The theory of hadronic parity violation

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Abstract. Parity-violating interactions between nucleons are the manifestation of an interplay of strong and weak interactions between quarks in the nucleons. Compared to the dominant parity-conserving part, the parity-violating component of the nuclear force is typically suppressed by approximately 6 to 7 orders of magnitude or more. Due to the short range of the weak interactions, however, it provides a unique probe of the strong dynamics that confine quarks into nucleons. An ongoing experimental program is mapping out this weak component of the nuclear force in few-nucleon systems. I will discuss recent theoretical progress based on effective field theory methods to analyze and interpret hadronic parity violation in few-nucleon systems, with a particular focus on two- and three-nucleon systems.

1. Introduction

The forces between nucleons are the manifestation of interactions between the quarks confined in these nucleons. While nucleon interactions are dominated by strong and electromagnetic effects, the confined quarks are also subject to the weak interactions. This leads to a parity-violating (PV) component in the interactions between nucleons. At the low energies typical of few-nucleon experiments, the PV part is expected to be suppressed by 6 to 7 orders of magnitude compared to the parity-conserving (PC) component. Effects due to the highly suppressed PV interaction can be isolated by considering observables that would vanish if parity was conserved. Typically, these are pseudoscalar observables involving a spin and a momentum vector, such as longitudinal and angular asymmetries and induced polarizations. The weak interactions are well understood at the quark level; however, when viewed at the hadronic level they are intertwined with the nonperturbative strong effects that confine the quarks in hadrons. Because of their short range, weak interactions are sensitive to quark-quark correlations at very short distances. PV nucleon interactions can therefore be viewed as a unique probe of nonperturbative strong effects.

PV effects can be enhanced by several orders of magnitude in some complex nuclei, e.g., by the presence of closely-spaced energy levels with opposite parity (see, e.g., Ref. [1]). While this is of great advantage experimentally, the theoretical description of such complex systems in terms of nucleon-nucleon interactions is difficult and can lead to uncontrolled theoretical errors. With the development of high-intensity neutron and photon sources and improved control over systematics, the measurement of PV effects in few-nucleon systems with $A \leq 5$ has become feasible, and an experimental program is underway to map out the PV component of the nucleon interactions. In these light systems, the theoretical description in terms of two- and three-nucleon systems is much more feasible. For recent reviews, see, e.g., Refs. [2, 3].

Traditionally, PV nucleon-nucleon interactions have been most commonly described in terms
of meson-exchange potentials with one PC and one PV meson-nucleon vertex. In particular, the formulation of Ref. [4] (often referred to as the DDH potential) has been used very frequently. More recently, effective field theory (EFT) methods, which have proven very successful in the PC sector, have been applied to PV observables. EFTs are model independent, treat PC and PV interactions as well as the coupling to external currents within a unified framework, and provide a method to estimate theoretical errors. The EFT analysis is performed at the hadronic level and does not directly relate the PV effects to interactions between standard model degrees of freedom. However, it provides a set of well-defined quantities with which any calculation performed at the quark level, such as potentially lattice QCD calculations, must be consistent.

2. Parity violation in pionless EFT

Many of the experiments in few-nucleon systems are performed at very low energies, e.g., using cold neutrons. At these energies, an EFT in which nucleons are the only dynamical degrees of freedom can be employed. In this so-called pionless EFT (EFT(\pi)) pions and all other degrees of freedom are integrated out and interactions between nucleons are described by contact terms with an increasing number of derivatives. All short-distance details of the interactions are encoded in the low-energy constants (LECs) of the theory that accompany each operator in the Lagrangian. The power counting of the theory provides a systematic method to determine observables order-by-order in terms of a small expansion parameter given by a ratio of typical scales involved in the reaction. For reviews of the highly successful application of EFT(\pi) in the PC sector, see, e.g., Refs. [5, 6, 7, 8].

