FORMATION AND EVOLUTION OF CLUMPY TIDAL TAILS AROUND GLOBULAR CLUSTERS

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ABSTRACT

We present some results of numerical simulations of a globular cluster orbiting in the central region of a triaxial galaxy on a set of “loop” orbits. Tails start forming after about a quarter of the globular cluster orbital period and develop, in most cases, along the cluster orbit, showing clumpy substructures as observed, for example, in Palomar 5. If completely detectable, clumps can contain about 7000 $M_\odot$ each, i.e., about 10% of the cluster mass at that epoch. The morphology of tails and clumps and the kinematical properties of stars in the tails are studied and compared with available observational data. Our finding is that the stellar velocity dispersion tends to level off at large radii, in agreement with that found for M15 and ω Cen.

Key words: galaxies: kinematics and dynamics — globular clusters: general — methods: n-body simulations

1. INTRODUCTION

Since Shapley’s pioneering work (Shapley 1918), globular clusters (GCs) have played a key role in our understanding of the Universe and of the manner in which our Galaxy formed. In the Milky Way, they are the oldest stellar systems found, with ages in the range 12–15 Gyr and so represent tracers of the formation history of the Galaxy.

They are the best systems for studying stellar dynamics, having relaxation times smaller than their age, so that, at least in the core, stars are expected to have lost memory of their initial conditions (Binney & Tremaine 1987). In the early 1980s, a number of approximated numerical studies of spherical self-gravitating systems (Cohn 1980) showed that the central density tends to increase dramatically over time, so that ultimately a power-law cusp is produced in the central region, even if these systems have an early evolutionary phase that resembles the King sequence of cluster models (King 1966). Together with the slow collapse of the core, stellar evaporation occurs: the approach to equipartition implies that more massive stars sink toward the center of the cluster, while lighter stars move outward. Core collapse can be halted by the presence of hard binaries, which, acting as energy sources, heat the central core by three-body encounters (Hénon 1961; Ostriker 1985).

Internal processes are not the only factor responsible for dynamical evolution in these systems: perturbations due to an external field (in particular, shocks due to the passage through the Galactic disk and to the interaction with the bulge) can significantly accelerate the evolution of a GC. Indeed, it is commonly accepted that the present GC population represents the survivor of an initially more numerous one depopulated by many disruptive processes (Murali & Weinberg 1997a, 1997b; Fall & Zhang 2001).

There is observational evidence that the globular cluster system (GCS) radial profile is less peaked than that of halo stars in our Galaxy, M31 (Capuzzo Dolcetta & Vignola 1997), M87, and M49 (Grillmair et al. 1986; McLaughlin 1995), as well as that in three galaxies of the Fornax Cluster (Capuzzo Dolcetta & Donnarumma 2001) and of 11 elliptical galaxies (Capuzzo Dolcetta & Tesseri 1999). This fact leads to the hypothesis (Capuzzo Dolcetta & Tesseri 1997) that the two systems (halo and GCS) originally had the same profile and that, afterward, the GCS evolved mainly because of two complementary effects: tidal interaction with the Galactic field (which causes less concentrated clusters to disintegrate more rapidly) and dynamical friction (which induces massive GCs to decay in the central Galactic region in less than $10^8$ yr; see Capuzzo Dolcetta & Vicari 2004). External tidal fields have the effect of inducing the evolution of the shape of the mass function (MF) of individual clusters because of the preferential depletion of low-mass stars (Baumgardt & Makino 2003) as a consequence of two-body relaxation. Strong evidence that the tidal field plays a fundamental role in the evolution of MFs was found from the discovery that their slopes correlate more strongly with the cluster location in the Milky Way than with the cluster metallicity (Djorgovski et al. 1993).

In the last decade, many observational evidences of the interaction of GCs with the tidal field have been found. First, Grillmair et al. (1995), using color-magnitude–selected star counts in 12 Galactic GCs, showed that in the outer parts of these clusters the stellar surface density profiles exceeded the prediction of King models, and also extended outside the tidal radius of the corresponding King model. Other results confirmed Grillmair et al.’s findings (Lehmann & Scholz 1997; Testa et al. 2000; Leon et al. 2000; Siegel et al. 2001; Lee et al. 2003); all these works suggest that many GCs are likely surrounded by halos or tails made up of stars that were tidally stripped from the system. This was the state of the art until the spectacular findings of two tidal tails emanating from the outer parts of the Palomar 5 GC and covering an arc of 10° on the sky, corresponding to a projected length of 4 kpc at the distance of the cluster (Odenkirchen et al. 2001, 2003), obtained by the Sloan Digital Sky Survey.¹

One of the relevant observational features of Pal 5 is the presence of well-defined clumps in the star distribution along the tails (Odenkirchen et al. 2001, 2003). NGC 6254 and Pal 12 also seem to show clumpy structures in their tails (Leon et al. 2000). This still deserves an exhaustive interpretation. The simulations of Combes et al. (1999) actually show the presence of small clumps (containing about 0.5% of the stars of the cluster) in the tidal tails. The authors attribute the formation of these

¹ See also http://www.sdss.org.
clumps to strong gravitational shocks suffered by the cluster. On another hand, Dehnen et al. (2004) were not able to reproduce the clumps observed in the tails of Pal 5, even after adopting a realistic Galactic model, so they argued that these structures could be due to the effect of Galactic substructures not accounted for in their simulations (giant molecular clouds, spiral arms, dark matter subhalos, or massive compact halo objects).

With the aim of better understanding the mechanism of interaction of GCs with the external field, in particular with the bulge, and investigating on the presence of clumps in tidal tails, we performed numerical simulations of GCs in orbit around a triaxial galaxy and also aimed at clarifying the morphological connection between the clusters’ tidal tails and their orbits.

