QCD Sum Rules for Heavy Flavour Physics

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Abstract. Uses of QCD sum rules for heavy flavoured hadrons are discussed. "Standard" applications such as the determination of the $b$, $c$ quark masses, the calculation of $f_B$, $f_D$ and of the heavy-to-light form factors are overviewed. Furthermore, a new approach to calculate the $B \to \pi \pi$ hadronic matrix elements from QCD light-cone sum rules is described.

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INTRODUCTION

Starting from the first work [1] the method of QCD sum rules was frequently applied to various problems of heavy flavour physics. Nowadays, different versions of the sum rule approach are used, all of them based on the general idea of calculating a quark-current correlation function and relating it to the hadronic parameters via dispersion relations.

The original version [1] (often called SVZ sum rules) employs the operator-product expansion (OPE) of correlation functions in terms of quark and gluon vacuum condensates. A typical and important application of this technique is the calculation [2, 3, 4] of the $B$-meson decay constant $f_B$ defined as

$$\langle 0 | m_b \bar{q} \gamma_5 b | B \rangle = f_B m_B^2, \quad q = u, d.$$  

One starts from the correlation function of two heavy-light currents

$$\Pi(q^2) = i \int d^4 x e^{ixq} \langle 0 | T \{ m_p \bar{q} \gamma_5 b(x), m_p \bar{b} \gamma_5 q(0) \} | 0 \rangle.$$  

Depending on the region of the momentum transfer $q$, the amplitude $\Pi(q^2)$ represents either a short-distance fluctuation (at $q^2$ far below $m_B^2$) or a complicated sum over hadronic states (at $q^2 \geq m_B^2$) starting from the ground-state $B$ meson. At $|q^2 - m_B^2| \gg \Lambda_{QCD}^2$, the correlation function (1) is approximately calculated in terms of the condensate expansion including the perturbative part (the loop and $O(\alpha_s)$ correction) and the quark-, gluon- and quark-gluon condensate contributions. A detailed derivation and the resulting expression can be found, e.g. in Ref. [5]. On the other hand, the correlation function (1)

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obeys the dispersion relation, schematically

\[ \Pi(q^2) = \frac{f_B^2 m_B^4}{m_B^2 - q^2} + \sum_{B_h} \frac{f_{B_h}^2 m_{B_h}^4}{m_{B_h}^2 - q^2}, \quad (2) \]

where the contribution of the ground-state \( B \) meson is shown explicitly and the sum over \( B_h \) represents the excited resonances and the continuum of hadronic states with the \( B \) meson quantum numbers. Matching the condensate expansion of \( \Pi(q^2) \) with the dispersion relation one obtains the primary sum rule. To achieve practical results one may proceed in different directions. Knowing the values of universal QCD parameters such as \( \alpha_s \), \( m_b \) and the condensate densities \( \langle G^2 \rangle \), \( \langle q\bar{q} \rangle \) it is possible to estimate \( f_B \) applying the quark-hadron duality approximation to the contribution of higher states in Eq. (2). A reverse way to use QCD sum rules is available in cases when the parameters of few lowest hadronic states in the dispersion relation are measured. Saturating the hadronic part of the sum rule it is then possible to determine the QCD parameters. This kind of analysis is accessible for the two-point correlation functions of \( b\gamma_\mu b \) or \( c\gamma_\mu c \) currents, where the hadronic states are \( \Upsilon \)- or \( \psi \)-resonances, respectively, with measured decay constants and masses.

A different version of QCD sum rules, the so called light-cone sum rules (LCSR) [6] are attracting a lot of attention in recent years. LCSR are used to calculate various hadronic transition matrix elements. The outline of this method can be illustrated taking the \( B \to \pi \) form factor calculation [7] as an example. The correlation function is in this case a vacuum-to-pion matrix element

\[ F_\lambda(p, q) = i \int d^4 x e^{ipx} \langle \pi^+(q) | T \{ \bar{u} \gamma_\mu b(x), m_b, \bar{b} \gamma_5 d(0) \} | 0 \rangle \]

