Escape rate of an active Brownian particle in a rough potential

Yating Wang and Zhanchun Tu*

Department of Physics, Beijing Normal University, Beijing 100875, China

E-mail: tuzc@bnu.edu.cn

Received 14 August 2022, revised 20 September 2022
Accepted for publication 21 September 2022
Published 30 November 2022

Abstract
We discuss the escape problem with the consideration of both the activity of particles and the roughness of potentials. We derive analytic expressions for the escape rate of an active Brownian particle in two types of rough potentials by employing the effective equilibrium approach and the Zwanzig method. We find that activity enhances the escape rate, but both the oscillating perturbation and the random amplitude hinder escaping.

Keywords: escape rate, active Brownian particle, rough potential, effective potential

(Some figures may appear in colour only in the online journal)

1. Introduction

The escape problem has attracted much attention from researchers in various fields [1–10]. The Arrhenius formula indicates that the rate of a chemical reaction depends exponentially on inverse temperature [3, 4]. Kramers presented the transition state method for calculating the rate of chemical reactions by considering a Brownian particle escaping over a potential barrier [5]. Subsequent studies on escape rate are summarized in [10]. All of the above studies merely involve passive particles. The research theme has been transferred to active particles with self-propulsion in recent years [11–24]. Active systems are intrinsically non-equilibrium since the detailed balance is broken. An effective equilibrium method has been developed to investigate active Brownian particles [25–28]. By using this method, Sharma et al discussed an escape problem of active particles in a smooth potential [29]. They found that introducing activity increases the escape rate.

The escape problem in the research mentioned above is simplified as a Brownian particle climbing over a smooth potential barrier. However, the potential is not always smooth in reality. Interface area scans of proteins imply that the protein surface is not smooth [30, 31]. The hierarchical arrangement of the conformational substrates in myoglobin indicates that the potential surface might be rough [32]. In addition, the inside of the cell is quite crowded. Thus, diffusion of substance in the cell may not be regarded as Brownian motion in smooth potential. From the biochemical point of view, it is valuable to consider the influence of the roughness of potential diffusion behaviors. The study of diffusion in rough potential offers insight into fields from the transport process in disordered media [33, 34] to protein folding [35, 36] and glassy systems [37, 38]. Zwanzig dealt with diffusion in a rough potential and found that the roughness slows down the diffusion at low temperatures [39]. Roughness-enhanced transport was also observed in ratchet systems [40–42]. Hu et al discussed diffusion crossing over a barrier in a random rough metastable potential [43]. By using numerical simulations, they demonstrate a decrease in the steady escape rate with the an increase of rough intensity. The activity of particles was not considered in these works.

There are a large number of active substances, biochemical reactions, and transport of substances in organisms. Therefore, it is of practical significance to discuss the escape problem with the consideration of both the activity of particles and the roughness of potentials. In order to describe the slow dynamics of a tagged particle in a dense active environment, Subhasish et al discussed the escape of a passive particle from an activity-induced energy landscape by using the activity-induced rugged energy landscape approach [44]. In this work, we calculate the escape rate of an active Brownian particle (ABP) in rough potentials by using the effective equilibrium approach [25–29] and the Zwanzig method [39]. The rest of this paper is organized as follows: In section 2, we briefly introduce the effective equilibrium approach. In section 3, we discuss the escape problems of...

* Author to whom any correspondence should be addressed.
ABPs in rough potentials with oscillating perturbation or random amplitude. We derive the effective rough potentials following the effective equilibrium approach. Then we analytically calculate the escape rates of ABPs in the effective rough potentials. We find that activity enhances the escape rate, but both the oscillating perturbation and the random amplitude hinder escaping. The last section is a brief summary.

2. Effective equilibrium approach

In this section, we briefly revisit the main ideas of the effective equilibrium approach [25–29].

