Hydrodynamic description of neutrino gas

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Abstract

The system of neutrino-antineutrino (ν̅ν) - plasma is considered taking into account their weak Fermi interaction. New fluid instabilities driven by strong neutrino flux in a plasma are found. The nonlinear stationary as well as nonstationary waves in the neutrino gas are discussed. It is shown that a bunch of neutrinos, drifting with a constant velocity across a homogeneous plasma, can also induce emission of lower energy neutrinos due to scattering, i.e. the decay of a heavy neutrino ν_H into a heavy and a light neutrino ν_L (ν_H → ν_H ν_L) in a plasma. Furthermore we find that the neutrino production in stars does not lead in general to energy losses from the neutron stars.

I. INTRODUCTION

Our understanding of the properties of neutrinos in a plasma has recently undergone some appreciable theoretical progress [1–5]. The interaction of neutrinos with a plasma particles, the creation of ν̅ν pairs, the emission of neutrinos due to the collapse of a star are of primary interests [1,6–9] in the description of some astrophysical events such as supernova
explosion. One of the key processes upon the explosions are the large-scale hydrodynamic instabilities as well as \( \nu \) driven plasma instabilities. These processes are also believed to have occurred during the lepton stage of the early universe. During the formation of a neutron star the collapsed core of the supernova is so dense and hot that \( \nu \) and \( \bar{\nu} \) are trapped and are thus unable to leave the core region of the neutron star. The rates of escape of \( \nu \) and \( \bar{\nu} \) are very small, inside the star an equilibrium state is reached, which includes the \( \nu \bar{\nu} \) concentration. Recently, the neutrino transport phenomenon during the Kelvin-Helmholtz phase of birth of a neutron star in the diffusion approximation was investigated by Pons et al. [10]. A detailed knowledge of transport properties of the neutrinos in these extreme environments must include the neutrino-electron Fermi weak interaction coupling. There are different mechanisms involved in the creation of the \( \nu \), as well as pairs. The collective effects of the stellar plasmas can significantly alter the production rate of neutrinos. It is widely thought that the most dramatic plasma process is the decay of photons as plasmons into neutrino pairs. It was pointed out by Adams et al. [11] that the neutrino pair radiation can be the dominant energy loss mechanism for the plasma of the very dense stars, as well as white dwarfs, red giants and supernovas. It has been also shown [12] that a \( \nu_H \) can undergo radiative decay into a photon and a \( \nu_L \) (\( \nu_H \rightarrow \gamma \nu_L \)) in the presence of a strong magnetic field, with the strength greater than the critical value given by \( H_{\text{cr}} = m_e^2 c^3 / e \hbar = 4.41 \cdot 10^{13} \) Gauss. The increase (decrease) of the \( \nu \) (\( \bar{\nu} \)) energy within the plasma was also demonstrated [4]. This may have a significant and potentially detectable effect. Observationally, the recent results from the Kamiokande group provide strong evidence for the existence of neutrino oscillations. These results, together with the increased confidence in ability to produce and manipulate intense muon beams makes feasible the future neutrino factories based on muon storage rings. A muon storage ring, as presently envisioned, would have energy in the 10-50 GeV range, and produce directed beams of intense neutrinos, and naturally such an intense stream of neutrinos can exist in astrophysical and cosmological plasmas. Thus, it is of interest to examine if new physical processes that are caused by the intense flux of neutrinos, can appear in such a plasma, i.e., of particular conceptual interest are effects
which have no counterpart in vacuum.

In this paper we consider a medium of neutrino gas and plasma in semitransparent regions, where collective process plays an important role. In our consideration we assume that the interaction between neutrinos is weak in comparison with the neutrino-electron interaction, i.e., the neutrino gas is ideal. We can also ignore the spin of neutrinos in an isotropic plasma, since in this case the energy of neutrinos is independent of the spin operator. With this modeling, here we show that there exists the opposite effect that has been proposed by others [2,11]: namely that the neutrinos production does not generally lead to energy losses from a hot and dense system.

II. BASIC EQUATIONS

In the following we shall demonstrate the excitation of longitudinal oscillations in neutrino flux-plasma medium. For this purpose, we make use of hydrodynamic equations for the neutrino gas, which have been derived by Tsintsadze et al. [4]. These equations for the neutrino density $n_\nu$ and the fluid velocity $\vec{u}_\nu$ take the following forms

\[
\frac{\partial n_\nu}{\partial t} + \text{div}(n_\nu \vec{u}_\nu) = 0 \tag{1}
\]

\[
\frac{\partial \vec{u}_\nu}{\partial t} + (\vec{u}_\nu \cdot \vec{\nabla})\vec{u}_\nu = -\frac{Uc^2}{\hbar <\omega_k>} \vec{\nabla} \frac{n_e}{n_{0e}}, \tag{2}
\]

where $U = \sqrt{2}G_F n_{0e}$ describes the universal Fermi weak interaction, $G_F = 10^{-49}\text{erg.cm}^4$ is the Fermi coupling constant, $\frac{n_\nu}{<\omega_k>} = \int \frac{d^3k}{(2\pi)^3} \frac{N_k}{\omega_k}$, and $n_e$ is the electron plasma density.

