Secrecy Fairness Aware NOMA for Untrusted Users

Sapna Thapar¹, Deepak Mishra², and Ravikant Saini¹
¹Department of Electrical Engineering, Indian Institute of Technology Jammu, India
²Department of Electrical Engineering (ISY), Linköping University, Sweden
Emails: 2018ree0019@iitjammu.ac.in, deepak.mishra@liu.se, ravikant.saini@iitjammu.ac.in

Abstract—Spectrally-efficient secure non-orthogonal multiple access (NOMA) has recently attained a substantial research interest for fifth generation (5G) networks. This work explores crucial security issue in NOMA which is stemmed from utilizing the decoding concept of successive interference cancellation. Considering untrusted users, we design a novel secure NOMA transmission protocol to maximize secrecy fairness among users. A new decoding order for two users’ NOMA is proposed that provides positive secrecy rate to both users. Observing the objective of maximizing secrecy fairness between users under given power budget constraint, the problem is formulated as minimizing the maximum secrecy outage probability (SOP) between users. In particular, closed-form expressions of SOP for both users are derived to analyze secrecy performance. SOP minimization problems are solved using pseudocoonvexity concept, and optimized power allocation (PA) for each user is obtained. Asymptotic expressions of SOPs, and optimal PAs minimizing these approximations are obtained to get deeper insights. Further, globally-optimized power control solution from secrecy fairness perspective is obtained at a low computational complexity and, asymptotic approximation is obtained to gain analytical insights. Numerical results validate the correctness of analysis, and present insights on optimal solutions. Finally, we present insights on global-optimal PA by which fairness is ensured and gains of about 55.12%, 69.30%, and 19.11%, respectively are achieved, compared to fixed PA and individual users’ optimal PAs.

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) is envisaged as a potential breakthrough for fifth generation (5G) networks because of the possibility of serving multiple users within same resource block [1]. Conversely, the broadcast nature of wireless communication at transmitter, and decoding concept of successive interference cancellation (SIC) at receiver, in NOMA, causes security challenge on the information-carrying signal. Therefore, the research on security issues in NOMA networks has attained great attention among 5G researchers. The integration of NOMA and physical layer security (PLS) has been observed as a new research frontier towards providing spectrally-efficient and secure wireless communication [2]. Still, despite merits, the design process includes security challenge of wiretapping in the presence of untrusted users.

A. Related Art

Motivated by the spectral efficiency improvement by NOMA, [1] has addressed research contributions in power-domain NOMA. Stimulated by potential of PLS, [3] has summarized existing research works on PLS techniques. Recently, innumerable researchers have concentrated on PLS in NOMA. PLS in large-scale networks has been studied in [2] where a protected zone around base station (BS) is designed to retain an eavesdropper-free region. Secure NOMA with multiple users against eavesdropper has been discussed in [4] for single-input single-output network. A joint beamforming scheme has been introduced in [5], where confidential data is transmitted to intended user only. Secrecy of a cooperative NOMA system with a decode-and-forward and an amplify-and-forward relay against eavesdropper has been analyzed in [6]. A secrecy beamforming scheme that exploits artificial noise (AN) to enhance secrecy of NOMA in the presence of eavesdropper has been presented in [7]. Besides eavesdroppers, NOMA itself has inherent security issue which is caused due to SIC based decoding at receiver. Regarding this, recently [8] has considered a system where near user is assumed to be trusted, whereas far user as untrusted, and investigated secrecy of only trusted user against untrusted node.

B. Research Gap and Motivation

As noted, existing works have considered different PLS techniques such as AN aided strategy [2], [7], optimal power allocation (PA) [4], beamforming [5], and cooperative relaying [6] to improve secrecy of NOMA against external eavesdroppers. Taking conventional decoding order concept in NOMA into account, two key steps are followed: (1) observing near user signal (with better channel conditions) as noise, far user (with poorer channel conditions) decodes its own signal first, and after decoding, it may apply SIC and decode signal of near user [8]; (2) near user first decodes signal associated to far user, applies SIC, and then decodes its own signal. As an outcome, near and far users, respectively, have access of far and near user which is a critical security concern in...
NOMA implementation between untrusted users. Considering this issue, [8] has assumed only far user as untrusted and analyzed the secrecy performance of trusted (near) user. As inferred, the system, assuming all users as untrusted is more challenging and practical scenario for designing a secure network. Untrusted users’ model is a more hostile situation, where all users do not have mutual trust and each user focuses on achieving secure communication from BS in the presence of untrusted users [9, 10, 11]. Towards this end, we investigate secure NOMA protocol from positive secrecy rates standpoint for both untrusted users, which to the best of our knowledge, has been an open problem in literature.

