The CSS parametrization for Hybrid Stars with the Field Correlator Method

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Abstract. We explore the structure of hybrid stars based on a nuclear matter equation of state (EoS) built with the microscopic Brueckner-Hartree-Fock many-body theory, and a quark matter EoS derived with the Field Correlator Method (FCM), which can be accurately represented by the CSS (constant speed of sound) parametrization. We find that the main features of the hadron-quark phase transition are directly related to the FCM parameters, i.e. the quark-antiquark potential $V_1$, the gluon condensate $G_2$ and the color-flavour superconducting gap $\Delta$, whose values range can be determined by the observational data on neutron star (NS) masses.

1. Introduction
The appearance of quark matter (QM) in neutron stars (NS) interiors and the corresponding equation of state are among the mostly debated issues in the physics of compact objects. Unfortunately, while the microscopic theory of the nucleonic EoS has reached a high degree of sophistication [1, 2, 3, 4, 5], the quark matter EoS is still poorly known at zero temperature and at the high baryonic density typical for NS. In fact the appropriate theoretical tool, i.e. the lattice formulation of the quantum chromodynamics (QCD) is inapplicable at large baryon densities and small temperature due to the so-called Sign Problem [6], due to its complicated nonlinear and nonperturbative nature. On the other hand, in the large temperature and small density region lattice QCD simulations have provided controlled results for the EoS as well as for the nature of the transition [7, 8].

The value of the maximum mass of NS is probably one of the physical quantities that is most sensitive to the presence of quark matter in NS. The recent observation of a large NS mass in PSR J0348+0432 [9] with mass $M = 2.01 \pm 0.04 \, M_\odot$ ($M_\odot = 2 \times 10^{33} \text{g}$) implies that the EoS of NS matter has to be stiff enough to keep the maximum mass at these large values. Purely nucleonic EoS are able to accommodate such large masses [10], but the likely appearance of non-nucleonic degrees of freedom, like hyperons and quarks, will eventually soften the EoS with respect to the purely nucleonic case. The resulting low maximum mass value would in this case be incompatible with observations. The large value of the mass could then be explained only if both hyperonic and quark matter EoS are stiffer than expected.

Many models of quark matter do exist, and they all contain a high degree of uncertainty. The best one can do is to compare the predictions of different models and to estimate the uncertainty of the results for the NS matter as well as for the NS structure and mass. In this paper we use the Field Correlator Method model for the quark EoS [11], which contains ab initio the property of confinement, which is expected to play a role as far as the stability of a neutron star is concerned.
[12], and in principle is able to cover the full temperature-chemical potential plane [13]. In ref. [14] we tested this model against NS observations, which could seriously constrain the model parameters, i.e. the quark-antiquark potential $V_1$, the gluon condensate $G_2$ and, more recently, also the color-flavour superconducting gap $\Delta$ [15]. In Sect.2 we illustrate the FCM model at finite temperature and density, with the inclusion of the color-flavour locking effect.

As far as the hadronic phase is concerning, we use two definite EoS, based on the Brueckner-Hartree-Fock many-body theory for nuclear matter, both in its non-relativistic and relativistic version. Those are briefly reviewed in the Sect.3. Sect.4 contains some numerical results on the hadron-quark phase transition as described by the CSS parametrization, and a discussion of the FCM mapping onto the CSS parametrization. Moreover, results for the mass-radius-central density relation for hybrid stars will be illustrated, while effects due to the inclusion of hyperons will be briefly mentioned. Conclusions are reported in Sect.5.

2. Quark Matter EoS : the Field Correlator Method
The approach based on the FCM provides a natural treatment of the dynamics of confinement in terms of the Color Electric ($D^E_1$ and $D^E_2$) and Color Magnetic ($D^H_1$ and $D^H_2$) Gaussian correlators, being the former one directly related to confinement, so that its vanishing above the critical temperature implies deconfinement [11]. The extension of the FCM to finite temperature $T$ and chemical potential $\mu_q = 0$ gives analytical results in reasonable agreement with lattice data thus allowing to describe correctly the deconfinement phase transition [16, 17]. In this work, we are interested in the physics of neutron stars, and therefore we extended the FCM to finite values of the chemical potential [16] and zero temperature.

