Abstract—A coding scheme based on irregular low-density parity-check (LDPC) codes is proposed to send secret messages from a source over the Gaussian wiretap channel to a destination in the presence of a wiretapper, with the restriction that the source can send only binary phase-shift keyed (BPSK) symbols. The secrecy performance of the proposed coding scheme is measured by the secret message rate through the wiretap channel as well as the equivocation rate about the message at the wiretapper. A code search procedure is suggested to obtain irregular LDPC codes that achieve good secrecy performance in such context.

I. INTRODUCTION

Physical-layer security provides secure communication between desired users by taking advantages of various physical channel characteristics. The study of physical-layer security dates back to Wyner’s seminal paper [1], in which he introduced the wiretap channel model. In a wiretap channel, a source tries to send secret information to a destination in the presence of a wiretapper. When the wiretapper channel is a degraded version of the destination channel, Wyner [1] described a code design based on group codes such that the source can transmit messages at a positive rate to the destination. This degradedness condition was removed in [2], which showed that a positive secrecy rate is possible for the case where the destination channel is “less noisy” than the wiretapper channel. Generalization of Wyner’s work to the Gaussian wiretap channel was considered in [3]. In [4], a code design based on coset codes was suggested for the type-II (the destination channel is error free) binary erasure wiretap channel. Reference [5] considered the design of secure nested codes for type-II wiretap channels. Recently, references [6] and [7] concurrently established the result that polar codes [8] can achieve the secrecy capacity of the degraded binary-input symmetric-output (BISO) wiretap channels.

Low-density parity-check (LDPC) codes have emerged as potential practical candidates for providing reliable and secure communication over wiretap channels. First, LDPC codes provide excellent “close-to-capacity” performance with reasonable encoding/decoding complexity [9], [10], [11]. Second, there exists powerful tools like density evolution [12] for asymptotic analysis and code design of LDPC codes. For example, the authors of [13] constructed LDPC based wiretap codes for the binary erasure channel (BEC) and the binary symmetric channel (BSC). In [14], multilevel coding/multistage decoding using LDPC codes has been proposed for the quasi-static Rayleigh fading wiretap channel. In [15], punctured LDPC codes were employed in a coding scheme that aims at reducing the security gap of the Gaussian wiretap channel. In [16], further reductions in the security gap are achieved using non-systematic LDPC code obtained by scrambling the information bits prior to LDPC encoding.

In this paper, we design LDPC codes for sending secret messages over the Gaussian wiretap channel with binary phase-shift keyed (BPSK) source symbols. This work is inspired by the results in [17] on secret key agreement over the same wiretap channel, with the availability of an additional public feedback channel. In particular, Theorem 2 of [17] can be modified to show the existence of regular LDPC code ensembles with increasing block lengths that achieve secrecy capacity [1], [3] of the BPSK-constrained Gaussian wiretap channel. Based on this observation, we propose a coding scheme which employs irregular LDPC codes [18] with finite block lengths to support practical secret transmission over the Gaussian wiretap channel. The proposed coding structure allows efficient design of irregular LDPC codes that give good secrecy performance as measured in terms of equivocation about the secret message at the wiretapper. This design constitutes the main technical contribution of the present paper. We note that codes suggested in [15] can be considered as unoptimized special cases of the proposed coding scheme. A more detailed comparison will be given in the sequel.

The outline of the paper is as follows. Section II introduces the BPSK-constrained Gaussian wiretap channel. The proposed coding scheme will be discussed in detail in Section III. In Section IV we describe a code search algorithm based on density evolution analysis to obtain good irregular codes for use in the proposed coding scheme. Finally, conclusions are drawn in Section V.

