Financial dynamics, economic state classification and optimal portfolio construction for unseen market conditions: learning from the past 60 years

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Abstract
Motivated by the current fears of a potentially stagflationary global economic environment, this paper uses new and recently introduced mathematical techniques to study multivariate time series pertaining to country inflation (CPI), economic growth (GDP) and equity index behaviours. We start by assessing the temporal evolution among various economic phenomena, and complement this analysis with "economic driver analysis", where we decouple country economic trajectories and determine what is most important in their association. Next, we study the temporal self-similarity of global inflation, growth and equity index returns to identify the most anomalous historic periods, and windows in the past that are most similar to current market dynamics. We then introduce a new algorithm to construct economic state classifications and compute an economic state integral, where countries are determined to belong in one of four candidate states based on their inflation and growth behaviours. Finally, we implement a decade-by-decade portfolio optimization to determine which equity indices and portfolio assets have been most beneficial in maximizing portfolio risk-adjusted returns in various market conditions. This could be of great interest to those looking for asset allocation guidance in the current period of high economic uncertainty.

Keywords: Nonlinear time series analysis, Nonlinear financial market dynamics, Stagflation, Economic modelling, Portfolio optimization

1. Introduction

The term stagflation is highly definitional, but generally refers to an economic scenario where inflation is high, and economic growth rate is stagnant (or slowing). Inflation itself has been relatively subdued for the past 20 years, providing an environment for business prosperity and growth in asset valuations among a range of asset classes, the most prominent of which is global equities. A current
question of great relevance is whether the currently elevated levels of inflation are transitory, or perhaps more structural (and in effect, pathological). This paper takes a data-driven and statistical approach to understand the drivers behind economic variables (growth, inflation and equity index returns), temporal self-similarity, the evolution of economic states/ regimes, and the most effective assets for portfolio risk-adjusted returns on a decade-by-decade basis.

Researchers have been interested in the relationship between economic growth and inflationary pressures for a long time [1, 2, 3, 4]. There is a substantial body of prior work that has focused on the application of principled econometric theory [5] to explore the evolutionary nature of inflationary behaviours. In the econophysics literature however, there is limited work exploring the interplay between inflation, economic growth and equity index performance. In this paper we use new and existing methodologies to better understand this relationship. This work could help economists and investment managers make better-informed decisions during the current economic climate, which is clouded by uncertainty surrounding the future of global inflation, growth and equity index dynamics.

Financial market dynamics, as a general topic of study, has been of great interest to researchers in applied mathematics, econophysics, econometrics and statistics. Temporal dynamics and correlation structures [6, 7, 8] has garnered the interest of many researchers. Such studies are often accompanied by the study of evolutionary dynamics with techniques such as principal components analysis (PCA) [7, 9, 10, 11], clustering [12, 13, 14], change point detection [15, 16, 17, 18] and various statistical modelling frameworks [19, 20]. Such methodologies have been applied to a wide variety of asset classes including equity markets [11], FX markets [21], cryptocurrencies [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 16] and debt-related instruments [36]. There is a litany of work where such techniques have been used in other domains including COVID-19 [37, 38, 39], extreme human behaviours [40] and epidemiology. Readers interested in recent work related to temporal dynamics with various societal impacts on the economy should see [41, 42, 43].

The topic of portfolio optimization has been of great interest to quantitative finance researchers for an extended period of time [44, 45]. The core optimization framework has been built upon in a variety of ways [46, 47, 48, 49, 50, 51]. In this work, we implement the portfolio optimization framework in a discrete, time-varying context where we may explore the effectiveness of various asset mixes in maximizing investor portfolio Sharpe ratios. In particular, this paper explores the impact additional commodity-related asset classes (often used for hedging purposes) may have in diversifying a portfolio of equities allocated between various country equity indices.

