Spontaneous Quantum Hall States and Novel Luttinger Liquids in Chiral Graphene

Fan Zhang, Jeil Jung, Allan H. MacDonald
Department of Physics, University of Texas at Austin, Austin TX 78712, USA
E-mail: zhangfan@physics.utexas.edu

Abstract. Chirally stacked neutral $N$-layer graphene systems with $N \geq 2$ exhibit spontaneous inversion symmetry breaking and interaction induced charge gaps due to interlayer exchange interactions. A variety of distinct broken symmetry states are distinguished by their charge, spin, and valley Hall conductivities, by their orbital magnetizations, and by their edge state properties. In the spinless case, quantum anomalous and valley Hall states are favored by a weak magnetic field and by an electric field between the graphene layers, respectively. At interfaces between different phases one dimensional gapless modes exhibit novel Luttinger liquid behaviors.

1. Introduction
Broken symmetry and topological order\cite{1, 2} in reduced dimensions have been important themes of condensed matter physics for decades. Recent success in isolating monolayer and few-layer graphene sheets\cite{3, 4} from bulk graphite, combined with progress in epitaxial growth methods, has provided us a new family of two-dimensional electron systems that can host broken symmetries and topological orders. In chirally stacked graphene systems\cite{5, 6, 7}, a new type of broken symmetry states\cite{8, 9} with momentum space vortices and real-space inversion symmetry breaking, first proposed theoretically within a Hartree Fork mean-field theory\cite{10}, appears likely on the basis of a perturbative renormalization group analysis\cite{8}, and recent experiments.\cite{11, 12}. These states have recently attracted a great deal of attention \cite{8, 9, 10, 11, 12, 13, 15, 14, 16, 17}. These spontaneous quantum Hall states\cite{9} are characterized by spin and valley dependent spontaneous layer polarization, driven by interlayer electron-electron interactions. A family of closely related broken symmetry states exhibit different orbital magnetizations and topological edge states\cite{9}, and are distinguished by their quantized charge, spin, and valley Hall conductances\cite{9}.

Quasiparticles in graphene are described by the massless Dirac fermion model\cite{3, 4} and the dispersion of the band remains linear down to the charge neutrality point. When these honeycomb layers are stacked, electronic properties are strongly modified in a way that is controlled by the specific stacking arrangement\cite{5, 6, 7}. Among all stacking possibilities, only the $ABC$ arrangement\cite{6, 7}, also referred to chiral stacking\cite{9} arrangements, extend the following features that make Bernal bilayer\cite{5} ($N = 2$) electronic structure particularly interesting to larger $N$-layer films: (i) two low-energy sublattice sites are localized in the outermost layers, and any inversion asymmetry, i.e. an electric field perpendicular to the film, is able to induce a energy gap;\cite{5, 6, 7} (ii) hopping between low-energy sites via high energy sites is an $N$-step
(a) QVH    (b) QAH                    (c) LAF                          (d) QSH (2D TI)    (e) “ALL”

Figure 1. For cases (a-e) the lower panel describes the sense of layer polarization for each spin-valley combinations while the upper panel schematically indicates the corresponding Hall conductivity contributions[9]. (a) a valley Hall insulator with a net layer polarization and a mass \( m\sigma_z \); (b) an anomalous Hall insulator with a valley-dependent mass \( m\tau_z\sigma_z \); (c) a layer-antiferromagnetic insulator with a spin-dependent mass \( m s_z\sigma_z \); (d) a quantum spin Hall (or 2D Topological) insulator with a valley and spin dependent mass term \( m\tau_z s_z\sigma_z \); (e) an exotic Hall state with a valley and spin dependent mass term \( m(1+\tau_z^2+\tau_z^2s_z)\sigma_z \).

process which leads to \( p^N \) dispersion at Dirac points, sublattice pseudospin chirality \( N \) and Berry phase \( N\pi;[6, 7] \) (iii) the density-of-state \( D(E) \sim E^{(2-N)/N} \) diverges[7] as \( E \) approaches zero for \( N > 2 \) whereas it remains finite for \( N = 2 \) and vanishes for \( N = 1 \). Because of the flat bands and the large pseudospin chirality, many-body effects become dominant[7] in the low-energy physics of a charge neutral chiral multilayer systems.

