On covariant quantization of M0-brane.
Spinor moving frame, pure spinor formalism
and hidden symmetries of D=11 supergravity

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Abstract

The covariant quantization of massless D=11 superparticle (M0–brane) in its twistor-like Lorentz harmonic formulation is used to clarify the origin and some properties of the Berkovits pure spinor approach to quantum superstring and to search for hidden symmetries of D=11 supergravity. In the twistor like Lorentz harmonic formulation, the SO(16) symmetry is seen already at the classical level. The quantization produces the linearized supergravity multiplet as $128 + \tilde{1}28 = 256$ component Majorana spinor of $SO(16)$ and also shows an indirect argument in favor of the possible $E_8$ symmetry.

1. Introduction

In this contribution we briefly review the results of [1, 2, 3] on the covariant quantization of the M0–brane (D=11 massless superparticle) [4, 5] in its twistor-like Lorentz harmonic or spinor moving frame formulation [6, 3]. We show how the covariant BRST quantization of this model [1, 2] explains origin and some properties of the Berkovits pure spinor approach [7] (see also e.g. [8, 9] and refs. therein), in the frame of which a significant progress in covariant loop calculations has been reached [10]. Then we discuss how the covariant quantization of physical degrees of freedom [3, 2] shows the tails of possible hidden symmetries of the eleven dimensional supergravity.

2. Brink–Schwarz superparticle and its $\kappa$–symmetry

The Brink-Schwarz massless superparticle action, $(\Pi^m := dx^m - id\theta^m \Gamma^m \theta = d\tau \tilde{\Pi}_m)$

\[ S_{BS} = \int_{W^1} \left( P_m \Pi^m - \frac{1}{2}d\tau e P_m P^m \right), \]

possesses the local fermionic $\kappa$-symmetry [11, 12]:

\[ \delta_\kappa x^m = i\delta_\kappa \theta^a \Gamma^m_{a\beta} \theta^\beta, \quad \delta_\kappa \theta^a = \tilde{\rho}^{a\beta} \kappa_\beta, \quad \delta_\kappa P^m = 0. \]
This is necessary to provide that the ground state of the model preserves a part (1/2) of supersymmetry and, thus, is a stable state - called BPS state (as they saturate a Bogomol’nyi–Prasad–Sommerfield bound). However this symmetry happens to be infinitely reducible. Indeed, the \( \kappa \)-symmetry parameter of the form \( \tilde{\kappa}_\beta = \tilde{P}^{\beta \gamma} \tilde{K}_\gamma \) does not produce any transformations of \( x^m(\tau) \) and \( \theta(\tau) \). First one observes that \( \tilde{P}^{\alpha \beta} \tilde{K}_\gamma = P_m P^m \kappa_\alpha \) which vanishes due to the mass shell constraint

\[
P_m P^m = 0
\]

which appears as equations of motion for the auxiliary einbein field \( e(\tau) \) in (1). In the terminology of [13] this is characterized by stating that the symmetry (\( \kappa \)-symmetry in our case) has the null vector \( (\kappa^\alpha = \tilde{P}^{\alpha \gamma} \tilde{K}_\gamma) \). The symmetry which has a null vector is called reducible. Provided this null vector did not have its own null vector, the reducibility would be of the first rank and the effective number of symmetry parameters would be equal to the number of manifest parameters minus the number of null vector. If the null vector had its own null vector, but the latter did not have the null vector of its own, the reducibility of the symmetry would be of the second rank and the effective number of symmetry parameters would be calculated as the manifest number of parameters minus number of null-vectors plus minus number of null-vectors for null vectors, and so on.

For the \( \kappa \)-symmetry, one notices that the null–vector \( \tilde{\kappa}_\alpha = \tilde{P}^{\alpha \gamma} \tilde{K}_\gamma \) has its own null-vector \( \tilde{\kappa}_\alpha = \tilde{P}^{\beta \gamma} \tilde{K}_\gamma \) which has null–vector \( \tilde{\kappa}_\alpha = \tilde{P}^{\beta \gamma} \tilde{K}_\gamma \) etc.. This chain is infinite as \( j + 2 \) null–vector coincides with the \( j \)-th one. Furthermore, all the (reducible) null-vectors have also the same dimension equal to the number of component \( n \) of the minimal spinor representation \( (\alpha, \beta = 1, ..., n) ; \ n = 32 \) for \( D = 11 \), \( n = 16 \) for \( D = 10 \), etc.). Then the effective number of supersymmetry parameter is defined as an infinite series

\[
n - n + n - n + ... = n \cdot \sum_{j=0}^{\infty} (-1)^n = n \cdot \lim_{q \to 1} \sum_{j=0}^{\infty} \frac{(-1)^n}{q^j} = n \cdot \lim_{q \to 1} \frac{1}{1-q} = \frac{n}{2},
\]

so that \( D = 11 \) model \( (n = 32) \) is invariant under 16 \( \kappa \)-symmetries and 32 supersymmetries.

