Adaptive grid-driven probability hypothesis density filter for multi-target tracking

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Abstract
The probability hypothesis density (PHD) filter and its cardinalised version PHD (CPHD) have been demonstrated as a class of promising algorithms for multi-target tracking (MTT) with unknown, time-varying number of targets. However, these methods can only be used in MTT systems with some prior information of multiple targets, such as dynamic model, newborn target distribution etc. Otherwise, the tracking performance will decline greatly. To solve this problem, an adaptive grid-driven technique is proposed based on the framework of the PHD/CPHD filter to recursively estimate the target states without knowing the dynamic model and the newborn target distribution. The grid size can be adaptively adjusted according to the grid resolution, and the dynamic tendencies of the grids can respond to the unknown dynamic models of each target, including arbitrary manoeuvring models. The newborn targets outside the grid area can be identified by analysing the measurements, and some new grids are generated around them. The experimental results show that the proposed algorithm has a better performance than the traditional particle filter-based PHD method in terms of average optimal sub-pattern assignment distance and average target number estimation for tracking multiple targets with unknown dynamic parameters and unknown newborn target distribution.

1 | INTRODUCTION

With the increasingly complex war environments, the requirements for multi-target tracking (MTT) techniques have increased and have gained wide attention, especially, for multiple targets with unknown, time-varying numbers. Generally, it is unknowable where the targets appear from and what dynamic models are used, which makes the MTT techniques a hot topic and an extremely difficult problem in the MTT field.

In recent years, different from the conventional data association techniques, the random finite set (RFS) theory [1–3] has been proposed as an elegant formulation for MTT and has generated substantial interest due to the development of the probability hypothesis density (PHD) filter [1] and the cardinalised PHD (CPHD) filter [2]. The PHD and CPHD filters can estimate the target states by recursively computing the first-order moment of the multi-target posterior probability distribution, avoiding the combinatorial problem that arises from complex data association. Furthermore, compared with the standard PHD filter, the CPHD filter can dramatically improve the accuracy of individual state estimates and cardinalised estimates due to the extra estimation of the cardinalised distribution. Now, the existing closed-form solutions of PHD and CPHD mainly include the particle filter PHD/CPHD (PF-PHD/CPHD) [4, 5], Gaussian mixture PHD (GM-PHD/CPHD) filter [6] and their modified versions [7–10]. However, these algorithms can exhibit a good MTT performance only when the model parameters of the tracking systems are knowable as a priori knowledge, such as the newborn intensity of the targets and the dynamic models. However, these parameters are usually unknowable in the real tracking scenarios, resulting in a serious decline in tracking performance.

For the unknown dynamic models in the MTT system, especially for the manoeuvring target tracking, the jump Markov system has proved to be efficient as it switches among a set of candidate models in a Markovian fashion [11, 12]. The closed-form solution for the non-linear jump Markov multi-target model is proposed by combining the linear fractional transformation and the unscented transform in [13, 14]. In [15], the best-fitting Gaussian approximation approach is employed in the GM-PHD filter with jump Markov models.
However, the Gaussian distribution of the PHD is assumed in these algorithms, which limits the scopes of the application. The multiple-model particle PHD/CPHD (MMP-PHD/CPHD) filter and the MMP-Multi-Bernoulli filter are proposed by implementing the sequential Monte Carlo method and their improved versions are presented in [16–18]. For the application of these MM-based filters, the difficulty lies in designing the model sets because the tracking accuracy depends on the matching degree of the prior designed model sets with the real target dynamic models. Moreover, if the process noises are very loud, the targets can also be considered arbitrary manoeuvring targets, and it is difficult for the non-manoeuvring models to track these targets. The variational Bayesian approximation method [19–21] is used to recursively estimate the joint state of multiple targets with process noises [22, 23], but in the tracking process, prior information about the newborn target is also needed, and when the target is manoeuvring, the tracking performance will be seriously affected.

In order to solve the above-mentioned problems, an adaptive grid-driven PHD/CPHD (GD-PHD/CPHD) filter algorithm is proposed under the framework of PHD and CPHD. The grid-based filter was first proposed by Bucy and Senne [24], followed by Kramer and Sorensen who further elaborated it [25, 26], but it generally uses the fixed grid points for filtering, which affects the filtering efficiency. The proposed algorithm can adaptively adjust the grid position and size according to the grid resolution and the estimated states. The main contributions of the proposed algorithm are as follows: (1) The newborn targets can be identified by analysing the measurements and the grid distribution; (2) the dynamic tendencies of the grids can respond to the unknown dynamic models of each target; (3) the weight update strategy for the grids is proposed when we compress and expand the grid regions according to the grid resolution.

The remainder of the article is organised as follows: Section 2 summarises the RFS model, the PHD and CPHD filters. Section 3 proposes the GD-PHD/CPHD algorithm and derives the grid particle solution. Simulation results are presented in Section 4. Finally, the conclusions are given in Section 5.

2 | BACKGROUNDS

2.1 | Random finite set model

For the MTT by using the PHD/CPHD-based filters, the multiple target state set and the measurement set are constructed as the RFSs $X_k = \{x_{k,1}, x_{k,2}, \ldots, x_{k,M_k}\}$ and $Z_k = \{z_{k,1}, z_{k,2}, \ldots, z_{k,N_k}\}$, $N_k$ and $M_k$ denote the number of targets and measurements, respectively. Suppose $X_{k-1}$ is the multiple target state set at time $k-1$, then $X_k$ and $Z_k$ can be expressed as

$$X_k = \{\cup_{x \in x_k} S_{x_{k-1}}(x)\} \cup \{\cup_{x \in x_k} B_{x_{k-1}}(x)\} \cup \Gamma_k \quad (1)$$

where $S_{x_{k-1}}(x)$ is the RFS of targets surviving from time $k-1$ to $k$, $B_{x_{k-1}}(x)$ is the RFS of targets spawned from $X_{k-1}$ and $\Gamma_k$ is the RFS of targets that appear spontaneously at time $k$. $\Theta_k(x)$ and $K_k$ are the RFSs of measurements originating from the targets in $X_k$ and the clutters, respectively.

