Quantum Gravity in Krein Space Quantization

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Abstract

Indefinite metric field quantization or "Krein" space quantization, is considered in this paper. Application of a new version of this quantization to the linear gravity in de Sitter space-time removes the persistent problem of field quantization, i.e. the non-renormalizability of quantum linear gravity. Pursuing this path the non uniqueness of vacuum expectation value of the product of field operator in curved space-time disappears as well. Contrary to previous methods in which the vacuum states in curved space arbitrarily altered the physical quantity, in this method the vacuum expectation value of the product of field operators can be defined properly and uniquely. In this method the Green function remains finite in the ultra-violet and infra-red limits and these result in automatic regularization of the quantum field theory.

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1 Introduction

One of the challenging goals in theoretical physics is achievement of construction of a proper covariant quantization of the gravitational field. The gravitational red-shift leads us to the conclusion that gravity could be explained through geometry as opposed to "force of gravity" [1]. Two dominant views in geometry have been utilized for this purpose. In the first perspective geometry is completely defined by the Riemannian curvature tensor $R_{\mu\nu\rho\sigma}$ or equivalently by the metric tensor $g_{\mu\nu}$ (due to the metric compatibility: $\nabla_\mu g_{\rho\nu} = 0$, and torsion free condition $\Gamma^\rho_{\mu\nu} = \Gamma^\rho_{\nu\mu}$). In this perspective the geometry (or gravitational field) can be explained by the Einstein field equation, confirmed relatively well by the experimental tests in the solar system scale. In this case the gravitational field is described by an irreducible rank-2 symmetric tensor field $g_{\mu\nu}$. This description for gravitational field is sustained in the present paper.

The other perspective for geometry discards the metric compatibility and torsion free condition [2, 3]. In this schema, geometry is defined by the connection coefficients and metric

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tensors \((\Gamma^\rho_{\mu\nu}, g_{\mu\nu})\)-or equivalently by physical quantities of Riemann curvature tensor, torsion tensor and metric tensor \((R^\rho_{\mu\nu\rho\sigma}, T^\rho_{\mu\nu}, g_{\mu\nu})\). In this view the gravitational field can not be described by merely an irreducible rank-2 symmetric tensor field \(g_{\mu\nu}\). Instead the physical triplet \(R^\rho_{\mu\nu\rho\sigma}, T^\rho_{\mu\nu}\) and \(g_{\mu\nu}\) \((RTg)\) determines the behaviour of the gravitational field. A special and simple case of this view is Weyl geometry [4, 5], in which the linear approximation of the gravitational field is described by a rank-3 mix-symmetry tensor field [6, 7, 8].

The quantum gravitation field theory is a combination of theory of gravitation and quantum theory, necessitated, by phenomena which could only be explained in both realm simultaneously. In the quantum dimension of the theory, it should be noted that natural phenomena such as radioactive decay, tunnelling processes, pair production and etc... are described by the probability amplitudes of physical phenomena. This probability amplitude could be calculated through various quantization methods such as canonical, covariant methods and etc.... This process however is problematic where the infra-red and ultraviolet divergences appear. These divergences not only may violate the principle of covariance and the gauge symmetry, but prevent the calculation of expectation value of physical quantities. Exertion of certain additional methods such as ”re-normalization” have been able to resolve a part of the above anomalies. These methods however have two-fold problems: (1) they are not a part of the fundamental theory of quantum mechanics and (2) they are not able to solve many of problems of quantum field theories such as quantum gravity.

Distortion of the concept of time and the principle of causality are the first obstacles in the process of quantization of gravitational field, both of which have been successfully removed in the background field method. Quantum linear gravity however is not re-normalizable in this method.

It should be noted that the problem of divergences are not inherent in gravitational field theory, but a defect inherited from quantum field theory. In other words even in the Minkowsky space-time the quantization of the field theory results in divergences which have to be removed arbitrarily, if possible, in order to preserve the compatibility with actual physical measurement. In the case of the general relativity, however, one can not eliminate the divergences, i.e. the theory is non-renormalizable. Clearly this defect is direct result of the quantization method used. We have shown that this anomaly in the QFT disappear in a new version of indefinite metric field quantization or ”Krein” space quantization.

This ”Krein” space quantization, for the first time, was applied to the minimally coupled scalar field in de Sitter space [9, 10, 11]. It was proved that a covariant quantization of the minimally coupled scalar field in de Sitter space demands addition of the negative frequency solutions of the field equation (or negative norm states) in order to eliminate infra-red and ultraviolet divergences, which maintain causality. In our method, unlike other indefinite metric quantizations which used in gauge QFT, the ”un-physical states” may have positive norms but their energies or frequencies are negative. An example is the spinor field in QFT that its negative energy states have positive norms [12]. Precisely for this reason that we are using quotation marks for Krein i.e. ”Krein” quantization.

Our choice of de Sitter space time is due to the recent cosmological observations. These observational data are strongly in favour of a positive acceleration of the present universe. In the first approximation, the background space-time that we live in, might be considered as a de Sitter space-time. In this paper properties of linear quantum gravity in de Sitter space has been studied and the necessity of the Krein space quantization for preserving the de Sitter invariance
is discussed. It should be noted that we maintain the covariance and leave the positivity in the same way that has been done for the minimally coupled scalar field in de Sitter space-time [11]. This theory was initially constructed in the Krein space instead of Hilbert space [13] and it has been proved that the use of an indefinite metric is an unavoidable feature if one wishes to preserve causality (locality) and covariance in quantum field theories. Elimination of positivity principle in the gauge QFT is due to the Lorentzian metric, rather than presence of the negative energy solutions of the field equation in our method. In our method the preserved principles of quantum field theory are: covariance, causality and existence of the vacuum [11]. One of the very interesting result of this construction is that the Green’s function at large distances does not diverge. As result the infra-red divergence previously presented in the theory disappears [11, 14]. Ultraviolet divergence disappears as well. This means that the quantum free scalar field in this method is automatically renormalized. The effect of “un-physical” states (negative energy states) appears in the physics as a natural renormalization procedure. For a technical consideration of the covariant quantization of the minimally coupled scalar field see [11].

This method was generalized to quantization of: 1) the massive free field in de Sitter space [11], 2) the interaction QFT in Minkowski space ($\lambda \phi^4$ theory) [15], 3) the calculation of the one-loop effective action for scalar field in a general curved space-time [16], 4) Casimir effect [17] and 5) QED in Minkowskian space time [12, 18], all of which resulted in a natural re-normalisation of the solution.

The linear quantum gravity in de Sitter space in the usual QFT was presented very well by Iliopoulos et al. [19]. Antoniadis, Iliopoulos and Tomaras [20] have shown that the pathological large-distance behaviour (infra-red divergence) of the graviton propagator on a de Sitter background does not manifest itself in the quadratic part of the effective action in the one-loop approximation. This means that the pathological behaviour of the graviton propagator may be gauge dependent and so should not appear in an effective way as a physical quantity. The linear gravity (the traceless rank-2 “massless” tensor field) on de Sitter space is indeed built up from copies of the minimally coupled scalar field [9]. It has been shown that one can construct a covariant quantization of the “massless” minimally coupled scalar field in de Sitter space-time, which is causal and free of any infra-red divergence [9, 10, 11]. The essential point of that paper is the unavoidable presence of the negative norm states. Although they do not propagate in the physical space, they play a renormalizing role. In the present paper, we shall show that this is also true for linear gravity (the traceless rank-2 “massless” tensor field). The linear quantum gravity in the new QFT was considered also in [21]. They calculated the propagator, which is covariant and free of any infra-red divergence.

The crucial point about the minimally coupled scalar field lies in the fact that there is no de Sitter invariant decomposition

$$\mathcal{H} = \mathcal{H}_+ \oplus \mathcal{H}_-,$$

where $\mathcal{H}_+$ and $\mathcal{H}_-$ are Hilbert and anti-Hilbert spaces respectively. For this reason our states contain negative frequency solutions and consequently the use of a Krein space (i.e. Hilbert $\oplus$ anti-Hilbert space) is necessitated. For the scalar massive field where such a decomposition exists as a de Sitter invariant, $\mathcal{H}_+$ as the usual physical state space ($\mathcal{H}_- = \mathcal{H}_+^*$) suffices [11]. Presence of negative states solutions plays the key role in the re-normalization procedure.

Although negative norm states appear in our method, by imposing the two conditions stated below, these states completely disappear and the theory becomes unitary:
i) The first condition is the "reality condition" in which the negative norm states do not appear in the external legs of the Feynmann diagram. This condition guarantees that the negative norm states only appear in the internal legs and in the disconnected parts of the diagram.

ii) The second condition is that the S matrix elements must be renormalized in the following form:

\[ S_{if} \equiv \text{probability amplitude} = \frac{< \text{physical states, in}|\text{physical states, out}>}{<0, \text{in}|0, \text{out}>}. \]

This condition eliminates the negative norm states in the disconnected parts.

