Current bounds on baryogenesis from complex Yukawa couplings of light fermions

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We calculate the contribution to the baryon asymmetry of the Universe (BAU) from a CP-violating source of the light quarks (charm, strange, down, up) and the electron, resulting from a dimension-six effective field theory term. We derive relevant bounds from the electric dipole moments of the electron and neutron to estimate the maximal contribution from each single flavor modification. Current bounds show that the charm quark can generate at most $\mathcal{O}(1\%)$ of the BAU, while the lighter quarks and the electron contribute at much lower levels.

**Introduction** The baryon asymmetry of the Universe (BAU) is defined and measured [1, 2] to be

$$Y_B \equiv \frac{n_B - n\bar{B}}{s} \approx (8.6 \pm 0.1) \times 10^{-11} \equiv Y_B^{\text{obs}},$$

where $n_B$ is the (anti-)baryon number density and $s$ is the entropy density of the Universe. A non-vanishing value can be either the result of initial conditions, or dynamically generated during the early Universe. The former requires fine tuning and is inconsistent with inflation. The latter, which is the more acceptable mechanism to address the asymmetry, is called Baryogenesis (See [3, 4], for reviews).

There are three necessary conditions, known as the Sakharov conditions [5], that are required from any theory in order to explain such an imbalance: Baryon number violation, $C$-symmetry and $CP$-symmetry violation and interactions out of thermodynamic equilibrium. Although the Standard Model (SM) meets all three criteria, the rate at which it contributes is far too small to account for the observed baryon asymmetry [6, 7] due to two factors: the smooth crossover of the electroweak phase transition and the suppression from the Kobayashi-Maskawa (KM) mechanism of $CP$ violation. Thus, if the baryon asymmetry was generated via electroweak baryogenesis, the electroweak phase transition had to be strongly first order and a new source of $CP$ violation must exist at, or at least not far above, the electroweak scale.

New Physics (NP) beyond the SM is highly motivated by several open questions in Physics (e.g. dark matter, neutrino masses). However, despite the efforts made to discover new particles, none were found up to the TeV scale [8, 9]. It is then plausible that NP is above the electroweak scale, and thus could be integrated out. This allows us to use Standard Model effective field theory (SMEFT) tools to explore higher order terms, without being model-dependent.

We add a $CP$-violating (CPV) phase using a dimension-six (dim-6) coupling of three Higgs fields to the SM charged fermions. The BAU is then proportional to the CPV source, which could be constrained by the Electric Dipole Moment (EDM) of both the electron and neutron and by the Higgs boson decay and production rates. This was previously done for the third generation particles [10–12] and the muon [13]. Of that list, it was shown that the $\tau$ is the only sole-contributor that can provide the entire observed value of the BAU [11, 12]. We applied this procedure to evaluate the contribution from all of the SM particles, including the light quarks (charm, strange, down, up) and the electron, and discuss the results here.

This Letter is organized as follows. First, we describe the SMEFT framework, including the complex dim-6 term and the CPV source it generates. We then outline key points in the process of electroweak baryogenesis, which are formulated by the two-step approach via the transport equations followed by the sphaleron process. Next, we present our numerical results for the contribution of a single flavor to the BAU. The contribution is later bounded using the experimental measurements of the electron and neutron EDMs and of various Higgs boson processes. Finally, we discuss our results and conclusions.

**SMEFT framework** We examine the implications of adding the following effective dim-6 terms to the SM,

$$\mathcal{L}_{\text{eff Yuk}} = - y_f |H|^2 \left( \frac{X_f^R + i X_f^I}{\Lambda^2} \right) \bar{\psi}_L f R_I \psi_R H + \text{h.c.},$$

where $y_f$ is the dimension-four (dim-4) Yukawa coupling, $H$ is the SM Higgs field $H \sim (1, 2)_{+1/2}$, $\Lambda$ is the NP scale, $X$ is the dim-6 Wilson coefficient and $\psi$ is a SM fermion. In our notation, the lower index, $f$, denotes the flavor whereas the upper index distinguishes between the real ($R$) and imaginary ($I$) coefficients. We find it useful to define [12, 13]

$$T_f^{R,I} \equiv \frac{v^2}{2\Lambda^2} \frac{X_f^{R,I}}{y_f},$$

where $v = 246$ GeV is the vacuum expectation value (VEV) of the Higgs background field ($h$) defined $H = \frac{1}{\sqrt{2}} (0, v + h)^T$. Accordingly, the mass ($m$) and effective

