Sigma-model anomalies in compact
D=4, N=1 Type IIB orientifolds and
Fayet-Iliopoulos terms

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Abstract

Compact Type IIB $D = 4$, $N = 1$ orientifolds have certain $U(1)$ $\sigma$-model symmetries at the level of the effective Lagrangian. These symmetries are generically anomalous. We study the particular case of $Z_N$ orientifolds and find that these anomalies may be cancelled by a generalized Green-Schwarz mechanism. This mechanism works by the exchange of twisted RR-fields associated to the orbifold singularities and it requires the mixing between twisted and untwisted moduli of the orbifold. As a consequence, the Fayet-Iliopoulos terms which are present for the gauged anomalous $U(1)$'s of the models get an additional untwisted modulus dependent piece at the tree level.
1 Introduction

The low-energy dynamics of moduli fields in string theory is described by a non-linear $\sigma$-model. The isometries of the corresponding target space appear as symmetries of the effective lagrangian. In general it is expected that these symmetries are not preserved by full-fledged string theory, since the properties of states in the massive tower depend non-trivially on the moduli. However, quite often a discrete version of these symmetries is valid for the complete string theory, including the massive stringy modes (or even non-perturbative states). The study of $\sigma$-model symmetries thus may provide interesting insights into deeper properties of string theory.

A simple example is provided by $D = 4 \ N = 1$ heterotic orbifold vacua. The classical low-energy lagrangian is invariant under a number of $SL(2, \mathbb{R})$ transformations acting on the untwisted moduli $T_i$ controlling the sizes of the compact dimensions. The masses of momentum and winding modes depend on these moduli, so the continuous symmetry is violated in the full string theory. However, the corresponding discrete $SL(2, \mathbb{Z})$ modular transformations correspond to an exact symmetry of the full theory, T-duality.

A new ingredient comes about when one realizes that the $\sigma$-model symmetries involve a non-trivial transformation of chiral fermions charged under gauge symmetries. This leads to potential $\sigma$-gauge (and $\sigma$-gravity) mixed anomalies, spoiling the quantum validity of the symmetry. In fact, direct computation shows that the triangle diagram contributions give a non-vanishing anomaly. Even though this is not of great concern for the continuous version of the symmetry, which is anyway broken by other effects, such anomalies would be clearly inconsistent with T-duality being an exact symmetry of the theory. Happily, the triangle contribution is cancelled by additional effects. First, the gauge kinetic functions have a non-trivial one-loop dependence (threshold correction \cite{1,2}) on the untwisted moduli associated to complex planes left unrotated by some orbifold group element. Second, a Green-Schwarz mechanism \cite{3} with dilaton exchange cancels the remaining anomaly \cite{4,5}. Notice that for complex planes rotated by all orbifold group elements, there is no threshold correction \cite{2}, so the GS counterterms cancel the anomalies not only for the discrete but also for the continuous version of the $\sigma$-model symmetry.

It is natural to consider similar questions for other $D = 4 \ N = 1$ vacua. In the present paper we center on type IIB orientifolds \cite{6-15}. The low-energy effective lagrangians for moduli are quite analogous to those of heterotic models, for instance they have $SL(2, \mathbb{R})$ $\sigma$-model symmetries for the untwisted moduli $T_i$. An important
difference, however, is that in these type of vacua T-duality is not related to these modular transformations. In fact, T-duality relates D-branes of different kinds, and so does not act within a given class of models, but maps one class of vacua to another. For instance, an orientifold with D9 branes and at a value $T_i$ for the $i^{th}$ complex plane modulus is equivalent to another kind of orientifold, with D7-branes and at a value $1/T_i$ of the $i^{th}$ complex plane modulus.

Since the low-energy $\sigma$-model symmetries are seemingly not related to exact symmetries of the full-string theory, it is not obvious to what extent these symmetries should be respected at the quantum level. In the present paper, however, we will argue that the triangle anomalies for these symmetries may cancel by a GS mechanism in certain specific cases.

This is suggested by the proposed duality \cite{[16]} between type IIB orientifold vacua and heterotic compactifications \cite{[4, 8, 12]}. For many of the models we will study, suitable heterotic orbifold duals have been identified. Since $\sigma$-model anomalies cancel in these heterotic models, it is reasonable to expect the corresponding anomalies to cancel also in the orientifold version. This does not imply, though, that the anomaly cancellation pattern is identical. In fact, the detailed analysis of the triangle anomaly we will perform shows that the anomaly indeed factorizes, but the GS mechanism required for the cancellation must involve not the dilaton, but closed string modes in twisted sectors (the dilaton plays a role only in the cancellation of $\sigma$-gravity mixed anomalies).

Finally, let us remark that we will center on the study of modular transformations associated to complex planes rotated by all elements in the orbifold group. Only for such planes we expect the GS mechanism to cancel the complete anomaly. For complex planes left unrotated by some element of the orbifold group, both the argument invoking duality with heterotic models, and the existence of threshold effects in $D = 4$ $N = 2$ type IIB orientifold \cite{[17]} suggest the anomaly cancellation may have additional sources beyond the GS mechanism.

The paper is organized as follows. In Section 2 we briefly review $\sigma$-model symmetries in heterotic orbifold vacua. We introduce the basic notation and discuss the formulae relevant to the cancellation of anomalies in these models.

In Section 3 we address the same problem in type IIB orientifold vacua. We start with a brief review of cancellation of anomalous gauge $U(1)$ symmetries, and the generation of Fayet-Iliopoulos terms, in section 3.1. In section 3.2 we derive general formulae for the triangle contributions to $\sigma$-model anomalies in type IIB orientifolds, and compute them explicitly for a set of models. The analysis of the factorization properties
of these anomalies is performed in sections 3.3 and 3.4, where we also discuss how a GS mechanism involving non-trivial shifts of the RR twisted sector axions may cancel the anomaly. Models with Wilson lines and/or non coincident branes are studied in section 3.5. Finally, in section 3.6 we study σ-gravitational mixed anomalies and show they can be cancelled through exchange of both the twisted RR axions and the universal axion (partner of the dilaton).

In Section 4 we comment on the mixing between the untwisted and twisted moduli required for the GS mechanism to work. An interesting consequence is the existence of an additional $T_i$-dependent contribution to the FI terms for the anomalous $U(1)$’s. Finally, Section 5 contains our conclusions.

2 Sigma-model anomalies in $D = 4$, $N = 1$ heterotic orbifolds

The Kähler potential dependence on the complex dilaton $S$ and the untwisted Kähler moduli $T_i$, $i = 1, 2, 3$ in this class of heterotic orbifolds is well known. These fields live in a coset $\sigma$-model with symmetry given by

$$\mathcal{M} = \left[ \frac{SU(1,1)}{U(1)} \right]_T^3 \otimes \left[ \frac{SU(1,1)}{U(1)} \right]_S.$$  \hspace{1cm} (2.1)

The Kähler potential is given by

$$K(S, S^*, T_i, T_i^*) = -\log(S + S^*) - \sum_{i=1}^{3} \log(T_i + T_i^*)$$  \hspace{1cm} (2.2)

The kinetic terms for all charged fields $A_\alpha$ in the orbifold, both untwisted and twisted may be written to first order in this fields as:

$$K_{\text{matter}} = \delta_{\alpha\beta} \prod_{i=1}^{3} (T_i + T_i^*)^{n_i^\alpha} A_\alpha \tilde{A}_\beta$$  \hspace{1cm} (2.3)

Here the $n_i^\alpha$, often called modular weights of the fields, are constants which depend on the conformal field theory sector corresponding to the field. For the untwisted matter fields associated to the $j^{th}$ complex plane one finds:

$$n_j^i = -\delta_j^i.$$  \hspace{1cm} (2.4)

\footnote{For particular orbifold models like the $Z_3$ or $Z_6$ there is an enlarged number of untwisted Kähler moduli and in some others like $Z_4$ or $Z_6'$ there may be complex structure scalars. We will concentrate for simplicity on the three Kähler moduli $T_i$ which are always present for any orbifold where the six-torus lattice can be decomposed as three two-dimensional lattices.}
For fields which are originated from a twisted sector with twist vector \( v = (v_1, v_2, v_3) \) (here we take all \( 0 \leq v_i \leq 1 \) and \( \sum_{i=1}^{3} v_i = 1 \)) one has  

\[
\begin{align*}
    n^i_\alpha &= -(1 - v_i), \quad \text{for } v_i \neq 0 \\
    n^i_\alpha &= 0, \quad \text{for } v_i = 0.
\end{align*}
\]  

(2.5)

The effective classical action presents a \( \sigma \)-model invariance under \( SL(2, R) \) transformations given by 

\[ T_i \rightarrow a_i T_i - ib_i \]

(2.6)

with \( a_i, b_i, c_i, d_i \in R \) and \( a_i d_i - b_i c_i = 1 \). Under these transformations the charged matter fields transform as: 

\[ A_\alpha \rightarrow A_\alpha \prod_{i=1}^{3} (ic_i T_i + d_i)^{n^i_\alpha} \]

(2.7)

so that the kinetic terms in (2.3) remain invariant. The transformation of the superpotential also compensates for the transformation of the \( T_i \)-dependent piece in the Kähler potential (2.2).

This continuous symmetry is in general expected to be violated by world-sheet effects. However, in the heterotic case we know that the discrete subgroup \( SL(2, Z)^3 \) of the above non-compact symmetry corresponds to the T-duality invariance of heterotic vacua. Thus this discrete subgroup has to remain as a symmetry even after world-sheet corrections are included.

