Task Agnostic Metrics for Reservoir Computing

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Physical reservoir computing is a computational paradigm that enables temporal pattern recognition to be performed directly in physical matter. By exciting non-linear dynamical systems and linearly classifying their changes in state, we can create highly energy-efficient devices capable of solving machine learning tasks without the need to build a modular system consisting of millions of neurons interconnected by synapses. The chosen dynamical system must have three desirable properties: non-linearity, complexity, and fading memory to act as an effective reservoir. We present task agnostic quantitative measures for each of these three requirements and exemplify them for two reservoirs: an echo state network and a simulated magnetic skyrmion-based reservoir. We show that, in general, systems with lower damping reach higher values in all three performance metrics. Whilst for input signal strength, there is a natural trade-off between memory capacity and non-linearity of the reservoir’s behaviour. In contrast to typical task-dependent reservoir computing benchmarks, these metrics can be evaluated in parallel from a single input signal, drastically speeding up the parameter search to design efficient and high-performance reservoirs.

I. INTRODUCTION:

Traditional information processing relies on discrete components such as transistors to sequentially manipulate data step-by-step. Although the step-by-step methods of computation have been successful, traditional computer architectures are reaching their fundamental limits in transistor density\(^1\)\(^2\)\(^3\). As demand for low cost and efficient devices increases, alternative architectures need to be explored. On top of the physical constraints, lots of problems are not easy to formulate symbolically, rendering them difficult to solve with step-by-step methods\(^4\).

Reservoir computing (RC) is an alternative model of computation inspired by nervous systems that learn how to solve problems rather than being told how to via explicit instructions. A reservoir computer consists of two components, a reservoir which is responsible for non-linear information processing and a readout mechanism responsible for learning\(^5\). Typically, reservoir computers are suited to pattern recognition problems dealing with temporally or spatially dependant information, such as audio or images.

To input information into a reservoir computer, one must first convert the information into a temporal signal. One then inputs the temporal signal into the reservoir in the form of an excitation. This excitation then changes the state of the reservoir, ideally resulting in unique state trajectories for unique inputs. The second component of the reservoir computer is the readout mechanism. After the information has been input to the reservoir, a finite \(N\)-dimensional representation of the reservoir’s state is measured, representing \(N\) individual readout nodes. By considering the value of each readout node as a dimension in a high-dimensional latent space, we create a mapping between the original input data and a single point in the high-dimensional latent space. One uses the results of previous mappings to train a linear model that can separate different classes of data within the latent space. The trained model is used to classify previously unobserved inputs.

Although one can use any dynamical system for reservoir computing, some are better than others. Solving complex tasks requires that the system has several fundamental properties: complexity, non-linearity and fading memory\(^6\). Unlike other machine learning paradigms such as neural networks, the non-linearity required for complex information processing of a reservoir computer comes from an untrained dynamical system with fixed parameters. By separating the training mechanism from the dynamical reservoir, it is possible to use physical systems to perform the non-linear information processing directly in matter. Thereby, one takes advantage of physics’s inherently parallel nature and opens doors to low cost, energy-efficient computations that are not constrained by the limitations of conventional hardware. A diverse set of physical dynamical systems\(^5\) based
upon optics, spintronics\cite{7} and even a bucket of water\cite{8} have already been proposed to implement the reservoir computing paradigm including magnetic skyrmion based systems\cite{9}\cite{10}\cite{11}.

While several works perform specific benchmark tasks to characterise the performance of their reservoir, direct evaluations and comparisons of different reservoirs are complex. In this work, we introduce task agnostic metrics to individually quantify the performance of reservoirs regarding non-linearity, complexity, and fading memory. By characterising dynamical systems in this manner, we can construct reservoirs to purpose by tuning the reservoir parameters to optimal metric values. In contrast to typical task-dependent data-hungry reservoir computing benchmarks, one can efficiently compute all three of these metrics in parallel from a single input signal. This quick evaluation is essential for numerically modelled reservoirs where the simulation time of the dynamical system is a significant bottleneck. Finally, we apply the three metrics to two different reservoirs, a class of recurrent neural networks known as echo-state networks and a skyrmion-based system.

