A Theory of Non-Abelian Tensor Gauge Field with Non-Abelian Gauge Symmetry $G \times G$

Chong-Sun Chu

Centre for Particle Theory and Department of Mathematical Sciences, Durham University, Durham, DH1 3LE, UK
E-mail: chong-sun.chu@durham.ac.uk

Abstract: The Chern-Simon action of the ABJM theory is not gauge invariant in the presence of a boundary. In the paper [1], this was shown to imply the existence of a Kac-Moody current algebra on the theory of multiple self-dual strings. In this paper we conjecture that the Kac-Moody symmetry induces a $U(N) \times U(N)$ gauge symmetry in the theory of $N$ coincident M5-branes. As a start, we construct a $G \times G$ gauge symmetry algebra structure which naturally includes the tensor gauge transformation for a non-abelian 2-form tensor gauge field. The gauge covariant field strength is constructed. This new $G \times G$ gauge symmetry algebra allows us to write down a theory of a non-abelian tensor gauge field in any dimensions. The $G \times G$ gauge bosons can be either propagating, in which case the 2-form gauge fields would interact with each other through the 1-form gauge field; or they can be auxiliary and carry no local degrees of freedom, in which case the 2-form gauge fields would be self-interacting nontrivially. We finally comment on the possible application to the system of multiple M5-branes. We note that the field content of the $G \times G$ non-abelian tensor gauge theory can be fitted nicely into $(1,0)$ supermultiplets; and we suggest a construction of the theory of multiple M5-branes with manifest $(1,0)$ supersymmetry.

Keywords: M-Theory, D-branes, M-branes, Gauge Symmetry.
1. Introduction

The low energy theory of $N$ coincident M5-branes is given by an interacting (2,0) superconformal theory in 6 dimensions. So far very little is known about this theory. As a first step, one would like to understand what kind of gauge symmetry structure underlies the worldvolume theory of multiple M5-branes. This is the primary goal of this paper.

Great progress has been made in the last couple of years for the case of multiple M2-branes. First a new class of (2+1)-dimensional superconformal field theories with maximal $\mathcal{N} = 8$ supersymmetry was constructed by Bagger and Lambert [2–4], and by Gustavsson [5]. The construction makes use of a new mathematical object called a Lie 3-algebra which is a generalization of the Lie algebra. However the application to describe multiple M2-branes has been hindered by a major difficulty that so far there is only one example of a Lie 3-algebra that could produce a well defined unitary quantum theory. Another proposal due to Aharony, Bergman, Jafferis and Maldacena [6] proposed a certain $\mathcal{N} = 6$ superconformal Chern-Simons-matter theory as the low energy theory of multiple M2-branes. In this construction, an ordinary $U(N) \times U(N)$ Lie algebra is used and the rank $N$ is arbitrary. It has been argued that the inclusion of a nonperturbative monopole sector enhances the supersymmetry to $\mathcal{N} = 8$ [7–10].
Much less is known for the theory of multiple M5-branes. A possible approach to this problem is to consider the M2-branes ending on the M5-brane(s) and to make use of the recently obtained knowledge of multiple M2-branes to learn about the physics of M5-brane(s) from the boundary dynamics of the M2-branes. This approach has been applied in [11] and [1]. In [11], the open BLG theory is considered and a novel kind of quantized geometry for M5-brane in a constant $C$-field is predicted. In [1], a system of open $N$ M2-branes described by the open ABJM theory is considered. Due to the gauge non-invariance of the Chern-Simon actions in the presence of a boundary, additional degrees of freedom must reside at the boundary of the M2-branes, which one could interpret as the worldsheet of a system of multiple self-dual strings. These degrees of freedom is govern by a WZW action on the group manifold $G = U(N) \times U(N)$ and admits a Kac-Moody $G_L \times G_R$ current algebra [12]. The existence of a Kac-Moody current algebra is interesting and naturally one wonders what it implies for the physics of the M5-branes, the spacetime of the self-dual strings. This form the motivation and the starting point of the analysis of this paper.

In the literature, there has been various attempts in constructing a non-abelian theory for the 2-form potential $B$. One class of attempts which also involve the use of 1-form gauge fields is to use a mathematical structure called non-abelian gerbes [13]. Our construction is different as some of the mathematical properties required in the non-abelian gerbes are not imposed in our construction. These properties (for example, the vanishing of the fake curvature as required in non-abelian gerbes in order to have a well defined parallel transport) are often well motivated mathematically, but their necessity are much less clear physically. As a result, the gauge transformations of the 2-form potential $B$ are different, for example. Another class of attempt is to use a lattice definition of the tensor gauge connection. Interestingly, this line of proposal also automatically contains a $G \times G'$ gauge structure [14]. For other recent works on the construction of the non-abelian (2,0) theory see [15–18].

In this paper we propose that the Kac-Moody currents generate a $G \times G$ ($G = U(N)$) gauge symmetry on the theory of $N$ coincident M5-branes. We also propose to identify this gauge symmetry with the non-abelian tensor gauge symmetry on multiple M5-branes. An immediate question is how could a tensor gauge symmetry get generated from Yang-Mills gauge symmetry? We find that if the $G \times G$ gauge symmetry is not of the usual form but admits a kind of “cross” structure, then a tensor gauge transformation is automatically included. This gauge symmetry structure allows us to write down immediately a theory of non-abelian tensor gauge fields in any spacetime dimensions. It is this $G \times G$ non-abelian tensor gauge symmetry algebra that we conjecture to be the symmetry of the low energy worldvolume theory of multiple M5-branes.

