Analysis of a subwavelength Z-shaped metamaterial

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Abstract. A Z-shaped easy made planar meta-atom is presented as an alternative design to the conventional electric-LC (ELC) resonator for achieving negative permittivity. Transforming the LC topology of the resonator helps to facilitate transposition of geometrical parameters for the optical regime and also to improve the metamaterial homogeneity. Our approach aims to simplify the resonator design and achieve a lower resonance frequency without being limited by fabrication techniques. The electromagnetic behaviour is investigated through both simulations and experiments in the microwave regime. Our results show that the developed meta-atom exhibits a strong electric response to normally incident radiation and can be used very effectively in producing materials with negative permittivity. The Z-shaped structure is also shown to present a higher effective medium ratio and can possibly find various applications in planar high frequency passive circuits due to the simplicity of fabrication.

1. Introduction

Metamaterials have gained lots of attention within the scientific community over the past decade [1-3] because of their capabilities which go beyond conventional materials [4] and because of their applications in novel class coordinate transformation based devices such as invisibility cloaks [5-9], rotators [10], retroreflectors [11], Luneburg lenses [12], waveguide tapers [13] and directive antennas [14-16]. A typical metamaterial is an artificially engineered structure made of a periodic array of subwavelength metallic or dielectric inclusions called meta-atoms. These inclusions can be presented as LC resonant elements with the inductance and capacitance defined by the shape and geometrical dimensions of the latter resonator. The resonators can exhibit permittivity and/or permeability values in the effective medium regime where the ratio between the operating wavelength and the unit cell size $\lambda/a$ must be greater than 4 [17]. With typical designs incorporating resonant and dispersive elements much smaller than the operating wavelength, a homogenization scheme is possible, which leads to meaningful interpretation of the effective permittivity $\varepsilon$, permeability $\mu$ and index $n$.

In this paper, we propose a practical way to change the LC equivalent circuit topology of the conventional ELC resonator [18] in the effective medium regime. It is reported that our transformation can lead to a better metamaterial homogeneity; in other words we can observe resonance at a lower frequency for similar geometrical dimensions. We investigate numerically and experimentally the electromagnetic behaviour of the Z-shaped structure evolved from the transformation and negative real parts of $\varepsilon$ are demonstrated in the microwave domain.

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2. Design of the Z-shaped meta-atom

We first consider the unit cell of the ELC resonator introduced in Ref. 18 and presented in figure 1. This resonator has both inductive and capacitive elements, where only the capacitive element couples strongly to the electric field. By continuous transformation, eliminating the y-oriented lateral wires, it can be reduced to an I-shaped resonator with a capacitive gap referred to as Igap resonator. We shall note that by removing these two side arms, we reduce the inductance of the element. On the counterpart, we add a lateral gap on both sides of the initial middle one. However, the parallel capacitances created by the two lateral gaps are supposed to be negligible compared to the middle one, which suggests that the Igap resonator will show a higher resonant frequency than the ELC one. Furthermore, without changing the lattice constant we compensate a part of the reduced inductance by transforming the Igap into a Zgap resonator. Adding physical length when passing from Igap to Zgap structure produces a higher phase delay and increases the total inductance of the metamaterial without requiring additional fabrication steps. Due to this increase in inductance, the Zgap resonator will present a resonance at a lower frequency. To facilitate the metamaterial fabrication process, we further transform the Zgap resonator into a Z-shaped one by removing the capacitive gap.

3. Simulations and experiments

The properties of the structures are characterized numerically using a Finite Integral Time Domain (FITD) Maxwell’s equations solver, Microwave Studio Suite by CST. A single unit cell is simulated together with necessary periodic boundary conditions. Floquet mode ports are used to launch an incident plane wave on the different resonators. The dielectric spacer used throughout this study is single face copper-cladded epoxy with a relative dielectric constant of 3.9, a tangential loss of 0.02 and a thickness $t_s = 0.4$ mm. Total sample size is $18 \times 18$ cells on a $120$ mm $\times$ $120$ mm dielectric board. For an electromagnetic wave incident with a wave vector and field polarization of figure 1, the different resonators will exhibit an electric resonance.

