Radiative Electroweak Symmetry Breaking from a Quasi-Localized Top Quark

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Abstract

We consider 5D supersymmetric $SU(3) \times SU(2) \times U(1)$ theories compactified at the TeV scale on $S^1/Z_2$ with supersymmetry broken by boundary conditions. Localizing the top quark at a boundary of a fifth dimension by a bulk mass term $M_t$, reduces the strength of radiative electroweak symmetry breaking. For $M_tR \approx 1–2$, the natural value for the top and bottom squark masses are raised to 500–1200 GeV, and all other superpartners may have masses of the compactification scale, which has a natural range of $1/R \approx 1.5–3.5$ TeV. The superpartner masses depend only on $1/R$, and are precisely correlated amongst themselves and with the mass of the Higgs boson, which is lighter than 130 GeV.
1 Introduction

While the past decade has been characterized by the success of the precision tests of the Standard Model (SM), the present decade, with the progression of the Tevatron runs and especially with the coming into operation of the LHC, should allow a thorough exploration of the physics of ElectroWeak Symmetry Breaking (EWSB), which remains a central unsettled problem in the theory of the fundamental interactions.

To say that EWSB is an unsettled problem does not quite do justice to the Standard Model. Although not directly proven experimentally, the SM most likely captures the essence of EWSB through the Higgs mechanism. The limitation of the SM in the EWSB sector is rather the lack of quantitative predictive power: the two parameters of the Higgs potential, $\mu$ and $\lambda$, are in one to one correspondence with the two physical observables, the Higgs mass and the Higgs vacuum expectation value (VEV), or the Fermi constant.

To improve on this situation is non-trivial, to say the least, due to the lack of crucial data so far. Among the different attempts, the one that goes farther involves supersymmetry, whose breaking triggers EWSB. Other than being supported by the success of gauge coupling unification, the standard supersymmetric picture of EWSB improves also on the predictive power of the SM. The quartic coupling $\lambda$ gets related to the gauge couplings, up to significant but controllable radiative corrections, thus implying a light (too light?) Higgs boson. Similarly the Fermi scale is given in terms of the soft supersymmetry-breaking parameters, $m_i$, in turn related to the superpartner masses, with only mild (logarithmic) dependence on the cutoff scale $\Lambda$,

$$G_F = G_F(m_i, \log \Lambda).$$  \hspace{1cm} (1)

The explicit form of this equation, although somewhat model dependent, is also the basis for expecting superpartners near the Fermi scale. Yet its structure and the number of parameters it generally involves has not made possible, so far, any precise statement on the superpartner masses: all quantitative predictions for them rest upon forbidding some predetermined level of fine-tuning among the different parameters. Although this is a plausible attitude to take, it does not avoid the most unpleasant feature of the current supersymmetric extensions of the SM. How should one judge, for example, in a truly objective way the significance of the failure to find any superpartner (or the Higgs itself) at LEP? Similarly, or conversely, how should one determine in a precise manner the discovery potential of supersymmetry at the Tevatron or even at the LHC?

This difficulty could be due to the inadequacy of the present understanding of supersymmetry breaking. For this reason and with the aim at drastically reducing the number of relevant parameters, a concrete model of supersymmetry breaking has been proposed in Ref. [1], whose basic content is to establish a precise connection between the Fermi scale and the inverse radius of a compactified fifth dimension $1/R$. 

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That this is possible at all looks at first rather surprising in view of the non-renormalizability of field theories in 5D. In this respect there are two crucial properties of this model: i) the existence of a residual local supersymmetry (and of a global $U(1)_R$ symmetry) that highly restrict the form of the Lagrangian with its possible counterterms; ii) the description of EWSB and of all fermion masses in terms of a single Higgs doublet as in the SM and unlike the case of the Minimal Supersymmetric Standard Model (MSSM). The cost of non-renormalizability of the 5D Lagrangian is only the requirement of a rather low cutoff scale $\Lambda$. This is not, however, a serious limitation in so far as one can show that the Higgs physics and the physics at $1/R$ have only weak ultraviolet sensitivity in an effective field theory sense.

One could object at this point that a low cutoff obscures the motivation coming from the gauge coupling unification and, therefore, any motivation at all for the entire program. Although we agree that the apparent lack of gauge unification in the model of Ref. [1] is a step backward, we think that achieving a description of EWSB which involves a naturally light Higgs perturbatively interacting up to a multi-TeV cutoff scale $\Lambda$ is both non-trivial and quite clearly motivated by the current status of the ElectroWeak Precision Tests (EWPT).\footnote{The low Confidence Level of the current SM fit of the EWPT has raised some questions on this last statement [2, 3]. We find the causes of the poor fit not particularly troublesome and, henceforth, the evidence for the lightness of the Higgs boson significant, although indirect and as such subject to obvious limitations. We thank Alessandro Strumia for help in clarifying this issue.}

The theories presented in this paper have the same motivation as the model of Ref. [1] and share with it the key properties that make possible a quantitative connection between $1/R$ and $G_F$. We study top quark hypermultiplets with bulk mass terms, so that the top quark wavefunctions are peaked close to a boundary of the fifth dimension, rather than being smoothly distributed throughout the bulk. This quasi-localization also yields controlled EWSB, but with the important result that the compactification scale is significantly larger, $1/R \simeq 1.5-3.5$ TeV, than in the Constrained Standard Model (CSM) of Ref. [1], giving a more natural agreement with EWPT. The cutoff scale is also increased and can be as large as 15 TeV in the entire range of $1/R$.

## 2 $U(1)_R$ Invariant Theories

A predictive theory of EWSB should have only a few parameters, and therefore as much symmetry as possible. We study theories of a single extra dimension, with gauge group $SU(3) \times SU(2) \times U(1)$, compactified to a line segment $(0, \pi R/2)$. We consider the possibility that the bulk Lagrangian has the following symmetries:

- 5D supersymmetry. The resulting bulk gauge interactions possess a $SU(2)_R$ symmetry. The physical line segment in the fifth dimension may be viewed as arising from an orbifold compactification corresponding to two orbifold symmetries: a translation by $\pi R$ involving a
rotation angle $\alpha$ inside $SU(2)_R$, and a reflection parity about a particular point, which we label $y = 0$. For the case of interest to us $\alpha \neq 0$, so that the compactification leaves no unbroken supersymmetry in the low energy equivalent 4D theory but a residual local supersymmetry in 5D.

- An additional unbroken symmetry, $U(1)_R$, with quantum numbers shown in Table 1. It arises from the special case of $\alpha = 1/2$, which corresponds to the $S^1/(Z_2 \times Z'_2)$ orbifold compactification introduced in Ref. [4]. This symmetry ensures that Majorana gaugino masses, $A$-terms or the $\mu$ of the Higgs sector are not generated from either compactification or brane interactions.

