Investigating the contour dynamics of the vortex eye on the laboratory model of a differentially rotating fluid

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Abstract. This article studies the contour dynamics of the vortex eye based on the results of laboratory experiments with a differentially rotating shallow fluid. This paper also describes the analysis of the energy processes inside the vortex center with variations in the velocity shift between the center of the system and the periphery in the framework of the instability representations due to the presence of inflection points on the velocity profile. Moreover, it is shown that the conditions for the development of instability arise only at fully determined values of the velocity shift to the width ratio of the shear zone. Finally, articles states that the formation of polygonal configurations of vortex contours with velocity shift is associated with energy transfer.

1. Introduction

In connection with the development of a general theory of atmospheric and ocean circulation special attention is paid to studies of the effects on the development of instability of large-scale oceanic currents of both baroclinic and barotropic processes caused by vertical or horizontal velocity shifts, respectively [1,2]. Jet flows refer to such class of movements in geo-environments in which horizontal velocity shifts can have a significant effect on the formation of the barotropic instability of hydrodynamic processes in the atmosphere and the ocean [3-5]. A definite contribution to the establishment of physical mechanisms of the prevailing influence on the dynamics of atmospheric and oceanic currents is introduced by laboratory hydrodynamic experiments [6-10].

It is not always possible to perform field observations and measurements and they usually do not provide answers to many questions due to limited research opportunities. In particular, this refers to the study of intense atmospheric vortices, the laws of formation, evolution and the impact on climatic oscillations, which despite years of research remain unanswered [11]. Recent qualitative data of aircraft measurements and satellite soundings have been added to the number of qualitative and quantitative questions of the influence of vortex structures forming on the boundary of the contour of the central region (eye) of the hurricane on its dynamics, stability and displacement [12]. It was found on the images of cloud fields of hurricanes taken from space that the outline of the eye changes its configuration in the form of a circle in the course of evolution from the usual shape to various
polygonal outlines, which are still unclear as to the intensity of the tropical cyclone and its trajectory [13].

It seems possible to use the results of laboratory experiments with a rotating shallow fluid with reference to the processes of barotropic instability in the central part of hurricanes. This refers to the phenomenon of polygonality of the contour "eye" in tropical cyclones and the double "eye" in hurricanes detected in radar observations from satellites [14]. A detailed study of these effects using laboratory models of shear instability can prove to be a useful auxiliary tool in constructing prognostic schemes for the creation and evolution of dangerous natural phenomena such as hurricanes. This paper on the basis of laboratory experiments considers the processes inside the central part of cyclonic vortices. At the same time, it is taken into account that the very fact of their existence presupposes a velocity shift between the center of the system and the periphery. In this formulation, laboratory modeling stimulates the analysis of nonlinear mechanisms of shear instability within the framework of the instability concepts caused by the presence of inflection points on the radial velocity profiles.

Aim of this paper is to study flow simulation for different values of the determining parameters and external conditions; and then perform analysis of the vortex structures forming in the central region of a differentially rotating shallow fluid. Moreover, this article studies the dynamics and energy of interaction of different-scale wave motions arising on horizontal shifts of the barotropic vortex velocity without taking into account the condensation heating, baroclinicity and vertical shift of the main flow velocity.

2. Experimental installation

The experiment was carried out by utilizing the method of mechanical excitation of differentially rotating flows of a homogeneous incompressible fluid. For this purpose a variant of the modernized plant of S.S. Lappo and V.M. Lushin is used. It is designed to study the geophysical patterns of general circulation in the atmosphere and the hydrosphere, as well as to solve problems of applied hydrodynamics [15].

