Cluster radioactivity: analysis, forecast, new factors of slowing the decay of atomic nuclei

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Abstract. Some regularities of cluster decays are revealed, which make it possible to predict the existence of a large number of medium, heavy and, possibly, superheavy f-active nuclei. The stability of the majority of non-magic nuclides (products of cluster radioactivity) is explained by the simultaneous action of the already known factors slowing (stabilizing) the radioactive decays of atomic nuclei and some additional factors. Among the additional factors, a weakly expressed effect of pairing of α-particles in light nuclei can be noted. In some cases, an additional particle (neutron, proton or light cluster) contributes to the stability of the nucleus by intensifying the forces of mutual nuclear attraction. Compact arrangement of α-clusters in another additional factor stabilizing the nuclei. One more factor is associated with compact placement of neutrons (possibly paired) between α-clusters within the of light nuclei.

1. Introduction. Baseline data for research
Evolutionary processes in nature, associated with the transmutation of atomic nuclei, proceed “towards the average”. The maximum average specific energy $\varepsilon_b$ of bonds correspond to mass numbers $A \approx 60$ ($^{58}\text{Fe}, \; ^{62}\text{Ni}$).

Cluster radioactivity (f-decay, f-activity, emission of a fragment of the nucleus) refers to the phenomenon of spontaneous emission of fragments (clusters) by nuclei, which is heavier than an α-particle (that is, with a charge of more than 2) and not related to spontaneous division. Clusters are stable compact coupled structures consisting of nucleons. In 1984, H. Rose and G. Jones first observed the spontaneous emission of $^{14}\text{C}$ radium nuclei [1]. In the known processes of cluster radioactivity, nuclei with a mass number $A = 221 \ldots 242$ (Fr, Ra, Ac, Th, Pa, U, Pu, Np, Am, Cm isotopes) emit a light cluster (from $^{12}\text{C}$ to $^{34}\text{Si}$) and heavy cluster radioactivity (isotopes of thallium, mercury, lead, bismuth) [2-5]. The exception is the decay of barium-114: $^{114}\text{Ba} \rightarrow ^{102}\text{Sn} + ^{12}\text{C}$ [2-5].

With the exception of barium-114, all f-active nuclides are α-active. (The probability of α-decay of $^{114}\text{Ba}$ is equal $9 \cdot 10^{-7}$ [2].) Cluster decays are closer to α-decays than to spontaneous fission. They obey the Geiger—Nutall law. Unlike fission, free neutrons are not emitted during f-decays. The condition of equilibrium of forces of nuclear, electromagnetic and weak interactions is characteristic for the most stable nuclei (optimal $N/Z$):

$$N/Z = 0.98 + 0.015 A^{2/3}. \quad (1)$$

Here $N$ is the number of neutrons, $Z$ is the number of protons, $A$ is the mass number, $A = N + Z$.

The works [2-7] were used as initial data for research. An analysis of these data made it possible to identify patterns of cluster decays and to predict the existence of a large number of medium and heavy...
f-active nuclei. Analysis of cluster decays allowed us to identify new factors for the stabilization of the nuclei and the (new factors slowing) radioactive decays.

2. Analysis and forecast of cluster decays

2.1. Elementary analysis. Basic patterns

The $^{58}$Fe and $^{62}$Ni (e → max) can be emitted as heavy clusters during f-decays. Therefore, f-decays of nuclei with $A > 58 \ldots 62 + A_1$ are possible, where $A_1$ is the mass number of the light cluster. If $A_1 = 12$, then $A = 70 \ldots 74$. The laws of conservation do not prevent the emission of a much lighter cluster than $^{12}$C. It means that f-decays can experience nuclei with a mass number slightly smaller than $70 \ldots 74$.

Magical nuclei and maximally deformed (in the ground energy state) nuclei are characterized by increased stability. For this reason, during the decays of superheavy elements, we can expect with $Z \approx 114 \pm 2$, $N \approx 114 \pm 2$, as well as the isotopes Th, U, Hs (or others with close Z values). Cluster and α-decays of neutron-deficient nuclei bring these nuclides closer to the optimal N/Z value. Cluster and α-decays of neutron-rich nuclei move these nuclides away from the optimal N/Z value. For this reason, α- and f-decays of medium nuclei are observed in neutron-deficient nuclides.

Elementary analysis revealed the following patterns.

In decays, the transition of a radioactive nuclide into the category of non-existent is practically impossible. Most likely are the directions of nuclear transmutation that bring them closer to the optimal N/Z (1).

All light products of the f-decay have an α-cluster structure. Among the products of known f-decays, there are no light nuclei consisting of less than three α-particles. Light clusters are often overloaded with neutrons. Moreover, $N$ is always even. In the f-decay process, an odd neutron or proton carries away a heavy cluster. Both f-decay products can be stable twice magic nuclei ($^{228}$Th → $^{208}$Pb + $^{20}$O).

