Reasoning over Permissions Regions in Concurrent Separation Logic

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PPLV seminar, UCL Dept of Computer Science

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Concurrent separation logic (CSL)

• Concurrent separation logic (CSL) is based upon the following concurrency rule:

\[
\begin{align*}
\{ A_1 \} \; C_1 \; \{ B_1 \} & \quad \{ A_2 \} \; C_2 \; \{ B_2 \} \\
\{ A_1 \otimes A_2 \} \; C_1 \; || \; C_2 \; \{ B_1 \otimes B_2 \}
\end{align*}
\]

• This rule says that concurrent threads behave compositionally with respect to separation (\( \otimes \)) between their respective memory resources.

• However, separation (\( \otimes \)) typically allows some sharing of read-only resources between threads, which can be controlled using fractional permissions.
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\begin{array}{c}
\{ A_1 \} \, C_1 \{ B_1 \} \quad \{ A_2 \} \, C_2 \{ B_2 \} \\
\{ A_1 \odot A_2 \} \, C_1 || C_2 \{ B_1 \odot B_2 \}
\end{array}
\]

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• Concurrent separation logic (CSL) is based upon the following concurrency rule:

\[
\{A_1\} C_1 \{B_1\} \quad \{A_2\} C_2 \{B_2\} \\
\{A_1 \odot A_2\} C_1 \parallel C_2 \{B_1 \odot B_2\}
\]

• This rule says that concurrent threads behave compositionally with respect to separation (\(\odot\)) between their respective memory resources.

• However, separation \(\odot\) typically allows some sharing of read-only resources between threads, which can be controlled using fractional permissions.
Fractional permissions

• Fractional permissions are intended to allow the division of memory into two or more “read-only copies”.

Separation \( \otimes \) denotes the division of a heap using this composition. E.g., we have
\[
\frac{3}{13} \rightarrow d \otimes \frac{5}{13} \rightarrow d \equiv \frac{5}{13} \rightarrow d.
\]
Fractional permissions

- **Fractional permissions** are intended to allow the division of memory into two or more “read-only copies”.

- **Permissions** can be represented e.g. as rationals in the open interval $(0, 1]$. 1 is the write permission and values in $(0, 1)$ are read-only permissions.
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- **Heaps** store a data value and permission at each location. Heaps can be composed provided they agree where they overlap; we add the permissions at overlapping locations.
Fractional permissions

- **Fractional permissions** are intended to allow the division of memory into two or more “read-only copies”.

- **Permissions** can be represented e.g. as rationals in the open interval \((0, 1]\). 1 is the write permission and values in \((0, 1)\) are read-only permissions.

- **Heaps** store a data value and permission at each location. Heaps can be composed provided they agree where they overlap; we add the permissions at overlapping locations.

- **Separation** \(\ast\) denotes the division of a heap using this composition. E.g., we have

\[
x \overset{0.5}{\mapsto} d \ast x \overset{0.5}{\mapsto} d \equiv x \mapsto d .
\]
Typical CSL proof structure

\[
\begin{align*}
\{ x \overset{0.5}{\rightarrow} d \} \\ foo(); \\
\{ x \overset{0.5}{\rightarrow} d \ast A \}
\end{align*}
\vline
\begin{align*}
\{ x \overset{0.5}{\rightarrow} d \} \\
bar(); \\
\{ x \overset{0.5}{\rightarrow} d \ast B \}
\end{align*}
\]
Typical CSL proof structure

\[
\{ x \mapsto d \}
\]

\[
\{ x \overset{0.5}{\mapsto} d \}
\]

\[
\text{foo();}
\]

\[
\{ x \overset{0.5}{\mapsto} d \}
\]

\[
\text{bar();}
\]

\[
\{ x \overset{0.5}{\mapsto} d \}
\]

\[
\{ x \overset{0.5}{\mapsto} d \}
\]

\[
\{ x \overset{0.5}{\mapsto} A \}
\]

\[
\{ x \overset{0.5}{\mapsto} B \}
\]

\[
\{ x \overset{0.5}{\mapsto} A \}
\]

\[
\{ x \overset{0.5}{\mapsto} B \}
\]

\[
\{ x \overset{0.5}{\mapsto} A \}
\]

\[
\{ x \overset{0.5}{\mapsto} B \}
\]

