Can the symmetry breaking in the SM be determined by the "second minimum" of the Higgs potential?

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The possibility that the spontaneous symmetry breaking in the Standard Model (SM) may be generated by the Top-Higgs Yukawa interaction (which determines the so called “second minimum” in the SM) is examined. A former analysis is extended about a QCD action only including the Yukawa interaction of a single quark with a scalar field. We repeat the calculation of the two loop effective action of the model for the scalar field. A correction of the evaluation allowed choosing a strong coupling \( \alpha(\mu, \Lambda_{\text{QCD}}) = 0.2254 \) GeV at an intermediate scale \( \mu = 11.63 \) GeV, in order to fix the minimum of the potential at a scalar field determining 175 GeV for the quark mass. A scalar field mass \( m = 44 \) GeV is following, which is of the order than the experimental Higgs mass. The effects of considering a running with momenta coupling are studied. For this, the finite part of the two loop potential contribution determined by the strong coupling, was represented as a momentum integral. Next, substituting in this integral the experimental values of the running coupling, the potential curve became very similar to the one for constant coupling. This happened after simply assuming that the low momentum dependence of the coupling is "saturated" to a constant value being close to its lowest experimental value.

I. INTRODUCTION

The so called "second minimum" of the Higgs field potential in the Standard Model is the result of the Yukawa interaction of the Higgs field with the Top quark. The presence of that minimum had been intrinsically related with the same construction of the model along the years. Special procedures of fixing the various parameters of the theory had to be designed in order to assure that the minimum is separated from the usual Higgs extremum by a potential barrier, being impossible to be tunnelled by the standard physical vacuum [1–3]. In addition proposals had been advanced that determine the Higgs mass from the condition for the two minima to coincide in values of the potential [3]. In Ref. [4] a simple massless QCD model including only one quark type (modeling the Top quark) and a scalar field (modeling the Higgs field) with a Yukawa interaction between them, was investigated. The aim of the study was to explore a suspicion about that the so called the "second minimum" could in fact be the responsible for the symmetry breaking in the SM. The idea was to evaluate the two loop effective potential for the scalar field, which in the SM is the responsible for the generation of the "second minimum" and to study the possibility of choosing the renormalization conditions to fix the value of the single fermion mass as equal to the top quark one 175 GeV. An idea strongly motivating this previous work, came after noting that this additional minimum was identified only after the SM calculations arrived up to the two loop order. Then, the question emerges about what could had been the result of an attempt to construct the SM around this new radiative corrections determined minimum, if it would had been known from the start in the SM construction. Up to our knowledge, there had not been attempts to answer this question in the past literature. The results in Ref. [4] were inconclusive, in spite of the fact the correct experimental values of the Higgs and the Top quark masses were able to be fixed by choosing a definite value of the strong coupling parameter. However, it happened, that the calculated value of this parameter was a high one: \( \alpha = 4\pi \) close to 1, which assuming the one loop formula for the relation between the coupling and the scale corresponded to a low momentum scale \( \mu = 0.49 \) GeV, being outside the region of measured experimental values of the couplings.

In the present work we extend the study done in reference [4]. The discussion starts by considering a new evaluation of the two loop effective potential for the mean value of the scalar field modeling the Higgs. The discussion will proceed in two main directions. 1) The first one is to reconsider the two loop evaluation done in [4] in order to search for
possible faults in those calculations, which could had altered the obtained numerical values of the couplings and the scale required for fixing the Top quark mass to its observed value.

2) In second place we will also consider to employ the running values of the coupling with the momentum in the evaluation, in order to check if the decreasing values of the coupling with momentum, also allows to justify the fixing of the potential minimum to reproduce the Top quark mass, which was attained at constant coupling.

In connection with the new evaluation of the potential, we present the results of the calculation of the three relevant loop integrals determining the effective potential for the scalar field. The revision allowed to detected a numerical error which slightly affected the calculated coupling and scale values for fixing the Top mass. The corrected results were employed to calculate the new values of the coupling and the scale. The change resulted a positive one: the new scale and coupling values (which were assumed to be related by the one loop formula for the coupling) resulted in values being larger for the scale: $\mu = 11.63$ GeV with respect to the value $\mu = 0.49$ GeV evaluated in [4]. For coupling values the new result was $\alpha = \frac{4\pi}{\mu} = 0.225445$, a smaller result than the high outcome of nearly $\alpha \approx 1$ following in the former work. Then, this first conclusion support the suspected possibility that the spontaneous symmetry breaking in the SM could be generated only by the Top quark-Higgs Yukawa interaction.

In order to consider the use of the running coupling with the momentum in evaluating the potential, we firstly reformulated the finite integral defining the quark-gluon effective potential contribution, which is directly determined by the strong coupling (the quark loop with a contracted gluon propagator). After substracting specially designed divergent parts of the relevant Feynman integral, it was possible to identically transform its finite part in the Minimal Subtraction scheme in an integral over the momenta. This technical result directly allowed to substitute the constant strong coupling by the running with the momentum one in the integral.

The result showed that by simply assuming that the coupling dependence on the momentum is "saturated" to a constant value for momentum smaller the smallest of the measured momenta at which the running coupling is experimentally measured, the calculated component of the effective potential becomes very close numerically to the one evaluated at the initial constant coupling $g(\mu, \Lambda_{QCD})$ at $\mu = 11.63$ GeV. This result indicates that the diminishing of the coupling with momentum does not alter the result for the effective potential, which shows a minimum at a scalar field mean value imposing a Top quark mass of 175 GeV.

Another important result, is that the new formula for the effective potential shows a second derivative at its minimum which predicts a scalar field mass of nearly $m = 44$ GeV. This result is smaller but yet close to the observed Higgs mass of 126 GeV. The new value corrects the one evaluated in Ref. [4]. We estimate this conclusion as one interesting outcome of the analysis. It means that once the Top quark mass is fixed, the spontaneous symmetry breaking pattern associated to the Top-Higgs Yukawa interaction (that is to the "second minimum") is able to determine a mass value of the scalar field being close to the experimentally measured mass of the Higgs particle. Therefore, it might be expected that the many new contributions to the curvature of the Higgs potential that will exist in a more realistic SM type of calculation make feasible to obtain the experimental value of 126 GeV for the Higgs mass. Therefore, the discussion in the work still sustain the expectation about the possibility of describing the full SM after considering an initial Lagrangian in which the classical Mexican hat potential may be absent. The exploration of this possibility will considered elsewhere.

The plan of the work is as follows. In Section 2, the model and its Feynman expansion are reviewed. Section 3 continues by presenting the new evaluation of the effective potential for the mean scalar, and discussing the changes with respect to the previous calculations in Ref. [4]. Section 4 exposes the determination of the new values of the scale parameter $\mu = 11.63$ GeV and its associated strong coupling value which allowed to fix the Top mass as equal to the experimental value. Next, Section 5 describes the derivation of the transformation of the effective potential contribution depending on the strong coupling, in a momentum integral. This allows to substitute the constant value of the strong coupling by the running with the momentum formula in Section 6. Finally, the results are reviewed at the Summary.

