Connecting Majorana phases to the Geometric Parameters of Majorana Unitarity Triangle in a model of Neutrino Mass Matrix

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We have investigated a possible connection between the Majorana phases and geometric parameters of leptonic unitarity triangle (LUT) in two-texture zero neutrino mass matrix. Such analytical relations can, also, be obtained for other theoretical models viz. hybrid textures, neutrino mass matrix with vanishing minors and have profound implications for geometric description of CP violation. As an example, we have considered two-texture zero neutrino mass model to obtain relation between Majorana phases and LUT parameters. In particular, we find that Majorana phases depend on only one of the three interior angles of LUT in each class of two-texture zero neutrino mass matrix. We have, also, constructed LUT for class A, B and C neutrino mass matrices. Non-vanishing areas and non-trivial orientations of these Majorana unitarity triangles indicate non-zero CP violation as a generic feature of this class of mass models.

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I. INTRODUCTION

Recent developments in experimental neutrino physics have catalyzed assiduous efforts to understand the origin of neutrino mass. The neutrino oscillation experiments have, now, revealed the dominant structure of the neutrino mass matrix with vanishing minors and have profound implications for geometric description of CP violation. What is the leptonic octant of atmospheric mixing angle? and what is neutrino mass hierarchy normal or inverted? What is the lightest neutrino Majorana or Dirac-particle? What is the octant of atmospheric mixing angle? and what is the leptonic CP violating phase? to name a few. The primary goal of the future neutrino experimental endeavours will be to address these questions by employing a miscellany of experimental configurations and techniques.

The observation of non-zero value of reactor mixing angle $\theta_{13}$, by oscillation experiments, provides unique opportunity for the possible measurement of CP violation in the leptonic sector. The flavor oscillations implies the existence of mixing in the weak charged current interaction

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{\nu}_L(x)\gamma_\alpha \nu_L(x) W^{\alpha\dagger}(x) + h.c.,$$

$$\nu_L(x) = \sum_{j=1}^{n} U_{ij} \nu_j(x),$$

where, $\nu_L(x)$ are flavor fields, $\nu_{jL}(x)$ are LH component of mass fields $\nu_j$ with mass $m_j$, and $U$ is unitary mixing matrix. The mixing matrix $U$ can be parameterised in terms of $\frac{1}{2} n(n-1)$ angles and $\frac{1}{2} (n-1)(n-2)$ phases. For Majorana neutrinos, there exist $(n-1)$ additional CP violating phases called “Majorana phases”. In the standard framework of three-neutrino the mixing matrix contains two CP violating phases $\rho$ and $\sigma$, in addition to one Dirac-type CP violating phase $\delta$. While the Dirac-type CP violating phase will, possibly, be measured in the neutrino oscillation experiments T2K, NO$\nu$A and DUNE, the information about Majorana-type CP violating phases can be extracted from the lepton number violating processes.

$\nu\nu$CP violation in the leptonic sector can either be studied through the construction of Lepton Unitarity Triangle (LUT) or by direct measurement of the CP violating phase $\delta$ in the neutrino oscillation experiments. The first approach has the advantage of being rephasing invariant description of the CP violation.

In the flavor basis, the unitary mixing matrix $V \equiv UP$, where $U$ is Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, is given by

$$U = \left( \begin{array}{ccc}
\frac{c_{12}c_{13}}{s_{12}c_{23}} & c_{12}s_{23} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} & c_{12}s_{23} & s_{13}e^{i\delta} \\
\frac{s_{12}s_{23}}{s_{13}} & -c_{12}s_{23} & c_{13} \end{array} \right),$$

where $U_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$ and $P = \text{diag}(1,e^{i\rho},e^{i(\sigma+\delta)})$. The unitarity of $V$ imposes six orthogonality conditions on the elements of $V$ viz.,

$$\Delta_{\eta} = V_{e1}V_{\mu1}^* + V_{e2}V_{\mu2}^* + V_{e3}V_{\mu3}^* = 0,$$

$$\Delta_{\mu\tau} = V_{\mu1}V_{\tau1}^* + V_{\mu2}V_{\tau2}^* + V_{\mu3}V_{\tau3}^* = 0,$$

