Self-referenced prism deflection measurement schemes with microradian precision

Rebecca Olson, Justin Paul, Scott Bergeson, and Dallin S. Durfee
Department of Physics and Astronomy, Brigham Young University, Provo, Utah 84602, USA
(Dated: March 31, 2022)

We have demonstrated several inexpensive methods which can be used to measure the deflection angles of prisms with microradian precision. The methods are self-referenced, using various reversals to achieve absolute measurements without the need of a reference prism or any expensive precision components other than the prisms under test. These techniques are based on laser interferometry and have been used in our lab to characterize parallel-plane beamsplitters, penta prisms, right angle prisms, and corner cube reflectors using only components typically available in an optics lab. Published in Applied Optics, Vol. 44, No. 22. ©2005 Optical Society of America.

INTRODUCTION

Reflecting prisms are key components in a variety of optical instruments. They can be used in place of mirrors to alter the direction of optical beams. Unlike mirrors, however, prisms can be used in such a way that the angle through which the beam is deflected does not change when the optic is rotated. For example, after a beam reflects off of the three perpendicular surfaces of a corner cube it will exit the prism travelling in precisely the opposite direction as the incoming beam. No careful alignment is needed to achieve this nearly perfect 180 degree deflection. Reflecting prisms are useful in situations where it is difficult to perform the initial alignment or when it is critical to maintain a particular beam deflection for a long period of time. One well known example is the use of corner reflectors for lunar ranging experiments. Our interest in prisms is to generate an extremely stable array of laser beams for use in an atom interferometer.

Since the beam deflection is determined by the angles between the prism surfaces rather than the alignment of the optic, it is extremely important that the prisms be made correctly. Several methods are commonly used to measure deflection angles of prisms. One class of techniques utilizes telescopes and autocollimators to image the separation of two beams at infinity. Our methods are based on a second class in which the angle between the two beams is ascertained using optical interference. Both types of measurements are limited by the size of the beam of light passing through the optics, in the first case by Rayleigh’s criterion, and in the second by the large fringe spacing resulting from nearly parallel beams. As such, both types of measurements have similar ultimate resolution limits. Techniques based on either type of measurement typically require a calibrated reference prism or other expensive optical components.

After purchasing a set of extremely high precision prisms for use in an atom interferometer, we began to have doubts as to whether the manufacturer had met our required specifications. Not having access to an instrument capable of measuring prism deflection angles to the necessary accuracy, we developed a set of techniques which allow prism deflection angles to be measured with accuracies of a few microradians. Our scheme is self referencing, requiring no calibrated prism. In addition to the prisms under test we only needed several standard-quality mirrors, lenses, and attenuators, an inexpensive alignment laser, a low quality surveillance camera, and for some measurements a piezoelectric actuator.

We characterized parallel plate beamsplitters (which generate two precisely parallel beams), penta prisms (which deflect light by 90 degrees), right angle prisms (which fold light by 180 degrees in the plane of the prism), and corner cubes.

Our methods utilize optical interferometry and bear similarity to the Jamin interferometer. Like several other schemes, in our methods the deflection angles of prisms are determined from the spacing between fringes formed by two interfering beams. Each of our designs produce similar intensities for the two interfering beams, resulting in high-contrast fringes for maximum sensitivity. Lenses and mirrors are only used before the beams are split or after the interference pattern is formed such that alignment or wavefront errors due to these optics have a negligible effect on the measurements.

