Computation of a Rising Bubble in an Enclosure Filled with Liquid Metal under Vertical Magnetic Fields

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Three-dimensional numerical computations for a single bubble rising in a liquid metal within a rectangular enclosure in a uniform vertical magnetic field are carried out. In this study, the bubble shape, the velocity field and the electric current density field interact with one another under the influence of the vertical magnetic field. This is a triple simultaneous problem. The pressure and the electric potential fields are obtained with an iterative procedure by the HSMAC method. The numerical results exhibit that the rising velocity of the bubble for the range of Hartmann number $0 < Ha < 75$ is slightly larger than that of $Ha = 0$ because of the drag reduction due to suppression effect on flow separation behind the bubble. On the other hand, the bubble rising velocity decreases monotonously with the increase in the Hartmann number for $Ha > 75$ owing to deceleration effect on the fluid flow by the magnetic field. The bubble elongates in the direction of the uniform magnetic field because of the modification of the pressure distribution by the Lorentz force.

KEY WORDS: bubble flow; free-surface flow; simulation; electromagnetic processing of materials; surface tension; liquid metal.

1. Introduction

The dynamics of bubble flows is complex owing to the existence of moving deformable interface at which the surface tension acts and density and viscosity vary steeply. The driving mechanism of bubble flows stems from buoyancy in a gravity or centrifugal force since the density of bubble is much smaller than that of liquid phase. The rising velocity of a single bubble is influenced by various factors such as bubble size, density difference, values of viscosities and surface tension as well as acceleration due to gravity. During rising, the bubble sometimes exhibits temporal change in its shape and velocity when Reynolds number is large. Such bubble flows appears not only in natural phenomena but also in many industrial processes.

For instance, in a continuous casting of steel making process, argon gas is injected into a nozzle between the tundish and the mold to avoid choking when the molten steel passes through the nozzle. Hence, the molten metal flow in the mold includes many bubbles. Removal of the bubbles from the molten metal flow is of great responsibility for the quality of solidified products. An external magnetic field applied to the mold effectively controls the molten metal bubble flow. Obviously, the Lorentz force does not act on the bubbles themselves but it acts on the molten metal around the bubbles. Therefore, the magnetic field indirectly affects the movement of the bubbles.

Magnetohydrodynamic (MHD) bubble flows are quite complex and have not been clarified sufficiently although several previous researches relevant to MHD two-phase flows have been reported. Chester,1) Chang,2) Ueno and Yasuda3) analytically studied characteristics of MHD flows past a solid body under a strong magnetic field. They suggested that the drag for the inertia-less regime is proportional to the Hartmann number when the magnetic field is parallel to the flow. Kyrlidis et al.4) solved the same problem, an axisymmetric Stokes equation including a Lorentz force term, by using a finite element method. Takatani5) developed a mathematical modeling for solving incompressible MHD flows with free surface and performed to compute various MHD free surface flows. One of his computations includes the flow around a rising bubble in a liquid metal either in a uniform vertical or horizontal magnetic field, and the influence of the direction of the magnetic field with a certain magnitude was briefly mentioned. Tagawa6) performed numerical simulations of a falling droplet into a liquid metal layer in the presence of a uniform vertical magnetic field. Concerning the numerical simulation of non-MHD bubble flows, Annaland et al.7) have developed their volume of fluid (VOF) model which simulates co-axial and oblique coalescence of two rising gas bubbles.

In the present study, three-dimensional numerical computations are carried out for a single bubble rising in a liquid metal within a rectangular enclosure in a uniform vertical magnetic field. The focus points in this study are

1) relation between the rising velocity of bubble and the magnitude of the magnetic field, and
2) deformation of the bubble shape during rising in the uniform vertical magnetic field.
2. Mathematical Modeling

2.1. Model for Analysis

Figure 1 shows the schematic drawing for the present problem. An electro-conducting fluid like mercury or molten steel is filled in a long rectangular enclosure. Its axis is parallel to the gravity and a spherical gas bubble is released near the bottom wall in the presence of a uniform vertical magnetic field. The bubble moves upwards owing to buoyancy caused by the hydrostatic pressure gradient within the liquid phase in the presence of the gravity force. The horizontal edges of the enclosure are twice the bubble diameter and the height of the enclosure is six times. All six walls are assumed to be electrically insulating.

