Generating Bragg solitons in a coherent medium

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In this Letter we discuss the possibility of producing Bragg solitons in an electromagnetically induced transparency medium. We show that this coherent medium can be engineered to be a Bragg grating with a large Kerr nonlinearity through proper arrangements of light fields. Unlike in previous studies, the parameters of the medium can be easily controlled through adjusting the intensities and detunings of lasers. Thus this scheme may provide an opportunity to study the dynamics of Bragg solitons. And doing experiments with low power lights is possible.

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Light propagating in a periodic medium is associated with many interesting phenomena. A fundamental change introduced by this periodicity is a forbidden band gap in the transmission spectrum. Light cannot transmit in this medium if its frequency falls into the band gap. However if the medium has some nonlinearity, in some cases light can transmit in it even though its frequency lies within the forbidden frequency band. An important example is that a periodic medium with Kerr nonlinearity may support a kind of solitary waves called Bragg solitons[3]. This kind of solitons was firstly studied by Chen and Mills[2], and later by many other researchers[3, 4, 5, 6, 7, 8]. Some experiments in nonlinear optical fibers were reported[9, 10]. In these experiments very high peak intensities were required because of the small value of the nonlinear coefficient.

On the other hand, electromagnetically induced transparency(EIT) is another fascinating phenomenon[11]. An optical opaque medium is rendered to be transparent for a probe light by a coupling field in a small frequency window. One important property is that in the same spectral region where there is a high degree of transmission the nonlinear response $\chi^3$ displays constructive interference, i.e., its value at resonance could be very large. This effect is also termed as giant Kerr effect[12]. Another attractive application is that one can make a controllable photonic band gap by properly arranging coupling lights[13, 14, 15, 16, 17].

In this Letter we try to combine these two applications. We will show that through a proper geometric configuration one can make an EIT medium to have both a tunable photonic band gap and a large Kerr nonlinearity. We propose a scheme to generate Bragg solitons in this medium with relatively low light power. Furthermore, unlike in the previous studies, the parameters of the medium can be easily controlled through adjusting the intensities and detunings of lasers. Thus this scheme may provide an opportunity to study the dynamics of gap solitons.

First consider an ensemble of atoms with energy diagram shown in Fig. 1. A probe laser with frequency $\omega_p$ is near resonant with transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$ (Here we assume $\omega_3 - \omega_1 \simeq \omega_4 - \omega_2$). The corresponding Rabi frequencies are $\Omega_s$ and $\Omega_{p'}$ respectively. A coupling field with frequency $\omega_c$ and Rabi frequency $\Omega_c$ is tuned to $|2\rangle \leftrightarrow |3\rangle$ transition. And a controlling field with frequency $\omega_s$ is tuned to $|2\rangle \leftrightarrow |5\rangle$ transition.

Fig.1

The nonlinear coefficient of the system can be obtained by calculating the coupled amplitude equations in the perturbative regime[18]. The susceptibility at the probe frequency is,

$$\chi(\omega_p) = \frac{K_0}{2} \frac{-4\delta \Delta_{42} \Delta_{52} + \tilde{\Delta}_{42} |\Omega_s|^2 + \tilde{\Delta}_{52} |\Omega_{p'}|^2}{4\delta \Delta_{42} \Delta_{52} - \Delta \Delta_{52} |\Omega_p|^2 - \Delta \Delta_{42} |\Omega_s|^2 - \Delta_{42} \Delta_{52} |\Omega_c|^2}, \quad (1)$$

