Dynamics of tachyonic preheating after hybrid inflation

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We study the instability of a scalar field at the end of hybrid inflation, using both analytical techniques and numerical simulations. We improve previous studies by taking the inflaton field fully into account, and show that the range of unstable modes depends sensitively on the velocity of the inflaton field, and thereby on the Hubble rate, at the end of inflation. If topological defects are formed, their number density is determined by the shortest unstable wavelength. Finally, we show that the oscillations of the inflaton field amplify the inhomogeneities in the energy density, leading to local symmetry restoration and faster thermalization. We believe this explains why tachyonic preheating is so effective in transferring energy away from the inflaton zero mode.

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I. INTRODUCTION

The dynamics of reheating after inflation have been studied intensively ever since the first models of inflation were formulated. In many cases, this process takes place non-perturbatively via a parametric resonance between the inflaton and matter fields. In this scenario, which is known as preheating, the long-wavelength matter modes are exponentially amplified to a very high effective temperature. This may have the consequence that certain symmetries, which are spontaneously broken at the reheat temperature, are temporarily restored during this non-thermal stage. When the fields thermalize, the symmetry breaks again, leading possibly to formation of topological defects. In models of electroweak scale inflation, the period of non-thermal symmetry restoration may also explain the baryon asymmetry of the universe.

However, it was argued more recently by Felder et al. that in hybrid inflationary models, this parametric resonance often fails to take place. In these models, inflation ends because a scalar field becomes unstable and its long-wavelength modes grow while the inflaton field rolls down the potential. The dynamics of this “tachyonic” or spinodal instability have been studied by many authors, using a range of different methods.

Naively, the inflaton field would be expected to start oscillating once it reaches the minimum of the potential. What Felder et al. observed was that the oscillations die away almost immediately, and coined the term “tachyonic preheating” to describe this phenomenon. This damping rules out the possibility of ordinary preheating, but the tachyonic instability itself still can lead to many similar effects, such as generation of baryon asymmetry, defect formation and particle production. Nevertheless, the physical origin of the damping has remained unclear until now.

In this paper, we use both detailed analytical calculations and non-perturbative numerical simulations to study the dynamics of tachyonic preheating after hybrid inflation. Analytical results show that the upper limit, $k_*$, of the range of momenta amplified during the instability has a simple dependence on the velocity $\dot{\phi}$ of the inflaton field at the instability point, $k_* \approx (2m\dot{\phi})^{1/3}$. This is essentially equivalent to calculations of the number of topological defects formed in a phase transition.

We confirm this result in our numerical simulations, in which we include also the inflaton as a dynamical field, by measuring the power spectrum directly and also by measuring the number of topological defects formed in the transition. Furthermore, we study the later evolution of the system, and show that because the potential in hybrid models is extremely flat above the critical value, the oscillations of the inflaton field form roughly spherical “hot spots”, inside which the energy density is very high and symmetry restoration can take place locally. Depending on the values of the couplings, these hot spots may appear already during the first oscillation of the inflaton field. We believe this explains the strong dissipation reported in Ref. [11], where the inflaton field was observed to settle down in its minimum after only few oscillations.

We carry out most of our simulations in two dimensions, because that allows us to treat a wider range of length scales in a single simulation, but in order to check that our results are not specific of two dimensions, we repeat some of the simulations in three dimensions. The breaking of global symmetries leads to the existence of massless Goldstone bosons, which are instead not present in the case of the breaking of local gauge symmetries. We recall that there is no experimental evidence for the existence of such scalars whose couplings to the particles of the Standard Model of Particle Physics are severely constrained by the present data. Therefore we study also the case in which the broken symmetry is a local gauge invariance.
The structure of the paper is the following: We start by reviewing the basic features of hybrid inflationary models in Section II. In Section III, we study the instability of the scalar field analytically both in the approximation that the mass term changes instantaneously and in the more realistic case of hybrid inflation. We present the details of our two-dimensional lattice simulations, together with the results for the power spectrum in Section IV, and the results for the defect density in Section V. In Section VI, we show that the oscillations of the inflaton field lead to “hot spots” with local symmetry restoration. In Section VII, we present results of three-dimensional field simulations, together with the results for the power spectrum in Section IV. We present the de

II. HYBRID INFLATION

In models of hybrid inflation [22], the inflaton field \( \varphi \) is coupled to another scalar field \( \chi \), which becomes unstable at a certain critical value of \( \varphi = \varphi_c \). This causes the slow roll conditions to break down, and inflation ends. The simplest realization of this idea has the potential

\[
V(\varphi, \chi) = \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} g^2 \varphi^2 \chi^2 + \frac{1}{4} \lambda \left( \chi^2 - v^2 \right)^2,
\]

where \( \varphi \) and \( \chi \) are real scalars. During inflation, the inflaton has a large value, \( \varphi \gg \varphi_c = m/g \), where we have defined \( m \equiv \lambda^{1/2} v \). Therefore the effective mass term of the \( \chi \) field, \( m^2 \chi^2 = -m^2 + g^2 \varphi^2 \), is positive and the symmetry \( \chi \rightarrow -\chi \) is restored. Eventually, \( \varphi \) reaches \( \varphi_c \), and \( m^2 \chi^2 \) becomes negative, implying that the \( \chi \) field becomes unstable, or tachyonic.

The COBE measurements require that at \( \varphi = \varphi_{\text{COBE}} \), when the fluctuations in the cosmic microwave background left the horizon,

\[
\frac{V(\varphi_{\text{COBE}})^{3/2}}{M_\text{P}^3 V(\varphi_{\text{COBE}})} \approx 5.3 \times 10^{-4},
\]

where \( M_\text{P} = (8\pi G)^{-1/2} \approx 2.4 \times 10^{18}\text{GeV} \) is the reduced Planck mass. Assuming that the value of the potential and its derivative do not change significantly between \( \varphi_{\text{COBE}} \) and \( \varphi_c \), we find

\[
V'(\varphi_{\text{COBE}}) \approx m^2 \varphi_c = \frac{m m^2}{g},
\]

and

\[
V(\varphi_{\text{COBE}}) \approx V_0 \equiv m^4/4\lambda.\n\]

Consequently,

\[
m^2 \varphi \approx \frac{1}{5.3 \times 10^{-4}} \frac{g}{8\lambda^{3/2} M_\text{P}^2} m^5,
\]

which is typically much less than \( m^2 \), and therefore we neglect it in the study of the dynamics of the system. If the slow roll condition \( V'(\varphi) M_P \ll V(\varphi) \) is satisfied, we have

\[
\frac{\dot{\varphi}}{3H} = \frac{V'}{\sqrt{3V}} \approx \frac{1}{5.3 \times 10^{-4}} \frac{V_0}{\sqrt{3} M_\text{P}^2} \approx (60H)^2,
\]

where \( H \) is the Hubble rate at the end of inflation. Even in GUT scale inflation, \( \dot{\varphi} \) is relatively low, and in models of electroweak scale inflation, we find \( \dot{\varphi} \sim (10^{-3}\text{eV})^2 \).