II. BPSK-CONSTRAINED GAUSSIAN WIRETAP CHANNEL

We consider the wiretap channel model in which the source tries to send a secret message to the destination via an additive white Gaussian noise (AWGN) channel in the presence of a wiretapper. The wiretapper intends to reconstruct the message by listening to the transmission through another independent AWGN channel. The secret message $M \in \{1,2,\ldots,2^k\}$ is encoded into a transmitted sequence $X^n = [X_1,X_2,\ldots,X_n]$. Let $Y^n$ and $Z^n$ denote the corresponding received sequences at the destination and wiretapper, respectively. We restrict the source to transmit only BPSK symbols, i.e. $X_i \in \{\pm 1\}$. In later sections, whenever appropriate, we implicitly employ the mapping $+1 \rightarrow 0$ and $-1 \rightarrow 1$, where 0 and 1 are the two usual elements in GF(2).
Then the BPSK-constrained Gaussian wiretap channel can be modeled as

\[ Y_i = \beta X_i + N_i \]
\[ Z_i = \alpha \beta X_i + \tilde{N}_i, \]

where \( N_i \) and \( \tilde{N}_i \) are independent and identically distributed (i.i.d.) zero-mean Gaussian random variables of variance \( \sigma^2 \). Note that \( \beta \) is the gain of the BPSK symbols transmitted by the source. We impose the source power constraint

\[ \frac{1}{n} \sum_{j=1}^{n} |X_j|^2 \leq P \]

such that \( \beta^2 \leq P \), where \( P \) is the maximum power available to the source. Also, \( \alpha \) is a positive constant that models the gain advantage of the wiretapper over the destination. Let the (noise) normalized gain be \( \beta = \beta/\sigma \), then the received signal-to-noise ratios (SNRs) at the destination and wiretapper are \( \beta^2 \) and \( \alpha^2 \beta^2 \), respectively.

Assuming a uniform message distribution, the rate of the secret message is \( R_s = \frac{b}{n} \). Let \( M \) denote the estimate of the message at the destination. The level of knowledge of the wiretapper possesses about the secret message can be quantified by the equivocation rate \( \frac{1}{n} H(M|Z^n) \). A rate-equivocation pair \((R_s, R_e)\) is achievable if for all \( \epsilon > 0 \), there exists a rate-\( R_s \) code sequence such that

1. \( \Pr\{M \neq \hat{M}\} < \epsilon \), and
2. \( R_e < \frac{1}{n} H(M|Z^n) + \epsilon \)

for sufficiently large \( n \). When the equivocation rate at the wiretapper is as large as the secret message rate, i.e. \( R_s = R_e \), we say that the equivocation-rate pair is achievable with perfect secrecy [1]. The capacity-equivocation region of a wiretap channel contains all achievable rate-equivocation pairs \((R_s, R_e)\). When \( \alpha \leq 1 \), specializing the result in [2] shows that the capacity-equivocation region of the BPSK-constrained Gaussian wiretap channel is given by

\[ 0 \leq R_e \leq C_b \]
\[ R_s \leq R_e \leq C \left( \frac{P}{\sigma^2} \right), \quad (1) \]

where

\[ C_b = \max_{0 < \beta < \sqrt{\frac{P}{\sigma^2}}} \left\{ C(\beta) - C(\alpha \beta) \right\}, \quad (2) \]

and

\[ C(t) = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(y-t)^2}{2}} \log_2 \left( e^{-2y^2} \right) \, dy \]

is the channel capacity of AWGN channel with BPSK input. The secrecy capacity of the wiretap channel is defined as the maximum secret message rate such that the condition of perfect secrecy is satisfied. For the BPSK-constrained Gaussian wiretap channel, the secrecy capacity is given by \( C_b \) if \( \alpha \leq 1 \).

We note that \( C_b \) is achieved when \( X_i \) is equiprobable; but it is not necessarily achieved by transmitting at the maximum allowable power \( P \). Fig. 1 shows the plot of \( C_b \), in units of bits per (wiretap) channel use (bpcu), versus the maximum allowable SNR \( P/\sigma^2 \) for \( \alpha^2 = -1.0, -2.5, -4.4, \) and \(-4.7 \) dB, respectively.