C. Key Contributions

Considering a system with one BS and two untrusted users, a novel secure NOMA protocol is designed to maximize secrecy fairness between untrusted users. Main contributions are as follows: (1) A new decoding order for two users’ NOMA is proposed that provides positive secrecy rate to both users. (2) Analytical expressions of secrecy outage probability (SOP) for both users, and their asymptotic approximations have been derived. (3) Closed-form expressions of optimal PA minimizing SOP, have been obtained using pseudoconvexity of SOP at both users. (4) Global-optimal PA solution to a problem minimizing the maximum SOP between users under given power budget constraint, is obtained. (5) Numerical results validate analytical derivations, present insights on optimal solutions, and analyze performance gains with proposed model.

II. PROPOSED SECURE NOMA PROTOCOL

A. System Model

We consider downlink NOMA system where BS communicates with two untrusted users (Fig. 1). Our two users consideration will shed light on the proof of concept, however, the protocol can be extended to a system with multiple users. Each node is equipped with single antenna [8]. We denote U1 and U2 as near and far user, respectively. hi denotes as Rayleigh fading channel gain coefficient from BS to Ui where \( i \in \{1, 2\} \). All the channels from BS to users are assumed to be independent, and follow small scale fading accompanying with path loss effects, such that channel power gain \( |h_i|^2 \) experience exponential distribution with mean \( \lambda_i = L_i d_i^{-n} \) where \( L_i, n, \) and \( d_i \), respectively, denote path loss constant, path loss exponent and distance between BS and Ui. Assuming statistical channel state information is known at BS, U1 and U2, respectively, are considered as strong and weak users. Channel power gains are sorted as \( |h_1|^2 > |h_2|^2 \). A fixed amount of transmit power \( P_t \) is allocated from BS to users. \( \alpha \) denotes the PA coefficient, i.e., the fraction of \( P_t \) allocated to U1. Remaining fraction, i.e., \( 1 - \alpha \) is allocated to U2.

Applying power-domain NOMA principle, BS broadcasts superposition of information signals \( x_1 \) and \( x_2 \) of U1 and U2, respectively, and then the transmitted signal is \( \sqrt{\alpha P_t} x_1 + \sqrt{(1 - \alpha) P_t} x_2 \) [8]. The received signals \( y_1 \) and \( y_2 \), respectively, at U1 and U2, from BS are given as [8]

\[
y_1 = h_1(\sqrt{\alpha P_t} x_1 + \sqrt{(1 - \alpha) P_t} x_2) + n_1,
\]

\[
y_2 = h_2(\sqrt{\alpha P_t} x_1 + \sqrt{(1 - \alpha) P_t} x_2) + n_2,
\]

where \( n_1 \) and \( n_2 \) denote additive white Gaussian noise (AWGN) with mean 0 and variance \( \sigma^2 \) at both users. We assume ideal SIC based decoding at receivers where interference from other user is perfectly cancelled at legitimate user. However, in real scenarios, perfect SIC cannot be readily satisfied due to various implementation problems such as error propagation. Therefore, imperfect SIC model where residual interference from imperfectly decoded user exists after SIC, is highly realistic to explore secure NOMA which has been considered in the extended version of this work [12].

B. Proposed Decoding Order for Untrusted NOMA

In secure NOMA protocol, signal of U2 must be protected from U1, and vice-versa. Before discussing secure protocol, we first present insights on how conventional decoding order is inefficient for providing secrecy at both users in untrusted scenario. Considering conventional NOMA (Section I(B)), the received signal-to-interference-plus-noise-ratio \( \Gamma_{ij} \) at Ui when signal of Uj is decoded by Uj (for \( i \in \{1, 2\}, j \in \{1, 2\} \)) is given as [8]

\[
\Gamma_{21} = \frac{(1 - \alpha)|h_1|^2}{|h_1|^2 + \frac{1}{\rho_1}}, \quad \Gamma_{22} = \frac{(1 - \alpha)|h_2|^2}{|h_2|^2 + \frac{1}{\rho_1}},
\]

\[
\Gamma_{11} = \alpha \rho_1 |h_1|^2, \quad \Gamma_{12} = \alpha \rho_1 |h_2|^2,
\]

