T-Flex: A fully flexure-based large range of motion precision hexapod

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A B S T R A C T

Six degree of freedom manipulation provides full control over position and orientation, essential for many applications. However, six degree of freedom parallel kinematic manipulators (e.g. hexapods) either have a limited range of motion combined with a good repeatability when comprising flexure joints, or they have limited repeatability with a large workspace when using traditional rolling- or sliding-element bearings. In this paper, the design and optimization of a fully flexure-based large range of motion precision hexapod robot is presented. The flexure joints have been specifically developed for the purpose of large range of motion and high support stiffness for this manipulator. The obtained system allows for ±100 mm of translational and more than ±10° of rotational range of motion in each direction combined with a footprint of 0.6 m² and a height of 0.4 m. Furthermore, a dedicated flexure-based design for the actuated joints combines high actuation forces with the absence of play and friction, allowing for accelerations exceeding 10 g. Experiments on a prototype validate the sub-micron repeatability, which is merely limited by the selected electronics.

1. Introduction

Various applications require six degree of freedom manipulators. These systems are used in industrial applications such as micro assembly robots, welding robots, vibration-isolation platforms, pick-and-place robots and optical alignment systems. For this purpose, often open kinematic serial robot configurations are used [1]. These serial manipulators are used for their large workspace with respect to their footprint. However, due to the open kinematic structure, errors at joints are accumulated and amplified at the end-effector, typically allowing for a repeatability up to about 0.01 mm [2–4]. Furthermore, due to the serial placement of the actuators, serial robots exhibit large moving masses, resulting in limitations on the maximum accelerations that can be achieved.

In contrast, parallel kinematic configurations consist of six independent kinematic chains connected to the end-effector. This allows for the placement of the actuators at the stationary base, resulting in a low moving mass allowing for high accelerations. From a kinematic point of view, parallel robots also allow for a higher repeatability as errors at the joints are averaged, instead of added cumulatively.

High precision parallel kinematic robots comprising traditional bearings allow for a repeatability up to a few tenths of a micrometer [5–7]. To improve the precision of parallel manipulators, it is required to eliminate sources of play and friction in the joints and actuators. Therefore, the traditional bearings are often replaced by flexure-based equivalents to enable motion, which rely on elastic deformation of slender elements, instead of tribological contacts [5,8]. However, state-of-the-art parallel manipulators comprising flexure joints suffer from a limited range of motion below 10 mm of translational travel range. Furthermore, high precision hexapods often rely on walk-drive-type or inertia-drive-type piezoelectric actuators, potentially suffering from wear, limiting the maximum actuation forces and restricting the maximum travel speed to about 10 mm/s [9,10].

Following recent developments in large stroke flexure joints and optimization strategies [11–17], it is anticipated that a flexure-based hexapod with a larger travel range, faster travel speed, higher load capacity and high repeatability can be realized. This paper considers the design and realization of a fully flexure-based six degree-of-freedom parallel manipulator with a large range of motion. A challenging combination of requirements is specified in order to realize a system with unprecedented performance. Flexures allow a competitive repeatability level of <0.1 μm, while the range of motion is aimed at a workspace of 100 × 100 × 100 mm³. Furthermore, high accelerations of at least 50 m/s² are targeted throughout the workspace, with a maximum speed of at least 1 m/s. Lastly, a maximum vertical load of 10 kg is specified combined with a mechanism volume below 250 dm³ (defined by the volume of the smallest enclosing cylinder). Compared to existing flexure-based precision hexapods, this represents an increase in travel
range of more than a factor ten with an unprecedented travel speed up to two orders of magnitude higher. An overview of the specifications of five state-of-the-art precision hexapods, a preview of the realized specifications for the T-Flex and the initially targeted specifications for the T-Flex are provided in Table 1.

In this paper, the design and optimization of a fully flexure-based large range of motion hexapod is presented. First, in section 2, a brief description of the hexapod layout is provided. Next, in section 3, an optimization strategy for the hexapod and the optimized design for each individual flexure-based joint is given. The optimization results for the hexapod are presented in section 4 and a design for the hexapod, based on the optimization results, is given in section 5. An analytic evaluation of the performance of the hexapod throughout the workspace is given in section 6. Lastly, a prototype and experimental validation is provided in section 7 and 8.

2. Hexapod layout

Various layouts for six degree of freedom parallel manipulators are described in literature [1], consisting of a combination of prismatic, rotational, universal and spherical joints. For the layout of the hexapod described in this paper, stationary single degree of freedom actuators positioned at the base are preferred to obtain low moving mass. Furthermore, the absence of cables attached to moving parts of the system prevents parasitic bending forces, potentially negating precision. A rotary direct drive actuator is selected as this proved to provide a more compact solution, especially when considering the required large range of motion. The design of the actuator including the flexure-based suspension is discussed in more detailed in Ref. [17] and in section 3.2.

For releasing the remaining five degrees of freedom of each arm of the hexapod, a passive universal and spherical joint is used connected to each other via an lower and upper arm, resulting in a 6-RUS type hexapod layout. The design of the spherical and universal joints consist of a folded leavespring based joint design, described in more detail in Refs. [16,18]. A schematic overview of the 6-RUS layout is provided in Fig. 1.

The range of motion of this system can be increased by choosing the 6-RSS hexapod layout instead. For this layout, the universal joints are replaced by spherical joints. In that case, the rotational motion of the upper-arm around its longitudinal axis is distributed over both spherical joints, resulting in an increase of the workspace mainly in the rotational direction around the vertical z-axis. However, this increased range of motion comes at the cost of internal (underconstrained) degrees of freedom of the upper arms. Since flexures provide small stiffness in their degrees of freedom, this underconstrained body can impose unwanted (low frequency) internal vibrations which may jeopardize controller bandwidth and positioning accuracy of the system.

For the spherical and universal joints of the hexapod, identical designs are chosen, with the exception of a single additional flexure for the universal joints to constrain one of the rotational degrees of freedom. As this additional flexure in the universal joint mainly affects the rotational stiffness at the upper arm along its longitudinal axis (the stiffness contributing to the internal eigenfrequency) without affecting other stiffness properties of the system, the behavior of a 6-RUS and a 6-RSS layout can be considered similar. The 6-RSS layout is selected for the optimization of the hexapod described in section 3, which can easily be converted to a 6-RUS layout.

3. Optimization

The pose-dependent stiffness and limited range of motion of flexure joints result in complex mechanical behavior of flexure-based systems [12,19,20], especially when considering a large range of motion. Therefore, manually designing a flexure-based hexapod is not straightforward. To overcome this, an optimization algorithm is used to optimize the design.

