Scalar Electrodynamics in Framework of Randall-Sundrum Model

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Considering the Randall-Sundrum background, we calculate the total cross-section for $\phi\phi^* \to G\gamma$ in the framework of the scalar electrodynamics.

I. INTRODUCTION

In [1,2], Randall and Sundrum (RS) proposed a new approach to extra dimensions for space-time to solve the hierarchy problem. In their model, it is assumed that there exists only one extra space-like dimension which is taken homeomorphic with an orbifold $S^1/Z_2$. This orbifold has two fixed points at $\varphi = 0$ and $\varphi = \pi$. At each fixed point, they put a 4-dimensional brane world. One of them which is located at $\varphi = \pi$ is called the visible world and is assumed we are living on it, and the other one is called hidden world.

In RS method same as the Arkani-Hamed, Dimopoulos and Dvali (ADD) approach [3,4], it is assumed that except the graviton (and also axions) all the Standard Model (SM) fields are confined in these two distinct worlds. The physical laws are the same on these two worlds but the masses and the coupling constants may differ.

In this model, the classical action is assumed to be:

\begin{equation}
S = S_{\text{gravity}} + S_{\text{vis}} + S_{\text{hid}},
\end{equation}

\begin{align*}
S_{\text{gravity}} &= \int d^4x \int d\varphi \sqrt{-G} \{ \Lambda + 2MR \}, \\
S_{\text{vis}} &= \int d^4x \sqrt{-g_{\text{vis}}} \{ \mathcal{L}_{\text{vis}} - V_{\text{vis}} \}, \\
S_{\text{hid}} &= \int d^4x \sqrt{-g_{\text{hid}}} \{ \mathcal{L}_{\text{hid}} - V_{\text{hid}} \},
\end{align*}

(1.1)

where $M$ and $\Lambda$ are the 5-dimensional Planck mass and the cosmological constant respectively, and $\mathcal{L}_{\text{vis}}$ ($\mathcal{L}_{\text{hid}}$) is the SM or any effective Lagrangian corresponding to matter and force fields except the gravity. The $V_{\text{vis}}$ and $V_{\text{hid}}$ are
vacuum expectations on the branes.

The classical solution of Einstein equation for the mentioned action is the following metric:

$$ds^2 = e^{-2\sigma(\phi)} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2,$$

where $\phi$ and $r_c$ are the coordinate and radius of $S^1$ in the orbifold $S^1/\mathbb{Z}_2$ and $\kappa = \sqrt{-\Lambda/24\pi M}$. By integrating out the fifth dimension, the coupling constant of the effective 4-dimensional action yields the 4-dimensional Planck scale (1.4):

$$M_P = \frac{M^3}{\kappa} \left(1 - e^{-2\kappa r_c \pi}\right).$$

It is found that a field on visible brane with the fundamental mass parameter $m_0$ will appear to have a physical mass $m = m_0 e^{-\kappa r_c \pi}$. By taking $\kappa r_c \simeq 12$, the observed scale hierarchy reproduces naturally by exponential factor and no additional large hierarchies arise [1,2].

At this stage, it is natural to search for any observable effects of this extra fifth dimension in RS model. Many efforts have been done to probe the effects of this extra dimension in ordinary particle interactions [5–7].

There would be two kinds of gravitons in this formalism; the first type is massless ordinary graviton, which is also confined to the 4-dimensional physical space-time, and the others are massive gravitons. In [6] it is shown that the effects of the massless gravitons in particles interactions are in order of $\frac{1}{M_P^2}$ where $M_P$ is the 4-dim Planck mass. However, the contributions of the massive ones are considerable and comparable with the weak scale of the Standard Model. The masses of gravitons come from Kaluza-Klein compactification of the fifth dimension. Due to non-factorization of the geometry, the masses of gravitons are $m_n = \kappa x_n e^{-\sigma(\pi)}$, where $x_n$’s are the roots of $J_1(x)$, the Bessel function of order one.

In this paper, we calculate the total cross-sections of scalar-scalar to photon and graviton fields ($\phi\phi^* \rightarrow G\gamma$). This process in Standard Model, without producing gravitons is forbidden by energy-momentum conservation.