Depending on the physical needs, the $G \times G$ gauge bosons can be constructed to be either propagating or non-propagating. In the first possibility, the gauge bosons may
obey, for example, a standard Yang-Mills term. In this case the 2-form gauge fields would interact with each other through the 1-form gauge field. This is similar to the interaction of fermions with gauge field in a minimally coupled theory. In the second possibility, the gauge fields carry no local degrees of freedom and are determined entirely in terms of the 2-form potentials and other fields of the theory, and the 2-form gauge fields would be self-interacting nontrivially. This second possibility is particularly interesting for the construction of the theory of multiple M5-branes as there is no room for a propagating gauge field in the worldvolume supersymmetric multiplet of M5-branes.

The plan and results of the paper are explained as follows. In section 2.1 we argue that the Kac-Moody currents generate a $G \times G$ ($G = U(N)$) gauge symmetry structure on the system of $N$ coincident M5-branes. In section 2.2, we introduce a set of $G \times G$ gauge bosons that is characterized by a new set of gauge transformation laws that are different from the standard direct-product structure of gauge groups. In fact the algebra of the gauge transformations does not close by itself and it is necessary to include the tensor gauge transformation. Thus tensor gauge transformation is naturally and automatically included. A gauge covariant and tensor gauge invariant non-abelian 3-form field strength is constructed. We also discuss the possible physical natures of the gauge fields. We also explain in what sense the full $G \times G$ gauge symmetry is essential in the construction of the non-abelian tensor gauge transformation. In section 3, we discuss couplings to matter fields and show how to construct an invariant action in general dimensions. We also show how our non-abelian tensor gauge symmetry can be used to construct a dual description of the 5-dimensional Yang-Mills gauge theory of 1-form gauge potential $A_\mu$ in terms of a non-abelian 2-form potential $B_{\mu\nu}$. The paper is concluded with some further discussions. In particular, we briefly comment on the application of our formalism of tensor gauge symmetry to the construction of the self-dual theory of multiple M5-branes. We also note that the field content of the $G \times G$ non-abelian tensor gauge theory can be fitted nicely into (1,0) supermultiplets and we suggest that it may be more feasible to write down the non-abelian (2,0) tensor theory of multiple M5-branes in terms of (1,0) supermultiplets. Progress in these directions will be reported elsewhere.

Note added: During the preparation of this manuscript, the preprint [36] appeared which overlaps with some of the ideas of this paper. For example, both papers draw on the similarity with the construction of ABJM and suggests to construct the multiple M5 theory using the (1,0) supermultiplets. Also, both papers make use of the ordinary

\[ \text{Note added:} \]
Lie algebra in describing the symmetry. However, the details of the constructions are different. For example, our construction is based on a special $G \times G$ ($G = U(N)$) gauge symmetry algebra that is not in [36] and the rank $N$ is allowed to be arbitrary.

**Note added in v3:** Recently, the authors of [36] (version 2) have checked that by switching off the 3-form gauge potential in the tensor hierarchy they proposed, the $G \times G$ symmetry proposed in this paper provides a non-trivial solution to their construction. They also obtained a set of (1,0) superconformal equations of motion from their general construction and found that these equations cannot be obtained from an action. Nevertheless it may still be possible to construct a supersymmetric action which is non-Lorentz covariant or Lorentz invariant if one allows for new auxiliary field of PST type in the action. These possibilities deserve further investigation [19, 20].

### 2. $G \times G$ Gauge Symmetry of Non-Abelian Tensor Gauge Field

#### 2.1 $U(N) \times U(N)$ gauge symmetry on M5-branes

Consider a system of $N$ open M2-branes ending on $N$ coincident M5-branes. This can be modelled with the $U(N) \times U(N) \equiv G$ ABJM theory with boundary, together with a certain coupling to the non-abelian $B$-field living on the M5-branes. The explicit form of this coupling is unknown, but the details are not necessary for our argument. It was shown in [1] that the gauge non-invariance of the boundary Chern-Simons couplings in the ABJM theory implies the existence of a $U(N) \times U(N)$ WZW action for the multiple self-dual string theory. In turn, this induces a $G_L \times G_R$ Kac-Moody symmetry on the worldsheet theory of $N$ self-dual strings. Here L/R signifies the fact the Kac-Moody symmetry is generated by the left/right chiral sector of the theory. We emphasis that the existence of this Kac-Moody symmetry is robust and is independent of supersymmetry or the details of the other part of the complete theory of the self-dual strings.

The existence of a Kac-Moody symmetry is intriguing. In the familiar case of the heterotic string, the existence of a group $G$ Kac-Moody symmetry in the left sector allows one to construct vertex operators which creates a Yang-Mills gauge symmetry $G$ in the spacetime. Now the spacetime of the self-dual strings is the worldvolume of the M5-branes. Although we don’t have a vertex operator, it is tempting to speculate that the Kac-Moody symmetry will similarly create a set of gauge bosons in the spacetime. However since we do not have the vertex operators, it is not clear whether a single left (or right) handed Kac-Moody current is enough to create a spacetime gauge bosons, or whether the left and right handed Kac-Moody currents must be taken together to create the gauge bosons. This corresponds to having a gauge symmetry of $G \times G$ or $G$ on the system of $N$ coincident M5-branes. As we will see in section 2.2, the gauge symmetry structure (equation (2.1)) that is needed for the construction of the non-abelian tensor...
gauge symmetry is different from the standard direct-product structure and suggests that the mechanism for creating the gauge bosons from the Kac-Moody current is different from the standard case. In any case, the correspondence between worldsheet global symmetry and spacetime gauge symmetry should be a rather general statement. All in all, we are motivated to conjecture that the (2,0) theory of a system of \( N \) coincident M5-branes is described by a \( U(N) \times U(N) \) tensor gauge symmetry algebra.

