Maximum integrator gain of PID on performing position control of DC motor in the presence of Stribeck friction: Kalman conjecture approach

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Abstract. The application of PID to perform position control in the presence of Stribeck friction is believed to provoke the system trapped in limit cycle oscillation. This research aims to investigate the formulation of PID that will stabilize the position control of DC motor in the presence of Stribeck friction. In order to diminish the limit cycle oscillation, the PID gain is set to meet global asymptotic stability in Lyapunov sense as Kalman conjecture is fully applied. The analysis shows that the stability of the system depends on the friction properties near Stribeck velocity. Furthermore, the proposed PID formulation is found to be easy to be applied as the maximum integrator gain applied to the system is directly proportional to the given proportional gain.

1. Introduction
The application of PID to control the motion of DC servo motor is frequent in manufacturing process for its simplicity. Unfortunately, while friction is introduced to the system, the system is prone to gain self-oscillation which is called as limit cycle oscillation in mathematics. The presence of limit cycle makes the system oscillate around its final position, yet the system never arrives at its final position. In this fashion, the position control system loses its accuracy.

The analysis of limit cycle due to friction for PID position control is dated back to the research led by Armstrong [1] who analyzed the existence of limit cycle via describing function and algebraic equation. The research of Armstrong indicated that the existence of limit cycle is related to the integration term of PID. Thus, Putra [2] proposed PD controller to stabilize position control of mechanical system. However, Putra found that PD controller makes the system possess steady state error. For this reason, PID cannot be easily replaced by PD controller.

In order to address the problem above, Sarot [3] devoted his research to find a good formulation of PID dealing with Stribeck friction. The observation of Sarot showed that a high derivative gain is required to reduce limit cycle oscillation. Furthermore, the computational issue of limit cycle of DC motor was investigated by Sin [4]. Marton [5] analyzed optimum PID gain via LQ design procedure. Nevertheless, Sarot, Sin, and Marton did not give any protocol to set PID gain which will remedy the limit cycle. This guideline was investigated by Jeon [6] who algebraically manipulated the trajectory of limit cycle of DC servo system under frictional atmosphere.

The attempt to stabilize the system with friction non-linearity can also be done by ensuring the system that satisfies the global asymptotic stability in Lyapunov sense. One way to achieve this condition is to make the system comply with absolute stability criterion. This approach was applied by Yang [7] who employed circle criterion on stabilizing the motion of mechanical system due to friction. Moreover, Bruin [8] tried to reduce the limit cycle on the rotor dynamic system via Popov criterion. For a system
of which the order is less than three, Kalman Conjecture is another alternative criterion [9]. However, the application of Kalman conjecture for a higher order system leads to a hidden limit cycle oscillation [10].

This research is the extension of the research led by Sarot [3] and Jeon [6]. It aims to find the PID rule that can reduce the limit cycle oscillation in permanent magnet DC servo motor due to Stribeck friction. In this research, such PID strategy is found by performing absolute stability criterion: Kalman conjecture. As it is built under Kalman conjecture, the PID strategy is very simple, straight forward and built in the sense of linear system. The analysis in this research shows that there is possibility to deal with Stribeck friction with only PI controller. The result of this research is very useful to the development of PID tuning based on genetic algorithm [11] since the genetic algorithm always needs a boundary of parameter which is the main concern of this research.

This manuscript is ordered in five sections. A brief introduction about DC servo motor and Stribeck friction is presented in section II. In section III, the analysis of Kalman conjecture on stabilizing the system is shown. Furthermore, a thorough numerical simulation to show the performance of the developed PID strategy is given in section IV. In section V, the conclusion of this decent research closes this manuscript.

2. PMDC motor and Stribeck friction
In the following, the main object of this research is explored: DC motor and Stribeck friction. The first part of this section is about the modelling of permanent magnet DC motor which is exposed by friction. Whereas, the Stribeck friction model employed on this research is reviewed in the second part.

2.1. Permanent magnet DC motor
Permanent magnet DC (PMDC) servo motor is a very simple mechatronic system that changes the electrical energy into a mechanical energy in the form of rotation. The simplicity is achieved by the fact that the transient response of the electrical system is much faster than the transient response of mechanical system. Under that condition, the inductance value of DC motor coil can be neglected and the system becomes as simple as the first order system. The general block diagram of DC motor performing position control with PID algorithm is given by figure 1. In figure 1, \( R_a, K_T, K_B, c_v \) and \( J \) stand for resistance of armature \( (R_a) \), torque constant \( (K_T) \), back EMF constant \( (K_b) \), viscous friction \( (c_v) \), and rotational inertia \( (J) \) of the motor respectively. Furthermore, \( K_P, K_I \) and \( K_D \) represent proportional gain \( (K_P) \), integrator gain \( (K_I) \) and derivative gain \( (K_D) \) of the controller. In this research, it is assumed that the PMDC motor changes all electrical energy to mechanical energy. For this reason, the value of \( K_T = K_B = K \).

