Interacting Majorana fermion model with spontaneous symmetry breaking and topological order: exact ground state with intertwined spin and pairing orders

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The exact ground state of the interacting Majorana fermion model on square lattice is obtained. The ground state exhibits the coexistence of spontaneous symmetry breaking and topological order. The \( Z_2 \) symmetry breaking leads to the intertwined spin and pairing orders. The topological order is reflected in the \( Z_2 \) quantum spin liquid ground state. The interacting Majorana fermion model is possibly realized on a Majorana-zero-mode lattice.

I. INTRODUCTION

Landau’s symmetry breaking theory establishes the first paradigm of phases of matters in condensed matter physics. Different phases are characterized by different symmetries. A phase transition is determined how symmetry changes across the critical point and can be described by the local order parameters that transform nontrivially under the symmetry transformation. Landau’s theory successfully accounts for the appearance of various low temperature orders due to spontaneous symmetry breaking, e.g. crystals and magnets. However, the discovery of fractional quantum Hall (FQH) effect\(^1\) and high-\( T_c \) superconductors\(^2\) provides the phenomena beyond the paradigm of Landau’s symmetry breaking theory. Anderson proposed the quantum spin liquid\(^3\)–\(^5\) (QSL) without symmetry breaking is the key to the mechanism of high-\( T_c \) superconductivity\(^6\). Wen found different chiral spin liquids have exactly the same symmetry\(^1\)^11, as well as different FQH states. These new orders without symmetry breaking and local order parameters are characterized by the topological invariants, e.g. ground state degeneracy on the torus and nontrivial edge states\(^1\). Dubbed topological orders, these nontrivial gapped disordered phases possess long-range entanglement at zero temperature\(^1\)^13–15. A new paradigm of topological phases of matters emerges and flourishes in the past decades\(^1\)^16. A interesting question comes that is it possible to unify the symmetry breaking and topological order frameworks into a single formalism.

The Majorana fermions are real counterparts of complex fermions\(^1\)^17–19. Not only can be realized experimentally, e.g. in the interface of s-wave superconductor and strong topological insulator\(^\ldots\)^20–24, but also have the potential to implement topological quantum computation\(^25\), Majorana fermions (zero modes) become the hot topics in condensed matter physics. Moreover, the strongly interacting model\(^26–28,42\) built from Majorana fermions can host exotic phenomena, such as Majorana dualities\(^29\), the emergence of supersymmetry\(^30–32\) and supersymmetry breaking\(^33\), Majorana surface code\(^34,35\), tricritical Ising point\(^36,37\), topological order\(^38,39\), SYK model with black hole physics\(^40,41\).

In this paper, we report a novel interacting Majorana fermion model on square lattice. The exact ground state exhibits the coexistence of spontaneous symmetry breaking and topological order. The \( Z_2 \) symmetry breaking leads to spin and pairing orders characterized by the local order parameters, meanwhile the ground state also hosts a \( Z_2 \) QSL state with topological order. This interacting Majorana fermion model provides the first concrete example to unify symmetry breaking and topological order.

II. INTERACTING MAJORANA FERMION MODEL

The Majorana fermion operators are used to construct the interacting Majorana fermion model on square lattice. We first introduce four \( \gamma \) Majorana fermions on each site \( r \), which are described by real operators \( (\gamma^r)^\dagger = \gamma^r \) obeying the anticommutation relations \( \{ \gamma^r, \gamma^{r'} \} = 2 \delta^{rr'} \). The four \( \gamma \) Majorana fermions can represent the complex fermion operators \( c_r \), describing the electrons with spin polarization \( s = \uparrow, \downarrow \).

\[
\begin{align*}
    c^r_\uparrow &= \frac{1}{2} \left( \gamma^r_1 - i \gamma^r_2 \right), \\
    c^r_\downarrow &= \frac{1}{2} \left( \gamma^r_3 - i \gamma^r_4 \right).
\end{align*}
\]

