Simplified approach for the optimal number of rotors and support structure design of a multi rotor wind turbine system

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Abstract. In this study different multi rotor wind turbine systems (MRSs) are designed in such a way that the space frame, forming the connection between rotor nacelle assemblies (RNAs) and tower, will be modeled and dimensioned as an ideal truss work. The main focus is on multi digit MRSs (MD-MRS) with a high number of rotors. To dimension the tube diameters and wall thicknesses, a simplified load case is used with an adjusted safety factor for loads. Parameters that are varied are the number of rotors, depth of the space frame, as well as the location of the fixed bearing for the yaw system. As such, the simplified approach is a preliminary step helping to choose a good design parameter combination for a more detailed and comprehensive analysis. Buckling is a main design driver, as well as the thrust forces. For a high number of rotors, a fixed bearing position and therefore tower height, halfway through the space frame height seems favorable regarding the cost.

1. Introduction
Due to ever growing wind turbines and their components, especially the blades, the wind industry is facing new challenges in manufacture and transportation, as well as loads and strength. A multi rotor wind turbine system (MRS) could overcome the obstacles of this growth trend.

Studies from the INNWIND project showed the potential of a 20 MW MRS with 45 rotors to reduce the levelized cost of energy (LCoE) compared to a power equivalent single rotor (SR) [1]. Results from Vestas’ four rotor MRS demonstrator revealed advantages in aerodynamic efficiency and in wake recovery when compared to a SR [2].

Due to the resulting lower rotor nacelle assembly (RNA) masses, based on the square cube law, and the load averaging effect [3], it should be more suitable to build up a MRS with a high number of small rotors, rather than a small number of large rotors. This MRS with a high number of small rotors can be categorized as a multi digit MRS (MD-MRS). To allow original equipment manufacturers (OEMs) to move towards MRSs by using their existing turbine portfolio, a medium term solution might be the use of few rotors. This single digit MRS (SD-MRS) would be built up of three to nine rotors in the megawatt range. In the long term, a MRS with a high number of rotors using a newly developed small RNA in the sub-megawatt range seems favorable.

A MRS support structure is composed of a tower and a space frame, connecting the RNAs among themselves and with the tower. The space frame consists of tubular steel connections.
In [4] a simplified support structure design and dimensioning procedure for MRSs was presented. SD-MRSs in the range from 14 MW to 28 MW were investigated and optimized with respect to geometric design parameters like the depth of the space frame and the yaw bearing position. The INNWIND MD-MRS design with 45 rotors [1] was also investigated regarding these design parameters. There, a space frame depth of around 10% of the space frame width and a fixed bearing position in the middle of the space frame height resulted in the lowest cost.

In this study the simplified support structure approach is used to investigate MRS designs for a given overall capacity of 20 MW and a variable number of rotors.

The assumptions and simplifications for the concept analysis are:
- simplified load case with adjusted safety factor: maximum thrust at steady rated wind speed on all rotors simultaneous as a worst case scenario for the space frame.
- no dead load of the space frame considered.
- no windage loads on space frame and tower considered.
- the space frame is assumed as an ideal truss work.
- simplified tower and space frame connection via a fixed/floating yaw bearing combination.
- thin shelled tubes with assumed thickness ratios for tower and space frame.
- dimensioning of space frame members based on yield strength and buckling.
- dimensioned masses result in material cost with assumed cost per mass factors.
- energy yield calculation with an assumed Rayleigh wind speed distribution (IEC turbine class I B [5]). In contrast to the loads, wind shear is considered in this calculation.
- material cost and the energy yield result in simplified cost of energy (SCoE).

An example of a hexagonal MRS design with a simplified yaw bearing connection is seen in Figure 1.

The geometric design parameters number of rotors, fixed bearing position, depth of the space frame and the slope of the back rows of the space frame are varied and the influence on space frame mass and SCoE investigated.

The objective of this study is to get a better understanding of the support structure regarding the loads and the main influences in the dimensioning process. To determine promising MRS
designs with respect to the number of rotors and the geometric design parameters, it is important to compare with other MRS designs, rather than a SR. Based on the simplicity of this analysis, a comparison between MRS and SR is not the main focus here.

2. Simplified support structure design

An MRS support structure consists of a tower and a space frame, connecting the RNAs among themselves and with the tower. The rotors are arranged in horizontal rows and equilateral triangles, guaranteeing a high packing density of the rotor areas. The distance of the blade tips of two neighboring rotors is set to three percent of the rotor diameter. A ground to blade tip clearance is set to 22 m. For each design all rotors on the MRS are of the same single capacity and therefore size. There is no mixture of different single rotors in one design.

