Decentralized Control of Multiple Strict-Feedback Systems With Unknown Control Directions

JUNMIN PENG, SHENPING XIAO, AND CHAO HUANG

Corresponding author: Shenping Xiao (xsp@hut.edu.cn)

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ABSTRACT This paper is concerned with the decentralized control problem of networked high-order nonlinear systems that can be transformed into strict feedback forms with parameter uncertainties and unknown control directions under a directed communication graph. A decentralized controller is designed recursively for each agent to realize output synchronization and guarantee the overall system to be bounded. Instead of a Lyapunov-based argument which is commonly used in the literature, an inductive contradiction argument is employed. Moreover, not like other existing works, we use the Nussbaum function in the analysis function form rather than its derivative, which not only facilitates the proof but also enlarges the schedule’s application scope. Simulation results are presented to verify the effectiveness of the proposed scheme.

INDEX TERMS Unknown control direction, output synchronization, strict feedback, Nussbaum.

I. INTRODUCTION

Due to its wide application, collaborative control of networked systems has attracted considerable attention among the control communities since the 21st century [1], [2]. The early relevant work focuses on networked agents with linear dynamics [2]–[7], or first-order nonlinear systems [8]. Currently, much attention has been paid to network agents with nonlinear dynamics, such as fractional-order systems [9], chained-form systems [10], stochastic systems [11], lower-triangular systems [12], nontriangular systems [13], Euler-Lagrange systems [14] and systems with quasi-one-sided Lipschitz nonlinear dynamics [15]. Besides, uncertainty and disturbance have been considered to reflect the actual situation of the agent, including unmeasured velocities [16], input constraints [17], unmodeled dynamics [18], [19], stochastic disturbance [20]. When the agents have high-order dynamics and uncertainty, people always change the objective from state consensus to output synchronization [21]–[23].

It is worth noting that the control direction or the sign of the high-frequency gain determines the characteristics of feedback, negative feedback, or positive feedback, which is important in controller design. Usually, it is assumed to be positive and known. However, this assumption becomes unrealistic in some cases, such as uncalibrated visual servo control in [24] and autopilot design of surface vehicles [25]. In these cases, the Nussbaum function has been proposed to solve the problem of stabilization or regulation for a single system with an unknown control direction [26]–[30]. However, rare work has been done for multiple systems with unknown control directions, because it is difficult to show cooperative behavior without sacrificing distributivity. Another basic requirement is that the networked system should remain bounded. Since the entire system may have multiple Nussbaum-type functions interacting simultaneously, analysis becomes extremely difficult. Our previous work [31] has studied the consensus of multiple first-order integrators, that is, the agent has a model of \( \dot{x}_i = b_i u_i \) with unknown high-frequency gain \( b_i \). After that, we have extended the agent’s dynamic to a high-order lower-triangular system [32], but the signs of all \( b_i \) are assumed to be the same. Recently, [33] has applied the same hypothesis about the unknown control directions, but with prior knowledge of their bounds. Motivated by the above discussion, this paper aims to solve the output synchronization problem in more complex situations, i.e., networked agents with unknown control directions and uncertainty, but no additional assumptions about their control directions are required. The objective of this paper is to design a decentralized controller for each agent whose dynamic is in the form of high-order
strict feedback, with parameter uncertainty and unknown control coefficient (sign and amplitude), to achieve output synchronization, and the entire system remains bounded.

The main contributions of this brief are summarized as follows:

i. Existing works have assumed that agents have known control directions [12] or unknown but identical control directions [32], [34], [35] or unknown and non-identical control directions but with a restriction on its bounds [33]. In this paper, all these restrictions are removed, that is, these agents can have non-identical unknown control directions, and the controller is designed without prior knowledge of the \( b_i \).

ii. The analysis function is crucial for recursively designing the decentralized controller. Usually, the Nussbaum function appears in the derivative of the Lyapunov function [26]–[28]. In this paper, the Nussbaum function appears in the analysis function itself rather than its derivative, which not only facilitates proofs based on contradictions but also expands the scope of application of the schedule.

