A possible cooling effect in high temperature superconductors

Anatoly A. Svidzinsky

Department of Physics, Stanford University, Stanford, CA 94305-4060

(March 22, 2022)

We show that an adiabatic increase of the supercurrent along a superconductor with lines of nodes of the order parameter on the Fermi surface can result in a cooling effect. The maximum cooling occurs if the supercurrent increases up to its critical value. The effect can also be observed in a mixed state of a bulk sample. An estimate of the energy dissipation shows that substantial cooling can be performed during a reasonable time even in the microkelvin regime.

Although the mechanism of high temperature superconductivity still remains unclear there is strong evidence that the order parameter in high temperature superconductors has nodes on the Fermi surface [1]. Such a property is also peculiar to some superconductors with heavy fermions. In this paper we show that presence of nodes of the order parameter can lead to a cooling effect. The cooling is reached by adiabatically increasing the supercurrent around a superconducting ring or a cylinder.

Currently the lowest temperature of a solid (\( T \sim 1\mu K \)) is achieved by using a method of adiabatic nuclear demagnetization [2]. Spontaneous magnetic ordering of the nuclear magnetic moments represents the low temperature limit for nuclear refrigeration. The ordering temperature due to nuclear dipole-dipole interaction is typically a fraction of a microkelvin and, therefore, to achieve temperatures lower than 0.1\( \mu K \) a new refrigeration technology is needed. In our cooling mechanism the conduction electrons are cooled during the adiabatic increase of the supercurrent instead of the nuclear spin system being cooled in the nuclear demagnetization process. Further development of the idea of cooling using the phenomena of superconductivity (superfluidity) could be promising to achieve the lowest temperature of solids.

Cooling effect in clean superconductors

Let us consider a superconducting ring (or a cylinder) made from a clean high temperature superconductor. For estimates one can assume the Fermi surface to be a cylinder and the order parameter has a simple \( d \)-wave form \( \Delta(p) = \Delta_0 \cos(2\phi) \), where \( \phi \) is the polar angle in the \( ab \) crystalline plane. Taking into account the real form of the Fermi surface might change numerical coefficients, but these details are not necessary for estimating the magnitude of the cooling effect. Let the \( c \)-axis of the superconductor be parallel to the symmetry axis of the ring and the temperature of the system is \( T \ll T_c \), where \( T_c \) is the superconducting transition temperature.

At low temperatures the main contribution to the electronic specific heat of the superconductor arises from quasiparticles that are activated above the gap in narrow vicinities of the nodes of the order parameter. The energy of these quasiparticles is given by

\[
E(p) = \sqrt{\xi(p)^2 + |\Delta(p)|^2},
\]

and the electronic specific heat is

\[
C(T) = 36\zeta(3)N_F \frac{T^2}{\Delta_0},
\]

where \( N_F \) is the density of states at the Fermi surface for normal metal and \( \zeta(3) \approx 1.202 \) is the Riemann zeta function (throughout this paper, apart from formula (2), we use units in which \( e_B = 1 \)). Therefore, the entropy of the system has the form:

\[
S(T) = \int_0^T C(T) \frac{dT}{T} = 18\zeta(3)N_F \frac{T^2}{\Delta_0}.
\]

Let us now consider the same superconducting ring with a supercurrent uniformly distributed across the cross section of the ring. This situation is possible if the thickness of the ring \( d \) is less than penetration depth of the magnetic field \( \lambda: d \lesssim \lambda \). In the presence of such superflow with a superfluid velocity \( v_s \), the quasiparticles can be considered quasiclassically with the energy [3,4]:

\[
E(p, v_s) = \sqrt{\xi(p)^2 + |\Delta(p)|^2} + p_{F\perp} \cdot v_s,
\]

where \( p_{F\perp} \) is the component of the Fermi momentum in the direction perpendicular to the \( c \)-axis. Due to the existence of the supercurrent, quasiparticle states with negative energy appear for momentum directions within the vicinity of nodes of the order parameter. These states are occupied even at zero temperature. The presence of these states gives a term, linear in \( T \), in the low temperature specific heat, which is dominant if \( p_{F\perp} v_s \gg T \). After a space averaging over the ring this term can be written in the form \( C(T) = 4\pi N_F T p_{F\perp} v_s / 3\Delta_0 \). As a result, the entropy in the presence of the supercurrent is:

\[
S(T) = \frac{4}{3} \pi N_F \frac{p_{F\perp} v_s}{\Delta_0} T.
\]

