Atomically thin nonreciprocal optical isolation

Xiao Lin1,2,3, Zuojia Wang1,2, Fei Gao3, Baile Zhang3,4 & Hongsheng Chen1,2

1State Key Laboratory of Modern Optical Instrumentation, Zhejiang University, Hangzhou 310027, China, 2The Electromagnetics Academy at Zhejiang University, Zhejiang University, Hangzhou 310027, China, 3Division of Physics and Applied Physics, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore 637371, Singapore, 4Centre for Disruptive Photonic Technologies, Nanyang Technological University, Singapore 637371, Singapore.

Optical isolators will play a critical role in next-generation photonic circuits, but their on-chip integration requires miniaturization with suitable nonreciprocal photonic materials. Here, we theoretically demonstrate the thinnest possible and polarization-selective nonreciprocal isolation for circularly polarized waves by using graphene monolayer under an external magnetic field. The underlying mechanism is that graphene electron velocity can be largely different for the incident wave propagating in opposite directions at cyclotron frequency, making graphene highly conductive and reflective in one propagation direction while transparent in the opposite propagation direction under an external magnetic field. When some practical loss is introduced, nonreciprocal isolation with graphene monolayer still possesses good performance in a broad bandwidth. Our work shows the first study on the extreme limit of thickness for optical isolation and provides theoretical guidance in future practical applications.

Results

Fig. 1 schematically shows the atomically thin and polarization-selective optical isolation based on graphene monolayer. Under an external magnetic field perpendicular to graphene plane, graphene (in the xy plane) becomes gyrotropic and can be described by an asymmetric permittivity tensor: 

\[
\varepsilon = \begin{pmatrix}
\varepsilon & i\varepsilon_g & 0 \\
-i\varepsilon_g & \varepsilon & 0 \\
0 & 0 & \varepsilon_z
\end{pmatrix}
\]

For
plane wave propagating along magnetization (+\(\hat{z}\)) direction, the eigenmodes in gyrotropic graphene are LCP and RCP. Without loss of generality, we analytically calculate the transmissivity of circularly polarized waves normally incident from a dielectric medium with relative permittivity \(\varepsilon_1\) onto the gyrotropic graphene layer, and transmitted into a second dielectric medium with relative permittivity \(\varepsilon_2\). Detailed calculation can be found in Methods. The calculated wavevectors for LCP and RCP in gyrotropic graphene are as follows:

\[
\begin{align*}
\kappa_{\text{LCP}^+} &= \omega / \sqrt{\varepsilon_1 - \mu_0 \varepsilon_2} & k_{\text{RCP}^+} &= \omega / \sqrt{\varepsilon_1 - \mu_0 \varepsilon_2} \\
\kappa_{\text{LCP}^-} &= \omega / \sqrt{-\varepsilon_1 - \mu_0 \varepsilon_2} & k_{\text{RCP}^-} &= \omega / \sqrt{-\varepsilon_1 - \mu_0 \varepsilon_2}
\end{align*}
\]

from which we see that for the same circularly polarized wave they have different phase velocities \((v_{\text{ph}} = \omega / k)\) in the forward and backward propagations. Moreover, since \(|k_{\text{LCP}^+}| = |k_{\text{RCP}^-}|\) and \(|k_{\text{LCP}^-}| = |k_{\text{RCP}^+}|\), the transmissivity of LCP in one propagation direction is the same with that of RCP in the opposite propagation direction. With this symmetry, we only need to analyze the forward propagating transmissivity.

The ideal case of graphene monolayer without loss is firstly discussed in Fig. 2(a). The parameters in Fig. 2(a) are as follows: the chemical potential \(\mu_c = 0.5\) eV, the cyclotron frequency \(\omega_c/2\pi = 10\) THz, the damping constant \(\gamma = 0\), and \(\varepsilon_1 = \varepsilon_3 = 1\). From the analytical details (are in Methods), we can get forward propagating transmissivity \(t_{\text{LCP}} = 0\) and \(t_{\text{RCP}} = 0.99\) at cyclotron frequency, indicating good isolation performance. The underlying mechanism of the remarkable contrast in transmissivity at cyclotron frequency is that the electron velocity in graphene under LCP incidence is largely different from that under RCP incidence, making gyrotropic graphene highly conductive and reflective for one circular polarization incidence while transparent for another circular polarization incidence. From a microscopic point of view, by using free electron model\(^9\), \(m_{\text{eff}} d\nu / dt = -e(\hat{E} + \hat{v} \times \hat{B})\), where \(m_{\text{eff}}\) is the effective electron mass and \(E\) is the electric field of incident wave being either RCP or LCP. Under time harmonic excitation, we can derive