Cluster decays are characteristic to the nuclei experiencing several consecutive α-decays. For the f-active nuclides, successive α-decays happen more often, sometimes alternating with β-decays. This brings the decomposition products closer to the band of stable nuclei. For example, the decay of $^{230}$Th → $^{206}$Hg + $^{24}$Ne is possible. Neon has an α-cluster structure (it consists of five α-particles and four neutrons). The α-decay chain is most likely: $^{230}$Th → $^{226}$Ra → $^{222}$Rn → $^{218}$Po → $^{214}$Pb → $^{210}$Hg. The $^{218}$Po, $^{214}$Pb, and $^{210}$Hg nuclei can undergo β-decays with subsequent α-decays. The final decay product is a stable lead-206. The product of an α-decay of thorium-230 is radium-226. It is also α- and f-active. The decay of $^{226}$Ra → $^{212}$Pb + $^{14}$C (three α-particles and two neutrons) is possible. The products of β-decay are radioactive. The chain of α- and β-decay of radium is most likely.

Spontaneous fission is characteristic to less stable nuclei. Cluster and α-decays are characteristic for the most stable nuclei (magical or maximally deformed in the ground energy state).

In extremely rare cases for heavy nuclei that are in the ground energy state, α-decay (with a probability close to 100%), spontaneous fission and f-decay are observed simultaneously. This is characteristic of $^{237}$Np and $^{240}$Pu. According to [2], the probability of spontaneous fission of $^{237}$Np is less than 2·10⁻¹², $^{240}$Pu is equal 5.7·10⁻⁸. The probability of emission of a $^{30}$Mg-cluster by the $^{237}$Np is less than 4·10⁻¹⁴. The probability of emission of $^{34}$Si by the $^{240}$Pu is less than 1.3·10⁻¹⁵.

2.2. Barium-114 decays

The main channel of radioactive decay of $^{114}$Ba is electron capture. The probability of α-decay of $^{114}$Ba is 9·10⁻⁷, probability of f-decay is 3.0·10⁻⁵ [2].

The decay $^{114}$Ba → $^{102}$Sn + $^{12}$C is an exception to all observed f-decays. This is due to the large deficiency of neutrons in the $^{114}$Ba. The symmetry energy ($Z \approx N$) contributes to an increase of half-life of $^{114}$Ba. The $^{114}$Ba has two possible directions of decay (Figure 1). The first one is the most probable (six consecutive electron captures) immediately toward the optimal N/Z, i.e., to a stable magic ($Z = 50$) $^{114}$Sn. The second is the “bypass” of $^{114}$Sn through magical neutron-deficient nuclides $^{102}$Sn, $^{106}$Sn or $^{110}$Sn. When “bypassing” through $^{102}$Sn (three α-decays or one f-decay), the decay products closely approach the boundary of proton stability. This is followed by a “turn” in the direction of the optimal
In the parent nucleus lead it to a doubly magic nuclide located on the boundary of the nucleon stability region. At the first stage, the radioactive transformation (one or several towards the boundary of the nucleon stability region) is a “bypass”, which is unlikely. Twice-magic nuclei $^{100}$Sn ($\pm 2$ nucleons) are the natural border of the “bypass” path.

![Diagram](image-url)

**Figure 1.** Barium-114 decays: arrows indicate the directions of $\alpha$-decays, the dotted arrows correspond to the unlikely $\alpha$-decays; $\beta^+$-decay (electron capture) are directed along the isobars ($A = \text{const}$) in the direction of increasing $N$; The $^{102}$Pd, $^{106}$Pd, $^{110}$Cd, $^{114}$Sn nuclei are stable.

2.3. Assumptions about possible cluster decays of neutron-deficient nuclei

If the transition from $^{114}$Ba to $^{102}$Pd through $^{102}$Sn is possible, then transition of stable tin isotopes (eg, $^{114}$Sn) to stable palladium isotopes (eg, $^{106}$Pd) along the band of stable nuclei parallel to the decay chain $^{114}$Ba $\rightarrow$ $^{102}$Sn (in $N$–$Z$ coordinates) cannot be ruled out. Also, an extremely slow $\alpha$-decay of $^{106}$Pd into stable $^{102}$Ru or two consecutive $\beta^+$-decay (electron capture) of stable nuclides $^{102}$Pd $\rightarrow$ $^{102}$Rh $\rightarrow$ $^{102}$Ru are not excluded. In both cases, the final decay product is $^{102}$Ru. This stable nucleus will slowly decay to $A \approx 60$ ($\epsilon \rightarrow \text{max}$). In the $N$–$Z$ coordinates from $^{114}$Sn to $^{102}$Ru there are three $\alpha$-decays. From $^{114}$Ba to $^{102}$Sn, there are also three consecutive $\alpha$-decays. The chains of decays $^{114}$Sn $\rightarrow$ $^{102}$Ru and $^{114}$Ba $\rightarrow$ $^{102}$Sn on the $N$–$Z$ diagram are parallel. If cluster decay of $^{114}$Ba (e.g., $^{114}$Ba $\rightarrow$ $^{102}$Sn + $^{12}$C) is possible, then the extremely slow decay of $^{114}$Sn ($^{114}$Sn $\rightarrow$ $^{102}$Ru + $^{12}$C) is possible.

