Abstract. The aim of this work is the study transient processes in nuclear reactors. The mathematical model of the reactor dynamics excluding reverse thermal coupling is investigated. This model is described by a system of integral-differential equations, consisting of a non-stationary anisotropic multispeed kinetic transport equation and a delayed neutron balance equation. An inverse problem was formulated to determine the stationary part of the function source along with the solution of the direct problem. The author obtained sufficient conditions for the existence and uniqueness of a generalized solution of this inverse problem.

1. Introduction.

The need for reliable, manageable and secure nuclear power plants is increased in recent years. This is an additional incentive for the development of the mathematical theory of nuclear reactors, which is relatively new and is constantly evolving. The proof can be found in a number of papers [1-10] devoted to this subject. While developing this theory, scientists together with technicians try to create a model reactor, which reliable work is less likely to be dependent on the human factor. The agenda raises questions of mathematical modeling of processes occurring in nuclear reactors. This is necessary both in the design and in the design and in the management and control of the operation of the reactor.

2. Formulation of linear inverse problem of the reactor dynamics.

We consider a mathematical model of the reactor dynamics [5], which describes a system of integral-differential equations:

1) non-stationary multispeed anisotropic kinetic transport equation

$$\frac{\partial u(x, v, t)}{\partial t} (x, v, t) + (v, \nabla_x) u(x, v, t) + \Sigma(x, v, t) u(x, v, t) = $$

$$= \int_V J(x, v, v', t) u(x, v', t) dv' + \sum_{k=1}^{N} z_k R_k(x, v, t) + F(x, v, t), \quad (1)$$

2) delayed neutron balance equation

$$\frac{\partial R_k(x, v, t)}{\partial t} (x, v, t) = -z_k R_k(x, v, t) + \int_V J_k(x, v, v', t) u(x, v', t) dv', \quad \forall k = 1, N, \quad (2)$$

$$(x, v, t) \in D = G \times V \times (0, T).$$
This model describes the function \( u(\mathbf{x}, \mathbf{v}, t) \) of the density distribution of neutrons that pass through a point \( \mathbf{x} \in G \) at a rate \( \mathbf{v} \in V \) in time \( t \in (0, T) \). Functions \( \Sigma(\mathbf{x}, \mathbf{v}, t), J(\mathbf{x}, \mathbf{v}, \mathbf{v}', t), F(\mathbf{x}, \mathbf{v}, t) \) characterize properties of a medium where a mass transfer process occurs. More specifically, \( \Sigma(\mathbf{x}, \mathbf{v}, t) \) is an absorption coefficient, \( J(\mathbf{x}, \mathbf{v}, \mathbf{v}', t) \) is a scattering phase function, and \( F(\mathbf{x}, \mathbf{v}, t) \) is a radiation source density. Here, \( G \) is a range of spatial coordinates, which is assumed to be a strictly convex domain, while \( V \) is a range of particles velocities \( \mathbf{v} \), which is a closed set contained in a spherical layer \( \{0 < v_0 \leq |\mathbf{v}| \leq v_1 < \infty\} \). Kernels \( J_k(\mathbf{x}, \mathbf{v}, \mathbf{v}', t) \) of scattering integrals characterize a density distribution of secondary neutrons, and a function \( R_k(\mathbf{x}, \mathbf{v}, t) \) is a density distribution of carriers neutron \( k \)-th group \( \forall k = 1, \ldots, N \).

Let us assume that the process under consideration occurs in the absence of external sources of particles; i.e., for example, external radiation does not pass through the reactor walls. This assumption is mathematically expressed by a boundary condition

\[
u(\mathbf{x}, \mathbf{v}, t) = 0, \quad (\mathbf{x}, \mathbf{v}, t) \in \gamma_- \times [0, T], \tag{3}\]

where \( \gamma_- = \{(\mathbf{x}, \mathbf{v}) \in \partial G \times V : (\mathbf{v}, n_x < 0)\} \), and \( n_x \) is an outward normal to a boundary \( \partial G \) of \( G \) at a point \( \mathbf{x} \).

Additionally, we set initial conditions

\[
u(\mathbf{x}, \mathbf{v}, 0) = \varphi(\mathbf{x}, \mathbf{v}), \quad (\mathbf{x}, \mathbf{v}, t) \in \overline{G} \times V, \tag{4}\]

\[
R_k(\mathbf{x}, \mathbf{v}, 0) = R_k(\mathbf{x}, \mathbf{v}), \quad \forall k = 1, \ldots, N, \quad (\mathbf{x}, \mathbf{v}, t) \in \overline{G} \times V. \tag{5}\]

The problem \((1) - (5)\) is a direct problem of the nuclear reactor dynamics.