In particular, the transformations (2.6), (2.7) induce chiral rotations in the massless fermions of the theory. They are associated to gauge transformations of a composite gauge vector potential involving the moduli fields \( T_i \). If we compute the triangle anomalies corresponding to this composite current and two gauge currents one finds in general an anomalous result. The coefficient of this anomaly can be computed to be given by [2, 4, 5]: 

\[
    b'^i_a = -C(G_a) + \sum_{R_a} T(R_a)(1 + 2n_{R_a}^i)
\]

(2.8)

Here \( C(G_a) \) is the quadratic Casimir of the gauge group \( G_a \) in the adjoint representation and \( T(R_a) \) is the quadratic Casimir in the representation \( R_a \) corresponding to a charged field. The sum extends over all fields charged under \( G_a \) and \( n_{R_a}^i \) is the modular weight along the complex plane \( i \) of each given field. In general this mixed \( \sigma - G^2 \) anomalies do not cancel. The gauge kinetic terms get one-loop (non-local) corrections [4, 5]: 

\[
    \mathcal{L}_{nl} = \sum_a \int d^2 \theta \frac{1}{4} W^a W^a \left\{ S - \frac{1}{32} \partial^2 \bar{D} D D \left[ \sum_{i=1}^{3} b'^i_a \log(T_i + \bar{T}_i) \right] \right\} + \text{h.c.}
\]

(2.9)

\(^{2}\)In the presence of twisted oscillators these formulae are slightly generalized. See ref [18] for details.
where $W^a$ are the field strength of gauge fields. Under $SL(2, \mathbb{R})_{T_i}$ transformations this action is not invariant due to the non-local piece. However we know that discrete $T$-duality transformations have to be a good symmetry also at the quantum level. The cancellation of the triangle anomaly comes about from two additional contributions:

1) Under the $\sigma$-model continuous transformations in (2.6) the complex dilaton $S$ gets also transformed as $\mathbb{I}, \mathbb{I}, \mathbb{I}$:

$$S \rightarrow S + k_a \sum_{i=1}^{3} \delta_{GS}^{i} \log(i c_i T_i + d_i).$$

(2.10)

Here $\delta_{GS}$ is a gauge group-independent coefficient which describes the one-loop mixing between the $S$ and the $T_i$ fields in these heterotic vacua, and $k_a$ is the Kac-Moody level of the gauge group. Since $S$ is the tree-level gauge function coefficient for all gauge groups, this transformation gives an additional contribution to the mixed $\sigma$-$G^2_a$ anomalies. In particular this transformation cancels all $\sigma$ model anomalies in $Z_N$, $N$ odd orbifolds with no twist leaving one complex plane unrotated ($Z_3$ and $Z_7$ standard heterotic orbifolds). This is also the mechanism which cancels anomalies corresponding to complex planes $i$ which are always rotated by the twists of the model. Thus, for example, that is the case of the anomalies with respect to the first two planes in the $Z_6$ orbifold generated by the twist $v = 1/6(1, 1, -2)$. Notice that this Green-Schwarz mechanism not only cancels discrete $T$-duality symmetry anomalies but continuous $\sigma$-model anomalies. Also notice that this mechanism is gauge group independent and hence the mixed anomalies should be equal for all gauge groups if they are to be cancelled only by this mechanism.

2) For heterotic orbifolds containing some complex plane $i$ left unrotated by some orbifold twist there is a $T_i$-dependent one-loop threshold correction to the gauge kinetic functions. This threshold correction is in general gauge-group dependent and was computed in ref. $\mathbb{I}$. For complex planes of this type, the discrete $T$-duality anomalies in (2.8) are cancelled by the transformation properties of these threshold corrections plus the Green-Schwarz mechanism above. Notice however thus, unlike the previous mechanism, the threshold correction explicitly violate the continuous $\sigma$ model symmetries and respect only the discrete subgroups associated to $T$-dualities.

Notice that in order for the $S$-dependent Kähler potential to be invariant under the transformation in (2.10) it has to be modified to:

$$K(S, S^*) = -\log(S + S^* + k_a \sum_i \delta_{GS}^i \log(T_i + T_i^*))$$

(2.11)

$^3$We define here $\text{Re}S = 8\pi^2/g^2$.  

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reflecting explicitly the one-loop mixing between $S$ and $T_i$ fields.

In addition to mixed $\sigma$-gauge anomalies there will be mixed $\sigma$-gravity anomalies. The corresponding triangle graph involving massless fermions gives an anomaly proportional to [18]:

$$u^i_{\text{grav}} = 21 + 1 + \delta^i_M - \text{dim} \ G + \sum_\alpha (1 + 2n^i_\alpha) \quad (2.12)$$

Here $21$ comes from the gravitino contribution, $1$ comes from the dilatino and $\delta^i_M$ represents the contribution from untwisted moduli. The other two terms come from the contribution of gauginos and charged chiral matter respectively. The same two mechanisms which we described above are also present in the cancellation of the corresponding duality anomalies.

Four-dimensional heterotic vacua do also often have one anomalous gauged $U(1)$ symmetry in their effective Lagrangian. Those anomalies are cancelled by a Green-Schwarz mechanism [3], very much like the $\sigma$-model anomalies discussed above. In this case under a gauge $U(1)$ transformation with gauge parameter $\Lambda(x)$, the dilaton transforms like:

$$S \rightarrow S + k_a \delta^X_{GS} \Lambda(x) \quad (2.13)$$

and this cancels all mixed anomalies. Notice that in an heterotic model with both an anomalous $U(1)$ and $\sigma$-model anomalies the $S$-dependent piece will thus take the form:

$$K(S, S^*) = -\log(S + S^* + k_a \sum_i \delta^i_{GS} \log(T_i + T_i^*) - k_a \delta^X_{GS} V_X) \quad (2.14)$$

where $V_X$ is the vector superfield of the anomalous $U(1)$. As is well known [19], there is also a Fayet-Iliopoulos (FI) term associated to the anomalous $U(1)$:

$$\xi_{\text{het}} \propto \left( \frac{\partial K}{\partial V_X} \right)_{V_X=0} = -k_a \delta^X_{GS} \quad (2.15)$$

This is of the order of the string scale for generic and realistic values of dilaton and moduli.

### 3 $\sigma$-model anomalies in compact $D = 4$, $N = 1$ Type IIB orientifolds

Let us recall the structure of $D = 4 N = 1$ [8] - [15] type IIB orientifolds [20, 21, 22, 23]. The models we will be centering on are constructed by modding out the toroidally

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4The result in eq.(2.13) is given in units of $M^2_{\text{Planck}}/8\pi$ since the Kahler potential in eq.(2.14) has in fact such an overall factor.
compactified type IIB theory by the joint action of a symmetry group $G_1$ of the six-torus together with the world-sheet parity operation $\Omega$ \cite{20, 21, 22, 23}. We will consider orientifold groups with the structure $G_1 + \Omega G_1$ and center on $G_1 = Z_N$ orbifold moddings. All possible twists consistent with spacetime $N = 1$ supersymmetry have been classified in \cite{24}, and their eigenvalues $v = (v_1, v_2, v_3)$ are shown in Table 1.

| $Z_3$ | $\frac{1}{3}(1, 1, -2)$ | $Z'_6$ | $\frac{1}{3}(1, 2, -3)$ | $Z'_7$ | $\frac{1}{3}(1, -3, 2)$ |
| $Z_4$ | $\frac{1}{4}(1, 1, -2)$ | $Z_7$ | $\frac{1}{4}(1, 2, -3)$ | $Z_{12}$ | $\frac{1}{12}(1, -5, 4)$ |
| $Z_6$ | $\frac{1}{6}(1, 1, -2)$ | $Z_8$ | $\frac{1}{6}(1, 3, -4)$ | $Z'_{12}$ | $\frac{1}{12}(1, 5, -6)$ |

Table 1: $Z_N$ actions in $D=4$.

The closed string sector is constructed by performing the orientifold projection to the spectrum of type IIB theory on the corresponding toroidal orbifold. In the untwisted sector, one obtains the $D = 4 \ N = 1$ supergravity multiplet, one chiral multiplet $S$ containing the dilaton, and a further set of moduli describing the geometry of the original torus. As in the heterotic case mentioned above, the number of such moduli is model-dependent, and we will center on the three moduli $T_i$, $i = 1, 2, 3$ corresponding to the three complex planes. The closed string twisted modes will also be relevant to our purposes. A fixed point $f$ will have associated chiral singlet fields $M^k_f$ for each $k$-twisted sector.

The consistency of the equations of motion for the RR potentials requires the cancellation of the corresponding tadpoles. This is implemented by introducing D branes whose RR charge cancels that of the orientifold planes. For $Z_N$, with $N$ odd, only D9-branes are required. They fill the full space-time and six dimensional compact space. For $N$ even, D5$_k$-branes, with world-volume filling space-time and the $k^{th}$ complex plane, may be required. This is so whenever the orientifold group contains the element $\Omega R_i R_j$, for $k \neq i, j$. Here $R_i$ ($R_j$) is an order two twist of the $i^{th}$ ($j^{th}$) complex plane. In what follows we consider cases with only one set of fivebranes, which, with the conventions for the twists in Table 1, wrap the third complex plane.

The action of an orientifold group element $g$ on $Dp$-branes is specified by a unitary matrix, $\gamma_{g,p}$. It turns out to be useful to introduce the vectors $V^p$ given in terms of the eigenvalues $e^{2\pi i V^p}$ of the matrices $\gamma_{\theta,p}$, corresponding to the generators of the orbifold group. Let us also define $w^p_k$ to be the number of times a given eigenvalue $V^p_k$ appears.

\footnotetext{In order for the orientifold projection to be a symmetry, the eigenvalues come in complex conjugate pairs. Thus we define $V^p$ to contain only the phases in $[0, \pi)$.}
For a given orientifold group one can obtain a set of constraints on the Chan-Paton matrices for these D branes. Some of them follow from the requirement that the matrices form a representation of the orientifold group, while others correspond to cancellation of twisted tadpoles (see e.g. \cite{12} for details). The Chan-Paton matrices determine the (open-string) spectrum of the model. For instance, the integers $w^p_i$ introduced above specify the ranks of the $i^{th}$ factor in the gauge group on the D$p$-branes. Instead of entering the details of their construction, Table 2 conveniently provides a list of the models we will consider, along with their spectra. The mentioned constraints ensure the consistency of the resulting models. In particular, they imply the cancellation of gauge and gravitational anomalies (see \cite{25} for a detailed discussion). It is known that out of the list of twists in table 1, the examples based on $Z_4$, $Z_8$, $Z_8'$ and $Z_12'$ have twisted tadpoles and hence are inconsistent \cite{9, 12}, at least with the standard GP-projection \cite{23} here discussed. We will thus focus in the remaining examples when we treat specific models.

3.1 Cancellation of $U(1)$ gauge anomalies and FI terms

Before studying $\sigma$-model anomalies in type IIB orientifolds, we recall the structure and cancellation of gauge $U(1)$ anomalies in these models. This will be useful because there is a close analogy between the GS anomaly cancellation mechanisms in both cases. Also, the mixing between untwisted and twisted moduli required to cancel $\sigma$-model anomalies implies an interesting contribution to the FI terms for the anomalous $U(1)$’s. Thus this section also provides the notation and formulae relevant to this issue.