The $N$–$Z$ diagram (Figure 2) shows schematically the possible directions of decays of neutron-deficient nuclides. On the $N$–$Z$ diagram, we add grid lines passing through $Z$ and $N$ corresponding to the most stable nuclei. Twice magic nucleus are located in the grid nodes.

There are two trends in the decay of barium-114 and neutron-deficient medium nuclei.

1. Striving towards the optimal $N/Z$ region (1). Formally, this is a single-criterium optimization problem. It may have one solution. Optimization of $N/Z$ ratio is achieved by successive $\beta$-decay (electron capture, $\beta^+$, $\beta^-$) of the parent nucleus.

2. The simultaneous pursuit of optimization of $N/Z$ ratio and $A$ ($A \rightarrow 60$). Formally, this is a two-criterium problem. It may have a finite set of solutions (since the number of nucleus is finite). Minimizing $A$ (to the optimal value $A \approx 60$) requires going beyond the rectangle in the $N$–$Z$ coordinates formed by magic $N$ and $Z$ to smaller $A$. To achieve this, it is necessary that the stabilization effect associated with filling of the nuclear shells is small. This is possible at a considerable distance from the optimal $N/Z$ towards the boundary of the nucleon stability region. In the decay chain, in this case, two stages can be distinguished. At the first stage, the radioactive transmutation (one or several $\alpha$ decays or $f$ decays) of the parent nucleus lead it to a doubly magic nuclide located on the $N$–$Z$ diagram, far from the optimal $N/Z$ at the vertex of the rectangle corresponding to the minimum $A$. For neutron-deficient nuclei this
condition is equivalent to the minimum $Z$, for neutron-rich nuclei — the minimum $N$. The twice-magic nuclei distant from the optimal $N/Z$ is by far less stable than the once-magic nuclei with the optimal (or close to optimal) $N/Z$ value. This is exactly what makes it possible to go beyond the boundary of the rectangle in which the parent nucleus is located. Transitions are possible through neutron-deficient radioactive magic isotopes.

The second stage is $N/Z$ optimization by a single criterion due to consequent $\beta$-decay (electron capture, $\beta^+$, $\beta^-$) of a short-lived, twice-magic nucleus with a non-optimal $N/Z$. This nucleus serves as the new parent nucleus. The first stage can be completed after the first (or second) $\alpha$-decay, when the nucleus has not yet reached the magical (or near the magical) values of $N$ and $Z$. In this case, there is no going beyond the limits of the rectangle in which the parent nucleus is located. An example is $\alpha$-decay of the $f$-active $^{114}$Ba in $^{110}$Xe with subsequent $\beta^+$-decay (electron capture) to the optimal $N/Z$ region. In decay, a transition is made through the border of the rectangle (the border of the grid cell) corresponding to the minimum $N$. In Figure 2 the notation is as follows: $1$ - optimal $N/Z$ curve; $2-4$ - directions of decay chains ($2$ - first trend, $3$ - first stage of the second trend, $4$ - second stage of the second trend). The numbers $2'$ and $2''$ denote the $\beta^+$-decay (electron capture) chains $^{110}$Xe and $^{106}$Sb ($a$), $^{208}$Po and $^{204}$Rn ($b$). Note that segments $2$, $2'$, $2''$ and $4$ in figure $2$ are parallel, $2$ and $3$ are perpendicular. The direction of decays of $3$ most closely approximates the maternal nucleus to the optimal $N/Z$. The nuclides $^{102}$Pd, $^{106}$Pd, $^{110}$Cd, $^{114}$Sn, $^{116}$Hg, $^{204}$Hg, $^{204}$Pb, $^{208}$Pb are stable.

Isotopes of barium and palladium exhibit the properties of magic nuclei while, not belonging to this class. The grid coordinate lines (Figure 2, $a$) can pass through $Z = 46$. Decays of stable or long-lived nuclei can be accelerated by an external influence (for example, by the collision of macro-objects containing stable or unstable nuclei) [8, 9]. As a result of the collision of stable tin ($^{114}$Sn) with material containing the nuclei with the highest $\epsilon$ ($^{58}$Fe, $^{62}$Ni, steel), $f$-decays of $^{114}$Sn can be observed with the formation of heavy clusters $^{106}$Pd or $^{102}$Ru (or near these nuclei) or $\alpha$-decay with the formation of $^{110}$Cd. The heavier the nucleus, the easier (with less external influences) should it be for such decays happen. The exception should be the most stable nuclei (magic and twice magic with optional $N/Z$). Their decay products should be the most stable compared with the neighboring nuclei of the $N$–$Z$ diagram.

