Introduction

A major challenge in cellular networks is interference. In most instances, the subscribers who are at the edge of the cell type must deal with lower Signal-to-Interference-plus-Noise-Ratio (SINR) figures. [1], [2] The impact is felt by the subscribers who are at the cell edge who will have a reduction in throughput because of lower modulation. Orthogonal Frequency Division Multiple Access (OFDMA) is a popular method used by LTE networks to reduce interference [3], [4]. The orthogonal nature of the subcarriers ensures a reduction in interference within the cell however, interference is still a major problem, particularly between cells. Fractional frequency re-use is a great technique that has been proposed as a great way to reduce inter-cell interference [5]. The cell structure is essentially subdivided into two sections. The outer region is essentially referred to as the partial re-use region, and the inner region is the full re-use region [3], [6]. The partial re-use region typically has a re-use factor of 3 as stated widely in literature while the inner region has a re-use factor of 1. The threshold level is a vital parameter to set in Fractional Frequency Reuse (FFR). The common problem in this research field is how to set the SINR threshold value. This further informs how the frequency resources are allocated between the inner and outer regions of the FFR scheme. The other challenge is how performance is measured. Most articles in literature focus on either throughput but fail to consider fairness. Our research clearly stipulates how the thresholding is done and considers both throughput and fairness to evaluate performance. Our focus in this paper is to look at two important thresholding techniques: Otsu’s technique, and Entropy thresholding which make use of histogram techniques to fashion the threshold value. These two methods are then compared with a dynamic FFR thresholding technique from literature. The performance...
metrics considered are edge throughput and fairness. A base station is mounted in the middle of the cell and is responsible for receiving the SINR values reported by the subscribers in the cell. The advantage of using global thresholding techniques is that one threshold value is set that clearly separates the inner and outer regions of the FFR cell network. This threshold is further updated periodically to match subscriber distribution to continuously match network conditions. The disadvantage of our proposal is that there are line propositions in literature where FFR schemes with more than two regions have been proposed. This leads to the need to have multiple threshold values to separate the subscribers in the various regions. Our approach is limited since only one threshold value can be set.

Our proposal is to create a dynamic FFR scheme in which periodically calculates the SINR threshold since the cell dynamics keep changing as subscribers move. The benefit is that an updated threshold is always used that matches the new situation in the cell to differentiate between outer regions and inner region users using their SINR values.

The motivation of this paper is to improve the network performance in terms of fairness and throughput by setting the best SINR threshold hence allocating the frequency resources optimally. To achieve this, we studied Image processing techniques and how they segment images. Image segmentation essentially entails separating an image into the background and the foreground [7]. Some of the most common methods used in image segmentation include; clustering techniques, edge detection, region based and thresholding. Thresholding determines the boundary of a greyscale image using specific set values such that any pixel value above the threshold constitutes the foreground and any value below the threshold becomes part of the background. Edge detection segmentation is done when pixel values change, which would suggest to image boundary. Any pixels which form the image will have a certain sort of relationship and not have any abrupt value changes [7].

In region-based segmentation techniques, the image gets divided into several regions, all of which have similar traits. The regions that are segmented are divided based selecting specific values referred to as seeds.

Clustering technique uses a cluster number that is initialized, resulting in centroid determination. Euclidean distance between each pixel and centroid is determined. The nearest centroid is then used to allocate the pixels, and after every pixel has been assigned, the process is repeated till a certain error value has been met [7].

Our research has focused on global thresholding techniques which are founded on a histogram. The thresholding method returns a single threshold value as opposed to adaptive thresholding where a single image can have multiple thresholds in various sections of the image [8]. In adaptive thresholding parts of the image that have strong illumination shadows or gradients are categorized into sections, with each section having its individual threshold values. Our research is mainly focused on having a single threshold value that inclines more towards global thresholding. The two main global thresholding techniques are Otsu’s method and entropy method. It is therefore necessary to test both global thresholding techniques to see how their SINR threshold outcomes impact on network throughput and fairness. The novelty in our work is the way we adapt these image processing techniques to set our SINR threshold in a wireless network. When dealing with images, the values reported are the intensity of pixel values. In image processing for example pixel values can range from 0–255. Based on the pixel values in the image, a threshold is set that states that all pixels with a value equal to or greater than the threshold value form the foreground. In our case we are using the reported SINR values to imitate pixel values. SINR values typically range from −5 dB to 15 dB. All subscribers that report an SINR value equal to or greater than the SINR threshold fall in the full reuse zone. We are therefore not dealing with images and pixel values but using the same image processing techniques but with SINR values.

