GENERAL EXPRESSION FOR THE DIELECTRONIC RECOMBINATION CROSS SECTION OF POLARIZED IONS WITH POLARIZED ELECTRONS

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Abstract

A general expression for the differential cross section of dielectronic recombination (DR) of polarized electrons and polarized ions is derived by using usual atomic theory methods and is represented in the form of multiple expansions over spherical tensors. The ways of the application of the general expressions suitable for the specific experimental conditions are outlined by deriving asymmetry parameters of angular distribution of DR radiation in the case of nonpolarized and polarized ions and electrons.

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1. Introduction

Dielectronic recombination (DR) and electron impact excitation are the basic excitation mechanisms of x-ray production from high temperature plasmas. For the case of non-equilibrium, anisotropic plasmas, the polarization and angular distribution of radiation can be useful for the investigation of the properties of electron distribution function and plasma diagnostics [1]. In tokamak an other laboratory devices, the highly charged ions can be aligned [1, 2], therefore the expression for DR cross section describing the polarization state of ions and electrons in the initial state and radiation in the final state are of importance. The alignment of the doubly excited state, i.e. a non-uniform occupation of magnetic sublevels, causes the anisotropic emission of a photon relative to the direction of an incident ion beam [3]. The values of calculated DR cross

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section for various directions can change up to several tens of percent when the anisotropy of DR radiation is taken into account \[4\].

Density matrix formalism \[5\] is the usual method for the derivation of the expressions for the differential cross sections of DR of polarized ions with polarized electrons \[4, 5\]. Recently in the case of fully relativistic treatment, the expression for the differential cross section of DR was derived \[3\] by using a projection operator formalism \[6\]. The aim of the present work is the derivation of general expression in nonrelativistic approximation for the differential cross section of DR process by applying alternative method based on the atomic theory methods \[7, 8, 9, 10\]. The polarization state of all particles in both the initial and final states are described in this approximation. The practical applications of the general expression for the specific experimental conditions are outlined. The expression for the asymmetry parameter of the angular distribution of radiation in DR of non-polarized and aligned ions with non-polarized and polarized electrons demonstrates the way of the derivation of more simple expressions.

2. General expression

The process of DR can be written as follows

$$A^+(\alpha_0 J_0 M_0) + e^- (p_m) \rightarrow A^{**}(\alpha_1 J_1) \rightarrow \begin{cases} A(\alpha_2 J_2 M_2) + h\nu (\epsilon_q, k_0), \\ A^+(\alpha_3 J_3 M_3) + e^- (p_1 m_1). \end{cases} \tag{1}$$

It is an example of two-step process. The first step is resonant electron capture that is inverse process to Auger decay. The next step is radiative or Auger decay those expressions for the differential probability were obtained \[12, 13\] in the case of orientated and aligned ions following photoionization of atoms. Two-step approximation for DR may be applied if the interference with the radiative recombination is neglected and the summation over intermediate states $J_1 M_1$, that usually occurs in second-order perturbation theory, is limited to a single resonance. Then, only a summation over the magnetic substates that are not registered is retained. DR process is finished when the photon is emitted.

In two-step approximation, the cross section for DR may be written as:

$$\frac{d\sigma(\alpha_0 J_0 M_0 p_m \rightarrow \alpha_1 J_1 \rightarrow \alpha_2 J_2 M_2 \epsilon_q k_0)}{d\Omega} = \rho_f \sum_{M_1, M'_1} \langle \alpha_2 J_2 M_2 \epsilon_q k_0 | H' | \alpha_1 J_1 M_1 \rangle \langle \alpha_1 J_1 M_1 | H^e | (\alpha_0 J_0 M_0 p_m) \rangle^* \delta_{M_1, M'_1} \times \frac{(E - E_1)^2 + \Gamma^2/4}{(E - E_1)^2 + \Gamma^2/4}^{-1}. \tag{2}$$
Here $H'$ and $H^c$ is the radiative decay and electrostatic interaction operators, respectively, $d\Omega$ is the solid angle of the emission of radiation, $E_1$ and $E$ is the energy of the intermediate and initial state of the system atom+electron, respectively, and $\Gamma$ denotes the decay width of the intermediate state that includes both radiative and nonradiative decay channels. In (2), $\rho_i$ and $\rho_f$ denote the flux of incoming electrons and the density of final states, respectively. Atomic system of units is used.

