G-Fluxes and Non-Perturbative Stabilisation of Heterotic M-Theory

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Abstract

We examine heterotic M-theory compactified on a Calabi-Yau manifold with an additional parallel M5 brane. The dominant non-perturbative effect stems from open membrane instantons connecting the M5 with the boundaries. We derive the four-dimensional low-energy supergravity potential for this situation including subleading contributions as it turns out that the leading term vanishes after minimisation. At the minimum of the potential the M5 gets stabilised at the middle of the orbifold interval while the vacuum energy is shown to be manifestly positive. Moreover, induced by the non-trivial running of the Calabi-Yau volume along the orbifold which is driven by the G-fluxes, we find that the orbifold-length and the Calabi-Yau volume modulus are stabilised at values which are related by the G-flux of the visible boundary. Finally we determine the supersymmetry-breaking scale and the gravitino mass for this open membrane vacuum.
1 Introduction and Summary

Eleven-dimensional heterotic M-theory \[1\],\[2\] exhibits two fundamental model-independent moduli. One, the length \(R\rho\) of the orbifold-interval \(S^1/Z_2\), determines the strength of the string-coupling. The other, which appears upon compactifying the theory on a further Calabi-Yau threefold (CY) down to four dimensions, is the CY volume \(Vv\). To make \(R\) and \(V\) dimensionless, we choose following \[3\]

\[
\rho = \frac{(2\kappa)^{2/9}}{\pi^{14/9}} \simeq 0.2\kappa^{2/9}, \quad v = \frac{\pi}{2^{1/3}} \kappa^{4/3} \simeq 2.5\kappa^{4/3} .
\] (1.1)

Phenomenological considerations of heterotic M-theory with just the two orbifold fixed-plane boundary sources \[4\],\[5\],\[6\],\[7\],\[8\] imply that

\[
R\rho \simeq 15\kappa^{2/9} \simeq \frac{7.5}{M_{\text{GUT}}}, \quad Vv \simeq 80\kappa^{4/3} \simeq \frac{1}{M_{\text{GUT}}^6},
\] (1.2)

or \(R \simeq 75, V \simeq 32\), where \(\kappa^{-2/9} \simeq 2M_{\text{GUT}}\) denotes the 11-dimensional Planck-scale and \(M_{\text{GUT}} = 3 \times 10^{16}\) GeV the grand unification scale. Therefore the orbifold-modulus is roughly an order of magnitude larger than the generic CY radius. It is, however, an important feature of adding a further parallel (to the boundaries) M5-brane that these tight phenomenological constraints on \(R\) and \(V\) become relaxed due to the extra freedom coming from the M5’s G-flux (see e.g. \[9\],\[10\]). In this case it is even possible to make \(R\) large enough such that \(R\rho\) approaches its experimental upper bound of one millimeter in a large extra dimension scenario. However this extreme case is highly unnatural and implies a hierarchy problem \[11\].

It is an intrinsic feature of heterotic M-theory that the magnetic sources for the G-flux which are its two boundaries lead to a variation of the CY volume along the orbifold direction. If one considers the theory from its four-dimensional effective point of view it is therefore necessary to average the CY volume over the orbifold-size which introduces a dependence of \(V\) on \(R\). Moreover, let us consider the situation with an additional parallel M5-brane located at the position \(x^{11} = x_{M5}\) along the orbifold-interval. This configuration guarantees that the M5 is compatible with the supersymmetry of the heterotic M-theory background and does not break it further. We assume that the M5 is space-time filling in the four external flat directions and wraps a holomorphic 2-cycle \(\Sigma_{M5}\) of the internal CY space. In this paper we will restrict ourselves to the case of \(h^{(1,1)} = 1\), which covers e.g. the case of the quintic. It means that \(\Sigma_{M5}\) can be expressed in terms of just one basis
holomorphic curve $\Sigma$ as $\Sigma_{M5} = \beta \Sigma$ with positive integer expansion coefficient $\beta$. One can understand $\beta$ as the number of wrappings of $\Sigma_{M5}$ around $\Sigma$.

The M5 induces an additional G-flux through the relation

$$\int_{\beta \Sigma} \omega_i = \int_{CY_3} \omega_i \wedge G ,$$

(1.3)

where $\omega_i$, $i = 1, \ldots, h^{(1,1)}$ is a basis of harmonic $(1, 1)$ two-forms and $G = \beta [\Sigma]$ is the four-form which is Poincaré-dual to $\Sigma_{M5} = \beta \Sigma$. Through its induced flux, the M5 has an influence on the $x^{11}$ dependence of the CY volume. Namely, the Bianchi identity in the presence of the M5 at position $x^{11} = x_{M5}$ becomes $[4],[7]

$$dG = -\frac{1}{2\sqrt{2}\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left[ \sum_{i=1,2} \left( \text{tr} F_{(i)}^2 - \frac{1}{2} \text{tr} R^2 \right) \delta(x^{11} - x^{(i)}) \right. 
\left. + 8\pi^2 \beta [\Sigma] \delta(x^{11} - x_{M5}) \right] \wedge dx^{11} ,$$

(1.4)

which in turn leads to the following expression for the CY volume (in units of $v$) as a function of the orbifold coordinate $\Sigma$ $[4],[12],[13]

$$V(x^{11}) = V_1 + \frac{2}{\rho} \left( -r_v x^{11} + r_{M5} \Theta(x^{11} - x_{M5})(x^{11} - x_{M5}) \right) .$$

(1.5)

The parameter $r_v$ is controlled by the G-flux integrated over the CY at the visible boundary

$$r_v = \frac{1}{8\pi \rho v} \left( \frac{\kappa}{4\pi} \right)^{2/3} \int_{CY_3} \omega \wedge \left( \text{tr}(F_{(1)} \wedge F_{(1)}) - \frac{1}{2} \text{tr}(R \wedge R) \right) ,$$

(1.6)

while the parameter $r_{M5}$ describes the G-flux coming from the M5 brane source

$$r_{M5} = \frac{\pi \rho}{v} \left( \frac{\kappa}{4\pi} \right)^{2/3} \beta \int_{CY_3} \omega \wedge [\Sigma]$$

(1.7)

Notice that both $r_v$ and $r_{M5}$ are positive quantities (for $r_v$ this holds as long as the “instanton number” $-\int_{CY_3} \omega \wedge \text{tr} F_{1}^2$ on the visible boundary exceeds the one of the hidden boundary).

The rhs of the Bianchi identity must be cohomologically trivial. Therefore one arrives by integration over the orbifold-coordinate at the following anomaly cancellation

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3For simplicity we will take $\Sigma$ to be isolated, such that we do not have to integrate over its moduli describing its position inside the CY threefold.

4The Heaviside step-function is defined as $\Theta(x \leq 0) = 0$ and $\Theta(x > 0) = 1$. 

2
constraint

\[ \sum_{i=1,2} \left( \text{tr}(F_{(i)} \wedge F_{(i)}) - \frac{1}{2} \text{tr}(R \wedge R) \right) + 8\pi^2 \beta [\Sigma] = 0 , \]

which holds at the cohomology level. In terms of the G-flux parameters a further integration over the CY renders this cohomology condition into an actual flux-equation

\[ r_v + r_h = r_{M5} , \]

where \( r_h \) gives the G-flux integrated over the hidden boundary (i.e. it is formally the same as \( r_v \) but with \( F_{(1)} \) substituted by \( F_{(2)} \)).

Because finally we will need the CY volume in the context of the four-dimensional effective theory, we have to average it over \( x^{11} \) between 0 and \( R\rho \) which gives

\[ V = V_1 - r(x)R , \quad r(x) = r_v - r_{M5}(1 - x)^2 \]

where we have expressed the M5-brane position \( x_{M5} \) through the dimensionless parameter \( x \)

\[ x_{M5} = xR\rho \]

with \( x \in [0, 1] \). Thus the magnitude of the slope in the expression (1.11) of the average CY volume hinges on both the boundary plus the M5-brane G-flux in an opposing way. Whereas the boundary flux tends to curve the volume dependence downwards, the M5 flux tends to bend it upwards. This counterbalance property will show up prominently in our stabilised solution later on.

