We investigate the uncertainties for current and future measurements of electroweak (EW) parameters at hadron colliders. These include the measurement of the mass of the top quark ($M_T$), the direct measurement of the mass of the W boson ($M_{W}^{\text{direct}}$), the measurement of the effective EW mixing angle $\sin^2 \theta_{\text{eff}}^{\text{lept}} (M_Z)$, and the measurement of the on-shell EW mixing angle $\sin^2 \theta_W = 1 - M_{W}^2/M_{Z}^2$ which is equivalent to an indirect measurement of the W mass ($M_{W}^{\text{indirect}}$). Reduction of a factor of 2 to 3 in the measurement errors is expected in the future.
1 Measurements of electroweak parameters.

With the discovery of the Higgs boson, the standard model is over constrained. Within the standard model (SM), measurements of the mass of the $Z$ boson ($M_Z$) and the mass of the top quark ($M_T$), in combination with the mass of the Higgs boson ($M_H$), can be used to predict the mass of the W boson ($M_W$).

Figure 1: (a) World average of all direct measurements of $M_W$ (CDF, D0, LEP2) versus the average of all $M_T$ measurements (CDF, D0, CMS, ATLAS) in 2014. Also shown is the expectation from the SM (with $M_H = 125.6 \pm 0.7$ GeV) in green. Supersymmetry models predict values which are above the SM line. (b) Same as (a) but with the CMS measurement of $M_T$ in 2015 as compared to the Tevatron measurement of $M_T$.

Fig.1 (a) (from ref.[1]) shows the current world average of direct measurements of $M_W=80.385 \pm 0.015$ GeV versus the average of the direct measurements of $M_T=173.34 \pm 0.76$ GeV. Also shown in green is the expectation from the SM with $M_H = 125.6 \pm 0.7$ GeV. The average of all direct measurements of $M_W$ is about 1.5 standard deviation higher than the prediction of the standard model. Predictions of supersymmetric models for $M_W$ are also higher than the predictions of the standard model. Therefore, more precise measurements of $M_W$ are of great interest.

Alternatively, $M_W$ can also be extracted indirectly from measurements of the on-shell electroweak mixing angle $\sin^2 \theta_W$ by the relation $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$. 

http://pdg.lbl.gov/2014/reviews/rpp2014-rev-standard-model.pdf
K.A. Olive et al. (PDG), Chin. Phys. C38, 090001 (2014) (http://pdg.lbl.gov)
In this communication we investigate the uncertainties in future measurements of electroweak (EW) parameters at hadron colliders. These measurements include $M_W$, $M_T$, the effective EW mixing angle $\sin^2\theta_{\text{eff}}(M_Z)$, and the on-shell EW mixing angle $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ which is equivalent to an indirect measurement of $M_W$.

1.1 Direct measurements of $M_T$

The average of the Tevatron measurements of $M_T$ in 2014 is $174.34 \pm 0.37 \pm 0.52$ GeV ($174.34 \pm 0.64$). When combined with the 2014 measurements of ATLAS and CMS in March 2014, the combined 2014 world average \cite{3} (CDF, D0, CMS, ATLAS) is $173.34 \pm 0.27 \pm 0.71$ GeV (or $173.34 \pm 0.76$ GeV).