The working part of the installation is a cylindrical vessel flat bottom, which is composed of a central disk and two concentric rings. On the circumference of the outer ring the side wall of the vessel that is made of Plexiglas is rigidly fixed. Rings and discs are installed and fixed on the upper ends of hollow shafts. The shafts are combined into one telescopic shaft. Bearings are used to ensure the rotation of the bottom sectors in hollow shafts. To seal the shafts it is envisaged to install cuffs to prevent leakage of fluid from the cavities formed between the disc, rings and hollow shafts. From the bottom of each shaft are installed pulleys which transfer the rotational motion. The vessel was filled with fluid to a certain level. Fresh water was used as a working fluid. To observe movements in differentially rotating shallow water, a digital video camera and IR thermal imager are used.

This model unlike other laboratory models creates feasibility to recreate and study a significant number of independent combinations of velocities and directions of rotation of circulation streams created in all parts of the bottom at one horizontal level. This is very important from the perspective of modeling the dynamic diversity of natural systems, both in terms of the scale of circulation, and in the velocities and directions of motion of the main shear flows. Another difference between the laboratory model and the cylindrical and annular installations used in other works was the absence of a forced general rotation of the entire system, in which the shift was set as a small perturbation of the mean motion.

To determine the flow velocity in a rotating system, it is proposed to use the method of analyzing the tracks of particle-flow indicators, including particles with local heating. Processing and interpretation of the results of the experiment is carried out using the software systems Matlab 9.

The diagram and general view of the laboratory setup are shown in the figure (figure 1).
Figure 1. Diagram (a) and general view of the experimental setup (b).

The working part of the installation with a diameter of 0.46 m and a height of 0.25 m is a cylindrical vessel (1), the flat bottom of which was composed of a central disk (2) and two concentric rings (3,4). On the circumference of the outer ring (4) the side wall of the vessel that is made of Plexiglas was rigidly fixed. The diameter of the central disk is 0.20 m; the width of the middle ring is 0.10 m, the width of the outer ring is 0.03 m. The rings and disks were installed and fixed on the upper ends of the hollow shafts. The shafts are combined into one telescopic shaft (5). To ensure rotation of the bottom sectors in hollow shafts, bearings (6) were installed. The outer shaft was attached and rotated in the support bearing (7) mounted on the mounting table (8). To seal the shafts, cuffs (9) were used to prevent leakage of fluid from cavities formed between the disc, rings and hollow shafts. From the bottom of each shaft, pulleys (10) were mounted with a dense nozzle, through which the rotation from the electric drives (11) was transmitted by means of a V-belt gear and gearbox systems. For each shaft, an individual engine was intended. To increase the possibilities of controlling the differential rotation modes, stepper motors were used, which made it possible to perform a discrete rotation of the shaft with a specified frequency of rotation and to reduce fluctuations in the rotational speed. These requirements were satisfied by stepping motors MEC MICHALOVCE OPC 100-2. Fluctuations in the speed of rotation did not exceed 1%.

The vessel (1) was filled with the fluid to a certain level, then at the electric drives (11) the corresponding fixed angular velocities of rotation for each sector of the bottom were set. By smoothly adjusting the voltage supply to each of the three engines the rotation frequencies of the bottom sectors were varied. After the motors were turned on, the rotation of the drives (11) was transmitted to the telescopic shaft (5); disc (2) rings (3, 4) fixed to the shafts, began to rotate at different angular velocities in the fluid layer. After a while, through the redistribution of inertial forces, centrifugal force and frictional forces of rings and a disc about fluid, the effect of differential rotation of the fluid in the vessel (1) was achieved.

The electrical block diagram of the experimental setup includes the following elements: a video camera; three electric motors; voltage control and monitoring unit; frequency meter, three photosensors; personal computer. The video camera was installed on the side wall of the vessel and allowed fixing the vortices in a coordinate system moving with the speed of rotation of the fluid.

