A Comprehensive Analysis of the Gravitational Wave Events with the Stacked Hilbert–Huang Transform: From Compact Binary Coalescence to Supernova

Chin-Ping Hu1, Lupin Chun-Che Lin2, Kuo-Chuan Pan3, Kwan-Lok Li4, Chien-Chang Yen5, Albert K. H. Kong3, and C. Y. Hui6

1 Department of Physics, National Changhua University of Education, Changhua, 50007, Taiwan; cphu0821@gm.ncue.edu.tw
2 Department of Physics, National Cheng-Kung University, Taiwan
3 Institute of Astronomy, National Tsing Hua University, Hsinchu 30013, Taiwan
4 Department of Physics, National Cheng Kung University, 70101 Tainan, Taiwan
5 Department of Mathematics, Fu Jen Catholic University, 24205 New Taipei City, Taiwan
6 Department of Astronomy and Space Science, Chungnam National University, Daejeon, Republic of Korea

Received 2022 May 5; revised 2022 July 4; accepted 2022 July 13; published 2022 August 23

Abstract

We analyze the gravitational wave signals with a model-independent time-frequency analysis, which is improved from the Hilbert–Huang transform (HHT) and optimized for characterizing the frequency variability on the time-frequency map. Instead of the regular HHT algorithm, i.e., obtaining intrinsic mode functions with ensemble empirical mode decomposition and yielding the instantaneous frequencies, we propose an alternative algorithm that operates the ensemble mean on the time-frequency map. We systematically analyze the known gravitational wave events of the compact binary coalescence observed in the first gravitational-wave transient catalog, and in the simulated gravitational wave signals from core-collapse supernovae (CCSNe) with our method. The time-frequency maps of the binary black hole coalescence cases show much more detail compared to those wavelet spectra. Moreover, the oscillation in the instantaneous frequency caused by mode-mixing could be reduced with our algorithm. For the CCSNe data, the oscillation from the proto-neutron star and the radiation from the standing accretion shock instability can be precisely determined with the HHT in great detail. More importantly, the initial stage of different modes of oscillations can be clearly separated. These results provide new hints for further establishment of the detecting algorithm and new probes to investigate the underlying physical mechanisms.

Unified Astronomy Thesaurus concepts: Gravitational wave astronomy (675); Core-collapse supernovae (304); Black holes (162); Neutron stars (1108)

1. Introduction

The successful detection of the gravitational wave signal from the Laser Interferometer Gravitational wave Observatory (LIGO), the Virgo interferometer, and the Kamioka Gravitational Wave Detector (KAGRA) collaboration opens a new window in both astronomy and physics (Abbott et al. 2016a). At the end of 2018 for the first and second observation runs by LIGO and Virgo (O1 and O2), the first gravitational-wave transient catalog (GWTC-1) has confirmed 11 gravitational wave (GW) transients, including 10 merging events from two massive stellar-mass black holes (BHs) and one from the coalescence of two neutron stars (NSs; Abbott et al. 2019). Until the end of the third observation run in March of 2020, the aVirgo and aLIGO network have confirmed 90 GW events (The LIGO Scientific Collaboration et al. 2021), 50 of which have been reported during the first half of the third observing run (O3; Abbott et al. 2021). The time-frequency properties of these GW events can be tracked by multiresolution techniques including wavelet Q-transform (Chatterji et al. 2004).

It has been suggested that the novel time-series analysis method, the Hilbert–Huang transform (HHT), could precisely describe the time-frequency properties of the GW signal owing to the high resolution of the HHT (Camp et al. 2007). Compared to other time-frequency analyses like spectrogram and wavelet analysis, the HHT does not depend on any prior basis. The GW signal can be extracted from the simulated data with a BH merging signal embedded in a noisy background (Stroeer et al. 2009). Similar work has been extended to investigate the simulated GW signal from the coalescence of binary NSs (Kaneyama et al. 2016).

The first and strongest event GW150914 has been analyzed with the HHT (Sakai et al. 2017). The signal is not able to be decomposed into a single component through empirical mode decomposition (EMD). The ensemble EMD (EEMD) has been proposed to overcome the so-called “mode-mixing” or “mode-splitting” problem (Wu & Huang 2004, 2009), but it is only valid for a signal with moderate-frequency variability because EMD is equivalent to a dyadic filter. This prevents getting a precise instantaneous frequency although the signal from different components with high Hilbert amplitudes could be marginally connected with each other. The entire time-frequency map could be consistent with the theoretical prediction although some frequency modulations can be clearly seen. A systematic investigation of known GW events using the HHT is carried out by Akhshi et al. (2021). A physical time delay could be derived from the reconstructed waveform.

