Analysis of students’ errors in solving probability based on Newman’s error analysis

T Triliana1, E C M Asih1
1Departement of Mathematics Education, Pascasarjana, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No. 229, Bandung 40154, Indonesia
Email: triliana@upi.edu

Abstract. Students’ errors in solving mathematical problems can come from several reasons. This research purposes to analyse students’ errors based on Newman’s error analysis in solving problems about probability using qualitative method. Newman’s error analysis are five stages of (1) reading, (2) comprehension, (3) transformation, (4) process skill, and (5) encoding. The steps consist of developing three tasks, validating the tasks, asking students to solve the tasks, and analysing students’ answer. The subjects of this research were five students at the eighth grade (13-14 year-olds). The result showed that students’ errors based on Newman’s error analysis often occurred in stages of reading, comprehension and process skills. Students made errors in choosing the formula to solve the tasks, understanding what the tasks asked and determining the events. Teachers should incorporate error analysis in their lesson designs as it will help making instructional based on students’ needs.

1. Introduction
Mathematics has an important role in life. In everyday activities, people cannot be separated from mathematical things. The mathematics aims to form logical, systematic, critical and creative thinking skills. In mathematics learning, students are required to be able to construct their own knowledge through various activities, so that students do not experience difficulties and even make mistakes in working on mathematical problems [1].

One of the subjects in mathematics is probability. Probability is defined as the study of possibility and uncertainty, and it involves in most of everyday decisions [2]. It is an interesting topic to many professions, such as psychologists for making judgments and decisions, doctors for interpreting risks of medical outcomes, political analyst for interpreting polls and elections, and journalists for explaining and critiquing statistical information in the media [3]. Solving students’ misconceptions about probability has been broadly acknowledged as an urgent instructional goal [4, 5]. The major misconception has occurred in the stages of providing formulas, rules, and procedures [5, 6, 7].

The research on misconceptions about probability have been made by Lai Huat Ang and Masitah Shahrill who have taken samples of 10-11 year-old students from secondary school in Brunei Darussalam [8]. Leonid Khazanov and Lucio Prado conducted a study about correcting students’ misconception on probability [9]. Those researches show that there are still many students who have difficulties and make mistakes in solving probability. But, in Indonesia, research on students’ errors in solving probability is still difficult to be found.

Errors in solving mathematical problems become one of the teachers’ concerns at school. Students’ errors need to be analysed to find out the variation of errors made by students. By pinpointing student
errors, the teacher can provide instruction targeted to students’ needs. Error analysis is a method that commonly used to identify the cause of students’ errors when they make consistent mistakes [10]. It is a process of reviewing a student’s work and then looking for patterns of misunderstanding. Errors in mathematics can be factual, procedural, or conceptual, and may occur for other reasons. Identification of students’ specific errors is especially important for students with less proficiency [11, 12]. One of the error analysis is the Newman’s error analysis. Newman suggested that students who want to solve mathematical problems must work through five stages, those are (1) reading; (2) comprehension; (3) transformation; (4) process skills; (5) encoding. These five stages can be used to find out where and why students make errors in solving mathematical problems [13].

This research aims to analyse students’ errors based on Newman’s error analysis in solving problems about probability. The result of this study can be used as supporting information to develop further learning plans.

2. Method
This research is qualitative method with descriptive design. The subjects of this research were five students at the eighth grade (13-14 year-olds). The students including high, medium, and low level in mathematics achievement, were selected by their teacher to participate in this study. The students came from a public school in Riau Province. This research aims to describe students’ error in solving probability tasks at the eighth grade. To do so, first, we developed three probability tasks (see Table 1). Next, the tasks were validated by two experts, consisting of a mathematics lecturer and a mathematics teacher. Then, we requested students to solve the tasks in 60 minutes. Finally, we analysed students’ answer based on Newman’s error analysis.

Table 1. Probability tasks

| No | Task |
|----|------|
| 1. | Ardi conducted experiment of rotating the unfair spinner with four colour. After conducting 25 experiments, Ardi got the following result: |
|    | Colour | Red | Yellow | Black | Blue |
|    | Frequency | 5 | 10 | 7 | 3 |
|    | If Ardi conducts 100 experiments, how many times does the arrow of spinner point to the yellow area. |
| 2. | There are 3 boxes of A, B and C. Each box contains 3 white balls and 2 red balls. We draw 3 balls with the given rules: |
|    | Stage 1: Draw a ball from box A. |
|    | Stage 2: |
|    | • If a white ball is drawn from stage 1, then the ball is put into box B. Next, draw another ball from box B. If a white ball is drawn, then the ball is put into box C. Meanwhile, if a red ball is drawn, the ball is put into box A. |
|    | • If a red ball is drawn from stage 1, then the ball is put into box C. Next, draw another ball from box C. If a white ball is drawn, then the ball is put into box A. Meanwhile, if a red ball is drawn, the ball is put into box B. |
|    | Stage 3: Draw a ball from each box. |
|    | What is the probability that all the drawn balls from stage 3 are red balls? |
| 3. | Rizki and Melati are watching a television. They struggle to obtain the remote of television because their favourite channels are different at the same time. Rizki has an idea to solve it by rolling 2 fair dices. If a total of 7 appears, then Melati wins, otherwise Rizki wins. Nevertheless, Melati has another idea to suggest tossing 2 fair coins. If the appearing coins are in a same side, then Rizki wins, otherwise Melati wins. What is the fairer idea to solve the problem? |
3. Result and Discussion
In this research, students were given three tasks about probability. For Task 1, it is found that three students made errors in answering the task. One of the students did not understand the concept well, as seen in Figure 1, he stated that he did not predict the solution. This indicates that the student did not know the definition of probability, whereas the core of probability is to predict. The probability is a subject to predict events in the future [2].