MEASURING THE ANGLE BETWEEN TWO BEAMS

When two monochromatic plane waves intersect they form an interference pattern. Because the spacing between interference fringes depends on the angle between the two wave vectors, it is possible to ascertain the angle between the two propagation directions by analyzing the fringe pattern. Using Fig. 1 and simple trigonometry it is easy to find a relationship between the fringe spacing $d$ and the angle between the $k$-vectors of the two plane waves $\Delta \theta$. In the small angle approximation, for two plane waves with wavelength $\lambda$ projected onto a screen at near normal incidence, the angle between the two beams is given by

$$\Delta \theta = \frac{\lambda}{d}. \quad (1)$$
Fitting to a Piece of a Fringe

Most of the optics we tested have a clear aperture of 2.5 cm. To prevent clipping we made our measurements using a helium-neon laser ($\lambda = 632$ nm) collimated to a diameter of about 1 cm, suggesting that we would only be able to measure fringe spacings if the fringes were less than 1 cm apart. According to Eq. 1 this limit on $d$ results in a minimum measurable $\Delta \theta$ of 0.13 milliradians. The optics we measured were specified to have angular tolerances of a few microradians. In order to make measurements with microradian precision we had to infer angles from images which contained much less than one fringe.

One method commonly used in this situation is phase shifting, in which intensity is measured at several points as the fringe pattern is scanned across the points by shifting the phase of one beam $\phi$. This method has several advantages over the spatial fringe-fitting method used in our experiments: it is less susceptible to wavefront distortion, it reveals the sign of the angle between the beams (not just the magnitude), and it can be used for other types of measurements (such as surface profiling) which cannot easily be done with the method we chose. But our spatial fringe-fitting method has the advantage that all of the data is recorded in a single moment, making it more robust in noisy environments. It also doesn’t require the incorporation of a phase-shifting device, reducing cost and complexity and eliminating potential errors due to phase shifter beam deflections, drifts, and hysteresis. Adding a phase shifter would have greatly complicated our scheme for the measurement of plate beamsplitters. In our other schemes it could have been implemented by scanning one prism with a piezoelectric actuator. As discussed later, we used piezoelectric actuators in some of our schemes for other purposes, but the piezos lacked sufficient stability for this purpose (see Fig. 7(a) in section ).

To find the angle between the two beams we simply curve-fit the intensity pattern on our camera. But to get accurate results when less than one fringe is visible we have to take into account the spatial profiles of the beams. To do this we first write down the expression for the electric field of a laser beam as a function of the position on the camera $r$ and time $t$. To simplify our analysis we assume that the interfering beams have the same polarization. We also assume that the two beams are well collimated such that the phase of each beam’s electric field is of the form $k \cdot r - \omega t + \phi$ where $k$ is the wave vector of the beam, $\omega$ is the angular frequency of the light field, and $\phi$ is a constant phase offset. With these assumptions, the electric field of each beam can be written as

$$E_n(r, t) = f_n(r) \cos(k_n \cdot r - \omega t + \phi_n), \quad (2)$$

where $f_n(r)$ is the amplitude of the electric field at position $r$ and the subscript $n$ is equal to 1 or 2 depending on which of the interfering beams we are describing.

The intensity of the interference pattern of two intersecting beams is related to the time average of the square of the sum of the two interfering electric fields. When the time average is evaluated and the equation is simplified it can be expressed as

$$I_{12}(r) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(k_{rel} \cdot r + \Delta \phi), \quad (3)$$

where $\Delta \phi = \phi_1 - \phi_2$ and $k_{rel} = k_1 - k_2$, and where $I_1$ and $I_2$ are the intensity patterns which would be measured on the camera if only one of the two interfering beams was present.

Without losing generality we can define the plane of the camera’s detector to be the $z = 0$ plane (such that $r$ has no $z$ component). Then we can write the dot product $k_{rel} \cdot r$ as $k_x x + k_y y$ where $x$ and $y$ are cartesian coordinates describing the location of pixels on our camera and $k_x$ and $k_y$ are the spatial frequencies of the interference.
pattern imaged by the camera. If both beams strike the camera near to normal incidence, then the z component of \( k_{\text{rel}} \) will be nearly zero and \( k_{\text{rel}} \) will be approximately equal to \( (k_x^2 + k_y^2)^{1/2} \). These definitions result in the following expression:

\[
\frac{I_{12} - I_1 - I_2}{2\sqrt{I_1 I_2}} = \cos (k_x x + k_y y + \Delta \phi) .
\] (4)

The left-hand side of this equation can be thought of as a “normalized” intensity.