The bubble shape, the velocity field and the electric current density field interact with one another under the vertical magnetic field. This is a triple simultaneous problem.

2.2. Governing Equations

In this paper, for the sake of simplicity, it is assumed that both the liquid metal and the gas in the bubble are incompressible Newtonian fluids and heat sources such as viscous dissipation and Joule heating are neglected. The induced pressible Newtonian fluids and heat sources such as viscous dissipation and Joule heating are neglected. The magnetic field is neglected since the magnetic Reynolds number of this problem is assumed to be much smaller than unity. Therefore, the following equations are employed.

\[ \frac{\partial \rho}{\partial t} + \vec{u} \cdot \nabla \rho = 0 \] ........................(1)

\[ \nabla \cdot \vec{u} = 0 \] ........................(2)

\[ \rho \left[ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} \right] = -\nabla p + \rho \vec{g} + \vec{j} \times \vec{b} + \vec{f}_{\nu} \] ........................(3)

Strain rate tensor:

\[
\dot{\varepsilon} = \begin{bmatrix}
\frac{\partial \nu}{\partial x} & \frac{1}{2} \left( \frac{\partial \nu}{\partial x} + \frac{\partial \mu}{\partial y} \right) & \frac{1}{2} \left( \frac{\partial \nu}{\partial z} + \frac{\partial \mu}{\partial x} \right) \\
\frac{1}{2} \left( \frac{\partial \nu}{\partial y} + \frac{\partial \mu}{\partial x} \right) & \frac{\partial \nu}{\partial y} & \frac{1}{2} \left( \frac{\partial \nu}{\partial z} + \frac{\partial \mu}{\partial y} \right) \\
\frac{1}{2} \left( \frac{\partial \nu}{\partial z} + \frac{\partial \mu}{\partial x} \right) & \frac{1}{2} \left( \frac{\partial \nu}{\partial z} + \frac{\partial \mu}{\partial y} \right) & \frac{\partial \nu}{\partial z}
\end{bmatrix}
\]

\[ \vec{J} = \sigma (-\nabla \psi \vec{u} + \vec{u} \times \vec{b}) \] ........................(4)

\[ \nabla \cdot \vec{J} = 0 \] ........................(5)

where \( \vec{u} = (u, v, w) \), \( \vec{J} = (j_1, j_2, j_3) \), \( \vec{b} \), \( \vec{g} \), \( \psi \), \( t \), \( \rho \), \( \mu \), \( \sigma \) and \( \vec{\varepsilon} \) denote velocity, electric current density, magnetic flux density, gravitational acceleration, pressure, electric potential, time, density, viscosity, electric conductivity and strain rate tensor, respectively. The term \( \vec{J} \times \vec{b} \) denotes the electromagnetic force (Lorentz force) induced in the liquid phase caused by the imposition of an external static magnetic field, and \( \vec{f}_{\nu} \) is an interfacial force acting per unit volume using the CSF model.8)

Generally, for the accurate computation of two-phase flows, not only liquid phase but also gas phase should be taken into consideration. Therefore, in this study, it is treated that both phases are simultaneously computed like a case of the single-phase flow on a fixed mesh system. Basically, the fluid velocity is obtained from Eq. (3) together with Eq. (2). Then, the local fluid density is obtained from Eq. (1). However, the sharpness of the density jump (interface) is gradually lost as time passes during the computation. In order to avoid this difficulty, we introduce the index function \( \phi \), which distinguishes whether the computational domain is the liquid phase (\( \phi = -0.5 \)), gas phase (\( \phi = 0.5 \)) or transitional region of interface (\( -0.5 < \phi < 0.5 \)) as performed in the recent paper.6,9)

In this study, the relationship between the density and the index function \( \phi \) within the transitional interface is assumed as follows:

\[
\rho = \frac{1}{\rho_L} \left[ \frac{1}{2} - H(\phi) \right] + \frac{1}{\rho_G} \left[ \frac{1}{2} + H(\phi) \right] \]

where \( \rho_L \) and \( \rho_G \) denote gas phase density and liquid phase density, respectively. The function \( H(\phi) \) denotes the approximated Heaviside function. This function is defined as follows:

\[
H(\phi) = \begin{cases} 
\frac{1}{2} & \phi \geq \frac{1}{2} \\
\frac{1}{2} - \frac{1}{2\pi} \sin(2\pi \phi) & -\frac{1}{2} < \phi < \frac{1}{2} \\
-\frac{1}{2} & \phi \leq -\frac{1}{2}
\end{cases}
\]

