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Abstract. A centimeter-scale micromixer was fabricated by two-photon polymerization inside a closed microchannel using direct laser writing. The structure consists of a repeating pattern of 20 μm × 20 μm × 155 μm acrylate pillars and extends over 1.2 cm. Using external ultrasonic actuation, the micropillars locally induce streaming with flow speeds of 30 μm s⁻¹. The fabrication method allows for large flexibility and more complex designs. © The Authors. Published by SPIE under a Creative Commons Attribution 3.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI. [DOI: 10.1117/1.JMM.14.2.023503]

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1 Introduction

Over the last decade, there has been a considerable development in the field of mixing in microfluidic devices.1–3 Mixing, passive or active, is part of many laboratory methods, therefore, adapting it to the microscale and nanoscale is important for lab-on-a-chip and microscale total analysis systems.4 Fluidics at this scale are typically dominated by viscous forces and turbulent mixing this is hard to achieve.

One class of active mixers is ultrasonic mixers, which use acoustic fields at ultrasonic frequencies. Typically, these mixers either excite a flow at the sidewalls of a microfluidic chamber5–7 or excite a flow inside the chamber using a bubble.6–8 In the case of homogeneous mixing, the chamber size is limited by the spatial extent of the induced flows, which subsequently limits the flow rate. Bubbles can be placed inside the volume and allow higher throughput compared with sidewall induced flows, enabling fast mixing.9 However, bubble-based micromixers also show serious challenges.10,11 with regard to stability. The excitation frequency must closely match the geometric resonance frequency,12 thereby, it is dependent on flow rate13 and is influenced by contamination and dissolving over time.

The mixer shown in this paper operates by locally exciting flow around pillars embedded inside the microchannel, providing the main advantage of bubble-based mixing without the mentioned drawbacks. This method allows, in principle, for a larger channel cross section and effective mixing dispersed over that cross section, enabling high flow rates combined with improved stability compared with bubble mixers. To compensate for lower induced mixing flow speed, many micropillars in a row are used. In this paper, we explore the use of in-chip direct laser writing (DLW) as a fabrication technique for the mixing structure, which requires accurate and repeatable pillar placement and shape over an extensive distance.

DLW is a flexible fabrication method for small batches of arbitrarily shaped microstructures. Using two-photon polymerization, only the resist in the focal volume is cured. By displacing the focus with respect to the substrate arbitrarily shaped three-dimensional (3-D) structures can be created. The height of the freestanding structures is limited by their structural strength.

The maximum fabricated size in previously reported work is limited to hundreds of micrometers.14,15 In this work, we explore a strategy for in-chip DLW fabrication of microstructures on a centimeter scale. An important aspect in this work is the choice of resist, enabling a faster and simpler workflow.16 The resist has a low viscosity, making it suitable for straightforward filling, development, and cleaning by flowing chemicals directly through the microsystem. The resist also does not require precleanup or postexposure baking. This, combined with the improved handling, enables a 10x faster preparation and development process (2.5 h versus 26 h) compared to earlier methods.16 Confocal laser scanning microscopy (CLSM) is used to nondestructively characterize the in-chip fabricated structures.17 As opposed to scanning electron microscopy, CLSM only requires optical access to the sample. Previously, DLW has been used to fabricate a wide variety of structures, such as nanochannels,18 “overpass”-structures for microfluidics,19 gold substrates for surface enhanced Raman scattering (SERS), and nanoshells.20

Integration into larger structures, as those needed for microfluidics,22 is not straightforward. Due to the difference in scale, DLW cannot easily be used to fabricate both the structures and the channels since that would take an unacceptably long time. As such microstructures fabricated by DLW are usually fabricated separately from the channels, requiring fine control of structure size during fabrication and accurate alignment during assembly. The same is true for strategies where the structure is partially embedded into a channel which is subsequently covered with a glass or polydimethylsiloxane layer.23 Examples of other in-chip fabrication techniques are 3-D patterning using laminar flows,24 patterning of surfaces using microchannel networks,25 and
structured elastomeric membranes. However, these techniques do not offer the flexibility of arbitrary shapes as is the case with DLW.

