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A New Formula for Calculation of Optimum Displacement and Its Effects

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Abstract

In maritime transport, fuel consumption is one of the biggest costs. So, various methods are used to reduce fuel consumption. The most common of these methods is to reduce the cruise speed of the ship. However, decreasing the voyage speed causes an increase in ship time. Nevertheless, the cruise speed is not only parameter which effects the fuel consumption. Weather condition, weight of the ship and even hull cleansing can affect the consumption. In this study, the effect of speed reduction and the effect of weight reduction were analyzed, and weight optimization was made for a ferry. In addition, cost of this reductions and amount of CO₂ emissions were compared. Finally, the advantages of weight optimization were revealed.

Keywords: Fuel Consumption, CO₂ Emission, Maritime Transport

Introduction

In maritime transport, all of the fuel and lube oil costs are referred to as bunker costs and this cost varies depending on the ship type and size. The main factors that technically affect this cost are the type, age, power in kilowatts of the ship's main engine, the type of fuel burned in the machine (HFO Heavy Fuel Oil, Marine Diesel Oil, etc.) (Beşik, Şıhmantepe, 2020). Considering that the fuel consumed on the ship accounts for more than 60-70 percent of the total cruising cost on average (Alexadridis et al., 2018). So, fuel consumption is the biggest expense item for ships. In addition, reducing fuel consumption can reduce CO₂ emission of a ship (Kiliç and Deniz, 2009).

Various methods are used to reduce fuel consumption. The most common of these methods is to reduce the cruise speed of the ship. However, decreasing the voyage speed causes an increase in ship time. Nevertheless, the cruise speed is not only parameter which effects the fuel consumption. Weather condition, weight of the ship and even hull cleansing can effect the consumption.

In this study, two conditions are analyzed. The effect of speed reduction and the effect of weight reduction. Of course, cargo weight can not be reduced because of cargo amount depends on the demand. So, weight of the bunker can only be reduced.

Literature Review

Alderton published a formula for consumption of a ship (Alderton, 1981). In this formula, weight of the ship was neglected. According to this formula, the fuel consumption was directly proportional to the cube of the speed. Then Ronen and Chrzanowski used this formula in their studies (Ronen, 1982; Chrzanowski, 1989). Barras published a formula for fuel consumption which does not neglect the weight of the ship (Barras, 2004). In this formula, displacement of the ship was added to Alderton’s formula. So, the formula was modified to

\[ C(v) = \lambda v^\frac{2}{3} V^2. \]

Where \( V \) is displacement tonnage of a ship. If the service speed is 20 kt or greater, it is more accurate when making comparisons, to change the power of velocity from being three to being four. Kim, Chang, Kim, and Kim determined amount of fuel and optimum vessel speed for a specific vessel route (Kim et al., 2012). The study was solved the problem by using epsilon-optimal algorithm. Considering more recent studies, Mersin et al. built up a new formula which does not neglect instant weight changing and showed that displacement tonnage at any time \( t \) is (Mersin et al., 2017);

\[ V(t) = \left( \frac{\sqrt{V(0)}}{3} - \frac{4v^2t}{3} \right)^3 \]

and fuel consumption for \( t \) day is

\[ C(t) = V(0) - \left( \frac{3}{\sqrt{V(0)}} - \frac{4v^2t}{3} \right)^3. \]

Bayırhan et al. analyzed the exhaust emissions generated by the ships of the local companies transporting in Strait of Istanbul (Bayırhan et al., 2019). Tokuşlu analyzed energy efficiency of a passenger ship
in Turkey (Tokuşlu, 2020). The Energy Efficiency Design Index (EEDI) of the ship was calculated. EEDI formula equations based on the study of passenger ships. Ülker et al. made a comparison between emissions of ro-ro and ferry lines (RFLs) in the Sea of Marmara and emissions of road transport (Ülker et al., 2020). Energy efficiency in terms of EEDI performance of sea buses which were operating in Istanbul Strait was analyzed. In terms of number sea buses, analysis showed over two thirds of the sea buses were not energy efficient. The analysis showed that speed reduction caused decrease in CO\textsubscript{2} emission and increase in energy efficiency (Tokuşlu, 2021-2022).