II. THE MODEL

Let us now start by reviewing the main elements of the model discussed in [4]. The generating functional of the Feynman expansion is based in an action including a singlet scalar field interacting with only one type of quark. The functional was chosen in the form

$$Z[j, \eta, \bar{\psi}, \xi, \bar{\xi}, \bar{\rho}] = \frac{1}{N} \int \mathcal{D}[A, \bar{\Psi}, \Psi, c, \phi] \exp[i \int S[A, \bar{\Psi}, \Psi, c, \phi]].$$

(1)

The action was taken in the form written below, in which in addition to the usual massless QCD Lagrangian, there were only considered a Yukawa interaction term of a quark with a one component scalar field and the corresponding
action term for the scalar. To simplify the discussion, the free action of the scalar field was defined as a massless free action in the absence of self-interaction. The action, after decomposed in its free and interaction parts, is written below

\[ S = \int dx (\mathcal{L}_0 + \mathcal{L}_1), \]

\[ \mathcal{L}_0 = \mathcal{L}^g + \mathcal{L}^{gh} + \mathcal{L}^q + \mathcal{L}^\phi, \]

\[ \mathcal{L}^g = -\frac{1}{4} (\partial_\mu A^a_{\nu} - \partial_\nu A^a_{\mu}) (\partial^\mu A^{a,\nu} - \partial^\nu A^{a,\mu}) - \frac{1}{2\alpha} (\partial_\mu A^{a,\alpha} (\partial^\nu A^a_{\nu}), \]

\[ \mathcal{L}^{gh} = (\partial_\mu \chi^{*a}) \partial_\mu \chi^a, \]

\[ \mathcal{L}^q = \bar{\Psi} i\gamma^\mu \partial_\mu \Psi, \]

\[ \mathcal{L}^\phi = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi, \]

\[ \mathcal{L}_1 = -\frac{g}{2} f^{abc} (\partial_\mu A^a_{\nu} - \partial_\nu A^a_{\mu}) A^{b,\mu} A^{c,\nu} - g^2 f^{abc} f^{cde} A^a_{\mu} A^b_{\nu} A^{c,\mu} A^{d,\nu} - g f^{abc} (\partial_\mu \chi^{*a}) \chi^b A^\mu_{\nu} + g \bar{\Psi} T^a \gamma^\mu \Psi A^a_{\mu} + y \bar{\Psi} \phi. \]

The dimensionless Yukawa coupling \( y \) will be assumed to have a value close to \( y = 1 \) as it had been estimated in the literature \[6\]. After constructing the Feynman expansion being associated to the above generating function and classical action, the evaluation of the effective potential as a function of an homogeneous scalar (Higgs resembling) field was considered in reference \[4\], up to the two loop approximation. All the notations for the quantum fields quantities, Minkowski metric, etc. used in this work closely follow the ones employed in reference \[5\].

III. TWO LOOPS EFFECTIVE POTENTIAL OF THE SCALAR FIELD

Let us consider again the evaluation of all the contributions to the effective potential \( V(\phi) \) for the scalar field \( \phi \), up to the two loop order. This is the quantity determining the spontaneous symmetry breaking predictions of the considered model and checking its calculation is central for to be sure about its physical predictions. We will see that numerical errors slightly affected the results of the previous work. The corrections will then allow to modify the results for the scale parameter \( \mu \) and the coupling \( g(\mu, \Lambda_{QCD}) \) values required for fixing the Top quark mass value for the fermion in the model.

A. The one loop term

The analytic expression for the one loop contribution shown in Fig. 1 which was evaluated in \[4\], was given by the classical logarithm of the fermion quark determinant as:

\[ \Gamma^{(1)}[\phi] = -V^{(D)} N \int \frac{dp^D}{i(2\pi)^D} \log [\text{Det} \left( G^{(0)rr'}_{ii'}(\phi, p) \right)], \]

\[ D = 4 - 2\epsilon, \]

where \( D \) is the space dimension of dimensional regularization and the free fermion propagator was written as before in the conventions of Ref. \[2\], which, as mentioned before, will be used also throughout this work. This propagator is defined as

\[ G^{(0)rr'}_{ii'}(\phi, p) = \delta^{ii'} \left( \frac{1}{-p_\mu \gamma^\mu + \phi} \right)^{rr'}, \]

\[ = -\frac{\delta^{ii'}}{p^2 - \phi^2} (p_\mu \gamma^\mu + \phi)^{rr'}. \]

As before, assuming the case of QCD with \( SU(N) \) symmetry for \( N = 3 \), and evaluating the spinor and color traces, the one loop expression is simplified to become

\[ \Gamma^{(1)}[\phi] = V^{(D)} \frac{N}{2} \int \frac{dp^D}{i(2\pi)^D} \log [(\phi^2 - p^2)^4]. \]
Taking the derivative over $\phi^2$ of $\Gamma^{(1)}[\phi]$ allows to write the easily integrable expression

$$\frac{d}{d\phi^2} \Gamma^{(1)}[\phi] = V(D) 2 N \int \frac{dp^D}{(2\pi)^D} \frac{1}{(p^2 + \phi^2)},$$

Making use of the identity

$$\int \frac{dp^D}{(2\pi)^D} \frac{1}{(p^2 + \lambda^2)} = \Gamma(1 - \frac{D}{2}) \frac{1}{(4\pi)^{D/2}} (\lambda^2)^{D/2 - 1},$$

and integrating the result back over $\phi^2$, gives the dimensionally regularized expression

$$\Gamma^{(1)}[\phi] = V(D) 2 N \frac{\Gamma(1 - \frac{D}{2})}{(4\pi)^{D/2}} \left(\phi^2\right)^{\frac{D}{2} - 1},$$

which coincides with the corresponding expression in [4]. Let us divide $\Gamma^{(1)}[\phi]$ by $\frac{V(D)}{\mu^2}$, in order to write the action density. The quantity $\mu$ in the denominator is the dimensional regularization scale parameter, and the divisor $\mu^{2\epsilon}$, which tends to one on removing the regularization, is introduced in order avoid results containing logarithms of quantities having dimension. Then, the one loop Lagrangian density takes the form

$$\gamma^{(1)}[\phi] = \frac{\Gamma^{(1)}[\phi]}{V(D)\mu^{2\epsilon}} = \frac{2 N \Gamma(\epsilon - 1)}{(4\pi)^{2-\epsilon}} \left(\phi^2\right)^{1 - 2\epsilon},$$

also coinciding with the former result in [4]. After deleting the pole part of the above expression according to the Minimal Subtraction rule, and taking the limit $\epsilon \to 0$, gives the finite part of the one loop action density as

$$\left[\gamma^{(1)}[\phi]\right]_{\epsilon \to 0}^{finite} = \frac{3\phi^4}{32\pi^2} \left(-3 + 2\gamma - 4 \log(2) - 2 \log(\pi) + 2 \log\left(\frac{\phi^2}{\mu^2}\right)\right),$$

where $\gamma = 0.57721\ldots$ is the Euler constant.

Finally, the one loop potential energy density is given by the negative of the above quantity

$$v^{(1)}[\phi] = -\frac{3\phi^4}{32\pi^2} \left(-3 + 2\gamma - 4 \log(2) - 2 \log(\pi) + 2 \log\left(\frac{\phi^2}{\mu^2}\right)\right),$$

$$= -\frac{3\phi^4}{32\pi^2} \left(-3 + 2\gamma + 2 \log\left(\frac{\phi^2}{4\pi\mu^2}\right)\right).$$
It can be noticed that one loop potential density is unbounded from below for increasing values of the scalar field, which is its main property determining the dynamical generation of the field \( \phi \) in the model.

**B. Quark-gluon two loop term**

Let us start now evaluating the two loop quark-gluon term which was calculated in reference [4] and is illustrated in Fig. 2. Again, after evaluating the color and spinor traces the analytic expression for this contribution was obtained in a coinciding form as follows

\[
\Gamma^{(2)}_g[\phi] = -V^{(D)}g^2(N^2-1) \int \frac{dp^D dq^D}{i^2(2\pi)^{2D}} \frac{D\phi^2 - (D-2)p.(p+q)}{q^2(p^2 - \phi^2)((p+q)^2 - \phi^2)}
\]  

(16)

where similarly as before \( g^2 \) is the QCD coupling constant in the dimensional regularization scheme, which introduces the scale parameter \( \mu \) according to

\[
g = g_0 \mu^{2-\frac{D}{2}} = g_0 \mu^\epsilon.
\]  

(17)

After repeating the same steps followed in [4], that is: symmetrizing the expression of \( \Gamma^{(2)}_g[\phi] \) under the change of sign in the momentum \( q \), by means of the integration variable shift \( p \rightarrow p - \frac{q}{2} \) and the use of the identity

\[
p^2 = (p + \frac{q}{2})^2 - \phi^2 + \phi^2 - \frac{q^2}{4} - q.p,
\]  

(18)

the quark-gluon term is written in the form

\[
\Gamma^{(2)}_g[\phi] = \Gamma^{(2,1)}_g[\phi] + \Gamma^{(2,2)}_g[\phi],
\]  

