Noncommutativity and Ramond-Ramond fields

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Abstract

We study the properties of the Ramond fields in superstring theory in the presence of a constant B-field and the subsequent implications of space-time noncommutativity for space-time fermions and space-time supersymmetry. We find that the noncommutativity of space-time coordinates leads to extra singularities in the spin field correlators and to appearance of the extra poles in the RR scattering amplitudes, carrying the opposite GSO parities.

1 Introduction

It is well known that the dynamics of open strings in a constant $B_{mn}$-field background implies the noncommutativity of space-time coordinates [4, 8]. At the same time, the influence of the B-field on the dynamics of the Ramond fields and physical consequences of the space-time noncommutativity for fermionic fields are not yet well understood. One particularly important question is how the scattering amplitudes of the Ramond fields are affected by the presence of the B-field background and what is their modification for the noncommutative space-time. In the bosonic case the modification is trivial: the correlators of the NS and NS-NS fields are unchanged, up to a constant “noncommutative phase” [2]. For the Ramond sector, however, the situation is different. The bosonization rules for the worldsheet fermions are modified in the B-field background and this affects the structure of the space-time spin operators and their mutual operator products. As a result, the correlators involving the
Ramond and Ramond-Ramond fields receive nontrivial corrections in the noncommutative θ parameter.

In this letter we analyze the example of the gravitational lensing of the RR-fields in noncommutative space, involving the two-point RR scattering amplitudes on a disc in the B-field background. We derive the modified bosonization relations for the spin fields in the noncommutative background and compute the relevant scattering amplitudes of the RR fields. We find that the change in bosonization leads to the nontrivial θ corrections to these amplitudes. In particular, the correction terms have a modified pole structure; namely, in the first order in the noncommutative θ parameter the correlator will be shown to receive contributions from the intermediate states of an opposite (odd) GSO sector. Thus an important physical consequence for the spin fields in the B-field background is that the noncommutativity of the space-time effectively mixes the operators from the opposite GSO sectors [6] if these operators involve the space-time spin fields.

2 Noncommutativity and bosonization of fermions

Let us recall how the noncommutativity of space-time appears in string theory. Firstly, the imposition of the B-field changes the worldsheet boundary conditions for an open string. For a string moving in a constant B-field background these boundary conditions are given by:

\[ g_{mn}(\partial - \bar{\partial})X^m + 2\pi\alpha' B_{mn}(\partial + \bar{\partial})X^n\big|_{z = \bar{z}} = 0. \]  

Here \( g_{mn} \) is the space-time metric and it is implied that the worldsheet surface is conformally mapped to the half-plane. The boundary conditions (1) interpolate between Dirichlet and Neumann conditions. If the B-matrix is invertible, the limit of \( B \to \infty \) corresponds to Dirichlet boundary conditions for the spatial directions in which the B-field is applied [4]. For this reason the imposition of an infinite B-field is similar to the T-duality transformation in the appropriate directions. The propagator corresponding to the boundary conditions (1) is given by [4]

\[ \langle X^m(z, \bar{z})X^n(w, \bar{w}) \rangle = \frac{-\alpha'}{g^{mn} \log |z - w| - g^{mn} \log |z - \bar{w}| + G^{mn} \log |z - w|^2 + \frac{1}{2\pi\alpha'} \theta^{mn} \log \left( \frac{z - \bar{w}}{\bar{z} - w} \right)} \]  

where

\[ G^{mn} = \left[ (g + 2\pi\alpha'B)^{-1}g(g - 2\pi\alpha'B)^{-1} \right]^{mn} \]
\[ \theta^{mn} = -2 \left[ (\alpha')^2(g + 2\pi\alpha'B)^{-1}B(g - 2\pi\alpha'B)^{-1} \right]^{mn} \]  