\[ = F((p + q)^2, p^2)q_\mu + \tilde{F}((p + q)^2, p^2)p_\mu, \quad (3) \]

correlating the \( b \to u \) weak current and the heavy-light current interpolating \( B_d \) meson. At large virtualities, that is at \( (p + q)^2 - m_b^2 \gg \Lambda_{QCD}^2 \) and \( p^2 \ll m_b^2 \) this correlator can be calculated employing OPE near the light-cone \( x^2 = 0 \). The result is expressed in terms of pion distribution amplitudes (DA) of growing twists, the most important one being the twist-2 pion DA defined as [8]

\[ \langle \pi^+(q) | \bar{u}(x)\gamma_\mu \gamma_5 d(0) | 0 \rangle = -i q_\mu f_\pi \int_0^1 du e^{iqux} \varphi_\pi(u). \quad (4) \]

Higher-twist contributions are suppressed by inverse powers of \((p + q)^2 - m_B^2\). Again, using the dispersion relation in the channel of the \( B \)-meson current, one obtains

\[ F((p + q)^2, p^2) = \frac{2 f_B f_{\pi \pi}^+ (p^2) m_B^2}{m_B^2 - (p + q)^2} + \sum_{B_h} \frac{2 f_{B_h} f_{B_h \pi}^+ (p^2) m_{B_h}^2}{m_{B_h}^2 - (p + q)^2}, \quad (5) \]

where the ground-state contribution contains a product of \( f_B \) and the \( B \to \pi \) form factor \( f_{B \pi}^+(p^2) \) defined in the standard way: \( \langle \pi^+(q) | \bar{u} \gamma_\mu b | B^0(p + q) \rangle = 2 f_{B \pi}^+(p^2) q_\mu + \ldots \)
Matching the result of the twist expansion for the amplitude $F$ with the dispersion relation one calculates the $B \to \pi$ form factor provided that $f_B$ is obtained from the SVZ sum rule as explained above. Conversely, one may use LCSR to estimate the parameters of the light-cone DA by saturating the sum-rule relations with the experimentally known form factors and decay constants. This is possible, e.g. for various vacuum-to-pion correlators of light-quark currents yielding LCSR for $\gamma \gamma^* \to \pi$ [9] or for the pion e.m. form factor [10]. Again, quark-hadron duality is employed in the same way as in SVZ sum rules. Note that in LCSR the meson DA (such as $\phi_{\pi}(u)$ with its normalization factor $f_{\pi}$) play the role of nonperturbative inputs, similar to the role the vacuum condensates play in SVZ sum rules.

For completeness, one has to mention that an independent version of QCD sum rules emerges when one takes the local limit of LCSR. The result is equivalent to the sum rules in the external field [11]. In particular, for the pion-to-vacuum correlation function (3) the local limit ($q \to 0$) corresponds to the external soft-pion field.

One should always bear in mind that QCD sum rules have a limited accuracy. Both OPE and the duality approximation should be kept under control by performing the Borel transformation or taking the power moments. The working region of the corresponding auxiliary parameters (Borel parameter or the number of moment) has to be restricted, so that the contributions of excited states and higher orders in OPE are simultaneously small.

Why QCD sum rules are in particular advantageous for heavy flavour physics? First of all, the presence of an intrinsic heavy-quark mass scale provides necessary conditions for applying the short-distance or light-cone OPE to the correlation functions. One has to emphasize that no infinite quark mass limit is necessary. The sum rules can be derived in full QCD for finite $b$ and $c$ quark masses. Moreover, since the correlation functions are Lorentz-covariant objects one does not need nonrelativistic approximations. On the other hand, the sum rule method is very flexible and can be applied in the frameworks of the effective QCD theories such as HQET and NRQCD. Finally, since QCD sum rules employ universal inputs (condensates, light-cone DA) there is a possibility to estimate theoretical uncertainties in the determination of heavy-flavour parameters such as $f_B$ or $f_{B^*}$.

