The Recurrent Temporal Discriminative Restricted Boltzmann Machines

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Abstract

The recurrent temporal restricted Boltzmann machine (RTRBM) has been successful in modelling very high dimensional sequences. However, there exists an unsolved question of how such model can be useful for discrimination tasks. The key issue of this is that learning is difficult due to the intractable log-likelihood. In this paper we propose the discriminative variant of RTRBM. We show that discriminative learning is easier than its generative counterparts where exact gradient can be computed analytically. In the experiments we evaluate RTDRBM on sequence modelling and sequence labelling which outperforms other baselines, notably the recurrent neural networks, hidden Markov models, and conditional random fields. More interestingly, the model is comparable to recurrent neural networks with complex memory gates while requiring less parameters.

1 Introduction

Modelling sequences is an important research topic with various applications in audio/music informatics, natural language processing and computer vision. While some work focus on synthesising time-series events [31, 27, 28], classification with sequential data also receives much attention [1, 20]. For such classification problem there are two questions of interest. The first one is known as sequence modelling which is concerned about how to predict the next events given the previous ones. This has been popular in natural language and musical domains [1, 4]. The second classification problem is how to label a sequence of events. This is referred to as sequence labelling and can be seen in wide range of applications such as optical character recognition (OCR) [30], part-of-speech (POS) tagging [29], name entity recognition [17]. In previous work, hidden Markov models [25], conditional random fields [29, 17], recurrent networks, and their variants have shown considerable success for both sequence modelling and sequence labelling. A recent sequence model named the recurrent temporal restricted Boltzmann machine (RTRBM) demonstrates that it can model very high dimensional sequences, i.e for generating look-like-real samples [27, 28].

RTRBM is constructed by rolling the restricted Boltzmann machines over time. Similar to recurrent neural networks (RNNs), RTRBM’s architecture has its hidden layer connected to itself by temporal weights. Besides that RTRBM have recurrent connections from the previous hidden state to the current observation (a.k.a visible state). The necessity of both temporal and recurrent connections, as has been explained in [28], is to reduce the difficulty of the inference since it is intractable if only temporal connections are available. Despite having connectionist architecture as RNN, RTRBM is a
probabilistic model which represents the distribution of sequences, similar as hidden Markov models [23]. This model has been shown being successful in motion simulation and modelling low resolution videos of bouncing balls [28, 18]. However, as far as we know there has not been any work to show how it can be applied for classification tasks. The key challenge here is that even though inference in RTRBM is “almost” tractable, it still problematic for learning RTRBM because the exact gradient can not be computed analytically. This necessitates the use of approximation algorithms.

In this paper, we show that different from generative learning of RTRBM, the discriminative learning of RTRBM is tractable. This is inspired by the use of RBM for classification [14, 26]. We call this discriminative variant of RTRBM as the recurrent temporal discriminative restricted Boltzmann machines (RTDRBM). The structure of RTDRBM is the same as RTRBM except that we split the visible layer to consist of input and class units. Also, for the sake of classification RTDRBM represents a conditional distribution rather than a joint distribution, similar as in conditional random fields [29, 17]. The key advantage of RTDRBM is that the log-likelihood function can be tractable and its gradient can be computed exactly. This helps the learning in RTDRBM is much easier than its generative counterpart. Furthermore, previous research have shown that discriminative learning is more effective than generative learning [19] with sufficient of training data. In the case of RBM, the former even requires less parameters [14].

For empirical evaluation, we show the effectiveness of RTDRBM in melody modelling, optical character recognition and POS tagging. Melody modeling is an interesting application which is closely related to language modelling [11]. Here we use the corpus of 7 datasets of folk melodies of different traditions, and one dataset of chorale melodies. The task is to model musical pitch sequences through prediction. The experimental results show that RTDRBM is better than other models such as n-gram [21], feed-forward neural network [3], restricted Boltzmann machines [4], recurrent neural networks. For the second application on OCR, we test RTDRBM on the MIT OCR dataset to show that it outperforms many models including multiclass SVM [7], structured SVM [32], max-margin Markov networks [30], averaged perceptron [6], an integration of search and learning algorithm (SEARN) [8], conditional random fields [13, 22], hidden Markov models [23], structured learning ensemble [20], and recurrent neural networks. Finally, for completeness we also compare RTDRBM with recurrent neural networks with complex memory gates, such as gated recurrent units (GRU) [5] and long short term memory (LSTM) [12]. We carry out the experiments on MIT OCR and Penn-Tree bank POS tagging datasets to show that in spite of having simpler structure RTDRBM still performs comparably with such complex models.

The remainder of the paper is organised as follow. In the next section, we review the theory behind the recurrent temporal restricted Boltzmann machines. After that we present the idea of extending discriminative representation in RBM for classification tasks to form the discriminative variant of RTRBM. In the Experiments section we perform empirical evaluation of our model, RTDRBM. The last section concludes the work and discusses future extensions.

2 The RTRBM

The recurrent temporal restricted Boltzmann machines [28] is a generative model for modelling high-dimensional sequences. RTRBM is constructed by rolling multiple RBMs over time while adding recurrent weights to ease the inference. In this section we first go over the salient properties of RBM and then reintroduce to RTRBM.