We must emphasize the fact that this method can be used to calculate physical observables in scenarios where the effect of quantum gravity (in the linear approximation) cannot be ignored. This method of quantization may be used as an alternative way for solving the non-renormalizability of linear quantum gravity in the background field method and is instrumental in finding a new method of quantization, compatible with general relativity.

It is important to note that imposition of the above additional conditions on the quantum states and the probability amplitude, one can circumvent the problem of propagation of negative energy states and obtain physical results for measurable quantities. The physical states or the external legs of Feynman diagram are all positive while the negative energy states only appear in the internal line. Therefore in calculating the S-matrix elements or probability amplitudes for the physical states, negative energy states only appear in the internal line and in the disconnected part of the Feynman diagram. The negative norm states, which appear in the disconnected part of the S-matrix elements, can be eliminated by renormalizing the probability amplitudes (the second condition).

In previous methods the choice of vacuum states directly affected the expectation values of energy momentum tensor. In the present method, however, vacuum expectation values are independent of choice of modes or vacuum states. Although the expectation value of the energy momentum tensor for physical states are dependent on the choice of modes but the expectation value of vacuum states remains uniquely the same.

## 2 Krein space quantization

Let us briefly describe our quantization of the minimally coupled massless scalar field in de Sitter space. de Sitter space-time can be identified by a 4-dimensional hyperboloid embedded in 5-dimensional Minkowskian space-time:

\[ X_H = \{ x \in \mathbb{R}^5; x^2 = \eta_{\alpha\beta}x^\alpha x^\beta = -H^{-2} \}, \quad \alpha, \beta = 0, 1, 2, 3, 4, \]

where \( \eta_{\alpha\beta} = \text{diag}(+1, -1, -1, -1, -1) \) and \( H \) is Hubble parameter. The de Sitter metrics is

\[ ds^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta |_{x^2 = -H^{-2}} = g^{\text{dS}}_{\mu\nu}dX^\mu dX^\nu, \quad \mu = 0, 1, 2, 3, \]

where \( X^\mu \) are 4 space-time intrinsic coordinates of de Sitter hyperboloid. Any geometrical object in this space can be written either in terms of the four local coordinates \( X^\mu \) (intrinsic space notation) or the five global coordinates \( x^\alpha \) (ambient space notation).
The minimally coupled massless scalar is defined by
\[ \Box_H \varphi(x) = 0, \tag{2.3} \]
where \( \Box_H \) is the Laplace-Beltrami operator on de Sitter space. \( \varphi \) is a “massless” minimally coupled scalar field with homogeneous degree \( \sigma = 0 \) or \(-3\). It is given in the form of a dS plane wave \[22\]
\[ \varphi(x) = (H x, \xi)^\sigma, \]
where \( \xi \) lies on the positive null cone \( C^+ = \{ \xi \in \mathbb{R}^5; \xi^2 = 0, \xi^0 > 0 \} \). Due to the zero mode problem (or constant solution \( \sigma = 0 \)), one cannot construct a covariant quantum field in the usual manner \[23\]. This problem has been studied and solved by introducing a specific Krein QFT \[11\]. Here we review the construction of minimally coupled massless field in de Sitter space, which will be used in linear quantum gravity in section 4. We return to intrinsic coordinates in order to restore the covariance in this case. The choices of bounded global coordinates \((X^\mu, \mu = 0, 1, 2, 3)\) is well suited for compactified de Sitter space, namely \( S^3 \times S^1 \) by
\[
\begin{align*}
  x^0 &= H^{-1} \tan \rho \\
  x^1 &= (H \cos \rho)^{-1} (\sin \alpha \sin \theta \cos \phi), \\
  x^2 &= (H \cos \rho)^{-1} (\sin \alpha \sin \theta \sin \phi), \\
  x^3 &= (H \cos \rho)^{-1} (\sin \alpha \cos \theta), \\
  x^4 &= (H \cos \rho)^{-1} (\cos \alpha),
\end{align*}
\tag{2.4}
\]
where \(-\pi/2 < \rho < \pi/2, 0 \leq \alpha \leq \pi, 0 \leq \theta \leq \pi \) and \(0 \leq \phi \leq 2\pi\). The de Sitter metrics now can be written as
\[ ds^2 = g_{\mu\nu}dX^\mu dX^\nu = \frac{1}{H^2 \cos^2 \rho} (d\rho^2 - d\alpha^2 - \sin^2 \alpha d\theta^2 - \sin^2 \alpha \sin^2 \theta d\phi^2). \tag{2.5} \]
The solution to the field equation (2.3) reads in this coordinate system (for \( L \neq 0 \)) \[11\]:
\[ \varphi_{Llm}(x) = X_L(\rho)Y_{Llm}(\Omega) \equiv \phi_k, \tag{2.6} \]
with
\[ X_L(\rho) = \frac{H}{2} [2(L + 2)(L + 1)L]^{-\frac{1}{2}} \left( L e^{-i(L+2)\rho} + (L + 2) e^{-iL\rho} \right). \tag{2.7} \]
And for \( L = 0 \), we have
\[ \phi_{000} = \phi_g + \frac{1}{2} \phi_s, \quad \phi_g = \frac{H}{2\pi} \phi_s = -i \frac{H}{2\pi} [\rho + \frac{1}{2} \sin 2\rho]. \]
The \( y_{Llm}(\Omega) \)'s are the hyper-spherical harmonics. As proved by Allen \[23\], the covariant canonical quantization procedure with positive norm states fails in this case. The Allen’s result can be reformulated in the following way: the Hilbert space generated by the positive modes, including the zero mode \( (\phi_{000}) \), is not de Sitter invariant,
\[ \mathcal{H} = \{ \sum_{k \geq 0} \alpha_k \phi_k; \sum_{k \geq 0} |\alpha_k|^2 < \infty \}. \]
This means that it is not closed under the action of the de Sitter group. Nevertheless, one can obtain a fully covariant quantum field by adopting a new construction \[10, 11\]. In order to
obtain a fully covariant quantum field, we add all the conjugate modes to the previous ones. Consequently, we have to deal with an orthogonal sum of a positive and negative inner product space, which is closed under an indecomposable representation of the de Sitter group. The negative values of the inner product are precisely produced by the conjugate modes: $\langle \phi^*_k, \phi^*_k \rangle = -1, \ k \geq 0$. We do insist on the fact that the space of solution should contain the un-physical states with negative norm. Now, the decomposition of the field operator into positive and negative norm parts reads

$$\varphi(x) = \frac{1}{\sqrt{2}} [\varphi_p(x) + \varphi_n(x)], \quad (2.8)$$

where

$$\varphi_p(x) = \sum_{k \geq 0} a_k \phi_k(x) + H.C., \quad \varphi_n(x) = \sum_{k \geq 0} b_k \phi^*_k(x) + H.C.. \quad (2.9)$$

The positive mode $\varphi_p(x)$ is the scalar field as was used by Allen. The crucial departure from the standard QFT based on CCR lies in the following on commutation relations requirement:

$$[a_k, a^\dagger_{k'}] = \delta_{kk'}, \quad [b_k, b^\dagger_{k'}] = -\delta_{kk'}. \quad (2.10)$$

The ground state is defined as the Gupta-Bleuler vacuum:

$$a_k |GBV> = 0, \quad b_k |GBV> = 0.$$

A direct consequence of these formulas is the positivity of the energy i.e.

$$\langle \vec{k}|T_{00}|\vec{k}\rangle \geq 0,$$

for any physical state $|\vec{k}\rangle$ (those built from repeated action of the $a^\dagger_k$’s on the vacuum),

$$|\vec{k}\rangle = |k_1^{n_1} \ldots k_j^{n_j}\rangle = \frac{1}{\sqrt{n_1! \ldots n_j!}} (a^\dagger_{k_1})^{n_1} \ldots (a^\dagger_{k_j})^{n_j} |0\rangle.$$

This quantity vanishes if and only if $|\vec{k}\rangle = |0\rangle$. Therefore the “normal ordering” procedure for eliminating the ultraviolet divergence in the vacuum energy, which appears in the usual QFT is not needed [11]. Another consequence of this formula is a covariant two-point function, which is free of any infrared divergence [14]. The un-physical state $|\vec{k}\rangle$ built from repeated action of the $b^\dagger_k$’s on the vacuum.

