Unparticle physics at hadron collider via dilepton production

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\textbf{ABSTRACT}

The scale invariant unparticle physics recently proposed by Georgi could manifest at low energies as non integral number $d_u$ of invisible particles. Unparticles if existing, could couple to the Standard Model fields and consequently affect the collider phenomenology. We consider the DY process to explore effects of the peculiar propagator of the scalar and tensor unparticle operators. To probe these effects at hadron collider one needs to go beyond LO in QCD and hence the quantitative impact of QCD corrections for unparticle physics at LHC is investigated. We present the K-factors at LHC. Inclusion of QCD corrections to NLO stabilises the cross section with respect to scale variations.
1 Introduction

Banks and Zaks [1] found a non-trivial zero of the $\beta$ function in the IR region of Yang-Mills theories with certain non-integral number of fermions, implying absence of a particle like interpretation. Georgi in a recent paper [2], termed this scale invariant stuff as unparticle and formulated a framework in which one could address its phenomenological consequences. The scheme proposed is that at high energies the Standard Model (SM) and the Banks-Zaks (BZ) fields interact via the exchange of particles of mass $M_U$

$$\frac{1}{M_U^k} O_{SM} O_{BZ} , \quad (1)$$

where $O_{SM}$ is the SM operators of mass dimension $d_{SM}$ and $O_{BZ}$ is the BZ operator of mass dimension $d_{BZ}$. At low energy what is relevant is the remanent unparticle and its interaction with the SM which is described in the effective field theory. With the onset of scale invariance in the BZ sector at some scale $\Lambda_U$, renormalisation effects induce dimensional transmutation. Below this scale, BZ operators match onto unparticle operators leading to a new set of interactions

$$C_U \frac{\Lambda_U^{d_{BZ} - d_U} M_U^k}{M_U^k} O_{SM} O_{U} , \quad (2)$$

where $C_U$ is a coefficient in the low energy effective theory and $O_U$ the unparticle operator with scaling dimension $d_U$. Further $M_U$ should be large enough such that its coupling to SM must be sufficiently weak, consistent with current experimental data. In general the effective coupling in Eq. (2) of the SM to the unparticle could be

$$\frac{\lambda_{S}}{\Lambda_{U}^{d_{BZ} - d_U}} T_{\mu}^\mu O_{U}, \quad \frac{\lambda_{V}}{\Lambda_{U}^{d_{BZ} - 1} \bar{\psi} \gamma_{\mu} \psi} O_{\mu}^\mu, \quad \frac{\lambda_{T}}{\Lambda_{U}^{d_{BZ} - 2} T_{\mu \nu}^\mu O_{\mu \nu} . \quad (3)}$$

The dimensionless coupling $\lambda_\kappa$ corresponds to the unparticle operator $O_{U}^\kappa$, where $\kappa = S, V, T$ refers to the scalar, vector and tensor operators respectively. $T_{\mu \nu}$ is the energy momentum tensor of the SM and $d_U$ the scaling dimension of the unparticle operators $O_{U}^\kappa$. These operators are Hermitian and $O_{U}^\mu$ and $O_{U}^{\mu \nu}$ are transverse. Given these effective interactions Eq. (3), unparticle stuff could be produced by a single insertion of the interaction from a SM process leading to a missing energy and momentum signals. The phase space for the unparticle production corresponding to the operators $O_{U}$ of scaling dimension $d_U$ is the same as the production of $d_U$ (non-integral number) invisible particles and is proportional to the factor [2]

$$A_{d_U} = \frac{16\pi^{5/2}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U + 1/2)}{\Gamma(d_U - 1) \Gamma(2d_U)} .$$

