What do heavy-light ($Qar{q}$) quark systems tell us about QCD vacuum properties?

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Abstract

Arguments in favor of a large magnitude (at least two-three times bigger than its factorized value) of the mixed vacuum condensate $\langle \bar{q} G_{\mu\nu}^a G_{\mu\nu}^a q \rangle$ are given. The analysis is based on the strict inequalities which follow from the QCD sum rules method and on very plausible phenomenological assumptions like $m_{B_s} > m_{B_u}$ for the few lowest exited $Q\bar{q}$ states in a heavy quark limit $m_Q \to \infty$. The same arguments show the suppression of the $SU(3)$ symmetry breaking effects for vacuum condensates when the additional gluon fields are included.

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1. Formulation of the problem

It is widely believed that the QCD vacuum has a very complex structure. To understand this structure is a very ambitious goal which assumes the solution of a whole spectrum of tightly connected problems, such as confinement, chiral symmetry breaking phenomenon, the $U(1)$ problem, the $\theta$ dependence problem and many, many others.

A less ambitious purpose is the phenomenological study of the QCD vacuum structure from the known experimental data. The most appropriate analytical approach which makes contact between fundamental theory, QCD, from the one side and observable variables from the other side, is the QCD sum rules [1],[2]. This method has been used extensively for about fifteen years for various purposes, mainly for the extraction of information on hadronic properties. It is important to stress that the enormous wealth of data referring to the low energy hadronic physics obtained in this way, is determined by only a few fundamental characteristics

$$\langle \bar{q}q \rangle \simeq -(0.25 GeV)^3$$

$$\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle \simeq 1.2 \cdot 10^{-2} GeV^4$$

(1)

$$\langle \bar{q} q G_{\mu\nu} G_{\lambda\sigma} \frac{\lambda^a}{2} q \rangle \simeq m_0^2 \langle \bar{q} q \rangle, \quad m_0^2 \simeq 0.8 GeV^2$$

Here $q$ describes any of the light quark fields $u, d, s$ and $G_{\mu\nu}$ is the field of colored gluons.

The value of the chiral condensate $\langle \bar{q} q \rangle$ has been known for a long time [3] and the value of the gluon condensate has been first extracted from the analysis on charmonium system [1]. The value of the mixed quark-gluon condensate has been extracted first from the analysis of the nucleon [4].

It is clear that the QCD vacuum is a very complicated system and a few global dimensional parameters (1) give only some insight on the scale of vacuum fluctuations, but do not at all exhaust all characteristics of the vacuum structure.

For studying of the fine aspects of the theory it is necessary to know the averages of more complicated operators like $\langle (\bar{q}q)^2 \rangle$, $\langle \bar{q} G_{\mu\nu} G_{\mu\nu} q \rangle$, $\langle G_{\mu\nu}^3 \rangle$, $\langle \bar{q} q \rangle$,
\langle G_{\mu \nu}^4 \rangle$, etc. Comparing these averages with global characteristics\(^1\) gives a quantitative information of the granulated structure of the vacuum, characterizes the correlation between vacuum fluctuations of large ($\sim R_{\text{conf}}$) and small ($\sim \text{GeV}^{-1}$) size, gives some insight on the role of the interaction of the various vacuum fluctuations, etc. For instance, if the vacuum would be homogeneous one, we would get more or less an uniform distribution for gluonic $G_{\mu \nu}$ and quark $\bar{q}q$ vacuum fields. The factorization hypothesis should work in this case perfectly well and predicts that $\langle G_{\mu \nu}^4 \rangle \simeq \langle G_{\mu \nu}^2 \rangle^2$, $\langle (\bar{q}q)^2 \rangle \simeq \langle \bar{q}q \rangle^2$. At the same time, some granulated distribution of the vacuum fluctuations, like instantons, gives a very strong deviation from this prediction. In particular, the instanton liquid model\(^2\) predicts the violation of the factorization hypothesis on the level of the factor 3-10 (numerical factor depends on color and Lorentz structure of the operator).

Let us note that the status of this hypothesis for four-quark condensates $\langle (\bar{q} \Gamma q)^2 \rangle$ (dimension six) has been examined in many different ways (see \cite{2} for review). Several channels have been studied with the special task of detecting deviations from the factorization for the four-quark operators. The conclusion is as follows: in vector and axial-vector cases ($\Gamma = \gamma_\mu, \gamma_\mu \gamma_5$) deviation from factorization formula cannot significantly exceed $\sim 20\%$. On the contrary, it can be shown\(^3\) that this situation is not universal and in exotic cases with ($\Gamma = \sigma_{\mu \nu}, \gamma_5, 1$), a noticeable violation of factorization does occur\(^4\).

The purpose of this letter is to obtain phenomenological, model independent information on mixed vacuum condensates of dimension seven $\langle \bar{q} G_{\mu \nu} G_{\mu \nu} q \rangle$, which are minimal possible operators where factorization procedure can be applied ($\langle \bar{q} G_{\mu \nu} G_{\mu \nu} q \rangle \sim \langle \bar{q} q \rangle \cdot \langle G_{\mu \nu} G_{\mu \nu} \rangle$) and checked. In addition, we study the $SU(3)$ breaking effects for these mixed condensates. Some motivation (apart from the academic interest concerning QCD vacuum structure) for the considering these VEVs will be explained in the conclusion.