In §§ 2–5, we show the results for GCs on “loop” orbits in an inner region of a triaxial galaxy. In particular, in § 2, the numerical methods adopted to perform the simulation are discussed. In § 3, the galaxy and cluster model adopted are presented. In §§ 4 and 5, we deal with the main results of our work, especially those concerning the formation of tidal tails around the cluster and their orientation with respect to the cluster orbit (§ 4.1); the radial density profiles of the cluster, as they evolve with time, and the presence of clumpy regions in the tails (§ 4.2); the velocity dispersion of stars in the cluster (§ 4.3); the estimate of the mass-loss rate (§ 5.1); and the evolution of the global MFs (§ 5.2). In § 6, all the findings are summarized and discussed.

2. NUMERICAL METHOD

All the simulations were performed by means of an implementation of a tree code, carried out mainly by one of the authors (P. M.). It is based on the algorithm described in Barnes & Hut (1986) and adopts multipolar expansions of the potential truncated at the quadrupole moment. It was parallelized to run on high-performance computers via MPI routines, employing an original parallelization approach (Miocchi & Capuzzo-Dolcetta 2002). The time integration of the “particles” trajectories is performed by a second-order leapfrog algorithm. This uses individual and variable time steps, according to the block time scheme (Aarseth 1985; Hernquist & Katz 1989), with additional corrections implemented in order to also keep the same order of approximation during the time-step change. The maximum time step allowed is \( \Delta t_{\text{max}} = 0.01t_c \) [where \( t_c \sim (r_{\text{core}}^3/GM)^{1/2} \) is the core-crossing time of the GC, in which \( M \) is its mass and \( r_{\text{core}} \) is its core radius], whereas the minimum is \( \Delta t_{\text{min}} = \Delta t_{\text{max}}/2^3 \); thus, the fastest particles may have a time step as small as \( \approx 4 \times 10^{-5}t_c \). The best criterion we found for choosing the time step of the \( i \)th particle is via the formula \((d_i/a_i)^{1/2}/d_i/v_i)/20\), where \( v_i \) is the velocity of the particle relative to its first neighbor, \( d_i \) is the distance from its first neighbor, and \( a_i \) is the modulus of the acceleration.

To avoid instability in the time integration, we smoothed the \( 1/r_{ij} \) interaction potential by substituting \( 1/r_{ij} \) with a continuous \( \beta \)-spline function that gives an exactly Newtonian potential for \( r_{ij} > \epsilon \) and a force that vanishes for \( r_{ij} \rightarrow 0 \) (Hernquist & Katz 1989). In all the runs, we set \( \epsilon = 1.4 \times 10^{-3}r_{\text{core}} \), so \( (\epsilon^3/GM)^{1/2} \sim \Delta t_{\text{min}} \). Note that such a value of \( \epsilon \) is much less than the typical interparticle distance. For the quality of the orbit’s time integration, we checked that the upper bound of the relative error in the energy conservation \( (\Delta E/E) \) is \( 10^{-8} \) per time step, even in the absence of the external field.

3. CLUSTER AND GALAXY MODELS

3.1. Galaxy Model

The external galactic field due to the bulge is represented by the potential of the Schwarzschild model (Schwarzschild 1979). The Schwarzschild model is a nonrotating, self-consistent triaxial ellipsoid with axis ratios 2:1.25:1, typical of many galaxies (Bertola et al. 1991).

Defining dimensionless units as

\[
x' = x/r_b, \quad y' = y/r_b, \quad z' = z/r_b,
\]

where \( r_b \) is the bulge core radius, the potential \( \Phi(x', y', z') \) is expressed as the sum of a spherically symmetric term, \( \Phi_1(r') \) \((r' = r/r_b)\) that corresponds to the potential given by a density distribution following the modified Hubble law \( \rho(r') = \rho_0 (1 + r'^2)^{-3/2} \), \( (\rho_0 \equiv M_b/r_b^3) \), plus two other terms, \( \Phi_2(x', y', r') \) and \( \Phi_3(x', y', r') \) so that

\[
\Phi(x', y', z') = A[\Phi_1(r') + \Phi_1(x', r') + \Phi_2(x', y', r')],
\]

where

\[
\begin{align*}
\Phi_1 &= - \frac{1}{r'} \ln \left( r' + \sqrt{1 + r'^2} \right), \\
\Phi_2 &= \frac{3}{2(1 + c_2 r'^2)^{3/2}}, \\
\Phi_3 &= \frac{3}{1 + c_4 r'^2)^{3/2}}, \\
A &= 4\pi \frac{GM_b}{r_b},
\end{align*}
\]

and \( M_b \) is the bulge mass.

The coefficients \( c_i \) have the values \( c_1 = 0.06408, \ c_2 = 0.65456, \ c_3 = 0.01533, \) and \( c_4 = 0.48067 \) (de Zeeuw & Merritt 1983; Pesce et al. 1992). They have been determined so to have density axial ratios roughly constant with \( r \).

Following Pesce et al. (1992), we consider \( M_b = 3 \times 10^9 M_\odot \) and \( r_b = 200 \) pc, but the results obtained in dimensionless variables (see Appendix A) are scalable in terms of \( r_b \) and \( M_b \) for given initial conditions for positions, velocities, and \( m_i/M_b \).

3.2. Cluster Model

For the initial cluster model, we chose a multimass King distribution (King 1966; Da Costa & Freeman 1976) with 10 mass classes ranging between 0.12 and 1.2 \( M_\odot \). To find the distribution function for each mass class, we integrated Poisson’s equation, as described in Appendix B.

