Property of Chiral Scalar and Axial-Vector Mesons in Heavy-Light Quark Systems

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Recently we have proposed a new level-classification scheme of hadrons with a manifestly covariant framework. In this scheme the requirement of chiral symmetry on the light quark leads to a prediction of existence of new type of scalars \(X_B, X_D\) and axial-vectors \(X_{B^*}, X_{D^*}\) as the chiral partners of ground state pseudoscalar \(B, D\) and vector \(B^*, D^*\) mesons, respectively. They belong to “relativistic S-wave states,” and are discriminated from the conventional \(P\)-wave mesons with \(j_q = 1/2\) appearing in the heavy quark effective theory. In this talk we examine the properties of these chiral mesons: The mass-splittings between the respective chiral partners are predicted to be equal, and the decay widths of one pion emission of \(X_B, X_D, X_{B^*}\) and \(X_{D^*}\) are to take the same value due to both chiral and heavy quark symmetries. Some experimental indications for existence of \(X_B\) and \(X_{D^*}\) are also given, which are consistent with the above prediction.

§1. Introduction

Recently, we have proposed a covariant level classification scheme of hadrons \(^1\), unifying the seemingly contradictory two viewpoints, non-relativistic one with \(LS\)-coupling scheme and relativistic one with chiral symmetry. In this scheme, it is expected that the hadron spectra are to show, concerning light quark constituents, the approximate \(\tilde{U}(12)_{SF}\) symmetry (including static \(SU(6)_{SF}\) as a subgroup) around the lower mass region, and is predicted the existence of many relativistic states, called “chiralons,” which are out of the framework of conventional \(LS\) coupling scheme in NRQM. Recently, the existence of a light scalar \(\sigma\) meson with the property as partner of \(\pi\) meson in the linear representation of chiral symmetry seems to be confirmed \(^2\), \(^3\).

In our classification scheme, this \(\sigma\) is naturally classified into the relativistic \(S\)-wave \(q\bar{q}\) state, which is to be discriminated from the \(3P_0\)-state appearing in NRQM.

In heavy-light quark \(n\bar{b}(=ub, d\bar{b})\) systems, the existence of chiralons, new scalar and axial-vector mesons, denoted as \(X_B = ^t(X_{B^+}, X_{B^0})\) and \(X_{B^*} = ^t(X_{B^{*+}}, X_{B^{*0}})\), is expected to exist as the chiral partners of the pseudoscalar \(B = ^t(B^+, B^0)\) and the vector \(B^* = ^t(B^{*+}, B^{*0})\) mesons, respectively. Similarly, in the \(u\bar{c}(=uc, d\bar{c})\) system, the scalar \(X_{\bar{D}} = ^t(X_{\bar{D}^0}, X_{\bar{D}^-})\) and the axial-vector \(X_{\bar{D}^*} = ^t(X_{\bar{D}^{*0}}, X_{\bar{D}^{*-}})\) mesons, is expected to exist as the chiral partners of the pseudoscalar \(\bar{D} = ^t(D^0, D^-)\) and the vector \(\bar{D}^* = ^t(D^{*0}, D^{*-})\) mesons, respectively. These chiral scalar and axial-vector mesons are classified to the relativistic \(S\)-wave states, which are discriminated from the \(P\)-wave mesons appearing in NRQM, or the scalars \(B^*_0, D^*_0\) and axial-vectors...
$B_s^*$, $D_s^*$ with $j_q = 1/2$ appearing in HQET.

In this talk we investigate the properties of these chiral scalar and axial-vector mesons by taking into account chiral symmetry for the light quark component, as well as HQS for the heavy quark component. A similar approach has already been done in Refs. 4, 5. However, they assigned as the respective chiral partners of S-wave state mesons to the $j_q = 1/2$ $P$-wave mesons. This assignment is crucially different from ours.

§2. Constraints on one pion emission process from HQS

We consider general constraints from HQS on the processes of one-pion emission $X_B \rightarrow B\pi, X_{B^*} \rightarrow B^*\pi, X_D \rightarrow D\pi$ and $X_{D^*} \rightarrow D^*\pi$, which are expected to be the main decay modes of the relevant mesons.