Below we will give an explicit construction for a theory of non-abelian tensor gauge fields based on a kind of \( G \times G \) gauge symmetry. The construction only works with this \( G \times G \) gauge symmetry structure and this is in support of the our conjecture that a \( G \times G \) gauge symmetry is relevant for the description of multiple M5-branes.

### 2.2 \( G \times G \) tensor gauge symmetry for non-abelian tensor gauge field

Gauge and tensor gauge transformations

Consider a gauge group \( G \times G' \) where \( G' = G \). Here \( G \) is general and does not need to be \( U(N) \). For notational convenience we denote the second gauge group and the associated quantities with a prime. The spacetime dimension \( D \) does not needed to be restricted to six.

Let \( T^a \) be the generators of the Lie algebra \( \mathfrak{g} \) of \( G \), \( a = 1, 2, \cdots, \text{dim} \mathfrak{g} \). In addition to the gauge fields \( A^a_\mu, A'^a_\mu \), we will include a 2-form tensor gauge field \( B^a_\mu \) in the adjoint representation of \( G \). Coupling to scalar fields and fermions is easy and will be considered in the next section. For now, we will concentrate on these fields.

Let us start with specifying the gauge transformations. We will take the gauge fields to transform under \( G \times G' \) as

\[
G: \quad \delta \Lambda A_\mu = \partial_\mu \Lambda + [A_\mu, \Lambda], \quad \delta \Lambda A'_\mu = [A'_\mu, \Lambda], \\
G': \quad \delta' \Lambda A'_\mu = [A'_\mu, \Lambda'], \quad \delta' \Lambda A_\mu = \partial_\mu \Lambda' + [A'_\mu, \Lambda'],
\]

(2.1)

where \( A_\mu = A^a_\mu T^a, \Lambda = \Lambda^a T^a, A'_\mu = A'^a_\mu T^a, \Lambda' = \Lambda'^a T^a \) are the Lie-algebra valued gauge fields and gauge parameters. Note that we have taken the gauge field \( A \) (resp. \( A' \)) of the gauge group \( G \) (resp. \( G' \)) to transform non-trivially in the adjoint representation of the other gauge group \( G' \) (resp. \( G \)). This is different from what one usually has in a standard Yang-Mills theory. As we will explain below, that this is a consistent choice is entirely due to the presence of a tensor gauge symmetry in the theory.

For the 2-form gauge fields, we will take their gauge transformation as

\[
\delta \Lambda B_\mu = [B_\mu, \Lambda] + \frac{1}{2} \left( [A'_\mu, \partial_\nu \Lambda] - [A'_\nu, \partial_\mu \Lambda] \right), \\
\delta' \Lambda B_\mu = [B_\mu, \Lambda'] - \frac{1}{2} \left( [A_\mu, \partial_\nu \Lambda'] - [A_\nu, \partial_\mu \Lambda'] \right).
\]

(2.2)
It is convenient to introduce the field

\[ B_{\mu\nu} := B_{\mu\nu} - \frac{1}{2}(F_{\mu\nu} - F'_{\mu\nu}), \quad (2.3) \]

where \( F = dA + A^2 \) and \( F' = dA' + A'^2 \) are the ordinary gauge field strengths. The field \( B \) transforms covariantly under \( G \times G' \):

\[ \delta_\Lambda B_{\mu\nu} = [B_{\mu\nu}, \Lambda], \quad (2.4) \]
\[ \delta'_\Lambda B_{\mu\nu} = [B_{\mu\nu}, \Lambda']. \quad (2.5) \]

In addition to the Yang-Mill gauge symmetry, there should also be a tensor gauge symmetry. In the case of a single tensor field, the tensor gauge transformation takes the form

\[ \delta_\Lambda B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu. \quad (2.6) \]

The question is how this should be generalized for the non-abelian theory. We propose the following tensor gauge transformations

\[ \delta_\Lambda B_{\mu\nu} = \frac{1}{2} \left[ (D_{\mu} + D'_{\mu}) \Lambda_\nu - (D_{\nu} + D'_{\nu}) \Lambda_\mu \right] \]
\[ = \left[ \partial_\mu + \frac{1}{2}(A_\mu + A'_\mu), \Lambda_\nu \right] - (\mu \leftrightarrow \nu), \quad (2.7) \]
\[ \delta_\Lambda A_{\mu} = \Lambda_\mu, \quad (2.8) \]
\[ \delta_\Lambda A'_{\mu} = -\Lambda_\mu. \quad (2.9) \]

This implies that \( B \) is tensor gauge invariant:

\[ \delta_\Lambda B_{\mu\nu} = 0. \quad (2.10) \]

In the free field limit where the commutator terms vanishes, the tensor gauge transformation \( (2.7) \) decouples from the gauge fields \( A, A' \) and reduces back to the \( (2.6) \).

