Theory of proximity effect in two-dimensional unconventional superconductor with Rashba spin-orbit interaction

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We study the anomalous proximity effect in diffusive normal metal (DN)/unconventional superconductor junctions, where the local density of states (LDOS) in the DN has a zero-energy peak due to the penetration of the odd-frequency spin-triplet s-wave pairing. In this study, we consider a two-dimensional unconventional superconductor on the substrate in the presence of a Rashba spin-orbit coupling (RSOC) $\lambda$, where the Rashba vector is parallel to the $z$-direction. The anomalous proximity effect, originally predicted in spin-triplet $p$-wave superconductor junctions, is sensitive to the RSOC. It disappears with the increase of $\lambda$. On the other hand, the anomalous proximity effect can be switched on by the large $\lambda$ values in the spin-singlet $d_{xy}$-wave superconductor junctions. The resulting zero-energy LDOS and the magnitude of the odd-frequency spin-triplet s-wave pair amplitude increase with the increase of $\lambda$.

I. INTRODUCTION

In unconventional superconductors (SCs), nodes of the energy gap and the change of the sign of the pair potential (gap function) are generated on the Fermi surface. It is known that the change of the sign of the pair potential produces in-gap zero-energy states localized near the surface, known as zero-energy Andreev bound states (ZESABS) [1–5]. The ZESABS manifests itself as a zero bias conductance peak in quasiparticle tunneling experiments [4, 6–9].

Nowadays, it is recognized that the ZESABS has deep physical meanings from the aspects of topology and symmetry in condensed matter physics [10]. It is known that the ZESABS is protected by the topological invariant defined in the bulk Hamiltonian [10–15]. Thus, unconventional superconductors hosting nodes can be classified as topological superconductors [13–15]. Besides this property, the ZESABS has a significant meaning from the aspect of the symmetry of Cooper pairs.

The breaking of translational invariance induces an odd-frequency pairing, i.e., spin-singlet odd-parity or spin-triplet even-parity, even if the symmetry of the bulk superconductor has a standard even-frequency pairing, i.e., spin-singlet even-parity or spin-triplet odd-parity [10–18]. Thus, the ZESABS always accompanies an odd-frequency pairing due to the breaking of translational invariance in the superconductor [20–22]. If the symmetry of the bulk superconductor is a spin-singlet $d_{xy}$-wave, the resulting ZESABSs have an odd-frequency spin-singlet odd-parity (OSO), see Table I. The resulting ZESABS manifests itself as a zero bias conductance peak in quasiparticle tunneling experiments [4, 6–9].

TABLE I. Possible pairing symmetry and symbols for symmetry of Cooper pair amplitude. Pairing symmetries of Cooper pair are classified into ESE (even-frequency spin-singlet even-parity), ETO (even-frequency spin-triplet odd-parity), OSO (odd-frequency spin-singlet odd-parity), and OTE (odd-frequency spin-triplet even-parity).

| Frequency | Spin | Parity | Symbol |
|-----------|------|--------|--------|
| Even      | Singlet | Even | ESE    |
| Even      | Triplet | Odd  | ETO    |
| Odd       | Singlet | Odd  | OSO    |
| Odd       | Triplet | Even | OTE    |

On the other hand, if the symmetry of the bulk superconductor is a spin-triplet $p$-wave, the resulting ZESABSs exhibit an odd-frequency spin-triplet even-parity (OTE), see Table I pairing symmetry [20–22, 25]. In the latter case, if the superconductor is contacted by a diffusive normal metal (DN), the OTE pairing can penetrate into the DN [25], as it has an s-wave component, which is robust against impurity scattering. The resulting local density of states (LDOS) in the DN has a zero-energy peak [25–29] in contrast to the conventional proximity effect, where the LDOS has a gap-like structure around zero-energy [30]. This unusual condition is known as an anomalous proximity effect [26–28, 31]. The anomalous proximity effect can occur in a DN/noncentrosymmetric superconductor junction when the spin-triplet pair potential is dominant [32].

The anomalous proximity effect, triggered by an odd-frequency spin-triplet $s$-wave pairing, shows several interesting physical properties: i) ZEP of the LDOS in the DN [26, 27, 33, 34], ii) ZEP of the LDOS at rough surface [35], iii) zero bias conductance peak in quasiparticle transport in the DN/spin-triplet $p$-wave superconductor junctions [26, 27, 31], iv) significant enhancement of the Josephson current at low temperatures in the DN/spin-triplet $p$-wave superconductor junctions [28, 31, 32] v) paramagnetic Meissner response [36, 43], and vi) anomalous surface impedance [44, 45]. It has been also shown that the anomalous proximity effect can occur in topologically designed hybrid systems based on conventional spin-singlet $s$-wave superconductor systems with spin-orbit coupling and Zeeman effect [31, 46, 47] and the anomalous proximity effect has been studied considering the classifica-
tion of the topological nature of the Hamiltonian \[48\]. In addition, there are several studies reporting that odd-frequency pairings appear as a Majorana fermion, which is a special type of the ZESABS in various topological superconducting systems \[46, 49–59\].

Although OTE \(s\)-wave pairing has been discussed in diffusive ferromagnet (DF)/conventional spin-singlet \(s\)-wave superconductor junctions, it is difficult to realize pure OTE \(s\)-wave pairing state in a DF \[60–71\]. Conversely, the demonstrated anomalous proximity effect generated by ZESABSs specific to unconventional superconductors is remarkable, as it can induce a purely odd-frequency pairing or at least a significant amount of it at low energy. Thus, it is interesting to understand the mechanism of effect of several external perturbations on the anomalous proximity effect. Among these, the Rashba spin-orbit coupling is very interesting, as it inevitably exists near the interface or the thin films of superconductor grown on the substrate. Here, we focus on the effect of the Rashba spin-orbit coupling (RSOC) on the anomalous proximity effect. As shown by a study of noncentrosymmetric superconductors, the RSOC can mix spin-triplet odd-parity pairing with the spin-singlet even-parity pairing \[72–80\]. It is possible that the anomalous proximity effect can be induced even in a spin-singlet \(d_{xy}\)-wave superconductor, when an OTE \(s\)-wave pairing can be induced from the RSOC.

In this paper, we study the anomalous proximity effect of DN/superconductor junctions with the RSOC by choosing a spin-triplet \(p_x\)-wave pairing with a \(d\)-vector parallel to \(z\)-axis and a spin-singlet \(d_{xy}\)-wave pairing. As the RSOC, we consider that it is proportional to \(\mathbf{g} \cdot \mathbf{\sigma}\) with \(\mathbf{g} = (\sin k_y, -\sin k_x, 0)\) which is originated from the inversion symmetry breaking in the \(z\)-direction and Pauli matrix \(\mathbf{\sigma}\) in the spin-space. We calculate the surface density of states, the spin-triplet \(s\)-wave component of the odd-frequency pairing, and the local density of states in the DN attached to superconductors, based on a tight-binding model. For the spin-triplet \(p_x\)-wave superconductor, as the ZESABS is fragile against the RSOC \[17, 81\], the magnitude of the OTE pairing is reduced and the resulting anomalous proximity effect is weakened by the RSOC. On the other hand, it is shown that the anomalous proximity effect can be switched on by the RSOC for spin-singlet \(d_{xy}\)-wave superconductor junctions. The magnitude of the LDOS at zero-energy in the DN increases with the increase of the RSOC. This indicates that the anomalous proximity effect can be detected for high \(T_c\) cuprate junctions.

The organization of this paper is as follows. In Section \[III\] we discuss the momentum resolved surface density of states and the OTE pairing for semi-infinite superconductors with the RSOC. In Section \[IV\] we consider the DN/superconductor junctions and discuss the LDOS and the OTE pairing in the DN. In Section \[V\] we summarize our results.

### II. SEMI-INFINITE SUPERCONDUCTOR

![FIG. 1. Schematic pictures corresponding to Table \[II\] (a) Bulk system without RSOC, (b) Bulk system with RSOC, (c) Semi-infinite system or junction without RSOC.]

| TS     | RSOC | spin-triplet | spin-singlet |
|--------|------|--------------|--------------|
| (a)    | YES  | NO           | ETO          |
| (b)    | YES  | YES          | ETO          |
| (c)    | NO   | NO           | ETO, OTE, ESE, ESE, ETO |

Before we start discussion, we explain about possible symmetry class of Cooper pair by the external symmetry breaking for spin-triplet \(p_x\)-wave superconductor and spin-singlet \(d_{xy}\)-wave superconductor in the bulk system. TS indicates translational symmetry in the \(x\)-direction. Symbols in the third and fourth columns are given in Table \[II\] (a), (b) and (c) in the first column correspond to those in Fig. \[I\].

### TABLE II. Possible symmetry of Cooper pair amplitude for spin-triplet \(p_x\)-wave superconductor and spin-singlet \(d_{xy}\)-wave superconductor in the bulk system. TS indicates translational symmetry in the \(x\)-direction. Symbols in the third and fourth columns are given in Table \[II\] (a), (b) and (c) in the first column correspond to those in Fig. \[I\].
and Table II (a). In the presence of the RSOC, ETO pairing mixes and the resulting pairing symmetry are ESE and ETO [Fig. II (b) and Table II (b)] 12. On the other hand, when only the translational invariance is broken without the RSOC, the symmetry of pair amplitude are ESE and OSO [Fig. II (c) and Table II (c)] 11.

In the case that the system does not have translational symmetry in the presence of the RSOC, all pairing symmetries: ETO, ESE, OTE and OSO exist both for spin-triplet \( p_x \)-wave and spin-singlet \( d_{xy} \)-wave superconductors.

