Gravitational form factors of a baryon with spin-3/2

June-Young Kim\textsuperscript{1,} and Bao-Dong Sun\textsuperscript{2,}\textsuperscript{†}

\textsuperscript{1}Ruhr-Universität Bochum, Fakultät für Physik und Astronomie, Institut für Theoretische Physik II, D-44780 Bochum, Germany
\textsuperscript{2}Key Laboratory of Particle Physics and Particle Irradiation (MOE), Institute of Frontier and Interdisciplinary Science, Shandong University, Qingdao, Shandong 266237, China

(Dated: November 3, 2020)

We define the form factors of the energy-momentum tensor (EMT) for a spin-3/2 hadron. The static EMT is related to the energy, angular momentum, pressure and shear force densities. We also studied the nucleon and Δ form factors of the energy-momentum tensor in the large $N_c$ limit within the SU(2) Skyrme model.

Keywords: energy-momentum tensor, gravitational form factor, pion mean fields, Skyrme model, large-$N_c$ limit, baryon Δ
I. INTRODUCTION

The EMT for the nucleon contains fundamental information on three mechanical properties mass, spin, and $D$-term. These mechanical properties are related to the gravitational form factors (GFFs) at zero momentum transfer, i.e., $A(0) =$ mass, $J(0) =$ spin, $D(0) =$ $D$-term. The mass and spin are well-known quantities, but the $D$-term related to the internal force inside a hadron experienced by constituents is not much-known \cite{1,2}. Therefore, measuring the $D$-term form factor is of great importance to take a grasp of the $D$-term. Unfortunately, GFFs were at first considered \cite{3,4} as a merely academic subject since it was realized that probing them is practically impossible through the graviton exchange. However, it sheds new light on this intriguing issue over few decades that the novel understanding of the GFFs for the nucleon as the second Mellin moments of the (unpolarized) generalized parton distributions (GPDs) \cite{5,6} which are accessible in hard exclusive reactions. On the experiment aspect, there are several ongoing or planned facilities that are ideal to measure GPDs, such as the newly upgraded 12GeV lepton-hadron facility CEBAF at Jefferson Lab (JLab) and the Electron-Ion Collider (EIC) at Brookhaven National Laboratory (BNL) \cite{7}, and the Electron-Ion Collider in China (EicC) \cite{8}. The EIC experiments are believed to be the next QCD frontier \cite{9}. Since the GPDs are $s$-$t$ crossed quantities of the generalized distribution amplitudes (GDAs) \cite{10}, the spacelike GFFs can also be obtained from GDAs by using the dispersion relation. For the pion case, the GDAs are firstly extracted from the actual experimental data reported by the Belle collaboration, the cross-section measurements of hadron-pair production process $\gamma^*\gamma \to \pi^0\pi^0$ at KEKB in Ref. \cite{11}, and thus the pion GFFs, gravitational densities, and its mass radius and mechanical (pressure and shear force) radius are all obtained. The future measurements of super-KEKB and international linear collider could provide more accurate experimental data for the GDAs study.

Another interesting object is the $\Delta$ baryon. This $\Delta$ is a first excited baryon with spin-3/2. One of the interesting subjects is its electromagnetic (EM) structure that can be accessed by the photon exchange. Its EM structure has been theoretically parametrized in terms of the multipole form factors for the $\gamma\Delta\Delta$ and $\gamma N\Delta$ vertices in Ref. \cite{12,13}, and experimentally measured at the various experimental facilities \cite{14,15}. While the numerous works on EM from factors has been done, there are merely few theoretical works—the symmetric tensor current is first introduced in Ref. \cite{16}, and the general properties of the pressure and shear forces are derived in the large-$N_c$ limit in Ref. \cite{17} for the $\Delta$ GFFs, even though the GFFs (second Mellin moments) are of equivalent importance as the EM form factors (first Mellin moments). One of the reasons is the experimental perspective on access to the GFFs. The GFFs of the nucleon are firstly extracted in Ref. \cite{18} through the GDAs which are accessed by the deeply virtual Compton scattering (DVCS), whereas it is experimentally rather difficult to get access to the GFFs of the $\Delta$ through GPDs as well as the graviton exchange because of the short-lived nature of the $\Delta$. Nevertheless, this topic is still interesting. Above all, the $\Delta$ GFFs can be measured on lattice QCD, and it is a more instructive topic for better understanding of the baryon mechanical properties than that for the nucleon by virtue of the affluent EMT structure of the $\Delta$. Compared with the nucleon case, the EMT for the $\Delta$ includes four more mechanical quantities in terms of multipole expansion: the mass quadrupole, the spin octupole, and two more generalized $D$-terms. In other words, the hadronic matrix elements of the EMT for the $\Delta$ can be expressed in terms of seven GFFs except for the EMT-nonconserving form factors. We emphasize that, to further understand the spin-3/2 baryon mechanical properties and perform further theoretical or lattice calculations, it firstly requires the parametrization of the GFFs in terms of multipole expansion and searching out the relations between the GFFs and the multipole structure of the EMT densities. This is the main goal of this work.

To verify the general requirements and relations of the GFFs proposed in this work, we adopt the SU(2) Skyrme model. The Skyrme model has some advantageous features. Firstly, the model respects the chiral symmetry, and thus it has a connection with the chiral perturbation theory. Secondly, a baryon is viewed as a soliton with an effective mesonic field provoked by QCD in the large-$N_c$ limit, therefore it respects the important properties of the low-energy QCD. Lastly, the model has explained the numerous observables for the baryons rather well \cite{19,20}, and succeeded in a description of the general requirements of the GFFs of the nucleon. Thus, the Skyrme model is thought to be good enough for this purpose.

There are various works on the GFFs for a hadron with different spins. The parametrizations of the GFFs for a hadron with various spin are discussed in Ref. \cite{21,22,23,24}. The GFFs of the spin-0 hadrons are investigated in the chiral quark model \cite{25,26}, the lattice QCD \cite{27}, the parameter fitting from experimental data \cite{28}, and the Nambu–Jona-Lasinio (NJL) model \cite{29}. For the spin-1/2 hadrons, there are results from the effective chiral theory \cite{30}, the chiral quark-soliton model \cite{31,32}, the SU(2) Skyrme model \cite{33,34}, the $\pi - \rho - \omega$ model \cite{35,36}, the bag model \cite{37}, the QCD sum rule \cite{38,39}, and the lattice QCD \cite{40,41}. The renormalization group properties of the nucleon’s twist-four gravitational form factor $\tilde{c}_{q,g} (t)$ \cite{42} which plays a role in the nucleon’s transverse spin sum rule, are studied in Ref. \cite{43}. In the case of the spin-1 hadrons, the relations between the GPDs and the GFFs are

---

1 $\tilde{c}_{q,g} (t) = 2M_N F_{3,0}^T (t)$ in the notation of Ref. \cite{20} where $M_N$ is the nucleon mass.
established in Ref. [31, 52] and the GFFs of the vector mesons are evaluated in Ref. [37, 52, 55]. For the higher spin hadrons, the works on the parametrization of the stress tensor are made in Ref. [27, 54].

We sketch the present work as follows: In Section II we define the hadronic elements of the EMT as the GFFs and reorganize the GFFs in terms of the multipole expansion. We also define the EMT densities of a baryon with spin-3/2 in terms of the multipole expansion and present the relations between the GFFs and the EMT densities. To verify the general requirements and relations proposed in Section II, the GFFs and the EMT multipole densities of the Δ are obtained within the Skyrme model in Section III and the numerical results are showed and discussed in Section IV. In Section V we make a summary.

II. GRAVITATIONAL FORM FACTORS OF A SPIN-3/2 HADRON

We use the covariant normalization \( \langle p', \sigma'|p, \sigma \rangle = 2p^0(2\pi)^3\delta_{\sigma',\sigma}\delta^{(3)}(p' - p) \) of one-particle states, and introduce kinematical variables \( P^\mu = (p^\mu + p'^\mu)/2, \Delta^\mu = p'^\mu - p^\mu \) and \( \Delta^2 = t \). For the GFFs of a spin-3/2 particle in QCD, the matrix elements of EMT current is given by \[ 26^2 \]

\[
\left\langle p', \sigma'|T^{\mu\nu}_a(0)|p, \sigma \right\rangle = -\pi^\nu(p', \sigma') \left[ \frac{P^\mu P^{\nu}}{m} \left( g_{\alpha\alpha} F^a_{1,0}(t) - \frac{\Delta_{\alpha}^\mu \Delta_{\alpha}^\nu}{2m^2} F^a_{1,1}(t) \right) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m} \left( g_{\alpha\alpha} F^a_{2,0}(t) - \frac{\Delta_{\alpha}^\mu \Delta_{\alpha}^\nu}{2m^2} F^a_{2,1}(t) \right) \right.
\]

\[
\left. + mg^{\mu\nu} \left( g_{\alpha\alpha} F^a_{3,0}(t) - \frac{\Delta_{\alpha}^\mu \Delta_{\alpha}^\nu}{2m^2} F^a_{3,1}(t) \right) \right] u^\alpha(p, \sigma) \]

where \( u^\alpha(p, \sigma) \) is the Rarita-Schwinger spinor, and it satisfies the Dirac equation \( (\gamma^\mu p - m)u^\alpha(p, \sigma) = 0 \) and the subsidiary conditions \( \gamma^\alpha u^\alpha(p, \sigma) = 0 \) and \( p_\alpha u^\alpha(p, \sigma) = 0 \). Here, \( \sigma, \sigma' \) is the initial (final) spin projections. The normalization of the Dirac spinors is taken to be \( \pi^\sigma(p)u_\sigma(p) = 2m\delta_{\sigma,\sigma'} \). The index \( a \) runs from a gluon to quark flavors. The quark and gluon form factors \( F^a_{i,k}(i = 1, 2, 4, 5) \) are individually conserved, whereas \( F^a_{3,k}(i = 3, 6) \) are not conserved. Note that we name the GFFs of a hadron with spin-3/2 according to Ref. [26] and reparametrize them to be analogous with those with spin-1/2 and spin-1. The separate quark and gluon GFFs depend on the renormalization scale \( \mu \), which is suppressed for simplicity. Because of the EMT conservation, the non-conservation terms \( F^a_{3,k}(i = 3, 6) \) have constraints, i.e., \( \sum_k F^a_{3,k} = 0 (i = 3, 6) \). The scale-invariant total GFFs are obtained as \( F_{i,k} = \sum_a F^a_{i,k} \) (\( i = 1, 2, 4, 5 \)).