Now we generalize Krein space quantization to the Minkowskian space. A classical scalar field $\varphi(x)$, which is defined in the 4-dimensional Minkowski space-time, satisfies the field equation

$$\Box \varphi(x) = 0 = (\eta^{\mu\nu} \partial_\mu \partial_\nu + m^2) \varphi(x), \quad \eta^{\mu\nu} = \text{diag}(1, -1, -1, -1). \quad (2.11)$$

Inner or Klein-Gordon product and related norm are defined by [24]

$$\langle \phi_1, \phi_2 \rangle = -i \int_{t=\text{cons.}} \phi_1(x) \overset{\leftrightarrow}{\partial_t} \phi^*_2(x) d^3x. \quad (2.12)$$

Two sets of solutions of (2.11) are given by:

$$\phi_p(k, x) = \frac{e^{i\vec{k} \cdot \vec{x} - iwt}}{\sqrt{(2\pi)^3 2w}} = \frac{e^{-ik \cdot x}}{\sqrt{(2\pi)^3 2w}}, \quad \phi_n(k, x) = \frac{e^{-i\vec{k} \cdot \vec{x} + iwt}}{\sqrt{(2\pi)^3 2w}} = \frac{e^{ik \cdot x}}{\sqrt{(2\pi)^3 2w}}, \quad (2.13)$$
where \( w(\vec{k}) = k^0 = (\vec{k}, \vec{k} + m^2)^{\frac{d}{2}} \geq 0 \). In Krein QFT the quantum field is defined as follows

\[
\varphi(x) = \frac{1}{\sqrt{2}} [\varphi_p(x) + \varphi_n(x)],
\]

where

\[
\varphi_p(x) = \int d^8 \vec{k} [a(\vec{k}) \phi_p(k, x) + a(\vec{k}) \phi_p^*(k, x)],
\]

\[
\varphi_n(x) = \int d^8 \vec{k} [b(\vec{k}) \phi_n(k, x) + b(\vec{k}) \phi_n^*(k, x)].
\]

\( a(\vec{k}) \) and \( b(\vec{k}) \) are two independent operators. The positive mode \( \phi_p \) is the scalar field as was used in the usual QFT. The time-ordered product propagator for this field operator is

\[
iG_T(x, x') = <0 | T \varphi(x) \varphi(x') | 0 > = \theta(t - t') W(x, x') + \theta(t' - t) W(x', x). \tag{2.15}
\]

In this case we obtain

\[
G_T(x, x') = \frac{1}{2} [G_F(x, x') + (G_F(x, x'))^*] = \Re G_F(x, x'). \tag{2.16}
\]

The Feynman Green function is \[24\]

\[
G_F(x, x') = \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-x')} \tilde{G}_F(p) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-x')} \tilde{G}_F(p) = -\frac{1}{8\pi} \delta(\sigma_0) + \frac{m^2}{8\pi} \theta(\sigma_0) J_1(\sqrt{2m^2\sigma_0}) - i N_1(\sqrt{2m^2\sigma_0}) \sqrt{2m^2\sigma_0}/\sqrt{-2m^2\sigma_0}, \tag{2.17}
\]

where \( \sigma_0 = \frac{1}{2}(x - x')^2 \). We have

\[
G_T(x, x') = \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-x')} PP \frac{1}{p^2 - m^2} = -\frac{1}{8\pi} \delta(\sigma_0) + \frac{m^2}{8\pi} \theta(\sigma_0) J_1(\sqrt{2m^2\sigma_0}) \sqrt{2m^2\sigma_0}, \quad x \neq x'. \tag{2.18}
\]

Contribution of the coincident point singularity \((x = x')\) merely appears in the imaginary part of \(G_F\)

\[
G_F(x, x) = -\frac{2i}{(4\pi)^2} \frac{m^2}{d-4} + G^\text{finit}_F(x, x),
\]

where \( d \) is space-time dimension and \( G^\text{finit}_F(x, x) \) is finite as \( d \to 4 \). By using the Fourier transformation of the Dirac delta function,

\[
-\frac{1}{8\pi} \delta(\sigma_0) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-x')} PP \frac{1}{p^2} = \frac{1}{8\pi} \frac{1}{\sigma_0} = -\int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-x')} \pi \delta(p^2),
\]

for the second part of the Green function, we obtain:

\[
\frac{m^2}{8\pi} \theta(\sigma_0) J_1(\sqrt{2m^2\sigma_0}) \sqrt{2m^2\sigma_0} = \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-x')} PP \frac{-m^2}{p^2(p^2 - m^2)}. \tag{2.19}
\]
In the previous paper we proved that in the one-loop approximation the Green function in Krein space quantization, which appears in the transition amplitude, is \[ \tilde{G}_T(p) \mid_{\text{one-loop}} = PP \frac{-m^2}{p^2(p^2 - m^2)}. \] (2.20)

This propagator was used by some authors in order to improve the ultra-violet behaviour in relativistic higher-derivative correction theory \[25, 26\]. Eq. (2.18) exhibits singularity on the light cone alone, \( x \neq x', \sigma_0 = 0 \). The quantum metric fluctuations remove the singularities of Green’s functions on the light cone. Therefore the quantum field theory in Krein space, including the quantum metric fluctuation \((g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu})\), removes all ultraviolet divergences of the theory \[27, 28\]:

\[
\langle G_T(x - x') \rangle = -\frac{1}{8\pi} \sqrt{\frac{\pi}{2\langle \sigma_1^2 \rangle}} \exp \left( -\frac{\sigma_0^2}{2\langle \sigma_1^2 \rangle} \right) + \frac{m^2}{8\pi} \theta(\sigma_0) J_1(\sqrt{2m^2\sigma_0}) \sqrt{2m^2\sigma_0},
\]

(2.21)

where \( 2\sigma = g_{\mu\nu}(x^\mu - x'^\mu)(x'^\nu - x'^\nu) \). In the case of \( 2\sigma_0 = \eta_{\mu\nu}(x^\mu - x'^\mu)(x'^\nu - x'^\nu) = 0 \), due to the quantum metric fluctuation \((h_{\mu\nu})\), \( \langle \sigma_1^2 \rangle \neq 0 \), and we have

\[
\langle G_T(0) \rangle = -\frac{1}{8\pi} \sqrt{\frac{\pi}{2\langle \sigma_1^2 \rangle}} + \frac{m^2}{8\pi} \frac{1}{2^4}.
\]

(2.22)

It should be noted that \( \langle \sigma_1^2 \rangle \) is related to the density of gravitons \[28\]. For the first part of the propagator (2.21) we have

\[-\frac{1}{8\pi} \sqrt{\frac{\pi}{2\langle \sigma_1^2 \rangle}} \exp \left( -\frac{(x - x')^4}{4\langle \sigma_1^2 \rangle} \right) = \int \frac{d^4p}{(2\pi)^4} e^{-ik.(x-x')} \tilde{G}_1(p).\]

Therefore we obtain

\[
< \tilde{G}_T(p) >= \tilde{G}_1(p) + PP \frac{-m^2}{p^2(p^2 - m^2)}.
\]

(2.23)

It should be noted that the tree order S-matrix elements do not change when it is applied to the Krein QFT since the un-physical state disappear in the external legs due to the reality conditions and

\[
\frac{1}{p^2 - m^2 + i\epsilon} = PP \frac{1}{p^2 - m^2}, \text{ when } p^2 \neq m^2.
\]

In the interaction case the S matrix elements can be written in terms of the time order product of the two free field operator (2.15) by applying the reduction formulas, Wick’s theorem, and time evolution operator \[29\]. Therefore in calculating the S-matrix elements for the physical states, negative energy states only appear in the internal line and in the disconnected part of the Feynman diagram. The negative norm states, which appear in the disconnected part can be eliminated by renormalizing the probability amplitudes (the second condition). Presence of negative norm states in the internal line plays the key role in the re-normalization procedure.
3 Scalar field in general curved space

In general curved space-time wave equation is

$$\Box \varphi + m^2 \varphi + \xi R \varphi = 0. \tag{3.1}$$

Here $R$ is the scalar curvature, and $\xi$ is a coupling constant. The inner product of a pair of their solutions is defined by:

$$\langle \phi_1, \phi_2 \rangle = i \int (\phi_2^* \partial_{\mu} \phi_1) d\Sigma^\mu, \tag{3.2}$$

where $d\Sigma^\mu = d\Sigma n^\mu$. $d\Sigma$ is the volume element in a given space-like hyper-surface, and $n^\mu$ is the time-like unit vector normal to this hyper-surface. Let $\{\phi_k\}$ be a set of solutions of positive norm states of Eq. (3.1),

$$\langle \phi_k, \phi_k' \rangle = \delta_{kk'}, \langle \phi_k^*, \phi_k'^* \rangle = -\delta_{kk'}, \langle \phi_k, \phi_k'^* \rangle = 0, \tag{3.3}$$

then $\{\phi_k^*\}$ will be a set of solutions of negative norm states. We have proved that the set $\{\phi_k\}$ is not a complete set of solutions for minimally coupled scalar field in de Sitter space [11]. Thus in general $\{\phi_k, \phi_k^*\}$ form a complete set of solutions of the wave equation in terms of which we may expand an arbitrary solution. The field operator $\varphi$ in Krein space quantization can be written as a sum of positive and negative field operators:

$$\varphi = \frac{1}{\sqrt{2}} (\varphi_p + \varphi_n) = \frac{1}{\sqrt{2}} \left[ \sum_k a_k \phi_k + a_k^\dagger \phi_k^* + \sum_k b_k \phi_k^* + b_k^\dagger \phi_k \right],$$

or

$$\varphi = \sum_k \left[ \frac{a_k + b_k^\dagger}{\sqrt{2}} \phi_k + \frac{a_k^\dagger + b_k}{\sqrt{2}} \phi_k^* \right],$$

where

$$[a_k, a_k^\dagger] = \delta_{kk'}, \quad [b_k, b_k^\dagger] = -\delta_{kk'},$$

and the other commutation relations are zero. The vacuum state $|0^\phi\rangle$ is defined such as

$$a_k |0^\phi\rangle = 0, \quad b_k |0^\phi\rangle = 0, \tag{3.4}$$

and the physical and un-physical states are:

$$a_k^\dagger |0^\phi\rangle = |k^\phi\rangle, \quad b_k^\dagger |0^\phi\rangle = |\bar{k}^\phi\rangle.$$