Note, however, that if the potential has a more complicated shape, \( \varphi \) can be higher, such as in inverted hybrid models [24]. Therefore, we treat \( \dot{\varphi} \) as a free parameter.

The dynamics of the fields are described by the equations of motion

\[
\begin{align*}
\dot{\varphi}^2(t, \vec{x}) &= -3H\dot{\varphi}(t, \vec{x}) - g^2 \chi^2(t, \vec{x}) \varphi(t, \vec{x}), \quad (6a) \\
\dot{\varphi}^2(t, \vec{x}) &= -3H\dot{\varphi}(t, \vec{x}) + (m^2 - g^2 \varphi^2(t, \vec{x})) \chi(t, \vec{x}) - \lambda \chi^3(t, \vec{x}). \quad (6b)
\end{align*}
\]

Let us first discuss the dynamics of the system in the simplest approximation, in which the field fluctuations are neglected altogether. The initial conditions are given by the field values at the end of inflation, when the inflaton \( \varphi \) has just reached its critical value \( \varphi_c \) and is rolling down the potential with velocity \( \dot{\varphi} \) and the \( \chi \) field is still at rest in its minimum,

\[
\begin{align*}
\varphi(0, \vec{x}) &= \varphi_c \quad \text{and} \quad \chi(0, \vec{x}) = 0 \quad \text{and} \quad \dot{\varphi}(0, \vec{x}) = 0. \quad (7)
\end{align*}
\]

When \( \varphi \) rolls further down, the defect field \( \chi \) is displaced out of the symmetric phase. As far as it reacts much faster than the inflaton field, we can assume that the field \( \chi(t, \vec{x}) \) reaches immediately the minimum at the given value of \( \varphi(t, \vec{x}) \) and is given by

\[
\chi^2(t, \vec{x}) = \frac{m^2 - g^2 \varphi^2(t)}{\lambda}. \quad (8)
\]

Substituting \( \chi(t, \vec{x}) \) in the potential \( V(\chi, \varphi) \) we obtain the effective potential for the inflaton field:

\[
V_{\text{eff}}(\chi, \varphi) \equiv \frac{g^2}{2\lambda} m^2 \varphi^2 - \frac{g^4}{4\lambda} \varphi^4. \quad (9)
\]

At late times, \( \varphi \) oscillates around zero, and when the amplitude is small enough, we can write down an approximate equation of motion

\[
\ddot{\varphi}(t) + 3H\dot{\varphi}(t) \approx -\omega_\varphi^2 \varphi(t), \quad (10)
\]

where \( \omega_\varphi^2 = m^2 g^2/\lambda \). Defining \( \tilde{\gamma} = 3H/2 \), we can write the solution as

\[
\varphi(t) = Ae^{-\tilde{\gamma}t} \cos \left( t \sqrt{\omega_\varphi^2 - \tilde{\gamma}^2} \right). \quad (11)
\]

Therefore the inflaton field shows a damped oscillatory behaviour with respect to \( t \).

III. INSTABILITY

We are interested in understanding the dynamics of the defect field just after the inflaton field has crossed the critical point \( \varphi_c \). We take into account the fluctuations of the defect field. As the mass for the defect field
is negative, the modes at low momenta experience a fast growth. We ignore the fluctuations in the inflaton field, because at early times, as we shall mainly be interested in the case in which $g^2 \ll \lambda$, the effects of the interactions between the fluctuations of the inflaton field and of the defect field can be neglected and, even if $g^2 \sim \lambda$, the homogeneous mode of the $\varphi$ field gives the dominant contribution. At late times, the linear approximation breaks down in any case and the results given in Sections IIIA and IIIB are not valid anymore.

A. Instantaneous quench

Let us first discuss the instability in the case when the mass parameter of the $\chi$ field changes instantaneously from zero to $-m^2$. This case has been studied by many authors (see, for instance, Refs. [11, 12, 16, 17]). For simplicity, we ignore the expansion of the universe.

In the linear approximation, the equation of motion for a Fourier mode $\chi(t, \vec{k})$ of wave number $\vec{k}$ is

$$\partial^2_0 \chi(t, \vec{k}) = (-\vec{k}^2 + m^2)\chi(t, \vec{k}).$$

(12)

Modes with $|\vec{k}| < m$ become unstable and grow exponentially.

In a quantum field theory, Eq. (12) is valid as an operator equation, and in a classical field theory as a field equation. In the quantum theory, the initial state is the vacuum, and at tree level, it is completely described by the two-point functions of the fields,

$$\left\langle \chi^\dagger(\vec{k}_1)\chi(\vec{k}_2) \right\rangle = \frac{1}{2|\vec{k}|} (2\pi)^3 \delta^3(\vec{k}_1 - \vec{k}_2),$$

$$\left\langle \pi^\dagger(\vec{k}_1)\pi(\vec{k}_2) \right\rangle = \frac{|\vec{k}|}{2} (2\pi)^3 \delta^3(\vec{k}_1 - \vec{k}_2),$$

(13)

where $\pi = \partial_0 \chi$. It is important to note that as long as the equation of motion is linear, the operator nature of $\chi$ only appears in these initial conditions, and therefore the classical field theory reproduces exactly the same time evolution, if the initial conditions are given by a Gaussian random field with the two-point function in Eq. (13). Thus, although we shall treat the theory as a classical system, the results of the linear approximation are identical to the quantum theory.

