Composite model of quark-leptons and duality

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Abstract

In the present investigation the model of preons and their composites is constructed in the framework of the superstring-inspired flipped $E_6 \times \tilde{E}_6$ gauge group of symmetry which reveals a generalized dual symmetry. Here $E_6$ and $\tilde{E}_6$ are non-dual and dual sectors of theory with hyper-electric $g$ and hyper-magnetic $\tilde{g}$ charges, respectively. Considering preons belonging to the 27-plet of $E_6$, we follow the old idea by J. Pati: we assume that preons are dyons, which in our model are confined by hyper-magnetic strings – composite $\mathbb{N} = 1$ supersymmetric non-Abelian flux tubes created by the condensation of spreons near the Planck scale. Investigating the breakdown of $E_6$ and $\tilde{E}_6$ at the Planck scale into the $SU(6) \times U(1)$ gauge group, we show that the six types of strings having fluxes $\Phi_n = n\Phi_0$ ($n = \pm 1, \pm 2, \pm 3$) produce three (and only three) generations of composite quark-leptons and bosons. Assuming the existence of the three types of hyper-magnetic fluxes and using Schwinger’s formula, we have given an explanation of hierarchies of masses established in the Standard Model (SM). The following values of masses obtained in our preonic model:

\[
\begin{align*}
    m_t &\approx 173 \text{ GeV,} & m_c &\approx 1 \text{ GeV,} & m_u &\approx 4 \text{ MeV,} \\
    m_b &\approx 4 \text{ GeV,} & m_s &\approx 140 \text{ MeV,} & m_d &\approx 4 \text{ MeV,} \\
    m_\tau &\approx 2 \text{ GeV,} & m_\mu &\approx 100 \text{ MeV,}
\end{align*}
\]

are in agreement with the experimentally known results.

The following left-handed neutrino masses were predicted:

\[
\begin{align*}
    m_1 &\approx 1.3 \times 10^{-3} \text{ eV,} & m_2 &\approx 9.2 \times 10^{-3} \text{ eV,} & m_3 &\approx 5.0 \times 10^{-2} \text{ eV} \\
    m_1 &\approx 0.73 \times 10^{-2} \text{ eV,} & m_2 &\approx 7.4 \times 10^{-2} \text{ eV,} & m_3 &\approx 5.5 \times 10^{-2} \text{ eV}
\end{align*}
\]

– for direct hierarchy,

\[
\begin{align*}
    m_1 &\approx 1.3 \times 10^{-3} \text{ eV,} & m_2 &\approx 9.2 \times 10^{-3} \text{ eV,} & m_3 &\approx 5.0 \times 10^{-2} \text{ eV} \\
    m_1 &\approx 0.73 \times 10^{-2} \text{ eV,} & m_2 &\approx 7.4 \times 10^{-2} \text{ eV,} & m_3 &\approx 5.5 \times 10^{-2} \text{ eV}
\end{align*}
\]

– for inverted hierarchy.

The compactification in the space-time with five dimensions and its influence on form-factors of composite objects are briefly discussed.
Contents:

1. Introduction: Superstring $E_8 \times E_8'$ theory.

2. Supersymmetric ‘flipped’ $E_6$-unification of gauge interactions.

3. A new preonic model:
   Preons are dyons confined by hyper-magnetic strings.

4. Naturalness of three generations.

5. An explanation of mass hierarchies in the Standard Model.

6. The prediction of neutrino masses.

7. Form factors as an indication of the compositeness of quark-leptons.
An explicit construction of model of preons and their composites, unified in the
supersymmetric flipped $E_6$ GUT, is prescribed in the framework of superstring-
inspired flipped $E_6 \times \tilde{E}_6$ gauge group of symmetry, which reveals a generalized dual
symmetry. Here $E_6$ and $\tilde{E}_6$ are non-dual and dual sectors of theory with hyper-
electric $g$ and hyper-magnetic $\tilde{g}$ charges, respectively.

Pati was first [1] who suggested to use the strong $U(1)$ magnetic force as the binding
force for preons-dyons making the composite objects. This idea was developed in
Refs. [2] and has an extension in our model in the light of recent investigations of
composite non-Abelian flux tubes in SQCD [3–6].

We assume that preons are dyons, which in our model [7] are confined by hyper-
magnetic strings – composite $N = 1$ supersymmetric non-Abelian flux tubes created
by the condensation of spreons near the Planck scale.