Before we proceed, let us describe briefly the main idea of the approach which we follow in this paper. As mentioned above, the QCD sum rules make contact between vacuum characteristics through their condensate values and some hadronic properties. It turns out that the heavy-light ($Q \bar{q}$) quark system is a very sensitive to the high dimensional condensates like $\langle \bar{q} G_{\mu \nu} \sigma_{\mu \nu} q \rangle$, $\langle \bar{q} G_{\mu \nu} G_{\mu \nu} q \rangle$... Thus, some hadronic characteristics are determined by these

\(^2\)Some qualitative arguments on why it should be so, are given in the last section.
vacuum expectation values (VEVs).

Once this is said, the standard line of reasoning (very roughly) is the following. The magnitude of the condensate gives, on one hand, the characteristic scale where deviation from the asymptotic behavior is still under control. On the other hand the same scale defines some dimensional hadronic parameters which, in principle, can be extracted by applying some fitting procedure.

We go in the opposite direction. We are not interested in calculation of any hadronic parameters in this paper. We are not calculating, in particular, $m_{B_s}$ or $m_{B_u}$ in the limit $m_Q \to \infty$. We are not going to prove the power of the method by demonstrating that $m_{B_s} - m_{B_u} \simeq 100\text{MeV}$ in agreement with experimental data. Instead, we assume that the QCD sum rules describe this system well enough. In particular they should explain the inequality $m_{B_s} > m_{B_u}$ which we assume is still valid in a heavy quark limit $m_Q \to \infty$. Because of the sensitivity of the corresponding sum rules (which we are going to consider) to the higher dimensional condensates, this assumption ($m_{B_s} > m_{B_u}$) gives a very nontrivial restriction on the absolute values of these VEVs. Such restrictions are the main result of this paper.

The next part of the paper is devoted to the description of the method we are going to use. We consider the well-known $\rho - \phi$ system and find the inequality which follows from the fact that $m_{\phi} > m_{\rho}$. The obtained restriction is a very weak and not interesting. The only justification for including this section to the paper is to formulate an idea with the simplest example. The third part of the paper is its main part where we apply the previously formulated idea to $Q\bar{q}$ system. Some comments are given in the conclusion.

2. The Method.

We introduce the $J_\mu^{(\phi)} = \bar{s}\gamma_\mu s$ current with the $\phi$ meson quantum numbers and define the corresponding polarization operator in a standard way:

$$i \int e^{iqx} dx \langle T \{ J_\mu^{(\phi)}(x), J_\nu^{(\phi)}(0) \} \rangle = (q_\nu q_\nu - q^2 g_{\mu\nu}) P^{(\phi)}(Q^2), \quad Q^2 \equiv -q^2. \quad (2)$$

The calculation of the correlator for $Q^2 \to \infty$ with the power corrections taken into account is done in the standard way, and after the Borelization
procedure the sum rule takes the following form \[1\]:

\[
\frac{1}{\pi} \int e^{-\frac{s}{M^2}} ImP^{(\phi)}(s)ds = \frac{M^2}{4\pi^2} \left\{ 1 - \frac{6m_s^2}{M^2} + \frac{8\pi^2 (m_s \bar{s}s)}{M^4} \right\},
\]

\[1\]

\[
+ \frac{\pi^2 \langle \frac{a_\mu G_{\mu\nu} G_{\mu\nu}}{M^4} \rangle}{3} - \frac{448}{81} \pi^2 \frac{\langle \bar{s}s \rangle^2}{M^6} + \ldots \}.
\]

(3)

The matrix elements entering to eq. (3) are discussed above and we assumed the factorization for four-quark operator with \( \Gamma = \gamma_\mu, \gamma_\mu \gamma_5 \) \[2\]. In principle, we could analyze this sum rule in a standard way in order to reproduce the well-known experimental data for \( \phi \) meson. It turns out that it is very useful to consider a slightly different sum rule by taking the first derivative with respect to \( M^2 \). In this case, the left hand side of eq.(3) is proportional to \[ \frac{1}{\pi} \int e^{-\frac{s}{M^2}} ImP^{(\phi)}(s)ds \] If the \( \phi \) meson would saturate these sum rules, we would get:

\[
m_\phi^2 \simeq \frac{1}{\pi} \int e^{-\frac{s}{M^2}} ImP^{(\phi)}(s)ds = \frac{M^4}{4\pi^2} \left\{ 1 + \text{power corrections} \right\}.
\]

(4)

The same procedure can be done for the \( \rho \) meson with substitution \( m_s \rightarrow m_{u,d} \) and \( \langle \bar{s}s \rangle \rightarrow \langle \bar{u}u \rangle \) \[1\]. Now it is time to introduce the new parameter \( R(M^2) \) defined as

\[
R(M^2) \equiv \frac{\frac{1}{\pi} \int e^{-\frac{s}{M^2}} ImP^{(\phi)}(s)ds}{\frac{1}{\pi} \int e^{-\frac{s}{M^2}} ImP^{(\rho)}(s)ds} = \frac{\frac{1}{\pi} \int e^{-\frac{s}{M^2}} ImP^{(\phi)}(s)ds}{\frac{1}{\pi} \int e^{-\frac{s}{M^2}} ImP^{(\rho)}(s)ds}
\]

(5)

If the lowest state would saturate the corresponding sum rule, we would get

\[
R(M^2) \simeq \frac{m_\phi^2}{m_\rho^2} > 1.
\]

(6)

The important thing for us in this paper is that \( R > 1 \). Intuitively it is clear and related to the fact that the mass of the strange meson (or meson built from the strange quarks) is bigger than the mass of its nonstrange

\[\footnote{we assume an exact isotopical invariance and do not distinguish \( \langle \bar{u}u \rangle \) and \( \langle \bar{d}d \rangle \).}
partner. More important, this inequality $R > 1$ is still correct even without
the assumption on saturating the dispersion integral by the lowest state.