The electron continuity equation takes the form:

\[
\frac{\partial n_e}{\partial t} + \text{div}(n_e \vec{v}_e) = 0. \tag{3}
\]

For the equation of motion of electrons in the presence of a neutrino gas, we have

\[
\frac{d\vec{p}_e}{dt} = e\nabla \phi - \sqrt{2}G_F \nabla n_\nu , \tag{4}
\]

where we have neglected the pressure term in comparison with the first term on the right hand side under assumption that $r^2_{De}/L^2 \ll 1$, where $r_{De}$ is the Debye length of the electrons,
and \( L \) is the characteristic length of the system. Note that the last term in Eq.(4) is the neutrino ponderomotive force. A more elaborate discussion on the neutrino ponderomotive force is given in [4].

The electrostatic potential \( \phi \) is determined from Poisson equation

\[
\nabla^2 \phi = 4\pi e(n_e - n_i) .
\]

(5)

We first consider a situation in which the ions are in an equilibrium state, i.e. \( n_i = n_{0i} \), where \( n_{0i} \) is the equilibrium value of the ion density. In this consideration the range of frequency of small perturbation of quantities of the medium is much less than the Langmuir frequency of the electrons. Thus, we can neglect the inertial term in Eq.(4), and define \( \phi \) from it. Finally, we obtain from the Poisson equation the expression for the electron density

\[
\frac{n_e}{n_{0e}} = 1 + \frac{\sqrt{2G_F n_{0\nu}}}{4\pi e^2 n_{0e}} \frac{\nabla^2 n\nu}{n_{0\nu}} ,
\]

(6)

where \( n_{0\nu} \) is the equilibrium value of the neutrino density. Substituting expression (6) into Eq.(2) we get the following equation, which except quantities of the neutrinos contains only the electron charge

\[
\frac{\partial \tilde{u}_\nu}{\partial t} + (\tilde{u}_\nu \cdot \nabla) \tilde{u}_\nu = -\beta \nabla^2 n\nu ,
\]

(7)

where \( \beta = G_F^2 c^2 n_{0\nu}/2\pi e^2 \hbar < \omega_k > \).

Equations (1) and (7) form a closed set of equations that describes the linear as well as nonlinear waves in the neutrino gas.

**III. CHERENKOV TYPE OF EMISSION OF LIGHT NEUTRINOS**

Let us consider the propagation of small perturbations in a homogeneous plasma-neutrino beam. To this end, we linearize Eqs.(1) and (7) with respect to the perturbations, which are represented as \( n_\nu = n_{0\nu} + \delta n_\nu \), \( \tilde{u}_\nu = \tilde{u}_{0\nu} + \delta \tilde{u}_\nu \), where the suffix 0 denotes the constant equilibrium value, and \( \delta n_\nu, \delta \tilde{u}_\nu \) are small variations in the wave. After linearization of
Eqs.(1) and (7), we will seek plane wave solutions proportional to \( \exp(i \vec{q} \cdot \vec{r} - \Omega t) \). We can then derive the following dispersion relation

\[
(\Omega - i q u_0)^2 = -\beta q^4 .
\]  

(8)

This leads to a solution of the form \( \Omega = \text{Re} \Omega + i \text{Im} \Omega \) such that:

\[
\text{Re} \Omega = q u_0 \cos \Theta
\]

(9)

and the growth rate for the unstable branch becomes:

\[
\text{Im} \Omega = \sqrt{\beta q^2} .
\]

(10)

This solution clearly describes the emission of the low-frequency neutrinos inside a resonance cone (\( \cos \theta = \frac{\text{Re} \Omega}{qu_0} \)), similar to the well-known Cherenkov emission of electromagnetic waves by charged particles moving in uniform medium with a velocity larger than the phase velocities of the emitted waves. On the other hand, strictly speaking the effect considered here is physically quite different from the usual Cherenkov emission by charged particles. The point is that the neutrino flux can no longer excite Langmuir waves and energetic (heavy) neutrino scatters of the electron density perturbation (caused by the ponderomotive force of neutrinos) and emits low (light) energy neutrino. We specifically note here that such decay process \( \nu_H \rightarrow \nu_H \nu_L \) is forbidden without a plasma.

