Dynamic modelling of covid-19 and the use of “Merah Putih” vaccination and herbal medicine treatment as optimal control strategies in Semarang city Indonesia

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Abstract. The Corona virus is a disease that threatens human population on earth in early 2020, the first infection began in China, is thought to be a virus that host in mammals and then able transmits into human body, this present paper will describe dynamic system of Covid-19, analyzing direction of the equilibrium, Disease Free Equilibrium (DFE) and Endemic Equilibrium (EE), which can be done by determining dimensionless number of Basic Reproduction Number ($R_0$). And then we put vaccine and treatment as control variables to reduce the infected population, we use the vaccinations and treatments recommended by the Indonesian government. Furthermore, we use Pontryagin Minimum Principle to find optimal solution of the control. Optimal control method in this paper is fixed time and fixed end point, means that we are determined the duration of the control and set goal for reaching the final state point. This optimal control will aim to minimize the number of infected population and values (cost) of control measures. Numerical calculations also performed to illustrate the graph.

1. Introduction
The use of dynamic system applications in biology has shown significant changes in mathematics and biosciences in recent years [1]. This dynamic system will use the optimal control laws to look for optimization of functional objectives for a certain period of time, to find the optimum control of the dynamic system, Pontryagin Minimum Principle will be used [2]. The type of optimal control in this discussion is fixed time and fixed end point which means that we are determined the duration of the control and set goal for reaching or achieving the final state point, the strength of this method (fixed time and fixed end point) is that we can determine the number of final point of the state [3-6] for example if the initial data from the state model let's say there are $\alpha_1$ infected people ($x(0) = \alpha_1$), then using this method we can determine that the final data (final point) from the state model should become $\alpha_2$ infected people during a certain period of time ($x(T) = \alpha_2$), with $\alpha_2 < \alpha_1$ which means that the control variable able to reduce the infected population in certain period of time $T$.

In 2019, in the city of Wuhan there was a corona virus outbreak that infected and killed thousands of people. and then Covid-19 is the name given by the world researcher and SARS Cov 2 as the name of the virus [7]. This outbreak began in the Hunan area in the seafood market. where many small mammals are traded especially bats, based on information from WHO [8]. Initially palm civets and raccoon dogs were thought by researchers to be the source of the infection, and the researchers suggested that the palm civet might be the secondary hosts [9]. In Hong Kong there is a sample from a
person that shows of antibodies against SARS-coronavirus, it means corona virus has already circumventing among human in 2003 [10]. The researchers later suggested that The Rhinolophus bats were the source of the virus [11].

In this paper we are going to simulate optimal control model of covid-19 using vaccination and treatment recommended by Indonesian government as control variable using data from official website of Semarang city government. The vaccination here is a vaccination called the “Merah Putih” vaccine, this “Merah Putih” vaccine was recommended by the Indonesian government and use of herbal medicine treatments [12-14] derived from native Indonesian plants such as “Daun Sembung”, “Daun Meniran”, “Jahe Merah”, and “Sambiloto”. The “Merah Putih” vaccine here based on the news from www.kompas.com and www.cnbindonesia.com.

2. Method
In this section we will divide the total population into three compartment, which is population of susceptible ($S$), infected ($I$), and recovered ($R$).

The mathematic model of Covid-19 spread without control is:

$$\dot{S}(t) = rN_s - \frac{\beta I(t)S(t)}{N_s} - \mu S(t)$$

$$\dot{I}(t) = \frac{\beta I(t)S(t)}{N_s} - \theta I(t) - \delta I(t) - \mu I(t)$$

$$\dot{R}(t) = \theta I(t) - \mu R(t)$$

(1)

The mathematic model of Corona virus spread based on Figure1 with control are:

$$\dot{S}(t) = rN_s - \frac{\beta I(t)S(t)}{N_s} - \mu S(t) - u_1(t)$$

$$\dot{I}(t) = \frac{\beta I(t)S(t)}{N_s} - \theta I(t) - \delta I(t) - \mu I(t) - u_2(t)$$

$$\dot{R}(t) = \theta I(t) - \mu R(t) + u_1(t) + u_2(t)$$

(2)

With $S(0) \geq 0$, $I(0) \geq 0$, $R(0) \geq 0$, as initial condition

$\dot{S}, \dot{I}, \dot{R}$ equal the rate of population change of the susceptible, infected, and recovered respectively, $\frac{\beta I(t)S(t)}{N_s}$ is rate of transmission from susceptible to infected, $N_s$ here is equal with number of the initial value of susceptible population, $rN_s$ is source of susceptible rate, $\theta I(t)$ is transmission rate from infected to recovery, $\mu S(t)$, $\mu I(t)$, $\mu R(t)$ is natural death rate of susceptible, infected and recovered population respectively, $\delta I(t)$ is death rate caused by disease with $\beta, r, \theta, \delta, \mu$ are the parameters. Dynamic system of (1) is taken from [15] and then modifying it by giving source of susceptible rate, natural death rate in each compartment, death rate caused by the disease, and given control function $u_1(t)$ and $u_2(t)$. Here $u_1(t)$ means how many susceptible people are required to be vaccinated and $u_2(t)$ means how many infected people are required to be treated simultaneously.

