The bosonic excitations induced by the $\omega$ meson propagation in dense nuclear matter is studied within the framework of random phase approximation. The collective modes are then analyzed by finding the zeros of the relevant dielectric functions. Subsequently we present closed form analytical expressions for the dispersion relations in different kinematical regime. Next, the analytical behaviour of the in-medium effective propagator for the $\omega$ meson is examined. This is exploited to calculate the full spectral function for the transverse (T) and longitudinal (L) mode of the $\omega$ meson. In addition, various sum rules are constructed for the $\omega$ meson spectral density in nuclear medium. Results are then discussed by calculating the residues at the poles and discontinuities across the cuts.

PACS numbers: 25.75.-q, 21.65. + f

Keywords : dispersion, sum-rule, nuclear matter

I. INTRODUCTION

One of the cardinal challenges facing the nuclear physics today is to determine the properties hadrons at finite temperature and/or chemical potential. Such studies are important both in the context of high energy heavy ion collisions (HIC) and fixed target electron scattering experiments [1,2]. Medium effects have also been found to be important to explain pion induced pion production data. It is seen that a clear enhancement of $\pi^+\pi^-$ pair in the low invariant mass region could be accounted for by invoking the in-medium $\sigma$ meson spectral function [3,4]. Similarly it is observed that the photoabsorption data in $\gamma + A$ system can be explained by invoking the medium modified $\rho$ meson spectral function [1].

In heavy ion collision, among others, the light vector mesons have acquired particular interest. This is because of their dileptonic decay channels. Dileptons, once produced leaves the medium without suffering further scattering and hence provide a penetrating probe to study the properties of the dense nuclear medium as has been discussed at length in the recent literatures [6]. Experimentally the dilepton invariant mass spectra measured at CERN/SPS suggests that in-medium properties of the vector mesons, in particular that of $\rho$ meson, are crucial to understand the low mass enhancement of the lepton pairs.

In the theoretical arena such studies were stimulated by the pioneering work of Brown and Rho [5] where it was proposed that, in hot and/or dense matter, vector meson masses would drop from their values in free space as a precursor phenomenon of chiral symmetry restoration. Since then, several theoretical models have been invoked to study the in-medium properties of $\omega, \rho$ or $\phi$ meson which include QCD sum-rule, Nambu-Jona-Lasinio model, Chiral perturbation theory or simple hadron effective Lagrangian approach [6–10].

In the present work we discuss the vector meson propagation in nuclear matter which could be visualized as meson waves traveling through dispersive medium. This concept is usually invoked to study the poinc collective effects in nuclear matter [11]. A characteristic feature of meson propagation in matter is that there exists two distinct scales depending upon its momenta; if the momenta of the meson is small compared to the Fermi momenta of the nuclear matter, the meson propagation becomes inseparable from the macroscopic collective oscillation of the system induced by the mesonic field, while on the other hand, for large momenta, as we shall see, the meson propagation can still be regarded as usual particle propagation with its dispersion characteristics very similar to that of free space. In particular we study the propagation of the $\omega$ meson in dense nuclear matter in the long-wave length limit i.e. in the regime when the typical characteristic length over which the meson field varies ($1/q$) is large compared to the the de Broglie wave-length of the nucleons $1/p_F$. Such a study was initially pursued by Chin [12] who showed that, in nuclear matter, $\omega$ meson picks up the collective oscillation of the system and becomes massive. However, in Ref. [12], the vacuum contribution (Dirac sea) was neglected on the ground that it only renders the coupling constant momentum dependent and in general, in the long-wave length limit (i.e. for low momentum transfer), the effect is only marginal. Later, in a seminal paper by Jean et al. [6] it was shown that, in presence of a mean scalar field (Walecka model), the vacuum part contributes substantially with opposite sign. This, therefore, plays a crucial role in determining the over all sign and magnitude of the net polarization insertion [12]. In the current work we include both the effects and present new and more complete results for the $\omega$ meson dispersion relations in nuclear matter by considering various
kinematical limits of the $\omega$ meson momentum compared to the density characterized by the Debye mass or the plasma frequency of the system. Moreover, we also recover the results presented in Ref. [12] in the appropriate limits.