Solving Eq. (12), we find that the Fourier mode $\chi(t, \vec{k})$ is given by

$$\chi(t, \vec{k}) = \frac{\chi(0, \vec{k}) \cosh \left( t\sqrt{m^2 - \vec{k}^2} \right)}{\sqrt{m^2 - \vec{k}^2}} + \frac{\pi(0, \vec{k})}{\sqrt{m^2 - \vec{k}^2}} \sinh \left( t\sqrt{m^2 - \vec{k}^2} \right).$$

(14)

Therefore, the variance of the fluctuations $\langle \chi^2(t) \rangle$ grows as

$$\langle \chi^2(t) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \langle \chi^\dagger(t, \vec{k})\chi(t, \vec{k}') \rangle$$

$$= \langle \chi^2(0) \rangle + \frac{1}{8\pi^2} \int_0^{m^2} dk^2 \frac{m^2}{m^2 - k^2} \sinh^2 \left( t\sqrt{m^2 - k^2} \right)$$

$$\sim \frac{m}{32\pi^2t} \exp(2mt),$$

(15)

where $\langle \chi^2(0) \rangle$ is a divergent “vacuum” contribution.

Another quantity we shall consider is the power spectrum of a given Fourier mode of the field $\chi$,

$$P(k)(2\pi)^3 \delta^3(\vec{k} - \vec{k}') = \langle \pi^\dagger(t, \vec{k})\pi(t, \vec{k}') \rangle.$$  

(16)

Using Eqs. (13) and (14), we find

$$P(k) = \frac{k^3}{2} + \frac{m^2}{2k} \sinh^2 t\sqrt{m^2 - k^2},$$

(17)

for $k < m$. This shows that modes with $k \leq m$ are amplified exponentially, or in other words, tachyonic preheating produces a power spectrum with an effective cutoff scale $k_\ast \approx m$ [11, 16].

B. Hybrid model

The assumption that the mass parameter changes instantaneously makes analytical calculations easy, but it is not a very good approximation for the instability at the end of hybrid inflation. During inflation, the inflaton field $\varphi$ is rolling down the potential slowly, and because it is only weakly coupled to the $\chi$ field, the effective mass parameter $m_{\chi, eff}(\varphi)$ changes typically very slowly. This case has been studied in Refs. [17] using the linear approximation, and also in Refs. [20, 21] in the context of defect formation.

We assume that the fluctuations of the inflaton field $\varphi$ are negligible, which means that we can describe it by its homogeneous part $\varphi(t)$. Near the instability point, we can approximate

$$\varphi(t) \approx \varphi_c - \dot{\varphi}t.$$  

(18)

We are interested in the time-dependence of the fluctuations $\chi(t, \vec{k})$ of the defect field, and at early times we can linearize Eq. (13),

$$\partial^2_0 \chi(t, \vec{k}) = (m^2 - g^2\varphi^2(t) - k^2)\chi(t, \vec{k})$$

$$\approx (2mg\dot{\varphi}t - k^2)\chi(t, \vec{k}).$$

(19)

The solution for $\chi(t, \vec{k})$ is given in terms of Airy functions:

$$\chi(t, \vec{k}) = c_A(\vec{k})Ai \left( \omega t - \frac{k^2}{\omega^2} \right) + c_B(\vec{k})Bi \left( \omega t - \frac{k^2}{\omega^2} \right),$$

(20)
where we have defined \( \omega = (2mg\ddot{\varphi})^{1/3} \).

This shows that each mode starts growing at the time \( t = t_k = k^2/\omega^3 \), when the effective mass becomes negative \( m_k^{eff} \equiv -\omega^3 t + k^2 < 0 \). We can express the coefficients \( c_A(\vec{k}) \) and \( c_B(\vec{k}) \) in terms of the values of \( \chi(\vec{k}) \) and \( \pi(\vec{k}) \) at the time \( t_k \) as

\[
\begin{align*}
c_A(\vec{k}) &= \frac{1}{2} \left\{ \frac{\chi(t_k, \vec{k})}{A(0)} + \frac{\pi(t_k, \vec{k})}{\omega A'(0)} \right\}, \\
c_B(\vec{k}) &= \frac{1}{2\sqrt{3}} \left\{ \frac{\chi(t_k, \vec{k})}{A(0)} - \frac{\pi(t_k, \vec{k})}{\omega A'(0)} \right\},
\end{align*}
\]

(21)

where \( A(0) = 3^{-2/3}\Gamma(2/3)^{-1} \approx 0.355 \) and \( A'(0) = -3^{-1/3}\Gamma(1/3)^{-1} \approx -0.259 \).

In the very early time regime, each Fourier mode behaves as

\[
\chi(t, \vec{k}) = \chi(0, \vec{k}) + (t - t_k)\pi(0, \vec{k}) + O((t - t_k)^3),
\]

(22)

which implies

\[
\langle \chi^2 \rangle(t) - \langle \chi^2 \rangle(0) \approx \int_0^{\sqrt{\omega t}} \frac{d^Dk}{(2\pi)^D} (t - t_k)^2 \frac{k^2}{2}
\]

\[
\approx 4C_D \frac{t^2}{(D + 1)(D + 3)(D + 5)},
\]

(23)

where \( D \) is the dimensionality of space and \( C_D = 2(4\pi)^{-D/2}\Gamma(D/2)^{-1} \). In the physical case \( D = 3 \),

\[
\langle \chi^2 \rangle(t) - \langle \chi^2 \rangle(0) \approx \frac{\omega^0 t^4}{96\pi^2}.
\]

(24)

The spinodal value of the field \( \chi \), on the other hand, grows linearly,

\[
\chi_{spinodal}^2(t) \equiv \frac{m^{2,eff}}{3\lambda} \approx \frac{\omega^3 t}{3\lambda},
\]

(25)

and therefore at early times,

\[
\langle \chi^2 \rangle(t) - \langle \chi^2 \rangle(0) \ll \chi_{spinodal}^2(t).
\]

(26)

This result shows that no matter how small \( \ddot{\varphi} \) is, there is always a period after the instability, during which the linear approximation is valid. Eqs. (24) and (26) seem to imply that the back reaction sets in when

\[
\tau_{spin}^{3D} \approx \left( \frac{32\pi^2}{\lambda} \right)^{1/3} \omega^{-1},
\]

(27)

but because Eq. (24) is a power series in \( \omega t \), the expansion has broken down by that time, and instead, we have to consider the asymptotic behaviour at late times.