A simplified design load case (DLC) is defined: maximum thrust at steady rated wind speed on all rotors simultaneous. Each space frame element and the tower are designed based on ultimate loads and global buckling. Design aspects that are neglected here: fatigue analysis, drag forces on the space frame, dead load of the space frame, dynamic behavior (modal analysis), local shell/plate buckling, full range of DLCs based on [5]. This is done due to simplicity and the nature of this study: a concept analysis, rather than the detailed design of a chosen concept.

The DTU 10 MW research wind turbine [6] is used in this study as a basis for up- and downscaling. This includes downscaling to the size of the rotors used for the MRS, as well as upscaling to a large SR with a capacity of 20 MW. Scaling is done under the assumption of similarity rules for wind turbines and a constant tip speed ratio [3]. The rotor diameters for the scaled turbines $D_{\text{rotor},i}$ are calculated with the rotor diameter of the DTU rotor $D_{\text{DTU}}$ as well as capacities $P_i$ and $P_{\text{DTU}}$:

$$D_{\text{rotor},i} = \sqrt{\frac{P_i}{P_{\text{DTU}}}} \cdot D_{\text{DTU}}.$$  \hspace{1cm} (1)

The masses of rotors and nacelles are scaled via:

$$m_i = \left(\frac{D_{\text{rotor},i}}{D_{\text{DTU}}}\right)^n \cdot m_{\text{DTU}},$$  \hspace{1cm} (2)

with the scaling exponent $n$. For upscaling an exponent of $n_{\text{up}} = 2.6$ is used, implying technological improvement. Whereas the downscaling exponent is set to $n_{\text{down}} = 3$, assuming a scaled state of the art small turbine with today’s knowledge in designing, materials science and manufacturing, without any new future improvements. To downscale with an exponent of $n_{\text{down}} = 2.6$ would lead to the turbines of the past, in terms of masses and therefore cost.

A finite element analysis (FEA) is used for the calculation of the space frame axial element forces. The space frame is modeled as an ideal truss work and therefore bar elements are used in the FEA. The bars are assumed to be thin shelled tubes with $t \ll D$, with tube diameter $D$ and wall thickness $t$. The relevant FEA solutions, the axial forces $F_{\text{bar},i}$, are independent of the initial values of $D$ and $t$, that are required for the simulation, since an ideal truss work is used. Thrust and gravitational forces resulting from the rotors are set as external forces acting on the rotor nodes. Safety factors, $f_{\text{thrust}} = 1.5$ and $f_{\text{gravitational}} = 1.35$ are applied. Due to the simplicity of the procedure $f_{\text{thrust}}$ is increased from the suggested value of $f_{\text{thrust}} = 1.35$ in [5]. The space frame is connected with the tower via a fixed/floating bearing combination, see Figure 1. Both are set with their respective degrees of freedom (DOFs) as boundary conditions in the FEA model. The bearings would allow the whole MRS to yaw, rather than each rotor itself. An additional floating bearing needs to be set to prevent this rotation of the space frame and is necessary to have a kinematically determined FEA model. How the space frame and tower connection through the bearings is designed in detail is of no importance in this preliminary concept design study.
Each space frame element is dimensioned on its own. Diameters are first dimensioned against yield strength. A ratio for the wall thickness to diameter is defined as $r_t = \frac{t}{D}$. This is set to $r_{t,b} = \frac{1}{120}$ for the space frame bars and $r_{t,t} = \frac{1}{250}$ for the tower. For the cross section follows under the assumption of a thin shell:

$$A \approx \pi \cdot D^2 \cdot r_t.$$  \hfill (3)

A construction steel with a yield strength of $\sigma_{\text{yield}} = 355 \text{ MPa}$ is used for both space frame and tower. Based on $\sigma = \frac{F}{A}$ and the axial forces $F_{\text{bar},i}$, including both $\gamma_{\text{f,gravitational}}$ and $\gamma_{\text{f,thrust}}$, the bar diameters $D_i$ can be calculated:

$$D_i = \sqrt{\frac{|F_{\text{bar},i}| \cdot \gamma_m}{\sigma_{\text{yield}} \cdot \pi \cdot r_{t,b}}}.$$  \hfill (4)

In case of a compression state, redimensioning against buckling (Euler buckling) is necessary:

$$D_{i,\text{buckling}} = \sqrt[3]{\frac{8 \cdot |F_{\text{bar},i}| \cdot \gamma_{m,\text{buckling}} \cdot l^2}{\pi^3 \cdot E \cdot r_{t,b}}},$$  \hfill (5)

with $\gamma_{m,\text{buckling}} = 1.2$ according to [5] and the column effective length factor set to the bar length $l_i$. The actual bar diameter is set to the maximum of $D_i$ and $D_{i,\text{buckling}}$.

