Learning Self-Game-Play Agents for Combinatorial Optimization Problems

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ABSTRACT

Recent progress in reinforcement learning (RL) using self-game-play has shown remarkable performance on several board games (e.g., Chess and Go) as well as video games (e.g., Atari games and Dota2). It is plausible to consider that RL, starting from zero knowledge, might be able to gradually approximate a winning strategy after a certain amount of training. In this paper, we explore neural Monte-Carlo-Tree-Search (neural MCTS), an RL algorithm which has been applied successfully by DeepMind to play Go and Chess at a super-human level. We try to leverage the computational power of neural MCTS to solve a class of combinatorial optimization problems. Following the idea of Hintikka’s Game-Theoretical Semantics, we propose the Zermelo Gamification (ZG) to transform specific combinatorial optimization problems into Zermelo games whose winning strategies correspond to the solutions of the original optimization problem. The ZG also provides a specially designed neural MCTS. We use a combinatorial planning problem for which the ground-truth policy is efficiently computable to demonstrate that ZG is promising.

KEYWORDS

Reinforcement Learning; neural MCTS; Self-game-play; Combinatorial Optimization; Tabula rasa

1 INTRODUCTION

The past several years have witnessed the progress and success of reinforcement learning (RL) in the field of game-play. The combination of classical RL algorithms with newly developed deep learning techniques gives a stunning performance on both traditional simple Atari video games ([12]) and modern complex RTS games (like Dota2 [14]), and even certain hard board games like Go and Chess. One common but outstanding feature of those learning algorithms is the tabula-rasa style of learning. In terms of RL, all those algorithms are model-free and learn to play the game with zero knowledge in the beginning. Such tabula-rasa learning can be regarded as an approach towards a general artificial intelligence.

We transform a family of combinatorial optimization problems (e.g., the Product Stress Testing Problem $HSR_{k,q}$) into games via a process called Zermelo Gamification, so that a game-play agent can be leveraged to play the transformed game and solve the original problem on a specific instance (e.g., $HSR_{7,7}$). In Fig. 5 one can see how two competitive agents, called P and OP, gradually, but with setbacks (as in AlphaZero [16] and [18]), improve and jointly arrive at the optimal strategy. The tabula-rasa learning converges and solves a non-trivial problem, although the underlying game is fundamentally different from Go and Chess. The trained game-play agent can be used to indicate the solution (or non-existence of a solution) of the original problem through competitions against itself based on the learned strategy. Hence solving a combinatorial problem in this way not only gives the solution to the problem, but also indications to the reason why it works/does not work. The indications are a benefit of the constructive nature of the semantic games defined by Hintikka’s approach to predicate logic.

One critical issue with those transformed games is that they usually have a large state space as well as a sparse reward function. Therefore, REINFORCE [21] style algorithms and even its modern-day advanced variation PPO [14] or DQN [12] can’t be applied to such kind of problems, because they all require a ‘dense’ reward function so that a feedback signal is generated after each action. However, one can hardly define a reward function on games like Sokoban or Sudoku where an agent has no idea whether the current action is correct or not until a much later phase or the end of the game. Imagination-Augmented Agents (I2As [20]), an algorithm invented by DeepMind, is used to handle such complex games. Although the algorithm has well performed, it is not model-free. Namely, one has to train, in a supervised way, an imperfect but adequate model first, then use that model to boost the learning process of a regular model-free agent. Even though I2As, along with a trained model, can solve games like Sokoban to some level, I2As can hardly be applied to games where even the training data is limited and hard to generate or label. In other words, a model-free learning algorithm is needed to play generally on different kinds of games. So we turn to the neural MCTS method used in AlphaZero by DeepMind. To our knowledge, this kind of neural MCTS is the only tabula-rasa style algorithm which can handle games with both large state spaces and sparse rewards as well.

We make three main contributions: (1.) We introduce the Zermelo Gamification which consists of two contributions (1.a) a way to generalize combinatorial problems to more comprehensive combinatorial problems in the form of Zermelo games following the approach in Hintikka’s Game-Theoretical Semantics [7]; (1.b) we implemented a variant of the neural MCTS algorithm specifically designed for those Zermelo games; 2. Evaluation: we evaluate our algorithm on a problem (i.e., $HSR$) for which a Bernoulli’s triangle shows the winning strategy efficiently. Our result shows that, for problems under a certain size, the $HSR$ Zermelo Gamification does find the optimal strategy, hence solving the original optimization problem in a tabula-rasa style. 3. Indications: We show how the

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winning strategy for both players of the Zermelo game provides indications that a given problem instance does (not) have a solution. Those indications are made possible through the generalization mentioned in contribution 1 and they are more useful then the simple answer: there is no solution.