When these band states are described in a pseudospin language[10], the broken symmetry state is characterized by a momentum-space vortex with vorticity \( N\) and a vortex-core which is polarized in the top-or-bottom layers. For a chiral (Bernal) bilayer, for example, interactions lead to a broken symmetry ground state with a spontaneous gap in which charge is transferred between top and bottom layers. As discussed in Ref. [8], the chiral bilayer can be viewed as a two-dimensional analog of the Luttinger liquid; however, the mean-field CDW-like instability is enhanced by quantum fluctuations[8] in bilayers, whereas it is canceled by the weak attractive BCS instability in Luttinger liquids.

In the rest of the current paper, we will first classify the broken symmetry states, which can be characterized by the signs of the valley and spin dependent mass terms[9] which appear in their mean-field Hamiltonians and by their quantized charge, valley and spin Hall conductances[9]. In section III we will explain why quantum anomalous[18, 19, 9, 16, 11, 12] and valley Hall[20, 21, 9] states are favored by a weak magnetic field and an electric field across the graphene layers, respectively. When the spin-degree of freedom is accounted for a topological insulator phase[22, 23, 9, 24] can also appear. In section IV we discuss the Luttinger liquid systems which appear at boundaries between different phases. We conclude with a brief summary.

2. Classification of Broken Symmetry States
We discuss the electronic properties of \( N \)-layer ABC-stacked systems in terms of ordered state mean-field Hamiltonians of the form[9],

\[
H_N = \frac{(v_0 p)^N}{(-\gamma_1)^{N-1}} [\cos(N\phi_p)\sigma_x + \sin(N\phi_p)\sigma_y] + m\sigma_z .
\]

We have used the notation \( \cos \phi_p = \tau_x p_x/p \) and \( \sin \phi_p = p_y/p \) where \( \tau_x = \pm 1 \) labels valleys \( K \) and \( K' \), the two inequivalent Brillouin zone corners. The Pauli matrices \( \sigma \) act on a which-layer
pseudospin degree-of-freedom. In Eq. (1) the first term is the low-energy \( \vec{k} \cdot \vec{p} \) band Hamiltonian for a single valley\[5, 7\]. Weak remote hopping processes have been dropped with the view that they do not play an essential role in the broken symmetry states. The second term is an interlayer interaction induced gap term which defines the direction of layer polarization in the momentum space vortex core\[10, 8\]. For each spin and valley, symmetry is broken by choosing a sign for \( m \)[8, 10]. We have dropped the momentum dependence of \( m \) because, as we see below, it does not play an essential role. In the mean-field Hamiltonians \( 2|m| \) is the size of the spontaneous gap, \( v_0 \) is the Fermi velocity in monolayer graphene, and \( \gamma_1 \sim 0.4 \text{ eV} \) is the inter-layer hopping energy. The \( p^N \) dispersion is a consequence of the \( N \)-step process in which electrons hop between low-energy sites in top and bottom layers via high-energy states.

When spin and valley degrees-of-freedom are taken into account, the system has sixteen distinct broken symmetry states in which the sign of \( m \) is chosen separately for \((K \uparrow), (K \downarrow), (K' \uparrow) \) and \((K' \downarrow)\) flavors\[9\]. We take the view that any of these states could be stable, depending on details\[17\] that are beyond current knowledge and might be tunable experimentally. The sixteen states can be classified according to their total layer-polarization which is proportional to the sum over spin-valley of the sign of \( m \). Six of the sixteen states have no net charge transfer between top and bottom layers and are likely to be lowest in energy in the absence of an external electric field. These six states can be separated into three doublets which differ only by layer inversion in every spin-valley. Thus three essentially distinct states compete for the broken symmetry ground state\[9\]: the anomalous Hall state in which the sign of \( m \) is valley-dependent but not spin-dependent \((m\sigma_z \rightarrow m\tau_z\sigma_z)\), the layer-antiferromagnetic state in which \( m \) is only spin-dependent \((m\sigma_z \rightarrow m\sigma_z\sigma_z)\) and the topological insulator (TI) state in which \( m \) is both spin and valley dependent \((m\sigma_z \rightarrow m\tau_z\sigma_z\sigma_z)\). Other interesting states\[9\] with net layer charge transfer between top and bottom layers can be favored by a potential difference between the layers: for example the valley Hall state in which \( m \) is neither valley nor spin dependent and the "All" state in which \( m \) depends on both valley and spin \((m\sigma_z \rightarrow m(1+\tau_z/2 + 1-\tau_z/2)s_z\sigma_z)\).[9]

