Generation Mass Hierarchy in Superstring Derived Models

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ABSTRACT

I discuss the problem of generation mass hierarchy in the context of realistic superstring models which are constructed in the free fermionic formulation. These models correspond to models which are compactified on $Z_2 \times Z_2$ orbifold. I suggest that the hierarchy among the generations results from horizontal symmetries, which arise from the compactification. In particular, I show that in a class of free fermionic standard–like models, the suppression of the mass terms for the lightest generation is a general, and unambiguous, characteristic of these models. I show that the mixing between the generations is suppressed due to the horizontal symmetries. I conclude that these models may potentially explain the generation mass hierarchy.

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1. Introduction

One of the most fundamental problems in high energy physics is the origin and hierarchy of the fermion masses. Why are the three fermion generations, which are universal in their gauge interactions, separated by orders of magnitude in their masses? In this respect the Standard Model, and point field theories in general, can only be considered as successful attempts to parameterize the observed mass spectrum. The Standard Model uses thirteen parameters to parameterize the observed spectrum. Grand Unified Theories (GUTs) reduce the number of free parameters and can explain inter family relations between some of the masses. However, GUTs do not explain the reason for the hierarchy between the generations nor the smallness of the mixing between different generations. One has to impose additional horizontal symmetries and specific choices of Higgs fields to try to explain the family hierarchies. Within the context of point field theories the problem of generation mass hierarchy looks rather arbitrary and some guiding principle is still missing.

Unlike point field theories, superstring theories provide a unique framework to understand the generations mass hierarchy in terms of symmetries which are derived in specific superstring models. The consistency of superstring theory imposes a certain number of degrees of freedom on the models. In the closed heterotic string [1], of the 26 right–moving bosonic degrees of freedom, 16 are compactified on a flat torus and produce the observable and hidden gauge groups. Six right–moving bosonic degrees of freedom, combined with six left–moving degrees of freedom, are compactified on Calabi–Yau manifold [2], or on an orbifold [3]. Alternatively, all the extra degrees of freedom, beyond the four space–time dimensions, can be taken as bosonic [4], or fermionic [5], internal degrees of freedom propagating on the string world–sheet. The different interpretations are expected to be related. Horizontal symmetries which arise from the compactification will be responsible for the generations mass hierarchy.

To illustrate how the compactified space may be responsible for creating the
generation mass hierarchy, I consider models which are constructed in the free fermionic formulation of the heterotic string. Of these, I focus mainly on a specific class of standard–like models [6,7,8]. The standard–like models have several unique characteristics. First, they have three and only three generations of chiral fermions. The chiral generation states are obtained from the three distinct twisted sectors of the corresponding orbifold model and none from the untwisted sector. Second, the standard–like models suggest an explanation for the heaviness of the top quark relative to the lighter quarks and leptons. At the trilinear level of the superpotential only the top quark gets a non vanishing mass term. The bottom quark and the lighter quarks and leptons get their mass terms from nonrenormalizable terms, which are suppressed relative to the leading cubic level terms. Finally, the standard–like models naturally evade the problem with proton decay from dimension four operators that usually exist in superstring models which are based on an intermediate GUT symmetry [8].

2. Realistic free fermionic models

In the free fermionic formulation of the heterotic string in four dimensions all the world–sheet degrees of freedom required to cancel the conformal anomaly are represented in terms of free fermions propagating on the string world–sheet. Under parallel transport around a noncontractible loop the fermionic states pick up a phase. A model in this construction is defined by a set of basis vectors of boundary conditions for all world–sheet fermions. These basis vectors are constrained by the string consistency requirements (e.g. modular invariance) and completely determine the vacuum structure of the model. The physical spectrum is obtained by applying the generalized GSO projections. The low energy effective field theory is obtained by S–matrix elements between external states. The Yukawa couplings and higher order nonrenormalizable terms in the superpotential are obtained by calculating correlators between vertex operators. For a correlator to be nonvanishing all the symmetries of the model must be conserved. Thus, the boundary condition vectors determine the phenomenology of the models.
The first five vectors (including the vector $\mathbf{1}$) in the basis are

\[ S = (1, \ldots, 1, 0, \ldots, 0 | 0, \ldots, 0), \]  
(1a)

\[ b_1 = (1, \ldots, 1, 0, \ldots, 0 | 1, \ldots, 1, 0, \ldots, 0), \]  
(1b)

\[ b_2 = (1, \ldots, 1, 0, \ldots, 0 | 1, \ldots, 1, 0, \ldots, 0), \]  
(1c)

\[ b_3 = (1, \ldots, 1, 0, \ldots, 0 | 1, \ldots, 1, 0, \ldots, 0). \]  
(1d)

with the choice of generalized GSO projections

\[ c \begin{pmatrix} b_i \\ b_j \end{pmatrix} = c \begin{pmatrix} b_i \\ S \end{pmatrix} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1, \]  
(2)

and the others given by modular invariance. This set is referred to as the NAHE\textsuperscript{*} set. The NAHE set is common to all the realistic models constructed in the free fermionic formulation [9,10,11,6,7,8,17] and is a basic set common to all the models which I discuss. The sector $S$ generates $N = 4$ space–time supersymmetry, which is broken to $N = 2$ and $N = 1$ space–time supersymmetry by $b_1$ and $b_2$, respectively. Restricting $b_j \cdot S = 0 \text{mod} 2$, and $c \begin{pmatrix} S \\ b_j \end{pmatrix} = \delta_{b_j}$, for all basis vector $b_j \epsilon B$ guarantees the existence of $N = 1$ space–time supersymmetry. The superpartners from a given sector $\alpha \epsilon \Xi$ are obtained from the sector $S + \alpha$. The gauge group after the NAHE set is $SO(10) \times E_8 \times SO(6) \times SO(6)$ with $N = 1$ space–time supersymmetry. The three $SO(6)$ symmetries are horizontal, generational dependent, symmetries.