The formulation of heterotic M-theory is only known as a perturbative expansion in \( \kappa^{2/3} \). The leading order \( \kappa^{2/3} \) terms give rise to the linear dependence of \( V(x^{11}) \) on the orbifold coordinate \( x^{11} \). Let us therefore now examine for which parameter values we can trust the linear approximation. Obviously, we can no longer trust it when the CY volume \( V(x^{11}) \) becomes negative, i.e. unphysical. A way out when this happens would be to go beyond the linear approximation and use results of the full non-linear treatment of the supersymmetric warped background geometry. This would give a manifestly positive quadratic volume thereby eliminating the negative volume problem \[12, 13\]. Unfortunately, due to the fact that in this paper, we will need the Kähler-potential later on, which is only known to first nontrivial \( \kappa^{2/3} \) order, we have to seek for stabilisation within the linear approximation framework and therefore have to check for its validity.
First, it is obvious that the linear approximation should not break down, i.e. encounter a negative CY volume, before having reached the M5 coming from the visible boundary. This then imposes the following parameter constraint ($x_{11}^0$ denotes the position where the volume might vanish)

$$x_{11}^0 \geq x_{M5} \iff V_1 \geq 2xRr_v .$$

(1.12)

Second, we should also make sure that a negative CY volume does not appear in the second region between the M5 and the hidden boundary at $x^{11} = R\rho$. In this second region two things can happen. Either one has a flux-relation

$$r_{M5} > r_v ,$$

(1.13)

which means that $V(x^{11})$ is increasing beyond the M5 and thus nullifies the negative volume problem for the second region. Or one could have

$$r_{M5} \leq r_v ,$$

(1.14)

which gives a constant or decreasing $V(x^{11})$ beyond the M5. To guarantee that $V(x^{11})$ in this second case does not become negative before the hidden boundary is reached means to constrain the slope of the running volume which is determined by the fluxes. Therefore, we have to require in addition that

$$x_{11}^0 \geq R\rho \iff V_1 \geq 2R(r_v - r_{M5}(1 - x)).$$

(1.15)

To summarise, we have to require either (1.12) with (1.13) or complementary (1.14) together with (1.17) (notice that (1.12) is implied by (1.14) and (1.15)) in order to trust the first order linear volume approximation.

Since the successful prediction of four-dimensional data, in particular Newton’s Constant [4],[12], hinges on the above values (1.2), the question arises of how to stabilise them. This will be the main concern of this paper. There are various non-perturbative effects which give rise to interesting potentials for $R$ and $V$. In the framework of the heterotic string the main non-perturbative mechanism for breaking supersymmetry has been gaugino condensation in a hidden sector [14]. In the context of heterotic M-theory gaugino condensation appears even more naturally as the gauge theory on the hidden boundary now becomes strongly coupled [4]. Moreover, with the geometrical separation of the two $E_8$ gauge groups there appears yet another class of non-perturbative objects. These are the open membranes (OM) which either connect one boundary with the other or with some intermediate M5-brane placed parallel to the boundaries along the orbifold-interval.
Furthermore, also M5-instantons and M2-instantons can appear. The former wrap the whole internal CY whereas the latter wrap a 3-cycle of the CY.

In [15] it was argued that through the combined effect of multi-gaugino condensation on the hidden wall together with parallel (to the boundaries) M2-instantons a phenomenologically satisfactory stabilisation of the R and V moduli could be achieved. While the parallel M2-instanton breaks all supersymmetry explicitly [16] and one cannot use supersymmetric tools to derive the potential, other non-perturbative sources like the mentioned orthogonal (to the boundaries) OM’s or M5-instantons are compatible with the supersymmetry of heterotic M-theory. They will break supersymmetry spontaneously.

In general there are two different stabilisation scenarios which have to be distinguished. They differ in the energy-scale at which stabilisation might occur. Either the theory could become stabilised above the threshold given by the inverse orbifold-size $1/R\rho = M_{GUT}/7.5$ or below. In the former case one would have to work with the eleven-dimensional formulation of heterotic M-theory if stabilisation even trespasses the CY compactification scale $M_{GUT}$ or otherwise with the effective five-dimensional action [17] between the two thresholds. This case offers the intriguing possibility that local supersymmetry gets broken via gaugino condensation [18],[19] only if energies become so low that the orbifold interval shrinks to a point. However it leads to the phenomenologically unsatisfactory situation that the mass of the gravitino

$$m_{3/2}^2 = M_{Pl}^2 e^{K/2}|W|$$

which is proportional to $\Lambda_{GC}^3$ becomes too high. (For a discussion of this case with an inverse orbifold-length at the intermediate scale $10^{12}$ GeV see [20]).

Therefore, subsequently we will search for a stabilisation in the energy-regime below the $M_{GUT}/7.5$ threshold, which necessitates a description of heterotic M-theory through its effective four-dimensional N=1 supergravity action. This had been derived in [21]. In particular we will analyse the case of vanishing charged scalar vacuum expectation values (vev’s).

As it turns out that in the regime where one can trust the perturbative formulation of the effective four-dimensional heterotic M-theory the non-perturbative M5-instantons and gaugino condensation appear exponentially suppressed, we will focus on the effect of OM-instantons in the presence of a parallel M5-brane which is the dominant one. By minimising the corresponding potential for the moduli, we find that OM-instantons do stabilise the M5 in the middle of the orbifold interval. Furthermore, the moduli $V$ and $R$
get stabilised at values

\[ V = \frac{V_1}{4}, \quad R = \frac{V_1}{r_v}. \]  

(1.16)

To find this minimum of the effective potential it is essential to have nontrivial G-fluxes caused by the boundaries and the M5. They trigger a dependence of \( V \) on \( R \) which is responsible for the stabilisation. Indeed, for consistency with the perturbative formulation of the theory, the G-fluxes integrated over the visible boundary and the M5 have to be equal

\[ r_v = r_{M5}. \]  

(1.17)

In the full eleven-dimensional picture this OM-instanton vacuum corresponds to a CY volume which falls off linearly and approaches zero in the middle of the interval where the M5 is located. For the second half of the interval it stays constant due to the flux-equality (1.17)

\[ V(x^{11}) = \begin{cases} V_1 - \frac{2}{\rho} r_v x^{11}, & 0 \leq x^{11} < \frac{R}{2} \\ 0, & \frac{R}{2} \leq x^{11} \leq R\rho \end{cases} \]  

(1.18)

We will however show that there is evidence that the full theory beyond the first order shifts the volume-zero on the second half-interval to a non-vanishing positive constant value. It is intriguing to see that the relationship between \( R \) and \( V \) is simply determined by the flux \( r_v \) coming from the visible boundary

\[ R = 4 \frac{V}{r_v}. \]  

(1.19)

Moreover, since at the minimum the leading order terms vanish it is important to include all first order \( \kappa^{2/3} \) corrections. This gives a manifest positive contribution to the vacuum energy possessing an interesting exponential suppression factor.

In the following table\(^5\) we give a quick impression of what will be the relevant data with OM-instantons present for integrated G-fluxes \( r_v \) and CY volumes \( V_1 \) on the visible boundary and what will be their respective influence on \( R, V, \) on the two heterotic M-theory expansion parameters \( \epsilon, \epsilon_R, \) on the vacuum energy \( U_{OM}, \) on the related supersymmetry-breaking scale \( M_{Susy} \) and finally on the gravitino mass \( m_{3/2}. \) A small \( r_v \) can be seen to ruin the smallness of \( \epsilon \) and thereby the reliability of the perturbative formulation of the

\(^5\)Here we have chosen a CY-intersection number \( d = 30 \) and \( |h| = \beta = 1. \) The meaning of \( h \) will become clear in the next section.
theory. Thus the \( r_v \) fluxes have to be considerable. At the same time a not too small \( r_v \) allows to bring the supersymmetry-breaking scale into the desired TeV region, however, simultaneously rises the vacuum energy \( U_{OM} \). It can be seen that an increasing \( V_1 \) has a similar effect on \( U_{OM} \), \( M_{Susy} \) and \( m_{3/2} \) as a decreasing \( r_v \). However, its influence on \( \epsilon \), \( \epsilon_R \) is rather modest. Generically, one obtains a \( V \) which is one or two magnitudes larger than \( R \). The basic reason for this is to keep the parameter \( \epsilon \) small enough.