In this paper, we comprehensively apply the Morlet wavelet analysis and the HHT on the known GWTC-1 events to test if the HHT is eligible to detect the trigger. Moreover, we propose an alternative method to obtain the Hilbert spectrum by stacking the time-frequency map instead of taking the ensemble mean of components in the data sets with artificial noises. Besides the coalescence events in the GWTC-1 data sets, we also tested the algorithm on the simulated GW signal from core-collapse supernovae (CCSNe). Section 2 describes the reduction and
The Astrophysical Journal, 935:127 (21pp), 2022 August 20

Hu et al.

Table 1
Data Files Of LIGO/Virgo Events Investigated in This Article

| Event     | Location  | Hanford H-H1_GWOSC_4KHZ_R1-1126259447-32.hdf5 |
|-----------|-----------|------------------------------------------------|
| GW150914  | Hanford   | H-H1_GWOSC_4KHZ_R1-1126259447-32.hdf5          |
|           | Livingston| L-L1_GWOSC_4KHZ_R1-1126259447-32.hdf5          |
| GW151012  | Hanford   | H-H1_GWOSC_4KHZ_R1-1128678885-32.hdf5          |
|           | Livingston| L-L1_GWOSC_4KHZ_R1-1128678885-32.hdf5          |
| GW151226  | Hanford   | H-H1_GWOSC_4KHZ_R1-1135136335-32.hdf5          |
|           | Livingston| L-L1_GWOSC_4KHZ_R1-1135136335-32.hdf5          |
| GW170104  | Hanford   | H-H1_GWOSC_4KHZ_R1-1167559921-32.hdf5          |
|           | Livingston| L-L1_GWOSC_4KHZ_R1-1167559921-32.hdf5          |
| GW170608  | Hanford   | H-H1_GWOSC_4KHZ_R1-1180922479-32.hdf5          |
|           | Livingston| L-L1_GWOSC_4KHZ_R1-1180922479-32.hdf5          |
| GW170729  | Hanford   | H-H1_GWOSC_4KHZ_R1-1185389792-32.hdf5          |
|           | Livingston| L-L1_GWOSC_4KHZ_R1-1185389792-32.hdf5          |
| GW170809  | Hanford   | H-H1_GWOSC_4KHZ_R1-1186302504-32.hdf5          |
|           | Livingston| L-L1_GWOSC_4KHZ_R1-1186302504-32.hdf5          |
| GW170814  | Hanford   | H-H1_GWOSC_4KHZ_R1-1186741846-32.hdf5          |
|           | Livingston| L-L1_GWOSC_4KHZ_R1-1186741846-32.hdf5          |
| GW170817  | Hanford   | H-H1_GWOSC_4KHZ_R1-1187008867-32.hdf5          |
|           | Livingston| L-L1_GWOSC_4HZ_R1-1187008867-32.hdf5           |
| GW170818  | Hanford   | H-H1_GWOSC_4KHZ_R1-1187058312-32.hdf5          |
|           | Livingston| L-L1_GWOSC_4KHZ_R1-1187058312-32.hdf5          |
| GW170823  | Hanford   | H-H1_GWOSC_4KHZ_R1-1187529241-32.hdf5          |
|           | Livingston| L-L1_GWOSC_4KHZ_R1-1187529241-32.hdf5          |

processing of the GWTC-1 data, and the mathematical algorithm used in this research. We present the analysis result of two cases, GW150914 and GW170817, in Section 3. The time-frequency maps of other events are shown in Appendix A. We describe the models and parameters used in generating the CCSN signal and show the corresponding analytical result of a typical CCSN case in Section 4. Cases with other rotational parameters and viewing geometries are presented in Appendix B.

2. Data Reduction and Time-frequency Analysis Algorithm

2.1. LIGO Data Reduction

In order to examine the capability of our numerical method, we check the performance of our algorithm on the known GW events in GWTC-1 in 2015-2017. We use the 32 s data of 4KHz released from the LIGO Open Science Center7 (Vallisneri et al. 2015) obtained from the Hanford and Livingston sites to perform our analysis. We notice that a strong glitch and a GW candidate event (i.e., GW170817) occurred almost simultaneously at the Livingston site, and therefore we downloaded the “clean” data after noise subtraction following the glitch model described in Abbott et al. (2017a).

Table 1 lists data sets of known GW events considered in this article, and all the signal processing was proceeded with the Python-based GW analysis library named GWPY v2.1.08 (Macleod et al. 2020). To deal with the GW data, we followed the standard whitening process to suppress the extra colored noise at low frequencies and spectral lines led by instrumental or background effects (see, e.g., Usman et al. 2016; Abbott et al. 2019). This operation might affect the nonlinearity of the GW signal. We note that the whitening process is parameter-dependent and the method is not unique (see, e.g., Tsukada et al. 2018). Further developments of the de-noising techniques would be important though this is beyond the scope of this research. We therefore whitened the time series to enhance the higher-frequency content with a fast Fourier transform (FFT) integration length of 4 s and 2 s overlap between FFTs for the given window of Hann smoothing. We also apply a bandpass filter to reserve the data within 30–450 Hz and then use notch filters to attenuate line noises because we want to suppress the effect from thermal noises of mirrors, quantum noise of laser, or any line noise arisen from detector resonances to dominate the spectral behavior.

