Simplified BB84 quantum key distribution secure against coherent attacks

Yu-Shuo Lu,1 Hua-Lei Yin,1,2, † and Zeng-Bing Chen1, ‡

1 National Laboratory of Solid State Microstructures and School of Physics, Nanjing University, Nanjing 210093, China
2 Zhongchuangwei Quantum Co., Ltd., Beijing 101400, China

Decoy-state quantum key distribution (QKD) has convincingly been shown the core solution to secure key exchange. While standard BB84 protocol needs to prepare and measure all states of two complementary bases, which seriously restricts its real life applications in photonic integration and high speed system. Fortunately, a simplified BB84, only sending and receiving three states, achieves the same asymptotic secret key rate compared with standard BB84 under coherent attacks. In this work, by skillfully exploiting uncertainty relation for smooth entropies, we provide the finite-key security bounds for decoy-state simplified BB84 QKD against coherent attacks in the universally composable framework. Simulation results show that it can also achieve almost the same performance compared with standard BB84 even the total number of pulses is as low as $10^8$. We conclude that simplified BB84 can completely replace standard BB84 to be the best choice for QKD in most common practical scenarios.

I. INTRODUCTION

Quantum key distribution (QKD) provides unconditionally secure secret keys between two remote users via the principles of quantum mechanics. In 1984, Bennett and Brassard proposed the first QKD protocol, called BB84 [1]. Since then, various QKD protocols have been presented, but BB84 is still the most practical one. Real-life implementations of BB84 protocol are mainly based on attenuated laser pulses. It is vulnerable to the photon-number-splitting attack [2], because the laser source occasionally generates multi-photon and the channel loss is inevitable. Fortunately, the development of the decoy-state method has successfully handled these issues and greatly improved the performance of QKD [3, 4]. Standard BB84 protocol usually includes preparation and measurement of four quantum states, i.e., Alice randomly prepares quantum states in $X$ or $Z$ basis while Bob randomly choose $Z$ or $X$ basis to measure these states.

Currently, the significant advances have been achieved in decoy-state BB84 QKD, which gets a lot of experimental demonstrations in various situation, including fiber [5–8], free-space [9, 10], chip [11–13], satellite [14–16] and high-dimension [17–19]. However, with the development of decoy-state QKD system towards high-frequency clock and miniaturization, the preparation of superposition quantum state $|−⟩$ based on $π$ phase difference is a bottleneck. For example, it is a big challenge that the high rate modulation in Si-based chip [20]. Besides, the passive basis detection with four detectors is usually adopted in high-frequency clock system, which will bring in the additional detector costs and complex calibration for detection-efficiency balanced. In order to overcome the preparation of state $|−⟩$, a three-state protocol has been proposed while the unconditionally secure secret key rate is very low compared with standard BB84 if one utilizes a special entangled state to prove security [21]. Afterwards, the asymptotic secret key rate is increased to the same level as standard BB84 using the loss-tolerant model [22]. However, it still needs to measure the four states. To further simplify the complexity of the experiment, a so-called simplified BB84 protocol that only has three measurement operators on the receiver is presented [23], which directly results in a record of transmission distance for point-to-point QKD [24]. Recently, a field trial of simplified BB84 has been demonstrated in backbone network [25].

Despite these amazing developments, the simplified BB84 proposed in Ref. [23] has some drawbacks that its security proof is only valid under the collective attack and requires basis-independent detection efficiency condition. A simple and direct security proof against coherent attacks is very meaningful, which avoids various estimation to lift the security from collective attacks to coherent attacks. Besides, the basis-independent detection efficiency condition usually cannot be satisfied in practical system unless at the cost of reduced key rate [26, 27]. Recently, some authors of us provide a security proof with simple phase error formula to solve the above problems in the asymptotic regime [28] via the phase error correction argument [29–31] and basis-independent source for single-photon. However, the experimental data is usually finite in the realization condition. Here, inspired by complementarity argument [31] and uncertainty relation for smooth entropies [32], we do not need to introduce a virtual entanglement-based protocol to provide security. We present a rigorous finite-key analysis for decoy-state simplified BB84 QKD, which is secure against coherent attacks in the universally composable framework [33, 34]. Numerical results indicate that the secret key rate of simplified BB84 is slightly smaller than that of standard BB84 protocol in finite size samples as low as $10^8$. The performances of two protocols are almost the same when the size of samples is increased to $10^{11}$. Besides, we also consider the transmission limit of decoy-state QKD, in which the secret key rate can achieve 0.25 bps over 472 km ultralow-loss fiber using 2.5 GHz system [24].
II. SECURITY ANALYSIS