| \( r_v \) | \( V_1 \) | \( R \) | \( V \) | \( \epsilon \) | \( \epsilon_R \) | \( U_{OM}^{1/4}/\text{TeV} \) | \( M_{Susy}/\text{TeV} \) | \( m_{3/2}/\text{TeV} \) |
|---|---|---|---|---|---|---|---|---|
| 90 | 3000 | 33 | 750 | 0.8 | 0.1 | \( 10^{-5} \) | \( 3 \times 10^{-4} \) | \( 2 \times 10^{-6} \) |
| 140 | 3000 | 21 | 750 | 0.52 | 0.18 | 91 | 1815 | 16 |
| 200 | 3000 | 15 | 750 | 0.36 | 0.25 | \( 5 \times 10^5 \) | \( 9 \times 10^6 \) | \( 10^5 \) |
| 200 | 4000 | 20 | 1000 | 0.4 | 0.2 | 39 | 782 | 6.7 |
| 200 | 5000 | 25 | 1250 | 0.4 | 0.2 | \( 10^{-3} \) | \( 3 \times 10^{-2} \) | \( 2 \times 10^{-4} \) |

The organisation of the paper is as follows. After presenting preparatory material and the relevant Kähler- and superpotential in section 2, we will derive in section 3 the four-dimensional \( N=1 \) supergravity potential for the OM-instanton background. It turns out to be positive. Its minimisation results in a minimum for which the leading order terms vanish and subleading terms become important. The M5 gets stabilised in the middle of the orbifold-interval and \( V, R \) obtain values depending on \( V_1 \) and the G-fluxes related to the visible boundary and the M5. In section 4 we analyse the constraints coming from the perturbative formulation of the theory and show that they require a flux-equality between those G-fluxes arising from the visible boundary and the M5. Moreover, we present the eleven-dimensional picture of the vacuum solution and give its vacuum energy. The final section 5 treats the issue of supersymmetry-breaking. We derive the supersymmetry-breaking scale and gravitino mass for the OM-vacuum studied before and compare the supersymmetry-breaking scale with its vacuum energy. Technical details related to the derivation of the potential and the determination of the supersymmetry-breaking scale plus gravitino mass appear in appendix A and B.

## 2 The Effective D=4 Potential

In the framework of the low-energy four-dimensional \( N=1 \) supergravity description, the moduli potential is obtained from the Kähler- and the superpotential by means of the
general formula

\[ (\kappa_4)^4 U = e^K \left( K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3W \bar{W} \right) + U_D \]  

(2.1)

where \( D_i W = \partial_i W + K_i W \) denotes the Kähler-covariant derivatives, \( K_i \equiv \partial_i K \) and \( U_D \sim \sum a (\bar{C} T^a C)^2 \) denotes the D-term contribution. The index \( i \) runs over all moduli. Note that we multiplied the potential \( U \) which has mass-dimension four by a factor \( (\kappa_4)^4 = 1/M_{Pl}^4 \) to render the right-hand-side of (2.1) and thereby \( W \) dimensionless. This is done to get rid of various onerous dimensionful powers of \( v, \rho \). For consistency we will also choose the moduli-fields dimensionless in the following.

Thus we need to know the superpotential \( W \) and the Kähler-potential \( K \). Besides the perturbative trilinear superpotential

\[ W_{(p)} = \lambda_{IJK} C^I C^J C^K , \]

(2.2)

where \( \lambda_{IJK} \) denotes the Yukawa-couplings there are various non-perturbative contributions. Recently, there appeared a detailed analysis of the contributions of open membrane instantons to the superpotential [3], [22]. Either the open membranes connect both boundaries with each other and give rise to a superpotential \( W_{(M^2)} \) or they connect the boundaries with the additional M5-brane located along the orbifold-interval giving a superpotential \( W_{(M^2,M^5)} \). In the latter case in order to have a supersymmetric configuration, the open membrane must have the geometry \( \Sigma \times I \), where \( I \) describes the interval in the orbifold direction and \( \Sigma \) denotes the same basis holomorphic curve on which also the M5 is wrapped. The superpotential is then given by [3], [21], [22]

\[ W = W_{(p)} + W_{(M^2,M^5)} + W_{(M^2)} , \]

(2.3)

with

\[ W_{(M^2,M^5)} = h \left( e^{-Z} + e^{Z-\beta T} \right) , \quad W_{(M^2)} = h' e^{-\beta T} , \]  

(2.4)

where the dimensionless complex prefactors \( h, h' \) are related to the complex structure moduli [3], [22]. We will not need their explicit expressions in the following.

The complex moduli fields are defined by

\[ S = V + \beta J x^2 + i\sigma , \quad T = J + i\chi , \quad Z = \beta J x + i\alpha , \]

(2.5)

where

\[ J = Ra , \]

(2.6)
with \( a = (6V/d)^{1/3} \) the Kähler-modulus of the CY and \( d = \frac{1}{v} \int_{\text{CY}} \omega_1^3 \) the CY-intersection number with \( \omega_1 \) the basis-element of harmonic (1,1)-forms (remember that we chose a CY with \( h^{(1,1)} = 1 \) such that the Kähler-form reads \( \omega = a\omega_1 \)). The axions \( \sigma \) and \( \chi \) arise from two different components of the eleven-dimensional 3-form potential \( C_{AB11} \) with one index tangent to the orbifold. \( \sigma \) is dual to \( C_{\mu\nu\rho} \)

\[
3V^2 \partial_{\mu} C_{\nu\rho |11} = \epsilon_{\mu\nu\rho\lambda} \partial^\lambda \sigma \tag{2.7}
\]

while \( \chi \) comes from

\[
C_{m\bar{m}11} = \chi \omega_{1,m\bar{m}} \tag{2.8}
\]

The axion \( \alpha \) is a combination of \( \chi \) and a scalar \( \mathcal{A} \) coming from the KK reduction of the M5’s 2-form potential \( A^{(2)} \)

\[
\alpha = \beta (x \chi - a\mathcal{A}) \tag{2.9}
\]

More precisely, if \( f \) denotes the holomorphic embedding of the curve \( \Sigma \) into the CY, then \( \mathcal{A} \) arises from the KK decomposition \( A^{(2)} = \pi \rho \mathcal{A} f^* (\omega) \) with \( f^* (\omega) \) the pullback of \( \omega \) to the cycle.

Geometrically \( \beta J \) gives the average volume occupied by an OM stretching from boundary to boundary while \( \beta Jx \) resp. \( \beta J(1-x) \) give the average volume of an OM connecting the M5 with the visible resp. hidden boundary. In \( S \) we included the higher-order correction \( \beta J x^2 \) which had been found in [3].

Next we have to specify the Kähler-potential \( K \), which is composed out of five pieces

\[
K = K_{(S,M5)} + K_{(T)} + K_{(C)} + K_{(cx)} + K_{(bd)} \tag{2.10}
\]

where

\[
K_{(S,M5)} = - \ln \left( S + \bar{S} - \frac{(Z + \bar{Z})^2}{\beta (T + \bar{T})} \right) , \quad K_{(T)} = - \ln \left( \frac{d}{6} (T + \bar{T})^3 \right) , \tag{2.11}
\]

\[
K_{(C)} = \left( \frac{3}{T + \bar{T}} + \frac{2\xi}{S + \bar{S}} \right) H_{IJ} C^I C^J + \mathcal{O}(C^3) , \quad K_{(cx)} = - \ln \left( \Pi^a G_a \right) , \tag{2.12}
\]

and the precise meaning of \( \xi, H_{IJ}, \Pi^a \) or \( G_a \) can be found in [3]. Unfortunately little is known about the Kähler-potential \( K_{(bd)} \) of the instanton gauge bundle moduli. It has

\(^6\) \( A, B = 0, \ldots, 9; \mu, \nu, \rho, \lambda = 0, \ldots, 3; \) (anti-)holomorphic CY-indices \( \bar{m}, m = 1, \ldots, 3. \)
to be noted that this Kähler-potential is only valid in a region where the two heterotic
M-theory expansion parameters

$$
\epsilon = \frac{2R}{V^{2/3}} \approx \frac{J}{V}, \quad \epsilon_R = \sqrt{\frac{\pi V^{1/6}}{2 R}} \approx \frac{\sqrt{V}}{J}
$$

are smaller than one. Note also that the Kähler-potential for the M5-brane moduli \[ 3, 24 \]

$$
K_{(M5)} = (Z + \bar{Z})^2 \frac{(S + \bar{S})}{(S + \bar{S})\beta(T + \bar{T})}
$$

is of subleading order $\epsilon$ relative to the leading piece $K_{(S)} = -\ln(S + \bar{S})$. It was shown in \[ 24 \] that because of supersymmetry they should be combined into the single expression $K_{(S,M5)}$ appearing above.

