Nucleon electromagnetic structure revisited

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Abstract

Unitary and analytic ten-resonance model of the nucleon electromagnetic (e.m.) structure with canonical normalizations and QCD (up to the logarithmic correction) asymptotics is constructed on the four-sheeted Riemann surface, which provides a superposition of vector-meson pole and continuum contributions in a very natural way. As a result it describes simultaneously all existing experimental space-like and time-like data on the proton e.m. form factors (ff’s) and on the neutron e.m. ff’s as well. A crucial factor in the latter achievement is the inclusion of a contribution of the fourth excited state of the $\rho(770)$ meson with the parameters $m_{\rho''''} = 2455 \pm 53$ MeV, $\Gamma_{\rho''''} = 728 \pm 2$ MeV and $(f^{(1)}_{\rho'''' NN}/f_{\rho''''}) = 0.0549 \pm 0.0005$, $(f^{(2)}_{\rho'''' NN}/f_{\rho''''}) = -0.0103 \pm 0.0001$. The pronounced effect of the two-pion continuum on the isovector spectral functions demonstrating a strong enhancement of the left wing of the $\rho(770)$-resonance close to two-pion threshold, which was revealed by Höhler and Pietarinen by means of the nucleon ff unitarity condition more than a quarter of the century ago, is predicted by the model automatically. The model gives large values of the $f^{(1,2)}_{\rho NN}$ coupling constants, thus indicating the violation of the OZI rule. Since in the framework of the considered model isoscalar ff’s above their lowest branch point $t_0^s = 9m^2_\pi$ are complex functions, the isoscalar spectral function behaviours are predicted as well.

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1 Introduction

The electromagnetic (e.m.) structure of the nucleons, as revealed first time in elastic electronnucleon scattering almost half a century ago, is completely described by four independent scalar functions of one variable called form factors (ff’s). They depend on the square momentum transfer $t = -Q^2$ of the virtual photon.

Nucleon e.m. ff’s can be chosen in a divers way, e.g. as the Dirac and Pauli ff’s, $F^p_1(t)$, $F^n_1(t)$ and $F^p_2(t)$, $F^n_2(t)$, or the Sachs electric and magnetic ff’s, $G^p_E(t)$, $G^n_E(t)$ and $G^p_M(t)$, $G^n_M(t)$, or isoscalar and isovector Dirac and Pauli ff’s, $F^s_1(t)$, $F^v_1(t)$ and $F^s_2(t)$, $F^v_2(t)$ and isoscalar and isovector electric and magnetic ff’s, $G^s_E(t)$, $G^v_E(t)$ and $G^s_M(t)$, $G^v_M(t)$, respectively.

The Dirac and Pauli ff’s are naturally obtained in a decomposition of the nucleon matrix element of the e.m. current into a maximum number of linearly independent covariants constructed from the four-momenta, $\gamma$-matrices and Dirac bispinors of nucleons as follows

$$\langle N | J_{\mu}^{e.m.} | N \rangle = e \bar{u}(p')\gamma_{\mu} F_N^1(t) + i \frac{\sigma_{\mu\nu}}{2m_N}(p' - p)_{\nu} F_N^2(t) \rangle u(p) \tag{1}$$

with $m_N$ to be nucleon mass.

On the other hand, the electric and magnetic ff’s are very suitable in extracting experimental information on the nucleon e.m. structure from the measured cross sections

$$\frac{d\sigma^{lab}(e^- N \rightarrow e^- N)}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \frac{1}{1 + \left(\frac{2m_N}{E}\right)^2 \sin^2(\theta/2)} \left[ G_E^2 - \frac{i}{4m_N} G_M^2 \right] - \frac{2}{4m_N^2} \frac{t}{G_M \tan^2(\theta/2)} \tag{2}$$

$$\alpha = 1/137, \ E\text{-the incident electron energy},$$

and

$$\sigma_{tot}^{c.m.}(e^+ e^- \rightarrow N \bar{N}) = \frac{4\pi \alpha^2 \beta_N}{3t} \left[ |G_M(t)|^2 + \frac{2m_N^2}{t} |G_E(t)|^2 \right], \ \beta_N = \sqrt{1 - \frac{4m_N^2}{t}} \tag{3}$$

or

$$\sigma_{tot}^{c.m.}(\bar{p}p \rightarrow e^+ e^-) = \frac{2\pi \alpha^2}{3\beta_{c.m.} \sqrt{t}} \left[ |G_M(t)|^2 + \frac{2m_N^2}{t} |G_E(t)|^2 \right], \tag{4}$$

($p_{c.m.}\text{-antiproton momentum in the c.m. system}$)

as there are no interference terms between them.
In the Breit frame, the Sachs ff’s give the distribution of charge and magnetization within the proton and neutron, respectively. From all four Sachs ff’s the neutron electric ff plays a particular role. Though the total neutron charge is zero, there is a nonvanishing distribution of charge, which leads to the nonvanishing neutron electric ff.

The isoscalar and isovector Dirac and Pauli ff’s are suitable for a construction of various phenomenological models of the nucleon e.m. structure. The most attractive of them is the Vector Meson Dominance (VMD) picture in the framework of which ff’s are simply saturated by a set of isoscalar and isovector vector meson poles on the positive real axis. However, this turns out to be practically an insufficient approximation and in a more realistic description of the data (especially in the time-like region) instability of vector-mesons has to be taken into account and the contributions of continua, to be created by n-particle thresholds, like, e.g., $2\pi$, $3\pi$, $K\bar{K}$, $N\bar{N}$ etc., together with the correct asymptotic behaviours and normalizations have to be included.

In recent years, abundant and very accurate data on the nucleon e.m. ff’s appeared. Most of the references concerning the nucleon space-like data can be found in [1]. More recent precise measurements are presented in [2-10]. Besides the latter, there are also very accurate data on the ratio of proton electric and magnetic ff’s [11] obtained at the Jefferson Lab for $-0.5\text{GeV}^2 < -3.5\text{GeV}^2$ by using polarization transfer and the new data on the neutron electric ff from BATES [12], MAMI [13-16] and NIKHEF [17].

For the time-like region data see [18-26]. There, in particular, the FENICE experiment in Frascati measured, besides the proton e.m. ff’s [25], the magnetic neutron ff in the time-like region [26] for the first time. There are also valuable results on the proton magnetic ff at higher energies measured at FERMILAB [21,22].

All this stimulated recent dispersion theoretical analysis [27,28] of the nucleon e.m. ff data in the space-like region and in the time-like region [29] as well. The latter works are an update and extension of historically the most competent nucleon ff analysis carried out by Höhler with collaborators [30]. However, the model does not allow one to describe all the time-like data consistently, while still giving good description of the data in the space-like region.

In this paper, we construct a ten-resonance unitary and analytic model of the nucleon e.m. structure, defined on the four-sheeted Riemann surface with canonical normalizations and QCD asymptotics, which provides a very effective framework for a superposition of complex conjugate vector-meson pole pairs on unphysical sheets and continua contributions in nucleon
e.m. ff’s. The model contains, e.g., an explicit two-pion continuum contribution given by the unitary cut starting with \( t = 4m^2_\pi \) and automatically predicts the strong enhancement of the left wing of the \( \rho(770) \) resonance in the isovector spectral functions to be consistent with the results of \([27,31]\).

Another result of the presented model is the prediction of parameters of the fourth excited state of the \( \rho(770) \) meson and the automatic prediction of isoscalar nucleon spectral function behaviours. At the same time, a description of all existing space-like and time-like nucleon e.m. ff data, including also FENICE (Frascati) results on the neutron from the \( e^+e^- \rightarrow n\bar{n} \) process, is achieved.

The paper is organized as follows. In Section 2., the unitary and analytic ten-resonance model of the nucleon e.m. structure with canonical normalizations and asymptotics as predicted by the quark model of hadrons is constructed. Evaluation of all free parameters of the model (however, with clear physical meaning) by a fit of all existing data is carried out in Section 3. In Section 4., we predict the isoscalar and isovector nucleon spectral function behaviours and various coupling constant ratio values, which appear not to be free parameters of the constructed model. The last section is devoted to conclusions and discussion.

