Price arbitrage using variable-efficiency energy storage

Benjamin Flamm, Annika Eichler, Roy S Smith and John Lygeros
Automatic Control Laboratory, ETH Zürich, Physikstrasse 3, 8092 Zürich, Switzerland
E-mail: flammb@control.ee.ethz.ch

Abstract. Necessary conditions for conducting arbitrage on electricity spot market prices are presented for large-scale, generic energy storage devices, as a function of device power, conversion efficiency, and per-period operating costs. A mixed integer linear program (MILP) is formulated that uses piecewise-affine approximations of potentially nonlinear energy conversion efficiency to maximize arbitrage over a fixed period. By initially removing storage limit constraints, the optimal solution to the MILP is shown to be a threshold policy, with device operating power a monotonic function of price level. The initial assumption of no storage limits is then revisited. The objective is found to be relatively insensitive to device storage limits, provided the storage size is within an order of magnitude of the optimal storage size for the given problem data.

1. Introduction

Electrical energy systems strive to match instantaneous energy supply and demand at all times. Unfortunately, the time profiles of energy production and consumption do not always coincide. For example, energy consumption follows diurnal and seasonal trends, while certain large energy generators like nuclear power plants are most efficient when run at a fixed power level for an extended period. In future energy systems with high levels of renewable energy infeeds, energy storage is an effective way to address the inherent spatial and temporal variability of renewable energy while satisfying a given demand.

Current energy storage methods vary extensively, with different storage capacities, levels of technological maturity, dynamic performance, and conversion efficiencies. Pumped hydro storage and lithium-ion batteries are good examples of large-scale energy storage, with roundtrip efficiencies of up to 85% [1], [2], [3]. Other promising storage technologies include power-to-gas hydrogen storage, compressed air energy storage, flywheels, and thermal storage [4].

Energy storage can help reduce the use of fossil fuel sources in the electrical grid. Potential applications of storage include providing primary and secondary frequency control [3], [5], as well as reducing charges on maximum demand from the grid [6]. Additionally, a body of research considers the optimal economic operation of storage in the face of supply and price uncertainty; see for example [6]-[11]. Threshold policies for conducting price arbitrage – that is, extracting a profit from electricity price differences over time – are considered in [1], [2], [6], [10], [11].

We derive necessary analytical conditions on discrete-time electricity prices to generate a profit through energy arbitrage, as in [2]. Unlike the above works, we consider power-dependent conversion efficiencies, and separate efficiencies for charging and discharging devices. We extend [2], which considers single conversion efficiencies and does not include per-period operating costs.
The treatment of per-period operating costs presented here differs from approaches like [1], where such costs are proportional to the device power (and thus do not require integer variables). Storage limits and coupling constraints between adjacent periods are initially ignored, as in [2], [6]. This approach gives insight into the solution structure, thus assessing the feasibility of arbitrage using given market and storage technologies. We then show that considering reasonable storage limits for seasonal storage devices only slightly changes the results.

2. Optimization Framework for Arbitrage using Energy Storage

Energy storage devices conduct arbitrage by buying and selling energy at specific times to extract a profit. Here, we formulate arbitrage maximization as a finite-horizon, mixed-integer linear program with piecewise-affine models for the energy conversion efficiencies of the storage.

2.1. Single Conversion Efficiency Storage Model

Energy storage efficiencies are commonly modeled using constant approximations [1], [2]. In this setting, the change in stored energy $\Delta E$ over a fixed time period, for given charging power $P_C \geq 0$ and discharging power $P_{DC} \geq 0$, can be written as

$$\Delta E(P_C, P_{DC}) = m_{C}P_C - m_{DC}P_{DC}, \quad (1)$$

where $m_C$ and $1/m_{DC}$ are the charging and discharging efficiencies respectively.