The rest of the paper is organized as follows. In Section II, we present some basic notions and preliminaries of the graph theory. Then, the problem formulation and control objective are proposed in Section III. In Section IV, the decentralized controller is proposed. A numerical example is proposed in Section V to illustrate the effectiveness of proposed decentralized controllers. Finally, the conclusion is drawn in Section VI.

II. PRELIMINARIES

A. NOTIONS

Throughout this paper, \( \mathbb{R}^{m \times n} \) denotes the family of \( m \times n \) real matrices. \( M \geq (\leq) 0 \) means that \( M \) is a positive (negative) semi-definite matrix, \( M > (\prec) 0 \) means that \( M \) is a positive (negative) definite matrix. Null(\( M \)) denotes the null space of matrix \( M \), sup(\( \cdot \)), inf(\( \cdot \)) denote the least upper bound and the greatest lower bound, respectively, and sign(\( \cdot \)) is the classical signum function. For a continuous differentiable function \( f : \mathbb{R}^n \rightarrow \mathbb{R} \), the row vector of \( \partial f / \partial x \) is \([\partial f / \partial x_1, \ldots, \partial f / \partial x_n]\).

B. GRAPH THEORY

Here, we introduce some graph terminologies that can also be found in [1]–[3]. A weighted graph is denoted by \( G = (V, E) \), where \( V = \{1, 2, \ldots, N\} \) is a nonempty finite set of \( N \) nodes, an edge set \( E \subseteq V \times V \) is used to model the communications among agents. The neighbor set of node \( i \) is denoted by \( N_i = \{j | j \in V, (i, j) \in E\} \). \( j \notin N_i \) means that there is no information flow from node \( j \) to node \( i \). A sequence of successive edges in the form \( (i, k_1), (k_1, k_2), \ldots, (m, j) \) is defined as a path from node \( i \) to node \( j \). For an undirected graph, it is said to be connected if there is a path from node \( i \) to node \( j \), for all the distinct nodes \( i, j \in V \).

A weighted adjacency matrix \( A = [a_{ij}] \in \mathbb{R}^{N \times N} \) where \( a_{ii} = 0 \) (\( \forall i \)) and \( a_{ij} > 0 \) (\( i \neq j \)) if \( (i, j) \in E \) and 0 otherwise. In an undirected graph, \( a_{ij} = a_{ji} \), where the information exchange is uniformly balanced. In what follows, we set \( a_{ij} = 1 \) when \( a_{ij} > 0 \), without loss of any generality. In addition, we define the in-degree of node \( i \) as \( d_i = \sum_j a_{ij} \) and \( D = \text{diag}(d_i) \in \mathbb{R}^{N \times N} \) is thus the in-degree matrix. Then, the Laplacian matrix of graph is \( L = D - A \). It is well-known that \( c_N \) is the null space of Laplacian matrix \( L \) when the communication graph has a spanning tree, with \( c \) is some constant and \( 1_N = [1, 1, \ldots, 1]^T \in \mathbb{R}^N \).

Barbalat Lemma: Consider the function \( f : \mathbb{R}_+ \rightarrow \mathbb{R} \). If \( f \) is uniformly continuous and \( \lim_{t \rightarrow \infty} f(t) = 0 \) exists and is finite, then, \( \lim_{t \rightarrow \infty} f(t) = 0 \).

C. NUSSBAUM-TYPE FUNCTION

A Nussbaum-type function \( N(\cdot) \) is the one with the following properties [26]

\[
\begin{align*}
\lim_{k \rightarrow \infty} \sup_{t} \frac{1}{k} \int_{0}^{t} N(\tau) d\tau &= +\infty \\
\lim_{k \rightarrow \infty} \inf_{t} \frac{1}{k} \int_{0}^{t} N(\tau) d\tau &= -\infty
\end{align*}
\]

(1)

Commonly used Nussbaum-type functions include \( e^{k^2 \cos(k)}, k^2 \sin(k) \) and \( k^2 \cos(k) \).