Considering the role of compactification we start, as in Ref. [8], with
$N = 1$ supersymmetric gauge theory in 5
$D$ dimensions with a local symmetry gauge group $G$
which in our case is equal to the flipped $E_6$. The matter fields transform accord-
ing to one of irreducible representation of $G$. For example, we have a 27-plet of
preons similar to the fundamental 27 representation for quarks and leptons, which
decompose under $SU(5) \times U(1)$ subgroup as follows:

$$27 \rightarrow (10, 1) + (\bar{5}, -3) + \left(\begin{array}{c}
(5, 2) + (5, -2) + (1, 5) + (1, 0),
\end{array}\right)$$

The first and second quantities in the brackets of Eq. (1) correspond to the $SU(5)$
representation and $U(1)_X$ charge, respectively.

The conventional SM family which contains the doublets of left-handed quarks $Q$
and leptons $L$, right-handed up and down quarks $u^c$, $d^c$, also $e^c$, is assigned to the
$(10, 1) + (\bar{5}, -3) + (1, 5)$ representations of the flipped $SU(5) \times U(1)_X$, along with
right-handed neutrino $N^c$. These representations decompose under

$$SU(5) \times U(1)_X \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X,$$

We consider charges $Q_X$ and $Q_Z$ in the units $1/\sqrt{40}$ and $\sqrt{3/5}$, respectively, using
assignments: $Q_X = X$ and $Q_Z = Z$ (see [9] for details).

The decomposition (2) gives the following content:

$$27 \rightarrow (10, 1) + (\bar{5}, -3) + \left(\begin{array}{c}
(5, 2) + (5, -2) + (1, 5) + (1, 0),
\end{array}\right)$$

$$\begin{align*}
(10, 1) & \rightarrow Q = \begin{pmatrix} u \\ d \end{pmatrix}, \\
(\bar{5}, -3) & \rightarrow \begin{pmatrix} u^c \\ \bar{d}^c \end{pmatrix}, \\
(1, 5) & \rightarrow \begin{pmatrix} e^c \end{pmatrix},
\end{align*}$$

$$\begin{align*}
(5, 2) & \rightarrow \begin{pmatrix} e \nu \end{pmatrix}, \\
(5, -2) & \rightarrow \begin{pmatrix} e \nu \end{pmatrix}, \\
(1, 0) & \rightarrow \begin{pmatrix} e \nu \end{pmatrix}.
\end{align*}$$

(3)
The remaining representations in Eq. (1) decompose as follows:

\[(5, -2) \rightarrow D \sim \left(3, 1, -\frac{1}{3}, -2\right),\]
\[h = \left(\begin{array}{c} h^+ \\ h^0 \end{array}\right) \sim \left(1, 2, \frac{1}{2}, -2\right).\]  
(6)

\[(5, 2) \rightarrow D^c \sim \left(3, 1, \frac{1}{3}, 2\right),\]
\[h^c = \left(\begin{array}{c} h^0 \\ h^- \end{array}\right) \sim \left(1, 2, -\frac{1}{2}, 2\right).\]  
(7)

The light Higgs doublets are accompanied by coloured Higgs triplets \(D, D^c\).

The singlet field \(S\) is represented by \((1,0)\):

\[(1, 0) \rightarrow S \sim (1, 1, 2, 2).\]  
(8)

It is necessary to notice that the flipping of our \(SU(5)\):

\[d^c \leftrightarrow u^c, \quad N^c \leftrightarrow e^c,\]  
(9)

distinguishes this group of symmetry from the standard Georgi-Glashow \(SU(5)\).

Supermultiplet \(\mathcal{V} = (A^M, \lambda^\alpha, \Sigma, X^a)\) in 5D-dimensional space (with \(M = 0, 1, 2, 3, 4\) space-time indices) contains a vector field:

\[A^M = A^{MJ}T^J,\]  
(10)

and a real scalar field:

\[\Sigma = \Sigma^J T^J,\]  
(11)

where \(J\) runs over the \(E_6\) group index values and \(T^J\) are generators of \(E_6\) algebra.

Two gauginos fields:

\[\lambda^\alpha = \lambda^{aJ}T^J,\]  
(12)

form a decuplet under the \(R\)-symmetry group \(SU(2)_R\) (with \(\alpha = 1, 2\)).