\[
\hat{v} = -e[\omega i \hat{E} + \omega_c \hat{B} \times \hat{E} - i \omega_c \hat{B} \omega_c ] / m_{\text{eff}}(\omega^2 - \omega_c^2)
\]

where \(\omega_c = -e\hat{B} / m_{\text{eff}}\). In this paper, we set the external magnetic field \(B\) along \(-\hat{z}\) direction, and the scalar cyclotron angular frequency is defined as \(\omega_c = [\omega_c]\). For the forward \((+\hat{z})\) propagation, we have

\[
\hat{v}_{\text{LCP}} = \frac{-i e \hat{E} + \hat{B}}{m_{\text{eff}}(\omega - \omega_c)}
\]

and \(\hat{v}_{\text{RCP}} = \frac{-i e \hat{E} + \hat{B}}{m_{\text{eff}}(\omega + \omega_c)}\) for RCP incidence with \(E_{\text{LCP}} = E_0(\hat{x} - i\hat{y})\) and \(E_{\text{RCP}} = E_0(\hat{x} + i\hat{y})\). Hence, the magnitude of Lorentz force acting on graphene electrons under LCP incidence \(|F_{\text{LCP}}| = \omega_c E_{\text{LCP}} / (\omega - \omega_c)|\) is much larger than that under RCP incidence \(|F_{\text{RCP}}| = \omega_c E_{\text{RCP}} / (\omega + \omega_c)|\) at \(\omega = \omega_c\). As a result, \(v_{\text{LCP}}\) can be theoretically very large and \(v_{\text{RCP}}\) is reduced to half at cyclotron frequency. Since the electric current density \(j \propto \hat{v}\) and \(j = e\hat{E}\), gyrotropic graphene becomes highly conductive under LCP incidence, indicating gyrotropic graphene behaves like a perfect electric conductor, and LCP is thus totally reflected. In

Figure 1 | Schematics of the atomically thin and polarization-selective optical isolation for circularly polarized waves. An external magnetic field is applied perpendicular to graphene plane. Only one circularly polarized wave can propagate through the gyrotropic graphene in one propagation direction. The arrows in the helices show the instantaneous electric field distributions of circularly polarized waves.

Figure 2 | Transmissivity of RCP \((t_{\text{RCP}}\)) and LCP \((t_{\text{LCP}}\)) through gyrotropic graphene monolayer in the forward propagation. (a) Comparison of \(t_{\text{RCP}}\) and \(t_{\text{LCP}}\) under the lossless \((2\pi \gamma / \omega_c = 0\)) and loss \((2\pi \gamma / \omega_c = 1\)) conditions with \(\omega_c / 2\pi = 10\) THz. (b) Loss influence on the isolation performance with different cyclotron frequencies. \(t_{\text{RCP}}/t_{\text{LCP}}\) increases as \(2\pi \gamma / \omega_c\) or \(\omega_c\) decreases. Other parameters are as follows: \(\mu_c = 0.5\) eV and \(\varepsilon_1 = \varepsilon_3 = 1\).
contrast, for RCP incidence, graphene has a finite negative permittivity and wave is evanescent inside of the graphene. As its skin depth (1/Im(κ_{RCP}) > 100 nm) is far larger than graphene’s atomic thickness, graphene is highly transparent to RCP incident wave.