We have to make a few very mild assumptions about the first $2 - 3$ lowest
states, whose contribution to the dispersion integral is not suppressed by the
weight factor $e^{-\frac{s}{M^2}}$. For those states we assume that the masses and the
corresponding coupling constants of the strange particles bigger than their
non-strange analogues (for example, $f_{K_i} > f_{\pi_i}$, $f_{\phi_i} > f_{\rho_i}$, $m_{\phi_i} > m_{\rho_i}$, $i = 1, 2, 3...$). An enormous wealth of experimental data and physics intuition tell
us that it is definitely should be true. In this case, we are quite sure that the
inequality $R > 1$ holds even when saturation by the lowest resonance does
not take place.

Taking into account the previous discussing and substituting the theor-etical parts of the corresponding sum rules into the expression (5) we get

$$R = 1 + \Delta R_1 + \Delta R_2$$

$$\Delta R_1 = 6 \frac{m_s^2}{M^2}, \quad \Delta R_2 = -16\pi^2 \frac{\langle m_s \bar{s}s \rangle}{M^4} - \frac{448}{27} \alpha_s \frac{\langle \bar{u}u \rangle^2 - \langle \bar{s}s \rangle^2}{M^6} + ...$$

We divided $\Delta R = \Delta R_1 + \Delta R_2$ on purpose. $\Delta R_1$ in the above equation
denotes the perturbative contribution to $\Delta R$ and $\Delta R_2$ denotes the correspond-
ing nonperturbative part. They have absolutely different physical meaning
and have different origin. It is clear that $\Delta R_1$ is related to the perturbative
contribution. The insertion (in order to calculate the polarization operator
of the nonzero quark mass ($m_s$) to the asymptotic quark loop clearly
decreases the phase volume. It automatically leads to the increasing of the
corresponding hadronic mass. The fact $\Delta R_1 > 0$ is the explicit demonstra-
tion of it. The contributions to $\Delta R_1$ come exclusively from the small distance
physics. In a sense, $\Delta R_1$ describes some trivial kinematical factors; there is
no deep, nonperturbative physics, involved with $\Delta R_1$.

In the rest of this paper, we will be interested in the nonperturbative
part of the decomposition (5), namely $\Delta R_2$. One may wonder whether the
decomposition (5) is unique. We refer to the original paper [1] for a detail

\footnote{We assume that the $SU(3)$ breaking effects are small enough in order to use an ex-
ansion with respect to this small parameter. It is clear that in the exact $SU(3)$ limit, the
ratio $R = 1$.}
discussion of Wilson operator expansion (OPE) in given context, but here we want to stress that the decomposition has the same status as the separation of all field fluctuations in large and small scales within an (OPE) and so it is well defined and based on solid ground.

There is no general theorem concerning the sign of \( \Delta R_2 \). However, the physical picture of the origin of the hadron mass assumes its nonperturbative nature. Let us note, in passing, that in the chiral limit, the mass of hadrons comes exclusively from nonperturbative physics.

Thus, it is natural to assume that the same, nonperturbative effects are responsible for the mass difference of a hadron and its partner with strange quarks. It is parametrically true in the chiral limit \( m_s \to 0 \) for the \( \rho - \phi \) system. In this case, \( \Delta R_1 \sim m_s^2, \Delta R_2 \sim m_s \gg \Delta R_1 \). As we know \( m_\phi^2 - m_\rho^2 \sim m_s \) and so, the mass difference in this case is determined exclusively by the nonperturbative part.

In \( Q\bar{q} \) system we are going to discuss in the next section, both parts \( \Delta R_1, \Delta R_2 \) have the same order of magnitude \( \sim m_s \). However, we believe that the nonperturbative effects, as well as perturbative ones, work in the \textit{same} direction in order to split the strange and nonstrange partners. All sum rules, I am aware of, satisfy this assumption, and the formal expression which is given by formula

\[
\Delta R_2 > 0. \tag{9}
\]

For \( \phi - \rho \) system this restriction leads to the following inequality:

\[
| m_s \langle \bar{s}s \rangle | > \frac{28 \pi \alpha_s}{27 M_0^2} (\langle \bar{u}u \rangle^2 - \langle \bar{s}s \rangle^2), \tag{10}
\]

where \( M_0^2 \simeq m_\phi^2 \simeq 1 GeV^2 \) is the characteristic scale when the corresponding sum rule does make sense. Let me briefly explain this point. As usual, the analysis of the sum rules is somewhat of an art. We could take parameter \( M_0 \) to be a very large; in this case the next power corrections could be definitely neglected, but the resulting inequality would be trivial and non-informative. If we take \( M_0 \) to be too small, there is no guarantee that a higher power corrections will not dominate. Our choice of \( M_0 \), when the main power correction is less then 20% of the asymptotically leading term, is

\footnote{The sign of \( \Delta R_1 \) is always positive by kinematics.}
the safe one in order to neglect other power corrections; usually this parameter numerically is very close to the position of the lowest resonance in a given channel. In particular, for the $\rho$ meson, $M_0^2(\rho) \simeq 0.6\text{GeV}^2$, for $K^*$ and $\phi$ it is $M_0^2(K^*) \simeq 0.8\text{GeV}^2$, $M_0^2(\phi) \simeq 1.0\text{GeV}^2$ respectively. Starting exactly from these points, the “theoretical” and “phenomenological” parts of the sum rules fit each other. From the other side, we are still in the transition region, and very sensitive to VEVs. To be safe enough for the numerical estimations, we will choose the parameter $M_0$ to be bigger than any lowest resonance involved in our analysis. In particular, for $\rho - \phi$ system we take $M_0 \simeq 1\text{GeV}^2$.