If the two beams are nearly parallel it can be shown that \( k_{\text{rel}} \approx 2\pi \Delta \theta / \lambda \). To find \( \Delta \theta \) we simply measure \( I_{12} \), \( I_1 \), and \( I_2 \) and numerically fit the left side of Eq. 4 to the right side to find \( k_x \) and \( k_y \) treating \( \Delta \phi \) as a free parameter. We then calculate \( k_{\text{rel}} \) and from that \( \Delta \theta \). The three intensity patterns needed to calculate the left side of Eq. 4 are measured by taking four images: one of the two interfering beams, one of beam 1 with beam 2 blocked, one of beam 2 with beam 1 blocked, and a “dark field” image with both beams blocked. An example of a set of images is shown in Figs. 2(a)-(d). We then subtract the dark field image from the other three to generate the three background-free intensity patterns \( I_{12} \), \( I_1 \), and \( I_2 \).

The separate \( I_1 \) and \( I_2 \) terms in Eq. 4 make this measurement technique work even if the fringes have low contrast due to mismatched power in the two interfering beams. Lower contrast does increase digitization noise, which is of special importance when using a low bit-depth camera. This technique also works if the interfering beams do not overlap perfectly, although misalignments can reduce the region of useful data (see Fig. 2(e)). Large overlap misalignments coupled with wavefront curvature in the beams can also add errors to the measurements.

Figure 2(e) shows the result of this calculation applied to the data in Figs. 2(a)-(d). Curve fits to find \( k_x \) and \( k_y \) from this data are illustrated in Figs. 3(a) and (b). Although the data in these figures is somewhat noisy we can still get accurate, repeatable results by applying the constraint that the “normalized interference pattern” on the left-hand side of Eq. 4 should oscillate with unity amplitude and zero offset. This is clearly evidenced by the consistency of the measurements shown in Fig. 7 in section .

**Experimental Subtleties**

When using this curve-fitting approach to measure deflection angles of prisms we often made small adjustments to the prism or beamsplitter alignment in order to shift the relative phase of the two interfering beams such that images were not centered on a light or dark fringe. Only small adjustments which did not affect the overlap of the interfering beams were needed. Capturing data between a light and a dark fringe results in a more precise fit to the data. Fitting data near an extremum of the cosine requires precise measurement of the curvature of the data. Near a zero crossing, however, simply extracting the slope of the data is enough to get a good measurement of \( k \).

In our treatment we have assumed a well-collimated laser beam and have ignored effects of wavefront curvature. To ensure good beam collimation we constructed a simple Michelson-Morley interferometer with mismatched arms, one arm being about 2 centimeters long and the other over one meter long. The interferometer was aligned to create a circular interference pattern. We then adjusted the the lenses used to telescope up the size of the laser beam until no interference rings were visible. When measuring prism deflection angles we made sure that the two optical paths were the same length on a millimeter scale and that the two interfering beams hit
the camera at nearly the same place. This made any residual wavefront curvature common to both field components such that it did not affect on our results.

The detector on the camera used in these experiments was smaller than the laser beam diameter. Since catching only part of the interference pattern limits sensitivity to small relative beam angles, we used a lens to demagnify the pattern. To account for the demagnification and to find the correct “effective size” of the camera pixels we placed a ruler in front of the lens. The ruler’s position was adjusted until it came into clear focus on the camera. We then took pictures of the ruler to determine the magnification due to the lens. We verified that this had been done correctly by using the lens’ focal length and the distance to the camera to calculate the position at which we would expect the ruler to come into focus and the expected magnification.

