Approximation method to compute domain related integrals in structural studies

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Abstract. Various engineering calculi use integral calculus in theoretical models, i.e. analytical and numerical models. For usual problems, integrals have mathematical exact solutions. If the domain of integration is complicated, there may be used several methods to calculate the integral. The first idea is to divide the domain in smaller sub-domains for which there are direct calculus relations, i.e. in strength of materials the bending moment may be computed in some discrete points using the graphical integration of the shear force diagram, which usually has a simple shape. Another example is in mathematics, where the surface of a subgraph may be approximated by a set of rectangles or trapezoids used to calculate the definite integral. The goal of the work is to introduce our studies about the calculus of the integrals in the transverse section domains, computer aided solutions and a generalizing method. The aim of our research is to create general computer based methods to execute the calculi in structural studies. Thus, we define a Boolean algebra which operates with ‘simple’ shape domains. This algebraic standpoint uses addition and subtraction, conditioned by the sign of every ‘simple’ shape (-1 for the shapes to be subtracted). By ‘simple’ shape or ‘basic’ shape we define either shapes for which there are direct calculus relations, or domains for which their frontiers are approximated by known functions and the according calculus is carried out using an algorithm. The ‘basic’ shapes are linked to the calculus of the most significant stresses in the section, refined aspect which needs special attention. Starting from this idea, in the libraries of ‘basic’ shapes, there were included rectangles, ellipses and domains whose frontiers are approximated by spline functions. The domain triangulation methods suggested that another ‘basic’ shape to be considered is the triangle. The subsequent phase was to deduce the exact relations for the calculus of the integrals associated to the transverse section problems. Thus we use a virtual rectangle which is framing the triangle, being generated supplementary right angled triangles. The sign of rectangle and the signs of the supplementary triangles are conditioned by the sign of the initial triangle. In this way, a generally located triangle for which we have direct calculus relations may be used to generate the discretization of any domain in transverse section associated integrals. A significant consequence of the paper is the opportunity to create modern computer aided engineering applications for structural studies, which use: intelligent applied mathematics background, modern informatics technologies and advanced computing techniques, such as calculus parallelization.

1. Introduction

Original models use analytic methods, as well as numerical solutions for the problems which have no exact mathematical solution, the computer being the instrument used to execute the calculi.
There are several situations in which numerical solutions are ‘embedded’ in the specific technical solution, being applied the principles on which a certain numerical method is based, rather than a ‘standard’ method or a function stored into software libraries. Furthermore, computer based modelling integrates analytic and numeric studies with experiments, being created hybrid models, which may offer precise results to intricate phenomena.

The development of original computer based models is a permanent concern of the authors, along the time being created software instruments stored in software libraries, which were used in many research projects, [1, 2].

A paramount aspect at the initiation of a software project which is the background that supports the research models to be developed is the data management model to be conceived, useful to have a facile development of the components, [3]. Once this problem is resolved, the researchers may focus on the scientific problem to be solved.

Beside the data management, another vector in the sustainable development of the computer based instruments for scientific research projects is the creation of some specialized libraries of programs that solve a certain mathematical problem using several methods. In this way, the researcher is able to use the data management facilities, as well as the mathematical methods and the calculus techniques, to invent efficient instruments of numerical programming to be used in several fields of science.

2. Specific details regarding the calculus of the integrals

A mathematical problem which had to be repeatedly solved in different contexts of numerous projects using several specific methods and data processing techniques was the calculus of the integrals.

