$h^0 \rightarrow c\bar{c}$ as a test case for quark flavor violation in the MSSM

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Abstract

We compute the decay width of $h^0 \rightarrow c\bar{c}$ in the MSSM with quark flavor violation (QFV) at full one-loop level adopting the DR renormalisation scheme. We study the effects of $c - t$ mixing, taking into account the constraints from the B meson data. We show that the full one-loop corrected decay width $\Gamma(h^0 \rightarrow c\bar{c})$ is very sensitive to the MSSM QFV parameters. In a scenario with large $c_{L,R} - t_{L,R}$ mixing $\Gamma(h^0 \rightarrow c\bar{c})$ can differ up to $\sim \pm 35\%$ from its SM value. After estimating the uncertainties of the width, we conclude that an observation of these SUSY QFV effects is possible at an $e^+e^-$ collider (ILC).
1 Introduction

The properties of the Higgs boson, discovered at LHC, CERN, with a mass of $125.15 \pm 0.24$ GeV (averaged over the values given by ATLAS [1] and CMS [2]) [3], are consistent with the prediction of the Standard Model (SM) [4]. Future experiments at LHC at higher energy ($\sqrt{s} = 14$ TeV) and higher luminosity will provide more precise data on Higgs boson observables, as Higgs production cross sections, decay branching ratios etc.. Even more precise data can be expected at a future $e^+e^-$ linear collider (ILC). This will allow one to test the SM more accurately and will give information on physics beyond the SM. The discovered Higgs boson could also be the lightest neutral Higgs boson $h^0$ of the Minimal Supersymmetric Standard Model (MSSM) [4,5].

The decays of $h^0$ are usually assumed to be quark flavor conserving (QFC). However, quark flavor violation (QFV) in the squark sector may significantly influence the decay widths of $h^0$ at one-loop level. In particular, the rate of the $h^0$ decay into a charm-quark pair, $h^0 \rightarrow c\bar{c}$, may be significantly different from the SM prediction due to squark generation mixing, especially that between the second and the third squark generations ($\tilde{c}_{L,R} - \tilde{t}_{L,R}$ mixing). This possibility will be studied in detail in the present paper.
It is well known that the mixing between the first and the second squark generations is strongly suppressed by the data on K physics [6]. Therefore, we assume mixing between the second and the third squark generation, respecting the constraints from B physics. In the MSSM this mixing was theoretically studied for squark and gluino production and decays at the LHC [7–15].

The outline of the paper is as follows: In Section 2 we shortly give the definitions of the QFV squark mixing parameters. In Section 3 we present the calculation of the width of $h^0 \rightarrow c \bar{c}$ at full one-loop level in the $\overline{\text{DR}}$ renormalisation scheme with quark flavor violation within the MSSM. In particular, we give formulas for the important one-loop gluino contribution. In Section 4 we present a detailed numerical analysis. In Section 5 we study the feasibility of observing the SUSY QFV effects in the decay $h^0 \rightarrow c \bar{c}$ at ILC by estimating the theoretical uncertainties. Section 6 contains our conclusions.

2 Definition of the QFV parameters

In the MSSM’s super-CKM basis of $\tilde{q}_0 = (\tilde{q}_1L, \tilde{q}_2L, \tilde{q}_3L, \tilde{q}_1R, \tilde{q}_2R, \tilde{q}_3R)$, $\gamma = 1, \ldots, 6$, with $(q_1, q_2, q_3) = (u, c, t), (d, s, b)$, one can write the squark mass matrices in their most general $3 \times 3$-block form [16]

$$M^2_{\tilde{q}} = \begin{pmatrix} M^2_{\tilde{q},LL} & M^2_{\tilde{q},LR} \\ M^2_{\tilde{q},RL} & M^2_{\tilde{q},RR} \end{pmatrix},$$

with $\tilde{q} = \tilde{u}, \tilde{d}$. The left-left and right-right blocks in eq. (1) are given by

$$M^2_{\tilde{q},LL} = V_{\text{CKM}} M^2_Q V_{\text{CKM}}^\dagger + D_{\tilde{q},LL} \mathbb{1} + \hat{m}^2_u,$$

$$M^2_{\tilde{q},RR} = M^2_Q + D_{\tilde{q},RR} \mathbb{1} + \hat{m}^2_d,$$

$$M^2_{\tilde{q},LR} = M^2_Q + \hat{m}^2_u, $$

$$M^2_{\tilde{q},RL} = v \sqrt{2} T_U - \mu^* \hat{m}_u \cot \beta,$$

where $M^2_{Q,U,D}$ are the hermitian soft SUSY-breaking mass matrices of the squarks and $\hat{m}_u,d$ are the diagonal mass matrices of the up-type and down-type quarks. Furthermore, $D_{\tilde{q},LL} = \cos 2\beta m^2_Z (T^q_3 - e_q \sin^2 \theta_W)$ and $D_{\tilde{q},RR} = e_q \sin^2 \theta_W \times \cos 2\beta m^2_Z$, where $T^q_3$ and $e_q$ are the isospin and electric charge of the quarks (squarks), respectively, and $\theta_W$ is the weak mixing angle. Due to the $SU(2)_L$ symmetry the left-left blocks of the up-type and down-type squarks in eq. (2) are related by the CKM matrix $V_{\text{CKM}}$. The left-right and right-left blocks of eq. (1) are given by

$$M^2_{\tilde{q},RL} = M^2_{\tilde{q},LR} = \frac{v_2}{\sqrt{2}} T_U - \mu^* \hat{m}_u \cot \beta,$$

$$M^2_{\tilde{q},RL} = M^2_{\tilde{q},LR} = \frac{v_1}{\sqrt{2}} T_D - \mu^* \hat{m}_d \tan \beta,$$

where $T_U,D$ are the soft SUSY-breaking trilinear coupling matrices of the up-type and down-type squarks entering the Lagrangian $\mathcal{L}_{int} \supset -(T_{Ua} \bar{\tilde{u}}_{Ra} \tilde{u}_{Lb} H_2^* + T_{Da} \bar{\tilde{d}}_{Ra} \tilde{d}_{Lb} H_1^*),$
\( \mu \) is the higgsino mass parameter, and \( \tan \beta \) is the ratio of the vacuum expectation values of the neutral Higgs fields \( v_2/v_1 \), with \( v_{1,2} = \sqrt{2} \left<H_{1,2}\right> \). The squark mass matrices are diagonalized by the \( 6 \times 6 \) unitary matrices \( R_{\tilde{q}} \), \( \tilde{q} = \tilde{u}, \tilde{d} \), such that

\[
R_{\tilde{q}}^2 M_{\tilde{q}}^2 (R_{\tilde{q}})^\dagger = \text{diag}(m_{\tilde{q}_1}^2, \ldots, m_{\tilde{q}_6}^2),
\]

with \( m_{\tilde{q}_1} < \cdots < m_{\tilde{q}_6} \). The physical mass eigenstates \( \tilde{q}_i, i = 1, \ldots, 6 \) are given by

\[
\tilde{q}_i = R_{\tilde{q}} \tilde{q}_0 \alpha_{\tilde{q}_0}.
\]

We define the QFV parameters in the up-type squark sector \( \delta_{\alpha\beta}^{LL}, \delta_{\alpha\beta}^{uRR} \), and \( \delta_{\alpha\beta}^{uRL} (\alpha \neq \beta) \) as follows [17]:

\[
\delta_{\alpha\beta}^{LL} \equiv \frac{M_{Q\alpha\beta}}{\sqrt{M_{Q\alpha\alpha}M_{Q\beta\beta}}},
\]

\[
\delta_{\alpha\beta}^{uRR} \equiv \frac{M_{U\alpha\beta}}{\sqrt{M_{U\alpha\alpha}M_{U\beta\beta}}},
\]

\[
\delta_{\alpha\beta}^{uRL} \equiv \frac{(v_2/\sqrt{2})T_{U\alpha\beta}}{\sqrt{M_{U\alpha\alpha}M_{Q\beta\beta}}},
\]

where \( \alpha, \beta = 1, 2, 3 \) denote the quark flavors \( u, c, t \). In this study we consider \( \tilde{c}_R - \tilde{t}_L, \tilde{c}_L - \tilde{t}_R, \tilde{c}_R - \tilde{t}_R \), and \( \tilde{c}_L - \tilde{t}_L \) mixing which is described by the QFV parameters \( \delta_{23}^{uRL}, \delta_{23}^{uLR} \equiv (\delta_{32}^{uRL})^*, \delta_{23}^{uRR}, \) and \( \delta_{23}^{uLL} \), respectively. We also consider \( \tilde{t}_L - \tilde{t}_R \) mixing described by the QFC parameter \( \delta_{33}^{uRL} \) which is defined by eq. (7) with \( \alpha = \beta = 3 \). All QFV parameters and \( \delta_{33}^{uRL} \) are assumed to be real.

\section*{3 \( h^0 \rightarrow c\bar{c} \) at full one-loop level with flavor violation}

We study the decay of the lightest neutral Higgs boson, \( h^0 \), into a pair of charm quarks (Figure 1) at full one-loop level in the general MSSM with quark flavor violation in the squark sector. The full one-loop decay width of \( h^0 \rightarrow c\bar{c} \) was first calculated within the QFC MSSM by [18].

