Current dependence of the ‘red’ boundary of superconducting single photon detectors in the modified hot spot model

D. Yu. Vodolazov

1 Institute for Physics of Microstructures, Russian Academy of Sciences, 603950, Nizhny Novgorod, GSP-105, Russia
2 Lobachevsky State University of Nizhny Novgorod, 23 Gagarin Avenue, 603950 Nizhny Novgorod, Russia
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We find the relation between the energy of the absorbed photon and the threshold current at which the resistive state appears in the current-carrying superconducting film with the probability about unity. In our calculations we use the modified hot spot model, which assumes different strength of suppression of the superconducting order parameter in the finite area of the film around the place where the photon is absorbed. To find the threshold current we solve the Ginzburg-Landau equation for superconducting order parameter, which automatically includes the current continuity equation and it allows us to consider the back effect of current redistribution near the hot spot on the stability of the superconducting state. We find quantitative agreement with the recent experiments, where we use the single fitting parameter which describes what part of the energy of the photon goes for the local destruction of the superconductivity in the film.

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I. INTRODUCTION

In 2001 it was demonstrated that the current-carrying superconducting film biased near its critical current can detect single photon in the visible and near infrared range of electromagnetic radiation (act of detection was seen in the experiment as an appearance of the voltage pulse in the film) [1]. The one of the main characteristics of such a superconducting single photon detector (SSPD) is the position of ‘red’ boundary, or the largest wavelength of the photon ($\lambda_{\text{max}}$) which could be detected with probability close to unity. In reality, the part of photons can miss the active element of SSPD - superconducting film, which automatically makes its system detection efficiency smaller than unity. This problem has a technical character and may be solved, for example, by putting a superconducting film in the form of meander to the resonator [2]. To characterize the intrinsic ability of superconducting film to detect the photons it was offered the new characteristic - intrinsic detection efficiency (IDE) [3], which has a meaning of probability of the photon detection after its absorption by the superconducting film (from this definition it follows that IDE \( \approx 1 \) when \( \lambda \leq \lambda_{\text{max}} \)).

In many experiments it was observed that $\lambda_{\text{max}}$ depends on the transport current (see for example [4-7]). Moreover at fixed $\lambda$ IDE does not drop suddenly when current becomes smaller some critical (threshold) value but it decreases gradually until it reaches unmeasurably small values. At the moment there are several theoretical models [4, 5, 8-12] which relates the threshold current $I_{\text{thr}}$ with $\lambda_{\text{max}}$. These models are based on the assumption that the absorbed photon creates the hot spot [4, 5, 8-11] or the hot belt [12] in the superconducting film (for their comparison see Ref. [8]). In the present work we use a bit different model for the hot spot (see section 2) than the one used in Ref. [8]. To study the situation when IDE\( \approx 1 \) we place the hot spot in the center of the film and identify the critical current of such a system as a threshold current. This assumption is based on our recent study [13] where we find that with current increase IDE gradually increases from \( \sim 0.05 \) (when only the photons absorbed near the edge of the film provide the instability of superconducting state) up to the unity (when the instability arises also from the photons absorbed in the center of the film). We make our calculations for the films with different width and the hot spots with different size, which correspond to the photons with different energies. We compare our results with recent experiments [6, 7, 14] and find good quantitative agreement with the only assumption that about 10% of the photon energy goes for the local suppression of the superconductivity.

II. MODEL

We consider a two-dimensional superconducting film with finite width and the hot spot is modelled by the local area (in the form of the circle - see insets in Fig. 1(a)) where the quasiparticle distribution function $f(\epsilon)$ is supposed to be far from the equilibrium. We assume that this nonequilibrium is created by the photon and it affects the stability of the superconducting state with transport current. To find the value of the critical current of the film with such a hot spot we numerically solve the Ginzburg-Landau equation for the superconducting order
parameter $\Delta = |\Delta| e^{i\phi}$.