In addition to the calculation of the dispersion relations, we also examine the analytical properties of the effective $\omega$ meson propagator for the longitudinal and transverse excitations and construct various sum rules for the relevant spectral densities ($\rho_{T,L}$). Later, the full spectral functions ($\rho_{T,L}$) are expressed in terms of the residues at the poles and cuts of the propagators. Such studies, as we shall see, provide insight to understand and interpret the collective excitations of the nuclear matter via the sum rules and the residues in different kinematical domains of the momenta.

The paper is organized as follows. We first outline the formalism and describe the collective modes in terms of the zeros of the dielectric functions involving density dependent $\omega$ meson polarization tensor. In section III, the concept of hard nucleon loop (HNL) approximation is introduced which allows for the analytical evaluation of the polarization insertions. The dispersion relations for various kinematical domains are then presented. Subsequently, in section IV, several sum rules for the $\omega$ meson spectral functions are constructed by exploiting the properties of dispersion relation techniques of complex variables. Finally the conclusion and summary is presented in section V.

II. FORMALISM

It is well known that the collective modes corresponding to density fluctuations are determined from the poles of the meson propagators. In Ref. [12] it has been demonstrated that in nuclear matter such collective modes are dominated solely by the vector interaction in the high-density limit. This, in turn, implies that the in-medium properties of the vector mesons are directly linked with the collective excitation set by their propagation.

The dielectric functions relevant for the $\omega$ meson propagation in dense nuclear matter can be calculated within the framework of random phase approximation (RPA) which essentially implies repeated insertion of the $\omega$ meson self-energy involving nucleon-nucleon loop as described in ref. [12,13].

Considering the interaction of the $\omega$ meson with the nucleon field, $L_I = g_V \bar{\psi}_\mu \gamma_\mu \omega^\nu$, the second order polarization tensor $\Pi_{\mu\nu}$, can be written as

$$\Pi^{\alpha\beta}_{\mu\nu} = \frac{1}{(2\pi)^4} \int d^4p \, Tr[i\gamma_\mu iG(p+q)i\gamma_\nu iG(p)]$$

where $(\alpha, \beta)$ are the isospin indices and $G(p)$ is the in-medium nucleon propagator [14]

$$G(p) = G_F(p) + G_D(p)$$

where

$$G_F(p) = (p_\mu \gamma^\mu + M^*)\left[\frac{1}{p^2 - M^2 + i\epsilon}\right]$$

and

$$G_D(p) = (p_\mu \gamma^\mu + M^*)\left[\frac{i\pi}{E^*(p)}\delta(p_0 - E^*(p))\theta(p_F - |\vec{p}|)\right].$$

The first term in $G(p)$, namely, $G_F(p)$, is the same as the free propagator of a spin 1/2 fermion, while the second part, $G_D(p)$, involving $\theta(p_F - |\vec{p}|)$, arises from Pauli blocking, describes the modification of the same in the nuclear matter at zero temperature. $E^*(p) = \sqrt{p^2 + M^2}$ and $M^*$ denotes the in-medium mass of the nucleon, which in the present context might be determined from the mean scalar density $\bar{\rho}$.

In a similar vein the polarization insertions can also be written as sum of two parts :

$$\Pi_{\mu\nu} = \Pi^F_{\mu\nu} + \Pi^D_{\mu\nu} \quad (5)$$

From the above set of equations we can write the real part of the density dependent $\omega$-meson self energy as [12,10]

$$\Pi^D_{\mu\nu} = \frac{g^2}{\pi^3} \int_0^{p_F} \frac{d^3p}{E(p)} \frac{\mathcal{P}_{\mu\nu} q^2 - Q_{\mu\nu}(p\cdot q)^2}{q^4 - 4(p\cdot q)^2} \quad (6)$$

where $\mathcal{P}_{\mu\nu} = (p_\mu - \frac{q_\mu}{q^2} q_\nu)(p_\nu - \frac{q_\nu}{q^2} q_\mu)$ and $Q_{\mu\nu} = (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})$. It is evident that the form of the polarization tensor conforms to the requirement of current conservation, i.e. $q^\mu \Pi^D_{\mu\nu} = \Pi^D_{nu} q^n = 0$. In order to evaluate $\Pi^D_{\mu\nu}$ conveniently,
we choose $\mathbf{q}$ to be along the $z$ axis i.e. $q = (q_0, 0, 0, |\mathbf{q}|)$, and $p,q = E(p)q_0 - |p||q| \chi$, where $\chi$ is the cosine of the angle between $\mathbf{k}$ and $\mathbf{q}$. On account of integration over the azimuthal angle, only six components survive. Then again for isotropic nuclear matter we have $\Pi^D_{11} = \Pi^D_{22}$ and $\Pi^D_{03} = \Pi^D_{30}$. Furthermore, the current conservation condition puts additional constraints leaving only two non-vanishing independent component of $\Pi^D_{\mu\nu}$, linear combinations of which gives us the longitudinal and transverse components of $\Pi^D_{\mu\nu}$, namely, $\Pi^D_\nu(q) = -\Pi^D_{00} + \Pi^D_{33}(q)$ and $\Pi^D_T(q) = \Pi^D_{11} = \Pi^D_{22}$.