- A local parity $P_5$, corresponding to a reflection about any point of the bulk (in the limit $R \to \infty$). This symmetry forbids both a Chern-Simons term and bulk masses for the hypermultiplets. However, with only one Higgs hypermultiplet containing a massless scalar $H_u$, as in Ref. [1], consistency of the theory requires the breaking of this symmetry [5, 6], while introducing a quadratically divergent brane-localized Fayet-Iliopoulos (FI) term [7, 8]. Hence, to consider this symmetry we must introduce a second Higgs hypermultiplet with boundary conditions that give rise to a second massless scalar $H_d$ with opposite hypercharge to $H_u$.

Table 1: Continuous $R$ charges for gauge, Higgs and matter multiplets. Here, $A_\mu$, $(\lambda, \lambda')$ and $\Sigma = (\sigma + i A_5)$ are the gauge field, two gauginos and the adjoint scalar inside a 5D gauge multiplet; $(h, h^c)$ and $(\tilde{h}, \tilde{h}^c)$ are two complex scalars and two Weyl fermions inside a 5D Higgs hypermultiplet; and $(m, m^c)$ and $(\tilde{m}, \tilde{m}^c)$ are two complex scalars and two Weyl fermions inside a 5D matter hypermultiplet, where $m$ represents $q, u, d, l$ and $e$.

Is it possible to construct a completely realistic theory with these three symmetries? Other than gauge interactions, the symmetries allow brane-localized Yukawa interactions. For the case of bulk matter these are:

$$\mathcal{L}_{\text{Yukawa}} = \delta(y)[\lambda_u Q U H_u + \lambda_d Q D H_d]_{\theta^2} + \delta(y - \pi R/2)[\lambda'_u Q' U' H'_{d}]_{\theta'^2},$$

(2)

where $Q, U, D$ and $H_{u,d}$ are chiral multiplets containing quark and Higgs-boson zero modes of the $N = 1$ supersymmetry acting at $y = 0$, while $Q', U', D'$ and $H'_{u,d}$, which also contain quark
and Higgs-boson zero modes, are the chiral multiplets of the $N = 1$ supersymmetry acting at $y = \pi R/2$. Even though one-loop radiative corrections lead to contributions to the soft mass terms $m_u^2 H_u H_u + m_d^2 H_d^* H_d + m_3^2 (H_u H_d + h.c.)$ in the Higgs potential, successful EWSB does not occur. The Yukawa contributions dominate $m_{u,d}^2$ and are large and negative, so that $m_u^2 + m_d^2 < 0$, giving an unbounded potential along the $D$-flat direction. It is interesting that the addition of an extra Higgs doublet hypermultiplet to the CSM destroys the theory. If the quark fields reside on a boundary, only one pair of the Yukawa couplings survive [9]. In this case the squark masses arise only at one loop, and we find that the corresponding two-loop top Yukawa contribution to $m_u^2$ is not sufficiently negative to overcome the positive contribution from the one-loop gauge radiative correction: $m_{u,d}^2$ are both positive, and $m_3^2 = 0$, so that there is no EWSB.

We conclude that we must give up either the bulk parity $P_5$ or the continuous $U(1)_R$ symmetry to construct realistic theories. Theories with $P_5$ but no $U(1)_R$ were constructed in Ref. [10]. They are theories with two Higgs doublet VEVs resulting from a scalar potential having terms induced by $U(1)_R$ breaking boundary operators. Here we pursue the alternative possibility of a $U(1)_R$ symmetric theory with $P_5$ broken. Once $P_5$ is given up, we can introduce bulk mass terms for the hypermultiplets [6]: $\mathcal{L} = [M_\Phi \Phi^\dagger]_{\theta^2}$. Since there is no $P_5$ symmetry, we study both one Higgs and two Higgs hypermultiplet versions of the theory. Note that the one Higgs version of the theory is precisely the theory introduced in Ref. [1] with the hypermultiplets having non-vanishing bulk masses. While the one Higgs theories have a quadratically divergent FI term, the two Higgs theories are less sensitive to unknown physics at the cutoff, as we discuss shortly.

From the viewpoint of EWSB the bulk mass terms of most importance are those of the third generation and the Higgs multiplets. For most of this paper we assume that the bulk mass for the $H_u$ hypermultiplet vanishes $M_{H_u} = 0$, and we concentrate on the bulk masses for the third generation quarks: $M_{Q,U,D}$. We consider values of $M_{Q,U}$ comparable to or larger than $1/R$, so that the corresponding zero-mode wavefunctions are peaked at the boundaries of the fifth dimension. In particular we choose $M_Q$ positive so that the left-handed top and bottom quarks are located near $y = 0$. To avoid large wavefunction suppression factors in the top quark mass we also choose $M_U$ to be positive. There is still freedom in $M_D$, which we allow to be positive, negative or even zero. In the theory with a single Higgs hypermultiplet, the $b$ quark must get its mass from a Yukawa coupling at $y = \pi R/2$, so that no matter which choice is made for $M_D$, the $m_b/m_t$ mass ratio receives a suppression of at least $\exp(-\pi M_Q R/2)$ due to the small value of the $q$ wavefunction at $y = \pi R/2$ [12]. It is significant that localization necessarily destroys the symmetry between up and down sectors, leading to a small value for $m_b/m_t$ without the need for a hierarchy of 5D Yukawa couplings.

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3Note that our sign convention is the opposite of the one used in [11].
In the theory with two Higgs hypermultiplets the situation is complicated by the possibility of four Yukawa couplings, as shown in Eq. (2), and two Higgs VEVs. Nevertheless, we require that localization naturally yield a hierarchy for $m_b/m_t$ and find that this can emerge in two ways:

- Introduce a global symmetry $U(1)_{H_d}$ which rotates the phase of only $H_d$, and therefore sets the Yukawa couplings $\lambda'_u = \lambda_d = 0$ as well as $m^2_3 = 0$. Only $H_u$ acquires a VEV, so that the physics of both EWSB and quark mass generation is identical to the case of the one Higgs theory.

- Break the symmetry between up and down sectors by requiring that $D$ not be localized at $y = 0$. In this case, even though all Yukawa couplings may be comparable, the radiatively generated value for $m^2_3$ is very small so that the VEV of $H_d$ is negligible, and the $t$ and $b$ quark masses arise dominantly from $\lambda_u$ and $\lambda'_d$. Furthermore the contributions of the other two Yukawa couplings to $m^2_{u,d}$ are also negligible.