At late times, \( A(\omega t - k^2/\omega^2) \) decreases exponentially while \( Bi(\omega t - k^2/\omega^2) \) shows an exponential growth. Therefore, we neglect the first term in the right hand side of Eq. (20). The asymptotic expansion of \( Bi(z) \) for large values of \( z \) is given by

\[
Bi(z) \sim \frac{1}{\sqrt{\pi}} z^{-1/4} \exp \left( \frac{2}{3} z^{3/2} \right).
\]

(28)

The variance of the defect field \( \chi(t, \vec{x}) \) at late times can therefore be computed as

\[
\langle \chi^2 \rangle(t) \approx \int_0^{\sqrt{\omega t}} \frac{d^Dk}{(2\pi)^D} \frac{d^Dk'}{(2\pi)^D} \langle \chi^2(t, \vec{k}) \chi(t, \vec{k}') \rangle
\]

\[
\approx C_D \int_0^{\sqrt{\omega t}} dkk'^{-1} \langle c_B(\vec{k}) c_B(\vec{k}) \rangle \times Bi(\omega(t - t_k))^2,
\]

(29)

where we have ignored the constant vacuum contribution \( \langle \chi^2 \rangle(0) \). Using Eqs. (13) and (21), we have

\[
\langle c_B(\vec{k}) c_B(\vec{k}) \rangle \approx \frac{1}{8k} \left[ 3^{3\frac{2}{3}} \left( \frac{2}{3} \right)^2 + 3^{3\frac{2}{3}} \left( \frac{1}{3} \right)^2 \frac{k^2}{\omega^2} \right]
\]

\[
\approx \frac{1}{8k} \left( 2.645 + 4.976 \frac{k^2}{\omega^2} \right),
\]

(30)

Therefore, we find

\[
\langle \chi^2 \rangle(t) \sim \frac{C_D}{8\pi} \int_0^{\sqrt{\omega t}} dkk'^{-2} 2.645 \sqrt{\omega(t - t_k)} \exp \left( \frac{4}{3} (\omega t)^{3/2} \right).
\]

(31)

which we can expand around small \( k \) to obtain the asymptotic behaviour

\[
\langle \chi^2 \rangle(t) \sim 2.645 \frac{C_D}{16\pi} \Gamma \left( \frac{D - 1}{2} \right) (\omega t)^{1 - D} \times
\]

\[
\times \left( \frac{\omega^3}{2} \right)^{\frac{D - 1}{2}} \exp \left[ \frac{4}{3} (\omega t)^{3/2} \right].
\]

(32)

In the physical case \( D = 3 \), this simplifies to

\[
\langle \chi^2 \rangle(t) \sim \frac{2.645 \omega}{64\pi} \frac{\omega}{t} \exp \left[ \frac{4}{3} (\omega t)^{3/2} \right].
\]

(33)

Using Eq. (20), it is also straightforward to write down the expression for the power spectrum \( P(k) \),

\[
P(k) = \frac{\omega^2}{8k} \left( \frac{Ai'((\omega(t - t_k)) + \frac{1}{\sqrt{3}} Bi'((\omega(t - t_k)) \right)^2
\]

\[
+ \frac{k}{8} \left( \frac{Ai'((\omega(t - t_k)) - \frac{1}{\sqrt{3}} Bi'((\omega(t - t_k)) \right)^2,
\]

(34)

At late times, \( t \gg t_k \), this behaves as

\[
P(k) \approx \frac{\sqrt{\omega t}}{24\pi k} \left( \frac{\omega^2}{0.355^2 + \frac{k^2}{0.259^2}} \right) e^{\frac{2}{3}(\omega(t - t_k))^{3/2}},
\]

(35)
but the high-\(k\) modes for which \(t < t_k\), are still in vacuum. Therefore, we conclude that that power spectrum has an effective cutoff: The energy density in modes with \(k \lesssim \sqrt{3} M_P\) is exponentially high, but modes with \(k \gtrsim \sqrt{3} M_P\) are still in vacuum.

The growth of the long-wavelength modes stops around the spinodal time \(t_{\text{spin}}\), which we can estimate by comparing Eq. (32) with Eq. (25) and ignoring all factors of order one,

\[
t_{\text{spin}} \approx \frac{1}{\omega} \left( \frac{3}{4} \ln \frac{\omega^3 - D}{\lambda} \right)^{2/3},
\]

which agrees with Eq. (30) of Ref. [12]. In contrast to Eq. (27), the dependence on \(\lambda\) is only logarithmic. In most cases, the logarithm can be ignored, and we find the simple result \(t_{\text{spin}} \approx \omega^{-1}\).

Assuming that the back reaction does not change the overall shape of the power spectrum \(P(k)\) significantly, we can see that the final value of the cutoff \(k\) is

\[
k_* \approx \sqrt{\omega^3 t_{\text{spin}}} \approx \omega \left( \frac{3}{4} \ln \frac{\omega^3 - D}{\lambda} \right)^{1/3}.
\]

Apart from the logarithmic factor, which is usually negligible, this result agrees with Ref. [13]. Ignoring the logarithmic factor and using the slow-roll condition (32), we can write Eq. (37) as

\[
k_* \approx \left( \frac{1}{5.3 \times 10^{-2}} \sqrt{3} g \frac{m^2}{M_P} \right)^{1/3} m.
\]

Consequently, the instability becomes weaker if the energy scale of inflation is low, and only non-relativistic particles are produced.

### IV. SIMULATIONS

In order to study the dynamics of the instability numerically, we approximate the system with a classical field theory, which is believed to be a good approximation when occupation numbers are large [23]. At early times, the quantum correlations may play an important role (see, e.g., [24, 25]), and while it would be interesting to study their consequences, that is beyond the scope of this paper.

For the scalar theory in Eq. (1), the discretization of space and time is straightforward: We replace the derivatives by finite differences. It is convenient to define the fields \(\chi\) and \(\varphi\) at the lattice sites and their time derivatives \(\pi\) and \(\tau\) at half way between two time steps. The discretized equations of motion for \(\chi\) and \(\pi\) are

\[
\frac{\pi(t + \delta t, \vec{x}) - \pi(t - \delta t, \vec{x})}{\delta t} = \Delta \chi(t, \vec{x}) - \frac{\partial V(\chi, \varphi)}{\partial \chi},
\]

\[
\frac{\chi(t + \delta t, \vec{x}) - \chi(t, \vec{x})}{\delta t} = \pi(t + \delta t/2, \vec{x}),
\]

and analogous equations for \(\varphi\) and \(\tau\). We have used here the lattice Laplacian

\[
\Delta f(\vec{x}) = \frac{1}{\delta \vec{x}^2} \sum_i \left[ f(\vec{x} + \vec{i}) - 2 f(\vec{x}) + f(\vec{x} - \vec{i}) \right],
\]

where \(i\) is a vector of length \(\delta \vec{x}\) in the direction \(i\).

