Massless monopole clouds and electric–magnetic duality

Xingang Chen 1

Physics Department, Columbia University, New York, NY 10027, USA

Received 7 July 2004; accepted 20 August 2004
Available online 7 September 2004
Editor: M. Cvetič

Abstract

We discuss the Montonen–Olive electric–magnetic duality for the BPS massless monopole clouds in \( \mathcal{N} = 4 \) supersymmetric Yang–Mills theory with non-Abelian unbroken gauge symmetries. We argue that these low energy non-Abelian clouds can be identified as the duals of the infrared bremsstrahlung radiation of the non-Abelian massless particles. After we break the \( \mathcal{N} = 4 \) supersymmetry to \( \mathcal{N} = 1 \) by adding a superpotential, or to \( \mathcal{N} = 0 \) by further adding soft breaking terms, these non-Abelian clouds will generally condense and screen the non-Abelian charges of the massive monopole probes. The effective mass of these dual non-Abelian states is likely to persist as we lower the energy to the QCD scale, if all the non-Abelian Higgs particles are massive. This can be regarded as a manifestation of the non-Abelian dual Meissner effect above the QCD scale, and we expect it to continuously connect with the confinement as we lower the supersymmetry breaking scale to the QCD scale.

1. Introduction

The \( \mathcal{N} = 4 \) supersymmetric Yang–Mills theory is conjectured to have a remarkable electric–magnetic duality [1–4]. A special form of this conjecture suggests that the electric theory is dual to the magnetic theory with a dual group and an inverse coupling constant.

This conjecture originated in the study of an \( SU(2) \) theory spontaneously broken to \( U(1) \) [1,2], where there is only one type of fundamental (anti-)monopole. The supersymmetric multiplet based on this monopole is dual to the massive gauge supermultiplet.

If the rank \( r \) of the gauge group is higher than one, when the non-Abelian gauge symmetry is maximally broken to \( U(1)^r \), the monopole configurations can be treated as superpositions of fundamental monopoles associated with simple roots [5]; while for elementary particles, each root of the dual group corresponds to a massive gauge supermultiplet. Studies of supersymmetric sigma models on the monopole moduli spaces [6] show that these supersymmetric fundamen-
Fig. 1. The root diagrams of $SO(5)$ and $Sp(4)$.

tal monopoles indeed form threshold bound states as predicted by the duality. To illustrate this, we use monopoles in $SO(5) \rightarrow U(1)^2$ theory. The root diagrams of $SO(5)$ and its dual group $Sp(4)$ are shown in Fig. 1. A unique normalizable anti-self-dual harmonic two-form is found [6] on the moduli space of the $\beta$ and $\gamma$ monopoles (we have chosen and labeled the simple roots as $\beta$ and $\gamma$). This corresponds to a threshold bound state. The supersymmetric multiplet associated with this bound state in $SO(5)$ is dual to the $\alpha^*$ gauge multiplet in $Sp(4)$ as predicted by the duality.

The situation is more subtle when a non-Abelian subgroup of the gauge symmetry remains unbroken. In such cases, BPS configurations of massless monopole clouds (or non-Abelian clouds) have been found [7,8]. These configurations describe massive monopoles surrounded by clouds of the non-Abelian fields, which form an overall magnetic color singlet. In addition to the usual moduli of positions and $U(1)$ phases of the massive monopoles, there are also ones describing the unbroken non-Abelian gauge group, as well as the sizes or shapes of the clouds. There have been many extended studies on both the BPS configurations [9–15] and the low energy classical dynamics [16–18] of such clouds. However, it remains unclear how such configurations and their properties should be properly placed in the context of the electric–magnetic duality of the $\mathcal{N} = 4$ theory and how it may be related to the properties of the QCD confinement. It is the purpose of this Letter to make some initial steps toward this direction. First we observe that these low energy non-Abelian clouds should be identified as the dual bremsstrahlung radiation of the non-Abelian massless particles. Then, after breaking the supersymmetry, we argue that the non-BPS properties of these cloud are the manifestation of the non-Abelian dual Meissner effect at weak electric coupling above the QCD scale. We expect it to continuously go to the non-Abelian dual Meissner effect in QCD confinement when we lower the supersymmetry breaking scale to the QCD scale.

