Parabose algebra as generalized conformal supersymmetry

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Abstract. The form of realistic space-time supersymmetry is fixed, by Haag-Lopuszanski-Sohnius theorem, either to the familiar form of Poincare supersymmetry or, in massless case, to that of conformal supersymmetry. We question necessity for such strict restriction in the context of theories with broken symmetries. In particular, we consider parabose $N = 4$ algebra as an extension of conformal supersymmetry in four dimensions (coinciding with the, so called, generalized conformal supersymmetry). We show that sacrificing of manifest Lorentz covariance leads to interpretation of the generalized conformal supersymmetry as symmetry that contains, on equal footing, two ”rotation” groups. It is possible to reduce this large symmetry down to observable one by simply breaking one of these two $SU(2)$ isomorphic groups down to its $U(1)$ subgroup.

PACS numbers: 11.30.Pb, 11.30.Ly

Keywords: generalized supersymmetry, parabose algebra, tensorial central charges, HLS theorem
1. Introduction

Prospect of finding larger symmetries that would embed observable Poincare symmetry and possibly some of the internal symmetries (in a non trivial way) has attracted physicists for a long time. Early attempts ended in formulation of the famous Coleman and Mandula theorem [1], but this no-go theorem was soon evaded by the idea of supersymmetry. The Coleman and Mandula theorem was then replaced by the Haag-Lopuszanski-Sohnius (HLS) theorem [2] that put the now standard super-Poincaré and super-conformal symmetries at the place of maximal supersymmetries of realistic models (up to multiplication by an internal symmetry group). However, the attempts to go around these no-go theorems never truly ceased, many of these trying to weaken the mathematical requirements of the theorems [3, 4].

We will here consider an extension of conformal supersymmetry (and thus also of the Poincaré supersymmetry) which does not meet one of the physical premises of the no-go theorem [1] - namely the "particle finiteness" premise: "for any finite M, there are only finite number of particle types with mass less than M”. Regarding this requirement, S. Coleman [5] comments: "We would probably be willing to accept a theory with an infinite number of particles, as long as they were spread out in mass in such a way that experiments conducted at limited energy could only detect a finite number of them”. Our point is that a proper symmetry breaking can, in principle, induce such mass splitting. Besides, this does not have to imply increasing of the complexity of a theory, since symmetry breaking is already an inescapable component of all supersymmetric models (and of the most of the contemporary physical models in general).

In this paper we will demonstrate that (non-extended) conformal superalgebra can be seen as a part of a very simple algebra, namely of $N=4$ parabose algebra. Relation of this algebra with the standard conformal superalgebra is rather interesting: it is the algebra that is obtained from non-extended conformal superalgebra when we remove the algebraic ”constraints” $\{Q_\eta, Q_\xi\} = 0$ (allowing these and adjoint anticommutators to be new symmetry generators) and appropriately close the algebra. All commutator relations, used to define the conformal superalgebra, remain the same within this bigger algebra, as well as all nonzero anticommutator relations (apart from value of one coefficient). It is intriguing that enlarging the conformal superalgebra in this way simplifies the algebra instead of complicating it, as the structural relations of the larger symmetry are determined by only two defining relations of parabose algebra. More importantly, we will show that the symmetry breaking necessary to reduce this symmetry to observable one is also very simple, both conceptually and by its mathematical form.

However, it is obvious that number of particles in a supermultiplet becomes infinite, since consecutive action of the same supersymmetry generator in this case no longer annihilates a state. Nevertheless, we argue that such an obvious disagreement with experimental data is a problem of the qualitatively same type as occurs in the standard Poincaré supersymmetry. Namely, whereas in models with Poincaré supersymmetry we...
need a symmetry breaking to induce mass differences between finite number of super-partners, here the symmetry breaking should provide ascending masses for infinite series of super-partners. We will demonstrate existence of simple form of symmetry breaking that reduces the symmetry down to the Poincaré group, altogether with providing the mass splitting.

