Charged lepton correction to tribimaximal lepton mixing and its implications to neutrino phenomenology

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Abstract

The recent results from Daya Bay and RENO reactor neutrino experiments have firmly established that the smallest reactor mixing angle $\theta_{13}$ is non-vanishing at the 5$\sigma$ level, with a relatively large value, i.e., $\theta_{13} \approx 9^\circ$. Using the fact that the neutrino mixing matrix can be represented as $V_{\text{PMNS}} = U_l^\dagger U_\nu P_\nu$, where $U_l$ and $U_\nu$ result from the diagonalization of the charged lepton and neutrino mass matrices and $P_\nu$ is a diagonal matrix containing the Majorana phases and assuming the tri-bimaximal form for $U_\nu$, we investigate the possibility of accounting for the large reactor mixing angle due to the corrections of the charged lepton mixing matrix. The form of $U_l$ is assumed to be that of CKM mixing matrix of the quark sector. We find that with this modification it is possible to accommodate the large observed reactor mixing angle $\theta_{13}$. We also study the implications of such corrections on the other phenomenological observables.

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I. INTRODUCTION

The results from various neutrino oscillation experiments firmly established the fact that neutrinos have a tiny but finite nonzero mass. Thus, analogous to the mixing in the down-quark sector, the three flavor eigenstates of neutrinos \( (\nu_e, \nu_\mu, \nu_\tau) \) are related to the corresponding mass eigenstates \( (\nu_1, \nu_2, \nu_3) \) by the unitary transformation

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
V_{e1} & V_{e2} & V_{e3} \\
V_{\mu1} & V_{\mu2} & V_{\mu3} \\
V_{\tau1} & V_{\tau2} & V_{\tau3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
\tag{1}
\]

where \( V \) is the \( 3 \times 3 \) unitary matrix known as PMNS matrix \([1]\), which contains three mixing angles and three CP violating phases (one Dirac type and two Majorana type). In the standard parametrization \([2]\), \( V_{PMNS} \) is expressed in terms of the solar, atmospheric and reactor mixing angles \( \theta_{12}, \theta_{23}, \theta_{13} \) and three CP-violating phases \( \delta_{CP}, \rho, \sigma \) as

\[
V_{PMNS} =
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}\ e^{-i\delta_{CP}} \\
-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\
s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23}
\end{pmatrix} P_{\nu} \equiv U_{PMNS} P_{\nu},
\tag{2}
\]

where \( c_{ij} \equiv \cos \theta_{ij}, \ s_{ij} \equiv \sin \theta_{ij} \) and \( P_{\nu} \equiv \{ e^{i\rho}, e^{i\sigma}, 1 \} \) is a diagonal matrix with CP violating Majorana phases \( \rho \) and \( \sigma \). The \( U_{PMNS} \) component of the mixing matrix describes the mixing of Dirac type neutrinos analogous to the CKM matrix of the quark sector. The neutrino oscillation data accumulated over many years allow us to determine the solar and atmospheric neutrino oscillation parameters with very high precision. Recently, the value of the smallest mixing angle \( \theta_{13} \) has been measured by the Daya Bay \([3]\) and RENO Collaborations \([4]\) with the best fit \((1\sigma)\) result as

\[
\sin^2 2\theta_{13} = 0.089 \pm 0.010 \text{(stat)} \pm 0.005 \text{(syst)}, \quad \text{Daya Bay}
\]

\[
\sin^2 2\theta_{13} = 0.113 \pm 0.013 \text{(stat)} \pm 0.019 \text{(syst)}, \quad \text{RENO}
\tag{3}
\]

which is equivalent to \( \theta_{13} \approx 8.8^\circ \pm 0.8^\circ \). This is \( 5.2\sigma \) evidence of nonzero value of \( \theta_{13} \) which confirms the previous measurements of T2K \([5]\), MINOS \([6]\) and Double Chooz \([7]\) experiments. The global analysis of the recent results of various neutrino oscillation experiments has been performed by several groups \([8,10]\), and the parameters which are used in this analysis are taken from Ref. \([10]\), are presented in Table-1.
The observation of this not so small reactor mixing angle $\theta_{13}$ has ignited a lot of interest to understand the mixing pattern in the lepton sector \cite{11}. It opens promising perspectives for the observation of CP violation in the lepton sector. The precise determination of $\theta_{13}$ in addition to providing a complete picture of neutrino mixing pattern, could be a signal of underlying physics responsible for lepton mixing and for the physics beyond standard model. It has been shown that if one includes some perturbative corrections to the leading order neutrino mixing patterns, such as bi-maximal (BM) \cite{12}, tri-bimaximal (TBM) \cite{13} and democratic (DC) \cite{14}, it is possible to explain the observed neutrino mixing angles \cite{15}. However, it should be noted that among these leading order mixing patterns i.e., BM, TBM and DC, the tri-bimaximal pattern, whose explicit form as given below

$$U_{TBM} = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\
-\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\
-\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}
\end{pmatrix},
$$