![Fig. 1. The secrecy capacity \( C_b \) of the BPSK-constrained Gaussian wiretap channel for different value of \( \alpha^2 \).](image)

### III. SECRET LDPC CODING SCHEME

In this section, we describe the proposed coding scheme for the BPSK-constrained Gaussian wiretap channel. Inspired by our previous work [17] on secret key agreement over the same wiretap channel, the proposed coding scheme employs irregular LDPC codes and its secrecy performance will be evaluated by measuring the equivocation rate of the secret message at the wiretapper.

Similar to [17], we start with an \((n, l, k)\) secret binary linear block code defined by the pair \((C, W)\), where \( C \) is an \((n, l)\) binary linear block code with \( 2^l \) distinct codewords of length \( n \) and \( W \) is an \((l-k)\)-dimensional subspace in \( C \). The ratios \( R_c = \frac{l}{n} \) and \( R_s = \frac{k}{n} \) will be referred to as the code rate and secret rate of \((C, W)\), respectively. The pair \((C, W)\) will be employed to support practical secret transmission over the wiretap channel. We take the following considerations into account when choosing \((C, W)\). First, the choice of \((C, W)\) should admit efficient encoder and decoder in order to make the proposed coding scheme practically implementable. Second, the choice of \((C, W)\) should avoid transmitting the secret message \( M \) “directly” through the channel since such “direct” transmission is undesirable. Third, the choice of \((C, W)\) should permit a design procedure based on which “good” codes can be obtained.

To satisfy these considerations, we choose an \((m, l)\) linear block code \( C' \) from an ensemble of irregular LDPC codes, where \( m = n + k \). Then the pair \((C, W)\) is chosen as follows. Let \( H \) be the parity-check matrix associated with \( C' \) and assume \( H = [A, B] \) where \( B \) is an \((m-l)\times(m-l)\) lower triangular matrix. Let \( X^{m} = [e^{k}, d^{l-k}, e^{m-l}] \) where \( e^{m-l} = [d^{k}, d^{l-k}, e^{m-l}] \). Then \((n, l)\) linear block code \( C \) is chosen to be set of codewords

2 This lower triangular matrix can be obtained through performing Gaussian elimination. If efficient encoding is necessary, an approximate lower triangular matrix as described in [11] can be used instead.
obtained by removing $e^k$ from $\tilde{X}^m$. That is, $C$ is a punctured version of $C'$. The subspace $W$ is chosen to be the subset of (punctured) codewords obtained by setting $c^k$ to zero. The coding scheme employing the pair $(C, W)$ is as follows:

1) **Encoding:** The source sets $c^k$ to be the $k$-bits secret message $M$ and chooses $d^{k-1}$ randomly according to a uniform distribution. Then it calculates $e^{m-1} = [e^k, d^{k-1}] A^T(B^{-1})^T$ and sends $X^m = [d^{k-1}, e^{m-1}]$ to the destination through the Gaussian wiretap channel.

2) **Decoding:** The destination performs belief propagation (BP) decoding to decode $\tilde{X}^m$ using its channel observation $Y^n$. The first $k$ bits of the decoded codeword give the estimate $M$ of the secret message.

We evaluate the secrecy performance of the proposed coding scheme in the context of achievable rate-equivocation pair defined in Section II. First, if the BP decoder at the destination achieves error probability $\epsilon_d$, then we have $\Pr\{M \neq \tilde{M}\} \leq \epsilon_d$. Hence, Condition 1 in Section II is satisfied if $\epsilon_d$ is small enough. Second, the uncertainty about the message $M$ at the wiretapper given his received sequence $Z^n$ is

$$H(M|Z^n) = H(X^n|M, Z^n) - H(X^n|Z^n).$$

Based on the memoryless nature of the source-to-wiretapper channel and the encoding process, we have $H(X^n, Z^n) \leq nC(\alpha\beta)$, $H(X^n) = I^m$ and $H(M|Z^n, X^n) \leq H(M|X^n) = 0$, respectively. Moreover, consider a fictitious receiver at the wiretapper trying to decode $X^n$ from observing $Z^n$ and $M$. Suppose that the average error probability achieved by this receiver is $\epsilon_w$. Then we have $H(X^n|M, Z^n) \leq 1 + (l-k)\epsilon_w$ by Fano’s inequality. Putting all these back to (3), we obtain

$$\frac{1}{n}H(M|Z^n) \geq R_c - C(\alpha\beta) - (R_c - R_s)\epsilon_w - \frac{1}{n}. \quad (4)$$