For this choice we are quite sure that the next power corrections are small enough and under control.

Now, a few remarks regarding eq. (10). First of all, as it was expected, both parts of this expression are proportional to $m_s$. In addition, because we know the $SU(3)$ breaking effects for the condensate (see reviews [6], [8], [9] and references on previous papers therein),

$$\Delta \equiv \frac{\langle \bar{u}u \rangle - \langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \simeq 0.2,$$

and one can see that the formula (11) is trivially satisfied. No new results can be obtained in this particular case. However, in the next section we will derive an analogous inequality where instead of $\langle \bar{u}u \rangle - \langle \bar{s}s \rangle$, there enters the higher dimensional operators like $\langle \bar{u}i\gamma \sigma_{\mu\nu}G_{\mu\nu}u \rangle - \langle \bar{s}i\gamma \sigma_{\mu\nu}G_{\mu\nu}s \rangle$ or $\langle \bar{u}g^2G_{\mu\nu}G_{\mu\nu}u \rangle - \langle \bar{s}g^2G_{\mu\nu}G_{\mu\nu}s \rangle$ which are less familiar. In this case we can study the influence of the gluonic fields on the $SU(3)$ breaking parameters and check the standard conjecture about weak influence of the gluonic fields on the $SU(3)$ breaking parameters, i.e.

$$\Delta = ? = \Delta_g = ? = \Delta_{gg} = ? = ...$$

$$\Delta_g \equiv \frac{\langle \bar{u}i\gamma \sigma_{\mu\nu}G_{\mu\nu}u \rangle - \langle \bar{s}i\gamma \sigma_{\mu\nu}G_{\mu\nu}s \rangle}{\langle \bar{u}i\gamma \sigma_{\mu\nu}G_{\mu\nu}u \rangle}, \quad \Delta_{gg} \equiv \frac{\langle \bar{u}g^2G_{\mu\nu}G_{\mu\nu}u \rangle - \langle \bar{s}g^2G_{\mu\nu}G_{\mu\nu}s \rangle}{\langle \bar{u}g^2G_{\mu\nu}G_{\mu\nu}u \rangle}.$$  

If we knew the $\Delta_g$ and $\Delta_{gg}$ parameters independently, the expression analogous to (11) would give us information on absolute value of the condensate. In the example under consideration it leads to

$$\begin{align*}
(0.25\text{GeV})^3 \simeq & \ |\langle \bar{u}u \rangle| < \frac{m_s M_0^2}{\Delta} \frac{27}{56\pi\alpha_s} \simeq (0.55\text{GeV})^3.
\end{align*}$$

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We expect an accuracy of such inequalities is about $\sim 20\%$, as the standard accuracy of the sum rules approach. The next section is devoted to $Q\bar{q}$ system where some more interesting operators (instead of $\langle \bar{u}u \rangle$) will enter into the game. In this case an inequality analogous to (13) will be less trivial and much more interesting.

### 3. Heavy-light quark system

Introduce the $J = \bar{Q}i\gamma_5 q$ current with the quantum numbers of the pseudoscalar meson and define the corresponding polarization operator in a standard way:

$$i \int e^{iqx} dx \langle T\{J(x)^+, J(0)\}\rangle = P(q^2). \quad (14)$$

We use below the technique proposed in [10], i.e. the energy $e$ is used instead of $q^2 : q^2 = (M_Q + e)^2, \ e \ll M_Q$. The advantage of this technique is its simplicity; the disadvantage is the existence of large $1/M_Q$ corrections for the real $D, B$-mesons. However, as we mentioned, we are not going to extract any information about real hadrons in this paper, we are going to study vacuum of QCD. In this case, with assumption that no qualitative changes will occur in the limit $m_Q \to \infty$, it is much easier to live in this imaginary world. Besides that, let us note, that the first calculations of $f_D, f_B$ within the sum rule approach have been done using exactly this technique [10],[11]; independently the same system has been investigated by the standard method in ref.[12]. In addition, this technique is very useful to make contact with heavy-quark effective theory [13], see recent paper [14].

A few remarks regarding new variable $e$ instead of $q^2$: as usual, we are going to calculate the correlation function in the nonphysical region, $e < 0$ (it is analogous to $q^2 < 0$ in the standard method). In the limit $E = -e \to \infty$, (but $E \ll M_Q$ in order to use heavy quark approximation), we can use the standard OPE and Borel transformation, defined by operator $\hat{B}$. In given case $\hat{B}$ takes the following form:

$$\hat{B} \equiv \lim_{E,n \to \infty; \frac{E}{n} = \text{const.}} \frac{1}{n!} E^n \left(-\frac{d}{dE}\right)^n \quad (15)$$

where the energy $E$ is positive and measured from the threshold. The physical meaning of the Borel transformation in this case much more clear than in
the relativistic case. Indeed, when we are passing from $E$ to $M$ we actually go from Green function at given energy to the time Green function. The contribution of each state with energy $E_i$ to the dispersion integral is given by formula $e^{-E_i/M}$ and $1/M$ plays the role of time $it$. It is clear that the OPE expansion $1/M^k$ at $M \to \infty$ corresponds to $t \to 0$, i.e. asymptotically small times. To study a low lying resonance we have to calculate the correlator at small enough $M$, which corresponds to large distance physics (large times $t$).