The most recent measurement of $M_T$ at the LHC are somewhat lower than at the Tevatron. The ATLAS result \cite{5} published 2015 is $M_T = 172.99 \pm 0.91$ GeV. The CMS \cite{6} 2015 measurement of $M_T = 172.44 \pm 0.13 \pm 0.47$ GeV ($172.44 \pm 0.48$ GeV) is the most precise measurement to date and supersedes all previous CMS results. There is about a 2 standard deviation tension between CMS measurement of $M_T$ in 2015 and the earlier Tevatron measurements. However, both are consistent with the world average. As shown in Fig. 1(b), a lower value of $M_T$ would imply a somewhat larger deviation of $M_W$ from the prediction of the SM.
As of 2015 CMS achieved\(^6\) a ±0.48 GeV uncertainty in the measurement of \(M_T\). Fig. 2(a) shows the projected uncertainty in the measurement of \(M_T\) (in GeV) at CMS using different methods for various integrated luminosities. Fig. 2(b) shows the various contributions to the error in \(M_T\) (in GeV) using the standard method for the measurement. In the long term, as shown in Fig. 2, a precision of ±0.2 GeV could be achieved\(^7\) with about 3000 fb\(^{-1}\) of data at 14 TeV. From the experimental point of view it is conceivable that an uncertainty of \(O(\Lambda_{QCD})\) is achieved at the end of HL-LHC. In the sub-GeV regime issues of theoretical interpretation become important, in particular the understanding of the relation of the top quark pole mass with that deployed in the simulations used for the calibration of the measurement of \(M_T\) in hadron colliders.

**1.2 Direct measurements of \(M_W\)**

Both D0 and CDF published direct measurements\(^2\) of \(M_W\) using data samples of the first 2.2 \(fb^{-1}\) of run II at the Tevatron. The uncertainty in the results from each experiment is about 20 MeV. The combined results of the two experiments has an error of 15 MeV. The sources of the uncertainty in the CDF\(^9\) published 2.2 \(fb^{-1}\) result are listed in Table 1. Analysis of the full 9.4 \(fb^{-1}\) Teatron run II sample is currently under way. One would expect a reduction in the statistical error of about a factor of 2 (from 12 to 6 MeV). The dominant systematic errors from the energy scale and Parton Distribution Functions (PDFs) could be reduced significantly as discussed later in this paper. Therefore, a 10 MeV uncertainty in the direct measurement of the W mass may be achievable with the full run II data sample at the Tevatron.

The current ongoing measurement of \(M_W\) at CMS is using the lower luminosity (lower pileup) sample taken at 7 TeV (with \(\approx\) 5 \(fb^{-1}\)). The current ongoing LHC analyses also aim at an error of about 10 MeV. Because of the sensitivity of the measurement of missing \(E_T\) to the effects of pileup, the higher integrated luminosities at 8 TeV and at 13-14 TeV may not be as useful for the direct measurement of \(M_W\). However, as discussed in following sections of this paper, data with higher luminosities can contribute to constraining PDFs and thus help reduce the PDF error in the measurement of \(M_W\) with existing low pileup LHC data samples.

**1.3 Tevatron measurements of \(\sin^2\theta_W\) and \(M_W^{indirect}\)**

Measurements of the forward-backward charge asymmetry \((A_{FB}(M,y)\) in Drell-Yan dilepton events produced at hadron colliders (in the region of the Z pole) have been used to measure the value of the effective electroweak (EW) mixing angle \(\sin^2\theta_{\text{eff}}(M_Z)\)\(^{10, 11, 15, 12, 16, 17}\). In addition, by incorporating electroweak radiative corrections in the analysis, the CDF collaboration has also extracted the on-shell
Source of uncertainty in CDF measurement of $M_W$ with 2.2 fb$^{-1}$ from Phys. Rev. Lett. 108, 151803 (2012)

| Source of Uncertainty                  | Uncertainty (MeV) |
|----------------------------------------|--------------------|
| Lepton energy scale and resolution     | 7                  |
| Recoil energy scale and resolution     | 6                  |
| Lepton removal                         | 2                  |
| Backgrounds                            | 3                  |
| $p_T(W)$                                | 5                  |
| Parton distributions (PDFs)            | 10                 |
| QED radiation                          | 4                  |
| W-bodon statistics                     | 12                 |
| Total                                  | 19                 |

Table 1: Sources of uncertainty in the CDF 2.2 fb$^{-1}$ direct measurement of $M_W$ at the Tevatron. Table from ref. [9] (Phys. Rev. Lett. 108, 151803 (2012)).

EW mixing angle $\sin^2 \theta_W = 1 - M_{W}^2/M_{Z}^2$ [10, 11, 12].

