Quantum Dilaton Gravity in Two Dimensions with Fermionic Matter

D. Grumiller and R. Meyer
Institut für Theoretische Physik, Universität Leipzig, Augustusplatz 10-11, D-04109 Leipzig, Germany
E-mail: grumil@lns.mit.edu, Rene.Meyer@itp.uni-leipzig.de

Abstract. Path integral quantization of generic two-dimensional dilaton gravity non-minimally coupled to a Dirac fermion is performed. After integrating out geometry exactly, perturbation theory is employed in the matter sector to derive the lowest order gravitational vertices. Consistency with the case of scalar matter is found and issues of relevance for bosonisation are pointed out.

PACS numbers: 04.60.Kz, 04.60.-m, 04.70.-s

1. Introduction

Two-dimensional gravity models retain many properties of their higher-dimensional counterparts but are considerably simpler and thus can be used as toy models for tackling the conceptual problems of quantum gravity while at the same time avoiding the technical difficulties that arise from the non-linear dynamics of gravity theories in higher dimensions. The most prominent of them, the Schwarzschild black hole, is not merely a toy model but of considerable interest for General Relativity in four (or higher) dimensions. Two-dimensional dilaton gravities [1] (cf. also [2] for earlier reviews), being not only classically integrable but even at the (non-perturbative) quantum level, cf. e.g. [3–8], especially fit that purpose, such that it is possible to discuss topics like background independence [9], the role of time [10] and the scattering on virtual black holes [11].

In this work we investigate the first order formulation [12] of two-dimensional dilaton gravities coupled to fermions in the “Vienna School approach” [1, 13]. Such models were first considered in the second order formalism classically in [14] and used later on in studies [15, 16] of the evaporation of charged CGHS black holes [17]. Interesting by itself because they have been a blind spot in the literature on quantum dilaton gravity until now, the main motivation of our work is to provide the grounds for an investigation of bosonisation, i.e. the quantum equivalence between the massive Thirring model and the Sine-Gordon model [18], in the context of quantum dilaton gravity. Originally, this equivalence has been stated for the corresponding field theories
on two-dimensional Minkowski space and used in recent studies [19–21] of charged black hole evaporation in two-dimensional dilaton gravity electrodynamics, i.e. on a fixed background with a quantized matter sector, where it is applicable in regions of small curvature (compared to the intrinsic length scale of the quantum theory). As a non-perturbative quantization of two-dimensional dilaton gravity theories coupled to scalar matter is already available [4–6], the question arises whether and how bosonisation carries over to the quantum gravity regime. A necessary prerequisite for answering this question is to perform the same constraint analysis [22] and – the main topic of this work – exact path integral quantization analogously to the scalar case. In a next step one can then compare the physical observables (e.g. S-matrix elements) on both sides of the correspondence. The underlying rationale is to integrate out geometry without split into background and fluctuations. The ensuing effective theory will be non-local and non-polynomial in matter degrees of freedom and can be studied with standard perturbation theory. To each order all gravitational backreactions are included automatically in a self-consistent way.

This paper is organised as follows: In section 2 the starting point, dilaton gravity in two dimensions with fermionic matter, is provided and our notation is introduced. Section 3 is devoted to a Hamiltonian analysis of constraints, the construction of the BRST charge and the gauge fixing procedure. The path integral quantization of geometry is performed non-perturbatively in section 4. Some applications of the general results are given in section 5, in particular the derivation of four-point vertices. We conclude in section 6 with a study of conformal properties, comments on bosonisation and an outlook to further possible applications.

2. Two-Dimensional Gravity with Fermions

Our starting point is the two-dimensional (2D) action

\[ S = S^{(1)} + S^{(\text{kin})} + S^{(\text{SI})} \]  

which comprises the first order action of 2D dilaton gravity,

\[ S^{(1)} = \int_{M^2} \left[ X_a T^a + X \mathcal{R} + \epsilon \mathcal{V}(X^a X_a, X) \right] , \]

the Dirac action\(^1\)

\[ S^{(\text{kin})} = \frac{i}{2} \int_{M^2} f(X) (\ast e^a) \wedge (\chi \gamma_a \overleftarrow{d} \chi) , \]

and fermion self-interactions

\[ S^{(\text{SI})} = - \int_{M^2} \epsilon h(X) g(\chi \overline{\chi}) . \]

The reader familiar with the notation used in [22] may skip the rest of this section which is devoted to detailed explanations of the latter and also provides a

\(^1\) The sign difference between [22] and this article for \(S^{(\text{kin})}\) stems from the differing sign of \(\epsilon^{ab}\).
brief recollection of some well-known results in 2D dilaton gravity. For background information and additional references the extensive review [1] may be consulted.

### 2.1. Notations and conventions

We collect first all notations relevant for the geometric action (2). To formulate this first order action one needs to introduce *Cartan* variables: \( e^a = e^a_{\mu} dx^\mu \) is the dyad one-form dual to \( e_{a} = e^\mu_{a} \partial_\mu \), i.e. \( e^a(e_b) = \delta^a_b \). Latin indices refer to an anholonomic frame, Greek indices to a holonomic one. The Levi-Civita tensor is given by

\[
\eta_{\mu\nu} = \det e^a_{\alpha} \tilde{\epsilon}_{\mu\nu} \quad \text{with} \quad \tilde{\epsilon}_{01} = +1.
\]

For calculations it is often convenient to express everything in light-cone gauge for the flat metric \( \eta_{ab} \), \( \eta^{+} - \eta^{-} = 1 = \eta^{-} + \eta^{+} \). (5)

The volume 2-form may be presented as

\[
\epsilon = -\frac{1}{2} \varepsilon_{ab} e^a \wedge e^b = e^+ \wedge e^- = (e) d^2 x, \quad (e) := \det e^a_{\mu} = e^+_0 e^-_1 - e^-_0 e^+_1, \quad (6)
\]

with the totally antisymmetric Levi-Civita symbol in tangent space \( \varepsilon_{ab} \) defined with the same sign as \( \tilde{\epsilon}_{\mu\nu} \). Consequently, \( \varepsilon^a_{b} \) is simply given by \( \varepsilon^\pm = \pm \varepsilon_{10} \). The Hodge star acts on the dyad as \( *e^a = -\varepsilon_{ab} e^b \) and on the volume 2-form as \( *\epsilon = 1 \). With these conventions the hermitian conjugate of the exterior derivative \( [23] \) reads \( d^\dagger = *d* \).

The one-form \( \omega \) represents the spin connection \( \omega^a_{\ b} = \varepsilon^a_{b} \omega \). The torsion two-form in light-cone gauge for the anholonomic frame is given by

\[
T^\pm = (d \pm \omega) \wedge e^\pm. \quad (7)
\]

The curvature two-form \( R^a_{\ b} \) can be represented by the two-form \( R \) defined by \( R^a_{\ b} = \varepsilon^a_{b} R \),

\[
R = d\omega. \quad (8)
\]

The Cartan variables, \( e^\pm \) and \( \omega \), are the gauge field 1-forms entering the action \[2\], together with their “field strengths” \[4\] and \[8\].

The fields \( X, X^a \) (or in light-cone gauge \( X, X^\pm \)) are zero forms and may be interpreted as Lagrange multipliers for curvature and torsion, respectively. The quantity \( \mathcal{V}(X^a X_a, X) \) is an arbitrary function of Lorentz invariant combinations of these Lagrange multipliers. Actually, for most practical purposes the potential takes the simpler form\(^2\)

\[
\mathcal{V}(X^a X_a, X) = X^+ X^- U(X) + V(X). \quad (9)
\]

The functions \( U, V \) are the crucial input defining the geometric part of the model, and several examples are presented below. The scalar field \( X \) is called “dilaton field” for reasons pointed out in the next subsection.

\(^2\) To the best of our knowledge the only exception appearing in the literature is the class of dilaton-shift invariant models introduced in [24] with \( \mathcal{V}(X^+X^-, X) = XU(X^+X^-/X^2) \).
We present now the missing pieces of notation required to comprehend the matter actions (3) and (4). Let us start with the Dirac matrices in 2D Minkowski space \((\gamma^\pm = (\gamma^0 \pm \gamma^1)/\sqrt{2})\)

\[
\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^+ = \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}, \quad \gamma^- = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}.
\] (10)

The analogue of the \(\gamma^5\) matrix is \(\gamma^* = \gamma_0 \gamma_1 = \text{diag}(+-)\). They satisfy \(\{\gamma^a, \gamma^b\} = 2\eta^{ab}\) and \(\{\gamma^*, \gamma^a\} = 0\). The Dirac conjugate is defined in the usual way, \(\bar{\chi} = \chi^\dagger \gamma^0\). For calculations in Euclidean space \(\gamma^0\) is defined as above, but \(\gamma^1 = \text{diag}(+-)\) and \(\gamma^* = \gamma_0 \gamma_1\), thus satisfying \(\{\gamma^a, \gamma^b\} = 2\delta^{ab}\). The Dirac matrices in Euclidean space are hermitian, \(\gamma^a = \gamma^a\dagger\), whereas \(\gamma^*\) becomes anti-hermitian. The derivative action on both sides in (3) is defined as \(\rightarrow \partial \leftarrow b = a\partial b - (\partial a)b\).

The functions \(f(X)\) and \(h(X)\) entail the coupling to the dilaton. If they are constant the fermions are called minimally coupled, and non-minimally coupled otherwise. Because of the Grassmann property of the spinor field the self-interaction \(g(\bar{\chi}\chi)\) may be Taylor expanded as

\[g(\bar{\chi}\chi) = c + m\bar{\chi}\chi + \lambda(\bar{\chi}\chi)^2.\] (11)

The constant contribution \(c\) can be absorbed into \(V(X)\). Thus, there is only a mass term (if \(m \neq 0\)) and a Thirring term (if \(\lambda \neq 0\)). Also, because there is only one generator of the Lorentz group, the fermion kinetic term does not include coupling to the spin connection, which together with the requirement that matter does not couple to the Lagrange multipliers for torsion \(X^a\) is crucial for the equivalence between the first and second order theories with matter [25] and, as will be seen below, simplifies the constraint structure significantly.

The action (1) depends functionally on the fields \(X, X^\pm, \omega, e^\pm\) and \(\chi\). Due to the presence of gauge symmetries the only propagating physical degree of freedom in our model is the two-component spinor \(\chi\). It should be noted that although in general the addition of fermions (as well as other matter fields) destroys classical integrability of first order gravity, some special cases still can be treated exactly, for instance chiral fermions [26].

