Calculation of the three-layer cylindrical shells taking into account the creep of the middle layer

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Abstract. The article deals with the problem of determining the stress-strain state of a three-layer cylindrical shell with axisymmetric loading, taking into account the creep of the middle layer. The derivation of the resolving equations is given, as well as the solution of the test problem for the construction with polyurethane foam filler. The problem is reduced to a system of two differential equations, which is solved by the finite difference method in combination with the Euler method.

1. Introduction

Three-layer structures in the form of plates and shells are widely used in various industries, including construction, shipbuilding, aircraft, etc. Such structures combine low weight and high rigidity. Polymeric materials are widely used as the middle layer of three-layer structures. For polymers, in addition to elastic properties, pronounced rheology is characteristic. Therefore, to calculate such structures, it is necessary to use the apparatus of the theory of creep.

There is a large number of publications on the creep calculation of three-layer structures, including [1-6]. In [2,5] for shallow shells, the effect of curvature on the growth of displacements in the creep process is investigated. It is established that for shells of greater curvature, the creep of the middle layer does not have a noticeable effect on the deflection. However, this result may be a consequence of the assumptions of the shallow shells theory. The purpose of this work is to study the stress-strain state of three-layer cylindrical shells in creep without simplifying assumptions of the shallow shells theory.

2. Derivation of resolving equations

We consider a three-layer cylindrical shell under the action of hydrostatic pressure (figure 1). The considered problem is axisymmetric.

Geometric equations will be obtained from the general Cauchy equations in cylindrical coordinates, having the form [7]:

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Figure 1. Axisymmetrically loaded three-layer cylindrical shell

\[ \varepsilon_x = \frac{\partial u}{\partial x}; \varepsilon_\theta = \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{w}{r}; \varepsilon_z = \frac{\partial w}{\partial r}; \]

\[ \gamma_{,\theta} = \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x}; \gamma_\theta = \frac{v}{r} + \frac{1}{r} \frac{\partial w}{\partial \theta}; \gamma_z = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial r}. \]  

(1)

To obtain the geometric equations of the cylindrical shell in (1), we should assume \( r = R + z \) and take into account that \( z \ll R \). Deformations of the carrier layers will take the form:

\[ \varepsilon_{x\theta}^{(+)} = \frac{\partial u}{\partial x}; \varepsilon_{,\theta}^{(+)} = \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{w}{r}; \varepsilon_z^{(+)} = \frac{\partial w}{\partial r}; \gamma_{,\theta}^{(+)} = \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x}. \]  

(2)

Index “+” in (2) corresponds to the lower carrier layer, and “−” corresponds to the upper carrier layer.

The shear deformations of the middle layer are written in the form:

\[ \gamma_{,\theta}^m = \frac{\partial u}{\partial z} \frac{v}{R} + \frac{1}{R} \frac{\partial w}{\partial \theta}; \gamma_z^m = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}. \]  

(3)

For the displacements \( u \) of the middle layer, we take the linear distribution in thickness:

\[ u^m = \frac{u^+ + u^-}{2} + \frac{u^+ - u^-}{h} z = u + \alpha z. \]  

(4)

Displacements \( v \) with axisymmetric loading are absent. The shear deformations of the middle layer from (3) and (4) are written in the form:

\[ \gamma_z^m = \alpha + \frac{dw}{dx}. \]  

(5)

Tangential stresses in the middle layer, taking into account creep, are defined as follows:

\[ \tau_z^m = G(\gamma_z^m - \gamma^*) = G(\alpha + \frac{dw}{dx} - \gamma^*), \]  

(6)
where \( \gamma^0, \gamma^* \), \( G \) are respectively the total shear deformation, creep deformation and shear modulus of the middle layer.

