Mathematical Model of Seat Arrangement in Large Gymnasium

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Abstract. In China, where the population ranks first in the world and the total economic output ranks second in the world, the construction of large-scale stadiums or Olympic sports centers has become a trend. When we arrange the seats on the viewing platform, we should not only consider the rationality, practicability and aesthetic value of the seat distribution, but also consider the safety of evacuation. Therefore, under the comprehensive consideration of the above factors, the mathematical model of periodic jump sequence is established, that is, an arithmetic sequence of periodical arithmetic jump, which provides an effective theoretical basis for engineering designers to use computers to process large data and arrange seats scientifically, and also provides an operable accurate model for later programming calculation and simulation technology research.

1. Introduction
The scale and level of urban public infrastructure construction are important indicators to measure a country's economic development, and also important aspects to reflect the comprehensive strength. In order to strengthen the consciousness of national culture and promote the harmonious development of its physical and mental health, the state should pay attention to the construction of infrastructure, because it has a profound impact on improving the overall quality of the nation. A country that is moving towards modern civilization should pursue the unity of science, practicality and beauty.

The world's largest stadium should be built in China, depending on China's population and economic trends, but a large stadium with a capacity of 100,000 people has yet to be built. The largest Beijing national Stadium, built in 2008, can hold 91000 people. Other major cities have built or are in the process of building some large stadiums. As shown in figures 1 and 2. Here are the real pictures of the seating arrangement of Hebei Olympic Sports Center and Dalian Sports Center respectively. Therefore, with the vigorous development of national economy and sports, China needs to build more and larger sports centers to meet the needs of people's lives. However, the layout of this kind of stadium audience seats should be practical, beautiful and safe. In order to pursue the scientific, practical, safe and beautiful overall effect, the corresponding mathematical model is established.

Mathematical modeling has a long history like mathematics. Since the beginning of the 20th century, with the penetration of mathematics into all fields with unprecedented breadth and depth, and the emergence and rapid development of electronic computers, mathematical modeling has a great potential in the field of engineering technology, high-tech and non-physical interdisciplinary fields[1]. As Marx said, "A science only when the successful use of mathematics, can be considered to have reached the point of perfection".

In order to set up the viewing platform and seats reasonably in the large public places of the Olympic Stadium, I first need to establish the corresponding mathematical model, and then use the
computer to process the big data and scientifically control it. In reality, the number of seats generally presents an arithmetic increase sequence from the first row to the back, and every few rows need to set up a corridor to facilitate the flow of people or safe evacuation, so this arithmetic increase sequence will jump periodically once. Therefore, the change of the number of seats actually corresponds to an arithmetic sequence of periodical jump. The following is a study of this sequence.

![Figure 1. Hebei Olympic Sports Center](image1.png)

![Figure 2. Dalian Olympic Sports Center](image2.png)

2. Study of the model of "sequence of periodic jump"

**Defined 1** If the sequence \( \{ a_n \} \) is an arithmetic sequence with a common difference \( d \) from item \((kt + 1)\)th to item \((kt + t)\)th \((k = 0, 1, 2, \ldots, t \) is a constant natural number)\), and 
\[
a_{(k + 1)t + 1} = a_{(k + 1)t} + e(e \text{ is any constant real number})
\]
Then the sequence \( \{ a_n \} \) is called an arithmetic sequence of periodical arithmetic jump, where \( t \) is jump period, \( e \) is its jump common difference[2], and \( d \) is its intra-period common difference. Its general form is as follows (notate as a sequence (1)):

\[
a_1, a_i + d, a_i + 2d, \ldots, a_i + (t - 1)d; a_i + e + (t - 1)d,
\]

\[
a_i + e + td, a_i + e + (t + 1)d, \ldots, a_i + e + (2t - 2)d;
\]

\[
a_i + 2e + (2t - 2)d, a_i + 2e + (2t - 1)d, a_i + 2e + 2td,
\]

\[
\ldots, a_i + 2e + (3t - 3)d; \ldots
\]

In particular, when \( t = 1 \), the sequence \( \{ a_n \} \) is an arithmetic sequence with a normal common difference \( e \); When \( t = 2 \), the sequence \( \{ a_n \} \) is a double arithmetic sequence [3].

**Conclusion 1** The general term formula for arithmetic sequence of periodic arithmetic jump (1) is

\[
a_n = a_i + \left[ \frac{n - 1}{t} \right] d + \left( n - 1 - \left[ \frac{n - 1}{t} \right] \right) e
\]

(2)

**Analysis of proof thinking** According to definition 1, the proof process of this conclusion 1 should be divided into two layers. The first level, in any period, the difference between the latter and its former is equal to \( d \); At the second level, the difference between the first item of any subsequent period and the last item of its former period is equal to \( e \).

