Tensor-to-Scalar Ratio in Eddington-inspired Born-Infeld Inflation

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We investigate the scalar perturbation of the inflation model driven by a massive-scalar field in Eddington-inspired Born-Infeld gravity. We focus on the perturbation at the attractor stage in which the first and the second slow-roll conditions are satisfied. The scalar perturbation exhibits the corrections to the chaotic inflation model in general relativity. We find that the tensor-to-scalar ratio becomes smaller than that of the usual chaotic inflation.

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I. INTRODUCTION

The Eddington-inspired Born-Infeld (EiBI) gravity was recently developed in Ref. [1]. The action in this theory is described by

$$\mathcal{S}_{\text{EiBI}} = \frac{1}{\kappa} \int d^4x \left[ \sqrt{-|g_{\mu\nu} + \kappa R_{\mu\nu}(\Gamma)|} - \lambda \sqrt{-|g_{\mu\nu}|} \right] + \mathcal{S}_M(g, \varphi), \quad (1)$$

where $\kappa$ is the only additional parameter of the theory, and $\lambda$ is a dimensionless parameter related with the cosmological constant by $\Lambda = (\lambda - 1)/\kappa$. This theory is based on the Palatini formalism in which the metric $g_{\mu\nu}$ and the connection $\Gamma^\rho_{\mu\nu}$ are treated as independent fields. The Ricci tensor $R_{\mu\nu}(\Gamma)$ is evaluated solely by the connection, and the matter field is coupled only to the gravitational field $g_{\mu\nu}$.

The inflationary universe in this theory driven by a massive scalar field was investigated in Ref. [2]. The matter action is in the usual form used for the chaotic inflation model [3] in general relativity (GR),

$$\mathcal{S}_M(g, \varphi) = \int d^4x \sqrt{-|g_{\mu\nu}|} \left[ -\frac{1}{2} g_{\mu\nu} \partial^\mu \varphi \partial^\nu \varphi - V(\varphi) \right], \quad V(\varphi) = \frac{m^2}{2} \varphi^2. \quad (2)$$

In EiBI gravity, there exists an upper bound in pressure due to the square-root type of the action. When the energy density is high, the maximal pressure state (MPS) is achieved, for which the scale factor exhibits an exponential expansion. It was investigated in Ref. [2] that this MPS is the past attractor from which all the classical evolution paths of the Universe originate. The energy density is very high in the MPS, but the curvature scale remains constant since the Hubble parameter becomes $H_{\text{MPS}} \approx 2m/3$. Therefore, quantum gravity is not necessary in describing the high-energy state of the early universe.

The MPS is unstable under the global perturbation (zero-mode scalar perturbation) and evolves to an inflationary attractor stage. The succeeding inflation feature is the same with the ordinary chaotic inflation in GR, but it is not chaotic at the high-energy state because the pre-inflationary stage can have a finite low curvature. Depending on the initial conditions, the evolution of the Universe can acquire the 60 $e$-foldings in the late-time inflationary attractor period. If the sufficient $e$-foldings are not acquired in this period, it must be complemented in the exponentially expanding period at the near-MPS in order to solve the cosmological problems.

The tensor perturbation in this model was investigated in Ref. [4]. For short wave-length modes, the perturbation is very similar to that of the usual chaotic inflation in GR, with a small EiBI correction. For long wave-length modes, however, there is a peculiar peak in the power spectrum originated from the near-MPS stage. This may leave a signature in the cosmic microwave background radiation.

In this paper, we investigate the scalar perturbation of this model. (The density perturbation has been studied in the EiBI universe filled with perfect fluid in Refs. [3,5]. Other works have been investigated in the cosmological and astrophysical aspects in Refs. [4,6].) From the very recent observational result of BICEP2 [22], there is an increasing interest in the tensor-to-scalar ratio of the various inflationary models. Therefore, we focus on the scalar

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power spectrum at the inflationary “attractor stage” at which the main band for the test of the tensor-to-scalar ratio is produced. We shall obtain the EiBI correction to the scalar power spectrum of the usual chaotic inflation. With the result of the tensor perturbation obtained in Ref. 4, we get the tensor-to-scalar ratio in the EiBI inflationary model.