This type of generalization of Poincaré and conformal supersymmetry, obtained by allowing $\{Q_\eta, Q_\xi\}$ anticommutators to be noncentral, has been already investigated, mostly in the context of branes and M theory \[6, 7, 8, 9, 10, 11, 12, 13\]. It has been pointed out that conclusions of HLS theorem are not applicable to considerations of extended objects \[7, 8, 13\]. In this context the anticommutators that are forbidden by HLS scheme (usually known as "tensorial central charges") can be interpreted as charges carried by domain walls \[6, 8, 10, 11, 12, 13\]. Most of the interest in these papers is focused on higher dimensional cases, but generalized algebras in four dimensions are also investigated \[11, 14, 15, 16, 17\], in some cases even in context of particles instead of branes \[14, 16, 17, 18\].

Our intention is to analyze the four dimensional case from purely algebraic point of view, considering all the algebra operators as generators of space-time symmetries. The idea is to explain the excess of symmetry generators by pointing out a type of symmetry breaking that could reconcile this large symmetry with observation. By using approach based on parabose algebra (which is determined essentially by only two defining relations) we would like to emphasize mathematical simplicity of the generalized conformal supersymmetry. We will show that sacrificing manifest Lorentz covariance leads to interesting interpretation of the generalized supersymmetry in four dimensions as symmetry of spacetime possessing two "rotational" groups existing on equal footing. Then we demonstrate that a breaking of one of these two $SU(2)$ isomorphic groups down to its $U(1)$ subgroup can lead to reduction of overall symmetry down to observable Poincaré symmetry. By underlining the simplicity of generalized supersymmetry and of the form of the required symmetry breaking, we would like to point out that potential significance of this symmetry is underestimated, in comparison to the attention given to the standard Poincaré and conformal supersymmetry hypothesis. As the generalized supersymmetry qualitatively differs from the standard supersymmetry (for example, by predicting infinite supermultiplets), existence of this alternative version of supersymmetry should be kept on mind when it comes to interpreting experimental data in future.

In order to establish connection of $N=4$ parabose algebra with conformal superalgebra in 4 space-time dimensions, we will need to introduce a special basis for expressing not only parabose operators themselves, but also for expressing their anticommutators. This basis, in which the connection becomes manifest, will be introduced in the next section. In section it the (bosonic) conformal algebra is recognized as a subalgebra of algebra of parabose anticommutators and the symmetry breaking is considered. Supersymmetry generators will be included in the picture in section \[14\]. In the last section some additional remarks will be given.
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Throughout the text, Latin indices $i, j, k, \ldots$ will take values 1, 2 and 3, Greek indices from the beginning of alphabet $\alpha, \beta, \ldots$ will take values from 1 to 4 and will in general denote Dirac-like spinor indices, $\eta$ and $\xi$ will be two-dimensional Weyl spinor indices, while Greek indices from the middle of alphabet $\mu, \nu, \ldots$ will denote Lorentz four-vector indices.

2. Special basis

Parabose algebra with $N$ degrees of freedom \cite{[19]} is determined by trilinear relations
\[
\{\hat{a}_\alpha, \hat{a}_\beta, \hat{a}_\gamma\} = 0, \quad \{\hat{a}_\alpha, \hat{a}_\beta^\dagger\} = -2\delta_\beta^\alpha\hat{a}_\alpha, \quad (1)
\]
connecting $N$ operators $\hat{a}_\alpha$ and their hermitian conjugates $\hat{a}_\alpha^\dagger$ (curly brackets denote anticommutator)\footnote{Relation $\{\hat{a}_\alpha, \hat{a}_\beta^\dagger\} = 2\delta_\beta^\gamma\hat{a}_\alpha + 2\delta_\alpha^\gamma\hat{a}_\beta^\dagger$ obtained from these two by generalized Jacobi identities, as well as relations obtained from these by hermitian conjugation, are also implied.} We will be interested in case $N = 4$. Being a generalization of an algebra of bosonic creation and annihilation operators, parabose algebra is, along with parafermi algebra, usually considered in context of parastatistics. This will not be the case in this paper.