We construct the on-site interaction \( H_U \) of \( \gamma \) Majorana fermions

\[
H_U = U \sum_r \left( \hat{c}^\dagger_r \hat{c}_r - \frac{1}{2} \right) \left( \hat{c}^\dagger_r \hat{c}_r - \frac{1}{2} \right),
\]

which is nothing but the on-site Hubbard interaction of electrons. We then introduce the four \( \chi \) Majorana fermions on each site \( r \) and construct their on-site interaction \( H_V \)

\[
H_V = V \sum_r \left( i \chi^r_1 \chi^r_2 \right) \left( i \chi^r_3 \chi^r_4 \right),
\]

which can alternatively be viewed as local constraints of \( \chi \) Majorana fermions. We also introduce the plaquette...
interaction $H_K$ of $\chi$ Majorana fermions

$$H_K = K \sum_r \left( i\chi_r^2 \chi_r^\dagger \right) \left( i\chi_{r+\hat{x}}^2 \chi_{r+\hat{x}}^\dagger \right) \times \left( i\chi_{r+\hat{y}}^2 \chi_{r+\hat{y}}^\dagger + i\chi_{r+\hat{x}+\hat{y}}^2 \chi_{r+\hat{x}+\hat{y}}^\dagger \right),$$

(5)

where four sites in each term of $H_K$ form a plaquette on square lattice and the plaquette interaction includes all $\chi$ Majorana fermions within the plaquette as shown in FIG. 1. We divide the square lattice into two sublattices $a$ and $b$ and introduce the nearest neighbor interaction $H_t$ coupling $\gamma$ and $\chi$ Majorana fermions

$$H_t = t \sum_{(r_a r_b)} \left( i\gamma_{r_a}^2 \gamma_{r_b}^\dagger + i\gamma_{r_b}^2 \gamma_{r_a}^\dagger \right) \left( i\chi_{r_a}^2 \chi_{r_b}^\dagger \right),$$

(6)

$$= t \sum_{(r_a r_b)} \left( i\gamma_{r_a}^2 \gamma_{r_b}^\dagger + i\gamma_{r_b}^2 \gamma_{r_a}^\dagger + H.c. \right) \left( i\chi_{r_a}^2 \chi_{r_b}^\dagger \right),$$

(7)

which describes the hopping and equal-spin-pairing of electrons meanwhile the electrons are coupled to the Majorana fermions $\chi^j$ and $\chi^k$ connecting nearest neighbor sites $(r_a, r_b)$. The full Hamiltonian of interacting Majorana fermion model is the sum of above interactions

$$H = H_U + H_V + H_K + H_t.$$

(8)

III. $V = 0$: EXACTLY SOLVABLE MODEL WITH SPONTANEOUS SYMMETRY BREAKING

In the limit $V = 0$, the interacting Majorana fermion model on square lattice reduces to an exactly solvable model $H_0 = H_U + H_K + H_t$. Note the on-site interaction $H_U$ and plaquette interaction $H_K$ are both composed of commuting projector Hamiltonian, thus are trivially exactly solvable. In the nearest neighbor interaction $H_t$, $\gamma^1$ and $\gamma^3$ on sublattice $a$ meanwhile $\gamma^2$ and $\gamma^4$ on sublattice $b$ are absent, we define the site operators in terms of $\gamma$ Majorana fermions

$$\hat{C}_r = \begin{cases} i\gamma_{r_a}^1 \gamma_{r_b}^3, & r = r_a \\ i\gamma_{r_a}^2 \gamma_{r_b}^4, & r = r_b \end{cases}$$

(9)

Since $[\hat{C}_r, H_0] = 0$, the site operators are constants of motion in the limit $V = 0$. Also $\hat{C}_r^2 = 1$, the eigenvalues of the site operators take $Z_2$ values $C_r = \pm 1$. The on-site interaction $H_U$ can be written in terms of site operators as

$$H_U = -\frac{U}{4} \sum_{r_a} \hat{C}_{r_a} \left( i\gamma_{r_a}^2 \gamma_{r_a}^4 + i\gamma_{r_a}^1 \gamma_{r_a}^3 \right) + \sum_{r_b} \hat{C}_{r_b} \left( i\gamma_{r_b}^2 \gamma_{r_b}^4 + i\gamma_{r_b}^1 \gamma_{r_b}^3 \right).$$

(10)

Similarly we define the bond operators in terms of $\gamma$ Majorana fermions

$$\hat{D}_{r,r'} = -\hat{D}_{r',r} = \begin{cases} i\chi_{r_a}^1 \chi_{r_b}^3, & r' = r + \hat{x} \\ i\chi_{r_a}^2 \chi_{r_b}^4, & r' = r + \hat{y} \end{cases}$$

(11)

