Flavour Changing Neutral Current Constraints from Kaluza-Klein Gluons and Quark Mass Matrices in RS1

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Abstract

We continue our previous study on what are the allowed forms of quark mass matrices in the Randall-Sundrum (RS) framework that can reproduce the experimentally observed quark mass spectrum and the CKM mixing pattern. We study the constraints the $\Delta F = 2$ processes in the neutral meson sector placed on the admissible forms found there, and we found only the asymmetrical type of quark mass matrices arising from anarchical Yukawa structures stay viable at the few TeV scale reachable at the LHC. We study also the decay of the first Kaluza-Klein (KK) excitation of the gluon. We give the decay branching ratios into quark pairs, and we point out that measurements of the decay width and just one of the quark spins in the dominant $\bar{t}t$ decays can be used to extract the effective coupling of the first KK gluon to top quarks for both chiralities. This provides further probe to the flavour structure of the RS framework.

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I. INTRODUCTION

The use of the warped extra-dimensional model of Randall and Sundrum (RS) as a framework for flavour physics has garnered a lot of attention ever since the model’s introduction. By implementing the split fermion scenario, the hierarchy in the Standard Model (SM) fermion masses can be understood geometrically in terms of the different localization of the SM fermions in the extra dimension. In such a set-up, the different fermion masses can be obtained without fine tuning the Yukawa couplings, in contrast to the usual four-dimensional theories.

Having fermions propagating in the extra dimension requires that the SM gauge symmetry be promoted to a bulk symmetry. Constraints then arise because of the electroweak precision tests (EWPTs). In particular, for the simplest model with just the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$, constraints on the $S$ and $T$ parameters and the $Zbb$ couplings are found to be difficult to satisfy without fine tuning. Since an $SU(2)_R$ symmetry is instrumental in ensuring the very accurate relation $\rho = 1$ in the SM, a natural way to satisfy the EWPTs would be to promote the $SU(2)_R$ to a bulk gauge symmetry, and this was done in Ref. [4].

In this work, we continue our study that began in Ref. [6] of the forms of quark mass matrices admissible in a minimal RS1 setting with custodial symmetry that can fit all the experimental data in the quark sector without having hierarchical Yukawa structures. It is well known that tree-level flavor changing neutral current (FCNC) interactions are generic in the RS flavour models. Processes mediated by the Kaluza-Klein (KK) excitations of the gauge bosons – in particular that of the gluons – can give rise to large FCNC effects, which are tightly constrained by the many low energy measurements in the neutral meson sector such as $\epsilon_K$ and $B^0_q-\bar{B}^0_q$ transition ($q = d, s$). We study in this work the impact these $\Delta F = 2$ FCNC constraints have on the admissibility of the forms found in Ref. [6]. In particular, as these FCNC constraints place stringent limits on the lowest KK gauge boson mass, $m_{\text{gauge}}^{(1)}$, which sets the scale of new physics (NP), we investigate which of the forms of the quark mass matrices can satisfy all the FCNC constraints at an NP scale reachable by the LHC.

Since the dominant contribution to the FCNCs comes from the KK gluons in the setting we

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1 Although having the custodial symmetry is still the surest way to satisfy the EWPT constraints, Ref. [5] has reported recently that they may also be satisfied by having a heavy Higgs boson alone.
study \(^2\), we concentrate on their effects below.

The paper is organized as follows. In Sec. [II] we give a brief outline of the minimal custodial RS (MCRS) model studied to set the notation. In Sec. [III] we study in the MCRS model the impact of FCNCs mediated by the tree-level exchange of KK gluons have on the \(\Delta F = 2\) processes in the meson sector. These place constraints on the scale of new flavour physics. In Sec. [IV] we evaluate the contribution due to the KK gluon exchanges in the neutral \(B\)-meson observables, and we calculate the branching ratios of the first KK gluon decaying into a pair of quarks. We point out that measuring even just one of the quark spin, such as in top decays which are the dominant decay mode, can be very useful in distinguishing the different models in the RS framework. We conclude in Sec. [V]. Appendix [A] contains asymmetrical quark mass matrices that are typical representations of the families of the admissible asymmetrical forms used in this work. In Appendix [B] we show that with the fermion representation we use in this work, the electroweak contributions neither displace the dominance of the KK gluon contributions, nor cause the current FCNC bounds to be violated if they are included as well.

II. THE MCRS MODEL

In this section, we describe briefly the basic set-up of the MCRS model to establish notations (see also Ref. [3]) relevant for studying the flavour changing processes in the meson sector. A more complete and detailed description can be found in, e.g. Ref. [4].

The MCRS mode is formulated on a slice of \(AdS_5\) space specified by the metric

\[
d s^2 = G_{AB} \, dx^A dx^B = e^{-2\sigma(\phi)} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2 ,
\]

where \(\sigma(\phi) = k r_c |\phi|\), \(\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)\), \(k\) is the \(AdS_5\) curvature, and \(-\pi \leq \phi \leq \pi\). The theory is compactified on an \(S_1/(Z_2 \times Z_2')\) orbifold, with \(r_c\) the radius of the compactified fifth dimension, and the orbifold fixed points at \(\phi = 0\) and \(\phi = \pi\) correspond to the UV (Planck) and IR (TeV) branes respectively. To solve the hierarchy problem, \(k \pi r_c\) is set to \(\approx 37\). The warped down scale is defined to be \(\tilde{k} = k e^{-k \pi r_c}\). Note that \(\tilde{k}\) sets the scale of the first KK gauge boson mass, \(m_{gauge}^{(1)} \approx 2.45 \tilde{k}\), which determines the scale of the new KK physics.

\(^2\) This is explicitly checked in the calculations below.
The MCRS model has a bulk gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$ under which the IR brane-localized Higgs field and transforms as $(1, 2, 2)_0$. The SM quarks are embedded into $SU(2)_L \times SU(2)_R \times U(1)_X$ via the five-dimensional (5D) bulk Dirac spinors

$$Q_i = \left( u_{iL} [+, +], d_{iL} [+, +] \right), \quad U_i = \left( u_{iR} [+, +], d_{iR} [−, +] \right), \quad D_i = \left( \bar{u}_{iR} [−, +], \bar{d}_{iR} [+, +] \right), \quad i = 1, 2, 3, \quad (2)$$

where $Q_i$ transforms as $(2, 1)_{1/6}$, and $U_i, D_i$ transform as $(1, 2)_{1/6}$. The parity assignment ± denote the boundary conditions applied to the spinors on the [UV, IR] brane, with + (−) being the Neumann (Dirichlet) boundary conditions. Only fields with the [+ ,+] parity contain zero-modes that do not vanish on the brane. These survive in the low energy spectrum of the 4D effective theory, and are identified as the SM fields.

