Thermally driven classical Heisenberg chain with a spatially varying magnetic field: thermal rectification and negative differential thermal resistance

Debarshee Bagchi
Condensed Matter Physics Division, Saha Institute of Nuclear Physics, Bidhan nagar, Kolkata, West Bengal 700064, India
E-mail: debarshee.bagchi@saha.ac.in

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Abstract. Thermal rectification and negative differential thermal resistance are two important features that have direct technological relevance. Here, we study the classical one-dimensional Heisenberg model, driven thermally by heat baths attached at the two ends of the system and in the presence of an external magnetic field that varies monotonically in space. Heat conduction in this system is studied using a local energy conserving dynamics. It is found that by suitably tuning the spatially varying magnetic field, the structurally homogeneous symmetric system exhibits both thermal rectification and negative differential thermal resistance. Thermal rectification, in some parameter ranges, shows interesting dependencies on the average temperature $T$ and the system size $N$—rectification improves as $T$ and $N$ are increased. Using the microscopic dynamics of the spins we present a physical picture to understand thermal rectification as exhibited by this system and provide supporting numerical evidence. Emergence of the negative response in this system can be controlled by tuning the external magnetic field alone, which can have possible applications in the fabrication of novel thermal devices.

Keywords: transport processes/heat transfer (theory), heat conduction
1. Introduction

Thermal rectification (TR) [1, 2] is an important property that has been extensively studied in a variety of nonlinear systems [3–12] in recent times. A thermally driven system can be so designed such that the thermal current through the system has unequal values when direction of thermal bias is reversed—the heat conduction is asymmetric. Thus the system behaves as a good heat conductor in one direction and a good insulator in the opposite direction. Thermal rectification owes its origins to the nonlinearity of the system and to its spatial asymmetry. Analogous to its electrical counterpart, the thermal rectifier is considered to be a crucial building block and therefore has an important role to play in the fabrication of sophisticated thermal devices.

Negative differential thermal resistance (NDTR) [1, 2] is a counterintuitive phenomenon, predicted in heat conduction studies, where the steady state thermal current decreases as the temperature difference across a system is increased. In recent decades, much attention has been devoted to studying NDTR in nonlinear lattices. However, in spite of enormous efforts, the underlying physical mechanism that generates NDTR in nonlinear systems is still not satisfactorily understood. Several mechanisms have been proposed and many unresolved questions regarding the emergence of NDTR, such as the mismatch of the phonon bands [13, 14], role of interface [15], ballistic-diffusive transport [16, 17], role of momentum conservation [18] presence of a critical system size and a transition from the exhibition to the nonexhibition of NDTR [19] and scaling [20] are still being explored. NDTR is considered to be of immense technological importance in the working of recently proposed thermal devices such as thermal transistors [13], thermal logic gates [14], thermal memory elements [21] etc. Theoretical studies using mostly numerical simulation have been employed extensively to study both these features in many nonlinear lattice systems, few examples are the Frenkel–Kontorova model [6, 19], the φ^4 model [15], the Fermi–Pasta–Ulam chain [7, 18], the Morse lattice [5].

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Linear systems, such as coupled harmonic oscillators, do not exhibit TR or NDTR. Surprisingly, it has been recently found that linear graded systems, such as a harmonic oscillator chain with linearly increasing masses, show both TR and NDTR [8, 9]. In fact, gradual mass-loaded carbon and boron nitride nanotubes have already been used effectively to fabricate a thermal rectifier [22]. These functional graded materials have been considered to be of huge technological relevance since these materials can be purposely manufactured and have many intriguing optical, electrical, mechanical and thermal properties [8].

Motivated by the concept of these functional graded materials, in this paper, we study thermal transport in the classical Heisenberg model [23, 24] coupled to heat baths and in the presence of a spatially varying magnetic field and investigate TR and NDTR. Both of these features have been shown to emerge in the Heisenberg spin chain previously [25] but by a different approach. In this paper, we present a new route to obtain these features.