Let $R_c = R_c - C(\alpha\beta)$, then Condition 2 in Section II is satisfied if $\epsilon_w$ is small enough and $n$ is large enough. Hence, $(R_s, R_c)$ is an achievable rate-equivocation pair through the BPSK-constrained Gaussian wiretap channel. Moreover, we note that the above lower bound is derived from the Fano’s inequality; thus it applies to any decoder at the fictitious receiver. In fact, the value of the bound depends on the choice of decoders only through $\epsilon_w$. In the next section, we perform computer simulation to estimate $\epsilon_w$ and then employ (4) to bound the equivocation rate achieved by the proposed coding scheme as described above. To get $\epsilon_w$, a BP decoder is implemented for the fictitious receiver at the wiretapper. In order to provide information about the secret message $M$ to the BP decoder, the intrinsic log-likelihood ratios (LLRs) of $c^k$ are explicitly set to $\pm \infty$ according to the true bit values.

### IV. Codes Design and Performance

As mentioned in Section I, reference [15] uses a systematic irregular LDPC code to encode the secret message $M$ (along with some random bits) and then punctures the secret message bits in the codeword prior to transmission in order to “hide” the secret message from the wiretapper. The puncturing pattern is designed to minimize the security gap. Such a coding scheme can be viewed as an unoptimized special case of our scheme proposed in Section III. We show in this section that the generalization in Section III allows us to systematically optimize the irregular LDPC code for good secrecy performance.

To that end, let us apply the code search process proposed in [17] to the present case. Our objective is to design the irregular LDPC code $C'$ and a puncturing scheme so that the secret LDPC code $(C, W)$ works well for both the channel from the source to the destination and the channel from the source to the wiretapper (given the secret message). Let us first consider uniform puncturing of the systematic bits of $C'$, with $p$ denoting the corresponding fraction of punctured variable nodes. Note that the secret rate $R_s = \frac{p}{1-p}$.

Express the variable- and check-node degree distribution polynomials of $C'$ as, respectively, $\lambda(x) = \sum_{i=2}^{d_v} \lambda_i x^{i-1}$ and $\rho(x) = \sum_{i=2}^{d_c} \rho_i x^{i-1}$, where $\lambda_i(\rho_i)$ represents the fraction of edges emanating from the variable (check) nodes of degree $i$ and $d_v (d_c)$ is the maximum variable (check) degree. For a fixed value of $R_s$ (which in turn fixes $p$), the design objective is to find $\lambda(x)$ and $\rho(x)$ that maximize the code rate $R_c' = 1 - \frac{1}{1-R_s} \sum_{x\in\mathcal{F}} \frac{\rho(x)\lambda(x)}{f(x)}$, subject to the constraint that both $\epsilon_d$ and $\epsilon_w$ vanish as the BP decoders iterate.

Fix $\rho(x)$, and let $\epsilon_d(\ell)$ and $\epsilon_w(\ell)$ denote the bit error probabilities obtained by the BP decoders at the destination and wiretapper, respectively, at the $\ell$th density evolution iteration [12], [19] when an initial $\lambda(x) = \sum_{i=2}^{d_v} \lambda_i x^{i-1}$ is used. Now, let $A_{\ell,j}$ denote the bit error probability obtained at the destination by running the density evolution for $\ell$ iterations, in which $\lambda(x)$ is used as the variable-node degree distribution for the first $\ell$ iterations and the variable-node degree distribution with a singleton of unit mass at degree $j$ is used for the final iteration. Let $B_{\ell,j}$ denote the similar quantity for the bit error probability obtained at the wiretapper. Then, we have $\epsilon_d(\ell) = \sum_{j=2}^{d_v} A_{\ell,j} \lambda_j$ and $\epsilon_w(\ell) = \sum_{j=2}^{d_c} B_{\ell,j} \lambda_j$. Note that the values of $A_{\ell,j}$ and $B_{\ell,j}$ are obtained via density evolution.