These states are distinguished\[9\] by their spin and valley dependent Hall conductivities and orbital magnetizations indicated schematically in Fig. 1 and summarized in Table 1. The broken symmetry states are distinguished by the signs of the Berry curvatures contributions\[9\] from near the \( K \) and \( K' \) valleys of \( \uparrow \) and \( \downarrow \) spin bands.

The Berry curvatures\[25, 9\] are non-zero only when inversion symmetry is spontaneously broken and act like an effective magnetic field in momentum space. In a gapped monolayer graphene, this effective magnetic field around one valley can be understood as a field generated by a magnetic monopole located at the valley center. The Berry curvature has opposite sign\[9\] near the two inequivalent Brillouin zone corners for a quantum valley Hall state as we see

| Fig. | \( K \uparrow \) | \( K \downarrow \) | \( K' \uparrow \) | \( K' \downarrow \) | \( \sigma^{(SH)} \) | \( \sigma^{(VH)} \) | \( \sigma^{(CH)} \) | \( \sigma^{(SHV)} \) | Insulator |
|------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------|
| 1(b) | T              | T              | B              | B              | 0              | 0              | 2N             | 0              | QAH      |
| 1(c) | T              | B              | T              | B              | 0              | 0              | 0              | 2N             | LAF      |
| 1(d) | T              | B              | B              | T              | 2N             | 0              | 0              | 0              | QSH      |
| 1(a) | T              | T              | T              | T              | 0              | 2N             | 0              | 0              | QVH      |
| 1(e) | T              | T              | T              | B              | N              | N              | N              | N              | All      |

Table 1. Summary\[9\] of spin-valley layer polarizations (T or B) and corresponding charge, spin, and valley Hall conductivities \( (e^2/h \text{ units}) \) and insulator types for the three distinct states (b-d) with no overall layer polarization, for a state in which every spin-valley is polarized toward the top layer (a), and for a state with partial layer polarization (e).
Figure 2. (Color online) Berry curvature (an effective magnetic field in momentum space) as a function of momentum-space position in the bilayer graphene Brillouin zone[9]. (a) For a quantum valley Hall state; (b) for a quantum anomalous Hall state. Large energy gaps are chosen deliberately in order to visualize the shape of the peaks and the trigonal warping effect.

in Fig. 2(a) but has the same sign[9] at both corners for a quantum anomalous Hall state as illustrated in Fig. 2(b). The Berry curvature not only has sharp peaks near all the Brillouin zone corners but at these large mass values exhibits large trigonal warping effect as well, reflecting the $C_3$ symmetry of the sublattice. In the presence of an in-plane electric field, an electron acquires an anomalous transverse velocity proportional to the Berry curvature, giving rise to an intrinsic Hall conductivity[25, 19]. Provided that the Fermi level lies in the mass gap, we find that the intrinsic Hall conductivity contribution[9] from a given valley and spin is

$$\sigma_H(\tau_z, s_z) = \tau_z \text{sgn}(m) \frac{Ne^2}{2h},$$

(2)
each spin and valley contributes $Ne^2/2h$ to the total Hall conductivity, with the sign given by $\tau_z \text{sgn}(m)$.

3. Competing Orders

In the spinless case, there are only two broken symmetry states, namely, the anomalous Hall state and the valley Hall state. In the case of a valley Hall state, valley $K$ and $K'$ have the same mass $m_0 \sigma_z$, and each valley is layer polarized in the same sense[9]. Mass terms with the valley Hall form can be generated simply by a potential difference between the layers, so the valley Hall state is an easily generated non-interacting electron state. The total Hall conductivity of this state is zero, i.e. $\nu = 0$ for bilayer graphene, with the $K$ and $K'$ valleys making Hall conductivity and magnetization contributions of opposite sign, preserving time reversal symmetry. The valley Hall states can be favored experimentally by applying a perpendicular electric field[9, 21], as shown in Fig. 3(a).

