Effect of Wind Turbine Wakes on the Performance of a Real Case WRF-LES Simulation

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Abstract. The main objective of this work is to estimate how much of the discrepancy between measured and modeled flow parameters can be attributed to wake effects. The real case simulations were performed for a period of 15 days with the Weather Research and Forecasting (WRF) model and nested down to a Large-Eddy Simulation (LES) scale of ~ 100 m. Beyond the coastal escarpment, the site is flat and homogeneous and the study focuses on a meteorological mast and a northern turbine subjected to the wake of a southern turbine. The observational data set collected during the Prince Edward Island Wind Energy Experiment (PEIWEE) includes a sonic anemometer at 60 m mounted onto the mast, and measurements from the two turbines. Wake versus free stream conditions are distinguished based on measured wind direction while assuming constant expansion for the wake of the southern turbine. During the period considered the mast and northern turbine were under the southern turbine wake ~ 16% and ~ 11% of the time, respectively. Under these conditions, the model overestimates the wind speed and underestimates the turbulence intensity at the mast but not at the northern turbine, where the effect of wakes on the model error is unclear and other model limitations are likely more important. The wind direction difference between the southern and northern turbines is slightly underestimated by the model regardless of whether free stream or wake conditions are observed, indicating that it may be due to factors unrelated to the wake development such as surface forcings. Finally, coupling an inexpensive wake model to the high-fidelity simulation as a post-processing tool drives the simulated wind speeds at the mast significantly closer to the observed values, but the opposite is true at the coastal turbine which is in the far wake. This indicates that the application of a post-processing wake correction should be performed with caution and may increase the wind speed errors when other important sources of uncertainty in the model and data are not considered.

1. Introduction
Atmospheric phenomena across a wide range of spatial and temporal scales affect wind plant aerodynamics. Due to computational limitations, the majority of wind turbine wake analyses so far have relied on semi-idealized simulations where some physical processes are disregarded and a simplified version of a complex system is then simulated. For example, the widely used Dynamic Wake Meandering model (DWM, [1]) solves a steady-state approximation of the Navier-Stokes equations and then superimposes to the solution a turbulence field using stochastic methods and assuming a form for the energy spectrum [2]. On the other hand, the National Renewable
Energy Laboratory Simulator for Wind Farm Applications (SOWFA, [3]) simulates unsteady velocity fields at a high resolution, but its computational requirements preclude the simultaneous inclusion of some relevant atmospheric phenomena (e.g., cloud physics) and limit the achievable simulation time to minutes instead of hours or days.

Ongoing advances in computational resources have only recently started to allow real case (i.e., non-idealized) numerical simulations of the atmosphere to be run at spatial resolutions on the order of a typical wind turbine (WT) rotor diameter. As a result, it has become imperative to consider the effect that individual WTs have on the flow and it has also become possible to use these high resolution simulations for wake analyses. Research in this field may be conducted using the Weather Research and Forecasting (WRF) model [5] which considers the full physics of the atmosphere and can be nested down to the meso and micro scales, and even be run in Large-Eddy Simulation (LES) mode by switching off parameterizations for grid resolutions \( \leq O(10^2) \) m. Parameterizations are simplified mathematical descriptions of processes based on a set of assumptions, and used to model quantities that are not resolved under a given grid resolution. Unlike SOWFA, WRF does not include an explicit WT model and the effect of WTs on the flow is instead added in the form of a drag parameterization. In other words, individual wind turbine wakes are not modeled.

The idea of a wind farm parameterization first appeared in the context of global models to investigate the large-scale effect of wind farms on the Earth’s climate [6]. Later, their impact on the regional climate was also considered [7] and more recently, shorter-term simulations using WRF are also seeking to include a description of wind farm effects [8, 9]. The wind farm parameterization currently distributed with WRF was developed for horizontal grid sizes higher than five rotor diameters [10] and real case WRF-LES simulations at a higher spatial resolution cannot benefit from it. Research is needed to investigate how the effect of WTs on the flow can be best modeled when performing WRF-LES simulations.

As a first step in this direction, the current work seeks to determine how the absence of such a parameterization affects the model performance. This is done by quantifying the role of wakes in explaining the discrepancy between simulated and observed flow conditions. The analysis consists in comparing WRF simulated wind and turbulence with observations from a meteorological mast and a northern wind turbine which are sometimes influenced by the presence of a wind turbine to the south. Finally, we investigate whether the application of an analytical wake model as a post-processing tool applied to the simulation output improves the degree of agreement between measured and simulated values. The use of this post-processing tool starts to allow for a quantification of wake error sources versus other errors sources in the model such as the level of accuracy and detail in the lower boundary conditions.