To account for the puncturing of $c^k$ at the destination’s BP decoder, the intrinsic LLLR distribution entered into the density evolution analysis for the destination’s BP decoder is set to be a mixture of the distribution of the channel outputs at the destination and an impulse at 0 with weights determined by the value of $p$. Let $0 > \epsilon > 0$ be a small prescribed error tolerance. Suppose that $\lambda(x)$ satisfies the property that $\epsilon_d(M_d) \leq \epsilon$ and $\epsilon_w(M_w) \leq \epsilon$, for some integers $M_d$ and $M_w$. Then, we can frame the $R_s$-maximizing code design problem as the
following linear program:

$$\max_{\lambda(x)} \sum_{j=2}^{d_v} \lambda_j \quad \text{subject to:}$$

$$\sum_{j=2}^{d_v} \lambda_j = 1, \quad \lambda_i \geq 0 \quad \text{for } 2 \leq i \leq d_v,$$

$$\sum_{j=2}^{d_v} A_{t,j} \lambda_j - e_s(\ell) \leq \max[0, \delta(e_s(\ell - 1) - e_s(\ell))]$$

$$\text{and} \quad \sum_{j=2}^{d_v} A_{t,j} \lambda_j \leq e_s(\ell - 1), \quad \text{for } 1 \leq \ell \leq M_s,$$

$$\sum_{j=2}^{d_v} B_{t,j} \lambda_j - e_w(\ell) \leq \max[0, \delta(e_w(\ell - 1) - e_w(\ell))],$$

$$\text{and} \quad \sum_{j=2}^{d_v} B_{t,j} \lambda_j \leq e_w(\ell - 1), \quad \text{for } 1 \leq \ell \leq M_w,$$

where $d_v$ here is the maximum allowable degree of $\lambda(x)$ and $\delta$ is a small positive number. The solution $\lambda(x)$ of the above linear program is then employed as the initial $\tilde{\lambda}(x)$ for the next search round. The search process continues this way until $e_d(M_d)$ or $e_w(M_w)$ becomes larger than $\epsilon$, or until $\lambda(x)$ converges. We can also fix $\lambda(x)$ and obtain a similar linear programming problem for $\rho(x)$. The iterative search can then alternate between the linear programs for $\lambda(x)$ and $\rho(x)$, respectively.

For illustration, we apply the above code search procedure to two different wiretap channel settings: (i) $P/\sigma^2 = 3.55$ dB and $\alpha^2 = -4.4$ dB, and (ii) $P/\sigma^2 = 1.0$ dB and $\alpha^2 = -1.0$ dB. In both cases, the code search process starts with the AWGN-optimized LDPC codes reported in [18]. Fig. 2 shows the secrecy performance of a rate-0.541 irregular LDPC code obtained by performing the code search process described above with $R_s = 0.076$. The degree distribution pair of this irregular LDPC code can be found in Table I. We observe that the pair $(R_s, \tilde{R}_{\epsilon}) = (0.33, 0.89)$ (denoted by the square marker) is achieved by this rate-0.541 LDPC code.

Next, we consider the more challenging case under the second channel setting, in which the wiretapper’s SNR is not much weaker than that of the destination. Fig. 3 shows the secrecy performance of a rate-0.505 irregular LDPC code obtained by performing the code search process described above with $R_s = 0.076$. The degree distribution pair of this irregular LDPC code with relatively good secrecy performance for different values of $\alpha^2$. A similar code search process can also be formulated to include optimization of the puncturing pattern. However, we have not been able to obtain significantly better codes with the modified search.

