Generation of pure, ionic entangled states via linear optics

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In this paper, we propose a novel scheme to generate two-ion maximally entangled states from either pure product states or mixed states using linear optics. Our new scheme is mainly based on the ionic interference. Because the proposed scheme can generate pure maximally entangled states from mixed states, we denote it as purification-like generation scheme. The scheme does not need a Bell state analyzer as the existing entanglement generation schemes do, it also avoids the difficulty of synchronizing the arrival time of the two scattered photons faced by the existing schemes, thus the proposed new entanglement generation scheme can be implemented more easily in practice.

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I. INTRODUCTION

Quantum superposition principle is a fundamental principle in quantum physics. When used in composite system, quantum superposition principle can induce an entirely new result, which is different from the classical physics, i.e. quantum entanglement \textsuperscript{\dagger}. After a long time debate on the completeness of quantum mechanics between Niels Bohr and Albert Einstein, it has been generally accepted that the entanglement between two systems exists. Entanglement state is the state that can not be expressed as the product state of the two systems \textsuperscript{\dagger}. In this sense, the quantum entanglement is used to disprove the local hidden variable theory \textsuperscript{\dagger}. Because of the non-locality feature of entanglement, entangled states have been widely used in quantum information processing, such as quantum cryptography \textsuperscript{1}, quantum computer \textsuperscript{1}, and quantum teleportation \textsuperscript{4}. All of the above applications are based on the entangled states, so the generation of entangled states plays a critical role in quantum information processing. Many theoretical and experimental schemes for the generation of entangled states have been proposed in Cavity QED \textsuperscript{6}, ion trap \textsuperscript{\dagger}, and NMR \textsuperscript{\dagger}. In photonic case, the polarization entangled photons have been generated in experiment by using Spontaneous Parametric-Down conversion \textsuperscript{\dagger}. For atomic case, the schemes for the generation of entangled atomic states have been proposed \textsuperscript{\dagger}. Shi-Biao Zheng and Guang-Can Guo have presented a realizable scheme for the generation of entangled atomic states, which is mainly based on the dispersive interaction between atoms and cavity modes. The obvious advantage of it is that the cavity is only virtually excited during the process and the requirement on the cavity quality is greatly loosened, which opens a promising future for quantum information processing \textsuperscript{\dagger}. This scheme has been realized in experiment by the S. Haroche group \textsuperscript{11}.

Alternatively, the entangled atomic states also can be generated via atomic interference \textsuperscript{12,13,14,15,16,17,18,14,20,21}. Most of the schemes work as follows: the two scattered(or leakage) photons from two spatially separated atoms(or cavities) will be mixed by a Bell State Analyzer(BSA) or Polarization Beam Splitter(PBS), and the two photons will be detected after the BSA or PBS. Because we can not distinguish from which atoms the two photons are scattered, the two atoms will be left in entangled states after the photon detection. The schemes of this type can entangle the spatially separated atoms, but there is still a serious difficulty. These schemes require the two photons reach the BSA or PBS simultaneously. Motivated by Xing-Xiang Zhou’s proposal on non-distortion quantum interrogation \textsuperscript{22}, we have proposed an entanglement purification scheme for arbitrary unknown ionic states via linear optics \textsuperscript{22}. In this paper, we will propose a novel entanglement generation scheme, which is free of the problem of simultaneity. In our scheme, the photon wave function of one incident circular polarized photon will be split into two parts, transmitted part and the reflected part, by an ordinary nonpolarizing 50–50 Beam Splitter(BS). Two multi-level ions will be pre-placed on the two possible paths of the photon. After interacting with the ions, the two parts of the photon wave function will be re-combined by the second ordinary nonpolarizing 50–50 BS. Through detecting the photon after the second BS, we can decide whether the entangled ionic pairs has been created or not. The main part of the setup can be regarded as a Mach-Zehnder interferometer(MZI). The BS is an ordinary one, and the relative phase problem inherent in the previous schemes has been avoided in our scheme. The photon detected in this scheme is a circular polarized one, and can be detected easier than the scattered one used in the previous schemes. It is not easy to make the two photons from two different ions interfere in the previous schemes. But in our scheme, the photon wave function has been split into two parts in MZI, and the coherent condition is satisfied naturally.