2.1 Restricted Boltzmann Machines

Restricted Boltzmann machines (RBMs) [25] is an undirected bipartite graphical model. It contains a set of visible units \( x \in \mathbb{R}^M \) and a set of hidden units \( h \in \mathbb{R}^N \) which make up its visible and hidden layers respectively. The two layers are fully inter-connected but there exist no connections between any two hidden units, or any two visible units. Additionally, the units of each layer are connected to a bias unit whose value is always 1. The edge between the \( i^{th} \) visible unit and the \( j^{th} \) hidden unit is associated with a weight \( w_{ij} \). All these weights are together represented as a weight matrix \( W \in \mathbb{R}^{M \times N} \). The weights of connections between visible units and the bias unit are contained in a visible bias vector \( b \in \mathbb{R}^M \). Likewise, for the hidden units there is a hidden bias vector \( c \in \mathbb{R}^N \). The RBM is characterised by an energy function:

\[
E(x, h) = -b^T x - c^T h - h^T W x .
\] (1)
The joint probability of the visible and hidden variables is modelled by the energy function above, given by:

\[
p(x) = \frac{1}{Z} \exp(-\mathcal{F}(x))
\]

where \(Z = \sum_x \exp(-\mathcal{F}(x))\) is the partition function, \(\mathcal{F}(x) = -\log \sum_h \exp(-E(x, h))\) is the free energy function. Learning in RBM can be carried out in a generative fashion by maximising the log-likelihood function of the joint probability distribution \(p(x)\). However, this learning is difficult because the exact gradient of the log-likelihood is intractable due to the complexity of the partition function. Therefore, approximation algorithm such as Contrastive Divergence is necessitated [10].

2.2 Recurrent Temporal Restricted Boltzmann Machine

We are now rolling (stacking) multiple RBMs over time to construct the recurrent temporal restricted Boltzmann machine (RTRBM) [28]. Note that this horizontal deep architecture is different from a deep network which stack RBMs vertically [11]. An RTRBM contains a sequence of RBMs, such that the RBM at time \(t\) is conditioned on itself at \((t - 1)\) through a set of time-dependent model parameters. In this paper we employ the generalised version of RTRBMs in [2], which considers the case where the visible and hidden layer biases \(b^{(t)}\) and \(c^{(t)}\) depend on the mean-field values of the hidden units \(\hat{h}^{(t-1)}\) at the previous time-step as follows:

\[
b^{(t)} = W_{hx} \hat{h}^{(t-1)} + b
\]
\[
c^{(t)} = W_{hh} \hat{h}^{(t-1)} + c
\]

This RTRBM is characterised by a set of parameters includes: \(W, W_{hh}, W_{hx}, \hat{h}^{(0)}, b\) and \(c\). Here \(W\) denotes the undirected weights between the visible and the hidden layers of the constituent RBM, \(W_{hh}\) and \(W_{hx}\) the directed weights between the hidden layer at time \((t - 1)\) and the hidden and visible layers at time \(t\) respectively, and \(\hat{h}^{(0)}\) is a vector of initial mean-field values of the hidden units. Here as well \(b\) and \(c\) are respectively the time-invariant visible and hidden layer biases of the RBM. The joint distribution of a sequence according to this model is formalised as:

\[
p(x^{(1:T)}, h^{(1:T)}) = \prod_t p(x^{(t)}|h^{(t-1)})p(h^{(t)}|x^{(t)}, h^{(t-1)}) .
\]

An interesting feature that makes RTRBM effective in modelling high dimensional sequences is that \(\hat{h}^{(t)}\) can be inferred exactly as:

\[
\hat{h}^{(t)} = \sigma(Wx^{(t)} + c^{(t)}) = \sigma(Wx^{(t)} + W_{hh} \hat{h}^{(t-1)} + c)
\]

Learning in RTRBM involves sampling of all variables for approximation of the gradients since its log-likelihood is intractable, as in the case of RBM. With the ease of the inference mentioned above one can combine Contrastive Divergence and backpropagation through time to efficiently estimate the model’s parameters [28, 2].

3 The RTDRBM

3.1 Graphical Structure

An RBM models the joint probability \(p(x)\) of the set of variables \(x\) represented in its visible layer. For classification one is often interested in explicitly learning a conditional distribution of the form \(p(y|x)\). This can be done in the RBM by assuming one subset of its visible units to be inputs \(x\), and the remaining a set of multinomial units \(y\) representing the class-conditional probabilities \(p(y|x)\) [14]. The RTDRBM is constructed by rolling this classification RBM over time as depicted in Figure 1. One will notice that it is identical in structure to RTRBM described in the previous section, where the
visible and hidden layers of the RBM at time $t$ are conditioned on the mean-field values of the hidden layer at time $(t - 1)$. The main motivation for proposing RTDRBM here is to learn the distribution $p(y^{(1:T)}|x^{(1:T)})$ over a sequence of input-label pairs $\{x^{(1:T)}, y^{(1:T)}\}$, rather than the joint probability of the entire sequence $p(y^{(1:T)}, x^{(1:T)})$ like RTRBM.