2. Data and Methods

2.1. Measurements

The measurements were collected at the Wind Energy Institute of Canada during the Prince Edward Island Wind Energy Experiment (PEIWEEx) [11]. A period of 15 days is considered (May 11-25, 2015). Wind measurements used herein were made at a frequency of 10 Hz with a 3-D Gill Windmaster Pro sonic anemometer mounted to a meteorological mast (compliant with International Electrotechnical Commission standards) at 60 m above ground. Additionally, 10-minute mean wind speed, nacelle position and power production data from two 2-MW DeWind D9.2 turbines are used. The WT hub height is 80 m and the rotor diameter (D) 93 m. The location of the turbines and the mast is shown in Fig. 1.

2.2. Simulation

The simulation was performed by running the Advanced Research core of the WRF model for a dynamical downscaling consisting of two well differentiated phases: meso scale and micro
Figure 1: Instrument locations at the measurement site: wind turbines (circles of diameter 93 m) and meteorological mast (star). Elevation contours [m] (a) and satellite imagery from Environmental Systems Research Institute (b). Shaded areas represent wake from SWT impacting MET (a) and NWT (b).

scale. Five domains were defined, centered at the meteorological mast location (see Section 2.1) following a set of telescopic nests from 9 km to 111 m under a two-way nesting approach. The domains were configured to have 70 points in both directions and 50 vertical levels distributed every \( \sim 10 \) m within the atmospheric boundary layer. The model source code was modified to enable meso-micro scale coupling based on the potential temperature perturbation method described in [12, 13]. Previous work has shown that this modification results in an improvement to simulations of wind speed and turbulence intensity under different meteorological regimes and terrain complexities [14, 15]. Details of the implementation constitute proprietary information and cannot be disclosed. The innermost domain was run on LES mode, switching off the physical parameterizations. The other domains (including the terra incognita [27] domain at a 333 m grid resolution) were run with the YSU [16] boundary layer scheme.

Initial and boundary conditions are taken from the Climate Forecast System Reanalysis [17] with a spatial resolution of \( \sim 38 \) km. The static data for the high resolution domains are taken from the Shuttle Radar Topography Mission (at a 90 m resolution) [18] for topography and from the GlobCover [19] for land use. Other fields are taken from the default data sets provided by the model. The model was run for a 15-day period and time series were saved at three points: the northern wind turbine (NWT), the meteorological mast (MET), and the southern wind turbine (SWT), whose locations are shown in Fig. 1. The model saves historic output with a temporal frequency of 4 Hz. These values are then used to compute 10-minute averages and standard deviation, considering seven heights between \( \sim 8 \) m and \( \sim 182 \) m.

2.3. Wake Periods
To quantify the effect of WT wakes on model performance, the simulated values were compared to observations in the presence and absence of wakes. Hereinafter these periods will be referred to as “waked” and “free”, respectively. The only wake considered is that of SWT which is located \( \sim 230 \) m (\( \sim 2.5 \) D) from MET and \( \sim 715 \) m (\( \sim 7.7 \) D) from NWT.

To determine the interval of nacelle positions \( \gamma \) in which the SWT wake impacts the downstream locations considered, the wake width \( w \) at any downstream distance \( x \) is
approximated as

\[ w(x) = D + 2kx \quad (1) \]

following [20] where \( D \) is the rotor diameter of the wake-generating turbine and \( k \) the wake expansion coefficient taken here as 0.05 (i.e., 5% of wake expansion). This value was chosen based on previous recommendations for offshore sites [21] because of the predominantly offshore footprint seen in the measurements collected at the island [11].

The range of waked sectors is shown for both downstream sites in Fig. 1. For MET (Fig. 1a) this interval is \( \gamma_{SWT} \in (140^\circ, 182^\circ) \) (where \( \gamma_{SWT} \) is the SWT 10-minute mean nacelle position) and represents \( \sim 16\% \) of the entire time series. For NWT (Fig. 1b) it is \( \gamma_{SWT} \in (140^\circ, 169^\circ) \) representing \( \sim 11\% \) of the data. The simulation reproduced well the frequency of occurrence and the magnitude of wind speeds in these directional sectors, as can be seen in Fig. 2. Note that the wake of NWT on the other locations is not considered because northwesterly winds were less frequent during the experiment as shown in Fig. 2.