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Yukawa coupling ($\lambda$) of each flavor can be defined and explicitly written as
\[
\mathcal{L}_{\text{eff Yuk}} \ni -m_f \bar{\psi}_L \psi_R f - \lambda_f \bar{\psi}_L \psi_R f h + \text{h.c.},
\]
\[
m_f = \frac{v h_f}{\sqrt{2}} \left(1 + T_f^R + i T_f^I\right), \quad \lambda_f = \frac{y_f}{\sqrt{2}} \left(1 + 3 T_f^R + i 3 T_f^I\right),
\]
In the mass basis, where $m_f \in \mathbb{R}$, they are given by [12, 13]
\[
m_f' = \frac{v h_f}{\sqrt{2}} \sqrt{\left(1 + T_f^R\right)^2 + T_f^I^2}, \quad \lambda_f' = \frac{y_f}{\sqrt{2}} \frac{(T_f^R + 1)(3 T_f^R + 1) + T_f^I(3 T_f^R + 2i)}{\sqrt{\left(1 + T_f^R\right)^2 + T_f^I^2}}.
\]
Throughout this work we will use the complex parameter $\kappa_f$, representing the deviation from the SM in the mass basis,
\[
\kappa_f \equiv \frac{\lambda_f'}{m_f'} = 3 - \frac{2}{1 + T_f^R + i T_f^I}, \quad \kappa_f' \equiv \text{Im}(\kappa_f) = \frac{2 T_f^I}{\left(1 + T_f^R\right)^2 + T_f^I^2}.
\]
Equivalently, one could express $\kappa_f = \frac{\lambda_f'}{\lambda_f^{\text{SM}}} \equiv \frac{m_f'}{m_f}$ where, as in the SM, $\lambda_f^{\text{SM}} \equiv m_f^2$. It is convenient to use $\kappa_f$ since the baryon asymmetry is proportional to the CPV source ($S_f$, Eqn. (A8)) and therefore linear in $\kappa_f'$ [12],
\[
Y_B \propto S_f \propto \text{Im}(m_f' m_f^*) \propto \kappa_f'.
\]
In the above, we use the VEV insertion approximation [14], to leading order.

**Electroweak baryogenesis** By adding a complex effective Yukawa coupling to one of the fermions of the SM we introduce a CPV source to the model. The source depends on the background Higgs boson field $h$, which acquires a VEV. Here we assume that the electroweak phase transition is strongly first order, and describe $h$ by the kink solution (Eqn. (A10)). This choice results with a CPV source that peaks mostly inside the non-vanishing VEV bubble. A schematic plot is given in Fig. 1. Namely, the CPV source generates a chiral asymmetry, mainly inside the broken phase. That asymmetry can be transformed into an abundance of baryons via the weak sphaleron, which is a non-perturbative effect of the SM. The rate of the weak sphaleron process is given by $\Gamma_{\text{us}} \sim e^{-\left(h\right)/T^2 T}$ [15], where $T$ is the temperature. Although the rate is exponentially suppressed inside the bubble, it is fast outside the bubble, during the early Universe. Therefore, for baryogenesis to occur at the electroweak phase transition, the chiral asymmetry should have propagated outside the bubble and into the symmetric phase [16]. Finally, as the bubble continues to expand, it eventually captures the resulting baryon asymmetry.

Two-step approach The dim-6 term described in Eqn. (2) will affect the dynamics of the number densities, which are defined as the difference between the number densities of particles and anti-particles. This effect is the first step of the approach and is described via a set of transport equations. We generalize the set given in [11] to include all the fermions of the SM in addition to the Higgs boson,
\[
\begin{align*}
\partial_t U_i &= -\Gamma_{M,Y} U_i + \Gamma_{\bar{U}} U_i + \Gamma_{ss} u_s + S_{U_i}, \\
\partial_t D_i &= -\Gamma_{D} D_i - \Gamma_{Y} \mu_Y D_i + \Gamma_{ss} \mu_{ss} + S_{D_i}, \\
\partial_t Q_i &= -\partial_t U_i - \partial_t D_i, \\
\partial_t E_i &= -\Gamma_{E} E_i - \Gamma_{Y} \mu_Y E_i + S_{E_i}, \\
\partial_t L_i &= -\partial_t E_i, \\
\partial_t h &= \sum_{i=1}^{3} \Gamma_{U} \mu_Y U_i - \sum_{i=1}^{3} \Gamma_{D} \mu_Y D_i - \sum_{i=1}^{3} \Gamma_{E} \mu_Y E_i,
\end{align*}
\]
where $U_R, D_R, Q_L$ are the SM quark fields, $E_R, L_L$ are the SM charged lepton fields (the chirality is implicit hereafter), $\Gamma_{M,Y}$ are the relaxation and Yukawa rates, respectively, and $S_f$ are the CPV sources. The chemical
approximately linear with various fermionic sources, given the term (See Eqn. (A8)), which is otherwise almost identical for different flavors of same type (either charged leptons or quarks) to the BAU is proportional to the ratio of the mass squared, up to 5%. For $f_1$, $f_2$ light flavors of same type, we get the following numerical result:

$$\frac{Y_{f_1}^B}{Y_{f_2}^B} \approx \left( \frac{m_{f_1}}{m_{f_2}} \right)^2 \kappa_{f_1}^I \kappa_{f_2}^J.$$