This paper is structured as follows. In Section 3 we investigate the temporal evolution in economic features’ global variability and the drivers behind country economic features’ association. In Section 4 we explore the temporal self-similarity of each economic feature, and identify the most anomalous periods in history as well as periods in the past which may resemble current market dynamics. In Section 5 we introduce a new algorithm to determine the evolution of various country’s economic profile, and explore stagflationary tendencies with
a novel economic state integral. We further investigate collective similarity by constructing a normalized inner product distance matrix and apply clustering to identify groupings among countries. In section 6, we apply a mean-variance portfolio optimizer to assess the effectiveness of various assets in maximizing portfolio Sharpe ratio. In Section 7, we conclude.

2. Data

In this paper we study quarterly and monthly time series data related to CPI inflation, GDP growth and equity index performance for a collection of high profile economies (Australia, Canada, France, Germany, Italy, Japan, UK and USA). In some experiments, monthly data is averaged over discrete quarters so time series in the data with a lower sampling rate may be compared fairly. In the final section of the paper, Section 6, we study a collection of assets which includes the aforementioned country equity indices, gold spot price, oil spot price and the CRB commodity index (on a monthly periodicity).

3. Economic feature similarity

Throughout this paper, the bulk of our analysis focuses on three multivariate economic time series pertaining to inflation (represented by Consumer Price Index, or CPI), economic growth (represented by Gross Domestic Product, or GDP) and country equity indices (representing each country’s equity market behaviours). We denote these time series \( c_i(t), g_i(t) \) and \( e_i(t) \) respectively, where \( i = 1, \ldots, N \) and \( t = 1, \ldots, T \) refer to \( N \) country indices and \( T \) time points.

3.1. Evolutionary norm analysis

In this section we explore the evolution in the total country variability for each economic feature. Let our norm of any candidate vector \( v \in \mathbb{R}^N \) be defined as \( \|v\| = \sum_{i=1}^{N} v_i \). Our norm of a length \( N \) vector at any point in time \( t \), provides an indication of the magnitude and direction of any candidate economic variable among the countries studied. Importantly in our norm computation, we do not sum over the modulus of each country’s values as we wish to capture directional information (any feature could conceivably be positive or negative). Given the stark differences in scale between countries’ equity indices, before studying their collective evolution, we compute each countries’ equity index log returns as follows:

\[
\hat{e}_i(t) = \log \left( \frac{e_i(t)}{e_i(t-1)} \right),
\]

We then construct three univariate time series, where each one corresponds to the global state of inflation, growth and equity index performance, respectively. This construction is implemented as follows:
Figure 1: Time-varying country vector norms for CPI inflation, GDP growth and equity index returns, denoted by $v^c(t)$, $v^g(t)$ and $v^\tilde{e}(t)$ respectively.

$$v^c(t) = \sum_{i=1}^{N} c_i(t)$$  \hspace{1cm} (2)  

$$v^g(t) = \sum_{i=1}^{N} g_i(t)$$ \hspace{1cm} (3)  

$$v^\tilde{e}(t) = \sum_{i=1}^{N} \tilde{e}_i(t)$$ \hspace{1cm} (4)  

where $v^c(t)$, $v^g(t)$ and $v^\tilde{e}(t)$ are time-varying functions, with the Frobenius norm being computed across all countries (at each point in time) for each economic feature.

Figure 1 shows the time-varying collective behaviours among each economic feature studied over the past 60 years. Each time series displays noteworthy, but varied, behaviours of interest. We start by investigating the evolution of global inflation displayed in Figure 1a. The figure tells an interesting story. After the significant levels of global inflation experienced during the 1970s and early 1980s, there is a material reduction in global inflation from the early 2000s onwards.
Although a small increase in global inflation occurred immediately prior to the GFC (Global Financial Crisis), the prolonged financial crisis led to an immediate and prolonged retreat in country CPI levels. The past several years of data highlight a rapid acceleration in global CPI levels, a trajectory which has not been experienced since the early 1970s. Of course, the high levels of global inflation depicted are unsurprising, as they are widely discussed by Central Banks, investment and retail banks, fund managers and market commentators.

