Axion Detection via Atomic Excitations

F.T. Avignone III, R. J. Creswick and J.D. Vergados

1 University of South Carolina, Columbia, SC 29208, USA
2 TEI of Western Macedonia, Kozani, Gr 501 00, Greece and KAIST, 291 Daehak-ro, Yuesong-gu, Daejeon 305-701, Republic of Korea

(Dated: January 18, 2018)

The possibility of axion detection by observing axion induced atomic excitations as recently suggested by Sikivie is discussed. The atom is cooled at low temperature and it is chosen to possess three levels. The first is the ground state, the second is completely empty chosen so that the energy difference between the two is close to the axion mass. Under the spin induced axion-electron interaction an electron is excited from the first to the second level. The presence of such an electron can be confirmed by exciting it further via a proper tunable laser beam to a suitably chosen third level, which is also empty, and lies at a higher excitation energy. From the observation of its subsequent de-excitation one infers the presence of the axion. In addition the presence of the axion can be inferred from the de-excitation of the second level to the ground state. The system is in a magnetic field so that the energies involved can be suitably adjusted. Reasonable axion absorption rates have been obtained.

PACS numbers: 93.35.+d 98.35.Gi 21.60.Cs

Keywords: Axion detection, Atomic excitations, WIMP, event rate

I. INTRODUCTION

In the standard model there is a source of CP violation from the phase in the Kobayashi-Maskawa mixing matrix. This, however, is not large enough to explain the baryon asymmetry observed in nature. Another source is the phase in the interaction between gluons ($\theta$-parameter), expected to be of order unity. The non observation of elementary electron dipole moment limits its value to be $\theta \leq 10^{-9}$. This has been known as the strong CP problem. A solution to this problem has been the P-Q (Pececi-Quinn) mechanism. In extensions of the S-M, e.g. two Higgs doublets, the Lagrangian has a global P-Q chiral symmetry $U_{PQ}(1)$, which is spontaneously broken, generating a Goldstone boson, the axion (a). In fact the axion has been proposed a long time ago as a solution to the strong CP problem \cite{1} resulting to a pseudo Goldstone Boson \cite{2,3}. The axion is a pseudo-Goldstone boson An initial displacement $a_i = \theta f_a$ of the axion field causes an oscillation with frequency $\omega = m_a$ and energy density $\rho_D = (\theta f_a m_a)^2/2$. The production mechanism varies depending on when SSB takes place, in particular whether it takes before or after inflation.

The axion field is homogeneous over a large de Broglie wavelength, oscillating in a coherent way, which makes it ideal cold dark matter candidate in the mass range $10^{-6}$ eV $\leq m_a \leq 10^{-3}$ eV. In fact it has been recognized long time ago as a prominent dark matter candidate by Sikivie \cite{16}, and others, see, e.g, \cite{17}. The axions are extremely light. So it is impossible to detect axions as dark matter particles via scattering them off targets. They are detected by their conversion to photons in the presence of a magnetic field (Primakoff effect), see Fig. 1(a),(b). The produced photons are detected in a resonance cavity as suggested by Sikivie \cite{16}.

In fact various experiments such as ADMX and ADMX-HF collaborations \cite{11,13,21} are now planned to

---

* Permanent address, University of Ioannina, Ioannina, Gr 451 10, Greece
1 Heavier axions with larger mass in the 1eV region produced thermally (such as via the $a\pi\pi\pi$ mechanism), e.g. in the sun, are also interesting and are searched by CERN Axion Solar Telescope (CAST) \cite{18}. Other axion like particles (ALPs), with broken symmetries
FIG. 1: The axion to photon interaction, axion photon coupling, (a) and the axion to photon conversion in the presence of a magnetic field, the Primakoff effect, (b). The axion absorption by an atom at low temperature (c). On the left the spectrum is as it appear in the absence of a magnetic field. On the right the energy splittings depend on the spin-orbit interaction and magnitude of the magnetic field. An electron is moved from an occupied initial level (anywhere the arrows begin) with energy $E_i$ to a level with energy $E_f$, which may be anywhere the arrows end, provided that this was initially empty. The third level discussed in text need not have the same structure and it is not shown.

not connected to QCD, and dark photons form dark matter candidates called WISPs (Weakly Interacting Slim Particles), see, e.g., [19], are also being searched.
search for them. In addition, the newly established center for axion and physics research (CAPP) has started an ambitious axion dark matter research program \cite{22}, using SQUID and HFET technologies \cite{23}. The allowed parameter space has been presented in a nice slide by Raffelt \cite{24} in the recent Multidark-IBS workshop and, focusing on the axion as dark matter candidate, by Stern \cite{20}, derived from Fig. 3 of ref \cite{20}.