When we evaluated the left-hand side of Eq. 4, we had to be careful to utilize only the parts of the images where sufficient laser light was present in both beams to avoid large errors due to division by small numbers (see Fig. 2(e)). We designed our software to prompt the user to select a region of interest to avoid regions of low intensity. The left-hand side of equation Eq. 4 is then computed in this region. The software then fits a horizontal row of data in the middle of the selected region to the function \( \cos(k_x x + \phi_x) \) and fits a vertical column of data in the middle of the region to the function \( \cos(k_y y + \phi_y) \). From these two one-dimensional fits it calculates \( k_{rel} \) and determines the angle between the beams.

**MEASURING PRISM BEAM DEFLECTIONS**

The following paragraphs discuss several methods which we used to characterize the properties of parallel-plate beamsplitters, penta prisms, right angle prisms, and corner cubes. We tested uncoated optics. Light intensity was lost due to imperfect transmission each time a beam entered or exited a prism. Much larger losses occurred due to missing reflective coatings on the beamsplitters and the penta prisms (right angle prisms and corner cubes do not require reflective coatings due to total internal reflection). But even with these losses we could still saturate the camera. Balancing the intensities of the two interfering beams was necessary to achieve high contrast fringes to get the most accuracy with the fixed bit-depth of our camera. Our methods have symmetric losses in each beam, resulting in well matched beam intensities.

**Absolute Beamsplitter Characterization**

The beamsplitters we measured were uncoated plates of BK7 glass with parallel surfaces. As shown in Fig. 4, when a laser beam passes through an uncoated piece of glass, surface reflections result in multiple beams exiting the glass. We are concerned only with the beam which passes through without reflecting and the nearly parallel beam resulting from one reflection from each surface (labelled 1 and 2 in the figure). If the two beamsplitter surfaces are exactly parallel, these two beams will emerge exactly parallel. Otherwise there will be an angle \( \theta \) between the two exiting beams (see Fig. 4). By measuring \( \theta \), the prism wedge angle \( \psi \) can be inferred.

The relationship between \( \theta \) and \( \psi \) can be found using Snell’s law and the law of reflection. If beam 1 in Fig. 4 defines the \( z \) axis and the \( x \) axis is defined such that the angle \( \gamma \) is in the \( x\-z \) plane, in the limit of small wedge angles the \( x \) component of \( \psi \) is related to the \( x \) component of \( \theta \) by

\[
\psi_x = \frac{\theta_x}{2} \sqrt{\frac{1 - \sin^2(\gamma)}{n^2 - \sin^2(\gamma)}}
\]

(5)
In this arrangement the interference pattern does not reveal the wedge angle of a single beamsplitter, but gives a combination of the wedge angles of both beamsplitters. To find the wedge angle of a single beamsplitter we make four measurements using different combinations of three beamsplitters and use the fact that flipping a beamsplitter over effectively reverses the sign of its wedge angle. The first and second measurements use beamsplitters “A” and “B” with beamsplitter “B” turned over between measurements. The third and fourth measurements use beamsplitters “A” and “C” with beamsplitter “C” turned over between them. In each of the four configurations we measure the \( k_x \) and \( k_y \) of the interference pattern to extract the magnitude of the \( x \) and \( y \) components of the angle between the outgoing interfering beams using the methods discussed previously.

If \( \theta_{Ax} \), \( \theta_{Bx} \), and \( \theta_{Cx} \) represent the \( x \) components of the relative deflection errors of beamsplitters “A,” “B,” and “C,” and the magnitudes of the \( x \) components of the angle between the interfering beams in the four measurements are represented by \( M_{1x} \), \( M_{2x} \), \( M_{3x} \), and \( M_{4x} \), the four measurements yield the following results:

\[
M_{1x} = \theta_{Ax} - \theta_{Bx}, \quad (6)
\]
\[
\pm M_{2x} = \theta_{Ax} + \theta_{Bx}, \quad (7)
\]
\[
\pm M_{3x} = \theta_{Ax} - \theta_{Cx}, \quad (8)
\]
\[
\pm M_{4x} = \theta_{Ax} + \theta_{Cx}. \quad (9)
\]

A similar set of equations can be written down for the \( y \) components. Fitting our data using Eq. 4 does not reveal the sign of the angle between the two interfering beams. But we can assume a convention in which the angle between the two interfering beams is defined to be positive for our first measurement. For the following measurements we must stick to the same convention. The \( \pm \) sign in the lower three relations therefore results from the uncertainty in the sign of the angle between the interfering beams when they are measured interferometrically.