Let us formulate an inverse problem where together with solving a direct problem, we simultaneously need to find some parameter of the equation \((1)\) by applying an additional condition. For example, we take a final condition, i.e., at \( t_1 \in (0, T) \)

\[
u(\mathbf{x}, \mathbf{v}, t_1) = \psi(\mathbf{x}, \mathbf{v}), \quad (\mathbf{x}, \mathbf{v}, t) \in \overline{G} \times V, \tag{6}\]

a condition which is called an over-determination condition.

From a physical point of view, the task is to find conditions in the input data of this problem under which a considered process can be driven from initial state \((4)\) to terminal state \((6)\) over a fixed time \( t_1 \in (0, T) \) by controlling a stationary part \( f(\mathbf{x}, \mathbf{v}) \) of a source function

\[
F(\mathbf{x}, \mathbf{v}, t) = f(\mathbf{x}, \mathbf{v})g(\mathbf{x}, \mathbf{v}, t), \tag{7}\]

where \( f(\mathbf{x}, \mathbf{v}) \) is an admissible control function (distributed time-invariant control) and \( g(\mathbf{x}, \mathbf{v}, t) \) is a given function (correction function).

**Inverse problem.** Find functions \( u(\mathbf{x}, \mathbf{v}, t), R_k(\mathbf{x}, \mathbf{v}, t), f(\mathbf{x}, \mathbf{v}) \), meeting the conditions \((1) - (7)\).

**Remark 1.** The role of initial conditions in such inverse problems for transient processes can be played by some steady reactor states.

### 3. Solution of the linear inverse problem of the reactor dynamics.

The inverse problem stated above will be studied in the following function spaces, which were introduced in \([7]\).

1) \( H_c(D) \) is the space of functions \( u \) that belong to a class \( L_c(D) \) together with their generalized derivatives \( u_t \) and \( (\mathbf{v}, \nabla_x)u \) on \( D \) and have traces \( u|_{\Gamma_-} \) on \( \Gamma_- = \gamma_- \times (0, T) \) from \( L_c(\Gamma_-) \), i.e.,

\[
H_c(D) = \left\{ u \in L_c(D) : \frac{\partial u}{\partial t} (\mathbf{v}, \nabla_x)u \in L_c(D), u|_{\Gamma_-} \in L_c(\Gamma_-) \right\};
\]
\( H_\infty(D) \) is a Banach space with respect to the norm
\[
\|u\|_{H_\infty} = [\|u\|_{\infty,D} + \|\frac{\partial u}{\partial t}\|_{\infty,D} + \|\langle v, \nabla_x \rangle u\|_{\infty,D} + \|u\|_{\Gamma_-} + \|u\|_{\Gamma_-}] < \infty.
\]

2) \( L^I_\infty(D) \) is the space of functions \( R_k \) that belong to a class \( L_\infty(D) \) together with their generalized derivatives \( \frac{\partial R_k}{\partial t} \) on \( D \), i.e.,
\[
L^I_\infty(D) = \left\{ R_k \in L_\infty(D) : \frac{\partial R_k}{\partial t} \in L_\infty(D) \right\}, \quad \forall k = 1, N;
\]
\[
L^I_\infty(D) \) is a Banach space with respect to the norm
\[
\|R_k\|_{L^I_\infty} = [\|R_k\|_{\infty,D} + \|\frac{\partial R_k}{\partial t}\|_{\infty,D}] < \infty, \quad \forall k = 1, N.
\]

Where \( \| \bullet \|_{\infty,D} \) is a norms in \( L_\infty(D) \).
The solution of the linear inverse problem under study can be provided by the following result.

