The $f_0(980)$ and $a_0(980)$ Productions in Two-Photon Collisions and Radiative $\phi$ Meson Decays

M. Uehara

Takagise-Nishi 2-10-17, Saga 840-0921, Japan

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Abstract

We study how the scalar $f_0(980)$ and $a_0(980)$ states are produced in the two-photon collisions and radiative decays of the $\phi$ meson through unitarized Born amplitudes with the charged pion and kaon loops followed by S-wave meson-meson scattering amplitudes. We found in a previous paper that $f_0(980)$ is generated as the bound state resonance, but $a_0(980)$ as the cusp, and find here that the nature of the generation of both states is consistent with the features of the production processes.

1 Introduction

It has long been expected that the nature of the scalar $f_0(980)$ and $a_0(980)$ mesons is revealed by the investigation of the production processes such as the two-photon collisions and the radiative decays of the $\phi$ meson\[1,2,3,4\]. This is because these reactions offer valuable information which cannot be obtained in the meson-meson scattering processes alone. For example, the $f_0(980)$ resonance is hidden in the broad $\pi\pi$ enhancement in the case of $\pi\pi$ scattering, but displays the peak in the two-pion invariant mass distributions in the cases of the radiative $\phi$ decay and the two-photon collisions. The magnitudes of the two-photon partial widths, $\Gamma_{\gamma\gamma}(f_0)$ and $\Gamma_{\gamma\gamma}(a_0)$, and the branching ratios, $B(\phi \rightarrow \gamma f_0)$ and $B(\phi \rightarrow \gamma a_0)$, are said to be useful to know the quark contents of the $f_0(980)$ and $a_0(980)$ states. The behavior of the amplitude $K^+K^- \rightarrow \pi^0\pi^0$ or $\rightarrow \pi^0\eta$ in the off-shell energy region below $KK$ threshold could be seen through these reactions.

So, we study the two-meson mass distributions by using the low energy S-wave scattering amplitudes obtained in a previous paper\[5\], which is referred to as I hereafter, where we showed that the amplitudes reproduce well, though not quantitatively, the low energy S-wave scattering behaviors. In order to give the scattering amplitudes we adopted the Oller-Oset-Peláez version\[6\] of the inverse amplitude method applied to the chiral perturbation theory\[7\]. We stressed in I that the $f_0(980)$ and $a_0(980)$ states are not twins, though the masses are very close to each other: The $f_0(980)$ state is generated as a typical bound state resonance grown from a bound state born in the $K\bar{K}$ channel, while the $a_0(980)$ state is not the bound state resonance but a typical cusp generated by the channel coupling between the $K\bar{K}$ and $\pi\eta$ channels, where since the channel coupling is not sufficiently strong, the cusp cannot develop into the resonance. It is interesting, therefore, to study how the two states are produced from the same initial $\phi$ meson and two-photon state, and to see the structure of the off-shell $K\bar{K} \rightarrow \pi\pi$ and $K\bar{K} \rightarrow \pi\eta$ amplitudes.

Through the study of the production processes we find the following results:

1. The clear $f_0(980)$ peak structure near $K\bar{K}$ threshold is seen in the $\pi^0\pi^0$ mass distribution of both processes, but its width is too narrow in our model, reflecting the too steeply rising behavior of the phase shift of the isoscalar $\pi\pi$ scattering amplitude in I.

\*e-mail: ueharam@cc.saga-u.ac.jp
2. We obtain $\Gamma_{\gamma\gamma}(f_0)B(f_0 \rightarrow \pi\pi) = 0.45$ keV by a rough estimate, where $B(f_0 \rightarrow \pi\pi)$ is the branching ratio of the $f_0$ decay to the isoscalar $\pi\pi$ channel, and $B(\phi \rightarrow \gamma\pi^0\pi^0) = 0.60 \times 10^{-4}$, where the dominant contribution comes from the $f_0$ peak.

3. The $\pi\eta$ mass spectrum shows a clear cusp-like peak in the two-photon collisions, but a round one with a precipice at the $K\bar{K}$ threshold in the $\phi$ meson decay. Thus the shape of the $\pi\eta$ mass distribution changes drastically depending on the reactions.

4. We obtain $\Gamma_{\gamma\gamma}(a_0)B(a_0 \rightarrow \pi\eta) = 0.35$ keV, and $B(\phi \rightarrow \gamma\pi^0\eta) = 1.15 \times 10^{-4}$.

