Computation of pion and kaon heavy ion multiplicities in a gluon-meson model

Pedro Bicudo
CFTP, Dep. Física, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

Francesco Giacosa and Elina Seel
Institute of Theoretical Physics, J. W. Goethe University, Max-von-Laue Str. 1, D-60438, Frankfurt am Main, Germany

In high energy Heavy Ion Collisions, the onset of the quark-gluon plasma is the colour glass condensate, dominated by gluons. The final state is hadronic, and dominated by pions and kaons. Here we investigate an effective approach of QCD with these bosonic fields and which can help to describe the transition of gluons into light mesons. Formally, our approach consists in integrating out the quark fields from the QCD path integral. In this way the fermionic fields are replaced by light mesons, such as the pions and sigma field. We apply our effective action to compute the number of pions and kaons per gluon emitted by a Boltzmann gluon gas, their multiplicities as a function of the gluon mass. We conclude that an effective gluon mass remains finite at $T = T_c$.

I. INTRODUCTION

In this work we develop a Lagrangian where only bosons, i.e., gluons and mesons, are the active degrees of freedom, and apply it to study the multiplicities in heavy ion collisions, the number of pions and kaons per gluon, and the gluon mass at the onset of the deconfinement/confinement phase transition $T_c$.

Effective approaches of QCD have been widely used to study the properties of strong interactions [2]. Quark models, meson effective models, or models combining both quarks with mesons are used thoroughly to explore hadronic physics. Although gluons have been proposed already in the 70’s together with the theory of strong interactions, QCD, in effective models it is common to assume that the gluons are integrated out, and only contribute indirectly through the quark or effective hadron interactions.

Nevertheless, there are two rapidly developing QCD domains where gluons are either easier to work with, or are phenomenologically more relevant. In many-body systems, the Grassmann variable nature of the quarks makes them technically much more difficult to address than the bosonic gluons. In particular, Lattice QCD first developed and applied pure gauge or quenched techniques since working with dynamical quarks is computationally very expensive [3,4]. Moreover, in high energy Heavy Ion Collisions [1], it was proposed successfully that the quark-gluon plasma is the colour glass condensate, dominated by gluons [3,5]. The final state is hadronic, and dominated by pions and kaons. For instance, in the many particle BAMPS set-up for heavy ion collisions, [4,10] the simulations are performed with gluons only form the onset, and mesons are included as final states of the hadronization. From the QCD perspective, in this case, quarks and not gluons are integrated out and effective mesons are included. Note also the recent work of Weinberg [11] where an effective Lagrangian with gluons, in addition to pions and quarks, was put forward.

Formally, the different effective approaches to QCD can be seen as the result of integrating out some degrees of freedom from the QCD Lagrangian [2]. For instance, when only gluons are integrated out, one obtains a NJL-like theory [12] or a quark model [13]. If, in addition, also quarks are integrated out from the NJL model, one is left with a purely linear $\sigma$ model. Moreover, as an intermediate step between the NJL and the $\sigma$ models a quark-meson model is obtained. Similarly, in the approach of Cahill and Roberts [14, 15] it was shown by using bilocal auxiliary fields (along the same line of the Hubbard-Stratonovich transformation [16]) how to integrate out quark and gluon degrees of freedom to obtain a purely mesonic Lagrangian. While these calculations were only performed at one loop order, it was an interesting approach to connect effective models, say the $\sigma$ model, directly to QCD. In addition, there are also lattice QCD approaches for effective meson theories [17], which deliver qualitatively similar results.

For the purpose of this paper it is necessary to chose a slightly different way, which consists of integrating out from the QCD Lagrangian the quarks only and then obtain a gluon-meson (fully bosonic) theory. From symmetry principles (colour gauge invariance and chiral symmetry) we expect at leading order the following tree-level coupling between the gluons, the pions and their chiral partner, the scalar $\sigma$ meson:

$$L_{\text{gluon-meson}} \propto (G^a_{\mu\nu} G^a_{\mu\nu})(\sigma^2 + \vec{\pi}^2 + ...),$$

where $G^a_{\mu\nu}$ is the gluonic field tensor and dots refer to other mesonic degrees, such as resonances with open and hidden strange-quark content (such as the kaons), vector resonances and, eventually, non-quarkonium resonances.