2. Massless monopole clouds and bremsstrahlung radiation

We use the same $SO(5)$ example. The gauge symmetry is now partially broken to $SU(2) \times U(1)$ by a Higgs expectation value $h$ orthogonal to the root $\gamma$ or $\gamma^*$ [19]. Correspondingly, the $\gamma$ monopoles or $\gamma^*$ elementary particles become massless. A spherically symmetric BPS magnetic monopole solution is found in [7]. It describes a massive monopole, embedded in the $SU(2)$ subgroup defined by the root $\beta$, surrounded by a non-Abelian cloud. There is a modulus $a$ characterizing the size of the cloud. We will be interested only in the non-Abelian fields which do not exponentially decay outside the massive monopole core $m^{-1}$:

$$A_{\gamma a}(r) = \epsilon_{\gamma a m} r_m G(r), \quad \phi_{\gamma a} = \hat{r}_a G(r),$$

(1)

where the subscripts $\gamma$ mean that the fields correspond to the triplet $SU(2)$ generators $\gamma_a (\gamma = 1, 2, 3)$ associated with the root $\gamma$, and

$$G(r) = \frac{1}{e r (1 + r/a)}.$$  

(2)

If the cloud size $a$ is infinite, we only have the massive monopole, carrying both Abelian and non-Abelian charges. If $a$ is finite, the cloud shields the non-Abelian charge of the massive one, so that the non-Abelian fields fall as $a/r^2$ outside of the radius $a$, as we can see from (1) and (2).

The metric for this massless monopole cloud can be obtained [8] by taking the zero reduced mass limit of the maximally broken case:

$$ds^2 = \frac{g^2}{8\pi} \left( \frac{da^2}{a} + a \sigma_1^2 + a \sigma_2^2 + a \sigma_3^2 \right).$$

(3)

where $g = 4\pi/e$ is the magnetic coupling and $\sigma_i (i = 1, 2, 3)$ are the one-forms describing the unbroken $SU(2)$. For this metric, the harmonic (anti-)self-dual form is not normalizable. So the massless monopole cloud is not bound. It has been a puzzle [8] why...
this configuration, which is dual to the $\alpha^*$ gauge multiplet
in the $Sp(4)$ theory, does not have a normalizable threshold
bound state as in the maximally broken case.

To answer that, we first look at the elementary
particles in the weakly coupled electric theory of
$Sp(4)$. Because the beta function vanishes in the
$\mathcal{N} = 4$ supersymmetric gauge theory [20],
the massless particles of the non-Abelian gauge multiplet
$\gamma^*$ in this weakly coupled theory are not confined.
Therefore, whenever a massive particle is coupled
to these massless ones, it emits non-Abelian infrared
bremsstrahlung radiation. For example, the massive $\alpha^*$
Higgs can become a massive $\beta^*$ Higgs by emitting
an infrared gauge or Higgs boson associated with the
root $\gamma^*$. Generalizing this, the massive gauge multi-
plets $\alpha^*$ and $\beta^*$ become indistinguishable through
the emission and absorption of the massless gauge super-
multiplet associated with $\gamma^*$.\(^2\)

These two descriptions for the monopoles and ele-
mentary particles are very different. The former de-
scribes a solitonic static field configuration, while
the latter describes massless elementary particles that
propagate in the speed of light. To see how they can be
dual to each other, we need analyze the low energy
supersymmetric quantum mechanics of the massless
monopole clouds on the moduli space.