The equations can be solved for the \( x \) component of the relative deflection angle of each beamsplitter as a function of the four measured angles. But without knowledge of the sign of the angle between the interfering beams, these expressions cannot be evaluated. Fortunately, the above system of four equations yields two independent expressions for \( \theta_{Ax} \), one in terms of \( M_{1x} \) and \( M_{2x} \), and the other in terms of \( M_{3x} \) and \( M_{4x} \). In most cases the requirement that \( \theta_{Ax} \) be the same as determined by both equations unambiguously determines the sign of each measurement term. Once the signs are determined, the wedge angle for each of the three beamsplitters can be determined. Using this technique we characterized several high-precision beamsplitters, measuring wedge angles from 1 to 6 \( \mu \text{rad} \).
Relative Penta Prism Characterization

Our application does not place tight requirements on the absolute angular deflection produced by our penta prisms. It does, however, require that pairs of penta prisms be precisely matched. As such we measured the relative deflection of each matched pair rather than the absolute deflection of individual prisms. We did this using the optical configuration shown in Fig. 6. In this configuration one of the plate beamsplitters, characterized using the methods described above, was used to generate two parallel beams. These beams were then folded at right angles using a pair of penta prisms. The two beams were then recombined using a second plate beamsplitter.

In this layout the two beam paths are symmetric, allowing us to make the two path lengths nearly the same and making for equal intensity losses in each beam as they reflect off of our uncoated prisms. We used the same angle of incidence for both beamsplitters to make the Fresnel coefficients equal. To get the two interfering beams to overlap we adjusted the separation of the penta prisms to make the spacing between the two beams entering the second beamsplitter equal to the spacing of the two beams exiting the first beamsplitter.

Penta prisms ensure deflection of a beam by a precise angle in the plane of the prism. If, however, one prism is tilted out of the plane defined by the other prism, the two interfering beams would be at an angle to one another determined not by the accuracy of the prisms but by their relative alignment. For small misalignments we can think of the light deflection by the second prism as a fixed deflection in the plane defined by the first prism plus an out-of-plane deflection due to misalignment. As such, the magnitude of the wave vector describing the sinusoidal interference pattern measured at the output would equal the quadrature sum of two orthogonal components: a component due to errors in the manufacture of the prisms and a component due to the relative alignment of the prisms, as shown in Eq. (10) below.

$$k_{rel} = \sqrt{k_p^2 + k_a^2}$$

Here $k_p$ represents the component due to error in the prism, and $k_a$ represents the component due to alignment error.

Because $k_{rel}$ is at a minimum when there is no alignment error (i.e., when $k_a = 0$), it is possible to measure $k_p$ by making measurements while adjusting the out-of-plane alignment of one prism. Rather than searching for a minimum value, we took several measurements at different alignments and fit our measurements to the form of equation Eq. (10) to extract an accurate value for $k_p$. To do this we mounted one of our prisms on a piezoelectric (PZT) mount which enabled fine alignment adjustments. We would manually adjust the alignment such that the minimum of $k_{rel}$ occurred near the middle of the range of our PZT actuator. We then took images as we scanned the PZT.