The motion of the ABP can be described by the following overdamped Langevin equations

\[
\dot{\mathbf{r}} = v_0 \mathbf{n} + \gamma^{-1} \mathbf{F} + \mathbf{\xi}(t),
\]

where \( \gamma \) is the friction coefficient and \( \mathbf{F}(t) \) is force on the ABP. \( \mathbf{n} \) represents the position of the particle. The particle is self-propelling with constant speed \( v_0 \) along orientations \( \mathbf{n} \). The dot \( \cdot \) above \( \mathbf{n} \) characterizes the derivative with respect to time \( t \). The stochastic vectors \( \mathbf{\xi}(t) \) and \( \eta(t) \) are white noise with correlations \( \langle \mathbf{\xi}(t) \mathbf{\xi}(t') \rangle = 2D_\perp \delta(t - t') \) and \( \langle \eta(t) \eta(t') \rangle = 2D_\parallel \delta(t - t') \), where \( D_\perp \) and \( D_\parallel \) are the translational and rotational diffusion coefficients, respectively. \( \mathbf{I} \) is the unit tensor.

We obtain \( \langle \mathbf{n}(t) \rangle = 0 \) and \( \langle \mathbf{n}(t) \mathbf{n}(t') \rangle = (1/3) \mathbf{I} e^{-2D_\perp |t - t'|} \) from equation (2). By substituting them into equation (1), we derive

\[
\dot{\mathbf{r}} = \gamma^{-1} \mathbf{F} + \mathbf{\chi}(t),
\]

where \( \mathbf{\chi}(t) \) is a white noise with correlation \( \langle \mathbf{\chi}(t) \mathbf{\chi}(t') \rangle = 2D_\perp \delta(t - t') + (v_0^2/3) \mathbf{I} e^{-2D_\perp |t - t'|} \).

A stochastic process with color-noise in equation (3) is non-Markovian. It is impossible to derive an exact Fokker–Planck equation for the time of the probability distribution. Nevertheless, using the Fox approximate method \cite{45, 46}, we may derive an approximate Fokker–Planck equation

\[
\frac{\partial \phi(r, t)}{\partial t} = -\nabla \cdot \mathbf{J}(r, t),
\]

where \( \phi(r, t) \) is the probability distribution. The current \( \mathbf{J}(r, t) \) is expressed as

\[
\mathbf{J}(r, t) = -D_t \nabla D(r) [\mathbf{F}(r) + \beta \mathbf{F}^{\text{eff}}(r)] \phi(r, t),
\]

where \( \mathbf{F}^{\text{eff}}(r) \) represents the effective force on the particle. \( \beta = (k_\text{B} T)^{-1} \), in which \( k_\text{B} \) is the Boltzmann constant and \( T \) is the temperature. The dimensionless effective diffusion coefficient \( D(r) = 1 + D_a/(1 - \gamma^{-1} \nabla \cdot \beta \mathbf{F}(r)) \), where \( \gamma^{-1} \beta D_\parallel /2D_\perp \). The activity parameter \( D_a = v_0^2/(6D_D D_\parallel) \). The effective force is given by

\[
\mathbf{F}^{\text{eff}}(r) = \frac{1}{D(r)} [\mathbf{F}(r) - \beta \nabla D(r)].
\]

The above three equations and the first three terms in equation (10) have been derived in [29].

The bare and effective rough potentials are schematically depicted in figure 1. \( x_0 \) and \( x_b \) correspond to the minimum and maximum of the potential, respectively. \( x_c \) is a point on the right of \( x_b \). Passing \( x_c \), the particle will not return. In the other words, it is an absorbing boundary condition at \( x_c \). In a stationary state, the current (5) can be rewritten as

\[
\mathbf{F}^{\text{eff}}(r) = -D_t \nabla D(r) e^{-\beta \mathbf{F}(r)} \frac{d}{dx} [e^{\beta \mathbf{F}(r)} \phi(x)].
\]

3. Escape rate of ABP in rough potentials

In this section, we will deduce the effective rough potential and escape rate of ABP in rough potentials. For simplicity, we only consider the case that the bare force depends merely on a one-dimension potential \( V = V(x) \). In this case, \( \mathbf{F} = -V(x) \mathbf{i} \), where \( \mathbf{i} \) is the unit vector of \( x \)-coordinate. The prime "\( ' \)" on the top right of a character represents the derivative with respect to position \( x \). From equation (6) we can obtain the effective potential

\[
\beta V^{\text{eff}}(x) = \ln D(x) + \int_0^x dy \frac{\beta V'(y)}{D(y)},
\]

with

\[
D(x) = 1 + \frac{D_a}{1 + \tau \beta V'(x)}.
\]