The initial cluster MF is chosen in the Salpeter form (Salpeter 1955), i.e., \( dN/dm \propto m^{-2.35} \) (see Appendix C for a discussion about the remnants of progenitor stars more massive than \( 1.2 \ M_\odot \)). We included mass segregation in the initial conditions of our cluster model because we wanted to simulate a dynamically relaxed cluster (supposed to be sufficiently massive as to have fractionally decayed in the central galactic region). The “initial” mass of the system is \( M_{\text{init}} = 3 \times 10^5 M_\odot = 10^{-4} M_b \), the initial concentration parameter is \( c = \log (r_c/r_{\text{core}}) = 1.1 \), and the central velocity dispersion is \( \sigma = 9.4 \) km s\(^{-1}\) \( = 0.036 r_b/t_{\text{cross}} \), where \( t_{\text{cross}} = (r_b^3/GM_b)^{1/2} \) is the bulge crossing time.

The system was then represented by a number \( N \) of “particles” lower than the number of stars in the real cluster,
with masses properly rescaled to give a total mass equal to $M_{\text{tot}}$. The cluster moves on the $y$-$z$ coordinate plane, following loop orbits of different ellipticity

$$e = \frac{R_a - R_p}{R_a + R_p},$$

(7)

where $R_a$ and $R_p$ are, respectively, the apocenter and pericenter distances (see Table 1 for orbital parameters).

4. RESULTS ON TIDAL TAILS

In the following, we present the main findings of our work concerning tidal tails’ structure and evolution. When we refer to the center of density of the cluster, we mean a mass density–weighted center as defined by Casertano & Hut (1985).

4.1. Tidal Tail Formation and Morphology

In all the simulations performed, the cluster starts moving around the galaxy center in a clockwise direction (seen from the positive $x$-axis). The different loop orbits have been followed for about $30t_{\text{cross}}$. In Figures 1–8, the formation and subsequent development of tails around the GC are shown. After about $8t_{\text{cross}}$, tidal tails have clearly formed. They continuously accrete the stars leaving the cluster, so that after $30t_{\text{cross}}$, in the case of the quasi-circular orbit ($e = 0.03$), they are elongated for more than $3r_b$ each and contain about 75% of the initial cluster mass. As is clearly visible from these figures, the degree of elongation of the tails along the cluster orbital path strongly depends on the ellipticity $e$ (eq. [7]) of the cluster orbit. Indeed, whereas in the case of the quasi-circular orbit, tails are a clear tracer of the cluster path, in the most eccentric orbit ($e = 0.57$), tails are strictly elongated along the orbital path only when the cluster is near the perigalacticon, whereas at the

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### Table 1

| $N$ \(1\) | $e$ \(2\) | $(x_0, y_0, z_0)$ \(3\) | $(v_{x,0}, v_{y,0}, v_{z,0})$ \(4\) |
|---------|---------|-----------------|-----------------|
| $1.6 \times 10^5$ | 0.03 | (0, 0, 2.50) | (0, 1.82, 0) |
| $1.6 \times 10^4$ | 0.03 | (0, 0, 2.50) | (0, 1.82, 0) |
| $1.6 \times 10^4$ | 0.27 | (0, 0, 3.50) | (0, 1.30, 0) |
| $1.6 \times 10^4$ | 0.57 | (0, 0, 7.50) | (0, 0.78, 0) |

Notes.—Orbital parameters of the cluster in the four simulations performed. The first column shows the total number of particles used in each simulation. The ellipticity $e$ is defined by eq. (7). Initial positions and velocities of the cluster with respect to the galaxy center (cols. [3] and [4]) have been expressed, respectively, in units of the galaxy bulge radius $r_b$ and of the bulge typical velocity dispersion $r_b/t_{\text{cross}}$. 

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Fig. 1.—First orbital period of a $3 \times 10^5 M_\odot$ GC in the potential described in § 3.1. The cluster moves on a quasi-circular orbit ($e = 0.03$) around the galaxy center in a clockwise direction (see text). Distances are in units of the galactic bulge radius $r_b$. Some snapshots are labeled with time, expressed in units of the galactic bulge crossing time $t_{\text{cross}}$.

Fig. 2.—Second orbital period of the GC described in Fig. 1.

Fig. 3.—Third orbital period of the GC described in Fig. 1.
apogalacticon, they tend to deviate from the cluster path. Nevertheless, in Miocchi et al. (2004), a remarkable tails-orbit alignment is found for clusters moving on quasi-radial orbits in the same bulge potential. However, it is important to stress that, in order to perform accurate predictions of the cluster orbit from observational detections of tidal tails, it is necessary to look at the spatial distribution of stars well outside the cluster (typically 2–3 times the cluster limiting radius). Indeed, in the vicinity of the GC, stars in the tails are not aligned with the cluster orbit, nor are they in the case of small ellipticity (see Fig. 9), but they are distributed along the peculiar S-shaped profile not aligned along the orbit.

The orbit ellipticity also influences the similarity between the two cluster tails. For the quasi-circular orbit, these structures are symmetric for the whole duration of the simulation, being elongated, at a given time, for the same length. For more eccentric orbits, the leading tail tends to be more elongated than the trailing one when going from the apocenter to the orbital pericenter, and vice versa, it is less elongated than the trailing tail when the cluster moves toward the apocenter. In any case, the tail that precedes the cluster always extends slightly below the orbit, whereas the trailing one lies slightly above the orbit, in agreement with what was observed for Pal 5.

