Mathematical model of the axial compressor blade in system assessing technical condition of the industrial gas turbine

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Abstract. The paper presents the key stages of developing a geometric model of the axial compressor blade. The model is based on Bezier curves, which are used to design the parts of the airfoil and allow making local changes to the airfoil and blade geometry. This model is designed to assess the technical condition of the axial compressor being a part of an industrial gas turbine or gas-turbine units, depending on the defects of axial compressor blade row (in the gas-flow channel) and the turbine as a whole. As a part of the study the algorithm, which describes the process of the blade formation, were presented. Although the study presents the main equations of the model based on the key geometric parameters of the airfoil, provides examples of airfoil, blade and stage modelling using the presented method, and shows the results of model verification using numerical simulation methods.

1. Introduction

Today, gas turbine units (GTU) are one of the most widely used machines for generating and converting energy. They cover almost the entire power range required: they are applied in power industry (as part of a combined cycle power unit), transport and other industries (oil and gas, chemical, etc.). In addition, the demand for gas turbines tends to increase across the world, which makes it important to maintain their technical condition and efficiency at a high level.

These parameters, however, largely depend on the technical condition of certain GTU components [1-3]. For instance, the axial compressor substantially contributes to the overall condition and performance of a GTU, since the power consumed by the former significantly exceeds half of the power generated by the turbine. The life cycle of both axial compressor and gas turbine unit will be determined in this case by the life cycle of the elements the most subjected to load – the blade row [4]. According to the statistics, blade damages are the second largest cause of accidents and shutdowns, while various geometric deviations of certain axial compressor blades occurring during its operation have a huge impact on the GTU performance. Making blade leading edge thinner may decrease the amount of losses (when operating at the rated speed) or increase them (when operating outside the optimal range of angles of attack), as well as decrease the operational stability margin of the axial compressor. Some more examples are presented in table 1. Almost all blade defects have such an inconsistent impact [5, 6], so their assessment and analysis, taking into account the operating mode of the stage, is a relevant research area [7-9].

For this purpose, a mathematical description is needed to model a wide range of airfoils, as well as to make various changes to the shape of the blade in all its regions and sections to simulate defects. The coordinates of airfoil points are then needed to be exported to CAE systems to numerically analyze the impact of these deviations on the performance of the airfoil, blade, axial compressor and GTU as a whole.
Table 1. Defects’ impact to the axial compressor and GTU performance/

| №  | Geometry change description | Impact description                                                                 | Possible impact to<sup>a</sup> |
|----|-----------------------------|------------------------------------------------------------------------------------|---------------------------------|
| 1  | Leading edge thinning       | 1. Leading edge loss level decrease; 2. Gas-dynamic stability margin decrease.        | $N_c \downarrow$ $\eta_{ad} \uparrow$ $N_e \uparrow$ $\eta_e \uparrow$ |
|    |                             |                                                                                    | $\pi \sim$ $G_{air} \sim$ |
| 2  | Trailing edge thickening    | 1. Vibration reliability increase 2. Stages mutual operation deterioration due to trailing edge vortices | $N_c \uparrow$ $\eta_{ad} \downarrow$ $N_e \downarrow$ $\eta_e \downarrow$ |
|    |                             |                                                                                    | $\pi \sim$ $G_{air} \sim$ |
| 3  | Bending angle increase      | 1. Suction side flow separation                                                    | $N_c \uparrow$ $\eta_{ad} \downarrow$ $N_e \downarrow$ $\eta_e \downarrow$ |
|    |                             |                                                                                    | $\pi \downarrow$ $G_{airr} \downarrow$ |
| 4  | Radial clearance increase   | 1. Secondary loss level increase due to overflows through the radial clearance 2. Probability of rotor grazing the stator decrease | $N_c \uparrow$ $\eta_{ad} \downarrow$ $N_e \downarrow$ $\eta_e \downarrow$ |
|    |                             |                                                                                    | $\pi \downarrow$ $G_{airl} \downarrow$ |
| 5  | Axial clearance decrease    | 1. Stages mutual operation deterioration due to trailing edge vortices             | $N_c \uparrow$ $\eta_{ad} \downarrow$ $N_e \downarrow$ $\eta_e \downarrow$ |
|    |                             |                                                                                    | $\pi \sim$ $G_{air} \sim$ |
| 6  | Airfoil thickness increase  | 1. Friction loss level increase 2. Blade passage configuration deterioration       | $N_c \uparrow$ $\eta_{ad} \downarrow$ $N_e \downarrow$ $\eta_e \downarrow$ |
|    |                             |                                                                                    | $\pi \downarrow$ $G_{air} \downarrow$ |

<sup>a</sup> $N_c$ – axial compressor power consumption; $\eta_{ad}$ – axial compressor adiabatic efficiency; $\pi$ – pressure ratio; $G_{air}$ – Mass flow rate; $N_e$ – GTU efficient power; $\eta_e$ – GTU efficient efficiency.