Deformations of the sheathings with axisymmetric loading will be written in the form:

\[
\varepsilon_{\alpha}^{(\cdot \cdot)} = \frac{du^{(\cdot \cdot)}}{dx}; \quad \varepsilon_{\theta}^{(\cdot \cdot)} = \frac{w}{R}.
\]

Taking the hypothesis of a uniform distribution of tangential stresses across the thickness of the middle layer, we write the transverse force as follows:

\[
Q_i = \tau^\theta h = Gh(\alpha + \frac{dw}{dx} - \gamma^*). \quad (8)
\]

The relationship between stresses and strains for the sheathings is written as:

\[
\sigma_{\alpha}^{(\cdot \cdot)} = \frac{E}{1 - \nu^2}(\varepsilon_{\alpha}^{(\cdot \cdot)} + \nu \varepsilon_{\theta}^{(\cdot \cdot)}); \quad \sigma_{\theta}^{(\cdot \cdot)} = \frac{E}{1 - \nu^2}(\varepsilon_{\theta}^{(\cdot \cdot)} + \nu \varepsilon_{\alpha}^{(\cdot \cdot)}). \quad (9)
\]

Assuming that the bending moment is completely perceived by the sheathings, we represent it in the form:

\[
M_x = \delta(\sigma_x^{(\cdot \cdot)} - \sigma_{\theta}^{(\cdot \cdot)}) \frac{h}{2} - \frac{E\delta h}{2(1 - \nu^2)}(\varepsilon_x^{(\cdot \cdot)} + \nu(\varepsilon_{\theta}^{(\cdot \cdot)} + \varepsilon_x^{(\cdot \cdot)})) = D \frac{d\alpha}{dx}, \quad (10)
\]

where \( D = \frac{E\delta h^2}{2(1 - \nu^2)} \) – is the cylindrical stiffness of a three-layer shell.

Axial forces are also fully perceived by the sheathings:

\[
N_x = (\sigma_x^{(\cdot \cdot)} + \sigma_{\theta}^{(\cdot \cdot)})\delta = \frac{E\delta}{1 - \nu^2}(\varepsilon_x^{(\cdot \cdot)} + \nu \varepsilon_{\theta}^{(\cdot \cdot)} + \varepsilon_x^{(\cdot \cdot)} + \nu \varepsilon_{\theta}^{(\cdot \cdot)}) = \frac{E\delta}{1 - \nu^2} \left( \frac{d(u_x^{(\cdot \cdot)} + u_{\theta}^{(\cdot \cdot)})}{dx} + 2\nu \frac{w}{R} \right). \quad (11)
\]

\[
N_{\theta} = \frac{E\delta}{1 - \nu^2}(\varepsilon_{\theta}^{(\cdot \cdot)} + \nu \varepsilon_x^{(\cdot \cdot)} + \varepsilon_{\theta}^{(\cdot \cdot)} + \nu \varepsilon_x^{(\cdot \cdot)}) = \frac{E\delta}{1 - \nu^2} \left( 2\frac{w}{R} + \nu \frac{d(u^{(\cdot \cdot)} + u_{\theta}^{(\cdot \cdot)})}{dx} \right). \quad (12)
\]

Equating the force \( N_x \) to zero and expressing the value \( \frac{d(u_x^{(\cdot \cdot)} + u_{\theta}^{(\cdot \cdot)})}{dx} \) through the deflection, we obtain the following formula for \( N_{\theta} \):

\[
N_{\theta} = \frac{2E\delta w}{R}. \quad (13)
\]

Equilibrium equations for axisymmetric loading can be written as [8]:

\[
\frac{dM_x}{dx} = Q_i; \quad \frac{dQ_i}{dx} - \frac{N_{\theta}}{R} + q = 0. \quad (14)
\]

The load \( q \) in the case of the hydrostatic pressure action is determined by the formula:

\[
q = \gamma(l - x). \quad (15)
\]

where \( \gamma \) is the specific fluid gravity.