If nothing has been done on the entanglement generation setup before the creation process, the probability of success is relative low because of the low scattering rate of photon. So after discussion on the original setup, we
FIG. 1: The level configuration of the ions. The ions in $|m_+\rangle$ (or $|m_-\rangle$) can be excited into the excited state $|e\rangle$ by absorbing one $\sigma^+$ (or $\sigma^-$) polarized photon, and then it will decay to the stable ground state $|g\rangle$ and scatter a photon rapidly. Here, we assume that the decay process is so rapid that the probability of excited emission can be neglected.

will make some modification on the main setup to enhance the efficiency, i.e. the MZI will be surrounded by an optical cavity resonant with the ionic transition. This cavity will enhance the scattering rate and the successful probability of entanglement generation. If we lengthen the two arms of the MZI, the setup can entangle the two ions spatially separated, because the polarization of photon can be preserved in a polarization-preserving fiber over a long distance.

In the long distance case, two separate ions can be in a product state or in a mixed state evolved from the pure entangled state before distribution. If the two ions initially in mixed state are placed on the setup, an pure maximally entangled state can be extracted, and the efficiency of it can exceed the product state case provided the mixed state satisfies some condition. In this sense, the scheme for the mixed state case looks like a entanglement purification process, but it is not the case, because entanglement purification involves only classical communication and local operations [24]. So we only denote it as “purification-like” generation scheme.

II. GENERATION SCHEME FOR THE PRODUCT INITIAL STATES CASE

Next, we will discuss the entanglement generation process in details. Here, we will consider two identical ions, and they are all multi-level systems. The level configuration of the ions has been depicted in Fig. 1.

Where $|m_+\rangle$ and $|m_-\rangle$ are two degenerate metastable states which are used to store quantum information. $|e\rangle$ is a excited state of ions and $|g\rangle$ is the stable ground state. Ions in states $|m_+\rangle$ (or $|m_-\rangle$) can be excited into the $|e\rangle$ state by absorbing one $\sigma^+$ (or $\sigma^-$) circular polarized photon with unit efficiency, then it will decay to ground state $|g\rangle$ and scatter a photon. This process can be expressed as:

$$\hat{a}^\dagger_\pm |0\rangle |m_\pm\rangle \longrightarrow |S\rangle |g\rangle.$$  (1)

![Diagram](image)

FIG. 2: The setup for the generation of two-ion entangled states. $BS_1$ and $BS_2$ denote the two identical nonpolarizing 50-50 BSs. $U$ and $L$ denote the two ions on the upper and the lower arm of the interferometer. $D_u$ and $D_l$ are two polarization sensitive single photon detectors at the output upper and lower port.

where $|S\rangle$ denotes the scattered photons which we assume will not be reabsorbed by the ions and can be filtered away from the detectors. Although this process does not always occur despite the photon impinging on the ions, we still consider the ideal case to demonstrate the process, and then we will consider how to enhance the scattering rate by adding an optical cavity. The setup for generation of maximally entangled ionic states is depicted in Fig. 2.

One MZI with two BSs is the main part of the generation setup. One $\sigma^+$ polarized photon enters the MZI from the left lower port. The MZI is initially adjusted (without ions) such that the upper detector($D_u$) registers photons with certainty. In the case of the existence of two ions in arbitrary superposition states of $|m_+\rangle$ and $|m_-\rangle$ at each arm of the MZI the two ions can be placed on the two arms of the MZI by using the trapping techniques [25], the upper and the lower detectors($D_u$ and $D_l$) all have the probability of fire. If we select the superposition coefficients of the initial states of the two ions appropriately, we can get the maximally entangled ionic states conditioned on the fire at $D_l$. Suppose that the two ions ($U, L$) are initially prepared in the following states:

$$|\Psi\rangle_U = \alpha |m_+\rangle_U + \beta |m_-\rangle_U,$$  (2a)

$$|\Psi\rangle_L = a |m_+\rangle_L + b |m_-\rangle_L,$$  (2b)