An error of $\pm 0.00030$ in the measurement of $\sin^2 \theta_W$ is equivalent to an indirect measurement of the W mass to a precision of $\pm 15$ MeV. Within the SM, the direct and indirect measurements of $M_W$ should agree with each other. Since the standard model is over constrained, any inconsistency between precise measurements of SM parameters would be indicative of new physics.

Similarly, in order to help resolve the long standing $3.2\sigma$ discrepancy [8] between the two most precise LEP/SLD Z pole measurements of $\sin^2 \theta_{\text{eff}}(M_Z)$, new measurements of $\sin^2 \theta_{\text{eff}}(M_Z)$ should have errors similar to SLD or LEP ($\approx \pm 0.00030$).

$$\sin^2 \theta_{\text{eff}}(\text{LEP}/\text{SLD} \ A_{FB}^{0,b}) = 0.23222 \pm 0.00029 \quad (1)$$

$$\sin^2 \theta_{\text{eff}}(\text{SLD} \ A_{LR}) = 0.23098 \pm 0.00026 \quad (2)$$

More precise extractions of $\sin^2 \theta_{\text{eff}}$ and the on-shell $\sin^2 \theta_W = 1 - M_{W}^2/M_{Z}^2$ using $A_{FB}(M, y)$ measurement for dilepton events produced in pp and pp collisions are now possible because of the following four novel techniques:

- A new technique [13] for calibrating muon and electron energy scales as a function of detector $\eta$ and $\phi$ (and sign), thus greatly reducing systematic errors from the energy scale. This technique is used in CDF, D0 and CMS.

- A new event angle weighting technique [14] for the measurement of $A_{FB}$. With this technique all experimental uncertainties in acceptance and efficiencies cancel (by measuring the $\cos \theta$ coefficient $A_4$ and using the relation $A_{FB} = (8/3)A_4$). Similarly, additional weights can be included for antiquark dilution, which
Figure 3: (a) Word measurements of $\sin^2 \theta_W$. (b) Word measurements $M_W^{\text{direct}}$ and $M_W^{\text{indirect}}$. Note that the value from LEP-1/SLD is an average of six measurements. Figures are from ref. [12].

makes the analysis independent of the acceptance in dilepton rapidity. This technique is used in CDF [10, 11, 12] and is currently being implemented at CMS.

- The implementation [10] of Z fitter Effective Born Approximation (EBA) electroweak radiative corrections into the theory predictions of POWHEG [13] and RESBOS [14] which allows for a measurement of both $\sin^2 \theta_{\text{eff}} (M_Z)$ and $\sin^2 \theta_W = 1 - M_W^2 / M_Z^2$. These EBA electroweak radiative corrections were implemented in CDF analyses [10, 11, 12] since 2013. Recently, a POWHEG version with electroweak radiative corrections has been released. Similarly, electroweak radiative corrections have been implemented in other theory predictions. Comparisons of different implementation of EW radiative corrections are now possible.

- The use of Drell-Yan $A_{FB}(M, \chi^2)$ (\chi^2 weighting) first proposed in ref. [20] for additional constraints on PDFs. The \chi^2 weighting technique reduces the PDF
error in the measurements of $\sin^2 \theta_{\text{eff}}^{\text{lept}}(M_Z)$, $\sin^2 \theta_W$, and in the indirect and direct measurements of $M_W$. This technique is now used in CDF\cite{12} and is currently being implemented in CMS.

Fig. 3(a) from ref. \cite{12} shows the CDF measurements of $\sin^2 \theta_{\text{eff}}^{\text{lept}}(M_Z)$ (extracted from the full run II 9.4 fm$^{-1}$ sample) compared to other measurements. Fig. 3(b) shows the corresponding CDF indirect measurements $M_W$ ($M_W^{\text{indirect}}$) compared to other indirect and direct measurements of $M_W$. .