The optimal Bayesian recursions for propagating the multi-target posterior probability density function (PDF) are expressed as shown in [6].

$$p_{k|k-1}(x_k|Z_{1:k-1}) = \int f_{k|k-1}(x_k|x) p_{k-1}(x|Z_{1:k-1}) \mu_k(dx) \quad (3)$$

$$p_{k|k}(x_k|Y_{1:k}) = \frac{\gamma_k(Y_k|x_k)p_{k|k-1}(x_k|Y_{1:k-1})}{\int \gamma_k(Y_k|x) p_{k|k-1}(x|Y_{1:k-1}) \mu_k(dx)} \quad (4)$$

where $\mu_k$ denotes the approximate state space Lebesgue measure, and $p_{k|k-1}(x_k|Y_{1:k-1})$ and $p_{k|k}(x_k|Y_{1:k})$ are the predicted PDF and the posterior PDF, respectively. $f_{k|k-1}(\cdot)$ is the state transition PDF and $\gamma_k(\cdot)$ is the measurement likelihood function.

Notice: Generally, the dynamic model $f_{k|k-1}(\cdot)$ and the process noise in the real scenarios are unknowable, that is, it is difficult to obtain them as a priori knowledge. Therefore, it is difficult to choose the right dynamic models for tracking the targets with arbitrary motions, resulting in the inaccurate estimations for the posterior PDF of the multi-target states. In the proposed algorithm, we will solve this problem by using the GD technique with grid extension. It is noteworthy that the grid extension can cover the targets with arbitrary motions.

2.2 | PHD filter

The PHD filter mainly includes two parts, that is, the prediction and the update. Assume $D_{k|k-1}$ denotes the PHD of multiple targets at the time $k-1$. The PHD recursive process can be expressed as follows [6]:

$$D_{k|k-1}(x_k|Z_{k-1}) = Y_k(x_k) + \int \phi(x|x_{k-1}) D_{k-1|k-1}(x_{k-1}|Z_{k-1}) dx_{k-1} \quad (5)$$

where $Y_k(x_k)$ denotes the intensity of the newborn target RFS at time $k$ and the transition kernel $\phi(x|x_{k-1})$ is expressed as

$$\phi(x|x_{k-1}) = P_{s,k}(x_{k-1}) P(x|x_{k-1}) + \beta_{k|k-1}(x|x_{k-1}) \quad (6)$$
where \( \beta_{k-1} \) is the intensity of the spawned target RFS at time \( k-1 \), \( P_{\delta,k}(x_{k-1}) \) is the survival probability of target \( x_{k-1} \), and \( P(x|x_{k-1}) \) is the state transition PDF of each target.

### 2.2.2 | PHD update

\[
D_{k|k}(x|Z_k) = \left[ (1 - P_{D,k}(x)) + \sum_{z \in Z_k} \lambda_k c_k(z) + D_{k|k-1}(x|Z_{k-1}) \right] \psi_{k,z}(x)
\]

where \( \lambda_k \) is the Poisson parameter used to represent the expected number of false alarms and \( c_k(z) \) is the probability distribution of clutters in the observation space. \( \langle \cdot, \cdot \rangle \) and \( \psi_{k,z}(\cdot) \) are expressed as

\[
\langle f, b \rangle = \int f(x)b(x)dx
\]

\[
\psi_{k,z}(x) = P_{D,k}(x)g_k(z|x)
\]

where \( g_k(z|x) \) is the measurement likelihood function and \( P_{D,k}(x) \) is the detection probability.

**Notice:** In the traditional PHD-based algorithms, the newborn intensity is usually assumed as a priori knowledge. However, it is unknowable in the real scenarios, that is, \( Y_k(x) \) is unknowable in Equation (5). Due to this the newborn targets cannot be estimated, decreasing the tracking accuracy. We will adaptively identify the newborn targets from the measurements in the proposed grid driven PHD (GD-PHD) algorithm.

### 2.3 | CPHD filter principle

The CPHD filter is a generalisation of the PHD recursion, which jointly propagates the intensity function and the cardinalised distribution. Compared with the PHD filter, the CPHD filter can dramatically improve the tracking accuracy and the number estimations. Assume \( D_{k|k-1}(x) \) and \( p_{k|k-1}(n) \) denote the multi-target intensity function and cardinalised distribution associated with predicted multi-target states at time \( k-1 \), and \( D_k(x) \) and \( p_k(n) \) denote the multi-target posterior intensity function and the cardinalised distribution at time \( k \), respectively. The prediction and updation steps of the CPHD method are briefly summarised as follows [5]:

#### 2.3.1 | CPHD prediction

The predicted intensity \( D_{k|k-1}(x) \) is the same as Equation (5), and the prediction of the cardinalised distribution is expressed as

\[
p_{k|k-1}(n) = \sum_{j=0}^{n} P_{1,k}(n-j) \prod_{i=1}^{k-1} [D_{i-1}, p_{i-1}](j)
\]

\[
\prod_{i=k}^{k-1} [D, p](j) = \sum_{l=j}^{\infty} C_{l-j}^{j}(P_{S}D^{j}(1-P_{S}D)^{l-j}) \frac{1}{(1,D)^{l}}
\]

where \( P_{1,k} \) denotes the cardinalised distribution of the newborn targets RFS, and \( \langle \cdot, \cdot \rangle \) denote the inner product operation. \( C_{j} = \Gamma(j)/j! \) denotes the binomial coefficient.

#### 2.3.2 | CPHD update

The updated intensity \( D_k(x) \) and the updated cardinalised distribution are expressed as

\[
D_k(x) = \frac{\langle 1 - P_D \rangle \langle \gamma_0[D_{k|k-1}, Z_k], P_{k|k-1} \rangle}{\langle \gamma_0[D_{k|k-1}, Z_k], P_{k|k-1} \rangle} D_{k|k-1}(x)
\]

\[
+ \sum_{z \in Z_k} \psi_{k,z}(x) \langle \langle j \rangle \rangle_{k|k-1}^{j=1}[D_{k|k-1}, Z_k \setminus \{z\}], P_{k|k-1} \rangle D_{k|k-1}(x)
\]

\[
p_k(n) = \frac{\gamma_{k}^{0}[D_{k|k-1}, Z_k]|(n)p_{k|k-1}(n)}{\langle \gamma_{k}^{0}[D_{k|k-1}, Z_k], P_{k|k-1} \rangle}
\]

where

\[
\gamma_{k}^{0}[D, Z](n) = \sum_{j=0}^{\min(|Z|, \eta)} \frac{|Z|-j}{j} p_{k|k-1}(|Z|-j)
\]

\[
p_{k|n}^{\eta} = \frac{(1 - P_d)D^{n-j+\eta}}{(1,D)^{\eta}} e_j(\Xi_k(D, Z))
\]

where \( P_d = \lambda_k \) is the cardinality of the measurement set, \( P_{k,0} \) denotes the cardinalised distribution of the clutter random set, and \( Z_k \setminus \{z\} \) denotes the remaining measurements of \( Z_k \) by deleting the measurements \( z \).

