Triviality Bounds in Two-Doublet Models

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Abstract

We examine perturbatively the two-Higgs-doublet extension of the Standard Model in the context of the suspected triviality of theories with fundamental scalars. Requiring the model to define a consistent effective theory for scales below a cutoff of $2\pi$ times the largest mass of the problem, as motivated by lattice investigations of the one-Higgs-doublet model, we obtain combined bounds for the parameters of the model. We find upper limits of 470 GeV for the mass of the light $CP$–even neutral scalar and 650–700 GeV for the other scalar masses.

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1 Introduction

In the Standard one-doublet Higgs Model of electroweak interactions the scalar potential is
\[ V = \frac{1}{2} m_0^2 \Phi^\dagger \Phi + \frac{1}{4} \lambda_0 (\Phi^\dagger \Phi)^2 \]  
(1)
where \( \Phi \) is a complex doublet and \( m_0^2, \lambda_0 \) are bare parameters. There are strong indications \cite{1, 2} that, in four dimensions and in the limit of vanishing gauge and Yukawa couplings, this defines a trivial field theory in the continuum limit. This means that for any physically acceptable value of the bare coupling \( \lambda_0 \), the renormalized self-coupling \( \lambda_R \) is forced to lie in a narrow range of values which shrinks to the point \( \lambda_R = 0 \) at the limit of infinite cutoff. Equivalently, a non-zero running coupling develops a Landau pole at a finite momentum scale. Yukawa and gauge couplings are not expected to alter this picture \cite{3, 4, 5}. Consequently the Standard one-doublet Model can only be accepted as an effective low energy theory valid up to some finite cutoff \( \Lambda \). The value of the renormalized coupling is thus allowed to be non-zero, but is bounded from above.

This can be illustrated perturbatively by integrating the one-loop \( \beta \)-function for the scalar self-coupling. The result, ignoring gauge and Yukawa couplings, is
\[ \frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \ln \frac{\Lambda}{\mu} \]  
(2)
Here \( \lambda(\Lambda) \) is the bare coupling and \( \mu \) is some low energy renormalization scale. Since \( \lambda(\Lambda) \geq 0 \), it follows that
\[ \lambda_R \equiv \lambda(\mu) \leq \frac{2\pi^2}{3} \left( \frac{1}{\ln(\Lambda/\mu)} \right) \]  
(3)
For a given cutoff \( \Lambda \), the mass \( M_H \) of the Higgs boson is also found to be bounded from above \cite{6, 2, 3, 7}. In lowest-order perturbation theory this is a consequence of the relation
\[ M_H^2 = 2\lambda_R v^2 \]  
(4)
where \( v \) is the vacuum expectation value of the Higgs field.

Various physically motivated choices of \( \Lambda \) have been made leading to different bounds on \( M_H \) \cite{8, 9, 10, 11, 12}. These bounds generally increase with decreasing \( \Lambda \). For the effective theory to make sense, the cutoff \( \Lambda \) must be at least of order \( M_H \) \cite{3}. This places an “absolute” upper bound on the mass of the Higgs boson, which has been estimated \cite{2, 4, 12} to be about 600–700 GeV, for small Yukawa and gauge couplings.
The purpose of this paper is to extend these considerations to models with two Higgs doublets and derive bounds on the masses of the scalar particles of these models. Our results are obtained using perturbative arguments. We believe they convey the right qualitative picture and, in the light of their agreement with other, non-perturbative approaches in the case of the one-Higgs model, we expect they may also have some quantitative validity.

In Section 2 we briefly review the two-doublet extension of the Standard Model. In Section 3 we describe our calculation and in Section 4 we present and discuss our results. For completeness, we list the renormalization group equations for the couplings of the model in the appendix.