A. Model

We consider two dimensional superconductors on a square lattice model with open boundary condition [Fig. 2] where the superconductor is located in \( j_x \geq 1 \) \( j = (j_x, j_y) \) is the coordinate of a lattice site] and discuss the RSOC dependence of a surface density of states and the odd-frequency spin-triplet s-wave pair amplitude. We compare the results for the spin-triplet \( p_x \)-wave superconductor and the spin-singlet \( d_{xy} \)-wave one. Throughout this paper, we use lattice constant as a unit of length. The Hamiltonian \( \mathcal{H}_{SC} \) is given by

\[
\mathcal{H}_{SC} = \mathcal{H}_t + \mathcal{H}_{SO} + \mathcal{H}_\Delta - \mu \sum_j n_j, \tag{1}
\]

\[
\mathcal{H}_t = -t \sum_{\langle i,j \rangle, \sigma} \left( c_{i,\sigma}^\dagger c_{j,\sigma} + H.c. \right), \tag{2}
\]

\[
\mathcal{H}_{SO} = \frac{\lambda}{2t} \sum_{j,\sigma,\sigma'} \left( c_{j,\sigma}^\dagger c_{j+1,\sigma} + e_y, \sigma' - c_{j,\sigma}^\dagger c_{j-1,\sigma}, \sigma' \right) (\sigma_1)_{\sigma,\sigma'} \tag{3}
\]

\[
- \frac{\lambda}{2t} \sum_{j,\sigma,\sigma'} \left( c_{j,\sigma}^\dagger c_{j+1,\sigma}, \sigma' - c_{j,\sigma}^\dagger c_{j-1,\sigma}, \sigma' \right) (\sigma_2)_{\sigma,\sigma'}, \tag{4}
\]

\[
\mathcal{H}_\Delta = \sum_{i,j,\sigma,\sigma'} \Delta_{ij} (c_{i,\sigma}^\dagger c_{j,\sigma'}^\dagger + H.c.), \tag{5}
\]

\[
n_j = \sum_{\sigma} c_{j,\sigma}^\dagger c_{j,\sigma} + H.c. \tag{6}
\]

where \( \{i,j\} \) in Eq. [2] denotes a sum of nearest-neighbor pairs, \( \sigma_i \) \( i = 1, 2, 3 \) is a Pauli matrix, and \( e_x, e_y \) are unit vector in the \( x (y) \)-direction i.e., \( e_x = (1, 0) \) and \( e_y = (0, 1) \). \( t \) is a hopping integral, \( \lambda \) is a Rashba spin-orbit coupling and \( \mu \) is a chemical potential. Here we assume \( \mu \leq \lambda \). We set \( \mu/t = -2.4 \) where we do not have to take into account the effect of the van Hove singularity. As the symmetry of the superconducting gap, we consider a spin-triplet \( p_x \)-wave pairing and a spin-singlet \( d_{xy} \)-wave one. For the spin-triplet \( p_x \)-wave case, the pair potential is

\[
\Delta_{ij} = \frac{\Delta_0}{t} \delta_{ij-e_x, \delta_{\sigma,-\sigma'}}, \tag{7}
\]

where we choose the direction of the \( d \)-vector parallel to the z-direction. For the spin-singlet \( d_{xy} \)-wave case, the pair potential is

\[
\Delta_{ij} = -\frac{\Delta_0}{2} f(\sigma)(\delta_{ij-e_x-e_y} - \delta_{ij-e_x+e_y})\delta_{\sigma,-\sigma'}, \tag{8}
\]

with \( f(\uparrow) = 1 \) and \( f(\downarrow) = -1 \). We set \( \Delta_0/t = 0.01 \) for both spin-triplet \( p_x \)-wave and spin-singlet \( d_{xy} \)-wave superconductor throughout this paper.

The energy dispersion of the periodic system for the spin-triplet \( p_x \)-wave superconductor is

\[
E(k) = \pm \sqrt{\xi_k^2 + (\Delta_0)^2} \sin^2 k_x \pm \lambda \sqrt{\sin^2 k_x + \sin^2 k_y}, \tag{9}
\]

with \( \xi_k = -2t(\cos k_x + \cos k_y) - \mu \) and the superconducting gap closes for \( \lambda \geq \Delta_0 \). For the spin-singlet \( d_{xy} \)-wave superconductor, the energy dispersion is

\[
E(k) = \pm \sqrt{[\xi_k \pm \lambda(\sin k_x + \sin k_y)]^2 + (\Delta_0 \sin k_x \sin k_y)^2}. \tag{10}
\]

B. Method to calculate local Green’s function

The surface DOS \( \rho(E, k_y) \) and the odd-frequency spin-triplet s-wave pair amplitude are calculated by using a surface Green’s function \( G_{j_x=1}^\infty(z, k_y) \). Here, \( G_{j_x=1}^\infty(z, k_y) \) is the Green’s function at the leftmost site [Fig. 2]. The derivation of \( G_{j_x=1}^\infty(z, k_y) \) is explained in Appendix A.

The momentum resolved surface DOS is obtained as

\[
\rho(E, k_y) = -\frac{1}{\pi} \text{Im} \left[ \text{tr} G_{j_x=1}^\infty(z = E + i\eta, k_y) \right], \tag{11}
\]

where \( \eta \) is an infinitesimally small constant (we set \( \eta/t = 10^{-5} \)) and trace is only taken in particle space. Then the surface DOS \( \rho(E) \) is given by

\[
\tilde{\rho}(E) = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk_y \rho(E, k_y). \tag{12}
\]

The odd-frequency spin-triplet s-wave pair amplitude
at the surface is defined as
\[
P_{\text{triplet}}(\omega_n) = P_{\uparrow\uparrow}(\omega_n),
\]
\[
P_{\text{triplet}}(\omega_n) = \frac{1}{2}\left[ P_{\uparrow\downarrow}(\omega_n) + P_{\downarrow\uparrow}(\omega_n) \right],
\]
\[
P_{\text{triplet}}(\omega_n) = P_{\downarrow\downarrow}(\omega_n),
\]
\[
p_{\text{triplet}}(\omega_n) = \sum_{s_z = -1,0,1} \left| P_{\text{triplet}}(\omega_n) \right|,
\]
with
\[
P_{\sigma,\sigma'}(\omega_n) = \frac{1}{N_{k_y}} \sum_{k_y} \tilde{F}_{\sigma,\sigma'}(z = i\omega_n, k_y),
\]
\[
G_{y=1}(z, k_y) = \left( \tilde{G}_{\sigma,\sigma'}(z, k_y) \tilde{F}_{\sigma,\sigma'}(z, k_y) \right),
\]
where $\omega_n$ is a Matsubara frequency. In Eq. (17), $\tilde{G}_{\sigma,\sigma'}(z, k_y)$, $\tilde{F}_{\sigma,\sigma'}(z, k_y)$, and $\tilde{G}_{\sigma,\sigma'}(z, k_y)$ are 2x2 matrices. $\tilde{F}_{\sigma,\sigma'}(z, k_y)$ and $\tilde{F}_{\sigma,\sigma'}(z, k_y)$ are anomalous Green’s functions. Note that $p_{\text{triplet}}(\omega_n)$ is odd in $\omega_n$.

C. Topological number

In this subsection, we consider a bulk system and explain about a winding number $W$ and one-dimensional $Z_2$ topological number $\nu$. $W$ and $\nu$ are topological numbers defined in a bulk and they predict the number of the ZESABSs known as bulk-boundary correspondence. The bulk Hamiltonian for the spin-triplet $p_x$-wave superconductor and that for the spin-singlet $d_{xy}$-wave superconductor are given by
\[
H_p(k) = H_1(k) + H_{SO}(k) + H^p_\Delta(k),
\]
\[
H_\nu(k) = H_1(k) + H_{SO}(k) + H^\nu_\Delta(k),
\]
with
\[
H_1(k) = -2t \left( \cos k_x + \cos k_y \right) - \mu \sigma_0 \tau_3,
\]
\[
H_{SO}(k) = \lambda \left( \sin k_y \sigma_1 \tau_0 - \sin k_x \sigma_2 \tau_3 \right),
\]
\[
H^p_\Delta(k) = \Delta_0 \sin k_x \sigma_1 \tau_1,
\]
\[
H^\nu_\Delta(k) = -\Delta_0 \sin k_x \sin k_y \sigma_2 \tau_2,
\]
where $\tau_i$ ($i = 1, 2, 3$) is a Pauli matrix in the particle hole space and $\tau_0$ is an identity matrix. Here $H_1(k)$ and $H_{SO}(k)$ are the Fourier transformed form of Eq. (2) and Eq. (3), respectively and $H^p_\Delta(k)$ and $H^\nu_\Delta(k)$ are the Fourier transformed form of Eq. (4) with Eq. (6) and Eq. (7), respectively.

1. Spin-triplet $p_x$-wave superconductor

In this subsection, we explain about the Topological numbers for the spin-triplet $p_x$-wave superconductor. The winding number can be defined in a bulk if there is a chiral operator $\Gamma$ which anticommutes with the bulk Hamiltonian ($\{H(k), \Gamma\} = 0$). The winding number is given by
\[
W(k_y) = \frac{i}{4\pi} \int_{-\pi}^{\pi} dk_y \text{Tr} \left[ \Gamma H^{-1}(k) \partial_{k_y} H(k) \right],
\]
where $k_\parallel = k_y$ and $k_\perp = k_x$ are wave numbers which are parallel and perpendicular to the surface, respectively.

Without the RSOC, the chiral operator is $\Gamma = \Gamma_p \equiv S_z CT = i\sigma_1 \tau_2$ ($S_z = \sigma_3$ is a spin-rotation operator around the $z$-axis, $C = \sigma_0 \tau_1 K$ is a charge conjugation operator, $T = i\sigma_2 \tau_0 K$ is a time-reversal operator and $K$ is a complex conjugation operator.) and it anticommutes with $H_1(k)$ and $H^\nu_\Delta(k)$:
\[
\{H_1(k), \Gamma_p\} = \{H^\nu_\Delta(k), \Gamma_p\} = 0.
\]
On the other hand, the RSOC term of the Hamiltonian does not anticommute with $\Gamma_p$:
\[
\{H_{SO}(k), \Gamma_p\} \neq 0.
\]
Therefore, the spin-triplet $p_x$-wave superconductor with non-zero value of the RSOC does not have chiral symmetry and the winding number cannot be defined. More precisely, the Hamiltonian Eq. (18) anticommutes with $\sigma_2 \tau_1$ but the winding number is always zero regardless of the value of the RSOC and it is not related to the number of the surface states. Then the ZESABSs are fragile against the RSOC given by Eq. (21) [17, 81]. This result does not depend on the direction of the $d$-vector. The winding number for the spin-triplet $p_x$-wave superconductor without the RSOC is
\[
W(k_y) = \begin{cases} 0 & k_y < -\alpha_-^p, \\ -2 & -\alpha_-^p < k_y < \alpha_+^p, \\ 0 & \alpha_+^p < k_y, \end{cases}
\]
for $-4t < \mu < 0$ and
\[
W(k_y) = \begin{cases} -2 & -\pi < k_y < -\alpha_+^p, \\ 0 & -\alpha_+^p < k_y < \alpha^p_+, \\ -2 & \alpha^p_+ < k_y < \pi, \end{cases}
\]
for $0 < \mu < 4t$ with $\alpha^p_+ = \arccos(-\mu/2t \pm 1)$.

Another topological number ($Z_2$ topological number [82]) can be defined for the spin-triplet $p_x$-wave superconductor in the presence of the RSOC for $k_y = 0$ or $\pi$. At $k_y = 0$ or $\pi$, the Hamiltonian given by Eq. (18) possesses time-reversal symmetry ($T = i\sigma_2 \tau_0 K$) and particle-hole symmetry ($C = i\sigma_0 \tau_1 K$):
where $I$ and $II$ indicate the Kramers degeneracy index, $\alpha$ denotes the band index and $\chi_{\alpha,n}(k_x)$ is a $U(1)$ phase factor. The Hamiltonian given by Eq. (18) with $k_y = 0$ or $\pi$ belongs to DIII class in one dimension and it is characterized by $\mathbb{Z}_2$ topological number. $\mathbb{Z}_2$ topological number is defined by the one of the Kramers pair as
\[(-1)^{\nu_1^I} = (-1)^{\nu_2^I} = \pm 1,\] (33)
with
\[\nu_1^I = \exp \left[ i \int_0^{2\pi} dk_x A^\beta_{\alpha}(k_x) \right],\] (34)
\[A^\beta_{\alpha}(k_x) = -i \sum_{\sigma_{\alpha\sigma}} \langle u_{n_{\sigma\alpha}}(k_x, k_y = \eta) \rangle \partial_{k_x} \| u_{n_{\sigma\alpha}}(k_x, k_y = \eta) \|.\] (35)

Here $\beta = I$ or $II$, summation in Eq. (35) runs over its occupied bands and $A^\beta_{\alpha}(k_x)$ is a Berry connection. The system is topologically trivial when $(-1)^{\nu_1^I} = 1$ and is topologically nontrivial when $(-1)^{\nu_1^I} = -1$. When $(-1)^{\nu_1^I}$ is $-1$, there are ZESABSs. $\mathbb{Z}_2$ topological number for the spin-triplet $p_x$-wave superconductor with the RSOC is
\[(-1)^{\nu_I^\beta} = \begin{cases} 1 & \mu > -4t \text{ or } 0 < \mu < \mu_0, \\ -1 & -4t < \mu < 0, \end{cases}\] (36)
for $k_y = 0$ and it is
\[(-1)^{\nu_I^\beta} = \begin{cases} 1 & \mu < 0 \text{ or } 4t < \mu, \\ -1 & 0 < \mu < 4t, \end{cases}\] (37)
for $k_y = \pi$.