A given 5D bulk fermion field, $\Psi$, can be KK expanded as

$$\Psi_{L,R}(x, \phi) = e^{3\sigma/2 \sqrt{kr_c \pi} \sum_{n=0}^{\infty} v_{L,R}^{(n)}(x) f_{L,R}^n(\phi)}, \quad (3)$$

where subscripts $L$ and $R$ label the chirality, and the KK modes $f_{L,R}^n$ are normalized according to

$$\frac{1}{\pi} \int_0^{\pi} d\phi f_{L,R}^{n*}(\phi) f_{L,R}^m(\phi) = \delta_{mn}. \quad (4)$$

The KK-mode profiles are obtained from solving the equations of motion. For the zero-modes, the RS flavor functions are given by

$$f_{L,R}^0(\phi, c_{L,R}) = \sqrt{\frac{k r_c \pi (1 \mp 2c_{L,R})}{e^{kr_c \pi (1 \mp 2c_{L,R})} - 1}} e^{(1/2 \mp c_{L,R}) kr_c \phi}, \quad (5)$$

where the $c$-parameter is determined by the bulk Dirac mass parameter, $m = c k$, and the upper (lower) sign applies to the LH (RH) label. Depending on the orbifold parity of the fermion, one of the chiralities is projected out.

After spontaneous symmetry breaking, the Yukawa interactions localized on the IR brane lead to mass terms for the fermions on the IR brane

$$S_{\text{Yuk}} = \int d^4x \frac{v_W}{k r_c \pi} \left[ \bar{\Psi}_u(x, \pi) \lambda_5^u \Psi_u(x, \pi) + \bar{\Psi}_d(x, \pi) \lambda_5^d \Psi_d(x, \pi) \right] + \text{h.c.} , \quad (6)$$

where $v_W = 174$ GeV is the VEV acquired by the Higgs field, and $\lambda_5^{u,d}$ are the (complex) dimensionless 5D Yukawa coupling matrices. For zero-modes, this gives the mass matrices for the SM quarks in the 4D effective theory

$$(M_{f_0}^{RS})_{ij} = \frac{v_W}{k r_c \pi} \lambda_{5,ij}^f f_{L}^0(\pi, c_{f,j}) f_{R}^0(\pi, c_{f,i}) \equiv \frac{v_W}{k r_c \pi} \lambda_{5,ij}^f F_L(c_{f,j}) F_R(c_{f,i}), \quad f = u, d, \quad (7)$$
where the label $f$ denotes up-type or down-type quark species. The up and down mass matrices are diagonalized by a bi-unitary transformation

\[
(U_L^{u,d})^\dagger M_{u,d}^{RS} U_R^{u,d} = \begin{pmatrix}
m_1^{u,d} & 0 & 0 \\
0 & m_2^{u,d} & 0 \\
0 & 0 & m_3^{u,d}
\end{pmatrix},
\]

where $m_i^{u,d}$ are the masses of the SM up-type and down-type quarks. The mass eigenbasis is then defined by $\psi' = U^\dagger \psi$, and the CKM matrix given by $V_{CKM} = (U_L^u)^\dagger U_L^d$.

### III. $\Delta F = 2$ PROCESSES IN THE MESON SECTOR

In extra-dimensional models, tree-level flavour changing neutral currents (FCNCs) arising from the KK-excitations of gauge bosons are generic. For $\Delta F = 2$ FCNCs, by virtue of the strength of the strong coupling constant, the largest and thus the most constrained contribution comes from processes mediated by the exchange of the KK gluons as depicted in Fig 1. Effective four-fermion operators are generated when the KK gluons are integrated out.

\[
\begin{align*}
\bar{f}_i & \quad \bar{f}_j \\
& \quad G^{(n)} \\
& \quad f_i \quad f_j \\
\end{align*}
\]

**FIG. 1:** Contributions to $\Delta F = 2$ processes from the tree-level exchange of KK gluons. The fermions are in the weak eigenbasis.

In the gauge (weak) eigenbasis, the coupling of the $n$-th level KK gluon, $G^{(n)}$, to zero-mode fermions is given by

\[
G^{(n)}_\mu \left[ \sum_i (g_f^n)^L_{ij} \bar{f}_i \gamma^\mu f_j + (L \leftrightarrow R) \right], \quad f = u, d,
\]

where $i$ is a generation index, and $(g_f^n)_{ij} = \text{diag}(g_{f_1}^n, g_{f_2}^n, g_{f_3}^n)$ is the weak eigenbasis coupling matrix with

\[
g_{f_i}^n = \frac{g_s}{\pi} \int_0^\pi d\phi |f^0(\phi, c_{f_i})|^2 \chi_\nu(\phi), \quad g_s = \frac{g_{5s}}{\sqrt{\alpha}}.
\]

$$
Here, $g_{5s}$ is the bulk 5D $SU(3)$ gauge coupling, $g_s$ that in the SM, and $\chi_n$ the profile of the n-th KK gluon. Note that the matching relation between $g_{5s}$ and $g_s$ can be changed by the presence of localized brane kinetic terms. As in Ref. [9], we have chosen here and for the analysis below UV boundary terms such that the bare kinetic terms cancel exactly the contribution coming from the one-loop running. The IR brane kinetic terms are small and can be neglected.

Going to the mass eigenbasis $f' = U^\dagger f$, the $G^{(n)} f' f'$ coupling reads

$$G^{(n)}_\mu \left[ \sum_{a,b} (\hat{g}_f^n)_{ab} \bar{f}_{aL} \gamma^\mu f_{bL} + (L \leftrightarrow R) \right], \quad f = u, d, \tag{11}$$

where

$$(\hat{g}_f^n)_{ab}^{L,R} = \sum_{i,j} (U^\dagger_{L,R})_{ai} (\hat{g}_f^n)_{ij}^{L,R} (U_{L,R})_{jb}. \tag{12}$$

The off-diagonal couplings $(\hat{g}_f^n)_{ab}$ appear because the diagonal weak eigenbasis couplings, $g_{fi}^n$, are not all equal.

In order to compute the coefficients of the effective four-fermion operators arising from the tree-level KK gluon exchanges, one has to perform (in the mass eigenbasis) sums of the form

$$S_{\omega,\xi}^{ab,cd} = \sum_{n=1}^{\infty} \frac{(\hat{g}_f^n)_{ab}^{\omega} (\hat{g}_f^n)_{cd}^{\xi}}{m_n^2}, \quad \omega, \xi = L, R, \tag{13}$$

where $m_n$ is the mass of the n-th KK gluon, and $\omega, \xi$ label the chirality. The sum over the KK gluon tower can be efficiently calculated with the help of the massive gauge 5D mixed position-momentum space propagators [4, 7, 8]. It can be computed in terms of the overlap integral,

$$G^{++}_{ff'}(c_i^\omega, c_j^\xi) = \frac{1}{\pi} \int_0^\pi d\phi |f^0_{\omega}(\phi, c_i^\omega)|^2 \tilde{G}^{(++)}_{p=0}(\phi, \phi') |f^0_{\xi}(\phi', c_j^\xi)|^2, \quad \omega, \xi = L, R, \tag{14}$$

where $\tilde{G}^{(++)}_{p=0}$ is the zero-mode subtracted gauge propagator evaluated at zero 4D momentum, and is given by [8]

$$\tilde{G}^{(++)}_{p=0}(\phi, \phi') = \frac{1}{4k(k \pi)} \left\{ \frac{1 - e^{-2kr_c\pi}}{kr_c\pi} + e^{2kr_c\phi} (1 - 2kr_c\phi) + e^{2kr_c\phi'} \left[ 1 + 2kr_c(\pi - \phi) \right] \right\}, \tag{15}$$

See also Ref. [9] for an equivalent way of summing up the gluon KK tower.