Further the exchange of a virtual unparticle corresponding to an operator $O_{U}^\kappa$ between the SM particles, would need the propagator for the unparticle. Using scale invariance and transverse properties of the vector and tensor operators $O_{U}^\kappa$, the unparticle propagator was obtained [3, 4]

$$\int d^4x e^{iP \cdot x} < 0 | T O_{U}^\kappa(x) O_{U}^\kappa(0) | 0 > = \frac{iA_{d_U}}{2 \sin(d_U \pi)} \frac{B_\kappa}{(-P^2 - i\epsilon)^{2-d_U}}, \quad (4)$$
where \( B_n \) depends on the Lorentz structure of the operator \( O_U \) as given below:

\[
\begin{align*}
O_U & = 1 \\
O_{U\rho} & = \eta_{\mu\nu}(P) = -g_{\mu\nu} + \frac{P_\mu P_\nu}{P^2} \\
O_{\rho\sigma} & = B_{\mu\nu\alpha\beta}(P) = \frac{1}{2} \left( \eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} - \frac{2}{n-1} \eta_{\mu\nu} \eta_{\alpha\beta} \right),
\end{align*}
\]

where, \( n \) number of space time dimension. The propagator of the unparticle is singular due to the \( \sin(d_U \pi) \) factor in the denominator and hence the \( d_U \) is constrained in the range \( 1 < d_U < 2 \). The phenomenological consequences of unparticles are dictated by the interactions given in Eq. (3), the nature of the unparticle phase space and the propagator. Given these sufficient informations, we can calculate various quantities of phenomenological interest, without actually having to understand the nature of the unparticle per se. Close to the scale \( \Lambda_{U} \), say of the order of a TeV the unparticle effects could manifest and hence be relevant at the LHC and ILC. There has been some activity to explore possible collider and flavour phenomenology of unparticle physics [3]-[16]. Unparticle effects on cosmology and astrophysics have been considered in [17].

In this paper we consider the Drell-Yan (DY) process at LHC and study the effects of unparticle corresponding to scalar and tensor operators which could also produce the dilepton pair. The contribution from the vector unparticle operator was considered in [4] for the DY process. At hadron colliders, a precise measurement of dilepton production cross sections is possible and hence could be sensitive to the physics beyond the SM through the exchange of unparticles. At hadron colliders such as Tevatron and LHC, the theoretical uncertainties coming from QCD effects due to initial state partons are quite sizable. The sources of these uncertainties are the renormalisation and factorisation scale dependence. Experience with next-to-leading order (NLO) QCD contributions to SM processes \textit{viz.} DY process and Higgs production at hadron colliders strongly suggests that the leading order (LO) results are quite unreliable. The NLO contributions are large and in addition the theoretical uncertainties are significantly reduced. To make any quantitative statement about unparticle searches at hadron colliders it is necessary to look at the NLO-QCD corrections to the DY process. In this analysis we have studied the effects of scalar and tensor unparticle propagator on the invariant mass distribution of the dilepton.

## 2 Drell-Yan process

Consider the production of dileptons \( \ell^+ \) and \( \ell^- \) in the collision of hadrons \( P_1 \) and \( P_2 \)

\[
P_1(p_1) + P_2(p_2) \rightarrow \ell^+(l_1) + \ell^-(l_2) + X(P_X), \tag{5}
\]

where the final inclusive hadronic state is denoted by \( X \) and it carries a momentum \( P_X \). In the QCD improved parton model, the hadronic cross section can be expressed in terms of partonic cross sections \( d\hat{\sigma}^{ab} \) convoluted with appropriate parton (of type \( a \)) distribution functions \( f_a^P(x) \) as follows

\[
2S \frac{d\hat{\sigma}^{P_1P_2}}{dQ^2}(\tau,Q^2) = \sum_{ab=q,g} \int_0^1 dx_1 \ dx_2 \ dz \ f_a^{P_1}(x_1) \ f_b^{P_2}(x_2) \ 2\hat{s} \ \frac{d\hat{\sigma}^{ab}}{dQ^2}(z,Q^2) \ \delta(\tau-zx_1x_2). \tag{6}
\]
The scaling variables are defined by \( k_1 = x_1 p_1, k_2 = x_2 p_2 \), where \( k_1, k_2 \) are the momenta of incoming partons,

\[
(p_1 + p_2)^2 \equiv S, \quad (k_1 + k_2)^2 \equiv \hat{s}, \quad (l_1 + l_2)^2 \equiv q.q \equiv Q^2,
\]

\[
\tau = \frac{Q^2}{S}, \quad z = \frac{Q^2}{\hat{s}}, \quad \tau = x_1 x_2 z. \quad (7)
\]