Because of the collision between electrons, loss is inevitable in practical graphene samples. In the following part, we studied the loss influence on the isolation performance. When loss is considered, the electron velocity in equation (2) becomes

\[ \tilde{\nu} = \frac{-e[\mu + i\nu]E_0 + \alpha_0 \times E}{\alpha_0 + i\Delta \nu E_0} \]

For the forward \((+\hat{z})\) propagation and by setting \(B = 0\) along \(-\hat{z}\) direction, we have

\[ t^{+LCP} = \frac{-i\nu E_0 + \alpha_0}{m_{\text{eff}} (\nu E_0 + \alpha_0)} \]

for LCP incidence and

\[ t^{+RCP} = \frac{-i\nu E_0 + \alpha_0}{m_{\text{eff}} (\nu E_0 + \alpha_0)} \]

for RCP incidence. Therefore, as long as \(2\pi\gamma/\omega_c\) is sufficiently small, \(t^{+LCP}/t^{+RCP}\) can still be theoretically very large, and \(t^{+LCP}\) decreases as \(2\pi\gamma/\omega_c\) increases. Figure 3(a) shows the isolation performance of LCP and RCP under the lossless \((2\pi\gamma/\omega_c = 0)\) and loss \((2\pi\gamma/\omega_c = 1)\) conditions. One can see that the loss degrades the performance of isolation. Figure 2(b) shows the transmissivity ratio between RCP and LCP at cyclotron frequency \((t^{+RCP}/t^{+LCP})\) as a function of \(2\pi\gamma/\omega_c\) with different cyclotron frequencies, from which one can see that, on one hand, \(t^{+RCP}/t^{+LCP}\) increases as \(2\pi\gamma/\omega_c\) decreases, on the other hand, with the same value of \(2\pi\gamma/\omega_c\), \(t^{+RCP}/t^{+LCP}\) increases as the cyclotron frequency decreases. Hence, better isolation performance can be obtained by keeping \(2\pi\gamma/\omega_c\), sufficiently small or setting smaller cyclotron frequency.

To verify the analytical prediction of the atomically thin isolation, Fig. 3 shows the numerical simulations of the lossless case at 10 THz with the use of Finite Element Method (COMSOL Multiphysics). In the simulation, the following parameters are used: \(\mu_c = 0.5\) eV, \(\omega_c/2\pi = 10\) THz, \(\gamma = 0\), and \(\epsilon_1 = \epsilon_3 = 1\). We set the incident field with unit magnitude. When propagating in the forward direction, RCP is almost totally transmitted through graphene, as shown in Fig. 3(a), featured with unit magnitude of maximum \(E_{\text{y}}\) at both sides of graphene. Meanwhile, LCP is totally reflected, as shown in Fig. 3(b), characterized with twice unit magnitude of maximum \(E_{\text{y}}\) at the left graphene side and near zero at the right side. Moreover, from Fig. 3(b), one can see that the total electric field at the graphene boundary is always zero, indicating that graphene exhibits zero impedance, verifying our previous theoretical analysis that in this case graphene behaves like a highly conductive material. When propagating in the backward direction, RCP is totally reflected in Fig. 3(c) and LCP is almost totally transmitted in Fig. 3(d), behaving inversely to the forward propagation.

The numerical verifications for the loss case are shown in Fig. 4. Based on previous practical graphene parameters, we set \(\omega_c/2\pi = 0.3\) THz, \(\mu_c = 0.5\) eV, and \(\gamma = 0.63\) THz in the demonstration. These correspond to \(B = -1\) Tesla, the 2D carrier density \(n_s = 2 \times 10^{14} \text{ cm}^{-2}\), and the electron mobility \(\mu = 28000 \text{ cmV}^{-1}\text{s}^{-1}\). These parameter settings are achievable in experiments, because that the 2D carrier density \(n_s\) in graphene is experimentally reported to be gated efficiently from very low \((10^{10} \text{ cm}^{-2})\) to very high values \((10^{12} \text{ cm}^{-2})\). When propagating in the backward direction, RCP wave passes graphene with a transmissivity of 31.5% in 4.8 THz bandwidth can be obtained with frequency ranging from 0.2 to 0.46 THz. Figure 4(c-e) show the numerical simulations of the nonreciprocal isolation at 0.3 THz. In the forward propagation, RCP wave passes graphene with a transmissivity of 31.5% in 4(d), while LCP is efficiently blocked with a trivial transmissivity of 1.23% in 4(d). When propagating in the backward direction, it behaves inversely in Fig. 4(e-f). From these demonstration, one see that an atomically thin wide-band THz nonreciprocal optical isolation is possible by using practical graphene samples, showing the appealing potential applications of graphene in ultrathin nonreciprocal device design.
Discussion