It is easy to check the following properties of operator $\hat{B}$:

$$\hat{B}(\frac{1}{E^k}) = \frac{1}{(k-1)!} (\frac{1}{M^k}), \quad \hat{B}(E^k \ln E) = (-1)^{k+1} k! M^k,$$

$$\hat{B}(\frac{1}{E + E_i}) = \frac{1}{M} \exp(-\frac{E_i}{M}).$$

After these few general remarks let us calculate the nonperturbative, Borel transformed part of $P_{np}(M)$ with strange $s$ quark in the current $J = \bar{Q}i\gamma_5 s$ :

$$P_{np} = \frac{1}{2M} \{-\langle \bar{s}s \rangle + \frac{1}{4M} \langle m_s \bar{s}s \rangle + \frac{1}{4M^2} \langle \bar{s}i\sigma_{\mu\nu}G_{\mu\nu}s \rangle\}, \quad P_p = \frac{1}{2M} \frac{3 \cdot 2! M^3}{\pi^2}.$$ (17)

We removed the common factor $1/2M$ on purpose—the contribution of any physical resonance to the dispersion integral has the same factor $1/2M$ and after Borelization takes the following form:

$$\hat{B} \frac{1}{\pi} \int \frac{ImP(s)ds}{s + Q^2} \to \frac{M_p^2}{M_Q^2} \frac{1}{2M} \frac{1}{M_Q} e^{-\frac{E_R}{M}}.$$ (18)

where we used the standard definition for $f_P$

$$\langle 0|\bar{Q}i\gamma_5 q|P_Q \rangle = f_P \frac{M_p^2}{M_Q}.$$ (19)

Besides that, the factor $1/M$ in (18) is related to Borel transformation (13), and $1/M_Q$ comes from nonrelativistic $\delta$ function $\delta(s - M_p^2) = \delta(2M_Q(E - E_R)) = \frac{1}{2M_Q} \delta(E - E_R)$. We included to eq. (17) the formula for perturbative, asymptotically leading part as well. The reason to do so just to make sure that
at the scale we are going to discuss, the power corrections are much smaller than the leading term. But essentially, the only information we need to know about this term is its dimension, which can be found by pure dimensional arguments. The inequalities we are interested in and which follow from the our main nonperturbative assumption (10) do not depend on the constant in front of $P_p$ and we will not discuss the perturbative contribution any more.

Let us note that $P(M)$ does not depend on $M_Q$. From this observation it is easy to reproduce the well known result $f_p^2 M_Q \sim 1$ of the effective heavy quark theory.

Now we are ready to repeat all steps of the previous section which lead us to the eq.(10) from the definition (5) and assumption (9). For the given correlator (14) the analogous formula looks as follows

$$\langle \bar{s}s - \bar{u}u \rangle - \frac{1}{3M} \langle m_s \bar{s}s \rangle - \frac{5}{12M^2} \langle \bar{s}i g \sigma_{\mu\nu} G_{\mu\nu} s - \bar{u}i g \sigma_{\mu\nu} G_{\mu\nu} u \rangle > 0,$$

which we would like to rewrite in the following form:

$$\frac{5m_0^2}{12M^2} \Delta g < \frac{m_s}{3} + M \Delta,$$

where $m_0^2 \simeq 0.8 GeV^2$ is parameter for mixed condensate (11); $m_s \simeq 0.15 GeV$ and $\Delta \simeq 0.2$ (11). Before we proceed with numerical estimations, let me make a few comments on (20). It is clear, that as $M \to \infty$ this inequality is trivially satisfied because $\langle \bar{s}s - \bar{u}u \rangle = \Delta |\langle \bar{q}q \rangle| > 0$. It becomes less trivial in the transition region. In this case the last term, proportional to $\Delta g$, contributes with the opposite sign. The requirement of positivity of $\Delta R_2$ impose some limitation on the absolute value of $\Delta g$. So, our main task is to find parameter $M_0$ where higher power corrections can be safely neglected, but inequality will be still interesting. As we said above, we are not going to extend the sum rules approach on the region below the first resonance. In given case, from the QCD sum rules analysis in the heavy quark limit (see recent papers [14], [17], [16]) and from experimental data (we expect that B system already very close to heavy quark limit), we can learn that $M_{B_u} - M_Q \simeq 0.5 GeV$ and $M_{B_s} - M_Q \simeq 0.6 GeV$. So, our choice

$$M_0 \simeq 0.6 GeV$$

\[\text{There is no doubt in sign } \Delta g [13].\]
should be considered as safe enough. Power corrections, as can be checked from (17) already small. With this remark in mind we will get the following (rather weak, again) restriction:

\[ \Delta g < 0.3. \]  

(23)

It is definitely in agreement with what we knew before [15], [18]. Moreover, from the first estimate [15], we could suspect that the SU(3) breaking effects for mixed condensate are much weaker than for the standard \langle \bar{q}q \rangle condensate. It would be very interesting to check whether it is indeed true by some independent analysis. In order to do so, we want to consider the correlators, which include derivatives. The reason for this choice will be clear soon.

