Exchange driven freeze out of dark matter

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Abstract: We introduce a novel mechanism where non number changing processes set the relic density of a thermal particulate dark matter. In a relatively degenerate multipartite dark sector if there is a considerable time lapse between the freeze out of various species then process like exchange between dark sector constituents can play the pivotal role of driving freeze out and setting dark matter relic density. We show that this unique mechanism can produce a viable GeV scale thermal dark matter. As a proof of principle we present simple scalar dark matter models to demonstrate this phenomenon.
1 Introduction

The overwhelming evidence from various astrophysical and cosmological observations establish that around 25% of the energy budget of the universe is made of dark matter (DM). The assumption that DM is a particulate thermal relic is one of the leading candidate. In this framework DM remains in thermal equilibrium at very early stage of the thermal history of the universe, however, due to combine effect of depletion in its number density through annihilation and expansion of the universe, it gets frozen out resulting in the DM relic density observable today.

The description of thermal freeze out of DM is usually driven by $2 \rightarrow 2$ number changing processes, where a pair of stable DM candidate annihilates to SM states. This framework has received considerable attention within a large class of particle physics models [1–3]. It is not surprising to expect these models to have a multiparticle dark sector whose stability is ensured by imposing one or more discrete symmetry. Depending on the number of particles and unbroken discrete symmetries in the theory either there will be a single or multi component DM. Assuming that, these dark sector particles couple to SM states in the primordial soup, then, the main underlying process that set the thermal DM relic density is shown in figure 1. To keep the discussion tractable we assume that there are two particles in the dark sector $\phi_1$ and $\phi_2$ with the former being the major component of the present day DM relic density. Depending on the stabilizing symmetry, $\phi_2$ may either be a minor component of the DM or an associated unstable state of the dark sector.

The chronology of the decoupling time of the processes, shown in figure 1, will determine freeze out behavior of the species under consideration. In the standard paradigm the freeze out of the processes in figure 1 happens almost simultaneously and the annihilation channels of individual species (figure 1a and 1c for $\phi_1$ and $\phi_2$ respectively) are primarily responsible for setting their relic density [4–9]. It has been already pointed out that, if there is a substantial gap in the freeze out time of these processes then non-annihilation channels may regulate the relic density of DM [10–12]. It is in this context, we propose for
the first time a mechanism where exchange process (shown in figure 1b) play the pivotal role in setting DM relic density.

In section 2, we describe basic formalism of exchange driven freeze out mechanism. In section 3, we present simple scalar DM model where the framework has been realized, before concluding in section 4.

2 Formalism

In this section we discuss basic formalism of exchange driven DM freeze out. Within this framework we will assume that the annihilation of the major DM constituent $\phi_1$, shown in figure 1a, decouples earliest in the thermal history compared to all the other relevant processes. This early freeze out implies a suppression in the annihilation cross section, which may be indicative of the null results in the DM experiments. The successful thermal freeze out of $\phi_1$ can still be achieved through the combined effect of the exchange $\phi_1\phi_1 \rightarrow \phi_2\phi_2$ and subsequent annihilation of the $\phi_2$ to SM states. The co-moving number density of $\phi_1$ freezes once either of both processes turns in-operational. Crucially the mode of freeze out hinges on the time ordering of decoupling of processes shown in figure 1b and 1c. If the interaction rate of exchange process goes below Hubble expansion rate before the other one then exchange process will determine the freeze out. This is an atypical feature of this framework where a non-number changing process drive the freeze out. While for the reverse time ordering annihilation of $\phi_2$ will regulate the freeze out of $\phi_1$. Interestingly here the decoupling of the annihilation of one species sets the relic abundance of a different state.

The evolution of the number density ($n$) of the dark sector species can be obtained by solving the coupled Boltzmann equation for the dark sector states, that can be schematically written as

$$\dot{n}_i + 3Hn_i = -\sum_j \left\{ \langle \sigma v \rangle_i^{\text{anh}} \left( n_i^2 - (n_i^{\text{eq}})^2 \right) + \langle \sigma v \rangle_i^{\text{ex}}_{i\rightarrow j} \right\}$$
\[
\left( n_i^2 - (n_i^{eq})^2 \right) \left( \frac{n_j}{n_j^{eq}} \right)^2 + \Gamma^{\text{dec}} \left( n_i - n_i^{eq} n_j^{eq} \right) \right), \tag{2.1}
\]

where \( i, j \) include all the particles in the spectrum, \( n_i^{eq} \) represents the equilibrium number density, \( H \) is the Hubble expansion rate. The first term in the RHS of equation (2.1) is related to annihilation processes, the second term is the contribution of exchange. The third term corresponds to the decay of \( \phi_2 \), which may or may not be present depending on underlying symmetry stabilizing the dark sector.

