Form-finding Design of Cable Network Antenna with Truss System by Force Density Method

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Abstract: In recent years, the design and development of space-borne large-scale deployable antennas represented by ring truss deployable mesh antennas has attracted the attention of scholars in the aerospace industry from all over the world. Force density is the most widely used form finding algorithm for mesh antennas. The force density method assumes that the truss nodes of the antenna are rigidly connected. In this paper, an integral form finding algorithm for cable truss antenna is proposed, and the difference between the two algorithms is compared.

1. Introduction
With the increasing complexity and diversity of space-based satellite applications such as resource monitoring, disaster assessment, electronic reconnaissance, and earth observation. Higher requirements are put forward for space-borne antennas, and urgently needs antenna with diameter of tens of meters or even larger[1,2]. Compared with other types of satellite-borne antennas, the cable truss mesh antenna has higher retraction ratio, smaller mass diameter ratio, good scalability and excellent surface accuracy, which makes it one of the best structures of large aperture space borne reflector antenna. [3]

Figure 1 Typical cable truss network reflector

Fig 1 shows a typical mesh reflector, which is composed of a ring truss and a pair of mirror symmetric front and rear cable networks that are mutually tensioned by low stiffness tie springs. This group of tensioned springs is called tie force. Usually the entire cable truss system is carefully designed to form a statically determinate structure. After the antenna is deployed, the front and rear networks are...
tensioned with the tie force springs, so that the cables on the entire net surface produce pretension. The front mesh surface forms an approximate standard paraboloid. If the pretension disappears, the cables will relax. A basic problem in designing a mesh reflector is to find a suitable mesh division and pretension to ensure that all ropes are tensioned so that the front rope network forms the designed parabolic shape. It has been studied extensively in the literature[4-6], and many form-finding methods have been formed, and many form finding methods have been proposed. Among these methods, the force density method (FDM) [7-9] introduces the concept of "force density". By defining the force and length of the cables as force density, the geometrically nonlinear problem of cable networks balance is converted into a linear problem, which greatly simplifies the cables tension solution process. So the force density method is widely used in the form finding of cable truss antenna.

The force density method usually assumes that the truss nodes connecting the edge of the cable network are rigidly fixed[9], that is, the force and deformation state of the ring truss is not considered. After the cable networks complete the force density form finding of the fixed truss node, the balance of the truss node is solved by the tension of the rope connected to the truss. However, we have studied the form-finding design of the cable net by the geometric method. This method considers the forces of the cable and the truss members at the same time, and the form-finding result is that the ring truss members are only subjected to pressure. Through comparison, it is found that there is a big difference between the results of cable-truss form finding entirety and fixed truss node form finding.

In this paper, based on the force density method, a form finding algorithm of cable truss Integral is proposed. The difference between the form finding results of fixed joints and that of trusses is compared to verify the difference between the two results, and the reasons for the difference are analyzed. The rest of this article is organized as follows. In Section 2, the form finding principle of force density method and the fixed node form finding algorithm of iterative force density method are introduced. In Section 3, the form finding method of force density method for truss with ring is proposed. Section 4 analyzes and summarizes the form finding results.

2. Force density method for cable network antenna form finding

The purpose of cable net form-finding is to find the position of the balance nodes under the pretension of the cables. Therefore, the mathematical formula of cable network form-finding mainly involves the balance equation of the free node, and this equation is a nonlinear equation. Force density method defines the ratio of rope tension and length as force density value, namely

\[ q_{ij} = \frac{T_{ij}}{L_{ij}} \]  

Where \( T_{ij} \) and \( L_{ij} \) are the tension and length of the cable connected to the node numbered \( i \) and \( j \) respectively. Then the balance of the nodes can be transformed into a linear equation

\[
\begin{align*}
C^T QCx &= P_x \\
C^T QCy &= P_y \\
C^T QCz &= P_z
\end{align*}
\]  

Where \( P_x = [p_{1x}, \ldots, p_{nx}]^T \), \( P_y = [p_{1y}, \ldots, p_{ny}]^T \) and \( P_z = [p_{1z}, \ldots, p_{nz}]^T \) are the nodal load vectors, \( x \), \( y \) and \( z \) are the node coordinate vectors, \( Q \) is the diagonal matrix of rope force density values, \( C \) is the topological matrix of the cable-truss structure.

