Study on RLS-Type Algorithm of GPS-Guided Range Correction

Jun Chen*, Zhonghua Du² and Jian Shen¹

¹The 28th Research Institute of China Electronics Technology Group Corporation, Nanjing, China
²Nanjing University of Science and Technology, Nanjing, China
Email: uniquecj@163.com

Abstract. Mitigating the error of GPS measurement signals is essential for the range precision of GPS-guided range correction projectile. In order to improve the range precision, an online method based on recursive least square algorithm (RLS) is applied to calculate the deployment time of brake. First, RLS is utilized to mitigate the error of the velocity, position, angle, etc., measured by GPS. Then, the deployment time can be calculated accurately by means of the appropriate pretreated data. To confirm the effectiveness of the method, a comparison to using least square algorithm and non-algorithm is done. The experimental results demonstrate that the method in this paper takes advantages in terms of online, real-time and the range correction accuracy.

Keywords. Recursive least square algorithm, range correction, deployment time, fitting, real-time.

1. Introduction

Trajectory correction projectile, as low cost correction ammunition, is easy to achieve. It basically makes use of “shoot far, correct near” method. Before the fire a slightly distant point away from actual target is aimed at. After the fire the detecting device measures one or several parameters of trajectory elements during the flight. Meanwhile based on these measured parameters, the controller in the projectile predicts the impact point real timely, calculates the deviation compared to actual target and depends on the magnitude of the deviation to form the control instruction. At last, the actuator is activated at the appropriate moment according to the control instruction and the brake is deployed to increase the air drag of the projectile during the flight so as to achieve the purpose of range correction. GPS, ground (shipboard) radar, inertial navigation system (INS) and GPS-INS are often applied to measure the trajectory elements. Because of accumulated error of the INS and high cost of the radar system, GPS becomes the mainstream choice of the trajectory correction projectile. As the “Beidou” positioning system networks perfectly, it is imminent to master the technology of the GPS-guided trajectory correction projectile.

Kalman filter technology is mostly used during the GPS dynamic navigation in order to improve the positioning precision. However, the measurement signals after filtering still exist a bigger error. If these measurement signals are used directly, the identification of trajectory would produce a very great error. It is necessary to pretreat GPS measurement signals received. Chen [6] showed that the phase observed value of the continuous epoch can form an approximately smooth curve, which can be fitted by means of several orthogonal polynomials, among which Chebyshev polynomial is a better algorithm. Xiong [7] by test obtained that using the Chebyshev polynomial can fit measurement data...
smoothly and eliminate the outliers in the group of observed value. Hou [8] took advantage of wavelet transform method to denoise GPS signals. Wang [5] proposed a method of reducing the fluctuation of GPS data by the longitudinal smoother of dynamic system based on the least squares algorithm. Combined with the principle of one dimensional trajectory correction projectile, Shen [9] presented that at a certain time-step GPS receiver accepts position and velocity measurement signals, which can be classified into 3 groups, \((t, v), (t, x)\) and \((t, y)\), then each group is fitted separately to a quadratic multinomial curve by the least square algorithm and according to the quadratic multinomial curve and mutual function relationship, \(v(t_p), x(t_p), y(t_p), \theta(t_p)\), which are treated as initial ballistic conditions, can be obtained. Although this method achieves well, it need take a period of time to accept GPS measurement signals, which makes this method lack of characteristics of real-time and online [10].

GPS-guided trajectory correction projectile utilizes the “controlled by itself” correction method and the preprocessing of the GPS measurement signals is required to be completed by the controller in the projectile, so a recursive estimation algorithm is needed to meet the real-time requirements. Kalman filter and recursive least square algorithm (RLS) are widely used as recursive estimation algorithm. Kalman filter has the ability of good tracking, but the algorithm requires predetermined statistical parameters on noise, large amounts of calculation, and the initial value of the gain and covariance matrix determines its convergence speed and the denoising performance [11, 12]. RLS not only need not statistically analyze noise, but also has the merits of smaller computational complexity, high precision and high speed, which is often used for filtering, data fitting, system identification and parameter estimation [13, 14]. Meng [15] proved that the estimation accuracy can be further improved in the method of using the least square algorithm to estimate the measurement data preprocessed by Kalman filter, thus putting forward the recursive least square estimation algorithm based on parallel Kalman filter. So RLS can be utilized to preprocess GPS measurement signals in order to further improve the measurement precision. In this paper, a method that GPS measurement signals are pretreated by the RLS on-line as initial ballistic conditions to calculate the deployment time of brake is showed. First GPS measurement signals are preprocessed to reduce the effects of random errors by RLS, then the pretreated velocity, position coordinate and trajectory angle at some moment are used to calculate the deployment time of brake. Finally, the comparison to the correction accuracy by the least squares estimation algorithm and non-pretreated data is done, the experimental results demonstrate the effectiveness of the method showed.