The transformation properties of the field \( B_{\mu\nu} \) makes itself a convenient ingredient for the construction of the covariant field strength. The field strength can be defined as

\[ H_{\mu\nu\lambda} \equiv [D_{\mu} , B_{\nu\lambda}] + (\mu\nu\lambda \text{ cyclic}). \quad (2.11) \]

where \( D_{\mu} = \partial_{\mu} + [A_{\mu}, \cdot] \) and \( A_{\mu} := A_{\mu} + A'_\mu \). \( H \) has the transformation properties

\[ \delta_\Lambda H_{\mu\nu\lambda} = [H_{\mu\nu\lambda}, \Lambda], \quad (2.12) \]
\[ \delta'_\Lambda H_{\mu\nu\lambda} = [H_{\mu\nu\lambda}, \Lambda'], \quad (2.13) \]
\[ \delta_\Lambda A_{\mu} = 0. \quad (2.14) \]

and satisfies the modified Bianchi identity

\[ D_{[\mu} H_{\nu\lambda]\rho]} = \frac{3}{2} [F_{\mu\nu}, B_{\lambda\rho}], \quad (2.15) \]
where $\mathcal{F} := dA + A^2$.

The above defined gauge transformations and tensor gauge transformations are consistent as they form a closed algebra. In fact, by acting on $B_{\mu\nu}, A_\mu$ or $A'_\mu$, it is easy to derive the following algebra of gauge and tensor gauge transformations:

$$\left[\delta^{(1)}_{\Lambda_\mu}, \delta^{(2)}_{\Lambda_\nu}\right] = 0,$$  \hspace{1cm} (2.16)

$$\left[\delta_{\Lambda_1}, \delta_{\Lambda_2}\right] = \delta_{[\Lambda_1, \Lambda_2]},$$  \hspace{1cm} (2.17)

$$\left[\delta^{\prime(1)}_{\Lambda'_1}, \delta^{\prime(2)}_{\Lambda'_2}\right] = \delta^{\prime}_{[\Lambda'_1, \Lambda'_2]},$$  \hspace{1cm} (2.18)

$$\left[\delta_{\Lambda}, \delta_{\Lambda'}\right] = \delta_{\Lambda_\mu} + \delta_{\Lambda'} - \delta'_{\Lambda},$$  \hspace{1cm} (2.19)

$$\left[\delta_{\Lambda}, \delta_{\Lambda_\alpha}\right] = -\delta_{\Lambda_\alpha},$$  \hspace{1cm} (2.20)

$$\left[\delta_{\Lambda'}, \delta_{\Lambda_\alpha}\right] = -\delta'_{\Lambda_\alpha}.$$  \hspace{1cm} (2.21)

Here in (2.19), the parameter for the tensor gauge transformation on the right hand side is given by

$$\Lambda_\mu \equiv \frac{1}{2} \left( [\partial_\mu \Lambda, \Lambda'] - [\Lambda, \partial_\mu \Lambda'] \right),$$  \hspace{1cm} (2.22)

and the parameters $\tilde{\Lambda}, \tilde{\Lambda}'$ for the $G$ or $G'$ gauge transformations on the right hand side are

$$\tilde{\Lambda} \equiv \frac{1}{2} [\Lambda, \Lambda'], \quad \tilde{\Lambda}' \equiv \frac{1}{2} [\Lambda', \Lambda];$$  \hspace{1cm} (2.23)

while in (2.20) and (2.21), the parameter for the tensor gauge transformation on the right hand side are given by

$$\tilde{\Lambda}_\alpha \equiv [\Lambda_\alpha, \Lambda], \quad \tilde{\Lambda}'_\alpha \equiv [\Lambda'_\alpha, \Lambda].$$  \hspace{1cm} (2.24)

We note from (2.19) that the commutator of a $G$-transformation and a $G'$-transformation results in a tensor gauge transformation. This explains why our proposed $G \times G$ gauge transformations (2.1) has not been considered before in an ordinary Yang-Mills gauge theory since a tensor gauge symmetry is absent. We will refer to the algebra (2.17)-(2.21) of the gauge and tensor gauge transformations as the $G \times G$ tensor gauge symmetry structure for our non-abelian tensor gauge theory.

For completeness, we remark that the above gauge symmetry algebra can also be written using a different basis in terms of the diagonal gauge field and the anti-diagonal gauge field

$$A_\mu := A_\mu + A'_\mu, \quad C_\mu := A_\mu - A'_\mu,$$  \hspace{1cm} (2.25)

and the diagonal and anti-diagonal gauge transformations

$$\delta^{(d)}_\Lambda := \delta_{\Lambda/2} + \delta'_{\Lambda/2}, \quad \delta^{(ad)}_\Lambda := \delta_{\Lambda/2} - \delta'_{\Lambda/2}.$$  \hspace{1cm} (2.26)
In terms of these, the gauge transformation rules read
\begin{align}
\delta_{\Lambda}^{(d)} A_\mu &= \partial_\mu \Lambda + [A_\mu, \Lambda], & \delta_{\Lambda}^{(ad)} A_\mu &= 0, & \delta_{\Lambda, r} A_\mu &= 0, \\
\delta_{\Lambda}^{(d)} C_\mu &= [C_\mu, \Lambda], & \delta_{\Lambda}^{(ad)} C_\mu &= \partial_\mu \Lambda, & \delta_{\Lambda, r} C_\mu &= 2\Lambda_\mu, \\
\delta_{\Lambda}^{(d)} C_{\mu} &= [C_{\mu}, \Lambda], & \delta_{\Lambda}^{(ad)} C_{\mu} &= \partial_\mu \Lambda, & \delta_{\Lambda, r} C_{\mu} &= 2\Lambda_\mu, \\
\delta_{\Lambda}^{(d)} B_{\mu\nu} &= [B_{\mu\nu}, \Lambda] - \frac{1}{4} ([C_\mu, \partial_\nu \Lambda] - [C_\nu, \partial_\mu \Lambda]), & \delta_{\Lambda}^{(ad)} B_{\mu\nu} &= \frac{1}{4} ([A_\mu, \partial_\nu \Lambda] - [A_\nu, \partial_\mu \Lambda]), \\
\delta_{\Lambda, r} B_{\mu\nu} &= \partial_{(\mu} + \frac{1}{2} A_{\mu\nu}, \Lambda_{\nu)} - (\mu \leftrightarrow \nu).
\end{align}