A. Gravitational form factors in Breit frame

Before discussing the GFFs, we define the n-rank irreducible tensors and the multipole operators. The n-rank irreducible tensors in coordinate (or momentum) space are given by

\[
Y_n^{i_1i_2...i_n}(\Omega_r) = \frac{(-1)^n}{(2n - 1)!!} r^{n+1} \partial^{i_1} \partial^{i_2} ... \partial^{i_n} 1 \quad \text{for} \quad Y_n^{i_1i_2...i_n}(\Omega_p) = \frac{(-1)^n}{(2n - 1)!!} p^{n+1} \partial^{i_1} \partial^{i_2} ... \partial^{i_n} 1 \quad \text{for} \quad p. \]

Thus, one gets the following expressions as

\[
Y_0(\Omega_r) = 1, \quad Y_1^i(\Omega_r) = \frac{r^i}{r}, \quad Y_2^{ij}(\Omega_r) = \frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij}, \quad Y_3^{ijk}(\Omega_r) = \frac{r^i r^j r^k}{r^3} - \frac{1}{5} \left( \frac{r^k}{r} \delta^{ij} + \frac{r^i}{r} \delta^{jk} + \frac{r^j}{r} \delta^{ik} \right). \]

\[1\] In order to be in line with the definition of the EMT current of the spin-1/2 baryon, we reparametrized the expressions given in Ref. [26] as \( (F^a_{1,0}, F^a_{1,1}, F^a_{2,0}, F^a_{2,1}, F^a_{3,0}, F^a_{3,1}, F^a_{4,0}, F^a_{4,1}, F^a_{5,0}, F^a_{5,1}) \) as \( (2F^a_{1,0}, 4F^a_{1,1}, F^a_{2,0}, 2F^a_{2,1}, F^a_{3,0}, F^a_{3,1}, F^a_{4,0}, F^a_{4,1}, 8F^a_{5,0}, 8F^a_{5,1}, 2F^a_{6,0}, 2F^a_{6,1}, 2F^a_{7,0}, 2F^a_{7,1}, 2F^a_{8,0}, F^a_{8,1}, F^a_{9,0}, F^a_{9,1}, F^a_{10,0}, F^a_{10,1}) \). Note that there is a typo in Ref. [26], and it should be corrected as \( g^{\mu\nu} \Delta_{\alpha}^\mu \Delta_{\alpha}^\nu \rightarrow 2g^{\mu\nu} \Delta_{\alpha}^\mu \Delta_{\alpha}^\nu \).
For a hadron with spin-3/2, the quadrupole- and octupole-spin operators $\hat{Q}^{ij}$ (rank 2 tensor) and $\hat{O}^{ijk}$ (rank 3 tensor)
are respectively defined in terms of the spin operator $\hat{S}$ as

$$
\hat{Q}^{ij} = \frac{1}{2} \left( \hat{S}^i \hat{S}^j + \hat{S}^j \hat{S}^i - \frac{2}{3} S(S + 1) \delta^{ij} \right),
$$

$$
\hat{O}^{ijk} = \frac{1}{6} \left( \hat{S}^i \hat{S}^j \hat{S}^k + \hat{S}^j \hat{S}^k \hat{S}^i + \hat{S}^k \hat{S}^i \hat{S}^j + \frac{2}{3} S(S + 1) \left( \delta^{ij} \hat{S}^k + \delta^{ik} \hat{S}^j + \delta^{jk} \hat{S}^i \right) \right),
$$

(4)

with $i, j, k = 1, 2, 3$, and the operators are symmetrized and traceless ($\hat{Q}^{ii} = 0$ and $\hat{O}^{ij} = \hat{O}^{ji} = \hat{O}^{ji} = 0$). The spin operators can be expressed in terms of the SU(2) Clebsch-Gordan coefficients in the spherical basis as

$$
\hat{S}_{a,\sigma}^a = \sqrt{S(S + 1)} \hat{O}^S_{S_a} \hat{O}^S_{\sigma_a} \quad \text{with} \quad (a = 0, \pm 1, \sigma, \sigma' = 0, \cdots, \pm S).
$$

(5)

In the Breit frame the average of the baryon momenta and the momentum transfer are respectively defined by $P^\mu = (p^\mu + p^\mu) / 2 = (E, 0, 0, 0)$ and $\Delta^\mu = p^\mu - p^\mu = (0, \Delta)$ with the initial (final) momentum $p(p')$. The momentum squared is defined as $\Delta^2 = -\Delta^2 = t = 4(m^2 - E^2)$ with the baryon mass $m$. The explicit expressions of the Rarita-Schwinger spinor and the polarization vector are given in Appendix A. In this frame, the matrix elements of the EMT current are expressed in terms of the gravitational multipole form factors (GMFFs) as

$$
\langle p', \sigma' | T_{a}^{\mu \nu}(0) | p, \sigma \rangle = 2mE \left[ \mathcal{E}_0(t) \delta_{\sigma' \sigma} + \left( \frac{\sqrt{-t}}{m} \right) \mathcal{E}_1(t) \right],
$$

$$
\langle p', \sigma' | \bar{T}_{a}^{\mu \nu}(0) | p, \sigma \rangle = 2mE \left[ \mathcal{E}_0(t) \delta_{\sigma' \sigma} + \left( \frac{\sqrt{-t}}{m} \right) \mathcal{E}_1(t) \right],
$$

$$
\langle p', \sigma' | \bar{T}_{a}^{\mu \nu}(0) | p, \sigma \rangle = 2mE \left[ \mathcal{E}_0(t) \delta_{\sigma' \sigma} + \left( \frac{\sqrt{-t}}{m} \right) \mathcal{E}_1(t) \right],
$$

(6)

with

$$
\mathcal{E}_0(t) = F^a_{0,0}(t) + F^a_{3,0}(t)
$$

$$
+ \frac{t}{6m^2} \left[ - \frac{5}{2} F^a_{1,0}(t) - \frac{3}{2} F^a_{2,0}(t) + 4 F^a_{5,0}(t) + 3 F^a_{4,0}(t) - F^a_{3,1}(t) - F^a_{4,1}(t) \right],
$$

$$
+ \frac{t^2}{12m^4} \left[ - \frac{5}{2} F^a_{1,0}(t) + F^a_{1,1}(t) + \frac{1}{2} F^a_{2,0}(t) + 4 F^a_{5,0}(t) - F^a_{3,1}(t) - F^a_{4,1}(t) + \frac{1}{2} F^a_{3,1}(t) \right],
$$

$$
+ \frac{t^3}{48m^6} \left[ - \frac{5}{2} F^a_{1,0}(t) - \frac{1}{2} F^a_{2,1}(t) + F^a_{4,1}(t) \right],
$$

$$
\mathcal{E}_1(t) = - \frac{1}{6} \left[ F^a_{1,0}(t) - F^a_{1,1}(t) - 4 F^a_{5,0}(t) + F^a_{3,0}(t) + F^a_{3,1}(t) + F^a_{6,0} \right],
$$

$$
+ \frac{t}{12m^2} \left[ - \frac{5}{2} F^a_{1,0}(t) + F^a_{1,1}(t) + \frac{1}{2} F^a_{2,0}(t) + 4 F^a_{5,0}(t) - F^a_{4,0}(t) - F^a_{4,1}(t) + \frac{1}{2} F^a_{3,1}(t) \right],
$$

$$
+ \frac{t^2}{48m^4} \left[ - \frac{1}{2} F^a_{1,0}(t) - \frac{1}{2} F^a_{2,1}(t) + F^a_{4,1}(t) \right],
$$

(7)
\[ J^a_1(t) = \frac{1}{3} \left[ F^a_{4,0}(t) - F^a_{6,0}(t) \right] \]
\[ - \frac{t}{15m^2} \left[ F^a_{4,0}(t) + F^a_{4,1}(t) + 5F^a_{5,0}(t) \right] + \frac{t^2}{60m^4} F^a_{4,1}(t), \]
\[ J^a_2(t) = -\frac{1}{6} \left[ F^a_{4,0}(t) + F^a_{4,1}(t) \right] + \frac{t}{24m^2} F^a_{4,1}(t), \] (8)

\[ D_0^a(t) = F^a_{2,0}(t) - \frac{16}{3} F^a_{5,0}(t) \]
\[ - \frac{t}{6m^2} \left[ F^a_{2,0}(t) + F^a_{2,1}(t) - 4F^a_{5,0}(t) \right] + \frac{t^2}{24m^4} F^a_{2,1}(t), \]
\[ D_2^a(t) = \frac{4}{3} F^a_{5,0}(t), \]
\[ D_0^a(t) = \frac{1}{6} \left[ -F^a_{2,0}(t) - F^a_{2,1}(t) + 4F^a_{5,0}(t) \right] + \frac{t}{24m^2} F^a_{2,1}(t). \] (9)

One can refer to Appendix [A] in detail. The sum of the quark and gluon contributions to the GMFFs is also scale-invariant:

\[ \mathcal{E}_{0,2}(t) = \sum_a \mathcal{E}^a_{0,2}(t), \quad \mathcal{J}_{1,3}(t) = \sum_a \mathcal{J}^a_{1,3}(t), \quad D_{0,2,3}(t) = \sum_a D^a_{0,2,3}(t). \] (10)

The static EMT \( T^{\mu\nu}(r, \sigma', \sigma) \) is given by the Fourier transform of the matrix element of the EMT current in momentum space:

\[ T^{\mu\nu}(r, \sigma', \sigma) = \sum_a T^{\mu\nu}_a(r, \sigma', \sigma) = \sum_a \int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\Delta \cdot r} \langle p', \sigma' | \hat{T}^{\mu\nu}_a(0) | p, \sigma \rangle. \] (11)

**B. Energy density**

The temporal component of static EMT \( T^{00}(r, \sigma', \sigma) \) is related to the energy density. The multipole expansion of the energy density is defined by

\[ T^{00}(r, \sigma', \sigma) = \sum_a \int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\Delta \cdot r} \langle p', \sigma' | \hat{T}^{00}_a(0) | p, \sigma \rangle = \varepsilon_0(r) \delta_{\sigma' \sigma} + \varepsilon_2(r) \hat{Q}^{ij}_{\sigma' \sigma} Y^i_2(\Omega_r), \] (12)

where the monopole and quadrupole densities \( \varepsilon_{0,2}(r) \) are respectively given by

\[ \varepsilon_0(r) = m \tilde{\varepsilon}_0(r), \quad \varepsilon_2(r) = -\frac{1}{m} \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{\varepsilon}_2(r), \] (13)

with

\[ \tilde{\varepsilon}_{0,2}(r) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot r} \varepsilon_{0,2}(t). \] (14)

