Additional Isospin-Breaking Effects in $\frac{\epsilon'}{\epsilon}$

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Abstract. Isospin-breaking effects, in particular those associated with electroweak-penguin contributions and $\pi^0$-$\eta, \eta'$ mixing, have long been known to affect the Standard Model prediction of $\frac{\epsilon'}{\epsilon}$ in a significant manner. We have found an heretofore unconsidered isospin-violating effect of importance; namely, the $u$-$d$ quark mass difference can spawn $|\Delta I| = 3/2$ components in the matrix elements of the gluonic penguin operators. Using chiral perturbation theory and the factorization approximation for the hadronic matrix elements, we find within a specific model for the low-energy constants that we can readily accommodate an increase in $\frac{\epsilon'}{\epsilon}$ by a factor of two.

INTRODUCTION

The recent measurement of a non-zero value of $\text{Re}(\frac{\epsilon'}{\epsilon})$ [1] establishes the existence of CP violation in direct decay and thus provides an important first check of the mechanism of CP violation in the Standard Model (SM). Nevertheless, the world average which emerges is $\text{Re}(\frac{\epsilon'}{\epsilon}) = (19.3 \pm 2.4) \cdot 10^{-4}$ [2], which is larger than the “central” SM prediction of $7.0 \cdot 10^{-4}$ [3,4] by nearly a factor of three. This compels us to scrutinize the SM prediction in further detail: we study isospin-violating effects arising from the $u$-$d$ quark mass difference.

Isospin violation plays an important role in the analysis of $\frac{\epsilon'}{\epsilon}$, for the latter is predicated by the difference of the imaginary to real part ratios in the $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ $K \to \pi\pi$ amplitudes. The differing charges of the $u$ and $d$ quarks engender $|\Delta I| = 3/2$ electroweak penguin contributions, whereas $\pi^0$-$\eta, \eta'$ mixing, driven by the $u$-$d$ quark mass difference, modifies the relative contribution of the $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ amplitudes in a significant way.

Here we describe isospin-breaking effects in the matrix elements of the gluonic penguin operators [5], such as $Q_6$. These operators have always been thought to

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induce exclusively $|\Delta I| = 1/2$ transitions, but this is true only in the limit of isospin symmetry. The difference in the up and down quark masses effectively distinguishes the interaction of gluons with up and down quarks, so that the $(\pi\pi|Q_6|K)$ matrix element possesses a $|\Delta I| = 3/2$ component as well [6].

Let us begin by showing why the numerical prediction of $\epsilon'/\epsilon$ is sensitive to the presence of isospin violation. The value of $\epsilon'/\epsilon$ is inferred from a ratio of ratios, namely

$$\text{Re} \left( \frac{\epsilon'}{\epsilon} \right) = \frac{1}{6} \left[ \frac{\eta_{++}}{\eta_{00}} \right]^2 - 1,$$

where

$$\eta_{++} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \approx \epsilon + \epsilon'; \quad \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \approx \epsilon - 2\epsilon'.$$

In the isospin-perfect limit, the two independent amplitudes present in $K \to \pi\pi$ decay are distinguished by the isospin of the final-state pions, namely $A_I \equiv A(K \to (\pi\pi)_I)$ with $I = 0, 2$. $\epsilon'/\epsilon$ can thus be written

$$\frac{\epsilon'}{\epsilon} = -\frac{\omega}{\sqrt{2}|\epsilon|} \xi (1 - \Omega),$$

with

$$\omega \equiv \frac{\text{Re} A_2}{\text{Re} A_0}; \quad \xi \equiv \frac{\text{Im} A_0}{\text{Re} A_0}; \quad \Omega \equiv \frac{\text{Im} A_2}{\omega \text{Im} A_0}.$$

In standard practice, $\omega \approx 1/22$ and $\text{Re} A_0$ are taken from experiment whereas $\text{Im} A_I$ is computed using the operator-product expansion [3,4], that is, via

$$\mathcal{H}_{\text{eff}}(|\Delta S| = 1) = 4 \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} C_i(\mu) Q_i(\mu) + \text{h.c.}$$