5. The $f_0(980)$ peak comes from the amplitudes $K^+K^- \rightarrow \pi^0\pi^0$ and $K^+K^- \rightarrow \pi^0\eta$ below the $K\bar{K}$ threshold. In spite of the too narrow $f_0$ shape, we get naturally the ratio $B(\phi \rightarrow \gamma\pi\pi)/B(\phi \rightarrow \gamma\pi^0\eta) > 1$.

6. The $K^0\bar{K}^0$ peaks just above the $K\bar{K}$ threshold in both two-photon and $\phi$ decay processes come dominantly from the isoscalar $K\bar{K}$ elastic amplitude, and we get $B(\phi \rightarrow \gamma K^0\bar{K}^0) = 4.21 \times 10^{-7}$.

In this paper we study the productions by the two-photon collisions in the next section, and by the radiative $\phi$ meson decays in Sec. 3, and finally we remark the relations of our findings to the experimental analyses and other theoretical works.

## 2 Two-meson production in the two-photon collisions

Many theoretical papers have already been published on the $\gamma\gamma \rightarrow M\bar{M}$ reactions, of which we cite a few recent ones; the elaborate amplitude analysis of $\gamma\gamma \rightarrow \pi\pi$ reactions by Boglione and Pennington and the calculations on various final $M\bar{M}$ states using the amplitudes obtained by the Bethe-Salpeter equation with chiral loops by Oller and Oset, and references therein. In the above $M\bar{M}$ denotes $\pi^+\pi^-$, $\pi^0\pi^0$, $\pi^0\eta$, $K^+K^-$ and $K^0\bar{K}^0$ final meson states.

In order to understand how the similarity or dissimilarity of the $f_0(980)$ and $a_0(980)$ states appears in the relevant reactions, we adopt the formalism developed by Mennessier, where only the charged pion and kaon loops are taken into account. According to Mennessier the production amplitudes are given as the Born term + unitarized Born amplitudes. The production amplitude $F_f(w)$ from the initial photon state with the helicity $(++)$ to the the final $(M_1M_2)_f$ state is written as

$$F_f(w) = 2e^2 \left[ B_f(w) + \sum_{i=1,2} T_{fi}(w)G_i(w) \right] \tag{2.1}$$

where $w$ is the total CM energy, $e$ the electric charge put outside the square brackets, and $i = 1$ for $\pi^+\pi^-$ and $i = 2$ for $K^+K^-$ channel. The term, $B_f(w)$, is the S-wave Born term, which is followed only by the final $f = \pi^+\pi^-$ and $K^+K^-$ states, and written as

$$B_f(w) = \frac{1 - \sigma_f^2(s)}{2\sigma_f(w)} \log \left( \frac{1 + \sigma_f(w)}{1 - \sigma_f(w)} \right), \tag{2.2}$$

$$\sigma_f(w) = \sqrt{1 - \frac{4m_f^2}{w^2}} \text{ for } f = \pi\pi \text{ and } K\bar{K} \tag{2.3}$$

We normalize $T_{ij}$ with the definite isospin so as to satisfy the following unitarity relation;

$$\text{Im}T^{(f)}_{ik} = -T^{(f)*}_{ik} \rho_k(w)T^{(f)}_{kj}, \tag{2.4}$$

$$\rho_k(w) = \frac{\sigma_k(w)}{16\pi} \theta(w - w_k) \tag{2.5}$$

with $w_k$ being the threshold of the $k$-th channel.

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1 For $k = \pi\eta, \sigma_k = \sqrt{1 - 2(m_\pi^2 + m_\eta^2)/w^2 + (m_\pi^2 - m_\eta^2)^2/w^4}$. 

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\( G_i(w) \) is the integral of the charged pion or kaon loop connecting the \( \gamma \gamma \) state with the helicity 0 to the S-wave \((M_1M_2)_f\) state, which is given as
\[
G_i(w) = \frac{1}{(4\pi)^2} \left\{ 1 + \frac{m_i^2}{w^2} \log^2 \left( \frac{\sigma_i(w) + 1}{\sigma_i(w) - 1} \right) \right\}.
\] (2.6)

The production amplitude \( F_f \) satisfies the final interaction theorem,
\[
\text{Im} F_f(w) = -T_{fi}(w)\rho_i(w)F_i^*(w),
\] (2.7)

since \( \text{Im} G_i(w) = -\rho_i(w)B_i(w) \) is hold. The S-wave two-meson production cross section in the \( f \)-final state is written as
\[
\sigma_f^S(w) = 2\pi\alpha^2 \left| \tilde{F}_f(w) \right|^2
\] (2.8)
with \( F_f = 2\alpha^2 \tilde{F}_f \), and \( k_f \) is the CM momentum.