To complement our understanding of the temporal evolution in the economy, we turn to Figure 1b which characterises the global effect of GDP growth. The two most notable attributes in the figure are the shocks corresponding to the GFC and the COVID-19 market crash (and its subsequent recovery). In particular, there is quite a clear distinction in the rate of the recovery following the GFC and the COVID-19 crash. The GFC market crash was significantly more prolonged and experienced a much slower recovery than that of the COVID-19 market crash. The COVID-19 market crash exhibits an "up-down-up" pattern, highlighting the initial economic crash associated with the pandemic, the associated revival of GDP growth and the economy, and a final pullback as GDP levels begin to taper off.

Finally, we wish to study the relationship between the underlying economic variables and the effect they have on equity index behaviours. The time-varying norm on country equity index log returns is shown in Figure 1c. The most obvious feature in the figure is the sharp drawdown in returns corresponding to the COVID-19 market crash. This drop is especially pronounced, and the fundamental drivers behind such an intense and short-lived crash must be considered when we consider the current state of affairs. It is likely that in addition to restricted mobility, business activity and the supply shocks experienced across many industries, a shift in investor composition toward more passive and factor-based investment products may have led to further indiscriminate selling during a time of crisis such as the pandemic.

Viewing these figures in aggregate creates a holistic, albeit complex view of
the economic and financial market's evolution. We have not seen a period in the last 60 years with such an unusual combination of features. That is,

1. High levels of inflation, coming from a relatively low base in recent times.
2. Economic growth (GDP) levels exhibiting such volatility post-pandemic.
3. Equity markets displaying such pronounced volatility on the back of such (arguably) stretched valuation levels.
4. An unprecedented level of investment manager money under management controlled by systematic, passive or factor-based investment strategies. It is likely that in times of economic and financial crisis, we will continue to see a widespread and somewhat mechanical selling of assets.

It is difficult to make an informed consideration of what may by likely moving forward, when the economic and financial circumstances surrounding a potential stagflationary environment are so unique.

3.2. Economic "driver" analysis

In this section, we seek to answer the following question: **is the country, or the specific economic feature, more important in driving cluster similarity?**

To address this question, we aggregate our three time series (inflation, growth and equity indices) together and construct a new time series \( f_j(t) \) where \( j = 1, ..., 3N \) and again, \( t = 1, ..., T \).

We let \( \|f_j\|_1 = \sum_{t=1}^{T} |f_j(t)| \) be the \( L_1 \) norm of the respective economic or financial time series, and subsequently wish to normalize each trajectory. We denote the proceeding normalization \( T^f_j = \frac{f_j}{\|f_j\|_1} \). The distance between two countries’ trajectory vectors highlight the relative similarity in changes over time. We compute an economic driver distance matrix as follows:

\[
\Omega^D_{ji} = \|T^f_i - T^f_j\|_1. \tag{5}
\]

The economic driver distance matrix reveals two interesting findings. First, it is clear that similarity between evolutionary trajectories is predominantly driven through economic behaviours, rather than specific countries’ behaviours. This highlights the systemic nature of such economic phenomena, and may be driven through the strength of the collective dynamics in the global economy. The second finding of note, is the varying degree of self-similarity among different economic features. Figure 3 shows that equity index returns clearly display the strongest degree of self-similarity, followed by inflation trajectories and then GDP growth trajectories. This finding may suggest that, at least among the economies studied in this paper, inflation behaviours are more globally systemic than economic growth patterns.
Figure 3: Economic driver distance matrix. The distance matrix shows 3 obvious groupings in evolutionary trajectories, driven by economic behaviours, rather than country features. That is, the inflation trajectories of two distinct countries are more likely to group together than the same countries’ varying economic features (inflation, growth, equity index behaviours, etc.).
4. Temporal self-similarity analysis