Table 2 lists the sources of uncertainty in the CDF 9.4 fb$^{-1}$ measurements of $\sin^2 \theta_{\text{eff}}^{\text{lept}}(M_Z)$ and $M_W^{\text{indirect}}$ at the Tevatron (ref. \cite{12}).

| Source of Uncertainty                        | Uncertainty in $\sin^2 \theta_{\text{eff}}^{\text{lept}}(M_Z)$ | Uncertainty in $M_W^{\text{indirect}}$ (GeV) |
|---------------------------------------------|-----------------------------------------------------------------|---------------------------------------------|
| Data: Statistics                            | $\pm 0.00042$ (stat)                                            | 0.020                                       |
| Data: Energy scale                          | $\pm 0.00003$ (syst)                                           | 0.001                                       |
| Data: Backgrounds                           | $\pm 0.00002$ (syst)                                           | 0.001                                       |
| Prediction: PDFs                            | $\pm 0.00016$ (syst)                                           | 0.008                                       |
| Prediction: QCD EBA (NLO minus LO)          | $\pm 0.00007$ (syst)                                           | 0.003                                       |
| Prediction: QCD scales                       | $\pm 0.00002$ (syst)                                           | 0.001                                       |
| All systematics                              | $\pm 0.00018$ (syst)                                           | 0.009                                       |
| Total: (stat+syst)                          | $\pm 0.00046$ (total)                                          | 0.023                                       |

Table 2: Sources of uncertainty in the CDF 9.4 fb$^{-1}$ measurements of $\sin^2 \theta_{\text{eff}}^{\text{lept}}(M_Z)$ and $M_W^{\text{indirect}}$ at the Tevatron.

The errors in the CDF and D0 measurements of $\sin^2 \theta_{\text{eff}}^{\text{lept}}(M_Z)$ are $\approx \pm 0.00046$. An official combination of the CDF ($e^+e^\mu\mu$) and D0 ($e^+e^-$) results has not yet been done, but the error in the combination of $\sin^2 \theta_{\text{eff}}^{\text{lept}}(M_Z)$ would be about $\pm 0.00038$ (which is equivalent to an error of 19 MeV in $M_W^{\text{indirect}}$). The error in the combination would be smaller when the analysis of the D0 $\mu\mu$ sample is completed.

### 1.3.1 Constraining PDFs through $\chi^2$ weighting

This technique which was first proposed in ref. \cite{20} has been implemented in the most recent CDF analysis\cite{12}. At the Tevatron the technique reduces the PDF error
by 20%. The reduction of the PDF error at the LHC is much more significant.

Fig. 4(a) shows the $\chi^2$ for the best fit value of $\sin^2 \theta_W$ at CDF for each of the 100 PDF replicas for the NNPDF 3.0 (NNLO) PDF set. As shown in ref. [20] different values of $\sin^2 \theta_W$ raise or lower $A_{FB}(M)$ for all values of dilepton mass. In contrast, PDFs which raise the value of $A_{FB}$ for dilepton mass above the mass of the Z boson, reduce $A_{FB}$ below the mass of the Z bosons. The sensitivity of $A_{FB}(M)$ to $\sin^2 \theta_W$ is very different from the sensitivity to PDFs. Therefore, PDFs with a high value of $\chi^2$ are less likely to be correct. As shown in ref. [20], this information can be incorporated into the analysis by weighting the PDF replicas by $e^{-\chi^2/2}$. This greatly reduces the weights of PDFs with large values of $\chi^2$.

In addition to the measurements of $\sin^2 \theta_{\text{lept}}^{\text{eff}}(M_Z)$, the measurement of the on-shell EW mixing angle $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$, and the indirect measurement of $M_W$, the CDF collaboration will be publishing the normalized $\chi^2$ weights for the set of 100 NNLO NNPDF3.0 replicas. These weights can then be used to reduce the PDF errors in other Tevatron measurements such as the direct measurement of $M_W$.

