Numerical Analysis of Key Influences Factors of Creep and Shrinkage Performance of Composite Beams

Chunxiu Han¹, Shuwei Lan², Kaicheng Yao³ and Jiuchang Zhang¹*

¹Department of Civil Engineering, Yunnan Minzu University, Kunming, Yunnan, 650504, China
²Faculty of Civil Engineering and Mechanics, Kunming University of Science and Technology, Kunming, Yunnan, 650224, China
*Corresponding author’s e-mail: zhangjiuchangyun@foxmail.com

Abstract. Based on concrete creep mechanics and finite element method, the key influence factors of creep and shrinkage are analysed numerically to evaluate the long-term mechanical properties of the steel-concrete composite beams. A constraint coefficient is adopted to judge the constraint degrees of the steel beam in the composite beam. Finite element analyses are conducted on 8 kinds of sections in the composite beams with different constraint degrees. The analyses indicate the axial and bending rigidities increase with the section height of the steel beam, which lead to an increase in both constraint degrees and distributed internal forces. The final internal forces of the concrete slab decrease and the internal forces of the steel beam increase, that is, the reduced internal forces in the concrete slab are transferred to the steel beam. The variations of deflections also increase with the constrain degrees, which show the same trend when the increase of constraints. It can be seen that the constraint degrees of the steel beam in the concrete are the key influence factors that cause stress redistribution and deformation change of the composite beams.

1. Introduction
Steel-concrete composite beam (referred to as composite beams for short) is a type of flexural member which works together by connecting concrete slab with steel beam through shear connectors. In the long-term service process, creep and shrinkage appearing in the concrete slab are constrained by the steel beam, resulting in stress redistribution on the section of composite beams[1-3]. The constraint degree of the steel beam on concrete is the key factor affecting creep and shrinkage of composite beams[4]. The whole process of this influence factor on the creep and shrinkage of composite beams can be obtained by numerical analysis through changing parameters, which has a good reference for revealing the long-term mechanical law of composite beams[5-6].

2. The influence factors of creep and shrinkage performance of composite beams
For plain concrete, the influence factors of creep and shrinkage include internal factors (types of cement, water-cement ratio, aggregate, member sizes, etc.) and external conditions (environmental temperature and humidity, loading age and load duration, etc.). The factors on creep and shrinkage are reflected in creep coefficient $\varphi$. According to the actual situation in a project to optimize the above factors, a reasonable concrete member can be designed, which has been studied very well. For composite beams, however, it is affected not only by creep coefficient, but also by different constraint
degree of the steel beam. The parameters of constraint degree include elastic modulus of concrete, width and thickness of flange, elastic modulus of steel, section height, section area of steel beam, which do not exist independently, but influence each other.

In this paper, the coefficient \( j = \frac{A_o I_{ao}}{A I_{i}} \) is used to judge constraint degrees of the steel beam on the concrete, where \( A_i \) denotes the area of steel section and \( A_o \) denotes the concrete area \( A \) converted into steel area with \( A_o = A \frac{E_c}{E_s} \), \( I_i \) represents the actual second moments of steel area and \( I_{ao} \) represents the actual second moments of concrete area \( I_c \) converted into steel area with \( I_{ao} = I_c \frac{E_c}{E_s} \), where \( E_c \) and \( E_s \) are the instantaneous elasticity moduli of concrete and steel, respectively. It can be seen from the composition of the coefficient \( j = \frac{A_o I_{ao}}{A I_{i}} \) that it covers the influence parameters such as the areas of concrete and steel section, elastic modulus and moment of inertia. With \( j = 0 \), the constraint is 0, that is, the beam is made up of concrete entirely. With \( j = \infty \), the constraint is the largest, that is, the beam is made up of steel entirely. The actual composite beams is between the two limits.