Let us assume that at an initial moment of time there is no superflow along the ring, and the ring is thermally isolated. A magnetic field is then turned on adiabatically so that the magnetic flux through the ring \( \Phi \) slowly increases in time. The induced electric field \( E \sim \Phi/lc \) (\( l \) is
the adiabatic increase of the supercurrent is proportional to $E$, and the energy dissipation can be estimated as
\[ \int_{t=0}^{t_f} I_{\text{diss}} E dt \propto \int_{t=0}^{t_f} \Phi^2 dt \sim \frac{\Phi_0^2}{t_f}. \]

The dissipation can be made negligibly small if the time $t_f$ to build up the magnetic field is big enough. In this process all the work which is produced by the induced electric field goes to increase the supercurrent around the ring; and the part of this work that dissipates into heat and changes the entropy can be made negligibly small. The necessary cooling time $t_f$ is estimated at the last section of this article.

The equation of entropy conservation in the adiabatic process of increasing the supercurrent has the form:
\[ 18\zeta(3)N_F \frac{T_0^2}{\Delta_0} = \frac{4}{3} \pi N_F \frac{p_{F\perp}v_s}{\Delta_0} T_f, \]

where $T_0$ and $T_f$ are the initial and the final temperatures of the ring. As a result, the final temperature $T_f$ after the adiabatic increase of the supercurrent is
\[ T_f = \frac{27\zeta(3) T_0^2}{2\pi p_{F\perp}v_s} = 5.2 \frac{T_0^2}{p_{F\perp}v_s} \]

This formula is applicable if $p_{F\perp}v_s \gg T_f$. The maximum cooling occurs when the supercurrent (and the superfluid velocity) is equal to its critical value, that is $p_{F\perp}v_s \sim 2\Delta_0 = 4.28T_c$ and
\[ T_{f(\text{min})} \approx \frac{T_0}{T_c} \]

So, to achieve substantial cooling, it is important to use a superconductor with high $T_c$. If, for example, $T_0 = 1K$ and $T_c = 100K$, after the adiabatic increase of the supercurrent, the ring cools down to $T_f = 0.01K$.

One should mention that we base our estimates on the assumption of uniform temperature distribution during the cooling process. However, the weight of quasiparticle states shifted from above the Fermi level to below depends on the angle the supercurrent makes with the nodal directions, which changes around the ring. This would provide a larger cooling effect at four points on the ring (at which $v_s$ constitutes $45^\circ$ angle with respect to $a, b$ axes) compared to elsewhere and in the ideal case of zero thermoconductivity the temperature distribution over the ring would be nonuniform with $T_{\text{max}}/T_{\text{min}} = \sqrt{2}$. However, finite thermoconductivity equalizes the temperature around the ring and nonuniform cooling would be very difficult to observe experimentally. Estimates show that nonuniform cooling can be detected if the cooling time is less than about $H^2/\nu_F^2\tau_N$, where $R$ is the radius of the ring, $v_F$ is the Fermi velocity and $\tau_N$ is the relaxation time in the normal state. For reasonable parameters we obtain that the necessary cooling time should be of the order of nanosecond. Real cooling time is much larger than nanosecond which guarantees approximately uniform temperature distribution during the cooling process. As a result the entropy dissipation due to the thermal current is negligible and does not modify our formulas.

Also we do not consider here the effect of fluxoid quantization which is negligible when the flux through the ring is much greater than the flux quantum. This gives a restriction on the superfluid velocity $v_s \gg v_F a/R$, where $a$ is the interatomic spacing. From the other hand, the critical superfluid velocity is of the order of $2\Delta_0/p_{F\perp} \approx v_F a/\xi$, where $\xi$ is the coherence length. So, we can omit the effect of fluxoid quantization in our problem if $R \gg \xi$.