### Table 1. Centralized dynamic allocation of resources to cell-edge users [9].

| Input: $f_e, f_t, M_e, r_d$ |
|-----------------------------|
| 1: Initialize: $M'_e, M'_t$ |
| 2: $S = \sum_{i=1}^{n} r_i^2 / \text{sum of normalized center area for cell } i$ |
| 3: for $i = 1$ to $f_t$ do |
| 4: $A'_i = r_i^2$ |
| 5: end for |
| 6: $\Delta = \min (A'_1, A'_2, \ldots, A'_n)$ |
| 7: $G = \frac{\Delta}{\sum_{i=1}^{n} r_i^2}$ |
| 8: $A_{\text{avg}} = \max (A'_1, A'_2, \ldots, A'_n)$ |
| 9: ind = find($r_d > 0.5$) |
| 10: if (length(ind) > 1) then |
| 11: $M'_e = A_{\text{avg}}$ |
| 12: else |
| 13: $M'_e = M_e + G$ |
| 14: end if |
| 15: $M'_t = M - M'_e$ |
| 16: for $i = 1$ to $f_t$ do |
| 17: $M'_i = M'_e, A'_i = 1 - A'_i / \text{cell-edge area for cell } i$ |
| 18: $Q_i = \frac{M'_i}{\theta}$ |
| 19: $Q_i = M'_t / \text{factor proportional to demand}$ |
| 20: $M'_i = Q_i M'_t$ |
| 21: end for |
| Output: $M'_e, M'_t, M'_i, M'_t$ |

### 1.1. Related work

Centralized dynamic frequency allocation technique: In the centralized dynamic allocation of frequencies for cell-edge demand satisfaction, the number of users in the cell-center and the received SINR are used to determine the distance threshold [9]. It has been assumed that users are uniformly distributed in every cell as the intensity is varied, thus allowing for dynamic cell portioning and resource allocation based on demand. Full frequency re-use has been implemented for cell-center regions where all the considered cells with varying requirements utilize the same sub-band for the cell-center users [9]. Depending on each cell’s center area and the re-use factor, an average of the available sub-bands is selected and used by every cell. If multiple cells have their cell-center areas surpassing the cell-edge areas, bands proportionate to the maximum normalized area, $A_{\text{avg}}$, the cell with the biggest normalized cell center radius is used as the cell centre radius for the network [9]. The threshold radius is represented by $r_d$. If $r_d > 0.5$ fails, the cell-center’s resource portion, $M_e$, is a sum of the average of the cell-center’s area allocation and finer allocation factor $G$.

The centralized dynamic allocation technique tends to over-allocate cell-center regions for cells with $r_d < 0.5$ while the cell-edge regions have limited bandwidth. In order to overcome this constraint, proportional allocation of sub-bands based on weight factors is performed on the cell-edge regions. The weight factor, $Q_i$, partitions the cell-edge band, $M_i$, into individual sub-bands, $M'_i$ [9]. This algorithm, as in Table 1, performs $f_t$ number of iterations twice and a search operation on the same number of participating cells where $f_t$ represents the cell-edge re-use factor.

Each base station (BS) determines the portioning radius that will ensure the maximum spectral efficiency (SE) of the cell. The information is then relayed to the central coordinator. The BS also computes the path loss, the spectral efficiency of its associated user, and the interference distance. The central coordinator then allocates resources based on the information from the BS. Therefore, more sub-carriers are allocated to the cell-center users while the cell-edge users are proportionally taken care of, resulting in cells with varying cell-center and cell-edge radii [9]. This research has shortcomings since it does not consider fairness as a performance metric. Secondly it does not continuously update the SINR threshold over time.
1.2. Paper organisation

The rest of the paper is organized as follows. Section 2 presents the multi-cell network model and equations for SINR, throughput, Jain’s Fairness index and proportionally fair scheduler. Section 3 shows the formulation of Otsu’s method and entropy thresholding techniques. Section 4 shows the methodology. Section 5 shows the results and section 6 shows the conclusion.

2. Cellular network model and channel characterization

2.1. Multi-cell network model

The hexagonal shape represents the coverage area, and each hexagon contains a base station at the center, as illustrated in Fig. 1 above. A circle with a similar area is used to estimate each cell. The radius of the circle is used to estimate each cell. The radius of the circle is connotated using \( R = \left( \frac{\sqrt{3} S}{\pi} \right) R_0 \). Thus, the total area of coverage is \( A^H = (R^2 - R_0^2) \pi \).

\( R_0 \) is the regular hexagon side. \( R_0 \) essentially represents the nearest possible point where a mobile station is found with reference to the spatial region \( H \) and serving eNodeB.