BUT... we hit problems when we use permissions to describe regions of memory and not just pointers.
Typical CSL proof structure

\[
\begin{align*}
\{x \mapsto d\} \\
\{x \mapsto d \odot x \mapsto d\} \\
\{x \mapsto d\} \quad \| \quad \{x \mapsto d\} \\
\text{foo();} \quad \| \quad \text{bar();} \\
\{x \mapsto d \ast A\} \quad \| \quad \{x \mapsto d \ast B\}
\end{align*}
\]
Typical CSL proof structure

\[
\begin{align*}
\{ x \mapsto d \} \\
\{ x \mapsto d \otimes x \mapsto d \} \\
\{ x \mapsto d \} & \quad | \quad \{ x \mapsto d \} \\
\text{foo();} & \quad | \quad \text{bar();} \\
\{ x \mapsto d \ast A \} & \quad | \quad \{ x \mapsto d \ast B \} \\
\{ x \mapsto d \otimes x \mapsto d \ast A \otimes B \}
\end{align*}
\]
Typical CSL proof structure

\[
\begin{align*}
\{ x \mapsto d \} \\
\{ x \overset{0.5}{\mapsto} d \ast x \overset{0.5}{\mapsto} d \}
\end{align*}
\]

\[
\begin{align*}
\{ x \overset{0.5}{\mapsto} d \} \\
\text{foo();} \\
\{ x \overset{0.5}{\mapsto} d \ast A \} \\
\{ x \overset{0.5}{\mapsto} d \ast B \}
\end{align*}
\]

\[
\begin{align*}
\{ x \overset{0.5}{\mapsto} d \ast x \overset{0.5}{\mapsto} d \ast A \ast B \}
\end{align*}
\]

\[
\begin{align*}
\{ x \mapsto d \ast A \ast B \}
\end{align*}
\]

BUT... we hit problems when we use permissions to describe regions of memory and not just pointers.
Typical CSL proof structure

\[ \{ x \mapsto d \} \]
\[ \{ x^{0.5} \mapsto d \otimes x^{0.5} \mapsto d \} \]
\[ \{ x^{0.5} \mapsto d \} \quad \text{foo}(); \quad \{ x^{0.5} \mapsto d \} \quad \text{bar}(); \]
\[ \{ x^{0.5} \mapsto d \ast A \} \quad \{ x^{0.5} \mapsto d \ast B \} \]
\[ \{ x^{0.5} \mapsto d \otimes x^{0.5} \mapsto d \ast A \otimes B \} \]
\[ \{ x \mapsto d \ast A \otimes B \} \]

BUT... we hit problems when we use permissions to describe regions of memory and not just pointers.
The first difficulty

Suppose we define linked list segments using $\ast$:

$$\text{ls } x y \overset{\text{def}}{=} (x = y \land \text{emp}) \lor (\exists z. x \mapsto z \ast \text{ls } z y).$$

Now consider traversal procedure $\text{foo}(x,y)$:

$$\text{foo}(x,y) \{ \text{if } x = y \text{ then return; else } \text{foo}([x],y) \}.$$ This satisfies the following Hoare triple:

$$\{ (\text{ls } x y) \} \text{foo}(x,y); \{ (\text{ls } x y) \}.$$ However, we will have difficulties proving so!
The first difficulty

Suppose we define linked list segments using ⊗:

\[ \text{ls} \ x \ y \ =_{\text{def}} (x = y \land \text{emp}) \lor (\exists z. x \mapsto z \otimes \text{ls} \ z \ y) . \]

Now consider traversal procedure \text{foo}(x,y):

\[
\text{foo}(x,y) \{ \text{if } x=y \text{ then return; else } \text{foo}([x], y); \}
\]
The first difficulty

Suppose we define linked list segments using ∗:

\[
\text{ls } x \, y \ =_{\text{def}} \ (x = y \land \text{emp}) \lor (\exists z. x \mapsto z \ast \text{ls } z \, y).
\]

Now consider traversal procedure \( \text{foo}(x, y) \):

\[
\text{foo}(x, y) \{ \text{ if } x = y \text{ then return; else } \text{foo}([x], y); \}
\]

This satisfies the following Hoare triple:

\[
\{(\text{ls } x \, y)^{0.5}\} \text{foo}(x, y); \{(\text{ls } x \, y)^{0.5}\}
\]