In the Hamiltonian formalism, the above nilpotent matrix \( \tilde{P}^{\alpha \beta} \) can be used to extract the (infinitely reducible) generator of the \( \kappa \)-symmetry, \( \tilde{P}^{\alpha \beta} d_\beta \), from the \( n \) fermionic primary constraints \( d_\alpha = \pi_\alpha + i P_{\alpha \beta} \theta^\beta \) which obey the algebra \( \{ d_\alpha , d_\beta \} = 2i P_{\alpha \beta} \).

\[
d_\alpha := \pi_\alpha + i P_{\alpha \beta} \theta^\beta \approx 0 , \quad \{ d_\alpha , d_\beta \} = 2i P_{\alpha \beta} \equiv 2i \Gamma_{\alpha \beta}^m P_m
\]

\( (\pi_\alpha := \frac{\partial L}{\partial \dot{\theta}^\alpha} \) and \( P_m := \frac{\partial L}{\partial \dot{x}^m} \) are fermionic and bosonic momenta canonically conjugate to \( \theta^\alpha(\tau) \) and \( x^m(\tau) \). Indeed, taking into account (3), one finds that \( \{ \tilde{P}^{\alpha \gamma} d_\gamma , d_\beta \} = 2i \delta_\beta^\alpha P^2 \approx 0 \), where \( \approx \) denotes the weak equality in the sense of Dirac [15]. On the other hand, the covariant extraction of the second class constraint from the fermionic primary constraint (4) and, hence, the covariant separation of the fermionic first and second class constraints, is not possible in the frame of classical Brink-Schwarz formulation, i.e. without introducing additional variables.

The infinite reducibility of the \( \kappa \)-symmetry and impossibility to separate covariantly the fermionic first and the second class constraints are also characteristics of the Green–Schwarz superstring. Many years these properties hampered the way to the covariant superstring quantization.\[1\]

\[1\]See [16] for an approach to quantization with infinitely many ghost for ghosts and its problems.
3. ‘Pure spinor’ BRST charge and its derivation in the superparticle spinor moving frame formulation

The problem of covariant superstring quantization is now considered to be resolved by the pure spinor formalism proposed by Berkovits in 2000 [7]. It is based on the intrinsically complex BRST charge

$$Q^B = \Lambda^\alpha d_\alpha ,$$

(5)

where \(d_\alpha\) are the fermionic constraints (4) and \(\Lambda^\alpha\) is the complex pure spinor which obeys

$$\Lambda \Gamma_a \Lambda = 0, \quad \Lambda^\alpha \neq (\Lambda^\alpha)^* , \quad \alpha = 1, \ldots, n \quad (n = 32 \text{ for } D = 11)$$

(6)

This constraint guarantees the nilpotency, \(\{Q^B, Q^B\} = 0\), of the BRST charge (5).

It is important that a solution of the constraint (6) is provided by [1]

$$\tilde{\Lambda}_\alpha = \tilde{\lambda}_p v_{ap}^-, \quad \tilde{\lambda}_p^+ \tilde{\lambda}_p^- = 0, \quad \{v_{ap}^-\} = \text{Spin}(1, D-1) \otimes \text{Spin}(D-2) \otimes \mathbb{R}_{(D-2)} = S^{D-2} ,$$

(7)

where \(\tilde{\lambda}_p^+\) is a complex \(n/2\) component \(SO(D - 2)\) spinor with zero norm, \(\tilde{\lambda}_p^+ \tilde{\lambda}_p^- = 0\), and \(v_{ap}^-\) are spinorial Lorentz harmonics [6, 17, 18] (see [1, 2, 3] for more references and discussion), a set of \(n/2\) constrained \(n\)-component \(D\)-dimensional bosonic spinors which, once the constraints are taken into account, provide the homogeneous coordinates for the \(D\) dimensional celestial sphere \(S^{D-2}\) (see Eq. (23) below).

The above equations are general, they are applicable, with \(n = 2, 4, 8, 16, 32\), for \(D = 3, 4, 6, 10, 11\). However, some properties of spinor moving frame formalism are dependent on \(D\). In particular, in \(D=11\) Eq. (7) provides a particular solution of the pure spinor constraint, while in \(D=10\) it gives the general solution [2].

