Article

Models of Molecular Structures of Hexa-Nuclear Al\textsubscript{n}Fe\textsubscript{m} Metal Clusters (n + m = 6): DFT Quantum-Chemical Design

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Abstract: By using the density functional theory (DFT) method at the OPBE/QZVP level, key parameters of molecular structures of six-atomic (heterobi)nuclear metal clusters with an Al\textsubscript{n}Fe\textsubscript{m} composition (n + m = 6) (bond lengths, bond angles, and torsion (dihedral) angles) were calculated. It was found that each of these clusters exists in a large number of structural isomers that differ substantially in terms of their total energy. Furthermore, the molecular structures of these structural isomers significantly differ regarding the geometric parameters and geometric form. In addition, the most stable structural isomers of these metal clusters also differ rather considerably in terms of the geometric form.

Keywords: metal cluster; aluminum; iron; molecular structure; DFT method

1. Introduction

In previous studies [1–33], quantum chemical calculations of metal clusters containing atoms of two variable \textit{p}- and \textit{d}-elements, known as (\textit{dd})- and (\textit{pd})-heterobimetallic metal clusters, were carried out using the density functional theory (DFT) method. In the works [1–14], the objects of study were (\textit{dd})-heterobimetallic metal clusters which included atoms of two different \textit{d}-elements, in particular, (Cu, Fe) [1], (Pd, Fe) [2], (Pd, Ag) [3–5], (Pt, Cu) [6], (Au, Fe) [7], (Au, Pd) [8], (Au, Ag) [9], (Au, Ir) [12], and (Pd, Ir) [14]. Some of these metal clusters have been applied in various fields of science and technology [10–14]. The (\textit{pd})-heterobimetallic metal clusters that include atoms of different metal categories, namely, \textit{p}- and \textit{d}-elements, are of greater interest than (\textit{dd})-heterobimetallic ones, because, theoretically, it can be expected that they will demonstrate new properties that are not inherent to metal clusters containing metal atoms of only one category of chemical elements.

Among the most important \textit{p}-elements is aluminum, which has a very wide industrial application; nevertheless, only a few (\textit{pd})-metal clusters containing this \textit{p}-element and any of the \textit{d}-metals are described in the literature [15–33]. On the other hand, the most important \textit{d}-element is iron, which plays a key role in the iron and steel industry. Metal clusters containing aluminum and iron are of undoubted interest, if only because they are structural units of bimetallic alloys such as “alfer” and “alphenol”, containing 13% and 16% aluminum, respectively, and having a high magnetic permeability in weak magnetic fields and significant electrical resistance. In addition, both of them have a fairly high hardness, strength, and wear resistance, due to which the first of these alloys is used for the manufacturing of electroacoustic (magneto-strictive) transducers, and the second, in the production of cores for recording and reproducing heads of magnetic recording equipment. Based on the specifics of the phase diagram of the (aluminum–iron) system and the composition of the (AlFe) intermetallic compounds formed within it, it may be assumed that the key structural units of these compounds are tetra-, penta-, and hexa-nuclear (AlFe)
metal clusters. However, quantum-chemical calculations (AlFe) of metal clusters containing four to six atoms have only been performed in a few works [23–33], and in all cases, the DFT method with the OPBE/TZVP basis set was used. In our opinion, hexa-nuclear aluminum–iron metal clusters with the general formula Al\(_n\)Fe\(_m\), where the values of \(n\) and \(m\) vary from 1 to 5, are of the greatest interest here. Considering this, this article will be devoted to the presentation and systematization of the quantum-chemical calculation results of such compounds’ molecular structures using a more advanced method in the framework of density functional theory (DFT), specifically DFT OPBE/QZVP.

2. Results

The quantum chemical calculations carried out by using the DFT method at the OPBE/QZVP level showed that each of the hexa-nuclear aluminum–iron metal clusters with the stoichiometric composition Al\(_n\)Fe\(_m\) (\(n + m = 6\)), namely, Al\(_5\)Fe, Al\(_4\)Fe\(_2\), Al\(_3\)Fe\(_3\), Al\(_2\)Fe\(_4\), and AlFe\(_5\), exists in a very significant number of structural isomers, the number of which varies from 19 in the case of Al\(_5\)Fe to 40 in the case of Al\(_3\)Fe\(_3\). The information of these metal clusters in detail is presented in the Tables 1–10. As may be seen from the Tables 1, 3, 5, 7 and 9, the relative total energies of these structural isomers for each of the metal clusters under examination also vary in a very wide range. The most energetically advantageous among these isomers for Al\(_5\)Fe, Al\(_4\)Fe\(_2\), Al\(_3\)Fe\(_3\), Al\(_2\)Fe\(_4\), and AlFe\(_5\) are shown in Figures 1–5. The molecular structure parameters of these structural isomers are given in Tables 2, 4, 6, 8 and 10, respectively. A complete assortment of molecular structures of all these metal clusters is presented in the Supplemental Materials. In terms of the numbering of metal clusters, the Arabic numeral in parentheses denotes the value of the spin multiplicity of the ground state (\(M_S\)), and the Roman numeral is the ordinal number of the metal cluster with \(M_S\) data in ascending relative energy.

Table 1. Relative energies and spin multiplicities of the ground states of various structural isomers of metal clusters with an Al\(_3\)Fe composition.

| Structure Designation | Relative Energy, kJ/mol | Structure Designation | Relative Energy, kJ/mol |
|-----------------------|-------------------------|-----------------------|-------------------------|
| Metal clusters with \(M_S = 2\) | Metal clusters with \(M_S = 4\) |
| Al\(_5\)Fe (2-I) | 0.0 | Al\(_5\)Fe (4-I) | 12.0 |
| Al\(_5\)Fe (2-II) | 4.4 | Al\(_5\)Fe (4-II) | 14.0 |
| Al\(_5\)Fe (2-III) | 23.3 | Al\(_5\)Fe (4-III) | 16.0 |
| Al\(_5\)Fe (2-IV) | 28.9 | Al\(_5\)Fe (4-IV) | 21.1 |
| Al\(_5\)Fe (2-V) | 32.3 | Al\(_5\)Fe (4-V) | 23.8 |
| Al\(_5\)Fe (2-VI) | 54.3 | Al\(_5\)Fe (4-VI) | 61.9 |
| Al\(_5\)Fe (2-VII) | 62.7 | Al\(_5\)Fe (4-VII) | 78.7 |
| Al\(_5\)Fe (2-VIII) | 104.4 | Al\(_5\)Fe (4-VIII) | 105.2 |
| Al\(_5\)Fe (2-IX) | 125.1 | Al\(_5\)Fe (4-IX) | 105.9 |
| Al\(_5\)Fe (2-X) | 126.3 |
Table 2. Key structural parameters of the most stable Al$_5$Fe metal clusters.