where the coefficients $\alpha, \beta, a, b$ satisfy $|\alpha|^2 + |\beta|^2 = 1$ and $|a|^2 + |b|^2 = 1$. These states can be prepared by a laser pulse focused on the ion. The effect of the BS on the input photon can be expressed as:

$$\hat{a}^\dagger_\pm |0\rangle \underset{BS}{\longrightarrow} \frac{1}{\sqrt{2}} (\hat{a}^\dagger_{\pm, a} \pm i\hat{a}^\dagger_{\pm, l}) |0\rangle,$$  (3a)

$$\hat{a}^\dagger_{\pm, l} |0\rangle \underset{BS}{\longrightarrow} \frac{1}{\sqrt{2}} (\hat{a}^\dagger_{\pm, a} \pm i\hat{a}^\dagger_{\pm, l}) |0\rangle.$$  (3b)

That is to say, the BS takes no effect on the polarization of the input photon, and reflects the wave function
with a $\pm \frac{\pi}{2}$ phase shift corresponding to the propagation direction of the photon 22.

Next, we will trace the input photon and give the evolution of the total system. After one $\sigma^+$ polarized photon entering the left lower port of the MZI, its wave function will be split into two parts (the upper arm and the lower arm) by BS. Because the two ions are placed on the two arms, they will interact with the different parts of the wave function. Then the two parts of the wave function will be combined by BS2. The total evolution of the system can be expressed as follow:

$$
\hat{a}_{+1}^+ |0\rangle (\alpha |m_+\rangle_U + \beta |m_-\rangle_U) + \beta |m_+\rangle_L + b|m_-\rangle_L) \\
\rightarrow \frac{1}{\sqrt{2}} \alpha |S\rangle |g\rangle_U (\alpha |m_+\rangle_L + b|m_-\rangle_L) \\
+ \frac{i}{\sqrt{2}} |S\rangle |L\rangle \alpha |m_+\rangle_U + \beta |m_-\rangle_U) \\
+ \frac{i}{2} \hat{a}_{+-+}^+ |0\rangle (\beta a|m_+\rangle_U |m_+\rangle_L + \beta b|m_-\rangle_U |m_-\rangle_L) \\
+ \frac{i}{2} \hat{a}_{+-L}^+ |0\rangle (\beta a|m_+\rangle_U |m_+\rangle_L - \beta b|m_-\rangle_U |m_-\rangle_L). \quad (4)
$$

From the above result, we can get that the two ions will be left in three possible states corresponding to three measurement results on the two output ports respectively. If the $D_1$ fires, we get two-ion entangled states: $\beta a|m_+\rangle_U |m_+\rangle_L - \beta b|m_-\rangle_U |m_-\rangle_L$. If we modulate the coefficients of the initial states to make $\alpha, \beta, a, b$ satisfy $|\alpha| = |a|$ and $|\beta| = |b|$, the two ions can be left in maximally entangled state $|\Psi\rangle_{UL} = \frac{1}{\sqrt{2}} (|m_+\rangle_U |m_+\rangle_L - |m_-\rangle_U |m_-\rangle_L$ with probability $P = \frac{1}{4} |a|^2 (1 - |a|^2)$. From this analysis, we conclude that the two ions must be prepared in the same superposition state initially, then we can get the two-ion maximally entangled state. The successful probability is a function of the modulus of the initial states.

Here we only discussed the ideal case where the ion decays are coherent. In fact, the ion decays will be essentially incoherent and very few of the photons will be in the ideal direction in the free space, so a resonant cavity must be introduced for each ion to achieve directional emission of the photons from each ion 23. To simplify the figures, we do not draw these cavities in the generation setups.

In addition, we have only discussed the ideal case where we suppose a photon impinging on an ion always leads to the process described by Eq. (4). But in most cases the photon will not be scattered by the ions. If the ions are placed inside the MZI, it would mean that detector $D_n$ will most likely fire as before but without any entanglement between the ions being created. To enhance the scattering rate, an optical cavity will be added to the MZI. This cavity encloses the MZI, and it is different from the resonant cavities surrounding the ions, so we denote the cavity enclosing the MZI as enclosure cavity. The modified setup is depicted in Fig. 3.