One should also note that value $v_s \sim 2\Delta_0/p_{F\perp}$ determines depairing current density $j_{\text{depair}}$. Because of the grain structure of high temperature superconductors the critical current density $j_c$ is usually much less than the depairing value. A misalignment of adjacent grains suppresses the order parameter at the grain boundary and results in Josephson tunneling behavior and a near-exponential decrease in $j_c$ with misorientation angle. If the current through the superconductor is increased to $j_c$, rather than $j_{\text{depair}}$, the estimate (6) for the cooling temperature should be multiplied by the factor $j_{\text{depair}}/j_c$. To reach $j_c$ close to the depairing value one should use materials with well-aligned grains, which can be achieved in thin films. For example, $YBa_2Cu_3O_7$ films were obtained with the critical current density $j_c \approx 40MA/cm^2$, so that $j_{\text{depair}}/j_c \approx 5$. It is worth to note that if the cooling is achieved by applying external magnetic field, rather than creating a supercurrent along the film, the grain structure would not be an obstacle. In such a way, the magnetic field induces screening supercurrent in each grain. The intragrain supercurrent is not limited by the small value of the Josephson critical current in weak links between the grains and can be close to $j_{\text{depair}}$.

It is interesting to estimate the value of the cooling effect for a bulk sample in a mixed state. In a magnetic field $H$ ($H_{c1}, H_{c2}/T^2 < 0.1 \ll H_{c2}$) the entropy of a clean superconductor with line(s) of nodes is $S \propto T\sqrt{H/H_{c2}}$. So, an adiabatic increase of magnetic field from $H_0$ to $H_f$ cools down the sample to the temperature
\[ T_f = T_0 \sqrt{\frac{H_0}{H_f}}. \]

One should note that in conventional $s$-wave superconductors the electronic contribution to the entropy is exponentially small (due to finite gap in the excitation spectrum) and thus not dominant at low temperatures. However, in a mixed state of $s$-wave superconductors normal regions in the vortex cores give the linear temperature contribution to the electron entropy $S \sim N_F TH/H_{c2}$, which can exceed the phonon contribution at low enough temperature $T$ (typically less then 1K)
\[ T \approx \frac{T_D}{4} \left( \frac{T_D}{T_F} \right)^{1/2} \left( \frac{H}{H_{c2}} \right)^{1/2}, \]  

where \( T_D \) and \( T_F \) are Debye and Fermi temperatures accordingly. In this temperature region an adiabatic increase of applied magnetic field \( H (H_{c1} < H < H_{c2}) \) results in the cooling effect \( T_f = T_0 H_0 / H_f \).

**Effect of impurities**

At very low temperatures (when \( T \lesssim \gamma \)) the effect of impurity-induced bound states becomes significant. Let us consider the cooling effect at temperatures when the regime \( T \lesssim \gamma \ll T_c \) is achieved. The energy scale \( \gamma \) is the bandwidth of quasiparticle states bound to impurities \([9,10]\). For an order parameter with a line of nodes, the bandwidth \( \gamma \) and density of the quasiparticle states at zero energy \( N(0) \) are finite for any nonzero concentration of impurities. The quantities \( \gamma \) and \( N(0) \) depend on the impurity concentration \( n_i \) and the scattering phase shift \( \delta_0 \). For example, for scattering in the unitary limit \( (\delta_0 = \pi/2) \) and the \( d \)-wave order parameter, \( \gamma \sim \sqrt{\pi T_0 \Delta_0^2 / 2} \), \( N(0) = 8 \gamma N_F \log (4 \Delta_0 / \gamma) / \pi \Delta_0 \), where \( \Gamma_u = n_i / \pi N_F \). In the opposite limit (Born approximation with \( \delta_0 \to 0 \)) the bandwidth \( \gamma \) is exponentially small \( \gamma \sim 4 \Delta_0 \exp (-\pi \Delta_0 / 2 \Gamma_u \delta_0^2) \) and, therefore, the regime \( T \lesssim \gamma \) can be very difficult to achieve experimentally. For most high temperature and heavy fermion superconductors the impurity scattering is strong (or an intermediate strength) rather than in the Born limit \([11–14]\).

