The research of statistical distribution of the image tristimulus values displayed on the avionics indication equipment

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Abstract. The problem of evaluation of statistical distribution of the image tristimulus values displayed on the avionics indication equipment screen is being studied. There are some mathematical expressions and simulation results which describe the behavior of the tristimulus values distribution probability density function. There are some histograms for tristimulus values statistical distribution in various combinations of the primary color codes which are used in the avionics indication equipment software.

1. Introduction

The development and exploitation experience of a color multi-function displays [1] made on the base of a LCD (liquid crystal display) panels by foreign manufacturers revealed that the same RGB program code (R – Red, G – Green, B – Blue) which defines the displayed image color causes different XYZ-tristimulus values and (x,y)-chromaticity coordinates [2, 3] in various samples of color multi-function display.

Difference in the values of the XYZ-tristimulus values and (x,y)-chromaticity coordinates can be explained by technological dispersion of the LCD panel parameters. The experiments with colorimeter show clearly that the color difference in the values of (x,y)-coordinates displayed on the different samples of the color multi-function display with the same RGB code may reach the marker of 0.01-0.03 units what can be perceived by the observer and the measuring instruments.

In that way a group of color multi-function displays manufactured with the same software and construction documentation when the mass production LCD panels are used and arranged in a row in the flight instruments panel of a civilian plane (up to 6 units) differ in the values of the tristimulus values (chromaticity coordinates) of the same displayed image. And that is unacceptable.

In that case the problem of research of the tristimulus values statistical distribution laws of the synthesizing images in order to establish the requirements for manufacturing quality to provide the acceptable level of the technological dispersion of the avionics mass production screens is an essential task.

2. Color spaces in avionics

Tristimulus values of the image displayed on a flight instruments LCD panel screen are defined by the
The tristimulus values distribution probability density function theoretical estimate

$$
\begin{align*}
X &= \begin{bmatrix} X_r, X_y, X_b \end{bmatrix} \\
Y &= \begin{bmatrix} Y_r, Y_y, Y_b \end{bmatrix} \\
Z &= \begin{bmatrix} Z_r, Z_y, Z_b \end{bmatrix}
\end{align*}
$$

(2)

where \( X, Y, Z \) – tristimulus values; \( X_r, X_y, X_b, Y_r, Y_y, Y_b, Z_r, Z_y, Z_b \) – screen profile coefficients; \( R, G, B \) – displayed color programed codes.

The LCD panel profile coefficients define the color gamut triangle on the \( XY \)-plane. The color gamut triangle includes the locus of all possible \((x, y)\)-chromaticity coordinates of the image displayed on the LCD panel. The values of the \((x, y)\)-chromaticity coordinates are defined through the \( XZ \)-tristimulus values:

$$
x = \frac{X}{X + Y + Z}, \quad y = \frac{Y}{X + Y + Z}.
$$

3. The LCD panel parameters technological dispersion model

The LCD panel colorimetric parameters technological dispersion model which takes into account the screen color reproduction stochastic effects may be represented as [4]:

$$
\begin{align*}
X &= R( X_r + \Delta \xi_r, X_y + \Delta \xi_y, X_b + \Delta \xi_b ) \\
Y &= G( Y_r + \Delta \xi_r, Y_y + \Delta \xi_y, Y_b + \Delta \xi_b ) \\
Z &= B( Z_r + \Delta \xi_r, Z_y + \Delta \xi_y, Z_b + \Delta \xi_b )
\end{align*}
$$

where

$$
\xi_r \in \left[ -X_r/2; +X_r/2 \right], \quad \xi_y \in \left[ -X_y/2; +X_y/2 \right], \quad \xi_b \in \left[ -X_b/2; +X_b/2 \right],
$$

$$
\xi_r \in \left[ -Y_r/2; +Y_r/2 \right], \quad \xi_y \in \left[ -Y_y/2; +Y_y/2 \right], \quad \xi_b \in \left[ -Y_b/2; +Y_b/2 \right],
$$

$$
\xi_r \in \left[ -Z_r/2; +Z_r/2 \right], \quad \xi_y \in \left[ -Z_y/2; +Z_y/2 \right], \quad \xi_b \in \left[ -Z_b/2; +Z_b/2 \right]
$$

— random values distributed with the uniform distribution law, \( \Delta \) – the model parameter which includes the presence of the screen colorimetric characteristics technological dispersion while the screens are being manufactured.