II. FIELD EQUATIONS

The EiBI theory can be formulated as a bimetric-like theory with the action,

$$ S[g, q, \varphi] = \frac{1}{2} \int d^4x \sqrt{|g_{\mu\nu}|} \left[R(q) - \frac{2}{\kappa} + \frac{1}{2} \int d^4x \left( \sqrt{|g_{\mu\nu}|} q^{\alpha\beta} g_{\alpha\beta} - 2 \sqrt{|g_{\mu\nu}|} \right) S_M[g, \varphi] \right], \tag{3} $$

where $g_{\mu\nu}$ is the metric and $q_{\mu\nu}$ is the auxiliary metric. When there is no cosmological constant ($\lambda = 1$), the action (3) is completely equivalent to the action (4) if one considers that $\Gamma$ is the affine connection of $q_{\mu\nu}$. The equations of motion are

$$ q^\mu{}^\nu = \frac{\sqrt{|q|}}{\sqrt{|g|}} q_{\mu\nu} - \kappa T^{\mu\nu}, \tag{4} $$

$$ q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}, \tag{5} $$

where $T^{\mu\nu}$ is the standard energy-momentum tensor. The ansätze for the auxiliary metric and the metric are

$$ q_{\mu\nu} dx^\mu dx^\nu = b^2(\eta) \left[ - \frac{d\eta^2}{z(\eta)} + \delta_{ij} dx^i dx^j \right], \tag{6} $$

$$ g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(\eta) \delta_{ij} dx^i dx^j = a^2(\eta) \left( -d\eta^2 + \delta_{ij} dx^i dx^j \right), \tag{7} $$

where $t$ is the cosmological time and $\eta$ is the conformal time for the metric. The derivatives are defined as $\dot{} \equiv d/dt$ and $\prime \equiv /d\eta$. In this paper, we shall denote $\mathcal{H} \equiv a'/a$, $H \equiv \dot{a}/a$, $h \equiv b'/b$, and $h_0 \equiv b/b$. The components of Eq. (4) give

$$ b^2 \sqrt{z} = (1 + \kappa \rho_0) a^2, \quad b^2 \sqrt{z} = (1 - \kappa \rho_0) a^2, \tag{8} $$

where we denote the subscript 0 for the unperturbed background scalar field, so $\rho_0 = \varphi^2_0/2a^2 + V(\varphi_0)$ and $p_0 = \varphi^2_0/2a^2 - V(\varphi_0)$. From Eq. (5), one gets $z = (1 + \kappa \rho_0)/(1 - \kappa \rho_0)$. The components of Eq. (5) provide dynamical equations,

$$ b^2 = 3\kappa z \left( b' \right)^2 + a^2(3 - z), \tag{9} $$

$$ b^2 = a^2 + \kappa z \left[ b'' \frac{b}{b} + \left( \frac{b'}{b} \right)^2 + \frac{1}{2} \frac{b'}{b} \frac{z}{z} \right]. \tag{10} $$

and the scalar field equation is given by

$$ \varphi''_0 + 2\mathcal{H}\varphi'_0 + a^2 \frac{dV}{d\varphi_0} = 0. \tag{11} $$

The background fields, $a$, $b$, $z$, and $\varphi_0$ are obtained by solving Eqs. (8)-(11).

Now let us consider the scalar perturbation. The perturbation fields for $q_{\mu\nu}$ and $g_{\mu\nu}$ are defined as

$$ ds^2_q = b^2 \left\{ - \frac{1 + 2\psi_1}{z} d\eta^2 + 2 \frac{B_{1,i}}{\sqrt{z}} d\eta dx^i + \left[ (1 - 2\psi_1) \delta_{ij} + 2E_{1,ij} \right] dx^i dx^j \right\}, \tag{12} $$

$$ ds^2_g = a^2 \left\{ -(1 + 2\psi_2) d\eta^2 + 2B_{2,j} d\eta dx^j + \left[ (1 - 2\psi_2) \delta_{ij} + 2E_{2,ij} \right] dx^i dx^j \right\}, \tag{13} $$

and the perturbation for the scalar field is given by $\varphi = \varphi_0 + \chi$. Therefore, there are nine perturbation fields in total. Let us denote them as $F_l$, where $l = 1 \sim 9$. With the perturbed metrics and the scalar field, one can expand the action (3) up to the second order in the perturbation fields. Then the second-order action can be collected as $S_s = S_1 + S_2 + S_3$ where $S_1$ contains the perturbation fields for $q_{\mu\nu}$, $S_2$ contains the perturbation fields for $g_{\mu\nu}$ and the mixing terms for $q_{\mu\nu}$, and $S_3$ contains the matter field perturbation (see Ref. [4]).
\[ S_1[\phi_1, B_1, \psi_1, E_1] = \frac{1}{2} \int d^4x \left\{ \frac{b^2}{\sqrt{2}} \left[ 4zh\psi_1^2 E_{1,ii} - 6z\psi_1^2 - 12zh(\phi_1 + \psi_1)\psi_1' - 2\psi_{1,i}(2\phi_{1,i} - \psi_{1,i}) ight. \\
- 4h\psi_1 B_{1,ii} + 6zh^2(\phi_1 + \psi_1)E_{1,ii} - 4\sqrt{\gamma}h(\phi_1 + \psi_1)(B_1 - \sqrt{\gamma}E_{1,ii}) \\
- 4\sqrt{\gamma}\psi_1'(B_1 - \sqrt{\gamma}E_{1,i}) \left. - 4\sqrt{\gamma}hE_{1,ii}(B_1 - \sqrt{\gamma}E_{1,jj}) + 4\sqrt{\gamma}zhE_{1,ii}B_{1,ji} \\
+ 3zh^2E_{1,ii}E_{1,jj} + 3zh^2B_{1,ii}B_{1,ji} - 9zh^2(\phi_1 + \psi_1)^2 \right] \\
- \frac{2b^4}{\kappa}\left[ \frac{3}{2}\psi_1^2 - 3\psi_{1,i} + \frac{1}{2}b_{1,ii} - \frac{1}{2}E_{1,ii}E_{1,jj} - \frac{1}{2}\phi_1^2 + E_{1,ii}(\psi_1' - \psi_1) \right] \right\}, \] (14)