(4)

is particularly very interesting. It corresponds to the three mixing angles of the standard parametrization as $\theta_{12} = \arctan(1/\sqrt{2}) \approx 35.3^\circ$, $\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$. Clearly, to accommodate the large value of $\theta_{13}$, one has to consider possible perturbations to the TBM mixing matrix. In this paper we would like to study the possible corrections arising from the charged lepton sector. The essential features of our analysis are as follows. We assume
the charged lepton mixing matrix to be of the same form as the CKM quark mixing matrix and the neutrino mixing matrix to be of the tri-bimaximal form. Furthermore, we use the Wolfenstein-like parametrization for the charged lepton mixing matrix and study its implications on various phenomenological observables. It should be noted that there have been several attempts made recently to understand the nonzero $\theta_{13}$ due to charged lepton correction \cite{16} and in the past also corrections to the leptonic mixing matrix due to charged leptons were considered in Ref. \cite{17}.

The paper has been organized as follows. The methodology of our analysis is presented in Section-II and the Results and Conclusion are discussed in Section-III.

II. METHODOLOGY

It is well known that the leptonic mixing matrix arises from the overlapping of the matrices that diagonalize charged lepton and neutrino mass matrices

$$U_{PMNS} = U_l^\dagger U_\nu .$$

Here we are focusing only the component of the mixing matrix which describes the mixing of Dirac type neutrinos. For the study of leptonic mixing it is generally assumed that the charged lepton mixing matrix as an identity matrix and the neutrino mixing matrix $U_\nu$ has a specific form dictated by a symmetry which fixes the values of the three mixing angles in $U_\nu$. The small deviations of the mixing angles from those measured in the PMNS matrix, are considered, in general, as perturbative corrections arising from symmetry breaking effects. A variety symmetry forms of $U_\nu$ have been explored in the literature e.g., BM/TBM/DC and so on. In this work we will consider the situation wherein the neutrino mixing matrix is described by the TBM matrix, i.e.,

$$U_\nu = U_{TBM} ,$$

and that the mixing angles induced by the charged leptons can be considered as corrections. Furthermore, we will neglect possible corrections to $U_{TBM}$ from higher dimensional operators and from renormalization group effects. In this approximation we will derive formulae which allow us to include corrections to neutrino mixing angles and to constrain the CP violating phase ($\delta_{CP}$) conveniently.
In our study, we use a simple ansatz for the charged lepton mixing matrix $U_l$, i.e., we assume that $U_l$ has the same structure as the CKM matrix connecting the weak eigenstates of the down type quarks to the corresponding mass eigenstates. This approximation is quite reasonable as we know that the CKM matrix is almost diagonal with the off diagonal elements strongly suppressed by the small expansion parameter $\lambda = \sin \theta_C$ ($\theta_C$, being the Cabibbo angle). Hence, such an assumption can naturally provide the small perturbations to the tri-bimaximal mixing pattern for neutrino mixing matrix. Furthermore, as discussed in Ref. [18], this approximation is quite acceptable as the mass spectrum of charged leptons exhibits similar hierarchical structure as the down type quarks, i.e., $(m_e, m_\mu) \approx (\lambda^5, \lambda^2)m_\tau$ and $(m_d, m_s) \approx (\lambda^4, \lambda^2)m_b$. This may imply that the charged lepton mixing matrix has a structure similar to the down type quark mixing and is governed by the CKM matrix.