Investigation of TR and NDTR in spin systems has been recently carried out only in a few other works such as the two-dimensional classical Ising model [26] and quantum spin systems [27]. These systems are quite simple and although they are helpful in understanding the underlying physical mechanism, nevertheless they are not very realistic. The Heisenberg model is a comparatively more realistic spin model for a magnetic insulator. Also, in both cases the system under consideration consists of two dissimilar segments coupled to each other. This scheme requires one to carefully fabricate the junction as it has been argued that TR and NDTR are crucially dependent on the junction properties [17, 19] which is difficult to implement in real systems. It was also initially believed that NDTR cannot be obtained in a symmetric system [28]. However later studies clearly showed that NDTR can be obtained from systems even without structural inhomogeneity [20,29].

The advantage of our proposal is that it is much simpler to implement, easy to manipulate over a wide range and should also be realizable in practice. Firstly, one does not need to specially design the system, unlike the case of two segment nonlinear lattices (with an interface) or graded systems mentioned above—one has to fabricate such a system with precise structural parameters which might be technologically more challenging and restrictive in applicability. Secondly, using spin systems one can control TR and NDTR over a wide range by tuning only an external magnetic field and so, in contrast to previous works, no special engineering of the system is required in our case. TR in this system also shows interesting anomalous dependences, as we shall discuss, that can be of technological relevance.

The organization of the rest of the paper is as follows. In the next section 2 we describe our model and the numerical scheme employed to study the system. Thereafter we present our results in section 3. Finally, in section 4 we conclude with a brief summary of our results and a discussion.

2. Model and numerical scheme

Consider classical Heisenberg spins \( \{ \vec{S}_i \} \) (three-dimensional unit vectors) on a one-dimensional lattice of length \( N \) \((1 \leq i \leq N)\) with nearest neighbor interaction.
The Hamiltonian of the system is
\[ \mathcal{H} = -K \sum_{i=1}^{N-1} \vec{S}_i \cdot \vec{S}_{i+1} - \sum_{i=1}^{N} \vec{h}_i \cdot \vec{S}_i \]  
where the spin–spin interaction is taken to interact ferromagnetically \( K > 0 \) (we have set \( K \) to unity for our results without any loss of generality). The second term in the equation (1) is due to a spatially varying magnetic field \( \vec{h}_i \) that acts on all the spins. The equation for the time evolution of the spin vectors can be written as
\[ \frac{d}{dt} \vec{S}_i = \vec{S}_i \times \vec{B}_i \]  
where \( \vec{B}_i = \vec{S}_{i-1} + \vec{S}_{i+1} + \vec{h}_i \) (with \( K = 1 \)) is the local molecular field experienced by the \( i \)th spin vector.

To drive the system out of equilibrium, we couple the ends of the system to two heat baths. This is implemented by introducing two additional spins at sites \( i = 0 \) and \( i = N+1 \). The bonds between \((\vec{S}_0, \vec{S}_1)\) and \((\vec{S}_N, \vec{S}_{N+1})\) at two extreme ends of the system behave as stochastic thermal baths. The left and right thermal baths are in equilibrium at their respective temperatures \( T_l \) and \( T_r \) and have Boltzmann distribution \( P(E) \sim \exp(-E/T) \). The average energies of the two extreme bonds read \( \langle E_i \rangle = -\mathcal{L}(T_l^{-1}) \) and \( \langle E_i \rangle = -\mathcal{L}(T_r^{-1}) \), \( \mathcal{L}(x) = \coth(x) - 1/x \) being the standard Langevin function.

We investigate the steady state transport properties of the Heisenberg model by numerically computing the steady state thermal current using the discrete time odd even (DTOE) dynamics \([25,30]\). The DTOE dynamics alternately update the spins belonging to the odd and even sites of the lattice using a spin precession dynamics given by
\[ \vec{S}_{i,t+1} = \left[ \vec{S} \cos \phi + (\vec{S} \times \vec{B}_t) \sin \phi + (\vec{S} \cdot \vec{B}_t) \vec{B}_t (1 - \cos \phi) \right]_{i,t} \]  
where \( \vec{B}_t = \vec{B}_i/|\vec{B}_i|, \phi_i = |\vec{B}_i| \Delta t \) and \( \Delta t \) is the time-step increment \([30]\).