In what follows I will overview the status of a few "standard" applications of QCD sum rules: the $m_b, m_c$ determination, the calculation of $f_B, f_D$ and of the $B \to \pi,K,...,D \to \pi,K,...$ form factors. Furthermore, I describe a new application of the method, the calculation of the $B \to \pi\pi$ matrix elements. I will not discuss many other interesting applications such as $B_c$ meson, $B - \bar{B}$ mixing parameter, properties of heavy baryons, sum rules in HQET. These issues as well as many other details concerning QCD sum rules can be found in the recent review [12].
TABLE 1. $b$-quark mass determination: QCD sum rules vs other methods; $m_b(\overline{m}_b)$ is the pole ($\overline{MS}$) mass

| $m_b$ (GeV) | $\overline{m}_b(\overline{m}_b)$ (GeV) | Ref. | Method |
|------------|--------------------------------------|------|--------|
| 4.72 ± 0.05 | [14] | SVZ |
| 4.62 ± 0.02 | [15] | “ |
| 4.827 ± 0.007 | [16] | |
| 4.84 ± 0.08 | 4.19 ± 0.06 | [17] | |
| 4.26 ± 0.10 | [18] | NRQCD |
| 4.88 ± 0.10 | 4.20 ± 0.06 | [19] | SR |
| 4.80 ± 0.06 | 4.21 ± 0.11 | [20] | |
| 4.97 ± 0.17 | 4.26 ± 0.10 | [21] | |
| 5.04 ± 0.09 | 4.44 ± 0.04 | [22] | |
| – | 4.24 ± 0.09 | [21] | \(\Upsilon\) |
| – | 4.21 ± 0.07 | [19] | +NRQCD |
| 4.21 ± 0.09 | 4.21 ± 0.025 | [23] | |
| – | 4.26 ± 0.11 | [24] | lattice av. |

"STANDARD" APPLICATIONS OF QCD SUM RULES

$b$ and $c$ quark masses

The heavy quark mass $m_Q$, $Q = b$ or $c$, can be determined if one considers the two-point correlation function of two $\bar{Q} \gamma_{\mu} Q$ currents and uses for the hadronic dispersion relation the experimental data on the masses and electronic widths of $J^P = 1^-$ quarkonium levels, $\Upsilon$ or $\psi$ resonances, respectively. In recent years there has been a lot of progress in the determination of the $b$ quark mass employing precise data on six $\Upsilon(nS)$ resonances obtained in $e^+ e^-$-annihilation. The emphasize was made on working with the highest possible power moments of the correlation function in which the region of small quark velocities $v_Q = \sqrt{1 - 4m_Q^2/q^2}$ dominates and the NRQCD approximation is valid. In this framework it is possible to sum over Coulomb $O(\alpha_s^n/v_Q^n)$ terms and to include relativistic corrections order by order (for a recent review, see e.g. Ref. [13]). An alternative approach is to stay within full QCD and to employ few first moments of the SVZ sum rules, determining $m_b$ in a purely relativistic way. In this case the Coulomb resummation is not accessible but also not that important. The price to pay is the sensitivity to the nonresonant tail of the hadronic spectral function in the dispersion relation which, in principle, can be reliably estimated from experimental data on the inclusive $e^+ e^- \rightarrow b\bar{b}$ cross section above resonances. In Table 1 the earlier SVZ and more recent NRQCD sum rule results are compared with the $m_b$ determinations using other approaches. The values of $m_b(\overline{m}_b)$ obtained by various methods agree within uncertainties. The potential of SVZ sum rules for $b$-quarkonium in full QCD is not yet thoroughly exploited, e.g. there is a possibility to include the already available $O(\alpha_s^2)$ corrections to the correlation function [25].

