THE GROWTH OF THE STELLAR SEEDS OF SUPERMASSIVE BLACK HOLES

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ABSTRACT

The collapse of baryons into extremely massive stars with masses \(\gtrsim 10^4 M_\odot\) in a small fraction of protogalaxies at \(z \gtrsim 10\) is a promising candidate for the origin of supermassive black holes (SMBHs), some of which grow to a billion solar masses by \(z \sim 7\). We determine the maximum masses such stars can attain by accreting primordial gas. We find that at relatively low accretion rates the strong ionizing radiation of these stars limits their masses to \(M_* \sim 10^3 M_\odot (M_{\text{acc}}/10^{-3} M_\odot \text{yr}^{-1})^{8/7}\), where \(M_{\text{acc}}\) is the rate at which the star gains mass. However, at the higher central infall rates usually found in numerical simulations of protogalactic collapse (\(\gtrsim 0.1 M_\odot \text{yr}^{-1}\)), the lifetime of the star instead limits its final mass to \(\sim 10^6 M_\odot\). Furthermore, for the spherical accretion rates at which the star can grow, its ionizing radiation is confined deep within the protogalaxy, so the evolution of the star is decoupled from that of its host galaxy. Ly\(\alpha\) emission from the surrounding H\(\text{II}\) region is trapped in these heavy accretion flows and likely reprocessed into strong Balmer series emission, which may be observable by the James Webb Space Telescope. This, strong He\(\text{II}\) \(\lambda 1640\), and continuum emission are likely to be the key observational signatures of the progenitors of SMBHs at high redshift.

Key words: accretion, accretion disks – cosmology: theory – early universe – galaxies: formation – H\(\text{II}\) regions – stars: formation

Online-only material: color figures

1. INTRODUCTION

The existence of \(10^8\)–\(10^9 M_\odot\) black holes (BHs) in massive galaxies by \(z \sim 7\), less than a billion years after the big bang (Fan et al. 2003; Willott et al. 2003; Mortlock et al. 2011), remains one of the great mysteries of cosmological structure formation. In the \(\Lambda\)CDM paradigm, early structure formation is hierarchical, with small dark matter halos at early epochs evolving into ever more massive ones by accretion and mergers through cosmic time. Hence, it is generally held that the supermassive black holes (SMBHs) of the \(z \sim 7\) Sloan Digital Sky Survey quasars grow from much smaller seeds at high redshifts. The origin of these seeds, and how they reach such large masses by such early times, remains to be understood. At least four main processes have been proposed for their formation (see Volonteri 2010; Alexander & Hickox 2011; Khoi 2010): the collapse of Population III stars into 100–300 \(M_\odot\) BH at \(z \sim 20\) (e.g., Madau & Rees 2001; Alvarez et al. 2009; Milosavljevi\'c et al. 2009; Tanaka & Haiman 2009), the direct collapse of extremely hydrogen molecule-poor primordial gas in \(10^8\)–\(10^9\) \(M_\odot\) dark matter halos into \(10^4\)–\(10^5\) \(M_\odot\) BH at \(z \sim 10\) (e.g., Bromm & Loeb 2003; Koushiappas et al. 2004; Begelman et al. 2006; Lodato & Natarajan 2006; Spaans & Silk 2006; Omukai et al. 2008; Regan & Haehnelt 2009; Sethi et al. 2010; Inayoshi & Omukai 2011, see also Colgate et al. 2003), the collapse of dense primaeval star clusters into \(10^4\)–\(10^6\) \(M_\odot\) BH (see, e.g., Djorgovski et al. 2008; Devecchi & Volonteri 2009), and the collapse of primordial overdensities in the immediate aftermath of the big bang (see, e.g., Mack et al. 2007; Carr et al. 2010).

The processes by which BHs form at high redshift and evolve into SMBHs must account for how they become so large by \(z \sim 7\) and why their numbers at that redshift are so small, about 1 Gpc\(^{-3}\). Population III seed BHs are plentiful at \(z \sim 20–30\) but must grow at the Eddington limit without interruption to reach \(10^5\)–\(10^6 M_\odot\) by \(z \sim 7\). This is problematic because they and their progenitors either expel all the baryons from the shallow potential wells of the halos that create them, so they are "born starving" (Whalen et al. 2004; Johnson & Bromm 2007; Pelupessy et al. 2007; Alvarez et al. 2009; Jeon et al. 2011), or they eject themselves from their halos, and thus their fuel supply, at hundreds of km s\(^{-1}\) if they are born in core-collapse supernova explosions (Whalen & Fryer 2011). Also, accretion onto Population III BH has been found to be inefficient on small scales, typically at most 20\% Eddington (Milosavljevi\'c et al. 2009; Park & Ricotti 2011), making the constant duty cycles required for sustained growth difficult (but see Li 2011).

If halos can instead congregate into primitive galaxies of \(\sim 10^6 M_\odot\) at \(z \sim 10–15\) that are devoid of the coolant molecular hydrogen (H\(_2\)), they reach virial temperatures of \(\sim 10^4\) K and begin to atomically cool. Numerical simulations of this process (Wise et al. 2008; Regan & Haehnelt 2009; Shang et al. 2010) find that analogous to Population III star formation in much smaller halos at higher redshifts (Nakamura & Umemura 2001; Bromm et al. 2002; Abel et al. 2002; O’Shea & Norman 2007; Yoshida et al. 2008; Turk et al. 2009; Stacy et al. 2010; Clark et al. 2011; Greif et al. 2011), baryons rapidly pool at the center of the halo and form a hydrostatic object. But this object is thought to become far more massive than Population III stars modeled to date because atomic line cooling and the deeper potential well of the halo lead to much higher infall rates at its center, 0.1–1 \(M_\odot\) yr\(^{-1}\) rather than the \(10^{-4} M_\odot\) yr\(^{-1}\) typical of primordial star-forming minihalos at \(z \sim 20\). Such objects could collapse into BHs that are far more massive than Population III BHs (e.g., Shibata & Shapiro 2002), with Bondi–Hoyle accretion rates that allow them to grow into SMBHs in

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less time (e.g., Wyithe & Loeb 2011; but see Dotan et al. 2011).

One difficulty with this scenario is that it is not yet understood how primitive galaxies can form in the high Lyman–Werner UV backgrounds needed to fully suppress H$_2$ molecule formation and quench Population III star formation in its constituent halos prior to assembly (1–10 times those expected at the epoch of reionization at $z \sim 6$; Dijkstra et al. 2008; Shang et al. 2010; Wolcott-Green et al. 2011; see also Agarwal et al., in preparation). Furthermore, it is not known if the object at the center of the protogalaxy even becomes a star, because at such high accretion rates it reaches very large masses on timescales that are short in comparison to Kelvin–Helmholtz times and the onset of nuclear burning (although Ohkubo et al. 2009 have modeled the evolution of Population III stars under accretion at much lower rates). It could be that the central object reaches such high entropies and densities that it is enveloped by an event horizon before reaching the main sequence (Fryer et al. 2001). If the central object becomes a star, which governs its rate of growth, its final mass, and thus the mass of the SMBH seed it becomes? Numerous authors have studied how stellar radiation regulates accretion onto Population III protostars, but at much lower inflow rates than those in the centers of collapsing protogalaxies (e.g., Stahler et al. 1986; Omukai & Palla 2001; Omukai & Inutsuka 2002; McKee & Tan 2008; Stacy et al. 2012a; Hosokawa et al. 2011). Do the much higher accretion rates in primitive galaxies quench radiative feedback?

We have performed extensive semi-analytical calculations of radiative feedback by Population III supermassive stars in collapsing protogalaxies at $z \sim 10$. In Section 2, we examine the primary forms of radiative feedback on accretion onto the star (deferring the processes that can be ignored to thorough examination in the Appendix). We also derive the maximum mass that the supermassive star, and hence the SMBH seed, can achieve as a function of accretion rate, accounting for its prodigious ionizing UV flux. In Section 3, we estimate the maximum mass that the star can reach if it is limited by its finite lifetime rather than by radiative feedback. We discuss the observational signatures of rapidly accreting supermassive protostellar objects in Section 4. Finally, in Section 5 we review the implications of final supermassive stellar mass for SMBH seed mass and the appearance of the first quasars in the universe.