**Methodology**

In this study, the Trozzi & Vaccaro method is used for calculating CO\textsubscript{2} emission. According to the method, 3 different situations should be examined while making calculations. They are cruise mod, manoeuvre mod and port mod. Despite the estimated emission factors created by the machine types according to the cruise modes of the ships (cruise, maneuver, hotelling), CO\textsubscript{2} emission is 3.20 for each mode and for each machine type. In the light of all these data, the formula for the total CO\textsubscript{2} emission of a ship is given below;

\[
E(t_{total}) = \sum_{i=1}^{3} C \times f \times t \times p_i \quad (\text{eq.1})
\]

Where,

- \(E(t_{total})\): The total amount of CO\textsubscript{2} emissions per passenger at t-day sailing.
- \(C\):Fuel consumption (tonne)
- \(f = 3200 \text{ kg/tonne(CO}_2\text{ emission factor)}\)
- \(t\): time (day)
- \(p_1\): Sailing mode multiplier (0.8)
- \(p_2\): Maneuver, mode multiplier (0.4)
- \(p_3\): Hotelling mode multiplier (0.2)

**Scenario Analysis**

In this part of the study, two scenarios were analyzed for M/V Spokane ferry. This jumbo class ferry sails between Edmonds and Kingston and properties of the ferry is given at Figure 1.

![M/V Spokane](www.wsdot.wa.gov, Retrieved 02.01.2021)

In the first scenario, “ship speed” was reduced and reduction of total emission of the ship was calculated. In the second scenario, the ship had fuel enough to complete the voyage and the emission of the ship was compared with “full tank” emission of the ship. Although, number of carried passenger is assumed 2000 and all passengers are adults (age 19-64).

**Scenario 1.**

In this scenario, the amount of fuel in the tank is assumed 130000 gallons=419,328 tonne (it means...
The distance between Edmonds and Kingston is 5.67 nm. Nevertheless, the ferry can take this route in 24 minutes. So, speed of the vessel is 5.67/0.4 = 14 kt. Fuel consumption can be calculated with the formula which is given below:

\[
C(t) = \nabla(0) - \left(\frac{2}{3} \sqrt[3]{\nabla(0)} - \frac{2v^3 t}{3}\right)^3
\]

(1)

Table 1. Effects of fuel consumption with variable speed per voyage.

| Speed (kt) | Time (h) | Consumption (tonne) | CO2 Emission (tonne) | Cost (USD) | Reducing Rate |
|------------|----------|---------------------|----------------------|------------|---------------|
| 14         | 0.40     | 0.109331747         | 0.279889271         | 54.66587328 | 0%            |
| 12.6       | 0.44     | 0.088558841         | 0.226710633         | 44.27942046 | 10%           |
| 11.2       | 0.50     | 0.699725070         | 0.179129618         | 34.98625337 | 20%           |
| 9.8        | 0.57     | 0.535727610         | 0.137146267         | 26.78638037 | 30%           |
| 8.4        | 0.67     | 0.039359618         | 0.100760622         | 19.67980885 | 40%           |
| 7          | 0.80     | 0.027333090         | 0.069972711         | 13.66654520 | 50%           |
| 2.8        | 2        | 0.004373301         | 0.011956561         | 2.186665068 | 80%           |

Table 2 Effects of reducing ship speed on daily income.

| Speed (kt) | Carried passenger | CO2 Emission (tonne) | Consumption (tonne) | Income (USD) | Cost (USD) | Profit (USD) |
|------------|-------------------|----------------------|---------------------|--------------|------------|--------------|
| 14         | 46000             | 8.046816547          | 6.437453238         | $416300      | $1257.315085 | $415042.6849 |
| 12.6       | 40000             | 5.667765819          | 4.334212655         | $362000      | $885.5884092 | $361114.4116 |
| 11.2       | 36000             | 4.030416388          | 3.22433311          | $325800      | $629.7525606 | $325170.2474 |
| 9.8        | 32000             | 2.742925350          | 2.194430428         | $289600      | $428.5820860 | $289171.4179 |
| 8.4        | 26000             | 1.637360969          | 1.309888077         | $235300      | $255.8375151 | $235044.1625 |
| 7          | 22000             | 0.962124782          | 0.769699826         | $199100      | $150.3319972 | $198949.668 |
| 2.8        | 8000              | 0.055978257          | 0.044782606         | $72400       | $8.74660272  | $72391.2534  |