In all these theories, the Higgs potential, and therefore EWSB, depends only on the unknown parameters $1/R, M_Q, M_U$ and $M_{H_u}$. The absence of any dependence on other bulk mass parameters, in particular $M_D$, is discussed in Appendix A. The top Yukawa coupling $\lambda_u$ enters, but is determined by $m_t$. In the next section we study the region of parameter space with $M_Q = M_U$ and $M_{H_u} = 0$ and find a restricted and therefore predictive region in which EWSB is successful. In section 4 we study the dependence on $M_U/M_Q$ and small values of $M_{H_u}$, and find that successful EWSB persists. The calculation of EWSB is identical for the one Higgs theory and for both types of two Higgs theories. The origin of EWSB is always a radiative contribution to $m^2_u$ from a top quark hypermultiplet which has a wavefunction peaked around $y = 0$.

Finally, we discuss the FI term. Since $P_5$ is broken, a brane-localized FI term for the hypercharge gauge interaction is allowed in our theories at tree level and can be generated by radiative corrections. However, by shifting the VEV of the scalar in the hypercharge gauge multiplet, we can always transform the FI term to bulk masses for the hypermultiplets [6]. Therefore, the FI term does not represent an additional parameter, as long as we consider all the bulk hypermultiplet masses in the analysis. The hypermultiplet masses in this paper are meant to be the ones after this transformation: they include both the tree and radiative contributions of the FI term. Then, even when the FI term is quadratically divergent, as in the case of the one Higgs theory, the resulting hypermultiplet masses are small relative to $1/R$. In the case $M_{Q,U} \ll 1/R$, the induced value for $M_{H_u}$ produces only a small perturbation to the theory [11]. However, in the case of a quasi-localized top quark, $M_{Q,U} \gtrsim 1/R$, the induced value for $M_{H_u}$ gives a mass squared to the lightest mode of $H_u$ of comparable size to the other radiative contributions. The quadratic divergence of the FI term can be canceled by a second Higgs hypermultiplet — indeed this may be a motivation for considering two Higgs theories. In the presence of hypermultiplet masses, however, a further condition arises from the ultraviolet insensitivity of the FI term, since there is a residual...
linear divergence proportional to $\text{Tr}[YM]$, where $Y$ and $M$ are hypercharge and bulk-mass matrices for the hypermultiplets. This may motivate interesting relations among the hypermultiplet masses, e.g., the case in which they are all equal, or $M_Q = M_U = M_D$ or $M_Q = M_U = -M_{H_u}$ and all other masses equal to zero. These relations make the radiative FI term identically vanishing. In fact, $M_{H_u} = 0$ then becomes a perfectly stable condition.

3 Electroweak Symmetry Breaking

In this section we study in detail the EWSB when $M_Q = M_U \equiv M_t$ and $M_{H_u} = 0$. We are mainly interested in the region where $M_t R \gtrsim 1$. The tree-level potential for the Higgs is

$$V_{\text{tree}}(\phi) = \frac{g^2 + g'^2}{8} |\phi|^4 = \frac{m_Z^2}{4 v^2} |\phi|^4,$$

where $\phi$ is the neutral component of $H_u$. EWSB is triggered by radiative corrections, which requires studying the corrected effective potential $V(\phi)$. One important point of our analysis is that a (quasi-)localized top quark naturally gives a large $1/R$ compared to the weak scale, as we show in subsection 3.1. The full detail of the effective potential is presented in subsection 3.2.

3.1 Localized matter and large $1/R$

To demonstrate a couple of important points in our analysis, it is best to discuss separately the corrections to the quadratic term,

$$V^{(2)}(\phi) = V^{(2)}_{\text{loop}}(\phi) + V^{(2)}_{\text{2loop}}(\phi),$$

from the rest of the potential, $\delta V(\phi)$: $V(\phi) = V^{(2)}(\phi) + \delta V(\phi)$. In the one-loop quadratic piece

$$V^{(2)}_{\text{loop}}(\phi) \equiv V^{(2)}_{\text{loop,gauge}}(\phi) + V^{(2)}_{\text{loop,top}}(\phi),$$

we include the $SU(2)$ and $U(1)$ gauge contributions to the Higgs squared mass

$$V^{(2)}_{\text{loop,gauge}}(\phi) = \frac{A_{SU(2)} + A_{U(1)}}{R^2} |\phi|^2 = 0.76 \frac{0.01}{R^2} |\phi|^2,$$

where

$$A_{SU(2)} = \frac{21 \zeta(3) g^2}{16 \pi^4} = 0.00688,$$

$$A_{U(1)} = \frac{7 \zeta(3) g'^2}{16 \pi^4} = 0.00069,$$

and the one-loop top-stop corrections at arbitrary $M_t R$, as computed in Ref. [11], which is given by

$$V^{(2)}_{\text{loop,top}}(\phi) = -f(M_t R) \frac{0.01}{R^2} |\phi|^2,$$
The function $f(M_t R)$ appearing in the one-loop top contribution, Eq. (9). The expen-

tential drop off of $f(M_t R)$ at large $M_t R$ is due to the recovery of a supersymmetric spectrum of

the top-stop towers in this limit, in particular, due to the recovery of the supersymmetric mass
degeneracy for the chiral multiplets made of the lightest modes, which get localized at $y = 0$ and

become massless \[11\].

where $f(M_t R)$ is plotted in Fig. 1.

The masses of the scalar components of the lightest modes (the left-handed and right-handed
stops) also receive a significant correction at one loop, due to the gauge and top-Yukawa interac-
tions. These corrections have been computed in Ref. \[14\] in the case of exact localization of the
chiral top multiplets with the result

\[
m_Q^2 = \frac{28\zeta(3)}{3\pi^3} \frac{\alpha_s}{R^2} + \frac{7\zeta(3)}{2\pi^3} \frac{\alpha_t}{R^2} = 0.052 \frac{R^2}{R^2}, \tag{10} \]

\[
m_U^2 = \frac{28\zeta(3)}{3\pi^3} \frac{\alpha_s}{R^2} + 2\frac{7\zeta(3)}{2\pi^3} \frac{\alpha_t}{R^2} = 0.062 \frac{R^2}{R^2}, \tag{11} \]

where $\alpha_s \equiv g_3^2/4\pi$ and $\alpha_t \equiv y_t^2/4\pi$ with $g_3$ and $y_t$ representing the QCD and top-Yukawa couplings

in 4D. The error from using exactly localized matter is expected to be small; in fact we have checked

that the $\alpha_t$ piece above deviates from the exact result by less then 15% at $M_t R \gtrsim 1$.