The mass squared is explicitly introduced to the source term (See Eqn. (A8)), which is otherwise almost identical for same type particles. However, it is non-trivial that the numerical solution of the transport equations is approximately linear with various fermionic sources, given they have different interaction rates. It is a consequence of the negligible difference between the light fermion rates (See Table III) and indeed, this relation does not hold for the third generation particles.

Moreover, the solution for every given $\kappa_f^I$ is centered around a cancellation between the dim-6 and dim-4 contributions to $m_f^I$. By rearranging the definition of $\kappa_f^I$ (Eqn. (8)), we get

$$(T_f^R + 1)^2 + (T_f^I - \frac{1}{\kappa_f^J})^2 = \left( \frac{1}{\kappa_f^J} \right)^2. \tag{14}$$

See inset of Fig. 2 for the geometrical interpretation. The center of the circle, at $T_f^R = -1$, implies that the mass of the fermion is effectively generated by the imaginary part of the dim-6 term (See Eqn. (6)). This point requires a fine-tuned cancellation between the dim-4 Yukawa coupling and the real part of the dim-6 term. Furthermore, we do not expect this tension to be relaxed by introducing higher order terms, as they have negligible contribution.

Finally, the desired $\kappa_f^I$ that saturates the contribution to the BAU to its observed value could correspond to an unfavorable solution, when demanding the theory to be perturbative. Let us denote the single flavor modification $\kappa_f^I$ which satisfies $Y_f^B = Y_{f}^{obs}$, according to Eqn. (12), by $\kappa_f^{I*}$. The solution $\kappa_f^{I*}$ corresponds to a circle in $(T_f^R, T_f^I)$-space. By setting the mass $m_f^I$ (See Eqn. (6)) to its measured value, we calculate the resulting dim-4 coupling $y_f$ for each point on the circle, as a function of its central angle from the positive horizontal direction ($\theta$). For some cases, depending on the particle and the position on the circle, it requires $y_f > 4\pi$ which in non-perturbative (See Fig. 2), rendering the analysis moot.

A theoretical upper bound on $|\kappa_f^I|$ is produced by requiring there exists $\theta_B$ for which $y_f(\theta_B) \leq 4\pi$. Although this constraint is fairly weak, we present the perturbativity bound in Table I for comparison reasons.

Note that $\theta = \frac{\pi}{2}$ ($\frac{3\pi}{2}$ for the top), which corresponds to $(-1,0)$ in $(T_f^R, T_f^I)$-space, is clearly un-physical and should be excluded. For large values of $\kappa_f^{I*}$, the radius of the circle, $1/|\kappa_f^{I*}|$, is too small to escape this critical region. We specifically point out the up and down quarks, for which there is no perturbative theory that can account for the observed BAU.

**Bounds** The implications of a non-zero $\kappa_f^I$ are manifold: In addition to the generation of baryon asymmetry, which was discussed above, we focus on the contribution of a single $\kappa_f^I$ to the EDMs of the electron and the neutron, and Higgs related measurements [22]. In this Section we use these experimental results to constrain the maximal value of $|\kappa_f^I|$, assuming $\kappa_f^I = 1$ for all $f \neq f$. This will allow us to infer the maximal contribution,
The contribution of the SM fermions, other than the top from a single flavor modification, to the BAU is mapped to (as a sole-source of the observed BAU. Perturbative throughout the entire range, which precludes them by [27]. The contribution of the electron to the eEDM is given by [26], The contribution of the top quark to the eEDM is given by [25], assuming \( \kappa_{t,c}^I = 0 \),

\[
|\kappa_t^I| \lesssim 7.4, \quad |\kappa_c^I| \lesssim 15.5. \tag{21}
\]

We used the weakest constraint, given for negative sign of the Weinberg-operator, with the short-distance theory uncertainty (in quadrature) for the charm (bottom).