In curved space-time, there is, in general, no unique choice of the mode solution $\{\phi_k, \phi_k^*\}$, and hence no unique notion of the vacuum state exist. This means that the notion of “particle” becomes ambiguous and it is not a good concept in general curved space time. Let $\{F_j, F_j^*\}$ be another solutions of the field equation. We may choose these sets of solutions to be orthonormal

$$(F_j, F_{j'}^*) = \delta_{jj'}, \quad (F_j^*, F_{j'}^*) = -\delta_{jj'}, \quad (F_j, F_{j'}^*) = 0. \tag{3.5}$$

We may expand the $\phi$-modes in terms of the F-modes:

$$\phi_k = \sum_j (\alpha_{kj} F_j + \beta_{kj} F_j^*). \tag{3.6}$$
Inserting this expansion into the orthogonality relations, Eqs. (3.3) and (3.5), leads to the conditions
\[ \sum_j (\alpha_{kj}^\ast \alpha_{kj} - \beta_{kj} \beta_{kj}^\ast) = \delta_{kk'}, \]  
(3.7)
and
\[ \sum_j (\alpha_{kj}^\ast \alpha_{kj} - \beta_{kj} \beta_{kj}^\ast) = 0. \]  
(3.8)
The inverse expansion is
\[ F_j = \sum_k (\alpha_{kj} \phi_k - \beta_{kj} \phi_k^\ast). \]  
(3.9)
The field operator \( \varphi \) in Krein space quantization may be expanded in terms of either of the two sets: \( \{ \phi_k, \phi_k^\ast \} \) or \( \{ F_j, F_j^\ast \} \):
\[ \varphi = \sum_k \left[ \frac{a_k + b_k^\dagger}{\sqrt{2}} \phi_k + \frac{a_k^\dagger + b_k}{\sqrt{2}} \phi_k^\ast \right] = \sum_j \left[ \frac{c_j + d_j^\dagger}{\sqrt{2}} F_j + \frac{c_j^\dagger + d_j}{\sqrt{2}} F_j^\ast \right]. \]  
(3.10)
The \( a_k \) and \( a_k^\dagger \) are annihilation and creation operators of physical state, respectively, in the \( \phi \)-modes, whereas the \( c_j \) and \( c_j^\dagger \) are the corresponding operators for the F-modes. \( b_k, b_k^\dagger \) and \( d_j, d_j^\dagger \) are annihilation and creation operators of the un-physical states in their respective mode solutions. The \( \phi \)-vacuum state, which is defined by (3.4) describes the situation when no particle (and un-physical state) is present in this state. The F-vacuum state is defined by \( c_j |0^F\rangle = 0, \quad d_j |0^F\rangle = 0 \forall j \), and describes the situation where no particle (and un-physical state) is present. Noting that \( a_k = (\varphi, \phi_k) \) and \( c_j = (\varphi, F_j) \), we may expand the two sets of creation and annihilation operator in terms of one another as
\[ a_k = \sum_j (\alpha_{kj}^\ast c_j - \beta_{kj}^\ast c_j^\dagger), \quad b_k^\dagger = \sum_j (\alpha_{kj} d_j^\dagger - \beta_{kj} d_j), \]  
(3.11)
and
\[ c_j = \sum_k (\alpha_{kj} a_k + \beta_{kj}^\ast a_k^\dagger), \quad d_j^\dagger = \sum_k (\alpha_{kj} b_k^\dagger + \beta_{kj} b_k). \]  
(3.12)
This is a Bogoliubov transformation, and the \( \alpha_{kj} \) and \( \beta_{kj} \) are called the Bogoliubov coefficients.

In Krein space quantization it is possible to describe the physical phenomenon of particle creation by a time-dependent gravitational field similar to the usual quantization. The physical number operator \( N_j^c = c_j^\dagger c_j \) counts particles in the F-modes. The \( \phi \)-vacuum state contains the particles of F-mode
\[ \langle 0^\phi | N_j^c | 0^\phi \rangle = \langle 0^\phi | c_j^\dagger c_j | 0^\phi \rangle = \sum_k |\beta_{kj}|^2, \]  
(3.13)
if any of the \( \beta_{kj} \) coefficients are non-zero, i.e. if any mixing of positive and negative frequency solutions occurs, then particles are created by the gravitational field.

One of the most fundamental problems of QFT in curved space-time is that the vacuum expectation value of physical quantities (such as \( T_{\mu\nu} \)) is depend on the choice of vacuum states. This is direct consequence of non-uniqueness of the vacuum states. In the Krein space quantization in-spite of non-uniqueness of the vacuum states, which results in ambiguity of the concept.
of particle, the choice of vacuum states does not affect the vacuum expectation value of physical quantities and they are defined uniquely:

$$\langle 0^\phi | T_{\mu\nu} | 0^\phi \rangle = 0 = \langle 0^F | T_{\mu\nu} | 0^F \rangle.$$  \hspace{1cm} (3.14)

This is due to the fact that the two point function is uniquely defined in the Krein space quantization

$$\langle 0^\phi | \varphi(x) \varphi(y) | 0^\phi \rangle = \langle 0^F | \varphi(x) \varphi(y) | 0^F \rangle.$$  \hspace{1cm} (3.15)

The expectation value of $T_{\mu\nu}$ on physical states is depended to the choice of mode

$$\langle k^\phi | T_{\mu\nu} | k^\phi \rangle \neq \langle j^F | T_{\mu\nu} | j^F \rangle.$$  \hspace{1cm} (3.15)

It is noted again that Krein space quantization was proved to remove the divergences of QFT [11, 12, 17, 18] and as well the linear quantum gravity, which will be considered in the next section.

In general relativity the physical quantities are independent of the specific choice of the metric since the Riemannian manifold is invariant under the gauge transformation of the metric (see equation (4.5)). In QFT in curved space-time the expectation value of the physical quantities depend on the choice of the metric, since quantum states do depend on the mode of solutions, which in turn are dependent on the choice of metric. This problem is resolved in Krein space quantization for the physical quantities are independent of the mode of the solution.

## 4 Linear quantum gravity

The quantization of the massless spin-2 field in dS space, without infrared divergence, suggests an excellent modality for understanding of quantum gravity and quantum cosmology. The metric tensor is expanded as

$$g_{\mu\nu} = g_{\mu\nu}^{dS} + h_{\mu\nu},$$  \hspace{1cm} (4.1)

where $g_{\mu\nu}^{dS}$ is the gravitational de Sitter background and $h_{\mu\nu}$ is the fluctuation part. An explicit construction of the covariant Krein quantization of the rank-2 “massless” tensor field $(h_{\mu\nu})$ on de Sitter space (linear covariant quantum gravity on a dS background) is presented here. The wave equation for “massless” tensor fields $h_{\mu\nu}(X)$ propagating in de Sitter space can be derived through a variational principle from the action integral

$$S = -\frac{1}{16\pi G} \int (R - 2\Lambda) \sqrt{-g} d^4X,$$  \hspace{1cm} (4.2)

where $G$ is the Newtonian constant and $\Lambda$ is the cosmological constant. $\sqrt{-g} d^4X$ is the $O(1, 4)$-invariant measure on $X_H$. Application of the variational calculus leads to the field equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0.$$  \hspace{1cm} (4.3)

The wave equation, obtained in the linear approximation, is [30]:

$$-(\Box_H + 2H^2)h_{\mu\nu} - (\Box_H + H^2)g_{\mu\nu}h' - 2\nabla_{(\mu} \nabla^\rho h_{\nu)\rho}$$
\[ + g_{\mu \nu} \nabla^\lambda \nabla^\rho h_{\lambda \rho} + \nabla_\mu \nabla^\nu h' = 0, \]  
\( (4.4) \)

where \( h' = h_\mu^\mu \). \( \nabla^\nu \) is the covariant derivative on dS space. As usual, two indices inside parentheses mean that they are symmetrized, i.e. \( T_{(\mu \nu)} = \frac{1}{2}(T_{\mu \nu} + T_{\nu \mu}) \). The field equation \((4.4)\) is invariant under the following gauge transformation

\[ h_{\mu \nu} \rightarrow h_{\mu \nu}^{\varphi\rho} = h_{\mu \nu} + 2 \nabla_{(\mu} \Xi_{\nu)} \],  
\( (4.5) \)

where \( \Xi_{\nu} \) is an arbitrary vector field. One can choose a general family of gauge conditions

\[ \nabla^\mu h_{\mu \nu} = \zeta \nabla_\nu h', \]  
\( (4.6) \)

where \( \zeta \) is a constant. If the value of \( \zeta \) is set to be \( \frac{1}{2} \) the relation between unitary representation and the field equation not only becomes clearly apparent but also reduces to a very simple form.

