ON $\sigma$-CONVEX SUBSETS IN SPACES OF SCATTEREDLY CONTINUOUS FUNCTIONS

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Abstract. We prove that for any topological space X of countable tightness, each $\sigma$-convex subspace F of the space SCp(X) of scatteredly continuous real-valued functions on X has network weight nw(F) ≤ nw(X). This implies that for a metrizable separable space X, each compact convex subset of the function space SCp(X) is metrizable. Another corollary says that two Tychonoff spaces X, Y with countable tightness and topologically isomorphic linear topological spaces SCp(X) and SCp(Y) have the same network weight nw(X) = nw(Y). Also we prove that each zero-dimensional separable Rosenthal compact space is homeomorphic to a compact subset of the function space SCp(ωω) over the space ωω of irrationals.

This paper was motivated by the problem of studying the linear-topological structure of the space SCp(X) of scatteredly continuous real-valued functions on a topological space X, addressed in [5, 6].

A function f : X → Y between two topological spaces is called scatteredly continuous if for each non-empty subspace A ⊂ X the restriction f|A : A → Y has a point of continuity. Scatteredly continuous functions were introduced in [3] (as almost continuous functions) and studied in details in [8, 4] and [7]. If a topological space Y is regular, then the scattered continuity of a function f : X → Y is equivalent to the weak discontinuity of f; see [3, 4, 4.4]. We recall that a function f : X → Y is weakly discontinuous if each subspace A ⊂ X contains an open dense subspace U ⊂ A such that the restriction f|U : U → Y is continuous.

For a topological space X by SCp(X) ⊂ R X we denote the linear space of all scatteredly continuous (equivalently, weakly discontinuous) functions on X, endowed with the topology of pointwise convergence. It is clear that the space SCp(X) contains the linear subspace Cp(X) of all continuous real-valued functions on X. Topological properties of the function spaces Cp(X) were intensively studied by topologists, see [2]. In particular, they studied the interplay between topological invariants of topological space X and its function space Cp(X).

Let us recall [10, 12] that for a topological space X its

- weight w(X) is the smallest cardinality of a base of the topology of X;
- network weight w(X) is the smallest cardinality of a network of the topology of X;
- tightness t(X) is the smallest infinite cardinal $\kappa$ such that for each subset $A \subset X$ and a point $a \in A$ in its closure there is a subset $B \subset A$ of cardinality $|B| \leq \kappa$ such that $a \in B$;
- Lindelöf number l(X) is the smallest infinite cardinal $\kappa$ such that each open cover of X has a subcover of cardinality $\leq \kappa$;
- hereditary Lindelöf number $lh(X) = \sup\{l(Z) : Z \subset X\}$;
- density $d(X)$ if the smallest cardinality of a dense subset of X;
- the hereditary density $hd(X) = \sup\{d(Z) : Z \subset X\}$;
- spread $s(X) = \sup\{|D| : D$ is a discrete subspace of $X\}$.

By [2, §I.1], for each Tychonoff space X the function space Cp(X) has weight $w(C_p(X)) = |X|$ and network weight $nw(SC_p(X)) = nw(X)$. For the function space SCp(X) the situation is a bit different.

Proposition 1. For any $T_1$-space X we have

$$s(SC_p(X)) = nw(SC_p(X)) = w(SC_p(X)) = |X|.$$

Proof. It is clear that $s(SC_p(X)) \leq nw(SC_p(X)) \leq w(SC_p(X)) \leq w(R^X) = |X|$. To see that $|X| \leq s(SC_p(X))$, observe that for each point $a \in X$ the characteristic function

$$\delta_a : X \to \mathbb{R} = \begin{cases} 1, & \text{if } x = a \\ 0, & \text{otherwise} \end{cases}$$

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of the singleton \{a\} is scatteredly continuous, and the subspace \(D = \{\delta_a : a \in X\} \subset SC_p(X)\) has cardinality \(|X|\) and is discrete in \(SC_p(X)\).

The deviation of a subset \(F \subset SC_p(X)\) from being a subset of \(C_p(X)\) can be measured with help of the cardinal number \(\text{dec}(F)\) called the \textit{decomposition number} of \(F\). It is defined as the smallest cardinality \(|C|\) of a cover \(C\) of \(X\) such that for each \(C \in C\) and \(f \in F\) the restriction \(f|C\) is continuous. If the function family \(F\) consists of a single function \(f\), then the decomposition number \(\text{dec}(F) = \text{dec}\{f\}\) coincides with the decomposition number \(\text{dec}(f)\) of the function \(f\), studied in [13]. It is clear that \(\text{dec}(C_p(X)) = 1\).