The free part is obtained by putting $G(p) = G_F(p)$ in Eq. 8 which is divergent and can be regularized using appropriate renormalization scheme \[12,14\]. The regularization procedure we adopt here is the following:

\[ \frac{\partial^n \Pi^F(q^2)}{\partial (q^2)^n}|_{M^* \to M, q^2 = m^2} = 0, \]

where, $(n = 0, 1, 2, \ldots, \infty)$. This yields

\[ \Pi^F_{\mu\nu} = Q_{\mu\nu} \frac{q^2}{\pi^2} q^2 \int_0^1 dx (1 - x) \log \frac{M^2 - q^2} {M^2 - m^2 x} x (1 - x). \]

It is evident that the free part does not distinguish between the longitudinal and transverse mode and we can take $\Pi^F_T = \Pi^F_L = \Pi_{22} = \Pi_{11}$. In order to study collective response of the nuclear system and to obtain the dispersion relation of the induced oscillation, one needs to find zeros of the dielectric function defined through

\[ \epsilon_{T,L}(q_0, |q|) = 1 - \frac{1}{q_0^2 - |q|^2 - m^2} \Pi_{T,L}(q_0, |q|) = 0, \]

where $\Pi_{T,L}(q_0, |q|) = \Pi^F_{T,L}(q^2) + \Pi^D_{T,L}(q_0, |q|)$. It is to be noted that the free part $\Pi^F_{L,T}$ depends only on the Lorentz scalar $q^2$ while the density-dependent part $\Pi^D_{L,T}$ involves both $q_0$ and $|q|$ individually. The equations $\epsilon_{T,L}(q_0, |q|) = 0$ in general have to be solved numerically. However, in the next section, we discuss appropriate approximation scheme which admits close form solutions.

### III. COLLECTIVE MODES AND DISPERSION RELATIONS

In this section we discuss first the approximation scheme to derive the polarization tensors analytically. Once the polarization insertions are determined one can solve Eq. 3 to find the dispersion relations for the $\omega$ meson in nuclear matter taking both the effect of free and dense part of the polarization function in the various kinematic limits as mentioned in the introduction. In addition, this also provides us a way to interpret the results in terms of the classical plasma oscillations.

First we recall that for collective excitations, the wavelength of the oscillations must be greater than the interparticle spacing. This means the meson momenta must be small compared to the nucleon momenta. Therefore, quantitatively, it is legitimate to assume that all the loop momenta (nucleon) are hard and the external momenta (meson) are soft i.e. $p \sim p_F, |q| \ll p_F$. We term this as hard nucleon loop (HNL) approximation. This allows us to drop $q^4$ compared to $4(p,q)^2$ in Eq. 3. Such an approximation, is, in effect, similar to what has been adopted in Ref. \[3\]. Under this assumption the integrations of Eq. 3 relevant for the longitudinal and transverse modes are performed easily to give following results:

\[ \Pi^D_L(\omega, k) = -3\Omega^2 (1 - \frac{\omega^2}{k^2 v_F^2})[-1 + \frac{\omega \log (\frac{\omega + kv_F}{\omega - kv_F})}{2kv_F}] \]

(10)

and

\[ \Pi^D_T(\omega, k) = \frac{3}{2} \Omega^2 \frac{\omega^2}{k^2 v_F^2} + (1 - \frac{\omega^2}{k^2 v_F^2}) \frac{\omega \log (\frac{\omega + kv_F}{\omega - kv_F})}{2kv_F} \]

(11)

where, $q_0 = \omega, |q| = k, v_F = p_F/\epsilon_F$ is the Fermi velocity with $\epsilon_F = \sqrt{p_F^2 + M^2}$ and $\Omega^2$ is identified as the plasma frequency given by $\Omega^2 = \frac{\epsilon_F}{\pi^2} \frac{1}{3} p_F^2$. This is related to the Debye screening mass as $\Omega^2 = \frac{3}{2} m_D$ \[3,10\].