\[
\Xi_k(D, Z) = \{ \langle D, \psi_{k,z} \rangle : z \in Z \}
\]

\[
\psi_{k,z}(x) = P_d \left( \frac{1}{K_k} g_k(z|x) \right)
\]

where \( e_j \) is a \( j \)-order elementary symmetric function.

\[
e_j(\{\rho_1, \rho_2, \ldots, \rho_m\}) = \frac{(-1)^j a_{m-j}}{a_m}
\]

where \( \{\rho_1, \rho_2, \ldots, \rho_m\} \) is the different root of the polynomial \( a_m x^m + a_{m-1} x^{m-1} + \ldots + a_1 x + a_0 \).
3 | GRID-DRIVEN PHD/CPHD FILTERING ALGORITHM

For the unknown dynamic model (e.g. movement parameters and process noise) and the unknown distributions of the newborn targets in the real MTT scenarios, the GD technique is proposed under the framework of the PHD and CPHD filters. The block diagram of the proposed algorithm is shown in Figure 1. First, the tracking area is uniformly divided into some small grids, which can be considered as grids with equal weights that are distributed on the tracking area at regular intervals. Then the arriving measurements are used to update the grid weights by computing the likelihoods between the grids and the measurements, and subsequently, to compress the grids and extract the target states according to the weights. The grid points are similar to the sample particles of the particle filter. The closer the grid point is to the target, the greater its weight and vice versa. Thus, some of the grids with small weights can be deleted to compress the grids. Finally, we expand the grid area according to the maximum speed of the target and re-divide the extension area as some predicted grids, according to the obtained grid resolutions, and their weights can be associatively calculated with the previous grids by using the kernel-based weight interpolation technique. The newborn targets and the clutterings can be identified from the measurements by analysing the associational information between the measurements and the survival grid area. If the measurements are not associated with the previous grids, they are considered as originating from the newborn targets or clutters.

3.1 | Grid-driven PHD filtering algorithm

3.1.1 | Grid initialisation

Assume that the observation area is a rectangular area. At the initial time $k = 0$, the observation area is uniformly divided into $N_0$ grids with the vertical resolution $d_y^i$ and the horizontal resolution $d_x^i$. The uppermost edge and the lowest edge coordinates are $d_y^{\max}$ and $d_y^{\min}$. The far right and the far left edge coordinates are $d_x^{\max}$ and $d_x^{\min}$. The initial grid set is expressed as $G_0 = \{g_0^j, w_0^j\}_{j = 1}^{N_0}$, where $g_0^j = [x_k^j, y_k^j]^T$ is the $i_{th}$ grids coordinate. Assume that at the initial time, there are $M_0$ targets with the same number as that of the initial measurement set $Z_1 = \{z_i\}_{i = 1}^{M_0}$, $|Z_1| = M_0$ denotes the cardinality of $Z_1$, and then we set the initial grid weights as $w_0^j = \frac{M_0}{N_0}$.

3.1.2 | Initial target state extraction

At time $k = 1$, the initial measurement set $Z_k$ is used to update the weight of each grid, for $i = 1, 2, \ldots, N_0$.

$$w_k^j = \left[\left(1 - P_{D_k}(g_0^j)\right) + \sum_{z_i \in z_k^j} \frac{\psi_{k}z_i(g_0^j)}{K_k(z) + C_k(z)}\right] w_0^j \quad (18)$$

where $K_k(z) = \lambda_k c_k(z)$ is the intensity of the clutter RFS, $\lambda_k$ is the clutter number obeying the Poisson distribution, and $c_k(z)$ is the PDF of the clusters.

$$C_k(z) = \sum_{j = 1}^{N_0} \psi_{k}z_i(g_0^j) w_0^j \quad (19)$$

Then the number of targets can be estimated by

$$\hat{M}_k = \text{round} \left(\sum_{j = 1}^{N_0} w_k^j\right) \quad (20)$$

where $\text{round}(\cdot)$ denotes the rounding-off operator. According to the estimated number $\hat{M}_k$ of the targets and their corresponding weights, the grids are clustered into $\hat{M}_k$ clusters expressed as $G_k = \{G_k^i = \{g_k^i, w_k^i\}_{j = 1}^{N_k^i}\}_{i = 1}^{\hat{M}_k}$, $N_k^i$ denotes the number of the grids belonging to the $i_{th}$ cluster and $g_k^i$ denotes the $i_{th}$ grid in the $i_{th}$ cluster. Figure 2 shows the clusters in which the red grids represent the grid clusters and the green ellipses in the clusters denote the grid area with high probability of the target state space.

The target states can be extracted by the weighted sum of the grids for each cluster expressed as

$$X_k = \{x_k^i\}_{i = 1}^{\hat{M_k}}$$

with $x_k^i = \sum_{j = 1}^{N_k^i} w_k^i g_k^i$.

3.1.3 | Grid shrinkage

Grid shrinkage is the pruning of the grids by deleting the grids with small weights. We arrange the weights $w_k^i$ in descending order for each grid in the clusters, and the first $L_k^i$ grids with larger weights are retained for subsequent filtering by deleting the grids with small weights. It is noted that the sum of weights for the deleted grids should be smaller than a threshold (e.g. 5% of all the weights sum in a cluster). The grids retained can
be expressed as \( G_k = \{ G^i_k, \tilde{w}^i_k \}_{j=1}^{M_k} \), where \( \tilde{w}^i_k \) is the grid weight which is renormalised by

\[
\tilde{w}^i_k = \frac{w^i_k}{\sum_{j=1}^{M_k} w^i_j}
\]

(22)

Assume that the resolution \( \alpha^i_k \) and \( \beta^i_k \) of the vertical and horizontal directions of the grids are equal and can be updated according to the measurement standard deviation \( \sigma_k \),

\[
\alpha^i_k = \beta^i_k = \eta \sigma_k, \quad i = 1, 2, \ldots, M_k
\]

(23)

where \( \eta \) is a scale coefficient.