After the installation is switched on the voltage is supplied to the control and voltage control unit which is switched from the computer to control the supply of voltage to the motors. Using the latter it was possible to reduce the voltage fluctuation in the circuit up to 1%, and also to control the supply of voltage to each engine by means of a computer. After setting the necessary voltage on the engines a frequency meter was turned on and the rotation frequency of the bottom sections of the installation was measured. Then, all the necessary quantities were recorded on the computer. Then the video camera was turned on and the measurement results were recorded on the computer.
3. Methods of processing the results

Methods of analyzing tracks of the particle-flow indicators are used to determine the flow velocity in a rotating system. The results of the experiment are processed on a computer. The video recording is fixed on a digital medium and the necessary framesets are cut out in the program of the video editor, and then all frames are formatted to a certain size suitable for analysis. To calculate the velocity of particle-indicators the following formula is used

\[ V = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \]

where \( \Delta \tau \) - is the time between two successive frames, \( x_1, y_1 \) - are the coordinates of the particle at the initial time, \( x_2, y_2 \) - are the coordinates of the particle through \( \Delta \tau \). The radial velocity component is calculated as

\[ V_{rad} = \sqrt{(x_1 - x_c)^2 + (y_1 - y_c)^2 - (x_2 - x_c)^2 + (y_2 - y_c)^2} \]

where \( x_c, y_c \) - the coordinates of the center around which the rotation of the vortex occurred. The tangential velocity component is calculated as

\[ V_{tan} = \sqrt{V^2 - V_{rad}^2} \]

In order to bring the calculated values to normal dimensions a normalization was carried out from the photograph of the ruler (from a distance that was the same for all the experiments) fixed on the central disk and the correspondence coefficient is \( K = \text{pixel} / \text{cm} \) according to formula:

\[ K = \frac{1}{\sqrt{(x_b - x_e)^2}} \]

where \( x_b \) - the beginning coordinate of a reference centimeter, \( x_e \) - the ending coordinate of a reference centimeter. Consequently, all previously calculated values were multiplied by the resulting coefficient. The average rotation speed of the entire system was calculated as

\[ \bar{\Omega}_{sys} = \frac{\sum_{i=1}^{N} \omega_k}{k} \]

where \( \omega_k \) - is the angular velocity of rotation of \( k \) - th sector of the bottom of the installation. The specific kinetic energy of the constituent vortices was calculated from the formula

\[ E_i = \frac{V_i^2}{2g} \]

where \( V_i \) - is the velocity of the corresponding vortex, \( g \) - is the acceleration due to gravity.

The average specific kinetic energy of the constituent vortices is given by the formula

\[ \bar{E} = \frac{\sum_{i=1}^{N} E_i}{N} \]

4. Discussion

In experiments with different values of the angular velocity of rotation of the central part of the fluid and the peripheral one a variety of stable patterns of streamlines were formed on the surface of the fluid. The pattern of fluid motion was visualized by weighted particles. With their help in the central part of the rotating stream polygonal shapes with different symmetry index were observed. The shape of the contours of particles in the form of different polygons resembles the patterns of cloud accumulation at the boundary of the eye of a tropical cyclone which are obtained by radar sounding [16,17].
Figure 2. Typical picture of the eye contour of a model vortex with different symmetry index (a); polygonal structures obtained during the radar sounding of Hurricane Wilma (b), [17].

In laboratory experiments, the figures are formed by clusters of particles floating on the water surface (figure 2. (a)). On the radar screen, such figures are associated with a signal reflected from cloud clusters in the zone of precipitation of the hurricane (figure 2. (b)). Qualitative correspondence indicates imitation of the effects of barotropic instability in the process of formation of the eye structure of a tropical cyclone.

The key parameters governing the occurrence and development of wave disturbances are: Rossby shift number $\text{Ro} = \frac{\delta \omega}{\Omega}$ (analog of the ratio of inertia and Coriolis forces). Here $\delta \omega$ is the angular velocity shift between the center and the periphery, $\Omega$ is the angular velocity of the total rotation of the entire fluid and the Ekman number $E = \frac{v}{\Omega H^2}$ (the viscosity and Coriolis force), where $v$ is the kinematic viscosity of water, $H$ is the characteristic size (the depth of the unperturbed fluid).