| Metal Cluster | Al$_5$Fe (2-I) | Al$_5$Fe (2-II) | Al$_5$Fe (4-I) | Al$_5$Fe (4-II) | Al$_5$Fe (4-III) |
|---------------|----------------|----------------|----------------|----------------|------------------|
| Distances between metal atoms, pm |
| Al1Al2 | 273.2 | 284.8 | 266.7 | 260.4 | 274.8 |
| Al1Al3 | 260.1 | 271.0 | 263.0 | 295.1 | 271.0 |
| Al2Al3 | 443.9 | 424.3 | 421.2 | 437.2 | 377.5 |
| Al1Fe1 | 242.4 | 238.0 | 240.0 | 240.5 | 242.8 |
| Al1Al4 | 279.2 | 251.4 | 262.0 | 277.2 | 272.5 |
| Al1Al5 | 279.9 | 303.3 | 266.5 | 262.9 | 377.7 |
| Al2Fe1 | 395.2 | 239.1 | 362.2 | 251.6 | 429.5 |
| Al2Al4 | 256.2 | 284.7 | 254.7 | 263.5 | 272.4 |
| Al2Al5 | 273.2 | 258.1 | 259.9 | 425.2 | 271.2 |
| Al3Fe1 | 246.8 | 442.1 | 241.2 | 247.4 | 429.5 |
| Al3Al4 | 431.7 | 270.8 | 432.6 | 260.0 | 267.7 |
| Al3Al5 | 260.7 | 259.3 | 254.8 | 255.6 | 251.5 |
| Fe1Al4 | 244.3 | 238.0 | 241.2 | 253.1 | 237.5 |
| Fe1Al5 | 242.5 | 415.0 | 362.1 | 251.6 | 429.5 |
| Al4Al5 | 279.1 | 303.2 | 421.0 | 422.2 | 267.5 |
| Plane angles between metal atoms, deg. |
| Al4Al1Al5 | 59.9 | 65.5 | 105.3 | 102.8 | 45.1 |
| Fe1Al1Al4 | 55.3 | 58.1 | 57.1 | 58.0 | 54.5 |
| Fe1Al2Al3 | 33.6 | 78.1 | 34.8 | 30.0 | 84.7 |
| Al4Al2Al3 | 70.4 | 39.0 | 75.1 | 33.1 | 45.2 |
| Al1Al2Al4 | 63.6 | 52.4 | 60.5 | 63.9 | 59.7 |
| Al4Al1Al2 | 55.3 | 63.8 | 57.5 | 58.6 | 59.7 |
| Al1Al2Al3 | 32.7 | 39.0 | 37.0 | 41.0 | 45.8 |
| Al2Al3Fe1 | 62.3 | 32.0 | 59.1 | 32.0 | 34.3 |
| Al3Fe1Al2 | 84.1 | 69.9 | 86.1 | 118.1 | 61.1 |
| Al5Al1Al2 | 59.2 | 51.9 | 58.3 | 119.7 | 45.8 |
| Fe1Al1Al5 | 54.8 | 99.4 | 91.1 | 59.8 | 84.7 |
| Fe1Al1Al2 | 99.9 | 53.5 | 91.1 | 63.0 | 55.5 |
| Al1Fe1Al2 | 42.9 | 73.3 | 47.4 | 62.2 | 68.9 |
| Al3Al4Al5 | 35.3 | 53.3 | 34.7 | 34.7 | 56.0 |
Table 3. Relative energies and spin multiplicities of the ground states of various structural isomers of metal clusters with an Al$_4$Fe$_2$ composition.

| Structure Designation | Relative Energy, kJ/mol | Structure Designation | Relative Energy, kJ/mol |
|-----------------------|-------------------------|-----------------------|-------------------------|
| Metal clusters with $M_S = 1$ |                         |                       |                         |
| Al$_4$Fe$_2$ (1-I)    | 0.0                     | Al$_4$Fe$_2$ (3-IV)   | 57.2                    |
| Al$_4$Fe$_2$ (1-II)   | 93.4                    | Al$_4$Fe$_2$ (3-VI)   | 59.6                    |
| Al$_4$Fe$_2$ (1-III)  | 95.0                    | Al$_4$Fe$_2$ (3-VII)  | 62.4                    |
| Al$_4$Fe$_2$ (1-IV)   | 101.9                   | Al$_4$Fe$_2$ (3-VIII) | 65.0                    |
| Al$_4$Fe$_2$ (1-V)    | 103.3                   | Al$_4$Fe$_2$ (3-IX)   | 82.2                    |
| Al$_4$Fe$_2$ (1-VI)   | 132.5                   | Al$_4$Fe$_2$ (3-X)    | 111.5                   |
| Al$_4$Fe$_2$ (1-VII)  | 135.9                   | Al$_4$Fe$_2$ (3-XI)   | 218.4                   |
| Al$_4$Fe$_2$ (1-VIII) | 139.5                   | Al$_4$Fe$_2$ (3-XII)  | 229.7                   |
| Al$_4$Fe$_2$ (1-IX)   | 147.7                   | Al$_4$Fe$_2$ (3-XIII) | 246.8                   |
| Al$_4$Fe$_2$ (1-X)    | 155.3                   | Metal clusters with $M_S = 5$ |                         |
| Al$_4$Fe$_2$ (1-XI)   | 169.2                   | Al$_4$Fe$_2$ (5-I)    | 22.1                    |
| Al$_4$Fe$_2$ (1-XII)  | 169.3                   | Al$_4$Fe$_2$ (5-II)   | 22.5                    |
| Al$_4$Fe$_2$ (1-XIII) | 183.4                   | Al$_4$Fe$_2$ (5-III)  | 30.8                    |
| Al$_4$Fe$_2$ (1-XIV)  | 205.7                   | Al$_4$Fe$_2$ (5-IV)   | 32.2                    |
| Al$_4$Fe$_2$ (1-XV)   | 211.8                   | Al$_4$Fe$_2$ (5-V)    | 32.8                    |
| Al$_4$Fe$_2$ (1-XVI)  | 254.3                   | Al$_4$Fe$_2$ (5-VI)   | 34.3                    |
| Metal clusters with $M_S = 3$ |                         |                       |                         |
| Al$_4$Fe$_2$ (3-I)    | 24.0                    | Al$_4$Fe$_2$ (5-VII)  | 51.0                    |
| Al$_4$Fe$_2$ (3-II)   | 25.0                    | Al$_4$Fe$_2$ (5-VIII) | 60.6                    |
| Al$_4$Fe$_2$ (3-III)  | 40.0                    | Al$_4$Fe$_2$ (5-IX)   | 64.6                    |

Table 4. Key structural parameters of the most stable Al$_4$Fe$_2$ metal clusters.

| Metal Cluster | Al$_4$Fe$_2$ (1-I) | Al$_4$Fe$_2$ (5-I) | Al$_4$Fe$_2$ (5-II) | Al$_4$Fe$_2$ (3-I) | Al$_4$Fe$_2$ (3-II) |
|---------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| Distances between metal atoms, pm |
| AI1AI2        | 256.6              | 265.0              | 288.8              | 432.8              | 261.9              |
| AI1AI3        | 362.9              | 277.3              | 335.9              | 277.8              | 275.6              |
| AI2AI3        | 256.6              | 277.2              | 443.2              | 267.9              | 275.8              |
| AI1Fe1        | 248.9              | 244.1              | 246.8              | 241.2              | 246.5              |
| AI1Fe2        | 248.9              | 244.1              | 246.8              | 252.7              | 246.5              |
| AI1AI4        | 256.6              | 277.3              | 443.2              | 429.9              | 275.6              |
| AI2Fe1        | 248.9              | 244.1              | 246.8              | 250.4              | 246.5              |
| AI2Fe2        | 248.9              | 244.1              | 246.8              | 244.6              | 246.5              |
| AI2AI4        | 362.9              | 277.3              | 336.3              | 266.3              | 275.6              |
| AI3Fe1        | 248.9              | 245.3              | 246.8              | 238.0              | 238.2              |
| AI3Fe2        | 248.9              | 403.9              | 246.8              | 249.9              | 399.8              |
Table 4. Cont.

| Metal Cluster | $\text{Al}_4\text{Fe}_2$ (1-I) | $\text{Al}_4\text{Fe}_2$ (5-I) | $\text{Al}_4\text{Fe}_2$ (5-II) | $\text{Al}_4\text{Fe}_2$ (3-I) | $\text{Al}_4\text{Fe}_2$ (3-II) |
|---------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Al3Al4        | 256.6                         | 483.5                         | 288.8                         | 264.5                         | 483.8                         |
| Fe1Fe2        | 340.8                         | 212.9                         | 217.5                         | 214.8                         | 213.1                         |
| Fe1Al4        | 248.9                         | 403.9                         | 246.8                         | 393.5                         | 399.8                         |
| Fe2Al4        | 248.9                         | 245.3                         | 246.8                         | 244.4                         | 238.2                         |

Plane angles between metal atoms, deg.

| Plane Angle | Value |
|-------------|-------|
| Al3Al2Fe2   | 59.0  |
| Fe1Al1Fe2   | 86.4  |
| Fe1Al2Al3   | 59.0  |
| Fe2Al2Al3   | 59.0  |
| Al1Al2Fe2   | 59.0  |
| Fe2Al1Al2   | 59.0  |
| Al1Al2Al3   | 90.0  |
| Al2Al3Fe1   | 59.0  |
| Al3Fe1Al2   | 62.1  |
| Al4Al1Al2   | 90.0  |
| Fe1Al1Al4   | 59.0  |
| Fe1Al1Al2   | 59.0  |
| Al1Fe1Al2   | 62.1  |
| Al3Fe2Al4   | 62.1  |

