The late infall of dark matter onto a galaxy produces structure (such as caustics) in the distribution of dark matter in the halo. We argue that such structure is likely to occur generically on length scales proportional to $l \sim t_0 v_{\text{rot}}$, where $t_0$ is the age of the universe and $v_{\text{rot}}$ is the rotation velocity of the galaxy. A set of 32 extended galactic rotation curves is analyzed. For each curve, the radial coordinate is rescaled according to $r \rightarrow \tilde{r} = r(t_0/v_{\text{rot}})$, where we choose $v_0 = 220$ km/s. A linear fit to each rescaled rotation curve is subtracted, and the residuals are binned and averaged. The sample shows significant features near $\tilde{r} = 40$ kpc and $\tilde{r} = 20$ kpc. This is consistent with the predictions of the self-similar caustic ring model of galactic halos.

Evidence for universal structure in galactic halos

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The rotation curves of most spiral galaxies are approximately flat, i.e. $v(R) \approx \text{constant}$ for $R$ much larger than the disk radius $R_0$, where $v(R)$ is the circular velocity of gas at radial coordinate $R$. The flatness of galactic rotation curves implies that galaxies are surrounded by halos of dark matter, and that the halo density falls off approximately as $\rho_{\text{DM}}(r) \propto 1/r^2$. The curves are far from exactly flat, however. They have all sorts of irregularities which may be referred to as “bumps”. When a bump occurs at $r >> R_0$ disk radius, it indicates structure in the dark matter distribution, i.e. a deviation from a perfect $\rho_{\text{DM}} \propto 1/r^2$ law. We are motivated by the possibility that such structures have universality and that a statistical analysis of well measured extended rotation curves may reveal that.

In refs. [2,3], 32 extended galactic rotation curves were selected and analyzed to test the validity of a modification of Newtonian dynamics (MOND) as an explanation of the “dark matter problem”, i.e. the discrepancy between the flatness of galactic rotation curves and the exponential fall-off of galactic luminous matter distributions. We analyze the same set of rotation curves with a different goal, namely to search for structure within dark matter halos. We adopt the conventional wisdom that a rotation curve is a tracer of the galactic mass distribution according to the laws of Newtonian dynamics. However, the selection criteria used by the authors of refs. [2,3] - that each rotation curve is an accurate tracer of the radial force law, and that it extends far beyond the edge of the luminous disk - are inappropriate from the point of view of our analysis. We do not know of another data set of comparable quality. We use the complete data set described there, without any cuts of our own.

One broad argument why structure may be present in galactic halos is that the halos form as a result of the infall of the dark matter surrounding the galaxy $\{n\}$. If the dark matter particles are collisionless, they oscillate back and forth numerous times before they are virialized by inhomogeneities in the galactic mass distribution $\{n\}$.

The non-virialized flows of dark matter produce structure within the halo, e.g. the caustics described below.

If structure is present, one would expect it to occur on length scales set by $t_0 v_{\text{rot}}$, where $t_0$ is the age of the galaxy and $v_{\text{rot}}$ is its rotation velocity. Indeed $v_{\text{rot}}$ sets the scale for the velocities with which all dark matter particles move about in the halo. Since all galaxies have nearly the same age, we expect that any regularity shared by the different galaxies of a set will reveal itself most readily after all lengths have been rescaled according to $\ell \rightarrow \ell' = \ell/v_{\text{rot}}$.

We are motivated in large part by the fact that caustics form in the non-virialized flows associated with the late infall of dark matter onto a galaxy. By definition, a caustic is a place in physical space where the dark matter density diverges at the caustics. Two types of caustic form: inner and outer. The outer caustics are 2-spheres surrounding the galaxy. One such caustic is located wherever an outflow of dark matter reaches its maximum radius before falling back in. The inner caustics are rings.

One caustic ring is located wherever the particles with the most angular momentum in a given inflow reach their distance of closest approach to the galactic center before moving back out.