At the same time, the energy multipole form factors \( \mathcal{E}_{0,2}(t) \) can be expressed in terms of the energy densities \( \varepsilon_{0,2}(r) \) in coordinate space:

\[ \varepsilon_0(t) = \frac{1}{m} \int d^3r j_0(r\sqrt{-t})\varepsilon_0(r), \]
\[ \varepsilon_2(t) = m \int d^3r j_2(r\sqrt{-t}) \frac{d}{dr} \varepsilon_2(r). \] (15)
For a particle of arbitrary spin, the general tensor quantity is introduced in Ref. [32, 54] by

$$M_{n}^{k_1...k_n}(\sigma', \sigma) = \sum_{a} \int d^{3}r r^{n} T_{n}^{k_1...k_n}(r, \sigma', \sigma).$$  \hspace{1cm} (16)$$

The monopole moment corresponds to the mass of a hadron, accordingly one arrives at the apparent relation

$$M_{0}(\sigma', \sigma) = \sum_{a} \int d^{3}r Y_{0}(\Omega_{r}) T_{a}^{00}(r, \sigma', \sigma) = \int d^{3}r \varepsilon_{0}(r) \delta_{\sigma'\sigma} = m F_{1,0}(0) \delta_{\sigma'\sigma},$$  \hspace{1cm} (17)

which gives the normalization

$$F_{1,0}(0) = \sum_{a} F_{1,0}^{a}(0) = 1.$$  \hspace{1cm} (18)

The constraint $F_{1,0}(0) = 1$ for spin-3/2 hadron coincides with that for the spin-1/2 and spin-1 hadrons. The gravitational quadrupole density of a hadron presents how the energy density is deformed from the spherically symmetric shape. This quantity does not appear in the spherically symmetric hadrons. It can be quantitatively estimated as

$$Q_{\sigma'\sigma}^{ij} = M_{2}^{ij}(\sigma', \sigma) = \sum_{a} \int d^{3}r r^{2} T_{a}^{00}(r, \sigma', \sigma) = \frac{2}{15} \hat{Q}_{\sigma'\sigma}^{ij} \int d^{3}r r^{2} \varepsilon_{2}(r)$$

$$= -\frac{2}{m} \hat{Q}_{\sigma'\sigma}^{ij} E_{2}(0) = \frac{1}{3m} \left[ F_{1,0}(0) + F_{1,1}(t) - 4F_{5,0}(0) \right] \hat{Q}_{\sigma'\sigma}^{ij} = \frac{1}{3m} \left[ 1 + F_{1,1}(0) - 4F_{5,0}(0) \right] \hat{Q}_{\sigma'\sigma}^{ij}.  \hspace{1cm} (19)$$

Another interesting property is the mass radius of a hadron. It can be derived by the $r^{2}$-weighted energy density in the Breit frame. The expression of the mass radius is found to be

$$\langle r^{2} \rangle = \frac{\sum_{a} \int d^{3}r r^{2} T_{a}^{00}(r)}{\sum_{a} \int d^{3}r T_{a}^{00}(r)} = \frac{1}{m} \int d^{3}r r^{2} \varepsilon_{0}(r)$$

$$= 6E_{0}^{*}(0) = 6F_{1,0}(0) + \frac{1}{m^{2}} \left[ -\frac{5}{2} F_{1,0}(t) - F_{1,1}(t) - \frac{3}{2} F_{2,0}(t) + 4F_{5,0}(t) + 3F_{4,0}(t) \right]_{t=0}.  \hspace{1cm} (20)$$

### C. Angular momentum density

The spin density is given by

$$J(r, \sigma', \sigma) = \sum_{a} J_{a}^{i}(r, \sigma', \sigma) = \epsilon^{ijk} r^{j} \sum_{a} T_{a}^{0k}(r, \sigma', \sigma)$$

$$= 2 \hat{S}_{\sigma'\sigma}^{ij} \int \frac{d^{3}r}{(2\pi)^{3}} e^{-i\Delta r} \left[ \left( \mathcal{J}_{1}(t) + \frac{2}{3} t \frac{d \mathcal{J}_{1}(t)}{dt} \right) Y_{0} \delta^{ij} - t \frac{d \mathcal{J}_{1}(t)}{dt} Y_{2}^{ij} \right]$$

$$+ \frac{2}{m^{2}} \hat{O}_{\sigma'\sigma}^{i} \int \frac{d^{3}r}{(2\pi)^{3}} e^{-i\Delta r} \left[ \left( t^{2} \frac{d \mathcal{J}_{2}(t)}{dt} \right) Y_{4}^{i} \delta^{ij} - \left( 2t \mathcal{J}_{3}(t) + \frac{4}{3} t^{2} \frac{d \mathcal{J}_{3}(t)}{dt} \right) \delta^{ij} \right] Y_{2}^{mn}.  \hspace{1cm} (21)$$

The angular momentum density is obtained from the $0k$-components of the static EMT, which is decomposed into the 0-, 2- and 4-multipole components (see also Ref. [55, 56]). The sum of the angular momentum contributions from quark and gluons to the spin-3/2 baryon is obtained by

$$J_{\sigma'\sigma}^{i} = \sum_{a} \int d^{3}r J_{a}^{i}(r, \sigma', \sigma) = 2 \mathcal{J}_{1}(0) \hat{S}_{\sigma'\sigma}^{ij} = \frac{2}{3} F_{4,0}(0) \hat{S}_{\sigma'\sigma}^{ij},  \hspace{1cm} (22)$$

which yields the spin operator of the baryon with the constraint $F_{4,0}(0) = 3/2$.

Since our interest lies in the monopole angular momentum density in this work, we separately define it as

$$J_{\text{mono}}^{i}(r, \sigma', \sigma) = 2 \hat{S}_{\sigma'\sigma}^{ij} \int \frac{d^{3}r}{(2\pi)^{3}} e^{-i\Delta r} \left[ \left( \mathcal{J}_{1}(t) + \frac{2}{3} t \frac{d \mathcal{J}_{1}(t)}{dt} \right) Y_{0} \delta^{ij} \right],  \hspace{1cm} (23)$$
accordingly the averaged angular momentum density is given by

\[
\frac{1}{\text{Tr}[S^2]} \sum_{\sigma',\alpha,i} \dot{S}^{ij}_{\sigma',\alpha} J^{\text{mono}}_{\text{mono}}(r, \sigma', \sigma) = \rho_J(r)/S, \quad \text{i.e.,} \quad \rho_J(r) = -r \frac{d}{dr} \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i \Delta \cdot r} \mathcal{J}_1(t),
\] (24)

with spin \( S = 3/2 \). The angular momentum form factor can be expressed in terms of the averaged angular momentum density as

\[
\mathcal{J}_1(t) = \int d^3r \frac{j_1(r\sqrt{-t})}{r\sqrt{-t}} \rho_J(r).
\] (25)

### D. Pressure and shear force densities

The pressure and shear force densities are related to the \( ij \)-components of the static EMT. These densities are firstly defined in Ref. \[31\] \[54\] and newly parametrized in Ref. \[27\] \[53\] to conveniently express the strong forces in a hadron acting on the radial area element. Following Ref. \[27\] \[53\] the stress tensor can be expressed in terms of the pressure and shear forces densities by

\[
T^{ij}(r, \sigma', \sigma) = \sum_{\alpha} \int \frac{d^3 \Delta}{2E(2\pi)^3} e^{-i \Delta \cdot r} (p', \sigma'|\dot{T}^{ij}_{\alpha}(0)|p, \sigma)
\]

\[
= p_0(r)\delta^{ij}\delta_{\sigma'\sigma} + s_0(r)Y_2^{ij}\delta_{\sigma'\sigma} + \left( p_2(r) + \frac{1}{3}p_3(r) - \frac{1}{9}s_3(r) \right) \hat{Q}^{ij}_{\sigma'\sigma}
\]

\[
+ \left( s_2(r) - \frac{1}{2}p_3(r) + \frac{1}{6}s_3(r) \right) 2 \left[ \hat{Q}^{jp}_{\sigma'\sigma} Y_2^{pj} + \hat{Q}^{jp}_{\sigma'\sigma} Y_2^{pi} - \delta^{ij} \hat{Q}^{pq}_{\sigma'\sigma} Y_2^{pq} \right]
\]

\[
+ \hat{Q}^{pq}_{\sigma'\sigma} Y_2^{pj} \left[ \frac{2}{3}p_3(r) + \frac{1}{9}s_3(r) \right] \delta^{ij} + \left( \frac{1}{2}p_3(r) + \frac{5}{6}s_3(r) \right) Y_2^{ij}.
\] (26)

From the EMT conservation \( \partial_t T^{ij}(r, \sigma', \sigma) = 0 \), the following equilibrium relations between the pressure and shear force densities are derived:

\[
\frac{2}{3} \frac{ds_n(r)}{dr} + \frac{2}{3} \frac{s_n(r)}{r} + \frac{dp_n(r)}{dr} = 0, \quad \text{with} \quad n = 0, 2, 3.
\] (27)

This differential equation guarantees the stability condition. The functions \( p_0(r) \) and \( s_0(r) \) correspond to the pressure and shear force densities appearing in the spherically symmetric hadrons. The functions \( p_2(r) \) and \( p_3(r) \) are named the quadrupole pressure densities, and the \( s_2(r) \) and \( s_3(r) \) are called the quadrupole shear force densities according to Ref. \[27\]. These densities \( p_n(r) \) and \( s_n(r) \) are respectively written as

\[
p_n(r) = \frac{1}{6m} \dot{\rho}^2 \tilde{\mathcal{D}}_n(r) = \frac{1}{6m} \frac{1}{r^2} \frac{d}{dr} \left( \frac{d}{dr} \tilde{\mathcal{D}}_n(r) \right),
\]

\[
s_n(r) = -\frac{1}{4m} \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{\mathcal{D}}_n(r),
\] (28)

with

\[
\tilde{\mathcal{D}}_0(t) = \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i \Delta \cdot r} \mathcal{D}_0(t),
\]

\[
\tilde{\mathcal{D}}_2(t) = \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i \Delta \cdot r} \mathcal{D}_2(t) + \frac{1}{m^2} \left( \frac{d}{dr} \frac{d}{dr} - \frac{2}{r} \frac{d}{dr} \right) \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i \Delta \cdot r} \mathcal{D}_3(t),
\]

\[
\tilde{\mathcal{D}}_3(t) = \frac{2}{m^2} \left( \frac{d}{dr} \frac{d}{dr} - \frac{3}{r} \frac{d}{dr} \right) \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i \Delta \cdot r} \mathcal{D}_3(t).
\] (29)