Since, however, the mass of the axion is not known, it is important to consider other processes, which may be accessible to a wider window of axion mass. In this paper we are going to discuss the possibility of axion detection by observing axion induced atomic excitations as recently suggested by Sikivie \cite{25}. The atom is cooled at low temperature and it is chosen to posses three levels. The first is the ground state. The second is completely empty, chosen such that the energy difference between the two is close to the axion mass. Under the spin induced axion-electron interaction an electron is excited from the first to the second level. The presence of such an electron there can be confirmed by exciting it further via a proper tunable laser beam to a suitably chosen third level, which is also empty, and lies at higher excitation energy. From the observation of its subsequent de-excitation one infers the presence of the axion (see Fig. 1c). In addition the presence of the axion can be inferred from the de-excitation of the second level to the ground state. The system is in a magnetic field so that the energies involved can be suitably adjusted.

II. OUR PROPOSAL

The axion $a$ is a pseudoscalar particle. We remind the reader that its coupling to the electron takes the form of an axial current interaction:

$$\mathcal{L} = \frac{g_e}{f_a} i \partial_\mu a \bar{\psi}(p', s) \gamma^\mu \gamma_5 \psi(p, s)$$

where $g_e$ is a coupling constant and $f_a$ a scale parameter with the dimension of energy. For an axion with mass $m_a$ described by a a plane wave it easy to show:

- the time component $\mu = 0$ is given by:

$$\mathcal{L} = \langle \phi | \Omega | \phi \rangle, \quad \Omega = \frac{g_e m_a}{2 f_a} \sigma_\mathbf{q}, \quad \mathbf{q} = p' - p$$

which is negligible for $m_a << m_e$.

- The space component, $\mu \neq 0$, in the non relativistic limit is given by

$$\mathcal{L}_{ace} = \langle \phi | \Omega | \phi \rangle, \quad \Omega = \frac{g_e}{2 f_a} \sigma_\mathbf{q}$$

where $\mathbf{q}$ is the axion momentum.

This interaction has been proposed by Sikivie as a way of detecting the axion by causing atomic excitations Thus the axion coupling to electrons in the non relativistic limit has the above form \cite{25}, where $f_a$ is the axion decay constant, $\mathbf{q}$ the axion momentum and $\sigma$ the spin of the electron. $g_e$ is the relevant coupling constant to be determined by experiment.

The target is selected so that there exist two levels, say $|j_1, m_1\rangle$ and $|j_2, m_2\rangle$ which result from the splitting of the atomic levels by the magnetic field and they are characterized by the same $n$ and $\ell$ so that they can be connected by the spin operator. The lower one $|j_1, m_1\rangle$ is occupied by electrons but the higher one $|j_2, m_2\rangle$ is completely empty at sufficiently low temperature. It can be populated only by exciting an electron to it from the lower one by the axion field. The occurrence of such an excitation is monitored by a tuned laser which excites such an electron from $|j_2, m_2\rangle$ to a higher state $|j_3, m_3\rangle$, which cannot be reached in any other way, by observing its subsequent decay. Since this is an one body transition the relevant matrix element takes the form:

$$\langle n\ell j_2 m_2 | q, \sigma | n\ell j_1 m_1 \rangle = \langle j_1 m_1, 1 m_2 - m_1 | j_2 m_2 \rangle \sqrt{(2j_1 + 1)3\ell + 1\sqrt{6} \left\{ \begin{array}{ccc} \ell & \frac{1}{2} & j_1 \\ \ell & \frac{1}{2} & \frac{1}{2} \end{array} \right\} \left( -1 \right)^{m_1 - m_2 q_{m_1 - m_2} I_{n\ell}(q) \right) \quad (1)$$
expressed in terms of the Glebsch-Gordan coefficient and the nine- \( j \) symbol.