One of the projects which demanded the calculus of the integrals was regarded the calculus of the rotations and of the deflections of some distinct points along a structure, [4], using the strain energy methods. The structure was made of straight and curved intervals, every interval having its own geometrical properties and material constants. The general relation used to calculate the deflections, relation provided by the Mohr-Maxwell method is:

\[
\delta = \sum \int \frac{N \cdot n}{E \cdot A} ds + \sum \int k_y \frac{T_y \cdot t_y}{G \cdot A_y} ds + \sum \int k_z \frac{T_z \cdot t_z}{G \cdot A_z} ds + \\
+ \sum \int \frac{M_x \cdot m_x}{G \cdot I_p} ds + \sum \int \frac{M_y \cdot m_y}{E \cdot I_y} ds + \sum \int \frac{M_z \cdot m_z}{E \cdot I_z} ds
\]  (1)

In the integral, \( n \), \( t_y \), \( t_z \), \( m_x \), \( m_y \) and \( m_z \) are the functions of the internal forces and moments produced by the unit force or the unit moment, positioned in the section where the displacement or rotation must be computed.

In this case, we have a plane problem and relation (1) becomes:

\[
\delta = \sum_{j=1}^{N_j} \left[ \frac{1}{E \cdot A} \int (N \cdot n) ds \right] + \sum_{j=1}^{N_j} \left[ k_z \cdot \frac{1}{G \cdot A_z} \int (T_z \cdot t_z) ds \right] + \sum_{j=1}^{N_j} \left[ \frac{1}{E \cdot I_y} \int (M_y \cdot m_y) ds \right] \]  (2)

where \( N_j \) is the total number of intervals of the system of straight and curved bars and \( j \) is the identifier of the current interval.

There was considered that the values of the deflections and of the rotations in every basic section of the structure, i.e. sections separating two intervals together with the sections in which there are located the external forces and moments, provide enough knowledge about the way how the structure is displaced.

Under these assumptions there was created a general model having in every basic section a vertical force, a horizontal force and a bending moment. Using these general loads there were deduced the general functions of the internal forces and moments. Substituting the real loads and considering the
other ones to be zero, we obtain the functions of internal forces and moments, i.e. \( N, T_z \) and \( M_y \). Next, we apply a single unit-force or unit-moment along the displacement or of the rotation to be computed, the other loads being zero. With this sets of loads we obtain the functions of the \( n, t_z \) and \( m_y \) internal forces and moment. The definition of the functions of the internal forces and moments was done algorithmically, therefore it resulted a flexible computation method, to be used in integrals shown in relation (2). These integrals had to be computed for both straight and curved intervals, so it was decided that a numerical approach is suitable. There was also conceived a method to minimize the number of calculi of the numerical computation of the integrals. A solver consisting of more than 6000 lines was developed for the automatic computations.

Another project in which a computer based method to compute integrals was employed, studied the behavior of a slender beam, [5]. According to the classic differential geometry, we have

\[
\frac{1}{\rho_z} = \frac{u''_z}{\left[1 + (u'_z)^2\right]^{\frac{3}{2}}}.
\]

Because the small displacement assumption is used, \( u'_z \) is small and \( (u'_z)^2 \) is considered nil, so

\[
\frac{1}{\rho_z} = u''_z(x).
\]

But we have the relation

\[
\frac{1}{\rho_z} = \frac{M_y(x)}{E \cdot I_y},
\]

so, by respecting the general principle ‘positive stresses are producing positive upper-level quantities (internal forces and moments, deflections aso.)’, the differential equation of the neutral fiber is

\[
\begin{align*}
\frac{u''_z(x)}{E \cdot I_y} &= -\frac{M_y(x)}{E \cdot I_y}, \\
\frac{u''_y(x)}{E \cdot I_y} &= \frac{M_y(x)}{E \cdot I_y}.
\end{align*}
\]

If we disregard the small displacements assumption, the general law of variation of the displacement is:

\[
u_z(x) = u_{Z_{ini}} + \varphi_{Y_{ini}} \cdot x + \int \left[ -\frac{M_y(x)}{E \cdot I_y} \right] \\
\sqrt{1 - \left[ \frac{M_y(x)}{E \cdot I_y} \right]^2}.
\]

There was developed a program of more than 400 lines which computes the integrals. In this way we obtain the value of the displacement using the classic method, the displacement using the exact
method, and the errors between these values. There was considered a cantilever having a rectangle transverse section. The vertical side of the rectangle was set to be a variable value, the beam being rigid for the first values and very flexible for the last ones. The results were analyzed to identify the extension of the small displacement hypothesis.