The reaction forces in the bearings are acting on the tower and result in reaction bending moments $M_{b,i}$. These bending moments are used to dimension the tower diameters for a number of sections over tower height. The reaction moment at tower top equals zero, so the tower top diameter is calculated using Equation 4 based on the gravitational force resulting from the weight of the RNAs and the dimensioned space frame. This tower top diameter is set for the sections that would result in smaller diameters based on the bending moment. The tower diameters $D_{\text{tower},i}$ follow to:

$$D_{\text{tower},i} = \sqrt{\frac{4 \cdot |M_{b,i}| \cdot \gamma_m}{\sigma_{\text{yield}} \cdot \pi \cdot r_{t,t}}}.$$  \hfill (6)

The masses for the space frame, tower and RNAs are multiplied with cost factors, see Table 1. These cost factors are based on a turbine cost splitting [3], a capital expenditure (CAPEX) assumption [7] and space frame cost estimation [1]. This results in the overall CAPEX of the MRS.

| Tower | 21.9 | 2.5 |
|-------|------|-----|
| Space frame | - | 5 |
| Rotor | 29.7 | 16 |
| Nacelle | 48.4 | 18 |

Table 1: Cost fractions after [3] and resulting cost per mass factors

With an assumed Rayleigh distribution for the wind speed, $c_P$-curve of the scaled DTU turbines and power law assumption for wind shear, the annual energy yield (AEP) is calculated. With an assumed wind turbine life time of 25 years the simplified LCoE (SCoE) can be calculated:
\[
SCoE = \frac{\sum \text{cost of RNAs, tower, space frame}}{n_1 \cdot \text{AEP}} = \frac{\text{CAPEX}}{n_1 \cdot \text{AEP}}. \tag{7}
\]

Simplified in that sense, that the operational expenditure (OPEX), the decommissioning expenditure (DECOMMEX), interest rate, as well as balance of plant (BoP) are not considered. This simplified approach allows fast computations of a large variety of different support structure designs. By variation of rotor number, space frame topology, space frame depth and the positioning of yaw bearings, it is possible to gain an understanding of the optimal MRS design. As such, the simplified approach is a preliminary step helping to choose a design parameter combination for a more detailed and comprehensive analysis.

3. Hexagonal MRS designs

To systematically compare different MRS designs with a variable number of rotors, a hexagonal design is chosen. Starting with seven rotors in three rows and subsequently adding one rotor to each side of the hexagon. This results in two additional rows in each new design. Figure 2 shows the layout of rotors for design no. 1 (7 rotors), design no. 2 (19 rotors) and design no. 6 (127 rotors).

The overall capacity for all designs is set to 20 MW. The overall dimensions, height and width of the MRS, are relatively constant for all hexagonal designs. Table 2 shows the numbering of the hexagonal designs with their number of rotors, rotor diameters \(D\) (see Equation 1) and rated power \(P_{\text{rated}}\) of the single turbines used in the MRS.

| hexagonal design no. | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|----------------------|----|----|----|----|----|----|----|----|
| number of rotors     | 7  | 19 | 37 | 61 | 91 | 127| 169| 217|
| \(D_{\text{rotor}}\) | [m]| 95.3| 57.9| 41.5| 32.3| 26.4| 22.4| 19.4| 17.1|
| \(P_{\text{rated}}\) | [kW]| 2857| 1053| 541| 328| 220| 157| 118| 92 |

The rotors are connected with respect to the spacing and equilateral triangles. This grid is also applied in the back, connected to the front with bars in each rotor node. Additional cross elements are needed for stability. An example of the space frame can be seen in Figure 3.

This simple version with mirrored front and back plane can be altered in the way that the outer sides of the hexagon only have one row each, the row in the front. This merged rows result in lesser bars.
The floating bearing is always located in the first row from the bottom. When there is no natural node/rotor position, an additional node and stiffening/connecting bars are needed. This is the case for an even number of rotors in a row. The position of the fixed bearing will be varied in the simulations and can be set in any row \( \geq 2 \) from the bottom to the top. The depth of the space frame can be varied with the parameters depth and angle \( \varphi \). The depth is defined as the depth of the top row, even in case of the merged top row, see Figure 4. For an angle of \( \varphi > 0 \), the depths of the rows below result in higher values. Figure 4 shows the geometric design parameters for the hexagonal design no. 3 with the six possible fixed bearing positions.

