LIMITS ON THE PRIMORDIAL FLUCTUATION SPECTRUM
VOID SIZES AND CMBR ANISOTROPY

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ABSTRACT

We suggest to use the common appearance of voids with a scale of 5000km/s in the galaxy distribution to estimate the power spectrum on this scale. We use a simple model for a gravitational formation of voids and we compare the results with the matter fluctuations as constrained by the CMBR observations of COBE. We find that a power spectrum \( P(k) \propto k^n \) with \( n \approx 1.25 \) is compatible both with COBE and with gravitational growth of large voids in an \( \Omega = 1 \) universe. A Harrison-Zel’dovich spectrum, \( n = 1 \), normalized to produce the observed CMBR fluctuations, does not have enough power for gravitational growth of voids with a diameter of 5000km/s in an \( \Omega = 1 \) Universe. Such a spectrum would be compatible if (i) void diameters are smaller (3500km/s), the voids are shallower or if the voids are rare (we assume that the universe is void filled); (ii) \( \Omega < 1 \) i.e. the Universe is open; (iii) galaxies do not trace matter on very large scale or if (iv) the voids do not grow gravitationally.

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I. INTRODUCTION

Recent observations of fluctuations in the cosmic microwave background radiation (CMBR) have provided strong support for the gravitational instability theory of structure formation. Moreover, since temperature fluctuations on scales larger than 1° are expected to arise from fluctuations in the potential field at the recombination era (Sachs and Wolfe 1967), these measurements can be used to directly constrain the primordial power–spectrum of matter fluctuations. On very large scales (≈ 1000h\(^{-1}\) Mpc), the CMBR anisotropy detected by the COBE DMR experiment (Smoot et al., 1992) can be used to normalize the power–spectrum of matter fluctuations. On smaller scales, more sensitive upper limits on the CMBR anisotropy have recently been reported by Gaier et al., (1992) based on the University of California Santa Barbara (UCSB) South Pole degree-scale experiment while Meyer (1993) has recently reported a detection on a comparable scale. This observation provides additional information on scales of about 100h\(^{-1}\) Mpc that combined with the COBE data can also be used to constrain the shape of the power–spectrum for long wavelengths.

These recent results of CMBR experiments have had a considerable impact on theories of structure formation. Until recently, the standard biased CDM model \((b = 1/\sigma_8 \approx 2.5)\) involving cold dark matter, Gaussian fluctuations and a Harrison-Zel’dovich spectrum had been one of the most successful models for the formation of structures in the universe. The CDM scenario was very appealing because of its simplicity, predictive power and the enormous success it had in accounting for a vast number of observations ranging from galaxy scales to about 10 h\(^{-1}\) Mpc. On larger scales, the model was less successful, having difficulties with the growing body of evidence indicating the need for extra amounts of large-scale power (Davis et al., 1992). The amplitude of the two–point correlation at large
angular scales for the APM sample (Maddox et al., 1990a,b), the amplitude of the variance in galaxy counts determined from deep redshift surveys of IRAS galaxies (Efstathiou et al., 1990, Saunders et al., 1992), the large coherent motions detected on scales of about $50h^{-1}$ Mpc or larger (Bertschinger et al., 1990, Courteau 1992), and the existence (and the frequency) of large voids in the galaxy distribution were some of the observations requiring more power on large scales.

The consequences of these observations and the new CMBR measurements to theories of large-scale structure formation are currently under investigation. For instance, Efstathiou, Bond and White (1992) find that normalizing the canonical CDM power–spectrum to the COBE measurements requires $\sigma_8 \approx 1$ and $b \approx 1$ and leads to conflicts with other observational data on small scales. This has motivated several authors to investigate different cosmological scenarios including the tilted CDM model (Cen et al., 1992), the mixed C+HDM model (Davis et al., 1992, Klypin et al., 1992) and low-Ω CDM models with non-zero cosmological constant (Efstathiou, Bond and White 1992, Kofman et al., 1992), all trying to reconcile the small and large scale properties, most based on the clustering properties of galaxies.

An alternative and more general approach to the problem is to consider a generic model–independent power-spectrum and examine the nature of the constraints imposed by combining the CMBR observations on large-scales, which are expected to be independent of the details relating galaxies to the matter density field, with observations on smaller scales. For instance, if large-scale structures grow gravitationally the amplitude of motions and the characteristics of voids will be sensitive to the fluctuations of the total density field and thus can be used to constrain the power-spectrum of the matter fluctuations on scales of the order of $50h^{-1}$ Mpc. Since the process of galaxy formation is rather complex and
models of the hydrodynamical and star formation processes are still very preliminary (Cen and Ostriker 1992a,b), it seems that properties that depend on matter fluctuations are more suitable to constrain the primordial power-spectrum than using clustering properties of galaxies.