2. Spin-singlet $d_{xy}$-wave superconductor

For the spin-singlet $d_{xy}$-wave case, ZESABSs can be understood by only using the winding number. The chiral operator is given by $\Gamma = \Gamma_d \equiv CT = -\sigma_2 \tau_1$, and $\Gamma_d$ anticommutes with the Hamiltonian:
\[\{ H_\chi(k), \Gamma_d \} = \{ H_{\text{SO}}^I(k), \Gamma_d \} = \{ H_{\text{SO}}(k), \Gamma_d \} = 0.\] (38)

Then, the ZESABSs for the spin-singlet $d_{xy}$-wave superconductor are robust against the RSOC. The winding number for the spin-singlet $d_{xy}$-wave superconductor with the RSOC is for the parameters we choose in this paper ($\lambda/\Delta_0 \leq 10$ with $\Delta_0/t = 0.01$ and $\mu/t = -2.4$), the winding number is
\[W(k_y) = \begin{cases} 0 & k_y < -\alpha_+^d, \\ -1 & -\alpha_+^d < k_y < -\alpha_-^d, \\ -2 & -\alpha_-^d < k_y < 0, \\ 2 & 0 < k_y < \alpha_+^d, \\ 1 & \alpha_+^d < k_y < \alpha_-^d, \\ 0 & \alpha_-^d < k_y, \end{cases}\] (39)
with
\[
\cos \alpha_\pm^d = \frac{1}{4t^2 + \lambda^2} \left[ -2t(2t + \mu) \pm \lambda \sqrt{\lambda^2 - 4t\mu - \mu^2} \right].
\] (40)

Other cases with different parameters are shown in the Appendix [13].

D. Results

We discuss the momentum resolved surface DOS, the surface DOS and the odd-frequency spin-triplet $s$-wave pair amplitude at the surface for the spin-triplet $p_x$-wave and the spin-singlet $d_{xy}$-wave superconductors.

1. Spin-triplet $p_x$-wave superconductor

At first, we discuss the surface DOS for the spin-triplet $p_x$-wave superconductor. In Fig. 3, we show the momentum resolved surface DOS $\rho(E, k_y)$ given by Eq. (10) [Figs. 3(a) to (d)] and the surface DOS $\rho(E)$ given by Eq. (11) [Figs. 3(e) to (h)]. As mentioned in Sect. II C, the zero-energy surface Andreev bound states (ZESABSs) appear as a flat band edge state for $-\alpha_+^d < k_y < \alpha_+^d$ with $\alpha_+^d = \arccos(-\mu/2t - 1)$ [Fig. 3(a)] due to the non-zero value of the winding number. $\rho(E)$ also exhibits sharp zero-energy peak (ZEP) as shown in Fig. 3(e). Since ZESABSs for the spin-triplet $p_x$-wave superconductor are fragile against the RSOC [17, 31] (see Sect. II C), ZESABSs split into two with the increase of the RSOC other than $k_y = 0$ [Figs. 3(b) and (f)]. As explained in Sect. II C at $k_y = 0$, $\mathbb{Z}_2$ topological number has a nontrivial value when $-4t < \mu < 0$ and the ZESABSs at $k_y = 0$ is topologically protected provided that the bulk superconducting gap opens ($\lambda < \Delta_0$). At $\lambda = \Delta_0$, the bulk gap closes [Eqs. (36) and Figs. 3(c) and (d)] and for $\lambda \gtrsim \Delta$, the corresponding $\rho(E)$ is almost independent of $E$ [Figs. 3(g) and (h)].

Next we discuss the odd-frequency spin-triplet $s$-wave pair amplitude at the surface $|P^{\text{triplet}}_S(\omega_n)|$ given in Eqs. (12) to (14) shown in Fig. 4. As can be seen in these graphs, $P^{\text{triplet}}_S(\omega_n)$ is odd in $\omega_n$ due to the Fermi-Dirac statistics. It is also noted that $P^{\text{triplet}}_S(\omega_n)$ satisfies $P^{\text{triplet}}_S(\omega_n) + P^{\text{triplet}}_S(-\omega_n) = 0$ within numerical accuracy. For the spin-triplet $p_x$-wave superconductor without the RSOC [Fig. 4(a)], $P^{\text{triplet}}_S(\omega_n)$ has a large value due to the translational symmetry breaking and $P^{\text{triplet}}_S(\omega_n)$ is zero due to the spin-rotational symmetry. $P^{\text{triplet}}_S(\omega_n)$ drastically decreases with the increase of $\lambda$ [Figs. 4(b)] since the zero-energy flat band disappears other than $k_y = 0$. On the other hand, divergent behavior very close to $\omega_n = 0$ remains due to ZESABS at $k_y = 0$. In Figs. 4(a) to (d) in the Appendix [C] we also show the odd-frequency $s$-wave pair amplitude at the surface with $\lambda/\Delta_0 = 0, 0.1, 1$ and 10.
FIG. 3. The momentum resolved surface DOS \(\rho(E, k_y)\) from (a) to (d) and the surface DOS \(\bar{\rho}(E)\) from (e) to (h) for the spin-triplet \(p_x\)-wave superconductor are plotted for several \(\lambda/\Delta_0\). Here \(\rho_N\) is a surface density of states at zero-energy with the normal state (\(\Delta_0 = 0\)). \(\rho(E, k_y)\) and \(\bar{\rho}(E)\) are calculated at \(j_z = 1\) (surface) in the system shown in Fig. 2. The figures in the same column have the same \(\lambda\). [(a), (e)] \(\lambda = 0\), [(b), (f)] \(\lambda/\Delta_0 = 0.1\), [(c), (g)] \(\lambda/\Delta_0 = 1\), and [(d), (h)] \(\lambda/\Delta_0 = 10\) with \(\Delta_0/t = 0.01\).

FIG. 4. The real part of the odd-frequency spin-triplet \(s\)-wave pair amplitude \(P_{S_{\lambda}}^{\text{triplet}}(\omega_n)\) at the surface is plotted as a function of \(\omega_n/\Delta_0\) for \(S_z = 1, 0, -1\) for the spin-triplet \(p_x\)-wave superconductor. (a) \(\lambda/\Delta_0 = 0\) and (b) \(\lambda/\Delta_0 = 0.1\). The imaginary part of \(P_{S_{\lambda}}^{\text{triplet}}(\omega_n)\) is zero within numerical accuracy.

The total odd-frequency spin-triplet \(s\)-wave pair amplitude at the surface \(P_{S_{\lambda}}^{\text{triplet}}(\omega_n)\) given by Eq. (15) with \(\omega_n/\Delta_0 = 10^{-3}\) is shown in Fig. 4. For the spin-triplet \(p_x\)-wave case, \(P_{S_{\lambda}}^{\text{triplet}}(\omega_n)\) has a large value at \(\lambda = 0\). On the other hand, with the non-zero value of the RSOC, \(P_{S_{\lambda}}^{\text{triplet}}(\omega_n)\) decreases drastically due to the absence of the ZESABS other than \(k_y = 0\). \(P_{S_{\lambda}}^{\text{triplet}}(\omega_n)\) has a peak approximately at \(\lambda/\Delta_0 = 1\) where the bulk energy gap closes. The surface DOS at zero-energy also exhibits a similar behavior [In the inset of Fig. 3] since the slope of the dispersive surface Andreev bound state at \(k_y = 0\) becomes smaller as \(\lambda/\Delta_0 = 1 - \delta\) approaches unity from \(\delta \to +0\).

FIG. 5. The odd-frequency spin-triplet \(s\)-wave pair amplitude \(P_{S_{\lambda}}^{\text{triplet}}(\omega_n/\Delta_0 = 10^{-3})\) given by Eq. (15) is plotted as a function of the RSOC \(\lambda\) for the spin-triplet \(p_x\)-wave superconductor and the spin-singlet \(d_{xy}\)-wave superconductor. In the inset, the surface DOS \(\bar{\rho}(E = 0)\) is plotted as a function of \(\lambda\). Here \(\Delta_0/t = 0.01\).

2. Spin-singlet \(d_{xy}\)-wave superconductor

In Fig. 5 the momentum resolved surface DOS \(\rho(E, k_y)\) and \(\bar{\rho}(E)\) for the spin-singlet \(d_{xy}\)-wave superconductor are shown. As explained in Sect. 1C ZESABSs for the spin-singlet \(d_{xy}\)-wave superconductor are robust against the RSOC shown in Figs. 3 (a) to (d). The regime for the ZESABSs are
FIG. 6. The momentum resolved surface DOS \( \rho(E, k_y) \) from (a) to (d) and the surface DOS \( \bar{\rho}(E) \) from (e) to (h) for the spin-singlet \( d_{xy} \)-wave superconductor are plotted for several \( \lambda/\Delta_0 \). Here \( \rho_N \) is a surface density of states at zero-energy with the normal state \( (\Delta_0 = 0) \). \( \rho(E, k_y) \) and \( \bar{\rho}(E) \) are calculated at \( j_x = 1 \) (surface) in the system shown in Fig. 2. The figures in the same column have the same \( \lambda/\Delta_0 \). [(a), (e)] \( \lambda = 0 \), [(b), (f)] \( \lambda/\Delta_0 = 1 \), [(c), (g)] \( \lambda/\Delta_0 = 5 \), and [(d), (h)] \( \lambda/\Delta_0 = 10 \) with \( \Delta_0/t = 0.01 \).

III. DIFFUSIVE NORMAL METAL/SUPERCONDUCTOR JUNCTION

In this section, we discuss the local DOS and the odd-frequency spin-triplet \( s \)-wave pair amplitude in the diffusive normal metal (DN) for DN/SC junctions with the RSOC (shown in Fig. 5).