The cancellation of $U(1)$ anomalies in $D = 4 \ N = 1$ type IIB orientifolds is quite interesting. In contrast with heterotic models, these theories have generically several $U(1)$’s with non-vanishing triangle anomalies. Moreover, the mixed anomalies with different non-abelian gauge factors and gravity are not in adequate ratios to be cancelled by a GS mechanism with exchange of the partner of the dilaton \cite{12}. However, in \cite{15} it was proposed that a different version of the GS mechanism, with exchange of RR twisted closed string modes, cancels these anomalies \cite{1}.

Let us consider for instance the mixed non-abelian anomaly. The anomaly in the field theory is reproduced by the string theory diagrams depicted in Figure 1, in the point-particle limit in the open-string channel. As mentioned above, the net contribution is non-vanishing, which would mean an inconsistency of the theory at the quantum level. However, the same string diagrams in Figure 1 give additional low-energy con-

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\footnote{This is the four-dimensional analog of the six-dimensional mechanism studied in \cite{26, 27}.}
Figure 1: The string theory diagrams contributing to the three point amplitude corresponding to the field theory anomaly.

Figure 2: The string theory diagram in figure 1c in the closed string channel. This contribution provides the GS counterterms which cancel the anomaly from the triangle diagrams. Closed string twisted models propagate in the closed string channel and mediate the GS mechanism.

tributions in the point-particle limit in the closed string channel. Notice that the close-string channel contributions corresponding to diagrams 1a and 1b cancel against each other, due to cancellation of disk and crosscap pieces at the end of the infinite tube (this is the statement of tadpole cancellation). On the other hand, the diagram 1c gives a non-vanishing closed-string channel contribution, schematically depicted in Figure 2. It is interpreted as the exchange of twisted RR fields, which propagate on the infinitely elongated cylinder. This contribution was determined in [15] to be

\[
A_{ij}^{pq} = \frac{1}{N} \sum_{k=1}^{N-1} C_k^{pq}(v) \, w^p_i \sin 2\pi k V^p_i \cos 2\pi k V^p_j
\]  

(3.1)

Here \(k\) runs over twisted \(Z_N\) sectors, \(p, q\) run over 5, 9 (meaning 5- or 9-brane origin of the gauge boson) and

\[
C_k^{pp} = \prod_{a=1}^{3} 2 \sin \pi k v_a \quad \text{for} \quad p = q
\]

\[
C_k^{59} = 2 \sin \pi k v_3
\]

(3.2)

In [13] it was checked that this contribution indeed cancels the mixed non-abelian anomalies in all orientifolds considered. Even though the structure of the amplitude (3.1) can be read off from Fig 1c, we would like to stress that the existence of factorization can be induced from the mere structure of the triangle anomalies. Indeed, a simple
strategy used in [15], and to be exploited below, is to write down the triangle anomaly for a generic orientifold in terms of the integers $w_i$, which define the Chan-Paton matrices and determine the field theory spectrum. Then one can perform a discrete Fourier transform to express the anomaly in terms of the Chan-Paton traces $\text{Tr} \gamma_k$. At this point, the anomaly exhibits a factorized form with the structure (3.1). Since consistency of string theory implies the total contribution from the triangle anomalies plus the GS terms must vanish, this technique allows to easily obtain the structure of the GS terms.

We would like to stress that this trick is in the spirit of the study of many other anomalies in string theory, where a preliminary low-energy analysis of the anomaly reveals the factorized structure, and suggests the corresponding GS counterterms. A further step, usually much more involved, is the explicit string theory computation of the required couplings.
The same mechanism can be employed to cancel mixed gravitational anomalies. Notice that in this case the graviton is a closed string mode, and so the relevant string diagrammatics is different. In particular, as shown in Fig 3, only two diagrams contribute and factorization is not as obvious as in the preceding case. Factorization is, however, strongly suggested by the structure of the triangle anomalies. This is done as sketched above: One writes the mixed gravitational anomalies for a general orientifold, and performs a discrete Fourier transform to rewrite them in terms of the Chan-Paton matrices. The triangle anomaly is given by a factorized expression, which can be cancelled by a GS term of the form

\[ A_i^\alpha = \frac{3}{4N} \sum_{\beta} \left( \sum_{k=1}^{N-1} C_{\alpha\beta}^k (v) w_i \sin(\pi k V_i^\alpha (\gamma_k^\beta)^{-1}) \right) \]  

(3.3)

In string theory factorization follows from the fact that the cylinder diagram provides two low-energy contributions, corresponding to the point-particle limit in the open and closed string channels (fig 4). Again, Figures 4a and 4b give the usual field theory triangle anomaly, whereas Figure 4c provides the GS contribution, which can be interpreted as exchange of twisted RR fields. This type of factorization will be relevant for the cancellation of \( \sigma \) model anomalies that we will study below.

Before doing that, let us point out some consequences of the coupling of twisted closed string modes to gauge fields for the effective low energy field theory Lagrangian. We will restrict ourselves in this discussion to \( Z_N \) orientifolds with \( N \) odd (\( Z_3 \) and \( Z_7 \) in the compact case) since the structure of their twisted closed string fields is much simpler. The even \( N \) case is discussed in the appendix. First, for this generalized Green-Schwarz mechanism to work there must be a modification of the gauge kinetic function. In particular one has for the gauge group \( G_b \):

\[ f_b = S + \frac{1}{N} \sum_{k=1}^{(N-1)/2} \cos(2\pi k V_b) \frac{1}{C_k} \sum_f M_f^k \]  

(3.4)

Here the sum on \( f \) goes over the number of fixed points = \( C^2_f \) whereas \( k \) labels the twisted sectors. \( M_f^k \) is a closed string chiral singlet field living on the fixed point \( f \) and corresponding to the twist \( \theta^k \). Notice that, since the D9-brane world-volume spans the complete compact space, the gauge fields couple to twisted fields from all fixed points. Now, under a \( U(1)_a \) gauge transformation with parameter \( \Lambda_a(x) \) the twisted closed string chiral fields \( M_f^k \) transform as:

\[ \text{Im} M_f^k \rightarrow \text{Im} M_f^k + w_a 2 \sin(2\pi k V_a) \Lambda_a(x) \]  

(3.5)

Notice how in this way the net effect of this shift combined with eq.(3.4) is the contribution (3.1) discussed above.
The fact that the fields $M^k_f$ are not gauge invariant means that the Kähler potential of those fields must have the general dependence:

$$K(M^k_f, M^{k*}_f) = K(M^k_f + M^{k*}_f - \sum_a \delta^a_{GS} V_a)$$  \hspace{1cm} (3.6)

where the sum on $a$ goes over the $U(1)$’s in the model and

$$\delta^a_{GS} = w_a^2 \sin(2\pi k V_a)$$  \hspace{1cm} (3.7)

Among the interactions generated upon expansion in components, a particularly interesting piece is a FI term for $U(1)_a$. If, as pointed out in ref.\cite{28} the Kähler potential for the fields $M_k$ is bilinear\footnote{The same qualitative conclusion follows for more general Kähler potentials, as long as they are non-singular at the orientifold point\cite{29}. We would like to thank E. Poppitz for comments on this point.}, given by $(M^i_k + M^{i*}_k)^2$, the FI term is

$$\xi_{IIB}^a = - \sum_f \sum_k w_a^2 \sin(2\pi k V_a) (M^k_f + M^{k*}_f)$$  \hspace{1cm} (3.8)

In models with 5-branes the results are analogous, and may be obtained starting from the results in the appendix.

A general property of this contributions to the FI-term is that it is controlled by the blow-up modes $Re M^k_f$, and can be adjusted at will. In particular, this contribution vanishes at the orbifold point. The above result concerning FI-terms will be revised in Section 4 in the light of our analysis of $\sigma$-model anomalies, since it implies substantial changes in several respects.

### 3.2 $\sigma$-model anomalies

Let us turn to the study of $\sigma$-model anomalies in $D = 4\ N = 1$ compact orientifolds. As mentioned in the introduction, heterotic/type I duality suggests that these anomalies should also cancel at least in some classes of type IIB orientifold. More precisely, there are a number of $D = 4,\ N = 1$ orientifolds for which specific candidate heterotic duals have been identified (see e.g. \cite{12} for a general discussion). That is the case for example of the $Z_3$ \cite{4, 8} and $Z_7$ \cite{30} orientifolds. Now, we already mentioned in the previous chapter that $\sigma$-model anomalies in the heterotic side are only cancelled for complex planes which are rotated by all twists in the orbifold group. Thus, properly speaking, we should expect cancellation of $\sigma$-model anomalies in the type IIB orientifold case only for this type of complex direction. Thus, they should cancel for any complex plane in $Z_N$ orientifolds with $N$ odd, and also along some complex planes of other type.
Figure 5: The two string theory diagrams contributing to the coupling of the $\sigma$-model composite connection with two non-abelian gauge bosons.

Figure 6: Factorization in this case is expected to follow the same pattern encountered for gravitational-U(1) gauge mixed anomalies.

of orientifolds, like e.g., the first two complex planes of the $Z_6$ and $Z_{12}$ orientifolds or the first complex plane in the $Z_6'$ orientifold. We will concentrate our study to those complex planes in which the candidate heterotic duals present cancellation of $\sigma$-model anomalies.