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In the limit when both \( z \) and \( w \) approach the real axis: \( z \to t_1, w = \bar{w} \to t_2 \), the propagator (2) becomes

\[
\langle X^n(t_1)X^n(t_2) \rangle = -\alpha' G^{mn} \log(t_1 - t_2)^2 + i\frac{\theta^{mn}}{2} \text{sign}(t_1 - t_2),
\]

(4)

implying the commutation relation:

\[
[X^m, X^n] = i\theta^{mn}
\]

(5)

after the regularization and the time ordering. The change in the propagator (4) and the commutation relation (5) give rise to the appearance of the noncommutative "star product" in the field-theoretic low energy effective action as \( \alpha' \to 0 \). Thus the space-time noncommutativity is related to the antisymmetric term in the propagator and the \( \theta^{mn} \) matrix defines the noncommutativity parameter.

The important question is what are the implications of the noncommutativity for the superspace and what are the properties of fermions in noncommutative space-time. Addressing this question from the string theory point of view means that one has to study the properties of the Ramond fields in the \( B \)-field background. For the NSR worldsheet fermions in the \( B \)-field background the boundary conditions, being the supersymmetric extension of (1), have the form:

\[
g^{mn}(\psi^n - \bar{\psi}^n) + 2\pi\alpha'B_{mn}(\psi^n + \bar{\psi}^n) = 0.
\]

(6)

The propagators corresponding to these boundary conditions are given by

\[
\langle \psi^m(z, \bar{z})\psi^n(w, \bar{w}) \rangle = \left( G^{mn} - \frac{1}{2}g^{mn} \right) \left( \frac{1}{z - w} + \frac{1}{\bar{z} - \bar{w}} \right) + \frac{1}{2}g^{mn} \left( \frac{1}{z - \bar{w}} + \frac{1}{\bar{z} - w} \right) + \frac{1}{2\pi\alpha'}\theta^{mn} \left( \frac{1}{z - \bar{w}} - \frac{1}{\bar{z} - w} \right).
\]

(7)

The propagator (7) implies that one can continue \( \psi(z, \bar{z}) \) to the entire complex plane with the corresponding correlators between \( \psi(z) \) and \( \bar{\psi}(\bar{z}) \) expressed as

\[
8)\langle \psi^m(z)\psi^n(w) \rangle = \frac{1}{2}g^{mn},
\]

\[
\langle \bar{\psi}^m(z)\bar{\psi}^n(w) \rangle = \frac{1}{2}g^{mn},
\]

\[
\langle \bar{\psi}^m(z)\psi^n(w) \rangle = \frac{G^{mn} - \frac{1}{2}g^{mn} + \frac{1}{2\pi\alpha'}\theta^{mn}}{z - w},
\]

\[
\langle \psi^m(z)\bar{\psi}^n(\bar{w}) \rangle = \frac{G^{mn} - \frac{1}{2}g^{mn} - \frac{1}{2\pi\alpha'}\theta^{mn}}{z - \bar{w}}.
\]

(8)

The important property of these propagators is the presence of the non-diagonal interactions \( < \psi^m\bar{\psi}_n > \) and \( < \bar{\psi}^m\psi_n > \) for \( n \neq 0 \). This fact will prove to be of importance for the properties of the space-time fermions in noncommutative space.
Now we are prepared to construct the Ramond spin fields in the presence of the B-field background. Of course our discussion is entirely restricted to the case of the space-type (magnetic) noncommutativity
\[ \theta_{ij} \neq 0, \quad i, j \neq 0, \] (9) since the formulation of string theory in backgrounds with time-like noncommutativity is problematic [5, 9]. The construction is quite analogous to the standard scheme [1]. As usual, one starts with constructing five complex worldsheet fermions out of ten \( \psi^m \)'s:

- \( \Lambda_1 = i\psi_0 + i\psi_9, \Lambda_1^* = i\psi_0 - i\psi_9; \)
- \( \Lambda_2 = \psi_1 + i\psi_2, \Lambda_2^* = \psi_1 - i\psi_2; \)
- \( \Lambda_3 = \psi_3 + i\psi_4, \Lambda_3^* = \psi_3 - i\psi_4; \)
- \( \Lambda_4 = \psi_5 + i\psi_6, \Lambda_4^* = \psi_5 - i\psi_6; \)
- \( \Lambda_5 = \psi_7 + i\psi_8, \Lambda_5^* = \psi_7 - i\psi_8; \) (10)

and analogously for the antiholomorphic part. Next, one bosonizes \( \Lambda_j \) as
\[ \Lambda_j = e^{i\phi_j}, \quad j = 1, \ldots, 5. \] (11)

Finally, one constructs the space-time spin operator \( \Sigma_\alpha, \alpha = 1, \ldots, 32 \) as
\[ \Sigma_\alpha(z) = \prod_{j=1}^{5} e^{\frac{2}{a_j} \phi_j}, \] (12)
with
\[ a_j = \pm 1 \] (13)
and each value of the spinor index \( \alpha \) corresponding to some particular combination of \( \{a_j\}, j = 1, \ldots, 5 \). In the absence of the B-field this spin operator has a conformal dimension \( 5/8 \) as its OPE with itself is given by:
\[ \Sigma_\alpha(z) \Sigma_\beta(w) \sim \frac{\delta_{\alpha\beta}}{(z-w)^{\frac{5}{4}}} + \sum_p \frac{\gamma_{\alpha\beta}^{m_1 \ldots m_p} \psi_{m_1} \ldots \psi_{m_p}}{(z-w)^{\frac{5}{4} - \frac{2}{p}}} + \text{derivatives}. \] (14)

Similarly, one can write the OPE between \( \Sigma_\alpha(z) \) and \( \bar{\Sigma}_\alpha(\bar{w}) \) on a disc using the relation:
\[ \bar{\Sigma}_\alpha(\bar{w}) = \gamma_{\alpha\beta}^{0m_1 \ldots m_q} \Sigma_\beta(\bar{w}), \] (15)
where \( q \) is the number of Dirichlet boundary conditions on the disc (out of the total number of 10). Let us now turn to the case of the nonzero B-field. In the presence of the B-field the OPE involving the matter spin operators with different chiralities, change significantly. First of all one needs to point out the relation between left and right modes of spin operators
on the disc in the noncommutative case. For simplicity, we consider the case when all the boundary conditions are Neumann. First, using the propagators (8) it is easy to see that the relation between $\psi_m$ and $\bar{\psi}_m$ is given by:

$$\bar{\psi}_n(\bar{z}) = \left( G_{mn} - \frac{1}{2} g_{mn} \right) \psi_m(\bar{z}) + \theta_{mn} \psi_m(\bar{z}). \quad (16)$$

For simplicity, let us consider for the time being the special case of $\theta = \theta_{13} \neq 0$, while setting all other components to zero; the result will be easily generalized to the arbitrary case.

In this case the corresponding relations between $\Lambda$ and $\bar{\Lambda}$, following from (16), are given by

$$\bar{\Lambda}_1(\bar{z}) = \Lambda_1(\bar{z}) + \theta_{13} \Lambda_2(\bar{z}), \quad \bar{\Lambda}_2(\bar{z}) = \Lambda_2(\bar{z}) - \theta_{13} \Lambda_1(\bar{z}),$$

$$\bar{\Lambda}_a(\bar{z}) = \Lambda_a(\bar{z}), \quad a = 3, 4, 5. \quad (17)$$

One can use these relations to find the correspondence between $\Sigma$ and $\bar{\Sigma}$ on the disc, provided that $\theta$ is small. Indeed, using:

$$\sqrt{x+\epsilon} : \sim : \sqrt{x} : + : \frac{\epsilon}{\sqrt{x}} : + O(\epsilon^2)$$