As $[\hat{D}_{r,r'}, H_0] = 0$ and $\hat{D}_{r,r'}^2 = 1$, the bond operators are also constants of motion in the limit $V = 0$ with $Z_2$ eigenvalues $D_{r,r'} = \pm 1$. The plaquette interaction $H_K$ and nearest neighbor interaction $H_t$ can be written in terms of bond operators as

$$H_K = -K \sum_r \hat{D}_{r, r+\hat{x}} \hat{D}_{r+\hat{x}, r+\hat{y}} \hat{D}_{r+\hat{x}+\hat{y}, r+\hat{y}} \hat{D}_{r+\hat{y}, r},$$

(12)

$$H_t = t \sum_{(r_a r_b)} \left( i\gamma_{r_a}^2 \gamma_{r_b}^3 + i\gamma_{r_b}^2 \gamma_{r_a}^3 \right) \hat{D}_{r_a, r_b}. \quad (13)$$

Therefore in the limit $V = 0$ the interacting Majorana fermion model reduces to the quadratic $\gamma$ Majorana fermions coupled to the static $Z_2$ gauge fields. The constants of motion $\hat{C}_r$ and $\hat{D}_{r,r'}$ serve as static $Z_2$ gauge fields. The exact solvability of the model $H_0$ is in the same spirit of the exactly solvable Kitaev honeycomb model.43

We can replace the operators $\hat{C}_r$ and $\hat{D}_{r,r'}$ by their eigenvalues in $H_0$ and the ground states of $H_0$ is determined by the configurations $\{C_r\}$ and $\{D_{r,r'}\}$ with lowest energy. The plaquette interaction $H_K$ is of the same form of Wegner’s Ising lattice gauge theory,44 where the $Z_2$ eigenvalues $D_{r,r'} = \pm 1$ act as classical Ising spins. We define the local gauge transformation $G_r$ on site $r$ that only the $D_{r,r'}$ connecting to site $r$ change sign meanwhile the $\gamma$ Majorana fermions on site $r$ change as

$$\begin{cases} \gamma_{r_a}^1 \to -\gamma_{r_a}^1, \quad \gamma_{r_a}^2 \to -\gamma_{r_a}^2, \quad r = r_a \\ \gamma_{r_b}^1 \to -\gamma_{r_b}^1, \quad \gamma_{r_b}^2 \to -\gamma_{r_b}^2, \quad r = r_b \end{cases}$$

(14)

The Hamiltonian $H_0$ is gauge invariant, thus the ground state configurations $\{D_{r,r'}\}$ include $2^{N}$ configurations that can be related to the uniform configuration...
\{D_{r,r'} = 1\} by all the local gauge transformations, where \(N\) is the total number of sites. To determine the ground state configurations \(\{C_{r}\}\), we consider the positive large-\(U\) limit and focus on the low-energy subspace with \(2^N\) states characterized by \(i\gamma_{r}^2\gamma_{r'}^4 = C_{r,r'}\) and \(i\gamma_{r}^1\gamma_{r'}^3 = C_{r.r}$. The nearest neighbor interaction \(H_I\) is treated as perturbation and we adopt the second-order perturbation theory to derive the effective Hamiltonian within the low-energy subspace

\[
H^{\text{eff}} = \frac{t^2}{U} \sum_{\{r, r'\}} C_{r} C_{r'}, \tag{15}
\]

which is nothing but the antiferromagnetic (AFM) Ising model. The ground state configurations \(\{C_{r} = -C_{r} = \pm 1\}\) with two-fold degeneracy. This conclusion is valid for arbitrary \(U\), which is confirmed numerically by exploring the configurations with lowest energy of quadratic \(\gamma\) Majorana fermions on finite size lattice.

The ground states of \(H_0\) exhibit the spontaneous \(Z_2\) symmetry breaking due to the two-fold degeneracy. The global \(Z_2\) symmetry interchanges \(\gamma\) Majorana fermions as

\[
\gamma_{r}^1 \leftrightarrow \gamma_{r}^3, \tag{16a}
\]

\[
\gamma_{r}^2 \leftrightarrow \gamma_{r}^4. \tag{16b}
\]