II. TASK AGNOSTIC METRICS:

We quantify non-linearity, short-term memory and complexity as metrics for reservoir computing systems. All metrics are calculated in parallel from the response of the reservoir $y(t)$ to a random input signal $u(t)$. In this work, we generate $u(t)$ from a uniform distribution over the interval minus one and one. The entire input-output relationship is required for the non-linearity and memory metrics, whilst the complexity and stability metrics only require analysis of the output signals.

A. Non-linearity

Dynamical systems with fading memory, such as a reservoir, can be modelled by a Volterra series, similar to how functions can be approximated by a Taylor series. We define the non-linearity of a reservoir as of how much its response $y(t)$ to an input signal $u(t)$ deviates from a Volterra series truncated to linear orders.

$$ y(t) = h^{(0)} + \int_{\tau=0}^{\infty} h^{(1)}(\tau) u(t-\tau) d\tau + y^{NL}(t). \quad (1) $$

Here, $y(t)$ describes the output vector of readout node values $y_n(t)$, $h^{(0)}$ is a constant, $h^{(1)}$ is a kernel that linearly couples past inputs to the present readout nodes, and $y^{NL}(t)$ represents the truncated nonlinear kernels.

We indirectly measure the non-linearity of the reservoir by measuring the quality of its representation using Eq. (1) truncated to linear order. For a standardised $u(t)$, i.e. $h^{(0)} = 0$, and discrete time, the linear components of the Volterra series simplify to a regression problem of the form:

$$ y^L(t) = \sum_{\tau=0}^{\infty} h^{(1)}_\tau u(t-\tau), \quad (2) $$

From this, we construct a distinct linear estimator $\hat{y}_n(t)$ for each readout node $y_n$.

$$ \hat{y}_n(t) = \sum_{\tau=0}^{k} w_\tau u(t-\tau), \quad (3) $$

where $w$ represents the estimator weights and $k$ is the time delay at which we truncate the kernel, chosen to be larger than the relaxation time of the reservoir. The estimator $\hat{y}_n(t)$ is trained to predict a readout node’s value $y_n(t)$ from all past input values up to $u(t-k)$ as shown in Fig. 1.

We use the known input and output samples to measure the quality of each output estimator $\hat{y}_n$ on a scale from zero to one using the $R^2$ correlation coefficient (see Appendix A for details). The estimator can perfectly represent the system for a coefficient value of one, meaning that it is an ideal linear time-invariant system. Whilst for a coefficient close to zero, the response of the system is dominated by non-linear terms. To convert this to an overall measure for non-linearity $NL$, we compute the mean estimator quality for all readout nodes and subtract from one.

$$ NL = 1 - \frac{\sum_{n=1}^{N} R^2[\hat{y}_n, y_n]}{N} \quad (4) $$

Thus $NL$ ranges from zero to one. 0 means completely linear behaviour, and one corresponds with purely non-linear behaviour.

B. Memory

We define the memory of a system as a measure of how much information about previous inputs is present in its current output state using the existing measure for linear memory capacity as defined in Ref. [12].

Here, for a certain temporal offset $\tau$ we measure how well the current state of the reservoir $y(t)$ can recall the input $u$ at time $t-\tau$. For each offset up to a chosen truncation, $k$ we construct a linear estimator of the form:

$$ \hat{u}(t-\tau) = \sum_{n=1}^{N} w_n y_n(t), \quad (5) $$

as shown in figure 2. Similarly to the non-linearity metric, we measure the quality of each estimator using the $R^2$ correlation coefficient. For each estimator, the result is a value between zero and one representing how well the current readout nodes can predict the past input for each offset up to $k$. We sum the quality of all estimators to get our overall metric for memory.
The larger the value of the memory capacity is, the longer the duration of patterns can, in principle, be recognised.

C. Complexity

In this work, we refer to complexity as the ability of a reservoir to produce a diverse set of output states, meaning that the individual outputs should be linearly separable. We propose a metric that quantifies the separability of output states by measuring the size of the latent space they span.