Models based on the NAHE set correspond to models that are based on $Z_2 \times Z_2$ orbifold. This correspondence is illustrated by extending the $SO(10)$ symmetry to

\[ * \text{ This set was first constructed by Nanopoulos, Antoniadis, Hagelin and Ellis (NAHE) in the construction of the flipped } SU(5) \text{ [9]. } nahe=\text{pretty, in Hebrew.} \]
Adding the vector

\[ X = (0, \cdots, 0 | 1, \cdots, 1, 0, \cdots, 0) \]  

(3)

to the NAHE set, extends the gauge symmetry to \( E_6 \times U(1)^2 \times SO(4)^3 \). The set \( \{1, S, I = 1 + b_1 + b_2 + b_3, X\} \) produces an \( E_8 \times E_8 \) toroidal compactification on a \( SO(12) \) lattice. The \( SO(12) \) symmetry is reproduced for special values of the metric and the antisymmetric tensor. The metric, \( g_{ij} \), is given by the cartan matrix of \( SO(12) \) and the antisymmetric tensor, \( b_{ij} \), is given by

\[
b_{ij} = \begin{cases} 
g_{ij} & ; i > j, \\ 0 & ; i = j, \\ -g_{ij} & ; i < j. \\
\end{cases} \]

(4)

The sectors \( b_1 \) and \( b_2 \) correspond to the \( Z_2 \times Z_2 \) twist and break the symmetry to \( SO(4)^3 \times E_6 \times U(1)^2 \times E_8 \). The fermionic states \( \{\chi^{12}, \chi^{34}, \chi^{56}\} \) and \( \{\bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} \) give the usual “standard--embedding”, with \( b(\chi^{12}, \chi^{34}, \chi^{56}) = b(\bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3) \). The \( U(1) \) current of the left–moving \( N = 2 \) world–sheet supersymmetry is given by \( J(z) = i \partial_z (\chi^{12} + \chi^{34} + \chi^{56}) \), and the \( U(1) \) charges in the decomposition of \( E_6 \) under \( SO(10) \times U(1) \) are given by the world–sheet current \( \bar{\eta}^1 \bar{\eta}^{1*} + \bar{\eta}^2 \bar{\eta}^{2*} + \bar{\eta}^3 \bar{\eta}^{3*} \). The sectors \( (b_1; b_1 + X), (b_2; b_2 + X) \) and \( (b_3; b_3 + X) \) each give eight 27 of \( E_6 \), and correspond to the twisted sectors of the orbifold model. The \( (NS; NS + X) \) sector gives in addition to the vector bosons and spin two states, three copies of scalar representations in \( 27 + \overline{27} \) of \( E_6 \). This sector corresponds to the untwisted sector of the orbifold model.

In this model the only internal fermionic states which count the multiplets of \( E_6 \) are the real internal fermions \( \{y, w|\bar{y}, \bar{w}\} \). This is observed by writing the degenerate vacuum of the sectors \( b_j \) in a combinatorial notation. The vacuum of the sectors \( b_j \) contains twelve periodic fermions. Each periodic fermion gives rise to a two dimensional degenerate vacuum \(|+\rangle \) and \(|-\rangle \) with fermion numbers 0 and
−1, respectively. The GSO operator, is a generalized parity operator, which selects states with definite parity. After applying the GSO projections, we can write the degenerate vacuum of the sector $b_1$ in combinatorial form

$$\left[ \binom{4}{0} + \binom{4}{2} + \binom{4}{4} \right] \left[ \binom{2}{0} \binom{5}{0} + \binom{5}{2} + \binom{5}{4} \right] \left( \binom{1}{0} + \binom{2}{0} \binom{5}{1} + \binom{5}{3} + \binom{5}{5} \right) \left( \binom{1}{0} + \binom{2}{2} \right)$$