**Theorem 1.** Let
\[
\Sigma, \frac{\partial \Sigma}{\partial t} \in L_\infty(D); \quad J, \frac{\partial J}{\partial t} \in L_\infty(D \times V); \quad \varphi, \langle v, \nabla_x \rangle \varphi \in L_\infty(G \times V), \varphi|_{\gamma_-} \in L_\infty(\gamma_-);
\]
\[
\psi, \langle v, \nabla_x \rangle \psi \in L_\infty(G \times V), \psi|_{\gamma_-} \in L_\infty(\gamma_-); \quad g, \frac{\partial g}{\partial t} \in L_\infty(D), \| g(x, v, t_1) \| \geq g_0 > 0;
\]
\[
J_k, \frac{\partial J_k}{\partial t} \in L_\infty(D \times V); \quad R_{k0} \in L_\infty(D), \quad \forall k = 1, N; \quad \text{and let the compatibility conditions}
\]
\[
\varphi(x, v) = 0 \quad \text{and} \quad \psi(x, v) = 0 \quad \text{for} \quad (x, v) \in \gamma_- \quad \text{are met.}
\]

If \( \text{diam}^G_{\infty} < b \) with a constant \( b \) depending on the norms \( \| \Sigma \|, \| J \|, \| g \|, \| \varphi \|, \| \psi \|, \)
\[
\| J_k \|, \| R_{k0} \|, \forall k = 1, N \) in the corresponding spaces and on numbers \( t_1, g_0, \) and \( m(V) \),

where \( m(V) \) is a measure of the set \( V \), then there exists a unique generalized solution of the inverse problem, i.e., there are functions \( u \in H_\infty(D), R_k \in L^I_\infty(D) \) and \( f \in L_\infty(G \times V) \), for which a process under study can transfer from initial state (4) to terminal state (6).

The idea behind the proof of this theorem is the reduction to a system of integral equations of the second kind for the functions \{\( u, R_k, f \)\}, considering the constraints as the smallness of \( \text{diam}^G_{\infty} < C \), where \( C \) is a constant depending on the norms \( \| \Sigma \|, \| J \|, \| g \|, \| \varphi \|, \| \psi \|, \| J_k \|, \| R_{k0} \|, \forall k = 1, N \) and on the numbers \( t_1, g_0, m(V) \). Therefore, the system of integral equations has a unique solution \( \{ u, R_k, f \} \in H_\infty(D) \times L^I_\infty(D) \times L_\infty(G \times V) \) and, hence, inverse problem is uniquely solvable.

4. Conclusion.
The linear inverse problem is studied in this paper. In terms of transient processes in nuclear reactors, a reactor can be transferred from one state to another by applying a time-invariant control to the source function or to an absorption coefficient of the non-stationary multispeed anisotropic kinetic transport equation. Sufficient conditions for existence and uniqueness of generalized solution of the above inverse problem were obtained. The proof of Theorem 1 implies that a desired generalized solution depends continuously on the input data of a problem and can be found by the method of successive approximations.

**Remark 2.** Inverse problem discussed in this paper, can be interpreted as a task of managing of transition processes in nuclear reactors. This function \( f(x, v) \) will play a role of a distributed control effect on the neutron sources.
References
[1] Marchuk G I The Methods of Calculation for Nuclear Reactors. Atomizdat, Moscow, Russia, 1961 (Russian). 667 p.
[2] Case K M, Zweifel P F Existence and Uniqueness Theorems for the Neutron Transport Equation. – J. Math. Phys., 1963, v. 4, N 11, p. 1376.
[3] Akcasu A Z, Lellouche G S and Shotkin L M Mathematical methods in nuclear reactor dynamics. N.-Y.: Academic Press, 1971.
[4] Shikhov S B Problems of the Mathematical Theory of Reactors: Linear Analysis, Atomizdat, Moscow (1973) (Russian). 376 p.
[5] Kryanev A V and Shikhov S B Questions in the Mathematical Theory of Reactors: Nonlinear Analysis, Atomizdat, Moscow (1983) (Russian). 280 p.
[6] Lewins J Nuclear reactor kinetics and control / Jefery Lewins. Pergamon Press, 1978. - 264 c.
[7] Germogenova T A Local Properties of Solutions to Transport Equations. Nauka, Moscow, Russia, 1986 (Russian). 272 p.
[8] Prilepko A I and Volkov N P Inverse problems of determining the parameters of a non-stationary kinetic transport equation for more information about the following desired function. - Differential Equations, 1988, v. 24, N 1, p. 136-146
[9] Volkov N P On Some Inverse Problems for Time-Dependent Transport Equation. // ILL-Posed Problems in Natural Sciences. Moscow: TVP-VSP, 1992, p. 431-438.
[10] Volkov N P Solvability of Certain Inverse Problems for the Nonstationary Kinetic Transport Equation. - Computational Mathematics and Mathematical Physics. Pleiades Publishing, 2016, v. 56, N 9, pp. 1598-1603.