Fig. 4(b) (from ref. [20]) shows $\chi^2$ versus $\sin^2 \theta_{\text{lept}}^{\text{eff}}(M_Z)$ for MC simulation of a CMS like detector with 15 fb$^{-1}$ at 8 TeV. At the LHC this technique yields a much more significant reduction in the PDF errors. More precise high statistics $A_{FB}(M,y)$ data place a more stringent constraints on PDFs. As the errors in $A_{FB}(M,y)$ become smaller at higher luminosity, the corresponding PDF errors are also reduced [20] and no longer limit the uncertainty in the extracted values of $\sin^2 \theta_{\text{lept}}^{\text{eff}}(M_Z)$, the on-shell $\sin^2 \theta_W$ and $M_W^{\text{indirect}}$.

### 1.4 LHC and HL-LHC measurements of $\sin^2 \theta_W$ and $M_W^{\text{indirect}}$

A summary of the various contribution to the errors in the CDF measurement of $\sin^2 \theta_{\text{lept}}^{\text{eff}}(M_Z)$ and $M_W^{\text{indirect}}$ is given in Table 2. Because $A_{FB}$ is an asymmetry, the QCD scale errors are small. CDF uses the difference between the results extracted using LO and NLO templates as a conservative estimate of the QCD EBA error. This small error will become even smaller when the differences between NLO and future NNLO analyses are used as the QCD EBA error. The most recent (2015) measurements of the electroweak mixing angle at the LHC are $\sin^2 \theta_{\text{lept}}^{\text{eff}}(M_Z) = 0.2398 \pm 0.0005$ (stat) $\pm 0.0006$ (syst) $\pm 0.0009$ (PDF), reported by ATLAS [16] using 7 TeV data, and $\sin^2 \theta_{\text{lept}}^{\text{eff}}(M_Z) = 0.23142 \pm 0.00073$ (stat) $\pm 0.00052$ (sys) $\pm 0.00056$ (theory/PDF) reported by LHCb [17] using both 7 and 8 TeV data.

The uncertainties in the ATLAS measurement of 0.2398 $\pm 0.00120$ (total) and in the LHCb measurement of 0.23142 $\pm 0.00110$ (total) are a factor of 2 larger than in the corresponding measurements at the Tevatron of 0.23146 $\pm 0.00047$ (D0), and 0.23222 $\pm 0.00046$ (CDF).

However if the novel techniques listed above are incorporated into the analyses, much more precise measurements of $\sin^2 \theta_{\text{lept}}^{\text{eff}}(M_Z)$, $\sin^2 \theta_W$ and $M_W^{\text{indirect}}$ can be ex-
Figure 4: (a) CDF data: Best $\chi^2$ versus $\sin^2 \theta_W$ (from ref. [12]). (b) Best $\chi^2$ versus $\sin^2 \theta_{\text{eff}}^\text{lept} (M_Z)$ for MC simulation of a CMS like detector with 15 fb$^{-1}$ at 8 TeV (from ref. [20] (arXiv:1507.02470)).

tracted from Drell-Yan $A_{FB}$ data at the LHC and HL-LHC. For example, the acceptance and efficiencies and sensitivity to pileup effects cancel to first order if the event angle weighting [19] technique is used. The sensitivity to pileup is further reduced if only track information is included in the isolation requirement for dimuons. When the new techniques are used, the analysis is not limited by detector systematics. In addition, as shown below, the PDF errors can be significantly reduced by incorporating $\chi^2$ weighting in the analysis and the total error is only limited by statistics. All of the novel techniques listed above are being incorporated in the current ongoing analyses of the CMS 8 TeV dimuon and dielectron data samples.

The LHC is a proton-proton collider and the direction of the dimuon pair is assumed to be the direction of the quark in the interaction. The asymmetry is diluted by the probability that the antiquark carries a higher momentum fraction than the quark. For rapidity close to zero, the dilution is maximal and the forward-backward asymmetry is zero. Therefore the PDF error from uncertainties in the antiquark distributions are much larger at the LHC than at the Tevatron. However, the use of $\chi^2$ weighting greatly reduces the PDF errors as shown below. Table [3] from ref. [20]
Table 3: Expected statistical and $\chi^2$ weighted PDFs errors in the measurements of $\sin^2\theta_W$ and $M_{W}^{\text{indirect}}$ with a CMS like detector for two samples. (1) A total of 15M reconstructed dilepton (8.2 M $\mu^+\mu^-$ and 6.8M $e^+e^-$) events, which is similar to the existing 19 fb$^{-1}$ data sample at 8 TeV (with a CMS like detector). (2) 120M reconstructed $\mu^+\mu^-$ events, which is the sample expected for 200 fb$^{-1}$ at 13-14 TeV. Table from ref. [20].