2. RADIATIVE FEEDBACK-LIMITED ACCRETION

We adopt an analytical approach to estimate the maximum mass that an accreting protostellar object can ultimately attain in a collapsing $10^7$–$10^9 M_\odot$ protogalaxy. For simplicity, we assume that accretion onto the star is constant and spherically symmetric. We first consider the ionizing UV radiation emitted by the star, which is the dominant form of radiative feedback on accretion in the ionized volume (first term on the right-hand side in Equation (3)) and the rate at which neutral atoms enter the H$_\text{II}$ region through the accretion flow (second term on the right-hand side in Equation (3)). Following Raiter et al. (2010), we note that the rate at which ionizations occur is higher than by a factor $P$, which accounts for corrections to the neutral hydrogen level populations at the high temperatures ($\geq 10^4$ K, e.g., Whalen et al. 2004; Alvarez et al. 2006) expected for the H$_\text{II}$ region of massive Population III stars. To account for this rate, we take it that the total ionization rate in the H$_\text{II}$ region is $Q_{\text{eff}} = P Q$, where $P$ is the rate of the average ionizing photon energy and the ionization potential for neutral hydrogen, 13.6 eV. Integrating over the density profile of the gas in Equation (1) to find the radius at which the number of recombinations in the enclosed volume balances the number of photoionizations, we arrive at the following expression for $r_{1H}$:

$$Q_{\text{eff}} \simeq \int_{r_c}^{r_{1H}} 4 \pi \alpha_B r^2 n^2 dr + \frac{M_{\text{acc}}}{\mu m_H}$$

$$= \int_{r_c}^{r_{1H}} 4 \pi \alpha_B r^2 \left[ \frac{M_{\text{acc}}}{4 \pi r^2 v(r) \mu m_H} \right]^2 dr + \frac{M_{\text{acc}}}{\mu m_H},$$

where $\alpha_B \simeq 1.4 \times 10^{-13}$ cm$^3$ s$^{-1}$ is the recombination coefficient for $\gtrsim 2 \times 10^4$ K photoionized primordial gas (Whalen et al. 2004; Osterbrock & Ferland 2006). Following Omukai & Inutsuka (2002), for simplicity we neglect the second term in our
calculations, as it is in general much smaller than $Q_{\text{eff}}$. This is true in particular for the solutions that we find for the minimum accretion rate onto a star of a given mass; at much higher rates, this term may become large, likely limiting the radius of the H II region.

Finally, once the accreting gas crosses the stationary ionization front (as shown in Figure 1), the equation that governs its deceleration due to momentum imparted by photoionizations is

$$\frac{dv}{dr} = -\frac{\alpha B h \nu}{\mu m_H c} = -\frac{\alpha B h \nu}{\mu m_H c} \left[ \frac{\dot{M}_{\text{acc}}}{4 \pi r^2 v(r) \mu m_H} \right],$$

where again we have assumed that the photoionization rate of hydrogen is balanced its recombination rate, $\alpha B h \nu$, and that the momentum transferred to the gas per photoionization is $h \nu / c$. We note again that here we have not included the force due to gravity or that due to Thomson scattering, as these two forces cancel one another since the star is assumed to radiate at the Eddington rate. For the massive primordial stars that we consider here, we take the effective surface temperature of the star to always be $\sim 10^5$ K, which yields an average energy per ionizing photon $h \nu \simeq 29$ eV. In turn, this implies $P \equiv h \nu / 13.6$ eV = 2.1, which we assume throughout our study. In Equation (4) we again choose case B, as we assume that recombinations to the ground level of hydrogen result in the emission of photons which do not deposit a net momentum to the gas (being emitted isotropically and contributing to the diffuse radiation field).\(^5\)

To find the solutions for steady state accretion, we must solve the equation of motion, Equation (4). In particular, to find the minimum accretion rate for which an inflow solution exists, we search for solutions for which the infall velocity of the gas goes to zero at the stellar surface $r_*$:

$$v(r_*) = 0.$$ (5)

That is, we must simultaneously solve Equations (3) and (4), under the constraints given by Equations (2) and (5). These constraints yield the maximum mass that the star can achieve by steady accretion before its intense ionizing radiation halts infall at $r > r_*$ and prevents its further growth. On the other hand, they also yield the lower bound for the final mass of the central object, since for $v(r_*) > 0$ radiative feedback fails to cutoff accretion and the star will grow even faster than when gas merely comes to a halt on its surface.

The solution to Equation (4) has the general form

$$v(r) = \left( \frac{3 \alpha B h \nu \dot{M}_{\text{acc}}}{4 \pi \mu m_H^2 c^2} \right) r^{-1} - K.$$ (6)

where $K$ is a constant whose value must satisfy the constraint that $v(r_*) = 0$. This yields for $K$:

$$K = \left( \frac{3 \alpha B h \nu \dot{M}_{\text{acc}}}{4 \pi \mu m_H^2 c^2} \right) r_*^{-1}.$$ (7)

The equation for the infall velocity in the H II region then becomes

$$v(r) = \left[ \frac{3 \alpha B h \nu \dot{M}_{\text{acc}}}{4 \pi \mu m_H c^2} \right] r^{-1} - \left[ \frac{3 \alpha B h \nu \dot{M}_{\text{acc}}}{4 \pi \mu m_H c^2} \right] r_*^{-1}.$$ (8)

As shown in Figure 2, these solutions give a relatively constant velocity for the gas in the H II region until it is very close to the stellar surface. In turn, Equation (1) implies that $n(r)$ is essentially $\propto r^{-3}$ in the H II region.

Applying this $v(r)$ (and hence $n(r)$) to the integral in Equation (3) and evaluating it yields $r_{\text{HII}}$:

$$r_{\text{HII}} = \left[ \frac{4 \pi M_*}{\alpha B h \nu \mu m_H} \right] \left( \frac{\dot{M}_{\text{acc}}}{c} \right)^2 r_*^{-1}.$$ (9)

Finally, we must satisfy the constraint given by Equation (2) to find the solution for the minimum accretion rate. Substituting Equations (8) and (9) into Equation (2) and rearranging the terms gives the following quadratic equation in $\dot{M}_{\text{acc}}^2$:

$$0 = \left( \frac{2 G M_*}{r_*} \right) \dot{M}_{\text{acc}}^4 - \left( \frac{Q_{\text{eff}} h \nu}{c} \right)^2 \dot{M}_{\text{acc}}^2 - \left( \frac{8 \pi G M_* Q_{\text{eff}}^2}{3 \alpha B} \right) \left( \frac{h \nu \mu m_H}{c} \right)^2,$$ (10)

whose solution is

$$\dot{M}_{\text{acc}}^2 = \left( \frac{Q_{\text{eff}} h \nu}{c} \right)^2 + \left( \frac{Q_{\text{eff}} h \nu}{c} \right)^4 + \left( \frac{8 \pi G M_* Q_{\text{eff}}^2}{3 \alpha B} \right) \left( \frac{h \nu \mu m_H}{c} \right)^2.$$ (11)
rates below those shown here is not possible, as in such cases partly because the vast majority of these photons are trapped within accreting onto primordial stars as a function of distance
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Figure 2. Infall velocity (top panel) and density (bottom panel) of the gas accreting onto primordial stars as a function of distance r from the center of the star for three stellar masses: 10^{2} (dark blue), 10^{4} (red), and 10^{6} M_{\odot} (light blue). Each star accretes at the minimum rate (as labeled) its strong ionizing flux permits, as given by Equation (11). The dotted lines show the radius \( \text{r}_{H} \) of the H II regions surrounding the stars, with their colors corresponding to those of the respective stellar masses. Outside the H II region, the gas is in free fall, but after crossing into the H II region boundary it is rapidly photoionized and decelerated until it arrives at the stellar surface with velocity \( v(\text{r}_{*}) = 0 \). The dashed lines show the trapping radius \( r_{\text{trapping}} \) for Ly\alpha photons from recombinations in the H II region (assuming a gas temperature of 4 \times 10^{4} K; see Equation (22)); partly because the vast majority of these photons are trapped within \( r_{\text{trapping}} \), we can neglect Ly\alpha scattering feedback as discussed in Section 2.2. Accretion at rates below those shown here is not possible, as in such cases \( r_{H} \rightarrow \infty \) and photoionization pressure halts infall at all radii.