# 2 Ten-resonance unitary and analytic model of nucleon e.m. structure

All four independent sets of four nucleon e.m. ff’s discussed in the Introduction are related by

\[
\begin{align*}
G^p_E(t) &= G^s_E(t) + G^v_E(t) = F^p_1(t) + \frac{t}{4m^2_p}F^p_2(t) = [F^s_1(t)] + [F^v_1(t)] + \frac{t}{4m^2_p}[F^s_2(t) + F^v_2(t)]; \\
G^p_M(t) &= G^s_M(t) + G^v_M(t) = F^p_1(t) + \frac{t}{4m^2_n}F^p_2(t) = [F^s_1(t) + F^v_1(t)] + [F^s_2(t) + F^v_2(t)]; \\
G^n_E(t) &= G^s_E(t) - G^v_E(t) = F^n_1(t) + \frac{t}{4m^2_n}F^n_2(t) = [F^s_1(t) - F^v_1(t)] + \frac{t}{4m^2_n}[F^s_2(t) - F^v_2(t)]; \\
G^n_M(t) &= G^s_M(t) - G^v_M(t) = F^n_1(t) + F^n_2(t) = [F^s_1(t) - F^v_1(t)] + [F^s_2(t) - F^v_2(t)],
\end{align*}
\]

and at the value \( t = 0 \) normalized as follows:

\[
\begin{align*}
(i) & \quad G^p_E(0) = 1; \quad G^p_M(0) = 1 + \mu_p; \quad G^n_E(0) = 0; \quad G^n_M(0) = \mu_n; \\
(ii) & \quad G^s_E(0) = G^v_E(0) = \frac{1}{2}; \quad G^s_M(0) = \frac{1}{2}(1 + \mu_p + \mu_n); \quad G^v_M(0) = \frac{1}{2}(1 + \mu_p - \mu_n); \\
(iii) & \quad F^p_1(0) = 1; \quad F^p_2(0) = \mu_p; \quad F^n_1(0) = 0; \quad F^n_2(0) = \mu_n.
\end{align*}
\]
$F_1^s(0) = F_1^v(0) = \frac{1}{2}; \; F_2^s(0) = \frac{1}{2}(\mu_p + \mu_n); \; F_2^v(0) = \frac{1}{2}(\mu_p - \mu_n),$

where $\mu_p$ and $\mu_n$ are the proton and neutron anomalous magnetic moments, respectively.

The ten-resonance unitary and analytic model will represent a consistent unification of the following three fundamental features (besides other properties) of the nucleon e.m. ff’s:

1. The experimental fact of creation of unstable vector-meson resonances in the $e^+e^-$-annihilation processes into hadrons.

2. The hypothetical analytic properties of the nucleon e.m. ff’s on the first (physical) sheet of the Riemann surface, by means of which just the contributions of continua are taken into account.

3. The asymptotic behaviour of nucleon e.m. ff’s as predicted [32] by the quark model of hadrons.

Here we would like to note that a further procedure will not mean any mathematically correct derivation of the unitary and analytic model, but only an (noncommutative) algorithm of its construction which is, however, generally valid also for any other strongly interacting particles.

In order to take into account the first feature, one starts with saturation of the isoscalar and isovector parts of the Dirac and Pauli ff’s by the isoscalar and isovector vector mesons possessing the quantum numbers of the photon. As there are no data on the nucleon e.m. ff’s for reliable determination of resonance masses and widths in the region $0 < t < 4m_N^2$ of manifestation of the majority of resonances under consideration, these parameters are fixed at the world averaged values. Then, their consistency with existing ff data in other regions is investigated.

In Review of Particle Physics [33] we find just 5 isoscalar resonances $\omega(782), \phi(1020), \omega'(1420), \omega''(1600), \phi'(1680)$ with the required properties. However, one finds there only 3 isovector resonances $\rho(770), \rho'(1450), \rho''(1700)$ with quantum numbers of the photon. On the other hand, we have gained experience in our previous analyses that the most stable description of existing data is achieved if an equal number of isoscalar and isovector vector meson resonances in the investigated models is taken into account. Therefore, in the isovector Dirac and Pauli ff’s we consider the third excited state of the $\rho$-meson, $\rho'''(2150)$, revealed in [34], and in order to achieve also a description of the time-like region data on proton at higher energies from Fermilab [21,22] and neutron data from Frascati [26], we also introduce
hypothetically the fourth excited state of the $\rho$-meson, $\rho'''$, the mass and width of which are left to be free parameters of the model.

As one will see later from the comparison of the unitary and analytic model with all existing data, those resonance parameters will be found to be quite reasonable, and a simultaneous description of the space-like and time-like nucleon ff data, including the FENICE (Frascati) results on the neutron, will be achieved.

With the aim of incorporation of the third feature of nucleon e.m. ff’s we transform VMD parametrizations of the isoscalar and isovector parts of the Dirac and Pauli ff’s into the common denominators. The explicit requirement of the normalizations \( (I) \) and the asymptotic behaviours

\[
i^{i+1} F_i^{e,v}(t)|_{t\to\infty} \sim \text{constant}, \quad i = 1, 2
\]  

lead (for more detail see Appendix A) again to the zero-width VMD parametrization of the isoscalar and isovector parts of the Dirac and Pauli ff’s

\[
F_i^{e}(t) = \frac{1}{2} \frac{m_{\omega,i}^2 m_{\omega,i}^2}{(m_{\omega,i}^2 - t)(m_{\omega,i}^2 - t)} + \\
+ \left\{ \frac{m_{\omega,i}^2 m_{\omega,i}^2}{(m_{\omega,i}^2 - t)(m_{\omega,i}^2 - t)} \frac{m_{\omega,i}^2 - m_{\omega,i}^2}{m_{\omega,i}^2 - m_{\omega,i}^2} - \frac{m_{\omega,i}^2 m_{\omega,i}^2}{(m_{\omega,i}^2 - t)(m_{\omega,i}^2 - t)} \frac{m_{\omega,i}^2 - m_{\omega,i}^2}{m_{\omega,i}^2 - m_{\omega,i}^2} \right\} (f_{\omega NN}^{(1)}/f_{\omega}) + \\
+ \left\{ \frac{m_{\omega,i}^2 m_{\omega,i}^2}{(m_{\omega,i}^2 - t)(m_{\omega,i}^2 - t)} \frac{m_{\omega,i}^2 - m_{\omega,i}^2}{m_{\omega,i}^2 - m_{\omega,i}^2} - \frac{m_{\omega,i}^2 m_{\omega,i}^2}{(m_{\omega,i}^2 - t)(m_{\omega,i}^2 - t)} \frac{m_{\omega,i}^2 - m_{\omega,i}^2}{m_{\omega,i}^2 - m_{\omega,i}^2} \right\} (f_{\phi NN}^{(1)}/f_{\phi}) - \\
- \left\{ \frac{m_{\omega,i}^2 m_{\omega,i}^2}{(m_{\omega,i}^2 - t)(m_{\omega,i}^2 - t)} \frac{m_{\omega,i}^2 - m_{\omega,i}^2}{m_{\omega,i}^2 - m_{\omega,i}^2} - \frac{m_{\omega,i}^2 m_{\omega,i}^2}{(m_{\omega,i}^2 - t)(m_{\omega,i}^2 - t)} \frac{m_{\omega,i}^2 - m_{\omega,i}^2}{m_{\omega,i}^2 - m_{\omega,i}^2} \right\} (f_{\phi NN}^{(1)}/f_{\phi}) + \\
+ \frac{m_{\omega,i}^2 m_{\omega,i}^2}{(m_{\omega,i}^2 - t)(m_{\omega,i}^2 - t)} (f_{\phi NN}^{(1)}/f_{\phi}) \right).
\]  

\[
F_i^{v}(t) = \frac{1}{2} \frac{m_{\phi,i}^2 m_{\phi,i}^2}{(m_{\phi,i}^2 - t)(m_{\phi,i}^2 - t)} + \\
+ \left\{ \frac{m_{\phi,i}^2 m_{\phi,i}^2}{(m_{\phi,i}^2 - t)(m_{\phi,i}^2 - t)} \frac{m_{\phi,i}^2 - m_{\phi,i}^2}{m_{\phi,i}^2 - m_{\phi,i}^2} - \frac{m_{\phi,i}^2 m_{\phi,i}^2}{(m_{\phi,i}^2 - t)(m_{\phi,i}^2 - t)} \frac{m_{\phi,i}^2 - m_{\phi,i}^2}{m_{\phi,i}^2 - m_{\phi,i}^2} \right\} (f_{\omega NN}^{(1)}/f_{\omega}) + \\
+ \left\{ \frac{m_{\phi,i}^2 m_{\phi,i}^2}{(m_{\phi,i}^2 - t)(m_{\phi,i}^2 - t)} \frac{m_{\phi,i}^2 - m_{\phi,i}^2}{m_{\phi,i}^2 - m_{\phi,i}^2} - \frac{m_{\phi,i}^2 m_{\phi,i}^2}{(m_{\phi,i}^2 - t)(m_{\phi,i}^2 - t)} \frac{m_{\phi,i}^2 - m_{\phi,i}^2}{m_{\phi,i}^2 - m_{\phi,i}^2} \right\} (f_{\phi NN}^{(1)}/f_{\phi}) \right) \]
\[
F_2^g(t) = \frac{1}{2} (\mu_p + \mu_n) \frac{m_{2\omega}^2 m_{2\omega}^2 m_{2\phi}^2}{(m_{2\omega}^2 - t)(m_{2\omega}^2 - t)(m_{2\phi}^2 - t)} + \frac{m_{2\omega}^2 m_{2\omega}^2}{(m_{2\omega}^2 - t)(m_{2\omega}^2 - t)} \left( f_{\omega NN}^{(1)} + \frac{m_{2\omega}^2 m_{2\phi}^2}{(m_{2\phi}^2 - t)(m_{2\phi}^2 - t)} \right) \left( f_{\phi NN}^{(1)} \right) \right) + 
\]
from the inverse transformations to (12), e.g.

\[ m^2_m m^2_m m^2_m \left( \frac{m^2_m - m^2_m}{(m^2_m - t)(m^2_m - t)(m^2_m - t)} \right) - \frac{m^2_m - m^2_m}{(m^2_m - t)(m^2_m - t)(m^2_m - t)} \] 

respectively, and a subsequent incorporation of the nonzer o values of vector meson widths.

