Neutrino diffusion in the pasta phase matter within the Thomas-Fermi approach

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The behavior and properties of neutrinos in non-uniform nuclear matter, surrounded by electrons and other neutrinos are studied. The nuclear matter itself is modeled by the non-linear Walecka model, where the so-called nuclear pasta phase is described using the Thomas-Fermi approximation, solved in a Wigner-Seitz cell. We obtain the total cross-section and mean-free path for the neutrinos, taking into account scattering and neutrino absorption, and compare the final results for two known kind of model parametrizations: one in which non-linear effects in the strong sector are explicitly written in the model Lagrangian and another one in which the coupling constants are density dependent. The solution for this problem is important for the understanding of neutrino diffusion in a newly born neutron star after a supernova explosion.

1. INTRODUCTION

Neutrinos are elementary particles that interact with other particles only through the weak force, which makes its scattering by matter very unlikely. On the other hand, this pure weak force scattering can reveal some aspects of the structure of matter that other stronger interactions can not. Also, this feature represents an advantage in the sense that neutrinos carry information about the target along great distances, as is the case of neutrinos produced in the interior of stars that reach earth detectors. For those reasons, the study of neutrino interaction with matter is at the same time very fruitful but very challenging. In what refers to the scattering of neutrinos by hadronic matter, despite of the experimental as well as theoretical problems, some important progress has been made as in the case of the Karmen collaboration and Los Alamos results. More recently, long-baseline experiments are under way, like MiniBoone, NOvA and other similar accelerator experiments. Although the main purpose is to obtain further information on Standard Models and on neutrino oscillations, a precise knowledge of the neutrino-hadron interaction as well as the hadronic structure of the targets is necessary.

Another important source of information comes from the physics of the supernova core collapse and the evolution of a neutron star. The important ingredient in this case is the propagation of neutrinos in such a medium, which contains not only baryons but other leptons. In particular, for baryonic densities below the nuclear saturation value, structures known as pasta phases are expected, which in some sense resemble the baryonic structure in nuclei, but now embedded in a neutron gas and an electronic distribution. Actually, a delicate competition between Coulomb and surface energies determine the most favorable final inhomogeneous structure. The presence of such structures presumably have a non-negligible role in the neutrino diffusion which is a key ingredient for the modeling and simulation of the core collapse supernovae mechanisms.

One way to obtain the pasta phase is to solve the problem considering charge neutral Wigner-Seitz cells of appropriate geometries containing neutrons, protons and electrons within a variational approach, using both relativistic and non-relativistic mean-field calculations. Most of the recent applications in this case have used the Thomas-Fermi approximation, but Hartree-Fock calculations can also be found in the recent literature, all within the Wigner-Seitz approximation. Recently, it was shown in that performing a pasta calculation taking a large enough cell to include several units of the pasta structures, different distributions of matter from the usual ones considered within the Wigner-Seitz (WS) approximation could be energetically favored in some density ranges. Another approach to this problem that also goes beyond WS approximation is based in the so-called quantum molecular dynamics.

Here we follow the Thomas-Fermi results as described in order to generate the pasta-phase structure, where a relativistic model lagrangian is the starting point. The self-consistent calculation is performed considering matter in β-equilibrium, where just protons, neutrons, electrons and neutrinos are present. The total neutrino cross-section for each kind of particle is calculated, as a function of the density, taken the corresponding geometry for the considered density, i.e., droplet, rod, slab, tube or bubble. The neutrino mean-free path (NMFP) energy and temperature dependence for selected values of the density is also presented.

The neutrino cross-section calculation includes the effects of strong interaction, accounted for in-medium mass...
and energy shifts and degeneracy effects, based on the formalism previously developed for homogeneous nuclear matter, with the difference that now the strong and electromagnetic potentials as well as the nucleon masses and energies are position dependent. Consequently, the result is similar to the free space transition amplitude, and the uniform matter result becomes, straightforwardly, a particular case of our expressions for the cross-section.