II. DIFFERENTIAL CROSS-SECTION FOR $\phi\phi^* \rightarrow G\gamma$

At the beginning, we consider the scalar electrodynamics as the effective theory of matter and forces in visible and hidden spaces. The action of this theory is:
\[ S_{\text{vis}} = S_{\text{SED}} = \int \left\{ g^{\mu\nu} (\partial_\mu - ieA_\mu)\phi^* (\partial_\nu + ieA_\nu)\phi - m_\phi^2 \phi^* \phi - \frac{1}{4} g^{\alpha\mu} g^{\beta\nu} F_{\alpha\beta} F_{\mu\nu} \right\} \sqrt{-g} \, d^4 x, \]  

where \( g^{\mu\nu} = g^{(0)\mu\nu} + \frac{1}{\Lambda} h^{\mu\nu} \) and \( g^{(0)\mu\nu} \) is the inverse of the classical metric (1.2). The factor \( \Lambda \) for massless graviton is equal to \( M_P \) and for the massive gravitons is \( e^{-\sigma(\pi)} M_P \). Inserting \( g^{\mu\nu} \) in eq. (2.1) and absorbing the conformal factor \( e^{-2\sigma(\pi)} \) in scalar fields and their mass, one can reduce the interaction part of Lagrangian up to the first order in \( h^{\mu\nu} \) to the following terms,

\[ \mathcal{L}_I = ieA^\mu (\partial_\mu \phi^* \phi - \phi^* \partial_\mu \phi) + e^2 A^\mu A_\mu \phi^* \phi \]
\[ + \frac{h^{\mu\nu}}{\Lambda} \partial_\mu \phi^* \partial_\nu \phi + ie \frac{h^{\mu\nu}}{\Lambda} A_\nu (\partial_\mu \phi^* \phi - \phi^* \partial_\mu \phi) \]
\[ - \frac{h^{\mu\nu}}{2\Lambda} (\partial^\lambda A_\nu \partial_\lambda A_\mu - 2 \partial^\lambda A_\nu \partial_\mu A_\lambda + \partial_\nu A^\lambda \partial_\lambda A_\mu) \]
\[ + e^2 \frac{h^{\mu\nu}}{\Lambda} A_\mu A_\nu \phi^* \phi. \]  

(2.2)

As it is pointed out in ref. [8], we can use \(-\mathcal{L}_I\) as \( \mathcal{H}_I \), for this theory. Since we are searching for amplitude of \( \phi\phi^* \rightarrow \gamma G \), the relevant terms of Hamiltonian to this interaction are,

\[ \mathcal{H}_1 = -ieA^\mu (\partial_\mu \phi^* \phi - \phi^* \partial_\mu \phi), \]
\[ \mathcal{H}_2 = -\frac{h^{\mu\nu}}{\Lambda} \partial_\mu \phi^* \partial_\nu \phi, \]
\[ \mathcal{H}_3 = -ie \frac{h^{\mu\nu}}{\Lambda} A_\nu (\partial_\mu \phi^* \phi - \phi^* \partial_\mu \phi), \]
\[ \mathcal{H}_4 = \frac{h^{\mu\nu}}{2\Lambda} (\partial^\lambda A_\nu \partial_\lambda A_\mu - 2 \partial^\lambda A_\nu \partial_\mu A_\lambda + \partial_\nu A^\lambda \partial_\lambda A_\mu), \]  

(2.3)

which contribute to the following diagrams,
Denoting in-state by \(| p_1, p_2 >\) where \(p_1\) and \(p_2\) are the momenta of scalar and anti-scalar particles respectively and out-state by \(| k, q >\), where \(k\) and \(q\) are the momenta due to the massive graviton and photon. Now, we are going to derive the \(S\)-matrix elements for the above diagrams in the tree level,

\[
< k, q | S | p_1, p_2 > = < k, q | T e^{-i \int \mathcal{H}_I dt} | p_1, p_2 > = i (2\pi)^4 \delta^{(4)}(q + k - p_1 - p_2) M_{tot},
\]