However, we will have difficulties proving so!
Failed proof attempt

\{ (ls x y)^{0.5} \}

foo(x,y) {
  if x=y then return;
  else

  foo([x],y);

  \}

\{ (ls x y)^{0.5} \}
Failed proof attempt

\{(lx y)^{0.5}\}

foo(x,y) {
  if x=y then return;  \{(lx y)^{0.5}\}
  else

  foo([x],y);

  }  \{(lx y)^{0.5}\}
Failed proof attempt

\[ \{(ls \ x \ y)^{0.5}\} \]

foo(x,y) {
  if x=y then return; \{ (ls \ x \ y)^{0.5} \}
  else \{ x \neq y \land (x \mapsto z \odot ls \ z \ y)^{0.5} \} 
}

foo([x],y);

\{ (ls \ x \ y)^{0.5} \}
Failed proof attempt

\{ (|s x y|)^{0.5} \} 

foo(x,y) { 
if x = y then return; \{ (|s x y|)^{0.5} \} 
else 
\{ x \neq y \land (x \mapsto z \circledast |s z y|)^{0.5} \} 
\{ x \neq y \land (x \overset{0.5}{\mapsto} z \circledast (|s z y|)^{0.5}) \} 

foo([x],y); 

} \{ (|s x y|)^{0.5} \}
Failed proof attempt

\[ \{ (ls\ x\ y)^{0.5} \} \]

foo(x,y) {
    if x=y then return; \[\{ (ls\ x\ y)^{0.5} \}\]
    else
        foo([x],y);

    \{ x \neq y \land (x \mapsto z \circ (ls\ z\ y)^{0.5}) \}
}

\[ \{ (ls\ x\ y)^{0.5} \} \]
Failed proof attempt

\[
\{ (ls \, x \, y)^{0.5} \} \\
\text{foo}(x,y) \{ \\
\text{if } x=y \text{ then return; } \{ (ls \, x \, y)^{0.5} \} \\
\text{else } \{ \{ x \neq y \land (x \mapsto z \odot (ls \, z \, y)^{0.5}) \} \\
\{ x \neq y \land (x \mapsto 0.5 z \odot (ls \, z \, y)^{0.5}) \} \} \\
\text{foo}([x],y); \\
\} \{ (ls \, x \, y)^{0.5} \} 
\]
Failed proof attempt

\[
\{ (ls x y)^{0.5} \} \\
\\
\text{foo}(x, y) \{ \\
\text{if } x = y \text{ then return; } \\
\text{else} \\
\text{foo}([x], y); \\
\} \quad \{ (ls x y)^{0.5} \}
\]
Failed proof attempt

\[
\{ (lx \ y)^{0.5} \} \\
\text{foo}(x, y) \ { \\
\text{if } x=y \text{ then return; } \{ (lx \ y)^{0.5} \} \\
\text{else } \{ \\
\{ x \neq y \land (x \mapsto z \odot lx \ y)^{0.5} \} \\
\{ x \neq y \land (x \xrightarrow{0.5} z \odot (lx \ y)^{0.5}) \} \\
\} \\
\text{foo}([x], y); \\
\} \ { \\
\{ (lx \ y)^{0.5} \} \\
\} \\
\]
Reason for failure

• The highlighted inference step is not sound:

\[ x^{0.5} \rightarrow z \otimes (ls \ z \ y)^{0.5} \not\models (x \rightarrow z \otimes ls \ z \ y)^{0.5} \]
Reason for failure

- The highlighted inference step is not sound:

\[ x^{0.5} \rightarrow z \odot (ls \ z \ y)^{0.5} \not\models (x \mapsto z \odot ls \ z \ y)^{0.5}. \]

- This is because the pointer and list segment can overlap on the LHS, but not on the RHS. In general,

\[ A^\pi \odot B^\pi \not\models (A \odot B)^\pi. \]
Reason for failure

- The highlighted inference step is not sound:
  \[ x \xrightarrow{0.5} z \odot (\text{ls } z \ y)^{0.5} \nvdash (x \mapsto z \odot \text{ls } z \ y)^{0.5}. \]

- This is because the pointer and list segment can overlap on the LHS, but not on the RHS. In general,
  \[ A^\pi \odot B^\pi \nvdash (A \odot B)^\pi. \]