In-chip DLW enables the direct integration of various functional structures into existing microchannels. Resist is flowed into the existing channels and structures are directly written and anchored into place. This ensures a good fit and reduces the amount of chemicals required. In-chip DLW was first used for enzymatic 3-D microreactors by Iosi et al. Amato et al. fabricated a porous filter into an existing microchannel and Serra et al. fabricated nematic liquid crystal cells. Wu et al. demonstrated fabrication of both channels and microstructures by combined DLW and direct femtosecond-laser writing.

DLW allows rapid prototyping of 3-D microstructures without the need for masks for each variation. The work presented in this paper provides the basis for more complicated structures, including structures which require full 3-D capability fabricated with DLW with an extensive length. At the same time, in-chip DLW provides good bonding of structures which is important for the robustness of the micromixer and mitigates the problem of alignment during assembly. In the next section, we will first explain the in-chip DLW fabrication process, after which we characterize the structure in terms of geometry and strength. We show the limits of this fabrication method for long microstructures and finally we show acoustic mixing around the micropillars.

2 Experimental Methods

2.1 Fabrication

The DLW system (Photonic Professional, Nanoscribe) uses a femtosecond pulsed laser at a 780-nm wavelength and an 80 MHz repetition rate. The system contains a Zeiss Axiovert inverted microscope with an autofocus/interface finding system and charge-coupled device (CCD) camera. The sample is moved using a positioning system consisting of a motorized stage and piezo stage. The motorized stage is used for coarse movements and has a lateral range of 100 mm. The piezo stage is used for the laser writing and has an extensive length of 1.2 cm. The acoustic mixing is excited using a piezo-element which is placed on top of a 500-μm thick borofloat glass wafer and then diced into glass-silicon microchannel chips measuring 6×1.5 cm (see Fig. 1). The length $L_w$, width $w_c$, and depth $h$ of the microchannel are 4 cm, 373 μm, and 155 μm, respectively.

The microchannel is filled with resist by connecting a tube to one of the access holes and manually filling the channel using a syringe. The tube is then removed and the chip is placed glass-down into the sample holder in the DLW system.

Sample tilt and rotation correction are performed using the interface finder and CCD. The writing of structures is performed from the silicon toward the glass (see Fig. 2) to prevent an aberrated structure due to diffraction from previously polymerized resist.

Large structures are written in sections using the piezo stage, then the motorized stage is used to move the sample to the beginning of the next section.

After writing, the sample is removed from the sample holder and tube connectors are connected to the access holes. Using a syringe pump (PHD 4400, Harvard Apparatus), first 20 mL propylene glycol monomethyl ether acetate (RER600, Fujifilm) developer is flowed through the channel at a flow rate of 20 mL h$^{-1}$, followed by 20 mL isopropanol at 20 mL h$^{-1}$. A gentle flow of air is applied over the access holes until all isopropanol has evaporated, which takes less than 30 s.

Fig. 1 (a) Photograph and (b) cross-sectional sketch of the mixing device (not to scale). The chip consists of a microchannel etched into silicon (gray), sealed with borofloat glass (blue). Access holes are etched into the backside of the chip on which tubing is connected (orange). The mixing structure fabricated using direct laser writing (green) consists of two parallel rows of square pillars, both rows extending over a region of 1.2 cm. The acoustic mixing is excited using a piezo-element which is placed on top of the chip (illustrated in purple in both photograph and sketch). See Fig. 2 for a cross-sectional sketch in the xz-direction.
2.2 Characterization

2.2.1 Structural analysis

For the 3-D analysis of the structures, we use CLSM. The channel is filled with index matching oil (Nikon immersion oil, n = 1.5) to limit image deterioration by diffraction. We use a home-built CLSM based on a Nikon TE300 inverted microscope, with a 25 mW, 488-nm laser (iFlex-Mustang 488, QiOptiq), spinning disk-based confocal scanner (Ultra View ERS, Perkin Elmer), and camera (C4742-95-12ERG, Hamamatsu). The axial z-position of the objective (S Plan Fluor ELWD 60X NA 0.7, Nikon) was scanned using an objective piezo scanner (PIFOC P-720, Physik Instrumente) with a range of 100 μm. Only the emitted autofluorescence from the structure was transmitted to the camera using a low-pass emission filter with a cut-off wavelength of 525 nm. For the overview image in Fig. 3, a regular fluorescence microscope (TE2000, Nikon) was used with a lower magnification objective (S Plan Fluor ELWD 45X NA 0.45, Nikon). An autofluorescence excitation-emission map of the cured polymer can be found in the Appendix.