The $^{118}$Ce contains 2 neutrons and 2 protons (i.e., an $\alpha$-particle) more than $^{114}$Ba. Cerium-118 is located near the proton-drip line. It is possible that four $\alpha$-particles are present in the $^{118}$Ce decay chain. It can be assumed that $^{118}$Ce $\rightarrow$ $^{106}$Sn + $^{16}$O. The $^{122}$Nd nucleus contains 2 neutrons and 2 protons more than $^{118}$Ce. There are no data on neodymium-122 decays (electronic capture is typical of $^{124}$Nd). If $^{122}$Nd is $\alpha$-active (against the background of proton activity), then up to five $\alpha$-particles may be present in the chain of its decays. In this case, $f$-decay can be not excluded: $^{122}$Nd $\rightarrow$ $^{108}$Sn + $^{20}$Ne. So, only stable nuclei can be the natural limit of decays, and, moreover, the most stable (long-lived) nuclei. These nuclei are characterized by a near-optimal $N/Z$ ratio, or by $N$ and $Z$ values close to magic numbers ($\pm$ 2 nucleons), or the maximum deformed nuclei. If, for example, a doubly magic nucleus is not stable, then a "transition" through it in the process of $f$-decay is possible.

Figure 2. Scheme of the transition through the magic nucleus during the decay of nuclei:

- $a$ through $Z = 50$ (decay of $^{115}$Ba, $^{110}$Xe, $^{106}$Sb),
- $b$ through $Z = 82$ (α-decay of $^{208}$Po, $^{204}$Rn, $^{208}$Ra, f-decay of $^{208}$Ra).
2.4. Dependence of the main decay channel on the stability of the nucleus

At the optimal and close to optimal N/Z values, two processes compete: α-decay and spontaneous fission. Alpha decays (hence, f-decays) prevail in the most stable nuclei, while spontaneous fission prevail in the least stable ones.

At non-optimal N/Z values, β-decay (electron capture, β⁺, β⁻) compete with one of the following processes: α-decay or spontaneous fission. Sometimes three processes compete at once (examples: 240Fm, 250Cm, 250Md, 252Fm, 253No, 254No, 255Es, etc.). The double magic nuclides are the most resistant. They are ball-shaped, always even-even, characterized by zero spin, zero value of the intrinsic (internal) quadrupole electric moment Q of the nucleus. The less stable nuclei, the more deformed in the main energy state. They are flattened or elongated with respect to the spherical shape, because for such nuclei |Q| → max. Nuclei with |Q (N)| → max and |Q (Z)| → max are less stable than twice magic nuclei, but more stable compared to the neighboring nuclides on the N–Z diagram. The following combinations are possible: the magic number of neutrons (protons) and the number of protons (neutrons) corresponding to the maximum deformation. Such nuclei are also characterized by increased stability. The N and Z values, corresponding to the maximum deformation, as a rule, lie in the middle between adjacent magic numbers. The period of the functions Q (N) and Q (Z) decreases with increasing N and (or) Z, and its amplitude increases.

The maxima and minima of the Q (N, Z) function are blurred. There are large regions of atomic nuclei, strongly deformed in the ground energy state. A large number of highly deformed nuclei are located in the N and Z = 34 ... 36; 42 ... 48; 52 ... 60; 64 ... 76; for N = 90 ... 120 and N > 142. As a result, stable nuclides in practice turn out to be much more than would be expected based on the theory of nuclear shells. Due to the pairing of neutrons (protons) in the nucleus, nuclides that differ from the least stable by two neutrons and (or) protons are also sufficiently (although less) resistant.

The least resistant nuclei are located on the N or Z axis between the maximally deformed and spherical nuclides. Since there are many deformed nuclei, the least stable nuclides are characterized by the maximum absolute value derivative dQ/dN and (or) dQ/dZ, provided Q ≠ 0.

The numbers N and Z, corresponding to the least stable nucleus, as a rule, lie in the middle between the magic number N (or Z) and the number N (or Z), corresponding to the maximum deformation of the nucleus. The regions of unstable nuclei are small. As a results, nuclides that differ from the least stable to two neutrons and (or) protons can be quite stable. Therefore, there are more stable nuclei than unstable ones, i.e., among heavy and superheavy nuclides, α-active nuclei are more than spontaneously dividing ones.

2.5. Predictions of cluster decays of heavy nuclei

The f-decays of heavy α-active neutron-deficient nuclei located near the proton stability boundary in the N–Z diagram cannot be ruled out. The 250No nucleus (Z = 102) is the starting point for a chain of eleven consecutive α-decays converting neutron-deficient heavy nuclei into neutron-excess: 250No → 248Fm → 244Cf → 238Cm → 234Pu → 230U → 226Th → 222Ra → 218Rn → 214Po → 210Pb → 206Hg. In this chain, the transition through the most deformed nuclei of uranium and thorium, characterized by the maximum half-life (as compared to the neighboring chain nuclides), is realized. Further α-decays are impossible, since the nuclide is more and more removed from the band of stable nuclei (from the optimal N/Z). Radium-222 is characterized by the optimal N/Z value. The known f-decay of 222Ra (222Ra → 208Pb + 14C) does not remove the heavy cluster (208Pb) from the band of stable nuclei.