Similarly, the relationship between the viscosity and the index function \( \phi \) within the transitional interface is as follows:

\[
\mu = \frac{1}{\mu_L} \left[ \frac{1}{2} - H(\phi) \right] + \frac{1}{\mu_G} \left[ \frac{1}{2} + H(\phi) \right]
\]
where $\mu_G$ and $\mu_L$ denote gas phase viscosity and liquid phase viscosity, respectively. By introducing Eq. (6), the following advection equation of $\phi$ is obtained from Eq. (1).

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0 \hspace{1cm} (9)$$

Equation (9) instead of Eq. (1) is computed. This replacement gives a remarkable advantage to compute iso-density, immiscible, two-phase flows. In order to implement computations of two-phase flow in the presence of a magnetic field, the electric conductivity $\sigma$ is also a function of $\phi$. We define the electric conductivity in the transitional region as follows:

$$\sigma = \frac{1}{2} (\sigma_L + \sigma_G) - (\sigma_L - \sigma_G) [H(\phi)] \hspace{1cm} (10)$$

where $\sigma_G$ and $\sigma_L$ denote gas phase electric conductivity and liquid phase electric conductivity, respectively.

### 2.3. Modeling of Surface Tension

In a case of isothermal field, according to the Brackbill et al.\(^9\)

$$\mathbf{f}_w = -\gamma \kappa \frac{dH}{d\phi} \nabla \phi \hspace{1cm} (11)$$

gives the interfacial body force acting on the transitional region per unit volume. Here, $\gamma$ represents the interfacial tension, the local value of mean curvature $\kappa$ is estimated by taking the divergence of the unit vector normal to the interface within the transitional region of interface.\(^8\)

$$\kappa = \nabla \cdot \left( \frac{\nabla \phi}{|\nabla \phi|} \right) \hspace{1cm} (12)$$

The interfacial body force is zero at the place where the index function $\phi$ is constant. Therefore, the body force due to surface tension acts only within the interfacial phase in the direction normal to the interface.

### 2.4. Dimensionless Equations

Since we cannot identify the velocity of rising bubble prior to the computation, the characteristic velocity is scaled as kinematic viscosity of gas divided by the characteristic length $\mu_v/(\rho_v d)$. Dimensionless equations for solving incompressible immiscible two-phase flows in the presence of a magnetic field are summarized as follows:

$$\mathbf{\nabla} \cdot \mathbf{\tilde{U}} = 0 \hspace{1cm} (13)$$

$$\frac{\partial \mathbf{\tilde{U}}}{\partial \tau} + (\mathbf{\tilde{U}} \cdot \mathbf{\nabla}) \mathbf{\tilde{U}} = -\frac{1}{\tilde{\rho}_e} \tilde{\mathbf{p}} + \frac{1}{\tilde{\rho}_e} \left[ \tilde{\mathbf{\nabla}} (2\mu_e \tilde{E}) \right]$$

$$- G \frac{\tilde{\mathbf{e}}_z + \tilde{\mu}}{\tilde{\rho}_e} H \mathbf{a} \cdot \tilde{\mathbf{B}} - \frac{\Gamma}{\tilde{\rho}_e} \tilde{\mathbf{\nabla}} \cdot \left( \frac{\tilde{\mathbf{\nabla}} \phi}{|\tilde{\mathbf{\nabla}} \phi|} \right) \frac{dH}{d\phi} \tilde{\mathbf{\nabla}} \phi \hspace{1cm} (14)$$

where $\tilde{\mathbf{B}} = (0,0,1)$ and

$$\tilde{\mathbf{E}} = \begin{bmatrix}
\frac{\partial U}{\partial X} & \frac{1}{2} \left( \frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y} \right) & \frac{1}{2} \left( \frac{\partial U}{\partial Z} + \frac{\partial W}{\partial X} \right) \\
\frac{1}{2} \left( \frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y} \right) & \frac{\partial V}{\partial Y} & \frac{1}{2} \left( \frac{\partial W}{\partial Y} + \frac{\partial V}{\partial Z} \right) \\
\frac{1}{2} \left( \frac{\partial U}{\partial Z} + \frac{\partial W}{\partial X} \right) & \frac{1}{2} \left( \frac{\partial W}{\partial Y} + \frac{\partial V}{\partial Z} \right) & \frac{\partial W}{\partial Z} 
\end{bmatrix}$$