In order to have a well-defined perturbative formulation of the theory, we have to require that $\epsilon \ll 1, \epsilon_R \ll 1$ which means that we have to restrict ourselves to the region of moduli space where

$$
J^2 \gg V \gg J \gg 1.
$$

This is the reason why we suppress gaugino condensation and M5-instanton effects in
the present work. Schematically their contribution to the potential will be exponentially
suppressed by $e^{-c_1V}$ whereas OM-instantons will exhibit a milder $e^{-c_2J}$ suppression and
hence dominate ($c_1, c_2$ are positive constants).

In order to gain a better understanding in which region of parameter space \( \{d, r(x), V_1\} \)
we obtain small $\epsilon$ and small $\epsilon_R$, we show some representative values in the following table
Here, we kept $R$ fixed (at $R = 40$) since it will be determined dynamically subsequently by minimizing the potentials whereas $\{d, r(x), V_1\}$ are regarded as free “input” parameters. From the table it can be seen that an increasing $V_1$ yields a decreasing $\epsilon$ and a slightly increasing $\epsilon_R$. On the other hand an increasing $r(x)$ yields the reversed effect. Finally a varying $d$ has no influence on $\epsilon$ and $\epsilon_R$ and merely affects the modulus $J$ which grows when $d$ decreases. Therefore, we will assume $d$ to be fixed at a value of 30 in the rest of this paper. In conclusion we should look for stabilisation in the parameter-region where $V_1$ is rather large, say $V_1 \gtrsim 2000$ while $r(x)$ should not be too big, say $r(x) \lesssim 100$.

In this paper we will examine the region of moduli space where charged scalar $C^I$ (which originate from the reduction of the ten-dimensional gauge-field) vev’s are absent or comparatively small. Basically this means that we look for stabilisation of heterotic M-theory at energies high enough such that the GUT gauge group is still (spontaneously) unbroken. For vanishing $C^I$ the perturbative contribution $W(p)$ to the superpotential and the charged scalar Kähler-potential $K_{(C)}$ can be neglected subsequently

$$C^I = 0 \quad \rightarrow \quad W(p) = 0 \, , \quad K_{(C)} = 0 \, . \quad (2.16)$$

It is important to note that in the case with $C^I = 0$ the sum $K_{(S,M5)} + K_{(T)}$ as given by (2.11) includes all corrections of order $\epsilon$ and order $\epsilon_R$ (see e.g. [23]). This is due to the fact that the subleading contributions to the leading order expressions for $K_{(S)}$ and $K_{(T)}$ are proportional to $C^IC^J$ and therefore vanish, while $K_{(M5)}$ is already of order $\epsilon$.  

| $d$ | $r(x)$ | $V_1$ | $V$   | $J$  | $\epsilon$ | $\epsilon_R$ |
|-----|--------|-------|-------|------|-------------|-------------|
| 30  | 10     | 500   | 100   | 108.6| 3.7         | 0.07        |
| 30  | 10     | 1000  | 600   | 197.3| 1.1         | 0.09        |
| 30  | 10     | 2000  | 1600  | 273.6| 0.6         | 0.11        |
| 30  | 10     | 5000  | 4600  | 389  | 0.3         | 0.13        |
| 30  | 10     | 5500  | 5100  | 402.7| 0.3         | 0.13        |
| 30  | 50     | 5500  | 3500  | 355.2| 0.4         | 0.12        |
| 30  | 100    | 5500  | 1500  | 267.8| 0.6         | 0.11        |
| 30  | 130    | 5500  | 300   | 156.6| 1.8         | 0.08        |
| 0.06| 10     | 2000  | 1600  | 2171.5| 0.6       | 0.11        |
| 0.6 | 10     | 2000  | 1600  | 1007.9| 0.6       | 0.11        |
| 6   | 10     | 2000  | 1600  | 467.8 | 0.6       | 0.11        |
| 60  | 10     | 2000  | 1600  | 217.2 | 0.6       | 0.11        |
Moreover for the case of $h^{1,1} = 1$ it is known that $W_{(M2)}$ vanishes \([28],[3]\)

$$h^{1,1} = 1 \quad \rightarrow \quad W_{(M2)} = 0 \quad (2.17)$$

The moduli-potential for the $h^{1,1} = 1$, $C^I = 0$ case which originates from the contributions

$$W = W_{(M2,M5)} \quad (2.18)$$

$$K = K_0 + K_T + K_{(ex)} + K_{(bd)} \quad (2.19)$$

has been first calculated in [3] and contains the following first and next-leading order terms

\[
(K^4)^4 U_{OM}^{(MPS)} = e^{K_{(ex)} + K_{(bd)}} \frac{3|h|^2}{4dJ^2} \left( e^{-2Jx} + e^{2J(1-z)} - 2e^{-J} \cos(2\alpha - \chi) \right) \\
+ \frac{2J}{3V} \left( 1 - 2x \right) e^{-2J(1-z)} + \frac{4Jx}{3V} e^{-J} \cos(2\alpha - \chi) \right) + \ldots
\]

Note that here where the $K_{(M5)}$ contribution has not been included the subleading terms in the second line can give a negative contribution. We will show in the next section that there are also $x^2$ terms at subleading order resulting from the inclusion of $K_{(M5)}$ rendering the potential manifestly positive. Moreover we will see that the leading order potential vanishes at its minimum and therefore carefully including all subleading contributions becomes essential.

### 3 The Moduli-Potential with OM-Instantons

Let us now extract the moduli-potential to subleading order resulting from the two OM instantons connecting the intermediate M5 with either boundary. The superpotential is given by [3]

$$W = W_{(M2,M5)} = h(e^{-Z} + e^{Z - \beta T}) \quad (3.1)$$

while the Kähler-potential up to subleading order reads

$$K = K_{(S,M5)} + K_T = -\ln \left( \frac{8}{3}dVJ^3 \right) + \mathcal{O}(\epsilon^2, \epsilon R, \epsilon R) \quad (3.2)$$

Notice the inclusion of the subleading $K_{(M5)}$ part. In the expression for the four-dimensional supergravity-potential \([2,1]\) we will consider only the covariant derivatives $D_i W$ with respect to the $i = S, T, Z$ moduli, i.e. we will neglect the dependence on complex-structure and bundle-moduli.
The potential is composed out of four structurally different parts which are hierarchically ordered in the \( J^2 \gg V \gg J \gg 1 \) region. We will examine now their order of magnitude in this moduli space region. From the superpotential one easily recognizes that partial derivatives of \( W \) with respect to the moduli fields do not generate further factors of \( V \) or \( J \). Thus for a determination of the magnitude of the four different parts, it is sufficient to take the leading behaviour of the Kähler-potential and its derivatives. This can be found in appendix A.

Let us start with the

\[ |W|^2 \]

term which is of \( \mathcal{O}(1) \) with respect to a counting of \( V \) and \( J \) prefactors. The second sort of contribution to the potential is of the form

\[ K^{ij} K_i \bar{W} K_j \bar{W}. \]

The expressions from the appendix show that these terms range between \( \mathcal{O}(1) \) and \( \mathcal{O}(J/V) \) in magnitude. A third class consists of mixed terms and is given by

\[ K^{ij} \partial_i W K_j \bar{W} \]

with ranges between \( \mathcal{O}(V) \), \( \mathcal{O}(J) \) and \( \mathcal{O}(J^2/V) \). Finally, the last class of terms is

\[ K^{ij} \partial_i W \partial_j \bar{W}. \]

This class dominates the three others since it exclusively gives the leading \( \mathcal{O}(JV) \) and subleading \( \mathcal{O}(J^2) \) contributions. Therefore it is enough to consider just this class to ensure that all leading and subleading contributions in \( \epsilon \) and \( \epsilon_R \) are taken into account. Notice that beyond this order the Kähler-potential of the theory is not known and therefore it would make no sense to include an incomplete set of terms at these lower orders coming from the three other classes of terms.

