Average formation length in string model

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The space-time scales of the hadronization process in the framework of string model are investigated. It is shown that the average formation lengths of pseudoscalar mesons, produced in semi-inclusive deep inelastic scattering (DIS) of leptons on different targets, depend from their electrical charges. In particular the average formation lengths of positively charged hadrons are larger than of negatively charged ones. This statement is fulfilled for all using scaling functions, for $z$ (the fraction of the virtual photon energy transferred to the detected hadron) larger than 0.15, for all nuclear targets and any value of the Bjorken scaling variable $x_{Bj}$. In all cases, the main mechanism is direct production of pseudoscalar mesons. Including in consideration additional mechanism of production in result of decay of resonances, leads to decrease of average formation lengths. It is shown that the average formation lengths of positively (negatively) charged mesons are slowly rising (decreasing) functions of $x_{Bj}$. The obtained results can be important, in particular, for the understanding of the hadronization process in nuclear environment.

PACS numbers: 13.87.Fh, 13.60.-r, 14.20.-c, 14.40.-n
Keywords: electroproduction, hadronization, Lund string model, formation length

Hadronization is a process that leads from partons produced in an initial hard interaction to the final hadrons observed experimentally. Two aspects of hadronization: (i) the spectra of hadrons produced and their kinematical dependences; (ii) the space-time evolution of the process, at present investigated not equally well. First of them has been studied extensively in $e^+e^-$ annihilation and lepton-nucleon DIS both experimentally and theoretically. As a result the spectra of hadrons and their kinematical dependences are rather well known. There are a few successful theoretical models, which give transition from initial partons to final hadrons through sets of fragmentation functions. The second aspect of hadronization, the space-time evolution of the process, despite on its importance, was investigated relatively little. Although the string model allows to study the space-time scale of the hadronization process, only a few works were performed in this direction [1,2]. In [2] it was claimed, that for hadrons (as composite particles) the very notion of formation length is ambiguous because different constituents of a hadron can be produced at different lengths. It is then an open and model-dependent question which of the two length scales plays more important role in hadronization process: (i) constituent formation length $l_c$ which is the distance between DIS point and the point where first constituent parton of the final hadron arises; (ii) the yo-yo formation length $l_y$ which is the distance between DIS point and point where two constituent partons of the final hadron meet first time and form particle with quantum numbers of final hadron but without its "sea". It is worth to note that these two length scales connected by simple way.

The string model can give information about space-time scale of hadronization, but it gives nothing about development of hadronic properties of string during hadronization process. At present it is supposed, that the unique way to get information about latter is the experimental and phenomenological study of hadronization process in atomic nuclei, where string interacts with nuclear medium. It is assumed that constituent formation length plays more important role in the hadronization process, because beginning with this scale the piece of string with a constituent parton on the slow end interacts with hadronic cross section. As was pointed out in Ref. [2] this result follows from the comparison with the data on fragmentation of $30 GeV$ pions into $p$ and $\bar{p}$ [3]. In [3] the formation lengths of pions were presented in form $l = (1 - w)l_c + w l_y$, where $w$ is the probability that formation length is $l_y$. Comparison with experimental data [4] showed, in case when $l_c$ and $l_y$ were calculated in the framework of standard Lund model, that $w = 0.1 - 0.17$. This result confirmed conclusion of Ref. [2] about importance of constituent formation length. Further we will consider it as a formation length. It is a function of variables $v$ and $z$ (the energy of virtual photon and the fraction of this energy carried away by the final hadron with energy $E_h (z = E_h/v)$) and can change, as we will see below, in wide region from zero to tens (may be even hundreds) femtometers.

In Refs. [4-5] it was shown that a ratio of multiplicities for the nucleus and deuterium can be presented in the form of a function of a single variable which has the physical meaning of the formation length (time) of the hadron. This scaling was verified for the case of charged pions by HERMES experiment [8]. Now HERMES experiment prepares two-dimensional analysis of nuclear attenuation data using more precise definition of the formation length of hadron presented in this work.

In the string model, for the construction of fragmentation functions, the scaling function $f(z)$ is introduced (see, for instance, Refs. [10-12]). It is defined by the condition that $f(z)dz$ is the probability that the first hierarchy (rank 1) primary meson carries away the fraction of energy $z$ of the initial string. We use three different scaling functions for calculations:

\begin{equation}
\text{The function is defined as:}
\end{equation}
(i) standard Lund scaling function \[ f(z) = (1 + C)(1 - z)^C, \] where \( C \) is the parameter which controls the steepness of the standard Lund fragmentation function; (ii) symmetric Lund scaling function \[ f(z) = N z^{-1}(1 - z)^a \exp(-b n^2 z^2), \] where \( a \) and \( b \) are parameters of model, \( m_\perp = \sqrt{m_h^2 + p_T^2} \) is the transverse mass of final hadron, \( N \) is normalization factor; (iii) Field-Feynman scaling function \[ f(z) = 1 - a + 3a(1 - z)^2, \] where \( a \) is parameter of model. We will specify parameters below, when will discuss the details of calculations.