To illustrate the things more explicitly, let us recall the values of the quark mixing angles in the standard PDG parametrization for the CKM matrix within 1σ range as [19]

$$\theta_{13}^q = 0.20^\circ \pm 0.01^\circ, \quad \theta_{23}^q = 2.35^\circ \pm 0.07^\circ, \quad \theta_{12}^q \equiv \theta_C = 13.02^\circ \pm 0.04^\circ.$$  \hspace{1cm} (7)

However, the leptonic sector is described by two large mixing angles $\theta_{23}^l$ and $\theta_{12}^l$ and the third mixing angle $\theta_{13}^l$, was expected to be very small. Recently, the third mixing angle $\theta_{13}^l$ has been measured by T2K, Double CHOOZ, Daya Bay and RENO Collaborations yielding the following mixing patterns in the lepton sector:

$$\theta_{13}^l = 8.8^\circ \pm 1.0^\circ, \quad \theta_{23}^l = 40.4^\circ \pm 1.0^\circ, \quad \theta_{12}^l = 34.0^\circ \pm 1.1^\circ.$$  \hspace{1cm} (8)

The different nature of the quark and lepton mixing angles can be inter-related in terms of the quark lepton complementarity (QLC) relations [20], as

$$\theta_{12}^q + \theta_{12}^l \simeq 45^\circ, \quad \theta_{23}^q + \theta_{23}^l \simeq 45^\circ.$$  \hspace{1cm} (9)

The QLC relations indicate that it could be possible to have a quark-lepton symmetry based on some flavor symmetry. The experimental result of this not-so-small reactor mixing angle $\theta_{13}^l$ has triggered a lot of interest in the theoretical community. Given the rather precise measurement of $\theta_{13}^l$, one may wonder whether $\theta_{13}^l$ numerically agrees well with the QLC relation, i.e.,

$$\theta_{13}^l = \frac{\theta_C}{\sqrt{2}} \approx 9.2^\circ.$$  \hspace{1cm} (10)
In particular, it is quite interesting to see whether this specific connection to $\theta_C$ can be a consequence of some underlying symmetry, which may provide a clue to the nature of quark-lepton physics beyond the standard model.

Starting from the fact that the mixing matrix of the up type quark sector can be almost diagonal and so the CKM matrix is mainly generated from the down type quark mixing matrix, we assume that the mixing matrix of the charged lepton sector is basically of the same form as that of down type quark sector. Consequently the lepton mixing matrix appears as the product of CKM like matrix (induced by charged lepton sector) and the TBM pattern matrix induced from the neutrino sector. As discussed before, in the limit of diagonal charged lepton mass matrix i.e., $U_l = 1$, and $U_\nu = U_{TBM}$, which gives the mixing angles at the leading order as

\[ \theta_{12}^0 = \arctan(1/\sqrt{2}) \simeq 35.3^\circ, \quad \theta_{13}^0 = 0^\circ, \quad \text{and} \quad \theta_{23}^0 = 45^\circ, \]  

(11)

deviate significantly from their measured values as

\[ |\theta_{12}^l - \theta_{12}^0| \simeq 2^\circ, \quad |\theta_{13}^l - \theta_{13}^0| \approx 9^\circ \quad \text{and} \quad |\theta_{23}^l - \theta_{23}^0| \simeq 5^\circ. \]  

(12)

These deviations are attributed to the corrections arising from the charged lepton sector.

We assume the charged lepton mixing matrix to have the form as the CKM matrix in the standard parametrization, i.e.,

\[ U_l = R_{23}U_{13}R_{12}, \]  

(13)

where the matrices $R_{23}, U_{13}$ and $R_{12}$ are defined by

\[
R_{12} = \begin{pmatrix}
\cos \theta_{12}^l & \sin \theta_{12}^l & 0 \\
-\sin \theta_{12}^l & \cos \theta_{12}^l & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad R_{23} = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta_{23}^l & \sin \theta_{23}^l \\
0 & -\sin \theta_{23}^l & \cos \theta_{23}^l
\end{pmatrix},
\]

\[
U_{13} = \begin{pmatrix}
\cos \theta_{13}^l & 0 & \sin \theta_{13}^l e^{-i\delta} \\
0 & 1 & 0 \\
-\sin \theta_{13}^l e^{i\delta} & 0 & \cos \theta_{13}^l
\end{pmatrix}.
\]  

(14)

Furthermore, as the mixing angle $\theta_{13}$ receives maximum deviation from the TBM pattern, we assume $\sin \theta_{13}^l = \sin \theta_C = \lambda$, where, $\lambda$ is a small expansion parameter analogous to the expansion parameter of Wolfenstein parametrization of the CKM matrix. The other two angles are assumed to be of the form

\[ \sin \theta_{23}^l = A\lambda^2, \quad \sin \theta_{12}^l = A\lambda^3, \]  