Numerically, the leftmost spin \( \vec{S}_0 \) is updated with the even spins and the rightmost spin \( \vec{S}_{N+1} \) is updated with the odd (even) spins for even (odd) \( N \). The bond energy between \( \vec{S}_0 \) and \( \vec{S}_1 \) is refreshed from a Boltzmann distribution and thereafter the spin \( \vec{S}_0 \) is reconstructed using the relation \( E_0 = -\vec{S}_0 \cdot \vec{S}_1 \). Note that during this update \( \vec{S}_1 \) is not modified (as it belongs to the odd sublattice). Similarly, the other end is also updated. This sets the temperatures of the two ends of the lattice to our desired values. A thorough discussion of the DTOE scheme and numerical implementation of the thermal baths can be found in \([30]\).

The computation of the steady state thermal current is done as described in the following. The energy of the \( i \)th bond \( E^o_i \) measured after the odd spin update is not equal to \( E^e_i \) measured after the subsequent even spin update, where \( E_i = -\vec{S}_i \cdot [\vec{S}_{i+1} + \vec{h}_i] \) is the energy density. This difference \( (E^e_i - E^o_i) \) is the measure of the energy crossing the \( i \)th bond in time \( \Delta t \) (we set \( \Delta t \) to unity \([30]\)). The steady state thermal current \( j \) (rate of flow of energy) is site independent and is computed in this scheme \([25,30]\) using
\[ j = \langle E^e_i - E^o_i \rangle \]  
Note that equation (4) is consistent with the definition of current obtained from the continuity equation \([30]\). We define a total current \( J = jN \) and all the results obtained are presented below in terms of this total current.

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where $\alpha$ thus the average temperature of the system is roughly linearly for $R$ system exhibits TR. $\tilde{h}_i$ is chosen as $(0,0,h_0)$ and $h_0 = h_0 + \alpha i/N$ in space. (a) Variation of the total thermal current $J$ with $\Delta$ in the range $-1 < \Delta < 1$ for different values of the parameter $\alpha$. The parameters used are $T = 1.0$ and system size $N = 500$. (b) Variation of the rectification ratio $R_\Delta$ (see text). The parameters used are $T = 1.0$ and system size $N = 500$.

3. Results

3.1. Thermal rectification

The temperature of the two thermal baths are set as $T_l = T(1 + \Delta)$ and $T_r = T(1 - \Delta)$, thus the average temperature of the system is $\frac{1}{2}(T_l + T_r) = T$. The spatially varying magnetic field $\tilde{h}_i$ is chosen as $(0,0,h_0^z)$ and $h_0^z = h_0 + \alpha i/N$ is a linearly varying field where $1 \leq i \leq N$; we set $h_0 = 1$ for all our results in this section. Starting from a random initial configuration of spins we let the system evolve using the DTOE dynamics until a steady state is reached and thereafter compute the thermal current using equation (4). For different $\alpha$ values of the thermal bias $\Delta$ we compute $R_\Delta$ and that in the backward bias $J_{-\Delta}$ are different in magnitude as can be seen from figure 1(a). We consider the system to be in forward bias for $\Delta > 0$ and in backward bias for $\Delta < 0$. The thermal current under the forward bias $J_\Delta$ and that in the backward bias $J_{-\Delta}$ are different in magnitude as can be seen from figure 1(a). Note that the system is structurally symmetric and homogeneous. The asymmetric heat conduction is completely brought about by the spatially varying magnetic field. We define the rectification ratio as $R_\Delta = |J_{-\Delta}/J_\Delta|$ which measures the amount of TR achieved. Thus for poor rectification $R_\Delta$ is close to unity and for good rectification $R_\Delta$ is very large (small) if $J_{-\Delta} \gg J_\Delta$ and $J_{-\Delta} \ll J_\Delta$. From figure 1(b), as expected $R_\Delta$ is found to increase as $\alpha$ is increased i.e. when the magnetic field varies more sharply across the system (apart from some discrepancies for large $\Delta$). Thus heat conduction is asymmetric, i.e. $J_\Delta \neq J_{-\Delta}$ and the system exhibits TR.