In the further study we will use the average value of the formation length defined as \( L_h^b = \langle l_h \rangle \).

The consideration is convenient to begin from \( L_h^b \) direct, \( f_h^{d(dir)} \), which takes into account the direct production of hadrons:

\[
L_h^{(dir)} = \int_0^\infty \int_0^\infty \ell \, d\ell \, D_h^b(L, z, l) / \int_0^\infty \int_0^\infty \ell \, d\ell \, D_h^b(L, z, l),
\]

where \( L = \nu / \kappa \) is the full hadronization length, \( \kappa \) is the string tension (string constant), \( D_h^b(L, z, l) \) is the distribution of the constituent formation length \( l \) of hadrons carrying fractional energy \( z \).

\[
D_h^b(L, z, l) = \left( C_{p1} f(z) \delta(l - L + z L) + C_{p2} \sum_{i=2}^n \delta(l - L + z L) \right) \theta(l) \theta(L - z L - l).
\]

The functions \( C_{p1}^h \) and \( C_{p2}^h \) are the probabilities that in electroproduction process on proton target the valence quark compositions for leading (rank 1) and subleading (rank 2) hadrons will be obtained. Similar functions were obtained in [13] for more general case of nuclear targets. In eq.(5) \( \delta \)- and \( \theta \)-functions arise as a consequence of energy conservation law. The functions \( D_h^b(L, z, l) \) are distributions of the constituent formation length \( l \) of the rank \( i \) hadrons carrying fractional energy \( z \). For calculation of distribution functions we used recursion equation from Ref. [2].

The simple form of \( f(z) \) for standard Lund model (eq.(1)) allows to sum the sequence of produced hadrons over all ranks \( n = \infty \). The analytic expression for the distribution function in this case is:

\[
D_c^b(L, z, l) = L(1 + C) \frac{L^C}{(l + z L)^{C+1}} \times \left( C_{p1}^h \delta(l - L + z L) + C_{p2} \frac{1}{l + z L} \right) \theta(l) \theta(L - z L - l).
\]

In [1, 2] (see also [14]) eq.(6) was obtained for the special case \( C_{p1}^h = C_{p2}^h = 1 \). Another special case was considered in Ref. [15]. It was supposed, that in electroproduction process only the \( u \) quarks are knocked out, which then turn into observed hadrons. It is clear, that this approximation is more or less valid for proton target in region of large enough values of Bjorken scaling variable. For positively charged mesons it does not differ from the first special case, for negatively charged ones it corresponds to special case \( C_{p1}^h = 0, C_{p2}^h = 1 \). Nevertheless, in this rough approximation it was obtained, that the average formation lengths for positively charged hadrons are larger than for negatively charged ones.

Unfortunately, in case of more complicated scaling functions presented in eqs.(2) and (3) the analytic summation of the sequence of produced hadrons over all ranks is impossible. In these cases we limited ourself by \( n = 10 \) in eq.(5).

As it is well known, essential contributions in the spectra of pions and kaons from the decays of vector mesons are expected. But so far, the formation lengths of pseudoscalar mesons were considered without taking into account this possibility.

We build the distributions for daughter mesons from the ones for parent resonances followed the method used in [10, 11] for the construction of fragmentation functions for daughter mesons.

The distribution function of the constituent formation length \( l \) of the daughter hadron \( h \) which arises in result of decay of parent resonance \( R \) and carries away the fractional energy \( z \) is denoted \( D_c^{R/h}(L, z, l) \). It can be computed from the convolution integral:

\[
D_c^{R/h}(L, z, l) = dR/h \int_{z_{\downarrow}^{R/h}}^{z_{\uparrow}^{R/h}} dz' D_c^{R}(L, z', l) \times f^{R/h}(\frac{z}{z'}),
\]

where \( z_{\uparrow}^{R/h} = \min(1, z/z_{\max}^{R/h}) \) and \( z_{\downarrow}^{R/h} = \min(1, z/z_{\min}^{R/h}) \), \( z_{\max}^{R/h} (z_{\min}^{R/h}) \) is maximal (minimal) fraction of the energy of parent resonance, which can be carried away by the daughter meson.