The shape and orientation of the tails can be easily understood in the case of a cluster moving on a circular orbit in an axisymmetric external field, using a rotating frame of reference.
with the origin in the barycenter of the cluster, the $X$-axis pointing toward the galactic center, the $Y$-axis parallel to the direction of motion of the cluster, and the $Z$-axis orthogonal to the orbital plane. In this reference frame, the galactic tidal field tends to accelerate stars along the $C6X$ directions (Heggie & Hut 2003), causing stars to escape from the system through the Lagrangian points $L_1$ and $L_2$ (which are the two equilibrium points located along the $X$-axis). However, the Coriolis acceleration tends to align escaping stars along the direction of motion of the cluster around the galaxy; this yields the peculiar $S$-shape just outside the cluster in the inner part of the tails. For a deeper discussion about the morphology of the $S$-shaped profile, see Di Matteo et al. (2005).

4.2. Density Profiles

In order to describe the tidal debris and to compare our findings with observations (Lehmann & Scholz 1997; Testa et al. 2000; Leon et al. 2000; Siegel et al. 2001; Lee et al. 2003; Odenkirchen et al. 2003), we studied the profile of the volume and surface densities as a function of the distance from the cluster center. Obviously, this description does not take into account the fact that stars lost from the cluster are not placed in a spherically symmetric structure, but it has the advantage of providing a global study of both the cluster and the tails that can be easily compared with observational data. In Figures 10 and 11, the volume density of the system is shown at different epochs for the various orbits. Once the tails have completely developed, outside the $S$-shape distribution, density clumps appear. They are symmetrically located in the two tails, as shown in Figure 12 for the cluster on the quasi-circular orbit in this case, the most prominent clumps are located at a distance from the cluster center between 0.25 and 0.4 $r_b$. The density profiles are very similar to that of Pal 5, where clumps are visible in the outer part of the cluster (Odenkirchen et al. 2003).

Of course, the possibility for observationally detecting these clumps is strongly related to the cluster position along its orbit. Indeed, we computed the contrast density ratio $\rho_{cl}/\rho_s$, where $\rho_{cl}$ is the local volume density in the clumps and $\rho_s$ is the background (i.e., the bulge) density around them. This ratio is at a maximum when the cluster is near apogalacticon and decreases when moving toward perigalacticon, as shown in Table 2. This is due to two complementary effects: when the cluster is near apogalacticon, $\rho_s$ is at a minimum (according to the galaxy model described in § 3.1); at the same time, the elongation of the tails tends to be compressed with respect to that at perigalacticon (see, for example, Figs. 7 and 8), and so $\rho_{cl}$ increases. If completely detectable, clumps can contain about 7000 $M_\odot$ each (i.e., about 10% of the cluster mass at that epoch), as in the case of the cluster moving on the quasi-circular orbit after $30t_{cross}$.

In order to study the mass distribution along the tails, we have also evaluated the “linear” density for the whole system.
Fig. 10.—Volume mass density of the cluster in the case of the quasi-circular orbit ($e = 0.03$) at four different epochs, as labeled. The dashed line in each panel represents the best King model fit at that epoch. The presence of clumps in the tails is clearly visible.
Fig. 11.—Volume mass density of the cluster moving on loop orbits with $e = 0.27$ (left) and $e = 0.57$ (right), at different epochs. The dashed line in each panel represents the best King model fit at that epoch. Note that in the case of the most eccentric orbit, clumps are not yet formed at $t = 19.2 t_{\text{cross}}$. 
in the quasi-circular orbit around the galaxy. This study is particularly well suited to investigate the mass distribution because tails form a long and thin structure. The upper panel of Figure 13 shows the linear mass density as a function of the curvilinear abscissa $s$ along the system: the absolute maximum in the plot corresponds to the cluster location, whereas the two symmetric relative maxima correspond to the two main clumps. These clumps are unbound structures (see also Di Matteo et al. 2004). We followed the motion of stars that, at a certain time, stay in the two clumps: they crowd in the clumps for some time and then move away to the outer parts of the tails. Once away from the clumps, these stars tend to disperse along the cluster tails. In addition, the symmetric location of these two clumps with respect to the cluster center makes it improbable that these structures can be due to local inhomogeneities in the gravitational field along the tails. These clumps are more likely related to kinematical properties of stars in their surroundings. The bottom panel in Figure 13 shows the velocity of stars, $v_{rel,a}$, as a function of the curvilinear abscissa $s$, in a rotating reference frame centered on the GC center and with an axis always pointing toward the galaxy center.

4.3. Velocity Dispersion

For the two Galactic GCs M15 and $\omega$ Cen, there is observational evidence that the stellar velocity dispersion remains constant at large radii (Scarpa et al. 2003; Drukier et al. 1998). Three hypotheses have been raised to justify these findings: (1) tidal heating, as suggested by Drukier et al. (1998) for M15; (2) the presence of a dark matter halo surrounding the clusters (Carraro & Lia 2000); and (3) a breakdown of Newton’s law of gravity in the weak acceleration regime (Scarpa et al. 2003).