Auxiliary fields:

\[X^a = X^{aJ}T^J\]  
(13)

form a triplet of \(SU(2)_R\) (with \(a = 1, 2, 3\))

After compactification, these fields are combined into the \(N = 1\) 4D fields:

vector supermultiplet \(V = (A^\mu, \lambda^1, X^3)\)

(here \(\mu = 0, 1, 2, 3\) are the space-time indices), and

a chiral supermultiplet \(\Phi = (\Sigma + iA^4, \lambda^2, X^1 + iX^2)\).
The matter fields are contained in the hypermultiplet:

\[ \mathcal{H} = (h^\alpha, \Psi, F^{\alpha}) , \]

where \( h^\alpha \) is a doublet of \( SU(2)_R \), Dirac fermion field \( \Psi = (\psi_1, \psi_2)^T \) is the \( SU(2)_R \) singlet and auxiliary fields \( F^{\alpha} \) also are the \( SU(2)_R \) doublet.

Then we have two \( N = 1 \) 4D chiral multiplets:

\[ H = (h^1, \psi_1, F^1) \quad \text{and} \quad H^c = (h^2, \psi_2, F^2) \]

transforming according to the representations \( R \) and anti-\( R \) of the gauge group \( G = E_6 \), respectively. Then the 5D-dimensional \( E_6 \)-symmetric action is given as follows:

\[ S = \int d^5x \int d^4\theta \left[ H^c e^V H^{c+} + H^+ e^V H \right] + \int d^5x \int d^2\theta \left[ H^c \left( \partial_4 - \frac{1}{\sqrt{2}} \Phi \right) H + h.c. \right] . \]

This theory is anomaly-free (see [8] and references therein).

Compactifying the extra fifth dimension \( x^4 \) in Eq. (14) on a circle of radius \( R_C \), the authors of Ref. [8] impose the Scherk-Schwarz boundary conditions to the preonic superfields and obtained the following result:

\[
\begin{align*}
P (x^m, x^4 + 2\pi R_C, \theta) &= e^{2\pi q_P} P (x^m, x^4, e^{i\pi (q_P + q_{\bar{P}})} \theta), \\
P^c (x^m, x^4 + 2\pi R_C, \theta) &= e^{2\pi q_{\bar{P}}} P^c (x^m, x^4, e^{i\pi (q_P + q_{\bar{P}})} \theta), \\
P_s (x^m, x^4 + 2\pi R_C, \theta) &= e^{2\pi q_s} P_s (x^m, x^4, e^{i\pi (q_s + q_{\bar{s}})} \theta), \\
P^c_s (x^m, x^4 + 2\pi R_C, \theta) &= e^{2\pi q_{\bar{s}}} P^c_s (x^m, x^4, e^{i\pi (q_s + q_{\bar{s}})} \theta),
\end{align*}
\]

where \( q_P, q_{\bar{P}} \) and \( q_s, q_{\bar{s}} \) are \( R \) charges of preons \( P, P^c \) and \( P_s, P^c_s \), respectively. The following conditions were obtained:

\[ q_P + q_{\bar{P}} = q_s + q_{\bar{s}}. \]

As a result of compactification, all fermionic preons are massive in 4D space-time. Supersymmetry is broken by the boundary conditions (15).

3. Why three generations exist in Nature? We suggest an explanation considering a new preonic model of composite SM particles. The model starts from the supersymmetric flipped \( E_6 \times \bar{E}_6 \) gauge group of symmetry for preons, where \( E_6 \) and \( \bar{E}_6 \) correspond to non-dual (with hyper-electric charge \( g \)) and dual (with hyper-magnetic charge \( \tilde{g} \)) sectors of theory, respectively. We assume that preons are dyons confined by hyper-magnetic strings in the region of energy \( \mu < M_P \).

**Preons are dyons bound by hyper-magnetic strings**

Considering the \( N = 1 \) supersymmetric flipped \( E_6 \times \bar{E}_6 \) gauge theory for preons in 4D-dimensional space-time, we assume that preons \( P \) and antipreons \( P^c \) are dyons with charges \( g \) and \( \tilde{g} \), respectively, resided in the 4D hypermultiplets \( \mathcal{P} = (P, P^c) \) and \( \bar{\mathcal{P}} = (\bar{P}, \bar{P}^c) \). Here “\( \bar{P} \)” designates spreons, but not the belonging to \( \bar{E}_6 \).
The dual sector $\tilde{E}_6$ is broken in our world to some group $\tilde{G}$, and preons and spreons transform under the hyper-electric gauge group $E_6$ and hyper-magnetic gauge group $\tilde{G}$ as their fundamental representations:

$$P, \tilde{P} \sim (27, N), \quad P^c, \tilde{P}^c \sim (\overline{27}, \bar{N}),$$

(17)

where $N$ is the $N$-plet of $\tilde{G}$ group. We also consider scalar preons and spreons as singlets of $E_6$:

$$P_s, \tilde{P}_s \sim (1, N), \quad P^c_s, \tilde{P}^c_s \sim (1, \bar{N}),$$

(18)

which are actually necessary for the entire set of composite quark-leptons and bosons.