Let us estimate the growth rate (10) for some typical neutrino parameters. Accordingly, we take the \( \nu_L \) energy to be about 100 keV, the \( \nu_H \) energy to be about 1-10 MeV, and \( n_\nu \sim 10^{32} \). It is important to emphasize that the growth rate does not depend on the density and temperature of a plasma. Then for the growth rate (10) we obtain \( \text{Im} \Omega \simeq 1.7 - 5.4 \cdot 10^8 \text{sec}^{-1} \). Comparing this with the collision frequency \( \delta \) of neutrinos, it can be seen the latter to be much less than the former. Indeed, for the given mean free path \( l_\nu = 2km(10/\varepsilon_\nu)^2 = 2 \cdot 10^5 \text{cm} \) [7], the collision frequency is \( \delta \sim \frac{n_\nu}{l_\nu} \sim \varepsilon_\nu \sim 10^5 \text{sec}^{-1} \), i.e. under these circumstances, the medium for the neutrino is "collisionless".
IV. NONLINEAR WAVES

We now discuss the nonlinear features of evolution of the unstable waves. If we consider the case, when the period of oscillations, $1/Re\Omega$, is much less than the wave amplitude growth time, $1/Im\Omega$, then the dispersion relation can be written in a one-dimensional perturbation as $\Omega = qu_0\nu$. This corresponds to the condition for the neglect of the third derivatives in Eq.(7). Doing so, we obtain

$$\frac{\partial u_\nu}{\partial t} + u_\nu \frac{\partial}{\partial z} u_\nu = 0,$$

which describes a beam of noninteracting neutrinos.

As was shown in [13], the equation type of Eq.(11) has solutions with the characteristic properties of nonlinear waves. It has been verified that the initial perturbation of the velocity $(u(z,0) \sim sinkz)$ in the phase plane $(u,z)$ after some time leads the wave $u(z,t)$ to break.

Thus, our results indicate that the neutrino beam of noninteracting particles, similar to a beam of plasma particles, has many properties of nonlinear system. Namely, it is subject to wave breaking and to generation of higher harmonics.

If we take into account the temperature $T_\nu$ of neutrinos and suppose that the neutrino gas is an ideal ($P_\nu = n_\nu T_\nu = P_{0\nu}(n_\nu/n_{0\nu})^\gamma$), then the equation of motion (7) modifies as

$$\frac{d\vec{u}_\nu}{dt} = -\frac{C^2_\nu(n_\nu)}{n_\nu} \hat{n}_\nu - \beta \nabla^2 n_\nu,$$

where $C^2_\nu(n_\nu) = C^2_{0\nu}(n_\nu/n_{0\nu})^{\gamma-1}$, $C_{0\nu}$ is the velocity of sound in the neutrino gas, and $\gamma$ is the effective adiabatic index.

Equations (1) and (12) are so called Boussinesq equations. We can see that the character of nonlinear processes strongly depends on the dispersion, i.e., the dependence of phase velocity on the wave number. The most important nonlinear effect here, as it was shown above, is that the wave front steepens. At this point the dispersion becomes significant. Waves having different wave number have different phase velocities, and the nonlinear steepening of front can be compensated by the dispersive spread. Therefore, in this case the stationary
waves can exist, which propagate with constant velocities without changing their shape (e.g.,
solitons). To note, Eqs.(1) and (12) describe also the nonstationary waves [14].

Equations type of Eqs.(1) and (12) have been thoroughly studied, and it has been shown
that these kind of equations have many periodic as well as solitary solutions. If we use
the common method for the weak dispersive medium, we obtain from Eqs.(1) and (12) the
Korteweg-de Vries equation for the neutrino velocity

\[ \frac{\partial u}{\partial t} + u \frac{\partial}{\partial z} u + \alpha \frac{\partial^3}{\partial z^3} u = 0 , \]

where \( u = \frac{(\gamma+1)}{2} u_\nu \), and \( \alpha = \frac{\beta}{2 c_0} \). This equation is well studied and we do not repeat all
solutions and conservation laws of it. Our aim was to prove that the neutrino gas exhibits
the same features as the waves in plasmas (e.g. electromagnetic waves, waves on the surface
of a heavy liquid, waves in biology, etc.).

V. ION DYNAMICS AND INSTABILITIES

We next consider the excitation of an ion-sound waves by the neutrino flux. Above we
studied the high-frequency oscillations neglecting the ion dynamics. However, the ion impact
turns out to be extremely important in the low-frequency waves which may be excited in a
strong non-isothermal plasma with hot electrons and cold ions. As for the electrons we may
assume that the electrons are in equilibrium under the conditions of low-frequency oscillation
and that their density is determined by the Boltzmann formula:

\[ n_e = n_{0e} e^{\frac{e\phi - \sqrt{2} G \delta n_\nu}{T_e}} , \quad \delta n_\nu = n_\nu - n_{0\nu} . \]

The ions can be described hydrodynamically by means of the HD equations \((T_i = 0)\):

\[ m_i \frac{dv_i}{dt} = -e \nabla \phi , \quad \frac{\partial n_i}{\partial t} + \text{div}(n_i \vec{v}_i) = 0 , \]

and the Poisson equation in this case is

\[ \nabla^2 \phi = 4\pi e \left( n_{0e} e^{\frac{e\phi - \sqrt{2} G \delta n_\nu}{T_e}} - n_i \right) . \]
After linearization of the set of Eqs. (1), (2), (15) and (16) we obtain the dispersion relation for the ion-sound oscillation in the presence of the monoenergetic neutrino beam as

\[(\Omega^2 - \omega_s^2)\left\{(\Omega - \bar{q}\bar{u}_0)\right)^2 + \frac{q^2\nu_D^2}{1 + q^2r_D^2}\alpha q^2c^2\right\} = \frac{\omega_s^2\alpha q^2c^2}{1 + q^2r_D^2}, \quad (17)\]

where \(\alpha = \frac{2G_F^2n_u n_0 \bar{n}_e}{\hbar <\omega_k>T_e}\), and \(\omega_s\) is simply the ion-sound frequency,

\[\omega_s = \frac{q v_s}{\sqrt{1 + q^2r_D^2}}, \quad (18)\]

here \(v_s = \sqrt{\frac{T_e}{m_i}}\) is the ion-sound velocity.