when $\epsilon$ and $x$ operators are independent, one has

$$e^{\frac{i}{2} \varphi_1}(\bar{z}) = (\Lambda_1(\bar{z}) - \theta_{13} \Lambda_2(\bar{z})) \frac{1}{2} = e^{\frac{i}{2} \varphi_1}(\bar{z}) - \frac{\theta_{13}}{2} e^{-\frac{i}{2} \varphi_1 + i \varphi_2} + O(\theta^2)$$

$$= e^{\frac{i}{2} \varphi_1}(\bar{z}) - \frac{\theta_{13}}{2} : \Lambda_2 e^{-\frac{i}{2} \varphi_1} : (\bar{z}) + O(\theta^2),$$

$$e^{\frac{i}{2} \varphi_2}(\bar{z}) = (\Lambda_2(\bar{z}) + \theta_{13} \Lambda_1(\bar{z})) \frac{1}{2} = e^{\frac{i}{2} \varphi_2}(\bar{z}) + \frac{\theta_{13}}{2} e^{-\frac{i}{2} \varphi_2 + i \varphi_1} + O(\theta^2)$$

$$= e^{\frac{i}{2} \varphi_1}(\bar{z}) + \frac{\theta_{13}}{2} : \Lambda_1 e^{-\frac{i}{2} \varphi_1} : (\bar{z}) + O(\theta^2),$$

$$e^{\frac{i}{2} \varphi_a}(\bar{z}) = e^{\frac{i}{2} \varphi_a}(\bar{z}). \quad (18)$$

Using these relations along with the definition (12) for the spin operators, it is straightforward to show that

$$\bar{\Sigma}_\alpha(\bar{z}) = \Sigma_\alpha(\bar{z}) + \frac{i}{2} \theta_{13} ((\gamma^1)_{\alpha \beta} \psi^3 - (\gamma^3)_{\alpha \beta} \psi^1) \Sigma_\beta(\bar{z}) + O(\theta^2).$$

This relation is straightforward to generalize to the case of arbitrary $\theta_{ij}$:

$$\bar{\Sigma}_\alpha(\bar{z}) = \Sigma_\alpha(\bar{z}) + \frac{i}{2} \theta_{ij} ((\gamma^i)_{\alpha \beta} \psi^j - (\gamma^j)_{\alpha \beta} \psi^i) \Sigma_\beta(\bar{z}) + O(\theta^2). \quad (19)$$

This important relation leads to some remarkable physical consequences. Upon multiplying $\Sigma$ by the ghost spin operator $e^{-\frac{i}{2} \varphi}$, it is easy to see that the full matter + ghost antiholomorphic spin operator is expressed in terms of the left-moving modes with mixed GSO parities; indeed, it is easy to check that the operators of the type $\sim e^{-\frac{i}{2} \varphi} \Sigma \psi$ are GSO-odd. Therefore it shows that the noncommutativity mixes the even and odd GSO sectors of the Ramond operators.
in the first order of $\theta$. Of course such a mixing does not reveal itself in the sphere amplitudes but becomes important on the disc when the left and the right modes mix with each other. In particular, this leads to the appearance of intermediate poles of opposite GSO parity in the two-point disc amplitude of the RR fields in noncommutative backgrounds.

## 3 RR scattering amplitudes in the B-field

Consider the two-point scattering amplitude of RR vertex operators on the disc in the case of $B_{ij} \neq 0, \theta_{ij} \neq 0$. The RR vertex operators taken at the standard $-1/2, -1/2$-picture are given by

$$V_{RR}(z, \bar{z}) = e^{-\frac{i}{2}\phi - \frac{i}{2}\phi} \sum_{a} \sum_{\beta} e^{ikX}(z, \bar{z})^{\gamma_{m_{1}...m_{q}}}F_{m_{1}...m_{q}}(k).$$

