Diffusion of hidden charm mesons in hadronic medium

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The drag and diffusion coefficients of a hot hadronic medium have been evaluated by using hidden charm mesons as probes. The matrix elements for the evaluation of these coefficients are calculated using an effective theory as well as from scattering lengths. Although the transport coefficients show a significant rise with temperature its effects on the suppression of \( J/\psi \) in hadronic matter is not significant.

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I. INTRODUCTION

The experimental evidence of \( J/\psi \) suppression by NA50\textsuperscript{1}, NA60\textsuperscript{2} as well as by the PHENIX\textsuperscript{3} collaboration has long been suggested as a signal of quark-gluon plasma formation in heavy ion collisions\textsuperscript{4,5}. However, other mechanisms based on heavy quark effective theory. The inelastic scattering rates of \( J/\psi \) have been investigated using effective hadronic interaction of heavy quark mesons and the role of hadronic matter in their suppression in heavy ion collisions\textsuperscript{6}. In addition, the opening of \( J/\psi \rightarrow D\bar{D} \) decay in the medium due to in-medium modification of D mesons\textsuperscript{7,8} may also play a significant role in \( J/\psi \) suppression in a hadronic environment.

Heavy quark transport in hadronic matter is a topic of high contemporary interest\textsuperscript{9,10,11,12}. The drag and diffusion of open charm\textsuperscript{13} and bottom\textsuperscript{14} mesons and the role of hadronic matter in their suppression in heavy ion collisions\textsuperscript{15} have been investigated using effective hadronic interactions based on heavy quark effective theory. The suppression of heavy flavour in the hadronic phase was found to be more significant at RHIC than at LHC suggesting that the characterization of QGP at LHC could be less complicated than at RHIC. Recently we have also obtained the drag and diffusion of the \( \Lambda_c \) baryon in hadronic matter\textsuperscript{20} and found them to be significant. In fact, the drag of the \( \Lambda_c \) being lower than that of the D mesons was seen to non-trivially affect the \( p_T \) dependence of the \( \Lambda_c/D \) ratio and thus the \( R_{AA} \) of single electrons originating from the decay of \( \Lambda_c \). Motivated by these interesting results we evaluate in this work the drag and diffusion of hidden charm mesons \( J/\psi \) and \( \eta_c \) and study their dependence with temperature.

In the next section we provide the formulae for the drag and diffusion coefficients followed by a discussion on the matrix elements of elastic scattering of the \( J/\psi \) with the light vector mesons in section III. Results are given in Section IV and finally a summary in Section V. We provide the squared matrix elements in the appendix.

II. FORMALISM

The drag (\( \gamma \)) and diffusion (\( D \)) coefficients of \( J/\psi \) and \( \eta_c \) are obtained from the elastic scattering of \( J/\psi \) with the light thermal hadrons (\( H \)) which constitute the equilibrated thermal medium. For the process \( J/\psi, \eta_c(p_1) + H(p_2) \rightarrow J/\psi, \eta_c(p_3) + H(p_4) \), the drag \( \gamma \) can be expressed as\textsuperscript{21}:

\[
\gamma = p_i A_i / p^2 ,
\]

where \( A_i \) is given by

\[
A_i = \frac{1}{2 E_{p_1}} \int \frac{d^3 p_2}{(2\pi)^3 E_{p_2}} \int \frac{d^3 p_3}{(2\pi)^3 E_{p_3}} \int \frac{d^3 p_4}{(2\pi)^3 E_{p_4}} \frac{1}{g_i(j/\psi, \eta_c)} \sum |M|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)
\]

\[
f(p_2) [1 \pm f(p_4)] [(p_1 - p_3)_i] \equiv \langle \langle (p_1 - p_3)_i \rangle \rangle .
\]

The \( g_i(j/\psi, \eta_c) \) is the statistical degeneracy of the probe particle, \( J/\psi \) or \( \eta_c \). The thermal distribution function \( f(p_2) \) of the hadron \( H \) in the incident channel takes the form of Bose-Einstein or Fermi-Dirac distribution depending on its spin and \( 1 \pm f(p_4) \) are their corresponding Bose enhanced or Pauli blocked phase space factor in their final states. The drag coefficient of Eq. (2) is just a measure of the thermal average of the momentum transfer, \( p_1 - p_3 \) weighted by the square of the invariant amplitude \( |M|^2 \).