**Proposition 2.** For a \(T_1\) topological space \(X\) the decomposition number \(\text{dec}(SC_p(X))\) is equal to the decomposition number \(\text{dec}(D)\) of the subset \(D = \{\delta_a : a \in X\} \subset SC_p(X)\) and is equal to the smallest cardinality \(\text{ddec}(X)\) of a cover of \(X\) by discrete subspaces.

**Proof.** It is clear that \(\text{dec}(D) \leq \text{dec}(SC_p(X)) \leq \text{ddec}(X)\). To prove that \(\text{dec}(D) \geq \text{ddec}(X)\), take a cover \(C\) of \(X\) of cardinality \(|C| = \text{dec}(D)\) such that for each \(C \in C\) and each characteristic function \(\delta_a \in D\) the restriction \(\delta_a|C\) is continuous. We claim that each space \(C \in C\) is discrete. Assuming conversely that \(C\) contains a non-isolated point \(c \in C\), observe that for the characteristic function \(\delta_c\) of the singleton \(\{c\}\) the restriction \(\delta_c|C\) is not continuous. But this contradicts the choice of the cover \(C\). Therefore the cover \(C\) consists of discrete subspaces of \(X\) and \(\text{ddec}(X) \leq |C| = \text{dec}(D)\).

In contrast to the whole function space \(SC_p(X)\) which has large decomposition number \(\text{dec}(SC_p(X))\), its \(\sigma\)-convex subsets have decomposition numbers bounded from above by the hereditary Lindelöf number of \(X\).

Following [1] and [15], we define a subset \(C\) of a linear topological space \(L\) to be \(\sigma\)-\textit{convex} if for any sequence of points \((x_n)_{n \in \omega}\) in \(C\) and any sequence of positive real numbers \((t_n)_{n \in \omega}\) with \(\sum_{n=0}^{\infty} t_n = 1\) the series \(\sum_{n=0}^{\infty} t_n x_n\) converges to some point \(c \in C\). It is easy to see that each compact convex subset \(K \subset L\) is \(\sigma\)-convex. On the other hand, each \(\sigma\)-convex subset of a linear topological space \(L\) is necessarily convex and bounded in \(L\).

The main result of this paper is the following:

**Theorem 1.** For any topological space \(X\) of countable tightness, each \(\sigma\)-convex subset \(F \subset SC_p(X)\) has decomposition number \(\text{dec}(F) \leq \text{hl}(X)\).

This theorem will be proved in Section 3. Now we derive some simple corollaries of this theorem.

**Corollary 1.** For any topological space \(X\) of countable tightness, each \(\sigma\)-convex subset \(F \subset SC_p(X)\) has network weight \(\text{nw}(F) \leq \text{nw}(X)\). Moreover,

\[
\text{nw}(X) = \max\{\text{nw}(F) : F \text{ is a } \sigma\text{-convex subset of } SC_p(X)\}
\]

provided the space \(X\) is Tychonoff.

**Proof.** By Theorem 1 each \(\sigma\)-convex subset \(F \subset SC_p(X)\) has decomposition number \(\text{dec}(F) \leq \text{hl}(X)\). Consequently, we can find a disjoint cover \(C\) of \(X\) of cardinality \(|C| = \text{dec}(F) \leq \text{hl}(X)\) such that for each \(C \in C\) and \(f \in F\) the restriction \(f|C\) is continuous.