Table 5. Relative energies and spin multiplicities of the ground states of various structural isomers of metal clusters of $\text{Al}_3\text{Fe}_3$.

| Structure Designation | Relative Energy, kJ/mol | Structure Designation | Relative Energy, kJ/mol |
|-----------------------|-------------------------|-----------------------|-------------------------|
| Metal clusters with $M_S = 2$ | $\text{Al}_3\text{Fe}_3$ (4-IV) | 102.4 |
| $\text{Al}_3\text{Fe}_3$ (2-I) | 51.6 | $\text{Al}_3\text{Fe}_3$ (4-V) | 103.4 |
| $\text{Al}_3\text{Fe}_3$ (2-II) | 72.4 | $\text{Al}_3\text{Fe}_3$ (4-VI) | 106.6 |
| $\text{Al}_3\text{Fe}_3$ (2-III) | 82.2 | $\text{Al}_3\text{Fe}_3$ (4-VII) | 114.5 |
| $\text{Al}_3\text{Fe}_3$ (2-IV) | 89.2 | $\text{Al}_3\text{Fe}_3$ (4-VIII) | 119.1 |
| $\text{Al}_3\text{Fe}_3$ (2-V) | 94.6 | $\text{Al}_3\text{Fe}_3$ (4-IX) | 122.4 |
| $\text{Al}_3\text{Fe}_3$ (2-VI) | 94.8 | $\text{Al}_3\text{Fe}_3$ (4-X) | 124.7 |
| $\text{Al}_3\text{Fe}_3$ (2-VII) | 102.9 | $\text{Al}_3\text{Fe}_3$ (4-XI) | 129.6 |
| $\text{Al}_3\text{Fe}_3$ (2-VIII) | 108.4 | $\text{Al}_3\text{Fe}_3$ (4-XII) | 147.2 |
| $\text{Al}_3\text{Fe}_3$ (2-IX) | 114.8 | $\text{Al}_3\text{Fe}_3$ (4-XIII) | 162.5 |
| $\text{Al}_3\text{Fe}_3$ (2-X) | 122.3 | Metal clusters with $M_S = 6$ | |
| $\text{Al}_3\text{Fe}_3$ (2-XI) | 125.6 | $\text{Al}_3\text{Fe}_3$ (6-I) | 0.0 |
| $\text{Al}_3\text{Fe}_3$ (2-XII) | 132.2 | $\text{Al}_3\text{Fe}_3$ (6-II) | 3.5 |
| $\text{Al}_3\text{Fe}_3$ (2-XIII) | 134.6 | $\text{Al}_3\text{Fe}_3$ (6-III) | 32.7 |
| $\text{Al}_3\text{Fe}_3$ (2-XIV) | 136.5 | $\text{Al}_3\text{Fe}_3$ (6-IV) | 57.4 |
Table 5. Cont.

| Structure Designation | Relative Energy, kJ/mol | Structure Designation | Relative Energy, kJ/mol |
|-----------------------|-------------------------|-----------------------|-------------------------|
| Al<sub>3</sub>Fe<sub>3</sub> (2-XV) | 137.1 | Al<sub>3</sub>Fe<sub>3</sub> (6-V) | 61.0 |
| Al<sub>3</sub>Fe<sub>3</sub> (2-XVI) | 154.0 | Al<sub>3</sub>Fe<sub>3</sub> (6-VI) | 65.9 |
| Al<sub>3</sub>Fe<sub>3</sub> (2-XVII) | 174.9 | Al<sub>3</sub>Fe<sub>3</sub> (6-VII) | 67.6 |

Metal clusters with $M_5 = 4$

| Structure Designation | Relative Energy, kJ/mol |
|----------------------|-------------------------|
| Al<sub>3</sub>Fe<sub>3</sub> (4-I) | 49.1 |
| Al<sub>3</sub>Fe<sub>3</sub> (4-II) | 81.2 |
| Al<sub>3</sub>Fe<sub>3</sub> (4-III) | 89.0 |

Table 6. Key structural parameters of the most stable Al<sub>3</sub>Fe<sub>3</sub> metal clusters.

| Metal Cluster | Al<sub>3</sub>Fe<sub>3</sub> (6-I) | Al<sub>3</sub>Fe<sub>3</sub> (6-II) | Al<sub>3</sub>Fe<sub>3</sub> (6-III) | Al<sub>3</sub>Fe<sub>3</sub> (4-I) | Al<sub>3</sub>Fe<sub>3</sub> (2-I) |
|---------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Distances between metal atoms, pm |
| Al1Al2 | 305.1 | 270.6 | 262.3 | 259.8 | 268.8 |
| Al1Al3 | 305.8 | 270.7 | 262.3 | 258.4 | 268.8 |
| Al2Al3 | 398.5 | 368.5 | 384.3 | 372.3 | 374.3 |
| Al1Fe1 | 236.0 | 241.8 | 271.4 | 245.7 | 241.0 |
| Al1Fe2 | 369.4 | 354.8 | 245.5 | 346.1 | 338.0 |
| Al1Fe3 | 236.0 | 241.8 | 245.5 | 243.7 | 243.8 |
| Al2Fe1 | 244.0 | 239.7 | 242.8 | 251.0 | 250.8 |
| Al2Fe2 | 242.3 | 241.8 | 389.6 | 252.5 | 236.8 |
| Al2Fe3 | 244.0 | 239.7 | 249.2 | 242.8 | 248.2 |
| Al3Fe1 | 244.0 | 239.7 | 242.8 | 248.5 | 250.8 |
| Al3Fe2 | 242.3 | 241.8 | 249.2 | 246.8 | 236.8 |
| Al3Fe3 | 244.0 | 239.7 | 389.6 | 253.0 | 248.2 |
| Fe1Fe2 | 222.3 | 226.8 | 219.1 | 221.5 | 235.4 |
| Fe1Fe3 | 270.9 | 305.7 | 219.1 | 330.4 | 328.7 |
| Fe2Fe3 | 222.3 | 220.8 | 253.4 | 246.3 | 222.9 |
| Plane angles between metal atoms, deg. |
| Fe2Al1Fe3 | 35.0 | 39.2 | 56.7 | 45.4 | 41.2 |
| Fe1Al1Fe2 | 35.0 | 39.2 | 49.8 | 39.6 | 44.2 |
| Fe1Al2Al3 | 35.2 | 39.8 | 37.7 | 41.6 | 41.7 |
| Fe2Al2Al3 | 34.7 | 40.4 | 37.6 | 41.2 | 37.8 |
| Al1Al2Fe2 | 84.1 | 87.4 | 38.3 | 85.0 | 83.6 |
| Fe2Al1Al2 | 40.7 | 42.9 | 100.2 | 46.6 | 44.1 |
| Al1Al2Al3 | 49.4 | 47.1 | 42.9 | 43.9 | 45.9 |
| Al2Al3Fe1 | 35.2 | 39.8 | 37.7 | 42.1 | 41.7 |
| Al3Fe1Al2 | 109.5 | 100.5 | 105.6 | 96.4 | 96.5 |
| Fe3Al1Al2 | 51.7 | 55.4 | 58.7 | 57.6 | 57.7 |
| Fe1Al1Fe3 | 70.1 | 78.4 | 49.8 | 84.9 | 85.4 |
| Fe1Al1Al2 | 51.7 | 55.4 | 54.1 | 59.5 | 58.6 |
| Al1Fe1Al2 | 78.9 | 68.4 | 61.0 | 63.1 | 66.2 |
| Al3Fe2Fe3 | 63.2 | 61.4 | 107.6 | 61.8 | 65.3 |
Table 7. Relative energies and spin multiplicities of the ground states of various structural isomers of metal clusters of Al₂Fe₄.