To see what happens, we need to find the spher-
ically symmetric eigenstates of the Laplacian
$\Delta = dd^t + d'd'$ corresponding to the metric (3) [21].
These non-normalizable scattering states can be described
by sixteen harmonic differential forms which are the du-
als of the gauge supermultiplet $\gamma^*$. Up to constant
factors, these are given by

0-form:

$$\frac{1}{\sqrt{a}} J_1(g\sqrt{Ea/2\pi})$$

(4)

1-forms:

$$\frac{1}{a} J_2(g\sqrt{Ea/2\pi}) da$$

2-forms:

$$\frac{1}{\sqrt{a}} J_1(g\sqrt{Ea/2\pi})(da \wedge \sigma_1 + a \sigma_2 \wedge \sigma_3),$$

and cyclic,

$$\frac{1}{\sqrt{a}} J_1(g\sqrt{Ea/2\pi})(da \wedge \sigma_1 - a \sigma_2 \wedge \sigma_3),$$

and cyclic,

(6)

(7)

where $E$ is the arbitrarily small energy of the mass-
less monopole cloud and $J$'s are Bessel functions.
The $\alpha_i$'s and $da$ correspond to fermionic excitations.
The 3-forms and 4-form are the Hodge duals of the
1-forms and 0-form, respectively. The $a$-dependence
of these wave functions are all similar. For example,
the 0-form wave function goes to a constant for
$a < \frac{2\pi}{\sqrt{Ea}}$ and falls as $a^{-3/4} \cos(g\sqrt{Ea/2\pi} - 3\pi/4)$
for $a \gg \frac{2\pi}{\sqrt{Ea}}$.

However the moduli space approximation for the
low energy solitons usually requires small velocities.
For the case of the massless monopole cloud, this re-
quires $[18] \dot{a} < 1$. From the metric (3), this imposes
the restriction $a < a_c = \frac{\sqrt{g^2}}{E}$. Beyond this region the
moduli space approximation fails and the cloud prop-
agates as a wavefront at the speed of light [18]. So
the wave function should be replaced by the spheri-
cal wave $\sim e^{iEa/a}$ as $a > a_c$, where $a$ becomes
the position of the wavefront. As we turn to the weak mag-
etic coupling (small $g$) limit, the duality conjecture
suggests that the monopoles and the elementary gauge
particles exchange roles. Indeed, as we can see, the ex-
tent of the solitonic wave function $a_c$ is much smaller
then the wavelength ($\sim 1/E$) of the wavefront and,
in addition, inside $a_c$ the wave function is nearly a
constant. Thus the solitonic phase is negligible. (See
Fig. 2.) The massless monopole always appears as in-
fraed radiation and the elementary local field description
takes over.

The above discussion is in accordance with the
classical dynamics discussed in [18], i.e., the pre-
diction of the moduli space approximation from (3) that
$a \sim \sqrt{E}/g^2$ is good only for a time period of order
$g^2/E$ during which the cloud speed $\dot{a} < 1$. According
to the uncertainty principle, for $g < 1$, it is quantum
mechanically unobservable.

\(^2\) It is interesting to compare this $SO(5)$ example to a single
massive fundamental monopole in $SU(3) \to SU(2) \times U(1)$ theory,
where the massless monopole cloud is absent. On the dual side, for
a single massive elementary particle in this $SU(3)$ theory, the non-
Abelian charge is unchanged (or gauge equivalent) after infrared
radiation.
3. Dynamics of non-BPS non-Abelian monopole clouds

Non-Abelian $\mathcal{N} = 1$ or $\mathcal{N} = 0$ supersymmetric gauge theories have the important property of confinement. Significant insights have been made by Seiberg and Witten in [22]. From the exact $\mathcal{N} = 2$ low energy theory, they explicitly show that a superpotential breaking the supersymmetry to $\mathcal{N} = 1$ causes the massless magnetic monopole field to condense. This confinement is described in a weakly coupled magnetic theory through the dual Meissner effect [23]. Related issues starting from $\mathcal{N} = 4$ have also been studied (see, e.g., [24] and references therein).