![Diagram of \( h^0 \rightarrow c\bar{c} \)](image)

Figure 1: \( h^0 \) decay into a pair of charm quarks.

The decay width of the reaction \( h^0 \rightarrow c\bar{c} \) including one-loop contributions can be written as

\[
\Gamma(h^0 \rightarrow c\bar{c}) = \Gamma^{\text{tree}}(h^0 \rightarrow c\bar{c}) + \delta \Gamma^{\text{loop}}(h^0 \rightarrow c\bar{c}).
\]
The tree-level decay width $\Gamma_{\text{tree}}(h^0 \rightarrow c\bar{c})$ reads

$$\Gamma_{\text{tree}}(h^0 \rightarrow c\bar{c}) = \frac{N_C}{8\pi} m_{h^0} (s^c_1)^2 \left( 1 - \frac{4m^2_c}{m^2_{h^0}} \right)^{3/2}, \quad \text{with } N_C = 3,$$

where $m_{h^0}$ is the on-shell (OS) mass of $h^0$ and the tree-level coupling $s^c_1$ is

$$s^c_1 = -g \frac{m_c \cos \alpha}{2m_W \sin \beta} = -\frac{h_c}{\sqrt{2}} \cos \alpha.$$

Here $\alpha$ is the mixing angle of the two CP-even Higgs bosons, $h^0$ and $H^0$ [19].

In the general MSSM at one-loop level, in addition to the diagrams that contribute within the SM, $\delta \Gamma_{1\text{loop}}(h^0 \rightarrow c\bar{c})$ also receives contributions from diagrams with additional Higgs bosons and supersymmetric particles. The contributions from SUSY particles are shown in Figure 2, neglecting the contributions from scalar leptons. The flavor violation is induced by one-loop diagrams with squarks that have a mixed quark flavor nature. In addition, the coupling of $h^0$ with two squarks $\tilde{u}_i \tilde{u}_j$ (see eq. (65) of Appendix A) contains the trilinear coupling matrices $(T_U)_{ij}$ which for $i \neq j$ break quark flavor explicitly.

The one-loop corrections to $\Gamma(h^0 \rightarrow c\bar{c})$ contain three parts, QCD ($g$) corrections, SUSY-QCD ($\tilde{g}$) corrections and electroweak (EW) corrections. In the latter we also include the Higgs contributions. In the following we will mainly give details for the QCD and SUSY-QCD corrections.

### 3.1 Renormalisation procedure

Loop calculations can lead to ultraviolet (UV) and infrared (IR) divergent result and therefore require renormalisation. In order to get UV finite result we adopt in our study the DR renormalisation scheme, where all input parameters in the tree-level Lagrangian (masses, fields and coupling parameters) are UV finite, defined at the scale $Q = 125.5$ GeV $\simeq m_{h^0}$, and the UV divergence parameter $\Delta = \frac{2}{\epsilon} - \gamma + \ln 4\pi$, where $\epsilon = 4 - D$ in a D-dimensional space-time and $\gamma$ is the Euler-Mascheroni constant, is set to zero. The tree-level coupling is defined at the given scale and thus does not receive further finite shifts due to loop corrections. In order to obtain the shifts from the DR masses and fields to the physical scale-independent masses and fields, we use on-shell renormalisation conditions. To ensure IR convergence, we include in our calculations the contribution of the real hard gluon/photon radiation from the final charm quarks assuming a small gluon/photon mass $\lambda$.

The one-loop corrected width of the process $h^0 \rightarrow c\bar{c}$ including hard gluon/photon radiation is given by

$$\Gamma(h^0 \rightarrow c\bar{c}) = \Gamma_{\text{tree}}(h^0 \rightarrow c\bar{c}) + \sum_{x=g,\tilde{g},\text{EW}} \delta \Gamma^x,$$

where $\delta \Gamma^x$ read

$$\delta \Gamma^g = \frac{3}{4\pi} m_{h^0} s^c_1 \text{Re}(\delta S^c_1 \tilde{g}) \left( 1 - \frac{4m^2_c}{m^2_{h^0}} \right)^{3/2},$$

$$\delta \Gamma^{\tilde{g}} = \frac{3}{4\pi} m_{h^0} s^c_1 \text{Re}(\delta S^{c,\tilde{g}}) \left( 1 - \frac{4m^2_c}{m^2_{h^0}} \right)^{3/2},$$

and

$$\delta \Gamma^{\text{EW}} = \sum_{x=g,\tilde{g}} \delta \Gamma^x + \sum_{\text{EW}} \delta \Gamma^x.$$
Figure 2: The main one-loop contributions with SUSY particles in $h^0 \rightarrow c\bar{c}$. The corresponding diagram to (e) with the self-energy contribution to the other charm quark is not shown explicitly.
\[ \delta \Gamma_{g/EW} = \frac{3}{4\pi} m_{h^0} s_1^c \text{Re}(\delta S_{1g/EW}^c) \left( 1 - \frac{4m_c^2}{m_{h^0}^2} \right)^{3/2} + \Gamma_{\text{hard}}(h^0 \to c\bar{c}g/\gamma). \] (13)

Note that all parameters in the tree-level coupling \( s_1^c \), eq. (10), are \( \overline{\text{DR}} \) running at the scale \( Q = 125.5 \text{ GeV} \). The renormalised finite one-loop amplitude of the process is a sum of all vertex diagrams, the amplitudes arising from the wave-function renormalisation constants and the amplitudes arising from the coupling counter terms. Note that in the \( \overline{\text{DR}} \) renormalisation scheme the counter terms contain only UV-divergent parts and have to cancel in order to yield a convergent result. The one-loop renormalised coupling correction can be written as

\[ \delta S_{1x}^c = \delta S_{1x}^c(v) + \delta S_{1x}^c(w) + \delta S_{1x}^c(0), \quad x = g, \tilde{g}, \text{EW}, \] (14)

where \( \delta S_{1x}^c(v) \) is the vertex coupling correction, \( \delta S_{1x}^c(w) \) is the wave-function coupling correction and \( \delta S_{1x}^c(0) \) is the coupling counter term. The tree-level interaction Lagrangian of the lightest Higgs boson \( h^0 \) and two charm quarks is given by eq. (63) in Appendix A. The renormalised Lagrangian \( L_{\text{ren}} \) is obtained after making the replacement \( L_{\overline{\text{DR}}} = L_{\text{ren}} + \delta L \), where \( \delta L = -\delta S_{1}^{c(v)} h^0 \bar{c}c \) describes all vertex-type interactions. The coupling correction due to wave-function renormalisation is given by

\[ \delta S_{1x}^c(w) = \frac{s_1^c}{2} \delta Z_{h^0} + \frac{s_2^c}{2} \delta Z_{h^0 H^0} + \frac{s_1^c}{4} \left( \delta Z_{c}^L + \delta Z_{c}^L + \delta Z_{c}^R + \delta Z_{c}^R \right), \] (15)

where \( s_2^c \) is the coupling of the heavier neutral Higgs \( H^0 \) and the charm quark, \( s_2^c = -\frac{h^0}{\sqrt{2}} \sin \alpha \). The charm quark wave-function renormalisation constants read

\[ \delta Z_{c}^L/R = -\tilde{\text{Re}} \Pi_{cc}^{L/R}(m_c) + \frac{1}{2m_c} \tilde{\text{Re}} \left( \Pi_{cc}^{S, L/R}(m_c) - \Pi_{cc}^{S, R/L}(m_c) \right) \]
\[ -m_c \tilde{\text{Re}} \left[ m_c \left( \hat{\Pi}_{cc}^{L/R}(m_c) + \hat{\Pi}_{cc}^{R/L}(m_c) \right) \right. \]
\[ + \hat{\Pi}_{cc}^{S, L/R}(m_c) + \hat{\Pi}_{cc}^{S, R/L}(m_c) \],

(16)

and the Higgs wave-function renormalisation constants for the case of \( h^0 - H^0 \) mixing are given by

\[ \delta Z_{h^0} = -\tilde{\text{Re}} \hat{\Pi}_{h^0 h^0}(m_{h^0}^2), \] (17)

\[ \delta Z_{h^0 H^0} = \frac{2}{m_{h^0}^2 - m_{H^0}^2} \left( \tilde{\text{Re}} \hat{\Pi}_{h^0 H^0}(m_{h^0}^2) - \delta t_{h^0 H^0} \right), \] (18)

with the tadpole contribution

\[ \delta t_{h^0 H^0} = -\frac{1}{v} \left[ \tau_{h^0} \left( \frac{s_2^c c_\alpha}{c_\beta} + \frac{c_2^c s_\alpha}{s_\beta} \right) + \tau_{H^0} \left( -\frac{c_2^c s_\alpha}{c_\beta} + \frac{s_2^c c_\alpha}{s_\beta} \right) \right], \] (19)
where \( c_\alpha = \cos \alpha \) and \( s_\alpha = \sin \alpha \). \( \tau_{\phi^0} \) and \( \tau_{H^0} \) are the loop corrections from the tadpole diagrams with \( h^0 \) and \( H^0 \), respectively. In eqs. (16), (17) and (18) \( \Re \) applied to the self-energies denoted by \( \Pi \) takes the real part of the loop integrals, but leaves the possible complex couplings unaffected. Finally, the coupling counter term \( \delta S_{c}^{(0)} \) is given by

\[
\delta S_{c}^{(0)} = \left( \frac{\delta g}{g} + \frac{\delta m_c}{m_c} - \frac{\delta m_W}{m_W} - \frac{\delta \sin \beta}{\sin \beta} + \frac{\delta \cos \alpha}{\cos \alpha} \right) \Delta \Delta H^0, \virt
\]

(20)

where the subindex \( \Delta \) means that only the part proportional to the UV divergence parameter \( \Delta \) is taken. The explicit expressions for the shifts of the parameters in (20) can be found in [20]. Note that \( \frac{\delta g}{g} = \frac{\delta e}{e} - \frac{\delta \sin \theta_W}{\sin \theta_W} \) is used.