To confirm the practicability of our methods, we also check the instantaneous frequency obtained from the simulated gravitational waveform. We generate the waveform using the effective-one-body (EOB) model. The simulated GW time series can be obtained by employing applications (LAL 6.19.1; LIGO Scientific Collaboration 2018) provided by the LIGO Algorithm Library.9 The basic physical parameters of GW150914 applied in the simulated waveform refer to those determined in GWTC-1 (Abbott et al. 2019).

2.2. Hilbert–Huang Transform

The Hilbert–Huang transform (HHT) is a novel time-frequency analysis algorithm that can trace frequency variability in great detail (Huang et al. 1998). This adaptive method needs no priori basis and has been widely used in many fields of science and engineering (see, e.g., Huang et al. 1999; Huang & Attoh-Okine 2005; Huang & Shen 2014). The core idea of the HHT is

---

7 https://www.gw Openscience.org/catalog/GWTC-1-confident/html/ 8 https://gwpy.github.io/docs/stable/
the Hilbert transform of a time series \( x(t) \):
\[
y(t) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{x(t')}{t - t'} dt',
\]
where \( \mathcal{P} \) indicates the Cauchy principal value, and \( y(t) \) is the Hilbert transform of \( x(t) \). With this definition, we can have an analytical signal
\[
z(t) = x(t) + iy(t) = a(t)e^{i\theta(t)},
\]
where \( a(t) = \sqrt{x^2(t) + y^2(t)} \) is the Hilbert amplitude, and \( \theta(t) = \tan^{-1}[y(t)/x(t)] \) is the instantaneous phase function. The time derivative of \( \theta(t) \)
\[
\omega(t) = \frac{d\theta(t)}{dt}
\]
can be defined as the instantaneous frequency. As the development of the HHT, a few more methods were also proposed to obtain the frequency, such as generalized zero-crossing, and direct quadrature (Huang et al. 2009). In our
study, we mainly utilize the normalized Hilbert transform by applying the Hilbert transform after normalizing the intrinsic mode functions (IMFs) to the range between $-1$ and $1$. This is designed to overcome the limitation imposed by the Bedrosian theorem (Bedrosian 1962; Xu & Yan 2006). In some cases the intrawave modulation, i.e., frequency modulation within one cycle that is caused by possible nonlinear effects of an oscillation system, is not physically important; then we use the generalized zero-crossing, which is a robust method to calculate the interwave modulation (i.e., the variation of a cycle length) frequency (Sekhar & Sreenivas 2004). Both methods yield a fully consistent result.

However, the instantaneous frequency is meaningful only if the input time series is an IMF, which must be symmetric with respect to zero, and the number of extrema and zero-crossing must be the same or at least differ by one. This criterion is difficult to be achieved in real data. Pre-processing is needed. Therefore, the HHT consists of two steps: (1) decompose the time-series data into finite IMFs and (2) obtain instantaneous frequencies for each IMF (Wu & Huang 2009).

The first method proposed for the decomposition is the empirical mode decomposition (EMD). EMD is an iterative process called “sifting” that extracts IMFs by removing the local mean values obtained from taking the average of the upper and lower envelope of a time series (Huang et al. 1998). It can separate time series into IMFs of different timescales and make IMFs symmetric with respect to zero.

In principle, there are infinite ways of decomposing the signal. For example, different sifting parameters, e.g., stoppage criteria, spline method. A confidence limit for the Hilbert spectrum can be obtained from an ensemble of valid IMFs resulting from different sifting parameters. The ideal case is that the raw time series $x(t)$ can be decomposed as

$$x(t) = \sum_{j=1}^{n} c_j(t) + r(t) = \sum_{j=1}^{n} a_j(t)e^{i\int \omega_j(t)dt} + r(t),$$

where $c_j(t)$ is the $j$-th IMF, and $r(t)$ is the nonoscillating residual.

A signal with a narrow modulation timescale is expected to be decomposed in a single IMF. However, it could be
decomposed into different IMFs using EMD if there are noises or intermittent signals. This “mode-mixing” problem can be solved by introducing white noise with finite amplitude into the data and taking the ensemble mean of corresponding IMFs obtained from all the simulated sets. This is the ensemble EMD (EEMD; Wu & Huang 2009). Based on the study of white noise, the EEMD is a dyadic filter that collates signals of similar scales together. The remaining problem is that the summation of IMFs is no longer an IMF. Therefore, a post-processing EMD is needed (Wu & Huang 2009). One can apply the post-processing EMD from arbitrary components of interest. Here we use it since the first IMF, the highest-frequency mode of the signal. The entire procedure is shown in Figure 1(a).