The HQ spin symmetry relates $B$, $X_B$, $D$ and $X_D$, respectively, to $B^*$, $X_{B^*}$, $D^*$ and $X_{D^*}$, and the HQ flavor symmetry relates $B$, $B^*$, $X_B$ and $X_{B^*}$, respectively, to $D$, $D^*$, $X_D$ and $X_{D^*}$, with the same velocity. Thus, the $S$-matrix elements of the relevant four decay modes are related with one another, and are represented by one universal amplitude $\xi$ as,

$$-i\langle\pi B^*(0,\epsilon(0))|U_f|X_{B^*}(0,\epsilon(0))\rangle = -i\langle\pi D^*(0,\epsilon(0))|U_f|X_{D^*}(0,\epsilon(0))\rangle$$

$$= \langle\pi B(0)|U_f|X_B(0)\rangle = \langle\pi D(0)|U_f|X_D(0)\rangle$$

$$= -\xi \frac{1}{(2\pi)^2} \frac{1}{(2\pi)^3 2E_\pi} i(2\pi)^4 \delta^{(4)}(P_{X_M} - P_M - p_\pi), \quad (2.1)$$

where $U_f$ is the translational operator of time from $-\infty$ to $+\infty$, $P_{X_M}$ ($P_M$) being the initial (final) heavy meson momentum, $p_\pi$ being the emitted pion momentum, and 0 represents the three velocity $v = 0$, the longitudinally polarized states of $B^*$, $D^*$ and $X_{B^*}$, $X_{D^*}$ appear, and a common factor of $\frac{1}{(2\pi)^3} \sqrt{\frac{1}{(2\pi)^3 2E_\pi}}$ has been introduced.

The decay widths of $X_M = X_B, X_{B^*}, X_D, X_{D^*}$ are given by

$$\Gamma_{X_M} = \frac{3}{2m_{X_M}} \frac{|p|}{4\pi m_{X_M}} (2\xi \sqrt{m_{X_M} m_M})^2 \approx 3 \frac{\sqrt{(\Delta m_M)^2 - m_\pi^2}}{8\pi} (2\xi)^2, \quad (2.2)$$

where we use the approximation $m_{X_M} \approx m_M, |p| = \sqrt{(\Delta m_M)^2 - m_\pi^2}$ is the pion CM momentum, and the factor 3 comes from the isospin degree of freedom of the final $|\pi M\rangle$ state. As expressed by Eq. (2.2), the decay widths of the relevant processes are dependent only upon the corresponding mass difference $\Delta m_M$.

§3. Chiral and Heavy Quark Symmetric Lagrangian

Here we construct the chiral and heavy quark symmetric Lagrangian of $B$ and $\bar{D}$ systems, by using the quark bi-spinor representation in the new classification scheme: $U^{(\pm)}_{\alpha}(v) = \sum_\phi (1/2\sqrt{2}) \Gamma_\phi \phi(1 + iv \cdot \gamma)_{\alpha\beta}$, where $\alpha(\beta)$ are light quark

5) Here the normalization of states $|B(0)\rangle \equiv a_{B(0)} |0\rangle$, etc. are used, where $[a_{B(0)}, a_{B(0)}^\dagger] = \delta^{(3)}(p - p')$. 
(heavy antiquark) spinor index, and the summation is taken for \( \phi = B, X_B, B_\mu^*, X_B^*; D, X_D, \bar{D}_\mu, X_D^* \); \( \Gamma_B = \Gamma_{\bar{D}} = i \gamma_5, \Gamma_{X_B} = \Gamma_{X_D} = \pm 1, \Gamma_{B_\mu^*} = \Gamma_{D_\mu^*} = i \gamma_\mu, \Gamma_{X_B^*} = \Gamma_{X_D^*} = \pm \gamma_5 \gamma_\mu \). The field \( \hat{\phi} \) is related with the ordinarily normalized field \( \phi \) as 
\[
\phi(X) = (1/\sqrt{2m_Q})e^{i m_Q \cdot X} \hat{\phi}(X),
\]
where \( m_Q = m_b(m_c) \) for \( B(\bar{D}) \) system. The chiral \( U_A(1) \) transformation for the light quark is given by \( U(\pm) \to e^{\pm i \alpha \gamma_3} U(\pm) \). By using the conjugate bispinor, defined by \( U(\pm) \equiv \gamma_4 U(\pm) \gamma_\nu \), the free Lagrangian is given by
\[
\mathcal{L}^{\text{free}} = \langle \hat{U}^- \rangle \left( iv \cdot \partial \hat{\phi} / 2 - m_q \right) U(+) = \sum_\phi \hat{\phi} \left( iv \cdot \partial \hat{\phi} / 2 - m_q \right), \tag{3.1}
\]
where \( \langle \cdot \rangle \) means the trace on spinor indices, and the light quark mass \( m_q \) is common for all \( B \) and \( D \) systems. The total meson mass \( m_M(M=B, D) \) is given by \( m_M = m_Q + m_q \), thus \( m_B = m_{X_B} = m_{B^*} = m_{X_{B^*}} = m_b + m_q \) and \( m_D = m_{X_D} = m_{D^*} = m_{X_{D^*}} = m_c + m_q \) in symmetric limit.