### 2.2. Some properties and examples of 2D dilaton gravity

The action (2) is equivalent to the frequently used second order action [27]

\[S^{(2)} = -\frac{1}{2} \int_{\mathcal{M}_2} \, \mathrm{d}^2 x \, \sqrt{-g} \left[ X R + U(X) \left( \nabla X \right)^2 - 2V(X) \right],\] (12)

with the same functions \(U, V\) as in (3). The curvature scalar\(^3\) \(R\) and covariant derivative \(\nabla\) are associated with the Levi-Civita connection related to the metric \(g_{\mu\nu}\), the

\(^3\) The sign of the curvature scalar \(R\) has been fixed conveniently such that \(R > 0\) for \(dS_2\). This is the only difference to the notations used in ref. [1].
Quantum Dilaton Gravity in Two Dimensions with Fermionic Matter

determinant of which is denoted by $g$. If $\omega$ is torsion-free $R = -2 * R$. In the presence of boundaries a York–Gibbons–Hawking-like boundary term has to be added to the actions (2) and (12), the precise form of which depends on what kind of variational principle one would like to employ [28]. In the present work boundaries will not be considered and hence all boundary terms will be dropped. Since (12) is a standard dilaton gravity action as encountered e.g. in low energy effective descriptions of string theory, the nomenclature “dilaton” for the field $X$ is evident. In this context it should be mentioned that often the string dilaton field $\phi$ is employed, with

$$X = e^{-2\phi}.$$  \hfill (13)

This brings (12) into the well-known form

$$S^{(2)} = -\frac{1}{2} \int_{M_2} d^2 x \sqrt{-g} e^{-2\phi} \left[ R + \hat{U}(\phi) (\nabla \phi)^2 + \hat{V}(\phi) \right],$$  \hfill (14)

where the new potentials $\hat{U}, \hat{V}$ are related to the old ones via

$$\hat{U} = 4e^{-2\phi}U(e^{-2\phi}), \quad \hat{V} = -2e^{2\phi}V(e^{-2\phi}).$$  \hfill (15)

Two prominent examples are the Witten black hole [17, 29] with

$$U(X) = -\frac{1}{X}, \quad V(X) = -2b^2 X, \quad \Rightarrow \hat{U}(\phi) = -4, \quad \hat{V} = +4b^2,$$  \hfill (16)

and the Jackiw-Teitelboim [30] model with

$$U(X) = 0, \quad V(X) = \frac{\Lambda}{2} X, \quad \Rightarrow \hat{U}(\phi) = 0, \quad \hat{V} = -\Lambda.$$  \hfill (17)

Other models are summarised in table 1.

Although first order gravity (2) is not conformally invariant, dilaton dependent conformal transformations

$$X^a \mapsto \frac{X^a}{\Omega}, \quad e^a \mapsto e^a \Omega, \quad \omega \mapsto \omega + X_a e^a \frac{d \ln \Omega}{dX}$$  \hfill (18)

with a conformal factor $\Omega = \exp \left[ \frac{1}{2} \int (U(y) - \hat{U}(y)) dy \right]$ map a model with potentials $(U(X), V(X))$ to one with $(\hat{U}(X), \hat{V}(X) = \Omega^2 V(X))$. Thus one can always transform to a conformal frame with $\hat{U} = 0$, which considerably simplifies the classical equations of motion, do calculations there and afterwards transform back to the original conformal frame. The expression

$$w(X) = \int_{X}^{X} e^{Q(y)} V(y) dy$$  \hfill (19)

is invariant under conformal transformations, whereas

$$Q(X) = \int_{X}^{X} U(y) dy$$  \hfill (20)
It turns out that there is an absolute (in space and time) conserved quantity,

\[ \mathcal{C}^{(g)} = e^{Q(X)} X^+ X^- + w(X), \quad d\mathcal{C}^{(g)} = 0, \tag{21} \]

which has been found in previous second order studies of dilaton gravity [49]. This local gravitational mass is nothing but the Misner-Sharp mass [50] for spherically reduced gravity. It also exists for first order gravity coupled to matter fields [51, 52], where the conservation law \( d(\mathcal{C}^{(g)} + \mathcal{C}^{(m)}) = 0 \) receives a matter contribution.

### 3. Constraint and BRST Analysis

In this section we will briefly review the constraint structure of our model (for details see [22]) and then obtain the BRST charge needed to construct the gauge fixed and
Quantum Dilaton Gravity in Two Dimensions with Fermionic Matter

BRST-invariant Hamiltonian, which in turn then is the starting point for path integral quantization of \( \mathcal{H} \). The constraint analysis for the special case of massless, non self-interacting and minimally coupled fermions was carried out in [53].

We will frequently denote the canonical coordinates and momenta by

\[
\overline{q}^i = (\omega_0, e_0^+, e_0^-), \quad \overline{q}^i = (\omega_1, e_1^+, e_1^-) \quad p_i = (X, X^+, X^-),
\]

with \( i = 1, 2, 3 \) and \( (\alpha = 0, 1, 2, 3) \)

\[
Q^\alpha = (\chi_0, \chi_1, \chi_0^*, \chi_1^*).
\]

The graded Poisson bracket is the usual one, i.e. \( \{q^i, p_j^f\} = \delta^j_i \delta(x^1 - x^{1'}) \) for the bosonic variables and \( \{Q^\alpha, P_\beta^f\} = -\delta_\beta^\alpha \delta(x^1 - x^{1'}) \) for the fermionic ones. The prime denotes evaluation at \( x^{1'} \), whereas quantities without prime are evaluated at \( x^1 \).

First order gravity has three gauge degrees of freedom, the local \( SO(1, 1) \) symmetry and two non-linear symmetries which on-shell correspond to space-time diffeomorphisms. Thus the system is anticipated to possess a corresponding number of first class constraints in the Hamiltonian formulation of the theory. Because (22) does not contain time (i.e. \( x^0 \)) derivatives of the \( \overline{q}^i \), their momenta \( \overline{p}_i \approx 0 \) weakly vanish. These are three additional first class constraints appearing in the Hamiltonian formalism of the first order theory (22) which generate shifts of the \( \overline{q}^i \) in the extended phase space that includes the pair \( (\overline{q}^i, \overline{p}_i) \).

Because the fermion kinetic term (23) is of first order in the derivatives, the momenta \( P_\alpha = \partial^L \mathcal{L}/\partial \dot{Q}^\alpha \) conjugate to the \( Q^\alpha \) yield second class constraints

\[
\Phi_0 = P_0 + \frac{i}{\sqrt{2}} f(p_1) q^3 Q^2 \approx 0, \quad \Phi_1 = P_1 - \frac{i}{\sqrt{2}} f(p_1) q^2 Q^3 \approx 0,
\]

\[
\Phi_2 = P_2 + \frac{i}{\sqrt{2}} f(p_1) q^3 Q^0 \approx 0, \quad \Phi_3 = P_3 - \frac{i}{\sqrt{2}} f(p_1) q^3 Q^1 \approx 0.
\]

To reduce the phase space to the surface defined by the second class constraints without explicitly solving them we introduce the Dirac bracket [54, 55]

\[
\{f(x), g(y)\}^* := \{f, g\} - \int dz dw \ \{f(x), \Phi_\alpha(z)\} C^{\alpha\beta}(z, w) \{\Phi_\beta(w), g(y)\}.
\]

The matrix-valued distribution

\[
C^{\alpha\beta}(x, y) = \frac{i}{\sqrt{2f(X)}} \begin{pmatrix}
0 & 0 & \frac{1}{e_1^+} & 0 \\
0 & 0 & 0 & -\frac{1}{e_1^+} \\
\frac{1}{e_1^-} & 0 & 0 & 0 \\
0 & -\frac{1}{e_1^-} & 0 & 0
\end{pmatrix} \delta(x^1 - y^1)
\]

is the inverse of the Dirac matrix \( C_{\alpha\beta}(z, w) = \{\Phi_\alpha(z), \Phi_\beta(w)\} \), viz.

\[
C_{\alpha\beta}(x, y) = i\sqrt{2f(X)} \begin{pmatrix}
0 & 0 & -e_1^+ & 0 \\
0 & 0 & 0 & e_1^- \\
-e_1^+ & 0 & 0 & 0 \\
0 & e_1^- & 0 & 0
\end{pmatrix} \delta(x^1 - y^1).
\]
Requiring constancy of the primary first class constraints under time evolution yields secondary first class constraints \( G_i = \{\bar{\Phi}_i, \mathcal{H}'\}^* \approx 0 \) which explicitly read
\[
\begin{align*}
G_1 &= G_1^q \\
G_2 &= G_2^q + \frac{i}{\sqrt{2}} f(X) (\chi_1^* \bar{\partial}_1 \chi_1) + e^+_1 h(X) g(\bar{\chi} \chi) \\
G_3 &= G_3^q - \frac{i}{\sqrt{2}} f(X) (\chi_0^* \bar{\partial}_1 \chi_0) - e^-_1 h(X) g(\bar{\chi} \chi).
\end{align*}
\]  

The constraints of the matterless theory [3] are
\[
\begin{align*}
G_1^q &= \partial_1 X + X^- e^+_1 - X^+ e^-_1 \\
G_2^q &= \partial_1 X^+ + \omega_1 X^+ - e^+_1 \mathcal{V} \\
G_3^q &= \partial_1 X^- - \omega_1 X^- + e^-_1 \mathcal{V}.
\end{align*}
\]

The Lagrangian and Hamiltonian densities of the combined system \((2), (3)\) and \((4)\) are related by Legendre transformation
\[
H = \oint d^1q \mathcal{H} = -\oint d^1q G_i \approx 0
\]
vanishes on the constraint surface, as expected for a generally covariant system [55], and the \( \bar{\Phi}^* \) serve as Lagrange multipliers for the secondary constraints. They form an algebra [22]
\[
\begin{align*}
\{G_i, G_j'\}^* &= 0, \quad i = 1, 2, 3, \\
\{G_1, G_2'\}^* &= -G_2 \delta(x^1 - x'^1), \\
\{G_1, G_3'\}^* &= G_3 \delta(x^1 - x'^1), \\
\{G_2, G_3'\}^* &= \left[ -\sum_{i=1}^3 \frac{dY}{dp_i} G_i + \left( gh' - \frac{h}{f} f' g' \cdot (\bar{\chi} \chi) \right) \right] \delta(x^1 - x'^1),
\end{align*}
\]
and are of first class because the right hand sides vanish weakly. The \( G_i \) are preserved under time evolution, \( \dot{G}_i = \{G_i, \mathcal{H}'\}^* = -\bar{\Phi}'^* \{G_i, G_j'\}^* \approx 0 \), so no ternary constraints are generated. The algebra closes with \( \delta \)-functions, resembling rather an ordinary gauge theory or Ashtekar’s approach to gravity [56] than the ADM approach [57] whose Hamiltonian and diffeomorphism constraints are recovered by linear combinations of the \( G_i \) and fulfil the classical Virasoro algebra [58]. The case of minimally coupled massless fermions [53] without self-interaction is reproduced. If the dilaton couplings \( f, h \) are proportional to each other, only a Thirring term \( \lambda (\bar{\chi} \chi)^2 \) contributes to the last term in (38). In contrast to the matterless case [59] the algebra generated by the \( G_i \) and \( p_i \) is no finite W-algebra [60] anymore (for a proof, cf. sec. 2.2.2 in [61]).