3.1. Hexapod optimization strategy

For the optimization, the relevant mechanical behavior is simulated in the most critically deflected configurations of the system. These critical configurations are given by the eight extrema of the ±50 mm of travel range in each degree of freedom (e.g. x: ±50 mm, y: ±50 mm, z: ±50 mm). However, including the full elastic behavior of all 18 flexure-based joints is not directly feasible. Therefore, a simplified linearized version is used. In this simplified version, the stiffness of the flexures is modeled as shown in Refs. [12,13,19,20], resulting in a system of the following equations.

\[
\begin{bmatrix}
\mathbf{K}_0 + \mathbf{K}_f(x) & \mathbf{K}_f(x) \\
\mathbf{K}_f(x) & \mathbf{K}_f(x)
\end{bmatrix}
\begin{bmatrix}
\mathbf{q} \\
\mathbf{q}
\end{bmatrix}
= \mathbf{K}_f(x)
\end{equation}

where \(\mathbf{K}_0\) is the initial stiffness matrix, \(\mathbf{K}_f(x)\) is the stiffness matrix resulting from the internal force vector due to flexures, \(\mathbf{q}\) is the displacement vector, and \(\mathbf{q}\) is the internal force vector. The optimization problem is then formulated as

\[
\min_{\mathbf{q}} \mathbf{q}^T \mathbf{K}_f(x) \mathbf{q}
\]

subject to

\[
\begin{aligned}
\mathbf{q} &\leq \mathbf{q}_{\text{max}} \\
\mathbf{q} &\geq \mathbf{q}_{\text{min}}
\end{aligned}
\]

where \(\mathbf{q}_{\text{max}}\) and \(\mathbf{q}_{\text{min}}\) are the maximum and minimum displacement vectors, respectively.

Table 1

| Manufacturer       | H-845.D51 | F-206.S | SmarPod110 | Commander6 | HH-30D | T-Flex University of Twente | T-Flex target specifications |
|--------------------|-----------|---------|------------|------------|--------|----------------------------|----------------------------|
| Flexure-based      | No        | Yes     | No         | Yes        | Yes    | Yes                        | Yes                        |
| Travel range X     | mm ±110   | ±8 to +6| 20         | ±4.5       | ±30    | ±100                       | > ±50°                     |
| Travel range Y     | mm ±110   | ±5.7    | 20         | ±4.5       | ±30    | ±105                       | > ±50°                     |
| Travel range Z     | mm ±50    | ±5.7    | 10         | ±3 to 2.1  | ±8     | ±100                       | > ±50°                     |
| Travel range \(\theta_1\) | deg ±15 | ±5.7    | 21         | ±3.9       | ±10    | ±21.5                      | > ±10                      |
| Travel range \(\theta_2\) | deg ±15 | ±6.6    | 24         | ±3.4       | ±10    | ±21.5                      | > ±10                      |
| Travel range \(\theta_3\) | deg ±30 | ±5.5    | 38         | ±7.3       | 360    | ±18                        | > ±10                      |
| Repeatability      | \(\mu\) m 2 – 10 | 0.3    | 0.2        | 0.05      | ±0.1   | 0.35 RMS                    |
| Max. speed         | mm/s 50  | 10      | 10         | n.a.      | 15-50  | 4000°                       |
| Max. vertical load | kg 400    | 1.5     | 0.25       | 0.1       | 5      | 20                         |
| Build volume       | dm³ 293   | 7.9     | 0.4        | 1.1       | n.a.   | 227                        |

1) Simultaneous travel range, showing the maximum travel range where all other axes are also at the maximum travel range.
2) Merely limited by the maximum travel range and maximum acceleration.
based joints over the full large range of motion would imply high complexity and large computational costs that renders optimization impractical. Furthermore, the high number of design variables involved will result in problems with respect to convergence to the optimal solution. To reduce complexity of the optimization problem, it is divided into two separate optimizations. First, the design of each flexure-based joint is individually optimized, taking into account the detailed elastic behavior of the flexures. Next, the geometric design of the hexapod is optimized by using a reduced model with the non-linear stiffness properties of each joint lumped between the links of the hexapod. The stiffness properties of the joints are derived from the earlier conducted optimization and simulation of the individual flexure joints. Parasitic motion of the flexure-based joints, which is unintended or nonideal motion that accompanies intended motion of the joints, is disregarded, as it is a deterministic effect that does not affect the repeatability of the system. In case absolute positioning accuracy is of interest, the parasitic motion being repeatable means it could be compensated for by calibration. The decrease in support stiffness caused by deflection in the degrees of freedom is taken into account as this contribution is significant.

As the designs of the joints are optimized separately and do not change in the optimization loop for the geometry of the hexapod, no computational effort is required to compute the joint’s stiffness properties during this second optimization. Therefore, the mechanism state, i.e. the rotations of all joints, can be derived from kinematics only and the computational cost for evaluating the performance of the hexapod for a specific set of design parameters is only small (i.e. a few seconds on a conventional computer system).

Lumping of the compliance properties of the flexure-based joints is done by modeling them as two-node elements. To obtain the compliance properties, which are a function of motion of the joints in the degrees of freedom, a third-order polynomial is fitted through the compliance values obtained from the initial optimization and simulation of the individual joints. The six deformation modes of the two-node elements are defined by

\[
\varepsilon_1 = x_1 - x_2 \\
\varepsilon_2 = y_2 - y_1 \\
\varepsilon_3 = z_2 - z_1 \\
\varepsilon_{\theta_1} = \theta_2 - \theta_1 \\
\varepsilon_{\theta_2} = \theta_3 - \theta_2 \\
\varepsilon_{\theta_3} = \theta_1 - \theta_3 \\
\] (1)

with the superscripts \( p \) and \( q \) indicating the two nodes of the element each connected to a different arm of the hexapod. Furthermore, \( x, y \) and \( z \) represent the position of each node and \( \theta_1, \theta_2 \) and \( \theta_3 \) its orientation in \( x-y-z \) Euler angles with respect to a local reference frame. Note that both nodes are initially aligned and positioned identically, yielding zero deformation. The corresponding stress resultants (reaction forces) are provided by

\[
\sigma_1 = \varepsilon_1 / C_1 = \varepsilon_{\theta_1} / C_{\theta_1} \\
\sigma_2 = \varepsilon_2 / C_2 = \varepsilon_{\theta_2} / C_{\theta_2} \\
\sigma_3 = \varepsilon_3 / C_3 = \varepsilon_{\theta_3} / C_{\theta_3} \\
\] (2)

with \( C \) the compliance in each degree of freedom of the element. For example, for a spherical joint, \( C_{\theta_1}, C_{\theta_2} \) and \( C_{\theta_3} \) represent the compliances in the three degrees of freedom of the joint and \( C_x, C_y \) and \( C_z \) provide the compliances in the supposedly stiff support directions. Note that the compliance values of the individual joints are not constant over the range of motion and depend on the deflection angle(s) in the degrees of freedom of the joints, which will be discussed in more detail in section 3.2 and 3.3. Furthermore, the compliance properties are evaluated with respect to the center of the joints, which approximates the center of compliance. Therefore, coupling between the deformation modes is only small and negligible.

3.2. Actuated revolute base joint optimization

3.2.1. Actuator selection

For the actuation of the revolute base joints, an actuator is required that does not jeopardize the pursued precision while allowing for a large range of motion, large velocities and large accelerations. For precision systems, piezo-based “resonating” or “walking” actuators can be used as they provide highly deterministic behavior combined with a high positioning resolution. However, these type of actuators typically suffer from a limited maximum motion velocity (typically \(< 10 \text{ mm/s} \)) and limited actuation forces (\(< 15 \text{ N} \)). Furthermore, they suffer from wear and a limited total travel range (e.g. \(< 10 \text{ km} \)), which limits their service life especially when considering large range of motion applications.