4. The tristimulus values distribution probability density function theoretical estimate

After the analysis of the equations (1) and (3) it is clear that the tristimulus values could be defined in the three combinations of the \( RGB \) code components conditions:

1. The color displayed on the LCD panel screen is formed with one of the \( RGB \) code components when the other two are equal to zero. In other words those are situations when \( R \neq 0, G = 0, B = 0 \) or \( R = 0, G \neq 0, B = 0 \) or \( R = 0, G = 0, B \neq 0 \).

In this case the tristimulus values are distributed [5, 6] according to the uniform law with boundaries defined by the absolute value of the \( RGB \) code non-zero component, corresponding LCD panel profile coefficient and the technological dispersion parameter \( \Delta \).

The expressions to evaluate the probability density functions (PDF) \( f_X(t), f_Y(t), f_Z(t) \) according to the \( X, Y, Z \) tristimulus values distribution are designated in the table 1. If the variable \( t \) is out of the intervals designated in the table 1 the PDF \( f(t) \) is equal to zero.
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2. The color is formed with a combination of all three non-zero codes of the RGB code when the third component is equal to zero. In other words those are situations when \( R \neq 0, G \neq 0, B = 0 \), \( R = 0, G \neq 0, B \neq 0 \) or \( R \neq 0, G = 0, B \neq 0 \). In this case the PDF \( f(t) \) of the tristimulus values distribution is a mathematical function of trapezoidal type [7, 8]. The function \( f(t) \) must be calculated as convolution of two PDF. For example for \( X \)-coordinate \( f_X(t) = f_{sxX}(t-s) \odot f_{sxX}(s) \) when \( R \neq 0, G \neq 0, B = 0 \):

\[
f_X(t) = f_{sxX}(t-s) \odot f_{sxX}(s) = \int_{-\infty}^{\infty} f_{sxX}(t-s)f_{sxX}(s)ds = \begin{cases} 0, & \text{if } a_1 + a_2, b_1 + b_2 \leq t \\ \frac{t - a_1 - a_2}{b_2 - a_2}, & \text{if } a_1 + a_2 \leq t < b_1 + b_2 \\ \frac{b_1 + b_2 - t}{b_1 - a_2}, & \text{if } b_1 + b_2 \leq t < b_1 + b_2 \\ \end{cases}, \tag{4}
\]

The system (4) is true when \( b_2 - a_2 > b_1 - a_2 \), if the inequality expression is not valid than the coefficient indices \( a_1, b_1 \) must be redefined. It could be shown that expressions of the system (4) for the PDF lead to different kind of functions in depends on the ratio of the interval lengths \( [a_1; b_1] \) and \( [a_2; b_2] \). In the table 2 there are some particular cases of all possible interval boundaries to evaluate the PDF distribution of the \( X, Y \) and \( Z \) tristimulus values. The cases \( b_1 - a_1 = b_2 - a_2 > 0 \) correspond to equal interval lengths \( [a_1; b_1] \) and \( [a_2; b_2] \) in (4) when the PDF \( f(t) \) of the trapezoidal type becomes of the triangular type. In the mathematical statistics when \( [a_1 = a_2 = 0; b_1 = b_2 = 0] \) the function \( f(t) \) defines the Simpson triangular distribution which is an Irwin-Hall distribution special case:

\[
f_\theta(t) = \frac{1}{2(n-1)!} \sum_{k=0}^{n} (-1)^k \binom{n}{k} (t-k)^{n-1} \text{sign}(t-k),
\]

for the sum \( n \) of the uniformly distributed \([0;1]\) random values when \( n=2 \).

3. The color is formed with a combination of all three non-zero codes of the primary color components. In other words it is general case when \( R \neq 0, G \neq 0, B \neq 0 \). In this case the probability density function \( f(t) \) of the tristimulus values distribution is a mathematical function of piecewise parabolic type [9].
The function \( f(t) \) has to be calculated again through the convolution of probability density functions. For the \( X \)-coordinate it is \( f_X(t) = f_{\xi_X}((t-s) \otimes f_{\xi_b}(s)) \), where a probability density function of trapezoidal type for example (4) is defined on the interval \([a_i + a_z; b_i + b_z]\) and a probability density function \( f_{\xi_{a_0}}(s) \) of uniform distribution is defined on its own interval \([a_i; b_i]\) (see the table 1). Statistically the function \( f(t) \) is a PDF of the sum of three uniformly distributed random values where each value is defined on its own interval \([a_i; b_i]\), \( b_i > a_i > 0 \), \( i = 1, 2, 3 \) and in general case \( a_1 \neq a_2 \neq a_3 \), \( b_1 \neq b_2 \neq b_3 \). Generally a piecewise parabolic type probability density function \( f(t) \) of the tristimulus value distribution (as in the example of the \( X \)-coordinate) must be represented as:

\[
f(t) = \begin{cases} 
  f_1(t), & a_1 + a_z + a_3 \leq t < a_1 + a_2 + b_3 \\
  f_2(t), & a_1 + a_2 + b_3 \leq t < a_1 + b_2 + a_3 \\
  f_3(t), & a_1 + b_2 + a_3 \leq t < a_2 + b_3 + a_3 \\
  f_4(t), & a_1 + b_2 + b_3 \leq t < b_1 + a_3 + a_3 \\
  f_5(t), & b_1 + a_3 + a_3 \leq t < b_1 + a_3 + a_3 \\
  f_6(t), & b_1 + a_3 + a_3 \leq t < b_1 + b_2 + a_3 \\
  f_7(t), & b_1 + b_2 + a_3 \leq t < b_1 + b_2 + b_3 \\
  0, & 0 \leq t < a_1 + a_2 + a_3, t \geq b_1 + b_2 + b_3 
\end{cases}
\]

where

\[
f_1(t) = \frac{1}{b_3 - a_3} \int_{a_i}^{a_i + a_z} f_{\xi_{a_0}}(s) \, ds + \frac{1}{b_3 - a_3} \int_{a_i + a_z}^{t - a_3} f_{\xi_{a_0}}(s) \, ds = \frac{(t - (a_i + a_2 + a_3))^2}{2(b_i - a_i)(b_2 - a_2)(b_3 - a_3)},
\]

\[
f_2(t) = \frac{1}{b_3 - a_3} \int_{a_i}^{a_i + a_2} \frac{s - (a_1 + a_2)}{b_1 - a_i} \, ds = \frac{(t - a_3)^2 + 2(a_1 + a_2)(a_3 - b_1) - (t - b_3)^2}{2(b_1 - a_i)(b_2 - a_2)(b_3 - a_i)}.
\]
\[ f_3(t) = \frac{1}{b_3 - a_3} \int_{b_3 - a_3}^{a_3 + b_3} s - (a_1 + a_2) \, ds + \frac{1}{b_3 - a_3} \int_{b_3 - a_3}^{a_3 - b_3} \frac{1}{b_3 - a_3} \, ds = \]
\[ = \frac{2(b_2 - a_2)(t - (a_1 + b_2 + a_1) + (a_2 + b_2)^2) + 2(a_1 + a_2)(t - (a_1 + b_2 + b_1) - (t - b_3)^2)}{2(b_1 - a_1)(b_2 - a_2)(b_3 - a_3)}, \]
\[ f_4(t) = \frac{1}{b_3 - a_3} \int_{b_1 - a_1}^{a_1 - b_1} \frac{1}{b_3 - a_3} \, ds = \frac{s}{(b_3 - a_3)} \bigg|_{b_1 - a_1}^{a_1 - b_1} = \frac{1}{b_1 - a_1}, \]
\[ f_5(t) = \frac{1}{b_3 - a_3} \int_{b_1 - a_1}^{a_1 - b_1} \frac{1}{b_3 - a_3} \, ds + \frac{1}{b_3 - a_3} \int_{a_1 - b_1}^{a_1 + b_3} \frac{1}{b_1 - a_1} \, ds = \]
\[ = \frac{2(b_2 - a_2)((b_1 + a_2 + b_3) - t) + (a_2 + b_2)^2 + 2(b_1 + b_2)(t - (b_1 + a_2 + a_3) - (t - a_3)^2)}{2(b_1 - a_1)(b_2 - a_2)(b_3 - a_3)}, \]
\[ f_6(t) = \frac{1}{b_3 - a_3} \int_{b_1 - a_1}^{a_1 - b_1} \frac{1}{b_1 - a_1} \, ds = \frac{(t - b_2)^2 + 2(b_1 + b_2)(b_3 - a_1) - (t - a_3)^2}{2(b_1 - a_1)(b_2 - a_2)(b_3 - a_3)}, \]
\[ f_7(t) = \frac{1}{b_3 - a_3} \int_{b_1 - a_1}^{a_1 - b_1} f_{\xi \xi_s \eta \eta_s}(s) \, ds + \frac{1}{b_3 - a_3} \int_{b_1 - a_1}^{a_1 - b_1} f_{\xi \xi_s \eta \eta_s}(s) \, ds = \frac{(t - (b_1 + b_2 + b_3))^2}{2(b_1 - a_1)(b_2 - a_2)(b_3 - a_3)}. \]