It is natural to ask what roles the non-Abelian clouds we have studied may play in this QCD confinement. To see this, we will focus on the energy region above the QCD scale $\Lambda_{\text{QCD}}$. Specifically, we start with a $\mathcal{N} = 4$ theory with a weak electric coupling at high energy. In this theory we have argued that, in the presence of certain massive monopoles, we can identify the low energy magnetic non-Abelian clouds as the dual infrared non-Abelian particles by exploring the duality conjecture. When we break the supersymmetry at low energy, we break the original electric–magnetic symmetry. But the dual states we identified should still exist and we will be interested in how they evolve as the supersymmetry is broken. As mentioned, we will focus mostly on the energy region above $\Lambda_{\text{QCD}}$, where the strongly coupled magnetic theory is described by non-BPS monopoles. Then we will discuss some implications for the low energy theory below $\Lambda_{\text{QCD}}$.

We explicitly break the supersymmetry to $\mathcal{N} = 1$ at low energy by adding a superpotential for the $\mathcal{N} = 1$ chiral multiplets. We expand the Higgs around those vacua where part of the non-Abelian symmetry is unbroken and use $\phi$ to represent the non-Abelian components of the deviations. Among all the terms in the expansion, we will study the quadratic terms

$$m_\phi^2 \text{tr}(\phi^2)$$

as examples. This gives an $\mathcal{N} = 4$ supersymmetry scale $m_\phi$. As mentioned, the fact that $m_\phi > \Lambda_{\text{QCD}}$ is guaranteed as long as the electric coupling is weak at the supersymmetry breaking scale $m_\phi$. We
will be interested in the limit where the non-Abelian Higgs masses \( m_\phi \) are much smaller than the massive gauge bosons \( m_W \). We also want the \( U(1) \) Higgs masses to be much smaller than the non-Abelian Higgs. By doing this, we effectively make the \( U(1) \) parts remain BPS so we can concentrate on the non-BPS properties of the non-Abelian parts only. This is why we have neglected the \( U(1) \) mass terms in (8).

To study the non-BPS monopoles, it is enough to add a superpotential in the direction of the non-zero Higgs. But for later purposes to connect with confinement, we will also add superpotentials for the other two chiral multiplets. This can be simply given by the mass terms with zero Higgs vev. It has no effect on the monopole properties we will discuss.

We first study the example in \( SO(5) \). The BPS fields are given in (1). When the non-BPS potential (8) is added, the non-Abelian Higgs field is exponentially cut off at a distance scale \( m_\phi^{-1} \). Outside of the region \( m_\phi^{-1} \) where the Brandt–Neri–Coleman (BNC) instability [25,26] applies, the gauge field decays to a magnetic-color neutral configuration, which corresponds to having a non-Abelian cloud inside \( m_\phi^{-1} \). Since \( m_\phi^{-1} \gg m_W^{-1} \), the BPS solution (1) is still a good approximation between \( m_\phi^{-1} \) and \( m_W^{-1} \). However, the cloud size is no longer a modulus. It is easy to see that, under the potential (8), it is classically energetically favored for the cloud to shrink. We can use the BPS solution to estimate this \( a \)-dependent potential. It is

\[
\frac{g^2}{8\pi} m_\phi^2 a. \tag{10}
\]

This should be a good approximation as the non-BPS potential is weak. The correction is given by factors of \( m_\phi a \). The potential change within the core, \( r < m_W^{-1} \), is negligible.

Using the metric (3) and this linear potential, we can study the quantum mechanics of this bounded non-Abelian cloud. This is non-supersymmetric, as the monopole breaks the \( N = 1 \) supersymmetry. For the purpose of this Letter, we simply note that the ground state of the cloud has a mass gap of order \( m_\phi \) and is concentrated in the region \( \langle a \rangle \sim g^{-2} m_\phi^{-1} \ll m_W^{-1} \), since the factor \( g^2 \) can be absorbed in the \( a \) in (3) and (10). Any multi-monopole configuration can be thought of as being a collection of these color singlets. Since we neglected the \( U(1) \) Higgs mass, there are no net long-range forces between the monopoles when they are separated further than \( m_\phi^{-1} \). Before discussing the physical interpretation of this result, we consider a case where the cloud encloses two massive monopoles.