Here, we review the simplified BB84 protocol [23, 28]. Alice sends to Bob two states |0⟩ and |1⟩ in Z basis or only one state |+⟩ in X basis using single-photon pulse. Bob measures the received pulses to project onto two states |0⟩ and |1⟩ in Z basis or only one state |−⟩ in X basis. The data of both Alice and Bob selecting Z basis is used to extract key rate while the data of Alice choosing Z (X) basis and Bob selecting X is utilized to monitor Eve’s eavesdrop. The phase error rate can be written as 
\[ e_p = \frac{Y_{x\bar{x}}}{Y_{z\bar{z}}} \] in the asymptotic regime [28], where \( Y_{x\bar{x}} \) (\( Y_{z\bar{z}} \)) is the yield given that Alice selects X (Z) basis and Bob chooses X basis.

By employing the entanglement distillation argument [29, 30], the simplified BB84 protocol has been proven secure against coherent attacks in the asymptotic limit [28]. Thereinto, one key observation is that the Bell-diagonal state is a superposition state of two joint quantum states with the same amplitude, such as \( |\Phi^\pm⟩ = (|+⟩ ± |−⟩) |+⟩/\sqrt{2} \). Therefore, the calculation of phase error does not need Bob to measure two states of X basis in principle. Besides, Alice also does not need to send two states of X basis due to the source basis-independent assumption, i.e., \( |0⟩0⟩ + |1⟩1⟩ = |+⟩1⟩ + |−⟩−⟩ \). Although the CSS quantum error correction code can be exploited to decouple the bit and phase error correction [30], resulting in a prepare-and-measure scheme, it’s still not easy to understand simplified BB84 due to existing virtual entanglement protocol in security proofs.

Different from the entanglement distillation argument, the complementarity argument [31] can decouple error correction and privacy amplification from the beginning, thereby relieving it from the constraint of the virtual entanglement protocol. We only need to estimate the leaked information from the error corrected key. Considering the equivalent time-reversed protocol, Alice measures the state transmitted by Eve in Z and X basis instead of preparing state. In a virtual protocol, for those states actually measured in Z basis, Alice hypothetically measures in X basis instead. She can always obtain state |−⟩ if there is perfect privacy. Then the phase error rate is defined as the ratio of acquired |+⟩ states [31], i.e., 
\[ e_p = \frac{Y_{x\bar{+}}}{Y_{x\bar{+}} + Y_{x\bar{−}}} = \frac{\rho_{xx}}{\rho_{xx} + \rho_{xx}} = \frac{Y_{x\bar{+}}}{Y_{x\bar{+}} + Y_{x\bar{−}}} \] Here, we utilize the basis-independent source and \( Y_{x\bar{+}} \) denotes the yield given that Alice sends |+⟩ and Bob obtains |−⟩.