The hyper-magnetic interaction is assumed to be responsible for the formation of $E_6$ fermions and bosons at the compositeness scale $\Lambda_s$. The main idea of the present investigation is an assumption that preons-dyons are confined by hyper-magnetic supersymmetric non-Abelian flux tubes which are a generalization of the well-known Abelian ANO-strings for the case of the supersymmetric non-Abelian theory developed in Refs. [3–6]. As a result, in the limit of infinitely narrow flux tubes (strings) we have the following bound states:

**i. quark-leptons (fermions belonging to the $E_6$ fundamental representation):**

$$Q^a \sim P^a(y) \left[ \mathcal{P} \exp \left( i\tilde{g} \int_x^y \tilde{A}_\mu dx^\mu \right) \right]^B_A (P^c_s)_B(x) \sim 27,$$

(19)

$$\tilde{Q}_a \sim (P^c_s)^A(y) \left[ \mathcal{P} \exp \left( i\tilde{g} \int_x^y \tilde{A}_\mu dx^\mu \right) \right]^B_A P_{AB}(x) \sim \overline{27},$$

(20)

where $a \in 27$-plet of $E_6$, $A, B \in N$-plet of $\tilde{G}$, and $\tilde{A}_\mu(x)$ are dual hyper-gluons belonging to the adjoint representation of $\tilde{G}$;

**ii. “mesons” (hyper-gluons and hyper-Higgses of $E_6$):**

$$M^a_b \sim P^a(y) \left[ \mathcal{P} \exp \left( i\tilde{g} \int_x^y \tilde{A}_\mu dx^\mu \right) \right]^B_A (P^c)^b_B(x)$$

$$\sim 1 + 78 + 650 \quad \text{of} \quad E_6,$$

(21)

$$S \sim (P^c_s)^A(y) \left[ \mathcal{P} \exp \left( i\tilde{g} \int_x^y \tilde{A}_\mu dx^\mu \right) \right]^B_A (P^c_s)_B(x) \sim 1,$$

(22)

$$\bar{S} \sim (P^c_s)^A(y) \left[ \mathcal{P} \exp \left( i\tilde{g} \int_x^y \tilde{A}_\mu dx^\mu \right) \right]^B_A (P^c_s)_B(x) \sim 1;$$

(23)
iii. “baryons”,

for $\tilde{G}$-triplet we have (see Section 5):

\begin{align}
D_1 & \sim \epsilon_{ABC} P^{aA'}(z) P^{bB'}(y) P^{cC'}(x) \left[ \mathcal{P} \exp \left( i \tilde{g} \int_X^{x} \tilde{A}_\mu dx^\mu \right) \right]_{A'}^A \times \\
& \left[ \mathcal{P} \exp \left( i \tilde{g} \int_X^{y} \tilde{A}_\mu dx^\mu \right) \right]_{B'}^B \left[ \mathcal{P} \exp \left( i \tilde{g} \int_X^{z} \tilde{A}_\mu dx^\mu \right) \right]_{C'}^C,
\end{align}

(24)

\begin{align}
D_2 & \sim \epsilon_{ABC} P^{aA'}(z) P^{bB'}(y) P^{cC'}(x) \left[ \mathcal{P} \exp \left( i \tilde{g} \int_X^{x} \tilde{A}_\mu dx^\mu \right) \right]_{A'}^A \times \\
& \left[ \mathcal{P} \exp \left( i \tilde{g} \int_X^{y} \tilde{A}_\mu dx^\mu \right) \right]_{B'}^B \left[ \mathcal{P} \exp \left( i \tilde{g} \int_X^{z} \tilde{A}_\mu dx^\mu \right) \right]_{C'}^C,
\end{align}

(25)

and their conjugate particles.

The bound states (19)–(25) are shown in Fig. 1 as unclosed strings (a) and “baryonic” configurations (b). It is easy to generalize Eqs. (19)–(25) for the case of string constructions of superpartners – squark-sleptons, hyper-gluinos and hyper-higgsinos. Closed strings – gravitons – are presented in Fig. 1(c). All these bound states belong to the $E_6$ representations and they are in fact the $N = 1$ 4D superfields.