Before proceeding further, few comments are in order. It is evident that for $k \neq 0$ in matter, the longitudinal and transverse self-energies are non-degenerate. This gives rise to splitting between these two collective modes in nuclear matter which could be attributed to the presence of $\mathcal{P}_{\mu\nu}$ in Eq. 3. However, they become identical in the static limit i.e. $\Pi_L(\omega, k \to 0) = \Pi_T(\omega, k \to 0) = \Omega^2$. 


Next we consider the free part (also known as Dirac sea contribution) given by Eq. 8. Following Ref. 6, this can also be simplified further by making a Taylor series expansion of Eq. 8 around \( M^* \). To the leading order the contribution is found to be \[ 11 \]

\[
\Pi_{L,T}^F(\omega,k) = \frac{g^2_v}{3\pi^2} M^* - M \omega^2 + \mathcal{O}(\frac{\omega^4}{4M^2})
\] (12)

As in nuclear matter effective nucleon mass is smaller than its free space value (\( M^* < M \)), it is clear from the last expression, that the free part reduces the effective \( \omega \) meson mass in the medium.

Eq. 9 in conjunction with Eq. 11, 13 and 14 define the the dispersion characteristics for the collective excitations:

\[
\omega^2 - k^2 - m_{\omega}^2 - \frac{g^2_v}{3\pi^2} M^* - M \omega^2 + \frac{3}{2} \Omega^2 \left[ \omega^2 \frac{(1 - \omega^2)}{k^2 v_F^2} + (1 - \omega^2) \frac{\omega \log[\frac{\omega + kv_F}{\omega - kv_F}]}{2kv_F} \right] = 0
\] (13)

for T mode and

\[
\omega^2 - k^2 - m_{\omega}^2 - \frac{g^2_v}{3\pi^2} M^* - M \omega^2 + 3\Omega^2 (1 - \omega^2)[1 - 1 + \frac{\omega \log[\frac{\omega + kv_F}{\omega - kv_F}]}{2kv_F} = 0,
\] (14)

for L mode.

Generally these transcendental equations, viz. Eq. 13 and 14, have to be solved numerically, however, analytical solutions could be obtained in two limiting cases viz., when \( k < \Omega \) and \( k > \Omega \) by expanding above equations in powers of \( k \) and solving the equations by successive iterations. To the first order, when \( k << \Omega \) one gets \( \omega_{L,T}^2 = m_{\omega}^2 + \Omega^2 \).

We note that the modes are degenerate in this limit and oscillations are independent of the wave vector \( k \). In the case of electron-ion plasma, this mode was identified as the 'plasma waves' or 'Langmuir' oscillation and \( \Omega^2 \) there is given by \( e^2 \rho/\mu^2 \) where \( \rho \) is the density of electron gas and \( m \) is the electron mass [18]. This justifies identifying \( \Omega \) as the plasma frequency in the present pretext. It should be mentioned that here we have absorbed the contribution of the free part as

\[
m_{\omega}^2 = m_{\omega}^2 (1 + \frac{g^2_v}{3\pi^2} \frac{M^* - M}{M})
\] (15)

Once the leading order solution is known, we can solve Eqs. 13 and 14 iteratively to obtain

\[
\omega_L^2 = \Omega^2 + k^2 + \frac{1}{5} k^2 v_F^2 \frac{\Omega^2}{m_{\omega}^2 + \Omega^2} + ...
\] (16)

\[
\omega_T^2 = \Omega^2 + \frac{3}{5} k^2 v_F^2 \frac{\Omega^2}{m_{\omega}^2 + \Omega^2} + ...
\] (17)

Evidently, the transverse mode lies above the longitudinal one and it requires more energy for excitation. However, the splitting in the low \( k \) region is very marginal. In the limit \( m_{\omega} \rightarrow 0 \) we recover the results presented in 12, i.e.

\[
\omega_L^2 = \Omega^2 + \frac{3}{5} v_F^2 k^2 + ...
\] (18)

\[
\omega_T^2 = \Omega^2 + \frac{1}{5} v_F^2 k^2 + ...
\] (19)

Furthermore, this taken with \( v_F \rightarrow 1 \) results in dispersion relations arising out of the photon propagation at finite temperature or density (where electron is considered to be ultrarelativistic) with the appropriate definition of the plasma frequency [16, 17]. It is worthy to note that the contribution of the density dependent part is opposite to that of the Dirac part of the polarization tensor. This is consistent with Ref. 3.

