Gravitational quasinormal radiation of higher-dimensional black holes.

R.A. Konoplya
Department of Physics, Dniepropetrovsk National University
St. Naukova 13, Dniepropetrovsk 49050, Ukraine
konoplya-roma@yahoo.com

Abstract

We find the gravitational resonance (quasinormal) modes of the higher dimensional Schwarzschild and Reissner-Nordstrom black holes. The effect on the quasinormal behavior due to the presence of the $\lambda$ term is investigated. The QN spectrum is totally different for different signs of $\lambda$. In more than four dimensions there excited three types of gravitational modes: scalar, vector, and tensor. They produce three different quasinormal spectra, thus the isospectrality between scalar and vector perturbations, which takes place for $D = 4$ Schwarzschild and Schwarzschild-de-Sitter black holes, is broken in higher dimensions. That is the scalar-type gravitational perturbations, connected with deformations of the black hole horizon, which damp most slowly and therefore dominate during late time of the black hole ringing.

1 Introduction and basic equations

Two sorts of small perturbations of black holes within general relativity are considered usually: the first consists in adding of a test field to a black hole space-time and thus can be described by a dynamical equation for this field in the background of a black hole. Second way to perturb a black hole is to perturb the metric itself. In this case in order to find the evolution equation one has to linearize the Einstein equations. These perturbations are called gravitational perturbations and are always paid more attention than the first ones. That is because gravitational radiation is much stronger than that of the external fields decaying near the black hole, and also the metric perturbations let us judge about stability of a black hole.

In four space-time dimensions the linearized Einstein equations for perturbations of Schwarzschild black hole can be treated separately for axial (invariant under the change $\phi \rightarrow -\phi$) and polar perturbations. After the separation of angular variables and supposing exponential dependence of the wave function on time ($\Psi_i \sim e^{-i\omega t}$), the final radial wave equations for these two kinds of perturbations have the form of the Schrodinger wave-like equations with some different effective potentials

$$\left(\frac{d^2}{dr^2} + \omega^2\right)\Psi_i(r) = V_i\Psi_i(r),$$

(1)
where the index \( i \) designates a type of perturbations.

It is a remarkable point that the spectra produced by these two potentials for \( D = 4 \) Schwarzschild black hole are the same. What occurs for higher dimensional black hole was the challenge up until now, when Ishibashi and Kodama \([1, 2, 3]\) managed to reduce the perturbation equations to three wave-like equations, one for each type of perturbations: scalar (reducing to polar at \( D = 4 \)), vector (reducing to axial at \( D = 4 \)), and "new" tensor perturbations. Such a reduction of the linearized Einstein equations to the wave-like form is not trivial work and requires decomposition of the wave functions into the corresponding harmonics which are connected with the symmetry of the background to be perturbed (see for example references in \([4]\)).

The metric of the spherically symmetric charged static black hole in a \( D \)-dimensional space-time with \( \lambda \) term has the form:

\[
 ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2d\Omega_{D-2}^2, \tag{2}
\]

where

\[
f(r) = 1 - \frac{2M}{r^{D-3}} + \frac{Q^2}{r^{2D-6}} - \lambda r^2 \tag{3}
\]

The effective potentials for scalar, vector, and tensor, gravitational perturbations respectively have the form:

\[
 V_s(r) = \frac{f(r)U(r)}{16r^2\left(m + \frac{d(d+1)x}{2} - \frac{d^2Q^2}{r^{2d-2}}\right)^2}, \tag{4}
\]

where

\[
 U(r) = \left[-d^3(d+2)(d+1)x^2 + 4d^2(d+1)(d(d^2 + 6d - 4)z + 3(d - 2)m)x -
12d^5(3d - 2)z^2 - 8d^2(11d^2 - 26d + 12)mz + 4(d - 2)(d - 4)m^2|\lambda r^2 +
d^4(d+1)^2x^3 + d(d+1)[-3d^2(5d^2 - 5d + 2)z + 4(2d^2 - 3d + 4)m +
d(d - 2)(d - 4)(d + 1)]x^2 + 4d[d^2(4d^3 + 5d^2 - 10d + 4)z^2 -
(d(34 - 43d + 21d^2)m + d^2(d + 1)(d^2 - 10d + 12))z - 3(d - 4)m^2 -
d(d + 1)(d - 2)m]x - 4d^5(3d - 2)z^3 + 12d^2(2(-6d + 3d^2 + 4)m +
d^2(3d - 4)(d - 2))z^2 + (4(13d - 4)(d - 2)m^2 + 8d^2(11d^2 - 18d + 4)m)z +
16m^3 + 4d(d + 2)m^2, \tag{5}
\]