In order to deal with the finite-key effect and make the security parameter computable, the smooth min-entropy is usually employed [33]. Here, we first exploit the uncertainty relation for smooth entropies [32] to prove the security of simplified BB84. For any \( \varepsilon > 0 \), let \( \rho \) be a tripartite quantum state among Alice, Bob and Eve. Obviously, the basis Z and X are two positive operator-valued measurements on Alice’s system. Using the uncertainty relations for smooth entropies [32], one has smooth min-entropy
\[
H^\varepsilon_{\min}(Z_A|E)_\rho \geq n^x - H^\varepsilon_{\max}(X_A|B)_\rho \geq n^x [1 - h(\varepsilon_p)],
\]
where \( Z_A \) is the raw key acquired by Alice, \( n^x \) is the size of \( Z_A \), \( X_A \) is the bit string Alice would have obtained if she had measured in the basis X instead. The smooth max-entropy \( H^\varepsilon_{\max}(X_A|B)_\rho \) quantifies the uncertainty that Bob has about \( X_A \). Note that the \( \rho \) of tripartite quantum state is arbitrary [32]. Inspired by the complementarity argument [31], we do not require the joint quantum state between Alice and Bob to have correlation in the virtual process, which is different from previous security proof [33]. For \( X_A \), Bob always guesses bit value is 1 (corresponding to |−⟩) in simplified BB84. Therefore, we have \( H^\varepsilon_{\max}(X_A|B)_\rho \geq n^z h(\varepsilon_p) \) with
\[ e_p = \frac{Y_{z\bar{+}}}{Y_{z\bar{+}} + Y_{z\bar{−}}} = \frac{Y_{z\bar{+}}}{Y_{z\bar{+}} + Y_{z\bar{−}}} = \frac{Y_{z\bar{+}}}{Y_{z\bar{+}} + Y_{z\bar{−}}} \] in the asymptotic limit, where \( h(x) = -x \log_2 x - (1 - x) \log_2 (1 - x) \) is the Shannon entropy. Then, we utilize the generalized chain-rule result of smooth entropies for phase-randomized coherent state and finite-size analysis for the decoy-state method [33–35] to provide the rigorous finite-key security bounds of decoy-state simplified BB84 QKD against the most general coherent attacks in the universally composable framework.

III. FOUR-INTENSITY DECOY-STATE QKD

Here, we consider a four-intensity decoy-state simplified BB84 QKD protocol. Although the number of intensity is optional, the four-intensity scheme usually shows better performance [26]. The probabilities of choosing basis and intensity are biased. Specially, intensities \( \mu \) and \( \nu \) are only prepared in Z basis while intensity \( \omega \) is sent in X basis.

1. Preparation. Alice randomly chooses an intensity \( k \in \{ \mu, \nu, \omega, 0 \} \) with probability \( p_k \) and prepares a phase-randomized weak coherent laser pulse with the intensity she has chosen. She randomly modulates qubit states |0⟩ and |1⟩ if the intensity is \( \mu \) or \( \nu \). She always sends qubit |+⟩ if the intensity is \( \omega \). N optical pulses are sent to Bob via insecure quantum channel.

2. Measurement. Bob utilizes basis B with probability \( q_b \) to measure the received pulses. In Z basis, he measures two states \( |0⟩ \) and \( |1⟩ \), but in X basis, he only measures state |−⟩. Bob records the effective events. For multiple clicks in passive basis detection, he randomly assigns a basis and a bit value.

3. Reconciliation. Through an authenticated classical channel, Bob publishes his basis choices of effective events while Alice announces the intensity information. They accumulate the data \( |\mathcal{E}_k| = n^E_k \) and \( |\mathcal{X}_k| = n^X_k \), where the \( \mathcal{E}_k \) (\( n^E_k \)) is set (number) of Bob choosing B basis when Alice sends \( k \) intensity.

4. Parameter estimation. Alice and Bob exploit a size of \( n^E_k + n^X_k \) in set \( Z_\mu \cup Z_\nu \) to obtain a raw key pair \( (Z_A, Z_B) \).
They calculate the observed number of vacuum events $s^z_0$, the observed number of single-photon events $s^z_1$ and the observed phase error rate associated with the single-photon events $\phi^x_1$ in $Z_A$.

5. Key distillation. Alice and Bob employ error-correction algorithm to correct errors in their raw key. This step consumes $\lambda_{EC}$ bit of information. Then they perform an error verification by applying a two-universal hash function to verify if their keys are identical, during which reveals $\log_2(2/\varepsilon_{cor})$ bits of information. Finally, Alice and Bob implement privacy amplification by adopting a two-universal hash function. The final extracted secret key has a length of $l$.

