Fractal dimension of premixed flames in multifractal turbulence

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Abstract
In turbulent premixed flames, the upper limit of the fractal dimension is argued to be $D = 8/3$ based on heuristic scaling arguments. However, such scaling arguments do not consider the effect of the multifractal nature of turbulent kinetic energy dissipation on the flame surface. Multifractal nature of turbulence modifies the inner cut-off scale and also causes fluctuations in the total flame area. When these effects are accounted for, it is possible to obtain two corrections to the upper-limit of fractal dimension of turbulent premixed flames: $D = 8/3 + 3/4(1 - D_{chem})$ and $D = 8/3 + 2/3(3 - D_{chem})$. In doing so, we explicitly quantify the effect of the multifractal nature of turbulence upon premixed combustion.

Keywords: Premixed flame, Fractal dimension, Multifractal, Turbulence

1. Introduction
The complex, non-Euclidean geometry of turbulent flows has led to the widespread use of fractal and related concepts to understand the phenomenology of turbulence [1, 2]. Most notably, scalar iso-surfaces in homogeneous, isotropic turbulence have been shown to have a fractal dimension of $D = 7/3$ using considerations of particle-pair diffusion and various scaling arguments [3, 4]. These predictions have been tested and validated in controlled experiments [4, 5]. The most renowned example of this scaling law is the fractal dimension of clouds which was shown to be $2.34 (= 7/3)$ by Lovejoy [6]. Observed deviations in the predicted value of the fractal dimension have since then been attributed to the phenomenon of small-scale intermittency of turbulent flows [4, 7]. Small-scale intermittency refers to the increasing non-Gaussian behaviour in dissipation quantities when one approaches scales close to Kolmogorov’s length scale. The small-scale intermittency is tied to the multifractality in the dissipation field, leading to extreme-value fluctuations in dissipation quantities, localized non-uniformly throughout space [8].

A closely related problem to the statistical description of scalar surfaces in turbulence is the description of propagating interfaces in turbulence. Propagating interfaces are frequently encountered in the study of premixed combustion and are of practical importance. In the limit where flow time scales are larger compared to combustion time scales (Da = $\tau_\delta / \tau_{chem} = (\eta/\delta_F)^3 > 1$), we can unambiguously define a flame surface by considering an iso-temperature or iso-concentration surface [9, 10]. In other words, internal flame processes are not affected by turbulent fluctuations, and the effect of turbulence is restricted only to wrinkles on the flame surface. Here, $\tau_\delta$ indicates the time scale with respect to a Kolmogorov scale ($\eta$), $\tau_{chem}$ the chemical processes associated with combustion and $\delta_F$ is the flame thickness.

Early studies considered the flame surface in the limit of $Da \gg 1$ as a passive interface with fractal dimension 7/3 similar to iso-surfaces in turbulence [11]. However, Kerstein [12] showed that the similarity in the estimate of fractal dimension to that of passive scalars in turbulence is only coincidental and the assumption of a fractal flame surface being passive is not physically plausible. It was shown that a dynamical balance between wrinkling due to turbulent convection and smoothing effects due to flame propagation is established at all length scales in the scaling range. The fractal dimension of 7/3 was recovered naturally by considering the balance between characteristic burning time governed by turbulent burning velocity and eddy turnover time. Later, Kerstein [13] considered the effect of intermittency in turbulent kinetic energy dissipation and arrived at a corrected value of the fractal dimension, which was quite similar to the correction obtained by Meneveau and Sreenivasan [7]. However, the two approaches were quite different and implied the possibility of geometrical equivalence of different surfaces in turbulent flows.

When the limit of $Da \gg 1$ is relaxed, Chatakonda et al. [14] showed that instead of a dynamical balance between wrinkling due to turbulence and turbulent flame propagation as considered by Kerstein [12], a balance between flame stretch at the lowest length scales is necessary. A balance of tangential flame strain and the effects of curvature and flame propagation leads to a modified inner cut-off scale known as the Obhukhov-Corrsin length scale ($\eta_{OC}$). The effect of this modified inner cut-off scale then leads to a modified prediction in the fractal dimension of low $Da$ ($Da \sim O(1)$) flames which is equal to $D = 8/3$. The fractal dimension of $D = 8/3$ was also suggested by Mandelbrot [1] for scalars in Gaussian random fields and possessing Kolmogorov spectra. Later, through an altogether different approach, Constantin et al. [15] theoretically derived the limit

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\(D = 8/3\) for non-reacting passive scalars and experimentally verified by considering scalar in the well-mixed region of a turbulent flow.