We studied the velocity dispersion profile of our simulated cluster as it would be detected if the system was seen along a line of sight perpendicular to the cluster orbital plane. In Figure 14, line-of-sight velocities of members of the cluster are plotted versus distance from the center, at four different epochs. At $t = 0$, the velocities decrease from the center of the cluster outward, as is expected from a King model with a tidal cutoff. As the system moves through the galaxy and loses stars, the velocity profile varies significantly: it tends to decrease to a limiting value and then increases again. This behavior is very similar to that found in M15 (cf. Fig. 8 in Drukier et al. 1998).
Fig. 14.—Stellar velocities along the x-axis vs. distance from the cluster center, for the case of the quasi-circular orbit ($e = 0.03$), plotted at four different epochs. Once stars begin to escape from the cluster, the velocity profile shows a minimum and then increases again.
Fig. 15.—Velocity dispersion profiles along the x-axis for the cluster in the quasi-circular orbit, at four different epochs.
In Figure 15 the line-of-sight velocity dispersion profile is shown. It is evident from the figure that in the outer part of the cluster, the dispersion tends to level off. This region corresponds to that characterized by a power-law volume density profile (see Figs. 10 and 11). Stars in this region are escaping from the system, and their motion is mostly oriented along the radial direction toward the galaxy center. Once escaped, they move around the galaxy weakly interacting with each other, with similar orbital parameters, so that the velocity dispersion found is similar to that of a set of particles moving in the adopted triaxial potential.

The second relevant finding is the decreasing velocity dispersion in the inner part of the cluster, which could be explained by the quick revirialization of the inner part as stellar mass is being lost. This is in accordance with the very low velocity dispersion of Pal 5 (Odenkirchen et al. 2002), a cluster that has suffered a great mass loss, as is now well established.

5. RESULTS ON MASS LOSS

5.1. Mass Loss

To estimate the mass loss from the cluster, we decided to use an “observational” definition. At any given time we compare the cluster local density $\rho_{GC}$ with the background stellar density $\rho_s$, assuming that a star actually belongs to the cluster if it is located in a region dense enough to make it distinguishable from the background, i.e., if

$$\frac{\Delta \rho}{\rho_s} \geq 1,$$  \hspace{1cm} (8)

where

$$\Delta \rho = (\rho_s + \rho_{GC}) - \rho_s.$$  \hspace{1cm} (9)

The limiting radius $r_L$ is then defined as the radius of the sphere (centered on the cluster density center) in which the cluster “emerges” from the stellar background. In Figure 16, the evolution of the cluster mass expressed in units of the initial mass $M_0$ is shown versus time for all four simulations performed.

In the three cases of orbits with relatively small apocenters ($\leq 3.5r_b$), the mass loss is dramatic: after about 30$r_{cross}$, the cluster loses about 75% of its mass. The best fit of the mass evolution as a function of time is given by

$$\frac{M(t)}{M(0)} = 0.77e^{-t/12} + 0.21,$$  \hspace{1cm} (10)

where $t$ is expressed in units of the bulge crossing time $t_{cross}$. In the remaining case, when the orbit extends up to 7.5$r_b$, the mass-loss rate considerably diminishes, and the cluster mass, after 30$r_{cross}$, is still about 60% of its initial value. As is evident in Figure 16, in this case, the mass-loss rate increases every time the cluster passes at the minimum distance from the galaxy center, and not all particles that become unbound at perigalacticon are still so when moving again to apogalacticon. It is possible to point out a region around the galaxy center inside which the cluster suffers more mass loss: in our case (cluster concentration equal to 1.1), this region corresponds roughly to $r \leq 4r_b$. This conclusion is in accordance with what is found in Miocchi et al. (2004), in which great mass loss occurs for clusters with comparable central density moving on quasi-radial orbits within such a region.

Finally, we want to stress that the choice of performing some of the simulations with a reduced number of particles ($N = 1.6 \times 10^4$) did not affect the mass-loss rate over the time interval of 30$r_{cross}$. Actually, Figure 16 clearly shows that, for the cluster in a quasi-circular orbit, the mass loss rate is the same using either $N = 1.6 \times 10^5$ (solid line) or an $N$ 10 times smaller (dashed line).

5.2. Mass Segregation and Mass Function

As explained in § 3.2, we adopted mass segregation in the GC initial conditions, because we aim at simulating a dynamically evolved cluster whose orbit had decayed in the inner galactic region because of dynamical friction. As the cluster begins to lose stars, the distribution of stars of different masses in the system starts evolving. This is shown in the left column of Figure 17, where the mean mass of stars populating three different spatial regions versus time is plotted. The first region corresponds to the sphere with radius $r = 0.016r_b$ (corresponding to $r = 3.22$ pc, with our choice of $r_b$) centered on the cluster, which initially contains 40% of the total mass of the system; the second region is the spherical shell with inner and outer radius $r = 0.016r_b$ and 0.036$r_b$, respectively, ($r = 7.18$ pc), which initially contains 80% of the cluster mass; and the third region is that outside $r = 0.036r_b$.

As time passes by, low-mass stars begin to escape from the system, and the mean stellar mass in the two inner regions starts to rise, whereas the external one remains quite constant. The increase of the mean stellar mass versus time in the central cluster regions is particularly evident in the case of the quasi-circular orbit and of the loop with ellipticity $e = 0.27$, because of the greater mass loss in these two cases. Plotting the mean stellar mass in the three regions defined above as a function of the fraction of mass lost, we see (Fig. 17, right) that the evolution of the mean mass depends mostly on the fraction of mass lost.
Fig. 17.—Left: Time evolution of the mean mass of stars in three different regions of space centered with the cluster. (a) Mean stellar mass inside $r = 0.016r_0$, in the case of the quasi-circular orbit (solid line), loop orbit with $e = 0.27$ (dashed line), and loop orbit with $e = 0.57$ (dot-dashed line). (c) Mean stellar mass between $r = 0.016r_0$ and $r = 0.036r_0$. (e) Mean stellar mass outside $r = 0.036r_0$. Right: Evolution of the mean mass of stars in three different regions of space as a function of the fraction of mass lost from the system. (b) Mean stellar mass inside $r = 0.016r_0$, in the case of the quasi-circular orbit (solid line), loop orbit with $e = 0.27$ (open circles), and loop orbit with $e = 0.57$ (filled circles). (d) Mean stellar mass between $r = 0.016r_0$ and $r = 0.036r_0$. (f) Mean stellar mass outside $r = 0.036r_0$. 