- But if we use strong separation \(*\), which enforces disjointness of heaps, to define our list segments, the proof above goes through (since \((A * B)^\pi \equiv A^\pi * B^\pi\)).
The second difficulty

The triple \( \{ ls \ x \ y \} \ foo(x,y); \ || \ foo(x,y); \{ ls \ x \ y \} \) is correct, but again the proof fails:

\[
\begin{align*}
\{(ls \ x \ y)^{0.5}\} & \ || \ {(ls \ x \ y)^{0.5}\} \\
foo(x,y); & \ || \ foo(x,y); \\
\{(ls \ x \ y)^{0.5}\} & \ || \ {(ls \ x \ y)^{0.5}\}
\end{align*}
\]
The second difficulty

The triple $\{ls\, x\, y\}$ $\text{foo}(x,y); \parallel \text{foo}(x,y); \{ls\, x\, y\}$ is correct, but again the proof fails:

$$\{ls\, x\, y\}$$

$$\{(ls\, x\, y)^{0.5}\} \parallel \{(ls\, x\, y)^{0.5}\}$$

foo$(x,y);$ $\parallel$ foo$(x,y);$
The second difficulty

The triple \( \{ \text{ls } x \ y \} \) \( \text{foo}(x,y); \ || \text{foo}(x,y); \{ \text{ls } x \ y \} \) is correct, but again the proof fails:

\[
\begin{align*}
\{ \text{ls } x \ y \} \\
\{(\text{ls } x \ y)^{0.5} \ast (\text{ls } x \ y)^{0.5}\} \\
\{(\text{ls } x \ y)^{0.5}\} & \ || \{(\text{ls } x \ y)^{0.5}\} \\
\text{foo}(x,y); & \ || \text{foo}(x,y); \\
\{(\text{ls } x \ y)^{0.5}\} & \ || \{(\text{ls } x \ y)^{0.5}\}
\end{align*}
\]
The second difficulty

The triple \( \{ls \, x \, y\} \text{foo}(x,y); \parallel \text{foo}(x,y); \{ls \, x \, y\} \) is correct, but again the proof fails:

\[
\begin{align*}
\{ls \, x \, y\} \\
\{ (ls \, x \, y)^{0.5} \odot (ls \, x \, y)^{0.5} \}
\end{align*}
\]

\[
\begin{align*}
\{ (ls \, x \, y)^{0.5} \} & \parallel \{ (ls \, x \, y)^{0.5} \} \\
\text{foo}(x,y); & \parallel \text{foo}(x,y); \\
\{ (ls \, x \, y)^{0.5} \} & \parallel \{ (ls \, x \, y)^{0.5} \}
\end{align*}
\]

\[
\{ (ls \, x \, y)^{0.5} \odot (ls \, x \, y)^{0.5} \}
\]
The second difficulty

The triple \( \{ls \ x \ y\} \text{foo}(x,y); \ || \ \text{foo}(x,y); \ \{ls \ x \ y\} \) is correct, but again the proof fails:

\[
\begin{align*}
&\{ls \ x \ y\} \\
&\{ (ls \ x \ y)^{0.5} \odot (ls \ x \ y)^{0.5} \} \\
&\{ (ls \ x \ y)^{0.5} \} \ || \ \{ (ls \ x \ y)^{0.5} \} \\
&\text{foo}(x,y); \ || \ \text{foo}(x,y); \\
&\{ (ls \ x \ y)^{0.5} \} \ || \ \{ (ls \ x \ y)^{0.5} \} \\
&\{ (ls \ x \ y)^{0.5} \odot (ls \ x \ y)^{0.5} \} \\
&\{ls \ x \ y\}
\end{align*}
\]
The second difficulty

The triple \( \{ls\ x\ y\} \ foo(x,y); \ || \ foo(x,y); \ {ls\ x\ y}\) is correct, but again the proof fails:

\[
\begin{align*}
\{ls\ x\ y\} \\
\{(ls\ x\ y)^{0.5} \odot (ls\ x\ y)^{0.5}\} \\
\{(ls\ x\ y)^{0.5}\} \ || \ \{(ls\ x\ y)^{0.5}\} \\
foo(x,y); \ || \ foo(x,y); \\
\{(ls\ x\ y)^{0.5}\} \ || \ \{(ls\ x\ y)^{0.5}\} \\
\{(ls\ x\ y)^{0.5} \odot (ls\ x\ y)^{0.5}\} \\
\xrightarrow{\ast} \{ls\ x\ y\}
\end{align*}
\]
Reason for second failure

