Effect of SUSY phases on the $B_d^0 - \overline{B}_d^0$ mixing in the minimal supergravity model

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Abstract

We investigate the effect of SUSY phases ($\theta_A$ and $\theta_\mu$) on the $B_d^0 - \overline{B}_d^0$ mixing in the minimal supergravity model. It is known that the complex phase $\theta_A = \text{arg}(A)$ ($A$ is the universal coefficient of the trilinear scalar couplings) is essentially unconstrained by the electric dipole moment experiment, while the phase $\theta_\mu = \text{arg}(\mu)$ ($\mu$ is the supersymmetric Higgsino mass) is strongly constrained to zero. We found that $\theta_A$ does not affect the phase of the $B_d^0 - \overline{B}_d^0$ mixing matrix element $M_{12}(B)$ by numerical analysis of the renormalization group equations. This means that the measurement of the $B_d^0 - \overline{B}_d^0$ mixing at the future B-factory could give the direct information on the parameters of the CKM matrix even in the framework of the minimal supergravity model with the SUSY phase $\theta_A$.

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1. Introduction

The minimal supergravity (SUGRA) model is expected to be the physics beyond the standard model. Due to the supersymmetry (SUSY), the quadratic divergence to the scalar (mass)\(^2\) cancels out and it helps the theory with elementary scalar fields to be natural. Furthermore the spontaneous breaking of the SUGRA can provide the preferable structure of soft SUSY breaking terms.

According to the current SUSY particle searches, masses of these particles are considered to be rather large. Even if the SUSY particles are too heavy to decay at presently working colliders, they may be detected through their radiative effects. Hence indirect tests for SUSY models are important.

CP violation in the neutral meson mixing is one of such indirect processes. Here we focus on the CP violation in the \(B^0_d - \overline{B}^0_d\) mixing, which is one of the main targets of B-factory experiments. In this case the effects of new physics can be extracted from \(\text{arg}[M_{12}(B)]\), where \(M_{12}(B)\) is the \(B^0_d - \overline{B}^0_d\) mixing matrix element. In general it seems that \(\text{arg}[M_{12}(B)]\) depends on SUSY parameters. If so, we cannot directly obtain the informations on the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

The prediction of the minimal SUGRA model for \(\text{arg}[M_{12}(B)]\) has been analysed. The result is that \(\text{arg}[M_{12}(B)]\) is the same as the standard model prediction independent of SUSY parameters. In these analyses, however, the SUSY parameters at the Planck (or GUT) scale are assumed to be real. This is because it seems natural for these parameters to be real in order to suppress the electric dipole moments (EDMs) of the neutron and the electron. On the other hand, it is found that it is possible for the SUSY parameter \(A\) ( the universal coefficient of the trilinear scalar couplings ) to have a complex phase of order one. On the \(K^0 - \overline{K}^0\) mixing it has already shown that the phase of \(A\) does not change the phase of the matrix element \(M_{12}(K)\) in the previous analysis where a mass insertion approximation is adopted.

In this letter we make an analysis of \(\text{arg}[M_{12}(B)]\) in the case of the complex \(A\) parameter. We have solved the renormalization group equations (RGEs) numerically including all the off-diagonal elements of the Yukawa coupling matrices, while they have been ignored in most of the previous works. Some phenomenological constraints on the SUSY parameters are considered. We take the effects of the right-handed external bottom quarks into account in evaluating \(M_{12}(B)\). QCD corrections below the weak scale are included. Implement for B-factory measurement is also mentioned.
2. The SUSY phases in the minimal SUGRA model

We examine the low energy effective theory of the minimal SUGRA model with chiral superfields for three generations of quarks ($Q_i$, $U_c^i$ and $D_c^i$) and leptons ($L_i$ and $E_c^i$), chiral superfields for two Higgs doublets ($H_1$ and $H_2$), and vector superfields for the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The superpotential is written as follows:

$$W = H_2 U^c \lambda_U Q + H_1 D^c \lambda_D Q + H_1 E^c \lambda_L L + \mu H_1 H_2,$$

where $\lambda_U$, $\lambda_D$ and $\lambda_L$ are Yukawa coupling matrices in the generation space. The generation indices ($i = 1, 2, 3$) are suppressed.