In [1, 2] it was shown how the complex BRST charge (6) for \(D = 11 \text{ (} n = 32 \text{) case can be obtained on the way of covariant BRST quantization of the } M0\text{-brane, i.e. eleven dimensional massless superparticle, in its spinor moving frame or twistor like Lorentz harmonic formulation [6] (described below, in Sec. 4; see [19, 20], [21] and [22] for spinor moving frame formulations of superstring, standard super-p-branes and super-Dp-branes). Namely, in [1, 2] we first constructed the Hamiltonian mechanics of this twistor-like formulation of the \(D = 11\) superparticle and, with the help of the spinorial Lorentz harmonics, separated covariantly the first and the second class constraints (see [20] for an analogous result for the Green-Schwarz superstring). Then we took into account the second class constraints by introducing Dirac brackets [15], and calculate the Dirac brackets algebra of the first class constraints. Further, following the pragmatic spirit of the Berkovit’s approach [7, 10], we take care of the part of the first class constraints separately (partially, by imposing them as a condition on the wavefunctions; one may also think on the gauge fixing at the classical level) and left with a set of 16 fermionic and 1 bosonic first class constraints, the generators of the fermionic \(\kappa\)-symmetry and its bosonic \(b\)-symmetry superpartner, the Dirac brackets of which represent the \(d = 1\), \(n = 16\) supersymmetry algebra (the origin of \(\kappa\)-symmetry as worldline supersymmetry was found in [23]). This set of constraints is described by the BRST charge [1, 2]

$$Q^{susy} = \lambda_q^+ D_q^- + ic_{++}^+ \partial_{++} - \lambda_q^+ \lambda_q^+ \frac{\partial}{\partial c_{++}}, \quad \{D_p^-, D_q^-\} = 2i\delta_{qp}\partial_{++},$$

(8)

Indeed, in \(D = 11\) the generic null spinor \(\Lambda_\alpha\) contains 23 complex or 46 real parameters [7], while eq. (7) provides its 39 parametric solution. In \(D=10\) the solution of Eq. (7) carries \(16+8-2=22\) degrees of freedom, the same number as the generic pure spinor.
including 16 real bosonic ghosts $\lambda^+_q$ and one real fermionic ghost $c^+$. 

An analysis of the cohomology of this BRST operator shows that it is trivial if the norm $\lambda^+_q \lambda^+_q$ of bosonic ghost $\lambda^+_q$ is nonvanishing. In other words, the nontrivial cohomology of $Q^\text{susy}$ has support on $\lambda^+_q \lambda^+_q = 0$. For a real spinor $\lambda^+_q \lambda^+_q = 0$ implies $\lambda^+_q = 0$. This produces a technical problem which is sorted out by means of a regularization which consists in allowing $\lambda^+_q$ to be complex, $\lambda^+_q \mapsto \tilde{\lambda}^+_q \neq (\lambda^+_q)^\star$. Furthermore, this implies the reduction of the cohomology problem for the regularized BRST operator $Q^\text{susy}$ to the search for cohomology at vanishing bosonic ghost, $\tilde{\lambda}^+_q = 0$, for the following complex BRST charge

$$
\tilde{Q}^\text{susy} = \tilde{\lambda}^+_q D_q^- + ic^+ \partial_{++}, \quad \tilde{\lambda}^+_q \tilde{\lambda}^+_q = 0, \quad \{D_p^-, D_q^-\} = 2i\delta_{qp}\partial_{++}.
$$

Now, taking into account that $D_q^-$ represents the constraint $d_q^- = v_q^{-\alpha}d_\alpha$, where $d_\alpha$ is the Brink–Schwarz fermionic constraint, one finds that this non-Hermitian $\tilde{Q}^\text{susy}$ operator is essentially (modulo additional $ic^+ \partial_{++}$ contribution) the Berkovits BRST operator $\tilde{Q}^\text{susy}$, but with composite pure spinor $\tilde{Q}^\text{susy}$.

Thus [1, 2] have shown (on the example of superparticle) the possible origin of the intrinsic complexity of the Berkovits pure spinor BRST charge: it appears on the stage of regularization in calculation of cohomology of the real BRST charge [3].

Let us stress that of all the cohomologies of the Berkovits-like BRST charge $\tilde{Q}^\text{susy}$ only the ones calculated (and remaining nontrivial) at $\tilde{\lambda}^+_q = 0$ describe the cohomology of the superparticle BRST operator $Q^\text{susy}$ [1, 2]. The full cohomology of $\tilde{Q}^\text{susy}$ is clearly reacher and is related with spinorial cohomologies of [24].