2.2. FFR network layout

Strict FFR and soft FFR are the two common FFR approaches, with each containing a unique characteristic [10, 6]. Strict FFR, for example, have the cells divided into inner regions referred to as the full re-use zones, with the outer region referred to as the partial re-use zone [10]. An SINR threshold is used to determine the two regions. In the event the received SINR is greater than a specific threshold, \( \Gamma_{ib} \), then it lies in the full re-use zone or else it is taken as an outer-cell and considered to be in partial re-use zone.

The cumulative bandwidth of the system, \( W \), contains the orthogonal subcarriers, \( S_I \) shown in equation (1), with each carrier containing a bandwidth, \( B \). The system bandwidth is partitioned into a group of \( W_I \) of inner region UEs together with \( W_O = W_T - W_I \) that are assigned to the outer zone. The \( W_O \) is split into three more identical portions \( W_{O1}, W_{O2}, W_{O3} \). The portions are alternately assigned to the outer cell regions, which ensures neighboring cells operate of distinct frequencies, thus mitigating against interference.

Hence,

\[ S_T = S_I + 3S_O, \quad (1) \]

where \( S_I \) represents subscribers in the inner part of the cell and \( S_O \) are subscribers in the outer neighborhood of the cell.

The parameter \( \rho_{FR} \) connotates a ratio determining the quantity of subscribers found in the inner region. Outer region subscribers are represented as \((1 - \rho_{FR})\) of the whole bandwidth of the system.

2.3. SINR

The pathloss, \( PL_{dB} \) represented in equation (2), between the base station (BS) and mobile station (MS) can be calculated using the log-distance path loss model, [11], [12], [13], [14]

\[ PL_{dB}(h_{i,b}) = R + a10\log_{10}(h_{i,b}) \quad (2) \]

The path-loss exponent is \( a \), and is dependent on the environment of operation e.g., free space, urban area or inside a building. \( R \) represents the path loss from the base station at a distance of one meter. \( h_{i,b} \) represents the distance between BS and MS \( b \).

Take for example MS \( b \) is situated in region \( A \) receiving services by a given Base Station, and \( A \) represents cell regions \( I \) or \( O \). The MS instantaneous SINR shown in equation (3) during scheduling period \( t \) of the \( n \)th subcarrier is

\[ y_{h_{i,b}}^A(t) = \frac{H_{h_{i,b}}(t)I_{A}(t)}{N_0F_nB + I_{h_{i,b}}^A(t)} \quad (3) \]

Each UE is assigned \( P_w \) as the power, and the BS antenna gain is \( G_T \).

Frequency response as a result of small scale fading within the channel that connects BS \( s \) to MS \( b \) within a period scheduling \( t \) in the \( n \)th subcarrier results is \( C_N(0,1) \sim H_{h_{i,b}}(t) \).

The noise power spectral density is given as \( N_0 \), and the noise factor of the receiver is \( F_n \) while interference term is \( I_{h_{i,b}}^A(t) \) shown in equation (4) represents the interference term, represented as:

\[ I_{h_{i,b}}^A(t) = \sum_{k \in \Phi_f} P_w |H_{h_{i,b}}(t)|^2 P_L (h_{i,b}) G_T \quad (4) \]

The set of interfering BSs is represented by \( \Phi_f \) shown in equation (5) and is directly related to the subcarrier and FFR region that the subcarrier is under.

From Fig. 1, we can obtain the set of interfering base stations

\[ \Phi_f = \begin{cases} \{ 1,2, \ldots, 18 \}, & \in W_I \\ \{ 18,10,12,14,16,18 \}, & \in W_O \end{cases} \]

Taking into account power allocation is homogeneous, every single subcarrier is allocated power using the equation (6) below where total transmit power in the BS is \( P_t \):

\[ P_w = \frac{P_t}{S_O + S_I} \quad (6) \]

2.4. Throughput

Shannon-Hartley theorem dictates the channel capacity represented by \( P \) as shown in equation (7) and measured in bits/s.

\[ P = \log_2 (y_{h_{i,b}}^A(t) + 1)W, \quad (7) \]

where, \( W \), represents the bandwidth and, \( y_{h_{i,b}}^A(t) \) represents the SINR between the BS and MS. The UE cell edge throughput is a vital FFR performance metric since the subscribers at the edge of the cell often have a poor SINR figure thus experience dismal throughputs [12].