2.6. Analysis of superheavy nuclei decays. Forecasts

So, for β-stable nuclei (i.e. nuclides with close to optimal N/Z value), α-activity and spontaneous fission compete. In Figure 3 shows the dependences between the probability of α decay (Pα) and spontaneous fission (Pfission) on one side and the mass number for nuclei with a charge from 90 to 108 with a close to optimal N/Z ratio on the other. (According to [2], transition from α-activity to spontaneous fission is very smooth.) With decreasing stability of the nucleus, the probability of spontaneous fission increases. The data presented on Figure 3 shows that the least stable heavy nuclei are located in the region of mass...
numbers between 243 and 266. For spontaneously fissioning nuclei from the ground energy state, $f$-decay is not observed. Thus, like the $\alpha$-decay, cluster radioactivity should be characteristic of more stable atomic nuclei. Figure 4 shows the dependence of the ratio of the probabilities of spontaneous fission and $\alpha$-decays ($P = P_{\text{fission}}/P_{\alpha}$) on the charge ($a$) and the mass number ($b$) of heavy nuclei ($Z = 90 \ldots 110$).

The maximum correlation between $P(Z)$ and $P(A)$ corresponds to the least stable, and the minimum to the most stable nuclei. This is most clearly manifested in Figure 4, $a$. The most resistant nucleus (shown on Figure 4) is thorium ($Z = 90$). Its isotopes are maximally deformed in the ground energy state. The second minimum is observed for the magic Fl ($Z = 114$). Data about fission of Fl are absent. Its isotopes with close to optimal $N/Z$ are pure $\alpha$-emitters. The least resistant are nuclei with a charge of 102 (maximum in Figure 4, $a$) located approximately in the middle on the $Z$ axis between the numbers of protons corresponding to the maximum deformed (90) and magic (114) nuclei. Nobelium-258 (the mass number corresponds to the maximum in Figure 4, $b$) has 156 neutrons. The number 156 is located approximately in the middle between two magic numbers 126 and 184 and must correspond to the maximum deformation of neutron matter in the nucleus.

In [10], $\alpha$-decay chains for neutron-deficient transactinides with even and odd $Z$ are given. For example, $^{118}\text{Og} \rightarrow \text{Lv} \rightarrow \text{Fl} \rightarrow \text{Cn} \rightarrow \text{Ds} \rightarrow \text{Hs} \rightarrow \text{Sg} \rightarrow \text{Rf}$ and $^{117}\text{Ts} \rightarrow \text{Ms} \rightarrow \text{Nh} \rightarrow \text{Rg} \rightarrow \text{Mt} \rightarrow \text{Bh} \rightarrow \text{Db}$. Comparing it with the decay chains of heavy f-active nuclei, it follows that instead of consistently emitting $\alpha$-particles, one cannot exclude the emission of a cluster (consisting of $\alpha$-particles). In this case, decays of Og $\rightarrow$ Cn + C, Lv $\rightarrow$ Hs + O, Lv $\rightarrow$ Rf + Mg, Ts $\rightarrow$ Db + Mg, Ts $\rightarrow$ Mt + O, etc. are not excluded. Cluster and $\alpha$-decays bring these nuclei closer to the optimal $N/Z$.

2.7. Findings
There should be much more $f$-active nuclei than we know at the moment.
Cluster (like any other) decays are realized under the condition that the nucleus approaches the region of maximum stability in the transmutation process. For heavy nuclei, a move towards the band of stable nuclei is realized. For medium nuclei, a move towards the stable middle nuclei shall happen ($\varepsilon \to \max$).

Some $f$-decays allow you to “go” through a stable nucleus (magic or maximally deformed) in order to get as close as possible to the “golden mean” ($A \approx 60$). In this case, the closest approach to the proton stability boundary is possible, i.e. to the most stable nucleus (twice magic, maximally deformed, etc.) located near this boundary.

The $f$-decay of neutron-deficient medium nuclei is characterized by the formation of a heavy cluster far from the optimal $N/Z$. During the decay of heavy nuclei with close to optimal $N/Z$, the formation of stable heavy clusters, close to twice the magic lead-208 and close to optimal $N/Z$ often happens. Like $\alpha$-decay chains, $f$-decays of transactinides can lead to formation of the least stable nucleus (compared to the neighboring nuclei on the $N-Z$ diagram). This nucleus is spontaneously fissioned. This is the way to achieve the "average" ($\varepsilon \to \max$).

The decay of nuclei with a charge of more than 110 happens in two stages. At the first stage, a cascade of $\alpha$-particles is emitted. At the second stage, spontaneous fission is observed.