In this section we wish to explore how similar each of economic feature is to periods in the past. We refer to this phenomenon as temporal self-similarity. The two primary motivating questions we seek to answer are as follows. First, which features exhibit, or have exhibited, the most extreme behaviours relative to current market conditions. Second, for each feature, which period in the past most closely resembles the current time period. We explore each economic feature (GDP growth, CPI inflation and country equity index returns) separately. First, we wish to study the similarity in vector norms between all possible points in time. For the sake of exposition, we detail how this would be computed on the inflation time series. We compute an $L_1$ distance (denoted $\| \cdot \|_1$) between all prospective country inflation vectors at every instance in time $1, \ldots, T$. The resulting distance matrix will be of dimension $T \times T$, as we explore the topology of the distances computed between all candidate points in time. We store these values in a distance matrix

$$d^c(s, t) = \|c^{(s)} - c^{(t)}\|_1 \forall s, t \in \{1, \ldots, T\}.$$ (6)

We compute analogous distance matrices for growth and equity index performance, which we denote $d^g(s, t)$ and $d^e(s, t)$.

Each distance matrix reveals the temporal self-similarity displayed by each economic/financial feature analyzed. We start by studying Figure 4 which shows the $L_1$ distance between vectors of country CPI values at all possible points in time. The figure reveals one notably anomalous period, corresponding to the high global inflation levels in the 1970s.
Figure 5: $d^r(s,t)$ distance matrix computing $L_1$ distance between vectors of country GDP values at all possible points in time. The two most anomalous periods correspond to the GFC and the COVID-19 market crash.

Figure 6: $d^\hat{r}(s,t)$ distance matrix computing $L_1$ distance between vectors of country equity index returns at all possible points in time. The two most anomalous periods correspond to the GFC and COVID-19 market crash.
1970s. This is consistent with our earlier discussion and the evolutionary norm analysis, as the 1970s was a period of especially high global inflation. The part of the distance matrix corresponding to this period in time is indicated by a red, dashed elliptical annotation. The second aspect of the distance matrix of considerable interest is the period of the late 1990s and early 2000s corresponding to low levels of global inflation. This is also consistent with our evolutionary norm analysis, which highlights (following a slow transition downward from the high levels of inflation during the 1970s) a subdued period of inflation from the early 2000s onward.

We then turn to Figure 5 which displays the $L_1$ distance between vectors of country GDP values at all possible points in time. There are two notably different periods of time, corresponding to the GFC and COVID-19 market crash. Again, these periods in time are marked by a red, dashed elliptical annotation. This finding is in agreement with the evolutionary norm analysis referred earlier. Outside these two periods of time, the temporal self-similarity in GDP growth is remarkably constant - with the distribution of global GDP levels exhibiting limited deviation at various points in time.

Finally we explore Figure 6 which shows the temporal self-similarity between vectors of country equity returns. Again, the two most prominently anomalous periods in time are the GFC and COVID-19 market crash. Both periods are marked by red, dashed elliptical annotations. This finding may be indicative of the increase in financial market correlations during crises over the past several decades. In particular, periods of economic distress are accompanied by a dramatic increase in the average correlation coefficient among equity market returns. There is a litany of work that discusses such phenomena.

When viewed together, these three figures highlight an interesting story. Both global GDP and country equity indices are characterised by relatively homogenous behaviours outside the two most recent and severe market crises (GFC and COVID-19). By contrast, these time periods did not impact global inflation levels, with significant homogeneity exhibited beyond the early 2000s. However, the unprecedented high levels of inflation experienced during the 1970s are shown to be materially different to all other periods in time under study.

5. Economic state classification

In this section, we develop a new algorithm to dynamically determine the economic state of a country at any candidate point in time. Given our particular emphasis on studying the time-varying impact of inflation and growth on country economies, we restrict our study to each country’s inflation (CPI) and growth (GDP) time series. We begin with the (strong) assumption, that a country may be in one of 4 potential states:

1. Ascending growth: Above threshold growth and positive inflation
2. Descending growth: Above threshold growth and declining inflation
3. Stagflation: Below threshold growth and declining inflation
4. Recession: Below threshold growth and declining inflation

Obviously, these states are highly dependent on the definition of "threshold growth". In our proceeding experiments, for any country \( i \), we define threshold growth to be \( \mu^g_i - \sigma^g_i \) where \( \mu^g_i \) and \( \sigma^g_i \) are the mean GDP level and standard deviation GDP level for each country. Intuitively, we are saying that threshold growth is approximately two thirds of the time, and that in approximately one third of cases we are in below threshold growth. Algorithm 1 (below) outlines the pseudocode for economic state classification, which is conditional on a country’s level of GDP (relative to its average) and CPI inflation.