$$\xi_{GL}^2 \left( \frac{\partial^2 \Delta}{\partial x^2} + \frac{\partial^2 \Delta}{\partial y^2} \right) + \left( 1 - \frac{T_{\text{bath}}}{T_c} + \Phi_1 \right) - \frac{\Delta^2}{\xi_{GL}^2} \Delta = 0$$

with the additional term $\frac{\Delta}{\Delta_{\text{GL}}}$.

$$\Phi_1 = \int_{|\Delta|}^{\infty} \frac{2(f^0(f) - f)}{\sqrt{\epsilon^2 - |\Delta|^2}} \, d\epsilon,$$ 

which describes the effect of nonequilibrium distribution function $f(\epsilon) \neq f^0(\epsilon) = 1/(\epsilon \exp(\epsilon/k_B T_{\text{bath}}) + 1)$. In Eq. (1) $\xi_{GL}^2 = \pi \hbar D/8k_B T_c$ and $\Delta_{\text{GL}}^2 = 8\pi^2(k_B T_c)^2/7\zeta(3) \approx 9.36 (k_B T_c)^2$ are the zero temperature Ginzburg-Landau coherence length and the order parameter correspondingly.

Note, that imaginary part of Eq. (1) leads to the current continuity equation $\text{div} j_\alpha = 0$ ($j_\alpha$ is a superconducting current density) which allows us to find the proper distribution of the current density in the film with the hot spot.

It is convenient to write Eq. (1) in dimensionless units (length is scaled in units of $\xi_{\text{GL}}$, time is scaled in units of $\xi_{\text{GL}}^2/|\Delta|$, and $\Delta$ in units of $\Delta_{\text{eq}} = \Delta_{\text{GL}}(1 - T_{\text{bath}}/T_c)^{1/2})$

$$\frac{\partial^2 \Delta}{\partial x^2} + \frac{\partial^2 \Delta}{\partial y^2} + \left( \alpha - |\tilde{\Delta}|^2 \right) \tilde{\Delta} = 0,$$ 

where $\alpha = (1 - T_{\text{bath}}/T_c + \Phi_1)/(1 - T_{\text{bath}}/T_c)$. In Eq. (3) the impact of the absorbed photon on the superconducting properties of the film is described by the single parameter $\alpha$, which in reality should vary in time and space due to $f(\epsilon, r, t)$ (in equilibrium $\alpha = 1$). In our model we use the static approach for the hot spot (with $\alpha(t) = \text{const}$ inside the hot spot). We can do it because of different times scales existing in this problem. Indeed, during initial very short time the hot quasiparticles (appearing after photon absorption - see for example [18]) undergo the downconversion cascade to the energy level just above $\Delta$ and the further relaxation process develops much longer. The low energy nonequilibrium quasiparticles diffuse in space (which is relatively slow process due to small group velocity of quasiparticles seating at the energies close to the energy gap), but simultaneously they suppress $\Delta$ locally below its equilibrium value. It results in the appearance of quasiparticles with the energy less than $\Delta_{\text{eq}}$ which already cannot diffuse and they become trapped by the hot spot (see more detailed discussion of this phenomena in Ref. [18]). These quasiparticles can relax to the equilibrium only via electron-phonon scattering with characteristic inelastic electron-phonon relaxation time $\tau_{\text{e-ph}}$. Therefore roughly this time governs the final stage of evolution of the hot spot. But the time change of $\Delta$ is much faster process and at low temperatures it is proportional to $\hbar/\Delta_{\text{eq}}$ which is much shorter than $\tau_{\text{e-ph}}$. Therefore on the time scale of change of $\Delta$ one may consider the quasiparticle distribution function as a static object (at the final stage of its evolution).