Concerning the last class of dominant terms, the \( \mathcal{O}(J^2) \) contributions which come from \( K^{\overline{T}\overline{T}}, K^{T\overline{Z}} \) are suppressed by \( \epsilon \) against the leading \( \mathcal{O}(JV) \) contribution from \( K^{Z\overline{Z}} \). This means that we have to include for the latter also its subleading corrections whereas for the former it is enough to consider merely their leading \( \epsilon, \epsilon_R \) behaviour.
3.1 The OM Potential

With help of the expressions collected in appendix A one derives the OM potential for the moduli. Including all leading and subleading $J/V$ corrections it reads

$$
(\kappa_4)^4 U_{OM} = \frac{3|h|^2}{4dJ^2} \left\{ \beta \left[ e^{-2J\beta x} + e^{-2J\beta(1-x)} - 2e^{-J\beta} \cos(2\alpha - \beta \chi) \right] + \frac{2J}{3V} \beta^2 \left[ e^{-2J\beta x}x^2 + e^{-2J\beta(1-x)}(1-x)^2 + 2x(1-x)e^{-J\beta} \cos(2\alpha - \beta \chi) \right] \right\}. \tag{3.3}
$$

The symmetry of the potential under the exchange $x \to 1-x$ originates from the symmetry of the OM-superpotential $W_{(M_2,M_5)}$ under the exchange of the corresponding moduli $Z \to \beta T - Z$ (the Kähler-potential is trivially symmetric as it does not depend on $x$). It is important to notice the sign-difference of the cosine term between leading and subleading order. It is this difference which prohibits $U_{OM}$ from becoming zero at its minimum and thereby leads to a spontaneous breaking of supersymmetry.

An immediate consequence is that this potential is bounded from below by a non-negative expression

$$
(\kappa_4)^4 U_{OM} > \frac{3|h|^2}{4dJ^2} \left\{ \beta \left[ e^{-2J\beta x} - e^{-2J\beta(1-x)} \right]^2 + \frac{2J}{3V} \beta^2 \left[ e^{-J\beta x}x - e^{-J\beta(1-x)}(1-x) \right]^2 \right\}. \tag{3.4}
$$

Since this lower bound can never be saturated, $U_{OM}$ has to be positive. Hence $D=4$, $N=1$ supersymmetry will be broken with a positive vacuum energy.

3.2 Minimisation

Let us now minimise $U_{OM}$. Minimisation with respect to the axion fields leads to $\sin(2\alpha - \beta \chi) = 0$ which is solved by

$$
2\alpha - \beta \chi = n\pi; \; n \in \mathbb{Z} \tag{3.5}
$$

and gives for the potential

$$
(\kappa_4)^4 U_{OM} = \frac{3|h|^2}{4dJ^2} \left\{ \beta \left[ e^{-2J\beta x} + (-1)^{n+1}e^{-J\beta(1-x)} \right]^2 + \frac{2J}{3V} \beta^2 \left[ e^{-J\beta x}x + (-1)^ne^{-J\beta(1-x)}(1-x) \right]^2 \right\}. \tag{3.6}
$$
The sectors with

\[ n \in 2 \mathbb{Z} \]  

result in a lower energy for the leading term and will be analysed subsequently. They give the manifestly positive expression

\[
(k_4)^4 U_{OM} = \frac{3|h|^2}{4dJ^2} \left\{ \beta \left[ e^{-J\beta x} - e^{-J\beta(1-x)} \right]^2 + \frac{2J}{3V} \beta^2 \left[ e^{-J\beta x}x + e^{-J\beta(1-x)}(1-x) \right]^2 \right\}.
\]

(3.8)

Furthermore, it is easy to see that the value

\[ x = \frac{1}{2} \]  

(3.9)

for the M5-brane position modulus \( x \) minimizes \( U_{OM} \). Hence, the parallel M5 becomes stabilised at the symmetric position in the middle of the orbifold-interval. This could have been anticipated since both the Kähler-potential and the OM superpotential are invariant under the symmetry which exchanges \( x \leftrightarrow 1 - x \). Thus, the OM-potential is mirror-symmetric with respect to the fixed-point \( x = 1/2 \) which means that it must exhibit a minimum or a maximum at the fixed-point. The explicit analysis confirms a minimum.

It is important to realize that for this value the leading-order part of the potential vanishes and it is the sub-leading term which contributes alone and hence becomes responsible for supersymmetry-breaking and a non-vanishing vacuum energy.

As an aside let us compare our result with the expression (2.20) of [3]. The difference lies in the additional \( x^2 \) terms which we have included in the subleading terms and lead to the complete squares. Their origin can be traced back to the subleading corrections coming from \( K^{ZZ} \). We therefore conclude that it is important to include the contribution from \( K_{(M5)} \) to the Kähler-potential which gives rise to \( K^{ZZ} \) and which seemingly had been omitted in the derivation of the potential in [3]. Finally, one could be inclined to view (3.8) as the begin of a series expansion which roughly could be summed up to \( \beta e^{-J\beta + \sqrt{\beta J/V} x} \). This then suggests that higher order in \( J/V \) contributions could not endanger the leading-order result when summed up as long as \( J/V \ll 1 \).

Before proceeding with the minimisation analysis let us briefly reflect on a consequence of (3.5) and (3.9). With the definition of \( \alpha \) inserted into (3.5) and setting \( x = 1/2 \), we
see that the axion $\chi$ cancels out and the minimisation condition \ref{eq:3.5} implies setting the scalar $A$ to

$$A = n \frac{\pi}{2} \frac{1}{\beta a}, \quad n \in \mathbb{Z}.$$ \hfill (3.10)

In particular $A$ can be zero.

Proceeding with the minimisation, let us set $x = 1/2$ and thus obtain for the OM-potential

$$(\kappa_4)^4 U_{OM} = \frac{(|h|\beta)^2}{2d J^V} e^{-J\beta}.$$ \hfill (3.11)

Notice once more that this comes from the subleading terms as the leading terms vanish. Because $J$ and $V$ are $R$ dependent, $U_{OM}$ becomes a function of $R$ which can be minimised with respect to $R$. However, it is more convenient to minimise with respect to $J$ instead since alternatively $V$ can be viewed as a function of $J$ once we have fixed $x = 1/2$. The vanishing of the first derivative of $U_{OM}$ with respect to $J$ leads to the condition

$$\frac{V_J}{V} + \frac{1}{J} + \beta = 0.$$ \hfill (3.12)

The derivative of the average linear volume with respect to $J$ is given by

$$V_J = \left( \frac{J}{3V} - \frac{1}{r_{OM}} \left( \frac{6V^d}{d} \right)^{1/3} \right)^{-1},$$ \hfill (3.13)

where we have defined the flux-parameter

$$r_{OM} \equiv r \left( \frac{1}{2} \right) = r_v - \frac{r_{M5}}{4}$$ \hfill (3.14)

which controls the “running” of $V$ with $R$

$$V = V_1 - r_{OM} R$$ \hfill (3.15)

for the case with the M5 located in the middle of the orbifold-interval.