The vanishing of $V^{(2)}_{1\text{loop,top}}(\phi)$ as $M_t R$ increases makes it necessary to compute the dominant
two-loop effects. The diagrams that contribute to the Higgs mass squared at order $\alpha_s \alpha_t$ and $\alpha_t^2$ are

shown in Fig. 2(a) and 2(b), respectively, in superfield notation. We compute them with localized

$Q$ and $U$ chiral multiplets. Note that these diagrams have to be ultraviolet finite without any

subtraction because in localized approximation for $Q$ and $U$ the Higgs squared mass has no $\alpha_t$

(nor $\alpha_s$) contribution which would generate a two-loop counterterm by renormalizing $\alpha_t$ (or $\alpha_s$).

The calculation can be done by means of the propagators for the various components of the
gauge and Higgs supermultiplets in mixed (4 momentum)-(5th coordinate) space, $G_i(k_4; y, y')$. 

Figure 1: The function $f(M_t R)$ appearing in the one-loop top contribution, Eq. (9). The exponent-
tial drop off of $f(M_t R)$ at large $M_t R$ is due to the recovery of a supersymmetric spectrum of

the top-stop towers in this limit, in particular, due to the recovery of the supersymmetric mass
degeneracy for the chiral multiplets made of the lightest modes, which get localized at $y = 0$ and

become massless \[11\].
Figure 2: The diagrams contributing to the Higgs mass squared at order (a) $\alpha_s\alpha_t$ and (b) $\alpha_t^2$.

Specifically, in localized approximation for $Q$ and $U$, one needs these propagators at $y = y' = 0$ given in Ref. [11]. Summing up the various contributions from the diagrams in Fig. 2 expressed in components and without eliminating the auxiliary fields, one finds

$$V_{2\text{loop}}(\phi) = -|\phi|^2 \left( 144\pi^3 R\alpha_t^2 + 256\pi^3 R\alpha_t\alpha_s \right) \int \frac{d^4p}{(2\pi)^8} \frac{d^4q}{(2\pi)^8} \frac{q}{p^4(p-q)^2} \left( \coth \frac{\pi Rq}{2} - \tanh \frac{\pi Rq}{2} \right),$$

(12)

where $p$ and $q$ are Euclidean 4 momenta.

As expected, Eq. (12) exhibits the characteristic exponential convergence of the integrand at high momenta relative to $1/R$. On the contrary there is an infrared divergence due to the masslessness of the stops, which we cut off by giving the masses given in Eqs. (10, 11) for the internal stop propagators. The overall result is

$$V_{2\text{loop}}(\phi) = -\frac{|\phi|^2}{R^2} \left[ \frac{3\alpha_t^2}{8\pi} \left( 2\eta(m\tilde{U}R) + \eta(m\tilde{Q}R) \right) + \frac{8\alpha_t\alpha_s}{8\pi} \left( \eta(m\tilde{U}R) + \eta(m\tilde{Q}R) \right) \right],$$

(13)

where

$$\eta(y) = \int_0^\infty dx \ x^2 \log \left( 1 + \frac{x^2}{y^2} \right) \left( \coth \frac{\pi x}{2} - \tanh \frac{\pi x}{2} \right),$$

(14)

which amounts to a negative contribution to the Higgs mass squared

$$V_{2\text{loop}}(\phi) = -0.49 \frac{0.01}{R^2} |\phi|^2.$$

(15)

Note that the sum of $V_{1\text{loop},\text{gauge}}(\phi)$ and $V_{2\text{loop}}(\phi)$, which is the only contribution for exactly localized matter, is positive. This shows, as anticipated in section 2 that no EWSB occurs in the case of matter localized on the boundary; theories with top quark located on the brane, such as those of Refs. [9, 14], do not work. To obtain a realistic theory, we need additional negative
contributions to the Higgs mass squared. A simple possibility is to slightly delocalize the top quark from the brane: to use $V_{1\text{loop},\text{top}}^{(2)}(\phi)$ to trigger EWSB. Then, since the delocalization is not perfect, the contribution from $V_{1\text{loop},\text{top}}^{(2)}(\phi)$ can still stay small and be comparable to those from $V_{\text{1loop,gauge}}^{(2)}(\phi)$ and $V_{2\text{loop}}^{(2)}(\phi)$, as explicitly shown in Eq. (9) and Fig. 1, naturally giving larger values for $1/R$ compared to the weak scale. This is one of the main results of our paper. We study various consequences of this scenario in the rest of the paper.

3.2 Full potential and EWSB

To complete the discussion of the EWSB, we need corrections to the remaining part of the potential:

$$\delta V(\phi) \equiv V_{\text{tree}}(\phi) + \delta V_{\text{1loop}}(\phi) + \delta V_{\text{2loop}}(\phi),$$

(16)

which are essential to obtain the physical Higgs-boson mass. At one loop we include the full top-stop contribution as a function of $M_t R$, which for large $1/R$ is given by

$$V_{1\text{loop},\text{top}}(\phi) = N_c \sum_{N=1}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \frac{(-1)^{N+1}}{N} \left( \frac{y_t |\phi|}{\eta^i_0 \eta^i_0} \right)^{2N} \left\{ \left[ G^U\psi(p; 0, 0)G^Q_F(p; 0, 0) \right]^N + \left[ G^Q(p; 0, 0)G^U_F(p; 0, 0) \right]^N - 2 \left[ G^U\psi(p; 0, 0)G^Q\psi(p; 0, 0) \right]^N \right\},$$

(17)

where $\eta^i_0$ is the wavefunction of the zero mode of particle $i$ evaluated at $y = 0$; the forms of $G_i(p; 0, 0)$’s are given in Appendix B. At two loops we consider the correction

$$\delta V_{\text{2loop}}(\phi) = \frac{3}{32\pi^2} y_t^4 |\phi|^4 \left[ \log (y_t^2 |\phi|^2 + m^2_{\text{tree}} + m^2_U) + \log (y_t^2 |\phi|^2 + m^2_{\text{tree}} + m^2_Q) - 2 \log (y_t^2 |\phi|^2 + m^2_{\text{tree}}) \right],$$

(18)

where $m_{\text{tree}}(M_t R)$ is the common tree-level mass of the lightest stops at finite $M_t R$ and $m^2_Q$, $m^2_U$ are the radiative masses given in Eqs. (10, 11). This is nothing but the top-stop radiative contribution to the quartic Higgs coupling in a standard MSSM effective potential with appropriate stop masses and no $A$-term, properly subtracted to avoid double countings with the one-loop top-stop corrections included in Eq. (5).