Next we consider the case when \( k > \Omega \) but still \( \omega > k \) and in fact \( \omega \sim kv_F \). The successive approximation for this asymptotic value of \( k \) yields solutions of the form

\[
\omega_L^2 = k^2 + m_{\omega}^2 + \frac{3}{5} \Omega^2 + ...
\] (20)

The corresponding longitudinal frequency takes little bit complicated form;

\[
\omega_T^2 = k^2 v_F^2 (1 + 4 \exp[-\frac{2}{3} k^2 v_F^2 + \frac{m_{\omega}^2}{3\Omega^2} \frac{v_F^2}{(v_F^2 - 1)} - 2])
\] (21)

Evidently, the longitudinal mode approaches the line \( \omega = kv_F \) and exponentially suppressed for very large values of \( k \). We have solved Eq. 13 and 14 numerically and checked that for meson momenta \( k \sim 1 GeV \) dispersion results are in good agreement with Eqs. 16 and 17.
IV. SPECTRAL FUNCTIONS AND SUM-RULES

Equipped with Eq. [10] and [11], we can proceed to calculate the spectral functions of the $\omega$ meson. As demonstrated already, the $\omega$ meson in matter has two modes. Confining our attention on the transverse sector first, we write the propagator as

$$\Delta_T(\omega, k) = \frac{-1}{\omega^2 - k^2 - m_{\omega}^2 + \frac{3}{2} \Omega^2 \left[ \frac{\omega^2}{(k\nu)^2} + (1 - \frac{\omega^2}{(k\nu)^2}) \frac{\omega}{2k\nu} \log \left( \frac{\omega + k\nu}{\omega - k\nu} \right) \right]}$$  \hspace{1cm} (22)

Next chain of steps would be to construct the sum rules by exploiting the analytic properties this effective propagator. First we note that the function $\Delta_T$ is analytic in the complex $\omega$-plane having a cut from $-k\nu$ to $+k\nu$; in addition to the poles $\omega = \pm \omega_T$. Having observed this one can make use of Cauchy’s theorem for $\Delta = \Delta_T$

$$\Delta(\omega, k) = \oint_{\Gamma'} \frac{d\omega'}{2\pi i} \frac{\Delta(\omega', k)}{\omega' - \omega} = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi i} \frac{\Delta(\omega', k)}{\omega' - \omega} + \oint_{\Gamma'} \frac{d\omega'}{2\pi i} \frac{\Delta(\omega', k)}{\omega'}$$ \hspace{1cm} (23)

In the above equation $\Gamma'$ is a circle with its radius pushed to infinity. This equation can be rewritten in terms of the spectral density $\rho(\omega, k)$ containing both the discontinuities across the cuts and the residues at the poles:

$$\rho(\omega, k) = 2Im\Delta(\omega + i\eta, k)$$ \hspace{1cm} (24)

as

$$\Delta(\omega, k) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\rho(\omega', k)}{\omega' - \omega} + \oint_{\Gamma'} \frac{d\omega'}{2\pi i} \frac{\Delta(\omega', k)}{\omega'}$$ \hspace{1cm} (25)

It is evident from Eq. [24] that $\Delta_T(\omega', k) \rightarrow \frac{1}{\omega'}$ as $\omega' \rightarrow \infty$, therefore, the integration over the contour $\Gamma'$ fails to contribute in this region. First sum rule is then immediately obtained by setting $\omega = 0$ in Eq. [25]

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\rho_T(\omega, k)}{\omega} = \Delta_T(0, k) = \frac{1}{k^2 + m_{\omega}^2}$$ \hspace{1cm} (26)

Other sum rules are obtained by examining the asymptotic behaviour of $\Delta_T(\omega, k)$ when $\omega \rightarrow \infty$: by taking the parity property of the spectral density i.e. $\rho_T(\omega, k) = -\rho_T(-\omega, k)$ into account, one can write from Eq. [25] for $\omega \rightarrow \infty$

$$\Delta_T(\omega, k) = -\frac{1}{\omega} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left( \frac{\omega'}{\omega} \right)^{2n+1} \rho_T(\omega', k)$$ \hspace{1cm} (27)