(A color version of this figure is available in the online journal.)

Noting that the second term in the square brackets is much larger than the first, we have

\[
\dot{M}_{\text{acc}} \approx \left( \frac{4\pi Q_{\text{eff}}^{3} r_{*}}{3 \alpha_{B}} \left( \frac{hv\mu m_{\text{H}}}{c} \right)^{3} \right)^{1/4}, \tag{12}
\]

which exactly matches the accretion rate for which the H II region breaks out to infinity (see Equation (9)) and therefore stops accretion entirely because photoionization pressure exerts a net outward force on gas at all radii.\(^{6}\) Thus, we see that the H II region is confined and that accretion proceeds at infall rates higher than those given by Equation (12) and that no inflow solutions exist for lower rates.\(^{7}\)

Now, by expressing \( Q \) (where again \( Q_{\text{eff}} = P Q \), with \( P = 2.1 \)) and \( r_{*} \) just in terms of the stellar mass \( M_{*} \), we can find the maximum stellar mass for which accretion onto the star is permitted (at \( M_{\text{acc}} \) in Equation (12)). We can take it that, if the star is thermally relaxed\(^{8}\) and radiating at the Eddington limit (Begelman 2010), then the following two equations relate the mass \( M_{*} \) of the star to its ionizing photon emission rate \( Q \) and radius \( r_{*} \):

\[
Q = 1.6 \times 10^{50} s^{-1} \left( \frac{M_{*}}{100 M_{\odot}} \right) \tag{13}
\]

and

\[
r_{*} = 3.7 R_{\odot} \left( \frac{M_{*}}{100 M_{\odot}} \right)^{1/2}. \tag{14}
\]

These are from Table 1 of Bromm et al. (2001) and are in good agreement with Schaerer (2002) and Begelman (2010). With these expressions, we find that the solution to Equation (12) is

\[
M_{*} \approx 10^{3} M_{\odot} \left( \frac{\dot{M}_{\text{acc}}}{10^{-3} M_{\odot} \text{yr}^{-1}} \right)^{8/7}, \tag{15}
\]

where \( M_{*} \) is the maximum stellar mass attainable under accretion of gas at a rate \( \dot{M}_{\text{acc}} \).

We plot this maximum stellar mass in Figure 3. As we shall see, the feedback due to ionizing radiation emitted by the star can limit its mass at relatively low accretion rates, which we term “feedback-limited accretion.” However, at higher accretion rates the finite lifetime of the star governs its maximum mass; we term this “time-limited accretion.” We discuss why this is so in Section 3.

Although we have focused on the pressure due to hydrogen photoionizations, it is straightforward to include He\,\text{i} photoionizations under the condition that within the H II region He\,\text{i} is also photoionized to He\,\text{ii}. For the relatively hard spectra of massive primordial stars this is a sound assumption because the number of He\,\text{i}-ionizing photons is comparable to the number of H\,\text{i}-ionizing photons (Schaerer 2002). The ratio of the radiation pressures from helium and hydrogen photoionization is then

\[
\frac{p_{\text{He\,\text{i}}}}{p_{\text{H\,\text{i}}}} \approx \frac{n_{\text{He}}}{n_{\text{H}}} \frac{hv_{\text{He\,\text{i}}}}{hv_{\text{H\,\text{i}}}} \frac{\alpha_{\text{He\,\text{i}}}}{\alpha_{\text{H\,\text{i}}}} \approx 0.14, \tag{16}
\]

where the first term is the ratio of helium and hydrogen number densities, which is \( \approx 0.1 \). The second term is the ratio of the average ionizing photon energies for He\,\text{i} and H\,\text{i}, which by integrating the stellar spectrum of a 10^{5} K primordial star we find to be 38 eV/29 eV \( \approx 1.3 \). The third term is the ratio of He\,\text{i} and H\,\text{i} case B recombination coefficients, which is \( \approx 1.1 \) (Osterbrock & Ferland 2006). While this is a relatively modest increase in the radiation pressure, for completeness we include it as a multiplicative coefficient when solving Equation (4). Other forms of radiation pressure are not so easily accommodated by our analytical approach but can be neglected without strongly affecting our conclusions, as we discuss in the next section and in the Appendix.

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\(^{6}\) While in our solution the H II region formally extends to infinity in this case, for accretion to be terminated it need only extend to the Bondi radius, outside of which gas cannot be gravitationally captured by the star.

\(^{7}\) Relatively small deviations from the solutions we have found for \( v(\text{r}) \) at \( \text{r}_{H} \) may result in a shock developing there. However, even for a strong shock for which there is a density jump of a factor of four, the minimum accretion rates in Equation (11) change by \( \lesssim 20 \) percent.

\(^{8}\) At the minimum accretion rates we find for a given stellar mass (Equation (15)), the “trapping” radius due to electron scattering (which is proportional to the accretion rate, e.g., Begelman 1978) is always smaller than the stellar radius given by Equation (14). Therefore, the main-sequence stars we consider here are thermally relaxed (see also Ohkubo et al. 2009). At higher accretion rates, the radiation emitted by the star can become trapped due to electron scattering, resulting in an expansion of the stellar photosphere. In this case, the star emits fewer ionizing photons than given by Equation (13) and the radiative feedback is thus weakened, as we discuss briefly in the Appendix.
emitted per second $Q$ to the stellar mass $M_*$ with Equation (13). The momentum of the accretion flow is

$$M_{\text{acc}} v \simeq 6 \times 10^{33} \text{ g cm}^{-2} \left( \frac{v}{10^3 \text{ km s}^{-1}} \right) \left( \frac{M_{\text{acc}}}{M_\odot \text{ yr}^{-1}} \right),$$

where we have normalized the infall velocity $v$ to its typical value at the edge of the H II region in the solutions shown in Figure 2, since this is where Lyman $\alpha$ photons will be most strongly coupled to the gas (indeed, these photons are trapped within the accretion flow at $r_{\text{HI}}$, as shown below). Equating these two expressions and using the relation between $M_{\text{acc}}$ and $M_*,\text{max}$ in Equation (15), we find the conditions under which Lyman $\alpha$ radiation pressure could halt accretion:

$$v \lesssim 600 \text{ km s}^{-1} \left( \frac{M_{\text{acc}}}{0.1 M_\odot \text{ yr}^{-1}} \right)^{1/7}. \quad (19)$$

Since the inflow velocity near the edge of the H II region is well above this value at the typical minimum accretion rates we find ($\lesssim 0.1 M_\odot \text{ yr}^{-1}$; see Figure 2), it is clear that Lyman $\alpha$ radiation pressure has only a small impact on accretion in comparison to photoionization pressure. In particular, over the range of stellar masses and critical accretion rates that we consider (Figures 2 and 3), we find that the infall velocity is $\sim 3$ times that in Equation (19). In other words, at the high accretion rates we find, the momentum of inflow is always roughly three times what could be countered by Ly$\alpha$ scattering.

It might be thought that if a Lyman $\alpha$ photon is scattered and then traverses the H II region, it could subsequently be scattered many times, thereby enhancing the momentum it imparts the gas (see Adams 1972; Milosavljević et al. 2009). However, upon scattering from an atom in the rapid ($\sim 10^3 \text{ km s}^{-1}$) accretion flow, the photon would be strongly blueshifted relative to the gas entering the opposite side of the H II region; as a result, it would couple to gas much less strongly thereafter, greatly limiting the degree to which it could add more momentum to the gas (see Dijkstra & Wyithe 2010 on how galactic outflows enhance Ly$\alpha$ escape by this process).