(19)

where \( \Gamma^{(2,1)}_g[\phi] \) and \( \Gamma^{(2,2)}_g[\phi] \) have the formulae

\[
\Gamma^{(2,1)}_g[\phi] = -V^{(D)}2\phi^2 g^2(N^2-1) \int \frac{dk_1^D dk_2^D}{i^2(2\pi)^{2D}} \frac{1}{k_1^2(k_2^2 - \phi^2)((k_1 + k_2)^2 - \phi^2)}
\]  

(20)

\[
\Gamma^{(2,2)}_g[\phi] = -V^{(D)}(D-2)g^2(N^2-1) \int \frac{dk_1^D}{i^2(2\pi)^{2D}} \frac{1}{k_1^2 - \phi^2}^2,
\]  

(21)

**FIG. 2:** The two loop contribution determined by the strong interaction. As before, the \( \phi \) dependence of the result is introduced though the free quark propagator.
in which the master two loop integral \( J_{111}(0, \phi, \phi) \) was evaluated making use of the results in Ref. [7], and its explicit form for the particular values of our arguments is:

\[
J_{111}(0, \phi, \phi) = \int dk_1^D dk_2^D \frac{1}{k_1^2(k_2^2 - \phi^2)((k_1 + k_2)^2 - \phi^2)}
\]

\[
= -\frac{A(\epsilon)\pi^{4-2\epsilon}}{\epsilon^2} (\phi^2)^{1-2\epsilon},
\]

(22)

\[
A(\epsilon) = \frac{(\Gamma(1 + \epsilon))^2}{(1 - \epsilon)(1 - 2\epsilon)}.
\]

Then, using the above definitions the following expression can be written for \( \Gamma_g^{(2,1)} \)

\[
\Gamma_g^{(2,1)}[\phi] = -\frac{V(D)g_0^2\mu^{2\epsilon}(N^2 - 1)A(\epsilon)\pi^{4-2\epsilon}}{2(2\pi)^{8-4\epsilon}} (\phi^2)^{1-2\epsilon}
\]

(23)

and coincides with the former result.

However, for the case of \( \Gamma_g^{(2,2)} \), we found that the result in [4] included an error after the square of the one loop integral in equation (21) was omitted. Thus, the correct expression for this term should be

\[
\Gamma_g^{(2,2)}[\phi] = -\frac{V(D)g_0^2\mu^{2\epsilon}(N^2 - 1)\epsilon}{2(2\pi)^{8-4\epsilon}} (\Gamma(\epsilon - 1))^2 \phi^4(\phi^2)^{1-2\epsilon}.
\]

(24)

Again dividing by \( \frac{V(D)}{\mu^{2\epsilon}} \) to evaluate the action densities gives

\[
\gamma_g^{(2,1)}[\phi] = \frac{\Gamma_g^{(2,1)}[\phi]}{V(D)\mu^{2\epsilon}} = -\frac{2g_0^2(N^2 - 1)\epsilon}{(2\pi)^{2D}} A(\epsilon)\pi^{4-2\epsilon} (\phi^2)^{1-2\epsilon}.
\]

(25)

and for the corrected term

\[
\gamma_g^{(2,2)}[\phi] = \frac{\Gamma_g^{(2,2)}[\phi]}{V(D)\mu^{2\epsilon}} = -\frac{V(D)g_0^2\mu^{2\epsilon}(N^2 - 1)\epsilon}{2(2\pi)^{8-4\epsilon}} (\Gamma(\epsilon - 1))^2 \phi^4(\phi^2)^{1-2\epsilon}.
\]

(26)

Therefore, repeating the process of substracting the divergent poles and taking the limit \( \epsilon \to 0 \) the total quark-gluon two loop finite contribution to the action density takes the expression

\[
\gamma_g^{(2)}[\phi] \bigg|_{finite}^{\epsilon \to 0} = -\frac{g_0^2}{64\pi^4} 30 - 28\gamma + 12\gamma^2 + \pi^2 + 56 \log(2) - 48 \log(2) + 48 \log(2)^2 + 28 \log(\pi) - 24 \gamma \log(\pi) + 48 \log(2) \log(\pi) + 12 \log(\pi)^2 + (24\gamma - 28 - 48 \log(2) - 48 \log(\pi)) \log(\phi^2/\mu^2) + 12 (\log(\phi^2/\mu^2))^2
\]

(27)

in which \( \gamma_g^{(2)}[\phi] \) defines the quark-gluon contribution to the effective potential.

It should be remarked, that the leading logarithm squared term in the action is negative, indicating that the contribution of the usual quark-gluon diagram to the potential (equal to minus the action) up to the two loop approximation remains being bounded from below as a function of \( \phi \) after the corrections are done.

The divergent contribution to the action follows in the form

\[
\gamma_g^{(2),\text{div}}[\phi] = -\frac{3g_0^2\phi^4}{32\pi^4\epsilon^2} + \frac{g_0^2\phi^4}{32\pi^4\epsilon} (-7 + 6\gamma - 6 \log(4\pi) + 12 \log(\phi/\mu)),
\]

(28)

which defines the Minimal Substraction making finite the quark-gluon two loops contribution.
Finally, let us repeat the evaluation of the two loop term being associated to the quark-scalar loop illustrated in Fig. 3. Due to the absence of spinor and color structures in the vertices the analytic expression for this term is again calculated to be

$$\Gamma_{Y}^{(2)}[\phi] = V^{(D)} 2N \int \frac{dp^D dq^D}{(2\pi)^{2D}} \frac{p^2 - q^2 + \phi^2}{q^2((p + \frac{\phi}{2})^2 - \phi^2)((p - \frac{\phi}{2})^2 - \phi^2)},$$

(31)

which in a close way as it was done for the quark-gluon term, was evaluated in the form

$$\Gamma_{Y}^{(2)}[\phi] = V^{(D)} \frac{4N}{i^2 (2\pi)^{2D}} J_{111}(0, \phi, \phi)$$

$$- V^{(D)} N \left( \int \frac{dk^D}{i(2\pi)^{D}} \frac{1}{k_1^2 - \phi^2} \right)$$

$$= \frac{V^{(D)} 4N}{(2\pi)^{8-4\epsilon}} \frac{A(\epsilon) \pi^{4-2\epsilon}}{\epsilon^2} \phi^4 (\phi^2)^{-2\epsilon}$$

$$- V^{(D)} \frac{N}{(2\pi)^{8-4\epsilon}} \pi^{4-2\epsilon} (\Gamma(\epsilon - 1))^2 \phi^4 (\phi^2)^{-2\epsilon}.$$  

(32)

It can be noted that the imaginary number included in the squared momentum integral, was now and before properly considered, avoiding in this way the error done in the former evaluation of the quark-gluon term. The division by the volume $\frac{V^{(D)}}{\mu^2}$ again allows to write for the action density, the formula

$$\gamma_{Y}^{(2)}[\phi] = \frac{4N}{(2\pi)^{8-4\epsilon}} \frac{A(\epsilon) \pi^{4-2\epsilon}}{\epsilon^2} \phi^4 (\phi^2)^{-2\epsilon}$$

$$- \frac{N}{(2\pi)^{8-4\epsilon}} \pi^{4-2\epsilon} (\Gamma(\epsilon - 1))^2 \phi^4 (\phi^2)^{-2\epsilon}. $$

(33)
Subtracting the divergent pole part in $\epsilon$, passing to the limit $\epsilon \to 0$ gives for the potential density

$$v_Y^{(2)}[\phi] = -\left[\gamma_Y^{(2)}[\phi]\right]_{finite}^{\epsilon \to 0}$$

$$= -\frac{3}{512\pi^4} \phi^4 (50 - 40\gamma + 12\gamma^2 + \pi^2 + 96 \log(2) - 64\gamma \log(2) + 64 \log(2)^2 + 8 \log(4\pi) + 8 \gamma \log(4\pi) - 4 \log(4\pi)^2 + (24\gamma - 40 - 64 \log(2) - 32 \log(\pi) + 8 \log(4\pi)) \log(\frac{\phi^2}{\mu^2}) + 12 (\log(\frac{\phi^2}{\mu^2}))^2).$$

(34)

It can be noted that this contribution, being a two loop one, also includes a squared logarithm term. However, its sign is contrary to the one appearing in the quark gluon loop.