The interaction Lagrangian (1) allows to study the transition of two gluons into a couple of mesons. If the latter are not stable, they further decay into pions or...
the generating functional of QCD can be written as a suitable gauge \[14, 15\], and with no fermion sources, the gauge-fixing and the Faddeev-Popov ghost terms. In \(\eta\) where

\[
\phi = S^a \mu^a + i P^a a^a ,
\]

\(L_\mu = L_\mu^a a^a = V_\mu + A_\mu ,
\]

\(R_\mu = R_\mu^a a^a = V_\mu - A_\mu ,
\]

\(R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu , L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu .
\]

In the previous expressions \(t^a\) are the \(N_f^2\) generators of \(U(N_f)\), \(S^a = \sqrt{2} q_i a^a q\) are the scalar, \(P^a = \sqrt{2} q_i a^a q\) are the pseudoscalar degrees of freedom, \(V_\mu = \sqrt{2} q_i a^a q\), \(A_\mu^a = \sqrt{2} q_i a^a q\) are the vector and axial-vector microscopic quark currents. For instance, in the case \(N_f = 2\) the fields are given by

\[
\phi = (\sigma + i \eta_N) t^0 + (\bar{a}_0 + i \pi) t ,
\]

where \(t^0 = I/2, t^i = \tau_i/2,\) and \(\tau_i\) are the Pauli matrices. Analogously, \(R^\mu = (\omega^\mu - f_\mu^I t^0 + (\bar{\rho}^\mu - \bar{\sigma}^\mu) t^I)\) represents the vector and \(L^\mu = (\omega^\mu + f_\mu^I t^0 + (\bar{\rho}^\mu + \bar{\sigma}^\mu) t^I)\) the axial vector degrees of freedom. The extension to the case \(N_f = 3\) is straightforward \[20\].

Clearly, upon Gaussian integration, Eq. (3) gives a constant factor and thus does not change the path integral in the generating functional Eq. (4).

To couple the mesons to fermions, the mesonic fields can be added as parallel transports in the fermion matrix similar to a mass term in the case of the scalar sigma and to a chiral transport in the case of the pion. The corresponding minimal coupling is given by

\[
\gamma_\mu D^\mu \rightarrow \tilde{D} = \gamma_\mu D^\mu - c_1 \Phi - c_2 (\gamma_\mu V^\mu + \gamma_\mu \gamma^5 A^\mu ) ,
\]

where \(c_1\) and \(c_2\) are free parameters, and \(A^\mu\) is the axial current. Finally, performing the Grassmann integration over the fermion fields, we obtain a purely bosonic theory

\[
Z = \int D\Phi DLDL_\mu D_\mu e^{i \int \mathcal{L}_{\text{meson}}} .
\]

The coupling of the mesonic and gluonic degrees of freedom resides in the fermion determinant \(\det [\tilde{D}]\), which cannot be computed analytically. However, by requiring local colour gauge invariance and chiral symmetry and restricting to lowest dimensionality of the interaction Lagrangian, we are left to the following Lagrangian,

\[
\mathcal{L}_{\text{gluon-meson}} = G^{a\mu\nu} G_{a\mu\nu} \text{Tr} \left[ a_1 \Phi + b \left( V^2 + A_3^2 \right) \right] ,
\]

where \(a\) and \(b\) are couplings with dimension Energy\(^{-2}\) and describe the transition from two gluons to two mesons. To obtain the relation between the parameters \(a, b\) and the parameters \(c_1, c_2\) introduced in Eq. (10),

\[
\mathcal{L}_{\text{mesonic sector}} = \text{Tr} \left[ \Phi^4 \Phi + \frac{1}{2} \Phi^2 \right] ,
\]
TABLE I. The parameters for each meson pair initially produced and then decaying into pions and kaons are the mass, degeneracy, number of pions produced by the meson, number of kaons produced by the meson, the family factor. For meson octets and their respective chiral partners we assume the same family factor.