Furthermore, another effect which dramatically lessens the impact of Ly$\alpha$ photons on accretion is that their large resonant scattering cross section ensures that they are trapped in the H II region and do not scatter out to larger radii. To see this, we follow the formula given by Begelman (1978) for the trapping radius due to Thomson scattering, $r_{\text{tr}} \equiv M_{\text{acc}} \sigma_T / 4\pi n_1 m_1 c$. We define the trapping radius for Ly$\alpha$ photons $r_{\text{tr}}$ by substituting the cross section for Lyman $\alpha$ scattering $\sigma_{\text{tr}}$ for the Thomson cross section $\sigma_T$ for electron scattering, and by accounting for the fraction of neutral hydrogen atoms off of which Ly$\alpha$ photons can scatter. This yields

$$\rho_{\text{tr}} \simeq 5 \times 10^{24} \text{ cm} \left( \frac{\dot{M}_{\text{acc}}}{M_\odot \text{ yr}^{-1}} \right) f_{\text{HI}} \left( \frac{T}{10^4 \text{ K}} \right)^{-1/2}, \quad (20)$$

where $f_{\text{HI}}$ is the neutral hydrogen fraction, $T$ is the temperature of the ionized gas, and we have used $\sigma_{\text{tr}} = 5.9 \times 10^{-14} \text{ cm}^2 (T/10^4 \text{ K})^{-1/2}$ (e.g., Milosavljević et al. 2009). The fraction of neutral hydrogen in the H II region can be estimated by assuming

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8 We note that in the high densities in the H II regions we consider (see Figure 2), frequent collisions can prevent the radiative decay of hydrogen via two photon emission, which raises the Ly$\alpha$ luminosity above that expected in the low density regime, where $\beta_{\text{Ly}\alpha} \simeq 0.68$ (Spitzer 1978).

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photoionization equilibrium\textsuperscript{10}, we obtain
\[ f_{\text{Hi}} \simeq \frac{4\pi \alpha \eta n(r) r^2}{Q_{\text{eff}} \sigma_{\text{Hi}}}, \]
\[ = 3 \times 10^{-5} \left( \frac{n}{10^{10} \text{ cm}^{-3}} \right) \left( \frac{r}{10^{15} \text{ cm}} \right)^2 \left( \frac{Q_{\text{eff}}}{10^{50} \text{ s}^{-1}} \right)^{-1}. \tag{21} \]
where \( \sigma_{\text{Hi}} = 6 \times 10^{-18} \text{ cm}^2 \) is the photoionization cross section for \( \text{H} \) (although it is likely slightly lower due to the relatively hard spectrum of the star; see, e.g., Johnson & Khochfar 2011). Then with this \( f_{\text{Hi}}, Q \) from Equation (13), \( n(r) \) from Equation (1), and \( M_{\text{acc}} \) in terms of \( M_* \) from Equation (15), we find
\[ r_{\text{Ly} \alpha} \simeq 10^{12} \text{ cm} \left( \frac{M_*}{100 M_\odot} \right)^{3/4} \]
\[ \times \left( \frac{\nu}{10^3 \text{ km s}^{-1}} \right)^{-1} \left( \frac{T}{10^4 \text{ K}} \right)^{-1/2}. \tag{22} \]

Assuming a temperature of \( 4 \times 10^4 \text{ K} \) for the photoionized primordial gas (Whalen et al. 2004; Kitayama et al. 2004), \( \text{Ly} \alpha \) trapping radii for three cases are shown in Figure 2. We see that \( r_{\text{Ly} \alpha} \) is \( \sim 2-6 r_* \) and is smaller than \( r_{\text{Hi}} \) by roughly an order of magnitude. While this is a modest fraction of the volume of the \( \text{H} \) region, because of its steep density profile \( (n \propto r^{-2}) \) most of the \( \text{Ly} \alpha \) photons originate from this region.

Integration of Equations (3) shows that the fraction of recombinations produced in the \( \text{H} \) region as a function of \( r \) is \( \sim (r_*^{-1} - r^{-1})/(r_*^{-1} - r_{\text{Hi}}^{-1}) \). Consequently, the fraction of \( \text{Ly} \alpha \) photons originating from within the trapping radius is \( (r_*^{-1} - r^{-1})/(r_*^{-1} - r_{\text{Hi}}^{-1}) \) or \( 
\geq 0.5 \). Thus, most of them are trapped deep in the \( \text{H} \) region and cannot propagate outward to slow accretion at greater radii. \( \text{Ly} \) scattering thus removes only \( \sim 15\% \) of the momentum of infall, not \( 30\% \), so we are justified in neglecting its impact on accretion flow.

At this point, we can show that \( \text{Ly} \alpha \) photons created outside \( r_{\text{Ly} \alpha} \) but within \( r_{\text{Hi}} \) are also trapped in the accretion flow at the boundary of the \( \text{H} \) region due to the huge optical depth of the neutral gas to these photons. This is easily shown by Equation (20), which for a largely neutral medium (i.e., \( f_{\text{Hi}} \gtrsim 1 \) ) yields a trapping radius on the order of \( 10^{21} \text{ cm} \) for the lowest accretion rates given by our solution (Equation (12)); as this is much larger than both \( r_{\text{Hi}} \) and the Bondi radius, the maximum distance from which the protogalactic gas in can accrete onto the star.\textsuperscript{11} Therefore, we can also safely conclude that \( \text{Ly} \alpha \) photons will not escape from the \( \text{H} \) region to affect the dynamics of the accretion flow at larger radii.

We note that although most \( \text{Ly} \alpha \) emission is trapped deep in the \( \text{H} \) region, the thermal state of the gas may not be greatly affected because cooling via other atomic transitions in hydrogen still occurs (see, e.g., Omukai 2001; Raiter et al. 2010; Schleicher et al. 2010). At the high densities within the \( \text{H} \) region atoms of both hydrogen and helium are readily excited to higher energy \((n > 1)\) states by both collisions and absorption of photons; in turn, Lyman series photons (principally \( \text{Ly} \alpha \)) are easily destroyed by ionizing these excited atoms (see, e.g., Osterbrock & Ferland 2006). However, the radiative decay of excited hydrogen atoms also results in significant emission of Balmer series photons as well as two-photon emission from the \( \text{H}(2s) \) state, to which the \( \text{H} \) is much less optically thick. The net result is that the energy in Lyman series photons is reprocessed largely into Balmer series and two-photon continuum emission that escapes the \( \text{H} \) region and cools the gas (see, e.g., Schleicher et al. 2010). Thus, photoionization heating of the gas can still be balanced by efficient radiative cooling, and even though much of the stellar radiation is reprocessed in the \( \text{H} \) region it still eventually emerges and may be observable (see Section 4).\textsuperscript{12} It follows that the luminosity of the outgoing radiation is always the same as that of the star; since the star is taken to shine at the Eddington limit, this in turn implies that the force due to electron scattering indeed balances the force due to gravity everywhere in the highly ionized \( \text{H} \) region, as we assumed in our calculation. This also suggests that the temperature of the gas is low enough to keep the flow supersonic so gas pressure cannot stop accretion, as we discuss in the Appendix.

Overall, we conclude that it is the momentum imparted to the accreting gas by ionizing photons, despite their being mostly converted to \( \text{Ly} \alpha \) photons that are in turn converted largely to Balmer series photons, that primarily regulates the growth of supermassive primordial stars.