Now, let us consider a rough potential

\[
\beta V(x) = \frac{1}{2} \kappa_0 x^2 - \alpha x^3 + \varepsilon V(x),
\]

where \( \kappa_0 \) and \( \alpha \) are positive constants. The first two terms in equation (9) provide a smooth background with a barrier. The last term in equation (9) is the superposed random or oscillating perturbation. The amplitude \( \varepsilon \) is assumed to be small, which represents a measure of the ‘roughness’ of the potential.

Now, we look for the effective rough potential \( \beta V^{\text{eff}}(x) \) corresponding to equation (9) from equation (7). Assuming \( \kappa_0 \tau \ll 1 \) and keeping the terms up to the linear order of \( \kappa_0 \tau \) and \( \varepsilon \), we obtain the effective rough potential

\[
\beta V^{\text{eff}}(x) \approx \frac{1}{2} \kappa_0 x^2 - \alpha x^3 + g(x) + \frac{\varepsilon V(x)}{1 + D_a},
\]

where

\[
\kappa_a = \kappa_0 \left[ \frac{1}{1 + D_a} + \frac{D_a \kappa_0 \tau}{(1 + D_a)^2} \right],
\]

\[
\alpha' = \alpha - \frac{1}{1 + D_a} + \frac{3D_a \kappa_0 \tau}{(1 + D_a)^2},
\]

\[
g(x) = \frac{6D_a \kappa_0 \tau x}{1 + D_a} + \frac{9D_a \kappa_0 \tau x^2}{2(1 + D_a)^2} e^x.
\]

The above three equations and the first three terms in equation (10) have been derived in [29].
where \( x_1 \leq x_e \leq x_2 \). The detailed derivation of this equation is shown in appendix A.

Considering the rough character of the potential, we use the Zwanzig method [39] to simplify equation (15). The rough potential (10) may be decomposed into two parts. One is the smooth skeleton

\[
\beta V^\text{eff}_0 (x) = \frac{1}{2} \kappa_a x^2 - \alpha' x^3 + g(x),
\]

and the other is the rough perturbation

\[
\beta V^\text{eff}_I (x) = \frac{\varepsilon V(x)}{1 + D_a}.
\]

Since \( V^\text{eff}_I (x) \) varies quickly with \( x \), we consider its average effect on escape rate in equation (15). Define \( \psi^+ (x) \) and \( \psi^- (x) \) such that

\[
e^{\psi^+(x)} = \langle e^{i \beta V^\text{eff}_I (x)} \rangle,
\]

where \( \langle \cdot \rangle \) denotes the spatial average during a small interval \( (x - \Delta/2, x + \Delta/2) \). Then equation (15) is transformed into

\[
e^{\psi^-(x)} = \int_{x_e}^{x_u} dy e^{-\beta V^\text{eff}_I (y)} e^{\psi^+(y)}
\times \int_{x_e}^{x_u} dz \frac{e^{\beta V^\text{eff}_I (z)}}{D_s D(z)} D_s D(z).
\]

Next we discuss the spacial situation that \( \psi^\pm (x) \) happens to be independent of \( x \). In this case, the above equation is transformed into

\[
e^{\psi^+(x)} = \int_{x_e}^{x_u} dy e^{-\beta V^\text{eff}_I (y)} \int_{x_e}^{x_u} dz \frac{e^{i \beta V^\text{eff}_I (z)}}{D_s D(z)}.
\]

By using the saddle-point approximation and considering \( \kappa_0 \tau \) is small, we derive the escape rate

\[
r^\text{pass} = \frac{D_s}{2 \pi} e^{-\beta E_0},
\]

where

\[
k_b = k_0 \left[ -\frac{1}{1 + D_a} + \frac{D_a k_0 \tau}{(1 + D_a)^2} \right],
\]

and

\[
\beta E_0 = \frac{k_0^3}{54 \alpha^2 (1 + D_a)} + \frac{2 D_a k_0 \tau}{1 + D_a}.
\]

The detailed derivation of equation (21) is displayed in appendix B.