\(^3\) a typo in Ref. \[53\] is corrected in Eq. (29).
Similarly, the form factors $D_{0,2,3}(t)$ can be expressed in terms of the pressure and shear force densities in coordinate space:

$$D_0(t) = 6m \int d^3r \frac{j_0(r \sqrt{-t})}{t} p_0(r),$$

$$D_2(t) = 2m \int d^3r \frac{j_2(r \sqrt{-t})}{t} \left( 2s_2(r) - \frac{1}{2} p_3(r) + \frac{2}{3} s_3(r) \right),$$

$$D_3(t) = 4m^3 \int d^3r \frac{j_4(r \sqrt{-t})}{t^2} \left( \frac{1}{2} p_3(r) + \frac{5}{6} s_3(r) \right).$$

(30)

The pressure densities $p_n(r)$ satisfying the relation given in Eq. (28) comply with the von Laue condition

$$\int d^3r p_n(r) = \frac{1}{6m} \int d^3r \partial^2 \hat{D}_n(r) = 0. \quad \text{with} \quad n = 0, 2, 3.$$  

(31)

Note that the dimensionless constants (generalized D-terms) are defined by [27]

$$D_n = \int d^3r \hat{D}_n(r) = m \int d^3r r^2 p_n(r) = -\frac{4}{15} m \int d^3r r^2 s_n(r), \quad \text{with} \quad n = 0, 2, 3.$$ 

(32)

The generalized D-terms $D_{0,2,3}$ introduced in Ref. [27] are related to the form factors $D_{0,2,3}(t)$ as follows:

$$D_0 = D_0(0), \quad D_2 = D_2(0) + \frac{2}{m^2} \int_{-\infty}^{0} dt D_3(t), \quad D_3 = -\frac{5}{m^2} \int_{-\infty}^{0} dt D_3(t).$$

(33)

Interestingly, the strong forces carried by constituents can be interpreted as a certain combination of pressure and shear force densities [1]. The spherical components of the strong forces ($dF_r, dF_\theta$ and $dF_\phi$) acting on the radial area element ($d\mathbf{S} = dS_r \mathbf{e}_r + dS_\theta \mathbf{e}_\theta + dS_\phi \mathbf{e}_\phi$) are expressed as follows [27]:

$$\frac{dF_r}{dS_r} = \delta_{\sigma' \sigma} \left( p_0(r) + \frac{2}{3} s_0(r) \right) + \hat{Q}_{\sigma' \sigma}^r p_2(r) + \frac{2}{3} s_2(r) + p_3(r) + \frac{2}{3} s_3(r),$$

$$\frac{dF_\theta}{dS_r} = \hat{Q}_{\sigma' \sigma}^\theta p_2(r) + \frac{2}{3} s_2(r), \quad \frac{dF_\phi}{dS_r} = \hat{Q}_{\sigma' \sigma}^\phi \left( p_2(r) + \frac{2}{3} s_2(r) \right).$$

(34)

Here, as defined in Ref. [2], the mechanical radius can be given by

$$\langle r_{\text{mech}}^2 \rangle = \frac{\int d^3r r^2 \left[ p_0(r) + \frac{2}{3} s_0(r) \right]}{\int d^3r \left[ p_0(r) + \frac{2}{3} s_0(r) \right]}.$$  

(35)

As for the unpolarized spin-3/2 hadron, since the normal force acting on the radial area element ($dF_r/dS_r$) is solely due to $p_0(r) + \frac{2}{3} s_0(r)$, it should comply with the local stability criterion given in Ref. [2] as

$$p_0(r) + \frac{2}{3} s_0(r) > 0.$$  

(36)

III. GRAVITATIONAL FORM FACTORS OF THE $\Delta$ IN THE SKYRME MODEL

The Skyrme Lagrangian density is given by

$$\mathcal{L} = \frac{F^2}{16} tr_F \left[ \partial_\mu U \partial^\mu U^\dagger \right] + \frac{1}{32e^2} tr_F \left[ (\partial_\mu U)U^\dagger, (\partial_\nu U)U^\dagger \right]^2 + \frac{m^2 F^2}{8} tr_F \left[ U - 1 \right].$$

(37)

where $U$ is the SU(2) chiral field, and $e$ stands for a dimensionless parameter and $tr_F$ is trace over flavors. The $F_\pi$ and the $m_\pi$ are the pion decay constant and the pion mass, respectively.

In the large-$N_c$ limit, we have the parameter scales $F_\pi = \mathcal{O}(N_c^{1/2})$, $e = \mathcal{O}(N_c^{-1/2})$, $m_\pi = \mathcal{O}(N_c^0)$, so as to have $\mathcal{L} = \mathcal{O}(N_c)$. In this limit, the chiral field is assumed to be a static one. Here, the static chiral field is written as $U(r) = \exp[i^{\tau^i}P(r)]$, with $i^{\tau^i} = \tau^i/|\tau|$ and the isospin Pauli matrices $\tau^i$, by adopting the hedgehog ansatz where $P(r)$ indicates a profile function with the boundary conditions $P(0) \to \pi$ and $P(\infty) \to 0$. 


The classical soliton mass is defined by $M_{\text{sol}} = -\int d^3r \mathcal{L}$, which is expressed with respect to the profile function and the model parameters by

$$M_{\text{sol}}[P] = 4\pi \int_0^\infty dr \, r^2 \left[ \frac{F_\pi^2}{8} \left( \frac{2\sin^2 P(r)}{r^2} + P'(r)^2 \right) + \frac{\sin^2 P(r)}{2e^2 r^2} \left( \frac{\sin^2 P(r)}{r^2} + 2P'(r)^2 \right) + \frac{m_\pi^2 F_\pi^2}{4} (1 - \cos P(r)) \right].$$

(38)

The profile function is obtained by minimizing the classical soliton mass and has an asymptotic behavior in the limiting case $r \rightarrow \infty$,

$$P(r) = \frac{2R_0^3}{r^2} (1 + m_\pi r) e^{-m_\pi r} \quad \text{(large } r),$$

(39)

where the constant $R_0$ is determined by the profile function derived by minimizing the classical soliton mass and is given in terms of the axial coupling constant in the chiral limit [41, 57].

In order to assign the quantum number to the soliton, the time-dependent chiral field should be considered as $U(r) \rightarrow A(t)U(r)A^\dagger(t)$, with $A(t) = a_0 + a \cdot \tau$. Here, we define the angular velocity $\Omega(t) = A^\dagger A (\dot{A} = dA/dt)$ with

$$\Omega^i = -\frac{i}{2} \text{tr}[\dot{A}^\dagger \tau^i].$$

(40)

Having quantized the collective coordinates ($\Omega^i \rightarrow \hat{J}^i/2I$), one obtains the collective Hamiltonian $H = M_{\text{sol}} + \frac{\hat{J}^2}{2I}$,

(41)

with the moment of inertia

$$I = \frac{2\pi}{3} \int dr \, r^2 \sin P(r) \left[ \frac{F_\pi^2}{8} + \frac{4P'(r)^2}{e^2 r^2} + \frac{4\sin^2 P(r)}{e^2 r^2} \right].$$

(42)

The collective Hamiltonian acts on the collective baryon wave functions given in Ref. [58]. In principle $F_\pi$ and $e$ are model parameters. However, the parameters are fixed to reproduce the following observables [41]

$$M_N + M_\Delta \equiv 2M_{\text{sol}} = 2171 \text{ MeV}, \quad M_\Delta - M_N \equiv \frac{3}{2I} = 293 \text{ MeV},$$

(43)

and the model parameters are consequently found to be

$$m_\pi = 138 \text{ MeV}, \quad F_\pi = 131.3 \text{ MeV}, \quad e = 4.628.$$

(44)

From the above parameters, the classical soliton mass and the moment of inertia are determined by

$$M_{\text{sol}} = 1085 \text{ MeV}, \quad I = 1.01 \text{ fm}.$$

(45)

In this work, we strictly follow the set of parameters used in Ref. [41] to keep consistency.

The canonical EMT in the Skyrme model can be derived by

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_a)} \partial^\nu \phi_a - g^{\mu\nu} \mathcal{L},$$

(46)

where $\phi_a$ is a time-dependent mesonic field with $U(t, r) = \phi_0 + i\tau \cdot \Phi$. The degree of freedom of the mesonic field is reduced to three ($a = 1, 2, 3$) by the constraint $\phi_0^2 + \Phi^2 = 1$. With the constraint, the canonical EMT in the Skyrme model is found to be symmetric. The respective components of EMT is expressed as

$$T^{00}(r, \sigma', \sigma) = \delta_\sigma'\sigma \left[ \frac{F_\pi^2}{8} \left( \frac{2\sin^2 P(r)}{r^2} + P'(r)^2 \right) + \frac{\sin^2 P(r)}{2e^2 r^2} \left( \frac{\sin^2 P(r)}{r^2} + 2P'(r)^2 \right) + \frac{m_\pi^2 F_\pi^2}{4} (1 - \cos P(r)) \right],$$

$$T^{ij}(r, \sigma', \sigma) = i\delta^{ij} \delta_\sigma'\sigma \left[ \frac{F_\pi^2}{8} \left( 2P'(r)^2 - \frac{2\sin^2 P(r)}{r^2} \right) + \frac{\sin^2 P(r)}{e^2 r^2} \left( P'(r)^2 - \frac{\sin^2 P(r)}{r^2} \right) \right]$$

$$+ \delta^{ij} \delta_\sigma'\sigma \left[ - \frac{F_\pi^2}{8} P'(r)^2 + \frac{\sin^2 P(r)}{2e^2 r^4} - \frac{m_\pi^2 F_\pi^2}{4} (1 - \cos P(r)) \right],$$

$$T^{0k}(r, \sigma', \sigma) = (\hat{J} \times \hat{n})^k_{\sigma'\sigma} \frac{\sin^2 P(r)}{4Ir} \left[ F_\pi^2 + \frac{4\sin^2 P(r)}{e^2 r^2} + \frac{4P'(r)^2}{e^2} \right].$$