2.2. Multiple Conversion Efficiency Storage Model

Using a single conversion efficiency to approximate a complex device can be inaccurate at certain power levels. Following [12], we use a piecewise-affine approximation of the conversion efficiency. The change in energy stored at charging and discharging powers $P_C$ and $P_{DC}$ is written as

$$\Delta E(P_C, P_{DC}) = \min_{i \in [1, n_C]} \left( b^{(i)}C + m^{(i)}C(P_C - a^{(i)}C) \right) - \max_{j \in [1, n_{DC}]} \left( b^{(j)}DC + m^{(j)}DC(P_{DC} - a^{(j)}DC) \right). \quad (2)$$

For the charging curve, the approximation is composed of $n_C - 1$ line segments that run sequentially through the $n_C$ breakpoints (including the origin, $n_C - 2$ intermediate breakpoints, and efficiency at maximum power). Breakpoint $i$ has coordinates $\{a^{(i)}C, b^{(i)}C\}$, and the line segment connecting it with the next breakpoint has slope $m^{(i)}C$. The construction for the discharging device is similar. To ensure the efficiency approximation has the proper convexity for the subsequent optimization problem, we constrain the approximation slopes to be non-increasing for the charging device, and non-decreasing for the discharging device. Furthermore, the approximations here and in Section 2.1 are constrained to have the energy conversion efficiency be less than 100%. Note that this method cannot model efficiency curves with nonconvexities.

2.3. Optimal Arbitrage: Problem Formulation

We formulate the following mixed integer linear program (MILP) to determine the optimal arbitrage strategy for a generic energy storage device.

$$\max_{P_C, P_{DC}, \delta_C, \delta_{DC}} \sum_{t=1}^{N} p[t](P_{DC}[t] - P_C[t] - f_C\delta_C[t] - f_{DC}\delta_{DC}[t]) \quad (3a)$$

subject to $$\sum_{i=1}^{N} \Delta E(P_{C}[i], P_{DC}[i]) = 0 \quad (3b)$$

$$0 \leq P_C[t] \leq P_{C,\text{max}}\delta_C[t] \quad \forall t \in [1, N] \quad (3c)$$
0 ≤ \( P_{DC}[t] \) ≤ \( P_{DC,max}\delta_{DC}[t] \) \( \forall \ t \in [1, N]. \) (3d)

The cost function (3a) seeks to maximize the profit from consuming and producing energy over a horizon \( N \) with timesteps of a given length, paying or receiving a predetermined electricity price \( p[t] \) for each period \( t \). Optimization is conducted over the vector of charging powers \( P_C \in \mathbb{R}_+^N \), discharging powers \( P_{DC} \in \mathbb{R}_+^N \), and binary states \( \delta_C \in \{0, 1\}^N \) and \( \delta_{DC} \in \{0, 1\}^N \) that specify whether the charging and discharging devices are turned on. When on, the devices can convert energy, but also consume an additional fixed nonnegative amount of power \( (f_C \text{ and } f_{DC} \text{ respectively}) \). The net energy stored (3b) is given as a function of input and output power by either (1) or (2), depending on how the conversion efficiency is modeled. The net energy stored is constrained to have a net-zero change over the optimization horizon. The limits on the input and output powers \( (P_{C,max} \text{ and } P_{DC,max} \text{ respectively}) \) are specified by (3c) and (3d).

Note that no upper or lower bounds on storage capacity are assumed in (3). In the presence of capacity constraints, it might not be possible to follow the generated optimal profile. The effect of this assumption will be further investigated in Section 4.

3. Solution Structure of Arbitrage Problem

We investigate the structure of the solution to the arbitrage problem (3) and provide a threshold on the prices at which the storage can be profitably charged and discharged.

3.1. Operating at Constraint Vertices

We consider the general case of a piecewise-affine approximation of the conversion efficiency, and derive several properties of the optimal solutions to the arbitrage problem.

Lemma 1. There exists an optimal solution to the arbitrage problem (3) that has the charging and discharging devices operating for each period at one of the efficiency breakpoints, except possibly one device operating at an intermediate power level for one period.

Proof. We first consider only the charging device. Assume, for the sake of contradiction, that there exists an optimal solution where the charging device operates for two periods at non-breakpoint power levels \( P_1 \) and \( P_2 \), with prices \( p_1 \) and \( p_2 \) respectively. At power levels \( P_1 \) and \( P_2 \), let the marginal amount of stored energy per input power be represented by positive slopes \( m_C^{(1)} \) and \( m_C^{(2)} \) respectively.

Since \( P_1 \) and \( P_2 \) are not at breakpoints, there exists an \( \epsilon \) such that for all \( \delta \in [-\epsilon, \epsilon] \), \( P_1 + \delta/m_C^{(1)} \) and \( P_2 - \delta/m_C^{(2)} \) both lie on the same line segment as \( P_1 \) and \( P_2 \) respectively.