\[
I_{nl}(q) = \int d^3p \phi_{nl}(p + q)\phi_{nl}(p)
\]

Since the momentum transfer \( q \) is small \( I_{nl}(q) \approx 1 \). So the matrix element becomes

\[
|ME(q)|^2 = \left( \frac{ge}{2f_a} \right)^2 \left( C_{\ell,j_1,m_1,j_2,m_2} \right)^2 \left( \delta_{m_1,m_2}q_0^2 + \frac{1}{2}(q_1^2 + q_2^2)(1 - \delta_{m_1,m_2}) \right)
\]

where

\[
C_{\ell,j_1,m_1,j_2,m_2} = (j_1 m_1, 1 m_2 - m_1 | j_2 m_2)\sqrt{(2j_1 + 1)(2\ell + 1)}\delta(\ell - \frac{\ell}{2}, j_1 - \frac{\ell}{2}, j_2 0 1 1) (-1)^{m_1 - m_2}
\]

The cross section becomes

\[
\sigma = \frac{1}{\nu} \frac{1}{2m_a} |ME(q)|^2 \int \int d^3p'\frac{(2\pi)^3}{(2\pi)^3} \delta(p + q - p')2\pi\delta(m_a + \frac{q^2}{2m_a} + E_i - E_f)
\]

or

\[
\sigma = \frac{1}{\nu} \frac{1}{2m_a} \left( \frac{ge}{2f_a} \right)^2 \left( C_{\ell,j_1,m_1,j_2,m_2} \right)^2 \left( \delta_{m_1,m_2}q_0^2 + \frac{1}{2}(q_1^2 + q_2^2)(1 - \delta_{m_1,m_2}) \right) 2\pi\delta(m_a + \frac{q^2}{2m_a} + E_i - E_f)
\]

Then the event rate is given by:

\[
R = \Phi_a \sigma = \frac{\rho_a}{m_a} \nu \sigma = \frac{\rho_a}{m_a} \frac{1}{2m_a} \left( \frac{ge}{2f_a} \right)^2 \left( C_{\ell,j_1,m_1,j_2,m_2} \right)^2 \left( \delta_{m_1,m_2}q_0^2 + \frac{1}{2}(q_1^2 + q_2^2)(1 - \delta_{m_1,m_2}) \right) 2\pi\delta(m_a(1 + \frac{1}{2}\nu^2) + E_i - E_f)
\]

We will assume that the axion velocity distribution with respect to the galactic center is of the Maxwell-Boltzmann type:

\[
f_a(v') = \frac{1}{v_0^3} \frac{1}{\pi\sqrt{\pi}} e^{-\left( \frac{v'}{v_0} \right)^2}
\]

In the local frame, ignoring the motion of the Earth, we have \( v' \rightarrow v + v_0 \hat{z} \)

\[
f_e(v) = \frac{1}{v_0^3} \frac{1}{\pi\sqrt{\pi}} e^{-\left( \frac{y^2 + 2y\xi + 1}{v_0} \right)}
\]

The integration over the velocity distribution takes the form:

\[
\Lambda = \int f(v) d^3v \left( \delta_{m_1,m_2}q_0^2 + \frac{1}{2}(q_1^2 + q_2^2)(1 - \delta_{m_1,m_2}) \right) 2\pi\delta(m_a(1 + \frac{1}{2}\nu^2) + E_i - E_f)
\]

\[
= \int \nu^2(m_a\nu)^22\pi\delta(m_a(1 + \frac{1}{2}\nu^2) + E_i - E_f)J \frac{1}{(v_0\sqrt{\pi})^3},
\]

\[
J = (v_0\sqrt{\pi})^3 \int f(v) d\Omega \left( \delta_{m_1,m_2}\xi^2 + \frac{1}{2}(1 - \xi^2)(1 - \delta_{m_1,m_2}) \right)
\]