The dileptons can be produced through the partonic cross sections given by \( a(k_1) + b(k_2) \rightarrow j(-q) + \sum_i X_i(-p_i) \), where \( j \) could be the usual SM photon, \( Z \)-boson or an unparticle stuff \( U \) which would decay to a leptonic pair in the final state. At LO in QCD, the contribution from the SM is only through the quark initiated process \( q + \bar{q} \rightarrow \gamma^*/Z^* \rightarrow \ell^+ + \ell^- \). When unparticle fields couple to SM fields, they could also decay to a pair of leptons through quark initiated process \( q + \bar{q} \rightarrow U \rightarrow \ell^+ + \ell^- \) and also through gluon initiated process \( g + g \rightarrow U \rightarrow \ell^+ + \ell^- \). As we indicated in the previous section, if the unparticle fields couple to the SM fields through vector interaction, then the LO hadronic cross section can get contribution only from quark, anti-quark initiated process. On the other hand, if the interaction of the unparticle fields with the SM is of scalar or tensorial in nature, then gluons can also produce unparticle fields and hence contribute to LO in QCD. Gluon initiated processes at LHC often give large contributions because its flux is the largest compared to other partonic fluxes that contribute to various cross sections. Since the higher order QCD effects can stabilise the cross section, we have incorporated all the NLO-QCD effects through the following processes

\[
q + \bar{q} \rightarrow \gamma^*/Z^* + g, \quad q + \bar{q} \rightarrow \gamma^*/Z^* + \text{one loop},
\]

\[
q + g \rightarrow \gamma^*/Z^* + q, \quad \bar{q} + g \rightarrow \gamma^*/Z^* + \bar{q}, \quad (8)
\]

in the SM and in unparticle theories,

\[
q + \bar{q} \rightarrow U + g, \quad q + \bar{q} \rightarrow U + \text{one loop},
\]

\[
q + g \rightarrow U + q, \quad \bar{q} + g \rightarrow U + \bar{q},
\]

\[
g + g \rightarrow U + g, \quad g + g \rightarrow U + \text{one loop}. \quad (9)
\]

In the above, we consider all the one loop QCD corrections to quark antiquark unparticle vertex and gluon gluon unparticle vertex.

The cross sections beyond LO involve the computation of one loop virtual corrections and real bremsstrahlung contributions to LO processes. The singularities we encounter in our computation are due to the soft and collinear divergences. We have used the dimensional regularisation to regulate both these singularities. The soft divergence coming from the virtual gluons and the bremsstrahlung contributions cancel exactly according to the Bloch-Nordsieck theorem. The remaining collinear divergences are removed by mass factorisation for which we use the \( \overline{MS} \) scheme. Hence the complete cross section after mass
factorisation is
\[
2S \frac{d\sigma_{P_1P_2}}{dQ^2} (\tau, Q^2) = \sum_q \mathcal{F}_{SM,q} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dz \frac{d\delta(\tau - zx_1x_2)}{dz}
\]
\[
\times \left[ H_{qq}(x_1, x_2, \mu_F^2) \left( \Delta_{qq}^{(0),\gamma/Z} (z, Q^2, \mu_F^2) + a_s \Delta_{qq}^{(1),\gamma/Z} (z, Q^2, \mu_F^2) \right) \right.
\]
\[
+ \left( H_{gq}(x_1, x_2, \mu_F^2) + H_{qg}(x_1, x_2, \mu_F^2) \right) a_s \Delta_{qq}^{(1),\gamma/Z} (z, \mu_F^2) \left. \right]
\]
\[
+ \sum_q \mathcal{F}_U \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dz \frac{d\delta(\tau - zx_1x_2)}{dz}
\]
\[
\times \left[ H_{qq}(x_1, x_2, \mu_F^2) \left( \Delta_{qq}^{(0),\mu} (z, Q^2, \mu_F^2) + a_s \Delta_{qq}^{(1),\mu} (z, Q^2, \mu_F^2) \right) \right.
\]
\[
+ \left( H_{gq}(x_1, x_2, \mu_F^2) + H_{qg}(x_1, x_2, \mu_F^2) \right) a_s \Delta_{qq}^{(1),\mu} (z, Q^2, \mu_F^2) \left. \right]
\]
\[
+ H_{gg}(x_1, x_2, \mu_F^2) \left( \Delta_{gg}^{(0),\mu} (z, Q^2, \mu_F^2) + a_s \Delta_{gg}^{(1),\mu} (z, Q^2, \mu_F^2) \right) \right] ,
\]
where $\Delta^{(ij)} (z, Q^2, \mu_F^2), j = \gamma^*/Z^*$, $U$ are the mass factorised coefficient functions of the SM and unparticle theories respectively to order $i$ in QCD. The constants $\mathcal{F}_{SM,q}$ and $\mathcal{F}_G$ are given by
\[
\mathcal{F}_{SM,q} = \frac{4\alpha_s^2}{3Q^2} \left[ Q^2 - \frac{2Q^2(Q^2 - M_Z^2)}{((Q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2)\Gamma_w^2 s_w^2} Q_q g_q V^q g_q V^q \right.
\]
\[
+ \frac{Q^4}{((Q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2)\Gamma_w^2 s_w^2} \left( (g_q^1)^2 + (g_q^4)^2 \right) \left( (g_q^1)^2 + (g_q^4)^2 \right),
\]
\[
\mathcal{F}^\kappa_U = \frac{C_\kappa \lambda_\kappa A_{d\ell}^2}{4\pi \sin^2(\pi d\ell) Q^2} \left( \frac{Q^2}{M_W^2} \right)^{2d\ell-m} ,
\]
where $\lambda_\kappa$ is varied from 0.4 to 0.9 for $\kappa =$ scalar (S), tensor (T). For the tensorial interaction, the constant $C_T = 1/80$ and $m = 0$, for the scalar interaction $C_S = 1/32$ for both quark anti-quark $(m = 2)$ as well as gluon $(m = 1)$ initiated processes.