### 3.2 One-loop gluon contribution

The one-loop virtual gluon contribution to \( \Gamma(h^0 \rightarrow c\bar{c}) \) is given by

\[
\delta \Gamma^g = \frac{3}{4\pi} m_{h^0} s_1^c \Re(\delta S_{c,g}^{(0)}) \beta^3, \quad (21)
\]

with \( \beta = (1 - 4m_{h^0}^2/m_{W^0}^2)^{1/2} \). \( \delta S_{c,g}^{(0)} \) contains terms originating from the vertex correction, the wave-function correction and the coupling correction due to gluon interaction,

\[
\delta S_{c,g}^{(0)} = \delta S_{c,g,V}^{(0)} + \delta S_{c,g,W}^{(0)} + \delta S_{c,g,0}^{(0)}. \quad (22)
\]

The individual contributions in \( \delta S_{c,g}^{(0)} \) are given by

\[
\delta S_{c,g,V}^{(0)} = \frac{2\alpha_s}{3\pi} s_1^c \left[ 2B_0 - r - (m_{h^0}^2 - 2m_c^2)C_0 - 4m_c^2C_1 \right], \quad (23)
\]

\[
\delta S_{c,g,W}^{(0)} = \frac{2\alpha_s}{3\pi} s_1^c \left[ -B_0 - B_1 + \frac{r}{2} + 2m_c^2(B_0 - \hat{B}_1) \right], \quad (24)
\]

\[
\delta S_{c,g,0}^{(0)} = \frac{2\alpha_s}{3\pi} s_1^c \left( B_0 - B_0 + \frac{r}{2} \right), \quad (25)
\]

where \( r = 0 \) in the DR scheme and \( r = 1 \) in the \( \overline{\text{MS}} \) scheme. \( B_k, \hat{B}_k \) and \( C_k \) are the two- and three-point functions

\[
B_k = B_k(m_c^2, 0, m_c^2), \quad (26)
\]

\[
\hat{B}_k = \frac{\partial B_k(p^2, \lambda^2, m_c^2)}{\partial p^2} \bigg|_{p^2 = m_c^2}, \quad (27)
\]

\[
C_k = C_k(m_c^2, m_{h^0}^2, m_c^2, \lambda^2, m_c^2, m_c^2), \quad (28)
\]

with \( k = 0, 1 \). Summing up eqs. (23)-(25) one can write \( \delta S_{c,g}^{(0)} \) in the form

\[
\delta S_{c,g}^{(0)} = \frac{2\alpha_s}{3\pi} s_1^c \Delta H^0,\virt(\beta). \quad (29)
\]
Furthermore, we will use the result for the hard gluon radiation, given in Appendix B. We can write eq. (69) in the form
\[ \Gamma_{\text{hard}}(h^0 \to c\bar{c}g) = \frac{3}{8\pi} m_{h^0} (s_1^c)^2 \frac{4}{3} \frac{\alpha_s}{\pi} \Delta_{\text{H,hard}}(\beta). \] (30)

Combining (21), (29) and (30) for the gluon one-loop corrected convergent width we obtain
\[ \Gamma^g(h^0 \to c\bar{c}) = \Gamma_{\text{tree}} + \delta \Gamma^g + \Gamma^g_{\text{hard}}(\beta) \]
\[ = \frac{3}{8\pi} m_{h^0} (s_1^c)^2 \beta^3 \left( 1 + \frac{4}{3} \frac{\alpha_s}{\pi} \Delta_{\text{H}}(\beta) \right) \] (31)
where \( \Delta_{\text{H}}(\beta) = \Delta_{\text{H,virt}}(\beta) + \Delta_{\text{H,hard}}(\beta) \) is the result of [21] and its explicit expression can be found therein or e.g. in [18, 22, 23]. Eq. (31) can be written in a compact form as
\[ \Gamma^g(h^0 \to c\bar{c}) = \Gamma_{\text{tree}}(m_c|_{\text{OS}}) \left( 1 + \frac{4}{9} \frac{\alpha_s}{\pi} \Delta_{\text{H}}(\beta) \right), \] (32)
where \( m_c|_{\text{OS}} \) denotes the on-shell (OS) charm quark mass. Note that the result for the photon one-loop corrected convergent width is obtained from (32) by making the replacement \( \frac{4}{3} \alpha_s \to e^2 \alpha \):
\[ \Gamma^\gamma(h^0 \to c\bar{c}) = \Gamma_{\text{tree}}(m_c|_{\text{OS}}) \left( 1 + \frac{16}{9} \frac{\alpha_s}{\pi} \Delta_{\text{H}}(\beta) \right), \] (33)
with \( \alpha = e^2/(4\pi) \).

For \( m_c \ll m_{h^0} (\beta \to 1) \)
\[ \Delta_{\text{H}} = -3 \ln \frac{m_{h^0}}{m_c|_{\text{OS}}} + \frac{9}{4} \] (34)
and from eq. (25) using eqs. (82) and (83) we get
\[ \frac{\delta m^g_c}{m_c} = \frac{\delta S_1^{c,(g,0)}}{s_1^c} = \frac{\alpha_s}{3\pi} \left( -6 \ln \frac{m_{h^0}}{m_c|_{\text{OS}}} + r - 5 \right). \] (35)

For \( \Gamma^g(h^0 \to c\bar{c}) \) in the limit \( m_c \ll m_{h^0} \) we obtain
\[ \Gamma^g(h^0 \to c\bar{c}) = \Gamma_{\text{tree}}(m_c|_{\text{SM}}) \left( 1 + \frac{16}{3} \frac{\alpha_s}{\pi} \right), \] (36)
where in (36) we have absorbed the logarithm of \( \delta m^g_c \) into
\[ m_c|_{\text{SM}} = m_c|_{\text{OS}} + \delta m^g_c. \] (37)

Combining eq. (35) with eqs. (36) and (37) one can see that the one-loop level \( \Gamma^g(h^0 \to c\bar{c}) \) does not depend on the parameter \( r \). In the numerical evaluation of \( m_c|_{\text{SM}} \) we follow the recipe given in [24], starting with eq. (4) and we use \( \alpha_s^{(2)}(Q) \) given therein. In all other cases we take \( \alpha_s(Q) \) from SPheno [25, 26], where it is calculated at two-loop level within the MSSM. In order to stay consistent, in our numerical calculations we have included in addition only the gluonic \( \alpha_s^2 \) contributions, taken from [23]. With these, \( \Gamma^g(h^0 \to c\bar{c}) \) will be denoted as \( \Gamma^{g,\text{impr}}(h^0 \to c\bar{c}) \),
\[ \Gamma^{g,\text{impr}}(h^0 \to c\bar{c}) = \Gamma_{\text{tree}}(m_c|_{\text{SM}}) + \delta \Gamma^g(m_c|_{\text{SM}}), \] (38)
3.3 One-loop gluino contribution and decoupling limit

The one-loop gluino contribution to $\Gamma(h^0 \to c\bar{c})$, Fig. 3 and Fig. 4, renormalised in the $\overline{\text{DR}}$ scheme reads

$$\delta\Gamma\tilde{g} = \frac{3}{4\pi} m_{h^0} s^c_1 \text{Re}(\delta S^c_{1,\tilde{g}}) \beta^3.$$ (39)

$\delta S^c_{1,\tilde{g}}$ acquires contributions from the vertex correction (Fig. 3), the wave-function correction (Fig. 4) and the coupling correction due to gluino interaction,

$$\delta S^c_{1,\tilde{g}} = \delta S^c_{1,\tilde{g},v} + \delta S^c_{1,\tilde{g},w} + \delta S^c_{1,\tilde{g},0}.$$ (40)

In the following we will use the abbreviations

$$\alpha_{ij} = U_{\tilde{u}_i}^* U_{\tilde{u}_j} + U_{\tilde{u}_j}^* U_{\tilde{u}_i}$$

and

$$\beta_{ij} = U_{\tilde{u}_i}^* U_{\tilde{u}_j} + U_{\tilde{u}_j}^* U_{\tilde{u}_i}.$$ Note that applying Einstein sum convention we get $\alpha_{ii} = 2$ and $\beta_{ii} = 0$. Neglecting the charm quark mass and the Higgs boson mass compared to the squark and gluino masses, one can write the individual contributions as

$$\delta S^c_{1,\tilde{g},v} = \frac{\alpha_s}{3\pi} \sum_{i,j=1}^{6} G_{i,j,\tilde{g}} \beta_{ij} C_{0,i}^j,$$ (41)

$$\delta S^c_{1,\tilde{g},w} = \frac{\alpha_s}{3\pi} s^c_1 \sum_{i=1}^{6} \left( \alpha_{ii} B^i_1 + 4m_{\tilde{g}} \beta_{ii} B^i_0 \right),$$ (42)

where the coupling $G_{i,j,\tilde{g}}$ is given in eq. (65) of Appendix A. For the following discussion of the gluino contribution in the large $m_{\tilde{g}}$ limit we give the charm mass counter term $\delta m^c_{\tilde{g}}$ in the OS scheme, which has a UV divergent and a finite contribution,

$$\delta m^c_{\tilde{g}} = -\frac{\alpha_s}{3\pi} \sum_{i=1}^{6} \left( m_c \alpha_{ii} B^i_1 + m_{\tilde{g}} \beta_{ii} B^i_0 \right).$$ (43)

Figure 3: (a) Gluino vertex contribution to $h^0 \to c\bar{c}$ and (b) examples of quark flavor mixing in the gluino vertex contribution.
Figure 4: (a) The gluino contribution to the charm quark self-energy and (b) examples of quark flavor mixing in the charm quark self-energy contribution with gluino.