For the total two loop effective potential it follows

$$V[\phi, \mu] = v^{(1)}[\phi] + v_g^{(2)}[\phi] + v_Y^{(2)}[\phi].$$

(35)

### IV. Fixing the Potential Minimum for $M_{Top} = 175$ GeV

Let us consider the sum of all the just evaluated contributions to the potential energy density $V(\phi)$. Its expression is a combination of terms of the form $\phi^4$, $\phi^4 \log(\frac{\phi}{\mu})$ and $\phi^4 (\log(\frac{\phi}{\mu}))^2$, with coefficients that only depend on the strong coupling $g_0$ in the present first analysis. Then, in order to approach the physical situation, we evaluated the potential $V(\phi)$ at the values of $g_0$ satisfying the one loop formula for the running coupling constant $g_0$.

$${g_0(\mu, \Lambda_{QCD}) = 2 \sqrt{\frac{\alpha}{\pi}} \sqrt{\frac{1}{\log(\frac{\mu}{\Lambda_{QCD}})}}}.$$ 

(36)

The $\Lambda_{QCD}$ constant was chosen to be the estimate $\Lambda_{QCD} = 0.217$ GeV. Here it should be remarked that for the determination of the one loop coupling we have assumed the number of fermions as equal to six, in place of to one, as it is proper for the model under discussion. This criterion was adopted in order to assume the strong coupling as more representative of the situation in the SM. In spite of this, also had evaluated the results for the case $N_f = 1$ and the qualitative conclusions of the work did not appreciably changed.

Next, we studied the potential curves in order to examine the behavior of their minimum as functions of $\phi$, when the scale $\mu$ is changed. It follows that when the strong coupling starts to increase as the scale diminish down to one GeV, the value of $\phi$ at the minima, which determines the quark mass also decreases. For the particular value of $\mu = 11.63$ GeV, the potential curve is shown in Fig. [4]. The particular value of $\mu$ chosen, fixes the position of the minimum at a field $\phi$ defining a quark mass of 175 GeV. The set of parameters for this curve are

$$\mu = 11.63 \text{ GeV},$$

$$g_0 = 1.68316 \ (\alpha = \frac{(g_0)^2}{4\pi} = 0.225445),$$

$$\Lambda_{QCD} = 0.217 \text{ GeV}.$$ 

(37) \hspace{1cm} (38) \hspace{1cm} (39)

#### A. The mass of the scalar field

Let us consider the mass of the scalar field to be defined in the present calculation. For evaluating it, we write the following approximate two loop action for the scalar field linear propagation modes

$$L^\phi = \frac{1}{2} \partial\mu \partial^\mu \phi - \frac{1}{2} \phi V''[0] \phi,$$

$$V''[0] = \frac{\partial^2 \phi}{\partial^2 \phi} V[\phi] \bigg|_{\phi=0}.$$

(40) \hspace{1cm} (41)

The Lagrange equation for the propagating scalar field waves $\phi = \exp(-i p.x)$ then writes

$$(\partial^\mu \partial_\mu + V''[0]) \phi = (-p^2 + V''[0]) \phi = 0.$$ 

(42)
FIG. 4: The effective potential of the mean field $\phi$ for the value of the scale $\mu$ determining $\phi$ at the potential minimum being equal to the top quark mass $m_{\text{top}} = 175$ GeV. The second derivative at the minimum gives for the scalar field a low mass $m_\phi = 44$ GeV, which is of the order the Higgs one 126 GeV. In this calculation the scale $\mu$ allowing the top mass fixation is within an intermediate energy region: $\mu = 11.63$ GeV, which gives a coupling value $\alpha = \frac{g^2}{4\pi} = 0.225445$.

Therefore the mass of the scalar field waves is given by

$$m_\phi = \sqrt{V''[0]}.$$  \hspace{1cm} (43)

That is, the mass of the scalar field in a first approximation is defined by the square root of the second derivative of the effective potential respect to the mean field. Therefore, the second derivative of the potential curve in Fig. 1, estimates for the mass of the scalar field $m_\phi = 45$ GeV. This value is smaller but of the order of the observed Higgs mass of 126 GeV. Then, after considering that by fixing the fermion mass to the top quark experimental mass, had determined a mass for the scalar field being close to the Higgs’s one, directly supports the possibility of generating the breaking of symmetry in the SM through the Yukawa interaction between the Top quark and the Higgs field. This possibility is also made plausible, by noticing that upon considering a similar evaluation, but in a model showing the same field content as the SM, there will exist a variety of additional particles. Some of them also have similar masses as the Top and Higgs (the W and Z bosons). Thus, their contribution to the Higgs potential could correct the resulting scalar field mass value to become close to the observed one.

As remarked before, the appeared single extremum of the potential is related with the existence of so called ”second minimum” of the Higgs potential in the SM, laying at large values of the Higgs field. That minimum is recognized to be produced precisely by the contributions of the top-quark Yukawa interaction term, which is of the same form that the one considered here [1–3].

V. POTENTIAL EVALUATION USING THE RUNNING COUPLING

In this section, we will investigate the stability of the previous evaluation of the effective potential for the scalar mean field by substituting the constant value of the coupling chosen at a given scale by the running coupling with momentum. The objective will be to check how robust can be the fixing of the Top quark mass, under the replacement of the constant value of the coupling by a momentum dependent one. For this purpose the expression for the finite part of the effective potential (obtained after employing the Minimal Substractions scheme) will be represented as a momentum integral, in which the replacement of the constant coupling can be afterwards implemented.

In fact this had been the most demanding technical part of the present work. The difficulty was determined by the employed dimensional regularization approach under the Minimal Substraction scheme. The obstacles were created by the fact that normally, the full divergence structure of the evaluated quantities near dimension equal to four, only appears after integrating over the momenta. But, for approximately substituting the constant coupling by the running one with momentum, it is required an expression for the finite part being represented as a momentum integral. To derive this expression is the main objective of most of the technical discussion to be presented below in this section.

The plan of the section is as a follows. First we will present the formula for the effective potential as represented by a momentum integral in terms of Appell series as functions of the momenta and $\epsilon = \frac{4-D}{2}$. But, since the Euclidean space integral has a volume differential of the form $dV = dq$ $q^{3-2\epsilon}$ (determined by the $D$ dimensional integration over
the momenta), it was noted that for making finite the integral, it is only needed to substract a specially designed asymptotic form at large values of $q$ of the factor $F(q)$ defining the full integrand as $F(q)dV$. The complex form of these terms complicated the discussion, because the Appell functions appearing do not show a pure Taylor expansion in powers of $\frac{1}{q}$. In fact, the expansion becomes a power series of $\frac{1}{q}$ with factors which are powers of $q^\epsilon$. These factors, although becoming equals to 1 for $\epsilon \to 0$, contribute to the final result due to the appearance of divergent pole terms in $\epsilon$.

After substracting the appropriate divergent terms, a formula for a momentum integral was obtained which became convergent at large momentum and showed a single divergent term at small momentum as $\frac{1}{q}$. However, we noted that after substituting few of these divergent factors as

$$\frac{1}{q} \to \frac{1}{\sqrt{q^2 + \delta^2}},$$

the integral became again convergent at large momentum, but also at zero momentum. This led to the introduction of a new parameter which afterwards played a helpful role. At this point it was possible to take the limit $\epsilon \to 0$ in the integral due to its finite character. Then, we passed to study the divergent contribution which was substracted to make the integral finite at large as well as for zero momenta. Since the momentum dependence of this substraction became simpler than the original one, it was possible to exactly evaluate the momentum integral, which allowed to determine its pole structure in $\epsilon$ as well as its finite part.

As it should result to be the case, the pole part of the divergent integral exactly reproduced the Minimal Substraction required to make the two loop quark-gluon term finite. As for its finite part, it resulted as a function of the scale parameter $\mu$, the mean field $\phi$, the strong coupling constant, but also of the new parameter $\delta$ introduced for making the substracted integral convergent at small momentum. At this point came the helpful character of the regularization parameter $\delta$: we selected its value as a function of $\mu$, $\phi$ and $g$ for to impose that the finite part of the divergent integral vanishes for all values of $\mu$, $\phi$ and $g$. This fixation of $\delta$ became possible and a real and positive solution exists for the variety of values of $\mu$, $\phi$ and $g$.