4. In the letter [10], it was shown that preonic $E_6$ can be broken by Higgses belonging to the 78-dimensional representation of $E_6$:

\[ E_6 \rightarrow SU(6) \times SU(2) \rightarrow SU(6) \times U(1), \]

(26)

where $SU(6) \times U(1)$ is the largest relevant invariance group of the 78.

$SU(6) \times U(1)$ group of symmetry existing near the Planck scale is described by theory of non-Abelian tubes in $N = 1$ SQCD which was developed recently in Refs. [3–6].

We assume the condensation of spreons near the Planck scale. Then theory leads to the topologically stable string solutions possessing both windings, in $SU(6)$ and $U(1)$. Now onwards we assume the dual sector of theory described by $\tilde{SU}(6) \times \tilde{U}(1)$ group of symmetry which produces hyper-magnetic fluxes. Then, according to the results obtained in Refs. [3–6], we have a nontrivial homotopy group:

\[ \pi_1 \left( \frac{SU(6) \times U(1)}{Z_6} \right) \neq 0, \]

(27)

and flux lines form topologically non-trivial $Z_6$ strings.

The model contains six scalar fields charged with respect to $U(1)$ and belonged to the 6-plet of $SU(6)$. Considering scalar fields of spreons:

\[ \tilde{P} = \{ \phi^{aA} \}, \quad \text{where} \quad a, A = 1, \ldots, 6, \]

(28)

5
we can construct condensation of spreons in vacuum:

\[ \tilde{P}_{\text{vac}} = \langle \tilde{P}^{aA} \rangle = v \cdot \text{diag}(1, 1, \ldots, 1), \quad a,A=1,\ldots,6. \quad (29) \]

The value \( v \) is the vacuum expectation value (VEV), which in theory of Refs. [3] is given by

\[ v = \sqrt{\xi} \gg \Lambda_4, \quad (30) \]

where \( \xi \) is the Fayet-Iliopoulos D-term parameter in the \( N = 1 \) supersymmetric theory and \( \Lambda_4 \) is its 4-dimensional scale. In our case:

\[ v \sim M_{Pl} \sim 10^{19} \text{ GeV}, \quad (31) \]

because spreons are condensed at the Planck scale.

Non-trivial topology (27) amounts to winding of elements of matrix (28), and we obtain string solutions:

\[ \tilde{P}_{\text{string}} = v \cdot \text{diag} \left( e^{i\alpha(x)}, e^{i\alpha(x)}, \ldots, 1, 1 \right) \quad \text{where} \quad x \to \infty. \quad (32) \]

Investigating string moduli space (see Ref. [4, 6]), we obtain the solutions for six types of \( Z_6 \)-flux tubes which are a non-Abelian analog of Abrikosov-Nielsen-Olesen (ANO)-strings.

Considering at the ends of strings preons \( P \) and \( P^c \) with hyper-magnetic charges \( n\tilde{g} \) and \( -n\tilde{g} \), respectively, we obtain six types of strings having the fluxes \( \Phi_n \) quantized according to the \( Z_6 \) center group of \( SU(6) \):

\[ \Phi_n = n\Phi_0, \quad n = \pm 1, \pm 2, \pm 3. \quad (33) \]

String tensions of these non-Abelian flux tubes also were calculated in Refs. [3]. The minimal tension is:

\[ T_0 = 2\pi \xi, \quad (34) \]

which in our preonic model is equal to:

\[ T_0 = 2\pi v^2 \sim 10^{38} \text{ GeV}^2, \quad (35) \]

that is, enormously large. This means that preonic strings have almost infinitely small \( \alpha' \to 0 \), where \( \alpha' = 1/(2\pi T_0) \) is a slope of trajectories in string theory [1]. Six types of preonic tubes give three types of \( k \)-strings having the following tensions:

\[ T_k = kT_0, \quad \text{where} \quad k = 1, 2, 3. \quad (36) \]

Thus, hyper-magnetic charges of preons and antipreons are confined by six flux tubes which are oriented in opposite directions, but have only three different tensions (36). Also preonic strings are enormously thin. As it was shown in the letter [10], the thickness of preonic strings given by the radius \( R_{\text{str}} \) of the flux tubes is very small:

\[ R_{\text{str}} \sim \frac{1}{m_V} \sim \frac{1}{gv} \sim 10^{-18} \text{ GeV}^{-1}. \quad (37) \]

Such infinitely narrow non-Abelian supersymmetric flux tubes remind us super-strings of Superstring theory.
5. In the previous item we have given a demonstration of a very specific type of the “horizontal symmetry”: three, and only three, generations of fermions and bosons exist in the superstring-inspired flipped $E_6$ theory. All bound states (3)–(8) form three generations – three 27-plets of $E_6$. We also obtain the three types of gauge bosons $A_i^{\mu}$ ($i = 1, 2, 3$ is the index of generations) Fig. 2 illustrates the formation of such hyper-gluons (Fig. 2(a)) and also hyper-Higgses of Fig. 2(b).