- The highlighted inference step is not sound:

\[(|s x y|^{0.5} \ast (|s x y|^{0.5}) \neq |s x y|).\]
Reason for second failure

• The highlighted inference step is not sound:

\[(\text{ls } x \ y)^{0.5} \otimes (\text{ls } x \ y)^{0.5} \not\models \text{ls } x \ y.\]

• This is because the list segments on the LHS might be (partially) non-overlapping. In general,

\[A^{0.5} \otimes A^{0.5} \not\models A.\]
Reason for second failure

• The highlighted inference step is not sound:

\[(\text{ls}\ x\ y)^{0.5} \otimes (\text{ls}\ x\ y)^{0.5} \nvdash \text{ls}\ x\ y.\]

• This is because the list segments on the LHS might be (partially) non-overlapping. In general,

\[A^{0.5} \otimes A^{0.5} \nvdash A.\]

• When splitting the list segment \text{ls}\ x\ y, we lost the info that the two formulas \((\text{ls}\ x\ y)^{0.5}\) are copies of the same region.
Proposed solution: nominal labels

- We introduce **nominal labels** (from hybrid logic), where a nominal $\alpha$ is interpreted as denoting a unique heap.
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• Any formula of the form $\alpha \land A$ then obeys the principle

\[(\alpha \land A)^\sigma \odot (\alpha \land A)^\pi \equiv (\alpha \land A)^{\sigma \oplus \pi}\]

where $\oplus$ is addition on permissions.
Proposed solution: nominal labels

- We introduce nominal labels (from hybrid logic), where a nominal $\alpha$ is interpreted as denoting a unique heap.

- Any formula of the form $\alpha \land A$ then obeys the principle

$$ (\alpha \land A)^\sigma \circledast (\alpha \land A)^\pi \equiv (\alpha \land A)^{\sigma \oplus \pi} $$

where $\oplus$ is addition on permissions.

- Thus we can repair the faulty CSL proof above by replacing every instance of $ls\, x\, y$ by $\alpha \land ls\, x\, y$ (and adding an initial step in which we introduce the fresh label $\alpha$).
What’s in the paper?

• We define an assertion language including both weak $\mathcal{\otimes}$ and strong $\mathcal{\star}$ separating conjunctions, and nominal labels $\alpha$.
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- We also include hybrid logic’s jump modality $@\alpha A$, meaning $A$ is true at $\alpha$, which is useful in treating more complex sharing examples.
What’s in the paper?

• We define an assertion language including both weak $\otimes$ and strong $\ast$ separating conjunctions, and nominal labels $\alpha$.

• We also include hybrid logic’s jump modality $@_{\alpha}A$, meaning $A$ is true at $\alpha$, which is useful in treating more complex sharing examples.

• We formally establish the needed principles, including

\[
(A \ast B)^\pi \equiv A^\pi \ast B^\pi \\
(\alpha \land A)^\sigma \otimes (\alpha \land A)^\pi \equiv (\alpha \land A)^{\sigma \oplus \pi}
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What’s in the paper?

• We define an assertion language including both weak ⊗ and strong ∗ separating conjunctions, and nominal labels α.

• We also include hybrid logic’s jump modality @αA, meaning A is true at α, which is useful in treating more complex sharing examples.

• We formally establish the needed principles, including

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\begin{align*}
(A \ast B)^\pi & \equiv A^\pi \ast B^\pi \\
(\alpha \land A)^\sigma \ast (\alpha \land A)^\pi & \equiv (\alpha \land A)^{\sigma \oplus \pi}
\end{align*}
\]

• Finally we show how our assertion language can be used in CSL to verify various concurrent programs with sharing.
Directions for future work

• Implementation and automation
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- Implementation and automation
- Specification inference and biabduction
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• Implementation and automation
• Specification inference and biabduction
• Identify tractable fragments
Thanks for listening!

James Brotherston, Diana Costa, Aquinas Hobor and John Wickerson.
Reasoning over Permissions Regions in Concurrent Separation Logic.
In Proc. CAV-2020.