The force density method of cable net antenna form-finding usually assumes that the nodes on the ring truss are rigidly fixed. As shown in the Fig. 2. Suppose the number of fixed nodes on the truss is \( n_f \), the coordinate matrix is \( (x_1, y_1, z_1) \), the number of free nodes is \( n_f \), and the coordinate matrix is \( (x_f, y_f, z_f) \). The topological matrix is also divided into two parts according to the fixed node and the
free node, namely $C_s$ and $C_f$. Taking the balance equation in the x direction as an example, the balance equation in the x direction of Equ. 2 can be expressed as

$$
\begin{bmatrix}
C_f^T \\
C_s^T
\end{bmatrix}
\mathbf{Q}
\begin{bmatrix}
x_f \\
x_s
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
P_{sg}
\end{bmatrix}
$$

(3)

Where, $P_{sg}$ is the constraint reaction force of the truss nodes. Equ. 3 can be simplified to

$$
C_f^T \mathbf{Q} C_f x_f + C_f^T \mathbf{Q} C_s x_s = 0
$$

(4)

$$
C_s^T \mathbf{Q} C_f x_f + C_s^T \mathbf{Q} C_s x_s = P_{sg}
$$

(5)

Given the cable force density and the topological connection of the cable and truss of the antenna, the x coordinate of the cable net nodes can be obtained by solving the linear Equ.4, and the constraint reaction force of the truss node in the x direction can be obtained by Equ.5. The y coordinate of the cable network nodes and the constraint reaction force of the truss node in the x direction can also be obtained. Since the cable network nodes are all located on the standard paraboloid, the z coordinate can be obtained by the xy coordinates of the nodes, and then the z-direction load of the nodes, that is, tie force tension, can be solved by the balance in the z direction of Equ. 2.

On the other hand, given the balance configuration requirements of the cable net, it is difficult to predict the rope force density distribution that meets the requirements. Because the cable network reflector antenna has high requirements for the position accuracy of the node after balance, and has specific requirements for the tension distribution of the cables, In order to obtain the cable force density value that meets the requirements of the node position or the tension distribution, the iterative force density method[10] is usually used to solve the cable network form-finding problem. In the iterative process, the force density value of each step is calculated according to the constraints of the cable network and the result of the previous step iteration.

3. Form finding of cable net reflector with truss

The physical basis of the form-finding problem of the cable-truss reflector is the statically determinate problem of the cable net truss system. $n_f$ is the number of internal the cable nodes. B is the number of members of the ring truss, which is equal to the number of truss nodes $n_s$, namely $b = n_s$, and c is the number of ropes. In addition to the 6 constraint degrees of freedom, $2 \times 3 \times (n_f + b) - 6$ force
density linear equations can be established for the balance of the cable and the truss nodes, and the
number of force density values of the cable, cross bar and vertical bars is \(2(c + b + b)\). The statically
determinate cable network satisfies Maxwell’s equation

\[
0 = 3(n_f + b) - (b + c + b + 3) = 3n_f + b - c - 3
\]

Equation 6, the number of linear equations of force density is equal to the number of force densities
of all cables and trusses. Therefore, if the tie force, the topology and geometry sharp of the cable net are
known. Then a group force density of cables and trusses that meet the requirements can be uniquely
determined by solving the linear equations. Equ.2 can be used to obtain the force of the cables and the
trusses in the static equilibrium state. The trusses obtained by this result are all subjected to pressure
only.

However, because the rope can only be subjected to tension, if the force density during tension is set
to be positive, when solving the above equations, it is necessary to ensure that the force density of the
rope is greater than 0. But the tension of any set of tie force may not be sufficient for a positive force
density of all cables. Therefore, the key problem of the integral form-finding of the cable rod is to find
a set of tension arrays that satisfy all the rope tension values.