The two-loop mass term $V_{\text{2loop}}^{(2)}(\phi)$ decreases slightly as $M_t R$ is brought down from infinity to $\sim 1$ to correctly induce EWSB. In order to include all such dependences on the bulk mass, the actual calculations were performed with the full, properly subtracted, two-loop potential. This necessitates cutting off the logarithm at some mass scale $M$ of order $1/R$, which is the scale where squark masses are generated. The cutoff dependent terms in the potential approximate the

4Another possibility is to completely delocalize the top quark, as in Refs. [1] [11], which leads to another interesting set of theories.
Figure 3: The physical Higgs-boson mass as a function of $1/R$, obtained by varying the top quark $\overline{MS}$ mass within the experimental $1\sigma$ uncertainty. The numbers written with dots on the curves represent the values of $M_tR$ at the corresponding points.

contributions from higher Kaluza-Klein (KK) modes making the total potential finite. The precise value of $M$ was determined by matching the quadratic term with the exact calculation in the localized limit and found to be about $1.07/R$. (This cutoff itself may have some $M_tR$ dependence, but since the potential is only logarithmically sensitive we neglect any such correction.)

With the complete expression of the effective potential, which depends upon $1/R$ and $M_t$, we can now minimize it at $|\phi| = 1/2(G_F/\sqrt{2})^{-1/2}$ and obtain a relation between these two parameters. As already mentioned, there is no EWSB in the localized approximation: the $|\phi|^2$ term in the potential is negative only for $M_tR \lesssim 1.92$. Note that if we were to (incorrectly) use the two-loop quadratic contribution in the localized limit, this value would rise to $M_tR \lesssim 2.28$. If $H_d$ is present, on the contrary, for too low values of $M_tR$, the negative $m_u^2$-term in Eq. (9) may exceed the positive $m_d^2$-term and lead to an unstable potential, as discussed in section 2. The largest value of $M_tR$ at which this can happen is where no hypermultiplet mass for $H_d$ is introduced, i.e. at $M_tR \approx 1.17$.

The resulting value of the physical Higgs-boson mass is shown in Fig. 3 as a function of $1/R$ in the described range, for three different values of the running top quark mass, which determines $y_t$. This plot also shows the relevant range of $1/R$, which exceeds 1 TeV at $M_tR \gtrsim 1.2$ because of the
cancellation in the quadratic term of the effective potential already introduced. This cancellation becomes even stronger as $M_t R$ approaches 1.92 but turns into a fine-tuning region as $M_t R$ gets closer to the value of no EWSB.

4 EWSB: Full Parameter Space

We now consider the full parameter space of our model, namely $(M_Q, M_U, M_{H_u})$. Let us begin by removing the previous restriction $M_Q = M_U$. This modifies the Higgs effective potential in a straightforward way. The mixed momentum-position propagators in $V_{\text{1loop,top}}$ now contain different bulk masses for fields in the $Q$ and $U$ hypermultiplets. Similarly, the $m_{\text{tree}}$ terms in $V_{\text{2loop}}$ differ for the left-handed and right-handed squarks. The physical Higgs-boson mass can be calculated in the same manner as described in section 3 with the result shown in Fig. 4.

Figure 4: Contours of the physical Higgs-boson mass in GeV as a function of $M_Q R$ and $M_U R$.

This shows that the previous discussion, with the condition $M_Q = M_U$, captures well the qualitative features of our model. These include a correlation between a localized top quark and a heavy Higgs boson. Specifically, increasing the degree of localization of either or both of the hypermultiplets causes the Higgs-boson mass to grow until the point of no EWSB is reached. This is in contrast to the case of small $M_Q R$ and $M_U R$ where the Higgs-boson mass decreases as the bulk masses grow. In the case of the two Higgs theory, the potential is unstable in this region.
Figure 5: Contours of $1/R$ in GeV as a function of $M_Q R$ and $M_U R$. 

of parameter space.

Deviation from $M_Q = M_U$ also leaves unaltered the trend seen earlier that the localization of the top quark yields an increased $1/R$, as seen in Fig. 5. This is again due to cancellation between the various contributions to the quadratic term in the Higgs potential. Therefore, we find that essential features of the model do not change by deviating from the condition $M_Q = M_U$: as $M_{Q,U}$ are increased approaching to the point of no EWSB, $1/R$ increases due to the cancellation in the Higgs quadratic term, resulting in a heavier physical Higgs-boson mass through a larger correction to the Higgs quartic coupling coming from heavier stops.

As either of the bulk masses, $M_Q$ or $M_U$, is decreased, $V_{1\text{loop, top}}^{(2)}$ becomes more negative; we do not have to reduce both $M_Q$ and $M_U$ to obtain sufficiently negative contribution from $V_{1\text{loop, top}}^{(2)}$. This opens up a new larger region of parameter space for which there is successful EWSB. In Fig. 6 we have shown the parameter region where successful EWSB occurs. This figure shows that Fig. 4 is somewhat misleading regarding the portion of parameter space that has been ruled out by direct Higgs searches. While more than half of the parameter space $1.2 \lesssim M_Q R, M_U R \lesssim 1.9$ is ruled out yielding a too light Higgs boson, there is now an additional region that is experimentally viable in which either $M_Q$ or $M_U$ (but not both) is larger than $1.9/R$.

Next we remove the restriction $M_{H_u} = 0$ while maintaining $M_Q = M_U = M_t$. The only significant effect of removing this restriction is to allow a tree-level mass for the Higgs doublet.
Figure 6: Region of $(M_Q R, M_U R)$ space in which electroweak symmetry is broken. Note that the ranges for $M_Q R$ and $M_U R$ are different from those of Figs. 4 and 5.

We consider only small tree-level mass squares (either positive or negative) so that the radiative effects are still dominant. Specifically, we wish to maintain the feature that as $M_t R$ approaches infinity, there is no EWSB. Fig. 7 demonstrates the case in which the magnitude of the tree-level mass squared is one half the sum $V_{1\text{loop, gauge}}^{(2)} + V_{2\text{loop}}^{(2)}$ in the localized limit. Such a mass is of the same order of magnitude as that expected from a linearly divergent FI term, which corresponds to 25% of the two-loop contribution in the localized limit. We find that the resulting effect is less than that arising from experimental uncertainty in the top quark mass. Incidentally, increasing the tree-level mass with the positive (negative) sign for $M_H u$ increases (decrease) the region of $M_t R$ in which electroweak symmetry is broken.

Localization of the top quark by hypermultiplet masses has also been discussed in Ref. [12]. They considered the limit of exact localization of $U$, $M_U R \to \infty$, and took a very high degree of localization of $Q$, $M_Q R = 2.6$, so that the $m_t/m_b$ ratio is entirely understood by the profile of $Q$. In this case they argue that EWSB is triggered by the two-loop top contribution, since the one-loop top term is negligible. However, our explicit two-loop calculation shows that their estimate of the two-loop contribution significantly exaggerates its effect, and that EWSB does not occur in this region.
Figure 7: The physical Higgs-boson mass as a function of $1/R$ for three values of the Higgs tree-level mass squared, given in units of $0.01/R^2$.