Within the FCM, the quark pressure for a single flavour is simply given by [16]

$$\frac{P_q}{T^4} = \frac{1}{\pi^2} \left[ \phi_\nu \left( \frac{\mu_q - V_1/2}{T} \right) + \phi_\nu \left( \frac{-\mu_q + V_1/2}{T} \right) \right]$$

where

$$\phi_\nu (a) = \int_0^\infty du \frac{u^4}{\sqrt{u^2 + \nu^2}} \left( \frac{1}{\exp[\sqrt{u^2 + \nu^2} - a] + 1} \right)$$

(2)

being $\nu = m_q/T$, and $V_1$, which indicates the large distance limit of the static $q\bar{q}$ potential, i.e.

$$V_1 = \int_0^{1/T} d\tau (1 - \tau T) \int_0^\infty d\chi D^E_{1}\left( \sqrt{\chi^2 + \tau^2} \right)$$

(3)

Therefore its value is an effective measure of the strength of the interaction when the particles are infinitely separated. The potential $V_1$ in Eq.(3) is assumed to be independent on the chemical potential, and this is partially supported by lattice simulations at very small chemical potential. The EoS is completely specified once the gluon contribution is added to the quark pressure, i.e.

$$\frac{P_g}{T^4} = \frac{8}{3\pi^2} \int_0^\infty d\chi \chi^3 \exp \left( \frac{1}{\chi^2 + \frac{9V_1^2}{8T}} \right) - 1$$

(4)

and therefore

$$P_{qq} = P_g + \sum_{j=u,d,s} P^j_q + \Delta \epsilon_{\text{vac}}$$

(5)

where $P_g$ and $P^j_q$ are respectively given in Eq. (1) and (4), and

$$\Delta \epsilon_{\text{vac}} \approx \frac{(11 \frac{2}{3} N_f)}{32} \frac{G_2}{2}$$

(6)
corresponds to the difference of the vacuum energy density between deconfined and confined phase, being $N_f$ the flavour number ($N_f = 3$ in this paper). $G_2$ is the gluon condensate whose numerical value, determined by the QCD sum rules, is known with large uncertainty [18]

$$G_2 = 0.012 \pm 0.006 \text{ GeV}^4$$

(7)

It is interesting to notice that $G_2$ appears only in the vacuum contribution to the pressure (6), and plays the same role of the bag constant in the MIT bag model. In addition, if one turns $V_1$ off, $P_q$ becomes the pressure of free quarks, and in this case the FCM reduces to the simplest version of the bag model. Therefore $V_1$ can be regarded as the main correction to the free quarks dynamics inside the bag. Therefore the EoS in Eq.(5) essentially depends on two parameters, namely the quark-antiquark potential $V_1$ and the gluon condensate $G_2$. A comparison with the lattice calculations of the Wuppertal-Budapest [19] and hotQCD [20] collaborations provides clear indications about their specific values at finite temperature and $\mu_B = 0$, but no direct relation is found with the corresponding values at finite $\mu_B$. Therefore, in our analysis we choose to keep $V_1$ and $G_2$ as a free parameters, and check what kind of indications can be extracted from the determination of the maximum mass of neutron stars. Additional effects that could induce changes in the FCM EoS are due to the colour superconducting pairing mechanism, which is expected to take place for extremely large values of the chemical potential [21, 22, 23]. We included these effects in our analysis [15] by adding to the full FCM pressure $P_{fg}$ the color-flavour locked (CFL) pressure contribution, i.e. $P_{cfl} = \Delta^2 \mu_B^2/3\pi^2$ only when the chemical potential is greater than $\mu_B = 3m^2/4\Delta$, as discussed for instance in [23]. The gap $\Delta$ is expected to be in the range $10-100$ MeV in the region of interest of $\mu_B$ for the NS, and we treat it as a third free parameter in the FCM model.