\[
x = \frac{2M}{r^{d-1}}, \quad d = D - 2, \quad m = l(l + d - 1) - d, \quad z = \frac{Q^2}{r^{2d-1}}. \tag{6}
\]

\[
 V_{v\pm}(r) = \frac{f(r)}{r^2}\left(l(l + d - 1) - 1 + \frac{d^2 - 2d + 4}{4} - \frac{d(d - 2)\lambda r^2}{4} + \frac{d(5d - 2)Q^2}{4r^{2d-2}} + \mu_\pm \right), \tag{7}
\]

\[
 \mu_\pm = -\frac{d^2 + 2}{2}M \pm \left((d^2 - 1)^2M^2 + 2d(d - 1)(l(l + d - 1) - d)Q^2\right) \tag{8}
\]

\[
 V_t(r) = f(r)\left(\frac{l(l + d - 1)}{r^2} + \frac{(d)(d - 2)}{4r^2}f(r) + \frac{d}{2r}f'(r)\right), \tag{9}
\]

2
The effective potential for tensor perturbations proved to be equivalent to that corresponding to decay of test scalar field in a black hole background \cite{5,6}. When $\lambda > 0$ ($< 0$) one has D-dimensional $\text{SdS}$ ($\text{SAdS}$) background. Note that the scalar-type perturbation equations for charged background together with the corresponding Maxwell equations can be reduced to the pair of equations for electromagnetic and gravitational perturbations (see eqs. (5.59) - (5.63) of \cite{3}), which we shall not write out here, since they are cumbersome.

When perturbing a black hole, its response has resonances at some complex discrete modes. If one denote the general solution at infinity as

$$\Psi = A_{in}\psi_{in} + A_{out}\psi_{out}, \quad r^* \to \infty$$

where $r^*$ is the tortoise coordinate ($dr^* = dr/f(r)$), $\psi_{in} \sim e^{-i\omega r^*}$, $\psi_{out} \sim e^{i\omega r^*}$ are radial solutions corresponding to incoming and outgoing radiation at infinity with some complex amplitudes $A_{in}$, $A_{out}$. Then the reflection coefficient $A_{out}/A_{in}$ is singular at the resonance frequencies. Therefore one can think that $A_{in}$ vanishes under the non-vanishing $A_{out}$, i.e. no incoming radiation is permitted at spacial infinity. Since no outgoing radiation can escape the horizon of a black hole, the modes are required to represent purely ingoing waves at the horizon. Thus under the choice of positive sign of the real part of $\omega$,

$$\omega = \omega_{Re} - i\omega_{Im}, \quad (\Psi_i \sim e^{-i\omega t})$$

the resonance (quasinormal) modes satisfy the boundary conditions:

$$\Psi \sim c_{\pm}e^{\pm i\omega r^*} \quad as \quad r^* \to \pm \infty.$$  

(11)

It is important that the quasinormal modes dominate during late-time stages of the gravitational response of a black hole to an external perturbation. This response consists of damping oscillations and is called quasinormal ringing. Note that the quasinormal modes do not depend on way in which they where excited and are determined by a black hole parameters only. Being calculated within the linear approximation of metric perturbations, the existence of quasinormal modes are confirmed in a non-linear analysis. They are important characteristic values of various dynamic processes, such as black hole collisions or decay of different fields in a BH background.

In addition it has been revealed recently that the quasinormal modes calculated in AdS gravity have a direct interpretation in the dual gauge theory \cite{7} and, also, help to predict the correct value of a black hole entropy in Loop quantum gravity \cite{8}. Al this stimulated considerable interest in QNMs (see for example \cite{9} and references therein). At the same time the higher dimensional brane models \cite{10}, created new motivations to calculate quasinormal modes of the higher-dimensional black holes \cite{11,5,12,13}.