5 Complete Spectrum

The value of $1/R$ determines in a simple way the spectrum of the towers of gauginos, Higgsinos and gauge bosons: up to small EWSB effects the lightest gauginos and Higgsinos are at $1/R$, whereas the first KK states of the vector towers are at $2/R$. The masses of the lightest scalars are of greater interest for experimental searches. For matter hypermultiplets with small hypermultiplet mass these are also at $1/R$, but as the hypermultiplet mass, $M$, increases the tree-level mass of the corresponding lightest scalar decreases, and vanishes exponentially for large $MR$. Hence in this limit the radiative and EWSB contributions to the mass become important. In the case of the theory with two Higgs hypermultiplets, there are additional scalars, $H^0$ and $H^+$ and their towers, from $H_d$. For $M_{H_d} = 0$, the scalars $H^0$ and $H^+$ are zero modes, acquiring mass from electroweak radiative corrections and from EWSB contributions. Possible radiative FI term contributions to scalar masses are taken to be included in $M$.

For any of these scalars, of charge $Q$ and hypercharge $Y$, we calculate the mass squared as

$$m^2 = m_{\text{tree}}^2 + m_{\text{rad}}^2 + Y m_Z^2 - Q m_W^2,$$

where $m_{\text{tree}}$ is the tree-level mass, dependent on the mass of the hypermultiplet, $M$, to which the particle belongs and including the effect of the Yukawa coupling, $m_{\text{rad}}$ is the one-loop mass
Figure 8: Physical masses for the squarks and sleptons from hypermultiplets with $M_Q = M_U = M_D = M_L = M_E = M$, and for the scalars of $H_d$ with $M_{H_d} = 0$. As $1/R$ increases so does $M$, so that the squark and slepton masses become dominated by the radiative contributions of Eqs. (20 – 24).

The masses which are unequivocally determined are those belonging to the $Q$ and $U$ hypermultiplets that play a crucial role in EWSB, $\tilde{t}_L, \tilde{t}_R, \tilde{b}_L$. They are given as functions of $1/R$ in Fig. 8 when $M_Q = M_U$. The radiative term dominates over all the other terms, hence a quasi-linear rise of the masses. A moderate variation in $M_U/M_Q$ has only a small influence on these masses, but a value larger than unity could reduce the mass of $\tilde{t}_R$ relative to that of $\tilde{t}_L, \tilde{b}_L$.

Fig. 8 shows masses for the other squarks and sleptons of the third generation in the case where all the matter hypermultiplet masses are taken equal to each other and $M_{H_d} = 0$. For large
values of $1/R$, these masses are dominated by the radiative contribution

\begin{align}
m_{\text{rad}}(\tilde{Q}) &= \frac{0.060}{R^2}, \quad (20) \\
m_{\text{rad}}(\tilde{U}) &= \frac{0.064}{R^2}, \quad (21) \\
m_{\text{rad}}(\tilde{D}) &= \frac{0.044}{R^2}, \quad (22) \\
m_{\text{rad}}(\tilde{L}) &= \frac{0.0075}{R^2}, \quad (23) \\
m_{\text{rad}}(\tilde{E}) &= \frac{0.0028}{R^2}, \quad (24)
\end{align}

computed here in the localized approximation, and including electroweak radiative corrections for the squarks as well as the sleptons. As any matter hypermultiplet mass is reduced, the corresponding scalar mass increases, asymptotically to $1/R$, due to the tree contribution. Only the matter with large hypermultiplet masses have light scalars. The values given in Eqs. (20–24) also apply to scalars of the first two generations if they originate in hypermultiplets with a large mass; the only difference is that the radiative top contribution of Eqs. (10, 11) must be subtracted for $\tilde{Q}$ and $\tilde{U}$.

For the two Higgs hypermultiplet theory, $M_{H_d} = 0$ ensures that the $H_d$-bosons are uniformly distributed in the bulk, so that the radiative masses are as in Eqs. (6–8). Note that, in this case, the mass of the neutral $H_d$ is below 100 GeV for $1/R \lesssim 1.4$ TeV, but, since $H_d$ has no VEV this is not presently excluded.

6 Phenomenology of Sparticle Production

The precise phenomenology of sparticle production will depend upon the choice of the hypermultiplet masses. There are, however, a few features of this phenomenology that have a universal character. The lightest superpartner is a squark or slepton, most likely charged, which is stable or practically stable.\footnote{Its instability can be due to a small $U(1)_R$-breaking effect or to its decay into a very light right-handed sneutrino. The latter case is motivated by having small Dirac neutrino masses accounted for by a Yukawa coupling at $y = 0$ to a $N_R$ hypermultiplet strongly localized at $\pi R/2$. The neutrino mass is exponentially small, so that values of $M_{N_R} R$ need only be 2–4 times larger than $M_{Q,U}$, depending on the localization of $L$.} This scalar is pair produced in a hadron collider, either directly or by cascade decay, via a strong interaction cross section determined by $1/R$.

If the squarks and sleptons of only the third generation are light enough to be relevant, there are three different cases to be considered:

- The lightest sparticles are $\tilde{t}_L, \tilde{t}_R$ and $\tilde{b}_L$, as is the case where $M_Q = M_U = -M_{H_d}$ and all other hypermultiplet masses are vanishingly small. In this case there are two essentially
degenerate super-hadrons $S^+$ and $S^0$, and their charge-conjugates, made of a squark, either $\tilde{t}$ or $\tilde{b}$, depending on which is the lightest, and an antiquark, which both appear as stable particles. The two other sparticles decay into them by emission of soft hadrons. The masses of these sparticles can be read off from Fig. 8 as functions of $1/R$. (The other superparticles shown in the figure, such as $\tilde{b}_R$ and the sleptons, are much heavier $\simeq 1/R$ in the present case.) The figure was drawn taking $M_D$ equal to $M_Q = M_U$, but the masses for $\tilde{t}_L, \tilde{t}_R$ and $\tilde{b}_L$ are practically unchanged even when $M_D = 0$.

- The lightest sparticle is $\tilde{b}_R$, as is the case for $M_Q = M_U = M_D$ with other hypermultiplet masses vanishingly small. The masses for the squarks in this case can be read off from Fig. 8. (The sleptons are much heavier of masses $\simeq 1/R$). Also in this case there are two degenerate super-hadrons $S^+$ and $S^0$, produced either directly or by cascade with a larger cross section than in the previous case. Furthermore, the heavier sparticles decay into them mostly in association with $b$-quarks or $W$-bosons.