The lattice approach can easily be used in an expanding universe, as well, by using conformal coordinates \(dt = dt/a\) and rescaled fields in which the expansion of the universe only appears as time-dependent mass terms \(m_x^2, m^2\) in the potential in Eq. (4). We used two-dimensional simulations, because we could reach larger system sizes and longer simulation times that way. As we shall discuss in Section VII, we also carried out three-dimensional simulations to make sure that, although the details of the back reaction are dependent on the dimensionality, the overall qualitative picture is not.

The corresponding three-dimensional result [14] for the potential in Eq. (5) is \(\delta m^2 \approx 0.452 g^2 / \delta x^2\).

To study the instability and growth of the long-wavelength \(\chi\) modes after the symmetry breakdown, we used two-dimensional simulations, because we could reach larger system sizes and longer simulation times that way. As we shall discuss in Section VII, we also carried out three-dimensional simulations to make sure that, although the details of the back reaction are dependent on the dimensionality, the overall qualitative picture is not.

In the two-dimensional simulations, we chose \(\chi\) to be a real one-component field \(\chi\). The lattice size was \(L = 216^2\), and in units in which the mass parameter was \(m^2 = 1\), the lattice spacing was \(\delta x = 0.5\). We used two set of couplings, \(g = 10^{-5}\) and \(g = 10^{-4}\), with \(\lambda = 200 g^2\) in both cases. In order to isolate the dependence on \(\varphi\), we used a Minkowskian space with \(H = 0\). We recall
satisfied the “quantum” two-point function, Eq. (13). We followed the time evolution of the field configuration by solving numerically the equations of motion, Eq. (33). The perturbative mass counterterm, Eq. (14), ensured that the inflaton $\varphi$ did not experience any significant acceleration or deceleration before it reached $\varphi_c$.

We measured the power spectrum $P(k)$ of the $\chi$ field defined in Eq. (16) at the instants when $\langle \chi^2 \rangle(t)$ was equal to some positive power of 10. In Fig. 1, we compare the simulated power spectra to the analytical result in Eq. (34) for the parameter values $g = 10^{-4}$ and $\varphi = 1.0$. We can see that at early times, the linear approximation works extremely well, apart from the very lowest momenta. However, these momenta correspond to wavelengths which are of the order of the system size, and therefore this discrepancy may well be due to finite-size effects. Another possibility is that the fluctuations, small as they are, give rise to enough friction to damp down the exponential growth of the modes with lowest $k$. Both of these effects are artifacts of our numerical technique, and would therefore be absent in the true quantum theory.

At later times, when $\langle \chi^2 \rangle \approx 1000$, a discrepancy appears at high momenta, due to the back reaction. Furthermore, when $\langle \chi^2 \rangle \approx 10000$, we can see that the linear approximation has broken down.

In Fig. 2, we show the power spectra measured at the instants when $\langle \chi^2 \rangle \approx 100$ and 10000 for different values of $\dot{\varphi}$. At the former time, the evolution is still linear and well described by the analytical formula, given in Eq. (14), while at the latter time, the non-linearities have set in. Nevertheless, the power spectrum retains its overall shape, with very high values at low momenta and a sharp cutoff around the same momentum scale as in the linear approximation. In particular, the back reaction does not change the cutoff scale $k_\ast$, and therefore the prediction $k_\ast \approx (2mg\varphi)^{1/3}$ of the linear theory in Eq. (17) remains valid.

\section{V. Defect Formation}

Depending on the number of components in the scalar field $\chi$, the potential $V(\chi, \varphi)$ is invariant under discrete or global symmetries, which get spontaneously broken when the inflaton $\varphi$ crosses its critical value $\varphi_c$, if $\chi$ obtains a non-zero expectation value. In general, this leads to the formation of topological defects \cite{24}. These defects may have several important effects on the late evolution of the universe \cite{24}, and in many cases the fact that they are formed can by itself rule out the model. On the other hand, defect formation is also important, because it is a convenient way of probing the dynamics of the transition.

In a scalar field theory, defect formation can be understood in terms of the Kibble mechanism \cite{26}. (For a recent review, see Ref. [28].) When the symmetry breaks, all directions on the vacuum manifold are equally probable, and whichever the system happens to choose is purely a matter of chance. However the correlation...
length of the $\chi$ field cannot be infinite at the time of the transition, and therefore the system cannot choose the same direction everywhere in space but only inside domains whose size is determined by the correlation length $\tilde{\xi}$ at the time of the transition.

This argument was made more accurate by Zurek [19], who related $\tilde{\xi}$ to the rate of the transition and the critical indices of the theory. The present case in which the defects are formed at the end of inflation is particularly simple, because, as the inflation has diluted away all the energy density, we can assume that the universe is at zero temperature. Therefore the equilibrium correlation length is given by the inverse of the mass, and near $\varphi_c$ we have simply

$$m^2_\chi(\varphi) = -m^2 + g^2\varphi^2 \approx -2g\varphi \dot{\varphi} = -\omega^3 t.$$  \hspace{1cm} (43)

Furthermore, the Lorentz invariance guarantees that the relaxation time of the field $\chi$ is equal to the correlation length, $r_\chi = \tilde{\xi} = m_\chi^{-1}$.

When $\varphi$ approaches $\varphi_c$, the equilibrium values of both $\xi$ and $\tau$ diverge, and eventually $\chi$ must fall out of equilibrium. This happens approximately at the time when the relaxation time $\tau$ is equal to $|t|$. Thus, we have

$$\dot{\tilde{\xi}} \approx \xi(-\tau) = (\omega^3 \dot{\xi})^{-1/2} = \omega^{-1}. \hspace{1cm} (44)$$

This determines the typical distance between the topological defects.

Comparing Eq. (44) with Eq. (37), we see that $\dot{\tilde{\xi}} \approx k_*^{-1}$, and this is obviously no accident. Indeed, a power spectrum with a cutoff $k_*$ means that the field changes strongly on distances larger than $1/k_*$, but is smooth at shorter distances. It is therefore clear that the correlation length is given by $1/k_*$. Thus, we are simply using different language to describe the same physical phenomenon.