\[ S_2[\phi_k, B_k, \psi_k, E_k] = \frac{1}{2} \int d^4x \left\{ \frac{a^2b^2}{\kappa}\left[ 2\sqrt{\gamma}B_1 B_{2,ii} + \phi_1 [(z - 1)(3\psi_1 - E_{1,ii}) - 6\psi_2 + 2E_{2,2ii} - 2z\phi_2] \\
+ \psi_1 [6\psi_2 - (z - 1)E_{1,ii} - 2E_{2,2ii} - 6\psi_2] - \frac{1}{2}(z - 1)(E_{1,ii}E_{1,jj} + B_1 B_{1,ii}) \\
+ \frac{3}{2}(\phi_1^2 + \psi_1^2)(z - 1) - 2E_{1,ii}(\psi_2 - 2\phi_2 + E_{2,2ii}) \right] \\
- \frac{2a^4}{\kappa}\left[ \frac{3}{2}\psi_2^2 - \frac{1}{2}\phi_2^2 + \frac{1}{2}B_1 B_{2,ii} E_{2,2jj} + (\phi_2 - \psi_2)E_{2,2ii} - 3\phi_2^2 \right] \right\}, \] (15)

\[ S_3[\phi_2, B_2, \psi_2, E_2, \chi] = \frac{1}{2} \int d^4x a^4 \left\{ \phi_0^2 \left( 4\phi_2^2 - B_{2,ii}B_{2,ii} \right) \\
+ (\phi_0^2 - 2V_0 a^2) \left[ \frac{1}{2}(3\psi_2^2 - \phi_2^2 + B_{2,ii}B_{2,ii} - E_{2,ii}E_{2,ii}) - 3\phi_2^2 + (\phi_2 - \psi_2)E_{2,2ii} \right] \\
- 2\phi_0^2\chi B_{2,ii} - 4\phi_0^2\chi \phi_2 + \chi^2 + 2(\phi_2 - 3\psi_2 + E_{2,ii})(\chi' \phi_0' - V_1 a^2 - \phi_2 \phi_0^2) - \chi \chi,ii - 2V_2 a^2 \right\}, \] (16)

where \( V_i \) is the \( i \)th-order potential from \( V = V_0(\phi_0) + V_1(\chi) + V_2(\chi) \). Using \( \rho_0 \) and \( p_0 \), \( S_3 \) can be recast into

\[ S_1[\phi_2, B_2, \psi_2, E_2, \chi] = \int d^4x a^4 \left\{ \rho_0 \left[ \frac{1}{2}(3\psi_2^2 - \phi_2^2 + B_{2,ii}B_{2,ii} - E_{2,ii}E_{2,ii}) - 3\phi_2^2 + (\phi_2 - \psi_2)E_{2,2ii} \right] \\
+ (\rho_0 + p_0) \left[ \frac{1}{2}\phi_2(\phi_2 - \chi X') - \frac{1}{2}B_{2,ii}(B_{2,ii} + 2\chi X,ii) + (\phi_2 - 3\psi_2 + E_{2,ii})(\chi X' - Y - \phi_2) \right] \\
+ \frac{1}{2a^2}(\chi^2 - \chi,ii) - \frac{m^2}{2} \right\}, \] (17)

where \( X \equiv 1/(a\sqrt{\rho_0 + p_0}) \) and \( Y \equiv m\sqrt{\rho_0 - p_0}/(\rho_0 + p_0) \).

