No parity anomaly in massless QED$_3$: A BPHZL approach

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In this letter we call into question the perturbatively parity breakdown at 1-loop for the massless QED$_3$ frequently claimed in the literature. As long as perturbative quantum field theory is concerned, whether a parity anomaly owing to radiative corrections exists or not will be definitely proved by using a renormalization method independent of any regularization scheme. Such a problem has been investigated in the framework of BPHZL renormalization method, by adopting the Lowenstein-Zimmermann subtraction scheme. The 1-loop parity-odd contribution to the vacuum-polarization tensor is explicitly computed in the framework of the BPHZL renormalization method. It is shown that a Chern-Simons term is generated at that order induced through the infrared subtractions – which violate parity. We show then that, what is called “parity anomaly”, is in fact a parity-odd counterterm needed for restoring parity.

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I. WARM UP: THE MASSLESS QED$_3$ AT 1-LOOP

It has been frequently addressed in the literature, since the work of Ref.[1], that even perturbatively, parity is broken in massless QED$_3$, such a breaking being achieved by a Chern-Simons term that appears in the 1-loop contribution to the vacuum-polarization tensor ($\Pi_{1\text{reg}}^{\mu\nu}$) through the use of the Pauli-Villars (PV) regularization scheme:

$$\Pi_{1\text{reg}}^{\mu\nu}(p) = \frac{e^2}{16} \eta^{\mu\nu}p^2 - p^\mu p^\nu + i\frac{e^2}{2\pi} \epsilon^{\mu\nu\rho\sigma} p_\rho \lim_{M\to\infty} \frac{M}{|p|} \arcsin \frac{|p|}{\sqrt{p^2 - 4M^2}}. \quad (1)$$

The only superficially divergent 1-loop graphs present in massless QED$_3$ are the vacuum-polarization tensor ($\Pi_{1\text{reg}}^{\mu\nu}$) and the massless fermion self-energy ($\Sigma_1$), with their degree of divergence given by $\delta(\Pi_1) = 1$ and $\delta(\Sigma_1) = 0$, respectively. In spite of their naïve degree of divergence, they are in fact finite, the 1-loop Feynman graphs give rise only to Speer-type integrals which do not develop poles in the analytic continuation from $d \to 3$ [2–4].

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The 1-loop Feynman graphs for the vacuum-polarization (in the Feynman gauge) and the fermion self-energy are given by:

\[ \Pi_1^{\mu\nu}(p) = -\text{Tr} \int \frac{d^3k}{(2\pi)^3} ie^{\gamma^\mu} \left[ i \frac{k_i}{k^2} \right] ie^{\gamma^\nu} \left[ i \frac{k_i - p_i}{(k - p)^2} \right], \]

\[ \Sigma_1(p) = \int \frac{d^3k}{(2\pi)^3} ie^{\gamma^\mu} \left[ i \frac{k_i}{k^2} \right] ie^{\gamma^\nu} \left[ -i \frac{\eta_{\mu\nu}}{(k - p)^2} \right]. \]

At this point it should be stressed that neither gauge invariance nor parity are broken by dimensional regularization (DR) [4, 5]:

\[ \Pi_1^{\mu\nu}(p) = -\frac{e^2}{16} \frac{\eta_{\mu\nu}p^2 - p^\mu p^\nu}{\sqrt{p^2}} \quad \text{and} \quad \Sigma_1(p) = \frac{e^2}{16} \frac{p}{\sqrt{p^2}}. \]