| CMS like detector | $f b^{-1}$ | $f b^{-1}$ |
|-------------------|------------|------------|
| Energy            | 8 TeV      | 13-14 TeV  |
| data sample       | current    | future     |
| Number of         |            |            |
| reconstructed      |            |            |
| events            |            |            |
| $\Delta \sin^2\theta_W$ CT10 PDF error | $\pm 0.00090$ | $\pm 0.00090$ |
| $\Delta \sin^2\theta_W$ NNPDF3.0 NNLO error | $\pm 0.00050$ | $\pm 0.00050$ |
| $\Delta \sin^2\theta_W$ $\chi^2$ Weighted PDF error | $\pm 0.00022$ | $\pm 0.00014$ |
| $\Delta \sin^2\theta_W$ statistical error | $\pm 0.00034$ | $\pm 0.00011$ |
| Stat+ $\chi^2$ weighted PDF error | $\pm 0.00040$ | $\pm 0.00018$ |
| $\Delta M_{W}^{\text{indirect}}$ | MeV | MeV |
| $\Delta M_{W}^{\text{indirect}}$ Statistical error | $\pm 17$ | $\pm 5$ |
| $\chi^2$ weighted PDF error | $\pm 11$ | $\pm 7$ |
| Stat+ $\chi^2$ weighted PDF error | $\pm 20$ | $\pm 9$ |

shows the expected statistical and $\chi^2$ weighted PDF errors in the measurements of $\sin^2\theta_W$ and $M_{W}^{\text{indirect}}$ with a CMS like detector for two samples.

1. A total of 15M reconstructed dilepton (8.2 M $\mu^+\mu^-$ and 6.8M $e^+e^-$) events for a CMS like detector. This sample is similar to the existing CMS 19 fb$^{-1}$ data sample at 8 TeV.

2. 120M reconstructed $\mu^+\mu^-$ events for a CMS like detector. This is similar to the sample expected for CMS with 200 fb$^{-1}$ at 13-14 TeV.

The PDF errors can also be reduced for other measurements (e.g. direct measurement of $M_W$) by using the same $A_{FB} \chi^2$ weighted constrained PDFs. Any additional constraints from new high precision measurements at the LHC (e.g. $W$ asymmetry), can be added to the $\chi^2$ weights, thus further reducing the PDF errors in the measurements of all EW parameters.
Figure 5: World average of all $M_T$ measurements (CDF, D0, CMS, ATLAS) in 2014, versus the most recent (2015) direct and indirect measurements of $M_W$. In the next few years we expect a factor of 2 to 3 reduction in the uncertainties in the measurements of $M_T$, $M_W^{\text{direct}}$, $\sin^2 \theta_{\text{eff}}^{\text{lept}}(M_Z)$ and $M_W^{\text{indirect}}$ (through $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$).

2 Conclusion

Fig. 5 shows the average of all $M_T$ measurements (CDF, D0, CMS, ATLAS) in 2014, versus the most recent (2015) direct and indirect measurements of $M_W$. In the future we expect reductions of a factor of 2 to 3 in the measurement errors of $M_T$, the effective mixing angle $\sin^2 \theta_{\text{eff}}^{\text{lept}}(M_Z)$, the measurement of the on-shell EW mixing angle $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$, and the direct and indirect measurement of $M_W$. Precise measurements of $A_{FB}(M,y)$ and $W$ boson asymmetry can be used to generate a set of constrained $\chi^2$ weighted PDFs that can be used to reduce PDF errors in the measurements of all EW parameters at the LHC.

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