Because our fringe analysis method utilizes data taken at a single moment in time, we were able to make precise measurements of $k_p$ even though our PZT actuator was unstable. Assuming that $k_a$ will be proportional to the voltage $V$ applied to the piezoelectric element, we can take the measured $k_{rel}$ as a function of $V$ and perform a curve fit to find $k_p$. This curve fit requires two free parameters (in addition to $k_p$): the voltage at which $k_{rel} = 0$ and the constant of proportionality between $V$ and $k_a$. As shown in Fig. 7(a), however, due to nonlinearity and drift in our piezoelectric mount the data does not fit the hyperbolic form of Eq. (10) well. But since the $k_p$ component was approximately in the horizontal plane of our camera and $k_a$ was in the vertical, we could perform much better fits when we plotted the total $k_{rel}$ vs. $k_y$, the vertical component of $k_{rel}$ extracted by our image analysis software. These fits had no free parameters. Typical curve fits are shown in figures 7(b) and (c).

The fit in Fig. 7(b) yields a $k_p$ of 64.4 rad/m corresponding to a relative deflection angle of 6.5 μrad for the two prisms with an RMS fit error corresponding to 0.38 μrad. Scanning the PZT had the side effect of moving the location of bright and dark fringes such that some images contained an extremum. But comparing the data points in 7(b) which contained an extremum to those which didn’t, it is clear that this did not significantly reduce the accuracy of the fits. A fit using just the data for which the image did not contain an interference minimum or maximum gives a relative deflection of 7.5 μrad. Although most of the information in the plots is contained in the lowest points where the hyperbola is dominated.

![Optical setup](image-url)
FIG. 7: Finding the relative deflection error of two penta prisms. The magnitude of the wave vector describing the interference pattern at different prism alignments is plotted vs. the PZT voltage (a) and vs. the $y$ component of the wave vector (b). The crosses and the asterisks represent the actual data extracted from the interference patterns. The asterisks represent the data points which should be the most accurate since the image happened to fall between a light and a dark fringe. The crosses represent data points for which the image contained a light or dark extremum. The lines represent equally weighted least-squares fits of the entire data set to Eq. 10. Data from a different set of prisms which did not meet our specifications is shown in (c).

by $k_p$, simply fitting to the two points at the extremes of the scan gives a reasonable relative deflection of 6.8 $\mu$rad, implying that only a small number of images are needed to get accurate results. Similar results were seen for our other prism pairs suggesting a repeatability of this method at the $\mu$rad level. Due to the known deflection error of the beamsplitters used in these measurements, the absolute accuracy of our measurements was limited to about 2 $\mu$rad.

The consistency of the data in Fig. 7 gives a good idea of the overall accuracy of our fringe measurement technique. One sign of self-consistency is the fact that the asymptotes of the hyperbola in Fig. 7(b) cross at a value of $k_{rel}$ which is very close to zero. In all of our measurements of precision prism pairs we measured offsets corresponding to angle measurement errors ranging from nearly zero to 1.02 $\mu$rad. Another indication of the accuracy of our fringe analysis is the low RMS error of the curve fits to Eq. 10. These ran from 0.40 to 1.24 $\mu$rad.

Right Angle Prism and Corner Cube Characterization

We characterized the relative deflection of pairs of right angle prisms using a scheme similar to the one we used for penta prisms. Because these prisms deflect light back towards the beamsplitter, an optical layout analogous with the one we used to measure penta prisms cannot be used — a beam reflected off of one prism would be occluded by the second prism. One approach would be to use a design in which the beams were deflected vertically back to a second beamsplitter placed above the first beamsplitter. To avoid the complications of multi-tiered optics, we instead used the layout shown in Fig. 8. In this design a single beamsplitter is used to split and recombine the two beams. Unlike the schemes described earlier in this paper, the intensities of the two interfering beams are not precisely balanced in this setup; while both paths involve one beamsplitter reflection, the path through the upper prism undergoes two more transmissions through beamsplitter surfaces than the path through the lower prism. Due to the low reflectivity of the uncoated beamsplitters we still achieved nearly 100% fringe contrast. This same set-up could also be used to characterize corner cubes.

