Investigations of energy dependence of saturation thickness of multiply backscattered gamma photons in elements and alloys - an inverse matrix approach

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Abstract. In Compton scattering experiments employing thick targets one observes that the numbers of multiply backscattered photons increases with increase in target thickness and then saturate at a particular target thickness called the saturation thickness. The energy of each of gamma ray photons continues to decrease as the number of scatterings, the photon undergoes, increases in the sample having finite dimensions. The present experiment is an independent study of energy and intensity distributions of 279-, 320-, 511-, 662 keV, and 1.12 MeV gamma rays multiply backscattered from targets of different atomic numbers and alloys of various thicknesses, and are carried out in a backscattering geometry. The backscattered photons are detected by a NaI(Tl) scintillation detector. The detector response unscrambling, converting the observed pulse-height distribution to a true photon energy spectrum, is obtained with the help of a 12x12 inverse response matrix. The present experimental results confirm that for thick targets, there is significant contribution of multiply backscattered radiations emerging from the targets, having energy equal to that of singly scattered Compton process. The measured saturation thickness (in units of mean free path) for multiply backscattering of gamma photons is found to be decreasing with increase in energy of incident gamma photons.

1. Introduction

The backscattering (or reflection) of gamma rays from the surface of a material is of fundamental importance in radiation shielding, radiation absorption and non-destructive testing of finite samples of industrial, medical and agricultural interest. Non-destructive examination of samples using backscattering technique has the advantage that the sample can be accessed from the same side; imaging is simple and also provides 3-dimensional physical information about the sample.

The experiments reported in literature on multiply backscattering of gamma rays provide useful information about intensity and energy distributions of gamma rays, experimentally backscattered from various materials as a function of primary gamma radiation energy and geometry. The quantity called albedo, characterizing the reflection probability of a material for gamma photon flux, is defined as the ratio of amount of radiation reflected from the slab in a certain time interval to the amount of radiation incident on the slab during this time. Hayward and Hubbell¹ developed a Monte Carlo method to determine the albedo factor, at 1 MeV photons scattered from semi-infinite slabs of water, aluminium, copper, tin and lead at various angles of incidence. Perkins² has developed Monte Carlo method to calculate the number and energy gamma ray albedo factors for a material of effective atomic number corresponding to concrete and aluminium. More recently, Asa’d et al³ investigated the intensity of backscattered photons by the use of GEANT3 Monte Carlo package and confirmed it
experimentally. He also used this technique for the contact-less measurement of the thickness of steel sections. Abdul-Majid and Tayyeb\(^\text{b}\) used gamma ray backscattering of 662 keV photons, Paramesh et al\(^\text{c}\) measured the saturation thickness (depth) of multiply scattered gamma rays, by subtracting the analytically evaluated contribution of singly scattered photons from the observed intensity distribution. Yuk et al\(^\text{d}\) developed a land mine detection system based on backscatter X-ray principle which relates the different backscattering X-ray characteristics of materials with different densities.

Multiple backscattering in finite volumes has been a major drawback in the extraction of information from scattered photon flux because during the interaction of gamma photons with material, these photons continue to decrease in energy as the number of scatterings increases in the target. These low energy gamma photons get registered in the spectrum along with the singly scattered events. So, the energy spectrum of such photons is broad and never completely separate from the singly scattered distribution. The problem is to quantitatively calculate the numbers of multiply backscattered gamma photons from the finite slabs of investigated samples. Our measurements\(^\text{e,f}\) provide Z-dependence of saturation thickness along with the survey of analytical and Monte Carlo simulation approaches to study the multiple backscattering, and available experimental data on these processes. The reported experimental data provide useful information about intensity and energy distributions of gamma rays experimentally backscattered from various materials as a function of primary gamma radiation energy.

Prior to our work on multiply backscattering of gamma photons, only one other group\(^\text{g}\) has studied the saturation thickness of multiply backscattered photons, and even then not at 180°. The present measurements provide saturation thickness of gamma rays of various energies multiply backscattered from the targets of elements and alloys. The response function of NaI(Tl) detector obtained in our earlier work\(^\text{h}\) employs an inverse matrix approach, and it does the required spectrum unfolding.