$$\frac{\partial \phi}{\partial \tau} + \tilde{\mathbf{U}} \cdot \mathbf{\nabla} \phi = 0 \hspace{1cm} (15)$$

$$\tilde{\mathbf{j}} = \sigma_e (- \tilde{\mathbf{\nabla}} \Phi + \tilde{\mathbf{U}} \times \tilde{\mathbf{B}}) \hspace{1cm} (16)$$

$$\tilde{\mathbf{v}} \cdot \tilde{\mathbf{j}} = 0 \hspace{1cm} (17)$$

The dimensionless physical properties are defined as follows:

$$\tilde{\rho} = \frac{\rho}{\rho_G} = \frac{1}{1 + \frac{1}{2} - H(\phi) + \frac{1}{2} + H(\phi)} \hspace{1cm} (18)$$

$$\tilde{\mu} = \frac{\mu}{\mu_G} = \frac{1}{1 + \frac{1}{2} - H(\phi) + \frac{1}{2} + H(\phi)} \hspace{1cm} (19)$$

$$\tilde{\sigma} = \frac{\sigma}{\sigma_L} = \frac{1}{2} \left( 1 + \tilde{\sigma} - (1 - \tilde{\sigma}) H(\phi) \right) \hspace{1cm} (20)$$

Here, the dimensionless variables are defined as follows:

$$\mathbf{\tilde{U}} = \frac{\mathbf{\tilde{u}}}{\rho_G (\rho_v d)}, \hspace{0.5cm} P = \frac{p}{\rho_G (\rho_v d)^2}, \hspace{0.5cm} \tau = \frac{1}{(\rho_v d)^2/\mu_G},$$

$$\bar{\Psi} = \frac{\psi}{(\rho_G b_0/\rho_v)}, \hspace{0.5cm} \tilde{B} = \frac{b}{b_0}, \hspace{0.5cm} \tilde{\mathbf{j}} = \frac{\mathbf{j}}{(\rho_G \mu_G b_0/\rho_v d)} \hspace{1cm} (21)$$

The dimensionless numbers are as follows:

$$Ha = \frac{\sigma_L}{\mu_G} b_0 d, \hspace{0.5cm} G = \frac{\mu_G d^3}{\rho_G^2}, \hspace{0.5cm} \Gamma = \frac{\gamma \rho_G d}{\mu_G},$$

$$\tilde{\rho} = \frac{\rho L}{\rho_G}, \hspace{0.5cm} \tilde{\mu} = \frac{\mu_L}{\mu_G}, \hspace{0.5cm} \tilde{\sigma} = \frac{\sigma_L}{\sigma_G}, \hspace{0.5cm} H = \frac{h}{d}, \hspace{0.5cm} L = \frac{l}{d} \hspace{1cm} (22)$$

where $b_0$ is the magnitude of the magnetic flux density. The characteristic length is the initial bubble diameter $d$.

The parameters of the present computations are the Galilei number $G$, the Hartmann number $Ha$, the dimensionless tension number $\Gamma$, and three physical property ratios of density, viscosity and conductivity. The electric conductivity of air is nearly equal to zero and therefore the definition of conductivity ratio and the Hartmann number is defined as shown above.
2.5. Numerical Method

The set of partial differential Eqs. (13) to (20) are discretized to finite difference equations on a staggered mesh system and solved numerically with a finite difference method. The pressure field is obtained with an iterative procedure by the HSMAC method. The electric potential field is obtained by use of the same procedure. The inertial term in Eq. (14) and the advective term in Eq. (15) are approximated with a third-order upwind scheme (UTOPIA). During the computation of Eq. (15), the value of index function are overwritten with 0.5 or \( \frac{1}{H^{1002}} \) if it is greater than 0.5 or less than \( \frac{1}{H^{1002}} \), respectively.

