Adjustments of correlated values by search method

G G Shevchenko 1*, M Ya Bryn2
1Kuban State Technological University, 2, Moskovskaya str., Krasnodar, 350072, Russia
2Emperor Alexander I St. Petersburg State Transport University, 9, Moskovsky pr., Saint Petersburg, 190031, Russia

E-mail: grettel@yandex.ru

Abstract. This article presents the sequence of triangulation networks adjustments by search method of non-linear programming, one of them has the horizontal distances measured and the other network has the correlated horizontal angles measured. Similar results of the both methods for triangulation networks adjustments refute an opinion that search method is impossible for correlated measurements adjustments.

Introduction

When there is a problem of optimizing (minimizing) of a quadratic objective function the non-linear programming methods are often used in geodesy and mainly the gradient method [1-5] and the Newton method [5, 6]. However, if problems contain a large number of variables, it is rather difficult to obtain derivatives in the form of analytical functions which are necessary for the gradient algorithm or the Newton method, so it is not always possible to get the minimum of the objective function.

The search method of nonlinear programming does not require regularity and continuity of the objective function and the existence of derivatives. Practical geodesy has many works [7–12] devoted to the usage of the search method for non-linear programming to adjust geodetic networks.

It should be noted that the other important task in geodesy is the adjustment of dependent values [13–20]. However, the adjustment of correlated measurements by the search method is not discussed in detail in the works listed above. Moreover, the author [8] indicates that the direct search algorithm does not work well if there is a dependence relationship between variables and this method cannot be recommended if the user does not work with the objective function with insignificant dependence relationship. Refutation of this opinion is given below.

Common Algorithm of Search Method

The search method is one of the optimization methods that does not use derivatives to determine the minimum of the objective function. This method determines the search for the minimization direction on the basis of successive calculations of the objective function. In fact they consist in changing one variable each time while the others remain constant until the minimum is achieved [8].

Assume that the given function $f(x)$ – is unimodal and has a single value $x^*$ so that $f(x^*)$ – is a minimum of $f(x)$. 
Then it is necessary to specify the initial values of the coordinates of the target points. They are determined by performing preliminary calculations or by an electronic map or paper-based maps [5].

The adjustment of the correlated measurements is supposed to be made according to the following algorithm:

Step 1. The initial value of the element $x(0)$ has an increment $\Delta x$.

Step 2. The value of the objective function $f(x(0))$ is calculated.

Step 3. The variable $x(0)$ is changed by the selected value $\Delta x$ and the function is $x(1) = x(0) + \Delta x$.

Step 4. The value of the objective function $f(x(1))$ is calculated.

Step 5. If $f(x(0)) > f(x(1))$, then the variable $x(1)$ changes by the selected value $\Delta x$ and is given as $x(2) = x(0) + 2\Delta x$ or $x(2) = x(0) + \Delta x$. If $f(x(0)) \leq f(x(1))$, then $x(2) = x(0) - \Delta x$ is assumed.

Step 6. The value of the objective function $f(x(2))$ is calculated.

Step 7. The variable $x^*$ is determined which corresponds to the minimum value of the objective function $f(x)$:

$$
    x^* = x(1) + \frac{\Delta x}{2} \left[ f(x(0)) + f(x(2)) \right] 
$$

Step 8. The value of the objective function $f(x^*)$ is calculated. So we get the minimum of the objective function.

**Operating of a Search Method Algorithm to Adjust Correlated Measurements**

The possibility of using the search method algorithm described above is supposed to be used to adjust correlated measurements. For this purpose we will model two triangulation geodetic networks, one of them has horizontal directions measured, the other has horizontal angles measured. Network diagrams with measurement values are presented in Figures 1 and 2.

It is known that adjacent horizontal angles are dependent values and their correlation coefficient is $r_{\beta_i \beta_j} = -0.5$. Thus, we have the following initial data.

**Problem 1:** Consider a triangulation network consisting of four points, two of which (t2 and t3) are taken as the initial ones, and points t1 and t4 as defined ones. 12 horizontal directions with a standard square error of measuring directions $m_N = 5.0^\prime$ are measured in the network.

**Problem 2:** Consider a triangulation network consisting of four points, two of which (t2 and t3) are taken as the initial ones, and points t1 and t4 are defined ones. The network has 8 measured horizontal angles with a standard square error of measuring angles $m_\beta = 7.1^\prime$.

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**Figure 1.** A triangulation network scheme (horizontal directions measured)

**Problem 2:** Consider a triangulation network consisting of four points, two of which (t2 and t3) are taken as the initial ones, and points t1 and t4 are defined ones. The network has 8 measured horizontal angles with a standard square error of measuring angles $m_\beta = 7.1^\prime$. 

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The initial points have the following coordinates: \( x_2 = 1964.947 \text{ m}; y_2 = 3418.836 \text{ m}; x_3 = 2013.231 \text{ m}; y_3 = 3151.480 \text{ m}. \)

The adjustment by the search method results in minimizing the objective function \( f(x) = V^TPV \), where \( V \) is the corrections to the measurement results, \( P \) is the weight matrix of the measurement results.

**The Triangulation Network Adjustment by Directions**

To adjust the network by the search method a program to find the minimum of the objective function was written, that implements the algorithm given above. This program works in *Microsoft Excel* as a macro written in the *VBA* language.

The program specifies the coordinates of the initial points and the approximate values of the defined points coordinates \( t_1 \) and \( t_4 \). An array of measured values of horizontal directions for the first task is given.