3. Numerical analysis of constraint degree affecting creep and shrinkage performance of composite beams

3.1 An example

The coefficient \( j = \frac{A_o I_{ao}}{A I_{i}} \) is affected by the moment of inertia directly proportional to the third square of section height. Therefore, the change of the section height has an important effect on the coefficient. The constraint degree of the steel beam on the concrete about creep and shrinkage is obtained by the numerical analysis of an example with the section height \( h \) from 400mm to 1800mm as shown in Figure 1. Suppose the section is taken from a simply supported composite beam with a span of \( L=20 \) meters, initial internal force with \( M_0=2.0 \times 10^3 \text{kN}\cdot\text{m} \) and \( N_0=0 \), considering the effects of creep and shrinkage with concrete grade C30, annual average humidity 30%, loading age or shrinkage age 7 days, creep coefficient \( \varphi = 4.0 \), shrinkage strain \( \varepsilon_{sh} = 20 \times 10^{-5} \), the elastic modulus of concrete and steel with \( E_c = 30000 \text{ MPa} \) and \( E_s = 210000 \text{ MPa} \), respectively. The detailed parameters of the section are shown in Table 1.

![Figure 1. The sectional dimensions of the composite beam](image-url)
3.2 Calculation of internal force

The influence of creep and shrinkage on composite beams is reflected in the changes of internal forces in the sections of concrete and steel beam, which finally leads to the deformation change of composite beams. The values of the redistribution internal forces and mid-span deflections of the 8 sections over time are shown in Table 2 to Table 4.

### Table 1. The parameters of the 8 sections with height $h_i$ from 400mm to 1800mm

| Section | $h_i$ (mm) | $A_0$ (10^2 mm^2) | $I_0$ (10^6 mm^4) | $A_i$ (10^2 mm^2) | $I_i$ (10^6 mm^4) | $A_0$ (10^2 mm^2) | $I_0$ (10^6 mm^4) | $S_0$ (10^4 mm^3) | $j$  |
|---------|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|------|
| 1       | 400       | 1142.9          | 380.9           | 300.8           | 712.6           | 1443.7          | 4539.8          | 905.9           | 2.031 |
| 2       | 600       | 1142.9          | 380.9           | 324.8           | 1790.1          | 1467.7          | 8927.4          | 1307.2          | 0.749 |
| 3       | 800       | 1142.9          | 380.9           | 348.8           | 3442.9          | 1491.7          | 15054.2         | 1732.4          | 0.363 |
| 4       | 1000      | 1142.9          | 380.9           | 372.8           | 5733.3          | 1515.7          | 23025.2         | 2180.3          | 0.200 |
| 5       | 1200      | 1142.9          | 380.9           | 396.8           | 8720.3          | 1539.7          | 32941.6         | 2649.9          | 0.126 |
| 6       | 1400      | 1142.9          | 380.9           | 420.8           | 12460.2         | 1563.7          | 44901.4         | 3140.1          | 0.083 |
| 7       | 1600      | 1142.9          | 380.9           | 444.8           | 17007.5         | 1587.7          | 58999.6         | 3650.1          | 0.058 |
| 8       | 1800      | 1142.9          | 380.9           | 468.8           | 22415.6         | 1611.7          | 75328.4         | 4178.9          | 0.041 |

### Table 2. The redistributed axial forces of the concrete slab (N)

| Time (day) | Section | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------|---------|---|---|---|---|---|---|---|---|
| 17         | $h_i=400$ | 31.61 | 158.42 | 194.12 | 207.13 | 213.23 | 216.99 | 220.72 |
| 27         | $h_i=600$ | 62.22 | 202.16 | 240.78 | 254.54 | 260.86 | 264.77 | 267.91 |
| 37         | $h_i=800$ | 84.79 | 231.45 | 271.38 | 285.35 | 291.68 | 295.59 | 298.78 |
| 47         | $h_i=1000$ | 102.43 | 253.27 | 293.90 | 307.92 | 314.19 | 318.05 | 321.24 |
| 57         | $h_i=1200$ | 118.15 | 272.08 | 313.17 | 327.16 | 333.34 | 337.13 | 340.30 |
| 67         | $h_i=1400$ | 131.61 | 287.83 | 329.19 | 343.12 | 349.19 | 352.91 | 356.04 |
| 77         | $h_i=1600$ | 143.97 | 302.05 | 343.59 | 357.42 | 363.83 | 367.02 | 370.11 |
| 87         | $h_i=1800$ | 155.14 | 314.71 | 356.35 | 370.08 | 375.93 | 379.49 | 382.53 |

