Designing quantum gates using the genetic algorithm

Karthikeyan S. Kumar and G. S. Paraoanu*
O.V. Launasmaa Laboratory, Aalto University – P.O. Box 15100, FI-00076 Aalto, Finland
Email: sorin.paraoanu@aalto.fi

Abstract. We demonstrate the usage of Genetic Algorithm (GA) to tailor the radio frequency pulses for producing unitary transformations in qubit systems. We find that the initial population converges to the optimal solution after 10 generations, for a one segment pulse corresponding to single qubit Hadamard gate. For a two qubit CNOT gate, we see the population convergence for a two segment pulse after 150 generations. This demonstrates that the method is suitable for designing quantum gates.

1. Introduction
The Genetic Algorithm (GA) is an artificial intelligence based search heuristic that is inspired from natural selection and evolution principles. The major principles operating in evolutionary biology are inheritance, mutation, selection, crossover and survival of the fittest. These principles have been adopted in genetic algorithms design and is used to obtain near exact or approximate solutions to optimization and search problems [1,2]. This particular class of evolutionary algorithms acquires the attributes of a global search due to the mutations of individual solutions, hence making it more versatile than the gradient ascent or descent based search methods which can precisely find the local maxima or minima. Here, we will consider a two-level system Hamiltonian, which can model a superconducting qubit or other mathematically equivalent spin-1/2 systems. We illustrate the usage of GA to tailor the propagators corresponding to quantum gates.

2. Quantum gates designed using the genetic algorithm
Numerical methods have been used to appropriately design microwave fields to manipulate superconducting quantum systems from entangling them [3] to decoupling them [4]. We demonstrate that GA is another suitable method to design the radio frequency pulses corresponding to qubit gates. The formulation of the pulses and the simulation results are presented below.

2.1. One qubit case:
Consider the simple case of a single two level system subject to external driving by a radio frequency field. The Hamiltonians are (for simplicity, we take \( \hbar = 1 \))

\[
H_{\text{sys}} = \frac{\omega}{2} \sigma^z,
\]

\[
H_{\text{RF}} = \Omega \cos(\omega_{\text{RF}} t + \varphi) \sigma^x.
\]

The effective Hamiltonian is simplified by choosing a rotating frame and applying the rotating wave approximation (RWA)
\[
H_{\text{eff}} = \frac{\delta}{2} \sigma^z + \frac{\Omega}{2} \left[ \cos(\phi) \sigma^x - \sin(\phi) \sigma^y \right]
\]

where the detuning is
\[
\delta = \omega - \omega_{RF}.
\]

We choose the single qubit Hadamard gate as the target.

\[
U_T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.
\]

The gate which the genetic algorithm parameterizes using a radio frequency pulse is given by,

\[
U_{exp} = \exp(-iH^0_{\text{eff}} \tau_{del}) \times \exp(-iH_{\text{eff}} \tau_{RF}),
\]

where

\[
H^0_{\text{eff}} = \frac{\delta}{2} \sigma^z.
\]

![Fig. 1: One segment pulse, used for parametrizing a single qubit gate](image)

We have used a rectangular pulse for simplicity, as shown in Fig 1. The five variables for the evolution of the solution are the qubit detuning \( \delta \), pulse amplitude \( \Omega \), phase \( \phi \), pulse on-time \( \tau_{RF} \) and pulse off-time \( \tau_{del} \) during which the system evolves under its internal Hamiltonian. GA calculates the fitness function iteratively according to the following expression.

\[
f = -\left[ \text{abs}(\text{trace}(\text{inv}(U_T)U_{exp}))/2 \right]^2.
\]

This limits the value of \( f \) to be within \(-1 \leq f \leq 0\). The simulation, for a population size of fifty and evolution for twenty generations for a single qubit Hadamard gate is shown in Fig. 2. The simulations are done using Matlab gatool function.