In our previous paper [8] in the numerical simulations we used the model with the local temperature of the quasiparticles and solved the heat conductance equation for space and temporal evolution of $T_{\text{loc}}$. This model oversimplifies the real situation because it does not take into account the aforementioned suppressed diffusion of the quasiparticles with $\epsilon \lesssim \Delta_{\text{eq}}$ and, besides, it assumes implicitly that at any moment in time the quasiparticles are in the local equilibrium which is rough approximation in the real SSPD where inelastic electron-electron relaxation time is comparable with inelastic electron-phonon relaxation time. But nevertheless it gives qualitatively the same results as using here the 'static' hot spot model. It is not surprising, because in the case when $f(\epsilon)$ is described by the Fermi-Dirac function with the local temperature $T_{\text{loc}}$ the our control parameter $\alpha(r, t) = (1 - T_{\text{loc}}(r, t)/T_c)/(1 - T_{\text{bath}}/T_c)$. Physically it corresponds to the step like distribution of $T_{\text{loc}}$ in space which provides qualitatively similar suppression of $\Delta$ as it does the Gaussian-like distribution of $T_{\text{loc}}$ following from the heat conductance equation (see for example Eq. (13) in Ref. [8]).

Note, that different $f(\epsilon)$ may provide the same value of $\alpha$, which is consequence of dependence of superconducting order parameter $\Delta$ on integral of $f(\epsilon)$ (with some weight function) over the energy. Finding $f(\epsilon)$ needs the solution of the kinetic equation which is very difficult problem [18] and it is beyond the scope of this paper.

But to have an insight on the possible values of $\alpha$ let as assume for simplicity that $f(\epsilon)$ is described by the Fermi-Dirac function with the local temperature $T_{\text{loc}}$. Due to diffusion of hot quasiparticles the region where $T_{\text{loc}} > T_{\text{bath}}$ grows but the most effectively the order parameter is suppressed in the area where $T_{\text{loc}} > T_c$ and $\alpha < 0$. At some moment, after the absorption of the photon, the region where $T_{\text{loc}} > T_c$ stops to grow and one can use this moment for determination of the effective size of the hot spot.

Based on above consideration we choose $\alpha = 0$ inside the spot in the form of circle with radius $R$ (we also checked what occurs at different values of $\alpha$). The radius $R$ and the energy of the photon $\hbar \omega/\lambda$ are roughly related as

$$\eta \frac{\hbar \omega}{\lambda} \simeq d \pi R^2 \frac{H_{\text{cm}}^2}{8\pi}$$

where $H_{\text{cm}} = \Phi_0/2 \sqrt{2 \pi \xi \lambda_L}$ is the thermodynamic magnetic field, $\Phi_0$ is the magnetic flux quantum, $\lambda_L$ is the London penetration depth, $d$ is the thickness of the film and $H_{\text{cm}}^2/8\pi$ is the superconducting condensation energy per unit of volume. Coefficient $0 < \eta < 1$ takes into account that not whole energy of the photon goes for suppression of $\Delta$ [8]. For example the large part of the energy of the photon is delivered to the energy of quasiparticles $E_q$. One can easily find it in the local temperature approximation (when $T_{\text{loc}}(r) = \text{const}$ inside the circle with
we plot contour plot of density (b) along the white lines marked in the insets (where $T$ figure (a) and at all radiuses the transport current is close to the threshold value. In comparison with Eq. (5) when $\Delta$ energy of quasiparticles in equilibrium (which is small order parameter at zero temperature and we neglect the

\[ \partial \Delta / \partial t \]

in a way how it was done in Ref. [8]) which allows us to find not only the value of the critical (threshold) current but also to find how the superconducting state becomes destroyed. As an edge effect during the nonstationary stage the normal current $j_n$ (and electrostatic potential) appears in the film and we find them by solving equation $\text{div}(j_n + j_s) = 0$.

III. RESULTS

In Fig. 1(a,b) we show distribution of $|\Delta|$ and current density across the hot spot and sidewalks. Note that despite of $\alpha = 0$ the order parameter is finite in the hot spot region (see Fig. 1(a)). It occurs due to proximity effect from the surrounding area and the same effect leads to partial suppression of $|\Delta|$ outside the region where we keep $\alpha$. Due to the current crowding effect the maximum of current density is located near the ‘edge’ of the hot spot (compare Fig. 1(a) and 1(b)).