3.1.4 Grid expansion and weight redistribution

The purpose of the grid expansion is to extend the grid area for each target to cover its measurement for the next time, which can keep the target identified and tracked for the next time. Suppose that the maximum speed of the targets is \( v_k \) and the minimum number of grids is set as \( E \), and update the uppermost edge coordinate \( \alpha^i_{max} \) and the lowest edge coordinate \( \alpha^i_{min} \) in the vertical direction for each cluster, \( i = 1, 2, \ldots, M_k \), that is,

\[
\alpha^i_{max} = min \left( \alpha^i_{max,1}, \alpha^0 \right)
\]

(24)

\[
\alpha^i_{max} = max_{j=1}^{L_k} \left( \alpha^j_i \right) + max \left( \alpha^i_{max,2}, v_k \cdot T \right)
\]

(25)

\[
\alpha^i_{min} = max \left( \alpha^i_{min,1}, \alpha^0 \right)
\]

(26)

\[
\alpha^i_{min} = min_{j=1}^{L_k} \left( \alpha^j_i \right) - max \left( \alpha^i_{min,2}, v_k \cdot T \right)
\]

(27)

Use the same method to update the far right edge coordinates \( \beta^i_{max} \) and the far left edge \( \beta^i_{min} \) for each cluster; \( i = 1, 2, \ldots, M_k \), that is,

\[
\beta^i_{max} = min \left( \beta^i_{max,1}, \beta^i_{max} \right)
\]

(28)

\[
\beta^i_{max,1} = max_{j=1}^{L_k} \left( \beta^j_i \right) + max \left( \beta^i_{max,2}, v_k \cdot T \right)
\]

(29)

\[
\beta^i_{min} = max \left( \beta^i_{min,1}, \beta^i_{min} \right)
\]

(30)

\[
\beta^i_{min,1} = min_{j=1}^{L_k} \left( \beta^j_i \right) - max \left( \beta^i_{min,2}, v_k \cdot T \right)
\]

(31)

The edge coordinates construct a box area for each cluster, and then we uniformly divide the box area into \( \tilde{N}^i_k \) grids. If \( \text{sum}(\tilde{N}^i_k) < E \), we reduce the vertical direction resolution \( \alpha^i_k \) and the horizontal direction resolution \( \beta^i_k \) by \( \sqrt{\frac{E}{\text{sum}(\tilde{N}^i_k)}} \) times to obtain new \( \tilde{N}^i_k \) grids. The newly generated grids are represented as \( G_k = \{ G^i_k, \tilde{w}^i_k \}_{j=1}^{\tilde{N}^i_k} \), where the weights \( \tilde{w}^i_k \) of the new grids need to be redistributed according to the original weights of the \( L_k \) grids due to the number difference of the grids before and after grid expansion. We propose to redistribute the weights \( \tilde{w}^i_k \) by weight interpolation calculation based on the kernel method, that is,

\[
\tilde{w}^i_k = \sum_{j=1}^{L_k} w^i_j b \left( \tilde{g}^i_k - g^i_k \right)
\]

(32)

\[
b \left( \tilde{g}^i_k - g^i_k \right) = e^{-\lambda \left( \frac{\parallel \tilde{g}^i_k - g^i_k \parallel}{s} \right)^2}
\]

(33)

where \( \lambda \) denotes the scale factor, and the weights are flattened and normalised according to Equations (34) and (35),

\[
\tilde{w}^i_k = \left( \tilde{w}^i_k \right)^a
\]

(34)

\[
\tilde{w}^i_k = \frac{\tilde{w}^i_k}{\sum_{j=1}^{\tilde{N}_k} \tilde{w}^i_k}
\]

(35)

where \( a < 1 \).

Figure 3 gives an example of shrunk grids and expanded grids of two targets (clusters). In Figure 3a, the red grids including the blue grid area are the initial grids of the targets, and the blue grids represent the shrunk grids of the red grid.
3.1.5 | Newborn target identification

Assume that the estimated grid set $G_k$ at time $k$ has been obtained. Then the current measurements corresponding to the survival (estimated) targets can be identified by matching the estimated grids, that is, if the measurements scatter in the grid area, they are considered as the measurements $\{z_k^j\}_{j=1}^{N_c}$ of the survival targets; otherwise, they are considered as the measurements $\{z_k^j\}_{j=1}^{N_c}$ of the newborn targets and/or clutters. Therefore, the measurement set is divided into two parts, that is, $Z_k = Z_k^1 \cup Z_k^2 = \{z_k^j\}_{j=1}^{N_c} \cup \{z_k^j\}_{j=1}^{N_c}$. Here we use $Z_k^2 = \{z_k^j\}_{j=1}^{N_c}$ to generate the newborn grids. First to extract the box area with the centre of each measurement of $Z_k^2$, the width and the height of each box area are set as 2 times the maximum speed $v_s$ and then the box area is uniformly divided into some small grids by the same vertical direction resolution $d_{k}^v$ and the horizontal direction resolution $d_{k}^h$ as that of the previous grids. Finally, we can obtain the new grids expressed as

$$
G_{new,k} = \left\{ \begin{array}{l}
G_{new,k}^i = \left\{ g_{new,k}^{ij}, w_{new,k}^{ij} = \frac{1}{N_{new,k}^i} \right\} \right\}_{i=1}^{N_{new,k}^i} \quad \text{where} \quad \begin{cases}
N_{new,k}^i = |z_k^i| \\
G_{new,k}^i = \left\{ g_{new,k}^{ij}, w_{new,k}^{ij} = \frac{1}{N_{new,k}^i} \right\} \end{cases}
$$