It is easy to compute directly the triangle anomalies in several models, and check that they do not cancel. Let us consider the simplest Type IIB $D = 4, N = 1$ orientifolds $Z_3, Z_7, Z_6, Z_6'$ and $Z_{12}$ whose spectra are given in Table 2 for a configuration with all 5-branes sitting at the origin (see ref. [12] for details). In order to compute the anomalies we need to know the “modular weights” $n_i$ of each field with respect to $SL(2, \mathbb{R})_{T_i}$ transformations. In other words, we need to know the $T_i$ dependence of the kinetic terms of each field in the orientifold. For this class of orientifolds this was discussed in ref. [12] and [31]. For models with only 9-branes the relevant piece of the Kähler potential is analogous to that of the untwisted sector of the heterotic orbifolds, namely

$$K(S, S^*, T_i, T_i^*) = - \log(S + S^*) - \sum_{i=1}^{3} \log(T_i + T_i^* - |C_i^9|^2)$$

(3.9)

where $C_i^9$ are the charged fields from the (99) sector of the orbifold corresponding to
the $i^{th}$ complex plane. Thus the “modular weights” of the $C_9^i$ field with respect to the $j^{th}$ complex plane is $n_{ij}^i = -\delta_{ij}^j$. For models with 5-branes, like the $Z_9$, $Z_6'$ and $Z_{12}$ in Table 2, the corresponding Kähler potential is given by:

$$K = -\log(S + S^* + |C_3^5|^2) - \log(T_3 + T_3^* + |C_3^5|^2)$$

$$- \log(T_1 + T_1^* + |C_1^9|^2 + |C_1^5|^2) - \log(T_2 + T_2^* + |C_2^9|^2 + |C_2^5|^2)$$

$$+ \frac{|C_{95}|^2}{(T_1 + T_1^*)^{1/2}(T_2 + T_2^*)^{1/2}}$$

(3.10)

Here the world-volume of 5-branes includes the third complex plane. Also, $C_{95}$ are the charged fields in the $(95)$ sector of the orientifold. The $S$ and $T_i$ dependence of the gauge kinetic functions is given by $f_9 = S$ and $f_5 = T_3$. Notice that, since the gauge kinetic function for the 5-brane gauge group is $T_3$, the $(55)$ gauge group explicitly breaks the $SL(2, \mathbb{R})$ symmetry associated to $T_3$ and hence cancellation of $\sigma$-model anomalies in the third complex plane are not in principle expected.

From the above formula one sees that the “modular weights” along the $j^{th}$ plane ($j = 1, 2$) are given by:

$$n_{9j}^i = -\delta_{ij}^j ; n_{5j}^i = \delta_{ij}^j - 1 ; n_{95}^1 = -1/2 ; n_{95}^2 = -1/2 ; n_{95}^3 = 0 .$$

(3.11)

The mixed Kähler-gauge anomalies with respect to the three complex planes can be computed now using eq.(2.8). The results are as follows:

$Z_3$ : The anomalies with respect to $SU(12)$ and $SO(8)$ are:

$$(b'_{a}) = \begin{pmatrix} -3 & 6 \\ -3 & 6 \\ -3 & 6 \end{pmatrix}$$

(3.12)

$Z_7$ : The mixed anomalies with respect to the $SU(4)^3 \times SO(8)$ yield:

$$(b'_{a}) = \begin{pmatrix} -3 & 1 & 3 & -2 \\ 3 & -3 & 1 & -2 \\ 1 & 3 & -3 & -2 \end{pmatrix}$$

(3.13)

$Z_6$ : The mixed anomalies with respect to the 9-brane group $SU(6)^2 \times SU(4)$ are:

$$(b'_{a}) = \begin{pmatrix} -1 & -1 & 2 \\ -1 & -1 & 2 \\ 2 & 2 & 8 \end{pmatrix}$$

(3.14)

$Z_6'$ : The mixed anomalies with respect to the 9-brane group $SU(4)^2 \times SU(8)$ are:

$$(b'_{a}) = \begin{pmatrix} 3 & 3 & -6 \\ 1 & 1 & -2 \\ 5 & 5 & 2 \end{pmatrix}$$

(3.15)
| Twist Group | Gauge Group | (99)/(55) matter | (95) matter |
|-------------|-------------|------------------|-------------|
| $Z_3$       | $SO(8) \times U(12)$ | $(8,12)_1 + (1,\overline{66})_1$ | - |
|            |             | $(8,12)_2 + (1,\overline{66})_2$ |              |
|            |             | $(8,12)_3 + (1,\overline{66})_3$ |              |
| $Z_7$      | $SO(8) \times U(4)^3$ | $(8,1,\overline{1},1)_1 + (1,4,\overline{1},1)_1 + (1,1,4,\overline{1})_1 + (1,1,1,6)_1$ | - |
|            |             | $(8,1,\overline{1},1)_2 + (1,4,1,\overline{4})_2 + (1,1,4,\overline{1})_2 + (1,6,1,1)_2$ |              |
|            |             | $(8,1,1,4)_3 + (1,\overline{4},1,\overline{4})_3 + (1,4,4,1)_3 + (1,1,6,1)_3$ |              |
| $Z_6$      | $[U(6) \times U(4) \times U(6)]^2$ | $(15,1,1)_1 + (\overline{6},4,1)_1 + (1,\overline{1},6)_1 + (1,1,4,\overline{1})_1$ | $(6,1,1;6,1,1) + (1,1,\overline{1};1,1,\overline{1})$ |
|            |             | $(15,1,1)_2 + (\overline{6},4,1)_2 + (1,\overline{1},6)_2 + (1,1,4,\overline{1})_2$ | $(1,1,6;1,\overline{4},1) + (1,\overline{1},1;1,1,6)$ |
|            |             | $(\overline{6},\overline{4},1)_3 + (6,1,6)_3 + (1,4,6)_3$ | $(\overline{6},1;1,4,1) + (1,4;1,6,1)$ |
| $Z_6'$     | $[U(4) \times U(8) \times U(4)]^2$ | $(6,1,1)_1 + (\overline{4},8,1)_1 + (1,\overline{8},4)_1 + (1,1,6)_1$ | $(\overline{4},1;1,4,1;1,1) + (1,1,4;1,1,4)$ |
|            |             | $(4,8,1)_2 + (\overline{4},1,4)_2 + (1,\overline{8},\overline{4})_2$ | $(1,1,\overline{4};1,8,1) + (1,8;1,1,\overline{4})$ |
|            |             | $(1,28,1)_3 + (1,\overline{28},1)_3 + (4,1,4)_3 + (\overline{4},1,\overline{4})_3$ | $(1,1,\overline{4};1,8,1) + (1,8;1,1,\overline{4})$ |
| $Z_{12}$   | $(U(3)^4 \times U(2)^2)^2$ | $(1,\overline{3},\overline{3},1,1,1,1)_1 + (3,1,1,1,1,2)_1 + (1,1,3,1,1,2)_1$ | $(\overline{3},1,1;1,1,\overline{1},1)_1 + (1,3,1,1,3,1)_1$ |
|            |             | $(3,1,1,1,1,1)_1 + (1,1,1,3,1,1)_1 + (\overline{3},1,1,\overline{3},1,1)_1$ | $(3,1;1,1,\overline{1},1,1)$ |
|            |             | $(1,1,3,1,1,2)_2 + (3,1,1,1,1,2)_2 + (1,1,3,1,1,1)_2$ | $(1,1,3,1,1,2)_2 + (1,1,3,1,1,2)_2 + (1,1,3,1,1,1)_2$ |
|            |             | $(\overline{3},1,\overline{3},1,1,1,1)_3 + (1,\overline{3},\overline{3},1,1,1)_3 + (3,1,1,1,1,1)_3$ | $(\overline{3},1,\overline{3},1,1,1,1)_3 + (1,\overline{3},\overline{3},1,1,1)_3 + (3,1,1,1,1,1)_3$ |
|            |             | $(1,1,3,1,1,2)_3 + (1,1,3,2,1,1)_3 + (1,3,1,1,2,1)_3$ | $(1,1,3,1,1,2)_3 + (1,1,3,2,1,1)_3 + (1,3,1,1,2,1)_3$ |
|            |             | + same with groups reversed | |

Table 2: Gauge group and charged chiral multiplets in some $Z_N$, $D=4$, $N=1$ type IIB orientifolds. The subindices denote the complex planes associated to the different matter fields. Underlining is used to indicate the spectrum contains the all permutations of the underlined representations.
The mixed anomalies with respect to the 9-brane group $SU(3)^4 \times SU(2)^2$ are:

$$
(b_{a}^{'i}) = \begin{pmatrix}
\frac{1}{2} & -\frac{3}{2} & -\frac{3}{2} & 1/2 & 1 & 1 \\
-\frac{3}{2} & 1/2 & 1/2 & -\frac{3}{2} & 1 & 1 \\
1 & 1 & 1 & 1 & 4 & 4
\end{pmatrix}
$$

(3.16)

In principle one expects cancellation of $\sigma$-model anomalies with respect to the three complex planes for $Z_3$, $Z_7$, with respect to the first two complex planes for $Z_6$ and $Z_{12}$ and only with respect to the first one for $Z_{6}^\prime$. Notice that the mixed anomalies with different gauge factors are *not* in appropriate ratios to be cancelled by a GS mechanism with exchange of the partner of the dilaton as in heterotic vacua.

This is hardly surprising, since we are already familiar with the fact that type IIB orientifolds GS interactions are typically mediated by exchange or RR fields in the closed string twisted sector. In the following we will argue that these interactions provide a mechanism to cancel $\sigma$-model anomalies.

In the particular case of mixed non-abelian anomalies, the relevant string diagrams, shown in figure 3 are analogous to those appearing in the study of gravitational-gauge $U(1)$ mixed anomalies, reviewed above. In particular, only two topologies contribute to the string theory amplitude. This is so because the composite gauge connections associated to the $\sigma$-model symmetries are constructed out of closed string moduli.

Thus, we expect a factorization pattern (see figure 3) analogous to that found in gravitational-gauge $U(1)$ anomalies.

Further support can be obtained through a detailed study of the triangle anomalies. As explained above, a simple technique to detect factorization is to write the general triangle anomaly and then perform a Fourier transform as we show in the next section.