The numerical value of $\epsilon'/\epsilon$ is driven by the matrix elements of the QCD penguin operator $Q_6$ and the electroweak penguin operator $Q_8$ [7]. Writing $\langle Q_i \rangle_I$ as $\langle (\pi\pi)_I | Q_i | K \rangle \equiv B_i^{(I+1/2)} \langle (\pi\pi)_I | Q_i |_K \rangle^{(\text{vac})}$, where “vac” indicates the use of the vacuum saturation approximation, one recovers the schematic formula [3]

$$\frac{\epsilon'}{\epsilon} = 13 \text{Im} \lambda_t \left[ B_6^{(1/2)} (1 - \Omega_{\eta+\eta'}) - 0.4 B_8^{(3/2)} \right].$$

Using $B_6^{(1/2)} = 1.0$, $B_8^{(3/2)} = 0.8$, and $\Omega_{\eta+\eta'} = 0.25$ yields the “central” SM value of $\epsilon'/\epsilon \sim 7.0 \cdot 10^{-4}$ [3], roughly a factor of three smaller than the measured value. Larger estimates of $B_6^{(1/2)}$, and hence of $\epsilon'/\epsilon$, exist [8–10]; we investigate sources
of $\Omega_{\eta+\eta'}$. Note that under $\Omega_{\eta+\eta'} \to -\Omega_{\eta+\eta'}$, $\text{Re}(\epsilon'/\epsilon) \to 2.2 \text{Re}(\epsilon'/\epsilon)$. We were
$|B_6^{(1/2)}| \gg |B_8^{(3/2)}|$, flipping the sign of $\Omega_{\eta+\eta'}$ would increase $\epsilon'/\epsilon$ by a factor of 1.7.

Let us consider possible sources of $\Omega_{\eta+\eta'}$. We replace $\Omega_{\eta+\eta'}$ by $\Omega_{IB}$, where

$$
\Omega_{IB} = \left( \frac{\sqrt{2}}{3\omega} \right) \frac{\text{Im}(A_P(K^0 \to \pi^+\pi^-) - A_P(K^0 \to \pi^0\pi^0))}{\text{Im} A_P(K^0 \to \pi\pi)}
$$

and $\text{Im} A_P(K^0 \to \pi\pi) = (\text{Im} A_P(K^0 \to \pi^+\pi^-) + \text{Im} A_P(K^0 \to \pi^0\pi^0))/2$. “$A_P$” denotes an amplitude induced by $(8_L, 1_R)$ (e.g., $Q_6$) operators — the empirical $|\Delta I| = 1/2$ rule suggests such operators dominate the isospin-violating effects. $\Omega_{IB}$ vanishes in the absence of isospin violation, i.e., if $m_u = m_d$, $e_u = e_d$. It can be generated by both strong-interaction and electromagnetic effects, mediated by $m_d \neq m_u$ and $e_u \neq e_d$ [11], respectively. We focus on $m_d \neq m_u$ effects. The latter include $\pi^0$-$\eta, \eta'$ mixing [12–14]; in $O(p^2, 1/N_c)$ this yields $\Omega_{\eta+\eta'} = 0.25 \pm 0.05$ [13,14], used in the analysis of Ref. [3]. However, $m_u \neq m_d$ effects can also spawn a $|\Delta I| = 3/2$ component in the matrix elements of the gluonic penguin operators [6], as illustrated in Fig. 1. We turn to a chiral Lagrangian analysis in order to estimate the size of this effect [5].