However, they are already automatically normalized and they govern the asymptotics (7) as predicted by QCD up to the logarithmic corrections.

Despite the latter properties the model is unable to reproduce the existing experimental information properly and only its unitarization, i.e., inclusion of the contributions of continua and instability of vector-meson resonances, leads to a simultaneous description of the spacelike and time-like data.

It is well known that the unitarity condition requires the imaginary part of the nucleon e.m. ff’s to be different from zero only above the lowest branch point \( t_0 \) and, moreover, it just predicts its smoothly varying behaviour (see e.g. [27,31]).

The unitarization of the model (8)-(11) can be achieved by application of the following special non-linear transformations

\[ t = t^s_0 - \frac{4(t^s_m - t^s_0)}{1/V - V^2} \] 

\[ t = t^v_0 - \frac{4(t^v_m - t^v_0)}{1/W - W^2} \] 

\[ t = t^v_0 - \frac{4(t^v_m - t^v_0)}{1/U - U^2} \] 

\[ t = t^v_0 - \frac{4(t^v_m - t^v_0)}{1/X - X^2} \] 

(12)

respectively, and a subsequent incorporation of the nonzero values of vector meson widths.

Here \( t^s_0 = 9m^2_m, t^v_0 = 4m^2_m, t^v_0 = 4m^2_m, t^v_0 = 4m^2_m, t^v_0 = 4m^2_m \) are square-root branch points, as it is transparent from the inverse transformations to (12), e.g.
\[ V(t) = \frac{1}{\sqrt{(t_s^1 - t_0^s)^{1/2} + (t - t_0^s)^{1/2}} - \sqrt{(t_s^1 - t_0^s)^{1/2} - (t - t_0^s)^{1/2}}} \]

and similarly for \( W(t), U(t) \) and \( X(t) \).

The interpretation of \( t_0^s = 9m^2_\pi \) and \( t_0^v = 4m^2_\pi \) is clear. They are the lowest branch points of isoscalar and isovector Dirac and Pauli ff’s on the positive real axis, respectively, as in the isoscalar case the 3-pion states and in the isovector case the 2-pion states are the lowest intermediate mass states in the unitarity conditions of the corresponding ff’s.

However, as it follows just from the unitarity conditions of ff’s, there is an infinite number of allowed higher mass intermediate states and as a result there is an infinite number of the corresponding branch points (and thus, an infinite number of branch cut contributions) in every of the considered nucleon ff’s.

Since, in principle, an infinite number of cuts cannot be taken into account in any theoretical scheme, we restrict ourselves in every isoscalar and isovector Dirac and Pauli ff to the two-cut approximation. The second one, an effective inelastic cut, in every isoscalar and isovector Dirac and Pauli ff is generated just by the square-root branch points \( t_{in}^1, t_{in}^v, t_{in}^2s, t_{in}^2v \), respectively. They are free parameters of the model and the data themselves, by a fitting procedure, will choose for them such numerical values that the contributions of the corresponding square-root cuts will be practically equivalent to the contributions of an infinite number of unitary branch cuts in every considered ff.

Some experts are suggesting to fix these square-root branch points at the \( NN \) threshold. However, it will be demonstrated in Section 3 that this can be done only in the case of isovector parts of Dirac and Pauli ff’s, but in none of the cases of the isoscalar parts of the Dirac and Pauli ff’s.

So, by application of (12) to (8)-(11), for every isoscalar and isovector Dirac and Pauli ff one gets one analytic function in the whole complex \( t \)-plane besides two right-hand cuts (see Appendix B) of the following forms:

\[ F_1^s[V(t)] = \left( 1 - \frac{V_0^2}{V_2^2} \right)^4 \left( \frac{1}{2} H_{\omega'}(V) \cdot L_{\omega'}(V) + H_{\omega''}(V) \cdot L_{\omega}(V) \cdot \frac{C_{1s}^{1s} - C_{1s}^{1s}}{C_{1s}^{1s} - C_{1s}^{1s}} - L_{\omega'}(V) \cdot L_{\omega}(V) \cdot \frac{C_{1s}^{1s} - C_{1s}^{1s}}{C_{1s}^{1s} - C_{1s}^{1s}} \right) \]

\[ (f_{\omega''}^{(1)}} f_\omega) + \]

\[ + \left( H_{\omega''}(V) \cdot L_\phi(V) \cdot \frac{C_{1s}^{1s} - C_{1s}^{1s}}{C_{1s}^{1s} - C_{1s}^{1s}} - L_{\omega'}(V) \cdot L_\phi(V) \cdot \frac{C_{1s}^{1s} - C_{1s}^{1s}}{C_{1s}^{1s} - C_{1s}^{1s}} \right) \]
\[- H_{\omega'}(V) \cdot L_{\omega'}(V) \left( f^{(1)}_{\phi NN} / f_\phi \right) - \left[ H_{\phi'}(V) \cdot H_{\omega'}(V) \frac{C_{\omega'}^{1s} - C_{\omega''}^{1s}}{C_{\omega''}^{1s} - C_{\omega'}^{1s}} - \right. \]
\[- H_{\phi'}(V) \cdot L_{\omega'}(V) \frac{C_{\omega'}^{1s} - C_{\omega''}^{1s}}{C_{\omega''}^{1s} - C_{\omega'}^{1s}} + H_{\omega''}(V) \cdot L_{\omega'}(V) \right] \left( f^{(1)}_{\phi NN} / f_\phi \right) \} \]

\[
F_{1}^{n}[W(t)] = \left( \frac{1 - W^2}{1 - W_N} \right)^4 \left\{ \frac{1}{2} L_{\rho''}(W) \cdot L_{\rho}(W) + \left[ L_{\rho''}(W) \cdot L_{\rho}(W) \frac{C_{\rho''}^{1s} - C_{\rho''}^{1s}}{C_{\rho''}^{1s} - C_{\rho''}^{1s}} - \right. \right.
\[- L_{\rho''}(W) \cdot L_{\rho}(W) \frac{C_{\rho''}^{1s} - C_{\rho''}^{1s}}{C_{\rho''}^{1s} - C_{\rho''}^{1s}} - L_{\rho''}(W) \cdot L_{\rho}(W) \right\} \left( f^{(1)}_{\rho NN} / f_\rho \right) + \]
\[+ \left[ H_{\rho''}(W) \cdot L_{\rho''}(W) \frac{C_{\rho''}^{1s} - C_{\rho''}^{1s}}{C_{\rho''}^{1s} - C_{\rho''}^{1s}} - H_{\rho''}(W) \cdot L_{\rho'}(W) \frac{C_{\rho''}^{1s} - C_{\rho''}^{1s}}{C_{\rho''}^{1s} - C_{\rho''}^{1s}} - \right. \]
\[- L_{\rho''}(W) \cdot L_{\rho''}(W) \left( f^{(1)}_{\rho NN} / f_\rho \right) - \right] \left. \left[ H_{\rho''}(W) \cdot L_{\rho''}(W) \frac{C_{\rho''}^{1s} - C_{\rho''}^{1s}}{C_{\rho''}^{1s} - C_{\rho''}^{1s}} - H_{\rho''}(W) \cdot L_{\rho'}(W) \frac{C_{\rho''}^{1s} - C_{\rho''}^{1s}}{C_{\rho''}^{1s} - C_{\rho''}^{1s}} + \right. \right.
\[+ L_{\rho''}(W) \cdot L_{\rho''}(W) \left( f^{(1)}_{\rho NN} / f_\rho \right) \right\} \]

\[
F_{2}^{n}[U(t)] = \left( \frac{1 - U^2}{1 - U_N^2} \right)^6 \left\{ \frac{1}{2} (\mu_p + \mu_n) H_{\omega''}(U) \cdot L_{\omega'}(U) \cdot L_{\omega}(U) + \right. \]
\[+ \left[ H_{\omega''}(U) \cdot L_{\phi}(U) \cdot L_{\omega}(U) \frac{C_{\omega''}^{2s} - C_{\omega''}^{2s}}{C_{\omega''}^{2s} - C_{\omega''}^{2s}} - \right. \]
\[+ H_{\omega''}(U) \cdot L_{\omega'}(U) \cdot L_{\omega}(U) \frac{C_{\omega''}^{2s} - C_{\omega''}^{2s}}{C_{\omega''}^{2s} - C_{\omega''}^{2s}} - \right. \]
\[- L_{\omega'}(U) \cdot L_{\phi}(U) \cdot L_{\omega}(U) \frac{C_{\omega''}^{2s} - C_{\omega''}^{2s}}{C_{\omega''}^{2s} - C_{\omega''}^{2s}} - \right. \]
\[- L_{\omega'}(U) \cdot L_{\phi}(U) \cdot L_{\omega}(U) \left( f^{(2)}_{\phi NN} / f_{\phi} \right) + \right. \]
\[+ \left[ H_{\phi'}(U) \cdot H_{\omega''}(U) \cdot L_{\omega'}(U) \frac{C_{\omega''}^{2s} - C_{\omega''}^{2s}}{C_{\omega''}^{2s} - C_{\omega''}^{2s}} - \right. \]
\[+ \left. H_{\phi'}(U) \cdot H_{\omega''}(U) \cdot L_{\omega}(U) \frac{C_{\omega''}^{2s} - C_{\omega''}^{2s}}{C_{\omega''}^{2s} - C_{\omega''}^{2s}} - \right. \]
\[+ \left. H_{\phi'}(U) \cdot L_{\omega'}(U) \cdot L_{\omega}(U) \frac{C_{\omega''}^{2s} - C_{\omega''}^{2s}}{C_{\omega''}^{2s} - C_{\omega''}^{2s}} - \right. \]
\[- H_{\omega''}(U) \cdot L_{\omega'}(U) \cdot L_{\omega}(U) \right\} \left( f^{(2)}_{\phi NN} / f_{\phi} \right) \}