In [4] a MD-MRS with 45 rotors and a layout based on the INNWIND report, see [1], was investigated. There, only the depth and fixed bearing position was varied. A minimal SCoE was found for a fixed bearing position in the middle of the structure and a depth of around 10% of the space frame width.
4. Results

In this chapter all SCoE values are normalized to the SCoE value of a 20 MW SR, that is dimensioned with the same assumptions and methods described in Chapter 2. Space frame mass values are normalized to the overall mass value of the SR. Tower mass values are normalized to the SR tower mass value.

Figure 5 shows the SCoE results of the eight hexagonal MRS designs. All have merged sides, a depth to width ratio of 10% and a $\varphi$ angle of zero degree. The three curves indicate three fixed bearing positions: bottom in the second row, middle in the exact middle of the hexagonal space frame and top at the highest row. For design no. 1 with seven rotors the bottom and middle position results in the same design, since there are only three rows overall. For all designs the middle fixed bearing position results in the lowest SCoE, followed by the bottom and top positions. Overall all curves are falling in a monotonic way, due to square cube law and the resulting lower overall RNA masses and cost. The cost of components over number of rotors can be seen in Figure 6 for the middle fixed bearing position. The RNAs are the main fraction of cost for small rotor numbers.

![Figure 5: normalized SCoE [-] for bottom, middle and top fixed bearing position](image)

![Figure 6: normalized SCoE [-], cost splitting for middle fixed bearing position](image)

In Figure 7 the space frame mass over number of rotors for the three fixed bearing positions bottom, middle and top can be seen. The highest masses are present for the bottom position. In this case all forces pass the whole space frame and are transferred at the bottom to the tower via the fixed bearing. A lot of members are in the compression state and need to be redimensioned against buckling after the initial stress dimensioning. Apart from the design no. 1 with seven rotors, the top position results in slightly less weight than the middle position. For a higher number of rotors the top position values are approaching the values of the middle position. Design no. 1 with seven rotors has no natural node for the bearing in the top row. Additional elements are needed and result in higher values than the middle position.

Since the fixed bearing position also indicates the tower height, the three curves for the tower masses in Figure 8 show large differences. The bottom and top positions change in height with the number of rotors based on the rotor diameter. This results in a monotonic falling curve for the bottom position, respectively a rising curve for the top position. For design no. 1 the positions bottom and middle are the same, since there are only three rows.
The influence of merging the space frame sides can be seen in Figure 9 on the SCoEs and space frame mass in Figure 10. The merging results in elimination of bars, see Figure 3, and therefore mass. The largest influence occurs by the merging of the sides, since four of the six hexagonal sides are affected. For higher number of rotors the influence is decreasing since the proportional number of rotors and bars affected by the merging is decreasing.

The following results are shown for hexagonal design no. 6 with 127 rotors in the base case version (fixed bearing position in the middle, a depth of 10% and $\varphi = 0^\circ$). These results and their characteristics are representative for the other hexagonal designs.

In Figure 11 the space frame mass is shown for two scenarios, the red curve for dimensioning against yield strength and buckling, the blue curve only based on yield strength. As seen, buckling is a main design driver of the space frame.
Figure 11: normalized space frame mass [-], dimensioned with and without buckling

Figure 12: normalized space frame mass [-], dimensioned with and without gravitational or thrust forces

Figure 12 shows the space frame masses for only gravitational forces, only thrust forces and both acting on the rotors of the space frame. As seen, the thrust forces are by far the main design driver in terms of forces for the space frame members.

Now the geometric design parameters depth respectively the depth to width ratio and angle \( \varphi \) are varied, again for the design no. 6. The depth to width ratio is varied from 1\% to 20\% and the angle \( \varphi \) from 0\(^{\circ}\) to 20\(^{\circ}\). In Figure 13a, a 2D view of the response surface of the middle fixed bearing position for the space frame mass is seen. In Figure 13b is the same response surface with an adjusted colorbar scale, to better visualize the minimum at around a depth of 10\% and an angle \( \varphi = 1^{\circ} \).