Thus within different contexts the quasinormal modes corresponding to decay of a free scalar field around the higher dimensional ($D > 4$) black holes have been considered recently in \cite{14,15,7,16,11,5,12}. We are interested here in calculating of the quasinormal modes of the D-dimensional S, $\text{SdS}$ and $\text{SAdS}$ black holes for gravitational perturbations. In Sec.II we calculate the QNMs of $D > 4$ SBH. In Sec.III the case of non-vanishing $\Lambda$ - term is investigating; this includes black holes in de-Sitter and Antide-Sitter space-times. Sec. IV is devoted to the effect of the black hole charge on the QN ringing.
Table I. The fundamental QN frequencies for $l = 2$ gravitational perturbations of SBH. The tensor-type gravitational mode for $D = 4$ black hole does not exist.

## 2 Schwarzschild

The effective potentials for vector and tensor potentials for Schwarzschild black holes (see formulas (7) and (9) at $Q = 0$) is constant at the infinity and at the horizon, and approach a maximum somewhere in between (see [2] for plots of the effective potentials). Together with the boundary conditions (11) this allows us to use the standard WKB approach of Schutz and Will [17] which have been recently extended to the 6th order [5]. Another case is the scalar-gravitational potential (11) which has negative pitch near the event horizon for some values of the parameters $D$ and $l$. Thus when analyzing this particular type of gravitational perturbations we shall be restricted by those values of $D$ and $l$ for which we have good potential barrier for $V_s$.

In [5] it was shown that the real part of quasinormal modes corresponding to decay of the free scalar field around the D-dimensional Schwarzschild black hole are roughly proportional to $D$ if measured in units $2r_0^{-1}$ where $r_0$ is the horizon radius, and for moderate and large $D$ this numerical linear dependence is very accurate.

These QNMs of free scalar field coincide completely with those of the tensor-type gravitational perturbations, which is naturally, since the effective potentials of these two perturbations are equivalent. To compute the QNMs we used the 6th order WKB formula [5]:

$$\frac{nQ_0}{\sqrt{2Q_0''}} - \sum_{i=2}^{i=6} \Lambda_i = n + \frac{1}{2},$$  \hspace{1cm} (12)

where the correction terms of the i-th WKB order $\Lambda_i$ can be found in [17] and [5], $Q = V - \omega^2$ and $Q_0$ means the i-th derivative of $Q$ at its maximum. The fundamental modes for vector and tensor $l = 2$ potentials are presented in Table 1 (the scalar potential for $l = 2$ has negative pitch in higher dimensions).

For $l = 3$ we can obtain with the help of the WKB method the QNMs for all three types of gravitational perturbations. Yet scalar-type effective potential has negative pitch at $D > 9$, and, what is more for WKB treatment, there is another maximum near the
Table II. The fundamental QN frequencies for $l = 3$ gravitational perturbations of SBH. The effective potential of scalar-type modes has negative pitch when $D > 9$ at $l = 3$, and therefore the WKB approach is inapplicable.

| D  | tensor-type | vector-type      | scalar-type      |
|----|-------------|------------------|------------------|
| 4  | -           | 0.5994 - 0.0927 i| 0.5994 - 0.0927 i|
| 5  | 1.4198 - 0.2516 i | 1.2197 - 0.2362 i | 1.345 - 0.2211 i  |
| 6  | 2.0471 - 0.3959 i | 1.7356 - 0.3742 i | 1.5085 - 0.3029 i |
| 7  | 2.6192 - 0.5272 i | 2.2218 - 0.5055 i | 1.8767 - 0.4548 i |
| 8  | 3.1621 - 0.6467 i | 2.6993 - 0.6262 i | 2.2887 - 0.5517 i |
| 9  | 3.6891 - 0.7555 i | 3.1723 - 0.7368 i | 2.6781 - 0.6435 i |
| 10 | 4.2082 - 0.8541 i | 3.6426 - 0.8391 i | -                |
| 11 | 4.7251 - 0.9427 i | 4.1116 - 0.9345 i | -                |
| 12 | 5.2440 - 1.0216 i | 4.5800 - 1.0241 i | -                |
| 13 | 5.7685 - 1.0907 i | 5.0483 - 1.1087 i | -                |
| 14 | 6.3017 - 1.1504 i | 5.5168 - 1.1890 i | -                |
| 15 | 6.8460 - 1.2009 i | 5.9857 - 1.2655 i | -                |

event horizon, so that one would have to consider the secondary scattering process near the peak of this sub-minimum. Nevertheless for sufficiently large $D$ we can anticipate the quasinormal behavior, since the horizon radius $r_0$ does not depend on $D$ in this regime, $f(r) \to 1$ and one can re-scale the wave equation, so that both real and imaginary QNMs are proportional to $D$.