So the integration over the angles yields:

- In the galactic frame

\[
J = e^{-\xi^2}2\pi \int d\xi \left( \delta_{m_1,m_2}\xi^2 + \frac{1}{2}(1 - \xi^2)(1 - \delta_{m_1,m_2}) \right) = \frac{4\pi}{3} e^{\nu^2}
\]

which is symmetric.
• in the local frame we get:

\[ J = e^{-y^2} 2\pi \int d\xi \, e^{2y\xi} \left( \delta_{m_1,m_2} \xi^2 + \frac{1}{2} (1 - \xi^2)(1 - \delta_{m_1,m_2}) \right) \]  

that is

\[ J = \frac{2\pi}{3} e^{-y^2} \left( \delta_{m_1,m_2} \frac{(2y^2 + 1) \sinh (2y) - 2y \cosh 2y}{2y^3} + (1 - \delta_{m_1,m_2}) \frac{2y \cosh 2y - \sinh 2y}{4y^3} \right) \]  

The integration over the magnitude of the velocity is trivial due to the \( \delta \) function appearing in Eq. 5. We thus get:

\[ \Lambda = 8 \sqrt{\pi} m_a N_{m_1,m_2} F_{m_1,m_2}(X), N_{m_1,m_2} = \begin{cases} \frac{1}{3} \sqrt{\pi} \text{erf}(1) & m_1 = m_2, \text{ local frame} \\ \frac{1}{e^{\sqrt{\pi} \text{erf}(1)+2}} & m_1 \neq m_2, \text{ local frame} \end{cases} \]  

Where \( F_{m_1,m_2}(X) \) is a normalized function of \( X \) given by

\[ F_{m_1,m_2}(X) = \frac{1}{N_{m_1,m_2}} X^2 \frac{2}{3} e^{-X^2} F^{(0)}_{m_1,m_2}(X) \]

\[ F^{(0)}_{m_1,m_2}(X) = \begin{cases} \frac{1}{e} \left( 2X^2 + 1 \right) \sinh (2X) - 2X \cosh 2X \quad m_1 = m_2, \text{ local frame} \\ \frac{1}{e} \left( 2X^2 \cosh 2X - \sinh 2X \right) \quad m_1 \neq m_2, \text{ local frame} \end{cases} \]  

with

\[ X = \frac{c}{v_0} \left( \sqrt{2 \left( \frac{E_f - E_i}{m_a c^2} \right) - 1} \right). \]  

The normalization factor \( N_{m_1,m_2} \), introduced to normalize \( F_{m_1,m_2}(X) \), takes the values 0.33, for the galactic frame, while in the local frame \( \approx 0.50 \) for \( m_1 = m_2 \) and \( \approx 0.36 \) for \( m_1 \neq m_2 \).

### A. The axion absorption widths

The above expression implies that the rate exhibits a resonance behavior in the variable \( X \), which depends on the energy difference. The latter depends on the magnetic quantum numbers \( m_1 \) and \( m_2 \) of the states involved. However there exists an additional dependence of \( F(X) \) on \( \Delta m \), which is mild. This behavior of the functions \( F(X) \) is exhibited in Fig. 2. In fact we find that the characteristics of the resonance are:

\[ \begin{align*}
\Gamma &= 1.16, \quad \langle X \rangle = \sqrt{\frac{2}{7}} = 1.22, \quad \text{galactic frame} \\
\Gamma &= 1.37, \quad \langle X \rangle = 1.71, \quad \text{local frame, } m_1 = m_2 \\
\Gamma &= 1.36, \quad \langle X \rangle = 1.49, \quad \text{local frame, } m_1 \neq m_2.
\end{align*} \]

This dependence of the widths on the magnetic quantum numbers of the states involved is a characteristic feature of the process and it can be exploited by experiments, if they reach adequate sensitivity in the measurement of the parameters of the width. Unfortunately the difference in the expected widths is small, about 1% but the location of the resonances can differ by more than 10% (see Fig. 2). In the latter case one needs to correct for the dependence of \( X \) on \( \Delta m \), since \( \Delta m \) is not the same for both types of transitions.