| meson | $M_m$ | $g$ | $N_\pi$ | $N_K$ | $f$ |
|-------|------|----|----------|--------|-----|
| $\pi$ | 138  | 3  | 1        | 0      | a   |
| $\eta$| 549  | 1  | 3        | 0      | a   |
| $\eta'$| 958  | 1  | 3        | 0      | a   |
| $K$   | 495  | 4  | 0        | 1      | a   |
| $\rho$| 775  | 9  | 2        | 0      | b   |
| $\omega$| 782  | 3  | 3        | 0      | b   |
| $\phi$| 1020 | 3  | 0        | 2      | b   |
| $K^+$| 892  | 12 | 1        | 1      | b   |
| $a_0$ | 985  | 3  | 4        | 0      | c   |
| $\sigma$| 600  | 1  | 2        | 0      | c   |
| $f_0$ | 980  | 1  | 2        | 0      | c   |
| $\kappa$| 800  | 4  | 1        | 1      | c   |
| $a_0$ | 1450 | 3  | 4        | 0      | a   |
| $f_0$ | 1370 | 1  | 4        | 0      | a   |
| $f_0$ | 1710 | 1  | 0        | 2      | a   |
| $\kappa$| 1430 | 4  | 1        | 1      | a   |
| $f_1$ | 1230 | 9  | 3        | 0      | b   |
| $f_1$ | 1282 | 3  | 4        | 0      | b   |
| $f_1$ | 1420 | 3  | 1        | 2      | b   |
| $K_1$ | 1272 | 12 | 1        | 1      | b   |

FIG. 1. Number of (top) pions $\Pi_\pi(E_{pair})$ and of (bottom) kaons $\Pi_K(E_{pair})$ produced per gluon as a function of the gluon energy in the centre of mass of the gluon pair. Here $a = b$, $c = 0$ are used.

III. ANALYTICAL AND NUMERICAL RESULTS

In our framework an effective gluon mass $M_g$ can be introduced. Notice the existence of a possible effective gluon mass [21], or pole in a gluon propagator, already at $T = 0$, has been under debate in QCD. While gauge invariance in a perturbative approach rules out a gluon mass and the existing lattice QCD glueballs suggest that the gluon propagator is transverse, nevertheless a run-
number of pions and kaons per gluon is calculated as the scalar channel below 1 GeV. Putting all together, the usual phase space factor relevant mesons used for the calculations. Moreover, we also include the usual phase space factor \( M_g^2 = \frac{24}{\pi} \alpha_s T^2 \), (13) which, say at \( T = T_c = 0.158 \text{ GeV} \) and \( \alpha_s \simeq 0.3 \) leads to a gluon mass of \( M_g \simeq 0.239 \text{ GeV} \).

For the following purposes we evaluate the Lorentz-invariant Mandelstam variable \( s \) for a system of two gluons with four-momenta \( p_i = (\sqrt{p_i^2 + M_i^2}, p_i) \), \( i = 1, 2 \). (14)

The Mandelstam variable \( s \) leads to the centre of mass energy \( E_{pair} \) of the gluon pair,

\[
s = (p_1 + p_2)^2 = 2M_g^2 + 2 \left( \sqrt{p_1^2 + M_g^2} \sqrt{p_2^2 + M_g^2} - p_1 \cdot p_2 \right) = E_{pair}^2.
\]

Due to the Lagrangian in Eq. (12), the two gluons convert into a meson-pair. Considering that only pions and kaons are regarded as stable, we must also take into account that the other resonances decay subsequently into kaons and pions. For instance, each \( \sigma \) meson decays into a two-pion pair, therefore the \( \sigma \sigma \) channel results into a final \( 4\pi \) state. Similarly, a pair of \( \rho \) mesons decays also into four pions. On the contrary, the vector state \( \phi \) decays predominantly into kaons. In Table II these conversion factors, expressed as \( n(\pi) \) and \( n(K) \), are listed for all the relevant mesons used for the calculations. Moreover, we also include the usual phase space factor \( \sqrt{E_{pair}^2 - M_k^2} \), where \( M_k \) is the mass of the \( k \)-th meson pair, and the corresponding degeneracy spin-isospin factor \( g_k \). Finally, we should also take into account the relative strength, which is equal to \( a^2 \) in the (pseudo)scalar channel, to \( b^2 \) in the (axial-)vector channel, and to \( c^2 \) in what concerns the scalar channel below 1 GeV. Putting all together, the number of pions and kaons per gluon is calculated as

\[
\Pi_\pi(E_{pair}) = \sum_k \sqrt{E_{pair}^2 - M_k^2} \theta(E_{pair} - 4M_k)g_k f_k^2 n_i^{(i)} \sum_k \sqrt{E_{pair}^2 - M_k^2} \theta(E_{pair} - 4M_k)g_k f_k^2
\]

where \( i = \pi, K \).