Algorithm 1 Economic State Classification and Economic Integral Computation

1: Initialize empty list for country state integral, \( \Delta^i \)
2: for \( i = 1 \) to \( N \) do
3: Initialize empty list for economic state time series, \( S_i(t) \)
4: Compute mean of country GDP time series, denoted \( \mu^g_i \)
5: Compute standard deviation of country GDP time series, denoted \( \sigma^g_i \)
6: for \( t = 1 \) to \( T \) do
7: if \( g_i(t) > (\mu^g_i - \sigma^g_i) \) and \( c_i(t) > 0 \) then
8: State declared as ascending growth
9: \( S_i(t) = 1 \). Append to list.
10: if \( g_i(t) > (\mu^g_i - \sigma^g_i) \) and \( c_i(t) \leq 0 \) then
11: State declared as descending growth
12: \( S_i(t) = 2 \). Append to list.
13: if \( g_i(t) \leq (\mu^g_i - \sigma^g_i) \) and \( c_i(t) > 0 \) then
14: State declared asstagflation
15: \( S_i(t) = 3 \). Append to list.
16: if \( g_i(t) \leq (\mu^g_i - \sigma^g_i) \) and \( c_i(t) \leq 0 \) then
17: State declared as recession
18: \( S_i(t) = 4 \). Append to list.
19: Compute \( \Delta^i = \left( \frac{1}{T} \times \sum_{t=1}^{T} S_i(t) \right) \)
20: Output each country’s economic state classification time series, \( S_i(t) \) and economic state integral \( \Delta^i \).

For each country \( i \), we compute \( S_i(t) \) and \( \Delta^i \), which correspond to the time-varying economic state classification and the state economic integral respectively. Given that higher integer state classifications are consistent with less prosperous economic conditions, the larger an economic state integral is indicative of more economic instability, or possibly an economy that is more prone to stagflationary, deflationary or recessionary tendencies. The economic state integral is computed
Figure 7: Country economic state classification time series $S_i(t)$ computed by Algorithm 1. (a) Australia, (b) Canada, (c) France, (d) Germany, (e) Italy, (f) Japan, (g) UK and (h) USA. Japan propagates toward State 4 most frequently, which is consistent with its high $\Delta^{(i)}$ score.
Economic State Integral

| Country  | ∆(i) |
|----------|------|
| Australia| 1.22 |
| Japan    | 1.47 |
| France   | 1.17 |
| Germany  | 1.24 |
| Italy    | 1.21 |
| UK       | 1.19 |
| Canada   | 1.22 |
| USA      | 1.27 |

Table 1: Economic state integral, ∆(i) computed from each countries’ economic state classification time series S_i(t). The scores indicate that Japan has been the economy with the strongest stagflationary or recessionary tendencies. This is consistent with the natural history of economic and financial market development over the past 60 years.

as follows

$$\Delta^{(i)} = \left( \frac{1}{T} \times \sum_{t=1}^{T} S_i(t) \right) \Delta \int_{t=1}^{T} S_i(t) dt,$$

with each value ∆(i) ∈ 1, ..., N corresponding to an individual country. The economic state classification results are shown in Table 1 below.

Table 1 has two primary takeaways. The first of which is the relative degree of similarity among most countries’ ∆(i) scores. This indicates the systemic nature of both CPI inflation and GDP growth, reflecting that most countries share a similar tendency for deflationary (et al.) market conditions. The most notable outlier is Japan. Japan’s economic state integral is materially different to the rest of the country collection, and may reflect the economic stagnation experienced by the country during the 1990s.