In the anomalous Hall state, on the other hand, one valley has a positive mass $m_3 \sigma_z$ while the other has a negative mass $-m_3 \sigma_z$. Correspondingly the two valleys spontaneously choose different layer polarizations[9]. Therefore the Hall conductivity and orbital magnetization contributions have the same sign for each valley[9]. This state breaks time reversal symmetry. The total Hall conductivity has the quantized value $\pm 2Ne^2/h$, i.e. $\nu = \pm 4$ in the case of bilayer graphene. In addition to its anomalous Hall effect, this state has a substantial orbital magnetization. The anomalous Hall state is probably most simply identified experimentally[11, 12] by observing a $\nu = 2N$ QHE which persists to zero magnetic field[9, 11, 12, 16], as depicted in Fig. 3(b).
The spectra of translationally invariant two-dimensional electron systems are quantized into Landau levels in the presence of a finite perpendicular magnetic field. In a gapless chirally stacked graphene \( N \)-layer, the zero energy Landau level (Fig. 3(c)) is \( 4N \) fold degenerate. In the case of a valley Hall insulator, the states around \( K \) and \( K' \) valleys have very large orbital magnetic moments with opposite sign, which couple to the out-of-plane magnetic field and gives rise to the relative energy shift between valley \( K \) and \( K' \). Consequently, the two-fold valley degeneracy is lifted and there is an asymmetry between those Landau levels around two Dirac points, as described in Fig. 3(d). The energy spectrum of a quantum valley Hall insulator is adiabatically connected to that of the \( \nu = 0 \) quantum Hall state. In contrast, as shown in Fig. 3(e)(f), the valley degeneracy is unbroken in the presence of magnetic field, because the nature of the orbital magnetic moments near the two Dirac points are the same. The energy gap in a quantum anomalous Hall state is adiabatically connected to that of the \( \nu = \pm 4 \) quantum Hall state where the sign is a choice of whether the magnetic field is in \( \hat{z} \) or \( -\hat{z} \) direction relative

**Figure 3.** (Color online) (a) Ribbon quantum valley Hall states and (b) ribbon quantum anomalous Hall states in the absence of magnetic fields; (c) Ribbon quantum Hall states for a gapless bilayer graphene; (d) \( \nu = 0 \) quantum Hall states for a gapped bilayer graphene in QVH phase; (e) and (f) \( \nu = \pm 4 \) quantum Hall states for a gapped bilayer graphene in QAH phase. To visualize the edge states, we use a zigzag ribbon, the intralayer and interlayer nearest neighbor hoppings are chosen as \( \gamma_0 = 1 \) and \( \gamma_1 = 0.3 \), respectively, and nonzero \( m \) is fixed as 0.15.
to $m_3$.

When spin is taken into account, the quantized Landau levels of the three additional phases[9] are obtained by each spin choosing to be a quantum valley Hall state or a quantum anomalous Hall state. For the case of a quantum spin Hall state, one spin flavor is the $\nu = 2$ QAH state and the other flavor is the $\nu = -2$ QAH state. For the layer-antiferromagnetic state, each spin flavor is a $\nu = 0$ QVH state but with the opposite layer polarization to the other flavor. In contrast, one spin flavor is a QVH state while the other is a QAH state in the "All" state.

By continuously lowering the magnetic field to zero, an anomalous Hall state is adiabatically identified[11, 12]. When a perpendicular electric field is switched on, the quantum valley Hall phase starts to compete with the quantum anomalous Hall phase. The sizes of the gap at the two valleys are $|m_0 + m_3|$ and $|m_0 - m_3|$, respectively. The state is still within the anomalous Hall phase but with two unequal gaps at $K$ and $K'$ for $|m_0| < |m_3|$ as seen in Fig. 4(a). Beyond the critical point where $|m_0| = |m_3|$ as depicted in Fig. 4(c), the state jumps to the valley Hall phase instead. In the quantum phase transition region as described by Fig. 4(b), the energy gap is enhanced at one valley while it closes at the other valley, where quantum anomalous and valley Hall effects coexist.