3.1.6 | Update grid weights and extract target states

At time $k+1$, the measurement set $Z_{k+1}$ is used to update the weight of each grid in $G_k$ and $G_{new,k+1}$ according to Equation (37), that is,

$$
\begin{align*}
\bar{w}_{k+1}^{ij} & = \left\{ \begin{array}{l}
\left( 1 - P_{D,k+1}(g_{k+1}^{ij}) \right) + \sum_{z \in Z_{k+1}} \frac{\psi_{k+1}(z_{k+1}|z)}{K_{k+1}(z) + C_{k+1}(z)} \bar{w}_{k}^{ij} \\
\left( 1 - P_{D,k+1}(g_{new,k}^{ij}) \right) + \sum_{z \in Z_{k+1}} \frac{\psi_{k+1}(z_{k+1}|z)}{K_{k+1}(z) + C_{k+1}(z)} \bar{w}_{new,k}^{ij} 
\end{array} \right\} \\
& \quad i = 1, \ldots, \hat{M}_k, \ j = 1, \ldots, \hat{N}_k
\end{align*}
$$

Then the number of the targets can be estimated by

$$
\hat{M}_{k+1} = \text{round} \left( \sum_{i=1}^{\hat{M}_k} \sum_{j=1}^{N_{new,k}^i} w_{k+1}^{ij} \right)
$$

According to the estimated number $\hat{M}_{k+1}$ of targets and their corresponding weights, the grids are clustered into $\hat{M}_k+1$ clusters expressed as $G_{k+1}^j = \{g_{k+1}^{ij}, \bar{w}_{k+1}^{ij} \}_{i=1}^{\hat{M}_k}$ in Figure 4 shows an example of the process of newborn target recognition. In Figure 4a, there are 2 survival clusters (the red grids) at time $k$ in the tracking area, and below the survival targets there are 2 purple grids generated by the unknown newborn targets or clutters. While at time $k+1$ in Figure 4b, when the new measurements arrive, the grid weights can be obtained by calculating the likelihood between the grids and the measurements. Then the grids with small weights that originated from the clutter are removed, and the grids originating from the newborn are kept as the survival target grids.

The steps of the GD-PHD algorithm are summarised in Table 1. Steps 3, 4 and 5 can be considered as the prediction stage of the traditional PHD filtering, and Step 6 belongs to the update stage. Step 7 is used to judge whether the tracking is terminated or not.
FIGURE 3 Shrink grids and expanded grids. (a) The shrink grids; (b) The expanded grids

FIGURE 4 Illustration for newborn target recognition. (a) two unknowable measurements; (b) one measurement identified from a newborn target

| TABLE 1 | Steps of the grid-driven PHD filtering algorithm |
|----------|--------------------------------------------------|
| 1.       | Grid initialisation, $G_0 = \{g_{0,j}^i, w_{0,j}^i\}_{j=1}^N$, $w_{0,j}^i = \frac{1}{N}$, $|Z_1| = M_0$. |
| 2.       | Extract initial target set $X_0$ according to Equation (21). |
| 3.       | Grid shrinkage by deleting the grids with small weights, $G_k = \{G_k = \{g_{k,j}^i, w_{k,j}^i\}_{j=1}^N, p_k\}$. |
| 4.       | Grid expansion according to Equations (24)–(31), and weight redistribution according to Equations (32)–(35). |
| 5.       | Newborn target identification and generate new grids $G_{new,k}$ according to Equation (36). |
| 6.       | Update grid weight and extract target states according to Equations (37) and (39), respectively. |
| 7.       | If the tracking does not end, go to Step (3) and repeat Steps (3)–(6), otherwise, terminate the tracking. |

3.2 Grid-driven CPHD algorithm

The GD-CPHD algorithm is similar to the GD-PHD algorithm in Section 3.1. The advantages of the CPHD algorithm are mainly reflected in the cardinalised distribution estimation of the targets, which improves the tracking accuracy of the algorithm. The steps of the GD-CPHD algorithm are briefly described as follows:

The grid initialisation is the same as that in Section 3.1, and then when the initial measurement set is arrived at time $k = 1$, the grid weights can be updated by

$$
\omega_k^{(i)} = \frac{(1 - P_0) \langle y_k[D_0, Z_k], p_0 \rangle \omega_0^{(i)}}{\langle y_k[D_0, Z_k], p_0 \rangle} + \sum_{z \in Z_k} \Psi_{k,z}(g_{0,j}) \frac{\langle y_k[D_0, Z_k \setminus \{z\}], p_0 \rangle \omega_0^{(i)}}{\langle y_k[D_0, Z_k \setminus \{z\}], p_0 \rangle} \tag{40}
$$

where the initial intensity $D_0$ can be approximated by the initial grids as $D_0(g) = \sum_{i=1}^{N_k} \omega_i^0 \delta(g - g_i^0)$. $p_0$ is the initial cardinalised distribution which is also assumed to equal the cardinality of
the initial measurement set, that is, the estimated number of target is \( \hat{M}_k = M_0 \), and the grids are clustered into \( \hat{M}_k \) clusters according to the weights and expressed as \( G_k = \{ G^i_k = \{ \mathbf{g}^{ij}_k, \mathbf{w}^{ij}_k \}_{j=1}^{N^i_k} \}_{i=1}^{M_k} \), where \( N^i_k \) denotes the number of the grids belonging to the \( i_{th} \) cluster and \( \mathbf{g}^{ij}_k \) denotes the \( j_{th} \) grid in the \( i_{th} \) cluster. The target states can be extracted according to Equation (39) by the weighted sum of the grids for each cluster.

Subsequently, the steps implemented for grid shrinkage, grid expansion, and newborn target identification are the same as those implemented for GD-PHD. After these steps, the grids \( G_k = \{ G^i_k = \{ \mathbf{g}^{ij}_k, \mathbf{w}^{ij}_k \}_{j=1}^{N^i_k} \}_{i=1}^{M_k} \) generated by the grid expansion and the newborn grids \( G_{new,k} = \{ G_{new,k}^i = \{ \mathbf{g}^{i,j}_{new,k}, \mathbf{w}^{i,j}_{new,k} \}_{j=1}^{N_{new,k}} \}_{i=1}^{M_{new,k}} \) are obtained, and at time \( k+1 \), their weights can be further updated as \( w^{i,j}_{k+1} \) when the latest measurements \( Z_{k+1} \) arrive, that is,

\[
\begin{align*}
\mathbf{w}^{i,j}_{k+1} &= (1-P_k) \left( \mathbf{w}^{i,j}_k \right) + \frac{P_k \left( \mathbf{g}^{i,j}_k, Z_{k+1} \right) \mathbf{w}^{i,j}_k}{\mathbf{g}^{i,j}_k, Z_{k+1}, P_k} \\
&\quad + \sum_{z \in Z_{k+1}} \mathbf{w}^{i,j}_k \left( \mathbf{g}^{i,j}_k \right)
\end{align*}
\]