3. Hadronic Phase: EoS in the Brueckner-Bethe-Goldstone theory

The BHF method for the nuclear matter EoS is based on the Brueckner-Bethe-Goldstone (BBG) many-body theory, which is a linked cluster expansion of the energy per nucleon of nuclear matter [24]. In this approach one systematically replaces the bare nucleon-nucleon (NN) interaction $V$ by the Brueckner reaction matrix $G$, which is the solution of the Bethe-Goldstone equation

$$G(\rho; \omega) = V + V \sum_{k_a k_b} \frac{|k_a k_b \rangle Q(k_a k_b \rangle}{\omega - e(k_a) - e(k_b)} G(\rho; \omega),$$

(8)

where $\rho$ is the nucleon number density, $\omega$ is the starting energy, and $|k_a k_b \rangle Q(k_a k_b \rangle$ is the Pauli operator. $e(k) = e(k; \rho) = \hbar^2 k^2/2m + U(k; \rho)$ is the single particle energy, and $U$ is the single-particle potential,

$$U(k; \rho) = \sum_{k' \leq k_F} \langle k' | G(\rho; e(k) + e(k')) | k' \rangle_a$$

(9)

The subscript “$a$” indicates antisymmetrization of the matrix element. In the BHF approximation the energy per nucleon is

$$\frac{E}{A}(\rho) = \frac{3}{5} \frac{\hbar^2 k_F^2}{2m} + \frac{1}{2A} \sum_{k,k' \leq k_F} \langle k k' | G(\rho; e(k) + e(k')) | k k' \rangle_a$$

(10)

The inclusion of nucleonic three- body forces (TBFs) in this approach allows to reproduce correctly the nuclear matter saturation point $\rho_0 \approx 0.16$ fm$^{-3}$, $E/A \approx -16$ MeV, and gives values of incompressibility and symmetry energy at saturation compatible with those extracted
from phenomenology [1]. In this work we choose the Argonne \( v_{18} \) nucleon-nucleon potential [25], supplemented by the so-called Urbana model [26] as three-body force. In the following we will show results obtained with the relativistic counterpart, i.e. the Dirac-Brueckner-Hartree-Fock scheme [27] where the Bonn A potential is used as NN interaction.

In this paper we adopt a conventional description of stellar matter as composed by neutrons, protons, and leptons in beta equilibrium. The chemical potentials of each species are the fundamental input for solving the equations of chemical equilibrium, charge neutrality and baryon number conservation, yielding the equilibrium fractions of all species. Once the composition of the \( \beta \)-stable, charge neutral stellar matter is known, one can calculate the equation of state, i.e., the relation between pressure \( P \) and energy density \( \varepsilon \) as a function of the baryon density \( \rho \). It can be easily obtained from the thermodynamical relation

\[
P = \frac{dE}{dV} = P_B + P_l
\]

and

\[
P_B = \rho^2 \frac{d(\varepsilon_B/\rho)}{d\rho}, \quad P_l = \rho^2 \frac{d(\varepsilon_l/\rho)}{d\rho}
\]

with \( E \) the total energy and \( V \) the total volume. The total nucleonic energy density \( \varepsilon_B \) is obtained by adding the energy densities of each species \( \varepsilon_i \). As far as leptons are concerned, at those high densities electrons are a free ultrarelativistic gas, whereas muons are relativistic. Hence their energy densities \( \varepsilon_l \) are well known from textbooks [28]. The numerical procedure has been often illustrated in papers and textbooks, and therefore it will not be repeated here.