We will not discuss in the present paper the important sources of suppression and enhancement due to in-medium correlation. Our main objective is to understand how inhomogeneous matter affects the neutrino cross section through the in-medium effects.

As in (19), we have considered elastic scattering and neutrino absorption in our derivation. In other words, neutral current and charge changing processes are included and their relative importance to the total cross-section is discussed. In what follows, an outline of the formalism is presented in section 2, numerical results and discussion are provided in section 3 and the conclusions are in the final section 4. The details of the pasta phase calculation can be found in the cited references and some details of the cross-section calculation are shown in the Appendix.

2. FORMALISM AND MODEL PARAMETRIZATION

We start with a model Lagrangian density that includes electrons, neutrinos, nucleons, the sigma, omega, rho and delta meson fields and the electromagnetic interaction, given by (10, 20):

$$\mathcal{L} = \sum_{i=p,n} \mathcal{L}_i + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\omega\rho} + \mathcal{L}_\delta + \mathcal{L}_\gamma + \mathcal{L}_e + \mathcal{L}_\nu, \quad (1)$$

where the nucleon Lagrangian reads

$$\mathcal{L}_i = \bar{\psi}_i [\gamma_i iD^\mu - M^*] \psi_i, \quad (2)$$

with

$$iD^\mu = i\partial^\mu - \Gamma_{\mu} V^\mu - \frac{\Gamma^\mu}{2} \tau \cdot b^\mu - e \frac{1 + \tau_3}{2} A^\mu, \quad (3)$$

$$M^* = M - \Gamma_{\delta} \phi - \Gamma_{\delta} \tau \cdot \delta. \quad (4)$$

The meson and electromagnetic Lagrangian densities are

$$\mathcal{L}_\sigma = \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 - \frac{k}{3} \phi^3 - \Lambda \phi^4 \right)$$

$$\mathcal{L}_\omega = \frac{1}{2} \left( -\frac{1}{2} \Omega_{\mu\nu} \Omega^{\mu\nu} + m^2 \phi V^\mu V^\mu + \frac{\zeta}{12} \phi^2 (V^\mu V^\mu)^2 \right)$$

$$\mathcal{L}_\rho = \frac{1}{2} \left( -\frac{1}{2} B_{\mu\nu} B^{\mu\nu} + m^2 \rho \cdot b^\mu \cdot b^\nu \right)$$

$$\mathcal{L}_\delta = \frac{1}{2} (\partial_\mu \delta^{\mu} \delta - m_3 \delta^2)$$

$$\mathcal{L}_\gamma = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\mathcal{L}_{\omega\rho} = \Lambda (g^2 \rho \cdot b^\mu) (g^2 (V^\mu V^\mu)),$$

where $\Omega_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $B_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu - \Gamma_{\mu}(b_\nu \times b_\nu)$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The electromagnetic coupling constant is given by $\gamma = \sqrt{4\pi/137}$ and $\tau$ is the isospin operator. Finally, the electron and neutrino Lagrangian densities read:

$$\mathcal{L}_e = \bar{\psi}_e [\gamma_\mu (i\partial^\mu + eA^\mu) - m_e] \psi_e$$

$$\mathcal{L}_\nu = \bar{\psi}_\nu [i\gamma_\mu \partial^\mu] \psi_\nu.$$

The weak boson contributions to the above Lagrangians affect the solutions of the corresponding Euler-Lagrange equations, in a completely negligible way, due to their huge masses and relatively small energy range, and are only relevant in order to obtain the neutrino cross sections, so they are not included in the above equations in the determination of the equations of state.