For the nine perturbation fields, \( F_i \), we introduce the corresponding Fourier modes as

\[ F_i(\eta, \vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} F_i(\eta, \vec{k})e^{i\vec{k}\cdot\vec{x}}. \] (18)

The gauge conditions have been studied precisely in Ref. \[ \Box \]. We choose the gauge conditions,

\[ \psi_1 = 0 \quad \text{and} \quad E_1 = 0. \] (19)

Then from the variation of \( S_2 \) and \( S_3 \) for \( \phi_2, \psi_2, E_2, \) and \( B_2 \), we get

\[ (1 - 2z)\phi_2 + z\phi_1 - 3z\psi_2 - k^2zE_2 - (1 - z)Y\chi' - (1 - z)Y\chi = 0, \] (20)

\[ 3\psi_2 + 3\phi_1 - 3z\phi_2 + k^2zE_2 - 3(1 - z)Y\chi' + 3(1 - z)Y\chi = 0, \] (21)

\[ k^2E_2 - \phi_1 + z\phi_2 - \psi_2 + (1 - z)Y\chi' - (1 - z)Y\chi = 0, \] (22)

\[ zB_2 - \sqrt{\gamma}B_1 - (1 - z)Y\chi = 0. \] (23)
From the variation of $S_1$ and $S_2$ for $\phi_1$ and $B_1$, we get
\[
(6\kappa h^2 - a^2)z\phi_1 + a^2z\phi_2 + 3a^2\phi_2 + k^2a^2E_2 - 2k^2\kappa h\sqrt{z}B_1 = 0,
\]
\[
a^2B_1 - 2k\kappa h\sqrt{z}\phi_1 - a^2\sqrt{z}B_2 = 0. \tag{25}
\]
From Eqs. (21) and (22), we have $E_2 = 0$. From Eqs. (20) and (22), we have
\[
\phi_2 = \frac{(z - 1)(3z + 1)X\chi' - (z - 1)(3z - 1)Y\chi + 4z\phi_1}{(z + 1)(3z - 1)}, \tag{26}
\]
and from Eqs. (23) and (25), we have
\[
\phi_1 = \frac{a^2(z - 1)X\chi}{2\kappa h z}. \tag{27}
\]
Then from Eqs. (21) and (26), we finally get
\[
\psi_2 = \frac{z - 1}{2\kappa h z(z + 1)(3z - 1)} \left[ -2k\kappa h(z - 1)X\chi' + a^2(z - 1)^2X\chi + 2k\kappa h(z - 1)Y\chi \right], \tag{28}
\]
which is expressed only by the background fields and the matter-field perturbation $\chi$. This quantity will be used later in evaluating the power spectrum from the comoving curvature,
\[
\mathcal{R} = \psi_2 + \frac{H}{\dot{\phi}_0} \chi. \tag{29}
\]

With the results of Eqs. (20)-(28), we can write the second-order action $S_s[\chi]$ expressed only by the matter-field perturbation $\chi$ and the background fields in the Fourier space,
\[
S_s[\chi] = \frac{1}{2} \int d^3k d\eta \left[ f_1(\eta, k)\chi'^2 - f_2(\eta, k)\chi^2 \right], \tag{30}
\]
where
\[
f_1(\eta, k) = a^2 + \frac{2a^2(z - 1)^2X^2 [a^2(z - 3) - 6\kappa h^2 z]}{\kappa \sqrt{z}(z + 1)(3z - 1)}, \tag{31}
\]
and
\[
f_2(\eta, k) = \frac{\beta}{8\kappa^3 h_0^2 z^{5/2}(z + 1)^2}. \tag{32}
\]
Here,
\[
\beta = a^2 \left[ \frac{\beta_1}{3z - 1} + \frac{\beta_2}{(3z - 1)^2} \right], \tag{33}
\]
where
\[
\beta_1 = (z + 1) \left\{ 8\kappa^3 h^2 z^2(3z - 1) \left[ k^2\sqrt{z} - 12h^2Y^2z + k^2z^{3/2} + 24h^2Y^2 z^2 - 12h^2Y^2 z^3 - 3k^2h^2X^2(z - 1)^2(z + 1) \right] \\
+ a^6X^2(z - 3)(z - 1)^3(3z - 2z + 3) + 4\kappa a^4 hX z(z - 1)^2 \left[ Y(z - 3)^2(3z - 1) - 3hX z(3z^2 - 6z - 1) \right] \\
+ 4\kappa^2 a^2 h^2 z^3(3z - 1) \left[ -6hX Y(z - 3)(z - 1)^2 z + X^2(z - 1)^2(z + 1)((k^2 + 9h^2)z - 3k^2) \\
+ 4Y^2 z(z - 3)(z - 1)^2 + 2\kappa m^2 z^{3/2}(z + 1) \right] \right\}, \tag{34}
\]
\[
\beta_2 = (z - 1) \left\{ a^2(z - 3) - 6k h^2 z \left[ a^4X^2(z - 1)^2(z - 1)(3z - 1)(3z^2 - 2z + 3) \\
+ 4\kappa^2 h^2 z^2(3z - 1)^2 \left[ 2(z - 1)(z + 1)(2(h + H)X Y + (X Y)') + X Y(z^2 - 6z + 1) \right] \\
+ 2\kappa a^2 hX \left[ z(z - 3)(z + 1)(3z - 1)(3z^2 - 2z + 3)((h + 4H)X + 2X') \\
+ X(9z^5 + 21z^4 - 34z^3 + 30z^2 + 9z - 3) \right] \right\} \right\}. \tag{35}
\]
The field $\chi$ in the action \[ \text{Eq. (30)} \] is not of the canonical form. Therefore, we introduce the canonical field $Q$ by the transformation $\chi = Q/\omega$ with introducing a new time coordinate $\tau$ by $d\eta = f_3d\tau$. Then the field equation becomes