We see that the the more the spin weight of the type of gravitational perturbations to be considered, the more the oscillation frequency and the more the damping rate. Thus that is the scalar type of perturbations that will dominate during the later stages of the quasinormal ringing. That seems to be connected with the fact that, the scalar-type gravitational perturbations describe the brane deformation while the other two are just a worldsheet diffeomorphism if one considers a vacuum brane. It can be explicitly shown that the fluctuation of a vacuum brane is completely described by the master variable for the scalar mode of gravitational perturbations.

The real part of $\omega$ behaves similar to that of the perturbations of the free scalar field showing for moderate and large $D$ linear dependence on $D$ with good accuracy.

3 A-term

3.1 Schwarzschild-de-Sitter

The effective potentials for Schwarzschild black holes in higher dimensional de-Sitter space-times behave similar to those for asymptotically flat black holes: they approach the constants both at infinity and at the horizon, and have maximum somewhere outside the horizon. The only exception is the scalar-type gravitational perturbations with some values of $D$ and $l$. Thus taking into account these exceptional cases, we are able to apply the WKB formula of the previous section again.

The presence of positive cosmological constant changes the spectrum of resonance oscillations in the following way: the real part of the oscillations and the damping rate
Figure 1: SdSBH: $\omega_{Re}$ as a function of $\Lambda$ for vector-type modes in the 6th order WKB approximation, $l = 3 \ n = 0; \ D = 5$ (bottom), 6, 7 (top).

are decreasing as the $\Lambda$ is growing from 0 to its extremal value $3(D - 3)/(D - 1)^{D-3} M^2$. We take here $M = 1$. The typical picture of dependence of the real and imaginary parts of $\omega$ as a function of $\Lambda$ is shown on Fig.1 and Fig.2.

The QN modes of the Schwarzschild-de-Sitter black hole have been considered in a number of papers (see for example [20], [14], [21], [22] and reference therein). In the regime of the near extremal value of $\Lambda$, which corresponds to the maximal mass of the black hole which can be embedded into de-Sitter space-time

$$M_{max} = \sqrt{\frac{(3/\Lambda)^{D-3}(D - 3)^{D-3}}{(D - 1)^{D-1}}},$$

(13)

the effective potential approaches the Pöschl-Teller potential and the quasinormal modes are best described by the Pöschl-Teller analytic formula [21]. It has been shown also that the sixth order WKB formula gives the values which are in excellent agreement with those of Pöschl-Teller in the near extremal limit [22].

In addition, similar to SBH behavior, the more the spin weight of the type of gravitational perturbations to be considered, the more the oscillation frequency and the more the damping rate.

### 3.2 Schwarzschild-anti-de-Sitter

In AdS space-time there are two parameters which determine the Schwarzschild black hole properties: the horizon radius $r_+$, and the AdS radius $R = \sqrt{3/\Lambda}$. As is known from an extensive previous study [4], [9], the quasinormal behavior of a black hole in anti-de-Sitter space-time crucially depends upon the size of the black hole relative to the AdS radius $R$. In particular, for large black holes ($r_+ \gg R$) the QNMs are proportional to $r_+$,
and, thereby, to the black hole temperature $T$ [7], while for small black holes ($r_+ \ll R$) they approach their pure anti-de-Sitter values as $r_+ \to 0$ [16]. Asymptotically, for large overtone number $n$, the QNMs do not depend on the spin of the field being considered and are evenly spaced in $n$, no matter the size of the black hole [24].

By rescaling of $r$ we can put $R = 1$. The effective potentials (11), (7), (9) are infinite at spatial infinity. This lead to natural boundary condition: $\Psi = 0$ at infinity. In the context of $\text{ADS}_{d+1}/\text{CFT}_d$ correspondence, this boundary condition, being suitable for free massless scalar field, does not guarantee the coincidence with the poles of the corresponding correlation function in the dual gauge theory for gravitational perturbations [23]. Thus which boundary conditions one should impose on gravitational perturbations for the purpose of the CFT is an open question now, [23], [20].