The \( \sigma \) and \( \pi \) fields are transformed as \( (\sigma + i \gamma_5 \tau \cdot \pi) \to e^{i \alpha \gamma_3} (\sigma + i \gamma_5 \tau \cdot \pi) e^{i \alpha \gamma_3} \), thus the chiral symmetric Yukawa coupling is given by
\[
\mathcal{L}^{\text{Yukawa}} = -\eta \langle \hat{U}^- \rangle (\sigma + i \gamma_5 \tau \cdot \pi) U(-), \tag{3.2}
\]
Through the spontaneous breaking of chiral symmetry the \( \sigma \) acquires vacuum expectation value \( \langle \sigma \rangle \equiv \sigma_0 (= f_\pi \text{ in SU}(2) \text{ linear } \sigma \text{ model}) \), which induces the mass splittings \( \Delta m \) between chiral partners. The \( \Delta m \) are universal in \( B \) and \( D \) systems:
\[
\Delta m \equiv m_{X_B} - m_B = m_{X_{B^*}} - m_{B^*} = m_{X_D} - m_D = m_{X_{D^*}} - m_{D^*}, \tag{3.3}
\]
which is given by \( \Delta m = 2 \eta \sigma_0 = 2 \eta f_\pi \) in Lagrangian (3.2). Even in the case, considering the contribution from all the other possible forms of effective chiral symmetric Lagrangian, the universality of mass splittings are shown to be preserved. Thus, following the argument given in the last sub-section, the decay widths of one pion emission also become universal,
\[
\Gamma(\Delta m) \equiv \Gamma_{X_B \to B\pi} = \Gamma_{X_D \to D\pi} = \Gamma_{X_{B^*} \to B^*\pi} = \Gamma_{X_{D^*} \to D^*\pi}, \tag{3.4}
\]
although the magnitude of \( \Gamma(\Delta m) \) cannot be predicted in the present framework. 

**§4. Experimental Evidence for \( X_B \) and \( X_{D^*} \)**

In order to examine phenomenologically whether these chiral mesons really exist or not, we analyze the \( B\pi \) mass spectra \(^6\) in \( 5.4\text{GeV} < m_{B\pi} < 5.9\text{GeV} \), obtained through \( Z^0 \)-boson decay by L3 \(^7\) and ALEPH \(^8\) collaborations. In the relevant

\(^*)\) In Lagrangian (3.2), \( \xi \) is given by \( \xi = \eta = \Delta m/(2f_\pi) \). By taking \( \Delta m \approx 300\text{MeV} \) in Eq. (2-2) as an example, \( \Gamma(\Delta m = 300\text{MeV}) = 331\text{MeV} \). We can also consider, as one of the possible forms of the effective Lagrangian, the \( \mathcal{L}^{(d)} = -k \langle \hat{U}^- \rangle [iv_\nu \partial_\nu (\sigma + i \tau \cdot \pi)] \langle U(+) \rangle \), where \( k \) is a coupling constant of \( O(1/m_q) \). In this case \( \xi = \eta + 2k v \cdot p_\pi \approx \eta - k \Delta m = (1 - 2k f_\pi) \Delta m/(2f_\pi) \). By taking a natural value of \( k \approx 2/m_\pi \) as an example, the \( \Gamma(\Delta m = 300\text{MeV}) = 88\text{MeV} \). As is seen in these examples, the magnitude of \( \Gamma(\Delta m) \) itself is largely dependent upon the value of \( k \).
mass region of $B\pi$ channel, the $X_B$, and the $P$-wave mesons, $B_2^0(j_q = 3/2)$ and $B_0^0(j_q = 1/2)$, are expected to be observed directly. We use the following forms of squared amplitude $|\mathcal{M}|^2$ and background $|\mathcal{M}_{BG}|^2$:

$$|\mathcal{M}|^2 = |r_1 e^{i\theta_3} \Delta_X(s) + r_2 e^{i\theta_2} \Delta_B(s)|^2 + |r_3 e^{i\theta_3} \Delta_{\text{other}}(s)|^2,$$