An example of this approach is the analysis of Gorski (1992) who investigated the bounds imposed by the new UCSB measurements to the deviations from a Hubble flow. In this paper we use a similar approach to investigate the implications to the nature of the primordial fluctuation spectrum, imposed by combining the CMBR measurements with the existence of large-scale voids. We suggest here a simple quantitative model that estimates the power required for gravitational formation of voids. Towards this end we extend the earlier work of Blumenthal et al., 1991 (paper I) who examined the conditions for the gravitational formation of voids from negative rms fluctuations in the primordial power-spectrum. Our approach is similar in nature to that of Gorski (1992) in the sense that our treatment is independent of the details of any specific structure formation model. Our primary goal is to develop the formalism and investigate how the observed properties of voids (size, underdensity and frequency) can be combined with the CMBR data to constrain the power-spectrum of matter fluctuations. In section 2 we review the observational background. The basic assumptions of our model and the condition for the gravitational formation of a void at a given scale today are reviewed in section 3. In section 4 we show how the CMBR data is used to normalize a power–law spectrum, while in section 5 we investigate the limits on the size of voids imposed by the CMBR measurements on large and small scales. A summary of our main conclusions is presented in section 6.

II. OBSERVATIONAL BACKGROUND
Recent observations of the galaxy distribution strongly suggest that galaxies tend to lie on wall-like features which bound large empty regions, forming a closely packed network of voids (Kirshner et al., 1981, 1983; De Lapparent, Geller & Huchra 1986; Geller & Huchra, 1989; da Costa et al., 1988). This property seems to be generic since it is present in all the slices in the northern and southern hemisphere that have been completed by the ongoing CfA2 and SSRS2 surveys (Geller & Huchra 1989, da Costa 1991). These surveys, which densely probe the galaxy distribution out to about 15000 km/s, also reveal that 5000 km/s voids are quite common. In fact, evidence that 5000 km/s may perhaps be a characteristic scale of the present-day large-scale structure has recently been provided by the redshift maps of the deep survey being conducted by Schectman et al. (1992). Combined these surveys are qualitatively suggestive of a void-filled Universe, with a characteristic scale of 5000 km/s. Below we investigate the consequences of this interpretation.

It is important to emphasize that no suitable algorithm is currently available to quantitatively study the properties of voids such as abundance, size distribution, filling factor and density profile, which depend on the amplitude of the large-scale density fluctuations and on the dynamical evolution of primordial low-density regions. Only some gross estimates such as typical size and underdensity can be inferred from the existing redshift surveys, which suggest a large abundance of voids ranging from 2500 to 5000 km/s in diameter (Kirshner, 1993) with a typical underdensity estimated at about 20% of the mean (see Dey, Strauss & Huchra, 1990, for a discussion the Boötes void).

Based on the qualitative evidence presented above, in this paper we adopt the picture that the Universe is volume filled by 5000 km/s voids with 20% underdensity. Although there is still some debate whether this is a fair description of the observed galaxy distribution (it is possible that a typical size is 2500 km/s and that 5000 km/s are rare) and the
underdensity is just a rough estimate based on the galaxies we feel that it is of interest to explore the theoretical implications of this picture. In sections V and VI we discuss how these various observational uncertainties affect our conclusions.

Finally, we point out that we interpret the common existence of these large voids as a consequence of non-linear effects in the evolution of the underlying matter distribution, which must also affect the galaxy distribution since the observed voids are delineated by galaxies. This may be the origin of the bump observed by Vogel et al. (1992) in the power-spectrum of the galaxy distribution at a characteristic scale of 5000 km/s.

III. GRAVITATIONAL GROWTH OF VOIDS

Following paper I, we focus our attention on the evolution of negative amplitude fluctuations to explain the formation of the observed voids in the galaxy distribution. We argue that the gravitational evolution of negative fluctuations offer a natural explanation for the observed geometry of the large-scale structure as they grow in size with time while positive fluctuations of the same scale collapse. The essence of our model is that the present–day voids result from the gravitational growth of small primordial negative fluctuations reaching shell crossing today and that the requirement that voids grow gravitationally can, in principle, yield a quantitative constraint on the primordial spectrum of fluctuation on the scale of the observed voids.

Ideally, a full $N$-body simulation with a large dynamical range would be required to determine the amplitude of the primordial fluctuations needed to produce gravitationally the observed voids. Since suitable simulations are not currently available, (see however, first attempts towards this direction by Van de Weygaert (1991) and Van de Weygaert & Van Kampen (1993)) here we follow paper I and consider the growth of an isolated spherical inverted top-hat void. Despite the simplicity of the adopted model, its main
features are supported by recent N-body experiments of simple configurations of several interacting voids carried out by Dubinski et al. (1993). In these experiments it was found that the condition for formation of an isolated void holds approximately, even for highly non–spherical perturbations and when other negative or positive perturbations are present. These numerical simulations also reveal that voids at shell–crossing are the most prominent being delineated by high-density contrast walls. Only tenuous traces of smaller scale voids, past the shell-crossing phase, are seen producing a distribution that roughly resembles that actually observed for the galaxies.