For the problem 1, the program calculates the values of the direction angles selecting the coordinates of the defining point. As the direction angles in the network have not been measured, the values of the orientation angles are calculated at the same time. Having adjusted the calculated values of the positional angles to the corresponding value of the orienting angle, we obtain the calculated values of the horizontal directions. The program gives the result in the form of deviation \( \nu_{Nij} \) of values of the calculated directions in selected coordinates from the measured directions. For deviation \( \nu_{Nij} \) we have: \( i \) – is the standing point and \( j \) – is the point of aligning.

The resulting deviations are the elements of the adjustment vector to the results of measurements \( V \). After each cycle the value of the objective function \( f(x) = V^TPV \) is calculated and it continues until the last two values of the function coincide. Thus, \( f(x) = V^TPV = \text{min} \) is calculated.

For the first problem, the vector of corrections to the measurement results \( V \) is equal to:

\[
V^T = [4.58 - 3.02 - 1.56 \ 3.99 \ 0.00 - 3.99 \ 4.77 - 0.66 - 4.11 \ 3.83 - 0.12 - 3.71].
\]

The correlation matrix of measurement results \( K_{\text{measured}} \) has a size of 12x12, the main diagonal of which has squares of mean square errors of horizontal directions measurement:

\[
P = K_{\text{measured}}^{-1}.
\]
Correspondingly the weight matrix of measurement results of $P$ according to the formula (2) is equal to:

\[
K_{\Delta \text{measured}} = \begin{bmatrix}
25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 25 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 25 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 25 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25 \\
0 & 0 & \end{bmatrix}.
\]

Thus, the program after performing several approximation cycles finds the minimum of the objective function $f(x)=V^TPV$ that makes 5.316. Adjusted values of the defining points coordinates as a result of the program are presented in Table 1.

**Table 1.** Adjusted values of the defining points coordinates of the triangulation network (problem 1)

| Point names | Coordinates, [m] |          |
|-------------|------------------|----------|
| $t_1$       | 2120.0256        | 3284.1375|
| $t_4$       | 2081.8053        | 3441.9930|
| $t_3$       | 2013.2310        | 3151.4800|
| $t_2$       | 1964.9470        | 3418.8360|

**The Triangulation Network Adjustment by the Corners**

The coordinates of the initial points and the approximate values of the coordinates of the defining points $t_1$ and $t_4$ are indicated in the program. An array of measured values of horizontal angles for the problem 2 is given.
For problem 2 the correlation matrix of the measurement results $K_{\Delta{\text{measured}}}$ will have a size of 8x8, the main diagonal of which has squares of mean square errors measurement of horizontal angles, and the corresponding correlation moments of $r_{\beta_{ij}}m_{\beta_{ij}}$ will be indicated in this matrix.

$$K_{\Delta{\text{measured}}} = \begin{bmatrix} 50 & -25 & 0 & 0 & 0 & 0 & 0 & 0 \\
-25 & 50 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 50 & -25 & 0 & 0 & 0 & 0 \\
0 & 0 & -25 & 50 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 50 & -25 & 0 & 0 \\
0 & 0 & 0 & 0 & -25 & 50 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 50 & -25 \\
0 & 0 & 0 & 0 & 0 & 0 & -25 & 50 \end{bmatrix}.$$

The weight matrix of measurement results of P according to the formula (2) is equal to:

$$P = \begin{bmatrix} 0.027 & 0.013 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.013 & 0.027 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.027 & 0.013 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.013 & 0.027 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.027 & 0.013 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.013 & 0.027 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.027 & 0.013 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.013 & 0.027 \end{bmatrix}.$$

Selecting the coordinates of the defining point the program calculates the values of the direction angles. The difference of the corresponding direction angles is used to calculate the values of the horizontal angles. Then the program gives the result in the form of deviation $\nu_{ij}$ of the values of the calculated angles according to selected coordinates from the measured angles. Thus, for the second problem the vector of corrections to the results of measurements $V$ is:

$$V^T = \begin{bmatrix} -6.14 & -1.46 & -4.00 & -3.99 & -5.42 & -3.45 & -3.95 & -3.59 \end{bmatrix}.$$  

So the program having performed several iteration cycles gives the minimum of the objective function $f(x)=V^TPV$, which is 5.316 and coincides with the minimum value of the objective function in problem 1. The adjusted values of the defining points coordinates after the program also coincide with the values of the adjusted coordinates from problem 1 which are presented in table 1.

In addition we should say that the following vector of corrections to the measured angles has been obtained as a result of the network adjustment by the angles without taking into account the correlation between adjacent angles

$$V^T = \begin{bmatrix} -5.47 & -1.53 & -4.20 & -3.59 & -5.84 & -4.10 & -3.89 & -3.38 \end{bmatrix}.$$  

The latter one as well as $V^TPV = 2.805$ did not coincide with the results of the previous two adjustments as it was expected.

Summary
As a result the triangulation network adjustment by the measured values of the horizontal directions and correlated values of the adjacent horizontal angles equal values of the minima of the quadratic objective function \( f(x) = V^2PV \) were obtained. The adjusted values of the coordinates of the defining points have coincided up to tenths of millimeters for the two problems described above. Thus it can be concluded that the search method of nonlinear programming can be successfully applied to adjust correlated measurements.

The advantages of the search method are concluded to be as follows – the regularity and continuity of the objective function are not required, derivatives are not used, not much time is required for the problem to be solved and it is simple enough to implement the algorithm for the problems solving.

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