4. The hadron-quark phase transition

Recently, we found that the FCM model can be expressed in the language of the "Constant Speed of Sound" (CSS) parametrization [29, 30], and we showed how its parameters can be mapped onto the CSS parameter space. We remind that the CSS scheme is a general parametrization suitable for classifying different models of quark matter and establishing connections among them, and for expressing experimental constraints in a model-independent way. We remind that it is applicable to high-density equations of state for which: (a) there is a sharp interface between nuclear matter and quark matter, (b) the speed of sound in the high-density phase is pressure-independent in the range between the first-order transition pressure up to the maximum central pressure of neutron stars. Given the nuclear matter EoS \( \varepsilon_{NM}(p) \), the high-density EoS can be expressed as

\[
\varepsilon(p) = \begin{cases} 
\varepsilon_{NM}(p) & p < p_{\text{trans}} \\
\varepsilon_{NM}(p_{\text{trans}}) + \Delta \varepsilon + \varepsilon_{QM}^2(p - p_{\text{trans}}) & p > p_{\text{trans}}
\end{cases}
\]

where the three parameters: the pressure \( p_{\text{trans}} \) at the transition, the discontinuity in energy density \( \Delta \varepsilon \) at the transition, and the speed of sound \( c_{QM} \) characterize completely the high-density phase. It has also been shown [29] that the CSS parametrization gives four different topologies of the mass-radius curve for compact stars, i.e. a hybrid branch connected to the nuclear branch (C), or disconnected (D), or both present (B) or neither (A). We use the term “hybrid branch” to refer to the part of the mass-radius relation of hybrid stars whose central pressure is above \( p_{\text{trans}} \), and so they contain a core of the high-density phase. The occurrence of these as a function of the CSS parameters \( p_{\text{trans}}/\varepsilon_{\text{trans}} \) and \( \Delta \varepsilon/\varepsilon_{\text{trans}} \) at fixed \( c_{QM}^2 \) is shown schematically in Fig. 1 (taken from Ref. [29]). The mass-radius curve in each region is depicted in inset plots, in which the thick green line is the hadronic branch, the thin solid red lines are stable hybrid stars, and the thin dashed red lines are unstable hybrid stars. In the phase diagram the solid red line shows the threshold value \( \varepsilon_{\text{crit}} \) below which there is always a stable hybrid star branch connected to the neutron star branch. This critical value is given by [31, 32, 33]

\[
\frac{\Delta \varepsilon_{\text{crit}}}{\varepsilon_{\text{trans}}} = \frac{1}{2} + \frac{3}{2} \frac{p_{\text{trans}}}{\varepsilon_{\text{trans}}}
\]
Figure 1. Schematic phase diagram [29] for hybrid star branches in the mass-radius relation of compact stars. The four regions are (A) no hybrid branch (“absent”); (B) both connected and disconnected hybrid branches; (C) connected hybrid branch only; (D) disconnected hybrid branch only.

Figure 2. (Color online) The squared speed of sound $c_{QM}^2$ (panel (a)) is displayed vs. quark matter pressure for several values of $V_1$ (in MeV) and $G_2$ (in GeV$^4$). In panel (b), the FCM energy density is represented by full symbols, whereas the full lines denote the CSS parametrization given by Eq.(13).

and was obtained by performing an expansion in powers of the size of the core of high-density phase. Eq.(14) is an analytic result, independent of $c_{QM}^2$ and the nuclear matter EoS. The dashed and dot-dashed black lines mark the appearance-disappearance of the connected or disconnected hybrid star branch. The position of these lines depends on the value of $c_{QM}^2$ and (weakly) on the accompanying nuclear matter EoS [29].

In Fig. 2 we show that the CSS parametrization is applicable to the FCM EoS, since the speed of sound depends only weakly on the density or pressure. The upper panel shows the speed of sound vs. pressure for different values of the FCM parameters, displayed in the lower panel. We see that the speed of sound varies by less than 5% over the considered range of pressures along each curve, and lies in the interval $0.28 < c_{QM}^2 < 1/3$. The value of $c_{QM}^2$ shows a weak dependence on $V_1$ and extremely weak on $G_2$. The transition pressure is more sensitive to the FCM parameters, increasing rapidly with $V_1$ and with $G_2$, whereas the energy density at a given pressure increases slightly with $V_1$ or $G_2$.