In this paper, we account for the effect of the multifractal nature of dissipation on the inner cut-off scale and, consequently, the total area of the corrugated flame surface. We show, in the same vein as Sreenivasan et al. [4] and Meneveau and Sreenivasan [7], that the intermittent nature of dissipation leads to corrections in the fractal dimension of low Da flames.

2. Estimate of fractal dimension of a stretched flame

We consider a corrugated flame surface propagating freely into a volume containing a combustible mixture. The flame surface divides the region of reactants and products. We assume that the wrinkles on the flame surface are induced by turbulent fluctuations in the inertial range. Hence, the wrinkles on the flame surface also follow dynamic self-similarity. The outer cut-off is conveniently defined by the integral length scale of the flow (\(\ell\)). We discuss the inner cut-off in more detail later on. The true area of any such a wrinkled flame, \(A_T\), depends on the measurement scale (\(\epsilon\)) as:

\[
A_T(\epsilon) = A_0(\epsilon/\ell)^{2-D},
\]

where \(A_0\) is some normalizing area proportional to \(\ell^2\) and \(D\) is the fractal dimension. The inner (outer) scale indicates the scale at which the measurement area no longer scales with an increase (decrease) in the resolution of the measurement.

For a stable and well-maintained flame surface in a turbulent flow field, turbulence induced tangential flame strain at the lowest length scales are balanced by the effects of curvature and flame propagation [16]. The tangential flame strain rate due to an eddy of size \(\epsilon_i\) is \(\alpha_T = u'_i/\epsilon_i\), where \(u'_i\) is the turbulent intensity at the scale \(\epsilon_i\). Further, turbulent intensity can be written as \(u'_i = (\langle \epsilon_i \epsilon_i \rangle)^{1/3}\), where \(\langle \epsilon_i \rangle\) is the rate of turbulent kinetic energy dissipation averaged over the volume \(\ell^3\). Thus, the tangential flame strain rate can be re-written as \(\alpha_T \sim (\langle \epsilon \epsilon \rangle)^{1/3}/\epsilon\).

The effect of flame propagation in balancing the tangential flame stretch is negligible for low Da flames, and is well-supported by theory [17] and DNS results [18]. In such a case, the equilibrium on the flame surface is maintained by the effect of curvature alone [16]. Curvature is quantified by the gradient of the surface normal \(\nabla \cdot \mathbf{N}\). The balance between the curvature and the tangential flame strain is, thus, \(\langle \nabla \cdot \mathbf{N} \rangle = \langle \alpha_T \rangle\), where \(\alpha_T\) is the diffusion and \(\langle \rangle\) indicates average weighted by the surface area of the flame surface [14]. The balance then leads to:

\[
\langle \nabla \cdot \mathbf{N} \rangle \sim \frac{1}{\epsilon^2} \frac{\epsilon^2}{D} \frac{(\epsilon x)}{\epsilon^3}.
\]

We can then define the Obukhov-Corrsin length scale (\(\eta_{OC}\)) based on the balance above as [14]

\[
\eta_{OC} \sim \epsilon_i \sim (D^3/\langle \epsilon \epsilon \rangle)^{1/4} \sim S \epsilon^{-3/4}\eta,
\]

where the Schmidt number \(S c\) is defined as the ratio of the kinematic viscosity (\(\nu\)) and the diffusivity \(\langle \mathcal{D} \rangle\), i.e., \(S c = \nu/D\).

Here, \(\eta\) is the Kolmogorov length scale and relates to kinematic viscosity and mean turbulent kinetic energy dissipation as \(\eta = (\nu^3/\langle \epsilon \epsilon \rangle)^{1/4}\).

We can assume that local isotropy is followed so that the length scale controlling the inner cut-off (\(\epsilon \sim \eta_{OC}\)) also controls the scalar gradient across the flame interface. The scalar gradient is thus \(\sim \delta c_{\eta_{OC}}/\eta_{OC}\), where \(\delta c_{\eta_{OC}}\) is the difference in scalar concentration associated with length scale \(\eta_{OC}\). For higher Reynolds number and in the limit of \(D a \sim O(1)\), the scalar difference can be related to the integral scale fluctuations \(\langle \epsilon \rangle\) of the scalar through Kolmogorov’s cascade argument such that \(\delta c_{\eta_{OC}} \sim \eta_{OC}/\eta_{OC}^3\epsilon\) [14].