Each side of the enclosure cavity has a very high reflection rate. One side($M_1$) is placed at the left lower port of the MZI, the other($M_2$) at the right upper port. Thus, if the photon has not been scattered by the ions, it will leave MZI at the right upper port. If it will be reflected into MZI by $M_2$, and analogously for the description of $M_1$. That is to say, if the photon is not scattered by the ions, it will vibrate in the enclosure cavity through MZI, which will enhance the scattering rate naturally. From Eq. (4), if the photon has been scattered by one ion, there is still possibility that one photon exit the MZI at the right upper port. Then the same process will repeat until the $D_1$ or $D_2$ register one photon, which indicates the entanglement generation process succeeds. During the process, if $D_1$ or $D_2$ do not record photons after an interval of the order of the lifetime of a photon, we must re-input one $\sigma^+$ polarized photon into the MZI through the cavity enclosure cavity side to repeat the entanglement generation process until the $D_1$ or $D_2$ register one photon. To clarify the evolution process, we suppose that one photon exiting the MZI at the right upper port indicates the process of the third term of the Eq. (4). This assumption is optimal because the vibration of photon in the cavity enclosure cavity enhance the chance of occurring the scattering process of Eq. (4).

Once the scattering process occurs the repetition of the third term of Eq. (4) will occur subsequently. After iteration of the above process, the probability for ions in scattered states is $\frac{1}{4} (|a|^2 + |a|^2) + \frac{1}{4} (|ab|^2 + |\beta a|^2)$, the probability for ions in product state $|m_+\rangle_U |m_-\rangle_L$ is $|\beta|^2$, and the probability for ions in entangled state $\beta a|m_+\rangle_U |m_+\rangle_L - \beta b|m_-\rangle_U |m_-\rangle_L$ is $\frac{1}{4} (|ab|^2 + |\beta a|^2)$. If the initial states of the two ions satisfy the condition $|\alpha| = |a|$ and $|\beta| = |b|$, the two ions will be left in maximally entangled state $|\Psi\rangle_{UL} = \frac{1}{\sqrt{2}} (|m_+\rangle_U |m_+\rangle_L - |m_-\rangle_U |m_-\rangle_L)$ with probability $\frac{1}{4} |a|^2 (1 - |a|^2)$. From the above result of iteration, we get that the added optical cavity enclosure cavity enhance the scattering rate and the efficiency of entanglement creation.
III. GENERATION SCHEME FOR THE MIXED INITIAL STATES CASE

Most of the previous preparation schemes prepare the entangled states at one location, and then the entangled particles will be distributed among different users for quantum communication purpose. But during the transmission of the particles, it will unavoidably couple with environments, and then the entanglement will degrade exponentially. So the entangled states after distribution are usually mixed ones, which need the purification process before use. We consider the generation from two ions that are initially in mixed state. For clarity, we will first give the evolution induced by the original setup in Fig. 2 then give some discussions on the process if we use the modified setup in Fig. 3.

Suppose that the initial mixed state is in the following form [26, 27]:

$$
\rho_{UL} = F|\Psi^+\rangle_{UL}\langle\Psi^+| + (1 - F)|\Phi^+\rangle_{UL}\langle\Phi^+|.
$$

where $|\Psi^+\rangle_{UL} = \frac{1}{\sqrt{2}}(|m_+\rangle_U|m_-\rangle_L + |m_-\rangle_U|m_+\rangle_L)$ and $|\Phi^+\rangle_{UL} = \frac{1}{\sqrt{2}}(|m_+\rangle_U|m_+\rangle_L + |m_-\rangle_U|m_-\rangle_L)$ are two Bell states of the two ions. To express the evolution clearly, we will consider the mixed state as the probabilistic mixture of pure two-ion entangled states, i.e. the state $|\Psi^+\rangle_{UL}$ with probability $F$ and the state $|\Phi^+\rangle_{UL}$ with probability $1 - F$. In the $|\Psi^+\rangle_{UL}$ case, if $D_u$ fires the two ions will be left in $\frac{1}{\sqrt{2}}(|m_+\rangle_U|m_-\rangle_L + |m_-\rangle_U|m_+\rangle_L)$ state. If $D_i$ fires the two ions will collapse into $\frac{1}{\sqrt{2}}(|m_-\rangle_U|m_+\rangle_L - |m_+\rangle_U|m_-\rangle_L)$ state. On the contrary, the $|\Phi^+\rangle_{UL}$ case only leads to fire at $D_u$ with the two ions in $|m_-\rangle_U|m_-\rangle_L$ state.