We notice that this production amplitudes do not involve any parameters which must be determined phenomenologically. Of course we could use generalized Born terms with vector meson exchanges between two photon vertices, but we ignore them. Fortunately, the contributions from the generalized Born terms are shown to be not significant below 1 GeV\(^2\), where our interest is focussed on. The calculated results on the mass distributions of the two meson final states are shown in Fig. 1 to 3. Our results are summarized as follows;

1. The \( f_0(980) \) peak is seen in both \( \pi^+\pi^- \) and \( \pi^0\pi^0 \) spectra. This peak comes from the off-shell amplitude \( K^+K^- \rightarrow \pi\pi \) as shown in Fig.1b and 2b, while the on-shell amplitude \( \pi^+\pi^- \rightarrow \pi\pi \) shows a dip similar to the \( \pi\pi \) scattering cross section. The size of the off-shell peak of the charged state is as twice as that of the neutral one, respecting the isospin factor, but the large interference between the pion and kaon loop-amplitudes makes the charged peak reduced to almost the same size as the neutral one. The shape of the peak is too narrow in our model.

2. The small bump centered at 500 MeV in the \( \pi^0\pi^0 \) mass distribution reflects straight that of the isoscalar \( \pi\pi \) cross section, for there are no contributions from the Born term and the \( K^+K^- \rightarrow \pi^+\pi^- \) amplitude almost vanishes there.

3. The cusp-type peak of the \( a_0(980) \) state appears in the \( \pi^0\eta \) final state. The height of the peak is about a half of the \( f_0(980) \) one in the \( \pi^0\pi^0 \) state.

4. The threshold peaks appear in both of the charged and neutral \( K\bar{K} \) states; the large charged peak is due to the Born term, while the neutral one is due to the behavior of the isoscalar and isovector \( K\bar{K} \) amplitudes near the threshold, where the isoscalar amplitude dominates over the isovector one.
After Ref. [3] we estimate roughly $\Gamma_{\gamma\gamma}(R)$ of the resonance $R = f_0(980)$ and $a_0(980)$ using the formula

$$\Gamma_{\gamma\gamma}(R) = \frac{\sigma(\gamma\gamma \rightarrow f)_{\text{peak}} m_R^2 \Gamma_{\text{tot}}}{8\pi B_f},$$

(2.9)

where $\sigma(\gamma\gamma \rightarrow f)_{\text{peak}}$ is the cross section at $m_R$ of the $R$ resonance, $\Gamma_{\text{tot}}$ the total width of $R$, and $B_f$ the branching ratio of $R$ to the final state $f$.

As to $a_0(980)$ we take values, $m_{a_0} = 0.990$ GeV, $\Gamma_{a_0} = 0.1$ GeV, though the $a_0$ is not well expressed by the Breit-Wigner formula, and $\sigma_{\text{at peak}} = 35$ nb. These value gives $\Gamma_{\gamma\gamma}(a_0) B(a_0 \rightarrow \pi\eta) = 0.35$ keV. It is difficult to estimate the branching ratio of the $a_0$ cusp to the $\pi\eta$ channel, so that 0.35 keV is the lower bound.

It seems rather difficult to estimate $f_0(980) \rightarrow \gamma\gamma$ partial width from the charged pion pair because of the large interference between the signal and the background as stated above, and then we use the cross section of the neutral pion pair, where the interference is small. Taking values $m_{f_0} = 0.985$ GeV, $\Gamma_{f_0} = 0.03$ GeV, $B_f = 1/3 \cdot B(f_0 \rightarrow \pi\pi)$ and $\sigma_{\text{at peak}} = 70$ nb, we have $\Gamma_{\gamma\gamma}(f_0) B(f_0 \rightarrow \pi\pi) = 0.63$ keV. Instead, if we add the charged and neutral peaks, that is $\sigma_{\text{at peak}} = 150$ nb, we get $\Gamma_{\gamma\gamma}(f_0) B(f_0 \rightarrow \pi\pi) = 0.45$ keV.
While we should notice that our values are tentative and should not be taken seriously, because we do not attempt to get the best fits of the data, we observe that the partial widths of $f_0(980)$ and $a_0(980)$ to the two-photon state are almost equal or the $f_0(980)$ partial width be larger than the $a_0(980)$ one within our model. The heights of the $f_0$ peaks in the charged and neutral pion pairs do not satisfy the isospin multiplicative relation $2 : 1$ because of the interference between the elastic $\pi\pi$ amplitudes including $l = 2$ component.

