Automated storage and retrieval system for Industry 4.0 concept

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Abstract. This paper presents the results of research that focused on the implementation of the intelligent design and development of subsystems meeting the requirements of the Industry 4.0 concept. The main emphasis is on a mathematical model of the digital interaction between two key components of the flexible automated production, which consists of warehouse adaptable to changing production and service equipped by robot with sensors for receiving the whole set of information. Mathematical formalisms were developed and computer calculations were carried out in order to meet these problems.

1. Introduction

Automatic warehouse (AW) is one of the most important elements largely determining the indices of the entire production. The tempo and character of the AW operation set limits on the productivity of the flexible manufacturing systems (FMS), output capacity and diversity of nomenclature, which influence the degree of the flexibility of the whole production and the basic technical-and-economical characteristics.

While the problem of the raising flexibility of the FMS basic equipment is solved [1] by both the development of the software and great diversity of the treating tools and fixtures, at present the automatic warehouses are busy mostly with perfecting algorithms of operation of the robot-stacker which has a certain limit caused by the rigidly determined structure of the great majority [2] of AW constructions. Since all goods arriving at AW are digitally labeled (barcode, QR codes, RFID) [3], which allowed machine-to-machine interaction (M2M), information about dimensions and weight of goods is available to the warehouse management server.

Nowadays, research and analysis of contemporary warehouses prove that the most rational solution is the implementation of automatic warehouses with rearrangeable dimensions cells, taking into consideration technical capabilities. For efficient development of those systems, it is necessary to elaborate methods of calculation their basic indices [4] in regard to main peculiarities of operation as a component of FMS [5,6].
2. The main formalisms
The volume $V$ of goods kept in the warehouse is variable and the range limits of its' change can be expressed through the relation $V_{\min} < V < V_{\max}$. Besides, the volumes of goods in the supply flow to those warehouses are represented by very stochastic values, which can be characterized by the density of the distribution $P_V(V)$. Using these parameters you can calculate $C_V$ - the number of cells required for arrangement of $N$ units of goods, $K_V$ - the busy volume of cells, $m$ - index of discreteness of a flexible rearrangeable automatic warehouse, $V_{m}^{(m)}$ - the volume of the warehouse required for arrangement of $N$ units of goods in a flexible warehouse, $R_s$ - the utilization factor of a cell of the rigid warehouse, where $q$ is the value of the ratio $V_{\max}/V_{\min}$, and $\gamma_{\min}, \gamma_{\max}$ - are limits of change to specific weight by volume of goods. So, $\overline{K}_V$ shows the specific volume weight spread versus loading capacity of the warehouse [7].

Now there is required total data for the calculation of the value $\Delta K_V$, that is the increment of the utilization factor of the AW volume, while changing to the flexible organization of its structure with rearrangeable dimensions of the cells.

3. Calculation
The value $\Delta K_V$ is the function of the volume distribution law, the discreteness index and the load characteristic of carrying capacity. In spite of the apparent complexity of the proposed equation [8], the results of the calculation can be represented graphically through the plot of influence of the values $R_s$, $q$, and $m$ on the increment $\Delta K_V$ in per cent (figure 1a, b).

![Figure 1a. Volume utilization factor versus values $m$ and $q$ with the known $R_s$.](image1a.png)

![Figure 1b. Volume utilization factor versus values $q$ and $R_s$ with the known $m$; dotted graphs relate to limitations corresponding to ideally flexible AW.](image1b.png)
The said dependencies are plotted for the normal law of distribution \( P_v(V) \) and with determinantal \( \gamma(V) = \text{const} \). The dependences shown in figure 1a can be conveniently used when it is necessary to select the degree of flexibility which determines the constructional complexity of an AW. The plots in figure 1b are more convenient for consideration of the problem of the functional efficiency of a flexible warehouse. From the curves shown in figure 1 it can be noted, that with the increase of \( m \), the relative increment of the volume utilization factor becomes less and less considerable, while enlargement of the range of spread of volumes results, on the contrary, in a higher efficiency of the flexible rearrangement, which can be explained by realization of capabilities of a more rational utilization of the AW volume.