By inputting a discrete random temporal signal of an equal number of samples to the number of output nodes $N$, we collect the values of each output node into a square matrix $M$ of dimensions $N \times N$. We use the effective rank of the matrix $M$, as defined in Ref. [13] as the complexity measure CP. It is a real value bound between 1 and $N$ where systems with a high complexity result in a reservoir with high dimensional latent space and vice versa.

D. Stability

We also consider the system’s stability as an additional metric for physical systems, as it is a prerequisite for its functional principle. For stable reservoirs, the system relaxes back to a consistent (meta-)stable state in the absence of inputs. Being stable is an essential property for physical reservoirs to ensure reproducibility and additionally prevents the reservoir from going into a chaotic regime.

To measure the stability, we first input a random signal into the reservoir. After the input has finished, the reservoir is relaxed until it reaches a (meta)-stable state. The distance between the system’s initial and final state characterises stability, where a low (high) distance refers to a high (low) stability. The details of the distance measure can be adjusted to the specific system.

For example, one could consider the mean square difference of the initial $y_{\text{initial}}$ and final $y_{\text{final}}$ output states, i.e.

$$S = \sum_{n=1}^{N} (y_{\text{initial}}^{n} - y_{\text{final}}^{n})^2.$$  \hspace{1cm} (7)

For online learning problems, where the reservoir weights are continuously updated, small instabilities may not be a problem since the change in system state can be re-learnt.

III. DEMONSTRATION OF METRICS:

To demonstrate the application of the above-introduced metrics, we now consider two reservoir computing systems. The first system is a mathematical model known as an echo state network (ESN), with our main results shown in Fig. 2. The second system is a numerically simulated, physical system based upon magnetic skyrmions with the results shown in Fig. 4.

A. Echo State Network

An ESN is a discrete-time recurrent neural network with fixed random weights, see Fig. 2(b) for a sketch. The network consists of $N$ discrete reservoir nodes described by the vector $y(t)$ and provides $K$ input nodes represented by the vector $u(t)$. An $N \times K$ weight matrix $V$ couples the input nodes to the reservoir nodes. Similarly, an $N \times N$ weight matrix $W$ couples the reservoir nodes to each other. Both matrices are initialised to random values and then remain fixed for the network’s lifetime. At each timestep, the state of the network is updated based upon the present input node values and the network’s weight matrices according to:
Metric Results for Echo State Network

\[ y(t) = \tanh(Vu(t) + Wy(t - 1)) + \sigma \]  

(8)

Where \( y(t) \) and \( u(t) \) represent the respective values of the reservoir nodes and input nodes at time \( t \). Additionally, \( \sigma \) represents a random noise vector with the purpose of regularisation. The values of \( y(t) \) are treated in the following as the reservoir's readout values.

The initial random weights for \( V \) and \( W \) are generated from a uniform random distribution \( U \) bound between minus one and one. The input weight matrix is initialised such that:

\[ V_{i,j} = sU \quad \forall i, j \in \mathbb{N} : i \leq N, j \leq K, \]  

(9)

where \( s \) is referred to as the input scaling parameter.

To initialise \( W \), we begin by creating a sparse matrix of random elements \( W' \):

\[ W'_{i,j} = SU \quad \forall i, j \in \mathbb{N} : i \leq N, j \leq N, \]  

(10)

where \( S \in \{0,1\} \) is a binary random variable such that \( P(S = 0) = \gamma \) where \( \gamma \) is the sparsity of \( W \). To get \( W \) we then re-scale the elements of \( W' \) to a chosen spectral radius \( R \).

\[ W = \frac{R}{\rho(W')}W', \]  

(11)

where \( \rho(W') \) is the computed spectral radius of \( W' \). By choosing a specific spectral radius \( R \), we control the duration that information resides in the system, thereby effectively tuning the damping of the ESN. In this paper, we refer to the damping of an ESN to be equal to one minus its spectral radius \( R \).