3 The radiative decay of the $\phi$ meson

Many theoretical works are based on the common production mechanism, in which the final S-wave two-meson state is produced following the charged kaon loop emitting a photon under the OZI rule in the $\phi$ meson decay. This is the same mechanism as the one of the two-photon collisions, since it reduces to the radiative decay of the $\phi$ meson if one of the initial photons becomes massive and the other an outgoing photon. The validity of the kaon-loop mechanism is discussed by Achasov\[15\]. The difference of the model depends on that of the amplitudes used, for example a unitary chiral approach\[16\], the Linear sigma Model\[17\], phenomenological approaches with the Breit-Wigner formula\[2, 18, 19\] and references therein. We use the S-wave amplitudes $T(K^+ K^- \to M_1 M_2)$ obtained in I as the same as in the preceding section. The interference with the $\phi \to \rho \pi^0 \to \gamma \pi^0 \pi^0$ sequential decay amplitude is ignored in this paper, though the experimental data do not exclude the contamination of the sequential processes. This is discussed in Refs.\[18, 17\].

According to Ref.\[16\] we write the decay amplitude of the $\phi$ meson to two-meson state $\gamma M_1 M_2$, where $M_1 M_2$ is $\pi^0 \pi^0$, $\pi^0 \eta$ and $K^0 \bar{K}^0$ as

$$F(\phi \to \gamma M_1 M_2) = 2 e g \{G_K(w) + f \frac{m_\phi^2 - w^2}{2m_\phi^2} g_K(w)\} T_{K^+ K^- \to M_1 M_2},$$

where $g$ and $f$ are the parameters defined as

$$g = \frac{G_V m_\phi}{\sqrt{2} f_\pi^2} \quad \text{and} \quad f = \frac{F_V}{2G_V} - 1 \quad (3.2)$$

with $G_V$ and $F_V$ being the constants in the chiral Lagrangian\[20, 21, 16\] and a factor $1/\sqrt{2}$ in $g$ is the ratio of the coupling constant $\rho \pi \pi$ to $\phi K \bar{K}$. We adopt the values $G_V = 55$ MeV and $F_V = 165$ MeV given in Ref.\[16\], which give $g = 4.69$ and $f = 0.5$. The triangle $K \bar{K}$ loop integral $G_K(W)$, which connects the $\phi$ meson to the S-wave two-meson state after emitting a photon, is given as

$$G_K(w) = \frac{1}{(4\pi)^2} \left\{ \frac{m_\phi^2}{w^2 - m_\phi^2} \left[ \frac{1}{\log^2 \left( \frac{\sigma_K(w) + 1}{\sigma_K(w) - 1} \right)} - \frac{1}{\log^2 \left( \frac{\sigma_K(\mu) + 1}{\sigma_K(\mu) - 1} \right)} \right] \right\} \quad (3.3)$$

and the loop integral $g_K(w)$ by

$$g_K(w) = \frac{1}{(4\pi)^2} \left\{ -1 + \log \left( \frac{m_K^2}{\mu^2} \right) + \sigma_K(w) \log \left( \frac{\sigma_K(w) + 1}{\sigma_K(w) - 1} \right) \right\}, \quad (3.4)$$

where we take $\mu = 1$ GeV as in I. One should note that the above triangle loop integral is the same as $I(a, b)(a - b)/8\pi^2$ with $I(a, b)$ being defined in Ref.\[22\], and that as $m_\phi \to 0$ this reduces to the triangle loop integral $G_\gamma(w)$ given in Eq.(2.4) for $\gamma \gamma \to M M$ used in the previous section.