The predictions for the $c$-quark mass obtained from QCD sum rules for the charmonium system are collected in Table 2, in comparison with the results of various other...
TABLE 2. \( m_c \) determination: QCD sum rules vs other methods.

| \( m_c \) (GeV) | \( \overline{m}_c(\overline{m}_c) \) (GeV) | Ref. | method          |
|-----------------|------------------|------|-----------------|
| 1.46 ± 0.05     |                  | [27] | SVZ             |
| 1.42 ± 0.03     | 1.23±0.02 ± 0.03 | [15] | "              |
| 1.70 ± 0.13     | 1.23 ± 0.09      | [26] | SVZ+NRQCD       |
| 1.70 ± 0.13     | 1.37 ± 0.09      | [28] | FESR            |
| 1.70 ± 0.13     | 1.21 ± 0.07 ± 0.065 | [23] | \( m_B - m_D \) + HQET |
| 1.525 ± 0.040 ± 0.125 | [29] | lattice QCD    |
| 1.33 ± 0.08     |                  | [30] | "              |
| 1.20 ± 0.04 ± 0.11 ± 0.2 | [31] | latt. NRQCD    |

methods. The most recent sum rule analysis [26] combines NRQCD at small \( v_c \) with the full QCD spectral function at large \( v_c \). The latter includes \( O(\alpha_s)^2 \) terms and \( d = 6,8 \) gluon condensates. Although the value of \( \overline{m}_c(\overline{m}_c) \) is in agreement with earlier sum rule determinations, the pole \( c \) quark mass is surprisingly large, being interpreted [26] as a result of large Coulomb corrections. At this point one has to note that for the charmonium levels the applicability of NRQCD is questionable due to the large average values of \( v_c \). An update of the full QCD SVZ sum rule at the \( O(\alpha_s^2) \) level remains a task which deserves attention. Summarizing the estimates given in Tables 1,2 I will adopt the following intervals of the pole quark masses obtained from QCD sum rules:

\[
m_b = 4.8 ± 0.1 \text{ GeV}, \quad m_c = 1.3 ± 0.1 \text{ GeV}.
\]  

\( f_B \) and \( f_D \)

Having determined the value of \( m_b \) with a certain accuracy it is possible to calculate \( f_B \) from the SVZ sum rule based of the dispersion relation (2). For the \( D \)-meson decay constant \( f_D \) the analogous sum rule is obtained by a simple \( b \to c \) (\( \bar{B} \to D \)) replacement in the \( f_B \) sum rule, together with the necessary adjustment of the normalization scale. Moreover, switching to \( q = s \) in Eq. (1) it is possible to predict also the ratios \( f_{Bs}/f_B \) and \( f_{Ds}/f_D \). Currently, the OPE of the correlation function (1) includes the \( O(\alpha_s) \) correction to the perturbative part which is rather large and all \( d \leq 6 \) condensate contributions.

The values of \( f_B \) and \( f_D \) determined from SVZ sum rules are quite sensitive to the \( b \) and \( c \) quark pole masses. Varying the latter in the intervals (6) one typically obtains (see, e.g. the review [12]):

\[
f_B = 170 \mp 30 \text{ MeV}, \quad f_D = 180 \mp 30 \text{ MeV},
\]

\[
f_{Bs}/f_B = 1.16 ± 0.09, \quad f_{Ds}/f_D = 1.19 ± 0.08.
\]

Here, the normalization scales \( \mu^2_B \sim m_B^2 - m_b^2 \) and \( \mu^2_c \sim m_D^2 - m_c^2 \), respectively are adopted. These scales are of the order of the corresponding Borel parameters reflecting the
average virtualities of the quarks in the correlators. Within uncertainties, the predictions (7) agree with the lattice determinations of the heavy meson decay constants.

Are improvements in $f_{B,D}$ determination still possible? Apart from narrowing the intervals for heavy quark masses one has to cope with the numerically large $O(\alpha_s)$ correction. Studies of $O(\alpha_s^2)$ effects, at least for an accurate fixing of the relevant scale in the $O(\alpha_s)$ term are necessary. An important progress in this direction is the calculation of the three-loop radiative corrections to the heavy-to-light correlator completed just recently [32]. The accuracy of the sum rule could be improved further if the $d=7$ corrections proportional to the heavy quark mass are calculated including both factorizable $\sim \langle G^2 \rangle \langle \bar{\psi} \psi \rangle$ and nonfactorizable $d=7$ condensates. On the hadronic side, it is important to get a better control over quark-hadron duality in the $B$ and $D$ channels. For the latter channel a valuable information could be provided by experimental studies of excited $D$ states in the semileptonic $B \rightarrow X_c \ell \nu$ decays. Knowing the positions of excited $D$ resonances one may try various alternative patterns for the hadronic spectral function in the $f_D$ sum rule and thereby test the validity and consistency of the duality approximation.