For the two-point disc amplitude one can take both RR vertices at this canonical picture. Using the bosonization formulae for spin operators (12) along with the relation (19) it is straightforward to compute the amplitude. First of all, the four-point correlator of matter spin operators (giving rise to the two-point disc amplitude) is given by:

$$\langle \Sigma_{a_{1}}(z) \Sigma_{a_{2}}(\bar{z}) \Sigma_{\beta_{1}}(w) \Sigma_{\beta_{2}}(\bar{w}) \rangle$$

$$= -\frac{\gamma_{\mu\alpha_{1}\beta_{1}}(\gamma_{1})_{\alpha_{2}\lambda}(z - \bar{z})^{4}(w - \bar{w})^{4}}{(z - w)^{4}(z - \bar{w})^{4}} \left( \delta_{\alpha_{2}\beta_{2}} + \frac{1}{2} \gamma_{ij}^{\beta_{2}} \theta_{ij} \left( \frac{\sqrt{w - \bar{w}}}{z - \bar{z}} + \frac{\sqrt{z - \bar{w}}}{w - \bar{w}} \right) \right)$$

$$+ \frac{\gamma_{\mu\alpha_{1}\beta_{1}}(\gamma_{1})_{\beta_{2}\lambda}(z - \bar{z})^{4}(w - \bar{w})^{4}}{(z - w)^{4}(z - \bar{w})^{4}} \left( \delta_{\beta_{2}\lambda} - \frac{1}{2} \gamma_{ij}^{\beta_{2}} \theta_{ij} \left( \frac{\sqrt{z - w}}{z - \bar{w}} + \frac{\sqrt{\bar{z} - w}}{\bar{z} - \bar{w}} \right) \right) + O(\theta^{2}).$$

In comparison with the $B = 0 = \theta$ case, there are the extra square root factors in the first and in the second terms, appearing in the first order in $\theta$. Using this correlator and noting that the B-field does not change the superconformal ghost propagator, one easily finds that the two-point Ramond-Ramond half-plane correlation function in the noncommutative case is given by:

$$\langle V_{RR}(z, \bar{z}; k)V_{RR}(w, \bar{w}; p) \rangle$$

$$= \left( -\gamma_{\mu\alpha_{1}\beta_{1}}(\gamma_{1})_{\alpha_{2}\lambda}(z - \bar{z})^{1}(w - \bar{w})^{1}(z - \bar{w})^{1}(w - \bar{w})^{1}(z - \bar{w})^{2}(w - \bar{w})^{2} \right.$$

$$\times \left( \delta_{\beta_{2}\lambda} + \frac{1}{2} \gamma_{ij}^{\beta_{2}} \theta_{ij} \left( \frac{\sqrt{w - \bar{w}}}{z - \bar{z}} + \frac{\sqrt{z - \bar{w}}}{w - \bar{w}} \right) \right)$$

$$+ \gamma_{\mu\alpha_{1}\beta_{1}}(\gamma_{1})_{\beta_{2}\lambda}(z - \bar{z})^{1}(w - \bar{w})^{1}(z - \bar{w})^{1}(w - \bar{w})^{1}(z - \bar{w})^{2}(w - \bar{w})^{2} \right. \times \left( \delta_{\beta_{2}\lambda} - \frac{1}{2} \gamma_{ij}^{\beta_{2}} \theta_{ij} \left( \frac{\sqrt{z - w}}{z - \bar{w}} + \frac{\sqrt{\bar{z} - w}}{\bar{z} - \bar{w}} \right) \right) \times \gamma_{\alpha_{1}\alpha_{2}}^{m_{1}...m_{p}} \gamma_{\beta_{1}\beta_{2}}^{n_{1}...n_{q}} F_{m_{1}...m_{p}}(k) F_{n_{1}...n_{q}}(p).$$

Here we denoted

$$\{a, b\} \equiv \alpha' \left( 2G_{mn} - g_{mn} + \frac{g_{mn}}{2\pi\alpha'} \right) a_{m}b_{n}, \quad (ab) = \alpha' a_{m}b^{m}$$