The mass dependence of the two-meson state in the radiative decay is given as

$$\frac{d\Gamma(w)}{dw} = \left( \frac{\alpha}{3\pi} \right) \left( \frac{g^2}{4\pi} \right) \frac{k_f(m_\phi^2 - w^2)}{m_\phi^2} \left| F(\phi \to \gamma M_1 M_2) \right|^2, \quad (3.5)$$
where $k_f$ is the momentum of the final $(M_1 M_2)_f$ state in the two-meson CMS, and we define $\tilde{F}$ as $F = 2eg\tilde{F}$. The $w$ dependence of the phase space factor $k_f(m_φ^2 - w^2)/m_φ^3$ has the maximum near the middle of the whole $w$-range, and vanishes at both of the ends. Furthermore, the loop integral part $G_K(w) + f(m_φ^2 - w^2)/(2m_φ^2)\cdot g_K(w)$ has a strong cusp behavior at the $K\bar{K}$ threshold, which is very near to $m_φ$, so that the mass distributions near 1 GeV are affected double by these kinematical factors. The term with $f$ in the loop integral plays a role to reduce the first $G_K$ term below the $K\bar{K}$ threshold for $f > 0$. The calculated mass distributions and production amplitudes are plotted in Figs. 4 to 6, in

Figure 4: (a) The $\pi^0\pi^0$ invariant mass distribution normalized by $\phi$ meson total width, $dB(\gamma\pi^0\pi^0)/dw \times 10^7$/MeV. Experimental data are taken from SND data[25]. (b) $F(\gamma\phi \rightarrow \pi^0\pi^0)$ amplitude; the real part is given by the dashed, the imaginary part by the dotted and the absolute value by the real line.

Figure 5: (a) $dB(\gamma\pi^0\eta)/dw \times 10^7$/MeV. Experimental data are taken from SND data[27]. (b) $F(\gamma\phi \rightarrow \pi^0\eta)$ amplitude; the real part is given by the dashed, the imaginary part by the dotted and the absolute value by the real line.

which we find the following results:

1. The $f_0(980)$ peak is manifest in the final $\pi^0\pi^0$ mass distribution as shown in Fig.4. The integrated branching ratio, $B(\phi \rightarrow \gamma\pi^0\pi^0) = 0.60 \times 10^{-4}$, and then we have $B(\phi \rightarrow \gamma\pi\pi) = 1.80 \times 10^{-4}$, where the dominant contribution comes from the $f_0$ peak.

2. Contrary to the $f_0(980)$ peak, the $\pi^0\eta$ mass distribution does not show the cusp like peak but the round peak accompanied with the precipice at the $K\bar{K}$ threshold. This is due to the kinematical factors pointed out in the above. The integrated branching ratio is $B(\gamma\pi^0\eta) = 1.15 \times 10^{-4}$.
Here we remark some relations of our results with the experimental analyses and other theoretical works.

4 Concluding Remarks

4.1 Two-photon collision processes

Some experimental groups measured the two-photon partial width of the \( f_0 \) state \( \Gamma_{\gamma\gamma}(f_0) \) through the fits of the \( \pi\pi \) mass spectra by the Breit-Wigner form,

\[
\sigma_{\gamma\gamma \to f_0 \to \pi\pi}(w) = 8\pi \left( \frac{m_R}{w} \right)^3 \frac{\Gamma_{\gamma\gamma}(f_0) B(f_0 \to \pi\pi) \Gamma_{\text{tot}}(w)}{(w^2 - m_R^2)^2 + m_R^2 \Gamma_{\text{tot}}(w)},
\]

where \( m_R \) is the \( f_0 \) mass, \( \Gamma_{\text{tot}} \) is the \( f_0 \) total width and \( B(f_0 \to \pi\pi) \) the branching ratio into \( \pi^0\pi^0 \) or \( \pi^+\pi^- \). Using \( m = 975 \text{ MeV} \), \( \Gamma_{\text{tot}}(m) = 33 \text{ MeV} \) and \( B(f_0 \to \pi^0\pi^0) = 1/3 \times 0.78 \), the Crystal Ball Collaboration gives \( \Gamma_{\gamma\gamma}(f_0) = 0.31 \pm 0.17 \text{ keV} \). JADE Collaboration reports \( \Gamma_{\gamma\gamma}(f_0) \approx 0.42 \text{ keV} \) from the \( \pi^0\pi^0 \) final state, though the significance of the \( f_0(980) \) signal is not high. From the \( \pi^+\pi^- \) final state Mark II detector group gives \( \Gamma_{\gamma\gamma}(f_0) = 0.29 \pm 0.11 \text{ keV} \), where \( m = 1012 \text{ MeV} \), \( \Gamma_{\text{tot}}(m) = 52 \text{ MeV} \) and \( B(f_0 \to \pi^+\pi^-) = 2/3 \times 0.78 \) are used. If we use the same mass and width of the \( f_0 \) state as the Crystal Ball Collaboration, we might have 0.17 keV instead of 0.29 keV, because \( \Gamma_{\gamma\gamma}(f_0) \) is proportional to \( m_R^2 \Gamma_{\text{tot}} \). The values 0.31 keV in the \( \pi^0\pi^0 \) spectrum and 0.17 keV in the \( \pi^+\pi^- \) one may indicate that the ratio of the peaks of the charged and neutral pion pairs could not be simply \( 2 : 1 \) as we encountered in Sec.2.