Due to many factors stabilizing radioactive nuclei and the superposition of these factors, the number of stable nuclei which can be products of cluster decays is large. The values of $N$ and $Z$, at which the nuclei have increased stability, significantly exceed the number of doubly magic and most deformed nuclei. Analysis of the products of known and suspected $f$-decays allows us to conclude that there are additional factors stabilizing the decays of atomic nuclei. We will consider these in the next chapter.

3. New factors of stabilization (slowing) of radioactive decays

3.1. The simplest mathematical task statement

Each factor of stabilization of radioactive decays of atomic nuclei can be put in accordance with any functional: $\varepsilon$, $Q$, the pairing energy, etc. This functionals are of different dimensions. Dimensionless (related to maximum values) functionals should therefor be considered. Let $i$ be the number of the nucleus, $i = 1, 2, \ldots, I$. Let $k$ be the number of functionals, $k = 1, 2, \ldots, K$. Define $F_{1,i} = q_{i}/q_{i}^{\max}$, $F_{2,i} = |Q_{i}|/|Q|^{\max}$. We formulate the problem of determining the most resistance nucleus among the stable, even-even, non-magic kernels as a discrete multicriteria problem of maximization of the $F_{k,i}$ functional. We reduce it to the problem of mathematical programming by formulating a single criterion of $F_{i}$ quality for each nucleus: $F_{i} = \lambda_{k,i} F_{k,i} \to \max$. Here $\lambda_{k,i}$ are the weight coefficients that determine the significance of the $k$-th criterion (functional) for the $i$-th nucleus. Summation is carried out by $k$ (from 1 to $K$). The coefficients $\lambda_{k,i}$ are chosen subjectively or are calculated as the relative values of the sensitivity coefficients of each quality criterion with respect to any functional characterizing the stability of the nucleus. As such a functionality, you can choose the half-life $T_{1/2}$, cross section or the rate of nuclear reactions in the field of hadrons (e.g. neutrons) etc. For clarity and simplicity, choose $T_{1/2,i}(i)$ — the number of the nucleus, $i = 1, 2, \ldots, I$). The significance of the functional $F_{k,i}$ with respect to $T_{1/2,i}$ in dimensionless form is defined as

$$\lambda_{k,i} = \left( F_{k,i} / dT_{1/2,i} \right) \cdot \left( dT_{1/2,i} / dF_{k,i} \right).$$

The coefficients $\lambda_{k,i}$ are the value functions of $F_{k,i}$ with respect to $T_{1/2,i}$. They can be calculated based on estimated nuclear data from [7] and known values of $F_{k,i}$.

3.2. Minimization of the number of functionals for the problem

Among the most significant factors of stabilization are optimal $N/Z$ (1), ad well as magic and even $Z$ and $N$ values. The last two factors are most noticeable when $N/Z$ ratio is close to optimal (1). We will exclude these three factors from our consideration. We will only analyze stable even-even non-magic nucleus.

The following stabilization factors are related to the maximum deformation of the nucleus (with respect to the spherical shape) in the ground energy state, the pursuit of the mean values of $A$ ($A \to 60$), and for light nuclei, clustering and symmetry energy ($N/Z \to 1$). The deformation can be characterized by the functional $Q = Q (N, Z)$, the pursuit of the average mass number - the functional $\varepsilon = \varepsilon (A)$. The
effect of symmetry energy on the binding energy is already taken into account in (1). The dependence $\epsilon (A)$ is also indirectly considered by (1). The function $\epsilon (A)$ is usually constructed along the curve of the optimal $N/Z$. Hence, the functionals $Q$ and $\epsilon$ can be excluded from the consideration. Thus, for the analysis of the stability of atomic nuclei there only relevant factors are those associated with the maximum deformation of the nucleus, the pursuit of the average values of the mass number, and for light nuclei — with static clustering. Perhaps the imposition of these factors and the influence of some other factors stabilizing decays.

### 3.3. Analysis of the simultaneous action of several decay stabilization factors

It follows from the known experimental dependence $Q (N, Z)$ that there are large regions of atomic nuclei strongly deformed in the ground energy state. A large number of strongly deformed nuclei are located in the $N$ and $Z = 34 \ldots 36; 42 \ldots 48; 52 \ldots 60; 64 \ldots 76; \text{for } N = 90 \ldots 120; \text{and for } N > 142$.

The mutual influence of the factor associated with the broadening of the band of stable nuclides due to the tendency of $A$ to the mean values, and the factor associated with a strong deformation of nuclei, led to the fact that a large number of stable isotopes have non-magical nuclei with a charge $Z = 34$ (6 stable isotopes), $36 (7)$, $40 (5)$, $42 (7)$, $44 (7)$, $46 (6)$, $54 (9)$, $56 (7)$, $60 (7)$, $62 (7)$, $64 (7)$, $66 (6)$, $70 (7)$, $72 (6)$, $74 (5)$, $76 (7)$, $78 (6)$. Increased resistance of only a few nuclei is associated with magic or nearly magic numbers $Z$ and $N$. As $Z$ increases, the optimal $N / Z$ ratio corresponds to a greater overcrowding of the nucleus with neutrons (compared to the case of $N = Z$). This is due to the need to compensate for the forces of Coulomb repulsion between the protons in the nucleus by adding additional neutrons (strengthening the mutual nuclear attraction, but not participating in the Coulomb interaction).