Outer caustic spheres and inner caustic rings are examples of the sort of structure we are looking for. Caustic rings are the more likely candidate for detection in an analysis of galactic rotation curves because they have the higher density contrast and because they are located closer to the galactic center, where the rotation curves are measured. Furthermore, a specific proposal [1] has been made for the radii $a_n$ of caustic rings:

$$a_n : n = 1, 2, ... \approx (39, 19.5, 13, 10, 8, ... ) \text{kpc}$$

$$\times \left( \frac{J_{\text{max}}}{0.25} \right)^{0.7} \left( \frac{h}{0.7} \right) \left( \frac{v_{\text{rot}}}{220 \text{ km/s}} \right)$$

where $h$ is the present Hubble rate in units of.
100 km/(s Mpc), and $j_{\text{max}}$ is the maximum of the dimensionless angular momentum distribution of the infalling dark matter particles in the self-similar infall model of galactic halo formation.

The infall of dark matter is called self-similar if it is time-independent after all distances are rescaled by a time-dependent scale $R(t)$ and all masses are rescaled by the mass $M(t)$ interior to $R(t)$. Usually $R(t)$ is taken to be the turn-around radius, defined as follows. Consider all particles which are about to fall onto the galaxy for the first time in their history at time $t$. Such particles are said to be at their ‘first turn-around’. In a spherical model, they have zero radial velocity then. Their distance to the galactic center is the turn-around radius $R(t)$ at that time. In the case of zero angular momentum and spherical symmetry, the infall is self-similar if the initial overdensity profile has the form

$$\delta M_i = \left( \frac{M_0}{M_i} \right)^\epsilon,$$

where $M_0$ and $\epsilon$ are parameters, $\epsilon$ must be in the range $0 \leq \epsilon \leq 1$. The rotation curve is flat if $0 \leq \epsilon \leq 2/3$. In models of large scale structure formation, $\epsilon$ is predicted to lie in the range $0.2$ to $0.35$. The self-similar infall model was generalized in ref. to include the effect of angular momentum. It was found that, in addition to the condition on the initial overdensity profile, self-similarity requires the angular momentum distribution $\ell(t)$ to have the time-dependence

$$\ell(t) = j \left( \frac{R^2(t)}{t} \right),$$

where $j$ is a dimensionless and time-independent distribution. The maximum of the $\ell$ distribution is the quantity $j_{\text{max}}$ which appears in Eq. (1). Good agreement of the self-similar model with the properties of our own galaxy was found for parameter values $\epsilon = 0.20$ to $0.35$, $j \simeq 0.2$ and $h \simeq 0.7$ where $j$ is the average of the $j$ distribution.

Eq. (1) assumes $\epsilon = 0.3$. For $\epsilon = 0.2$,

$$a_1 \simeq 36 \text{ kpc} \left( \frac{j_{\text{max}}}{0.25} \right) \left( \frac{0.7}{h} \right) \left( \frac{v_{\text{rot}}}{220 \text{ km/s}} \right),$$

but the ratios $a_n/a_1$ are almost the same as in the $\epsilon = 0.3$ case. The ratios happen to be close to $a_n/a_1 = 1/n$ over the range $(0.2 \leq \epsilon \leq 0.35)$ of interest.

Eq. (1) predicts the caustic ring radii of a galaxy in terms of its first ring radius $a_1$. If the caustic rings lie close to the galactic plane they cause bumps in the rotation curve at the caustic ring radii. As a possible example of this effect, consider the rotation curve of NGC3198, one of the best measured and a member of the set selected in refs. It has three faint bumps at radii: 28, 13.5 and 9 kpc, assuming $h = 0.75$. The ratios happen to be consistent with Eq.(1) assuming the bumps are caused by the first three $(n = 1, 2, 3)$ ring caustics of NGC3198. Moreover, since $v_{\text{rot}} = 150 \text{ km/s}$, $j_{\text{max}}$ is determined in terms of $\epsilon$. For $\epsilon = 0.3$, $j_{\text{max}} = 0.28$. Note that the uncertainty in $h$ is a systematic effect that can be corrected for when determining $j_{\text{max}}$ because the bump radii scale like $1/h'$ where $h'$ is the Hubble rate assumed by the observer in constructing the rotation curve, and the caustic ring radii scale as $1/h$. Rises in the inner rotation curve of the Milky Way were also interpreted as due to caustics $n = 6, 7, 8, 9, 10, 11, 12$ and 13. This determined the value of $j_{\text{max}}$ of our own galaxy to be $0.263$ for $\epsilon = 0.3$, the value we assume henceforth. The first five caustic ring radii in our galaxy are then predicted to be: 41, 20, 13.3, 10, 8 kpc.