**CHIRAL LAGRANGIAN ANALYSIS**

The weak chiral Lagrangian for $K \to \pi\pi$ decay is written in terms of the unitary matrix $U = \exp(i\phi/f)$ and the function $\chi$, both of which transform as $U \to RUL^\dagger$ under the chiral group $SU(3)_L \times SU(3)_R$. The function $\phi$ represents the octet of pseudo-Goldstone bosons, i.e., $\phi = \sum_{a=1,...,8} \lambda_a \phi_a$. In the absence of external fields, $\chi = 2B_0M$ with $M = \text{diag}(m_u, m_d, m_s)$ and $B_0 \propto \langle \bar{q}q \rangle$. The leading-order, $O(p^2)$, weak chiral Lagrangian contains no mass-dependent terms [15], so that $m_d \neq m_u$ effects in the hadronization of the gluonic penguin operators first appear in $O(p^4)$. This is illustrated in Fig. 2.

![Figure 1](image_url)  
**FIGURE 1.** Quark line diagram illustrating the “strong penguin” $s \to d\bar{q}q$ transition in the Standard Model. Note that $\bar{q}q \in \bar{u}u, \bar{d}d$; in the isospin-perfect limit, $m_u = m_d$ and only $|\Delta I| = 1/2$ transitions are generated. If $m_u \neq m_d$, the $K \to \pi\pi$ matrix element associated with this operator contains a $|\Delta I| = 3/2$ component as well.
Let us enumerate the possible isospin-violating effects which occur in $O(m_d - m_u)$ and $O(p^4)$:

i) $\pi^0$-$\eta$ mixing realized from the $O(p^2)$ strong chiral Lagrangian, in concert with the $O(p^2)$ weak chiral Lagrangian, computed to one-loop order.

ii) $\pi^0$-$\eta$ mixing, realized from the $O(p^2)$ strong chiral Lagrangian, combined with the isospin-conserving vertices of the $O(p^4)$ weak chiral Lagrangian.

iii) $\pi^0$-$\eta$ mixing as realized from the strong chiral Lagrangian in $O(p^4)$, combined with the $O(p^2)$ weak chiral Lagrangian. The $\pi^0$-$\eta'$ mixing effects included in Refs. [13,14] are this effect.

iv) Isospin violation in the vertices of the $O(p^4)$ weak chiral Lagrangian. This serves as our focus here, for it contains the qualitatively new effects we argue.

We use the octet terms in the $O(p^4)$, CP-odd weak chiral Lagrangian of Ref. [16]. Collecting the $\chi$-dependent terms as per iv), working to $O(m_d - m_u)$, and dropping terms suppressed by $M_\pi^2/M_K^2$, we find

$$\Omega_p = \frac{2\sqrt{2}}{3\omega} \frac{M_{K^0}^2}{M_{K^0}^2 - M_\pi^2} \frac{B_0(m_d - m_u)}{c_2} \tilde{E}^- \approx \frac{0.12\text{GeV}^2}{c_2^2} \tilde{E}^-$$

(8)

with $\tilde{E}^- = 2E_1^- - 2E_3^- - 4E_4^- - E_{10}^- - E_{11}^- - 4E_{12}^- - E_{15}^-$. Note that $c_2^-$ is the low-energy constant associated with the $O(p^2)$, $(8_L, 1_R)$ weak chiral Lagrangian [16]. As per ii), $\pi^0$-$\eta$ mixing in $O(p^2)$ also enters when combined with the isospin-conserving vertices of the $O(p^4)$ weak chiral Lagrangian. Including the $\chi$-dependent octet terms, we find

$$\Omega^{(4)}_{\eta + \eta'} = \frac{2\sqrt{2}}{3\omega} \frac{M_{K^0}^2}{M_{K^0}^2 - M_\pi^2} \frac{B_0(m_d - m_u)}{c_2} E_{\eta + \eta'}^- \approx \frac{0.12\text{GeV}^2}{c_2^2} E_{\eta + \eta'}^-$$

(9)

**FIGURE 2.** Isospin violation in $K \to \pi\pi$ decays. The square box represents the weak $|\Delta S| = 1$ transition at low energies, whereas the “×” represents the presence of $m_d \neq m_u$ effects. Mass effects do not occur in the weak transition in leading order in chiral perturbation theory, so that only the left-hand diagram occurs in $O(p^2)$ and in $O(m_d - m_u) - \pi^0$-$\eta$ mixing, mediated by $m_d - m_u$ effects in the strong chiral Lagrangian, can occur. In $O(p^4)$ both diagrams are present; the new effect we discuss is associated with the right-hand diagram.
with
\[E_{\eta + \eta'} = -2(E_3 - E_5) + (E_{10} - E_{11})/2 - 2E_{12} + E_{14} + 3E_{15}/2\]
so that no manifest cancellation with the terms of Eq. (8) occurs.