where $\Delta_1 = \omega_p - \omega_3$, $\Delta_2 = \omega_c - \omega_2$, $\Delta_4 = \omega_p - \omega_4$, $\Delta_5 = \omega_3 - \omega_5$, $\Delta = \Delta_1 + \frac{1}{2}\Gamma_3$, $\tilde{\Delta} = \Delta_1 - \Delta_2 + \frac{1}{2}\Gamma_2$, $\tilde{\Delta}_{42} = \Delta_1 - \Delta_2 + \Delta_4 + \frac{1}{2}\Gamma_4$, $\Delta_{52} = \Delta_1 - \Delta_2 + \Delta_5 + \frac{1}{2}\Gamma_5$, $\Gamma_3$, $\Gamma_4$ and $\Gamma_5$ are relaxation rates for energy level 2, 3, 4 and 5 respectively. $K_0 = \rho |\mu_3|^2 / \hbar \epsilon_0$, $\rho$ is the density of the atoms. Here we assume that $(\Gamma_3, \Gamma_4, \Gamma_5, |\Omega_c|) \gg (|\Omega_p|, |\Omega_{p'}|)$. This susceptibility has three components $\chi^{(1)}(\omega_p)$, $\chi^{(3)}(\omega_p; \omega_s, -\omega_s, \omega_p)$ and $\chi^{(3)}(\omega_p; \omega_s, -\omega_s, \omega_p)$. By defining $\chi_a = \chi^{(1)}(\omega_p) + \chi^{(3)}(\omega_p; \omega_s, -\omega_s, \omega_p) |E_s|^2$ and noting $|\Omega_{p'}|^2 << |\Omega_c|^2$, $(\Delta, \delta) \ll (\Delta_{42}, \Delta_{52})$ we have,

$$\chi_a(\omega_p) = \frac{K_0}{2} \frac{-4\tilde{\Delta}_{42} + |\Omega_s|^2}{4\delta \Delta_{42} \Delta_{52} - \Delta_{52} |\Omega_c|^2}, \quad (2)$$
\[ \chi^{(3)}(\omega', \omega_p, -\omega_p, \omega_p) = \frac{K_1}{2} \frac{1}{4\delta\Delta\Delta_42 - \Delta_42 |\Omega_c|^2}, \tag{3} \]

where \( K_1 = \rho |\mu_{13}|^2 |\mu_{24}|^2 / c_0 \hbar^3 \). Eq. (3) governs the response of the medium to the incident probe field. An important problem about practical concern is that absorption of the medium to probe light must be kept small. Note when \(|\Omega_c|^2 > \Gamma_2 \Gamma_3\), there is transparent window near the two photon resonance. The width of the transparent window is given by \( \Delta \omega_{\text{trans}} \approx \frac{\Delta_42 |\Omega_c|^2}{\rho \sqrt{\mu_{13}^2 |\mu_{24}|^2}} \), where \( \sigma = 3\lambda^2 / 2\pi \) is the absorption cross section of an atom and \( L \) is length of the sample. If we choose a \( \omega_p \) inside this window, the probe field will experience a negligible small absorption while the medium still maintains a large Kerr nonlinearity. Fig. 2 gives the real and imaginary part of \( \chi_a \) and \( \chi^{(3)} \). We can see near the two-photon resonance absorptions are quite small, while \( \text{Re}(\chi^{(3)}) \) has a relative large value.

**Fig. 2**

Next we concentrate on engineering a medium, which has a periodic refraction index and a Kerr nonlinearity. Consider geometric configuration shown in Fig. 1(b). \( \vec{E}_p \) and \( \vec{E}_c \) are co-propagating probe and coupling fields. A standing wave is formed by \( \vec{E}_{sf} \) and \( \vec{E}_{sb} \). So \( \Omega_c \) is spatially modulated and has the form \( |\Omega_c(z)|^2 = \Omega_1^2 \cos^2(k_s \cdot \vec{r}) \), where \( k_s \) is wave vector of the controlling field. A proper angle between \( \vec{E}_s \) and \( \vec{E}_a \) is chosen so that \( k_s \cos \phi = k_B \approx k_p \) is fulfilled. This angle is small when the wavelengths of probe field and controlling field are close. Under this assumption we can work with a simplified 1D model. We choose the direction of wave propagation as \( z \) axis. Substitute \( \Omega_a(z) \) into (2) and noting \( n^2 = 1 + \chi \), we have

\[
n^2(z) = 1 + \chi_a(\omega) + \chi^{(3)}(\omega_p, \omega_p, -\omega_p, \omega_p) |E_p|^2, = 1 + \chi_a + \delta \chi \cos(2k_Bz) + \chi^{(3)} |E_p|^2, \tag{4}
\]