The tensor field notation \( K_{\alpha \beta}(x) \) (ambient space notation) is adapted to establish this relation.

\[ K_{\alpha \beta}(x) = \partial x^\alpha \partial x^\beta \partial x^\mu h_{\mu \nu}(x(x)), \quad K_{\alpha \beta}(x) = \frac{\partial x^\nu}{\partial x^\alpha} \partial x^\mu \partial x^\beta h_{\mu \nu}(X(x)). \]  
\( (4.7) \)

The field \( K_{\alpha \beta}(x) \) defined on de Sitter space-time is a homogeneous function in the \( IR^5 \)-variables \( x^\alpha \):

\[ x^\alpha \partial x^\alpha K_{\beta \gamma}(x) = x \cdot \partial K_{\beta \gamma}(x) = \sigma K_{\beta \gamma}(x), \]  
\( (4.8) \)

where \( \sigma \) is an arbitrary degree of homogeneity. It also satisfies the conditions of transversality [31]

\[ x \cdot K(x) = 0, \quad i.e. \quad x^\alpha K_{\alpha \beta}(x) = 0, \quad \text{and} \quad x^\beta K_{\alpha \beta}(x) = 0. \]  
\( (4.9) \)

In order to obtain the wave equation for the tensor field \( K \), we must use the tangential (or transverse) derivative \( \bar{\partial} \) on de Sitter space

\[ \bar{\partial}_\alpha = \theta_{\alpha \beta} \bar{\partial}^\beta = \partial_\alpha + H^2 x_\alpha x \cdot \partial, \quad x \cdot \bar{\partial} = 0, \]  
\( (4.10) \)

where \( \theta_{\alpha \beta} = \eta_{\alpha \beta} + H^2 x_\alpha x_\beta \) is the transverse projector. To express tensor field in terms of the ambient space coordinates, transverse projection is defined [32, 33]

\[ (Trpr K)_{\alpha_1 \ldots \alpha_i} \equiv \theta_{\alpha_1 \beta_1} \cdots \theta_{\alpha_i \beta_i} K_{\beta_1 \ldots \beta_i}. \]

The transverse projection guarantees the transversality in each index. Therefore the covariant derivative of a tensor field, \( T_{\alpha_1 \ldots \alpha_n} \), in the ambient space notation becomes

\[ Trpr \bar{\partial}_{\beta} K_{\alpha_1 \ldots \alpha_n} \equiv \nabla_{\beta} T_{\alpha_1 \ldots \alpha_n} \equiv \bar{\partial}_{\beta} T_{\alpha_1 \ldots \alpha_n} - H^2 \sum_{i=1}^{n} x_{\alpha_i} T_{\alpha_1 \ldots \alpha_{i-1} \beta \alpha_{i+1} \ldots \alpha_n}, \]

so we have

\[ \nabla_\mu h_{\nu \rho} \rightarrow \theta_{\alpha \beta}^\mu \theta_{\alpha \gamma}^\nu \theta_{\beta \gamma}^\rho \partial_\alpha \partial_\beta K_{\gamma \gamma}. \]  
\( (4.11) \)

The field equation for \( K \) from \((4.4)\) is shown to be [30, 33, 34]

\[ B[(Q_2 + 6)K(x) + D_2 \partial_2 \cdot K] = 0, \]  
\( (4.12) \)
where \( BT = T - \frac{1}{2} \theta T' \) with \( T' := \eta^{\alpha\beta} T_{\alpha\beta} \). \( Q_2 \) is the Casimir operator of the de Sitter group and the subscript 2 in \( Q_2 \) shows that the carrier space encompasses second rank tensors [34]. The operator \( D_2 \) is the generalized gradient

\[
D_2 K = H^{-2} S(\bar{\partial} - H^2 x) K, \tag{4.13}
\]

where \( S \) is the symmetrizer operator. The generalized divergence is defined by \( \partial_2 \):

\[
\partial_2 \cdot K = \partial^T \cdot K - \frac{1}{2} H^2 D_1 K' = \partial \cdot K - H^2 x K' - \frac{1}{2} \bar{\partial} K', \tag{4.14}
\]

where \( \partial^T \cdot K = \partial \cdot K - H^2 x K' \) is the transverse divergence. One can invert the operator \( B \) and hence write the equation (4.12) in the form

\[
(Q_2 + 6) K(x) + D_2 \partial_2 K = 0. \tag{4.15}
\]

This equation is gauge invariant, i.e. \( K'^{\mu} = K + D_2 \Lambda_g \) is a solution of (4.15) for any vector field \( \Lambda_g \) as far as \( K \) is. The equation (4.15) can be derived from the Lagrangian density

\[
\mathcal{L} = -\frac{1}{2x^2} K..(Q_2 + 6) K + (\partial_2 \cdot K)^2. \tag{4.16}
\]

The gauge fixing condition (4.6) reads in our notations as

\[
\partial_2 \cdot K = (\zeta - \frac{1}{2}) \bar{\partial} K'. \tag{4.17}
\]

For the value of \( \zeta = 1/2 \) chosen by Christensen and Duff [35], we have

\[
\partial_2 \cdot K = 0. \tag{4.18}
\]

Similar to the flat space QED, gauge fixing is accomplished by adding to (4.16) a gauge fixing term:

\[
\mathcal{L} = -\frac{1}{2x^2} K..(Q_2 + 6) K + (\partial_2 \cdot K)^2 + \frac{1}{\alpha}(\partial_2 \cdot K)^2. \tag{4.19}
\]

The variation of \( \mathcal{L} \) then leads to the equation [36]

\[
(Q_2 + 6) K(x) + c D_2 \partial_2 \cdot K = 0. \tag{4.20}
\]

where \( c = \frac{1+\alpha}{\alpha} \) is a gauge fixing term. Actually, the simplest choice of \( c \) is not zero, as it will be shown later.

In the general gauge condition (4.17) the gauge fixing Lagrangian is

\[
\mathcal{L} = -\frac{1}{2x^2} K..(Q_2 + 6) K + \frac{1}{\alpha}(\partial_2 \cdot K)^2 + \frac{1}{\alpha}(\partial_2 \cdot K - (\zeta - \frac{1}{2}) \bar{\partial} K')^2. \tag{4.21}
\]

The field equation which derives from this Lagrangian becomes

\[
(Q_2 + 6) K(x) + D_2 \partial_2 \cdot K + \frac{1}{\alpha} [D_2 \partial_2 \cdot K + (\zeta - \frac{1}{2})^2 \eta(\bar{\partial})^2 K' - (\zeta - \frac{1}{2})^2(D_2 \bar{\partial} K' - S \bar{\partial} \partial_2 \cdot K)] = 0.
\]

Clearly this equation is more complicated than (4.20) obtained by the choice of \( \zeta = \frac{1}{2} \). In the following we shall work with the choice \( \zeta = \frac{1}{2} \) only.
4.1 dS-field solution