2.2.2 Acoustic mixing

To visualize the acoustic mixing, we use the same CLSM. The channel is filled with glycerol water mixture (80:20 ratio by weight), in which an 18% volume fraction of fluorescent silica tracer particles is suspended. The mixture is index matched to the silica particles. The spheres have an outer diameter of 2r = 1 μm and a core of 0.5-μm diameter filled with fluorescein isothiocyanate fluorophore. The fundamental acoustic resonance of the channel at f = 1.92 MHz, corresponding to a standing wave along the channel width, is excited using a piezo plate (PZ 26-302, Ferroperm) connected to an amplifier and function generator. The piezo plate is attached on top of the chip using a drop of paraffin oil as the coupling liquid.

Assuming a plane wave resonance, the magnitudes of the acoustic radiation force and the Stokes drag are

$$F_{\text{rad}} = 4\pi \Phi kr^3 E_{\text{ac}} \sin(2kx),$$

$$\Phi = \frac{1}{3} \left[ \frac{5}{\rho_p} - 2 \frac{1}{\rho_m} + \frac{k_p}{k_m} \right],$$

$$k = \frac{2\pi f}{c_m},$$

$$F_{\text{drag}} = 6\pi \eta rv_p.$$
\[ \Lambda_1 = 30 \ \mu m \] is used for characterization, the second version with \( \Lambda_2 = 70 \ \mu m \) is used for the acoustic mixing. Both structures have a total length of 1.2 cm.

We used a distance between written lines and planes of 400 nm and 2 \( \mu m \), respectively, writing at a speed of 1 mm s\(^{-1} \) parallel with the channel. Writing is started at an anchoring depth of approximately 7.5 \( \mu m \) into the silicon. The rows are written using the piezo stage in sections of nine subsequent pillars for \( \Lambda_1 \) and four subsequent pillars for \( \Lambda_2 \), alternating between the top and bottom rows after each section and moving to the next section using the motorized stage. Writing time per pillar is 295 s and the total time is approximately 66 and 29 h for \( \Lambda_1 \) and \( \Lambda_2 \), respectively. The average power at the sample is estimated to be 70 mW.

### 3.2 Characterization

The confocal measurements for the structural analysis were done using steps of 100 \( \mu m \) in the \( z \)-direction using the objective scanner, 100 ms integration time per slice, and 25 mW of laser power.

With exception of the last 1.1 mm, the fabricated structure closely matches the design with some minor deviations which are discussed in the below sections. The last segment shows strong irregularities, which will be investigated in Sec. 3.2.4.

#### 3.2.1 Pillar placement

A typical section is shown in Fig. 3. The average side-to-wall distance between the top row and nearest wall is 84.0 \( \mu m \) and for the bottom row this distance is 81.2 \( \mu m \). The designed value is 82.3 \( \mu m \), so the structure is offset from the center of the channel by approximately 2 \( \mu m \). The average angle between the rows and the walls is 6 mdeg. Corrected for the angle, the standard deviation in distance to the wall is 1.4 \( \mu m \), which can be explained by the accuracy of the motorized stage. Within rows of nine consecutively written pillars (i.e., structures written without the use of the motorized stage), the standard deviation in side-to-wall distance is on average 0.3 \( \mu m \) reflecting the accuracy of the piezo stage using a relatively high writing speed.

The horizontal placement of the pillars is less accurate, reflecting an average side-to-side separation of 7.1 \( \pm 1.1 \) \( \mu m \). Again, between consecutively written pillars, the variation is smaller with 0.8 \( \mu m \). The significant deviation from the designed separation of 10 \( \mu m \) is mostly explained by the consistently wider pillars, leaving less space in between. The variation in the width is 0.7 \( \mu m \), explaining most of the variation between consecutively written pillars. The additional variation is caused by the motorized stage, but also by the writing direction: because of the relatively high speed, the piezo positioning in this direction is less accurate.

