We examine the calculation of the solar neutrino flux based on the standard solar model (SSM). It is found that the solar neutrino data (KAMIOKANDE experiment) can be well described by the SSM with careful employment of nuclear data of $^7\text{Be}(p, \gamma)^8\text{B}$. The main point is that the simple-minded product ansatz of Coulomb plus nuclear parts should have a few percent uncertainties which induce the large reduction of the neutrino flux from $^8\text{B}$. Also, if the electron capture of $^7\text{Be}$ inside the sun is suppressed, then the GALLEX experiment can be understood by the SSM calculation.
1. Introduction

The solar neutrino problem is a long standing puzzle. The discrepancy between theoretical predictions of the neutrino flux by the standard solar model (SSM) and the observed data is still believed to be a factor of 2 or more [1]. This problem, however, has produced many different kinds of refinements of solar internal structure model as well as new ideas in neutrino physics such as neutrino oscillations [2].

In this paper, we reexamine the calculation of the standard solar model by carefully considering the nuclear reaction data. To this claim, we may face criticisms that the nuclear reaction parts must have already been examined very carefully by all of the previous calculations. This is certainly right. The nuclear reaction data have been improved a lot and only those refined data have been employed.

However, there is one important point which is required to reconsider in the previous calculations. That is, the Coulomb part calculated by the WKB method. The Coulomb coefficients can be calculated quite reliably if it is only one body problem. However, if it involves many body nature in the nuclear reaction, it is not very clear to what accuracy one can believe the WKB results even though we know that they cannot be very bad.

Also, one knows in nuclear physics that the Coulomb problem is not as simple as one at first thinks. The Nolen-Schiffer anomaly is a good example [3]. The Coulomb displacement energy is not well described if one wants to discuss it to a very high accuracy [4].

Here, the problem is that the high energy neutrino flux is very sensitive to the Coulomb coefficients. In fact, the few percent change of the Coulomb coefficients may sometimes induce a large effect on the neutrino flux, leaving most of the solar structure quantities unchanged. In particular, the nuclear reaction data of $^7Be(p, \gamma)^8B$ is most sensitive to the high energy part of the solar neutrino flux. As we will see below, a few percent increase of the Coulomb coefficient is enough to reduce the neutrino flux by a factor of 5.
Furthermore, the choice of the new Coulomb coefficient is perfectly consistent with the existing reaction data of $^7Be(p, \gamma)^8B$ [5]. At the same time, we reproduce all of the physical quantities of the solar internal structure at the same level of accuracy as the previous calculations of SSM.

To summarize our results of the neutrino flux, we obtain the following neutrino capture rates for GALLEX [6], KAMIOKANDE [7], SAGE [8] and Homestake (Davis et al. [9]). Here, BP95 and DS96 denote the recent calculations by Bahcall and Pinsonneaul [10], and Dar and Shaviv [11], respectively. We present the two different calculations (Case I and Case II) which will be explained below in detail.

| Neutrino Flux | Present cal. | BP95 | DS96 | Experiment |
|---------------|-------------|------|------|------------|
|               | I           | II   | BP95 | DS96       | Experiment |
| Homestake(SNU)| 4.5         | 3.4  | 9.3 ± 1.4 | 4.1 ± 1.2  | 2.55 ± 0.17 ± 0.18 |
| KAMIOKANDE ($10^6$cm$^{-2}$sec$^{-1}$)| 2.9 | 1.9  | 6.62 | 2.49 | 2.73±0.17±0.34 |
| GALLEX(SNU)   | 116         | 114  | 137±8 | 115±6 | 77.1±8.5$^{+4.4}_{-5.1}$ |
| SAGE(SNU)     | 116         | 114  | 137±8 | 115±6 | 69±10$^{+5}_{-7}$ |

2. The Standard Solar Model

The internal structure of the sun is by now described reasonably well by the standard solar model. The chain of nuclear reactions is well understood. The description of the sun reduces to several couples of differential equations which should be solved mostly by numerical calculations. Among the
parameters that enter in the equations, the opacity coefficient must be most ambiguous. However, recent studies to refine the SSM enable us to remove the ambiguity of the opacity fairly well. In connection with the solar neutrino problems, the ambiguity of the opacity may lead to a correction of a few tens of percents to the neutrino flux. In this respect, we have only a very little freedom left for neutrino flux [1].