Previous graphene based traditional Faraday isolators\(^{14,16}\) are generally composed by a graphene sheet, a resonating structure and two wire grids. In that Faraday isolator structure, the gyrotropic graphene only serves as a Faraday rotator, the thickness of the resonating structure is around half of the operating wavelength\(^{14}\), and the two wire grids serving as the polarizers are indispensable to realize isolation. When considering the thickness of the resonating structure and wire grids, the total thickness of previous graphene based Faraday isolators\(^{14,16}\) is usually in the order of operating wavelength, far larger than our atomic thickness. Hence, previous graphene based Faraday isolators\(^{14,16}\) are different from the demonstration in this work where no external wire grids or other resonant structures are needed. Flexible tunability is a potential advantage of graphene based devices\(^{26-27}\). Since the best performance of the ultrathin graphene isolation exists near cyclotron frequency, the isolation working frequency range should also be tunable by external magnetic field.

In conclusion, we have theoretically proposed the atomically thin and polarization-selective nonreciprocal optical isolation for circular polarizations. The mechanism underlying the atomically thin optical isolator is revealed from a microscopic viewpoint. Our work shows a feasible solution to control the propagation of circularly polarized waves in graphene and to manipulate light nonreciprocally at atomic scale, which opens up new possibilities for further innovations in functional ultrathin nonreciprocal optics.

Methods

Graphene monolayer is treated as a homogeneous film with thickness \(d = 1\) nm\(^{28}\) and characterized by an anisotropic permittivity tensor diag\((\varepsilon_x, \varepsilon_y, \varepsilon_z)\). Based on Kubo formula, \(\varepsilon_{eq}\) can be cast to the Drude model\(^{10,20,28,29}\),

\[
\varepsilon_{eq} = 1 + \frac{G_{\text{Gauss}} + G_{\text{inter}}}{\varepsilon_0 d} = \varepsilon_{\text{inter}} + \frac{G_{\text{Gauss}}}{\varepsilon_0 d} = \varepsilon_0 \left(1 + \frac{G_{\text{Gauss}}}{\varepsilon_0 d}\right)
\]

where \(G_{\text{Gauss}} = \frac{i e^2 k_g T}{\pi \hbar (\varepsilon_0 + \varepsilon_f)} \left(\frac{\mu_i k_g T}{\pi \hbar} + 2 \ln\left(e^{-\mu_i k_g T} + 1\right)\right)\) and \(G_{\text{inter}} = 1 + G_{\text{Gauss}} \varepsilon_0 d\), \(\mu_i k_g T = \frac{i e^2 k_g T}{\pi \hbar (\varepsilon_0 + \varepsilon_f)} \left(\frac{\mu_i k_g T}{\pi \hbar} + 2 \ln\left(e^{-\mu_i k_g T} + 1\right)\right)\) is the plasma frequency, \(\mu_i\) is the inverse of relaxation time, and \(\varepsilon_0, \varepsilon_f, \varepsilon_g\) and \(\varepsilon_0 d\) are the dielectric permittivity in vacuum, electron charge, reduced Planck constant, and Boltzmann constant, respectively. When setting \(\mu_i = 0.5\) eV and temperature \(T = 300\) K, we have the effective electron mass \(m_{\text{eff}} = 0.1 \times 0.91 \times 10^{-30}\) kg and \(\varepsilon_0/2m = 410\) THz. At the operation frequency below 1 THz, \(\varepsilon_{eq}\) is equal to 3.9.