$$\ddot{Q} + \left( \frac{f_1}{f_3} - \frac{f_3}{f_3} - 2\frac{\dot{\omega}}{\omega} \right) Q + \left[ f_2f_2 - \frac{\dot{\omega}}{\omega} \left( \frac{f_1}{f_3} - \frac{f_3}{f_3} - 2\frac{\dot{\omega}}{\omega} \right) - \frac{\dot{\omega}}{\omega} \right] Q = 0. \quad (36)$$

For the canonical field, the $\dot{Q}$-term vanishes, and thus we get $\omega^2 = f_1/f_3$. The field equation then becomes

$$\ddot{Q} + \left( \frac{f_1f_2}{\omega^4} - \frac{\dot{\omega}^2}{\omega^2} \right) Q \equiv \ddot{Q} + \left( c_s^2k^2 - \frac{\dot{\omega}^2}{\omega^2} \right) Q = 0, \quad (37)$$

where the speed of sound is identified by $c_s^2 \equiv f_1f_2/k^2\omega^4$. We assume a Bunch-Davies vacuum described by the plane wave, requiring $c_s^2 \to 1$ in the limit of $k \to \infty$. Then $\omega$ is determined as

$$\omega^4 = \frac{a^4}{2\kappa z^2(z+1)(3z-1)} \left\{ a^2\lambda^2(z-3)(z-1)^2 - 2\kappa z \left[ 3h^2\lambda^2(z-1)^2 - \sqrt{z} \right] \right\} \times \left\{ 2a^2\lambda^2(z-3)(z-1)^2 - \kappa\sqrt{z} \left[ 12h^2\lambda^2\sqrt{z}(z-1)^2 - 3z^2 - 2z + 1 \right] \right\}. \quad (38)$$

For the canonical field $Q$, the normalization condition is given by

$$QQ^* - Q^* \dot{Q} = i. \quad (39)$$

For the initial perturbation produced in the Bunch-Davies vacuum ($k \to \infty$ and $c_s^2 \to 1$), we impose the minimum-energy condition which picks up the positive mode solution of Eq. \[ \text{Eq. (39)} \]. The normalization condition \[ \text{Eq. (39)} \] fixes the coefficient, and the solution becomes

$$Q(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}. \quad (40)$$

### III. PERTURBATION AT ATTRACTOR STAGE

At the attractor stage, both of the first and the second slow-roll conditions are satisfied. The background evolution was found \[ \text{Eq. (2)} \] to be approximately the same with that of the usual chaotic inflation in GR. In this paper, we focus on the scalar perturbation at the attractor stage, and investigate the EiBI correction in the power spectrum.

At the attractor stage, the background scalar field and scale factor are given by

$$\varphi_0(t) \approx \varphi_i + \sqrt{\frac{2}{3}}mt, \quad a(t) \approx a_i e^{[\varphi_i^2 - \varphi_0^2(t)]/4}, \quad (41)$$

where $\varphi_i < 0$ is the value of the scalar field in the beginning of the attractor. (We consider the scalar field rolling down the potential at $\varphi < 0$.) For 60 e-foldings, $|\varphi_i| \gtrsim 15$ is required. From observational data, $m \sim 10^{-5}$ for the standard inflationary model. At the early stage of the attractor, $m^2t^2 \ll mt$. (We set $t = 0$ as the beginning of the attractor stage.) Then the scale factor is further approximated as

$$a(t) \approx a_i e^{-\varphi_i mt/\sqrt{6} - m^2t^2/6} \approx a_i e^{-\varphi_i mt/\sqrt{6}}. \quad (42)$$

In this background, we can approximate $z$ as

$$z = \frac{1 + \kappa \rho_0}{1 - \kappa \rho_0} = 1 + \kappa \frac{\varphi_0^2}{2} + m^2 \frac{\varphi_0^2}{2} = 1 + \frac{\kappa \varphi_0^2}{2} = \frac{1}{1 - \kappa \varphi_0^2 - 2m^2 \varphi_0^2/2} \approx 1 + \frac{2\kappa m^2/3}{1 + \kappa m^2 \varphi_0^2/2}. \quad (43)$$