In order to overcome these limitations, a permanent magnet torque motor with iron core is selected (Tecnotion’s QTR-A-133-60-N torque motor \([21]\)). This actuator allows for a high ultimate\(^1\) output torque of 55 Nm and a non-restrictive maximum velocity of 230 rad/s. The flexure-based revolute joint supporting the rotor of the actuator is designed to deal with the parasitic reaction forces caused by the attraction between the iron core and permanent magnets. This parasitic force increases with the misalignment between the central axis of the rotor and the stator. Therefore, a joint design with low parasitic motion is required to maintain good alignment over the full range of motion and minimize this parasitic force. For this purpose, the butterfly hinge design is selected, which will be discussed in more detail in the next section.

Finally, the absence of self locking in the selected actuator, or in fact the absence of any friction in the drive train, renders the system backdrivable. Therefore, this actuator enables the sensing of interaction forces with respect to the environment by using the motor current and position feedback only, which is not possible for self-locking piezo-based systems.

The rotation of the rotor is measured via Heidenhain’s LIC 4119 encoder with a 1 nm measuring step, resulting in a rotational resolution of 13 mrad. The actuator is controlled with Kollmorgen’s industrial class AKD-P00306 servo drive which can provide a 1 mA current feedback. In combination with the motor constant of the actuator, this allows for an 6 Nnm torque feedback resolution.

3.2.2. Base joint optimization

The actuated revolute joint at the base is based on the butterfly hinge design \([22]\) and optimized for maximum off-axis radial support stiffness. The butterfly hinge design is selected as it features a low shift of the rotation axis, which is required to maintain good alignment between the rotor and the stator. A detailed design drawing of this flexure-based actuator suspension is provided in Fig. 2, consisting of two butterfly hinges placed in parallel at each side of the rotor of the actuator.

For the optimization of the butterfly hinges, an adapted Nelder-Mead based shape optimization algorithm \([15]\) is used. In the optimization loop, the performance is evaluated over the range of motion with the flexible multidof body software SPACEAR \([23]\). The hinge is modeled with a series of interconnected nonlinear 3D finite beam elements, which include the geometric non-linearities to capture the relevant behavior of the flexures. For the optimization objective, the off-axis radial support stiffness is selected in order to maximize support stiffness for the lower arm and to provide resistance to the magnetic forces between the permanent magnets and the iron core. A stainless tooling steel is selected with an E-modulus of 200 GPa and a maximum allowable fatigue stress of 600 MPa (about 40% of the yield stress). Furthermore, a range of motion of \(\pm 25^\circ\) is considered, which is slightly less than the \(\pm 30^\circ\) considered in Ref. \([17]\). The range of motion is decreased in order to ensure all joints utilize their full range of motion throughout the

\(^1\) At ultimate torque the temperature of the coil increases by 20 °C/s. The maximum duration at which this torque can be maintained depends on thermal properties of the coils housing.
considered workspace. The width of each butterfly hinge (the dimension measured parallel to the rotation axis) is limited to 50 mm. Furthermore, a center to center distance of 160 mm between both butterfly hinges is considered, which is required for the space claim for the actuator. A schematic overview of the butterfly hinge with the optimized dimensions is provided in Fig. 3, with each flexure given the same dimensions and $P$ indicating the location of the rotation axis.

3.2.3. Base joint compliance properties

The compliance of an individual butterfly hinge is evaluated with respect to the center of the joint (at point $P$, Fig. 3), which results in off-diagonal terms in the compliance matrix that are at least an order of magnitude smaller and therefore negligible. Hence, the compliance of a single butterfly hinge can be approximated by diagonal compliance matrix. As the resulting compliance of the butterfly hinge depends strongly on the rotation angle of the hinge in the degree of freedom ($\theta_r$), a third-order polynomial is fitted through the numerically obtained compliance values to capture the variation. As the behavior of the butterfly hinge is equal for positive and negative rotation angles, each individual compliance term is approximated by

$$C = p_0 + p_1|\theta_r| + p_2|\theta_r|^2 + p_3|\theta_r|^3$$

(3)

with $\theta_r$ the rotation angle in degrees. An overview of the fitted coefficients is provided in Table 2. The compliance values for the translational and rotational directions are provided in Fig. 4. Note that the compliance in the degree of freedom is almost constant and of a different order of magnitude (0.27 rad/Nm), and therefore not provided in this Table 2. Interpolation values for the compliance in the support directions of the base joint with $R^2 > 0.99$

| $C_x$ (N/m) | $C_y$ (N/m) | $C_z$ (N/m) | $C_{\theta_x}$ (Nm/rad) | $C_{\theta_y}$ (Nm/rad) |
|-------------|-------------|-------------|------------------------|------------------------|
| 2.06 $\cdot 10^{-6}$ | 8.63 $\cdot 10^{-9}$ | 7.22 $\cdot 10^{-11}$ | 3.80 $\cdot 10^{-6}$ |
| 2.10 $\cdot 10^{-8}$ | 5.00 $\cdot 10^{-12}$ | 8.14 $\cdot 10^{-8}$ | 3.25 $\cdot 10^{-10}$ |

Fig. 2. Exploded view of actuator suspension. 1) Butterfly hinge 2) Slaving mechanism 3) Rotor-hub 4) Permanent magnets of the rotor 5) Body coupling intermediate bodies of both butterfly hinges 6) Encoder 7) Mechanical stop 8) Optical switch 9) Actuator’s stator 10) Stator housing 11) Cooling channels [17].

Fig. 3. Schematic overview of the optimized dimensions for the butterfly hinge.

Fig. 4. a) Translational and b) rotational compliance values of the optimized revolute joint.
3.3. Spherical/universal joint optimization

The designs of the spherical and universal joints are based on the two stage serially stacked spherical joint concept described in Ref. [16], optimized to maximize support stiffness in the load carrying direction (along the longitudinal direction of the upper arm, Fig. 1). A range of tip-tilt motion of $\pm 25^\circ$ is considered, combined with a $\pm 10^\circ$ of pan motion for the spherical joint. These values for the maximum deflection angles are deduced from the maximum range of motion which could be obtained by this joint design while maintaining high support stiffness combined with a small envelope ($< 90$ mm diameter). A detailed design drawing of the flexure-based universal and spherical joint is provided in Fig. 5. It has to be noted that the design of the universal and spherical joint is identical, with the exception of a single additional flexure constraining one of the rotational degrees of freedom for the universal joint.

For optimization of the spherical joint the same optimization strategy as for the revolute base joint is used. For the optimization objective, the stiffness along the symmetry axis is considered as this stiffness contributes to the observed stiffness at the end-effector of the hexapod. For material, stainless tooling steel is again selected with an E-modulus of 200 GPa and a maximum allowable stress of 600 MPa. To constrain the footprint of the spherical joint, the diameter is limited to 90 mm. A schematic overview of the spherical joint with the optimized dimensions is provided in Fig. 6, with each flexure given the same dimensions.