A large number of stable isotopes have magic and near-magic nuclei with a charge of $48 (8 \text{ isotopes})$, $50 (10)$, $52 (8)$, $80 (7)$. There is a sharp transition from strongly deformed nuclei to spherical nuclei ($|dT_{1/2}/dQ| \rightarrow \text{max}$). The narrowing of the band of stable nuclides with further increase in $Z$ or $A$ leads to a noticeable decrease in the number of stable isotopes of the last stable elements: lead (4 isotopes) and bismuth (1).

The band of stable nuclei is located asymmetrically with respect to the line $N/Z = 1$. Its width along the $N$ axis is noticeably greater than along the $Z$ axis, i.e. at $Z \geq 19$, $\Delta N > \Delta Z$ as a rule. As a result, the number of stable isotopes ($N = \text{const}$) of a nuclide almost always exceeds the number of stable isotons ($Z = \text{const}$) of the same nuclide. As a result of the simultaneous action of stabilization factors associated with the parity of $N$ and $Z$, with the pursuit of the mean values of $A$, as well as with the maximum deformation, we obtain entire regions in the diagram $N–Z$, containing a large number of stable even-even non-magic nuclides.

### 3.4. The additional factor of stabilization of the light nuclei decay

It is known that $^4\text{He}$ nuclei (α-particles) are characterized by high average specific binding energy (7.15 MeV) which is abnormal for a light nucleus. Some light products of cluster decay ($^{12}\text{C}, ^{16}\text{O}, ^{20}\text{Ne}, ^{24}\text{Mg}, ^{28}\text{Si}$) consist of α-clusters.

For light nuclei, there is a factor of decay stabilization associated with the energy of symmetry. A neutron-rich (relative to $N/Z = 1$) nucleus may contain clusters of $^4\text{He}$ and $^4\text{He}$ and (or) neutrons not bound in clusters (e.g. $^6\text{Be} = 2 ^4\text{He} + ^2\text{n}$). Other static clusters ($^3\text{H}, ^3\text{He}$, binetron, biproton), formed in the atomic nucleus, are characterized by a much lower binding energy of nucleons in the cluster. (The pairing effect is equivalent to clustering.) However, such loosely coupled clusters, under certain conditions, can make a decisive contribution to the stabilization of decays.

The bubble structure of certain nuclei was investigated since the 1970s. The clusters in the nucleus can be located at the nodes of a triangle or in the shape of a ring. In the central part of such nuclei a cavity may be located. According to experimental data [11], the density of neutron and proton matter reaches its maximum in the center of the $^{32}\text{Si}$ nucleus. At the center of the $^{34}\text{Si}$ nucleus, the density of neutron matter is maximal, and the density of proton matter has a local minimum (depending on the radius of the nucleus). Bubble nuclei ($Z \approx 400$) can have not only an external but also an internal surface layer, where $N/Z > 1$. Light α-cluster nuclei $^{28}\text{Si}$ have a cavity (bubble) in the central part [11]. The
surface layer can fill this entire cavity, i.e. pure neutron matter (one or two paired neutrons) can exist in the central part of the $^{34}$Si nucleus. Excess neutrons can be concentrated not only in the surface layer, but also in the central cavity of the neutron-excess nucleus. Other excess neutrons in heavy isotopes of light nuclei may also be present in heavy $^6\alpha$-clusters.

Compact placement of neutron matter in nuclei with optimal $N/Z > 1$ is a factor of decay stabilization.

3.5. Pairing (pair interaction) as a fundamental property of matter

The reasons for the pairing of particles can vary, however the results is the same: formation of a stable pair. The effects of pairing are observed in the presence of any even arbitrarily small additional forces of attraction between the two particles. Mating forces are of different nature (depending on the types of fundamental interactions responsible for these forces) and manifest themselves in different ways. The Cooper pairs of electrons and $^3$He is known. Correlated pairs of protons ($^2$He) and neutrons ($n-n$) do not exist freely. However, such pairs exist in the nucleus. For the existence of a stable pair of $d$-quarks, an additional particle ($u$-quark) is needed. The result is a neutron ($T_{1/2} \approx 11$ min). For the existence of a stable pair of $u$-quarks, an additional particle ($d$-quark) is needed. The result is a stable proton. For the existence of a stable pair of protons or neutrons, an additional particle (neutron and proton, respectively) is also required. The $^3$He ($2p + n$) is stable, and the $^3$H ($p + 2n$) is relatively long-lived ($T_{1/2} \approx 12$ years).