III. PROBLEM FORMULATION

Consider a network of \( N \) agents with the dynamic of agent \( i \) described by

\[
\begin{align*}
\dot{x}_i &= x_2 + \phi_i^T(x_i)\theta_i \\
\dot{x}_2 &= x_3 + \phi_i^T(x_1, x_2)\theta_i \\
&\vdots \\
\dot{x}_{i,n-1} &= x_n + \phi_i^T(x_{i,n-1})\theta_i \\
\dot{x}_n &= b_i u_i + \phi_i^T(x_i)\theta_i \\
y_i &= x_1, i = 1, 2, \ldots, N
\end{align*}
\]

(2)

where \( x_i = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n \), \( u_i \in \mathbb{R}, y_i \in \mathbb{R} \) are the state, input and output of agent \( i \), respectively. \( b_i \neq 0 \) is the control direction, i.e., high-frequency gain, whose sign and amplitude are both unknown. \( \theta_i \in \mathbb{R}^p \) is a vector of unknown parameters. \( \phi_i(x_1), \ldots, \phi_i(x_m) \in \mathbb{R}^p \) are sufficiently smooth known functions bounded for all \( x_1 \), where \( \bar{x}_m = [x_{i1}, \ldots, x_{im}]^T, m = 1, \ldots, n \).

Control objective: Our goal is to design \( u_i \) for all agents in graph \( G \), such that their outputs are asymptotically synchronized, i.e., \( \lim_{t \rightarrow \infty} \|y_i(t) - y_j(t)\| = 0, \forall i, j \in V \), while the overall system is guaranteed to be bounded.

Remark 1: In existing works [12], [33]–[38], the coefficients \( b_i \) (sign and amplitude) are assumed to be known or unknown but identical or some assumptions have been made on \( b_i \). In this paper, the signs of \( b_i \) can be nonidentical, which means that some agents may have positive high-frequency gains while others may have negative high-frequency gains. In the circumstance, the voltage equalization control of power source which is composed by multiple ultra-capacitors can be formulated [39], where the reversal of positive and negative
poles may happen occasionally, either by misoperation in assembling or inappropriate use.

IV. MAIN RESULT
A. DECENTRALIZED CONTROLLER DESIGN

Next, we have the following main result of this paper. For the sake of simplicity, in what follows, \( \hat{x}_i \) is used to denote the information of \( x_{jk}(\forall j \in N_i) \), i.e., \( \hat{x}_i \) is a state vector of \( x_{jk} \) for all \( j \in N_i \).

**Theorem 1:** For agent \( i \) described by (2), there is a decentralized controller

\[
u_i = N_0(s_i)x_{i,n+1}^d
\]

with the Nussbaum function \( N_0(s_i) = s_i^2 \cos(s_i) \) and \( s_i \) is updated by

\[
s_i = -x_{i,n+1}^d \xi_{in}
\]

where

\[
\begin{align*}
\xi_{i1} &= \sum_{j \in N_i} a_j(x_{i1} - x_{j1}) \\
x_{i2}^d &= -x_{i2}^d \xi_{i1} - \psi_{i1}^T \hat{\theta}_i \\
\xi_{i2} &= x_{i2} - x_{i2}^d \\
x_{i3} &= -a_2 \xi_{i2} - \phi_2 - a_{i2}^T \hat{\theta}_i - \sum_{j \in N_i} \phi_2(a_{ij}^T \hat{\theta}_j) \\
\vdots \\
x_{i,m+1}^d &= x_{i,m+1} - x_{i,m+1}^d \\
x_{i,m+1} &= -\xi_{i,m+1} - a_{im} \xi_{im} - \phi_{im} - w_{im}^T \hat{\theta}_i \\
\xi_{im} &= x_{im} - \xi_{im}^d \\
x_{i,n+1}^d &= -\xi_{i,n+1} - a_n \xi_{in} - \phi_{in} - w_{in}^T \hat{\theta}_i - \sum_{j \in N_i} \phi_{in}(a_{ij}^T \hat{\theta}_j)
\end{align*}
\]