(47)
The rotational corrections $O(1/N_c)$ to the EMT are obtained by
\[
\delta_{\text{rot}} T^{00}(r, \sigma', \sigma) = \left[ \mathbf{J}^2 - (\mathbf{J} \cdot \mathbf{\hat{r}})^2 \right]_{\sigma' \sigma} \frac{\sin^2 P(r)}{8I^2} \left[ F_\pi^2 + \frac{4P'(r)^2}{e^2} + \frac{4\sin^2 P(r)}{e^2} \right],
\]
\[
\delta_{\text{rot}} T^{ij}(r, \sigma', \sigma) = \hat{r}^i \hat{r}^j \left[ \mathbf{J}^2 - (\mathbf{J} \cdot \mathbf{\hat{r}})^2 \right]_{\sigma' \sigma} \sin^2 P(r) \left[ \frac{8P'(r)^2}{e^2} + \frac{8\sin^2 P(r)}{e^2} \right]
\]
\[
+ \delta^i j \left[ \mathbf{J}^2 - (\mathbf{J} \cdot \mathbf{\hat{r}})^2 \right]_{\sigma' \sigma} \sin^2 P(r) \left[ \frac{4P'(r)^2}{e^2} + \frac{4\sin^2 P(r)}{e^2} \right]
\]
\[
- \left[ 2\mathbf{J}^2 \hat{r}^i \hat{r}^j - \hat{r}^i \hat{r}^j \{ \mathbf{J}^2, \mathbf{\hat{r}}^2 \} - \hat{r}^i \hat{r}^j \{ \mathbf{J}^2, \mathbf{\hat{r}}^2 \} + \{ \mathbf{J}^2, \mathbf{\hat{r}}^2 \} \right]_{\sigma' \sigma} \sin^4 P(r) \left[ \frac{2}{2I^2e^2} \right].
\] (48)

The rotational corrections of the 0k-components of the EMT is found to be null in the current Skyrme Lagrangian. The corrections appear in the higher-order derivative terms which generate $\Omega^3 \sim O(1/N_c^3)$. However, the corrections are strongly suppressed in the large-$N_c$ expansion. Thus, we can safely neglect the rotational corrections to the 0k-component of the EMT.

As given in Eqs. (12), (24) and (26), the multipole densities can be extracted from the EMT in Eq. (47):
\[
\varepsilon_0(r) = \left[ \frac{F_\pi^2}{8} \left( \frac{2\sin^2 P(r)}{r^2} + P'(r)^2 \right) + \frac{\sin^2 P(r)}{2e^2} \left( \sin^2 P(r) + 3P(r)^2 \right) + \frac{m_\pi^2 F_\pi^2}{4} (1 - \cos P(r)) \right],
\]
\[
\rho_j(r) = \frac{\sin^2 P(r)}{4I} \left[ F_\pi^2 + \frac{4\sin^2 P(r)}{e^2} \right],
\]
\[
p_0(r) = -\frac{F_\pi^2}{24} \left( \frac{2\sin^2 P(r)}{r^2} + P'(r)^2 \right) + \frac{\sin^2 P(r)}{6e^2} \left( \sin^2 P(r) + 3P(r)^2 \right) - \frac{m_\pi^2 F_\pi^2}{4} (1 - \cos P(r)),
\]
\[
s_0(r) = \left( \frac{F_\pi^2}{4} + \frac{\sin^2 P(r)}{e^2} \right) \left( P'(r)^2 - \frac{\sin^2 P(r)}{r^2} \right).
\] (49)

In the same manner, the rotational corrections are derived from Eq. (48):
\[
\delta_{\text{rot}} \varepsilon_0^{(J)}(r) = J(J + 1) \frac{\sin^2 P(r)}{12I^2} \left[ F_\pi^2 + \frac{4P'(r)^2}{e^2} + \frac{4\sin^2 P(r)}{e^2} \right],
\]
\[
\delta_{\text{rot}} P_j^{(J)}(r) = O(1/N_c),
\]
\[
\delta_{\text{rot}} P_0^{(J)}(r) = J(J + 1) \frac{\sin^2 P(r)}{12I^2} \left[ F_\pi^2 + \frac{4P'(r)^2}{3e^2} + \frac{4\sin^2 P(r)}{3e^2} \right],
\]
\[
\delta_{\text{rot}} S_0^{(J)}(r) = J(J + 1) \frac{\sin^2 P(r)}{12I^2} \left[ - \frac{8P'(r)^2}{e^2} + \frac{4\sin^2 P(r)}{e^2} \right].
\] (50)

Interestingly, in the chiral soliton picture the rotational corrections of the densities in Eq. [50] are related to the quadrupole densities, which was firstly found in Ref. [27]. The quadrupole energy density $\varepsilon_2(r)$ is found to be
\[
\delta_{\text{rot}} \varepsilon_2^{(J)}(r) = -\frac{2}{3} J(J + 1) \varepsilon_2(r).
\] (51)

In other words, the quadrupole energy density $\varepsilon_2(r)$ is related to the energy densities of the nucleon and the $\Delta$
\[
\varepsilon_2(r) = -\frac{1}{2} \left[ \varepsilon_0^{\Delta}(r) - \varepsilon_0^N(r) \right],
\] (52)
with
\[
\varepsilon_0^{\Delta N}(r) \equiv \left[ \varepsilon_0(r) + \delta_{\text{rot}} \varepsilon_0^{(\frac{1}{2}, \frac{3}{2})}(r) \right].
\] (53)

Also, the usual sizes of the both densities $\delta_{\text{rot}} \varepsilon_0^{(\frac{1}{2}, \frac{3}{2})}(r)$ and $\varepsilon_2(r)$ are estimated by
\[
\int d^3r [\varepsilon_0^{\Delta}(r) - \varepsilon_0^N(r)] = \int d^3r [\delta_{\text{rot}} \varepsilon_0^{(\frac{1}{2}, \frac{3}{2})}(r) + \delta_{\text{rot}} \varepsilon_0^{(\frac{1}{2}, \frac{3}{2})}(r)] = -2 \int d^3r \varepsilon_2(r) = \frac{3}{2I} \sim O(1/N_c).
\] (54)
As for the quadrupole pressure and shear force densities, the remarkable general relation in a chiral soliton picture is derived in Ref. [27]:

\[ p_2(r) + \frac{2}{3} s_2(r) = 0. \]  

(55)

By comparing Eq. (26) with Eq. (48), we reproduce this relation in the Skyrme model. The above relation, together with the EMT conservation [27], implies the nullification of the quadrupole densities \( s_2(r) \) and \( p_2(r) \), and the similar relations to Eq. (51) for the pressure and shear force densities are obtained [27]:

\[ p_2(r) = s_2(r) = 0, \quad \delta_{\text{rot}} p_0^J(r) = -\frac{2}{3} J(J + 1)p_3(r), \quad \delta_{\text{rot}} s_0^J(r) = -\frac{2}{3} J(J + 1)s_3(r). \]  

(56)

Therefore, we arrive at the similar expressions as Eq. (52)

\[ p_3(r) = -\frac{1}{2} \left[ p_0^A(r) - p_0^N(r) \right], \quad s_3(r) = -\frac{1}{2} \left[ s_0^A(r) - s_0^N(r) \right], \]  

(57)

with

\[ p_0^{N,\Delta}(r) = \left[ p_0(r) + \delta_{\text{rot}} p_0^{(\frac{1}{2}, \frac{3}{2})}(r) \right], \quad s_0^{N,\Delta}(r) = \left[ s_0(r) + \delta_{\text{rot}} s_0^{(\frac{1}{2}, \frac{3}{2})}(r) \right]. \]  

(58)

Here, one has to bear in mind that due to the EMT densities with the induced rotational corrections the pressure \( p_3(r) \) and the shear force \( s_3(r) \) densities do not uniquely exist and should be reasonably determined. Let us first recall that the leading-order (LO) result of \( p_0(r) \) satisfies the von Laue condition which is equivalent to satisfying the equation of motion [11]. However, once one considers the next-leading-order (NLO) result of \( p_0(r) \), it breaks the von Laue condition. Thus, one might introduce the “variation after projection” method—minimizing the baryon mass after projecting the soliton on the quantum number—as a prescription. However, this method also has a drawback: the chiral symmetry in the large-\( r \) region is not satisfied [27]. To preserve the chiral symmetry and satisfy the von Laue condition, we first adopt the “projection after variation” method—projecting the soliton on the quantum number after minimizing the soliton mass—and treat the NLO result as a small perturbation. Of course, the von Laue condition is broken. Thus, instead of directly using the model result of \( p_0(r) \), we solve the differential equation given in Eq. (27) with the shear force density \( s_0(r) \) and reconstruct the pressure \( p_0(r) |_{\text{reconst}} \), which then automatically complies with the stability condition [27]. We also derive the reconstructed quadrupole pressure \( p_3(r) |_{\text{reconst}} \) in the same manner.

Before discussing GFFs, it is important to discuss the large distance properties of the EMT densities. The large distance behaviors of the EMT densities for the spherically symmetric baryon were investigated and presented within the Skyrme model in Ref. [11, 57]. For completeness, we present the large distance properties of the quadrupole densities in the chiral limit as

\[ \varepsilon_2(r) \sim -\frac{F_\pi^2 R_0^4}{2T^2 r^4} \cdots, \quad p_3(r) = -\frac{F_\pi^2 R_0^4}{2T^2 r^4} \cdots, \quad s_3(r) = \frac{56}{15} \frac{R_0^{10}}{T^2 c^2} \cdots, \quad p_3(r) |_{\text{reconst}} = \frac{392}{15} \frac{1}{T^2 c^2} \frac{R_0^{10}}{r^{16}} \cdots. \]  

(59)

The \( \cdots \) indicates the contributions strongly suppressed in the large-\( r \) region. Interestingly, the quadrupole densities \( \varepsilon_2(r) \) and \( p_3(r) \) are weakly suppressed in the large distance, which have the analogous behavior with the angular momentum density \( p_3 \approx \frac{1}{r} \). Keep in mind that in order to respect the chiral physics and the stability condition, we discard the result of \( p_3(r) \) and adopt the newly reconstructed \( p_3(r) |_{\text{reconst}} \) by solving the differential equation (27). As a result, the discrepancy between \( p_3(r) |_{\text{reconst}} \) and \( p_3(r) \) arises from the fulfillment of both the chiral physics and the stability condition and is inevitable. For finite pion mass, the densities are exponentially suppressed.

