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Modelling of resonant droplet detachment in laser metal droplet generation

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Abstract

In laser generation of droplets from a metal wire, it has been observed experimentally that droplet detachment can be influenced by interaction of the laser pulse frequency and eigen dynamics of a pendent droplet. In laser pulse frequency range between 70 Hz and 150 Hz, the dynamics of the pendent droplet resembles that of a spring-mass system. Based on experimental observations a nonlinear model of droplet generation in the form of spring-mass oscillator is developed. Model based simulation of pendent droplet dynamics, and especially predicted time of droplet detachment and detached droplet diameter are in high agreement with experimental results.

Keywords: Laser droplet generation; resonant droplet detachment; mathematical modelling

1. Introduction

Laser metal droplet generation (LDG) is a process where laser light is used to melt the tip of a vertically fed metal wire and then to detach the formed pendent droplet [1]. The resulting metal droplets have many potential applications in joining [2], especially joining of dissimilar materials [3] or welding of temperature sensitive parts, for example zinc coated steel sheets [4]. For the above mentioned joining applications beside the drop-on-demand generation also a sequential generation of droplets is required.

Recently, experiments of a sequential LDG were realized by feeding a metal wire at constant velocity and using rectangular laser pulses at selected constant frequency. Dependent on the laser pulse frequency, four different detachment mechanisms were experimentally observed [5]. In the laser pulse frequency range between 70 Hz and 150 Hz a distinct vertical droplet oscillation was noticed, resembling the oscillation of a spring-mass system. When in resonance with the laser pulse frequency this oscillation was

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observed to cause the droplet detachment. An analogue phenomena was also observed in gas metal arc droplet generation [6] and was modelled by a second order spring-mass system [7].

The aim of this work is to model the LDG process as a spring-mass system in order to predict the oscillation of the growing pendent droplet, droplet detachment time, and the diameter of the detached droplet. Beside the droplet temperature, the diameter of the detached droplet is of a high importance to the applications mentioned above. The spring-mass model is based on the experimental observations of the LDG process by a high speed infrared (IR) camera images. The experiment and its results are presented in the following section. In the third section, the spring-mass model of droplet growth and detachment is described. In the fourth section preceding the conclusions the results of the model are presented and discussed in comparison to experimental data.

2. Experimental observations

A scheme of experimental setup [8] is shown in Fig. 1a. The conical laser beam is focused coaxially on the tip of a vertically fed metal wire. The laser light pulses melt the wire that is fed downwards with a constant velocity to produce a pendent droplet. Repetitive melting of the wire by laser pulses provides driving for the pendent droplet vertical oscillation.

The diameter $d_w$ of the nickel wire was 0.25 mm. The wire was fed downwards with a constant feeding velocity $v_f$ of 0.06 m/s. The laser pulse frequency $f_p$ values in the experiments were 70 Hz, 80 Hz, 100 Hz, and 150 Hz. The duration $\tau$ of rectangular laser pulses was 0.8 ms. The laser pulse power was adjusted to keep the average laser power equal to 120 W at different laser pulse frequency values. The process was characterized by a high speed IR camera. The IR camera recording frequency at 64x128 image resolution was 1445 frames per second with 0.04 ms integration time.

Fig. 1. (a) scheme of experimental setup; (b) four selected examples of recorded IR images. The droplet detaches directly before the last image; (c) the corresponding time series $z(t)$ of droplet centroid vertical position. The droplet detaches at $t = 0.67$ s
Examples of the recorded IR intensity images are shown in Fig. 1b. The leftmost image shows the droplet at early beginning of its growth. The middle two images illustrate the upper and lower extreme positions of the laser pulse induced resonant droplet vertical oscillation. The rightmost image shows the droplet directly after detachment with a visible hot wire end above the droplet.

Based on the IR image analysis a droplet centroid vertical position time series $z(t)$ are generated. An example of the $z(t)$ time series observed at laser pulse frequency $f_p = 70$ Hz is presented in Fig. 1c. The $z(t)$ time series is nonstationary and exhibits significant nonlinear oscillation that is induced by a pulsed laser light melting the wire. Namely, during the laser pulse the laser light rapidly melts the wire above the pendent droplet. The melted volume merges with the droplet as the droplet is pulled up by surface tension. These pull-ups are rapid and distinctive in the beginning part of $z(t)$ time series. Between the pull-ups, the droplet vertical oscillation is superimposed on a linearly decreasing droplet centroid vertical position $z(t)$ which corresponds to the constant wire feeding velocity $v_f$. The increasing droplet mass is reflected in increasingly smoother oscillation and decreasing natural oscillation frequency of the droplet. The oscillation becomes smoother also because a large and hot droplet contains enough energy to melt the wire by itself between the laser pulses and thus the amplitude of pull-ups is decreased. The abrupt end of $z(t)$ time series oscillation with a subsequent monotonous decrease near the end of the time series is caused by detachment of the droplet followed by a free fall.