$$\Delta_{\tau\tau} = V_{\tau1}V_{\tau1}^* + V_{\tau2}V_{\tau2}^* + V_{\tau3}V_{\tau3}^* = 0.$$
obtained from multiplying two rows of \( V \) and
\[
\Delta_{22} \equiv V_{\mu 1} V_{\mu 2}^* + V_{\tau 1} V_{\tau 2}^* = 0, \\
\Delta_{23} \equiv V_{\mu 2} V_{\mu 3}^* + V_{\tau 2} V_{\tau 3}^* = 0, \\
\Delta_{31} \equiv V_{\tau 3} V_{\tau 1}^* + V_{\mu 3} V_{\mu 1} = 0,
\]
where, \( M_\nu = \text{Diag}\{m_1, m_2, m_3\} \). There are total 15 possible patterns of two-texture zero Majorana neutrino mass matrices. Seven out of these patterns, are found to be consistent with the neutrino mixing parameters. These are \( A_1, A_2, B_1, B_2, B_3, B_4 \) and \( C \) (Table I). Since \( M \) is symmetric it has total six independent complex elements. Any two vanishing elements of \( M \), i.e., \( M_{\mu \tau} = 0, M_{\tau \mu} = 0 \), where \( s, t, x \), and \( y \) can take values \( e, \mu, \tau \), result in two complex constraining equations viz; \( \sum_{i=1}^3 V_{\alpha i} m_{\alpha i} = 0, \sum_{i=1}^3 V_{\alpha i} V_{\beta i} m_{\beta i} = 0 \). These two relations involve nine free parameters i.e., \( m_1, m_2, m_3, \theta_{12}, \theta_{13}, \theta_{23} \) and three \( CP \)-violating phases \( \rho, \sigma \) and \( \delta \), where \( \rho, \sigma \) are two Majorana-type \( CP \) violating phases and \( \delta \) is Dirac-type \( CP \) violating phase. We solve these constraining equations to obtain mass ratios\( (\frac{m_1}{m_2}, \frac{m_1}{m_3}) \) and Majorana phases\( (\rho, \sigma) \) as,
\[
\frac{m_1}{m_2} = \frac{U_{x2} U_{y2} U_{y3} - U_{x2} U_{y2} U_{y3}}{U_{x1} U_{y2} U_{y3} - U_{x3} U_{y1} U_{y2}}, \\
\frac{m_1}{m_3} = \frac{U_{x3} U_{y3} U_{y2} - U_{x3} U_{y3} U_{y2}}{U_{x1} U_{y2} U_{y3} - U_{x2} U_{y1} U_{y2}},
\]
and
\[
\rho = -\frac{1}{2} \text{Arg}\left( \frac{U_{x2} U_{y2} U_{y3} - U_{x2} U_{y2} U_{y3}}{U_{x1} U_{y2} U_{y3} - U_{x3} U_{y1} U_{y2}} \right), \\
\sigma = -\frac{1}{2} \text{Arg}\left( \frac{U_{x3} U_{y3} U_{y2} - U_{x3} U_{y3} U_{y2}}{U_{x1} U_{y2} U_{y3} - U_{x2} U_{y1} U_{y2}} - \delta(9) \right)
\]
respectively. Using Eqs. (2), (6) and (7), we have shown these mass ratios upto first order in \( s_{13} \) in Table II. These relations will be useful while studying the phenomenology of two-texture zero neutrino mass matrices. In our numerical analysis, we have used global data\( \text{12} \) to obtain the allowed parameter space of neutrino mixing parameters in two-texture zero neutrino mass model. The best fit point(bfp) and 1σ range of these parameters are tabulated in Table III. We have, also, calculated the Jarlskog\( \text{14} \) \( CP \) invariant \( J_{CP} \) sensitive to Dirac phase \( \delta \)
\[
J_{CP} = \text{Im}\{V_{e1} V_{\mu2} V_{e3}^* V_{\mu1}^* \},
\]
and \( s_1, s_2 \) \( CP \) invariants\( \text{15, 16} \)
\[
s_1 = \text{Im}\{V_{e1} V_{\mu3}^* \}, \\
s_2 = \text{Im}\{V_{e2} V_{\mu3}^* \}
\]
sensitive to Majorana phases \( \rho \) and \( \sigma \) (Table III).