In order to obtain a gauge fixed Hamiltonian density, we follow the method of Batalin, Vilkovisky and Fradkin [62] and first construct the BRST charge \( \Omega \). With three gauge symmetries generated by the \( G_i \) the phase space has to be enlarged by three pairs of ghosts and antighosts \((c_i, p_i^c)\) and equipped with a Poisson structure
obeying the same (anti)commutation relations for the \( q^i, p_i, Q^\alpha, P_\alpha \) as above together with \( \{ c^i, p^j \} = -\delta^i_j \delta(x^1 - x'^1) \) for the ghost sector. The Dirac bracket is still defined as in \[25\], but with the new Poisson structure. The BRST charge has to fulfil four requirements: First it has to act on functions on the enlarged phase space through the Dirac bracket, \( \Omega F(q, p, Q, P, c, p^c) := \{ \Omega, F \}^* \). Second, it has to be nilpotent, \( \Omega^2 F = 0 \), which by virtue of the Jacobi identity is equivalent to

\[
\{ \Omega, \Omega \}^* = 0 .
\]

Third, it should act on functions on the non-extended phase space as gauge transformations, i.e. through \( G_i \) and, fourth, is required to have ghost number one, which leads to the Ansatz \( \Omega = c^i G_i + \text{higher ghost terms} \). Constructed in this way it is unique up to canonical transformations of the extended phase space \[55\]. Evaluating \[39\] yields \( (C_{ij}^k)^* \) are the structure functions of the algebra \( \{ G_i, G_j \} = C_{ij}^k G_k \delta(x^1 - x'^1) \)

\[
\Omega = c^i G_i + \frac{1}{2} c^i c^j C_{ij}^k p_k^c .
\]

The homological perturbation series terminates at Yang-Mills level, i.e. \[39\] holds for \[40\] without the necessity of introducing higher order ghost terms. BRST invariant functionals with total ghost number zero are then sums of a BRST closed and a BRST exact part \[63\]. The gauge fixed Hamiltonian should be BRST invariant and thus is of form

\[
\mathcal{H}_{gf} = \mathcal{H}_{BRST} + \{ \Omega, \Psi \}^* .
\]

Choosing the gauge fixing fermion \[6\]

\[
\Psi = p_c^2
\]

and \( \mathcal{H}_{BRST} = 0 \) yields the gauge fixed Hamiltonian density

\[
\mathcal{H}_{gf} = \{ \Omega, \Psi \}^* = -G_2 - C_{2i}^k c^i p_k^c
\]

in Eddington-Finkelstein (or Sachs-Bondi) gauge

\[
(\omega_0, e_0^-, e_0^+) = (0, 1, 0) .
\]

The gauge fixed Lagrangian

\[
\mathcal{L}_{gf} = \dot{Q}^a P_\alpha + \dot{q}^i p_i + G_2 + p_c^k M^k_{\ i} c^i
\]

contains the Faddeev-Popov operator

\[
M = \begin{pmatrix}
\partial_0 & 0 & \frac{\partial \nu}{\partial X} - \left( gh' - \frac{h}{f} g' \cdot (X \chi) \right) \\
-1 & \partial_0 & \frac{\partial \nu}{\partial X^+} \\
0 & 0 & \partial_0 + \frac{\partial \nu}{\partial X}
\end{pmatrix} .
\]
4. Integrating out Geometry Non-perturbatively

In this section we will perform the path integration over the (anti)ghosts \((c^i, p^c_i)\) and the geometric variables \((q^i, p_i)\) non-perturbatively. We introduce external sources for the latter and the fermion,

\[
\mathcal{L}_{\text{src}} = J^i p_i + j_i q^i + \bar{\eta} \chi + \bar{\chi} \eta.
\]

The generating functional of Green functions is formally given by the path integral with the action (45) and (47) \((N\) is a normalisation factor)

\[
Z[J, j, \eta, \bar{\eta}] = N \int \mathcal{D} \mu \left[ \mathcal{Q}, \mathcal{P}, q, p, c, p^c \right] \exp \left( i \int d^2 x \left( \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{src}} \right) \right)
\]

and the measure

\[
\mathcal{D} \mu \left[ \mathcal{Q}, \mathcal{P}, q, p, c, p^c \right] = \mathcal{D} \chi \mathcal{D} \bar{\chi} \prod_x \frac{1}{|q^3|^2} \prod_{i=1}^3 \mathcal{D} p_i \mathcal{D} q^i \prod_{i=0}^3 \mathcal{D} p_i \delta(\Phi_i) \prod_{j=1}^3 \mathcal{D} c^j \mathcal{D} p^c_j.
\]

The delta functional in the measure restricts the integration to the surface defined by the second class constraints [55]. The local measure factor for the fermion integration has been chosen such that general covariance is retained in the quantum theory [64].

From the point of view of the phase space path integral it is for minimal coupling \((f(X) = 1)\) composed of a well-known [65] factor \([-g^{00}] = 2q^2/q^3\) and a factor \(\sqrt{\text{sdet} C_{\alpha\beta}} = (\text{det} C_{\alpha\beta})^{-1/2} = (4(q^2 q^3)^2)^{-1/2}\), where the latter results from rewriting the path integral over the surface of second class constraints as a path integral over the whole phase space [55, 66]. For non-minimal coupling the question of which measure is the “right” covariant one is subtle and still not completely settled (for a review cf. e.g. [67]).

Integrating over the ghost sector yields the functional determinant of the Faddeev-Popov operator [43],

\[
\Delta_{\phi^\Pi} = \text{Det} \left( \partial_0^2 (\partial_0 + U(X) X^+) \right),
\]

which will be cancelled during the \(p_i\)-integration later on. Integration of the fermion momenta \(P_i\) is trivial because of the delta functionals in [43] and the \(P_i\)-linearity of the second class constraints [24], and yields an effective Lagrangian

\[
\mathcal{L}_{\text{eff}}^{(1)} = p_i \dot{q}^i + G_2 + \frac{i}{\sqrt{2}} f(p_1) \left[ q^3 (\chi^*_0 \dot{\partial}_0 \chi_0) - q^2 (\chi^*_1 \dot{\partial}_0 \chi_1) \right] + \mathcal{L}_{\text{src}}.
\]

Equation (51) is linear in the \(q^i\), and without the non-linearity introduced by the covariant matter measure in [43] the \(q^i\)-integral could be evaluated immediately. We thus replace [5] the measure factor \([q^3]^{-2}\) by introducing a new field \(F\),

\[
Z[J^i, j_i, \eta, \bar{\eta}] = N \int \mathcal{D} F \delta \left( F - \frac{1}{i \delta J^3} \right) \tilde{Z}
\]

\[
\tilde{Z}[F, J^i, j_i, \eta, \bar{\eta}] = \int \mathcal{D} \chi \mathcal{D} \bar{\chi} \prod_x F^{-2} \mathcal{D} p_i \mathcal{D} q^i \Delta_{\phi^\Pi} \exp \left( i \int d^2 x \mathcal{L}_{\text{eff}}^{(1)} \right).
\]
Now we use the $q^i$-linearity of (51) (with (29) and (32)) which upon functional integration yields three $\delta$-functionals containing partial differential equations for the $p_i$,

\begin{align}
\partial_0 p_1 &= j_1 + p_2 \\
\partial_0 p_2 &= j_2 - \frac{i}{\sqrt{2}} f(p_1)(\chi^*_1 \partial_0 \chi_1) \\
(\partial_0 + U(p_1)p_2)p_3 &= j_3 + \frac{i}{\sqrt{2}} f(p_1)(\chi^*_0 \partial_0 \chi_0) + h(p_1)g(\bar{\chi}\chi) - V(p_1). 
\end{align}

Performing the $p_i$-integration now amounts to solving these equations for given currents $j_i$ and matter fields and substituting the solutions $p_i = \hat{B}_i$ back into the effective action obtained after $q^i$-integration, yielding an effective Lagrangian

$$L^{(2)}_{\text{eff}} = j^i \hat{B}_i + \bar{\eta}\chi + \chi\eta + \frac{i}{\sqrt{2}} f(\hat{B}_1)(\chi^*_1 \partial_1 \chi_1).$$

During this integration the Faddeev-Popov determinant (50) cancels, because the differential operators on the right hand sides of (54)-(56) combine to a factor $\text{Det}(\partial_0^2 + \partial_0 U(X)X^-)^{-1}$.

Equations (54) and (55) can, for general non-minimal couplings $f(p_1)$, be solved order by order in the weak matter approximation [61], i.e. for matter configurations with total energy several orders of magnitude below the Planck scale. The most common case of non-minimal coupling is the linear one, $f(X) = -X$, arising from spherical reduction of in four dimensions minimally coupled matter [69]. In the weak matter approximation it can be solved with the Ansatz

$$p_i = \hat{B}_i = \sum_{n=0}^{\infty} p_i^{(n)} \quad i = 1, 2,$$

i.e. assuming $p_1^{(n)}$ to be of order $n$ in fermion bilinears $\chi^*_1 \partial_0 \chi_1$. The vacuum solutions $p_1^{(0)}$ are given by (63) and (64) below with $\kappa = 0$, and the higher order terms by the recursion relations

\begin{align}
p_2^{(n)} &= \frac{i}{\sqrt{2}} \nabla_0^{-1} \left( (\chi^*_1 \partial_0 \chi_1) p_1^{(n-1)} \right) \quad n \geq 1 \\
p_1^{(n)} &= \nabla_0^{-1} p_2^{(n)}. 
\end{align}

With

$$\hat{Q}(\hat{B}_1, \hat{B}_2) = \nabla_0^{-1}(U(\hat{B}_1)\hat{B}_2)$$

the third equation (56) is solved by

$$p_3 = \hat{B}_3 = e^{-\hat{Q}(\hat{B}_1, \hat{B}_2)} \left[ \nabla_0^{-1} e^{\hat{Q}(\hat{B}_1, \hat{B}_2)} \left( j_3 - V(\hat{B}_1) + h(\hat{B}_1)g(\bar{\chi}\chi) \\
+ \frac{i}{\sqrt{2}} f(\hat{B}_1)(\chi^*_0 \partial_0 \chi_0) \right) + \hat{p}_3 \right].$$
For minimal coupling $f(p_1) = -\kappa = \text{const.}$ the solution even can be given non-perturbatively,

$$\begin{align*}
P_1 &= \hat{B}_1 = \nabla_0^{-1}(j_1 + \hat{B}_2) + \tilde{p}_1 \\
P_2 &= \hat{B}_2 = \nabla_0^{-1}
\left(j_2 + \kappa \frac{i}{\sqrt{2}}(\chi_1 \overleftrightarrow{\partial_0} \chi_1)\right) + \tilde{p}_2.
\end{align*}
$$

The quantities $\tilde{p}_i$ are homogeneous solutions of $\nabla_0 \tilde{p}_i = 0$ with the regularised time derivative $\nabla_0 = \partial_0 - i(\mu - i\varepsilon) = \partial_0 - i\tilde{\mu}$, where we applied the regularisation prescription of [5]. The integral operator $\nabla_0^{-1}$ is the Green function of $\nabla_0$. This regularisation provides proper infrared and asymptotic behaviour of the Green function. In the next section we will however use another strategy [68] to obtain the lowest order interaction providing proper infrared and asymptotic behaviour of the Green function. In the next section we will however use another strategy [68] to obtain the lowest order interaction vertices by directly imposing boundary conditions and solving the equations (57)–(58), such that no additional regularisation is necessary.