(23)
for any space-time vectors $a$ and $b$. For simplicity, let us consider the rank $q$ even, i.e. the type IIB case. Then the second term linear in $\theta$ in (22) is absent as its gamma-matrix trace factor vanishes. The correlator (22) now needs to be integrated over the worldsheet to obtain the amplitude. The zero order in $\theta$ part of the correlator is the standard one, giving rise to the usual commutative gravitational lensing [10]. From now on, let us therefore concentrate just on the term linear in $\theta_{ij}$, proportional to $\sqrt{z - \bar{z}}$, which is the one contributing to the GSO parity mixing. To integrate it over the worldsheet with the disc topology, it is convenient to conformally map the half-plane to the disc using the transformation $(z, \bar{z}) \rightarrow (u, \bar{u})$ with

$$z = \frac{i u + i}{2 u - i}$$

(24)

with $(u, \bar{u})$ now being the disc coordinates. To calculate the scattering amplitude it is convenient to place one of the vertex operators at the origin of the disc (corresponding to the point $w = -\frac{i}{2}$ on the halfplane) and to integrate over the position of another one. Writing $u = re^{i\lambda}$ and using

$$|z - w|^2 = \frac{r^2}{r^2 + 1 - 2r \sin \lambda}, \quad |z - \bar{w}|^2 = \frac{1}{r^2 + 1 - 2r \sin \lambda},$$

$$z - \bar{z} = \frac{i(r^2 - 1)}{r^2 + 1 - 2r \sin \lambda}, \quad w - \bar{w} = -i,$$

(25)

we have:

$$A(k, p) = \langle V_{RR}(k)V_{RR}(p)\rangle\big|_{NC}$$

$$= \frac{1}{2} \theta_{ij} Tr(\gamma^{\mu_1...\mu_q}\gamma^{\nu_1...\nu_q}\gamma_{\mu_1...\mu_q}\gamma_{\nu_1...\nu_q})F_{\mu_1...\mu_q}(k)F_{\nu_1...\nu_q}(p)$$

$$\times \int_0^{2\pi} d\lambda \int_0^\infty dr r (r^2 + 1 - 2r \sin \lambda)^{-\{k,k\} - \{k,p\} - (kp) + \frac{3}{2}(r^2 - 1)\{k,k\} + \frac{1}{2} r^{2(kp)} - 2}.$$  

First of all, let us calculate the angular integral over $\lambda$. We have:

$$\int_0^{2\pi} (r^2 + 1 - 2r \sin \lambda)^{-\{k,k\} - \{k,p\} - (kp) + \frac{3}{2}} \equiv \int_0^{2\pi} (r^2 + 1 - 2r \sin \lambda)^\rho = 2\Gamma\left(\frac{1}{2}\right) F(\rho, 1, r^2).$$

(27)

where $F$ is hypergeometric function and we denoted $\rho = -\{k,k\} - \{k,p\} - (kp) + \frac{3}{2}$. The amplitude is then reduced to the radial integral

$$A(k, p) \sim 2\Gamma\left(\frac{1}{2}\right) \int_0^{1} dr (r^2 - 1)^{\{k,k\} - \frac{3}{2} r^{2(kp)} - 2} F(\rho, 1, r^2)$$

$$= \Gamma\left(\frac{1}{2}\right) \int_0^{1} dx (x - 1)^{\{k,k\} + \frac{3}{2} x^{(kp)} - 1} F(\rho, 1, x),$$

(28)

where we have introduced the new integration variable $x = r^2$. In addition, here and everywhere below we suppress the gamma-matrix trace for the sake of shortness. The evaluation
of the radial integral over $x$ involving the hypergeometric function depends on the value of the parameters $(kp)$ and $\{k,k\}$. In the case of

$$(kp) > 0; \{k,k\} > -\frac{1}{2}$$

the evaluation of the integral gives

$$A(k,p) \sim \frac{\Gamma\left(\frac{1}{2}\right)\Gamma((-kp))\Gamma(\{k,k\} + \frac{1}{2})}{\Gamma((-kp) + \{k,k\})} \ {}_3F_2(\rho;\rho;(kp);1;(kp) + \{k,k\} + \frac{3}{2};1),$$

