Performance measurement model for ranking of educational institutes: a fuzzy reasoning approach

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Abstract. Ranking of educational institutes starts gaining a significant attention among all the stakeholders to know the comparative status of the institutes. This paper presents a comprehensive ranking framework based on fuzzy logic using multiple criteria proposed in National institutional ranking framework (NIRF). The proposed model presents a hierarchical structure of metrics proposed by NIRF to measure the performance attributes and then the overall performance of the institute. Mamdani fuzzy reasoning approach has been adopted to address the inexact causal relationship between the leading (input) and the lagging (output) metrics. Finally, the behaviour of the responses produced by the proposed model has been analyzed and also a comparative study has been conducted between National institutional ranking framework (NIRF) and rankings with proposed rankings system for various institutes.

Keywords: Performance Measurement, Fuzzy logic, Mamdani Fuzzy Inference System, NIRF.

1. Introduction

Educational institutes have the profound impact on building a nation. The overall acceptability of an educational institute depends on its performance. Ranking is a very important part of this performance measurement (PM) [1] system in global educational territory. Ranking of an institute keeps the competitive environment in education circuit [2]. In view of the importance of a unique ranking framework government of India has established National Institutional Ranking Framework (NIRF) to rank the educational institutes on their merit. Ranking system was mainly introduced to ensure that all educational institutes should take participation in the system and lift their performance to compete globally [3]. Several parameters have been assimilated by NIRF to define ranking of an institute. A large scale of data provided by the institutes such as approved intake, actual student strength, financial utilization, placement and higher education etc. plays a major role to measure of those parameters.

Several researches have been carried out over the years to measure the performance of an educational institute [4-5]. It has been observed that several performance attributes defined by NIRF are more often linguistic in nature and cannot be expressed by crisp numbers [6]. Those attributes can only be addressed by the experience and knowledge of experts. Due to frequent use of subjective judgment by the expert(s), vagueness/fuzziness becomes an integral part of the PM system. Fuzzy set theoretic approach [7] has gained a significant attention in capturing vagueness/fuzziness present in the system. Moreover fuzzy logic [8] has the proficiency in addressing the casual relationship presents between input and output parameters in the PM system [9, 10].
In this paper an outline of fuzzy performance measurement (FPM) model for ranking of educational institutes has been suggested considering the performance attributes defined by NIRF. In order to measure those performance attributes several metrics have been defined based on their casual relationship. This paper presents a hierarchical structure of leading and lagging metrics connected with casual relationship to measure those performance attributes. Based on their casual relationship the performance score of output (lagging metrics) have been estimated from the input (leading metrics) values. Mamdani fuzzy inference system has been applied in order to deal with the casual relationship between the predefined metrics.

2. NIRF
NIRF was introduced by honourable ministry of human resource development (MHRD), Government of India on 29th September, 2015 to address the comparative status of the Indian educational institutes and also encourage them in global competition [11]. This ranking framework ranks the educational institutes based on five parameters defined in NIRF by MHRD are as follows [11]:

- Teaching learning and resources (TLR)
- Graduation outcomes (GO)
- Outreach and inclusivity (OI)
- Research and professional practice (RP)
- Perception (PR)

NIRF measures those parameters based on the data supplied by various institutes and finally calculates the overall score by a weighted sum method.

3. Fuzzy set theory
The fuzzy set theory [12] has gained ample attention in representing uncertain or vague information evolved from the judgment of decision makers. In decision making problems subjective judgments are made in the form of a linguistic term such as low, medium, high. Each of these can be defined by a fuzzy set, and characterized by membership function on the universe of discourse. In the present study, trapezoidal type fuzzy set has been employed as it is the general form of all types of linear fuzzy set. The membership function which characterizes a trapezoidal number \( \tilde{A} \) is as follows [7, 8]:

\[
\mu_A(x; a, b, c, d) = \begin{cases} 
\frac{x-a}{b-a}, & \text{for } a \leq x \leq b \\
1, & \text{for } b \leq x \leq c \\
\frac{d-x}{d-c}, & \text{for } c \leq x \leq d \\
0, & \text{otherwise}
\end{cases}
\]  

Moreover, Mamdani fuzzy inference system (MFIS) [13, 14] based on the concept of fuzzy logic has the proficiency to define the reasoning of linguistic values of input/output variables. With the help of IF-THEN fuzzy rules, fuzzy inference system allows the practitioners to develop computer aided systems with human-like thinking capability.