The required amplitude for the matter fluctuations at horizon crossing to form a void at the present epoch in a flat universe ($\Omega = 1$) was derived in paper I. We assume that at some initial time $t_i$ (e.g. at horizon crossing) there is an inverted top-hat density distribution; i.e., for $r \leq R_i$, $\rho = \rho_c(1 - \epsilon_i)$ where $\rho_c$ is the critical density and $\epsilon_i = 1 - \Omega_i$, and for $r > R_i$, $\rho = \rho_c$. Initially this perturbation grows linearly. During this phase the depth of the void changes with time while its size grows only with the expansion of the universe. Eventually the perturbation becomes non-linear and its size increases faster than the expansion of the universe. Finally at a time $t_{sc}$ given by $\epsilon_i(t_{sc}/t_i)^{2/3} \approx \delta_{crit}$, (i.e. when $\epsilon_i(1 + z_i)/(1 + z_{sc}) \approx \delta_{crit}$) shell crossing begins (with the shell just inside the boundary being the first to cross). We define the instant of shell crossing as the moment at which the void forms. This definition might seems somewhat arbitrary. However, at $t_{sc}$ the co-moving radius $R_{sc}$ has grown and it is 1.7 times the initial comoving radius $R_i$. The density in the void is then $1/1.7^3 \approx 0.2$ of the average density, which is the frequently quoted value for the galaxy underdensity in the observed voids. The requirement for shell crossing may perhaps be too extreme, although it naturally leads to underdensity values which are roughly comparable to observations. Relaxing this condition and requiring the
underdensity to be 30% of the mean (which corresponds to an expansion over the comoving scale by a factor of 1.45) implies that the amplitude of the initial perturbation could be decreased by a factor of 1.5.

We considered, in paper I, pure density fluctuations for which $\delta_{\text{crit}} = 4.5$. However, if the perturbation cross the horizon in a pure growing mode the density perturbation will be accompanied by a velocity perturbation and for such a perturbation $\delta_{\text{crit}} = 2.7$, as pointed out by Dubinski et al. (1993). It is not clear what is the exact nature of the perturbations as they enter the horizon. We decided to adopt here the common growing mode assumption, keeping in mind the corresponding uncertainty in $\delta_{\text{crit}}$.

Two other properties are critical to our model. First, the largest voids at any epoch are those reaching shell crossing at that epoch, since after shell-crossing voids grow relatively slowly. As discussed in paper I, this is satisfied for a wide range of power-spectra. Second, Dubinski et al. (1993) argue that if the fluctuations are Gaussian-distributed then when rms negative fluctuations on a given scale reach shell crossing the corresponding voids will also occupy a large fraction of the volume of the universe, resembling a void-filled universe. We use this fact to identify $\epsilon$ with $\sigma$, the rms mass fluctuation on the scale of voids. It might be that in a more realistic model 1.5$\sigma$ or 2$\sigma$ fluctuations will be sufficient to produce a void-filled universe. This introduces an additional uncertainty in our model which is difficult to estimate (see section VI).

The condition for shell crossing implies an initial amplitude $\epsilon_i = \delta_{\text{crit}}(1+z_{\text{sc}})/(1+z_i)$. If we take $z_{\text{sc}} = 0$ and $z_i = z_{\text{rec}}$ we have a condition for the amplitude of the mass fluctuations at recombination, $\epsilon_{\text{rec}}$. These fluctuations are also responsible for the CMBR fluctuations measured today. Therefore for a given form of the power-spectrum we can use the observed fluctuations of the CMBR on large scales to normalize the power-spectrum.
and determine the comoving scale that satisfies the condition $\epsilon_{\text{rec}} = \delta_{\text{crit}}/(1 + z_{\text{rec}})$. This would then provide the size of voids today, consistent with the CMBR fluctuations.

Instead of working at the recombination era we can determine the comoving scale of voids in terms of the linear power spectrum today, $P_L(k) = (1 + z_{\text{rec}})^2 P_{\text{rec}}(k)$. We notice that $\epsilon_i(1 + z_i)/(1 + z_{\text{sc}})$ can also be viewed as the amplitude of the perturbation $\delta M/M_L$ had it continued to grow linearly until today. Since we are interested in the rms linear fluctuation in the density field $\sqrt{\langle (\delta M/M)^2 \rangle}$ we can express it in terms of $P_L(k)$ as:

$$\langle (\frac{\delta M}{M})^2 (R) \rangle = \frac{1}{2\pi^2} \int_0^\infty dk \, k^2 P_L(k) \, W^2(kR) \, ,$$

(1)

where $W(kR)$ is a window function. Using this we can directly relate the appearance of voids to the linear amplitude of the matter perturbations measured by COBE via the Sachs-Wolfe effect, in exactly the same way as it was done in paper I.