If we detect a photon at $D_u$, the two ions will be left in a mixed state whose fidelity (with respect to $|\Psi^+\rangle_{UL}$) is lower than the initial one. So we consider this result as garbage. If $D_i$ registers one photon, the two ions are left in a pure maximally entangled state $|\Psi^+\rangle_{UL} = \frac{1}{\sqrt{2}}(|m_-\rangle_U|m_+\rangle_L - |m_+\rangle_U|m_-\rangle_L)$ with probability $P^I = \frac{1}{4}$. So long as the initial fidelity satisfies $F > \frac{1}{2}$, the successful probability of the mixed states case will be larger than the pure product states case. This point can be understood easily. The pure states case starts from a product state, but the mixed states case from a partially entangled state. Naturally, the probability of the later case is larger than the former one.

Then if the enclosure cavity used in the pure product case state has been added in the MZI of the mixed state case, the scattering rate and the generation efficiency all can be enhanced. Through analysis, we get that the successful probability of getting pure maximally entangled states after iteration of the process is $\frac{1}{4}F$, which is larger than that of the one round case $\frac{1}{4}$ with an increased scattering rate.

Compared to the previous generation schemes, our scheme has the following advantages: (1) The relative phase problem has been avoided successfully in our scheme by using MZI, and the relative phase in our scheme is adjusted to zero and will not change in the process. The common phase of the state takes no effect on the entanglement of the generated entangled states. (2) The photon we want to detect is the input circular polarized photon, which makes it easier to be registered than the scattered ones, because the input photon has a better directionality than the scattered one. (3) In the previous schemes, the BSA is a necessity, but in our scheme, only two ordinary BS are needed. So the current scheme is simpler than the previous ones. (4) The simultaneity of the two scattered photons is a main difficulty of the preceding schemes. But in our scheme, the simultaneity will be satisfied naturally because of the MZI.

IV. DISCUSSION

After discussion on the generation scheme itself, we will consider the feasibility of the current scheme. Singly positively charged alkaline ions, which have only one electron outside a closed shell, are commonly used in the quantum information experiments using trapped ions [31, 37]. Here we discuss a possible implementation of our generation scheme using $^{40}\text{Ca}^+$ as example. The relevant levels of $^{40}\text{Ca}^+$ has been depicted in Fig. 4 [21].

$D_{5/2}$ and $D_{3/2}$ are two metastable levels of $^{40}\text{Ca}^+$ with lifetimes of the order of $1s$. $s_1$ and $s_2$ are two sublevels of $D_{3/2}$ with $m = -5/2$ and $m = -1/2$, and this two sublevels are coupled to $|e\rangle$ by $\sigma_-$ and $\sigma_+$ light at $854nm$. Here $e, S_1, S_2, S_{1/2}$ correspond to $e, m_-, m_+, g$ in Fig. 4 respectively. i.e. we use the $S_{1/2}$ as stable ground state, $S_1, S_2$ as two degenerate metastable state and $P_{3/2}$ as excited state. Arbitrary superposition state of this two degenerate metastable states can be prepared by applying a laser pulse of appropriate length, and this process can be realized in a few microsecond. The $^{40}\text{Ca}^+$ in state $S_1$ or $S_2$ can be excited into the excited state $P_{3/2}$ by applying one $\sigma_-$ or $\sigma_+$ light at $854nm$. Then decay from $|e\rangle$ to $S_1, S_2$, to $D_{3/2}$ and to $S_{1/2}$ are all possible. But the branching ratio for $P_{3/2} \rightarrow D_{5/2}(854nm)$ versus $P_{3/2} \rightarrow S_{1/2}(393nm)$ can be estimated as 1:30, giving

![Diagram](https://example.com/diagram.png)
$0.5 \times 10^7/s$ for the transition probability $^{21, 31}$. So in most case, the $^{40}$Ca$^+$ in the excited state will decay into the stable ground state $S_{1/2}$. The detection of the internal states of $^{40}$Ca$^+$ can be realized by using a cycling transition between $S_{1/2}$ and $P_{1/2}(397nm)$ $^{30, 31}$.