where $4 = \{y^3, y^4, y^5, y^6\}$, $2 = \{\psi^\mu, \chi^{12}\}$, $5 = \{\psi^{1,\cdots,5}\}$ and $1 = \{\eta^1\}$. The combinatorial factor counts the number of $|−\rangle$ in the degenerate vacuum of a given state. The two terms in the curly brackets correspond to the two components of a Weyl spinor. The $10 + 1$ in the $27$ of $E_6$ are obtained from the sector $b_j + X$. From Eq. (5) it is observed that the states which count the multiplicities of $E_6$ are the internal fermionic states $\{y^{3,\cdots,6} | \bar{y}^{3,\cdots,6}\}$. A similar result is obtained for the sectors $b_2$ and $b_3$ with $\{y^{1,2}, \omega^{5,6} | \bar{y}^{1,2}, \bar{\omega}^{5,6}\}$ and $\{\omega^{1,\cdots,4} | \bar{\omega}^{1,\cdots,4}\}$ respectively, which suggests that these twelve states correspond to a six dimensional compactified orbifold with Euler characteristic equal to 48. The number of fixed points in the $Z_2 \times Z_2$ orbifold on a $SO(12)$ lattice is 48 and matches twice the number of generations in the fermionic model. The correspondence between the fermionic models and the $Z_2 \times Z_2$ orbifold will be discussed further in Ref. [12]. The important point to realize is that in the fermionic formulation the 12 internal fermionic states, $\{y, w | \bar{y}, \bar{\omega}\}$, play the role of the six dimensional “compactified space” of the orbifold. The boundary conditions, assigned to these internal fermions, determine many of the properties of the low energy spectrum.

Turning back to the NAHE set. At the level of the NAHE set, the sectors $b_1$, $b_2$ and $b_3$ each produce 16 chiral generations. There exists a permutation symmetry between these sectors. The $SO(6)^3$ horizontal symmetries constrain the possible interactions.

The number of generations is reduced by adding three additional vectors to the NAHE set. The standard–like models use $Z_2 \times Z_2 \times Z_4$, where the $Z_4$ twist is used
to break $SO(2n) \to SU(n) \times U(1)$. The boundary conditions of the gauge sector are fixed by requiring that the $SO(10)$ symmetry breaks to the Standard Model, and by modular invariance constraints [8]. The assignment of boundary conditions to the set of real internal fermions $\{y, \omega|\bar{y}, \bar{\omega}\}$ distinguishes between different models. The possible assignments are constrained by requiring a net chirality of three generations. One half of the generations is projected out by the $Z_4$ twist. The assignment of boundary conditions to the real fermions, $\{y, \omega|\bar{y}, \bar{\omega}\}$, is constrained by requiring that the combinatorial factor in Eq. (5) reduces to one, for each of the $b_1, b_2$ and $b_3$ sectors. At the same time the three $SO(6)$ horizontal symmetries are broken to horizontal $U(1)$ symmetries. Three $U(1)$s, $U(1)_{r_j}$ ($j = 1, 2, 3$), correspond to the world–sheet currents $\bar{\eta}_1\eta_1^*, \bar{\eta}_2\eta_2^*$ and $\bar{\eta}_3\eta_3^*$. These $U(1)$ symmetries are a generic feature of realistic free fermionic models [9,10,11,6,7,8]. Additional $U(1)$ symmetries arise from complexification of real right–moving fermions from the set $\{\bar{y}, \bar{\omega}\}$. In the standard–like models, requiring that the Higgs doublets from the Neveu–Schwarz sector survive the GSO projections, imposes at least three additional $U(1)$ symmetries [8]. One for each sector $b_1, b_2$ and $b_3$. In the models of tables 1 (model 1) [6] and 2 (model 2) [7], they correspond to the right–moving world–sheet currents $\bar{y}_3\bar{y}_6, \bar{y}_1\bar{\omega}_5$ and $\bar{\omega}_2\bar{\omega}_4$, denoted by $U(1)_{r_j+3}$ ($j = 1, 2, 3$). Other choices for the low energy gauge group do not impose such a restriction. In addition to these symmetries, for every right–moving $U(1)$ symmetry correspond a left–moving global $U(1)$ symmetry. The first three, $U(1)_{\ell_j}$ ($j = 1, 2, 3$), correspond to the charges of the supersymmetry generator $\chi^{12}, \chi^{34}$ and $\chi^{56}$. These are common to all the realistic free fermionic models. Additional global $U(1)$ symmetries arise from additional complexified left–moving fermions. In the standard–like models of tables 1 and 2, the last three, $U(1)_{\ell_{j+3}}$ ($j = 1, 2, 3$), correspond to the complexified left–moving fermions $y^3y^6$, $y^4\omega^5$ and $\omega^2\omega^4$. Finally, the models contain Ising model sigma operators, which are obtained by pairing a left–moving real fermion with a right–moving real fermion. In the standard–like models [6,7] there are six Ising model operators, $\sigma^1_\pm = \{\omega^1\bar{\omega}^1, y^2\bar{y}^2, \omega^3\bar{\omega}^3, y^4\bar{y}^4, y^5\bar{y}^5, \omega^6\bar{\omega}^6\}_\pm$. These symmetries are additional horizontal symmetries, which constrain the possible F–terms in the
superpotential. Thus, each sector $b_1$, $b_2$ and $b_3$ produces one generation, with horizontal symmetries. The notation used in tables 1 and 2, emphasizes the division of the internal world-sheet fermions among the three generations.