### 2. Experimental set-up and method of measurements

In the present measurements, we have used a special geometry\(^\text{i,k}\) which does not have any dead space of measurements. In the experimental arrangement\(^\text{i,k}\) small isotropic radioactive sources (sealed in the form of thin disks in which the position of source was \(\approx 1\) mm inside the shielding) of \(^{203}\)Hg, \(^{51}\)Cr, \(^{22}\)Na, \(^{137}\)Cs and \(^{65}\)Zn of order of micro curies are placed adjacent to the surface of the target (different elements and alloys). The backscattered radiations are detected by a properly shielded NaI(Tl) scintillation detector. The NaI(Tl) crystal, optically coupled to photomultiplier tube, is cylindrical in shape having dimensions 51 mm in diameter and 51 mm thickness. The backscattered beam is collimated by a cylindrical collimator made of lead and lined with aluminium having opening of radius 10 mm and thickness 30 mm. The axes of the gamma ray detector, radioactive source and cylindrical collimator pass through the centre of target. The rectangular targets of different elements and alloys of dimensions 80 mm length and 40 mm breadth (thickness varying from 5 mm to 30 mm) have been placed at a distance of 390 mm from the detector collimator face so that the solid angle subtended by detector collimator on center of the target is 0.002 Sr. The field of view of NaI(Tl) detector is confined to target only. The gamma ray spectrometer is calibrated by using standard calibration gamma sources. Each of the calibration sources is placed at the target’s position and its spectrum is recorded. From the recorded spectra FWHM and photo-fraction values corresponding to the observed full energy peaks are measured. The experimental data are accumulated on a PC based ORTEC Mastreo-32 Multi channel analyzer (MCA). The measuring time for each thickness of sample is 10 ks with the background also recorded for the same duration of time. To evaluate the true scattered spectrum for each thickness, the spectra are taken with and without the target in the primary incident beam. In order to determine the contribution due to multiply backscattered photons only, the spectrum of singly backscattered photons is reconstructed analytically, described in detail in our recent work\(^\text{i}\). The subtraction of this reconstructed normalized singly backscattered spectrum from the observed experimental spectrum provides the numbers of multiply backscattered photons under the full energy peak. However to take into account the contribution due to low pulse-height counts resulting from the partial absorption of higher energy photons, we make use of inverse matrix approach\(^\text{i}\), thereby using an inverse response matrix which shifts these low pulse-height counts into
their photo-peak energy region by unscrambling the pulse-height distribution recorded by NaI(Tl) gamma ray detector.

3. Results and discussion

A typical observed pulse-height distribution (curve-a) from the carbon target (thickness 30 mm) exposed to 279 keV gamma photons is given in Figure 1. The observed pulse-height distributions are a composite of the singly and multiply backscattered photons along with background events. The curve-b is the observed pulse-height distribution recorded after removal of target out of the primary beam. The subtraction of events under curve-b from those under curve-a results in backscattered spectrum (curve-c) corresponding to events originating from the interaction of 279 keV primary gamma photons with the target material and subsequent events such as multiple scattering, bremsstrahlung, Rayleigh scattering etc. The spectrum, (curve-c) consists of an intensity distribution of singly as well as multiply backscattered photons. The singly scattered events under the backscattered peak are obtained by reconstructing analytically the singly backscattered peak using the experimental parameters, such as FWHM, efficiency of the gamma ray detector corresponding to the backscattered energy, counts at the photo-peak and Gaussian distribution of backscattered peak. The analytically reconstructed singly scattered peak is shown by curve-d of Figure 1. The experimental pulse-height distribution (curve-c) is converted to a photon energy spectrum with the help of an inverse response matrix. The solid curve-e is the resulting calculated histogram in units of number of photons. Low pulse-height counts resulting from partial absorption of higher energy photons are shifted to the backscattered peak energy. The events under the histogram in the Compton continuum accounts for photons of reduced energy (less than that of backscattered peak) originating from multiple interactions in the target and finally escaping in the direction of gamma ray detector. The events under the calculated histogram corresponding to end points energies of the backscattered peak accounts for singly and multiply scattered radiations having energy equal to that of backscattered ones. The events under curve-d of Figure 1 are divided by peak-to-total ratio, $\gamma(E)$, of the gamma ray detector. Then their subtraction from the events under the calculated histogram (curve-e) in the specified energy range results in events originating from multiple scattering but having energy equal to singly backscattered photons. These residual events are divided by intrinsic (crystal) efficiency of the scintillation crystal. A correction for the iodine escape peak, and absorptions in the Al-window of the detector and in air present between the target and detector, provides the emergent flux from the target at 180° having energy in the range of backscattered peak distribution. This procedure is repeated for targets of different thicknesses and atomic numbers to evaluate the intensity of multiply backscattered photons, corresponding to respective end points energies of the backscattered peak, when exposed to different gamma photons energies using different gamma energy sources.

The plots of observed number of multiply backscattered events (having energy equal to singly scattered ones) for different incident photon energies as a function of target thickness are shown in Figure 2. The solid curves provide best-fitted curves to the present experimental data. The present experimental results show that for each of the incident photon energy, the numbers of multiply backscattered events increases with increase in target thickness and then saturate after a particular value of target thickness, called the saturation thickness (depth). The saturation of multiply backscattered photons is due to the fact that as the thickness of target increases, the number of scattered events also increases but on the other hand enhanced self-absorption results in decrease of the number of photons coming out of the target. So a stage is reached when the thickness of the target becomes sufficient to compensate the above increase and decrease of the number of photons. The measured saturation thickness values, in targets of different elements and alloys for various incident gamma photon energies, are given in column 5 (in mm) and in column 6 (in mean free path) of Table 1. The column 4 in the table provides the mean distance which a photon of a given energy travels in a given element is known as the mean free path ($\lambda$) given by
\[ \lambda = \frac{1}{\Sigma_i} = \frac{M}{N_A \rho \sigma_i}, \]

Where \( \Sigma_i \), \( M \), \( N_A \), \( \rho \) and \( \sigma_i \) are macroscopic cross-section, molecular weight, Avogadro number, density and total cross-section of the medium respectively. The measured saturation thickness (in mean free path) for multiply backscattering of gamma ray photons is found to be decreasing with increase in incident gamma photons energy. This is because the penetration of gamma ray photons increases with increase in incident gamma ray energy, so the backscattered radiation has to propagate through a large thickness and the flux of multiply backscattered photons having energy equal to the singly backscattered photons reduces. The present experiments are also simulated with the Monte Carlo package (Figure 2) developed by Bauer and Pattison. 