An important final point we would like to remark is the relationship between the $\sigma$-model anomalies and the conformal anomaly in this type of orientifolds. If we consider the diagonal $SL(2, \mathbb{R})_T$ transformation corresponding to the identification $T_1 = T_2 = T_3 = T$, the corresponding anomaly coefficient is given by:

$$
b_a^T = \sum_{i=1}^{3} b_{a}^{'i} = \beta_a
$$

(3.17)

i.e., it equals the one-loop $\beta_a$ function. This happens because (unlike the heterotic case) all charged fields have overall “modular weights” $n^T = -1$. Thus for all conformal theories (like, e.g. subsectors of the theory corresponding to branes away from orientifold planes) the overall anomalies cancel identically.
3.3 Green-Schwarz cancellation of $\sigma$-model anomalies: odd $N$, $Z_N$ orientifolds

The only odd order $Z_N$ twists which act cristalographically in six compact dimensions are $Z_3$ and $Z_7$, but we will write general expressions for $Z_N$ with $N = 2P + 1$ with arbitrary $P$. The gauge group in this class of orientifolds is given by

$$SO(w_0) \times \prod_{j=1}^{P} U(w_j)$$  \hspace{1cm} (3.18)

and the charged chiral fields from the $(99)$ sector are given by:

$$\sum_{i=1}^{3} \left( \sum_{a=0}^{2P} \frac{1}{2}(w_{a} - w_{a-l_i}) + A_{N-l_i} \frac{1}{2} + A_{l_i} \frac{1}{2} \right)$$  \hspace{1cm} (3.19)

where $v_i = l_i/N$, and $w, A$ denote the fundamental and two-index antisymmetric representations. The sum over $a$ goes only over $2a \neq l_i$, mod $N$ and a negative subindex for a representation implies conjugation. For fractional subindices the corresponding antisymmetric representations are absent. Starting from this spectrum and using eq.(2.8) one obtains the following $\sigma$-$G_a^2$ anomalies with respect to the $i^{th}$ complex plane:

$$b^i_a = -w_a + 2\delta_{a,0} - \frac{1}{2}(w_{a+l_i} + w_{a-l_i}) + \delta_{2a+l_i} + \delta_{2a-l_i} + \frac{1}{2} \sum_{j \neq i} (w_{a+l_j} + w_{a-l_j}) - \sum_{j \neq i} (\delta_{2a+l_j} + \delta_{2a-l_j})$$  \hspace{1cm} (3.20)

$$+ \frac{1}{2} \sum_{j \neq i} (w_{a+l_j} + w_{a-l_j}) - \sum_{j \neq i} (\delta_{2a+l_j} + \delta_{2a-l_j})$$  \hspace{1cm} (3.21)

Now we want to re-express this in terms of the Chan-Paton twist matrices. The general action of the twists $k = 1, \ldots, N$ on the 9-branes is given by the matrix:

$$\gamma_k = \text{diag} (I_{w_{a}}, \alpha_k I_{w_1}, \ldots, \alpha_k^j I_{w_j}, \ldots, \alpha_k^{N-1} I_{w_{N-1}})$$  \hspace{1cm} (3.22)

with $\alpha_k = e^{2i\pi k/N}$. Notice that the orientifold symmetry requires $w_a = w_{N-a}$. The trace of this matrix is given by

$$\text{Tr} \gamma_k = \sum_{a=1}^{N} e^{\frac{2\pi ik}{N}} w_a$$  \hspace{1cm} (3.23)

Now we can perform an inverse Fourier transform to re-express the $w_a$ in terms of the traces of $\gamma_k$'s. Plugging it back in eq.(3.21) one obtains, after some simple algebra, the result:

$$b^i_a = \frac{1}{N} \sum_{k=1}^{N-1} \tilde{\alpha}_k^i \cos(4\pi kV_a)$$  \hspace{1cm} (3.24)

with

$$\tilde{\alpha}_k^i = \frac{1}{2} C_{2k}(v) \cotg (2\pi kv_i) \text{Tr} \gamma_k - C_k(v) \cotg (\pi kv_i)$$  \hspace{1cm} (3.25)
where \( C_k \) was defined in eq.(3.2). Notice that this expression has the structure expected from the diagrams in Fig.5. Indeed, the factor \( \cos(4\pi kV_a) = \text{Tr} (\gamma_2 k \lambda_a^2) \) comes from the insertion of two gauge bosons in the outer boundaries of figures 5a and 5b. Then the cylinder graph should give rise to the \( \text{Tr} \gamma_2 k \) factor in eq.(3.25) whereas the other term corresponds to the Moebius strip graph 5a. It is nice to recover the expected structure starting merely from the massless spectrum. Twisted tadpole constraints in odd \( N \) orientifolds further require \([7, 8, 9, 12]\):

\[
\text{Tr} \gamma_2 = 32 \prod_{i=1}^{3} \cos(\pi k v_i) \quad (3.26)
\]

and plugging this back into eq.(3.23) one finally gets the simple result:

\[
\tilde{\alpha}_i^k = -C_k(v) \tan(\pi k v_i) \quad (3.27)
\]

where we have made use of the fact that \( C_{2k} = C_k \prod_i 2 \cos(\pi k v_i) \). Thus altogether the mixed \( \sigma \)-gauge anomalies for odd order orientifolds can be written as:

\[
b_i^j = -\frac{2}{N} \frac{(N-1)/2}{2} \sum_{k=1}^{C_k(v)} \cos(\pi k v_i) \cos(4\pi k V_a) = -\frac{2}{N} \sum_f \sum_{k=1}^{1} \frac{1}{2 \cos(\pi k v_i)} \cos(4\pi k V_a) \quad (3.28)
\]

A comparison with the equivalent result for mixed gauged \( U(1) \) anomalies in eq.(3.1) shows how analogous these expressions are. This is highly suggestive that indeed, as it happened in the \( U(1) \) case, \( \sigma \)-model anomalies are cancelled by a Green-Schwarz mechanism in which twisted RR fields are exchanged in the closed string channel. From the field theory point of view, the mechanism works in analogy with the discussion following eqs (3.4) and (3.5). In the present case the twisted fields corresponding to the fixed point \( f \) would transform with respect to a \( \sigma \)-model transformation along the \( i^{th} \) complex plane like:

\[
\text{Im} M_i^k \to \text{Im} M_i^k + 2 \tan(\pi k v_i) \log(i c_i T_i + d_i) \quad (3.29)
\]

which combined with (3.4) would exactly cancel the anomaly (recall \( C_k^2 \) gives the number of fixed points in these compact orbifolds).

Let us finally comment that using eqs.(3.17) and (3.28) one can write a simple expression for the \( \beta \)-functions of the gauge groups in these models:

\[
\beta_a = -\frac{2}{N} \frac{(N-1)/2}{2 \cos(\pi k v_i)} \cos(4\pi k V_a) = -\frac{2}{N} \sum_f \sum_{k=1}^{1} \frac{1}{2 \cos(\pi k v_i)} \cos(4\pi k V_a) \quad (3.30)
\]

where \( f \) label the fixed points.
For the compact $Z_3$ and $Z_7$ orientifolds under consideration, if one sets all $M^k_f = M$ the gauge kinetic function (3.4) may be written in the simple form

$$f_b = S \pm \frac{\beta_a}{2} M$$

(3.31)

for $Z_3$ and $Z_7$ respectively. Under an $SL(2, \mathbb{R})_T$ transformation one has $ImM \rightarrow ImM \pm 2 \log(icT + d)$, for $Z_7$ and $Z_3$ respectively and this cancels the overall modulus anomaly. Indeed the variation cancels the contribution from the second term in (2.9).

This is an interesting expression since the disk coupling of the twisted field $M$ looks like a one-loop factor, in the sense that it is proportional to the $\beta$-function. This fact had already been observed in ref.[31], where it was used to achieve precocious unification of gauge couplings constants.

3.4 Green-Schwarz cancellation of $\sigma$-model anomalies: even $N$, $Z_N$ orientifolds

Here we will discuss the particular case of even order $Z_N$ orientifolds with only one sector of 5-branes, with world-volume in the third complex direction. As we mentioned above, in this case $SL(2, \mathbb{R})_{T_3}$ is explicitly broken at the classical level by the gauge kinetic terms of the 5-brane gauge group so we will not discuss anomaly cancellations along the third complex plane. In addition, if the complex plane is left unrotated in some twisted sector, we know that in the heterotic dual anomalies are not cancelled only by a GS mechanism. Thus we will discuss anomaly cancellation only along complex planes which are rotated by all twists in the model.

We will consider the case of the standard Gimon-Polchinski projection leading to a embedding “without vector structure” in the gauge degrees of freedom. The prototype models we have in mind here are the $Z_6$ and $Z_6'$ orientifolds mentioned above.

Let us consider arbitrary $Z_N$ ($N = 2P$) twists with eigenvalues given by $\frac{1}{N}(l_1, l_2, l_3)$, with $l_1 + l_2 + l_3 = 0$ and $l_3$ an even integer (thus $l_1, l_2$ are odd). As we said, we concentrate on models without vector structure. The general Chan-Paton matrix for D9-branes has the form

$$\gamma_{k,9} = \text{diag}(\alpha_k I_{w_1}, \cdots, \alpha_k^{(2j-1)} I_{w_j}, \cdots, \alpha_k^{(2P-1)} I_{w_P}, \alpha_k^{-1} I_{w_1}, \cdots, \alpha_k^{-1} I_{w_j}, \cdots, \alpha_k^{-1} I_{w_P})$$

(3.32)

with $\alpha_k = e^{i\pi k/N}$. Here we have already imposed the orientifold symmetry $w_j = w_{N-j+1}$. The matrices for D5-branes are analogous with the replacement of $w_j$ by $u_j$ as the number of eigenvalues $\alpha_k^{(2j-1)}$. These matrices correspond to the shifts

$$V^P = \frac{1}{2N}(1, \cdots, 1, \cdots, 2j - 1, \cdots, 2j - 1, \cdots, 2P - 1, \cdots, 2P - 1)$$

(3.33)
with \( w_j \) \((u_j)\) entries \((2j - 1)\) for D9- and D5-branes, respectively, and \( j = 1, \ldots, P \).