Suppose the power level in the first period is changed to \( P_1 + \delta/m_C^{(1)} \), and the power in the second period is changed to \( P_2 - \delta/m_C^{(2)} \). The original solution is not changed for any other period, and the energy conservation constraint (3b) is still satisfied.

The original solution incurs a cost of \( p_1P_1 + p_2P_2 \), while the modified solution costs \( p_1(P_1 + \delta/m_C^{(1)}) + p_2(P_2 - \delta/m_C^{(2)}) \). Letting \( \eta_1 = p_1/m_C^{(1)} \) and \( \eta_2 = p_2/m_C^{(2)} \), the net change in cost is \( \delta(\eta_1 - \eta_2) \). There are two possibilities:

(i) \( \eta_1 = \eta_2 \): The net change in cost is zero, and charging power can be freely shifted from one time period to the other until an efficiency breakpoint power level is hit, leading to another optimal solution with only one device operating away from a breakpoint.

(ii) \( \eta_1 \neq \eta_2 \): Taking \( \delta > 0 \) (if \( \eta_1 < \eta_2 \)) or \( \delta < 0 \) (otherwise) leads to a feasible solution with lower cost, contradicting the optimality of our original solution.

The case of the discharging device is symmetric. Note that this argument also holds for the case of one device operating for more than two periods at intermediate power, since the above logic can be applied iteratively to pairs of periods at intermediate power.
Finally, suppose the charging and discharging devices each operate for a single period at intermediate powers, with prices $p_C$ and $p_{DC}$, and slopes $m_C$ and $m_{DC}$ respectively. Similar to the above argument, if the power in the charging device changes by $\delta/m_C$ and the power in the discharging device changes by $\delta/m_{DC}$, the energy conservation constraint (3b) is satisfied. The net change in cost is equal to $\delta(p_C/m_C - p_{DC}/m_{DC})$. Comparing $\eta_C = p_C/m_C$ to $\eta_{DC} = p_{DC}/m_{DC}$, by shifting power between periods as above, we see that an optimal solution exists with only one device operating at an intermediate power for at most one period.

3.2. Price Threshold Policy is Optimal
The fact that the total energy constraint (3b) is the only constraint coupling different time periods of problem (3) implies that optimal solutions have the following property:

**Lemma 2.** For optimal solutions to the arbitrage problem (3), the charging power is non-increasing as prices increase. Likewise, the discharging power is non-decreasing as prices increase.

**Proof.** Consider first the charging device. Assume, for the sake of contradiction, that an optimal solution exists with a device charging for two given periods at powers $P_1 \geq 0$ and $P_2 = P_1 + \delta > P_1$, with corresponding prices $p_1 < p_2$. This incurs a cost of $p_1P_1 + p_2(P_1 + \delta)$. Switching the periods when the device charges at the given power levels leads to a new feasible solution with a cost of $p_2P_1 + p_1(P_1 + \delta)$. The cost change of $\delta(p_1 - p_2)$ is negative since $p_1 < p_2$. Thus, the overall cost is reduced, contradicting the optimality assumption. The argument for the discharging device is symmetric.

Finally, since it is assumed in Section 2.2 that the conversion efficiency approximations have efficiencies less than 100%, it is suboptimal for both devices to operate during the same period – such a situation would be more costly than simply charging or discharging the net difference in power. Therefore, we need only to consider operating one device per period, as done above.

3.3. Profitable Price Ratio
We now consider solving (3) using the single conversion efficiency case (1), and derive a lower bound on the ratio of selling to buying price needed for arbitrage to be profitable, as a function of device power, conversion efficiency, and per-period operating costs.