The solution of the corresponding Euler-Lagrange equations are obtained in the mean field self-consistent Thomas-Fermi (TF) approximation, as explained in references (10, 17). We have considered two types of parametrizations: one in which the couplings $G_i$ are taken to be constants (NL) and another one in which they are density dependent (DD). In the last case, the terms proportional to $\kappa$, $\lambda$, $\zeta$ and $\Lambda$ are set equal to zero. Within TF the system is considered locally uniform and the main output are the densities, which are position dependent. Explicitly, we have for the baryonic, scalar, isoscalar, scalar-isoscalar, electron and neutrino densities:

$$\rho(r) = \rho_p(r) + \rho_n(r) = \langle \bar{\psi}^\dagger \psi \rangle;$$

$$\rho_s(r) = \rho_{sp}(r) + \rho_{sn}(r) = \langle \bar{\psi}^\dagger \gamma_3 \psi \rangle;$$

$$\rho_3(r) = \rho_p(r) - \rho_n(r) = \langle \bar{\psi}^\dagger \tau_3 \psi \rangle;$$

$$\rho_{3s}(r) = \rho_{sp}(r) - \rho_{sn}(r) = \langle \bar{\psi}^\dagger \tau_3 \psi \rangle;$$
\[
\rho_e(r) = \left\langle \bar{\psi}_e \gamma_5 \psi_e \right\rangle;
\]
\[
\rho_\nu(r) = \left\langle \bar{\psi}_\nu \gamma_5 \psi_\nu \right\rangle;
\]
with
\[
\rho_i(r) = \frac{\gamma}{(2\pi)^3} \int d^3k \ (\eta_{ki}(T) - \bar{\eta}_{ki}(T)), \quad i = p, n, e, \nu;
\]
\[
\rho_{si}(r) = \frac{\gamma}{(2\pi)^3} \int d^3k \ \frac{M_i^*}{E_i^*} (\eta_{ki}(T) + \bar{\eta}_{ki}(T)), \quad i = p, n;
\]
where \(E^* = \sqrt{k^2 + M^2} \), \(k\) is the momentum and \(\gamma = 2\) is the spin multiplicity. For a given temperature \(T\), the distributions are:
\[
\eta_{ki}(T) = \frac{\exp \left[ (E_i - \mu_i)/T \right]}{1 + \exp \left[ (E_i - \mu_i)/T \right]},
\]
\[
\bar{\eta}_{ki}(T) = \frac{\exp \left[ (\bar{E}_i - \bar{\mu}_i)/T \right]}{1 + \exp \left[ (\bar{E}_i - \bar{\mu}_i)/T \right]},
\]
with \(E_i (\bar{E}_i)\) and \(\mu_i (\bar{\mu}_i)\) being the particle (antiparticle) energy and chemical potential, respectively. The particle (antiparticle) energy depends on the mesonic fields and is position dependent.

Once the densities are determined we proceed for the calculation of the total neutrino cross section \(\sigma\). We follow here the procedure discussed in [18]. For a collision \(1 + 2 \rightarrow 3 + 4\) we may write:
\[
\sigma = G_F^2 \int d^3r \int \frac{d^3k_2}{(2\pi)^3} \int \frac{d^3k_3}{(2\pi)^3} \int \frac{d^3k_4}{(2\pi)^3} \frac{(2\pi)^4}{|v_1 - v_2|} \delta^4(P_1 + P_2 - P_3 - P_4) \eta_R(T)(1 - \eta_3(T))(1 - \eta_4(T)) \cdot \\
\{ (V + A)^2(1 - v_2\cos(\theta_{12}))(1 - v_4\cos(\theta_{14})) + (V - A)^2(1 - v_4\cos(\theta_{14}))(1 - v_2\cos(\theta_{23})) - \frac{M_4^* M_4^-}{E_2^* E_4^-} (V^2 - A^2)(1 - \cos(\theta_{13})) \}.
\]

In the above expression \(P_i\) is the particle four momentum, \(v_i = |k_i|/E_i^*\), \(G_F\) is the Fermi constant and \(V, A\) are the weak current vector and axial vector couplings, respectively, and depend on the target particle and on the exchanged weak boson. For the neutrino-electron(neutrino) cross section we replace \(M^*\) by \(m_\nu(\text{zero})\). The factors \((1 - \eta_3(T))\) and \((1 - \eta_4(T))\) are due to final state Pauli blocking effects. We have considered here scattering through neutral current and charge-changing processes as well as neutrino absorption by the neutrons. The explicit expressions and definitions for each case are shown in the Appendix. Through the analysis of those expressions we may conclude that the integrand in the cross section is \(r\) dependent, once the potentials and the effective mass are position dependent. Note that the same expression can be used for the calculation of the cross section in the infinite nuclear matter, for which the potentials and effective mass are not \(r\) dependent.