At the low energies of interest, PV interactions between nucleons can be described by transitions between S- and P-wave states [9, 10]. Incorporating isospin, the leading-order (LO) PV EFT(\hat{\pi}) Lagrangian contains five independent operators [11, 12, 13]:

\[
\mathcal{L}_{PV} = - \begin{bmatrix}
g^{(3S_1-1P_0)} d_t^{\dagger} & d_t \left( N T^2 \sigma_2 \tau_2 \gamma_i \gamma_j \right)
+ g^{(1S_0-3P_0)} d_s^{A\dagger} \left( N T^2 \sigma_2 \tau_2 \gamma_i \gamma_j \right)
+ g^{(3S_0-3P_0)} d_s^{A\dagger} \left( N T^2 \sigma_2 \tau_2 \gamma_i \gamma_j \right)
+ g^{(3S_1-3P_1)} d_t^{\dagger} \left( N T^2 \sigma_2 \tau_2 \gamma_i \gamma_j \right)
\end{bmatrix} + \text{h.c.}
\]

Here, \(d_t\) and \(d_s\) are auxiliary fields representing two-nucleon states in the \(3S_1\) and \(1S_0\) channel, respectively [14], \(a \sigma \nabla^2 b = a \partial^2 b - (D_a) \partial b\), with \(\nabla\) a nucleon covariant derivative and \(\sigma\) some spin-isospin-operator, and the isospin matrix \(I = \text{diag}(1, 1, -2)\). The \(g^{(X-Y)}\) are the PV LECs encoding the short-distance physics. They cannot be determined within the EFT itself, but are related to standard model parameters though nonperturbative QCD calculations. Given the complexity of such calculations, another approach is to fit the LECs to data. However, this requires the existence of a sufficient number of experimental results and the calculation of the corresponding observables in the EFT(\hat{\pi}) framework.

3. Parity violation in two-nucleon systems

Because transition amplitudes can be determined analytically, two-nucleon systems present the simplest environments to determine PV observables theoretically, and a number of reactions
The scattering of a longitudinally polarized nucleon beam on an unpolarized nucleon target, a longitudinal asymmetry $A_L$, can be defined as

$$A_L^{NN} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-},$$

where $\sigma_\pm$ represents the cross section for scattering of a nucleon beam with helicity $\pm 1$. Results of a measurement of $A_L^{pp}$ in proton-proton scattering at an energy low enough for EFT($\hat{g}$) to be applicable are reported in Ref. [15]. Neglecting Coulomb effects in the proton-proton case and adjusting conventions to be consistent with Eq (1), the LO EFT($\hat{g}$) results are given by [16, 17, 12]

$$A_L^{pp} = -\frac{32M}{\pi} p \left( g_{(\Delta I=0)}^{(1S_0^{-3}P_0)} - g_{(\Delta I=1)}^{(1S_0^{-3}P_0)} + g_{(\Delta I=2)}^{(1S_0^{-3}P_0)} \right),$$

$$A_L^{np} = -\frac{32M}{\pi} p \left( \frac{d^3s_0}{dt^3} + 3 \frac{d^3s_1}{dt^3} \left( g_{(\Delta I=0)}^{(1S_0^{-3}P_0)} - 2g_{(\Delta I=2)}^{(1S_0^{-3}P_0)} \right) \right),$$

$$A_L^{np} = -\frac{32M}{\pi} p \left( \frac{d^3s_0}{dt^3} + 3 \frac{d^3s_1}{dt^3} \left( g_{(\Delta I=0)}^{(3S_1^{-1}P_1)} + 2g_{(\Delta I=2)}^{(3S_1^{-1}P_1)} \right) \right),$$

with $p$ the momentum in the center-of-mass frame, $M$ the nucleon mass, and $d\sigma/d\Omega$ the PC differential cross section in the corresponding channel. At the energy of the experiment of Ref. [15], Coulomb effects are shown to be of the order of approximately 3% and can therefore be safely neglected given the theoretical uncertainties of a LO calculation.