2. The Exterior Ballistics of GPS-guided Range Correction Projectile

In order to meet the need of calculating real-timely, the exterior ballistics of GPS-guided one-dimensional trajectory correction projectile is based on the point mass trajectory, which can be optimized according to the practical application [16, 17].

2.1. The Exterior Ballistics of One-Dimensional Trajectory Correction Projectile

According to the point mass trajectory model, the following assumptions are made: (1) The projectile is axisymmetric body; (2) The surface is a plane; (3) The value of gravity acceleration is nearly constant and the direction is always vertical downward; (4) The Coriolis’ acceleration is zero; (5) The angle of attack is small and negligible.

The exterior ballistics of one-dimensional trajectory correction projectile may be written as:

\[
\begin{align*}
\frac{dv}{dt} &= \frac{c_p v^2 S}{2m} - g \sin \theta \\
\frac{d\theta}{dt} &= -\frac{g \cos \theta}{v} \\
\frac{dv}{dt} &= v \sin \theta \\
\frac{dx}{dt} &= v \cos \theta
\end{align*}
\]  
\tag{1}
where \( c = \begin{cases} c_{x0}(Ma), & t \in [0, t_c) \\ \lambda \cdot c_{x0}(Ma), & t \in [t_c, T] \end{cases} \), \( t_c \) is the deployment time of brake, \( T \) is the time of impact point, \( c_{x0}(Ma) \) is the drag coefficient before the brake deployment, \( \rho \) is air density, \( S \) is projectile reference area, \( m \) is projectile mass, \( \lambda \) is drag-added coefficient, which is the increased multiple of drag coefficient after the brake deployment. For a given projectile and brake, drag coefficient is approximately constant in a certain speed range.

2.2. The Effect Produced by GPS Error on Range Correction

How to use GPS measurement signals to achieve one-dimensional trajectory correction, is shown in figure 1. Before launching the projectile the local ephemeris is set into the GPS receiver, the origin of the trajectory is fixed, and a slightly distant point \( x(t_{end}) \) away from actual target \( x_{target} \) is aimed at. After launching, the GPS receiver in the projectile starts to accept real-time GPS positioning signals (position coordinate signals and speed signals), the received GPS measurement signals at some moment \( t \) or during a period as the initial ballistic condition for the controller in the projectile. Then according to the target distance \( x_{target} \), the deployment time of brake \( t_c \) can be calculated. From figure 1, GPS measurement accuracy can directly determine the accuracy of trajectory correction projectile. GPS measurement signals exist random error, so the problem of how to reduce the error and improve the measurement accuracy of ballistic data is needed to solve urgently.

GPS itself has measurement error. If the initial ballistic conditions only consist of GPS measurement signals at a single moment, \( t_c \) solved may have a very big fluctuation of error. The example in the paper is certain type of rocket widely used. Non-pretreated GPS measurement signals at a single moment as initial ballistic conditions of the correction trajectory (the action time error of brake is ignored), are utilized directly. According to the GPS receiver provided, the \( x \) error consists of 100 random values between \(-3 \) m to \( 3 \) m, the \( y \) error consists of 100 random values between \(-6 \) m to \( 6 \) m, the error of \( v_x \) and \( v_y \) separately consists of 100 random values between \(-0.1 \) m/s to \( 0.1 \) m/s, \( x_{target} = 800 \) m and \( \lambda = 7 \). Then the error of range correction is shown in figure 2.
From figure 2, the $t_c$ calculated has the big error, which has reached one hundred milliseconds and led to the distance error of more than fifteen meters by non-pretreated GPS measurement signals at a single moment as initial ballistic conditions. These completely go against the original intention of one-dimensional trajectory correction projectile. Therefore, a method meeting the requirement of preprocessing GPS measurement signals real-timely need to be introduced.