2 The two-doublet model

The scalar sector contains two electroweak doublets $\Phi_1$, $\Phi_2$, both with hypercharge $Y = 1$. A discrete symmetry must be imposed in order to eliminate flavor changing neutral currents at tree level. The two-doublet models fall in two broad categories according to the way this discrete symmetry is implemented [13]:

- Model I : $\Phi_2 \rightarrow -\Phi_2$ ; $d_{Ri} \rightarrow -d_{Ri}$
- Model II : $\Phi_2 \rightarrow -\Phi_2$

\[(d_{Ri} \ (i = 1, 2, 3) \text{ are the right-handed negatively charged quarks.}) \]

The Lagrangian is

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_Y - V$$

where $\mathcal{L}_{kin}$ contains all the covariant derivative terms, $V$ is the scalar potential and $\mathcal{L}_Y$ contains the fermion-scalar interactions. The form of the latter is the following:

- Model I

$$\mathcal{L}_Y = g_{ij}^{(u)} \psi_L^i \Phi_1^c u_{Rj} + g_{ij}^{(d)} \psi_L^i \Phi_2 d_{Rj} + \text{h.c.} + \text{leptons}$$

- Model II

$$\mathcal{L}_Y = g_{ij}^{(u)} \psi_L^i \Phi_2^c u_{Rj} + g_{ij}^{(d)} \psi_L^i \Phi_1 d_{Rj} + \text{h.c.} + \text{leptons}$$

i.e. in Model I $\Phi_1$ gives mass to up-type quarks and $\Phi_2$ to down-type quarks while in Model II only $\Phi_1$ couples to quarks.
The results we present were derived using Model II. Since the dominant fermion effects are due to the top quark whose couplings are the same in both models, no substantial changes are expected in Model I.

The scalar potential is

\[
V = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\
+ \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2]
\]

Note that by absorbing a phase in the definition of \( \Phi_2 \), we can make \( \lambda_5 \) real and negative:\[\lambda_5 \leq 0\] (9)

The most interesting case arises when both doublets acquire non-zero vacuum expectation values (vevs). To avoid spontaneous breakdown of the electromagnetic \( U(1) \), the vacuum expectation values must have the following form:

\[
\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}
\]

where \( v_1^2 + v_2^2 \equiv v^2 = (246 \text{ GeV})^2 \). The choice (9) ensures that \( v_1 \) and \( v_2 \) are relatively real. \( v_1 \) can be chosen to be real by an \( SU(2) \times U(1) \) rotation.) This configuration is indeed a minimum of the tree level potential if

\[
\begin{align*}
\lambda_1 &\geq 0 \\
\lambda_4 + \lambda_5 &\leq 0 \\
4\lambda_1\lambda_2 &\geq (\lambda_3 + \lambda_4 + \lambda_5)^2
\end{align*}
\]

The spectrum of the scalar sector contains three Goldstone bosons, to be eaten by the \( W \)’s and the \( Z \); two neutral \( CP \)–even scalars, denoted by \( h, H \); one neutral \( CP \)–odd scalar \( \zeta \); and two charged scalars \( G^\pm \). It is customary to introduce two angles \( \alpha \) and \( \beta \): \( \beta \) \( (0 < \beta < \pi/2) \) rotates the \( CP \)–odd and the charged scalars into their mass eigenstates while \( \alpha \) \( (-\pi/2 \leq \alpha < \pi/2) \) rotates the neutral scalars into their mass eigenstates. The tree level expressions for the masses and angles are the following:

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1. This pushes all potential CP violating effects into the Yukawa sector.
\[ \tan \beta = \frac{v_2}{v_1} \quad (12) \]
\[ \sin \alpha = -(\text{sgn} \, C) \left[ \frac{1}{2} \sqrt{(A - B)^2 + 4C^2 - (B - A)} \right]^{1/2} \quad (13) \]
\[ \cos \alpha = \left[ \frac{1}{2} \sqrt{(A - B)^2 + 4C^2 + (B - A)} \right]^{1/2} \quad (14) \]
\[ M_{G^\pm}^2 = -\frac{1}{2} (\lambda_4 + \lambda_5) \, v^2 \quad (15) \]
\[ M_\zeta^2 = -\lambda_5 \, v^2 \quad (16) \]
\[ M_{H,h}^2 = \frac{1}{2} \left[ A + B \pm \sqrt{(A - B)^2 + 4C^2} \right] \quad (17) \]

where
\[ A = 2\lambda_1 \, v_1^2 ; \quad B = 2\lambda_2 \, v_2^2 ; \quad C = (\lambda_3 + \lambda_4 + \lambda_5) \, v_1 \, v_2 \]

We emphasize that, as is the case in the one-doublet model, all masses get their scales from the vevs, with multiplicative factors that are functions of the quartic couplings. If considerations of triviality put bounds on the couplings (which they do), then these will automatically translate into bounds for the masses. The two-doublet models are described by 7 independent parameters which can be taken to be \( \alpha, \beta, M_{G^\pm}, M_\zeta, M_h, M_H \) and the top quark mass given by
\[ M_t = g_t \, v \cos \beta \quad (18) \]

where \( g_t \) is the top quark Yukawa coupling. The light quark and lepton couplings are inessential to our analysis and we ignore them.