#### 3.2.2 Pillar shapes and cross sections

At the glass-pillar interface, the pillars have an average width in direction parallel to the channel of 22.5 \( \pm 0.7 \) \( \mu m \) and a width normal to the channel of 21.5 \( \pm 0.6 \) \( \mu m \). The pillars are not perfectly square and are also slightly larger than the designed value of 20 \( \mu m \). In both the parallel and normal directions, most of the extra width is due to nonzero lateral size of the DLW focal volume. In the parallel direction, additional width comes from the fact that this is the laser writing direction. Given the small variations in the width, both effects could be corrected for in the design.

All pillars extend over the full depth of the channel. The change in cross section over the full height is less than 3 \( \mu m \) in both widths compared with the values at the glass interface. The displacement with respect to the glass surface is less than 4 \( \mu m \), meaning that the sidewall angle of the pillars is less than 1.5 deg.

#### 3.2.3 Structural strength

The fabricated pillars are sufficiently strong for microfluidic applications, such as an ultrasonic micromixer. Forces due to developer and solvent flow, interfacial tension due to evaporation of the solvent and the force due to the ultrasonic pressure resonance have no influence on the structure.

The Reynolds number for the flow is

\[
\text{Re} = \frac{\rho \cdot \bar{v} \cdot D_H}{\mu},
\]

where \( \rho \) is the density, \( \mu \) is the dynamic viscosity of the fluid, \( \bar{v} \) is the average flow speed, and \( D_H \) is the hydraulic diameter. For our case of a completely filled channel with rectangular cross section, \( D_H \) equals \( 2h \cdot w/(h + w) \). The average flow speed is the flow rate divided by the cross section and equals 97 mm s\(^{-1} \). The Reynolds number for the developer and solvent flows is 1.7 and 0.8, respectively, which sufficiently low to treat the flows as laminar.

Due to the presence of the walls, the drag on a single cylinder \( F_D \) does not depend on the Reynolds number, but on geometry only and has to be calculated using a wall correction factor \( \lambda = F_D/\mu \cdot \bar{v}_{\text{max}} \), with \( \bar{v}_{\text{max}} \) the maximum flow speed in the channel.\(^{31,32} \) For a single circular cylinder in our geometry, \( \lambda \approx 4 \) and the drag force is estimated to be \( F_D = 1 \) mN.\(^{33} \) Estimation of the drag via shear stresses, using a laminar flow profile and no-slip yields the same order of magnitude. Comparing this estimation with our case of square cylinders placed in a row shows that this value should be taken as a lower estimate, since both a square cross section as placing pillars closely in a row increase the drag compared a single round cylinder.\(^{34,35} \)

Based on the curvatures of the solvent surface surrounding the pillars during evaporation, we estimate a maximum Laplace pressure of −0.8 kPa inside the solvent, corresponding with a peak inward faced capillary force on the outermost pillar of approximately 3 \( \mu N \).

#### 3.2.4 Fabrication defects

The last segment of the structure covers the last 1.1 mm of the double row structure. Here, the structure shows strong deviations from the designed characteristics, potentially limiting the scale on which we can use in-chip DLW.

Some pillars are misplaced toward neighboring pillars, sometimes even touching completely. Toward the end of the section, some pillars are missing, implying that the flow forces during development were potentially the cause. CLSM measurements of pillars close to the beginning of the second segment are shown in Figs. 4 and 5.

Figure 4 shows the cross section of three pillars at 128 \( \mu m \) from the glass interface. The cross sections of the pillars are intact, showing that pillars extend at least to this depth. The left pillar is wider in the direction parallel to the channel than
expected, suggesting that the pillar is bent. The side-to-side distance between the first pillar and second is around 16 and is 8.5 μm between the second and third, again indicating a writing error or bending in the first pillar. The right pillar is cut-off due to optical reflections and seems narrower than in reality.

A cross section along the height of the pillar is shown in Fig. 5. The pillar visible in the top left is bent at an angle of 5 deg and seems partially detached from the silicon interface. To the left is another pillar partially visible, showing the same problem.