<sup>b</sup> «↑» – increase; «↓» – decrease; «~» d – doesn’t change or difficult to assess the impact.

<sup>c</sup> Presented impact is observed for nominal operating mode.

2. Materials and Methods
In view of the above, a geometric model of the GTU axial compressor blade was developed, which allows taking into account changes in certain geometric parameters using the data on real post-operational and post-repair geometric deviations of certain axial compressor blades. The developed airfoil mathematical description is based on the use of key geometric parameters (pitch angle ($\beta_u$), chord ($b$), radii of the leading and trailing edge ($R_{in}$; $R_{out}$) and their edge angles ($\phi_1$; $\phi_2$), blade angles at leading and trailing edge ($\beta_{1b}$; $\beta_{2b}$).
to find the position of the control points of Bezier curves (figure 1). This approach ensures highly precise design of the initial airfoil and allows flexibly changing its geometry [6, 10, 11]. Moreover, it has previously proven itself in solving airfoil optimization problems [10, 12, 13]. The developed mathematical description uses various Bezier curves – second-order Bezier curves for edge modeling, and third-order Bezier curves for the suction and pressure sides of the blade. Fourth-order Bezier curves were also used to describe the center of the mass line of the blade airfoil to set the tangent offset (figure 4-b) [14].

![Figure 1. Airfoil description.](image)

### 3. Results

According to figure 2, the initial stage of modelling is to set the initial airfoil data.

![Figure 2. Axial compressor gas-flow channel outline.](image)

The starting point for modeling process is the circular arc center of the leading edge – its vertical and horizontal coordinates are assumed to be equal to the spherical radius of the leading edge $R_{inr}$.
which help clearly set the position of the leading edge and use it as a basis for airfoil design. The present paper assumes that the smallest distance between the coordinates of the circular arc centers of the leading and trailing edges is equal to the chord minus the radii of the airfoil edges. The next stage is to find the position of the control points of Bezier curves. The coordinates of A1 (1) are intersection points of tangents to the leading edge:

\[
\begin{align*}
X_{A1} &= R_{in} - \frac{R_{in}}{sin\left(\frac{\phi_1}{2}\right)} \times sin\beta_{1b} \\
Y_{A1} &= R_{in} - \frac{R_{in}}{sin\left(\frac{\phi_1}{2}\right)} \times cos\beta_{1b}.
\end{align*}
\] (1)

The coordinates of control points $B_0^{ps}$ (2) and $B_3^{ss}$ (3):

\[
\begin{align*}
X_{B_0^{ps}} &= X_{A1} + R_{in} \times ctg\left(\frac{\phi_1}{2}\right) \times sin\left(\beta_{1b} + \frac{\phi_1}{2}\right) \\
Y_{B_0^{ps}} &= Y_{A1} + R_{in} \times ctg\left(\frac{\phi_1}{2}\right) \times cos\left(\beta_{1b} + \frac{\phi_1}{2}\right); \quad (2)
\end{align*}
\]

\[
\begin{align*}
X_{B_3^{ss}} &= X_{A1} + R_{in} \times ctg\left(\frac{\phi_1}{2}\right) \times sin\left(\beta_{1b} - \frac{\phi_1}{2}\right) \\
Y_{B_3^{ss}} &= Y_{A1} + R_{in} \times ctg\left(\frac{\phi_1}{2}\right) \times cos\left(\beta_{1b} - \frac{\phi_1}{2}\right). \quad (3)
\end{align*}
\]

The coordinates of A2 (4) are intersection points of tangents to the leading edge:

\[
\begin{align*}
X_{A2} &= \left[R_{in} + (b - R_{in} + R_{out}) \times sin\beta_y\right] + \frac{R_{out}}{sin\left(\frac{\phi_2}{2}\right)} \times sin\beta_{2b} \\
Y_{A2} &= \left[R_{in} + (b - R_{in} + R_{out}) \times cos\beta_y\right] + \frac{R_{out}}{sin\left(\frac{\phi_2}{2}\right)} \times cos\beta_{2b}. \quad (4)
\end{align*}
\]