II. TWO-TEXTURE ZERO NEUTRINO MASS MATRICES

In the flavor basis, where the charged-lepton mass matrix \( M_\ell \) is diagonal, the Majorana neutrino mass matrix \( M \) is given by
\[
M = V M_\ell V^T,
\]
where, \( M_\nu = \text{Diag}\{m_1, m_2, m_3\} \). There are total 15 possible patterns of two-texture zero Majorana neutrino mass matrices. Seven out of these patterns, are found to be consistent with the neutrino mixing parameters. These are \( A_1, A_2, B_1, B_2, B_3, B_4 \) and \( C \) (Table I). Since \( M \) is symmetric it has total six independent complex elements. Any two vanishing elements of \( M \), i.e., \( M_{\mu \tau} = 0, M_{\tau \mu} = 0 \), where \( s, t, x \), and \( y \) can take values \( e, \mu, \tau \), result in two complex constraining equations viz; \( \sum_{i=1}^3 V_{\alpha i} m_{\alpha i} = 0, \sum_{i=1}^3 V_{\alpha i} V_{\beta i} m_{\beta i} = 0 \). These two relations involve nine free parameters i.e., \( m_1, m_2, m_3, \theta_{12}, \theta_{13}, \theta_{23} \) and three \( CP \)-violating phases \( \rho, \sigma \) and \( \delta \), where \( \rho, \sigma \) are two Majorana-type \( CP \) violating phases and \( \delta \) is Dirac-type \( CP \) violating phase. We solve these constraining equations to obtain mass ratios\( (\frac{m_1}{m_2}, \frac{m_1}{m_3}) \) and Majorana phases\( (\rho, \sigma) \) as,
\[
\frac{m_1}{m_2} = \frac{U_{x2} U_{y2} U_{y3} - U_{x2} U_{y2} U_{y3}}{U_{x1} U_{y2} U_{y3} - U_{x3} U_{y1} U_{y2}}, \\
\frac{m_1}{m_3} = \frac{U_{x3} U_{y3} U_{y2} - U_{x3} U_{y3} U_{y2}}{U_{x1} U_{y2} U_{y3} - U_{x2} U_{y1} U_{y2}},
\]
and
\[
\rho = -\frac{1}{2} \text{Arg}\left( \frac{U_{x2} U_{y2} U_{y3} - U_{x2} U_{y2} U_{y3}}{U_{x1} U_{y2} U_{y3} - U_{x3} U_{y1} U_{y2}} \right), \\
\sigma = -\frac{1}{2} \text{Arg}\left( \frac{U_{x3} U_{y3} U_{y2} - U_{x3} U_{y3} U_{y2}}{U_{x1} U_{y2} U_{y3} - U_{x2} U_{y1} U_{y2}} - \delta(9) \right)
\]
respectively. Using Eqs. (2), (6) and (7), we have shown these mass ratios upto first order in \( s_{13} \) in Table II. These relations will be useful while studying the phenomenology of two-texture zero neutrino mass matrices. In our numerical analysis, we have used global data\( \text{12} \) to obtain the allowed parameter space of neutrino mixing parameters in two-texture zero neutrino mass model. The best fit point(bfp) and 1σ range of these parameters are tabulated in Table III. We have, also, calculated the Jarlskog\( \text{14} \) \( CP \) invariant \( J_{CP} \) sensitive to Dirac phase \( \delta \)
\[
J_{CP} = \text{Im}\{V_{e1} V_{\mu2} V_{e3}^* V_{\mu1}^* \},
\]
and \( s_1, s_2 \) \( CP \) invariants\( \text{15, 16} \)
\[
s_1 = \text{Im}\{V_{e1} V_{\mu3}^* \}, \\
s_2 = \text{Im}\{V_{e2} V_{\mu3}^* \}
\]
sensitive to Majorana phases \( \rho \) and \( \sigma \) (Table III).

