Identifying students' understanding of missing angles in parallel lines: a case study of year 7 students in The United Kingdom

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Abstract. This study aims to identify students' understanding of missing angles in parallel lines by considering what is arbitrary and what is necessary. Data were collected from observing a lesson in a year 7 classroom about the role of angles in parallel lines and interviewing the students to identify their understandings. The results reveal that there are three levels of students' understanding: (1) the students can remember arbitrary things, but not be aware of the missing angles; (2) the students can remember arbitrary things and sometimes can be aware of necessary things, but get confused when solving a different problem; (3) the students can remember arbitrary things, can be aware of the missing angles although the problem is different from what they usually do, but they do not remember the terms introduced by the teacher during the lesson. It is suggested that teachers have such skill to identify if their students can memorize what is arbitrary and be aware of what is necessary, to assist their students' memories and to help to educate their awarenesses.

1. Introduction
It is common to describe understanding as "knowing how," instead of "knowing that" [1, 2]. To illustrate, a student who understands a mathematical concept must know how to use the concept to solve problems that are conceptually related, but a student who knows how to solve the problem might not understand the concept as potentially he or she just memorize the procedure. Hence, understanding can also be categorized into relational understanding, which is "knowing both what to do and why," and instrumental understanding, which is "rules without reasons" [3, 4]. In this case, instrumental understanding seems not the real understanding because it is more like "knowing that," where students might memorize rules without having relational understanding.

In the West, memorizing and understanding are often related to “surface” and “deep” approaches respectively, whereas in the East, to achieve the best result, it is believed that memorizing and understanding should be combined [5, 6, 7]. Therefore, I believe that combining both memorizing and understanding can help students to connect a concept that they have grasped to address a problem related to the concept. For instance, if a student wants to understand how to find the volume of a cube, he or she has to remember or know the definitions of volume and cube. Knowing the definitions can help him/her to relate and to understand how to find the volume of the cube. All in all, memorizing and understanding must be working together for the student to have a full understanding. As such, students have to memorize arbitrary things and be aware of necessary things [8].
2. Methods

2.1. The framework of students’ understanding

In mathematics, there are names, labels, and conventions, which are arbitrary for students. As a consequence, the students have to memorize and remember them so that they can communicate their ideas to others. One example of arbitrary things is “a straight line has 180 degrees”. Teachers cannot explain why it is 180 degrees since it is a convention, which is why there is no reason behind arbitrary things. Therefore, the teachers can only inform the students about arbitrary things in a way that they go to the realm of students’ memory.

On the other hand, teachers should not inform their students about the necessary things. Otherwise, the students will memorize them (received wisdom). Students are expected to work out on what is necessary so that they are aware of it and able to provide reasons. For example, the students can figure out a missing angle of a triangle, in which two angles are given. To figure out the angle, the students have to be aware of it, and as such, the necessary things are in the realm of awareness.

When identifying students’ understanding based on what is arbitrary and what is necessary for teaching mathematics, there will be possible conditions that the students may experience, as shown in Figure 1.

![Figure 1. The possible conditions of students’ understanding based on arbitrary and necessary things (boxes labeled 1, 2, and 3 are not described in Hewitt’s papers)](image)

It has been described previously that arbitrary things have to be in the realm of memory while necessary things are in the realm of awareness. Besides, there are possibilities where students neither memorize nor find out what is arbitrary, and they are not aware of what is necessary.

2.2. Research Methodology

This case study used a qualitative approach, in which the data were collected from observation, interviews, and documentation. Firstly, a lesson delivered by a mathematics teacher about missing angles in parallel lines was observed in a year 7 classroom in the United Kingdom. The school, the teacher, and the students’ names are anonymous. In the classroom, there were 14 students consisting of nine boys and five girls. The lesson’s aim was to discover missing angles in parallel lines with a straight line crossing them. One week after the observation, six students (four boys and two girls) were interviewed to identify their understanding based on arbitrary and necessary things in the lesson. Also, some worksheets done by the students were collected during the observation and the interviews.
Finally, the collected data were analyzed thematically to reveal the students’ understanding of missing angles in parallel lines by considering how they memorized what is arbitrary and became aware of what is necessary.

3. Result and Discussion
3.1. Observation
First of all, the teacher wrote down the aim of the lesson, which is to discover missing angles in parallel lines with a straight line crossing them. Next, the teacher started the lesson by drawing four pairs of lines on the whiteboard, as shown in Figure 2.

![Figure 2. Pairs of lines provided by the teacher](image)

The teacher asked, "How would you describe each set of lines?". “They are equal, they are symmetrical, and they are parallel," a student answered. After that, a student said, "The last two are not parallel because parallel lines never touch." After successfully recalling students’ understanding of parallel lines, the teacher then moved to Worksheet 1.

3.1.1. Worksheet 1
The teacher instructed the students to measure each angle in parallel lines with a straight line crossing them by using a protractor. Several students could easily measure the angles, while others still needed assistance. Fortunately, a teacher assistant was there to help them. After measuring the angles, a student realized that some sets of angles were the same. The teacher then asked every angle the students found while drawing them on the whiteboard, as shown in Figure 3. Eventually, they realized that a straight line is equal to 180º and opposite angles are the same.