Having in our preonic model supersymmetric strings with $\alpha' \to 0$ we obtain, according to the description [1], only massless ground states: spin 1/2 fermions (quarks and leptons), spin 1 hyper-gluons and spin 2 massless graviton, as well as their superpartners. The exited states belonging to these strings are not realized in our world as very massive: they have mass $M > M_{Pl}$.

Using our preonic model, we give a simple explanation why quarks and leptons of three SM generations have such different masses. We show that the hierarchy of the SM masses is connected with the string construction of preon bound states.

New preon-antipreon pairs can be generated in the flux tubes between preons. Our assumption is that the mechanism of preon-antipreon pair production in these tubes is similar to that of $e^+e^-$ pair production in the uniform constant electric field considered in one space dimension (see [7] and references therein).

Almost thirty years ago J. Schwinger obtained the following expression for the rate per unit volume that a $e^+e^-$ pair will be created in the constant electric field of strength $\mathcal{E}$:

$$W = \frac{(e\mathcal{E})^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left( -\frac{\pi m_e^2 n}{e\mathcal{E}} \right),$$  \hspace{1cm} (38)

where $e$ is the electron charge and $m_e$ its mass.

A number of authors have applied this result to hadronic productions considering a creation of quark-antiquark pairs in the QCD tubes of colour flux.

Considering that the probability $W_k$ of one preon-antipreon pair production in a unit space-time volume of the $k$-string tube with tension (36) is given by Schwinger’s expression:

$$W_k = \frac{T_k^2}{4\pi^3} \exp \left( -\frac{\pi M_k^2}{T_k} \right), \quad \text{where} \quad T_k = kT_0, \quad k = 1, 2, 3,$$  \hspace{1cm} (39)

we assume that Yukawa couplings $Y_k$ for the three generations of quarks ($t, c$ and $u$) are proportional to the square root of probabilities $W_k$. Then we obtain the following ratios:

$$Y_t : Y_c : Y_u :: W_1^{1/2} : W_2^{1/2} : W_3^{1/2}. \hspace{1cm} (40)$$

In Eq. (39) the values $M_k$ are the constituent masses of preons produced in the $k$-strings. Yukawa couplings $Y_k$ are proportional to the masses of quarks $t, c, u$, and we have:

$$m_t : m_c : m_u :: \frac{T_0}{2\pi^{3/2}} \exp \left( -\frac{\pi M_t^2}{2T_0} \right) : \frac{2T_0}{2\pi^{3/2}} \exp \left( -\frac{\pi M_c^2}{4T_0} \right) : \frac{3T_0}{2\pi^{3/2}} \exp \left( -\frac{\pi M_u^2}{6T_0} \right).$$  \hspace{1cm} (41)
Assuming that $M_k$ are proportional to $k$:

$$M_k = kM_0,$$  \hspace{1cm} (42)

we get the following result:

$$m_t : m_c : m_u = m_0 \left( w : 2w^2 : 3w^3 \right),$$  \hspace{1cm} (43)

where

$$w = \exp \left( -\frac{\pi M_0^2}{2T_0} \right),$$  \hspace{1cm} (44)

and $m_0$ is a mass parameter.

For parameter

$$w = w_1 \approx 2.9 \cdot 10^{-3}$$  \hspace{1cm} (45)

we obtain the following values for masses of $t, c, u$-quarks:

$$m_t \approx 173 \text{ GeV}, \quad m_c \approx 1 \text{ GeV} \quad \text{and} \quad m_u \approx 4 \text{ MeV}.$$  \hspace{1cm} (46)

For $b, s, d$-quarks we have different parameters $M_0$, $m_0$ and $w = w_2$. The value:

$$w_2 \approx 1.7 \cdot 10^{-2}$$  \hspace{1cm} (47)

gives the following masses of $b, s, d$-quarks:

$$m_b \approx 4 \text{ GeV}, \quad m_s \approx 140 \text{ MeV} \quad \text{and} \quad m_d \approx 4 \text{ MeV}.$$  \hspace{1cm} (48)

