Recent work on random field Ising model is described briefly emphasising exact solutions of the model in simple cases and their relevance in understanding equilibrium and non-equilibrium properties of systems with quenched disorder.

I. INTRODUCTION:

Extended systems with quenched random disorder often possess a large number of nearly degenerate states separated by high energy barriers. This kind of energy landscape gives rise to very complex relaxation phenomena at low temperatures as well as hysteresis in the system. Even a weak disorder may modify the system significantly and destroy the long range order in the equilibrium state of the system. One needs a simple theoretical model to make sense of the rich, complex, and vast amount of experimental data in this field. The random field Ising model is perhaps the simplest minimal model that fits the bill. Although very simple to state, the model is not exactly solvable except in a few special cases. Computer simulations of the model are helpful, but are often unable to resolve the key questions because they suffer from metastability and slow relaxation rates in the system just as much as the laboratory experiments. For specificity, we focus on disordered magnets. Some of the important questions one would like to be answered are the following: What is the lower critical dimensionality below which any disorder, no matter how small, would destroy the long range order in the system at zero temperature? Could the disorder cause a first order jump discontinuity in the response of the system to an applied field? If so, in what circumstances? In recent years, we have examined these questions in some exactly solved cases of the model. Due to the limitation of space, we only indicate the scope and status of our studies, and mention the results obtained so far. Interested readers should consult the references for more technical details.

II. THE MODEL

The model is defined on a lattice. Each site is labeled by an integer \( i \), and carries an Ising spin \( S_i \) ( \( S_i = \pm 1 \) ), a quenched random magnetic field \( h_i \), and an externally applied uniform field \( h \). The quenched fields \( \{h_i\} \) are independent identically distributed random variables with a continuous probability distribution \( \phi(h_i) \). For convenience, we assume \( \phi(h_i) \) to be Gaussian with mean value zero, and variance \( \sigma^2 \). The nearest neighbor interaction can be ferromagnetic (\( J > 0 \)) or anti-ferromagnetic (\( J < 0 \)). The Hamiltonian of the system may be written as,

\[
H = -J \sum_{i\neq j} S_i S_j - \sum_i h_i S_i - h \sum_i S_i \tag{1}
\]

The spin \( S_i \) experiences a net field \( f_i \) on it that is given by,

\[
f_i = J \sum_j S_j + h_i + h \tag{2}
\]

The Glauber dynamics of the system at temperature \( T \) is specified by the rate \( R_i \) at which a spin \( S_i \) flips to \( -S_i \)

\[
R_i = \frac{1}{\tau} \left[ 1 - S_i \tanh(f_i/(k_B T)) \right], \tag{3}
\]

Here \( \tau \) sets the basic time scale for the relaxation of individual spins. The energy of the spin \( S_i \) is equal to \( -f_i S_i \). If \( f_i \) and \( S_i \) have the same sign, we say that the spin is aligned along the net field at its site. The energy of a spin is the lowest if it is aligned along the net field at its sight. We are interested in the properties of the model at zero temperature (\( T = 0 \)) and on time scales much larger than \( \tau \). In this limit, the dynamics simplifies to the following rule: choose a spin at random, and flip it only if it is not aligned along the net field at its site. Repeat the process till all spins are aligned along the net fields at their respective sites. The dynamics described above always brings the system to a fixed point state that is stable against single spin flips. The fixed point corresponds to a local minimum of the energy of the system. The system possesses a thermodynamically large number of fixed point states each marking a domain of attraction in the phase space of the system. The particular local minima that the system reaches depends on the domain that contained the initial state of the system.

Although the fixed points are stable, they can be thought to represent the metastable states of the system in the sense that they are local minima of energy. In this context, it is useful to consider the time scales of interest in physical systems, and the appropriateness of the zero-temperature dynamics as a model. In a typical experiment on non-equilibrium behavior of a physical system, there are at least four time scales: (i) time \( \tau \) that
an individual spin takes to relax, (ii) time \( \tau_1 \) that the system takes to relax to a metastable state, (iii) time \( \tau_2 \) over which the applied field changes, and (iv) life time \( \tau_3 \) of the metastable state. In complex systems, \( \tau_1, \tau_2, \) and \( \tau_3 \) may each contain an entire spectrum of time scales. In physical systems relevant to our model, the shortest time is \( \tau \) which may be taken to be unity to set the scale. The next larger time is \( \tau_1 = \nu \times \tau \), where \( \nu \) is the number of iterations of the dynamics to reach a fixed point. The applied field is assumed to vary very slowly (driving frequency goes to zero!) so that \( \tau_2 \gg \tau_1 \). Specifically, it means that the applied field is held constant during the relaxation of the system. The time \( \tau_3 \) is infinite. Thus, the present model is applicable if \( \tau_3 \gg \tau_2 \gg \tau_1 \gg \tau \). These conditions fit a wide class of complex magnetic materials that have a large number of metastable states separated from each other by barriers much larger than the available thermal energy.