3.3.1. Spherical/universal joint compliance properties

The compliance properties of the universal and spherical joint are also identified by fitting third-order polynomials through the numerically obtained compliance values. However, as motion in multiple degrees of freedom is allowed, the compliance properties are a function of this multi-dimensional rotation. In Ref. [16] it was observed that the support stiffness along the symmetry axis of the spherical joint was mostly affected by the tip or tilt angle, which we have captured in a single tilt angle perpendicular to the symmetry axis irrespective of pan motion. Therefore, the compliance can be approximated by

$$C = p_0 + p_1 \theta_t + p_2 \theta_t^2 + p_3 \theta_t^3$$

(4)

with $\theta_t$ the tilt angle in degrees. An overview of the fitted coefficients is provided in Table 3. The compliance values for the translational directions are shown in Fig. 7. Note that the compliance values in $x$- and $y$-direction are taken equal due to the rotational symmetry of the spherical joint. Furthermore, as the frame compliance of the spherical joint has a significant contribution to the overall compliance, additional frame compliance is taken into account. To assess this frame compliance, compliance of the joint in undeflected state with a rigid and a flexible frame is evaluated by means of a FEM simulation. By comparing both compliance values and by taking the frame and flexure compliance in series, the frame compliance is estimated at $1.25 \times 10^{-6}$ m/N in the vertical direction and $1.25 \times 10^{-5}$ m/N in the directions perpendicular to it.

3.4. Hexapod design optimization

For high positioning accuracy of the hexapod, a high controller bandwidth is desired to obtain good dynamic performance and to suppress disturbances, e.g. current noise caused by the motor drivers. As the mass and inertia properties of the individual components are not known beforehand and depend strongly on the geometry of hexapod, accurately evaluating this parasitic eigenfrequency in the optimization loop is hard. However, as the stiffness observed at the end-effector is strongly related to the critical parasitic eigenfrequency, this stiffness can be used instead.
Furthermore, when the system is controlled via the encoders on the actuators, this stiffness is crucial for obtaining high precision at the end-effector when subjected to external forces and disturbances. Therefore, the translational compliance of the end-effector (given locked actuators in the actuated direction) in the most compliant direction is used as the optimization objective. Furthermore, it is worth noting that this optimization objective also avoids (near) singular mechanism configurations as these configurations would suffer from strongly decreased support stiffness in specific directions.

For evaluating the performance of the hexapod given a specific set of design parameters, first an inverse kinematic rigid body analysis is conducted for which the end-effector is positioned at each extremum of the workspace of $100 \times 100 \times 100$ mm$^3$. Hereby, the pose of the hexapod and the deflection angle of each individual joint is determined. Next, the obtained support stiffness at the end-effector is evaluated by taking into account the lumped compliance properties of all flexure-based joints by modeling them as the two-node flexible elements according to Eqs. (1) and (2). Note that the stiffness values of the joints strongly depend on their deflection angles in the degrees of freedom of the joints, which are obtained from the inverse kinematics. A schematic overview of this lumped model is provided in Fig. 8 with each spring representing a six dimensional lumped stiffness. Compliance in the frame parts is assumed negligible with respect to the compliance of the flexure-based joints, with exception of the frame parts of the spherical joints whose stiffness has proven to have a significant contribution.

For the optimization of the geometry of the hexapod, the optimization strategy provided in section 3.2 is used. The geometry of the hexapod is described by eight design parameters ($R_1, R_2, R_3, \beta_1, \beta_2, \beta_3, H_1, H_2$) which provide the position of each of the joints in the parallel chains, schematically illustrated in Fig. 9. The base joints are placed on a circle with radius $R_1$, with each pair of arms separated by angle $2\beta_1$ between them. Similarly, the elbow and wrist joints are placed on circles with radii $R_2$ and $R_3$, which are separated by each other with angles $2\beta_2$ and $2\beta_3$. Lastly, the height of the elbow and wrist joints are specified by $H_1$ and $H_2$.

Furthermore, constraints are specified for the optimization in order to limit the deflection angle of the individual joints, limiting the rotation angle of the revolute base joints to $25^\circ$ and the rotations of the elbow and wrist joints to $25^\circ$ tip-tilt. Rotation around the symmetry axis of the spherical joint (the wrist joint), providing pan motion, is limited to $10^\circ$. Collision between the six parallel chains of the hexapod is avoided by putting a constraint on the minimum distance between each of the joints. A minimum distance between the centers of the base joints of 150 mm is required which is dictated by the size of the actuators and their housing. Furthermore, given the diameter of 90 mm of the universal and spherical joints, a minimal distance of 90 mm between them is required. Note that the distance between the elbow joints depends on the pose of the hexapod, and therefore needs to be evaluated through the workspace of the hexapod. Lastly, a limitation on the maximum mechanism volume (in undeformed state) of 0.25 m$^3$ is added, defined by the volume of the smallest enclosing cylinder including the diameter of the joints.

4. Optimization results

An initial optimization is conducted by taking into account all eight design parameters independently. However, considering each design parameter as an independent degree of freedom has shown to result in inconvenient values that complicate the mechanical design. Hereby, the design becomes more complex, costly and possibly suffers from increased mass and reduced stiffness of the frame parts. Furthermore, optimizations on the full set of design parameters result in a wide range of local optima with almost similar performance, which indicates redundancy in the design freedom. This means that the number of independent design parameters can be reduced without excluding good mechanism designs from the optimization problem. For this purpose, design parameters are iteratively eliminated by choosing convenient constraint relations between some of the design parameters, which will be detailed in the next section. For each iteration, the optimal solution is provided in Table 4.

4.1. Iterations

4.1.1. Iteration 1

The first optimization, considering all eight independent design parameters, provides solutions at which the base and wrist joints (Fig. 9) of each arm pair are positioned in a single vertical plane. In other words, $R_1 \cos(\beta_1) = R_3 \cos(\beta_3)$, which can be rewritten to a constraint function for $R_3$

$$R_3 = R_1 \frac{\cos(\beta_1)}{\cos(\beta_3)} \quad (5)$$

Table 4

| Iteration | 0  | 1  | 2  | 3  | 4  | 5  |
|-----------|----|----|----|----|----|----|
| Performance (N/mm) | 340 | 346 | 341 | 350 | 346 | 299 |
| $R_1$ (mm) | 286 | 266 | 281 | 271 | 266 | 255 |
| $R_2$ (mm) | 413 | 415 | 488* | 427* | 417* | 387* |
| $R_3$ (mm) | 285 | 291* | 279* | 265* | 259* | 248* |
| $\beta_1$ (deg) | 16.5 | 16.8 | 15.8 | 16.0 | 16.4* | 17.2* |
| $\beta_2$ (deg) | 52.5 | 52.2 | 52.9 | 52.3 | 52.4 | 50.9 |
| $\beta_3$ (deg) | 13.8 | 9.5 | 14.3 | 9.8* | 10.0* | 10.6* |
| $H_1$ (mm) | 79 | 80 | 63 | 71 | 81 | 100 |
| $H_2$ (mm) | 183 | 206 | 193 | 278 | 195 | 190 |
eliminating $R_3$ as an independent design variable.