In order to demonstrate connection of parabose algebra with conformal supersymmetry, we will rewrite relations (1) in a different, rather complicated basis of operators.

As a first step of this change of basis, we will switch from operators $\hat{a}_\alpha$ and $\hat{a}_\alpha^\dagger$ to their hermitian combinations, defined as:
\[
S^\alpha = (a_\alpha + a_\alpha^\dagger), \quad Q_\alpha = -i(\hat{a}_\alpha - \hat{a}_\alpha^\dagger). \quad (2)
\]

Relations (1) imply the following six relations:
\[
\{Q_\alpha, Q_\beta\}, Q_\gamma\} = 0, \quad \{S^\alpha, S^\beta\}, S^\gamma\} = 0, \quad (3)
\]
\[
\{Q_\alpha, Q_\beta\}, S^\gamma\} = -4i\delta^\gamma_\alpha Q_\beta - 4i\delta^\gamma_\beta Q_\alpha, \quad \{S^\alpha, S^\beta\}, Q_\gamma\} = 4i\delta^\beta_\alpha S^\alpha + 4i\delta^\alpha_\beta S^\beta, \quad (3)
\]
\[
\{S^\alpha, Q_\beta\}, S^\gamma\} = 4i\delta^\delta_\gamma S^\alpha, \quad \{Q_\alpha, S^\beta\}, Q_\gamma\} = 4i\delta^\beta_\gamma Q_\alpha.
\]

These relations express commutators between basic anticommutators $\{Q_\alpha, Q_\beta\}$, $\{Q_\alpha, S^\beta\}$, $\{S^\alpha, S^\beta\}$ and operators $S^\alpha$ and $Q_\alpha$ themselves. We can rewrite these relations using some other basis of anticommutators, i.e. using some linear combinations $A^{\alpha\beta}\{Q_\alpha, Q_\beta\}$, $B^{\alpha\beta}\{Q_\alpha, S^\beta\}$ and $C_{\alpha\beta}\{S^\alpha, S^\beta\}$, where matrices of coefficients $A$, $B$ and $C$ take values from some basis set of 4 by 4 real matrices. For that purpose we introduce the following matrix basis: we choose a set of six real matrices $\sigma_i$ and $\tau_2$, $i, j, k = 1, 2, 3$ satisfying
\[
[\sigma_i, \sigma_j] = 2\varepsilon_{ijk}\sigma_k, \quad [\tau_2, \tau_2] = 2\varepsilon_{ijk}\tau_k, \quad [\sigma_i, \tau_2] = 0, \quad (4)
\]
\footnote{Normalization is so chosen to simplify later comparison with the standard form of supersymmetry relations.}
as a basis of antisymmetric four by four real matrices, and we choose nine matrices $\alpha_{ij} \equiv \tau_2 \sigma_j$, plus the unit matrix denoted as $\alpha_0$, as a basis for symmetric matrices. (Notice that we distinct tau indices from sigma indices by underlining the former.) Matrices $\tau$ and $\sigma$ are four dimensional analogs of Pauli matrices and they are here defined to be anti-hermitian, satisfying $\sigma_i^2 = \tau_i^2 = -1$.

Using these matrices we define new basis for expressing anticommutators of $Q$ and $S$:

$$\begin{align*}
J_i &\equiv \frac{1}{8}(\sigma_i)_{\alpha}^{\beta} \{Q_\alpha, S^\beta\}, \\
Y_j &\equiv \frac{1}{8}(\tau_2)_{\alpha}^{\beta} \{Q_\alpha, S^\beta\}, \\
N_{ij} &\equiv \frac{1}{8}(\alpha_{ij})_{\alpha}^{\beta} \{Q_\alpha, S^\beta\}, \\
D &\equiv \frac{1}{8}(\alpha_0)_{\alpha}^{\beta} \{Q_\alpha, S^\beta\}, \\
P_{ij} &\equiv \frac{1}{8}(\alpha_{ij})_{\alpha \beta} \{Q_\alpha, Q_\beta\}, \\
K_{ij} &\equiv -\frac{1}{8}(\alpha_{ij})_{\alpha \beta} \{S^\alpha, S^\beta\}, \\
K_0 &\equiv \frac{1}{8}(\alpha_{0})_{\alpha \beta} \{S^\alpha, S^\beta\}. 
\end{align*}$$