Equation (67) as it stands is not the whole effective action, but has to be supplemented with ambiguous terms [5, 68]. These arise from the source terms $J^i \hat{B}_i$ in the following way: In expressions like $\int J \nabla^{-1} A$ the inverse derivative acts after changing the order of integration on the source $J$, giving rise to another homogeneous contribution $\int \tilde{g}A$, while the homogeneous functions in $A$ have been made explicit already in the solutions $\hat{B}_i$. Thus the action gets supplemented by three terms

$$\begin{align*}
\mathcal{L}_{\text{amb}} &= \sum_{i=1}^{2} \tilde{g}_i K_i(\nabla_0^{-1}, (\chi_1 \overleftrightarrow{\partial_0} \chi_1), j_1, j_2) \\
&\quad + \tilde{g}_3 e^Q \left(j_3 - V(\hat{B}_1) + h(\hat{B}_1)g(\chi_0) + f(\hat{B}_1)\frac{i}{\sqrt{2}}(\chi_0 \overleftrightarrow{\partial_0} \chi_0)\right).
\end{align*}
$$

The expressions $K_i$ can be read off from the solutions $\hat{B}_{1/2}$ up to the desired order in matter contributions. The homogeneous solutions $\tilde{g}_i$ are fixed by asymptotic conditions on the expectation values of the Zweibeine. For instance if $\tilde{g}_3 = 1$ then

$$\langle e_1^+ \rangle = \frac{1}{i\delta j_3} e^{i\int d^2x(\mathcal{L}_{\text{eff}} + \mathcal{L}_{\text{amb}})} \bigg|_{j_1 = j_2 = 0} = e^{\hat{Q}((\hat{B}_1|_{j_1 = j_2 = 0})}
$$

is just the correct asymptotic expression $e^Q = \sqrt{-g}$ in Eddington-Finkelstein gauge, cf. (68) below, if the fermion field obeys an appropriate fall-off condition. That these ambiguous terms are necessary and can not be omitted can also be seen from (57), which is independent of the source $j_3$. Because of the measure factor $F^{-2}$ in (52), the generating functional (52) and (53) would be ill-defined without the last term in (65). Also, for the special case of the Katanaev-Volovich model without matter the integration can be carried out in the “natural” order [70], i.e. first over $p_i$ and then over $q^i$, while never introducing sources $J^i$, yielding an effective action exactly of the type of the $\tilde{g}_3$-term in (65).

Thus after integrating out the whole ghost and geometric sector, the partition function reads ($\bar{\chi} = \sqrt{F} \chi$)

$$Z[J^i, j_i, \eta, \bar{\eta}] = \mathcal{N} \int \mathcal{D}F \delta \left(F - \frac{1}{i\delta j_3}\right) \int \mathcal{D}\bar{\chi} \mathcal{D}\chi \exp \left(i \int d^2x(\mathcal{L}_{\text{eff}}^{(2)} + \mathcal{L}_{\text{amb}})\right),
$$
Quantum Dilaton Gravity in Two Dimensions with Fermionic Matter

with $\mathcal{L}_{\text{eff}}^{(2)}$ from (57) and $\mathcal{L}_{\text{amb}}$ from (65). It should be emphasised that $Z$ includes all gravitational backreactions, because the auxiliary field $F$ upon integration is equivalent to the quantum version of $e_1^+$. 

5. Matter Perturbation Theory

The remaining matter integration in (67) is carried out perturbatively. One first splits the effective action (57) and (65) into terms independent of the fermions, in those quadratic in the spinor components and in higher order terms summarised in an interaction Lagrangian $\mathcal{L}_{\text{int}}$. The solutions of (54) and (55) up to quadratic fermion terms are for general non-minimal coupling ($B_{1/2}$ are the zeroth order solutions (63) and (64) with $\kappa = 0$)

$$\hat{B}_1 = B_1 - \frac{i}{\sqrt{2}} \nabla_0^{-2}(f(B_1)(\chi_1^* \partial_0 \chi_1)) + \mathcal{O}(\chi^4)$$

$$\hat{B}_2 = B_2 - \frac{i}{\sqrt{2}} \nabla_0^{-1}(f(B_1)(\chi_1^* \partial_0 \chi_1)) + \mathcal{O}(\chi^4).$$

Expanding4 (61),

$$\hat{Q}(\hat{B}_1, \hat{B}_2) = Q_x(B_1, B_2) - \frac{i}{\sqrt{2}} \int_y G_{xy} f(B_{1y})(\chi_1^* \partial_0 \chi_1)_y + \mathcal{O}(\chi^4)$$

$$G_{xy} = \int_z \nabla_0^{-1} [U_z B_{2z} \nabla_0^{-2} + U_z \nabla_0^{-1}],$$

and (62) yields

$$\hat{B}_{3x} = B_{3x} + \frac{i}{\sqrt{2}} \int_y H_{xy} f_y(B_1)(\chi_1^* \partial_0 \chi_1)_y + e^{-Q_x} \int_y \nabla_0^{-1} e^{Q_y} \left( \frac{i}{\sqrt{2}} f(B_1)(\chi_0^* \partial_0 \chi_0) - m h(B_1) \overline{\chi} \right)_y + \mathcal{O}(\chi^4)$$

$$H_{xy} = e^{-Q_x} \int_z \nabla_0^{-1} e^{Q_z} \left( [G_{xy} - G_{zy}](j_3 - V)_z + V_z' \nabla_0^{-2} \right) + \tilde{p}_{3x} e^{-Q_y} G_{xy}.$$

A similar expansion of the ambiguous terms (64) yields for the whole effective action

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}^{(0)} + \mathcal{L}_{\text{eff}}^{(2)} + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{\text{eff}}^{(0)} = J' B_1 + \tilde{g}_3 e^Q (j_3 - V(B_1)) + \tilde{g}_1 (j_1 + B_2) + \tilde{g}_2 j_2$$

$$\mathcal{L}_{\text{eff}}^{(2)} = \frac{i}{\sqrt{2}} f(B_1) \left[ (\chi_1^* \partial_1 \chi_1) - E_1 (\chi_1^* \partial_0 \chi_1) + F^{(0)}(\chi_0^* \partial_0 \chi_0) \right] + F^{(0)} h(B_1)m \overline{\chi} + \overline{\eta} \chi + \overline{\chi} \eta$$

4 Prime denotes differentiation with respect to the argument of $U(B_1)$ and the space-time points where the functions are evaluated as well as the space-time integration variables are denoted in subscript.
Quantum Dilaton Gravity in Two Dimensions with Fermionic Matter

\[ E_{1x}^- = \int_y \left[ J_y^1 \nabla_{0yx}^{-2} + (J_y^2 + \tilde{g}_{1y}) \nabla_{0yx}^{-1} - J_y^3 H_{yx} \right. \\
\left. + \tilde{g}_{3y} e^{Q_y} (G_{yx}(j_3 - V)_y - V'_y \nabla_{0yx}^{-2}) \right] + \tilde{g}_{2x} \]  

(77)

\[ E_{1x}^+(0) = e^{Q_x} \left[ \int_y J_y^3 e^{-Q_y} \nabla_{0yx}^{-1} + \tilde{g}_{3x} \right] =: F^{(0)}. \]  

(78)

One recognises in \( L_{\text{eff}}^{(2)} \) the kinetic term \( \tilde{L} \) of fermions on a curved background in Eddington-Finkelstein gauge \((44)\) with a background metric

\[ g_{\mu\nu} = F^{(0)} \begin{pmatrix} 0 & 1 \\ 1 & 2E_{1x}^- \end{pmatrix}. \]  

(79)

This background solely depends on sources \((j^i, J^i)\) for the geometric variables and the zeroth order solutions \(B_i\). We redefine the interaction part of the Lagrangian density such that the background \((79)\) depends on the full \(E_{1x}^+(x) = F(x) = \frac{\delta}{\delta j_3(x)} \int d^2 z L_{\text{eff}}(z)\) instead of its matter-independent part \(F^{(0)}\), i.e. take into account backreactions onto the metric determinant to all orders in the fermion fields. The generating functional\(^5\) then becomes

\[ \tilde{Z} = \exp \left( i \int d^2 x L_{\text{int}}^{(0)} + L_{\text{int}} \left[ \frac{1}{i} F^{-\frac{1}{2}} \frac{\delta L}{\delta \eta} \right] \right) \times \\
\times \int D\tilde{\chi} D\chi \exp \left( i \int d^2 x \frac{i}{2} f(B_1) \varepsilon_{ab} \varepsilon^{\mu
u} E^b_{\mu} (\bar{\chi} \gamma^\nu \partial_\nu \chi) + \bar{\eta} \chi + \bar{\chi} \eta \right). \]  

(80)

5.1. Vertices

In this section we are interested in the gravitationally induced scattering and thus set the sources for the geometric variables \(j^i, J^i\) to zero. For simplicity massless and non self-interacting fermions \(g(\overline{\chi}\chi) = 0\) are considered. The non-polynomial structure of the effective action \(L_{\text{eff}} = (57) + (65)\) gives rise to scattering vertices with arbitrary even numbers of external legs, which can in principle be extracted from the effective action by expanding it order by order in spinor bilinears. Because of the non-locality of the effective action such calculations are very cumbersome, so we will adopt a strategy first introduced in \([5]\) which is based on the local quantum triviality of first order gravity \([3]\), i.e. the absence of local quantum corrections from the geometric sector of the theory. This implies that the expectation values \(\langle q^i \rangle = \frac{\delta}{\delta j^i} S_{\text{eff}}\) and \(\langle p_i \rangle = \frac{\delta}{\delta J^i} S_{\text{eff}}\) fulfil the

\(^5\) There is one subtlety to address in connection with this interpretation of fermions on an effective background: The effective metric and spin connection both become complex if one chooses the regularisation of the inverse derivatives as in \([5]\), and the kinetic term of the fermions \((8)\) will depend on the imaginary part of the latter. The imaginary parts however are proportional to the regularisation parameter and thus vanish in the limit where the regularisation is removed. This subtlety plays no role for the method employed in subsection \(5.1\).
Quantum Dilaton Gravity in Two Dimensions with Fermionic Matter

equations of motion for the \((q^i, p_i)\) following from (51):

\[
\begin{align*}
\partial_0 p_1 &= p_2 \\
\partial_0 p_2 &= -2f(p_1) \Phi_0 \\
\partial_0 p_3 &= -U(p_1) p_2 p_3 - V(p_1) + 2f(p_1) \Phi_1 \\
\partial_0 q^1 &= q^3(U'(p_1) p_2 p_3 + V'(p_1)) + 2f'(p_1) [q^2 \Phi_0 - q^3 \Phi_1 - \Phi_2] \\
\partial_0 q^2 &= -q^1 + q^3 p_3 U(p_1) \\
\partial_0 q^3 &= q^3 p_2 U(p_1),
\end{align*}
\]

with off-shell matter contributions denoted by

\[
\Phi_0 = \frac{i}{2\sqrt{2}}(\chi_1 \partial_0 \chi_1), \quad \Phi_1 = \frac{i}{2\sqrt{2}}(\chi_0 \partial_0 \chi_0), \quad \Phi_2 = \frac{i}{2\sqrt{2}}(\chi_1 \partial_1 \chi_1).
\]

That this indeed holds can be seen from (54)-(56) with \(j_i = J^i = 0\) and \(g(\chi \chi) = 0\), which are just (81)-(83). From (57) and (65) three four-point vertices,

\[
V^{(4)} = \int \int [V_\xi(x, z) \Phi_0(x) \Phi_0(z) + V_\zeta(x, z) \Phi_0(x) \Phi_2(z) + V_c(x, z) \Phi_0(x) \Phi_1(z)],
\]

can be expected. Solving (81)-(86) with matter contributions

\[
\Phi_i(x) = c_i \delta^2(x - y), \quad i = 0, 1, 2
\]

localised at a space-time point \(y\) is possible. The crucial observation now is that inserting the solutions back into the interaction terms of (51), expanding up to \(O(c^2)\) and reading off the corresponding coefficients is equivalent to taking functional derivatives \(\delta^2 S_{\text{eff}} / \delta \Phi_i(x) \delta \Phi_j(y)\) and thus yields the correct four-point interaction vertices.