A typical Mamdani type IF-THEN fuzzy rule with \( n \) input linguistic variables \( x_1, x_2, \ldots, x_n \) and one output linguistic variable \( y \) has the form

\[
\text{IF } x_1 \text{ is } \tilde{A}_1 \text{ and } x_2 \text{ is } \tilde{A}_2, \ldots, x_n \text{ is } \tilde{A}_n, \text{THEN } y \text{ is } \tilde{B}
\]  

where \( \tilde{A}_i, i = 1, 2, \ldots, n \) are linguistic values in the antecedent and \( \tilde{B} \) is a linguistic value in the consequent. The max-min inference method has been employed for fuzzy reasoning process and it has been shown with two IF-THEN rules in figure 1.
4. Methodology

In the proposed model, a hierarchical structure has been developed on the basis of the key performance attributes taken from NIRF with their derived metrics (see figure 2). The casual relationship between the leading and lagging metrics has been addressed by MFISs. The overall process has been depicted in the following steps.

**Step 1: Selection of Performance Metrics.** Identification of performance attributes and defining the appropriate metrics (leading and lagging) in order to measure them, is a critical issue in any PM model. It primarily depends on the purpose of study. In the present study, TLR, GO, OI, RP, PR are selected as key performance indicator to measure the overall performance of the educational institute, whereas these KPIs have been measured through the derived metrics. A hierarchical structure of leading and lagging metrics has been developed based on their cause-effect relationship.

**Step 2: Identification of Universe of Discourse.** One of the crucial parts of the FPM model is to identify the universe of discourse for leading and lagging metric. Based on the available information and data collected from NIRF [11], universe of discourse has been defined for each metric (leading and lagging) throughout the model for the present study.

**Step 3: Development of Membership Functions.** Based on the experts’ judgment and knowledge the input/output domain of each metric has been covered by well defined fuzzy sets and the corresponding membership functions (MFs) has been characterized accordingly.

**Step 4: Generation of Fuzzy Rule-base.** The casual relationship between lagging and leading metrics has been addressed by a set of IF-THEN fuzzy rules based on experts knowledge and judgment.

**Step 5: Development of Inference Engine.** In the proposed model, for inference engine of all MFISs, min operator has been used as implication operator to get fuzzy output for each rule where as max operator has been used as aggregation operator to generate the overall fuzzy output.

**Step 6: Development of Defuzzification Interface.** Defuzzification process is used mainly to convert the overall fuzzy output into a crisp number. In the present study, to compute the overall output, centre of area (COA) method [13] has been used as a defuzzification process.

The architecture of the proposed FPM model has been depicted in figure 2.
5. Results and discussions
In this section, the proposed model has been verified by measuring the performance of India’s top 20 engineering institute and comparing the result with NIRF system to ensure its acceptability. The linguistic terms and the corresponding fuzzy sets those are implemented in the proposed model have been depicted in table 1. The results obtained by the proposed model, have been presented in table 2.
Table 1. Membership functions for the leading and lagging metrics.