Trilinear and nonrenormalizable contributions to the superpotential are obtained by calculating correlators between vertex operators $[13,14]$,

$$A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle,$$

(6)

where $V_i^f$ ($V_i^b$) are the fermionic (scalar) components of the vertex operators. The non vanishing terms are obtained by applying the rules of Ref. [14]. To obtain the correct ghost charge some of the vertex operators are picture changed by taking

$$V_{q+1}(z) = \lim_{w \to z} \exp(c)(w)T_F(w)V_q(z),$$

(7)

where $T_F$ is the super current and in the fermionic construction is given by

$$T_F = \psi^\mu \partial_\mu X + i \sum_{I=1}^{6} \chi_i y_i \omega_i = T_F^0 + T_F^{-1} + T_F^{+1}$$

(8)

with

$$T_F^{-1} = e^{-i\chi_1^2} \tau_{12} + e^{-i\chi_3^4} \tau_{34} + e^{-i\chi_5^6} \tau_{56} \quad ; \quad T_F^{-1} = (T_F^{+1})^*$$

(9)

where $\tau_{ij} = \frac{i}{\sqrt{2}}(y^i \omega^j + y^j \omega^i)$ and $e^{i\chi^j} = \frac{1}{\sqrt{2}}(\chi^j + i\chi^j$).

Several observations simplify the analysis of the potential non vanishing terms. First, it is observed that only the $T_F^{+1}$ piece of $T_F$ contributes to $A_N$ [14]. Second, in the standard–like models [6,7] the pairing of left–moving fermions is $y^1 \omega^5$, $\omega^2 \omega^4$ and $y^3 y^6$. One of the fermionic states in every term $y^i \omega^j$ ($i = 1, \ldots, 6$) is complexified and therefore can be written, for example for $y^3$ and $y^6$, as

$$y^3 = \frac{1}{\sqrt{2}}(e^{iy^3 y^6} + e^{-iy^3 y^6}), \quad y^6 = \frac{1}{\sqrt{2}}(e^{iy^3 y^6} - e^{-iy^3 y^6}).$$

(10)

Consequently, every picture changing operation changes the total $U(1)_\ell = U(1)_{\ell_4} + U(1)_{\ell_5} + U(1)_{\ell_6}$ charge by $\pm 1$. An odd (even) order term requires an even (odd)
number of picture changing operations to get the correct ghost number \([14]\). Thus, for \(A_N\) to be non vanishing, the total \(U(1)_\ell\) charge, before picture changing, has to be an odd (even) number, for even (odd) order terms, respectively. Similarly, in every pair \(y_i\omega_i\), one real fermion, either \(y_i\) or \(\omega_i\), remains real and is paired with the corresponding right–moving real fermion to produce an Ising model sigma operator. Every picture changing operation changes the number of left–moving real fermions by one. This property of the standard–like models significantly reduces the number of potential non vanishing terms.

3. Higgs mass matrix

There are two types of Higgs doublets, from two distinct sectors, common to all the realistic free fermionic models \([9,11,6,7,8]\). The first type are Higgs doublets form the Neveu–Schwarz sector. They correspond to Higgs doublets from the untwisted sector in the orbifold language. In the standard–like models \([10,6,7,8]\) and the \(SO(6) \times SO(4)\) \([11]\) models, the presence of Higgs doublets from the untwisted sector in the massless spectrum, is correlated with the additional \(U(1)_{r_j+3}\) \((j = 1, 2, 3)\) horizontal symmetries, which arise from pairing real right–moving fermions \([8]\). In all the realistic standard–like models that are based on the NAHE set, there are three pairs of Higgs doublets \(h_1, \bar{h}_1\ h_2, \bar{h}_2\) and \(h_3, \bar{h}_3\), from the untwisted sector. Each pair \(h_j, \bar{h}_j\) carries \(U(1)_{r_j}\) charge and therefore can couple at the cubic level only to the states from the sector \(b_j\).

The second type of Higgs doublets is obtained from a combination of the two \(Z_2 \times Z_2\) vectors, which are used to reduce the number of generations, and some combination of \(b_1, b_2\) and \(b_3\). In the flipped \(SU(5)\) \([9]\) and the \(SO(6) \times SO(4)\) model \([11]\) the combination is \(b_4 + b_5\). In the standard–like models of tables 1 and 2, the combination is \(\zeta = b_1 + b_2 + \alpha + \beta\). In this vector, \(\zeta_R \cdot \zeta_R = \zeta_L \cdot \zeta_L = 4\). Therefore, the massless states are obtained by acting on the vacuum with one right–moving fermionic oscillator. The states in this sector transform only under the observable gauge group. The presence of these states in the massless spectrum, and consequently of the vector combination in the additive group is essential for the
application of the Dine–Seiberg–Witten (DSW) mechanism [16] and for obtaining realistic phenomenology. Requiring the existence of this vector combination in the additive group is an additional strong constraint on the allowed basis vectors, which extend the NAHE set. These two \( Z_2 \times Z_2 \) basis vectors play an important role in generating the generation mass hierarchy. Their combination is symmetric with respect to \( b_1 \) and \( b_2 \). However, they brake the cyclic symmetry between \( b_1 \), \( b_2 \) and \( b_3 \).