2.5. Jain’s fairness index

Other than throughput, fairness is an equally important metric that needs to be considered since networks have the tendency to display high throughput figures, which in some instances is often limited to just a few subscribers who have better SINR figures. The danger with subscribers
who have poor SINR values is that they might end up being overlooked. Jain’s fairness index is a metric that is used to rate how bandwidth that is equal is allocated between users. The index is represented as shown in equation (8) [15], [16]:

$$J(x) = \frac{\left(\sum_{n=1}^{N} x_n^2\right)^{1/2}}{N^{1/2} \sum_{n=1}^{N} x_n^2}.$$  

(8)

Throughput of rth users is represented by \(x_r\). \(N\) in each area of coverage represents the number of mobile users.

2.6. Proportionally Fair (PF) scheduling

Scheduling involves resource allocation to subscribers guided by a formula. Different scheduling techniques exist from proportionally fair, Best Channel Quality Indicator, and Round Robin technique [16], [17]. Proportionally fair scheduler is a channel-aware scheduler because it considers the SINR figures which subscribers report to determine how to perform the scheduling. Proportionally fair, aims at balancing to maximize the overall throughput available to the network while at the same time ensuring every single subscriber gets to receive a specific bare minimum service level.

Prioritization coefficient \(c_p(t)\) as shown in equation (9) is used to determine which MSs are to be scheduled first.

$$c_p(t) = 1/\mu_p(t)$$

(9)

where \(\mu_p(t)\) as shown in equation (10) represents the CSI average evolution in the short term, which is often calculated with the help of a moving average of scheduling periods in window \(W\).

$$\mu_p(t) = \sum_{n \in F_p} \mu_p(t-1) \left( 1 - \frac{1}{W} \right) + \frac{\gamma_{W_p}(t)}{W}.$$  

(10)

\(\gamma_{W_p}(t)\) as shown in equation (11) helps in indicating MS \(q\) can communicate during the period \(t\) the scheduling period in the \(n\)th subcarrier, i.e.

$$\gamma_{W_p}(t) = \begin{cases} 1, & \text{if MS } q \text{ is scheduled} \\ 0, & \text{else} \end{cases}$$  

(11)

Instantaneous SINRs information is utilized by the PF scheduler and experienced by every MSs, i.e., \(\in M_p\).

The subcarrier \(nL_q\) gets assigned to MS \(z\) \(\in M_z\) as shown in equation (12) ensuring the condition:

$$z = \max_{p \in M_z} \left\{ u_p(t) \gamma_{W_p}(t) \right\}.$$  

(12)

is met. The total number of MSs in specific cell area \(A\) is represented by \(M_z\).

3. Global thresholding techniques

3.1. Otsu’s technique

Otsu’s thresholding mechanism splits picture elements into background and foreground by utilizing a single threshold value. It is based on either diminishing the intraclass variance or augmenting the interclass variance [8].

3.1.1. Formulation

For any picture with \(M\) gray levels \([1, 2, \ldots, M]\), the number of pixels at each level, \(j\), is denoted by \(m_j\), and the number of pixels, \(Q\), in the picture can be given as \(Q = \sum_{j=1}^{M} m_j\) [8]. In addition, the histogram of the picture can be normalized and regarded as a probability distribution given as per equation (13):

$$r_j = m_j/Q, \quad r_j \geq 0, \quad \sum_{j=1}^{M} r_j = 1.$$  

(13)

where \(j\) denotes the pixel level with \(m_j\) being number of pixels and \(r_j\) being its relative probability. The pixels are then split into two groups, \(A_1\) and \(A_2\), representing the background and foreground, respectively using the selected threshold level \(\tau\). \(A_0\) comprises pixel values \([1, \ldots, \tau]\) while \([\tau + 1, \ldots, M]\) corresponds to \(A_1\) [8]. Further, the class mean levels denoted as \(\mu_j\) and the class occurrence probability, \(\omega_j\) can be represented as per equation (14)–(19).

$$\omega(t) = \Pr\left( A_0 \right) = \sum_{j=1}^{\tau} r_j = \omega_0$$

(14)

$$1 - \omega(t) = \Pr\left( A_1 \right) = \sum_{j=\tau+1}^{M} r_j = \omega_1$$

(15)

$$\mu_0 = \sum_{j=1}^{\tau} r_j/\omega_0 = \mu(t)/\omega(t)$$

(16)

$$\mu_1 = \sum_{j=\tau+1}^{M} r_j/\omega_1 = (\tau - \mu(t))/\left(1 - \omega(t)\right)$$

(17)

where

$$\omega(t) = \sum_{j=1}^{\tau} r_j$$

(18)

and

$$\mu(t) = \sum_{j=\tau+1}^{M} r_j$$

(19)