Substituting (10) and (8) into the first equilibrium equation in (14), we get:

\[
D \frac{d^2\alpha}{dx^2} = Gh(\alpha + \frac{dw}{dx} - \gamma^*). \quad (16)
\]
Substituting (8) and (13) into the second equilibrium equation in (14), we get:

\[
\frac{d^2 w}{dx^2} + \frac{d\alpha}{dx} - \frac{d\gamma}{dx} - \frac{1}{Gh} \frac{2E\delta w}{R^2} + \frac{q}{Gh} = 0. 
\]  

(17)

Thus, the problem of calculating a three-layer cylindrical shell under axisymmetric loading has been reduced to a system of two differential equations (16) and (17) with respect to the functions \( \alpha \) and \( w \).

For a shell rigidly clamped at the base, the boundary conditions are:

at \( x = 0 \): \( \alpha = 0, w = 0 \);

at \( x = l \): \( M = 0 \Rightarrow \frac{d\alpha}{dx} = 0, \ Q = 0 \Rightarrow \alpha + \frac{dw}{dx} = 0 \).

(18)

The solution of the system of equations (16) and (17) can be performed numerically by the finite difference method in combination with the Euler or Runge-Kutta method for determining the creep deformations.

3. Results and discussion

The shell with polyurethane foam middle layer was calculated with the following initial data: \( l = 3 \) m, \( R = 2 \) m, \( G = 4.85 \) MPa, \( E = 2 \cdot 10^5 \) MPa, \( \nu = 0.3 \), \( h = 8 \) cm, \( \gamma = 10 \) kN / m\(^3\), \( \delta = 1 \) mm. As the creep law, we used the nonlinear Maxwell-Gurevich equation, which has the form [1]:

\[
\frac{\partial \varepsilon_{xx}^*}{\partial t} = \left( \frac{3}{2} \tau_{xx} - E_{xx} \varepsilon_{xx}^* \right) \frac{1}{\eta^*},
\]

(19)

where \( \varepsilon_{xx}^* = \frac{1}{2} \gamma^* \), \( E_{xx} \) – is the high elasticity modulus, \( \eta^* \) – defines relaxation viscosity, which is nonlinearly dependent on stress:

\[
\frac{1}{\eta^*} = \frac{1}{\eta_{0}^*} \exp \left( \frac{|f^*|}{m^*} \right),
\]

(20)

where \( \eta_{0}^* \) – is the initial relaxation viscosity, \( m^* \) – is the velocity module, \( f^* = \frac{3}{2} \tau_{xx} - E_{xx} \varepsilon_{xx}^* \) – is the stress function.

Rheological parameters of polyurethane foam [1] are: \( E_{xx} = 27.38 \) MPa, \( \eta_{0}^* = 1.43 \cdot 10^4 \) MPa·hours, \( m^* = 0.0218 \) MPa·a.
Figure 2. The change in time of the shell deflection maximum value

Figure 2 shows the graph of the deflection maximum value change obtained as a result of the calculation. From the presented graph, it can be seen that the creep of the middle layer does not have a noticeable effect on the shell displacements. Similar results were obtained by the authors earlier in [2,5] for the shallow three-layer shells.

From Figure 3, showing the change in time of the maximum tangential stresses in the middle layer, it can be seen that the stress relaxation occurs in the middle layer. Bending moments also decrease with time, as shown in Figure 4. Since the ring force is proportional to the deflection \( w \), it is constant in time. Thus, in general, the creep of the middle layer has a positive effect on the stress-strain state of the structure under consideration.

Figure 3. The change in time of the greatest tangential stresses in the middle layer
4. Summary
A system of two differential equations is obtained for calculating the three-layer cylindrical shells under axially symmetric loading taking the creep of the middle layer into account. The calculation of a three-layer shell with polyurethane foam filler was performed. It is established that the creep of the middle layer does not affect the displacements. In the process of creep, shear stresses in the middle layer and bending moments decrease.

The obtained results show that the effects of the three-layer shell curvature on a creep of structure revealed in the papers [2,5] are not a consequence of the shallow shells theory hypotheses.

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