with \( a_2, \cdots, a_n > 0 \) and \( \phi_{im}, w_{im}, \psi_{jm} \) are calculated by

\[
\begin{align*}
\phi_{i2} &= -\frac{\partial x_{i2}^d}{\partial x_{i1}} x_{i2} - \sum_{j \in N_i} a_j \frac{\partial x_{i2}^d}{\partial x_{j1}} x_{j2} + x_{i1} - \gamma_1 \frac{\partial x_{i1}^d}{\partial \hat{\theta}_i} T_{i2} \\
\phi_{im} &= -\sum_{k=1}^{m-1} a_k \frac{\partial x_{im}^d}{\partial x_{i1}} x_{i1,k+1} - \sum_{j \in N_i} \sum_{l=1}^{m-1} a_{ij} \frac{\partial x_{im}^d}{\partial x_{jl,i,l+1}} + \xi_{i,m-1} \\
\psi_{jm} &= -\gamma_1 \sum_{k=2}^{m-1} a_k \frac{\partial x_{im}^d}{\partial \hat{\theta}_i} w_{im} \xi_{ik} - \gamma_2 \sum_{j \in N_i} a_{ij} \sum_{l=2}^{m-1} \frac{\partial x_{im}^d}{\partial \hat{\theta}_j} \psi_{jm} \xi_{il} \\
\psi_{jm} &= -\gamma_1 \frac{\partial x_{im}^d}{\partial \hat{\theta}_i} \tau_{im} - \gamma_2 \sum_{j \in N_i} a_{ij} \frac{\partial x_{im}^d}{\partial \hat{\theta}_j} \psi_{jm} \xi_{im} \\
w_{i2}^T &= \psi_{i2}^T - \frac{\partial x_{i2}^d}{\partial x_{i1}} \hat{\theta}_1 \\
w_{im}^T &= \psi_{im}^T - \sum_{k=1}^{m-1} a_k \frac{\partial x_{im}^d}{\partial x_{ik}} \hat{\theta}_k
\end{align*}
\]

such that output synchronization of the network can be achieved, i.e., \( \lim_{t \to \infty} ||y_i(t) - y_j(t)|| = 0 \) and the overall system is guaranteed to be bounded provided that the digraph has a spanning tree.

**Proof:** Now, we start the step-by-step design procedure. The proof is carried out at the same time.

**Step 1:** Define

\[
V_{i1} = \frac{1}{2} x_{i1}^2
\]

The time derivative of \( V_{i1} \) along (2) is

\[
\dot{V}_{i1} = x_{i1}(x_{i2} + \psi_{i1}^T(x_{i1}\hat{\theta}_1))
\]

Rewrite (10) as

\[
\dot{V}_{i1} = x_{i1}(x_{i2} + \psi_{i1}^T(x_{i1}\hat{\theta}_1)) + x_{i1}(x_{i2} - x_{i2}^d)
\]

with \( x_{i2}^d = -\xi_{i2}^d x_{i1} - \psi_{i2}^T(x_{i1}\hat{\theta}_1) \) where \( \xi_{i1} = \sum_{j \in N_i} \gamma_1 (y_j(t) - y_{i,j}(t)) \) is the neighborhood error of agent \( i \). Define \( \hat{\theta}_1 = \theta_1 - \hat{\theta}_1 \) and \( \xi_{i2} = x_{i2} - x_{i2}^d \) (11) is then equal to

\[
\dot{V}_{i1} = -x_{i1}^2 \xi_{i1}^2 + \psi_{i1}^T \hat{\theta}_1 x_{i1} + x_{i1} \xi_{i2}
\]