The value of the coefficients \( a_i, b_i \) in the system (5) are different for the probability density function of the X-coordinate \( f_x(t) \), Y-coordinate \( f_y(t) \) and Z-coordinate \( f_z(t) \). There are some exact expressions in the table 3 to calculate the coefficients \( a_i, b_i \) of the system (5).

**Table 3.** The expressions to calculate the tristimulus values distribution PDF coefficients.

| Coefficients | \( f_x(t) \) | \( f_y(t) \) | \( f_z(t) \) |
|--------------|-------------|-------------|-------------|
| \( a_1 \)    | \( RX_x - RX_x, \Delta / 2 \) | \( RY_x - RY_x, \Delta / 2 \) | \( RZ_x - RZ_x, \Delta / 2 \) |
| \( b_1 \)    | \( RX_x + RX_x, \Delta / 2 \) | \( RY_x + RY_x, \Delta / 2 \) | \( RZ_x + RZ_x, \Delta / 2 \) |
| \( a_2 \)    | \( GX_g - GX_g, \Delta / 2 \) | \( GY_g - GY_g, \Delta / 2 \) | \( GZ_g - GZ_g, \Delta / 2 \) |
| \( b_2 \)    | \( GX_g + GX_g, \Delta / 2 \) | \( GY_g + GY_g, \Delta / 2 \) | \( GZ_g + GZ_g, \Delta / 2 \) |
| \( a_3 \)    | \( BX_y - BX_y, \Delta / 2 \) | \( BY_y - BY_y, \Delta / 2 \) | \( BZ_y - BZ_y, \Delta / 2 \) |
| \( b_3 \)    | \( BX_y + BX_y, \Delta / 2 \) | \( BY_y + BY_y, \Delta / 2 \) | \( BZ_y + BZ_y, \Delta / 2 \) |

It is important to notice that the system (5) is true if: \( b_1 - a_1 \geq b_2 - a_2 \geq b_3 - a_3 > 0 \). To comply with this condition the distribution intervals could be previously ranked with the indices coefficients \( a_i, b_i \) redefinition.

On the figure 1 are presented some PDF \( f_x(t) \), \( f_y(t) \), \( f_z(t) \) evaluations for the XYZ-tristimulus values distribution obtained through simulation and also some theoretical estimations of probability density functions \( f_x(t), f_y(t), f_z(t) \) of the XYZ-tristimulus values distribution which are built according to the system (5) when the RGB code components values are non-zero. The analysis of (4) and (5) shows that function \( f(t) \) depends on technological dispersion parameter \( \Delta \) of the LCD panel profile coefficients \([10]\) and the RGB code components values. It means that under some non-zero values of the RGB code components the \( f(t) \) distribution of the XYZ-tristimulus values is a trapezoidal (triangular) function and under the others non-zero values it is a piecewise parabolic function.
5. Conclusion
The expressions from the tables 1 to 3 and systems (4) and (5) are presented analytically and can be used to define the tristimulus values distribution probability density functions for both the International Commission on Illumination standards of 1931 and 1964 with different white balance standards: D-75, D-65, D-55, D-50 etc. It’s only need to define corresponding values \([10]\) of screen profile components \(X_r, X_g, X_b, Y_r, Y_g, Y_b, Z_r, Z_g, Z_b\) of the basic conversions (1) and (2).

The obtained analytical expressions for tristimulus values distribution probability density functions may be used not only in colorimetry. The mathematical expressions of the system (5) expand the applicability boundaries of the Irwin-Hall distribution law dealing with sum of uniformly distributed random values \(\xi \in [0;1]\) on the situation of sum of three uniformly distributed random values with different boundaries \(\xi_i \in [a_i; b_i], a_i \neq a_2 \neq a_3, b_1 \neq b_2 \neq b_3, b_i > a_i > 0\).

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