![Figure 1](image)

**Figure 1.** Block diagram of simplified DC motor position control with PID. The simplification is realized as the inductance of DC motor is neglected.

2.2. Stribeck friction
Stribeck friction is a friction model that combines several phenomena: Coulomb friction, Stribeck effect, and viscous friction. The formula of Stribeck friction in this research is similar to the work of [3] and [4]. In this formulation, viscous friction is not taken into account due to its linear behavior. In fact, the
viscous friction is already considered in the linear system as shown in figure 1. The Stribeck friction formula which is applied on this research is shown in equation (1). In this formulation, \( \tau_s, \tau_c, \omega_n, \varepsilon, \) and \( n \) represent static friction (\( \tau_s \)), coulomb friction (\( \tau_c \)), Stribeck angular velocity coefficient (\( \omega_n \)), angular velocity threshold (\( \varepsilon \)), and Stribeck coefficient (\( n \)) respectively.

\[
\tau_f(\omega) = \begin{cases} \tau_s \omega & \text{for } |\omega| < \varepsilon \\ \tau_s + (\tau_c - \tau_s) \exp \left[ - \left( \frac{|\omega| - \varepsilon}{\omega_n} \right)^n \right] \text{sign}(\omega) & \text{for } |\omega| \geq \varepsilon \\ \end{cases}
\]  

(1)

The observation of equation (1) shows the pre-sliding behavior occurs when \( |\omega| < \varepsilon \), whereas the sliding is developed while \( |\omega| \geq \varepsilon \). In the sliding regime, the friction torque is dropped from static friction torque value to the value of Coulomb friction torque. This phenomenon is called as Stribeck effect. The severity of Stribeck effect can be represented by the minimum value of \( d\tau/d\omega \) as shown in equation (3).

Meanwhile, the nature of pre-sliding is mathematically represented by \( \tau_s/\varepsilon \). These two formulations will be applied in the stability analysis via Kalman conjecture.

\[
\frac{d\tau_f}{d\omega} = \begin{cases} \frac{\tau_s}{\varepsilon} & \text{for } |\omega| < \varepsilon \\ -n(\tau_s - \tau_c) \left( \frac{|\omega| - \varepsilon}{\omega_n} \right)^{n-1} \exp \left[ - \left( \frac{|\omega| - \varepsilon}{\omega_n} \right)^n \right] & \text{for } |\omega| > \varepsilon \\ \end{cases}
\]

(2)

\[
\frac{d\tau_f}{d\omega}_{\min} = -n \frac{\tau_s - \tau_c}{\omega_n} \left( \frac{n-1}{n} \right)^{\frac{n}{n}} \exp \left( \frac{1-n}{n} \right)
\]

(3)

3. Kalman conjecture

The basic idea of Kalman conjecture is intuitive and built under the linear stability criterion. For the given system shown on figure 1, it can be stated as follow. Suppose there exists \( \{k_1, k_2\} \) \( \in \) Real such that \( k_1 < \frac{d\tau_f}{d\omega} < k_2 \). The system will achieve global asymptotic stability if the system is linearly stable for any friction gain \( k_{fric} \) where \( k_1 < k_{fric} < k_2 \). Based on this preposition, the system will be stable when \( G(s) \) is stable.

\[
G(s) = \frac{\theta_{out}}{\theta_{ref}} = \frac{\frac{K}{R_s}K_p + \frac{K}{R_n}K_1}{J s^3 + \frac{\frac{K}{R_s}(K + K_D) + c_v + k_{fric}}{R_n} s^2 + \frac{\frac{K}{R_s}K_p s + \frac{K}{R_n}K_1}{}}
\]

where

\[
\frac{d\tau_f}{d\omega} < k_{fric} < \frac{\tau_s}{\varepsilon}
\]

where

\[
J s^3 + \frac{\frac{K}{R_s}(K + K_D) + c_v + k_{fric}}{R_n} s^2 + \frac{\frac{K}{R_s}K_p s + \frac{K}{R_n}K_1}{}
\]