To measure the effect of damping and input scaling, we consider a \( N = 256 \) node ESN with a sparsity of 0.9 and \( \sigma \) drawn from the random variable \( 0.001U \) such that the noise is sufficiently smaller than the input. We use a grid search to compute each metric, varying the network damping over the interval \([0.01, 0.5]\) and input scaling over \([0.3, 3]\) and using a one thousand element signal for the input. Fig. 2(a) shows a summary of the analysis, explained in more detail below, with the individual resulting metric values shown in Fig. 2(c)-(e).

For the non-linearity metric, Fig. 2(c), the reservoir behaves like a linear system for high damping values. This behaviour is due to the network's activation function \( \tanh \), which is approximately linear for small values. The activation function also explains the linear behaviour for small input scaling \( s \) for the same reason. However, for more significant input scaling, the activation function is no longer approximately linear and therefore, the reservoir behaviour is in the non-linear regime.
Memory, Fig. 2(d), is also maximised for low damping, as expected since the state of the system is more dependent on inputs in the distant past. However, unlike the other two measures, the highest memory is found for low input scaling. This compromise reveals the trade-off between order and disorder that one has to deal with when working with reservoir computing. The behaviour is also consistent with the observation that a recurrent neural network can learn most efficiently at the edge of chaos, i.e. when the damping is sufficient to prevent the system from being unstable for a given input scaling.

Complexity, Fig. 2(e), for the ESN example, is again maximised for low damping. For the output states to have a large latent space, then non-linear transformations must occur on the input signal. As mentioned previously, the non-linear transformations occur for high input scaling, where the reservoir node values approach the non-linear regime of the activation function. However, when the input scaling becomes too high, above 1.5 in this case, the input scaling term of Eq. (8) dominates and the reservoir output is approximated as a linear transformation of the input signal.

While metrics alone are not sufficient to optimally parameterise a reservoir for a task, as different tasks may require different metrics to be optimised, we can use the measures as a guide. For the ESN, it is desirable to have low damping and a balanced input scaling depending on the task at hand. We show a combination of all three metrics in Fig. 2(a), where we score the reservoir from zero to one based using the following equation:

$$\text{SCORE} = \min(\hat{\text{NL}}, \hat{\text{CP}}, \hat{\text{MC}}) \quad (12)$$

Where $\hat{X}$ means is the value of the corresponding metric $X$, normalised by its maximal value shown in Fig. 2(c)-(e), respectively. The idea of the figure is to give a guideline as to regions in which all three metrics are significant compared to other choices of reservoir parameters.

### B. Physical Skyrmion Reservoir

We next demonstrate how the metrics apply to a physical system based on magnetic skyrmions [14]. Skyrmion and other spintronic based reservoirs [7, 9, 15, 16] are of particular interest due to their ability to integrate into existing CMOS devices [17], their nanoscale size, and their responsiveness to low power excitations.

For our specific model, we consider an inhomogeneous magnetic thin-film populated with magnetic skyrmions in the presence of a magnetic field and an interfacial Dzyaloshinskii-Moriya interaction (DMI); see Fig. 3. We introduce inhomogeneities in the form of grain defects to stabilise the system, as shown in Fig. 3(b). Each grain in the system has a different DMI strength, as in Ref. [14], resulting in a complex energy landscape, causing differing regions of the film to be more or less favourable for skyrmions. One consequence of this is that specific arrangements of skyrmions become metastable states, allowing one to excite the system and have it return to the same initial state. We give precise details of the model and how we obtain the metastable state, shown in Fig. 3(a), in the Appendix.

We implement the reservoir input as a time-varying electrical excitation in the form of a unidirectional electrical current across the film characterised by the current density $j(t) = j_{in}u(t)\mathbf{x}$ with a maximal strength $j_{in}$. To convert the discrete input signal $u(t)$ into an analogue time series, we feed each data point into the reservoir as sequential 0.2ns pulses as seen in Fig. 3(c). The electrical current interacts with the magnetic skyrmions via spin-torques, resulting in a time-dependant deformation of the magnetic texture.