The total scalar flux across the flame interface is, thus, proportional to the total area \(\langle A_T \rangle\), the diffusivity \(\langle \mathcal{D} \rangle\), and the scalar gradient \(\delta c_{\eta_{OC}}/\eta_{OC}\).

\[
F \sim A_T D(\delta c_{\eta_{OC}}/\eta_{OC}) \sim A_0 D c(\eta_{OC}/\epsilon)^{2-D}(\eta_{OC}/\epsilon)^{3/3}\eta_{OC}^{-1}
\]

Using the definition of \(\eta_{OC}\) from Eq. 3 and the scaling relation in the universal subrange, \(\eta/\ell \sim (Re)^{-3/4}\) where, \(Re = u'\ell/\nu\), we obtain:

\[
F \sim A_0 c u'(S c)^{3/4-D/3}(Re)^{3/4-D/3/3}
\]

Thus, we get the scalar flux in terms of only the Schmidt number and the Reynolds number. In the inertial subrange, the total scalar flux (mass, momentum, and concentration) is independent of the kinematic viscosity and Reynolds number. The independence of flux properties of the kinematic viscosity and energy inducing scale is referred to as the Reynolds number similarity. Thus, for \(Re\) independence, the fractal dimension must be \(D = 8/3 = 2.67\). This result was derived in Chatakonda et al. [14]. For a mixing layer, through similar scaling arguments in the calculation of flux across two interfaces, Sreenivasan et al. [4] showed that the fractal dimension of the interface separating the two mixing regions has a fractal dimension of \(D = 7/3 = 2.33\). Thus, \(D = 7/3\) is observed in entraining layers of a passive scalar surface [4], and \(D = 8/3\) for passive scalars in well-mixed regions [15] and for low Da flames [14, 16].

3. Corrections to the fractal dimension of the flame due to multifractal dissipation

3.1. Corrections in the inner cut-off scale

Insofar, we have assumed that the rate of turbulent kinetic energy dissipation remains uniform throughout the flow field, which allowed us to define a volume averaged dissipation \(\langle \epsilon \rangle\) and subsequently the inner cut-off scales \(\eta\) and \(\eta_{OC}\). However, turbulent flows are multifractal with large variation in the local dissipation rate. In such an inhomogeneous dissipation field, the dissipation in a local region of size \(r^3\) can be approximated by the generalized power law [4]:

\[
\langle \epsilon_q \rangle = \langle \epsilon \rangle^{q/3} (r/\ell)^{l_{\epsilon_q}(q-1)-1},
\]

where, \(D_{\epsilon_q}\) indicates the generalized dimension of order \(q\) and \(\langle \epsilon \rangle\) is the averaged dissipation in box of size \(\ell^3\). Thus, if \(\epsilon\) varies
locally according to Eq. 6, \( \eta \) and \( \eta_{oc} \) will vary which would change the total scalar flux across the flame surface. As is clear from Eq. 3, \( \eta_{oc} \) has a 1/4 dependence on the dissipation. Thus, we replace \( q = 1/4 \) in Eq. 6 and get the local dissipation as:

\[
(\varepsilon_1^{1/4}) \sim (\eta_{oc}^{1/4}) = (\varepsilon)^{1/4}/(\eta_{oc}^{1/4}(\tau)^{1/4-1/D})_1.
\] (7)

Now, instead of using an inner cut-off defined on averaged dissipation \( \langle \varepsilon \rangle \) as done in Eq. 3, we use the dependence of \( \eta_{oc} \) on the local dissipation value. Thus, we substitute in Eq. 3 the local dissipation in a box of size \( \eta_{oc} \) i.e.,

\[
\eta_{oc} \sim (D^3/\varepsilon_{loc})^{1/4}.
\] (8)

Substituting Eq. 7 in Eq. 8:

\[
\frac{\eta_{oc}}{\ell} - \frac{1}{\ell} \left( \frac{D^3}{\varepsilon_{loc}} \right)^{1/4} \sim (ScRe)^{A},
\] (9)

where, \( A = -(3/4)[1 + 3/4(1 - D_1)] \). Substituting Eq. 9 in Eq. 3 and carrying out the algebra, it is straightforward to see that the flux becomes:

\[
F = A_0 c u' (ScRe)^B,
\]

where,

\[
B = \frac{3}{4}[3 - \frac{3}{4}(1 - D_1)].
\] (10)

3.2. Fluctuations in the area of the interface

As mentioned above, the multifractal nature of dissipation leads to fluctuations in the inner cut-off \( \eta_{oc} \) (Eq. 8). From Eq. 1 we observe that the total area depends on the inner cut-off. Thus, any fluctuations in the inner cut-off would lead to fluctuations in the total flame area. We show that this leads to the second correction to the fractal dimension of a stretched flame.