(4)

In order to make \( G(s) \) be linearly stable, \( G(s) \) has to follow Hurwitz criterion which requires the following condition.

\[
K_1 < \frac{1}{J} \frac{\frac{K}{R_s}(K + K_D) + c_v + k_{fric}}{R_n} > 0 \forall \left( \frac{d\tau_f}{d\omega} < k_{fric} < \frac{\tau_s}{\varepsilon} \right) \Leftrightarrow \left( \frac{\frac{K}{R_s}(K + K_D) + c_v + \frac{d\tau_f}{d\omega}}{R_n} \right) > 0
\]

(5)

\[
K_1 < \frac{1}{J} \frac{\frac{K}{R_s}(K + K_D) + c_v + k_{fric}}{R_n} > 0 \forall \left( \frac{d\tau_f}{d\omega} < k_{fric} < \frac{\tau_s}{\varepsilon} \right) \Leftrightarrow \left( \frac{\frac{K}{R_s}(K + K_D) + c_v + \frac{d\tau_f}{d\omega}}{R_n} \right) > 0
\]

(6)

The elaboration of equation (5) also shows the possibility of the system to have PI controller \((K_D=0)\) while maintaining the stability. PI controller is possible to be employed only when a light Stribeck phenomenon presented in the system so that equation (7) is fulfilled. In this condition, the system will be stable as long as the value of \( K_i \) is less than its maximum value \((K_i, \text{Kalman PI})\) which is given in equation (8).

\[
\left( \frac{\frac{K}{R_s} + c_v + \frac{d\tau_f}{d\omega}}{R_n} \right) > 0
\]

(7)
Given equation (7) is not fulfilled, the minimum $K_D$ has to be fed-back to the system denoted by $K_D-min$, shown in equation (9). Having $K_D > K_D-min$, the system will achieve global asymptotic stability if $K_I$ less than its maximum value ($K_{I-Kalman PID}$) that is shown in equation (10).

$$K_I < K_{I-Kalman PID} = \frac{1}{J} \left( \frac{K^2}{R_a} + c_v + \frac{dr_f}{d\omega}_\text{min} \right) K_P$$

(8)

$$\left( \frac{K^2}{R_a} + c_v + \frac{dr_f}{d\omega}_\text{min} \right) < 0 \Rightarrow K_D > K_D-min = -\frac{R_a}{K} \left( \frac{dr_f}{d\omega}_\text{min} + c_v \right) - K$$

(9)

$$K_I < K_{I-Kalman PID} = \frac{1}{J} \left( \frac{K}{R_a} (K + K_D) + c_v + \frac{dr_f}{d\omega}_\text{min} \right) K_P$$

(10)

The elaboration above shows that Kalman conjecture highlights the Stribeck effect in the stability analysis. It is pointed out as the minimum value of $dr_f/d\omega$ determines the possibility of PI controller as well as the minimum $K_D$ has to be applied. Moreover, the relationship between maximum $K_I$ that can be applied is proportional to $K_D$ and $K_P$. It is noted that Kalman conjecture assures the stability of the system but it does not confirm the instability of the system. The system that does not agree with Kalman conjecture is not guaranteed to be unstable. The inaccuracy to determine the real maximum $K_I$ before the system experiencing the limit cycle oscillation is the main weakness of this approach.

4. Illustrative example and discussion

In order to understand the application of the analysis above more in depth, an illustrative example is given. The parameter applied in this illustrative example is shown in table 1. The DC motor parameter presented in table 1 is the characteristic of UGTMEM 06 LD series. As shown in table 1, the value of $K_T$ is equal to $K_B$ as it is assumed on the modelling section. There are four cases which will be evaluated in this section. For all cases, the value of $K_P$ is set to 10, while the value of $K_I$ and asperity of Stribeck effect are varying.

| Table 1. Simulation Parameter |
|-----------------------------|
| Parameter | Unit | Value |
|----------|------|-------|
| $J$ | gm$^2$ | 0.176 |
| $K_T$ | Nm/A | 0.274 |
| $K_B$ | V/(rad/s) | 0.274 |
| $R_a$ | Ω | 2.800 |
| $c_v$ | mN.m/(rad/s) | 0.216 |
| $r_e$ | Nm | 0.039 |
| $\omega_s$ | rad/s | 0.100 |
| $\epsilon$ | rad/s | 0.005 |
| $n$ | - | 1.000 |