Figure 1: A graphical depiction of RTDRBM

Here, $x$ is the input vector, and $y$ is a vector representing a class-label in which all values are set to 0 except at the position corresponding to a label $y$, which is set to 1. It is worth noting that the structure in Figure 1 is less detailed than one in [28] as it does not depict the binary hidden layer $H'$. In fact, this structure is similar to the generalised version of RTRBM in [2].

### 3.2 Inference

With the existence of class variables the inference in RTDRBM is slightly different from that in RTRBM as we need to infer the state of hidden layer while, at the same time, performing prediction. As has been shown earlier in previous section, inference of hidden layer is easy given the state of visible layer. Similarly, in this RTDRBM once the class layer is known then one can infer the hidden layer straightforward. We now show that inference of class layer is also easy given the state of hidden layer. In particular, from mean-field values of the hidden layer in previous time step we can infer the class units using the following conditional distribution:

$$p(y^{(t)}|x^{(t)}, \hat{h}^{(t-1)}) = \frac{\exp \left( -F(x^{(t)}, y^{(t)}, \hat{h}^{(t-1)}) \right)}{\sum_{y'} \exp \left( -F(x^{(t)}, y', \hat{h}^{(t-1)}) \right)}$$  \hfill (7)

with

$$F(x^{(t)}, y, \hat{h}^{(t)}) = -b_y \sum_j \log(1 + \exp(w_j^T x^{(t)} + u_{jy} + c_j))$$  \hfill (8)

where $w_j$ is the $j^{th}$ column of the weight matrix $W$ between the hidden units and the units of the visible layer corresponding to the inputs, $u_{jy}$ is an element of the weight matrix $U$ between the hidden units and the units in the visible layer corresponding to the class-labels. For simplicity we only need the visible bias for label units which is denoted in this section as: $b^{(t)} = W_{hy} \hat{h}^{(t-1)} + b$.

From (7) we can predict the value of the class. Once $y^{(t)}$ is known we use it to infer the mean-field values $\hat{h}^{(t)}$ as:

$$\hat{h}^{(t)} = \sigma(W x^{(t)} + U y^{(t)} + W_{hh} \hat{h}^{(t-1)} + c^{(t)})$$  \hfill (9)

Starting from $t = 1$, the conditional distribution $p(y^{(1)}|x^{(1)}, \hat{h}^{(0)})$ can be computed exactly by marginalising out the hidden variable $h^{(1)}$ while having $\hat{h}^{(0)}$ as parameters. In order to pass the uncertainty of the prediction in previous step to the next step we do not use the predicted value of $y^{(1)}$, instead we use the distribution to infer the mean-field value $\hat{h}^{(1)}$. This, then would be passed to next prediction step, and so on. The details of inference algorithm with RTDRBM is described as below.

### 3.3 Learning

With RTDRBM we are usually interested in learning the conditional distribution as follow:

$$p(y^{(1:T)}|x^{(1:T)}) = \frac{p(x^{(1:T)}, y^{(1:T)})}{\sum_y p(x^{(1:T)}, y^{(1:T)})}$$  \hfill (10)
Algorithm 1 Inference with RTDRBM

Data: Input: $x^{(1:T)}$

Result: Output: $y^{(1:T)}$

for $t = 1 : T$
do
  set $\hat{y}^{(t)} = p(y^{(t)}|x^{(t)}, \hat{h}^{(t-1)})$
  set $y^{(t)} = \arg \max_h y^{(t)}$
  set $\hat{h}^{(t)} = \sigma(Wx^{(t)} + U\hat{y}^{(t)} + Wh\hat{h}^{(t-1)} + c^{(t)})$
end

However, it is difficult to marginalise out all hidden variables $h^{(1:T)}$ to compute exactly the distribution. Moreover, the complexity would increase exponentially with the length of the sequence due to the need to sum over all possible classes at every time step. So, instead of computing the distribution directly we simplify it by marginalising out the hidden variable at each time $t$ using the expectation of the hidden state at previous step $t - 1$. Let us consider:

$$p(x^{(1:T)}, y^{(1:T)}) = \sum_{h^{(1:T)}} p(y^{(1:T)}, x^{(1:T)}, h^{(1:T)})$$

$$= \sum_{h^{(1:T)}} \prod_{t=1}^T p(y^{(t)}, x^{(t)}, h^{(t)}|h^{(t-1)})$$

(11)

If we first compute the expectation of $h^{(t-1)}$ given the previous input states $x^{(1:t-1)}$ and $y^{(1:t-1)}$, which is equivalent to minimising the total energy function of RTDRBM, then we have:

$$q(x^{(1:T)}, y^{(1:T)}) = \prod_{t=1}^T p(y^{(t)}, x^{(t)}|\hat{h}^{(t-1)})$$

(12)

One can see that as an expectation step that would be followed by an optimisation step which maximises the log-likelihood of this simplified distribution:

$$q(y^{(1:T)}|x^{(1:T)}) = \frac{\prod_{t=1}^T p(y^{(t)}, x^{(t)}|\hat{h}^{(t-1)})}{\sum_{y^{(1:T)}} \prod_{t=1}^T p(y^{(t)}, x^{(t)}|\hat{h}^{(t-1)})}$$

$$= \frac{\prod_{t=1}^T p(y^{(t)}, x^{(t)}|\hat{h}^{(t-1)})}{\prod_{t=1}^T \sum_{y^{(t)}} p(y^{(t)}, x^{(t)}|\hat{h}^{(t-1)})}$$