**Heavy-to-light form factors**

The procedure of obtaining LCSR for the $B \rightarrow \pi$ form factor is briefly outlined in the Introduction. More detailed discussion can be found in Ref. [5]. The most recent LCSR prediction [33] for $f_{B\pi}^+$ is presented in Fig. 1. This calculation includes twist 2 (LO and $O(\alpha_s)$ NLO) and twist 3,4 effects. The inputs used to calculate $f_{B\pi}^+$ from LCSR are: 1) the values of $f_B, m_b$ and the duality threshold, all taken from the SVZ sum rule for $f_B$ and 2) the pion DA of twist 2,3,4. The shapes of the latter are largely fixed by their asymptotic forms whereas the sensitivity of LCSR to the nonasymptotic effects in DA turns out to be very mild.
The LCSR result [33] is parametrized in a form suggested in Ref. [34]:

\[ f^+_B(p^2) = \frac{f^+_B(0)}{(1 - p^2/m^2_B)(1 - \alpha_B p^2/m^2_B)} \]  

(8)

with \( f^+_B(0) = 0.28 \pm 0.05 \) and \( \alpha_B = 0.32^{+0.21}_{-0.07} \). The theoretical uncertainties are estimated by varying all sum rule parameters within allowed regions and adding them up linearly. The accuracy of the form factor calculation can still be improved if the twist 3 \( \hat{O}(\alpha_s) \) correction is calculated and if the pion DA are better constrained, e.g. from LCSR for the pion form factors or from lattice QCD studies.

With the form factor (8) it is possible to calculate the semileptonic \( \bar{B}^0 \to \pi^+ l^- \) width (\( l = e, \mu \)) and to extract the CKM parameter \( |V_{ub}| \) using the current experimental data:

\[ BR(B \to \pi l\nu) = (1.8 \pm 0.4 \pm 0.4) \times 10^{-4} \text{ (CLEO [35]) and } BR(B \to \pi l\nu) = (1.28 \pm 0.20 \pm 0.26) \times 10^{-4} \text{ (Belle, preliminary [36]).} \]

The results are:

\[ |V_{ub}| = (4.0 \pm 0.6 \pm 0.7) \times 10^{-3} \text{ (CLEO), and } |V_{ub}| = (3.4 \pm 0.4 \pm 0.6) \times 10^{-3} \text{ (Belle),} \]

where the first error is experimental and the second one is caused by the theoretical uncertainty of the form factor calculation.

Replacing the pion with the kaon in the correlation function (3) leads to LCSR for the \( B \to K \) form factor, including the effects of the \( SU(3) \)-flavour symmetry breaking such as \( f_K/f_\pi \neq 1 \) and the asymmetry in the kaon DA \( \phi_K(u) \). Interestingly, the predicted ratio \[ f^+_B(0)/f^+_K(0) = 1.28^{+0.18}_{-0.10}, \]

(9)

is mainly sensitive to the value of the strange quark mass \( m_s(1\text{GeV}) = 150 \pm 50 \text{ MeV}. \) This result indicates that the rate of \( SU(3) \) breaking could be quite noticeable, an important message for studies of CP-violation in hadronic \( B \) decays where various \( SU(3) \) relations are frequently used. The semileptonic form factors in the charmed sector are also predicted from LCSR, e.g. [33]: \( f^+_{D \to \pi}(0) = 0.65 \pm 0.11 \) and \( f^+_{D \to K}(0)/f^+_{D \to \pi} = 1.20 \) (at \( m_s(1 \text{ GeV}) = 150 \text{ MeV}, \) in a good agreement with both experiment and lattice QCD.