Therefore, it followed that the defined finite integral over the momenta exactly coincides with the effective potential when the strong coupling is constant. Hence, the obtained formula for effective potential can be used to explore the effects of considering that the strong coupling of the gluons with the quarks runs with the exchanged momenta, within the contribution associated to the quark self energy loop, contracted with the gluon propagator.

### A. The two loops quark-gluon contribution to the effective potential as a momentum integral

As it was mentioned in the past subsection, we want now to investigate the effects that could have on the results to assume that the strong coupling varies with the magnitude of the exchanged momentum $q$. The evaluation of the effective potential done in the past section seems amenable of being influenced by assuming the coupling to run with the momentum.

We followed a specific path in order to derive a momentum integral for the finite part of the quark-gluon contribution to the effective action. The outcome, in one hand coincides with the result when the coupling is momentum independent and then it was employed to investigate the effects of substituting the strong coupling by a running with the momentum expression in the following section.

We start by considering that the effective action is given by the contraction of the gluon polarization tensor and free gluon propagator. Then, the polarization tensor was expressed through the formula derived in the page 374 of reference [5]

$$\Pi^{ab\mu\nu}(q) = -\frac{4g^2 \delta^{ab}}{(4\pi)^2} \epsilon(\epsilon) (q^2 g_{\mu\nu} - q_\mu q_\nu) \int_0^1 dx \ x(1-x)(\phi^2 - x(1-x)q^2)^{-\epsilon}.$$  \(\text{(45)}\)

After contracting the tensor with the gluon propagator

$$D^{ab\mu\nu}(q) = \frac{\delta^{ab}}{q^2} (g_{\mu\nu} - (1 - \alpha) \frac{q_\mu q_\nu}{q^2}),$$  \(\text{(46)}\)

the quark-gluon contribution to the effective action for the scalar field (minus the effective potential), after evaluating
the integral over the variable $x$ got the expression

$$
\Gamma_g^{(2)}(\epsilon, g, \phi, \mu) = \int_0^\infty dq \int_0^1 dx \frac{2^{-3+4\epsilon} q^2 q^{3-2\epsilon} \pi^{-4+2\epsilon} (-3 + 2\epsilon) \Gamma(\epsilon)(1 - x) x (1 - q^2 (-1 + x)x)^{-\epsilon} \phi^4 \left( \frac{\phi}{\mu} \right)^{-4\epsilon}}{\Gamma(2 - \epsilon)},
$$

where it was defined the momentum integrand $L_g^{(2)}(q, \epsilon, g, \phi, \mu)$ having the explicit form

$$
L_g^{(2)}(q, \epsilon, g, \phi, \mu) = \frac{1}{3\Gamma(2 - \epsilon)} g^2 q^{3-2\epsilon} 4^\epsilon (2\pi)^{-4+2\epsilon} (3 - 2\epsilon) \phi^4 \left( \frac{\phi}{\mu} \right)^{-4\epsilon} \Gamma(\epsilon) \times
$$

$$
\left( -3 \text{AppellF}_1[2, \epsilon, 3, \frac{-q^2 + \sqrt{q^2(4 + q^2)}}{2}, \frac{-q^2 - \sqrt{q^2(4 + q^2)}}{2}] + \right)$$

$$
\left( -2 \text{AppellF}_1[3, \epsilon, 4, \frac{-q^2 + \sqrt{q^2(4 + q^2)}}{2}, \frac{-q^2 - \sqrt{q^2(4 + q^2)}}{2}] \right),
$$

in terms of the Appell functions \[8, 9\]

$$
\text{AppellF}_1[a, b_1, b_2, c, x, y] = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(a)_{m+n}(b_1)_m(b_2)_n}{m!n!(c)_{m+n}} x^m y^n,
$$

in which the Pochhammer symbols are defined as

$$
(a)_n = \frac{\Gamma(a + n)}{\Gamma(a)}.
$$

The $L_g^{(2)}$ function is not convergent at large momentum, and the pole parts of its momentum integral as functions of the $\epsilon$ parameter define the Minimal Substraction required to make the result finite. However, as it was mentioned before, by substracting the asymptotic behavior of the Appell functions at large momentum the integral can be made finite. The resulting integrand after this substraction can be written as

$$
L_{\text{sub}}(q, \epsilon, g, \phi, \mu, \delta) = \frac{1}{3\Gamma(2 - \epsilon)} g^2 q^{3-2\epsilon} 4^\epsilon (2\pi)^{-4+2\epsilon} (3 - 2\epsilon) \phi^4 \left( \frac{\phi}{\mu} \right)^{-4\epsilon} \Gamma(\epsilon) \times
$$

$$
\left( -3 \left( \text{AppellF}_1[2, \epsilon, 3, \frac{-q^2 + \sqrt{q^2(4 + q^2)}}{2}, \frac{-k^2 - \sqrt{q^2(4 + q^2)}}{2}] \right) + \right)$$

$$
\left( -2 \left( \text{AppellF}_1[3, \epsilon, 4, \frac{-q^2 + \sqrt{q^2(4 + q^2)}}{2}, \frac{-q^2 - \sqrt{q^2(4 + q^2)}}{2}] \right) \right) +$$

where the substractions done are defined by the large momentum asymptotic form of the two entering Appell functions
given by the formulæ

\[
\text{Appel23Sub}[k, \epsilon, \delta] = \frac{2^{-1+2r} \pi^{\frac{4}{3}} q^{-2\epsilon} \csc(\pi(1-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(\epsilon)} + \\
\frac{(1}{q^2})^{1-\epsilon} q^{-2\epsilon} \left( \frac{2^{-1+2r} \pi^{\frac{4}{3}} q^{-2\epsilon} \csc(\pi(1-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(\epsilon)} \right) + \\
\frac{2\pi \csc(\pi(1-\epsilon))}{\Gamma(2-\epsilon)\Gamma(\epsilon)} + \frac{4^{1+\epsilon} \pi^{\frac{4}{3}} q^{-2\epsilon} \csc(\pi(1-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(1+\epsilon)} + \\
\frac{1}{q^{2+\delta^2}} \left( \frac{4^{1+\epsilon} \pi^{\frac{4}{3}} q^{-2\epsilon} \csc(\pi(1-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(\epsilon)} \right) + \\
\frac{4^{1+\epsilon} \pi^{\frac{4}{3}} q^{-2\epsilon} \csc(\pi(1-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(\epsilon)} + \frac{8\pi \csc(\pi(1-\epsilon))}{\Gamma(2-\epsilon)\Gamma(\epsilon)} - \\
\frac{12\pi \csc(\pi(1-\epsilon))}{\Gamma(3-\epsilon)\Gamma(\epsilon)} + \frac{8\pi \csc(\pi(1-\epsilon))}{\Gamma(3-\epsilon)\Gamma(\epsilon)} - \\
\frac{4^{1+\epsilon} \pi^{\frac{4}{3}} q^{-2\epsilon} \csc(\pi(1-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(1+\epsilon)} + \frac{4^{1+\epsilon} \pi^{\frac{4}{3}} q^{-2\epsilon} \csc(\pi(1-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(1+\epsilon)} + \\
\frac{3 \times 4^{1+\epsilon} \pi^{\frac{4}{3}} q^{-2\epsilon} \csc(\pi(1-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(2+\epsilon)} + \frac{3 \times 4^{1+\epsilon} \pi^{\frac{4}{3}} q^{-2\epsilon} \csc(\pi(1-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(2+\epsilon)} ,
\]