The remainder of this paper is organized as follows. Section 2 presents essential preliminaries on neural MCTS and certain combinatorial optimization problems. Section 3 introduces a general way to transform certain combinatorial optimization problems into two-player games, where we specifically discuss the transforming of the HSR game. Section 4 gives our correctness measurement and presents experimental results. 5 and 7 made a discussion and conclusions.

2 PRELIMINARIES

2.1 Monte Carlo Tree Search

The PUCT algorithm implemented in AlphaZero [17, 18] is essentially a neural MCTS algorithm which uses PUCB [2, 13] as its confidence upper bound [1, 10] and uses the neural prediction \( P_\theta(a|s) \) as the predictor [13]. The algorithm usually proceeds through 4 phases (S.E.R.B) during each iteration:

1. SELECT: At the beginning of each iteration, the algorithm select a path from the root (current game state) to a leaf (either a terminal state or a unvisited state) in the tree according to the PUCB. Specifically, suppose the root is \( s_0 \), we have 1:

   \[
   a_{i-1} = \arg \max_a Q(s_{i-1}, a) + cP_\theta(a|s_{i-1}) \sqrt{\sum_{a'} N(s_{i-1}, a') / N(s_{i-1}, a) + 1} \]

   \[
   Q(s_{i-1}, a) = \frac{W(s_{i-1}, a)}{N(s_{i-1}, a) + 1} \]

   \( s_i = \text{next}(s_{i-1}, a_{i-1}) \)

2. EXPAND: Once the select phase ends at a non-terminal leaf, the leaf will be fully expanded and marked as an internal node of the current tree. All its children nodes will be considered as leaf nodes during next iteration of selection.

3. ROLL-OUT: Normally, starting from the expanded leaf node chosen from previous phases, the MCTS algorithm uses a random policy to roll out the rest of the game [5]. The algorithm simulates the actions of each player randomly until it arrives at a terminal state which means the game has ended. The result of the game (winning information or ending score) is then used by the algorithm as a result evaluation for the expanded leaf node.

   However, a random roll-out usually becomes time-consuming when the tree is deep. A neural MCTS algorithm, instead, uses a neural network \( V_\theta \) to make a prediction of the result evaluation so that the algorithm saves the time on rolling out.

   1Theoretically, the exploratory term should be \( \sqrt{\sum_{a'} N(s_{i-1}, a') / N(s_{i-1}, a) + 1} \), however, the AlphaZero used the variant \( \sqrt{\sum_{a'} N(s_{i-1}, a') / N(s_{i-1}, a)} \) without any explanation. We tried both in our implementation, and it turns out that the AlphaZero one performs much better.

4. BACKUP: This is the last phase of an iteration where the algorithm recursively back-ups the result evaluation in the tree edges. Specifically, suppose the path found in the Select phase is \( \{s_0, a_0, (s_1, a_1), ... (s_{j-1}, a_{j-1}), (s_j, \_ \}_ \} \), then for each edge \( (s_i, a_j) \) in the path, we update the statistics as:

   \[
   W^{\text{new}}(s_i, a_j) = W^{\text{old}}(s_i, a_j) + \rho(s_j)
   \]

   \[
   N^{\text{new}}(s_i, a_j) = N^{\text{old}}(s_i, a_j) + 1
   \]

   However, in practice, considering the +1 smoothing in the expression of \( Q \), the following updates are actually applied:

   \[
   Q^{\text{new}}(s_i, a_j) = \frac{Q^{\text{old}}(s_i, a_j) \times N^{\text{old}}(s_i, a_j) + V_\theta(s_j)}{N^{\text{old}}(s_i, a_j) + 1}
   \]

   \[
   N^{\text{new}}(s_i, a_j) = N^{\text{old}}(s_i, a_j) + 1
   \]

   Once the given number of iterations has been reached, the algorithm returns a vector of action probabilities of the current state (root \( s_0 \)). And each action probability is computed as \( \pi(a|s_0) = \frac{N(s_i, a)}{\sum_{a'} N(s_i, a')} \) The real action played by the MCTS is then sampled from the action probability vector \( \pi \). In this way, MCTS simulates the action for each player alternately until the game ends, this process is called MCTS simulation (self-play).

An MCTS algorithm often consists of several simulations (self-play). In each simulation, the algorithm will play one complete game step by step. Each step was chosen by running several iterations of S.E.R.B. as mentioned above. And for a neural MCTS structure, the MCTS usually needs hundreds of simulations to generate enough training data.