Although there is seen only a subtle signal of the \( f_0(980) \) peak in the experimental two-pion mass spectra masked by the large tail of the \( f_2(1279) \) resonance, the theoretical works including us predict the existence of the clear \( f_0(980) \) peak over the \( f_2 \) background. For example, the \( f_0(980) \) peak in the peak solution of the \( I = 0 \) cross section is about \( 180 \text{ nb} \) as seen in Fig.17 of Ref.\cite{3}. If the pion-loop dominates the whole energy region below 1 GeV, the \( f_0(980) \) peak does not appear reflecting the structure of the \( \pi\pi \) scattering amplitude, but we have to include the kaon-loop into the game. The kaon-loop is followed by the \( K\bar{K} \to \pi\pi \) transition amplitude, which displays a peak related to the \( f_0(980) \) state below the \( K\bar{K} \) threshold as shown in Figs.1b and 2b. Indeed such a peak is observed in the radiative \( \phi \) decay into \( \gamma\pi\pi \). The shape by our calculation is too narrow, but we hardly avoid the peak structure, because the \( f_0(980) \) state strongly couple to the \( K\bar{K} \) channel. Theoretical estimate of the partial width of \( f_0 \) gives 0.13 \( \rightarrow 0.36 \text{ keV} \) in \cite{3} and 0.20 keV in \cite{3}. But if we make the same rough estimate for \( \sigma_{\text{peak}} = 180 \text{ nb} \), we have \( \Gamma_{\gamma\gamma}(f_0) B(f_0 \to \pi\pi) \approx 0.54 \text{ keV} \).

3. The \( K^+K^- \to K^0\bar{K}^0 \) amplitude is dominated by the isoscalar \( K\bar{K} \) elastic amplitudes and is not small compared with the amplitudes at the \( K\bar{K} \) threshold as shown in Fig. (6b). We get \( B(\phi \to \gamma\gamma K^0\bar{K}^0) = 4.21 \times 10^{-7} \).

Figure 6: (a) \( dB(\gamma\gamma K^0\bar{K}^0)/dw \times 10^7/\text{MeV} \). (b) Lines are the same as in previous Figs.
The two-pion mass spectrum produced by two-photon collisions is a useful field to investigate the off-shell $K\bar{K} \to \pi\pi$ scattering amplitude and the production mechanism. We expect, therefore, that the partial wave analysis of the two-pion mass spectrum in the region of 1 GeV is performed.

As to the $a_0(980)$ state the experimental data by Crystal Ball and JADE Collaborations give a clear peak in the $\pi\eta$ spectrum\[^{12,13}\]. The peak of the cross section is about 40 nb, which is very similar to ours. The two-photon partial width is $\Gamma_{\gamma\gamma}(a_0)B(a_0 \to \pi\eta) \approx 0.19$ keV\[^{12}\] and $\approx 0.28$ keV\[^{13}\], which are a little smaller than our rough estimate, 0.35 keV. They include a non-resonant smooth background under the the $a_0$ peak, however, while our calculation uses the full cross section without any background. Further, the $a_0(980)$ state could be expressed not by the Breit-Wigner form but by a cusp, whose peak is sit just at the $K\bar{K}$ threshold. These may give an effect on the values of $\Gamma_{\gamma\gamma}(a_0)$.

The threshold peak of the $\gamma\gamma \to K^0\bar{K}^0$ cross section is about 7 nb, that is not so small compared with the $\pi\pi$ and $\pi\eta$ states. There are experimental data for $K^0\bar{K}^0$ final state, which indicate about 2 nb at 1.1 GeV with a large error\[^{14}\], with which our prediction could be consistent.