III. MAJORANA UNITARITY TRIANGLES IN TWO-TEXTURE ZERO MODEL

In Majorana triangles(Eq. (I)), all the terms are rephasing invariant and hence do not rotate in complex plane under rephasing transformations. The study
of these triangles, is of great physical significance due to their dependence on Majorana phases \[17, 18\]. The MTs \( \Delta_{12} \) and \( \Delta_{31} \) are sensitive to \( \rho \) and \( \sigma \), respectively, whereas, \( \Delta_{23} \) is sensitive to both Majorana phases \( (\rho, \sigma) \). In general, the sides and angles of MT can be expressed as,

\[
(S_1, S_2, S_3) = (|V_{ef} V_{f'i'}|, |V_{\mu f} V_{\mu'f'}|, |V_{\tau f} V_{\tau'f'}|),
\]

(13)
TABLE III. The CP invariants in two-texture zero neutrino mass matrices.

| Type of Texture | Hierarchy | $J_{CP}(\text{bfp} \pm 1\sigma)$ | $s_1(\text{bfp} \pm 1\sigma)$ | $s_2(\text{bfp} \pm 1\sigma)$ |
|-----------------|-----------|----------------------------------|-------------------------------|-------------------------------|
| $A_1$           | NH        | $(2.67 \pm 1.16) \times 10^{-2}$ | $(-0.10 \pm 0.04) \times 10^{-2}$ | $(-0.14 \pm 0.06) \times 10^{-2}$ |
| $A_2$           | NH        | $(1.60 \pm 0.80) \times 10^{-2}$ | $(-0.28 \pm 0.03) \times 10^{-2}$ | $(0.40 \pm 0.03) \times 10^{-2}$ |
| $B_1$           | NH        | $(3.21 \pm 0.63) \times 10^{-2}$ | $(-1.38 \pm 0.98) \times 10^{-5}$ | $(-8.30 \pm 3.60) \times 10^{-5}$ |
|                 | IH        | $(3.22 \pm 0.85) \times 10^{-2}$ | $(0.64 \pm 0.17) \times 10^{-2}$ | $(0.43 \pm 0.11) \times 10^{-2}$ |
| $B_2$           | NH        | $(3.29 \pm 0.34) \times 10^{-2}$ | $(-0.66 \pm 0.06) \times 10^{-2}$ | $(0.44 \pm 0.04) \times 10^{-2}$ |
|                 | IH        | $(3.22 \pm 0.62) \times 10^{-2}$ | $(-4.97 \pm 1.79) \times 10^{-5}$ | $(-1.43 \pm 0.44) \times 10^{-5}$ |
| $B_3$           | NH        | $(3.22 \pm 0.58) \times 10^{-2}$ | $(-6.54 \pm 1.19) \times 10^{-3}$ | $(4.41 \pm 0.80) \times 10^{-3}$ |
|                 | IH        | $(3.29 \pm 0.68) \times 10^{-2}$ | $(-1.65 \pm 0.65) \times 10^{-4}$ | $(-1.01 \pm 0.65) \times 10^{-4}$ |
| $B_4$           | NH        | $(3.32 \pm 0.53) \times 10^{-2}$ | $(-1.99 \pm 0.94) \times 10^{-6}$ | $(-1.20 \pm 0.87) \times 10^{-5}$ |
|                 | IH        | $(3.23 \pm 0.55) \times 10^{-2}$ | $(-6.50 \pm 1.10) \times 10^{-3}$ | $(4.30 \pm 0.75) \times 10^{-3}$ |
| $C$             | IH        | $(2.95 \pm 0.38) \times 10^{-2}$ | $(-6.19 \pm 0.64) \times 10^{-3}$ | $(5.25 \pm 0.59) \times 10^{-3}$ |

IV. STATUS OF NEUTRINOLESS DOUBLE BETA DECAY ($0\nu\beta\beta$) IN TWO-TEXTURE ZERO NEUTRINO MASS MODEL

The sensitivities of current and future experiments to Lepton Number Violating (LNV) processes such as, $K^+ \rightarrow \mu^+\mu^+\pi^-$ decay [19 21], the nuclear muon to positron [22 23], tri-onium production in neutrino muon scattering [24], the process $e^+p \rightarrow \nu l_1^+ l_2^+ X$ [25] and $0\nu\beta\beta$ decay [26 28], have been extensively studied in the literature and the existence of which will demonstrate the Majorana nature of the neutrinos. The sensitivities of the current experiments searching for these processes are much less in comparison to experimental sensitivity to $0\nu\beta\beta$ decay process. $0\nu\beta\beta$ decay is the most promising probe of observing lepton number violation and will shed light on mechanism of neutrino mass generation. The effective Majorana mass, $\langle m \rangle_{ee}$, may vanish for normal hierarchical neutrino masses, however, for inverted hierarchical neutrino masses there exist a lower bound on $\langle m \rangle_{ee}$ providing bright prospects for observation of $0\nu\beta\beta$ decay. Although, it will be difficult to measure Majorana phases in these experiments but a correlation between them can, always, be obtained. One can uniquely determine both Majorana phases for the case of vanishing $\langle m \rangle_{ee}$ [29]. The decay width of the $0\nu\beta\beta$ is proportional to the effective Majorana mass, $\langle m \rangle_{ee}$, given by [30]