An interesting feature of TR in this system is the variation of the rectification ratio $R_\Delta = |J_{-\Delta}/J_\Delta|$ with the parameter $\alpha$. The rectification ratio does not increase indefinitely as $\alpha$ is increased but rather shows an intriguing nonmonotonic $\alpha$ dependence. For different values of the thermal bias $\Delta$ we compute $R_\Delta$ as a function of $\alpha$ as shown in figure 2(a). For $\alpha$ in the range $0 < \alpha \lesssim \alpha_0$, we find $R_\Delta > 1$ initially whereas for $\alpha \gtrsim \alpha_0$, $R_\Delta < 1$, where $\alpha_0$ lies roughly in the range $3.5 < \alpha \lesssim 4.0$ (figure 2(a)). For small $\Delta$, $R_\Delta$ increases roughly linearly for $\alpha < \alpha_0$, then drops abruptly below $R_\Delta = 1$ and then increases linearly towards unity again. For larger $\Delta$, $R_\Delta$ has an even more complicated nonmonotonic $\alpha$ dependence but it jumps from $R_\Delta > 1$ to $R_\Delta < 1$ at the same $\alpha = \alpha_0$.

*Alternatively, one can define the rectification ratio as $R_\Delta = \max |J_{\Delta}, J_{-\Delta}|/\min |J_{\Delta}, J_{-\Delta}|$, so that $R \geq 1$ always.*
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Figure 2. Variation of the rectification ratio $R_\Delta$ with $\alpha$ for different values of (a) thermal bias $\Delta$ for $T = 1.0$ and $N = 500$; (b) temperature $T$ with $\Delta = 0.5$ and $N = 500$; (c) system size $N$ with $T = 1.0$ and $\Delta = 0.5$.

We look into the temperature and size dependencies of $R_\Delta$ which are also very unusual. Generally rectification is found to deteriorate as the average temperature and the system size are increased [1,2,25,31]. However, in our case, TR for higher temperature in certain ranges of $\alpha$ is actually higher than that for lower temperature as can be seen in figure 2(b). Also note that $\alpha_0$ shifts to higher $\alpha$ values as the average temperature is increased. Similar nontrivial dependence is seen when one studies the variation of $R_\Delta$ with the system size $N$. In some $\alpha$ regime, $R_\Delta$ decreases as $N$ is increased whereas in some other regime we get an anomalous size dependence as can be seen from figure 2(c)—for $\alpha = 6.0$ the rectification ratio increases approximately by a factor of 1.65 when the system size is increased from $N = 100$ to $N = 1000$. A very recent study [31] on the size dependence of rectification in a mass-graded $\beta$-Fermi–Pasta–Ulam ($\beta$-FPU) oscillator system shows an increase in rectification for very large system sizes $N > 10^4$. In our case, by tuning the magnetic field suitably, the increase in rectification can be obtained for $N$ values even two orders of magnitude less than what has been reported for the $\beta$-FPU system mentioned above. For other functional forms of the spatially varying magnetic external magnetic field we speculate a substantial improvement in TR similar to [31]. Thus, depending on which $\alpha$ range one is in, the $T$ and $N$ dependencies can be normal ($R_\Delta$ approaching unity as $T$ and $N$ increases) or anomalous. These dependencies seem to be due to a complicated interplay of the imposed thermal bias and the spatially varying magnetic field.

To understand this, we look into the individual currents $J_\Delta$ and $J_{-\Delta}$. For relatively small values of the magnetic field the current $J$ is higher when it flows from a higher magnetic field region to a lower magnetic field region. This is due to the fact that the magnetic field tries to restrict the motion of the spins and thereby inhibits the flow of energy through the system. Now according to our definition, in the forward bias (figure 3(a)) the magnetic field increases as one approaches the colder bath—thus the motion of the spins nearer to the right end of the system is doubly restricted—one because of the low temperature and the other due to the higher magnetic fields. For the backward bias (figure 3(b)) however the effect of the higher magnetic field is somewhat compensated by the hotter bath and the spins are relatively more free to rotate in this case and therefore the system has a higher current. Since in the steady state the current through the system is a site independent constant (a consequence of the equation of continuity) the overall current of the system is dictated by the current carrying capacity of the weakest bond (corresponding to the most restricted spin) and therefore the current in the forward bias.
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Figure 3. Schematic diagram of the system in (a) forward and (b) backward bias conditions according to our definition. The horizontal line represents the spin chain attached between the two baths with bath temperatures $T_1$ and $T_2$ where $T_1 > T_2$. The vertical arrows represent the spatially varying magnetic field which grows monotonically as one moves from the left end of the system towards the right end.