Let \(Z = \oplus C = \{(x, C) : x \in X \times C \subset X \times C\}\) be the topological sum of the family \(C\), and \(\pi : Z \to X\), \(\pi : (x, C) \mapsto x\), be the natural projection of \(Z\) onto \(X\). Since the cover \(C\) is disjoint, the map \(\pi : Z \to X\) is bijective and hence induces a topological isomorphism \(\pi^* : \mathbb{R}^X \to \mathbb{R}^{\mathbb{R}}\), \(\pi^* : f \mapsto f \circ \pi\). The choice of the cover \(C\) guarantees that \(\pi^*(F) \subset C_p(Z)\). By (the proof of) Theorem 1.1.3 of [2], \(\text{nw}(C_p(Z)) \leq \text{nw}(Z)\) and hence

\[
\text{nw}(F) = \text{nw}(\pi^*(F)) \leq \text{nw}(C_p(Z)) \leq \text{nw}(Z) \leq |C| \cdot \text{nw}(X) = \text{hl}(X) \cdot \text{nw}(X) = \text{nw}(X).
\]

If the space \(X\) is Tychonoff, then the “closed unit ball”

\[
B = \{f \in C_p(X) : \sup_{x \in X} |f(x)| \leq 1\} \subset C_p(X)
\]

is \(\sigma\)-convex and has network weight \(\text{nw}(B) = \text{nw}(X)\) according to Theorem 1.1.3 of [2]. So,

\[
\text{nw}(X) = \max\{\text{nw}(F) : F \text{ is a } \sigma\text{-convex subset of } SC_p(X)\}.
\]

\[\square\]
In the same way we can derive some bounds on the weight of compact convex subsets in function spaces $SC_p(X)$.

**Corollary 2.** For any topological space $X$ of countable tightness, each compact convex subset $K \subset SC_p(X)$ has weight $w(K) \leq \max\{hl(X), hd(X)\}$. Moreover,

$$hl(X) \leq \sup\{w(K) : K \text{ is a compact convex subset of } SC_p(X)\} \leq \max\{hl(X), hd(X)\}. $$

**Proof.** Given a compact convex subset $K \subset SC_p(X)$, use Theorem 1 to find a disjoint cover $\mathcal{C}$ of $X$ of cardinality $|\mathcal{C}| = \text{dec}(K) \leq hl(X)$ such that for each $C \in \mathcal{C}$ and $f \in K$ the restriction $f|C$ is continuous. Let $Z = \oplus \mathcal{C}$ and $\pi : \oplus \mathcal{C} \to X$ be the natural projection, which induces a linear topological isomorphism $\pi^* : \mathbb{R}^X \to \mathbb{R}^Z$, $\pi^* : f \mapsto f \circ \pi$, with $\pi^*(K) \subset C_p(Z)$. It follows that the topological sum $Z = \oplus \mathcal{C}$ has density $d(Z) \leq \sum_{C \in \mathcal{C}} d(C) \leq |\mathcal{C}| \cdot hd(X) \leq \max\{hl(X), hd(X)\}$, and so we can fix a dense subset $D \subset Z$ of cardinality $|D| = d(Z) \leq \max\{hl(X), hd(X)\}$. Since the restriction operator $R : C_p(Z) \to C_p(D)$, $R : f \mapsto f|D$, is injective and continuous, we conclude that

$$w(K) = w(\pi^*(K)) = w(R \circ \pi^*(K)) \leq w(R^D) = |D| \cdot \aleph_0 \leq \max\{hl(X), hd(X)\}. $$

Next, we show that $hl(X) \leq \tau$ where

$$\tau = \sup\{w(K) : K \text{ is a compact convex subset of } SC_p(X)\}. $$

Assuming conversely that $hl(X) > \tau$ and using the equality $hl(X) = \sup\{|Z| : Z \subset X \text{ is scattered}\}$ established in [12], we can find a scattered subspace $Z \subset X$ of cardinality $|Z| > \tau$. It is easy to check that each function $f : X \to [0, 1]$ with $f(X \setminus Z) \subset \{0\}$ is scatteredly continuous, which implies that the subset

$$K_Z = \{f \in C_p(X) : f(Z) \subset [0, 1], f(X \setminus Z) \subset \{0\}\} $$

is compact, convex and homeomorphic to the Tychonoff cube $[0, 1]^Z$. Then $\tau \geq w(K_Z) = \sup\{|Z| \} = |Z| > \tau$ and this is a desired contradiction that completes the proof. \hfill $\Box$

**Corollary 3.** For a metrizable separable space $X$, each compact convex subspace $K \subset SC_p(X)$ is metrizable.

Finally, let us observe that Corollary 1 implies:

**Corollary 4.** If for Tychonoff spaces $X, Y$ with countable tightness the linear topological spaces $SC_p(X)$ and $SC_p(Y)$ are topologically isomorphic, then $nw(X) = nw(Y)$.