2.2 Combinatorial Optimization Problems

The combinatorial optimization problems studied in this paper can be described with the following logic statement:

\[
\forall n : \{G(n) \land (\forall n' > n \rightarrow \neg G(n'))\}
\]

\[
G(n) := \exists x \forall y : \{F(x, y; n)\}
\]

or

\[
G(n) := \exists y \forall x : \{F(x, y; n)\}
\]

In this statement, \( n \) is a natural number and \( x, y \) can be any instances depending on the concrete problem. \( F \) is a predicate on \( n, x, y \). Hence the logic statement above essentially means that there is a maximum number \( n \) such that for all \( x \), some \( y \) can be found so that the predicate \( F(x, y; n) \) is true. Formulating those problems as interpreted logic statements is crucial to transforming them into games (Hintikka [7]). Next, we will introduce two specific examples of problems which can be transformed into the given logic form.

2.2.1 Silver Ratio. Our first example is a classical function optimization problem which involves to approximate the saddle point of a 2-variable function: \( f_d(x, y) = xy + (1 - x)(1 - y^2); x, y \in I_d = [0, 1, \frac{1}{d}, \frac{2}{d}, ..., \frac{d-1}{d}, 1], d \in \mathbb{N}^+ \). The problem can be defined as:

\[
z_d = \min_{x \in I_d} \max_{y \in I_d} f_d(x, y), d \in \mathbb{N}^+
\]

Therefore the corresponding logic statement of this problem can be formulated as:

\[
\exists z_d : \{G(z_d) \land (\forall z' > z_d \rightarrow \neg G(z'))\}
\]
We have the Testing Problem: consider throwing jars from a specific rung of a ladder, the jars could either break or not. One has identical jars and test chances to throw those jars, can one locate the highest safe rung on the ladder?

The solution to the original HSR problem is a number \( n = N(k, q) \) such that \( \exists k \in \mathbb{N}, \forall n' > n \exists G_k, q(n') \). Theorem 1 states that if one knows the highest safe rung, the ladder could have hundreds of rungs while there are only a few jars and test chances so that the resources are obviously insufficient. Now we define a predicate \( \exists k \in \mathbb{N} : \{G_k, q(n') \land (\forall n' > n \sim G_k, q(n'))\} \)

The reason behind the recursion is that if one knows the highest safe rung which one can be handled by \((k - 1, q - 1)\) and \((k, q)\), then one can just concatenate those two ladders together to get a higher ladder which can exactly be handled by \((k, q)\). On the other hand, if one has been given a ladder of \( N(k, q) \) rungs, one can divide it into \( N(k - 1, q - 1) \) and \( N(k, q - 1) \) by testing the first jar on the concatenating point. If the jar broke, then we got \( k - 1 \) jars and \( q - 1 \) chances left, otherwise, we still got \( k \) jars left but \( q - 1 \) chances.

Summarizing the analysis above, the solution for the HSR problem can be represented efficiently with a Bernoulli’s Triangle (Fig. 1).

3 ZERMELO GAMIFICATION

3.1 General Formulation

We introduce the Zermelo Gamification (ZG) to transform a combinatorial optimization problem \( X \) into a Zermelo game. We call this the X Zermelo Gamification. For now, the X Zermelo Gamification is a manual compilation process which takes as input a combinatorial optimization problem \( X \), translates it into a semantic game (Zermelo game) that is fit for being used by a specifically designed neural MCTS algorithm to find a winning strategy for the Zermelo gamification. The winning strategy can be translated into a solution of the input combinatorial problem \( X \) for a specific instance. We will illustrate the Zermelo Gamification by deriving the HSR Zermelo Gamification.

A Zermelo game is defined to be a two-player, finite, and perfect information game with only one winner and loser, and during
the game, players move alternately (i.e., no simultaneous move). Leveraging the logic statement (see section 2.2) of the problem, the Zermelo game is built on the Game-Theoretical Semantic approach (by Hintikka [7]). We introduce two roles: the Proponent (P), who claims that the statement is true, and the Opponent (OP), who argues that the statement is false. The original problem can be solved if and only if the P is able to propose some optimal number $n$ so that a perfect OP cannot refute it. To understand the game mechanism, let’s recall the logic statement in section 2.2:

$$\exists n : (G(n) \wedge (\forall n' > n \neg G(n')))$$

This statement implies the following Zermelo game (Fig. 2):