Therefore, some claims found in the literature that Pauli-Villars regularization must be used because it does not break gauge invariance, contrary to the dimensional regularization case, are not true in the light of perturbation theory for massless QED. Moreover, Pauli-Villars breaks parity, dimensional regularization does not, both preserve gauge invariance, which of them is more suitable in the quantization of the massless QED? Such a question makes no sense if both schemes are properly used. Fortuitous breakings of gauge symmetry happens when there is no invariant regularization scheme available, however, the Quantum Action Principle [6-8] guarantees that they can be completely eliminated, when gauge anomaly is absent, by the introduction of noninvariant local counterterms at each perturbative order. The use of DR is quite well analyzed by Delbourgo and Waites for the three-dimensional QED [4]. Pimentel and Tomazelli ask in Ref.[9], “what’s wrong with Pauli-Villars regularization in QED?”, there they present the correct use, which has to satisfy some necessary consistency conditions, of PV regularization in three space-time dimensions. The important issue of “how superrenormalizable interactions cure their infrared divergences” is analyzed by Jackiw and Templeton [10]. As will be shown in a subsequent work [11], through the use of the algebraic renormalization method [8], the massless QED is in fact perturbatively finite, no regularization scheme is required at all. Furthermore, an anomaly is not an ambiguity, it does not depend on which kind of regularization scheme is made use of.

**II. BPHZL: 1-LOOP VACUUM POLARIZATION**

In order to clarify the matter in an unambiguous way, we will perform an explicit 1-loop computation in the Zimmermann’s momentum space subtraction scheme – BPHZ [13], which does not use any regularization procedure. Due to the presence of massless fields (\(\psi\) and \(A_\mu\)), the momentum subtraction scheme modified by Lowenstein and Zimmermann – BPHZL [14] – has to be adopted in order to deal with the infrared (IR) divergences that otherwise would arise in the process of ultraviolet (UV) subtractions.

The action for the massless QED, with the gauge invariant BPHZL mass terms added, is given by:

\[ \Sigma^{(s-1)} = \int d^3x \left\{ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{\mu}{2} (s - 1) e^{\mu\rho \gamma} A_\mu \partial_\rho \gamma - m(s - 1) \psi \bar{\psi} - \frac{1}{2\alpha} (\partial_\mu A^\mu)^2 \right\}, \]

where \(\bar{\psi} \equiv (\bar{\theta} + ie\bar{A})\psi\) and \(e\) is a dimensionful coupling constant. The BPHZL parameter \(s\) lies in the interval \(0 \leq s \leq 1\) and plays the role of an additional subtraction variable (as the external momentum) in the BPHZL renormalization program. The massless theory is recovered by putting \(s = 1\) at the end of the calculations. At the classical level, the massless classical action for QED is recovered for \(s = 1\):

\[ \Sigma_{\text{QED}} = \Sigma^{(s-1)}|_{s=1}. \]

Let us now establish some conventions and useful identities necessary to compute the 1-loop parity-odd contribution to the vacuum-polarization tensor:

\(\eta_{\mu\nu} = \text{diag}(+ - -)\), \(\gamma^\mu = (\sigma_x, i\sigma_y, -i\sigma_z)\),

\[ \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\rho} \mathbf{1}, \quad \gamma^\mu e^{\gamma^\rho} = \eta^{\mu\nu} \mathbf{1} + e^{\gamma^\nu} \gamma^\rho, \quad e^{\beta\gamma^\nu} e_{\beta\rho} = \delta_\nu^\alpha \delta_\rho^\beta - \delta_\rho^\alpha \delta_\nu^\beta \quad \text{and} \quad \text{Tr}(\gamma^\mu \gamma^\rho) = i\eta_{\mu\rho} \mathbf{1}. \]

1 The authors of [4] have already drawn attention to that and to the absence of a parity anomaly as well, this was also pointed out by Rao and Yahalom [12].
For our 1-loop computations we need the fermion propagator, it reads
\[ \langle \bar{\psi}(k)\psi(k) \rangle = i \frac{k + m(s-1)}{k^2 - m^2(s-1)^2}. \]  

The 1-loop vacuum-polarization tensor, \( \Pi_1^{\mu\nu}(p, s) \):
\[ \Pi_1^{\mu\nu}(p, s) = \int \frac{d^3k}{(2\pi)^3} I_1^{\mu\nu}(p, k, s) \]
\[ = -e^2 \text{Tr} \int \frac{d^3k}{(2\pi)^3} \gamma^\mu \frac{k + m(s-1)}{k^2 - m^2(s-1)^2} \gamma^\nu \frac{k - \phi + m(s-1)}{(k - p)^2 - m^2(s-1)^2}, \]
has the following UV and IR subtraction degrees, \( \delta(\Pi) = 1 \) and \( \rho(\Pi) = 1 \), respectively.