As mentioned before, the codes suggested in [15] are "unoptimized" special cases of the coding scheme described here. In particular, a rate-0.5 irregular LDPC code with $p = 0.3$ is employed in [15], resulting in secret rate $R_s = 0.43$. The secrecy performance of the coding scheme in [15] is evaluated by the security gap. In our notation, that is to find the values $\beta$ and $\alpha$ such that the decoding (bit) error probability of the secret message at the destination is smaller than a prescribed value, and the decoding (bit) error probability of the secret message at the wiretapper is close to 0.5. The security gap is

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**TABLE I**

| Rate-0.541 | Rate-0.508 | Rate-0.505 |
|------------|------------|------------|
| $\lambda_2$ | 0.3013 | 0.2762 | 0.2599 |
| $\lambda_3$ | 0.1846 | 0.2804 | 0.2837 |
| $\lambda_4$ | 0.1510 | 0.0284 | 0.0284 |
| $\lambda_5$ | 0.0614 |
| $\lambda_{10}$ | 0.3017 | 0.4424 | 0.4283 |
| $\rho_2$ | 0.3892 | 0.6086 | 0.6315 |
| $\rho s$ | 0.6054 | 0.3914 | 0.2532 |
| $\rho_{10}$ | 0.0054 | 0.0153 |

Fig. 2. Plot of $(R_s, \tilde{R}_{\epsilon})$ pairs achieved by the proposed coding scheme and by the coding scheme in [15] when $P/\sigma^2 = 3.55$ dB and $\alpha^2 = -4.4$ dB. The solid curve traces the boundary of the capacity-(fractional) equivocation region.
In summary, the proposed coding scheme and code search operate at this rate if we target to achieve perfect secrecy.

In this paper, we developed a coding scheme for sending secret messages over the BPSK-constrained Gaussian wiretap channel. The proposed coding scheme employs punctured systematic irregular LDPC codes in which secret message bits are punctured. To systematically address the secret code design problem, we presented a density-evolution based linear program to search for good irregular LDPC codes to be used in the proposed coding scheme. Simulation results showed that the irregular LDPC codes obtained from our search can achieve secrecy performance relatively close to the boundary of the capacity-equivocation region of the BPSK-constrained Gaussian wiretap channel.

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Fig. 3. Plot of the $(R_s, \tilde{R}_e)$ pair achieved by the proposed coding scheme when $P/\sigma^2 = 1.0$ dB and $\alpha^2 = -1.0$ dB. The solid curve traces the boundary of the capacity-(fractional) equivocation region.

then defined as the ratio of the SNR of the destination to that of the wiretapper, i.e. $\frac{R_s}{\sigma^2}$. As reported in [15], the security gap, with uniform puncturing over all variable nodes of different degree for $p = 0.3$ is about 4.4 dB.

To compare with our optimized codes, Fig. 2 shows the secrecy performance of the rate-0.5 code in [15] with $p = 0.3$ evaluated by using [4] as before under channel setting (i). The pair $(R_s, \tilde{R}_e) = (0.43, 0.68)$ (denoted by the circle marker) is achieved by this code. We also perform a code search under this channel setting with $R_s = 0.43$ for comparison. The pair $(R_s, \tilde{R}_e) = (0.43, 0.70)$ (denoted by the diamond marker) is achieved using the resulting rate-0.508 irregular LDPC code. We see that the irregular LDPC code obtained from the proposed code search process also slightly outperforms the “unoptimized” one used in [15] in terms of equivocation rate.

Consulting back to Fig. 1, we see that for $\alpha^2 = -4.4$ dB, the secrecy capacity of the BPSK-constrained Gaussian wiretap channel never exceeds 0.34 bpcu. Hence, the fractional equivocation $\tilde{R}_e$ is strictly below 1 at $R_s = 0.43$. In fact, the highest achievable $\tilde{R}_e$ at $R_s = 0.43$ under this channel setting is only 0.78 (cf. Fig 2). That means, we should not operate at this rate if we target to achieve perfect secrecy.

In summary, the proposed coding scheme and code search process provide a much more systematic and flexible means to designing irregular LDPC codes for the BPSK-constrained wiretap channel than the approach in [15].

**V. CONCLUSIONS**

In this paper, we developed a coding scheme for sending secret messages over the BPSK-constrained Gaussian wiretap channel. The proposed coding scheme employs punctured systematic irregular LDPC codes in which secret message bits are punctured. To systematically address the secret code design problem, we presented a density-evolution based linear