To support this computation, we compute a normalized inner product distance matrix between each of our country’s state classification time series as follows:

$$\Omega^{S}_{ij} = \frac{\langle S^{(i)}_j, S^{(j)}_j \rangle}{\|S^{(i)}_j\|_2 \|S^{(j)}_j\|_2}.$$  

This yields an N × N distance matrix, which captures the similarity in the evolution of the interplay between GDP growth and inflation among our countries under investigation. We term this distance matrix an economic state distance matrix. To associate countries with similar evolutionary behaviours, we apply hierarchical clustering to our matrix Ω^S, and analyze the estimated number of clusters |kS|, and the association between various countries. The resulting dendrogram is shown in Figure 8.
Figure 8: Hierarchical clustering applied to our economic state distance matrix $\Omega^S$. The algorithm determines the existence of $|k_S| = 2$ clusters, with Japan and Australia displaying materially different behaviours to the remaining countries. This is most likely in relation to these countries’ inflationary behaviours, which has been studied in previous work.

5.1. Markov transition matrices

We build upon the state classification time series of each country $S_i(t)$ by studying the transition probability matrix of each country. For each country $i$, we generate a transition probability matrix $P^{(i)}$ between our $r = 4$ states which is given by

$$P^{(i)} = \begin{bmatrix}
p_{11}^{(i)} & p_{12}^{(i)} & p_{13}^{(i)} & p_{14}^{(i)} 
p_{21}^{(i)} & p_{22}^{(i)} & p_{23}^{(i)} & p_{24}^{(i)} 
p_{31}^{(i)} & p_{32}^{(i)} & p_{33}^{(i)} & p_{34}^{(i)} 
p_{41}^{(i)} & p_{42}^{(i)} & p_{43}^{(i)} & p_{44}^{(i)}
\end{bmatrix}. \quad (9)$$

As is standard in the stochastic processes literature, the rows of our state transition matrices all sum to one. That is, $p_{jk} \geq 0$, and for all rows $j$,

$$\sum_{k=1}^{K} p_{jk} = 1 \quad (10)$$

To highlight the difference in economic state behaviours, we contrast the transition probability matrices of Australia and the USA.
Both countries display broad similarity in their tendency to stay in state 1 of ascending growth, with Australia having $P(S_{t+1} = 1|S_t = 1) = 0.9438$ which is comparable (although slightly lower) than the USA $P(S_{t+1} = 1|S_t = 1) = 0.9543$. The second major difference is the distinction in Australia and the United States’ apparent ‘stickiness’ in State 4 - periods of recession. The contrasting transition probability matrices suggest that the United States tends to have more prolonged recessions, with $P(S_{t+1} = 4|S_t = 4) = 0.67$ and $P(S_{t+1} = 3|S_t = 4) = 0.33$. By contrast, Australia exhibits a transition probability $P(S_{t+1} = 3|S_t = 4) = 1$, implying that whenever Australia has experienced a period of recession, it is quick to transition into a more positive economic state. These transition probabilities are consistent with the natural history of these two countries’ economies over the last several years, in particular. For example, the GFC had a much more significant impact on the United States’ economy than that of Australia, and can be seen in Figure 7h. During the global recession, the Australian economy was largely protected due to pre-existing strong governance across the big four financial institutions (CBA, ANZ, WBC and NAB) and a boom in the resources sector where the Australian economy is heavily leveraged.