For the window function $W(x)$ in equation (1) we use the Fourier transform of a Gaussian which has been normalized to have the same spatial volume as a top hat of radius $R$:

$$W(x) = \exp\left[-\frac{1}{2}x^2(2/9\pi)^{1/3}\right] \, .$$

(2)

This choice of a window function assures the convergence of the integrals in Eq. (1).

Before turning to the comparison between the value of $\delta M/M_L$ needed for the formation of voids and the limits imposed by the CMBR measurements it is worthwhile to emphasize that voids undergo shell crossing well after the perturbation has become non-linear. At such late stages of development the actual power spectrum bears no resemblance to the linear power spectrum, $P_L(k)$. The scale of an underdense region, once the evolution becomes non-linear, increases in comoving coordinates by a factor of 1.7 at shell crossing, and an overdense ridge (formally, a caustic) forms at the edge of the shell. The linear power spectrum, $P_L(k)$, which we use in Eq. (1) should not be confused with neither
the present non-linear \( P(k) \) for the matter distribution nor with the observed \( P_g(k) \), the power-spectrum of the galaxy distribution today. Both will be modified by the highly non-linear evolution (see e.g. Couchman & Carlberg 1992). In addition, \( P_g(k) \) will also be affected by biasing factors (reflecting the relationship between the distribution of galaxies and matter) determined by dissipational processes which are difficult to model.

On the other hand, we note that although the relation between \( P_g(k) \) and \( P(k) \) is undetermined in our model, we expect that the former will show some signature of the non-linear evolution since the galaxies do delineate the observed voids. One possibility would be the presence of a peak in \( P_g(k) \) on the scale of the voids. It is noteworthy that Vogeley et al. (1992) detect a feature near the 5000 km/s in the power-spectrum of the galaxy distribution, which may be the signature of the characteristic scale of the voids. We point out, however, that it is not clear how well defined this signature should be. As an experiment we have computed the power-spectrum for a Voronoi tessellation, which vaguely resembles the observed galaxy distribution. We find that although the power-spectrum peaks at the characteristic length of the cells, the peak is broad and may not be discernible in a more realistic distribution with more power on small scales.

Similarly, \((\delta M/M)_L\) should not be confused with the expected \( \delta M/M \) for the matter distribution today. In particular, we emphasize that we cannot compare it to the recent measurements of \( \delta M/M_g \) by Efstathiou et al. (1990). In fact, very non-linear models such as the Voronoi tessellation yield values of \( \delta M/M \) which are small (< 1), indicating that the statistic \( \delta M/M \) is not a suitable indicator of the dynamical phase of the underlying matter distribution. In conclusion, the existing measurements of \( P_g(k) \) and \( \delta M/M_g \) cannot rule out the existence of excess large-scale power and that non-linear effects maybe taking place beyond \( 8h^{-1} \) Mpc.
IV. LIMITS ON THE POWER-SPECTRUM FROM CMBR DATA

Smoot et al. (1992) describe the results of the DMR experiment on the COBE satellite. The experiment has a beam width of $3.5^o$ and the data extends from $\approx 5^o$ to $180^o$. DMR measures the temperature correlation function, $C_{\text{obs}}(\theta) = \langle \Delta T(0) \Delta T(\theta) \rangle$ of the CMBR. Smoot et al. (1992) present the observed quadrupole moment: $C_2^{\text{obs}} = 13 \pm 4 \mu K$ of the CMBR fluctuations and a correlation function $\tilde{C}$ from which the monopole, the dipole and the quadrupole terms were removed. The tilde denotes the subtraction of those moments.

The fluctuations in the CMBR on scales larger than one degree, which are of interest here, are dominated by the Sachs-Wolfe effect (Sachs & Wolfe, 1967). In a flat ($\Omega = 1$) universe, $C(\theta)$ can be expressed in terms of the current linear power spectrum, $P_L(k)$, as:

$$C(\theta)_{T^2} = \frac{H_o^4}{8\pi^2 c^4} \int_0^\infty \frac{dk}{k^2} P_L(k) \sin \frac{k\zeta}{k\zeta} ,$$  \hspace{1cm} (3)

(see e.g. Gorski, 1991). The angular dependence of $C(\theta)$ is expressed through

$$\zeta = 2R_h \sin(\theta/2) \hspace{1cm} (4)$$

where $R_h = 2c/H_o$ is the horizon size.

After removing the monopole, dipole and quadrupole moments from $C(\theta)$ we obtain:

$$\frac{\tilde{C}(\theta)}{T^2} = \frac{H_o^4}{8\pi^2 c^4} \int_0^\infty \frac{dk}{k^2} P_L(k) \left\{ \frac{\sin k\zeta}{k\zeta} - \left( \frac{\sin kR_h}{kR_h} \right)^2 - \frac{3\cos \theta}{(kR_h)^2} \left( \frac{\sin kR_h}{kR_h} - \cos kR_h \right)^2 \right\}$$

$$-5 \left( \frac{3}{2}\cos^2 \theta - \frac{1}{2} \right) \left\{ \frac{3}{(kR_h)^2} - \left( \frac{\sin kR_h}{kR_h} \right)^2 \left( 2 + \frac{3}{(kR_h)^2} \right) \right\}$$

$$- \frac{3}{(kR_h)^2} \left( \frac{\sin kR_h}{kR_h} - \cos kR_h \right)^2 \left( 1 - \frac{3}{(kR_h)^2} \right) \right\} \right\} \right\}$$