To find the corrections, we find the total flux by integrating boxes along the flame interface in the manner detailed in Menveneu and Sreenivasan [7]. We assume that the flame is contained in a domain of size \( \ell^3 \). We cover the entire domain in cubic boxes of size \( \eta_{oc} \), which is the inner cut-off scale for our problem. Thus, the total flux is due to the sum of contributions of each of the boxes along the entire interface. The contributions of each box again depends on the area of the element \((\eta_{oc}^2)^2\), the diffusivity \(D\), and the scalar gradient \(\partial c_{loc}/\partial c_{oc}\). Thus, the total flux after substituting the appropriate scalings associated with the gradient, we obtain:

\[
F \sim \sum_i \eta_{oc}^2 D \partial c_{loc}^2 \sim c u' \ell^2 (ScRe)^{-1} \sum_i \eta_{oc}^2 / (\ell)^4/3 (11)
\]

Locally, for a domain of length scale \( \eta_{oc} \), the dissipation depends on the local singularity strength \( \alpha_i \), through the following relation:

\[
\varepsilon_{loc} \sim (\varepsilon)(\eta_{oc}/\ell)^{\alpha_i-3}
\] (12)

The local singularity strength \( \alpha_i \) is essentially the fractal dimension of the singularity associated with dissipation in a box of size \( \eta_{oc}^i \), and changes when \( \eta_{oc}^i \) changes. We can then express the inner cut-off \( \eta_{oc}^i \) in terms of singularity exponent \( \alpha_i \) alone by substituting Eq. 12 to Eq. 8 to obtain:

\[
\frac{\eta_{oc}}{\ell} \sim (\eta/\ell)^{1/4-1/(\alpha_i+1)} (Sc)^{-3/(\alpha_i+1)}.
\] (13)

Now, we calculate the number of cubic boxes of size \( \eta_{oc} \) having local singularity exponent \( \alpha = \alpha_i \). The total number of boxes containing singularity exponent \( \alpha_i \) in the box \( \ell^3 \) is defined by the scaling relation:

\[
N(\alpha_i) \sim (\eta_{oc}/\ell)^{(\alpha_i)}.
\] (14)

Here, \( f(\alpha_i) \) is the fractal dimension associated with counting the number of boxes of size \( \eta_{oc} \) containing a given value of singularity exponent \( \alpha_i \). Substituting Eq. 13 in Eq. 14, we obtain

\[
N(\alpha_i) \sim (\eta_{oc}/\ell)^{(\alpha_i)}.
\] (15)

In order to make further progress, we need to determine the total number of boxes of size \( \eta_{oc} \) and singularity exponent \( \alpha_i \), only along the flame interface. Let \( S_1 \) and \( S_2 \) indicates the set containing the fractal flame element of dimension \( D \) and singularity \( \alpha_i \) of dimension \( f(\alpha_i) \) in 3-dimensional space \( \mathbb{R}^3 \), respectively. The co-dimension of \( S_1 \) and \( S_2 \) in \( \mathbb{R}^3 \) is thus \( 3 - D \) and \( 3 - f(\alpha_i) \). The additive property of sets \( S_1 \) and \( S_2 \) stipulates: co-dim of \( S_1 \) + co-dim of \( S_2 \) < dim of \( \mathbb{R}^2 \), which is the condition of intersection of sets \( S_1 \) and \( S_2 \) in d-dimensional space [1]. Physically, this implies that the flame surface intersects, or interacts, with the dissipation in the turbulent flow field. Thus, we have:

\[
(3 - f(\alpha_i)) + (3 - D) < 3 \Rightarrow f(\alpha_i) + D > 3.
\]

The dimension of the intersection of \( S_1 \) and \( S_2 \), i.e., the set of singularities \( \alpha_i \) along the interface of the fractal flame structure, follows from the condition of intersection as \( D = f(\alpha_i) + D > 3 \). Thus, the total number of boxes where \( \alpha = \alpha_i \) along the interface is given by the dimension of the intersection of the sets \( S_1 \) and \( S_2 \) so that Eq. 15 gets modified to

\[
N(\alpha_i) \sim (\eta_{oc}/\ell)^{(\alpha_i)} (Sc)^{D/(\alpha_i+1)}.
\] (16)