In a similar way, the diffusion coefficient \( D \) can be defined as:

\[
D = \frac{1}{4} \left[ \langle p^2 \rangle - \langle (p_1 \cdot p_2)^2 \rangle / p_1^2 \right] .
\]
III. DYNAMICS

The hot hadronic matter produced in the later stages of relativistic heavy ion collisions is populated by light pseudoscalars and vector mesons like π, K, η, ρ, ω and φ. The magnitude of such scatterings are estimated either by introducing different perturbative or non-perturbative approach at quark level \[22, 23\] or by using an effective Lagrangian to calculate Feynman diagrams. Concerning the latter approach, SU(4) is the smallest possible symmetry group which includes the charmonium state explicitly along with the light and heavy pseudoscalar and vector mesons.

The corresponding pseudoscalar and vector meson matrices and as well as the chiral Lagrangian is explicitly given in works like \[24, 25\]. However since SU(4) symmetry is badly broken by the large mass of the charmed meson, terms involving hadron masses are included to the chiral Lagrangian using the experimentally determined values.

Since pions are identified with the Nambu-Goldstone bosons of QCD their interaction strength with other particles should abruptly decrease in the chiral limit. We recall the standard relation \[26, 27\] with other particles should abruptly decrease in the Goldstone bosons of QCD their interaction strength the experimentally determined values.

\[
\langle T(p_1) \rangle = \frac{1}{512\pi^4 E_{p_1}} \int_0^{\infty} \frac{p_2^2 dp_2 d(cos \chi)}{E_{p_2}} \lambda(s, m_{p_1}^2, m_{p_2}^2) \int_1^{\infty} d(cos \theta_{c.m.})\]

\[
\frac{1}{g} \sum_{i} |M|^2 \int_0^{2\pi} d\phi_{c.m.} T(p_3), \tag{4}
\]

where \(\lambda(s, m_{p_1}^2, m_{p_2}^2) = (s - m_{p_1}^2 - m_{p_2}^2)^2 - 4m_{p_1}^2 m_{p_2}^2\) is the triangular function and \(|M|^2\) is the square of the invariant amplitude for the elastic scattering of \(J/\Psi\) or \(\eta_c\) with thermal hadrons.

When the other hadron is \(J/\Psi\) or \(\eta_c\), this term in \[11\] vanishes exactly, not only in the chiral limit but also for finite pion mass (because \(\vec{I}_\pi \cdot \vec{I}_{J/\Psi} = 0\)). Hence the contribution of the pion in the charmonium scattering length starts from \(\mathcal{O}(m_\pi^2)\), which shows that at least at low energy the pion-charmonium interaction should be very weak. To support this argument we refer to the calculations of the meson exchange model of Haglin et.al \[24\]. From their calculations we can see that the elastic channels of \(J/\Psi\) interaction involving the light pseudoscalars are significantly smaller in comparison with the vector mesons. They suggest that π, η, and k elastic cross sections with \(J/\Psi\) are of order 100 fb, 1 nb and 100 nb respectively. On the other hand the contributions for elastic scattering with ρ, ω and φ mesons are quantitatively much larger, up to about a few mb. We hence consider the elastic scattering of the heavy charmonium states like \(J/\Psi\) and \(\eta_c\) with vector mesons only. These processes involve vector-vector-pseudoscalar interactions which are not present in the chiral Lagrangian. The relevant effective interaction describing \(J + V \rightarrow \eta_c \rightarrow J/\Psi + V\) processes \[24\] is

\[
\mathcal{L}_{J V \eta_c} = g_{J V \eta_c} \epsilon_{\alpha \beta \gamma \delta} \{ \partial^\alpha J/\Psi^\beta \} \{ \partial^\gamma V^\delta \} \eta_c, \tag{7}
\]

where \(g_{J V \eta_c} = 2.44 \text{ GeV}^{-1}, 7.03 \text{ GeV}^{-1}\) and 4.51 \text{ GeV}^{-1} for \(V = \rho, \omega \text{ and } \phi\) respectively \[24\].