### 4.2 The radiative decays of the $\phi$ meson

As to the final $\gamma\pi^0\pi^0$ state, all the experimental results have a peak near 970 MeV and a small tail below 600 MeV in the invariant $\pi^0\pi^0$ mass distributions. The branching ratio $B(\phi \to \gamma\pi^0\pi^0)$ over the whole mass range is given $(1.158 \pm 0.093 \pm 0.052) \times 10^{-4}$ by the SND detector group\[^{23}\], $(1.08 \pm 0.19 \pm 0.09) \times 10^{-4}$ by the CMD-2 group\[^{24}\] and $(1.09 \pm 0.03 \pm 0.05) \times 10^{-4}$ by KLOE collaboration. These values are consistent with each other. The theoretical work\[^{10}\] related to ours gives $B(\phi \to \gamma\pi^0\pi^0) = 0.8 \times 10^{-4}$, which is a little larger than, but not inconsistent with our calculation.

The results become strongly model-dependent, however, when we extract the $B(\phi \to \gamma f_0(980))$ from the $\phi \to \gamma\pi^0\pi^0$ mass distribution. The SND group assumes that the whole mass distribution is dominantly given by the $f_0(980)$ resonance with $m_{f_0} = 969.8 \pm 4.5$ MeV and $\Gamma_{f_0 \to \pi\pi} \approx 200$ MeV, and then gives

$$B(\phi \to \gamma f_0) = (3.5 \pm 0.3^{+1.2}_{-0.5}) \times 10^{-4}. \quad (4.2)$$

The CMD-2 group gives

$$B(\phi \to \gamma f_0(980)) = (3.05 \pm 0.25 \pm 0.72) \times 10^{-4} \quad (4.3)$$

under the single $f_0(980)$ fit, but

$$B(\phi \to \gamma f_0(980)) = (1.5 \pm 0.5) \times 10^{-4} \quad (4.4)$$

under the two-resonance fit with $f_0(980)$ and $f_0(1200)$, which is smaller by a factor of 2 than the one under the single resonance fit. On the other hand KLOE collaboration fits the $\pi^0\pi^0$ mass distribution in terms of the $\gamma(f_0 + \sigma)$ mode, and gives

$$B(\phi \to \gamma f_0) = 3 \times (1.49 \pm 0.07) \times 10^{-4} \approx 4.47 \times 10^{-4}, \quad (4.5)$$

where the group uses $m_\sigma = 478$ MeV and $\Gamma_\sigma = 324$ MeV, which are set in Ref.\[^{30}\]. The values of $B(\phi \to \gamma f_0)$ are, thus, scattered depending on the assumption adopted by each group, though the invariant mass distributions coincide with each other.

The last result by KLOE collaboration is much larger than others. According to their analysis the $\sigma$ contribution gives a bump near 500 MeV, but the experimental mass distribution is very small there, so that they are forced to erase the bump by the destructive interference with the $f_0$ resonance with a large width, where $m_{f_0} = 973$ MeV and $\Gamma(f_0 \to \pi\pi) = 260$ MeV. This enlarges the $\gamma f_0$ branching ratio. On the other hand, Achasov and Gubin\[^{18}\] analyze the Novosibirsk data\[^{22,24}\] by assuming the $f_0(980)$ and $f_0(1500)$ resonances with the background phase $\delta_{b\ell}(w) = b\sigma_1(w)$ with $b$ being a positive constant, which describe the $I = 0$ $\pi\pi$ scattering phase shift rather well, and give $B(\phi \to \gamma f_0) = 3 \times (0.78 \sim 1.01) \times 10^{-4} \approx (2.34 \sim 3.03) \times 10^{-4}$. Since the off-shell amplitude $T(K^+K^- \to \pi^0\pi^0)$ must have the same phase as the pure isoscalar $T^{(0)}(\pi\pi \to \pi\pi)$ amplitude below the $K\bar{K}$ threshold by the unitarity, it
should be verified that such a sum of the $\sigma$ and $f_0(980)$ states can really give the isoscalar phase shift of $\pi\pi$ elastic scattering.

We note that the QCD sum rule give $B(\phi \rightarrow \gamma f_0) = (2.7 \pm 1.1) \times 10^{-4}$ and $(3.5 \times (1 \pm 0.3)) \times 10^{-4}$.