3. TIME-LIMITED ACCRETION

As noted by Begelman (2010), the lifetime of a very massive primordial star burning nuclear fuel at the Eddington rate \( L_{\text{Edd}} \) and growing by accretion at a constant rate is \( t_{\text{life}} \gtrsim 4 \text{ Myr} \), twice the lifetime of non-accreting Population III stars of similar mass.\textsuperscript{13} However, the stellar lifetime is in fact shorter than this for two reasons. First, nuclear fuel must be burned at a higher rate than the Eddington luminosity \( L_{\text{Edd}} \) in order to support the star against its constantly increasing mass. Thus, the star consumes fuel at the rate given by the sum of the Eddington rate and the rate at which the binding energy of the star increases (Begelman 2010):
\[ L_{\text{acc}} \simeq L_{\text{Edd}} + \frac{G M_* M_{\text{acc}}}{r_*} = L_{\text{Edd}} \left( 1 + \frac{r_{\text{trap}}}{r_*} \right), \tag{23} \]
where \( r_* \) is the radius of the star (Equation (14)), \( r_{\text{trap}} \equiv M_{\text{acc}} \sigma T/4\pi m_{\text{H}} c = 6 \times 10^{13} \text{ cm} \) \((M_{\text{acc}}/M_\odot \text{ yr}^{-1}) \) is the radius at which radiation is trapped in the accretion flow by electron scattering, and \( L_{\text{Edd}} \equiv 4\pi G M_* m_{\text{H}} c / \sigma T = 1.2 \times 10^{40} \text{ erg s}^{-1} \) \((M_*/100 M_\odot) \) is the Eddington luminosity for a star of mass \( M_* \). This effect of super-Eddington nuclear burning contributes to the gradual turnover of the stellar lifetime-limited maximum stellar mass curve shown in red in Figure 3.

While Equation (23) is valid when radiation can escape from the stellar surface at \( r_* \) and the star is thus thermally...
relaxed, it ceases to be when radiation is trapped in the accretion flow within a radius $r_{\text{trap}}$ above the star due to Thomson scattering. This occurs for accretion rates above $M_{\text{acc,trap}} \simeq 4.3 \times 10^{-3} M_\odot \, \text{yr}^{-1} \left( M_\odot / 100 M_\odot \right)^{1/2}$. As radiation is trapped in the accretion flow, the accreting material cannot radiate its energy within $r_{\text{trap}}$. Upon reaching the stellar surface it therefore deposits energy at a rate $G M_\odot M_{\text{acc}} / r_{\text{trap}}$, instead of at the higher rate $G M_\odot M_{\text{acc}} / r_*$ assumed in Equation (23). Therefore, with this substitution is Equation (23), at accretion rates $M_{\text{acc}} > M_{\text{acc,trap}}$ the star consumes fuel as twice the Eddington rate. This results in a decreased lifetime $t_{\text{life}}$, which in turn leads to the sharp turnover of the time-limited maximum stellar mass curve in Figure 3.

The second reason that the stellar lifetime is shortened is also related to the trapping of radiation above the surface of the star, as when this occurs the stellar envelope ceases to be convective (Belgeman 2010). This results in the envelope material never reaching the core and being available for fusion. The fuel supply of the star is thus limited to the mass within its convective core, whose fraction of the total mass is (Equation (26) of Belgeman 2010)

$$\frac{M_{\text{conv}}}{M_*} \simeq 0.54 \left( \frac{r_{\text{trap}}}{r_*} \right)^{2/3} \left( \frac{M_{\text{acc}}}{M_\odot \, \text{yr}^{-1}} \right)^{-2/3} \left( \frac{M_*}{10^6 M_\odot} \right)^{1/3},$$

(24)

where we have assumed a central temperature of $10^8$ K. This second effect comes into play on the right-hand side of the dashed line separating the fully convective regime from the partially convective regime (also separating the regimes in which $r_* > r_{\text{trap}}$ and $r_* < r_{\text{trap}}$).

As shown in Figure 3, these two effects ultimately limit the final mass of the star to $\sim 10^6 M_\odot$ at the highest accretion rates ($\sim 1 M_\odot \, \text{yr}^{-1}$) found in cosmological simulations of the formation of supermassive stars via direct protogalactic collapse (e.g., Wise et al. 2008; Shang et al. 2010; Johnson et al. 2011). That said, we emphasize that our calculations have been done with significant simplifying assumptions, some of which we briefly discuss in Section 5. In particular, we have adopted a simple model for the evolution of rapidly accreting supermassive stars; however, to be fully confident in our results will require dedicated stellar evolution calculations that account for the continued growth of the star over a large portion of its lifetime (A. Heger et al., in preparation).

Finally, we note that “dark stars,” which are powered by dark matter annihilation, could grow to considerably higher masses than those powered by fusion (e.g., Freese et al. 2008). This is because the cooler surface temperatures of such stars exert much less radiative feedback on accretion and dark matter fuel may last for a much longer time than nuclear fuel. However, recent high-resolution cosmological simulations show that primordial stars forming at the centers of dark matter halos may not remain there because of dynamical interactions, as required for the continual capture and annihilation of dark matter in their interiors (Stacy et al. 2012b; Greif et al. 2011). Therefore, it may be that the final masses of dark stars are not so different from those expected for standard primordial stars (see also Ripamonti et al. 2010). If, however, dark stars do grow to be more massive they would exhibit spectral properties distinct from those of solely fusion-powered supermassive Population III stars (Zackrisson et al. 2010; Ilie et al. 2011), as we discuss next.

4. OBSERVATIONAL SIGNATURES

As supermassive stars are intense sources of radiation that could be detected by current and future surveys, we now examine their observable signatures. The fact that radiative feedback in most cases cannot terminate accretion onto supermassive Population III stars in collapsing protogalaxies implies that their H II regions will be confined deep in their host halos for most of their lives. It follows that their ionizing radiation is largely reprocessed into nebular emission instead of escaping the halo and reionizing the intergalactic medium (IGM). As discussed in Section 2.2, because the accretion flow is optically thick to the Lyα photons, they cannot directly exit the halo and be observed. These photons are instead further reprocessed largely into Balmer series photons which do escape the H II region and halo and propagate into the IGM. Therefore, one likely signature of rapidly accreting, isolated supermassive Population III stars in high-redshift protogalaxies is a strong Balmer line flux accompanied by a conspicuous lack of Lyα emission. A detailed radiative transfer calculation is necessary to quantitatively predict luminosities for all the Balmer lines in hydrogen, but we can place lower limits on the Hα flux, which is expected to be the dominant line.

Following Schaerer (2002) and Raiter et al. (2010), we compute the luminosity $L_{H\alpha}$ in Hα as a function of $Q_{\text{eff}}$ as discussed in Section 2.1. Relating this ionizing photon emission rate to stellar mass with Equation (13) yields $L_{H\alpha} \approx 4 \times 10^{38} \left( M_\odot / 100 M_\odot \right) \, \text{erg s}^{-1}$; we again emphasize that this is a lower limit because we exclude reprocessing of Lyα into Hα, which may dramatically boost Hα luminosities above those estimated here. The Hα flux at redshift $z$ is

$$f_{H\alpha} = \frac{L_{H\alpha}}{4\pi D_L^2} \approx 10^{-20} \text{ erg s}^{-1} \text{ cm}^{-2} \left( \frac{L_{H\alpha}}{10^{40} \text{ erg s}^{-1}} \right) \left( \frac{1+z}{10} \right)^{-2},$$

(25)

where $D_L(z)$ is the luminosity distance to redshift $z$. We plot this flux as a function of stellar mass and redshift in Figure 4, with the arrows on the Hα curves signifying lower limits.

The hard spectrum of ionizing radiation from hot primordial stars also creates an H II region from which a large luminosity in H II $\lambda 1640$ emission is expected (Oh et al. 2001; Tumlinson et al. 2001; Bromm et al. 2001; Schaerer 2002; Johnson et al. 2009). Since the optical depth to this line is low, we also estimate how much of its flux exits the halo. For the large ionization rates and densities in our study, we can apply the standard model of Raiter et al. (2010) to compute the luminosity of this line, which we find to be $L_{1640} \approx 2 \times 10^{38} \left( M_\odot / 100 M_\odot \right) \, \text{erg s}^{-1}$, where we again have used Table 1 of Bromm et al. (2001) for the number of He II-ionizing photons produced as a function of stellar mass. Then, replacing $L_{H\alpha}$ with $L_{1640}$ in Equation (34), we derive the flux in the He II $\lambda 1640$ line, which we plot with $f_{H\alpha}$ in Figure 4 as a function of stellar mass and redshift. Also shown in Figure 4 are detection limits for two instruments that will be on board the James Webb Space Telescope (JWST): the Near-Infrared Spectrograph (NIRSpec) and the Mid
of rapidly accreting supermassive stars is strong continuum emission below the Lyman limit, which is the sum of the stellar continuum and the nebular continuum (e.g., Raifer et al. 2010); the latter would be substantial due to the exceptionally high densities expected in the H ii regions of these stars. Indeed, it is possible that the sum of the stellar and nebular emission could be detected, for example, in the Deep-Wide Survey to be carried out with the Near-Infrared Camera on the JWST.