$$|\mathcal{M}_{BG}|^2 = P_1(m_{B^+_2} - P_2)P_3 e^{P_4(m_{B^+_2} - P_2) + P_5(m_{B^+_2} - P_2)^2 + P_6(m_{B^+_2} - P_2)^3},$$  \hspace{1cm} (4.1)

where $P_1$-$P_6$ are parameters and $\Delta_R(s) = -m_R \Gamma_R/(s - m_R^2 + i m_R \Gamma_R)$; $\Delta_X(s)$ ($\Delta_B(s)$) are the Breit-Wigner amplitude for $X_B (B_0^0)$ mesons, and $\Delta_{\text{other}}$ represents contributions from all the other possible resonances including $B_2^+$. Preliminary results of the fit are given in FIGURE 1. Both data show a dip at the same energy $m_{B^+_2} \approx 5.55 \text{GeV}$, which is reproduced by the interference between the $X_B$ with a narrow width and the $B_0^0$ with a wide width. The mass and width of $X_B$ is given by $m_X = 5540 \text{MeV}$ and $\Gamma_X = 21 \text{MeV}$, and the obtained values of $\chi^2$ is $\tilde{\chi}^2 = 22.92/20 = 1.15$. We also tried the fit without $X_B$, and obtain almost the same value of $\tilde{\chi}^2 = 26.280/24 = 1.0$, thus, only by this analysis, we cannot obtain the definite conclusion on existence of $X_B$.

Similar analysis is also done$^9$ on the $D^*\pi$ mass spectra by CLEO and DELPHI collaborations, and we obtained the preliminary result on existence of $X_{D^*}$ with $m_{X_{D^*}} = 2306 \text{MeV}$ and $\Gamma_{X_{D^*}} = 21 \text{MeV}$.

Here it may be worthwhile to note that the preliminary values above obtained on properties of $X_B$ and $X_{D^*}$ are consistent with our predictions by heavy quark symmetry and chiral symmetry, Eqs. (5) and (6). $m_{X_B} - m_B = 261 \text{MeV} \approx m_{X_{D^*}} - m_{D^*} = 296 \text{MeV}$, $\Gamma_{X_B} = 21 \text{MeV} = \Gamma_{X_{D^*}} = 21 \text{MeV}$.

§5. Concluding Remarks

In the new classification-scheme, the chiral scalars $X_B, X_D$ and axial-vectors $X_{B^*}, X_{D^*}$, which are the chiral partners of $B, D$ and $B^*, D^*$, respectively, are predicted to exist, which are to be discriminated from the conventional $P$-wave state mesons. Preliminary analyses of experimental data give some indications of possible existence of both $X_B$ and $X_{D^*}$, besides $B_0^0$ and $D_1^0$, with the properties consistent with the theoretical prediction.

This fact seems to suggest that the new level-classification scheme is actually realized in heavy-light quark meson systems.

$^9$ In these experiments, the final photon was not detected, and so the following states decaying into $B^*\pi$ are also to be seen in $B\pi$ spectra indirectly through the successive $B^* \to B\gamma$ decay: $B^*_2(\to B^*\pi \to \gamma B\pi)$, $B_1(j_q = 3/2)(\to B^*\pi \to \gamma B\pi)$, $B_1^*(j_q = 1/2)(\to B^*\pi \to \gamma B\pi)$, and $X_B(\to B^*\pi \to \gamma B\pi)$. The observed $m_{B^*}$-values of these resonances become smaller than their real values by the missing photon energy $E_\gamma = m_{B^*} - m_B$. In Eq. (4.1) $\Delta_{\text{other}}$ are meant as including, in addition to direct $B^*_2(\to B\pi)$, the above mentioned indirect $B^*_2(\to B^*\pi)$ and $B_1^*(\to B^*\pi)$, which are $D$-wave decays. The indirect $B_1^*(\to B^*\pi)$ and $X_B(\to B^*\pi)$, which are $S$-wave decays interfering with each other, are considered to be included in $\Delta_{B^*_2}$ and $\Delta_{X_B}$, respectively.
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Fig. 1. $B\pi$ mass spectra obtained in (a) L3\(^7\) and (b) ALEPH\(^8\) collaborations. Contributions from the individual Breit-Wigner amplitudes are shown by dotted lines. In (a) the background contribution is subtracted. The $m_{X_B}$ and $\Gamma_{X_B}$ are determined only through CLEO data, since ALEPH data have much less statistics.

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