Collider constraints The signal strength \( \mu_{h \to \ell \ell} \) can be written in terms of the production rate \( \sigma \) and branching ratio \( B \) as

\[
\mu_{h \to \ell \ell} \equiv \frac{\sigma_{i}(pp \to h)B(h \to f \ell)}{\sigma_{i}^{SM}(pp \to h)B^{SM}(h \to f \ell)}. \tag{22}
\]

We first use measurements of \( \mu_{h \to \ell \ell} \) to constraint \( |\kappa_\ell| \) directly, for the charged leptons and bottom quark, denoted \( \ell = \tau, \mu, e, b \). Since the contribution of light fermions to the production rate of the Higgs boson is insignificant already at dim-4, we can safely neglect their effect via the dim-6 term. Regarding the bottom quark, this approximation neglects its 1% loop contribution to gluon-gluon fusion (ggF) [30]. However, the eEDM bound, in this case, turns out to be more significant [12]. We therefore approximate

\[
\mu_{h \to \ell \ell} \approx \frac{B(h \to \ell \ell)}{B^{SM}(h \to \ell \ell)}. \tag{23}
\]

It is then straightforward to translate the U.B. of the signal strength to the maximal value of \( |\kappa_\ell| \) using [27]

\[
\mu_{h \to \ell \ell} = \frac{B^{SM}(h \to \ell \ell)}{1 + \left(|\kappa_\ell|^2 - 1\right)B(h \to \ell \ell)^{SM}}. \tag{24}
\]

The next class is that of the light quarks \( q = u, c, d, s \), which could only be bounded via its effect on the total decay width of the Higgs boson. When NP interacts only with \( q \), i.e. \( \kappa_f = 1 \) for all \( f \neq q \), the signal strength of \( f \) is modified as

\[
\mu_{h \to f f} = \frac{1}{1 + \left(|\kappa_q|^2 - 1\right)B(h \to q q)^{SM}}. \tag{25}
\]
As the lower bound of the signal strength $\mu_{h \to ff}$ tends to one, the upper bound on $|\kappa_f|$ gets stronger. Currently, the experimental lower bound closest to unity is that of the bottom, $\mu_{h \to bb} = 1.04 \pm 0.20$ [31] (see Table I). Therefore, we use $\mu_{h \to bb}$ to solve the above equation and constrain $|\kappa_f|$. Lastly, the single flavor modification of the top quark is bounded using the dominant, top mediated, production modes of the Higgs boson: ggF and $ttH$. The top affects both the production rate $\left(\sigma/\sigma^{SM} = |\kappa_t|^2\right)$, as well as the total decay width of the Higgs boson (Eqn. (25)). The overall effect can be written as [12]

$$\mu_{ggF+ttH} = \frac{|\kappa_t|^2}{1 + (|\kappa_t|^2 - 1) B(h \to gg)^{SM}}. \quad (26)$$

### Table I. The BAU calculated following the full set of transport equations. $Y_B$ is the BAU resulting from a single source $S_f$. Collider constraints are at ~ 95% C.L. [12, 30–35] (for details see Table II). EDM constraints are at 90% C.L. [23, 28] for all, except for the bottom and charm, for which the nEDM constraints are at 68% C.L. [25]. Perturbativity bounds are calculated by setting $m_f^\prime$ to its measured value and demanding that there exists $\theta_p$ for which $y_f(\theta_p) = 4\pi$.

| $S_f$, $f$ | BAU, $Y_B^f$ | Collider | eEDM | nEDM | Perturbativity | $Y_B^{max}$ | $Y_B^{obs}$ |
|---|---|---|---|---|---|---|---|
| $\tau$ | $-9.9 \cdot 10^{-10}$ | 1.1 | 0.3 | - | $2 \cdot 10^{-5}$ | 3.37 |
| $\mu$ | $-1.0 \cdot 10^{-11}$ | 1.3 | 31 | - | $4 \cdot 10^{-1}$ | 0.16 |
| $b$ | $-2.1 \cdot 10^{-11}$ | 1.7 | 0.2 | 7.4 | $1 \cdot 10^{-3}$ | $5.8 \cdot 10^{-2}$ |
| $t$ | $+2.4 \cdot 10^{-9}$ | 1.1 | $1.2 \cdot 10^{-3}$ | - | 25 | $3.3 \cdot 10^{-2}$ |
| $c$ | $-2.7 \cdot 10^{-12}$ | 3.9 | 0.4 | 15.5 | $3 \cdot 10^{-3}$ | $1.1 \cdot 10^{-2}$ |
| $s$ | $-1.6 \cdot 10^{-14}$ | 30 | 109 | 4.5 | $5 \cdot 10^{-4}$ | $8 \cdot 10^{-4}$ |
| $d$ | $-3.8 \cdot 10^{-17}$ | 621 | $2.3 \cdot 10^4$ | 0.14 | $9 \cdot 10^{-5}$ | $6 \cdot 10^{-8}$ |
| $u$ | $-8.2 \cdot 10^{-18}$ | 1326 | $2.2 \cdot 10^4$ | 0.6 | $2 \cdot 10^6$ | $6 \cdot 10^{-8}$ |
| $e$ | $-2.5 \cdot 10^{-16}$ | 265 | $2.2 \cdot 10^3$ | - | $9 \cdot 10^6$ | $6 \cdot 10^{-9}$ |