4. Spinor moving frame of formulation of massless superparticle

Since the constraint [3] is algebraic, it may be substituted into the action (1), which gives $S_{M0}^\prime = \int_{W_1} P_m \Pi^m |P_m|_{P_m = 0}$. Thus, if the general solution of (3) is known, one may substitute it for $P_m$ in (1) and obtain a classically equivalent formulation of the Brink–Schwarz superparticle. It is easy to solve the constraint (3) in a non-covariant manner: in a special Lorentz frame a solution with positive energy reads as, e.g., $\tilde{P}_m = \frac{P}{2} (1, \ldots, -1) = \frac{\not{P}}{2} (\delta_0^\alpha - \delta_0^{\#\alpha})$. The solution in an arbitrary frame follows from this by making a Lorentz transformation,

$$
P_m := U_m (a) \tilde{P}_m (a) = \frac{P}{2} (u_m^0 - u_m^{\#}), \quad U_m (a) := (u_m^0, u_m^i, u_m^{\#}) \in SO(1, D - 1).
$$

Since $P_m = P_m(\tau)$ is dynamical variable in the action (1), the same is true for the Lorentz group matrix, $U_m (a) = U_m (a)(\tau) = (u_m^0(\tau), u_m^i(\tau), u_m^{\#}(\tau))$ in Eq. (10). Such moving frame variables [19] are called Lorentz harmonics (see [25, 16, 17, 11, 2] and refs. therein).

Substituting (10) for $P_m$ in (1), one arrives at the action

$$
S = \int_{W_1} \frac{1}{2} \rho^{++} u_m^- \Pi^m, \quad u_m^- u_m^- = 0
$$

where the light–likeness of the vector $u_m^- = u_m^0 - u_m^{\#}$, follows from the orthogonality and normalization of the timelike $u_m^0$ and spacelike $u_m^{\#}$ vectors which, in their turn, follow from $U := \{u_m^0, u_m^i, u_m^{\#}\} = \{u_m^0 + u_m^-, u_m^i, \frac{u_m^0 - u_m^-}{2}\} \in SO(1, 10)$.

An important property of the action (11) is that it hides the twistor–like action

$$
S : = \int_{W_1} \frac{1}{2} \rho^{++} u_m^- \Pi^m = \int_{W_1} \frac{1}{32} \delta_{\alpha\beta} v_{\alpha q} \tilde{\Pi}_{\alpha q} \tilde{\Xi}_m^\alpha \beta,
$$

(12)
where, for \( D = 11 \) case \( \alpha = 1, 2, \ldots, 32 \) (in general), \( q = 1, \ldots, 16 \) (in general) and \( m = 0, \ldots, 9, \# \), \((\#) = 10, (D - 1)\) in general). The first from of the action (12) coincides with (11); the second form is twistor-like, i.e. it generalizes the Ferber–Schirafuji (FS) action [20] to arbitrary \( D \); the original \( D = 4 \) FS action is reproduced from \( n = 4 \) version of (12) after writing the \( D = 4 \) Majorana spinors in terms of two Weyl ones. In \( D = 11 \), instead of two–component unconstrained Weyl spinor in [26], the action of Eq. (12) includes the set of 16 bosonic 32–component Majorana spinors \( \nu_{\alpha q} \) which satisfy the following kinematical constraints (see [19, 20, 6]),

\[
\begin{align*}
2\nu_{\alpha q} \nu_{\beta q}^- &= u_m^- \Gamma_{\alpha\beta}^m \quad (a), \\
v_q^- \Gamma_m v_p^- &= \delta_{qp} u_m^- \quad (b), \\
v_{\alpha q} \Gamma^{\alpha\beta} v_{\beta q}^- &= 0 \quad (c), 
\end{align*}
\]

Although, in principle, one can study the dynamical system using just the kinematical constraints (13), for many problems, including the covariant quantization, it is more convenient to treat the set of 16 constrained \( SO(1,10) \)–spinors \( \nu_{\alpha q}^- \) as part of the corresponding \( Spin(1,10) \)–valued matrix describing the spinor moving frame,

\[
V_\alpha^{(\beta)} = (\nu_{\alpha q}^-, \nu_{\alpha q}^+) \in Spin(1,10) \quad (\in Spin(1,D - 1) \text{ in general}),
\]

These spinor moving frame variables, \( \nu_{\alpha q}^- \), \( \nu_{\alpha q}^+ \), are also called spinor Lorentz harmonics.