The Hamiltonian for the SC \( (j_x \geq 1) \) is given by Eq. (10) and the Hamiltonian for the DN \( (-49 \leq j_x \leq 0) \) is

\[
\hat{H}^{l_s}_{DN} = -t \sum_{(i,j),\sigma} \left( c_{i,\sigma}^\dagger c_{j,\sigma} + H.c. \right) + \sum_{-49 \leq j_x \leq 0} \left( -\mu + V_j^{l_s} \right) n_j,
\]

FIG. 7. The real part of the odd-frequency spin-triplet \( s \)-wave pair amplitude \( P_{S_z}^{\text{triplet}}(\omega_n) \) at the surface is plotted as a function of \( \omega_n/\Delta_0 \) for \( S_z = 1, 0, -1 \) for the spin-singlet \( d_{xy} \)-wave superconductor with \( \lambda/\Delta_0 = 5 \). The imaginary part of \( P_{S_z}^{\text{triplet}}(\omega_n) \) is zero within numerical accuracy.
where the summation in the $x$-direction run over from $-49$ to $0$ ($-49 \leq i_x, j_x \leq 0$), and those in the $y$-direction run over from $1$ to $L_y$ ($1 \leq i_y, j_y \leq L_y$). $V_{j_x}^{\delta} (-t \leq V_{j_x}^{\delta} \leq t$ with $t = 1, 2, \ldots, N_{\text{sample}}$) is a randomly chosen onsite impurity potential in the DN and there is no RSOC in the DN. The Hamiltonian which connects the DN and the SC is

$$H_{\text{connect}} = -t \sum_{j_y, \sigma} \left( c_{(0,j_y),\sigma}^\dagger c_{(1,j_y),\sigma} + \text{H.c.} \right).$$  

The local Green’s function is obtained from the same procedure explained in Sect. 11B. The LDOS in the DN is given by

$$\tilde{\rho}(E, j_x) = \frac{1}{2N_{k_y}L_yN_{\text{sample}}} \sum_{j_y = j_x, k_y, l_x} \frac{1}{\pi} \text{Im} \left[ \text{tr} \tilde{g}_{j_x}^{\text{DN}, l_x}(z = E + i\eta, j_y, j_y', k_y) \right],$$  

where we use a super unit cell $k_y = \pi m/(N_{k_y}L_y)$ with $m = -N_{k_y} + 1, \ldots, N_{k_y}$. $N_{k_y} = 10$ and $L_y$ is a length of the super unit cell in the $y$-direction used in numerical calculation. Here, trace is only taken in particle space and we set $\eta/t = 10^{-5}$. The Green’s function in the DN $\tilde{g}_{j_x}^{\text{DN}, l_x}(z, j_y, j_y', k_y)$ in Eq. (43) is given in the Appendix 2 [Eq. (12)]. To calculate the LDOS for the spin-triplet $p_x$-wave superconductor junction, we take $L_y = 120$ which is the length of the system in the $y$-direction and to reduce the system size effect, 60 samples for $V_{j_x}^{\delta}$ ($j_x = 1, 2, \ldots, 60$, i.e., $N_{\text{sample}} = 60$) are used to calculate the averaged odd-frequency pair amplitude in the DN and the odd-frequency pair amplitude in the DN. We take $L_y = 240$ and 12 samples ($N_{\text{sample}} = 12$) are used to the impurity average for the spin-singlet $d_{xy}$-wave junction since size effect for the spin-singlet $d_{xy}$-wave case is larger than that for the spin-triplet $p_x$-wave case since the pair potential for the spin-singlet $d_{xy}$-wave superconductor depends on $k_y$ but that for the spin-triplet $p_x$-wave superconductor does not depend on $k_y$.

To discuss the relation between the anomalous proximity effect and the odd-frequency spin-triplet $s$-wave pair amplitude, we calculate its averaged value in the DN region given by

\begin{align}
\tilde{P}_{\text{DN,triplet}}^{\text{triplet}}(\omega_n) &= \frac{1}{N} \sum_{j_x = N_i}^{N_f} P_{j_x, \uparrow, \uparrow}(\omega_n), \\
\tilde{P}_{\text{DN,triplet}}^{\text{triplet}}(\omega_n) &= \frac{1}{\sqrt{2N}} \sum_{j_x = N_i}^{N_f} \left[ P_{j_x, \uparrow, \downarrow}(\omega_n) + P_{j_x, \downarrow, \uparrow}(\omega_n) \right], \\
\tilde{P}_{\text{DN,triplet}}^{\text{triplet}}(\omega_n) &= \frac{1}{N} \sum_{j_x = N_i}^{N_f} P_{j_x, \downarrow, \downarrow}(\omega_n),
\end{align}

with

\begin{align}
P_{j_x, \sigma, \sigma'}(\omega_n) &= \frac{1}{2N_{k_y}L_yN_{\text{sample}}} \sum_{j_y, j_y', k_y} \tilde{f}_{j_x, \sigma, \sigma'}(i\omega_n, j_y, j_y', k_y),
\end{align}

where we average the odd-frequency pair amplitude in the DN: $N_i = -49$, $N_f = 0$ and $N = N_i - N_f + 1 = 50$. Here $2L_y \times 2L_y$ matrix $\tilde{f}_{j_x, \sigma, \sigma'}(z = i\omega_n, j_y, j_y', k_y)$ is an anomalous Green’s function of $g_{j_x}^{\text{DN}, l_x}(z, j_y, j_y', k_y)$ in Eq. (43) defined as the same manner as in Eq. (11B). To calculate the odd-frequency spin-triplet $s$-wave pair amplitude, we take the same system size in the $y$-direction and the same number of the samples are used to the impurity average.

\section{B. Results}

We discuss the LDOS and the odd-frequency spin-triplet $s$-wave pair amplitude in the DN for the spin-triplet $p_x$-wave and the spin-singlet $d_{xy}$-wave superconductor junction.

\subsection{1. Spin-triplet $p_x$-wave superconductor junction}

Firstly, we discuss the LDOS in the DN corresponding to Fig. 5. LDOS for the spin-triplet $p_x$-wave superconductor junction without the RSOC has a sharp ZEP as shown in Fig. 5(a) consistent with the previous results in quasiclassical regime [20]. It is called anomalous proximity effect [26–28]. The ZEP on LDOS in the DN stems from ZESABSs as a flat band zero-energy state in semi-infinite spin-triplet $p_x$-wave superconductor. The present ZESABS accompanies OTE $s$-wave pairing, then it can penetrate into the DN. However, after switching on $\lambda$ in the spin-triplet $p_x$-wave superconductor, the ZEP of the LDOS in the DN is suppressed by $\lambda$, as shown in Fig. 5(b). In this case, if we look at the semi-infinite spin-triplet $p_x$-wave superconductor without the DN, the zero-energy flat bands at the surface split into two [Figs. 6 (b) and (f)] and the surface DOS at zero-energy is suppressed. This is the reason why the LDOS at zero-energy

FIG. 8. Schematic illustration of DN without RSOC ($-49 \leq j_x \leq 0$)/semi-infinite ($j_x \geq 1$) SC with RSOC junction.
FIG. 10. The averaged value of the real part of the odd-frequency spin-triplet pair amplitude in the DN $\bar{P}_{S_z=0}^{\text{triplet}}(\omega_n)$ is plotted as a function of $\omega_n$ for several strength of the RSOC for the spin-triplet $p_x$-wave superconductor junction. The amplitude of the RSOC is (a) $\lambda/\Delta_0 = 0$, (b) 0.1, (c) 1, and (d) 10 with $\Delta_0/t = 0.01$.

FIG. 10. The averaged value of the real part of the odd-frequency spin-triplet $s$-wave pair amplitude in the DN $\bar{P}_{S_z=0}^{\text{triplet}}(\omega_n)$ is plotted as a function of $\omega_n$ for several strength of the RSOC for the spin-triplet $p_x$-wave superconductor junction with $S_z = 1, 0$ and $-1$. The imaginary part of $\bar{P}_{S_z=0}^{\text{triplet}}(\omega_n)$ is zero within numerical accuracy. (a) $\lambda/\Delta_0 = 0$, (b) 0.1, (c) 1, and (d) 10.

Corresponding to the suppression of the height of ZEP in the DN, the averaged value of the odd-frequency spin-triplet $s$-wave pair amplitude $\bar{P}_{S_z=0}^{\text{triplet}}(\omega_n)$ given by Eq. 45 in the DN becomes smaller with the increase of the RSOC [Figs. 11 (a) and (b)] and this behavior of $\bar{P}_{S_z=0}^{\text{triplet}}(\omega_n)$ is similar to that for the semi-infinite system [Figs. 4 (a) and (b)]. The odd-frequency spin-triplet $s$-wave pair amplitude $\bar{P}_{S_z=1}^{\text{triplet}}(\omega_n)$ is zero for $\lambda = 0$ due to the spin-rotational symmetry and it has non-zero value for $\lambda > 0$ [Figs. 11 (b) and (c)] and it also satisfies $\bar{P}_{S_z=1}^{\text{triplet}}(\omega_n) + \bar{P}_{S_z=-1}^{\text{triplet}}(\omega_n) = 0$ within numerical accuracy. For large magnitude of $\lambda$, all kinds of $\bar{P}_{S_z}^{\text{triplet}}(\omega_n)$ becomes almost zero [Fig. 10 (d)]. This result is similar to that for semi-infinite system shown in Fig. 4 and Figs. 15 (a) to (d) in the Appendix C.

2. Spin-singlet $d_{xy}$-wave superconductor junction

For the spin-singlet $d_{xy}$-wave superconductor without the RSOC, the LDOS in the DN is nearly constant as a function of $E$ and $j_x$ [Fig. 11 (a)]. It is known from the quasicalssical theory of proximity effect in unconventional superconductor, that the LDOS is reduced to that in the normal state due to the absence of the proximity effect in the DN $\bar{S}_z$. The slight energy dependence is due to the finite size effect in the numerical calculation of the lattice model. A similar behavior also appears for $\lambda/\Delta_0 = 1$ [Fig. 11 (b)]. However, for a large RSOC values $\lambda/\Delta_0 = 5$ and 10, the LDOS has a ZEP in the DN region [Figs. 11 (c) and (d)]. In Fig. 12, we show the LDOS at the center of the DN ($j_x = -25$) and for $\lambda/\Delta_0 \geq 2$, the ZEP appears. Therefore, it may be possible to detect the anomalous proximity effect for the spin-singlet $d$-wave superconductor if the amplitude of the RSOC becomes larger than that of the pair potential. System size and the number of samples dependence of the LDOS are discussed in the Appendix C.

At $\lambda = 0$, the odd-frequency spin-triplet $s$-wave pair amplitude $\bar{P}_{S_z}^{\text{triplet}}(\omega_n)$ is zero due to the spin-rotational symmetry [Figs. 13 (a)]. The amplitude of $\bar{P}_{S_z}^{\text{triplet}}(\omega_n)$ increases as the increase of $\lambda$ as shown
Also Figs. 15 (e) to (h) in the Appendix C. The odd-frequency spin-singlet $p$-wave superconductor junction. The amplitude of the RSOC is (a) $\lambda/\Delta_0 = 0$, (b) 1, (c) 5, and (d) 10 with $\Delta_0/t = 0.01$.

![FIG. 11. The LDOS $[\bar{\rho}(E, j_x)]$ in the DN is plotted as functions of $j_x$ and $E/\Delta_0$ for several strength of the RSOC for the spin-singlet $d_{xy}$-wave superconductor junction. The amplitude of the RSOC is (a) $\lambda/\Delta_0 = 0$, (b) 1, (c) 5, and (d) 10 with $\Delta_0/t = 0.01$.](image)

The LDOS $[\bar{\rho}(E, j_x)]$ in the DN is plotted as functions of $j_x$ and $E/\Delta_0$ for several strength of the RSOC for the spin-singlet $d_{xy}$-wave superconductor junction. The amplitude of the RSOC is (a) $\lambda/\Delta_0 = 0$, (b) 1, (c) 5, and (d) 10 with $\Delta_0/t = 0.01$.