For a passive Brownian particle moving in a smooth potential, equation (21) is degenerated into

\[
r^\text{pass} = \frac{D_s}{2 \pi} e^{-\beta E_0},
\]

where \( \beta E_0 = k_0^3/(54 \alpha^2) \). This is exactly the Kramers rate for the passive particle escaping from a smooth barrier [5]. The escape rate of ABP in rough potential may be further expressed as

\[
r^\text{pass} = r^\text{pass} \left[ \frac{\varepsilon \sin(q \alpha x)}{1 + D_a} \right]^{-1},
\]

Obviously, the above equation implies the escape rate

\[
r^\text{pass} = r^\text{pass} \left[ \frac{\varepsilon \sin(q \alpha x)}{1 + D_a} \right]^{-1},
\]

for APB in a smooth potential [29] since \( \psi^+ = \psi^- = 1 \) for the smooth potential. Then, equation (25) can be further expressed as

\[
r^\text{pass} = r^\text{pass} \left[ \frac{\varepsilon \sin(q \alpha x)}{1 + D_a} \right]^{-1}.
\]

### 3.1. Oscillating perturbation of rough potential

We consider the oscillating perturbation, \( V_I (x) = \sin(q \alpha x) \), and that the effective barrier decreases with the increase of the activity parameter. Thus, the introduction of activity lowers the effective barrier height so that the particle easily escapes the barrier.

From equation (28), we obtain

\[
e^{\psi^+(x)} = I_0 \left( \frac{\varepsilon}{1 + D_a} \right),
\]

where \( I_0 \) is the modified Bessel function [36]. Substituting equation (29) into equation (25), we obtain the escape rate

\[
r^\text{pass} = r^\text{pass} \left[ \frac{\varepsilon \sin(q \alpha x)}{1 + D_a} \right]^{-2}.
\]

Since the modified Bessel function is always larger than 1, we have \( r^\text{pass} = r^\text{pass} \left[ \frac{\varepsilon \sin(q \alpha x)}{1 + D_a} \right]^{-2} \). That is, the roughness due to oscillating perturbation hinders escaping. Figure 2 shows the dependence of \( r^\text{pass} / r^\text{pass} \) on activity and
roughness, $r_{pass}^{act}/r_{pass}$ increases with the increase of activity, but decreases with the increase of roughness.

### 3.2. Random amplitude of rough potential

Considering the random amplitude of rough potential $V_1$ with a Gaussian distribution

$$
\rho(V_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\sqrt{2}}{\sigma^2} V_i^2}, \quad (31)
$$

where $\sigma$ is the standard deviation. The effective rough potential $\beta V_{eff}(x)$ is

$$
\beta V_{eff}(x) \approx \frac{1}{2} \kappa_a x^2 - \alpha x^3 + g(x) + \frac{\varepsilon V_i}{1 + D_a}. \quad (32)
$$

From equation (18), we obtain

$$
e^{-\frac{\varepsilon^2}{\kappa a}} = e^{-\frac{\varepsilon^2}{\kappa a x^2}}. \quad (33)
$$

Substituting equation (33) into equation (25), we obtain the escape rate

$$
r_{act}^{pass} = r_{pass} e^{\frac{\alpha x^3 - g(x)}{1 + D_a}} e^{-\frac{\varepsilon^2}{\kappa a x^2}}. \quad (34)
$$

Since $e^{-\frac{\varepsilon^2}{\kappa a x^2}}$ is always less than 1, we have $r_{act}^{pass} < r_{act}^{pass}$. That is, the random amplitude hinders escaping.

Figure 3 shows the dependence of $r_{act}^{pass}/r_{pass}$ on activity and roughness. $r_{act}^{pass}/r_{pass}$ increases with the increase of activity, but decreases with the increase of roughness.