**Proof** 1° When \( n = k + 1, k + 2, \ldots, k + t \) \((k = 0, 1, 2, \ldots)\), 
\[
\left[ \frac{n - 1}{t} \right] = k;
\]
the general formula of the \((k + 1)\)th period of the sequence (1) be obtained by the formula (2) is as follows

\[
a_n = a_i + ke + \left( n - 1 - k \right) d;
\]

When \( kt + 1 \leq n < kt + t \),

\[
a_n + 1 - a_n = \left( a_i + ke + \left( n - k \right) d \right) - \left( a_i + ke + \left( n - 1 - k \right) d \right) = d.
\]
That is, \{a_n\} is an arithmetic sequence with a common difference \(d\) in the \((k+1)\)th period of the sequence \{a_n\}. In particular, its last item is
\[
a_{k+1} = a_1 + ke + ((k+1)t - (k-1))d.
\]

2° when \(n = (k+1)t + 1\), \(\left\lceil \frac{n-1}{t} \right\rceil = k+1\), at this time, \(a_n\) is the first item of the \((k+2)\)th period in the sequence \{a_n\}. Obtain the following expression from (2)
\[
a_n = a_{(k+1)t+1} = a_1 + (k+1)e + (k+1)t - (k+1)d.
\]

So, the difference between the first item in the \((k+2)\)th period and the last item in the \((k+1)\)th period in the sequence \{a_n\} is
\[
\left(a_{(k+1)t+1} - a_{k+1}\right).
\]

By synthesizing 1°, 2°, we know that formula (2) is a general term formula that satisfies definition 1.

Conclusion 2 The sequence formed by the sum of \(t\) items in each period of arithmetic sequence (1) of periodical arithmetic jump is \(a_n\) arithmetic sequence with the first item is \(ta_1 + \frac{t(t-1)}{2}d\), and the common difference is \(te + t \left( t - 1 \right)d\). Then the sequence \{a_n\} is called a sequence of within-cycle sum.

Proof If the sum of \(t\) items in the \(k\)-th period of the sequence (1) is denoted by \(S^{(k)}(k = 1, 2, \ldots)\),
\[
S^{(k)} = ta_1 + \frac{t(t-1)}{2}d.
\]
And the first item in the \(k\)-th period of the sequence (1) is
\[
a_{(k-1)t+1} = a_1 + (k-1)e + ((k-1)t - (k-1))d.
\]
\[
S^{(k)} = ta_{(k-1)t+1} + \frac{t(t-1)}{2}d = ta_1 + t(k-1)e + \frac{2k-1}{2}t(t-1)d
\]
the first item in the \((k+1)\)th period of the sequence \{a_n\} is \(a_{(k+1)t+1} = a_1 + ke + (kt-k)d\),
\[
S^{(k+1)} = ta_{(k+1)t+1} + \frac{t(t-1)}{2}d = ta_1 + tke + \frac{2k+1}{2}t(t-1)d
\]
\[
S^{(k+1)} - S^{(k)} = te + t(t-1)d,
\]
therefore, conclusion 2 is established.

In the following, we'll find the formula for the sum \(S_n\) of the first \(n\) items of the sequence (1).

For the first \(n\) items of the sequence (1). It contains all the items in the first \(\left\lfloor \frac{n}{t} \right\rfloor\) periods and the first \(\left\lfloor \frac{n}{t} \right\rfloor\) items in the \(\left\lceil \frac{n}{t} \right\rceil + 1\) th period. So we can divide the first \(n\) items of the sequence into the first \(\left\lfloor \frac{n}{t} \right\rfloor\) items of the sequence of within-cycle sum and the first \(n - \left\lfloor \frac{n}{t} \right\rfloor\) items in the \(\left\lceil \frac{n}{t} \right\rceil + 1\) th period. From conclusion 2, it is known that the sum of the first \(\left\lceil \frac{n}{t} \right\rceil\) items of the sequence of within-cycle sum is
\[(ta_i + \frac{t(t - 1)}{2}d) \left[ \frac{n}{t} \right] + \frac{n}{t} \left( \frac{n}{t} - 1 \right) \left[ \frac{n}{t} \right] - 1 \right) (te + t(t - 1)d) \]

\[= \left[ \frac{n}{t} \right] ta_i + \frac{n}{t} \left( \frac{n}{t} - 1 \right) te + \frac{t(t - 1)}{2} \left[ \frac{n}{t} \right]^2 d.\]

And the first item of the \((n + 1)th\) period of the sequence (1) is

\[a_{\left( \frac{n}{t} + 1 \right)} = a_i + \left[ \frac{n}{t} \right] e + \left[ \frac{n}{t} \right] (t - 1)d.\]

\[\therefore\] The sum of the first \(n\) items of the \((n + 1)th\) period is

\[(a_i + \left[ \frac{n}{t} \right] e + \left[ \frac{n}{t} \right] (t - 1)d) (n - \left[ \frac{n}{t} \right] t) + \frac{(n - \left[ \frac{n}{t} \right] t) (n - \left[ \frac{n}{t} \right] t - 1)}{2} d.\]

The following conclusions can thus be obtained.