\[
F_{2}^{n}[X(t)] = \left( \frac{1 - X^2}{1 - X_N^2} \right)^6 \left\{ \frac{1}{2} (\mu_p - \mu_n) L_{\rho''}(X) \cdot L_{\rho'}(X) \cdot L_{\rho}(X) + \right. \]
\[ L_r(V) = \frac{(V_N - V_r)(V_N - V_r^*)(V_N - 1/V_r)(V_N - 1/V_r^*)}{(V - V_r)(V - V_r^*)(V - 1/V_r)(V - 1/V_r^*)}; \]

\[ C_r^{1s} = \frac{(V_N - V_r)(V_N - V_r^*)(V_N - 1/V_r)(V_N - 1/V_r^*)}{-(V - 1/V_r)(V - 1/V_r^*)}; \quad r = \omega, \phi, \omega', \phi'; \]

\[ H_l(V) = \frac{(V_N - V_l)(V_N - V_l^*)(V_N + V_l)(V_N + V_l^*)}{(V - V_l)(V - V_l^*)(V + V_l)(V + V_l^*)}; \]

\[ C_l^{1s} = \frac{(V_N - V_l)(V_N - V_l^*)(V_N + V_l)(V_N + V_l^*)}{-(V_l - 1/V_l)(V_l^* - 1/V_l^*)}; \quad l = \omega^*, \phi^*; \]

\[ L_k(W) = \frac{(W_N - W_k)(W_N - W_k^*)(W_N - 1/W_k)(W_N - 1/W_k^*)}{(W - W_k)(W - W_k^*)(W - 1/W_k)(W - 1/W_k^*)}; \]

\[ C_k^{1s} = \frac{(W_N - W_k)(W_N - W_k^*)(W_N - 1/W_k)(W_N - 1/W_k^*)}{-(W_k - 1/W_k)(W_k^* - 1/W_k^*)}; \quad k = \rho, \rho', \rho''; \]

\[ H_n(W) = \frac{(W_N - W_n)(W_N - W_n^*)(W_N + W_n)(W_N + W_n^*)}{(W - W_n)(W - W_n^*)(W + W_n)(W + W_n^*)}; \]

\[ C_n^{1s} = \frac{(W_N - W_n)(W_N - W_n^*)(W_N + W_n)(W_N + W_n^*)}{-(W_n - 1/W_n)(W_n^* - 1/W_n^*)}; \quad n = \rho'', \rho''' \]

\[ L_r(U) = \frac{(U_N - U_r)(U_N - U_r^*)(U_N - 1/U_r)(U_N - 1/U_r^*)}{(U - U_r)(U - U_r^*)(U - 1/U_r)(U - 1/U_r^*)}; \]

\[ C_r^{2s} = \frac{(U_N - U_r)(U_N - U_r^*)(U_N - 1/U_r)(U_N - 1/U_r^*)}{-(U - 1/U_r)(U - 1/U_r^*)}; \quad r = \omega, \phi, \omega', \phi'; \]

\[ H_l(U) = \frac{(U_N - U_l)(U_N - U_l^*)(U_N + U_l)(U_N + U_l^*)}{(U - U_l)(U - U_l^*)(U + U_l)(U + U_l^*)}; \]

\[ C_l^{2s} = \frac{(U_N - U_l)(U_N - U_l^*)(U_N + U_l)(U_N + U_l^*)}{-(U_l - 1/U_l)(U_l^* - 1/U_l^*)}; \quad l = \omega^*, \phi^*; \]
Expressions (14)-(17), together with relations (5), represent just the ten-resonance unitary and analytic model of the nucleon e.m. structure with canonical normalizations (6) and the correct asymptotic behaviours as predicted by the quark model of hadrons. In the next section this model is used to analyze all existing nucleon e.m. ff data and obtain further predictions.

3 Analysis of all existing space-like and time-like data

Taking into account the discussion in Section 2 and applying the asymptotic conditions [35] together with ff normalizations, the ten-resonance unitary and analytic model of the nucleon e.m. structure depends (see Appendix A) on the following parameters:

\begin{align*}
L_k(X) &= \frac{(X_N - X_k)(X_N - X_N^*)(X_N - 1/X_k)(X_N - 1/X_N^*)}{(X - X_k)(X - X_k^*)(X - 1/X_k)(X - 1/X_k^*)}; \\
C_k^{2v} &= \frac{(X_N - X_k)(X_N - X_k^*)(X_N - 1/X_k)(X_N - 1/X_k^*)}{-(X_k - 1/X_k)(X_k^* - 1/X_k^*)}; \\
H_n(X) &= \frac{(X_N - X_n)(X_N - X_n^*)(X_N + X_n)(X_N + X_n^*)}{(X - X_n)(X - X_n^*)(X + X_n)(X + X_n^*)}; \\
C_n^{2v} &= \frac{(X_N - X_n)(X_N - X_n^*)(X_N + X_n)(X_N + X_n^*)}{-(X_n - 1/X_n)(X_n^* - 1/X_n^*)}; \\
\end{align*}

\begin{align*}
k &= \rho, \rho', \rho''; \\
n &= \rho'''', \rho''''
\end{align*}

Here we would like to note that criticism has been expressed by some specialists that the presented unitary and analytic model of nucleon e.m. structure comprises too many (though with clear physical meaning) free parameters. We believe that the responsibility for such a situation rests with various experimental groups confirming [33] nowadays so many vector-meson resonances possessing the quantum numbers of the photon, as a substantial majority of free parameters of the model are coupling constants just of these vector-mesons under consideration with nucleons. Once these vector-meson resonances exist, they can not be ignored in any theoretical considerations, including theoretical models of the nucleon e.m. structure.

A solution for lowering the number of free parameters could be true numerical values of these coupling constants, however, brought from outside of the considered model. Neverthe-
Figure 1: A simultaneous optimal fit of all existing data on proton electric ff

less, for the determination of coupling constants there is no reliable theory up to now and as
a consequence, one is forced to consider them to be free parameters of the model.

For numerical evaluation of the parameters (24) we have collected 512 experimental points
which have been discussed in more detail in the Introduction. The data have been analyzed
by relations (3) and (14)-(17) by using the CERN program MINUIT. The best description
of them was achieved with $\chi^2/ndf = 1.46$ and the following values of free parameters:

\[
\begin{align*}
t_{1s}^1 &= 2.6012 \pm 0.6391 \text{ GeV}^2 & t_{1v}^1 &= 3.5220 \pm 0.0059 \text{ GeV}^2 \\
t_{2s}^2 &= 2.7200 \pm 0.6271 \text{ GeV}^2 & t_{2v}^2 &= 3.6316 \pm 0.6235 \text{ GeV}^2 \\
(f_{\omega NN}/f_\omega)^{(1)} &= 1.1112 \pm 0.0030 & (f_{\rho NN}/f_\rho)^{(1)} &= 0.3843 \pm 0.0043 \\
(f_{\phi NN}/f_{\phi})^{(1)} &= -0.9389 \pm 0.0056 & (f_{\rho'''' NN}/f_{\rho''''})^{(1)} &= -0.0840 \pm 0.0008 \\
(f_{\phi' NN}/f_{\phi'})^{(1)} &= -0.3255 \pm 0.0047 & (f_{\rho'''' NN}/f_{\rho''''})^{(2)} &= 0.0299 \pm 0.0003 \\
(f_{\phi'' NN}/f_{\phi''})^{(2)} &= -0.2659 \pm 0.0287 & m_{\rho''''} &= 2455 \pm 53 \text{ MeV} \\
(f_{\phi''' NN}/f_{\phi'''})^{(2)} &= 0.1190 \pm 0.0032 & \Gamma_{\rho'''''} &= 728 \pm 2 \text{ MeV} \\
(f_{\rho''' NN}/f_{\rho'''})^{(1)} &= 0.0549 \pm 0.0005 & & \\
(f_{\rho''' NN}/f_{\rho'''})^{(2)} &= -0.0103 \pm 0.0001.
\end{align*}
\]

