Resistance scaling for Composite Fermions in the presence of a density gradient

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Resistance scaling for Composite Fermions (CFs) of the same flavor (or the same in CF transport is found in the scaling of the resistivity resolved.

speculated typically by a factor of 3 and more. Although it was are always smaller than the theoretically predicted ones, of the impurities from the 2DES. However, it has long samples the 2DES often becomes insulating beyond an almost perfect linear relationship between $R_{xx}$ and $B$ emerges over the whole magnetic field range except for spikes at the integer quantum Hall states. This linear $R_{xx}$ cannot be understood within the Composite Fermion model, but can be explained through the existence of a density gradient in our sample.

The Composite Fermion (CF) model has been very successful in explaining the Fermi liquid like behavior at even denominator Landau level fillings in a two dimensional electron system (2DES). Within this model, a CF is formed by attaching 2 flux quanta to each electron at filling factor $\nu = 1/2\phi$, where $\phi$ is positive integer. Due to this flux attachment, the CFs see a zero effective $B$ field. Furthermore, since the strong electron-electron interaction is effectively removed by forming the CFs, the resulting CF-CF interaction is very weak. Consequently, at low temperatures, their transport properties can be well described by Fermi liquid theory.

Over the years, the CF model has been tested and verified in many types of experiments. However, a few unresolved issues remain. Among them is the resistivity of CFs. Unlike ordinary electrons, whose main scattering mechanism is Coulomb scattering from residual impurities within the samples, it is believed that for the CFs the main scattering mechanism is due to the gauge field fluctuations introduced by the same impurities. Therefore, the resistivity of CFs is given by the formula: $\rho_{xx}^{CF} \approx (n_{imp}/n) \times (\pi \phi^2/k_f d_s e^2)$, where $n_{imp}$ is the density of residual impurities, $n$ is the 2DES density, $k_f$ is the CF Fermi wavevector, and $d_s$ is the spacing of the impurities from the 2DES. However, it has long been noticed that the experimental values at $\nu = 1/2$ are always smaller than the theoretically predicted ones, typically by a factor of 3 and more. Although it was speculated that this discrepancy might be related to the specifics of the density inhomogeneity the issue was never resolved.

Another discrepancy between theory and experiment in CF transport is found in the scaling of the resistivity for CFs of the same flavor (or the same $\phi$), and/or of different flavors. According to the above equation, one would expect $R_{\nu=1/2} = 3^{1/2} \times R_{\nu=3/2}$, and $R_{\nu=1/4} = 4 \times R_{\nu=1/2}$. So far, this CF resistivity scaling, especially the one between $\nu = 1/2$ and $1/4$, could not be clearly tested. The primary reason is that even in very high mobility samples the 2DES often becomes insulating beyond $\nu = 1/2$. Consequently, the resistivity at $\nu = 1/4$ becomes very large and a comparison becomes meaningless.

Recently many high order FQHE states, e.g., the $\nu = 4/11, 10/21$ and $10/19$ states around $\nu = 1/2$ and the $\nu = 6/23$ and $6/25$ states around $\nu = 1/4$, were observed. Such ultra high quality specimen allows for a reliable and in depth investigation of the resistivity of CFs at the even-denominator fillings, i.e., $\nu = 1/4, 1/2, 3/4, 3/2$. As it turns out, the resistivity at even-denominator filling factors is found to be linear in $B$ field, which is at variance from standard CF transport theory. This linear magnetoresistance (MR) becomes very pronounced at a high temperature of $T \approx 1.2$ K, where only a few sharp spikes from the integer quantum Hall states disrupt an otherwise strictly linear relationship between $R_{xx}$ and $B$. Such a linear MR is not consistent with the resistivity scaling from the CF model. However, all such features can be understood assuming a small electron density gradient within the 2DES.

The sample consists of a symmetrically doped quantum well of width 500 Å. The setback distance of the modulation doping is $d_s = 2200$ Å. An electron density of $n \approx 1 \times 10^{13}$ cm$^{-2}$ and a mobility $\mu \approx 1 \times 10^7$ cm$^2$/Vs were achieved after illumination of the sample at low temperatures by a red light-emitting diode (LED). A self-consistent calculation shows that at this density only one electrical subband is occupied. Conventional low-frequency ($\sim 7$ Hz) lock-in amplifier techniques were em-
ployed to measure the diagonal magnetoresistance $R_{xx}$ and Hall resistance $R_{xy}$.