We have seen that we have resonance behavior in the variable \( X \). Regarding the energy parameters we find

\[ \Gamma_E = m_a c^2 \left( 1 + \frac{1}{2} \left( \Gamma E \right) \right), \quad \langle E \rangle = m_a c^2 \left( 1 + \frac{1}{2} \left( \langle X \rangle \right)^2 \right) \]

Both are very close to the axion mass, since \( (v_0/c)^2 \approx 0.5 \times 10^{-6} \)

It is, of course, more practical to exhibit the function \( F(X) \) as a function of the the energy \( (E_f - E_i)/m_a c^2 \). This is exhibited in Fig. 3.
FIG. 2: (a) The normalized distribution $F(X)$ as a function of $X$ with $X = \frac{1}{v_0} \left( \sqrt{2 \left( \frac{E_f - E_i}{m_a c^2} - 1 \right)} \right)$ and $v_0$ is the velocity of the sun around the center of the galaxy, 220 km/s. The thick solid line corresponds to a Maxwell-Boltzman distribution with respect to the galactic center. The other two lines correspond to the local frame taking into account the motion of the sun: The solid line holds for for $m_1 = m_2$, while the dashed line for $m_2 = m_1 \pm 1$. The motion of the Earth was ignored. The widths are 1.16, 1.37 and 1.36 for the thick solid, the solid and the dashed curves. The corresponding values of $\langle X \rangle$ are $\sqrt{\frac{3}{2}}$, 1.71 and 1.49 respectively. The dispersion for the last two cases is 0.569 and 0.550 respectively. (b) For comparison the normalized distribution $F(X)$ $X = \frac{1}{v_0} \left( \sqrt{2 \left( \frac{\omega}{m_a c^2} - 1 \right)} \right)$, with $\omega$ the photon energy, in the case of the standard axion to photon conversion is presented, obtained with the same halo parameters as in (a), in the galactic frame (solid curve) and local frame. (dashed curve)
FIG. 3: The cross section exhibits resonance behavior. Shown is $F(X)$ as a function of $\left(\frac{E_f - E_i}{m_a c^2} - 1\right)$. It is seen that the function remains constant and equal to 0.1 for $m_a c^2 \leq E_f - E_i \leq 3 \times 10^{-6} m_a c^2$.

B. Modulated widths due to the annual motion of the Earth

The width exhibits annual modulation due to the motion of Earth. This can be simply be included by making in the local frame we make the replacement:

$$F_0(X) \rightarrow e^{-\delta \cos \alpha - \delta^2} F_0(X) \left(\frac{1}{2} \delta \cos \alpha + 1\right)$$

We thus get a time variation of the width shown in Fig. 4. We see that the effect is small, the difference between the maximum and the minimum is less than 3%, almost the same with that obtained in the axion to photon conversion \[26\]. We note, however that, in addition to the seasonal dependence we have a dependence on the magnetic quantum numbers of the states involved. The variation in the case of $m_1 \neq m_2$ is almost twice as large compared to that with $m_1 = m_2$. This, in principle, can be exploited by the experiments.

It is amusing to know that the dispersion $\sigma = \sqrt{\langle X^2 \rangle - \langle X \rangle^2}$ also exhibits a time dependence (see Fig. 5).

III. SOME ESTIMATES ON THE EXPECTED RATES

Using the formulas of the previous section we can cast the event rate in the form:

$$R = N \frac{\rho_a}{m_a} \left(\frac{g_e}{f_a}\right)^2 \sqrt{\pi} (C_{j_1, m_1, j_2, m_2, \ell})^2 N_{m_1, m_2} F_{m_1, m_2}(X)$$

(13)

with $N$ is the number of atoms in the target. We separated out the resonance, which we have discussed above. It is affected by the assumed velocity distribution and the energy difference of the states involved. It depends mildly on the magnetic numbers of the states involved. The above expression can be cast in the form:

$$R = R_0 g_e^2 (C_{j_1, m_1, j_2, m_2, \ell})^2 N_{m_1, m_2} F_{m_1, m_2}(X)$$

(14)

with $g_e$ to be determined (the limit has been set by CAST \[27\] only involves its product with $g_{a\gamma}$).

