See-saw type mixing and $\nu_\mu \rightarrow \nu_\tau$ oscillations

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Abstract

We consider $\nu_\mu \rightarrow \nu_\tau$ oscillations under the assumption that there is a see-saw type mixing of the light neutrinos with heavy Majorana particles. It is shown that the existing data, including the recent LEP data, do not exclude the possibility that the additional terms in the transition probability due to this mixing could be of the same order of magnitude as the usual oscillating term. Detail investigations of $\nu_\mu \rightarrow \nu_\tau$ transitions in future CERN and Fermilab experiments could allow to get informations not only about the neutrino masses and mixing but also about the mixing of neutrinos with heavy Majorana particles.

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The problem of neutrino masses and mixing is one of the main challenges of today’s physics. Many experiments devoted to this problem have been performed recently. Up to now the only indications in favour of the existence of a neutrino mass come from the solar neutrino data \[1\] and, possibly, from the atmospheric neutrino data \[2\].

At present a new generation of solar neutrino experiments \[3\], experiments for search of neutrinoless double beta decay, short and long baseline neutrino oscillation experiments with reactor and accelerator neutrinos \[4\] and other experiments are under development. Two new $\nu_\mu \rightarrow \nu_\tau$ oscillation experiments with a high sensitivity to small mixing angles are under preparation at CERN \[5,6\]. Analogous experiments are planned at Fermilab \[7\].

Having in mind these future experiments, we consider here $\nu_\mu \rightarrow \nu_\tau$ oscillations under the assumption that the light neutrinos mix with heavy Majorana particles. This scenario corresponds to many models beyond the standard model.

In accordance with the neutrino mixing hypothesis \[8\], the fields $\nu_{\ell L}$ which appear in the standard charged and neutral weak currents

\[
\begin{align*}
{j^W}_\alpha & = 2 \sum_{\ell=e,\mu,\tau} \bar{\nu}_{\ell L} \gamma_\alpha \ell_L + \ldots \\
{j^Z}_\alpha & = \sum_{\ell=e,\mu,\tau} \bar{\nu}_{\ell L} \gamma_\alpha \nu_{\ell L} + \ldots
\end{align*}
\]

(1)

are mixtures of the left-handed components $\nu_{aL}$ of the massive neutrino fields:

\[
\nu_{\ell L} = \sum_a U_{\ell a} \nu_{aL} ,
\]

(2)

where $U$ is a mixing matrix which satisfy the unitarity relation

\[
\sum_a U_{\ell a} U_{\ell' a}^* = \delta_{\ell \ell'} .
\]

(3)

From the LEP data it follows that the number of light neutrino flavours is equal to 3 \[9\]. As for the number of massive neutrinos, the LEP data do not allow to exclude different theoretical possibilities. If massive neutrinos are Dirac particles (Dirac mass term), their number is equal to the number of lepton flavours \[8\]. In the case of massive Majorana neutrinos, there are two different possibilities. In the simplest case (Majorana mass term) the number of neutrinos with definite mass is equal to the number of lepton flavours. In the most general case of neutrino mixing (Dirac and Majorana mass term) the number of massive Majorana particles is more than 3. This general type of mixing has a great theoretical interest and arises in many models beyond the standard model \[8,10\]. The see-saw mechanism \[11\], which seems to be the most plausible mechanism for the generation of neutrino masses, could be realized in this scheme. In accordance with this mechanism, the smallness of the masses of the light neutrinos is due to their mixing with very heavy Majorana fermions which have masses much larger than the masses of the charged fermions (quarks or leptons). There are many models in which the see-saw mechanism for the generation of neutrino masses take place \[10\]. They differ mainly in the assumptions about the heavy Majorana sector.
Here we consider neutrino oscillations under the assumption that the current fields $\nu_{\ell L}$ are mixtures of 3 fields of light Majorana neutrinos with masses $m_i$ ($i = 1, 2, 3$) and some fields of heavy Majorana particles with masses $m_a$ ($a \geq 4$) at least larger than the mass of the $Z$ boson. We do not assume any specific see-saw model and consider the non-diagonal matrix elements $U_{\ell a}$ ($a \geq 4$) (which are small in most see-saw models) as parameters. It is clear that any information about these matrix elements which can be obtained from experiment is very important for understanding the nature of the neutrino mass.