To complete our discussion on heavy-to-light form factors one has to mention various \( B \to V \) form factors, \( V = K^{*0}, \rho, \phi \), relevant for \( B \to V l\nu \) and \( B \to V \gamma \) decays. Their most accurate calculation is in Ref. [37]. Using LCSR it is also possible to estimate the amplitudes of \( B \to \rho \gamma \) weak annihilation [38, 39] and the \( B \to \mu \nu \gamma \) width [38] employing the photon DA. The list of heavy-to-light semileptonic and radiative processes treated with the help of LCSR can be enlarged to include also \( B \to a_{0,1,2}, B \to K^0_s, K_1, K_2^* \) transition form factors if the corresponding DA of these light mesons are worked out. Another potentially interesting application is to employ the two-pion DA [40] in studying \( B \to \pi\pi l\nu \) decay. The first step in this direction was done in Ref. [41].

### \( D^*D\pi \)-coupling, QCD sum rules vs experiment

Recently the total width of \( D^* \) meson was measured by CLEO collaboration [42]:

\[ \Gamma_{tot}(D^*) = 96 \pm 4 \pm 22 \text{ keV.} \]  

This remarkable measurement yields the strong \( D^*D\pi \) coupling \( g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9 \) defined as in Ref. [43]. Among many theoretical
predictions for this coupling obtained by various methods I would like to single out the LCSR prediction [43] updated in Ref. [44] by including the $O(\alpha_s)$ correction to the twist $2$ term: $g_{D^* D\pi} = 10 \pm 3.5$. The sum rule is derived [43] from the correlator (3) employing the double dispersion relation. An estimate in the same ballpark is obtained from the QCD sum rules in the soft pion limit [43, 45]. Note that the theoretical uncertainty quoted above includes variation of all inputs within reasonable limits, therefore it is difficult to push the LCSR prediction above its upper limit $g_{D^* D\pi} = 13.5$ which is still $25\%$ lower than the central value of the CLEO measurement. If the currently observed discrepancy between the LCSR prediction and experiment persists in future one might suspect that the simple quark-hadron duality ansatz which works in the one-variable dispersion relations is too crude for the double dispersion relation.

Let me make one parenthetical remark. It is often claimed that knowing the value of the $D^* D\pi$ coupling one fixes the effective scale-independent coupling in HQET defined as $\hat{g} = f_{\pi H} g_{H \pi} / 2m_H$, $H = B, D$. However, in the charmed sector there are large $1/m_H$ corrections to the HQET limit. Indeed, as shown in Refs. [43, 46] where both $D^* D\pi$ and $B^* B\pi$ couplings are calculated from LCSR, they can be fitted to a single effective $\hat{g}$ only by adding a substantial $1/m_H$ correction: $g_{H^* H\pi} = 2m_H \hat{g} / f_\pi (1 + \Delta / m_H)$, with $\Delta \sim 1$ GeV. Therefore, expressing $g_{D^* D\pi}$ in terms of $\hat{g}$ is not a straightforward procedure.

**LIGHT-CONE SUM RULES FOR HADRONIC $B$ DECAYS**

The CP-violation studies are nowadays concentrated on the two-body hadronic $B$ decays. In order to fully explore experimental data one needs reliable theoretical predictions on hadronic matrix elements of the operators $O_i$ entering the effective weak Hamiltonian, $H_W = \frac{\hat{g}}{\sqrt{2}} \sum \lambda_i^{CKM} c_i(\mu) O_i$. The solution of this tremendously difficult problem can only be achieved within approximate QCD methods, such as the recently developed QCD factorization [47].

Here I will shortly outline a new approach to this problem [48] which is based on LCSR and allows to calculate the hadronic matrix elements in the same framework as the $B \rightarrow \pi$ form factor. As a study case the matrix elements $\langle \pi^- \pi^+ | O_{1,2} | B \rangle_{\text{Emission}}$ of the current-current operators $O_{1,2}$ for $B^0 \rightarrow \pi^+ \pi^-$ in the emission topology are considered, where $O_1 = (d\bar{\Gamma}_\mu(t)(\bar{\pi}\Gamma^\mu b)$ and $O_2$ is replaced by a combination of $O_1$ and the colour-octet operator $\tilde{O}_1 = (d\bar{\Gamma}^\mu(\bar{\pi}\Gamma^\mu b)$.