When applying EEMD, two more parameters are needed to be considered: the amplitude of the input noise \( \sigma_n \) and the number of ensemble trials \( N \). Unlike EMD, the completeness of the signal is not guaranteed in EEMD due to the input noise. Therefore, a large \( N \) is needed to minimize the incompleteness of the output signal, which costs high computing resources. To reduce the computing time, the EEMD has been improved by adding a pair of positive and negative noises. This operation is also named as complementary EEMD (CEEEMD; Yeh et al. 2010). With this operation, a good completeness can be achieved compared to the original version of EEMD (see, e.g., Figure 6 in Yeh et al. 2010). With our parameter setting, the deviation between the summation of IMFs and the original time series is \( \sim 10^{-15} \) times the deviation of the original time series. This algorithm has been included in the Matlab fast EEMD package as the default EEMD algorithm, which is developed by the Research Center for Adaptive Data Analysis at National Central University (Wang et al. 2014). We have successfully explored the time-frequency properties of X-ray binaries and active galactic nuclei using this package (see, e.g., Hu et al. 2014, 2019; Su et al. 2015). A pywrapper of this package\(^{10}\) is also available (Su et al. 2017). We also tried to perform the following analysis using the complete ensemble empirical mode decomposition with adaptive noise\(^{11}\) and found that the result is fully consistent with that obtained from the fast EEMD algorithm (Torres et al. 2011).

To decide the best value of \( \sigma_n \), we tried different level of and choose the best \( \sigma_n \) to minimize the orthogonality index OI, which is defined as

\[
OI = \frac{\sum_{t} \sum_{j} \alpha_{ij} \bar{c}_{ij}(t) c_{ij}(t)}{\sum \bar{x}^2(t)}.
\]

The OI can be defined specifically for any IMF of interest or between all IMFs. Because the GW signal usually spread in a wide frequency range, we minimize the overall OI presented in Equation (5).

However, the GW signal of the coalescence of two compact objects usually spans a wide frequency range, which is difficult to be decomposed into a single IMF with EEMD. This makes it difficult to trace the evolution of the signal in detail, especially near the boundary when the signal migrates from one IMF to another. For visualization purposes, we establish a stacked Hilbert spectrum algorithm to trace the time-frequency property of GW signals.

The idea of our algorithm is demonstrated in Figure 1(b). Similar to EEMD, we first add finite noises into the signal and create \( N \) simulated time series. Then, we apply EMD or EEMD on the noise-inserted time series to obtain IMFs. After this, we did not take the ensemble mean of IMFs. Instead, we calculate the instantaneous frequency and amplitudes of individual IMFs in each simulated data set. Finally, we stack all the Hilbert spectra on the time-frequency map. The basic idea is that the variability of the instantaneous frequency introduced by the mode-splitting effect would be different in each set of added white noise. As a result, the true frequency evolution would be enhanced on the stacked map, and the spurious variability would be smeared out. The detailed mathematical proof and statistical study of this method will be presented in another paper (C.-C. Yen et al. 2022, in preparation).

3. Analysis of the LIGO O1/O2 Data

3.1. GW150914

We obtain the LIGO data and whiten them using the data reduction pipeline described in Section 2.1. We first input white noises with \( \sigma = 1.0 \) times the deviation of the data and perform \( 10^4 \) times of such simulations (step 1 in Figure 1). This is helpful to obtain confidence intervals because the whitened data can be treated as the Gaussian white noise except for a short time interval that contains GW signal. Then, we try both EMD and EEMD on the simulated time series (step 2 in Figure 1). Further input of noise in EEMD is needed, and we observe the variability of the OI with different noise levels between 0 and 5 times the deviation of the data (see Figure 2).

\(^{10}\) https://github.com/HHTpy/HHTpywrapper

\(^{11}\) https://github.com/macolominas/CEEMDAN
We found that the OI could be minimized with $\sigma \approx 0.3$, and therefore we use it for the following analysis. We then stack the Hilbert spectra on the two-dimensional map to smear the boundary effect when the GW signal was split into two nearby IMFs.

Figure 3 shows the decomposition result and the Hilbert spectra of GW150914 observed with LIGO Hanford (left) and Livingstone (right). Panel (a) shows the whitened data and the summation of IMF3–5. Because the GW signal spans from $\sim 50$ to $\sim 300$ Hz, it matches the characteristic timescales of IMFs 3 ($\sim 340$ Hz)–IMF 5 ($\sim 80$ Hz) based on the fact that the EEMD is a dyadic filter, and the sampling rate is 4096 Hz. In this case, the summation of IMFs 3–5 can greatly reproduce the waveform. This operation is equivalent to applying an EEMD bandpass filter to the data. We plotted the decomposed IMFs 3–5 in Figures 3(b)–(d), respectively. It clearly shows that the GW signal first appears in IMF 5, then migrates toward the high-frequency components, IMF4 and IMF3.