The phase-randomized coherent state source can be securely regarded as a mixture of photon-number states. For the vacuum state, Eve cannot acquire any information. For the single-photon state, the leaked information can be bounded by the smooth min-entropy in Eq. (1). Besides, Eve is assumed to have all information of multi-photon states. Conditioned on passing the error-verification step, the length of the final key with $\varepsilon_{sec}$-secret and $\varepsilon_{cor}$-correct in $Z_A$ is \[ l = s^z_0 + s^z_1[1 - h(\phi^x_1)] - \lambda_{EC} - 6 \log_2 \frac{2^{35}}{\varepsilon_{cor}}, \]

where $\tau$ and $x$ represent the upper and lower bounds of observed value $x$. Note that the final secret keys can be securely utilized in any cryptographic task. Our protocol is universally composable guaranteed by two security criteria $\varepsilon_{sec}$-secret and $\varepsilon_{cor}$-correct [33].

The expected numbers of vacuum events and single-photon events in $Z_A$ satisfy

\[ s^z_0 \geq \left( e^{-\mu} p_\mu + e^{-\nu} p_\nu \right) \frac{n^*_0}{p_0}, \]

and

\[ s^z_1 \geq \frac{\mu e^{-\mu} p_\mu + \mu e^{-\nu} p_\nu}{\mu - \nu^2} \frac{\mu^2}{\mu - \nu^2} \left( \frac{e^{\mu^2} \mu^2}{p_\mu} - \frac{e^{\mu^2} \nu^2}{p_\nu} \right) - \frac{\mu^2}{\nu^2} \frac{\mu^2}{\nu^2} \frac{n^*_1}{p_0}, \]

where $x^*$ denotes the expected value. Parameter $n^*_k$ can be acquired by the variant of Chernoff bound \[ \tau = x + \sqrt{2\beta x + \beta^2} \] and $\omega = x - \frac{\beta}{2} + \sqrt{2\beta x + \frac{\beta^2}{4}}$ with $\beta = \ln \frac{\sum e^{\mu^2}}{\varepsilon_{sec}}$. Besides, the observed number of error bit associated with the single photon events in $X_A$ is

\[ \tau^x_1 \leq n^*_0 - \frac{x^*}{p_0}, \]

where the expected value is $\tau^x_1 = \frac{e^{-\mu} p_\mu}{p_0} - \frac{x^*}{p_0}$. Given an expected value $x^*$, the corresponding observed value $x$ can be given by the Chernoff bound \[ \tau = x + \frac{\beta}{2} + \sqrt{2\beta x + \frac{\beta^2}{4}} \] and $x = x^* - \sqrt{2\beta x^*}$. In order to acquire the phase error rate, we need to calculate the number of single-photon events $s^x_1$ corresponding to the error events $\tau^x_1$. The corresponding expected value can be written as

\[ s^x_1 \geq \frac{\mu \omega e^{-\omega} p_\omega}{\mu - \nu^2} \left( \frac{e^{\mu^2} \mu^2}{p_\mu} - \frac{e^{\mu^2} \nu^2}{p_\nu} \right) - \frac{\mu^2}{\nu^2} \frac{\mu^2}{\nu^2} \frac{n^*_1}{p_0}. \]

Finally, the phase error rate can be acquired via the random sampling without replacement [35]

\[ \phi^x_1 \leq \tau^x_1 \leq \frac{\tau^x_1}{2\tau^x_1} + \gamma U(2\tau^x_1, 2\tau^x_1, \frac{\tau^x_1}{2\tau^x_1}, \frac{\varepsilon_{sec}}{22}), \]

where we have

\[ \gamma U(n, k, \lambda, \epsilon) = \frac{(1-2\lambda)AG}{n+k} + \sqrt{\frac{A^2 G}{(n+k)^2} + 4\lambda(1-\lambda)G}, \]

with $A = \max(n, k)$ and $G = \frac{n+k}{2\pi n k \lambda (1-\lambda) \epsilon^2}$.