The light Higgs spectrum is determined by the massless eigenstates of the doublet Higgs mass matrix. The doublet mass matrix consists of the terms \( h_i \bar{h}_j \langle \Phi^n \rangle \), and is defined by \( h_i (M_h)_{ij} \bar{h}_j \), \( i, j = 1, 2, 3, 4 \) where \( h_i = (h_1, h_2, h_3, h_{45}) \) and \( \bar{h}_i = (\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_{45}) \). At the cubic level of the superpotential the Higgs doublets mass matrix is given by,

\[
M_h = \begin{pmatrix}
0 & \Phi_{12} & \Phi_{13} & 0 \\
\Phi_{12} & 0 & \Phi_{23} & 0 \\
\Phi_{13} & \Phi_{23} & 0 & \Phi_{45} \\
0 & 0 & \Phi_{45} & 0
\end{pmatrix}.
\]

At the cubic level this form of the Higgs mass matrix is common to the flipped \( SU(5) \) string model [9] and to the realistic standard–like models, which are based on the NAHE set [6,7,8]. These models contain an anomalous \( U(1) \) symmetry, which breaks supersymmetry at the Planck scale, and destabizes the vacuum. Supersymmetry is restored by giving a VEV to some singlets in the spectrum, along F and D flat directions [16]. In the flipped \( SU(5) \) string model and the standard–like models, it has been found that we must impose \([18,6,7,19],\)

\[
\langle \Phi_{12}, \bar{\Phi}_{12} \rangle = 0,
\]

and that \( \Phi_{45} \), and \( \bar{\Phi}_{13} \) or \( \bar{\Phi}_{23} \), must be different from zero. From this result it follows that in any flat F and D solution, \( h_3 \) and \( \bar{h}_3 \) obtain a Planck scale mass. This result is not surprising. It is a consequence of the symmetry of the sectors \( \alpha \).
and $\beta$ with respect to the $b_1$ and $b_2$ sectors. The implication is that $h_3$ and $\bar{h}_3$ do not contribute to the light Higgs representations. Consequently, the mass terms for the states from the sector $b_3$ will be suppressed.

The matrix $M_h$ is diagonalized by $SM_h T^\dagger$ where $S$ and $T$ are two unitary matrices and $(SM_h T^\dagger)_{ij} = m_i \delta_{ij}$. It follows that $SMM^\dagger S = TM^\dagger MT = |m|^2$. The $h$ and $\bar{h}$ mass eigenstates are obtained by evaluating the eigenvalues and eigenstates of $MM^\dagger$ and $M^\dagger M$, respectively. At the cubic level of the superpotential there are two pairs of light Higgs states. Additional vanishing terms in the cubic level Higgs mass matrix depend on specific F and D flat solutions. At the nonrenormalizable level of the superpotential, additional non vanishing entries in the Higgs mass matrix can appear. For example in the standard–like model of table 1, at the quintic level we get,

$$h_2 \bar{h}_{45} \Phi_{45} H_{25} H_{26}; \quad \bar{h}_2 h_{45} \Phi_{45} H_{23} H_{27}. \quad (13a, b)$$

These additional terms reduce the number of light Higgs pairs to one pair. For example, if $\langle H_{25} \rangle \sim \langle H_{26} \rangle \sim 10^{14}$ GeV, one of the light pairs receives a mass of $O(10^{10}$ GeV). The light eigenstates are $\bar{h}_2$ and $h_{45}$. A VEV for $\Phi_{45}$ of the order of $\langle \Phi_{45} \rangle \sim O(10^{10}$ GeV), produces the mixing between the two light Higgs eigenstates. At order $N = 7$ we obtain additional terms of the form $h_1 h_2 V_i V_j \phi^3$, where $V_i V_j$ is a condensate of the hidden $SU(5)$ gauge group. These terms make the extra pair massive without breaking $U(1)_{Z'}$. They are proportional to $(\Lambda_5/M)^2$, where $\Lambda_5$ is the scale at which the hidden $SU(5)$ group is strongly interacting. The remaining light combinations depend on the specific entries in the Higgs mass matrix which become non zero and depend on specific F and D flat solutions. However, already at this stage, and without knowledge of the specific solution, we can see how the symmetries of the spin structure are reflected in the generation mass hierarchy.

Among the realistic free fermionic models, the standard–like models [10,6,7,8] have the unique property that the generations from the twisted sectors $b_1$, $b_2$ and $b_3$ are the only light generations. There aren’t additional generations and mirror generations, which become massive at some high scale. Thus, the identification
of the light generations is unambiguous. Below, I focus entirely on this class of standard–like models.

4. Generation mass hierarchy

The class of superstring standard–like models have two unique properties that restrict the fermion mass terms. A unique property of the standard–like models is the possible connection between the requirement of a supersymmetric vacuum at the Planck scale, via the DSW mechanism, and the heaviness of the top quark relative to the lighter quarks and leptons [8]. The only standard–like models which were found to admit a solution to the set of F and D constraints are models in which only $+\frac{2}{3}$ charged quarks obtain trilevel Yukawa couplings [8]. Trilevel Yukawa couplings for $+\frac{2}{3}$ or $-\frac{1}{3}$ charged quarks are selected by the assignment of boundary conditions for the real fermions in the vector $\gamma$. They are determined by [8],

$$\Delta_j = |\gamma(U(1)_{\ell_j+3}) - \gamma(U(1)_{r_j+3})| = 0, 1 \quad (j = 1, 2, 3). \quad (14)$$

$\Delta_j = 0$ gives a trilevel Yukawa coupling for $-\frac{1}{3}$ charged quarks and $\Delta_j = 1$ gives a Yukawa coupling for $+\frac{2}{3}$ charged quarks. The only standard–like models that admit F and D flat solution are models with $\Delta_j = 1$ for the three sectors $b_1$, $b_2$ and $b_3$. The second property unique to the standard–like models is the unambiguous identification of the light generations. There are only three twisted generations from the sectors $b_1$, $b_2$ and $b_3$ and none from the untwisted sector.