where \( \rho_1 = P_t/\sigma^2 \) is BS transmit signal-to-noise ratio (SNR). Secrecy rates \( R_{s1} \) and \( R_{s2} \) for U1 and U2 can be given by

\[
R_{s1} = R_{11} - R_{12}, \quad R_{s2} = R_{22} - R_{21},
\]

where \( R_{11}, R_{12}, R_{21} \) and \( R_{22} \) respectively, are given as [13]

\[
R_{11} = \log_2(1 + \Gamma_{11}), \quad R_{12} = \log_2(1 + \Gamma_{12}),
\]

\[
R_{22} = \log_2(1 + \Gamma_{22}), \quad R_{21} = \log_2(1 + \Gamma_{21}).
\]

The condition \( R_{11} > R_{12} \) required for positive secrecy rate at U1, simplified as \( \Gamma_{11} > \Gamma_{12} \) gives a feasible condition \( |h_1|^2 > |h_2|^2 \). This ensures positive secrecy rate at U1. Next, for positive \( R_{s2} \) at U2, \( R_{22} > R_{21} \), simplified as \( \Gamma_{22} > \Gamma_{21} \) results an infeasible condition \( |h_2|^2 > |h_1|^2 \) because channel power gains are assumed as \( |h_1|^2 > |h_2|^2 \). Thus, positive secrecy rate is not achieved at U2. Hence, the conventional decoding order cannot be considered for untrusted NOMA.

Now, with the goal of providing positive secrecy rate to both users, we propose a new decoding order, according to which both U1 and U2 first decode signal associated to other user, and then decode its own signal after performing SIC. Compared to the conventional NOMA, the decoding order is changed for the far user only. As a result, \( \Gamma_{ij} \) are

\[
\Gamma_{21} = \frac{(1 - \alpha)|h_2|^2}{|h_2|^2 + \frac{1}{\rho_1}}, \quad \Gamma_{12} = \frac{\alpha|h_2|^2}{(1 - \alpha)|h_2|^2 + \frac{1}{\rho_1}},
\]

\[
\Gamma_{11} = \alpha \rho_1 |h_1|^2, \quad \Gamma_{22} = (1 - \alpha) \rho_1 |h_2|^2.
\]

For positive \( R_{s1}, R_{11} > R_{12} \), simplified as \( \Gamma_{11} > \Gamma_{12} \) gives

\[
\alpha < 1 + \frac{|h_1|^2 - |h_2|^2}{|h_1|^2 |h_2|^2 \rho_1}.
\]
Thus, positive secrecy rate can be ensured at U1. Similarly, for positive $R_{s2}, R_{22} > R_{21}$, simplified as $\Gamma_{22} > \Gamma_{21}$ gives
\[
\alpha > \left(\frac{|h_1|^2 - |h_2|^2}{|h_1|^2 |h_2|^2 \rho_1}\right).
\] (8)

Observing (7) and (8), it can be concluded that proposed decoding order is efficient to provide positive secrecy rate to both users in untrusted NOMA, provided $|h_1|^2 - |h_2|^2 < \alpha < 1$.

### III. Secrecy Performance Analysis

Next we derive analytical expressions of SOP and investigate optimal PAs minimizing SOPs for both U1 and U2.

#### A. Exact Secrecy Outage Probability

The SOP is defined as the probability that maximum achievable secrecy rate at each user falls below a target secrecy rate. Denoting $s_{o1}$ as SOP for U1, now we derive SOPs analytically.

1) **Near user**: Considering achievable and target secrecy rate of U1 as $R_{s1}$ and $R_{th1}$, respectively, $s_{o1}$ is given as
\[
s_{o1} = \Pr\{R_{s1} < R_{th1}\} = \Pr\left\{\log_2\left(1 + \frac{1 + \Gamma_{22}}{1 + \Gamma_{21}}\right) < \Pi_1\right\},
\] (9)
where $\Pr\{\cdot\}$ is denoted for the probability measure, $\Pi_1 \triangleq 2^{\alpha t_1}, A_1 \triangleq \frac{\Pi_1 - 1}{\alpha t_1}, F_{h_2}(x)$ and $f_{h_2}(x)$ are the cumulative distribution function (CDF) and probability density function (PDF), respectively, of exponentially distributed random channel power gain $|h_1|^2$ and $|h_2|^2$, respectively.