IV. PERFORMANCE

In order to show the performance of the final secret key rate and the maximum secure transmission distance of decoy-state simplified BB84 QKD, we simulate a fiber-based QKD system with the related parameters as listed in Table I. For a given experiment, we can directly obtain the experimental parameters $n^*_k$ and $n^*_0$. In this simulation, we utilize the formulas $n^*_k = N p_k Q_k^{-1}$.
and $n_k^b = N p_k Q_k^x$, where $Q_k^b$ is the gain of Bob selecting $B$ basis when Alice sends $k$ intensity. Without loss of generality, these gain parameters can be given by $Q_k^a = \frac{1}{2} [1 - (1 - p_d) e^{-aq_k \eta^d}][1 + (1 - p_d) e^{-aq_k \eta^d}]$, $Q_k^b = \frac{1}{2} [1 - (1 - p_d) e^{-bq_k \eta^d}][1 + (1 - p_d) e^{-bq_k \eta^d}]$ and $Q_k^c = \frac{1}{2} [e_d^{x_k} + (p_d - e_d^{x_k}) e^{-aq_k \eta^d}][1 + (1 - p_d)^2 e^{-aq_k \eta^d}]$, where $a \in \{\mu, \nu, 0\}$ and $\eta^b = \eta^b_{\text{ins}} \times 10^{-\alpha L / 10}$ is the overall efficiency with the fiber length $L$ in $B$ basis selected by Bob. $\eta_d$ and $p_d$ are the detector efficiency and dark count rate of single-photon detector, $\alpha$ is the attenuation coefficient of fiber, $e_d^x$ is the misalignment rate of $B$ basis. $\eta^b_{\text{ins}}$ is the extra insert loss of Bob’s system in $B$ basis. In order to estimate the parameter $\lambda_{EC}$, we assume that $\lambda_{EC} = (n_{\mu}^x + n_{\nu}^x) f h (\frac{m_{\mu}^x + m_{\nu}^x}{n_{\mu}^x + n_{\nu}^x})$ and $f$ is the efficiency of error correction. The number of error bit events can be expressed as $m_{\mu}^x = N p_a E_{\mu}^x Q_{\mu}^a$ with $E_{\mu}^x Q_{\mu}^a = e_d^{x_k} Q_{\mu}^a + (1 - e_d^{x_k}) \frac{1}{2} [1 - (1 - p_d)^2] e^{-aq_k \eta^d} [1 + (1 - p_d) e^{-aq_k \eta^d}]$. We numerically optimize the secret key rate $R := l / N$ with the free parameters $\{\mu, \nu, \omega, p_{\mu}, p_{\nu}, p_{\omega}, q_{\lambda}\}$. For a fair comparison, we also simulate the performance of the four-intensity decoy-state QKD with standard BB84 [27]. As shown in Fig. 1, the solid lines represent simplified BB84 and dashed lines denote standard BB84 in the four-intensity decoy-state QKD. There is a negligible difference of secret key rate between simplified and standard BB84 even in the case of large attenuation.

Furthermore, in order to assess the maximum transmission of point-to-point decoy-state QKD, we simulate the secret key rate under coherent attacks using a state-of-the-art system [24]. The related simulation parameters are shown in Table II. The simulation results are shown in Fig. 2, where we fix the security bounds to $\varepsilon_{\text{sec}} = 10^{-10}$ and $\varepsilon_{\text{cor}} = 10^{-15}$. The maximal number of pulses will cost one day for a 2.5 GHz system [24]. Here, we use the reported optimal attenuation coefficient $\alpha = 0.163$ dB/km for the ultralow-loss fiber. The solid and dashed lines denote the simplified and standard BB84 in four-intensity decoy-state QKD, respectively. The secret key rate and secure transmission distance of two protocols have no significant difference in the finite size samples as low as $N = 10^8$. The secure transmission distance of our protocol can be pushed up to 472 km, where the secret key rate is 0.25 bps under coherent attacks for the case of one day data accumulation.