(13)

Since $p(y^{(t)}|x^{(t)}, \hat{h}^{(t-1)})$ is tractable as we show in (7), we can compute the above distribution exactly. Now one can thus learn the model by maximising the log-likelihood function:

$$\ell = \sum_{y^{(1:T)}, x^{(1:T)}} \sum_{t=1}^T \log q(y^{(t)}|x^{(t)}, \hat{h}^{(t-1)})$$

(14)

Similar to other time-series connectionist models such as RNN and RTRBM, we train the model using back-propagation through time [33]. The update of the model parameters is shown below. We use $\theta$ as general denotation of the parameters.

$$\nabla \theta = \sum_{t=1}^T \frac{\partial \log p(y^{(t)}|x^{(t)}, \hat{h}^{(t-1)})}{\partial \theta}$$

$$+ \sum_{t=1}^{T-1} \frac{\partial \hat{A}^{(t)}}{\partial \hat{h}^{(t)}} \frac{\partial A}{\hat{h}^{(t)}} + \frac{\partial \log p(y^{(1)}|x^{(1)}, \hat{h}^{(0)})}{\partial \hat{h}^{(0)}}$$

(15)
where
\[
\frac{\partial A}{\partial \mathbf{h}^{(t)}} = W^T_{hh} \mathbf{h}^{(t+1)} (1 - \mathbf{h}^{(t+1)}) \frac{\partial A}{\partial \mathbf{h}^{(t+1)}} + \frac{\partial \log p(y^{(t+1)} | x^{(t+1)}, \mathbf{h}^{(t)})}{\partial \mathbf{h}^{(t)}}
\]  \hspace{1cm} (16)

Besides the tractability issue, another motivation for the need of a discriminatively learned variant of RTRBM is the desire for better classification performance. It has been shown in [19] that with sufficient training examples, a discriminative learning tends to do better on the task it is optimized for than its generative counterpart in the same model. Such inspiration has been reinforced by the findings in [14]. This work confirms specifically that discriminative learning of RBM is not only more efficient in terms of having less parameters but also better for a classification task than learning RBM generatively.

4 Experiments

4.1 Music Modelling

The task of music prediction addressed here has been seen more generally as sequence modelling which also includes building language models [15]. The goal here is to model the joint probability distribution of subsequences of words (music pitches) occurring in a language \( S \). A statistical language model (SLM) can be represented by the conditional probability of the next word \( s^{(T)} \) given all the previous ones \( [s^{(1)}, \ldots, s^{(T-1)}] \) (written \( s^{(1:T-1)} \)). In previous work, the most commonly used SLMs are \( n \)-gram models. Their properties are based on an assumption that the probability of a word in a sequence depends only on the a fixed number of immediately preceding words [15]. The group of these words is called context and its length is called context-length.

The melody modelling task treats notes in a monophonic melody analogous to words in the above language example. This is inspired by a similar analogy was made between sequences of characters in the English language and notes in music. We use an event-based representation of music, where the occurrence of each note is treated as a musical event. Much in the same way as an SLM, a system for music prediction models the conditional distribution \( P(s^{(t)} | s^{(1:t-1)}) \) given a sequence \( s^{(1:T)} \) of musical events [21][4] from a musical language \( S \), such that \( s^{(t)} \in [S] \), where \([S]\) is the set of symbols (musical pitch values) in \( S \). The cardinality of \( S \) is given by \(|S|\). For each prediction, context information is obtained from the events \( s^{(t+i+1)} \) immediately preceding \( s^{(t)} \). The case where \( l = t \) corresponds to a model which takes the entire sequence from its beginning into account while making a prediction. In order to encode the task to RTRBM, the one-hot encoding of the musical event \( s^{(t)} \) is substituted by the label \( y^{(t)} \) and the most recent event from the context \( s^{(t-1)} \) are substituted by \( x^{(t)} \).

4.1.1 Dataset

The corpus considered here contains 7 datasets of folk melodies of different traditions, and one dataset of chorale melodies. The data in this corpus can be obtained in the **kern format from the Centre for Computer Assisted Research in Humanities** at Stanford University, California [http://www.ccarh.org](http://www.ccarh.org) and the **Music Cognition Laboratory** at Ohio State University [http://kern.humdrum.net](http://kern.humdrum.net).

The datasets comprise the corpus are as follows (a summary is available in Table 1). Six of these are from the Essen Folk Song Collection (EFSC) [24] which consists of folk melodies, mostly from Europe and China, with a few from other parts of the world. The present work uses a subset of the EFSC containing 119 Yugoslavian folk melodies, 91 Alsatian folk melodies, 93 Swiss folk melodies, 104 Austrian folk melodies, 213 German folk melodies (dataset kinder), and 237 Chinese folk melodies. The number of predictable musical events in these ranges between between 2,691 for the smallest one (Yugoslavian folk songs) and 11,056 for the largest (Chinese folk melodies). Another subset of the corpus consists of 152 folk songs and ballads from Nova Scotia, Canada. And the rest are 185 chorale melodies harmonized by J. S. Bach (BWV 253 to BWV 438).
| Dataset                  | \(n_{melodies}\) | \(n_{events}\) | \(|S|\) |
|-------------------------|------------------|----------------|-------|
| Yugoslavian folk songs  | 119              | 2,691          | 25    |
| Alsatian folk songs     | 91               | 4,496          | 32    |
| Swiss folk songs        | 93               | 4,586          | 34    |
| Austrian folk songs     | 104              | 5,306          | 35    |
| German folk songs       | 213              | 8,393          | 27    |
| Canadian folk songs     | 152              | 8,553          | 25    |
| Chorale melodies        | 185              | 9,227          | 21    |
| Chinese folk songs      | 237              | 11,056         | 41    |

Table 1: The folk and chorale melody datasets used for evaluation in the melody modelling task with their respective total number of melodies, predictable events and number of prediction classes.