For the gluino contribution we have $\delta S_c^{(\tilde{g}, \tilde{g})}/s_1^c = \delta m_{\tilde{g}}/m_c$. Therefore, with eq. (42) we get

$$\delta S_c^{(\tilde{g}, \tilde{g})}/s_1^c = \frac{\alpha_s}{3\pi} s_1^c \sum_{i=1}^6 \left( \frac{m_{\tilde{g}}}{m_c} \beta_{ij} B_0^i + \frac{\delta m_{\tilde{g}}}{m_c} B_0^i \right).$$

(44)

In the DR scheme we need only the UV divergent part of (44) which is

$$\delta S_c^{(\tilde{g}, \tilde{g})}/s_1^c = 6\frac{\alpha_s}{3\pi} s_1^c \Delta.$$

(45)

$\Delta$ is the UV divergence factor. In eqs. (41)-(44) $B_k^i$, $\dot{B}_0^i$ and $C_0^i$ are the two- and three-point functions

$$B_k^i = B_k(0, m_{\tilde{g}}, m_{\tilde{u}_i}), \quad k = 0, 1, \ldots, 6,$$

(46)

$$\dot{B}_0^i = \frac{\partial B_0(p^2, m_{\tilde{g}}, m_{\tilde{u}_i})}{\partial p^2} \bigg|_{p^2=0}, \quad i = 1, \ldots, 6,$$

(47)

$$C_0^i = C_0(0, 0, m_{\tilde{g}}, m_{\tilde{u}_i}, m_{\tilde{u}_j}), \quad i = 1, \ldots, 6.$$

(48)

The total correction $\delta S_1^{(\tilde{g}, \tilde{g})}$ (eq. (40)) is given by

**DR scheme:**

$$\delta S_1^{(\tilde{g}, \tilde{g})} = \frac{\alpha_s}{3\pi} \sum_{i,j=1}^6 \left( m_{\tilde{g}} \beta_{ij} \left( G_{ij,1} C_0^{ij} + 4 s_1^c \delta_{ij} \dot{B}_0^i + \delta m_{\tilde{g}} \frac{m_{\tilde{g}}}{m_c} \beta_{ii} B_0^i + \Delta \right) \right).$$

(49)

**OS scheme:**

$$\delta S_1^{(\tilde{g}, \tilde{g})} = \frac{\alpha_s}{3\pi} \sum_{i,j=1}^6 \left( m_{\tilde{g}} \beta_{ij} \left( G_{ij,1} C_0^{ij} + 4 s_1^c \delta_{ij} \dot{B}_0^i - \delta m_{\tilde{g}} \frac{m_{\tilde{g}}}{m_c} \beta_{ii} B_0^i \right) \right).$$

(50)

As $B_0^i \to -\Delta/2$ and thus $\alpha_{ii} B_0^i \to -\Delta$, (49) is UV convergent. As $\beta_{ii} B_0^i \to 0$, also (50) is UV convergent.

In the limit $m_{\tilde{g}} \to \infty$, from (94) it follows $m_{\tilde{g}} C_0^{ij} \to 0$ and from (87) it follows $\dot{B}_0^i \to 0$. However, in this limit (78) and (79) become independent of the index $i$ and grow with $\ln \frac{m_{\tilde{g}}^2}{m_{\tilde{u}_i}^2}$. Therefore, $\beta_{ii} B_0^i \to 0$ guarantees decoupling of the gluino loop contribution in the OS scheme.
In the $\overline{\text{DR}}$ scheme for $m_\tilde{g} \to \infty$, we get
\[ \delta S^{c,\tilde{g}}_1 \sim \frac{2\alpha_s}{3\pi} s^c_i B^i_1 \quad \text{with} \quad B_1 \sim \ln \frac{m_\tilde{g}^2}{m_{h^0}^2}. \] (51)
At first sight it seems that the gluino contribution does not decouple for $m_\tilde{g} \to \infty$. However, the tree-level coupling $s^c_i$ (eq. (10)) contains a factor $m_c$. We have
\[ m_c(m_{h^0})|_{\overline{\text{DR}}} = m_c(m_c)|_{\overline{\text{MS}}} + \delta m^g_c + \ldots, \] (52)
where we take $m_c(m_c)|_{\overline{\text{MS}}} = 1.275$ GeV as input [27]. $\delta m^g_c$ is due to the self-energy contributions with gluino (see Figs. 4(a) and 4(b)). We get
\[ \delta m^g_c \sim -\frac{2\alpha_s}{3\pi} m_c B^i_1. \] (53)
Thus the sum $\Gamma^{\text{tree}} + \delta \Gamma^g$ is indeed decoupling for $m_\tilde{g} \to \infty$. Analogously, this also holds for the chargino and neutralino contributions.

### 3.4 Total result for the width at full one-loop level

Finally, we want to sum up all contributions to get the total result for $\Gamma(h^0 \to c\bar{c})$ at full one loop level.

The one-loop result including gluino and EW contributions reads
\[ \Gamma^{g+\text{EW}}(h^0 \to c\bar{c}) = \Gamma^{\text{tree}}(m_c) + \delta \Gamma^g(m_c) + \delta \Gamma^{\text{EW}}(m_c), \] (54)
where $\Gamma^{\text{tree}}$, $\delta \Gamma^g$ and $\delta \Gamma^{\text{EW}}$ are given by eqs. (9), (39) and (13), respectively. Note that eq. (54) is a series expansion around $\Gamma^{\text{tree}}(m_c) = \Gamma^{\text{tree}}(m_c(m_{h^0})|_{\overline{\text{DR}}})$.

However, the improved result with gluon contribution (eq. (38)) given by
\[ \Gamma(h^0 \to c\bar{c})^{g,\text{impr}} = \Gamma^{\text{tree}}(m_c|_{\text{SM}}) + \delta \Gamma^g(m_c|_{\text{SM}}) \] (55)
is a series expansion around $\Gamma^{\text{tree}}(m_c|_{\text{SM}})$. In order to combine eqs. (54) and (55) in a consistent way we write:
\[ \Gamma^{\text{tree}}(m_c|_{\text{SM}}) = \Gamma^{\text{tree}}(m_c) \frac{m^2_c|_{\text{SM}}}{m^2_c}, \] (56)
and therefore
\[ \Gamma^{\text{tree}}(m_c|_{\text{SM}}) = \Gamma^{\text{tree}}(m_c) - \Gamma^{\text{tree}}(m_c) \frac{m^2_c - m^2_c|_{\text{SM}}}{m^2_c}. \] (57)

Thus, our total result can be written in the form
\[ \Gamma(h^0 \to c\bar{c}) \equiv \Gamma^{\text{impr}}(h^0 \to c\bar{c}) = \Gamma^{\text{tree}}(m_c) + \delta \Gamma^g + \delta \Gamma^g + \delta \Gamma^{\text{EW}}, \] (58)
where the new gluon contribution $\delta \Gamma^g$ is given by
\[ \delta \Gamma^g = \delta \Gamma^g(m_c|_{\text{SM}}) - \Gamma^{\text{tree}}(m_c) \frac{m^2_c - m^2_c|_{\text{SM}}}{m^2_c}. \] (59)
4 Numerical results

In order to demonstrate clearly the effect of QFV in the MSSM, we have explicitly chosen a reference scenario with a rather strong \( \tilde{c} - \tilde{t} \) mixing. The MSSM parameters at \( Q = 125.5 \) GeV \( \approx m_{h^0} \) are given in Table 1.