The time-frequency analysis of the $z(t)$ time series shows a series of resonances at multiples of laser frequency $f_p$ experienced by the droplet during its growth. It has been observed that the last resonance, where the droplet natural frequency equals the laser pulse frequency $f_p$, triggers the droplet detachment. To model the pendent droplet as a spring-mass system the effective spring stiffness $k$ is calculated at each resonant frequency $f_{res}$ by using the corresponding droplet mass $m_{res}$ and the spring-mass natural frequency equation:

$$k = 4\pi^2 f_{res}^2 m_{res}. \quad (1)$$

The calculated effective spring stiffness $k$ shows a weak dependence on the droplet mass which can be described by a power law:

$$k(m) = 0.186 \text{ N/m} \cdot \left( m_{[\text{kg}]} \right)^{0.26}. \quad (2)$$

3. Model formulation

Based on the experimental observations, a spring-mass theoretical model of the LDG is formed. In the model, the pendent droplet system with its complex dynamics is reduced to a one-dimensional nonlinear time dependent spring-mass system that still includes the essential phenomena governing the droplet oscillation and detachment.

The correspondence between the droplet and the spring-mass system is illustrated in Fig. 2. The droplet centroid vertical position corresponds to the position $z$ of the point mass and the position of the droplet neck corresponds to the spring attachment position $z_s$.

The model equation is a second order differential equation with time-varying mass $m(t)$ and coefficients $c(t)$ and $k(t)$:

$$m(t)\ddot{z}(t) + c(t)\dot{z}(t) + k(t)z(t) + m(t)g = F_{\text{ext}}(t) = k(t)z_s(t). \quad (3)$$
Here, $c$ denotes the damping coefficient, $z_s$ the spring attachment position, $k$ the spring stiffness, and $g$ the acceleration of gravity. The external driving force $F_e$ is introduced by displacement of the spring attachment position $z_s$ which follows the droplet neck displacement.

In the following, the time dependent mass $m(t)$ and coefficients $c(t)$ and $k(t)$ are defined based on the experimental observations. The droplet mass $m$ is increased during each laser pulse by adding the melted portion of the wire. The corresponding mass increase $\Delta m$ equals the mass of the wire that is fed through the laser focus plane in one period $T$ of the laser: $\Delta m = \rho v_f T\pi d_w^2 / 4$. Here, $\rho$ denotes density of liquid nickel ($\rho = 7905 \text{ kg/m}^3$ [9]), $v_f$ is wire feeding velocity, $T = 1/f_p$ is period of the laser and $d_w$ is wire diameter. A simple linear increase of droplet mass during the laser pulse duration and a constant droplet mass value between the pulses are presumed (see Fig. 3a):

$$m(t) = \begin{cases} m_0 + n \Delta m; & nT \leq t < (n+1)T - \tau, \\ m_0 + n \Delta m + \frac{t - [(n+1)T - \tau]}{\tau}; & (n+1)T - \tau \leq t < (n+1)T. \end{cases} \tag{4}$$

Here, $m_0$ is initial mass, $\tau$ is duration of the laser pulse, and $n = 0, 1, 2, \ldots$ is a number of the completed laser periods $T$. It is presumed that the laser pulse occurs at the end of each period $T$. The viscous damping coefficient time dependence $c(t)$ is estimated by [10]:

$$c(t) = \mu 4\pi r(t) = \mu 4\pi \sqrt{\frac{3m(t)}{4\pi \rho}}. \tag{5}$$

where $\mu$ is dynamic viscosity of liquid nickel ($\mu = 1.66 \cdot 10^{-4} \text{ Pa} \cdot \text{s}$ [9]) and $r$ is droplet radius. To reproduce the experimentally observed dynamics, the damping coefficient (5) has to be multiplied by a factor of 1300. The observed high damping may be a result of rotational melt flow inside the droplet [11]. The spring stiffness time dependence $k(t)$ is expressed by combining Eqs. (2) and (4): $k(t) = k(m(t))$.