This suggests that the sample tilt remaining after the tilt correction is approximately 0.04 deg, resulting in a slowly diminishing anchoring depth over the length of the rows and causing only partial anchoring in the last segment. The forces during the development phase, as discussed earlier, then cause bending or complete displacement of pillars if the adhesion at the glass is insufficient. In the most extreme cases pillars are removed altogether. This shows the importance of correct tilt and rotation correction for large-scale microstructures.

### 3.2.5 Fabrication time

The total fabrication time is long when compared to other rapid prototyping techniques, although those do not provide the same flexible 3-D fabrication capability. A possibility to speed up the process is to increase the line and plane separation, which was set to 400 nm and 2 μm, respectively. Considering the DLW focal spot size, the plane separation could have been increased to 4 μm, diminishing the writing time by almost 50%. Furthermore, a strategy can be employed where only the walls of the structure are written, trapping liquid resist inside that can be cured with an additional illumination step.21

### 3.3 Acoustic mixing

The confocal measurements were done at 10 μm from the glass channel interface, 100 ms integration time per frame, and 25 mW of laser power. The structure has a center-to-center pillar spacing Λ of 70 μm and is, in all other aspects, the same as the structure characterized in Sec. 3.2. The piezo excitation voltage is 16 Vpp.

The local acoustic field, composed of the fundamental channel resonance and the locally scattered fields, induces...
Rayleigh–Schlichting streaming at the pillar walls.\textsuperscript{36,37} The streaming is visualized by fluorescent tracer particles around the pillars (see Fig. 6). The asymmetry of the streaming is likely to be caused by the slow bulk flow going to the right. The speed of the induced flow $u_{ac}$ is approximately 30 $\mu$m s$^{-1}$ and leads to rigorous mixing of the suspension in the vicinity of the pillars. To give an estimate of mixing performance, we assume a mixing flow around each pillar of $u_{ac}$ and a flow distance required for mixing of $w_c/2$ (because of two rows). The structure would then require a mixing time of approximately $t = 6$ s. Since pillars over a length of $l = 12$ mm contribute to the mixing, this leads to an estimated flow rate of 7 $\mu$L min$^{-1}$ which is comparable to other acoustic mixers,\textsuperscript{6,38,39} but not as fast as the fastest bubble mixer.\textsuperscript{11}

4 Conclusion

We have demonstrated the fabrication of a 1.22-cm long micropillar micromixer using in-chip DLW. Most of the structure matches the designed double row very well. The largest source of deviation is caused by the motorized stage. Pillars that were consecutively written are accurately placed with micrometer precision. The fabricated pillars show good bonding with both the glass and silicon and are strong enough to withstand forces in the order of millinewtons.

The intrinsic fluorescence of the acrylic-based resist allows for straightforward optical characterization. Both fluorescence microscopy as well as CLSM was employed to successfully assess the fabricated structure.

We have demonstrated the proof-of-principle of a novel type of ultrasonic micromixer, based on locally inducing flows on micropillars embedded inside the microchannel. The local streaming flow speed in the vicinity of the pillars is 30 $\mu$m s$^{-1}$, which would yield an estimated flow rate of approximately 7 $\mu$L min$^{-1}$ with homogeneous mixing. The fabrication technique explored is able to precisely fabricate a long mixing structure, which is important for high flow rate mixing. This combined with further research on the effect of pillar placement and 2-D/3-D shape on the induced streaming flow and mixing efficiency will likely lead to improvement of the micropillar-based micromixer, which combines robustness and stability with higher throughput flows.

Appendix: Autofluorescence Map of Cured IP-L

Using the autofluorescence of the polymerized resist only the structures are visualized, providing background-free 3-D imaging. Although the fluorescence is used to our advantage, in other experiments, the autofluorescence can be highly interfering (i.e., biofluorescence experiments). Therefore, it is useful to provide a fluorescence map of the developed resist as a reference for future work. The autofluorescence excitation-emission map of cured IP-L resist was measured (F900 fluorometer, Edinburgh Instruments) and is shown in Fig. 7.

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