Table 1: Reference QFV scenario: shown are the basic MSSM parameters at \( Q = 125.5 \) GeV \( \approx m_{h^0} \), except for \( m_{A^0} \) which is the pole mass (i.e. the physical mass) of \( A^0 \), with \( T_{\tilde{c}\tilde{t}} = -2050 \) GeV (corresponding to \( \delta_{33}^{uRL} = -0.2 \)). All other squark parameters not shown here are zero.

| \( M_1 \) | \( M_2 \) | \( M_3 \) |
|---|---|---|
| 250 GeV | 500 GeV | 1500 GeV |

| \( \mu \) | \( \tan \beta \) | \( m_{A^0} \) |
|---|---|---|
| 2000 GeV | 20 | 1500 GeV |

| \( \alpha = 1 \) | \( \alpha = 2 \) | \( \alpha = 3 \) |
|---|---|---|
| \( M_{Q\alpha\alpha}^2 \) \( (2400)^2 \) GeV\(^2 \) | \( (2360)^2 \) GeV\(^2 \) | \( (1850)^2 \) GeV\(^2 \) |
| \( M_{U\alpha\alpha}^2 \) \( (2380)^2 \) GeV\(^2 \) | \( (1050)^2 \) GeV\(^2 \) | \( (950)^2 \) GeV\(^2 \) |
| \( M_{D\alpha\alpha}^2 \) \( (2380)^2 \) GeV\(^2 \) | \( (2340)^2 \) GeV\(^2 \) | \( (2300)^2 \) GeV\(^2 \) |

\[
\begin{array}{cccc}
\delta_{23}^{LL} & \delta_{23}^{uRR} & \delta_{23}^{uRL} & \delta_{23}^{uLR} \\
0.05 & 0.2 & 0.03 & 0.06 \\
\end{array}
\]

Table 2: Physical masses in GeV of the particles for the scenario of Table 1.

\[
\begin{array}{cccccccc}
m_{\tilde{\chi}_1^0} & m_{\tilde{\chi}_2^0} & m_{\tilde{\chi}_3^0} & m_{\tilde{\chi}_4^0} & m_{\tilde{\chi}_1^+} & m_{\tilde{\chi}_2^+} \\
260 & 534 & 2020 & 2021 & 534 & 2022 \\
\end{array}
\]

\[
\begin{array}{cccc}
m_{h^0} & m_{H^0} & m_{A^0} & m_{H^+} \\
126.08 & 1498 & 1500 & 1501 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
m_{\tilde{g}} & m_{\tilde{u}_1} & m_{\tilde{u}_2} & m_{\tilde{u}_3} & m_{\tilde{u}_4} & m_{\tilde{u}_5} & m_{\tilde{u}_6} \\
1473 & 756 & 965 & 1800 & 2298 & 2301 & 2332 \\
\end{array}
\]

The resulting physical masses of the particles are shown in Table 2. The flavor decomposition of the two lighter squarks \( \tilde{u}_1 \) and \( \tilde{u}_2 \) can be seen in Table 3. This scenario satisfies all present experimental and theoretical constraints given in Appendix D. For calculating
Table 3: Flavor decomposition of $\tilde{u}_1$ and $\tilde{u}_2$ for the scenario of Table 1. Shown are the squared coefficients.

|    | $\tilde{u}_L$ | $\tilde{c}_L$ | $t_L$ | $\tilde{u}_R$ | $\tilde{c}_R$ | $t_R$ |
|----|---------------|---------------|-------|---------------|---------------|-------|
| $\tilde{u}_1$ | 0             | 0.0004        | 0.012 | 0             | 0.519         | 0.468 |
| $\tilde{u}_2$ | 0             | 0.0004        | 0.009 | 0             | 0.480         | 0.509 |

the masses and the mixing, as well as the low-energy observables, especially those in the B meson sector (see Table 4), we use the public code SPheno v3.3.3 [25, 26]. The width $\Gamma(h^0 \to c\bar{c})$ at full one-loop level in the MSSM with QFV is calculated on the basis of the formulas given above with the help of FeynArts [28] and FormCalc [29]. We also use the SSP package [30]. In the following plots we show the QFV parameter dependences of the full one-loop level width $\Gamma(h^0 \to c\bar{c})$ of eq. (58) around the reference point of Table 1.

In Figs. 5(a) and 5(b) we show the dependence of the width $\Gamma(h^0 \to c\bar{c})$ on the QFV parameters $\delta_{23}^{LL} (\tilde{c}_L - \tilde{t}_L$ mixing) and $\delta_{23}^{RR} (\tilde{c}_R - \tilde{t}_R$ mixing), with the other parameters fixed as in Table 1. In Fig. 5(a) we show the width in MeV as a function of $\delta_{23}^{LL}$ and $\delta_{23}^{RR}$. The white area is the region allowed by all the constraints of Appendix D, with the reference point of Table 1 indicated by X. In the allowed region this width can vary from 0.1 MeV to 0.14 MeV. As can be seen, there is a rather strong dependence on $\delta_{23}^{RR}$.

In Fig. 5(b) we show the deviation of the $\Gamma(h^0 \to c\bar{c})$ from the SM width $\Gamma_{SM}(h^0 \to c\bar{c}) = 0.118$ MeV [6]. This deviation varies between -15% and 20%. It is interesting to mention that we obtain $\Gamma_{QFC}(h^0 \to c\bar{c}) = 0.116$ MeV for the full one-loop width in the QFC MSSM case for our reference scenario corresponding to Table 1. This means that the QFC supersymmetric contributions change the width $\Gamma(h^0 \to c\bar{c})$ by only $\sim -1.5\%$ compared to the SM value. Comparing our QFC one-loop result with FeynHiggs-2.10.2 [31] we have a difference less then 1%. Note that the mass of the lightest squark $\tilde{u}_1$ can vary in the allowed region between 650 GeV and 850 GeV, as seen in Fig. 5(c). Note also that in Figs. 5(a) and 5(b) the QFV parameter $-0.3 < \delta_{23}^{RR} < 0.3$ is not restricted by the constraints from the B sector, but from the mass of the lightest stop (corresponding to the lightest squark mass shown in Fig. 5(c)) and the lightest neutralino (see Table 2) in the context of simplified MSSM with QFC [32]. In principle, this experimental restriction on the lightest stop mass does not hold for the case of QFV, and a wider range of $\delta_{23}^{RR}$ is allowed [33].

In Figs. 6(a) and 6(b) we show the dependence of the width $\Gamma(h^0 \to c\bar{c})$ on the QFV parameters $\delta_{23}^{LL}$ and $\delta_{23}^{LR} (\tilde{c}_L - \tilde{t}_R$ mixing) with the other parameters fixed as in Table 1. In the allowed range the width can vary between 0.08 MeV and 0.15 MeV. The deviation of $\Gamma(h^0 \to c\bar{c})$ from the SM value $\Gamma_{SM}(h^0 \to c\bar{c})$ lies between -30% and 25% (Fig. 6(b)). Fig. 6(c) shows the dependence of the mass $m_{\tilde{u}_1}$.

In analogy we show in Fig. 7 the corresponding plots for the dependences on the QFV parameters $\delta_{23}^{RR}$ and $\delta_{23}^{RL}$. As seen in Fig. 7(a), the width $\Gamma(h^0 \to c\bar{c})$ varies in the allowed region between 0.07 MeV and 0.15 MeV. The deviation from the SM value...
Figure 5: Dependence on the QFV parameters $\delta_{uLL}^{23}$ and $\delta_{uRR}^{23}$ of the width (a) $\Gamma(h^0 \to c \bar{c})$ in MeV, (b) $\Gamma(h^0 \to c \bar{c})/\Gamma_{SM}^{h^0 \to c \bar{c}}$ and (c) the mass of the lightest squark $\tilde{u}_1$ in GeV. The gray region is excluded by the constraint from the $B_s \to \mu^+\mu^-$ data.

$\Gamma_{SM}^{h^0 \to c \bar{c}}$ is between -35% and 30% (see Fig. 7(b)). The mass of $\tilde{u}_1$ varies between 600 GeV and 850 GeV, as seen in Fig. 7(c).

In Fig. 8 we show the dependence of $\delta\Gamma_X/\Gamma_{SM}^{h^0 \to c \bar{c}}$ on the QFV parameters $\delta_{uRR}^{23}, \delta_{uLR}^{23}$ and $\delta_{uRL}^{23}$, where $\delta\Gamma_X$ denotes the individual contribution of $X = (g, \text{impr}), \tilde{g}, \text{EW}$ (including the EW MSSM contributions) to the width $\Gamma(h^0 \to c \bar{c})$ (see eq.(58)). As can be seen, the gluino loop contribution $\delta\Gamma_{\tilde{g}}$ depends significantly on $\delta_{uRR}^{23}$ and $\delta_{uLR}^{23}$ with the dependences on $\delta_{uLL}^{23}$ and $\delta_{uRL}^{23}$ being somewhat weaker. The gluino loop contribution $\delta\Gamma_{\tilde{g}}/\Gamma_{SM}^{h^0 \to c \bar{c}}$ can go up to 45% (see Figs. 8(a) and 8(d)). It can also be seen
Figure 6: Dependence on the QFV parameters $\delta_{23}^{LL}$ and $\delta_{23}^{uLR}$ of the width (a) $\Gamma(h^0 \to c \bar{c})$ in MeV, (b) $\Gamma(h^0 \to c \bar{c})/\Gamma^{SM}(h^0 \to c \bar{c})$ and (c) the mass of the lightest squark $\tilde{u}_1$ in GeV. The light and dark gray regions are excluded by the constraints from the $B(B_s \to \mu^+ \mu^-)$ and $B(b \to s\gamma)$ data, respectively.

that the electroweak loop contributions $\delta\Gamma^{EW}$ cannot be neglected with $\delta\Gamma^{EW}/\Gamma^{SM}$ being around 5%. Clearly, its dependence on the QFV parameters is weak.