We consider the best-possible case in Lemma 1, where neither device operates at an intermediate power level. We thus assume that the constraints (3c) and (3d) are tight, with $P_C[t] = P_{C,\text{max}}\delta_C[t]$ and $P_{DC}[t] = P_{DC,\text{max}}\delta_{DC}[t]$. The problem reduces to whether to, for each $t \in [1, N]$, operate a device at zero or full power. Combining (3b) with (1) implies that

$$m_CP_{C,\text{max}} t_C = m_{DC} P_{DC,\text{max}} t_{DC},$$

where $t_C = \sum_{t=1}^N \delta_C[t]$ and $t_{DC} = \sum_{t=1}^N \delta_{DC}[t]$. To make a profit, the income from selling energy should be greater than the cost of buying energy. Let all energy be bought and sold at fixed prices $p_{\text{buy}}$ and $p_{\text{sell}}$. Arbitrage is possible if

$$t_{DC} p_{\text{sell}}(P_{DC,\text{max}} - f_{DC}) > t_C p_{\text{buy}}(P_{C,\text{max}} + f_C).$$

Solving for the ratio $\frac{t_C}{t_{DC}}$ in (4) and substituting into (5), a storage system is profitable if

$$\frac{p_{\text{sell}}}{p_{\text{buy}}} > m^{-1} \frac{P_{DC}}{P_C} \frac{(P_C + f_C)}{(P_{DC} - f_{DC})},$$

where $m = m_C/m_{DC}$ is the roundtrip conversion efficiency.
Figure 1: (left) Optimal arbitrage profile for 2014 Swiss day-ahead spot market prices. 100 kW power-to-gas system, with efficiency approximated by five breakpoints. Note that early in the year, storage level is negative due to high prices. (right) Equivalent price-sorted policy.

**Example.** Suppose that the round-trip conversion efficiency $m = 0.5$. Assume that the charging and discharging powers are equal ($P_{C_{\text{max}}} = P_{D_{\text{C, max}}}$), and there are no per-period device operating costs ($f_{C} = f_{DC} = 0$). Then (6) states that the sell/buy price ratio must be $> 2$.

4. Numerical Results and Discussion

As a case study inspired by a power-to-gas system, we solve problem (3) to determine the yearly arbitrage opportunity of an energy storage device operating on the 2014 hourly Swiss day-ahead spot market prices [13]. The storage device can charge and discharge at 100 kW, with per-period charging and discharging device operating powers of 2.4 kW and 0.36 kW respectively.

Figure 1 (left) shows the solution to the arbitrage problem (3), where the nonlinear conversion efficiency curves are approximated by five breakpoints. Note that early in the year, the amount of storage becomes negative. When the solution is reordered by price in Figure 1 (right), the structure of the optimal solution as noted in Lemmas 1 and 2 becomes apparent, and is similar to that observed in [2]. The devices run at breakpoint power levels, and as prices become less extreme, the devices operate at more efficient, lower-power breakpoints.

In response to the unrealistic initial negative storage volume, we analyze the sensitivity of the arbitrage potential to storage limits. We add the constraint

$$0 \leq S_0 + \sum_{t=1}^{t=N} \Delta E(P_{C[t]}, P_{D_{C}[t]}) \leq S_{\text{max}}$$

to (3), for a given $S_{\text{max}}$ and for all $t \in [1, N]$. Here, the starting volume $S_0 \in [0, S_{\text{max}}]$ is an additional optimization variable. Note that the problem remains an MILP and can hence be solved to optimality. Due to the additional constraints coupling different time periods, however, the optimal solution may not be a threshold policy.

Figure 2: Arbitrage potential over the 2014 day-ahead Swiss spot market yearly price, as a function of storage capacity.
The effect on arbitrage potential of a reduced $S_{\text{max}}$ over a 365 day period is illustrated in Figure 2. We find that taking $S_{\text{max}}$ of 600 m$^3$ (approximately 20 days at maximum power) suffices to reach the maximum arbitrage potential. Assuming an optimal starting storage level, a 50% reduction of the optimal storage limit of 600 m$^3$ results in a 7.6% reduction in arbitrage potential. Though the specific reduction may depend on the temporal cycles present within the price data, this suggests that the arbitrage potential of a large-scale seasonal storage system operating in the Swiss spot market is relatively insensitive to storage limits.

5. Conclusion
This paper presents an optimization framework and conditions for conducting arbitrage on time-varying electricity prices. Several simplifying assumptions are made: temporal and storage limit constraints are not present, the storage plant operates as a price taker on the market, and future prices are known for the given horizon. While these assumptions often do not hold in practice, the problem still gives intuition and an upper bound on the arbitrage potential.

A more realistic setting would operate in real time, consider stochastic prices, and conduct arbitrage in conjunction with the provision of other revenue-generating services (such as peak shaving or primary frequency control). These extensions are currently under investigation.

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