(2.4)

where \(M_{tot}\) is invariant amplitude which is the sum of the following amplitudes (see figs.),

\[
\mathcal{M}_1 = -\frac{e}{\Lambda} \epsilon_{\mu}(q) e^{\mu\nu}(k) (p_1 - p_2)_{\nu},
\]

\[
\mathcal{M}_2 = -\frac{e}{\Lambda} \epsilon^{\lambda}(q) e^{\mu\nu}(k) \frac{(p_1 - k)_{\mu} p_{1\nu}(p_2 - p_1 + k)_{\lambda}}{(p_1 - k)^2 + m_{\phi}^2},
\]

\[
\mathcal{M}_3 = -\frac{e}{\Lambda} \epsilon^{\lambda}(q) e^{\mu\nu}(k) \frac{(p_2 - k)_{\mu} p_{2\nu}(p_2 - p_1 - k)_{\lambda}}{(p_2 - k)^2 + m_{\phi}^2},
\]

\[
\mathcal{M}_4 = \frac{e}{\Lambda (p_1 + p_2)^2 + i\epsilon} e^{\mu\nu}(k) \left( (p_1 - p_2)_{\nu} \epsilon_{\mu}(q \cdot (p_1 + p_2)) - (p_1 - p_2)_{\nu} \epsilon_{\mu}(q \cdot (p_1 + p_2)) - \epsilon_{\nu}(p_1 + p_2)_{\mu} (p_1 - p_2) \cdot q + q_{\nu}(p_1 + p_2)_{\mu} (p_1 - p_2) \cdot \epsilon \right).
\]

(2.5)

In the above equation \(\epsilon^{\mu}\) and \(e^{\mu\nu}\) are the polarization of photon and graviton respectively. To calculate the unpolarized cross-section, we should make summation over these polarizations. We have,

\[
\sum_{pol.} \epsilon_{\mu\nu}(k) e_{\alpha\beta}(k) = f_{\mu\nu\alpha\beta}(k),
\]

\[
\sum_{pol.} \epsilon_{\mu}(q) \epsilon_{\nu}(q) = -g_{\mu\nu},
\]

(2.6)
in which $f_{\mu\nu\alpha\beta}$ for a massive graviton is \[9\],

\[
f_{\mu\nu\alpha\beta}(k) = \frac{1}{2} \left\{ g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta} \right\} \\
+ \frac{1}{2} \left\{ \frac{k_{\mu}k_{\nu}}{m^2} + \frac{k_{\mu}k_{\alpha}}{m^2} + \frac{k_{\nu}k_{\alpha}}{m^2} + \frac{k_{\nu}k_{\beta}}{m^2} \right\} \\
+ \frac{2}{3} \left( \frac{1}{2} g_{\mu\nu} - \frac{k_{\nu}k_{\mu}}{m^2} \right) \left( \frac{1}{2} g_{\alpha\beta} - \frac{k_{\alpha}k_{\beta}}{m^2} \right),
\]

(2.7)

and for a massless graviton,

\[
f_{\mu\nu\alpha\beta}(k) = \frac{1}{2} \left\{ g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - \frac{2}{3} g_{\mu\nu}g_{\alpha\beta} \right\}.
\]

(2.8)

Using the above relations, it is straightforward to calculate the cross-section in the center of mass frame of the incident particles for the massive gravitons. The differential cross-section of the mentioned process in the center of mass frame is \[10\],

\[
\frac{d\Sigma}{d\Omega}_{\text{CM}} = \frac{1}{256\pi E^2} \left| \frac{\vec{k}}{p_1} \right| |M_{\text{tot}}|^2,
\]

(2.9)

where $M_{\text{tot}} = M_1 + M_2 + M_3 + M_4$, and $2E$ is the center of mass energy of incident particles. The contribution of the massless graviton can be neglected, as it is pointed out in \[6\]. For a massive graviton of mass $m$, we have calculated in the Appendix the terms in $\sum_{\text{pol.}} |M_{\text{tot}}|^2$. To obtain the total cross-section, as a function of $E$, one should integrate the (2.9) over the scattering angles and sum over all massive gravitons which their masses are less than $2E$. For the calculations the computer program Mathematica version 3.0 was used.

The final result is the following graph, where the solid curve shows the behavior of the total cross-section versus the energy, $E$. Here, we take $m_\phi = 0.2\, TeV$, $\sigma(\pi) = 12\pi$ and $\kappa = 10^{16} TeV$. According to the mass formula, $m_n = \kappa x_n \exp(-12\pi)$, we obtain the first four gravitons’ masses, $m_1 = 1.6\, TeV$, $m_2 = 2.9\, TeV$, $m_3 = 4.2\, TeV$ and $m_4 = 5.6\, TeV$.