We examine the dispersion relation (17) for two interesting cases. Let us first discuss the small wavelength case, \(qr_D \gg 1\) and \(\Omega \ll \omega_s\). In this case Eq. (17) reduces to

\[(\Omega - \bar{q}\bar{u}_0)^2 + \alpha q^2c^2 = 0, \quad (19)\]

which has a solution of the form \(\Omega = Re\Omega + iIm\Omega\) with

\[Re\Omega = qu_0\cos\theta \quad \text{and} \quad Im\Omega = \sqrt{\alpha q c}. \quad (20)\]

Hence, this solution describes the emission of low-frequency neutrinos inside a resonance cone \((\cos\theta)\).

Next, for the long wavelengths, \(qr_D \ll 1\), the dispersion relation (17) casts in to the form

\[(\Omega^2 - q^2v_s^2)(\Omega - \bar{q}\bar{u}_0)^2 - \alpha q^2v_s^2q^2c^2 = 0. \quad (21)\]

Here we look for the solution with coincide roots, i.e. \(\Omega = qv_s + \Gamma\) and \(\Omega = \bar{q}\bar{u}_0 + \Gamma\), and get for the growth rate of the oscillatory instabilities the following expression

\[Im\Gamma = \frac{\sqrt{3}}{2}\left(\frac{v_s}{c}\alpha\right)^{1/3}q c. \quad (22)\]

**VI. FORMATION OF SHOCK WAVES**

Let us now investigate the nonlinear low-frequency waves. To this end, we add the ion pressure to the equation of motion of the ions, Eq. (15), and since \(\Omega \ll \omega_s\) neglect the inertial term. Then solution of this equation is
\[ n_i = n_0 e^{-\frac{e\phi}{T_i}}. \]

If we further impose the quasi-neutrality, \( n_e \simeq n_i \), we obtain

\[ e\phi = \sqrt{2}G_F \delta n \frac{T_i}{T_e + T_i}. \]

Substituting (24) into the Boltzmann distribution function of the electrons (14), we get

\[ \frac{n_e}{n_0 e} = e^{-\frac{\sqrt{2}G_F \delta n}{T_e + T_i}}. \]

For the neutrino gas the natural requirement is \( G_F \delta n \nu \ll T_e \), so that we have

\[ \frac{n_e - n_0 e}{n_0 e} = -\frac{\sqrt{2}G_F \delta n \nu}{T_e + T_i}. \]

This equation when substituted into Eq.(2) yields the equation of motion of the neutrinos

\[ \frac{\partial \vec{u}_\nu}{\partial t} + (\vec{u}_\nu \cdot \vec{\nabla}) \vec{u}_\nu = \delta \frac{\partial}{\partial z} n_\nu n_{0\nu}, \]

where \( \delta = \frac{2G_F e^2 n_0 \nu n_0 e}{h <\omega_k> (T_e + T_i)} \). If we linearize this equation and the equation of density (1), then we obtain the dispersion equation (19) for \( T_i = 0 \).

We now consider the one-dimensional nonlinear waves, for this case equations are

\[ \frac{\partial n_\nu}{\partial t} + \frac{\partial}{\partial z} n_\nu u_\nu = 0 \]

\[ \frac{\partial u_\nu}{\partial t} + u_\nu \frac{\partial}{\partial z} u_\nu = \delta \frac{\partial}{\partial z} n_\nu n_{0\nu}. \]

These equations have exact solutions for the initial profile of the neutrino density

\[ n_\nu(z, 0) = \frac{n_{0\nu}}{ch^2 \frac{z}{z_0}}, \]

and in nondimensional quantities are written as

\[ Q = (1 + Q^2 r^2) ch^{-2}(\xi - V \tau) \]

\[ V = -2Q \tau \theta h(\xi - V \tau), \]