| Structure Designation | Relative Energy, kJ/mol | Structure Designation | Relative Energy, kJ/mol | Structure Designation | Relative Energy, kJ/mol |
|-----------------------|-------------------------|-----------------------|-------------------------|-----------------------|-------------------------|
| Metal clusters with \(M_S = 1\) |                         |                       |                         |                       |                         |
| Al₂Fe₄ (1-I)          | 15.3                    | Al₂Fe₄ (1-XX)         | 378.3                   | Al₂Fe₄ (3-X)          | 77.3                    |
| Al₂Fe₄ (1-II)         | 15.9                    | Al₂Fe₄ (1-XXI)        | 378.7                   | Al₂Fe₄ (3-XI)         | 86.8                    |
| Al₂Fe₄ (1-III)        | 16.1                    | Al₂Fe₄ (1-XII)        | 394.4                   | Al₂Fe₄ (3-III)        | 89.4                    |
| Al₂Fe₄ (1-IV)         | 17.2                    | Al₂Fe₄ (1-XXII)       | 381.8                   | Al₂Fe₄ (3-IV)         | 91.2                    |
| Al₂Fe₄ (1-V)          | 30.1                    | Al₂Fe₄ (1-XXIII)      | 402.1                   | Al₂Fe₄ (3-VI)         | 121.1                   |
| Al₂Fe₄ (1-VI)         | 41.9                    | Al₂Fe₄ (1-XXIV)       | 403.0                   | Al₂Fe₄ (3-VII)        | 201.7                   |
| Al₂Fe₄ (1-VII)        | 53.8                    | Al₂Fe₄ (1-XXV)        | 403.3                   | Metal clusters with \(M_S = 5\) |                        |
| Al₂Fe₄ (1-VIII)       | 83.1                    | Al₂Fe₄ (1-XXVI)       | 409.8                   | Al₂Fe₄ (5-I)          | 0.0                     |
| Al₂Fe₄ (1-IX)         | 129.3                   | Al₂Fe₄ (1-XXVII)      | 426.0                   | Al₂Fe₄ (5-II)         | 16.8                    |
| Al₂Fe₄ (1-X)          | 142.6                   | Metal clusters with \(M_S = 3\) | Al₂Fe₄ (5-III) | 24.4                   |
| Al₂Fe₄ (1-XI)         | 187.3                   | Al₂Fe₄ (3-I)          | 15.0                    | Al₂Fe₄ (5-IV)         | 29.0                    |
| Al₂Fe₄ (1-XII)        | 231.1                   | Al₂Fe₄ (3-II)         | 15.7                    | Al₂Fe₄ (5-V)          | 41.2                    |
| Al₂Fe₄ (1-XIII)       | 241.9                   | Al₂Fe₄ (3-III)        | 26.6                    | Al₂Fe₄ (5-VI)         | 45.2                    |
| Al₂Fe₄ (1-XIV)        | 259.0                   | Al₂Fe₄ (3-IV)         | 27.6                    | Al₂Fe₄ (5-VII)        | 119.8                   |
| Al₂Fe₄ (1-XV)         | 259.4                   | Al₂Fe₄ (3-V)          | 37.6                    | Al₂Fe₄ (5-VIII)       | 131.1                   |
| Al₂Fe₄ (1-XVI)        | 330.8                   | Al₂Fe₄ (3-VI)         | 44.2                    | Al₂Fe₄ (5-IX)         | 204.2                   |
| Al₂Fe₄ (1-XVII)       | 350.3                   | Al₂Fe₄ (3-VII)        | 56.8                    | Al₂Fe₄ (5-X)          | 212.1                   |
| Al₂Fe₄ (1-XVIII)      | 352.4                   | Al₂Fe₄ (3-VIII)       | 57.5                    |                        |                         |
Table 8. Key structural parameters of the most stable Al$_2$Fe$_4$ metal clusters.

| Metal Cluster | Al$_2$Fe$_4$ (5-I) | Al$_2$Fe$_4$ (3-I) | Al$_2$Fe$_4$ (1-I) | Al$_2$Fe$_4$ (3-II) | Al$_2$Fe$_4$ (1-II) |
|---------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Fe1Fe2        | 333.1             | 244.1             | 217.0             | 329.9             | 218.5             |
| Fe1Fe3        | 246.1             | 213.4             | 252.9             | 241.9             | 379.5             |
| Fe2Fe3        | 226.5             | 326.5             | 335.6             | 214.7             | 254.4             |
| Fe1Al1        | 249.7             | 249.2             | 242.0             | 246.6             | 242.9             |
| Fe1Al2        | 249.7             | 249.2             | 350.2             | 246.6             | 242.9             |
| Fe1Fe4        | 246.1             | 326.5             | 248.5             | 234.3             | 395.9             |
| Fe2Al1        | 242.9             | 247.6             | 247.7             | 248.7             | 250.5             |
| Fe2Al2        | 242.9             | 247.7             | 252.6             | 248.7             | 250.5             |
| Fe2Fe4        | 226.5             | 220.2             | 253.0             | 242.1             | 389.9             |
| Fe3Al1        | 242.9             | 249.2             | 252.7             | 248.7             | 246.1             |
| Fe3Al2        | 348.7             | 249.2             | 247.8             | 248.7             | 246.1             |
| Fe3Fe4        | 231.8             | 244.1             | 217.0             | 330.0             | 216.7             |
| Al1Al2        | 270.1             | 374.5             | 262.6             | 369.0             | 277.5             |
| Al1Fe4        | 348.6             | 247.7             | 350.2             | 246.6             | 248.5             |
| Al2Fe4        | 242.8             | 247.7             | 242.0             | 246.6             | 248.4             |

Distances between metal atoms, pm

| Plane angles between metal atoms, deg. |
|---------------------------------------|
| Al2Fe1Fe4                             | 58.6 | 48.7 | 43.7 | 61.6 | 36.8 |
| Al1Fe1Al2                             | 65.5 | 97.4 | 48.4 | 96.9 | 69.7 |
| Al1Fe2Fe3                             | 62.2 | 49.1 | 48.5 | 64.4 | 58.3 |
| Al2Fe2Fe3                             | 95.9 | 49.1 | 47.3 | 64.4 | 58.3 |
| Fe1Fe2Al2                             | 48.3 | 60.9 | 96.1 | 48.0 | 61.9 |
| Al2Fe1Fe2                             | 46.6 | 60.3 | 45.8 | 48.5 | 65.5 |
| Fe1Fe2Fe3                             | 47.6 | 40.8 | 48.9 | 47.1 | 106.5|
| Fe2Fe3Al1                             | 62.2 | 48.7 | 47.3 | 64.4 | 60.0 |
| Fe3Al1Fe2                             | 55.6 | 82.2 | 84.2 | 51.1 | 61.6 |
| Fe4Fe1Fe2                             | 42.8 | 42.4 | 65.4 | 47.2 | 72.4 |
| Al1Fe1Fe4                             | 89.4 | 48.7 | 91.1 | 61.6 | 36.8 |
| Al1Fe1Fe2                             | 46.6 | 60.3 | 65.0 | 48.5 | 65.5 |
| Fe1Al1Fe2                             | 85.1 | 58.9 | 52.6 | 83.5 | 52.6 |
| Fe3Al2Fe4                             | 41.5 | 58.9 | 52.6 | 83.5 | 52.0 |
Table 9. Relative energies and spin multiplicities of the ground states of various structural isomers of metal clusters of AlFe5.

| Structure Designation | Relative Energy, kJ/mol | Structure Designation | Relative Energy, kJ/mol |
|-----------------------|-------------------------|-----------------------|-------------------------|
| Metal clusters with $M_S = 2$ | | | |
| AlFe5 (2-I) | 49.1 | AlFe5 (2-III) | 18.7 |
| AlFe5 (2-II) | 57.6 | AlFe5 (2-IV) | 65.5 |
| AlFe5 (2-III) | 69.7 | AlFe5 (2-V) | 123.0 |
| AlFe5 (2-V) | 139.1 | Metal clusters with $M_S = 6$ | |
| AlFe5 (2-VI) | 177.4 | AlFe5 (6-I) | 33.8 |
| AlFe5 (2-VII) | 178.4 | AlFe5 (6-II) | 61.2 |
| AlFe5 (2-VIII) | 254.7 | AlFe5 (6-III) | 69.4 |
| Metal clusters with $M_S = 4$ | | | |
| AlFe5 (4-I) | 0.0 | AlFe5 (6-IV) | 105.1 |
| AlFe5 (4-II) | 36.0 | AlFe5 (6-V) | 189.8 |
| AlFe5 (4-III) | | AlFe5 (6-VI) | |