### 4. Conclusions

In this work, we have discussed the escape rate of ABPs in rough potentials by using the effective equilibrium approach and the Zwanzig method. We find that activity usually enhances the escape rate. Both the oscillating perturbation and the random amplitude of rough potential hinder escaping. We only discussed the influence of two types of rough potential on the escape rate of ABP. According to equation (21), ‘rough potential hinders the escape’ may not hold in all cases. However, ‘activity enhances the escape rate’ should be generally applicable. In the theoretical derivation, we need the amplitude $\varepsilon$ and $\kappa_0$ to be small. Our theory is not applicable to large $\varepsilon$ and $\kappa_0$. We will develop a new theoretical approach to deal with these situations in the future.

### Acknowledgments

The authors are grateful for financial support from the National Natural Science Foundation of China (Grant No. 11975050 and No. 11735005). We are very grateful for the help of Xiu-Hua Zhao.

### Appendix A. Detailed derivation of equation (15)

Following the Kramers method in [47], we derive the inverse escape rate of ABP in the effective rough potential.

Assume $\beta \varepsilon^0 \gg 1$. In this situation, the system stays the quasi-stationary state such that the probability current $J_{act}^{pass}$ is approximately independent of $x$. By integrating equation (14) between $x_b$ and $x_a$ and considering an absorbing boundary condition $x = x_a$, we obtain

$$
J_{act}^{pass} = D_e e^{\beta V_{eff}(x_a)} \phi(x_a) \int_{x_b}^{x_a} \frac{e^{-\beta V_{eff}(x)}}{D_e} dx. \quad (A1)
$$

Because the barrier is high, $\phi(x)$ near $x_a$ may be approximately given by the stationary distribution

$$
\phi(x) \approx \phi(x_a) e^{-(\beta V_{eff}(x) - \beta V_{eff}(x_a))}. \quad (A2)
$$

The probability $p$ to find ABP near $x_a$ is

$$
p = \int_{x_1}^{x_2} \phi(x) dx = \phi(x_a) e^{\beta V_{eff}(x_a)} \int_{x_1}^{x_2} e^{-\beta V_{eff}(x)} dx, \quad (A3)
$$

where $x_1 \leq x_2 \leq x_b$. Finally, we can derive equation (15) by using $p/r_{pass}^{act}$. 

![Figure 2](image1.png)

**Figure 2.** Dependence of $r_{pass}^{act}/r_{pass}$ on amplitude $\varepsilon$ and active parameter $D_a$, where $\alpha \kappa_0^{-3/2} = 0.1, \tau = 0.02$.

![Figure 3](image2.png)

**Figure 3.** Dependence of $r_{pass}^{act}/r_{pass}$ on amplitude $\sigma$ and active parameter $D_a$, where $\alpha \kappa_0^{-3/2} = 0.1, \tau = 0.02$. 


Appendix B. Saddle-point approximation

The integral expression in equation (20) may be obtained via the saddle-point approximation at $x_a$ and $x_b$, respectively.

The effective smooth potential of nearly $x_b$ can be expanded nearby $x_b$ as:

$$\beta V_{\text{eff}}^0 (x) = \beta V_{\text{eff}}^0 (x_b) - \frac{1}{2} \kappa_0 (x - x_b)^2. \quad (A4)$$

The second integral of smooth potential on the right-hand side of equation (20) is expressed as

$$\int_{x_a}^{x_b} \frac{dx}{D(x)} \int_{x_a}^{x} \frac{dx}{D(x)} e^{\beta \tilde{V}^0_{\text{eff}}(x)} = \int_{x_a}^{x_b} \frac{dx}{D(x)} e^{\beta \tilde{V}^0_{\text{eff}}(x)} \frac{1}{2} \kappa_0 (x - x_b)^2. \quad (A5)$$

According to the spirit of saddle-point approximation, equation (A5) is transformed into

$$\int_{x_a}^{x_b} \frac{dx}{D(x)} e^{\beta \tilde{V}^0_{\text{eff}}(x)} \frac{1}{2} \kappa_0 (x - x_b)^2 = \frac{2 \pi}{\kappa_0} e^{\beta \tilde{V}^0_{\text{eff}}(x_b)} \frac{1}{D(x_b)}. \quad (A6)$$