In Figure 1a, we plot the $R_{xx}$ data at $T \sim 35$ mK. This data was shown earlier in Ref. [7] in the context of the discovery of new FQHE states. Here, we focus on the extended straight sections around $\nu = 1/4$ and $\nu = 1/2$. In most previous experiments the data beyond $\nu = 1/3$ experienced a considerable increase in resistivity, often tending towards infinity as $T$ towards zero. Such divergent behavior is closely correlated with sample quality as measured by mobility and is generally attributed to magnetic field induced localization, which is furthered by increased disorder. The absence of such a rising background in our data and the lack of a temperature dependence in the $\nu = 1/4$ regime attests to the ultra-high quality of our sample. It renders this sample an excellent candidate for a study of CF transport behavior. In Fig. 1b, the resistance at the even-denominator fillings $\nu = 3/2$, 3/4, 1/2, and 1/4 from three different cool-downs are plotted versus $B$ field. A linear dependence on $B$ field is clearly observed.

This linear $B$ field dependence becomes more extended at higher temperatures as seen in Fig. 2, where we show $R_{xx}$ and $R_{xy}$ at $T = 1.2 K$. At such high temperature, $R_{xy}$ behaves practically classically and is linear in $B$ field. What is surprising is that $R_{xx}$ also shows a linear $B$ dependence over the whole $B$ field range, except at those positions where the integer quantum Hall states start to form.

![FIG. 2: Diagonal resistance $R_{xx}$ and Hall resistance $R_{xy}$ at $T \sim 1.2 K$. IQHE states are forming at the Landau level fillings $\nu = 1$, 2, and 4](image)

This linear MR is puzzling. First, it is inconsistent with the semiclassical theory of Lifshitz-Azbel-Kaganov, which states that in a Fermi liquid with a close Fermi surface, MR should saturate when $\omega_c \tau = \mu B \gg 1$, where $\omega_c = \hbar B/m^*$ is the cyclotron frequency, $\tau$ is the scattering time, and $m^*$ is the electron effective mass. Therefore, in our specimen with $\mu \sim 1 \times 10^7$ cm$^2$/Vs, $R_{xx}$ should be constant beyond $B \sim 10^{-3}$ T. Second, the linear MR cannot be understood within the CF model, which requires $\rho_{xx}^{CF} \approx (n_{imp}/n) \times (\pi \delta^2/\kappa f d q e^2)$. Consequently, one would expect that $R_{1/4} = 4 \times R_{1/2}$ and $R_{1/2} = 3^{1/2} \times R_{1/2}$. The experimental data, however, indicate $R_{1/4} = 2 \times R_{1/2}$, and $R_{1/2} = 3 \times R_{3/2}$, which is clearly different. Linear MR in a 2DES has been observed before$^{9,10,11}$. In fact, in one publication$^{10}$, the authors suggested that it might be related to a density inhomogeneity.

To understand the physical origin of the linear MR, we
draw from our recent publication\textsuperscript{12} on the empirical resistivity rule\textsuperscript{13}. There it was shown that $R_{xx}$ data in the first and second Landau levels turned out to be merely a reflection of $R_{xy}$. This relationship was caused by an unintentional electron density gradient, $\Delta n$, in the sample and expressed by an earlier, empirical resistivity rule\textsuperscript{13}

$$R_{xx} = R_{xy}(n) - R_{xy}(n + \Delta n) = c \times B \times dR_{xy}/dB \quad (1)$$

where the constant is now determined to be $c = \Delta n/n$. Following this recent insight we examine our data in this light. Obviously, an $R_{xy} \propto B$ leads to a linear $R_{xx}$, suggesting that its origin is again an unintentional electron density gradient in our sample. Moreover, if we calculate $R_{xx}$ directly from the $R_{xy}$ data according to Eq.(1) we reach practically perfect agreement with our $R_{xx}$ data (as shown in Fig.3), assuming a relative density gradient of $c = \Delta n/n = 0.5\%$. This is very strong evidence that the linear MR is a result of $R_{xy}$ via the resistivity rule, which has been traced back to a density gradient.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{Comparison of $R_{xx}$ (black trace) and $\Delta R_{xy}$ (noisy gray trace).}
\end{figure}

It is remarkable that three apparently unrelated transport features, the linear MR, $R_{xx}$ quantization\textsuperscript{14} and the resistivity rule\textsuperscript{13}, can all be explained by the empirical density gradient model, and that the explanation holds well over an extremely wide temperature range, from 6 mK to 1.2 K and may well hold beyond this range. This simple, classical explanation of $R_{xx}$ data in terms of $R_{xy}$ raises the question as to its connection to the intrinsic resistivity $\rho_{xx}$. Recent theoretical work is addressing this relationship\textsuperscript{15}.