2.1. Basic Reproduction Number
Basic Reproduction Number $R_0$ is defined as the expected number of secondary infection produced by a single infection in a completely susceptible population. The idea of $R_0$ is developed by Driessche P and Watmough J [16-17]. Basically $R_0$ is used to determine the spread level of the disease. Basic Reproduction Number can be determine using next generation matrix, let $F_j(x)$ define rate of new infection and $V_j(x)$ define rate of individual displacement, then define:
\[
F = \left[ \frac{BS}{N_\sigma} \right], \quad V = [\theta + \mu + \delta], \quad \text{Jacobian} \quad F = \bar{F} = \left[ \frac{BS}{N_\sigma} \right], \quad \text{Jacobian} \quad V = \bar{V} = [\theta + \mu + \delta]
\]

Then
\[
\bar{F}V^{-1} = \left[ \frac{BS}{N_\sigma(\theta + \mu + \delta)} \right]
\]

So that:
\[
R_0 = \frac{\beta r}{\mu(\theta + \mu + \delta)}, \quad \text{with} \quad S^0 = \frac{r}{\mu} N_\sigma
\]

2.2. Disease Free Equilibrium

**Theorem 1.** If \( R_0 > 1 \), the disease free equilibrium point of system (1) is locally asymptotically stable, while if \( R_0 < 1 \), \( L^0 \) is unstable.

**Proof:**
Define \( L^0 = (S^0, I^0, R^0) \) with \( S^0 = \frac{r}{\mu} N_\sigma \), \( I^0 = 0 \), \( R^0 = 0 \)

Jacobian \( L^0 = \)

\[
\begin{bmatrix}
  -\mu & -\frac{\beta r}{\mu} & 0 \\
  0 & \frac{\beta r}{\mu} - \theta - \mu - \delta & 0 \\
  0 & \theta & -\mu
\end{bmatrix}
\]

The characteristic equation
\[
(\lambda + \mu)^2(\lambda + a_1) = 0
\]

where
\[
a_1 = (1 - R_0)(\mu + \delta + \theta)
\]

There are two eigenvalues from equation (7) which have negative real parts \( \lambda_2 = \lambda_1 = -\mu \). Then we need to consider the other eigenvalue
\[
(\lambda + a_1) = 0
\]

According to the Routh-Hurwitz criteria [18], the eigenvalues of (9) have negative real parts if and only if \( a_1 > 0 \), using equation in \( a_1 \) eigenvalues of (10) will be negative if \( R_0 > 1 \)

2.3. Endemic Equilibrium

**Theorem 2.** If \( R_0 > 1 \), the endemic equilibrium point of system (1) is locally asymptotically stable, while if \( R_0 < 1 \), \( L^* \) is unstable.

**Proof:**
Define: \( L^* = (S^*, I^*, R^*) \) with \( S^* = \frac{(\theta + \mu + \delta)N_\sigma}{\beta} \), \( I^* = \frac{(R_0 - 1)\mu N_\sigma}{\beta} \), \( R^* = \frac{(R_0 - 1)\theta N_\sigma}{\beta} \)

Jacobian \( L^* = \)

\[
\]
The characteristic equation
\[(\lambda + \mu)(\lambda^2 + c_1 \lambda + c_2) = 0\]  
(11)

Where
\[c_1 = \mu + \frac{(R_0 - 1)\mu N_\sigma}{\beta}, \quad c_2 = (R_0 - 1)\mu\]  
(12)

There is one of the eigenvalues from equation (11), which have negative real parts \(\lambda_1 = -\mu\), Then we need to consider the other eigenvalue
\[(\lambda^2 + c_1 \lambda + c_2) = 0\]  
(13)

The eigenvalues of (13) have negative real parts if and only if \(c_1, c_2 > 0\), using equation in \(c_2\) eigenvalues of (13) will be negative if \(R_0 > 1\).

2.4. Optimal Control Analysis
To control the outbreak of Corona virus infection, we apply optimal control method, the purpose is to minimizing susceptible (S) and infected (I) population and the value (cost) of vaccination and treatment represent by \(u_1\) and \(u_2\) respectively.