| MFIss | Metrics       | Low linguistic terms and their trapezoidal membership functions |
|-------|---------------|---------------------------------------------------------------|
|       |               | **Low** | **Medium** | **High** |
|       |               | a  b  c  d | a  b  c  d | a  b  c  d |
| MFIS 1 | Leading Metrics | SS 0 0 4 10 | 4 10 10 16 | 10 16 20 20 |
|        | Lagging Metrics | FSR 0 0 6 15 | 6 15 15 24 | 15 24 30 30 |
| MFIS 2 | Leading Metrics | FQE 0 0 4 10 | 4 10 10 16 | 10 16 20 20 |
|        | Lagging Metrics | FRU 0 0 5 15 | 5 15 15 25 | 15 25 30 30 |
| MFIS 3 | Leading Metrics | TLR 0 0 30 50 | 30 50 50 70 | 50 70 100 100 |
|        | Lagging Metrics | PU 0 0 5 17.5 | 5 17.5 17.5 30 | 17.5 30 35 35 |
| MFIS 4 | Leading Metrics | IPR 0 0 2.5 7.5 | 2.5 7.5 7.5 12.5 | 7.5 12.5 15 15 |
|        | Lagging Metrics | QP 0 0 8 20 | 8 20 20 32 | 20 32 40 40 |
| MFIS 5 | Leading Metrics | GPH 0 0 5 20 | 5 20 20 35 | 20 35 40 40 |
|        | Lagging Metrics | GUE 0 0 2.5 7.5 | 2.5 7.5 7.5 12.5 | 7.5 12.5 15 15 |
|        | Leading Metrics | GMS 0 0 4.5 12.5 | 4.5 12.5 12.5 20.5 | 12.5 20.5 25 25 |
|        | Lagging Metrics | GPHD 0 0 4 10 | 4 10 10 16 | 10 16 20 20 |
|        | Leading Metrics | GO 0 0 15 50 | 15 50 50 85 | 50 85 100 100 |
|        | Lagging Metrics | RD 0 0 6 15 | 6 15 15 24 | 15 24 30 30 |
|        | Leading Metrics | ESCS 0 0 4 10 | 4 10 10 16 | 10 16 20 20 |
|        | Lagging Metrics | PCS 0 0 3 10 | 3 10 10 17 | 10 17 20 20 |
| MFIS 5 | Leading Metrics | OI 0 0 25 50 | 25 50 50 75 | 50 75 100 100 |
|        | Lagging Metrics | TLR 0 0 30 50 | 30 50 50 70 | 50 70 100 100 |
|        | Leading Metrics | RP 0 0 20 50 | 20 50 50 80 | 50 80 100 100 |
|        | OI 0 0 25 50 | 25 50 50 75 | 50 75 100 100 |
|        | PR 0 0 30 50 | 30 50 50 70 | 50 70 100 100 |
|        | Overall performance | 0 0 15 50 | 15 50 50 85 | 50 85 100 100 |
Table 2. Result obtained by FPM model.

| COLLEGE NAME                                      | TLR  | RPP  | GO   | OI   | PR   | FPM SCORE | FPM RANK |
|--------------------------------------------------|------|------|------|------|------|-----------|----------|
| Indian Institute of Technology Madras            | 79.84| 81.69| 80.8 | 72.78| 100  | 81.83     | 1        |
| Indian Institute of Technology Delhi             | 79.84| 81.69| 79.48| 79.84| 94.46| 81.61     | 2        |
| Indian Institute of Technology Kharagpur          | 78.37| 78.78| 79.42| 66.67| 89.31| 80.5      | 3        |
| Indian Institute of Technology Bombay             | 79.13| 81.69| 79.65| 59.18| 92.51| 80.27     | 4        |
| Indian Institute of Technology Hyderabad          | 78.78| 58.66| 73.79| 75.31| 60.42| 79.48     | 5        |
| Indian Institute of Technology Kanpur             | 79.84| 81.2 | 82.45| 62.74| 85.78| 79.39     | 6        |
| Indian Institute of Technology Roorkee            | 78.01| 78.65| 82.45| 70.79| 60.55| 76.42     | 7        |
| Anna University                                   | 73.54| 46.41| 79.85| 50   | 68.24| 75.96     | 8        |
| Amrita School of Engineering                      | 77.96| 58.56| 45.6 | 79.67| 28.01| 74.38     | 9        |
| Vellore Institute of Technology                   | 68.14| 63.87| 75.47| 78.33| 46.29| 73.75     | 10       |
| Indian Institute of Technology Guwahati           | 77.76| 74.22| 80.55| 68.25| 62.45| 73.26     | 11       |
| Jadavpur University                               | 67.16| 58   | 80.41| 41.61| 51.61| 73.15     | 12       |
| National Institute of Technology Tiruchirappalli  | 78.73| 44.02| 79.28| 78.39| 63.68| 71.28     | 13       |
| Indian Institute of Technology Indore             | 79.2 | 47.8 | 74.61| 69.2 | 27.15| 70.48     | 14       |
| Institute of Chemical Technology                  | 74.14| 64.17| 81.41| 44.97| 25.98| 68.89     | 15       |
| National Institute of Technology Rourkela         | 75.29| 42.84| 79.51| 64.04| 30.48| 68.23     | 16       |
| Indian Institute of Technology (Indian School of Mines) | 77.1 | 59.16| 70.93| 68.36| 39.78| 66.54     | 17       |
| Indian Institute of Technology Varanasi           | 78.65| 31.78| 80.34| 65.76| 53.49| 58.92     | 18       |
| National Institute of Technology Karnataka        | 75.34| 30.93| 79.33| 75.25| 55.59| 58.17     | 19       |
| National Institute of Technology Warangal         | 79.79| 25.87| 66.63| 79.26| 36.91| 54.93     | 20       |