We performed the normalized Hilbert transform to calculate the Hilbert energy, which is defined as the square of the amplitude, and instantaneous frequency of each IMFs, and stacked those obtained from IMFs 3–5 to minimize the contribution from other frequency ranges. This operation is done for all $10^4$ simulated data sets (see Figure 1), and thus we obtained the mean value and deviation of the bandpass filtered Hilbert energy (Figure 3(e)). This instantaneous Hilbert energy can be used to detect the GW signal. We use the Bayesian block technique to detect occurrence times of significant change in the Hilbert energy (Scargle et al. 2013). GW150914 is the strongest event in the LIGO O1/O2 catalog where the boundaries can be easily detected if we set the null hypothesis probability $p_0$ to be $10^{-80}$.

The stacked Hilbert spectra with the EEMD and EMD algorithms with $10^4$ times of simulation are presented in Figures 3(h) and 3(i), respectively. We also performed weighted wavelet analysis (WWA) of the amplitude spectrum (Figure 3(f)) and the Z-transform spectrum (WWZ; Figure 3(g)) for comparison. The Morlet wavelet,

$$f(\tau) = e^{-\omega^2(\tau-\tau_0)^2} \left[e^{i\omega(\tau-\tau_0)} - e^{-i/\omega}\right]$$  

is used in this analysis, where $\omega$ is the scale factor, and $\tau$ is the time shift. Parameter $c$ determines how rapidly the wavelet or the size of the Gaussian envelope decays. We set it as $c = 0.0125$ (Foster 1996). These four algorithms yielded consistent results in the frequency evolution. However, the wavelet analysis has a trade-off between the frequency and the temporal resolutions by setting the width of the Gaussian window. The time-frequency property would be smeared out.
In comparison, the Hilbert spectrum shows a strikingly good resolution on the time-frequency map and indicates the precise time of the incoming GW signal. The mode-mixing may introduce a few nonphysical modulations when the signal crosses two IMFs. Except for this, HHT would be an ideal tool to study the detailed time-frequency structure of the GW signals induced by two BHs coalescence.

To compare the analysis result between different algorithms and the model template, we simulated the GW waveforms using the Python-based open software to explore astrophysical sources of GWs, PyCBCv2.0.112 (Usman et al. 2016), with the approximant given by the EOB (Taracchini et al. 2014; Pürrer 2016; Bohé et al. 2017) NR model. The instantaneous frequency of the template can be directly calculated via Hilbert transform without decomposing it into IMFs because the template fits the IMF criterion during the merging phase. The resulting instantaneous frequency of the template, the WWZ spectrum, and the stacked Hilbert spectrum are plotted in Figure 4. The time-frequency map of the HHT can clearly describe the instantaneous frequency in better detail compared to that of the wavelet spectrum. To demonstrate this, we choose three slices in the (a) low-frequency, (b) intermediate-frequency, and (c) high-frequency regimes. The slices of the HHT and WWZ spectra, as well as the instantaneous frequency obtained from the template, are shown in panels (a)–(c) of Figure 4. The gray color map denotes the density of the Hilbert energy-frequency pairs of each trial. The HHT result at epoch (b) is most ambiguous because the GW signal switches from one IMF to another at this time. However, the performance of the HHT remains comparable to that of the WWZ.

GW150914 is one of the strongest signals recorded in GWTC-1. We analyzed all the BH–BH coalescence events in GWTC-1 and present the results in the appendix (see Figures A1–A9).

These signals have been analyzed in a combination with the HHT and de-noise algorithm using a network array (Akhshi et al. 2021). The instantaneous frequency can be detected but large variability in the instantaneous frequency is occasionally seen (see, e.g., Figure 4 in Akhshi et al. 2021). These signals could be minimized by our stacking method.

### 3.2. GW170817

GW170817 is the first confirmed NS–NS coalescence event (Abbott et al. 2017a). A γ-ray burst and multiwavelength afterglow is a benchmark of the multimessenger astronomy era (Abbott et al. 2017b). Compared to other BH–BH coalescence obtained in GWTC-1 (Abbott et al. 2019; van Putten et al. 2019), the signal of GW170817 has a much longer coherence time, but the amplitude is much weaker. We tested both the WWZ and the HHT algorithms on this event. The signal can be seen on the waveform map with c = 0.00125, 10 times smaller than the c value for BH–BH coalescence events. Compared to the signals of BH–BH events, the amplitude of the NS–NS signal is much lower, and the signal keeps its coherence for a long while. The nearly coherent signal before the merging makes it possible to be detected with this setting, but the time-frequency property near the merging phase is smeared in time due to the large Gaussian window. In contrast, the signal is difficult to be resolved in the HHT time-frequency map (see Figure 5). Because the HHT is sensitive to both the instantaneous variability of the frequency and the noise, the signal-to-noise ratio (S/N) of this event is insufficient for the HHT to identify its intrinsic time-frequency property. Therefore, the HHT could be powerful for future NS–NS coalescence events with improved GW observatories in the next generation that could have a high S/N.