### 4.1. Vector and spinor Lorentz harmonics. Spinor moving frame

The relation between vector Lorentz harmonics \( u_m^{\pm} \), \( u_m^{i} \) [25], which, in the \( D=11 \) case are elements of the \( SO(1,10) \) Lorentz group matrix

\[
U_m^{(a)} = (u_m^-, u_m^+, u_m^i) \in SO(1,10) \quad (\in SO(1,D - 1) \text{ in general}),
\]

and the spinor harmonics [17] or spinor moving frame variables [19, 20, 21, \( \nu_{\alpha q}^\pm \) Eq. (14), are defined by the Dirac matrices conservation

\[
VT^{(a)} V = \Gamma^m U_m^{(a)} \quad (a), \quad VT \Gamma_m V = U_m^{(a)} \Gamma^{(a)} \quad (b),
\]

and also by conservation of the charge conjugation matrix, if this exists,

\[
V CV^T = C \quad , \quad V^T C^{-1} V = C^{-1}.
\]

In this sense one says that the spinorial harmonics are ‘square roots’ of the associated vector harmonics.

Eqs. (16) implies Eqs. (15), (14) modulo scaling factor \((U \mapsto e^{2\gamma} U, V \mapsto e^{\gamma} V)\). The fact that \( U \in SO(1,10) \) implies the following set of constraints

\[
U^T \eta U = \eta \iff \begin{cases}
u_m^- - u_m^- = 0, & u_m^+ - u_m^i = 0, \\
u_m^+ - u_m^i = 2, & u_m^i = -\delta_{ij}.
\end{cases}
\]

or, equivalently, \( \delta_n = \frac{1}{8} u_m^- u_m^i + \frac{5}{2} u_m^- u_m^i - u_i u_m^i \) (\( \Rightarrow U \eta U^T = \eta \)).

The relations (16), (17) reproduce the constraints (13). Indeed, using the Dirac matrices realization with diagonal \( \Gamma^0 \) and \( \Gamma^\# \), one may check that (13a) coincides the \((a) = (--) \equiv (0) - (\#)\) component of Eq. (16a) ; Eq. (13b) comes from the upper diagonal block of Eq. (16b); finally, one of the diagonal blocks of (17) gives rise to (13c).
4.2. Gauge symmetries of the spinor moving frame action

The action (12) possesses the irreducible \( \kappa \)-symmetry\(^3\)

\[
\delta_{\kappa} x^m = i \delta_{\kappa} \theta^m \Gamma_m^{ab} \theta^b, \quad \delta_{\kappa} \theta^a = \kappa^{+} v_{a}^{\alpha}, \quad \delta_{\kappa} v_{a q}^- = 0 = \delta_{\kappa} u_{m}^-;
\]

as well as its superpartner called \( b \)-symmetry \(^{11}\), which is the tangent space copy of the worldvolume reparametrization symmetry, \( \delta_{b} x^m = b^{++} u_{-m}^-, \quad \delta_{b} \theta^a = 0, \quad \delta_{b} v_{a q}^- = 0 = \delta_{b} u_{m}^- \), and a scaling \( GL(1, \mathbb{R}) \) symmetry

\[
\rho^{++} \mapsto e^{2\alpha} \rho^{++}, \quad u_{-m}^- \mapsto e^{-2\alpha} u_{-m}^-, \quad v_{a q}^- \mapsto e^{-\alpha} v_{a q}^-,
\]

with the wait determined by the sign indices \( ++, -- \) and \(-\), which we prefer to identify as \( SO(1,1) \) subgroup of \( SO(1,D-1) \). The action (12) is also invariant under the \( Spin(9) \) symmetry acting on the \( q \) index of the constrained bosonic spinor variable \( v_{a q}^- \),

\[
v_{a q}^- \mapsto v_{a q}^- S_{pq}, \quad S_{pq} \in Spin(9) \quad \Leftrightarrow \quad \begin{cases} S^T S = \mathbb{I}_{16 \times 16}, \\ S \gamma^l S^T = \gamma^l U J, \quad U^T U = \mathbb{I}_{9 \times 9} \end{cases},
\]

Notice that the nine dimensional charge conjugation matrix is symmetric and can be identified with the Kroneker delta symbol, \( \delta_{qp} \), so that the contraction \( v_{a q}^- v_{b q}^- \), entering the action, is \( Spin(9) \) invariant.