The symmetry of the vectors $\alpha$ and $\beta$ with respect to the vectors $b_1$ and $b_2$, forces $h_3$ and $\tilde{h}_3$ to get a Planck scale mass. Nonrenormalizable terms have the form $cg^{N-2}f_if_jh\phi^{N-3}(2\alpha')^{N-3}$, or $cg^{N-2}f_if_j\tilde{h}\phi^{N-3}(2\alpha')^{N-3}$, where $f_i, f_j$ are two fermions from the sectors $b_1$, $b_2$ and $b_3$. $h$ and $\tilde{h}$ are Higgs doublets which are combinations of $(h_1, h_2, h_{45})$ and $(\tilde{h}_1, \tilde{h}_2, \tilde{h}_{45})$, respectively. The coefficients, $c = O(1)$, can be obtained by calculating the nontrivial correlators between the vertex operators, and $g$ is the gauge coupling constant. The combination $\phi^{N-3}$ is a combination
of fields that get a VEV. Using the relation $\frac{1}{2}g\sqrt{\alpha'} = \sqrt{8\pi}/M_{Pl}$, the nonrenormalizable terms take the form, $cg_{ij}f_{jl}(\frac{\langle \phi \rangle}{M})^{N-3}$. Thus, the nonrenormalizable terms become effective trilinear terms, suppressed by $(\frac{\langle \phi \rangle}{M})^{N-3}$ relative to the trilevel terms, where $M \equiv \frac{M_{Pl}}{2\sqrt{8\pi}} \sim 10^{18}GeV$ [14]. In the standard–like models several scales contribute to these generalized VEVs: (a) The leading scale is the scale of singlet VEVs, $\langle \phi \rangle/M$, which are used to cancel the D–term equation of the anomalous $U(1)_A$. These are typically of the order of $\langle \phi \rangle/M \sim 10^5$. (b) The scale of hidden sector condensates, $(\frac{\langle \phi \rangle}{M})^{N-3}$. The hidden sector contains two non abelian gauge groups, $SU(5) \times SU(3)$, with $\Lambda_5 \gg \Lambda_3$, and $\Lambda_5 \geq 10^{14}GeV$. Thus, the leading terms are proportional to hidden $SU(5)$ condensates, and the analysis focuses on these terms. (c) The scale of $Z'$ breaking. In Ref. [19], it was shown that VEVs that break $U(1)_{Z'}$ violate the cubic level F flat solution and therefore break supersymmetry at the Planck scale. Therefore, $\Lambda_{Z'} \leq \Lambda_5$.

I now turn to examine the fermion mass terms in the standard–like models. At the cubic level the only potential terms are $u_1Q_1\bar{h}_1$ and $u_2Q_2\bar{h}_2$ [6,7]. Below the intermediate scale $\bar{h}_1$ or $\bar{h}_2$ obtain a large mass and one term remains. Thus, only the top quark has a cubic level mass term, and only its mass is characterized by the electroweak scale. This property is common to all the standard–like models which admit supersymmetric, F and D flat, solutions at the Planck scale.

The quartic and quintic orders mass terms differ between the models of tables 1 and 2. This again is a consequence of the assignment of boundary conditions in the vectors $\alpha$ and $\beta$ [8]. The boundary conditions in the gauge sector, $\{\bar{\psi}^{1,\ldots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\ldots,8}\}$, are identical in the two models. The two models differ by the boundary conditions of the internal fermions $\{y, w|\bar{y}, \bar{w}\}$. This is reflected in the nonvanishing higher order mass terms.

In model 1 there are no potential quark and lepton mass terms at the quartic order. At the quintic order we get the nonvanishing terms,

$$d_2Q_2h_{45}\bar{\Phi}_2^-\xi_1, \ e_2L_2h_{45}\bar{\Phi}_2^+\xi_1$$ (15a)
\[
d_1 Q_1 h_{45} \Phi_1^+ \xi_2, \quad e_1 L_1 h_{45} \Phi_1^- \xi_2
\]
(15b)

\[
u_2 Q_2 (\bar{h}_{45} \Phi_2 + \bar{h}_1 \Phi_i^+ \Phi_i^-)
\]
(15c)

\[
u_1 Q_1 (\bar{h}_{45} \Phi_1 + \bar{h}_2 \Phi_i^+ \Phi_i^-)
\]
(15d)

\[(u_2 Q_2 h_2 + u_1 Q_1 h_1) \frac{\partial W}{\partial \xi_3}.
\]
(15e)

These terms can produce mass terms for the charm quark and for the two heavier generations of $-\frac{1}{3}$ charged quarks and for charged leptons.