V. CONCLUSION

In this paper, inspired by complementarity argument [31] and uncertainty relation for smooth entropies [32], we develop a rigorous finite-key analysis for decoy-state QKD with simplified BB84 against coherent attacks in the universally composable framework. In order to show the advantage of our protocol, we compare the performance of it with standard BB84. The simulation results show that the performance of two protocols are almost the same even in the finite size samples. For an acquisition time of one day, the secret key rate is 0.25 bps over 472 km fiber at 2.5 GHz system, which still has practical utility. Moreover, compared with the standard BB84 protocol, an extra advantage is that the requirement of efficiency balance in $X$ basis is removed directly because only one state needs to be detected. The insert loss of $X$ basis can be decreased (circularator not needed) compared with standard BB84 if using Faraday-Michelson interferometer. Therefore, the decoy-state simplified BB84 QKD is more practical and should be a preference scheme in photonic integrated and high-frequency clock system. It will be interesting to combine our results with the measurement-device-independent [36], high-dimensional encoding [37] and reference-frame-independent scheme [38] to reduce experiment requirements.

VI. ACKNOWLEDGMENTS

We gratefully acknowledge support from the National Natural Science Foundation of China under Grant No. 61801420 and the Fundamental Research Funds for the Central Universities.
[1] C. H. Bennett and G. Brassard, Proceedings of the Conference on Computers, Systems and Signal Processing pp. 175–179 (IEEE, New York, 1984).
[2] G. Brassard, N. Lütkenhaus, T. Mor, and B. C. Sanders, Phys. Rev. Lett. 85, 1330 (2000).
[3] X.-B. Wang, Phys. Rev. Lett. 94, 230503 (2005).
[4] H.-K. Lo, X. Ma, and K. Chen, Phys. Rev. Lett. 94, 230504 (2005).
[5] A. Tanaka, M. Fujiwara, S. W. Nam, Y. Nambu, S. Takahashi, W. Maeda, K.-i. Yoshino, S. Miki, B. Baek, Z. Wang, et al., Opt. Express 16, 11354 (2008).
[6] Z.-Q. Yin, Z.-F. Han, W. Chen, F.-X. Xu, Q.-L. Wu, and G.-C. Guo, Chin. Phys. Lett. 25, 3547 (2008).
[7] Y. Liu, T.-Y. Chen, J. Wang, W.-Q. Cai, X. Wan, L.-K. Chen, J.-H. Wang, S.-B. Liu, H. Liang, L. Yang, et al., Opt. Express 18, 8587 (2010).
[8] B. Fröhlich, M. Lucamarini, J. F. Dynes, L. C. Coman dar, W. W.-S. Tam, A. Plews, A. W. Sharpe, Z. Yuan, and A. J. Shields, Optica 4, 163 (2017).
[9] T. Schmitt-Manderbach, H. Weier, M. Fürst, R. Ursin, F. Tiefenbacher, T. Scheidl, J. Perdigues, Z. Sodnik, C. Kurtsiefer, J. G. Rarity, et al., Phys. Rev. Lett. 98, 010504 (2007).
[10] J.-Y. Wang, B. Yang, S.-K. Liao, L. Zhang, Q. Shen, X.-F. Hu, J.-C. Wu, S.-J. Yang, H. Jiang, Y.-L. Tang, et al., Nat. Photonics 7, 387 (2013).
[11] C. Ma, W. D. Sacher, Z. Tang, J. C. Mikkelsen, Y. Yang, F. Xu, T. Thiessen, H.-K. Lo, and J. K. Poon, Optica 3, 1274 (2016).