1. **Weakly discontinuous families of functions**

In this section we shall generalize the notions of scattered continuity and weak discontinuity to function families.

A family of functions $F \subset Y^X$ from a topological space $X$ to a topological space $Y$ is called

- scatteredly continuous if each non-empty subset $A \subset X$ contains a point $a \in A$ at which each function $f|A : A \to Y$, $f \in F$ is continuous;

- weakly discontinuous if each subset $A \subset X$ contains an open dense subspace $U \subset A$ such that each function $f|U : U \to Y$, $f \in F$ is continuous.

The following simple characterization can be derived from the corresponding definitions and Theorem 4.4 of [1] (saying that each scatteredly continuous function with values in a regular topological space is weakly discontinuous).

**Proposition 3.** A function family $F \subset Y^X$ is scatteredly continuous (resp. weakly discontinuous) if and only if so is the function $\Delta F : X \to Y^F$, $\Delta F : x \mapsto (f(x))_{f \in F}$. Consequently, for a regular topological space $Y$, a function family $F \subset Y^X$ is scatteredly continuous if and only if it is weakly discontinuous.

Propositions 4.7 and 4.8 [4] imply that each weakly discontinuous function $f : X \to Y$ has decomposition number $\text{dec}(f) \leq hl(X)$. This fact combined with Proposition 3 yields:
Corollary 5. For any topological spaces $X,Y$, each weakly discontinuous function family $F \subset Y^X$ has decomposition number $\text{dec}(F) \leq h\ell(X)$.

2. Weak discontinuity of $\sigma$-convex sets in function spaces

For a topological space $X$ by $SC^*_p(X)$ we denote the space of all bounded scatteredly continuous real-valued functions on $X$. It is a subspace of the function space $SC_p(X) \subset \mathbb{R}^X$. Each function $f \in SC^*_p(X)$ has finite norm $\|f\| = \sup_{x \in X} |f(x)|$.

Theorem 2. For any topological space $X$ with countable tightness, each $\sigma$-convex subset $F \subset SC^*_p(X)$ is weakly discontinuous.

Proof. By Proposition 3 the weak discontinuity of the function family $F$ is equivalent to the scattered continuity of the function $\Delta F : X \to \mathbb{R}^F$, $\Delta F : x \mapsto (f(x))_{f \in F}$. Since the space $X$ has countable tightness, the scattered continuity of $\Delta F$ will follow from Proposition 2.3 of [3] as soon as we check that for each countable subset $Q = \{x_n\}_{n=1}^\infty \subset X$ the restriction $\Delta F|Q : Q \to \mathbb{R}^F$ has a continuity point. Assuming the converse, for each point $x_n \in Q$ we can choose a function $f_n \in F$ such that the restriction $f_n|Q$ is discontinuous at $x_n$.

Observe that a function $f : Q \to \mathbb{R}$ is discontinuous at a point $q \in Q$ if and only if it has strictly positive oscillation

$$\text{osc}_q(f) = \inf \sup \{|f(x) - f(y)| : x, y \in O_q\}$$

at the point $q$. In this definition the infimum is taken over all neighborhoods $O_q$ of $q$ in $Q$.

We shall inductively construct a sequence $(t_n)_{n=1}^\infty$ of positive real numbers such that for every $n \in \mathbb{N}$ the following conditions are satisfied:

1) $t_1 \leq \frac{1}{2}$, $t_{n+1} \leq \frac{1}{2} t_n$, and $t_{n+1} \cdot \|f_{n+1}\| \leq \frac{1}{2} t_n \cdot \|f_n\|$,  

2) the function $s_n = \sum_{k=1}^n t_k f_k$ restricted to $Q$ is discontinuous at $x_n$,  

3) $t_{n+1} \cdot \|f_{n+1}\| \leq \frac{1}{4} \text{osc}_{x_n}(s_n|Q)$.

We start the inductive construction letting $t_1 = 1/2$. Then the function $s_1|Q = t_1 \cdot f_1|Q$ is discontinuous at $x_1$ by the choice of the function $f_1$. Now assume that for some $n \in \mathbb{N}$ positive numbers $t_1, \ldots, t_n$ has been chosen so that the function $s_n = \sum_{k=1}^n t_k f_k$ restricted to $Q$ is discontinuous at $x_n$.