Since the fixed truss node form-finding and the integral form-finding cable network in the same
form-finding case have the same geometry sharp and topology, the only difference is the treatment of
truss node. Therefore, we take the value of the tie force obtained by the fixed truss node form-finding
into the form-finding integral force density equation, and the force densities are all positive, which meets
the form-finding requirements.

In summary, the integral form-finding force density method can be completed by the process shown
in Fig. 3. This process will be demonstrated in the next section with a frontfed case and an offset case,
and compare the results of fixed truss node form-finding.

4. Example of form finding of cable truss reflector

4.1 Form Finding of Frontfed Mesh Reflectors
Here, the proposed integral form-finding method was used to find the balanced forces of frontfed mesh
reflect. An frontfed mesh reflectors example has a circular aperture of \(D = 12\) m and a focal length of \(f = 7.5\) m.

The reflection surface is meshed with 30 border nodes and 121 inner nodes on the paraboloid. The
projections of those node on the xy plane are arranged to make sure that the nodes, located within the
inner regular hexagon of the circle, are uniformly distributed.

When form finding, it is first assumed that the 30 nodes of the ring truss are rigidly fixed and the
force state of the truss is not considered, as shown in Fig. 2, the force density iteration method of Equ.4 and Equ.5 is used to complete the fixed truss nodes antenna network form finding design, and the terminated condition of the iteration is that the maximum error between the form-finding cable length and the target cable length is less than 0.01mm. Then the tie force obtained from the fixed truss nodes is brought into the integral form-finding, and the integral form-finding cables and bars force density is obtained. Finally, the cables tension is calculated according to the cables force density and the cables length. The results of iterative form-finding and form-finding with truss cable net are shown in Figure 4 (a~c).

![Figure 4 Comparison of form finding results of frontfed reflector](image)

4.2 Form Finding of Offset Mesh Reflectors

For an offset cable net antenna, its reflector surface is the paraboloid intersects the cylinder. The rotation axis of the cylinder is parallel to the rotation axis of the paraboloid, but rotation axis of the cylinder is an offset distance from the rotation axis of the paraboloid.

The offset mesh reflectors form-finding is basically the same as the frontfed antenna. Take the offset antenna with a focal length of f=7.5m paraboloid as an example, the cylinder diameter is 12m, and the offset of the cylinder is D=9m. In this case, the ring truss node on the reflector surface is located on an ellipse with semi-minor axis d/2=6m and semi-major axis D/2=6.99m. Similar to the previous frontfed example. The results of iterative form-finding and form-finding with truss cable net are shown in Fig. 5 (a~c).
5. Conclusions

Comparing the results of the fixed truss node form-finding and the integral form-finding in the above examples, since the tie force value of integral form-finding is the result of the fixed truss nodes form-finding, that is, the tie force value of the two form-finding methods exactly the same. However, the cables tensions calculated by the two form-finding methods are quite different. The hinges of the integral form-finding are equivalent to spherical hinges. Therefore, all cables of the cable truss system are in tension state and all cables are only in pressure state.

For the cable-truss system of statically determinate structure, a certain set of tie force corresponds to a set of unique solutions.

In engineering, ring truss members often use fixed-angle hinges to fix the angle between adjacent truss members of the ring truss for structural stability. For the cable-truss system of statically determinate structure, a certain set of tie force corresponds to a set of unique solutions. It can be judged that the bars of the fixed truss nodes form-finding will be affected by the bending moment. Since the bars of the truss are slender thin-walled composite materials. The deformation under pressure is much smaller than the deformation under bending moment, so the fixed node form-finding may lead to a larger deformation of the entire ring truss, and the reliability of the members under bending is also lower than that under normal pressure. Therefore, compared with the fixed truss nodes form-finding, the form-finding of the statically determinate structure system composed of the cable net-truss as a whole can be solved to obtain better form-finding results. The cable truss integral form-finding can be solved to obtain better form-finding results.
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