where \( Q = \frac{n_\nu(z, t)}{n_{0\nu}}, \tau = \omega_{pi} t, \xi = \omega_{\nu} \frac{z}{\sqrt{\theta}}, V = \frac{u_\nu}{\sqrt{\theta}}, \) with \( \omega_{pi} = \left( \frac{4\pi e^2 n_0}{m_i} \right)^{1/2} \) being the Langmuir frequency of ions.
From Eqs. (31) and (32), we can readily derive
\[ \xi = \arccosh \sqrt{\frac{1 + Q^2 \tau^2}{Q}} - 2Q \tau \sqrt{1 - \frac{Q}{1 + Q^2 \tau^2}}. \]  
(33)
Examining Eq. (33), we now discuss the question how the maximum of the initial density changes in time. For this purpose, we take a derivative of Eq. (33) by \( Q \), i.e. \( \frac{d\xi}{dQ} \big|_{\xi=0} \), and allow it to be infinite. From this condition, we find the expression for the maximum density
\[ Q_{\text{max}}(0, \tau) = \frac{1}{2\tau^2}(1 - \sqrt{1 - 4\tau^2}) . \]  
(34)
One can see that at \( \tau \to 0 \), \( Q_{\text{max}} \to 1 \), \( n_\nu(0, 0) = n_{0\nu} \). Then with increase of time (\( \tau \)) the shape of the density changes, namely becomes narrow. Also, the center of the density (\( \xi = 0 \)) increases with rise of \( \tau \) and reaches another maximum, \( Q_{\text{max}} = 2 \) at \( \tau = 1/2 \). After a certain value \( \tau = \tau_0 \), the shock waves are formed, and derivatives are \( \frac{dQ}{d\xi} \big|_{\xi=0, \tau=\tau_0} = \infty \), \( \frac{d^2Q}{d\xi^2} \big|_{\xi=0, \tau=\tau_0} = \infty \). However, the density itself remains continuous. Using these conditions from Eq. (33) follows the relation \( Q_0 \tau_0^2 = 3/2 \), where \( \tau_0 > 0.5 \) and \( Q_0^2 \tau_0^2 \gg 1 \).

3D numerical simulation which underscores the above picture is carried out. Namely, we consider 3D problem, when the initial profile of density of neutrinos is the Gaussian distribution
\[ n_\nu(\vec{r}, 0) = n_{0\nu} \exp\left\{-\frac{x^2}{2a_0^2} - \frac{y^2}{2a_0^2} - \frac{z^2}{2z_0^2}\right\} . \]  
(35)
Numerical studies have shown that the dynamics of non-linearity is determined by the dimensionality of problem and of the initial profile of the neutrino density.

In order to confirm the validity of 1D analytical calculation, we first discuss 3D numerical analysis of Eqs. (1) and (27) for pancake-shaped beams, i.e., \( a_0 = 2 \), and \( z_0 = 0.5 \). Figures 1a and 1b show that the width of the density becomes shorter in time along the propagation direction (\( z \)-axis). The density of neutrinos is concentrated in the small region (on the \( z \)-axis), and at \( t = 4 \) starts to form the shock wave. Whereas Figures 1c and 1d show the insignificant expansion of the pancake beams in the transverse direction. Therefore, we can conclude that the 1D analytical calculation is very good approximation for the 3D pancake-shaped beams.
Quite different processes develop when the initial distribution of neutrinos has a form of the bullet, i.e., when $a_0 = 0.5$ and $z = 2$. We can see that there are two special competitive regimes: one, which is due to the self-focusing, is the compression of the neutrino density in the transverse direction (see Figures 2a and 2b), and the second is the expansion of the density along the propagation direction (see Figures 2b and 2c). The redistribution of neutrinos in the transverse direction is accompanied by the formation of sharp maximum of density near the $z$-axis ($r = \sqrt{x^2 + y^2} \to 0$), and as Figure 2a shows, the formation of the shock wave takes place at $t=2.8$.

VII. SUMMARY AND DISCUSSIONS

We have studied the problem of weak Fermi interaction of a neutrino beam with a plasma. Novel fluid instabilities driven by the neutrino flux in a plasma are observed. The range of wavelengths corresponding to several instabilities, discussed in this paper, for relevant parameters of the neutrino gas and plasma is $\lambda \sim 1 - 10^{-9} cm$. We have found that for the neutrino beam there exists a Cherenkov type of emission of low energy neutrinos; this is different from the usual Cherenkov effect, though. It should be emphasized that in our case the physical mechanism is that the high energy neutrino does not just excite the plasma wave but also can scatter from density perturbations of electrons and can emit the low energy neutrino. We also found that in the usual hot plasma environments such as supernova explosions, the effect considered above prevents neutrino escape. Since the low energy neutrinos cannot stream away from the central regions of the plasma one can have an accumulation of such neutrinos in a dense stellar medium. Thus, the above discussed processes can considerably modify certain phases of evolution in a stellar model. It should be emphasized that unlike the previous models [11], [16] on neutrino pair emission by a stellar plasma, the emission of low energy neutrinos by high energy neutrinos, as discussed in this paper, does not depend on the density and temperature of a plasma. The growth rate found for some typical neutrino parameters is $5.4 \cdot 10^8 sec^{-1}$. Which is much larger than
the collision frequency of neutrinos, $10^5 \text{sec}^{-1}$. Thus in this case the medium for the neutrino is "collisionless".