Within accepted Dirichle boundary condition, the wave function vanishes at infinity and satisfies the purely in-going wave condition at the black hole horizon. Then one can compute the quasinormal frequencies stipulated by the above potentials following the procedure of G.Horowitz and V.Hubeny [7]. The main point of that approach is to expand the solution to the wave equation $\Psi$ around $x_+ = \frac{1}{r_+}$ ($x = 1/r$):

$$\psi(x) = \sum_{n=0}^{\infty} a_n(\omega)(x - x_+)^n$$  \hspace{1cm} (14)

and to find the roots of the equation $\Psi(x = 0) = 0$ following from the boundary condition at infinity. In fact, one has to truncate the sum (14) at some large $n = N$ and check that for greater values of $n$ the roots converge.

Since one can anticipate the QN behavior for small black holes, and for higher overtones [24], we shall consider here only lowest overtones of large black holes. The data for fundamental QNMs $D = 5$ BH are collected in Table 1. The more $D$ the more time con-
| $r_+$ | scalar ($n = 0$) | vector ($n = 0$) | vector ($n = 1$) | tensor ($n = 0$) |
|-------|-----------------|-----------------|-----------------|-----------------|
| 100   | 312.028 - 274.581 i | 0 - 0.012489 i  | 311.969 - 274.663 i | 311.982 - 274.659 i |
| 150   | 467.973 - 411.944 i | 0 - 0.008329 i  | 467.933 - 411.999 i | 467.942 - 411.996 i |
| 200   | 623.930 - 549.292 i | 0 - 0.006249 i  | 623.902 - 549.333 i | 623.909 - 549.331 i |
| 250   | 779.896 - 686.634 i | 0 - 0.00500 i   | 779.872 - 686.667 i | 779.878 - 686.666 i |
| 300   | 935.863 - 823.974 i | 0 - 0.004165 i  | 935.843 - 824.001 i | 935.848 - 824.000 i |

Table III. The fundamental quasinormal frequencies, $l = 2$, $n = 0$ for large 5-dimensional SAdS black hole.

Assuming is computing of the quasinormal modes if requiring good accuracy, since one has truncate the sum (14) at larger $N$. Estimations show that for higher $D$ the $n = 0$ modes of scalar, tensor types of gravitational perturbations and $n = 1$ of the vector-type perturbations are roughly the same for large black holes. Since the purely decaying vector-type $n = 0$ mode do not contribute into the black hole ringing, i.e. do not oscillating, we find that, similar to SBH and SdS BH ringing behavior, the scalar-type oscillating modes have the least damping rate and thereby dominate at late times of the ringing.

For $D = 4$ SAdS black hole it was found an analytic expression for the fundamental modes of vector-type (odd) gravitational perturbations [24]:

$$\omega_{n=0} = \frac{(l-1)(l+2)}{3r_+}i.$$  

(15)

From our numerical data, for arbitrary dimensional AdS black hole this formula can be generalized as follows

$$\omega_{n=0} = \frac{(l-1)(l+D-2)}{(D-1)r_+}i.$$  

(16)

4 Charge

When perturbing a charged black hole there excited two kinds of scalar-type and vector-type gravitational modes: first induced by the potential $V_+$ reducing at $Q = 0$ to the potential for electromagnetic perturbations, and second, induced by $V_-$ reducing to the potential for purely gravitational perturbations at $Q = 0$. At $Q \neq 0$, which is the case of RN BH background, there is no quasinormal mode which is purely electromagnetic or purely gravitational; each quasinormal mode will be accompanied by the emission of both electromagnetic and gravitational radiation. Another interesting property of the RN BH ringing was observed in [25]: it appeared that the quasinormal modes of gravitational waves with multi-pole index $l$ coincide with those of electromagnetic waves with multi-pole index $l - 1$ in the extremal limit. Then it was observed that this is connected with the fact that the extremally charged RN black hole preserves super-symmetry, and thus responses in a similar way on fields of different spin [26]. We do not observe similar effect in the case for the higher dimensional black holes (see for example Table VII.)

In the charged background with $D > 4$ we have generally two effective potentials for scalar-type modes, two potentials for vector modes, and one potential for tensor-type modes [3]. Yet the scalar-type potentials for some value of $l$ and $D$ have, similar to the case of the neutral black hole, negative pitch near the black hole horizon.
In order to compare the results obtained through the 6th order WKB approximation and numerical QN modes for charged black hole we give in the Appendix the fundamental QN modes for 4-dimensional RN black hole. The agreement for tensor and $V_+$ vector modes with numerical data is very good, while for $V_-$ the agreement is worse. It is general for WKB approach that the more multi-pole index $l$, the better accuracy it provides.