![Figure 3. The angles that the students were supposed to find](image)

Unexpectedly, the picture drawn by the teacher was different from that on students’ worksheets, which made some students confused. After realizing this situation, the teacher corrected the image so that it was the same as that on the spreadsheet 1.
Interestingly, a couple of students could not precisely measure the angles. As a consequence, they discovered that the straight line was equal to 179° (Figure 4a), which might also be caused by unprecise pictures given on the worksheet 1, and the corresponding angles are different (Figure 4b).

### 3.1.2. Worksheet 2
After that, the teacher instructed the students to figure out the missing angles on the worksheet without using a protractor, but they only knew one angle. Some students could easily find the missing angles, while others faced some difficulties. Therefore, the teaching assistant assisted them, who could not figure out the missing angles.

Before moving to the last worksheet, the teacher asked the students to pay attention and then started introducing alternate angles and corresponding angles by drawing pictures on the whiteboard, as shown in Figure 5.

![Figure 5. Alternate angles (left) and corresponding angles (right)](image)

### 3.1.3. Worksheet 3
The last worksheet is about exercises, where the students were given 24 questions about missing angles in parallel lines. However, some of them did not accomplish all the questions on the worksheet because the lesson had ended.

### 3.2. Interviews
During the interviews, two similar, but different problems (Figure 6) were given to the students.
Based on my observation during the interviews, when the students were given a different picture from what they used to do, they became confused. That is why I think that they might memorize the necessary things (received wisdom). From the interviews, I categorized the students’ understandings into three levels based on how they memorized arbitrary things and how they became aware of necessary things.

3.2.1. **Level 1**
The students at this level could remember what is arbitrary such as “a straight line has 180°”, but they could not figure out the missing angles in the problems. The evidence is when I asked a student “if this is 60°, what about this angle?”, He said, "Because it is a 180° line so it would be, em, can I use a paper, please?". I gave him a piece of paper, and he worked on it. He then answered "40°" (the answer should be 120°). When I asked him the opposite angle of the 60°, he replied: "I do not know." Also, the other characteristic of the students is that they tended to guess and to estimate the angles because when I asked “Why are this 40° and this 30°?”, he said, “Erm, I don't know, just guessing."

3.2.2. **Level 2**
At this level, the students also could remember that “a straight line has 180°” and figure out the missing angles, but they became confused when given different problem as shown in Figure 7.

3.2.3. **Level 3**
The students at this level were also able to memorize the common arbitrary thing that “a straight line has 180°”, but they were not able to remember the arbitrary things introduced by the teacher at the end of worksheet 2, which are alternate angles and corresponding angles (none of the students interviewed could memorize it). Even though the students at this level could not memorize the terms, they were
able to figure out the missing angles confidently. The evidence can be seen when I asked them to find the alternate angle of the first problem that I gave him, and they answered: "I’m pretty sure that would be 60° because it is opposite or parallel angles have the same, I think, but I can’t remember the name of it”.

3.3. Discussion
The aforementioned evidence reveals that memorizing arbitrary things does not imply students' awarenesses of necessary things, but their understanding levels do. This is because all of the students interviewed could remember “a straight line has 180°”, but only a few of them could completely be aware of the missing angles. However, understanding what is arbitrary can potentially help students to become aware of what is necessary.

At level 1 of students understanding, the students remembered that a straight line has 180°, but they could not figure out the missing angles. This might be caused by the obstacle they experienced when attempting to measure the angles during the lesson that I observed, in which a few of them found that a straight line made 179° (figure 4). On the other hand, the teacher said that a straight line is equal to 180°. As a consequence, they became confused because they believed and memorized what the teacher said, but not understanding what the arbitrary thing means. As such, they were not able to become aware of the missing angles (the necessary things). Also, some students seemed to lack of number sense and ability to estimate the missing angles even though they knew that a straight line makes 180°[9, 10]. All in all, it is suggested that teachers have to be careful when approaching their students using discovery learning that is related to measurement and estimation.

The next level of understanding is where the students seemed aware of the necessary thing (figuring out the missing angles), but in fact, they were not aware. When the students at level 2 were given a common problem, they could solve it. However, they became confused when given the different (uncommon) problem, even though the problems were similar. This evidence shows that the students lacked awareness of alternate and corresponding angles [11]. They did not fully understand the arbitrary things.

Moreover, they forgot the terms. The students just memorized the necessary things (received wisdom) because they just followed the procedures. As a result, although they could solve the problems procedurally, they probably did not relationally understand the concept [12-14]. To improve their understanding, I think they need more guidance and practice more similar problems.

The top level of understanding in this study is the condition in which the students could remember and completely understand some arbitrary things. Besides, they could be aware of the necessary things regardless of the fact that they forgot the terms “alternate angles” and “corresponding angles” introduced by the teacher during the lesson. The students were not able to remember the terms because the teacher used them only once. It requires more repetitions to remember arbitrary things [15]. Therefore, it is suggested to unpack the arbitrary things to engage the students with the mathematics concept.

In conclusion, these three levels of students' understanding give an insight that memorizing and understanding cannot be separated for students to have a full understanding of a concept. Memorizing arbitrary things is not enough to be aware of necessary things. Students should also understand arbitrary things. However, although understanding arbitrary things is enough to make students aware of necessary things, remembering arbitrary things will help students to communicate their ideas and improve their understanding levels [16]. This study suggests that mathematics teachers should have a skill in identifying their students' levels of understanding by considering what is arbitrary and what is necessary.

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