![Figure 4.](image). (Color online) The competition between the quantum anomalous Hall state (intra-valley edge state) and the quantum valley Hall (inter-valley edge state) state in broken symmetry bilayer graphene. (a) a quantum anomalous Hall state with unequal gaps at $K$ and $K'$; (b) a critical point with an enhanced gap at $K$ and a closed gap at $K'$, where quantum anomalous and valley Hall effects coexist; (c) a quantum valley Hall state with a reopened gap at $K'$. To visualize the edge states, we use a zigzag ribbon with the same $\gamma_0$ and $\gamma_1$ values as in Fig. 3.

In the quantum spin Hall state[9], the helical edge modes are likely to localize in a $N$-even system, due to the possible backscattering process in which $N$ left movers and $N$ right movers scatter into each other allowed by time reversal symmetry. Therefore, the QSH phase is topologically protected in chirally stacked odd number of graphene layers. Besides the effective spin-orbit coupling induced by interlayer interaction, Rashba interaction $\lambda_R(\tau_x\sigma_x s_x - \sigma_y s_y)$ is possibly induced by inversion symmetry breaking, and a small staggered potential $m_S\sigma_z$ is also present due to coupling to substrates. These interactions lift the degenerate zero energy to $m_{SH} \pm \lambda_S$, $-m_{SH} \pm \sqrt{4\lambda_R^2 + \lambda_S^2}$. The topological-nontrivial phase persists as long as the effective spin-orbit gap is not closed, or in other words,

$$\left(\frac{\lambda_R}{m_{SH}}\right)^2 + \left|\frac{\lambda_S}{m_{SH}}\right| \leq 1,$$  \quad (3)
which is independent of layer number $N$ and determines the phase diagram. Trigonal warping and other remote hopping terms are time reversal invariant, not disfavoring the TI phase.

4. Edge States at Domain Walls
At zero temperature, in a clean chirally stacked few-layer graphene system, there are 16 possible broken symmetry states and they are classified as 5 distinct phases, as discussed in the section II. In the presence of disorder or thermal fluctuations, different phase are likely to appear locally in different parts of the system. There are 240 possible domain walls and they can be classified into 16 distinct types. In the spinless case, as only valley Hall phase and anomalous Hall phase are allowed, there are 2 types of intra-phase and 1 type of inter-phase domain walls. At each type of domain wall, we argue that a unique Luttinger liquid emerges; the spinless bilayer examples of which are illustrated in Fig. 5.

In a quantum valley Hall state, the Chern numbers[2, 9] of the two valleys are $\pm N/2$, respectively. For the case of a quantum anomalous Hall state, the Chern number[2, 9] is $N/2$ at both valleys with a uniform sign. These features are equivalently shown in Fig. 2. At the domain wall separating two quantum valley Hall regions with the opposite layer polarization, the Chern numbers change by $\pm N$ for a single valley and hence $N$ parallel zero modes appear at each valley as depicted in Fig. 5(a). These one dimensional zero modes form $N$ copies of full Luttinger liquids and the valley-pseudospin becomes exactly the left-or-right chirality. This QVH-QVH domain walls and the generated Luttinger liquids are likely to be formed in the electron-hole puddles, and can be easily realized and tuned by an external electric field[26].

At a domain wall separating two quantum anomalous Hall regions with opposite total Hall conductance, the change of Chern number is $N$ (neglecting spin) for both valleys. Therefore, we expect that $N$ parallel zero modes appear at each valley as seen in Fig. 5(b). Each valley has a copy of a purely chiral ”spin”-like $N/2$ like Luttinger liquid. At the domain wall between a quantum valley Hall and a quantum anomalous Hall regions, the chern number is changed by $N$ for one valley while it is preserved for the other. Thus the edge states at the interface are purely chiral at one valley while they completely disappear at the other. We expect the Luttinger liquid behaviors of the one dimensional zero modes at the spinfull domain walls are much more exotic.