Table IV. The fundamental ($n = 0$) quasinormal frequencies, $l = 2$, for vector and tensor perturbations of 5-dimensional RN black hole. *ext is near extremal value of $Q$ at which the QN mode converge to some its limiting value (usually it is $Q = 0.999999M$ or closer to 1 M).

| Q/M | tensor-type | vector-type $V_-$ | vector-type $V_+$ |
|-----|-------------|------------------|------------------|
| 0.2 | 1.07084 - 0.25259 i | 0.80166 - 0.23717 i | 1.04554 - 0.24930 i |
| 0.4 | 1.07942 - 0.25139 i | 0.80271 - 0.23585 i | 1.06791 - 0.24927 i |
| 0.5 | 1.08623 - 0.25028 i | 0.80363 - 0.23471 i | 1.08579 - 0.24898 i |
| 0.7 | 1.10613 - 0.24608 i | 0.80643 - 0.23094 i | 1.13963 - 0.24668 i |
| 0.8 | 1.12007 - 0.24209 i | 0.80816 - 0.22799 i | 1.17960 - 0.24329 i |
| 0.99 | 1.15571 - 0.22565 i | 0.81072 - 0.22002 i | 1.30465 - 0.21604 i |
| ext | 1.15786 - 0.22424 i | 0.81082 - 0.21953 i | 1.31425 - 0.21213 i |

Table V. The fundamental ($n = 0$) quasinormal frequencies, $l = 2$, for vector and tensor perturbations of 6-dimensional RN black hole. *ext is near extremal value of $Q$ at which the QN mode converge to some its limiting value.

| Q/M | tensor-type | vector-type $V_-$ | vector-type $V_+$ |
|-----|-------------|------------------|------------------|
| 0.2 | 1.59863 - 0.39798 i | 1.2255 - 0.38118 i | 1.57851 - 0.39361 i |
| 0.4 | 1.60531 - 0.39551 i | 1.22485 - 0.37859 i | 1.60416 - 0.39225 i |
| 0.5 | 1.61054 - 0.39333 i | 1.22429 - 0.37596 i | 1.62477 - 0.39075 i |
| 0.7 | 1.62528 - 0.38578 i | 1.2237 - 0.37109 i | 1.68592 - 0.38487 i |
| 0.8 | 1.63479 - 0.37950 i | 1.22077 - 0.36765 i | 1.72882 - 0.37972 i |
| 0.99 | 1.65469 - 0.36053 i | 1.21656 - 0.36048 i | 1.85359 - 0.34648 i |
| ext | 1.65572 - 0.35926 i | 1.21633 - 0.36003 i | 1.86325 - 0.34165 i |

Table VI. The fundamental ($n = 0$) quasinormal frequencies, $l = 2$, for vector and tensor perturbations of 7-dimensional RN black hole. *ext is near extremal value of $Q$ at which the QN mode converge to some its limiting value.

| Q/M | tensor-type | vector-type $V_-$ | vector-type $V_+$ |
|-----|-------------|------------------|------------------|
| 0.2 | 2.10134 - 0.53092 i | 1.65365 - 0.509963 i | 2.08732 - 0.52520 i |
| 0.4 | 2.10623 - 0.52798 i | 1.65193 - 0.506922 i | 2.11625 - 0.52210 i |
| 0.5 | 2.11012 - 0.52525 i | 1.65054 - 0.504657 i | 2.14023 - 0.51872 i |
| 0.7 | 2.12073 - 0.51570 i | 1.64641 - 0.498804 i | 2.21054 - 0.50807 i |
| 0.8 | 2.12678 - 0.50825 i | 1.64359 - 0.495383 i | 2.24964 - 0.50566 i |
| 0.99 | 2.13650 - 0.48935 i | 1.63746 - 0.488578 i | 2.32953 - 0.50006 i |
| ext | 2.13692 - 0.48824 i | 1.63714 - 0.488215 i | 2.34040 - 0.494896 i |
Table VII. The fundamental \((n = 0)\) quasinormal frequencies, for vector and tensor perturbations of 5-dimensional extremal RN black hole.