3.1.1 Proposal Game. In the initial phase of the Zermelo game player P will propose a number $n$. Then the player OP will decide whether to accept this $n$, or reject it. OP will make his decision based on the logic statement: $A \wedge B, A := G(n), B := \forall n' > n \neg G(n')$. Specifically, the OP tries to refute the P by attacking either on the statement $A$ or $B$. The OP will accept $n$ proposed by the P if she confirms $A = \text{False}$. The OP will reject $n$ if she is unable to confirm $A = \text{False}$. In this case, the OP treats $n$ as non-optimal, and proposes a new $n' > n$ (in practice, for integer $n$, we take $n' = n + 1$) which makes $B = \text{False}$). To put it in another way, $B = \text{False}$ implies $\neg B = \text{True}$ which also means that the OP claims $G(n')$ holds. Therefore, the rejection can be regarded as a role-flip between the two players. In order to make the Zermelo non-trivial, in the following game, we require that the P has to accept the new $n'$ and tries to figure out the corresponding $y$ to defeat the OP. Notice that since this is an adversarial game, the OP will never agree with the P (namely, the OP will either decide that the $n$ is too small or too large because the OP has to regard the move made by the P as incorrect). Therefore, the OP is in a dilemma when the P is perfect, i.e., the P chooses the optimal $n$.

3.1.2 Refutation Game. This is the phase where the two players actually search for evidence and construct strategies to attack each other or defend themselves. Generally speaking, regardless of the role-flip, we can treat the refutation game uniformly: the P claims $G(n)$ holds for some $n$, the OP will refute this claim by giving some instances of $x$ (for existential quantifier) so that $\neg G(n)$ holds. If the P successfully figures out the exceptional $y$ (for universal quantifier) which makes $F(x, y; n)$ hold, the OP loses the game, otherwise, the P loses.

The player who takes the first move is decided by the order of the quantifiers, namely, for $G(n) := \exists x \forall y : \{F(x, y; n)\}$ the P will take the first move; for $G(n) := \forall y \exists x : \{F(x, y; n)\}$ the OP will take the first move. The game result is evaluated by the truth value of $F(x, y; n)$, specifically, if the P takes the last move then she wins when $F(x, y, n)$ holds; otherwise, if the OP takes the last move then she wins when $F(x, y, n)$ doesn’t hold. Also notice that, in an extreme case where $G(n)$ is true and both of the players are perfect, the P can always win the game while the OP always loses the game. In other words, the OP is in a dilemma when the P is perfect, i.e., the P always chooses the optimal $m$. Vice versa, for a problem without any solution, the P is in a dilemma when the OP is perfect.

Notice that the general form of the Zermelo game can already handle most cases of combinatorial optimization problem which has simple parameters (i.e. $x, y$). It is obvious that for the Silver

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{A general Zermelo game where white nodes stand for the P’s turn and black nodes stand for the OP’s turn. A role-flip happened after OP’s rejecting of $n$. The refutation game can be treated uniformly where, depending on the order of the quantifiers, the OP takes the first move. The OP wins if and only if the P fails to find any $y$ to make $F(x, y; n)$ holds, hence the game result $R = \neg F(x, y, n)$.}
\end{figure}

Ratio example, it is trivial to transform it into this form of gameplay. However, for problems which have complex parameters (e.g. HSR where we have a policy $\pi$ under the existential quantifier), a further transform and refinement of the refutation game have to be applied, which we will discuss in section 3.3.

3.2 Indications

Zermelo Gamification helps one to explain the output of the neural network and hence to better understand the problem. A correctly trained neural network provides constructive answers both for a positive problem as well as a negative problem (which does not have a solution). For a positive problem, a solution is found when the P can maintain her winning position. For a negative problem, the neural network behaves oppositely so that the OP maintains her winning position, which indicates that any attempt by the P to construct a solution results in a contradiction. An indication is more useful than the answer no since the indication is given during an attempt to construct a solution and requires a “tightrope walk” of the neural network to produce a contradiction. Indications are not proofs but small steps in a case-by-case proof.

3.3 HSR Game

In this section we introduce the HSR Game, our experiment subject, which comes from the HSR problem (section 2.2.2). By simply following the Zermelo Gamification in section 3.1, one can derive the game immediately (Fig. 3). However, it is an abstract game because the refutation phase requires a search of $\pi$ from some policy space, which is complex. Therefore, a further transformation and
Notice that in this new expression, \( x \) becomes irrelevant to the recursion. This is because all \( x \) chosen by the OP only lead to two test results. Hence instead of searching all possible \( x \), she can directly enumerate the two results: break or not break. Therefore \( G_{k,q}(n) \) can be further simplified as:

\[
G_{k,q}(n) = \begin{cases} 
\text{True, if } n = 0 \\
\text{False, if } n > 0 \land (k = 0 \lor q = 0) \\
\exists m \leq n : \{G'_{k-1,q-1}(m-1) \land G_{k,q}(n-m)\}
\end{cases}
\]

With \( G'_{k,q}(n) \), we refine the abstract refutation game into a concrete game where the P and OP move alternately. In this new refutation game (Fig. 4), during each round, the P will pick a testing point \( m \), then the OP will reply “break” or “not break”. Specifically, in each round, the P claims \( \exists m \leq n : \{G'_{k-1,q-1}(m-1) \land G'_{k,q}(n-m)\} \) holds by giving an instance of \( m \). Then the OP refutes the claim by either refuting \( G'_{k-1,q-1}(m-1) \) or \( G'_{k,q}(n-m) \). In this way, both the policy \( \pi \) and highest safe rung \( x \) are constructed implicitly and none of them need to be searched explicitly.