If, on the basis of suggestions of some experts, the parameters $t_{1s}^1, t_{1v}^1, t_{2s}^2, t_{2v}^2$ are fixed
at the $N\bar{N}$ threshold, the best description of existing data is achieved with a worse value of $\chi^2/ndf = 1.82$ and the rest parameters as follows:

\[
\begin{align*}
(f^{(1)}_{\omega NN}/f_{\omega}) &= 0.9916 \pm 0.0112 \quad (f^{(1)}_{\rho NN}/f_{\rho}) = 0.3746 \pm 0.0159 \\
(f^{(2)}_{\rho NN}/f_{\rho}) &= -1.1209 \pm 0.0125 \quad (f^{(1)}_{\rho'''' NN}/f_{\rho''''}) = -0.0799 \pm 0.0006 \\
(f^{(1)}_{\phi' NN}/f_{\phi'}) &= -2.6079 \pm 0.0384 \quad (f^{(2)}_{\rho'''' NN}/f_{\rho''''}) = 0.0324 \pm 0.0002 \\
(f^{(1)}_{\phi NN}/f_{\phi}) &= -4.4532 \pm 0.1020 \quad (f^{(2)}_{\phi'' NN}/f_{\phi''}) = 0.3617 \pm 0.0502 \\
&\quad \quad \quad m_{\rho'''''} = 2461 \pm 38 \, MeV \\
&\quad \quad \quad \Gamma_{\rho'''''} = 728 \pm 44 \, MeV \\
(f^{(1)}_{\rho''''' NN}/f_{\rho'''''}) &= 0.0542 \pm 0.0004 \\
(f^{(2)}_{\rho''''' NN}/f_{\rho'''''}) &= -0.0112 \pm 0.0001.
\end{align*}
\]

From the comparison of the numerical values of (28) with (27) we come to the conclusions that by fixing the second branch points of the isoscalar and isovector Dirac and Pauli ff’s at the nucleon-antinucleon threshold the coupling constant ratios of isovector vector-mesons to nucleons (the mass and the width of $\rho'''''$ as well) are almost unchanged, but the coupling constant ratios of isoscalar vector-mesons to nucleons (especially of higher vector-mesons) are remarkably out of order.

This fact has explanation in the values of parameters of (27) where the thresholds $t_{in}^{1s, 1v, 2s, 2v}$
Figure 3: A simultaneous optimal fit of all existing data on neutron electric ff

were left to be found in a fitting procedure of data. In the isovector case, they were determined around the value corresponding to the nucleon-antinucleon threshold and, as a result, it does not matter if they are fixed at the $N\bar{N}$ threshold or they are found almost at the same position in a fitting procedure. However, in the isoscalar case they are found at much lower values than the $N\bar{N}$ threshold. So, those values indicate (unlike the isovector ff’s) that between the lowest $t_0^\pm = 9m^2_\pi$ branch point and the $N\bar{N}$ threshold there is some allowed intermediate mass state in the unitarity condition generating an important cut contribution which cannot be neglected in a description of the nucleon e.m. structure. We know from other considerations that it is just the $K\bar{K}$ threshold.

Compilation of world nucleon ff data and their description by our ten-resonance unitary and analytic model with parameters (27) is graphically represented in Figs. 1-4. One can see from Fig. 4 that unlike papers [27,29] the ten-resonance unitary and analytic model is able to describe FENICE time-like data on neutron [26] quite well. The same is also valid for the FERMILAB proton time-like data [21,22] (see Fig. 2). The latter was possible to achieve by introducing a hypothetical fourth excited state of the $\rho(770)$-meson the parameters of which were found in a fitting procedure of all existing data to be quite reasonable (see (27)). Its existence, however, has to be proved by being identified also in other processes and not only $e^+e^- \rightarrow N\bar{N}$. 
Of particular interest is determination of the electric and magnetic, and also Dirac and Pauli radii of nucleons. They are given in Table 1 where for comparison the results of papers [27] and [30] are presented too.

Table 1: Electric and magnetic, and also Dirac and Pauli radii of the proton and neutron

|                | \( r_E^p [fm] \) | \( r_M^p [fm] \) | \( r_M^n [fm] \) | \( r_1^n [fm] \) | \( r_2^n [fm] \) | \( r_3^n [fm] \) |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| our results    | 0.827           | 0.860           | 0.891           | 0.752           | 0.914           | 0.883           |
| Ref. [27]      | 0.847           | 0.836           | 0.889           | 0.774           | 0.894           | 0.893           |
| Ref. [30]      | 0.836           | 0.843           | 0.840           | 0.761           | 0.883           | 0.876           |

Here we would like to stress that the neutron charge radius is predicted by the model to be negative automatically and its value \( < r_{E,n}^2 > = -0.130 [fm^2] \) is more or less compatible with the newest experimental result \( < r_{E,n}^2 > = -0.113 \pm 0.003 \pm 0.004 [fm^2] \) [36].

In order to demonstrate explicitly substantial deviations from the dipole fit in all channels and at the same time the violation of the nucleon ff scaling, particularly at large momentum transfer, we show in Figs. 5 the electric and magnetic proton and neutron ff’s in the space-like region normalized to the dipole formula \( G_D(t) = (1 - t/0.71)^{-2} \).
Figure 5: Ratios of appropriately normalized electric and magnetic proton and neutron ff’s in the space-like region to the dipole formula.

In relation to the data on decreasing ratio of proton electric and magnetic ff’s [11] obtained recently at the Jefferson Lab for $-3.5 GeV^2 < t < -0.5 GeV^2$ by using polarization transfer, one can say that the unitary and analytic ten-resonance model at the same values of momentum transfers gives almost negligible falling around the value 1.02. However, the latter result does not mean that the model presented here is in disagreement with the Jefferson Lab data, but all experimental points measured up to present time (see Figs. 1 and 2) in the interval $-3.5 GeV^2 < t < -0.5 GeV^2$ are in contradiction with them. If the data on the electric and magnetic proton ff in the space-like region are consistent with the new Jefferson Lab data, then the behaviours of $G_E^p(t)$ and $G_M^p(t)$ predicted by the unitary and analytic model will be consistent with them too.
4 Other predictions of unitary and analytic model of the nucleon e.m. structure

The unitary and analytic ten-resonance model of the nucleon e.m. structure constructed in this paper reflects all known nucleon ff properties and thus, it gives one analytic function, i.e. one smooth function on the whole real axis for every nucleon e.m. ff. As a result, one can then believe the predicted behaviours of these nucleon e.m. ff’s to be realistic also outside the regions of existing experimental data.

Valuable is the predicted existence of the fourth excited state of the $\rho(770)$-meson with the resonance parameters $m_{\rho''''}=2455 \pm 38 \ MeV$ and $\Gamma_{\rho''''}=728 \pm 2 \ MeV$ without which one could not achieve a satisfactory description of the FENICE time-like neutron data [26] and also of eight FERMILAB proton experimental points [21,22] at higher energies.

Taking into account the numerical results [27] for the parameters of the model and the transformed relations for additional coupling constant ratios (A.6)-(A.8) from Appendix A

\begin{equation}
I. \quad (f_{\omega''''\omega''''}^{(1)}/f_{\omega''''}) = \frac{1}{2} \frac{C_1^{1s}_{1\omega''''}}{C_1^{1s}_{1\omega''''} - C_1^{1s}_{\omega''''}} - (f_{\omega''''\omega''''}/f_{\omega''''}) \frac{C_1^{1s}_{\omega''''} - C_1^{1s}_{1\omega''''}}{C_1^{1s}_{1\omega''''} - C_1^{1s}_{\omega''''}} + (f_{\omega''''\omega''''}/f_{\omega''''}) \frac{C_1^{1s}_{1\omega''''} - C_1^{1s}_{\omega''''}}{C_1^{1s}_{1\omega''''} - C_1^{1s}_{\omega''''}} \tag{29}
\end{equation}

\begin{equation}
II. \quad (f_{\omega''''\omega''''}^{(1)}/f_{\omega''''}) = \frac{1}{2} \frac{C_1^{1v}_{1\rho''''}}{C_1^{1v}_{1\rho''''} - C_1^{1v}_{\rho''''}} - (f_{\omega''''\omega''''}/f_{\omega''''}) \frac{C_1^{1v}_{\rho''''} - C_1^{1v}_{1\rho''''}}{C_1^{1v}_{1\rho''''} - C_1^{1v}_{\rho''''}} + (f_{\omega''''\omega''''}/f_{\omega''''}) \frac{C_1^{1v}_{1\rho''''} - C_1^{1v}_{\rho''''}}{C_1^{1v}_{1\rho''''} - C_1^{1v}_{\rho''''}} \tag{30}
\end{equation}