5 Appendix. Quasinormal modes of \(D = 4\) RN BH

| Q/M  | numerical       | third order WKB          | six order WKB          |
|------|-----------------|--------------------------|------------------------|
| 0.2  | 0.46296 - 0.09537 i | 0.46251 - 0.09543 i | 0.46296 - 0.09538 i    |
| 0.4  | 0.47993 - 0.09644 i | 0.47949 - 0.09649 i | 0.47992 - 0.09645 i    |
| 0.5  | 0.49368 - 0.09719 i | 0.49325 - 0.09723 i | 0.49367 - 0.09720 i    |
| 0.7  | 0.53651 - 0.09877 i | 0.53613 - 0.09879 i | 0.53651 - 0.09878 i    |
| 0.8  | 0.57013 - 0.09907 i | 0.56978 - 0.09908 i | 0.57013 - 0.09908 i    |
| 0.99 | 0.69275 - 0.08864 i | 0.69243 - 0.08863 i | 0.69275 - 0.08864 i    |
| **ext** | 0.70430 - 0.085973 i | 0.70398 - 0.08596 i | 0.704305 - 0.085972 i  |

Table III. The fundamental quasinormal frequencies, \(l = 2, n = 0, Z_+\) 4-dimensional RN black hole; \(*ext\) is near extremal value of \(Q\) at which the QN mode converge to some its limiting value (usually it is \(Q = 0.999999M\) or closer to 1 M). The numerical results for this limiting value is taken from the paper of H.Onozawa et. al. [25].

| Q/M  | numerical       | third order WKB          | six order WKB          |
|------|-----------------|--------------------------|------------------------|
| 0.2  | 0.37475 - 0.08907 i | 0.37423 - 0.08933 i | 0.37469 - 0.08900 i    |
| 0.4  | 0.37844 - 0.08940 i | 0.37792 - 0.08965 i | 0.37838 - 0.08933 i    |
| 0.5  | 0.38168 - 0.08916 i | 0.38115 - 0.08985 i | 0.38162 - 0.08954 i    |
| 0.7  | 0.39250 - 0.08990 i | 0.39191 - 0.09009 i | 0.39248 - 0.08982 i    |
| 0.8  | 0.40122 - 0.08964 i | 0.40053 - 0.08978 i | 0.40125 - 0.08953 i    |
| 0.99 | 0.42930 - 0.08427 i | 0.42810 - 0.08431 i | 0.42955 - 0.08396 i    |
| **ext** | 0.43134 - 0.083460 i* | 0.43013 - 0.08349 i | 0.431612 - 0.083139 i  |

Table III. The fundamental quasinormal frequencies, \(l = 2, n = 0, Z_-\) 4-dimensional RN black hole; \(*ext\) is near extremal value of \(Q\) at which the QN mode converge to some its limiting value. The numerical results for this limiting value is taken from the paper of H.Onozawa et. al. [25].

6 Discussion

We have been studied the gravitational quasinormal frequencies for higher dimensional S, SdS, and SAdS black holes. The quasinormal behavior crucially different in these three cases: for \(\Lambda = 0\) the quasinormal modes are inversely proportional to the black hole size (the horizon radius), the presence of \(\Lambda > 0\) leads to decreasing of the real oscillation frequency and to increasing of the damping time. The presence of \(\Lambda < 0\) lead to totally
different behavior: the quasinormal modes of large and intermediate AdS black holes are proportional to the black hole size (and thereby proportional to the temperature in this regime), and goes to pure AdS limit when the radius of the event horizon goes to zero [16]. In all these points the higher dimensional gravitational modes mimic the behavior of the four dimensional modes. Different features in quasinormal behavior of the higher dimensional black holes is connected with existence of the three types of perturbations: scalar, vector and tensor types of gravitational modes. These three types of perturbations excite three different spectra of quasinormal modes. In particular, the more the spin weight of the corresponding harmonics in which the perturbations are expanded, the greater the real oscillation frequencies and the greater the damping rate. Thus that is the scalar-type modes which damp most slowly and thereby must dominate at later stages of quasinormal ringing. And this is the case of charged background, and also asymptotically de-Sitter or Anti-de-Sitter backgrounds. All found here modes are damping what supports the stability of the above black holes.

Unfortunately, with the help of existent semi-analytical methods, we are not able to find the quasinormal modes of scalar-type gravitational perturbations for a full range of parameters, since the corresponding effective potentials have negative pitch near the black hole horizon. Nevertheless in some of these cases (such as the case of large $D$) one can anticipate the quasinormal behavior.

Another interesting step in the direction of investigating of gravitational radiation from higher dimensional black holes is the considering the quantum effects such as hawking radiation (see [27] and references therein).