For $E < 1.45\, TeV$, only the first graviton mode contributes to the total cross-section. For $E \geq 1.45\, TeV$, the dashed curve shows the contribution of this first mode to the $\Sigma_{\text{tot}}$. The individual behavior of the other graviton modes are similar to this dashed curve which shows a monotonic increasing behavior. This increasing behavior is expected due to the non-renormalizability of the quantum gravity. The peaks on the solid curve show the resonance behavior according to creation of the graviton modes.
Total cross section of $\phi^* \phi \rightarrow G\gamma$ versus energy of one incident particles, with $m_\phi = 0.2 \text{TeV}$. The peaks correspond to the productions of massive gravitons.

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APPENDIX

In this appendix, all the ten terms of the scattering diagrams up to the order of $\frac{\epsilon}{M_p}$ for $m_\phi = 0.2 \text{TeV}$ in terms of the mass of graviton, $m$, and energy, $E$, have been calculated.

$$\frac{A^2}{e^2} \sum_{\text{pol.}} |M_1|^2 = \frac{5 \left(4 E^2 - m^2\right) \left(4 c^2 + (-0.16 + 4 E^2) m^2\right)}{3072 E^3 \sqrt{0.04 + E^2 m^2 \pi^2}},$$

$$\frac{A^2}{e^2} \sum_{\text{pol.}} (M_1 M_2^* + M_1^* M_2) = \frac{-1}{3072 E^3 \sqrt{0.04 + E^2 (4 E^2 - 4 c - m^2)} m^4 \pi^2} \left((4 E^2 - m^2) (32 c^4 - 16 E^6 m^2)ight. - 16 c^3 m^2 + 0.64 c m^4 + (0.0512 + 0.04 m^2) m^4 + c^2 m^2 (-2.56 + 2 m^2) + E^2 (-64 c^3 + 96 c^2 m^2 + \ldots)$$
\[
\frac{\Lambda^2}{e^2} \sum_{\text{pol.}} (M_1 M_3^* + M_3^* M_4) = \frac{1}{E^3 \sqrt{-0.04 + E^2} (4 E^2 + 4 c - m^2^2) m^4} (0.000527714 (4 E^2 - m^2) (-2.4^2 + 1. E^6 m^2 - c^3 m^2 + c^2 (0.16 - 0.125 m^2) m^2 + 0.04 c m^4 + (-0.0032 - 0.0025 m^4) m^4 + E^4 (-2. c^2 - 4. c m^2 - 0.04 - 2.5 m^2) m^2) + E^2 (-4. c^3 - 6. c^2 m^2 + (0.16 - 1. m^2) m^2 + (0.18 + 0.0625 m^2) m^4)),
\]
\[
\frac{\Lambda^2}{e^2} \sum_{\text{pol.}} (M_1^* M_2^* + M_2^* M_3) = \frac{E (0.000527714 + 0.000989465 m^2)}{E^2 \sqrt{-0.04 + E^2} m^2} \frac{8.24554 \times 10^{-7} m^4}{E^5 \sqrt{-0.04 + E^2} m^2} \frac{5 E^3}{384 \sqrt{-0.04 + E^2} m^2 \pi^2} + c^2 (0.000659643 - 0.000329822 m^2) + 0.0000206138 (-7.60416 + m^2) m^2 (0.0841644 + m^2)
\]
\[
\frac{\Lambda^2}{e^2} \sum_{\text{pol.}} |M_2|^2 = \frac{1}{98304 E^3 \sqrt{-0.04 + E^2} m^4 (-4 E^2 + 4 c + m^2)^2 \pi^2} (4096. E^{12} + E^{10} (-163.84 - 20480. c + 2048. m^2) + E^8 (4096. c^2 + c (655.36 - 7168. m^2) + (-450.56 - 256. m^2) m^2)
\]
\[
\frac{\Lambda^2}{e^2} \sum_{\text{pol.}} (M_3 M_2^* + M_2^* M_4) = \frac{1}{49152 E^3 \sqrt{-0.04 + E^2} m^4 (16 E^4 - 16 c^2 - 8 E^2 m^2 + m^4) \pi^2} (256. E^8 + 256. c^4 - 1280. E^6 m^2 + c^2 (-20.48 - 32. m^2) m^2 + 1. m^4 (0.171487 + m^2)