In the absence of available data on the decays of stable nucleides $^{12}$C (3 $\alpha$-clusters), $^{16}$O (4 $\alpha$), $^{20}$Ne (5 $\alpha$), $^{24}$Mg (6 $\alpha$), $^{28}$Si (7 $\alpha$), $^{32}$S (8 $\alpha$), it is possible to compare their unstable neutron-redundant isotopes containing $^4\alpha$ - and possibly $^6\alpha$-clusters. Dependence of the half-life on the number of $\alpha$-clusters in the nucleus is presented in Figure 5.

Comparing $^4\alpha$-cluster nuclei with nuclei that have an extra neutron and (or) proton shows that odd-odd isotopes consisting of two $^4\alpha$ particles and additional proton and a neutron (one each), characterized by $N = Z$, turn out to be much longer-lived than neighboring odd-even neutrons in the number of $(N - 1)$ at $N < Z$. This may mean that either the role of symmetry energy in stabilizing decays is quite important, or there is an additional stabilization factor associated with the pairing of “extra” neutrons and protons as well as with the presence of an additional particle, $^2$H-cluster, in the nucleus. The analysis is difficult due to the small size of the region of existing light nuclei.

**Figure 5.** Dependence of the half-life on the number of $\alpha$-clusters in the nucleus.

**Figure 6.** Dependence of $\sigma_{n, \gamma}$ on $A$ for some nuclei (according to [6]): the composition of natural carbon is $^{nat}$C = 98.89 % $^{12}$C + 1.11 % $^{13}$C; 1 b (barn) = $10^{-28}$ m$^2$.

At a distance from the optimal $N/Z$ curve, neither the effect of pairing of $\alpha$-clusters (nor the effect of pairing of nucleons) in the nucleus is noticeable. The pairing of $^3\alpha$ and $^4\alpha$-clusters within a single nucleus does not happen. The pairing of $^3\alpha$-clusters ($^3\alpha + ^3\alpha$) in the nucleus with $^4\alpha$-clusters ($^4\alpha + ^4\alpha$) is noticeable.

For the most stable nuclei, the cross sections for inelastic processes are minimal and the threshold for nuclear reactions in the field of hadrons (for example, neutrons) is maximum.

Figure 6 shows the microscopic cross section of the ($n$, $\gamma$)-reactions $\sigma_{n, \gamma}$ (where “$n$” is thermal neutron) for the $\alpha$-cluster nuclei. As $Z$ increases, the deficit of neutrons in the nucleus increases, and the nucleus tends to attach free neutrons, the cross section increases. The exceptions are the twice magic nuclei $^{16}$O and $^{40}$Ca, for which, as expected, $\sigma_{n, \gamma} \rightarrow$ min.
3.6. Findings
There are many factors stabilizing the decays of atomic nuclei. The role of these factors in different areas of the $N$–$Z$ diagram is not the same. There is a weakly pronounced effect of pairing of $\alpha$-particles in the nucleus. Despite the small energy of pairing of $\alpha$-particles in the nucleus, this is an additional factor of stabilization of the radioactive decays of light atomic nuclei. Like the neutron (proton) pairing effect, it is noticeable only near the optimal $N/Z$ values. In some cases (for example, for $\alpha$-cluster nuclei), an additional particle (neutron, proton, cluster) in the composition of a micro-object (nucleus) contributes to an increase in stability due to the intensification of the forces of mutual nuclear attraction. This is also an additional stabilization factor. An increased stability of the nuclei is observed with compact arrangement of $\alpha$-clusters within the atomic nucleus. Compact placement of neutrons in neutron-rich nuclei with optimal $N/Z > 1$ is another stabilization factor for radioactive decays. As a result of the simultaneous action of stabilization factors related to the parity of $N$ and $Z$, with a tendency towards the mean values of $A$, as well as with maximum deformation, whole regions are obtained on the $N$–$Z$ diagram containing a large number of stable even-even non-magic nuclides.

4. Conclusion
The revealed regularities of cluster decays allow us to conclude about the possibility of the existence of a much larger number of f-active nuclei than is known to date.

Assumptions about f-decays of $\alpha$-active transactinides are based on their relatively high stability. The transition to the region of medium nuclei ($\epsilon \rightarrow \max$) is possible in two stages. The first is associated with the formation of a heavy unstable nucleus (heavy f-decay cluster). The second is associated with the spontaneous division of this nucleus.

The resilience of most nuclei is explained by the simultaneous action of various factors stabilizing the radioactive decays. The new factors of slowing (stabilizing) radioactive decays are identified. There is a weakly pronounced effect of pairing of $\alpha$-particles in light $\alpha$-cluster nuclei (products of f-decays). Often, an additional nucleon or static cluster in the composition of the nucleus contributes to increasing stability of the nucleus due to the intensification of the forces of mutual nuclear attraction. The compact arrangement of $\alpha$-clusters in the nucleus, as well as the compact arrangement of neutrons (including paired ones) between $\alpha$-clusters in the composition of light nuclei, can be considered as an additional factor of stabilization of decays.

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