The nonlinear stationary as well as nonstationary waves in the neutrino gas are also discussed. The dispersive effect, which is responsible for the existence of the nonlinear stationary waves, e.g. solitons, in the neutrino gas is due to the electron density modulation by the neutrino ponderomotive force [4]. Our results indicate that a neutrino beam is subject to wave breaking and shock wave formation. Thus, high energy neutrinos will lose energy also by wave breaking in addition to Cherenkov emission, leading to plasma heating (and neutrino cooling). Whereas, the shock waves can produce a relativistic energy flow. In addition, the solitons can also be a potential candidates for the generation of relativistic particles. These particles then release the energy and produce the observed radiation in gamma-ray bursts (GRBs). Hence, one has a origin of radiation in the modeling of burst sources, which requires a discussion of particle acceleration processes. Since the simplest, most conventional, and practically inevitable interpretation of the observations of GRBs is that GRBs result from the conversion of the kinetic energy of ultra-relativistic particles to radiation. It is well accepted that many of them originate in the very distant, early universe. In the early universe, the processes discussed in the paper can also lead to formation of nonlinear structures, contributing to the formation of the large scale structure of the Universe [15]. Which, is believed, grew gravitationally out of small density fluctuations [17]. The effect of this density variation in the early universe was left on the cosmic microwave background radiation in the form of spatial temperature fluctuations. It should be emphasized that the gravity, however, cannot produce these fluctuations, but increase alone. Therefore a discussion of the physical models of generation of the initial matter density fluctuations is very crucial. There is no comprehensive model at present that can explain their origin. The weak Fermi interaction in a plasma, as discussed in this paper, can be an alternative and a new source for the required density perturbations. As demonstrated, a long lived nonlinear structures, which carry large amount of mass and energy, are generated in such a system. Since an initial localization of mass and energy is exactly that the gravity needs for eventual structure formation, weak
Fermi interaction may have provided a decisive element in the formation of a large scale map of the observable Universe.

**APPENDIX A: THE GENERALIZATION OF THE WIGNER-MOYAL EQUATION**

The Wigner-Moyal equation in quantum kinetic theory is the analogy of the one-particle Liouville equation in kinetic theory. In order to derive this equation, we start with a dispersion relation of a single neutrino, taking into account the universal weak Fermi interaction between plasma electrons and neutrinos \( U = \sqrt{2}G_F n_e \)

\[
(E - U)^2 = p^2 c^2 + m_{\nu} c^4 ,
\]

(A1)

where \( E \) is the energy, \( p \) is the momentum, \( m_{\nu} \) is the neutrino rest mass. This equation has two distinct energy solutions

\[ E = U \pm c \left( p^2 + m_{\nu} c^2 \right)^{1/2} . \]  

(A2)

The physical meaning of this result is best understood when we are in the rest frame of the background medium, which creates the potential energy \( U \). The positive sign in Eq.(A2) corresponds to the neutrino solution and it shows that, for a given magnitude of the particle momentum, the energy of the neutrino is increased by an amount \( U \) with respect to its value in vacuum. The other solution in Eq.(A2), corresponding to the negative sign, is that of an antineutrino solution.

As mentioned in the Introduction, in an isotropic plasma one can ignore the spin of neutrinos, and therefore we can consider a scalar wave function \( \psi_{\nu} \), associated with the neutrino gas, and we can write in equilibrium

\[
\left\{ \omega - c(k^2 + k_0^2)^{1/2} - \omega_F \right\} \psi_{\nu} = 0 .
\]

(A3)

where \( \omega = \frac{E}{\hbar}, \overline{k} = \frac{\vec{p}}{\hbar}, k_0 = \frac{m_{\nu} c}{\hbar} \) and \( \omega_F = \frac{U}{\hbar} \) is the Fermi frequency.
On the other hand, if the neutrino gas is not in equilibrium, the amplitude of $\psi_\nu$ will slowly change in space and time due to the interaction of neutrinos with the background plasma fluctuations. We can then use the geometrical optic approximation, $\omega \to \omega - i \partial / \partial t$, $k \to \tilde{k} + i \tilde{\nabla}$ and $n_e = n_{0e} + \delta n_e (\omega \gg | \frac{\partial}{\partial t} |, | \tilde{k} | \gg | \nabla |)$.

Doing expansion

$$\sqrt{(\tilde{k} + i \tilde{\nabla})^2 + k_0^2} = \sum_{j=0}^{N} \binom{N}{j} (k^2 + k_0^2)^{N-j} (2i\tilde{k}\tilde{\nabla} - \nabla^2)^j$$

(A4)

and neglecting higher order derivatives greater than $\nabla^2$ in Eq.(A4), we arrive from Eq.(A3) to

$$i \left( \frac{\partial}{\partial t} + \tilde{v}_g \tilde{\nabla} \right) \psi_\nu - \frac{\tilde{v}_g}{2k} \left( \nabla^2 - \left( \frac{\tilde{v}_g}{c} \tilde{\nabla} \right)^2 \right) \psi_\nu - \omega \frac{\delta n_e}{n_{0e}} \psi_\nu = 0,$$