Define cost function:
\[J(u_1, u_2) = \min \int_0^T \left[ 0.5(S(t)^2 + I(t)^2 + u_1^2(t) + u_2^2(t)) \right] dt\]  
(14)

Subject to:
\[
\dot{S}(t) = r N_\sigma - \frac{\beta I(t)S(t)}{N_\sigma} - \mu S(t) - u_1(t)
\]
\[
\dot{I}(t) = \frac{\beta I(t)S(t)}{N_\sigma} - \theta I(t) - \delta I(t) - \mu I(t) - u_2(t)
\]
\[
\dot{R}(t) = \delta I(t) - \mu R(t) + u_1(t) + u_2(t)
\]  
(15)

With initial condition \(S(0) \geq 0, I(0) \geq 0, R(0) \geq 0\)

Our purpose is to find control function of \(u_1(t)\) and \(u_2(t)\) such that:
\[J(u_1, u_2) = \min \left\{ J(u_1, u_2) \mid u_1, u_2 \in U \right\}\]  
(16)

subject to (2), and control set is given by:
\[U = \left\{ u_1, u_2 \mid u_i \text{ is Lebesque measurable on } [0, T], i = 1, 2 \right\}\]  
(17)

Note that, for the solution of the optimal control should exist non-negative initial conditions, positive bounded solutions to the system (2) and control are bounded Lebesque measurable. There exists an optimal control which minimizes \(J(u_1, u_2)\) if the following conditions are satisfied:

i. The variables of state and variables of control are positive.

ii. The control set \(U\) is closed and convex.

iii. The right hand side of the state system is continuous, is bounded above by a linear combination of the control and state, and can be written as a linear function of \(u\) with coefficients defined by the time and the state.
iv. The integrand of the objective functional is convex on $U$.

1. Matrix of Controllability

The controllability of system is needed in order to stabilize the system. In addition, solutions to an optimal control problem may not be obtained if the system concerned is not able to be controlled. Thus, it needs to be analyzed about the control of the system. Controllability can be analyzed by forming a control matrix and determining the number of ranks of the matrix [19]. Then define dynamic model of system (2) as:

$$\dot{x} = f(x) + g_1u_1 + g_2u_2$$

(18)

$$\dot{x} = \begin{bmatrix} \dot{S} \\ \dot{i} \\ \dot{R} \end{bmatrix}, \quad f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} = \begin{bmatrix} rN_{\sigma} - \frac{\beta S}{N_{\sigma}} - \mu S \\ \frac{\beta I}{N_{\sigma}} - \theta I - \delta - \mu \theta \\ \theta I - \mu R \end{bmatrix}, \quad g_1 = \begin{bmatrix} g_{1_1} \\ g_{1_2} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \quad g_2 = \begin{bmatrix} g_{2_1} \\ g_{2_2} \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

(19)

Then the Controllability matrix

$C = [g_1, g_2, [f, g_1], [f, g_2]]$

Where $[f, g_1], [f, g_2]$ are Lie bracket operation [12] and defined as:

$$[f, g_1] = \frac{\partial g_1}{\partial x} f(x) - \frac{\partial f(x)}{\partial x} g_1$$

$$[f, g_2] = \frac{\partial g_2}{\partial x} f(x) - \frac{\partial f(x)}{\partial x} g_2$$

(20)

Then

$$\frac{\partial f(x)}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial I} & \frac{\partial f_3}{\partial R} \end{bmatrix} = \begin{bmatrix} -\frac{\beta I}{N_{\sigma}} - \mu & -\frac{\beta S}{N_{\sigma}} & 0 \\ \frac{\beta I}{N_{\sigma}} & \frac{\beta S}{N_{\sigma}} & -\theta - \delta - \mu & 0 \\ 0 & \theta & -\mu \end{bmatrix}$$

(21)

$$\frac{\partial g_1(x)}{\partial x} = \begin{bmatrix} \frac{\partial g_{1_1}}{\partial S} \\ \frac{\partial g_{1_1}}{\partial I} \\ \frac{\partial g_{1_1}}{\partial R} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(22)

$$\frac{\partial g_2(x)}{\partial x} = \begin{bmatrix} \frac{\partial g_{2_1}}{\partial S} & \frac{\partial g_{2_1}}{\partial I} & \frac{\partial g_{2_1}}{\partial R} \\ \frac{\partial g_{2_2}}{\partial S} & \frac{\partial g_{2_2}}{\partial I} & \frac{\partial g_{2_2}}{\partial R} \\ \frac{\partial g_{2_3}}{\partial S} & \frac{\partial g_{2_3}}{\partial I} & \frac{\partial g_{2_3}}{\partial R} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(23)
These adjoin variables will maximize or minimize the state variable with respect to the state function. Then the optimal variables of control:

\[ \frac{\partial g_1(x)}{\partial x} = \begin{bmatrix} \frac{\partial g_{1x}}{\partial S} & \frac{\partial g_{1x}}{\partial I} & \frac{\partial g_{1x}}{\partial R} \\ \frac{\partial g_{2x}}{\partial S} & \frac{\partial g_{2x}}{\partial I} & \frac{\partial g_{2x}}{\partial R} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 \end{bmatrix} \] (24)

\[ [f, g_1] = \begin{bmatrix} -1 & \frac{\beta I}{N_\sigma} - \mu \\ -1 & \frac{\beta I}{N_\sigma} - \mu \\ 1 & \mu \end{bmatrix}, \quad [f, g_2] = \begin{bmatrix} -\frac{\beta S}{N_\sigma} \\ -\frac{\beta S}{N_\sigma} - (\theta + \delta + \mu) \end{bmatrix} \] (25)

Then the controllability matrix:

\[ C = \begin{bmatrix} -1 & 0 & -\frac{\beta I}{N_\sigma} - \mu & -\frac{\beta S}{N_\sigma} \\ 0 & -1 & \frac{\beta I}{N_\sigma} & \frac{\beta S}{N_\sigma} - (\theta + \delta + \mu) \\ 1 & 1 & \mu & \mu \end{bmatrix} \] (26)

has rank 3, so that system (1) is controllable.

2. Convexity
Convexity of the integrand of the objective functional with respect to the control variables is necessary to ensure that there is a global minimum value solution, meaning that any Pontryagin solution is optimal.

**Theorem 3.** The objective functional integrand of equation (14) is convex.

**Proof**
As we can see in the equation (14) the objective functional integrand equal

\[ L(S, I, u_1, u_2) = 0.5\left(S(t)^2 + I(t)^2 + u_1^2(t) + u_2^2(t)\right) \] (27)

are consist of addition of quadratic function, quadratic function is convex and the addition of quadratic function also convex.

2.5. Optimality Condition
In order to find the optimal solution, will be used Pontryagin Minimum Principle (PMP), then we define Hamiltonian function of the control:

\[ H = 0.5\left(S(t)^2 + I(t)^2 + u_1^2(t) + u_2^2(t)\right) + \lambda_1 \dot{S} + \lambda_2 \dot{I} + \lambda_3 \dot{R} \] (28)

Here \( \lambda_1, \lambda_2, \lambda_3 \), are adjoin variables that satisfies:

\[
\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial S} = -S + \lambda_1 \mu + \frac{\beta I (\lambda_1 - \lambda_2)}{N_\sigma}
\]

\[
\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial I} = -I + \frac{\beta S (\lambda_1 - \lambda_2)}{N_\sigma} + \lambda_2 (\delta + \mu + \theta)
\]

\[
\frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial R} = \lambda_3 \mu
\] (29)

These adjoin variables will maximize or minimize the state variable with respect to the state function.
\[
\frac{\partial H}{\partial u_1} = u_1 - \lambda_1 + \lambda_3 = 0 \implies u_1^* = \lambda_1 - \lambda_3 
\]
\[ (30) \]
with \( u_1 = u_1^* \)
\[
\frac{\partial H}{\partial u_2} = u_2 - \lambda_2 + \lambda_3 = 0 \implies u_2^* = \lambda_2 - \lambda_3 
\]
\[ (31) \]
with \( u_2 = u_2^* \)

3. Result and Discussion

The numerical analysis of the optimal control is formulated as boundary value problem and performed with MATLAB programming. Later on we will compare the graph of real data from Semarang city government official website with the graph of dynamic system (2) with fixed time \((T = 88\text{ days})\) and fixed end point \((x(0)\text{ and } x(T))\) are determined) optimal control, where \(x\) is the state variable \((x = S, I, R)\).

In this case of fixed time and fixed end point we should targeted on the 88th day the susceptible and infected population should reach a third of the final point (value) data from official website of Semarang city government, for example the initial data of susceptible population from website is 301 \((S(0) = 301)\) individuals and initial data of infected population from website is 36 \((I(0) = 36)\) individuals, final point (value) data of susceptible population on the 88th day from website is 606 \((S(88) = 606)\) individuals and final point (value) data of infected population on the 88th day from website is 955 \((I(88) = 955)\) individuals, so we need to targeted that on the 88th day the susceptible population should become 202 individuals (a third of 606), and infected population should become 319 individuals (a third of 955). To hit the targeted population, we need to conducting vaccination to a portion of susceptible individuals and treatment to a portion of infected individuals correspond with time \((t)\) using numerical calculation of optimal control.