\( \text{Figure 1.} \) A typically observed pulse-height distribution with (curve-a) and without (curve-b) carbon target of thickness 30 mm in the primary 279 keV gamma beam recorded for time duration of 10 ks. Experimentally observed pulse-height distribution (curve-c) obtained after subtracting background events (unrelated to target). Normalized analytically reconstructed singly backscattered full energy peak (curve-d) and resulting calculated histogram (curve-e) of converting pulse-height distribution to a true photon spectrum.
Figure 2. The experimentally measured (solid-symbol) and Monte Carlo simulated (hollow-symbol) numbers of multiply backscattered events from targets of different Z-numbers for 279-, 320-, 511-, and 662 keV incident gamma photons. The measured statistical uncertainties lie within the radii of experimental data points.

4. Conclusions
Our experimental results have confirmed that for thick targets, there is significant contribution of multiply backscattered radiation emerging from the target, having energy equal to that of a singly scattered Compton process. The intensity of multiply backscattering increases with increase in target thickness and saturates beyond a particular value, called the saturation thickness, thus supporting the work reported in references 5, 7-8. It has also been concluded that the saturation thickness (in units of mean free path) decreases with increase in incident gamma photon energy. Monte Carlo calculations support the results of present experiments. It is further required to perform the experiment using HPGe detector, which provides a more faithful reproduction of the shape of distribution under the observed spectra owing to its better resolution in comparison to scintillation detectors, for better understanding of the process of multiply backscattered photons.
Table 1: Experimentally measured values of saturation thickness in different elements and alloys for different gamma photon energies. The systematic error in the target thickness and measured saturation thickness comes to be 1 mm.

| Energy (MeV) | Elements     | Backscattered photon energy , keV | 1 m.f.p. (mm) | Measured saturation thickness (mm) in mean free path |
|--------------|--------------|-----------------------------------|---------------|----------------------------------------------------|
| 0.279        | Carbon       | 38.8                              | 31.0          | 0.79                                               |
|              | Aluminium    | 34.6                              | 25.0          | 0.72                                               |
|              | Copper       | 9.40                              | 17.5          | 1.86                                               |
|              | Tin          | 7.50                              | 10.5          | 1.40                                               |
|              | Bakelite     | 76.72                             | 31.0          | 0.42                                               |
|              | Perspex      | 76.39                             | 30.0          | 0.39                                               |
| 0.320        | Carbon       | 41.09                             | 29.0          | 0.70                                               |
|              | Aluminium    | 36.30                             | 22.0          | 0.60                                               |
|              | Copper       | 10.40                             | 17.0          | 1.63                                               |
|              | Tin          | 9.10                              | 10.0          | 1.09                                               |
|              | Bakelite     | 72.72                             | 29.0          | 0.39                                               |
|              | Perspex      | 76.39                             | 28.0          | 0.36                                               |
| 0.511        | Carbon       | 49.40                             | 27.0          | 0.54                                               |
|              | Aluminium    | 44.20                             | 20.0          | 0.45                                               |
|              | Copper       | 13.40                             | 16.0          | 1.19                                               |
|              | Tin          | 14.80                             | 9.50          | 0.64                                               |
|              | Bakelite     | 92.59                             | 27.0          | 0.29                                               |
|              | Perspex      | 97.26                             | 26.0          | 0.26                                               |
| 0.662        | Carbon       | 55.30                             | 25.0          | 0.45                                               |
|              | Aluminium    | 49.50                             | 19.0          | 0.38                                               |
|              | Copper       | 15.30                             | 15.5          | 1.01                                               |
|              | Tin          | 18.30                             | 9.5           | 0.51                                               |
|              | Bakelite     | 103.62                            | 25.0          | 0.24                                               |
|              | Perspex      | 108.85                            | 24.0          | 0.22                                               |
| 1.12         | Carbon       | 73.17                             | 27.5          | 0.37                                               |
|              | Aluminium    | 63.74                             | 20.0          | 0.31                                               |
|              | Iron         | 22.45                             | 17.0          | 0.75                                               |
|              | Zinc         | 25.01                             | 16.0          | 0.63                                               |
|              | Tin          | 25.34                             | 10.0          | 0.39                                               |
|              | Brass        | 21.40                             | 16.5          | 0.77                                               |
|              | Bronze       | 22.06                             | 12.0          | 0.54                                               |
|              | Glass        | 68.96                             | 22.0          | 0.31                                               |
|              | Bakelite     | 132.89                            | 25.5          | 0.19                                               |
|              | Perspex      | 139.59                            | 25.0          | 0.17                                               |

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