### Table 3. The redistributed bending moment of the steel beam (N·mm)

| Time (day) | Section | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------|---------|---|---|---|---|---|---|---|---|
| 17         | $h_i=400$ | 101.84 | 127.76 | 152.59 | 177.71 | 203.40 | 229.75 | 256.78 | 284.50 |
| 27         | $h_i=600$ | 123.21 | 155.22 | 185.67 | 216.29 | 247.50 | 279.42 | 312.11 | 345.58 |
| Time (day) | Section | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|-----------|---------|----|----|----|----|----|----|----|----|
| 0         | h_{i=400} | 87.41 | 44.45 | 26.36 | 17.23 | 12.05 | 8.84 | 6.73 | 5.27 |
| 17        | h_{i=600} | 115.77 | 58.61 | 35.15 | 23.38 | 16.67 | 12.50 | 9.72 | 7.79 |
| 27        | h_{i=800} | 121.72 | 61.66 | 37.06 | 24.72 | 17.68 | 13.29 | 10.37 | 8.33 |
| 37        | h_{i=1000} | 125.54 | 63.62 | 38.29 | 25.58 | 18.32 | 13.80 | 10.78 | 8.67 |
| 47        | h_{i=1200} | 128.32 | 65.05 | 39.18 | 26.21 | 18.79 | 14.17 | 11.08 | 8.93 |
| 57        | h_{i=1400} | 130.68 | 66.26 | 39.95 | 26.74 | 19.19 | 14.48 | 11.34 | 9.14 |
| 67        | h_{i=1600} | 132.63 | 67.27 | 40.58 | 27.18 | 19.52 | 14.74 | 11.55 | 9.32 |
| 77        | h_{i=1800} | 134.38 | 68.17 | 41.14 | 27.57 | 19.82 | 14.97 | 11.74 | 9.47 |
| 87        | j=0.203 | 135.92 | 68.97 | 41.64 | 27.92 | 20.08 | 15.18 | 11.90 | 9.61 |
| 97        | j=0.749 | 137.27 | 69.66 | 42.07 | 28.23 | 20.31 | 15.35 | 12.05 | 9.73 |
| 107       | j=0.363 | 138.61 | 70.35 | 42.51 | 28.53 | 20.53 | 15.53 | 12.19 | 9.85 |
| 117       | j=0.200 | 143.35 | 72.80 | 44.04 | 29.59 | 21.33 | 16.16 | 12.70 | 10.27 |
| 127       | j=0.126 | 146.53 | 74.44 | 45.06 | 30.31 | 21.86 | 16.57 | 13.04 | 10.56 |
| 137       | j=0.083 | 149.14 | 75.78 | 45.90 | 30.89 | 22.30 | 16.91 | 13.31 | 10.78 |
| 147       | j=0.058 | 151.17 | 76.83 | 46.56 | 31.35 | 22.64 | 17.18 | 13.53 | 10.96 |
| 157       | j=0.041 | 152.83 | 77.69 | 47.09 | 31.72 | 22.91 | 17.39 | 13.70 | 11.11 |
| 167       | Final multiply | 154.29 | 78.44 | 47.56 | 32.05 | 23.16 | 17.58 | 13.86 | 11.23 |
| Final multiply | (w_{i=0}/ w_{i})×100% | 1.77 | 1.77 | 1.80 | 1.86 | 1.92 | 1.99 | 2.06 | 2.13 |

### 3.3 Analysis of influence factors

It can be seen from table 2 to table 4, the axial and bending rigidities of the steel beam increase with the section height $h_i$ which result in an increment of constraint degree and distributed internal force. Then the final internal forces of the concrete slab decrease and the internal forces of the steel beam increase, that is, the reduced internal forces in the concrete slab are transferred to the steel beam. The
variations of deflections also increase with the constraint, for example, the final variation multiple of the deflections before and after creep and shrinkage is 1.77-2.13, which shows the same trend as the increase of constraint.