To solve the six first order differential equations (81)-(86) unambiguously [37] one fixes the six integration constants in the asymptotic region \(x^0 > y^0\) by imposing the following conditions:

- \(p_1 |_{x^0 > y^0} = x^0\) and \(p_2 |_{x^0 > y^0} = 1\), i.e. the dilaton is identified with the \(x^0\)-coordinate. This corresponds to a fixing of the residual gauge freedom [61]. Consequently the \(x^0\) direction is a “radial” direction and \(x^1\) corresponds to retarded time.
- \(C^{(g)} |_{x^0 > y^0} = C_\infty\) fixes the integration constant in \(p_3\), cf. (94) below.
- \(q_3 |_{x^0 > y^0} = c \delta^{(x^0)}\) then solves (86) and defines the asymptotic unit of length.
- The remaining two integration constants entering \(q^2 |_{x^0 > y^0}\), which is the solution of the second order partial differential equation (91), are called \(m_\infty\) and \(a_\infty\) because for spherical reduced gravity (cf. the fifth model in table 1) they correspond to the Schwarzschild mass and a Rindler acceleration.

The remaining three integration constants \(m_\infty, C_\infty\) and \(a_\infty\) are not independent from each other, because for describing a physical asymptotic region the solutions of (81)-(86) also have to fulfil the first order gravity constraints (81)-(83). The Lorentz constraint (31) then requires

\[
m_\infty = C_\infty, \quad a_\infty = 0.
\]
Equation (82) even yields a vacuum solution for the spin connection, which however is not necessary for finding the vertices, and (33) is fulfilled identically.

Another differentiation of (85) with respect to \(x^0\) and use of the other equations of motion yields a second order differential equation for \(q^2\),

\[
\partial_0^2 q^2 = -w''(p_1) - 2f'(p_1) [q^2\Phi_0 - q^3\Phi_1 - \Phi_2] + 2f(p_1)q^3U(p_1)\Phi_1. \tag{91}
\]

Solving (81)-(83), (86) and (91) in the vacuum regions \(x^0 \neq y^0\) and patching the solutions according to their continuity properties implied by (89) \((p_2, p_3, \partial_0q^2\) jumping at \(x^0 = y^0\) and \(p_1, q^3, q^2\) continuous\) yields with \(h_i = c_i\theta(y^0 - x^0)\delta(x^1 - y^1)\)

\[
p_1 = x^0 + 2f(y^0)(x^0 - y^0)h_0 \tag{92}
\]

\[
p_2 = 1 + 2f(y^0)h_0 \tag{93}
\]

\[
p_3 = e^{-Q(p_1)} \left[m_\infty - w(p_1) + 2f(y^0)h_0(w(x^0) - w(y^0)) - 2f(y^0)e^{Q(p_0)}h_1\right] \tag{94}
\]

\[
q^2 = m_\infty - w(p_1) + 2h_0 \left[2f(y^0)(w(x^0) - w(y^0)) + m_\infty f'(y^0) - (fw)'|_{y^0}(x^0 - y^0)\right]
- 2\left(f e^{Q(y^0)}\right)'(x^0 - y^0)h_1 - 2f'(y^0)(x^0 - y^0)h_2 \tag{95}
\]

\[
q^3 = e^{Q(p_1)}. \tag{96}
\]

From the last two equations the asymptotic line element

\[
ds^2 = 2e^{Q(w^0)}dx^1 [dx^0 + (m_\infty - w(x^0))dx^1] \tag{97}
\]

reveals that \(q^2|_{x^0 > y^0} = m_\infty - w(x^0)\) is the Killing norm, and its zeros correspond to Killing horizons.

Inserting the solutions (92)-(96) into the interaction terms of (81) (cf. also (87) and (89)),

\[
S_{\text{int}} = -2\int f(p_1) [q^2\Phi_0 - q^3\Phi_1 - \Phi_2], \tag{98}
\]

expanding up to \(\mathcal{O}(\epsilon^2)\), replacing the coefficients \(c_i\) with the fermion bilinears (87) and integrating over \(y\) yields three four-point vertices (depicted in figure 1), namely the symmetric one

\[
V_a = -4\int\int \Phi_0(x)\Phi_0(y)\theta(y^0 - x^0)\delta(x^1 - y^1)f(x^0)f(y^0) \times
\]

\[
\times \left[2(w(x^0) - w(y^0)) - (x^0 - y^0)(w'(x^0) + w'(y^0))
- (x^0 - y^0)\left(\frac{f'(x^0)}{f(x^0)}(w(x^0) - m_\infty) + \frac{f'(y^0)}{f(y^0)}(w(y^0) - m_\infty)\right)\right] \tag{99}
\]

and two non-symmetric ones

\[
V_b = -4\int\int \Phi_0(x)\Phi_2(y)\delta(x^1 - y^1)|x^0 - y^0|f(x^0)f'(y^0) \tag{100}
\]

\[
V_c = -4\int\int \Phi_0(x)\Phi_1(y)\delta(x^1 - y^1)|x^0 - y^0|f(x^0)(fe^{Q})'|_{y^0}. \tag{101}
\]
Interestingly, the vertices $V_a$ and $V_b$ are the same as for a real scalar field coupled to first order gravity [37], and only $V_c$ is new. They share some properties with the scalar case, namely

(i) They are local in the coordinate $x^1$, and non-local in $x^0$.
(ii) They vanish in the local limit ($x^0 \to y^0$) and $V_b$ vanishes for minimal coupling.
(iii) They respect a $\mathbb{Z}_2$ symmetry $f(X) \mapsto -f(X)$.
(iv) The symmetric vertex depends only on the conformally invariant combination $w(X)$ (cf. (19)) and the asymptotic value $m_\infty$ of the geometric part of the conserved quantity $\mathcal{Q}$. $V_b$ is independent of $U$, $V$ and $m_\infty$. Thus if $m_\infty$ is fixed in all conformal frames both vertices are conformally invariant.

By contrast, the new vertex $V_c$ is not conformally invariant.

5.2. Asymptotic Matter States

For calculation of the S-matrix one needs to determine the quantities $\Phi_i$ in (99)-(100) from the asymptotic matter states. They fulfil the Dirac equation

\[ \bar{D}_\chi = i E^a \gamma_a \nabla_\mu \chi = 0 , \quad \nabla_\mu = \partial_\mu - \frac{1}{2} \hat{\omega}_\mu \gamma_5 \]

on the asymptotic background (97). If they form a complete and (in an appropriate sense) normalisable set, an asymptotic Fock space can be constructed. The components of the “Dirac equation” obtained from (3) decouple,

\[ \partial_0 \chi_0 = - \frac{1}{2} \left( \frac{f'(x^0)}{f(x^0)} + U(x^0) \right) \chi_0 \]

\[ (\partial_1 - q^2(x^0) \partial_0) \chi_1 = \frac{1}{2} \left( \frac{f'(x^0)}{f(x^0)} q^2(x^0) + q^2(x^0) \right) \chi_1 . \]
These equations can be written in the standard form (102), with a spin connection
\[ \hat{\omega}_0 = -U(x^0) - \frac{f'(x^0)}{f(x^0)} \]
(105)
\[ \hat{\omega}_1 = -U(x^0)q^2(x^0) - q^2(x^0) - 2\frac{f'(x^0)}{f(x^0)}q^2(x^0). \]
(106)
Non-minimal coupling thus enters the connection by a torsion-like contribution. For minimal coupling \( f'(X) = 0 \) (105) and (106) reduce to the Levi-Civita connection \( \omega = e_a * (de^a) \) calculated from (97), which reflects the fact that in two dimensions the fermions do not couple to the spin connection and thus do not feel torsion. Iterating (102) yields
\[ \Phi^2 \chi = -g^{\mu\nu} \nabla_\mu \nabla_\nu \chi - \frac{R}{4} \chi + e^\mu_a (**T^a**) \nabla_\mu \chi; \]
(107)
with the torsion two-form \( T^a = de^a + \hat{\omega}e^a_b \wedge e^b \).

The equation for \( \chi_1 \) is conformally invariant, but the one for \( \chi_0 \) contains the potential \( U(X) \) and thus transforms while changing between conformal frames. To solve (104) one introduces coordinates \( (C_\pm \) are constants)
\[ v_\pm(x^0, x^1) = x^1 \pm \int^0 \frac{dz}{q^2(z)} + C_\pm \]
(108)
with lower integration limits such that the integrals are defined, which is the case in the asymptotic region (97) outside horizons. In this way both coordinates are locally orthogonal to each other, \( \partial_{v_\pm}v_\mp = 0 \). With the Ansatz \( \chi_i = R_i e^{i\phi_i} \) one finds that \( R_0 \) has to fulfil (103), \( R_1 \) obeys
\[ \partial_{v_\pm}R_1 = \frac{1}{4} \left( \frac{f'(x^0)}{f(x^0)}q^2(x^0) + q^2(x^0) \right) R_1 \bigg|_{x^0 = x^0(v_-, v_+)} \]
(109)
and the phases \( \phi_i \) obey \( \partial_0 \phi_0 = 0 \) and \( \partial_{v_-} \phi_1 = 0 \). Solutions can be given explicitly even for general non-minimal coupling,
\[ \chi_0(x^0, x^1) = \frac{F(x^1)}{\sqrt{f(x^0)}} \exp \left[ i\phi_0(x^1) - \frac{Q(x^0)}{2} \right] \]
(110)
\[ \chi_1(x^0, x^1) = \tilde{F}(v_+) e^{i\phi_1(v_+)} \times \]
\[ \times \exp \left[ \frac{1}{4} \int^{v_-} dv'_- \left( \frac{f'(x^0)}{f(x^0)}q^2(x^0) + q^2(x^0) \right) \bigg|_{x^0 = x^0(v'_-, v^1)} \right], \]
(111)
with \( F(x^1) \) and \( \tilde{F}(v_+) \) being arbitrary real integration functions. The conformal transformation properties of these solutions are consistent with table 2 below: (110) includes a factor \( e^{-Q/2} \) and thus transforms with weight \(-1\), while (111) only depends on the conformally invariant combination \( e^{QV} \) and the mass \( m_\infty \) of the asymptotic spacetime. Thus if \( m_\infty \) is fixed for all conformal frames, the solution for \( \chi_1 \) is conformally invariant. The fermion bilinears (87) which enter the tree-level scattering vertices (99)- (100) read on-shell
\[ \Phi_0(x) = -\frac{|\chi_1|}{\sqrt{2}} \frac{\phi_1(v_+)}{q^2(x^0)} = \frac{\Phi_2(x)}{q^2(x^0)}, \quad \Phi_1(x) = 0. \]
(112)
5.3. One-Loop Effects