Figure 4. (a) Variation of the individual currents $J_\Delta$ (filled) and $|J_{-\Delta}|$ (open) with $\alpha$ for two values of the thermal gradient $\Delta = 0.1$ and 0.9. Average temperatures $T = 1.0$. (b) Variation of $J_\Delta$ with $\alpha$ for different average temperatures $T = 0.5, 1.0$ and 2.0. Here $\Delta = 0.5$ and $N = 500$ in both the cases. (c) Variation of $J_\Delta$ (scaled by $N$) with $\alpha$ for different system size $N = 200, 500$ and 1000. Here $T = 1.0$ and $\Delta = 0.5$. For (b) and (c) $|J_{-\Delta}|$ has similar variation as in (a).

is lower than that in the backward bias. This explains why $|J_{-\Delta}| > J_\Delta$ for $\alpha < \alpha_0$, as can also be seen in figure 4(a) and the rectification ratio $R_\Delta > 1$; $R_\Delta$ increases in this region as $\alpha$ is increased because of increased asymmetry of the system. As the magnetic field increases, the current starts to decrease since the orientational stiffness of the spins increases which restricts energy passage through the system. As the magnetic field becomes high the system goes into a magnetic field dominated regime which limits the current carrying capacity of the system—the weaker current $J_\Delta$ still continues to decrease but eventually it too attains a saturation (figure 4(a)) (note that in figure 4 the $y$-axis is a logarithmic scale in all the figures).
Thermal rectification and NDTR

Figure 5. Variation of $J_\Delta$ and $J_{-\Delta}$ as a function of the system size $N$ for (a) $\alpha = 1.0$ and (b) $\alpha = 6.0$ with $T = 1.0$ and $\Delta = 0.5$.

For a lower temperature the spins of the system are more orientationally stiff and thus this domination of the magnetic field commences at a lower value of $\alpha$. The current saturates at lower $\alpha$ (figure 4(b)) and this explains the decrease of $\alpha_0$ as the average temperature is decreased in figure 2(b). With regards to the value of $\alpha_0$ there is no appreciable variation as the system size $N$ is altered as can be seen from figure 4(c) and also previously in figure 2(c). The system of smaller size is closer to the ballistic limit and carries slightly more current than a system of larger size [30].

The system is known to approach the Fourier (diffusive transport) regime as the temperature $T$ or the size $N$ is increased [30], but in the forward bias condition the approach is obviously slower than in the backward bias. This difference in the approach rates between the forward and the backward current becomes considerably large in presence of the high magnetic field and this is the reason that one obtains an improvement of rectification ($R_\Delta$ moves away from unity as in figures 2(b) and (c)) as $T$ or $N$ is increased in the $\alpha > \alpha_0$ regime. This disparity in the approach to the Fourier regime (where $J \equiv jN$ is independent of $N$, which implies that the conductivity is an intensive quantity) can be clearly observed in figure 5 as the system size $N$ is varied (the $T$ variation should be similar and is not shown here). For $\alpha = 1.0$, both $J_\Delta$ and $J_{-\Delta}$ approach the diffusive limit almost equally fast (figure 5(a)). But for larger magnetic fields $\alpha = 6.0$, $J_{-\Delta}$ reaches the diffusive regime much faster (figure 5(b)). The system size dependence of thermal rectification has been a matter of grave concern since its inception. Our system is one of the very few systems that shows an improvement in rectification as $N$ is increased and this could be technologically useful. Also, as we shall show in the following, it is the motion of the $N$th spin that decides the value of $\alpha_0$ which therefore is independent of the length of the system to which it belongs. This is why $\alpha_0$ remains essentially unchanged as the system size $N$ is altered and changes only when the average temperature $T$ is changed.