### III. RELEVANCE TO EXPERIMENTS

At an intuitive level, quenched random fields may be thought to arise from impurities and imperfections in disordered magnetic materials. A more precise connection between random field Ising model and experiments was made by a theoretical argument due to Fishman and Aharony \[2\] suggesting that the critical behavior of a weakly diluted anti-ferromagnet in a uniform external field should be in the same universality class as that of a ferromagnet in a random external field. The essential idea is that an anti-ferromagnet without dilution has two sub-lattices on which the spins are oriented opposite to each other. One of the sub-lattices is aligned with the applied field. In the presence of dilution, locally the sub-lattice with most spins tends to align with the applied field in competition with the global anti-ferromagnetic order in the absence of dilution. As a result the applied field acts as an effective random field coupling to the anti-ferromagnetic order parameter i.e. the staggered magnetization. The effective random field produced by the applied field is proportional to the applied field. The strength of the random field is therefore easily controlled. One can do scaling studies with varying strengths of disorder in a system by simply tuning the applied field rather than making fresh samples with different degrees of dilution. This feature has a far reaching significance in investigating the theoretical model experimentally, and the interaction between theory and experiment has helped the field develop considerably, and clarified many fine points of the model \[3, 4\]. \( \text{Rb}_2\text{CoF}_4 \) is a good two dimensional Ising anti-ferromagnet. It consists of layers of magnetic ions with a single dominant intralayer exchange interaction, and an interlayer interaction which is smaller by several orders of magnitude. It is very anisotropic so that the spins can be well represented as Ising spins. The material can be magnetically diluted by introducing a small fraction of manganese ions in place of cobalt ions. Thus crystals of \( \text{Rb}_2\text{Co}_x\text{Mn}_{1-x}\text{F}_4 \) are good examples of a two dimensional diluted anti-ferromagnet suitable for an experimental realization of the two dimensional random field Ising model. In three dimensions, the most studied diluted anti-ferromagnet is \( \text{Fe}_x\text{Zn}_{1-x}\text{F}_2 \) crystal. In the pure ferrous fluoride \( \text{FeF}_2 \) crystal, the ferrous ions ( \( \text{Fe}^{++} \)) are situated approximately on a body centered tetragonal lattice. Each ferrous ion is surrounded by a distorted octahedron of flurine ( \( F^- \)) ions. The predominant interactions are a large single-ion anisotropy, and an anti-ferromagnetic exchange between nearest neighbor ferrous ions. The magnetic moments of ferrous ions on the corners of the tetragonal cell are anti-parallel to the magnetic moments of \( \text{Fe}^{++} \) ions on the body centers. The large crystal field anisotropy persists as the magnetic spins are diluted with \( \text{Zn} \). The diluted crystal remains an excellent Ising anti-ferromagnet for all ranges of magnetic concentration \( x \). Furthermore crystals with excellent structural quality can be grown for all concentrations \( x \) with extremely small concentration variation \( \delta x < 10^{-3} \). These attributes combine to make \( \text{Fe}_x\text{Zn}_{1-x}\text{F}_2 \) the popular choice for experiments on diluted anti-ferromagnets, although experiments have been done on several other materials as well \[3\].

### IV. DOMAINS IN RANDOM FIELDS

In one of the earliest studies of the random field Ising model, Imry and Ma \[1\] argued that quenched random fields in a system may cause a uniform ferromagnetic state to break into domains. The argument is essentially as follows. Consider a uniform ferromagnetic state at zero temperature. In the presence of random fields, a strategically placed domain of linear size \( L \) may turn over and gain an energy of the order of \( \sigma L^{d/2} \). However, this would create a domain wall that would cost an energy of the order of \( JL^{d-1} \). If \( d/2 > (d - 1) \), i.e. if \( d < 2 \), then for any \( \sigma \), there will be a characteristic length over which the bulk energy gain will overcome the cost of the surface energy. In other words, domains will occur spontaneously if \( d < 2 \). The above argument is intuitively appealing, but nonrigorous. It is also inconclusive at the lower critical dimensionality because the gain and the cost of energy both scale linearly with \( L \) at \( d = 2 \). The situation in two dimensions can be clarified by taking into account the roughness of the domain wall. The work of Binder \[5\] supplemented with numerical simulation of a toy model predicts that the gain in energy scales as \( \sigma L \log L \) in \( d = 2 \). Thus the domain argument predicts the absence of long range order in two dimensions.