We use the minimal symmetry breaking model of \( SU(3) \) [9]. When the two massive monopoles are far apart, so that the non-Abelian Higgs has decayed exponentially, the relative orientation of their non-Abelian gauge charges is self-adjusted to minimize the energy [26]. The charges are then given by

\[
\frac{1}{\sqrt{2}} \text{diag}(1, 0, -1), \quad \frac{1}{\sqrt{2}} \text{diag}(0, 1, -1), \tag{11}
\]

respectively. Here, the first two entries of the matrices correspond to the unbroken \( SU(2) \). Since only the non-Abelian part is non-BPS, these two monopoles are attracted by the Coulomb potential

\[
-\frac{g^2}{16\pi l^2} (l > m_\phi^{-1}), \tag{12}
\]

Note we have a non-standard kinetic term for \( a \) from Eq. (3).
where \( l \) is the monopole separation. Here a factor of \(-\frac{1}{4}\) is from the inner product of the non-Abelian part of (11), and the Abelian part is neglected because it is approximately BPS under our mass conditions mentioned before.

When the two monopoles stay inside the range \( m_\phi^{-1} \), we can approximate the near-BPS fields outside of the massive cores by the superposition of two \( SU(2) \) monopoles at positions \( r_1 \) and \( r_2 \). This gives the Higgs fields at \( r \) as

\[
\text{diag} \left( t_1 + \frac{1}{\sqrt{2}er_1}, t_1 + \frac{1}{\sqrt{2}er_2}, t_3 - \frac{1}{\sqrt{2}er_1} - \frac{1}{\sqrt{2}er_2} \right)
\]

if there were no non-Abelian cloud and

\[
\text{diag} \left( t_1, t_1 + \frac{1}{\sqrt{2}er_1} + \frac{1}{\sqrt{2}er_2}, t_3 - \frac{1}{\sqrt{2}er_1} - \frac{1}{\sqrt{2}er_2} \right)
\]

with a minimal size non-Abelian cloud, where \( \text{diag}(t_1, t_1, t_3) \) is the vacuum and \( r_i = |r - r_i| (i = 1, 2) \). In the latter case, the non-Abelian field is cancelled at a length scale bigger than the monopole separation \( l \). Therefore, under the potential (8), it is energetically favored to have a minimal size non-Abelian cloud surrounding the massive monopoles. However the non-Abelian Higgs field is still present within the separation scale \( l \). Integrating (8) over the spatial region up to \( m_\phi^{-1} \), we obtain an attractive potential

\[
\frac{g^2}{32\pi} m_\phi^2 l + O\left(g^2 m_\phi^2 l^2\right) \quad (l < m_\phi^{-1}).
\]

4. Non-Abelian monopole clouds and dual Meissner effect

The energy scale \( m_\phi \) and the linear property of the potentials (10) and (15) may receive corrections from the higher order terms neglected in (8). However, the following qualitative features do not depend on these terms and the specific examples. Within the \( \mathcal{N} = 4 \) supersymmetry length scale \( m_\phi^{-1} \) around the massive monopoles, the appearance of the non-BPS Higgs raises the energy above the vacuum due to the non-BPS potential; outside of this scale, we have the BNC instability; so, whenever the topology is allowed, the non-Abelian clouds will always contract to cancel the non-Abelian fields of the enclosed massive monopoles.

In our discussion, because the massive monopoles carry non-Abelian magnetic charges, they actually serve as probes so that we can study the properties of the dual non-Abelian states. Unlike the Coulomb-like phase in \( \mathcal{N} = 4 \) as we saw in Section 2, these dual states now have effective masses and the non-Abelian magnetic charges are screened. In other words, in this intermediate energy region where we describe the magnetic theory by solitons, breaking the supersymmetry by a superpotential (but maintaining the non-Abelian nature of the vacuum) in the weakly coupled electric theory causes the magnetic theory to be in an analogous dual Higgs phase. In the following, we will discuss the possibility of this phenomenon continuously going to the dual Meissner effect when we lower the energy scale \( m_\phi \) to that of the vacuum state (AQCD), where the test massive solitonic monopole becomes the test elementary particle.