The nucleon internal structure was taken into account in our calculation, just multiplying the couplings \(V\) and \(A\) by the vector and axial-vector nucleon form factors as explained, for instance, in [21].

The neutrino mean-free-path is then obtained considering the cross-section for a Wigner-Seitz cell divided by its volume
\[
\lambda = \left( \frac{\sigma}{V} \right)^{-1}.
\]

3. RESULTS AND DISCUSSION

In what follows we show our main results for the pasta phase, using both Lagrangian parametrizations with constant and density dependent couplings, as explained before. For the parametrizations with non-linear terms in the mesonic sector and constant couplings, we choose the GM3 [22], NL3 [23], NL3\(\omega\rho\) [24] and the FSUGold [25] sets of parameters. The sets NL3 and NL3\(\omega\rho\) only differ on the density dependence of the symmetry energy and will allow the discussion of the effect of this quantity on the neutrino mean-free-path. For the density dependent case we take the TW [26] and the van Dalen et al. [27] sets. The last one was recently taken for neutrino mean-free-path studies [11]. Table 1 shows the properties of the models for symmetric nuclear matter at saturation density \(\rho_0\) and zero temperature, and in Fig. 1 we have plotted the symmetry energy of the models versus the density, a quantity that will be necessary to discuss the results in the following. Models Dalen, GM3 and NL3 have a smaller symmetry energy in all or part of the density range below 0.1 fm\(^{-3}\). A smaller symmetry energy allows the system to have a larger isospin asymmetry. As we will see next these three models are precisely the ones with smaller proton fractions in the pasta phase matter, and this will induce a noticeable effect on the neutrino mean-free-path.

The free energy per particle obtained with the FSUGold parametrization for \(\beta\)-equilibrium matter with trapped neutrinos for a fixed fraction of leptons \(Y_L = 0.4\) is shown in Fig. 2 where the different geometries are identified by different colors. The uniform matter result is also shown, and, as expected, has a larger free energy than the pasta phases. Although the transition densities between geometries may differ slightly, depending on the parametrization used [28], the main behavior does not change significantly compared to the case shown.

Figures 3(a) and 4 display, respectively, the proton fraction and the chemical potential for all particles involved using again the FSUGold parametrization to describe neutrino trapped \(\beta\)-equilibrium matter. All these results were obtained for a temperature \(T = 1\) MeV. One
TABLE 1: Properties of the models parametrizations for infinite symmetric nuclear matter at zero temperature at saturation density: the saturation density $\rho_0$, the binding energy $E_B/A$, the incompressibility $K$, the nucleon effective mass $M^*$, the symmetry energy $a_{sym}$ and its slope $L$.

|       | $\rho_0$ (fm$^{-3}$) | $E_B/A$ (MeV) | $K$ (MeV) | $M^*/M$ | $a_{sym}$ (MeV) | $L$ (MeV) |
|-------|----------------------|---------------|-----------|---------|----------------|-----------|
| FSUGold | 0.148                | 16.299        | 271.76    | 0.6     | 37.4           | 60.4      |
| GM3    | 0.153                | 16.3          | 240       | 0.78    | 32.5           | 89.66     |
| NL3    | 0.148                | 16.3          | 272       | 0.6     | 37.4           | 118.3     |
| NL3$\omega\rho$ ($l_v = 0.3$) | 0.148         | 16.3          | 272       | 0.6     | 31.7           | 55.2      |
| Dalen  | 0.178                | 16.25         | 337       | 0.68    | 32.11          | 57        |
| TW     | 0.153                | 16.247        | 240       | 0.555   | 33.39          | 55.3      |

FIG. 1: Symmetry energy as a function of the total baryonic density for all the models used in the present work.