5. Discussion
The trigonal warping effect is sometimes ignored in theoretical efforts to identify the broken symmetry physics based on mean field theory or renormalization group analysis. This is reasonably justified in the bilayer case for the following two reasons. (i) The trigonal warping effect dominates only below 1 meV[5], compared to relevant band broadening not much smaller than 1 meV. Trigonal warping effect are therefore likely to be smeared and become inessential, due to residue disorder and fluctuations at higher energies. (ii) The infrared cutoff of the RG flows can be reasonably set at where the quadratic band dispersion overwhelms the trigonal warping effect. For chirally stacked graphene with the layer number $N > 2$, the trigonal warping energy scale is increased by almost a factor of 10[7] while the even flatter gapless bands are much more unstable to interlayer interactions. Interaction effects are expected to dominate at low carrier densities[7] and to drive the spontaneous inversion symmetry breaking, but samples that are clean enough to reveal its interaction physics have not yet been studied. We comment that the broken symmetry is likely to occur only in low disorder and high quality samples, since large disorder can destroy the perfect nesting conditions and reduce the parameter space of the RG flows.

Recent experiments[11, 12] in bilayers appear to provide definitive proof that, at $\nu = \pm 4$, the ground state at very weak external magnetic fields is the quantized anomalous Hall state. (At $\nu = 0$, LAF state (or called SDW state) and QSH state are competing for the ground state at zero magnetic field.) Even though a QAH state does not have a finite spin-polarization, the orbital
magnetic moment close to each Dirac point has a symmetric sharp peak\cite{9} at which individual states carry moments twenty times larger than $\mu_B$, and a total orbital magnetization per area goes as $\sim (N\lambda n_e/2\pi\hbar^2) \ln(\gamma_1/|\lambda|)\mu_B$, that is $\sim 0.002\mu_B$ per carbon atom for $|m| = 10$ meV\cite{9}. Thus the energy of the quantized anomalous Hall state will be lowered by a perpendicular external magnetic field. Lattice mean field theory shows the size of the gap can be as large as 30 meV\cite{17} but the actual ground state is dependent on subtle correlation and microscopic physics issues. We estimate that a magnetic field of the order of 0.004 T is sufficient to favor the QAH state over the QVH state\cite{17}. Increasing the magnetic field further results in quantum Hall ferromagnetism\cite{27, 28, 29}. The fully layer polarized QVH state will be favored by an external electric field which produces a potential difference between the layers. The turning point of the band gap is approximately 7 mV/nm for a perpendicular electric filed\cite{17}.

The quantum spin Hall effect we discuss in this paper is in several respects different from that discussed in the well known papers\cite{22, 23} which foreshadowed the identification of topological insulators. (i) The broken symmetry occurs only for $N \geq 2$, rather than in the single-layer systems\cite{22, 23}. (ii) The quantum spin Hall effect is driven by broken symmetries produced by weak repulsive interaction instability, rather than by spin-orbit interactions\cite{22, 23}. The interaction induced effective spin-orbit coupling $\lambda s_z\sigma_z\sigma_z$ can be $10^4$ times larger\cite{9, 17} than the intrinsic one\cite{30}.

We close this discussion by pointing out that the edge states of a broken symmetry state have physical significance\cite{9}. (i) The edge states for QAH phase are inter-valley one dimensional gapless modes while the zero modes are intra-valley like for QVH phase\cite{9}. (ii) The states with anomalous Hall effects have $N$ topologically protected robust chiral edge states associated with the QHE\cite{9}. (iii) For the valley Hall effects, in general we expect $[N/2]$ chiral edge state branches at each valley in an $N$-layer stack\cite{9}; the full $e^2/h$ unit of Hall conductance requires the two valleys to act in concert; the additional half quantum Hall effect from each valley in

---

**Figure 5.** (Color online) Three distinct Luttinger liquids at domain walls in spinless bilayer graphene with broken symmetry. The red lines denotes the zero modes localized at domain walls between (a) two quantum valley Hall regions with opposite layer polarization; (b) two quantum anomalous Hall regions with opposite total Hall conductance; (c) a quantum valley Hall region and a quantum anomalous Hall region. The green lines represent the edge states on the outermost zigzag boundaries and note that they are doubly degenerate in (a) and (b) for the two zigzag boundaries. To visualize the edge states, we use a zigzag ribbon with the same $\gamma_0$ and $\gamma_1$ values as in Fig. 3 and $|m| = 0.25$.
the \(N\)-odd layers is insufficient to produce a new chiral edge state branch. This is a novel manifestation of the chiral anomaly in condensed matter systems. Of course valley Hall edge states are topologically protected only when the edge-direction projections of \(K\) and \(K'\) valleys are not coincident and inter-valley scattering due to disorder is absent.