The time-varying spring attachment position $z_s(t)$ which is a source of external driving is presumed to consist of linear sections. Between the laser pulses, the attachment position $z_s$ descends with the velocity of feeding $v_f$. During the laser pulse, $z_s$ linearly rises to reach the initial value. The amplitude $\Delta z_s$ of this oscillation equals the length of the wire that is fed through the laser focus plane in the time $T - \tau$ between the consecutive laser pulses: $\Delta z_s = v_f (T - \tau)$. This amplitude value is experimentally observed at the very beginning of the droplet growth. However, in time of $t_s = 0.28s$ the experimental $\Delta z_s$ value is gradually
reduced to 50% of its initial value due to melting of the wire between the laser pulses by the large and hot droplet. The spring attachment position time dependence $z_s(t)$ is thus defined as (see Fig. 3b):

$$z_s(t) = \begin{cases} 1 - 0.5 \frac{t}{t_s}; & t < t_s \\ 0.5; & t \geq t_s \end{cases} \left\{ \begin{array}{l} \Delta z_s \left( 1 - \frac{t_n T - \tau}{T - \tau} \right); & nT \leq t < (n+1)T - \tau, \\ \Delta z_s \frac{t_n - (nT - \tau)}{\tau}; & (n+1)T - \tau \leq t < (n+1)T. \end{array} \right.$$  \hspace{1cm} (6)

To avoid unrealistic discontinuities in $z_s(t)$, the above function is smoothed by a running average filter with a length of 0.13 $T$.

To model the detachment, two necessary detachment conditions are set. The first condition takes into account the limited surface tension force which holds the droplet attached to the wire. For droplet detachment it is required that the maximum retaining force which equals the surface tension force minus the droplet weight is exceeded by the upward droplet acceleration force:

$$m \ddot{z} > \gamma \pi d_w^2 \cdot mg.$$  \hspace{1cm} (7)

Here, $\gamma$ denotes the surface tension of liquid nickel ($\gamma = 1.78$ N/m [9]). The second condition takes into account the possibility of reattachment in the case of a positive droplet velocity $\dot{z}$ at the time of detachment. It is experimentally observed that if the droplet moves upwards after detachment, it could reattach which renders the detachment unsuccessful. The second condition therefore requires that at the time of detachment the vertical droplet velocity $\dot{z}$ should be equal or less than a predefined value $v_{det}$:

$$\dot{z} \leq v_{det}.$$  \hspace{1cm} (8)

A natural choice of $v_{det}$ value is 0 which requires that the droplet does not move upwards at the time of detachment. Beside the value $v_{det} = 0$, also $v_{det} = \infty$ is employed in the calculation to automatically fulfil the second condition. Namely, with $v_{det} = \infty$, the second detachment condition is fulfilled for any velocity $\dot{z}$ of the droplet and the detachment takes place as soon as the first detachment condition is fulfilled.

**Fig. 3.** (a) the droplet mass $m$ time dependence. $m_0$ denotes the droplet initial mass, $\Delta m$ denotes the droplet mass increase during the pulse duration $\tau$ and $T$ denotes the laser period; (b) the unsmoothed spring attachment position $z_s$ time dependence (schematically). The amplitude of the spring attachment position oscillation is denoted by $\Delta z_s$. 
The LDG model given by differential equation (3) is numerically solved using a Runge-Kutta method for the laser pulse frequency $f_p$ values between 50 Hz and 200 Hz in steps of 5 Hz. At each frequency value, the solution is calculated for a set of 50 equally spaced initial mass $m_0$ values from the interval between $0.04\cdot10^{-7}$ kg and $2.4\cdot10^{-7}$ kg which corresponds to the experimental initial mass values. The initial position $z_0$ of the point mass is assumed to be the equilibrium position of the initial mass $m_0$ suspended from a spring. The initial velocity $z_0$ of the point mass is assumed to equal the velocity of feeding $v_f$. The detachment is modelled separately for two cases: $v_{det} = 0$ and $v_{det} = \infty$.

4. Results and discussion

In Fig. 4, examples of $z(t)$ time series calculated by the model (thin black line) are compared to the experimental droplet centroid vertical position time series, extracted from the IR images (thick grey line) at laser pulse frequencies $f_p$ of 70 Hz, 80 Hz, 100 Hz, and 150 Hz. In the figure, the modelled time of detachment is denoted by a circle for $v_{det} = 0$ and a triangle for $v_{det} = \infty$. The experimental time of detachment is denoted by a square. The initial mass $m_0$ value used to model the presented time series is $1.18\cdot10^{-7}$ kg.