Another approach to calculate the integrals uses the Monte Carlo method, [6]. Thus, the random variables are extensively used in the computer based modeling of the economic events, as well as in electrical, mechanical and structural engineering. The level of randomness of some original pseudo-random numbers generators was evaluated using various criteria, i.e. the differences between the integration based on the Monte-Carlo method and the, so to say, classic computation methods.

3. Comparison between the methods employed to approximate the integrals

Automatic calculus of the geometrical properties was another project which demanded the calculus of the integrals.

There were conceived two methods to compute the integrals which define the geometrical properties.

First method, uses the general function

$$ f(y, z) = y^u \cdot z^v, $$

where $u, v \in \mathbb{N}$ and $u + v \leq 2$. This function is included in the integral

$$ I = \int_a^b \left( \int_{f(y)} f(y, z) \, dz \right) \, dy, $$

the frontiers of the inner integral being modeled using spline functions.

This integral was resolved mathematically and the solution uses series. The same integral was solved using a numerical method.

This method precisely approximated the domain using spline functions and it was verified for various shapes of the transverse sections. Furthermore, the precision of the approximation of the curved frontiers of the domain was verified to select the optimum number of nodes. If the number of nodes is small, then the approximation may become unstable on some intervals. If we use too many nodes, the execution time of the according program will increase.

![Figure 1. Division of a section in a set of hollow or solid 'simple' shapes.](image-url)
The second method to compute the geometrical properties is to divide the surface of the transverse section in so-to-say ‘basic’ shapes, hollow or solid, [8]. We define the ‘simple’ shapes as shapes for which there are direct calculus relations of the geometrical properties. We assign $\text{sgn}_j = -1$ if the $j$-th simple shape is hollow and $\text{sgn}_j = +1$ if it is solid. In this way we create a Boolean algebra which operates using simple shapes. To conclude, the geometrical property of a section may be computed as the algebraic summation of the geometrical properties of the simple shapes. In reference [7] the libraries of simple shapes included only rectangles and ellipses.

4. Discussion

Based on the above mentioned methods, one can settle that the principle to divide the section in sub-domains may also include ‘basic’ shapes whose frontiers may be defined using spline functions.

Another simple shape which may be extensively used for the division of a transverse section is the triangle. Once the principal calculus relations are deduced, [8], there may be conceived a general algorithm which may be used for generally located triangle, figure 2. In this way, a triangle may be included in a rectangle. The surface between the rectangle and the initial triangle includes right angled triangles. For both, the rectangle and the right angled triangles, there are direct calculus relations. In this way it was conceived an algorithm which uses direct calculus relations for a general triangle. Furthermore, any transverse section may be divided in triangles. To conclude, the triangle is a general shape which may be included in the libraries of ‘basic’ shapes.

5. Conclusions

Calculus of the integrals in engineering has many particularities with respect to the specificity of the problem to be solved and the data processing technique to be used. Most of the solutions are based on approximations, the examples about the calculus of the geometrical properties being significant to identify methods to compute integrals, which may be based on: exact analytical methods, division of the domain in a set of simple shapes and direct calculus relations. If several discretizations of a domain are used, i.e. for a transverse section, there may be evaluated the precision of the solution for every division. A benefit of this approach is the general definition of the set of shapes of a domain, being possible to use advanced computing techniques, like calculus parallelization. Other strength is the
aspect that the ‘simple’ shapes to be used for the calculus of the geometrical properties are associated to the set of nodes of the transverse section in which the significant stresses may be located. In this way, the choice about the ‘simple’ shapes in which the transverse section is divided may influence the time used to compute the results and their precision.

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