\begin{equation}
III. \quad (f_{\omega''''\omega''''}^{(2)}/f_{\omega''''}) = \frac{1}{2} (\mu_p + \mu_n) \frac{C_2^{2s}_{1\omega''''} C_2^{2s}_{\omega''''}}{(C_2^{2s}_{1\omega''''} - C_2^{2s}_{\omega''''})(C_2^{2s}_{\omega''''} - C_2^{2s}_{1\omega''''})} - (f_{\omega''''\omega''''}/f_{\omega''''}) \frac{C_2^{2s}_{1\omega''''} C_2^{2s}_{\omega''''}}{(C_2^{2s}_{\omega''''} - C_2^{2s}_{1\omega''''})(C_2^{2s}_{\omega''''} - C_2^{2s}_{1\omega''''})} - (f_{\omega''''\omega''''}/f_{\omega''''}) \frac{C_2^{2s}_{1\omega''''} C_2^{2s}_{\omega''''}}{(C_2^{2s}_{\omega''''} - C_2^{2s}_{1\omega''''})(C_2^{2s}_{\omega''''} - C_2^{2s}_{1\omega''''})} -
\end{equation}
the following coupling constant ratio numerical values are predicted

\[
\begin{align*}
(f^{(2)}_{\omega NN}/f_{\omega'}) &= 0.5045 & (f^{(2)}_{\rho NN}/f_{\rho'}) &= 0.7647 \\
(f^{(1)}_{\omega NN}/f_{\omega'}) &= 0.1482 & (f^{(1)}_{\rho NN}/f_{\rho'}) &= -0.6199
\end{align*}
\]
\[
(f_{\omega NN}^{(2)}/f_{\omega}) = 0.1712 \quad (f_{\rho NN}^{(2)}/f_{\rho}) = 3.0530
\]
\[
(f_{\omega' NN}^{(2)}/f_{\omega'}) = -0.02455 \quad (f_{\rho' NN}^{(2)}/f_{\rho'}) = -1.6790
\]
\[
(f_{\omega'' NN}^{(2)}/f_{\omega''}) = -0.05992 \quad (f_{\rho'' NN}^{(2)}/f_{\rho''}) = 1.0040.
\]

The universal vector meson coupling constants \(f_s\) and \(f_v\) are determined from the leptonic decay widths by the relation
\[
\frac{f_v^2}{4\pi} = \frac{\alpha^2 m_v}{3 \Gamma(\nu \rightarrow e^+ e^-)}.
\]

Then, numerical values
\[
f_\rho = 5.0320 \pm 0.1089; \quad f_\omega = 17.0499 \pm 0.2990; \quad f_\phi = -12.8832 \pm 0.0824
\]
are found from the corresponding world averaged lepton widths [33] and the universal \(\omega'\), \(\omega''\) and \(\rho', \rho''\) meson coupling constants
\[
f_{\omega'} = 47.6022 \pm 7.5026; \quad f_{\omega''} = 48.3778 \pm 7.5026
\]
and
\[
f_{\rho'} = 13.6491 \pm 0.9521; \quad f_{\rho''} = 22.4020 \pm 2.2728
\]
have been determined from the leptonic widths estimated by Donnachie and Clegg [37].

As a result, the following numerical values of the corresponding coupling constants are predicted
\[
\begin{align*}
 f_{\omega NN}^{(1)} &= 18.9527; \quad f_{\rho NN}^{(1)} = 1.9335; \\
 f_{\phi NN}^{(1)} &= 12.0956; \quad f_{\rho NN}^{(1)} = 10.4375; \\
 f_{\omega NN}^{(1)} &= 24.0153; \quad f_{\rho NN}^{(1)} = -13.8870; \\
 f_{\omega NN}^{(1)} &= 7.1696; \\
 f_{\omega NN}^{(2)} &= 2.9189; \quad f_{\rho NN}^{(2)} = 15.3627; \\
 f_{\phi NN}^{(2)} &= 3.4251; \quad f_{\rho NN}^{(2)} = -22.9168; \\
 f_{\omega NN}^{(2)} &= -1.1686; \quad f_{\rho NN}^{(2)} = 22.4916; \\
 f_{\omega NN}^{(2)} &= -2.8988.
\end{align*}
\]

Their squares divided by \(4\pi\) are reviewed in Table 2 and Table 3, where for comparison also values obtained by other authors [27,30] are presented.
Table 2: Coupling constants of the isoscalar vector mesons to nucleons

|                   | $f_{\omega NN}^{(1)}/4\pi$ | $f_{\phi NN}^{(1)}/4\pi$ | $f_{\omega' NN}^{(1)}/4\pi$ | $f_{\omega'' NN}^{(1)}/4\pi$ |
|-------------------|-----------------------------|---------------------------|-----------------------------|-------------------------------|
| our results       | 28.58                       | 11.64                     | 45.89                       | 4.09                          |
| Ref. [27]         | 34.6                        | 6.7                       | -                           | -                             |
| Ref. [30]         | 24.0                        | 5.1                       | -                           | -                             |

|                   | $f_{\omega NN}^{(2)}/4\pi$ | $f_{\phi NN}^{(2)}/4\pi$ | $f_{\omega' NN}^{(2)}/4\pi$ | $f_{\omega'' NN}^{(2)}/4\pi$ |
|-------------------|-----------------------------|---------------------------|-----------------------------|-------------------------------|
| our results       | 0.67                        | 0.93                      | 0.11                        | 0.67                          |
| Ref. [27]         | 0.9                         | 0.3                       | -                           | -                             |
| Ref. [30]         | -                           | 0.2                       | -                           | -                             |

Table 3: Coupling constants of the isovector vector mesons to nucleons

|                   | $f_{\rho NN}^{(1)}/4\pi$ | $f_{\rho NN}^{(2)}/4\pi$ | $f_{\rho' NN}^{(1)}/4\pi$ |
|-------------------|---------------------------|---------------------------|---------------------------|
| our results       | 0.30                      | 8.67                      | 15.35                     |
| Ref. [27]         | -                         | 40.27                     | 793.53                    |
| Ref. [30]         | 0.55                      | -                         | -                         |

|                   | $f_{\rho NN}^{(2)}/4\pi$ | $f_{\rho NN}^{(2)}/4\pi$ | $f_{\rho' NN}^{(2)}/4\pi$ |
|-------------------|---------------------------|---------------------------|---------------------------|
| our results       | 18.78                     | 41.79                     | 40.26                     |
| Ref. [27]         | -                         | 143.97                    | 304.07                    |
| Ref. [30]         | 24.0                      | 11.5                      | -                         |

One can immediately notice large value of the $f_{\rho NN}^{(1,2)}$ coupling constants which may indicate violation of the OZI rule [38].

Using the numerical values one can predict the $\omega - \phi$ mixing angle employing the relation

$$\sqrt{3} \frac{f_{\rho NN}^{(1)}}{\cos \vartheta} \frac{f_{\omega NN}^{(1)}}{\tan \vartheta} = \frac{f_{\rho NN}^{(1)}}{f_{\omega NN}^{(1)}}.$$  \hspace{1cm} (39)

It takes the value $\vartheta = 0.7175$ which is very close to the ideal mixing.

Nevertheless, the most important predictions of the unitary and analytic model of the nucleon e.m. structure are the isovector spectral function behaviours (see Fig. 6) to be consistent with the predictions of Höhler and Pietarinen [31] and Mergel, Meißner and Drechsel [27], which have been carried out on the basis of the Frazer and Fulco [39] unitarity relation
by using the pion e.m. ff $F_\pi(t)$ and the $P$-wave $\pi\pi \to N\bar{N}$ partial wave amplitudes obtained by an analytic continuation of experimental information on $\pi N$-scattering into the unphysical region.

The method of our prediction of the latter consists in the following. The ten-resonance unitary and analytic model of the nucleon e.m. structure constructed in this paper contains an explicit two-pion continuum contribution given by the unitary cut starting with $t = 4m_\pi^2$ from where just the isovector spectral functions start to be different from zero. Then, despite the fact that the unstable $\rho$-meson is taken into account as a pole shifted from the real axis into the complex plane on the second Riemann sheet of the four-sheeted Riemann surface, the model predicts the strong enhancement on the left wing of the $\rho(770)$ resonance in the isovector spectral functions automatically. Just agreement of our predictions with those obtained by means of the Frazer and Fulco unitarity relation convinces us that our model constructed in this paper is really unitary.

Another result of the presented model is the prediction of the isoscalar nucleon spectral function behaviours (see Fig. 6), as the model contains an explicit three-pion continuum contribution given by the unitary cut starting with $t = 9m_\pi^2$ from where just the isoscalar spectral functions start to be different from zero.
5 Conclusions

We have constructed the unitary and analytic ten-resonance (5 isoscalars and 5 isovectors) model of the nucleon e.m. structure which represents a harmonic unification of all known nucleon ff properties, like analyticity, reality condition, experimental fact of creation of vector-meson resonances in electron-positron annihilation processes, normalization and the asymptotic behaviour as predicted for nucleon e.m. ff’s by the quark model of hadrons. It depends only on parameters with clear physical meaning. They are four effective square-root branch points representing contribution of all other higher thresholds given by the unitarity condition, the mass and width of the hypothetical fourth excited state of the $\rho(770)$-meson and coupling constants of some resonances under consideration. They all are numerically evaluated by analyzing all existing space-like and time-like nucleon ff data.