A general solution of Equation (4.20) can be constructed by a scalar field and two vector fields. Let us introduce a tensor field $K$ in terms of a five-dimensional constant vector $Z = (Z_\alpha)$ and a scalar field $\phi_1$ and two vector fields $K$ and $K_g$ by putting

\[
K = \theta \phi_1 + S \tilde{Z}_1 K + D_2 K_g, \tag{4.22}
\]

where $\tilde{Z}_{1\alpha} = \theta_{\alpha\beta} Z_1^\beta$. Substituting $K$ into (4.20), using the commutation rules and following algebraic identities [36, 37, 38, 33]:

\[
Q_2 \theta \phi = \theta Q_0 \phi, \tag{4.23}
\]

\[
\partial_2 \theta \phi = -H^2 D_1 \phi, \tag{4.24}
\]

\[
Q_2 D_2 K_g = D_2 Q_1 K_g, \tag{4.25}
\]

\[
\partial_2 D_2 K_g = -(Q_1 + 6) K_g, \tag{4.26}
\]

\[
Q_2 S \tilde{Z} K = S \tilde{Z} (Q_1 - 4) K - 2H^2 D_2 x. Z K + 4 \theta Z K, \tag{4.27}
\]

\[
\partial_2 S \tilde{Z}_1 K = \tilde{Z}_1 \partial_1 K - H^2 D_1 Z.K - H^2 x Z.K + Z. (\tilde{\partial} + 5 H^2 x) K, \tag{4.28}
\]

we find that $K$ obeys the wave equation

\[
(Q_1 + 2) K + c D_1 \partial.K = 0, \quad x.K = 0,
\]

where $D_1 = H^{-2} \tilde{\partial}$. $Q_0$ and $Q_1$ are the Casimir operators of the de Sitter group and the subscript 0 or 1 reminds us that the carrier spaces do encompass scalar or vector fields respectively. If we impose the supplementary condition $\partial.K = 0$, we get

\[
(Q_1 + 2) K = 0, \quad x.K = 0 = \partial.K. \tag{4.29}
\]

The vector field $K$ as a consequences of their conditions could be transform as unitary irreducible representation of de Sitter group [39, 40]. The further following choice of condition

\[
\phi_1 = -\frac{2}{3} Z_1 . K, \tag{4.30}
\]

results that $K_g$ can be also determined in terms of $K$

\[
(Q_1 + 6) K_g = \frac{c}{2(c - 1)} H^2 D_1 \phi_1 + \frac{2 - 5c}{1-c} H^2 x. Z_1 K + \frac{c}{1-c} (H^2 x Z_1 . K - Z_1 . \tilde{\partial} K). \tag{4.31}
\]

Then the scalar field $\phi_1$ satisfies the following wave equation

\[
Q_0 \phi_1 = 0, \tag{4.32}
\]

where $\phi_1$ is a “massless” minimally coupled scalar field. If we chose $c = \frac{2}{5}$, then we get the simplest form for $K_g$.

\[
K_g = \frac{1}{9} \left( H^2 x Z_1 . K - Z_1 . \tilde{\partial} K + \frac{2}{3} H^2 D_1 Z_1 . K \right). \tag{4.33}
\]
In conclusion, if we know the vector field $K$, we also know the tensor field $\mathcal{K}$.

The vector field $K$ in de Sitter space was explicitly considered in previous papers [39, 40]. It can be written in terms of two scalar field $\phi_2$ and $\phi_3$:

$$K_\alpha = Z_{2\alpha} \phi_2 + D_{1\alpha} \phi_3. \quad (4.34)$$

Applying $K_\alpha$ to equation (4.29) result in the following equations

$$Q_0 \phi_2 = 0,$$

$$\phi_3 = -\frac{1}{2} [Z_2. \bar{\partial} \phi_2 + 2H^2 x.Z_2 \phi_2], \quad (4.35)$$

where $\phi_2$ is also a “massless” minimally coupled scalar field. Therefore we can construct the tensor field $\mathcal{K}$ in terms of two “massless” minimally coupled scalar fields $\phi_1$ and $\phi_2$. But both fields are related by (4.30). Therefore one can write

$$\mathcal{K}_{\alpha\beta}(x) = \mathcal{D}_{\alpha\beta}(x, \partial, Z_1, Z_2) \phi, \quad \phi = \phi_2, \quad (4.36)$$

where

$$\mathcal{D}(x, \partial, Z_1, Z_2) = \left( -\frac{2}{3} \theta Z_1 + S Z_1 + \frac{1}{9} D_2 (H^2 x Z_1. - Z_1. \bar{\partial} + \frac{2}{3} H^2 D_1 Z_1. ) \right)$$

$$\left( \bar{Z}_2 - \frac{1}{2} D_1 (Z_2. \bar{\partial} + 2H^2 x.Z_2) \right), \quad (4.37)$$

and $\phi$ is a “massless” minimally coupled scalar field, which was given by equation (2.6). The solution (4.36) can be written as

$$\mathcal{K}_{\alpha\beta}(x) = \mathcal{D}_{\alpha\beta}(x, \partial, Z_1, Z_2) \phi_{Llm}(x) = \mathcal{D}_{\alpha\beta}(x, \partial, Z_1, Z_2) X_L(\rho) y_{Llm}(\Omega). \quad (4.38)$$

$Z_1$ and $Z_2$ are two constant vectors. We choose them in such a way that in the limit $H = 0$, one obtains the polarization tensor in the Minkowskian space

$$\lim_{H \to 0} \mathcal{D}_{\alpha\beta}(x, \partial, Z_1, Z_2) X_L(\rho) y_{Llm}(\Omega) \equiv \epsilon_{\mu\nu}(k) \frac{e^{ik.x}}{\sqrt{k_0}}, \quad (4.39)$$

where $\epsilon_{\mu\nu}(k)$ is the polarization tensor in the Minkowski space-time [41]:

$$k^\mu \epsilon_{\mu\nu}(k) - \frac{1}{2} k_\nu \epsilon^\nu(k) = 0,$$

$$\epsilon_{\mu\nu}(k) = \epsilon_{\nu\mu}(k), \quad k^\nu k_\nu = 0. \quad (4.40)$$

Finally, we can write the solution under the form

$$\mathcal{K}_{\alpha\beta}(x) = \mathcal{D}^\lambda_{\alpha\beta}(x, \partial) \phi_{Llm}(\rho, \Omega) \equiv \mathcal{E}^\lambda_{\alpha\beta}(\rho, \Omega, Llm) \phi_{Llm}(\rho, \Omega), \quad (4.41)$$

where $\mathcal{E}$ is the generalized polarization tensor and the index $\lambda$ runs on all possible polarization states. The explicit form of the polarization tensor is actually not important here. Indeed, one can find the two-point function by just using the recurrence formula (4.22). In order to
determine the generalized polarization tensor $E$ we let the projection operator $D$ act on the scalar field (2.6) and taking the Minkowskian limit (4.39).

The solution (4.22) is traceless $K' = 0$. Let us now consider the pure trace solution (conformal sector)

$$K^\nu = \frac{1}{4} \theta \psi. \quad (4.42)$$

Implementing this to equation (4.20), we obtain

$$(Q_0 + 6) \psi + \frac{c}{2} Q_0 \psi = 0,$$

or

$$(Q_0 + \frac{12}{2 + c}) \psi = 0. \quad (4.43)$$

On the other hand, any scalar field corresponding to the discrete series representation of the dS group obeys the equation

$$(Q_0 + n(n + 3)) \psi = 0. \quad (4.44)$$

Hence we see that the value $c = \frac{2}{5}$ does not correspond to a unitary irreducible representation of the dS group. But there exists a nonunitary representation corresponding to that $c = \frac{2}{5}$ for the conformal sector:

$$(\Box_H - 5H^2) \psi = 0.$$

Difficulties arise when we wish to quantize these fields where the mass square has negative values (conformal sector with $c > -2$ and discrete series with $n > 0$). The two-point functions for these fields have a pathological large-distance behaviour. If we choose $c < -2$ this pathological behaviour for the conformal sector disappears but it is still present in the traceless part. In the following sections the advantage of Krein space quantization vividly shows itself were the pathological large-distance behaviour disappears in the traceless part. The conformal linear gravity in this notation was previously considered [33, 7, 42]. In the next section, we shall consider merely the traceless part since it bears the physical states.

### 4.2 Two-point function and Field operator

The quantum field theory of the “massive” spin-2 field (divergenceless and traceless) have been already constructed from the Wightman two-point function $\mathcal{W}_{a\beta \alpha' \beta'} [34]$:

$$\mathcal{W}^{\nu}_{a\beta \alpha' \beta'}(x, x') = \langle \Omega, K_{a\beta}(x)K_{\alpha' \beta'}(x')\Omega \rangle, \quad \alpha, \beta, \alpha', \beta' = 0, 1, ..., 4, \quad (4.45)$$

where $x, x' \in X_H$ and $| \Omega \rangle$ is the Fock vacuum state. We have found that this function can be written under the form

$$\mathcal{W}^{\nu}_{a\beta \alpha' \beta'}(x, x') = D_{a\beta \alpha' \beta'}(x, x') \mathcal{W}^{\nu}(x, x'), \quad (4.46)$$

where $\mathcal{W}^{\nu}(x, x')$ is the Wightman two-point function for the massive scalar field and $D_{a\beta \alpha' \beta'}(x, x')$ is the projection tensor. Of course, we crudely could replace $\nu$ (principal-series parameter) by $\pm \frac{3}{2} l$ (discrete-series parameter) in order to get the “massless” tensor field associated to linear quantum gravity in dS space. However this procedure leads to appearance of two types of
singularity in the definition of the two-point function \( W_{\alpha\beta\gamma\delta}(x, x') \). The first one appears in the projection tensor \( D_{\alpha\beta\gamma}\delta(x, x') \) and it disappears if one fixes the gauge \( c = \frac{2}{3} \). The other one appears in the scalar two-point function \( W^{\pm \frac{3}{2}}(x, x') \) which corresponds to the minimally coupled scalar field and it disappears if one uses the Krein space quantization.