4.1.2 Evaluation Measure

For the evaluation we use cross entropy to measure the mean divergence between the entropy calculated from the predicted distribution and the correct prediction label. Given a test data \(D_{test}\), the cross entropy can be computed over all the events belonging to different sequences as:

\[
H_{c}(p_{mod}, D_{test}) = - \frac{\sum_{s \in D_{test}} \sum_{t=1}^{T_s} \log_2 p_{mod}(s^{(t)} | s^{(1:t-1)})}{\sum_{s \in D_{test}} T_s}
\]  

where \(p_{mod}\) is the probability assigned by the model to the pitch of the musical event \(s^{(t)}\) in the melody \(s \in D_{test}\) given its preceding context, and \(T_s\) is the length of \(s\).

4.1.3 Baselines

We compare the performance of the proposed RTDRBM with the state-of-the-art approaches on this data that includes RTRBM and five other related connectionist models. The details of these models are as follows:

- Feed-forward neural network (FNN): A FNN contains a single hidden layer and a softmax output layer. The model predicts a distribution of the future pitch in its output layer given the context in the input [3].
- Restricted Boltzmann machine (RBM): RBM have been shown very effective in melody modelling [4]. In this case, we apply two styles of learning: generative and discriminative. The first uses the Contrastive Divergence (CD) algorithm [10], an approximation method which work effectively in practice. The second employs discriminative approach as in [14].
- \(n\)-gram Models: We use two best models from [21], namely C2I and C*I. The first one is a bounded order \(n\)-gram model of order 2 which employs Witten-Bell interpolated smoothing with update exclusion. The other is the unbounded order \(n\)-gram.
- Recurrent neural network (RNN): The RNN considered here contains a single hidden layer with hyperbolic tangent units and a set of softmax output units. The model is learned on entire melodies using backpropagation through time [35].

4.1.4 Experimental Results

For fair comparison we partition the data as the same as in previous work [21] [4]. In particular, a resampling method is employed for evaluation on the 8 datasets [9]. For model selection, we use a small part of the training set (5%) in each case as the validation set. In order to determine the most effective hyper-parameters for a model a grid-like search was carried out. Such hyper-parameters include the learning rate (denoted as \(\eta\)), the regularization (both \(L_1\) and \(L_2\), denoted as \(\lambda_1\) and \(\lambda_2\) respectively) and the number of hidden units (denoted as \(n_{hid}\)). For each of the models, the learning rate \(\eta\) was searched from 0.0001 to 0.1 in log-scale, and \(n_{hid}\) was selected among \(\{10, 25, 50, 100, 200, 500, 1000\}\). For simplicity, both \(L_1\) and \(L_2\) hyper-parameters were set to identical values, i.e. \(\lambda_1 = \lambda_2 = \lambda\), and were from 0.00001 to \(\lambda = 0.001\) in log-scale plus the “off”
Table 2: Predictive performance of different models in the evaluation.

| Model         | Context | Cross Entropy (bits) |
|---------------|---------|----------------------|
| RTDRBM \(n_h = 100, h_{act} = \text{LogSigU}\) | N/A     | 2.712                |
| RTRBM \(n_h = 100, h_{act} = \text{LogSigU}\) | N/A     | 2.738                |
| RNN \(n_h = 100, h_{act} = \text{TanSigU}\)       | N/A     | 2.787                |
| DRBM \(n_h = 25, h_{act} = \text{LogSigU}\)       | 5       | 2.819                |
| FNN \(n_h = 200, h_{act} = \text{ReLU}\)          | 7       | 2.878                |
| n-gram (u)    | N/A     | 2.878                |
| n-gram (b)    | 2       | 2.957                |

value \((\lambda = 0)\). We apply this grid-like search for the models in the experiments. If there no apparent optima is found we expand the search. We also perform the search over different types of hidden units in models where applicable.

Table 2 contains the best predictive performance of each of the connectionist models (both non-recurrent and recurrent) considered here, together with the equivalent n-grams from (Pearce and Wiggins, 2004). The results are averaged across all 8 datasets. The table show the progressive improvement in the best-case performance from the n-gram models, to the non-recurrent and recurrent connectionist models, with RTDRBM outperforming the rest. The main influencing factor with regard to the predictive performance of the different classes of models was the hidden layer size.

It was found both in the case of RTRBM and RTDRBM that a hidden layer size of 100 units resulted in the best predictive performance. In the case of the RNNs, there was an additional factor which influenced the performance of the models, namely the activation type of the hidden units. A paired t test between the two confirmed that the difference in performance was indeed significant \[ t(79) = 5.54, p < 0.001 \]. This was also the case for the difference in performance between the RNN and RTRBM \[ t(79) = 5.03, p < 0.001 \].