From the proposed ranking framework the new scores and ranks are calculated. In the table 3 it is observed that Indian Institute of Technology Madras and Indian Institute of Technology Delhi are in same rank as in NIRF ranking. Similarly there are some other institutes where the new rank is very close to their NIRF ranking. But, for some other institutes, comparatively a big shift has been observed in their ranking. These changes in the proposed ranking system is mainly due to the fuzzy decision making process which completely relies on the expert’s view instead of linear weights. The score range in NIRF is 32.56 whereas that in FPM is 27.2 for these twenty institutes which indicates that they are close to each other. The highest score in the proposed model is much less than that of the NIRF score which indicates that a high scope is there for all the institutes to grow in the coming years. In nut-shell it is a positive sign for the education sector of India.
6. Conclusion
This paper presents a FPM model for addressing the ranking of an educational institute. A hierarchical structure of leading and lagging metrics has been proposed to evaluate performance of a particular educational institute. To capture the vagueness and uncertainty present in the subjective judgment of decision makers, MFIS has been implemented. The FPM model helps the practitioners to analyze the cause effect relationship between the input and output metrics. Keeping the future scope of study in mind, a weighted fuzzy reasoning method can be applied to improve the proposed model.

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| COLLEGE NAME                                           | SCORE  | RANK  | RANK  |
|--------------------------------------------------------|--------|-------|-------|
| Indian Institute of Technology Madras                  | 89.93  | 81.83 | 1     | 1     |
| Indian Institute of Technology Delhi                   | 88.08  | 81.61 | 2     | 2     |
| Indian Institute of Technology Kharagpur                | 80.56  | 80.5  | 5     | 3     |
| Indian Institute of Technology Bombay                   | 85.08  | 80.27 | 3     | 4     |
| Indian Institute of Technology Hyderabad                | 66.44  | 79.48 | 8     | 5     |
| Indian Institute of Technology Kanpur                   | 82.18  | 79.39 | 4     | 6     |
| Indian Institute of Technology Roorkee                 | 76.29  | 76.42 | 6     | 7     |
| Anna University                                        | 59.89  | 75.96 | 14    | 8     |
| Amrita School of Engineering                            | 57.37  | 74.38 | 20    | 9     |
| Vellore Institute of Technology                        | 59.32  | 73.75 | 15    | 10    |
| Indian Institute of Technology Guwahati                 | 74.9   | 73.26 | 7     | 11    |
| Jadavpur University                                    | 59.23  | 73.15 | 17    | 12    |
| National Institute of Technology Tiruchirappalli       | 64.1   | 71.28 | 9     | 13    |
| Indian Institute of Technology Indore                  | 62.88  | 70.48 | 10    | 14    |
| Institute of Chemical Technology                       | 58.7   | 68.89 | 18    | 15    |
| National Institute of Technology Rourkela               | 59.29  | 68.23 | 16    | 16    |
| Indian Institute of Technology (Indian School of Mines)| 62.06  | 66.54 | 12    | 17    |
| Indian Institute of Technology Varanasi                 | 62.54  | 58.92 | 11    | 18    |
| National Institute of Technology Karnataka             | 61.3   | 58.17 | 13    | 19    |
| National Institute of Technology Warangal              | 57.76  | 54.93 | 19    | 20    |
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