The \(s\) and \(u\) channel diagrams for the process \(J/\Psi + V \rightarrow \eta_c \rightarrow J/\Psi + V\) are shown in the panels (A) and (B) of Fig. 1. The matrix elements for the...
two channels are respectively given by,

\[ M_s = -g_{J\Psi}\eta_c [\varepsilon^\beta(p_1)\varepsilon^\gamma(p_2)\varepsilon^{*\delta}(p_3)\varepsilon^{*\epsilon}(p_4) \epsilon_{\alpha\beta\gamma\delta} p_1^\alpha p_2^\beta p_3^\gamma p_4^\delta] / (s - m_{\eta_c}^2) \] (8)

and

\[ M_u = -g_{J\Psi}\eta_c [\varepsilon^\beta(p_1)\varepsilon^\gamma(p_2)\varepsilon^{*\delta}(p_3)\varepsilon^{*\epsilon}(p_4) \epsilon_{\alpha\beta\gamma\delta} p_1^\alpha p_2^\beta p_3^\gamma p_4^\delta] / (u - m_{\eta_c}^2) \] (9)

The modulus squared of the spin averaged total amplitude i.e. \[ |M|^2 \] are given in the Appendix.

The s and u channel diagrams of the \( \eta_c \) meson scattering with the thermalized vector mesons by exchanging \( J/\Psi \) are shown in the panels (C) and (D) of Fig. 1. The respective matrix elements are given by

\[ M_s = -g_{J\Psi}\eta_c [\varepsilon^\beta(p_1)\varepsilon^\gamma(p_2)\varepsilon^{*\delta}(p_3)\varepsilon^{*\epsilon}(p_4) \epsilon_{\alpha\beta\gamma\delta} p_1^\alpha p_2^\beta p_3^\gamma p_4^\delta] / (s - m_{\eta_c}^2) \] (10)

and

\[ M_u = -g_{J\Psi}\eta_c [\varepsilon^\beta(p_1)\varepsilon^\gamma(p_2)\varepsilon^{*\delta}(p_3)\varepsilon^{*\epsilon}(p_4) \epsilon_{\alpha\beta\gamma\delta} p_1^\alpha p_2^\beta p_3^\gamma p_4^\delta] / (u - m_{\eta_c}^2) \] (11)

Using these scattering amplitudes in eq. (4) we obtain the drag and diffusion coefficients of the \( J/\Psi \) and \( \eta_c \) mesons in hadronic matter.

The low-energy interactions of \( J/\Psi \) as well as \( \eta_c \) with \( \pi, \rho \) or \( N \) have been investigated by Yokokawa et al [27] in the quenched lattice framework. From the scattering lengths, \( a \) (say) of \( J/\Psi \) or \( \eta_c \) interacting with light hadrons \( H \) (where \( H = \pi, \rho \) and \( N \)) we can extract the dimensionless threshold, the \( T \)-matrix element by using the relation

\[ T = 4\pi|m_{(J/\Psi, \eta_c)} + m_H|a \] (12)

Using these \( |T|^2 \) in place of \( |M|^2 \) in Eq. (4), we can get an alternative estimation of the diffusion and drag coefficients of \( J/\Psi \) as well as \( \eta_c \) mesons in hadronic matter [10]. The extracted values of \( T \) from \( a \) in fm are given in Table I.

**IV. RESULTS**

We begin this section by plotting in Fig. 2 the drag coefficients of the \( J/\Psi \) (solid line) and \( \eta_c \) mesons (dashed line) as a function of temperature. In Fig. 3 we show the corresponding results for the case where the amplitudes are extracted from scattering lengths. Not much difference is seen between the \( J/\Psi \) and \( \eta_c \) in this case. As mentioned before the drag is a measure of the momentum transfer between the \( J/\Psi \) (or \( \eta_c \)) and the thermal hadrons weighted by the interactions implemented through \( |M|^2 \). The average momentum of the bath particles increase with temperature. Therefore, the thermal hadrons gain the ability to transfer larger momentum through interactions as the temperature of the bath increases. This causes the rise of drag at high temperatures both for \( J/\Psi \) and \( \eta_c \). Let us now show results for the diffusion coefficient \( D \). This is plotted against \( T \) in Figs. 4 and 5 respectively corresponding to the effective Lagrangian and scattering length approaches. In addition to the results of direct calculation using eq. (4), the results from the fluctuation-dissipation theorem (FDT) are also shown by red lines in Figs. 4 and 5.
and 5. As for the earlier case of the drag coefficient, the diffusion in the scattering length approach is similar for the \( J/\psi \) and \( \eta_c \) mesons. The rise of diffusion coefficients with increasing temperature has the same origin as that of drag coefficients as explained above. The momentum suppression of \( J/\psi \) and \( \eta_c \) at high momenta in nuclear collisions compared to proton-proton collision may be approximately estimated as, \( R_{AA} \sim e^{-\Delta \tau \gamma} \) [13], where \( \Delta \tau \) is typically the life time of the hadronic phase. Taking \( \Delta \tau \sim 5 \) fm/c and \( \gamma \sim 10^{-4} \) one finds that \( R_{AA} \) is close to unity. Thus the hadronic phase appears to play no significant role in the suppression of \( J/\psi \) or \( \eta_c \) at high \( \langle p_T \rangle \). Therefore, if a significant suppression is observed that will indicate the presence of QGP in the evolving fireball produced in heavy ion collisions at relativistic energies.