An important obstacle to finding these objects is that their numbers could be very small. As discussed by previous authors (Bromm & Loeb 2003; Dijkstra et al. 2004, 2008; see also Agarwal et al., in preparation), supermassive stars cannot be so abundant that the BHs they create exceed observed limits on the BH mass density and X-ray background. This suggests that finding the BHs may be easier than detecting their progenitors. First, they could accrete material for at least $10^8$ yr and be luminous for far longer than the stars that created them. Second, future X-ray missions such as the Joint Astrophysics Nascent Universe Satellite (Roming 2008; Burrows et al. 2010), LOBSTER (Gorenstein 2011), SVOM (Götz et al. 2009), and the Energetic X-ray Imaging Survey Telescope (Grindlay 2005) will perform all sky surveys with far greater coverage than the JWST. However, if the nascent BH does not have an accretion disk it may only emit weakly in X-rays at birth (Fryer et al. 2001; Fryer & Heger 2011; Saum & Ioka 2011; but see also Komissarov & Barkov 2010). If so, the SMBH seed does not become visible until it begins to accrete surrounding protogalactic gas (Hahnelt & Rees 1993; Kuhlen & Madau 2005; Li et al. 2007; Volonteri & Begelman 2010; Johnson et al. 2011).

Supermassive stars could also be detected if they explode as luminous supernovae. However, previous studies have concluded that $\gtrsim 10 M_\odot$ Population III stars collapse to BHs without an explosion (Fuller et al. 1986; Fryer & Heger 2011; Montero et al. 2012; but see Ohkubo et al. 2006). At $140-260 M_\odot$, however, pair-instability supernovae occur (Heger et al. 2003) and may be observable by future missions such as the JWST (Wisec & Abel 2005; Scannapieco et al. 2005; Weinmann & Lilly 2005; Jogerst & Whalen 2011; Kasen et al. 2011). Finally, we note that much of the continuum and line emission from rapidly accreting supermassive stars will appear in the near infrared background (NIRB) today. Although their contribution to the NIRB may be small if they are rare, it could be detected by missions such as the Cosmic Infrared Background Experiment (e.g., Bock et al. 2006), which is designed to find signatures of primordial galaxy formation at $z \gtrsim 10$.

5. DISCUSSION AND CONCLUSIONS

We find that the masses of the stellar seeds of SMBH forming from baryon collapse in early protogalaxies are primarily governed by their intense ionizing UV flux and their finite lifetimes. For spherically symmetric accretion at constant rates $M_{\text{acc}} \lesssim 0.1 M_\odot$ yr$^{-1}$, the maximum mass the star can reach is governed by radiative feedback and is $M_* \lesssim 10^2 (M_{\text{acc}}/10^{-3} M_\odot$ yr$^{-1})^{8/7} M_\odot$. At higher masses, the H ii region breaks out to large radii and terminates accretion. We have verified that other forms of feedback, such as gas pressure, radiation pressure from trapped line emission, photodissociation of H, and scattering of Lyα photons are much less effective at slowing accretion (see Section 2.2 and the Appendix).

At accretion rates above $\gtrsim 0.1 M_\odot$ yr$^{-1}$ the lifetime of the star, not radiative feedback, determines its final mass by limiting the time for which gas can accumulate on the star. Radiative feedback limits supermassive Population III stars to final masses
of $\sim 3 \times 10^5 M_\odot$ and time-limited accretion limits them to
$\sim 10^6 M_\odot$ at the highest accretion rates ($\sim 1 M_\odot$ yr$^{-1}$) found in numerical simulations of protogalactic collapse (Wise et al. 2008; Shang et al. 2010; Johnson et al. 2011).

We caution that our analytical calculations do not account for all conceivable effects that could stem the growth of supermassive stars. For instance, McKee & Tan (2008) note that rotation of infalling gas leads to lower circumstellar densities and larger H$\text{\textsc{ii}}$ regions with greater radiative feedback (see also Hosokawa et al. 2011; Stacy et al. 2012a), implying lower final stellar masses. Our spherically symmetric calculation excludes rotation, so the mass limits we find for a given accretion rate (Figure 3) are upper limits. We do, however, find agreement with (Omukai & Inutsuka 2002), who performed a similar calculation, although they only considered the growth of stars up to $\sim 10^5 M_\odot$. Furthermore, we also note that the spherical symmetry and constant accretion in our models do not address accretion that is episodic or clumpy and self-shielded from ionizing radiation from the star (Whalen et al. 2008, 2010; Krumholz et al. 2009). These processes could cause accretion to proceed at lower time-averaged rates than those predicted here (but see Kuiper et al. 2012).

Another process that could truncate the growth of supermassive stars well before feedback or stellar lifetimes is the onset of a general relativistic instability in the core of the star that causes it to collapse when it becomes sufficiently massive (Chandrasekhar 1964). Such instabilities are predicted to set in once the star has grown to $\sim 10^5 M_\odot$ (Iben 1963; Fowler 1964), but stellar rotation could stabilize it against collapse up to much larger masses (Fowler 1966; Bisnovatyi-Kogan 1998; Baumgarte & Shapiro 1999). While our models ignore rotation, the accreting gas is likely to have some (e.g., Colgate et al. 2003); if so, the supermassive star will inherit the angular momentum of the gas from which it formed and perhaps bypass the general relativistic instability. Additional insight into the processes that limit the growth of supermassive stars will be gleaned from stellar evolution calculations accounting for the continual accretion of mass at high rates (A. Heger et al., in preparation).

If supermassive stars formed and grew to masses of $\gtrsim 10^5 M_\odot$ in the early universe, recombination emission from their H$\text{\textsc{ii}}$ regions may be bright enough to be detected by future missions such as JWST. In particular, Ly$\alpha$ photons trapped in the accretion flow are reprocessed into Balmer series photons that could escape into the IGM. Consequently, the formation of these objects is accompanied by distinctive strong He emission together with strong continuum and He$\text{\textsc{ii}} \lambda 1640$ emission, the latter arising from the hard spectrum of hot Population III stars. However, their BHs may be easier to discover in observational surveys, given the small numbers and brief lifetimes of their progenitors. Indeed, these BHs may be the very ones that have already been found at the centers of massive galaxies and quasars at high redshifts.

Current numerical simulations of SMBH seed formation in early protogalaxies are now at an impasse because the formation of the hydrostatic supermassive protostar restricts Courant times to values that are too short to evolve central flows for even one dynamical time (but see Johnson et al. 2011 for an alternative approach using the sink particle technique). Our results suggest that in the early and intermediate stages of the growth of the star its evolution is essentially decoupled from flows on even slightly larger spatial scales. Consequently, it should now be possible to retreat from the extreme spatial resolutions previously applied to the protostar and evolve flows at the center of the galaxy over enough dynamical times to capture its structure at the time of the death of the star and breakout of X-rays from the BH seed into the IGM. Thus, it will soon be possible to witness the births of the first quasars in the universe with supercomputers.

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APPENDIX

NEGLECTED FEEDBACK PROCESSES

Here, we discuss several processes which have a negligible impact on our estimate of the final maximum stellar mass given by Equation (15).