The general soft SUSY breaking consists of the following terms:

(i) scalar masses:

$$\tilde{q}_L^\dagger M^2_{\tilde{q}} \tilde{q}_L + \tilde{u}_R^\dagger M^2_{\tilde{u}} \tilde{u}_R + \tilde{d}_R^\dagger M^2_{\tilde{d}} \tilde{d}_R + \tilde{\ell}_L^\dagger M^2_{\tilde{\ell}} \tilde{\ell}_L + \tilde{e}_R^\dagger M^2_{\tilde{e}} \tilde{e}_R + M_{\tilde{H}_1}^2 |h_1|^2 + M_{\tilde{H}_2}^2 |h_2|^2.$$

(ii) A-terms: $h_2 \tilde{u}_R^\dagger A_U \tilde{q}_L + h_1 \tilde{d}_R^\dagger A_D \tilde{d}_L + h_1 \tilde{e}_R^\dagger A_L \tilde{\ell}_L + \text{h.c.}$

(iii) B-terms: $B_U h_1 h_2 + \text{h.c.}$

(iv) gaugino masses: $\frac{1}{2} M_1 \tilde{\lambda}_1 \tilde{\lambda}_1 + \frac{1}{2} M_2 \tilde{\lambda}_2 \tilde{\lambda}_2 + \frac{1}{2} M_3 \tilde{\lambda}_3 \tilde{\lambda}_3 + \text{h.c.}$

Here $\tilde{q}_L$, $\tilde{u}_R$, $\tilde{d}_R$, $\tilde{\ell}_L$, $\tilde{e}_R$, $h_1$ and $h_2$ are the scalar components of $Q$, $U^c$, $D^c$, $L$, $E^c$, $H_1$ and $H_2$, respectively. The fields $\tilde{\lambda}_\alpha$ ($\alpha = 1, 2, 3$) denote gauginos.

These soft SUSY breaking terms which result from the couplings to the hidden sector of $N = 1$ SUGRA have universal structure at the Planck scale ($M_P \sim 10^{19}$ GeV), if we assume that the hidden sector is flavor-blind. In this analysis we put the following boundary conditions at the GUT scale ($M_X \sim 10^{16}$ GeV) for simplicity, ignoring the RGE running effects between $M_P$ and $M_X$:

(i) universal scalar masses:

$$M_{\tilde{Q}}^2 = M_{\tilde{U}}^2 = M_{\tilde{D}}^2 = M_{\tilde{L}}^2 = M_{\tilde{E}}^2 = m_0^2 1, \ M_{\tilde{H}_1}^2 = M_{\tilde{H}_2}^2 = m_0^2,$$

(ii) universal A-terms: $A_U = A m_0 \lambda_U$, $A_D = A m_0 \lambda_D$, $A_L = A m_0 \lambda_L$,

(iii) universal gaugino masses: $M_1 = M_2 = M_3 = M_g$.

These universal structures are required in order to suppress the flavor changing neutral current processes. The last relation for gaugino masses is derived if we assume the supersymmetric grand unification, which is strongly suggested from the precise measurements of the gauge coupling constants at LEP. In our analysis we assume
the realization of the grand unification which is consistent with the negative results of the proton decay experiments in the wide parameter region, though we don’t specify the unified gauge group\(^\dagger\).

It is known that the low energy effective theory of the minimal SUGRA model has four physical phases\(^7\): (i) the phase \(\delta_{\text{CKM}}\) in the CKM matrix, (ii) the phase \(\theta_A = \text{arg}(A)\), (iii) the phase \(\theta_\mu = \text{arg}(\mu)\) and (iv) the QCD vacuum parameter \(\overline{\theta}_{\text{QCD}}\). Here we have taken such a convention that \(B_\mu\) is real at the weak scale by phase rotation of the Higgs fields. Then the vacuum expectation values of the Higgs fields \(h_1\) and \(h_2\) are found to be real. The universal gaugino mass \(M_g\) is made real by an R-rotation. Throughout our analysis we assume \(\bar{\theta}_{\text{QCD}} = 0\). The two phases \(\theta_A\) and \(\theta_\mu\) are peculiar to SUSY models, hence we call them 'SUSY phases'.