(15)
where the parameter $A = \mathcal{O}(1)$. With these values, one can obtain the Wolfenstein-like parametrization for $U_i$ (upto order $\lambda^3$) as

$$
U_i = \begin{pmatrix}
1 - \lambda^2/2 & A\lambda^3 & \lambda e^{-i\delta} \\
-A\lambda^3(1 + e^{i\delta}) & 1 & A\lambda^2 \\
-\lambda e^{i\delta} & -A\lambda^2 & 1 - \lambda^2/2
\end{pmatrix} \tag{16}
$$

Thus, with the help of Eqs. (5), (6) and (16), one can schematically obtain the PMNS matrix up to order of $\lambda^3$ as

$$
U_{\text{PMNS}} = U_{\text{TBM}} + \Delta U, \tag{17}
$$

with

$$
\Delta U = \begin{pmatrix}
\frac{\lambda e^{-i\delta} - \lambda^2 + A\lambda^3(1 + e^{-i\delta})}{\sqrt{2}} & \frac{-\lambda e^{-i\delta} + \lambda^2/2 + A\lambda^3(1 + e^{-i\delta})}{\sqrt{3}} & \frac{-\lambda e^{-i\delta} - A\lambda^3(1 + e^{-i\delta})}{\sqrt{2}} \\
\frac{-A\lambda^2(1 + 2\lambda)}{\sqrt{6}} & \frac{-A\lambda^2(1 - \lambda)}{\sqrt{3}} & \frac{-A\lambda^2}{\sqrt{2}} \\
\frac{2\lambda e^{i\delta} - A\lambda^2 + \lambda^2/2}{\sqrt{6}} & \frac{\lambda e^{i\delta} + A\lambda^2 - \lambda^2/2}{\sqrt{3}} & \frac{-A\lambda^2 + \lambda^2/2}{\sqrt{2}}
\end{pmatrix} + \mathcal{O}(\lambda^4), \tag{18}
$$

which allows one to obtain the elements of the PMNS matrix as

$$
|U_{e1}| = \sqrt{\frac{2}{3}} \left[ 1 + \frac{1}{2} \lambda \cos \delta - \frac{1}{8} \lambda^2(3 + \cos^2 \delta) + \frac{1}{16} \lambda^3 \left( 8A(1 + \cos \delta) - \cos \delta \sin^2 \delta \right) \right],
$$

$$
|U_{e2}| = \sqrt{\frac{1}{3}} \left[ 1 - \lambda \cos \delta - \frac{1}{2} \lambda^2 \cos^2 \delta - \frac{1}{2} \lambda^3(2A(1 + \cos \delta) - \cos \delta \sin^2 \delta) \right],
$$

$$
|U_{e3}| = \frac{\lambda}{\sqrt{2}} \left[ 1 - A\lambda^2(1 + \cos \delta) \right],
$$

$$
U_{\mu1} = -\frac{1}{\sqrt{6}} \left[ 1 - A\lambda^2 - 2A\lambda^3 \right],
$$

$$
U_{\mu2} = \frac{1}{\sqrt{3}} \left[ 1 - A\lambda^2 + A\lambda^3 \right],
$$

$$
|U_{\mu3}| = \frac{1}{\sqrt{2}} \left( 1 + A\lambda^2 \right),
$$

$$
U_{\tau1} = -\frac{1}{\sqrt{6}} \left( 1 - 2\lambda e^{i\delta} + \frac{1}{2} \lambda^2(2A - 1) \right),
$$

$$
U_{\tau2} = \frac{1}{\sqrt{3}} \left( 1 + \lambda e^{i\delta} + \frac{1}{2} \lambda^2(2A - 1) \right),
$$

$$
|U_{\tau3}| = \frac{1}{\sqrt{2}} \left( 1 - \frac{1}{2} \lambda^2 - A\lambda^3 \right). \tag{19}
$$

From Eq. (2), one can express the neutrino mixing parameters in terms of the PMNS mixing matrix elements as

$$
\sin^2 \theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2}, \quad \sin^2 \theta_{23} = \frac{|U_{\mu3}|^2}{1 - |U_{e3}|^2},
$$