The zeroth and first-order cumulative statistics for an \(M^{th}\) level histogram are denoted by \(\omega(k)\) and \(\mu(k)\), respectively. The picture’s average pixel level is given as per equation (20).

$$\mu_{\text{total}} = \mu(M) = \sum_{j=1}^{M} r_j$$

(20)

Thus, for any value of \(t\) as per equation (21).

$$\mu_{\text{total}} = \omega(t) \mu_0 + (1 - \omega(t)) \mu_1.$$  

(21)

The class variances \(\sigma_0^2\) and \(\sigma_1^2\) based on second-order cumulative statistics are represented as per equation (22) and (23).

$$\sum_{j=1}^{\tau} \Pr\left( j | A_0 \right) (j - \mu_0)^2 = \sigma_0^2$$

(22)

$$= \sum_{j=\tau+1}^{M} r_j/\omega_1 \left( j - \mu_0 \right)^2$$

$$\sigma_1^2 = \sum_{j=\tau+1}^{M} (j - \mu_1)^2 \Pr\left( j | A_1 \right)$$

(23)

The within, total, and between class variance levels are respectively given as shown in equations (24)–(26).

$$\sigma_{\text{total}}^2 = \sum_{j=1}^{M} (j - \mu_{\text{total}})^2 r_j$$

(24)

$$\sigma_{\text{within}}^2 = \sigma_0^2 \omega_1 + \sigma_1^2 \omega_0.$$  

(25)

$$\sigma_{\text{between}}^2 = \omega_1 (\mu_1 - \mu_{\text{total}})^2 + \omega_0 (\mu_0 - \mu_{\text{total}})^2$$

$$= \sigma_0^2 \omega_1 (\mu_1 - \mu_0)^2$$

(26)

The measures of class separability can be used to test the effectiveness of the threshold is represented as follows:
\[
k = \frac{\sigma_{t\text{otal}}^2}{\sigma_{\text{within}}^2}, \quad \eta = \frac{\sigma_{\text{between}}^2}{\sigma_{\text{within}}^2}, \quad \lambda = \frac{\sigma_{\text{between}}^2}{\sigma_{\text{within}}^2}
\]

Finding the best threshold value in order to clearly distinguish the classes poses an optimization challenge. This is because, either of the objective functions represented in equations (24)–(26) can be utilized and the relation in equation (27) is always valid:

\[
\sigma_{\text{total}}^2 = \sigma_{\text{between}}^2 + \sigma_{\text{within}}^2
\]

maximizing either criterion in equation (27) is analogous to maximizing the rest. The parameters \(\sigma_{\text{between}}^2\) and \(\sigma_{\text{within}}^2\) are functions of the threshold value \(t\), while \(\sigma_{\text{between}}^2\) is independent of \(t\). \(\sigma_{\text{between}}^2\) is a first-order statistical measure based on class meanwhile \(\sigma_{\text{within}}^2\) is a second-order statistical measure based on class variance. The parameter \(\eta\) is used to find out the threshold quality at level \(t\).

Therefore, in order to realize the optimum threshold value that would maximize \(\eta\) or \(\sigma_{\text{between}}^2\), a meticulous approach using equations (28) and (29) is undertaken where:

\[
\eta(t) = \frac{\sigma_{\text{between}}^2(t)}{\sigma_{\text{total}}^2}
\]

\[
\sigma_{\text{between}}^2(t) = \frac{\left[ \mu_p - \mu(t) \right]^2}{\sigma(t)[1 - \sigma(t)]}.
\]

The optimal threshold \(t^*\) is then given as equation (30)

\[
\sigma_{\text{between}}^2(t^*) = \max_{1 \leq t < \lambda} \sigma_{\text{between}}^2(t).
\]

3.2. Entropy method

In order to determine the entropy of a histogram, a global thresholding approach is used whereby a pair of probability distributions are obtained from the grey level pixel distribution of a given image [18].

The image segmentation process begins by determining the total entropy of the image. For maximum entropy thresholding, maximum image information measure is done between the foreground and background. A system is said to have a low entropy if it is highly harmonized. On the other hand, a system with minimum order is said to have high entropy. Thus, entropy is based on the spread of states in a system whereby a low-entropy system is characterized by fewer states while a high-entropy system is identified as one with a wide-ranging number of states.

In a given image, the states are of \(t\), analogous to the gray levels acquired by each pixel. There are 256 states for an 8-bit pixel, and if all states are utilized, yielding a uniformly distributed gray-level histogram, then the spread of states will achieve maximum entropy. However, if the gray-scale image utilizes only a pair of states, either black or white, then the entropy is low. Finally, the entropy is zero if all pixel values are equal.