As is usually the case, if the major DM component \( \phi_1 \) is the lightest state in the dark sector, then the \( \phi_1 \phi_1 \rightarrow \phi_2 \phi_2 \) exchange process is endothermic and its thermally averaged cross section has an exponential suppression \([12]\).

\[
\langle \sigma v \rangle_{1 \rightarrow 2}^{\text{ex}} \approx \left( \frac{m_2}{m_1} \right)^3 e^{-2x\delta} \langle \sigma v \rangle_{2 \rightarrow 1}^{\text{anh}}, \tag{2.2}
\]

where \( x = m_1/T \), and \( \delta = (m_2 - m_1)/m_1 \) is a measure of the degree of degeneracy in the dark sector spectrum.

An approximate analytical solution of the coupled Boltzmann equation (2.1) in the exchange and \( \phi_2 \) annihilation dominated regime may be obtained by comparing the relevant event rate \( n_{1/2} \langle \sigma v \rangle_{\text{ex/an}} \) with the Hubble parameter. This gives an estimate of the freeze out temperature \( (x_f) \) of the \( \phi_1 \) up to a normalizing factor \( (c) \) in the event rate. The relic density can now be computed assuming that co-moving number density of the \( \phi_1 \) remains constant since freeze out. The approximate expression for the major component of DM relic density for the two regime can be written as,

\[
\Omega_h^2 \approx \begin{cases} 
0.16 \text{ pb} \frac{c x_f e^{x_f(2\delta-1)}(1 - 3\delta)}{\langle \sigma v \rangle_{\text{ex}}^{\phi_1} \sqrt{g_{\text{eff}}(x_f)}} , & \text{exchange} \\
0.16 \text{ pb} \frac{c x_f e^{x_f\delta}(1 - 3\delta/2)}{\langle \sigma v \rangle_{\text{anh}}^{\phi_2} \sqrt{g_{\text{eff}}(x_f)}} , & \text{\( \phi_2 \) annihilation}
\end{cases}, \tag{2.3}
\]

Note that, in equation (2.4) \( g_{\text{eff}} \) is the effective number of relativistic degrees of freedom at the time of freeze out. Matching with full numerical solution of equation (2.1) imply \( c \sim \mathcal{O}(1) \). For typical values of the parameters of interest in this paper, we find that \( x_f \sim 20 \), implying DM freeze out in the non-relativistic regime. This signify that exchange driven and \( \phi_2 \) annihilation driven freeze out yield a cold dark matter as preferred from the large scale structure of the universe.

### 3 Minimal setup

To construct the minimal multiparticle dark sector we augment the SM with two real scalar SM singlet fields \( \phi_1 \) and \( \phi_2 \). Depending on the stabilizing symmetry of the dark sector one can obtain a single component or two component DM. Regions of parameter space of these generic constructions demonstrate the phenomenon of exchange driven freeze out.
3.1 Toy Model 1

The most economic Lagrangian is obtained when one considers two discrete symmetries \( \mathbb{Z}_2 \) and \( \mathbb{Z}_2' \) under which \( \phi_1 \) and \( \phi_2 \) are charged respectively while the SM remains even under both [14]. The most general renormalizable Lagrangian including the Higgs portal coupling to the SM can be written as

\[
-L_{\text{int}} = \frac{\lambda_1}{2} \phi_1^2 H^\dagger H + \frac{\lambda_1}{4!} \phi_1^4 + \frac{\lambda_2}{4!} \phi_2^4 + \frac{\lambda_2}{2} \phi_2^2 H^\dagger H + \frac{\lambda_e}{4} \phi_2 \phi_1^2, \tag{3.1}
\]

where \( H \) denotes the SM Higgs doublet. Clearly, both \( \phi_1 \) and \( \phi_2 \) will be stable leading to a two component DM.