A.1. He$\text{\textsc{ii}}$ Photoionization

To assess the relative importance of He$\text{\textsc{ii}}$ photoionization in slowing the accretion flow in the He$\text{\textsc{iii}}$ region, we follow the argument in Section 2.1 for He$\text{\textsc{i}}$ photoionization. For the ratio of He$\text{\textsc{ii}}$ photoionization pressure to H$\text{\textsc{i}}$ photoionization pressure, we have

$$\frac{\rho_{\text{He}\text{\textsc{ii}}}}{\rho_{\text{H}\text{\textsc{i}}}} \approx \frac{n_{\text{He}}}{n_{\text{H}}} \frac{\hbar v_{\text{He}\text{\textsc{ii}}}}{\hbar v_{\text{H}\text{\textsc{i}}}} \frac{\alpha_{\text{B,He}\text{\textsc{ii}}}}{\alpha_{\text{B,H}\text{\textsc{i}}}} \approx 1.1,$$  \hspace{1cm} (A1)

where we have taken it that $\hbar v_{\text{He}\text{\textsc{ii}}}/\hbar v_{\text{H}\text{\textsc{i}}} \approx 54.4$ eV/29 eV $\approx 1.8$, and that $\alpha_{\text{B,He}\text{\textsc{ii}}}/\alpha_{\text{B,H}\text{\textsc{i}}} \approx 6.4$ (Osterbrock & Ferland 2006). Therefore, within the He$\text{\textsc{iii}}$ region, the radiation pressures due to He$\text{\textsc{ii}}$ and H$\text{\textsc{i}}$ photoionizations are comparable. This is convenient because in the He$\text{\textsc{iii}}$ region it is recombination emission from He$\text{\textsc{ii}}$, not stellar photons, which largely keeps hydrogen photoionized (Osterbrock & Ferland 2006). Therefore, while diffuse recombination emission does not transfer appreciable outward momentum to the gas, the stellar photons that ionize He$\text{\textsc{ii}}$ transfer roughly the same momentum to the gas that they would in ionizing H$\text{\textsc{i}}$. As a consequence, solving in detail for the pressure due to He$\text{\textsc{ii}}$ photoionization would result in an almost identical solution to the one we have found by just treating the H$\text{\textsc{i}}$ region.

A.2. H$^{-}$ Photodetachment

In order for very high accretion rates onto a supermassive star in a primordial protogalaxy to be realized, the H$^{-}$ fraction in the accreting protogalactic gas must be very low (Bromm & Loeb 2003; Lodato & Natarajan 2006; Spaans & Silk 2006; Omukai et al. 2008; Wise et al. 2008). Indeed, the $\gtrsim 10^4$ K virial temperatures at the centers of $10^5-10^6 M_\odot$ halos heavily suppress H$^{-}$ fractions, so it is a good approximation to take it that the opacity due to absorption of photons by H$^{-}$ is also low. However, this does not imply that H$^{-}$ fractions are negligible because it forms from reactions whose abundances...
are not suppressed by H$_2$-dissociating backgrounds or high virial temperatures:

$$\text{H} + \text{e}^- \rightarrow \text{H}^- + \gamma,$$  \hspace{1cm} (A2)

where $\gamma$ denotes the emission of a photon. In principle, the absorption of photons with energies $\geq 0.75$ eV by photodetachment of H$^-$ could be an important channel by which momentum can be imparted to inflow. To determine the rate at which momentum that could be transferred to the gas, it suffices to estimate the equilibrium rate of H$^-$ formation, since this is also the rate at which H$^-$ will be destroyed and momentum will be acquired by the gas (see Abel et al. 1997; Chuzhoy et al. 2007). Using the density profiles from our calculations, which assume that the gas falls toward the star at the free-fall velocity above $r_{\text{Hi}}$ (Figure 2), we have $n(r) \lesssim 10^{12} (r/r_{\text{Hi}})^{-3/2} \text{ cm}^{-3}$. Numerical simulations of protogalactic collapse predict central free electron fractions $f_e \sim 10^{-4}$ (e.g., Shang et al. 2010). With these as upper limits along with the rate coefficient $k_{\text{H}^-}$ for H$^-$ formation, integration over the density profile of the halo yields the H$^-$ formation rate $Q_{\text{H}^-}$ outside the H II region:

$$Q_{\text{H}^-} \lesssim k_{\text{H}^-} \int_{r_{\text{Hi}}}^{\text{outer}} 4\pi r^2 f_e n^2 dr \sim 10^{44} \text{ s}^{-1},$$  \hspace{1cm} (A3)

where $k_{\text{H}^-} \sim 10^{-14} \text{ cm}^3 \text{ s}^{-1}$ (Wishart 1979; Abel et al. 1997). We assume that hydrogen is predominantly neutral outside the H II region and neglect the small H$^-$ fractions that may form in the H II region. Finally, to ensure that we have found a strong upper limit, we integrate this profile out to $r_{\text{outer}} = 10^{21} \text{ cm}$, which is approximately the virial radius of the atomically cooled halos at high redshift that host the supermassive stars we are studying. Even allowing for the high average photon energy for photodetachment $h\nu_{\text{H}^-} \sim 10\text{ eV}$ for supermassive Population III stars, the maximum momentum that could be transferred to the gas per unit time is

$$\frac{h\nu_{\text{H}^-} Q_{\text{H}^-}}{c} \lesssim 10^{22} \text{ g cm s}^{-2}.$$  \hspace{1cm} (A4)

This is orders of magnitude smaller than the momentum of the accretion flow in Equation (18), so H$^-$ destruction does not contribute to radiative feedback.

### A.3. Radiation Pressure from Trapped Ly$\alpha$ and Balmer Series Lines

In Section 2.2, we raised the possibility that Ly$\alpha$ photons may slow down accretion by scattering from neutral atoms at the edge of the H II region. While we showed that this is unlikely to alter the flow, mostly because Ly$\alpha$ photons are confined to $r_{\text{Ly}\alpha}$, radiation pressure due to nebular emission lines in optically thick gas may be sufficient to reduce accretion. As also mentioned in Section 2.2, trapped Ly$\alpha$ photons may be largely converted to Balmer series photons, in some cases after being destroyed by absorption by excited hydrogen or helium (Osterbrock & Ferland 2006). These Balmer photons can then propagate outward and efficiently cool the gas. The high temperatures, densities, and Ly$\alpha$ photons trapped in the H II region cause a large fraction of the hydrogen atoms to be excited to the $n = 2$ state there. From this state, either collisions or absorptions will further excite the atoms, which often later decay by emitting a Balmer series photon.

To evaluate the degree to which radiation pressure from trapped line photons alters the dynamics of accretion, we first estimate the optical depth to Balmer photons both inside and outside the H II region. Within $r_{\text{Hi}}$, the density of neutral hydrogen atoms can be estimated from Equation (21) and assuming that $n \propto r^{-2}$, as implied by the nearly constant velocity profile of the gas in the H II region (Figure 2). The neutral hydrogen column density $N_{\text{H}}$ through the H II region is then

$$N_{\text{H}}(< r_{\text{Hi}}) \simeq \int_{r_{\text{Hi}}}^{r} n(r) dr$$

$$= \frac{f_{\text{Hi}}(r_{\text{Hi}}) n(r_{\text{Hi}})}{r_{\text{Hi}}} \simeq 10^{17} \text{ cm}^{-2},$$  \hspace{1cm} (A5)

which, from the scaling in the second equation, can be shown to have only a weak dependence on the stellar mass for $M_* = 10^2 - 10^6 M_\odot$ for our solutions to the minimum accretion rate. The cross sections for absorption of Balmer series photons by H in the $n = 2$ state are 19, 3.5, and $1.3 \times 10^{-17} \text{ cm}^2$ for H$\alpha$, H$\beta$, and H$\gamma$, respectively. The higher energy Balmer series cross sections are all below these values. With these cross sections and $N_{\text{H}}(< r_{\text{Hi}})$, we can express the optical depth through the H II region solely as a function of the relative populations $N_2/N_1$ of the $n = 1$ and $n = 2$ levels of neutral hydrogen in the H II region.