2) **Far user**: Considering $R_{s2}$ and $R_{th2}$ as achievable and target secrecy rate, respectively, of U2, $s_{o2}$ is stated as
\[
s_{o2} = \Pr\{R_{s2} < R_{th2}\} = \Pr\left\{\log_2\left(1 + \frac{1 + \Gamma_{22}}{1 + \Gamma_{21}}\right) < R_{th2}\right\},
\] (10)
where $\Pi_2 \triangleq 2^{\alpha t_2}, A_2 \triangleq \frac{\Pi_2 - 1}{\alpha t_2}, F_{h_2}(x)$ and $f_{h_2}(x)$ are the CDF and PDF of $|h_2|^2$ and $|h_1|^2$, respectively.

#### B. Secrecy Outage Probability Minimization

1) **Near User**: The SOP minimization problem for U1, considering $s_{o1}$ as a function of $\alpha$, can be stated as
\[
(J1) : \text{minimize } \alpha \text{, subject to } (C1) : 0 < \alpha < 1.
\] (11)
The optimality of problem (J1) is asserted by Lemma 1.

**Lemma 1**: $s_{o1}$ is pseudoconvex function of $\alpha$.

**Proof**: Denoting integrand of $s_{o1}$ as $I_1$, we obtain
\[
I_1 = \frac{1}{\lambda_2} \exp\left\{-\frac{\Pi_1 y_1}{(1 - \alpha) \rho_1 y_1 + 1} - \frac{y_1}{\lambda_2} - \frac{(\Pi_1 - 1)}{\alpha \rho_1 \lambda_1}\right\}.
\] (12)
The second-order derivative of $\log(1)$ with respect to $\alpha$ is
\[
\frac{d^2 \log(1)}{d\alpha^2} = \left(-\frac{2(\Pi_1 - 1)}{\rho_1 \lambda_1 \alpha^3} + \frac{2\Pi_1^2 \rho_2^2 y_1^2}{\lambda_1 ((1 - \alpha) \rho_1 y_1 + 1)^3}\right),
\] (13)
which is decreasing and shows $I_1$ is a logarithmically concave function. Because log-concavity is preserved under integration [14], the integral function in (9) is also log-concave function. Considering pseudoconcave property [15] Lemma 5 of log-concave function, the integral function of (9) is pseudoconcave. The negative of pseudoconcave function is pseudoconvex function [16]. Hence, $s_{o1}$ is pseudoconvex function of $\alpha$.

We apply golden section search (GSS) algorithm [17] to find optimal solution $\alpha^*_1$ which minimizes $s_{o1}$. GSS algorithm considers pseudoconvex function $s_{o1}$, lower and upper bounds of $\alpha$, i.e., $\alpha_{lb}$ and $\alpha_{ub}$ respectively, as input. It provides optimized solution $\alpha^*_1$ and corresponding minimized $s_{o1}$ as outputs. Firstly, $\alpha_{lb} = 0$ and $\alpha_{ub} = 1$ are considered and algorithm searches along $\alpha$ with $\epsilon << 1$, where $\epsilon$ is acceptable tolerance. Algorithm terminates when the search length is less than a pre-determined tolerance level [17].

2) **Far User**: $s_{o2}$ minimization problem can be stated as
\[
(J2) : \text{minimize } s_{o2}, \text{ subject to } (C1).
\] (14)
The feasibility of unique solution is proved in Lemma 2.

**Lemma 2**: $s_{o2}$ is a pseudoconvex function of $\alpha$.

**Proof**: Denoting integrand of $s_{o2}$ as $I_2$, we obtain
\[
I_2 = \frac{1}{\lambda_2} \exp\left\{-\frac{\Pi_2 y_2}{(\alpha \rho_2 y_2 + 1) \lambda_2} - \frac{y_2}{\lambda_2} - \frac{(\Pi_2 - 1)}{(1 - \alpha) \rho_2 \lambda_2}\right\}.
\] (15)
Observing $\frac{d^2 \log(1)}{d\alpha^2}$ is monotonically decreasing, $I_2$ is also a log-concave function, and similar to the proof in Lemma 1, $s_{o2}$ is also a pseudoconvex function of $\alpha$.

The optimal solution $\alpha^*_2$ of (J2) is evaluated using GSS algorithm by considering $s_{o2}$ function as input.

#### C. Asymptotic Approximations: SOP and Optimization

In aforementioned analysis, SOP minimization problems have been solved numerically due to the complexity of derived expressions. Next, we present asymptotic approximations of SOPs and optimal PAs to gain analytical insights.