$$R_0 = 7.5 \times 10^4 \text{ per mol-\gamma} \left(\frac{\rho_a}{\text{GeV/cm}^3}\right) \left(0.6 \times 10^{-4}\text{eV/GeV}\right) \left(\frac{10^{11}\text{GeV}}{f_a}\right)^2 \left(\frac{N}{N_A}\right)$$

where $N_A$ is Avogadro’s number $\approx 6.0 \times 10^{23}$. The coefficients $(C_{j_1, m_1, j_2, m_2, \ell})^2$ can easily be calculated and are given in tables \[III\] for some cases of practical interest. We should mention that $R_0$ does not contain the dependence on
the sub-states $m_1$ and $m_2$ as well as the factor 0.1 coming from $F(X)$, see Fig. 3. The expected rate may thus be reduced, if one takes into account such a dependence and the selected states are not $s$-states.

The coupling $g_e$ is not known and, hopefully, it can be extracted from experiment. The coupling of axion to matter has been investigated [28, 29], in particular in the context of the DFSZ axion models [7, 9]. This leads to:

$$g_e = \frac{1}{3} \left( 1 - \frac{\tan^2 \beta}{1 + \tan \tan^2 \beta} \right), \tan \beta = \frac{v_2}{v_1}$$

Where $\tan \beta$ is the ratio of the vacuum expectation values of the two doublets of the model. This parameter is not known. It can be one, but it be quite large. Thus we find $0 < g_e < \frac{1}{6}$. Adopting the higher value we find the scale of the expected rate to be:

$$R_0 g_e^2 \approx 2.0 \times 10^3 \text{ per mol-yr} \left( \frac{\rho_a}{\text{GeV/cm}^2} \right) \left( \frac{0.6 \times 10^{-4} \text{eV}}{m_a} \right) \left( \frac{10^{13} \text{GeV}}{f_a} \right)^2 \left( \frac{N}{N_A} \right)$$

(16)
FIG. 5: The time variation of the dispersion $\sigma = \sqrt{\langle X^2 \rangle - \langle X \rangle^2}$ for axion absorption by an atom due to the motion of the Earth. The notation for the curves is the same as in Fig. 4.

IV. LOW TEMPERATURE REQUIREMENTS

To achieve the condition that the second level must be essentially free of electrons the target material should be brought at low temperatures. The critical temperature depends on the axion mass to be explored. The average number of electrons expected to be at any given at the excited state is $N_A$ times the probability of occupancy which is given by the Boltzmann distribution probability $e^{-m_a/kT}$. The number of electrons per year is expected to be $e^{-m_a/kT}N_A3.156 \times 10^7$. Let us take as expected number of real events those estimated above reduced by a factor of 100 due to angular momentum effect etc, i.e. $7.5 \times 10^4 \times 10^{-2} \left( \frac{0.6 \times 10^{-4} \text{eV}}{m_a} \right)$. Thus

$$7.5 \times 10^4 \times 10^{-2} \left( \frac{0.6 \times 10^{-4} \text{eV}}{m_a} \right) \geq e^{-m_a/kT}N_A3.156 \times 10^7$$

The condition on the temperature is given in Fig. 6.

V. SOME EXPERIMENTAL CONSIDERATIONS

Thus in the search for for light axions one has to develop detectors involving materials, which at these low temperatures exhibit atomic structure. Ordinary atoms do not suffice. The ions of the crystal still exhibit atomic structure involving the bound electrons, as, e.g., the CUORE detector of Crystalline $^{130}\text{TeO}_2$ at low temperatures. The electronic states probably won’t carry all the important quantum numbers as their corresponding neutral atoms. So one may prefer to consider targets which contain appropriate impurity atoms in a host crystal, e.g chromium in sapphire. It is, also, possible that one may be able to employ at low temperatures some exotic materials used in quantum technologies (for a recent review see [30]) like nitrogen-vacancy (NV), i.e. materials characterized by spin $S = 1$, which in a magnetic field allow transitions between $m = 0, m = 1$ and $m = -1$. These states are spin symmetric. Antisymmetry requires the space part to be antisymmetric, i.e. a wave function of the form

$$\psi = \phi_{n\ell}^2(r) \begin{cases} [L = \text{odd}, S = 1] & J = L - 1, L, L + 1 \end{cases}$$

Of special interest are the cases:

$$\psi = \phi_{n\ell}^2(r)^3P_J, \phi_{n\ell}^2(r)^3F_J$$
FIG. 6: The temperature in degrees Kelvin to be achieved is the region below the above curve, so that the population of the excited state by thermal electrons can be neglected.