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3 See also Ref. [8] for an equivalent way of summing up the gluon KK tower.
where $\phi_\ell = \min(\phi, \phi')$, $\phi_\ell = \max(\phi, \phi')$. The sum over the KK tower is then given by

$$S_{\omega, \xi}^{ab, cd} = g_s^2 \sum_{i,j} (U_\omega^a U_\xi^b)_{i,j} G^{++}(c_i^\omega, c_j^\xi)(U_\omega^a)_{c_i} (U_\xi^b)_{c_j}, \quad \omega, \xi = L, R. \quad (16)$$

The most general effective Hamiltonian for the $\Delta F = 2$ processes beyond the SM can be written as

$$H_{\text{NP}}^{\text{eff}} = \sum_{i=1}^5 C_i(\Lambda) Q_{i,ab}^{ab} + \sum_{i=1}^3 \tilde{C}_i(\Lambda) \tilde{Q}_{i,ab}^{ab}, \quad (17)$$

where $\Lambda$ is the scale of new physics (NP), and

$$Q_{i}^{ab} = \bar{\psi}_a^{\alpha_L} \gamma_\mu \psi_b^{\alpha_L} \bar{\psi}_a^{\beta_L} \gamma_\mu \psi_a^{\beta_L},$$

$$Q_{2}^{ab} = \bar{\psi}_a^{\alpha_L} \gamma_\mu \psi_b^{\alpha_L} \bar{\psi}_a^{\beta_L} \gamma_\mu \psi_b^{\beta_L},$$

$$Q_{3}^{ab} = \bar{\psi}_a^{\alpha_L} \gamma_\mu \psi_b^{\alpha_L} \bar{\psi}_a^{\beta_R} \gamma_\mu \psi_b^{\beta_R},$$

$$Q_{4}^{ab} = \bar{\psi}_a^{\alpha_R} \gamma_\mu \psi_b^{\alpha_R} \bar{\psi}_a^{\beta_L} \gamma_\mu \psi_b^{\beta_L},$$

$$Q_{5}^{ab} = \bar{\psi}_a^{\alpha_R} \gamma_\mu \psi_b^{\alpha_R} \bar{\psi}_a^{\beta_R} \gamma_\mu \psi_b^{\beta_R}, \quad (18)$$

with $\alpha, \beta$ the colour indices, and $a, b$ the generation indices $^4$. The operators $\tilde{Q}_{1,2,3}^{ab}$ are obtained from $Q_{1,2,3}^{ab}$ by the $L \leftrightarrow R$ exchange. All operators are given in the mass eigenbasis here. In the MCRS model, only $Q_{1,4,5}^{ab}$ and $\tilde{Q}_{1}^{ab}$ arise from the tree-level exchange of KK gluons, and their coefficients are given by

$$C_{1}(\Lambda) = \frac{1}{6} S_{ab,ab}^{LL}, \quad \tilde{C}_{1}(\Lambda) = \frac{1}{6} S_{ab,ab}^{RR}, \quad C_{4}(\Lambda) = -S_{ab,ab}^{LR}, \quad C_{5}(\Lambda) = -\frac{1}{3} C_{4}. \quad (19)$$

Note that here the NP scale is the scale where the KK excitations first come in, hence $\Lambda \sim m_1$.

Recently, a model independent global analysis of the physical observables in the $\Delta F = 2$ processes have been performed by the UTfit collaboration $^{[11]}$. Bounds on the NP scale Wilson coefficients $C_i(\Lambda)$ above have been found with the Renormalization Group evolution fully taken into account. Given these bounds, an immediate question with regard to the admissible forms of quark mass matrices found in Ref. $^{[6]}$ is whether they remain viable, as they govern the form of the rotation matrices, $U_\omega$, that determine the Wilson coefficients in the MCRS model (see Eqs. $^{[16]}$ and $^{[19]}$).

$^4$ The so-called supersymmetric (SUSY) basis of operators $^{[10]}$ is used here. Other basis can be obtained via the appropriate Fierz identities.
Two types of mass matrix structures were found in Ref. \[6\] that reproduce well the observed patterns of quark masses and CKM mixings, and are compatible with non-hierarchical and perturbative Yukawa structures (|\lambda_5| < 4 \[12\]) in the RS framework. In one type, mass matrices have a symmetrical texture that is a slight deformation of the ansatz proposed by Koide \textit{et. al.} \[13\]. In the other, there are no symmetries \textit{a priori}. The form of the mass matrices is characterized by the localizations of the fermions in the 5D bulk that are admissible under the electroweak constraints, and each particular realization of the form arise from Yukawa structures that are completely anarchical. For each type of the quark mass matrices, we calculate below the resulting Wilson coefficients for the $\Delta F = 2$ processes due to KK gluon exchanges, and we compare them to the UTfit bounds.

For the symmetrical Koide-type form of quark mass matrix, we begin by focusing on the kaon sector where the constraints are most stringent \[11\]. At $\Lambda = 4$ TeV, while the imaginary part of the resulting kaon sector Wilson coefficients are all very much smaller than the bounds listed, we find the real parts are all larger than the respective bounds by three orders of magnitude. As a result, insisting that the symmetrical type pass the UTfit bounds would require one to push the NP scale up to $\mathcal{O}(100)$ TeV \[^5\].

For asymmetrical forms, we demonstrate that each of the asymmetrical configurations discussed in Ref. \[6\] remain viable at the few TeV scale. In Table I, we list the UTfit bounds on the relevant Wilson coefficients, and we give their values for a typical “solution” – admissible set of up and down quark mass matrices which give the observed quark masses and mixings, and satisfy all electroweak and FCNC bounds – at $\Lambda = 4$ TeV (corresponding to $\tilde{k} = 1.65$ TeV where $m_1 \simeq 4$ TeV) in each of the asymmetrical configurations, and we see that they are all well within the bounds. The details of the specific quark mass matrices used are given in Appendix A. In all calculations, we have explicitly checked that the KK gluons do indeed give rise to the dominant contributions in the tree-level $\Delta F = 2$ process under study. We show in Appendix B that the contributions from the electroweak sector are small as expected, and would not lead to violations of the UTfit bounds if included with the KK gluon contributions.

Note that in Table I, only one of the many admissible solutions we found are given

\[^5\] As can be seen from Eq. \[13\], the mass of the lightest mode sets the suppression scale for the four-fermion operator. To make up for a factor of $\mathcal{O}(10^3)$ at $m_1 \simeq \Lambda = 4$ TeV would require a factor of $\mathcal{O}(30)$ increase in $m_1$. 