For a perpendicularly polarized neutron beam passing through an unpolarized target, PV interactions cause a rotation of the neutron spin. For a proton target with target density $\rho$, the rotation angle per unit length to next-to-leading (NLO) order is given by [18]

$$\frac{1}{\rho} \frac{d\phi}{dt} = 4\sqrt{2\pi M} \left( 2g_{(\Delta I=0)}^{(3S_1^{-1}P_1)} + g_{(\Delta I=1)}^{(3S_1^{-1}P_1)} Z_t + 1 \cdot \frac{1}{\gamma_t} + g_{(\Delta I=2)}^{(3S_1^{-1}P_1)} Z_s + 1 \cdot \frac{1}{\gamma_s} \right),$$

where $Z_{t/s} = (1 - \gamma_{t/s} r_{t/s})^{-1}$, and $\gamma_{t/s}$ ($r_{t/s}$) are the poles in the NN S-wave scattering amplitude (effective ranges) in the $^3S_1/^1S_0$ channel. An order-of-magnitude estimate of the size of the rotation angle per unit length for a typical hydrogen target is in the range $10^{-7}$ rad/m to $10^{-6}$ rad/m.

Deuteron photodisintegration and radiative neutron capture on protons provide several PV observables. Of particular experimental interest is the angular asymmetry $A_\gamma$, in the capture of polarized neutrons ($\vec{n}p \rightarrow d\vec{\gamma}$), which has been the goal of a measurement at the Spallation Neutron Source at Oak Ridge National Laboratory [19]. The LO EFT($\hat{g}$) result is

$$A_\gamma = 4 \sqrt{\frac{2}{\pi} \frac{M^3}{\kappa_1(1 - \gamma_t a_s)}} g_{(\Delta I=0)}^{(3S_1^{-1}P_1)} ,$$

where $\kappa_1$ is the nucleon isoscalar anomalous magnetic moment and $a_s$ the $^1S_0$ scattering length.

With ongoing developments in high-intensity photon sources, there is renewed interest in the measurement of the asymmetry $A_L^\gamma$ in deuteron photodisintegration with polarized photons ($\vec{\gamma}d \rightarrow np$),

$$A_L^\gamma = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-},$$
where $\sigma_+\gamma$ is the breakup cross section for a photon beam with helicity $\pm 1$. For exactly reversed kinematics, this asymmetry is identical to the induced photon polarization $P_\gamma$ in unpolarized neutron capture ($np \rightarrow d\gamma$). It provides independent and complementary information to $A_\gamma$, as can be seen from the LO result for $P_\gamma$ [13, 20],

$$P_\gamma = -2\sqrt{\frac{2}{\pi}} \frac{M_0^3}{\gamma} \left[ \left( 1 - \frac{2}{3}\gamma_t a_s \right) g^{(3S_1 - P_1)} + \frac{\gamma_t a_s}{3} \left( g^{(1S_0 - P_0)} - 2g^{(1S_0 - P_0)} \right) \right].$$

In particular, $P_\gamma$, or equivalently $A^L_\gamma$, is one of a very limited number of observables sensitive to the isotensor coupling $g^{(1S_0 - P_0)}$. Reference [21] determined the energy dependence of $A^L_\gamma$ to NLO as well as a very simplistic figure of merit to determine at which energy to best perform the corresponding experiment. Because the PV LECs are currently not determined, Ref. [21] used different model estimates and found that the figure of merit is maximized for photon energies close to threshold, $2.259\text{MeV} < k < 2.264\text{MeV}$. However, these results only represent order-of-magnitude estimates because of the uncertainty in determining the LECs.

### 4. Parity violation in three-nucleon systems

In systems with three or more nucleons, three-nucleon (3N) forces have to be taken into account. The EFT power counting rules based on dimensional analysis indicate that 3N interactions are suppressed relative to NN interactions. However, in the PC sector of the theory a cutoff dependence is present in the LO solutions of the scattering amplitude for neutron-deuteron scattering in the spin doublet channel [22]. This unphysical dependence on the cutoff can be removed by the introduction of a 3N contact term at LO. The accompanying LEC can be related to a three-body observable, such as a nucleon-deuteron scattering length or a three-body binding energy. Given the difficulty in obtaining a sufficient number of experimental results to pin down the PV two-nucleon LECs, an analogous enhancement of a 3N interaction in the PV sector would present a significant obstacle in describing hadronic parity violation in few-nucleon systems.