In the following we consider massless and minimally coupled \( f(X) = h(X) \equiv 1 \) fermions. A self interaction term \( g(\overline{\chi}\chi) = \lambda(\overline{\chi}\chi)^2 \) can be rewritten by introducing an auxiliary vector potential in the action

\[
S_{SI}[\chi, g, A] = \frac{\lambda}{2} \int d^2x \left( FA_a^2 + 2A_a \overline{\chi} \gamma^a \chi \right),
\]

which is integrated over in the path integral. The last term is absorbed into the Dirac operator, such that the fermion couples in the standard minimal way to the vector potential. After replacing \( \partial_\mu \) in (80) with the metric compatible and torsion-free covariant derivative \( \nabla_\mu \) partially integrating the kinetic term in (80), completing the square and evaluating the Gaussian integral yields

\[
\tilde{Z} = \exp \left( i \int d^2x \left( \mathcal{L}^{(0)}_{\text{eff}} + \mathcal{L}_{\text{int}} \left[ -i F^{-1} \frac{\delta^L}{\delta \eta} \right] \right) \right)
\]

\[
\int \mathcal{D}A \text{Det} \mathcal{D} \exp \left( i \int d^2x \left( \frac{\lambda}{2} FA_a^2 - \overline{\eta} \mathcal{D}^{-1} \eta \right) \right),
\]

(114)

with the Dirac operator (cf. (102)) \( \mathcal{D} = iE_\mu^a \gamma^a (\nabla_\mu - i\lambda A_\mu) \). The determinant of the Dirac operator is most easily calculated in Euclidean space (where \( \mathcal{D} \) is essentially self-adjoint) using zeta-function regularisation and heat kernel methods [71]. The square of the Euclidean Dirac operator (for Euclidean \( \gamma \)-matrix conventions see [2]) is of Laplace type,

\[
\mathcal{D}^2 = -(g^{\mu \nu} \nabla_\mu \nabla_\nu + E), \quad E = \frac{R}{4} + \frac{\lambda}{2} \epsilon^{\mu \nu} F_{\mu \nu}.
\]

(115)

Here \( F = dA \) denotes the electromagnetic field strength and \( R \) is the curvature scalar with respect to the Levi-Civita connection. The conformal anomaly has the same value as in the scalar case\(^6\),

\[
T^\mu_\mu := g^{\mu \nu} \langle T_{\mu \nu} \rangle = -\frac{R}{24\pi}.
\]

(116)

The chiral symmetry also becomes anomalous at one-loop level, yielding the consistent chiral anomaly

\[
\mathcal{A}(\varphi) = \delta_\varphi W^{\text{ren}} = -\frac{\lambda}{2\pi} \int d^2x \sqrt{g} \varphi \epsilon^{\mu \nu} F_{\mu \nu},
\]

(117)

where \( \delta_\varphi \) is an infinitesimal chiral transformation and \( W^{\text{ren}} \) denotes the zeta-function renormalised one-loop effective action. Note that although the mass-like coupling of the vector potential to the auxiliary field \( F \) in (114) breaks U(1) gauge invariance, the determinant of the Dirac operator is still gauge invariant and therefore the methods above are applicable.

Both anomalies can be integrated (with initial condition \( W[g_{\mu \nu} = \eta_{\mu \nu}] = 0 \) for the conformal anomaly), yielding a one-loop effective action comprising a Polyakov [72] and

\(^6\) The field strength term in \( E \) does not contribute because of \( \text{tr}_{\mathbb{C}^2}(\gamma_+) = 0 \).
Quantum Dilaton Gravity in Two Dimensions with Fermionic Matter

a Wess-Zumino part [73]

\[
W_{\text{loop}} = - \ln \det \mathcal{D} = W_{\text{Pol}} + W_{\text{WZ}}
\]

\[
W_{\text{Pol}} = \frac{1}{96\pi} \int_{M} d^2x \sqrt{-g} R \frac{1}{\Delta} R
\]

\[
W_{\text{WZ}} = \frac{\lambda}{4\pi} \int_{M} d^2x \sqrt{-g} \left( \ast F \right) \frac{1}{\Delta} \left( \ast F \right),
\]

with the Laplacian on 0-forms \( \Delta = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu \) and the Green function of the Laplacian defined as \( \Delta_x \Delta_x^{-1} = \delta^{(2)}(x - y) \).

What remains to be evaluated is the path integral over the vector potential in (114), now with the highly non-local integrand (120). The nature of the application should thus decide whether it is favourable to use this form, or to treat the Thirring term directly as an interaction vertex. On the other hand it may well be that the Wess-Zumino action becomes local in some special gauges, as happens for the Polyakov action in conformal gauge. Otherwise the Polyakov and Wess-Zumino action can be written in local form

\[
W_{\text{Pol}} = \frac{1}{48\pi} \int_{M} d^2x \sqrt{-g} \left[ \frac{1}{2} (\nabla \Phi)^2 + \Phi R \right]
\]

\[
W_{\text{WZ}} = \frac{\lambda}{2\pi} \int_{M} d^2x \sqrt{-g} \left[ \frac{1}{2} (\nabla Y)^2 + Y(\ast F) \right]
\]

by introducing two auxiliary scalars \( \Phi \) and \( Y \) which have to be integrated over in the path integral (114). These expressions coincide with the ones in [15, 16] up to notational differences.

6. Discussion & Conclusions

One of the main results of this work is the non-perturbative and background independent quantization (in the sense that no split of the metric into a fiducial and a fluctuation part is assumed) of the dilaton gravity sector of the theory. Although fixing the residual gauge freedom by imposing asymptotic boundary conditions on the momenta \( p_i \) as in section 5.1 (cf. (92)-(94)) determines the geometry in the asymptotic region, the metric is still subject to quantum fluctuations in the interior of the space-time manifold. This results in an effective background consistently including quantum backreactions onto the geometry. Another key result is the perturbative quantization of the Dirac field in that framework. The gravitationally induced four-point vertices as well as asymptotic fermion states have been calculated, both of which are necessary prerequisites for S-matrix calculations. Finally, known results for the one-loop effective action have been recovered.

6.1. Conformal Properties of the Effective Action

The action of conformal transformations \( g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu} \) after fixing the gauge and specifying asymptotic values of the \( p_i \) is slightly non-trivial and requires some discussion
Quantum Dilaton Gravity in Two Dimensions with Fermionic Matter

even for massless non self-interacting fermions. This is necessary to understand the conformal properties of the vertices and the scattering matrix. By assumption it will be required that the asymptotic values $\tilde{p}_i$ and the gauge fixing conditions $[44]$ are invariant. This implies in particular that neither $e^+_0$ nor $X^+$ transform and that $e^-_1$ has the same conformal weight as the metric. Furthermore, the conserved quantity $[21]$ is required to be conformally invariant. This leads to different conformal weights (listed in table 2) as compared to the situation before gauge fixing, equation (18). The conformal weight $\sigma(A)$ of a field monomial $A$ transforming homogeneously is defined by its transformation behaviour $A \mapsto \Omega^{\sigma(A)} A$. Conformal weights thus add for products of field monomials, $\sigma(AB) = \sigma(A) + \sigma(B)$.

| Weight 2 | Weight 1 | Weight 0 | Weight -1 | Weight -2 |
|----------|----------|----------|-----------|-----------|
| $g_{\mu\nu}, e^+_1, J^3$ | $\eta_1$ | $e^-_0, e^-_1, X, X^+, \tilde{p}_i, \tilde{g}_i, \chi_1, J^{1/2}, j_{1/2}, \eta_0, A^+$ | $\chi_0$ | $X^-, j_3, A^-$ |

Table 2. Conformal weights for Eddington-Finkelstein gauge

The gauge fixed spin connection transforms inhomogeneously,

$$\omega_0 \rightarrow \tilde{\omega}_0 = \omega_0 = 0, \quad \omega_1 \rightarrow \tilde{\omega}_1 = \omega_1 + (X^+ e^{-}_1 + X^- e^+_1) \frac{d \ln \Omega}{dX}, \quad (123)$$

and the dilaton potentials transform as described in section 2 below equation (18). Consequently the effective action (57) and (65) is conformally invariant at tree-level, i.e. before performing the path integration over $\chi$ and thus before taking into account quantum corrections from the fermions. For (57) and the first two ambiguous terms (65) this is evident as all terms are invariant by themselves. The third term in (65) requires some explanation: The four contributions in the bracket have to transform with a weight opposite to $e^Q$; obviously, this is true for the first two entries; for the third it holds if and only if the mass term is absent; for the last term it holds because $\chi_0$ has an appropriate weight and the derivative terms cancel.

For four-fermi scattering the tree-level S-matrix is indeed conformally invariant: Although the new vertex (101), by contrast to the two known ones (99) and (100), is not invariant, the vanishing of the fermion bilinear $\Phi_1$ (cf. (112)) implies that the four-fermi Feynman diagram corresponding to $V_c$ vanishes after attaching external legs, as well as all (tree-level and loop) diagrams with two outer $\chi_0$-legs attached to the same vertex $V_c$. This argument extends to all scattering vertices with external $\chi_0$-legs as all of them are generated by the last term in (65) and thus depend on $\Phi_1$. Diagrams like figure 2 with internal $\chi_0$-propagators could contribute to the S-matrix in a conformally non-invariant way. However, they are always (at least) one-loop diagrams and at one-loop level conformal invariance is broken already by the conformal anomaly (116), such that for one-loop effects a specific choice of the potential $U$ is necessary. In particular, two conformally related models (12) with the same $w$ (cf. (13)) but different $U$ lead to different results for certain physical observables, for instance the specific heat of black holes [74].
The occurrence of these new interactions also imposes a stronger requirement on
scattering triviality, namely minimal coupling and $U(X) = 0$ and $V(X) = \text{const}$, in
contrast to the scalar case where $w'(X) = \text{const}$. was sufficient for the vanishing of all
tree-level vertices [37]. The CGHS and Rindler ground state models (cf. table 1) thus
could exhibit non-trivial fermion scattering.