**Step 2:** Note that \( \xi_{i2} = x_{i2} - x_{i2}^d \), it is shown that

\[
\dot{\xi}_{i2} = \dot{x}_{i2} - \dot{x}_{i2}^d = x_{i2} + \psi_{i2}^T \hat{\theta}_1 - \frac{\partial x_{i2}^d}{\partial \hat{\theta}_i} \hat{\theta}_1 \\
\dot{\xi}_{i2} = -a_2 \xi_{i2} - \phi_2 - a_{i2}^T \hat{\theta}_i - \sum_{j \in N_i} \phi_2(a_{ij}^T \hat{\theta}_j)
\]

with \( \phi_2, \psi_{i2}^T, \psi_{i2}^T, \tau_{i2} \) and \( \xi_{i2}^d \) are chosen as (6), and \( x_{i3} = -a_3 \xi_{i2} - \phi_2 - a_{i2}^T \hat{\theta}_i - \sum_{j \in N_i} \phi_2(a_{ij}^T \hat{\theta}_j) \). Then, (13) is reduced to

\[
\dot{\xi}_{i2} = -x_{i1} - a_2 \xi_{i2} + \xi_{i3} + a_{i3} \hat{\theta}_i + \sum_{j \in N_i} \phi_2(a_{ij}^T \hat{\theta}_j)
\]
\[ \frac{\partial \xi_m}{\partial \mathbf{t}_m} = \gamma_1 \mathbf{t}_m + \gamma_2 \mathbf{t}_2 + \frac{1}{2} \xi_m \mathbf{t}_m \]

(14)

where 41 > 0 are control gains.

Step m: Note that \( \dot{\xi}_m = x_m - x_m^d \), hence

\[ \dot{\xi}_m = \dot{x}_m - \dot{x}_m^d = x_{m,m+1} + \psi_m \mathbf{t}_m - \sum_{l=1}^{\text{m-1}} \frac{\partial \psi_m}{\partial \mathbf{t}_m} (x_{m,m+1} + \psi_{m,m} \mathbf{t}_m) \]

(17)

(\( a_{ij} \) and \( b_{ij} \) are chosen as (6) and (8).

\[ x_m = -\xi_{m,m-1} - a_m \xi_m - \psi_m - w_m \mathbf{t}_m \]

(17)

such that (17) is equal to

\[ \dot{\xi}_m = -\xi_{m,m-1} - a_m \xi_m + \xi_{m,m+1} + \psi_m \mathbf{t}_m + \frac{1}{2} \xi_m \mathbf{t}_m \]

(17)

Define

\[ V_m = \dot{V}_m = -\xi_m \mathbf{t}_m - \frac{1}{2} \xi_m \mathbf{t}_m \]

(19)

We can see that

\[ \dot{V}_m = \dot{V}_m = -\xi_m \mathbf{t}_m - \frac{1}{2} \xi_m \mathbf{t}_m \]

(20)

To obtain (21),

\[ V_m = V_{m-1} + \frac{1}{2} \xi_m \mathbf{t}_m \]

(19)

(21)

Define

\[ V_m = V_{m-1} + \frac{1}{2} \xi_m \mathbf{t}_m + \frac{1}{2} \xi_m \mathbf{t}_m \]

(19)

\[ \gamma_1 \frac{\partial x_m}{\partial \mathbf{t}_m} - \dot{\xi}_m \]

(17)

(\( \gamma_1 \) and \( \gamma_2 \) are chosen as (6) and (8).

\[ x_m = -\xi_{m,m-1} - a_m \xi_m - \psi_m \]

(17)

(\( a_{ij} \) and \( b_{ij} \) are chosen as (6) and (8).

\[ x_m = -\xi_{m,m-1} - a_m \xi_m - \psi_m - w_m \mathbf{t}_m \]

(17)

such that (17) is equal to

\[ \dot{\xi}_m = -\xi_{m,m-1} - a_m \xi_m + \xi_{m,m+1} + \psi_m \mathbf{t}_m + \frac{1}{2} \xi_m \mathbf{t}_m \]

(17)