- The lightest sparticle is a slepton, most likely charged, as in the case when all hypermultiplet masses are equal (see Fig. 8). More precisely, in this case the lightest sparticle is a charged slepton, which can be pair produced by the Drell-Yan mechanism with an electroweak cross section or from the cascade decay of the heavier states, always with at least a charged lepton and, in most cases, with a $t$- or a $b$-quark. Note that the lightest sparticle could also be a sneutrino, as in the case $M_Q = M_U = -M_L$ and $M_E = 0$. A sneutrino would give rise to a missing energy signal in association with $t$- or $b$-quarks from the cascade decays.

Further light scalars could result from large hypermultiplet masses for the first two generations.

7 Bound on $1/R$

The quasi-localization of some matter hypermultiplets gives rise to new interactions which limit $1/R$ from below in a definitely stronger way than in the case of matter homogeneously spread throughout the bulk, as in Ref. [1]. The strength of these interactions critically depends on the localization of the first two generations, i.e. on the hypermultiplet masses $M_{1,2}$, and also depends in general on the gauge quantum numbers inside one generation.

Correspondingly, the strongest limits arise in two cases:

1. when the first generation is mostly localized, from 4-fermion operators generated by exchanges of KK gauge bosons.

2. when the first two generations have their hypermultiplet masses different from each other or from the one of the third generation, from their Flavor Changing Neutral Current (FCNC) effects.
In view of this, for definiteness, we consider two different cases:

(i) \( M_1 = M_2 = M_3 \) for the entire family multiplets,

(ii) \( M_1 = M_2 = 0 \), with \( M_3 \) as in section 3

When all masses are equal, the case (i), there is no new FCNC effect. On the contrary, a new significant effect occurs through the couplings of the KK \( W \)-bosons, of masses \( 2n/R \), to the first generation lepton-doublet \( L_1 \), described by the effective Lagrangian

\[
L_{\text{eff}} = \frac{A}{16} g^2 R^2 (L_1 \gamma_\mu \tau^a L_1)^2.
\]

Here, \( A \) is a normalization factor depending upon \( M_1 R \) through the zero-mode wavefunctions, which vanishes for \( M_1 R = 0 \) and is close to unity for \( M_1 R \approx 1-2 \). In order not to disturb the success of EWPT fit, for \( M_1 R = 1-2 \) a bound on \( 1/R \) of 1.4 TeV arises [15].

Suppose now that we consider the case where the first two generations, unlike the third one, are uniformly spread in the bulk \( (M_1 = M_2 = 0) \), so that this effect is absent. In this case it is the difference between the couplings of the KK gluons to the first two generations and to the third one that gives the largest effect, as calculated in Ref. [16]. If one assumes mixing angles and phases of the down-quark Yukawa-coupling matrix comparable to those of the Cabibbo-Kobayashi-Maskawa matrix, the strongest bound arises from \( CP \)-violating \( \epsilon \)-parameter in \( K \) physics and is about \( 1/R \gtrsim 2 \) TeV.

From these considerations we conclude that the range of values for \( 1/R \) compatible with EWSB, \( 1/R = 1.5–3.5 \) TeV, could give rise to some interesting indirect effects either in EWPT or in flavor physics.

8 Conclusions

We have constructed a theory of ElectroWeak Symmetry Breaking (EWSB) with supersymmetry broken by boundary conditions in a fifth dimension, which is determined to have a scale \( 1/R \approx 1.5–3.5 \) TeV. The only particles beyond those of the standard model which must be lighter than \( 1/R \) are the three squarks \( \tilde{t}_L, \tilde{b}_L \) and \( \tilde{t}_R \), which have masses approximately proportional to \( 1/R \) and in the range 500–1200 GeV.

In Table 2 we compare this theory with the Standard Model (SM), the Minimal Supersymmetric Standard Model (MSSM) and the Constrained Standard Model (CSM). By the constrained standard model, we mean the theory introduced in Ref. [1] together with the possibility of small hypermultiplet masses [8, 11]. The only crucial difference of the model in the present paper is that the top quark has a large hypermultiplet mass, causing it to be approximately localized at a boundary of the fifth dimension. The first row of Table 2 shows whether each model, considered as an effective field theory below some cutoff scale \( \Lambda \), provides a physical theory of EWSB. Can
Table 2: A comparison of models for EWSB.

|                  | SM | MSSM | CSM | This paper |
|------------------|----|------|-----|------------|
| EWSB             | −  | +    | +   | +          |
| Direct searches  | +  | ?    | +   | +          |
| EWPT             | +  | +    | ?   | +          |
| Gauge coup. unif.| −  | +    | −   | −          |

the electroweak scale, $v$, be computed in terms of some parameters of the effective theory in a way which is relatively insensitive to physics at the cutoff and therefore to $\Lambda$? This of course is the great failing of the SM, motivating the introduction of the other models. The second row shows whether any of these models predicts new particles which would have already been discovered by direct searches. Strictly speaking none does, but in the case of the MSSM this is because the parameter space of the model allows cancellations so that the superpartners can be made unnaturally heavy. The third row shows that none of the theories is in conflict with ElectroWeak Precision Tests (EWPT), although excessive contributions to the $\rho$ parameter might have been expected in the CSM, as it has a low cutoff scale, $\Lambda \approx 2$ TeV. High scale gauge coupling unification is only possible in the SM and MSSM, where $\Lambda$ is above the unification scale, and is successful only for the MSSM, as shown in the fourth row.

The MSSM has a very plausible physical origin for EWSB: a negative Higgs mass-squared, $m_{\phi}^2$, induced radiatively by the top-quark Yukawa coupling. However, the top Yukawa coupling is large and, even though it is radiative, this effect is very powerful

$$m_{\phi}^2 \approx -\frac{\ln(\Lambda/\tilde{m})}{30}\tilde{m}^2, \quad (26)$$

partly because of the large logarithm. Consequently, the scale $\tilde{m}$ of colored superpartners is expected to be close to $v$, and this is especially problematic when the supersymmetry breaking leads to non-colored superpartners significantly lighter than the colored ones. In the CSM the calculation is extended to include the KK modes of the top quark, with the result that the Higgs mass squared is finite and independent of $\Lambda$

$$m_{\phi}^2 \approx -9\left(\frac{0.01}{R^2}\right). \quad (27)$$

The masses for both colored and non-colored superpartners are predicted to be at $1/R \approx 400$ GeV; the issue of fine-tuning does not arise, and these superpartners could be readily discovered or excluded. However, such low values of $1/R$, and therefore $\Lambda$, might have been discovered at LEP in EWPT.