Table 1 shows the dimensionless numbers for the computations in this study. This problem has many dimensionless numbers, thus it is difficult to cover wide range of the parameter space. Therefore, the dimensionless numbers except Hartmann number are fixed.

| Dimensionless numbers | Values | Definitions |
|-----------------------|--------|-------------|
| Hartmann number, \( Ha \) | 0, 25, 50, 75, 100, 200 | \( Ha = \frac{\mu_0 H}{\rho u_d d} \) |
| Galilei number, \( G \) | \( 4.0 \times 10^4 \) | \( G = \frac{\rho u_d d}{\mu_0} \) |
| Tension number, \( T \) | \( 2.0 \times 10^4 \) | \( T = \frac{\gamma \theta_0 d}{\mu_0^2} \) |
| Magnetic Reynolds number, \( Rm (Ha=100) \) | \( 8.6 \times 10^{10} \) | \( Rm = \mu_0 \sigma_\epsilon w_{tr} d \) |
| Density ratio, \( \rho \) | 250 | \( \rho = \frac{\rho_g}{\rho_0} \) |
| Viscosity ratio, \( \mu \) | 100 | \( \mu = \frac{\mu_g}{\mu_0} \) |
| Conductivity ratio, \( \sigma \) | \( 2.5 \times 10^{-7} \) | \( \sigma = \frac{\sigma_g}{\sigma_0} \) |
| Dimensionless height of enclosure, \( H \) | 6 | \( \frac{Height \ of \ enclosure}{Initial \ bubble \ diameter} \) |
| Dimensionless width of enclosure, \( L \) | 2 | \( \frac{Length \ of \ enclosure}{Initial \ bubble \ diameter} \) |

\( \mu_\epsilon \): Permeability of free space, \( w_{tr} \): Terminal velocity of the bubble

2.6. Initial and Boundary Conditions

The initial conditions are as follows:

\[
\bar{U} = 0 \quad \text{for all the computational domain}
\]

\[
\phi = \begin{cases} 
\frac{1}{2} & \text{for } \phi_0 \geq 0.3 \\
\phi_0 & \text{for } -0.3 < \phi_0 < 0.3 \\
-\frac{1}{2} & \text{for } \phi_0 \leq -0.3 
\end{cases} \quad \text{(gas phase)}
\]

\[
\phi = \frac{1}{\alpha} \tan^{-1} \left[ \frac{1}{\alpha} \left( \frac{X-1}{R} \right)^2 + \left( \frac{Y-1}{R} \right)^2 + \left( \frac{Z-1}{R} \right)^2 - 1 \right] \]

Here, \( R \) represents the dimensionless radius of the initial bubble. The thickness of the interface is proportional to \( \alpha \). In this calculation, \( R \) is 1/2 and \( \alpha \) is 1/7. Extensive trial-and-error tests to obtain the best conservation of the bubble volume during the computation were performed. The value of \( \alpha \) was determined to be 1/7.

The boundary conditions for velocity, electric current density, and index function are summarized as follows:

\[
\bar{U} = 0 \quad \text{at all six walls (No-slip condition for viscous fluid)}
\]

\[
J_z = 0 \quad \text{at all six walls (Condition for electrically insulating wall)}
\]

\[
\frac{\partial \phi}{\partial n} = 0 \quad \text{at all six walls (Supposition of 90 degree contact angle between interface and the wall)}
\]

where \( n \) denotes the normal component or the normal direction. The boundary conditions for the pressure and electric potential are not required for the computations using the HSMAC method.

Fig. 2. Instantaneous profiles of the bubbles at \( \tau = 0.003 \) and 0.03 (at \( \tau = 0.03 \) only for \( Ha = 200 \)). A uniform vertical magnetic field is applied. The number of meshes is \( 60 \times 60 \times 180 \).
3. Numerical Results

3.1. Overview of Computational Results

Figure 2 shows the computational results for various Hartmann numbers from 0 to 200. The number of meshes is $60 \times 60 \times 180$. The lower and upper bubbles in each enclosure correspond to their locations at $\tau=0.003$ and 0.03, respectively, except for the case of $Ha=200$ in which only $\tau=0.03$ is drawn to avoid overlap of the two bubbles. With increase in the strength of magnetic field, the shape of bubble tends to become longer and the rising velocity is substantially retarded.

3.2. Distribution of Velocity and Electric Current Density

Figure 3 shows an instantaneous distribution of velocity vectors when the bubble passes through the middle plane $Z=3$ of the enclosure for $Ha=0$, 25 and 200. The vectors in $Ha=200$ are scaled by five times as large as those in $Ha=0$ and $Ha=25$.