1) **Near User**: Asymptotic expression of $s_{o1}$, i.e., $\hat{s}_{o1}$, can be obtained by setting $(1 - \alpha) \rho_1 y_1 + 1 \approx (1 - \alpha) \rho_1 y_1$ for $\rho_1 \gg 1$ in (9). Accordingly, $\hat{s}_{o1}$ can be given as
\[
\hat{s}_{o1} = -\exp\left\{-\frac{\Pi_1}{\alpha \rho_1 \lambda_1} - \frac{(\Pi_1 - 1)}{\alpha \rho_1 \lambda_1}\right\} \int_0^{\infty} \exp\left(-\frac{y_1}{\lambda_2}\right) dy_1,
\] (16)
The \( \hat{s}_{o1} \) minimization problem can be formulated as
\[
(J3) : \text{minimize } \alpha \hat{s}_{o1}, \text{ subject to } (C1). \tag{17}
\]
Lemma 3 gives the optimal solution for (J3).

**Lemma 3:** The asymptotic optimal PA \( \hat{\alpha}_1 \), that minimizes \( \hat{s}_{o1} \), can be given as
\[
\hat{\alpha}_1 = -(\Pi_1 - 1) + \sqrt{\Pi_1 \Pi_1(\Pi_1 - 1)}. \tag{18}
\]

**Proof:** Since \( \hat{\alpha}_1 \) is obtained by minimizing \( \hat{s}_{o1} \), second-order derivative of \( \hat{s}_{o1} \) with respect to \( \alpha \) is given as
\[
\frac{d^2 \hat{s}_{o1}}{d\alpha^2} = \left\{ \frac{2\Pi_1}{\lambda_1 \rho_1 (1 - \alpha)^3} - 2 \left( 1 - \Pi_1 \right) \lambda_1 \rho_1 \alpha^3 \right. \\
- \left. \left( \frac{1 - \Pi_1}{\lambda_1 \rho_1 \alpha^2} - \frac{\Pi_1}{\lambda_1 \rho_1 (1 - \alpha)^2} \right)^2 \right\} \times \exp \left\{ \frac{1 - \Pi_1}{\lambda_1 \rho_1 (1 - \alpha)^2} \right\}, \tag{19}
\]
which does not imply monotonic behaviour. We set \( \frac{d^2 \hat{s}_{o1}}{d\alpha^2} = 0 \), and obtain \( \hat{\alpha}_1 = -(\Pi_1 - 1) + \sqrt{\Pi_1 \Pi_1(\Pi_1 - 1)} \). Note that \( \hat{\alpha}_1 = -(\Pi_1 - 1) - \sqrt{\Pi_1 \Pi_1(\Pi_1 - 1)} \) is negative, and therefore infeasible. Hence, \( \hat{\alpha}_1 \) minimizing \( \hat{s}_{o1} \) is given as \( 18 \).

2) **Far User:** Asymptotic approximation of \( s_{o2} \), i.e., \( \hat{s}_{o2} \) obtained using \( (\alpha \rho_2 y_2 + 1) = \hat{\alpha} \rho_2 y_2 \) in (10) for high \( \rho_1 \), is
\[
\hat{s}_{o2} = 1 - \exp \left\{ \frac{\Pi_1 - 2}{\alpha \rho_2 \lambda_2 - (1 - \alpha) \rho_1 \lambda_2} \right\} \int_0^\infty \frac{\exp \left( -\frac{x^2}{2} \right)}{\lambda_1} dy_2, \\
= 1 - \exp \left\{ \frac{\Pi_1 - 2}{\alpha (1 - \alpha) \rho_1 \lambda_2} \right\}. \tag{20}
\]
Now \( \hat{s}_{o2} \) minimization problem for U2 can be stated as
\[
(J4) : \text{minimize } \hat{s}_{o2}, \text{ subject to } (C1). \tag{21}
\]
Optimal solution for minimizing \( \hat{s}_{o2} \) is given by Lemma 4.

**Lemma 4:** The asymptotic optimal PA \( \hat{\alpha}_2 \) minimizing \( \hat{s}_{o2} \) can be given as
\[
\hat{\alpha}_2 = \Pi_2 - \sqrt{\Pi_2 \Pi_2(\Pi_2 - 1)}. \tag{22}
\]

**Proof:** By setting, \( \frac{d \hat{s}_{o2}}{d \alpha} = 0 \), \( \hat{\alpha}_2 = \Pi_2 + \sqrt{\Pi_2(\Pi_2 - 1)} \) is obtained. Here \( \hat{\alpha}_2 = \Pi_2 + \sqrt{\Pi_2(\Pi_2 - 1)} \) is infeasible, because it forces \( R^{\phi}_{th} < 0 \) which is infeasible since secrecy rate cannot be negative. Hence, \( \hat{\alpha}_2 \) for U2 is given as \( 22 \).