where \( \chi_a = \frac{\chi_0}{2} \frac{4\delta\Delta_{52} + \Omega_1^2 / 2}{4\delta\Delta_{52} - \Delta_{52} |\Omega_c|^2} \). Thus we obtain a Bragg grating with a Kerr nonlinearity. The modulation depth of \( \chi_a \) is controlled by \( \Omega_c \). And the amplitude of \( \chi^{(3)}(\omega_p, \omega_p, -\omega_p, \omega_p) \) can be controlled by \( \Omega_c \) and \( \Delta_{42} \). As a result of Floquet-Bloch theorem the periodic refraction index should produce a band gap in the transmission spectrum known as photonic band gap. However there are two important differences between this Bragg grating and the traditional grating formed in an optical fiber. The first one is that the refraction index and modulation depth is strongly frequency dependent. The second one is the effect of absorption should be taken into account. Generally speaking the absorption can make the edge of the band gap blur or even vanish \[14, 20\]. We can see this effect more clearly from the numerical result that follows. We use the transfer matrix method to get the reflection coefficient and the dispersion relation \[21\].

**Fig. 3**

Fig. 3 is the calculated reflectivity of the sample and dispersion relation. We can see there is a band gap inside the EIT window. The width of the band gap is much less than the traditional case because of the frequency dependence of \( \chi_a \) and \( \delta \chi \). This is not surprising. The modulation of refraction index comes from the interaction between atoms with light fields, when the probe light is far off resonant this modulation should disappear. Therefore strong refraction index modulation exists only in the vicinity of \( \Delta_4 = 0 \) where two-photon detuning is small. Consequently the band gap is considerably narrowed. Note the edges of the gap are blurred due to small absorption. As a comparison we also give the reflectivity and dispersion relation when absorption is not taken into account. From the discussion above we can conclude that a relatively strong coupling field is needed for the photonic band gap to survive.

Now we will consider the propagation of light with frequency near or falls into the band gap. The propagation of probe light is governed by Maxwell equations,

\[
\frac{\partial^2 E_p}{\partial z^2} - \frac{n^2(z) \partial^2 E_p}{c^2 \partial t^2} = 0. \tag{5}
\]

Decompose \( E_p \) into a forward and backward wave, i.e.,

\[
E_p = A_+(z, t) e^{i(k_p z - \omega_p t)} + A_-(z, t) e^{-i(k_p z - \omega_p t)}, \tag{6}
\]

where \( A_+(z, t) \) and \( E_-(z, t) \) are the envelopes of forward and backward waves respectively. \( k_p = n_0 \omega_p / c \) is the wave vector. Substitute Eq. (6) into Eq. (5). After applying slowly varying envelopes approximation and expanding \( \chi_a, \delta \chi \) and \( \chi^{(3)} \) around \( \omega_p \), we have,

\[
\partial_z A_+ + v_g^{-1} \partial_t A_+ - i \kappa e^{-2i\Delta k z} A_- - i \gamma (|A_+|^2 + 2 |A_-|^2) A_+ = 0, \tag{7}
\]

\[
-\partial_z A_- + v_g^{-1} \partial_t A_- - i \kappa e^{2i\Delta k z} A_+ - i \gamma (|A_-|^2 + 2 |A_+|^2) A_- = 0. \tag{8}
\]

where, \( \Delta k = k_p - k_B \), \( v_g = \left( \frac{\hbar}{c} + \frac{\omega_p}{2\hbar \nu_\omega \frac{\partial \chi}{\partial \omega}} \right)^{-1}, \kappa = \frac{\delta \chi}{4\hbar c \omega_p}, \gamma = \frac{\omega_p}{2\hbar c} \chi^{(3)} \). We also used \( \delta \chi \gg \frac{\chi}{\partial \omega}, \chi^{(3)} \gg \frac{\partial \chi^{(3)}}{\partial \omega} \).
in deriving Eq. (7), \( v_g \) is group velocity of the light pulse inside the medium without the light induced grating. \( \kappa \) is coupling strength between the front propagating wave \( A_+ \) and back propagating wave \( A_- \). \( \gamma \) represents the self phase modulation (SPM) and cross phase modulation (XPM) due to Kerr nonlinearity. This coupled mode equation has been studied extensively. These equations are related to the well known massive Thirring model of quantum field theory. Although it is non-integrable when the SPM term is non-zero, shape-preserving solitary waves can still be obtained\(^2\). The solution is,