The same method can be used to update the weights of the newborn grids, where the intensity function \( D_k \) can be expressed as

\[
D_k(g) = \sum_{i=1}^{M_k} \sum_{j=1}^{N^i_k} \delta (g - \mathbf{g}^{ij}_k)
\]

The number estimation of the targets can be calculated by

\[
\hat{M}_{k+1} = \sum_{j=1}^{\infty} j P_k(j)
\]

According to the estimated number \( \hat{M}_{k+1} \) of targets and their corresponding weights, the grids are clustered into \( \hat{M}_{k+1} \) clusters expressed as \( G_{k+1} = \{ G^i_{k+1} = \{ \mathbf{g}^{ij}_{k+1}, \mathbf{w}^{ij}_{k+1} \}_{j=1}^{N^i_{k+1}} \}_{i=1}^{M_{k+1}} \). The target states can be extracted by the same method as that of GD-PHD. If the tracking does not end, we can jump to the grid shrinkage step; otherwise, we terminate the tracking.

## 4 | SIMULATIONS

In order to verify the effectiveness of the proposed GD-PHD and GD-CPHD algorithms, the traditional PF-PHD [4] and MM-PHD algorithms [16] are chosen as the comparison algorithms. It is noted that the proposed algorithm is based on the grid driven to recursively estimate the target states without knowing the dynamic model and the newborn target distribution, while the comparative algorithms are based on the dynamic models. Therefore, we assume that the constant velocity (CV) model and the coordinated turn (CT) model, respectively, are expressed as follows:

\[
X^i_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} X^i_{k-1} + \nu_k^i
\]

\[
X^i_k = F(\omega)X^i_{k-1} + \nu_k^i
\]

where \( \nu_k^i \) denotes the zero-mean white Gaussian noise with covariance \( Q \).

The measurement model of each sensor can be expressed as

\[
X^i_k = \tan^{-1} \left( \frac{y_k - y^i}{x_k - x^i} \right) + \nu_k^i
\]

where \( \nu_k^i \) denotes the position of the \( i_{th} \) sensor, \( i = 1, 2, 3 \). \( \nu_k^i \) denotes the zero-mean Gaussian noise with variance \( \sigma_w^2 = 0.05 \text{rad}^2 \).

It is noted that the traditional PF-PHD and MM-PHD algorithms should know the newborn target distribution, while the proposed algorithm should not; thus we assume that
the newborn intensity for the PF-PHD and MM-PHD algorithms is

$$I_k^{(i)}(x) = \sum_{i=1}^{4} 0.1N(x; m_i^{(i)}, p_i^{(i)})$$ (49)

where $i = 1, 2, 3, 4$, $(m_i^{(i)}, p_i^{(i)})$ denotes the newborn component parameter of the $i_{th}$ target. The experiment was performed on an ASUS PC with Core i5-7300 processor and 16 GB memory using the MATLAB 2018 simulation software. The optimal sub-pattern assignment (OSPA) metric [27] is employed to evaluate the state estimate precision of each algorithm with cut-off parameter $c = 100$ and order parameter $p = 2$. Moreover, the average number estimates and their root mean square error (RMSE) are used for performance evaluation of the number estimates. The simulation results are obtained from Monte Carlo experiments of 100 ensemble runs.

### 4.1 Cross multi-target tracking scenario

In this experiment, assume that there are three targets making a crossing motion. Targets 1 and 2 start moving from the initial positions $(20, -20)$ m with a velocity $(-1, 1)$ m/s and $(-40, -20)$ m with a velocity $(1, 1)$ m/s, and finally 60 s. Target 3 starts moving at the 20th second from the initial position $(30, 0)$ m with a velocity $(2, 0)$ m/s, and disappears at the 50th second. Their motion trajectories are shown in Figure 5. The clutter is modelled as a Poisson distribution with the average clutter rate $r = 3$ over the observation space. The variance of the process noise is assumed as $\sigma_k^2 = 0.8 \text{ m}^2/\text{s}^{-3}$. The survival probability and detection probability of the target are set as $P_{S,k} = 0.99$ and $P_{D,k} = 0.98$, respectively. It is assumed that the number of particles for the PF-PHD filter is 1500, and the experimental results are shown in Figures 6, 7, and 8.

Figure 6 shows the comparison of the OSPA distances of the proposed GD-PHD and GD-CPHD filters and the traditional PF-PHD filter. It is clear that the tracking accuracy of the proposed algorithms, GD-PHD and GD-CPHD, is significantly better than the traditional PF-PHD filter, and the proposed algorithm, relative to the PF-PHD filter, can adaptively identify newborn targets. It does not need a priori information of newborn targets and has a better application ability for unknown scenes. In addition, since the proposed algorithm GD-CPHD has a better cardinalised distribution estimation ability, it is obviously superior to the GD-PHD filtering algorithm in estimation accuracy. It is noted that when the third target disappears at the 50th second, the OSPA distance of the GD-CPHD algorithm is higher than that of the other two algorithms. The reason is that the missed detection problem [17] has been cured in the CPHD-based method, which is beneficial when missed detection really occurs but is harmful when targets really disappear.
Figure 7 shows the cardinalised estimation of the proposed GD-PHD and GD-CPHD filters and the traditional PF-PHD filter, and Figure 8 shows the average number estimates of targets for the different algorithms. As can be seen, the proposed algorithms are superior to the PF-PHD algorithm in cardinalised estimation.

In order to further verify the stability of the proposed algorithm, Table 2 gives a comparison of the tracking accuracy of different algorithms with different process noises. It is clear that the proposed algorithms, GD-PHD and GD-CPHD, are not sensitive to the process noise, while PF-PHD is sensitive. As the process noise increases, the average OSPA distance of PF-PHD also increases significantly.