![Instantaneous distribution of velocity vectors](image)

Fig. 3. Instantaneous distribution of velocity vectors when the bubble passes through the middle plane $Z=3$ of the enclosure for $Ha=0$, 25 and 200. The vectors in $Ha=200$ are scaled by five times as large as those in $Ha=0$ and $Ha=25$.

3.3. Bubble Shape and Pressure Field

Figure 7 shows the pressure distribution around the bubble in the cross-section $X=1$ and the surface of the bubble for $Ha=200$ during transition period to the terminal velocity. Low-pressure areas exist in the upper and the lower parts of the bubble surface. On the other hand, the pressure in the side part of the bubble surface is higher than the upper and the lower-part pressure. Therefore, the bubble is deformed into a long spheroidal shape in the direction of the Z-axis (vertical direction).

3.4. Rising Velocity of the Bubble

Figure 8 shows the transient response of rising velocity of the bubble for typical magnetic field strengths $Ha=0$ and $Ha=100$. Each bubble velocity has a flat part as shown in Figs. 8(a) and 8(b). Therefore, the bubble reaches enough to the terminal velocity in this enclosure. In $Ha=200$, the bubble does not exhibit any flat part of the rising velocity since the bubble begins to decelerate before reaching a terminal velocity.
Figure 9 shows the bubble terminal velocity in each magnetic field strength. According to Chester,1) Chang,2) and Ueno and Yasuda,3) the terminal velocity of a solid spherical body is $\frac{5}{3}\times 10^4 Ha^{-1}$ when there is no appreciable interaction between the moving body and the enclosure, i.e. the enclosure is much larger than the body size. The curve in Fig. 9 shows the theoretical curve obtained for the solid spherical body. For the high Hartmann number cases, the tendency of the present results and the theory agrees qualitatively, but there is a certain difference in their values. This difference is possibly attributed to the influence of the top and the bottom walls of enclosure as well as the influence of deformation of the bubble. The terminal velocity is considerably weakened under the strong magnetic field compared with the case of no magnetic field.

Apart from the case of the strong magnetic field, the bubble terminal velocity in the range of $0 < Ha < 75$ is slightly larger than the case without the magnetic field. When the Hartmann number is small, the effect of the separation is predominant for the drag. On the other hand, when the Hartmann number is 100 or larger, the MHD braking effect is predominant. In the range of $0 < Ha < 75$, these two effects are in competition to control the flow field. In that case, the magnetic field has not only the immediate MHD braking effect but also the separation suppression effect. Hence, the flow in the range of $0 < Ha < 75$ is quite complex, and thus the relationship of the Hartmann number and the drag reduction has not been clarified yet. Further research is necessary for the rising velocity of the bubble when the magnetic field is applied.

4. Conclusions

The electric current circulates axisymmetrically with re-
spect to the Z-axis and therefore, the resulting Lorentz force in liquid phase acts in radial direction in the X–Y cross-section. This Lorentz force controls the flow of the liquid metal. The separation of flow behind the bubble tends to disappear as the magnetic field is higher. Moreover, at the

Fig. 6. Instantaneous distribution of velocity $W$ around the bubble in the cross-section of $X=1$.

Fig. 7. Instantaneous distribution of pressure around the bubble in the cross-section of $X=1$ and the surface of the bubble for $H_a=200$ ($\tau=0.008$, $\tau=0.028$ and $\tau=0.038$).

Fig. 8. The transient response of rising velocity of the bubble for typical magnetic field strength, (a) $H_a=0$ and (b) $H_a=100$.

Fig. 9. The bubble terminal velocity in each magnetic field strength.
same time, a remarkable change on the pressure distribution takes place because of the Lorentz force.

During transition period to the terminal velocity, the bubble has a relatively low-pressure area in the upper and the lower parts of the bubble surface when the magnetic field is strong. Therefore, the bubble is elongated in the bubble-moving direction.

The rising velocity of the bubble for $0 < Ha < 75$ is slightly larger than that of $Ha = 0$. On the other hand, for $Ha > 75$, the bubble rising velocity decreases monotonously with the increase in the Hartmann number because of the deceleration effect on the fluid flow by the magnetic field (MHD braking effect).

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