The low-energy constants \(E_i\) are unknown, so that we turn to the factorization approximation to proceed. The construction relevant to \((8_L, 1_R)\) transitions in \(K^0 \to \pi\pi\) decay is \[L_P = \frac{G_F}{\sqrt{2}} V^{\star}_{us} V_{ud} C_6 \left( -8(\bar{s}_L q_R)(\bar{q}_R d_L) \right) + \text{h.c.} \]
\[\to \frac{G_F}{\sqrt{2}} V^{\star}_{us} V_{ud} C_6 32B_0^2 \delta L_{\text{str}} \delta L_{\text{str}} + \text{h.c.}, \]
where \(L_{\text{str}}\) is the strong chiral Lagrangian. To generate terms of \(O(p^4)\) in \(L_P\) requires terms of both \(O(p^4)\) \[18\] and \(O(p^6)\) \[19\] in \(L_{\text{str}}\). Unfortunately, the low-energy constants of the latter are also unknown; the use of “resonance saturation” allows us to estimate some of them. We explicitly consider the scalar nonet of resonances as per Ref. [20]. An example of the manner in which the scalar resonances can generate contributions to the \(E_i\) is illustrated in Fig. 3. Integrating out the scalar resonances for \(p^2 \ll M_S^2\), we find two terms which contribute to the scalar densities in the bosonization of \(Q_6\) \[20\],
\[L_S^{(6)} = \frac{d_m c_m^2}{2M_S^4} \langle \chi_3^2 \rangle + \frac{c_d c_m d_m}{M_S^4} \langle \chi_2^2 L^2 \rangle , \]
yielding contributions to \(E_1\) and \(E_{10}\) in terms of \(d_m, c_m, c_d,\) and \(M_S\). The parameter \(d_m\) is ill-known; we find \(d_m \sim -2.4(-0.76)\). The sign of \(d_m\) and thus of \(\Omega_P\) in our model results from the mass of the lowest-lying strange scalar being greater than that of the lowest-lying isovector scalar. As per our earlier classification, \(\Omega_{iB}^{(4)} = \Omega_{iB}^{(4),i} + \Omega_{iB}^{(4),ii} + \Omega_{iB}^{(4),iii} + \Omega_{iB}^{(4),iv}\), so that with \(d_m = -2.4(-0.76)\), we

**FIGURE 3.** A contribution to the right-hand diagram of Fig. 2 in \(O(m_d - m_u)\), estimated in the factorization approximation with explicit, scalar-resonance degrees of freedom. The “⊗” represents a bosonized current; the open parentheses denote contributions from the vertices of the \(O(p^6)\) model Lagrangian of Ref. [20]. In this case the isospin-violating contribution is driven by \(a_0-f_0\) mixing.
have $\Omega_{IB}^{(4),iv} = -0.79 (-0.21)$. Estimating $\Omega_{IB}^{(4),ii}$ using the $\chi$-dependent $E_i^-$ yields $\Omega_{IB}^{(4),ii} = -0.12 (-0.03)$. $\Omega_{IB}^{(4),iii}$ has been partially determined through the inclusion of $\pi^0-\eta'$ mixing in $\Omega_{\eta+\eta'} = 0.25 \pm 0.05$ [13,14]. Using the result $\Omega_{IB}^{(2)} + \Omega_{IB}^{(4),iii} = 0.16 \pm 0.03$ [21] and neglecting $\Omega_{IB}^{(4),i}$, as the ill-known $E_i^-$ do not warrant such a calculation, we estimate, finally, that $\Omega_{IB} = \Omega_{IB}^{(2)} + \Omega_{IB}^{(4)} \sim -0.05 \rightarrow -0.78$. For reference, note that $\Omega_{IB}^{(2)} \sim 0.13$. The large value of $\Omega_{IB}^{(4)}$ is driven by the numerical prefactor of Eqs. (8,9) — the contributions in $\Omega_{IB}^{(4)}$ are “naturally” of the same size as $\Omega_{IB}^{(2)}$. Thus we find a very large correction to the value of $\Omega_{\eta+\eta'} = 0.25 \pm 0.05$, used in “central value” of $\epsilon'/\epsilon$. The large negative change in $\Omega_{IB}$ found in $\mathcal{O}(p^4)$ generates a substantial increase in $\epsilon'/\epsilon$.