Thus to summarize, there are two regimes: (a) a temperature (or thermal bias) dominated regime in the parameter range $0 < \alpha < \alpha_0$ in which the spins are more orientationally free and the current steadily decrease as $\alpha$ increases and (b) a magnetic field dominated regime for $\alpha > \alpha_0$ in which the spins have a restricted motion and the current through the system changes negligibly as $\alpha$ is varied. It is the complicated interplay of these two mutually opposing factors (temperature and magnetic field) that gives rise to the interesting features that are observed in our system.

In order to further validate the picture just presented, we now look into the microscopic dynamics of the individual spins. Since the most restricted spin dictates the current...
(as discussed previously) we look into the $S_z$ component of the $N$th spin (since the $N$th spin experiences the largest magnetic field amongst all the spins) and show its distribution $P_N(S_z)$ for different values of the parameter $\alpha$ in figure 6(a). Since $S_z \equiv \cos \theta$ ($\theta$ is the polar angle and the spins being of unit magnitude), $S_z$ lies in the range $[-1, 1]$. For a spin which is completely free to orient itself in all possible directions, the distribution would be uniform in the range $[-1, 1]$. However in the presence of a finite temperature and an external magnetic field $P_N(S_z)$ has an exponential form. Note that as $\alpha$ increases, the slope of the distribution $\frac{d}{dS_z} \ln P_N(S_z)$ increases which signifies that the spin motion becomes more and more restricted. From the distribution, we compute the average $\langle S_z \rangle$ and the standard deviation $\sigma = (\langle S_z^2 \rangle - \langle S_z \rangle^2)^{-1/2}$ for the $N$th spin. These two quantities are shown in figures 6(b) and (c) respectively. The average $\langle S_z \rangle$ approaches unity as $\alpha$ increases both for the forward and backward currents and $\langle S_z \rangle_\Delta > \langle S_z \rangle_{-\Delta}$. The standard deviation $\sigma$ is an indicator of how freely a spin can rotate about the magnetic field. Thus, the larger the $\sigma$ is, the more the current $J$ that the spin allows to pass through. Figure 6(c) shows that $\sigma$ for forward and backward bias decreases as $\alpha$ increases although not always monotonically for all parameters. (For the chosen parameters in figure 6(c), $\sigma$ approximately fits to the form $\sigma^{-1} = m\alpha + c$, where $m$ and $c$ are constants—both $m$ and $c$ values are higher for $J_\Delta$ than $J_{-\Delta}$.)

To obtain a numerical estimate of $\alpha_0$ where the rectification ratio $R_\Delta = |J_{-\Delta}/J_\Delta|$ shows a jump and the current attains saturation, we compute the ratio of the two $\sigma$ values, $r_\Delta = \sigma_{-\Delta}/\sigma_\Delta$ and study its variation as a function of $\alpha$. It is found that $r_\Delta$ increases steadily till some $\alpha$ value in the range $3.5 < \alpha < 4.0$ and thereafter it attains
Figure 7. Variation of $r_\Delta = \sigma_- / \sigma_\Delta$ with $\alpha$ for (a) different $T = 1.0, 2.0$ with $\Delta = 0.5$ and $N = 500$; the arrows mark the onset of the magnetic field dominated regimes. (b) different system sizes $N = 100, 500$—here $\Delta = 0.5$ and $T = 1.0$; (c) different $\Delta = 0.5, 0.9$ with $T = 1.0$ and $N = 500$. The arrows in (a) mark the onset of the magnetic field dominated regimes.

Thus, thermal rectification as exhibited by this system, shows several intriguing features that have not been observed or investigated (at least not in the manner that we have studied here) in any of the cited works of rectification and can have many technological implications in the fabrication of thermal devices.

3.2. Negative differential thermal resistance

Next, we turn our attention to the emergence of NDTR in this system. Note that the current $J$ in a strictly non-decreasing function of $\Delta$ as has been obtained in figure 1. To
make this system exhibit NDTR, we keep the temperature of one bath fixed and change the temperature of the other bath; we set $T_l = T$ and $T_r = T - \Delta$. The magnetic field is chosen to be linearly varying in space $h_z = h_0 + \alpha i / N$ as in the previous section.