2.4. Post-simulation wake correction

We propose and evaluate a post-simulation wake correction method which seeks to drive the model results closer to the observations after the simulations have been conducted, for sites where wind turbines are present. The method is built on the Jensen wake formulation [20], widely used in industry and the basis of the well-known Park model [22]. Following this approach, the expected velocity deficit (VD) at a point can be estimated as

\[ VD(t) = \frac{1 - \sqrt{1 - C_T(t)}}{\left(1 + \frac{kx}{R}\right)^2} \quad (2) \]

where \( C_T \) [-] is the thrust coefficient of the WT generating the wake, \( k \) [-] the wake expansion factor (also known as decay coefficient), \( x \) [m] the downstream distance between the WT

Figure 2: Wind roses at SWT during 15-day period considered (measurement height \( z = 80 \text{ m} \)) using 10-minute mean measurements of wind speed and nacelle position in (a) and 10-minute mean simulated wind speed and direction at the grid cell containing SWT in (b). Color shading distinguishes between horizontal wind speed bins starting at 0 m s\(^{-1}\) in 5 m s\(^{-1}\) intervals. Radial axis gives normalized frequencies [%]. Dashed lines represent sectors in which MET (a) and NWT (b) are subjected to the SWT wake.
generating the wake and the location of interest, and $R \, [m]$ the WT rotor radius. We consider unsteady $C_T$ values calculated from the SWT wind speed measurements and thrust curve. A constant expansion factor is assumed as recommended for the first turbines in the row of a wind farm [23]. From Eq. (2) and a free stream velocity $U_\infty$, a velocity corrected for the wake can be estimated as $U = U_\infty (1 - VD)$.

2.5. Variables and Notation
All of the variables considered throughout the analysis are 10-minute means. We focus on quantities of relevance to wind energy such as power $P$, horizontal wind speed $U$, wind direction $\beta$, nacelle position $\gamma$, and turbulence intensity $I = \sigma_U/U$ (where $\sigma_U$ is the standard deviation of $U$ over a 10-minute period). Wind direction measurements at the turbines were not provided. We assume negligible yaw misalignment at the two turbines so that nacelle position measurements can be treated as wind direction measurements (i.e., $\beta \sim \gamma$). The predicted power for the simulations is obtained from the simulated 10-minute mean wind speeds at hub height and the turbine power curve.

The analysis is predominantly based on calculated differences $\delta$ between the simulated and measured variables such that $\delta = x_{\text{sim}} - x_{\text{obs}}$, evaluated at each 10-minute mean time stamp for which observation were available. Absolute differences $|\delta|$ are used to determine the overall magnitude of the model error, and averages over a given time series are symbolized as $\bar{\delta}$ so that a mean absolute error is $|\bar{\delta}|$. The subscripts “w” and “f” (i.e., $\delta_w$ versus $\delta_f$) differentiate between values in waked versus free directional sectors following the definition given in Section 2.3. Note that only coinciding time stamps are included in analyses that consider both observational and simulation data. To minimize the uncertainty, model values are not interpolated to different heights to conduct the analysis. Because there is a high density of values in the vertical ($z$) direction in the model, we simply consider the model level ($z_{\text{sim}}$) that is closest to the observation height ($z_{\text{obs}}$). At MET (NWT) the measuring height is $z_{\text{obs}} = 60 \, m$ (80 m) and the simulation height $z_{\text{sim}} \sim 52 \, m$ ($\sim 80 \, m$).

3. Results
The results presented herein are divided into three sections. First, a general evaluation of the model performance during waked conditions (Section 3.1). Second, a comparison of model performance under waked versus free stream conditions (Section 3.2). Finally, an assessment of the post-simulation wake correction technique is given (Section 3.3).

3.1. Model performance under “waked” conditions
When operational, WTs convert a portion of the mean kinetic energy into electricity. The momentum deficit downstream leads some of this mean energy to cascade down the spectrum increasing the turbulent kinetic energy budget [24] and the ambient turbulence intensity. It is therefore expected that a simulation which fails to account for the effect of WTs on the flow will overestimate $U$ and underestimate $I$. In terms of wind speed (Fig. 3, left) this expectation was only confirmed at the site MET. At NWT the model mostly underestimated the wind speed with a small mean error of $\bar{\delta} \sim -0.1 \, m \, s^{-1}$. For $I$ (Fig. 3, center) the expectations are confirmed at both sites, with mean underestimations of $\bar{\delta} \sim -0.06$ at NWT and $\bar{\delta} \sim -0.15$ at MET.