where we used the first slow-roll condition $\varphi_0^2/2 \ll m^2 \varphi_0^2/2$ in the last step. At the attractor stage, therefore, the value of $z - 1$ is a small quantity proportional to $\kappa$, which is responsible for the “EiBI correction” in the power spectrum as we will see in the next section.
Now, let us evaluate the involved quantities in approximation. From the background field equations in Eq. (8), we have

\[ b = (1 + \kappa \rho_0)^{1/4}(1 - \kappa \rho_0)^{1/4}a \approx (1 + \kappa \rho_0)^{1/2}a \approx a \sqrt{1 + \frac{1}{2} \kappa m^2 \varphi_0^2}, \]

where we used the first slow-roll condition for the approximation. Then the scalar factor \( h_b \) can be approximated by

\[ h_b = \frac{\dot{b}}{b} = \frac{\dot{a}}{a} + \frac{\kappa \sqrt{z} \varphi_0}{b^2(z - 1)} \approx -\frac{m \varphi_1}{\sqrt{6}} + \frac{\kappa m^2 \varphi_0}{\sqrt{6}} \approx -\frac{m \varphi_1}{\sqrt{6}} = \frac{\dot{a}}{a} = H, \]

which we assumed \( \kappa m^2 \ll 1 \). Therefore, the terms containing \( \kappa h_b^2 \approx \kappa m^2 \varphi_0^2 / 6 \ll 1 \) can be ignored in the approximations. Using Eqs. (5) and (43), we get

\[
\chi^2 = \frac{1}{a^2(\rho_0 + \rho_0)} = \frac{\kappa \sqrt{z}}{b^2(z - 1)} \approx \frac{\kappa \sqrt{z}}{a^2(z - 1)} \left(1 + \frac{1}{2} \kappa m^2 \varphi_0^2\right)^{-1} \approx \frac{\kappa \sqrt{z}}{a^2(z - 1)} \left(1 - \frac{1}{2} \kappa m^2 \varphi_0^2\right),
\]

\[
\gamma = m \frac{\sqrt{\rho_0 - \rho_0}}{\rho_0 + \rho_0} \approx m \left[ \frac{z + 1}{\kappa \sqrt{z}} \left(1 + \frac{1}{2} \kappa m^2 \varphi_0^2\right) - 2_{\kappa}^1 \right]^{1/2} \left[ \frac{z - 1}{\kappa \sqrt{z}} \left(1 + \frac{1}{2} \kappa m^2 \varphi_0^2\right) \right]^{-1},
\]

and \( \omega \) in Eq. (38) can be approximated as

\[ \omega \approx a S, \quad \text{where} \quad S \equiv \left[ \frac{(z^2 - 2z + 3)(5z^2 - 6z + 5)}{2z(z + 1)(3z - 1)} \right]^{1/4}. \]

Plugging Eq. (46) into Eq. (31) and reminding of \( \kappa m^2 \ll 1 \) and \( \kappa h^2 = \kappa h_b^2 a^2 \ll a^2 \), we get

\[ f_1 \approx a^2 \frac{5z^2 - 6z + 5}{(z + 1)(3z - 1)}. \]

Then from Eqs. (38) and (49), we get

\[ f_3 = \frac{f_1}{\omega^2} \approx \left[ \frac{2z(5z^2 - 6z + 5)}{(z + 1)(3z - 1)(z^2 - 2z + 3)} \right]^{1/2}. \]

Now let us keep the lowest-order correction that is proportional to \( \kappa m^2 \). Then from Eqs. (49) and (50), we have

\[ f_1 \approx a^2 \left(1 - \frac{2}{3} \kappa m^2\right), \quad f_3 \approx \frac{1}{2} (\kappa m^2)^2 \approx 1, \quad \text{and} \quad \omega^4 \approx a^4 \left(1 - \frac{4}{3} \kappa m^2\right). \]

Using the results for \( \chi \) and \( \gamma \) in Eqs. (46) and (47), we can get

\[ f_2 \approx a^2 \left\{ k^2 \left(1 - \frac{2}{3} \kappa m^2\right) - m^2 a^2 \left[1 + 2 \kappa m^2 (\varphi_0^2 - 1)\right] \right\}. \]

For the time transformation at the attractor stage, we have then

\[ d\eta = f_3 d\tau \approx d\tau. \]

**IV. POWER SPECTRUM**

In this section, we evaluate the scalar power spectrum at the attractor stage using the quantities that we obtained in the previous section. We shall focus on the corrections from the EiBI theory, and compare the result with the power spectrum in GR. Finally we will get the EiBI correction in the tensor-to-scalar ratio.
Let us express the scale factor $a$ in terms of $\tau$. Using $a(t)$ in Eq. (41), the time coordinates are transformed by

$$
\frac{dt}{f_3} = \frac{dt}{f_3a} \approx \frac{dt}{a} \Rightarrow \int_{\tau_i}^{\tau} d\tau' = \int_{0}^{t} \frac{dt'}{a(t')} \Rightarrow \tau - \tau_i = \frac{\sqrt{6}}{\phi_i m} \left( \frac{1}{a} - \frac{1}{a_i} \right),
$$