Also it is
\begin{align}
B_{\mu\nu} &= B_{\mu\nu} - \frac{1}{2} \left( [\partial_\mu + \frac{1}{2} A_\mu, \Lambda_\nu] - (\mu \leftrightarrow \nu) \right).
\end{align}

Nature of the fields $C_\mu$ and $A_\mu$

So far our construction involves the fields $A_\mu$ and $C_\mu$ in addition to the 2-form potential $B_{\mu\nu}$. The gauge field $C_\mu$ is transformed by a shift under the tensor gauge transformation $\delta_{\Lambda}^{(d)}$ and so can be gauged away if one fixes the tensor gauge symmetry completely. Since part of the $B$-field can also be gauged away by using the tensor gauge symmetry, that means if one does not want to introduce extra pure gauge modes, one should identify the field $C_\mu$ with part of $B_{\mu\nu}$. This can be achieved in a gauge invariant way (with the gauge symmetries as well as the tensor gauge symmetry all intact) by imposing the constraint
\begin{align}
B_{\mu 5} &= 0, \quad \mu \neq 5,
\end{align}
where 5 is an arbitrary fixed spacelike direction $^2$ of the $D$-dimensional spacetime. Superficially the constraint $\text{(2.32)}$ breaks the Lorentz symmetry to $SO(D-2,1)$. But it is possible that the theory processes an additional modified Lorentz symmetry mixing the $5-\mu$ directions even if the theory is formulated with manifest $(D-1)$-dimensional Lorentz invariance. For example, this is the case in the Perry-Schwarz construction $[23,24]$ of the single M5-brane theory. A special feature of the PS construction is that it is based on a $5 \times 5$ tensor gauge fields with $B_{\mu\nu}, \mu = 0,1,2,3,4$. The components $B_{\mu 5}$ is completely missing in the formulation. This may appear “artificial” but is in fact extremely natural in the manifestly Lorentz covariant formulation of Pasti-Sorokin-Tonin (PST) $[25–27]$ where the field $B_{\mu\nu}$ is extended to $B_{MN}, M = 0,1,2,3,4,5$. In addition an auxiliary scalar field $a$ is introduced with new gauge symmetries that allow one to choose the gauge $B_{\mu 5} = 0$ and $a = x 5$. In this gauge, the Perry-Schwarz action is recovered. Recently, a

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$^2$One could equally take a timelike direction.
non-abelian generalization of the Perry-Schwarz action was constructed in [19]. In this construction, a Yang-Mills gauge symmetry \( G = U(N) \) is present. It is envisaged that the full \( G \times G \) formalism would be needed in the PST-like formulation of the theory and the condition (2.32) would then be a gauge fixing condition.

As for the nature of \( A_\mu \), there are two possibilities. The first possibility is for the gauge field \( A \) to be propagating. In this case, for a standard kinetic term \( \text{Tr} H^2 \), one see that the 2-form \( B \) field interacts with each other via the interaction through \( A \). This is similar to the familiar situation that with a standard kinetic term \( \bar{\psi} D \psi \) for fermions, \( \psi \) interacts with each other only via the gauge field. A more non-trivial possibility is for the gauge field \( A \) to be auxiliary and be determined in terms of the other fields of the theory. In this case there is direct nonlinear self interaction among the 2-form gauge field \( B \) even within a single \( H \); and our construction (2.2), (2.7) and (2.11) is a non-trivial generalization of the usual non-abelian gauge transformation \( \delta A = d \Lambda + [A, \Lambda] \) and the Yang-Mills field strength \( F = dA + A^2 \) which would be impossible to write down if only the field \( B \) was allowed to appear. An example of these kind of constraint is

\[
X^2 F_{\mu\nu} = H_{\mu\nu\lambda} D^\lambda X, \tag{2.33}
\]

where \( X \) is a scalar.

3. Dynamics

In this section, we discuss dynamics of the 2-form gauge field. We will also include couplings to matter fields such as scalar fields and fermions and construct actions that are invariant under the gauge and tensor gauge transformations. We consider the generic case where the construction is valid for general dimensions and self-duality for \( H_{\mu\nu\lambda} \) is not assumed. In the discussion section, we comment on the construction of the theory of multiple M5-branes using our formalism of \( G \times G \) tensor gauge symmetry algebra.

3.1 Matter couplings

The covariance and invariance (2.12) - (2.14) of the field strength \( H \) allow one to write down an invariant kinetic term \( \text{Tr} H^2_{\mu\nu\lambda} \) immediately. Next let us include the coupling to matter fields. Consider first fields that are neutral under tensor gauge transformation. A simple example is for a field \( f = (f^a) \) to transform covariantly under the gauge transformations as

\[
\delta^{(d)}_A f = [f, \Lambda], \quad \delta^{(ad)}_A f = 0 \tag{3.1}
\]

and is invariant under the tensor gauge transformation

\[
\delta_{\Lambda_\alpha} f = 0. \tag{3.2}
\]
For these kind of fields, it is easy to construct their covariant derivatives

\[ \mathcal{D}_\mu f = \partial_\mu + [A_\mu, f]. \quad (3.3) \]

Note that these kind of matter fields does not interact with the tensor gauge field minimally; non-minimal interaction is possible, see (3.16) below.