By expressing relations (3) in terms of these linear combinations of anticommutators, we obtain:

$$\begin{align*}
[J_i, Q_a] &= -i(\sigma_i)_{\alpha}^{\beta} Q_\beta, \\
[Y_j, Q_a] &= -i(\tau_2)_{\alpha}^{\beta} Q_\beta, \\
[N_{ij}, Q_a] &= i(\alpha_{ij})_{\alpha}^{\beta} Q_\beta, \\
[K_0, Q_a] &= i(\alpha_0)_{\alpha \beta} S^\beta, \\
[P_{ij}, S^\alpha] &= -i(\alpha_{ij})_{\alpha}^{\beta} S^\beta, \\
[K_{ij}, S^\alpha] &= -i(\alpha_{ij})_{\alpha \beta} S^\beta, \\
[D, Q_a] &= i(\tau_2)Q_a, \\
[D, S^\alpha] &= -i(\frac{1}{2})S^\alpha.
\end{align*}$$

These relations, combined with definitions (2) and (5), are equivalent to two starting relations of parabose algebra (11). The extreme superficial complexity of these numerous relations stems only from the complicated choice of variables, i.e. of basis operators.

In the following sections we will clarify connection of relations (6) with conformal superalgebra.

3. Connection of $N = 4$ parabose algebra with conformal algebra and the symmetry breaking

It is not difficult to see that set of all anticommutators of starting parabose operators forms an algebra, to be denoted as $A_2$. It has 36 generators and is isomorphic to $sp(2n)$ algebra, where $n = 4$. Operators defined by (5) represent a particular basis of this algebra. In this basis structural relations of algebra $A_2$ have the following form:

$$\begin{align*}
[J_i, J_j] &= i\varepsilon_{ijk} J_k, \\
[Y_i, Y_j] &= i\varepsilon_{ijk} Y_k, \\
[J_i, N_{jk}] &= i\varepsilon_{ikl} N_{jl}, \\
[Y_i, N_{jk}] &= i\varepsilon_{ijl} N_{lk},
\end{align*}$$

One possible realization of such matrices is, for example: $\sigma_1 = -i\sigma_y \times \sigma_z$, $\sigma_2 = -iI_2 \times \sigma_y$, $\sigma_3 = -i\sigma_y \times \sigma_z$, $\tau_1 = i\sigma_x \times \sigma_y$, $\tau_2 = -i\sigma_z \times \sigma_y$, $\tau_3 = -i\sigma_y \times I_2$, where $\sigma_x$, $\sigma_y$ and $\sigma_z$ are standard two dimensional Pauli matrices and $I_2$ is a two dimensional unit matrix.
We will now show that algebra $\mathcal{A}_2$ has conformal algebra as a subalgebra, and that the reduction from the corresponding group to the conformal subgroup can be seen as a consequence of symmetry breaking of one $SU(2)$ group to its $U(1)$ subgroup.

To obtain conformal subalgebra let us discard all operators from $\mathcal{A}_2$ basis $[5]$ with underlined index having values $\mathbf{1}$ and $\mathbf{2}$. What we are left with is a subalgebra isomorphic with conformal algebra $c(1,3)$ plus one additional generator that commutes with the rest of the subalgebra. The remaining operators that generate $c(1,3)$ algebra are:

$$J_k, N_i \equiv N_{\underline{2}i}, D, P_i \equiv P_{\underline{2}i}, P_0, K_i \equiv K_{\underline{2}i}, K_0,$$  

(8)

playing roles of rotation generators, boost generators, dilatation generator, momenta and pure conformal generators, respectively. The additional remaining operator is $Y_{\underline{2}}$ which commutes with all of the conformal generators.