Breaking of the \(SU(2)_L \times U(1)_Y\) gauge symmetry is realized radiatively through the large Yukawa coupling constant of the top quark\(^12\). At the weak scale we minimize the Higgs potential to determine \(|\mu|\) and \(|B|\) by the condition that the vacuum expectation values of the neutral Higgs bosons ( \(\langle h_1^0 \rangle \equiv v_1\) and \(\langle h_2^0 \rangle \equiv v_2\) ) give the correct weak boson mass by \(m_W^2 = g_2^2(v_1^2 + v_2^2)/2\).

3. Constraint on the SUSY phases from EDM experiments

Nonvanishing particle EDMs are indications of CP violation. The current experiments for EDMs give stringent limits especially for the neutron and the electron: \(|d_n| \lesssim 1 \times 10^{-25} e \cdot \text{cm}\)\(^13\) and \(|d_e| \lesssim 1 \times 10^{-26} e \cdot \text{cm}\)\(^14\), respectively. In principle these bounds put severe constraints on \(\theta_A\) and \(\theta_\mu\). However it was found that \(\theta_A\) is essentially unconstrained, while \(\theta_\mu\) is strongly constrained to be vanishing\(^6\). The reason is as follows. The EDMs receive three contributions: (i) chargino-squark loop, (ii) gluino-squark loop and (iii) neutralino-squark loop. The results of EDM calculation are given in Ref.\(^{15}\)\(^{16}\). It was found that the chargino contribution \(d^{(C)}\) is dominant in the minimal SUGRA model\(^{16}\). The gluino contribution \(d^{(G)}\) is subdominant and the neutralino contribution \(d^{(N)}\) is the smallest. With the relation \(d^{(C)} \sim \text{Im}(\mu)\), the phase \(\theta_\mu\) is strongly bound to zero. On the other hand \(\theta_A\) comes in the subdominant gluino contribution \(d^{(G)} \sim \text{Im}(A m_0 + \mu \tan \beta)\), therefore \(\theta_A\) does not have such a severe constraint. These statements on \(\theta_A\) and \(\theta_\mu\) hold independent of \(\delta_{\text{CKM}}\) because the diagrams exchanging the first generation of squarks/sleptons are dominant.

We have confirmed the statement in Ref.\(^6\). In Fig. \(^[1]\) we present the allowed

\(^1\) Therefore we have not included the proton decay analysis in this letter.
region on the \((\theta_{\mu}, \theta_A)\) plane, where we have fixed the parameters as \(|A| = 0.5\), \(m_0 = 300\ GeV\), \(M_g = 100\ GeV\) and \(\tan \beta \equiv v_2/v_1 = 3\). We take the CKM parameters as \(|V_{us}| = 0.221\), \(|V_{cb}| = 0.041\), \(|V_{ub}|/|V_{cb}| = 0.08\) and \(\delta_{\text{CKM}} = \pi/3\) in the standard parametrization\[17\]. The result is almost independent of these CKM parameters. The region to satisfy the \(d_n\) bound \(|d_n| < 1 \times 10^{-25} e \cdot cm\) is shown between the two solid lines. The region to satisfy the \(d_e\) bound \(|d_e| < 1 \times 10^{-26} e \cdot cm\) is shown between the two dashed lines. The allowed regions are obtained by combining the two constraints. This figure shows that \(\theta_{\mu}\) has the strong constraint: \(\theta_{\mu} \lesssim 0.01\pi\), while \(\theta_A\) does not have such a strong constraint around the small \(\theta_{\mu}\) region. Figure 2 is a similar result for \(|A| = 0.5\), \(m_0 = 700\ GeV\), \(M_g = 300\ GeV\) and \(\tan \beta = 3\). In this case we don’t have any constraint on \(\theta_A\) around the small \(\theta_{\mu}\) region.

4. Effect of \(\theta_A\) on the phase of \(M_{12}(B)\)

From the above discussion, it follows that the phase \(\theta_A\) may be large in the small \(\theta_{\mu}\) region. Now we wish to examine whether a large \(\theta_A\) can affect the complex phase of \(M_{12}(B)\).