6.2. Comments on Bosonisation

Frolov, Kristjansson and Thorlacius exploited bosonisation of minimally coupled
fermions to study the semi-classical geometry of charged Witten black holes [19–21].
This was possible because they quantized the fermions on a fixed background and
assumed backreactions to be small. In the present work we have avoided such a split and
integrated out geometry exactly. We collect now some evidence in favour of bosonisation
in this more general context, but we emphasise that it is by no means conclusive.

The constraint algebra (35)–(38) resembles the one for non-minimally coupled scalar
matter [1]. In fact, if the coupling functions in (3) and (11) coincide, $f(X) = h(X)$, and
only the Thirring term $g = \lambda (\bar{\chi} \chi)^2$ is present in (11), then the structure functions of the
two constraint algebras are identical, provided that

$$\phi^\pm = j^\pm \quad \Rightarrow \quad \epsilon^{\mu\nu}(\partial_\mu \phi) e^{\pm}_\nu = \bar{\chi} \gamma^\pm \chi$$

where $\phi^\pm = *(d\phi \wedge e^\pm)$ are the (anti-)self dual components of the scalar field $\phi$ and
$j^\pm = \bar{\chi} \gamma^\pm \chi$ are the (anti-)chiral fermion currents. It should be noted that (124) is the
curved version of Coleman’s bosonisation map $j_\mu \propto -\epsilon_\mu \varphi$ [18].

The generating functional (114) also exploits the identification (124) since (113)
yields $\chi \gamma^a \chi = j^a = -A^a$ for the bosonic auxiliary field $A_\alpha$. This trick is applicable even

7 The coupling constant $\lambda$ is absorbed in the corresponding coupling function $\tilde{f}(X) \propto f(X)$ multiplying
the kinetic term of the scalar field.
after quantizing the dilaton gravity sector because the effective action (57) and (65) still allows the interpretation of fermions propagating on an effective background including quantum backreactions on the geometry.

Using perturbation theory in the matter sector we found that two of the lowest order vertices, (99) and (100), coincide with the vertices found in the scalar theory [37], and the new vertex (101) does not contribute to tree-level Feynman diagrams.

In conclusion, the results of this work indicate the presence of bosonisation in 2D quantum dilaton gravity beyond fixed background quantization. For further investigation of bosonisation along the lines of the present work it is necessary to calculate S-matrix elements and compare with the scalar case.

6.3. Outlook

As all tools are available now, the next natural step is the evaluation of the four-fermi S-matrix elements, by analogy to minimally [68] and non-minimally [75] coupled scalar fields. Similar effects that led to the prediction of decaying s-waves ought to be present for fermions. Moreover, by analogy to [76] one may also check CPT invariance. Note that although spherical reduction of four-dimensional Einstein-Hilbert gravity with a minimally coupled Dirac fermion [69] yields spherical reduced gravity with two non-minimally coupled Dirac fermions plus local interaction terms, the quantization procedure given in this work is still applicable. The additional Dirac spinor and the corresponding interactions will, however, lead to additional matter terms in (81)-(86) and thus give rise to new gravitational interaction vertices involving both fermion generations. From the four-dimensional point of view only such a setting is truly spherically symmetric. Obviously, the calculation of scattering amplitudes will be more involved and new physical effects may be expected due to the presence of these additional interactions.

We mention again that conformal invariance of S-matrix elements, which holds at tree-level to all orders, is no longer true for one-loop amplitudes. By contrast to the case of scalar matter there exists a tree level vertex breaking conformal invariance, $V_c$ in our nomenclature (cf. (101)), which however cannot contribute to tree-level amplitudes as a consequence of conformal invariance of the effective action (57) and (65).
At one-loop level corrections to the specific heat of black hole solutions are expected. In particular, the CGHS model [17] – which exhibits scattering triviality for scalar fields but not for fermions – should allow a straightforward application of our results, by analogy to [74]. In addition to the contributions from the Polyakov loop calculated there, corrections from the Feynman diagram depicted in figure 3 will arise. If the Thirring term is present the one-loop effective action receives an additional contribution (120) from the chiral anomaly.

Higher order vertices can be obtained with essentially the same algorithm as above, and it could be of interest to implement such algorithms in some computer algebra system, in order to obtain expressions for arbitrary 2n-point vertices. With such a result available possible (partial) resummations of the perturbation series could allow a discussion of bound states, i.e., black holes as long-living intermediate states, in a Bethe-Salpeter like manner. Other possible applications are outlined in [1, 11].

Finally, we stress that the exact path integration over geometry performed in this work appears to be possible in two dimensions only. Despite of this fact we can point out some observations which may serve as lessons for higher-dimensional quantum gravity:

(i) It has been crucial from a technical point of view to employ the Vielbein formalism rather than the metric formalism.

(ii) Choosing Minkowskian signature from the beginning has led to simplifications by enabling us to choose light-cone gauge.

(iii) Our path integral contains a sum over all configurations, including singular ones, and also a sum over both “East Coast” and “West Coast” sign conventions. The sum over singular configurations includes those where $e_1^+$ vanishes, which up to a sign is the determinant of the Zweibein in our gauge. The sum over both signatures is a consequence of summing over configurations with either positive or negative $e_1^+$ and, for spherically reduced gravity, additionally summing over positive and negative values of the dilaton field. So in the terminology of [77] our quantization procedure may be dubbed “bicoastal”.

(iv) The asymptotic conditions we had to impose in order to fix the residual gauge freedom led to a breaking of “bicoastalness” and to a unique asymptotic line element. However, the concept of some smooth “background geometry” makes sense only in the asymptotic region.

(v) From experience with the case of real scalars coupled to dilaton gravity and from the fact that we only use standard quantum field theory methods we expect the S-matrix to be unitary despite the emergence of “virtual black hole” intermediate states [11, 61].

Acknowledgements

We are deeply indebted to Dima Vassilevich for numerous helpful discussions. Additionally we are grateful to Luzi Bergamin, Max Kreuzer and Wolfgang Kummer
for helpful remarks.

This work has been supported by project J2330-N08 of the Austrian Science Foundation (FWF), by project GR-3157/1-1 of the German Research Foundation (DFG) and during its final stage by project MC-OIF 021421 of the European Commission. RM has been supported financially by the MPI MIS Leipzig and expresses his gratitude to J. Jost in this regard. We would like to thank the Vienna University of Technology for the hospitality while part of this work was conceived.

References

[1] D. Grumiller, W. Kummer, and D. V. Vassilevich, *Phys. Rept.* **369** (2002) 327–429, [hep-th/0204253](https://arxiv.org/abs/hep-th/0204253).

[2] J. Brown, *Lower Dimensional Gravity*. World Scientific, 1988.

J. A. Harvey and A. Strominger, “Quantum aspects of black holes,” in *Recent directions in particle theory: from superstrings and black holes to the standard model (TASI - 92)*. 1992, [hep-th/9209055](https://arxiv.org/abs/hep-th/9209055).

N. Ikeda and K. I. Izawa, *Prog. Theor. Phys.* **90** (1993) 237–246, [hep-th/9304012](https://arxiv.org/abs/hep-th/9304012).

L. Thorlacius, “Black hole evolution,” *Nucl. Phys. Proc. Suppl.* **41** (1995) 245–275, [hep-th/9411020](https://arxiv.org/abs/hep-th/9411020).

S. B. Giddings, “Quantum mechanics of black holes,” *Trieste HEP Cosmology* (1994) 0530–574, [hep-th/9412138](https://arxiv.org/abs/hep-th/9412138).

A. Strominger, “Les Houches lectures on black holes,” [hep-th/9501071](https://arxiv.org/abs/hep-th/9501071). Talk given at NATO Advanced Study Institute.

Y. N. Obukhov and F. W. Hehl, “Black holes in two dimensions,”, in: “Black Holes: Theory and Observations”, *Proc. of 179.WE-Heraeus Seminar Bad Honnef, Germany, 18-20 Aug 1997*, Eds. F.W. Hehl, C. Kiefer, and R.J.K. Metzler, *Lect. Notes in Phys.* **514** (Springer: Berlin, 1998) 289-316, [hep-th/9807101](https://arxiv.org/abs/hep-th/9807101).

H.-J. Schmidt, *Gen. Rel. Grav.* **31** (1999) 1187–1210, [gr-qc/9905051](https://arxiv.org/abs/gr-qc/9905051).

S. Nojiri and S. D. Odintsov, *Int. J. Mod. Phys.* **A16** (2001) 1015–1108, [hep-th/0009202](https://arxiv.org/abs/hep-th/0009202).

T. Strobl, “Gravity in two spacetime dimensions,” [hep-th/0011240](https://arxiv.org/abs/hep-th/0011240) Habilitation thesis.

[3] W. Kummer, H. Liebl, and D. V. Vassilevich, *Nucl. Phys.* **B493** (1997) 491–502, [gr-qc/9612012](https://arxiv.org/abs/gr-qc/9612012).

[4] W. Kummer, H. Liebl, and D. V. Vassilevich, *Nucl. Phys.* **B513** (1998) 723–734, [hep-th/9707115](https://arxiv.org/abs/hep-th/9707115).

[5] W. Kummer, H. Liebl, and D. V. Vassilevich, *Nucl. Phys.* **B544** (1999) 403–431, [hep-th/9809168](https://arxiv.org/abs/hep-th/9809168).

[6] D. Grumiller, *Quantum dilaton gravity in two dimensions with matter*. PhD thesis, Technische Universität Wien, 2001, [gr-qc/0105078](https://arxiv.org/abs/gr-qc/0105078).

[7] L. Bergamin, D. Grumiller, and W. Kummer, *JHEP* **05** (2004) 060, [hep-th/0404004](https://arxiv.org/abs/hep-th/0404004).

L. Bergamin, “Quantum dilaton supergravity in 2D with non-minimally coupled matter,” [hep-th/0408229](https://arxiv.org/abs/hep-th/0408229).

[8] D. Louis-Martinez, J. Gegenberg, and G. Kunstatter, *Phys. Lett.* **B321** (1994) 193–198, [gr-qc/9309018](https://arxiv.org/abs/gr-qc/9309018).

K. V. Kuchař, *Phys. Rev.* **D50** (1994) 3961–3981, [gr-qc/9403003](https://arxiv.org/abs/gr-qc/9403003).

D. Cangemi, R. Jackiw, and B. Zwiebach, *Ann. Phys.* **245** (1996) 408–444, [hep-th/9505161](https://arxiv.org/abs/hep-th/9505161).

K. V. Kuchař, J. D. Romano, and M. Varadarajan, *Phys. Rev.* **D55** (1997) 795–808, [gr-qc/9608011](https://arxiv.org/abs/gr-qc/9608011).
Quantum Dilaton Gravity in Two Dimensions with Fermionic Matter

[9] D. Grumiller and W. Kummer, “How to approach quantum gravity: Background independence in 1+1 dimensions,” in What comes beyond the Standard Model? Symmetries beyond the standard model, N. M. Borstnik, H. B. Nielsen, C. D. Frogbatt, and D. Lukman, eds., vol. 4 of Bled Workshops in Physics, pp. 184–196, EURESCO. Portoroz, Slovenia, July, 2003. gr-qc/0310068 based upon two talks.