The associated gauge group is

\[
\prod_{j=1}^{P} U(w_j) \times \prod_{j=1}^{P} U(u_j)
\]

(3.34)

The complete massless spectrum for this class of models can be found in ref. [25]. Using that spectrum and the modular weights given in eq. (3.11), one finds the following result for the mixed anomalies with the non-Abelian gauge symmetries from the 9-brane sector:

\[
b_{i}^{'a} = -w_a - \frac{1}{2}(w_{a+l_i} + w_{a-l_i}) + \delta_{2a+l_i+1} + \delta_{2a-l_i+1} + \frac{1}{2} \sum_{j \neq i} (w_{a+l_j} + w_{a-l_j}) - \sum_{j \neq i} (\delta_{2a+l_j-1} + \delta_{2a-l_j-1})
\]

(3.35)

The trace of the twist matrices are given by

\[
\text{Tr} \gamma_{k,9} = \sum_{a=1}^{N} e^{i \pi (2a-1)k} w_a
\]

(3.36)

(and analogously for 5-branes). Again, we perform an inverse discrete Fourier transform to express the \( w_a \) in terms of the Chan-Paton traces and substitute them in (3.36). The result is

\[
b_{i}^{'a} = \frac{1}{2N} \sum_{k=0}^{N-1} \tilde{\alpha}_k^i \cos(4\pi k\bar{V}_a)
\]

(3.37)

where

\[
\tilde{\alpha}_k^i = \sum_{j \neq i} \cos(4\pi k v_j) - \cos(4\pi k v_i) - 1 \text{ Tr} \gamma_{2k,9} - 4 \left( \sum_{j \neq i} \cos(2\pi k v_j) - \cos(2\pi k v_i) \right) + \delta_3^i \cos(2\pi k v_3) \text{ Tr} \gamma_{2k,5}
\]

(3.38)

After some trigonometry and rearrangement of terms one can rewrite this formula as:

\[
\tilde{\alpha}_k^i = \frac{1}{2} C_{2k}(v) \cotg(2\pi k v_i) \text{ Tr} \gamma_{2k,9} - 2 C_k(v) \cotg(\pi k v_i) + \delta_3^i \cos(2\pi k v_3) \text{ Tr} \gamma_{2k,5}
\]

(3.39)

where the sum in eq. (3.37) is now extended only from \( k = 1 \) to \( k = N - 1 \). Now, if we restrict to the case of complex planes \( i \) which are rotated by all twists in the model, the last term drops, leaving

\[
\tilde{\alpha}_k^i = \frac{1}{2} C_{2k} \cotg(2\pi k v_i) \text{ Tr} \gamma_{2k,9} - 2 C_k \cotg(\pi k v_i)
\]

(3.40)

To proceed further we need to impose the twisted tadpole cancellation conditions for \( \text{Tr} \gamma_{2k,9} \), which are model dependent. For the \( Z_6 \) and \( Z_6' \) orientifolds we have the
condition [3]:
\[ \text{Tr} \gamma_{2k,9} = (-1)^k 32 \prod_{i=1}^{3} \cos(\pi k v_i) \] (3.41)

After substitution one finally gets:
\[ \tilde{\alpha}_k^i = 2 \left[ (-1)^k C_{4k} \cotg(2\pi k v_i) - C_k \cotg(\pi k v_i) \right] \] (3.42)

The results for \( Z_6 \) and \( Z_6' \) for the different twists \( k \) are shown in Table 3.

For the \( Z_{12} \) orientifold with all the D5-branes at the origin we have the following twisted tadpole cancellation conditions [12, 25]:
\[ \text{Tr} \gamma_{k,9} = \text{Tr} \gamma_{k,5} = 0 ; \ k = 1, 2, 3, 5, 7, 9, 10, 11 \] (3.43)
\[ \text{Tr} \gamma_{4,9} = \text{Tr} \gamma_{4,5} = 4 \]
\[ \text{Tr} \gamma_{8,9} = \text{Tr} \gamma_{8,5} = -4 \] (3.44)

Plugging these traces back into eq.(3.40), one finally gets the \( \tilde{\alpha}_k^i \). For the \( k=\text{odd} \) contribution the first term in eq.(3.40) vanishes, leaving only:
\[ \tilde{\alpha}_k^i = -2C_k \cotg(\pi k v_i) , \ i = 1, 2 , \ k = \text{odd} \] (3.45)

For the \( k=\text{even} \) sectors the contribution of \( \text{Tr} \gamma_{2k} \) is non-vanishing. The additional contribution for these sectors is of the form:
\[ \pm 2C_{2k} \cotg(2\pi k v_i) , \ i = 1, 2 , \ k = \text{even} \] (3.46)

The sum of the two terms gives the \( \tilde{\alpha}_k^i \) shown in Table 3.

As we see, the structure we obtain for the \( \sigma \) model anomalies along complex planes which are always rotated by the twists is very analogous to that we found for odd orientifolds. The result in both cases shows the anomaly can be cancelled by a GS mechanism mediated by the exchange of twisted RR fields. Comparing these results with those obtained for gauged anomalous \( U(1)'s \), one observes that the role of the \( \sin(2\pi k V_a) \) factors in \( U(1) \) anomaly cancellation is here played by the \( \cotg \) and \( \cotg \) factors displayed in Table 3. Just like \( \sin(2\pi k V_a) \) measures the mixing of the anomalous \( U(1)'s \) with twisted moduli, in the present case those trigonometric factors should measure the mixing of untwisted moduli \( T_i \) with twisted moduli. It would be interesting to confirm these couplings by a direct computation in string theory.

\^8The difference in sign compared to eq.(3.26) is due to the fact that here \( \gamma_{1,9}^N = -1 \) since the embedding has no vector structure.
\[ \tilde{\alpha}_i^k \]

| \( Z_3, Z_7 \) | \(-C_k \tan(\pi k v_i)\) | \( i = 1, 2, 3 \) , \( k = 1, \ldots, (N - 1)/2 \) |
| \( Z_6 \) | 0 | \( i = 1, 2 \) , \( k = 1, 3, 5 \) |
| \( Z_6 \) | \(-C_k (\tan(\pi k v_i) + \cotan(\pi k v_i))\) | \( i = 1, 2 \) , \( k = 2, 4 \) |
| \( Z_6 \) | \(-2C_k \cotan(\pi k v_i)\) | \( i = 1 \) , \( k = 1, 5 \) |
| \( Z_12 \) | 0 | \( i = 1 \) , \( k = 2, 3, 4 \) |
| \( Z_12 \) | \(-2C_k \cotan(\pi k v_i)\) | \( i = 1, 2 \) , \( k = \text{odd} \) |
| \( Z_12 \) | \(2 (C_{2k} \cotan(2\pi k v_i) - C_k \cotan(\pi k v_i))\) | \( i = 1, 2 \) , \( k = 2 \mod 4 \) |
| \( Z_12 \) | \(-2 (C_{2k} \cotan(2\pi k v_i) + C_k \cotan(\pi k v_i))\) | \( i = 1, 2 \) , \( k = 0 \mod 4 \) |

Table 3: \( \tilde{\alpha}_i^k \) coefficients for some orientifolds.

### 3.5 Models with Wilson lines/non coincident branes

Let us briefly comment on how the same anomaly cancellation mechanism works in models with Wilson lines or with branes sitting at different points in the compact space (both possibilities are related by T-duality). To keep the discussion simple, we present two concrete examples with the \( Z_3 \) orientifold as starting point. In order to make the construction more intuitive, we will perform a T-duality along the six compact dimensions, thereby transforming the Wilson lines on the D9-branes into positions of the T-dual D3 branes. The resulting models have orientifold group \( Z_3 + Z_3 R_1 R_2 R_3 \Omega(-1)^F \) and contain 32 D3-branes and no D7-branes.

#### i) An example with a conformal subsector

The first model we consider is analogous to that studied in [11]. It is obtained upon placing 20 D3-branes at the origin, which is a \( Z_3 \) orientifold point, and the remaining 12 D3-branes, in two groups of six, at two of the other \( Z_3 \) fixed points (these are \textit{orbifold} rather than \textit{orientifold} points), related by the orientifold projection. The spectrum is

\[
SO(4) \times U(8) \times U(2)_1 \times U(2)_2 \times U(2)_3 \\
3[(4, 8; 1, 1, 1) + (1, \overline{28}; 1, 1, 1) + (1, 1; 2, 2, 1) + (1, 1; 1, 2, 2) + (1, 1; 2, 1, 2)] (3.47)
\]

Notice that the model with all branes at the origin is continuously connected to this theory, by moving 12 D3 branes off the origin to the \( Z_3 \) orbifold points. Thus,

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*Even though the anomaly cancellation works analogously for the model in [11], the theory we consider is slightly more illustrative for this particular issue.*
it is expected the pattern of anomaly cancellation will be similar in both models. In checking that this is so we will learn an interesting bit or information concerning the behaviour of twisted modes of orbifold (rather than orientifold) singularities in the anomaly cancellation.

It is easy to check that the mixed $\sigma$-gauge anomalies with respect to the different factors are

$$b_{SO(4)}^i = 6, \quad b_{SU(8)}^i = -3, \quad b_{SU(2)_a}^i = 0, \quad i = 1, 2, 3$$

We see that the structure of $\sigma$-model anomalies reveals the existence of two well-defined sectors. The modular anomalies related to gauge groups for D3-branes at the origin ($SO(4)$ and $SU(8)$) are exactly the same as in the model with all branes at the origin. On the other hand, the triangle mixed anomalies with respect to the $SU(2)$ groups on the D3-branes at the orbifold singularity vanish automatically, as expected since this sector is conformal (see section 3.2).

It is straightforward to check that the anomalies for D3-branes at the origin cancel through the GS mechanism exactly as in the model with coincident branes. For the D3 branes at the orbifold singularity triangle anomalies vanish, and no GS mechanism is required. We thus learn that closed string twisted modes of orbifold (rather than orientifold) singularities do not generate GS counterterms.

### ii) A further example

The second example we would like to consider has appeared in [14]. In this model 23 D3-branes sit at the origin, and the remaining 9 D3-branes are stuck at different points fixed under $R_1 R_2 R_3 \Omega(-1)^{F_L}$, i.e. O3-planes. Notice that this theory is not continuously connected to the model with all branes at the origin, and so anomaly cancellation is not obvious a priori.

The spectrum of the model is

$$SO(5) \times U(9)$$

$$3 \left[ (5, 9) + (1, \overline{36}) \right]$$

Explicit computation of the triangle anomalies reveals that

$$b_{SO(5)}^i = 6, \quad b_{SU(9)}^i = -3$$

The anomaly is exactly cancelled by the GS mechanism, as discussed above.

Notice the D3-branes stuck at the O3-planes do not contribute any gauge factors or matter multiplets, but this fact is not essential for the GS mechanism to work.
Actually, it is easy to construct related models with several D3-branes at each O3-plane (at the expense of reducing the number of D3-branes at the origin), so that the model contains additional subsectors. The triangle anomalies with respect to these gauge factors vanishes automatically, so no GS contribution is required. This is expected since the O3-plane is not a fixed point with respect to the $Z_3$ orbifold group and thus does not contain the appropriate twisted fields. The triangle anomaly associated to D3-branes at the origin is exactly as above and cancels through the usual GS mechanism.