The energy of the sun is governed by the nuclear reaction cross sections. The energy production rate $\epsilon_{12}$ for $1 + 2 \rightarrow 3 + 4 + Q$ reaction is described by

$$\epsilon_{12} = \frac{QN_1N_2 <\sigma v>}{(1 + \delta_{12}) \rho} \text{ erg/g \cdot s}$$

where $N_1$ and $N_2$ are the number of particles in the reactions.

Further, the $<\sigma v>$ can be parametrized for nuclear reactions in the following way except $^7Be(e, \nu_e)^7Li$ reaction,

$$N_A <\sigma v> = C_1 T_9^{-\frac{2}{3}} \exp \left(-C_2 T_9^{-\frac{1}{3}} - \left(T_9/T_0\right)^2\right)$$

$$\times \left(1 + C_3 T_9^{\frac{1}{3}} + C_4 T_9^{\frac{2}{3}} + C_5 T_9 + C_6 T_9^{\frac{4}{3}} + C_7 T_9^{\frac{5}{3}}\right) + C_8 T_9^{-\frac{8}{3}} \exp \left(-C_9 T_9^{-1}\right)$$

(2.1)

For the $^7Be(e, \nu_e)^7Li$ reaction, we employ the following form,

$$N_A <\sigma v> = 1.34 \times 10^{10} T_9^{-\frac{1}{2}}$$

$$\times \left(1 - 0.537 T_9^{\frac{1}{3}} + 3.86 T_9^{\frac{2}{3}} + 1.2 T_9 + 0.002 T_9 \exp \left(\frac{0.002515}{T_9}\right)\right)$$

(2.2)

Here, $N_A$ denotes Avogadro number. $T_9$ is measured by $10^9$ K.

The values of the parameters $C_1, ..., C_9, T_0$ are determined from the nuclear reaction data and are listed in ref.[1,12].

The temperature $T$ and the density $\rho$ of the sun are determined by solving the following coupled equations,
\[
\frac{dP}{dr} = -\frac{GM\rho}{r^2} \quad (2.3a)
\]
\[
\frac{dM}{dr} = 4\pi r^2 \rho \quad (2.3b)
\]
\[
\frac{dL}{dr} = 4\pi r^2 \rho \epsilon \quad (2.3c)
\]
\[
\frac{dT}{dr} = -\frac{3\kappa L}{16\pi acr^2 T^3} \quad \text{for radiative} \quad (2.3d)
\]
\[
\frac{dT}{dr} = \frac{1}{(n + 1)_{ad}} \frac{T}{P} \frac{dP}{dr} \quad \text{for convective} \quad (2.3e)
\]

where \(P, M\) and \(L\) denote the pressure, the interior mass and the luminosity, respectively. Also, \(a\) and \(c\) are radiation density constant and the velocity of light. Further, \(\kappa\) and \((n + 1)_{ad}\) denote the opacity and adiabatic coefficient, respectively.

3. \(^7Be(p, \gamma)^8B\) reaction

Now, we want to discuss the nuclear reaction \(^7Be(p, \gamma)^8B\) since this is obviously the most important reaction to produce high energy neutrinos. In particular, we want to focus on the penetration factor \(P_{\text{Coul}}\). This is expressed in terms of the WKB calculation as

\[
P_{\text{Coul}} = \frac{C_0}{\sqrt{E}} \exp(-2\pi \eta) \quad (3.1)
\]

where \(C_0\) is a constant and \(\eta\) can be described as
\[ \eta = \frac{Z_1 Z_2 e^2}{\hbar v} = \frac{Z_1 Z_2 e^2 \sqrt{\mu}}{\hbar \sqrt{2}} \frac{1}{\sqrt{E}} \] (3.2)

where \( \mu \) denotes the reduced mass of the interacting particles. To the first order approximation, one may assume that the cross section can be described as a product of \( P_{\text{Coul}} \) and the nuclear part \( P_{\text{Nucl}} \) that is connected to the probability to make nuclear reactions.

\[ \sigma(E) = \frac{1}{\nu} P_{\text{Coul}} P_{\text{Nucl}} = \frac{S(E)}{E} \exp(-2\pi\eta) \] (3.3)

where \( S(E) \) is a nuclear spectroscopic factor.