Table 10. Key structural parameters of the most stable AlFe5 metal clusters.

| Metal Cluster | AlFe5 (4-I) | AlFe5 (2-I) | AlFe5 (6-I) | AlFe5 (4-II) | AlFe5 (4-III) |
|---------------|-------------|-------------|-------------|-------------|---------------|
| Distances between metal atoms, pm | | | | | |
| Fe1Fe2 | 223.6 | 224.4 | 233.1 | 221.4 | 251.4 |
| Fe1Fe3 | 252.1 | 248.2 | 249.8 | 382.4 | 251.5 |
| Fe2Fe3 | 252.8 | 329.9 | 335.1 | 393.7 | 342.7 |
| Fe1Al1 | 246.1 | 353.0 | 358.0 | 219.5 | 351.8 |
| Fe1Fe4 | 231.2 | 243.6 | 250.5 | 249.2 | 243.7 |
| Fe1Fe5 | 335.8 | 224.4 | 223.9 | 258.7 | 251.5 |
| Fe2Al1 | 350.9 | 243.6 | 250.5 | 249.2 | 243.7 |
| Fe2Fe4 | 223.5 | 245.9 | 238.1 | 224.2 | 252.6 |
| Fe2Fe5 | 252.7 | 222.1 | 228.8 | 388.9 | 225.1 |
| Fe3Al1 | 252.2 | 252.1 | 249.4 | 247.4 | 243.8 |
| Fe3Fe4 | 335.8 | 218.0 | 212.5 | 255.7 | 252.7 |
| Fe3Fe5 | 212.9 | 245.7 | 250.8 | 211.7 | 225.1 |
| Al1Fe4 | 246.1 | 252.1 | 244.8 | 273.7 | 254.4 |
| Al1Fe5 | 252.1 | 243.6 | 251.1 | 252.1 | 248.4 |
| Fe4Fe5 | 252.0 | 329.9 | 323.1 | 254.0 | 330.0 |
| Plane angles between metal atoms, deg. | | | | | |
| Fe4Fe1Fe5 | 48.6 | 88.4 | 87.0 | 61.9 | 90.7 |
| Al1Fe1Fe4 | 62.0 | 45.6 | 43.0 | 69.0 | 45.8 |
| Al1Fe2Fe3 | 45.9 | 49.4 | 47.8 | 37.4 | 45.4 |
| Fe4Fe2Fe3 | 89.5 | 41.4 | 39.1 | 37.6 | 47.3 |
| Fe1Fe2Fe4 | 62.3 | 63.5 | 62.7 | 63.1 | 49.4 |
Table 3. Relative energies and spin multiplicities of the ground states of various structural isomers of metal clusters with the general formula AlnFem, where the values of n and m vary from 1 to 5. The comparison is made in ascending relative energy.

| Metal Cluster | AlFe5 (4-I) | AlFe5 (2-I) | AlFe5 (6-I) | AlFe5 (4-II) | AlFe5 (4-III) |
|---------------|-------------|-------------|-------------|--------------|---------------|
| Fe4Fe1Fe2     | 58.9        | 62.5        | 59.7        | 59.0         | 65.6          |
| Fe1Fe2Fe3     | 63.6        | 48.8        | 48.2        | 70.6         | 47.1          |
| Fe2Fe3Al1     | 88.0        | 47.2        | 48.1        | 37.7         | 45.3          |
| Fe3Al1Fe2     | 46.0        | 83.4        | 84.2        | 104.9        | 89.3          |
| Fe5Fe1Fe2     | 48.8        | 59.3        | 60.1        | 108.0        | 53.2          |
| Al1Fe1Fe5     | 48.4        | 43.1        | 44.0        | 59.4         | 44.9          |
| Al1Fe1Fe2     | 90.6        | 43.1        | 44.1        | 63.6         | 43.8          |
| Fe1Al1Fe2     | 39.8        | 39.0        | 40.4        | 52.7         | 45.6          |
| Fe3Fe4Fe5     | 39.3        | 48.1        | 50.9        | 49.1         | 42.9          |

Figure 1. Molecular structures of the five most energetically stable Al5Fe metal clusters.

Figure 2. Molecular structures of the five most energetically stable Al4Fe2 metal clusters.
Figure 3. Molecular structures of the five most energetically stable Al$_3$Fe$_3$ metal clusters.

Table 6. Key structural parameters of the most stable Al$_3$Fe$_3$ metal clusters.

| Metal Cluster | Distances between metal atoms, pm |
|---------------|-----------------------------------|
| Al$_3$Fe$_3$ (6-I) | Al$_1$Al$_2$ 305.1, Al$_1$Al$_3$ 305.8, Al$_2$Al$_3$ 398.5, Al$_1$Fe$_1$ 236.0, Al$_1$Fe$_2$ 369.4 |
| Al$_3$Fe$_3$ (6-II) | Al$_1$Al$_2$ 270.6, Al$_1$Al$_3$ 270.7, Al$_2$Al$_3$ 368.5, Al$_1$Fe$_1$ 241.8, Al$_1$Fe$_2$ 354.8 |
| Al$_3$Fe$_3$ (6-III) | Al$_1$Al$_2$ 262.3, Al$_1$Al$_3$ 262.3, Al$_2$Al$_3$ 384.3, Al$_1$Fe$_1$ 271.4, Al$_1$Fe$_2$ 245.5 |
| Al$_3$Fe$_3$ (4-I) | Al$_1$Al$_2$ 259.8, Al$_1$Al$_3$ 258.4, Al$_2$Al$_3$ 372.3, Al$_1$Fe$_1$ 245.7, Al$_1$Fe$_2$ 346.1 |
| Al$_3$Fe$_3$ (2-I) | Al$_1$Al$_2$ 268.8, Al$_1$Al$_3$ 268.8, Al$_2$Al$_3$ 374.3, Al$_1$Fe$_1$ 241.0, Al$_1$Fe$_2$ 338.0 |

Figure 4. Molecular structures of the five most energetically stable Al$_2$Fe$_4$ metal clusters.

Table 8. Key structural parameters of the most stable Al$_2$Fe$_4$ metal clusters.

| Metal Cluster | Distances between metal atoms, pm |
|---------------|-----------------------------------|
| Al$_2$Fe$_4$ (5-I) | Fe$_1$Fe$_2$ 333.1, Fe$_1$Fe$_3$ 246.1, Fe$_2$Fe$_3$ 226.5, Fe$_1$Al$_1$ 249.7, Fe$_1$Al$_2$ 249.7 |
| Al$_2$Fe$_4$ (3-I) | Fe$_1$Fe$_2$ 244.1, Fe$_1$Fe$_3$ 213.4, Fe$_2$Fe$_3$ 326.5, Fe$_1$Al$_1$ 249.2, Fe$_1$Al$_2$ 350.2 |
| Al$_2$Fe$_4$ (1-I) | Fe$_1$Fe$_2$ 217.0, Fe$_1$Fe$_3$ 252.9, Fe$_2$Fe$_3$ 335.6, Fe$_1$Al$_1$ 242.0, Fe$_1$Al$_2$ 242.0 |
| Al$_2$Fe$_4$ (3-II) | Fe$_1$Fe$_2$ 329.9, Fe$_1$Fe$_3$ 241.9, Fe$_2$Fe$_3$ 214.7, Fe$_1$Al$_1$ 246.6, Fe$_1$Al$_2$ 246.6 |
| Al$_2$Fe$_4$ (1-II) | Fe$_1$Fe$_2$ 218.5, Fe$_1$Fe$_3$ 379.5, Fe$_2$Fe$_3$ 254.4, Fe$_1$Al$_1$ 242.9, Fe$_1$Al$_2$ 242.9 |

Plane angles between metal atoms, deg.

| Angle | Al$_2$Fe$_1$Fe$_4$ 58.6, Al$_1$Fe$_1$Al$_2$ 65.5, Al$_1$Fe$_2$Fe$_3$ 62.2 |