It is also possible to include matters that are charged under the tensor gauge transformation. For simplicity take \( G = U(N) \) and consider a \( N \times N \) Hermitian matrix of scalar field \( \varphi \) which transforms under a tensor gauge transformation as

\[ \varphi \to U\varphi, \quad (3.4) \]

where \( U = U(\Lambda_\mu) \) is some function of the tensor gauge parameter \( \Lambda_\mu \). Let us assume that \( U \) does not depend on the derivatives of \( \Lambda_\mu \) and has the form

\[ U = e^{\alpha_\mu \Lambda_\mu}, \quad (3.5) \]

for some matrix function \( \alpha_\mu \). Infinitesimally

\[ \delta_{\Lambda_\mu} \varphi = \alpha^\mu \Lambda_\mu \varphi. \quad (3.6) \]

It is convenient to introduce the field

\[ \tilde{\varphi} = (1 + \beta^\mu C_\mu)\varphi := C\varphi, \quad (3.7) \]

where \( \beta^\mu \in U(1) \) and is independent of \( C_\mu \). The neutral case is included with \( \beta = 0 \). \( \tilde{\varphi} \) is invariant under tensor gauge transformation if

\[ \beta^\mu + C\alpha^\mu = 0, \quad (3.8) \]

or

\[ \alpha^\mu = -C^{-1} \beta^\mu. \quad (3.9) \]

This is well defined generically and the transformation (3.6) is a field dependent one. As for the gauge transformations, we take them to be

\[ \delta^{(d)}_{\Lambda} \varphi = [\varphi, \Lambda], \quad \delta^{(ad)}_{\Lambda} \varphi = \alpha^\mu \partial_\mu \varphi. \quad (3.10) \]

It is easy to check that the transformations (3.6), (3.10) obey the algebra of transformations (2.16) - (2.21).

As a result, for a scalar field which transforms as (3.6) under the tensor gauge transformation and (3.10) under gauge transformations, the covariant derivative \( \mathcal{D}_\mu \tilde{\varphi} \) is either invariant or covariant:

\[ \delta_{\Lambda_\mu}(\mathcal{D}_\nu \tilde{\varphi}) = 0, \quad \delta^{(d)}_{\Lambda}(\mathcal{D}_\mu \tilde{\varphi}) = [\mathcal{D}_\mu \tilde{\varphi}, \Lambda], \quad \delta^{(ad)}_{\Lambda}(\mathcal{D}_\mu \tilde{\varphi}) = 0. \quad (3.11) \]
Therefore the action
\[ S_\varphi = \int \text{Tr}(D_\mu \tilde{\varphi})^2 \] (3.12)
is invariant under the $G \times G$ gauge symmetry algebra. Notice that the coupling of $\varphi$ to $C_\mu$ is rather non-standard. This is because the gauge field $C_\mu$ does not transform in the standard way under the $G \times G$ gauge transformation, therefore it is not surprising that its coupling to the matter fields is non-standard. We also note that the coupling of the field $\varphi$ to the tensor gauge field $B$ can either go through $A$ in case $A$ is propagating, or nontrivially through $A$ as a function of $B$ in case $A$ is auxiliary. The construction for fermions goes in the same way.

We remark that the construction of gauge invariant coupling in ordinary Yang-Mills theory can also be proceeded by introducing an invariant field. Consider for example a scalar field $\varphi$ which transforms as $\varphi \to U^{-1} \varphi U$ under gauge transformation $A_\mu \to U^{-1} A_\mu U + U^{-1} \partial_\mu U$. Introduce
\[ \tilde{\varphi} := W \varphi W^{-1}, \] (3.13)
where $W$ is a Wilson line which transforms as $W \to WU$ under gauge transformation. $\tilde{\varphi}$ is gauge invariant. It is
\[ \partial_\mu \tilde{\varphi} = W(D_\mu \varphi)W^{-1} \] (3.14)
and so the gauge invariant Lagrangian
\[ \text{Tr}(\partial_\mu \tilde{\varphi})^2 = \text{Tr}(D_\mu \varphi)^2 \] (3.15)
is indeed the same as the standard one constructed using covariant derivatives. Our construction above for $H_{\mu\nu\lambda}$ and $\tilde{\varphi}$ was inspired and guided by this observation since the direct construction of a covariant derivative for the tensor gauge transformation is met with immediate difficulty.

### 3.2 Generic action in arbitrary dimensions

Invariant action can be constructed readily. For example, an action that is quadratic in the matter field is:
\[ S = \int d^D x \text{Tr} \left[ (D_\mu \tilde{\varphi})^2 + \bar{\psi} \Gamma^\mu D_\mu \tilde{\psi} + \frac{1}{4g_1^2} F_{\mu\nu}^2 + \frac{1}{g_2^2} B_{\mu\nu}^2 + \frac{1}{g_3^2} H_{\mu\nu\lambda}^2 
+ g_4 H_{\mu\nu\lambda\psi\bar{\psi}} \Gamma^{\mu\nu\lambda} \tilde{\psi} + g_5 F_{\mu\nu} \tilde{\psi} \Gamma^{\nu} \tilde{\bar{\psi}} \right]. \] (3.16)