[Graphs showing time evolution of mean mass for different regions and mass loss over time intervals of $2.9t_{cross}$ for loop orbits with higher ellipticities]
lost from the system rather than on the number of stars populating the cluster and on the cluster orbital path.

The differential mass loss obviously influences the shape of the MF at different times. In Figure 18 the MF of stars belonging to the cluster is shown at three different epochs, when the cluster has lost, respectively, 20%, 35%, and 75% of its initial mass. As the cluster loses stars in the galactic field, the MF evolves toward flatter configurations, because of the preferential loss of low-mass stars, which, according to the initial mass segregation, are located mostly in the external regions of the cluster. The evolution of the MF appears to be driven by the fraction of mass loss rather than by other parameters (like the total number of stars in the system and the orbital type of the parent cluster), confirming the findings of Baumgardt & Makino (2003). This is evident from the fact that, for a given fraction of mass lost, the curves found for the different orbits, in practice, coincide.

6. CONCLUSIONS
The main results of our work are as follows:

1. Stars are lost from the system along a direction that is the composition of the direction toward the galactic center and the cluster velocity around the galaxy, thus leading to the peculiar S-shape found in the outermost region of the cluster. Once formed, tidal tails are elongated as to remain parallel to the cluster orbit, with a trailing tail that lies slightly inside the orbit and a leading tail slightly outside it. As previously found by Dehnen et al. (2004), tails are excellent tracers of the cluster orbit near the pericenter, whereas at the apocenter, they tend to deviate from the orbital path.

2. Tidal tails have a clumpy structure that cannot be associated with an episodic mass loss or tidal shocks with galactic compact substructures, since stars are lost from the cluster continuously and the interaction with the bulge is not episodic.

These clumps are not bound self-gravitating systems; they are rather due to a local deceleration of the motion of the stars along the tails.

3. The observational evidence found for M15 and ω Cen that the velocity dispersion increases and then remains constant at large radii is explained in terms of the so-called “tidal heating”: the stars that evaporate outside the tidal radius of the cluster mainly undergo the interaction with the external field, thus acquiring the higher velocity dispersion pertaining to that field.

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APPENDIX A

ADIMENSIONALIZATION OF THE EQUATIONS

The equations of motion of the \( j \)th star of the cluster, interacting with all the other cluster members and with the bulge, are

\[
\ddot{r}_j = \sum_{i=1}^{N} \frac{G m_i}{r_{ij}^3} (r_i - r_j) + \nabla U_b(x_j, y_j, z_j),
\]  

(A1)

where \( r_j = (x_j, y_j, z_j) \) is the position vector, \( r_{ij} \) is the distance between the \( i \)th and the \( j \)th particle, and \( U_b \) is the bulge potential. In the case of the Schwarzschild (1979) model,

\[
U_b(x, y, z) = 4\pi G M_b \left\{-\frac{1}{r} \ln \left( \frac{r}{r_b} + \sqrt{1 + \left(\frac{r}{r_b}\right)^2} \right) + c_1 \left( \frac{3}{2} \frac{x^2 - y^2}{r_b^3} \right) + c_2 \left( \frac{3}{2} \frac{y^2 - z^2}{r_b^3} \right) - 3 c_3 \left( \frac{x^2 - y^2}{r_b^3} \right) + 2 \frac{1 + c_2 r_b^2}{3} \right\},
\]  

(A2)

where \( r_b \) and \( M_b \) are the bulge core radius and the bulge mass, respectively. Equation (A2) can be rewritten as a product of a dimensional factor and a dimensionless function as

\[
U_b(x', y', z') = 4\pi G M_b \left\{-\frac{1}{r'} \ln \left( \frac{r'}{r_b} + \sqrt{1 + \left(\frac{r'}{r_b}\right)^2} \right) + c_1 \left( \frac{3}{2} \frac{x'^2 - y'^2}{r_b^3} \right) + 2 \frac{1 + c_2 r_b^2}{3} \right\},
\]  

(A3)

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\(^2\) See http://inaf.cineca.it.
where
\[ r' = \frac{r}{r_b}, \quad x' = \frac{x}{r_b}, \quad y' = \frac{y}{r_b}, \quad z' = \frac{z}{r_b}. \]  
(A4)

The first term on the right side of equation (A1) can also be written as the product of a dimensional and a dimensionless factor:
\[ \sum_{i=1}^{N} \frac{Gm_i}{r_{ij}^2} (r_i - r_j) = \frac{G M_b}{r_b^2} \sum_{i=1}^{N} m_i' \left( r_i' - r_j' \right), \]  
(A5)
with \( m_i' = m_i / M_b \).

Finally, defining a dimensionless time
\[ t' = \frac{t}{t_{\text{cross}}}, \]  
(A6)

where \( t_{\text{cross}} = (r_b^3 / GM_b)^{1/2} \) is the bulge crossing time, equation (A1) may be written as
\[ \frac{d^2r_i'}{dt'^2} = \sum_{j \neq i}^{N} \frac{Gm_i'}{r_{ij}'} \left( r_i' - r_j' \right) + 4\pi \nabla U'. \]  
(A7)

This implies that, once the initial conditions \( r_i'(0) \) and \( v_i'(0) \) are assigned, the existence of a unique solution for equation (A7) ensures that all the results obtained can be scaled in terms of the ratios \( r_i / r_b, m_i / M_b \), and \( t / t_{\text{cross}} \).