We tested the prediction in Eq. (44) in the two-dimensional simulations described in Section IV. Because $\chi$ is a real one-component field, the broken symmetry is $Z_2$ and the topological defects are domain walls. With the parameters we were using, Eq. (44) predicts that the typical distance between walls is $\tilde{\xi} \approx 0.79(g\dot{\varphi})^{-1/3}$. We can also estimate the total length of domain walls immediately after the phase transition to be

$$L_{wall}^{\text{pred}} = L^2/\tilde{\xi} \approx 1.28 \times 10^6 (g\dot{\varphi})^{1/3}. \hspace{1cm} (45)$$

In our simulations, we measured the total length of domain walls $L_{wall}$ in the system by counting all pairs of neighbouring lattice sites where $\chi$ has different signs and multiplying the result by the lattice spacing $\delta x$. We carried out these measurements at various times after the transition, and the results are shown in Fig. 3 for different values of $\dot{\varphi}$. In the figure we plot the ratio of the measured value to the predicted one given by Eq. (45), and therefore a horizontal line corresponds to a power law with the predicted exponent $1/3$. Each data point is an average of eight different runs with different random initial conditions, but we used the same set of eight initial conditions for all values of $\dot{\varphi}$.

Testing the prediction in Eq. (44) is not entirely straightforward, because, as the domain wall network evolves with time, the measured value of $L_{wall}$ depends on when the measurement was carried out. This choice is more or less arbitrary, and we have used two different
ways of determining the time of the measurement: Either when \((\chi^2)\) crosses a certain threshold value, or when \(\langle \varphi \rangle\) crosses zero. These choices can be interpreted as different ways of giving the quantity \(L_{\text{wall}}\) a precise definition.

When \((\chi^2)\) is small, domain walls are ill-defined objects, because the sign of \(\chi\) can vary simply due to random fluctuations. Therefore the measurements at \((\chi^2) = 10^3\) give high values and do not agree with the predicted power law. Later, when \((\chi^2) = 10^4\) or \(10^5\), the correspondence between the measurements and the prediction \(13\) is good at high values of \(\dot{\varphi}\), apart from a constant factor of around five. Because the prediction was intended to give the order of magnitude only, we can conclude that the agreement is good.

However, we can see that at very low values of \(\dot{\varphi}\), the predicted power law breaks down. The most likely explanation for this is that these extra domain walls are generated by vacuum fluctuations, which are not strongly enough suppressed when \((\chi^2)\) is relatively small. This means that the corresponding definition of \(L_{\text{wall}}\) is not suitable for such slow transitions. Of course, we cannot rule out the possibility that the prediction \(13\) breaks down at low \(\dot{\varphi}\) in the full quantum theory, either.

We also tested how \(L_{\text{wall}}\) depends on the strength of the couplings. The open circles in Fig. \(3\) show \(L_{\text{wall}}/L_{\text{pred}}\) in simulations with \(g = 10^{-3}\) measured at the time when \((\chi^2) = 10^4\). This corresponds to \((\chi^2) = 10^5\) in the simulations with \(g = 10^{-4}\), and we see that they agree reasonably well, and therefore the dependence on \(g\) predicted by Eq. \(13\) seems to be correct. Furthermore, it seems that the velocity \(\dot{\varphi}\) below which the prediction breaks down is independent of \(g\).

Finally, we also measured \(L_{\text{wall}}\) at the time when \(\varphi\) reaches zero for the first time, which is typically much later than any of the other measurements. We can see that at high \(\varphi\), the power-law prediction breaks down in this case. This all illustrates that measuring \(L_{\text{wall}}\) accurately is rather difficult, because the result depends on the time when the measurement is carried out. Nevertheless, our results show that the prediction \(13\) works reasonably well in a wide range of velocities \(\dot{\varphi}\).

VI. LOCAL SYMMETRY RESTORATION

In ordinary thermal phase transitions, the defect network formed in the transition evolves mostly dissipatively, and the domain wall loops would simply collapse and disappear with time. However, in the case of hybrid inflation, the dynamics of the inflaton field may have significant effects on the later evolution of the system. Eq. \(11\) shows that if the \(\chi\) field were completely homogeneous, the inflaton would oscillate with roughly the frequency \(\omega_0^2 \approx V''_\infty(0) = (g^2/\lambda)n^2\). However, the presence of a defect network means that \(\chi\) is far from being homogeneous: It vanishes in the cores of the defects, and furthermore, when the defects annihilate, they release energy, which heats up the system locally and leads to inhomogeneities in \(\chi\). In this case, the frequency of \(\varphi\) becomes dependent on the position in space.

In regions where the effective \(\omega_{\varphi}\) is lower than the average value, the \(\varphi\) field will start to lag behind, and a gradient energy starts to build up. We can study this effect in more detail in a simple model of one scalar field with an inhomogeneous mass term, with the equation of motion

\[
\partial_0^2 \varphi - \nabla^2 \varphi + M^2(\vec{x}) \varphi = 0, \tag{46}
\]

where we adopt a Gaussian shape of width \(\Delta\) for \(M^2\),

\[
M^2(\vec{x}) = M_0^2 \left[ \frac{1 - \exp \left( -\frac{x^2}{2\Delta^2} \right) }{1 - \epsilon} \right]. \tag{47}
\]

We assume that the spatial extent of the perturbation is large compared with the mass, \(\Delta \gg M^{-1}\).

We assume that initially, the field has the value \(\varphi(0, \vec{x}) = \varphi_0\) and is at rest. At the spatial infinity, \(|\vec{x}| \to \infty\), the field oscillates with a constant amplitude

\[
\varphi(t, \infty) = \varphi_0 \cos \left( \frac{M_0 t}{\sqrt{1 - \epsilon}} \right). \tag{48}
\]

Because \(\Delta\) is large, \(M(\vec{x})\) is slowly varying, and it makes sense to treat the spatial gradient term in Eq. \(46\) as a small perturbation. Therefore, we write

\[
\varphi(t, \vec{x}) = \varphi_0(t, \vec{x}) + \delta \varphi(t, \vec{x}), \tag{49}
\]

where \(\varphi_0\) satisfies the ordinary differential equation

\[
\partial_0^2 \varphi_0(t, \vec{x}) + M^2(\vec{x}) \varphi_0(t, \vec{x}) = 0, \tag{50}
\]

and has the solution

\[
\varphi_0(t, \vec{x}) = \varphi_0 \cos(M(\vec{x})t). \tag{51}
\]