3.2.1. Formulation

The probability distribution of gray levels can be denoted as \(r_1, r_2, r_3, \ldots, r_n\) whereby two probability distributions, \(J\), and \(K\) as shown in equations (31) and (32) can be obtained [15]. \(J\) is used to define for discrete values 1 to \(x\) and \(K\) for values \(x+1\) top. The two distributions are represented as shown in equations (33) and (34):

\[
J = \frac{r_1}{r_x}, \frac{r_2}{r_x}, \ldots, \frac{r_x}{r_x}
\]

\[
K = \frac{r_{x+1}}{1 - r_x}, \frac{r_{x+2}}{1 - r_x}, \ldots, \frac{r_p}{1 - r_x}
\]

\[
H_p = -\sum_{j=1}^{p} r_j \ln r_j
\]

\[
H_x = -\sum_{j=1}^{x} r_j \ln r_j
\]

Their associated entropies are given by equations (35)–(37):

\[
H(J) = -\sum_{j=1}^{x} \frac{r_j}{r_x} \ln \frac{r_j}{r_x}
\]

\[
H(J) = -\sum_{j=1}^{x} \frac{r_j}{r_x} \ln r_j - \frac{r_x}{r_x} \ln r_x
\]

\[
H(K) = -\sum_{j=1}^{x} \frac{r_j}{1 - r_x} \ln \frac{r_j}{1 - r_x}
\]

\[
H(K) = -\sum_{j=1}^{x} \frac{r_j}{1 - r_x} \ln r_j - \frac{r_x}{1 - r_x} \ln (1 - r_x)
\]

\[
\Phi(x) = \ln (1 - r_x) + \frac{H_p - H_x}{r_x} + \frac{H_p - H_x}{1 - r_x}
\]

\[
\Phi(x)\text{ is maximized to determine the threshold.}
\]

4. Methods

System performance was tested using the Vienna LTE-A Downlink System Level Simulator v2.0_Q3_2018. The simulation system performs LTE MATLAB-based simulations, used to analyze network performance. [19], [20]. Downlink is considered by the system traffic. Various configurations are taken into consideration through variation of UE numbers starting from having 10 UEs in every cell and incrementally increasing the number to 50 UEs in every cell. FFR threshold is constantly updated after 50 transmissions with every change occurring in the cell.

The number of the inner zone as a fraction of the total UEs inform the \(\beta_{\text{opt}}\) setting. System performance uses Edge user equipment fairness and throughput. The Table 2 illustrates the different parameters used to obtain results in the simulations. The assumptions made are that there is a fixed number of subscribers per cell which allows for easy comparison of Otsu’s method with entropy method and centralized dynamic frequency allocation technique. No shadow fading is also assumed in order to characterize a cell boundary using an SIN threshold as shown in Fig. 1.

| Parameter | Value |
|-----------|-------|
| Network Shape | Regular hexagonal grid |
| Inter-eNodeB path length | 500 [14] |
| Number of cells | 21 |
| User Equipment per cell | 10, 20, 30, 40, 50 |
| Transmission bandwidth | 20 MHz (100 resource blocks) |
| Antennas \((N_{TX} \times N_{RX})\) | 4 \times 2 |
| Feedback | AMC: CQI, MIMO: RI, and PMI |
| Feedback delay | 3 ms |
| Channel model | TU |
| User Equipment speed | 5 km/h |
| Model of the receiver | Zero forcing |
| Noise spectral density | –174 dBm/Hz |
| Path loss model | TS 36.942 – Urban area, 70 dB MCL [17] |
| Shadow fading | None |
| Minimum coupling loss | 70 dB [14] |
| Antenna pattern | TS 36.942 |
| Transmitted power (Macrocell eNodeB) | 40 W |
| Simulation distance | 50 subframes (TTIs) |
| Traffic model | Proportional fair |

| Table 2. LTE FFR Simulation Variables. |
Table 3. Optimal frequency allocation and thresholding algorithm.