IV. **Maximization of Secrecy Fairness**
Next we formulate secrecy fairness optimization problem and, investigate globally-optimized PA to maximize fairness.

A. **Optimization Formulation**
Using (9) and (10), the secrecy fairness maximization problem which minimizes the maximum SOP between users under BS transmit power budget constraint can be stated as
\[
(J5) : \text{minimize } \max [s_{o1}, s_{o2}], \text{ subject to } (C1). \tag{23}
\]
Using \( x_c = \max [s_{o1}, s_{o2}] \), (J5) is formulated equivalently as
\[
(J6) : \text{minimize } x_c, \text{ subject to } (C1), \tag{24}
\]
\[
(C2) : s_{o1} \leq x_c, \quad (C3) : s_{o2} \leq x_c, \tag{24}
\]
where (C2) and (C3) comes from the definition of \( \max [\cdot] \).

B. **Power Control for Optimizing min-max Secrecy Outage**
Since \( (J6) \) is nonconvex problem because of (C2) and (C3) nonconvex constraints, we solve it by analyzing optimal candidates that are characterized by Karush-Kuhn-Tucker (KKT) conditions \([18]\). Global-optimal PA is given by Lemma 5.

**Lemma 5:** The global-optimal solution \( \alpha^*_{sop} \) of (J6), which minimizes the maximum SOP between users, is given as
\[
\alpha^*_{sop} = \arg\min_{a \in \{a_1,a_2,a_3\} \sim \alpha} \max [s_{o1}, s_{o2}], \tag{25}
\]
where \( a^*_1, a^*_2, a^*_3 \) are obtained using GSS by minimizing \( s_{o1}, s_{o2} \) (Section III(B)), and solving \( s_{o1} = s_{o2} \) respectively.

**Proof:** We consider boundary constraint (C1) implicit and associate Lagrange multipliers \( \eta_1 \) with (C2) and \( \eta_2 \) with (C3). Hence, Lagrangian function \( L \) of (J6) can be given as
\[
L \triangleq x_c + \eta_1 [s_{o1} - x_c] + \eta_2 [s_{o2} - x_c]. \tag{26}
\]
The corresponding KKT conditions are given by constraints (C1), (C2) and (C3). The dual feasibility conditions are given as
\[
\eta_1 \geq 0 \text{ and } \eta_2 \geq 0.
\]
The subgradient conditions are obtained as
\[
\alpha^*_{sop} \triangleq \arg\min_{a \in \{a_1,a_2,a_3\} \sim \alpha} \max [s_{o1}, s_{o2}].
\]
Here exists three cases. Case 1: \( \eta_1 > 0 \) and \( \eta_2 = 0 \), implies \( \frac{ds_{o1}}{dx} = 0 \) which results same solution of \( s_{o1} \) minimization, i.e., \( \alpha = \alpha^*_1 \). Case 2: \( \eta_2 > 0 \) and \( \eta_1 = 0 \), implies \( \frac{ds_{o2}}{dx} = 0 \), which results same solution of \( s_{o2} \) minimization \( \{14\} \) of U2, i.e., \( \alpha = \alpha^*_2 \). Case 3: \( \eta_1 > 0 \) and \( \eta_2 > 0 \), \( s_{o1} = s_{o2} \), which shows equal SOP for both users, and gives \( \alpha = \alpha^*_3 \). Thus, \( (J6) \) has three candidates, i.e., \( \alpha^*_1 \) and \( \alpha^*_2 \) for minimizing \( s_{o1} \) and \( s_{o2} \), respectively, and \( \alpha^*_3 \) is obtained from \( s_{o1} = s_{o2} \) condition. As a result, global-optimal \( \alpha^*_{sop} \) to \( (J6) \) problem is obtained at the optimal candidate for which maximum SOP between users is minimum.