### 3 Triviality and stability constraints

We wish to determine when a given set of parameters \( \{\alpha, \beta, M_{G^\pm}, M_\zeta, M_H, M_h, M_t\} \) defines a valid, consistent low energy effective theory. By 'valid' we mean the following: Suppose \( \Lambda \) is a finite cutoff scale beyond which new phenomena appear. Any physical quantity
calculated using the two-doublet model as described in Section 2, will differ from its ‘true’ value by terms of order $p_i^2/\Lambda^2$, $M_j^2/\Lambda^2$ where $p_i$ are typical external momenta of the processes under consideration and $M_j$ are the masses of the particles in the problem. We shall define our theory to be a valid effective theory if all masses satisfy

$$\frac{M_j}{\Lambda} \leq \frac{1}{2\pi}$$

This convention corresponds to a Higgs correlation length $M_H^{-1} = 2$ (in lattice units), and is widely used in lattice investigations of the problem of triviality and Higgs mass bounds [2]. (The external momenta $p_i$ should also satisfy a similar relation, but this is irrelevant here.) Thus, given a set of parameters, we define a cutoff

$$\Lambda = 2\pi \max \{M_{G\pm}, M_\zeta, M_H, M_h, M_t, M_Z\}$$

$M_Z$ being the $Z$-boson mass, and require, for consistency of the theory, the following conditions to be true:

(i) No coupling should develop a Landau pole at a scale less than $\Lambda$;
(ii) The effective potential should be stable for all field values less than $\Lambda$.

The last requirement is satisfied if

$$\lambda_1(\mu) \geq 0$$
$$\lambda_2(\mu) \geq 0$$
$$\tilde{\lambda}(\mu) \geq -2\sqrt{\lambda_1(\mu)\lambda_2(\mu)}$$

for all $\mu \leq \Lambda$, where

$$\tilde{\lambda}(\mu) = \begin{cases} \lambda_3(\mu) + \lambda_4(\mu) + \lambda_5(\mu) & \text{if } \lambda_4(\mu) + \lambda_5(\mu) < 0 \\ \lambda_3(\mu) & \text{if } \lambda_4(\mu) + \lambda_5(\mu) \geq 0 \end{cases}$$

Our numerical procedure was the following: a set of parameters $\{\alpha, \beta, M_{G\pm}, M_\zeta, M_H, M_h, M_t\}$ was chosen at random. By inverting the relations (12)–(18) the scalar and Yukawa couplings were calculated. It was assumed that the tree-level expressions (12)–(18) approximate best the physical values when the renormalization scale at which the couplings are evaluated is taken to be

$$\mu = \max \{M_{G\pm}, M_\zeta, M_H, M_h, M_t, M_Z\}$$

For field values greater than $\Lambda$ the cutoff effects are large and the renormalized effective potential is meaningless. If a one-component Higgs-Yukawa system is well defined as a bare theory, then it does not develop a vacuum instability [4]. If this is the case in this model too, then the inequalities (21) are equivalent to the condition that the theory exists as a bare theory.
Note that (11) are automatically satisfied if all masses are real.

The coupled renormalization group equations [14] for the scalar, gauge and top Yukawa couplings were evolved up to the scale defined by eq. (20). (In practice, \( \Lambda \) was taken to be at least 1 TeV which is the lowest scale at which one would expect new phenomena.) If any of the couplings became unbounded during this evolution or if the stability constraints (21) were violated, this set of parameters was rejected; otherwise it was accepted. Subsequently a new set was chosen and the procedure repeated. In the end, a large set of randomly generated ‘points’ in parameter space was accumulated. An envelope to these points represents the combined bounds we are seeking.