4.1.2. Iteration 2

Optimization results using constraint equation (5) yield solutions with near parallel rotation axis of the base joints of a single arm-pair. By demanding both rotation axes to be parallel, the design of the actuators can be simplified. Ensuring the rotation axis of the two base hinges of an arm pair to be parallel can be obtained by

$$R_2 = R_1 \frac{\cos(\beta_1)}{\cos(\beta_2)}$$

eliminating $R_2$ as an independent design variable.

4.1.3. Iteration 3

Optimization results using constraint equations (5) and (6) show convergence to solutions for which the wrist joints of a single arm-pair are placed as close to each other as possible. Taking into account the required offset of 90 mm between the spherical joints at the platform, we can eliminate $\beta_3$ as an independent design variable by

$$\beta_3 = \arcsin \left( \frac{90 \text{ mm}}{2R_3} \right)$$

Note that the design freedom obtained this way (the design space) shows strong similarities with the Stewart platform [1] when considering the location of the joints. More specifically, the universal and spherical joint of a single arm are in a vertical plane perpendicular to the ground. Furthermore, the spherical joints at the end-effector of each arm pair are positioned close together whereas the lower spherical joints are positioned at close distance to the neighboring arm-pair.

4.1.4. Iteration 4

Optimization results using constraint equations (5)–(7) show that the system benefits from base joints located as close together as possible. Therefore, the distance between the base joints can be chosen equal to 150 mm by means of

$$\beta_1 = \arcsin \left( \frac{150 \text{ mm}}{2R_1} \right)$$

eliminating $\beta_1$ as an independent variable.

4.2. Discussion

From the obtained optimization results shown in Table 4 it can be concluded that the additional constraint equations do not result in a penalty on the performance of the overall design, despite the strong reduction in the freedom space of the optimization problem. For some of the iterations, even a small increase in performance is obtained because the convergence improves when the set of design parameters is reduced. In the optimization process and system model, practical aspects such as manufacturability are not considered. These are accounted for by hand after the optimization. The final design parameters are provided in the last column of Table 4. Although these resulting design alterations come at the apparent cost of a performance loss, the improved manufacturability is considered worth it.

5. System design

Based on the geometry obtained from the optimizations, a prototype of the hexapod has been designed. A design drawing is provided in Fig. 10. Furthermore, a front and top view with the main dimensions are provided in Figs. 11 and 12.

Each pair of actuated revolute base joints with parallel rotation axes is combined into a single assembly. The electronics for each actuator pair are placed in a single electronic box placed underneath the actuators. Furthermore, two steel balance masses are attached to each actuator to counteract gravity and to ensure that the neutral position of the hexapod is approximately at the center of its workspace (balance masses are tuned for a mechanism without payload). The balance masses do add to the inertia of the lower arms, lowering the achievable accelerations. However, additional inertia simultaneously reduces the sensitivity to current noise, making the trade-off worthwhile.

The lower arm is attached to the revolute base joint. It provides the connection with the first elbow joint and consists of a hollow aluminum structure. The connection between the lower arm and this joint is equipped with a mechanical overload mechanism fitted between the frame part of the lower arm (part 4, Fig. 10) and the frame part of the spherical joint (part 3). Those parts are attached to each other by means of a kinematic coupling of the Maxwell type consisting of three V-shaped grooves oriented to the center of the part, mated to a counter part with three spherical surfaces preloaded by a permanent magnet. The preload force is adjusted such that is disconnects at excessive moments in order to prevent damage to the spherical joints when they exceed their
maximum deflection angle. A micro switch (part 5) is placed parallel to the overload protection to detect a disconnected spherical joint and to shutdown the system.

Furthermore, the lower spherical joint is connected to the second “upper” spherical joint by means of a hollow thin-walled carbon tube (diameter: 28 mm, thickness: 1 mm, length: 250 mm). The upper spherical joint provides the connection with the end-effector, which is made from aluminum sheet material to provide a lightweight end-effector. The connection with the folded leafsprings of each upper spherical joint consists of a solid aluminum part (comparable to part 3), which is an integral part of the frame of the end-effector.

6. Detailed system analysis

6.1. Workspace and eigenfrequencies

To evaluate the dynamic performance of the system, the eigen-frequencies of the system are analyzed by taking into account the mass and inertia properties of the frame components and the non-flexure parts of the joints. An overview of the mass and inertia properties is provided in Appendix A. For the dynamic performance of the system, the eigen-frequency that limits the maximum bandwidth of the controller is of direct interest. This eigenfrequency is given by the seventh eigenfrequency of the system with “free” actuators (the first six eigenfrequencies are related to the degrees of freedom of the system). To visualize the frequencies as a function of the end-effector location through the workspace, a 2D plane throughout the workspace is selected for displaying this parasitic frequency. The first parasitic frequency as a function of the \( z \)-position and for equal \( x \)- and \( y \)-position is provided in Fig. 13. In this figure, the total displacement in the xy plane is indicated by \( d = \sqrt{x^2 + y^2} \) with \( x = y \). Furthermore, the boundaries of the workspace of \( 100 \times 100 \times 100 \text{ mm}^3 \) are given by the dashed lines. Frequencies outside the workspace, which is determined by the maximum deflection angle of each of the joints, are not displayed. Eigenfrequencies throughout other segments of the workspace, including rotations of the end-effector, are provided in Appendix B.

From the results it can be observed that the required workspace of \( 100 \times 100 \times 100 \text{ mm}^3 \) is within the range of motion of the hexapod. Furthermore, even larger displacements in the simultaneous directions are allowed, resulting in an actual range of motion of \( \pm 105 \text{ mm} \) in \( x \)-direction, \( \pm 100 \text{ mm} \) in \( y \)-direction and \( \pm 95 \text{ mm} \) to \( \pm 105 \text{ mm} \) in \( z \)-direction. The rotational range of motion is \( \pm 12.5^\circ \) rotation around the \( x \)-axis, \( \pm 11^\circ \) around the \( y \)-axis and \( \pm 15^\circ \) around the \( z \)-axis. When instead the internal degree of freedom around the longitudinal axis of the upper arm is not constrained (6-\( \mathbf{R} \)-\( \mathbf{S} \)-\( \mathbf{S} \) layout), the rotational range of motion is increased to \( \pm 21^\circ \) rotation around the \( x \)- and \( y \)-axis and \( \pm 18^\circ \) around the \( z \)-axis. The 6-\( \mathbf{R} \)-\( \mathbf{S} \)-\( \mathbf{S} \) layout does suffer from an additional internal degree of freedom, resulting in an extra parasitic eigenfrequency (one for each arm) at approximately 32 Hz, which is nearly constant over the range of motion.

At the center of the workspace the first parasitic frequency is computed at 87 Hz. Within the workspace of \( 100 \times 100 \times 100 \text{ mm}^3 \), this frequency drops only slightly to about 74 Hz at the extrema of this workspace. Over the entire range of motion, the frequency drops further to 60 Hz at the worst-case location in the workspace (e.g. at \( -105 \text{ mm} \) displacement in \( z \)-direction). A visualization of the modeshape of the critical eigenfrequency at the center of the workspace is provided in Fig. 14. It consists of a translational motion in the \( x \)-direction combined with a rotational motion around the \( y \)-axis. Throughout the workspace, a similar modeshape for the critical parasitic eigenfrequency is found.
It has to be noted that the compliance properties of the frame parts were not included in the computation of the eigenfrequencies. Despite the high rigidity of the frame parts with respect to the flexures, it can be expected that the eigenfrequencies of the actual system are slightly lower, particularly in the neutral position.