**Results** We present our results in Table I. The first prominent result is that no light charged fermion could give the dominant contribution to the BAU. Of all the SM charged fermions only the $\tau$ could produce 100% of the observed BAU [11, 12]. The next in importance can be the $\mu$ [13], which brings us to consider the relatively negligible effect of the light quarks. In addition to their low contribution to the BAU, the bounds on light quarks are comparable to those of the leptons, and thus their maximal percentage is relatively small.

That being said, one could consider two flavor modification, in which case the electron has a special feature. Assuming the numerical result is linear with multiple CPV sources of different species, the electron could cancel the contribution of some particles to the eEDM, while leaving the contribution to the BAU essentially unchanged. When combined, the interference allows particles that are constrained mostly by the eEDM, such as the top [12], to account for the BAU. For example, for $\kappa_t^f = 0.04$, the top generates $Y_B^{obs}$, while $\kappa_t^{f \ast} = -0.06$ cancels the top’s contribution to the eEDM (See Fig. 3). Because of possible cancellation, it is much harder to exclude such an elusive hypothetical scenario.

**Conclusions** We consider the CPV source resulting from a dim-6 SMEFT term which couples three Higgs fields to the SM fermions. We apply the procedure described in Ref. [17] to evaluate the complete set of single flavor modifications from all of the SM charged fermions. We deduce that although a larger dim-4 Yukawa coupling enhances the CPV source, quarks have more washout than leptons and are therefore less favorable candidates to produce the BAU. Moreover, to saturate $Y_B^{obs}$, some of the particles require non-perturbative dim-4 Yukawa couplings, e.g. the up and down quarks, and are therefore unequivocally ruled out as sole-contributors via this mechanism.

Constrained by U.B.s of the electron and neutron EDMs and measurements of various Higgs boson processes, we evaluate the maximal contribution from each single flavor modification (See Table I). We conclude that the $\tau$ is the only candidate able to produce the observed BAU [11, 12]. Other than the $\mu$, which could provide up to 16% $Y_B^{obs}$ [13], the rest of the charged fermions produce negligible contributions (less than 6% $Y_B^{obs}$). An interplay of different flavors could relax current bounds to allow the observed baryon asymmetry be accounted for by a CPV Yukawa coupling. Specifically, the interplay of the top and the electron could allow the top
Table II. Collider limits on the signal strength $\mu_{h\rightarrow ff}$ from which we evaluate the U.B. at $\sim 95\%$ C.L. Combined with the SM prediction for the branching ratio, $B_{SM}^{h\rightarrow ff}$, we constrain $|\kappa_f|$ using Eqn. (24) for $\ell = \tau, \mu, e, b$, Eqn. (25) for $q = u, c, d, s$ and Eqn. (26) for the top. We extrapolated $B_{SM}^{h\rightarrow ff}$ for the electron (up and down quarks) from that of the muon (strange), using $B_{SM}^{h\rightarrow f\bar{f}} \propto m_f^2$.

| Channel | Experiment | $\text{best fit} \ \mu_{h\rightarrow ff}$ | $\text{U.B.} \ \mu_{h\rightarrow ff}$ | $B_{SM}^{h\rightarrow ff}$ |
|---------|-------------|---------------------------------|---------------------------------|------------------------|
| $h \rightarrow \tau\tau$ | ATLAS+CMS | 0.91 $\pm$ 0.13 $[12]$ | 1.1 | 6.3 $\cdot 10^{-2}$ $[30]$ |
| $h \rightarrow \mu\mu$ | ATLAS | 1.2 $\pm$ 0.6 $[32]$ | 1.8 | 2.2 $\cdot 10^{-4}$ $[30]$ |
| $h \rightarrow bb$ | CMS | 1.19 $\pm$ 0.44 $[33]$ | 1.4 | 0.58 $[30]$ |
| $ggF + t\bar{t}h$ | ATLAS+CMS | 1.09 $\pm$ 0.08 $[12]$ | 1.2 | $B(h \rightarrow gg)_{SM}$ = 8.2 $\cdot 10^{-2}$ $[30]$ |
| $h \rightarrow c\bar{c}$ | ATLAS+CMS | using $\mu_{h\rightarrow sb} \geq 0.71$ $[31]$ | 2.9 $\cdot 10^{-2}$ $[30]$ |
| $h \rightarrow s\bar{s}$ | using $\mu_{h\rightarrow sb} \geq 0.71$ $[31]$ | 4.40 $\cdot 10^{-4}$ $[34]$ |
| $h \rightarrow dd$ | | | 1.1 $\cdot 10^{-6}$ $[34]$ |
| $h \rightarrow u\bar{u}$ | | | 2.4 $\cdot 10^{-7}$ $[34]$ |
| $h \rightarrow c\bar{c}$ | ATLAS | $B_{U.B.} = 3.6 \cdot 10^{-4}$ $[35]$ | 7.0 $\cdot 10^4$ | 5.1 $\cdot 10^{-9}$ $[30]$ |