We find that even for $N_2/N_1$ as high as $\sim 0.05$, the H II region is optically thin to all Balmer series photons, while the largest optical depth possible for H$\alpha$ is $r_{\text{Hi}} \sim 20$. However, this is still far below the optical depth for Ly$\alpha$ photons, and Balmer series photons will eventually leak out of the H II region, although in some cases only after a number of scatterings. Therefore, while Ly$\alpha$ photons in general will not escape the H II region, Balmer series photons will escape and allow the gas to radiatively cool, as we noted in Section 2.2.

Next, we consider the optical depth to Balmer series lines outside the H II region. In this region, we set $f_{\text{Hi}} = 1$ and adopt a free-fall density profile $n \propto r^{-3/2}$ for the gas, as in Section 2.1. The neutral hydrogen column density beyond $r_{\text{Hi}}$ is then

$$N_{\text{H}}(> r_{\text{Hi}}) \simeq \int_{r_{\text{Hi}}}^{\infty} n(r) dr$$

$$= 2n(r_{\text{Hi}}) r_{\text{Hi}} \simeq 10^{24} \text{ cm}^{-2},$$  \hspace{1cm} (A6)

which again is not strongly dependent on stellar mass.

Because the gas outside the H II region is well below the critical density at which excited levels in hydrogen would be populated and kept in equilibrium by collisions (and because Ly$\alpha$ photons are mostly trapped in the H II region and cannot excite neutral hydrogen beyond $r_{\text{Hi}}$), the relative population of the $n = 2$ level of hydrogen is expected to be very small. In this case, Schleicher et al. (2010) find that in general $N_2/N_1 \lesssim 10^{-10}$. With this as an upper limit and the cross sections for Balmer series photon absorption above, we find that the optical depths to Balmer series lines are $\tau_{\text{Hi}} \sim 4 \times 10^{-2}$, $\tau_{\text{H}\beta} \sim 8 \times 10^{-3}$, and $\tau_{\text{H}\gamma} \sim 3 \times 10^{-3}$. At such low optical depths, Balmer series photons will propagate largely unimpeded through the inflow and, most likely, exit the halo with minimal effect on accretion. As these photons also move freely through the IGM, accreting supermassive stars probably have a distinctive observational signature, as we discussed in Section 4.

Thus, because outside the H II region the optical depth to Balmer series photons is small and Ly$\alpha$ photons cannot propagate across the H II region boundary, we conclude that radiation pressure due to trapped emission lines will not be appreciable beyond $r_{\text{Hi}}$. However, it may be that this pressure is substantial within the H II region. To estimate its magnitude,
we compare it to the ram pressure of the accretion flow. For the latter, we have
\[
P_{\text{ram}} = n \mu m_{\text{H}} v^2 
\]
\[\simeq 50 \left( \frac{M_{\text{acc}}}{M_\odot \text{yr}^{-1}} \right) \left( \frac{r}{10^{15} \text{ cm}} \right)^{-2} \left( \frac{v}{100 \text{ km s}^{-1}} \right) \text{ dyn cm}^{-2}, \tag{A7}\]
where we have used Equation (1) to express \( n \) in terms of \( v \) and \( M_{\text{acc}} \). We use the prescription of Braun & Dekel (1989) for the radiation pressure (see also Elitzur & Ferland 1986):
\[
P_{\text{line}} = \left( \frac{4\pi}{9c} \right) \frac{2 h \nu_{\text{tran}}^3}{c^2} \frac{\Delta 
abla_{\text{tran}}}{N_{i+1}/N_i}, \tag{A8}\]
where \( N_{i+1}/N_i \) is the ratio of the upper and lower level populations for the given transition, whose frequency is \( \nu_{\text{tran}} \). The line width \( \Delta 
abla_{\text{tran}} \simeq 2 \times 10^{11} \left( T/10^4 \text{ K} \right)^{1/2} \left( \ln \tau \right)^{1/2} \text{ s}^{-1} \), where \( \tau \) is the optical depth of the line.

Assuming optical depths \( \tau_{\text{Ly}\alpha} = 10^4 \) and \( \tau_{\text{Balmer}} = 20 \), both of which are rough upper limits in the \( \text{H} \text{II} \) region, and \( N_{i+1}/N_i = 1 \) in both cases, we find strong upper limits of \( \sim 6 \) dyn cm\(^{-2}\) and \( 2 \times 10^{-2} \) dyn cm\(^{-2}\) for the \( \text{Ly}\alpha \) and Balmer line radiation pressures, respectively. Comparing these pressures to \( P_{\text{ram}} \) at \( r_{\text{HI}} \), which is a lower limit for the \( \text{H} \text{II} \) region due to its strong dependence on \( r \) in Equation (A7), we find that it is always at least a factor of \( \sim 4 \) larger than radiation pressure from optically thick lines. Therefore, lines will not play a large role in slowing accretion in the \( \text{H} \text{II} \) region.

### A.4. Gas Pressure

Because infall is highly supersonic, gas pressure cannot decelerate the gas (Omukai & Inutsuka 2002). We have verified that this holds even when a gas pressure term is included in the equation of motion. We find that only for extremely high sound speeds corresponding to temperatures of at least \( 10^6 \) K would gas pressure begin to impact the dynamics of the accretion flow. However, temperatures this high are not found in numerical simulations of protogalactic collapse (Wise et al. 2008; Regan & Haehnelt 2009; Shang et al. 2010; Johnson et al. 2011) or in \( \text{H} \text{II} \) regions because the Balmer thermostat limits ionized gas temperatures to at most a few times \( 10^4 \) K.

### A.5. Accretion Luminosity

Our conclusions regarding the maximum mass of the supermassive star rest on the assumption that the critical accretion rate for a given stellar mass (Equation (12)) separates two regimes: below this rate, accretion is suppressed by radiative feedback and above this rate accretion proceeds. As shown in Section 2.1, it is clear that accretion is prevented for inflow rates below the critical rate because the ionization front breaks out to infinity and photoionization pressure halts infall at all radii. For accretion rates above the critical rate, the flow will arrive at the star with a velocity \( v(r_\ast) > 0 \). This implies that the accreting material will have some kinetic energy that must be dissipated at the stellar surface, some portion of which will be radiated. This radiation could slow accretion within the \( \text{H} \text{II} \) region by electron scattering or photoionizations.

Let us assume that all of the kinetic energy of the flow is converted to radiation that propagates outward on impact with the star at some velocity \( v(r_\ast) \). The luminosity thus generated is
\[
L_{\text{acc}} = \frac{M_{\text{acc}} v^2}{2}, \tag{A9}\]
which yields a total momentum in photons of \( L_{\text{acc}} = M_{\text{acc}} v^2 / 2c \). Comparing this to the momentum of the infalling gas, \( M_{\text{acc}} v \), we see that the momentum in the accretion luminosity is a factor of \( v/2c \) lower than that of the gas. Therefore, accretion radiation cannot halt the flow in the steady-state approximation. However, bouts of massive accretion could in principle generate enough radiation to slow infall at larger radii where gas has a lower momentum, like the episodic accretion onto BHs in the early universe discussed by Milosavljević et al. (2009).

We note that an additional effect of accretion at very high rates (much higher than those in Equation (12)) is the trapping of radiation by electron scattering or other sources of opacity in the flow (e.g., Begelman 1978, 2010; Wyithe & Loeb 2011). As discussed by Omukai & Palla (2003; see also Stahler et al. 1986) in the context of primordial protostellar accretion, radiation trapping can lead to rapid expansion of the stellar photosphere that halts the growth of the star. We note, however, that such expansion also causes the effective temperature of the star to fall with the increase in surface area of the photosphere,\(^{16}\) which in turn leads to a softening of the emitted radiation and a lower ionizing photon emission rate \( Q \). This, and the fact that accretion rates greater than those in Equation (12) result in smaller \( \text{H} \text{II} \) regions, suggests that the flow cannot be stopped, at least not by photoionization pressure.

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\(^{16}\) At constant luminosity, the effective temperature at the stellar photosphere scales as \( T_{\text{eff}} \propto r^{-1/2} \).