In the BPHZL scheme the subtracted integrand, \( R_1^{\mu\nu}(p, k, s) \), is written in terms of the unsubtracted one, \( I_1^{\mu\nu}(p, k, s) \), as
\[ R_1^{\mu\nu}(p, k, s) = (1 - t^0_{p,s-1})(1 - t^1_{p,s})I_1^{\mu\nu}(p, k, s) \]
\[ = (1 - t^0_{p,s-1} - t^1_{p,s} + t^0_{p,s-1}t^1_{p,s})I_1^{\mu\nu}(p, k, s), \]
where \( t^d_{x,y} \) is the Taylor series about \( x = y = 0 \) to order \( d \) if \( d \geq 0 \).

Thus, for our purposes, by assuming \( s = 1 \), the subtracted integrand, \( R_1^{\mu\nu}(p, k, s) \), reads
\[ R_1^{\mu\nu}(p, k, 1) = I_1^{\mu\nu}(p, k, 1) - p^\rho \partial_{p^\rho} I_1^{\mu\nu}(0, k, 0), \]
\[ \text{parity-even} \quad \text{parity-even} \quad \text{parity-odd terms} \]

where
\[ I_1^{\mu\nu}(p, k, s) = -e^2 \text{Tr} \gamma^\mu \frac{k + m(s-1)}{k^2 - m^2(s-1)^2} \gamma^\nu \frac{k - \phi + m(s-1)}{(k - p)^2 - m^2(s-1)^2}, \]
\[ I_1^{\mu\nu}(p, k, 1) = -e^2 \text{Tr} \gamma^\mu \frac{k}{k^2} \gamma^\nu \frac{k - \phi}{(k - p)^2}, \]
\[ I_1^{\mu\nu}(0, k, 1) = -e^2 \text{Tr} \gamma^\mu \frac{k}{k^2} \gamma^\nu \frac{k}{k^2}, \]
\[ p^\rho \partial_{p^\rho} I_1^{\mu\nu}(0, k, 0) = -e^2 \text{Tr} \gamma^\mu \frac{k - m}{k^2 - m^2} \gamma^\nu \left[ -\frac{\phi}{k^2 - m^2} + 2p \cdot k \frac{k}{(k^2 - m^2)^2} \right]. \]

Bearing in mind that in the intermediary steps of the BPHZL subtraction scheme parity is explicitly broken, when \( s = 0 \) is assumed, a Chern-Simons term could be expected as a parity-odd 1-loop counter-term due to the parity-noninvariant nature of the BPHZL renormalization scheme. In fact, such a CS term, generated at the 1-loop correction to the vacuum-polarization tensor, arises in the process of IR subtractions performed so as to subtract IR divergences induced by the UV subtractions.

The parity-odd contribution, \( R_{1,\text{odd}}^{\mu\nu}(p, k, 1) \), stemming from the subtracted integrand, is given by
\[ R_{1,\text{odd}}^{\mu\nu}(p, k, 1) = e^2 m \epsilon^{\mu\nu\rho} p^\rho \frac{1}{(k^2 - m^2)^2}. \]

Then, the parity-odd 1-loop subtracted (finite) vacuum-polarization tensor, \( \Pi_{1,\text{odd}}^{\mu\nu}(p, 1) \), reads
\[ \Pi_{1,\text{odd}}^{\mu\nu}(p, 1) = e^2 m \epsilon^{\mu\nu\rho} p^\rho \int \frac{d^3k}{(2\pi)^3} \frac{1}{(k^2 - m^2)^2} \]
\[ = \frac{e^2 m}{8\pi |m|} \epsilon^{\mu\nu\rho} p^\rho \]
\[ . \]

The appearence of this contribution may however be compensated by adding to the action a gauge invariant local Chern-Simons counterterm:
\[ \frac{e^2 m}{8\pi |m|} \int d^3x \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho . \]
Therefore, there is no parity-odd 1-loop contribution to the vacuum-polarization tensor $\Pi^{\mu(\text{sub})}_1(p, 1)$.