Figure 4. Molecular structures of the five most energetically stable Al$_2$Fe$_4$ metal clusters.
Table 9. Relative energies and spin multiplicities of the ground states of various structural isomers of metal clusters of AlFe5.

| Structure designation | Relative energy, kJ/mol |
|-----------------------|-------------------------|
| AlFe5 (4-III)         | 49.1                    |
| AlFe5 (2-I)           | 57.6                    |
| AlFe5 (4-IV)          | 183.7                   |
| AlFe5 (2-II)          | 69.7                    |
| AlFe5 (4-VII)         | 162.1                   |
| AlFe5 (2-III)         | 139.1                   |
| AlFe5 (2-IV)          | 65.5                    |
| AlFe5 (2-V)           | 123.0                   |
| AlFe5 (2-VI)          | 132.2                   |
| AlFe5 (2-VII)         | 178.4                   |
| AlFe5 (2-VIII)        | 254.7                   |
| AlFe5 (2-IX)          | 254.7                   |
| AlFe5 (6-I)           | 33.8                    |
| AlFe5 (6-II)          | 61.2                    |
| AlFe5 (6-III)         | 69.4                    |
| AlFe5 (6-IV)          | 105.1                   |
| AlFe5 (4-I)           | 0.0                     |
| AlFe5 (4-II)          | 36.0                    |
| AlFe5 (4-III)         | 189.8                   |

Figure 5. Molecular structures of the five most energetically stable AlFe5 metal clusters.

Table 11. Standard thermodynamic parameters of formation for the most energetically stable (heterobi)hexa-nuclear metal clusters of AlₙFeₘ (n + m = 6).

| Metal Cluster | ∆_H^θ (298 K), kJ/mol | S^θ (298 K), J/mol K | ∆_G^θ (298 K), kJ/mol |
|---------------|------------------------|----------------------|-----------------------|
| Al₄Fe (2-I)   | 805.0                  | 457.5                | 718.9                 |
| Al₄Fe₂ (1-I)  | 789.0                  | 445.8                | 706.1                 |
| Al₃Fe₃ (6-I)  | 807.5                  | 469.0                | 717.4                 |
| Al₂Fe₄ (5-I)  | 890.1                  | 480.3                | 796.3                 |
| AlFe₅ (4-I)   | 927.6                  | 476.1                | 834.7                 |
Table 12. Charge distribution (in units of electron charge) of various Al and Fe atoms for the most stable Al\textsubscript{n}Fe\textsubscript{m} metal clusters (n + m = 6) according to NBO analysis data.

|      | Al\textsubscript{5}Fe (2-I) |      |      |      |      |      |      |      |
|------|-----------------------------|------|------|------|------|------|------|------|
| Al1  | +0.0719                     | Al2  | +0.1187 | +0.2723 | +0.2682 | +0.0716 | −0.8027 |
|      |                              | Al3  |      |      |      |      |      | −0.8027 |
|      |                              | Al4  |      |      |      |      |      |      |
|      |                              | Al5  |      |      |      |      |      |      |
|      |                              | Fe1  |      |      |      |      |      |      |
|      |                              | Fe2  |      |      |      |      |      |      |

|      | Al\textsubscript{4}Fe\textsubscript{2} (1-I) |      |      |      |      |      |      |      |
|------|---------------------------------------------|------|------|------|------|------|------|------|
| Al1  | +0.1861                                    | Al2  | +0.1861 | +0.1861 | +0.1861 | −0.3722 | −0.3722 |
|      |                              | Al3  |      |      |      |      |      |      |
|      |                              | Al4  |      |      |      |      |      |      |
|      |                              | Fe1  |      |      |      |      |      |      |
|      |                              | Fe2  |      |      |      |      |      |      |

|      | Al\textsubscript{3}Fe\textsubscript{3} (6-I) |      |      |      |      |      |      |      |
|------|---------------------------------------------|------|------|------|------|------|------|------|
| Al1  | +0.5766                                    | Al2  | +0.6292 | +0.6312 | −0.7980 | −0.2411 | −0.7979 |
|      |                              | Al3  |      |      |      |      |      |      |
|      |                              | Fe1  |      |      |      |      |      |      |
|      |                              | Fe2  |      |      |      |      |      |      |
|      |                              | Fe3  |      |      |      |      |      |      |

|      | Al\textsubscript{2}Fe\textsubscript{4} (5-I) |      |      |      |      |      |      |      |
|------|---------------------------------------------|------|------|------|------|------|------|------|
| Al1  | +0.3197                                    | Al2  | +0.3197 | −0.1446 | −0.2312 | −0.1316 | −0.1320 |
|      |                              | Fe1  |      |      |      |      |      |      |
|      |                              | Fe2  |      |      |      |      |      |      |
|      |                              | Fe3  |      |      |      |      |      |      |
|      |                              | Fe4  |      |      |      |      |      |      |

|      | AlFe\textsubscript{5} (4-I) |      |      |      |      |      |      |      |
|------|-----------------------------|------|------|------|------|------|------|------|
| Al1  | +0.2554                     | Fe1  |      |      |      |      |      |      |
|      |                              | Fe2  |      |      |      |      |      |      |
|      |                              | Fe3  |      |      |      |      |      |      |
|      |                              | Fe4  |      |      |      |      |      |      |
|      |                              | Fe5  |      |      |      |      |      |      |

3. Discussion

As can be seen from the presented data, the Al\textsubscript{5}Fe metal cluster exists in 19 structural isomers, the Al\textsubscript{4}Fe\textsubscript{2} metal cluster in 39 structural isomers, the Al\textsubscript{3}Fe\textsubscript{3} metal cluster in 40 structural isomers, the Al\textsubscript{2}Fe\textsubscript{4} metal cluster in 53 structural isomers, and the AlFe\textsubscript{5} metal cluster in 23 structural isomers. There is no regularity in these numbers; there is only a tendency for them to first increase their number with an increase in the number of iron atoms (m) in the structural unit of the metal cluster (in the range m = 1–4), and then to decrease (in the range m = 4–5). For any of the considered metal clusters in the series Al\textsubscript{5}Fe–Al\textsubscript{4}Fe\textsubscript{2}–Al\textsubscript{3}Fe\textsubscript{3}–Al\textsubscript{2}Fe\textsubscript{4}–AlFe\textsubscript{5}, in general, a very significant variety of molecular structures is characteristic—from pseudo-octahedral [Al\textsubscript{2}Fe\textsubscript{4} (3-V)] to strictly planar [Al\textsubscript{4}Fe\textsubscript{2} (3-XIII)]. Both of these structures are unique and are not observed in any of the other Al\textsubscript{n}Fe\textsubscript{m} metal clusters (n + m = 6). For the overwhelming majority of the presented metal cluster structures, a pronounced asymmetry with an almost complete absence of any symmetrical elements is typical. Therefore, none of these metal clusters has a symmetry axis of the third or higher order, and only some of them have a plane of symmetry and a symmetry center (see Supplementary Materials). This fully applies to the most energetically stable (i.e., those with the minimum total energy values) metal clusters presented in Figures 1–5. It is important to note that in pairs of so-called “inverted” metal clusters, where the numerical values of m and n for Al and Fe atoms are interchanged [(Al\textsubscript{5}Fe, AlFe\textsubscript{5}) and (Al\textsubscript{4}Fe\textsubscript{2}, Al\textsubscript{2}Fe\textsubscript{4})], there is no noticeable similarity between the assortments of molecular structures in either quantitative or qualitative relations.