The comparison of internal forces and deflections are shown in figure 2 to figure 5. For the change of $j = \frac{A_{c0} / I_{c0}}{A_{i} / I_{i}}$, is proportional to the third square of the section height $h_i$, the change in $j$ is much faster than the change in $h_i$. As can be seen from figure 2, the redistributed axial forces of concrete $N_c$, are proportional to $j$. As $j$ decreases, namely, the proportion of concrete stiffness in composite beams decreases, the constraint of the steel beam increases, and the redistributed axial forces of concrete increase accordingly. It can also be seen from figure 3, the redistributed bending moment variations of steel beam are proportional to the section height $h_i$, which increases uniformly with $h_i$.

Under the same load, the beam with small stiffness ($j$ large) has larger initial deflections, and the development of the deflections is the same as that of $j$. It can be seen from figure 4, the beam of $j \leq 0.2$ has close deflections. Using the deflections of each time to compare with the initial deflections, the growth multiple of the deflections is obtained. It can be seen from figure 5, with the increase of constraint stiffness of the steel beam, the growth multiple of deflections increases uniformly, which has the same trend as the change of the redistributed bending moments of the steel beam when the constraint is increased.

It can be seen from the above analysis that the constraint degree of the steel beam on the concrete is the key factor causing stress redistribution and deformation change of composite beams.  

**Figure 2. The redistributed axial force of the concrete slab on the 8 sections**

**Figure 3. The redistributed bending moment of the steel beam on the 8 sections**

**Figure 4. The mid-span deflections of the composite beam with 8 sections**

**Figure 5. The variation multiple on deflections of the composite beam with 8 sections**

4. Conclusions
This article presents a numerical analysis of key influence factors of creep and shrinkage performance of composite beams. The constraint coefficient $j$ is used to judge constraint degrees of the steel beam on the concrete in composite beams, and the finite element analysis of 8 kinds of sections of composite beams with different constraint degree is carried out with an example. The analysis indicates the axial and bending rigidities of the steel beam increase with the section height $h$, which leads to an increase in both constraint degree and distributed internal force. The final internal forces of the concrete slab decrease and the internal forces of the steel beam increase. The variations of the midspan deflections also increase with the constrain degree, which shows the same trend as the increase of constraint. The constraint degree of the steel beam on the concrete is the key influence factor causing stress redistribution and deformation change of composite beams.

Acknowledgments
The work presented in this paper was financially supported by the National Natural Science Foundation of China (Grant Nos. 51708486 and 51668027).

References
[1] Ban, H., Uy, B., Pathirana, S.W., et al. (2015) Time-dependent behaviour of composite beams with blind bolts under sustained loads. Journal of Constructional Steel Research, 112:196-207.
[2] Nguyen, Q.H., Hjiaj, M.(2016) Nonlinear Time-dependent behavior of composite steel-concrete beams. Journal of Structural Engineering, 142: 151-157.
[3] Gattesco, N., Macorini, L., Fragiacomo, M. (2010) Moment redistribution in continuous steel-concrete composite beams with compact cross section. Journal of Structural Engineering, 136:193-202.
[4] Han, C.X., Zhou, D.H., Yang, Y.H., et al. (2016) Use of an algebraic constitutive equation to calculate the stress redistribution due to creep and shrinkage of composite beams. Journal of Harbin Engineering University, 37:1041-1049.
[5] Virtuoso, F., Vieira, R. (2004) Time dependent behaviour of continuous composite beams with flexible connection. Journal of Constructional Steel Research, 60:451-63.
[6] Gara, F., Ranzi, G., Leoni, G. (2006)Time analysis of composite beams with partial interaction using available modelling techniques: a comparative study. Journal of Constructional Steel Research, 62:917-30.