and

\[
\text{Appel34Sub}[q, \epsilon, \delta] = \frac{3}{2q^2(1-\epsilon)} + \frac{3}{4} \text{Appel23Sub}[q, \epsilon, \delta] - \\
\frac{3 \times 2^{-4+2r} \pi^{\frac{4}{3}} q^{-2\epsilon} \csc(\pi(2-\epsilon))}{(-1+\epsilon)\Gamma(\frac{2}{3} - \epsilon)\Gamma(-1+\epsilon)} + \frac{2^{-2\epsilon} (1}{q^2})^{1-\epsilon} \left( \frac{3 \times 2^{-4+2r} \pi^{\frac{4}{3}} q^{-2\epsilon} \csc(\pi(2-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(-1+\epsilon)} \right) - \\
\frac{3 \times 2^{-3+4r} \pi^{\frac{4}{3}} q^{-4\epsilon} \csc(\pi(2-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(\epsilon) + \frac{2^{-2\epsilon} q^{2\epsilon}}{(q^2 + \delta^2)} \left( \frac{3 \times 2^{-1+4r} \pi^{\frac{4}{3}} q^{-4\epsilon} \csc(\pi(2-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(-1+\epsilon)} \right) + \\
\frac{3 \times 2^{2\epsilon} \pi q^{-2\epsilon} \csc(\pi(2-\epsilon))}{(-1+\epsilon)\Gamma(3-\epsilon)\Gamma(-1+\epsilon)} + \frac{3 \times 2^{-1+4r} \pi^{\frac{4}{3}} q^{-4\epsilon} \csc(\pi(2-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(-1+\epsilon)} - \\
\frac{9 \times 2^{-3+4r} \pi^{\frac{4}{3}} q^{-4\epsilon} \csc(\pi(2-\epsilon))}{\Gamma(\frac{2}{3} - \epsilon)\Gamma(1+\epsilon)} ,
\]

In the above two expressions, it should be noted that the before defined quantity \( \delta \) is appearing in the denominators of the form \((q^2 + \delta^2)\). They, appeared after making the substitution \( q \rightarrow \frac{1}{\sqrt{q^2 + \delta^2}} \) in a \( q^2 \) denominator of the only term diverging as \( \frac{1}{q} \) at zero momentum. This procedure eliminates the mentioned zero momentum divergence, furnishes a simple momentum dependence of the result and makes the result a function of the quantity \( \delta \). The substitution, on another hand does not disturbed the large momentum convergence of the considered integral. Therefore, the substracted divergent expression has the form

\[
L_{\text{count}}(q, \epsilon, g, \phi, \mu, \delta) = \frac{1}{3\Gamma(2-\epsilon)} q^2 q^{3-2\epsilon} 4\pi (2\pi)^{-4+2\epsilon} (3-2\epsilon) \Gamma(\epsilon) \phi^4 (\frac{\phi}{\mu})^{-4\epsilon} \times \\
(-3 \text{Appel23Sub}[q, \epsilon, \delta] + 2\text{Appel34Sub}[q, \epsilon, \delta]).
\]

Now, as remarked above, the relative simplicity of the obtained momentum dependence of the substracted term,
allows to exactly perform the momentum integrals to obtain the result

\[ S_{\text{count}}(\epsilon, g, \phi, \mu, \delta) = \int_0^\infty dq \ L_{\text{count}}(q, \epsilon, g, \phi, \mu, \delta) \]

\[ = (4^{-5+3\epsilon} g^2 \pi^{-\frac{3}{2}+2\epsilon} \frac{1}{\delta^2})^\epsilon (-3 + 2\epsilon) \phi^4 \left(\frac{\phi}{\mu}\right)^{-4\epsilon} \csc(\pi \epsilon)^2 \times \]

\[ \left( -4^{2+\epsilon} \pi^{\frac{3}{2}} \left(\frac{1}{\delta^2}\right)^\epsilon (-2 + \epsilon) (-1 + 2\epsilon) \csc(2\pi \epsilon) \Gamma(2 - 2\epsilon) \Gamma(1 + \epsilon) - \right. \]

\[ 2^{1+4\epsilon} \pi^{\frac{3}{2}} \left(\frac{1}{\delta^2}\right)^\epsilon (-2 + \epsilon) (-1 + 2\epsilon) (3 + 4\epsilon) \csc(2\pi \epsilon) \Gamma(3 - 2\epsilon) \Gamma(1 + \epsilon) - \]

\[ 4^\epsilon \pi^{\frac{3}{2}} \left(\frac{1}{\delta^2}\right)^\epsilon (-1 + 2\epsilon) (1 + 4\epsilon) \csc(2\pi \epsilon) \Gamma(5 - 2\epsilon) \Gamma(1 + \epsilon) - \]

\[ 64 (-2 + \epsilon) (-1 + 2\epsilon)^2 \Gamma(2 - 2\epsilon) \Gamma\left(\frac{5}{2} - \epsilon\right) \Gamma(-1 + \epsilon) \Gamma(1 + \epsilon) + \]

\[ 64 (-1 + 2\epsilon)^3 \Gamma(2 - 2\epsilon) \Gamma\left(\frac{5}{2} - \epsilon\right) \Gamma(-1 + \epsilon) \Gamma(1 + \epsilon) + \]

\[ 4^{1+\epsilon} \pi^{\frac{3}{2}} \left(\frac{1}{\delta^2}\right)^\epsilon (1 + 2\epsilon) \csc(2\pi \epsilon) \Gamma(5 - 2\epsilon) \Gamma(2 + \epsilon)) / (\Gamma\left(\frac{3}{2} - \epsilon\right) \Gamma(2 - 2\epsilon) \Gamma\left(\frac{5}{2} - \epsilon\right) \Gamma(3 - \epsilon) \Gamma(-1 + \epsilon) \Gamma(1 + \epsilon)). \]  

(55)

But, expanding this relation in series of the \( \epsilon \) parameter leads to the result

\[ S_{\text{count}}(\epsilon, g, \phi, \mu, \delta) = \sum_{n=-\infty}^\infty S_{\text{count}}^{(n)}(g, \phi, \mu, \delta) \epsilon^n \]

\[ = -\frac{3g^2 \phi^4}{32 \pi^4 \epsilon^2} + \frac{g^2 \phi^4}{32 \pi^4 \epsilon} \left(-7 + 6 \gamma - 6 \log(4\pi) + 12 \log\left(\frac{\phi}{\mu}\right)\right) + \]

\[ \frac{g^2 \phi^4}{64 \pi^4} \times (-21 + \pi^2 + 10\gamma - 48 \log(2)^2 - 28 \log(4\pi) + \]

\[ 6(\gamma^2 + \gamma(3 - 2\gamma) - \log(\pi) \log(256\pi^2) + \gamma(-\gamma + \log(256\pi^4))) - \]

\[ 18 \log\left(\frac{1}{\delta^2}\right) + 6 \log\left(\frac{1}{\delta^2}\right)^2 + 8(7 - 6\gamma + 6 \log(4\pi) - 6 \log\left(\frac{\phi}{\mu}\right) \log\left(\frac{\phi}{\mu}\right)) + \]

\[ + O^{(1)}(\epsilon), \]  

(56)

where \( O^{(1)}(\epsilon) \) is a function vanishing when \( \epsilon \to 0 \).

In the above formula, it should be remarked that the pole part, which defines the divergent contribution, exactly coincides with the Minimal Substraction term \( (30) \) required to make finite the quark-gluon contribution to the effective action. Now, it can noticed that the expression for the integral \( S_{\text{count}}(\epsilon, g, \phi, \mu, \delta) \) (which was substracted from the momentum integral of the term \( L(q, \epsilon, g, \phi, \mu) \) to obtain a finite remaining integral) has a finite part when \( \epsilon \to 0 \). But, this finite part is depending on the regularization parameter \( \delta \) which was used to make convergent the momentum integral around the zero momentum. This circumstance opens the interesting possibility of choosing this value of \( \delta \) precisely to force the finite part to vanish for all the value of the scale \( \mu \) and the mean field. Then, imposing this condition for determining \( \delta \), we may write

\[ \frac{g^2 \phi^4}{64 \pi^4} \times (-21 + \pi^2 + 10\gamma - 48 \log(2)^2 - 28 \log(4\pi) + \]

\[ 6(\gamma^2 + \gamma(3 - 2\gamma) - \log(\pi) \log(256\pi^2) + \gamma(-\gamma + \log(256\pi^4))) - \]

\[ 18 \log\left(\frac{1}{\delta^2}\right) + 6 \log\left(\frac{1}{\delta^2}\right)^2 + 8(7 - 6\gamma + 6 \log(4\pi) - 6 \log\left(\frac{\phi}{\mu}\right) \log\left(\frac{\phi}{\mu}\right)) = 0. \]  

(57)

One helpful property of this equation is the fact that it does not involve the values of the strong coupling. This means that \( \delta \) is only a function of the scale \( \mu \) and the mean field \( \phi \). The equation for \( \delta \) has a real and positive solution
defined for all the values of the ratio $\frac{\phi}{\mu}$ which can be expressed as follows

$$\delta(\phi, \mu) = \frac{1}{\sqrt{\exp(f_1 (36 + 4 \mathcal{F}(\phi, \mu)))}}$$

(58)

$$\mathcal{F}(\phi, \mu) = \sqrt{f_2 + f_3 \log\left(\frac{\phi}{\mu}\right)} + f_4 \log\left(\frac{\phi}{\mu}\right)^2,$$

(59)

\[f_1 = 0.041666,\quad f_2 = 750.872709,\quad f_3 = -898.696871,\quad f_4 = +288.00.\]

(60) \hspace{1cm} (61) \hspace{1cm} (62) \hspace{1cm} (63)

The dependence of $\delta$ of the ratio $\frac{\phi}{\mu}$ is depicted in figure 5.