To do this, we first note that, although the \( \mathcal{N} = 1 \) non-Abelian vacuum has the energy scale \( \Lambda_{\text{QCD}} \), we have only seen the non-Abelian clouds at \( m_\phi \) because we rely on the presence of massive non-Abelian monopoles. To look at these non-Abelian clouds at a lower energy scale \( m_\phi \) (\( m_\phi > m_\phi > \Lambda_{\text{QCD}} \)) with a corresponding bigger electric coupling \( \tilde{e} \) (according to the asymptotic freedom), we should change the setup by lowering the supersymmetry breaking scale to \( \tilde{m}_\phi \) and choose the \( \mathcal{N} = 4 \) theory above it to have the corresponding coupling \( \tilde{e} \). By the same argument we see that, after the supersymmetry breaking, the non-Abelian clouds of the \( \mathcal{N} = 1 \) theory with coupling \( \tilde{g} \)}
are Higgsed and get a mass $\sim m_{\phi}$. The same reasoning can go all the way to $e \lesssim 1$ ($g \gtrsim 1$).

From $e \sim g \sim 1$ around $\Lambda_{\text{QCD}}$, the solitonic description we used in the magnetic theory starts to deviate from being a good approximation. For the massive monopoles, the Compton wavelength begins to exceed the monopole core size. For the non-Abelian clouds, the potential becomes too shallow. Only one bound state can exist, with a mass gap $g^2 m_{\phi}$ determined by the depth of the potential. This bound state has a wavelength of order $g^{-3} m_{\phi}^{-1}$, which begins to exceed the range of the potential $m_{\phi}^{-1}$ as $g \lesssim 1$. (Outside of $m_{\phi}^{-1}$, we still have the BNC instability in the Higgs direction we are considering.) So as mentioned before, below $g \sim 1$ we should switch the roles of elementary particles and the solitons between the electric and magnetic theories. These analyses also suggest that the masses (which should be of order $\Lambda_{\text{QCD}}$ from the last paragraph) of the dual non-Abelian fields are likely to vary continuously, rather than abruptly vanish, at $g \sim 1$.\footnote{For the solitonic description of the magnetic theory at big $g$, the massive monopoles can only probe one Higgs direction since the non-Abelian clouds are non-zero in only one of the Higgs fields. There it is enough that we have a superpotential for one chiral multiplet. But in order for this screening effect to continuously go to the case $g \lesssim 1$ where the non-Abelian Higgs can oscillate in all directions, the superpotential in the other two complex directions of the Higgs should also be present as we mentioned in footnote 3.}

This meets the expectation that the usual weakly coupled dual Higgs mechanism starts to take effect. Nielsen–Olesen electric flux tubes\cite{28} appear as solitonic objects and this causes confinement of non-Abelian electric charge and electric fields. The quantum fluctuations of these tubes are of order $g$ times the thickness of the flux tubes\cite{28}. Here we comment that for big $g$ above $\Lambda_{\text{QCD}}$, these fluctuations are much bigger than the size of the electric flux. This is consistent with the fact that the electric fields are not confined above $\Lambda_{\text{QCD}}$, despite of the analogous dual Higgs mechanism.

The coupling stops running soon after the magnetic perturbation theory starts to become valid, since all the non-Abelian magnetically charged particles obtain masses of order $\Lambda_{\text{QCD}}$ through this dual Higgs mechanism.

Since so far all the non-BPS properties of the $\mathcal{N} = 1$ theory that we have used are shared by the non-supersymmetric theory, we can further break the $\mathcal{N} = 1$ supersymmetry by adding some soft breaking terms. For example, we can add a non-Abelian gaugino mass term with mass equal to the supersymmetry breaking scale $m_{\phi}$ and get the same picture.

Acknowledgements

I would like to thank Brian Greene, David Tong, Piljin Yi and especially Dan Kabat and Erick Weinberg for many helpful discussions and comments on the manuscript. This work was supported in part by the US Department of Energy.