(54)

where we assumed that the attractor stage begins at $\tau = \tau_i > 0$ ($t = 0$). [We assume that the Universe begins at the near-MPS stage at $\tau = 0$ ($t \to -\infty$).] Setting $t = 0$ for the beginning of the attractor stage fixes the arbitrariness of the scale factor, $a(t = 0) = a_i$. From Eq. (54), the scale factor can be obtained as

$$
a(\tau) = \frac{a_i(\tau_i - \tau_0)}{\tau - \tau_0}, \quad \tau_0 \equiv \tau(t \to \infty) = \tau_i - \frac{\sqrt{6}}{\phi_i m}.
$$

(55)

Let us consider the corrections for $c_s^2$ and $\ddot{\omega}/\omega$, in the field equation,

$$
\ddot{Q} + \Omega_k^2 Q = 0, \quad \text{where} \quad \Omega_k^2 = c_s^2 k^2 - \frac{\ddot{\omega}}{\omega}.
$$

(56)

For the speed of sound, from the approximated quantities in Eqs. (51) and (52), we get

$$
c_s^2 = 1 - \frac{m^2 a^2}{k^2} \left[ 1 + 2\kappa m^2 \left( \phi_i^2 - \frac{2}{3} \right) \right].
$$

(57)

Here, the $a^2$-dependence originates from the non-conventional form of the action, and the $\kappa$-dependence is the EiBI correction. Using the last expression for $z$ in Eq. (58) for $\omega$ in Eq. (59) and the time transformation between $t$ and $\tau$ in Eq. (54), we get

$$
\ddot{\omega} \approx (1 - \kappa^2 m^4) \frac{\phi_i^2 m^2 a^2}{3} \approx \frac{\phi_i^2 m^2 a^2}{3},
$$

(58)

where we neglected the $\kappa$-dependent EiBI correction in the last step since it is the higher-order in $\kappa$. Therefore, the field equation (58) can be approximated by

$$
\Omega_k^2 \approx k^2 - \frac{\phi_i^2 m^2 a^2}{3} \left[ 1 + \frac{3}{\phi_i^2} + 6\kappa m^2 \left( 1 - \frac{2}{3\phi_i^2} \right) \right] \approx k^2 - \frac{2}{(\tau - \tau_0)^2},
$$

(59)

where we neglected the last three terms in the bracket as $\phi_i \sim \mathcal{O}(10)$ and $\kappa m^2 \ll 1$. Therefore, there is no significant correction in the field equation, and thus the normalized positive-energy mode solution to the field equation (50) becomes the usual one in GR,

$$
Q \approx \frac{e^{-ik(\tau - \tau_0)}}{\sqrt{2k}} \left[ 1 - \frac{i}{k(\tau - \tau_0)} \right].
$$

(60)

Now let us evaluate the comoving curvature. Using Eqs. (50) and (51), the most dominant term for $\psi_2$ is the last term in Eq. (57). Then we have

$$
\psi_2 \approx \frac{1}{2\kappa m^2} \phi_i \chi \quad \Rightarrow \quad R = \psi_2 + \frac{H}{\phi_0} \chi \approx \frac{\kappa m^2 - 1}{2} \phi_i \chi.
$$

(61)

Here, $\psi_2$ results purely from the EiBI correction. When $\kappa \to 0$, we have $\psi_2 \to 0$ which indicates that our choice of gauge condition ($\psi_1 = 0$ and $E_1 = 0$) corresponds to the spatially flat gauge ($\psi_2 = 0$ and $E_2 = 0$) in the GR limit.

With the field $Q$ and the comoving curvature $R$ obtained in Eqs. (60) and (61), the power spectrum is evaluated as

$$
P_R = \frac{k^3}{2\pi^2} R^2 = \frac{k^3}{2\pi^2} \left( \psi_2 + \frac{H}{\phi_0} \chi \right)^2 \approx (1 - \kappa m^2)^2 \frac{k^3 \phi_i^2}{8\pi^2} \chi^2 = (1 - \kappa m^2)^2 \frac{k^3 \phi_i^2}{8\pi^2} \left( \frac{Q}{\omega} \right)^2
$$

(62)

$$
\approx \frac{(1 - \kappa m^2)^2}{(1 - 4\kappa^2 m^4/3)^{1/2}} \frac{k^3 \phi_i^2}{8\pi^2} \left( \frac{Q}{\omega} \right)^2
$$

(63)

$$
\approx \frac{(1 - \kappa m^2)^2}{(1 - 4\kappa^2 m^4/3)^{1/2}} \frac{m^2 \phi_i^2}{96\pi^2} \times k^2(\tau - \tau_0)^2 \left[ 1 + \frac{1}{k^2(\tau - \tau_0)^2} \right].
$$

(64)
At the end of inflation ($\tau \to \tau_0$), finally we get

$$P_R = \frac{(1 - \kappa m^2)^2}{(1 - 4\kappa m^2/3)^{1/2}} \times \frac{m^2 \varphi_i^4}{96\pi^2} = \frac{(1 - \kappa m^2)^2}{(1 - 4\kappa m^2/3)^{1/2}} \times P_R^{GR} \approx \left(1 - \frac{4}{3}\kappa m^2\right) P_R^{GR},$$

(65)

where $P_R^{GR} = m^2 \varphi_i^4/96\pi^2$ is the power spectrum in GR.