The standard two component DM framework with \( \mathbb{Z}_2 \times \mathbb{Z}_2' \) symmetry is mostly disfa- vored by the results of direct detection experiments except in the tuned Higgs resonance region or for DM mass \( \gtrsim 500 \text{ GeV} \) [14]. Interestingly, in the regime where the annihilation cross section of the lighter species, say \( \phi_1 \), is quite small the freeze out of \( \phi_1 \) can still be achieved by the combined effect of conversion of \( \phi_1 \) to \( \phi_2 \) and subsequent annihilation of \( \phi_2 \) to SM states. In this region where \( \lambda_1 \ll \lambda_2, \lambda_e \) the model demonstrate the phenomenon of exchange driven freeze out explained above. Depending on relative strength and hence the freeze out chronology, either the exchange process or the \( \phi_2 \) annihilation will set the relic density of \( \phi_1 \).

Assuming that \( \phi_1 \) is the major constituent of the DM the two distinct possibilities have been depicted in figure 2 through a detailed numerical solution of the full coupled Boltzmann equation involving number densities of both \( \phi_1 \) and \( \phi_2 \) [15]. Throughout the discussion the annihilation cross section of \( \phi_1 \) has been kept small by fixing it at \( \lambda_1 = 10^{-5} \).

**Figure 2**: Different phases of freeze out for the models. For model 1 in both the left and right panel red solid, dashed, and dot dashed lines represent total relic density (\( \Omega h^2 \)), the contribution of \( \phi_1 \) (\( \Omega_1 h^2 \)) and, the contribution of \( \phi_2 \) (\( \Omega_2 h^2 \)) respectively while the blue dashed lines correspond to model 2. We assume \( \lambda_1 = 10^{-5} \), \( m_1 = 100 \text{ GeV} \), and \( \delta = 1\% \). The black solid band represent central value of DM relic density, measured by Planck [13].

(a) Exchange driven freeze out regime in \( \Omega h^2 - \lambda_e \) plane. The chosen value of \( \lambda_2 \) is 0.5.

(b) \( \phi_2 \) annihilation dominated freeze out regime in \( \Omega h^2 - \lambda_e \) plane. The chosen value of \( \lambda_e \) is 0.5.
**Figure 3:** Phenomenological consequences of model 1 in $\lambda_2 - \lambda_e$ plane for $m_1 = 100$, 300, and 500 GeV are shown by light blue, orange, and green lines respectively. The solid lines represent $\Omega_ch^2 = 0.12$ contours [13]. The dashed lines illustrate upper limit on the couplings for the aforementioned masses from XENON1T experiments. The three shaded regions show the allowed parameter space from both the over-closure of relic density and direct detection. For the hatched region the exchange process drive the freeze out of $\phi_1$ whereas for the non-hatched region annihilation of $\phi_2$ determine the same. The numerical value of the other relevant parameters are mentioned in figure 2.

In the left and right panel we show variation of relic abundance with $\lambda_e$ and $\lambda_2$ respectively. In each plot there are three distinctive regions, the region in between two gray vertical lines represents the part of the parameter space where the relic density of $\phi_1$ and hence that of DM is set by exchange (figure 2a) or $\phi_2$ annihilation (figure 2b) processes. While outside this band the total DM relic abundance becomes insensitive to these parameters signifying that the exchange and annihilation of $\phi_2$ no longer drive the freeze out of $\phi_1$.

In the exchange driven freeze out regime the large value of $\lambda_2$ results in a large spin-independent $\phi_2$–nucleon cross section [16, 17]. This leads to a strong constraint from direct detection limits inspite of the fact that the $\phi_2$ is a minor constituent of the total DM relic density. For instance, the limit on such Higgs portal coupling from the recent results of XENON1T experiment [18] effectively rules out this possibility in the $\sim$ 100 GeV mass scale within the minimal model. However, in the $\phi_2$ annihilation dominated region the lower rate of $\phi_2$ annihilation cross-section together with the reduction factor $\Omega_2/\Omega_t \ll 1$, is enough to suppress the effective direct detection coupling to an extent to be able to evade the present XENON1T bound. This has been demonstrated in figure 3 where the light blue, orange, and green lines correspond to DM relic density contours for $m_1 = 100$, 300, and 500 GeV respectively. The dashed lines of figure 3 show direct detection upper limit on $\lambda_2$ for the aforementioned masses. The three shaded regions are consistent with both the direct detection and relic density over closure bounds. Further, in figure 3 the pink hatched patch denotes the exchange driven freeze out region which remains forbidden by the direct detection bound. Interestingly the allowed $\phi_2$ annihilation
region resurrect the two component $\mathbb{Z}_2 \times \mathbb{Z}_2'$ model with DM masses in the 100 GeV range, which is otherwise ruled out in conventional scenarios \[14\]. It is to be noted that in the above discussion we have chosen $\delta = 1\%$ signifying the amount of fine tuning required in this framework.