C. **Closed-form Approximation of Optimal Power Allocation**
In above analysis, the min-max SOP optimization problem has been solved numerically. To gain analytical insights, next the asymptotic closed-form approximation of global-optimal PA for high SNR is derived. Here, the asymptotic secrecy fairness maximization problem can be formulated as
\[
(J7) : \text{minimize } \max [\hat{s}_{o1}, \hat{s}_{o2}], \text{ subject to } (C1). \tag{28}
\]
Considering \( \hat{x}_c = \max [\hat{s}_{o1}, \hat{s}_{o2}] \), (J7) can be rewritten as
\[
(J8) : \text{minimize } \hat{x}_c, \text{ subject to } (C1), \tag{29}
\]
\[
(C4) : \hat{s}_{o1} \leq \hat{x}_c, \quad (C5) : \hat{s}_{o2} \leq \hat{x}_c,
\]
where (C4) and (C5) also comes from definition of \( \max [\cdot] \). Globally optimized solution of (J8) is given in Lemma 6.

**Lemma 6:** Asymptotic global-optimal PA \( \hat{\alpha}_{sop} \) of min-max problem (J8) that maximizes secrecy fairness is given by
\[
\hat{\alpha}_{sop} = \arg\min_{a \in \{a_1,a_2,a_3\} \sim \alpha} \max [s_{o1}, s_{o2}], \tag{30}
\]
where \( \hat{\alpha}_1, \hat{\alpha}_2, \) are obtained by minimizing \( \hat{s}_{o1}, \hat{s}_{o2} \) (Section III(C)), respectively, and \( \hat{\alpha}_3 \) is derived by solving \( \hat{s}_{o1} = \hat{s}_{o2} \).

Proof: Associating lagrange multipliers \( \mu_1 \) and \( \mu_2 \), respectively, with (C4) and (C5), the Lagrangian function \( \hat{\mathcal{L}} \) can be written as

\[
\hat{\mathcal{L}} = \hat{x}_c + \mu_1[\hat{s}_{o1} - \hat{x}_c] + \mu_2[\hat{s}_{o2} - \hat{x}_c].
\]

The corresponding KKT conditions are obtained by constraints (C4) and (C5). The dual feasibility conditions are given as \( \mu_1 \geq 0 \) and \( \mu_2 \geq 0 \) using (29) and (31). The subgradient conditions are given as \( \frac{d\hat{\mathcal{L}}}{d\alpha} = 1 - \mu_1 - \mu_2 = 0, \) \( \frac{d\hat{\mathcal{L}}}{d\alpha} = \mu_1 \frac{d\hat{s}_{o1}}{d\alpha} + \mu_2 \frac{d\hat{s}_{o2}}{d\alpha} = 0 \). The complementary slackness conditions are given as

\[
\mu_1[\hat{s}_{o1} - \hat{x}_c] = 0, \quad \mu_2[\hat{s}_{o2} - \hat{x}_c] = 0.
\]

Similar to the numerical proof (Section IV(B)), here also three cases exist by analyzing KKT conditions. Case 1: \( \mu_1 > 0 \) and \( \mu_2 = 0 \), implies \( \frac{d\hat{s}_{o1}}{d\alpha} = 0 \) and results \( \alpha = \hat{\alpha}_1 \) \( \Box \) from \( \hat{s}_{o1} \) minimization. Case 2: \( \mu_2 > 0 \) and \( \mu_1 = 0 \), implies \( \frac{d\hat{s}_{o2}}{d\alpha} = 0 \). This gives \( \alpha = \hat{\alpha}_2 \Box \) as from \( \hat{s}_{o2} \) minimization. Case 3: \( \mu_1 > 0 \) and \( \mu_2 > 0 \), implies \( \hat{s}_{o1} = \hat{s}_{o2} \) from (32) and it gives \( \hat{\alpha}_3 \) which is obtained as

\[
\hat{\alpha}_3 = \frac{\Pi_2 \lambda_1 + \lambda_2 (1 - \Pi_1)}{\lambda_1 + \lambda_2},
\]

Since three candidates exist for minimization problem (78), the global-optimal solution \( \hat{\alpha}_{sop} \) is obtained at the candidate for which maximum SOP between users is minimum.

V. NUMERICAL INVESTIGATIONS

For generating numerical results, downlink of NOMA system with a BS and two users is considered. Near and far user distances from BS are considered as \( d_1 = 50 \) meter and \( d_2 = 100 \) meter, respectively. Noise signal for both users follows Gaussian distribution with a noise power of \(-60 \) dBm. Small scale fading follows exponential distribution with 1 mean value \( \mathbb{E} \). \( L_c = 1 \) and \( n = 2.5 \) are taken. The simulation results are sampled over \( 10^6 \) randomly generated channel realizations utilizing Rayleigh distribution for both the users. For GSS algorithm, \( \epsilon = 0.01 \). \( \rho_r \) is assumed as received SNR in decibels (dB) at U2. SOP is considered as performance metric to evaluate system performance.