The coordinates of control points $B_3^{ps}$ (5) and $B_0^{ss}$ (6):

\[
\begin{align*}
X_{B_3^{ps}} &= X_{A2} - R_{out} \times ctg\left(\frac{\phi_2}{2}\right) \times sin\left(\beta_{2a} + \frac{\phi_2}{2}\right) \\
Y_{B_3^{ps}} &= Y_{A1} + R_{out} \times ctg\left(\beta_{1a} + \frac{\phi_2}{2}\right); \quad (5)
\end{align*}
\]

\[
\begin{align*}
X_{B_0^{ss}} &= X_{A1} - R_{out} \times ctg\left(\frac{\phi_2}{2}\right) \times sin\left(\beta_{1a} - \frac{\phi_2}{2}\right) \\
Y_{B_0^{ss}} &= Y_{A1} + R_{out} \times ctg\left(\beta_{1a} - \frac{\phi_2}{2}\right). \quad (6)
\end{align*}
\]

The coordinates of control points $B_2^{ss}$ (7) and $B_1^{ss}$ (8):
\[
\begin{align*}
X_{B_2} &= \frac{2}{3} (X_{1} - X_{B_2}) + X_{B_3} \\
Y_{B_2} &= \frac{2}{3} (Y_{1} - Y_{B_2}) + Y_{B_3} \\
X_{B_1} &= \frac{1}{3} (X_{B_0} - X_{1}) + X_{1} \\
Y_{B_1} &= \frac{1}{3} (Y_{B_0} - Y_{1}) + Y_{1} \\
\end{align*}
\] (7)
\[
\begin{align*}
X_{B_3} &= \frac{1}{3} (X_{B_0} - X_{1}) + X_{1} \\
Y_{B_3} &= \frac{1}{3} (Y_{B_0} - Y_{1}) + Y_{1} \\
\end{align*}
\] (8)

Where \(X_{A} \) and \(Y_{A}\) (9) are the coordinates of the auxiliary point, calculated as follows:
\[
\begin{align*}
X_{1_{ps}} &= \frac{\left(Y_{B_3} - \tan \left(90° + \beta_2 + \frac{\varphi_2}{2}\right) * X_{B_3}\right) - \left[Y_{B_0} - \tan \left(90° - \beta_1 - \frac{\varphi_1}{2}\right) * X_{B_0}\right]}{\left[\tan \left(90° - \beta_1 - \frac{\varphi_1}{2}\right) - \tan \left(90° + \beta_2 + \frac{\varphi_2}{2}\right)\right]} \\
Y_{1_{ps}} &= \left[\tan \left(90° + \beta_2 + \frac{\varphi_2}{2}\right) * X_{1_{ps}}\right] + \left[Y_{B_3} - \tan \left(90° + \beta_2 + \frac{\varphi_2}{2}\right) * X_{B_3}\right] \\
\end{align*}
\] (9)

The coordinates of control points \(B_2^{ps}\) (10) and \(B_1^{ps}\) (11):
\[
\begin{align*}
X_{B_2} &= \frac{2}{3} (X_{1_{ps}} - X_{B_0}) + X_{B_0} \\
Y_{B_2} &= \frac{2}{3} (Y_{1_{ps}} - Y_{B_0}) + Y_{B_0} \\
X_{B_1} &= \frac{1}{3} (X_{B_0} - X_{1_{ps}}) + X_{1_{ps}} \\
Y_{B_1} &= \frac{1}{3} (Y_{B_0} - Y_{1_{ps}}) + Y_{1_{ps}} \\
\end{align*}
\] (10)
\[
\begin{align*}
X_{B_3} &= \frac{1}{3} (X_{B_0} - X_{1_{ps}}) + X_{1_{ps}} \\
Y_{B_3} &= \frac{1}{3} (Y_{B_0} - Y_{1_{ps}}) + Y_{1_{ps}} \\
\end{align*}
\] (11)

Where \(X_{A^{ps}} \) and \(Y_{A^{ps}}\) (12) are the coordinates of the auxiliary point:
\[
\begin{align*}
X_{1_{ps}} &= \frac{\left(Y_{B_3} - \tan \left(90° + \beta_2 + \frac{\varphi_2}{2}\right) * X_{B_3}\right) - \left[Y_{B_0} - \tan \left(90° - \beta_1 - \frac{\varphi_1}{2}\right) * X_{B_0}\right]}{\left[\tan \left(90° - \beta_1 - \frac{\varphi_1}{2}\right) - \tan \left(90° + \beta_2 + \frac{\varphi_2}{2}\right)\right]} \\
Y_{1_{ps}} &= \left[\tan \left(90° + \beta_2 + \frac{\varphi_2}{2}\right) * X_{1_{ps}}\right] + \left[Y_{B_3} - \tan \left(90° + \beta_2 + \frac{\varphi_2}{2}\right) * X_{B_3}\right] \\
\end{align*}
\] (12)