Figure 3. The interplay between the electron and the top could allow the top to saturate the observed BAU via $\kappa_t = 1 + i\kappa_t^*$, while canceling the contribution of $\kappa_e^*$ to the eEDM using $\kappa_e = 1 + i\kappa_e^{**}$. Accordingly, $\kappa_e^{**}$ is set such that $d_e$, i.e. the sum of equations (17) and (18), equals zero. Since the next leading bound on both particles is orders of magnitude weaker, such cancellation would be difficult to detect.

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Appendix A: Benchmark parameters

The input used for this work is the following:

- Coupling constant at nucleation temperature [10]:
  \( g_s = 1.23 \), \( g = 0.65 \), \( g' = 0.36 \). (A1)
- Bubble wall velocity and width [11]:
  \( v_w = 0.05 \), \( L_w = 0.11 \) GeV\(^{-1} \). (A2)
- VEV during nucleation [10] and at 0 temperature:
  \( v_N = 152 \) GeV, \( v_0 = 246 \) GeV. (A3)
- The SM fermion masses were taken from [1].
- The Mass and Yukawa rates are given in Table III. These interaction rates are calculated for \( T_n = T = T_R = 0 \).
- The weak sphaleron rate [36]
  \[
  \Gamma_{ws} = 6 \cdot \kappa \cdot \frac{g_w}{\sin^2 \theta_w} \cdot \frac{T}{120} \left( \frac{g^2}{4\pi} \right)^5 \approx 4.5 \cdot 10^{-4} \text{ GeV}. \quad (A4)
  \]

Table III. Relaxation rates (for the broken phase), and Yukawa rate (for both phases), calculated from [17].

| Particle | \( \Gamma_M/M_0 \) (GeV) | \( \Gamma_Y \) (GeV) |
|----------|----------------|----------------|
| \( \tau \) | \( 4.9 \cdot 10^{-3} \) | \( 5.6 \cdot 10^{-4} \) |
| \( \mu \) | \( 1.7 \cdot 10^{-5} \) | \( 2.0 \cdot 10^{-6} \) |
| \( e \) | \( 3.9 \cdot 10^{-10} \) | \( 4.4 \cdot 10^{-11} \) |
| \( t \) | \( 102 \) | \( 2.6 \) |
| \( c \) | \( 4.7 \cdot 10^{-3} \) | \( 1.6 \cdot 10^{-4} \) |
| \( u \) | \( 1.4 \cdot 10^{-8} \) | \( 4.7 \cdot 10^{-10} \) |
| \( b \) | \( 5.3 \cdot 10^{-2} \) | \( 1.7 \cdot 10^{-3} \) |
| \( s \) | \( 2.7 \cdot 10^{-5} \) | \( 9.0 \cdot 10^{-7} \) |
| \( d \) | \( 6.5 \cdot 10^{-8} \) | \( 2.1 \cdot 10^{-9} \) |

- The strong sphaleron rate [37]
  \[
  \Gamma_{ss} = 14 \alpha_s^4 T \approx 0.26 \text{ GeV}. \quad (A6)
  \]

- Thermal width [38]:
  \[
  \Gamma_{\text{leptons}} \approx 2 \cdot 10^{-3} T, \quad \Gamma_{\text{quarks}} \approx 0.16 T. \quad (A7)
  \]

The temperature during nucleation is \( T_N = 88 \) GeV, and for the SM we use \( R = \frac{13}{4} \).