Table 1 shows the speed scales, horizontal and vertical dimensions of the central region of tropical cyclones (TC) [13]. Scales of analogous parameters of the physical model are also given there. The table shows a satisfactory coincidence in terms of the relative magnitudes of the determining characteristics. Here $L$ is the radius of the maximum winds; $R$ - the outer boundary of the central area of the TC; $V_{\text{max}}$ is the velocity in the region of the maximum horizontal wind shear; $V_r$ is the velocity on the radius $R$; $h$ is the vertical scale.

Table 1. The scale of the quantities characterizing the central region of the TC and the parameters of the laboratory model.

| Typical unit | Tropical cyclones | Laboratory model |
|--------------|-------------------|-----------------|
| $L$          | 14 - 90 km        | 10 cm           |
| $V_{\text{max}}$ | 33 - 113 m/s   | 20 - 110 cm/s   |
| $R$          | 150 - 200 km      | 23 cm           |
| $V_r$        | 10-25 m/s         | 18 - 40 cm/s    |
| $h$          | 15 km             | 2,5 cm          |
| $V_{\text{max}}/ V_r$ | 1,3-11,3   | 1,1-2,7         |
| $L/R$        | 0,6 – 0,7         | 0,43            |
| $h/L$        | 0,11-1,7          | 0,25            |

The values of the dimensionless dynamic parameters of the Rossby-Obukhov Ro and Ekman E numbers of the central region of the hurricanes Inec, Hilda, Ellen [18] and laboratory movements are given in Table 2. Here, the order of the turbulent kinematic viscosity coefficient is $10^2$ m$^2$ / s for TC and $10^6$ m$^2$ / s for the laboratory model.

Table 2. The scale of the quantities characterizing the central region of the
TC and the parameters of the laboratory model.

| Parameters | Hurricane | Laboratory model |
|------------|-----------|------------------|
|            |  Ines     |  m= 3            |
| Ro         |  0,42     |  0,42            |
| E          |  3,14 10^-4 |  3,14 10^-4     |
| h/L        |  10,7 10^-1 |  2,5 10^-1      |
|            |  Hilda    |  m = 1           |
| Ro         |  0,63     |  0,68            |
| E          |  3,61 10^-4 |  3,61 10^-4     |
| h/L        |  7,9 10^-1 |  2,5 10^-1      |
|            |  Ellen    |  m= 2            |
| Ro         |  0,51     |  0,53            |
| E          |  3,5 10^-4 |  3,5 10^-4      |
| h/L        |  7,9 10^-1 |  2,5 10^-1      |

The comparison was carried out as follows. According to the parameters (E, R) estimated for the hurricanes considered in Table 2, there was a corresponding point on the experimental stability diagram, shown in article [19], which determines the mode number m of the system and the speed of rotation of the shear flows. The obtained relationships indicate that the dynamic instability of the region of maximum winds in hurricanes is reflected in the polygonal structure of the cyclone eye configuration. From the same positions, it is possible to consider the rain bands in hurricanes.

From the experimental stability diagram it follows that for a simultaneous discrete change in the magnitude of the velocity shift $\delta \omega$ and the rotation speed $\Omega$ of the overall system was observed two characteristic cases of instability. The increasing velocity shift, with increasing rotation, is a stabilizing factor preventing the increase in the number of vortices. The decrease in the intensity of the shift with increasing rotation acts as a destabilizing factor, contributing to the dissipation of energy entering the rotating system. The fact that the number of vortex structures decreases with an increase in the velocity shift and an increase in the total rotation from the literature is well known [20]. However, the results obtained by us indicate the following important fact. Conditions for the development of instability due to dissipation of energy by molecular motions that prevail over the generation of low-frequency hydrodynamic motions arise only at completely determined values of the ratio of the velocity shift to the width of the shear zone, i.e. by varying this ratio, it is possible to create situations in which intensification processes are developed in the system, characterized by the enlargement of the vortex structures and a decrease in their number. In this case, the nonlinear situations of perturbation development are limited by the range of variation of the ratio of the disturbance frequency to their amplitude. The latter circumstance means that an effective conversion of the external energy at a relatively low value of the total rotation can be maintained by increasing the spatial horizontal velocity shift.