Introduce the \( J_\mu = \bar{Q}i\gamma_5 \vec{D}_\mu q \) current with the quantum numbers of the pseudoscalar meson and define the corresponding polarization operator in a standard way:

\[ i \int e^{iqx} dx \langle T \{ J_\mu(x)^+, J(0) \} \rangle = q_\mu P^{(1)}(q^2). \]  

(24)

Here \( \vec{D}_\mu = \partial_\mu - igA_\mu^a \frac{\lambda^a}{2} \) is the covariant derivative acting on the light quark. The physical sense of the corresponding matrix element

\[ \langle 0 | \bar{Q}i\gamma_5 \vec{D}_\mu q | P_Q(q_\mu) \rangle \]  

(25)

can be understood from the observation that \( \langle x_q \rangle \) determines the mean momentum fraction carried by the light quark in the meson \( P_Q \) with total momentum \( q_\mu \). This information is very important for description of the asymptotic behavior of exclusive processes; we refer to original paper [11] on this subject, but now we want to remark that \( \langle x_q \rangle \sim \frac{M_P}{M_Q} \ll 1 \) and so, the corresponding common factor \( \frac{1}{M_Q} \) will occur in phenomenological as well as in the theoretical parts. It leads, of course, to the increasing dimension of the \( P^{(1)}(q^2) \) on one unit in comparison with \( P(q^2) \) from (14).

As was mentioned in [11] the main power correction to the theoretical part of (24) is determined by the mixed operator in the chiral limit. So the corresponding correlation function is very sensitive to this operator and we have a good chance to improve our limit (23). This is the main motivation to consider (24) in this paper.
By repeating again all steps which lead us to the formula (17), we get the following Borel transformed expression for $P^{(1)}$:

$$
P^{(1)}_{np} = \frac{1}{2MMQ} \left\{ -\frac{4\alpha_sM}{3\pi} \langle \bar{s}s \rangle \left( 1 - e^{-\frac{s_0}{M}} \right) + \frac{1}{4} \langle m_s \bar{s}s \rangle + \frac{1}{8M} \langle \bar{s}g\sigma_{\mu\nu}G_{\mu\nu}s \rangle - \\
\frac{m_s}{32M^2} \langle \bar{s}g\sigma_{\mu\nu}G_{\mu\nu}s \rangle \right\}, \quad P^{(1)}_{(p)} = \frac{1}{2MMQ} \frac{3 \cdot 3!M^4}{\pi^2}.
$$

(26)

Few comments are in order. First of all, as was expected, the leading at $M \to \infty$ term proportional to chiral condensate $\langle \bar{q}q \rangle$ is suppressed by the loop correction $\sim \frac{\alpha_s}{\pi}$ and numerically small. This fact increases sensitivity to the next VEV $\langle \bar{s}g\sigma_{\mu\nu}G_{\mu\nu}s \rangle$. As the second remark, let us note the appearance of the continuum threshold parameter $s_0$ in the nonperturbative part $P^{(1)}_{np}$. This is the standard parameter which accompany all sum rules, but usually it occurs only in the perturbative parts. Its physical meaning can be seen from the standard model for the spectral density as the resonance plus continuum which starting at $s_0$. The reason of appearing such parameter in our case is the high dimensions of our currents. The consequence is the growth of the spectral density $\text{Im} P^{(1)}_{np}(s)$ with $s$. From the QCD sum rules analysis in the heavy quark limit [14],[17], [16], we can learn that

$$
s_0 = m_R + 0.5\text{GeV}.
$$

(27)

It is interesting to note that this “mnemonic rule” works amusingly well in all known massive channels. Anyhow, because the condensate contribution comes with the small factor $\sim \frac{\alpha_s}{\pi}$, the influence of $s_0$ is negligible. As usual we included to the formula (26) the perturbative part as well in order to demonstrate the smallness of the power correction at the scale $M_0 \simeq 0.6\text{GeV}$.

Let us rewrite the final inequality, analogous to (20) and which follows from our main conjecture [4]:

$$
\frac{4M\alpha_s}{\pi} \left( 1 - e^{-\frac{s_0}{2M}} - \frac{s_0}{3M} e^{-\frac{s_0}{2M}} \right) \langle \bar{s}s - \bar{u}u \rangle - \langle m_s \bar{s}s \rangle - \\
\frac{5}{8M} \langle \bar{s}g\sigma_{\mu\nu}G_{\mu\nu}s \rangle + \frac{3m_s}{16M^2} \langle \bar{s}g\sigma_{\mu\nu}G_{\mu\nu}s \rangle > 0.
$$

(28)
It can be rewritten in much more clear way, using our notations for $\Delta$ and $\Delta_g$ \cite{12}:

$$\frac{5m_0^2}{8M} \Delta_g < m_s + \Delta \frac{4M\alpha_s}{\pi} \left(1 - e^{-s_0\frac{m}{M}} - \frac{s_0}{3M} e^{-s_0\frac{m}{M}}\right) - \frac{3m_sm_0^2}{16M^2},$$

(29)

For numerical estimations we use the standard set of parameters already normalized at the low normalization point: $\mu \simeq 1 GeV; \ m_s \simeq 0.15 GeV; \ m_0^2 \simeq 0.8 GeV^2; \ \alpha_s(\mu) \simeq 0.34$. Exactly these parameters have been used in the analysis of $\bar{q}Q$ system in the heavy quark limit \cite{16} with the special attention on the loop corrections and we prefer not to change it. With these remarks in mind we will get the following very strong restriction on $\Delta_g$:

$$\Delta_g < 0.15.$$

(30)

This is in agreement with our feeling gathered from the absolutely independent consideration \cite{17}. The relation (30) means that the gluon insertion into the chiral condensate makes $\langle \bar{s}ig\sigma_{\mu\nu}G_{\mu\nu}s \rangle$ numerically very close to $\langle \bar{u}ig\sigma_{\mu\nu}G_{\mu\nu}u \rangle$.