## III. CONCLUSION

In the BPHZL renormalization scheme used in the present work, the odd term (17) appears as a consequence of the IR subtractions, performed at $s = 0$, which explicitly break parity. However, as we have seen, we can remove it through the introduction of a Chern-Simons local counterterm and thus recover parity symmetry. This is more an illustration of the reestablishment of a symmetry which has been broken by the subtraction or regularization procedure used. This is well known for the case of continuous symmetry [7, 8]. To the best of our knowledge, this is new for a discrete symmetry.

In short, such a Chern-Simons term could never be interpreted as a “parity anomaly”. We thus conclude that no parity anomaly exists perturbatively in massless QED$_3$.

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[1] A.N. Redlich, Phys.Rev.Lett. 52 (1984) 18 and Phys.Rev. D29 (1984) 2366.
[2] E.R. Speer, J.Math.Phys. 15 (1974) 1 and Ann.Inst.Henri Poincaré XXII (1975) 1;
    G. Giavarini, C.P. Martin and F. Ruiz Ruiz, Nucl.Phys. B381 (1992) 222;
    F. Ruiz Ruiz and P. van Nieuwenhuizen, Nucl.Phys. B486 (1997) 443.
[3] G. Leibbrandt, Rev.Mod.Phys. 47 (1975) 849 and Noncovariant Gauges: Quantization of Yang-Mills and Chern-Simons Theory in Axial Type Gauges, World Scientific (Singapore), 1994.
[4] R. Delbourgo and A.B. Waites, Phys.Lett. B300 (1993) 241 and Austral.J.Phys. 47 (1994) 465.
[5] C. Bollini and J.J. Giambiagi, Phys.Lett. B40 (1972) 566;
    G.M. Cicuta and E. Montaldi, Nuovo Cimento Lett. 4 (1972) 329;
    G. ’t Hooft and M. Veltman, Nucl.Phys. B44 (1972) 189.
[6] J.H. Lowenstein, Phys.Rev. D4 (1971) 2281 and Comm.Math.Phys. 24 (1971) 1;
    Y.M.P. Lam, Phys.Rev. D6 (1972) 2145 and Phys.Rev. D7 (1973) 2943;
    T.E. Clark and J.H. Lowenstein, Nucl.Phys. B113 (1976) 109;
    P. Breitenlohner and D. Maison, Comm.Math.Phys. 52 (1977) 55.
[7] C. Becchi, A. Rouet and R. Stora, Comm.Math.Phys. 42 (1975) 127 and Ann.Phys.(N.Y.) 98 (1976) 287;
    O. Piguet and A. Rouet, Phys.Rep. 76 (1981) 1.
[8] O. Piguet and S.P. Sorella, Algebraic Renormalization, Lecture Notes in Physics, m28, Springer-Verlag (Berlin-Heidelberg), 1995; see also references therein.
[9] B.M. Pimentel and J.L. Tomazzelli, Prog.Theor.Phys. 95 (1996) 1217.
[10] R. Jackiw and S. Templeton, Phys.Rev. D23 (1981) 2291.
[11] O.M. Del Cima, D.H.T. Franco and O. Piguet, On the finiteness of massless QED$_3$, in progress.
[12] S. Rao and R. Yahalom, Phys.Lett. B172 (1986) 227.
[13] W. Zimmermann, Comm.Math.Phys. 15 (1969) 288 and Lectures on Elementary Particles and Quantum Field Theory, 1970 Brandeis lectures, eds. S. Deser, M. Grisaru and H. Pendleton, MIT Press (Cambridge-USA), 1971.
[14] J.H. Lowenstein and W. Zimmermann, Nucl.Phys. B86 (1975) 77;
    J.H. Lowenstein, Comm.Math.Phys. 47 (1976) 53 and Renormalization Theory, eds. G. Velo and A.S. Wightman, D. Reidel (Dordrecht-Holland), 1976.