It remained unclear for several years if the results obtained from the domain argument were correct. The controversy was generated by the existence of another argument based on a field theoretic method (dimensional reduction) that predicted the lower critical dimensionality of random field Ising model to be three. Initially, com-
puter simulations of the model as well as experimental observations did not help to resolve the controversy. It took several years to realize the error in the dimensional reduction argument. The difficulty with both theory and experiment was in the interpretation of results. Concentration gradients in the diluted anti-ferromagnetic samples tend to round off a transition and affect the measurements of critical behavior drastically. Further, the majority of experiments were performed on samples prepared in two separate ways: (i) cooling the sample in zero magnetic field, and (ii) cooling it in a magnetic field, and turning the field off at the end. Experiments on samples prepared in the two ways yield different results. Three dimensional field cooled samples show no long range order and were first thought to show that the lower critical dimensionality of the model is three. But three dimensional samples cooled in zero field showed long range order. It was only after several years of controversy that the experimental situation resolved itself in favor of the domain argument, i.e. no long range order in two dimensions, but long range order in three dimensions. The main point that was clarified by theoretical work is that the field cooled state is not an equilibrium state. It relaxes logarithmically slowly, and one should not expect to see an equilibrium ordered state in a field cooled sample over any reasonable experimental time scale.

V. HYSTERESIS

Permanent magnets are typically a two phase solid material with fine magnetic particles of one phase embedded in the other phase. The precipitation is carried out in a liquid matrix. Instead, the magnetic domains inside the matrix, the needles can not physically turn over as in a liquid matrix. If the field is reversed after the material is set in a solid magnetic field, and needle like magnetic particles are oriented with their long axis parallel to the field direction. If the field is reversed after the material is set in a solid matrix, the needles can not physically turn over as in a liquid matrix. Instead, the magnetic domains inside the particles have to reverse themselves. This requires a threshold field that varies from particle to particle. Experimentally observed magnetization of the material in a smoothly increasing applied field may look smooth on a macroscopic scale but on a microscopic scale, it is made of steps of irregular widths and heights. This is known as Barkhausen noise. Hysteresis in such materials is some-what different in character from the usual hysteresis that characterizes the ferromagnetic model on a Bethe lattice of coordination \( z \) \( \geq 3 \). For \( z \leq 3 \) there is no jump discontinuity in the hysteresis loops for any amount of disorder. For \( z \geq 4 \), there is a critical value of \( \sigma \) that characterizes the Gaussian random field distribution. If \( \sigma \) is less than the critical value \( \sigma_c \), the magnetization in increasing field has a macroscopic first-order jump at an applied field \( h_c \) > J. As \( \sigma \) increases to \( \sigma_c \), \( h_c \) decreases to J, and the first-order jump in magnetization reduces to zero. The system shows non-equilibrium critical behavior at \( h = h_c \), and
\( \sigma = \sigma_c \). For \( z=4 \), \( \sigma_c = 1.78 \) approximately. The value of critical disorder \( \sigma_c \) increases with the coordination number \( z \) of the lattice. At the critical point the Barkhausen noise shows a power law distribution. The probability of avalanches of size \( s \) scales as \( s^{-3/2} \). Nonequilibrium critical point phenomena and its relationship with the coordination number of the lattice appears to hold on other lattices as well. Numerical simulations and theoretical arguments on several periodic lattices embedded in two and three-dimensional space show that hysteresis on periodic lattices with \( z \geq 4 \) is qualitatively different from that on lattices with \( z < 4 \). Although there are some similarities between Bethe lattices and periodic lattices of the same coordination number, there are differences as well. The differences are related to the bootstrap percolation instability on some periodic lattices [15].

Minor hysteresis loops have also been obtained, and their calculation reveals an interesting feature of the model that has an experimental significance. Consider two halves of the major hysteresis loop connecting states of saturated magnetization in opposite directions. If the applied field is reversed while the system is on one of these halves, the magnetization trajectory branches off and heads towards the other half of the major loop. We find that the trajectory in reversed field meets the other half of the major loop exactly when the field has been reversed by an amount \( 2J \) irrespective of the point of reversal. This result provides an interesting possibility for measuring the exchange interaction \( J \) in a hysteresis experiment.

We have also studied hysteresis in the anti-ferromagnetic random field Ising model at zero temperature [14]. Hysteresis in the anti-ferromagnetic model is qualitatively different from that in the ferromagnetic model because it does not show Barkhausen noise. On account of the anti-ferromagnetic interactions, a spin turning up in an increasing field blocks its nearest neighbors from turning up. Thus there is no microscopic avalanche of up-turned spins as in the case of ferromagnetic interactions. A spin that turns up in increasing field in the anti-ferromagnetic model, occasionally causes its nearest neighbor (that had turned up earlier) to turn down. As the applied field increases from \(-\infty\) to \(+\infty\), a small fractions of sites flip three times, first up, then down, and finally up again. Also, the dynamics of the anti-ferromagnetic model is non-Abelian. These features make the anti-ferromagnetic dynamics rather complex, and an exact solution of the model becomes difficult. So far the anti-ferromagnetic model had been solved exactly in one dimension only, and that too for a rectangular distribution of the random field of width \( 2\Delta \), where \( \Delta \leq |J| \).

Recently, we have been able to extend the calculation of hysteresis in a one dimensional random field Ising model to an arbitrary continuous distribution of the random field [16].

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