\[
E_q = \frac{d\pi R^2 4N_0}{\Delta^2} \int_0^\infty \epsilon f(\epsilon) d\epsilon = \frac{d\pi R^2 N_0}{3} 0.67 T^2_{\text{loc}} \Delta_0^2
\]

where $N_0$ is a one-spin density of states of quasiparticles at the Fermi level, $\Delta_0 = 1.76k_BT_c$ is the superconducting order parameter at zero temperature and we neglect the energy of quasiparticles in equilibrium (which is small in comparison with Eq. (5) when $\Delta_{\text{eq}} > k_B T_{\text{bath}}$ and $T_{\text{loc}} \gg T_{\text{bath}}$). One can see that when $T_{\text{loc}} = T_c$, $E_q$ is twice larger than the condensation energy (because at low temperatures $H^2_{\text{eq}}/8\pi \sim N_0 \Delta_0^2/2$) and when $T_{\text{loc}} = 2T_c$, $E_q$ is larger more than in eight times.

In numerical calculations we consider the film of finite width $w$ and length $L = 4w$ with the hot spot (region where $\alpha < 1$) placed in the center of the film - see insets in Fig. 1a. We also add to the right hand side of Eq. (3) the term with time derivative $\partial \Delta / \partial t$.

\[
\frac{I_{\text{th}}}{I_{\text{dep}}} \bigg|_{R = \text{max}} = 0.97 I_{\text{th}} \quad \text{and} \quad \frac{I_{\text{th}}}{I_{\text{dep}}} \bigg|_{R = \text{min}} = 0.99 I_{\text{th}}
\]

FIG. 1: Distribution of $|\Delta|$ (a) and superconducting current density (b) along the white lines marked in the insets (where we put $\alpha = 0$ is bordered by the white circle in the insets in figure (a) and at all radiuses the transport current is close to the threshold value.

In Fig. 1(a,b) we show distribution of $|\Delta|$ and current density across the hot spot and sidewalks. Note that despite of $\alpha = 0$ the order parameter is finite in the hot spot region (see Fig. 1(a)). It occurs due to proximity effect from the surrounding area and the same effect leads to partial suppression of $|\Delta|$ outside the region where we keep $\alpha$. Due to the current crowding effect the maximum of current density is located near the ‘edge’ of the hot spot (compare Fig. 1(a) and 1(b)).

\[
\frac{I_{\text{th}}}{I_{\text{dep}}} \bigg|_{R = \text{max}} = 0.97 I_{\text{th}} \quad \text{and} \quad \frac{I_{\text{th}}}{I_{\text{dep}}} \bigg|_{R = \text{min}} = 0.99 I_{\text{th}}
\]

One can see that when the hot spot located not in the center of the film the free vortex motion starts at lower currents. In Fig. 2 we present dependence of $I_{\text{th}}/I_{\text{dep}}$ on the radius of the spot when $\alpha = 0$. In the same figure we present dependence $I_{\text{th}}(R)$ (solid curves) following from the London model (see Eq. (12) in Ref. [8]). The quantitative difference between these results is not surprising, because in the analytical model of Ref. [8] the gradual variation of $|\Delta|$ at the edge of the hot spot was...
neglected and semi-quantitative criteria for unbinding of vortex-antivortex pair was used.

In Fig. 2 we also plot $I_{thr}(R)$ (dashed line) for the film with $w = 30\xi$ which follows from the model with spatially uniform current distribution in the sidewalks [5]. One can see that the current crowding effect leads to smaller value of $I_{thr}$ when $\xi \ll R \ll w$.

In Fig. 3 we show the same dependencies but instead of radius of the spot, as one of the coordinate, we use the difference between the free energies of the states of the film with and without the hot spot

$$\delta E = F_{spot} - F_{no-spot} - \frac{\hbar}{2e} I \delta \varphi.$$  \hspace{1cm} (6)

where

$$F = -\frac{H^2 d}{8\pi} \int \left( \frac{|\Delta|}{\Delta_{eq}} \right)^4 dS,$$  \hspace{1cm} (7)

and $\delta \varphi$ is the extra phase difference due to presence of the hot spot. At the derivation of Eq. (7) from the Ginzburg-Landau free energy functional we take into account that $|\Delta|$ is the solution of the Ginzburg-Landau equation and we do not consider difference in the energy of quasiparticles because it strongly depends on explicit dependence $f(\varphi)$. In Fig. 3 (where the energy is normalized in units of $E_0 = H^2 d^4 \xi^2 / 2 = \Phi_0 d / 16 \pi^2 \lambda_I^2$) for the film with $w = 20\xi$ we also show the results for the hot spots with different values of $\alpha$. Surprisingly, the difference is not large in the used coordinates (notice, that for larger $\alpha$ the same value $I_{thr}$ is reached for larger radius of the hot spot, but the energies of these states are turned out to be close to each other).