3. The Control Algorithm Process

RLS has the advantages of fast speed, high precision and great ability of online. First, GPS measurement signals, which are accepted at a single moment, may be preprocessed by RLS algorithm to fit ballistic curve. Then the data on the fitted curve is selected into the algorithm of calculating the deployment time. The effectiveness of GPS error can be mitigated by this way.

3.1. RLS Algorithm and Its Implementation

RLS algorithm is a very important parameter estimation algorithm, often used for linear and nonlinear curve fitting. RLS algorithm is a method of extending least square algorithm adaptive, whose purpose is that after obtaining new data, a new estimate result can be recalculated based on the original estimate result and the new data, while not recalculated with all data [18].

For the model, $f(x) = W^T \Phi(x)$, the mechanism of RLS with forgetting factor is to minimize the weighted mean square error (MSE), $J(n)$:

$$ J(n) = e^T \Psi e = \sum_{i=1}^{n} \rho^{n-i} |e_i|^2 $$

where $e_i = y_i - \hat{W}_i \Phi(x_i)$, $y_i$ is the actual output, $e$ is error matrix, $e = [e_1, e_2, \ldots, e_n]^T$, $\Psi$ is a diagonal weighted matrix, $\Psi = \text{diag}(\rho^{-1}, \rho^{-2}, \ldots, \rho^{-n}, \rho)$, $\rho$ is a forgetting factor, $0 < \rho \leq 1$, and $\hat{W}_i$ is the estimated parameter at $n$-th step.

The forgetting factor can let the error close to moment $n$ with greater weight and the error far from moment $n$ with smaller weight, which may ensure that observation data in some past period can be “forgotten” [13].

In order to let MSE least, RLS algorithm after derivation is summarized as follows:
where \( K \) is the gain vector, \( P \) is the inverse of the correlation vector, \( \Phi \) is kernel function vector.

If \( \rho = 1 \), equation (3) becomes the standard RLS.

If the environment on the occasion of using the RLS algorithm is unstable and the model is changed as time, variable weighted forgetting factor matrix \( \psi' \) is needed, \( \psi' = \text{diag} [\rho_1, \rho_2, \ldots, \rho_n, \rho_s] \). In non-stationary conditions, the error at only nearby moment need to function, so the algorithm can quickly track the local trend of the non-stationary signals [19].

### 3.2. RLS-Type Algorithm of GPS-Guided One-Dimensional Correction Trajectory

According to the RLS algorithm and the principle of GPS-guided One-dimensional Trajectory Correction projectile, the proposed algorithm can be summarized as follows:

Step 1: select the original parameters, \( a_0, \bar{a}_0, \rho \) and \( \Phi(x_o) \), where the quadratic multinomial curve fitting can meet the requirement based on the ballistic characteristics, \( \Phi(x_o) = [1 \ x \ x^2] \) [9].

Step 2: obtain the GPS measurement data, \( v_i(t), v_i(t), x(t) \) and \( y(t) \) at the moment \( t \) as initial ballistic conditions, utilize the algorithm of the deployment time of brake to calculate \( t_w \). The deployment time \( t_w \) is not accurate, but it can be regarded as reference value of the end time, tend, when the last GPS measurement data is accepted. \( t_{end} < t_w \) and \( n = (t_{end} - t) / f_{gps} + 1 \), where \( f_{gps} \) is the acceptance frequency of GPS.

Step 3: plugging the GPS measurement data, \( v_i(t), v_i(t), x(t) \) and \( y(t) \), \( t \leq t \leq t_{end} \), into the RLS algorithm in turn yields \( \bar{w} \) at each iteration.

Step 4: calculate \( f(x), f_y(x), f_x(x) \) and \( f_x(x) \) at the moment \( t \) in the function of \( f(x) = W^T \Phi(x) \), as initial ballistic conditions to insert to the algorithm of the deployment time to solve the actual deployment time \( t \).

RLS algorithm applies the cyclic inversion method to calculate inverse of matrix, which can reduce the computational burden significantly compared to calculating inverse of matrix directly. RLS algorithm can deal with the GPS measurement signals in turn and greatly make full use of processor’s online and real time capability, compared with the least squares estimation.