### 4 EXPERIMENT

#### 4.1 Neural MCTS implementation

In this section, we will discuss our neural MCTS implementation on the HSR game. Since the Zermelo game has two phases and the learning tasks are quite different between these two phases, we applied two independent neural networks to learn the proposal game and refutation game respectively. The neural MCTS will access the first neural network during the proposal game and then the second

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**Figure 3:** An abstract HSR game where the parameter \( \pi \) is complex since it is searched through a policy space. Also notice that the order of player in the refutation game depends on the order of quantifiers: the P will move first in this game. Notice that \( F(x, \pi, n; k, q) \) means \( F_{k,q}(x, \pi, n) \) in this figure. 

**Figure 4:** Refined design of the HSR refutation game, which is also the one used in our experiment. There are between 2 and 2q moves before a game ends. Notice that \( G(n; k, q) \) means \( G_{k,q}(n) \) in this figure.
neural network during the refutation game. There are also two independent replay buffers which store the self-play information generated from each phase, respectively.

Our neural network consists of four layers of 1-D convolution neural networks and two dense layers. The input is a tuple \((k, q, n, m, r)\) where \(k, q\) are resources, \(n\) is the number of rungs on the current ladder, \(m\) is the testing point and \(r\) indicates the current player. The output of the neural network consists of two vectors of probabilities on the action space for each player as well as a scalar as the game result evaluation.

During each iteration of the learning process, there are three phases: 1. 100 episodes of self-play will be executed through a neural MCTS using the current neural network. Data generated during self-play will be stored and used for the next phase. 2. the neural networks will be trained with the data stored in the replay buffer. And 3. the newly trained neural network and the previous old neural network are put into a competition to play with each other. During the competition phase, the new neural network will first play as the OP for 20 rounds, then it will play as the P for another 20 rounds. We collect the correctness data for both of the neural networks during each iteration.

We shall mention that since it is highly time-consuming to run a complete Zermelo game on our machines, to save time and as a proof of concept, we only run the complete game for \(k = 7, q = 7\) and \(n \in [1...130]\). Nevertheless, since the refutation game, once \(n\) is given, can be treated independently from the proposal game, we run the experiment on refutation games for various parameters.

4.2 Correctness Measurement

Informally, an action is correct if it preserves a winning position. It is straightforward to define the correct actions using the Bernoulli Triangle (section 2.2.2).

4.2.1 P’s correctness. Given \((k, q, n)\), correct actions exist only if \(n \leq N(k, q)\). In this case, all testing points in the range \([n - N(k, q - 1), N(k - 1, q - 1)]\) are acceptable. Otherwise, there is no correct action.

4.2.2 OP’s correctness. Given \((g, k, n, m)\), When \(n > N(k, q)\), any action is regarded as correct; when \(n \leq N(k, q)\), the OP should take the action “not break” if \(m < n - N(k, q - 1)\) and take action “break” if \(m > N(k - 1, q - 1)\). Otherwise, there is no correct action.

4.3 Complete Game

In this experiment, we run a full Zermelo game under the given resources \(k = 7, q = 7\). Since there are two neural networks which learn the proposal game and the refutation game respectively, we measure the correctness separately: Fig. 5 shows the ratio of correctness for each player during the proposal game. And Fig. 6 shows the ratio of correctness during the refutation game. The horizontal axis is the number of iterations and it can be seen that the correctness converges extremely slow (80 iterations). It is because, for \(k = 7, q = 7\) the game is relatively complex so the neural MCTS can hardly find the optimal policy. Even though it takes roughly 90 hours to get this result, the experimental result still serves as a proof-of-concept.

![Figure 5](image)

Figure 5: Correctness ratio measured for the proposal game on \(k = 7, q = 7\). The legend “New_OP” means that the newly trained neural network plays as an OP; “Old_P” means that the previously trained neural network plays as a P. The same for the following graphs.

![Figure 6](image)

Figure 6: Correctness ratio measured for the refutation game on \(k = 7, q = 7\).