for each asymmetrical configurations. Moreover, the configurations of fermion localizations themselves are just three of many that we found which lead to admissible solutions. Indeed, we have found that parameter space generically exists in the RS1 setting where quark mass and mixing data and $\Delta F = 2$ FCNC bounds can be satisfied at the few TeV scale with asymmetrical quark mass matrices that arise from underlying anarchical Yukawa structures. This does not, however, contravene the conclusion reached in Ref. 9 that a KK scale of 10 to 20 TeV is necessary to satisfy the $\Delta F = 2$ FCNC constraints in the kaon sector. The higher NP scale is required if one wants to ensure that the FCNC bounds is generically satisfied for any given quark mass matrices that give the pattern of the observed quark mass hierarchy and CKM mixings. Our point here is that a subset of these consisting of asymmetrical quark

| Parameter | 95% allowed range | Config. I | Config. II | Config. III |
|-----------|-------------------|-----------|------------|-------------|
| Re $C^1_K$ | $[-9.6, 9.6] \times 10^{-13}$ | $4.3 \times 10^{-17}$ | $1.8 \times 10^{-15}$ | $-4.2 \times 10^{-15}$ |
| Re $C^4_K$ | $[-3.6, 3.6] \times 10^{-15}$ | $-1.4 \times 10^{-16}$ | $-2.8 \times 10^{-16}$ | $-1.8 \times 10^{-15}$ |
| Re $C^5_K$ | $[-1.0, 1.0] \times 10^{-14}$ | $4.6 \times 10^{-17}$ | $9.4 \times 10^{-17}$ | $6.0 \times 10^{-16}$ |
| Im $C^1_K$ | $[-4.4, 2.8] \times 10^{-15}$ | $2.6 \times 10^{-18}$ | $1.8 \times 10^{-15}$ | $-1.0 \times 10^{-15}$ |
| Im $C^4_K$ | $[-1.8, 0.9] \times 10^{-17}$ | $1.5 \times 10^{-19}$ | $8.8 \times 10^{-18}$ | $-1.8 \times 10^{-18}$ |
| Im $C^5_K$ | $[-5.2, 2.8] \times 10^{-17}$ | $-4.9 \times 10^{-20}$ | $-2.9 \times 10^{-18}$ | $6.0 \times 10^{-19}$ |
| $|C^1_D|$ | $< 7.2 \times 10^{-13}$ | $1.3 \times 10^{-13}$ | $3.1 \times 10^{-13}$ | $1.6 \times 10^{-14}$ |
| $|C^4_D|$ | $< 4.8 \times 10^{-14}$ | $1.7 \times 10^{-15}$ | $8.8 \times 10^{-15}$ | $4.0 \times 10^{-14}$ |
| $|C^5_D|$ | $< 4.8 \times 10^{-13}$ | $5.7 \times 10^{-16}$ | $2.9 \times 10^{-15}$ | $1.3 \times 10^{-14}$ |
| $|C^1_{B_a}|$ | $< 2.3 \times 10^{-11}$ | $7.5 \times 10^{-13}$ | $7.7 \times 10^{-14}$ | $4.8 \times 10^{-13}$ |
| $|C^4_{B_a}|$ | $< 2.1 \times 10^{-13}$ | $1.9 \times 10^{-13}$ | $4.8 \times 10^{-14}$ | $1.7 \times 10^{-13}$ |
| $|C^5_{B_a}|$ | $< 6.0 \times 10^{-13}$ | $6.2 \times 10^{-14}$ | $1.6 \times 10^{-14}$ | $5.6 \times 10^{-14}$ |
| $|C^1_{B_s}|$ | $< 1.1 \times 10^{-9}$ | $9.0 \times 10^{-11}$ | $4.1 \times 10^{-11}$ | $4.0 \times 10^{-11}$ |
| $|C^4_{B_s}|$ | $< 1.6 \times 10^{-11}$ | $9.4 \times 10^{-12}$ | $7.6 \times 10^{-13}$ | $5.8 \times 10^{-12}$ |
| $|C^5_{B_s}|$ | $< 4.5 \times 10^{-11}$ | $3.1 \times 10^{-12}$ | $2.5 \times 10^{-13}$ | $1.9 \times 10^{-12}$ |

TABLE I: The 95% allowed range of the Wilson coefficients [11] contributing in the $\Delta F = 2$ tree-level gluon exchange processes, and their typical values at $\Lambda = 4$ TeV in each of the asymmetrical configurations given in Ref. [6]. All values above are given in units of GeV$^{-2}$. 

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mass matrices exists such that the experimental quark masses and mixings are reproduced to a high accuracy, and at the same time the lower, few TeV scale is still viable. \(^6\) We emphasize here that this subset does not contain isolated singular points in the parameter space, but generic solutions throughout all the parameters space.

Now one may worry that radiative correction may spoil our results, as there are loop induced corrections to the brane localized Yukawa couplings, and loop induced brane kinetic mixing terms that can introduce additional flavour violations. This is however not so. First, we are not calculating theoretically the Yukawa couplings in the RS framework; to do that requires a UV completion of the theory. There will be radiative corrections to the Yukawa matrices, but they will not change the form of the 4D effective mass matrices given in Eq. (7) even if it is derived at tree-level. Thus if the physical (or renormalized) Yukawa matrices take any of the forms that we found, the FCNC bounds will be satisfied, the form of the mass matrices we give should therefore be viewed as “physical” and the corresponding Yukawa matrices renormalised. Next, the brane kinetic mixings, which is loop suppressed, lead to a correction to the gauge-fermion couplings of order \(\delta \sim |\lambda_{5D}|^2/4\pi^2\) as can be estimated from NDA (naive dimensional analysis) \(^{15}\). In the search for solutions, we have set \(|\lambda_{5D}| \lesssim 2\), consequently \(\delta \ll 1\) and the flavour violating contribution from the brane kinetic mixing terms is small (\(\mathcal{O}(0.01)\) of the KK gluon contributions), which do not impact the Wilson coefficients calculated.

IV. EXPERIMENTAL OBSERVABLES

A. \(B_q^0 - \bar{B}_q^0\) mixings

One very sensitive probe to NP in the meson sector comes from the \(B_q^0 - \bar{B}_q^0\) mixing \((q = d, s)\), which has received much theoretical attention, and has now an extensive body of experimental data from the \(B\) factories and FNAL. The contribution of NP to \(\Delta B = 2\) transitions can be parametrized in a model-independent way as the ratio of the full (SM +

\(^6\) The few TeV NP scale can also be achieved one imposes additional symmetries. For recent works in this direction see e.g. Refs. \(^{14, 15}\).
NP) amplitude to the SM one \[11\] 7:

\[
\frac{\langle B_q^0 | \mathcal{H}_{\text{eff}}^{\text{NP}} | B_q^0 \rangle}{\langle B_q^0 | \mathcal{H}_{\text{eff}}^{\text{SM}} | B_q^0 \rangle} = 1 + \frac{\langle B_q^0 | \mathcal{H}_{\text{eff}}^{\text{NP}} | B_q^0 \rangle}{\langle B_q^0 | \mathcal{H}_{\text{eff}}^{\text{SM}} | B_q^0 \rangle} \equiv C_q e^{2i\phi_q}, \quad q = d, s, \quad (20)
\]

The SM amplitude arise mainly from the one-loop box diagram, which is dominated by the top quark exchanges. It can be written as

\[
\langle B_q^0 | \mathcal{H}_{\text{eff}}^{\text{SM}} | B_q^0 \rangle = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B m_{B_q} \bar{B}_{B_q} (V_{tq}^* V_{tb})^2 S_0(x_t), \quad x_t \equiv \frac{m_t^2}{m_W^2}, \quad (21)
\]

where \(\eta_B = 0.552\) is a short distance QCD correction \([17]\), and \(S_0\) is an “Inami-Lim” function \([18]\) with \(m_t(m_t) = 163.6 \text{ GeV} \quad (19)\). We take for the CKM mixings \([16]\)

\[
|V_{td}^* V_{tb}| = 8.6 \cdot 10^{-3}, \quad |V_{ts}^* V_{tb}| = 41.3 \cdot 10^{-3}, \quad (22)
\]

for the decay constants \([20]\)

\[
f_{B_d} = 197 \text{ MeV}, \quad f_{B_s} = 240 \text{ MeV}. \quad (23)
\]

and for the renormalization invariant bag parameter \([21]\)

\[
f_{B_d} \sqrt{\hat{B}_{B_d}} = 244 \text{ MeV}, \quad f_{B_s} \sqrt{\hat{B}_{B_d}} = 295 \text{ MeV}, \quad (24)
\]

All other input parameters take their values from the PDG \([22]\).