### 4. Simulation Experiments

Certain type of rocket widely used is taken as example to demonstrate the effectiveness of the proposed RLS algorithm. The simulation flow diagram is shown in figure 3.

Random number generator based on the actual GPS measurement error, at every moment, generates \( j \) groups of the error of \( (v_x, v_y, x, y) \). In each simulation, homologous group of the error of \( (v_x, v_y, x, y) \), is selected to add to the corresponding standard value, which forms the “true” GPS measurement data at these moments. Through statistically analyzing the group of \( x_{end} \), the average value, variance and intensity (probability of relative error) are obtained.

\( \lambda \) of this rocket is 7, according to the measurement accuracy of certain GPS, \( \sigma_x = 3 \) m, \( \sigma_y = 6 \) m, \( \sigma_{x_{end}} = 0.1 \) m/s, \( f_{gps} = 10 \) Hz and \( t_r = 0.6 \) s. Initialize \( P_0 = 1000 \cdot I \), \( \bar{w}_0 = 0 \) and \( \Phi(x_o) = 0 \). If the initial ballistic conditions of one-dimensional trajectory correction projectile are determined, this model is generally stable, so choose \( \rho = 1 \).
Assuming that the target distance is 800 m, the deployment time $t_c$ obtained is 3.827 s according to standard trajectory. So the moment $t_p$ when the GPS measurement data is accepted is from 0.6 s to 3.7 s and $n=(t_{end}-t_{1})/f_{GPS}+1=32$. Compared to the non-pretreated data and the least square fitting data, the comparison of 100 groups of $x_{end}$ at different moment $t_p$ is shown in figure 4.

**Figure 3.** The simulation flow diagram.

**Figure 4.** The comparison of different algorithm.
Figure 4 shows (1) With non-preprocessed GPS data, \( t_c \) calculated has a big error and the impact point distribution is very large, while with the least square estimation and RLS algorithm to pretreat GPS measurement data, the impact point distribution is smaller; (2) Because its ability of real-time is better than the least square estimate, RLS algorithm can replace the least squares estimate on occasions with high requirements of real-time.

The simulation results in the conditions of different \( x_{target} \) and \( t_p \) are shown in table 1.

| \( x_{target} \) (m) | \( t_p \) (s) | Average value \( x_{ave} \) | Variance \( \sigma \) | Intensity \( E_i \) |
|---------------------|-------------|-----------------|----------------|----------------|
| RLS /m              | Non-pretreated /m | RLS | Non-pretreated | RLS | Non-pretreated |
| 600 0.8             | 601.05       | 602.31          | 2.81           | 4.47           | 0.0032      | 0.0050      |
| 600 1.2             | 600.38       | 601.64          | 1.57           | 4.56           | 0.0018      | 0.0051      |
| 600 1.4             | 600.82       | 601.04          | 2.21           | 4.25           | 0.0025      | 0.0048      |
| 650 0.8             | 650.84       | 652.46          | 1.91           | 4.35           | 0.0020      | 0.0045      |
| 650 1.2             | 650.55       | 651.79          | 2.01           | 4.46           | 0.0021      | 0.0046      |
| 650 1.6             | 650.71       | 650.24          | 2.65           | 4.21           | 0.0027      | 0.0044      |
| 650 2.0             | 650.96       | 651.10          | 3.95           | 4.86           | 0.0041      | 0.0050      |
| 700 1.0             | 700.58       | 701.86          | 1.41           | 4.31           | 0.0014      | 0.0041      |
| 700 1.4             | 700.47       | 701.38          | 1.74           | 4.26           | 0.0017      | 0.0041      |
| 700 1.8             | 700.48       | 701.33          | 3.049          | 4.36           | 0.0029      | 0.0042      |
| 700 2.2             | 700.31       | 700.87          | 3.60           | 4.92           | 0.0035      | 0.0047      |
| 800 1.0             | 800.39       | 802.43          | 1.27           | 4.72           | 0.0011      | 0.0040      |
| 800 1.4             | 800.39       | 801.84          | 1.50           | 4.70           | 0.0013      | 0.0040      |
| 800 1.8             | 800.05       | 801.74          | 0.98           | 4.89           | 0.0008      | 0.0041      |
| 800 2.2             | 799.98       | 801.32          | 1.07           | 5.76           | 0.0009      | 0.0049      |

From table 1, the method of preprocessing the GPS measurement signals with RLS algorithm, can greatly improve impact point accuracy of GPS guided one-dimensional trajectory correction projectile.