We would like to stress that, even though we have discussed only two simple examples, the mechanism for the cancellation of anomalies remains valid for more complicated models.

### 3.6 Mixed $\sigma$-model-gravitational anomalies

The $\sigma$-model symmetries we have studied in the preceding sections have mixed anomalies not only with the non-abelian gauge factors, but also with gravity. In this section we address the cancellation of these gravitational anomalies. For simplicity, we will restrict ourselves to the case of odd order compact orientifolds, but the analysis holds in general with suitable modifications of the relevant couplings.

The main difference between these anomalies and those studied above is that the triangle anomalies also contain the contribution of massless closed string states, running in the loop. Thus we will split the triangle anomaly in two pieces:

$$b^i_{\text{grav}} = b^i_{\text{closed}} + b^i_{\text{open}}$$

The first contribution $b^i_{\text{closed}}$ has the form:

$$b^i_{\text{closed}} = 21 + 1 + \delta^i_T + \delta^i_M$$

As we discussed in the heterotic case, the $21 + 1$ come from the gravitino and dilatino (partner of $S$) fields. The third term $\delta^i_T$ represent the contribution of the untwisted moduli fields themselves. It is easy to check that $\delta^i_T = -3, -1$ for $Z_3$ and $Z_7$ respectively. The fourth term $\delta^i_M$ represent the contribution of the twisted closed string states. Those have only non-linear transformations with respect to the $SL(2, \mathbb{R})$ transformations, and so have zero "modular weights". Their contribution is equal to the number of twisted chiral fields.

This $b^i_{\text{closed}}$ piece of the anomaly is analogous to the one appearing in the heterotic models, which also involves closed strings. As we mentioned in section 2, in the heterotic case the contribution from the field theory triangle diagrams is cancelled exactly by a one-loop diagram which mixes the $S$ and $T_i$ fields.
Figure 7: The field-theory limits of the torus (a and c) and Klein bottle (b and d) string world-sheets contributing to the mixed $\sigma$-gravitational anomalies. The ‘cut’ in the handles of diagrams b) and d) represent a gluing that reverses the orientation. Diagrams a) and b) represent closed string massless fields running though a loop, and reproduce the field theory triangle anomaly. Diagrams c) and d) represent the exchange of the dilaton multiplet along the infinite tube, and its one-loop coupling to the moduli $T_i$. They provide the GS amplitudes that cancel the triangle anomaly.

The fact that the closed string contribution to the anomaly in the type I case has the same structure suggest that this piece is cancelled in a similar fashion. Namely, we propose that $b^\text{closed}_i$ is cancelled by a one-loop mixing between the dilaton multiplet and the untwisted moduli $T_i$. Thus, unlike what happens with mixed $U(1)$ or $\sigma$-model anomalies for which the dilaton $S$ plays no role in anomaly cancellation, in the case of mixed $\sigma$-model-gravity anomalies the $S$ field gets transformed at one loop under an $SL(2,\mathbb{R})$ transformation.

The diagrammatic explanation for this behaviour is depicted in Figure 7. The first two diagrams show the field theory triangle anomaly due to type I closed string modes. The diagrams c) and d) show the additional low-energy contributions corresponding to these topologies. They are interpreted as the exchange of the dilaton multiplet along the infinite tube, and its coupling to the untwisted moduli through a one-loop subdiagram.

Notice that this non-trivial transformation of the dilaton leads to no contradiction with our previous results concerning $\sigma$-model-gauge anomalies. This follows from the different dilaton dependence of the $F \wedge F$ and $R \wedge R$ terms in Type I string theory. The contribution arising from the coupling $SF \wedge F$ coupling upon the one-loop transformation of the $S$-field is a term of higher order in perturbation theory, as compared with the analogous coupling with gravity.\footnote{This can be rephrased in string diagrammatics as follows. A string topology combining the one-loop mixing of the $S$ and $T_i$ fields and the coupling of $S$ to the gauge bosons will include one handle}. In summary, the triangle anomaly from...
closed string states will be cancelled by the one-loop mixing of $S$ and $T_i$ fields.

The remaining contribution, $b^i_{\text{open}}$, is on the other hand cancelled through exchange of twisted closed string modes in the already familiar fashion. The relevant diagrams providing the field theory triangle anomaly and the GS terms are analogous to those involved in the discussion of mixed $\sigma$-gauge anomalies (see figures 3, 6), with the difference that the graviton vertex operator should be attached in the interior of the world-sheet. To support the existence of this cancellation mechanism one can use techniques similar to those used for mixed gauge anomalies. For the odd $Z_N$ we are considering, using eqs.(3.19) and (2.12) one gets

$$b^{i}_{\text{open}} = \frac{1}{2} \sum_{a=1}^{(N-1)/2} w_a^2 - \frac{w_0(w_0-1)}{2}$$

(3.53)

$$- \frac{1}{4} \sum_{a=0}^{N-1} (w_a(w_a+t_i + w_{a-t_l}) + w_a(\delta_{2a+t_l} + \delta_{2a-t_l}))$$

(3.54)

$$+ \frac{1}{4} \sum_{j \neq i} w_a(w_{a+t_j}+w_{a-t_j}) - \sum_{j \neq i} w_a(\delta_{2a+t_j} + \delta_{2a-t_j}))$$

(3.55)

As before, doing an inverse discrete Fourier transform and substituting the $w$’s one gets

$$b^{i}_{\text{open}} = \frac{1}{2N} \left\{ \sum_k [ \text{Tr} \gamma_k \text{Tr} \gamma_{-k} - \text{Tr} \gamma_{2k}] (\sum_{j \neq i} \cos(4\pi kv_j) - \cos(4\pi kv_i) - 1) \right\}$$

(3.56)

After some trigonometry one finds

$$b^{i}_{\text{open}} = \frac{1}{4N} \sum_{k=1}^{N-1} \tilde{\omega}_k^i (\text{Tr} \gamma_k \text{Tr} \gamma_{-k} - \text{Tr} \gamma_{2k})$$

(3.57)

with

$$\tilde{\omega}_k^i = C_k \cotg(\pi kv_i)$$

(3.58)

Indeed, the result in eq.(3.57) suggests the two contributions from annulus and from Moebius strip. Also it shows an structure which is compatible with its cancellation by the exchange of twisted closed string massless states coupling simultaneously to untwisted moduli $T_i$ and gravitons.

and one boundary. If $g,b$ and $c$ are the number of handles, boundaries and crosscaps, we will have a dilaton dependence with a power $(2-2g-b-c) = -1$, whereas one-loop effects (like the diagrams in Fig. 3) have a vanishing power ($g = 0, b = 2, c = 0$, or $g = 0, b = 1, c = 1$).
4 Fayet-Iliopoulos terms and untwisted/twisted moduli mixing

The presence and cancellation of $\sigma$-model anomalies have certain implications for the structure of the effective low-energy action of Type IIB compact orientifolds. We already observed how in the case of heterotic orbifolds some mixing (eq. (2.11)) must appear between the field which transforms non-linearly under the $SL(2, R)_{T_i}$ symmetries (i.e., $S$ in the heterotic case) and the untwisted moduli $T_i$. Something analogous must occur in the Type IIB orientifold case, but now it will be the twisted fields $M_{f}^{k}$ which mix with the untwisted moduli. If, in addition there are also anomalous $U(1)$ symmetries, as is generically the case, the Kähler potential of the twisted fields will be of the form

$$K(M_{f}^{k}, M_{f}^{k*}) = K(M_{f}^{k} + M_{f}^{k*} - \sum_{a} \delta_{GSk}^{a} V_{a} + \sum_{i=1}^{3} \delta_{GSk}^{i} \log(T_{i} + T_{i}^{*})) \quad (4.1)$$

in order to have both $U(1)_{a}$ gauge invariance and $\sigma$-model invariance with respect to the $i^{th}$ complex direction. Thus the above Kähler potential would be invariant respectively under the transformations (we consider here the case of odd $N$ orientifolds for simplicity):

$$\begin{align*}
Im M_{f}^{k} &\rightarrow Im M_{f}^{k} + \delta_{GSk}^{f} (a) \Lambda_{a}(x) \\
Im M_{f}^{k} &\rightarrow Im M_{f}^{k} + \delta_{GSk}^{i} \log(ic_{i}T_{i} + d_{i})
\end{align*} \quad (4.2)$$

where

$$\begin{align*}
\delta_{GSk}^{f} (a) &= w_{a} 2 \sin(2\pi k V_{a}) ; \delta_{GSk}^{i} &= 2\tan(\pi k v_{i}) \quad (4.3)
\end{align*}$$

For a quadratic Kähler potential for the $M_{f}^{k}$ fields, eq.(4.1) gives rise to a FI-term corresponding to the $U(1)_{a}$ field :

$$\xi_{a} = -\sum_{f} \sum_{k} \delta_{GSk}^{a} [M_{f}^{k} + M_{f}^{k*} + \sum_{i=1}^{3} \delta_{GSk}^{i} \log(T_{i} + T_{i}^{*})] \quad (4.4)$$

This is an interesting result since it shows that in compact Type IIB orientifolds the Fayet-Iliopoulos terms are controlled not only by the blowing up modes of the orbifold singularities but also by the untwisted moduli $T_{i}$. The FI-term in fact vanishes at the points with:

$$2Re M_{f}^{k} = -\sum_{i=1}^{3} \delta_{GSk}^{i} \log(T_{i} + T_{i}^{*})) \quad (4.5)$$

This corresponds to the SUSY-preserving vacuum when non-Abelian gauge symmetry remains unbroken (FI-terms =0).
As we remarked in the previous section, the cancellation of mixed $\sigma$-gravitational anomalies requires also the presence of a mixing between the complex dilaton $S$ and the untwisted moduli, very much analogous to the heterotic case. Thus one expects a form for this mixing (again in the case of odd orientifolds):

$$K(S, S^*) = -\log(S + S^* + \sum_i \delta^i_{\text{closed}} \log(T_i + T_i^*)) \quad (4.6)$$

The additional untwisted moduli-dependence of FI-terms in compact Type IIB, $D = 4, N = 1$ orientifolds have interesting implications which will be discussed in more detail elsewhere [32]. Notice, for example, that it changes previous discussions about matching of $Z_3$ and $Z_7$ orientifolds with their corresponding heterotic duals [29] (for example, for generic compact radii both the orientifold model and the heterotic dual will have non-vanishing FI-terms associated to their anomalous $U(1)$'s), or about the process of blowing up the orientifold singularities [33].