Let us now solve \ref{eq:3.12} in the moduli-region $J^2 \gg V \gg J \gg 1$. By neglecting the $1/J$ against the $\beta$ term and employing \ref{eq:3.13}, one obtains upon again neglecting $1/\beta$ against $J/3$

$$\frac{1}{r_{OM}} \left( \frac{6V^d}{d} \right)^{1/3} = \frac{J}{3}.$$ \hfill (3.16)
Hence we have to constrain $r_{OM}$ to positive values. Since $V \gg J$, the validity of this equation requires a rather large $r_{OM}d^{1/3} \gg 1$. With $J = (6V)^{1/3}(V_1 - V)/(r_{OM}d^{1/3})$ it is then easy to arrive at the final solution which gives the stabilised values of the moduli

$$V = \frac{V_1}{4}, \quad R = \frac{3V_1}{4r_{OM}}.$$  (3.17)

To actually show that the above solution corresponds to a minimum of the potential and not just to an extremum, we have to show that the second derivative of the potential with respect to $R$ is positive. Because

$$U_{OM,RR} = \frac{d^2J}{dR^2}U_{OM,J} + \left(\frac{dJ}{dR}\right)^2U_{OM,JJ}$$  (3.18)

and both $d^2J/dR^2$ and $U_{OM,J}$ are negative, it suffices to show that $U_{OM,JJ}$ is positive in order to establish a minimum. Explicitly, the second derivative of the potential is proportional to

$$U_{OM,JJ} \propto e^{-J\beta} \left[ \left(\frac{V_J}{V} + \frac{1}{J} + \beta\right)^2 + \frac{1}{J^2} + \left(\frac{V_J}{V}\right)^2 - \frac{V_{JJ}}{V} \right]$$  (3.19)

The first term in square brackets vanishes at the extremal point by using the extremality condition (3.12). The remaining terms are manifestly positive except for the last one containing the second derivative of $V$. However, with the help of (3.12) it can be written as

$$-\frac{V_{JJ}}{V} = \frac{1}{3} \left[ \beta + \frac{1}{J} \right]^2 \left( 1 + \left[ \beta + \frac{1}{J} \right] \left[ J + \frac{1}{r_{OM}}(\frac{6V^4}{d})^{1/3} \right] \right),$$  (3.20)

which shows that the second derivative of the potential is positive at its extremal point which therefore represents a minimum of the potential.

We emphasize that this minimum of the potential only occurs because we have a non-constant CY volume whose running along the orbifold-interval is caused by the non-trivial G-flux. In contrast a constant CY volume and thereby an $R$ independent average $V$ would lead to the well-known runaway-behaviour for (B.11).

4 Properties of the OM Instanton Stabilised Vacuum
4.1 G-Fluxes and the Validity of the First Order Approximation

Let us now check for what values of $r_v$ and $r_{M5}$ we can trust the obtained solution, i.e. the first order approximation. Evaluating the corresponding constraints for the vacuum (3.17), either (1.12) with (1.13) or (1.14) together with (1.15), both lead to the flux-equality

$$r_v = r_{M5} \Rightarrow r_{OM} = \frac{3}{4} r_v$$

(4.1)

which can be used to express the obtained stabilised value for $R$ purely in terms of visible boundary data

$$R = \frac{V_1}{r_v}.$$  

(4.2)

Thus the solution saturates the bound $R \leq V_1/r_v$ imposed by (1.12) which means that the CY volume $V(x^{11})$ becomes zero at the location of the M5.

Thus compatibility of the stabilised solution with the first order approximation gives a precise relationship between the fluxes on the visible boundary and the M5. Taken together with the anomaly cancellation constraint (1.9), one obtains

$$r_h = 0$$

(4.3)

and hence the following relationships

$$\text{tr} F^2_{(2)} = \frac{1}{2} \text{tr} R^2, \quad -\left(\text{tr} F^2_{(1)} - \frac{1}{2} \text{tr} R^2\right) = 8\pi^2 [\Sigma].$$

(4.4)

These lead to a relation between the “instanton-numbers” on the two boundaries

$$-\int_{\text{CY}_3} \omega \wedge \text{tr} F^2_{(1)} + \int_{\text{CY}_3} \omega \wedge \text{tr} F^2_{(2)} = 8\pi^2 \int_{\Sigma} \omega = 8\pi^2 \text{Vol}(\Sigma),$$

(4.5)

their difference being determined by the G-flux jump coming from the M5. Notice that the rhs is proportional to

$$8\pi^2 \epsilon \int_{\text{CY}_3} \omega \wedge [\Sigma] \equiv W_G,$$

(4.6)

where $W_G$ is the tree-level superpotential generated by the G-flux of the M5 brane (see e.g. [7], [27], and also [28], [31] for the CY fourfold case). This is what one could have expected on account of energy-conservation reasoning [28], namely that the G-flux from the M5 leads to a flux jump which is responsible for the difference between the boundary
G-fluxes. We remark that it was not necessary to include in (3.1) this type of superpotential or a related one stemming from the dimensional reduction of the Chern-Simons term, $C \wedge G \wedge G$, of eleven-dimensional supergravity for the following reason. As has been shown in [7] they are of higher order in $\epsilon$ than the leading contributions considered in (3.1).

Eventually, we have to verify that the expansion parameters $\epsilon$ and $\epsilon_R$ stay small. For the above solution this requires that

$$\epsilon = 32^{1/3} \frac{V_1^{1/3}}{r_{OM}} < 1 , \quad \epsilon_R = \sqrt{\frac{\pi}{2}} \frac{2^{5/3} r_{OM}}{3V_1^{5/6}} < 1 .$$

(4.7)

In particular this implies that $V_1 > 8\pi \simeq 25.1$ and $r_{OM} > 3(16\pi)^{1/3} \simeq 11.1$. To show that these two constraints actually do have a common solution, we have plotted in fig.4 and fig.5 in appendix C the two expansion parameters, $\epsilon$ and $\epsilon_R$ in the region $525 \leq V_1 \leq 5000$, $80 \leq r_v \leq 250$ with $d = 30$. The average CY volume chosen is the one appropriate for the OM case (i.e. with $x = 1/2$).

4.2 The Eleven-Dimensional Picture

We can also infer to which kind of eleven-dimensional geometry this flux relation corresponds to. It is easy to see that the obtained stabilised $V$ and $R$ moduli values together with the equality of the G-fluxes imply that in the eleven-dimensional picture the variation of the CY volume (not its average) with the orbifold coordinate $x^{11}$ is as follows (see

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1}
\caption{The CY volume dependence on the orbifold coordinate $x^{11}$ in the eleven-dimensional picture which is implied by the stabilised moduli and G-flux values found within the four-dimensional effective description.}
\end{figure}

...
Figure 2: The CY volume behaviour which is found beyond leading order under the assumptions that $x = 1/2$ and $r_v = r_{M5}$ remain true in the full theory. Over the first half of the orbifold-interval the volume varies quadratically and stays constant over the second half. The zero volume interval gets lifted to a positive value $V_1/4$.

\[ V(x^{11}) = \begin{cases} V_1 - \frac{2}{\rho}r_v x^{11}, & 0 \leq x^{11} < \frac{R_\rho}{2} \\ 0, & \frac{R_\rho}{2} \leq x^{11} \leq R_\rho \end{cases} \quad (4.8) \]

It might seem bizarre that the eleven-dimensional geometry exhibits a zero CY volume along an interval. There is however reason to believe that this is so only in the first order approximation but no longer the case in a full treatment of heterotic M-theory. To explain this, let us assume that beyond the first order approximation two features of the stabilised vacuum remain true. First, the $x \leftrightarrow 1 - x$ exchange symmetry should remain valid since nothing distinguishes one of the OM’s against the other. This means that $x = 1/2$ would remain the equilibrium position of the M5. Second, let us assume that in addition the equality of the fluxes on the visible boundary and the M5 remains valid. With these two assumptions, it is possible to use the result of [12] to obtain the eleven-dimensional CY volume behaviour in the full non-linear treatment\[ \]

\[ V(x^{11}) = \begin{cases} (1 - \frac{r_v}{V_1 \rho} x^{11})^2 V_1, & 0 \leq x^{11} < x_{M5} = \frac{R_\rho}{2} \\ (1 - \frac{r_M^5}{2V_1 \rho})^2 V_1, & x_{M5} \leq x^{11} \leq R_\rho \end{cases} \quad (4.9) \]

Over the first part of the interval the CY volume varies quadratically while over the second part it stays constant as a consequence of the flux-equality. It is interesting now to substitute for $R$ the value found for the stabilised OM vacuum

\[ R = \frac{V_1}{r_v} \quad (4.10) \]

\[^8\]One has to identify the flux $S_1$ in the notation of [12] with $r_v/(V_1 \rho)$ in the notation used here. Similarly $S_{M5}$ there has to be identified with $-r_{M5}/(V_1 \rho)$ here.
Figure 3: The logarithm of the OM potential, \( \ln((\kappa_4)^4 U_{OM}) \), is depicted as a function of the orbifold modulus \( R \) for parameters \(|h| = \beta = 1, V_1 = 3000, r_{OM} = 200, d = 30\). It exhibits a minimum at \( R = 11 \). At \( R = 15 \) the average CY volume \( V \) vanishes thus leading to the steep increase there. The reason for this is that the CY volume becomes negative to the right of the minimum and one can strictly trust the potential only up to its minimum. The possibility of a saddle point at \( R = 11 \) is however excluded since the potential exhibits a positive second derivative there.

which gives no longer a vanishing but positive value \( V(x^{11}) = V_1/4 \) for the second part of the orbifold-interval (see fig.2).