The results (46) and (48) are in agreement with experimentally established quark masses published in Ref. [12]:

$$m_t \approx 174 \pm 5.1 \text{ GeV}, \quad m_c \approx 1.15 \text{ to } 1.35 \text{ GeV} \quad \text{and} \quad m_u \approx 1.5 \text{ to } 4 \text{ MeV}.$$  \hspace{1cm} (49)

and

$$m_b \approx 4.1 \text{ to } 4.9 \text{ GeV}, \quad m_s \approx 80 \text{ to } 130 \text{ MeV} \quad \text{and} \quad m_d \approx 4 \text{ to } 8 \text{ MeV}.$$  \hspace{1cm} (50)

The value of $w$-parameter:

$$w_3 \approx 2.5 \cdot 10^{-2}$$  \hspace{1cm} (51)

leads to the following values for masses of $\tau, \mu$-leptons and electron:

$$m_\tau \approx 2 \text{ GeV}, \quad m_\mu \approx 100 \text{ MeV} \quad \text{and} \quad m_e \approx 3.5 \text{ MeV},$$  \hspace{1cm} (52)

which are comparable with the results of experiment [12]:

$$m_\tau \approx 1.777 \text{ GeV}, \quad m_\mu \approx 105.66 \text{ MeV} \quad \text{and} \quad m_e \approx 0.51 \text{ MeV}.$$  \hspace{1cm} (53)

We see that our preonic model explains the hierarchy of masses existing in the SM.
6. The method permits to predict masses of the left-handed neutrinos [13,14]. Experimental results on solar neutrino and atmospheric neutrino oscillations from Sudbury Neutrino Observatory (SNO Collaboration) and the Super-Kamiokande Collaboration have been used to extract the following parameters [15]:

\[
\Delta m^2_{\odot} = m_2^2 - m_1^2 \approx 8.3 \times 10^{-5} \text{eV}^2,
\]

\[
\Delta m^2_{\text{atm}} = |m_3^2 - m_2^2| \approx 2.4 \times 10^{-3} \text{eV}^2,
\]

where \(m_1, m_2, m_3\) are the hierarchical left-handed neutrino effective masses for three families.

The solution of Eqs. (54) and (55) leads to the following result:

\[
m_0 \approx 0.540 \text{eV}, \quad w \approx 0.092.
\]

This solution gives the direct hierarchy masses of the left-handed neutrinos:

\[
m_1 \approx 1.3 \times 10^{-3} \text{eV}, \quad m_2 \approx 9.2 \times 10^{-3} \text{eV}, \quad m_3 \approx 5.0 \times 10^{-2} \text{eV}.
\]

For the case of inverted hierarchy of neutrino masses we have obtained the following prediction:

\[
m_1 \approx 0.73 \times 10^{-2} \text{eV}, \quad m_2 \approx 7.4 \times 10^{-2} \text{eV}, \quad m_3 \approx 5.5 \times 10^{-2} \text{eV}.
\]

7. Non point-like behaviour of the fundamental quarks and leptons is related with the appearance of form factors describing the dependence of cross sections in 4-momentum squared \(q^2\) for different elementary particle processes, for example, in reactions:

\[e^+e^- \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-, \gamma\gamma,\]

or

\[e^+e^- \rightarrow \text{hadrons},\]

or in the deep inelastic lepton scattering experiments, etc.

Considering the form factor \(F(q^2)\) as a power series in \(q^2\), we find:

\[
F(q^2) = 1 + \frac{q^2}{\Lambda^2} + O \left(\frac{1}{\Lambda^4}\right), \quad \text{where} \quad q^2 \ll \Lambda^2.
\]

If \(\Lambda = \Lambda_4\) (see item 4) is the energy scale associated with the quark-lepton compositeness scale \(\Lambda_s\) of the preon interaction, then our model predicts:

\[
\Lambda \sim \Lambda_s \sim 10^{18} \text{GeV}.
\]

Of course, such a scale is not available even for future high energy colliders.