The wind direction errors (Fig. 3, right) were similar at both sites, with the largest occurrences in the bin $\beta \in (0^\circ, 30^\circ)$ at NWT and $\beta \in (-30^\circ, 0^\circ)$ at MET. It is important to note that these direction errors can be brought on not only by the presence of wind turbine wakes, but also by the presence of other roughness elements at the site, which can lead to the development of internal boundary layers over roughness changes and to a modification in the displacement height [25]. More details on the effect of roughness and topography changes on near-surface flow
Figure 3: Probability density of model errors during waked conditions at MET (solid) and NWT (dashed) for time series of 10-minute means: wind speed (left), turbulence intensity (center) and wind direction (right). For wind direction, the standard 30° bin width is chosen.

Table 1: Temporal mean of model absolute errors at MET and NWT when considering waked ($|\delta_w|$) and free ($|\delta_f|$) conditions. Note that power error values are only available at the NWT site, and do not consider time stamps in which the NWT was not producing power.

| Variable          | Unit  | MET   | NWT   |
|-------------------|-------|-------|-------|
| Wind Speed $U$    | [m s$^{-1}$] | 3.1   | 2.3   | 2.1   |
| Wind Direction $\beta$ | [deg]  | 35.5  | 34.7  | 40.2  | 18.6  |
| Turbulence Intensity $I$ | [-]    | 0.16  | 0.07  | 0.08  | 0.06  |
| Power $P$         | [kW]  | 495.4 | 385.8 |

can be found in [26]. More details for the present site can be found in [11]. In the next section, the role of wakes in explaining this discrepancy is estimated.

3.2. Model performance under waked versus free conditions

To estimate how much the presence of WT wakes affects the model performance, mean absolute errors $|\delta|$ can be computed separately over the entire free and waked periods as given in Table 1. Note that in these averages, only absolute values are considered and therefore the information given in Table 1 differs from that in Fig. 3 where the sign of the error values was kept.

The results show that $|\delta_w| > |\delta_f|$ consistently. For wind speed, the error was $\sim 35\%$ higher at MET during the waked period but remained the same for both periods at the NWT site. These results along with those shown in Fig. 3 indicate that the model performance at MET was more affected by the presence of a wake than at NWT. It is important to note that although the wind speed errors under both flow scenarios are indistinguishable at NWT, the same is not true for power. Relatively small differences in wind speed may result in large differences in estimated power depending on the magnitude of the wind speeds. The power estimated from the simulation data presented larger errors under waked conditions, and consider only periods in which the NWT was operating.

The turbulence intensity error differences between waked and free conditions are especially large at MET, where the model failed to simulate high turbulence periods (Fig. 4a). These high turbulence periods in the measurements coincide with waked periods where sometimes the variation in wind speeds over 10 minutes is very large due to the intermittence and inhomogeneity of the flow under these conditions. At NWT (Fig. 4b) the model better reproduced the variation
of turbulence with wind speed, but still performed better during free conditions.

As mentioned in Section 3.1 and verified in Fig. 4, the model consistently underestimated the turbulence levels not only under waked but also under free conditions. The underestimation of turbulence by the model should be interpreted with caution because it can be brought on by data and model limitations that are not necessarily related to the absence of a wind turbine wake model. For example, the presence of an escarpment also contributes to enhanced turbulence at this site for the heights considered [11] and limitations in the spatial resolution of the lower boundary conditions could partly explain the \( I \) underestimation seen also during free stream periods. Moreover, a portion of this underestimation of turbulence intensity can likely be attributed to the numerical treatment of the transition from the meso to the micro scale, and the method chosen to simulate turbulence in the terra incognita [27] domain (see Section 2.2).

Fig. 4 also shows that the simulated turbulence intensity was slightly higher for the free sectors which include not only an onshore but also an offshore footprint, adding to the flow complexity as it transitions over different roughness and terrain regimes. Note that error metrics given in Table 1 are not necessarily representative of the overall model skill, which should be evaluated with longer (e.g., one year) simulations. These investigations are beyond the scope of the present work, which seeks to differentiate the model performance under the two scenarios considered.

The wind direction differences between the SWT and NWT locations was also underestimated by the model as shown in Fig. 5. However, the model errors are very similar under free and waked periods, indicating that this was not an inability to simulate wake-induced meandering and rather a consequence of other model limitations likely relating to surface phenomena as such as the spatial resolution of the lower boundary conditions (i.e., roughness elements, terrain elevation, land-sea mask, sea surface temperatures) and the surface layer treatment to ensure the development of internal boundary layers both for on and offshore flow depending on the wind direction.