(\( a_{ij} \) and \( b_{ij} \) are chosen as (6) and (8).

\[ x_m = -\xi_{m,m-1} - a_m \xi_m - \psi_m \]

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\[ x_m = -\xi_{m,m-1} - a_m \xi_m - \psi_m \]

(17)
\[-(\dot{s}_i + x_{i,n+1}\dot{\xi}_{in}) + b_i(u_i\dot{\xi}_{in} + N_0(s_i)\dot{s}_i)\]  \hspace{1em} (24)

with \(\phi_{in}, w_{in}^T, \psi_{jn}^{(i)}T, \tau_{in}\) and \(\sigma_{jn}^{(i)}\) are chosen as (6) and (8),
\[x_{i,n+1} = -\dot{s}_{i,n-1} - a_is_{in} - \phi_{in}w_{in}^T - \sum_{j\in N_i}a_{ij}\psi_{jn}^{(i)}\dot{\theta}_j^{(i)}.\]

Choose the control protocol \(u_i\) as (3) where \(s_i, \theta_i\) and \(\dot{\theta}_j^{(i)}\) are updated by (4) and (7). Then, (24) is reduced to
\[\dot{V}_{in} = -x_{i1}^2\dot{s}_i^2 - \sum_{k=2}^n a_k x_{ik}^2 \leq 0\]  \hspace{1em} (25)

Next, we will prove the existence of the closed-loop system solution on the time interval \([0, +\infty)\).

Let us denote by \([0, t_f]\) that the maximum interval of existence of the closed-loop system solution. First, assume on the contrary that, at \(t_f\), the state \(s_{i1}\) escapes to the positive infinity, no matter what the sign of \(b_i\) is, there is a strictly increasing, infinite sequence \(s_{i1}^1, s_{i1}^2, \cdots\), with the property \(\lim_{k\to+\infty}s_{i1}^k = t_f\), such that
\[\lim_{k\to+\infty} b_i \int_0^{s_{i1}^k} N_0(\sigma)d\sigma - s_{i1}^k = +\infty\]
which means \(\lim_{k\to+\infty}V_{in}(s_{i1}^k) = +\infty\). On the other hand, (25) shows that \(V_{in}(t) \leq 0\), which contradicts \(\lim_{t\to+\infty}V_{in}(s_{i1}^k) = +\infty\). Therefore, \(s_i\) is within a compact set on \([0, t_f]\), i.e., \(s_i\) is bounded on \([0, t_f]\). Similarly, if we assume that, at \(t_f\), the state \(s_{i1}\) escapes to the negative infinity, the same conclusion can be given due to the property of the Nussbaum function \(N_0(\cdot)\), see (1).

Second, assume on the contrary that, at \(t_f\), the state \(x_{i1}\) escapes to infinity. In that case, \(\lim_{t\to t_f}V_{in}(t) \geq \frac{1}{2}x_{i1}^2 + C_{i1}^{(i)} = +\infty\), where \(C_{i1}^{(i)}\) is some constant number, while the escape of \(x_{i1}\) to the positive infinity after a certain time \(T < t_f\), which contradicts \(\dot{V}_{in}(t) \leq 0\). Hence, \(x_{i1}\) is bounded on \([0, t_f]\). By inductive argument, \(\xi_{i2}, \cdots, \xi_{in}, \theta_i, \dot{\theta}_j^{(i)}\) are bounded on, i.e., the closed loop system is bounded. Integrating (25), one has
\[\int_0^{t_f} \dot{V}_{in}(\sigma)d\sigma = \int_0^{t_f} \left(-x_{i1}^2\dot{s}_i^2 - \sum_{k=2}^n a_k x_{ik}^2\right)d\sigma = V_{in}(t) - V_{in}(0) \leq 0\]  \hspace{1em} (26)

Moreover,
\[\int_0^{t_f} x_{i1}^2\dot{s}_i^2d\sigma \leq V_{in}(0) - V_{in}(t)\]  \hspace{1em} (27)