FIG. 5: The figure shows the plot of the real and positive solution for $\delta(\phi, \mu)$ as a function of the ratio $\frac{\phi}{\mu}$. The dashed plot is the same curve in which the horizontal axes is magnified in a factor of 20, to show the behavior near the origin.

Further, the function $\delta(\phi, \mu)$ was substituted in the integrand $L_{sub}$ defined in equation (51) which upon integration
furnishes the finite integral. The result for the integrand of the finite integral can then be written in the form,

\[ L_{MS}(q, g, \phi, \mu) = L_{sub}(q, \epsilon, g, \phi, \mu, \delta)_{\delta \to \delta(\phi, \mu), \epsilon \to 0} \]

\[
= \frac{g^2 \phi^4}{32\pi^2 q} (12.9266 - 12 q^2 + 2.1789 q^4 - 12 \gamma + 2 q^4 + \frac{0.643577 - 0.597445 \gamma - 0.597445 \log(\frac{1}{\gamma})}{(0.22313 + \exp(0.166667 \sqrt{750.873 + \log(\frac{\mu}{\Lambda})} (-898.697 + 288 \log(\frac{\Lambda}{\mu})))) q^2}) - 12 \log(\frac{1}{q^2}) + 6 q^2 \log(\frac{1}{q^2}) + \frac{-5.76862 + 5.35512 \gamma + 5.35512 \log(\frac{1}{\gamma})}{0.22313 + \exp(0.166667 \sqrt{750.873 + \log(\frac{\mu}{\Lambda})} (-898.697 + 288 \log(\frac{\Lambda}{\mu})))) q^2} + 12 q^2 \log(q) - 4 q^4 \log(q) + q^4 (-6 \text{ AppellF}_1^{(0,0,1,0,0,0)}(2,0,0,3,1_2(-q^2 + \sqrt{q^2(4 + q^2)}), \frac{1}{2}(-q^2 - \sqrt{q^2(4 + q^2)})) + 4 \text{ AppellF}_1^{(0,0,1,0,0,0)}(3,0,0,4,\frac{1}{2}(-q^2 + \sqrt{q^2(4 + q^2)}), \frac{1}{2}(-q^2 - \sqrt{q^2(4 + q^2)})) - 6 \text{ AppellF}_1^{(0,1,0,0,0,0)}(2,0,0,3,\frac{1}{2}(-q^2 + \sqrt{q^2(4 + q^2)}), \frac{1}{2}(-q^2 - \sqrt{q^2(4 + q^2)})) + 4 \text{ AppellF}_1^{(0,1,0,0,0,0)}(3,0,0,4,\frac{1}{2}(-q^2 + \sqrt{q^2(4 + q^2)}), \frac{1}{2}(-q^2 - \sqrt{q^2(4 + q^2)})) \right), \]

where a superindex of the form \(n_1, n_2, n_3, n_4, n_5, n_6\) in the appearing Appell functions represents the numbers of \(n_i\), \(i = 1, \ldots, 6\) of the derivatives over the corresponding six arguments of the Appell functions. It should be noticed that in writing this expression, the limit \(\epsilon \to 0\) was also chosen, as allowed by the finiteness of the integral. Therefore, we have arrived to an expression of the finite part of the effective action in the form

\[ \mathcal{L}(g, \phi, \mu) = \int dq\ L_{MS}(q, g, \phi, \mu). \] (66)

The figure 6 shows the evaluations of the quark-gluon contribution of the effective potential (minus the action) through both formulae (29) and (66). The coincidence of the two plots checks the equivalence between the two expressions. The plots are done for values of the scale parameter \(\mu = 11.63\ \text{GeV}\) and of the coupling \(g = g_0(11.63, \Lambda)\). The solid curve shows the values of \(-\mathcal{L}(g, \phi, \mu)\) and the dotted one the values of \(v_2^{(2)}[\phi]\) in (29) as functions of the mean field.

FIG. 6: The plots illustrate the coincidence of the calculated momentum integral representation for the quark-gluon two loop contribution to the effective action with the result of the direct evaluation of the same quantity. The solid curve indicates the values of the momentum integral representation in (66) and the dotted one the evaluation of the two loop quark gluon contribution in formula (29).
In expression (66), it is possible now to replace the up to now constant value of the strong coupling by its running expression with momentum. The following section will discuss some properties of this substitution.

VI. THE USE OF THE RUNNING COUPLING WITH MOMENTUM

In this section, the discussion starts by defining the running coupling to be considered. Since it is known that the values of couplings are not well defined at low momenta of the order of 1 GeV, we defined values of the running coupling "saturated" at low momentum. That is, in a neighborhood of zero momentum they were assumed to be constant. Then, we explored the values of the scale \( \mu \) for which the substitution of the running coupling in the obtained finite integral, approximately coincide with the value attained at constant value of the coupling for the same scale. Interestingly, it followed that for the determined before value for the scale \( \mu = 11.63 \text{ GeV} \), the effect of the diminishing of the coupling with the momentum mainly does not affect the obtained spontaneous symmetry breaking pattern.

For the purpose of substituting the constant coupling by its running counterpart we analyzed two variants of couplings. The first of them was the expression for the one loop renormalization coupling as a function of momentum

\[
g_o(q) = \begin{cases} 
\sqrt{\frac{1}{b_o \log\left(\frac{q^2}{\Lambda^2}\right)}} , & q^2 > \exp\left(\frac{16\pi^2}{g_{sat}^2}\right) \\
 g_{sat} , & q^2 < \exp\left(\frac{16\pi^2}{g_{sat}^2}\right)
\end{cases},
\]

(67)

\[
g_{sat} = 2.06702.
\]

(68)

In this expression the couplings for momenta smaller than the value at which they become equal to the highest measured coupling \( g_{sat} = 2.06702 \), are assumed to remain constant, and equal to their "saturation" values \([10]\).