Substituting $x_b$ into equation (8) and considering $\kappa_0 \tau$ is small, we obtain

$$\int_{x_a}^{x_b} \frac{dx}{D(x)} e^{\beta \tilde{V}^0_{\text{eff}}(x)} = \sqrt{\frac{2 \pi}{\kappa_0 \tau}} \frac{e^{\beta \tilde{V}^0_{\text{eff}}(x_b)}}{1 + D_\tau}. \quad (A7)$$

Similarly, the first integral of smooth potential on the right-hand side of equation (20) may also be obtained by a saddle-point approximation at $x_a$.

References

[1] McGinley E and Crim F 1986 Overtone vibration initiated unimolecular reaction of tetramethyldioxetane in a free jet expansion: a comparison with RRKM theory J. Chem. Phys. 85 5748
[2] Bohr N and Wheeler J A 1939 The mechanism of nuclear fission Phys. Rev. 56 426
[3] Arrhenius S 1889 Über die Dissoziationswärme und den Einfluss der Temperatur auf den Dissoziationsgrad der Elektrolyte Z. Phys. Chem. 4 96
[4] Arrhenius S 1889 Influence of temperature on the rate of inversion of sucrose Z. Phys. Chem. 4 226
[5] Kramers H A 1940 Brownian motion in a field of force and the diffusion model of chemical reactions Physica 7 284
[6] Brinkman H 1956 Brownian motion in a field of force and the diffusion theory of chemical reactions, II Physica 22 149
[7] Weiss G H 1967 First passage time problems in chemical physics Adv. Chem. Phys. 13 1–18
[8] Ao P et al 1989 Influence of dissipation on the Landau–Zener transition Phys. Rev. Lett. 62 3004
[9] Aslangul C, Pottier N and Saint-James D 1985 Quantum ohmic dissipation: transition from coherent to incoherent dynamics Phys. Lett. A 110 249
[10] Hänggi P, Talkner P and Borkovec M 1990 Reaction-rate theory: fifty years after Kramers Rev. Mod. Phys. 62 251
[11] Vicsek T, Czikor A, Ben-Jacob E, Cohen I and Sochet O 1995 Novel type of phase transition in a system of self-driven particles Phys. Rev. Lett. 75 1226
[12] Dombrowski C, Cisneros L, Chatkaew S, Goldstein R E and Kessler J O 2004 Self-concentration and large-scale coherence in bacterial dynamics Phys. Rev. Lett. 93 098103
[13] Howse J R, Jones R A, Ryan A J, Gough T, Vafabakhsh R and Golestanian R 2007 Self-motive colloidal particles: from directed propulsion to random walk Rev. Lett. 99 048102
[14] Ebbens S J and Howse J R 2010 In pursuit of propulsion at the nanoscale Soft Matter 6 726
[15] Schaller V, Weber C, Semmrich C, Frey E and Bausch A R 2010 Polar patterns of driven filaments Nature 467 73
[16] Valeriani C, Li M, Novosel J, Arlt J and Marenduzzo D 2011 Colloids in a bacterial bath: simulations and experiments Soft Mater 7 5228
[17] Fily Y and Marchetti M C 2012 Athermal phase separation of self-propelled particles with no alignment Phys. Rev. Lett. 108 235702
[18] Romanczuk P, Bar M, Ebeling W, Lindner B and Geier L S 2012 Active brownian particles—from individual to collective stochastic dynamics Phys. J. Spec. Top. 202 1–162
[19] Cates M E and Tailleur J 2015 Motility-induced phase separation Annu. Rev. Condens. Matter Phys. 6 219
[20] Dibyendu M, Katherine K, Weese D and Michael R 2017 Entropy production and fluctuation theorems for active matter Phys. Rev. Lett. 119 258001
[21] Burkholder E W and Brady J F 2020 Nonlinear microrheology of active Brownian suspensions Soft Matter 16 1034
[22] Caprini L, Marconi U M B, Puglisi A and Vulpiani A 2019 Active escape dynamics: The effect of persistence on barrier crossing J. Chem. Phys. 150 024902