Let us briefly recall the required conditions for the “massless” bi-tensor two-point function \( W \), which is defined by

\[
W_{\alpha\beta\gamma\delta}(x, x') = \langle GBV|K_{\alpha\beta}(x)K_{\gamma\delta}(x')|GBV \rangle,
\]

where \( |GBV \rangle \) is the Gupta-Bleuler vacuum \([11]\). These functions entirely encode the theory of the generalized free fields on dS space-time \( X_H \). They have to satisfy the following requirements:

a) **Indefinite sesquilinear form**

for any test function \( f_{\alpha\beta} \in \mathcal{D}(X_H) \), we have an indefinite sesquilinear form that is defined by

\[
\int_{X_H \times X_H} f^{*\alpha\beta}(x) W_{\alpha\beta\gamma\delta}(x, x') f^{\alpha\beta\gamma\delta}(x') d\sigma(x)d\sigma(x'),
\]

where \( f^* \) is the complex conjugate of \( f \) and \( d\sigma(x) \) denotes the dS-invariant measure on \( X_H \). \( \mathcal{D}(X_H) \) is the space of rank-2 tensor functions \( C^\infty \) with compact support in \( X_H \).

b) **Locality**

for every space-like separated pair \( (x, x') \), i.e. \( x \cdot x' > -H^{-2} \),

\[
W_{\alpha\beta\gamma\delta}(x, x') = W_{\alpha\beta\gamma\delta}(x', x).
\]

c) **Covariance**

\[
(g^{-1})^\gamma_{\alpha}(g^{-1})^\delta_{\beta} W_{\gamma\delta\gamma'\delta'}(g x, g x') g^{\gamma\delta'}_{\alpha\beta} = W_{\alpha\beta\gamma\delta}(x, x'),
\]

for all \( g \in SO(0, 1, 4) \).

d) **Index symmetrizer**

\[
W_{\alpha\beta\gamma\delta}(x, x') = W_{\beta\alpha\gamma\delta}(x, x').
\]

e) **Transversality**

\[
x^\alpha W_{\alpha\beta\gamma\delta}(x, x') = 0 = x^\alpha W_{\gamma\delta\gamma'\delta'}(x, x').
\]

f) **Tracelessness**

\[
W_{\alpha\beta\gamma\delta}(x, x') = 0 = W_{\alpha\beta\gamma\delta}(x, x').
\]

The two-point function \( W_{\alpha\beta\gamma\delta}(x, x') \), which is a solution of the wave equation (4.20) with respect to \( x \) and \( x' \), can be found simply in terms of the scalar two-point function. Let us try the following formulation for a transverse two-point function:

\[
W_{\alpha\beta\gamma\delta}(x, x') = \theta_{\alpha\beta} \theta_{\gamma\delta} W_0(x, x') + SS'_{\alpha} \theta_{\alpha}' W_{1\beta}(x, x') + D_{2\alpha} \theta_{2\alpha}' W_{y\beta}(x, x'),
\]

note that \( D_2 D_2' = D_2' D_2 \) and \( W_1 \) and \( W_y \) are transverse bi-vector two-point functions which will be identified later. The calculation of \( W_{\alpha\beta\gamma\delta}(x, x') \) could be initiated from either \( x \) or \( x' \).
without any difference. This means either choices result in the same equation for $\mathcal{W}_{\alpha\beta\alpha'}(x, x')$. By imposing this function to obey equation (4.20), with respect to $x$, it is easy to show that:

\[
\begin{align*}
(Q_0 + 6)\theta'\mathcal{W}_0 &= -4S'\theta'.\mathcal{W}_1, \\
(Q_1 + 2)\mathcal{W}_1 &= 0, \\
(Q_1 + 6)D'_2\mathcal{W}_g &= \frac{c}{1-c}H^2D_1\theta'\mathcal{W}_0 + H^2S'\left[\frac{2-5c}{1-c}(x.\theta')\right. \\
&\quad+ \left.\frac{c}{1-c}\left(D_1\theta'. + x\theta'. - H^{-2}\theta'.\bar{\partial}\right)\right]\mathcal{W}_1,
\end{align*}
\]

where the condition $\partial.\mathcal{W}_1 = 0$, is used. By imposing the following condition

\[
\theta'\mathcal{W}_0(x, x') = -\frac{2}{3}S'\theta'.\mathcal{W}_1(x, x'),
\]

the bi-tensor two-point function (4.54) can be written explicitly in terms of bi-vector two-point function $\mathcal{W}_1$. The bi-vector two-point function $\mathcal{W}_1$ can be written in the following form [39, 40]

\[
\mathcal{W}_1 = \theta.\theta'\mathcal{W}_2 + D_1D'_1\mathcal{W}_3,
\]

where $\mathcal{W}_2$ and $\mathcal{W}_3$ are bi-scalar two-point functions. They satisfy the following equation

\[
D'_1\mathcal{W}_3 = -\frac{1}{2}\left[2H^2(x.\theta')\mathcal{W}_2 - \theta'.\bar{\partial}\mathcal{W}_2\right],
\]

\[
Q_0\mathcal{W}_2 = 0.
\]

This means that $\mathcal{W}_2$ is a massless minimally coupled bi-scalar two-point function. Putting $\mathcal{W}_2 = \mathcal{W}_{mc}$, we have

\[
\mathcal{W}_1(x, x') = \left(\theta.\theta' - \frac{1}{2}D_1[\theta'.\bar{\partial} + 2H^2x.\theta']\right)\mathcal{W}_{mc}(x, x').
\]

Using (4.56) in (4.55-III) we have

\[
\begin{align*}
(Q_1 + 6)D'_2\mathcal{W}_g &= \frac{cH^2}{1-c}H^2S'\left[\frac{2-5c}{c}(x.\theta')\mathcal{W}_1 + \frac{2}{3}D_1(\theta'.\mathcal{W}_1) + x(\theta'.\mathcal{W}_1) - H^{-2}\theta'.\bar{\partial}\mathcal{W}_1\right].
\end{align*}
\]

Using the following identities [39, 38]

\[
\begin{align*}
(Q_0 + 6)^{-1}(x.\theta')\mathcal{W}_1 &= \frac{1}{6}\left[\frac{1}{9}D_1(\theta'.\mathcal{W}_1) + (x.\theta')\mathcal{W}_1\right], \\
(Q_1 + 6)\theta'.\bar{\partial}\mathcal{W}_1 &= 6\theta'.\bar{\partial}\mathcal{W}_1 + 2H^2D_1(\theta'.\mathcal{W}_1), \\
(Q_1 + 6)D_1\theta'.\mathcal{W}_1 &= 6D_1(\theta'.\mathcal{W}_1), \\
(Q_1 + 6)x\theta'.\mathcal{W}_1 &= 6x(\theta'.\mathcal{W}_1),
\end{align*}
\]

one can obtain

\[
D'_2\mathcal{W}_g(x, x') = \frac{cH^2}{6(1-c)}S'\left[\frac{2 + c}{9c}D_1\theta'.\mathcal{W}_1 + \frac{2 - 5c}{c}x.\theta'\mathcal{W}_1 + x\theta'.\mathcal{W}_1 - H^{-2}\theta'.\bar{\partial}\mathcal{W}_1\right].
\]
Using equations (4.56–58) it turns out that the bi-tensor two-point function can be written in the following form \( \left(c = \frac{2}{5}\right) \) \[43\]

\[
W_{\alpha\beta\alpha'\beta'}(x, x') = \Delta_{\alpha\beta\alpha'\beta'}(x, x') W_{mc}(x, x'), \tag{4.59}
\]

where

\[
\Delta(x, x') = -\frac{2}{3} S' \theta \theta' \left( \theta \theta' - \frac{1}{2} D_1 [2H^2 x \theta' + \theta'.\bar{\partial}] \right) + SS' \theta \theta' \left( \theta \theta' - \frac{1}{2} D_1 [2H^2 x \theta' + \theta'.\bar{\partial}] \right) + \frac{H^2}{9} S'D_2 \left( \frac{2}{3} D_1 \theta' + x \theta' - H^{-2} \theta'.\bar{\partial} \right) \left( \theta \theta' - \frac{1}{2} D_1 [2H^2 x \theta' + \theta'.\bar{\partial}] \right), \tag{4.60}
\]

and \( W_{mc} \) is the two-point function for the minimally coupled scalar field. The biscalr two-point function \( W_{mc} \) in the “Gupta-Bleuler vacuum” state is \[14\]

\[
W_{mc}(x, x') = i\frac{H^2}{8\pi^2} \epsilon(x^0 - x'^0) \left[ \delta(1 - \mathcal{Z}(x, x')) + \vartheta(\mathcal{Z}(x, x') - 1) \right], \tag{4.61}
\]

where \( \vartheta \) is the Heaviside step function and

\[
\epsilon(x^0 - x'^0) = \begin{cases} 
1 & x^0 > x'^0, \\
0 & x^0 = x'^0, \\
-1 & x^0 < x'^0.
\end{cases} \tag{4.62}
\]

\( \mathcal{Z} \) is an invariant object under the isometry group \( O(1, 4) \). It is defined for \( x \) and \( x' \) on the dS hyperboloid by:

\[
\mathcal{Z} \equiv -x.x' = 1 + \frac{1}{2}(x - x')^2.
\]

Note that any function of \( \mathcal{Z} \) is dS-invariant, as well. For obtaining this two-point function we implement the Krein space quantization preserving the de Sitter-invariance thoroughly \[11, 14\].