$$
\sin \theta_{13} = |U_{e3}|. \tag{20}
$$
Thus, from Eqs. (19) and (20), one can obtain the solar neutrino mixing angle $\theta_{12}$, up to order $\lambda^3$, as
\[
\sin^2 \theta_{12} \simeq \frac{1}{3} \left( 1 - 2\lambda \cos \delta + \frac{\lambda^2}{2} - \lambda^3 [2A(1 + \cos \delta) + \cos^3 \delta] \right),
\]
(21)
Clearly, when $\cos \delta$ approaches zero we observe a tiny deviation from $\sin^2 \theta_{12} = 1/3$. Following similar approach, one can obtain the atmospheric neutrino mixing angle $\theta_{23}$ as
\[
\sin^2 \theta_{23} \simeq \frac{1}{2} \left( 1 + \frac{\lambda^2}{2}(1 + 4A) \right),
\]
(22)
which also shows a small deviation from the maximal mixing pattern i.e., $\sin^2 \theta_{23} = 1/2$. The reactor mixing angle $\theta_{13}$ can be obtained as
\[
\sin \theta_{13} = \frac{\lambda}{\sqrt{2}} \left( 1 - A\lambda^2(1 + \cos \delta) \right).
\]
(23)
Thus, we have a non-vanishing large $\theta_{13}$. This in turn implies that it could be possible to observe CP violation in the lepton sector analogous to the quark sector, which could be detected through long base-line neutrino oscillation experiments. The Jarlskog invariant, which is a measure of CP violation, for the lepton sector has the expression
\[
J_{\text{CP}}^\ell \equiv \text{Im}[U_{e1}U_{\mu2}U_{\mu1}^*U_{\tau2}^*] = -\frac{\lambda \sin \delta}{6} \left( 1 - \frac{\lambda^2}{2} - A\lambda^2 \right) + \mathcal{O}(\lambda^4),
\]
(24)
which is sensitive to the Dirac CP violating phase.

The Dirac CP phase $\delta_{\text{CP}}$ can be deduced by using the PMNS matrix elements and the neutrino mixing parameters as [18]
\[
\delta_{\text{CP}} = -\arg \left( \frac{U_{e1}^*U_{\mu3}U_{\mu1}^*U_{\tau2}^*}{\frac{c_{12}c_{13}c_{23}}{s_{12}s_{23}} + c_{12}c_{23}s_{13}} \right).
\]
(25)
With Eqs. (4), (17) and (18), this yields the correlation between the two CP violating phases (Dirac type CP violating phase and the phase $\delta$ introduced in the charged lepton mixing)
\[
\delta_{\text{CP}} = -\arctan \left( \frac{-\lambda(1 - (A + \frac{1}{2}\lambda^2)) \sin \delta}{\lambda \left( A(1 - \lambda^2) - \frac{5}{2}\lambda^2 \cos \delta - \left( \frac{3}{2}\lambda + A\lambda^2 \right) + \lambda^2(1 + \lambda \cos \delta) \right)} \right).
\]
(26)
Three mass-dependent neutrino observables are probed in different types of experiments. The sum of absolute neutrino masses $\sum_i m_i$ is probed in cosmology, the kinetic electron neutrino mass in beta decay ($M_\beta$) is probed in direct search for neutrino masses, and the
effective mass \( (M_{ee}) \) is probed in neutrino-less double beta decay experiments with the decay rate for the process \( \Gamma \propto |M_{ee}|^2 \). In terms of the bare physical parameters \( m_i \) and \( U_{\alpha i} \), the observables are given by \[21\]

\[
\begin{align*}
\sum_i m_i &= m_1 + m_2 + m_3, \\
M_{ee} &= \sum_i U_{ei}^2 m_i, \\
M_\beta &= \sqrt{\sum_i |U_{ei}|^2 m_i^2}.
\end{align*}
\] (27)

The absolute values of neutrino masses are currently unknown. Recently the Planck experiment on Cosmic Microwave Background (CMB) \[22\] has reported an interesting result for the sum of three neutrino masses with an assumption of three species of degenerate neutrinos as

\[
\sum_i m_i \leq 0.23 \text{ eV} \quad \text{(Planck + WP + highL + BAO)}
\] (28)

The most stringent upper bound on electron-antineutrino mass has been measured in the Troitsk experiment \[23\] on the high precision measurement of the end-point spectrum of tritium beta decay as

\[ M_\beta < 2.05 \text{ eV} \quad 95\% \text{ C.L.} \] (29)

In our analysis we ignore the Majorana phases \( (\rho, \sigma) \) and consider the normal hierarchy scenario for the neutrino mass spectrum in which the neutrino masses \( m_2 \) and \( m_3 \) can be expressed in terms of the lightest neutrino mass \( m_1 \) and the measured solar and atmospheric mass-squared differences \( \Delta m^2_{\text{sol}} \) and \( \Delta m^2_{\text{atm}} \) as

\[
\begin{align*}
m_2 &= \sqrt{m_1^2 + \Delta m^2_{\text{sol}}}, \\
m_3 &= \sqrt{m_1^2 + \Delta m^2_{\text{sol}} + \Delta m^2_{\text{atm}}}.
\end{align*}
\] (30)