IV. COMPARISON WITH THE EXPERIMENT

To relate the theoretical results present in Figs. 2,3 with the recent experiments [6,7,14] we suppose that only fraction of the photon’s energy $\hbar \omega / \lambda$ goes for destruction of superconductivity inside the hot spot. In Figs. 4,5 we compare our results with the experimental results present in Refs. [6,7,14]. In Fig. 4 we use $\xi = 7\text{nm}$ and the square resistance $R_{sq} = 400\text{Ohm}$ for all theoretical curves. Depairing current for TaN meander from Ref. [6] is calculated with the Kupriyanov-Lukichev correction [10]. In Ref. [7] this correction was omitted [20] and it leaded to slightly smaller value of $I_{dep}$ (compare Fig. 4 with the inset in Fig. 9 of Ref. [6]). In Ref. [14] authors did not present parameters of the NbN bridge ($R_s$ or $\lambda_L$ and $T_c$) and we take these parameters from Ref. [22] for NbN film with the same thickness as in Ref. [14].

In Figs. 4,5 the only fitting parameter is the coefficient $\eta$. It has almost the same value about 0.1 for different TaN meanders. For NbN bridge the best fitting is obtained for $\eta = 0.17$ (note, that other hot spot models give $\eta \approx 0.1 - 0.4$ after their comparison with the experiments [4,6,7]). We should mention that in Ref. [14] the IDE was fixed at the level $\sim 0.25$ (this number can be estimated using the experimental results for detection probability $p_n$ in Fig. 3 of Ref. [14]). From Fig. 3 of Ref. [14] one can see that $p_n$ saturates (and IDE $\rightarrow 1$) at larger currents. It means that the experimental points in Fig. 5 should be shifted upwards and in this case the theoretical results would fit the experimental ones practically at the same value of $\eta$ as for TaN meanders.

There is also another work [21], where the photon detection by the NbN bridge (in the same geometry as in
In this work we exploit the modified hot spot model to find the relation between the energy of the absorbed photon and the threshold current at which the resistive state appears with probability about unity in the current-carrying superconducting film. The hot spot is modelled as a finite region in the center of the film where the quasiparticle distribution function is far from the equilibrium and it leads to the partial suppression of the superconducting order parameter. We find that the resistive state starts from the nucleation and unbinding of the vortex-antivortex pair inside the hot spot. When the energy of the photon goes to zero, the suppression of the order parameter in the hot spot becomes small and the threshold current rapidly approaches the depairing current. Comparison of found results with recent experiments shows good agreement if one assumes that only about 10% of the energy of the photon goes for the local destruction of the superconductivity.

The main differences of our model with the previous models are following: i) we solve the current continuity equation \( \nabla j_s = 0 \) in the film with the hot spot (which gives us correct distribution of the current density in the superconductor); ii) we take into account the back effect of the current redistribution around the hot spot on the superconducting order parameter. Because we did not solve the kinetic equation and we did not find nonequilibrium distribution function we do not know the actual value of \( \alpha \) and we cannot relate quantitatively the energy of the photon with the size of the hot spot and how strong the superconductivity is suppressed inside it. But we find that the relation between the threshold current and the energy needed for suppression of the superconductivity inside and around the hot spot (which does not include the energy of hot quasiparticles and phonons) weakly depends on the these parameters. This result encourage us that the found results are rather general and have direct relation to the dependence of the ‘red’ boundary of SSPD on the trans-

V. CONCLUSION

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port current.

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