### 4.3 Tensor Field operator

As it was discussed in the construction of two-point function in the section 4.2, we crudely could replace \( \nu \) (principal-series parameter) by \( \pm \frac{3}{2} i \) (discrete-series parameter) in order to get the “massless” tensor field associated to linear quantum gravity in dS space from massive case. However this procedure leads to appearance of singularity in the definition of the field operator [in \[34\] equation (3.12)]. This singularity is actually due to the divergencelessness condition needed to associate the tensor field with a specific UIR of the dS group. To solve this problem, the divergencelessness condition must be dropped and as a result we cannot associate this field with a UIR’s of the dS group. To maintain the covariance condition in field quantization we must use an indecomposable representation of the dS group. In this case however we do not have the positivity condition and there appear some unphysical states. In order to get rid of the unphysical parts we are led to impose new conditions.

The explicit knowledge of \( \mathcal{W} \) allows us to make the quantum field formalism work. The tensor fields \( \mathcal{K}(x) \) is expected to be an operator-valued distributions on \( X_H \) acting on a complex vector space \( V \) with an indefinite metric. In terms of complex vector space and field-operator, the properties of the two-points functions \( \mathcal{W} \) are equivalent to the following conditions:
1. **Existence of an indecomposable representation of the dS group**

2. **Existence of at least one “vacuum state”** $|GBV>$, cyclic for the polynomial algebra of field operators and invariant under the representation of dS group.

3. **Existence of a complex vector space $V$**
   with an indefinite sesquilinear form that can be described as the direct sum
   \[ V = V_0 \bigoplus_{n=1}^{\infty} S \mathcal{V}_1 \otimes^n. \] (4.63)
   $S$ denotes the symmetrization operator and $V_0 = \{ \lambda|0>, \lambda \in \mathbb{C} \}$. $\mathcal{V}_1$ is defined with the indefinite sesquilinear form
   \[ (\Psi_1, \Psi_2) = \int_{X_H \times X_H} \Psi_1^* \alpha\beta(x) \mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x') \Psi_2^{\alpha'\beta'}(x') d\sigma(x) d\sigma(x'), \] (4.64)
   where $\Psi_{\alpha\beta} \in \mathcal{D}(X_H)$.

4. **Covariance**
   of the field operators under the representation of dS group,
   \[ U(g)K_{\alpha\beta}(x)U(g^{-1}) = g^\gamma_\alpha g^\delta_\beta K_{\gamma\delta}(gx). \] (4.65)

5. **Locality**
   for every space-like separated pair $(x, x')$
   \[ [K_{\alpha\beta}(x), K_{\alpha'\beta'}(x')] = 0. \] (4.66)

6. **Transversality**
   \[ x \cdot K(x) = 0. \] (4.67)

7. **Index symmetrizer**
   \[ K_{\alpha\beta} = K_{\beta\alpha}. \] (4.68)

8. **Tracelessness**
   \[ K_{\alpha}^\alpha = 0. \] (4.69)

We now define the field operator, which result to the two-point function (4.59) and satisfy the above conditions. Using the Eqs. (4.41) and (2.8), the field operator in Krein space is defined as
\[ K_{\alpha\beta}(x) = \sum_{\Lambda Llm} a_{Llm}^\Lambda \mathcal{E}_{\alpha\beta}^\Lambda (\rho, \Omega, Llm) \phi_{Llm}(\rho, \Omega) + H.C. + \sum_{\Lambda Llm} b_{Llm}^\Lambda \left[ \mathcal{E}_{\alpha\beta}^\Lambda (\rho, \Omega, Llm) \phi_{Llm}(\rho, \Omega) \right]^* + H.C., \quad \forall \ 0 \leq l \leq L, \quad -l \leq m \leq l. \] (4.70)
The Gupta-Bleuler vacuum is defined by
\[ a_{\lambda Llm}^\dagger GBV >= 0, \quad b_{\lambda Llm}^\dagger GBV >= 0. \] (4.71)

The commutation relation between the annihilation and creation operators are:
\[ [a_{\lambda Llm}^\dagger, a_{\lambda' L'l'm'}^\dagger] = f(\lambda) \delta_{\lambda\lambda'} \delta_{LL'} \delta_{ll'} \delta_{mm'}, \]
\[ [b_{\lambda Llm}^\dagger, b_{\lambda' L'l'm'}^\dagger] = -f(\lambda) \delta_{\lambda\lambda'} \delta_{LL'} \delta_{ll'} \delta_{mm'}, \]
where \( f(\lambda) \) is a sign function (positive or negative) defined as:
\[ f(\lambda) \equiv \begin{cases} 1 \quad \text{for } \lambda = 1, \ldots, 6, \\ -1 \quad \text{for } \lambda = 7, \ldots, 10. \end{cases} \] (4.72)

By imposing the same conditions on polarization tensor \( \mathcal{E}^\lambda_{\alpha\beta}(\rho, \Omega, Llm) \) obtained from the minkowskian limit [34], the field operator (4.40) results in the two point function (4.59).

In this case we have 20 polarization states, amongst which two of them are physical (transverse traceless positive frequency mode). These modes can be defined by:
\[ (a_{\lambda Llm}^\dagger)|0 >= |1_{\lambda Llm}^{(a)} >= |\text{physical state }>, \quad \lambda = 1, 2. \]

There are two type of un-physical states. The first type is the usual mode, which appear due to the gauge invariance of the field equation. These un-physical positive frequency modes are:
\[ (a_{\lambda Llm}^\dagger)|0 >= |1_{\lambda Llm}^{(a)} >= |\text{unphysical state }>, \quad \lambda = 3, \ldots, 8, \]
four of which have negative norms. The others un-physical states are due to the Krein space quantization with negative frequency mode:
\[ (b_{\lambda Llm}^\dagger)|0 >= |1_{\lambda Llm}^{(b)} >= |\text{unphysical negative frequency state }>, \quad \lambda = 1, \ldots, 10. \]

Among these un-physical states exist the polarization modes with the positive norms. For scalar fields, the auxiliary negative frequency states have negative norms as well. For the tensor field the auxiliary states have the either negative or positive norms. Let us insist here that the Krein procedure allows us to avoid the pathological large-distance behaviour of the graviton propagator and preserve the de Sitter invariant.

5 Conclusion and outlook

The negative frequency solutions of the field equation are needed for proper quantization of the minimally coupled scalar field in de Sitter space. Contrary to the Minkowski space, the elimination of de Sitter negative norm in the minimally coupled states breaks the de Sitter invariance. In order to restore the de Sitter invariance, one needs to take into account the negative frequency solution or negative norm states \( \text{i.e.} \) the Krein space quantization. It is found that perusing this method the quantum field theory is automatically re-normalized.

Unitarity of the S matrix reflects the fundamental principle of probability conservation. Even though we have introduced the artificial device of indefinite metric quantization, the
physical quantities always refer to states with positive norms—a principle that is preserved through the time evolution of the theory. We conclude that with the conditions i) and ii) in page 4, the un-physical states do not contribute to the S matrix elements and the unitarity is preserved. Even though un-physical states have disappeared from the physical subspace, their impact in automatic regularization of probability amplitudes remains as an excellent tool for resolving the problem of divergences in QFT and consequently in quantum gravity. In this method the gravitational field can be quantized in the background field method.

In the conformal gravity (or in the gauging of the conformal group $SO(2, 4)$), the gravitational field can be described by a rank-3 mix-symmetry tensor field [6, 7, 8, 44]. In Krein space quantization this field can be quantized and in its linear approximation, graviton is indeed an elementary particle! This case will be considered explicitly in the forthcoming papers.

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