4.4 Refutation Game

In order to test our method further, we focus our experiment only on refutation games with a given \(n\). We first run the experiment on an extreme case where \(k = 7, q = 7\). Using the Bernoulli Triangle (Fig. 1), we know that \(N(7, 7) = 2^7\). We set \(n = N(k, q)\) so that the learning process will converge when the P has figured out the optimal winning strategy which is binary search: namely, the first testing point is \(2^6\) then \(2^5, 2^4\) and so on. Fig. 7 verified that the result is as expected. Then we run the same experiment on a resource-insufficient case where we keep \(k, q\) unchanged and set \(n = N(k, q) + 1\). In this case, theoretically, no solution exists. Fig. 8, again, verified our expectation and one can see that the P can never find any winning strategy no matter how many iterations it has learned.

In later experiments, we have also tested our method in two more general cases where \(k = 3, q = 7\) for \(n = N(3, 7)\) (Fig. 9) and
n = N(3, 7) − 1 (Fig. 10). All experimental results are conforming to the ground-truth as expected.

Figure 7: Refutation game on k = 7, q = 7, n = 128

Figure 8: Refutation game on k = 7, q = 7, n = 129. Notice that in this game, the P is doomed for there is no winning strategy exists.

Figure 9: Refutation game on k = 3, q = 7, n = 64

The HSR_{k,q} game is also intrinsically asymmetric in terms of training/learning because the OP always takes the last step before the end of the game. This fact makes the game harder to learn for the P. Specifically, considering all possible consequences (in the view of the P) of the last action, there are only three cases: win-win, win-lose, and lose-lose. The OP will lose the game if and only if the consequence is win-win. If the portion of such type of consequence is very small, then the OP could only focus on learning the last step while ignoring other steps. However, the P has to learn every step to avoid possible paths which lead him to either win-lose or lose-lose, which, theoretically, are more frequently encountered in the end game.

5 DISCUSSION

5.1 State Space Coverage

Neural MCTS is capable to handle a large state space [18]. To be efficient, it is necessary for such an algorithm to search only a small portion of the state space and make the decisions on those limited observations. To measure the state space coverage ratio, we recorded the number of states accessed during the experiment, specifically, in the refutation game k = 7, q = 7, n = 128, we count the total number of states accessed during each self-play, and we compute the average state accessed for all 100 self-plays in each iteration. It can be seen in Fig. 11 that the maximum number of state accessed is roughly 1500 or 35% (we have also computed the total number of possible states in this game, which is 4257). As indicated in Fig. 11, at the beginning of the learning, neural MCTS accessed a large number of states, however, once the learning converged, it looked at a few numbers of state and pruned all other irrelevant states. It can also be seen that the coverage ratio will bounce back sometimes, which is due to the exploration during self-play. Our experimental result indicates that changes in coverage ratio might be evidence of adaptive self-pruning in a neural MCTS algorithm, which can be regarded as a justification of its capability of handling large state spaces.

5.2 Perfectness

This discussion is in the context where the ground truth is known. Since the correct solution is derived from the optimal policy, it is important to question whether the players are perfect after the training converged (i.e., the correctness of each player becomes flat without further changes). The experimental result shows that, after convergence, for a problem which has a solution, the P always
keeps 100% correctness while the OP rests at 0%. On the other hand, for a problem which has no solution, the opposite happens. Notice that a consistent 100% correctness indicates that the player is perfect because, otherwise, the other player will quickly find out the weakness in her adversary. However, there is no guarantee that a consistent 0% correctness player is also perfect. Since after one player becoming perfect, the other one will always lose no matter what decisions have been made. In this case, all game results are the same and there is no reward to be gained from further training. Even though, from our experimental observation, the doomed loser is still a strong sub-optimal player after being competitively trained from tabula rasa. The question of when to stop training and how to guarantee that both P and OP become perfect are further topics for future research.

5.3 Asymmetry
One can observe some asymmetry in the charts we presented in section 4, and notice that it is always the case that during the beginning iterations the OP is dominating until the P has gained enough experience and learned enough knowledge. This asymmetry is caused by two facts: 1. the action space of the P is quite different from the one of the OP. 2. the OP always takes the last step before the end of the game. These two facts make the game harder to learn for the P but easier for the OP.

5.4 Limitations
The neural MCTS algorithm is known to be time-consuming. It usually takes a large amount of time to converge. In order to make the algorithm run faster, we have to use more resources (more CPUs, distributed parallel computing) to trade for time. That’s the reason why we don’t experience the amazing performance of AlphaZero for Chess and Go on huge game trees. Another limitation is that, in order to learn the correct action in a discrete action space, the neural MCTS algorithm has to explore all possible actions before learning the correct action. This fact makes the action space a limitation to MCTS like algorithms: the larger the action space, the lower the efficiency of the algorithm.