As to the resistivity at even denominator fillings and their theoretical relationships it is no longer surprising that they are not borne out in experiment. $R_{xx}$ is the result of a density gradient and really only a reflection of $R_{xy}$. Therefore, typical $R_{xx}$ data say little about $\rho_{xx}$ and hence little about the scattering behavior of the CFs. The extraction of reliable $\rho_{xx}$ values from $R_{xx}$ data on 2DEGs will require either the reduction or elimination of residual electron density gradients or the application of appropriate correction formulae, based on realistic models for the density distribution as they are presently being developed\textsuperscript{16}.

Before concluding we would like to put our data and their interpretation in the context of a wider scope: Recently a semiclassical approach has been proposed to explain the linear magnetoresistance in 2DES\textsuperscript{15}. Within this model, a linear MR arises from a competition between the long-range and the short-range disorder potentials. This model was tested in a 2DES with an antidot array, to create the short-range disorder potential\textsuperscript{16}, and provided general support. However, it appears unlikely that this model would also apply to our setting, since our sample is unpatterned. Moreover, in antidots\textsuperscript{16}, the linear MR was strong only at high temperatures, whereas, in our sample, the linear MR was observed from 35 mK to our maximum temperature of 1.2 K. Finally, linear MR was frequently observed in the past in many three-dimensional simple metals\textsuperscript{17}. It is now largely accepted that density inhomogeneities are responsible for this behavior. In a broader sense, this is similar to the present 2D case. However, it is not known whether in 3D metals this special $R_{xx}$ dependence can be related to $R_{xy}$ in the same way as it seems to be related in our 2DES specimens. Maybe such an observation will require a special form of inhomogeneity, e.g., a simple density gradient.

In summary, in an ultra high quality two-dimensional electron system, a linear magnetoresistance is observed, which implies particular ratios for the resistivity of CFs at different, even-denominator filling factors. We show that such a linear $B$ field dependence of $R_{xx}$ cannot be understood within the scattering model for CFs. Rather, it can be reproduced by the Hall resistance $R_{xy}$, based on an empirical density gradient model.

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1. J.K. Jain, Phys. Rev. Lett. 63, 199 (1989).
2. A. Lopez and E. Fradkin, Phys. Rev. B 44, 5246 (1992).
3. B.I. Halperin, P.A. Lee, and N. Read, Phys. Rev. B 47, 7312 (1993).
4. J.K. Jain, Physics Today 53, 39 (2000).
5. B.I. Halperin, Chapter 6 in Perspectives in Quantum Hall Effects, S. Das Sarma and A. Pinczuk (Eds.), Wiley, New York (1996).
6. H.W. Jiang, R.L. Willett, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 65, 633 (1990).
7. W. Pan, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. Lett. 90, 016801 (2003).
8. I.M. Lifshitz, M. Azbel, and M.I. Kaganov, JETP 4, 41 (1957).
9. T. Rötger, G.J.C.L. Bruls, J.C. Maan, P. Wyder, K. Ploog, and G. Weimann, Phys. Rev. Lett. 62, 90 (1989).
10. H. Hirai, S. Komiyama, S. Sasa, and T. Fujii, Solid State Communications 72, 1033 (1990).
11. H.L. Stormer, K.W. Baldwin, L.N. Pfeiffer, and K.W. West, Solid State Communications 84, 95 (1992).
12. W. Pan, J.S. Xia, H.L. Stormer, D.C. Tsui, C.L. Vicente, E.D. Adams, N.S. Sullivan, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. Lett. 95, 066808 (2005).
13. S.H. Simon and B.I. Halperin, Phys. Rev. Lett. 73, 3278 (1994), and references therein.
14. N. Cooper and A. Stern, private communications.
15. D.G. Polyakov, F. Evers, A.D. Mirlin, and P. Wölfle, Phys. Rev. B 64, 205306 (2001).
16. V. Renard, Z.D. Kvon, G.M. Gusev, and J.C. Portal, Phys. Rev. B 70, 033303 (2004).
17. See, for example, M.M. Parish and P.B. Littlewood, Nature 426, 162 (2003); and references therein.