From a phenomenological point of view it is quite important to understand whether the compactification scale lowers down \(\Lambda\) in Eq. (59) to energies accessible for the future accelerators.
Proposing the presence of extra space-time dimensions at small distances \[8\] we have a hope that the radius \(R_C\) of compactification given by Eq. (15) is larger than the radius \(R_s\) of compositeness:

\[ R_C \gg R_s, \tag{61} \]

and

\[ \Lambda \sim \Lambda_C \ll 10^{18} \text{ GeV}. \tag{62} \]

According to Ref. \[8\] the \(R\) charges for the composite states \((19)-(22)\) are:

\[ Q \sim q_P + q_s, \quad \bar{Q} \sim q_{\bar{P}} + q_s, \quad M \sim q_P + q_{\bar{P}}, \quad S \sim q_s + q_{\bar{s}}. \tag{63} \]

However, this problem needs the more detailed investigations in higher dimensional theories.

**Conclusions**

In this talk we have presented:

i. Supersymmetric \(E_6\) content in five space-time dimensions and the problem of compactification.

ii. Starting with \(N = 1\) supersymmetric \(E_6 \times \tilde{E}_6\) preonic model of composite quark-leptons and bosons, we have assumed that preons are dyons confined by hyper-magnetic strings in the region of energies \(\mu \lesssim M_{Pl}\). This approach is an extension of the old idea by J. Pati \[1\] who suggested to use the strong magnetic forces to bind preons-dyons in composite particles – quark-leptons and bosons. Our model is based on the recent theory of composite non-Abelian flux tubes in SQCD which was developed in Refs. \[3, 4\].

iii. Considering the breakdown of \(E_6\) and \(\tilde{E}_6\) at the Planck scale into the \(SU(6) \times U(1)\) gauge group we have shown that six types of \(k\)-strings – composite \(N = 1\) supersymmetric non-Abelian flux tubes – are created by the condensation of spreons near the Planck scale.

iv. We have obtained that six types of strings-tubes have six fluxes of hyper-magnetic fields quantized according to the \(Z_6\) center group of \(SU(6)\):

\[ \Phi_n = n\Phi_0, \quad n = \pm 1, \pm 2, \pm 3, \]

and these fluxes produce three (and only three) generations of composite quark-leptons and bosons.

v. It was shown that in the present model preonic strings are very thin, with radius

\[ R_{str} \sim 10^{-18} \text{ GeV}^{-1}, \]

and their tension is enormously large:

\[ T \sim 10^{38} \text{ GeV}^2. \]

They remind superstrings of Superstring theory.
vi. By the existence of three types of hyper-magnetic fluxes, using Schwinger’s formula, we have given an explanation of hierarchies of masses established in the SM. Our result:

\[ m_t \approx 173 \text{ GeV}, \quad m_c \approx 1 \text{ GeV} \quad \text{and} \quad m_u \approx 4 \text{ MeV}, \]
\[ m_b \approx 4 \text{ GeV}, \quad m_s \approx 140 \text{ MeV} \quad \text{and} \quad m_d \approx 4 \text{ MeV}, \]
\[ m_{\tau} \approx 2 \text{ GeV}, \quad m_{\mu} \approx 100 \text{ MeV} \quad \text{and} \quad m_e \approx 3.5 \text{ MeV}, \]

is comparable with experimentally established results (see Ref. [12]).

vii. The following left-handed neutrino masses were predicted:

\[ m_1 \approx 1.3 \times 10^{-3} \text{ eV}, \quad m_2 \approx 9.2 \times 10^{-3} \text{ eV}, \quad m_3 \approx 5.0 \times 10^{-2} \text{ eV} \]

– for direct hierarchy,
\[ m_1 \approx 0.73 \times 10^{-2} \text{ eV}, \quad m_2 \approx 7.4 \times 10^{-2} \text{ eV}, \quad m_3 \approx 5.5 \times 10^{-2} \text{ eV} \]

– for inverted hierarchy. These results are in agreement with experiment [12, 15].

viii. We have considered form factors described the compositeness of quark-leptons. We have shown that in our preonic model the scale \( \Lambda_s \) of the quark-lepton compositeness is very large:

\[ \Lambda_s \sim 10^{18} \text{ GeV}, \]

what is not accessible even for future high energy colliders. Only the compactification procedure of extra space-time dimensions can help to lower the scale \( \Lambda_s \).

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Fig. 1: Preons are bound by hyper-magnetic strings: (a,b) correspond to the string configurations of composite particles belonging to the 27-plet of the flipped $E_6$ gauge group of symmetry; (c) represents a closed string describing a graviton.
Fig. 2: Vector gauge bosons belonging to the 78 representation of the flipped $E_6$ and Higgs scalars – singlet of $E_6$ – are composite objects created (a) by fermionic preons $P, P^c$ and (b) by scalar preons $P_s, P_s^c$. Both of them are confined by hyper-magnetic strings.