6.2. Maximum accelerations

Given the maximum torque of the actuators, the mass and inertia of the system, and the pose of the mechanism, the maximally achievable accelerations can be computed. Due to reaction forces to counteract gravity and the stiffness of the flexures, the maximum accelerations which can be obtained will be direction dependent (accelerations are either assisted or counteracted by the elastic reaction and gravity forces). In this section, the worst-case direction is considered for which the forces are counteracting accelerations.

An example of the maximum accelerations in $z$-direction as function of the $z$- and $d$-position is provided in Fig. 15. Accelerations exceeding $120 \text{ m/s}^2$ can be achieved within the ultimate torque range of the actuators over the workspace. The highest accelerations are possible in the center of the workspace, allowing for accelerations up to $180 \text{ m/s}^2$. The maximum accelerations in $x$- and $y$-direction and the rotational accelerations throughout the workspace are provided in Appendix C, resulting in translational accelerations in the range of $75-150 \text{ m/s}^2$. The maximum rotational accelerations are in the range of $540-850 \text{ rad/s}^2$ for rotations around the $x$- and $y$-axis and $300-550 \text{ rad/s}^2$ for rotations around the $z$-axis.

6.3. Maximum load capacity

Additional payload placed on the end-effector of the system will result in an increase in reaction forces. The limiting factor in the maximum force that the end-effector can handle is not the maximum torque of the actuators, but the stress in the flexures of the spherical and universal joint. For example, when accepting a $250 \text{ MPa}$ increase in Von Mises stress at the worst-case deflection angle of $25^\circ$, giving a total stress of about $60\%$ of the yield stress, a payload up to $100 \text{ N}$ is allowed in the support direction. For smaller deflection angles higher payloads are permitted [16].

The load on the end-effector is distributed over the six legs and the distribution depends on the pose of the mechanism. The worst-case position for static gravity forces is given for the end-effector positioned at the bottom of the workspace, which results in a worst-case reaction force in the spherical joints of approximately half of the reaction force exerted on the end-effector. Therefore, considering the allowed stress increase of $250 \text{ MPa}$, a payload of $200 \text{ N}$ of vertical force is allowed, not including any safety factor.

Note that additional payload also results in increased reaction forces and increased stress levels when subjected to accelerations. Therefore, payload can result in reduced maximum accelerations, either due to the larger torque requirements for the actuators, or due to the increased reaction forces and finite load capacity of the joints.

6.4. Overview

A complete overview of the specifications of the T-Flex is listed in Table 5.

7. Prototype design

7.1. Mechanical design

A photograph of the completed prototype is provided in Fig. 16. For this prototype, the aluminum frame parts of the spherical joints are anodized in red and some of the stationary frame parts at the base are anodized in black. Furthermore, the steel counter masses are painted dark grey. Note that the presented prototype in the figures is of the type 6-RSS with the additional flexures required to constrain the internal degrees of freedom of the upper arms detached.

7.2. System identification

For evaluating the dynamic performance and designing the controller, first the behavior of the system is identified by evaluating the frequency response function from actuator current to rotations. The
Frequency response is evaluated by applying a pseudo random binary signal on the provided current (once for each actuator) and measuring the rotation. The results are shown in Fig. 17 considering an end-effector position in the center of the workspace.

From the results we can see the first natural frequency in the degree of freedom of the actuators at around 2 Hz. Some variation in the first natural frequency can be observed between the actuators, presumably related to initial stress in the flexures induced by minor miss-alignments and variations in the manufacturing process. Furthermore, each of the actuators shows dominant second-order behavior with a nearly identical frequency response for each of the actuators. The transfer function of the dominant second order behavior of the plant \( P(s) \) is identified as

\[
P(s) = \frac{1}{m_{eq}s^2 + d_{eq}s + k_{eq}} = \frac{1}{0.10s^2 + 0.48s + 12.0}
\]  

The first parasitic eigenfrequency of the system can be identified from the strongly excited eigenfrequency just below 70 Hz. Some other resonances with smaller magnitude can be observed at 30–40 Hz, which can be related to the 6 internal eigenfrequencies of the upper arms for the 6-RSS layout. Note that the torque provided by the actuators does not result in significant reaction forces in this underconstrained degree of freedom, producing only low energy input and small excitations. For the same reason, parasitic motion/vibration of this internal degree of freedom does not result in significant error motions at the actuator and end-effector.

A measurement of the first parasitic eigenfrequency throughout the workspace is provided in Fig. 18. Due to more strict safety limits on the actuator positions during the prototyping phase, the outer edges of the workspace are excluded for the experimental results displayed in Fig. 18. At the center of the workspace, the eigenfrequency is just below 70 Hz and drops to about 56 Hz near the edges. This frequency distribution throughout the workspace is in agreement with the simulated eigen-frequencies provided in Fig. 13, although the frequency values are about 10–20% lower. This can be attributed to the additional compliance introduced by the frame parts, which have been excluded in the simulations.

### Table 5

| Simultaneous travel range | |
|---------------------------|--|
| Travel range X            | ±50 mm |
| Travel range Y            | ±50 mm |
| Travel range Z            | ±50 mm |

| Individual travel range | |
|-------------------------|--|
| Travel range X          | ±100 mm |
| Travel range Y          | ±105 mm |
| Travel range Z          | ±100 mm |
| Travel range \( \theta_x \) | ±12.5° (±21.5°) |
| Travel range \( \theta_y \) | ±11.5° (±21.5°) |
| Travel range \( \theta_z \) | ±10° (±18°) |
| Translational workspace | 5.5 dm³ |

| Positioning performance | |
|-------------------------|--|
| Max. acceleration XY    | 75 – 150 m/s² |
| Max. acceleration Z      | 120 – 180 m/s² |
| Max. acceleration \( \theta_x \) | 540 – 850 rad/s² |
| Max. acceleration \( \theta_y \) | 300 – 550 rad/s² |
| Max. velocity XYZ       | up to 4 m/s |

| Miscellaneous            | |
|--------------------------|--|
| Actuator resolution      | 13 mrad |
| Actuator torque feedback resolution | 6 Nmm |
| Back-drivable            | Yes |
| Footprint (radius)       | 430 mm |
| Height (excl. electronics)| 420 mm |
| Max. vert. payload       | 20 kg |
| Mechanism volume         | 0.24 m³ |

* Merely limited by the maximum travel range and maximum acceleration according to distance-acceleration

** Resulting in stress up to 60% of the yield stress in the flexures.

Fig. 16. Photograph of the T-Flex with the end-effector tilted by 10° around the x-axis and positioned at \( z = 30 \) mm.

Fig. 17. Transfer function from actuator current (mA) to rotor position (rad) at the undeflected state (black lines provide the transfer function from current to rotor position between different actuators).

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position control loop, 7.3. Controller design

It has been shown that the main source of positioning error originates from noise on the current signal produced by the motor drivers (d1 ≈ ± 5 mA RMS white noise). Disturbances on the position sensor are in the order of magnitude of the sensor resolution and negligible (d2 ≈ 0 μrad).