Alternatively, we could have obtained conformal subalgebra by keeping operators with underlined index equal to $\mathbf{1}$ or $\mathbf{2}$, instead of $\mathbf{2}$. As the matter in fact, if we pick any linear combination of operators $Y_{\underline{2}}$, or of operators $J_i$, the subalgebra of $\mathcal{A}_2$ that commutes with the chosen operator will be $c(1,3)$ isomorphic. On the other hand, operators $Y_{\underline{2}}$ and $J_i$ constitute two, mutually commuting $su(2)$ isomorphic subalgebras (a consequence of $so(4) = su(2) \oplus su(2)$ identity). The two corresponding $SU(2)$ isomorphic groups act, respectively, on underlined and on non-underlined indices of algebra operators. Furthermore, if we consider the way in which the recognized conformal subalgebra fits into the larger algebra $\mathcal{A}_2$, we see that spatial momenta, being equal to $\frac{1}{8}(\alpha_{\underline{2}i})^{\alpha\beta}\{Q_\alpha, Q_\beta\}$ naturally fit into a set of nine operators $\frac{1}{8}(\alpha_{\underline{2}i})^{\alpha\beta}\{Q_\alpha, Q_\beta\}$, spatial components of pure conformal generators fit into a set of nine $-\frac{1}{8}(\alpha_{\underline{2}i})^{\alpha\beta}\{S^\alpha, S^\beta\}$ and boosts into set of nine $\frac{1}{8}(\alpha_{\underline{2}i})^{\alpha\beta}\{Q_\alpha, S^\beta\}$. Overall, the situation slightly looks like as if we had two independent rotation groups, generated by $Y_{\underline{2}}$ and $J_i$, while "momenta", "boosts" and "pure conformal operators" were here determined by two independent three-vector directions, each related to its own "rotation" group. And the symmetry reduction from $\mathcal{A}_2$ group to its conformally isomorphic subgroup can be therefore understood as a consequence of symmetry breaking of one of these two $SU(2)$ subgroups. Without loss of generality, we have assumed breaking of the group generated by $Y_{\underline{2}}$, with
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As a more concrete example of such symmetry breaking, we can assume existence of effective potential being an increasing function of absolute value of $Y_3$ [e.g. proportional to the $(Y_3)^2$]. If the potential is sufficiently strong, all low energy physics would be constrained to subspace of $Y_3$ eigenvalue equal to zero, and the remaining symmetry would be conformal symmetry. Moreover, since such a potential would have to break dilatational symmetry, overall symmetry would be reduced to the observable Poincaré group. (This is just the simplest of all possibilities. For example, the minimum of potential does not have to be at $Y_3 = 0$, or potential could be function of some additional observables combined with $Y_3$. As long as the potential changes with the change of $Y_3$, the extra generators $P_{1i}, P_{2i}, K_{1i}, K_{2i}, N_{1i}, N_{2i}, Y_1, Y_2$ will be broken. After all, a preferred direction with respect of the $Y$ rotation group could be introduced in some completely different way.)

Note that such symmetry breaking also automatically fixes metric of space-time (i.e. of the remained symmetry) to be Minkowskian [the remaining $J_i$ and $N_{3i}$ hermitian operators generate exactly an $so(1,3)$ algebra]. It is also interesting that the energy operator $P_0$ singles out among other momentum operators (i.e. among the rest of operators quadratic in $Q$) even before the symmetry reduction. Indeed, this operator, being the sum of squares of $Q_\alpha$, stands out as a positive operator, and there is no algebra automorphism that takes any other "momentum component" $P_{2j}$ into the $P_0$ or vice-versa. This gives us some right to interpret the full group generated by $\mathcal{A}_2$ as a symmetry that differs from the observable space-time symmetry in the first place by existence of two "spatial-like" rotations, whereas it possesses something that looks like unique role of one axis (to be interpreted as the time axis). And the symmetry breaking only gets us rid of one of the "rotation-like" groups.