Table 3: Cross entropies of the best instance of each model class on the different datasets, and the average of these values in the bottom row. The rightmost column lists the average cross-entropy of all the models on each of the datasets.

| Dataset        | FNN | RBM | DRBM | RNN | RTRBM | RTDRBM | Average |
|----------------|-----|-----|------|-----|-------|--------|---------|
| jugoslav       | 2.715 | 2.722 | 2.705 | 2.681 | 2.676 | 2.655 | 2.692 |
| elsass         | 3.054 | 3.002 | 3.060 | 2.970 | 2.945 | 2.897 | 2.988 |
| schweiz        | 3.035 | 3.024 | 3.017 | 2.988 | 2.961 | 2.932 | 2.993 |
| oesterrh       | 3.365 | 3.310 | 3.418 | 3.309 | 3.296 | 3.259 | 3.326 |
| kinder         | 2.389 | 2.377 | 2.398 | 2.424 | 2.313 | 2.301 | 2.367 |
| nova-scotia    | 2.750 | 2.727 | 2.738 | 2.679 | 2.635 | 2.609 | 2.690 |
| bach           | 2.420 | 2.441 | 2.423 | 2.451 | 2.378 | 2.362 | 2.413 |
| shanxi         | 2.827 | 2.798 | 2.795 | 2.792 | 2.703 | 2.685 | 2.767 |
| Average        | 2.819 | 2.799 | 2.819 | 2.787 | 2.738 | 2.712 |

Table 3 lists the performance of best of each class of models on each of the datasets. One will notice a consistent trend across different models in the best possible cross entropies achievable by each of them on the datasets. Given a dataset, the prediction cross entropy can be used to compare the performance of different models. On the other hand, given a model, the best achievable cross entropy on a given dataset by it sheds light into the nature of the dataset as well. For instance, the greater the number of symbols the greater the number of permutations of these that are possible in the sequences that make up the dataset. Those of the permutations that appear in the melodies are determined by style or musical tradition represented by the dataset.

4.2 OCR

4.2.1 Dataset

The MIT OCR dataset\[^3\] is a widely used benchmark dataset for evaluating sequence labelling algorithms. It contains 6,877 words, which corresponds to 52,152 English characters. The data

\[^3\]http://www.seas.upenn.edu/~taskar/ocr/
are sequences of isolated characters, where each character is represented by a binary 128-dimensional vector (of $16 \times 8$ images) and belongs to one of 26 classes. The dataset is divided into 10 folds of training, validation and testing samples. We select the model based on its performance on the validation sets and report the average accuracy of test sets from all folds.

### 4.2.2 Baseline Models

We compare the performance of RTDRBM on the sequence labelling task with the 8 baseline models, including:

- Multiclass support vector machines (SVM-multiclass) \[7\].
- Structured support vector machines (SVM-struct) \[32\].
- Max-margin Markov network (M3N) \[30\].
- Averaged Perceptron \[6\].
- Search-based structured prediction model, a.k.a SEARN \[8\].
- Conditional random field (CRF) \[13, 22\].
- Hidden Markov model (HMM) \[23\].
- Structured Learning Ensemble (SLE) \[20\] which selects and combines predictions by a subset of all the models above.

### 4.2.3 Evaluation Measure

In this task, each model is expected to predict the correct label corresponding to the image of a character. All the models in this task are evaluated using the average loss per sequence $E(y, y^*)$, given by:

$$E(y, y^*) = \frac{1}{N} \sum_{i=1}^{N} \left[ \frac{1}{L_i} \sum_{j=1}^{L_i} I((y_i)_j \neq (y^*_i)_j) \right]$$

where $y$ and $y^*$ are the predicted and the true sequence respectively, $N$ is the total number of test examples, $L_i$ is the length of the $i^{th}$ sequence, and $I$ is the $0-1$ loss function.

### 4.2.4 Results

In order to determine the best model for the task, a grid-like search was carried out. For fair comparison, we use the best results of the baselines (except RNN) reported in \[20\] on the same dataset. For RTDRBM and RNN the hyper-parameters include: the learning rate and the number of hidden unit. The former was searched from $0.00001$ to $1$ in log scale while the latter was searched within $\{10, 20, 50, 100, 500, 1000, 2000\}$. Note that the search was extended if there is no apparent optima is found. We also use early stopping in which the performance of the model on a validation set was determined after every 10 epochs. If the performance in one of these checks happened to be worse than the previous best one for 5 consecutive checks the training was stopped. In the sequence learning, stochastic gradient descent with backpropagation through time was used. Table 4 shows the comparative performance of RTDRBM. It is evident that RTDRBM clearly outperforms the other models by a large margin.