As usual in the sum rule derivation, one constructs a suitable correlation function:

$$F^{(O)}_\alpha(p, q, k) = -\int d^4 x e^{-i(p-q) x} \int d^4 y e^{i(p-k) y} \langle 0 | T \{ \bar{u} \gamma_\alpha \gamma_5 d(y) O(0) m_b \bar{b} \gamma_\alpha d(x) \} | \pi^- (q) \rangle .$$

(10)

Here the effective operator $O = O_1$ or $\tilde{O}_1$ is correlated with the currents interpolating $B$ meson and pion. In the above, a fictive momentum $k$ is attributed to the weak vertex to avoid certain technical difficulties in the dispersion relations. Furthermore, we put $p^2 = k^2 = 0$ and consider the kinematical region of large spacelike external momenta $| (p-k)^2 | \sim (p-q)^2 | \sim | P^2 | \gg \Lambda^2_{QCD}$, where $P = p - k - q$. Due to the large $b$ quark mass it is possible to apply the light-cone OPE to the correlator (10) in this region. The lowest-order diagram for the operator $O_1$ is shown in Fig. 2a. It factorizes.
FIGURE 2. Diagrams corresponding to the leading order of the correlator (10) for $O = O_1, \tilde{O}_1$: the cross indicates the point of gluon emission in the second similar diagram.

into a simple light-quark loop and the vacuum-to-pion correlation function similar to Eq. (3). The OPE of the correlator (10) with the operator $\tilde{O}_1$ starts from the diagrams containing a one-gluon nonfactorizable exchange: either a soft (low virtuality) gluon which is absorbed by the pion DA (Fig. 2b) or a hard gluon exchanged between the light-quark loop and the heavy-light part of the correlator. The latter effect corresponds to the $O(\alpha_s)$ two-loop diagrams which demand technically difficult calculation. In what follows we concentrate on the soft nonfactorizable effect represented by the diagrams in Fig. 2b. Their calculation involves twist 3 and 4 quark-antiquark-gluon DA of the pion. The key nonperturbative parameters determining these DA are the matrix elements

\[
\langle 0 | \bar{u} \gamma_\mu \gamma_5 g_s G_{\alpha \beta} \gamma_\mu d | \pi \rangle
\]

and

\[
\langle 0 | g_s \bar{u} \tilde{G}_{\alpha \beta} \gamma_\mu d | \pi \rangle
\]

estimated from SVZ sum rules [49].

The procedure of the sum rule derivation from Eq. (10) is more complicated than in the $B \to \pi$ form factor case. It can be shortly summarized as follows:

1. The dispersion relation in the pion-current channel with the momentum $(p - k)$ is employed together with the quark-hadron duality approximation, allowing one to obtain an analytic expression for the hadronic matrix element

\[
\Pi_{\pi \pi}^{(O)}((p - q)^2, P^2) = i \int d^4 x e^{-i(p - q) x} \langle \pi^- (p - k) | T \{ O(0) m_b \bar{d} i \gamma_5 d \} | \pi^- (q) \rangle.
\]  

(11)

This matrix element resembles the pion form factor at large spacelike momentum transfer $P^2$ where, instead of a simple e.m. vertex, one has a more complicated short-distance part with a virtual $b$ quark.

2. Analytic continuation of Eq. (11) in the variable $P^2$ to the large timelike $P^2 = m_B^2$ is performed. This procedure is analogous to the transition from large spacelike to large timelike momentum transfers for the pion e.m. form factor. Note that an imaginary part may emerge as a result of this continuation. It has to be interpreted as a strong final state interaction phase.

3. The dispersion relation for $\Pi_{\pi \pi}^{(O)}((p - q)^2, m_B^2)$ in the variable $(p - q)^2$ (in the $B$-meson channel) is written down and the duality ansatz for the higher $B$ states is applied. As a final result, one obtains LCSR for the on-shell $\bar{B}_d \to \pi^+ \pi^-$ matrix element of the operator $O$ where the fictive momentum $k$ vanishes due to $P^2 = m_B^2$. As usual, in order...
to suppress the higher states and to reduce the sensitivity to the duality approximation in both pion and $B$ meson channels, two independent Borel transformations are performed in the variables $(p - k)^2$ and $(p - q)^2$, respectively. Note that all parameters entering the obtained sum rules are fixed either from the SVZ sum rules for two-point correlation functions or from LCSR for $f_{B^+}^\pi$.