V. SUMMARY AND DISCUSSIONS

We have estimated the drag and diffusion coefficients of \( J/\psi \) and \( \eta_c \) in a hot hadronic medium using effective field theory and \( T \) matrices obtained within the ambit of quenched lattice calculations. The values of these transport coefficients turn out to be small compared to the values obtained for open charmed hadrons for the temperature range relevant for the hadronic phase expected to be formed in the later stages of the evolving matter produced in nuclear collisions at RHIC and LHC energies. We find that the momentum suppression of \( J/\psi \) and \( \eta_c \) are not significant in the hadronic phase. Therefore, such a suppression if observed experimentally will possibly indicate creation of QGP in heavy ion collisions at relativistic energies.

VI. APPENDIX

The modulus square of the spin averaged total amplitude for the processes of \( J/\psi + V \rightarrow \eta_c \rightarrow J/\psi + V \) is given by the following expression,

\[
|M|^2 = |M_s|^2 + |M_u|^2 + 2|M_s|M_u^* \tag{13}
\]

where the respective terms in the expression are given by

\[
|M_s|^2 = \frac{g^4_{J\psi\eta_c}}{36(s - m_{\eta_c}^2)^2} \lambda^2(s, m_{J/\psi}^2, m_V^2)
\]

\[
|M_u|^2 = \frac{g^4_{J\psi\eta_c}}{36(u - m_{\eta_c}^2)^2} \lambda^2(u, m_{J/\psi}^2, m_V^2)
\]

\[
M_s M_u^* = \frac{g^4_{J\psi\eta_c}}{9(s - m_{\eta_c}^2)(u - m_{\eta_c}^2)} f.
\]
where,

\[
I = \frac{1}{8} [m_y^2 + s^4 + 2s^3(t - 2m_y^2) + 2m_y^2(t - 2m_y^2) - 4m_y^2(s + m_y^2) + m_y^4(t^2 + m_y^2) + s^2(t^2 - 4t m_y^2 + 6m_y^2) + m_y^4(6s^2 + t^2 + 6m_y^2 + 2s(t + 2m_y^2)) - 2m_y^2(2s^3 + 2s(t - m_y^2)(s + m_y^2) + m_y^2(t^2 + 2m_y^2))].
\]

and \(\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz\) is the triangular function. Next, the spin averaged modulus square of total amplitude for the processes \(\eta_c + V \rightarrow J/\psi \rightarrow \eta_c + V\) are given by

\[
|\mathcal{M}|^2 = |\mathcal{M}_s|^2 + |\mathcal{M}_u|^2 + 2\mathcal{M}_s \mathcal{M}_u^* \tag{14}
\]

where

\[
|\mathcal{M}_s|^2 = \left( \frac{g_{4V\eta_c}}{3} \right)^2 \frac{1}{4} (t - 4m_y^2)(s + m_y^2 - m_{\pi c})^2 + \frac{1}{8}(s + m_y^2 - m_{\pi c})^4 + \frac{s^2}{4} (t^2 - 4t m_y^2 + 8m_y^2)/(s - m_y^2)^2.
\]

and

\[
M_s^2 M_u^* = \left( \frac{g_{4V\eta_c}}{3} \right)^2 \frac{1}{4} (t - 4m_y^2)(s + m_y^2 - m_{\pi c})^2 + \frac{1}{8}(s + m_y^2 - m_{\pi c})^4 + \frac{s^2}{4} (t^2 - 4t m_y^2 + 8m_y^2)/(s - m_y^2)^2.
\]

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