In the MCRS model, the NP contribution to the \(\Delta B = 2\) transition amplitude is dominated by the tree-level exchanges of KK gluons, as the coupling strength for the strong interactions is much larger than that for the electroweak interactions. Evolving down from the NP scale \(\Lambda\) to the hadronic scale \(\mu = m_t\), the KK gluon contribution is given by

\[
\langle B_q^0 | \mathcal{H}_{\text{eff}}^{\text{NP}} | B_q^0 \rangle = \langle B_q^0 | \sum_r C_r(\mu) Q_r^{(q)}(\mu) + \sum_s \tilde{C}_s(\mu) \tilde{Q}_s^{(q)}(\mu) | B_q^0 \rangle, \quad (25)
\]

where

\[
C_r(\mu) = \sum_{i,j} \left( b_{ij}^{(r,i)} + \eta c_{ij}^{(r,i)} \right) \eta^{\alpha_i} C_i(\Lambda), \quad \eta = \frac{\alpha_s(\Lambda)}{\alpha_s(m_t^{\text{pole}})}, \quad (26)
\]

are the Wilson coefficients at the hadronic scale, with \(\tilde{C}_r\) defined similarly with the same coefficients as for \(C_r\), and \(m_t^{\text{pole}} = 171.4 \text{ GeV} \quad (22)\). The magic numbers \(\alpha_j, b_{ij}^{(r,i)}, c_{ij}^{(r,i)}\), and the operator matrix elements can be found in Ref. \([23]\) \(8\).

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7 See also Ref. \[16\].

8 Note that Ref. \[23\] works in the Landau RI-MOM scheme \([24]\); for magic numbers in the MS (NDR) scheme, see Ref. \[25\]. For consistency, all running quark masses used in Eq. \(25\) should be in the same scheme as the operator matrix elements. The relevant quark masses in the RI-MOM scheme are \(m_b(m_b) = 4.6 \text{ GeV}, m_s(m_b) = 87 \text{ MeV}, \) and \(m_d(m_b) = 5.4 \text{ MeV}\.\)
In Table II we give the values of the parameters $C_q$ and $\phi_q$ for each of the three asymmetrical configurations solutions used in Table. The values of $C_q$ and $\phi_q$ agree well with the UTfit values at 95% probability (and mostly at 68% as well; see Table 3 in Ref.) as expected, since the physical observables fitted here are the same that go into the analysis for the meson sector flavour bound on the NP Wilson coefficients listed in Table. As above, we have checked that the electroweak contributions is small – they are much less than the standard error given by the UTfit collaboration at 68% probability – and do not cause large shifts that would violate the UTfit bounds. We note for the configurations of solutions given here, KK gluons are not manifest in the $B^0_q$-$\bar{B}^0_q$ mixing, and the SM effects are expected to be dominant.

### Table II: Parameters determining the NP contributions to $B^0_q$-$\bar{B}^0_q$ mixings in the MCRS model with mass matrices from the three asymmetrical configurations given in Ref.

| Parameter | Config. I   | Config. II  | Config. III |
|-----------|-------------|-------------|-------------|
| $C_d$     | 1.13        | 1.02        | 1.08        |
| $\phi_d [^\circ]$ | -2.48       | -0.24       | -3.02       |
| $C_s$     | 1.68        | 1.36        | 1.29        |
| $\phi_s [^\circ]$ | 0.61        | 0.12        | 0.04        |

The B. KK gluon top decays

In RS models, a distinguishing property of the KK gluons is that their couplings to the LH and RH fermions (in the mass eigenbasis), $\hat{g}_L$ and $\hat{g}_R$, are in general not the same. For all the asymmetrical quark mass matrix solutions that we found, this is true. To test this experimentally, one way is to measuring both the decay width and the spin of the top in the decays of gluons into top pairs as we show below. We will also be concentrating on the first KK gluon, $G^{(1)}$, which has the highest potential of being within the reach of the LHC.

As $G^{(1)}$ couples strongly to states localized near the IR brane, and the large top mass requires that either $Q_3$ or $t_R$ be IR localized, top decays are expected to be dominant modes of decay. The partial width of $G^{(1)}$ decaying into quarks in the mass eigenbasis, $\bar{q}_a q_b$, can
be written as

$$ \Gamma(G^{(1)} \rightarrow \bar{q}_a q_b) = \frac{m_1}{48\pi} \lambda(1, x_a^2, x_b^2)^{1/2} \times \left\{ \frac{1}{2} (|\hat{g}_L^1|^2 + |\hat{g}_R^1|^2) \left[ 2 - x_a^2 - x_b^2 - (x_a^2 - x_b^2)^2 \right] + 6 \text{Re}[\hat{g}_L^1(\hat{g}_R^1)^*] x_a x_b \right\}$$

$$\rightarrow \frac{m_1}{48\pi} ((|\hat{g}_L^1|^2 + |\hat{g}_R^1|^2) \quad (x_a = x_b \ll 1), \quad x_{a,b} = \frac{m_{a,b}}{m_1}, \quad (27)$$

where $m_1$ is the mass of $G^{(1)}$, $\hat{g}_{L,R}^1$ denote the mass eigenbasis couplings, $(\hat{g}_f^1)_{ab}^{L,R}$, given in Eq. (12), and $\lambda(u,v,w) = (u - v - w)^2 - 4vw$. For the three asymmetrical configuration solutions used in Table I and for $m_1 = 4.0$ TeV, the widths into the $\bar{t}t$ pairs are:

769.3 GeV (Config. I), 635.4 GeV (Config. II), 747.4 GeV (Config. III). (28)

In Table III we give the branching ratios of $G^{(1)}$ into top, bottom, and all other modes involving at least one light quark (jets) for the same three asymmetrical solutions.

| Branching ratios | Config. I | Config. II | Config. III |
|------------------|-----------|------------|-------------|
| Top quarks       | 0.83      | 0.83       | 0.84        |
| Bottom quarks    | 0.16      | 0.16       | 0.15        |
| Light quarks     | 0.01      | 0.01       | 0.01        |

TABLE III: Branching ratios of $G^{(1)}$ into $\bar{q}_a q_b$ pair in the MCRS model with mass matrices from the three asymmetrical configurations given in Ref. [6]. The term “light quarks” here denotes all modes (flavour changing included) that involve at least one light quark (jet).