5. Conclusion
Due to the limitations of GPS itself, it is hard to improve the precision of measurement signals. And because GPS-guided one-dimensional trajectory correction projectile utilizes the “controlled by itself” correction method, GPS measurement data preprocessing need be completed in the projectile and meet the requirements of online and real-time. In this paper, RLS is utilized to mitigate the error of the velocity, position, angle, etc., measured by GPS. Then, the deployment time can be calculated accurately by means of the appropriate pretreated data. The comparison to using least square algorithm and non-algorithm is done has demonstrated that the method in this paper takes advantages in terms of online, real-time and the range correction accuracy.

References
[1] Zhang I-Q, Liu D-F, Wang D-M and Pang Y-K 2010 A summary for trajectory correction projectiles Acta Armamentarii 31 (2) 127-130.
[2] Holls M S L 1996 Preliminary design of a rang correction module Army Research Laboratory (3) ARL-MR-298.
[3] Qiu R-J, Tao J-W and Wang M-L 2009 A summary for range correction projectiles Technology Foundation of National Defence (8) 45-48.
[4] Yuan D-B, Cui X-M, Fan D-L, Feng W-G and Yu Y-J 2010 Application of Kalman filter method to the date processing of GPS of deformation monitoring Second International
Workshop on Education Technology and Computer Science (ETCS 2010) pp 269-272.

[5] Wang Q, Wan D-J and Li Z-G 1996 The longitudinal smoother of dynamic system and its application in GPS data processing Journal of Data Acquisition & Processing 11 (3) 191-194.

[6] Chen C-M 2003 Applications of orthogonal polynomials in GPS data processing Journal of Geodesy and Geodynamics 23 (3) 74-76.

[7] Xiong Y-Q 1997 Optimum orthogonal polynomials in GPS data Bulletin of Surveying and Mapping (8) 2-5.

[8] Hou L-F, Xiao B-L, Hu Y-J, Sun X-B and Liu Z-C 2008 Application of wavelet transform threshold de-noising method in GPS data processing GNSS World of China 58-62.

[9] Shen Q, Ge M, Zhang J-X and Li J 2009 Analysis on the precision of a GPS-based trajectory identification method by simulation Transactions of Beijing Institute of Technology 29 (2) 100-102.

[10] Shen Q, Ge M, Peng B and He X 2009 Parameters estimation algorithm by kalman filtering based on GPS measurement for projectile trajectory Transactions of Beijing Institute of Technology 29 (12) 1049-1051.

[11] Dash P K, Jena R K and Panda G 2000 An extended complex kalman filter for frequency measurement of distorted signals IEEE Transactions on Instrumentation and Measurement 49 (4) 746-753.

[12] Dash P K, Pradhan A K and Panda G 1999 Frequency estimation of distorted power system signals using extended complex Kalman filter IEEE Trans on Power Delivery (14) 761-766.

[13] Munjal A, Aggarwal V and Singh G 2008 RLS Algorithm for acoustic echo cancellation Proceedings of 2nd National Conference on Challenges & Opportunities in Information Technology (COIT-2008) (RIMT-IET, Mandi Gobindgarh).

[14] Moustakides G V 1997 Study of the transient phase of the forgetting Factor RLS IEEE Trans Signal Processing 45 (10) 2468-2476.

[15] Meng Z, Yu J-Y and Yan Y-P 2011 A least squares frequency measurement algorithm based on the parallel Kalman filter Microelectronics & Computer 28 (3) 1-6.

[16] Chang Y, Wang X-B and Gao M 2005 Research on the modified method of GPS one-dimensional trajectory correction fuze Journal of Ordnance Engineering College 17 (4) 16-18.

[17] Pu F 1986 Exterior Ballistics (Beijing:National Defence Industry Press).

[18] Luo X-C 1995 The real-time weighted recursive algorithm of least squares estimation and its initial applications Missiles and Space Vehicles (1) 40-49.

[19] Xiao Y, Tadokoro Y and Shida K 1999 Adaptive algorithm based on least mean-power error criterion for fourier analysis in additive noise IEEE Trans on Signal Processing 47 (4) 1172-1181.