$$\langle m \rangle_{ee} = \sum_{i=1}^{3} V^2_{ei} m_i,$$

$$= m_1 |U^2_{e1}| + m_2 |U^2_{e2}| e^{2i\delta} + m_3 |U^2_{e3}| e^{2i\sigma}. (15)$$

Using the allowed parameter space obtained for two-texture zero neutrino mass matrices shown in Table III we have obtained the $1\sigma$ range of effective Majorana mass, $\langle m \rangle_{ee}$, for each type of two-texture zero neutrino mass model which has been tabulated in Table IV. For class $A$ type neutrino mass models, the effective Majorana mass, $\langle m \rangle_{ee}$, is vanishing thus observation of $0\nu\beta\beta$ decay will rule out these class of neutrino mass models. The sensitivities of the $0\nu\beta\beta$ experiments like KamLAND-ZEN [31 32], GERDA [33], CUORE [34 35], NEXT [36 37] and EXO-200 [38 39] have set strong bounds on effective Majorana mass, $\langle m \rangle_{ee}$, tabulated in Table V. Recently, the strongest constraint has been obtained by KamLAND-ZEN [32]. The predictions for effective Majorana mass, $\langle m \rangle_{ee}$, for class $B$ and $C$ are found to be well within the sensitivity reach of current and future $0\nu\beta\beta$ decay experiments (Table VI). However, non-observation of $0\nu\beta\beta$ decay will favour class $A$ and class $B$ (with NH) type neutrino mass models.

V. RESULTS AND DISCUSSION

In class $A$, $\cos\delta$ should be positive (negative) for $A_1(A_2)$ because $\frac{m_1}{m_2} < 1$. So, $\delta$ can be in I or IV quadrant for $A_1$ and in II and III quadrant for $A_2$ type mass matrix. These theoretical predictions are consistent with the obtained best-fit values of $\delta$ shown in Table IV.

The class $B$ yield a quasi-degenerate spectrum of neutrino masses. For mass ratio $\frac{m_1}{m_2} < 1$, $\cos\delta$ should be negative (positive) for $B_1$ and $B_4(B_2$ and $B_3)$. We find that
\( \delta = 267.10^\circ, 266.91^\circ, 267.80^\circ, 268.50^\circ (268.87^\circ, 269.00^\circ, 269.47^\circ, 266.82^\circ) \) for \( B_1, B_2, B_3 \) and \( B_4 \) with NH(III), respectively. The important point here is to note that the best-fit values of \( \delta \) comes out to be nearly equal to \( \frac{3\pi}{2} \) for class B, which is in accordance with the experimental value of \( \delta \) predicted by the combined analysis of T2K and Daya Bay experiments [41].

In class C, for \( m_{\nu_3}/m_{\nu_1} < 1 \), the factor \( \tan\theta_{23}\cos\delta \) should be positive and \( \delta \) must lie either in I or IV quadrant. This prediction is found to be consistent with best-fit value of \( \delta = 292.80^\circ \) (Table [II]). In Table [III] we have obtained the expressions for Majorana phases in terms of the interior angle of Majorana unitarity triangle for class \( A, B \) and \( C \). We observe that the Majorana phases depend only on one independent geometric parameter of MT for each class of two-texture zero neutrino mass model. The orientation of these tri-
TABLE V. $|\langle m \rangle_{ee}|$ (eV) for each type of two-texture zero neutrino mass matrices.