The resulting sum rule for the matrix element of $O_1$ in the leading order simply factorizes into a product of SVZ sum rule for $f_\pi$ and the LCSR for $B \to \pi$ form factor: 

$$
\langle \pi^- \pi^+ \mid O_1 \mid B \rangle_E = i f_\pi f_{B^+}^\pi(0)_{LCSR} m_B^2 \}
$$

thereby reproducing the factorization approximation in the limit of the heavy quark mass. The LCSR for the matrix element of the colour-octet operator $\tilde{O}_1$ calculated from the diagrams in Fig. 2b quantifies the soft nonfactorizable effect. Importantly, at $m_b \to \infty$ it is $1/m_b$ suppressed with respect to the factorizable part, in accordance with QCD factorization [47]. Numerically, 

$$
\frac{\langle \pi^- \pi^+ \mid \tilde{O}_1 \mid B \rangle}{\langle \pi^- \pi^+ \mid O_1 \mid B \rangle} \equiv \frac{\lambda_E}{m_B}, \quad \lambda_E = 50 \div 150 \text{ MeV},
$$

that is, the soft nonfactorizable effect due to $\tilde{O}_1$ is small and does not contain imaginary part. At the same time, the soft effect is as important as the small $O(\alpha_s)$ hard nonfactorizable effects calculated in the QCD factorization approach.

The $\bar{B}_d \to \pi^+ \pi^-$ decay amplitude obtained from LCSR

$$
\mathcal{A}(\bar{B}_d \to \pi^+ \pi^-) = i \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* f_\pi [f_{B^+}^\pi(0)]_{LCSR} m_B^2 \left\{ c_1(\mu) + \frac{c_2(\mu)}{3} 
\right.
\left. + 2 c_2(\mu) \left( \frac{\lambda_E}{m_B} + O(\alpha_s) \right) \right\} + ...
$$

has to be completed by calculating the $O(\alpha_s)$ nonfactorizable effects. After including the latter in the decay amplitude the scale $\mu$ dependence has to be partially canceled and the imaginary part will emerge at $O(\alpha_s)$. It is also important to calculate one by one the contributions of annihilation, penguin topologies for $O_{1,2}$ as well as the matrix elements of penguin operators $O_n$, $n \geq 3$ denoted by ellipses in the above. The LCSR approach can be generalized to other channels such as $B \to K \pi, KK, D \pi, J/\psi K$.

**SUMMARY**

The aim of this minireview was to demonstrate that employing QCD sum rules one is able to determine various heavy-flavour parameters starting from the most fundamental ones, the heavy quark masses, and ending with the most complicated ones, the hadronic matrix elements of nonleptonic $B$ decays. The method is selfsufficient, that is, extracting a certain parameter from a sum rule, one uses the result in the other sum rules to calculate more complicated parameters. The following hierarchy can be traced:

$$
m_b \to f_B \to f_{B^+}^\pi \to \langle \pi \pi \mid O_i \mid B \rangle.
$$
Summarizing, QCD sum rules remain a reliable approximate approach well equipped to attack various topical problems in the physics of heavy flavours.

**Note added:**
After this meeting, two new QCD sum rule calculations of the heavy meson decay constants have been published, both including the $O(\alpha_s^2)$ correction [32] to the heavy-light correlator. The first analysis is done in HQET[50] and predicts $f_B = 206 \pm 20$ MeV, $f_D = 195 \pm 20$ MeV. The second one [51] uses $M_{\overline{S}}$-mass of the $b$ quark instead of the pole mass yielding $f_B = 197 \pm 23$ MeV and $f_{B_s} = 232 \pm 25$ MeV. Within uncertainties, both results [50, 51] agree with the intervals in Eq. (7).

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