The mass dimensions of the fields are: \([A] = [A'] = 1, [B] = 2, [\varphi] = D/2 - 1, [\psi] = (D-1)/2\) and for the couplings: \([g_1] = [g_2] = 2 - D/2, [g_3] = 3 - D/2, [g_4] = -2 - D, [g_5] = -1 - D\). It is straightforward to introduce a multiplet of scalars and fermions to account for internal symmetry. Also one may adjust the field content, the couplings and to include additional terms such as Yukawa couplings to construct supersymmetric action. In this action, the gauge field $A_\mu$ is propagating.
3.3 A dual formulation of 5d Yang-Mills

In this subsection, we demonstrate how our framework of non-abelian tensor gauge symmetry could be used to construct a dual description of the 5d Yang-Mills gauge theory in terms of a three form field strength $H_{\mu\nu\lambda}$. Consider the action of the non-abelian two-form $B_{\mu\nu}$ and one-form gauge field $A_{\mu}$ with the Lagrange multiplier field $\lambda_{\mu\nu}$,

$$ S = \int \text{tr} \left[ \tilde{H}_{\mu\nu}^2 + (\tilde{H}_{\mu\nu} - F_{\mu\nu})\lambda_{\mu\nu} \right], \quad (3.17) $$

where

$$ \tilde{H}_{\mu\nu} := \frac{1}{6} \epsilon^{\mu\nu\rho\lambda\sigma} H_{\rho\lambda\sigma} \quad (3.18) $$

is the Hodge dual of $H_{\mu\nu\lambda}$ of (2.11). Integrating out $\lambda_{\mu\nu}$, we get the constraint

$$ F_{\mu\nu} = \tilde{H}_{\mu\nu}. \quad (3.19) $$

If we replace $H_{\mu\nu\lambda}$ in the integrated action using the constraint, we get the Yang-Mills description

$$ S = S(A) = \int \text{tr} F_{\mu\nu}^2. \quad (3.20) $$

If we instead solve the constraint (3.19) for $A_{\mu} = A_{\mu}(B)$, then we get the description

$$ S = S(B) = \int \text{tr} \tilde{H}_{\mu\nu}^2. \quad (3.21) $$

This provides an equivalent formulation of the Yang-Mills action in terms of a non-abelian 2-form $B_{\mu\nu}$. The action is formulated in terms of the tensor gauge invariant variables $B_{\mu\nu}$. Both (3.20) and (3.21) carries the same Yang-Mills gauge symmetry.

4. Discussions

In this paper we have constructed a $G \times G$ tensor gauge symmetry algebra which includes the gauge and tensor gauge transformations all together in a natural way. This $G \times G$ structure allows us to write down readily a theory of non-abelian tensor gauge field in any dimensions, with or without self-duality, and with matters neutral or charged under the tensor gauge symmetry.

We have also shown that one can construct an action of the 2-form gauge potential $B$ where the Yang-Mills gauge dynamics is completely fixed in terms of $B$ and the other matter fields. In this way, we obtain a 3-form field strength with non-trivial non-linear terms of the 2-form gauge potential packaged nicely in terms of a 1-form gauge field. Attempts to write down this formula directly in the beginning by using $B$ only would be impossible. These are the main results of the paper.
Our construction of the non-abelian tensor gauge symmetry relies on the use of one-form gauge fields with a non-trivial kind of “cross” gauge transformations. It should be possible to construct higher tensor gauge theory with non-abelian symmetry in a similar manner. These may be relevant for the studies of higher spin fields and duality [28–33].

4.1 Comments on M5-branes: self-duality

With the application of M5-branes in mind, the most interesting case is six dimensions. The low energy worldvolume theory of multiple M5-branes is given by an interacting 2-form tensor gauge field theory with (2,0) supersymmetry. The 3-form field strength has to satisfy a self-duality condition as required by (2,0) supersymmetry. A sensible self-duality condition is of the form

\[ H_{\mu\nu\lambda} = \Phi_{\mu\nu\lambda}, \]  

(4.1)

where

\[ H_{\mu\nu\lambda}^{\pm} := \frac{1}{2} (H_{\mu\nu\lambda} \pm H_{\mu\nu\lambda}^{*}). \]  

(4.2)

Here \( H_{\mu\nu\lambda}^{*} = \frac{1}{6} \epsilon_{\mu\nu\lambda\alpha\beta\gamma} H^{\alpha\beta\gamma} \) is the Hodge dual and \( H^{+} \) (resp. \( H^{-} \)) is the self-dual (resp. anti-self-dual) part of \( H \). \( H_{\mu\nu\lambda} \) is a quantity which is required to be tensor gauge invariant and is covariant under gauge transformation in order for (4.1) to make sense. In the theory of multiple M5-branes, the self-duality condition (4.1) is a part of the supermultiplet of equations of motion and \( \Phi \) may need to be non-trivial.

In addition to a self-dual \( H \), the (2,0) supermultiplet of M5-branes also contains 5 scalars and 8 fermions on-shell. The self-duality of the tensor gauge field makes it difficult to write down an action with standard \( SO(5,1) \) Lorentz symmetry. The problem for a single M5-brane with the self-duality equation of motion

\[ \hat{H}_{\mu\nu\lambda} := H_{\mu\nu\lambda} - H_{\mu\nu\lambda}^{*} = 0 \]  

(4.3)

is solved with a formulation where the Lorentz symmetry is realized in a non-standard manner [23–27]. Consider the action

\[ S_B = \int d^6 x \left( -\frac{1}{6} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + \frac{1}{2 u^2} (u \hat{H})_{\mu\nu} (u \hat{H})^{\mu\nu} \right), \]  

(4.4)

where

\[ (u \hat{H})_{\mu\nu} = u^\rho \hat{H}_{\rho\mu\nu} \]  