Finally, when \( v_{a q}^- \) are considered as Lorentz harmonics the fact of absence of the other \( 16 \times 32 \) block \( v_{a q}^+ \) of the spinor moving frame matrix \(^{14}\) can be formulated as the statement of \( K_9 \) symmetry,

\[
\delta v_{a q}^- = 0, \quad \delta v_{a q}^+ = k^{++} \gamma^l_{qp} v_{a q}^-, \quad i = 1, \ldots, 9.
\]

The \( \left[ SO(1,1) \otimes SO(D-2) \right] \otimes K_{D-2} \) is the Borel subgroup of \( SO(1, D-1) \) so that \(^{17}\) the coset \( \frac{SO(1,1) \otimes SO(D-2)}{SO(1, D-1)} \) is compact; moreover it is isomorphic to the sphere \( S^{D-2} \) which can be identified as celestial sphere of the \( D \)-dimensional observer \(^{17}\).

Thus, using the \( Spin(9) \), \( SO(1,1) \) and \( K_9 \) symmetry, Eqs. (21), (20) and (22), as an identification relation, the spinor harmonics \( v_{a q}^- \), explicitly present in the action (12), can be identified as homogeneous coordinates of the celestial sphere \( S^9 \) \( (S^{D-2}) \) of the eleven-dimensional \( (D\)-dimensional) observer \(^{17}\) \(^{18}\),

\[
\{ v_{a q}^- \} = \frac{Spin(1,10)}{Spin(1,1) \otimes Spin(9)} \otimes K_9 = S^9 \left( = \frac{Spin(1,D-1)}{Spin(1,1) \otimes Spin(D-2)} \otimes K_{D-2} = S^{D-2} \right).
\]

4.3. On \( O(16) \) gauge symmetry of the M0–brane action

However, when the action (12) with the variable \( v_{a q}^- \) subject only to the constraints (13) is considered (we call them \( \tilde{v}_{a q}^- \) to distinguish from the harmonics \( v_{a q}^- \)), one notices that neither constraints nor the action involve the \( d = 9 \) gamma matrices; all the contractions are made with \( 16 \times 16 \) Kroneker \( \delta_{qp} \) only. This implies that the \( D = 11 \) action (12), when considered as constructed from 16 32-component spinors \( \tilde{v}_{a q}^- \) restricted by (13) only,

\[
S = \int_{W^{1}} \frac{1}{32} \rho^{++} \tilde{v}_{a q}^- \tilde{v}_{b q}^- \Gamma_{m}^{\alpha \beta}, \quad \begin{cases} 2 \tilde{v}_{a q}^- \tilde{v}_{b q}^- = \frac{1}{16} \tilde{v}_{p}^- \Gamma_{p}^{m} \tilde{v}_{a q}^\alpha \Gamma_{m}^{\alpha \beta}, \quad (a) \\ \tilde{v}_{q}^- \Gamma_{m} \tilde{v}_{p}^- = \delta_{qp} \frac{1}{16} \tilde{v}_{p}^- \Gamma_{m} \tilde{v}_{p}^- = \delta_{qp} \tilde{v}_{q}^- \Gamma_{m} \tilde{v}_{q}^- = \delta_{qp}, \quad (b) \\ \tilde{v}_{a q}^- \Gamma_{m} \tilde{v}_{b q}^- = 0, \quad (c) \end{cases}
\]

\(^3\)Let us stress that the possibility to reformulate the \( \kappa \)-symmetry in the irreducible form is due to the presence of the constrained bosonic spinor variables \( v_{a q}^- \), spinorial harmonics (see \(^6\) \(^{19}\)).
actually possesses the local \( SO(16) \) symmetry acting on the \( q = 1, \ldots, 16 \) indices of \( \tilde{v}_{aq}^- \):

\[
\tilde{v}_{aq}^- \mapsto \tilde{v}_{aq}^- O_{pq} \ , \quad O_{pq} \in O(16) \iff O^T O = I_{16 \times 16} .
\]  

(25)

The relation between spinorial harmonic \( v_{aq}^- \), which transforms under \( Spin(9) \) symmetry, and the above \( \tilde{v}_{ap}^- \), carrying the \( SO(16) \) index \( p \) is given by \[3, 2\]

\[
\tilde{v}_{ap}^- = v_{aq}^- L_{qp} \ , \quad L_{qp} \in O(16) \iff L^T L = I_{16 \times 16} ,
\]  

(26)

where \( L_{qp} \) is an arbitrary orthogonal \( 16 \times 16 \) matrix.

Eq. (26) provides the general solution of the constraints \[13\text{-d}] \[3, 2\]. As \( \tilde{v}_{ap}^- \tilde{v}_{bp}^- = v_{aq}^- v_{bq}^- \), substituting (26) for \( \tilde{v}_{ap}^- \) in (24), one observes the cancelation of the contributions of the matrix \( L_{qp} \). This shows the \( O(16) \) symmetry of the action \( (24) \) with variable restricted only by the constraints presented there explicitly. Furthermore \[3, 2\], this (seemingly fictitious) \( SO(16) \) symmetry of the M0–brane, which we have observed studying different versions of (deferent treatment of the variable in) its twistor–like formulation, reappears inevitably in the quantization of the physical degrees of freedom.