\[
A_+(z,t) = a_+ \text{sech}(\zeta - i\psi/2)e^{\theta}, \\
A_-(z,t) = a_- \text{sech}(\zeta + i\psi/2)e^{\theta},
\]

where

\[
a_\pm = \pm(1 \pm \nu)/4 \sqrt{\frac{\kappa(1-\nu^2)}{\gamma(2-\nu^2)}} \sin \psi, \\
\zeta = \frac{z - V_G t}{\sqrt{1 - \nu^2}} \kappa \sin \psi, \\
\theta = \frac{\nu(z - V_G t)}{\sqrt{1 - \nu^2}} \kappa \cos \psi - \frac{4\nu}{3 - \nu^2} \tan^{-1} \left( \frac{\cot \frac{\psi}{2}}{\coth \zeta} \right).
\]

This solution represents a two-parameter family of Bragg solitons. The parameter \( \nu \) is in the range \(-1 < \nu < 1\), while the parameter \( \psi \) can be chosen anywhere in the range \(0 < \psi < \pi\). The velocity of the Bragg soliton is given by \( V_G = v_g \nu \).

The scenario here is quite different from that in nonlinear fiber gratings. In fibers the group velocity \( v_g \) is always comparable with \( c \). But in our case there is a strong normal dispersion near the two-photon resonance beside the dispersion induced by the grating. So \( v_g \) can be considerably less than \( c \). The ultra slow group velocity will cause the width of the band gap to decrease sharply because the band gap width \( \Delta \nu \) is given by \( 2 |v_g c| \). When there is no Kerr nonlinearity, the group velocity and the group velocity dispersion (GVD) are given by \( V_g = v_g \sqrt{1 - \kappa^2/\delta_{\omega}} \) and \( \beta_2 = -2\delta_{\omega}(\delta_{\omega})^{3/2}/\gamma(2-\nu^2) \), where \( \delta_{\omega} = v_g^{-1}(\omega - \omega_p) \). Here the GVD is enhanced due to the small \( v_g \).