The individual mean-free-path contribution for each particle type is shown in Fig. 5 as a function of $\rho_B$, for the FSUGold and $T = 1$ MeV. The curves labeled proton, neutron and neutrino are the contributions for elastic neutral current scattering, while the curve labeled electron has also an elastic contribution from charged current scattering. It is clear the dominance of the absorption process for the total cross section.

The sensitivity of the total mean-free-path to the parametrization used is presented in Fig. 6 together with the uniform matter result obtained with the FSUGold parameter set. At low densities the models do not differ much. This is expected since all models have similar behaviors, in particular, the proton fraction is almost the same in all of them, see [28]. The differences occur precisely after the onset of the non-spherical pasta structures, $\rho_B > 0.02$ fm$^{-3}$, which is a range of densities that is sensitive to the density dependence of the symmetry energy. GM3 and Dalen have the lowest mean-free paths: a smaller proton fraction (see Fig. 3) corresponds to a smaller electron fraction, which increases the neutrino fraction and the neutrino chemical potential. All these facts favor the absorption process, and consequently the NMFP is smaller. Comparing NL3 and NL3$\omega\rho$ we also conclude that the softer density dependence of the symmetry energy gives rise to larger mean-free paths.

Considering the scattering of the neutrinos from the individual nucleons of the pasta, as done in the present calculation, we see that the mean-free path increases a lot comparatively to the homogeneous matter case, mainly at low densities, when the clustered matter presents much larger proton fractions, and, therefore, the absorption process is less likely to occur. Other reason is that the processes are strongly suppressed by Pauli blocking effects inside the cluster, where the Fermi energy is larger.
while outside the cluster the space is almost empty. As expected, the larger the neutron and proton densities of the background gas the closer get the pasta and homogeneous matter mean-free paths.

The dependence of the results with the temperature can be seen in Fig. 4. Temperature has a strong effect on the pasta structures which start to melt. Although within a TF calculation pasta structures still exist at $T > 10$ MeV, according to [29], if thermal fluctuations are considered the Wigner-Seitz cell structure is supposed to melt for $T > 7$ MeV. Temperature increases drastically the background gas of dripped particles in the Wigner-Seitz cells and, therefore, the larger the temperature the closer come the pasta mean-free path to the homogeneous matter ones. The reduction of the mean-free path with the increase of the temperature is also expected, because the Fermi-Dirac distributions are smoothed when the temperature increases, making possible more reactions.

In Fig. 5, the NMFP is plotted as a function of the total baryonic density using the FSUGold parametrization for the pasta phase matter. Individual contributions for the electrons (red), neutrinos (green), protons (blue) and neutrons (purple). The absorption contribution is shown in light blue and the total value is shown in black. The temperature was taken as $T = 1$ MeV and $E_\nu = \mu_\nu$. 

FIG. 3: Proton fraction as a function of the total baryonic density, (a) using the FSUGold parametrization for uniform matter (black) and the pasta phase matter within droplet (red), rod (green), slab (blue), tube (purple) and bubble (light blue) geometries, (b) for all the models and the pasta phase matter. The temperature was taken as $T = 1$ MeV.

FIG. 4: Chemical potential as a function of the total baryonic density using the FSUGold parametrization for the pasta phase (points) compared to homogeneous matter (lines) for the neutrons (green), protons (red), electrons (blue) and neutrinos (black). The temperature was taken as $T = 1$ MeV.

FIG. 5: Mean-free-path as a function of the total baryonic density using the FSUGold parametrization for the pasta phase matter. Individual contributions for the electrons (red), neutrinos (green), protons (blue) and neutrons (purple). The absorption contribution is shown in light blue and the total value is shown in black. The temperature was taken as $T = 1$ MeV and $E_\nu = \mu_\nu$. 