4 Results and discussion

In Figures 2–8 we display projections of the allowed volume of parameters on selected two-dimensional planes. For comparison, in Fig. 1 we show the bounds for the Standard one-doublet Model particles obtained using the same method\(^3\). The absolute bounds on the masses of the scalar particles in the two-doublet model are about 650–700 GeV (roughly the same as the one-doublet model Higgs mass bound), with the exception of the light neutral scalar which is constrained to be lighter than about 470 GeV. Upper bounds on the top quark are somewhat looser than in the Standard one-doublet Model. We estimate the numerical errors in the calculation of the bounds to be not more than a few GeV, which is insignificant given the largely qualitative nature of our computation. Experimental and other theoretical bounds are not shown in these figures. The upper limits on some splittings among the scalar masses that arise from the precise measurement of the electroweak \( \rho \)-parameter [15, 16] are hardly more stringent than our triviality bounds. Most other reported bounds are lower bounds and do not interfere with our conclusions.

It is not possible to give a description of the exact shape of the bounding surface in the parameter space. We will simply mention some broad qualitative features: The bounds depend strongly on the angle \( \beta \); because of (18) the stability (lower) bounds become stricter as \( \beta \) becomes large at fixed \( M_t \). It is also found that for both small and large \( \beta \) the triviality bounds are stricter than they are for moderate \( \beta \); the precise way in which this happens depends on the values of the other parameters. The dependence on \( \alpha \) is not as strong. Stability bounds on the scalars are strictest when \( \alpha \) takes values close to zero (for a fixed top quark mass.) The bounds on \((M_{G^\pm}, M_\zeta)\) are largely insensitive to the values of \((M_H, M_h)\) for a large range of these values, but shrink sharply outside that

\(^3\)Note the close agreement with the results of ref.[2] where a relation equivalent to (19) was used.
range—and vice-versa—much like fig. 6 shows.

The angle $\alpha - \beta$ is of phenomenological significance since it governs the couplings of the neutral scalars to the $W$'s and the $Z$. We examined the bounds on the neutral scalar masses as a function of $\cos^2(\alpha - \beta)$, projecting out all the other parameters, and found no significant variation.

There is a way in which most of these bounds can be avoided, still within the context of two-doublet models. A quadratic term

$$\mu_3^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}$$

can be added to the scalar potential (8). This violates the discrete symmetry (5) but only softly, so that flavor changing neutral currents still do not appear at tree level. In this case all scalar particle masses but $M_h$ are increasing functions of $|\mu_3^2|$; since $\mu_3^2$ is not constrained from triviality considerations, we can only impose bounds on $M_h$. As $|\mu_3^2|$ grows from zero, we expect the bounds on $M_{G^\pm}$, $M_\zeta$ and $M_H$ to become gradually weaker. For large $|\mu_3^2|$ there is a hierarchy between the scales $M_h^2$ and $|\mu_3^2|$; the latter determines the other scalar masses. Below $|\mu_3|$ the theory looks like the one-Higgs model; insisting that the theory makes sense as a two-doublet model requires an effectively Standard Model quartic coupling to remain finite up to a scale of order $2\pi|\mu_3|$ rather than $2\pi M_h$; hence we expect much stricter bounds than those exhibited in Fig. 1. We have not examined intermediate values of $|\mu_3^2|$ in more detail.

Bounds on the scalar particle masses from triviality considerations have previously been reported in the literature. The authors of ref. [17, 18] concentrate on very large cut-offs while in ref. [19] a different definition of triviality, closely associated with perturbative unitarity, is used. Our bounds are generally stricter than those imposed by perturbative unitarity [19, 20]. The authors of ref. [21] adopt a similar, but stricter, approach than ours and obtain a bound of 475 GeV for the charged scalar mass $M_{G^\pm}$.

According to triviality constraints, the scalar sector of the one-Higgs model is not allowed to become strongly interacting; even the heaviest possible Higgs will be light enough to be detected as a relatively narrow resonance at the SSC. We are currently investigating the implications of the triviality and stability constraints on the phenomenology of two-doublet models.