The matrix element \(M_{12}(B)\) is estimated by the usual box diagram calculation. In the minimal SUGRA model there are five contributions to \(M_{12}(B)\):

(i) W-boson and up-type quark loop (SM),

(ii) charged Higgs and up-type quark loop,

(iii) chargino and up-type squark loop,

(iv) gluino and down-type squark loop,

(v) neutralino and down-type squark loop or neutralino, gluino and down-type squark loop.

Among these the neutralino contribution (v) is expected to be much smaller than the gluino contribution (iv) due to smallness of gauge coupling constants and down type Yukawa coupling constants. Therefore we neglect it in this analysis. We have included the diagrams with the external right-handed bottom quarks, though they are subdominant. An analytic expression for \(M_{12}(B)\) is found in Ref.[3] and we have also considered QCD corrections following the method described in Ref.[4].

At first we give a brief summery in the case of \(\theta_A = \theta_{\mu} = 0\). In this case it is known that \(\text{arg}[M_{12}(B)]\) in the minimal SUGRA model is the same as the prediction.
of the standard model\[^2\]: \[ \arg[M_{12}(B)]_{\text{SUGRA}} = \arg[M_{12}(B)]_{\text{SM}} = \arg[\xi^2], \]
where \( \xi_i \equiv V_{ai}^* V_{ib} \) \((i = u, c \text{ and } t)\). The reason is that due to the degeneracy between the first two generations of squarks their contributions sum up to the terms proportional to \( \xi_t (\xi_u + \xi_c) \) or \( (\xi_u + \xi_c)^2 \), which are equal to \(-\xi^2_t\) and \(\xi^2_t\) respectively because of the unitarity of the CKM matrix.

Now we consider the effect of \( \theta_A \) and present our numerical result. In the following analysis, the phase \( \theta_\mu \) is fixed at zero in order to satisfy the EDM constraints. We investigate the three dimensional parameter space \{\(|A|, m_0, M_g\}\} in the range of 

\(-5 < |A| < 5, \ 0 < m_0 < 2\text{ TeV} \) and \(0 < M_g < 2\text{ TeV}\). Moreover we require the phenomenological constraints in the following:

(i) The mass of any charged superparticle is larger than 45 GeV\[^18\]\[^19\].

(ii) The gluino mass is larger than 100 GeV\[^20\].

(iii) All the sneutrino masses are larger than 41 GeV\[^21\].

(iv) The CLEO result for the \( b \to s\gamma \) inclusive branching ratio: \(1 \times 10^{-4} < \text{Br}(b \to s\gamma) < 4.2 \times 10^{-4}\)\[^22\]\[^23\].

(v) The bounds from the neutralino search at LEP on the decay width \( \Gamma(Z^0 \to \chi\chi) \) < 8.4 MeV and the branching ratios \( \text{Br}(Z^0 \to \chi\chi'), \text{Br}(Z^0 \to \chi'\chi') < 2 \times 10^{-5} \), where \( \chi \) is the lightest neutralino and \( \chi' \) is any neutralino other than \( \chi \)\[^24\]\[^19\].

(vi) The lightest superparticle (LSP) is neutral\[^25\].

(vii) The condition to avoid the color and charge breaking vacua\[^26\].

In Fig. \[3\], we show the complex quantity \( M_{12}(B) \) normalized so as to remove the uncertainty of low energy hadron physics. In this figure we take \( \tan\beta = 3 \) and we plot the SUGRA predictions for allowed SUSY parameters. The cross represents the standard model prediction. The SUSY phases are fixed as \( \theta_A = \pi/2 \) and \( \theta_\mu = 0 \). We have fixed the CKM parameters as \( |V_{us}| = 0.221, |V_{cb}| = 0.041, |V_{ub}|/|V_{cd}| = 0.08 \) and \( \delta_{\text{CKM}} = \pi/3 \). The top quark mass is fixed at \( m_t = 175\text{ GeV} \)\[^27\]. One finds from Fig. \[3\] that the complex phase of \( M_{12}(B) \) is not affected by the phase \( \theta_A \) as well as by the soft SUSY breaking parameters \(|A|, m_0 \text{ and } M_g\). We have also confirmed that the similar results are obtained for another choices of \( \theta_A \) and \( \tan\beta \). Though the phase of \( M_{12}(B) \) depends, of course, on the CKM parameters, it holds even for another choice.
of the CKM parameters that the phase is the same as the corresponding standard model prediction.