(A5)

where use was made of the relation $\omega = c(k^2 + k_0^2)^{1/2} + \omega_F(n_{0e})$, and the following notations were introduced

$$\tilde{v}_g = \frac{c\tilde{k}}{(k^2 + k_0^2)^{1/2}}, \quad \left( \frac{\partial \omega_F}{\partial n} \right)_{n=n_{0e}} = \frac{\omega_F(n_{0e})}{n_{0e}},$$

(A6)

We now write the product of two wave functions $\psi_\nu$, which are defined at two distinct points and instants of time

$$F = \psi_\nu(\vec{r}_1, t_1)\psi^*_\nu(\vec{r}_2, t_2) = \psi_\nu\left( \vec{R} + \frac{\vec{r}}{2}, t + \frac{\tau}{2} \right)\psi^*_\nu\left( \vec{R} - \frac{\vec{r}}{2}, t - \frac{\tau}{2} \right),$$

(A7)

where we have introduced new space and time variables defined by

$$\vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2), \quad \vec{r} = \vec{r}_1 - \vec{r}_2, \quad t = \frac{1}{2}(t_1 + t_2), \quad \tau = t_1 - t_2.$$

Let us make a Fourier transform of (A7) on the variables $\vec{r}, \tau$ in order to introduce the Wigner function $f(\vec{R}, t, k, \omega)$

$$f(\vec{R}, t, k, \omega) = \frac{1}{(2\pi)^3} \int d\vec{r} \int d\tau \psi_\nu\left( \vec{R} + \frac{\vec{r}}{2}, t + \frac{\tau}{2} \right)\psi^*_\nu\left( \vec{R} - \frac{\vec{r}}{2}, t - \frac{\tau}{2} \right) e^{-i(\vec{k}\vec{r} - \omega \tau)}.$$

(A8)

This is the generalized Wigner distribution function. Integration of (A8) by the frequency $\omega$ leads to the ordinary expression of the Wigner distribution function [18] as

$$f^w(\vec{R}, t, k) = \frac{1}{(2\pi)^3} \int d\vec{r} \psi_\nu\left( \vec{R} + \frac{\vec{r}}{2}, t \right)\psi^*_\nu\left( \vec{R} - \frac{\vec{r}}{2}, t \right) e^{-i\vec{k}\vec{r}}.$$

(A9)
From (A8) also follows important relation

\[ |\psi_\nu|^2 = \int \frac{d\vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} f(\vec{R}, t, \vec{k}, \omega) . \]  

(A10)

Assuming that the neutrinos verify the dispersion relation (A2), we can say that for a given value \( \vec{k} \) we always have a well-defined value of the frequency, \( \omega = \omega(k) \). This means that we can write

\[ f(\vec{R}, t, \vec{k}, \omega) = 2\pi f(\vec{R}, t, \vec{k}) \delta(\omega - \omega(k)) . \]  

(A11)

Integration over the wave vector spectrum leads to the density of neutrinos

\[ n_\nu(\vec{R}, t) = |\psi_\nu|^2 = \int \frac{d\vec{k}}{(2\pi)^3} f(\vec{R}, t, \vec{k}) . \]  

(A12)

Following the procedure described in previous works [18–21,4], we can derive an equation for the function (A7), the result is

\[ i\left\{ \frac{\partial}{\partial t} + \vec{v}_g \cdot \vec{\nabla}_R \right\} F - \frac{v_g}{2k} \left\{ \vec{\nabla}_R \cdot \vec{\nabla}_r - \left( \frac{v_g}{c} \vec{\nabla}_R \right) \left( \frac{v_g}{c} \vec{\nabla}_r \right) \right\} F - \frac{\omega F}{n_0} (\delta n_1 - \delta n_2) F = 0 . \]  

(A13)

Now making the Fourier transformation of Eq.(A13), we obtain an evolution equation for function (A8) in the form

\[ \left( \frac{\partial}{\partial t} + \vec{v}_{eff} \cdot \vec{\nabla}_R \right) f(\vec{R}, t, \vec{k}, \omega) = \frac{\omega F}{n_0} \int d\vec{r} \int d\tau (\delta n_1 - \delta n_2) F e^{-i(\vec{k}_r - \omega \tau)} = 0 , \]  

(A14)

where

\[ \vec{v}_{eff} = \vec{v}_g \left( 1 + \frac{k^2_0}{k^2 + k_0^2} \right) . \]

If all derivatives of \( \delta n \) exist, we can expand \( \delta n_1(\vec{R} + \frac{\vec{r}}{2}, t + \frac{\tau}{2}) - \delta n_2(\vec{R} - \frac{\vec{r}}{2}, t - \frac{\tau}{2}) \) around \((\vec{R}, t)\), i.e.

\[ \delta n_1 - \delta n_2 = \sum_{l=0}^{\infty} \frac{1}{l!} \left( \frac{\vec{r}}{2} \cdot \vec{\nabla}_R + \tau \frac{\partial}{\partial t} \right)^l [1 - (-1)^l] \delta n(\vec{R}, t) \]  

(A15)

and obtain the generalized Wigner-Moyal equation

\[ \left( \frac{\partial}{\partial t} + \vec{v}_{eff} \cdot \vec{\nabla}_R \right) f(\vec{R}, t, \vec{k}, \omega) = \frac{\omega F}{n_0} \sin \left( \vec{\nabla}_R \cdot \vec{\nabla}_k - \frac{\partial}{\partial t} \cdot \frac{\partial}{\partial \omega} \right) \cdot \delta n(\vec{R}, t) f(\vec{R}, t, \vec{k}, \omega) . \]  

(A16)
Integration of Eq.(A16) by the frequency leads to the Wigner-Moyal equation for the Wigner function (A9) without the last term on the right hand side of Eq.(A16).