We see from Table III that most decays are into top pairs, with negligible fraction into light quarks. Compared to Ref. [26] (see Table I), the branching ratio into top pairs from each of our asymmetrical configurations is slightly lower at about 80\% instead of around 90\%, which is due to the difference in the quark mass matrices and the localization parameters used. Note that the branching ratios are stable across the different configurations. This is because the couplings of KK gluons to quarks are dominated by that to the third generation quarks, which varied little across the configurations. It is thus fairly robust that in the RS scenario, the KK gluon will decay predominantly into top pairs, and then into b-jets with a much smaller, but non-negligible rate. Other light quark modes are negligible and certainly no leptons. However, as can be seen from Eq. (28), the top pair width is not small.
as $\Gamma_{tt}/m_1 \sim 0.2$. Thus looking for signals in the resonant productions will require good top identification. Detail discussions of the discovery potential at the LHC can be found in Ref. [26]. We note that the bottom mode should not be overlook and can be used as a check if not the primary discovery tool.

If a KK gluon is found at the LHC, it will certainly be important to measure the spin of the top in its $\bar{t}t$ decays. The differential decay rate with only one of the top spin measured but with the other top spin summed over is given by

$$\frac{d\Gamma_s}{d\cos\theta} = \frac{m_1}{192\pi} \sqrt{1 - 4x_t^2} \left\{ (|\hat{g}_L|^2 + |\hat{g}_R|^2)(1 - x_t^2) + 6 \text{Re} (\hat{g}_L \hat{g}_R^*) x_t^2 + 2(|\hat{g}_R|^2 - |\hat{g}_L|^2) x_t \sqrt{1 - 4x_t^2} \cdot \hat{s} \cdot \hat{p} \right\},$$

(29)

where

$$\hat{s} \cdot \hat{p} = \frac{\cos\theta}{\sqrt{1 - (1 - 4x_t^2) \cos^2\theta}}, \quad \cos\theta \equiv \hat{s} \cdot \hat{p},$$

(30)

with $\hat{s}$ the measured top spin three-vector, and $\hat{p}$ the three-momentum of the same top quark in the rest frame of $G^{(1)}$. From this we see that a measurement of the angular dependence together with the decay width can allow one to extract out $\hat{g}_L$ and $\hat{g}_R$. The feasibility of doing this at the LHC requires detailed numerical simulations which are beyond the scope of the present work (see Ref. [27] for work in this direction).

V. CONCLUSION

Previously in Ref. [6] we have studied the phenomenologically allowed form of quark mass matrices in the MCRS model, and we have found admissible both a symmetrical form, and many distinctive asymmetrical configurations with Yukawa structures non-hierarchical and anarchical that satisfy all EWPTs. The benchmark warped down scale was chosen at $\tilde{k} = 1.65$ TeV implying an equivalently NP scale of $\Lambda = 4$ TeV, since a higher scale will prevent the KK gauge bosons being detectable at the LHC initially at least. A much higher scale will also create its own hierarchy problem which one would like to avoid. We continue the study in this work the constraints that $\Delta F = 2$ processes in the neutral meson sector impose. We found from these constraints that for the symmetrical mass quark matrices, the viable scale is pushed up to $\mathcal{O}(100)$ TeV. However, for the asymmetrical quark mass matrices, $\Lambda = 4$ TeV is still viable. This is a consequence of the fact that in the asymmetrical cases there is freedom in the LH and RH rotations being very different – rather than being
locked into a specific pattern as in the symmetrical case – which can supply the suppression required to pass the meson sector $\Delta F = 2$ constraints. This underscores the importance of the quark mass matrices in the RS framework both phenomenologically and theoretically for identifying any family symmetries that may be hidden.

At the $\Lambda = 4$ TeV scale, discovery of the first KK gluon state at the LHC is possible. This can be achieved through a resonance search in the $\bar{t}t$ channel which we predict to have a branching ratio of $\approx 0.8$. Note that the dominance of the $\bar{t}t$ decays is a characteristic of the RS1 scenario. The $\bar{b}b$ mode has a branching fraction of about 0.15, and should not be overlooked. This mode consists mainly of LH pairs because $b_L$ is an $SU(2)$ partner to $t_L$, which has a large overlap with $G_{KK}$. Thus this channel can be useful as a diagnostic tool if the expected background can be handled. All other decay modes involving light quarks are negligible. Finally, if one can also measure in the $\bar{t}t$ decays at least one of the quark spins, it will help to unravel $\hat{g}_L$ and $\hat{g}_R$, and provide further an invaluable probing into the flavour structure of the RS scenario.

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Note added: At the time when this work was completed, Ref. [28] came out which also considered some of the same issues as us. There, the bulk gauge group contains an additional discrete left-right parity group. Consequently the fermion matter contents are embedded in a different representation than the one in our work, resulting in electroweak contributions to the $\Delta F = 2$ FCNCs that are far larger than what we have found. We have checked that both works agree whenever direct comparisons can be made.
APPENDIX A: TYPICAL SOLUTIONS FOR THE ASYMMETRICAL CONFIGURATIONS

In this appendix, we give the details of the quark mass matrices of the typical solution used in Table I in each of the three asymmetrical configurations given in Ref. [6]. Although the bound on $Zb_L\bar{b}_L$ used there was that given in the PDG [22], many generic solutions from generic configurations exist with localization parameters that can easily accommodate the more stringent bound found in e.g. Ref. [29].

Parameterizing the complex 5D Yukawa couplings as $\lambda_{5,ij} = \rho_{ij}e^{i\phi_{ij}}$, admissible mass matrices of the forms given by Eq. (7) are found with $\rho_{ij}$ and $\phi_{ij}$ randomly and uniformly generated in the intervals $(0, 2)$ and $[-\pi, \pi)$ respectively. In the following, we list the complex mass matrices in the form of $M_f = |M_f|e^{i\theta_f}$, the magnitude and phase of the 5D Yukawa couplings, and the mass eigenvalues for both the up and down sector. All values are given to six significant figures. The mass eigenvalues agree with the quark masses at 1 TeV found in Ref. [19] to within two standard deviations quoted.

- Configuration I:

$$c_Q = \{0.633604, 0.556171, 0.256293\},$$
$$c_U = \{-0.663816, -0.535621, 0.185413\},$$
$$c_D = \{-0.641469, -0.572479, -0.616085\}. \quad (A1)$$

$$|M_u| = \begin{pmatrix}
0.00136839 & 0.0770365 & 1.19782 \\
0.0778813 & 0.560874 & 2.93683 \\
0.24404 & 8.1122 & 147.741
\end{pmatrix}, \quad \theta_u = \begin{pmatrix}
1.59621 & 2.80118 & -2.65001 \\
-2.34319 & -0.190895 & -0.644161 \\
-1.61289 & 0.584021 & 0.07447
\end{pmatrix} \quad (A2)$$

$$\rho_u = \begin{pmatrix}
1.52494 & 1.57620 & 1.56165 \\
0.765990 & 1.01280 & 0.337923 \\
1.46664 & 0.895098 & 1.03875
\end{pmatrix}, \quad \phi_u = \begin{pmatrix}
1.39426 & 1.49660 & 1.50005 \\
0.716676 & 0.984072 & 0.332161 \\
1.39602 & 0.884794 & 1.03875
\end{pmatrix} \quad (A3)$$

$$m_u^1 = 0.369308 \text{ MeV}, \quad m_u^2 = 0.409125 \text{ GeV}, \quad m_u^3 = 147.999 \text{ GeV}. \quad (A4)$$
\[ M_d = \begin{pmatrix} 0.00205044 & 0.0096169 & 0.0025584 \\ 0.00702768 & 0.0985925 & 0.0173996 \\ 0.242765 & 2.33774 & 0.76264 \end{pmatrix}, \quad \theta_d = \begin{pmatrix} -0.184947 & 2.04673 & 1.12293 \\ -1.04910 & 1.68206 & -2.47164 \\ 0.00506372 & -2.31542 & 3.06043 \end{pmatrix} \]  
(A5)

\[ \rho_d = \begin{pmatrix} 1.07943 & 0.555546 & 0.583531 \\ 0.326515 & 0.502659 & 0.350252 \\ 0.689207 & 0.728280 & 0.938063 \end{pmatrix}, \quad \phi_d = \begin{pmatrix} 0.993588 & 0.522050 & 0.541247 \\ 0.307557 & 0.483365 & 0.332447 \\ 0.660453 & 0.712475 & 0.905822 \end{pmatrix} \]  
(A6)

\[ m_1^d = 2.25527 \ \text{MeV}, \quad m_2^d = 47.9153 \ \text{MeV}, \quad m_3^d = 2.47254 \ \text{GeV}. \]  
(A7)