Acknowledgement
This work has been supported by Welch Foundation under grant TBF1473, NRI-SWAN, DOE grant Division of Materials Sciences and Engineering DE-FG03-02ER45958, and ARO grant W911NF-09-1-0527. We acknowledge helpful discussions with C. Lau, B. Halperin, A. Yacoby, K. Novoselov, W. Bao, J. Velasco and C. Schönenberger. When this manuscript is about to publish, some experiments in graphene bilayers\cite{bilayer_exp1, bilayer_exp2} and trilayers\cite{trilayer_exp} appear online, and a gapped \(\nu = 0\) state has been observed in bilayer graphene at zero magnetic field\cite{bilayer_exp1}.

References
[1] K. Von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. 45, 494 (1980).
[2] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. Dennijs, Phys. Rev. Lett. 49, 405 (1982).
[3] A. K. Geim and A. H. MacDonald, Phys. Today 60(8), 35(2007).
[4] A. H. Castro Neto et al., Rev. Mod. Phys. 81, 109 (2009).
[5] E. McCann and V. I. Fal’ko, Phys. Rev. Lett. 96, 086805 (2006).
[6] Hongki Min and A. H. MacDonald, Phys. Rev. B 77, 155416 (2008).
[7] F. Zhang, B. Sahu, H. Min, A. H. MacDonald, Phys. Rev. B 82, 035409 (2010).
[8] F. Zhang, H. Min, M. Polini and A. H. MacDonald, Phys. Rev. B 81, 041402(R) (2010).
[9] F. Zhang, J. Jung, G. A. Fiete, Q. Niu and A. H. MacDonald, Phys. Rev. Lett. 106, 156801 (2011).
[10] H. Min, G. Borghi, M. Polini and A. H. MacDonald, Phys. Rev. B 77, 041407(R) (2008).
[11] J. Martin et al., Phys. Rev. Lett. 105, 256806 (2010).
[12] R. T. Weitz et al., Science 330, 812 (2010).
[13] R. Nandkishore and L. Levitov, Phys. Rev. Lett. 104, 156803 (2010).
[14] O. Vafek and K. Yang, Phys. Rev. B 81, 041401(R) (2010).
[15] F. Guinea, Physics 3, 1 (2010).
[16] R. Nandkishore and L. Levitov, Phys. Rev. B 82, 115124 (2010).
[17] J. Jung, F. Zhang, A. H. MacDonald, Phys. Rev. B 83, 115408 (2011).
[18] F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988).
[19] N. Nagaosa, J. Sinova, S. Onoda, A. H. MacDonald and N. P. Ong, Rev. Mod. Phys. 82, 1539 (2010).
[20] D. Xiao, W. Yao and Q. Niu, Phys. Rev. Lett. 99, 236809 (2007).
[21] J. Li, I. Martin, M. Buttiker, A. F. Morpurgo, Nature Phys. 7, 38 (2011).
[22] C. L. Kane and E. J. Mele, Phys. Rev. Lett. 95, 226801 (2005).
[23] S. Raghu, X. Qi, C. Honerkamp, and S. Zhang, Phys. Rev. Lett. 100, 156401 (2008).
[24] E. Prada, P. San-Jose and L. Brey, arXiv:1007.4910 (2010).
[25] D. Xiao, M. Chang and Q. Niu, Rev. Mod. Phys. 82, 1959 (2010).
[26] I. Martin, Y. M. Blanter, and A. F. Morpurgo, Phys. Rev. Lett. 100, 036804 (2008).
[27] Y. Barlas, R. Cote, K. Nomura and A. H. MacDonald, Phys. Rev. Lett. 101, 097601 (2008).
[28] B. Feldman, J. Martin and A. Yacoby, Nature Phys. 5, 889 (2009).
[29] Y. Zhao, P. Cadden-Zimansky, Z. Jiang and P. Kim, Phys. Rev. Lett. 104, 066801 (2010).
[30] H. Min et al., Phys. Rev. B 74, 165310 (2006).
[31] J. Velasco Jr. et al., to appear soon (2011).
[32] F. Freitag et al., arXiv:1104.3816 (2011).
[33] W. Bao et al., arXiv:1103.6088 (2011).