4. Supersymmetry generators

Next, we turn attention to the role of operators $Q$ and $S$. First we note that under the action of generators from conformal subalgebra [8], they transform exactly as supersymmetry generators in the standard conformal superalgebra. To see this more clearly, we can introduce the following (Majorana) representation of Dirac matrices:

$$\gamma_0 = i\tau_2, \quad \gamma_i = \gamma_0 \alpha_i \sigma_i = i\tau_1 \sigma_i, \quad \gamma_5 = -i\gamma_0 \gamma_1 \gamma_2 \gamma_3 = i\tau_3.$$

(9)

Relations ([8]) [more precisely, that part of these relations with conformal subalgebra operators [8]], expressed by using these matrices, gain the form familiar from the standard conformal superalgebra. For example, commutators of Lorentz subalgebra generators with parabose operators $Q_\alpha$ can be now written in the standard form $[M_{\mu\nu}, Q_\alpha] = -i(\frac{1}{4}[\gamma_\mu, \gamma_\nu])_\alpha^\beta Q_\beta$ (where $M_{ij} = \varepsilon_{ijk}J_k$, $M_{i0} = N_{3i}$), and so on. In particular, we conclude that hermitian operators $Q$ and $S$ are Majorana spinors, written in Majorana basis of Dirac matrices ([9]).
Relations (5) defining our "basis", used for expressing anticommutators of parabose operators, can be inverted to yield the following identitie s:

$$\{Q_\alpha, Q_\beta\} = (\alpha_0)_{\alpha\beta} P_0 + (\alpha_{ij})_{\alpha\beta} P_{ij},$$

$$\{S^\alpha, S^\beta\} = (\alpha_0)^{\alpha\beta} K_0 - (\alpha_{ij})^{\alpha\beta} K_{ij},$$

$$\{S^\alpha, Q_\beta\} = (\alpha_0)^{\alpha\beta} D + (\alpha_{ij})^{\alpha\beta} N_{ij} + (\sigma_i)^{\alpha\beta} J_i + (\tau_i)^{\alpha\beta} Y_i.$$

(10)

These relations can be compared, using representation of Dirac matrices (9), to the anticommutator relations of the standard conformal superalgebra. An obvious difference is appearance of additional bosonic generators $P_1, P_2, K_1, K_2, N_1, N_2, Y_1, Y_2$ in (10), which do not exist in the standard conformal superalgebra (these operators, i.e. these linear combinations of $Q$ and $S$ anticommutators are, in the standard superalgebra, defined to be zero). However, apart from this, it turns out that the only difference is in the coefficient multiplying operator $Y_3$. Namely, by comparing the commutation relations of the two algebras, we recognize that operator $2Y_3$ plays the role of chiral $R$-charge. (It is interesting that, in this picture, chiral $R$-charge becomes part of an $su(2)$ subalgebra. This subalgebra appears in unbroken symmetry on the same footing as the rotational subalgebra.) The corresponding $R$ coefficient in the case of conformal superalgebra has a different value that equals 3, fixed there only by the generalized Jacoby identities.

To summarize this comparison, the transition from $N = 4$ parabose algebra to non-extended conformal superalgebra is achieved by setting $P_1 = P_2 = K_1 = K_2 = N_1 = N_2 = Y_1 = Y_2 = 0$ and by replacing the value of coefficient multiplying $Y_3$ operator.

The connection in the opposite direction (from conformal superalgebra to this extension) can be established if we notice that anticommutator of two left-handed $Q$ operators $\{Q_\eta, Q_\xi\}$, or of two right-handed operators $\overline{Q}_\eta, \overline{Q}_\xi$, yields linear combination of operators $P_1$ and $P_2$ (and similarly, such anticommutators of $S$ operators yield combinations of $K_1$ and $K_2$). Graded algebra consisting of parabose operators and their anticommutators [isomorphic to $osp(1, 8)$] can be seen as a special non-extended conformal superalgebra where all anticommutators of supersymmetry generators are allowed to be nonzero operators (so called "generalized conformal algebra" [12, 16]).