![FIG. 12. The LDOS for the spin-singlet $d_{xy}$-wave superconductor junction at $j_x = -25$ (center of the DN) is plotted as a function of $E/\Delta_0$ for several $\lambda/\Delta_0$ with $\Delta_0/t = 0.01$.](image)

The LDOS for the spin-singlet $d_{xy}$-wave superconductor junction at $j_x = -25$ (center of the DN) is plotted as a function of $E/\Delta_0$ for several $\lambda/\Delta_0$ with $\Delta_0/t = 0.01$.

In Figs. 13 (b) to (d). Here, non-zero value of the odd-frequency spin-triplet $s$-wave pair amplitude in the DN means that it is robust against the impurity scattering in the DN [spatial dependence is shown in Fig. 20 in the Appendix]. Qualitative behavior of the odd-frequency spin-triplet $s$-wave pair amplitude in the DN as a function of the RSOC [Figs. 13 (a) to (d)] is the same as that for the semi-infinite SC system at the surface i.e., it increases as the increase of the RSOC [see also Figs. 15 (e) to (h) in the Appendix C]. The odd-frequency spin-triplet $s$-wave pair amplitude also satisfies $F_{S_z=1}^{DN,triplet}(\omega_n) + F_{S_z=-1}^{DN,triplet}(\omega_n) = 0$ within numerical accuracy. The odd-frequency spin-singlet $p_y$-wave pair amplitude rapidly decays in the DN since it is fragile against impurity scattering [Appendix H].

### IV. SUMMARY

In this paper, we have studied the proximity effect in the DN/superconductor junctions, focusing on the anomalous proximity effect, where the LDOS in the DN has a zero-energy peak. For the spin-triplet $p_x$-wave superconductor case, when the direction of the $d$-vector is parallel to the $z$-direction, ZESABSs are suppressed by the RSOC $\lambda$ and the anomalous proximity effect disappears for large magnitude of $\lambda$.

It should be also noted that the anomalous proximity effect is switched on by the RSOC in spin-singlet $d_{xy}$-wave superconductor junctions. The resulting zero-energy LDOS and the magnitude of the odd-frequency spin-triplet $s$-wave pair amplitude is enhanced with the increase of the magnitude of $\lambda$. This indicates that high $T_c$ cuprate junctions can be used to detect the anomalous proximity effect. Another candidate material is CeCoIn$_5$, where the promising pairing symmetry is spin-singlet $d$-wave [85]. It has been reported that the amplitude of the RSOC exceeds that of the superconducting gap $\Delta_0$ in the superlattices of CeCoIn$_5$.

We comment on the effect of surface roughness on the odd-frequency spin-triplet $s$-wave pair amplitude for the spin-singlet $d_{xy}$-wave superconductor. It is known that in the presence of SABSs, pair potential decays near the surface [87, 90]. It has been clarified that the surface roughness influences on the SABS and the resulting height of ZEP is suppressed by roughness [91, 92] since the induced odd-frequency pairing near the interface has an odd-parity with $p_{y}$-wave like symmetry [93]. On the other hand, in the presence of the RSOC, we can generate odd-frequency spin-triplet $s$-wave pair amplitude near the interface. The present pair amplitude is robust against the surface diffusive scattering [95]. In order to clarify this point, we have calculated spin-triplet $s$-wave pair amplitude in diffusive layer attached to spin-singlet $d_{xy}$-wave superconductor junction with the RSOC in the presence of the spatial depletion of the pair potential near the surface [Appendix D]. We have checked the robustness of the odd-frequency spin-triplet $s$-wave pair amplitude against diffusive scattering. Even if the spatial depletion...
of the pair potential near the surface, the magnitude of the odd-frequency spin-triplet s-wave pair amplitude does not almost change as compared to the case with the constant pair potential up to the surface.

In this paper, we assume that a Rashba spin-orbit coupling exists in the two-dimensional superconductor uniformly along the z-direction, as superconductor is assumed to be thin. In addition to this effect, spin-orbit coupling also exists near the interface, on the plane parallel to the interface. In this case, the direction of the Rashba vector is different. For spin-singlet pair potentials, the direction of the spin-orbit coupling only affects the direction of the spin of the induced odd-frequency spin-triplet pair amplitude due to the spin-rotational symmetry. On the other hand, for spin-triplet pair potentials, the relative angle of the d-vector and the Rashba spin-orbit coupling affects surface Andreev bound states like diffusive ferromagnet/superconductor junctions. It is also interesting to study this type of spin-orbit coupling.

We comment on the case that the amplitude of the RSOC is much larger than the hopping integral for the spin-singlet $d_{xy}$-wave superconductor. In our study, we consider that the amplitude of the RSOC is smaller than the hopping integral $t$ and the odd-frequency spin-triplet s-wave pair amplitude increases with the increase of the amplitude of the RSOC. On the other hand, when the amplitude of the RSOC is much larger than the hopping integral, the ZESABS disappears when the value of the chemical potential is non-zero as can be seen in the Appendix A and the odd-frequency spin-triplet s-wave pair amplitude disappears as increasing $\lambda/t$ for the large amplitude of the RSOC.

A ZESABS is also produced by a Dresselhaus spin-orbit interaction and the Zeeman field. The Dresselhaus spin-orbit interaction breaks the spin-rotational symmetry and the anomalous proximity effect is also expected for the spin-singlet $d_{xy}$-wave superconductors.

Recently, relation between crossed Andreev reflection (CAR) and odd frequency pair amplitude is pointed out. On the other hand, in ballistic transport regime, CAR with mixed parity state is calculated. It is an interesting future problem to calculate CAR in diffusive regime.

In this paper, we focus on the localized odd-frequency pairing accompanied by the ZESABS, which is generated by the symmetry breaking from the bulk conventional even-frequency pair potential. On the other hand, several studies have been pursuing the realization of a bulk odd-frequency superconductor so far. Although it is difficult to express the bulk odd-frequency superconducting states consistent with various conditions, a recently proposed odd-frequency gap function in the two channel Kondo lattice model is a promising system. The calculation of charge transport and Josephson effect has already started. It is interesting to study proximity effects in the DN/TCKL system.

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where \( n \) is the number of sites in the \( x \)-direction and periodic boundary condition in the \( y \)-direction is given by

\[
\mathcal{H}_{n,SC} = \sum_{k_y} C^\dagger_{n,k_y} H_n(k_y) C_{n,k_y},
\]

(A1)

\[
H_n(k_y) = \begin{pmatrix}
\hat{u} & \hat{i} \\
\hat{i}^\dagger & \hat{u}^\dagger
\end{pmatrix},
\]

\[
\hat{u} = \begin{pmatrix}
\varepsilon_{k_y} & \lambda \sin k_y \\
\lambda \sin k_y & \varepsilon_{k_y}
\end{pmatrix},
\]

\[
\hat{i} = \begin{pmatrix}
-\varepsilon_{k_y} & \lambda \sin k_y \\
\lambda \sin k_y & -\varepsilon_{k_y}
\end{pmatrix},
\]

(A2)

where \( n \) is the number of sites in the \( x \)-direction, \( \varepsilon_{k_y} = -2t \cos k_y - \mu \), \( \Delta_{12} \) and \( \Delta_{21} \) are given by

\[
\Delta_{12}(k_y) = \frac{\Delta_0}{(2i)},
\]

\[
\Delta_{21}(k_y) = \frac{\Delta_0}{(2i)},
\]

(A5)

(A6)

for the spin-triplet \( p_x \)-wave case, \( \Delta_{12} \) and \( \Delta_{21} \) are given by

\[
\Delta_{12}(k_y) = -i \frac{\Delta_0 \sin k_y}{2},
\]

\[
\Delta_{21}(k_y) = i \frac{\Delta_0 \sin k_y}{2},
\]

(A7)

(A8)

for the spin-singlet \( d_{xy} \)-wave case and

\[
C_{n,k_y} = \begin{pmatrix}
c_{1,k_y,\uparrow} & c_{1,k_y,\downarrow} & c^\dagger_{1,-k_y,\uparrow} & c^\dagger_{1,-k_y,\downarrow} & \cdots & c_{n,k_y,\uparrow} & c_{n,k_y,\downarrow} & c^\dagger_{n,-k_y,\uparrow} & c^\dagger_{n,-k_y,\downarrow}
\end{pmatrix}^T.
\]

(A9)
Let Möbius transformation be
\[ \hat{A}_* \hat{z} = \left( \hat{a} \hat{z} + \hat{b} \right) \left( \hat{c} \hat{z} + \hat{d} \right)^{-1}, \] (A10)

with
\[ \hat{A} = \begin{pmatrix} \hat{a} & \hat{b} \\ \hat{c} & \hat{d} \end{pmatrix}, \] (A11)

where \( \hat{a}, \hat{b}, \hat{c}, \hat{d} \) and \( \hat{z} \) are \( m \times m \) matrices. The Möbius transformation satisfies following relation:
\[ \hat{A}_* \hat{B}_* \hat{z} = (\hat{A} \hat{B})_* \hat{z}. \] (A12)

We also define the elements of the Green’s function as
\[ [z - H_n(k_y)]^{-1} = \begin{pmatrix} [g_n(z, k_y)]_{1,1} & [g_n(z, k_y)]_{1,2} \cdots \\ [g_n(z, k_y)]_{2,1} & [g_n(z, k_y)]_{2,2} \cdots \\ \vdots & \ddots \end{pmatrix}, \] (A13)

with \( 4 \times 4 \) matrix \( [g_n(z, k_y)]_{j,x',j',x} \). Then the recurrence relation for the surface Green’s function for the \( n \)-site system at the leftmost site \( [g_n(z, k_y)]_{1,1} \) and the \( n+1 \)-site one \( [g_{n+1}(z, k_y)]_{1,1} \) is given by
\[ [g_{n+1}(z, k_y)]_{1,1} = \left( (z - \hat{u}) - \hat{t} [g_n(z, k_y)]_{1,1} \hat{t}^\dagger \right)^{-1} \]
\[ = X_\bullet [g_n(z, k_y)]_{1,1}, \] (A14)

with
\[ X = \begin{pmatrix} \hat{0} & (\hat{t}^\dagger)^{-1} \\ -\hat{t} & (z - \hat{u})(\hat{t}^\dagger)^{-1} \end{pmatrix}. \] (A15)

Here, \( \hat{0} \) is a \( 4 \times 4 \) matrix with all the elements equal to zero. It is noted \( [g_1(z, k_y)]_{1,1} \) satisfies
\[ [g_1(z, k_y)]_{1,1} = X_\bullet \hat{0}, \] (A16)

and \( [g_n(z, k_y)]_{1,1} \) is written as
\[ [g_n(z, k_y)]_{1,1} = (X^n)_\bullet \hat{0}. \] (A17)