**Acknowledgements**

We thank A. Cohen, K. Lane and Y. Shen for reading the manuscript. D. K. would like to thank H. Larralde and C. Rebbi for suggestions in the computational part of the work and V. Koulovassilopoulos and Y. Shen for discussions. R.S.C. acknowledges the support
of an Alfred P. Sloan Foundation Fellowship, an NSF Presidential Young Investigator Award, a DOE Outstanding Junior Investigator Award, and a Superconducting Super Collider National Fellowship from the Texas National Research Laboratory Commission. This work was supported in part under NSF contract PHY-9057173 and DOE contract DE-FG02-91ER40676, and by funds from the Texas National Research Laboratory Commission under grant RGFY92B6.
Appendix

In this appendix we include the coupled renormalization group equations for the couplings of the two-doublet model [14]. The gauge couplings for the $SU(3), SU(2)$ and $U(1)$ groups are $g_c$, $g$ and $g'$ respectively. For the other couplings we use the notation of the text. We use the notation

\[ D \equiv 16\pi^2 \mu \frac{d}{d\mu} \]

\[ Dg_c = -7g_c^3 \]
\[ Dg = -3g^3 \]
\[ Dg' = 7g'^3 \]
\[ Dg_t = g_t \left( -\frac{17}{12}g'^2 - \frac{9}{4}g^2 - 8g_c^2 + \frac{9}{2}g_t^2 \right) \]
\[ D\lambda_1 = 24\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \lambda_5^2 - 3\lambda_1(3g^2 + g'^2) + 12\lambda_1g_t^2 + \frac{9}{8}g^4 + \frac{3}{4}g^2g'^2 + \frac{3}{8}g'^4 - 6g_t^4 \]
\[ D\lambda_2 = 24\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \lambda_5^2 - 3\lambda_2(3g^2 + g'^2) + \frac{9}{8}g^4 + \frac{3}{4}g^2g'^2 + \frac{3}{8}g'^4 \]
\[ D\lambda_3 = 4(\lambda_1 + \lambda_2)(3\lambda_3 + \lambda_4) + 4\lambda_3^2 + 2\lambda_3^2 + 2\lambda_5^2 - 3\lambda_3(3g^2 + g'^2) + 6\lambda_3g_t^2 + \frac{9}{4}g^4 - \frac{3}{2}g^2g'^2 + \frac{3}{4}g'^4 \]
\[ D\lambda_4 = 4\lambda_4(\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4) + 8\lambda_5^2 - 3\lambda_4(3g^2 + g'^2) + 6\lambda_4g_t^2 + 3g^2g'^2 \]
\[ D\lambda_5 = \lambda_5(4\lambda_1 + 4\lambda_2 + 8\lambda_3 + 12\lambda_4 - 3(3g^2 + g'^2) + 6g_t^2) \]
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Figure captions

1. Triviality and stability bounds for the Standard Model Higgs and top quark masses $M_H, M_t$. The allowed region is inside the curve.

2. Triviality and stability bounds in the two-doublet model, for the heavy neutral scalar $H$ and the top quark $t$. All other parameters are projected on the $(M_H, M_t)$ plane: the region outside the curve is excluded whatever the values of the parameters not shown on the graph. Constraints from the weak interaction $\rho$-parameter suggest that $M_t \lesssim 250 \text{ GeV}$ \[22\].

3. Same as fig. 2, but projecting on the $(M_h, M_t)$ plane.

4. Same as fig. 2, but projecting on the $(M_{G^\pm}, M_t)$ plane. A similar graph is obtained in the $(M_\zeta, M_t)$ plane, the bound on $M_\zeta$ being slightly higher than the one on $M_{G^\pm}$.

5. Same as fig. 2, but projecting on the $(M_H, M_h)$ plane.

6. Same as fig. 2, but projecting on the $(M_H, M_\zeta)$ plane. A similar plot is obtained for the $(M_H, M_{G^\pm})$ plane, with the bounds on $M_{G^\pm}$ slightly lower than those on $M_\zeta$.

7. Same as fig. 2, but projecting on the $(M_{G^\pm}, M_\zeta)$ plane.

8. Same as fig. 2, but projecting on the $(M_h, M_\zeta)$ plane; as in figures 4 and 6, the bounds on $M_\zeta$ are slightly higher than those on $M_{G^\pm}$.