where $\ {}_3F_2(\rho;\rho;(kp);1;(kp) + \{k,k\} + \frac{3}{2};1)$ is the generalized hypergeometric function. The constraints (29) insure that the amplitude has no physical poles in this case. When the conditions (29) are not fulfilled, the value of the integral (28) is different. Expanding $F(\rho,\rho,1,x)$ in series we can write this integral as

$$A(k,p) \sim \Gamma\left(\frac{1}{2}\right) \int_0^1 dx x^{(kp)-1}(x-1)^{\{k,k\}+\frac{1}{2}} \left(1 + \sum_{n=1}^{\infty} \frac{\rho(\rho+1)\ldots(\rho+n-1)}{n!} x^n\right)$$

$$= \Gamma\left(\frac{1}{2}\right) \left\{B\left((kp),\{k,k\} + \frac{1}{2}\right) + \sum_{n=1}^{\infty} \frac{\rho(\rho+1)\ldots(\rho+n-1)}{n!} B\left((kp) + n,\{k,k\} + \frac{3}{2}\right)\right\}. $$

The crucial property of this amplitude is that it has extra physical poles corresponding to

$$\{k,k\} = -\frac{3}{2} - m,$$

where $m$ is non-negative integer. These poles correspond to the appearance of the GSO-odd intermediate states. As it is easy to notice, these extra poles, leading to the GSO-parity change, originate from the square roots in the unintegrated amplitude, creating cuts on the worldsheet surface. In the commutative case these poles are of course absent since $\{k,k\} = 0$ when $B = 0$.

4 Conclusions

In this letter we have studied the properties of Ramond and Ramond-Ramond fields in constant B-field backgrounds and the influence of the string theory noncommutativity on the space-time fermions and bispinors. We have found that properties of spin operators are significantly changed in the noncommutative case. Namely, we found that in noncommutative space-time the relation between left and right modes of spin operators on the disc involves the mixing of the operators with opposite GSO parities. The effect of the GSO parity change becomes more transparent when one considers the Ramond-Ramond scattering amplitudes on the disc in the noncommutative space-time, where the ”anomalous” interaction between
left and right-moving fields brings about significant physical consequences. In the two-point RR disc amplitude, describing the "gravitational lensing" of the RR particles off the noncommutative plane, we observe the appearance of physical poles corresponding to intermediate GSO-odd states. Next, because of the extra gamma-matrix factor of $\gamma^{ij}$, the first order $\theta$-correction appears as if the boundary conditions in the given i-j directions are changed from Neumann to Dirichlet, or the T-duality transform is applied to these directions, imitating a scattering off a $D8$-brane spanned in the directions transverse to i and j. This is consistent with the fact that the noncommutative boundary conditions effectively mix the Dirichlet and Neumann boundary conditions, and, in the limit of a large B-field, introducing the noncommutativity is similar to the T-duality transformation applied in the directions of B. The effects described in this paper may have several interesting applications. Particularly interesting is the limit of the large B-field. In this case the interactions of fermions with opposite chiralities become dominant and the physical sector of the theory is shifted into the GSO-odd sector. At the same time, the axionic term becomes dominant in the worldsheet NSR action. It is known [3] that the worldsheet action with the axionic term $\sim B_{ij} \partial X^i \bar{\partial} X^j$ and with the graviton absent is zigzag invariant, i.e. they may describe the confining QCD string. In view of the above, one may speculate that the confining phase of the QCD may be relevant to the dynamics of the GSO-odd sector of the NSR superstring theory. Another important issue is the role of the tachyon in noncommutative space. Because of the GSO parity change, the tachyon becomes a physical GSO-projected state in the $B \to \infty$ limit. Changes in conformal dimensions may insure that noncommutative tachyons are still consistent with the vacuum stability. We hope to address this and the related issues in the future works.

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