III. RESULTS AND DISCUSSION

For numerical estimation we need to know the values of the three unknown parameters \( A, \lambda \) and \( \delta \). In this analysis we assume the small expansion parameter \( \lambda \) to have the same value as that of the quark sector \[19\]:

\[
\lambda = 0.22535 \pm 0.00065.
\] (31)
Now with Eq. (22) and using the experimental value of $\sin^2 \theta_{23}$ as input parameter, we obtain the $1\sigma$ ($3\sigma$) range of $A$ as

$$A = (-2.4 \rightarrow -1.2) \quad (1\sigma)$$

$$= (-3.4 \rightarrow 3.0), \quad (3\sigma)$$

and we treat the CP violating phase $\delta$ as a free parameter, i.e., we allow it to vary in its entire range $0 \leq \delta \leq 2\pi$. Now varying these input parameters in their $3\sigma$ ranges, and using Eqs. (21) and (23), we present the variation of the solar and reactor mixing angles ($\sin^2 \theta_{12}$ and $\sin \theta_{13}$) with the CP violating phase $\delta$ in Figure-1. From the figure, it can be seen that in this formalism, it is possible to accommodate simultaneously the observed value of the reactor mixing angle $\theta_{13}$ and solar mixing angle $\theta_{12}$. The correlation plots between the solar and atmospheric mixing angles with $\theta_{13}$ is shown in Figure-2. In Figure-3, we show the variation of the Jarlskog Invariant $J_{CP}$ with $\delta$ and $\theta_{13}$. From the figure it can be seen that it could be possible to have large CP violation $\mathcal{O}(10^{-2})$ in the lepton sector. The correlation between the Dirac CP violating phase $\delta_{CP}$ and the CP violating parameter $\delta$ of the charged lepton mixing matrix is shown in Figure-4. The variation of $M_{ee}$ with the lightest neutrino mass $m_1$ (for Normal Hierarchy) and the variation of $M_\beta$ with $\sum m_i$ (where the parameters are varied in their $1\sigma$ range) are shown in Figure-5. Thus, for $m_1$ below $\mathcal{O}(10^{-2})$ eV, one can get $M_{ee} \leq 1.2 \times 10^{-2}$ eV and $M_\beta \leq 1.4 \times 10^{-2}$ eV.

![FIG. 1: Variation of $\sin^2 \theta_{12}$ with the CP violating phase $\delta$ (left panel) and $\sin \theta_{13}$ on the right panel. The horizontal lines (in both panels) represent the $3\sigma$ allowed range.](image)
FIG. 2: Correlation plot between solar (left panel) and the atmospheric mixing angle (right panel) with $\theta_{13}$.

FIG. 3: Variation of $J_{CP}$ with $\delta$ (left panel) and with $\theta_{13}$ (right panel).

To summarize, to accommodate the observed value of relatively large $\theta_{13}$, we consider the corrections due to the charged lepton mixing matrix to the TBM pattern of neutrino mixing matrix. Based on the possible inter-relation between the charged lepton and the quark mixing structures we constructed the lepton mixing matrix to have the form of the CKM-like matrix (induced from the charged lepton sector) times the TBM matrix induced from the neutrino sector. Our result showed that in this formalism, it is possible to accommodate the observed reactor mixing angle $\theta_{13}$ along with the other mixing parameters within their experimental range. We have also found that sizable leptonic CP violation characterized by the Jarlskog invariant $J_{CP}$, i.e., $|J_{CP}| \leq 10^{-2}$ could be possible in this scenario. The
FIG. 4: The correlation plot between the Dirac CP violating phase $\delta_{CP}$ and $\delta$.

FIG. 5: Variation of $M_{ee}$ with the lightest neutrino mass $m_1$ (left panel) and the variation of $M_\beta$ with $\sum m_i$ (right panel).

observation of CP violation in the upcoming long base-line neutrino experiments would be a smoking gun signal of this formalism. We have also shown that the measured value of $\theta_{13}$ along with other mixing parameters can be used for constraining the value of the Dirac CP violating phase $\delta_{CP}$. The upper limits on $M_{ee}$ and $M_\beta$ are found to be $\mathcal{O}(10^{-2})$, if the mass of the lightest neutrino $m_1 \leq 0.01$ eV.

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