\]
\[
\frac{\Lambda^2}{e^2} \sum_{\text{pol.}} (M_2 M_3^* + M_3^* M_4) = \frac{1}{24576 E^5 \sqrt{-0.04 + E^2} (4 E^2 - 4 c - m^2) m^4 \pi^2} (-1024. E^{12} - 7.10543 \times 10^{-15} c^4 m^2
\]
\[
+ 3.55271 \times 10^{-15} c^3 m^6 + c^2 (-1.92 - 4.44089 \times 10^{-16} m^2) m^6 + (0.0384 + 0.12 m^2) m^8 + \]
\[
\]
\[
\frac{\Lambda^2}{c^2} \sum_{\text{pol.}} |M_4|^2 = \frac{1}{98304 E^3 \sqrt{-0.04 + E^2 \sum_{\text{pol.}} |M_3|^2}} (4096. E^{12} + E^{10} (-163.84 + 20480. E^2 + 2048. m^2) + 2048. m^2 + E^2 (4096. c^5 + c^4 (-163.84 - 2048. m^2) + c^3 (-327.68 - 2560. m^2) m^2 - 512. c^2 (-0.492029 + m^2) m^2 (0.0520294 + m^2) + 8. (-0.64 + m^2) m^4 (0.0626408 + m^2) \times (0.817359 + m^2) + 16. c m^4 (0.0812907 + m^2) (5.03871 + m^2)) + E^4 (20480. c^4 + c^2 (-1228.8 - 3072. m^2) m^2 - 640. c (-0.276264 + m^2) m^2 (0.148264 + m^2) - 16. \times (3.07951 + m^2) m^4 (0.199512 + m^2) + c^3 (-655.36 + 2048. m^2)) + E^6 (40960. c^2 - (450.56 + 256. m^2) m^2 + c (-655.36 + 7168. m^2)) + E^6 (40960. c^3 - c (1310.72 + 1536. m^2) m^2 - 256. (-0.189783 + m^2) m^2 (0.269783 + m^2) + c^2 (-983.04 + 8192. m^2)) m^2 (-1024. c^5 + c^4 (40.96 - 768. m^2) + c^3 (122.88 - 128. m^2) m^2 + 12. c (-0.64 + m^2) m^4 (0.426667 + m^2) + 32. c^2 (-0.0849806 + m^2) m^2 (1.20498 + m^2) + 1. m^4 (0.16 + m^2) (0.4096 - 1.28 m^2 + 4 m^4)),
\]
\[
\frac{\Lambda^2}{c^2} \sum_{\text{pol.}} (M_3 M_4^* + M_4^* M_3) = \frac{(0.16 - 4 E^2 - 2 c) \left( E - \frac{m^2}{2 E} \right) (c - \frac{m^2}{3}) + E^4 \left( \frac{c^2}{3} + E^4 \left( 6 + \frac{16 c^2}{3 m^2} - \frac{8 c}{3 m^2} \right) \right)}{512 E^4 \sqrt{-0.04 + E^2 \sum_{\text{pol.}} |M_3|^2}} (c (4 E^2 - m^2) (-8 E^6 (2 c + 3 m^2) + m^2 (-4. c^3 - 2. c^3 m^2 + c (0.24 - 2.25 m^2) m^2 + 0.06 m^4) + E^2 (-16. c^3 + 8. c^2 m^2 + c (0.96 - 11. m^2) m^2) + (-0.48 - 1.5 m^2) m^4) + E^4 (-32. c^2 - 12. c m^2 + m^2 (0.96 + 12. m^2)))) - \frac{1}{3072 E^5 \sqrt{-0.04 + E^2 \sum_{\text{pol.}} |M_3|^2}} (c (4 E^2 - m^2) (-8 E^6 (2 c + 3 m^2) + m^2 (-4. c^3 - 2. c^3 m^2 + c (0.24 - 2.25 m^2) m^2 + 0.06 m^4) + E^2 (-16. c^3 + 8. c^2 m^2 + c (0.96 - 11. m^2) m^2) + (-0.48 - 1.5 m^2) m^4) + E^4 (-32. c^2 - 12. c m^2 + m^2 (0.96 + 12. m^2)))) - \frac{1}{12288 E^5 \sqrt{-0.04 + E^2 \sum_{\text{pol.}} |M_3|^2}} ((4 E^2 - m^2) (4 E^2 + m^2) (-8 E^6 (2 c + 3 m^2) + m^2 (-4. c^3 - 2. c^3 m^2 + c (0.24 - 2.25 m^2) m^2 + 0.06 m^4) + E^2 (-16. c^3 + 8. c^2 m^2 + c (0.96 - 11. m^2) m^2) + (-0.48 - 1.5 m^2) m^4) + E^4 (-32. c^2 - 12. c m^2 + m^2 (0.96 + 12. m^2)))) + \]