(5)

To estimate $\tilde{C}(\theta)$ we choose a power spectrum, $P_L(k)$, of the form $P_L(k) = A_n k^n$. This might be justified since we are interested in extrapolation only from the scale of
COBE measurements to the scale of the voids. For each \( n \) we define \( F(\theta) \) such that:

\[
\frac{C(\theta)}{T^2} \equiv \frac{2}{\pi^2} \left( \frac{H_o}{2c} \right)^{n+3} A_n F(\theta) .
\] (6)

The ratio \( \langle \tilde{C}_{\text{obs}}/T^2 \tilde{F} \rangle \), averaged over \( \theta \), yields the amplitude \( A_n H_o^{n+3}/(2^{n+2}\pi^2c^{n+3}) \). Explicitly, we use \( \langle \tilde{C}_{\text{obs}}/T^2 \tilde{F} \rangle = 0.5[\tilde{C}_{\text{obs}}(10^o)/T^2 \tilde{F}(10^o) + \tilde{C}_{\text{obs}}(20^o)/T^2 \tilde{F}(20^o)] \) to obtain the results presented in Table I. These results do not change significantly if we average over different values of \( \theta \) in the same range. An alternative way to estimate this ratio is to compare the quadrupole moment of the function \( F \), which we denote by \( F_2 \), with the observed quadrupole moment measured by COBE, \( C_2(\text{obs}) = 13\mu K \). The resulting ratio is also shown in Table I. It is slightly lower than \( \langle \tilde{C}_{\text{obs}}/T^2 \tilde{F} \rangle \), in agreement with the conclusions of Smoot et al. (1992) and Wright et al. (1992) that the quadrupole moment is somewhat lower than expected from \( \tilde{C}(\theta) \). Note that there is a better agreement for \( n \) at the middle of the allowed range. Again, in agreement with Smoot et al. (1992) and Wright et al. (1992), a comparison of the analytical curves and the observed data show that \( 0.6 \leq n \leq 1.6 \). As a final note we should mention that recently Gould (1993) has argued that a proper analysis of the errors in the estimates of the quadrupole moment suggest that the best fit to the spectral index is \( n = 1.5 \) rather than \( n = 1.1 \) found by Smoot et al. (1992).

The COBE data constrains the power-spectrum on very large scales. On smaller scales Gaier et al. (1992) recently reported a 95 % confidence level upper limit of \( \Delta T/T \) for a Gaussian model on a scale of \( 1.2^o \) of \( 1.4 \times 10^{-5} \). If we extrapolate the \( C(\theta) \) to this angular scale (using the amplitude given in Table I from the fit to COBE) we find \( \sqrt{C(1.2^o)} = 2.4 \times 10^{-5} \) for \( n = 1.5 \) and \( \sqrt{C(1.2^o)} = 1.6 \times 10^{-5} \) for \( n = 1 \). These values suggest that \( n \leq 1 \). However, Gaier et al. (1992) describe their results in terms of \( \Delta T/T \) for a Gaussian model which is not directly comparable to \( C(\theta) \) (Blumenthal et al., 1992;
Gorski, 1992) and consequently it is not clear how one should interpret this constraint. In an attempt to compare our fit to COBE with Gaier et al. results we present in Table I the values for $C_{1.2}(0)$, where $C_{1.2}$ is convolved with a FWHM beam of 1.2° (Martínez-González & Sanz, 1989). We find that for $n = 1$ $\sqrt{C_{1.2}(0)} = 1.4 \times 10^{-5}$. If this is the correct interpretation the constraint is even more severe if we recall that on the scale of 1° additional effects such as peculiar velocities (Doppler effect) contribute to $\Delta T/T$, while our $C(\theta)$ includes only the Sachs–Wolfe effect.

A more direct constrain on small scales has recently become available from the work of Meyer (1993), who report a detection of $\Delta T/T = 1.4 \pm 0.5 \times 10^{-5}$ for an experiment with a FWHM of 3.5°. Similar analysis of our predicted values (see Table I) shows that in this case $n = 1$ agrees well with this measurement. In fact, the entire range of power-indexes considered $n = 0.5$ to $n = 1.5$ is within the quoted error bars of this experiment.