The strong dependences of the width $\Gamma(h^0 \to c \bar{c})$ on the QFV parameters shown in this section can be explained as follows. First of all, the scenario chosen is characterised by large QFV parameters, which in our case are the large $\tilde{c}_{L,R} - \tilde{t}_{L,R}$ mixing parameters $\delta_{23}^{LL}, \delta_{23}^{uRR}, \delta_{23}^{uRL}, \delta_{23}^{uLR}$, and particularly large QFV trilinear couplings $T_{U32}, T_{U32}$ (Note that $\delta_{23}^{uRL} \sim T_{U32}$ and $\delta_{23}^{uLR} \sim T_{U23}$). In such a scenario, the lightest up-type squarks $\tilde{u}_{1,2}$ are strong admixtures of $\tilde{c}_{L,R} - \tilde{t}_{L,R}$, and, hence, the couplings $\tilde{u}_{1,2} \tilde{u}_{1,2}^* h^0(\sim \text{Re}(H_2^0))$ in Fig. 3...
Figure 7: Dependence on the QFV parameters $\delta_{23}^{uRR}$ and $\delta_{23}^{uLR}$ of the width (a) $\Gamma(h^0 \to c \bar{c})$ in MeV, (b) $\Gamma(h^0 \to c \bar{c})/\Gamma^{SM}(h^0 \to c \bar{c})$ and (c) the mass of the lightest squark $\tilde{u}_1$ in GeV.

are strongly enhanced, see eq. (65). In addition, large $\tilde{t}_L - \tilde{t}_R$ mixing due to the large QFC trilinear couplings $T_{i33}$ occurs. Moreover, the $\tilde{t}_L \tilde{t}_L^* h^0$ and $\tilde{t}_R \tilde{t}_R^* h^0$ couplings are proportional to the top quark mass squared (see eq. (65)), which additionally enhances the $\tilde{u}_{1,2} \tilde{u}_{1,2}^* h^0$ couplings and thus also the vertex gluino contributions of Fig. 3 in case of QFV.
Figure 8: Dependences on the QFV parameters of the one-loop $\tilde{g}$, EW and improved $g$ contributions to the width $\Gamma(h^0 \to c\bar{c})$. Note that for the $\tilde{g}$ and EW contributions only the one-loop scale independent part is shown as in the DR scheme the scale dependent part cancels with the tree-level scale dependent part.

5 Observability of the deviation of $\Gamma(h^0 \to c\bar{c})$ from its SM value at ILC

Observation of any significant deviation of the width $\Gamma(h^0 \to c\bar{c})$ from its SM prediction signals new physics beyond the SM. It is important to estimate the uncertainties of the SM prediction reliably in order to confirm such a deviation. Once the deviation is discovered, one has to work out the new physics candidates suggesting it.

The uncertainties of the SM prediction come from two sources [34–37]. One is the parametric uncertainty and the other is the theory uncertainty. The former is due to the
Figure 9: Renormalisation-scale dependence of $\Gamma(h^0 \to c\bar{c})$. $\Gamma_{\text{impr}}(h^0 \to c\bar{c})$ is the improved one-loop corrected width of eq. (58). The vertical line shows $Q = m_{h^0}$.

The errors of the SM input parameters such as $m_c(m_c)_{\overline{\text{MS}}}$ and $\alpha_s(m_Z)_{\overline{\text{MS}}}$, and the latter is due to unknown higher order corrections. The theory uncertainty is estimated mainly by renormalisation-scale dependence uncertainties which are indicative of not knowing higher order terms in a perturbative expansion of the corresponding observable. These scale dependence uncertainties are estimated by varying the scale $Q$ from $Q/2$ to $2Q$ [34–36]. (Note that in our case $Q = m_{h^0}$.)

In order to estimate the uncertainty of the width $\Gamma(h^0 \to c\bar{c})$ in the MSSM with QFV at our reference point we proceed in an analogous way. We calculate the parametric uncertainty in the width $\Gamma(h^0 \to c\bar{c})$ due to errors in the inputs $m_c(m_c)_{\overline{\text{MS}}}$ and $\alpha_s(m_Z)_{\overline{\text{MS}}}$ following [38]

$$\frac{\delta \Gamma}{\Gamma} = \left| \frac{m_c}{\Gamma} \frac{\partial \Gamma}{\partial m_c} \right| \frac{\delta m_c}{m_c} \oplus \left| \frac{\alpha_s}{\Gamma} \frac{\partial \Gamma}{\partial \alpha_s} \right| \frac{\delta \alpha_s}{\alpha_s},$$

(60)

where as input we take $m_c(m_c)_{\overline{\text{MS}}} = 1.275$ GeV with $\delta m_c/m_c = 2\%$ [39], and $\alpha_s(m_Z) = 0.1185$ with $\delta \alpha_s/\alpha_s = 0.5\%$ [40]. $\delta X/X$ denotes the relative error of the quantity $X$. At our reference point of Table 1 we get

$$\frac{\delta \Gamma}{\Gamma} = |2.6| \frac{\delta m_c}{m_c} \oplus | -4.0| \frac{\delta \alpha_s}{\alpha_s} = 5.2\% + 2\%.$$ 

(61)

Note that the parametric uncertainties due to errors of the other SM input parameters, such as $m_b$, are negligible.

The theory uncertainty of the width for our reference point is shown on Fig. 9. We have: $\delta \Gamma/\Gamma(h^0 \to c\bar{c}) = ^{+0.11\%}_{-0.46\%}$, where $\Gamma(h^0 \to c\bar{c})$ is the improved one-loop corrected width of eq. (58). Thus, for this uncertainty we take $\sim 0.5\%$.

For the total error in the width at our reference point we get

$$\sqrt{5.2^2 + 2^2 + 0.5^2 \approx 6.1\%},$$

(62)
where the parametric uncertainties are added quadratically and the theory uncertainty is added to them linearly. The obtained total uncertainty (62) at our reference point is \( \sim \pm 6.1\% \) (at 68\% CL), which is in good agreement with the estimated total uncertainty of \( \Gamma^{\text{SM}}(h^0 \rightarrow c\bar{c}) \), see Table 13 of [36]. Note that the uncertainty in the coupling is half of the uncertainty in the width.

As seen in Section 4, the deviation \( \Gamma(h^0 \rightarrow c\bar{c})/\Gamma^{\text{SM}}(h^0 \rightarrow c\bar{c}) \) can be as large as \( \sim \pm 35\% \). Such a large deviation can be observed at ILC (500 GeV) with 1600 (500) fb\(^{-1} \), where the expected experimental error in the width is \( \sim 3\% \) (5.6\%) [41, 42] . A measurement of \( \Gamma(h^0 \rightarrow c\bar{c}) \) at LHC (even with the high luminosity upgrade) is demanding due to uncertainties in the charm-tagging.

6 Conclusions

We have calculated the width \( \Gamma(h^0 \rightarrow c\bar{c}) \) at full one-loop level within the MSSM with quark flavor violation. In particular, we have studied \( \tilde{c}_{R,L} - \tilde{t}_{R,L} \) mixing, taking into account the experimental constraints from B-physics, \( m_{h^0} \) and SUSY particle searches. The width \( \Gamma(h \rightarrow c\bar{c}) \) turns out to be very sensitive to \( \tilde{c}_{R,L} - \tilde{t}_{R,L} \) mixing.

In our calculation we have used the DR renormalisation scheme. In particular, we have derived the explicit formula for the dominant gluino loop contribution. We also have performed a detailed numerical study of the QFV parameter dependence of the width. Whereas the width \( \Gamma(h^0 \rightarrow c\bar{c}) \) in the QFC MSSM case is only slightly different from its SM value, in the QFV case this width can deviate from the SM by up to \( \sim \pm 35\% \).

We have estimated the theoretical uncertainties of \( \Gamma(h^0 \rightarrow c\bar{c}) \) and have shown that the SUSY QFV contribution to this width can be observed at the ILC.