\[
+ E^{10} (40.96 + 2048. m^2) + E^8 (1.13687 \times 10^{-13} c^2 + c (-81.92 - 4096. m^2))
\]
\[
+ (-112.64 - 640. m^2) m^2 + E^4 (1024. c^4 + 1024. c^3 m^2 + c^2 (-30.72 - 512. m^2) m^2
\]
\[
+ c (-35.84 - 128. m^2) m^4 + 44. m^4 \left( 0.0220983 + m^2 \right) (0.210629 + m^2))
\]
\[
+ E^2 m^2 (-256. c^4 - 128. c^3 m^2 - 3. m^4 (0.106667 + m^2) (0.8 + m^2) + c m^4 (1.28 + 8. m^2)
\]
\[
+ c^2 m^2 (12.8 + 64. m^2)) + E^6 (-2048. c^3 - 96. (-0.247773 + m^2) m^2 (0.0344401 + m^2)
\]
\[
+ c^2 (40.96 + 1024. m^2) + c m^2 (143.36 + 1280. m^2))\).
\]
\[ \frac{\Lambda^2}{c^2} \sum_{\text{pol}} |\mathcal{M}_4|^2 = \]
\[ + \frac{1}{24576 E^5 \sqrt{-0.04 + E^2}} \frac{1}{(4 E^2 + 4 c - m^2)^2 m^4 \pi^2} \left( (-4 E^2 + m^2)^2 \left(32 c^4 - 16 E^6 m^2 + 16 c^3 m^2 - 0.64 c m^4 + (0.0768 - 0.6 m^2) m^4 + c^2 m^2 (-3.2 + 18. m^2) + E^2 (64. c^3 + 96. c^2 m^2 + m^4 (-3.52 + 15. m^2) + c m^2 (-2.56 + 16. m^2)) + E^4 (32. c^2 + 64. c m^2 + m^2 (0.64 + 40. m^2))) \right), \]
\[ + \frac{5 c^2 (4 E^2 - m^2)^3 (4 c^2 + (-0.16 + 4 E^2) m^2)}{786432 E^7 \sqrt{-0.04 + E^2} m^2 \pi^2} + \frac{5 c^2 (-4 E^2 + m^2)^2 (4 E^2 + m^2)}{98304 E^7 \sqrt{-0.04 + E^2} m^2 \pi^2} \]
\[ + \frac{5 c^2 (4 E^2 - m^2)^3}{196608 E^7 \sqrt{-0.04 + E^2} m^2 \pi^2} \left( \frac{1}{256} (4 E^2 - m^2) \left(256 E^8 - 256 E^6 m^2 - 16 E^2 m^6 + m^8 + E^4 (96 m^4) \right) \right) \]
\[ + \frac{1}{98304 E^3 \sqrt{-0.04 + E^2} m^4 \pi^2} \left( \frac{983216 E^7 m^4 \pi^2}{32} \right), \]
\[ - \frac{1}{983216 E^7 \sqrt{-0.04 + E^2} m^4 \pi^2} \left( \frac{393216 E^7 m^4 \pi^2}{32} \right), \]
\[ + \frac{(-4 E^2 + m^2)^2 (48 E^6 m^2 - 0.12 m^6 + E^4 (64 c^2 + (-1.92 - 24 m^2) m^2))}{393216 E^7 \sqrt{-0.04 + E^2} m^4 \pi^2} \]
\[ + \frac{(-4 E^2 + m^2)^2 (4 m^4 c^2 + E^2 m^2 (-16 c^2 + m^2 (0.96 + 3 m^2)))}{393216 E^7 \sqrt{-0.04 + E^2} m^4 \pi^2} \]
\[ - \frac{(-4 E^2 + m^2)^2 \left(6 c^2 m^4 + 0.08 m^6 + c^2 m^6 - 32 E^6 (2 c + m^2) + E^4 m^2 (1.28 - 56 c + 16 m^2) \right)}{393216 E^7 \sqrt{-0.04 + E^2} m^4 \pi^2} \]
\[ - \frac{(-4 E^2 + m^2)^2 \left(E^2 (24 c^2 m^2 + (-0.64 - 2 m^2) m^4 - 8 m^4 c) \right)}{393216 E^7 \sqrt{-0.04 + E^2} m^4 \pi^2}, \]

where \( c = \vec{p}_1 \cdot \vec{k} = |\vec{p}_1||\vec{k}| \cos \theta. \)

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