In model 2 we get at the quartic order potential mass terms for $-\frac{1}{3}$ charged quarks and for charged leptons, from the sectors $b_1$ and $b_2$,

\[
d_L^c Q_1 h_{45} \Phi_1, \quad e_L^c L_1 h_{45} \Phi_1, \quad d_L^c Q_2 h_{45} \Phi_2, \quad e_L^c L_2 h_{45} \Phi_2,
\]
(16)

while there are no non vanishing quartic order terms for $+\frac{2}{3}$ charged quarks.

At order $N = 5$ potential mass terms appear for the charm quark of the form $u_2 Q_2 (\bar{h}_{45} \phi^2 + \bar{h}_1 \phi^2 + \bar{h}_1' \phi^2)$ where $\phi^2$ represent combinations of singlet VEVs. There are no potential quintic order mass terms for $-\frac{1}{3}$ charged quarks and for charged leptons. At order $N = 6$, there are additional terms of the form $e_i L_i h \phi^3$ and $d_i Q_i h \phi^3$ ($i=1,2$), which produce possible diagonal mass terms for the strange quark and for the $\mu$ lepton.

At this stage it is seen that the mass terms for the $b_1$ and $b_2$ sectors come from terms which are suppressed by powers of $(\langle \phi \rangle M)^{N-3}$, where $\phi$ are singlets VEVs that are used to cancel the anomalous D–term equation. The split between these two sectors in terms of the boundary condition vectors is still not transparent.

As discussed above the vectors $\alpha$ and $\beta$ are symmetric with respect to $b_1$ and $b_2$. Thus, the symmetry can be broken by the vector $\gamma$. Examination of tables 1 and 2 reveals that this is the case in model 2, while in model 1 the vector $\gamma$ is symmetric with respect to $b_1$ and $b_2$. Thus, in this model the symmetry between the sectors $b_1$ and $b_2$ has to be broken by the choice of generalized GSO phases or by specific choices of flat directions.
Rather than the presence of potential leading mass terms for \( G_1 \) and \( G_2 \), the most important aspect of nonrenormalizable terms is the absence of such terms for \( G_3 \). As argued above the Higgs doublets \( h_3 \) and \( \bar{h}_3 \) get a Planck scale mass and do not contribute to the light Higgs representations. Similarly, requiring F and D flat solution to the anomalous D–term equation imposes [6,7,19],

\[ \langle \Phi_{12}, \bar{\Phi}_{12}, \xi_3 \rangle \equiv 0. \quad (17) \]

The potential leading terms for \( G_3 \) have the form \( f_3 f_3 h \phi_{N-3} \) or \( f_3 f_3 \bar{h} \phi_{N-3} \), where \( f_3 \) are fermions from the sectors \( b_3 \), \( h \) and \( \bar{h} \) are combinations of \( \{ h_1, h_2, h_{45} \} \) and \( \{ \bar{h}_1, \bar{h}_2, \bar{h}_{45} \} \) respectively, and \( \phi_{N-3} \) is a combination of singlets VEVs. However, each \( f_3 \) carries \( U(1)_{\ell_3} = \frac{1}{2} \) to give a total of \( U(1)_{\ell_3} = 1 \). The only singlets that do not break \( U(1)_{Z'} \) and which have \( U(1)_{\ell_3} \) charge are \( \Phi_{12}, \bar{\Phi}_{12} \) and \( \xi_3 \). Consequently, all the potential leading mass terms for \( G_3 \) vanish identically to all orders. A possible way to get diagonal mass terms for \( G_3 \) is to couple them with the states from the \( b_j + 2\gamma \) sectors, which generate \( SU(5) \) condensates, or with VEVs that break \( U(1)_{Z'} \). For example, in model 1 these states come from the sectors \( b_1 + b_3 + \alpha \pm \gamma + (I) \) and \( b_2 + b_3 + \beta \pm \gamma + (I) \). In this model at the quintic level we get a term \( u_3 Q_3 \bar{h}_{45} H_{17} H_{24} \). However, as argued above \( \Lambda_{Z'} \leq \Lambda_5 \). Therefore, the diagonal mass terms for \( G_3 \) are suppressed by at least \( (\frac{\Lambda_{Z'}}{M})^2 \lambda_t \), where \( \lambda_t \) is the top Yukawa coupling. Thus, the states from \( G_3 \) are identified with the lightest generation. The suppression of their mass terms is a consequence of the symmetries of the vectors which extend the NAHE set. I would like to emphasize that this is a general result which will be applicable to all the realistic standard–like models [6,7,8], and may be a general result of realistic free fermionic models. A similar result holds in the flipped \( SU(5) \) string model [18,20]. However, there, one needs to avoid identifying \( G_3 \) with the lightest generation because of problems with dimension four operators, which mediate rapid proton decay [20].