On the other hand, one can expand Eq. [22] in powers of $\omega^{-1}$; comparing this with Eq. [27] one writes for $n = 0$

$$\int_{-\infty}^{\infty} \frac{\omega d\omega}{2\pi} \rho_T(\omega, k) = 1$$ \hspace{1cm} (28)

and similarly for $n = 1$ one observes

$$\int_{-\infty}^{\infty} \frac{\omega^3 d\omega}{2\pi} \rho_T(\omega, k) = k^2 + m_{\omega}^2 + \Omega^2$$ \hspace{1cm} (29)

Thus we obtain other two sum rules. Of particular importance is Eq. [28]. This does not depend on $k$ which could be shown to be a consequence of the canonical commutation relation. Therefore, it is natural to expect that such a relation will also be satisfied by the transverse $\omega$ meson in nuclear matter.

The longitudinal propagator is denoted as

$$\Delta_L(\omega, k) = \frac{\omega^2 - k^2}{k^2} \left[ \frac{-1}{\omega^2 - k^2 - m_{\omega}^2 + 3\Omega^2 (1 - \frac{\omega^2}{(k\nu)^2}) \left[ -1 + \frac{\omega}{2k\nu} \log \left( \frac{\omega + k\nu}{\omega - k\nu} \right) \right] \right]$$ \hspace{1cm} (30)
Contrary to the case of transverse propagator, we now have non-vanishing contribution from the contour denoted by \( \Gamma' \) in Eq. (23) as in the limit \( \omega \to \infty \) the logarithmic term \( \to 1 \). Hence in this limit the longitudinal propagator goes like \( \Delta_L(\omega, k) \to -1/k^2 \). Therefore, a subtraction, unlike the transverse propagator, is necessary. The subtracted dispersion relation gives the following sum rule essentially in the same manner as in the transverse case.

\[
\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \rho_L(\omega, k) = \frac{1}{k^2} + \Delta_L(0, k) = \frac{3\Omega^2 + m_\omega^2}{k^2(k^2 + m_\omega^2 + 3\Omega^2)}
\]

(31)

Similarly to the asymptotic behaviour of the longitudinal propagator one can write,

\[
\Delta_L(\omega, k) = -\frac{1}{k^2} - \frac{1}{\omega} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left(\frac{\omega'}{\omega}\right)^{2n+1} \rho_T(\omega', k)
\]

(33)

This is identical with Eq. (27) except here we have an additional factor of \(-1/k^2\) for reason mentioned in the previous paragraph. Likewise, we now can derive the sumrules by first setting \( n = 0 \) to get

\[
\int_{-\infty}^{\infty} \frac{\omega d\omega}{2\pi} \rho_L(\omega, k) = \frac{m_\omega^2 + \Omega^2}{k^2}
\]

(34)

and \( n = 1 \) to obtain

\[
\int_{-\infty}^{\infty} \frac{\omega^3 d\omega}{2\pi} \rho_L(\omega, k) = \frac{(\Omega^2 + m_\omega^2)(k^2 + m_\omega^2 + \Omega^2)}{k^2} - \frac{5k^2\Omega v_F^2}{\pi}
\]

(35)

We can write down the full spectral function of the transverse and longitudinal modes for the \( \omega \) meson in nuclear matter in terms of the poles (and residues at the poles) together with the cuts of the propagator.

\[
\frac{1}{2\pi} \rho_{T,L}(\omega, k) = Z_{T,L}(k) [\delta(\omega - \omega_{T,L}) - \delta(\omega + \omega_{T,L})] + \beta_{T,L}(\omega, k) \theta(k^2 - \omega^2)
\]

(36)

where, \( \beta_{T,L} \) is given by

\[
\beta_{T,L}(\omega, k) = \frac{Im \Pi_{T,L}}{\pi} |\Delta_{T,L}(\omega, k)|^2
\]

(37)

The imaginary part of the self-energies \( \Pi_{T,L} \) correspond to Landau damping and relevant only for the space-like momenta of the meson. Physically this refers to the energy loss due to scattering of the \( \omega \) meson with hard nucleons in the medium. They are given by the imaginary part of Eq. (10) and Eq. (11).