The quasi-localized top quark studied in this paper has the virtue of exponentially suppressing any one-loop top contribution, such as Eqs. (26, 27). Relevant contributions to the Higgs mass
squared are then provided by electroweak gauge interactions at one loop:

\[ m_\phi^2 \simeq 0.76 \left( \frac{0.01}{R^2} \right), \tag{28} \]

and two-loop diagrams involving the top-quark Yukawa interaction

\[ m_\phi^2 \simeq -0.49 \left( \frac{0.01}{R^2} \right). \tag{29} \]

The sum of these contributions is 30 times smaller than the unsuppressed one-loop top contribution of Eq. (27), and therefore leads to an increase in \( 1/R \) from the CSM value by about a factor of 6 to the region of 2.5 TeV. However, this sum is positive: an exactly localized top quark does not lead to any EWSB. This means that the localization must not be complete: the one-loop top contribution must not be negligible. At first sight this looks like another fine tune, but it is not: the one-loop top contribution is suppressed by a factor \( \exp(-\pi M_t R) \) which is in the desired range of \( 10^{-1} - 10^{-2} \) for \( M_t R \approx 1 \sqrt{2} \).

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Appendix A

In this appendix, we discuss how many bulk mass parameters the theory possesses and how many of them enter the EWSB calculation. Without loss of generality the bulk mass matrix for each charge sector can be taken diagonal, so that there is a separate mass parameter for each matter and Higgs hypermultiplet. The masses for the scalar and fermion components take the form

\[
\mathcal{L}_m = \cdots - \psi^c \partial_y \psi - M \eta(y) (\psi^c \psi + \text{h.c.}) - M^2 (|\phi|^2 + |\phi^c|^2) + 2M (\delta(y) + \delta(y - \pi R/2)) (|\phi|^2 - |\phi^c|^2)
\]

where \( \eta(y) = +1 (-1) \) for \( y > 0 \) \((< 0)\), and the \( \partial_y \) piece is included because what matters is a relative sign between \( \partial_y \) and \( M \). These mass terms have a brane contribution to maintain the form

\[ 20 \]
of the unbroken local supersymmetry \( [11] \). Despite the presence of so many parameters, the physics of EWSB is sensitive to only three of them: \( M_{Q_3}, M_{U_3} \) and \( M_{H_u} \). It is perhaps obvious that the masses for the lighter generation quarks are irrelevant — they have small Yukawa couplings which give only small radiative contributions to the scalar potential — but it is not obvious that \( M_{D_3} \) is irrelevant. A large value for \( M_{Q_3} \) localizes \( b_L \) largely on the brane distant from the bottom Yukawa coupling, so that the 5D bottom Yukawa coupling must be large to overcome the wavefunction suppression. Nevertheless, we find that the radiative contribution to the Higgs potential through the bottom Yukawa coupling is always suppressed by \((m_b/m_t)^2\).

We consider here only the case of a single Higgs hypermultiplet. Everything that follows may be directly generalized to the two Higgs case for moderate value of \( M_Q R \). The relevant part of the Lagrangian for studying the contribution of the radiative correction from the bottom quark Yukawa coupling to the scalar potential is:

\[
L = \lambda_t \delta(y)(\bar{q} F_U h + F_{\tilde{Q}} \tilde{h} h - q t h) - \lambda_b \delta(y - \pi R/2)(\bar{q}^c F'_D h^* + F'_{\tilde{Q}} \tilde{b}^c h^* - q b h^*). \tag{31}
\]

Here, the chiral supermultiplets under the \( N = 1 \) supersymmetry acting at \( y = 0 \) are given by

\[
H = (h, \tilde{h}, F_H), \tag{32}
\]
\[
Q_3 = (\tilde{q}, q, F_Q), \tag{33}
\]
\[
U_3 = (\tilde{t}, t, F_U). \tag{34}
\]

and the chiral supermultiplets under the \( N = 1 \) supersymmetry acting at \( y = \pi R/2 \) are given by

\[
H^c = (-h^*, \tilde{h}^c, F'_H), \tag{35}
\]
\[
Q'_3 = (\tilde{q}^c, q, F'_Q), \tag{36}
\]
\[
D'_3 = (\tilde{b}^c, b, F'_D). \tag{37}
\]

The Lagrangian in Eq. (31) can be derived from the superpotential term \( W = \lambda_t \delta(y)(Q_3 U_3 H) + \lambda_b \delta(y - \pi R/2)(Q'_3 D'_3 H^c) \).

In terms of mixed momentum-position propagators, the ratio of bottom to top Yukawa contributions in the Higgs mass squared clearly depends on the ratios \( G_q(k_4; 0, 0)/G_q(k_4; \pi R/2, \pi R/2) \), \( G_{\tilde{q}}(k_4; 0, 0)/G_{\tilde{q}}(k_4; \pi R/2, \pi R/2) \) and \( G_{F_Q}(k_4; 0, 0)/G_{F_{\tilde{Q}}}(k_4; \pi R/2, \pi R/2) \). These ratios of propagators are all equal in the infrared, which dominates the loop integral, and given by \( \exp(-\pi M_Q R) \). This exactly cancels the enhancement of the 5D bottom Yukawa coupling due to the small wavefunction overlap. Therefore, the contribution to the Higgs mass squared due to the bottom Yukawa interaction is down by a factor of \((m_b/m_t)^2\) and can be safely neglected. We can similarly neglect all the other Yukawa contributions relative to the top one. The most general such theory of EWSB is therefore parameterized by a three dimensional space spanned by \((M_{Q_3}, M_{U_3}, M_{H_u})\).
Appendix B

In this appendix we list propagators $G_i(p; 0, 0)$ for various components of a matter hypermultiplet with a bulk mass $M$:

$$G_\varphi(p; 0, 0) = \frac{\sinh[\sqrt{p^2 + M^2 \pi R/2}]}{\sqrt{p^2 + M^2} \cosh[\sqrt{p^2 + M^2 \pi R/2}] - M \sinh[\sqrt{p^2 + M^2 \pi R/2}]}, \quad (38)$$

$$G_\psi(p; 0, 0) = \frac{\sqrt{p^2 + M^2}}{\sqrt{p^2 + M^2} \coth[\sqrt{p^2 + M^2 \pi R/2}] + \frac{M}{\sqrt{p^2 + M^2}}}, \quad (39)$$

$$G_F(p; 0, 0) = \frac{\cosh[\sqrt{p^2 + M^2 \pi R/2}] + \frac{M}{\sqrt{p^2 + M^2}} \left(1 + \frac{p^2}{2M^2}\right) \sinh[\sqrt{p^2 + M^2 \pi R/2}]}{\frac{1}{2M} \cosh[\sqrt{p^2 + M^2 \pi R/2}] + \frac{1}{2\sqrt{p^2 + M^2}} \sinh[\sqrt{p^2 + M^2 \pi R/2}]}, \quad (40)$$

where $\varphi, \psi$ and $F$ represent the scalar, fermion and auxiliary field components, respectively; $p$ is an Euclidean momentum.
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