Let \( U \) be a matrix which diagonalizes \( X \) as
\[ U^{-1} X U = \text{diag} (\lambda_1, \lambda_2, \ldots, \lambda_8), \] (A18)

with \( |\lambda_1| \leq \cdots \leq |\lambda_4| < |\lambda_5| \leq \cdots \leq |\lambda_8| \). Then the surface Green’s function for a semi-infinite system at the leftmost site \( G_{j=1}^\infty (z, k_y) \) is given by
\[ G_{j=1}^\infty (z, k_y) = \lim_{n \to \infty} [g_n(z, k_y)]_{1,1} \] (A19)
\[ = \lim_{n \to \infty} \left[ U (U^{-1} X U)^n U^{-1} \right]_\bullet \hat{0} \]
\[ = U_\bullet \hat{0} \]
\[ = U_{12}(U_{22})^{-1}, \] (A20)

where \( U_{ij} \) is a \( 4 \times 4 \) matrix defined as
\[ U = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix}, \] (A21)

and we use Eq. (A10).
Appendix B: Winding number for spin-singlet $d_{xy}$-wave superconductor with RSOC

From Eq. (9), point nodes are on $k_x(y) = 0$ or $k_x(y) = \pi$, since $\Delta_0 \sin k_x \sin k_y = 0$ is satisfied at these points. We consider only $k_x = 0$ or $k_x = \pi$ case because Eq. (9) is symmetric with respect to replacing $k_x$ by $k_y$. Point nodes also satisfy $\xi \pm |\lambda(\sin k_x + \sin k_y)| = 0$. For $k_x = 0$, point nodes are located at

$$k_y = k_{0,\pm} = \arccos \left\{ \frac{1}{4t^2 + \lambda^2} \left[ -2t(2t + \mu) \pm \lambda \sqrt{\lambda^2 - 4t\mu - \mu^2} \right] \right\}, \quad (B1)$$

with $\lambda^2 - 4t\mu - \mu^2 \geq 0$ and for $k_x = \pi$, point nodes are located at

$$k_y = k_{x,\pm} = \arccos \left\{ \frac{1}{4t^2 + \lambda^2} \left[ -2t(-2t + \mu) \pm \lambda \sqrt{\lambda^2 + 4t\mu - \mu^2} \right] \right\}, \quad (B2)$$

with $\lambda^2 + 4t\mu - \mu^2 \geq 0$. Here we suppose $\lambda \geq 0$. Then the number of point nodes is

$$\begin{cases} 8 & -2t - \sqrt{4t^2 + \lambda^2} < \mu < +2t - \sqrt{4t^2 + \lambda^2}, \\ 16 & +2t - \sqrt{4t^2 + \lambda^2} < \mu < -2t + \sqrt{4t^2 + \lambda^2}, \\ 8 & -2t + \sqrt{4t^2 + \lambda^2} < \mu < +2t + \sqrt{4t^2 + \lambda^2}, \end{cases} \quad (B3)$$

with the non-zero value of the RSOC.

As shown in Fig. 14, there are nine regimes from A to I. The region A in Fig. 14, the winding number is zero for any $k_y$.

The winding number for $-2t - \sqrt{4t^2 + \lambda^2} < \mu < +2t - \sqrt{4t^2 + \lambda^2}$ [region B in Fig. 14] is

$$W(k_y) = \begin{cases} 0 & -\pi < k_y < -k_{0,-}, \\ -1 & -k_{0,-} < k_y < -k_{0,+}, \\ -2 & -k_{0,+} < k_y < 0, \\ 2 & 0 < k_y < k_{0,+}, \\ 1 & k_{0,+} < k_y < k_{0,-}, \\ 0 & k_{0,-} < k_y < \pi. \end{cases} \quad (B4)$$

\[\text{FIG. 14. Zero-energy flatband is characterized by the region A to I.} \]

\[f_+/(\lambda/t) = \pm 2 + \sqrt{4^2 + (\lambda/t)^2}, \text{ and } g(\lambda/t) = \frac{2}{\sqrt{\lambda/t}} \sqrt{(\lambda/t)^2 - 4^2}.\]
The winding number for \(+2t - \sqrt{4t^2 + \lambda^2} < \mu < 0\) and \(\lambda < \sqrt{2(1 + \sqrt{5})t}\) [region C in Fig. 14] is

\[
W(k_y) = \begin{cases} 
0 & \pi < k_y < k_{0,0}, \\
-1 & k_{0,0} < k_y < k_{0,0}, \\
1 & k_{0,0} < k_y < k_{0,0}, \\
2 & k_{0,0} < k_y < k_{0,0}, \\
2 & \pi < k_y < 0, \\
1 & k_{0,0} < k_y < k_{0,0}, \\
2 & k_{0,0} < k_y < k_{0,0}, \\
0 & k_{0,0} < k_y < \pi.
\end{cases} \tag{B5}
\]

The winding number for \(+2t - \sqrt{4t^2 + \lambda^2} < \mu < -\frac{2t}{\lambda}\sqrt{\lambda^2 - 4t^2}\) and \(\lambda > \sqrt{2(1 + \sqrt{5})t}\) [region D in Fig. 14] is

\[
W(k_y) = \begin{cases} 
0 & \pi < k_y < k_{0,0}, \\
-1 & k_{0,0} < k_y < k_{0,0}, \\
0 & k_{0,0} < k_y < k_{0,0}, \\
-1 & k_{0,0} < k_y < k_{0,0}, \\
-2 & k_{0,0} < k_y < 0, \\
1 & k_{0,0} < k_y < k_{0,0}, \\
0 & k_{0,0} < k_y < k_{0,0}, \\
0 & k_{0,0} < k_y < \pi.
\end{cases} \tag{B6}
\]

The winding number for \(-\frac{2t}{\lambda}\sqrt{\lambda^2 - 4t^2} < \mu < 0\) [region E in Fig. 14] is

\[
W(k_y) = \begin{cases} 
0 & \pi < k_y < k_{0,0}, \\
-1 & k_{0,0} < k_y < k_{0,0}, \\
0 & k_{0,0} < k_y < k_{0,0}, \\
-1 & k_{0,0} < k_y < k_{0,0}, \\
-2 & k_{0,0} < k_y < 0, \\
1 & k_{0,0} < k_y < k_{0,0}, \\
0 & k_{0,0} < k_y < k_{0,0}, \\
0 & k_{0,0} < k_y < \pi.
\end{cases} \tag{B7}
\]

The winding number for \(0 < \mu < -2t + \sqrt{4t^2 + \lambda^2}\) and \(\lambda < \sqrt{2(1 + \sqrt{5})t}\) [region F in Fig. 14] is

\[
W(k_y) = \begin{cases} 
-2 & \pi < k_y < k_{0,0}, \\
-1 & k_{0,0} < k_y < k_{0,0}, \\
-2 & k_{0,0} < k_y < k_{0,0}, \\
-1 & k_{0,0} < k_y < k_{0,0}, \\
0 & k_{0,0} < k_y < k_{0,0}, \\
1 & k_{0,0} < k_y < k_{0,0}, \\
2 & k_{0,0} < k_y < k_{0,0}, \\
0 & k_{0,0} < k_y < \pi.
\end{cases} \tag{B8}
\]
The winding number for $0 < \mu < -2t + \sqrt{4t^2 + \lambda^2}$ and $\lambda > \sqrt{2(1 + \sqrt{5})t}$ [region G in Fig. 14] is

$$W(k_y) = \begin{cases} 
-2 & -\pi < k_y < -k_{\pi,-}, \\
-1 & -k_{\pi,-} < k_y < -k_{0,-}, \\
0 & -k_{0,-} < k_y < -k_{0,+}, \\
-1 & -k_{0,+} < k_y < -k_{\pi,+}, \\
0 & -k_{\pi,+} < k_y < k_{\pi,+}, \\
1 & k_{\pi,+} < k_y < k_{0,+}, \\
0 & k_{0,+} < k_y < k_{0,-}, \\
1 & k_{0,-} < k_y < k_{\pi,-}, \\
2 & k_{\pi,-} < k_y < \pi.
\end{cases} \tag{B9}$$

The winding number for $0 < \mu < \frac{2\lambda}{\pi}\sqrt{\lambda^2 - 4t^2}$ [region H in Fig. 14] is

$$W(k_y) = \begin{cases} 
-2 & -\pi < k_y < -k_{0,-}, \\
-1 & -k_{0,-} < k_y < -k_{\pi,-}, \\
0 & -k_{\pi,-} < k_y < -k_{0,+}, \\
-1 & -k_{0,+} < k_y < -k_{\pi,+}, \\
0 & -k_{\pi,+} < k_y < k_{\pi,+}, \\
1 & k_{\pi,+} < k_y < k_{0,+}, \\
0 & k_{0,+} < k_y < k_{\pi,-}, \\
1 & k_{\pi,-} < k_y < k_{0,-}, \\
2 & k_{0,-} < k_y < \pi.
\end{cases} \tag{B10}$$

The winding number for $-2t + \sqrt{4t^2 + \lambda^2} < \mu < +2t + \sqrt{4t^2 + \lambda^2}$ [region I in Fig. 14] is

$$W(k_y) = \begin{cases} 
-2 & -\pi < k_y < -k_{\pi,-}, \\
-1 & -k_{\pi,-} < k_y < -k_{\pi,+}, \\
0 & -k_{\pi,+} < k_y < k_{\pi,+}, \\
1 & k_{\pi,+} < k_y < k_{\pi,-}, \\
2 & k_{\pi,-} < k_y < \pi.
\end{cases} \tag{B11}$$

**Appendix C: Odd-frequency spin-triplet s-wave pair amplitude for semi-infinite system**

Frequency dependence of the odd-frequency spin-triplet s-wave pair amplitude at the surface of the semi-infinite SC is plotted for $S_z = 1, 0$ and $-1$ in Fig. 15.

In Figs. 15 (a) to (d), the odd-frequency spin-triplet s-wave pair amplitude for the spin-triplet $p_x$-wave superconductor are shown. Without the RSOC, $P_{S_z}^{\text{triplet}}(\omega_n)$ with $S_z = \pm 1$ is zero due to the spin-rotational symmetry but $P_{S_z}^{\text{triplet}}(\omega_n)$ with $S_z = 0$ has large value due to translational symmetry braking and it diverges near $\omega_n = 0$ [Fig. 15 (a)]. At $\lambda/\Delta_0 = 0.1$ shown in Fig. 15 (b), $P_{S_z}^{\text{triplet}}(\omega_n)$ with $S_z = 0$ greatly suppressed and $P_{S_z}^{\text{triplet}}(\omega_n)$ with $S_z = \pm 1$ has non-zero value due to the spin-rotational symmetry breaking. $P_{S_z}^{\text{triplet}}(\omega_n)$ diverges very close to $k_y = 0$ since there is ZESABS at $k_y = 0$. For $\lambda/\Delta_0 \gtrsim 1$, $P_{S_z}^{\text{triplet}}(\omega_n)$ with $S_z = 0$ almost vanishes [Figs. 15 (c) and (d)] and all the components of $P_{S_z}^{\text{triplet}}(\omega_n)$ are almost zero for $\lambda/\Delta_0 \gtrsim 10$ shown in Fig. 15 (d). For $\lambda/\Delta_0 = 10$, we cannot see divergent behavior for $|\omega_n/\Delta_0| > 5 \times 10^{-6}$.