The first two cases show the application of Kalman conjecture for the system under a severe Stribeck phenomenon. This condition is remarked by the low coulomb friction torque compared to the static friction torque. The coulomb friction is assumed to be 0.02 Nm which is about one half of the static friction torque. Thus, the system needs derivative gain of which the minimum value is 1.67 per derivative equation (9). In this illustrative example, the value of $K_D$ is set to 2.00. For the given $K_P$ and $K_D$, the maximum integrator gain of the system without considering friction nonlinearity is about 1.27e4. However, the maximum $K_I$ could be applied regarding the presence of Stribeck friction is only about 1.86e3 per equation (10). The value of integrator gain in case 1 is set to its maximum value, $K_{I-Kalman PID}$. Thus, it would be stable. Meanwhile the value of $K_{I-set}$ in case 2 is set to be 1.01e4. This $K_{I-set}$ value is eighty percent of maximum $K_I$ while friction nonlinearity is not considered. As it is much higher than $K_{I-Kalman PID}$, the system is predicted to be unstable.

The last two cases show the possibility of PI controller in stabilizing the system with light Stribeck phenomenon. In these cases, the value of coulomb friction torque is about 0.037 Nm which is only 5% lower of the static friction torque. This condition makes the PI controller is viable per equation (7). Without considering friction nonlinearity, the maximum integrator gain that could be employed is about
Nevertheless, as the Stribeck friction nonlinearity is considered, the maximum $K_I$ that could be applied in this system is only 3.99e2 per equation (8). The $K_{I,set}$ value in case 3 is set to $K_{I,Kalman}$. Hence, case 3 would be stable. Meanwhile, the integrator gain in case 4 is set to 1.23e3 which is about 80% of the maximum integrator gain without considering friction nonlinearity. Since the system in case 4 does not meet Kalman conjecture, the system is predicted to be unstable. The resume of those four cases is shown in Table 2.

### Table 2. Simulation Parameter

| Case | Coulomb friction torque $\tau_c$ [N.m] | $K_P$ | $K_{I,set}$ | $K_D$ | Prediction | Justification | Condition |
|------|--------------------------------------|------|------------|------|------------|---------------|------------|
| 1    | 0.020                                 | 10   | 1.86e3     | 2    | equation (10) | Stable       |
| 2    | 0.020                                 | 10   | 1.01e4     | 2    | equation (10) | Unstable     |
| 3    | 0.037                                 | 10   | 3.99e2     | 0    | equation (8)  | Stable       |
| 4    | 0.037                                 | 10   | 1.23e3     | 0    | equation (8)  | Unstable     |

The result of all cases is presented in Fig. 2. This result is generated via ODE 45 algorithm with relative error of 1e-6. The calculation is done in Scilab 6.0.0. The simulation results for case no 1, 2, 3 and 4 are orderly presented in Fig. 2 (a), (b), (c), and (d). It is shown in Fig. 2 (a) and (c) that case 1 and case 3 are stable although it is slowly decaying. This condition matches the prediction shown in Table 2. Moreover, Fig. 2 (b) and (d) show that case 2 and case 4 are trapped in the limit cycle oscillation as it violates Kalman conjecture. The amplitude of limit cycle in case 2 is about 0.006 rad, while the amplitude of limit cycle in case 4 is about 0.003 rad.

![Figure 2](image_url)

**Figure 2.** Simulation result. (a) Response of system in case 1. The system is stable as it complies with Kalman conjecture. (b) Response of system in case 2. The system is trapped in limit cycle oscillation due to the $K_I$ applied is higher than $K_{I,Kalman}$. (c) Response of system in case 3. The system is stable since it follows Kalman conjecture. (d) Response of system in case 4. The system is unstable since the $K_I$ employed is higher than $K_{I,Kalman}$.  

### 5. Conclusion
In this research the PID formulation to stabilize the DC motor position control under Stribeck friction atmosphere has been well examined via Kalman conjecture. Based on the analysis, the Kalman conjecture focuses on the properties of Stribeck effect of friction. The analysis shows that the PI controller is also possible to be applied for the system with a light Stribeck phenomenon. However, for the system with severe Stribeck phenomenon, derivative gain is needed. The minimum $K_D$ for this kind of system is well formulated. Moreover, the maximum $K_I$ that can be applied to the system is
proportional to $K_P$. It has to be noted that the PID formulation proposed in this research guarantees the stability of the system in the perspective of Kalman conjecture. However, the formulation is not the boundary of instability. This inaccuracy is the weakness of this research.

Acknowledgments
This work is supported by ITB under P3MI research grant 2017.

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