The temperature was taken as $T = 1$ MeV.
FIG. 6: Mean-free-path as a function of the total baryonic density comparing various parametrizations for the pasta phase matter. TW (red), Dalen (green), GM3 (blue), NL3 (pink), NLωρ (grey) and FSUGold (orange). The FSUGold for homogeneous matter (black), is also shown. The temperature was taken as $T = 1$ MeV and $E_\nu = \mu_\nu$.

FIG. 7: The neutrino mean-free-path as a function of the total baryonic density within FSUGold parametrization for the pasta phase (points) compared to homogeneous matter (lines) for $T = 1$ MeV (red), $T = 3$ MeV (green), $T = 5$ MeV (blue) and $T = 8$ MeV (black) and $E_\nu = \mu_\nu$.

FIG. 8: Neutrino mean-free-path as a function of the neutrino incident energy for the pasta phase with $\rho_B = 0.018$ fm$^{-3}$, comparing the parametrizations TW(red), Dalen(green), GM3(blue), NL3(pink), NLωρ(grey), FSUGold(orange) and FSUGold homogeneous matter(black). The temperature was taken as $T = 1$ MeV (upper panel) and $T = 5$ MeV (lower panel). The triangles indicate the NMFP at the neutrino chemical potential, $\mu_\nu$, of the respective model.

The neutrino incident energy for the pasta phase with $\rho_B = 0.018$ fm$^{-3}$. We also include the homogeneous matter result obtained within FSUGold. With triangles, we select the values of $\lambda$ at the neutrino chemical potential, at the two temperatures shown, $T = 1$ and 5 MeV. We expect that the neutrinos with energy around the chemical potential will provide the main contribution to the dispersion of energy in the system, because at low temperatures the system is practically degenerate and Pauli blocking factors will weaken other contributions. The highly degenerate regime is expected for $\mu_i/T \gg 1$, and this condition is essentially true for temperatures below the pasta melting temperature. The differences between the two temperatures is mainly explained by the drip of nucleons out of the clusters increasing the background gas, and reducing the differences between the pasta and homogeneous matter results. Temperature also decreases the mean-free path by almost one order of magnitude.
both for the pasta and the homogeneous matter calculation and this is explained by the opening of new transitions that are Pauli blocked at $T$ close to zero.

In Fig. 9 we consider a larger density corresponding to the slab geometry at $T = 1$ MeV. For these densities the gas of dripped neutrons is denser and the proton fraction smaller (see the blue region in Fig. 3 in comparison with the red region). Consequently, the NMFP decreases and comes closer to the homogeneous matter result.

4. CONCLUSIONS

We have studied the effect of the pasta phase, occurring in the inner crust of a neutron star, on the neutrino mean-free-path (NMFP). The pasta phases have been obtained within a self-consistent Thomas–Fermi approximation, as explained in references [10, 17]. Several relativistic nuclear models have been used to describe the pasta phase, both with nonlinear meson terms and constant couplings, and with density-dependent coupling constants. In particular, we were interested in discussing whether the properties of the models, such as the density dependence of the symmetry energy, would have some influence on the NMFP. We have also studied the effect of the temperature. It should be stressed, however, that the present work is restricted to the density and the temperature range for which the pasta phases exist, e.g. $T \lesssim 8$ MeV and $\sim 0.0002\text{fm}^{-3} < \rho < \rho_c$ where the density at the crust-core transition $\rho_c \sim \rho_0/2$. We have considered both charged and neutral current reactions. It has been shown that the absorption process dominates the NMFP, but other processes can not be discarded.