Let us consider neutrinos with momentum $\vec{p}$ which are produced together with charged leptons $\ell^+$ ($\ell = e, \mu, \tau$) in some charged-current weak decay. In accordance with Eq.(2), the neutrinos are described by the state

$$|\nu_{\ell}\rangle = \sum_{i=1,2,3} U_{\ell i}^* |i\rangle,$$

where $|i\rangle$ are the states of the light neutrinos with mass $m_i \ll p$. Let us stress that the sum in Eq.(4) is only over the light mass eigenstates (the heavy mass eigenstates cannot be produced). After some time $t$ the state of the beam is given by

$$|\nu_{\ell}\rangle_t = \sum_{i=1,2,3} U_{\ell i}^* e^{-iE_i t} |i\rangle,$$

where $E_i \simeq p + \frac{m_i^2}{2p}$. Neutrinos are analyzed with weak interaction processes. So the transition amplitude for $\nu_{\ell} \rightarrow \nu_{\ell'}$ is given by

$$A(\nu_{\ell} \rightarrow \nu_{\ell'}) = \langle \nu_{\ell'} | \nu_{\ell}\rangle_t = \sum_{i=1,2,3} U_{\ell i}^* e^{-iE_i t} U_{\ell i}^*.$$

This expression has the same form as the usual expression for the transition amplitude [8]. The essential difference is that the sum in Eq.(6) is only over the indices which correspond to the light neutrinos. In order to see more clearly this difference, let us rewrite Eq.(6) as

$$|A(\nu_{\ell} \rightarrow \nu_{\ell'})| = \sum_{i=2,3} U_{\ell i}^* U_{\ell i}^* \left[ \exp \left(-i \frac{\Delta m^2_{ij} R}{2p} \right) - 1 \right] + \delta_{\ell \ell} - \Omega_{\ell \ell},$$

where $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$, $R \simeq t$ is the distance between the source and the detector,

$$\Omega_{\ell \ell} = \sum_{a \geq 4} U_{\ell a}^* U_{\ell a}^*.$$

and the relation (3) was used. If all the squared mass differences are so small that $\frac{\Delta m^2_{ij} R}{p} \ll 1$, then the transition amplitude is constant and given by

$$|A(\nu_{\ell} \rightarrow \nu_{\ell'})| \simeq |\delta_{\ell \ell} - \Omega_{\ell \ell}| = |\langle \nu_{\ell'} | \nu_{\ell}\rangle|.$$
So the part of the transition amplitude which does not depend on \( R \) and \( p \) is connected with the non-orthogonality of the states in Eq. (1), which describe neutrinos taking part in weak interactions in the case of mixing among light and heavy Majorana particles. Different phenomenological aspects of this non-orthogonality have been considered in Refs. [12,13]. Here we discuss the implications of neutrino masses, mixing and non-orthogonality for neutrino oscillations.

The transition amplitude in Eq. (7) contains many unknown parameters. In the following we make some general assumptions about the neutrino masses and the elements of the mixing matrix which reduce the number of relevant parameters. We assume that there is a hierarchy of light neutrino masses

\[ m_1 \ll m_2 \ll m_3 . \]

The masses of all known fundamental fermions satisfy this type of hierarchy. If the neutrino masses are generated through the see-saw mechanism, then \( m_i \simeq \frac{m_{f_i}^2}{M_i} \), where, for each generation \( i \), \( m_{f_i} \) is the mass of the up-quark or charged lepton and \( M_i \) is the mass of the heavy Majorana fermion. In this case, the hierarchy in Eq. (10) is obviously satisfied. We assume also that the masses \( m_1 \) and \( m_2 \) are too small to be relevant for terrestrial neutrino oscillation experiments. The masses \( m_1 \) and \( m_2 \) could be responsible for the MSW resonant transition of solar neutrinos, which can explain all existing experimental data, including the new GALLEX data.

There are some indications at present that the mass \( m_3 \) could be in the electronvolt region. One indication come from the analysis of the recent COBE data and the observations of the large scale distribution of galaxies. Another indication comes from the MSW explanation of the solar neutrino data together with a see-saw formula for the neutrino masses.