With warehouse loading limitations, the increment of the volume utilization factor \( \Delta K_v \) becomes less considerable, because of saturation of the loading space and limited number of goods, however this does not mean, that higher flexibility [9] is not rational in this case: the complex evaluation of the warehouse efficiency must include its performance considered below, which depends on the flexibility and has different values with one and the same \( K_v \).

Thus, the dependences obtained by equations for \( \Delta K_v \) and plotted in figure 1, permit to describe the benefit, that can be obtained due to reduction of the warehouse volumes, which in several cases additionally provides for either their arrangement in a more convenient place nearer to the equipment, or installation of an additional robot-stacker, etc.

4. Mathematical model

From the results obtained, it is possible to derive one more, practically important index of increment of the amount of goods simultaneously kept in the warehouse with rearrangeable sizes of the cells. It will allow to define probable directions of realization of potential capabilities of the equipment productivity, increase in output of production, and, which is most important, to enlarge the flexibility of the whole manufacturing process due to the possibility of mastering the production [10] requiring more capacious AW in the process of manufacture in the FMS.

In order to calculate the amount of goods which can be stored in a rigid warehouse with the general volume \( V_0 \) containing accordingly \( c_0 = V_0 / V_{\text{max}} \) of cells, one can use the results obtained above, whence it follows, that with sufficiently great capacity of the warehouse \( N_0^{(r)} \), the general amount of goods in a rigid warehouse is equal to (taking into account the limitations to the AW loading):

\[
N_0^{(r)} = \min \left\{ \frac{V_0}{V_{\text{min}} \sum_{i=1}^{r} \int_{V_{i-1}}^{V_i} VP_\gamma(V) dV}, \frac{M}{\int_{V_{\text{min}}}^{V_{\text{max}}} \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} VP_\gamma(V) P_\gamma(\gamma) d\gamma dV} \right\}
\]

The amount of goods in a flexible warehouse of the same volume as the rigid one will be designated \( N_0^{(m)} \) and calculated as follows:

\[
N_0^{(m)} = \min \left\{ \frac{V_0}{\sum_{i=1}^{n} \sum_{j=1}^{m} \int_{V_{i-1}}^{V_i} VP_\gamma(V) dV}, \frac{M}{\int_{V_{\text{min}}}^{V_{\text{max}}} \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} VP_\gamma(V) P_\gamma(\gamma) d\gamma dV} \right\}
\]

Since there is an interest in the increment of the amount of goods in warehousing \( \Delta N \), the following equation can be used:
\[ \Delta N = V_0 \left[ V_{\text{max}} \sum_{i=1}^{n-1} \frac{1}{V_{\text{max}}} P_i(V) dV - \frac{1}{V_{\text{max}}} \int V P_i(V) dV \right] \left[ V_{\text{max}} \int V P_i(V) dV \sum_{i=1}^{n-1} \frac{1}{V_{\text{max}}} P_i(V) dV \right] \] (1)

The value \( \Delta N \) is the function of the value \( V_0 \), laws of distribution of volumes [11] and arrangement of goods.

5. Application of results

For calculations by the equations (1) the same data is used, which is used for estimation of the increment of the volume utilization factor, that is the normal distribution of goods volumes and the determinants volumetric density.

In such way that a series of plots shown should be obtained in figure 2a, b.

![Figure 2](image_url)

Figure 2. Plot of growth of the amount of goods in warehousing: (a) direction is shown of plots displacement with increase of spread of volumetric weights; (b) values are shown of bending points of characteristic with limitations on \( K_v \).