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A Interaction Lagrangian

- The interaction of the lightest neutral Higgs boson, \( h^0 \), with two charm quarks is given by

\[
\mathcal{L}_{h^0 c\bar{c}} = s_1^c h^0 c\bar{c},
\]

where the tree-level coupling \( s_1^c \) is given by eq. (10).
In the super-CKM basis, the interaction of the lightest neutral Higgs boson, \( h^0 \), with two up-type squarks is given by

\[
L_{h^0\tilde{u}_i\tilde{u}_j} = G_{ij1}^\tilde{u} h^0 \tilde{u}_i^\dagger \tilde{u}_j, \quad i, j = 1, \ldots, 6.
\]  

(64)

The coupling \( G_{ij1}^\tilde{u} \) reads

\[
G_{ij1}^\tilde{u} = -\frac{g}{2m_W} \left[ -m_W^2 \sin(\alpha + \beta) \left( 1 - \frac{1}{3} \tan^2 \theta_W \right) \times (U^\tilde{u})_{j(k)(k+3)} (U^\tilde{u})_{i(k+3)} \right]
\]

\[
+ \frac{2 \cos \alpha}{\sin \beta} \left( (U^\tilde{u})_{j(k)(k+3)} m_{u,k} (U^\tilde{u})_{ik} + (U^\tilde{u})_{j(k+3)} m_{u,k} (U^\tilde{u})_{ik} \right)
\]

\[
+ \frac{\sin \alpha}{\sin \beta} \left[ \mu^* (U^\tilde{u})_{j(k+3)} m_{u,k} (U^\tilde{u})_{ik} + \mu (U^\tilde{u})_{j(k+3)} m_{u,k} (U^\tilde{u})_{ik} \right]
\]

\[
+ \frac{\cos \alpha \sin \beta}{\sqrt{2}} \left( (U^\tilde{u})_{j(k+3)} (T_U)_{kl} (U^\tilde{u})_{il} + (U^\tilde{u})_{j(k+3)} (T_U)_{ik} (U^\tilde{u})_{il} \right) \right],
\]  

(65)

where the sum over \( k, l = 1, 2, 3 \) is understood. Here \( U^\tilde{u} \) is the mixing matrix of the up-type squarks

\[
\tilde{u}_{iL} = (U^\tilde{u})_{ik} \tilde{u}_k, \quad \tilde{u}_{iR} = (U^\tilde{u})_{(i+3)k} \tilde{u}_k, \quad i = 1, 2, 3, \quad k = 1, \ldots, 6.
\]  

(66)

Note that \( (T_U)_{kl} \) in (65) are given in the SUSY Les Houche Accord notation [43].

The interaction of gluino, up-type squark and a charm quark is described by

\[
L_{\tilde{g}\tilde{u}_i c} = -\sqrt{2} g_s T_{\rho\sigma}^{\alpha} \left[ \bar{\tilde{g}}^{\alpha} (U_{i2}^\tilde{u} e^{-i\frac{\phi_3}{2}} P_L - U_{i5}^\tilde{u} e^{i\frac{\phi_3}{2}} P_R) c^s \bar{u}_i^{\dagger} \gamma^s \bar{u}_i^\dagger \gamma^s \right]
\]

\[
+ \bar{c}^{\tau} (U_{i2}^\tilde{u} e^{i\frac{\phi_3}{2}} P_R - U_{i5}^\tilde{u} e^{-i\frac{\phi_3}{2}} P_L) \bar{g}^{\alpha} \bar{u}_i \gamma^\tau \gamma^\tau \right],
\]  

(67)

where \( T^{\alpha} \) are the SU(3) colour group generators and summation over \( r, s = 1, 2, 3 \) and over \( \alpha = 1, \ldots, 8 \) is understood. In our case the parameter \( M_3 = m_{3/2} e^{i\phi_3} \) is taken as real, \( \phi_3 = 0 \).
B Hard gluon/photon Bremsstrahlung

The convergent one-loop gluon/photon corrected decay width in the limit of vanishing gluon/photon mass, $\lambda = 0$, is given by

$$\Gamma^{g/\gamma}(h^0 \rightarrow c\bar{c}) = \Gamma^{\text{tree}} + \delta\Gamma^{g/\gamma} + \Gamma^{\text{hard}}(h^0 \rightarrow c\bar{c}/\gamma).$$

The hard gluon radiation width reads

$$\Gamma^{\text{hard}}(h^0 \rightarrow c\bar{c}) = \frac{2\alpha_s|s|^2}{\pi^2m_{h^0}} \left[ J_1 - (m_{h^0}^2 - 4m_c^2)(J_2 - (m_{h^0}^2 - 2m_c^2)J_3) \right],$$

with the integrals [44]

$$J_1 = \frac{1}{8m_{h^0}^2} \left( (\kappa^2 + 6m_c^4) \ln \beta_0 - \frac{3}{2}\kappa(m_{h^0}^2 - 2m_c^2) \right),$$

$$J_2 = \frac{1}{4m_{h^0}^2} \left( 2\kappa \ln(\frac{\kappa^2}{\lambda m_{h^0} m_c^2}) - 4\kappa - m_{h^0}^2 \ln \beta_0 \right),$$

$$J_3 = \frac{1}{2m_{h^0}^2} \left( -\ln(\frac{\lambda m_{h^0} m_c^2}{\kappa^2}) \ln \beta_0 + \ln^2 \beta_0 - \ln^2 \beta_1 + \text{Li}_2(1 - \beta_0^2) - \text{Li}_2(1 - \beta_1^2) \right),$$

where $\text{Li}_s(z)$ is the polylogarithm function, defined by the infinite sum

$$\text{Li}_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s},$$

$$\beta_0 = \frac{m_{h^0}^2 - 2m_c^2 + \kappa}{2m_c^2}, \quad \beta_1 = \frac{m_{h^0}^2 - \kappa}{2m_{h^0} m_c},$$

$$\kappa \equiv \kappa(m_{h^0}^2, m_c^2, m_{h^0}^2) = m_{h^0} \sqrt{1 - \frac{4m_c^2}{m_{h^0}^2}}.$$

The expression for the hard photon radiation width $\Gamma^{\text{hard}}(h^0 \rightarrow c\bar{c}\gamma)$ is obtained from (69) by making the replacements $C_F = 4/3 \rightarrow e_c^2 = 4/9$ and $\alpha_s \rightarrow \alpha = e^2/(4\pi)$.

C Simplified formulas for the two- and three-point functions

In our analytic calculations we neglect the squared masses of the charm quark and the lightest neutral Higgs boson, $m_c^2$ and $m_{h^0}^2$, in comparison to the squared masses of the
scalar quarks and the gluino, \( m_{\tilde{q}_i}^2 \) and \( m_{\tilde{g}}^2 \). In the following we list the simplified expressions for the two- and three-point functions for this case.

\[
B_0(0, m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2) = \Delta + 1 + \frac{m_{\tilde{q}_1}^2 \ln \frac{m_{\tilde{q}_1}^2}{Q^2} - m_{\tilde{q}_2}^2 \ln \frac{m_{\tilde{q}_2}^2}{Q^2}}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2} \tag{76}
\]

\[
B_1(0, m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2) = -\frac{\Delta}{2} + \frac{1}{2} \left( \ln \frac{m_{\tilde{q}_1}^2}{Q^2} - \frac{m_{\tilde{q}_1}^4}{(m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2)^2} \ln \frac{m_{\tilde{q}_1}^2}{m_{\tilde{q}_1}^2} + \frac{m_{\tilde{q}_2}^2 - 3m_{\tilde{q}_1}^2}{2(m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2)} \right) \tag{77}
\]

\[
B_0(0, m^2, 0) = \Delta + 1 - \ln \frac{m^2}{Q^2} \tag{78}
\]

\[
B_1(0, m^2, 0) = -\Delta + \ln \frac{m^2}{Q^2} - \frac{3}{4} \tag{79}
\]

\[
B_0(0, m^2, m^2) = B_0(0, m^2, 0) - 1 = \Delta - \ln \frac{m^2}{Q^2} \tag{80}
\]

\[
B_1(0, m^2, m^2) = -\frac{1}{2} B_0(0, m^2, m^2) \tag{81}
\]

\[
B_0(m^2, 0, m^2) = \Delta + 2 + \ln \frac{Q^2}{m^2} \tag{82}
\]

\[
B_1(m^2, 0, m^2) = -\frac{1}{2} \left( \Delta + 1 + \ln \frac{Q^2}{m^2} \right) \tag{83}
\]

with \( \Delta \) the UV divergence factor and \( Q \) the renormalisation scale.

\[
\dot{B}_0(0, m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2) = \frac{m_{\tilde{q}_1}^4 - m_{\tilde{q}_2}^4 + 2m_{\tilde{q}_2}^2 m_{\tilde{q}_1}^2 \ln \frac{m_{\tilde{q}_2}^2}{m_{\tilde{q}_1}^2}}{2(m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2)^3} \tag{84}
\]

\[
\dot{B}_1(0, m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2) = -\frac{2m_{\tilde{q}_1}^6 + 3m_{\tilde{q}_2}^2 m_{\tilde{q}_1}^4 - 6m_{\tilde{q}_2}^4 m_{\tilde{q}_1}^2 + m_{\tilde{q}_2}^6 + 6m_{\tilde{q}_2}^2 m_{\tilde{q}_1}^4 \ln \frac{m_{\tilde{q}_2}^2}{m_{\tilde{q}_1}^2}}{6(m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2)^4} \tag{85}
\]

\[
\dot{B}_0(0, m^2, m^2) = \frac{1}{6m^2} \tag{86}
\]

\[
\dot{B}_0(0, m^2, 0) = \frac{1}{2m^2} \tag{87}
\]
\[
\hat{B}_0(m^2, \lambda^2, m^2) = -\frac{1}{2m^2} \left( 2 - \ln \frac{m^2}{\lambda^2} \right) \tag{88}
\]

\[
\hat{B}_1(m^2, \lambda^2, m^2) = -\frac{1}{2m^2} \tag{89}
\]

\[
C_0(m_1^2, m_2^2, m_1^2, \lambda^2, m_1^2, m_1^2) = \frac{1}{m_2^2} \left[ \ln \frac{1 + \beta}{1 - \beta} \ln \frac{m_2^2}{\lambda^2} - \frac{2\pi^2}{3} - 2\text{Li}_2 \left( \frac{1 + \beta}{1 - \beta} \right)^{1/2} - 2 \ln^2 \left( \frac{1 + \beta}{1 - \beta} \right)^{1/2} + \ln \beta \ln \frac{1 + \beta}{1 - \beta} \right] \tag{90}
\]

\[
C_1(m_1^2, m_2^2, m_1^2, \lambda^2, m_1^2, m_1^2) = \frac{1}{m_2^2} \ln \frac{1 + \beta}{1 - \beta} \tag{91}
\]

where \(\beta = (1 - 4m_1^2/m_2^2)^{1/2}\) and \(\text{Li}_s(z)\) is defined with (73).