6 RELATED WORK
By formulating a combinatorial problem as an MDP, Ranked reward [11] binarized the final reward of an MDP based on a certain threshold, and improves the threshold after each training episode so that the performance is forced to increase during each iteration. However, this method can hardly be applied to problems that already have a binary reward (such as a zero-sum game with reward \(-1, 1\)). Even though, the idea that improves the performance threshold after each learning iteration has been used also in AlphaZero as well as our implementation.

Pointer networks [19] have been shown to solve certain combinatorial NP problems with a limited size. The algorithm is based on supervised attention learning on a sequence to sequence RNN. However, due to its high dependency on the quality of data labels (which could be very expensive to obtain), Bello et al. [4] improved the method of [19] to the RL style. Specifically, they applied actor-critic learning where the actor is the original pointer network but the critic is a simple REINFORCE [21] style policy gradient. Their result shows a significant improvement in performance. However, this approach can only be applied to sequence decision problem (namely, what is the optimal sequence to finish a task). Also, scalability is still a challenge.

Graph neural networks (GNNs) [3] are a relatively new approach to hard combinatorial problems. Since some NP-complete problems can be reduced to graph problems, GNNs can capture the internal relational structure efficiently through the message passing process [6]. Based on message passing and GNNs, Selsam et al. developed a supervised SAT solver: neuroSAT [15]. It has been shown that neuroSAT performs very well on NP-complete problems within a certain size. Combining such GNNs with RL [8] could also be a potential future work direction for us.

7 CONCLUSION
Can the amazing game playing capabilities of the neural MCTS algorithm used in AlphaZero for Chess and Go be applied to Zermelo games that have practical significance? We provide a partial positive answer to this question for a class of combinatorial optimization problems which includes the HSR problem. We show how to use Zermelo Gamification (ZG) to translate certain combinatorial optimization problems into Zermelo games: We formulate the optimization problem using predicate logic (where the types of the variables are not “too” complex) and then we use the corresponding semantic game [7] as the Zermelo game which we give to the adapted neural MCTS algorithm. For our proof-of-concept example, HSR Zermelo Gamification, we notice that the Zermelo game is asymmetric. Nevertheless, the adapted neural MCTS algorithm converges on small instances that can be handled by our hardware and finds the winning strategy (and not just an approximation). Our evaluation counts all correct/incorrect moves of the players, thanks to a formal HSR solution we have in the form of the Bernoulli triangle which provides the winning strategy. In addition, we discussed the coverage ratio and transfer learning of our algorithm. We hope our research sheds some light on why the neural MCTS works so well on certain games. While Zermelo Gamification currently is a manual process we hope that many aspects of it can be automated.
REFERENCES

[1] P. Auer, N. Cesa-Bianchi, and P. Fischer. 2002. Finite-time Analysis of The Multiaimed Bandit Problem. Machine learning 47, 2 (2002), 235–256.

[2] David Auger, Adrien Courtois, and Olivier Teysaud. 2013. Continuous Upper Confidence Trees with Polynomial Exploration - Consistency.. In ECML/PKDD (1) (Lecture Notes in Computer Science), Vol. 8188. Springer, 194–209.

[3] Peter W Battaglia, Jessica B Hamrick, Victor Bapst, Alvaro Sanchez-Gonzalez, Vinicius Zambaldi, Mateusz Malinowski, Andrea Tacchetti, David Raposo, Adam Santoro, Ryan Faulkner, et al. 2018. Relational inductive biases, deep learning, and graph networks. arXiv preprint arXiv:1806.01261 (2018).

[4] Irwan Bello, Hieu Pham, Quoc V Le, Mohammad Norouzi, and Samy Bengio. 2016. Neural combinatorial optimization with reinforcement learning. arXiv preprint arXiv:1611.09940 (2016).

[5] Cameron Browne, David Jack Powley, Daniel Whitehouse, Simon M. Lucas, Peter I. Cowling, Philipp Rohlfshagen, Stephen Tavener, Diego Perez Liebana, Spyridon Samothrakis, and Simon Colton. 2012. A Survey of Monte Carlo Tree Search Methods. IEEE Trans. Comput. Intellig. and AI in Games 4, 1 (2012), 1–43.

[6] Justin Gilmer, Samuel S Schoenholz, Patrick F Riley, Oriol Vinyals, and George E Dahl. 2017. Neural message passing for quantum chemistry. In Proceedings of the 34th International Conference on Machine Learning-Volume 70. JMLR. org, 1263–1272.