Now, the question is to what accuracy we can believe the product ansatz of eq.(3.3) even though the WKB estimation is taken to be reliable. This is connected to the fact that the nuclear reaction of \(^7\text{Be}(p, \gamma)^8\text{B}\) should be treated as a many body problem. Recent calculations by Brown et al.[13] show rather a large value of \( S(0) \).

On the other hand, Xu et al. [14] claim that the \( S(0) \) value extracted from \(^8\text{B} \rightarrow p + Be \) decay vertex constant is consistent with the observed value of Filippone et al. \((S(0) \sim 17.5 \text{ eV}) \) [5]. Thus, it is still far beyond determining the Coulomb coefficient to the accuracy of a few percent in the realistic nuclear many body calculations.

Here, we do not want to rely on the simple-minded product ansatz of eq.(3.3). Instead, we assume the following form for \( \sigma(E) \),

\[ \sigma(E) = \frac{B}{E} \exp \left[ -\frac{A}{\sqrt{E}} \right] \] (3.4)

where \( A \) and \( B \) are free parameters which should be determined by reproducing the nuclear reaction data. We stress that our aim is not to reproduce theoretically the cross section data, but to find out some parameter sets that reproduce the observed cross section [5].

In fig.1, we show the comparison of the observed cross section of \(^7\text{Be}(p, \gamma)^8\text{B}\) with that reproduced by eq.(3.4) with two choices of the parameter set \( A \).
and $B$. The first case (Case I) is the best fit to the nuclear cross section with the fixed value of $A$ which is estimated by the WKB method. They are $A = 4.70 \times 10^{-3}$ erg$^{1/2} \text{ and } B = 17.5 \text{ eV}$. In the second case (Case II), we make the best fit to the nuclear cross section varying the values of the parameters $A$ and $B$ freely. We find that the best fit values of $A$ and $B$ are $A = 4.80 \times 10^{-3}$ erg$^{1/2} \text{ and } B = 20.0 \text{ eV}$.

As can be seen, there are obviously some ambiguities which arises from the difficulty of the Coulomb cross sections once we want to understand it to a very high accuracy. With these two cases of the parameters, we can calculate the neutrino flux in the sun.

4. The solar structure

In the previous section, we have determined the parameters of the cross section $< \sigma v >$ for the $^7\text{Be}(p, \gamma)^8\text{B}$ reaction. For other reaction cross sections, we have used the same values of parameters as those used in the calculation of Bahcall et al [15].

Here, we want to show our calculated result of the solar structure quantities. In fig.2, we show the luminosity and the temperature of the sun as the function of the solar radius. The solid lines are the calculated results where the reaction cross section of $^7\text{Be}(p, \gamma)^8\text{B}$ is used with the parameters $A$ and $B$ (Case I) as determined above. All the other nuclear data are the same as those used in the calculations of Bahcall et al. On the other hand, the dashed lines indicate the calculated luminosity and temperature by Bahcall et al. As can be seen from these figures, the shape of the luminosity and the temperature are almost the same between the two calculations.

Therefore, we can conclude that the solar structure quantities are not so much influenced by the change of nuclear reaction data of $^7\text{Be}(p, \gamma)^8\text{B}$, as expected.
5. The neutrino flux

Since we know now how many reactions occur inside the sun, we can calculate the neutrino flux.

In table 1, we show the neutrino fluxes as well as the capture rates at the Earth for GALLEX, KAMIOKANDE and Homestake experiments. In table 1a, we show the calculated results by Bahcall et al. [15] while, in table 1b, the calculations by Dar and Shaviv [11] are shown. In table 1c, we show our calculated results with the Case I while, in table 1d, the results with the Case II are shown.