If we just retain the first term on the right hand side of Eq.(A16), i.e., the term corresponding to \( \sin x \approx x \), we obtain the well-known Liouville-Vlasov kinetic equation for the distribution function (A8), but with an additional time derivative term

\[
\left( \frac{\partial}{\partial t} + \vec{v}_g \cdot \vec{\nabla} \right) f(\vec{R}, t, \vec{k}, \omega) - \frac{\omega F}{n_0} \left( \vec{\nabla} \delta n \cdot \vec{v}_k f - \frac{\partial \delta n}{\partial t} \cdot \frac{\partial f}{\partial \omega} \right) = 0 .
\]

(A17)

The stationary solution of Eq.(A17) is the Fermi distribution function

\[
f_F = \left( e^{\frac{\hbar c}{\nu} \sqrt{k^2 + k_0^2 + U_T}} + 1 \right)^{-1}.
\]

(A18)

From Eq.(A17) we can now derive a set of fluid equations for the neutrino gas. To this end we shall introduce the principal definitions of quantities which describe the state of the neutrinos in a plasma. Namely, the total number of neutrinos is defined as

\[
N_\nu = \int d\vec{R} \int \frac{d\vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} f(\vec{R}, t, \vec{k}, \omega) = \int d\vec{R} n_\nu(\vec{R}, t) ,
\]

(A19)

the total energy of the neutrino gas

\[
E_{\text{total}} = \int d\vec{R} \int \frac{d\vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \hbar \omega f(\vec{R}, t, \vec{k}, \omega) = \int d\vec{R} \epsilon(\vec{R}, t) ,
\]

(A20)

and the neutrino mean velocity

\[
\vec{u}_\nu = \frac{1}{n_\nu} \int \frac{d\vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{\vec{k} \epsilon^2}{\omega} f(\vec{R}, t, \vec{k}, \omega) .
\]

(A21)

Having definitions (A19-A21) and using the usual momentum procedure, from Eq.(A17) we obtain a set of fluid equations

\[
\frac{\partial n_\nu}{\partial t} + \text{div}(n_\nu \vec{u}_\nu) = 0 ,
\]

(A22)

\[
\frac{\partial \vec{u}_\nu}{\partial t} + (\vec{u}_\nu \cdot \vec{\nabla}) \vec{u}_\nu = -\frac{\omega_F \epsilon^2}{\epsilon_k} \left( \vec{\nabla} + \frac{\vec{u}_\nu}{\epsilon \frac{\partial}{\partial t}} \right) \frac{\delta n_e}{n_0} - \frac{\nabla P_\nu}{n_\nu} ,
\]

(A23)

\[
\frac{\partial \epsilon_\nu}{\partial t} + \vec{\nabla} \cdot \vec{p}_\nu = \hbar \omega_F n_\nu \frac{\partial \delta n_e}{\partial t} n_0 ,
\]

(A24)
where we used the definitions of the momentum density

$$\bar{p}_\nu = \hbar \int \frac{d\vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \vec{k} f(\vec{R}, t, \vec{k}, \omega) , \quad (A25)$$

and

$$\frac{n_\nu}{<\omega_k>} = \int \frac{d\vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} f(\vec{R}, t, \vec{k}, \omega) \frac{\omega}{2\pi} . \quad (A26)$$

The quantity $P_\nu$ plays role of the neutrino pressure and is defined as

$$P_\nu = \frac{1}{3} \int \frac{d\vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left( \frac{\vec{k} c^2}{\omega} - \vec{u}_\nu \right)^2 f(\vec{R}, t, \vec{k}, \omega) . \quad (A27)$$

Equations (A22-A24) together with equations of the plasma, given in this paper, describe the collective process, which plays a more important role than the pair collision effect.
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FIGURES

FIG. 1. a) and b) The time evolution of the neutrino density along the z - axis in the case of the pancake-shaped beams
c) and d) The time evolution of the neutrino density along the x - axis in the case of the pancake-shaped beams

FIG. 2. a) The time evolution of the neutrino density along the x - axis in the case of the neutrino bullet
b) The neutrino density at t=2.8 in (z,x) space
c) The time evolution of the neutrino density along the z - axis in the case of the neutrino bullet
Fig. 1a Tsintsadze
Fig. 1b Tsintsadze
Fig.1c Tsintsadze
Fig. 1d Tsintsadze
Fig. 2a Tsintsadze
Fig. 2b Tsintsadze