4.4. Supertwistor representation of the M0-brane action

The spinor moving frame superparticle action \( (12) \) can be written in the following equivalent form \[3\]:

\[
S = \int_{W^1} (\lambda_{aq} d\mu^a_q - d\lambda_{aq} \mu^a_q - id\eta_q \eta_q) ,
\]  

(27)

were the sixteen 32-component spinors \( \lambda_{aq} \) are taken to be proportional to the spinor harmonics \( v_{aq}^- \) in product with an arbitrary \( SO(16) \) valued matrix,

\[
\lambda_{aq} := \sqrt{\rho^{++}} v_{aq}^- L_{pq} , \quad LL^T = I_{16 \times 16} .
\]  

(28)

Hence these 16 bosonic spinorial variables obey the constraints (see \[13\] or \[24\text{-a-c}]\)

\[
2\lambda_{aq} \lambda_{bq} = p_m \Gamma^m_{\alpha \beta} , \quad \lambda_q \Gamma_m \lambda_p = \delta_{qp} p_m , \quad C^{\alpha \beta} \lambda_{aq} \lambda_{bq} = 0 , \quad p_m p^m = 0 ,
\]  

(29)

with a light-like vector \( p_m = \rho^{++} u_m^- \) which can be identified as a massless particle momentum. On account of \( \rho^{++} \) in (28), and due to Eq. (23), the \( \{ \lambda_{aq} \} \) parametrize the \( \mathbb{R}_+ \times S^9 \) manifold (all the degrees of freedom in the \( SO(16) \) matrix \( L_{pq} \) are pure gauge); furthermore due to (13), this is identified as the space parametrized by the light–like momentum \( p_m , \ p^2 = 0 , \)

\[
\{ \lambda_{aq} \} = \mathbb{R}_+ \times S^9 = \{ p_m : p_m p^m = 0 \} .
\]  

(30)

The variables \( \mu^a_q , \eta_q \) in (27) are related to the superspace coordinates by the following generalization of the Penrose incidence relation,

\[
\mu^a_q := \frac{1}{32} x^m \Gamma_m^{\alpha \beta} \lambda_{aq} - \frac{i}{2} \theta^a \theta^\beta \lambda_{aq} , \quad \eta_q := \theta^\beta \lambda_{aq} .
\]  

(31)

Together with \( \lambda_{aq} \), the \( \mu^a_q \) and \( \eta_q \) in (31) define a set of sixteen constrained \( OSp(1|64) \) supertwistors (see [3]), \( \Upsilon_{\Sigma q} := \{ \lambda_{aq} , \mu^a_q , \eta_q \} \). The action (27) can be written as

\[
S = \int_{W^1} d\Upsilon_{\Sigma q} \Omega^\Sigma\Pi \Upsilon_{\Pi q} \text{ where } \Omega^\Sigma\Pi = -(-)^{(\Sigma+1)(\Pi+1)} \Omega^{\Sigma\Pi} \text{ is the orthosymplectic } OSp(1|64) \text{ invariant tensor (including the symplectic } \Omega^{\Sigma\beta} = -\Omega^{\beta\alpha} \text{ invariant of } Sp(32)).
\]
5. Supertwistor covariant quantization of M0–brane and hidden symmetries of D=11 supergravity

The supertwistor quantization of D=11 superparticle has been performed in [3]; there it was firstly motivated that, in the purely bosonic limit, the wavefunction is just an arbitrary function on the $\mathbb{R}^+ \otimes S^9$ space, which allows for identification with the space of light–like momenta, although appears as parametrized by its ”square root”, Eqs. (29), provided by the highly constrained coordinates $\lambda_{aq}$, Eqs. (30),

$$\Phi|\theta_q=0 = \Phi_0(\mathbb{R}^+ \otimes S^9), \quad \{(v^a_{\alpha q}, \rho^{++})\} = \mathbb{R}^+ \otimes S^9 = \{(p_m : p^2 := p_m p^m = 0)\}. \quad (32)$$

Then, beyond the purely bosonic limit, the pure supertwistor form (27) of the superparticle action contains the set of 16 free fermionic fields $\eta_q$ which, upon quantization, become the $\text{Cl}^{16}$ Clifford algebra valued variables,

$$\{\hat{\eta}_q, \hat{\eta}_p\} = \frac{1}{2} \delta_{qp}, \quad q = 1, 2, \ldots, 16. \quad (33)$$