Note that we have cast the contribution of each box of size \( \eta_{oc} \) to the total flux in terms of the distribution of singularity only along the interface. Thus, the summation in the contribution of all the individual boxes defined in Eq. 11 can be replaced with an integral over the entire spectrum of singularity exponent \( \alpha \). The flux can then be calculated as

\[
F \sim c u' \ell^2 (ScRe)^{-1} \int N(\alpha)(\eta_{oc}(\alpha))/\ell)^{4/3} da.
\] (17)

Substituting the expression for \( N(\alpha) \) from Eq. 16 and \( \eta_{oc} \) from Eq. 13 in Eq. 17, we obtain

\[
F \sim c u' \ell^2 (ScRe)^{-1} \int (\eta/\ell)(Sc)^{-3/4} da.
\] (18)
where, \( C = -4(\beta - 4/3)/(a + 1) = -4[f(a) - 13/3 + D]/(a + 1) \). Equation 18 can be solved using the method of steepest descent in the limit of small \( \eta < \ell \) and \( Sc > 1 \). The latter is generally true for premixed air and hydrocarbon fuel mixtures. The saddle point is determined from \( \partial C/\partial a = 0 \), which leads to:

\[
df/da = [f(a) - 13/3 + D]/(a + 1).
\]  

(19)

Now, we know that \( q = df/da \) which relates the order of the generalized dimension \( q \) and the singularity spectrum \( f(a) \). Further, the generalized dimension \( D_q \) is related to the \( f(a) \) through the following relation [19]

\[
D_q = 1/(q - 1)[aq - f(a)].
\]  

(20)

Now, whenever Eq. 19 is satisfied for a given \( q \), we assign it the value \( Q \) such that

\[
df/da = [f(a) - 13/3 + D]/(a + 1) = Q = -C/4.
\]  

(21)

From here, we can work out the relation for \( f(a) \), which is

\[
f(a) = Q(a + 1) + 13/3 - D.
\]  

(22)

The integral in Eq. 18 is evaluated at the saddle point where, in the power of the integrand, we substitute \( C = -4Q \). In such a case, the total flux across the flame surface can be re-written, after substituting \( \eta/\ell \sim Re^{-3/4} \), as:

\[
F \sim cu L^2 (ScRe)^{-1+3Q}.
\]  

(23)

Further, from Eqs. 20, 21 and Eq. 22 we get

\[
DQ = \frac{Qa - f(a)}{Q - 1},
\]

which after substituting Eq. 22 leads to

\[
D = 13/3 + Q + (Q - 1)DQ.
\]  

(24)

Now Reynolds similarity argument stipulates that the flux \( F \) be independent of \( Re \), which upon enforcing in Eq. 23 we obtain \(-1 + 3Q = 0 \) which yields \( Q = 1/3 \). Consequently, we find from Eq. 24 the correction in the fractal dimension of the flame front as

\[
D = \frac{8}{3} + \frac{2}{3}(3 - D_{1/3}).
\]  

(25)

Equations 10 and 25 are analogous to the correction for a passive scalar interface in a mixing layer derived by Sreenivasan et al. [4] and Meneveau and Sreenivasan [7], respectively. The fractal dimension we derive here quantifies the effect of multifractal nature of turbulence on the properties of premixed combustion through corrections to the dimension of the flame surface.

Experimentally measuring the multifractal spectrum of turbulent kinetic energy dissipation in turbulent reacting flows is a significant challenge as it involves the determination of two-point velocity correlations. The generalized dimension \( D_Q \) which appears in Eqs. 10 and 25 can then only be measured from high fidelity DNS data. From the literature on turbulence, we envisage the corrections to \( D \) to be small. However, to the best of our knowledge, we did not find any study on turbulent combustion which have measured these dissipation quantities \((D_Q, f(a) \text{ vs } a, \text{ etc.})\). So, we do not know the extent of variation in the estimate of \( D \). The determination of such quantities from DNS data appears worthwhile as it has the potential to lead to better closure models for flamelets used in LES, which are utilized quite extensively in combustion literature.

4. Conclusions

In this study, we derive corrections to the fractal dimension of premixed flames in the limit of \( Da \sim O(1) \). In this limit, as the flame is no longer a passive surface in turbulence, we consider the balance between kinematic viscosity and scalar diffusion and define the so-called Obhukov-Corrsin length scale. Turbulent flows are intermittent, and the rate of turbulent kinetic energy dissipation displays multifractality. Consequently, the effect of multifractality in the dissipation rate affects the inner cut-off and the total area of the premixed flame surface, which leads to two different corrections to the fractal dimension of the premixed flame. Thus, we analytically quantify the effect of multifractal turbulence upon premixed combustion.

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