- **Configuration II:**

\[ c_Q = \{0.628524, 0.546221, 0.285007\}, \]

\[ c_U = \{-0.662224, -0.550397, 0.0801805\}, \]

\[ c_D = \{-0.579521, -0.628656, -0.626738\}. \]  
(A8)

\[ M_u = \begin{pmatrix} 0.000705160 & 0.0296351 & 1.25154 \\ 0.00391734 & 0.303462 & 4.75543 \\ 0.157250 & 8.57855 & 148.068 \end{pmatrix}, \quad \theta_u = \begin{pmatrix} 3.03996 & 0.107148 & 2.03582 \\ 1.79158 & -1.76781 & -2.88842 \\ 2.07507 & -0.648895 & 3.02998 \end{pmatrix} \]  
(A9)

\[ \rho_u = \begin{pmatrix} 0.576867 & 0.721635 & 1.44261 \\ 0.259565 & 0.598525 & 0.443977 \\ 0.908033 & 1.47451 & 1.20473 \end{pmatrix}, \quad \phi_u = \begin{pmatrix} 3.03996 & 0.107148 & 2.03582 \\ 1.79158 & -1.76781 & -2.88842 \\ 2.07507 & -0.648895 & 3.02998 \end{pmatrix} \]  
(A10)

\[ m_1^u = 1.05432 \ \text{MeV}, \quad m_2^u = 0.399582 \ \text{GeV}, \quad m_3^u = 148.398 \ \text{GeV}. \]  
(A11)
\[
M_d = \begin{pmatrix}
0.00418127 & 0.000860589 & 0.00186071 \\
0.0663893 & 0.0619168 & 0.0228064 \\
2.43751 & 0.183510 & 0.140323
\end{pmatrix}, \quad \theta_d = \begin{pmatrix}
-2.87101 & 1.39416 & 2.70561 \\
-2.90716 & 1.17362 & 2.90809 \\
-1.80892 & -2.30267 & 2.33582
\end{pmatrix}
\] (A12)

\[
\rho_d = \begin{pmatrix}
0.237597 & 0.231857 & 0.470886 \\
0.305562 & 1.35114 & 0.467477 \\
0.977694 & 0.348985 & 0.250662
\end{pmatrix}, \quad \phi_d = \begin{pmatrix}
-2.87101 & 1.39416 & 2.70561 \\
-2.90716 & 1.17362 & 2.90809 \\
-1.80892 & -2.30267 & 2.33582
\end{pmatrix}
\] (A13)

\[
m_1^d = 1.41124 \text{ MeV}, \quad m_2^d = 66.9487 \text{ MeV}, \quad m_3^d = 2.44931 \text{ GeV}.
\] (A14)

- **Configuration III:**

\[
c_Q = \{0.627322, 0.570679, 0.272429\},
\]

\[
c_U = \{-0.517935, -0.664365, 0.180466\},
\]

\[
c_D = \{-0.576159, -0.610047, -0.638422\},
\] (A15)

\[
M_u = \begin{pmatrix}
0.147921 & 0.00223583 & 0.70694 \\
0.787783 & 0.00477027 & 4.06577 \\
8.66694 & 0.201339 & 145.112
\end{pmatrix}, \quad \theta_u = \begin{pmatrix}
-2.80680 & 2.86302 & 2.43167 \\
-0.23652 & -1.20710 & -1.23730 \\
1.00216 & 0.0966827 & 0.0
\end{pmatrix}
\] (A16)

\[
\rho_u = \begin{pmatrix}
1.53467 & 1.88939 & 0.723485 \\
1.38068 & 0.680969 & 0.702896 \\
0.641530 & 1.21401 & 1.05965
\end{pmatrix}, \quad \phi_u = \begin{pmatrix}
-2.80680 & 2.86302 & 2.43167 \\
-0.23652 & -1.20710 & -1.23730 \\
1.00216 & 0.0966827 & 0.0
\end{pmatrix}
\] (A17)

\[
m_1^u = 1.49993 \text{ MeV}, \quad m_2^u = 0.553929 \text{ GeV}, \quad m_3^u = 145.430 \text{ GeV}.
\] (A18)
\[ M_d = \begin{pmatrix}
    0.0122178 & 0.00379117 & 0.00346894 \\
    0.0813964 & 0.0316802 & 0.0033306 \\
    2.33248 & 0.899976 & 0.488706
\end{pmatrix}, \quad \theta_d = \begin{pmatrix}
    2.54815 & 2.37217 & -1.79028 \\
    0.769324 & -0.385483 & 0.262617 \\
    0.348142 & 2.10335 & 0.0
\end{pmatrix} \] (A19)

\[ \rho_d = \begin{pmatrix}
    0.603011 & 0.537917 & 1.23789 \\
    0.678640 & 0.759331 & 0.200775 \\
    0.821415 & 0.911140 & 1.24436
\end{pmatrix}, \quad \phi_d = \begin{pmatrix}
    2.54815 & 2.37217 & -1.79028 \\
    0.769324 & -0.385483 & 0.262617 \\
    0.348142 & 2.10335 & 0.0
\end{pmatrix} \] (A20)

\[ m_1^d = 2.38820 \text{ MeV}, \quad m_2^d = 60.8655 \text{ MeV}, \quad m_3^d = 2.54821 \text{ GeV}. \] (A21)

**APPENDIX B: ELECTROWEAK CONTRIBUTIONS TO THE TREE-LEVEL ∆F = 2 FCNCs**

The electroweak contributions to ∆F = 2 FCNCs come from the tree-level processes mediated by the KK photons, the Z boson, and the heavy Z′ boson that arise due to the SU(2)_R in the MCRS model [4] \(^9\). As we show below, the electroweak contributions are small due to the suppression of the (much) smaller electroweak interaction strength relative to that of the strong interaction (at the NP scale Λ).

The electroweak gauge bosons contribute to the ∆F = 2 processes in two ways. They contribute either directly through the four-fermion process analogous to that in Fig. 1 or they modify the gauge-fermion vertex through mixings with gauge and fermion KK modes as discussed in Ref. [6]. In the former case, all electroweak gauge KK modes can contribute, while the latter only happens via the mixing of the Z zero mode with the Z′ KK modes and the KK fermions.