In section II and section III, we have discussed the effects of the cavities enclosing the MZI (enclosure cavities) on the generation scheme, and next we will discuss the effects of the resonant cavities surrounding the ions on the generation scheme. To achieve directional emission of the photons from the ions, we introduced an optical resonant cavity to surround each ion. We will introduce a cavity on the $S_{1/2}$ to $P_{3/2}$ transition to enhance the emission of the photons from atom transition $P_{3/2}$ to $S_{1/2}$. Then the following two items will affect the emission efficiency of the photon from the ions: (1) The coupling between cavity mode and the $P_{3/2} \rightarrow S_{1/2}(393nm)$ transition; (2) Decaying from $P_{3/2}$ to $D_{5/2}$ (3) Cavity decay. The probability $p_{\text{cav}}$ for a photon to be emitted into the cavity mode after excitation to $|e\rangle$ can be expressed as $p_{\text{cav}} = \frac{4\pi\gamma^2(L\lambda/4)^2}{(\gamma+1)(\gamma+4\Omega)^2}$, where $\gamma = 4\pi c/L\lambda V$ is the decay rate of the cavity, $L\lambda V$ its finesse, $L$ its length, $\Omega = \sqrt{\frac{\hbar c}{2\gamma a}}$ is the coupling constant between the transition and the cavity mode, $D$ the dipole element, $\lambda$ the wavelength of the transition, $V$ the mode volume (which can be made as small as $L^2\lambda/4$ for a confocal cavity with waist $\sqrt{L\lambda/\pi}$), and $\Gamma$ is the non-cavity related loss rate $^{21, 33}$. From the discussion of Ref. $^{21}$, the photon package is about 100ns, and such a long coherence time makes it easy to achieve good overlap for the wave function of the photon on the beam splitter.

When calculating the total efficiency of the generation scheme, we must consider the following items:

- The emission efficiency of photon: $p_{\text{cav}}$, which has included the cavity decay: To maximize the $p_{\text{cav}}$, we have chosen $F_{\text{cav}} = 19000$, $L = 3nm$. Then $\gamma = 9.9 \times 10^6/s$, $p_{\text{cav}} = 0.01^{21}$;
- The efficiency of the photon detectors is expressed as $\eta$. Here we let the detection efficiency $\eta = 0.7$, which is a level that can be reached within the current technology.
- Coupling the photon out of the cavities will introduce another error $\xi$, which can be modulated to be close to unit.

In addition, because the two ions have been placed on the MZI symmetrically, the different transition times for the ions and the consequent pulse broadening will affect the efficiency of the scheme slightly. To complete the generation scheme, we suppose that the state maker has held two ionic ensembles. After considering the above factors, the total success probability can be expressed as follow (considering the modified schemes as example):

- $P = \frac{F}{4} \times p_{\text{cav}} \times \eta$ for mixed state, that is to say, if we input photon with the rate of 5000/s, we can get four pairs of pure maximally entangled $^{40}$Ca$^+$ ions per second for $F = 0.7$.
- $P = \frac{2a^2(1-a^2)}{4} \times p_{\text{cav}} \times \eta$ for product initial states, that is to say, if we input photon with the rate of 5000/s, we can get five pairs of pure maximally entangled $^{40}$Ca$^+$ ions per second for $a^2 = 0.7$.

From the experimental point of view, because the efficiency of the current scheme would be greatly enhanced if there were enough photons in the resonant system to induce stimulated emission from the ions, we will input enough photons into the MZI simultaneously. That is to say, the current scheme becomes more realizable.

V. CONCLUSION

In conclusion, we have proposed an entanglement generation scheme, which can entangle two ions by using MZI plus an optical enclosure cavity. Pure maximally entangled states can be generated from either product states or mixed states. Single photon detection can give us the signal indicating whether the generation process succeeds or not. The added optical enclosure cavity enhances the generation efficiency. Because the simultaneity problem inherent in the preceding schemes does not appear in our scheme, ours is more realizable than the previous ones.

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