The tensor-to-scalar ratio is obtained with the result of the tensor power spectrum obtained in Ref. [4],

$$r = \frac{P_T}{P_R} \approx \frac{P_T^{GR}/(1 + \kappa m^2 \varphi_i^2/2)}{(1 - 4\kappa m^2/3) P_R^{GR}} \approx \left(1 - \frac{1}{2}\kappa m^2 \varphi_i^2 + \frac{4}{3}\kappa m^2\right) r^{GR},$$

(66)

where $r^{GR} \sim 0.131$ for 60 e-foldings. The EiBI correction of the tensor spectrum lowers the value of $r$, while that of the scalar spectrum raises the value. As $\varphi_i \sim O(10)$, the effect of the tensor spectrum is larger and the whole EiBI corrections lower the value of $r$.

V. CONCLUSIONS

Recently the gravitational waves produced in the inflationary stage of the early Universe attract much attention due to the observational result of BICEP2 [23]. The result tells that the tensor-to-scalar ratio is very high, $r \sim 0.2$. Although its validity requires further examinations, for example, from the PLANCK observational results [20], it is very interesting to discuss how the various inflationary models predict the value of the tensor-to-scalar ratio.

In this paper, we investigated the scalar perturbation in a newly suggested inflationary model driven by a massive scalar field in Eddington-inspired Born-Infeld gravity [2]. With the result of the tensor perturbation investigated in Ref. [4], we evaluated the tensor-to-scalar ratio. As it was investigated in Ref. [2], there are two exponentially expanding stages of the Universe in this inflationary model. The one is the near-MPS stage, and the other is the attractor stage. We mainly focused on the attractor stage since the main band for the test of the tensor-to-scalar ratio is related with this stage. (The near-MPS stage affects mostly the very long wave-length modes.)

The background evolution at the attractor stage is very similar to that of the chaotic inflation in GR. We assumed that the attractor stage maintained sufficiently long, and investigated the scalar perturbation produced at this stage. We assumed that the Bunch-Davies vacuum for the initial production of the perturbation mode $k \to \infty$. The speed of sound $c_S$ was then obtained accordingly for the arbitrary $k$-modes. We imposed the minimum-energy condition for the intial perturbation which picks up the positive-energy mode.

For the arbitrary $k$-modes, we obtained the EiBI corrections in terms of $\kappa m^2$ which is supposedly small. (The value of the EiBI theory parameter $\kappa$ is constrained to be small from the study of the star formation [10, 11], and $m \sim 10^{-5}$ from observational data.) The correction for the canonical perturbation field $Q$ is very tiny and minor, so $Q$ is of the same form with that of the $\varphi_i^2$ chaotic inflation model in GR. The main EiBI correction comes from two sources. The one is from the relation $\chi = Q/\phi \equiv Q/aS$ between the matter field perturbation $\chi$ and its canonical form field $Q$. In GR, $S = 1$ while in EiBI $S = 1 - \kappa m^2/3$. The other is from the metric perturbation field $\psi_2$ in the comoving curvature $\mathcal{R} = \psi_2 + (H/\dot{\varphi}_0)\chi$. This is related with the gauge. In the spatially flat gauge in GR, $E_2 = 0$ and $\psi_2 = 0$. In EiBI, we imposed the gauge conditions, $E_1 = 0$ and $\psi_1 = 0$, which results in $E_2 = 0$ and $\psi_1 = \kappa m^2 \varphi_i \chi/2$.

With these corrections, the scalar power spectrum $P_S$ is smaller than that in GR. With the tensor power spectrum $P_T$ obtained in Ref. [4], we observe that the tensor-to-scalar ratio in EiBI gravity becomes smaller than that ($r^{GR} \sim 0.131$) in GR. If we stick only to our study of the density perturbation in EiBI gravity (i.e., if we do not take the constraint for $\kappa$ from the study of the star formation into account), the value of $\kappa m^2 \varphi_i^2$ can be nonnegligible, and the reduction in $r$ in the final result (66) can be considerable while keeping the approximation valid. This reduction is affirmative in considering the dispute between the BICEP2 and the PLANCK results in the literature.

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