Now there are the results of calculations of the cells dimensions’ rearrangement probability coefficient influence on the volume utilization factor and on the amount of goods in warehousing. However, as mentioned earlier, the said parameters do not give exhaustive information [12] on the results of the flexible organization of the automatic warehouses, because, for example, the same goods, arranged differently, leave the values \( K_v \) and \( N \) unchanged, however the tempo of transfer of articles increases due to a more convenient arrangement, and therefore in addition to the information on \( K_v \) and \( N \), it is necessary to calculate the influence of the possibility of cells dimensions flexible rearrangement on the performance of the automatic warehouses [13], which will be understood as the number of transfers of goods between the cells and the goods issue/reception point with the help of the robot-stacker. The value of performance is designate as \( \nu \) so that the index for both rigid and flexible organization of a warehouse will be evaluated. In that it is mostly used in practice [14] and adopted for the calculation model in the present paper, the automatic warehouses have rectangular configuration with similar cells, whose number in vertical line is assumed equal to \( a \) and horizontally equal to \( b \), then the total number of cells in a rigid warehouse \( N^{(r)} \) is equal to: \( N^{(r)} = ab \). The average time of the robot-stacker displacement by one step vertically is assumed as \( t_v \) and horizontally to \( t_h \), (not taking into account in each case the start-up, braking and so on). The influence of the location of the goods issue/reception point on the value of \( \nu \) will be calculated. In such way that the values \( a \) and \( b \) of coordinates of the goods issue/reception point relative to the cells of the warehouse have to be introduced.

Then \( T_z \) is the summary time of goods transfer from all the cells to that point (or the time of loading, assuming the travel speeds equal in one direction) and will be equal to:

\[ T_z = bt_v \sum_{j=1}^{a} j + at_h \sum_{j=1}^{b} j + bt_h \sum_{j=1}^{a} j + at_v \sum_{j=1}^{b} j \]
Assuming the number of cells sufficiently great, one can find the most rational location of the goods issue/reception point:

\[
\frac{\partial T_x}{\partial a_n} = 0 ; \quad \frac{\partial T_x}{\partial b_n} = 0
\]

whence its coordinates are received as: \( a_n = \text{ent}(a/2) , \quad b_n = \text{ent}(b/2) \).

In this case there is a formula: \( T_a = N_0^{(r)} \left[ t_a \left( \frac{a}{2} + 1 \right) + t_b \left( \frac{b}{2} + 1 \right) \right] \).

Thus the performance of the rigid warehouse is equal to: \( v^{(r)} = 2l (bt_a + at_b) \).

In order to determine the maximum performance \([15,16]\), the optimum configuration of the warehouse, determined by the expressions for its dimensions is defined: \( \overline{a} = \sqrt{Nt_a / t_a} \quad \overline{b} = \sqrt{Nt_b / t_b} \).

With this, the performance is equal to: \( v^{(r)} = l / \sqrt{Nt_a / t_b} \).

For calculating the performance for the flexible organization, one can use the data on increase of the volume utilization factor. With this, the time of the robot-stacker displacement by one step in both horizontal and vertical directions will amount to the following accordingly: \( t_a \sqrt{1 - \Delta K_v} ; \quad t_b \sqrt{1 - \Delta K_v} \), so, the summary time of transfers will be decreased by the following value:

\[
\Delta T = \frac{N}{4} \left[ \left( \frac{Nt_a}{4t_a} - 1 \right) t_a \left( 1 - \sqrt{1 - \Delta K_v} \right) + \left( \frac{Nt_b}{4t_b} - 1 \right) t_b \left( 1 - \sqrt{1 - \Delta K_v} \right) \right]
\]

This is the reserve of time \([17]\) during which the robot-stacker can perform operations of reception and issue of goods. Using the introduced probabilistic model of goods flow, one can determine the increase in performance of the warehouse.

6. Conclusions
Apply of the obtained results would provide an opportunity to increase the degree of flexibility \([18,19]\) of automatic warehouses and make it more relevant to other elements of FMS. It is possible due to elaboration of optimum control algorithms \([20,21]\). Technical developers can determine the most important parameter of the warehouse, that is the volume utilization factor. Under these conditions several units of goods of smaller volume can be warehoused in one cell. For calculation of the AW limitation, an additional function describing the distribution of density of probability of the specific volume weight against the volume of goods is introduced. The dependences obtained in article, permit to describe the reserve of time during which the robot-stacker can perform operations of reception and issue of goods and the benefit, that can be obtained due to reduction of the warehouse volumes, which in several cases additionally provides for either their arrangement in a more convenient place nearer to the equipment, or installation of an additional robot-stacker.

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