Gyrotropic graphene can be described by an asymmetric permittivity tensor

\[
\epsilon = \begin{pmatrix}
0 & -i\epsilon_x & 0 \\
-i\epsilon_x & \varepsilon_0^2(\varepsilon_f + \varepsilon_0) + \frac{\mu_i k_g T}{\pi \hbar} & i\epsilon_y \\
0 & i\epsilon_y & \varepsilon_0^2(\varepsilon_0 + \varepsilon_f) + \frac{\mu_i k_g T}{\pi \hbar} \\
\end{pmatrix}
\]

where \(\epsilon = \varepsilon_{\text{inter}} - \varepsilon_{eq} = \varepsilon_0^2(\varepsilon_f + \varepsilon_0) + \frac{\mu_i k_g T}{\pi \hbar} \) and \(\epsilon_x = -\varepsilon_{\text{inter}} - \varepsilon_{eq} = \varepsilon_0^2(\varepsilon_0 + \varepsilon_f) + \frac{\mu_i k_g T}{\pi \hbar}\). Hence, \(\varepsilon = \varepsilon_{\text{inter}} - \varepsilon_{eq} = \varepsilon_0^2(\varepsilon_f + \varepsilon_0) + \frac{\mu_i k_g T}{\pi \hbar}\) and \(\varepsilon_x = \varepsilon_{\text{inter}} - \varepsilon_{eq} = \varepsilon_0^2(\varepsilon_0 + \varepsilon_f) + \frac{\mu_i k_g T}{\pi \hbar}\). The scalar cyclotron angular frequency \(\omega_c\) is defined as \(\varepsilon_0\), when the external magnetic field \(B\) is along \(-z\) direction.

Wave polarizations can be viewed either by the temporal or spatial view points. In the case of a circularly polarized wave, from the temporal viewpoint, the tip of the electric field vector describes a circle at a fixed point in space as time progresses. If the right-hand (left-hand) thumb points in the direction of propagation while the fingers point in the direction of the tip motion, the wave is defined as right-hand (left-hand) circularly polarized. From the spatial viewpoint, the instantaneous electric field vector of the RCP (LCP) wave describes a left-hand (right-hand) helix along the direction of propagation, as shown in Fig. 1.

To achieve the condition of good isolation for circular polarizations, the transmissivity of the forward propagation in Fig. 1 are analytically calculated in the following. The calculation of backward propagation can be done by the same methodology. Setting \(\varepsilon_{\text{Gauss}} = \varepsilon_{\text{inter}} = \varepsilon_0 d\) in the \(p\)th region with \(l = 2\) standing for gyrotropic graphene and \(l = 1.3\) for isotropic dielectric. After calculation, we have:

\[
\begin{align*}
\varepsilon_{\text{Gauss}}^{\text{LCP}} &= \varepsilon_{\text{Gauss}}^{\text{RCP}} = \varepsilon_0 d \\
A_1^{\text{LCP}} &= A_1^{\text{RCP}} = \frac{1}{\varepsilon_{\text{Gauss}}^{\text{LCP}}} = \frac{1}{\varepsilon_{\text{Gauss}}^{\text{RCP}}} = \frac{1}{\varepsilon_0 d} \\
\varepsilon_{\text{Gauss}}^{\text{LCP}} &= \varepsilon_{\text{Gauss}}^{\text{RCP}} = \varepsilon_0 d \\
A_1^{\text{LCP}} &= A_1^{\text{RCP}} = \frac{1}{\varepsilon_{\text{Gauss}}^{\text{LCP}}} = \frac{1}{\varepsilon_{\text{Gauss}}^{\text{RCP}}} = \frac{1}{\varepsilon_0 d} \\
\end{align*}
\]