Few words on the accuracy. We expect that the next power corrections can not exceed 10 – 20% anyhow. Besides that, we keep only linear $SU(3)$ breaking terms $\sim m_s$. In particular, $\langle \bar{u}u - \bar{s}s \rangle$ is treated as $\Delta$ and not as $\Delta(1 + \Delta)$. The term $\sim \Delta^2$ is order $m_s^2$ and should be discarded (or we must gather all these corrections at once). We expect that these next corrections $\sim m_s^2$ can not exceed 10 – 20%. Finally, the reason for our belief in (30) is that the analysis \cite{19} of the quite different system of Goldstone bosons, gives the same conclusion.

As a last remark concerning formula (29), we want to stress one more time that this is not a sum rule in the usual sense. We are not fitting it in order to extract some information on hadron properties. Instead, we do believe that the corresponding sum rules reproduce mass spectrum correctly, in particular, strange hadron heavier than its non-strange partner. It leads to some relations between VEVs. We expect that the obtained inequalities should hold in the region of variable $M$ where sum rules are supposed to be working.

With this remark in mind and in order to get some feeling on the high dimension condensate $\langle \bar{q}g^2G_{\mu\nu}G_{\mu\nu}q \rangle$, let us introduce the current $J_{\mu\nu} = \bar{q}g^2G_{\mu\nu}G_{\mu\nu}q$,
\[ \bar{Q}i\gamma_5 i\vec{D}_\mu i\vec{D}_\nu q \] and consider polarization operator

\[ i \int e^{iqx} dx \langle T \{ J^\mu(x)^+, J(0) \} \rangle = q_\mu q_\nu P^{(2)}(q^2) + g_{\mu\nu} P_\perp. \] (31)

We will be interested in consideration of \( q_\mu q_\nu \) kinematical structure only. The corresponding matrix element

\[ \langle 0 | \bar{Q}i\gamma_5 i\vec{D}_\mu i\vec{D}_\nu q | PQ(q_\mu) \rangle = q_\nu q_\mu f_P \frac{M_P^2}{M_Q} \langle x_q^2 \rangle + g_{\mu\nu} \ldots. \] (32)

determines the mean square of the longitudinal momentum carried by light quark. It is clear that \( \langle x_q^2 \rangle \sim \frac{M^2}{M_Q} \) and so we expect an additional suppression \( \frac{1}{M_Q} \) of the correlator in comparison with the previous case and increasing of its dimension on one unit more. An explicit calculation supports this expectation and the corresponding expressions for the perturbative \( P^{(2)}_p \) and nonperturbative \( P^{(2)}_{np} \) parts of (31) take the form

\[ P^{(2)}_{np} = \frac{1}{24M} \left\{ \frac{2\alpha_s M}{3\pi} \langle m_{q}\bar{s}s \rangle (1 - e^{-\frac{M^2}{\Lambda^2}}) + \frac{4\pi}{81M} \langle \sqrt{\alpha_s \bar{s}s} \rangle^2 - \frac{1}{24M} \langle \bar{q}g\sigma_{\mu\nu}G_{\mu\nu}s \rangle - \frac{1}{24M^2} \langle O_q \rangle \right\}, \quad P^{(2)}_p = \frac{1}{24M} \frac{4 \cdot 4! M^5}{\pi^2}. \] (33)

In this formula the operator \( \langle O_q \rangle \) has dimension seven and defined as follows

\[ \langle O_q \rangle = \langle \bar{q}g^2 G_{\mu\nu} G_{\mu\nu} q \rangle - \frac{1}{4} \langle \bar{q}g^2 \sigma_{\mu\nu} G_{\mu\nu} \sigma_{\mu\nu} G_{\mu\nu} \rangle \] (34)

where as usual \( G_{\mu\nu} \equiv G_{\mu\nu}^a \frac{\lambda^a}{2} \). The factorized value of this condensate is given by

\[ \langle O_q \rangle_f = \langle g^2 G_{\mu\nu}^a \rangle \cdot \langle \bar{q}q \rangle \cdot \left( \frac{1}{6} + \frac{1}{12} \right) \] (35)

where the coefficient \( \frac{1}{6} \) comes from the factorization of the first term (34) and \( \frac{1}{12} \) comes from the last one. Let us note that the sign of \( \langle O \rangle \) is determined uniquely from the inequality obtained below and independently from (19); it coincides with its factorized value (35). Thus we define

\[ \langle O \rangle = \langle O \rangle_f \cdot K, \quad K > 0 \] (36)