Along with stationary rotation modes, modes of oscillations (transitions) from one polygonal configuration to another were detected. These $\Omega$ rotation modes were formed at the Ro and E numbers at the boundary of the stability curve. figure 3 shows a picture of the development of disturbances during the transformation of rotation modes from mode 4 to mode 3 and from 3 to 4.
Figure 3. The oscillations of the polygonal configuration of the eye of a model vortex between the rotational modes and the mode 4\to 3 \text{ and } 3\to 4.

To study the energy balance between the vortex structures forming behind the polygonal contour of the eye, the values of the specific kinetic energy $E_i$ were calculated from the measurements of the pulsation rate, their mean values $\overline{E}$ and the deviation from the average value $E_i - \overline{E}$ of the individual vortices during the observation time, which was 6 minutes. During this period, the eye contour along with the swirling vortices made about 40 revolutions. Figure 4 (a) shows a typical dependence of the specific kinetic energy $E_i$ of individual vortices (for $i = 1$) on time for different contour configurations. Further, for each pair $(i, n)$ of interacting vortices, the ratio of the deviation from the mean values was calculated:

$$
\sigma_{i,n} = \frac{E_i - \overline{E}}{E_n - \overline{E}}.
$$
Figure 4. The dependence of the kinetic energy of the first vortex behind the contour of the central part of the differential-rotating shallow fluid on time, the mode $m = 4$, (a); the time dependence of the deviations from the mean values of the kinetic energy ratios of the individual interacting vortex structures with the number $i = 1, 2, 3$, the mode $m = 3$, (b) and $i = 1, 2, 3, 4$, the mode $m = 4$, (c).

The time dependence of the deviations from the average values of the kinetic energy ratios of individual vortex structures $i = 1, 2, 3, 4$ beyond the polygonal configuration of the eye with modes $m = 3, 4$, shown in figure 4 (b, c), shows that in the process of evolution system there is an unbalanced energy exchange between its constituent parts. This may be the reason for the change in the polygonal configuration of the eye, when the outflow in a particular vortex will significantly exceed the influx of energy. In this case, the formation of asymmetric rotation modes with a different configuration of the eye contours is due to the predominant transfer of the pulsation energy to the mean motion. From the analysis of the results, it is impossible to draw a conclusion about the reasons for the imbalance of energy. Apparently, the non-equivalence of energy exchange is due to the dependence of the amount of energy contained in each vortex and its linear dimensions.

5. Conclusion

It is established that a variety of stable patterns of streamlines are formed on the surface of the fluid. They created in the central part of the visualizing particles a variety of figures with different indices of symmetry. Crests and troughs of waves were formed in the peripheral area of the rotating shallow fluid. Their axes were oriented along the radius or at some angle to it. The wave system was a chain of vortices with anticyclonic rotation clockwise. Their number coincided with the number of sides of polygonal structures rotating around the axis of the vessel. With a simultaneous discrete change in the magnitude of the velocity shift and the rotation speed of the system as a whole, two different cases of instability development were observed. An increase in the shear intensity led to a decrease in the number of vortices, and an increase in the total rotation and, correspondingly, a decrease in the width of the shear zone led to an increase in the number of vortices while maintaining the dimensionless wave number.

The balance of turbulent energy in model vortices is studied. It is shown that the formation of asymmetric rotation modes with different configurations of the eye contours is due to the predominant transfer of pulsation energy to the mean motion. During this experiment regimes of periodic transitions of the eye contours from one configuration to the other under constant conditions of excitation are experimentally discovered.

Acknowledgments
This research was supported by the Russian Ministry of Education and Science (agreement 14.613.21.0083, project ID RFMEFI61317X0083)

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