Choose any positive number $\tilde{t}_{n+1}$ such that

$$\tilde{t}_{n+1} \leq \frac{1}{2} t_n \cdot \tilde{t}_{n+1} \cdot \|f_{n+1}\| \leq \frac{1}{2} t_n \cdot \|f_n\| \quad \text{and} \quad \tilde{t}_{n+1} \cdot \|f_{n+1}\| \leq \frac{1}{4} \text{osc}_{x_n}(s_n|Q),$$

and consider the function $\tilde{s}_{n+1} = s_n + \tilde{t}_{n+1} f_{n+1}$. If the restriction of this function to $Q$ is discontinuous at the point $x_{n+1}$, then put $t_{n+1} = \tilde{t}_{n+1}$ and finish the inductive step. If $\tilde{s}_{n+1}|Q$ is continuous at $x_{n+1}$, then put $t_{n+1} = \frac{1}{2} \tilde{t}_{n+1}$ and observe that the restriction of the function

$$s_{n+1} = \sum_{k=1}^{n+1} t_k f_k = s_n + \frac{1}{2} \tilde{t}_{n+1} f_{n+1} = \tilde{s}_{n+1} - \frac{1}{4} \tilde{t}_{n+1} f_{n+1}$$

to $Q$ is discontinuous at $x_{n+1}$. This completes the inductive construction.

The condition (1) guarantees that $\sum_{n=1}^\infty t_n \leq 1$ and hence the number $t_0 = 1 - \sum_{n=1}^\infty t_n$ is non-negative. Now take any function $f_0 \in F$ and consider the function

$$s = \sum_{n=0}^\infty t_n f_n$$

which is well-defined and belongs to $F$ by the $\sigma$-convexity of $F$. 


The functions $f_0, s \in F \subset SC_p(X)$ are weakly discontinuous and hence for some open dense subset $U \subset Q$ the restrictions $s|U$ and $f_0|U$ are continuous. Pick any point $x_n \in U$. Observe that

$$s = t_0f_0 + s_n + \sum_{k=n+1}^{\infty} t_kf_k$$

and hence

$$s_n = s - t_0f_0 - \sum_{k=n+1}^{\infty} t_kf_k = s - t_0f_0 - u_n,$$

where $u_n = \sum_{k=n+1}^{\infty} t_kf_k$. The conditions (1) and (3) of the inductive construction guarantee that the function $u_n$ has norm

$$\|u_n\| \leq \sum_{k=n+1}^{\infty} t_k\|f_k\| \leq 2t_{n+1}\|f_{n+1}\| \leq \frac{1}{4}\text{osc}_{x_n}(s_n|Q).$$

Since $s_n = s - t_0f_0 - u_n$, the triangle inequality implies that

$$0 < \text{osc}_{x_n}(s_n|Q) \leq \text{osc}_{x_n}(s|Q) + \text{osc}_{x_n}(t_0f_0|Q) + \text{osc}_{x_n}(u_n) \leq 0 + 0 + 2\|u_n\| \leq \frac{1}{2}\text{osc}_{x_n}(s_n|Q)$$

which is a desired contradiction, which shows that the restriction $\Delta F|Q$ has a point of continuity and the family $F$ is weakly discontinuous. \qed

3. Proof of Theorem 1

Let $X$ be a topological space with countable tightness and $F$ be a $\sigma$-convex subset in the function space $SC_p(X)$. The $\sigma$-convexity of $F$ implies that for each point $x \in X$ the subset $\{f(x) : f \in F\} \subset \mathbb{R}$ is bounded (in the opposite case we could find sequences $(f_n)_{n \in \omega} \in F^\omega$ and $(t_n)_{n \in \omega} \in [0,1]^\omega$ with $\sum_{n=0}^{\infty} t_n = 1$ such that the series $\sum_{n=1}^{\infty} t_nf_n(x)$ is divergent). Then $X = \bigcup_{n=1}^{\infty} X_n$ where $X_n = \{x \in X : n \leq \sup_{f \in F} |f(x)| < n+1\}$ for $n \in \omega$.

It follows that for every $n \in \omega$ the family $F|X_n = \{f|X_n : f \in F\}$ is a $\sigma$-convex subset of the function space $SC^*_p(X_n)$. By Theorem 2 the function family $F|X_n$ is weakly discontinuous and by Corollary 5 $\text{dec}(F|X_n) \leq \text{hl}(X_n)$. Then $\text{dec}(F) \leq \sum_{n=0}^{\infty} \text{dec}(F|X_n) \leq \sum_{n=0}^{\infty} \text{hl}(X_n) \leq \text{hl}(X)$.