5 Conclusions

In this paper we have addressed the issue of $\sigma$-model anomalies in $D = 4, N = 1$ type IIB orientifold vacua. We have presented evidence suggesting that anomalies associated to certain modular transformations (those corresponding to complex planes rotated by all elements in the orbifold group) are cancelled by a GS mechanism mediated by the exchange of RR twisted closed string modes (for mixed gravitational anomalies, the dilaton also plays a non-trivial role).

The main a priori reason to expect such cancellation is the duality of these orientifolds with certain heterotic vacua, in which these anomalies cancel. In heterotic models, this cancellation is required since a discrete version of the $\sigma$-model symmetries corresponds to T-duality, which is an exact symmetry of the full string theory. It would be desirable to gain a better insight of $\sigma$-model symmetries in type I string theory, in order to understand whether the cancellation we have discussed follows as a consequence from a similarly deep property of string theory.

The mechanism for the cancellation of $\sigma$-model anomalies that we have uncovered is analogous to the GS mechanism which cancels $U(1)$ gauge anomalies, in that the exchanged fields are closed string twisted modes. In particular, this has the consequence that the mixed anomalies are allowed to be highly non-universal with respect to the different gauge factors and gravity. This differs markedly from the behaviour in heterotic vacua, and may be used to relax certain constraints on the low-energy spectrum of phenomenologically interesting string vacua.
Finally we would like to stress that our conclusions have been based on a detailed analysis of the triangle anomalies. In particular we have found that rewriting the anomalies in terms of Chan-Paton traces is an extremely useful trick which automatically exhibits the factorization properties of the anomaly. In particular, it shows clearly the contribution of the RR twisted modes to the GS mechanism to all these amplitudes, and, in the case of the mixed gravitational anomaly, shows the necessity of having a non-trivial transformation of the dilaton multiplet.

The anomaly cancellation mechanism requires the existence of interesting tree-level mixings between the $T_i$ and the twisted closed string modes, and a one-loop mixing between the $T_i$ and the dilaton. The consequences of the existence of these couplings should be further explored. In particular, they lead to an interesting modification for the FI terms for the anomalous $U(1)$’s, which for compact models do not vanish at the orbifold point. This fact had been overlooked in previous studies of the consequences of the FI terms.

Type IIB orientifolds constitute an extremely interesting set of models, with properties often differing from the well-known behaviour of heterotic vacua. As such, they are worthy of detailed exploration. We hope our analysis helps in adding some useful information to our present knowledge of the perturbative structure of type IIB orientifold vacua.

**Acknowledgements**

We are thankful to G. Aldazabal, M. Klein, B. Ovrut, E. Poppitz and F. Quevedo for useful discussions. A. M. U. is grateful to M. González for encouragement and support, and to the Center for Theoretical Physics of M. I. T. for hospitality. L. E. I. and R. R. thank CICYT (Spain) and the European Commission (grant ERBFMRX-CT96-0045) for financial support. The work of A. M. U. is supported by the Ramón Areces Foundation (Spain).
6 Appendix

In this appendix we discuss the contribution of the different fixed points to the cancellation of $U(1)$ anomalies in models with D5- and D9-branes.

For a given twist $k$ there are $C^2_k (= \prod_{i=1}^{3} 4 \sin^2 \pi k v_i)$ fixed points. Let us label by an index $p$ the $4 \sin^2 \pi k v_3$ fixed points, located at the origin in the first two complex planes and anywhere in the third. Since in our models the D5-branes sit at the origin in the two complex planes, it will be a combination of these twisted fields the one responsible for the cancellation of $U(1)$ anomalies in the D5-branes. On the other hand, D9-branes fill the compact space completely and couple to twisted modes from all the $C^2_k$ fixed points, which we label by an index $f$. Let us define

$$B^k_5 = \frac{1}{2 \sin \pi k v_3} \sum_p M^k_p$$

$$B^k_9 = \frac{1}{\prod_{i=1}^{3} 2 \sin \pi k v_i} \sum_f M^k_f$$  \hspace{1cm} (6.1)

Under a gauge transformation of the $a$th ($b$th) $U(1)$ factor in the sector of the D5-branes (D9-branes), with parameters $\Lambda^a_5$ ($\Lambda^b_9$), the axion fields in $M^k_f$ transform as follows

$$\text{Im } M^k_f \rightarrow \text{Im } M^k_f + w_b \frac{1}{N} \frac{N/2}{2} \sum_{k=1}^{N/2} \cos 2\pi k V^5_b \Lambda^b_9$$

(6.2)

(no sum in $a$, $b$ implied). Here the second contribution is only present if the fixed point labeled by $f$ couples to the D5-branes. This behaviour induces the following non-trivial transformation on the fields (5.1)

$$\text{Im } B^k_5 \rightarrow \text{Im } B^k_5 + C^5_{55} w_a 2 \sin 2\pi k V^5_a \Lambda^a_5 + C^5_{9} w_b 2 \sin 2\pi k V^9_b \Lambda^b_9$$

$$\text{Im } B^k_9 \rightarrow \text{Im } B^k_9 + C^9_{55} w_a 2 \sin 2\pi k V^5_a \Lambda^a_5 + C^9_{9} w_b 2 \sin 2\pi k V^9_b \Lambda^b_9$$  \hspace{1cm} (6.3)

Notice the manifest symmetry between the couplings to D5- and D9-branes. This exhibits the T-duality of this type of models.

The gauge kinetic functions for gauge fields on the D5- and D9-branes are given by

$$f^5_{a'} = T_3 + \frac{1}{N} \sum_{k=1}^{N/2} \cos 2\pi k V^5_{a'} B^k_5$$

$$f^9_{b'} = S + \frac{1}{N} \sum_{k=1}^{N/2} \cos 2\pi k V^9_{b'} B^k_9$$  \hspace{1cm} (6.4)

It is easy to check that the transformations (6.3) then generate the GS counterterms that cancel the triangle anomalies.
References

[1] V. S. Kaplunovsky, Nucl. Phys. B307 (1988) 145, Erratum Nucl. Phys. B382 (1992) 436, hep-th/9205070.

[2] L. J. Dixon, V. Kaplunovsky, J. Louis, Nucl. Phys. B355 (1991) 649.

[3] M. Green and J. Schwarz, Phys. Lett. B149 (1984) 117.

[4] J.-P. Derendinger, S. Ferrara, C. Kounnas, F. Zwirner, Nucl. Phys. B372 (1992) 145; Phys. Lett. B271 (1991) 307.

[5] G. Lopes Cardoso, B. A. Ovrut, Nucl. Phys. B369 (1992) 351; Nucl. Phys. B392 (1993) 313.

[6] M. Berkooz and R. G. Leigh, Nucl. Phys. B483 (1997) 187, hep-th/9605049.

[7] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Ya.S. Stanev, Phys. Lett. B385 (1996) 96, hep-th/9606169.

[8] Z. Kakushadze, Nucl. Phys. B512 (1998) 221, hep-th/9704059.
Z. Kakushadze and G. Shiu, Phys. Rev. D56 (1997) 3686, hep-th/9705163.
Z. Kakushadze and G. Shiu, Nucl. Phys. B520 1998 75, hep-th/9706051.

[9] G. Zwart, Nucl. Phys. B526 (1998) 378, hep-th/9708040.

[10] D. O’Driscoll, hep-th/9801114.

[11] L.E. Ibáñez, hep-th/9802103.

[12] G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, FTUAM-98/4, hep-th/9804026.

[13] Z. Kakushadze, hep-th/9804110; hep-th/9806044.

[14] J. Lykken, E. Poppitz and S. Trivedi, Nucl. Phys. B543 (1999) 105, hep-th/9806050.

[15] L. E. Ibáñez, R. Rabadán and A. Uranga, hep-th/9808139.

[16] J. Polchinski, E. Witten, Nucl. Phys. B460 (1996) 525.

[17] C. Bachas, C. Fabre, Nucl. Phys. B476 (1996) 436, hep-th/9605028.

[18] L. E. Ibáñez, D. Lüst, Nucl. Phys. B382 (1992) 305, hep-th/9202046.
[19] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B289 (1987) 585
J. Attick, L. Dixon and A. Sen Nucl. Phys. B292 (1987) 109,
M. Dine, I. Ichinoise and N. Seiberg, Nucl. Phys. B293 (1987) 253.

[20] A. Sagnotti, in Cargese 87, Strings on Orbifolds, ed. G. Mack et al. (Pergamon
Press, 1988) p. 521.

[21] P. Horava, Nucl. Phys. B327 (1989) 461; Phys. Lett. B231 (1989) 251;
J. Dai, R. Leigh and J. Polchinski, Mod.Phys.Lett. A4 (1989) 2073;
R. Leigh, Mod.Phys.Lett. A4 (1989) 2767.

[22] G. Pradisi and A. Sagnotti, Phys. Lett. B216 (1989) 59;
M. Bianchi and A. Sagnotti, Phys. Lett. B247 (1990) 517.

[23] E. Gimon and J. Polchinski, Phys.Rev. D54 (1996) 1667, hep-th/9601038

[24] L. Dixon, J.A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B274 (1986) 285.

[25] G. Aldazabal, D. Badagnani, L. E. Ibáñez, A. M. Uranga, hep-th/9904071.

[26] A. Sagnotti, Phys. Lett. B294 (1992) 196, hep-th/9210127.

[27] M. Berkooz, R. G. Leigh, J. Polchinski, J. H. Schwarz, N. Seiberg, E. Witten,
Nucl.Phys.B475(1996)115, hep-th/9605184.

[28] E. Poppitz, Nucl. Phys. B542 (1999) 31, hep-th/9810010.

[29] Z. Lalak, S. Lavignac, H.-P. Nilles, hep-th/9903160.

[30] Z. Kakushadze and G. Shiu, Phys. Rev. D56 (1997) 3686, hep-th/9705163.

[31] L. E. Ibáñez, C. Muñoz, S. Rigolin, hep-ph/9812397.

[32] L. E. Ibáñez, to appear.

[33] M. Cvetič, L. Everett, P. Langacker, J. Wang, hep-th/9903051.