Hence, \(x_{i1}^2\dot{s}_i^2\) is integrable. Notice that the derivative of \(x_{i1}^2\dot{s}_i^2\) is a polynomial that composed by the states of the closed loop system. Hence, the derivative of \(x_{i1}^2\dot{s}_i^2\) is also bounded, i.e., \(x_{i1}^2\dot{s}_i^2\) is uniformly continuous. By Barbalat Lemma, one has \(\lim_{t\to+\infty}x_{i1}^2\dot{s}_i^2 = 0\), i.e., \(\lim_{t\to+\infty}x_{i1}(t) = 0\) or \(\lim_{t\to+\infty}\dot{s}_{i1}(t) = 0\). No matter which case, when the directed communication graph has a spanning tree, one has \(\lim_{t\to+\infty}\dot{s}_{i1}(t) = 0 \Rightarrow \lim_{t\to+\infty}[y_i(t) - y_j(t)] = 0, \forall i, j \in \mathcal{V}\), which completes the proof.

**Remark 2:** Compared with the existing works [32], [33], the main contribution of this paper is that, the unknown \(b_i\) in agent’s model can be with different signs without other additional assumptions. It is a significant progress for many practical problems. For example, in the circumstance, the voltage equalization control of ultracapacitor-type power source can go on wheels even in fault mode.

Usually, when combined with backstepping design, the Nussbaum item appears in the derivative of the Lyapunov function, see [26], [28], [31], [33]. But in this paper, the Nussbaum item has been added in the analysis function itself, see (22). There are two reasons for this. Firstly, it facilitates the contradiction argument when the Nussbaum item appears in the derivative of the Lyapunov function, see [26], [28], [31], [33]. But in this paper, the Nussbaum item has been added in the analysis function itself, see (22). Secondly, it is essentially an extension of the known control direction case. A stabilizer for a single system is firstly proposed in our previous work [40] in this way. To the best of our knowledge, it is the first time that this method is applied in a multi-agent system.

**Remark 3:** By applying the above scheme, the outputs \(y_i(t)\) are driven to the equilibrium eventually. If we alter the controller slightly, outputs of agents can be synchronized to other desired trajectories as well. For example, by setting \(x_{i1}^2 = -\Phi^2(\cdot)\dot{s}_{i1} - \psi_i^{(i)}\dot{\theta}_i\), then, \(y_i(t) \to y_j(t) \to \Phi(\cdot)\), where \(\Phi(\cdot)\) is the desired trajectory.

**Remark 4:** Distributed control and decentralized control are applicable to large scale systems composed by multiple subsystems. Each subsystem can only acquire partial information. The main difference lies in their control objectives. In distributed control, researchers mainly focus on subsystems’ behaviors, such as consensus or synchronization of the multi-agent system. But in decentralized control, not only subsystems’ behavior but also the overall system’s behavior are considered, just like the control objective of this paper, i.e., output synchronization as well as the boundedness of the overall system are the points of focus. Typically, in traditional decentralized control, the partial information needed for subsystem i is not formulated by a communication graph, i.e., subsystem i may receive redundant information which does not appear in its controller. Motivated by the cooperative control, in this paper, we formulate the information exchange by the graph theory, such that not only a graph condition has been added in the result but also the information subsystem i received is used in its controller.

**V. SIMULATION RESULTS**

In this section, an example is presented to verify the effectiveness of the proposed controller in Theorem 1. To this end, we consider the output synchronization problem of a group of three agents, denoted by ‘1’-‘3’ in Fig 1. For the sake of simplicity, \(a_{ij}\) are set to be 1 when \(a_{ij} > 0\). The dynamic of agent \(i\) is the same as that in [28]

\[
\begin{align*}
\dot{x}_{i1} &= x_{i2} + \theta_i \sin(x_{i1}) \\
\dot{x}_{i2} &= b_i u_i \\
y_i &= x_{i1} \\
\end{align*}
\]  \hspace{1em} (i = 1, 2, 3)
with $x_i = [x_{i1}, x_{i2}]^T$, $y_i$ and $u_i$ are the state, output and input of agent $i$, respectively. $\theta_i$ is the unknown parameter.