We would like to note that by means of the model presented in this paper all existing nucleon ff data, including FENICE neutron time-like data and FERMILAB proton eight points at higher energies, are reasonably described. In this effect, existence of the $\rho^\prime\prime\prime\prime(2500)$ resonance with the parameters $m_{\rho^\prime\prime\prime\prime} = 2455$ $MeV$ and $\Gamma_{\rho^\prime\prime\prime\prime} = 728$ $MeV$ plays a crucial role. So, there is challenge to experimental physicists to confirm existence of this resonance also in other processes than $e^+e^- \rightarrow NN$.

The unitary and analytic ten-resonance nucleon ff model gives several reasonable predictions. However, the most important among them are isoscalar and isovector spectral function behaviours which coincide also with the predictions obtained in the framework of heavy baryon chiral perturbation theory [40].

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Appendix A

The isoscalar and isovector parts of the Dirac and Pauli ff’s are saturated by the isoscalar and isovector vector-mesons as follows:

\[
\begin{align*}
F_1^s(t) &= \sum_{\omega,\phi,\omega',\phi'} \frac{m_s^2}{m_s^2 - t} (f_{sNN}^{(1)}/f_s); \\
F_1^v(t) &= \sum_{\vartheta,\vartheta',\vartheta''} \frac{m_v^2}{m_v^2 - t} (f_{vNN}^{(1)}/f_v); \\
F_2^s(t) &= \sum_{\omega,\phi,\omega',\phi'} \frac{m_s^2}{m_s^2 - t} (f_{sNN}^{(2)}/f_s); \\
F_2^v(t) &= \sum_{\vartheta,\vartheta',\vartheta''} \frac{m_v^2}{m_v^2 - t} (f_{vNN}^{(2)}/f_v), 
\end{align*}
\]
where \( m_s \) and \( m_v \) are the isoscalar and isovector vector-meson masses, \( f_{sNN}^{(1)} \), \( f_{vNN}^{(1)} \) and \( f_{sNN}^{(2)} \), \( f_{vNN}^{(2)} \) are the vector and tensor vector-meson-nucleon coupling constants and \( f_s, f_v \) are the universal vector-meson coupling constants to be determined in a vector-meson decay into two charged leptons. The explicit requirement of normalizations (6) and asymptotic behaviours (7) in (A.1) leads to four systems of algebraic equations [35]

I. \[ \sum_{ω,φ,ω′,ω″,φ′} (f_{sNN}^{(1)}/f_s) = \frac{1}{2} \]
\[ \sum_{ω,φ,ω′,ω″,φ′} (f_{sNN}^{(1)}/f_s)m_s^2 = 0 \] (A.2)

II. \[ \sum_{σ,φ,σ′,σ″,φ′} (f_{vNN}^{(1)}/f_v) = \frac{1}{2} \]
\[ \sum_{σ,φ,σ′,σ″,φ′} (f_{vNN}^{(1)}/f_v)m_v^2 = 0 \] (A.3)

III. \[ \sum_{ω,φ,ω′,ω″,φ′} (f_{sNN}^{(2)}/f_s) = \frac{1}{2}(\mu_p + \mu_n) \]
\[ \sum_{ω,φ,ω′,ω″,φ′} (f_{sNN}^{(2)}/f_s)m_s^2 = 0 \] (A.4)
\[ \sum_{ω,φ,ω′,ω″,φ′} (f_{sNN}^{(2)}/f_s)m_s^4 = 0 \]

IV. \[ \sum_{σ,φ,σ′,σ″,φ′} (f_{vNN}^{(2)}/f_v) = \frac{1}{2}(\mu_p - \mu_n) \]
\[ \sum_{σ,φ,σ′,σ″,φ′} (f_{vNN}^{(2)}/f_v)m_v^2 = 0 \] (A.5)
\[ \sum_{σ,φ,σ′,σ″,φ′} (f_{vNN}^{(2)}/f_v)m_v^4 = 0 \]

for \( (f_{sNN}^{(1)}/f_s) \), \( (f_{vNN}^{(1)}/f_v) \), \( (f_{sNN}^{(2)}/f_s) \) and \( (f_{vNN}^{(2)}/f_v) \). The solutions of (A.2)-(A.5) can be chosen in the following form:

I. \[ (f_{ϕ′′NN}/f_{ϕ′}) = \frac{1}{2} \frac{m_{ϕ′′}^2 - m_{ϕ′}^2}{m_{ϕ′′}^2 - m_{ϕ′}^2} - (f_{ϕNN}/f_ϕ) \frac{m_{ϕ′′}^2 - m_{ϕ′}^2}{m_{ϕ′′}^2 - m_{ϕ′}^2} - \]
\[ - (f_{ϕNN}/f_ϕ) \frac{m_{ϕ′′}^2 - m_{ϕ′}^2}{m_{ϕ′′}^2 - m_{ϕ′}^2} + (f_{ϕ′NN}/f_{ϕ′}) \frac{m_{ϕ′′}^2 - m_{ϕ′}^2}{m_{ϕ′′}^2 - m_{ϕ′}^2} \] (A.6)

II. \[ (f_{ϕ′′NN}/f_{ϕ′}) = -\frac{1}{2} \frac{m_{ϕ′′}^2 - m_{ϕ′}^2}{m_{ϕ′′}^2 - m_{ϕ′}^2} + (f_{ϕNN}/f_ϕ) \frac{m_{ϕ′′}^2 - m_{ϕ′}^2}{m_{ϕ′′}^2 - m_{ϕ′}^2} + \]
\[ + (f_{ϕNN}/f_ϕ) \frac{m_{ϕ′′}^2 - m_{ϕ′}^2}{m_{ϕ′′}^2 - m_{ϕ′}^2} - (f_{ϕ′NN}/f_{ϕ′}) \frac{m_{ϕ′′}^2 - m_{ϕ′}^2}{m_{ϕ′′}^2 - m_{ϕ′}^2} \]
II. \( \frac{f^{(2)}_{\omega'NN/f_\omega'}}{f^{(1)}_{\phi'NN/f_\omega'}} = \frac{1}{2} \left( \frac{m^2_{\omega'}}{m^2_{\omega'} - m^2_\phi} - \frac{m^2_{\omega''}}{m^2_{\omega''} - m^2_\phi} \right) \)

\[ + \left( f^{(1)}_{\phi'NN/f_\omega'} \right) \frac{m^2_{\omega'} - m^2_\phi}{m^2_{\omega'} - m^2_\phi} \]

\[ + \left( f^{(1)}_{\phi''NN/f_\omega''} \right) \frac{m^2_{\omega''} - m^2_\phi}{m^2_{\omega''} - m^2_\phi} + \left( f^{(1)}_{\phi'''NN/f_\omega'''} \right) \frac{m^2_{\omega'''} - m^2_\phi}{m^2_{\omega'''} - m^2_\phi} \]

(A.7)

III. \( \frac{f^{(2)}_{\omega NN/f_\omega}}{f^{(1)}_{\phi NN/f_\omega}} = \frac{1}{2} \left( \mu_p + \mu_n \right) \left( \frac{m^2_{\omega}}{m^2_{\omega} - m^2_\phi} \right) \)

\[ - \left( f^{(2)}_{\phi NN/f_\omega} \right) \frac{m^2_{\omega'} - m^2_\phi}{m^2_{\omega'} - m^2_\phi} \]

\[ - \left( f^{(2)}_{\phi' NN/f_\omega'} \right) \frac{m^2_{\omega''} - m^2_\phi}{m^2_{\omega''} - m^2_\phi} \]

\[ - \left( f^{(2)}_{\phi'' NN/f_\omega''} \right) \frac{m^2_{\omega'''} - m^2_\phi}{m^2_{\omega'''} - m^2_\phi} \]

(A.8)

IV. \( \frac{f^{(2)}_{\phi NN/f_\omega}}{f^{(2)}_{\phi' NN/f_\omega'}} = \frac{1}{2} \left( \mu_p - \mu_n \right) \left( \frac{m^2_{\phi}}{m^2_{\phi} - m^2_\omega} \right) \)

\[ - \left( f^{(2)}_{\phi'' NN/f_\omega''} \right) \frac{m^2_{\omega''} - m^2_\phi}{m^2_{\omega''} - m^2_\phi} \]

\[ - \left( f^{(2)}_{\phi''' NN/f_\omega'''} \right) \frac{m^2_{\omega'''} - m^2_\phi}{m^2_{\omega'''} - m^2_\phi} \]

\[ - \left( f^{(2)}_{\phi' NN/f_\omega'} \right) \frac{m^2_{\omega'} - m^2_\phi}{m^2_{\omega'} - m^2_\phi} \]

\[ - \left( f^{(2)}_{\phi'' NN/f_\omega''} \right) \frac{m^2_{\omega''} - m^2_\phi}{m^2_{\omega''} - m^2_\phi} \]

\[ - \left( f^{(2)}_{\phi''' NN/f_\omega'''} \right) \frac{m^2_{\omega'''} - m^2_\phi}{m^2_{\omega'''} - m^2_\phi} \]

\[ - \left( f^{(2)}_{\phi' NN/f_\omega'} \right) \frac{m^2_{\omega'} - m^2_\phi}{m^2_{\omega'} - m^2_\phi} \]
\[ + \left( f_{\ell}^{(2)}/f_{\ell}^{(m)} \right) \frac{(m_{\ell}^2 - m_{\ell}^2)(m_{\ell}^2 - m_{\ell}^2)}{(m_{\ell}^2 - m_{\ell}^2)(m_{\ell}^2 - m_{\ell}^2)} + \]

\[ + \left( f_{\ell}^{(2)}/f_{\ell}^{(m)} \right) \frac{(m_{\ell}^2 - m_{\ell}^2)(m_{\ell}^2 - m_{\ell}^2)}{(m_{\ell}^2 - m_{\ell}^2)(m_{\ell}^2 - m_{\ell}^2)} \]

\[ (f_{\ell}^{(2)}/f_{\ell}^{(m)}) = \frac{1}{2} \left( \frac{\mu_p - \mu_n}{m_p - m_n} \right) \frac{m_p^2 m_n^2}{(m_p^2 - m_n^2)(m_p^2 - m_n^2)} \]

which transform the original parametrizations (A.1) of the isoscalar and isovector Dirac and Pauli nucleon ff’s just into the normalized zero-width VMD expressions (8)-(11) with asymptotics (7).