For readout nodes, we computationally observe the changes in magnetic texture by taking a low-resolution snapshot of the film’s out-of-plane magnetisation component. We choose the resolution to be of similar magnitude to the sizes of individual skyrmions as indicated in Fig. 3(a). This readout method yields the spatial regions of the film that contribute the most to each metric, allowing us to identify magnetic structures or dynamics that are predominantly responsible for the system's information processing and memory.

We measure the value of each metric over a range of system parameters that we consider analogous to damping and input scaling for an ESN. The first parameter we consider for our skyrmion reservoir is the magnetic Gilbert damping strength $\alpha$. By increasing (decreasing) $\alpha$, we decrease (increase) the time it takes for the magnetic texture to relax after an input. The second parameter we consider is the strength of the input current density $j_{in}$. We vary $\alpha$ over the interval $[0.025, 0.25]$ and input scaling $j_{in}$ over $[30A/\mu m^2, 300A/\mu m^2]$.

Since the skyrmion reservoir is a physical system, we also consider its stability as an additional metric. To do this, we compare the initial and final states of the reservoir after receiving its input. For each cell, we take the

![FIG. 3. Example arrangement of magnetic skyrmions, with 16nm × 16nm readout node size shown at the origin in red (left) and the corresponding grain inhomogeneities (right). The light and dark regions represent high and low DMI parameter respectively, with precise details given in the appendix.](image)
The memory capacity of the skyrmion reservoir is maximal for around $\alpha = 0.05$, rather than the lowest tested Gilbert damping at $\alpha = 0.025$. Although, on the one hand, low Gilbert damping stops information from old inputs decaying, when the Gilbert damping is too low, information from past inputs becomes 'washed-out'. The comparison of memory capacity and non-linearity shows the trade-off between memory and efficient information processing taking place.

For stability, the system is unstable for low Gilbert damping and high input current strengths. As expected, in this regime, the magnetic texture becomes unstable as the pinning threshold is exceeded. Low stability impacts the memory capacity negatively because the irreversible deformations that occur in unstable systems change the future response of the system. Since the weights for the memory recall metric task were trained on the system’s response before the instability occurred, the same weights are no longer ideal for the task with the updated response. Note that the measure for non-linearity is no longer valid for the regions with imperfect stability, as the calculation for non-linearity assumes perfect stability.

We show a combination of all four metrics in Fig. 4(a), where we use the same scoring as for the ESN case but additionally setting the score to zero for all stability mea-
Spatial Analysis for Skyrmion Reservoir

![Spatial Analysis for Skyrmion Reservoir](image)

**FIG. 5.** Spatially resolved metrics for Skyrmion Reservoir with parameters $\alpha = 0.05$, current strength $= 210 \text{ A/µm}^2$; (a) configuration of the Skyrmion Reservoir (b) non-linearity of each simulation cell (c) memory capacity contribution of each simulation cell.

We then spatially analyse the contributions of the skyrmion reservoir Fig 5(a) with parameters $\alpha = 0.05$ & current strength $= 210 \text{ A/µm}^2$, chosen to be a reservoir on the edge of stability. We then calculate the spatial contributions of non-linearity and memory for each cell in the simulation. For complexity, note that the rank of the readout value matrix depends on all readout nodes, and therefore does not allow for a localised measure of complexity in this manner. Fig 5(b) reveals the regions of most substantial non-linearity by showing the individual non-linearity of each cell in the simulation. Typically, small spots of high non-linearity are at the centre of each skyrmion and the boundaries between them. The regions of typically low non-linearity towards the right of Fig 5(b) contain strongly pinned skyrmions that have comparatively small deformations than the rest, see complementary movie file.

To spatially analyse memory capacity, we repeat the memory capacity task, using all individual cells as readout nodes with standardisation preprocessing. We then calculate the average relative weights of each node up to a delay of $k = 10$, yielding a measure between zero and one representing the relative memory contribution of each cell. The areas of high memory contribution, shown in Fig 5(c), overlap with the highly non-linear regimes where skyrmion are free to deform.