At low density, $\rho_B < 0.02\text{fm}^{-3}$, where the pasta phases obtained within the different parametrizations have similar properties [25], the NMFP has a small dependence on the parametrization. As the density increases, namely at the layers close to the upper border of the inner crust, the dependence on the parametrization becomes more important. With a larger proton fraction, the absorption process is less likely to occur. Consequently, models with larger proton fraction result in smaller NMFP, and since the proton fraction is closely related to the density dependence of the symmetry energy, its effect on the NMFP is not negligible. We have shown that the NMFP is larger in the presence of pasta phases than considering homogeneous $\beta$-equilibrium matter at the same density. Besides the fact that the pasta phases have a larger proton fraction, the processes are strongly suppressed by Pauli blocking effects inside the cluster, where the Fermi energy is larger, while outside the cluster the space is almost empty. When the density increases, the closer get the pasta and homogeneous matter mean-free paths.

As the temperature increases, the pasta phase starts to melt and the pasta phase EOS approaches the infinity nuclear matter EOS, and, therefore, the NMFP in both cases get closer. At the temperatures we have considered the NMFP neutrinos are degenerate, and, therefore, the Pauli blocking factors ensure that only neutrinos close to the Fermi energy are involved at the dispersion of energy in the system. Comparing with the homogeneous case, we expect that neutrinos with less energy are involved in the presence of pasta phases, which give rise to larger NMFP.

The NMFP as function of the neutrino energy for the different models do not differ much at low densities and temperatures, because the models have similar properties in this region of phase space. With the increase of the temperature and density, the dependence on the models starts to appear. The great variation of the NMFP with the neutrino energy around the neutrino chemical potential is attributed to Pauli blocking effects.

Our results imply that the effects of the pasta phase can not be neglected when calculating the NMFP at baryonic densities below $0.1\text{fm}^{-3}$ and temperatures below $10\text{MeV}$. The parametrization used to described the pasta phase has also influence on the results, in particular due to the density dependence of the symmetry energy.

We have also not considered coherent scattering of the neutrinos from the pasta clusters as done in [7, 17]. These processes are important when all neutrons respond coherently, for scattering by neutrinos with a wavelength of the size of the cluster, or energies $E_\nu \lesssim 75 \text{ MeV}$. The scattering processes discussed in the present work become important when the inner constitution of the clusters is distinguished by the neutrinos, and, therefore, for neutrino with an energy above that value.

![FIG. 9: Neutrino mean-free-path as a function of the neutrino incident energy for the pasta phase with $\rho_B = 0.05\text{fm}^{-3}$ and $T = 1$ MeV, comparing the parametrizations TW(red), Dalen(green), GM3(blue), NL3(pink), NL3(pink), FSUGold(dark red) and FSUGold homogeneous matter(black). The triangles indicate the NMFP at the neutrino chemical potential, $\mu_\nu$, of the respective model.](image-url)
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5. APPENDIX

According to equation (3), with \( k_1 = k_1 \hat{z} \) and after the \( k_4 \) integration in the scattering case:

\[
\sigma_{\text{scat}} = \frac{G_F^4}{(2\pi)^5} \int d^3 r \int d^3 k_2 \int_0^{2\pi} sen(\theta_3) d\theta_3 \int_0^{2\pi} d\phi_3 k_3^2 \left[ \frac{\eta_3(T)(1 - \eta_3(T))(1 - \eta_4(T))}{\sqrt{|v_1 - v_2|}} \right] \frac{k_1 + k_3}{k_1 + E_2^* - k_3} \left( k_1 - \cos(\theta_3) \right) \left( k_2 + \cos(\theta_3) \right) \left( k_4 - \cos(\theta_3) \right) \left( 1 - \cos(\theta_3) \right) \]

with

\[
f_{123} = \frac{k_1 \cos(\theta_3) + k_2 \cos(\theta_23) - k_3}{E_4},
\]

\[
g_{123} = \frac{k_1 + k_2 \cos(\theta_23) - k_3 \cos(\theta_3)}{E_2^*},
\]

\[
k_3 = \frac{k_1 (E_2^* - k_2 \cos(\theta_4))}{k_1 + E_2^* - k_1 \cos(\theta_3) - k_2 \cos(\theta_23)}.\]