**Appendix B**

Incorporation of the assumed analytic properties of the nucleon e.m. ff’s into the normalized zero-width VMD model (8)-(11) can be achieved by application of the nonlinear transformations (12) and a subsequent installation of the nonzero values of vector meson widths.

There are also other expressions utilized for the vector meson masses squared

\[ m_s^2 = t_0^2 - \frac{4(t_{1s}^1 - t_{0s}^0)}{1/V_{s0} - V_{s0}^2}, \quad m_s^2 = t_0^2 - \frac{4(t_{2s}^1 - t_{0s}^0)}{1/U_{s0} - U_{s0}^2}, \]

\[ m_v^2 = t_0^2 - \frac{4(t_{1v}^1 - t_{0v}^0)}{1/W_{v0} - W_{v0}^2}, \quad m_v^2 = t_0^2 - \frac{4(t_{2v}^1 - t_{0v}^0)}{1/X_{v0} - X_{v0}^2}, \]

and identities

\[ 0 = t_0^2 - \frac{4(t_{1s}^1 - t_{0s}^0)}{1/V_N - V_N^2}, \quad 0 = t_0^2 - \frac{4(t_{2s}^1 - t_{0s}^0)}{1/U_N - U_N^2}, \]

\[ 0 = t_0^2 - \frac{4(t_{1v}^1 - t_{0v}^0)}{1/W_N - W_N^2}, \quad 0 = t_0^2 - \frac{4(t_{2v}^1 - t_{0v}^0)}{1/X_N - X_N^2}, \]

following from (12) where \( V_{s0}, W_{v0}, U_{s0}, X_{v0} \) are the zero-width (therefore, they have a subindex 0) VMD poles and \( V_N, W_N, U_N, X_N \) are the normalization points (corresponding to \( t = 0 \)) in the \( V, W, U, X \) planes, respectively.

Really, relations (12), (B.1), (B.2) first transform every \( t \)-dependent term and every constant term consisting of a ratio of mass differences in (8)-(11) into a new form as follows. For instance, the term \( m_s^2/(m_s^2 - t) \) in (8) is transformed into the following form:

\[ \frac{m_s^2}{m_s^2 - t} = \left( \frac{1 - V^2}{1 - V_{s0}^2} \right)^2 \frac{(V_N - V_{s0})(V_N + V_{s0})(V_N - 1/V_{s0})(V_N + 1/V_{s0})}{(V - V_{s0})(V + V_{s0})(V - 1/V_{s0})(V + 1/V_{s0})}. \]
The constant mass terms, e.g. \( (m^2_\omega - m^2_\omega)/(m^2_{\omega'} - m^2_{\omega'}) \), also from \([8]\), becomes:

\[
\frac{m^2_\omega - m^2_\omega}{m^2_{\omega'} - m^2_{\omega'}} = \frac{(m^2_\omega - 0) - (m^2_\omega - 0)}{(m^2_{\omega'} - 0) - (m^2_{\omega'} - 0)} = \\
= \frac{(V_N - V_{\omega_0})(V_N + V_{\omega_0})(V_N - 1/V_{\omega_0})(V_N + 1/V_{\omega_0})}{(V_{\omega_0} - 1/V_{\omega_0})^2} - \\
- \frac{(V_N - V_{\omega_0})(V_N + V_{\omega_0})(V_N - 1/V_{\omega_0})(V_N + 1/V_{\omega_0})}{(V_{\omega_0} - 1/V_{\omega_0})^2} \\
= \frac{(V_N - V_{\omega_0})(V_N + V_{\omega_0})(V_N - 1/V_{\omega_0})(V_N + 1/V_{\omega_0})}{(V_{\omega_0} - 1/V_{\omega_0})^2} - \\
- \frac{(V_N - V_{\omega_0})(V_N + V_{\omega_0})(V_N - 1/V_{\omega_0})(V_N + 1/V_{\omega_0})}{(V_{\omega_0} - 1/V_{\omega_0})^2} = \\
= \frac{C_{1\omega_0}^{1s} - C_{1\omega_0}^{1s}}{C_{1\omega_0}^{1s} - C_{1\omega_0}^{1s}}.
\]

Then by utilization of the relations between complex and complex conjugate values of the corresponding zero-width VMD pole positions in the \( V, W, U, X \) planes

\[
V_{\omega_0} = -V^{*}_{\omega_0}; V_{\omega_0} = -V^{*}_{\omega_0}; V_{\omega_0} = -V^{*}_{\omega_0}; V_{\omega_0} = 1/V^{*}_{\omega_0}; V_{\omega_0} = 1/V^{*}_{\omega_0}; \\
W_{\omega_0} = -W^{*}_{\omega_0}; W_{\omega_0} = -W^{*}_{\omega_0}; W_{\omega_0} = -W^{*}_{\omega_0}; W_{\omega_0} = 1/W^{*}_{\omega_0}; W_{\omega_0} = 1/W^{*}_{\omega_0}; \quad (B.5) \\
U_{\omega_0} = -U^{*}_{\omega_0}; U_{\omega_0} = -U^{*}_{\omega_0}; U_{\omega_0} = -U^{*}_{\omega_0}; U_{\omega_0} = 1/U^{*}_{\omega_0}; U_{\omega_0} = 1/U^{*}_{\omega_0}; \\
X_{\omega_0} = -X^{*}_{\omega_0}; X_{\omega_0} = -X^{*}_{\omega_0}; X_{\omega_0} = -X^{*}_{\omega_0}; X_{\omega_0} = 1/X^{*}_{\omega_0}; X_{\omega_0} = 1/X^{*}_{\omega_0}
\]

following from the fact that in a fitting procedure one finds

\[
m^2_\omega - \Gamma^2_\omega/4 < t^{1s}_{in}; m^2_\omega - \Gamma^2_\omega/4 < t^{1s}_{in}; m^2_{\omega'} - \Gamma^2_{\omega'}/4 < t^{1s}_{in}; \\
m^2_{\omega'} - \Gamma^2_{\omega'}/4 > t^{1s}_{in}; m^2_{\omega'} - \Gamma^2_{\omega'}/4 > t^{1s}_{in}; \\
m^2_\omega - \Gamma^2_\omega/4 < t^{2s}_{in}; m^2_\omega - \Gamma^2_\omega/4 < t^{2s}_{in}; m^2_{\omega'} - \Gamma^2_{\omega'}/4 < t^{2s}_{in}; \\
m^2_{\omega'} - \Gamma^2_{\omega'}/4 > t^{2s}_{in}; m^2_{\omega'} - \Gamma^2_{\omega'}/4 > t^{2s}_{in}; \quad (B.6) \\
m^2_\omega - \Gamma^2_\omega/4 < t^{1v}_{in}; m^2_\omega - \Gamma^2_\omega/4 < t^{1v}_{in}; m^2_{\omega'} - \Gamma^2_{\omega'}/4 < t^{1v}_{in}; \\
m^2_{\omega'} - \Gamma^2_{\omega'}/4 > t^{1v}_{in}; m^2_{\omega'} - \Gamma^2_{\omega'}/4 > t^{1v}_{in}; \\
m^2_\omega - \Gamma^2_\omega/4 < t^{2v}_{in}; m^2_\omega - \Gamma^2_\omega/4 < t^{2v}_{in}; m^2_{\omega'} - \Gamma^2_{\omega'}/4 < t^{2v}_{in}; \\
m^2_{\omega'} - \Gamma^2_{\omega'}/4 > t^{2v}_{in}; m^2_{\omega'} - \Gamma^2_{\omega'}/4 > t^{2v}_{in};
\]

and subsequent introduction of the non-zero values of vector-meson widths \( \Gamma \neq 0 \) by the substitutions

\[
m^2_s \rightarrow (m_s - \frac{\Gamma_s}{2})^2; \quad m^2_v \rightarrow (m_v - \frac{\Gamma_v}{2})^2, \quad (B.7)
\]

one comes to \([14]-[17]\).
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