In Figs. 15 (e) to (h), the odd-frequency spin-triplet s-wave pair amplitude for the spin-singlet $d_{xy}$-wave superconductor are shown. $P_{S_z}^{\text{triplet}}(\omega_n)$ is zero for $\lambda/\Delta_0 = 0$ due to the spin-rotational symmetry [Fig. 15 (e)]. For $\lambda/\Delta_0 > 0$, absolute value of $P_{S_z}^{\text{triplet}}(\omega_n)$ increases as the increase of $\lambda/\Delta_0$ [Figs. 15 (f) to (h)]. The absolute value of $P_{S_z}^{\text{triplet}}(\omega_n)$ with $S_z = \pm 1$ is larger than that with $S_z = 0$ since the even-frequency spin-triplet with $S_z = 0$ pair amplitude is absent in the bulk [see also the Appendix E]. In these graphs, the absolute value of $P_{S_z}^{\text{triplet}}(\omega_n)$ increases as the increase of $\lambda$. 
FIG. 15. The real part of the odd-frequency spin-triplet $s$-wave pair amplitude $P_{S_z}^\text{triplet}(\omega_n)$ is plotted as a function of $\omega_n$ for $S_z = -1, 0, 1$. The imaginary part of $P_{S_z}^\text{triplet}(\omega_n)$ is zero within numerical accuracy. The odd-frequency pair amplitude of the spin-triplet $p_x$-wave superconductor is shown in (a) $\lambda/\Delta_0 = 0$, (b) 0.1, (c) 1 and (d) 10. The odd-frequency pair amplitude of the spin-singlet $d_{xy}$-wave superconductor is shown in (e) $\lambda/\Delta_0 = 0$, (f) 1, (g) 5 and (h) 10. In all cases, we set $\Delta_0/t = 0.01$.

Appendix D: Odd-frequency spin-singlet $p_y$-wave pair amplitude for spin-singlet $d_{xy}$-wave superconductor

In Fig. 16, the odd-frequency spin-singlet $p_y$-wave pair amplitude at the surface is shown for the semi-infinite spin-singlet $d_{xy}$-wave superconductor. The definition of the odd-frequency spin-singlet $p_y$-wave pair amplitude is given by

$$P_{p_y}^\text{singlet}(\omega_n) = \frac{1}{2N_{k_y}} \sum_{k_y} \left[ \hat{F}_{j_x=1, \downarrow, \downarrow}(i\omega_n, k_y) - \hat{F}_{j_x=1, \downarrow, \uparrow}(i\omega_n, k_y) \right] \sin k_y,$$

(D1)

where $\hat{F}_{j_x, \sigma, \sigma'}(i\omega_n, k_y)$ is the anomalous Green’s function given by Eq. (17). The amplitude does not almost depend on the value of $\lambda$.

FIG. 16. The real part of the odd-frequency spin-singlet $p_y$-wave pair amplitude is plotted as a function of $\omega_n/\Delta_0$ for (a) $\lambda/\Delta_0 = 0$, (b) 1, (c) 5, and (d) 10 with $\Delta_0/t = 0.01$. The imaginary part of the odd-frequency spin-singlet $p_y$-wave pair amplitude is zero within numerical accuracy.
Appendix E: Anomalous Green’s function for $d_{xy}$-wave superconductor for bulk system

The bulk Hamiltonian with $d_{xy}$-wave superconductor is given by

$$H_{\text{bulk}} = \sum_k \tilde{C}_k^\dagger H(k) \tilde{C}_k,$$

$$H(k) = \begin{pmatrix} E(k) & \Delta(k) \\ \Delta^\dagger(k) & E^\dagger(-k) \end{pmatrix},$$

$$E(k) = E_0 \sigma_0 + E_1 \sigma_1 + E_2 \sigma_2,$$

$$E_0 = [-2t (\cos k_x + \cos k_y) - \mu] \sigma_0,$$

$$E_1 = + \lambda \sin k_y \sigma_1,$$

$$E_2 = \lambda \sin k_x \sigma_2,$$

$$\Delta(k) = \Delta \sin k_x \sin k_y \sigma_2$$

$$= \Delta \sigma_2,$$

$$\tilde{C}_k = \begin{pmatrix} \tilde{c}_{\uparrow,k} & \tilde{c}_{\downarrow,k} & \tilde{c}_{\uparrow,-k} & \tilde{c}_{\downarrow,-k} \end{pmatrix}.$$  \hfill (E9)

The anomalous Green’s function for bulk system with the spin-singlet $d_{xy}$-wave superconductor is given by

$$G_{12}(z,k) = \frac{1}{\alpha_0^2 - \alpha_2^2 - \alpha_3^2} (\alpha_0 \sigma_0 - \alpha_2 \sigma_2 - \alpha_3 \sigma_3),$$  \hfill (E10)

$$\alpha_0 = \frac{2E_0 E_2}{\Delta},$$  \hfill (E11)

$$\alpha_2 = - \tilde{\Delta}^* + \frac{z^2}{\Delta} - \frac{1}{\Delta} \left( E_0^2 + E_1^2 + E_2^2 \right),$$  \hfill (E12)

$$\alpha_3 = - \frac{2iE_0 E_1}{\Delta},$$  \hfill (E13)

with

$$G(z,k) = [z - H(k)]^{-1},$$  \hfill (E14)

$$G(z,k) = \begin{pmatrix} G_{11}(z,k) & G_{12}(z,k) \\ G_{21}(z,k) & G_{22}(z,k) \end{pmatrix},$$  \hfill (E15)

where $G_{ij}(z,k) (i,j = 1,2)$ is a $2 \times 2$ matrix. From Eq. (E10), we can see that the spin-triplet component with $S_z = 0$ is absent since $S_z = 0$ component is proportional to $\sigma_1$.

Appendix F: Matrix elements of Green’s function for SC/DN junction

In this Appendix, we explain the derivation of the Green’s function in the DN in DN/superconductor junctions where we use super unit cell in the $y$-direction ($-\pi/L_y < k_y \leq \pi/L_y$) with periodic boundary condition. The Hamiltonian in the DN is

$$\mathcal{H}_{\text{DN}}^i = \sum_{k_y} \tilde{C}_y^\dagger(k_y) \begin{pmatrix} \tilde{u}\left((-49),l_{s}\right) & \tilde{t} & \tilde{u}\left((-48),l_{s}\right) & \tilde{t} \\ \tilde{t} & \tilde{u}\left((-47),l_{s}\right) & \tilde{t} & \tilde{u}\left((-46),l_{s}\right) \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{t} & \tilde{u}(0,l_{s}) & \tilde{t} & \tilde{u}\left((-1),l_{s}\right) \end{pmatrix} \tilde{C}(k_y).$$  \hfill (F1)
with $4L_y \times 4L_y$ matrices

$$
\mathbf{\tilde{q}}^{(j_\tau, L_y)} = \begin{pmatrix}
A^{(j_\tau, 1), L_y} & A_2 & \hat{0} & \cdots & \hat{0} & A_2^\dagger \\
A_2^\dagger & A^{(j_\tau, 2), L_y} & A_2 & \hat{0} & \cdots & \hat{0} \\
\hat{0} & A_2^\dagger & A^{(j_\tau, 3), L_y} & A_2 & \cdots & \cdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\
\hat{0} & \cdots & \cdots & \hat{0} & A_2^\dagger & A^{(j_\tau, L_y - 1), L_y} & A_2 \\
A_2 & \hat{0} & \cdots & \hat{0} & A_2^\dagger & A^{(j_\tau, L_y), L_y} \\
\end{pmatrix} ,
$$

(F2)

$$
\mathbf{\tilde{t}} = \begin{pmatrix}
B_{12} & 0 & \cdots & 0 & B_{21} \\
B_{21} & B_1 & B_{12} & 0 & 0 \\
0 & B_{21} & B_1 & B_{12} & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & B_{21} & B_1 & B_{12} \\
B_{12} & 0 & \cdots & 0 & B_{21} & B_1 \\
\end{pmatrix} ,
$$

(F3)

and

$$
\bar{C}_k = (\bar{c}_{(-49,1), k_y}, \bar{c}_{(-49,2), k_y}, \cdots, \bar{c}_{(-49, L_y), k_y}, \bar{c}_{(48,1), k_y}, \bar{c}_{(48,2), k_y}, \cdots, \bar{c}_{(0, L_y), k_y})^T ,
$$

(F4)

$$
\bar{c}_{j,k_y} = (\bar{c}_{j,k_y}, \bar{c}_{j,k_y+1}, \bar{c}_{j-1,k_y}, \bar{c}_{j-1,k_y+1}) .
$$

(F5)

Here, $A_1^{(j_\tau, j_y), L_y}$ and $A_2$ are $4 \times 4$ matrices given by

$$
A_1^{(j_\tau, j_y), L_y} = \begin{pmatrix}
-\mu + V_{j_\tau, j_y}^{(l_x)} & -\mu + V_{j_\tau, j_y}^{(l_z)} & \mu - V_{j_\tau, j_y}^{(l_x)} & \mu - V_{j_\tau, j_y}^{(l_z)} \\
-\mu + V_{j_\tau, j_y}^{(l_z)} & -\mu + V_{j_\tau, j_y}^{(l_x)} & \mu - V_{j_\tau, j_y}^{(l_z)} & \mu - V_{j_\tau, j_y}^{(l_x)} \\
-\mu + V_{j_\tau, j_y}^{(l_x)} & \mu - V_{j_\tau, j_y}^{(l_z)} & -\mu + V_{j_\tau, j_y}^{(l_x)} & \mu - V_{j_\tau, j_y}^{(l_z)} \\
-\mu + V_{j_\tau, j_y}^{(l_z)} & \mu - V_{j_\tau, j_y}^{(l_x)} & \mu - V_{j_\tau, j_y}^{(l_z)} & -\mu + V_{j_\tau, j_y}^{(l_x)} \\
\end{pmatrix} ,
$$

(F6)

$$
A_2 = \begin{pmatrix}
-t e^{ik_y} & -i\lambda e^{ik_y} / 2 \\
-i\lambda e^{ik_y} / 2 & -te^{ik_y} \\
-t e^{ik_y} / 2 & -i\lambda e^{ik_y} / 2 \\
-i\lambda e^{ik_y} / 2 & -te^{ik_y} \\
\end{pmatrix} ,
$$

(F7)

where $-t < V_{j_\tau}^{(l_x)} < t$ is a random potential at $j = (j_\tau, j_y)$-th site. $B_1$, $B_{12}$ and $B_{21}$ are $4 \times 4$ matrices and they are given by

$$
B_1 = \begin{pmatrix}
t & \lambda / 2 & -i\Delta_0 / 2 \\
-\lambda / 2 & -t & -i\Delta_0 / 2 \\
-i\Delta_0 / 2 & -i\Delta_0 / 2 & -\lambda / 2 \\
\end{pmatrix} ,
$$

(F8)

$$
B_{12} = B_{21} = 0,
$$

(F9)

for the spin-triplet $p_x$-wave superconductor. For the spin-singlet $d_{xy}$-wave superconductor, $B_1, B_{12}$ and $B_{21}$ are given by

$$
B_1 = \begin{pmatrix}
-t & \lambda / 2 \\
-\lambda / 2 & -t \\
\lambda / 2 & -\lambda / 2 \\
\end{pmatrix} ,
$$