Thermal functions:

- The source is given by [14, 39]
  \[
  S_f(z;T) = \frac{v_wN_f^f}{\pi^2} \Im(m_F^f m_{F}^f) \Gamma_f(T)
  = \frac{v_wN_f^f \gamma_{SM}^f}{2\pi^2 v_0^2} J_f(T) h^3(z) h'(z) \times \kappa_f, \quad (A8)
  \]
  \[
  J_f(T) = \int_0^\infty dk \frac{n_f^f (\xi_L^f - \xi_R^f)}{\omega_L^f \omega_R^f} \Im \left[ \frac{\xi_L^f - \xi_R^f}{2} \left( \xi_L^f \xi_R^f - k^2 \right) \right] \quad (A9)
  \]

For the background Higgs boson field we use the kink solution:

\[
  h = \frac{v_N}{2} \left( 1 + \tanh \left( \frac{z}{T_w} \right) \right). \quad (A10)
  \]

- The frequencies, energies and Fermi-Dirac distributions are
  \[
  \omega_{L,R}^f(k) = \sqrt{k^2 + \text{Re}(\delta m_{f_i,L,R}^2(T))}, \quad \xi_L^f = \omega_{L,R}^f(k) - i \Gamma_{f_i},
  \]
  \[
  n_f(\xi) = \frac{1}{e^{\xi/T} + 1} \quad (A11)
  \]
\[ \text{The thermal masses are given by [40] } \]

\[ \text{Re}(\delta m^2_{\ell_{\tau}}(T)) = \left( \frac{3}{32} g^2 + \frac{1}{32} g^2 + \frac{1}{16} y_i^2 \right) T^2 \equiv a^2_{\ell_{\tau}} T^2, \]

\[ \text{Re}(\delta m^2_{\epsilon_{e\tau}}(T)) = \left( \frac{1}{6} g^2 + \frac{1}{2} g^2 + \frac{1}{8} y_i^2 \right) T^2 \equiv a^2_{\epsilon_{e\tau}} T^2, \]

\[ \text{Re}(\delta m^2_{Q_{d\tau}}(T)) = \left( \frac{1}{6} g^2 + \frac{3}{32} g^2 + \frac{1}{288} g^2 + \frac{1}{16} y_i^2 \right) T^2 \equiv a^2_{Q_{d\tau}} T^2, \]

\[ \text{Re}(\delta m^2_{u_{i\tau}}(T)) = \left( \frac{1}{6} g^2 + \frac{1}{18} g^2 + \frac{1}{8} y_i^2 \right) T^2 \equiv a^2_{u_{i\tau}} T^2, \]

\[ \text{Re}(\delta m^2_{d_{i\tau}}(T)) = \left( \frac{1}{6} g^2 + \frac{1}{72} g^2 + \frac{1}{8} y_i^2 \right) T^2 \equiv a^2_{d_{i\tau}} T^2, \]

\[ \text{Re}(\delta m^2_{\delta f}(T)) = \left( \frac{3}{16} g^2 + \frac{1}{6} g^2 + \frac{1}{12} \sum_{i=c,\tau} y_i^2 \right) T^2 \equiv a^2_{\delta f} T^2. \]

\[ \text{where we used Fick’s first law and the diffusion approximation. } \]

\[ b. \ \text{Reduction of order} \quad \text{We can solve this set of } N = 16-\text{second order differential equations by reduction of order:} \]

\[ g_j, = f_j', \quad f_j = (U_R \bar{D}_R \bar{Q}_L \bar{E}_R \bar{L}_L \ h), \quad \bar{X} = (f \ g_j)^T. \]

\[ \bar{X}' = \left( \begin{array}{c} 0 \\ \mathbb{1}_{N \times N} \end{array} \right) \bar{X} + \bar{S} \equiv \bar{K} \bar{X} + \bar{S}. \quad (B2) \]

We are left with \( 2N \)-first order differential equations. Then, \( \bar{K} \) can be diagonalized numerically, e.g. by using MATLAB.

\[ c. \ \text{Numerical diagonalization} \quad \text{The solution for the symmetric phase is given by} \]

\[ \bar{X}^S = \sum_{i=1}^{2N} C_i e^{\lambda_i^S t} \bar{u}_i^S \equiv \bar{\Phi}^S(z) \bar{C}^S, \]

\[ \bar{\Phi}^{X,(i)}_j (z) = e^{\lambda_{ij} t} (\bar{u}_i^X)_j, \quad (B3) \]

where \( C_j \)'s are constants, \( \lambda_i \) are the eigenvalues, and \( \bar{u}_i \)'s are the eigenvectors. We define

\[ \hat{\lambda} = \text{diag}(\lambda_i), \quad i = [1 : 2N]. \quad (B4) \]

\[ \hat{\phi} = \left( \begin{array}{cc} \bar{u}_1 & \bar{u}_2 & \ldots & \bar{u}_{2N} \end{array} \right) \quad (B5) \]

Accordingly,

\[ \bar{\Phi}_{t,j} (z) = \hat{\phi}_{t,j} e^{\lambda_{ij} t} = \left( e^{\lambda_{1j}} \bar{u}_1 \ e^{\lambda_{2j}} \bar{u}_2 \ldots e^{\lambda_{2Nj}} \bar{u}_{2N} \right) \quad (B6) \]