Table 3. Effects of fuel consumption with variable fuel amounts.

| Fuel Amount (tonne) | Displacement (tonne) | Consumption (tonne) | CO2 Emission (tonne) | Cost (USD) | Reducing Rate |
|---------------------|----------------------|---------------------|----------------------|------------|---------------|
| 419.328             | 4859                 | 0.109331747         | 0.279889271         | 54.66587328 | 0%            |
| 377.3952            | 4817.0672            | 0.108701820         | 0.278276658         | 54.35090978 | 10%           |
| 335.4624            | 4775.1344            | 0.108070062         | 0.276659358         | 54.03503102 | 20%           |
| 293.5296            | 4733.2016            | 0.107436453         | 0.275037318         | 53.71822625 | 30%           |
| 251.5968            | 4691.2688            | 0.106800969         | 0.273410481         | 53.40048455 | 40%           |
| 209.664             | 4649.3360            | 0.106163589         | 0.271778789         | 53.08179471 | 50%           |
According to the Table 3, reducing the amount of fuel in the tank reduces fuel consumption. But, this reducing has to be stopped at an optimum fuel amount. Because the ship must have fuel enough to complete the voyage. The question is “what is the optimum amount of fuel to complete the voyage?”

**Theorem:** Let W be the total weight of the cargo and light ship weight and F(0) be the weight of the fuel that the vessel has at time t=0. The optimum amount of bunker that vessel should have is

\[
F(0) = \left( \sqrt{\frac{W}{3}} + \frac{\lambda v^3 t}{3} \right)^3 - W
\]

**Proof:**
It could be calculated that the displacement of a ship at any time t is \(\nabla(t) = \left( \sqrt{\nabla(0)} - \frac{\lambda v^3 t}{3} \right)^3\) where \(\nabla(0)\) is the displacement of the ship at time t=0. So, fuel consumption of the ship for t-day is \(C(t) = \nabla(0) - \nabla(t)\). In this part of the proof, \(W+F\) will represent the displacement of the ship where \(W=\) the weight of the cargo + weight of the light ship and \(F\) is the weight of the fuel that vessel has. So,

\[
C(t) = \nabla(0) - \nabla(t) = \nabla(0) - \left( \sqrt{\nabla(0)} - \frac{\lambda v^3 t}{3} \right)^3 = (W+F(0)) - \left( \sqrt{(W+F(0))} - \frac{\lambda v^3 t}{3} \right)^3
\]

It is obvious that \(W\) is constant and \(F\) is variable during the voyage. So, the above formula can be rewritten as Table 4. Effect of optimum fuel amount.

| Fuel Amount (tonne) | Displacement (tonne) | Consumption (tonne) | CO₂ Emission (tonne) | Cost (USD) | Reducing Rate |
|---------------------|----------------------|--------------------|---------------------|------------|---------------|
| 419.328             | 4859                 | 0.109331747        | 0.279889271         | 54.66587328 | 0%            |
| 377.3952            | 4817.0672            | 0.108701820        | 0.278276658         | 54.35090978 | 10%           |
| 335.4624            | 4775.1344            | 0.108070062        | 0.276659358         | 54.03503102 | 20%           |
| 293.5296            | 4733.2016            | 0.107436453        | 0.275037318         | 53.71822625 | 30%           |
| 251.5968            | 4691.2688            | 0.106800969        | 0.273410481         | 53.40048455 | 40%           |
| 209.664             | 4649.3360            | 0.106163589        | 0.271778789         | 53.08179471 | 50%           |
| **0.109**           | **4439.781**         | **0.102949115**    | **0.263549735**     | **51.47455771** | **99%**       |