V. VOIDS AND CMBR ANISOTROPY

Using the results of the preceding sections we can now examine the size of typical voids today which are consistent with the constrains on the power-spectrum imposed by the CMBR experiments. Using the COBE data alone we find that for a given power-law spectrum, we can integrate equation (3), determine the appropriate normalization constant $A_n$, and integrate equation (1) to obtain an estimate of $\delta M/M_L$ as a function of the power index $n$:

$$\frac{\delta M}{M_L}(v) = \sqrt{2^n \left(\frac{9\pi}{2}\right)^{(n+3)/6} \Gamma \left(\frac{n + 3}{2}\right) \sqrt{\frac{\langle \tilde{C}_{\text{obs}} \rangle}{T^2 F}} \left(\frac{\tilde{C}}{v}\right)^{(n+3)/2}}.$$  (7)

Here we have used the velocity, $v = H_0 R$, instead of the radius, $R$, to obtain a formula that is independent of the Hubble constant. We also remind the reader that since at shell
crossing $R_{sc} \approx 1.7R_i$, a typical void with a diameter of $2R_{sc}$ today corresponds to an initial perturbation with a comoving radius of $R_i$.

The condition imposed by the existence of voids that evolve gravitationally from rms fluctuations can be expressed by equating equation (7) to the value of $\delta_{crit}$ discussed in section III. We adopt in the following the value $\delta_{crit} = 2.7$ but remind the reader that $\delta_{crit}$ could be as low as 1.8 (if we consider underdensity of 30% in the voids or if $1.5\sigma$ rather than $1\sigma$ fluctuations produce the typical voids) or as high as 4.5 (if we adopt a pure density initial perturbations).

Our results are summarized in figure 1, where we show the variation of $\delta M/M_L$ as a function of $v_f = 3.4 \times v$, where $v$ corresponds to the comoving size in km/s of the unevolved primordial perturbation. We also represent the likely range of power required to form voids, $\delta_{crit}$. A summary of our results is also presented in Table I in columns (4) and (5) where we list for each $n$ the value of $\delta M/M_L$ for $v = 1500$ km/s, (which can lead to the formation of 5000 km/s voids) and the scale at which the void formation condition $\delta M/M_L = 2.7$ is satisfied, respectively.

Inspection of figure 1 shows that if we assume that the most likely value for $\delta_{crit}$ is 2.7 than in order to form typical voids of 5000 km/s in diameter we must consider a power-spectrum with $n \approx 1.25$. This could be the case if the primordial spectrum is steeper than expected from inflation. Since there is no physical reason to expect an amplification of a primordial scale–independent spectrum somewhere between the scale of measurements of COBE and the scale of the voids a spectrum that steepens at short wavelengths would have had to be imprinted in the initial conditions.

These results suggest that a void-filled distribution with characteristic scale of 5000 km/s while it agrees with the results of COBE and Meyer (1993) may already be in conflict
with the upper limits of the CMBR anisotropy at the $1^\circ$ scale. However, it is unclear how the Gaier \textit{et al.} (1992) limits apply since, as we discussed earlier, these results are described in terms of $\Delta T/T$ for a Gaussian model for the correlation function which is not directly comparable to the computed $C(\theta)$ (Blumenthal \textit{et al.}, 1992; Gorski, 1992).

For a Harrison-Zel’dovich spectrum ($n = 1$), consistent with the COBE observations and the South Pole anisotropy upper-limit, we find that $\delta M/M_L \approx 1.2$ at the scale of 5000 km/s, and thus such voids are not expected to be typical. As can be seen from the figure, the typical diameter of volume-filling voids should be $\approx 3500$ km/s. Voids as large as 5000 km/s could still be formed from $2\sigma$ fluctuations and they would represent the high-end tail of the distribution of void sizes.

From figure 1 we can also see that the tilted CDM model considered by Cen \textit{et al.} (1992) ($n = 0.7$) can produce gravitationally only small voids, with diameters of about 2500 km/s. Voids with diameters of 5000 km/s would be very rare representing $5\sigma$ events, in apparent contradiction with the observations. This is to be expected since this model was proposed to reconcile a high-bias CDM model with the COBE results. We remind the reader that the standard CDM model ($n = 1$ on large scales) normalized to COBE leads to an unbiased galaxy distribution with $\delta M/M_L \approx 1$ at 800 km/s, in which case voids should have small diameters, typically $\ll 2500$ km/s.

If the perturbations crossing the horizon are pure density fluctuations then more power is needed to form voids. The required initial amplitude is 4.5 and for a Harrison-Zel’dovich spectrum only $4\sigma$ fluctuations will reach shell crossing on the 5000 km/s scale. Clearly this is too rare to yield enough seeds to form the apparent close-packed system of 5000 km/s voids. The discrepancy that we find is far larger than the uncertainty introduced from the fit of the observed COBE data to the analytical curves. In this case, only voids
with diameters of about 2500 km/s could grow gravitationally. We also note that for \( n < 1 \) only very small voids, with diameters of about 1800 km/s, would form.

Of course, some of the difficulties of forming large voids could be alleviated if we consider the possibility that voids are essentially formed when the underdensity is of the order of 30%. In this case the required amplitudes are decreased by a factor of 1.5 and voids with diameters of 4200 km/s can form from 1σ fluctuations for \( \delta_{\text{crit}} = 2.7 \), (3500 km/s for \( \delta_{\text{crit}} = 4.5 \)) for a Harrison-Zel’dovich spectrum. We note, however, that in this case we would not expect them to fill in the volume. We also expect the voids to have shallower profiles and less sharp boundaries. Unfortunately, the available data is far from adequate to address these details.