Because the cross talk is relatively low in the frequency range up to the first parasitic eigenfrequency, six identical single input single output (SISO) controllers are designed. The goal of the control loops is to minimize the positioning error e given by:

$$e(s) = \frac{1}{1 + CKP} r(s) + KP \frac{1}{1 + CKP} d(s)$$

(10)

For the transfer function on the left-hand side, we typically aim for a high crossover frequency for good tracking performance combined with sufficient phase margin for a stable system. Below the controller bandwidth (\(|CP| \approx 1\)), the transfer function on the right-hand side can be approximated by \(1/C\). Therefore, a higher crossover frequency provides more disturbance rejection. In the higher frequency range (\(|CP| \ll 1\)), the system can be approximated by \(KP\), where additional roll-off provided by the pole of \(K\) can provide additional suppression of the higher frequency peaks in \(P\). As a result, we aim for a controller with high crossover frequency combined with a stable closed loop over the entire workspace, with additional roll-off in the current control loop to suppress high frequency peaks of the plant. Note that the balance masses attached to the rotor provide a reduction in the magnitude of \(P\).

For the current controller \(K\), the built-in current controller of the motor driver is used, which is structured as a low-pass filter. For the position control loop a PID controller is used. The controller parameters are:

$$C = \frac{k_p}{\tau_p s + 1}$$

(11)

$$K = \frac{1}{\tau_s s + 1}$$

(12)

with

$$k_p = 382.9 \quad \tau_p = 0.0014$$

$$\tau_s = 0.0495$$

(13)

The controller settings are chosen to maintain a stable system throughout the workspace while achieving good performance. The provided controller settings provide a crossover frequency at 20 Hz and a phase margin of 44° for the undeflected state of the mechanism. Throughout the workspace, the cross-over frequency varies between 19.6 and 23.6 Hz and the phase margin between 41 and 45° by the slight variation of the effective inertia felt by the actuators. Note that the effect of end-effector payload has been neglected. As the focus of this paper is on the structural optimization, mechanical design and experimental validation of this system, the controller design is not described in more detail.

8. Experimental validation

8.1. The standstill performance

As the positioning performance of the system is strongly affected by current noise produced by the motor drivers, the influence of current noise on the actuator and end-effector position is evaluated. To measure the position of the end-effector, three capacitive displacement sensors (Lion Precision C6, resolution < 1 nm) are placed in a circular pattern in an external frame measuring the z-position of the end-effector. By combining the sensor data of the three sensors, the z-position and rotations around the x- and y-axis can be evaluated simultaneously.

To analyze the influence of current noise on maintaining a stationary end-effector position as a function of frequency, the results in this section are provided as the square root of the cumulative sum of the power density multiplied with the spectral resolution. According to Parseval’s theorem, the complete sum provides the total RMS error according to:

$$e_{\text{RMS}} = \sqrt{\sum S \Delta f}$$

(14)

with S the power spectral density of the time-signal and \(\Delta f\) the spectral resolution.

The resulting cumulative positioning error of the actuators, measured at the actuators including feedback control for a stationary target position, is provided in Fig. 20. These results show an error between 1.5 and 3.5 μrad RMS. The observed frequency spectrum of the actuator position corresponds with the relation between the current disturbance and the error of Eq. (10). As discussed in section 7.3, this transfer is approximately \(1/C\) below the cross-over frequency. In the frequency range from 3 Hz (\(\tau_s\)) to 20 Hz (the bandwidth \(\sqrt{\tau_s}\)), the controller’s transfer function is approximately flat at \(k_p\), explaining the approximately linear increase of the cumulative RMS of the error. Beyond the bandwidth the relation between the current noise and the error is approximately equal to \(KP\), which decreases with frequency by the third order, which explains the almost flat increase of error beyond the bandwidth, except for the resonance peak just below 70 Hz.

The measured position at the end-effector is provided in Fig. 21. The results show a 0.34 μm RMS error in the z-direction and a 4–5 μrad RMS error in the rotational x- and y-direction. Note that the contribution at the parasitic eigenfrequency mainly affects the error in the \(\theta_x\) and \(\theta_y\) direction, which corresponds to the expected modeshape as visualized in Fig. 14. It has to be pointed out that the eigenfrequency related to modes of the upper arms for the 6-RSS layout at 30–40 Hz does not mitigate precision at the end-effector.

Fig. 19. Controller overview with: reference signal (r), position error (e), disturbances (d), current reference signal (i_d), actual measured current (i_m) and actuator angle (\(\theta\)).

Fig. 20. Cumulative RMS error of the actuator position including feedback control.
8.2. Repeatability

For evaluating the repeatability of the system, the capacitive displacement sensors described earlier are used to measure the $z$-position of the end-effector. For this test, the end-effector is moved repeatedly in $z$-direction with a bi-directional displacement of $\pm 100 \mu m$ at an interval of 2.5 s combined with a large displacement of $+50$ mm at an interval of 17.5 s. The measurement results, provided in Fig. 22, show highly repeatable behavior well below the magnitude of the position fluctuations caused by disturbances. It can be observed that the repeatability is independent of the direction of movement, which would be typical for a system which observes friction, play or hysteresis.

Additionally, the averaged long-term end-effector position for a period of 400 s is provided in Fig. 23, showing fluctuations of $\pm 0.25 \mu m$ of the mean end-effector position. For reference, the rotation of one of the actuators is provided in Fig. 24, showing a constant rotation angle. The measured values are in range of expectation considering dimensional changes due to temperature variations in the environment and heat input from the actuators and electronics. For example, when considering the linear temperature expansion coefficient for aluminum and steel of $10^{-25} \mu m/(m^\circ C)$ and a roughly estimated linear dimension of 0.1 m, a 0.1 $^\circ C$ temperature change can already results in 0.25 \mu m change in length. Note that the base for the actuators have been equipped with cooling channels which allow for active cooling if deemed necessary. For the measurements in this section, no active cooling is employed.

9. Discussion

Assuming a fixed relative magnitude of the disturbances in the provided current with respect to the ultimate current, the standstill performance is directly affected by the motor constant of the actuators combined with the inertia of the system. Therefore, the influence of input disturbances can be adjusted in order to meet specified criteria. For example, motor torque (the motor constant) can be sacrificed to reduce sensitivity to disturbances. However, the designer will always face a trade-off between the standstill performance and the maximum torque which can be delivered, directly affecting the maximum accelerations. Moreover, as the standstill performance is currently restricted by limitations of the electronics, it can be improved by using a better...
electronics design. The currently used industrial class PWM-based motor drivers provide a peak current to noise ratio <1000, which can be considered typical for a motor driver. To further improve this ratio, an amplifier with higher performance is a necessity, e.g. an linear amplifier. As this is expensive especially considering the high power requirements for the actuators, and as the mechanical design is the main focus of this research, this has not been improved on.