(F10)

$$
B_{12} = -B_{21} = \begin{pmatrix}
\Delta_0 e^{ik_y} / 4 & \Delta_0 e^{ik_y} / 4 \\
\Delta_0 e^{ik_y} / 4 & \Delta_0 e^{ik_y} / 4 \\
\end{pmatrix} .
$$

(F11)
The Green’s function in the DN is then obtained as

$$\tilde{g}_{j_x}^{DN, t_s}(z, j_y, j'_y, k_y) = \left\{ g_{j_x}^{(L), t_s}(z, j_y, j'_y, k_y) \right\}^{-1} - \ell g_{j_x+1}^{(R), t_s}(z, j_y, j'_y, k_y) \ell^{-1},$$

where $g_{j_x}^{(L), t_s}(z, j_y, j'_y, k_y)$ and $g_{j_x+1}^{(R), t_s}(z, j_y, j'_y, k_y)$ are given by

$$g_{j_x}^{(L), t_s}(z, j_y, j'_y, k_y) = \left( X_{l_s}^{j_x} \cdots X_{l_s}^{j_y} = -48 X_{l_s}^{j_y} = -49 \right) \tilde{0},$$

$$g_{j_x+1}^{(R), t_s}(z, j_y, j'_y, k_y) = \left( Y_{l_s}^{j_x} = j_y + 1 Y_{l_s}^{j_y} = j_y + 2 \cdots Y_{l_s}^{j_y} = 0 \right) \tilde{G}^{\infty}(z, j_y, j'_y, k_y),$$

with $4L_y \times 4L_y$ matrix $\tilde{0}$ where all the elements of $\tilde{0}$ are zero. Here $X_{l_s}$ and $Y_{l_s}$ are given by

$$X_{l_s}^{j_x} = \left( \tilde{0} \quad \tilde{t}^{-1} \quad \tilde{z}^{-1} \right) \tilde{t}^{-1},$$

$$Y_{l_s}^{j_x} = \left( \tilde{0} \quad \tilde{z}^{-1} \quad \tilde{t}^{-1} \right),$$

and $\tilde{G}^{\infty}(z, j_y, j'_y, k_y)$ is given by

$$\tilde{G}^{\infty}(z, j_y, j'_y, k_y) = \sum_{m=0, 1, \ldots, N_y-1} e^{i\pi m (j_y - j'_y)} G_{j_y=1} \left( z, k_y + \frac{2\pi m}{N_y} \right),$$

where $k_y$ is $k_y = -\frac{\pi (N_y-1)}{N_y L_y}, -\frac{\pi (N_y-2)}{N_y L_y}, \ldots, \frac{\pi}{L_y}$, $\tilde{G}^{\infty}_{j_y=1}(z, k'_y)$ with $k'_y = k_y + 2\pi m/N_y$ is given by Eq. [A20].

Appendix G: Size dependence of LDOS in DN for spin-singlet $d_{xy}$-wave superconductor junction

In this Appendix, we check the size and the number of sample dependence of LDOS at the center of the DN ($j_x = -L_x/2$ where $L_x$ is a length of the DN in the $x$-direction) for the spin-singlet $d_{xy}$-wave superconductor junction with $\lambda/\Delta_0 = 50$.

![FIG. 17. Zero energy LDOS at the center in the DN $\tilde{\rho}(E = 0, j_x = -25)$ is plotted as a function of the inverse of the size in the $y$-direction $1/L_y$. Standard deviation (SD) estimated by 12 and 60 samples is also plotted. The length of the DN in the $x$-direction is $L_x = 50$. SD (error bar in main panel) is plotted in the inset as a function of $1/L_y$.](image)

In Fig. 17 we plot LDOS at zero-energy averaged over 12 and 60 samples as a function of $1/L_y$. As we can see in this figure, the extrapolated value of the ZEP for $L_y \to \infty$ is much larger than the background value ($\sim 0.2$). In the...
inset of Fig. 17, we plot the corresponding standard deviation. Although the magnitude of the standard deviation is not small, it decreases with the increase of $L_y$.

In Fig. 18, we plot the value of ZEP of LDOS as a function of the number of samples. The height of ZEP is almost independent of the number of samples for all values of $L_y$. The value of standard deviation becomes smaller with the increase of $L_y$.

In Fig. 19, we change $L_x$ from 10 to 100 and plot LDOS at $x = -L_x/2$. Although, it is difficult to estimate the peak width of LDOS due to the large value of standard deviation, it decreases with the increase of $L_x$.

Appendix H: Spatial dependence of odd-frequency pair amplitude for the DN/SC junction

In this Appendix, we discuss the spatial dependence of the odd-frequency spin-triplet $s$-wave pair amplitude for the DN/$d_{xy}$-wave superconductor junction. The odd-frequency spin-triplet $s$-wave pair amplitudes at each site ($j_x$) given
are shown in Fig. 20 where \( P_{j_x,S_z=1}(\omega_n) \) is given by Eq. (I3). The odd-frequency spin-singlet \( p_y \)-wave (onsite) one given by

\[
P_{j_x,p_y}(\omega_n) = \frac{1}{2N_{s_{1,1}}} \sum_{k_y j_y j_x} \frac{1}{2\sqrt{2}} \left\{ \begin{array}{c} f_{j_x,+,-}(i\omega_n,j_y,j_y,1,k_y) - f_{j_x,-,+}(i\omega_n,j_y,j_y,1,k_y) \\ + f_{j_x,+,-}(-i\omega_n,j_y,j_y,1,k_y) - f_{j_x,-,+}(-i\omega_n,j_y,j_y,1,k_y) \end{array} \right\} e^{ik_y} ,
\]

is shown in Fig. 21. Here \( f_{j_x,\sigma,\sigma'}(z,j_y,j_y,j_y,1,0) \) is the anomalous Green’s function of \( \hat{g}^\text{DN,Js}(z,j_y,j_y,j_y,1) \) given by Eq. (IP2).

![Graph](image)

FIG. 20. The real part of the odd-frequency spin-triplet \( s \)-wave pair amplitude with \( \omega_n/\Delta_0 = 10^{-3} \) \( (\Delta_0/t = 0.01) \) is plotted as a function of site \( j_x \) for (a) \( S_z = 1 \), (b) \( S_z = 0 \) and (c) \( S_z = -1 \) with several \( \lambda (\lambda/\Delta_0 = 0, 1, 5, \) and 10) for the DN/spin-singlet \( d_{xy} \)-wave superconductor junction. The imaginary part of the odd-frequency spin-triplet \( s \)-wave pair amplitude is zero within numerical accuracy.

The odd-frequency spin-triplet \( s \)-wave pair amplitude \( P_{j_x,S_z=1}(\omega_n/\Delta_0 = 10^{-3}) \) is almost constant in the DN and \( P_{j_x,S_z=-1}(\omega_n) + P_{j_x,S_z=1}(\omega_n) = 0 \) holds within numerical accuracy. The odd-frequency spin-singlet \( p_y \)-wave pair amplitude rapidly decays in the DN since \( p \)-wave pair amplitudes are fragile against impurity scattering. \( p_x \)-wave components are almost zero in the DN and the SC.

### Appendix I: Surface roughness for spin-singlet \( d_{xy} \)-wave superconductor

In order to study the effect of roughness, we study a thin diffusive layer attached to the surface of \( d_{xy} \)-wave superconductor shown in Fig. 22. The effect of diffusive scattering on the SABS has been studied within quasiclassical approximation [61, 62]. Without RSOC, induced odd-frequency pair amplitude is suppressed since it has an odd parity [63]. On the other hand, in the presence of the RSOC, the odd-frequency spin-triplet \( s \)-wave pair amplitude is induced near the surface [52]. Here, we take into account of the spatial depletion of the pair potential near the surface. The assumed spatial dependence is shown in Eq. (15). The Hamiltonian is given by...
spin-singlet $d_{xy}$-wave superconductor DN/SC junction

![Graph showing the imaginary part of the odd-frequency spin-singlet $p_y$-wave pair amplitude](image)

**FIG. 21.** The imaginary part of the odd-frequency spin-singlet $p_y$-wave pair amplitude with $\omega_n/\Delta_0 = 10^{-3}$ ($\Delta_0/t = 0.01$) is plotted as a function of site ($j_x$) with several $\lambda$ ($\lambda/\Delta_0 = 0, 1, 5,$ and $10$) for the DN/spin-singlet $d_{xy}$-wave superconductor junction. The real part of the odd-frequency spin-singlet $p_y$-wave pair amplitude is zero within numerical accuracy.

![Graph showing the real part of the odd-frequency spin-singlet $p_y$-wave pair amplitude](image)

**FIG. 22.** Schematic of the SC with impurity potential ($1 \leq j_x \leq N_d = 20$). Green colored region indicates the area with impurity potential. Pair potential $\tilde{\Delta}(j_x)$ is also plotted as a function of site ($j_x$).

\[
\mathcal{H} = \mathcal{H}_t + \sum_{1 \leq j_x \leq N_d, j_y} V_{jy} n_j + \mathcal{H}_{SO} - \mu \sum_j n_j + \mathcal{H}_\Delta,
\]

(11)

\[
\tilde{\mathcal{H}}_\Delta = -\frac{1}{2} \sum_j \tilde{\Delta}(j_x) \left( c_{j,\uparrow}^\dagger c_{j+e_x+e_y,\downarrow}^\dagger - c_{j,\downarrow}^\dagger c_{j+e_x+e_y,\uparrow}^\dagger - c_{j,\uparrow}^\dagger c_{j+e_y-e_x,\downarrow}^\dagger + c_{j,\downarrow}^\dagger c_{j+e_y-e_x,\uparrow}^\dagger \right),
\]

(12)

\[
\tilde{\Delta}(j_x) = \begin{cases} 
\Delta_0, \\
\Delta_0 \tanh[(j_x - 1)/100],
\end{cases}
\]

(13)

In Eq. (13), we take into account the fact that the coherence length is about 100 site ($\Delta_0/t = 0.01$). Here we impose impurity potential $V_j$ for $1 \leq j_x \leq N_d = 20$. In Fig. 22 absolute value of the odd-frequency spin-triplet $s$-wave pair amplitude given by

\[
|P_{\text{triplet}}(\omega_n)| = \sum_{S_z = 1, 0, -1} |P_{j_x, j_y, S_z}^{\text{triplet}}(\omega_n)|,
\]

(14)

is shown for $\lambda/\Delta_0 = 1, 5$ and $10$. Here $P_{\text{triplet}}(\omega_n)$ is given by Eqs. (H1) to (H3). From Fig. 23 we can conclude that the spatial dependence of the pair potential near the surface almost does not affect the magnitude of the odd-frequency
FIG. 23. The absolute value of the odd-frequency spin-triplet $s$-wave pair amplitude is plotted as a function of site ($j_x$) for the spin-singlet $d_{xy}$-wave superconductor with diffusive layer (green colored region). $P_{j_x}^{\text{triplet}}(\omega_n/\Delta_0 = 10^{-3})$ for $\tilde{\Delta}(j_x) = \Delta_0$ and $\Delta_0 \tanh[(j_x - 1)/100]$ are shown for (a) $\lambda/\Delta_0 = 1$, (b) $\lambda/\Delta_0 = 5$ and (c) $\lambda/\Delta_0 = 10$.

spin-triplet $s$-wave pair amplitude as compared to the case with constant pair potential.