[7] Jaakko Hintikka. 1982. Game-theoretical semantics: insights and prospects. Notre Dame J. Formal Logic 23, 2 (04 1982), 219–241.

[8] Elias Khalil, Hanjun Dai, Yuyu Zhang, Bistra Dilkina, and Le Song. 2017. Learning combinatorial optimization algorithms over graphs. In Advances in Neural Information Processing Systems. 6348–6358.

[9] Jon Kleinberg and Éva Tardos. 2006. Algorithm Design. Addison-Wesley.

[10] Lev Recht, Kun Huang, Alex Bellamy, Justin Gilmer, Oriol Vinyals, and Veli-Pekka Smolik. 2017. Bandit Based Monte-Carlo Planning.. In ECML (Lecture Notes in Computer Science), Vol. 4212. Springer, 282–293.

[11] Alexandre Laterre, Yunguan Fu, Mohamed Khalil Jabri, Alain-Sam Cohen, David Kas, Karl Hajjar, Torbjorn S Dahl, Amine Kerkem, and Karim Beguer. 2018. Ranked Reward: Enabling Self-Play Reinforcement Learning for Combinatorial Optimization. arXiv preprint arXiv:1807.01672 (2018).

[12] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A. Rusu, Joel Veness, Marc G. Bellemare, Alex Graves, Martin Riedmiller, Andreas K. Fidjeland, Georg Ostrovski, Stig Petersen, Charles Beattie, Amir Sadik, Ioannis Antonoglou, Helen King, Dharshan Kumaran, Daan Wierstra, Shane Legg, and Demis Hassabis. 2015. Human-level control through deep reinforcement learning. Nature 518, 7540 (Feb. 2015), 529–533.

[13] Christopher D. Rosin. 2011. Multi-armed bandits with episode context. Annals of Mathematics and Artificial Intelligence 61, 3 (mar 2011), 203–230.

[14] John Schulman, Filip Wolski, Prafulla Dharia, Alec Radford, and Oleg Klimov. 2017. Proximal Policy Optimization Algorithms. (2017). arXiv:arXiv:1707.06347

[15] Daniel Selsam, Matthew Lamm, Benedikt Bunz, Percy Liang, Leonardo de Moura, and David L Dill. 2018. Learning a SAT Solver from Single-Bit Supervision. arXiv preprint arXiv:1802.03685 (2018).

[16] David Silver, Thomas Hubert, Julian Schrittwieser, Ioannis Antonoglou, Matthew Lai, Arthur Guez, Marc Lanctot, Laurent Sifre, Dharshan Kumaran, Thore Graepel, et al. 2018. A general reinforcement learning algorithm that masters Chess, Shogi, and Go through self-play. Science 362, 6419 (2018), 1140–1144.

[17] David Silver, Thomas Hubert, Julian Schrittwieser, Ioannis Antonoglou, Matthew Lai, Arthur Guez, Marc Lanctot, Laurent Sifre, Dharshan Kumaran, Thore Graepel, Timothy P. Lillicrap, Karen Simonyan, and Demis Hassabis. 2017. Mastering Chess and Shogi by Self-Play with a General Reinforcement Learning Algorithm. CoRR abs/1712.01815 (2017).

[18] David Silver, Julian Schrittwieser, Karen Simonyan, Ioannis Antonoglou, Aja Huang, Arthur Guez, Thomas Hubert, Lucas Baker, Matthew Lai, Adrian Bolton, Yutian Chen, Timothy Lillicrap, Fan Hui, Laurent Sifre, George van den Driessche, Thore Graepel, and Demis Hassabis. 2017. Mastering the game of Go without human knowledge. Nature 550 (Oct. 2017), 354.

[19] Oriol Vinyals, Meire Fortunato, and Naveen Jaitly. 2015. Pointer Networks. In Advances in Neural Information Processing Systems 28. C. Cortes, N. D. Lawrence, D. D. Lee, M. Sugiyama, and R. Garnett (Eds.) Curran Associates, Inc., 2692–2700.

[20] Thore Graepel, Sebastien Racaniere, David P. Reichert, Lars Buesing, Arthur Guez, Danilo Jimenez Rezende, Adria Puigdomenech Badia, Oriol Vinyals, Nicolas Heess, Yujia Li, Razvan Pascanu, Peter Battaglia, Demis Hassabis, David Silver, and Daan Wierstra. 2017. Imagination-Augmented Agents for Deep Reinforcement Learning. (2017) arXiv:arXiv:1707.06283

[21] R. J. Williams. 1992. Simple statistical gradient-following algorithms for connectionist reinforcement learning. Machine Learning 8 (1992), 229–256.