Polignac: New Conjecture

Leichsenring IG*
Department of Mathematics, Karting International Aldeia da Serra, Aldeia da Serra, Barueri, São Paulo 06428-180, Brazil

Abstract
The intent of this essay is not to try to prove that the twin primes are infinite. We would just like to add another way so that others interested in Number Theory can help in elucidating this mystery.

The conjecture of Polignac states that each natural pair is equal to the difference of two primes; but this conjecture, it seems, has not yet been proven. However, we note that there is a certain correlation of that thesis with the foundations of our previous study, as proposed in "Goldbach - New Conjecture", which led to this monograph on twin primes.

Introduction
Initially we will summarize the proposal equivalent to Goldbach’s conjecture, which can be examined [1].

All natural >1 can be represented by the mean of two primes \( p \) and \( q \) equidistant from a natural \( n \), through an integer index \( k \), such that [2]:

\[
\begin{align*}
  n &\equiv (p + q)/2, \\
  p &\equiv n - k, \\
  q &\equiv n + k.
\end{align*}
\]

There is symmetry involving \( n \) and both primes \( p \) and \( q \) with amplitude [3]

\[
3 \cdots n \cdots 23n.
\]

We will use these concepts as a foundation for the study that we will present about the twin primes, the pairs \((g, h)\), with \[4\]

\[
| h - g | \leq 2.
\]

So we have, within our formulation, for a given \( k \) [5]:

\[
\begin{align*}
  g &\equiv (p + q)/2, \\
  p &\equiv g - k, \\
  q &\equiv g + k.
\end{align*}
\]

And for a given \( k \):

\[
\begin{align*}
  h &\equiv (p + q)/2, \\
  p &\equiv h - k, \\
  q &\equiv h + k.
\end{align*}
\]

For example, below are some symmetries with the pair \((71,73)\) (Table 1).

According to the conjecture, both symmetries exist -individually, of course and therefore the index \( k \) behaves randomly [6,7], as seen in the examples with

\[
\begin{align*}
  k &\equiv 12 \text{ and } k \equiv 5 \text{ or} \text{ or} \\
  k &\equiv 5 \text{ and } k \equiv 36.
\end{align*}
\]

Without connection between \( g \) and \( h \).

However, we observed the possibility of finding in each pair chosen for testing, many cases where index \( k \) could be unique, as seen in two other examples [8,9], with

\[
\begin{align*}
  k &\equiv 30 \text{ or} \text{ or} \\
  k &\equiv 66.
\end{align*}
\]

And there is a link between \( g \) and \( h \).

This condition \(-k \equiv 5 \text{ or} \text{ or} k \equiv 5\) is the basis of this study and we are interested only when and if it can occur; in this situation [10].

For any pair \((g, h)\) we can do

\[
\begin{align*}
  (g - k) &\equiv 5, \\
  (g + k) &\equiv 30, \\
  (h - k) &\equiv 5, \\
  (h + k) &\equiv 66.
\end{align*}
\]

Therefore we will have:

\[
\begin{align*}
  g &\equiv (p + q)/2, \\
  h &\equiv (p + q)/2.
\end{align*}
\]

| Pair \((g, h)\) | \(k\) | \(p\) | \(q\) |
|---------------|------|------|------|
| \((71,73)\)   |      |      |      |

\[
\text{Table 1: Symmetries pair of (71,73).}
\]

*Corresponding author: Leichsenring IG, Department of Mathematics, Karting International Aldeia da Serra, Aldeia da Serra, Barueri, São Paulo 06428-180, Brazil, Tel: 55 11 99898-5544; E-mail: ivan@apex.eti.br

Received August 03, 2018; Accepted September 12, 2018; Published September 20, 2018

Citation: Leichsenring IG (2018) Polignac: New Conjecture. J Appl Comput Math 7: 415. doi: 10.4172/2168-9679.1000415

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Looking only for these solutions we had some success with several tests, which induced us to the theory that follows and to distinguish the twin primes in that \( k_g \neq k_h \) of others in that \( k_g \neq k_h \) we adopt the following concept.

**Identical twin primes**

Are those in which at least one \( k \), simultaneously, satisfies a pair \((g, h)\).

Therefore, among the symmetries of the previous examples, only the following identities can be considered **identical twin primes** (Table 2).

In iterative surveys with \( k \), we were able to conduct symmetry in this way for identical twin primes of small magnitude, and we realized that it was possible to obtain them many times. In Table 3 we have the result of the first pairs.

However, as we can see in the table, we have already started with two pairs where we cannot obtain simultaneous symmetry and, later, we stop at pair \((197, 199)\), also in the same situation; that is, there are impossible cases if we require \( n>k \).

| Pair \((g, h)\) | \( k \) | \( p \) | \( q \) |
|-------------|-----|-----|-----|
| \((5, 7)\)   | 30  | 5 41| 5 101|
|             | 66  | 5 53| 5 103|

Table 2: Symmetries of identical twin primes.

At this point we will pause in our study of twin primes.

Let’s revisit the original conjecture considering what would happen if we could expand the symmetry to negative values, that is, if we could make \( k>n \) possible.

Without restriction for \( k \), one immediately observes symmetry with infinite amplitude.

Similarly, as in the initial conjecture, equalities are maintained:

- \( n \approx (p+q) \approx 2 \), being
- \( p \approx n 2 k \) and \( q \approx n 1 k \);

Where are primes:

- \(|p|\) and \(|q|\).

Note that any integers can now be obtained, and that, in particular:

- \( n \approx 0 \) with any primes, for \( p \approx q \approx 5 \);
- \( n \approx 1 \) with any pairs of twin primes;
- \( n < 0 \) it is a reflection of \( n > 0 \).

The search iteration can be obtained as follows:

For \( n \) even:

- \( k \approx 1, 3, 5, \cdots \infty \).

For \( n \) odd:

- \( k \approx 2, 4, 6, \cdots \infty \).

But, let’s return to our study, when we have **identical twin primes**.

The proposition assumes the bond between twin primes \( g \) and \( h \), when and if

\[ k \approx 5 \approx k_g = k_h. \]

And, except for the pair \((3, 5)\), we have the iteration of \( k \) boils down to:

\[ k \approx 5, 12, 18, \cdots \infty \]

Until simultaneously appear the primes:

- \(|p|\) and \(|q|\);
- \(|p| 2\) and \(|q| 2\).

In summary, we have:

- \( p = 5 (g 2 k) \), \( q = 5 (g 1 k) \),
- \( p = 5 (g 2 k 1 2) \) and \( q = 5 (g 1 k 1 2) \).

Without restriction for \( k \) let’s see those impossible identities of Table 4.

Interesting; it is possible to obtain symmetry.

In addition, among the set of the first 1048576 odd primes we have:

- 3199 identities representing the identical twin primes \((5, 7)\);
- 1669 identities for \((197, 199)\).

Curiously, even with infinite amplitude, there is only one identity...
Then, reiterating, if

\[(g, h)\] are identical twin primes, we have:

\[g \equiv 2 \pmod{3}\]
\[h \equiv 1 \pmod{3}\]

And as a consequence, are also twin primes the pairs:

\[(p, p + 2)\]
\[(q, q + 2)\].

Therefore, under these conditions, each pair of identical twin primes leads to other twin primes, however not necessarily identical!

But by exploring the previous question:

- If we could ensure that all twin primes can also be identical and
- If there were one last pair of identical twin primes \((g_u, h_u)\).

It would mean that the last pair of identical twin primes would forward to the another pair of identical twin primes of greater magnitude, which would be an incongruity.

**Conclusion**

If it were so, forcibly, the twin primes numbers would be infinite.

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