| Textures | NH(bfp ±1σ) | IH(bfp ±1σ) |
|----------|-------------|-------------|
| $A_1$    | $|\langle m \rangle_{ee}| = 0$ | -           |
| $A_2$    | $|\langle m \rangle_{ee}| = 0$ | -           |
| $B_1$    | $|\langle m \rangle_{ee}| = 0.059 ± 0.016$ | $|\langle m \rangle_{ee}| = 0.181 ± 0.066$ |
| $B_2$    | $|\langle m \rangle_{ee}| = 0.173 ± 0.088$ | $|\langle m \rangle_{ee}| = 0.080 ± 0.014$ |
| $B_3$    | $|\langle m \rangle_{ee}| = 0.061 ± 0.016$ | $|\langle m \rangle_{ee}| = 0.033 ± 0.015$ |
| $B_4$    | $|\langle m \rangle_{ee}| = 0.180 ± 0.113$ | $|\langle m \rangle_{ee}| = 0.084 ± 0.016$ |
| $C$      | -           | $|\langle m \rangle_{ee}| = 0.065 ± 0.043$ |

TABLE VI. Sensitivities to effective Majorana mass, $|\langle m \rangle_{ee}|$, of various $0\nu\beta\beta$ decay experiments.

| Experiment            | $|\langle m \rangle_{ee}|$(eV) |
|-----------------------|-------------------------------|
| EXO-200(4 yr)         | 0.075-0.2                     |
| nEXO(5 yr)            | 0.012-0.029                   |
| nEXO(5 yr + 5 yr w / Ba tagging) | 0.005-0.011               |
| KamLAND-ZEN(300 kg, 3 yr) | 0.045-0.11               |
| GERDA phase II        | 0.09-0.29                     |
| CUORE(5 yr)           | 0.051-0.133                   |
| SNO+                  | 0.07-0.14                     |
| SuperNEMO             | 0.05-0.15                     |
| NEXT                  | 0.03-0.1                      |
| MAJORANA Demonstrator | 0.06-0.17                    |

In conclusion, we have investigated CP violation in neutrino mass models with two-texture zeros. In particular, we have obtained possible connection between Majorana phases ($\rho, \sigma$) and independent geometric parameters of Majorana triangle (MT). We find that Majorana phases depend on one interior angle of MT (i.e. $\gamma_{12}$ in $A_1$, $\beta_{12}$ in $A_2$, $\beta_{23}$ in $B_1$, $\gamma_{23}$ in $B_2$, $\gamma_{31}$ in $B_3$, $\beta_{31}$ in $B_4$ and $\alpha_{31}$ in class $C$ type neutrino mass matrix). This analysis is important in light of the future neutrino mass experiments focusing on measuring Dirac-type CP violation phase $\delta$. We have, also, obtained the best-fit and $\pm 1\sigma$ values of neutrino oscillation parameters and CP rephasing invariants for class $A$, $B$ and $C$ neutrino mass matrices. We find that for class $B$, the best-fit value lies very close to $\delta \approx \frac{3\pi}{2}$, which is in accordance with T2K and Daya Bay experiments. The non-zero area and non-trivial orientation of Majorana triangles shows that two-texture zero neutrino mass matrices are necessarily CP violating.

The current and future $0\nu\beta\beta$ decay experiments can shed light on two-texture zero neutrino mass models. As long as fixing of parameters (absolute neutrino mass and Majorana phases) is concerned, the non-observation of $0\nu\beta\beta$ decay is more predictive than the situation in which $0\nu\beta\beta$ decay is observed. The predictions for effective Majorana mass, $|\langle m \rangle_{ee}|$, for class $B$ and $C$ are found to be well within the sensitivity reach of current and future $0\nu\beta\beta$ decay experiments (Table VI). The non-observation of $0\nu\beta\beta$ decay will favour class $A$ and class $B$ (with NH) type two-texture zero neutrino mass matrices while class $C$ will be ruled out.