The variation of the total thermal current $J$ with $\Delta$ for different values of the parameter $\alpha$ is shown in figure 8(a). When $\alpha = 0$ i.e. with an uniform magnetic field throughout the system, the current $J$ sharply increases as $\Delta$ is increased and there is no NDTR (data not shown). As $\alpha$ is increased, the system exhibits NDTR for some nonzero value of $\alpha$. Thus by simply tuning the external magnetic field, one obtains NDTR in this system without the need to manipulate parameters of the system. For a fixed nonzero value of $\alpha$ one can also obtain NDTR by tuning $h_0$ as has been shown in figure 8(b). The physical mechanism that gives rise to NDTR in our system is the obstruction to the current by the magnetic field as has been discussed in detail in a previous work [25]. A similar trapping mechanism (which causes an effective caging in the phase space) is being thought to be responsible for the origin of such negative responses in nonequilibrium systems in recent studies [32, 33]. The generality and many other issues pertaining to such a mechanism, however, need to be explored in detail. Note that when the magnetic field is increased further (either by increasing $\alpha$ or $h_0$) the current becomes very small and the NDTR regime disappears.

The temperature dependence of NDTR is described in figure 9(a). It is seen that the point of emergence of NDTR $\Delta_m$ shifts to larger values of $\Delta$ as temperature increases. The value of the energy current increases too, as the temperature is increased. From the main figure we find that the $J \sim \Delta$ curves show an excellent data collapse when the axes are rescaled as $J/T^\nu$ and $(\Delta - \Delta_m)/T$; for the chosen set of parameters $\nu = 2.0$ and $\Delta_m$ is the point where the NDTR regime commences corresponding to the maximum value of current $J_m$.

As has been commonly seen in previous works, NDTR becomes more pronounced as the system size $N$ is decreased. Here, too, we find that the point of commencement of NDTR gradually shifts towards larger values of $\Delta$ as the system size is increased. This is depicted in figure 9(b). The decrease of the NDTR regime due to an increase in system size can however be compensated for by decreasing the temperature or increasing the magnetic field suitably. We have also verified the emergence of NDTR in this system for other spatial dependence of the magnetic field. With an exponentially varying magnetic

![Figure 8. Variation of the total thermal current $J$ with $\Delta$ for different values of (a) $\alpha$ with $h_0 = 2$ and (b) $h_0$ with $\alpha = 2$. The temperatures are chosen as $T_l = T$ and $T_r = T - \Delta$. For both panels $T = 0.5$ and $N = 500$.](image-url)
field of the form $h_z = h_0 \exp(\alpha i/N)$ we find a clear NDTR regime as $\alpha$ is increased from zero (data not shown).

4. Discussion

To summarize our main results, we have studied thermal rectification (TR) and negative differential thermal resistance (NDTR) in the one-dimensional classical Heisenberg model under thermal bias with a spatially varying magnetic field. Systematic analysis of TR with respect to system parameters reveals intriguing dependencies with respect to temperature and system size. For proper tuning of the magnetic field NDTR can also be observed. Thus both the features emerge and can be controlled by the external magnetic field unlike previous works where one had to prepare the system with specific structural parameters. Heat transport in a magnetic system assisted by classical spin waves was described several years ago [34] and has also been experimentally observed recently in yttrium iron garnet [35,36]. Transport studies in spin systems have also been of active experimental interest in recent times [37,38]. Actual chemical compounds [39–41] that mimic classical spin interactions, such as TMMC((CH$_3$)$_4$NMnCl$_3$) and DMMC((CH$_3$)$_2$NH$_2$MnCl$_3$), have already been known for quite some time now. Apart from carbon nanotubes which are considered suitable for fabricating thermal devices, this present work (and also [25]) suggests these spin materials to be another promising candidate. Hopefully, with the recent advancement in low dimensional experimental techniques, these theoretical predictions would be verified experimentally and lead to efficient thermal management in the future.

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