Let us now discuss the main features of the hadron-quark phase transition. Fig. 3 shows numerical results for the pressure as a function of the baryon chemical potential $\mu_B$ in hadronic matter and quark matter in beta equilibrium. In particular, the green (red) solid curves represent the BHF (DBHF) EoS, whereas the remaining curves are the results for the FCM model with different choices of the quark-antiquark potential $V_1$ (expressed in MeV). In the left, middle and right panels the value assumed for the gap $\Delta$ is respectively equal to 0, 50 and 100 MeV. All calculations shown in Fig. 3 are performed taking $G_2 = 0.006$ GeV$^4$. We notice that with increasing the value of $V_1$ the transition point is shifted to larger values of the chemical potential, hence of the baryon density. However, the exact value depends also on the stiffness of the hadronic EoS at those densities. In this case, being the DBHF EoS stiffer than the BHF, the transition takes place at smaller values of the density. We notice that the transition point is affected also by the value of the gap $\Delta$, and is shifted toward smaller $\mu_B$ for larger value of the gap.
Figure 3. The pressure is displayed vs. the baryon chemical potential $\mu_B$ for the FCM quark matter and the purely hadronic matter. All calculations for FCM have been performed for $G_2 = 0.006$ GeV$^4$, and several values of $V_1$. The solid curves represent the BHF (green) and DBHF (red) EoS. In each panel results for different values of the gap $\Delta$, i.e. 0, 50, and 100 MeV are shown.

Figure 4. The mass as function of the radius (left panel) and of the central density (right panel) is displayed for several values of $V_1$, $G_2$ and $\Delta$. The labels (A), (B), (C) and (D) indicate the specific topologies of the hybrid star branch. The BHF EoS is used for the hadronic phase.
The resulting EoS, for the several cases discussed, is the main input for solving the well-known Tolman-Oppenheimer-Volkoff equations [28] for spherically symmetric NS, thus obtaining the mass-radius-central density relation. This is illustrated in Fig. 4, where the relation between mass and radius (left panel) and central density (right panel) in units of the saturation density $\rho_0$ is displayed. Results are plotted for different values of $V_1$, $G_2$ and $\Delta$ and the BHF EoS is used for hadronic matter. The largest value of the maximum mass in this example is obtained for $V_1 = 200$ MeV, $\Delta = 100$ and $G_2 = 0.01$, and it is compatible with the largest mass observed up to now, i.e. $(2.01 \pm 0.04)M_\odot$ in PSR J0348+0432 [9]. In Fig. 4 we show also the topology of the mass-radius curve. For example, for unpaired quark matter and $V_1 = -50$ MeV, the mass-radius relation obtained with $G_2 = 0.01$ GeV$^4$ (blue dashed line) exhibits a branch of stable hybrid stars disconnected (D) by the hadronic branch. With increasing $V_1$ the transition point moves to larger values of the pressure and the energy density, and as a consequence we explore regions of the phase diagram where the topology changes. For instance, for $V_1 = 0$ we can get both (B) connected and disconnected hybrid star branches, whereas for $V_1 = 100$ MeV connected (C) hybrid star branches are present and, for the largest value of $V_1 = 200$ MeV the hybrid branch is absent (A). This is clearly shown by a cusp in the mass-radius relation, and all configurations with radii smaller than the one characterizing the cusp are unstable. Therefore only purely nucleonic stars do exist in this case. We observe a similar topology when the DBHF EoS is used for the hadronic phase. In both cases we see that the largest possible values of the maximum mass are obtained only for values of $V_1 > 100$ MeV, and that only in the DBHF case maximum masses can be well above the observational limit. In fact, the heaviest BHF+FCM hybrid star has a mass of $2.03M_\odot$, and the heaviest DBHF+FCM hybrid star has a mass of $2.31M_\odot$ [15]. Those values are indicated by an orange cross in Fig. 5, where we display the mapping between the FCM and CSS parameters. In the upper (lower) panels we show results for the BHF (DBHF) hadronic EoS, whereas in the left, middle and right panels calculations are displayed for different values of the gap $\Delta = 0$, 50, 100 MeV respectively. Along each line we keep $V_1$ constant and vary $G_2$. For the BHF case the maximum hybrid star mass has value $V_1 = 240$ MeV, $G_2 = 0.0024$ GeV$^4$, and for the DBHF case $V_1 = 255$ MeV, $G_2 = 0.0019$ GeV$^4$. In Fig. 5 the dashed black contour delimits the region accessible by the FCM calculation, and the symbols connected by solid lines show the CSS parametrization of the FCM quark matter EoS. The vertical black dashed lines indicate the parameter regions accessible by the FCM and consistent with the measurement of a $M = 2M_\odot$. Hybrid stars with mass heavier than $2 M_\odot$ lie on a very small connected branch on the right side of the vertical black dashed lines, and cover a small range of central pressures, having a very tiny quark core, with mass and radius similar to those of the heaviest purely hadronic star, as was already discussed in Ref. [30]. We have found that for a hybrid star with $M = 2 M_\odot$ the hadronic layer occupies the largest portion of the star, and is characterized by a radius of about 10 km, whereas the crust radius is always smaller than 1 km.