In two step approximation ($E \approx E_1$), the general expression for DR (2) in the case of the interaction of polarized ion with polarized electron may be obtained by applying the methods described in [10, 12] and is as follows:

$$
\frac{d\sigma(\alpha_0 J_0 M_0 p m \rightarrow \alpha_1 J_1 \rightarrow \alpha_2 J_2 M_2 \epsilon_q, k_0)}{d\Omega} = \frac{\rho_f}{\rho_i} \sum_{K_1 N_1} W^c_{K_1 N_1} (\alpha_0 J_0 M_0 p m \rightarrow \alpha_1 J_1) \times \frac{dW^r_{K_1 N_1} (\alpha_1 J_1 \rightarrow \alpha_2 J_2 M_2 \epsilon_q, k_0)}{d\Omega} \left[ (E - E_1)^2 + \Gamma^2/4 \right]^{-1}.
$$

(3)

The resonant electron capture cross section $W^c$ is reversed to that of Auger decay and is defined by [12]

$$
W^c_{K_1 N_1} (\alpha_1 J_1 \rightarrow \alpha_0 J_0 M_0 p m) = \sum_{K, K_0, K_1} A^K (K_1, K_0, K, K_1, K_s, K_s) \sum_{N, N_0, N_1, N_2} \left[ \begin{array}{cccc} K_1 & K & N & N_1 \\ K & K_0 & N & N_0 \\ K_1 & K_0 & N & N_2 \\ K_1 & K_0 & N & N_2 \end{array} \right] T^K_{N_0} (J_0, J_1, M_0 | \hat{J}_0) T^K_{N_2} (s, s, m|\hat{s}) \sqrt{4\pi} \sqrt{\gamma_{K_1 K}} (\theta, \phi).
$$

(4)

Here

$$
T^K_{N_0} (J, J', M, | \hat{J}) = (-1)^{J' - M} \left[ \begin{array}{ccc} 4\pi & 2J + 1 \\ 2J + 1 & 0 \end{array} \right]^{1/2} \left[ \begin{array}{ccc} J & J' & K \\ M & -M & 0 \end{array} \right] Y_{K N} (\hat{J}),
$$

(5)

$$
A^K (K_1, K_0, K, K_s, K) = 2\pi \sum_{\lambda_1, \lambda_2} \lambda_1 \lambda_2 \langle \alpha_0 J_0 \epsilon \lambda_1 (j_1) | J_1 || H || \alpha_1 J_1 \rangle \langle \alpha_0 J_0 \epsilon \lambda_2 (j_2) | J_2 || H || \alpha_1 J_1 \rangle^* \times (2J_1 + 1) [(2\lambda_1 + 1)(2\lambda_2 + 1)(2j_1 + 1)(2j_2 + 1)(2J_0 + 1)(2s + 1)(2K + 1)]^{1/2} \times \left\{ \begin{array}{c} J_0 \\ J_1 \\ J_2 \\ K_0 \\ K_1 \end{array} \right\} \left\{ \begin{array}{c} j_1 \\ j_1 \\ j_2 \\ K_0 \\ K_1 \end{array} \right\} (-1)^{\lambda_2} \left[ \begin{array}{ccc} \lambda_1 & \lambda_2 & K_1 \\ 0 & 0 & 0 \end{array} \right].
$$

(6)

The expression for the radiative decay probability $dW^r/d\Omega$ is [13]:

$$
\frac{dW^r_{K_1 N_1} (\alpha_1 J_1 \rightarrow \alpha_2 J_2 M_2 \epsilon_q k_0)}{d\Omega} = \sum_{K_r, K_2, k, k'} A^K (K_1, K_r, K_2, k, k') \times \sum_{N_r, N_2} \left[ \begin{array}{ccc} K_1 & K_r & K_2 \\ N_1 & N_r & N_2 \end{array} \right] T^{K_r}_{N_2} (J_2, J_2, M_2 | \hat{J}_2) T^{K'_r}_{N_1} (k, k', q | \hat{k}_0),
$$

(7)

$$
A^K (K_1, K_r, K_2, k, k') = C(k, k') \langle \alpha_2 J_2 || Q(k') || \alpha_1 J_1 \rangle \langle \alpha_2 J_2 || Q(k') || \alpha_1 J_1 \rangle^*
$$
King angular distribution of radiation in the case of nonpolarized ions and electrons, one needs to insert the laboratory distribution of DR radiation (the angle \( \delta \) is measured from the direction of electrons) is applied. In the case of electrical dipole radiation, \( k = k' = 1 \), and the expression for \( \beta_2 \) coincides with (9) from [11].

Below the general expressions (4), (6), (7) and (8) are used to obtain some special expressions for specific experimental conditions.

3. Special cases

The tensor (5) describes the orientation of the angular momentum with respect to the laboratory \( z \) axis. In the case of the magnetic components are not registered, the summation over them of (5) leads to \( \delta(K, 0)\delta(N, 0) \). Thus, to obtain the expression of DR cross section describing angular distribution of radiation in the case of nonpolarized ions and electrons, one needs to insert \( K_0 = N_0 = K_s = N_s = K_2 = N_2 = 0 \) and \( K_1 = K_{\lambda} = K = K_r = \text{even} \) into (4).