### 4. Core-collapse Supernovae

Detection of GW signals from a nearby CCSN will be the next milestone of gravitational wave astrophysics (Abbott et al. 2016b, 2020). It would be crucial for multimessenger astronomy because the possible co-detection of gravitational waves and neutrinos would allow astronomers to probe the activity of the center of a massive star during the final evolutionary stage. Thanks to the growth of computing power, our knowledge of the explosion engine of a CCSN improved dramatically. Owing to the asymmetric and complex fluid motion and the oscillation/evolution of the proto-neutron star (PNS), complex gravitational signals with multiple components are expected to be investigated from three-dimensional high-fidelity self-consistent CCSN simulations (Janka et al. 2016; Burrows & Vartanyan 2021). CCSN waveforms depend on several physical quantities, such as the core angular momentum and the composition of the collapsar, the nuclear equation of state, and the dynamics of fluid instabilities. The future multimessenger observations will place meaningful constraints on the supernova engines and the microphysics.

Nevertheless, recent theoretical works have reached consensuses on a few main structures of the GW signal on the time-frequency domain. These provide a strong basis to improve searching algorithms. On the other hand, the HHT is likely the most precise tool to investigate the detailed time-frequency properties of the CCSNe. Here we adopted the gravitational waveforms from the self-consistent three-dimensional CCSN simulations with neutrino transport in Pan et al. (2021).
4.1. Waveforms and Simulations

The gravitational waveforms of CCSNe simulations used in this project are adopted from the three-dimensional self-consistent CCSN simulations with the isotropic diffusion source approximation (Liebendorfer et al. 2009) for the neutrino transport, using the FLASH code (Fryxell et al. 2000) and are presented in Pan et al. (2021). The data are derived from a 40 $M_\odot$ progenitor at zero-age main sequence with solar metallicity from Woosley & Heger (2007) with three different initial rotational speeds ($\Omega_0 = 0, 0.5$ and 1 rad s$^{-1}$) at the onset of core collapse. A post-Newtonian treatment with the effective general relativity potential (Marek et al. 2006) and the Lattimer–Swesty (LS220) equation of state are considered (Lattimer & Swesty 1991).

The GW strains are evaluated by the first time derivative of the mass quadrupole moment with an assumption of a fixed distance $d = 10$ kpc to an observer. The second time derivative of the mass quadrupole moment is calculated by the finite difference in post-processing, using the formula described in Scheidegger et al. (2008) and Murphy et al. (2009). We simulated two viewing angles, where the observers are in the infinity in the direction of the equatorial plane or the rotational axis.

4.2. Analysis of Simulated CCSNe Waveforms

Here we take the gravitational waveform of the nonrotating model with an observer viewed from the equatorial direction as an example for our HHT analysis. The gravitational waveforms of other models with different parameters are shown in the appendix (Figures B10–B20). Figure 6 (a) shows the simulated strains, followed by the wavelet spectra (b)–(c) and the stacked HHT spectra (d)–(e). The waveform starts from the core collapse and reaches the core bounce at $t = 0$ s. As this model is nonrotating and the proto-neutron star convection happens after the core bounce, the gravitational strain shows a loud bounce signal at time zero, and then it is followed by a low-frequency peak PNS oscillation, which could be caused by the g-mode oscillation although the nature remains controversial (Müller et al. 2013; Pan et al. 2021). Multiple low-frequency oscillation signals dominate the first $\sim 0.15$ s, and they can be well resolved with the HHT. In comparison, the limitation of the wavelet spectrum prevents us to identify them. As the frequency of the oscillation increases with time, another high-frequency mode oscillation signal appears with similar strength as the peak PNS mode. The low-frequency PNS oscillation vanishes at $t \sim 0.5$ s. Then, interestingly, it followed by the quadrupole radiation of the standing accretion shock instability (SASI Blondin et al. 2003) signal, which was suggested to be seen in the GW data (Kuroda et al. 2016; Pan et al. 2021). The detailed waveform of the SASI signal is more complex, but its time-frequency property has been shown in a few analysis either with the spectogram or the HHT (see, e.g., Andreassen et al. 2017; Mezzacappa et al. 2020; Takeda et al. 2021). In our analysis, the SASI signal can be seen on both the wavelet and the HHT spectra. The wavelet and the HHT results for other simulated CCSN events are shown in the Appendix B.

The Hilbert spectra of all the data sets show much better resolution than the wavelet spectra.