As can be seen from the table 1, the present calculations with the Case I are very similar to the ones by Dar and Shaviv. Therefore, it is confirmed that the KAMIOKANDE experiment is indeed consistent with the SSM calculations with the careful employment of the nuclear reaction cross section of $^7\text{Be}(p, \gamma)^8\text{B}$.

Further, the case II indicates that the ambiguity of the coulomb coefficient is so large that one has to be very careful for drawing any conclusions on the solar neutrino problems. At least, the result of the case II suggests that, once the $^7\text{Be}$ neutrino flux is suppressed, then there is a fairly good chance that all the neutrino experiments fall into the range of the SSM predictions.

For the Case II, one sees that the cross section of $^7\text{Be}(p, \gamma)^8\text{B}$ is best fitted. Here, the Coulomb coefficient is slightly different from the WKB value. In this parameter set, we find that the neutrino flux for KAMIOKAMDE is a little bit too small compared to the data. Instead, the Homestake and GALLEX experiments will be in the range of the present calculation once the $^7\text{Be}$ neutrino flux is suppressed.

Also, in the Case II, the $S(0)$ value is found to be $S(0) = 20\ eV$. This suggests that the $S(0)$ factor depends on the factorization ansatz of eq.(3.3).
6. Conclusions

We have presented a new calculation of the standard solar model with the emphasis on the careful considerations of the nuclear reactions of $^7Be(p, \gamma)^8B$.

We show here that the solar neutrino capture rates are consistent with the observed data for the KAMIOKANDE experiments. We believe that possible refinements may improve the accuracy of the neutrino capture rates by $20 \sim 30\%$ so that the GALLEX experiments may well be in the range of the SSM picture. In particular, the suppression of the $^7Be$ electron capture inside the sun will lead to the understanding of the GALLEX and Homestake experiments in a natural way.

Therefore, we conclude that the solar neutrino fluxes are mostly consistent with the standard solar model with careful considerations of the nuclear reactions of $^7Be(p, \gamma)^8B$.

In the course of the present study, we received a preprint of the new calculation by Dar and Shaviv which shows very similar results to the present calculations. This confirms that the present result does not so much depend on the modeling of the sun as far as we take into account the gross structure of the sun.

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Table captions

Table 1: We plot the calculated neutrino flux from various nuclear reactions together with the experiments. Table 1a shows the calculation by Bahcall and Ulrich [15] while Table 1b plots the calculation by Dar and Shaviv [11]. Tables 1c and 1d are the present calculations with the parameter sets of Case I and Case II, respectively.

Figure captions

Fig.1: We show the nuclear cross section of $^7Be(p,\gamma)^8B$. The black circles with error bars are the observed data by Filippone et al [5]. The solid line is our calculation with the Case I parameters while the dashed line with the Case II parameters.

Fig.2: The properties of the internal structure of the sun are shown as the function of the radius. The solid lines show the present calculations while the dashed lines the ones by Bahcall et al [15]. The $L$, $M$, $T$, $\rho$ and $P$ denote the luminosity, the mass, the temperature, the density and the pressure of the sun.
### Table 1a

| Source | Flux (cm⁻²·sec⁻¹) | Homestake (SNU) | GALLEX (SNU) | SAGE (SNU) | KAMIOKANDE (10⁶ cm⁻²·sec⁻¹) |
|--------|------------------|----------------|-------------|------------|-----------------------------|
| pp     | 6.0 × 10¹⁰       | —              | 70.8        | 70.8       | —                           |
| pep    | 1.4 × 10⁸        | 0.2            | 3.0         | 3.0        | —                           |
| ⁷Be    | 4.7 × 10⁹        | 1.1            | 34.3        | 34.3       | —                           |
| ⁸B     | 5.8 × 10⁶        | 6.1            | 14.0        | 14.0       | 5.8                         |
| ¹³N    | 6.1 × 10⁸        | 0.1            | 3.8         | 3.8        | —                           |
| ¹⁵O    | 5.2 × 10⁸        | 0.3            | 6.1         | 6.1        | —                           |
| Total  |                  | 7.9            | 132         | 132        | 5.8                         |
| Experiment |              | 2.55 ± 0.25    | 77.1 ± 8.5±4.4 | 69 ± 10±5 | 2.73 ± 0.17 ± 0.34 |