In addition to the two beams we are interested in, a third beam travelling through the upper prism in the opposite direction can have an effect on the interference pattern. This beam undergoes two additional beamsplitter reflections and is therefore much less intense. When measuring right angle prisms, the prisms can be tilted vertically to walk this stray beam out of the interference pattern. With the vertical alignment walked off, the distance between the beamsplitter and the prisms will have to be adjusted to achieve good overlap of the interfering beams. As with the penta prism measurements, to
measure the difference in the intrinsic deflection angles of two right angle prisms we scanned the vertical angle of one of the prisms and then fit the measured relative beam angles to Eq. 10. Using this method we measured the relative deflection angle of pairs of high-quality right angle prisms. The repeatability of these measurements was similar to what we achieved with our penta prism measurements.

To measure the absolute deflection angle of a single right angle prism or corner cube we used the scheme illustrated in Fig. 9. Unlike the other schemes presented in this paper this scheme requires that the beamsplitter angles be chosen carefully. Simpler designs using one or two beamsplitters had problems with stray reflections which resulted in interference of more than two paths and unequal intensities of interfering beams. The three beamsplitter design allows us to control stray reflections but requires a different angle of incidence at each beamsplitter. This results in different Fresnel reflection coefficients at each beamsplitter. Also, like our scheme for relative measurements of right angle prisms, in this setup one of the beams undergoes two more transmissions through a beamsplitter surface than the other beam. By carefully choosing the beamsplitter angles one can make the two pathways overlap and be equal in intensity at the camera. This is easily done with a knowledge of the beamsplitter thickness and index of refraction.

We used this method to measure the absolute deflection angles of several high-quality right angle prisms as well as a low-quality right angle prism and a high-quality corner cube. The high-quality prisms and the corner cube deflection angles were typically found to deviate from 180 degrees by a few microradians. The deflection angle of the cheap right angle prism was found to be much less accurate. Once again we found repeatability at the µrad level.

FIG. 8: Optical setup for measuring the relative deflection of two right angle prisms or corner cubes.

FIG. 9: Optical setup for absolute measurement of right angle prism and corner cube deflection angles.

COMPONENTS USED

Our measurements used only the prisms under test and parts available in our lab. The laser was an inexpensive ≈ 1 mW helium-neon alignment laser [JDS Uniphase Model 1507P]. Because the prisms and beamsplitters did not have reflective coatings, only about 0.01 to 1 percent of the laser light reached the camera depending on the type of prisms being measured. Even so we still needed significant attenuation to avoid saturating the camera. The laser had a good spatial mode and a coherence length long enough to produce good interference fringes on the asymmetric Michelson-Morley interferometer mentioned previously. A laser with poorer spatial and temporal qualities could also have been used. The required spatial mode can easily be achieved by spatial filtering, especially considering the low power needed. If the two optical paths are made equal within about 1 mm when measuring the prisms, a short-term linewidth of
tens of GHz would be sufficient to produce high-contrast fringes. Although a short coherence length would not allow collimation to be tested using an asymmetric interferometer, there are many other ways to ensure good collimation.

The camera was a $156 closed circuit surveillance camera connected to a computer frame grabber card. The low-quality camera resulted in three significant difficulties. First was the camera’s nonlinear response. Our camera was not designed for scientific work and its response function was not well calibrated. As a result, in our first measurements the “cosine” function in Eq. 4 did not oscillate between -1 and 1. We attempted to characterize the camera’s response (surveillance cameras usually have a response in which the value of each pixel is proportional to the intensity of light on the pixel raised to some power $\gamma$). But we found that, even with a fixed iris setting, at high intensities the signal reported on one pixel depended on the intensity present on other parts of the chip! But for sufficiently low intensities the camera response was fairly linear. So our solution was to reduce light intensities by adding attenuators in front of the camera until the highest value reported at any pixel was 80 counts (out of a maximum of 255 counts for the 8-bit camera).