5. Discussion

A Morlet wavelet consists of a complex exponential carrier multiplied by a Gaussian window. It provides a window size for both the low- and high-frequency regime that contains the same number of waves. The time-frequency map is distorted, and the signal is smeared if the frequency of a signal changes drastically within a few cycles (see, e.g., Figures 3(f)–(g) and Figures 6(b)–(c)). A narrower width of the Gaussian envelope can result in a better time resolution, although the frequency resolution would be sacrificed.

On the other hand, the Hilbert analysis calculates the instantaneous frequency of a signal. This helps trace the signal’s detailed evolution of the time-frequency property. Moreover, the complex multicomponent, low-frequency signals of CCSNe can be well resolved with the HHT. For example, the SASI signal in Figure 6 and a few multiple low-frequency signals in Figures B12–B20 can only be clearly resolved in the Hilbert spectra.

Nevertheless, the HHT can decompose a fast-changing signal into different IMFs. Fake frequency modulation in the instantaneous frequency is seen when the signal crosses two components. This can be reduced with the stacking algorithm suggested in this study, although they cannot be entirely removed. Moreover, the signal cannot be seen for low S/N events. For example, the NS–NS coalescence event GW170817 can be clearly detected with the wavelet analysis with a large Gaussian width. This is because the signal keeps its coherence for a few more cycles compared to the BH–BH coalescence events, but the S/N within a cycle is much lower.

Before stacking the time-frequency map, the evolution of Hilbert energy has given hints of the GW signal. A detailed time delay among instruments can be further characterized by cross correlating the EMD filtered profiles (Akhshi et al. 2021). In addition, the special pattern of the GW signal across IMFs (see, e.g., Figures 3(b)–(d)) would form a remarkable feature in the two-dimensional series where one dimension is time and another is the IMF index. This would be helpful for detecting the GW signal using convolution neural network.

The GW signal from a CCSN is believed to contain multiple components, where they could spread across a large range in both the frequency and the energy. The high resolutions of the HHT in both the time and frequency domains are helpful in distinguishing them from each other, especially the low-frequency transient signals within $\sim 0.4$ s after the core bounce.

6. Summary

The HHT is a novel technique for time-frequency analysis although the mode-splitting problem is severe when working with signals with huge frequency variability like GW signals. We propose a method that could minimize the unreal frequency modulation by stacking the time-frequency map instead of taking the ensemble mean of the IMFs. This helps visualize the evolution of the instantaneous frequency. We carried out an analysis for GWTC-1 events and found that most of their frequency evolution can be well tracked with our algorithm. Except for the known GW sources, we also tested our algorithm on the simulated CCSN events. We found that multiple GW components can be obtained with the HHT. Compared to the wavelet analysis, the unprecedented resolutions in both the time and frequency domains of the HHT are an ideal tool for further investigating the time-frequency properties of a signal. This provides evidence that the HHT can be a powerful tool in detecting CCSNe events for future multi-messenger campaigns.
Appendix A
Resulting Spectra of LIGO Events

All the remaining GWTC-1 events and corresponding time-frequency maps of BH–BH mergers are shown in this section (Figures A1–A9).

Figure A1. Analysis results of GW151012 observed with LIGO Hanford (left) and Livingstone (right). Panels (a)–(i) have the same definition as those in Figure 3.
Figure A2. Analysis results of GW151226 observed with LIGO Hanford (left) and Livingstone (right). Panels (a)–(i) have the same definition as those in Figure 3.
Figure A3. Analysis results of GW170104 observed with LIGO Hanford (left) and Livingstone (right). Panels (a)–(i) have the same definition as those in Figure 3.
Figure A4. Analysis results of GW170608 observed with LIGO Hanford (left) and Livingstone (right). Panels (a)–(i) have the same definition as those in Figure 3.
Figure A5. Analysis results of GW170729 observed with LIGO Hanford (left) and Livingstone (right). Panels (a)–(i) have the same definition as those in Figure 3.
Figure A6. Analysis results of GW170809 observed with LIGO Hanford (left) and Livingstone (right). Panels (a)–(i) have the same definition as those in Figure 3.
Figure A7. Analysis results of GW170814 observed with LIGO Hanford (left) and Livingston (right). Panels (a)–(i) have the same definition as those in Figure 3.
Figure A8. Analysis results of GW170818 observed with LIGO Hanford (left) and Livingston (right). Panels (a)–(i) have the same definition as those in Figure 3.
Figure A9. Analysis results of GW170823 observed with LIGO Hanford (left) and Livingstone (right). Panels (a)–(i) have the same definition as those in Figure 3.
Appendix B

Resulting Spectra of CCSNe Events

All the remaining simulated CCSN GW data, corresponding wavelet, and Hilbert spectra are shown in this section (Figures B10–B20).

Figure B10. (a) Cross mode of the gravitational waveform of a CCSN with an initial stellar mass of 40 $M_\odot$ and without rotation viewed from the polar direction. Panels (b)–(e) show the wavelet and Hilbert maps with the same definition as those in Figure 6.