### Table 1b

| Source | Flux (cm⁻²·sec⁻¹) | Homestake (SNU) | GALLEX (SNU) | SAGE (SNU) | KAMIOKANDE (10⁶ cm⁻²·sec⁻¹) |
|--------|------------------|----------------|-------------|------------|-----------------------------|
| pp     | 6.1 × 10¹⁰       | —              | 72.0        | 72.0       | —                           |
| pep    | 1.43 × 10⁸       | 0.20           | 3.06        | 3.06       | —                           |
| ⁷Be    | 3.71 × 10⁹       | 0.87           | 27.1        | 27.1       | —                           |
| ⁸B     | 2.49 × 10⁶       | 2.62           | 6.01        | 6.01       | 2.49                        |
| ¹³N    | 3.82 × 10⁸       | 0.06           | 2.38        | 2.38       | —                           |
| ¹⁵O    | 3.74 × 10⁸       | 0.22           | 4.39        | 4.39       | —                           |
| Total  |                  | 4.1            | 115         | 115        | 2.49                        |
| Experiment |              | 2.55 ± 0.25    | 77.1 ± 8.5±4.4 | 69 ± 10±5 | 2.73 ± 0.17 ± 0.34 |
Table 1c

| source | Flux (cm$^{-2}$sec$^{-1}$) | Homestake (SNU) | GALLEX (SNU) | SAGE (SNU) | KAMIOKANDE (10$^6$cm$^{-2}$sec$^{-1}$) |
|--------|---------------------------|-----------------|--------------|------------|-------------------------------------|
| $pp$   | $5.7 \times 10^{10}$       | —               | 67.3         | 67.3       | —                                   |
| $pep$  | $1.4 \times 10^8$          | 0.20            | 3.00         | 3.00       | —                                   |
| $^7$Be | $4.7 \times 10^9$          | 1.10            | 34.3         | 34.3       | —                                   |
| $^8$B  | $2.9 \times 10^6$          | 3.05            | 7.00         | 7.00       | 2.9                                 |
| $^{13}$N | $3.7 \times 10^8$      | 0.06             | 2.30         | 2.30       | —                                   |
| $^{15}$O | $2.2 \times 10^8$     | 0.13             | 2.58         | 2.58       | —                                   |
| Total  |                            | 4.5             | 116          | 116        | 2.9                                 |
| Experiment |                    | $2.55 \pm 0.25$ | $77.1 \pm 8.5^{+4.4}_{-5.4}$ | $69 \pm 10^{+5}_{-7}$ | $2.73 \pm 0.17 \pm 0.34$ |

Table 1d

| source | Flux (cm$^{-2}$sec$^{-1}$) | Homestake (SNU) | GALLEX (SNU) | SAGE (SNU) | KAMIOKANDE (10$^6$cm$^{-2}$sec$^{-1}$) |
|--------|---------------------------|-----------------|--------------|------------|-------------------------------------|
| $pp$   | $5.7 \times 10^{10}$       | —               | 67.3         | 67.3       | —                                   |
| $pep$  | $1.4 \times 10^8$          | 0.20            | 3.00         | 3.00       | —                                   |
| $^7$Be | $4.7 \times 10^9$          | 1.10            | 34.3         | 34.3       | —                                   |
| $^8$B  | $1.9 \times 10^6$          | 1.95            | 4.47         | 4.47       | 1.9                                 |
| $^{13}$N | $3.7 \times 10^8$      | 0.06             | 2.30         | 2.30       | —                                   |
| $^{15}$O | $2.2 \times 10^8$     | 0.13             | 2.58         | 2.58       | —                                   |
| Total  |                            | 3.4             | 114          | 114        | 1.9                                 |
| Experiment |                    | $2.55 \pm 0.25$ | $77.1 \pm 8.5^{+4.4}_{-5.4}$ | $69 \pm 10^{+5}_{-7}$ | $2.73 \pm 0.17 \pm 0.34$ |
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