This $O(16)$ covariant Clifford algebra $\text{Cl}^{16}$ has a finite dimensional representation by $256 \otimes 256$ sixteen dimensional gamma matrices $\hat{\eta}_q = \frac{1}{2} (\Gamma_q) A^B (A, B = 1, \ldots, 256, \ q = 1, \ldots, 16)$. The choice of this representation in the M0-brane quantization implies that the wavefunction is to be the $256$ Majorana spinor representation of $SO(16)$ (see [3]),

$$\Phi_A := \begin{pmatrix} \Phi_A \\ \Psi^B \end{pmatrix} = \begin{pmatrix} h_{IJ} \\ A_{IJK} \end{pmatrix}, \quad \begin{cases} h_{IJ} = h_{(IJ)} , \quad h_{II} = 0 , \\ A_{IJK} = A_{[IJK]} , \\ \Psi_{Iq} \gamma^I_{qp} = 0 \end{cases} \quad (34)$$

(see [2]), and this describes the linearized $D=11$ supergravity multiplet with $h_{IJ} = h_{(IJ)}, A_{IJK} = A_{[IJK]}$ and $\Psi_{Iq}$ restricted by $h_{II} = 0 = \Psi_{Iq} \gamma^I_{qp}$ (see [5]).

This indicates the $SO(16)$ symmetry of the linearized $D=11$ supergravity multiplet and suggests [3] the origin of the $SO(16)$ symmetry of (uncompactified) $D=11$ supergravity observed by Nicolai in [14]. Notice that our spinor moving frame formulation (12) makes this the $SO(16)$ symmetry manifest already at the classical level.

Furthermore, as it is well-known, $E_8$ exceptional group Lie algebra can be written in terms of the generators of $SO(16)$ and 128 bosonic generators carrying the Majorana spinor (128) representation of $SO(16)$ [27],

$$E_8 : \ [J_{qp}, J_{q'p'}] = 4 \delta_{[q' [q} J_{p'] p}, \quad [J_{qp}, Q_A] = \frac{1}{2} \sigma_{pqAB} Q_B , \quad [Q_A, Q_B] = \sigma_{pqAB} J_{pq}. \quad (35)$$

This makes tempting to speculate [2] on that the $E_8$ symmetry might be characteristic of the $D = 11$ supergravity itself rather than of its reduction to $d = 3$ only. A check of whether this is the case is an interesting subject for future study. See [2] for more discussion.

6. Outlook

Probably the most important conclusion of the study of the M0–brane covariant quantization in [12] is that the twistor-like Lorentz harmonic approach [19] [3] is able to produce a simple and practical BRST charge. This suggests a similar investigation of the $D = 10$ Green–Schwarz superstring case. The natural guess is that such a quantization of e.g.
type IIB superstring should produce (after some stages of reduction/simplification) the Berkovits BRST charge

\[ Q_{IIB}^B = \int \Lambda^{\alpha_1} d_{\alpha} + \int \Lambda^{\alpha_2} d_{2\alpha}, \quad \Lambda^{\alpha_1} \sigma^{\alpha}_{\alpha_3} \Lambda^{\beta_1} = 0 = \Lambda^{\alpha_1} \sigma^{\alpha}_{\alpha_3} \Lambda^{\beta_1}, \]  

but with composite pure spinors \( \Lambda^{\alpha_1} \) and \( \Lambda^{\alpha_2} \) given by

\[ \tilde{\Lambda}^{\alpha_1} = \tilde{\lambda}^{+\alpha} + p, \quad \tilde{\Lambda}^{\alpha_2} = \tilde{\lambda}^{-\alpha} + p, \quad \tilde{\lambda}^{+\alpha} + p = \tilde{\lambda}^{-\alpha} + p = 0. \]  

Here, the \( \tilde{\lambda}^{\pm\alpha} \) are two complex eight component \( SO(8) \) spinors and the stringy harmonics \( v^{\pm\alpha}_{p} \) are the homogeneous coordinates of the non–compact 16–dimensional coset

\[ \{ V^{(\alpha)} \} = \{ (v^{+\alpha}_{p}, v^{-\alpha}_{p}) \} = \frac{Spin(1,9)}{SO(1,1) \otimes SO(8)}, \]  

characteristic for the spinor moving frame formulation of the (super)string \([19, 20]\).

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**Notice added.** When the present work was finished, the author became aware of the work \([28]\) in which the possible hidden \( E_8 \times SO(16) \) symmetry of D=11 supergravity was conjectured for the first time.

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