Another form of the analyzed running coupling was given by an interpolation of the set of experimental values reported in reference \([10]\). The expression describing the data was obtained in the form

\[
g_{exp}(q) = \begin{cases} 
\log\left(\frac{23.4103}{q_{sat}^2}\right)^{-1} , & q > 1.6042285 \\
 q_{sat} , & q < 1.6042285
\end{cases},
\]

(69)

where \( g_{sat} \) is the same saturation value defined before, that is, the maximal value of the experimentally measured couplings given in reference \([10]\). Both coupling’s momentum behavior are plotted in figure 7. As it can noticed the

![Figure 7](image)

FIG. 7: The figure shows the values of the fitting formula for a number of experimentally measured values of the strong coupling according to reference \([10]\). This curve is the lower one and the fitted experimental points are indicated by the dots. The higher plot shows the values of the one loop renormalization group running coupling.

values of the observations are systematically smaller than the one loop determined values. Therefore, in what follows we decided to employ the fitting curve of the experimental values for substituting the constant coupling in formula (66).
A. The quark-gluon effective potential evaluation using the running coupling

It is possible now to substitute the expression for the experimental value of the running coupling $g_{\text{exp}}(q)$ in the (66) to define the quark-gluon contribution to the effective potential as evaluated at the running coupling values, in the form

$$V_{\text{run}}(\phi, \mu) = \int dq \ L_{\text{MS}}(q, g, \phi, \mu)|_{g=g_{\text{exp}}(q)}.$$  \hspace{1cm} (70)

The resulting formula for this contribution to the effective potential takes the form

$$V_{\text{run}}(\phi, \mu) = \int_{0}^{\infty} dq \ 2\pi^{4} q \left[ (6.7504 - 12 q^2 + 2.1789 q^4 - 12 \gamma + 2 \gamma q^4 + \frac{0.643577 - 0.597445 \gamma - 0.597445 \log(\frac{1}{q})}{0.22313 + \exp(0.166667 \sqrt{750.873 + \log(\frac{\phi}{\mu})}(-898.697 + 288 \log(\frac{\phi}{\mu}))) q^2}) \right] 12 \log(q) + 6 q^2 \log(q) +$$

$$+ \left[ -5.76862 + 5.35512 \gamma + 5.35512 \log(\frac{1}{q}) \right] 0.22313 + \exp(0.166667 \sqrt{750.873 + \log(\frac{\phi}{\mu})}(-898.697 + 288 \log(\frac{\phi}{\mu}))) q^2 \right] -12 q^2 \log(q) - 4 \gamma q^4 \log(q) + q^4 \left[ -6 \text{AppellF}_1(0,0,1,0,0,0,0) \left( 2,0,0,3, \frac{1}{2}, (-q^2 + q^2(4 + q^2))^2 \right) + \text{AppellF}_1(0,0,1,0,0,0) \left( 3,0,0,4, \frac{1}{2}, (-q^2 + q^2(4 + q^2))^2 \right) - 4 \text{AppellF}_1(0,1,0,0,0,0) \left( 2,0,0,3, \frac{1}{2}, (-q^2 + q^2(4 + q^2))^2 \right) + \text{AppellF}_1(0,1,0,0,0,0) \left( 3,0,0,4, \frac{1}{2}, (-q^2 + q^2(4 + q^2))^2 \right) \right] \right).$$  \hspace{1cm} (71)

Next, it possible to evaluate the effects of the running on the calculation of the effective potential. Before, we have been able to fix the Top quark mass by fixing the minimum of the potential (after calculated at constant strong coupling) at the scale parameter value $\mu = 11.63$ GeV. We then firstly calculated $V_{\text{run}}(\phi, \mu)$ at this scale. The result of the evaluation of the total effective potential

$$V_{\text{total}}(\phi, \mu) = v(1)[\phi, \mu] + v(2)[\phi, \mu] + V_{\text{run}}(\phi, \mu),$$  \hspace{1cm} (73)

as a function of the mean scalar field in which the quark-gluon term is calculated using the above formula for $V_{\text{run}}(\phi, \mu)$ is shown in figure. Note that the one loop and scalar two loop contributions are defined by the same formulae (15) and (33) which were used in calculating the potential for constant coupling, since they do not depend on the running coupling.

The figure shows an interesting result. The solid curve represents the values of the potential evaluated by using the running coupling and the dashed one the potential values calculated by employing constant coupling values, chosen at the scale $\mu = 11.63$ GeV. As it can be noticed the substitution of the constant coupling by the running one, had not drastically modified the minimum position at the mean field determining a Top mass value near 175 GeV. Therefore, a suspected beforehand possibility of a distortion of the spontaneous symmetry breaking pattern obtained at constant coupling, was not realized: the pattern is mainly unaffected by the consideration that the couplings varies with the momentum scale.

After evaluating the mass of the scalar field the result was close to the corresponding outcome associated to a constant coupling

$$m_{\phi} = \sqrt{-\frac{V_{\text{total}}[\phi, \mu]}{\mu}},$$  \hspace{1cm} (74)

It should be remarked that the scalar field mass values obtained here are smaller than the observed Higgs particle mass of 126 GeV. After thinking about this outcome, we consider that it does not represents a direct negative result
FIG. 8: The figure shows the plot of the values of the total two-loop effective potential for the scalar field (solid curve) after the running coupling is employed for calculating the quark-gluon contribution. The dashed curve shows the similar potential as evaluated for the constant values of the strong coupling. Note that the results indicate that the momentum dependence of the coupling does not appreciable disturb the arising spontaneous symmetry breaking pattern.

in connection of the studied possibility of basing the SM on a symmetry breaking associated to the Yukawa Top-Higgs interaction. This conclusion is determined by the following reasoning. If we consider a similar calculation of the Higgs mass in the framework of the more complex SM, then, there will be few more fields having similar values of mass as the Top quark (as the $W$ and $Z$ bosons). Then, the contributions to the Higgs potential at two loops of those modes can be expected to appreciably change the scalar field mass value which could be determined only by considering the Top and the Higgs fields (which are the only two fields included in the model). Therefore, the most important outcome of the present work, should be considered as this one: to determine that the spontaneous symmetry breaking generated by a single quark and a scalar (upon fixing the observed quark top mass) produces a mass for the scalar field being smaller and close to the Higgs one. We estimate that the results of the present work directly suggest the interest of attempting to construct the SM upon the here investigated spontaneous symmetry effect.

Summary

We have explored the possibility for that the spontaneous symmetry breaking effect in the SM could be implemented thanks to the Yukawa interaction of the Top quark with the Higgs field. For this purpose a formerly proposed simple model was reconsidered. The previous work although indicating in some sense the possibility investigated, was inconclusive due to the arising in it of a small value of the renormalization scale (smaller than 1 GeV ) in order to allow the Top quark to get the observed value of its mass.

In the present work, we reevaluated the effective potential created by the system for the scalar field, and found calculational errors. Their correction then, led to a picture in which it is also possible to fix the Top quark mass value, but at an intermediate value of the scale $\mu = 11.63$ GeV. The value of the scalar particle mass now emerging was of nearly 45 GV which is close but smaller than the Higgs mass. However, being an amount of the same order still allows for the possibility that the $W$ and $Z$ contributions, to be added in a calculation done in the framework of the SM, could rise the result up to the observed Higgs mass of 126 GeV.

The work also investigate the stability of the result for the spontaneously symmetry pattern by considering the effect of employing the running with momentum coupling in the calculation. For this purpose, the finite expression in dimensional regularization of the quark-gluon contribution two loop effective potential for the Higgs fields was expressed as a momentum integral through a specially designed subtraction procedure. The difficult in attaining this formula, was produced by the use of dimensional regularization. In this scheme, the divergences are normally substracted after integrating over the momenta. However, we required to conserve the momentum integral in order to allow the substitution of the constant coupling by the running one. Then a momentum integral was retained by firstly substracting to the momentum integrand a relatively simple expression, which makes the momentum integral finite. Afterwards, the integral that was substracted was exactly evaluated in dimensional regularization. This allowed to determine the divergent pole part and also the finite part which is also dependent of a parameter just introduced for eliminating a remaining zero momentum divergence. The divergent part just reproduced the minimal subtraction counterterm of the quark-gluon contribution to the two loop effective action. Finally the mentioned
additional parameter was fixed by imposing that the finite part of the subtracted integral vanishes for all values of the scalar field and scale parameter.

Further, the obtained formula for the potential (influenced by substituting the running coupling) was calculated for the same value of scale parameter for which the potential was before evaluated at a constant coupling. The formula for the running coupling employed was a fit to the available data for the measured couplings. The values of the coupling at small momenta were assumed to be constant when the momentum value is smaller than the one associated to the maximal value of the measured coupling. Surprisingly, the results for the potential became very close to the ones evaluated for constant couplings at the same scale. This outcome allows to conclude that the reduction of the coupling with momenta does not disturb the arising spontaneous symmetry pattern.

In a future extension of this work we plan to start from a Lagrangian being practically equivalent to the SM’s one, in which all the Higgs field terms associated to the usual scalar doublet will be present, but in which only the negative mass squared term creating the Mexican Hat potential will not be considered. The idea will be to attempt use the many parameters in this slightly modified SM model, for implementing a symmetry breaking patterns being similar to the one discussed here.

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