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and our task now is to find some restrictions on the coefficient of nonfactorizability $K$. To do so we follow the standard procedure which bring us to eqns. (21,29):

$$\frac{7\pi^2}{6M^2}\langle G^2_{\mu\nu}\rangle \cdot K \cdot \Delta_{gg} > -m_s \cdot 32M\alpha_s \frac{\langle 1 - e^{-\frac{M}{\mu}} - \frac{s_0}{3M} e^{-\frac{M}{\mu}} \rangle +}{5\pi}$$

$$\frac{m_s m_s^2}{M} - \frac{64\pi\alpha_s}{27M} |\langle \bar{q}q \rangle|.$$  

(37)

With the standard input parameters it gives:

$$K \cdot \Delta_{gg} > 0.38.$$  

(38)

As we argued above, the insertion of an additional gluon field into the condensate $\langle \bar{q}q \rangle$, leads to the suppression of the $SU(3)$ symmetry breaking effects in the obtained mixed condensate $\langle \bar{q}Gq \rangle$ of approximately one quarter: $\Delta_g/\Delta \simeq 0.75$. We expect that the same suppression holds for each of the next gluon insertions. Thus, for $0.75\Delta_g < \Delta_{gg} < \Delta_g$ we obtain:

$$K > 2.5 (\text{for } \Delta_{gg} = \Delta_g); \quad K > 3.4 (\text{for } \Delta_{gg} = 0.75\Delta_g),$$  

(39)

which is our main result. The independent analysis of the different system (Goldstone bosons, not $\bar{Q}q$ mesons) supports this result and demonstrates the violation of factorization by a factor 3.

4. Conclusion

Thus we have found two qualitative phenomena from the analysis of the $\bar{Q}q$ system (very sensitive to vacuum structure) in the heavy quark limit $M_Q \to \infty$:

- *SU(3) breaking effects are being suppressed with the insertion of gluon fields into condensate. In particular, we expect that the following link of inequalities is correct $\Delta > \Delta_g > \Delta_{gg} > ...$. I do not know whether any QCD vacuum models can (at least) qualitatively describe this behavior.*

- The factorization hypothesis does not work for mixed, dimension seven operator $\langle \bar{q}GGq \rangle$. It is not a big surprise, because we faced the analogous phenomenon early for the four-fermion condensates with an "exotic" Lorentz structure.

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7 This term has been borrowed from the Shifman Introduction to the book ☞
In conclusion, I want to give some qualitative remark which elucidates the reason why some VEVs must violate factorization. First of all, let me remind you that in some special cases the QCD sum rules do not work in principle. In particular, in the pseudoscalar quark channel

$$i \int e^{iqx} dx \langle T\{J^{(\pi)}(x), J^{(\pi)}(0)\} \rangle = P^{(\pi)}(Q^2), \quad Q^2 \equiv -q^2, \quad J = \bar{u}i\gamma_5 d$$

the standard sum rule cannot reproduce the residue $\langle 0 | J^{\pi} | \pi \rangle = f_{\pi} \cdot 2 GeV$ which is known exactly from PCAC, and which has much bigger scale than QCD sum rule can provide. The reason for that became clear very soon after QCD sum rules were invented and related to the fact that the limited dynamical information encoded by means of the different VEVs is not sufficient for describing the vacuum $0^\pm$ channels. So called direct instanton contributions \cite{20} play a decisive role in these channels. Let us call these currents as “exotic” ones, to emphasize their difference in comparison with the “normal” currents. The scale provided by QCD sum rules analysis in the “normal” cases does describe all residues correctly.

Now let us consider any nondiagonal correlator, constructed from the “normal” and “exotic” currents. On the one hand, there are no direct instanton contributions; on the other hand, the “exotic” matrix element $\langle 0 | J^{\pi} | \pi \rangle = f_{\pi} \cdot 2 GeV$ is several times larger than the normal scale $f_{\pi} \cdot m_{\rho}$, which the sum rule can provide. Thus, the information about big scale connected with contributions of direct instantons, clearly filters through into $\text{Im} P(s)$ via the residue $r_{\pi} \sim \langle |J^{\text{exotic}}(\pi)\rangle_{\pi} |J^{\text{normal}}(\pi)\rangle$. The question arises, in which way can one guarantee a large scale of the quantity $r_{\pi}$, if the corresponding sum rules do not allow direct instanton contribution? From our point of view the answer is that some VEVs which enter into the corresponding correlator become numerically large. Precisely in such a way (through enhancement of the vacuum condensates) the information of large value of the scale filters through into correlators, which do not allow direct instanton contributions.

- Few words on applications obtained results. First of all, as was explained in the text, these mixed VEVs determine the momentum distribution between $q$ and $Q$ quarks in the heavy-light system. In any calculation with $qQ$ system involved, this information is very useful, see e.g. \cite{21}.

As a second remark, I would like to note that the transverse quark distribution in the light mesons is determined mainly by high-dimensional condensates studied above, see \cite{13} for a more details. As is known, the transverse
5. Acknowledgements

The present analysis and the introducing of the parameter \( R \) \cite{22} was motivated by paper Ben Grinstein \cite{23} where he considered the double ratio \( R_1 = \frac{f_{P_1}}{f_{P_2}} \). I want to thank him for conversations. This work is supported by the Texas National Research Laboratory Commission under grant \# 528428.

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The main task of this paper was the calculation of the matrix element \( \langle N | \bar{s}s | N \rangle \) and the neutron dipole moment in the Weinberg model of CP violation. This problem is reduced to the evaluation of the vacuum characteristic \( \Delta_g \). In the appendix of the paper, we made a rough estimation of this VEV: \( \Delta_g \approx 0.15 \). A more careful analysis \[18\], exploiting the same idea, confirmed this estimate.

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