[23] Caprini L, Cecconi F and Marconi U M B 2019 Transport of active particles in an open-wedge channel J. Chem. Phys. 150 144903
[24] Caprini L, Cecconi F and Marconi U M B 2021 Correlated escape of active particles across a potential barrier J. Chem. Phys. 155 234902
[25] Maggi C, Marconi U M B, Gian N and Leonardo R D 2015 Multidimensional stationary probability distribution for interacting active particles Sci. Rep. 5 1
[26] Farage T F, Krinnering P and Brader J M 2015 Effective interactions in active Brownian suspensions Phys. Rev. E 91 042310
[27] Wittmann R and Brader J M 2016 Active Brownian particles at interfaces: an effective equilibrium approach EPL 114 68004
[28] Marconi U M B, Paoluzzi M and Maggi C 2016 Effective potential method for active particles Mol. Phys. 114 2400
[29] Sharma A, Wittmann R and Brader J M 2017 Escape rate of active particles in the effective equilibrium approach Phys. Rev. E 95 012115
[30] Wodak S J and Janin J 1980 Analytical approximation to the accessible surface area of proteins Proc. Natl. Acad. Sci. USA 77 1736
[31] Janin J and Wodak S J 1983 Structural domains in proteins and their role in the dynamics of protein function Proc. Biophys. Mol. Biol. 42 21
[32] Ansari A, Berendzen J, Bowne S F, Frauenfelder H, Iben I, Sauter T B, Shyamsunder E and Young R D 1985 Protein states and proteinquakes Proc. Natl. Acad. Sci. USA 82 5000
[33] Bouchaud P and Georges A 1990 Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications Phys. Rep. 195 127
[34] Berlin Y A and Burin A L 1996 Diffusion in one-dimensional disordered systems Chem. Phys. Lett. 257 665
[35] Onuchic J N, Schulen Z L and Wolynes P G 1997 Theory of protein folding: the energy landscape perspective Annu. Rev. Phys. Chem. 48 545
[36] Yu H, Dec D R, Liu X, Brigley A M, Sosova I and Woodside M T 2015 Protein misfolding occurs by slow diffusion across multiple barriers in a rough energy landscape Natl. Acad. Sci. USA 112 8308
[37] de Souza V K and Wales D J 2008 Energy landscapes for diffusion: analysis of cage-breaking processes J. Chem. Phys. 129 164507
[38] Niblett S, Souza V D, Stevenson J and Wales D 2016 Dynamics of a molecular glass former: energy landscapes for diffusion in ortho-terphenyl J. Chem. Phys. 145 024505
[39] Zwanzig R 1988 Diffusion in a rough potential Proc. Natl. Acad. Sci. USA 85 2029
[40] Li Y, Xu Y and Kurths J 2017 Roughness-enhanced transport in a tilted ratchet driven by Lévy noise Phys. Rev. E 96 052121
[41] Li Y, Xu Y and Kurths J 2019 First-passage-time distribution in a moving parabolic potential with spatial roughness Phys. Rev. E 99 052203
[42] Archana G R and Barik D 2021 Roughness in the periodic potential enhances transport in a driven inertial ratchet Phys. Rev. E 104 024103
[43] Hu M and Bao J D 2018 Diffusion crossing over a barrier in a random rough metastable potential Phys. Rev. E 97 062143
[44] Chaki S and Chakrabarti R 2020 Escape of a passive particle from an activity-induced energy landscape: emergence of slow and fast effective diffusion Soft Matter 16 7103
[45] Fox R F 1986 Functional-calculus approach to stochastic differential equations Phys. Rev. A 33 467
[46] Fox R F 1986 Uniform convergence to an effective Fokker–Planck equation for weakly colored noise Phys. Rev. A 34 4525
[47] Risken H 1966 The Fokker-Planck Equation (Berlin: Springer) 123-125