Figure 13: response surface for geometric design parameter, hexagonal design no.6, middle fixed bearing position
The optimized and normalized SCoE values for all designs are shown in Figure 14 in comparison to the base case version with the fixed bearing position in the middle, a depth of 10% and $\varphi = 0^\circ$. The optimum for each design regarding the geometric design parameters is determined manually rather than through the use of an overarching optimization strategy. Since the hexagonal base case is already a good design, the optimizations of the parameters result only in a small change in SCoEs. Also included is the SCoE value of the optimized INNWIND design with 45 rotors. This design was dimensioned with the same assumptions as the hexagonal designs. The INNWIND SCoE value is seen as a small kink in the optimized curve. This indicates that there is further potential to reduce SCoE apart from the hexagonal design investigated here.

![Figure 14: base case and optimized, normalized SCoE [-] for each design](image)

In Figure 15 a sensitivity study on the SCoE for the design no. 6 base case regarding several input parameters can be seen. The main influence is the downscaling exponent resulting in the largest changes in SCoE. In Chapter 2 it was reasoned why a downscaling exponent of $n_{\text{down}} = 3$ seems appropriate for MRSs in general and this study. Figure 16 shows the normalized SCoE for all base case designs and different downscaling exponents. As seen in Figure 15, there is a large influence of the exponent. But the SCoE curves are progressing in the same way for all exponents and the objective of this study was to compare the MRS designs with themselves, rather than to compare with a SR. The downscaling exponent of $n_{\text{down}} = 3.2$ would lead to a new small rotor with assumed future improvement in designing, materials science and manufacturing technology.

5. Conclusions and outlook
A simplified approach to dimension MRS support structure designs was used to determine the influence of parameters like the number of rotors, the space frame depth and the fixed bearing position.
A fixed bearing position, and therefore tower height in the relative middle of the space frame height seems favorable. A space frame depth to width ratio of \( \approx 10\% \) and a small angle \( \varphi \) leads to reduced cost.

Buckling is a major design driver. The use of additional stiffening elements that would lead to a better load distribution and also to reduced buckling lengths, is a topic that needs to be considered in future studies.

The influence on the dimensioning resulting from the RNA weights is negligible compared to the influence from the thrust forces, even for a small number of rotors. This justifies the simplification to neglect the dead load of the space frame, although this was initially done due to the chosen dimensioning process.

The simplified cost, the SCoE, are declining for higher number of rotors. Based on this study and simplifications, the optimal design and number of rotors would be the design no. 8 with 217 rotors.

However, for a high number of rotors the complexity of the space frame rises. It needs to be considered, if this would influence the assembly process, like welding, and therefore the cost. Additionally, the OPEX should be considered in relation to the number of rotors.

The aerodynamic efficiency has been assumed to be independent of the rotor size in this paper. For very small rotors with low Reynolds numbers the aerodynamic efficiency is likely to decrease. This effect might be one among others shifting the optimum to slightly lower rotor numbers compared to the findings in this paper.

For further studies with additional load cases and more design parameters, an approach as part of a formal optimization problem needs to be considered.

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References

[1] P. Jamieson, M. Branney, K. Hart, P.K. Chaviaropoulos, G. Sieros, S. Voutsinas, P. Chasapogiannis, and J. M. Prospathopoulos. Innovative Turbine Concepts - Multi-Rotor System. INNWIND Deliverables, 1.33, 2017.

[2] M. P. van der Laan, S. J. Andersen, N. Ramos García, N. Angelou, G. R. Pirsch, S. Ott, M. Sjöholm, K. H. Sørensen, J. X. Vianna Neto, M. Kelly, T. K. Mikkelsen, and G. C. Larsen. Power curve and wake analyses of the Vestas multi-rotor demonstrator. Wind Energy Science, 4(2):251–271, 2019.

[3] Peter Jamieson. Innovation in Wind Turbine Design. John Wiley & Sons, New York, 2018.

[4] S. Störtenbecker, P. Dalhoff, M. Tamang, and R. Anselm. Simplified support structure design for multi rotor wind turbine systems. https://wes.copernicus.org/preprints/wes-2020-46/, 2020.

[5] IEC 61400-1 Ed. 4, Wind energy generation systems - Part 1: Design requirements. Guideline, International Electrotechnical Commission (IEC), 2019.

[6] Christian Bak, Frederik Zahle, Robert Bitsche, Taeseong Kim, Anders Yde, Lars Christian Henriksen, Anand Natarajan, and Morten Hansen. Description of the dtu 10 mw reference wind turbine. DTU Wind Energy Report-I-0092, 5, 2013.

[7] Fraunhofer ISE. Stromgestehungskosten: Erneuerbare Energien. 2018.