The renormalised partonic distributions are
\[
H_{qq}(x_1, x_2, \mu_F^2) = f_q^{P_1}(x_1, \mu_F^2) f_q^{P_2}(x_2, \mu_F^2) + f_{\bar{q}}^{P_1}(x_1, \mu_F^2) f_{\bar{q}}^{P_2}(x_2, \mu_F^2),
\]
\[
H_{gq}(x_1, x_2, \mu_F^2) = f_g^{P_1}(x_1, \mu_F^2) \left( f_{\bar{q}}^{P_2}(x_2, \mu_F^2) + f_{\bar{g}}^{P_2}(x_2, \mu_F^2) \right),
\]
\[
H_{qg}(x_1, x_2, \mu_F^2) = H_{qg}(x_2, x_1, \mu_F^2),
\]
\[
H_{gg}(x_1, x_2, \mu_F^2) = f_g^{P_1}(x_1, \mu_F^2) f_g^{P_2}(x_2, \mu_F^2).
\]
3 Discussions

In this section, we present the impact of unparticle fields at NLO level in QCD to the dilepton production at LHC with the center of mass energy $\sqrt{S} = 14$ TeV. Similar effects can be easily studied at Tevatron using our results and we postpone the detailed study for future publication. We have not used the existing Tevatron data to constrain the parameters of unparticle theories, instead we have demonstrated the observable effects of this new theory using reasonable choices of parameters. Our predictions are less sensitive to renormalisation and factorisation scale uncertainties. The SM parameters are $\alpha = 1/137.03604$, $M_Z = 91.1876$ GeV, $\Gamma_Z = 2.4952$ GeV. For the parton density sets, we have used in the leading order the MRST 2001 LO ($\Lambda = 0.1670$ GeV) and in the next-to-leading order the MRST 2001 NLO ($\Lambda = 0.2390$ GeV). Recent MRST sets available in the literature are not expected to change our findings.

The invariant mass distributions for scalar (both quark-antiquark and gluon initiated processes) and tensor interactions are plotted in the Fig. (1-3) for the range $150$ GeV $< Q < 1100$ GeV and for the coupling parameters $\lambda_k = 0.4$ (lowest), 0.8 (middle), 0.9 (upper). We choose $d_U = 1.01$ for all the plots. It is clear from the Figs. (123), the unparticle effects will be visible only for larger values of $\lambda_k$. We also find that the unparticle fields tend to increase the cross section in the large invariant mass region, due to the factor $(Q^2 d_U^2)$ in $F_k$. This effect is very similar to the four Fermi interaction or $Z'$ exchange or tower of Kaluza-Klein excitations in extra-dimensional models (see [18]). The nature of these interactions are similar but for the overall scale dependent couplings.