Alternatively, if 1.5σ fluctuations are sufficient to produce a volume filled Universe, (or if the voids are not volume filling) this will also reduce by a similar factor the required amplitude for formation of voids and 4200 km/s voids could form with a \( n = 1 \) spectrum. However, we would have to relax both assumptions – i.e. to require that the observed voids are shallower and that they are produced by 1.5σ events – to explain the gravitational formation of 5000 km/s voids with such a spectrum.

VI. DISCUSSION

The new measurements of CMBR temperature anisotropy, especially as measured by COBE, have offered the unique opportunity to normalize the primordial power-spectrum of the matter distribution. In this paper we have investigated how the properties of voids observed in the galaxy distribution are constrained by the CMBR anisotropy measurements.

The basic assumption of our model is that the “voidy” nature of the galaxy distribution is a natural consequence of negative rms amplitude fluctuations which grow in size during
the evolution of the universe producing an essentially void-filled distribution. Although the details of our model are uncertain and the information on voids is still sketchy, our formalism can be used to quantitatively assess the likelihood of a given model of structure formation to generate voids. The underlying assumption of our approach is that voids grow gravitationally, that light traces matter at large scales and therefore the large voids in the galaxy distribution correspond to real voids in the matter distribution. One might resort to biasing and suggest that voids in the galaxies do not correspond to voids in the matter. However, this would require a galaxy density which is not proportional to the matter density on the scale of voids. The standard biasing schemes do not achieve this. These schemes can amplify, in the galaxy distribution, small amplitude perturbations in the dark matter. However, for biasing to create a large void which does not follow the dark matter distribution, it will also have to erase structures corresponding to large amplitude dark matter perturbations within the void.

Another basic assumption in our analysis is that the universe is flat. The dominant modification in an open Universe is the increase in $R_h$ (which is proportional to $\Omega^{-1}$). This results in a much larger separation between the scale of COBE ($\geq 10^9$) and the scale of voids ($5000\text{ km/sec}$). Thus the discrepancy that we have pointed out between the $n = 1$ power spectrum, the CMBR observations and the appearance of voids may disappear in an open universe. The standard inflationary paradigm can thus be preserved at the cost of introducing a non-zero cosmological constant.

Under the above general assumptions, our main conclusions can be summarized as follows:

1) With our standard assumptions that $\delta_{crit} = 2.7$ and that $1\sigma$ fluctuations produce the observed voids (corresponding to a growing mode initial fluctuation and to voids with
an underdensity of 20%) we find:

(a) A power law of $n = 1.25$ is required to produced a 5000 km/s void filled Universe. This power spectrum gives $C$, on the scale of $1^\circ$ slightly above the limits of Gaier et al. (1992), but in agreement with the results of Meyer (1993).

(b) A Harrison-Zel’dovich ($n=1$) spectrum can produce gravitationally, only 3500 km/s voids. Larger voids can still be formed but are not expected to be typical.

2) If we relax either one of these requirements (underdensity for voids is 30% or voids are produced by $1.5\sigma$ fluctuations) the required rms amplitude decreases by 1.5 and 5000 km/s voids can be produced with $n = 1.1$, while $n = 1$ leads to 4200 km/s voids.

3) If we relax both assumptions we find that $n = 1$ power spectrum can produce 5000 km/s voids.

4) The “common” existence of voids on scales ranging from 2500 to 5000 km/s requires, if they form gravitationally, more power in the range 800 to 1500 km/s than predicted from the standard unbiased CDM model. This does not mean that occasionally voids of this size or even larger cannot be produced by this model, as demonstrated by some $N$-body simulations. However, they cannot be as common as those observed in the galaxy distribution, in which they seem to be volume filling. We can also rule out gravitational formation of voids in the tilted CDM model with $n = 0.7$, since for this model there is even less power on large scales than in standard CDM.

5) The COBE data alone constrains the largest possible void to about 6000 km/s in diameter. If we also consider the limits imposed at the $1^\circ$ scale then the size of the largest void is $< 5000\text{km/s}$ . Voids as large as those suggested by Broadhurst et al. (1991) cannot form gravitationally and they cannot correspond to voids in the matter distribution.

Based on our model, we believe that as long as voids grow gravitationally, the existence
and abundance of voids with sizes of about 5000 km/s, as suggested by several redshift surveys, require considerable power on scales larger than about 800 km/s. This implies that non-linear effects should be important in the evolution of perturbations on these scales. This contrasts with the conventional idea that scales beyond about 8 $h^{-1}$ Mpc are still in the linear regime.