\[
C_0(0, 0, 0, m_1^2, m_2^2, m_3^2) = \frac{B_0(0, m_1^2, m_3^2) - B_0(0, m_2^2, m_3^2)}{m_1^2 - m_2^2} = \frac{m_2^2 m_1^2 \ln \frac{m_2^2}{m_1^2} + m_2^2 m_3^2 \ln \frac{m_3^2}{m_1^2} + m_2^2 m_1^2 \ln \frac{m_3^2}{m_1^2}}{(m_1^2 - m_2^2)(m_2^2 - m_3^2)(m_1^2 - m_3^2)} \tag{92}
\]

\[
C_0(0, 0, 0, m_1^2, m_2^2, m_2^2) = \frac{m_1^2 - m_2^2 + m_1^2 \ln \frac{m_2^2}{m_1^2}}{(m_1^2 - m_2^2)^2} \tag{93}
\]

For \(m_3 = m_2 \ll m_1\) we get

\[
C_0(0, 0, 0, m_1^2, m_2^2, m_2^2) = \frac{1 + \ln \frac{m_2^2}{m_1^2}}{m_1^2} = \frac{1}{m_1^2} - \frac{\ln m_1^2}{m_1^2} + \frac{\ln m_2^2}{m_1^2} \tag{94}
\]

\[
C_0(0, 0, 0, m_1^2, m_2^2, m_2^2) = -\frac{1}{2m^2} \tag{95}
\]

Note, that the expression (94) vanishes for fixed \(m_2\) and \(m_1 \to \infty\).

### D Theoretical and experimental constraints

Here we summarize the experimental and theoretical constraints taken into account in the present paper. The constraints on the MSSM parameters from the B-physics experiments and from the Higgs boson measurement at LHC are shown in Table 4.

The BaBar and Belle collaborations have reported a slight excess of \(B(B \to D \tau \nu)\) and \(B(B \to D^* \tau \nu)\) [45–47]. However, it has been argued in [48] that within the MSSM...
this cannot be explained without being at the same time in conflict with $B(B_d \to \tau \nu)$. Using the program SUSY.FLAVOR [49] we have checked that in our MSSM scenarios no significant enhancement occurs for $B(\bar{B} \to D \tau \nu)$. However, as pointed out in [50], the theoretical predictions (in the SM and MSSM) on $B(B \to D l \nu)$ and $B(B \to D^* l \nu)$ ($l = \tau, \mu, e$) have potentially large theoretical uncertainties due to the theoretical assumptions on the form factors at the $B D W^+$ and $B D^* W^+$ vertices (also at the $B D H^+$ and $B D^* H^+$ vertices in the MSSM). Hence the constraints from these decays are unclear. Therefore, we do not take these constraints into account in our paper.

In [51] the QFV decays $t \to q h$ with $q = u, c$, have been studied in the general MSSM with QFV. It is found that these decays can not be visible at the current LHC runs due to the very small decay branching ratios $B(t \to q h)$.

For the mass of the Higgs boson $h^0$, taking the naive combination of the ATLAS and CMS measurements [1, 2] $m_{h^0} = 125.15 \pm 0.24$ GeV [3] and adding the theoretical uncertainty of $\sim \pm 2$ GeV [52] linearly to the experimental uncertainty at $2 \sigma$, we take $m_{h^0} = 125.15 \pm 2.48$ GeV.

| Observable          | Exp. data                  | Theor. uncertainty | Constr. (95%CL) |
|---------------------|----------------------------|--------------------|-----------------|
| $\Delta M_{B_s}$ [ps$^{-1}$] | 17.768 ± 0.024 (68% CL) [53] | ±3.3 (95% CL) [54,55] | 17.77 ± 3.30    |
| $10^4 \times B(b \to s \gamma)$ | 3.40 ± 0.21 (68% CL) [39]    | ±0.23 (68% CL) [56] | 3.40 ± 0.61     |
| $10^6 \times B(b \to s \ell^+ \ell^-)$ | 1.60 ±0.48 (68% CL) [57]     | ±0.11 (68% CL) [58] | 1.60 ±0.97      |
| ($l = e$ or $\mu$)  | 2.9 ± 0.7 (68%CL) [59–61]  | ±0.23 (68% CL) [62] | 2.90 ± 1.44     |
| $10^4 \times B(B_s \to \mu^+ \mu^-)$ | 1.15 ± 0.23 (68% CL) [63–65] | ±0.29 (68% CL) [63] | 1.15 ± 0.73     |
| $m_{h^0}$ [GeV]     | 125.03 ± 0.30 (68% CL)(CMS) [2], 125.36 ± 0.41 (68% CL)(ATLAS) [1] | ±2 [52] | 125.15 ± 2.48   |

In addition to these constraints we also require our scenarios to be consistent with the following experimental constraints:

(i) The LHC limits on the squark and gluino masses (at 95% CL) [32,66–87]:

In the context of simplified models, gluino masses $m_{\tilde{g}} \lesssim 1$ TeV are excluded at 95% CL. The mass limit varies in the range 1000-1400 GeV depending on assumptions. First and second generation squark masses are excluded below 900 GeV. Bottom squarks are excluded below 600 GeV. A typical top-squark mass limit is $\sim 700$ GeV. In [86, 87] a limit for the mass of the top-squark $m_{\tilde{t}} \gtrsim 500$ GeV for $m_{\tilde{t}} - m_{\text{LSP}} = 200$ GeV is quoted. Including mixing of $\tilde{c}_R$ and $\tilde{t}_R$ would even lower this limit [33].
(ii) The LHC limits on $m_{\tilde{\chi}_1^\pm}$ and $m_{\tilde{\chi}_2^0}$ from negative searches for charginos and neutralinos mainly in leptonic final states \[32, 88, 89\].

(iii) The constraint on $(m_{A^0, H^+}, \tan \beta)$ from the MSSM Higgs boson searches at LHC \[1, 2, 90\].

(iv) The experimental limit on SUSY contributions on the electroweak $\rho$ parameter \[91\]: $\Delta \rho_{(SUSY)} < 0.0012$.

Furthermore, we impose the following theoretical constraints from the vacuum stability conditions for the trilinear coupling matrices \[92\]:

$$|T_{U_{\alpha\alpha}}|^2 < 3 Y_{U\alpha}^2 (M_{Q_{\alpha\alpha}}^2 + M_{U_{\alpha\alpha}}^2 + m_1^2),$$

$$|T_{D_{\alpha\alpha}}|^2 < 3 Y_{D\alpha}^2 (M_{Q_{\alpha\alpha}}^2 + M_{D_{\alpha\alpha}}^2 + m_1^2),$$

$$|T_{U_{\alpha\beta}}|^2 < Y_{U\gamma}^2 (M_{Q_{\alpha\alpha}}^2 + M_{U_{\beta\beta}}^2 + m_2^2),$$

$$|T_{D_{\alpha\beta}}|^2 < Y_{D\gamma}^2 (M_{Q_{\alpha\alpha}}^2 + M_{D_{\beta\beta}}^2 + m_2^2),$$

where $\alpha, \beta = 1, 2, 3$, $\alpha \neq \beta$; $\gamma = \text{Max}(\alpha, \beta)$ and $m_1^2 = (m_{H^+}^2 + m_Z^2 \sin^2 \theta_W) \sin^2 \beta - \frac{1}{2} m_Z^2$, $m_2^2 = (m_{H^+}^2 + m_Z^2 \sin^2 \theta_W) \cos^2 \beta - \frac{1}{2} m_Z^2$. The Yukawa couplings of the up-type and down-type quarks are $Y_{U\alpha} = \sqrt{2} m_{ua}/v_2 = \frac{g}{\sqrt{2} m_W \sin \beta} m_{ua}$ ($u_\alpha = u, c, t$) and $Y_{D\alpha} = \sqrt{2} m_{da}/v_1 = \frac{g}{\sqrt{2} m_W \cos \beta} m_{da}$ ($d_\alpha = d, s, b$), with $m_{ua}$ and $m_{da}$ being the running quark masses at the weak scale and $g$ being the SU(2) gauge coupling. All soft SUSY-breaking parameters are given at $Q = 125.5$ GeV. As SM parameters we take $m_Z = 91.2$ GeV and the on-shell top-quark mass $m_t = 173.3$ GeV \[93\]. We have found that our results shown are fairly insensitive to the precise value of $m_t$.

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