The large-\( N_c \) expansion is valid in the region \( |t| \ll M_A^2 \). In this limit, we have following large-\( N_c \) behaviors

\[ M_{N,\Delta} \sim \mathcal{O}(N_c^0), \quad I \sim \mathcal{O}(1/N_c), \quad t \sim \mathcal{O}(N_c^0), \]  

(60)

and the scales of the GMFFs are found to be

\[ E_0(t) \sim \mathcal{O}(N_c^0), \quad E_2(t) \sim \mathcal{O}(N_c^0), \quad J_0(t) \sim \mathcal{O}(N_c^0), \quad J_3(t) \sim \mathcal{O}(N_c^0), \]  

\[ D_0(t) \sim \mathcal{O}(N_c^2), \quad D_2(t) \sim \mathcal{O}(N_c^2), \quad D_3(t) \sim \mathcal{O}(N_c^2), \]  

(61)

or according to Ref. [27] the generalized \( D \)-terms have the scales as

\[ D_0 \sim \mathcal{O}(N_c^2), \quad D_2 \sim \mathcal{O}(N_c^0), \quad D_3 \sim \mathcal{O}(N_c^0). \]  

(62)

It is found that the GMFFs have the order of \( \sim N_c^0 \) except for the form factors \( D_0(t) \) and \( D_3(t) \), which have the order of \( \sim N_c^2 \), whereas the generalized \( D \)-terms \( D_0, D_2 \) and \( D_3 \) have the orders of \( \sim N_c^2, \sim N_c^0 \) and \( \sim N_c^0 \), respectively.
IV. NUMERICAL RESULTS

In this section, we discuss the numerical results. We first examine the monopole and quadrupole energy densities appearing in the 00-component of the static EMT. The LO monopole energy densities of the nucleon and the $\Delta$ are degenerate. To lift the degeneracy, we take into account the rotational corrections $\Omega^J \sim O(1/N^2)$. Thus, the integrations of the NLO monopole energy densities over space yield masses of the nucleon ($M_N = 1159$ MeV) and the $\Delta$ ($M_{\Delta} = 1452$ MeV) projected on quantum numbers $S = I = \frac{1}{2}, \frac{3}{2}$, respectively, and they satisfy the constraint given in Eq. \[18.\]

$$
\frac{1}{M_{\text{sol}}} \int d^3 r \varepsilon_0(r) \bigg|_{LO} = \frac{1}{M_{N,\Delta}} \int d^3 r \varepsilon_0^{N, \Delta}(r) \bigg|_{NLO,S=\frac{1}{2}, \frac{3}{2}} = F_{1,0}(0) = 1.
$$

(63)

![Graph showing energy densities of classical soliton, nucleon, and $\Delta$.](image)

FIG. 1. The left panel presents the monopole energy densities of the classical soliton (LO), the nucleon (NLO) and the $\Delta$ (NLO) as a function of radius $r$. The right panel depicts the quadrupole energy density of the $\Delta$ as a function of radius $r$. The dashed, dotted and solid curves draw the classical soliton, the nucleon and the $\Delta$.

In the left panel of Fig. 1 the monopole energy densities of the classical soliton (LO), the nucleon (NLO) and the $\Delta$ (NLO) as a function of radius $r$ are presented. The center of monopole energy density is found to be $\varepsilon_0(0) = \varepsilon_0^{N, \Delta}(0) = 2.27$ GeV $\cdot$ fm$^{-3}$. The monopole density of the $\Delta$ is slightly broader than that of the nucleon and the classical soliton. As expected, the rotational corrections to the monopole energy density are suppressed by $O(1/N^2)$. Those effects on the monopole energy densities can be quantitatively observed by calculating the mass radii given in Eq. \[20.\]

$$
\langle r^2 \rangle_\varepsilon = 0.54 \text{ fm}^2 \text{ (LO)}, \quad \langle r^2 \rangle_\varepsilon = 0.57 \text{ fm}^2 \text{ (NLO, Nucleon)}, \quad \langle r^2 \rangle_\varepsilon = 0.64 \text{ fm}^2 \text{ (NLO, } \Delta). 
$$

(64)

The rotational corrections make the monopole energy density broader as the spin quantum number increases. In the chiral limit, the NLO mass radii for the nucleon and the $\Delta$ diverge, i.e., $\langle r^2 \rangle_\varepsilon \propto \int_0^\infty dr \varepsilon_0^{N, \Delta}(r)$, because of $\delta_{\text{rot}} \varepsilon_0^{(J)}(r) \propto \frac{1}{r}$, given in Eq. \[56.\] and Eq. \[59.\]. In the right panel of Fig. 1 the quadrupole energy density of the $\Delta$ is drawn as a function of radius $r$. It has a peak around 0.4 fm with a negative sign and its strength is marginal compared with the monopole energy density. With Eq. \[54.\], one can obtain the numerical result as

$$
\int d^3 r \varepsilon_2(r) = -\frac{1}{2} \int d^3 r [\varepsilon_0^{\Delta}(r) - \varepsilon_0^N(r)] = -147 \text{ MeV}. 
$$

(65)

The value of the integration of the quadrupole energy density over $r$ is approximately $10\% \sim O(1/N^2)$ of that of the monopole energy density. This result exactly complies with the relation obtained in Ref. \[27.\]. Another interesting property is the mass quadrupole moment given in Eq. \[19.\]. Its value exhibits how the energy density is deformed from the spherically symmetric shape and is found to be

$$
Q_{ij}^s = \frac{2}{15} Q_{ij}^s \int d^3 r r^2 \varepsilon_2(r) = -0.0181 Q_{ij}^s \text{ GeV} \cdot \text{fm}^2.
$$

(66)
In the chiral limit, the mass quadrupole moment diverges for the same reason of the NLO mass radius, i.e., $Q_{\sigma/\sigma}^{ij} \propto \int_0^\infty dr r^4 \varepsilon_\omega(r)$ with $\delta_{\text{rot}}(J)(r) \propto \frac{1}{r}$.

Fig. 2 shows the averaged angular momentum density $\rho_J(r)$ normalized by the corresponding particle spin $S$. As shown in Eq. (21) and Eq. (24), $\rho_J(r)$ is related to the $0k$-components of the static EMT. In QCD, the total angular momentum of a hadron comes from the orbital angular momentum and the spin of constituents, i.e., $\int d^3r \rho_J(r) = S$.

For the mean square radius $\langle r_J^2 \rangle$, one gets the following degenerate result for the nucleon and the $\Delta$,

$$\langle r_J^2 \rangle_{N,\Delta} = \frac{\int d^3r r^2 \rho_J(r)}{\int d^3r \rho_J(r)} = 0.92 \text{ fm}^2.$$  \hfill (67)

Since the absence of the higher-order corrections $O(1/N_c^2)$ given in Eq. (50), the averaged angular momentum density of the $\Delta$ merely differs from that of the nucleon by factor three, and the mean square radius of the $\Delta$ is the same as that of the nucleon. As mentioned in the previous section, the higher-order corrections of the averaged angular momentum arise from the higher derivative terms generating $\Omega^3 \sim O(1/N_c^3)$ in the Skyrme Lagrangian. In principle, the corresponding expressions can be derived from the effective chiral Lagrangian incorporating the infinite order of derivatives terms [59]. However, we neglect the corrections in this work.

FIG. 2. The angular momentum densities of the nucleon and the $\Delta$ as a function of $r$ normalized by its spin $S$.

FIG. 3. The left (right) panel depicts the pressures (shear forces) of the classical soliton, the nucleon, and the $\Delta$ as a function of radius $r$. The dashed, dotted and solid curves draw the classical soliton, the nucleon, and the $\Delta$. Note that in the case of NLO results the pressures are reconstructed from Eq. (27) by using the approximated shear forces to comply with the von Laue condition.

\footnote{Note that in the chiral limit the averaged angular momentum density $\rho_J(r)$ decrease as $r^{-4}$ in large distance, so the radius diverges.}
The left panel of Fig. 3 shows the pressures of the classical soliton (LO), the nucleon (NLO), and the Δ (NLO) as a function of radius \( r \). The LO pressure naturally complies with the von Laue condition that is equivalent to taking the variation of the soliton mass. However, as discussed in the previous section the NLO pressures do not satisfy the stability condition. Thus, we adopt a strategy, preserving the chiral symmetry and satisfying the stability condition, that we first take the variation of the soliton mass, and then project the soliton on the quantum numbers. By treating the rotational corrections as a small perturbation, we afterward obtain the approximated shear force density \( s_0(r) \). With \( s_0(r) \), we reconstruct the pressure \( p_0(r) \) from Eq. (27) so that the pressure meets the stability condition. Thus, the LO and NLO pressures in Fig. 3 satisfy the stability condition. In the meantime, the satisfaction of the stability condition implies that the pressure has at least one nodal point \( (r_0) \) where the pressure vanishes. The nodal points of the pressures are located at

\[
 r_0 = 0.64 \text{ fm (LO)}, \quad r_0 = 0.65 \text{ fm (NLO, Nucleon)}, \quad r_0 = 0.83 \text{ fm (NLO, Δ)}.
\]  

(68)

The rotational corrections make the pressure density weaker and spreading more widely as the spin quantum number increases. In the right panel of Fig. 3 approximated shear forces are presented.

![FIG. 4. The left (right) panel depicts the quadrupole pressures (shear forces) of the Δ as a function of radius \( r \). The quadrupole pressure density \( p_3(r) \) is reconstructed from Eq. (27) by using the approximated quadrupole shear force density \( s_3(r) \) in order to comply with the von Laue condition.](image)

Fig. 3 shows the quadrupole pressure (shear force) of the Δ. The quadrupole pressure \( p_3(r) \) is reconstructed as if the NLO pressures \( p_{0,Δ}^N(r) \) were derived from Eq. (27). Thus, \( p_3(r) \) complies with the stability condition given in Eq. (31), and has a nodal point located at \( r_0 = 0.56 \text{ fm} \). The shapes of \( p_3(r) \) and \( s_3(r) \) show a similar tendency with those of \( p_0(r) \) and \( s_0(r) \), respectively.

The left panel of Fig. 4 shows normal force components acting on the radial area element for the classical soliton, the nucleon and the unpolarized Δ. When it comes to a spherically symmetric baryon, the normal force can be directly as if the NLO pressures \( p_3(r) \) and \( s_3(r) \) show a similar tendency with those of \( p_0(r) \) and \( s_0(r) \), respectively.