6. Decade-by-decade portfolio optimization

We conclude our analysis with a dynamic portfolio optimization exercise whereby we wish to determine the most essential assets in maximizing an investor’s Sharpe ratio in varying market conditions. Let our collection of asset prices be $a_k(t)$ where $k = 1, \ldots, K$ refer to our $K$ assets under study. This collection includes the country indices of Japan, France, Germany, Italy, UK, Canada and the USA, gold, oil and the CRB Commodity index. We sequentially apply our optimization algorithm for 10-year periods between 1960-1970, 1970-1980,...,2010-2020. One must note that due to limitations in our dataset, oil is omitted from the first two decades’ optimization experiments. We begin by computing the log returns for each asset as follows:

$$\tilde{a}_k(t) = \log \left( \frac{a_k(t)}{a_k(t-1)} \right)$$

(13)

As mentioned above, we wish to conduct our experiment on a decade-by-decade basis, so we choose a window of $\tau = 120$ to evaluate our optimization
Dynamic optimal portfolio weights

| Time     | Gold | Oil | CRB | JPN | FR  | GER | ITY | UK  | CAN | USA |
|----------|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1960-1970| 0.025| N/A | 0.025| 0.33| 0.025| 0.025| 0.18| 0.34| 0.025|     |
| 1970-1980| 0.4  | N/A | 0.22 | 0.18| 0.025| 0.025| 0.07| 0.025| 0.025|     |
| 1980-1990| 0.025| 0.025| 0.025| 0.4 | 0.025| 0.17 | 0.25| 0.025| 0.025|     |
| 1990-2000| 0.025| 0.082| 0.025| 0.025| 0.025| 0.025| 0.34| 0.025| 0.4  |     |
| 2000-2010| 0.4  | 0.025| 0.39 | 0.025| 0.025| 0.025| 0.025| 0.025| 0.036|     |
| 2010-2020| 0.35 | 0.025| 0.025| 0.062| 0.025| 0.025| 0.025| 0.025| 0.025| 0.4  |

Table 2: Decade-by-decade optimal portfolio weights. Abbreviations in the table heading are as follows: CRB: CRB Commodity index, JPN - Japan equity index, FR - French equity index, GER - German equity index, ITY - Italy equity index, UK - UK equity index, CAN - Canada equity index, USA - USA equity index.

results. Intuitively, this length is chosen so that we examine the trailing 10 years of data, and study the evolution in optimal portfolio weights as we propagate forward in time. The optimization objective function is constructed as follows:

$$\arg\max_{w_1,\ldots,w_K} \frac{\mathbb{E}(R_p[t - \tau : t])}{\sigma_p^2[t - \tau : t]}, \forall t \in \{120, 240, \ldots, 720\},$$

where

$$\mathbb{E}(R_p) = \sum_{k=1}^{K} w_k \tilde{a}_k,$$

and

$$\sigma_p^2 = w^T \Sigma w.$$ (16)

We restrict each asset’s portfolio weight $0.025 \leq w_k \leq 0.4$, $k = 1, \ldots, K$ and assume that the portfolio is always fully invested in a long-only capacity $\sum_{k=1}^{K} w_k = 1$. At each point in time, the optimization generates a vector of optimal portfolio weights which we denote $w_k^*$. We apply these constraints to simulate the dynamics of a global asset allocator, who may seek to diversify their investments geographically and have a long-term holding period with respect to candidate investments. The results in the table show the evolution in optimal portfolio weights when seeking to maximize the portfolio Sharpe ratio.

Table 2 tells an interesting story as to the most important countries and commodities to allocate capital toward in various market conditions. During the 1960-1970 period, the two equity indices that should have received the largest capital allocation were Japan and Canada, with 33% and 34% of total money invested, respectively. During this time period, both equity indices experienced strong growth. During the 1970-1980 period, however, optimal weight allocations were markedly different. This decade was characterised by high inflation, and
unsurprisingly the two assets that receive the largest optimal capital allocation are gold and the CRB Commodity index, comprising 40% (the upper bound of the weight constraint) and 22% respectively. During the 1980-1990 and 1990-2000 decades the most effective assets in maximizing portfolio Sharpe ratio were the Japan and USA equity indices, both comprising 40% of total allocated capital. During the 2000-2010 period, more than 79% of total capital should be allocated toward gold (40%) and commodities (39%) respectively. Finally, during the 2010-2020 period, the two most critical assets in maximizing risk-adjusted returns were the US equity index (40%) and gold (35%) respectively.