4. Some Open Problems

The presence of the condition of countable tightness in Theorem 1 and its corollaries suggests the following open problem.

**Problem 1.** Is it true $w(K) \leq nw(X)$ for each topological space $X$ and each compact convex subset $K \subset SC_p(X)$?

By Theorem 2, for each topological space $X$ of countable tightness, each compact convex subset $K \subset SC^*_p(X)$ is weakly discontinuous.

**Problem 2.** For which topological spaces $X$ each compact convex subset $K \subset SC_p(X)$ is weakly discontinuous?

According to Corollary 5 each compact convex subset $K \subset SC_p(\omega^\omega)$ is metrizable.

**Problem 3.** Is a compact subset $K \subset SC_p(\omega^\omega)$ metrizable if $K$ is homeomorphic to a compact convex subset of $\mathbb{R}^\omega$?

Let us recall that a topological space $K$ is *Rosenthal compact* if $K$ is homeomorphic to a compact subspace of the space $B_1(X) \subset \mathbb{R}^X$ of functions of the first Baire class on a Polish space $X$. In this definition the space $X$ can be assumed to be equal to the space $\omega^\omega$ of irrationals.
Problem 4. Is each Rosenthal compact space homeomorphic to a compact subset of the function space $SC_p(\omega^n)$?

This problem has affirmative solution in the realm of zero-dimensional separable Rosenthal compacta.

Theorem 3. Each zero-dimensional separable Rosenthal compact space $K$ is homeomorphic to a compact subset of the function space $SC_p(\omega^n)$.

Proof. Let $D \subset K$ be a countable dense subset in $K$. Let $A = C_D(K, 2)$ be the space of continuous functions $f : K \to 2 = \{0, 1\}$ endowed with the smallest topology making the restriction operator $R : C_D(K, 2) \to 2^D$, $R : f \mapsto f|D$, continuous. By the characterization of separable Rosenthal compacta [11], the space $A$ is analytic, i.e., $A$ is the image of the Polish space $X = \omega^n$ under a continuous map $\pi : X \to A$. Now consider the map $\delta : K \to 2^A$, $\delta : x \mapsto (f(x))_{f \in A}$. This map is continuous and injective by the zero-dimensionality of $K$. The map $\pi : X \to A$ induces a homeomorphism $\pi^* : 2^A \to 2^X$, $\pi^* : f \mapsto f \circ \pi$. Then $\pi^* \circ \delta : K \to 2^X$ is a topological embedding.

We claim that $\pi^* \circ \delta(\pi(X)) \subset SC_p(\omega^n)$ and $\delta(\pi(X))$ is scatteredly continuous. It will be convenient to denote the function $\delta(x) \in 2^A$ by $\delta_x$. This function assigns to each $f \in A = C_p(K)$ the number $\delta_x(f) = f(x)$ for each $x \in K$.

By [13, 9], the Rosenthal compact space $K$ is Fréchet-Urysohn, so there is a sequence $(x_n)_{n \in \omega} \in D^\omega$ with $\lim_{n \to \infty} x_n = x$. Then the function $\delta_x : A \to 2$, $\delta_x : f \mapsto f(x)$, is the pointwise limit of the continuous functions $\delta_{x_n}$, which implies that $\delta_x$ is a function of the first Baire class on $A$ and $\delta_x \circ \pi : X \to 2$ is a function of the first Baire class on the Polish space $X$. Since this function has discrete range, it is scatteredly continuous by Theorem 8.1 of [4]. Consequently, $\pi^* \circ \delta(x) \in SC_p(X)$ and $K$ is homeomorphic to the compact subset $\pi^* \circ \delta(K) \subset SC_p(X)$.

A particularly interesting instance of Problem 4 concerns non-metrizable convex Rosenthal compacta. One of the simples spaces of this sort is the Helly space. We recall that the Helly space is the subspace of $B_1(1)$ consisting of all non-decreasing functions $f : I \to I$ of the unit interval $I = [0, 1]$.

Problem 5. Is the Helly space homeomorphic to a compact subset of the function space $SC_p(\omega^n)$?

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