If we assume that rich cluster form from positive density fluctuations with the same initial amplitude as the negative perturbtions that form voids, we expect that in our model there will be a relationship between the abundance of voids and rich clusters. For a void-filled universe with a scale of 5000 km/s the abundance of dark matter clusters with masses of $5 \times 10^{15} M_\odot$ (corresponding roughly to galaxy clusters of $5 \times 10^{14} M_\odot$) will be of the order of one per $(50 h^{-1} \text{Mpc})^3$. This should be compared with an Abel richness 1 cluster density of one per $(55 h^{-1} \text{Mpc})^3$ and with the density of Abell clusters of richness 2, one per $(95 h^{-1} \text{Mpc})^3$ (Bahcall and Cen, 1992). It seems that our model predicts more rich clusters than observed. However, given all the uncertainties involved in the model and the data available for clusters we cannot discard the possibility of non-linearity on the basis of the existing data on cluster abundance. Specifically, if voids form from $1.5 \sigma$ fluctuations the symmetry between positive and negative fluctuations is broken, reducing the predicted cluster density.

We would argue that the mounting evidence for the common existence of large voids points out the need to pursue more evolved N–body simulations, allowing for larger values of $\sigma_8$ (e.g. Couchman & Carlberg 1992). These simulations would also be important to confirm if our model for the formation of voids from primordial underdense regions is correct. We note that for $n = 1$, $\sigma_8$ can be as high as 4.5 and still be consistent with the newly established limits of the UCSB experiment. Turning the argument around, if the
value of $\Delta T/T$ at the few degree scales is of the same order as the current upper-limits this will definitely establish that the matter distribution is much more evolved than originally thought. The challenge will then be to account for the properties of galaxy clustering in small scales and the relationship between the galaxy and the matter distribution.

Although we have only considered generic power-law spectra, our approach can also be used to predict the size of typical voids for any specific model of structure formation. It would also be interesting to examine how the power requirement for void formation compares to that necessary to account for the bulk motion on comparable scales. It is interesting that the latter seems to be the most critical test for all alternative models proposed to replace standard CDM. As in the case of voids, the observed bulk motions seem to require extra power on large–scales, which as discussed by Gorski (1992) conflicts with the upper limit imposed the UCSB experiment. It should be pointed out that since the existence of large voids seems to indicate that non-linear effects may be important on scales as large as 5000 km/s , the underlying assumptions used to compare the theoretical predictions for the amplitude of bulk motions with the observations may not be valid. This demonstrates that CMBR measurements, bulk motion data and size of voids may offer complementary information on the nature of the primordial power–spectrum and are important independent tests which may help to discriminate amongst competing models of structure formation.

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Figure Captions

Fig. 1: Predicted $\delta M/ M_L$ vs the void diameter (measured in km/sec) for various power laws spectrum, whose amplitude fits COBE.

Table I

| $n$  | $\sqrt{\frac{\langle C_{obs} \rangle}{T^2 P}}$ | $\sqrt{\frac{C_{2,obs}^2}{T^2 F}}$ | $\delta M / M_L$ | $v^2$ | $\sqrt{C_{1,2}(0)}$ | $\sqrt{C_{3,8}(0)}$ |
|------|--------------------------------|--------------------------------|-----------------|-------|----------------|----------------|
| .50  | $1.3 \cdot 10^{-5}$            | $.98 \cdot 10^{-5}$            | 0.34            | 1500  | $1.3 \cdot 10^{-5}$ | $1.2 \cdot 10^{-5}$ |
| .75  | $.1 \cdot 10^{-5}$             | $.85 \cdot 10^{-5}$            | 0.63            | 2400  | $1.4 \cdot 10^{-5}$ | $1.3 \cdot 10^{-5}$ |
| 1.00 | $.89 \cdot 10^{-5}$            | $.74 \cdot 10^{-5}$            | 1.2             | 3400  | $1.6 \cdot 10^{-5}$ | $1.4 \cdot 10^{-5}$ |
| 1.25 | $.74 \cdot 10^{-5}$            | $.64 \cdot 10^{-5}$            | 2.4             | 4800  | $1.9 \cdot 10^{-5}$ | $1.6 \cdot 10^{-5}$ |
| 1.50 | $.65 \cdot 10^{-5}$            | $.54 \cdot 10^{-5}$            | 4.7             | 6500  | $2.4 \cdot 10^{-5}$ | $2.0 \cdot 10^{-5}$ |

1) $\delta M/ M_L$ on a scale that corresponds to voids with a diameter of $v = 5000 km/sec$ today.

2) Diameter of the voids (using $\delta M/ M_L = 2.7$) in km/sec.

3) $\sqrt{C(0)}$ convolved with a FWHM beam of $1.2^0$. This should be compared with the $2\sigma$ upper limit of Gaier et al. (1992) of $1.4 \cdot 10^{-5}$.

4) $\sqrt{C(0)}$ convolved with a FWHM beam of $3.8^0$. This should be compared with the detection of Meyer (1993) of $1.4 \pm 0.5 \cdot 10^{-5}$.