Other geometric parameters used in the proposed description are the radius of the root section \(R_r\) and the height of the blade \(l\). By setting them, a model of the blade is formed from a certain number of airfoils. In this case, the vertical coordinate \(Z\) (13) of the airfoil points is defined as follows:
\[
Z = R_r + nl; \\
\] (13)
where \( n \in [0; 1] \) is a parameter that determines the position of the section (for \( n = 0 \) the airfoil will be located at the root of the blade, and for \( n = 1 \) – in its periphery). The proposed description allows for specifying 5 or more blade sections, which ensures high accuracy of its design.

The description provides for setting the airfoil axial displacement (along the \( x \) and \( y \) axes) in order to form the flow by offsetting all airfoils which constitute one blade, as well as to ensure that the blade offset can be adjusted (by offsetting certain airfoils). The center of the mass line of the blades was also described with the help of a fourth-order Bezier curve. This algorithm is analyzed in detail in [14].

![Image](image.png)

**Figure 3.** Introducing airfoil defects

The developed description provides for two ways of changing the airfoil point coordinates in order to introduce defects. Firstly, if the geometry of a certain airfoil (a set of coordinates) and the defect parameters (for example, the depth and location of the nick) are known, the coordinates can be manually changed in a certain part of the airfoil – in one of the 8 zones according to figure 3. Secondly, it is possible to change the coordinates of several points simultaneously by changing the position of the corresponding Bezier curve control point, which can be useful, for example, in the case of bended edges. Defects for a blade 3D model can be introduced likewise at the stage of remodeling a separate airfoil, or when working with the 3D model.

### 4. Discussion

The derived mathematical description allows flexibly changing the airfoil shape. The possibility to make a large number of sections (for example, 10 or more), in turn, helps create the most precise model of a blade. The coordinates of airfoil points can be exported to various CAD and CAE systems for creating blade and flow 3D models, as well as for performing numerical studies. Figure 4-a shows how airfoil forming a single blade are displayed in the developed description. Figure 4-b shows a 3D model of the blade and a scheme for offsetting. Figure 4-c shows an example of a typical axial compressor stage model.

To verify the geometry using the developed description, numerical studies of the NACA-65 airfoils were conducted [15]. Figure 5-a shows an example of a NACA 65-(27)10 airfoil model. Figure 5-b shows a comparison of its design and experimental performance. Figure 5-c shows the distribution of the pressure coefficient \( S \) along the airfoil for the angle of attack \( i = 33 \). The problem was solved in a quasi-three-dimensional formulation: using the selected airfoil, a blade was formed and located at the maximum possible distance from the axis of rotation. The number of blades, as well as the boundary conditions (total pressure and temperature, flow direction at the inlet, flow rate at the outlet) were selected according to the conditions presented in the report [15].

According to the results obtained, the model can be considered accurate: the design performance almost completely coincides with the experimental one. The increase in the performance difference stems from the failures on the suction side of the blade in the region of 0.6-0.8\( b \) occurring during the calculations. However, no failures were observed during the experiment [15]. The maximum deviation was 7.8% (absolute). This result can be associated primarily with the modeling assumptions.
The authors aim to emphasize that the verification of the computational model parameters is the most important task of numerical studies, but it is not a priority in the current work. Demonstrating the capabilities of the developed approach is of greater importance.

5. Conclusion
The present paper presents the key results of developing a mathematical description of the airfoil using Bezier curves. The coordinates of the control points are calculated based on the key geometric parameters of the airfoil. The paper provides equations for calculating the coordinates of Bezier curve points and describes the algorithm of blade and flow modeling. Finally, this work shows the results of the verification of the developed mathematical description of the blade geometry using numerical modeling of the cascade flow.
In general, the model meets the specified requirements, providing for flexible airfoil and blade modeling, as well as making local changes in their geometry. It is designed to become the main tool for the numerical analysis of the defect influence on the characteristics of the axial compressor stage, gas-flow channel or GTU as a whole. This model can be used during the blade row repair or subsequently, with the sufficient data accumulated, to predict the development of various defects of the blades based on the degradation characteristics of the axial compressor and GTU while in operation.

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