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Appendix: Majorana phases in terms of interior angles of the leptonic unitarity triangle

Using Eq. (16), along with values of mixing parameters as shown in Table IV, the Majorana triangles have been constructed as shown with solid lines in Figs. 11-13. For reference, the dashed triangles in Figs. 11-13 are obtained assuming $\rho$ and $\sigma$ equal to zero. In general, we find that the Majorana triangles provide an indubitable signal towards non-zero CP violation in this class of models because none of the sides of the triangle is parallel to the axis and all the Majorana triangles having non-vanishing area.

$$
\Delta_{12} = e^{i\rho} \left( U_{e1}U_{e2}^* + U_{\mu1}U_{\mu2}^* + U_{\tau1}U_{\tau2}^* \right),
\Delta_{23} = e^{i(\beta - \sigma - \rho)} \left( U_{e2}U_{e3}^* + U_{\mu2}U_{\mu3}^* + U_{\tau2}U_{\tau3}^* \right),
\Delta_{31} = e^{i(\beta + \sigma)} \left( U_{e3}U_{e1}^* + U_{\mu3}U_{\mu1}^* + U_{\tau3}U_{\tau1}^* \right). \quad (16)
$$

We have obtained relations of Majorana phases $\rho$ and $\sigma$ in terms of geometric angles of Majorana triangle for each type of two-texture zero neutrino mass matrices (Table IV). The method for type $A_1$ has been elaborated here. We choose triangle as shown in Fig. 4 with $f = 1, f' = 2$ and angles of triangle as shown in Eq. (14).
\[
\rho = -\frac{1}{2} \left( \text{Arg} \left( \frac{U_{c2}U_{\mu 2}U_{c3}^2 - U_{c2}U_{c3}U_{\mu 3}}{U_{c3}U_{\mu 3}U_{c2}^2 - U_{c3}U_{\mu 3}U_{c1}} \right) \right)
\]

\[
= -\frac{1}{2} \left( \text{Arg} \left( \frac{U_{c2}U_{\mu 2}}{U_{c3}U_{\mu 3}} \right) \text{Arg} \left( \frac{U_{c3}U_{\mu 3} - U_{c2}U_{\mu 3}}{U_{c1}U_{\mu 1}} \right) \right)
\]

\[
= -\frac{1}{2} \left( \text{Arg} \left( \frac{U_{c2}U_{\mu 2}}{U_{\mu 1}U_{c1}} \right) + \text{Arg} \left( \frac{U_{c3}U_{\mu 2}U_{\mu 1} - U_{c2}U_{\mu 3}}{U_{\mu 2}U_{\mu 1}U_{\mu 3}U_{c1}} \right) \right)
\]

\[
= -\frac{1}{2} \left( \text{Arg} \left( \frac{U_{c2}U_{\mu 2}}{U_{\mu 1}U_{c1}} \right) + \text{Arg} \left( \frac{U_{c3}U_{\mu 2}U_{\mu 1} - U_{c2}U_{\mu 3}}{U_{\mu 2}U_{\mu 1}U_{\mu 3}U_{c1}} \right) \right)
\]

\[
= -\frac{1}{2} \left( \text{Arg} \left( \frac{U_{c2}U_{\mu 2}}{U_{\mu 1}U_{c1}} \right) + \text{Arg} \left( \frac{U_{c3}U_{\mu 2}U_{\mu 1} - U_{c2}U_{\mu 3}}{U_{\mu 2}U_{\mu 1}U_{\mu 3}U_{c1}} \right) \right)
\]

\[
= -\frac{1}{2} \left( \text{Arg} \left( \frac{U_{c2}U_{\mu 2}}{U_{\mu 1}U_{c1}} \right) + \text{Arg} \left( \frac{U_{c3}U_{\mu 2}U_{\mu 1} - U_{c2}U_{\mu 3}}{U_{\mu 2}U_{\mu 1}U_{\mu 3}U_{c1}} \right) \right)
\]

\[
= -\frac{1}{2} \left( \text{Arg} \left( \frac{U_{c2}U_{\mu 2}}{U_{\mu 1}U_{c1}} \right) + \text{Arg} \left( \frac{U_{c3}U_{\mu 2}U_{\mu 1} - U_{c2}U_{\mu 3}}{U_{\mu 2}U_{\mu 1}U_{\mu 3}U_{c1}} \right) \right)
\]