A remarkable feature of all the Al\textsubscript{n}Fe\textsubscript{m} metal clusters is the relatively small number of metal–metal bonds—no more than 8; at the same time, there is no apparent correlation between the total number of these bonds in the metal cluster and its energy stability (i.e., the value of its relative energy). It is very curious that the formation of Al–Al bonds in molecular structures of these metal clusters is much rarer than the formation of Fe–Fe bonds, and this phenomenon even takes place in the case of the Al\textsubscript{4}Fe\textsubscript{2} metal cluster, where the number of aluminum atoms in each of the 39 molecular structures is twice as high as the number of iron atoms (in the most energetically stable Al\textsubscript{4}Fe\textsubscript{2} metal cluster, namely Al\textsubscript{4}Fe\textsubscript{2} (1-I), there is also no Al–Al bond (Figure 2)). Among the molecular structures of the Al\textsubscript{2}Fe\textsubscript{4} metal cluster, there is not even one structure in which such a bond is present; the most energetically stable of them—Al\textsubscript{2}Fe\textsubscript{4} (5-I)—is no exception (Figure 4). Moreover,
Figures 2–4 show that, among the five most stable structural isomers of each of the metal clusters $\text{Al}_2\text{Fe}_4$, $\text{Al}_3\text{Fe}_3$, and $\text{Al}_2\text{Fe}_4$, there is not a single one in which the molecular structure contains at least one Al–Al bond. Molecular structures with the absence of Fe–Fe bonds only occur in the case of the $\text{Al}_3\text{Fe}_3$ metal cluster (where such a bond cannot exist in principle) and the $\text{Al}_4\text{Fe}_2$ metal cluster, so they are in the minority compared to structures where the Fe–Fe bond is present (16 out of 39 molecular structures). Al–Fe bonds, as was expected, are present in each of the structural isomers of each metal cluster under consideration. It should be noted that the length of any of the metal–metal bonds in any of these metal clusters is more than 200 pm (see Tables 2, 4, 6, 8 and 10). The Al–Al bonds are the longest on average, the Fe–Fe bonds are the shortest, and the Fe–Al bonds have an intermediate length between the lengths of bonds formed by two aluminum atoms and two iron atoms. This relationship seems to be real due to the atomic radii of Al (143 pm) and Fe (126 pm). As for the plane bond angles formed by metal–metal bonds, they are, as a rule, relatively small and almost always less than $90^\circ$; a similar situation occurs for planar non-bonded angles, as well as for dihedral (torsion) angles.

As can be seen from the data in Tables 1, 3, 5, 7 and 9, the most energetically stable metal clusters $\text{Al}_5\text{Fe}$ (2-I) and $\text{Al}_4\text{Fe}_2$ (1-I) are low-spin, while the most energetically stable metal clusters $\text{Al}_3\text{Fe}_3$ (6-I) and $\text{Al}_2\text{Fe}_4$ (5-I) are high-spin; the $\text{Al}_5\text{Fe}_5$ (4-I) metal cluster occupies an intermediate position. This fact is quite understandable because the number of unpaired electrons in a neutral Al atom (the electronic configuration of the ground state $3s^23p^1$) is significantly lower than the number of unpaired electrons in a neutral Fe ($4s^23d^6$) atom and with an increase in the number of Fe atoms (and, accordingly, a decrease number of Al atoms) in the molecular structure of the $\text{Al}_n\text{Fe}_m$ metal cluster ($n + m = 6$), logically, there should be an increase in the total number of unpaired electrons and a higher probability of the realization of the high-spin ground state compared to the low-spin one. Such a prediction, however, is not fully justified when going from $\text{Al}_4\text{Fe}_4$ to $\text{Al}_5\text{Fe}_5$, because, contrary to our expectations, the ground state of the most stable $\text{Al}_5\text{Fe}_5$ metal cluster, which is $\text{Al}_5\text{Fe}_5$ (4-I), was not in a high-spin state with $M_S = 6$, but a state with $M_S = 4$. The reason for such a metamorphosis is apparently related to the fact that in the smallest homohexa-nuclear metal cluster $\text{Fe}_6$, as the calculation shows, the ground state is the spin singlet ($M_S = 1$), and the spin multiplicity of the ground state for $\text{Al}_5\text{Fe}_5$ should thus be intermediate, between those of $\text{Al}_2\text{Fe}_4$ and $\text{Fe}_6$, which is confirmed by the calculation results. According to NBO analysis data, the charges on the Al and Fe atoms that make up the investigated metal clusters, on the whole, are relatively small and do not exceed 1.00 in absolute value. As should be expected, for the metal clusters presented in Table 11, the charges on the Al atoms are positive, and those on the Fe atoms are negative. This is also quite understandable since the electronegativity of Al on the Pauling scale (1.5) is lower than the electronegativity of Fe (1.8). An interesting exception is the $\text{Al}_5\text{Fe}_5$ (4-I) cluster, in which one of the Fe atoms, namely Fe2, has a positive charge (Table 12). As a rule, the magnitudes of the charges on different atoms of the same element are different, which is also quite obvious because of the asymmetry of most structural isomers of these metal clusters.

According to our calculation, the values of the Gibbs free energy of formation $\Delta_f G^0$ (298 K), even for the most stable $\text{Al}_n\text{Fe}_m$ metal clusters ($n + m = 6$), are positive (Table 11), which means that none of them can be obtained by direct interactions between metallic iron and metallic aluminum. However, the situation changes radically if gaseous aluminum and iron are taken as starting materials, i.e., the following reactions (1)–(5) are used:

$$\begin{align*}
\text{5Al(gas)} + \text{Fe(gas)} & \rightarrow \text{Al}_5\text{Fe(gas)} \\
\text{4Al(gas)} + 2\text{Fe(gas)} & \rightarrow \text{Al}_4\text{Fe}_2(gas) \\
\text{3Al(gas)} + 3\text{Fe(gas)} & \rightarrow \text{Al}_3\text{Fe}_3(gas) \\
\text{2Al(gas)} + 4\text{Fe(gas)} & \rightarrow \text{Al}_2\text{Fe}_4(gas)
\end{align*}$$
\[
\text{Al(gas)} + 5\text{Fe(gas)} \rightarrow \text{AlFe}_{5}\text{(gas)} \quad (5)
\]
each of which, according to the calculations, can be thermodynamically resolved and belongs to the number of chemical processes occurring with the so-called enthalpy factor (Table 13).

Table 13. Calculated values of the parameters \(\Delta_r H^0\) (298 K) and \(\Delta_r S^0\) (298 K) for the Reactions (1)–(5).

| Parameter          | \(\text{Al}_2\text{Fe}\) | \(\text{Al}_4\text{Fe}_2\) | \(\text{Al}_3\text{Fe}_3\) | \(\text{Al}_2\text{Fe}_4\) | \(\text{AlFe}_5\) |
|--------------------|---------------------------|-----------------------------|-----------------------------|-----------------------------|---------------------|
| \(\Delta_r H^0\) (298 K), kJ   | -1257.1                   | -1360.5                     | -1429.4                     | -1434.2                     | -1484.1             |
| \(\Delta_r S^0\) (298 K), J/K | -545.1                    | -572.7                      | -565.5                      | -570.1                      | -590.2              |

As can be seen from these values, for each of the reactions (1)–(5) in the gas phase, the values of both of these parameters are negative for any of the considered metal clusters; hence, all these reactions are thermodynamically allowed at relatively low temperatures and forbidden at high ones. It is noteworthy that all of these reactions are exothermic. Moreover, the thermal effect of each of them is quite significant. In the simplest version using the Gibbs–Helmholtz Equation (6) for the isobaric process,

\[
\Delta_r G^0 (T) = \Delta_r H^0 (298\text{ K}) - T \Delta_r S^0 (298\text{ K})
\]

where \(\Delta_r H^0\) (298 K) and \(\Delta_r S^0\) (298 K) are the changes in the enthalpy and entropy as a result of a chemical process in standard conditions, respectively; \(T\) is the process temperature in K; and \(\Delta_r G^0(T)\) is the dependence of the Gibbs free energy on the temperature \(T\). It is easy to find the temperature at which one or another of the Reactions (1)–(5) will not take place due to thermodynamic prohibition. Actually, this parameter is the temperature indicating the beginning of thermal destruction of a metal cluster \(T_{td}, \text{K}\) or \(t_{td}, \text{°C}\) in the gas phase; the values of this parameter for each of the most energy-stable metal clusters, namely, \(\text{Al}_5\text{Fe}\) (2-I), \(\text{Al}_4\text{Fe}_2\) (1-I), \(\text{Al}_3\text{Fe}_3\) (6-I), \(\text{Al}_2\text{Fe}_4\) (5-I), and \(\text{AlFe}_5\) (4-I), are displayed in Table 14.