## 7 Acknowledgments

I wish to thank Akihiro Ishibashi for stimulating discussions.

### References

1. A.Ishibashi and H.Kodama, [hep-th/0305147](http://arxiv.org/abs/hep-th/0305147) (2003) (see also A.Ishibashi and H.Kodama, [hep-th/0004160](http://arxiv.org/abs/hep-th/0004160) (2000))

2. A.Ishibashi and H.Kodama, [hep-th/0305185](http://arxiv.org/abs/hep-th/0305185) (2003)

3. A.Ishibashi and H.Kodama, [hep-th/0308128](http://arxiv.org/abs/hep-th/0308128) (2003)

4. R.A.Konoplya, Phys. Lett. A268, 37 (2000)

5. R. A. Konoplya, Phys.Rev D 68, 024018 (2003)

6. G.Gibbons and S.Hartnoll, Phys.Rev D 67, 064024 (2002)

7. G. T. Horowitz and V. E. Hubeny Phys. Rev. D62, 024027 (2000).

8. S. Hod, Phys. Rev. Lett. 81, 4293 (1998); O. Dreyer, Phys. Rev. Lett. 90, 081301 (2003);
[9] S. F. J. Chan and R. B. Mann, Phys. Rev. D 55, 7546 (1997);
V. Cardoso and J. P. S. Lemos, Phys. Rev. D 63, 124015 (2001);
D. Birmingham, I. Sachs and S. N. Solodukhin, Phys. Rev. Lett. 88, 151301 (2002);
W. T. Kim and J. J. Oh, Phys. Lett. B 514, 155 (2001);
R. A. Konoplya, Phys. Rev. D 66, 084007 (2002);
S. Musiri and G. Siopsis, hep-th/0301081
E. Berti, K. D. Kokkotas, hep-th/0303029
R. A. Konoplya, Phys. Lett. B 550, 117 (2002);
R. A. Konoplya, Gen. Relativ. Grav. 34, 329 (2002);
S. Fernando, hep-th/0306214
D. Birmingham, hep-th/0306004 (2003)
J. Oppenheim, gr-qc/0307089 (2003)
[10] S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. 87, 161602 (2001);
[11] V. Cardoso, O. J. C. Dias and J. P. S. Lemos, Phys. Rev. D (in press) hep-th/0212168
[12] D. Ida, Y. Uchida and Y. Morisawa, Phys. Rev. D 67 084019 (2003)
[13] K. Berti, K. D. Kokkotas and E. Papantonopoulos, gr-qc/0306106
[14] C. Molina, gr-qc/0304053 (2003)
[15] L. Motl and A. Neitzke, hep-th/0301173 (2003)
[16] R. A. Konoplya, Phys. Rev. D 66, 044009 (2002).
[17] B. F. Schutz and C. M. Will Astrophys. J. Lett 291 L33 (1985); S. Iyer and C. M. Will, Phys. Rev. D 35 3621 (1987)
[18] O. B. Zaslavskii, private communication
[19] A. Ishibashi and T. Tanaka, gr-qc/0208006 (2002)
[20] I. G. Moss and J. P. Norman, Class. Quant. Grav. 19, 2323-2332 (2002).
[21] V. Cardoso and J. P. S. Lemos, Phys. Rev. D (in press), gr-qc/0301078
[22] A. Zhidenko, gr-qc/0307012 (2003)
[23] A. O. Starinets, hep-th/0207133
D. T. Son, A. O. Starinets, J.H.E.P. 0209:042, (2002);
A. Nunez and A. O. Starinets, Phys.Rev.D67, 124013, (2003);
[24] V. Cardoso, R. A. Konoplya and J. P. S. Lemos, Phys. Rev. D in press, gr-qc/0305037
[25] H. Onozawa, T. Mishima, T. Okamura, and H. Ishihara, *Phys. Rev.* D53 7033 (1996); [gr-qc/9603021]

[26] H. Onozawa, T. Okamura, T. Mishima, and H. Ishihara, *Phys. Rev.* D55 4529 (1997)

[27] P. Kanti, J. March-Russell, *Phys. Rev.* D67:104019, 2003 e-Print Archive: [hep-ph/0212199]; P. Kanti, J. March-Russell, *Phys. Rev.* D66:024023, 2002 e-Print Archive: [hep-ph/0203223]; C. M. Harris, Panagiota Kanti, e-Print Archive: [hep-ph/0309054].