| Step | Description |
|------|-------------|
| 1.   | **START**
|      | AFTER EVERY 50 TTIs DO
|      | DRAW histogram of user equipment SINR distribution, \( H_f \) |
| 2.   | EXECUTE Otsu’s method to calculate threshold, \( T \)
|      | \( T = \text{otsuthresh}(H_f) \) |
| 3.   | SCALE \( T \) TO \( \Gamma_\alpha \)
|      | \( \Gamma_\alpha = H_f \text{thresh} + (H_{\text{max}} - H_{\text{thresh}}) \times T \) |
| 4.   | FFR UE MAPPING
|      | FR\_zone\_UEs = 0
|      | PR\_zone\_UEs = 0
| 5.   | IF user equipment SINR \( \geq \Gamma_\alpha \)
|      | SET user equipment to FR zone
|      | FR\_zone\_UEs ++ |
| 6.   | ELSE
|      | SET user equipment to PR zone
|      | FR\_zone\_UEs ++ |
| 7.   | FIND \( \beta_F \)
|      | \( \beta_F = FR\_zone\_UEs / (FR\_zone\_UEs + PR\_zone\_UEs) \)
|      | SET new values of \( \Gamma_\alpha \) and \( \beta_F \)
| APPLY | entropy thresholding to calculate threshold, \( T \) (replace with Otsu threshold when using Otsu method)
| 1.   | **START**
|      | AFTER EVERY 50 TTIs DO
|      | DRAW histogram of user equipment SINR distribution, \( H_f \) |
| 2.   | EXECUTE Otsu’s method to calculate threshold, \( T \)
|      | \( T = \text{otsuthresh}(H_f) \) |
| 3.   | SCALE \( T \) TO \( \Gamma_\alpha \)
|      | \( \Gamma_\alpha = H_f \text{thresh} + (H_{\text{max}} - H_{\text{thresh}}) \times T \) |
| 4.   | FFR UE MAPPING
|      | FR\_zone\_UEs = 0
|      | PR\_zone\_UEs = 0
| 5.   | IF user equipment SINR \( \geq \Gamma_\alpha \)
|      | SET user equipment to FR zone
|      | FR\_zone\_UEs ++ |
| 6.   | ELSE
|      | SET user equipment to PR zone
|      | FR\_zone\_UEs ++ |
| 7.   | FIND \( \beta_F \)
|      | \( \beta_F = FR\_zone\_UEs / (FR\_zone\_UEs + PR\_zone\_UEs) \)
|      | SET new values of \( \Gamma_\alpha \) and \( \beta_F \)

4.1. System algorithm

Table 3 shows how the proposed algorithm works. After every 50 TTIs, SINR values in the cell are used to develop a histogram \( H_f \). Otsu method and entropy thresholding methods are then used to determine the optimal threshold.

Steps in the algorithm:

1. The subscribers in the network report their SINR values to the Base Station. These SINR values are used to generate a histogram. The histograms are as shown in Figs. 2–6.
2. Otsu’s method is then run to determine the threshold. The threshold calculated by Otsu’s method is given as a ratio referred to as \( T \) or otsuthresh.
3. Scaling \( T \) to form actual SINR threshold. Since the SINR values don’t start from 0 dB, the actual SINR threshold is calculated as follows: The lower limit SINR value is added to the product of the difference between the highest SINR value and lowest SINR value and the ratio value stated earlier.
4. The subscribers in the inner region and outer region are initially set to zero.
5. Based on the SINR threshold, the inner region (FR) is populated from those subscribers with an SINR value equal to or greater than the threshold.
6. The remaining subscribers that have SINR values lower than the threshold fall in the outer region (PR).

Table 4. Otsu and Entropy simulation configurations.

| Number of user equipment per cell | 1 | 2 | 3 | 4 | 5 |
|-----------------------------------|---|---|---|---|---|
| Total number of UEs              |   |   |   |   |   |
| SINR threshold (dB - Otsu)       | 1.5 | 5.00 | 3.00 | 3.00 | 3.00 |
| SINR threshold (dB - Entropy)     | 0.56 | 0.33 | 0.48 | 0.47 | 0.47 |
| SINR threshold (dB - Entropy)     | 4.50 | 5.00 | 5.00 | 5.00 | 5.00 |
| SINR threshold (dB - Entropy)     | 0.33 | 0.33 | 0.35 | 0.33 | 0.34 |

The Otsu’s and Entropy’s configuration parameters are listed in Table 4. In Figs. 2–5, histograms representing the user equipment wideband SINR distributions are presented. The Otsu’s and Entropy techniques are then applied to find out the threshold level. The dotted line on the histogram represents the threshold value with the Otsu’s method giving the best threshold values.

5. Results and analysis

The Otsu’s and Entropy’s configuration parameters are listed in Table 4. In Figs. 2–5, histograms representing the user equipment wideband SINR distributions are presented. The Otsu’s and Entropy techniques are then applied to find out the threshold level. The dotted line on the histogram represents the threshold value with the Otsu’s method giving the best threshold values.

5.1. Wideband SINR distribution for two equipment

The Figs. 2–6 represent histograms where thresholding is done. The dotted line shows the optimal thresholding value for a particular distribution. The histograms are constructed in intervals of 50 TTIs thus the threshold value is dynamic. The vertical axis of the histogram shows the number of UEs or subscribers. The horizontal axis shows bins of the various SINR values reported. The UEs with the same SINR value are binned or grouped together and their total number is reflected in the vertical axis.