Calculated reflection and transmission spectra of a single layer structure are presented in figure 2. Microwave measurements have been done on fabricated prototypes in an anechoic chamber using an Agilent 8722ES network analyzer and two standard FLANN® 2-18 GHz wideband band horn antennas serving as the source and the receiver. Phase referencing and normalization have been performed in transmission and reflection. In the transmission measurements, the microwaves are incident normal to the sample surface. Transmission measurements are calibrated to the transmission between the horns with the sample removed. The reflection measurements are done by placing the source and receiving horns on the same side of the sample and bouncing the microwave signal off the sample. The source and receiver horns are each inclined with an angle of about 4-5° with respect to normal on the sample surface. The reflection measurement is calibrated using a sample-sized sheet of copper as a reflecting mirror. Measured reflection and transmission coefficients are compared to the simulated ones. There is a very good qualitative agreement between simulations and measurements.
Figure 2. Computed and measured characteristics of the different resonators.

Calculated and measured magnitudes of $S_{21}$ show clearly a resonance dip for each type of resonator. The ripples that appear in the measured responses are due to noise in the signals. They can be eliminated by a time domain gating of the frequency domain $S$-parameters. The Igap resonator presents a resonance frequency at 13.6 GHz, compared to 6.0 GHz for ELC. This shift toward higher frequencies was predictable since the inductance is greatly reduced when transforming ELC into Igap. To decrease the resonance frequency of Igap, we therefore increase its inductance by using slant lines on each side of the gap instead of vertical ones, therefore converting it to Zgap. This procedure causes a frequency downward shift of approximately 33%, which suggests that the capacitance created by the gap acts mainly to drive the LC resonance and that the LC resonance frequency is mainly based on the inductance of the structure. When cancelling the middle capacitive gap to design the Z-shaped resonator, the resonance frequency is considerably decreased. This is because two capacitances are connected in series in the Zgap structure. One is created between two consecutive cells due to the periodicity of the structure and another one is formed by the gap in the middle of the Z. In the Z-shaped structure, only the capacitance between two consecutive cells is present. According to the dimensions $w_g$ and $l_g$ used for the gap, the capacitance of the latter gap is almost half of the one created by the periodicity. Thus the resonance frequency of the Z is found to be 1.8 times smaller than that of the Zgap, if we consider similar inductance in both structures. And finally, in terms of effective medium ratio $\lambda/a$, an improvement of 1.9 can be observed for the Z-shaped resonator compared to the conventional ELC.

4. Effective parameters

Using reflection and transmission responses from a single layer of the sample, effective parameters can be extracted using the retrieval procedure described in [19]. This is possible since the structure period along the propagation direction is very small compared to the working wavelength. In performing the retrieval, we assume a z-direction size $a_z = 6 \text{ mm}$ corresponding to a cubic unit cell as proposed in [18]. Extracted permittivity $\varepsilon$ and permeability $\mu$ are shown for ELC, Zgap and Z resonators in the various parts of figure 2. Concerning Igap resonator, parameters extraction is not possible since the ratio between the operating wavelength and the unit cell size, i.e. the effective medium ratio $\lambda/a$ is low (~3.7). As stated above, the resonators present an electric response but no magnetic response, i.e., the real part of $\mu$ is close to unity over the frequencies of interest. However, the retrieved parameters do not strictly comply with this principle. The real part of $\mu$ becomes anti-resonant. In fact, this anomaly is an artefact introduced during parameter extraction in a frequency range where the medium is highly spatially dispersive [18]. From figure 2, we observe a strong electric resonance, with the real part of the permittivity ranging from positive to negative values in the vicinity
of the resonance. The values issued from measurement data agree very well with those calculated from numerical simulations. It must be noted that the extraction from the measured spectra are presented without any fitting. From the simulated and measured real parts of $\varepsilon$, it can also be noted that both Z-shaped meta-atoms (with and without gap) present a wider negative permittivity frequency band compared to the ELC resonator. The proposed Z-shaped resonator can be very useful in applications where permittivity gradient is desired, such as in devices designed from transformation optics concept [5-16].

5. Conclusion
In summary, we have presented an experimental demonstration of a negative permittivity from a Z-shaped meta-atom in the microwave domain. This proposed electric resonator allows to considerably improve the effective medium ratio $\lambda/\alpha$ observed from a conventional electric LC (ELC) resonator. Due to the simplicity of the structure and the absence of capacitive gap, a transposition of geometrical parameters can be considered for the optical regime.

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