In Ref. [23] the ultraviolet behavior of the PV neutron-deuteron scattering amplitude for S-P wave transitions is analyzed. It is shown that no divergent contribution appears at LO, while at NLO the spin-isospin structures of the allowed PV 3N operators are different from those of possibly divergent terms. Therefore, up to and including NLO, no PV 3N operators are required, and up to an accuracy of roughly 10% two-nucleon interactions should be sufficient to analyze parity violation in few-nucleon systems.

Considering a deuteron target, the spin rotation angle of a polarized neutron beam to NLO is $\text{EFT}(\hat{\theta})$ is given by [18] (also see Ref. [24] for a LO calculation)

$$\frac{1}{\rho} \frac{d\sigma^\text{PV}}{dl} = \left( \left[ 16 \pm 1.6 \right] g^{(3S_1 - P_1)} + \left[ 34 \pm 3.4 \right] g^{(3S_1 - 3P_1)} \right) \frac{\text{rad}}{\text{MeV}^2}.$$ 

(10)

Theoretical uncertainties are estimated using three different methods, and the reported errors are the most conservative of these three estimates. An order of magnitude estimate of the rotation angle per unit length is again in the range of $10^{-7} \text{rad/m}$ to $10^{-6} \text{rad/m}$.

### 5. Parity violation in chiral EFT

The range of applicability of $\text{EFT}(\hat{\theta})$ is restricted to very low energies and to few-nucleon systems. To analyze processes involving higher energies and/or more complex nuclei, the

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1 Ref. [20] includes a formally higher-order contribution.
theoretical description has to include pions as dynamical degrees of freedom. Effective field theory methods can also be applied in this case. The LO PV pion-nucleon Lagrangian was constructed in Refs. [25, 26]. These results can be used to derive a PV nucleon-nucleon potential based on chiral symmetry [16, 27]. In addition to a one-pion-exchange contribution at LO, at NLO contact terms analogous to those in EFT(\#) and two-pion-exchange contributions appear. In addition to two-nucleon observables, chiral EFT has been used to determine the longitudinal asymmetry in the charge-exchange reaction \( \vec{n} \ ^3\text{He} \rightarrow p^\prime \ ^3\text{H} \) [27, 28].

6. Parity-violating interactions in the large-\( N_c \) expansion

Given the challenges in obtaining experimental results to determine the PV LECs, additional theoretical constraints on the couplings can be very valuable. QCD in the limit of the number of colors \( N_c \) becoming large [29, 30] provides one method of arriving at such constraints. In order to apply the large-\( N_c \) analysis to two-nucleon interactions, the NN potential is defined by [31]

\[
V(p_-, p_+) = \langle (p'_1, C), (p'_2, D) | \hat{H} | (p_1, A), (p_2, \beta, B) \rangle,
\]

where \( A, B, C, D \) collectively denote nucleon spin and isospin quantum numbers. The momenta \( p_\pm \) are related to the momenta of the incoming and outgoing nucleons through \( p_\pm \equiv p' \pm p \), where \( p' = p'_1 - p'_2 \) and \( p = p_1 - p_2 \). The Hartree Hamiltonian is given by [30, 31]

\[
\hat{H} = N_c \sum_n \sum_{s,t} v_{stn} \left( \frac{\hat{S}^s}{N_c} \right)^n \left( \frac{\hat{I}^t}{N_c} \right)^{n-s-t} \frac{1}{G^t} \left( \frac{N_c}{N} \right)^{n-s-t},
\]

where

\[
\hat{S}^s = q^\dagger \frac{\sigma^s}{2} q, \quad \hat{I}^t = q^\dagger \frac{\tau^t}{2} q, \quad \hat{G}^{st} = \hat{q}^\dagger \frac{\sigma^s \tau^t}{4} \hat{q},
\]

and \( \hat{q} \) denotes the light-quark doublet. The matrix elements of these operators scale as [32, 33]

\[
\langle N' | \hat{S} | N \rangle \sim \langle N' | \hat{I} | N \rangle \sim 1, \quad \langle N' | \hat{G} | N \rangle \sim \langle N' | \Pi | N \rangle \sim N_c.
\]