The $\Omega_{IB}$ we calculate impacts $\epsilon'/\epsilon$ in a significant manner. Our estimate of $\Omega_{IB}$ from the specific $m_d \neq m_u$ effects we consider ranges from $-0.05 \rightarrow -0.78$; this range exceeds the central value, $\Omega_{\eta+\eta'} = 0.25 \pm 0.05$, used in earlier analyses and reflects a variation in $\epsilon'/\epsilon$ of more than a factor of two. The presence of unknown low-energy constants implies that we lack a reliable way to calculate the effects we consider. Such limitations, however, underscore the need for a larger uncertainty in the Standard Model prediction of $\epsilon'/\epsilon$.

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REFERENCES

1. A. Alavi-Harati et al. (KTeV Collaboration), Phys. Rev. Lett. 83, 22 (1999); V. Fanti et al. (NA48 Collaboration), Phys. Lett. B465, 335 (1999).
2. A. Ceccucci, CERN seminar, February 29, 2000, http://www.cern.ch/NA48/.
3. S. Bosch et al., Nucl. Phys. B565, 3 (2000).
4. A. Buras, hep-ph/9806471, to appear in Probing the Standard Model of Particle Physics, F. David and R. Gupta, eds. (Elsevier Science B. V., Amsterdam, 1998).
5. S. Gardner and G. Valencia, Phys. Lett. B466, 355 (1999).
6. S. Gardner, Phys. Rev. D59, 077502 (1999); hep-ph/9906269.
7. A. J. Buras, M. Jamin, and M. E. Lautenbacher, Nucl. Phys. B408, 209 (1993).
8. S. Bertolini, M. Fabbrichesi, and J. Eeg, hep-ph/0002234, and references therein.
9. T. Hambye, G. O. Kohler, E. A. Paschos, and P. H. Sokdan, Nucl. Phys. B564, 391 (2000).
10. J. Bijnens and J. Prades, hep-ph/0005189.
11. V. Cirigliano, J. F. Donoghue, and E. Golowich, Phys.Rev. D61, 093001 (2000); Phys.Rev. D61, 093002 (2000).
12. J. Bijnens and M. Wise, Phys. Lett. 137B, 245 (1984).
13. J. F. Donoghue et al., Phys. Lett. 179B, 361 (1986).
14. A. J. Buras and J. M. Gerard, Phys. Lett. 192B, 156 (1987).
15. J. A. Cronin, Phys. Rev. 161, 1483 (1967).
16. J. Kambor, J. Missimer, and D. Wyler, Nucl. Phys. B346, 17 (1990).
17. S. Chivukula, J. Flynn, and H. Georgi, *Phys. Lett.* **B171**, 453 (1986).
18. J. Gasser and H. Leutwyler, *Nucl. Phys.* **B250**, 465 (1985).
19. H. W. Fearing and S. Scherer, *Phys. Rev.* **D53**, 315 (1996); J. Bijnens, G. Colangelo, and G. Ecker, *JHEP* **9902**, 020 (1999).
20. G. Amorós, J. Bijnens, and P. Talavera, *Nucl.Phys.* **B568**, 319 (2000).
21. G. Ecker, G. Müller, H. Neufeld, and A. Pich, *Phys. Lett.* **B477**, 88 (2000).