The other two problems with the camera were related to its low signal-to-noise ratio and to an uncoated window on the front of the camera. The signal-to-noise problem was overcome by averaging 50 frames to produce each image. This took less than 2 seconds on our 30 frames-per-second video camera. The uncoated window affixed to the camera produced low-contrast interference fringes in our data (see Figs. 2 and 3). We were unable to remove this window. But by tilting the camera we were able to make the spatial frequency of these fringes high enough that they did not confuse the fitting routines when fitting the much broader fringes due to the relative angle of the two interfering beams. Note that Eq. 4 was derived under the assumption that the beams strike the camera near to normal incidence. The equation is still approximately correct when we tilt the camera, especially when the camera is tilted around an axis which is nearly perpendicular to the fringes. Tilting our camera, therefore, did not change the way that we analyzed our images and the residual error due to the camera tilt was negligible.

The lenses and mirrors we used were standard research-quality optics which were already available in the lab. Two lenses were used to telescope and collimate the laser beam before entering the interferometer. These had to be of reasonable quality to prevent significant wavefront distortion of the laser. Any distortion due to these lenses is common to both of the interfering beams and should have a reduced impact on the measured fringes. A third lens was used to demagnify the interference pattern to fit onto the camera. This lens simply images the interference pattern. Small wavefront errors at this lens do not have an effect on the measurement, and only its imaging characteristics need to be considered. Like the lenses, the mirrors were only employed before the optical beam was split or after the two paths had been recombined such that wavefront distortions were common to both paths.

After verifying the quality of our parallel-plate beam-splitters, these optics were used in the evaluation of the other prisms. Therefore our measurements were limited to the accuracy of the beamsplitters. It should be possible to remove this offset by careful characterization of the beamsplitters used and by making two sets of measurements with the beamsplitters flipped over between them. But given the $\lambda/10$ surface quality of the beamsplitters, it is possible that the deflection angles for two different 1 cm sized spots on a beamsplitter will differ at the microradian level even if the optic has no overall deflection error. Lowering this systematic error, therefore, would require either beamsplitters with better surface flatness or calibration at the precise locations at which beams enter and leave the beamsplitters.

**CONCLUSIONS**

In conclusion, we have demonstrated several relatively simple and inexpensive techniques to characterize the deflection angles of parallel-plate beamsplitters, penta prisms, right angle prisms, and corner cubes. We have achieved accuracies at the level of 2 $\mu$rad (0.4 arcseconds), approaching what is possible in high-end commercial devices. Better results are likely to be possible by calibrating and removing effects due to imperfect beam-splitters and by using a higher quality detector.

We acknowledge the contributions of Rebecca Merrill and Elizabeth Cummings. This work was supported by the Research Corporation and the National Science Foundation.

* Currently at: Department of Physics, University of Maryland, College Park, Maryland 20742, USA

[1] J. O. Dickey, P. L. Bender, J. E. Faller, X. X. Newhall, R. L. Ricklefs, J. G. Ries, P. J. Shelus, C. Veillet, A. L. Whipple, J. R. Wiant, J. G. Williams, and C. F. Yoder, “Lunar Laser Ranging: A Continuing Legacy of the Apollo Program,” Science 265, 482-490 (1994).

[2] M. V. Mantravadi, “Newton, Fizeau, and Haidinger Interferometers,” in Optical Shop Testing, 2nd ed., D. Malacara, ed., (John Wiley and Sons, Inc., New York, 1992), pp. 1-50.

[3] D. Malacara, “Twyman-Green Interferometer,” in Optical Shop Testing, 2nd ed., D. Malacara, ed., (John Wiley and Sons, Inc., New York, 1992), pp. 51-94.
ed., (John Wiley and Sons, Inc., New York, 1992), pp. 715-742.

[5] M. V. Mantravadi, “Lateral Shearing Interferometers,” in *Optical Shop Testing*, 2nd ed., D. Malacara, ed., (John Wiley and Sons, Inc., New York, 1992), pp. 123-172.

[6] J. E. Greivenkamp and J. H. Bruning, “Phase Shifting Interferometers,” in *Optical Shop Testing*, 2nd ed., D. Malacara, ed., (John Wiley and Sons, Inc., New York, 1992), pp. 501-598.