4.2 Manoeuvring a multi-target tracking scenario

Suppose there are four manoeuvring targets in the experimental scenario. The manoeuvring parameters of the turn rate are set as $\omega = \pm 0.1$ rad/s, and the initial positions of the targets 1 and 2 are $(10, 30)$ and $(-55, -30)$ m. Target 3 starts moving at the $20^{th}$ second and disappears at the $50^{th}$ second. Target 4 starts moving at the $10^{th}$ second and disappears at the $60^{th}$ second. Their initial states and covariance can be described as $m^{(1)}_0 = [10 m, -1.5 m/s, 30 m, -1.5 m/s]$, $m^{(2)}_0 = [-55 m, 1.5 m/s, -30 m, -1.5 m/s]$, $m^{(3)}_0 = [-60 m, 1 m/s, 50 m, 1.5 m/s]$, $m^{(4)}_0 = [-30 m, 0 m/s, -60 m, 2 m/s]$.

The real trajectories of the four targets are shown in Figure 9. It is noted that the proposed algorithm can track the targets with arbitrary motion trajectories, since the proposed algorithm is not disturbed by the motion model and unknown newborn targets. The clutter is modelled as a Poisson distribution with an average rate $r = 3$ in the observation space. The variance of the process noise is assumed as $\sigma^2_x = 0.01$ m$^2$/s$^{-2}$. The probabilities of survival and detection of the targets are $P_{S_k} = 0.99$ and $P_{D_k} = 0.98$, respectively. 1500 particles are implemented in the PF-PHD algorithm. For the MM-PHD algorithm, assume the model set includes three different models, that is, CV and CT with turning rate $+\omega$ and $-\omega$, respectively. For testing the different tracking performance of the MM-PHD, we set the turning rate $\omega$ as $\pm 0.01$, $\pm 0.05$, $\pm 0.1$, $\pm 0.2$, and $\pm 0.5$ for different model combinations. The experimental results are shown in Figures 10, 11 and 12, and Tables 3 and 4.

Figure 10 shows the OSPA distances of GD-PHD, GD-CPHD, MM-PHD and PF-PHD. It is clear that the proposed GD-PHD and GD-CPHD algorithms have a higher tracking accuracy than PF-PHD. For MM-PHD, when the model manoeuvre parameters are accurate (e.g. $\omega = \pm 0.1$), the tracking accuracy of the model is higher than that of the others; moreover, the tracking accuracy is also affected by the mismatched models in the model set. If the models cannot be matched with the real manoeuvring models, the tracking accuracy will decrease. Therefore, the proposed GD-PHD and GD-CPHD algorithms have a better tracking performance than the MM-PHD algorithm without having prior information about the dynamic model. In addition, the proposed algorithms also do not require a priori information of the newborn targets due to their good adaptive capability for arbitrary MTT. Compared to the GD-PHD, the GD-CPHD
has a higher accuracy due to its additional estimation of the cardinalised distribution.

Figure 11 shows the cardinalised estimation of the proposed GD-PHD and GD-CPHD algorithms and the MM-PHD and PF-PHD filters. Figure 12 shows the RMSE of the number estimate. As can be seen, the proposed algorithm is also significantly better at cardinalised estimation than the MM-PHD and PF-PHD algorithms. For the PF-PHD and MM-PHD algorithms, some targets are missed estimations due to the mismatched models when the targets make a manoeuvring motion; therefore, the RMSEs of the two methods are higher than the proposed methods.

Table 3 shows a comparison of the running time of the different algorithms. The computational cost of GD-CPHD is higher than that of GD-PHD due to the extra calculation required for cardinalised distribution estimation, but it is lower than that of MM-PHD that involves interactive operation of multiple models. The PF-PHD method has the fastest operation speed. This is there is no extra computation of model interaction and/or cardinalised distribution estimation in the PF-PHD filter. However, it has the worst tracking performance for manoeuvring target tracking compared to the other algorithms because only the linear model is used in the PF-PHD filter, and it does not have the tracking capability for tracking manoeuvring targets.

| Algorithm  | GD-PHD | GD-CPHD | PF-PHD | MM-PHD |
|------------|--------|---------|--------|--------|
| Time (s)   | 17.8378 | 20.7282 | 5.2942 | 25.2974 |

Abbreviations: CPHD, cardinalised PHD; GD-PHD, grid driven PHD; MM-PHD, multiple-model PHD; PF-PHD, particle filter PHD; PHD, probability hypothesis density.

| Algorithm | The variance $\sigma^2$ of the process noise (m$^2$s$^{-2}$) |
|----------|-------------------------------------------------|
|          | 0.01       | 0.05       | 0.1        | 0.3        | 0.5        |
| GD-PHD   | 8.3247     | 8.5614     | 9.0647     | 9.1478     | 9.3678     |
| GD-CPHD  | 7.2896     | 7.5214     | 8.0278     | 8.2471     | 8.3567     |

Abbreviations: CPHD, cardinalised PHD; GD-PHD, grid driven PHD; MM-PHD, multiple-model PHD; PF-PHD, particle filter PHD; PHD, probability hypothesis density.

| Algorithm | Clutter rate |
|----------|--------------|
|          | 3     | 5     | 10    | 20    |
| GD-PHD   | 8.3247 | 8.7637 | 9.4012 | 11.9501 |
| GD-CPHD  | 7.2896 | 7.7968 | 8.5630 | 10.6144 |

Abbreviations: CPHD, cardinalised PHD; GD-PHD, grid driven PHD; MM-PHD, multiple-model PHD; PF-PHD, particle filter PHD; PHD, probability hypothesis density.
In order to further verify the stability of the proposed algorithm, Table 4 shows a comparison of the tracking accuracy for the GD-PHD and GD-CPHD algorithms under a maneuvering scenario with different process noises. It is clear that the average OSPA error is stable without big oscillations for different process noises, demonstrating that the proposed algorithms, GD-PHD and GD-CPHD, are not sensitive to process noise, with a good tracking performance.

Table 5 shows the performance of the proposed methods under different clutter levels. The clutter rates are set as $r = 3, 5, 10, 20$, $\sigma^2_c = 0.01$ m$^2$s$^{-3}$, and $P_{D_k} = 0.98$. Table 6 shows the performance of the proposed methods under different detection probabilities. The detection probabilities are set as $P_{D_k} = 0.98, 0.9, 0.8, 0.7$ and $\sigma^2_c = 0.01$ m$^2$s$^{-3}$, $r = 3$. As can be seen, although the clutter rate increases dramatically or the detection probability decreases sharply, the average OSPA distances of GD-PHD and GD-CPHD increase slightly, which demonstrates that the proposed algorithms, GD-PHD and GD-CPHD, are also not sensitive to the different clutter rates and the different detection probabilities.