For \( T \lesssim \gamma \) and in the absence of superflow the electronic specific heat is given by \( C(T) = \pi^2 N(0) T / 3 \). So, instead of \([9]\), the entropy of the system is

\[ S(T) = \frac{\pi^2}{3} N(0) T. \]  

After the adiabatic process of creating the supercurrent around the ring with \( p_{F\perp} v_s \gg \gamma \), the entropy of the system is given approximately by the same expression \([9]\) as for the clean superconductor. As a result, the equation of the entropy conservation gives the following formula for the final temperature:

\[ T_f = \frac{\pi}{4} N(0) \frac{\Delta_0}{p_{F\perp} v_s} T_0 \]  

(13)

For maximum cooling \( (p_{F\perp} v_s \sim 2 \Delta_0) \)

\[ T_{f_{\text{min}}} = \frac{\pi}{8} N(0) T_0 \]  

(14)

For scattering in the unitary limit we get:

\[ T_{f_{\text{min}}} \approx \frac{\gamma}{4 \Delta_0} \log \left( \frac{4 \Delta_0}{\gamma} \right) T_0 \sim \frac{\gamma}{T_c} T_0 \]  

(15)

In the case of a bulk superconductor in a mixed state the presence of impurities results in appearance of a crossover field \( H^* \) when square-root magnetic field dependence of the specific heat changes to a dependence of the form \( H \ln H \) \([15]\). If \( H > H^* \) the influence of impurities is negligible and the cooling effect is given by Eq. \([11]\). However, if \( H_0 < H^* < H_f \) then \( T_f \approx T_0 \sqrt{H^*/H_f} \).

**Estimation of the energy dissipation**

If there is a time-dependent current in a superconductor, the normal electrons give a finite amount of dissipation because the supercurrent is not a zero-impedance shunt at nonzero frequency \( \omega \) (in a two-fluid model \( I m \sigma = n_s e^2 / m \omega \)). This dissipation increases the entropy of the system. Given an imposed ac current density \( j = j_0 \sin (\omega t) \), the power dissipated per unit volume at low frequencies is \([16]\):

\[ \frac{dW_{\text{diss}}}{dt} = \Re \left( \frac{1}{\sigma} \right) j^2 \approx \frac{\Re \sigma}{(I m \sigma)^2} j^2 = \frac{16 \pi^2 \lambda^4}{c^4} \omega^2 \Re \sigma j^2, \]  

(16)

where \( \sigma \) is the conductivity of the superconductor, \( \lambda \) is the penetration depth. To estimate the change of the entropy \( \Delta S \) in the process of increasing the supercurrent from \( j = 0 \) to \( j = j_0 \) one can integrate Eq. \([17]\) in the limits \( 0 < t < t_f = \pi / 2 \omega \). Generally speaking, the conductivity \( \sigma \) can depend on the magnitude of the supercurrent in the superconductor. However for estimation one can take \( \sigma \) to be equal to its maximum value (at a maximum value of the current). As a result, we obtain the following estimate for the change of the entropy per unit volume:

\[ \Delta S = \int_{0}^{t_f} \frac{1}{T} \frac{dW_{\text{diss}}}{dt} \approx \int_{0}^{t_f} \sin (\omega t) \Re \sigma j^2 dt \approx \frac{16 \pi^2 \lambda^4}{3T f c^4} \Re \sigma j_0^2. \]  

(17)

The energy dissipation gives rise to a heating of the superconductor and changes the final temperature \( T_f \). To estimate the magnitude of this effect one should add \( \Delta S \) to the left side of Eq. \([17]\). It is possible to obtain substantial cooling even if \( \Delta S \) is larger than the initial entropy. However, to make the cooling effective, one should increase the supercurrent slowly enough to ensure that the dissipation does not significantly change the entropy: \( \Delta S \lesssim 4 \pi N_F T_f p_{F\perp} v_s / 3 \Delta_0 \). Taking into account \( j_0 \approx e v_s c = mc^2 v_s / 4 \pi e \lambda^2 \), we see that dissipation can be omitted if the time of increasing the supercurrent from zero to its critical value satisfies the following condition

\[ t_f \gtrsim \frac{\Re \sigma \Delta_0^2}{2 \pi^2 N_F v_s^2 T_f c^4}. \]  