It is not reasonable to expect the large contribution from the $f_0(980)$ to the low mass region such as 500 MeV from the elastic $\pi\pi$ scattering behavior, so that $B(\phi \rightarrow \gamma f_0)$ could be much reduced to $\sim 1.7 \times 10^{-4}$, if the $f_0(980)$ contribution is restricted to the mass region larger than 900 MeV in the data of SND group\[23\]. It is dangerous, therefore, to discuss the nature of the $f_0(980)$ and $a_0(980)$ using the too model-dependent values.

As to the $\gamma\pi^0\eta$ final states, two Novosibirsk groups SND\[27\], CMD-2\[26\] and KLOE collaboration\[29\] give $B(\phi \rightarrow \gamma\pi^0\eta)$ a consistent value $(0.80 \sim 0.90) \times 10^{-4}$ within the errors. The shapes of the invariant $\pi^0\eta$ mass distribution seem to be different, though the number of events maybe not enough. The shape of the KLOE collaboration does not show a clear peak near the $KK$ threshold, that is very similar to ours and other theoretical works\[17\]. The integrated branching ratio in Ref.\[10\] is $0.87 \times 10^{-4}$, which is a little smaller than ours, but not inconsistent with ours.

Since there are no clear resonant states other than the $a_0(980)$ state below 1 GeV the branching ratio for $\gamma\pi^0\eta$ could be assigned to the $a_0(980)$ state as a whole, irrespective of the assumed mass and width of the $a_0$ state. On the other hand our calculation on the integrated branching ratio $B(\phi \rightarrow \gamma\pi^0\pi^0)$ comes almost from the $f_0$ peak, so that $B(\phi \rightarrow \gamma f_0) \sim 3 B(\phi \rightarrow \gamma\pi^0\pi^0) = 1.8 \times 10^{-4}$. Thus, it is not unreasonable to say that $B(\phi \rightarrow \gamma f_0) > B(\phi \rightarrow \gamma a_0) \sim 1 \times 10^{-4}$.

The decay into the $\gamma K^0\bar{K}^0$ state may reveal the structure of the $KK$ elastic amplitudes just above the $KK$ threshold. Our result of the branching ratio is $4.2 \times 10^{-7}$, that is 10 times as large value as the one predicted by the $f_0(980)\pm a_0(980)$ resonance model written in terms of the Breit-Wigner forms\[31\].

We have studied how the $f_0(980)$ and $a_0(980)$ are produced in the two-photon collisions and the radiative decays of the $\phi$ meson. The structure of the production amplitudes common to both of the processes is that the production proceeds through the charged pion- and kaon-loop diagrams. It is understood by the same mechanism that the two-pion spectrum in the $J/\psi \rightarrow \phi\pi\pi$ show a peak near $f_0(980)$ resonance as like as the $\phi \rightarrow \gamma\pi\pi$ decay process, if we respect the OZE rule at the decay vertices and $\phi$ be a spectator. Similarly, the bump peaked at 500 MeV in the two-pion mass spectrum in the $J/\psi \rightarrow \omega\pi\pi$ comes from the pion-loop followed by $\pi\pi$ elastic scattering amplitude. Thus, these processes offer a unique field to investigate the off-shell $KK \rightarrow \pi\pi$ and $\rightarrow \pi\eta$ amplitudes accompanied by the pion- and kaon-loop diagrams.

In I we studied how these scalar states are generated through a unitarized chiral perturbation theory. There we pointed out that the generation mechanism may be different from each other, and the $f_0$ state is the typical bound state resonance, but the $a_0$ the cusp generated by the channel coupling. We find that this nature of the generation is consistent with the features of the production processes. We, thus, conclude within our scheme that we have naturally 1 keV $> \Gamma_{\gamma\gamma}(f_0) > \Gamma_{\gamma\gamma}(a_0)$ and $B(\phi \rightarrow \gamma f_0) > B(\phi \rightarrow \gamma a_0) \sim 10^{-4}$.

The experimental $\pi^0\pi^0$ invariant mass spectrum in the two-photon collisions show a small but broad bump centered at 500 MeV, though this bump does not play any leading role in our production processes in contrast to the $f_0$ and $a_0$ states. The bump is naturally reproduced by our model with S-wave $\pi\pi$ scattering amplitudes following the charged pion loop. We regard the bump not as the resonant state but as the vacuum excitation owing to the spontaneous breakdown of chiral symmetry. We point out that the $\sigma$ resonance with a mass about 480 MeV and a large width would not be consistent with these production processes nor the scattering processes.

We cannot find any reason to change our point of view of the low mass scalar mesons developed in I through the study of the photon related production processes.

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