Let us consider the two-body isotropic decay of resonance \( R, R \rightarrow h_1 h_2 \), and denote the energy and momentum of the daughter hadron \( h (h = h_1 \text{ or } h_2) \), in the rest system of resonance, \( E_h^{(0)} \) and \( p_h^{(0)} \), respectively. In the coordinate system where resonance has energy and momentum equal \( E_R \) and \( p_R \),

\[
z_{\max}^{R/h} = \frac{1}{m_R} \left( E_h^{(0)} + \frac{p_R}{E_R} p_h^{(0)} \right),
\]

\[
z_{\min}^{R/h} = \frac{1}{m_R} \left( E_h^{(0)} - \frac{p_R}{E_R} p_h^{(0)} \right),
\]

where \( m_R \) is the mass of resonance \( R \). In the laboratory (fixed target) system the resonance usually fastly moves,

[12]
[17]
i.e. $p_R/E_R \to 1$.

The constants $d^{R/h}_c$ can be found from the branching ratios in the decay process $R \to h$. We will present their values for interesting for us cases below.

The distributions $f^{R/h}(z)$ are determined from the decay process of the resonance $R$, with momentum $p$ into the hadron $h$ with momentum $z p$. We assume that the momentum $p$ is much larger than the masses and the transverse momenta involved.

In analogy with eq.(4) we can write the expression for the average value of the formation length $L^{R/h}_c$ for the daughter meson $h$ produced in result of decay of the parent resonance $R$ in form:

$$L^{R/h}_c = \int_{0}^{\infty} dl D^{R/h}_c(L, z, l)/\int_{0}^{\infty} dl D^{R/h}_c(L, z, l) . (10)$$

Here it is need to give some explanations. We can formally consider $L^{R/h}_c$ as the formation length of daughter meson $h$ for two reasons: (i) the parent resonance and daughter hadron are the hadrons of the same rank $i$, which have common constituent quark; (ii) according to above discussions, beginning from this distance the chain consisting from hadron, resonance and final meson $h$ interacts (in nuclear medium) with hadronic cross sections.

The general formula for $L^h_c$ for the case when a few resonances contribute can be written in form:

$$L^h_c = \int_{0}^{\infty} dl \left( \alpha_p D^h_c(L, z, l) + \alpha_v \sum_{R} D^{R/h}_c(L, z, l) \right)/$$

$$\int_{0}^{\infty} dl \left( \alpha_p D^h_c(L, z, l) + \alpha_v \sum_{R} D^{R/h}_c(L, z, l) \right) , (11)$$

where $\alpha_p$ ($\alpha_v$) is the probability that $q \bar{q}$ pair turns into pseudoscalar (vector) meson. Following Refs. $[11, 12]$ we use condition $\alpha_p = \alpha_v = 1/2$.

Let us now discuss the details of model, which are necessary for calculations. We will cosider four kinds of pseudoscalar mesons $\pi^+, \pi^-, K^+$ and $K^-$ electro-produced on proton, neutron and nuclear targets. The scaling function $f(z)$ in eq.(1) has single free parameter $C$. It is known $[12]$ that comparison with experimental data gives limitation on its possible values $C = 0.3 - 0.5$. Our calculations showed that changing of parameter $C$ from the minimal value $C = 0.3$ to the maximal value $C = 0.5$ leads to the small increasing of the average formation lengths. Therefore further we will present results for standard Lund model obtained for $C = 0.3$ only. For symmetric Lund scaling function we use parameters $[17]$ $a = 0.3, b = 0.58 GeV^{-2}$, and for Field-Feynman scaling function $[11]$ the value $a = 0.77$ for the single parameter $a$. Next parameter, which is necessary for the calculations in the framework of string model is the string tension. It was fixed at a static value determined by the Regge trajectory slope $[16, 17]$ $\kappa = 1/(2\pi a') = 1 GeV/fm$ . (12)

Now let us turn to the functions $C^{h}_{pi}$ and $C^{h}_{p2}$. For pseudoscalar mesons they have form:

$$C^{h+}_{pi} = \frac{1}{2} u(x_{Bj}, Q^2) + \frac{1}{2} \tilde{u}(x_{Bj}, Q^2) \sum_{q=u,d,s} c^2_q q(x_{Bj}, Q^2) \gamma_q ,$$

$$C^{h-}_{pi} = \frac{1}{2} \tilde{u}(x_{Bj}, Q^2) + \frac{1}{2} d(x_{Bj}, Q^2) \sum_{q=u,d,s} c^2_q q(x_{Bj}, Q^2) \gamma_q ,$$