According to the self-similar caustic ring model, each galaxy has its own value of $j_{\text{max}}$. Over the set of 32 galaxies selected in refs., $j_{\text{max}}$ has some unknown distribution. However, the fact that the values of $j_{\text{max}}$ of NGC3198 and of the Milky Way happen to be close to one another, within 7%, suggests that the $j_{\text{max}}$ distribution may be peaked near a value of 0.27. Our strategy is to rescale each rotation curve according to

$$r \rightarrow \bar{r} = r \left( \frac{220 \text{ km/s}}{v_{\text{rot}}} \right)$$

and to add them together in a way made precise below. Since Eq. (1) predicts the $n$th caustic radius $a_n$ to be distributed like $j_{\text{max}}$ for all $n$, and it fixes the ratios $a_n/a_1 \simeq 1/n$, the sum of rotation curves should show the $j_{\text{max}}$ distribution, once for $n = 1$, then at about half the $n = 1$ radii for $n = 2$, then at about $1/3$ the $n = 1$ radii for $n = 3$, and so on. If the $j_{\text{max}}$ distribution is broad, the sum of rotation curves is unlikely to show any feature. However, if it is peaked, then the sum should show a peak for $n = 1$ at some radius, then again at $1/2$ that radius for $n = 2$, at $1/3$ the radius for $n = 3$, and so on. If the $j_{\text{max}}$ distribution is peaked at 0.263 (the value for the Milky Way), the peaks in the sum of rotation curves should appear at 41 kpc, 20 kpc, 13.3 kpc, and so forth.

We search for overdensities in the halo dark matter distribution as manifested by upward fluctuations in velocity relative to the underlying flat rotation curve of the galaxy. Unfortunately, rotation curves are never exactly flat. They often rise or fall systematically with increasing radius, and all show a sharp drop in rotation velocity near the galactic center. Furthermore, individual sources (i.e. gas) in a galaxy have peculiar velocities independent of any caustic structure in the galaxy. We require a definite procedure for subtracting out the background rotation of the galaxy, and for quantifying the “noise” due to the peculiar velocities of sources. The procedure we adopt is as follows. Each rotation curve is expressed as a set of radii $r_i$ in kpc and rotation velocities $v_i$ in km/sec. Each $(r_i, v_i)$ is given by a data point in the
rotation curves published in refs. 2,3. Now, in the case of a flat rotation curve, \( v_{\text{rot}} \) is just the average velocity of the outer rotation curve, which we denote \( \bar{v} \). In the case of a rising or falling rotation curve, the “overall” rotation velocity \( v_{\text{rot}} \) is less precisely defined. We adopt the prescription \( v_{\text{rot}} \equiv \bar{v} \) in all cases. Thus the rescaled radii \( \tilde{r}_i \) are

\[
\tilde{r}_i \equiv r_i \left( \frac{220 \text{ km/s}}{\bar{v}} \right),
\]

where \( \bar{v} \) is calculated according to the following procedure. We are interested in the outer, approximately flat portion of the rotation curve, so we remove the inner points by specifying a cut: all points with rescaled radii \( \tilde{r}_i < 10 \text{ kpc} \) are thrown out. The remaining points in the rotation curve are then fitted to a line, and \( \bar{v} \) is the corresponding average. A subtlety arises because the average velocity \( \bar{v} \) is calculated after the cut is applied, but the cutoff is defined in rescaled coordinates and therefore depends on \( \bar{v} \). The cutoff procedure is in fact performed iteratively: we start with a guess for \( \bar{v} \), calculate the cutoff radius, recalculate \( \bar{v} \) based on the cutoff radius, and so on until both quantities converge (usually after one or two iterations).