(18)

The electrical conductivity for a superconductor with an order parameter that vanishes along a line of nodes has a
universal limiting value which is independent of the concentration of impurities and the scattering phase shift \[17\]. For the two dimensional d-wave order parameter and an isotropic 2D Fermi surface the universal value has the form \[18\]: \(\text{Re}\sigma(\omega \rightarrow 0, T \ll \gamma) \sim e^2 N_F v_F^2 \tau_0\), where \(\tau_\Delta \simeq h/\pi \Delta_0\) is a universal transport time. However, the superflow modifies this result and necessarily increases the conductivity in the region interesting for cooling (i.e. when \(p_{F\perp} v_s \gg \gamma\)).

To estimate the electrical conductivity in the presence of the superflow, one can use the same semiclassical approach as for the case of thermal conductivity \[4\]. At \(\omega \rightarrow 0\) and \(\gamma \ll p_{F\perp} v_s\) the final result is given by (cf. \[20\], \[18\]):

\[
\text{Re}\sigma_{ij} = \frac{ne^2}{2m} \int_{-\infty}^{+\infty} d\omega \frac{\tau(\omega, v_s)}{\cosh^2 \left(\frac{\omega}{2T}\right)} \left|\vec{\omega} - p_{F\perp} v_s \right| \sqrt{\left(\vec{\omega} - p_{F\perp} v_s\right)^2 - |\Delta(\phi)|^2}.
\]

(19)

where \(\tau(\omega, v_s)\) is the relaxation time for quasiparticle scattering on impurities in the superconducting state, \(n\) is the electron density. For the d-wave order parameter we obtain for \(T \ll p_{F\perp} v_s \ll \Delta_0\), \(\text{Re}\sigma_{ii} = ne^2 \tau_N/m\), in the Born approximation: \(\text{Re}\sigma_{ii} = ne^2 \tau_N (1 + |\cos \phi_i|) p_{F\perp}^2 v_s^2 / 4 m \Delta_0^2\), in the unitary limit, where \(\tau_N\) is the relaxation time in the normal state, \(\phi_i\) is the angle between \(v_s\) and the \(i\)-axis, \(i = a, b\). For the supercurrent close to its critical value we get the estimate \(\text{Re} \sigma \sim ne^2 \tau_N / m = e^2 N_F v_F^2 \tau_\Delta\) which is \(\tau_N / \tau_\Delta \sim \Delta_0^2 / \gamma^2 \gg 1\) times larger than the universal value. If we substitute this estimate into \(18\), we obtain the following limitation for the necessary cooling time

\[
t_f \geq \tau_N T_c^2 / T_f^2.
\]

(20)

In the unitary limit \(\tau_N = \pi n \Delta_0 / 4 k_B^2 \gamma^2 \sim h T_c / k_B^2\gamma^2\) and the limitation can be rewritten as

\[
t_f \geq \frac{T_c^2}{\gamma^2} \frac{h T_c}{k_B T_f^2}.
\]

(21)

If, for example, \(T_c \sim 100 K, \gamma = 0.01 T_c, T_0 = 0.01 K, T_f \sim 10^{-4} K\) the estimate for the necessary cooling time is \(t_f \geq 12\) min. In principle, the cooling time can be reasonable even in a submicrokelvin region. In this region unconventional superconductors with low \(T_c\) like heavy fermion systems can be used. One of the possible candidates is \(UPT_3\), which has an order parameter with a line of nodes and \(T_c \approx 0.5 K\). If, for example, \(T_c = 0.5 K, \gamma = 0.1 T_c, T_0 = 10^{-4} K, T_f \sim 10^{-7} K\) then from Eq. \[21\] we obtain \(t_f \geq 10h\). One should note, however, that applicability of the method is limited by ultra low temperatures. At microkelvin temperatures the entropy of the quasiparticles near nodes is small as compared to the nuclear entropies and an estimate of the Korringa constant for \(UPT_3\) from the value of pure \(Pt\) (\(K = 30 m K/s\)) shows that in 10 hours the electrons are not thermally disconnected from the nuclear system.