The left panel of Fig. 4 shows normal force components acting on the radial area element for the classical soliton, the nucleon and the unpolarized Δ. When it comes to a spherically symmetric baryon, the normal force can be directly related to a local stability criterion (36), which signifies the positivity of the normal force. In other words, the normal force should be directed outwards. As for the polarized Δ, it has the quadrupole contributions to the normal force as shown in Eq. (34). Except for the nullified densities \( s_2(r) \) and \( p_2(r) \), the quadrupole contribution to the normal force is shown in the right panel of Fig. 4. For the large distance in the chiral limit, the unpolarized normal force \( p_{0,Δ}^N \) keep the positivity which means that the local stability condition is satisfied, and also the quadrupole normal force \( p_3(r) \) has the positive sign within the Skyrme model

\[
 0 < p_{0, Δ}^N (r) \bigg|_{\text{reconst}} + \frac{2}{3} s_0^N, Δ (r) = \frac{R_1^3}{R_6} \cdots, \quad 0 < p_3 (r) \bigg|_{\text{reconst}} + \frac{2}{3} s_3 (r) = \frac{56}{15} \frac{1}{R_6^2} \frac{R_1^3}{R_6} \cdots .
\]  

(69)

However, for the polarized Δ we do not know how the quadrupole densities are related to the local stability conditions as of now. Another interesting quantity is the mechanical radius. As defined in Ref. 2, the mechanical radius is obtained by

\[
 〈r_0^2\rangle_{\text{mech}} = 0.61 \text{ fm}^2 (\text{LO}), \quad 〈r_0^2\rangle_{\text{mech}} = 0.63 \text{ fm}^2 (\text{NLO, Nucleon}), \quad 〈r_0^2\rangle_{\text{mech}} = 0.85 \text{ fm}^2 (\text{NLO, Δ}).
\]  

(70)

Similar to Eqs. (64) and (68), the rotational corrections make the mechanical radius getting larger as the spin quantum number increases. Futhermore, we boldly defined the quadrupole mechanical radius in Eq. (65), and it is found to be

\[
 〈r_3^2\rangle_{\text{mech}} = 0.33 \text{ fm}^2.
\]  

(71)
Finally, we discuss the GFFs and GMFFs. As shown in Sec. II, the GMFFs are expressed in terms of the GFFs. In the meanwhile, the GMFFs can also be expressed in terms of the EMT densities as shown in Eq. (15), Eq. (25) and Eq. (30). The results of the Δ GMFFs and GFFs are shown in Fig. 6 and Fig. 7. In the Skyrme model, the energy the meanwhile, the GMFFs can also be expressed in terms of the EMT densities as shown in Eq. (15), Eq. (25) and Eq. (30). The results of the Δ GMFFs and GFFs are shown in Fig. 6 and Fig. 7. In the Skyrme model, the energy quadrupole form factor $J_3(t)$ and angular momentum form factor $J_3(t)$ form factors satisfy the constraints $E_0(0) = F_{1,0}(0) = 1$ and $J_0(0) = \frac{1}{3} F_{4,0}(0) = \frac{1}{2}$. Besides the octupole angular momentum form factor $J_3(t)$ is assume to be zero, i.e., $J_3(0) = -\frac{1}{6} [F_{4,0}(0) + F_{4,1}(0)] = 0$, because the corresponding density is suppressed by $\Omega^3 \sim O(1/N_c^3)$ in the large-$N_c$ expansion. As a result, we get the relation $F_{4,1}(0) = -F_{4,0}(0) = -3/2$. There is no additional constraint on $E_2(t), D_0(t), D_2(t)$ and $D_3(t)$. Therefore, we determine the moments of the GMFFs from Eq. (15), Eq. (25) and Eq. (30) as

$$E_2(0) = -\frac{M_\Delta}{15} \int d^3 r r^2 \varepsilon_2(r) = -\frac{1}{6} [F_{1,0}(0) + F_{1,1}(0) - 4 F_{5,0}(0)] = 0.34,$$

$$D_0(0) = M_\Delta \int d^3 r r^2 p_0^\Delta(r) = -\frac{4}{15} M_\Delta \int d^3 r r^2 s_0^\Delta(r) = F_{2,0}(0) - \frac{16}{3} F_{5,0}(0) = -3.53 < 0,$$

$$D_2(0) = \frac{2}{3} M_\Delta \int d^3 r r^2 p_2(r) = -\frac{8}{75} M_\Delta \int d^3 r r^2 s_2(r) = \frac{4}{3} F_{5,0}(0) = -0.20,$$

$$D_3(0) = -\frac{1}{140} M_\Delta^3 \int d^3 r r^4 p_3(r) = \frac{2}{735} M_\Delta^3 \int d^3 r r^4 s_3(r) = \frac{1}{6} [F_{2,0}(0) - F_{2,1}(0) + 4 F_{5,0}(0)] = 0.24. \quad (72)$$

The energy quadrupole form factor $E_2(0)$ is obtained as 0.34, and it can be related to the mass quadrupole moment given in Eq. (10). The value of the $D$-term $D_0(0)$ of the $\Delta$ is found to be $-3.53$. The quadrupole form factors $D_2(0)$ and $D_3(0)$ turn out to be $-0.20$ and $0.24$, respectively. As introduced in Eq. (32), the generalized $D$-terms are determined as

$$D_0^\Delta = -3.53, \quad D_0^N = -3.63, \quad D_2 = 0, \quad D_3 = -0.50. \quad (73)$$

The generalized $D$-terms of the present work are comparable with that of the Ref. [27, 57]. We restrict ourselves in the range of $0 < (-t) < 1$ GeV$^2$ because of the validity of the large-$N_c$ expansion, i.e., $|t| \ll M_\Delta^2$. We find that the quadrupole form factors $E_2(t)$ and $D_2(t)$ are relatively small in comparison with the monopole form factors $E_0(t)$ and $D_0(t)$. It seems that there are conflicts with the large-$N_c$ behavior. For instance, the sizes of the numerical values for $D_3(0)$ and $D_0(0)$ with the same order of $\sim N_c^2$ are not comparable. However, in nature, the number of color $N_c$ is just three, and these conflicts arise from the different orders of the multipole structures. The fact indicates that the $N_c$ counting is not valid between the different orders of the multipole form factors.

5 By assuming that the pressure $p_n(r)$ and shear force $s_n(r)$ densities vanish at large-$r$ faster than any power of $r$, one can arrive at the general relation, arising from the differential equation [27], between them as follows $\int_0^\infty dr r^N s_n(r) = -\frac{2}{3(N+1)} \int_0^\infty dr r^N p_n(r)$ for $N > -1$.

6 The discrepancy of $D$-term between this work and Ref. [27, 57] arises because of the different masses. They have used the LO mass, whereas we take the NLO masses to keep a consistency in this work.
V. SUMMARY AND CONCLUSIONS

In the present work, we aim at investigating the formalism for the GMFFs of the spin-3/2 baryon and verifying the general requirements of the GFFs based on the Skyrme model. The possible structure of the EMT current was sorted out, and the GFFs were defined in Ref. [26]. In addition, the multipole structure of the stress tensor for the spin-3/2 baryon was investigated, and the general relations between GFFs in the large-\( N_c \) limit were derived in Ref. [27]. In this work, the GFFs defined in Ref. [26] are classified into the GMFFs, and also the energy, spin, pressure and shear force densities are given in terms of the multipole expansion.

As derived in Ref. [27, 57], we have shown that all general requirements of the EMT are satisfied up to the leading order of \( N_c \) expansion. However, in order to go beyond the leading order of \( N_c \) the stability condition should be broken. To cure this problem, we first quantize the soliton after minimizing its mass while the rotational corrections are treated.
FIG. 7. Gravitational form factors of the $\Delta$ as a function of squared momentum transfer $t$. 
as a small perturbation. With the approximated shear force densities with the induced rotational corrections, we then reconstruct the pressure densities. Utilizing this strategy, we can satisfy all the requirements.

We obtain the large-$N_c$ behavior of the GMFFs, and it is found that the GMFFs have the order of $\sim N_c^0$ except for the form factors $D_0(t)$ and $D_2(t)$, which have the order of $\sim N_c^2$. Since the large-$N_c$ approach is valid in the region $|t| \ll M^2_{\Delta}$, we predict the $t$ dependence of the GMFFs in the region of $0 < (-t) < 1$ GeV$^2$. The moments of the GMFFs are evaluated—the mass quadrupole moment, the generalized $D$-terms, the mass radius, etc. We find that the quadrupole form factors $E_2(t)$, $D_2(t)$ and $D_3(t)$ are relatively small in comparison with the monopole form factors $E_0(t)$ and $D_0(t)$.

As suggested in Ref. [27] the lattice measurements of $\Delta$ GFFs can be used to check whether $\Delta$-baryon is rotating soliton. Here, we provided the first numerical estimates of corresponding GFFs in the soliton picture using the Skyrme model. Thus, we expect that the results of the lattice QCD or the theoretical model will soon come out.

ACKNOWLEDGMENTS

The authors want to express M. V. Polyakov, H.-Ch. Kim and R.-H. Fang for valuable discussions. J.-Y. Kim is supported by DAAD doctoral scholarship. B.-D. Sun is supported by the National Natural Sciences Foundations of China under the grant No.11947228 and China Postdoctoral Science Foundation under Grant No. 2019M662316. This work is also supported by the BMBF (Grant No. 05P18PCFP1).