where \(\varepsilon_{\text{Gauss}}^{\text{LCP}} = \frac{1}{\varepsilon_{\text{Gauss}}^{\text{RCP}}} = \frac{1}{\varepsilon_0 d} \).
25. Lin, X.
24. Dean, C. R.
23. Bolotin, K. I.
22. Efetov, D. K. & Kim, P. Controlling electron-phonon interactions in graphene at large scale monolayer graphene. Nano Lett. 12, 2470–2474 (2012).
21. Das, A.
20. Ju, L.
19. Kong, J. A.
18. Lin, X.
17. Nair, R. R.
16. Sounas, D. L. & Caloz, C. Electromagnetic nonreciprocity and gyrotropy of monolayer graphene. Appl. Phys. Lett. 98, 021911 (2011).
15. Zhou, Y.
14. Sounas, D. L. & Caloz, C. Graphene-based non-reciprocal spatial isolator. Paper presented at Proc. IEEE International Symposium on Antennas and Propagation (APSURSI) 1597–1600, Spokane, WA, USA. DOI: 10.1109/APS.2011.5996606 (2011, July 3–8). Available at: http://ieeexplore.ieee.org/xpl/login.jsp?tp=&arnumber=5996606&url=http%3A%2F%2Fieeexplore.ieee.org%2Fiel5%2F5981577%2F5996606.pdf%3Farnumber%3D5996606 (Accessed: 2014, Jan. 17).
13. Sounas, D. L. & Caloz, C. Gyrotropic properties of graphene and subsequent microwave applications. Paper presented at Proc. 41st European Microwave Conference (EuMC) 1142–1145 Manchester (2011, Oct. 10–13). Available at: http://ieeexplore.ieee.org/xpl/articleDetails.jsp?tp=&arnumber=6102014&url=http%3A%2F%2Fieeexplore.ieee.org%2Fpaps%2Fabs_all.jsp%3Farnumber%3D6102014 (Accessed: 2014, Jan. 17).
12. Crassee, I.
11. Crassee, I.
10. Fang, Z.
9. Hanson, G. W. Dyadic Green’s functions and guided surface waves for a surface conductivity model of graphene. J. Appl. Phys. 103, 064302 (2008).
8. Vakil, A. & Engheta, N. Transformation optics using graphene. Science 332, 1291–1294 (2011).
7. Fang, Z. et al. Active tunable absorption enhancement with graphene nanodisk arrays. Nano Lett. 14, 299–304 (2014).
6. Fang, Z. et al. Tunable terahertz plasmons and magnetoplasmons in large scale monolayer graphene. Nano Lett. 12, 2470–2474 (2012).
5. Lin, X., Wang, Z.J., Gao, F., Zhang, B.L. & Chen, H.S. Atomically thin graphene plasmonics for tunable terahertz Metamaterials. Nat. Nanotech. 6, 630–634 (2011).
4. Das, A. et al. Monitoring dopants by Raman scattering in an electrochemically top-gated graphene transistor. Nat. Nanotech. 3, 210–215 (2008).
3. Efetov, D. K. & Kim, P. Controlling electron-phonon interactions in graphene at ultrahigh carrier densities. Phys. Rev. Lett. 105, 256805 (2010).
2. Bolotin, K. I. et al. Ultrahigh electron mobility in suspended graphene. Solid State Commun. 146, 351–355 (2008).
1. Dean, C. R. et al. Boron nitride substrates for high-quality graphene electronics. Nat. Nanotech. 5, 722–726 (2010).

Acknowledgments
This work was sponsored by the National Natural Science Foundation of China under Grants No. 61322501, No. 61275183, and No. 60990322, the National Youth Top-notch Talent Support Program, the Foundation for the Author of National Excellent Doctoral Dissertation of PR China under Grant No. 200950, the Fundamental Research Funds for the Central Universities under Grant No. 2011QNA5020, the Chinese Scholarship Council Foundation under Grant No. 2011833070, the Program for New Century Excellent Talents (NCET-12-0489) in University, the K. P. Chao’s High Technology Development Foundation, the Nanyang Technological University, and the Singapore Ministry of Education under Tier 1 Ref. No. RG 27/12 and Grant No. MOE2011-T3-1-005.

Author contributions
X.L., B.Z. and H.C. conceived the idea of the study. X.L. performed the analysis and numerical simulations. Z.W. and F.G. contributed in the calculation and interpretation. H.C. and B.Z. supervised the project. All of the authors interpreted the results. X.L., B.Z. and H.C. wrote the manuscript with input from others.

Additional information
Competing financial interests: The authors declare no competing financial interests.

How to cite this article: Lin, X., Wang, Z.J., Gao, F., Zhang, B.L. & Chen, H.S. Atomically thin nonreciprocal optical isolation. Sci. Rep. 4, 4190; DOI:10.1038/srep04190 (2014).

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 Unported license. To view a copy of this license, visit http://creativecommons.org/licenses/by-nc-nd/3.0