For other common single qubit gates such as Pauli-X,Y,Z and phase gates such as S and T gates, the convergence of population to the optimal solution occurred after 10 generations.
2.2. Two qubit case:

For the two qubit system, characterized by driving and coupling, the Hamiltonians are given by

$$H_{\text{sys}} = \sum_{j=1,2} \frac{\omega_j}{2} \sigma_j^z,$$

$$H_{\text{RF}} = \sum_{j=1,2} \Omega_j \cos(\omega_{\text{RF}} t + \varphi_j) \sigma_j^x,$$

$$H_c = \omega^{\text{sc}} \sigma_1^x \sigma_2^x.$$

Similar to the single qubit case, the Hamiltonian in rotating frame with RWA, is given by

$$H_{\text{eff}} = \sum_j \left( \delta \sigma_j^z + \frac{\Omega_j}{2} \left[ \cos(\varphi_j) \sigma_j^x - \sin(\varphi_j) \sigma_j^y \right] \right) + \frac{\omega^{\text{sc}}}{2} \left( \sigma_1^\dagger \sigma_2^x + \sigma_1^x \sigma_2^\dagger \right) = H_{\text{eff}}^0 + H_{\text{ext}},$$

where $H_{\text{eff}}^0 = \sum_j \delta \sigma_j^z + \frac{\omega^{\text{sc}}}{2} \left( \sigma_1^\dagger \sigma_2^x + \sigma_1^x \sigma_2^\dagger \right)$ and

$$H_{\text{ext}} = \sum_j \frac{\Omega_j}{2} \left[ \cos(\varphi_j) \sigma_j^x - \sin(\varphi_j) \sigma_j^y \right].$$

Now we choose the target gate as a CNOT gate represented in matrix form as

$$U_T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

In this case, two pulse segments were used, such that

$$U_{\text{exp}} = \prod_{k=1}^2 U_{\text{seg},k}.$$
where each $U_{\text{seg},k} = U_{\text{eff},k}^0 \times U_{\text{eff},k}$, which are given by

$$U_{\text{eff},k}^0 = \exp\left(-iH_{\text{del},k}^0 \tau_{\text{del},k}\right) \quad \text{and} \quad U_{\text{eff},k} = \exp\left(-iH_{\text{del},k} \tau_{\text{del},k}\right)$$

The number of variables per segment is two detunings, two amplitudes, two phases and pulse on-time and off-time, making a total of eight. The two segments require a total of sixteen variables. The GA evolution is simulated by calling the gatool function in Matlab, and the simulation results are shown in Fig. 3.

![Fitness vs Generation](image)

**Fig. 3:** GA simulation results. Population size = 200, Generations = 400, Target: CNOT gate.

For a two qubit gate, the population did not always converge to the optimal solution for a single segment pulse. Tailoring a pulse sequence with multiple segments, allowed us to see the convergence of solution for the common two qubit gates.

### 3. Conclusion

As the number of degrees of freedom increases in a quantum circuit, the parameters to manipulate the same become excessively complicated to solve. Interesting phenomena such as the build-up of entanglement in the stationary state has been predicted to occur [5]. Falling into the category of global search algorithms, genetic algorithms are optimal candidates for solving the parameters that can effectively manipulate quantum systems. GA can also be used in hybrid search, where the final population at the termination of GA can be used with a gradient ascent or descent algorithm, which are maximally efficient in finding the local minima or maxima.

### References

[1] Weile, DS and Michielssen, E, Genetic Algorithm Optimization Applied to Electromagnetics: A review, IEEE Transactions, Vol 45, 3, (1997)

[2] Deaven, DM and Ho, KM, Phys. Rev. Lett. 75, 288-291 (1995)

[3] J. Li, K. Chalapat, and G. S. Paraoanu, Phys. Rev. B 78, 064503 (2008)

[4] G. S. Paraoanu, Phys. Rev. B 74, 140504(R) (2006)

[5] J. Li and G. S. Paraoanu, New J. Phys. 11, 113020 (2009)