The reason for $\theta_A$ independence is as follows. The phase $\theta_A$ comes mainly through the squark left-right mixing in the box diagram calculation. Hence the diagrams exchanging the W-boson or the charged Higgs are trivially independent of $\theta_A$. In the chargino contribution, it follows that the squark left-right mixing appears only in the combination of $\sim |A m_0 + \mu \cot \beta|^2$. Therefore the phase cancels out and $\theta_A$ does not change the phase of $M_{12}(B)$. However $\theta_A$ can affect the absolute value of $M_{12}(B)$ because $A m_0$ is only a part of the left-right mixing. As for the gluino contribution, the effect of $\theta_A$ is negligible because the left-right mixing of the down-type squarks is quite small.

We have also investigated the effect of $\theta_\mu$. From our numerical evaluations it is found that $\theta_\mu$ also does not affect $\text{arg}[M_{12}(B)]$ even if we remove the EDM constraints and give a large complex phase to $\mu$.

From our analysis we conclude that the SUSY phase $\theta_A$ does not change the phase of the $B^0_d - \bar{B}^0_d$ matrix element $M_{12}(B)$. Combined with the previous results for $\theta_A = \theta_\mu = 0$, it follows that $\text{arg}[M_{12}(B)]$ is completely determined by the CKM parameters. It means that the measurement of CP asymmetry at the future B-factory could give the direct information on the parameters of the CKM matrix even in the framework of the minimal SUGRA model with the SUSY phase $\theta_A$.

5. Summery

In this letter we have examined the effect of the CP violating SUSY phase $\theta_A$ on the phase of the $B^0_d - \bar{B}^0_d$ mixing matrix element $M_{12}(B)$ in the minimal SUGRA model. We have solved the RGEs numerically including all the off-diagonal elements of the Yukawa coupling matrices\cite{8-10,11,5}, while they have been ignored in most of the previous works. It is found that the phase $\theta_A$ does not change the phase of $M_{12}(B)$ and it is completely determined by the CKM parameters. This means that the measurement of the $B^0_d - \bar{B}^0_d$ mixing at the future B-factory could give the direct information on the parameters of the CKM matrix even in the framework of the minimal SUGRA model with the SUSY phase $\theta_A$.

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Figure 1: The allowed region on the $(\theta_\mu, \theta_A)$ plane from the experimental constraints of $d_\mu$ and $d_e$ in the minimal SUGRA model. The parameters except these phases are fixed as $|A| = 0.5$, $m_0 = 300$ GeV, $M_g = 100$ GeV and $\tan \beta = 3$. The region to satisfy the bound $|d_\mu| < 1 \times 10^{-25} e \cdot cm$ is shown between the two solid lines. The region to satisfy the bound $|d_e| < 1 \times 10^{-26} e \cdot cm$ is shown between the two dashed lines. The allowed regions are obtained by combining the two constraints. We have denoted $\theta_\mu$ as $\theta_\mu(M_X)$. This is because the phase of $\mu$ does not run as seen from the RGE of the $\mu$ parameter: $\dot{\mu} \sim (\text{real}) \times \mu$. On the other hand, the phases of $A_U$, $A_D$ and $A_L$ run due to the contributions of the gaugino masses.
Figure 2: The same as Fig. 1 for $|A| = 0.5$, $m_0 = 700$ GeV, $M_g = 300$ GeV and $\tan \beta = 3$. 
The complex value of $M_{12}(B)$ in the minimal SUGRA model. The SUSY phases are fixed as $\theta_A = \pi/2$ and $\theta_\mu = 0$. We have scanned the three dimensional parameter space $\{ |A|, m_0, M_g \}$ in the range of $-5 < |A| < 5$, $0 < m_0 < 2\text{TeV}$ and $0 < M_g < 2\text{TeV}$. This is the result for $\tan \beta = 3$ and $\delta_{\text{CKM}} = \pi/3$. The cross represents the standard model prediction. The axes are normalized so as to remove the uncertainty of low energy hadron physics. The constants $f_B$, $M_B$ and $B_B$ are the decay constant, the mass and the bag parameter of the B-meson respectively. The constant $\eta_B$ is the QCD correction factor: $\eta_B = \alpha_3(m_W)^{6/23}$. 

Figure 3