where we have used the identities, \( \text{Arg} \left( \frac{z_1}{z_2} \right) = \text{Arg}(z_1) - \text{Arg}(z_2) \), \( \text{Arg}(z_1 z_2) = \text{Arg}(z_1) + \text{Arg}(z_2) \), \( \text{Arg} \left( \frac{1}{z} \right) = -\text{Arg}(z) \), \( \text{Arg}(-z) = \text{Arg}(z) + \pi \), \( \text{Arg}(z^n) = n \text{Arg}(z) \). Note that all these equalities hold modulo 2πn (n ∈ ℤ).
\[
\sigma = -\frac{1}{2} \left( \text{Arg} \left( \frac{U_{e3} U_{\mu 3} U_{e2}}{U_{e2} U_{\mu 2} U_{\mu 1}} - U_{e3} U_{e2} U_{\mu 2} U_{\mu 1} \right) \right) - \delta
\]

\[
= -\frac{1}{2} \left( \text{Arg} \left( \frac{U_{e2} U_{\mu 2}}{U_{e1} U_{\mu 1}} \right) + \text{Arg} \left( \frac{U_{e2} U_{\mu 2} U_{e3}}{U_{e2} U_{\mu 2}} - \frac{U_{e3} U_{e2}}{U_{e1} U_{\mu 1}} \right) \right) - \delta
\]

\[
= -\frac{1}{2} \left( \text{Arg} \left( \frac{U_{e2} U_{\mu 2}}{U_{e1} U_{\mu 1}} \right) + \text{Arg} \left( \frac{U_{e2} U_{\mu 2} U_{e3}}{U_{e2} U_{\mu 2}} - \frac{U_{e3} U_{e2}}{U_{e1} U_{\mu 1}} \right) \right) - \delta
\]

\[
= -\frac{1}{2} \left( \text{Arg} \left( \frac{U_{e1} U_{e2}^{\ast}}{U_{\mu 1} U_{\mu 2}} \right) + \text{Arg} \left( \frac{U_{e2} U_{\mu 2}}{U_{e1} U_{\mu 1}} \right) + \text{Arg} \left( \frac{U_{e2} U_{\mu 2} U_{e3}}{U_{e2} U_{\mu 2}} - \frac{U_{e3} U_{e2}}{U_{e1} U_{\mu 1}} \right) \right) - \delta
\]

\[
= -\frac{1}{2} \left( \text{Arg} \left( \frac{U_{e1} U_{e2}^{\ast}}{U_{\mu 1} U_{\mu 2}} \right) + \text{Arg} \left( \frac{U_{e2} U_{\mu 2}}{U_{e1} U_{\mu 1}} \right) + \text{Arg} \left( \frac{U_{e2} U_{\mu 2} U_{e3}}{U_{e2} U_{\mu 2}} - \frac{U_{e3} U_{e2}}{U_{e1} U_{\mu 1}} \right) \right) - \delta
\]

\[
= -\frac{1}{2} \left( \gamma_{12} - \pi + \text{Arg} \left( \frac{U_{e2} U_{\mu 2} U_{e3}}{U_{e2} U_{\mu 2} U_{\mu 1}} - \frac{U_{e3} U_{e2}}{U_{e1} U_{\mu 1}} \right) \right) - \delta
\]

\[
= -\frac{1}{2} \left( \gamma_{12} - \pi + \text{Arg} \left( \frac{U_{e2} U_{\mu 2} U_{e3}}{U_{e2} U_{\mu 2} U_{\mu 1}} - \frac{U_{e3} U_{e2}}{U_{e1} U_{\mu 1}} \right) \right) - \delta
\]

\[
= -\frac{1}{2} \left( \gamma_{12} - \pi + \text{Arg} \left( \frac{U_{e2} U_{\mu 2} U_{e3}}{U_{e2} U_{\mu 2} U_{\mu 1}} - \frac{U_{e3} U_{e2}}{U_{e1} U_{\mu 1}} \right) \right) - \delta.
\]

In similar way, we can obtain expressions for other type of textures $A_2, B_1, B_2, B_3, B_4$ and $C$.

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FIG. 1. Majorana triangles for class A neutrino mass matrices with normal hierarchical (NH) neutrino masses. MT for type $A_1(A_2)$ with NH in left(right).

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FIG. 2. Majorana triangles for class $B$ neutrino mass matrices with normal and inverted hierarchical (IH) neutrino masses. MT for $B_1, B_2, B_3$ and $B_4$ with NH(left) and IH(right).
FIG. 3. Majorana triangles for class C neutrino mass matrices with inverted hierarchical neutrino masses.

FIG. 4. Majorana Unitarity Triangle in complex plane.