[10] P. Schaller and T. Strobl, Class. Quant. Grav. 11 (1994) 331–346, hep-th/9211054.

[11] D. Grumiller, Int. J. Mod. Phys. D13 (2004) 1973–2002, hep-th/0409231.

[12] P. Schaller and T. Strobl, Mod. Phys. Lett. A9 (1994) 3129–3136, hep-th/9405110.

[13] W. Kummer and D. J. Schwarz, Phys. Rev. D45 (1992) 3628–3635; Nucl. Phys. B382 (1992) 171–186.

[14] M. Cavaglia, L. Fatibene, and M. Francaviglia, Class. Quant. Grav. 15 (1998) 3627–3643, hep-th/9801155.

[15] S. Nojiri and I. Oda, Phys. Lett. B294 (1992) 317–324, hep-th/9206087.

[16] A. Ori, Phys. Rev. D63 (2001) 104016, gr-qc/0011248.

[17] C. G. Callan, Jr., S. B. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. D45 (1992) 1005–1009, hep-th/9111056.

[18] S. R. Coleman, Phys. Rev. D11 (1975) 2088. cf. also M. Stone (Ed.) “Bosonization”, World Scientific, 1994.

[19] A. V. Frolov, K. R. Kristjansson, and L. Thorlacius, Phys. Rev. D72 (2005) 021501, hep-th/0504073.

[20] A. V. Frolov, K. R. Kristjansson, and L. Thorlacius, “Global geometry of two-dimensional charged black holes,” hep-th/0604041.

[21] L. Thorlacius, “Cosmic censorship inside black holes,” hep-th/0607048.

[22] R. Meyer, “Constraints in two-dimensional dilaton gravity with fermions,” hep-th/0512267.

[23] M. Cavaglia, L. Fatibene, and M. Francaviglia, Class. Quant. Grav. 15 (1998) 3627–3643, hep-th/9801155.

[24] D. Grumiller and D. V. Vassilevich, JHEP 11 (2002) 018, hep-th/0210069.

[25] M. O. Katanaev, W. Kummer, and H. Liebl, Phys. Rev. D53 (1996) 5609–5618, gr-qc/9511009.

[26] W. Kummer, “Deformed iso(2,1) symmetry and non-Einsteinian 2d-gravity with matter”, Lecture at Hadron Structure 92, Stara Lesna, Czechoslovakia, Sep 6-11, 1992, TUW-92-12.

[27] J. G. Russo and A. A. Tseytlin, Nucl. Phys. B382 (1992) 259–275, hep-th/9201021.

[28] J. G. Russo and A. A. Tseytlin, Nucl. Phys. B382 (1992) 259–275, hep-th/9201021.

[29] C. Teitelboim, Phys. Lett. B126 (1983) 41.

[30] R. Jackiw, Nucl. Phys. B252 (1985) 343–356.

[31] P. Thorn, B. Isaak, and P. Hájíček, Phys. Rev. D30 (1984) 1168.

[32] J. P. S. Lemos and P. M. Sa, Phys. Rev. D49 (1994) 2897–2908, gr-qc/9311008.

[33] A. Fabbri and J. G. Russo, Phys. Rev. D53 (1996) 6995–7002, hep-th/9510109.

[34] D. Grumiller, JCAP 05 (2004) 005, gr-qc/0307008.

[35] M. O. Katanaev, W. Kummer, and H. Liebl, Nucl. Phys. B486 (1997) 353–370, gr-qc/9602040.

[36] Y. Nakayama, Int. J. Mod. Phys. A19 (2004) 2771–2930, hep-th/0402009.
[37] D. Grumiller, W. Kummer, and D. V. Vassilevich, *European Phys. J.* C30 (2003) 135–143, hep-th/0208052.

[38] H. Reissner, *Ann. Phys.* 50 (1916) 106.

[39] S. W. Hawking and D. N. Page, *Commun. Math. Phys.* 87 (1983) 577.

[40] M. O. Katanaev and I. V. Volovich, *Phys. Lett.* B175 (1986) 413–416.

[41] A. Achucarro and M. E. Ortiz, *Phys. Rev.* D48 (1993) 3600–3605, hep-th/9304068.

[42] G. Gurahnik, A. Iorio, R. Jackiw, and S. Y. Pi, *Ann. Phys.* 308 (2003) 222–236, hep-th/0305117.

[43] D. Grumiller and W. Kummer, *Ann. Phys.* 308 (2003) 211–221, hep-th/0306036.

[44] L. Bergamin, “Constant dilaton vacua and kinks in 2d (super-)gravity,” hep-th/0509025.

[45] M. Katanaev and I. Volovich, *Phys. Lett.* B175 (1986) 413–416.

[46] A. Achucarro and M. E. Ortiz, *Phys. Rev.* D48 (1993) 3600–3605, hep-th/9304068.

[47] G. Guralnik, A. Iorio, R. Jackiw, and S. Y. Pi, *Ann. Phys.* 308 (2003) 222–236, hep-th/0305117.

[48] M. O. Katanaev and I. V. Volovich, *Phys. Lett.* B175 (1986) 413–416.

[49] D. Grumiller and W. Kummer, *Ann. Phys.* 308 (2003) 211–221, hep-th/0306036.

[50] M. O. Katanaev, *Nucl. Phys.* B416 (1994) 563–605, hep-th/0101168.

[51] H. Grosse, W. Kummer, P. Presnajder, and N. Toumbas, *JHEP* 03 (2004) 017, hep-th/0312208.

[52] W. Kummer and P. Widerin, *Phys. Rev.* D52 (1995) 6965–6975, gr-qc/9502031.

[53] W. Kummer and G. Tieber, *Phys. Rev.* D59 (1999) 044001, hep-th/9807122.

[54] R. B. Mann, *Phys. Rev.* D47 (1993) 4438–4442, hep-th/0604049.

[55] C. W. Misner and D. H. Sharp, *Phys. Rev.* 136 (1964) B571.

[56] R. Meyer, *Classical and Quantum Dilaton Gravity in Two Dimensions with Fermions,* Diploma Thesis, University of Leipzig, 2006, gr-qc/0607062.

[57] E. S. Fradkin and G. A. Vilkovisky, *Phys. Lett.* B55 (1975) 224–2247; *Phys. Rev.* D36 (1987) 1587–1602.

[58] R. A. Batalin and G. A. Vilkovisky, *Phys. Lett.* B69 (1977) 309–312.

[59] S. Arnowitt, S. Deser, and C. W. Misner in *Gravitation: An Introduction to Current Research,* L. Witten, ed. Wiley, New York, 1962. gr-qc/0405109.

[60] H. Grosse, W. Kummer, P. Presnajder, and N. Toumbas, *JHEP* 03 (2004) 017, hep-th/0312208.

[61] W. Kummer and G. Tieber, *Phys. Rev.* D59 (1999) 044001, hep-th/9807122.

[62] R. Meyer, “Classical and Quantum Dilaton Gravity in Two Dimensions with Fermions,” Diploma Thesis, University of Leipzig, 2006, gr-qc/0607062.

[63] E. S. Fradkin and G. A. Vilkovisky, *Phys. Lett.* B55 (1975) 224.

[64] I. A. Batalin and G. A. Vilkovisky, *Phys. Lett.* B69 (1977) 309–312.

[65] E. S. Fradkin and T. E. Fradkina, *Phys. Lett.* B72 (1978) 343.

[66] S. Weinberg, *The Quantum Theory of Fields,* vol. II. Cambridge University Press, 1995.

[67] K. Fujikawa, U. Lindstrom, N. K. Nielsen, M. Rocek, and P. v. Neveuwenhuizen, *Phys. Rev.* D37 (1988) 391.

[68] M. Henneaux and A. Slavnov, *Phys. Lett.* B416 (1994) 563–605, hep-th/0101168.

[69] H. Grosse, W. Kummer, P. Presnajder, and D. J. Schwarz, *Czech. J. Phys.* 42 (1992) 1325–1329.

[70] J. de Boer, F. Harmans, and T. Tijn, *Phys. Rept.* 272 (1998) 3–214, hep-th/9503151.

[71] R. Meyer, “Classical and Quantum Dilaton Gravity in Two Dimensions with Fermions,” Diploma Thesis, University of Leipzig, 2006, gr-qc/0607062.

[72] E. S. Fradkin and G. A. Vilkovisky, *Phys. Lett.* B55 (1975) 224.

[73] I. A. Batalin and G. A. Vilkovisky, *Phys. Lett.* B69 (1977) 309–312.

[74] E. S. Fradkin and T. E. Fradkina, *Phys. Lett.* B72 (1978) 343.

[75] S. Weinberg, *The Quantum Theory of Fields,* vol. II. Cambridge University Press, 1995.

[76] K. Fujikawa, U. Lindstrom, N. K. Nielsen, M. Rocek, and P. v. Neveuwenhuizen, *Phys. Rev.* D37 (1988) 391.

[77] D. J. Toms, *Phys. Rev.* D35 (1987) 3796.

[78] M. Basler, *Fortschr. Phys.* 41 (1993) 1–43.

[79] E. S. Abers and B. W. Lee, *Phys. Rept.* 9 (1973) 1–141.

[80] M. Henneaux and A. Slavnov, *Phys. Lett.* B338 (1994) 47–50, hep-th/9406161.

[81] W. Kummer and D. V. Vassilevich, *Annalen Phys.* 8 (1999) 801–827, gr-qc/9907041.

[82] D. Grumiller, W. Kummer, and D. V. Vassilevich, *Annalen Phys.* 8 (1999) 801–827, gr-qc/9907041.

[83] D. Grumiller, W. Kummer, and D. V. Vassilevich, *Annalen Phys.* 8 (1999) 801–827, gr-qc/9907041.

[84] H. Balasin, C. G. Bohmer, and D. Grumiller, *Gen. Rel. Grav.* 37 (2005) 1435–1482.

[85] F. Haider and W. Kummer, *Int. J. Mod. Phys.* A9 (1994) 207–220.
[71] D. V. Vassilevich, *Phys. Rept.* **388** (2003) 279–360, [hep-th/0306138](http://arxiv.org/abs/hep-th/0306138).
[72] A. M. Polyakov, *Phys. Lett.* **B103** (1981) 207–210.
[73] J. Wess and B. Zumino, *Phys. Lett.* **B37** (1971) 95.
[74] D. Grumiller, W. Kummer, and D. V. Vassilevich, *JHEP* **07** (2003) 009, [hep-th/0305036](http://arxiv.org/abs/hep-th/0305036).
[75] P. Fischer, D. Grumiller, W. Kummer, and D. V. Vassilevich, *Phys. Lett.* **B521** (2001) 357–363, [gr-qc/0105034](http://arxiv.org/abs/gr-qc/0105034). Erratum ibid. **B532** (2002) 373.
[76] D. Grumiller, *Class. Quant. Grav.* **19** (2002) 997–1009, [gr-qc/0111097](http://arxiv.org/abs/gr-qc/0111097).
[77] M. J. Duff and J. Kalkkinen, [hep-th/0605274](http://arxiv.org/abs/hep-th/0605274).