We proceed by computing the average optimal portfolio weight for each asset $a_k$. This calculation reveals that gold has the highest average optimal portfolio weight $\sim 20.4\%$, which may suggest that it remains an essential inclusion in equity portfolios during varying market dynamics. Although the Japan equity index has an average optimal weight allocation of $\sim 17\%$, the optimal allocation has declined significantly in recent decades due to the significant economic pressures the country has faced, and continues to face.

7. Conclusion

This paper uses various data, mathematical techniques and frameworks to provide a holistic view of inflation, and its impact on equity markets and (predominantly) equity market investors. We believe this is the first work to explore this topic from a macroeconomic, financial market dynamics and portfolio optimization perspective.

In Section 3 we conduct two sets of experiments. First we study the evolution in the $L_1$ norms of inflation, economic growth and equity index return time series. This analysis demonstrates the natural history of each economic feature. Inflation is characterised by significant levels globally during the 1970s, followed by a steady decline and 20 years of relatively subdued levels throughout the period from 2000-2020. The most notable features in the GDP growth evolution are the precipitous drops corresponding to the GFC and COVID-19 market crash. The global equity index returns demonstrate a marked decline corresponding to the COVID-19 crash, also. In the second section, we aggregate all these time series in an effort to determine what the the economic driver is in the ultimate association between economic features. Our analysis indicates that country economic features group more predominantly with similar economic features, rather than other economic features belonging to the same country.

In Section 4 we examine the temporal self-similarity exhibited by CPI inflation, GDP growth, and country equity index time series. Our analysis indicates variable temporal self-similarity among the features studied. The inflation distance matrix shows the highly anomalous nature of the elevated inflation levels throughout the 1970s, which preceded the relative homogeneity from the late 1990s onward. GDP growth and equity index returns both display broad temporal similarity, with the GFC and COVID-19 market crises displaying material difference in distance between all other candidate periods in time.
Next, we introduce a new algorithm to classify countries into one of four possible states corresponding to: i) ascending growth, ii) descending growth, iii) stagflation and iv) recession. Having computed a time-varying economic state classification time series, \( S_i(t) \), we produce an economic state integral \( \Delta^{(i)} \) which is computed by averaging state classifications over time. Our new methods indicate that Japan’s economy is most capable of exhibiting stagflationary / recessionary tendencies. We further investigate the collective similarity in these states among our collection of countries by computing a normalized inner product distance matrix, and then apply hierarchical clustering. This analysis confirms the existence of \( |k_S| = 2 \) clusters, with Australia and Japan determined to be most dissimilar to the rest of the collection.

Finally, we implement a decade-by-decade portfolio optimization to determine optimal country (and other) asset allocation for maximizing portfolio Sharpe ratio. Our study demonstrates that optimal portfolio weights vary significantly from decade-to-decade, and this is largely driven by varying economic conditions experienced during different windows in time. One prominent example is the Japanese equity index which averages 30.3% optimal capital allocation during the first three decades, and declines to an average of .037% in the final three decades. This reinforces the importance of continued financial portfolio monitoring and asset reallocation based on macroeconomic factors. The second primary finding is related to that of gold’s importance in producing optimal risk-adjusted portfolio returns. Over the 6 decades studied, gold yields an average optimal allocation of \( \text{sim} 20.4\% \). Furthermore, during the extreme inflation during the 1970s, and the two decades containing the GFC and COVID-19 market crash gold’s optimal portfolio allocation was 40%, 40% and 35% (bearing in mind the upper bound weight constraint throughout the optimization experiments is 40%). These experiments demonstrate the importance gold may play in investor portfolios during a period of heightened economic uncertainty such as one we are now experiencing.

There are various opportunities for future work. First, one could study more economic variables over a longer period of time. Second, one could further develop the economic state classification algorithm and test the sensitivity for various thresholds. This algorithm is only an initial idea, and could be a basic building block for further statistical and mathematical modelling regarding economic regime classification. Finally the portfolio optimization section could be further developed to include further investor considerations such as portfolio leverage and shorting.

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