In the equation for \(\delta \varphi\), we ignore its gradient term and get

\[
\partial_0^2 \delta \varphi + M^2(\vec{x}) \delta \varphi = -\varphi_0 \left[ t(\nabla M)^2 \cos(Mt) + \nabla^2 M \sin(Mt) \right]. \tag{52}
\]

This ordinary differential equation has the solution

\[
\delta \varphi(t, \vec{x}) = -\frac{\varphi_0 t}{12M} \left[ 2(\nabla \ln M)^2 M^2 t^2 \sin(Mt) - 3(\nabla^2 \ln M)(Mt \cos(Mt) - \sin(Mt)) \right]. \tag{53}
\]

At the origin \(\vec{x} = 0\), the gradient \(\nabla \ln M\) vanishes and Eq. \(53\) simplifies to

\[
\delta \varphi(t, 0) = \frac{\varphi_0 t D}{8 \Delta^2 M_0} \frac{\epsilon}{1 - \epsilon} \left[ \sin(M_0 t) - M_0 t \cos(M_0 t) \right], \tag{54}
\]

where \(D\) is the dimensionality of space. This behaviour is shown in Fig. \(4\) for the parameter values \(D = 1\), \(\varphi =\)
the frequency is lower, and the amplitude of oscillations grows gradually at points where the potential becomes flat, and $|\phi|$ may therefore exceed $\varphi_c$ at certain localized points. Above that value, the potential becomes flat, and $|\varphi|$ can shoot to very high values.

A somewhat similar phenomenon has been discussed by McDonald [29], who showed using a linear analysis that the self-interaction of the inflaton field may lead to the formation of “condensate lumps”. In our case, the interaction with the matter field $\chi$ plays a more important role, because the inflaton-matter coupling is stronger than the inflaton self-coupling and because the instability at the end of inflation generates large inhomogeneities in the $\chi$ field.

If the damping of the oscillations caused by the expansion of the universe or by interactions with other fields is very weak, this overshoot may happen already when $\varphi$ reaches its turning point for the first time. A very rough way of estimating how likely this is consists in calculating the damping of the amplitude during the first oscillation,

$$\frac{\varphi(t_{\text{osc}})}{\varphi(0)} \sim \exp \left( -\frac{3\pi H}{\omega_\varphi} \right) \sim \exp \left( -\frac{3m}{gM_p} \right).$$

Thus, if $m \ll gM_p$, this localized overshoot is likely to happen at the first turning point.

Even if there is no overshoot at that time, the amplitude of oscillations grows gradually at points where the frequency is lower, and $|\varphi|$ may therefore exceed $\varphi_c$ at some later time. We can see this effect in our two-dimensional simulations, even when we take into account the expansion of the universe with the Hubble rate $H \approx 0.0024$, which corresponds to $m/gM_p \approx 0.1$. First, the domain wall network is evolving, with cusps and loop collapses releasing shock waves of energy. After a few oscillations of $\varphi$, however, the energy density and the amplitude of the oscillations of $\varphi$ in certain localized regions start to grow. Eventually, $|\varphi|$ exceeds $\varphi_c$ in these regions, and the $Z_2$ symmetry is locally restored. Because the potential of $\varphi$ is extremely flat above $\varphi_c$, the force that pulls $\varphi$ back towards its minimum, effectively disappears at this point.

A snapshot of the energy density at this stage is shown in Fig. 6. The bright, almost circular disks are regions in which the energy density is very high. In this particular case, all these “hot spots” are located at a domain wall, but this is not always the case. In some simulations, they appeared in places where shock waves form cusps or loop collapses hit each other.

We stress that for the existence of these hot spots, it is crucial that in hybrid inflationary models, the inflaton potential is extremely flat above the critical value. In a model with a convex potential, they would only appear as regions with a slightly higher amplitude. Even with the flat potential, the gradient energy pulls $\varphi$ towards the origin, and therefore it is slowly oscillating around zero with a high amplitude. Of course, there is nothing that would stabilize these hot spots, and therefore they eventually die away having radiated away their energy. It is, nevertheless, interesting to speculate whether they could have significant cosmological consequences, because the local energy density inside them is much higher than the reheat temperature. In more complicated models in which the inflaton $\varphi$ is charged under a continuous global symmetry, the hot spots may, in fact, become stable Q-
are cosmic strings. The potential of the theory is
\[ V(\varphi, \chi) = \frac{1}{2} m_\varphi^2 \dot{\varphi}^2 + g^2 |\chi|^2 \varphi^2 + \lambda \left( |\chi|^2 - v^2 \right)^2 , \] (56)
and the continuum equations of motion in conformal coordinates are [6]
\begin{align}
\partial_0^2 \varphi &= \partial_i \partial_i \varphi - 2g^2 |\chi|^2 \varphi, \quad (57a) \\
\partial_0^2 \chi &= \partial_i \partial_i \chi + (2\lambda v^2 a^2 + g^2 \varphi^2) \chi - 2\lambda |\chi|^2 \varphi. \quad (57b)
\end{align}

The time derivatives are with respect to the conformal time \( \eta \), and we have assumed that the universe is radiation dominated so that the scale factor behaves as \( a = 1 + H \eta \).

The discretization of the field equations was carried out in the same way as in Sect. [V] with lattice size 256\(^2\), lattice spacing \( \delta x = 1.0 \) and time step \( \delta t = 0.05 \) in units with \( m^2 = 2\lambda v^2 = 1 \).

To compare with the results in Ref. [11], we carried out a simulation with the same coupling values \( g = 0.01, \lambda = 0.01, m = 10^{15}\) GeV. In our units, the Hubble rate becomes \( H = 0.0012 \). The initial velocity of the inflaton field was \( \dot{\varphi} = -0.00814 \), and its initial value was such that it reaches \( \varphi_c \) at time \( t = 10 \), i.e., \( \varphi(0) = 100.0814 \). As in Sect. [V], the fields also had Gaussian “quantum” fluctuations in the initial configuration.