References

[1] C. Montonen, D.I. Olive, Phys. Lett. B 72 (1977) 117; P. Goddard, J. Nuyts, D.I. Olive, Nucl. Phys. B 125 (1977) 1.
[2] H. Osborn, Phys. Lett. B 83 (1979) 321.
[3] A. Sen, Phys. Lett. B 329 (1994) 217, hep-th/9402032.
[4] L. Girardello, A. Giveon, M. Porrati, A. Zaffaroni, Phys. Lett. B 334 (1994) 331, hep-th/9406128; C. Vafa, E. Witten, Nucl. Phys. B 431 (1994) 3, hep-th/9408074.
[5] E.J. Weinberg, Nucl. Phys. B 167 (1980) 500.
[6] J.P. Gauntlett, D.A. Lowe, Nucl. Phys. B 472 (1996) 194, hep-th/9601085; K. Lee, E.J. Weinberg, P. Yi, Phys. Lett. B 376 (1996) 97, hep-th/9601097; G.W. Gibbons, Phys. Lett. B 382 (1996) 53, hep-th/9603176.
[7] E.J. Weinberg, Phys. Lett. B 119 (1982) 151.
[8] K. Lee, E.J. Weinberg, P. Yi, Phys. Rev. D 54 (1996) 6351, hep-th/9605229.
[9] A.S. Dancer, Commun. Math. Phys. 158 (1993) 545; A.S. Dancer, Nonlinearity 5 (1992).
[10] P. Irwin, Phys. Rev. D 56 (1997) 5200, hep-th/9704153.
[11] K. Lee, C. Lu, Phys. Rev. D 57 (1998) 5260, hep-th/9709080.
[12] E.J. Weinberg, P. Yi, Phys. Rev. D 58 (1998) 046001, hep-th/9803164.
[13] C. Lu, Phys. Rev. D 58 (1998) 125010, hep-th/9806237.
[14] C.J. Houghton, E.J. Weinberg, Phys. Rev. D 66 (2002) 125002, hep-th/0207141.
[15] X.g. Chen, E.J. Weinberg, Phys. Rev. D 67 (2003) 065020, hep-th/0212328.
[16] A. Dancer, R. Leese, Proc. R. Soc. London A 440 (1993) 421; A.S. Dancer, R.A. Leese, Phys. Lett. B 390 (1997) 252.
[17] P. Irwin, hep-th/0004054.
[18] X. Chen, H. Guo, E.J. Weinberg, Phys. Rev. D 64 (2001) 125004, hep-th/0108029; X. Chen, E.J. Weinberg, Phys. Rev. D 64 (2001) 065010, hep-th/0105211.
[19] E.J. Weinberg, Nucl. Phys. B 203 (1982) 445.
[20] M.F. Sohnius, P.C. West, Phys. Lett. B 100 (1981) 245.
[21] E. Witten, Nucl. Phys. B 202 (1982) 253;
J.P. Gauntlett, Nucl. Phys. B 400 (1993) 103, hep-th/9205008.
[22] N. Seiberg, E. Witten, Nucl. Phys. B 426 (1994) 19;
N. Seiberg, E. Witten, Nucl. Phys. B 430 (1994) 485, hep-th/9407087, Erratum.
[23] S. Mandelstam, Phys. Rep. 23 (1976) 245;
A.M. Polyakov, Nucl. Phys. B 120 (1977) 429;
G. ’t Hooft, Nucl. Phys. B 190 (1981) 455.
[24] M.J. Strassler, Prog. Theor. Phys. Suppl. 131 (1998) 439, hep-lat/9803009.
[25] R.A. Brandt, F. Neri, Nucl. Phys. B 161 (1979) 253.
[26] S.R. Coleman, in: A. Zichichi (Ed.), The Unity of the Fundamental Interactions, Plenum, New York, 1983.
[27] C.L. Gardner, J.A. Harvey, Phys. Rev. Lett. 52 (1984) 879.
[28] H.B. Nielsen, P. Olesen, Nucl. Phys. B 61 (1973) 45.