For the absorption, we find:

\[
\sigma_{\text{abs}} = \frac{G_F^4}{(2\pi)^5} \int d^3 r \int d^3 k_2 \int_0^{2\pi} sen(\theta_3) d\theta_3 \int_0^{2\pi} d\phi_3 k_3^2 \left[ \frac{\eta_3(T)(1 - \eta_3(T))(1 - \eta_4(T))}{\sqrt{|v_1 - v_2|}} \right] \frac{Ck_3 - F \sqrt{m_c^2 + k_3^2}}{\sqrt{m_c^2 + k_3^2}} \left( C - \sqrt{m_c^2 + k_3^2} \right) \left( g_{123} - \frac{M_2^* M_4^*}{E_2^* E_4} \left( g_2^2 - g_A^2 \right)(1 - \cos (\theta_3)) \right)
\]

where now

\[
k_3 = -\frac{2F(C + m_e^2 - D)}{4(k_1 \cos (\theta_3) + k_2 \cos (\theta_23))^2 - 4C^2} - \frac{\sqrt{16m_c^2C^2 F^2 + 4C^2(C^2 + m_c^2 - D)^2 - 16m_e^2 C^4}}{4F^2 - 4C^2}.
\]

\[
D = (k_1 + k_2)^2 + M_r^2;
\]

\[
C = k_1 + E_2^* - g_r \rho_0;
\]

\[
F = k_1 \cos (\theta_3) + k_2 \cos (\theta_23).
\]

The constants are:

neutrino-proton: \( c_V = 1/2 - 2 \text{sen}^2(\theta_w) \); \( c_A = 1.23/2 \)
neutrino-neutron: \( c_V = -1/2 \); \( c_A = -1.23/2 \)
neutrino-electron: \( c_V = 1/2 + 2 \text{sen}^2(\theta_w) \); \( c_A = 1/2 \)
neutrino-neutrino: \( c_V = \sqrt{2} \); \( c_A = \sqrt{2} \)

absorption: \( \gamma V = C \); \( \gamma A = -1.23C \)

with \( C = 0.973 \) and \( \text{sen}^2(\theta_w) = 0.230 \). The collisions neutrino-electron and neutrino-neutrino can be represented by two (first-order) Feynman diagrams each one, in such a way that the values of \( c_V \) and \( c_A \) can be re-defined to accommodate the different cross section contributions in a single expression, as given above. Also, we define:

\[
E^* = \sqrt{k^2 + M^*}.\]

For the particle energy we have for the nucleon:

\[
E_i = E^* + \Gamma_{V_0}(r) \pm \frac{1}{2} \Gamma_{\rho_0}(r) + eA_0(r),
\]

in the NL parametrization case and

\[
E_i = E^* + \Gamma_{V_0}(r) \pm \frac{1}{2} \Gamma_{\rho_0}(r) + eA_0(r)
\]

\[
+ \frac{\partial \Gamma_{\rho_0}}{\partial \rho_B} \rho_B V_0 + \frac{\partial \Gamma_{\rho_0}}{\partial \rho_0} \rho_0 \rho_B V_0 - \frac{\partial \Gamma_{\rho_0}}{\partial \rho_0} \rho_0 \rho_B V_0,
\]

in the DD case. The plus sign in the equations above has to be chosen for the proton and the minus sign for the neutron. Also, \( V_0, \rho_0, \rho_B \) and \( \delta_0 \) are the time-like components of the meson fields. For the electron:

\[
E_e = \sqrt{k^2 + m_e^2} - eA_0(r); \quad M^* \to m_e
\]

and for the neutrino:

\[
E_{\nu} = k; \quad M^* \to 0.
\]

Finally, in order to take in to account the finite size of the nucleon, we have multiplied the constants \( \gamma V \) (\( \gamma A \)), or \( c_V \) (\( c_A \)), by the corresponding form factor. Using the parametrizations as explained in [21], we have taken a common structure factor given by:

\[
\left( 1 + \frac{4 \text{q} r^2}{4M^2} \right)^{-2},
\]

where \( q = |q| = |p_1 - p_3| \).
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