$$C^{h+}_{p2} = C^{h-}_{p2} = \gamma_q ,$$

$$C^{K+}_{pi} = \frac{1}{2} u(x_{Bj}, Q^2) + \frac{1}{2} \tilde{u}(x_{Bj}, Q^2) \sum_{q=u,d,s} c^2_q q(x_{Bj}, Q^2) \gamma_q ,$$

$$C^{K-}_{pi} = \frac{1}{2} \tilde{u}(x_{Bj}, Q^2) + \frac{1}{2} d(x_{Bj}, Q^2) \sum_{q=u,d,s} c^2_q q(x_{Bj}, Q^2) \gamma_q ,$$

$$C^{K+}_{p2} = C^{K-}_{p2} = \gamma_q \gamma_s ,$$

where $x_{Bj} = Q^2/2m_p^2$ is the Bjorken’s scaling variable; $Q^2 = -q^2$, where $q$ is the 4-momentum of virtual photon; $m_p$ is proton mass; $q(x_{Bj}, Q^2)/(q(x_{Bj}, Q^2))$, where $q = u, d, s$ are quark (antiquark) distribution functions for proton. Easily to see, that functions $C^{h}_{pi}$ for hadrons of higher rank $(n > 2)$ coincide with ones for second rank hadron $C^{h}_{p2} = C^{h}_{p2}$. This fact was already used for construction of eq.(5). For neutron and nuclear targets more general functions $C^{h}_{pi}(i = 1, 2)$ from $[13]$ are used. Functions for resonances can be built in analogy with the above equations. For example, $C^{h+}_{pi} = C^{h-}_{pi}$, where $i = 1, 2$.  

All calculations were performed at fixed value of $\nu$ equal $10 GeV$. Calculations of $z$ dependence were performed at fixed value of $Q^2$ equal $2.5 GeV^2$, which give $x_{Bj} \approx 0.133$. For quark (antiquark) distributions in proton the parameterization in leading order parton distribution functions from $[18]$ was used. We assume, that new $q \bar{q}$ pairs are $u\bar{u}$ with probability $\gamma_{u}\alpha \bar{d}d$ with probability $\gamma_d$ and $s\bar{s}$ with probability $\gamma_s$. It is followed from isospin symmetry that $\gamma_u = \gamma_d = \gamma_s$. We use two sets of values for $\gamma$: for Lund model $[17]$ $\gamma_u : \gamma_d : \gamma_s = 1 : 1 : 0.3$ and for Field-Feynman model $\gamma_u : \gamma_d : \gamma_s = 1 : 1 : 0.5$.

We take into account that part of pseudoscalar mesons can be produced from decays of resonances. As possible sources of $\pi^+, \pi^-, K^+$ and $K^-$ mesons we consider $\rho^+, \rho^0, \omega, K^{*+}$ and $K^{*0}; \rho^-, \rho^0, \omega, K^{*-}$ and $K^{*0}; K^{*+}, K^{*0}$ and $\phi; K^{*-}, K^{*0}$ and $\phi$ mesons, respectively. The contributions of other resonances are neglected.

In Ref. $[10]$ were presented simple expressions for $f^{R/h}$ which are close enough to the experimental data. In case of pions we interested in $f^{0/\pi}, f^{0/\pi}$ and $f^{K^0/\pi}$, $f^{K^0/\pi}$, $f^{K^0/\pi}$.
in case of kaons in \( f^{K^+}/K \) and \( f^{\phi}/K \). (Sometimes we omit the charge symbols in our expressions, but everywhere it is imply that the different necessary charge states for parent and daughter hadrons are taken into account.) The function \( f^{\pi^+}/\pi \) has form \( f^{\pi^+}/\pi(z) = 2(1 - z) \). For the other functions common expression \( f^{R/h}(z) = 1/(z^{R/h}_{max} - z^{R/h}_{min}) \) is used. The values of \( z^{R/h}_{max} \) and \( z^{R/h}_{min} \) it is easily to obtain from eqs. (8) and (9). For instance, for the \( \rho \) meson decay into pions we receive \( z^{\rho/\pi}_{max} \approx 0.965 \) and \( z^{\rho/\pi}_{min} \approx 0.035 \).

For \( \pi^+ \) and \( \pi^- \) mesons we have \( d^{\rho/\pi} = 3/2 \), \( d^{\omega/\pi} = 0.3 \) and \( d^{K^+}/\pi = 1/3 \). For \( K^+ \) and \( K^- \) mesons we have \( d^{K^+/K^+} = d^{K^-/K^-} = 1/3 \); \( d^{K^+/K^-} = d^{K^-/K^+} = 1/3 \); \( d^{\rho/K} = 1/4 \).