And on the 88th day we also targeted that recovered population should become twice of the final point (value) data from official website of Semarang city government, for example the initial data of recovered population from website is 10 \((R(0) = 10)\) individuals, final point (value) data of recovered population on the 88th day from website is 980 \((R(88) = 980)\) individuals, so we need to targeted that on the 88th day the recovered population should become 1960 individuals (twice of 980).

Table 1 shows parameter data of transmission coefficient, mortality rate due to Covid-19 infection, recovery rate and source of susceptible rate of the dynamic system (1) and (2).

| Parameter | Description                        | Values | Reference |
|-----------|------------------------------------|--------|-----------|
| \(\beta\) | Transmission coefficient           | 0.25   | [15]      |
| \(\delta\) | Mortality rate due to infection    | 0.18   | Assumed   |
| \(\theta\) | Rate from infected to recovery     | 0.1    | [15]      |
| \(r\)    | Source of susceptible rate         | 0.3    | Assumed   |

Table 2 shows initial value (point) data of dynamic system (1) and (2) and final value (point) data, this final data will be compared with the iteration results of system (2).

| Parameter | Description                        | Values |
|-----------|------------------------------------|--------|
| \(S(0)\) | Initial value (point) of Susceptible | 301    |
| \(I(0)\) | Initial value (point) of Infected   | 36     |
Table 3 shows parameter data of natural death rate of the dynamic system (1) and (2).

| Parameter | Description | Values  
|-----------|-------------|---------|
| $\mu$     | Natural death rate | 0.00706 |

Source: https://semarangkota.bps.go.id.

Figure 1. Dynamic of susceptible population with optimal control compare to the real data

Figure 1 shows the dynamic of susceptible population of system (2) (blue line) compare to the real data of susceptible population from Semarang government official website (blackline).

Figure 2. Dynamic of infected population with optimal control compare to the real data
Figure 2 shows the dynamic of infected population of system (2) (blue line) compare to the real data of infected population from Semarang government official website (black line).

Figure 3 shows the dynamic of recovered population from of system (2) (blue line) compare to the real data of recovered population from Semarang government official website (black line).

After numerical calculations, as we can see in the simulation graph in Figs. 1 to 3 controls (vaccination and treatment) are applied to the system, and the controls significantly decrease the population of susceptible (Fig.1), and minimizing the population of the infected (Fig.2) at a predetermined final point (value) which is 202 individuals for the susceptible and 319 individuals for the infected, and also the number of population of the recovered increases at a predetermined final point (value) which is 1960 individuals (Fig.3).

The terms of \(-u_1(t)\) and \(-u_2(t)\) in the system equation (2) means that there is a reduction in the rate of change of susceptible population by \(u_1(t)\) and a reduction in the rate of change of infected population as much as \(u_2(t)\), using MATLAB software it is known that \(u_1(3) = 96.52\) with \(t = 3\) days. Means, when the rate of the susceptible population at \(t = 3\) days \(\dot{S}(3)\) must be reduced by 97 individuals (rounding up from 96.52). For the infected individuals, since \(u_2(3) = -60.3\), means we didn’t have to conducting treatment on the third day to the infected class.

For a further description, on the third day, we should target to vaccinate 97 susceptible individuals, when this is done, the susceptible and infected population will decrease on the 88th day rather than not being vaccinated and treated, and then the number of recovered populations will increase because people who have been vaccinated and treated are cured (considered cured). These vaccination and treatments should continue for the rest of the time until the targeted population achieved on the 88th day, with notes that the amount of \(u_1(t)\) and \(u_2(t)\) vary depending on time \(t\) (depending on numerical calculations of the dynamic system (2) and (29)). So this control value is used as a benchmark for how many people must vaccinated and treated on the third day.

The depiction of the use of the optimal control here in this paper can be seen in figure 4 below:
4. Conclusion
The use of control function with the Pontryagin Minimum Principle method which is applied to the dynamic modeling of Corona virus spread, can be seen in the numerical simulation in figure 1 and 2 there is a reduction in the population of susceptible and infected populations, which means conducting vaccination and treatment has an impact on reducing the population of susceptible and infected. In the end, use of control function in the dynamic model are expected to provide advice to the public health authorities to handle and control covid-19 outbreak in Semarang city.

Acknowledgement
Authors thank the lecturers who have provided lessons and suggestions for writing this article material

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