### 4.3 RTDRBM vs LSTM & GRU

For completeness, we now compare our model with different recurrent neural networks, especially with ones having complex memory gates in hidden units such as gated recurrent units (GRU) and long short term memory (LSTM). We use a traditional RNN with tanh hidden units (denoted as RNN), HMMs, and CRFs as baselines. The additional weights of GRU and LSTM over RNN are $2N(M+N+1)$ and $3N(M+N+1)$ respectively, where $M$ is the dimension of visible layer and $N$ is the dimension of the hidden layer. In RTDRBM we only need $NK$ more weights than RNN, with $K$ is the class size (we omit the parameter $\hat{h}^{(0)}$). It is worth noting that the class size is usually much smaller than the dimension of hidden layer and therefore we need much less parameters for RTDRBM in comparison with GRU and LSTM.
Table 4: Test errors of different models in the evaluation.

| Model  | Error (%) |
|--------|-----------|
| RTDRBM | 15.46     |
| RNN    | 22.92     |
| Ensemble | 20.58  |
| SVM-struct | 21.16 |
| HMM    | 23.70     |
| M3N    | 25.08     |
| Perceptron | 26.40 |
| SEARN  | 27.02     |
| SVM-multiclass | 28.54 |
| CRF    | 32.30     |

Table 5: Test errors of hidden Markov model, conditional random field, RNN: recurrent neural network with \( \tanh \) hidden units, GRU: recurrent neural network with gated recurrent units, LSTM: recurrent neural network with long short term memory units.

|       | OCR   | POS500 | POS1000 | POS2000 |
|-------|-------|--------|---------|---------|
| HMM   | 23.70 | 23.460 | 19.950  | 17.960  |
| CRF   | 32.30 | 16.530 | 12.510  | 09.840  |
| RNN   | 22.92 | 12.220 | 10.480  | 09.207  |
| GRU   | 15.85 | 11.663 | 09.892  | 09.011  |
| LSTM  | 13.33 | 12.012 | 10.168  | 09.273  |
| RTDRBM| 15.46 | 12.119 | 10.061  | 09.178  |

We test both GRU and LSTM on the OCR dataset above. Furthermore, we also perform experiments with POS tagging dataset from Penn Treebank [16]. The data is partitioned into different training sets of 500, 1000, and 2000 samples. The models are selected by using a held-out 10% samples of training sets which then are evaluated in a test set of \( \sim 1600 \) sentences. The challenge of this data is that the lexical features are very large, around 450,000. The results are shown in Table 5. We can see that in both case of OCR and POS datasets RTDRBM outperforms RNNs, CRFs, and HMMs while having comparable performance as GRU and LSTM. In particular, RTDRBM is better than GRU in the OCR dataset and better than LSTM in the POS dataset. The LSTM seems to beat our RTDRBM in OCR dataset, however it needs 2000 hidden units for that best result while RTDRBM only requires 100 hidden units.

5 Conclusions

In this paper, we show how to use the recurrent temporal discriminative restricted Boltzmann machine for classification with sequences. The model is constructed by rolling restricted Boltzmann machines with class layer over time. Inference with RTDRBM is done by performing prediction of the class and computing mean-field values of hidden layer at each time \( t \), consecutively. We learn RTDRBM by first compute the expectation of hidden units at time \( t - 1 \) for each element of the conditional distribution and then maximising the simplified distribution which is tractable. In the experiments, we show the effectiveness of RTDRBM in sequence modelling with a melody dataset and in sequence labelling with OCR and POS datasets. We also show the simplicity advantage of RTDRBM over GRU and LSTM, two types of recurrent neural networks with complex memory gates in hidden units.

This paper serves as proof-of-concept of a time-series connectionist model for classification task. Further study on inference and learning are left for future work which are hoped for more improvement. It would be also interesting to see how this model can be applied in other application domains, especially with such modifications as: stack of more hidden layers, bidirectional variants, and having more complex memory gates.

References

[1] Yoshua Bengio, Rejean Ducharme, Pascal Vincent, and Christian Jauvin. A Neural Probabilistic Language Model. Journal of Machine Learning Research, 3:1137–1155, 2003.
[2] Nicolas Boulanger-Lewandowski, Yoshua Bengio, and Pascal Vincent. Modeling Temporal Dependencies in High-Dimensional Sequences: Application to Polyphonic Music Generation and Transcription. In \textit{International Conference on Machine Learning (ICML)}, pages 1159–1166, 2012.

[3] John S Bridle. Probabilistic interpretation of feedforward classification network outputs, with relationships to statistical pattern recognition. In \textit{Neurocomputing}, pages 227–236. 1990.

[4] Srikanth Cherla, Tillman Weyde, Artur S d’Avila Garcez, and Marcus Pearce. A Distributed Model for Multiple-Viewpoint Melodic Prediction. In \textit{International Society for Music Information Retrieval Conference}, pages 15–20, 2013.

[5] Kyunghyun Cho, Bart van Merrienboer, Caglar Gulcehre, Dzmitry Bahdanau, Fethi Bougares, Holger Schwenk, and Yoshua Bengio. Learning phrase representations using rnn encoder-decoder for statistical machine translation. In \textit{Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing}, pages 1724–1734, 2014.

[6] Michael Collins. Discriminative Training Methods for Hidden Markov Models: Theory and Experiments with Perceptron Algorithms. In \textit{ACL Conference on Empirical Methods in Natural Language Processing}, pages 1–8. Association for Computational Linguistics, 2002.

[7] Koby Crammer and Yoram Singer. On the Algorithmic Implementation of Multiclass Kernel-based Vector Machines. \textit{The Journal of Machine Learning Research}, 2:265–292, 2002.