**APPENDIX B**

**THE CONSTRUCTION OF A MULTIMASS KING MODEL**

As described in King (1966), in a single-mass isotropic King model, the phase-space stellar distribution function is given by
\[ f(r, v) = \begin{cases} \alpha \left[ \exp \left( -\frac{E}{m \sigma^2} \right) - \exp \left( -\frac{C}{\sigma^2} \right) \right] & E \leq -mC, \\ 0 & \text{otherwise,} \end{cases} \]  
(B1)

where
\[ E = \frac{1}{2} mv^2 + m\psi(r) \]  
(B2)
is the energy of a star, \( \psi(r) \) is the mean gravitational potential generated by the cluster, and \( \alpha \) is a normalization constant. The "global" parameter \( C \) is related to the tidal radius \( r_t \) by the implicit relation
\[ \psi(r_t) + C = 0. \]  
(B3)

The mass density can be found integrating the distribution function \( f(r, v) \) over the velocity, obtaining an explicit relation for \( \rho \) as a function of the potential \( \psi \)
\[ \rho = \int_{E \leq -mC} f(r, v) 4\pi v^2 dv = 4\pi m \alpha e^{C/m \sigma^2} (2\sigma^2)^{3/2} \]
\[ \times \left[ \frac{1}{2} \left( -\psi + C \right) \sqrt{\sigma^2} \right] \]
\[- \frac{1}{3} \left( \left. \psi + C \right|^{1/2} \right) \]
\[ = 4\pi m \alpha e^{C/m \sigma^2} (2\sigma^2)^{3/2} \]
\[ \times \left[ \frac{1}{2} \sqrt{-U} + \frac{\sqrt{\pi}}{4} \exp(-U) \right] \left( \sqrt{-U} \right) \]
\[- \frac{1}{3} \left( -U \right)^{1/2} \]
\[ \equiv k\bar{\rho}(U), \]
(B7)

where \( U \equiv (\psi + C)/\sigma^2 \) is the dimensionless potential, \( k = 4\pi m \alpha e^{C/m \sigma^2} (2\sigma^2)^{3/2} \), and \( \bar{\rho} \) is the dimensionless density, which explicitly depends only on \( U \). Once initial conditions for the potential \( \psi \) and its derivative \( \psi' \) in \( r = 0 \) have been assigned, the Poisson equation
\[ d^2\psi / dr^2 = 4\pi G \rho, \]
with boundary conditions
\[ \psi(0) = \psi_0, \]
\[ \psi'(0) = 0, \]
can be rewritten in terms of the dimensionless potential as
\[ d^2U / d\tilde{r}^2 = 9\rho(U) / \rho(0) = 9\bar{\rho}(U) / \bar{\rho}(0), \]
\[ U(0) = U_0, \]
\[ U'(0) = 0, \]
where \( \rho(0) = \rho(U_0) \) and \( \tilde{r} = r / r_{\text{core}} \), with
\[ r_{\text{core}}^2 = \frac{9\sigma^2}{4\pi G \rho_0} \]
(B8)

being the King radius.

Once the initial parameters \( U_0 \) and \( U_0' \) have been assigned, the Poisson equation can be integrated, obtaining the dimensionless potential \( U(\tilde{r}) \), the dimensionless mass density \( \bar{\rho}(\tilde{r}) \), and the tidal radius \( \tilde{r}_t \), which is \( \tilde{r}_t = r_t / r_{\text{core}} \), with \( r_{\text{core}} \) yet not determined. To determine the core radius \( r_{\text{core}} \) and the constant \( k \) (which depends on the normalization constant \( \alpha \), among others), it is possible to proceed as follows.
Once the total mass of the cluster \( M_{\text{tot}} \) and the velocity dispersion \( \sigma \) of stars in the system have been assigned as initial parameters, using the following relations

\[
M_{\text{tot}} = \int_0^r 4\pi \rho(r) r^2 dr = 4\pi k r^3 \text{core} \int_0^{\tilde{r}} \tilde{\rho}(\tilde{r}) \tilde{r}^2 d\tilde{r} \quad (B9)
\]

and equation (B8), it is possible to calculate \( r_{\text{core}} \) and \( k \) and hence obtain \( \rho(r) \), \( r_{\text{core}} \), and \( r_1 = \tilde{r} r_{\text{core}} \).

In a multimass isotropic King model, as described in Da Costa & Freeman (1976), stars are first grouped in \( n \) different mass classes, each characterized by a mass \( m_i \). The phase-space stellar distribution function for the \( i \)th mass class is given by

\[
f_i(r, v) = \begin{cases} 
\alpha_i \left[ \exp \left( -\frac{E_i}{m_i \sigma_i^2} \right) - \exp \left( \frac{C}{\sigma_i^2} \right) \right] & \text{if } E_i \leq -m_i C, \\
0 & \text{otherwise},
\end{cases}
\]

where \( E_i = \frac{1}{2} m_i v^2 + m_i \psi(r) \) (B11)