### 4.3 Vacuum Energy

To illustrate graphically that the obtained extremising solution (3.17) actually corresponds to a minimum of the potential we have plotted the logarithm of the OM potential in fig.3 for the choice of parameters (which are representative for the orders of magnitude needed to obey the constraints (4.7))

\[
|h| = \beta = 1 , \quad V_1 = 3000 , \quad r_{OM} = 200 , \quad d = 30 .
\]

(4.11)

Indeed, the OM potential exhibits a minimum around \( R = 11 \) in agreement with (3.17). Due to the exponential suppression by the factor \( e^{-J\beta} \) the contribution to the vacuum energy can be remarkably low. Indeed, e.g. by choosing parameter values like

\[
|h| = \beta = 1 , \quad V_1 = 5400 , \quad r_{OM} = 100 , \quad d = 30 .
\]

(4.12)

it is possible to lower this contribution to the vacuum energy to the order of

\[
U_{OM} \simeq 10^{-121} M_{Pl}^4 \simeq \text{meV}^4 ,
\]

(4.13)
which is the observed scale of the cosmological constant. One has to note, however, that the complete vacuum energy will also comprise the quantum fluctuations of other fields like the gauge fields for example. These are not suppressed likewise and therefore one still faces the cosmological constant problem. To suppress them likewise another mechanism like e.g. a suppression by higher-dimensional warp-factors might be a prospect (see [29] for a purely geometrical approach). In the context of the still fictitious full M-Theory one also has to keep in mind that T-duality can change the value of the cosmological constant and thus there is some arbitrariness in its definition as long as one does not “fix” this duality.

In the last section when we come to the issue of the scale of supersymmetry-breaking, it will turn out that to achieve $M_{\text{Susy}} \simeq \text{TeV}$ requires smaller values for $V_1$ and/or larger values for $r_{OM}$ than those given in (4.12) and thus the vacuum energy contribution becomes much bigger. For the solution found the OM instanton contribution to the vacuum energy reads

$$
(\kappa_4)^4 U_{OM} \simeq 2.3 \left( \frac{|h| \beta}{a^{1/2}} \right)^2 r_{OM} \frac{V_1}{V_1^{7/9}} e^{-0.9 \frac{\beta V_1^{4/3}}{r_{OM} a^{7/3}}} \text{.} 
$$

(4.14)

5 Gravitino-Mass and Supersymmetry-Breaking Scale

We have seen that in the effective four-dimensional description of heterotic M-theory OM-instantons generically break supersymmetry (for earlier considerations of supersymmetry-breaking in heterotic M-theory by gaugino condensation see [32]). In order to determine the supersymmetry-breaking scale $M_{\text{Susy}}$, we have to calculate the F-terms of the respective chiral moduli supermultiplets and determine their vev’s [33]. The F-terms are given by

$$
F^i = e^{K/2} D^i W = e^{K/2} K^{ij} D_j W \text{, } i = S, T, Z \text{.} 
$$

(5.1)

In terms of them and the generalized Kähler-potential $G$, given by $e^G = e^K |W|^2$, the potential of four-dimensional N=1 supergravity can be expressed as

$$
(\kappa_4)^4 U = K_{ij} F^i F^j - 3e^G \text{.} 
$$

(5.2)

The scale $M_{\text{Susy}}$ of the supersymmetry-breakdown is given by the vev of $F^i$ through

$$
M_{\text{Susy}}^2 = M_{Pl}^2 |\langle F^i \rangle| = M_{Pl}^2 e^{K/2} |D^i W| \text{.} 
$$

(5.3)
The other interesting quantity related to supersymmetry-breaking is the value of the gravitino mass $m_{3/2}$ which is given by

$$m_{3/2}^2 = M_{Pl}^2 e^K |W| = M_{Pl}^2 e^F.$$  \hspace{1cm} (5.4)

Thus, in order to determine $M_{Susy}$ and $m_{3/2}$, we have to know $e^K$, $|W|$ and $|D^i W|$ for the OM stabilised vacuum examined previously. This is derived in appendix B and leads to the following expressions

$$m_{3/2} \simeq M_{Pl} \sqrt{|h|} \left( \frac{6e^{-J}}{dVJ^3} \right)^{1/4} \hspace{1cm} (5.5)$$

$$M_{Susy} \simeq J m_{3/2} = M_{Pl} \sqrt{|h|} \left( \frac{6J}{dV} e^{-J} \right)^{1/4}. \hspace{1cm} (5.6)$$

We have equal F-terms for $S$, $T$, and $Z$ all giving rise to the same $M_{Susy}$.

### 5.1 Comparison of Vacuum Energy with $M_{Susy}$

It is interesting to compare the vacuum energy of the OM case with its supersymmetry breaking scale. We obtained for the vacuum energy

$$U_{OM}^{1/4} \simeq M_{Pl} \sqrt{|h|} \left( \frac{6e^{-J}}{dV} \right)^{1/4}. \hspace{1cm} (5.7)$$

From phenomenological reasoning one would like to have

$$M_{Susy} \gg U_{OM}^{1/4}. \hspace{1cm} (5.8)$$

With the above formula for $M_{Susy}$ this translates into

$$\sqrt{J} \gg 1. \hspace{1cm} (5.9)$$

It is satisfying to see that this is true in the considered region of moduli space, where $J \gg 1$. However, to become more realistic a huge value of $J \simeq 10^{30}$ would be needed to bridge the gap between the observed meV vacuum energy and a TeV supersymmetry breaking scale. This is however far beyond the values of $J$ considered in this paper which had to be rather small to guarantee the reliability of the perturbative formulation of heterotic M-theory.
5.2 $M_{\text{Susy}}$ for the OM Vacua

Let us finally evaluate $M_{\text{Susy}}$ for the OM-instanton vacuum in terms of the CY data $V_1, d, r_{OM} = 3r_v/4$. Using the vacuum given by (3.17) we obtain to leading order

$$M_{\text{Susy}} = 2.3 M_P\sqrt{|h|} \left( \frac{1}{r_v} \left( \frac{V_1}{d^4} \right)^{1/3} e^{-\frac{1}{r_v} \left( \frac{3V_1^4}{2d^4} \right)^{1/3}} \right)^{1/4}.$$  (5.10)

In fig.6 in appendix C we plot $M_{\text{Susy}}$ as a function of $V_1$ and $r_v$ in the region $525 \leq V_1 \leq 5000$, $80 \leq r_v \leq 250$ for fixed values $|h| = 1$, $d = 30$. As evident from fig.4 and fig.5 (see appendix C) in this region of parameter space we can trust the perturbative approach, since both $\epsilon$ and $\epsilon_R$ stay smaller than one throughout this region and thereby guarantee that higher order contributions are sufficiently suppressed. From fig.6 it can be seen that in order to reach the TeV scale with $M_{\text{Susy}}$, rather large values for $r_v$ are required in order to diminish the huge $V_1$ contribution in the exponent.

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A K"ahler-Potential and its Derivatives

\[ K = K_{(S,M5)} + K_{(T)} \]  
\[ = - \ln \left[ S + \overline{S} - \frac{(Z + \overline{Z})^2}{\beta(T + \overline{T})} \right] - \ln\left[ \frac{d}{6} (T + \overline{T})^3 \right] + \ln \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \]  
\[ = - \ln \left[ \frac{8}{3} dV J^3 + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \right] \]

from which it follows that

\[ e^K = \frac{3}{8dV J^3} + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \].  