[12] P. Silsson, C. Erven, M. Godfrey, S. Miki, T. Yamashita, M. Fujiwara, M. Sasaki, H. Terai, M. G. Tanner, C. M. Natarajan, et al., Nat. Commun. 8, 13984 (2017).
[13] D. Bunandar, A. Lentine, C. Lee, H. Cai, C. M. Long, N. Boynton, N. Martinez, C. DeRose, C. Chen, M. Grein, et al., Phys. Rev. X 8, 021009 (2018).
[14] S.-K. Liao, W.-Q. Cai, W.-Y. Liu, L. Zhang, Y. Li, J.-G. Ren, J. Yin, Q. Shen, Y. Cao, Z.-P. Li, et al., Nature 549, 43 (2017).
[15] H. Takenaka, A. Carrasco-Casado, M. Fujiwara, M. Kimamura, M. Sasaki, and M. Toyoshima, Nat. Photonics 11, 502 (2017).
[16] S.-K. Liao, W.-Q. Cai, J. Handsteiner, B. Liu, J. Yin, L. Zhang, D. Rauch, M. Fink, J.-G. Ren, W.-Y. Liu, et al., Phys. Rev. Lett. 120, 030501 (2018).
[17] N. T. Islam, C. C. W. Lim, C. Cahall, J. Kim, and D. J. Gauthier, Sci. Adv. 3, e1701491 (2017).
[18] Y. Ding, D. Bacco, K. Dalgaard, X. Cai, X. Zhou, K. Rottwitt, and L. K. Oxenløwe, npj Quantum Inf. 3, 1 (2017).
[19] G. Cañas, N. Vera, J. Carine, P. González, J. Carde nas, P. Connolly, A. Przysezenia, E. Gómez, M. Figueroa, G. Vallone, et al., Phys. Rev. A 96, 022317 (2017).
[20] J. Wang, F. Sciarrino, A. Laing, and M. G. Thompson, Nat. Photonics pp. 1–12 (2019).
[21] C.-H. F. Fung and H.-K. Lo, Phys. Rev. A 74, 042342 (2006).
[22] H. Takenaka, A. Carrasco-Casado, M. Fujiwara, M. Kimamura, M. Sasaki, and M. Toyoshima, Nat. Photonics 11, 502 (2017).
[23] D. Bunandar, A. Lentine, C. Lee, H. Cai, C. M. Long, N. Boynton, N. Martinez, C. DeRose, C. Chen, M. Grein, et al., Phys. Rev. X 8, 021009 (2018).
[24] A. Boaron, G. Boso, D. Rusca, C. Valliez, C. Autebert, M. Caloz, M. Perrenoud, G. Gras, F. Bussières, M.-J. Li, et al., Phys. Rev. Lett. 121, 190502 (2018).
[25] D. Bacco, I. Vagniluca, B. Da Lio, N. Biagi, A. Della Freera, D. Calonico, C. Toninelli, F. S. Cataliotti, M. Bellini, L. K. Oxenløwe, et al., EPJ Quantum Technol. 6, 5 (2019).
[26] Z.-W. Yu, Y.-H. Zhou, and X.-B. Wang, Phys. Rev. A 93, 032307 (2016).
[27] H.-L. Yin, P. Liu, W.-W. Dai, Z.-H. Ci, J. Gu, T. Gao, Q.-W. Wang, and Z.-Y. Shen, arXiv:2002.10668 (2020).
[28] H.-L. Yin and Z.-B. Chen, Opt. Lett. 45, 1627 (2020).
[29] H.-K. Lo and H. F. Chau, Science 283, 2050 (1999).
[30] P. W. Shor and J. Preskill, Phys. Rev. Lett. 85, 441 (2000).
[31] M. Koashi, New J. Phys. 11, 045018 (2009).
[32] M. Tomamichel and R. Renner, Phys. Rev. Lett. 106, 110506 (2011).
[33] M. Tomamichel, C. C. W. Lim, N. Gisin, and R. Renner, Nat. Commun. 3, 634 (2012).
[34] C. C. W. Lim, M. Curty, N. Walenta, F. Xu, and H. Zbinden, Phys. Rev. A 89, 022307 (2014).
[35] H.-L. Yin, M.-G. Zhou, J. Gu, Y.-M. Xie, Y.-S. Lu, and Z.-B. Chen, arXiv:2002.06530 (2020).
[36] H.-K. Lo, M. Curty, and B. Qi, Phys. Rev. Lett. 108, 130503 (2012).
[37] L. Sheridan and V. Scarani, Phys. Rev. A 82, 030301 (2010).
[38] A. Laing, V. Scarani, J. G. Rarity, and J. L. O'Brien, Phys. Rev. A 82, 012304 (2010).