**Conclusion 3** The sum \(S_n\) of the first \(n\) items of the arithmetic sequence of periodical arithmetic jump is

\[S_n = na_i + \frac{n}{t} \left( \frac{n}{t} - 1 \right) te + \frac{t(t - 1)}{2} \left[ \frac{n}{t} \right]^2 d + \left( \frac{n}{t} e + \frac{n}{t} (t - 1)d \right) \left( n - \left[ \frac{n}{t} \right] t \right) + \frac{(n - \left[ \frac{n}{t} \right] t) (n - \left[ \frac{n}{t} \right] t - 1)}{2} d.\]

In particular, when \(t | n\) and \(\frac{n}{t} = k\), \(S_n = na_i + \frac{k(k - 1)}{2} te + \frac{t(t - 1)}{2} k^2 d.\)

3. **Model application and its theory Promotion**

**Example** There are 72 rows of seats in one stand of a large gymnasium. There are 180 seats in the first row, 26 more seats in each row than in the front row, one aisle every 13 rows, and 48 more seats in the row behind each aisle than in the front row.

**Solution** From the meaning of the question, we know that the number of seats in each row here forms arithmetic sequence of periodical arithmetic jump. The first item of these is \(a_i = 180\), jump period is \(t = 13\), jump common difference is \(e = 48\), intra-period common difference is \(d = 26\), \(n = 72\).

\[\left[ \frac{n}{t} \right] = 5\]

So there are \(5\) complete periods in the sequence, and there are the first 7 items in the 6th period. We know from conclusion 3 that the total number of seats in this stand is
By analogy, it is natural to deduce geometric sequence of periodical geometric jump sequence and its related conclusions from the arithmetic sequence of periodical arithmetic jump.

**Defined 2** If the sequence from the item \((kt + 1)\)th to the item \((kt + t)\)th \((k = 0, 1, 2, \ldots)\) is a geometric sequence with common ratio \(q\), and \(a_{(k+1)t+1} = a_{(k+1)t}p\) \((p \text{ is any constant})\), then the sequence is said to be a geometric sequence of periodical geometric jump, where \(t\) \((t \text{ is a constant natural number})\) is the jump period, \(p\) is the jump common ratio, and \(q\) is the common ratio in a period. The form is as follows (recorded as a sequence (3))

\[
a_i, a_iq, a_iq^2, \ldots, a_iq^{t-1}; a_ipq^{t-1}, a_ipq^t, a_ipq^{t+1}, \ldots
\]

\[
a_ipq^{2t-2}; a_ipq^2, a_ipq^{2t-2}, a_ipq^3, a_ipq^{2t-2}, \ldots
\]

In particular, when \(t = 1\), the sequence \(\{a_n\}\) is an ordinary geometric sequence of periodical geometric jump with a common ratio \(p\); When \(t = 2\), the sequence \(\{a_n\}\) is a double geometric sequence.

**Conclusion 4** The general term formula of the geometric sequence (3) of periodical geometric jump is

\[
a_i(1 - q^{t})
\]

sequence (3) of periodical geometric jump is a geometric sequence with the first term \(a_i\) and the common ratio \(pq^{t-1}\).

**Conclusion 5** The sequence formed by the sum of \(t\) items in each period of the geometric sequence (3) of periodical geometric jump is a geometric sequence with the first item \(a_i(1 - q^{t})\) and the common ratio \(pq^{t-1}\).

**Conclusion 6** The sum \(S_n\) of the first \(n\) items of the geometric sequence (3) of periodical geometric jump is

\[
S_n = \frac{a_i(1 - q^{t})}{1 - pq^{t-1}} \left(1 - \left(pq^{t-1}\right)^{\frac{n}{t}}\right)
\]

In particular, when \(t|n\) and \(t\)

The proof of the conclusion 4, 5, 6 is omitted.

4. **Summary**

The above theory is a scientific and effective mathematical model for engineering designers. They can determine the corresponding parameter values according to the actual needs of the specific site, and substitute them into the model for calculation.

Computers have brought great convenience to big data processing. People can achieve the final results by programming, just like dealing with problems in engineering, sports, economy and other fields, first establish mathematical models, and then use computer language programming or direct use of relevant software to achieve. In paper[4], the author has established the equation for calculating the equilibrium composition of the furnace gas accurately. When the atmosphere in the furnace changes...
very much, the data or the parameters of the mathematical model need to be adjusted according to the actual problems, which provides convenience for the computer to process the data.

In the later research, I will use computer simulation technology, not only to achieve the calculation of large data, but also to control the change of data, in order to correct and update the model in various situations, and promote its application in the solution of practical problems with similar characteristics. As mentioned in paper [5], the author can infer the nature and motion change of the flight control system of UAV through online interactive simulation technology, and update and correct the flight control system of UAV through simulation model and update and correct model, so as to ensure the accuracy and precision of the flight control system of UAV.

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