Fitting the outer rotation curve to a line may seem an arbitrary choice. In practice, it works quite well: deviations from a linear fit are typically less than 10 km/s in galaxies with a typical rotation velocity of 200 km/s. However, to verify the robustness of our conclusions, we also performed the analysis with quadratic polynomial fits to the rotation curves, with no substantial change in the results. (Of course, a fit to a high enough order polynomial will remove any features present in the rotation curve!)

Once the linear fit is calculated, this background rotation is subtracted off the rotation curve, leaving a set of peculiar velocities \( \delta v_i \). It is then straightforward to calculate an rms “noise” \( \sqrt{\langle \delta v^2 \rangle} \) for each galaxy,

\[
\sqrt{\langle \delta v^2 \rangle} \equiv \sqrt{\frac{1}{N - N_{\text{fit}}} \sum_i (\delta v_i)^2},
\]

where \( N \) is the number of points and \( N_{\text{fit}} \) is the number of degrees of freedom in the fit (\( N_{\text{fit}} = 2 \) for a fit to a line). We adopt \( \sqrt{\langle \delta v^2 \rangle} \) as the size of the error bar on the \( \delta v_i \) for a given galaxy. This error is a measure of the intrinsic peculiar velocities of the sources, which we assume to be random. It is considerably more conservative than the quoted observational errors on the points in the rotation curve. Finally, the peculiar velocities are expressed in dimensionless units \( \delta \bar{v}_i \equiv v_i / \sqrt{\langle \delta v^2 \rangle} \), and the sample of galaxies is averaged into radial bins:

\[
b_i \equiv \frac{1}{N_i} \sum_{j=1}^{N_i} \delta \bar{v}_j,
\]

where \( N_i \) is the number of data points in the \( i \)-th bin. The assigned error on each \( b_i \) is then simply \( 1 / \sqrt{N_i} \). Figure 1 shows the complete set of 32 galaxies averaged into 2 kpc bins.

There are two features evident at roughly 20 and 40 kpc. A fit to two Gaussians (with amplitude, width and mean left as free parameters) plus a constant indicates features at 19.4 \( \pm \) 0.7 kpc and 41.3 \( \pm \) 0.8 kpc, with overall significance of 2.4\( \sigma \) and 2.6\( \sigma \), respectively. Figure 1 also shows the fitted curve. When the same fit is applied to the same data in 1 kpc bins, the significance of the two peaks is 2.6\( \sigma \) and 3.0\( \sigma \) respectively. The locations of the features agrees with the predictions of the self-similar caustic ring model with the \( j_{\text{max}} \) distribution peaked at 0.27. The use of Gaussians to fit the peaks in the combined rotation curve was an arbitrary choice in the absence of information on the \( j_{\text{max}} \) distribution.

*These curves are heavily processed and do not represent the velocities of individual sources. See, for example, Ref. 10 and references therein for a discussion of the observational details relating to one galaxy in our sample, IC2574. In the case of the galaxy M33, whose published rotation curve ref 10 is so densely sampled that the data points overlap with one another, we choose to sample the published curve at intervals of 0.25 kpc, a sufficient resolution for our purpose here.

FIG. 1. Binned data for 32 galaxy sample, with peaks fit to Gaussians

We have shown that there is evidence for universal
structure in the dark halos of spiral galaxies, in that the rotation velocity has a statistically significant tendency to fluctuate upward at rescaled radii $\tilde{r} \sim 20$ and 40 kpc. While the significance of these peaks in the data set analyzed is not overwhelming, roughly $2.5\sigma$ for each peak when considered with appropriately conservative error bars, our analysis provides a tantalizing indication that the structure is real and that it is within observational reach of a more detailed survey of galactic rotation curves.

It is particularly striking that the positions of the peaks coincide with the radii of the $n = 1$ and $n = 2$ caustic rings in the self-similar infall model for $j_{\text{max}} \sim 0.27$. This strongly suggests that the $j_{\text{max}}$ distribution is indeed peaked near the value of 0.27, as the earlier estimates of the $j_{\text{max}}$ values of NGC3198 (0.28) and the Milky Way (0.265) had suggested.

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