TABLE I: the coefficients $(C_{j_1, m_{j_1}, j_2, m_{j_2}, \ell})^2$ connecting via the spin operator a given initial state $|i\rangle = |n\ell, j_1, m_1\rangle$ with all possible states $|f\rangle = |n\ell, j_2, m_2\rangle$, for $\ell = 0, 1$. Note s-states are favored.

\[
\begin{pmatrix}
\ell & j_1 & m_{j_1} & j_2 & m_{j_2} & C_{j_1, m_{j_1}, j_2, m_{j_2}, \ell}^2
\end{pmatrix}
\]

Then the spin matrix element takes the form:

\[
\langle P_jm_2|\sigma|P_jm_1\rangle^2 = \frac{1}{\sqrt{2J_2 + 1}}(J_1m_1, 1m_2 - m_1|J_2m_2\rangle\langle 3L_2|\sigma|3L_1\rangle, L = P, F
\]

The reduced matrix elements are given in table [III] the full matrix element $\langle P_jm_2|\sigma|P_jm_1\rangle^2$ for the most important case is also shown in [III].

[1] R. Peccei and H. Quinn, Phys. Rev. Lett 38, 1440 (1977).
[2] S. Weinberg, Phys. Rev. Lett. 40, 223 (1978).
[3] F. Wilczek, Phys. Rev. Lett. 40, 279 (1978).
[4] J. E. Kim, Phys. Rev. Lett. 43, 137 (1979).
[5] M. A. Shifman, A. Vainshtein, and V. I. Zakharov, Nuc. Phys. B 166, 493 (1980).
[6] M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. B 104, 199 (1981).
TABLE II: The same as in Table II the coefficients \((C_{j_1, m_1, j_2, m_2, \ell})^2\) for \(\ell = 2\)

| \(\ell\) | \(j_1\) | \(m_1\) | \(j_2\) | \(m_2\) | \((C_{j_1, m_1, j_2, m_2, \ell})^2\) |
|---|---|---|---|---|---|
| 2 | 2 | 2 | 1 | 1 | \(\frac{1}{2}\) |
| 2 | 2 | 2 | 1 | 1 | \(\frac{1}{2}\) |
| 2 | 2 | 2 | 1 | 1 | \(\frac{1}{2}\) |
| 2 | 2 | 2 | 1 | 1 | \(\frac{1}{2}\) |
| 2 | 2 | 2 | 1 | 1 | \(\frac{1}{2}\) |
| 2 | 2 | 2 | 1 | 1 | \(\frac{1}{2}\) |
| 2 | 2 | 2 | 1 | 1 | \(\frac{1}{2}\) |
| 2 | 2 | 2 | 1 | 1 | \(\frac{1}{2}\) |
| 2 | 2 | 2 | 1 | 1 | \(\frac{1}{2}\) |
| 2 | 2 | 2 | 1 | 1 | \(\frac{1}{2}\) |

TABLE III: The coefficients \((3P_{j_2}||\sigma||3P_{j_1})\), \((3F_{j_2}||\sigma||3F_{j_1})\) and \((3P_{j_2m_2}||\sigma||3P_{j_1m_1})^2\). For the notation see text.