Taking into account all these arguments, the probability of \( \nu_\mu \rightarrow \nu_\tau \) transition is given by

\[
P(\nu_\mu \rightarrow \nu_\tau) = 2 |U_{\tau3}|^2 |U_{\mu3}|^2 \left[ 1 - \cos \left( \frac{\Delta m_{31}^2 R}{2p} \right) \right] - 2 |U_{\tau3}| |U_{\mu3}| |\Omega_{\tau\mu}| \left[ \cos \left( \frac{\Delta m_{31}^2 R}{2p} \pm \chi \right) - \cos \chi \right] + |\Omega_{\tau\mu}|^2 ,
\]

(11)

where

\[ \chi \equiv \text{Arg} \left\{ U_{\tau3} U_{\mu3}^* \Omega_{\tau\mu}^* \right\} . \]

(12)

The following comments are in order:

1. If there is a hierarchy of couplings in the lepton sector

\[ |U_{e3}|^2 \ll |U_{\mu3}|^2 \ll |U_{\tau3}|^2 , \]

(13)

\footnote{Let us notice that in the case of non-orthogonality the usual notion of flavour neutrinos lose its meaning. For example, a neutrino which is created with a muon and is described by the state \( |\nu_\mu \rangle \) can produce besides a muon also an electron or a tau.}
analogous to the hierarchy in the quark sector, then the $\nu_\mu \rightarrow \nu_\tau$ transition probability is the largest one [13].

2. It can be seen from Eq.(11) that if there is no $\Omega$ term in the amplitude, the $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ transition probabilities are equal. If there is a mixing among light and heavy Majorana particles and CP is violated in the lepton sector (the mixing matrix is complex), the $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ transition probabilities could be different. So a comparison between these transition probabilities could be a test for the mixing of light neutrinos with heavy Majorana particles and CP violation in the lepton sector.

3. Besides the usual $\frac{R_p}{p}$-dependent term \[1 - \cos \left( \frac{\Delta m^2_{31} R}{2p} \right)\], the transition probability in Eq.(11) contains an additional $\frac{R_p}{p}$-dependent term with a phase shift $\chi$. Notice that this additional term is proportional to $|U_{\mu 3}|$, which is much bigger than the $|U_{\mu 3}|^2$ in the usual term if there is a hierarchy relation Eq.(13). So a measurement of the energy dependence of the transition probability allows in principle to get information about the mixing of the light neutrinos with heavy Majorana particles.

If the squared mass difference $\Delta m^2_{31}$ is so small that $\frac{\Delta m^2_{31} R}{p} \ll 1$, then

$$P \left( \nu^(-)_\mu \rightarrow \nu^(-)_\tau \right) \simeq |\Omega_{\tau \mu}|^2 = |\langle \nu_\tau | \nu_\mu \rangle|^2 . \quad (14)$$

In this case the transition probability does not depend on $\frac{R_p}{p}$ and it is determined by the non-orthogonality of the $\nu_\mu$ and $\nu_\tau$ states. Such a situation was discussed in Ref.[13].

4. An upper bound for the value of $|\Omega_{\tau \mu}|$ in Eq.(11) is given by the Schwartz inequality

$$|\Omega_{\tau \mu}| \leq \sqrt{|\Omega_{\mu \mu}| |\Omega_{\tau \tau}|} . \quad (15)$$

Information about the values of $|\Omega_{\mu \mu}|$ and $|\Omega_{\tau \tau}|$ can be obtained from the analysis of different weak interaction processes. Such analysis was done in Refs.[13,16]. Using the values obtained in Ref.[16] we have the following $2\sigma$ upper bounds

$$|\Omega_{\mu \mu}| \lesssim 4 \times 10^{-3} ,$$

$$|\Omega_{\tau \tau}| \lesssim 2 \times 10^{-1} . \quad (16)$$

From Eqs.(13) and (16) we have

$$|\Omega_{\tau \mu}| \lesssim 2.8 \times 10^{-2} . \quad (17)$$
The upper bound of the value of $|\Omega_{\tau\mu}|$ can be estimated also from the LEP measurement of the number of neutrino flavours. In the case under consideration the number of neutrino flavours is given by [16]

$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_0} = 3 - 2 \sum_\ell \Omega_{\ell \ell} + \sum_{\ell \ell'} |\Omega_{\ell \ell'}|^2 .$$

(18)

Using the relation

$$|\Omega_{\tau\mu}| \leq \frac{1}{\sqrt{2}} \sum_\ell \Omega_{\ell \ell} .$$

(19)

and the latest LEP data $N_\nu = 2.99 \pm 0.04$ [9], we obtain

$$|\Omega_{\tau\mu}| \lesssim 3 \times 10^{-2} .$$

(20)

Let us compare the value of the coefficients of the first and second terms of the transition probability in Eq.(11). Using Eqs.(17) and (20) for their ratio we have

$$R = \frac{|\Omega_{\tau\mu}|}{|U_{\tau 3}| |U_{\mu 3}|} \leq \frac{3 \times 10^{-2}}{|U_{\tau 3}| |U_{\mu 3}|} .$$

(21)