The Hamiltonian \(H_0\) is invariant under the \(Z_2\) transformation while the the site operators \(C_{r}\) change sign. Thus the expectation values of \(\hat{C}_r\) naturally serve as order parameters. Nevertheless we shall define the order parameters in terms of the electron operators by introducing the spin and charge operators

\[
\hat{S}_{r}^\alpha = \frac{1}{2} \begin{pmatrix} \hat{c}_r^\dagger \hat{c}_r^\dagger & \hat{c}_r & \hat{c}_r^\dagger \end{pmatrix} \tau^\alpha \begin{pmatrix} \hat{c}_r^\dagger \\ \hat{c}_r \\ \hat{c}_r^\dagger \end{pmatrix}, \tag{17}
\]

\[
\hat{Q}_{r}^\alpha = \frac{1}{2} \begin{pmatrix} \hat{c}_r^\dagger & \hat{c}_r & \hat{c}_r^\dagger \end{pmatrix} \tau^\alpha \begin{pmatrix} \hat{c}_r^\dagger \\ \hat{c}_r \\ \hat{c}_r^\dagger \end{pmatrix}, \tag{18}
\]

where \(\tau^\alpha\) are Pauli matrices with \(\alpha = x, y, z\). The operator \(2\hat{Q}_{r}^\alpha = \hat{c}_r^\dagger \hat{c}_r + \hat{c}_r \hat{c}_r^\dagger - 1\) measures the charges with respect to half-filling. Since

\[
\hat{S}_{r}^\alpha + \hat{Q}_{r}^\alpha = -\frac{1}{2} \hat{C}_r, \tag{19}
\]

\[
\hat{S}_{r}^\alpha - \hat{Q}_{r}^\alpha = -\frac{1}{2} \hat{C}_r, \tag{20}
\]

the order parameters are defined as \(S_{r}^\alpha = \langle \hat{S}_{r}^\alpha \rangle\) and \(Q_{r}^\alpha = \langle \hat{Q}_{r}^\alpha \rangle\). We calculate the order parameters under the uniform configuration \(\{D_{r,r'} = 1\}\), in which case the model \(H_0\) is equivalent to the exactly solvable BCS-Hubbard model\(^{45}\). Without loss of generality, the order parameters are calculated for the ground state \(\{C_{r} = -1\}\)

\[
S_{r}^0 = (-)^\gamma \frac{1}{4} \left( 1 + \frac{1}{N} \sum_{k} \frac{U}{E_k} \right), \tag{21}
\]

\[
Q_{r}^\gamma = \frac{1}{4} \left( 1 - \frac{1}{N} \sum_{k} \frac{U}{E_k} \right), \tag{22}
\]

where \(E_k = \frac{1}{2} \sqrt{U^2 + 16t^2\gamma_k^2}\) is the quasiparticle dispersion of \(\gamma\) Majorana fermions and \(\gamma_k = 2(\cos k_x + \cos k_y)\) is the form factor on square lattice. The summation \(\sum_k\) is over the folded Brillouin zone. \(S_{r}^0\) and \(Q_{r}^\gamma\) characterize the spin and pairing orders respectively. The staggered factor in \(S_{r}^0\) indicates the spin order is the AFM order meanwhile the pairing order is uniform as shown in FIG. 2. If we define the complex fermion operators on sublattice \(b\) as

\[
c_{r,\uparrow} = \frac{1}{2} (\gamma_{r}^2 - i\gamma_{r}^1), \tag{23a}
\]

\[
c_{r,\downarrow} = \frac{1}{2} (\gamma_{r}^4 - i\gamma_{r}^3), \tag{23b}
\]

the staggered factor will also appear in front of pairing order. Then the pairing order is nothing but the \(\eta\)-pairing\(^{46,47}\) in the Hubbard model \(\eta^y = \sum_{r} e^{i\mathbf{Q}.r} Q_{r}^y = \frac{i}{2} \sum_{r} e^{i\mathbf{Q}.r} (c_{r,\uparrow} c_{r,\downarrow}^\dagger + H.c.)\) with \(Q = (\pi, \pi)\). The two orders coexist as they break the same \(Z_2\) symmetry. However the repulsive Hubbard interaction favors the spin order while the attractive Hubbard interaction favors the pairing order. Thus the two orders also compete with each other that lead to the waxing and waning pattern of magnitudes of order parameters in FIG. 3. Such coexistence and competition of orders in a exactly solvable model provide a concrete example of intertwined orders in highly correlated electron systems\(^{48}\).