Figure B11. (a) Plus mode of the gravitational waveform of a CCSN with an initial stellar mass of 40 $M_\odot$ and without rotation viewed from the equatorial direction. Panels (b)–(e) show the wavelet and Hilbert maps with the same definition as those in Figure 6.

Figure B12. (a) Plus mode of the gravitational waveform of a CCSN with an initial stellar mass of 40 $M_\odot$ and without rotation viewed from the polar direction. Panels (b)–(e) show the wavelet and Hilbert maps with the same definition as those in Figure 6.

Figure B13. (a) Cross mode of the gravitational waveform of a CCSN with an initial stellar mass of 40 $M_\odot$ and slow rotation (0.5 rad s$^{-1}$) viewed from the equatorial direction. Panels (b)–(e) show the wavelet and Hilbert maps with the same definition as those in Figure 6.

The Astrophysical Journal, 935:127 (21pp), 2022 August 20

Hu et al.
(a) Cross mode of the gravitational waveform of a CCSN with an initial stellar mass of $40 M_\odot$ and and slow rotation ($0.5 \text{ rad s}^{-1}$) viewed from the polar direction. Panels (b)–(e) show the wavelet and Hilbert maps with the same definition as those in Figure 6.

Figure B14.

(a) Cross mode of the gravitational waveform of a CCSN with an initial stellar mass of $40 M_\odot$ and slow rotation ($0.5 \text{ rad s}^{-1}$) viewed from the polar direction. Panels (b)–(e) show the wavelet and Hilbert maps with the same definition as those in Figure 6.

Figure B15.

(a) Plus mode of the gravitational waveform of a CCSN with an initial stellar mass of $40 M_\odot$ and slow rotation ($0.5 \text{ rad s}^{-1}$) viewed from the equatorial direction. Panels (b)–(e) show the wavelet and Hilbert maps with the same definition as those in Figure 6.

Figure B16.

(a) Plus mode of the gravitational waveform of a CCSN with an initial stellar mass of $40 M_\odot$ and fast rotation ($1 \text{ rad s}^{-1}$) viewed from the equatorial direction. Panels (b)–(e) show the wavelet and Hilbert maps with the same definition as those in Figure 6.

Figure B17.
Figure B18. (a) Cross mode of the gravitational waveform of a CCSN with an initial stellar mass of 40 $M_\odot$ and fast rotation (1 rad s$^{-1}$) viewed from the polar direction. Panels (b)–(e) show the wavelet and Hilbert maps with the same definition as those in Figure 6.

Figure B19. (a) Plus mode of the gravitational waveform of a CCSN with an initial stellar mass of 40 $M_\odot$ and fast rotation (1 rad s$^{-1}$) viewed from the equatorial direction. Panels (b)–(e) show the wavelet and Hilbert maps with the same definition as those in Figure 6.

Figure B20. (a) Plus mode of the gravitational waveform of a CCSN with an initial stellar mass of 40 $M_\odot$ and fast rotation (1 rad s$^{-1}$) viewed from the polar direction. Panels (b)–(e) show the wavelet and Hilbert maps with the same definition as those in Figure 6.

ORCID IDs

Chin-Ping Hu @ https://orcid.org/0000-0001-8551-2002
Lupin Chun-Che Lin @ https://orcid.org/0000-0003-4083-9567
Kuo-Chuan Pan @ https://orcid.org/0000-0002-1473-9880
Kwan-Lok Li @ https://orcid.org/0000-0001-8229-2024
Chien-Chang Yen @ https://orcid.org/0000-0002-5105-344X
C. Y. Hui @ https://orcid.org/0000-0003-1753-1660

References

Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2016a, PhRvL, 116, 061102
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2016b, PhRvD, 94, 102001
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017a, PhRvL, 119, 161101
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017b, ApJL, 848, L15
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2019, PhRvX, 9, 031040
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2020, PhRvD, 101, 084002
Abbott, R., Abbott, T. D., Abraham, S., et al. 2021, PhRvX, 11, 021053
Akhshi, A., Alimohammadi, H., Baghram, S., et al. 2021, NatSR, 11, 20507
Andersen, H., Müller, B., Müller, E., & Janka, H. T. 2017, MNRAS, 466, 2032
Bedrosian, E. 1962, A Product Theorem for Hilbert Transforms (Santa Monica, CA: RAND Corporation)
Blondin, J. M., Mezzacappa, A., & DeMarino, C. 2003, ApJ, 584, 971
Bohé, A., Shao, L., Taracchini, A., et al. 2017, PhRvD, 95, 044028
Burrows, A., & Vartanyan, D. 2021, Nat, 589, 29
Camp, J. B., Cannizzo, J. K., & Numata, K. 2007, PhRvD, 75, 061101
Chatziioannou, E., Blackburn, L., Martin, G., & Katsavounidis, E. 2004, CQGra, 21, S1809