As already announced in the introduction, by relaxing the "constraint" $\{Q_\eta, Q_\xi\} = 0$, supermultiplets become infinite. Nevertheless, the simple symmetry breaking assumption, discussed in the previous section, breaks not only extra bosonic generators, but also the supersymmetry generators $Q_\alpha$ and $S^\alpha$. Since action of operators $Q_\eta$ and $Q^\eta$ change value of $Y_3$ for $\frac{1}{2}$, each following member of a supermultiplet would gain higher and higher mass, whereas the low-energy space-time symmetry would be given by the Poincaré group.

¶ For general and more formal treatment of connection between parabose algebras with Lie (super)algebras see, for example, [20] [21].
5. Conclusion

In this paper we analyzed generalized conformal supersymmetry in $D = 4$ from algebraic point of view, constructing it using parabose operators. By considering the way the conformal subalgebra fits into the parabose algebra, we offered interpretation that the whole symmetry should correspond to a space-time with two "rotational groups" existing \textit{a priori} on equal footing, of which one should be broken in order to obtain observable symmetry. This aspect of generalized supersymmetry is invisible unless we sacrifice manifest Lorentz covariance. For example, the existence of two "rotation" groups generating algebra automorphisms is obscured if we write the first of relations (10) in a more standard Lorentz covariant way \cite{7, 8, 10, 14}:

$$\{Q_\alpha, Q_\beta\} = (C^{\gamma \mu})_{\alpha \beta} P_\mu + (C^{\gamma \mu \nu})_{\alpha \beta} Z_{\mu \nu},$$

with Lorentz antisymmetric tensor $Z_{\mu \nu}$ denoting the components of generalized momentum other than four-momentum. Notice that $P_\mu$ and $Z_{\mu \nu}$, which were in our case connected by $Y$ rotations, even have different number of Lorentz indices. The emergence of Minkowskian metric is, in this picture, also a consequence of the symmetry breaking (the metric need not be introduced by hand).

It is interesting that, although the analyzed symmetry is higher and mathematical structure thus richer, the algebra relations are actually simplified. Namely, commutators of bosonic with fermionic operators (6) are nothing more than simple relations of parabose algebra written in a complicated basis. Moreover, the fermionic anticommutators (10) are relations that describe this new basis, so these relations can be seen as a specific naming convention for linear combinations of $Q$ and $S$ anticommutators. The idea is that this complicated basis becomes physically relevant due to the symmetry breaking, analyzed in section 3. The relatively simple symmetry breaking is therefore responsible not only for reduction of the starting symmetry and for introduction of mass splitting, but also for superficial complexity that hides simplicity of the starting parabose algebra. Bosonic algebra $A_2$ relations (7) are direct consequence of (10) and (6).

From the perspective of this higher symmetry, those relations of standard conformal superalgebra that set some of the anticommutators to zero appear as a kind of artificial constraints – constraints that are, in this picture, consequences of a symmetry breaking. This fact, that some linear combinations of anticommutators are zero (in standard superalgebra) makes it impossible to see anticommutators of fermionic generators simply as a naming convention, as it was possible for (10).

Transition from the non-extended standard conformal superalgebra to the symmetry discussed here can be done by allowing all anticommutators of supersymmetry generators to be nonzero operators. By doing so we end up with an algebra determined by only two parabose relations (1).

We remind that at the present point supersymmetry is still only a theoretical construct still awaiting for experimental confirmation, and that, in particular, we possess no experimental data that would suggest that supersymmetry, if exists, should be of
the form of the standard Poincaré (or conformal) type. Thus, by putting forward the simplicity of the generalized supersymmetry algebra in parabose formulation, as well as by demonstrating simplicity of the required form of symmetry breaking, we would like to point out that generalized supersymmetry should be seriously considered as possible candidate for real space-time supersymmetry, together with the conventional supersymmetry obeying the HLS conclusions.

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