![Fig. 4. Examples of $z(t)$ time series calculated by the model (thin black line) compared to the experimental $z(t)$ time series (thick grey line) at selected values of the laser pulse frequency $f_p$. The modelled time of droplet detachment is denoted by a circle for $v_{det} = 0$ and a triangle for $v_{det} = \infty$. The experimental time of detachment is denoted by a square.](image-url)
According to Fig. 4, the model successfully reproduces the vertical oscillation of a pendent droplet except in the interval of a few laser pulse periods in the second quarter of the time series. For example, in the case of laser pulse frequency $f_p = 70$ Hz this discrepancy between the modelled and experimental time series $z(t)$ is the most expressed around $t = 0.25$ s, and at $f_p = 100$ Hz around $t = 0.13$ s. The cause for this disagreement is a superimposed pendulum-like lateral oscillation of the pendent droplet. The lateral oscillation could be initiated by a small deviation from the axial symmetry, for example by a small misalignment between the wire and the laser beam. We believe that the observed non-translational oscillation is one of the resonant motions of the elastic pendulum [12]. Obviously, such motion cannot be described by a proposed spring-mass model. However, at the time of the elastic pendulum resonance the modelled vertical oscillation undergoes a resonance at the double laser pulse frequency which is also observed in experimental vertical oscillation and is consistent with the elastic pendulum resonance.

The modelled time of droplet detachment at condition $v_{det} = 0$ (circles in Fig. 4) corresponds well to the experimental time of droplet detachment (squares) with a difference of up to two laser periods $T$. These detachments are triggered by the resonance between the pendent droplet vertical oscillation and the frequency of the laser pulses $f_p$.

The time of detachment directly influences the detached droplet diameter which is of a high importance to LDG applications. Namely, the volume of the detached droplet is proportional to the time of detachment due to the constant velocity of wire feeding. In Fig. 5 the detached droplet diameter $d$ vs. the laser pulse frequency $f_p$ is presented as determined by the experiment (squares) and the model (circles for $v_{det} = 0$ and triangles for $v_{det} = \infty$).

In experiment, the majority of droplets detach due to resonance at the laser pulse frequency. The droplet diameters of these droplets constitute the upper branch of squares in Fig. 5. A few cases of earlier detachment are also experimentally observed. The corresponding droplet diameters constitute the lower branch of squares in Fig. 5. In these cases the detachment is triggered by the above mentioned elastic pendulum resonant motion. It seems that the lateral component of resonant oscillation can critically contribute to the detachment by adding to the total oscillation energy of the droplet which results in larger acceleration forces and enhance the possibility of a droplet to detach.

The dashed lines in Fig. 5 indicate bifurcation and the simultaneous occurrence of two experimental droplet diameters at the laser pulse frequency of 100 Hz and below. The smaller droplets at that frequencies seem to occur irregularly during the experimental laser droplet sequence generation. Their occurrence could be influenced by various factors including the droplet initial mass value, misalignment between the wire and the laser beam, the temperature of the droplet, and the elastic pendulum resonance, which could not be fully controlled by the present experimental setup.

![Fig. 5. Detached droplet diameter $d$ vs. the laser frequency $f_p$. Experimental values are denoted by squares, modelled values are denoted by circles for $v_{det} = 0$ and triangles for $v_{det} = \infty$.](image-url)
The modelled detached droplet diameters for $v_{\text{det}} = 0$ (circles in Fig. 5) reproduce the upper experimental branch very well with the exception of the lowest two laser pulse frequency values where the diameters fall on the lower experimental branch. In attempt to consider the effect of lateral droplet oscillation which facilitates the detachment, the second detachment condition is omitted by using $v_{\text{det}} = \infty$. This way, the double laser pulse frequency resonance that coincides with the elastic pendulum resonance can trigger the detachment provided that the first detachment condition is fulfilled. The resulting modelled detached droplet diameters are denoted by triangles in Fig. 5 (see also the position of triangles in Fig. 4). The modelled detached droplet diameters at $f_p = 120$ Hz and below fall to the lower branch as the detachment in these cases is triggered by the double laser pulse frequency resonance. Depending on the initial mass, the resulting diameters populate both branches at laser pulse frequencies 65–70 Hz and 115–120 Hz. Above the frequency of 120 Hz only the upper branch is populated as the detachment is triggered by the laser pulse frequency resonance. At laser pulse frequencies above 120 Hz the model therefore predicts a stable LDG process resulting in only one detached droplet diameter value which is consistent with the experiment.

5. Conclusion

In the paper a spring-mass model is proposed to describe LDG. The presented spring-mass model of LDG process is able to successfully reproduce the basic dynamics of the pendent droplet and to predict the time of droplet detachment as well as the detached droplet diameters. What is more, the model reproduces the region of LDG process stability above the laser pulse frequency of 120 Hz and also the detached droplet diameter bifurcation and a simultaneous occurrence of two detached droplet diameter values at laser pulse frequencies below 120 Hz. In future, the model will be used to simulate laser droplet sequence generation and to determine the experimental process parameter values.

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