in the large-\( N_c \) limit. The coefficients \( v_{stn} \) contain factors of momentum, and momenta scale as \( p_- \sim 1 \) and \( p_+ \sim N_c^{-1} \) [32, 33, 31]. When applied to PV interactions, the large-\( N_c \) analysis leads to a hierarchy of terms [34, 35]. The LO terms are given by

\[
V^p_{N_c} = N_c \left[ U^1_p(p_2^2) p_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \vec{\tau}_1 \cdot \vec{\tau}_2 + U^2_p(p_2^2) p_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)[\vec{\tau}_1 \vec{\tau}_2]^{\pm 2} \right],
\]

with \([\ldots]_2\) the symmetric and traceless rank-two tensor, and the coefficient functions \( U^i_p(p_2^2) \) scaling as \( O(1) \). There are four terms at NLO in the large-\( N_c \) expansion that are all isovector, while the six operators at next-to-next-to-leading order are isoscalar and isotensor like the LO terms.

These results have important implications when combined with the EFT(\#) approach [35]. While in the EFT power counting all five LECs in Eq. (1) are expected to be of the same size, the large-\( N_c \) analysis implies that only two of these terms are dominant in a combined EFT(\#) and large-\( N_c \) expansion. In addition, the large-\( N_c \) analysis implies a relation between the two isoscalar LECs that should hold up to corrections of order \( N_c^{-2} \), or roughly 10%. The analysis of Ref. [35] shows that these results are not inconsistent with existing measurements.

The reduction in the number of dominant terms and the relation between the LECs in the large-\( N_c \) limit provide significant constraints on PV interactions. In particular the fact that the isotensor term is LO in the combined EFT and large-\( N_c \) analysis indicates the importance
of finding corresponding experimental constraints and to perform a lattice QCD calculation of this term. While in principle the existing measurement of the longitudinal asymmetry in $\bar{p}p$ scattering at 221 MeV [36, 37] could provide information on the isotensor coupling, this energy is far outside the range of applicability of EFT($\chi$). At low energies, a measurement of the PV asymmetry in $\vec{\gamma}d \to np$ could put important experimental constraints on the isotensor component of the PV interaction.

7. Conclusions

Hadronic parity violation originates in an interplay of weak and nonperturbative strong interactions between quarks inside hadrons. It therefore provides a unique probe of our understanding of the standard model. While experimentally more accessible in complex nuclear systems, the recent focus has been on parity violation in few-nucleon systems, which can be more readily described in terms of two- and three-nucleon interactions. Effective field theories provide a model independent framework to systematically analyze and interpret experimental results and to provide theoretical error estimates. At LO in EFT($\chi$), which is applicable at the very low energies relevant to many of the recent experiments, parity violation is described in terms of five S-P transition operators. A variety of observables has been calculated within this framework. In addition, the large-$N_c$ expansion of QCD provides important theoretical constraints on the PV interactions. In a combined EFT($\chi$) and large-$N_c$ expansion, the number of LO terms reduces from five to two, and a relation between the two isoscalar LECs is predicted. These results provide strong motivation to determine the isotensor component of the PV interactions, both from a measurement of the longitudinal asymmetry in $\vec{\gamma}d \to np$ and from lattice QCD.