Next I turn to examine the mixing between the generations. The mixing terms have the form \( f_i f_j h \phi^n \) and \( f_i f_j \bar{h} \phi^n \), where \( i \neq j \), \( h \) and \( \bar{h} \) are light Higgs combinations and \( \phi^n \) is a combination of generalized VEVs.
The fermion states from each sector $b_j$ carry $U(1)_{\ell_j+3} = \pm \frac{1}{2}$. The singlets from the NS sector and the sector $b_1 + b_2 + \alpha + \beta$, all have $U(1)_{\ell_j+3} = 0$. Every picture changing operation changes the total $U(1)_{\ell} = U(1)_{\ell_4} + U(1)_{\ell_5} + U(1)_{\ell_6}$ by $\pm 1$. Thus, to construct nonrenormalizable terms which are invariant under $U(1)_{\ell}$, we must tag to $f_i f_j h$ additional fields with $U(1)_{\ell_j+3} = \pm \frac{1}{2}$. For example, examining model 1 [6], we observe that the only available states are from the sectors $b_j + 2\gamma$. These states transform under the hidden $SU(5) \times SU(3)$ gauge group in the fundamental representations, 5 and $\overline{5}$ of $SU(5)$, and 3 and $\overline{3}$ of $SU(3)$. Thus, the mixing terms are suppressed by at least $(\frac{\Lambda_5}{M_p})^2$ relative to the leading diagonal terms.

5. Conclusions

In this paper I examined the texture of fermion mass matrices that emerges in realistic superstring derived standard–like models. These models are constructed in the free fermionic formulation and correspond to superstring models which are based on $Z_2 \times Z_2$ orbifold compactification. Among the realistic free fermionic models the standard–like models possess a unique property. They have three and only three chiral generations. There are no additional generations and mirror generations that become massive at a large scale. Therefore, the identification of the light generations is unambiguous. The light generations come from three distinct sectors, which correspond to the three distinct twisted sectors of the corresponding orbifold model. The light generations carry, generational dependent, $U(1)$ charges and Ising model operators. These symmetries restrict the allowed F–terms in the superpotential, and are responsible for creating the generations mass hierarchy.

Requiring space–time supersymmetry at the Planck scale restricts the possible standard–like models. The only models that were found to admit a supersymmetric vacuum at the Planck scale, are models that allow trilevel mass terms only for $+\frac{2}{3}$ charged quarks. Similarly, the requirement of space–time supersymmetry gives a generic choice of vanishing VEVs, and forces the Higgs doublets of the third generation to be superheavy. Consequently, the mass terms for the third generation are suppressed. This result, like the result for trilevel Yukawa coupling, is a gen-
eral characteristic of these models. Therefore, these models give an unambiguous explanation for the lightness of the lightest generation relative to the two heavier generations. The suppression of the mass terms for the lightest generation states is independent of the specific choice of flat directions in the cancellation of the anomalous D–term equation.

The following general texture emerges for the fermion mass matrices in these models,

\[ M_U = \begin{pmatrix} \epsilon, a, b \\ \tilde{a}, A, c \\ \tilde{b}, \tilde{c}, \lambda_t \end{pmatrix} \]  
\[ M_D = \begin{pmatrix} \epsilon, d, e \\ \tilde{d}, C, f \\ \tilde{e}, \tilde{f}, D \end{pmatrix} \]  
\[ M_E = \begin{pmatrix} \epsilon, i, j \\ \tilde{i}, E, k \\ \tilde{j}, \tilde{k}, F \end{pmatrix} \]  

(18a)  
(18b)  
(18c)

where \( \lambda_t = \sqrt{2} y = O(1) \). The entries in capital letters are diagonal terms which are suppressed by powers of singlet VEVs. The entries in small letters represent terms which are suppressed by \( (\frac{\Lambda_5}{M})^2 \). The diagonal terms for the lightest generation are suppressed by \( (\frac{\Lambda_5}{M})^2 \). The traditional GUT relations among quark and lepton masses are broken at various levels of nonrenormalizable terms. At the cubic level the \( SU(5) \) relations are maintained. In model 1, at the quartic level, the \( SU(5) \) relation \( m_b = m_\tau \) is obeyed, while in model 2, at the quintic level it is obeyed only for specific choices of flat directions. The mixing terms between the generations, in general, are obtained at different levels of nonrenormalizable terms [19]. Therefore, the unsuccessful GUT relations for the lighter generation can be cured in the context of the superstring models.

The analysis presented in this paper provides a qualitative understanding of the fermion mass and mixing spectrum. The texture of the fermion mass matrices, exhibited in Eqs. (17), is expected to be a general characteristic of the
class of superstring standard–like models under consideration. In particular, it is independent of the specific choice of singlet VEVs, which are used to cancel the D–term equation of the anomalous $U(1)$. To make progress on a more quantitative analysis, we must take several steps. First, the nontrivial correlators of the non-renormalizable terms have to be calculated by using well known conformal field theory techniques. Second, the dynamics of the hidden $SU(5)$ group has to be examined. The scale $\Lambda_5 > 10^{14} GeV$ depend on the number of fundamental $SU(5)$ representations that are light bellow the Planck scale. The hidden $SU(5)$ condensates can then be approximated and the mixing between the generations can be estimated. Finally, the problem of SUSY breaking in the context of the standard–like models must be addressed and specific choices of flat directions have to be made, in a phenomenologically realistic way. The standard–like models provide a highly constrained laboratory to study these questions, and to address the question of the origin of fermion masses and mixing in the context of superstring theory.

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