In many models (see for example Ref.[10]) the elements of the lepton mixing matrix $U$ are approximately equal to the elements of the Cabibbo-Kobayashi-Maskawa mixing matrix of quarks. In this case $|U_{\tau 3}| |U_{\mu 3}| \simeq (4.3 \pm 0.7) \times 10^{-2}$ [17], leading to

$$R \lesssim 1 .$$

(22)

In a recent paper [18] the elements of the lepton mixing matrix were calculated in the context of a flipped SU(5) model. The value of $|U_{\tau 3}| |U_{\mu 3}|$ in this model depends on the top quark mass $m_t$ and is predicted to lie in the interval

$$0.9 \times 10^{-2} (m_t = 150 \text{ GeV}) \leq |U_{\tau 3}| |U_{\mu 3}| \leq 2 \times 10^{-2} (m_t = 90 \text{ GeV}) .$$

(23)

From Eq.(23) the upper bound for the ratio $R$ is given by

$$R \leq 3.3 \quad \text{for} \quad m_t = 150 \text{ GeV} ,$$

$$R \leq 1.5 \quad \text{for} \quad m_t = 90 \text{ GeV} .$$

(24)

These estimates show that the coefficient of the second term of the transition probability in Eq.(11) could be comparable to the coefficient of the “main” term.

From our estimation also follows that the third term of the transition probability in Eq.(11), which does not depend on $\frac{R}{p}$, is bounded by

$$|\Omega_{\tau\mu}|^2 \lesssim 10^{-3} .$$

(25)
Therefore, if the neutrino squared mass difference is so small that the first and second terms in Eq. (11) vanish, the third term, which is due to the non-orthogonality of the neutrino states, could give a constant $\nu_\mu \to \nu_\tau$ transition.

In conclusion, our considerations show that a detailed investigation of $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ transitions in future CERN and Fermilab experiments could allow to get informations not only about neutrino masses and mixing but also about the possibility of a mixing of the light neutrinos with heavy Majorana particles.

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References

[1] R. Davis, in Neutrino '88, Proceedings of the XIIIth International Conference on Neutrino Physics and Astrophysics, ed. J. Scheps et al. (World Scientific 1989) p. 518; K.S. Hirata et al., Phys. Rev. Lett. 65 (1990) 1297, Phys. Rev. Lett. 65 (1990) 1301; GALLEX Collaboration, Phys. Lett. B 285 (1992) 390.

[2] K.S. Hirata et al., Phys. Lett. B 280 (1992) 146; D. Casper et al., Phys. Rev. Lett. 66, 2561 (1991).

[3] For a review see: D. Sinclair, Nucl. Phys. B (Proc. Suppl.) 19 (1991) 100.

[4] J. Schneps, Talk presented at the 15th International Conference on Neutrino Physics and Astrophysics (NEUTRINO 92), Granada, Spain, June 1992.

[5] CHORUS Collaboration, CERN-SPSC/90-42 (1990).

[6] NOMAD Collaboration, CERN-SPSC/91-21 (1991).

[7] R. Bernstein et al., Neutrino Physics after the Main Injector Upgrade, FNAL 1991.

[8] See the reviews: S.M. Bilenky and B. Pontecorvo, Phys. Rep. 41 (1978) 225; S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59 (1987) 671.

[9] The LEP Collaborations, Phys. Lett. B 276 (1992) 247.

[10] P. Langacker, Lectures presented at TASI-90, Boulder, June 1990, UPR 0470T.

[11] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, ed. F. van Nieuwenhuizen and D. Freedman, North Holland, Amsterdam 1979, p. 315; T. Yanagita, Prog. Theor. Phys. B 135 (1987) 66.

[12] J. Bernabeu et al., Phys. Lett. B 187 (1987) 303; S.L. Glashow Phys. Lett. B 187 (1987) 367; J.W.F. Valle, Phys. Lett. B 199 (1987) 432.

[13] P. Langacker and D. London, Phys. Rev. D 38 (1988) 886, Phys. Rev. D 38 (1988) 907.

[14] M. Davis, F.J. Summers and D. Schlegel, Nature 359 (1992) 393; A.N. Taylor and M. Rowan-Robinson, Nature 359 (1992) 396.

[15] S.M. Bilenky, M. Fabbrichesi and S.T. Petcov, Phys. Lett. B 276 (1992) 223.

[16] S.M. Bilenky, W. Grimus and H. Neufield, Phys. Lett. B 252 (1990) 119.

[17] Review of Particle Properties, Phys. Rev. D 45 (1992) Part II.

[18] J. Ellis, J.L. Lopez and V. Nanopoulos, CERN-TH.6569/92, June 1992.