| \(J_1\) | \(J_2\) | \((3P_{j_2}||\sigma||3P_{j_1})\) | \(J_1\) | \(J_2\) | \((3F_{j_2}||\sigma||3F_{j_1})\) | \(J_1\) | \(J_2\) | \((3P_{j_2m_2}||\sigma||3P_{j_1m_1})^2\) |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | \(\sqrt{\frac{12}{5}}\) | 2 | 2 | \(\sqrt{\frac{10}{3}}\) | 0 | 0 | \(m_2\) |
| 1 | 1 | \(\sqrt{\frac{12}{5}}\) | 2 | 3 | \(\sqrt{\frac{12}{5}}\) | 1 | -1 | 1 | 1 |
| 1 | 2 | \(\sqrt{\frac{12}{5}}\) | 3 | 3 | \(\sqrt{\frac{12}{5}}\) | 1 | 0 | 0 | 0 |
| 2 | 2 | \(\sqrt{\frac{12}{5}}\) | 4 | 4 | \(\sqrt{\frac{12}{5}}\) | 1 | 0 | 1 | 1 |

[7] A. Zhitnisky, Sov. J. Nuc. Phys. 31, 260 (1980), in Russian.
[8] L. F. Abbott and P. Sikivie, Phys. Lett. B120, 133 (1983).
[9] M. Dine and W. Fischler, Phys. Lett. B120, 137 (1983).
[10] J. Preskill, M. B. Wise, and F. Wilczek, Phys. Lett. B120, 127 (1983).
[11] S. J. Asztalos et al., Phys. Rev. Lett. 104, 041301 (2010), the ADMX Collaboration, arXiv:0910.5914 (astro-ph.CO).
[12] L. Duffy et al., Phys. Rev. Lett. 95, 09134 (2005), for the ADMX Collaboration.
[13] A. Wagner et al., Phys. Rev. Lett. 105, 171801 (2010), for the ADMX collaboration; arXiv:1007.3766 (astro-ph.CO).
[14] I. G. Irastorza and J. A. Garca, JCAP 1210, 022 (2012), arXiv:1007.3766 (astro-ph.IM).
[15] D. J. E. Marsh, Phys. Rep. D 643, 1 (2016).
[16] P. Sikivie, Phys. Rev. Lett. 51, 1415 (1983).
[17] J. Primack, D. Seckel, and B. Sadoulet, Ann. Rev. Nuc. Par. Sc. 38, 751 (1988).
[18] S. Aune et al., Phys. Rev. Let. 107, 261302 (2011), arXiv:1106.3919; CAST Collaboration.
[19] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper, and J. March-Russell, Phys. Rev. D 81, 123530 (2010), arXiv:0905.4720.
[20] I. P. Stern, ArXiv 1403.5332 (2014) physics.ins-det, on behalf of ADMX and ADMX-HF collaborations, Axion Dark Matter Searches.
[21] G. Rybka, The Axion Dark Matter Experiment, IBS MultiDark Joint Focus Program WIMPs and Axions, Daejeon, S. Korea October 2014.
[22] Center for Axion and Precision Physics research (CAPP), Daejeon 305-701, Republic of Korea. More information is available at http://capp.ibs.re.kr/html/capp_en/.
[23] S. J. Asztalos et al., Nucl. Instr. Meth. in Phys. Res. A656, 39 (2011), arXiv:1105.4203 (physics.ins-det).
[24] G. Raffelt, Astrophysical Axion Bounds, IBS MultiDark Joint Focus Program WIMPs and Axions, Daejeon, S. Korea October 2014.
[25] P. Sikivie, Phys. Rev. Lett. 113, 201301 (2014).
[26] J. D. Vergados and Y. Semertzidis, Nuc. Pys. B 897, 821 (2016), arXiv:1601.04765 (hep-ph).
[27] K. Barth et al., JCAP 1305, 010 (2013), arXiv:1302.6283.
[28] G. G. di Cortona, E. Hardy, J. P. Vega, and G. Villadoro, JHEP 01, 034 (2016), arXiv:1511.02867, (hep-ph), (hep-ex), (hep-lat).
[29] A. Ringwald and K. Saikawa, Phys. Rev. D 93, 085031 (2016), arXiv:1512.06436 [hep-ph].
[30] M. W. Doherty, N. B. Manson, P. Delaney, F. Jelezko, J. Wrachtrup, and L. C. L. Hollenberg, Phys. Rep. 528, 1 (2013), arXiv:1302.3288.