IV. SUMMARY

In this work, we have demonstrated how three independent, task agnostic metrics can be employed to quantify the quality of a reservoir computer in terms of non-linearity, complexity and memory. All metrics can be calculated in parallel using a small random dataset allowing for a rapid evaluation of reservoir models, in contrast to typical time-consuming and extensive benchmark tasks. This methodology allows for efficient screening of reservoir parameters for 'good' reservoir behaviour and tuning reservoir performance to purpose.

Additionally, for physical systems, one can use spatially resolved analysis of the metrics to identify regions of behaviour with interesting dynamics, giving a better understanding of what environments lead to memory and non-linearity in the system. By creating conditions that replicate such environments, one can parameterise reservoirs in a guided manner to optimise a balance between non-linearity, memory and complexity. This methodology also provides a path to design the layout of readout nodes for reservoirs by choosing node locations based on local contributions to memory and non-linearity. Having fewer effectively placed readout nodes can reduce the number of model parameters, potentially decreasing the amount of data needed to train the reservoir and reducing over-fitting.

For the particular reservoir computing models studied - the ESN and the skyrmion reservoir - we find that the ideal dynamics for both systems are typically found for low damping, where information decays slowly, and the system response is highly non-linear with a trade-off between high and low input scaling. For the physical skyrmion system, we see that the best behaviour is on the edge of stability, so one would have to be care-
ful to develop such a device further. Despite the potential stability problems for the skyrmion reservoir, it manages to achieve decent scores in memory, complexity, and non-linearity in a comparatively small physical area than would be required to implement an equivalent ESN with transistors.

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Appendix A: Measuring Estimator Quality

To measure the quality of the linear estimators, as for the non-linearity, Eq. (3) and memory Eq. (5), in the main text, we use the $R^2$ coefficient of determination: Consider the following linear estimator:

$$\hat{y}(t) = \sum_i w_i u(t)$$

We train the weights $w$ of the estimator using the first 75% of our input $u(t)$ and output $y(t)$ samples, minimising the residual sum of squares error. Using the remaining 25% of input samples, we compare the corresponding estimated values $\hat{y}(t)$ to the true value $y(t)$ with:

$$R^2[\hat{y}, y] = \frac{\text{Cov}^2(\hat{y}(t), y(t))}{\sigma(\hat{y}(t))\sigma(y(t))}.$$  \hspace{1cm} (A2)

$R^2[\hat{y}, y]$ is a measure for the estimators quality, ranging from zero (no correlation) to one (perfect prediction). In this work we use a total of 1000 data points for training and testing to measure estimator quality.

Appendix B: Simulation Details for Skyrmion Reservoir

The skyrmion reservoirs in Sec. III were simulated using MuMax3\cite{muMax}. Each thin film was modelled with a 256 by 256 cell grid with a cell size of 1 nm, smaller than the exchange length calculated for the material parameters below.

The material parameters were as follows: saturation magnetisation $M_{sat} = 956 \text{ kAm}^{-1}$, exchange coupling constant $A_{ex} = 10 \text{ pJm}^{-1}$, interfacial DMI $D_{\text{ind}} = -3 \times 10^{-3} \text{ Jm}^{-2}$, uniaxial anisotropy constant $K_{U1} = 0.8 \text{ MJm}^{-3}$, perpendicular external field $B_{ext} = 200 \text{ mT}$, temperature $T=0K$, and we used an electrical current polarisation of 1. The DMI grain inhomogeneities were generated using a Voronoi tessellation\cite{voronoi} and have average size of 20nm and DMI variance of 40% (see Fig. 3b of the main text), similar to the experimentally observed DMI grains in [20]. In the simulations, we do not include anisotropic magneto-resistance effects and instead assume the material resistance to be constant.

We initialise the magnetic texture by first creating a skyrmion lattice and letting it relax to a meta-stable state. To further stabilise the system, we excite the texture with a random input current for 200ns, then allow it to relax. This is to remove any low energy, initial instabilities in the reservoir and to obtain a meta-stable state with a higher energy barrier suitable for RC.
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