is the energy of a star in the \( i \)th mass class, \( \psi(r) \) is the mean gravitational potential generated by the whole cluster, \( \alpha_i \) is a normalization constant, and \( C \) is related to the cluster tidal radius \( r \) by equation (B3). Once again, the mass density for the \( i \)th mass class can be found integrating the distribution function \( f_i(r, v) \) over velocities, obtaining an explicit relation for \( \rho_i \) in terms of the dimensionless potential \( U \) defined above and the ratio \( \sigma^2/\sigma_i^2 \): \n
\[
\rho_i = \int_{E_i \leq -m_i C} f_i(r, v) 4\pi v^2 dv = 4\pi m_i \alpha_i e^{C/\sigma_i^2} (2\sigma_i^2)^{3/2} \\
\times \left[ -\frac{1}{2} \left( -\frac{\psi + C}{\sigma_i^2} \right)^{1/2} + \frac{\sqrt{\pi}}{4} \exp \left( -\frac{\psi + C}{\sigma_i^2} \right) \text{erf} \left( \sqrt{\frac{\psi + C}{\sigma_i^2}} \right) - \frac{1}{3} \left( -\frac{\psi + C}{\sigma_i^2} \right)^{3/2} \right] \\
= 4\pi m_i \alpha_i e^{C/\sigma_i^2} (2\sigma_i^2)^{3/2} \\
\times \left[ -\frac{1}{2} \sqrt{-U \sigma_i^2} + \frac{\sqrt{\pi}}{4} \exp \left( -U \sigma_i^2 \right) \text{erf} \left( \sqrt{-U \sigma_i^2} \right) - \frac{1}{3} \left( -U \sigma_i^2 \right)^{3/2} \right] \\
\equiv k_i \tilde{\rho}_i (U, \sigma_i^2/\sigma_i^2), \quad (B13)
\]

where \( k_i = 4\pi m_i \alpha_i e^{C/\sigma_i^2} (2\sigma_i^2)^{3/2} \) and \( \tilde{\rho}_i \) is the dimensionless density. Note that the density profiles \( \rho_i \) of the \( i \)th mass class are related to the “global” density distribution \( \rho \) by the relation

\[
\rho(r) = \sum_{i=1}^{n} \rho_i(r). \quad (B16)
\]

To distribute stars in the cluster according to this isotropic multimass King model, we proceeded in the following way:

Once we assigned \( U_0, U_i' \), the total cluster mass \( M_{\text{tot}} \), and the velocity dispersion \( \sigma \), we integrate the Poisson equation as in the case of a single-mass model previously described, obtaining the dimensionless potential \( U(r) \), the “global” mass density \( \rho \), the core radius \( r_{\text{core}} \), and the tidal radius \( r_t \).

We then assigned the mass \( m_i \) of stars in the \( i \)th mass class and the total mass \( M_{\text{tot}, i} \) of each mass class (i.e., \( M_{\text{tot}, i} = n_i m_i \), where \( n_i \) is the number of stars populating the \( i \)th mass class). In our case, we chose to set the masses \( M_{\text{tot}, i} \) according to the Salpeter MF. Given a value of the ratio \( \sigma^2/\sigma_i^2 \) (all the other \( \sigma^2/\sigma_i^2 \) values were obtained according to energy equipartition using the relation \( m_i \sigma_i^2 = m_i \sigma_i^2 \) for \( i \geq 2 \)), allowing the calculation of \( \tilde{\rho}_i(U, \sigma_i^2/\sigma_i^2) \) and then of the coefficients \( k_i \), using equation (B9) applied to the \( i \)th mass class.

We varied the ratio \( \sigma^2/\sigma_i^2 \) until the relation (B16) is satisfied with the desired accuracy.

Finally, stellar velocities were generated according to equation (B10).

**APPENDIX C**

**THE GC INITIAL MASS FUNCTION**

The GC we considered in our simulations is supposed to have an age of \( t_{\text{GC}} \sim 10^{5} \) yr; thus, only stars more massive than \( \sim 1.2 M_\odot \) at that time have evolved to a compact remnant (Straniero et al. 1997; Domínguez et al. 1999). For this reason, we considered only stars distributed according to the Salpeter MF with masses in the range \( 0.12 M_\odot \leq m \leq 1.2 M_\odot \). Moreover, we assumed that the contribution to the low-mass population due to the mentioned remnants is practically negligible.

Indeed, according to the Salpeter MF, the ratio between the number of remnants whose progenitor had a mass around \( m_p \) and the number of stars with mass around \( m \) is

\[
\frac{N_{\text{remn}}}{N_m} = \left( \frac{m}{m_p} \right)^{2.35}. \quad (C1)
\]

Supposing that such progenitors are those giving rise to remnants with mass \( m \), then, from the estimates in Straniero et al. (1997) and Domínguez et al. (1999), \( m_p(M_\odot) \sim 9.5(m - 0.45) \), with \( m > 0.45 M_\odot \), because stars with lower mass remnants cannot evolve in a Hubble time. Thus, substituting in equation (C1),

\[
\frac{N_{\text{remn}}}{N_m} = \begin{cases} 
\left( \frac{0.11m}{m - 0.45} \right)^{2.35} & \text{if } m \geq m_1, \\
0 & \text{otherwise},
\end{cases} \quad (C2)
\]

where \( m_1 \) is the lowest mass a remnant can have at the assumed cluster age. From fitting the above-cited estimates, this lower limit turns out to be \( m_1 \sim (0.45 \log t_{\text{GC}} - 1.2)^{-3.1} + 0.45 \sim 0.59 M_\odot \).

One can see that the ratio in equation (C2) is monotonically decreasing for \( m \geq m_1 \); hence, the maximum takes place for the lowest mass class we used in the model, i.e., \( m = 0.72 M_\odot \), for
which $N_{\text{rem}}/N_{0,72} \approx 0.05$. Since in our numerical representation $N_{0,72}/N \approx 0.01$ ($N$ is the total number of particles), in this class there should have been about $5 \times 10^{-3}N$ remnants. Thus, bearing in mind that the less-populated mass class contains $\sim 2 \times 10^{-3}N$, we can reasonably affirm that the MF we assumed for the initial conditions was substantially affected by stellar evolution neither at the initial time $t_{GC}$ nor later during the simulation (because it lasts much shorter than $t_{GC}$).

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