The functions \( \Pi_\pi(E_{pair}) \) and \( \Pi_K(E_{pair}) \) as a function of the gluon energy and as a function of the Boltzmann temperature are both depicted in Fig. 1

\[
N_i(T) = \int dp_1 dp_2 f_B(p_1^2, T) f_B(p_2^2, T) \Pi_i(E_{pair}) \, , \quad (17)
\]

We are interested in comparing our results with the parametrization of the BAMPS set-up [9], since, like our framework, BAMPS also consists of gluons, decaying into pions. In BAMPS, 1.5 to 2.0 pions are produced per gluon pair. This production takes place in different conditions than ours, at non-chemical equilibrium with local and dynamical many-gluon simulations. Nevertheless, if we compare with our approach, we notice we can easily obtain \( N_\pi \sim 1 \), but the increase of \( N_\pi \) above unity can only be achieved at the price of including a sizeable effective gluon mass. This confirms the importance of including in our framework an effective gluon mass, simulating a finite non-perturbative scale characteristic of the gluon plasma.

We now evaluate the emitted number of pions \( N_\pi(T) \) and kaons \( N_K(T) \) per gluon as a function of the temperature \( T \). We denote the multiplicities \( N_i(T) \) by employing a Boltzmann distribution of each gluon, thus leading to

\[
N_i(T) = \int dp_1 dp_2 f_B(p_1^2, T) f_B(p_2^2, T) \Pi_i(E_{pair}) \, , \quad (17)
\]
where \( f_B(p^2, T) = N_c e^{-\sqrt{p^2 + m^2_{gluon}}/T} \) is the normalized Boltzmann distribution. The integral in Eq. (1) can be simplified to a three-dimensional integration, and we compute it with a numerically accurate c++ code. In Fig. 2 the functions \( N_{\pi}(T) \) and \( N_K(T) \) are plotted for different values of the gluon mass \( M_g \) and for both choices \( a = b, c = 0 \) and \( a = b = c \). For the intermediate value \( M_g = 400 \text{ MeV} \) we have roughly one pion per each gluon for each \( T \), while \( N_K(T) \) is a rapidly increasing function with \( T \).

In Fig. 3 we present the ratio \( N_K(T)/N_{\pi}(T) \) for the temperature of \( T = T_c = 0.158 \text{ GeV} \) as a function of the gluon mass. The critical temperature \( T_c \) for the confinement and chiral crossover was measured in lattice QCD to be in the range \( T_c \in [0.145, 0.165] \text{ GeV} \) [26-29]. This was achieved in very precise full lattice QCD simulations with dynamical fermions. This temperature range is consistent with the freeze-out temperature of the quark-gluon plasma measured in Heavy ion collisions. The freeze-out temperature in heavy ion collisions can be determined from the inverse slope of the hadronic species multiplicity as a function of the transverse momentum. Recent analysis of heavy ion collisions indicate that the freeze-out temperature is in the range of \([0.150, 0.170]\) GeV with results between \(0.150 \text{ GeV} \) and \(0.160 \text{ GeV} \) [30] and results between \(0.160 \text{ GeV} \) and \(0.170 \text{ GeV} \) [31, 32]. Both ranges are compatible, and we consider in our computations the mean value of \( T = 0.158 \text{ GeV} \).

In Fig. 4 we compare the ratio with the experimental data \( (N_K/N_{\pi})_{\text{exp}} \in [1.13, 1.22] \) measured by the PHENIX, STAR, BRAHMS, E866 and NA49 collaborations and extrapolated by the UrQMD 2.0, UrQMD 2.1 and HSD transport approaches [32, 52] for the ratio of the pion and kaon multiplicity in the most central collisions.