Furthermore, the position stability of the end-effector position appears to be limited, possibly due to temperature variations in the system. It is expected that a homogeneous temperature change of only 0.1 °C can already cause dimensional changes up to 0.25 μm. An additional source of disturbances can be found in the small, but not negligible, amount of hysteresis in flexures. For flexure mechanisms with clamped flexures, the magnitude of hysteresis has been measured in the order of magnitude of 0.1% [24]. For flexure mechanisms with monolithic flexures, which is the case for the T-Flex, the magnitude of hysteresis can be expected to be even smaller. This means that the elastic reaction forces in the degrees of freedom of the joints can vary (e.g. 1/1.000 Nm variation). Combined with the stiffness of the system, this can result in deformations in the order of magnitude of tens of nanometers.

A commonly used approach to improve positioning performance in high precision systems is the use of an additional feedback loop with respect to the end-effector position [25]. With this additional feedback, low frequent disturbances caused by hysteresis and temperature changes can be easily compensated for. Alternatively, additional effort can be put in stabilizing the temperature of both the environment and the system [26].

10. Conclusion

A large range of motion flexure-based hexapod has been designed. The design has been optimized by considering each elastic joint as an isolated component, lumping the nonlinear compliance behavior of the flexure-based joints. Combined with some practical symmetry relations, a fast model with a small set of design parameters has been created for efficient optimization.

The optimization results have led to a fully flexure-based hexapod design, the T-Flex, which provides a worst-case translational end-effector stiffness of 300 N/mm. Furthermore, the first parasitic eigen-frequency at 56–70 Hz throughout the workspace allows for a 20 Hz control bandwidth. The maximum translational range of motion is about ±100 mm in each direction with a combined workspace of 5.5 dm³. Furthermore, the specifically designed elastically suspended actuators permit high payload, high actuation torque and back-driveability, resulting in torque feedback with a resolution of 6 Nmm. Due to the high torque actuators, the system is able to achieve high accelerations (up to 18 g), unprecedented travel speed (exceeding 1 m/s) and a large vertical payload of 20 kg. This is a large improvement over existing high-precision parallel manipulators, since the travel range is up to an order of magnitude larger and the maximum travel speed has been increased by to two orders of magnitude.

Since the design is completely free from play, friction and cable disturbances, the repeatability of the system is only limited by the electronics. This results in disturbances on the provided current signal to the actuators, leading to 0.35 μm RMS position noise on the end-effector. Further increasing the precision and the implementation of feed forward control strategies to improve tracking accuracy will be subject of future research.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Mass and inertia properties

A schematic overview of the locations of the center of masses of the frame parts and the orientation of the axis used to define the inertia properties is provided in Fig. A.25. The center of mass of the end-effector (superscript e) and the upper arms (superscript u) are positioned in the center of the parts. The center of mass of the lower arms (superscript l) are located at a vertical distance of 56 mm and at a horizontal distance of 23 mm with respect to the rotation axis of the revolute joint.
An overview of the mass and inertia properties is provided in Table A.6.

### Table A.6

| Part          | $m$ (kg) | $I_{xx}$ (kgm$^2$) | $I_{yy}$ (kgm$^2$) | $I_{zz}$ (kgm$^2$) |
|---------------|----------|---------------------|---------------------|---------------------|
| Lower arm     | 6.83     | 0.031               | 0.025               | 0.038               |
| Upper arm     | 0.27     | 0.007               | 0.007               | 0.0002              |
| End effector  | 0.91     | 0.019               | 0.019               | 0.038               |

### Appendix B. First parasitic eigenfrequency
(a) First parasitic eigenfrequency for end-effector displacements in the positive $x$-, $y$- and $z$-direction

(b) First parasitic eigenfrequency for end-effector displacements in the positive $x$- and $y$-direction and the negative $z$-direction

(c) First parasitic eigenfrequency for end-effector rotations for the 6-RUS hexapod layout

(d) First parasitic eigenfrequency for end-effector rotations for the 6-RSS hexapod layout

Fig. B.26. First parasitic eigenfrequency throughout the workspace.

Appendix C. Maximum accelerations
Fig. C.27. Maximum acceleration throughout the workspace.

References

[1] Merlet JP. Parallel robots. Spring Science & Business Media; 2006. doi:10.1007/1-4020-4133-0. arXiv:arXiv:1011.1669v0. 3.

[2] Koubela T, Pochyly A, Singule V. Investigation of position accuracy of industrial robots and online methods for accuracy improvement in machining processes. In: 2015 international conference on electrical drives and power electronics (EDPE). IEEE; 2015. p. 385-8. doi:10.1109/EDPE.2015.7325325.

[3] Robots Universal, brochure E-Series. URL, https://www.universal-robots.com/media/1802432/e-series-brochure.pdf; 2021.

[4] Kuric I, Tlach V, Ságová Z, Gitsuk I. Measurement of industrial robot pose repeatability. MATEC Web of Conferences 2018;244:1–9. doi:10.1051/matecconf/201824401015.

[5] Physik Instrumente (PI). Hexapod positioning systems. URL, https://static.physikinstrumente.com/fileadmin/user_upload/physik_instrumente/files/CAT/PI-CAT136-Hexapod-Positioning-Systems.pdf; 2019. https://static.physikinstrumente.com/fileadmin/user_upload/physik_instrumente/files/CAT/PI-CAT136-Hexapod-Positioning-Systems.pdf.

[6] SmarAct. SMARPOD 110.45. URL, https://www.smaract.com/smarpod/product/smarpod-110-45; 2019. https://www.smaract.com/smarpod/product/smarpod-110-45.

[7] Carpenter A. ALIO HH brochure 2019. URL, https://www.optoprim.it/wp-content/uploads/2020/02/ALIO-HH-Brochure-2019.pdf.

[8] Pyschny NK. Auslegung und Optimierung von festkörpergekoppelten Parallelkinematiken für die Montage von optischen Komponenten, Ph.D. thesis. Rheinisch-Westfälischen Technischen Hochschule Aachen; 2014.

[9] K. Spanner, B. Koc, Piezoelectric motors, an overview, Actuators 5 (1). doi:10.3390/act5010006.

[10] Li J, Huang H, Morita T. Stepping piezoelectric actuators with large working stroke for nano-positioning systems: a review. Sens Actuators, A 2019;292(April):39–51. https://doi.org/10.1016/j.sna.2019.04.006.

[11] Henein S. Flexure delicacies. Mechanical Sciences 2012;3:1–4. https://doi.org/10.5194/ms-3-1-2012.

[12] Folkersma KGP, Boer SE, Brouwer DM, Herder JL, Soemers H. A 2-DOF large stroke flexure based positioning mechanism. In: ASME 2012 international design engineering technical conferences and computers and information in engineering conference. American Society of Mechanical Engineers; 2012. p. 221–8.

[13] Gunnink K, Aarts R, Brouwer DM. Performance optimization of large stroke flexure hinges for high stiffness and eigenfrequency. In: Proceedings of the 28th annual meeting of the American society for precision engineering (ASPE), saint Paul, MN; 2013. p. 20-5.

[14] Wiersma D, Boer S, Aarts R, Brouwer D. Design and performance optimization of large stroke spatial flexures. J Comput Nonlinear Dynam 2014;9(1):011016. https://doi.org/10.1115/1.4025669.

[15] Naves M, Brouwer D, Aarts R. Building block-based spatial topology synthesis method for large-stroke flexure hinges. J Mech Robot 2017;9(4):041006. https://doi.org/10.1115/1.4036225.