3.3. Post-simulation wake correction

In this section, the effect of the SWT wake is added to the simulated wind speed sampled at MET and NWT after the simulation has been completed. We seek to determine whether this
approach can improve the results with respect to local wind speed and turbine power prediction. Such an approach is of great value to the scientific community for being easy to implement and for eliminating the need for the costly alternative of fully coupling fluid and structural dynamics solvers, but it does not affect the turbulence and wind direction estimates. The method employed is described in Section 2.4.

We find that the wake correction method improves the error metrics by bringing the MET wind speed values closer to the observations during waked periods. Namely, the mean error goes from $\sim 3.1 \text{ m s}^{-1}$ (Table 1) at this site before the wake correction to $\sim 2.0 \text{ m s}^{-1}$ after the correction is applied (Fig. 6). The sensitivity of the results to different wake expansion factors was evaluated as shown in Fig. 6. All $k$ values resulted in a reduction of the mean error at MET, but the largest improvement was seen for $k = 2\%$ where $|\delta_u|$ was reduced by $\sim 38\%$ relative to the mean error for the non-corrected $U$ values. This result agrees with expectations for the site during the period considered, where atmospheric conditions were mostly stable [11]. This expansion factor is close to the WAsP recommended value of $k = 5\%$ for offshore conditions [21] and is in agreement with the expectations for the measurements site (i.e., flat and narrow island with flow characteristics that often resemble those of offshore environments [11]).

At the NWT site, the post-processing wake correction had the opposite effect and increased the mean model error. This is expected following the analyses in Sections 3.1 and 3.2 which showed that the model already underestimated $U$, demonstrating that errors at this location are not necessarily related to the absence of a WT wake representation, but are rather a result of a combination of factors including limitations in the data and model, and in the post-processing technique itself which assumes a constant wind direction.

4. Conclusion
When wind turbines and their wakes are not represented in atmospheric models, the simulation performance is compromised. In this work, we quantify this effect by comparing model and observational data separately during free stream and wake conditions. Three unique data sets are considered which include a sonic anemometer mounted onto a meteorological mast compliant with the International Electrotechnical Commission standards [28], two utility-scale wind turbines, and cutting-edge WRF-LES simulations at a spatial resolution of 111 m. We focus on flow parameters of relevance to wind energy: wind speed, wind direction, and turbulence intensity. The wake of one turbine on two downstream sites is considered, namely a meteorological mast and another turbine.
Figure 6: Mean absolute errors $|\Delta w|$ for horizontal wind speed $U$ [m s$^{-1}$] during waked periods without the wake correction (horizontal lines) and with the wake correction (lines with markers) at MET (solid) and NWT (dashed) sites, as a function of the expansion factor $k$ [-] used in the Jensen wake expression (Eq. (2)).

We find that the model consistently underestimates turbulence intensity at both sites, but the relationship of this result with the absence of a wake description is only evident at one of the sites, where the model wind speed error in the presence of a wake is $\sim 35\%$ larger than during free stream conditions. At the other site, the wind speed is underestimated indicating that these errors are related to other model and data limitations, such as the resolution and fidelity of the lower boundary conditions. The model also consistently underestimates the wind direction difference between the turbines regardless of whether a wake is present, evidence that this is not an inability of the model to simulate wake-induced meandering. Despite these shortcomings, it is important to keep in mind that the simulation considered is not long enough to undertake a thorough evaluation of the model skill which is beyond the scope of the work.

Finally, we find that a post-simulation wake correction reduces the error in wind speed estimates by $\sim 38\%$ at the site that presented larger errors under wake conditions, but increases the errors at the other site which is in a coastal location, and in the far wake. The wake correction is done using the Jensen wake formulation and the best results are found using a wake expansion parameter of $k = 2\%$. This value is lower than has typically been found for offshore environments [29] but is consistent with the strongly stable conditions observed during the PEIWEE field campaign [11]. These preliminary results alert to the dangers of blindly applying a post-processing tool without consideration for other model error and uncertainty sources, which under certain conditions (e.g., complex terrain) may be larger in magnitude than the errors brought on by the absence of a wind turbine model. This analysis also reiterates the need for a wind turbine wake parameterization for high resolution WRF-LES which will improve simulations of wind speed, wind direction and turbulence intensity within and around wind power plants.

**Acknowledgments**

This work was partly funded by National Science Foundation 1565505, and the U. S. Department of Energy DE-EE0005379 and DE-SC0016438. The model data was provided by Vortex FDC, and the turbine data by WEICAN.

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