### CONCLUSIONS

In order to overcome the shortcomings of traditional PHD and CPHD filtering algorithms for MTT, with an unknown dynamic model and the newborn target distribution, improved GD algorithms are proposed, that is, the GD-PHD and GD-CPHD filtering algorithms, which can adaptively adjust the position and the size of the grids and identify newborn targets according to the measurements and the grid resolution. The dynamic tendency of the grids through the shrinkage and expansion operator can respond to the unknown arbitrary dynamic models. Experimental results show that the proposed algorithms have a better tracking performance than the traditional PF-PHD and M-CPHD filtering algorithms for arbitrary motion targets, and they do not require a priori intensity information for unknown newborn targets.

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### REFERENCES

1. Mahler, R.: Multitarget Bayes filtering via first-order multitarget moments. IEEE Trans. Aero. Electron. Syst. 39(4), 1152–1178 (2004)
2. Mahler, R.: PHD filters of higher order in target number. IEEE Trans. Aero. Electron. Syst. 43(4), 1523–1543 (2007)
3. Vo, B.T., Vo, B.N., Cantoni, A.: Bayesian filtering with random finite set observations. IEEE Trans. Signal Process. 56(4), 1313–1326 (2008)
4. Vo, B.N., Singh, S., Doucet, A.: Sequential Monte Carlo methods for multitarget filtering with random finite sets. IEEE Trans. Aero. Electron. Syst. 41(4), 1224–1245 (2005)
5. Vo, B.T., Vo, B.N., Cantoni, A.: Analytic implementations of the cardinalized probability hypothesis density filter. IEEE Trans. Signal Process. 53(7), 3553–3567 (2007)
6. Vo, B.N., Ma, W.K.: The Gaussian mixture probability hypothesis density filter. IEEE Trans. Signal Process. 54(11), 4091–4104 (2006)
7. Schlangen, L., et al.: A second-order PHD filter with mean and variance in target number. IEEE Trans. Signal Process. 66(1), 48–63 (2018)
8. Li, C., et al.: PHD and CPHD filtering with unknown detection probability. IEEE Trans. Signal Process. 66(14), 3784–3798 (2018)
9. García-Fernández, Á.F., Svensson, L.: Trajectory PHD and CPHD filters. IEEE Trans. Signal Process. 67(22), 5702–5714 (2019)
10. Jortner, F., Hernández, S., Vergara, D.: Probability hypothesis density filter using determinantal point processes for multi-object tracking. Comput. Vis. Image Understand. 183, 33–41 (2019)
11. Hernandez, M.I., et al.: Performance measure for Markovian switching systems using best-fitting Gaussian distributions. IEEE Trans. Aero. Electron. Syst. 44(2), 724–747 (2008)
12. Li, X.R., Jilkov, V.P.: Survey of maneuvering target tracking. Part V: multiple-model methods. IEEE Trans. Aero. Electron. Syst. 41(4), 1255–1321 (2005)
13. Pasha, S.A., et al.: A Gaussian mixture PHD filter for jump Markov system models. IEEE Trans. Aero. Electron. Syst. 45(3), 919–936 (2009)
14. Pasha, S.A., Tuan, H.D., Apkarian, P.: The LFT based PHD filter for nonlinear jump Markov models in multi-target tracking. In: Proceedings of the 48th IEEE Conference on Decision and Control, pp. 5478–5483. IEEE, Shanghai (2009)
15. Li, W.L., Jia, Y.M.: Gaussian mixture PHD filter for jump Markov models based on best-fitting Gaussian approximation. Signal Process. 91(4), 1036–1042 (2011)
16. Punithakumar, K., Kirubarajan, T., Sinha, A.: Multiple-model probability hypothesis density filter for tracking maneuvering targets. IEEE Trans. Aero. Electron. Syst. 44(1), 87–98 (2008)
17. Yang, J.L., Ji, H.B., Ge, H.W.: Multi-model particle cardinality-balanced multi-target multi-Bernoulli algorithm for multiple maneuvering target tracking. IET Radar, Sonar Navig. 7(2), 101–112 (2013)
18. Rajiv, S., et al.: Multiple model spline probability hypothesis density filter. IEEE Trans. Aero. Electron. Syst. 52(3), 1210–1226 (2016)
19. Sarkka, S., Nummenmaa, A.: Recursive noise adaptive Kalman filtering by variational Bayesian approximations. IEEE Trans. Automat. Contr. 54(3), 596–600 (2009)
20. Li, W.L., Jia, Y.M.: State estimation for jump Markov linear systems by variational Bayesian approximation. IET Control Theory & Appl. 6(2), 319–326 (2012)
21. Gao, X., et al.: Multi-sensor centralized fusion without measurement noise covariance by variational Bayesian approximation. IEEE Trans. Aero. Electron. Syst. 47(1), 718–727 (2011)
22. Yang, J.L., Ge, H.W.: Adaptive probability hypothesis density filter based on variational Bayesian approximation for multi-target tracking. IET Radar, Sonar Navig. 7(9), 959–967 (2013)
23. Zhang, G., et al.: An improved PHD filter based on variational Bayesian method for multi-target tracking. In: Proceedings of the 17th International Conference on Information Fusion, pp. 1–6. IEEE, Salamanca (2014)

24. Bucy, R.S., Senne, K.D.: Digital synthesis of non-linear filters. Automatica. 7(3), 287–298 (1971)

25. Kramer, S.C., Sorenson, H.W.: Recursive Bayesian estimation using piece-wise constant approximations. Automatica. 24(6), 789–801 (1988)

26. Yang, Y., Zhao, Y., Kyas, M.: RBGF: recursively bounded grid-based filter for indoor position tracking using wireless networks. IEEE Commun. Lett. 18(7), 1234–1237 (2014)

27. Schuhmacher, D., Vo, B.T., Vo, B.N.: A consistent metric for performance evaluation of multi-object filters. IEEE Trans. Signal Process. 56(8), 3447–3457 (2008)

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