(4.5)

and \( u^\mu \) is a fixed constant vector in \( U(1) \). The action has manifest \( SO(1,4) \) or \( SO(5) \) Lorentz symmetry depending on whether \( u \) is spacelike or timelike. It is straightforward to check that the action is invariant under the symmetry

\[ \delta B_{\mu\nu} = \frac{1}{2} u_{[\mu} \varphi_{\nu]}, \quad \partial_{[\mu} u_{\nu]} = 0. \]  

(4.6)
The equation of motion for $B_{\mu\nu}$ is
\[ \epsilon^{\mu\nu\rho\sigma\alpha\beta} \partial_\mu \left( \frac{1}{u^2} u_\nu (\hat{H} u)_{\rho\sigma} \right) = 0. \] (4.7)

One can show that the most general solution of it is of the form
\[ (\hat{H} u)_{\lambda\rho} = u^2 \partial_{[\lambda} \varphi_{\rho]} + u_{[\lambda} \partial_{\rho]} \varphi_{\alpha} u^\alpha + u^\alpha \partial_\alpha \varphi_{[\lambda} u_{\rho]}, \] (4.8)

for an arbitrary function $\varphi_\mu$. Since the right handed side of (4.8) has precisely the same form as the transformation of $(\hat{H} u)_{\lambda\rho}$ under the transformation (4.6) and so one can use (4.6) to gauge fix it to zero
\[ (\hat{H} u)_{\lambda\rho} = 0. \] (4.9)

This implies the whole $H_{\mu\nu\lambda}$ is zero and so the equation of motion (4.7) is exactly the same as the self-duality condition (4.3) after using the gauge symmetry. It turns out that for the non-abelian case, a similar action can be constructed which gives the self-duality equation as the equation of motion [19]. One may also write down constraint on $A_\mu$ that determines its in terms of $B_{\mu\nu}$ and other fields of the theory and leaves no local degree of freedom in it. In this way, the non-abelian field strength $H_{\mu\nu\lambda}$ contains non-trivial non-linear self interaction of the 2-form gauge potential packaged nicely in terms of the 1-form gauge field. We note that non-propagating gauge fields also play an essential role in the BLG or ABJM theory for multiple M2-branes. There the gauge fields are auxiliary due to their Chern-Simons kinetic term and they are essential for a supersymmetric construction. Here we would like to propose that they are essential in the construction of the non-abelian tensor gauge theory of M5-branes. A detailed discussion of the action principle for the self-duality equation and of the auxiliary nature of the gauge field $A_\mu$ will be the subject of [19].

We remark that another interesting approach [34,35] makes use of canonical variables has the advantage of being local, Lorentz invariant and polynomial in the fields. It will be interesting to see whether one can use similar ideas to construct an action for a non-abelian 2-form potential with a self-duality condition.

4.2 Comments on M5-branes: supersymmetry

Apart from the self-duality equation of motion, the inclusion of (2,0) supersymmetry is another important aspect of the theory of multiple M5-branes. Let us also comment on the supersymmetry. In analogy with the situation of the ABJM theory where the full $\mathcal{N} = 8$ supersymmetry is supposed to be seen only nonperturbatively after including the monopole sector, it might be possible that only a fraction of the (2,0) supersymmetry, i.e. (1,0) supersymmetry, is visible and full supersymmetry can be seen only nonperturbatively. Therefore let us consider the possibility of constructing the theory of multiple M5-branes using (1,0) supersymmetry.
In (1,0) supersymmetry, we expect the following supermultiplets to be useful:

Tensor multiplet: \((B^+_{\mu\nu}, X, \chi)\),
Hyper-multiplet: \((\phi_i, \psi)\),
Yang-Mills multiplet: \((A_\mu, \lambda)\).

Here \(B_{\mu\nu}\) has a self-dual 3-form field strength (hence the superscript +), \(X\) and \(\phi_i(i = 1, \cdots 4)\) are scalar fields and \(\chi, \lambda, \psi\) are fermions. It is understood that all the fields carry the same non-abelian indices \(a\). With respect to (1,0) supersymmetry, a (2,0) tensor multiplet is simply the sum of a (1,0) tensor multiplet and a (1,0) hyper-multiplet. However in order to have a non-abelian tensor gauge symmetry, it is necessary to have two additional gauge fields in our construction. Therefore it seems natural to use the following (1,0) multiplets:

\[1 \times \text{Tensor} + 1 \times \text{Hyper} + 2 \times \text{Yang-Mills}\]  

in the construction of the theory of multiple M5-branes in the (1,0) language. The Yang-Mills multiplet should be auxiliary and governed by a suitable constraint. The construction of the (2,0) non-abelian M5 theory in this way seems feasible [20].

Recently, it has been proposed that the theory of multiple M5-branes compactified on a circle is nonperturbatively given by the D4-branes SYM theory by including instantons [17,18]. The fact that our \(G \times G\) tensor gauge symmetry could be gauge fixed to a \(G\) gauge symmetry provides a possible connection of the gauge symmetries of the two theories which may be useful in understanding this proposal. Very recently, a way to include the KK modes in the D4-branes theory and an action of the multiple M5-branes in \(C\)-field in terms of a 1-form gauge potential was proposed in [21]. As a theory of 1-form, there is no tensor gauge symmetry and the proposed action has only a \(G = U(N)\) gauge symmetry, which is precisely the same symmetry we would obtain here if the tensor gauge symmetry is fixed. Due to the coincidence of symmetries, it is natural to wonder if this theory admits a dual formulation in terms of our framework \(G \times G\) tensor gauge field. It will be very interesting to clarify the possible connection.

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