[8] Hal Daumé III, John Langford, and Daniel Marcu. Search-based Structured Prediction. \textit{Machine learning}, 75(3):297–325, 2009.

[9] Thomas G Dietterich. Approximate Statistical Tests for Comparing Supervised Classification Learning Algorithms. \textit{Neural computation}, 10(7):1895–1923, 1998.

[10] Geoffrey E Hinton. Training Products of Experts by Minimizing Contrastive Divergence. \textit{Neural Computation}, 14(8):1771–1800, 2002.

[11] Geoffrey E. Hinton, Simon Osindero, and Yee-Whye Teh. A fast learning algorithm for deep belief nets. \textit{Neural Comput.}, 18(7):1527–1554, July 2006.

[12] Sepp Hochreiter and Jürgen Schmidhuber. Long short-term memory. \textit{Neural Comput.}, 9(8):1735–1780, November 1997.

[13] John Lafferty, Andrew McCallum, and Fernando CN Pereira. Conditional Random Fields: Probabilistic Models for Segmenting and Labeling Sequence Data. In \textit{International Conference on Machine Learning}, pages 282–289, 2001.

[14] Hugo Larochelle and Yoshua Bengio. Classification using Discriminative Restricted Boltzmann Machines. In \textit{International Conference on Machine Learning}, pages 536–543, 2008.

[15] Christopher D Manning and Hinrich Schütze. \textit{Foundations of Statistical Natural Language Processing}. MIT Press, 1999.

[16] Mitchell P Marcus, Mary Ann Marcinkiewicz, and Beatrice Santorini. Building a Large Annotated Corpus of English: The Penn Treebank. \textit{Computational linguistics}, 19(2):313–330, 1993.

[17] Andrew McCallum and Wei Li. Early results for named entity recognition with conditional random fields, feature induction and web-enhanced lexicons. In \textit{Proceedings of the Seventh Conference on Natural Language Learning}, pages 188–191, Stroudsburg, USA, 2003.

[18] Roni Mittelman, Benjamin Kuipers, Silvio Savaresi, and Honglak Lee. Structured recurrent temporal restricted boltzmann machines. In Eric P. Xing and Tony Jebara, editors, \textit{International Conference on Machine Learning}, volume 32, pages 1647–1655, Beijing, China, 22–24 Jun 2014.

[19] Andrew Y Ng and Michael I Jordan. On Discriminative vs. Generative Classifiers: A Comparison of Logistic Regression and Naive Bayes. In \textit{Advances in Neural Information Processing Systems}, pages 841–848, 2001.

[20] Nam Nguyen and Yunsong Guo. Comparisons of Sequence Labeling Algorithms and Extensions. In \textit{International Conference on Machine Learning}, pages 681–688. ACM, 2007.

[21] Marcus Pearce and Geraint Wiggins. Improved Methods for Statistical Modelling of Monophonic Music. \textit{Journal of New Music Research}, 33(4):367–385, 2004.
[22] Fuchun Peng and Andrew McCallum. Information extraction from research papers using conditional random fields. *Information processing & management*, 42(4):963–979, 2006.

[23] Lawrence R. Rabiner. Readings in speech recognition. chapter A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition, pages 267–296. San Francisco, USA, 1990.

[24] Helmut Schaffrath and D Huron. The Essen Folksong Collection in the Humdrum Kern Format. 1995.

[25] Paul Smolensky. Parallel Distributed Processing: Explorations in the Microstructure of Cognition, vol. 1. chapter Information Processing in Dynamical Systems: Foundations of Harmony Theory, pages 194–281. MIT Press, 1986.

[26] Cherla Srikanth, Tran Son, N., Weyde Tillman, and Garcez Artur. Generalising the discriminative restricted boltzmann machines. In *International Conference on Artificial Neural Networks*, 2017.

[27] Ilya Sutskever and Geoffrey E. Hinton. Learning multilevel distributed representations for high-dimensional sequences. In *AISTATS*, volume 2 of *JMLR Proceedings*, pages 548–555, 2007.

[28] Ilya Sutskever, Geoffrey E Hinton, and Graham W Taylor. The Recurrent Temporal Restricted Boltzmann Machine. In *Advances in Neural Information Processing Systems (NIPS)*, pages 1601–1608, 2009.

[29] Charles Sutton and Andrew McCallum. An introduction to conditional random fields. *Found. Trends Mach. Learn.*, 4(4):267–373, April 2012.

[30] Ben Taskar, Carlos Guestrin, and Daphne Koller. Max-margin Markov Networks. *Advances in neural information processing systems*, 16:25, 2004.

[31] Graham W. Taylor, Geoffrey E Hinton, and Sam T. Roweis. Modeling human motion using binary latent variables. In *Advances in Neural Information Processing Systems 19*, pages 1345–1352. 2007.

[32] Ioannis Tsochantaridis, Thorsten Joachims, Thomas Hofmann, and Yasemin Altun. Large Margin Methods for Structured and Interdependent Output Variables. In *Journal of Machine Learning Research*, pages 1453–1484, 2005.

[33] Paul J Werbos. Backpropagation Through Time: What It Does and How to Do It. *Proceedings of the IEEE*, 78(10):1550–1560, 1990.