A. Validation of Analysis

We first validate the closed-form expressions of SOP derived in section III. Fig. 2 presents validation results of \( s_{o1} \) with \( R_{th}^{th} \) for various \( \rho_r \), \( \alpha = 0.5 \) is taken. A close match between analytical and simulation results confirms the accuracy of analysis of \( s_{o1} \) with a RMSE of the order of \( 10^{-4} \). We observe from results that increasing \( R_{th}^{th} \) increase \( s_{o1} \). Considering the definition that the outage happens when the users’ maximum achievable secrecy rate falls below a target rate, it is obvious that increasing target secrecy rates at user increases SOP. Also, we observe that increasing \( \rho_r \) decreases \( s_{o1} \). This is because the achievable secrecy rates at users increase by increasing SNR, and hence, for a fixed target secrecy rate, SOP decreases.

B. Impact of variation of far user distance

Fixing \( d_1 = 50 \) meter, the impact of variation of \( d_2 \) from BS on achievable SOP is presented in Fig. 3. Fig. 3(a), demonstrate the effect of increasing \( d_2 \) on \( s_{o1} \), show that \( s_{o1} \) decreases with the increase in \( d_2 \). The is because, increasing \( d_2 \) implies a decrease in achievable data rate at U2 which results an improvement in secrecy rate at U1, and hence, SOP at U1 decreases. Also, decrease in data rate at U2 implies decrease in secrecy rate at U2 which increases SOP for U2 as shown in Fig. 3(b). It is noted that increasing the distance from BS to U2 has an contradicting effect on \( s_{o1} \) and \( s_{o2} \). Hence, we conclude that achievable SOP depends on distances of users.

C. Optimal Design Insights

Now optimal SOP is investigated in Fig. 4(a) and Fig. 4(b), which validate pseudoconvex nature of \( s_{o1} \) and \( \max [s_{o1}, s_{o2}] \), respectively, with \( \alpha \). The numerical optimal PAs are obtained.
using GSS algorithm. Asymptotic analysis is also verified with numerical results at high SNR, i.e., $\rho_r \geq 20$ dB. Here we observe that $\alpha$ decides PA to users, which highly effects SOPs. Hence, for given system parameters, the appropriate PA to users can ensure optimal secure communication system. Next, global-optimal $\alpha_{s_op}$ that provides secrecy fairness between users is shown in Fig. 5(a) as a function of $R_s^{th}$ for various $R_s^{th}$. Results indicate that there exist one and only one optimal $\alpha$ for each target secrecy rate pair $(R_s^{th}, R_s^{th})$. We observe that increasing $R_s^{th}$, $\alpha_{s_op}$ decreases, whereas the optimal SOP obtained from min-max optimization problem increases as shown in Fig. 5(b). It is also noted that lower value of $R_s^{th}$ compared to $R_a^{th}$ provides an improvement in SOP. Hence, we conclude that $\alpha_{s_op}$ that provides secrecy fairness to users highly depends on target secrecy rate pair $(R_s^{th}, R_a^{th})$.

D. Performance Comparison

To analyze the performance gain obtained by the proposed protocol for maximizing secrecy fairness, Fig. 6 demonstrates the performance comparison of globally optimized PA $\alpha_{s_op}$ with fixed PA $\alpha = 0.33$, and individual users’ optimal PAs $\alpha_1^*$ and $\alpha_2^*$, respectively, obtained by minimizing $s_{a1}$ and $s_{a2}$. Results indicates the percentage gain which depicts that $\alpha_{s_op}$ obtains best SOP performance, because of ensuring secrecy fairness between users. Note that the average percentage improvement by $\alpha_{s_op}$ over fixed PA, optimal PAs $\alpha_1^*$ and $\alpha_2^*$ are approximately 55.12%, 69.30% and 19.11%, respectively.

VI. CONCLUDING REMARKS

This paper has proposed a novel decoding order for a NOMA system with two untrusted users, that can provide positive secrecy rate to both users. With the objective of secrecy fairness between users, globally-optimized PA to minimize the maximum SOP between users is presented. Asymptotic solution is also obtained to gain analytical insights. Also, individual PAs minimizing SOPs for both the users, along with closed-form asymptotic expressions are presented. Numerical results are conducted to verify the correctness of analytical expressions as well as to provide insights on optimal performance and significant performance gains.

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