The residues at the poles which are used to determine the full spectral function of the \( \omega \) meson in nuclear matter can be determined from the one-loop effective propagators. For instance the residue at the pole for the transverse excitation is determined by

\[
Z_T = -((\partial \Delta_T^{-1}/\partial \omega)|_{\omega=\omega_{T(k)}}))^{-1}
\]

(38)

\[
= \frac{\omega_T(k^2v_F^2 - \omega_T^2)}{\omega _T^2(\omega^2 - 3(\Omega_T^2 + k^2 + m_\omega^2)) + k^2(k^2 + \omega_T^2 + m_\omega^2)v_F^2}
\]

(39)

In a similar way we also calculate the reside at the pole for the longitudinal oscillation which is found to be

\[
Z_L = \frac{-(k^2 - \omega_L^2)(-\omega_L^2 + \omega_L k^2 v_F^2)}{k^2[\omega_L^2(-3k^2 - 3m_\omega^2 + \omega_L^2 - 3\Omega^2) + k^2(k^2 + m_\omega^2 + \omega_L^2 + 3\Omega^2)v_F^2]}
\]

(40)

We now present the limiting values of these residues at the poles given in section III. First considering \( k < \Omega \) we have for the transverse \( \omega \)

\[
Z_T(k) \approx \frac{1}{2\sqrt{\Omega^2 + m_\omega^2}} - \frac{k^2}{20(\Omega^2 + m_\omega^2)^{5/2}}
\]

(41)

while for large \( k \), i.e. for \( k > \Omega \) we have
\[ Z_T(k) \simeq \frac{1}{2\sqrt{k^2 + m^2}} \]  

(42)

Similar results are obtained for the longitudinal mode also. They are given by, for \( k < \Omega \)
\[ Z_L(k) \simeq \frac{\sqrt{\Omega^2 + m^2}}{2k^2} - \frac{25(m^2 + \Omega^2) - 22v_F^2\Omega^2}{20(\Omega^2 + m^2)^{3/2}} \]  

(43)

and for \( k > \Omega \)
\[ Z_L(k) \simeq \frac{4k v_F}{3\Omega^2} \exp \left( \frac{2}{3} \frac{k^2 v_F^2}{\Omega^2} + \frac{m^2 v_F^2}{3\Omega^2 (v_F^2 - 1)} - 2 \right) \]  

(44)

It is now evident from Eq. 28 and Eq. 42 that the transverse spectral function for large meson momentum reduces to that of the \( \omega \) meson propagation in free space, except its mass is modified because of the presence of the mean scalar field. Longitudinal mode in this limit decouples from the system. For small momenta, the sum rules are almost saturated by the pole contributions. This could be checked numerically.

V. SUMMARY AND CONCLUSION

To conclude and summarize, in the present work, we investigate the collective modes induced by the \( \omega \) meson propagation in dense nuclear matter. We then determine the dispersion relations for the \( \omega \) meson by considering both the effect of the Fermi sea and Dirac vacuum within the scheme of HNL approximation. It is observed that in the appropriate limit, i.e. for meson momenta small compared to the plasma frequency of the system, the results are consistent with the previous calculations [6,12]. The \( \omega \) meson mass, in presence of a mean field, decreases even at normal nuclear matter while for free Fermi gas the effect of nucleon loop is to increase the \( \omega \) meson mass proportional to the characteristic plasma frequency of the system.

We also present analytical results for the dispersion relation in the limit \( k > \Omega \) which shows that for the large value of the momenta the medium effects die off and the transverse mode propagates like a free meson. The longitudinal mode in this limit decouples from the system. We also construct various sum rules satisfied by the \( \omega \) meson spectral density in nuclear matter by investigating the analytic properties of the relevant effective propagator at finite density. It is observed that that sum rules are mostly saturated by the pole contributions for small momenta. The residues are also presented in the different limit of the meson momenta to understand the collective modes and their behavior. Results pertinent to the classical electron plasma oscillations are also recovered.

Similar studies can also be pursued for other light vector mesons. For instance, it is known that the \( \rho \) nucleon interaction is very similar to the \( \omega \) nucleon interaction, where, in addition to the vector interaction, one has to include tensor interaction as well. We, therefore, believe that the \( \rho \) meson propagation in cold nuclear matter can also be studied within the framework HNL approximation. This would allow for an analytical derivation for the spectral density of the \( \rho \) meson. Hence , sum rules similar to what we present here, relevant for the \( \rho \) meson spectral function at finite density can also be constructed in a parallel approach. Such studies are in progress and shall be reported elsewhere.

VI. ACKNOWLEDGMENTS

The author would like to thank A. G. Williams and J. Piekarewicz for useful private communications.

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