First derivatives of \( K \) with respect to the moduli:

\[ K_S = K_{\overline{S}} = -\frac{1}{2V} + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \]  
\[ K_T = K_{\overline{T}} = -\frac{3}{2J} \left[ 1 + \frac{x^2 \beta}{3} \frac{J}{V} + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \right] \]  
\[ K_Z = K_{\overline{Z}} = \frac{x}{V} + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \]

Second derivatives:

\[ K_{S\overline{S}} = \frac{1}{4V^2} + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \]  
\[ K_{S\overline{T}} = \frac{x^2 \beta}{4V^2} + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \]  
\[ K_{S\overline{Z}} = -\frac{x}{2V^2} + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \]  
\[ K_{T\overline{T}} = \frac{3}{4J^2} \left[ 1 + \frac{2x^2 \beta}{3} \frac{J}{V} + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \right] \]  
\[ K_{T\overline{Z}} = -\frac{x}{2V J} \left[ 1 + x^2 \beta \frac{J}{V} + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \right] \]  
\[ K_{Z\overline{Z}} = \frac{1}{2\beta V J} \left[ 1 + 2x^2 \beta \frac{J}{V} + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \right] \]

Note that the second terms in the square brackets are of order \( \epsilon \) and are kept since we are analyzing the potential to subleading order.

The inverse of the second derivatives K"ahler-matrix exact to subleading order obeys

\[ K^{-1} K = 1 + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \]  

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and can be obtained as follows. Let us split the Kähler-matrix into its leading and subleading part

\[ K = K_0 + \epsilon K_1 + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) . \]  

(A.10)

It is easy to show that \( M_0 \), the inverse to \( K_0 \) at leading order

\[ M_0 K_0 = 1 + \mathcal{O}(\epsilon, \epsilon_R) \]  

(A.11)

is given by

\[
\begin{align*}
K_{SS} &= 4V^2 , & K_{ST} &= \frac{4}{3} \beta J^2 x^2 , & K_{SZ} &= 4\beta JV x , \\
K_{T\bar{T}} &= \frac{4}{3} J^2 , & K_{T\bar{Z}} &= \frac{4}{3} \beta J^2 x , & K_{Z\bar{Z}} &= 2\beta JV ,
\end{align*}
\]  

(A.12)

with the missing entries related to the ones given by \( K^{ij} = K^{ji} \) symmetry. The additional subleading piece, \( M_1 \), which completes the inverse Kähler-matrix at subleading order

\[ K^{-1} = M_0 + \epsilon M_1 + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) \]  

(A.13)

is given by

\[ M_1 \equiv -(M_0 K_1 + \Delta) M_0 , \]  

(A.14)

where \( \Delta \) measures the deviation of \( M_0 K_0 \) from the identity at subleading order

\[ \epsilon \Delta = M_0 K_0 - 1 + \mathcal{O}(\epsilon^2, \epsilon_R^2, \epsilon \epsilon_R) . \]  

(A.15)

Following these steps gives us finally the inverse Kähler-matrix, \( K^{-1} \), correct up to subleading order

\[
\begin{align*}
K_{SS} &= 4V^2 \left(1 + 2x^2 \beta \frac{J}{V}\right) , & K_{ST} &= \frac{4}{3} \beta J^2 x^2 , \\
K_{SZ} &= 4\beta JV x \left(1 + \frac{1}{3} x^2 \beta \frac{J}{V}\right) , & K_{T\bar{T}} &= \frac{4}{3} J^2 , \\
K_{T\bar{Z}} &= \frac{4}{3} \beta J^2 x , & K_{Z\bar{Z}} &= 2\beta JV \left(1 + \frac{2}{3} x^2 \beta \frac{J}{V}\right)
\end{align*}
\]  

(A.16)

Again the symmetry \( K^{ij} = K^{ji} \) gives the remaining matrix entries.
B  Technical Details for Deriving $M_{\text{Susy}}$ and $m_{3/2}$

We will present in this appendix those expressions which are needed for the computation of the supersymmetry-breaking scale $M_{\text{Susy}}$ and the gravitino mass $m_{3/2}$.

From appendix A we see that the exponential involving the Kähler-potential is given by

$$e^{\frac{K}{2}} = \left(\frac{3}{8dVJ^3}\right)\frac{1}{2}.$$  \hspace{1cm} (B.1)

Next, we have to calculate the modulus of the OM superpotential

$$W = h \left( e^{-Z} + e^{Z-\beta T} \right)$$  \hspace{1cm} (B.2)

which turns out to be

$$|W| = |h| \left( e^{-2j\beta x} + e^{-2j\beta(1-x)} + 2e^{-\beta J} \cos(2\alpha - \beta \chi) \right)^{\frac{1}{2}}.$$  \hspace{1cm} (B.3)

This has to be evaluated for the OM vacuum derived in the main text. For this we have to use the axion minimisation condition (3.5) together with $n$ even and M5 position modulus $x = 1/2$. This leads to the vacuum expression

$$|W| = 2|h|e^{-\frac{j}{2}}.$$  \hspace{1cm} (B.4)

The last ingredient is the absolute value of the Kähler-covariant derivatives $|D^i W|$. Let us start from the derivatives with lower indices first. Their leading orders\footnote{These are sufficient to determine $|D^i W|$ including all $JV$ and $J^2$ contributions which give the leading expressions for $M_{\text{Susy}}$ and $m_{3/2}$.} are given by

$$D_{S}W = -\frac{h}{2V}(e^{-Z} + e^{Z-\beta T})$$  \hspace{1cm} (B.5)

$$D_{T}W = -\beta he^{Z-\beta T}$$  \hspace{1cm} (B.6)

$$D_{Z}W = h(-e^{-Z} + e^{Z-\beta T}).$$  \hspace{1cm} (B.7)

The next step is to calculate from these the upper-index derivatives $D^i W = K^{ij}D_j W$. The general structure of the $D^i W$ can be parameterised as $(i = S, T, Z)$

$$D^i W = A_i \overline{he}^{-Z} + B_i \overline{he}^{Z-\beta T},$$  \hspace{1cm} (B.8)
where the specific coefficients read

\[ A_{S} = -4JV\beta x\left(1 + \frac{\beta x^2 J}{V}\right), \quad B_{S} = -A_{S} - \frac{4}{3}\beta^2 x^2 J^2, \]

\[ A_{T} = -\frac{4}{3}\beta x J^2, \quad B_{T} = -\frac{4}{3}\beta(1 - x)J^2, \]

\[ A_{Z} = -2\beta JV\left(1 + \frac{2}{3}\beta x^2 J/V\right), \quad B_{Z} = -A_{Z} - \frac{4}{3}\beta^2 x J^2. \]  \hspace{1cm} (B.9)

Its absolute value can then be figured out to be

\[ |D^iW| = |h|(A_i^2 e^{-2J\beta x} + B_i^2 e^{-2J\beta(1-x)} + 2A_i B_i e^{-\beta J} \cos(2\alpha - \beta \chi))^\frac{1}{2}. \]  \hspace{1cm} (B.10)

Again, to evaluate this expression for the OM vacuum, we use the axion minimisation condition (3.5) together with \(n\) even and \(x = 1/2\) which gives

\[ |D^iW| = |h||A_i + B_i|e^{-\frac{J}{2}}. \]  \hspace{1cm} (B.11)

Specifically, this leads to the following vacuum expressions

\[ |D^{S}W| = \frac{\beta}{4}|D^{T}W| = \frac{1}{2}|D^{Z}W| = \frac{\beta^2}{3}|h|J^2e^{-\frac{J}{2}}. \]  \hspace{1cm} (B.12)

Hence we obtain the succinct result

\[ |D^iW| \approx |h|J^2e^{-\frac{J}{2}}. \]  \hspace{1cm} (B.13)

\[ \textbf{C} \quad \textbf{Plots} \]
Figure 4: The figure shows that $\epsilon$ stays smaller than one in the $(V_1, r_v)$ parameter region given by $525 \leq V_1 \leq 5000$, $80 \leq r_v \leq 250$ and $d = 30$.

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Figure 5: The figure shows that also $\epsilon_R$ stays smaller than one in the same $(V_1, r_v)$ parameter region as considered in the previous figure.

Figure 6: The figure shows the dependence of $\ln(M_{\text{Susy}}/\text{TeV})$ on the two visible boundary CY volume and flux parameters $V_1$ and $r_v$. The remaining parameters are set to the values $|h| = 1$, $\beta = 1$, $d = 30$. 
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