In Fig. 1 the average formation lengths for electroproduction of different pseudoscalar mesons on proton, normalized on \( L \), are presented as a function of \( z \). The formation lengths for pions (panels a, c and e) and kaons (b, d, f) are presented. The contributions of direct hadrons as well as of the sum of direct and produced from decay of resonances hadrons are presented. Upper curves represent formation lengths of positively charged hadrons and lower curves of negatively charged ones. Results for standard Lund model are presented on panels a, b; for symmetric Lund model on panels c, d; for Field-Feynman model on panels e, f, respectively. The values of parameters using in calculations are presented also. Of course results of different models are quantitatively differ, but qualitatively they have the same behavior as functions of \( z \). Therefore further, for illustration, we will use the results of symmetric Lund model only.

Let us briefly discuss why the average formation lengths of positively charged hadrons are larger than of negatively charged ones. It happens due to the large probability to knock out \( \nu \) quark in result of DIS (even in case of neutron target). The knocked out quark enter in the composition of leading hadron, which has maximal formation length. The \( K^+ \) meson has average formation length larger than \( \pi^+ \) meson, because in first case the influence of resonances is smaller. The \( K^- \) meson has average formation length smaller than \( \pi^- \) meson, because it is constructed from “sea” quarks and practically can not be leading hadron, whereas \( \pi^- \) meson can be leading hadron due to \( d \) quark entering in its composition.

In Fig. 2 the average formation lengths for electroproduction of \( \pi^+, \pi^- \), \( K^+ \) and \( K^- \) mesons on different targets in symmetric Lund model, normalized on \( L \), as a function of \( z \). The contributions of direct hadrons as well as of the sum of direct and produced from decay of resonances hadrons are presented. Upper curves represent formation lengths of positively charged hadrons and lower curves of negatively charged ones. Results for standard Lund model are presented on panels a, b; for symmetric Lund model on panels c, d; for Field-Feynman model on panels e, f, respectively.
between $l_{cL}^h$ of positively and negatively charged hadrons is maximal on proton target and minimal on neutron one. It is worth to note, that results for deuteron coincide, in our approach, with results for any nuclei with $Z = N$, where $Z$ ($N$) is number of protons (neutrons). Average formation lengths of hadrons on krypton nucleus, which has essential excess of neutrons, do not differ considerably from the ones on nuclei with $Z = N$.

In Fig.3 the average formation lengths for electro-production of $\pi^+$, $\pi^-$, $K^+$ and $K^-$ mesons on proton target in symmetric Lund model, normalized on $L$, as a function of $x_{Bj}$, are presented. It is taken into account, that mesons can be produced directly or from decay of resonances.

We obtained, for the first time, the average formation lengths for different pseudoscalar mesons in the electro-production process on proton, neutron, deuteron and krypton. Main conclusions are: (i) positively charged pseudoscalar mesons ($\pi^+$ and $K^+$) have the formation lengths larger than negatively charged ones ($\pi^-$ and $K^-$) on all targets; (ii) contribution from the decay of resonances is maximal for $\pi^+$ mesons (reach $\sim 20\%$ in case of symmetric Lund model), for $\pi^-$ and $K^+$ mesons it reaches a few percents, the formation length of $K^-$ mesons practically does not feel contribution from resonances; as it was expected, in case of pions maximal contribution gives $\rho$ meson, in case of kaons $K^+$ meson.

It is worth to note that in string model the formation length of the leading (rank 1) hadron $l_{cL} = (1 - z)\nu/\kappa$ does not depend from type of process, kinds of hadrons and target. We want to stress, that obtained result depends from the type of process, kinds of targets and observed hadrons mainly due to presence of higher rank hadrons. Including in consideration hadrons produced from decay of resonances diminishes constituent formation length. It happens because for producing of hadron with fractional energy $z$ we must have resonance with $z'$ larger than $z$. The larger is the fractional energy, the shorter is the formation length. Of course this statement is right for the large enough $z$ (for instance $z > 0.2$).

Calculations performed with different scaling functions: standard Lund [12], Field-Feynman [11] and symmetric Lund [12] showed, that although the numerical values of average formation lengths slightly shift, qualitatively they have the same behavior (see Fig.1).

FIG. 3: Average formation lengths for electroproduction of $\pi^+, \pi^-, K^+$ and $K^-$ mesons on proton target in symmetric Lund model, normalized on $L$, as a function of $x_{Bj}$.

ACKNOWLEDGEMENTS

I am grateful for stimulating discussions to N.Akopov, H.P.Blok, G.Elbakian and I.Lehmann who read the paper and made many useful comments.

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