In passing, we note that one can consider the heavier of the two species as the major DM constituent which would ease the exponential suppression in equation (2.2). However, such a framework seems to be mostly ruled out from direct detection constraints.

### 3.2 Toy Model 2

In a setup where the two SM singlet scalars $\phi_1$ and $\phi_2$ are charged under the same $\mathbb{Z}_2$ symmetry keeping the rest of the SM even, leads to a multipartite dark sector with a single stable DM candidate \[19, 20\]. The most general renormalizable Lagrangian consistent with the charge assignment is given by

\[
-L_{\text{int}} = \left( \frac{\lambda_1}{2} \phi_1^2 + \frac{\lambda_2}{2} \phi_2^2 + \lambda_{12} \phi_1 \phi_2 \right) H^2 H + \frac{\lambda_{12}}{4!} \phi_1^4 \\
+ \frac{\lambda_2}{4!} \phi_2^4 + \frac{\lambda_{e1}}{4!} \phi_1^2 \phi_2^2 + \frac{\lambda_{e2}}{3!} \phi_1^3 \phi_2 + \frac{\lambda_{e3}}{3!} \phi_1 \phi_2^3,
\]

where $H$ denotes the SM Higgs doublet. The lighter state represents the stable DM candidate. In our discussion, we will assume $m_1 < m_2$, therefore, $\phi_1$ represents the lightest stable particle (LSP) and is the sole DM candidate within this setup while $\phi_2$ is the next to lightest stable particle (NLSP) that can decay to the LSP. In addition to the processes shown in figure 1, co-annihilation is the major feature that distinguishes this from model 1. Conventionally, the annihilation and co-annihilation set the DM relic density. However, here we will assume that these processes are suppressed enough to induce a considerable time lapse between the freeze-out of the LSP annihilation process with the exchange and NLSP annihilation. This enables the exchange processes and NLSP $\phi_2$ annihilation to drive the freeze-out of LSP $\phi_1$. Further to keep the discussion tractable we assume $\lambda_{e1} = \lambda_e$.

The different phases of the freeze-out shown by the blue dashed lines of figure 2 which is obtained using a full numerical simulation of the dark sector, is represented by the Lagrangian given in equation (3.2). Clearly, when $\lambda_1, \lambda_{12} < \lambda_e/2 \sim \lambda_{2/e}$ then either exchange processes or the annihilation of $\phi_2$ predominantly drive the thermal freeze-out of the DM, hence we set $\lambda_1 = \lambda_{12} = 10^{-5}$. The relic density allowed contours for DM masses 100, 300, and 500 GeV has been depicted in figure 4. The hatched region denotes the exchange driven freeze-out parameter space while for the non-hatched region annihilation of $\phi_2$ drive the freeze-out.

The spin-independent direct detection cross section of the DM would be considerably small due to the smallness of the Higgs portal coupling $\lambda_1$. However, in both the exchange and $\phi_2$ annihilation driven regime the lifetime of $\phi_2$ can be quite large, having potential implication for early universe cosmology.
Figure 4: $\Omega h^2 = 0.12$ contours for model 2 in $\lambda_2 - \lambda_e$ plane for $m_1 = 100$, 300, and 500 GeV are depicted by light blue, orange, and green lines respectively. For the hatched region exchange processes drive the DM freeze out whereas for the non-hatched region annihilation of $\phi_2$ determine the same. We have taken $\lambda_1 = \lambda_{12} = 10^{-5}$, and $\delta = 5\%$.

4 Conclusion:

In this paper, we propose a new paradigm of thermal freeze out of DM. In a relatively degenerate dark sector if annihilation cross section of the major constituent of DM relic density is suppressed then exchange process within the dark sector and subsequent annihilation of other species drive the freeze out. Depending on time ordering of freeze out, either exchange processes or annihilation of second lightest species can set the relic density of DM. We have demonstrated that for simple multiparticle Higgs portal DM model this novel mechanism can be readily realized to saturate the DM relic density constraints while being consistent with direct detection bounds.

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