To be more specific, the initial values of agents are chosen as $x_1 = [1 \ 0]^T$, $x_2 = [-3 \ 0]^T$, $x_3 = [3 \ 0]^T$, and all the initial values of updaters are set to be 0. Then, we set control gains $\alpha_2 = 2$, $\gamma_1 = 0.2$, $\gamma_2 = 0.2$ and system parameters $b_1 = 1$, $b_2 = -1$, $b_3 = 2$, $\theta_1 = 0.5$, $\theta_2 = 0.25$, $\theta_3 = 0.75$. $N_0(s_i) = s_i^2 \cos(s_i)$ is chosen here as the Nussbaum function.

By Theorem 1, a decentralized controller (3) together with parameter updaters (7) are designed for each agent. The simulation results are shown in Fig. 2 to Fig. 6. It can be seen in Fig. 2 that the outputs of all agents in the network are asymptotically synchronized, i.e., $\lim_{t \to \infty} ||y_i(t) - y_j(t)|| = 0 \ (\forall i, j \in V)$. Fig. 3 shows that $x_{i2}$ ($\forall i \in V$) are bounded and Fig. 4 shows that all $s_i$ are bounded. It can be seen
in Fig. 5 and Fig. 6 that both parameter updaters, i.e., $\tilde{\theta}_i$ for agent $i$ itself and $\tilde{\theta}_i^{(1)}$ for its neighbors, are also guaranteed to be bounded. So, the overall system is bounded. Thus, the simulation results well confirm the theoretical issues in Theorem 1. Compared with the related references [32], [33], [35], the proposed controller results in faster convergence of the parameter estimators.

Our future research will focus on the convergence speed of such a networked system with time varying delays [41] and agents in non-strict feedback forms [42].

VI. CONCLUSION

This paper investigates the output synchronization problem of multiple strict feedback systems with parameter uncertainties and unknown control directions, in the circumstance of directed communication graph and agents may have different control directions, which is the first highlight of this paper. A decentralized controller is designed recursively for each agent such that output synchronization can be achieved. Meanwhile, the overall system maintains bounded. A new analysis method is proposed, in which the Nussbaum item appears in the analysis function itself directly rather than its derivative, such that the proof can be simplified significantly, which is the second highlight of the paper. The simulation example shows the efficiency of the presented scheme.

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JUNMIN PENG received the B.S. degree in automation and the Ph.D. degree in control theory and control engineering from Zhejiang University, Hangzhou, China, in 2009 and 2014, respectively. From 2015 to 2019, he was a Postdoctoral Fellow with CRRC Zhuzhou Locomotive Company Ltd., Zhuzhou. He has been with the School of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou, China, where he is currently an Associate Professor of automatic control engineering. His current research interests include nonlinear systems, networked control systems, and voltage equalization control of ultracapacitor-type power source.

SHENPING XIAO received the B.S. degree in automation from Northeastern University, Shenyang, China, in 1988, the M.S. degree in computer science from the Central South University of Forestry and Technology, Changsha, China, in 2002, and the Ph.D. degree in control theory and control engineering from Central South University, Changsha, in 2008. He has been with the School of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou, China, where he is currently a Professor of automatic control engineering. His current research interests include time-delay systems, networked control systems, and robust control in power systems.

CHAO HUANG received the B.S., M.S., and Ph.D. degrees from Zhejiang University, in 2010, 2012, and 2015, respectively, all in electrical engineering. In 2016, he was a Postdoctoral Research Fellow with the School of Engineering, Australian National University. From 2017 to 2019, he was with the School of Automation, Hangzhou Dianzi University, as a Lecturer. He is currently an Assistant Professor with the School of Electronics and Information Engineering, Tongji University. His research interests include nonlinear and adaptive control and multi-agent systems.

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