Our results point to a solution corresponding to a possible effective gluon mass, in a range of \( M_g \in [0.28, 0.37] \) GeV. Remotely, a second less likely mass of circa 0.8 GeV may be possible. We notice that the solution points to a gluon mass at \( T_c \) of the order of circa 0.4 of the gluon effective mass of 0.5 to 1.0 GeV at \( T = 0 \) resulting from different gluon calculations. The solution for the gluon mass is also consistent with the Debye mass [3] of the gluon at finite \( T \).

Finally we test the robustness of our results checking the parameter dependence of the gluon-meson model. In Fig. 4 we compare the ratio \( N_K/N_{\pi} \) for different couplings \((c = 0\text{ with } c = 1)\), different temperatures \((T = 0.145 \text{ GeV} \text{ with } T = 0.170 \text{ GeV})\) and different statistical distributions (Boltzmann and Bose-Einstein). All the tests we performed with plausible changes of our parameters suggest our results are robust.

The small effect of changing the coupling of gluons to mesons is quite relevant for our results, since the meson properties are not yet established. Strong coupled channel effects, tetraquarks, and glueballs have been shown to affect the meson spectrum and the mesonic couplings. Here we utilize the meson properties listed in the Particle Data Group [34], but other meson data would yield similar results. We utilize the sigma model for the meson production, but other hadronic models would again yield similar results.

The robustness of our results occurs because a gluon in the freeze-out of the plasma has a low energy \((T \approx 160 \text{ MeV})\) and an effective mass of \(M_g \approx 320 \text{ MeV}\), much lower than the energy of a gluon in any glueball typical of lattice QCD simulations or of constituent gluon model estimations. Thus our results do not really depend on the details of the meson spectrum and of the meson couplings above that low energy, and escape the problem of understanding higher energy reactions such as the glueball decays.

IV. CONCLUSIONS

In this work we develop an effective Lagrangian which connects gluons to mesons. We then utilize this approach to calculate the emitted number of pions and kaons per gluon out of a gluon gas at temperature \( T \). We assume the gluons are in a thermal bath at \( T = T_c \) whereas the mesons are produced at vanishing temperature.

The fact that our effective Lagrangian consists of bosonic degrees of freedom only might be also interesting for lattice QCD applications. In fact, the Grassmann variable nature of the quarks makes them technically much more difficult to address (see for instance Ref. [2] and refs. therein) than the bosonic gluons and mesons.

Our approach represents an attempt to link directly and in an understandable way the gluon-dominated physics of the quark gluon plasma, as suggested by the colour glass condensate and BAMPS approaches, to the
Further developments of our approach could include meson mass modifications in the medium, effects of the freeze-out boundary, different meson-gluon couplings, missing resonances such as the tensor mesons, final state interactions among mesons, and, last but not least, glue-ball fields, which directly couple to gluonic fields. Nevertheless, all the tests we performed with plausible changes of our parameters suggest that our results are stable. Thus we regard our effective Lagrangian as a pilot study for a better understanding of the rich physics of QCD and of Heavy Ion Collisions.

Interestingly, to reproduce the experimental results of the PHENIX, STAR, BRAHMS, E866 and NA49 collaborations for the pion and kaon multiplicities, we have to include a finite scale for the gluon energy at $T_\text{c}$. Our results point to a possible effective gluon mass, in a range of the order of circa 0.4 and 1.0 of the gluon effective mass of 0.5 to 1.0 GeV at $T_\text{c}$ resulting from different gluon calculations. An $M_g \in [0.25, 0.39]$ is also close to the Debye gluon mass at $T = T_\text{c}$. Notice this solution corresponds to an absolute pion multiplicity of circa 0.9 pions per gluon.

If the gluon mass is related to confinement, say as in superconductors, our result is consistent with the QCD order parameters at $T_\text{c}$. Both for a first order phase transition where the order parameter is discontinuous and in a crossover where the order parameter remains finite, the scale of confinement should not simply vanish at $T = T_\text{c}$. Here we present an evidence, based in the experimental Heavy Ion data and in our simple and robust gluon-meson model that a finite scale of 0.25 to 0.39 GeV exists in the gluon sector of QCD at $T = T_\text{c}$.

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