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References

[1] Bowman C, Bowman J and Yuan V 1989 Phys.Rev. C39 1721–1724
[2] Haxton W C and Holstein B R 2013 Prog.Part.Nucl.Phys. 71 185–203 (Preprint 1303.4132)
[3] Schindler M R and Springer R P 2013 Prog.Part.Nucl.Phys. 72 1–43 (Preprint 1305.4190)
[4] Desplanques B, Donoghue J F and Holstein B R 1980 Annals Phys. 124 449
[5] Beane S R, Bedaque P F, Haxton W C, Phillips D R and Savage M J 2000 In *Shifman, M. (ed.): At the frontier of particle physics, vol. I* 133–269. (Preprint nucl-th/0008064)
[6] Bedaque P F and van Kolck U 2002 Ann. Rev. Nucl. Part. Sci. 52 339–396 (Preprint nucl-th/0203055)
[7] Platter I 2009 Few Body Syst. 46 139–171 (Preprint 0904.2227)
[8] Vanasse J 2016 Int. J. Mod. Phys. E25 1641002 (Preprint 1609.03086)
[9] Danilov G 1965 Phys.Lett. 18 40–41
[10] Danilov G 1971 Phys.Lett. B35 579–580
[11] Girlanda L 2008 Phys. Rev. C77 067001 (Preprint 0804.0772)
[12] Phillips D R, Schindler M R and Springer R P 2009 Nucl. Phys. A822 1–19 (Preprint 0812.2073)
[13] Schindler M R and Springer R P 2010 Nucl. Phys. A846 51–62 (Preprint 0907.5358)
[14] Kaplan D B 1997 Nucl. Phys. B494 471–484 (Preprint nucl-th/9610052)
[15] Eversheim P D, Schmitt W, Kuhn S E, Hinterberger F, von Rossen P et al. 1991 Phys.Lett. B256 11–14
[16] Zhu S L, Maekawa C M, Holstein B R, Ramsey-Musolf M J and van Kolck U 2005 Nucl.Phys. A748 435–498 (Preprint nucl-th/0407087)
[17] Holstein B R 2005 Fizika. B14 165–216 (Preprint nucl-th/0607038)
[18] Grießhammer H W, Schindler M R and Springer R P 2012 Eur.Phys.J. A48 7 (Preprint 1109.5667)
[19] Gericke M T, Alarcon R, Balascuta S, Barron-Palos L, Blessinger C et al. 2011 Phys.Rev. C83 015505
[20] Shin J, Ando S and Hyun C 2010 Phys.Rev. C81 055501 (Preprint 0907.3995)
[21] Vanasse J and Schindler M R 2014 Phys. Rev. C90 044001 (Preprint 1404.0658)
[22] Bedaque P F, Hammer H W and van Kolck U 2000 Nucl. Phys. A676 357–370 (Preprint nucl-th/9906032)
[23] Grießhammer H W and Schindler M R 2010 Eur. Phys. J. A46 73–83 (Preprint 1007.0734)
[24] Vanasse J 2012 Phys.Rev. C86 014001 (Preprint 1110.1039)
[25] Kaplan D B and Savage M J 1993 Nucl. Phys. A556 653–671 [Erratum-ibid. A 570, 833 (1994)] [Erratum-ibid. A 580, 679 (1994)]
[26] Savage M J and Springer R P 1998 Nucl.Phys. A644 235–244 erratum-ibid. A 657, 457 (1999) (Preprint nucl-th/9807014)
[27] Viviani M, Baroni A, Girlanda L, Kievsky A, Marcucci L E and Schiavilla R 2014 Phys. Rev. C89 064004 (Preprint 1403.2267)
[28] Viviani M, Schiavilla R, Girlanda L, Kievsky A and Marcucci L 2010 Phys.Rev. C82 044001 (Preprint 1007.2052)
[29] ’t Hooft G 1974 Nucl. Phys. B72 461
[30] Witten E 1979 Nucl. Phys. B160 57
[31] Kaplan D B and Manohar A V 1997 Phys.Rev. C56 76–83 (Preprint nucl-th/9612021)
[32] Dashen R F, Jenkins E E and Manohar A V 1995 Phys. Rev. D51 3697–3727 (Preprint hep-ph/9411234)
[33] Kaplan D B and Savage M J 1996 Phys. Lett. B365 244–251 (Preprint hep-ph/9509371)
[34] Phillips D R, Samart D and Schat C 2015 Phys. Rev. Lett. 114 062301 (Preprint 1410.1157)
[35] Schindler M R, Springer R P and Vanasse J 2016 Phys. Rev. C93 025502 (Preprint 1510.07598)
[36] Berdoz A R et al. (TRIUMF E497) 2001 Phys. Rev. Lett. 87 272301 (Preprint nucl-ex/0107014)
[37] Berdoz A et al. (TRIUMF E497 Collaboration) 2003 Phys.Rev. C68 034004 (Preprint nucl-ex/0211020)