5.2. Throughput ECDF

With different number of UEs per cell as shown in Fig. 7–11, the Empirical Cumulative Distribution Curves (ECDFs) are used to measure the network performance. An empirical cumulative distribution function is a probability model for data. While a CDF is a hypothetical model of a
distribution, the ECDF models empirical (i.e., observed) data. For a set of experimental (observed) data $x_1, x_2, \ldots, x_n$, the ECDF will give you the fraction of sample observations less than or equal to a particular value of $x$ as shown in equation (42).

$$F_n = \frac{1}{n} \sum_{i=1}^{n} 1_{x_i \leq t}$$  \hspace{1cm} (42)

Where $1$ is the indicator function. [21]

ECDF curves are a measure of the probability or percentage of the number of UEs achieve a particular throughput or less of the desired throughput chosen.

The graph below in Fig. 12 shows the peak UE throughput for the different thresholding techniques at the edge. This simply represents the highest value of throughput recorded by the Edge UEs. The graph in Fig. 13 represents the average edge throughput for all the UEs at the edge of the cell for the various techniques. The final bar graph in Fig. 14 represents the fairness of the different thresholding techniques.

### 6. Conclusion

In terms of peak edge throughput, Otsu’s method has achieved the highest values in all the cases examined from 10 UEs per cell to 50 UEs per cell. The peak value achieved is 0.81 Mbps when there are 10 UEs per cell compared to 0.59 Mbps for Entropy and 0.44 Mbps for
DFFR. This shows that Otsu outperforms Entropy by 27% and outperforms DFFR by 45.7%. The case of 50 UEs per cell still gives Otsu the lead with 0.21 Mbps compared to Entropy with 0.14 Mbps and DFFR at 0.13 Mbps. Otsu does better by 33.3% compared to Entropy and 38% compared to DFFR. Perhaps a more valuable throughput figure to use is the average Edge UE throughput. This would give a more reflective overall figure as it will consider all edge UEs throughput performances. Otsu’s method at 10 UEs per cell performs better with 0.66 Mbps compared to Entropy at 0.51 Mbps and DFFR at 0.44 Mbps. Otsu therefore is 22.7% better than Entropy and 33.3% better than DFFR. For the case of 50 UEs per cell, Otsu still does better with 0.18 Mbps compared to Entropy 0.12 Mbps and DFFR at 0.11 Mbps. Otsu does better by 33.3% compared to Entropy and 38.9% better than DFFR. Using average cell throughput, it can be therefore concluded that Otsu’s threshold performance is higher than the other approaches and it does better in a more loaded cell scenario. The fairness metric is used to measure which approach allocates the frequency as effectively as possible even to UEs with poor SINR. Otsu’s method performs better for 10 UEs per cell with a fairness of 0.73, Entropy at 0.56 and DFFR at 0.49. Otsu therefore outperforms Entropy by 23.3%, and outperforms DFFR by 32.9%. This performance is more or less repeated for the other numbers of UEs. The only exception is for 20 UEs where Otsu and Entropy approaches set the same SINR threshold. Therefore, Otsu’s method is the most superior technique in setting the threshold. For the case of the ECDF curves, they can also be used to determine the probability of UEs achieving a certain throughput level. The ECDF curves also show that Otsu’s method outperforms entropy method and centralized dynamic frequency allocation technique. The performance of the different approaches can be checked as follows. For example, in Fig. 10 if we pick the throughput value of 0.16 Mbps, the probability of achieving that throughput is 0.32 for Otsu or 32%. Meaning 68% of UEs achieve throughputs higher than 0.16 Mbps. The performance of Entropy at this point is 0.6 or 60% and achieves 0.16 Mbps. Meaning only 40% of UEs achieve throughputs higher than 0.16 Mbps. Whereas for Centralized dynamic frequency allocation technique from literature all the UEs fall below 0.16 Mbps. Meaning Otsu’s method outperforms Entropy thresholding and the DFFR approach from literature. The limitation of the proposed approach is that it sets only one global threshold where two regions are formed per cell of full re-use zone and partial re-use zones. Some proposals in literature have been made to form four-layer FFR networks within each cell where more than one threshold is required. In such
a case, our proposed approach will be limited since it only sets one threshold.

**Declarations**

**Author contribution statement**

Antony Onim: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Stephen Musyoki; Peter Kihato: Conceived and designed the experiments; Analyzed and interpreted the data.

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