Now we will discuss under what conditions would we observe the optical solitary waves in a cold atomic clouds. The low power limit (\( \gamma P_0 \ll \kappa \)) is of particular interest because in this limit the coupled-mode equations reduced to the nonlinear Schrödinger equation (NLS), where \( P_0 \) is the peak power of the pulse propagating inside the grating. In this case the Bragg soliton is actually reduced to the fundamental NLS solitons and is found to be stable. We take some typical values for an \(^{87}\)Rb atom in our estimation. \( \mu_{13} = 2.5 \times 10^{-20}\text{C-m}, N = 10^{12}\text{cm}^{-3}, \gamma = 0.01\gamma_a, \Gamma_3 = \Gamma_4 = \Gamma_5 = \gamma_c, \Delta_1 = \Delta_2 = 0, \Delta_3 = 5\gamma_\alpha, \Delta_4 = 20\gamma_\alpha, \Omega_c = 10\gamma_\alpha \) and \( \Omega_1 = 10\gamma_\alpha \) where \( \gamma_\alpha = 6\text{MHz} \). The results are \( v_g \approx 4200\text{m/s} \), \( \kappa \approx -2600\text{m}^{-1}, \gamma \approx -0.60\mu\text{W} \). In order to form a band gap, \( |\kappa| L \geq 2 \) is required. This can be trivially fulfilled since the typical size of the atomic cloud is about 1 - 5 mm corresponding to \( |\kappa| L = 2.6 \sim 13 \). The width of the band gap is about 0.67. Since the bandwidth of the input pulse should be much less than the width of the band gap, we can also estimate the minimum width of the input pulse \( T_{\text{FWHM}} > 1/\Delta \nu = 0.29\mu\text{s} \). The possibility of observing Bragg solitons depends on the soliton order \( N \) and the soliton period \( \Delta \nu \). These parameters are given by \( N^2 = \frac{(\Delta - \nu^2)^2(\Delta - \nu^2)}{2(1 - \nu^2)^2} P_0 \) and \( \gamma = \frac{1}{4} \kappa^2 \gamma P_0 \), where \( T_0 \) is related to the FWHM of the input pulse as \( T_{\text{FWHM}} = 1.76T_0 \). A Bragg soliton can form only if \( N \gg 1 \). So the peak power \( P_{\text{in}} \) required to excite the fundamental Bragg soliton can be estimated through relation \( P_{\text{in}} = \frac{4(1 - \nu^2)^{3/2}}{v(3 - \nu^2)^{3/2} \epsilon_0 c} \). Here we use \( P_{\text{in}} = P_{\text{in}}(\nu) \). Another important parameter is \( Z_0 \) since it sets the length scale over which optical soliton evolve. It gives the minimal length of the grating. Note \( P_{\text{in}} \) and \( Z_0 \) depend only on \( \nu \) and \( T_0 \) when the power and detuning of the coupling laser is fixed. Fig. 4 gives \( P_{\text{in}}(\nu) \) and \( T_0(\nu) \) with different \( T_0 \). The region where we can observe Bragg soliton is given by \( P_{\text{in}} \leq P_c/10 \leq P_0 \) and \( Z_0 \leq L \), where \( P_c \) is the power of coupling laser, \( L \) is the length of the cold atomic sample. When \( T_0 = 2\mu\text{s} \) the workable region of \( \nu \) is \( 0.05 < |\nu| < 0.25 \). When \( T_0 = 10\mu\text{s} \), the workable region of \( \nu \) is \( 0.0005 < |\nu| < 0.05 \). So we can work in different range of parameter \( \nu \) by choosing different pulse width \( T_0 \). The lower limit of \( T_0 \) is set by the width of the band gap. One thing worth noting is the input power \( P_{\text{in}} \) is very low. Compared with the experiments done in nonlinear fiber gratings (typical peak intensity required is about 10 GW/cm\(^2\)), the power requirement here is modest. This advantage is due to the ultra slow group velocity \( v_g \) caused by the EIT effect and the larger Kerr coefficient caused by the giant Kerr effect.

Fig.4

In conclusion we have discussed the possibility of using a coherent medium to produce Bragg solitons. We have shown that by using a proper geometric configuration one can make an EIT medium to have both a tunable photonic band gap and a large Kerr nonlinearity. This scheme has two advantages. One is it requires very low light power. The other is it provides a large controllability over the properties of the medium. The modulation depth of refraction index and Kerr nonlinearity can be tuned by varying \( \Omega_1 \) and \( \Omega_2 \).

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**Figure Captions**

Fig. 1 (a) Energy diagram of the atom. (b) Geometric configuration of the lights. Coupling beam and probe beam are co-propagating. A standing wave is formed by a forward and a backward controlling fields $E_{sf}$ and $E_{sb}$. Small angle $\phi$ between the standing wave and probe beam is chosen so that $k_s \cos \phi = k_p$ is fulfilled.

Fig. 2 Real and imaginary parts of $\chi_0$ and $\chi^{(3)}$ versus $\Delta_1/\gamma_a$ (see the text for parameters). Near the two-photon resonance the absorption is small while the Kerr nonlinearity is large.

Fig. 3 (a) Reflectivity of the medium (see text for parameters). A band gap appears near the two-photon resonance. Solid lines are results when absorption is included while dash lines are results when there is no absorption. (b) Dispersion relation (with absorption). $d$ is the period of the Bragg grating. (c) Dispersion relation (without absorption).

Fig. 4 $P_{in}(\nu)$ and $T_0(\nu)$ with different $T_0$. 