Table 5 Effects of reducing fuel amount on daily income

| Fuel Amount (tonne) | Carried passenger | CO₂ Emission (tonne) | Consumption (tonne) | Income (USD) | Cost (USD) | Profit (USD) |
|---------------------|-------------------|----------------------|--------------------|--------------|------------|--------------|
| 419.328             | 46000             | 8.046816547          | 2.51430171         | $416300      | $1257.315085 | $415042.6849 |
| 377.3952            | 46000             | 8.000453919          | 2.500141850        | $416300      | $1250.070925 | $415049.9291 |
| 335.4624            | 46000             | 7.953956565          | 2.485611427        | $416300      | $1242.805713 | $415057.1943 |
| 293.5296            | 46000             | 7.907322904          | 2.471038408        | $416300      | $1235.519204 | $415064.4808 |
| 251.5968            | 46000             | 7.860551325          | 2.456422889        | $416300      | $1228.211145 | $415071.7889 |
| 209.664             | 46000             | 7.813640181          | 2.441762557        | $416300      | $1220.881278 | $415079.1187 |
| **0.109**           | **46000**         | **7.577054895**      | **2.367829655**    | **$416300**  | **$1183.914827** | **$415116.0852** |
In this scenario, carried passenger and income do not depend on reducing rate. So, this method can be more profitable than reducing speed method. Table 5 shows the effects of reducing fuel amount on daily income.

**Discussion and Conclusion**

In this study, two different scenarios’ performances had been illustrated in which environmental and financial impacts were taken into consideration. Performances of these scenarios were evaluated through amount of CO₂ emission release, profit and fuel cost in this paper. Independent variables of the scenarios were speed and initial tank fuel amount. While one of the independent variables was kept as constant in each scenario, a set of values was assigned to the other independent variable. Analysis carried out on the values of amount of CO₂ emission release amount, profit and fuel cost. Both single voyage and daily based values were subject to analyze. It was seen that amount of CO₂ emission release and fuel cost should be evaluated together. In the initial scenario, the effect of ship speed was analyzed. It had been observed that the percentage change in speed, CO₂ emission release amount and fuel cost were moving in the same direction. In addition, the percentage change in CO₂ emission amount and fuel cost were equal due to the formula in which they are being calculated. For any given speed value, voyage based percentage change values for CO₂ emission amount and fuel cost were higher than daily figures. It was stand out as a result of keeping the speed constant. In the relevant scenario, it had been seen that the profit values move parallel to the speed value. It was a natural result of decreasing in engine operation times. The decrease in operation times. For CO₂ emission amount and fuel cost, exact parallelism between daily and voyage based values could not be observed on percentage change values. It was due to slight difference in engine operation times. The decrease in speed resulted a negative impact on the profit. In the relevant scenario, it had been seen that the profit values move parallel to the speed value. It was a natural result of decrease in the number of trips made at decreased speed. As a result of decrease in number of trips on daily basis yielded significant drop on sales figures. Decrease in the number of trips caused the decrease in fuel cost which had a positive impact on profit. It had been observed that the percentage drop in profit was greater than percentage drop value in speed. The most important point to mention in this issue was the CO₂ emission tax. CO₂ emissions could be taxed at certain countries. The tax rate in Finland, British Columbia and BAAQMD, California were $30, $0.045 and $9.50 per metric ton CO₂ or CO₂ equivalent respectively in 2008(Sumner et al.,2009). Decrease in CO₂ emission would result in decrease in related tax which would result in profit increase. In the second scenario, while speed was taken as constant initial tank fuel amount was taken as independent variable. As a result of keeping the speed constant, the number of trips and sales value during the day was constant for all alternatives. Daily and voyage based percentage change values for CO₂ emission amount and fuel cost were same for all initial fuel amount alternatives. This was another result of keeping speed and number of daily trips constant. Reducing the initial fuel quantity to the minimum did not make any significant effect in profit. Operating with minimum fuel quantity bring operational load. The result of the effort yield %6 drop on CO₂ emission and fuel cost. Even though the figure was relatively low compared to available drop in previous scenario, it should be noted that the reduction is obtained without any profit sacrifice. Any reduction on possible CO₂ tax payable could have a positive in impact on profit amount. The study had shown that among the two scenarios, speed reduction yields significant drop in CO₂ emission amount. The minimum fuel tank scenario would be more desirable option when not only CO₂ emission and fuel cost reduction aimed but also profit increase was desired

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