For the direct electroweak contribution, the Wilson coefficients have similar forms as those for the KK gluons given in Eq. (19), but with appropriate changes in the numerical

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\(^9\) Note unlike the Z field which has [+,-] boundary conditions, the Z′ field has [-,+] boundary conditions, which give rise to KK excitations only.
coefficients (no colour factor 1/3), the interaction strengths, and gauge boson wavefunctions:

\[ C_1(\Lambda)^{EW} = \frac{1}{2} C_{ab,ab}^{LL}(A_{KK}), \]
\[ \tilde{C}_1(\Lambda)^{EW} = \frac{1}{2} C_{ab,ab}^{RR}(A_{KK}), \]
\[ C_4(\Lambda)^{EW} = 0, \]
\[ C_5(\Lambda)^{EW} = -2 C_{ab,ab}^{LR}(A_{KK}), \]  \hspace{1cm} (B1)

where \( A = \gamma, Z, Z' \). Note that without the colour structure, there is no electroweak contribution to the Wilson coefficient \( C_4 \) at tree-level.

For the photon and the \( Z \) (and their respective KK excitations), their couplings to the fermions are the same as in the SM. For the \( Z' \), its coupling to the fermions depends on \( g_R \), the gauge coupling constant of \( SU(2)_R \). Since it is commonly assumed in the literature that the coupling constants of \( SU(2)_{L,R} \) are equal, for the purpose of comparison we take \( g_R = g_L \) also. The \( Z'ff \) coupling is then given by \( g_{Z'} Q_{Z'}(f) \), where

\[ g_{Z'} = \sqrt{c g_Z}, \quad Q_{Z'}(f) = T_3^f - s^2 c^2 Y_f, \]  \hspace{1cm} (B2)

with the usual definitions \( g_Z = e/(sc) \), \( s = e/g_L \), and \( c = \sqrt{1 - s^2} \).

In Table [IV], we list the ratio of electroweak contribution to the Wilson coefficients at \( \Lambda = 4 \) TeV to that due to KK gluons alone for each source of the direct electroweak tree-level four-fermion processes. Note that because the difference between the overlap integrals for the \([+,+] \) and \([-,+\]) gauge bosons is very small as the respective bulk profiles are almost the same in regions where large overlap happens, the ratios listed in Table [IV] are essentially just

| Wilson Coefficient Ratio | Fermion Type | KK \( \gamma \) | KK \( Z \) | KK \( Z' \) | Total |
|--------------------------|--------------|----------------|-----------|-------------|-------|
| \( C_1(\Lambda)^{EW}/C_1(\Lambda)^{QCD} \) | u | 0.13 | 0.18 | 0.0054 | 0.32 |
| | d | 0.033 | 0.28 | 0.0054 | 0.32 |
| \( \tilde{C}_1(\Lambda)^{EW}/\tilde{C}_1(\Lambda)^{QCD} \) | u | 0.13 | 0.044 | 0.14 | 0.32 |
| | d | 0.033 | 0.011 | 0.28 | 0.32 |
| \( C_5(\Lambda)^{EW}/C_5(\Lambda)^{QCD} \) | u | -0.27 | 0.18 | 0.056 | -0.033 |
| | d | -0.067 | 0.11 | -0.077 | -0.033 |

TABLE IV: Ratio of direct electroweak contributions to KK gluon contributions in the Wilson coefficients. The fermion type “u” (“d”) denotes that up-type (down-type) quarks are involved in the \( \Delta F = 2 \) FCNC process. All quarks have SM quantum numbers.
that of the respective charge factors and electroweak gauge coupling constants. Note also that the total electroweak contribution for up-type and down-type quarks are the same, and that for $C_1$ and $\tilde{C}_1$ are the same. This can be seen most easily in the gauge interaction basis where the magnitude of the gauge charges are the same for both up-type and down-type quarks, and the $SU(2)_{L,R}$ quark quantum numbers are the same. Since the direct processes depend on the square of the gauge charges, the conclusion follows.

We remark here that the contributions to the Wilson coefficients due to KK gluon and KK photon are universal for all RS models with bulk fermions, but those due to $Z$ and $Z'$ are not. This is because the coupling of the $Z$ and $Z'$ to fermions depend on the representation in which the fermions are embedded in the gauge group of the model. Throughout this work and in Table IV, the bulk fermions are embedded such that the SM LH doublets (singlets) are $SU(2)_R$ singlets (doublets) so that they have SM quantum numbers (see Eq. (2)). However, ratios different from those listed in Table IV would arise if different fermion representation is used. For example, we list the electroweak to KK gluon ratios in Table IV in the case where the SM LH doublets are embedded as bifundamentals in the $SU(2)_L \times SU(2)_R$, and the up-type (down-type) singlets as $SU(2)_R$ singlets (triplets) (see e.g Ref. [30]) so that there is a left-right parity [31]. Note that only the KK $Z'$ contributions are different in changing to

| Wilson Coefficient Ratio | Fermion Type | KK $\gamma$ | KK $Z$ | KK $Z'$ | Total |
|--------------------------|--------------|-------------|--------|---------|-------|
| $C_1(\Lambda)^{EW}/C_1(\Lambda)^{QCD}$ | u | 0.13 | 0.18 | 0.36 | 0.67 |
| | d | 0.033 | 0.28 | 0.56 | 0.87 |
| $\tilde{C}_1(\Lambda)^{EW}/\tilde{C}_1(\Lambda)^{QCD}$ | u | 0.13 | 0.044 | 0.087 | 0.26 |
| | d | 0.033 | 0.011 | 1.43 | 1.47 |
| $C_5(\Lambda)^{EW}/C_5(\Lambda)^{QCD}$ | u | -0.27 | 0.18 | 0.35 | 0.26 |
| | d | -0.067 | 0.11 | -1.78 | -1.74 |

TABLE V: Ratio of direct electroweak contributions to KK gluon contributions in the Wilson coefficients. The fermion type “u” (“d”) denotes that up-type (down-type) quarks are involved in the $\Delta F = 2$ FCNC process. Here the SM LH doublets, and u-type and d-type singlets transform as $(2, \bar{2})_{2/3}$, $(1, 1)_{2/3}$, and $(1, 3)_{2/3}$ under $SU(2)_L \times SU(2)_R \times U(1)_X$ respectively.

this representation because only $Q'_Z$ is sensitive to the different assignment of the $SU(2)_R$
For the electroweak contributions due to mixings, the effects are no longer universal – the suppression factors are no longer determined by the electroweak charges and coupling constants alone – as there is now dependence on the fermion localization parameters and the quark mixing matrices. However, they are generically expected to be small as they are $\mathcal{O}(v^4/\Lambda^4)$ compared to the direct contributions \(^{10}\), although non-generic enhancement may happen depending on the particular quark mixing matrices involved, which typically do not exceed $\mathcal{O}(0.01)$ of the KK gluon contributions. We have checked in each case that the combined effect of the direct and mixing electroweak contributions do not appreciably alter the KK gluon contributions to the Wilson coefficients, and that they are still well within the UTfit bounds.

\(^{10}\) The zero mode-KK mode mixing happens through interactions with the Higgs, hence the effective coupling of the fermions to the zero mode of $Z$ is $\mathcal{O}(v^2/\Lambda^2)$ compared to the direct fermion couplings to the KK modes of $Z$ and $Z'$. More details can be found in Ref [6].

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