu$ is shown separately.

From Fig. 1, most of the $b \to u$ transition events survive the cut on the hadronic invariant mass, $m_X < m_D$, contrary to the more conventional cut on the electron
energy. In fact, one can relax the condition $m_X < m_D$ because the $m_X$ distribution in $b \to cl\nu$ is completely dominated by contributions by three resonances $D, D^*$ and $D^{**}$, which are essentially like delta functions, Eq. (2). In other words, one can use the $b \to u$ events in the region even above $m_X = m_D$, excluding small regions in $m_X$ around $m_X = m_D, m_D^*, m_D^{**}$. The cut on the $m_X$ is more effective than the cut on the electron energy by factor of $\sim 10$, and therefore we have much better statistics. Furthermore, theoretical understanding of exclusive decay modes of $B \to X_u l\nu$ is rather poor, as we discussed in Section 3. Two different models lead to vastly different predictions for $X_u = \pi$ and $\rho$. This would induce theoretical uncertainties in determination of $V_{ub}$ from the measurement of an exclusive semileptonic decay of $B$ mesons. On the other hand, the inclusive decay is better understood, so it would be more reliable to calculate the $m_X$ distribution for inclusive $b \to u$ transitions.

At future $B$-factory with microvertex detectors (symmetrical or asymmetrical), one expects that the efficiency for the event reconstruction will be improved (might 30 % efficiency). Then, among $10^8$ $B\bar{B}$ events, $\sim 10^5$ events without any constraint on $m_X$ may be reconstructed. For more details on the problems of experimental reconstruction and continuum background, see Ref. [3].

Even without a full event reconstruction, one may have good measurements of missing energy and momentum of missing neutrino, $E_\nu$ and $\vec{p}_\nu$ satisfying the mass–shell condition,

$$E^2_\nu - \vec{p}^2_\nu = m^2_\nu = 0.$$ 

In this case, the hadronic mass $m_X$ is not fully constructed, but it is bounded by

$$m^2_X < m^2_{X,max} = m^2_B + m^2_\nu - 2\gamma m_B(E_l\nu - \beta p_l\nu).$$  \hspace{1cm} (20)

Here, $m_B$ is the $B$-meson mass, and $\gamma = (1 - \beta^2)^{-1/2} = m_\Upsilon/2m_B$ for the symmetric $B$ factory with $e^+e^- \to \Upsilon(4S) \to B\bar{B}$. Since $\beta$ is very small, $m^2_{X,max}$ is close to $m^2_X$, and we lose very little efficiency. It turns out that $\sim 80\%$ events for $b \to u$ transitions survive the cut on the $m^2_{X,max}$ [3]:

$$m_{X,max} < m_D.$$ 

In any case, studying the hadronic mass distributions in inclusive semileptonic $b \to u$ transition is experimentally viable. It’s also theoretically better described, so theoretical uncertainties in determining $|V_{ub}|$ would be less compared to the $|V_{ub}|$ determined from studies of exclusive decay modes. In summary, we would have better
statistics in extracting $|V_{ub}|$ by measuring the $m_X$ distributions in inclusive $b \to u$ semileptonic decays, and have better theoretical handles over the inclusive decays rather than exclusive decays.

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**Figure Captions**

**Fig. 1** The $m_X$ distributions in $B \to X_{c,u}\ell\nu$ with $|V_{ub}/V_{cb}| = 1$. The $b \to c$ transition is dominated by the $X_c = D, D^*, D^{**}$, and can be reliably calculated in the HQET. The regions between the arrows are predicted rate in the unit of $10^{13}$ sec$^{-1}$ when the $dm_X$ integration over the delta function is performed. On the other hand, the $b \to u$ transition is largely nonresonant. The cases with $X_u = \pi, \rho$ are shown explicitly for two different models. For $X_u = \pi$, the region between the open triangles are predictions by Hagiwara et al., and the region between the closed triangles are predictions by Casalbuoni et al.. For $X_u = \rho$, the regions between the lower two curves are predictions by Hagiwara et al., whereas the region between the upper two curves are predictions by Casalbuoni et al.. The inclusive $m_X$ distribution for $b \to u$ was obtained from the ACCMM model with hadronic mass constraint of $m_X \sim 2m_\pi$. 
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