The full solution in the broken phase is obtained by variation of parameters to be

\[ \bar{X}^B = \bar{\Phi}^B(z) \bar{C}^B + \bar{\Phi}^B(z) \int_0^z (\bar{\Phi}^B(x))^{-1} \bar{S}(x) \ dx. \quad (B7) \]

\[ d. \ \text{Boundary conditions} \]

\[ \text{The integration constants of the divergent modes in the symmetric phase (correspond to } \lambda_i^S \leq 0 \text{) are set to zero,} \]

\[ C_{0-}^S = 0. \quad (B8) \]

\[ \text{The positive eigenvalues in the broken phase, } C_{j+}^B \text{ (correspond to } \lambda_j^B > 0 \text{), are chosen such that they cancel the divergent part of the full solution at infinity:} \]

\[ \bar{C}^B_+ = - \int_0^\infty (\bar{\Phi}^B_+(x))^{-1} \bar{S}(x) \ dx. \quad (B9) \]

\[ \text{Appendix B: Solving the set of transport equations: the two-step approach} \]

In this appendix we summarize the analytic techniques used to calculate the produced baryon asymmetry, similarly to Ref. [17]. We used the two-step approach: First we solve the set of transport equations, given in Eqn. (10). Then, the BAU is obtained by summing over the left-handed number densities and considering the weak sphaleron process.

\[ a. \ \text{Reduction of dimensions} \quad \text{The left hand side of Eqn. (10) can be written as a one dimensional - second order differential equation with respect to the bubble wall dimension denoted } z \ [18]: \]

\[ \partial f \equiv \partial_{\mu} f^\mu = \frac{\partial f^0}{\partial t} - \vec{v} \cdot \vec{f}' = \frac{\partial f^0}{\partial t} + \vec{\nabla} \cdot (-D_f \vec{\nabla} f^0) = v_w f' - D_f \vec{\nabla}^2 f^0 = v_w f' - D_f f'' , \quad (B1) \]

\[ \text{where } v_w \text{ is the number of degrees of freedom, and } +(-) \text{ is for fermions (the Higgs boson).} \]
• We demand continuity at $z = 0$.
  
  - In the symmetric phase we have
    \[
    \tilde{\chi}^S_i (z \rightarrow 0^-) = \hat{\phi}^+_i \tilde{C}^S_j = \hat{\phi}^+_i \tilde{C}^S_+ \, .
    \] (B10)
  
  - In the broken phase we have
    \[
    \tilde{\chi}^B_i (z \rightarrow 0^+) = \hat{\phi}^B_i \tilde{C}^B_j = \hat{\phi}^B_0 \tilde{C}^B_0 + \hat{\phi}^B_+ \tilde{C}^B_+ \\
    = \hat{\phi}^B_0 + b_i \, .
    \] (B11)

  Continuity is then
  \[
  \hat{\phi}^S_0 \tilde{C}^S_+ \equiv \hat{\phi}^B_0 \tilde{C}^B_0 + b_i \longrightarrow \hat{\phi}^S_0 \tilde{C}^S_+ - \hat{\phi}^B_0 \tilde{C}^B_0 \equiv b_i \, . \tag{B12}
  
  We obtain a linear set of equations,
  \[
  \hat{\phi}_{SB} \equiv \left( \begin{array}{c} \hat{\phi}^+_i \\ \hat{\phi}^B_0 \\ \hat{\phi}^B_+ \end{array} \right), \quad \tilde{C}_{SB} \equiv \left( \begin{array}{c} \tilde{C}^S_i \\ \tilde{C}^B_0 \\ \tilde{C}^B_+ \end{array} \right),
  \]
  \[
  \hat{\phi}_{SB} \tilde{C}_{SB} \equiv b_i \, . \tag{B13}
  
  Solving it sets the rest of the coefficients.

  e. The solution  The BAU is then given by
  \[
  Y_B = \frac{3 \Gamma_{ws}}{2 D_q \alpha + \delta} \int_0^{-\infty} e^{-\alpha - \frac{1}{\alpha} n_L(x)} \, dx \, , \tag{B14}
  \]
  where $n_L$ is the density of left handed particles in the symmetric phase (where the weak sphaleron process is efficient),
  \[
  n_L(z) = \sum_{i=1}^3 (Q_L_i(z) + L_L_i(z)) \, , \tag{B15}
  \]
  and
  \[
  \alpha = \frac{1}{2 D_q} \left( v_w \pm \sqrt{4 D_q \Gamma_{ws} R + v_w^2} \right) \, . \tag{B16}
  
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