Translation:
I do not know, because I cannot procedurally determine the task. It can be more or less as my prediction. If Ardi conducted 25 experiments and he got 10 times in yellow, then yellow area in the spinner is more than the others, so that yellow area is the most chosen.

Figure 1. Representative example of student’s error in Task 1

For Task 2, it is found that all students cannot answer correctly. Some of them answer nothing. This means that they did not understand the task. Meanwhile, one of them gave the answer by representing the task in the simpler form, but he cannot determine probability correctly (see Figure 3). We consider that students cannot use all the information from the task. This Task 2 needs the deep analysis so that students have difficulties in predicting the possibilities and following the stages from
Task 2. It also means that students’ ability is low to interpreting data. We conclude from Task 2, students’ errors based on Newman’s error analysis are in the stages of reading and process skills.

For Task 3, it is found that some students did not understand the task, so that they write nothing. The other students understand the given information in the task but they cannot understand to what the task asked. The other one answered the task with all procedures, but he made an error in solving the task in determining the number of event where a total of 7 appears (see Figure 4). This means he cannot differ events between (1,6) and (6,1); or (2,5) and (5,2); or (3,4) and (4,3). We conclude from Task 3, students’ errors based on Newman’s error analysis are in the stages of reading, comprehension, and process skills.

![Figure 3.](image1.png)

**Figure 3.** Representative example of student’s error in Task 2

![Figure 4.](image2.png)

**Figure 4.** Representative example of student’s error in Task 3

Based on the students’ answers, there are still many students’ errors in solving probability. This shows that the students’ understanding about probability is not well developed. To solve students’ errors, teachers should give students tasks with various difficulty levels, in order to habituate students understanding to what the tasks asked. Teachers should teach the concepts about elements of probability, such as probability of an event, frequency of an event, sample spaces, sample points, etc.
4. Conclusion
The research question addressed in this paper concerns about students’ errors in solving probability task. The results lead to the following conclusions. Students’ errors based on Newman’s error analysis in Task 1 consist of comprehension, transformation, and encoding, Task 2 consist of reading and process skills, and Task 3 consist of reading, comprehension, and process skills. Students’ errors often occurred in stages of reading, comprehension and process skills. In comprehension stage, students read all the words in the problem accurately but does not understand the overall problem or specific terms within the problem. In transformation stage, students understand what the problem requires but is unable to identify the operation or the sequence of operations needed to solve the problem. We suggest that teachers should design the instructional emphasising concepts of probability. Teachers also should give sundry tasks.

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References
[1] Umam M D 2014 Kesalahan siswa dalam menyelesaikan soal cerita matematika materi operasi hitung pecahan Mathtedunesa Jurnal Ilmiah Pendidikan Matematika 3(3) pp 131-134
[2] Hirsch L S and O'Donnell A M 2001 Representativeness in statistical reasoning: Identifying and assessing misconceptions Journal of Statistics Education 9(2) pp 1-22
[3] Garfield J and Ben-Zvi D 2007 How students learn statistics revisited: A current review of research on teaching and learning statistics International Statistical Review 35(2) pp 372-396
[4] Shaughnessy J M 2007 Second Handbook of Research on Mathematics Teaching and Learning (Charlotte: Information Age Publishing)
[5] Khazanov L 2008 Addressing students' misconception about probability during the first years of college Mathematics and Computer Education 42(3) pp 180-192
[6] Konold C 1995 Issues in assessing conceptual understanding in probability and statistics Journal of Statistics Education 3(1) pp 1-9
[7] Sharma S 2006 Personal experiences and beliefs in probabilistic reasoning: Implications for research International Electronic Journal of Mathematics Education 1(1) pp 33-54
[8] Ang L H and Shahrill M 2014 Identifying students' specific misconceptions in learning probability International Journal of Probability and Statistics 3(2) pp 23-29
[9] Khazanov L and Prado L 2010 Correcting students' misconceptions about probability in an introductory college statistics course ALM International Journal 5(1) pp 23-35
[10] Lai C F 2012 Error analysis in mathematics (Oregon: Behavioral Research and Teaching University of Oregon)
[11] Fuchs L S, Fuchs D, and Hamlett C L 1994 Strengthening the connection between assessment and instructional planning with expert systems Exceptional Children 61(2) pp 138-146
[12] Salvia J and Ysseldyke J E 2004 Assessment (9th ed.) (Boston: Houghton Mifflin)
[13] White A L 2010 Numeracy, literacy, and Newman’s error analysis Journal of Science and Mathematics Education in Southeast Asia 33(2) pp 129-148