Taking into account the \( z \) axis coinciding with the direction of electrons \( (N_1 = 0) \) we can write DR cross section in well known form [11]:

\[
\frac{d\sigma}{d\Omega}(\alpha_0 J_0 \rightarrow \alpha_1 J_1 \rightarrow \alpha_2 J_2 k_0) = \frac{\sigma(\alpha_0 J_0 \rightarrow \alpha_1 J_1 \rightarrow \alpha_2 J_2)}{4\pi} \left[ 1 + \sum_{K_1 > 0} \beta_{K_1} P_{K_1}(\cos \theta) \right].
\]

Here \( \sigma(\alpha_0 J_0 \rightarrow \alpha_1 J_1 \rightarrow \alpha_2 J_2) \) is DR cross section, and the asymmetry parameter of the angular distribution of DR radiation (the angle \( \theta \) is measured from the direction of electrons) is defined by

\[
\beta_{K_1} = \sum_{k,k'} (-1)^{k-q} \sqrt{(2k+1)(2K_1+1)} \begin{vmatrix} k & k' \cr q & -q \end{vmatrix} K_1 \frac{A_0^\alpha(K_1,0,K_1,0,K_1)}{A_0^\alpha(0,0,0,0)} \frac{A_r(K_1,K_1,0,k,k')}{A_r(0,0,0,k,k')}. \]

In the case of electrical dipole radiation, \( k = k' = 1 \), and the expression for \( \beta_2 \) coincides with (9) from [11].

To obtain an expression for the dependence of angular distribution of DR radiation on the polarization state of ions we need to perform summation of (4) over \( m = \pm 1/2 \) states of the electron and \( M_2 \) of the final state of recombined ion that lead to \( K_s = N_s = K_2 = N_2 = 0, K_{\lambda} = K = \text{even}, K_1 = K_r \). The choice of the laboratory \( z \) axis along the direction of electrons \( (N_1 = 0) \) gives \( N_{\lambda} = 0 \) because of \( Y_{K_{\lambda}N_1}(0,0) = [(2K_{\lambda} + 1)/4\pi]^{1/2} \delta(N_{\lambda}, 0) \). Then

\[
\frac{d\sigma}{d\Omega}(\alpha_0 J_0 M_0 \rightarrow \alpha_1 J_1 \rightarrow \alpha_2 J_2 k_0) = \frac{\rho_f}{2\rho_i[(E - E_1)^2 + \Gamma^2/4]} \sum_{K_1, N_1, k, k'} A_r(K_1, K_1, 0, k, k') T^{K_1}_{-N_1}(k, k', q|k_0) \]

\[
\times \left[ \frac{(2K_1 + 1)(2J_2 + 1)(2k + 1)}{2K_2 + 1} \right]^{1/2} \begin{vmatrix} J_1 & K_1 & J_1 \ k & K_r & k' \\
 J_2 & K_2 & J_2 \end{vmatrix}. \tag{8}
\]
\[
\sum_{K_0, K_\lambda} A^a(K_1, K_0, K_\lambda, 0, K_\lambda) \begin{bmatrix}
K_1 & K_0 & K_\lambda \\
-N_1 & N_1 & 0
\end{bmatrix}
T^s_{N_1}(J_0, J_0, M_0|J_0). \tag{12}
\]

In the case when the final state of an ion is not registered and laboratory \( z \) axis chosen along the direction of incoming electrons, the expression for DR of nonpolarized ions with polarized electrons can be obtained by averaging with respect of magnetic states of ion and summation over \( M_2 \) of (4). Then \( K_0 = N_0 = K_2 = N_2 = 0, K_1 = K = K_r, N_\lambda = 0, \) and

\[
d\sigma(\alpha_0 J_0 m \rightarrow \alpha_1 J_1 \rightarrow \alpha_2 J_2 \mathbf{k}_0) = \frac{\rho_f}{(2J_0 + 1)\rho_i[(E - E_1)^2 + \Gamma^2/4]} \sum_{K_1, N_1, k, k'} A^r(K_1, K_1, 0, k, k')
\]

\[
\times T_{-N_1}^{K_1}(k, k', q|\mathbf{k}_0) \sum_{K_S, K_\lambda} A^a(K_1, 0, K_\lambda, K_S, K_1) \begin{bmatrix}
K_s & K_1 & K_\lambda \\
N_1 & -N_1 & 0
\end{bmatrix}
T^s_{N_1}(s, s, m|s). \tag{13}
\]

The last sums in (12) and (13) can be called differential alignments similar to that introduced in [13].

4. Concluding remarks

The general expression for DR differential cross section of polarized ions and polarized electrons is obtained in two-step approximation. It is presented in the form of the multipole expansions over the states of all particles participating in the process that is very convenient for investigations, since the geometrical and dynamical parts are separated. This form coincides with that of the expansion over state multipoles (statistical tensors) widely used in the density matrix formalism. A simple way to derive the expressions for the special cases by using the general expression for the DR is described. The expressions for the asymmetry parameter of the angular distribution of radiation is obtained in the case of DR of nonpolarized atoms with nonpolarized and polarized electrons.
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