In this case \( m/gM_p \approx 0.04 \ll 1 \), and indeed, the inflaton \( \varphi \) overshoots \( -\varphi_c \) in a significant fraction of the space, as shown in Fig. 3a. We have also plotted in the same figure an isosurface of \( |\chi|^2 \) at the time when \( \varphi \) crosses zero. It shows the cosmic strings formed in the transition. The regions with \( \varphi < -\varphi_c \) follow closely the shape of the cosmic string network, in agreement with the discussion in Sect. [V]. In these regions, the inflaton field starts to lag behind and this effect destroys the homogeneity of the oscillations. Indeed, Fig. 8 shows that the oscillations of the spatial average of \( \varphi \) die away during the first period, as observed earlier in Ref. [11]. In fact, a sign of this overshoot can be seen in Fig. 3 of Ref. [11]: at \( t = 130 \), the histogram has two peaks, one in the broken phase, corresponding to the bulk of the space, and one at \( \varphi < -\varphi_c \).

Even though the spatial average does not oscillate, the space is full of oscillating hot spots even at late times, as shown in Fig. 3b. The energy is concentrated in these hot spots, and although \( \varphi \) oscillates with a high amplitude inside each hot spot, the phases are uncorrelated and do not contribute to the spatial average of \( \varphi \).

Eq. (55) predicts that if the mass scale \( m \) is increased, the probability of an overshoot decreases, and the damping is therefore weaker. We studied this by repeating the simulation with \( m = 3 \times 10^{15} \) GeV, which corresponds to \( H = 0.00361 \). The initial values of the inflaton field and its velocity were \( \varphi(0) = 100.732 \) and \( \dot{\varphi} = -0.0732 \). For these parameters, \( m/gM_p \approx 0.12 \), and Fig. 3a confirms that \( \varphi \) exceeds \( -\varphi_c \) only in some small, isolated regions. The spatial average of \( \varphi \) is shown Fig. 8 and
Spontaneous breakdown of a continuous global symmetry gives rise to massless Goldstone bosons, which have not been observed in nature, and therefore it is interesting to consider the case in which the broken symmetry is a local gauge invariance. We need to couple the field $\chi$ to an Abelian gauge field by replacing the derivatives in Eq. (57) by covariant derivatives $D_i = \partial_i + ieA_i$, and introducing the equations of motion for the gauge field

\begin{align}
\partial_0 E_i &= \partial_j F_{ij} + 2e \text{ Im} \chi^* D_i \chi, \\
\partial_i E_i &= 2e \text{ Im} \chi^* \partial_0 \chi,
\end{align}

where $E_i = -\partial_0 A_i$, $F_{ij} = \partial_i A_j - \partial_j A_i$. The initial conditions for the gauge field are analogous to Eq. (13), except that they have to satisfy the Gauss constraint Eq. (58b).

We carried out the discretization of the gauge field equations in the standard way as discussed, say, in Ref. [34], with the scalar fields defined at the lattice sites and the gauge field on links between the sites. The scalar couplings were $g = 0.01$, $\lambda = 0.01$ and $m = 3 \times 10^{15}$ GeV, and the gauge coupling was $e = 0.1$. For the Hubble rate, these parameters correspond to the value $H = 0.00361$. The initial values of the inflaton field and its velocity were $\varphi = 100.732$ and $\dot{\varphi} = -0.0732$.

From Fig. 8 we can see that the gauge field does not change the dynamics significantly. Therefore, the existence of Goldstone modes is not crucial for the damping.

In thermal phase transitions the gauge field plays a crucial role in defect formation [28, 33], but its effect depends on the temperature, and as the transition takes place at zero temperature, the gauge fields can be neglected. Indeed, the total length of string immediately after the transition is similar for global and local theories. Therefore we expect Eq. (14) to be valid in gauge field theories, as well. Because global strings have long-range interactions while local strings do not, the string network decays much faster in the global case.

The result that Eq. (14) applies to gauge theories, supports the analytic estimates given in Ref. [14] for baryon density in a scenario in which the baryon asymmetry is generated by unwinding electroweak textures formed by the Kibble mechanism at the end of electroweak-scale hybrid inflation. The predicted number density of these “knots” is $\rho_H \approx \xi^{-3} \sim k^3 \approx 2mg\dot{\varphi}$. Once formed, they can decay either by changing the topology of the Higgs or the SU(2) gauge field configuration, and as pointed

\begin{align}
\partial_0 E_i &= \partial_j F_{ij} + 2e \text{ Im} \chi^* D_i \chi, \\
\partial_i E_i &= 2e \text{ Im} \chi^* \partial_0 \chi,
\end{align}

where $E_i = -\partial_0 A_i$, $F_{ij} = \partial_i A_j - \partial_j A_i$. The initial conditions for the gauge field are analogous to Eq. (13), except that they have to satisfy the Gauss constraint Eq. (58b).

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The result that Eq. (14) applies to gauge theories, supports the analytic estimates given in Ref. [14] for baryon density in a scenario in which the baryon asymmetry is generated by unwinding electroweak textures formed by the Kibble mechanism at the end of electroweak-scale hybrid inflation. The predicted number density of these “knots” is $\rho_H \approx \xi^{-3} \sim k^3 \approx 2mg\dot{\varphi}$. Once formed, they can decay either by changing the topology of the Higgs or the SU(2) gauge field configuration, and as pointed
out by Turok and Zadrozy [34, 35], CP violation biases the decay of the winding configurations. Thus, if CP violation is present, this mechanism leads to baryon asymmetry. For a reliable calculation of the baryon density, one has to understand the dynamics of the gauge field in the presence of CP violation.

The further evolution of the system confirms that the hot spots with local symmetry restoration appear also in the same way as in thermal phase transitions. We also studied the dynamics of the system at later times, and showed that oscillations of the inflaton field coupled to the inhomogeneities of the matter field lead to hot spots of high energy density, inside of which the symmetry can become temporarily restored. Depending on the coupling constants, this effect may be very strong, in which case it damps down the homogeneous oscillations of the inflaton field. We believe this explains the high effectiveness of tachyonic preheating observed in Ref. [36]. Further work is required for relating the density and sizes of these hot spots to the parameters of the model, and for understanding their other possible consequences.

VIII. CONCLUSIONS

In this paper, we have studied tachyonic preheating in detail using both analytical and numerical techniques, and taking the full dynamics of the inflaton field into account. We simulated the non-perturbative dynamics after the spinodal time in two and three dimensional scalar field theories and a three-dimensional gauge field theory, and showed that at early times, the instability is well described by a linear approximation. When non-linearities set in, the long-wavelength fluctuations form topological defects, and our results show that their number density is related to the cutoff scale $k_*$ in the same way as in thermal phase transitions.

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