Finally we briefly discuss the effects of adding the hyperon degrees of freedom in the BHF approach, which should produce a softening of the EoS with a strong decrease of the maximum mass [34, 35], with the maximum mass remaining well below the $2M_\odot$, at variance with the observational constraint. In ref.[15] we found that when $V_1$ is small, the masses remain below 1.5 $M_\odot$, whereas with larger $V_1$, the mass of the NS grows up to 1.95 $M_\odot$, which is reasonably close to the observational constraint of $2 M_\odot$. Therefore only with a special choice of the FCM parameters, we are able to reach sufficiently high NS masses in the presence of hyperon degrees of freedom.

5. Conclusions
We have studied the structure of hybrid NS employing for the hadronic phase the non-relativistic BHF EoS and its relativistic counterpart, the DBHF EoS. For the quark matter phase we have
used the FCM extension at finite $T$ and $\mu_B$, which provides a very simple description of the quark dynamics in terms of two parameters, namely the potential $V_1$, which summarizes the interaction corrections to the free quark and gluon pressure, and the gluon condensate $G_2$, that parametrizes the vacuum pressure and energy density. For completeness, we have included the effects of color superconductivity through the CFL mechanism, which amounts to the addition of a new free energy contribution in terms of the gap $\Delta$.

With this new set of more refined calculations, we confirm the trend already observed in [14, 36, 30], i.e. the maximum mass of hybrid stars grows with the two parameters $V_1$ and $G_2$ while it decreases when $\Delta$ is increased. We have also extended the mapping developed in [30] among the parameters of the FCM and those defining the CSS parametrization, by displaying the effect of the gap $\Delta$, thus concluding that a particular configuration with mass around or above two solar masses can be realized in the FCM by different pairs of $G_2$ and $V_1$, depending on the specific value assigned to $\Delta$. The inclusion of the hyperons induces dramatic changes in this picture. In fact, a regular transition from nuclear to quark matter with a stable quark phase up to very high chemical potential requires a particular tuning of the FCM parameters that leads to values of the maximum mass up to $1.95 M_\odot$.  

Figure 5. The mapping of the FCM quark matter model onto the CSS parametrization. The green curves are the phase boundaries for the occurrence of connected and disconnected hybrid branches. The dashed black line delimits the region yielded by the FCM model, and the symbols give CSS parameter values as $G_2$ is varied at constant $V_1$ (given in MeV). The (orange) cross denotes the parameters for the heaviest FCM hybrid star. Results are obtained using the BHF (upper panels) and DBHF (lower panels) nuclear matter EoS.
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6. References
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