Appendix A: Breit frame formulae

In the Breit frame the initial momentum $p$ and final momentum $p'$ have the relations $P^\mu = (p^\mu + p'^\mu)/2 = (E, 0, 0, 0)$ and $\Delta^\mu = p'^\mu - p^\mu = (0, \Delta)$. The momentum squared is defined as $\Delta^2 = -\Delta^2 = t = 4(m^2 - E^2)$. Definition of the Rarita-Schwinger spinor is given by

$$u^\mu = \sum C^2_{\lambda 4, 5} s(p) c^\mu_\lambda, \quad \text{with} \quad u_s(p) = \sqrt{m + E} \left( \sigma_{\lambda s} p + \phi_s \right),$$

(A1)

with the two component Dirac spinor $\phi_s$. The spin-1 vector $c^\mu_\lambda$ is defined by

$$c^\lambda_\mu(p') = \left( -\frac{\Delta \cdot \hat{e}_\lambda}{2m}, \hat{e}_\lambda + \frac{\Delta \cdot \hat{e}_\lambda}{4m(m + E)} \right), \quad c^\mu_\lambda(p') = \left( \frac{\Delta \cdot \hat{e}_\mu}{2m}, \hat{e}_\lambda + \frac{\Delta \cdot \hat{e}_\lambda}{4m(m + E)} \right),$$

(A2)

with

$$\hat{e}_+ = \sqrt{\frac{1}{2}} (-1, -i, 0), \quad \hat{e}_0 = \sqrt{\frac{1}{2}} (0, 0, 1), \quad \hat{e}_{-1} = \sqrt{\frac{1}{2}} (1, -i, 0).$$

(A3)

There are useful relations for Dirac field:

$$\pi_{\lambda s'}(p') \gamma_0 u_s(p) = 2m \delta_{\lambda s'},$$
$$\pi_{\lambda s'}(p') u_s(p) = 2E \delta_{\lambda s'},$$

$$\pi_{\lambda s'}(p') \frac{\gamma_0 \sigma_{00} p \Delta_\rho + p^0 \sigma_{00} p \Delta_\rho}{2} u_s(p) = \Delta^2 E \delta_{\lambda s'},$$

$$\pi_{\lambda s'}(p') \frac{\gamma_0 \sigma_{00} p \Delta_\rho + p^0 \sigma_{00} p \Delta_\rho}{2} u_s(p) = 2imE \hat{e}^j \hat{e}^k \delta_{\lambda s'} \Delta^k,$$

$$\pi_{\lambda s'}(p') \frac{\gamma_0 \sigma_{00} p \Delta_\rho + p^0 \sigma_{00} p \Delta_\rho}{2} u_s(p) = 0. \quad (A4)$$
For vector field:

\[(\hat{\epsilon}_\lambda^*_i \cdot \Delta)(\hat{\epsilon}_\lambda^* \cdot \Delta) = -\frac{t}{3} \delta_{\lambda \lambda} - \hat{Q}^{(1)kl}_{\lambda \lambda} \Delta^k \Delta^l,\]

\[\epsilon_{\lambda^*} \cdot \epsilon_\lambda = \left( \frac{t}{6m^2} - 1 \right) \delta_{\lambda \lambda} + \frac{1}{2m^2} \hat{Q}^{(1)kl}_{\lambda \lambda} \Delta^k \Delta^l,\]

\[\epsilon_{\lambda^*}^0 \epsilon_\lambda = \frac{t}{12m^2} \delta_{\lambda \lambda} + \frac{1}{4m^2} \hat{Q}^{(1)kl}_{\lambda \lambda} \Delta^k \Delta^l,\]

\[\epsilon_\lambda \cdot \Delta = -\frac{E}{m} \hat{\epsilon}_\lambda^* \cdot \Delta,\]

\[\epsilon_\lambda \cdot \Delta = -\frac{E}{m} \hat{\epsilon}_\lambda^* \cdot \Delta,\]

\[(\epsilon_\lambda^* \cdot \Delta)(\epsilon_\lambda \cdot \Delta) = -\frac{E^2}{m^2} \left( \frac{t}{3} \delta_{\lambda \lambda} + \hat{Q}^{(1)kl}_{\lambda \lambda} \Delta^k \Delta^l \right),\]

\[c_{\lambda \lambda}^{(0)}(\epsilon_\lambda \cdot \Delta) + c_{\lambda \lambda}^{(1)}(\epsilon_\lambda \cdot \Delta) = 0,\]

\[\epsilon_{\lambda^*}^0 \epsilon_\lambda + \epsilon_{\lambda^*}^0 \epsilon_\lambda = \frac{i}{2m} \epsilon^{kl}_{\lambda \lambda} \Delta^k \Delta^l,\]

\[\epsilon_{\lambda^*}^{ij} \Delta^i(\epsilon_\lambda \cdot \Delta) + \epsilon_{\lambda^*}^{ij} \Delta^j(\epsilon_\lambda \cdot \Delta) = \frac{2E^2}{3m^2} \Delta^i \delta_{\lambda \lambda} + \frac{2E}{m} \hat{Q}^{(1)jk}_{\lambda \lambda} \Delta^k \Delta^l + \frac{E}{2m^2(m + E)} \hat{Q}^{(1)kl}_{\lambda \lambda} \Delta^k \Delta^l \Delta^i \Delta^j,\]

\[\epsilon_{\lambda^*}^{ij} + \epsilon_{\lambda^*}^{ij} = \left( \frac{2}{3} \delta^{ij} + \frac{\Delta^i \Delta^j}{6m^2} \right) \delta_{\lambda \lambda} - 2 \hat{Q}^{(1)ij}_{\lambda \lambda} - \frac{1}{(m + E)^2} \left( \Delta^i \Delta^k \hat{Q}^{(1)jk}_{\lambda \lambda} + \Delta^j \Delta^k \hat{Q}^{(1)jk}_{\lambda \lambda} \right).\]

The dipole- and quadrupole-spin operators for the spin-3/2 field are defined by

\[\hat{S}^{ij}_{\sigma \sigma} = \sum C^{3}_{1X \lambda^{\sigma} s} C^{3}_{1X \lambda^{\sigma} s} \left( -i \epsilon^{ijkl}_{\lambda \lambda} \hat{\epsilon}^{k}_{\lambda \lambda} \delta_{\lambda \lambda} + \frac{\sigma^i_{s} s}{2} \delta_{\lambda \lambda} \right),\]

\[\hat{Q}^{ij}_{\sigma \sigma} = \sum C^{3}_{1X \lambda^{\sigma} s} C^{3}_{1X \lambda^{\sigma} s} \left[ -\frac{1}{2} \left( \epsilon^{ij*}_{\lambda \lambda} \epsilon_{\lambda \lambda} - i \epsilon^{ijkl}_{\lambda \lambda} \delta_{\lambda \lambda} \right) - i \epsilon^{ijkl}_{\lambda \lambda} \epsilon_{\lambda \lambda} \frac{\sigma^i_{s} s}{2} - i \epsilon^{ijkl}_{\lambda \lambda} \epsilon_{\lambda \lambda} \frac{\sigma^j_{s} s}{2} \right].\]
M. Kotulla, J. Ahrens, J. R. M. Annand, R. Beck, G. Caselotti, L. S. Fog, D. Hornidge, S. Janssen, B. Krusche and J. C. McGeorge, et al., Phys. Rev. Lett. 89, 272001 (2002).

J. Ahrens et al. [GDH and A2], Eur. Phys. J. A 21, no.2, 323-333 (2004).

S. Stave et al. [A1], Phys. Rev. C 78, 052509 (2008).

N. Sparveris, S. Stave, P. Achenbach, C. Ayerbe Gayoso, D. Baumann, J. Bernauer, A. M. Bernstein, R. Bohm, D. Bosnar and T. Botto, et al. Eur. Phys. J. A 49, 136 (2013).

S. Cotogno, C. Lorcé, P. Lowdon and M. Morales, Phys. Rev. D 101, no.5, 056016 (2020).

J. Y. Panteleeva and M. V. Polyakov, Phys. Lett. B 809, 135707 (2020).

V. D. Burkert, L. Elouadrhiri and F. X. Girod, Nature 557, no.7705, 396-399 (2018).

I. Zahed and G. E. Brown, Phys. Rept. 142, 1-102 (1986).

G. Holzwarth and B. Schwesinger, Rept. Prog. Phys. 49, 825 (1986).

M. V. Polyakov and B. D. Sun, Phys. Rev. D 100, no.3, 036003 (2019).

W. Cosyn, S. Cotogno, A. Freese and C. Lorcé, Eur. Phys. J. C 79, no.6, 476 (2019).

B. R. Holstein, Phys. Rev. D 74, 084030 (2006).

W. Broniowski and E. Ruiz Arriola, Phys. Rev. D 78, 094011 (2008).

H. D. Son and H.-Ch. Kim, Phys. Rev. D 90, no.11, 111901 (2014).

P. E. Shanahan and W. Detmold, Phys. Rev. D 99, no.1, 014511 (2019).

A. Freese and I. C. Cloét, Phys. Rev. C 100, no.1, 015201 (2019).

H. Alharazin, D. Djukanovic, J. Gegelia and M. V. Polyakov, [arXiv:2006.05890 [hep-ph]].

K. Goeke, J. Grabis, J. Ossmann, M. V. Polyakov, P. Schweitzer, A. Silva and D. Urbano, Phys. Rev. D 75, 094021 (2007).

J. Y. Kim, H.-Ch. Kim, M. V. Polyakov and H. D. Son, [arXiv:2008.06652 [hep-ph]].

C. Cebulla, K. Goeke, J. Ossmann and P. Schweitzer, Nucl. Phys. A 794, 87-114 (2007).

H.-Ch. Kim, P. Schweitzer and U. Yakhshiev, Phys. Lett. B 718, 625-631 (2012).

J. H. Jung, U. Yakhshiev and H.-Ch. Kim, J. Phys. G 41, 055107 (2014).

J. H. Jung, U. Yakhshiev, H.-Ch. Kim and P. Schweitzer, Phys. Rev. D 89, no.11, 114021 (2014).

M. J. Neubelt, A. Sampino, J. Hudson, K. Tezgin and P. Schweitzer, Phys. Rev. D 101, no.3, 034013 (2020).

I. V. Anikin, Phys. Rev. D 99, no.9, 094026 (2019).

K. Azizi and U. Özdem, Eur. Phys. J. C 80, no.2, 104 (2020).

P. E. Shanahan and W. Detmold, Phys. Rev. Lett. 122, no.7, 072003 (2019).

W. Detmold et al. [USQCD], Eur. Phys. J. A 55, no.11, 193 (2019).

Y. Hatta, A. Rajan and K. Tanaka, JHEP 12, 008 (2018) [arXiv:1810.05116 [hep-ph]].

W. Cosyn, A. Freese and B. Pire, Phys. Rev. D 99, no.9, 094035 (2019).

Z. Abidin and C. E. Carlson, Phys. Rev. D 77, 095007 (2008).

B. D. Sun and Y. B. Dong, Phys. Rev. D 101, no.9, 096008 (2020).

M. V. Polyakov and P. Schweitzer, PoS SPIN 2018, 066 (2019).

C. Lorcé, L. Mantovani and B. Pasquini, Phys. Lett. B 776, 38 (2018).

P. Schweitzer and K. Tezgin, Phys. Lett. B 796, 47 (2019).

I. A. Perevalova, M. V. Polyakov and P. Schweitzer, Phys. Rev. D 94, no.5, 054024 (2016).

G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. B 228, 552 (1983).

D. Diakonov, V. Y. Petrov and P. V. Pobylitsa, Nucl. Phys. B 306, 809 (1988).