The higher order QCD effect is quantified by the K factor defined by

$$K = \left[ \frac{d\sigma_{LO}(Q)}{dQ} \right]^{-1} \left[ \frac{d\sigma_{NLO}(Q)}{dQ} \right].$$

(13)

We have plotted the K factor for both scalar and tensor interactions in Fig. (4-6). We find that the K factor in the unparticle theory for quark anti-quark initiated process is very similar to that of SM. On the other hand, the gluon initiated processes receive large K factor for both scalar and tensorial interactions.

Finally, we have shown how NLO corrections improve the scale uncertainties by plotting the following ratios at LO and NLO

$$R_{LO} = \left[ \frac{d\sigma_{LO}(Q, \mu = \mu_0)}{dQ} \right]^{-1} \left[ \frac{d\sigma_{LO}(Q, \mu)}{dQ} \right] \bigg|_{Q=700 \text{ GeV}},$$

$$R_{NLO} = \left[ \frac{d\sigma_{NLO}(Q, \mu = \mu_0)}{dQ} \right]^{-1} \left[ \frac{d\sigma_{NLO}(Q, \mu)}{dQ} \right] \bigg|_{Q=700 \text{ GeV}}.$$  

(14)

From the plots in Fig. (79), we find that our NLO corrected predictions are less sensitive to renormalisation and factorisation scales making our predictions stable under QCD radiative corrections that are important at hadron colliders.

To summarize our study, we find that both scalar and tensorial interactions of unparticle fields to SM fields can lead to sizable observable effect in the invariant mass distributions of
of dilepton pairs at hadron colliders in the large invariant mass region. Since QCD effects at hadron colliders are important, we have incorporated its effect through next-to-leading order corrections. We found that unlike quark initiated processes, the gluon initiated processes give large K factor which has to be taken into account when extracting the parameters of the unparticle theory from the experimental data. We have also demonstrated how our QCD improved next-to-leading order results improve the predictions by reducing the uncertainties coming from the factorisation scale.

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Figure 1: Invariant mass distribution of the dilepton which includes the scalar unparticles coupling to quark-antiquark. The lowest set corresponds to $\lambda_S = 0.4$, middle $\lambda_S = 0.8$ and upper $\lambda_S = 0.9$.

Figure 2: The scalar unparticles coupling to gluons in the initial state to produce a dilepton pair with invariant mass $Q$. The lowest set corresponds to $\lambda_S = 0.4$, middle $\lambda_S = 0.8$ and upper $\lambda_S = 0.9$. 
Figure 3: The tensor unparticles coupling to quarks and gluons in the initial state to produce a dilepton pair with invariant mass $Q$. The lowest set corresponds to $\lambda_T = 0.4$, middle $\lambda_T = 0.8$ and upper $\lambda_T = 0.9$.

Figure 4: $K$ factor for the invariant mass distribution of dileptons at LO and NLO for the scalar unparticle from the quark-antiquark channel for $\lambda = 0.9$. 
Figure 5: The K-factor for the invariant mass distribution of dilepton production in which the scalar unparticle is exchanged from the gluon-gluon channel for $\lambda = 0.9$.

Figure 6: The K-factor for the invariant mass distribution of dilepton production in which the tensor unparticle is exchanged from the quarks and gluons in the initial state for $\lambda = 0.9$. 
Figure 7: Factorisation scale variation of the invariant mass distribution of dileptons at LO and NLO for the scalar unparticle from the quark-antiquark channel for $\lambda = 0.9$.

Figure 8: Factorisation scale variation of LO and NLO invariant mass distribution of dileptons from the gluon initiated process which couple to the scalar unparticle for $\lambda = 0.9$. 
Figure 9: Factorisation scale variation of LO and NLO dilepton invariant mass distribution which includes the tensor unparticle. This includes both the quark-antiquark and gluon channels for $\lambda = 0.9$. 