IV. \(V \neq 0\): EXACT GROUND STATE WITH TOPOLOGICAL ORDER

Even though the on-site interaction \(H_V\) spoils the exact solvability of \(H_0\), we can still obtain the exact
ground state of $H$. Define the operators $\hat{G}_r = \chi_r^{1,2,3,4}$, constituting the on-site interaction $H_V$, which anticommute the bond operators that sharing the same site $\{\hat{G}_r, \hat{D}_{r,r'}\} = 0$. Thus the bond operators are no longer constants of motion. Note the operator $\hat{G}_r$ implements the local gauge transformation $G_r$. Recall the ground state configurations of $H_0$ are $2^N$ configurations related by all the local gauge transformations. Thus the ground state of $H$ is the equal weight linear superposition of $2^N$ gauge equivalent configurations, which is an example of Anderson’s resonating valence bond state. The on-site interaction $H_V$ doesn’t alter the symmetry breaking ground state of $\gamma$ Majorana fermions as the Hamiltonian $H$ is also gauge invariant.

The on-site interaction $H_V$ can be alternatively treated as local constraints $\chi_r^{1,2,3,4} = \text{sign} V$, where $\text{sign} V$ is the sign of on-site interaction $H_V$. We define the Pauli spin operators in terms of $\chi$ Majorana fermions

$$\sigma^x_r = i \chi_r^{1,2} = \text{sign} V i \chi_r^{3,4}, \quad (24a)$$

$$\sigma^y_r = i \chi_r^{1,3} = \text{sign} V i \chi_r^{2,4}, \quad (24b)$$

$$\sigma^z_r = \text{sign} V i \chi_r^{1,4} = i \chi_r^{3,2}, \quad (24c)$$

where the local constraints are used in the second equality. The four $\gamma$ Majorana fermions with local constraints give a faithful representation of Pauli spin operators, such as the identity $\sigma^x_r \sigma^y_r \sigma^z_r = i = \text{sign} V i \chi_r^{1,2,3,4}$. The plaquette interaction $H_K$ in the Pauli spin representation becomes

$$H_K = -K \sum_r \sigma^x_r \sigma^y_{r+x} \sigma^z_{r+y} + \hat{y} \sigma^z_{r+\hat{y}}. \quad (25)$$

which is just the Wen plaquette model, that is equivalent to the toric code hosting the $Z_2$ QSL ground state with topological order. Note different sign $V$’s lead to the same topological order.

The phase diagram of interacting Majorana fermion model for $V \neq 0$ is sketched in FIG. 4. Due to the bidirectionality of square lattice, the sign of nearest neighbor interaction is irrelevant and we set $t > 0$. For positive $U$, the spin order dominates while for negative $U$, the pairing order dominates. According to the projective symmetry group classification, the ground state is $Z2A$ and $Z2B$ QSL for positive and negative $K$ respectively.

**V. CONCLUSION AND DISCUSSION**

In summary, we study the interacting Majorana fermion model on square lattice and obtain the exact ground state with spontaneous symmetry breaking and topological order. The spin and pairing orders breaking the same $Z_2$ symmetry coexist meanwhile compete with each other. The $Z_2$ QSL ground state indicates the topological order therein. The interacting Majorana fermion model is equivalent to Wegner’s Ising gauge theory, AFM Ising model, BCS-Hubbard model, Wen plaquette model and toric code in various limits.

The interacting Majorana fermion model can be easily generalized to other lattices and higher dimensions. Simply let the number of $\gamma$ Majorana fermions on each site equal to the site coordination number. We can also introduce $2m \gamma$ Majorana fermions with $m > 2$ to include more degrees of freedom, e.g. orbit. Thus the construction of interacting Majorana fermion model provides a large class of models with spontaneous symmetry breaking and topological order.

Formally, $\gamma$ Majorana fermions act as matter fields and $\chi$ Majorana fermions act as gauge fields. The $\gamma$ and $\chi$ Majorana fermions are coupled in a gauge invariant way, thus the interacting Majorana fermion model can be viewed as lattice gauge theory nevertheless purely composed of Majorana fermions. The matter fields can have spontaneous global symmetry breaking that lead to local order parameters. However the gauge symmetry of gauge fields can never be broken due to Elitzur theorem, but gauge fields can host topological order. The interacting Majorana fermion theory can unify these two frameworks in a single formalism.
Recently, the Majorana-zero-mode lattice has been realized in a tunable way, which provides a natural platform to implement interacting Majorana fermion model composed of local interactions only. A theoretically interacting question is the doping effect, that is doping on $\gamma$ or $\chi$ Majorana fermions to study the effect on AFM order or quantum spin liquid, which may provide clues to high-$T_c$ cuprates.

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