To make effective refrigerating storage based on the method discussed here, it is not sufficient to use a single ring or a cylinder because the allowed width of the ring is confined by the penetration depth \(\lambda\), which is usually a fraction of a micron. Instead of a single ring, one can make a coil from a superconducting wire or a tape so that the width of the tape is of the order of \(\lambda\), but the total width of the coil can be made much larger. The superconducting current in the tape can be created by applying an external magnetic field to the coil or by applying voltage to the tape ends. Another possibility to obtain a cooling effect is to use a bulk sample in a mixed state.

In conclusion, we have shown that an adiabatic increase of the supercurrent through a superconductor with an order parameter with line(s) of nodes on the Fermi surface results in a cooling effect. For maximum cooling, the initial temperature \(T_0\) decreases to \(T_f^2 / T_c\) for clean superconductors and to \(T_0 / \gamma / T_c\) for \(T_0 < \gamma\). The effect can also occur in a mixed state of a bulk sample. To observe the cooling effect it is important to be in the low temperature region in which conduction electrons give the main contribution to the system’s entropy.

I would like to thank A. Fetter and K. Moler for useful discussions. This work was partially supported by the National Science Foundation, Grant No. DMR 99-71518, and by Stanford University.

[1] See, e. g., C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. 72, 969 (2000).
[2] W. Wendler, T. Herrmannsdörfer, S. Rehmann and F. Pobel, Europhys. Lett. 38, 619 (1997).
[3] W. Wendler, T. Herrmannsdörfer, S. Rehmann and F. Pobel, J. Low Temp. Phys. 111, 99 (1998).
[4] See, e. g., Yu. S. Barash and A. A. Svidzinsky, Phys. Rev. B 53, 15254 (1996).
[5] D. Xu, S.-K. Yip and J. A. Sauls, Phys. Rev. B 51, 16233 (1995).
[6] T. P. Sheehan, “Introduction to High-Temperature Superconductivity”, Plenum Press, New York, 1994; Chrs. 13, 16.
[7] D. T. Verebelyi et al., Appl. Phys. Lett. 78, 2031 (2001).
[8] G. E. Volovik, Pis’ma Zh. Eksp. Teor. Fiz. 58, 457 (1993) [JETP Lett. 58, 6 (1993)].
[9] H. Shiba, Progr. Theor. Phys. 40, 435 (1968).
[10] P. Hirschfeld, D. Vollhardt, P. Wölfle, Solid State Commun. 59, 111 (1986).
[11] L. Taillefer, B. Lissier, R. Gagnon, K. Behnia and H. Aubin Phys. Rev. Lett. 79, 483 (1997).
[12] E. Schachinger and J. P. Carbotte, Phys. Rev. B 57, 7970 (1998).
[13] C. J. Pethick and D. Pines, Phys. Rev. Lett. 57, 118.
(1986).

[14] S. Schmitt-Rink, K. Miyake and C. M. Varma, Phys. Rev. Lett. 57, 2575 (1986).

[15] Yu. S. Barash, V. P. Mineev, and A. A. Svidzinskii, Pis’ma Zh. Eksp. Teor. Fiz. 65, 606 (1997) [JETP Lett. 65, 638 (1997)].

[16] M. Tinkham, Introduction to Superconductivity, McGraw-Hill, Inc. (1996), pp. 39-40.

[17] P. A. Lee, Phys. Rev. Lett. 71, 1887 (1993).

[18] P. Hirschfeld, W. Putikka, D. Scalapino, Phys. Rev. Lett. 71, 3705 (1993).

[19] Yu. S. Barash and A. A. Svidzinsky, Phys. Rev. B 58, 6476 (1998).

[20] P. Hirschfeld et al., Phys. Rev. B 40, 6695 (1989).