Table 14. Temperatures of the beginning of thermal destruction \(T_{td}, \text{K}\) and \(t_{td}, \text{°C}\) of the most energetically-stable metal clusters of \(\text{Al}_n\text{Fe}_m\) \((n + m = 6)\).

| Metal Cluster | \(T_{td}, \text{K}\) | \(t_{td}, \text{°C}\) |
|---------------|---------------------|-----------------------|
| \(\text{Al}_2\text{Fe}\) (2-I) | 2306.18 | 2008.02 |
| \(\text{Al}_4\text{Fe}_2\) (1-I) | 2375.58 | 2077.42 |
| \(\text{Al}_3\text{Fe}_3\) (6-I) | 2527.67 | 2229.51 |
| \(\text{Al}_2\text{Fe}_4\) (5-I) | 2515.57 | 2217.41 |
| \(\text{AlFe}_5\) (4-I) | 2514.57 | 2216.41 |

As can be seen from the given table, this temperature is very high and for each of the most stable \(\text{Al}_n\text{Fe}_m\) metal clusters \((n + m = 6)\), it exceeds 2000 °C, so at least in this aggregation state, all of them are very resistant to thermal effects. These data also demonstrate that the most stable in this respect is \(\text{Al}_3\text{Fe}_3\) (6-I), and the least stable is \(\text{Al}_5\text{Fe}\) (2-I).

It seems appropriate to compare the results for the \(\text{Al}_3\text{Fe}_3\) and \(\text{Al}_2\text{Fe}_4\) metal clusters presented in this article and the results for the same metal clusters obtained using a simpler version of the DFT method, namely, DFT OPBE/TZVP (they are presented in a recently published review [33], as well as in earlier original publications [30–32]). According to these data, the \(\text{Al}_3\text{Fe}_3\) metal cluster exists in 20 different structural isomers, whereas the \(\text{Al}_2\text{Fe}_4\) metal cluster only exists in nine different structural isomers. These numbers of structural isomers are in sharp contrast to the analogous values for the data of metal clusters obtained using the DFT OPBE/QZVP method (40 and 53, respectively). It should be noted that for
the Al$_4$Fe$_2$ metal cluster, DFT OPBE/TZVP gives practically the same number of possible structural isomers (38) as for the DFT OPBE/QZVP method (39); a similar coincidence can be observed for the Al$_3$Fe metal cluster (any data on these two metal clusters, however, were not published by the authors of this article or any other researchers). Therefore, the difference in the number of possible structural isomers obtained by these two methods increases with an increase in the number of 3D-element atoms (in our case, Fe) in the structural unit of the Al$_m$Fe$_n$ metal cluster. This difference is quite understandable because the DFT OPBE/QZVP method better takes into account the specifics of the electronic structure and wave functions of 3D-element atoms than DFT OPBE/TZVP, and the larger the fraction of Fe atoms in the structure, the more significant the difference in the data should be. It should be noted that, in this regard, the molecular structures of the most stable metal clusters Al$_3$Fe$_3$ and Al$_2$Fe$_4$, as well as the spin multiplicities of their ground states, obtained by the DFT OPBE/TZVP method, are similar to the analogous data obtained by the DFT OPBE/QZVP method and presented in this article.

4. Calculation Method

The quantum-chemical calculations of Al$_m$Fe$_n$ metal clusters were carried out using the density functional method (DFT) combining the standard extended split-valence QZVP basis [34,35] and the OPBE functional [36,37]. The data of works [34–41] gave us reason to assert that the given method allows the most accurate estimation of the ratio between energies of the high-spin state and low-spin state to be obtained and, at the same time, rather reliably predicts the key geometric parameters of molecular structures for various compounds of 3p- and 3d-elements. To build quantum chemical models of the molecular structures of the metal clusters under examination, GAUSSIAN09 software was used [42]. As in [16–22], the accordance of the found stationary points to the energy minima was confirmed by the calculation of the second derivatives with respect to the atomic coordinates. Additionally, all equilibrium structures corresponding to the minima at the potential energy surface only revealed real positive frequency values. Parameters of the molecular structures for spin multiplicities ($M_S$) higher than 1 were determined using the so-called unrestricted method (UOPBE), and those for $M_S = 1$, using the so-called restricted method (ROPBE). Along with this, the unrestricted method in conjunction with the GUESS = Mix option was used for the cases when $M_S$ was equal to 1. Moreover, NBO analysis of these metal clusters (Natural Population, Natural Electron Configurations, and Natural Atomic Orbital Occupancies) was carried out according to the procedure described in the works [43,44]. NBO 3.0 version built-in GAUSSIAN09 was used. The energetically most favorable structure has always been checked with the STABLE = OPT procedure; in all cases, the wave function corresponding to it was stable. The standard thermodynamic parameters of the formation of metal clusters under study, and namely $\Delta H^0$ (298 K), $S^f$ (298 K), and $\Delta G^0$ (298 K) for the metal clusters under examination, were calculated using the method described in [45].

5. Conclusions

As can be seen from the calculated data, each of the Al$_m$Fe$_n$ hexa-nuclear metal clusters ($n + m = 6$) under examination has a rather significant number of structural isomers, which differ very significantly in terms of their structural and geometric parameters and relative total energies; the total number of such structural isomers increases from Al$_3$Fe to Al$_2$Fe$_4$, while from Al$_3$Fe$_4$ to AlFe$_3$, it decreases. It is also noteworthy that in the case of the so-called “inverted” metal clusters [(Al$_3$Fe, AlFe$_3$) and (Al$_4$Fe$_2$, Al$_2$Fe$_4$)], the number of structural isomers in AlFe$_3$ and Al$_2$Fe$_4$ is noticeably higher than the number of structural isomers in Al$_3$Fe and Al$_4$Fe$_2$, respectively (although, given the uniformity of their composition in each of these two pairs, the same number of these same isomers would be expected). As a rule, structural isomers of the analyzed metal clusters are either completely devoid of symmetry elements, or only have one plane of symmetry; at the same time, it is remarkable that, in most of these compounds, Al–Al chemical bonds are absent. For the spin multiplicity of the most stable metal clusters’ ground state in the series Al$_3$Fe–Al$_4$Fe$_2$–Al$_3$Fe$_3$–Al$_2$Fe$_4$–
AlFe₅, the tendency to transition from the low-spin ground state in Al₅Fe and Al₄Fe₂ to the high-spin number of unpaired atoms in Al₃Fe₃ and Al₂Fe₄ is quite clearly expressed. Taking into account the presence of a greater number of unpaired electrons in the Fe atom in comparison with that in the Al atom, this seems to be quite natural. The most stable AlₙFeₘ metal clusters (n + m = 6) are characterized by a very high thermal stability, although none of them can be directly obtained from metallic iron and aluminum (since, for any of them, the values of the standard Gibbs energies of formation ΔfG₀ (298 K) > 0).

In conclusion, the (AlFe) metal clusters considered in the given article could be primarily used in the creation of new composite materials and alloys based on polynuclear nanoparticles in the future; it is quite possible that such alloys may have extraordinary magneto-chemical and physical-mechanical characteristics. Other possible areas of their application may be the doping of traditional alloys based on both non-ferrous and ferrous metals, metal complex catalysis, the creation of specific electrochemical systems, and semiconductor technology. It is also highly probable that they can be used as potential so-called quantum dots, the possibilities of technologies with the use of which are still far from exhausted.

**Supplementary Materials:** These data are available online at https://www.mdpi.com/1996-1944/14/3/597/s1. Figure S1: Molecular Structures of Hexanuclear Al₅Fe Metal Clusters calculated by DFT OPBE/QZVP method, Figure S2: Molecular Structures of Hexanuclear Al₄Fe₂ Metal Clusters calculated by DFT OPBE/QZVP method, Figure S3: Molecular Structures of Hexanuclear Al₃Fe₃ Metal Clusters calculated by DFT OPBE/QZVP method, Figure S4: Molecular Structures of Hexanuclear Al₂Fe₄ Metal Clusters calculated by DFT OPBE/QZVP method, Figure S5: Molecular Structures of Hexanuclear AlFe₅ Metal Clusters calculated by DFT OPBE/QZVP method.

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