Chapter

Bases of Combustion Instability

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Abstract

Combustible systems generally consist of two types of chemically interacting components during combustion: an oxidizing agent (oxygen, fluorine, chlorine, their compounds) and fuel (hydrogen, hydrocarbons, nitrogen and hydrogen compounds, aluminum, etc.). The chemical properties of the components, their phase state, and their physical structure are essential when choosing the methods for supplying the components and organizing the processes in the combustion chambers, but they relatively weakly affect the basic laws of combustion processes. In the theory of combustion, the problems of burning homogeneous, premixed, gaseous components are studied in most detail. The concepts and methods of the theory of combustion are used in other areas of science and technology when considering exothermic processes with high heat generation. The separation of the issues of flame stability into diffusion-thermal and hydrodynamic problems, which is often encountered in theoretical works, is conditional and is caused by the desire to reduce the mathematical difficulties that arise when solving the problem in the general formulation. In fact, flame instability is determined by the influence of both transport processes in the flame (diffusion-thermal processes), depending on its structure, and hydrodynamic processes, i.e., the effects of gas flow. The determination of the concentration limits of flame propagation, ignition, and extinction, spontaneous instability of the flame front, the transition of combustion to detonation, and the excitation of oscillations during combustion are practical problems of the theory of combustion. Acoustic combustion instability can be considered as a self-oscillating process in which the feedback providing the energy necessary for maintaining undamped wave motions from a nonperiodic heat source (combustion process) is realized through the action of sound (acoustic) waves on combustion; in this case, the parameters of the wave motions, amplitude, waveform, and frequency, are determined by the internal properties of the system itself. This chapter provides a sequence of parametric estimates of acoustic instability during combustion in cylindrical chambers.

Keywords: combustion instability, intracameral instability, acoustic self-oscillations, modes

1. Introduction

Chemical transformations during combustion are accompanied by a strong increase in the temperature of the reacting substances, since the initial mixture has a large supply of chemical energy. These transformations usually occur in a thin (less than 1 mm), intensely luminous combustion zone, propagating at a certain speed (of the order of $10^3$ cm/s) through the initial system. The combustion zone (flame) is a wave of exothermic chemical reaction—a combustion wave [1].
The most important feature of combustion processes is the strong exponential dependence of the rate of a chemical reaction on temperature. Because of this, at a usual initial temperature, the reaction rate in the initial system is negligible, and the system is in a stable, quasi-equilibrium state (although its reacted components correspond to its state of chemical equilibrium). With a significant increase in temperature, an exothermic reaction occurs, which can spread through the medium. As a result of a large heat release, the nearest layers of the combustible mixture are heated, initiating a quick reaction in them, and so a self-propagating combustion process occurs. Moreover, in the combustion zone that separates the cold initial mixture from the hot combustion products, large temperature and concentration gradients are formed, causing heat transfer through heat conduction and diffusion of the initial mixture and combustion products. Thus, the development of an exothermic chemical reaction during combustion is accompanied by physical processes of heat and substance transfer, which, in turn, have a strong influence on the intensity of the reaction and on the speed of flame propagation. Combustion in power plants and engines is usually carried out in a moving medium, and this affects the nature of the movement. In turn, the movement of the medium affects the processes of heat and substance transfer and, consequently, combustion. The influence of physical factors is especially pronounced when the fuel components are separately supplied to the combustion chamber—fuel and oxidizer, which is widely used in engines. In this case, the combustion process is dependent on mixture formation—evaporation (in the case of liquid components) and mixing of the components, as a result of which the combustion characteristics in many cases are determined mainly by physical processes. If one of the components of the fuel during combustion is in the condensed phase, then there may be heterogeneous combustion in which a chemical reaction occurs at the interface. Then the combustion characteristics are determined mainly by heat exchange processes, the rate of supply of reacting substances to the interface between molecular and convective diffusion. Thus, although chemical transformation is the basis of combustion, the physical processes of heat transfer, transfer of reacting substances, and gas-dynamic conditions play an important role in it. The physical interpretation of the combustion processes makes it possible to summarize the extensive experimental data and observations, establish the general laws of combustion, and obtain practical results. The concepts and methods of the theory of combustion are also used in various fields of science and technology when considering exothermic processes with high heat release. The phase state and physical structure of the starting components can be different: it is possible to burn gaseous components, burn a liquid atomized component in a gaseous stream of another component, burn liquid components and combustion of solid components, etc. The chemical properties of the components, their phase state, and physical structure are essential in selecting methods for feed components and organizing process in the combustion chamber but relatively little effect on the basic laws of combustion processes [2–6].

2. Chemical combustion reactions and relaxation processes in the combustion zone

In the combustion processes that have to be dealt with in technology, characterized by a large increase in temperature due to the release of chemical energy of the starting components in the form of heat, the combustion mechanism is thermal in nature, determined by the accumulation of heat in the reacting system. An increase in temperature in this case leads to an increase in the rate of each of the stages of the reaction, which increases the rate of the total reaction and the rate of heat release.
The resulting intermediate reaction products—atoms and radicals—play the role of active centers in accordance with the kinetic mechanism of the reaction; their effect on combustion cannot be separated from the effect of increasing temperature. In the case of complex combustible systems, widely used in power plants and engines, the chemical reaction mechanism includes a large number of stages and possible intermediate products. Clarification of the detailed kinetic scheme of the total chemical process during the combustion of complex systems is not always possible. Therefore, in most cases, one has to confine oneself to the simplest concepts of the kinetics of chemical combustion reactions. The speed of a homogeneous chemical reaction can be represented as a change in the concentration of the initial gas mixture per unit time:

\[
W = \frac{-dC}{dt}
\]

The rate of simple exothermic reactions considered in the theory of combustion depends only on temperature and on the concentration of the starting materials [4, 5]:

\[
W = kC^n \exp\left(-\frac{E}{RT}\right),
\]

where \(k\) is a constant coefficient (preexponential factor), \(C\) is the concentration of the initial gas mixture at a given moment, \(n\) is an indicator depending on the order of the reaction (for a first-order reaction, \(n = 1\); second-order \(n = 2\)), \(E\) is the activation energy, \(T\) is the gas temperature, and \(R\) is the gas constant.

The main influence on the course of combustion processes and on the conditions for the transition of combustion from one level to another (ignition, extinction) is exerted by the ratio \(E/R\). When burning gases, according to experimental data, \(E/R \sim 15-10^3\) K, and the temperature dependence of the rate of a chemical reaction is very significant. The exponential dependence of the rate of a chemical reaction on temperature is nonlinear, and this determines significant differences in the implementation of combustion. Irreversible processes occurring in the combustion zone—physical and chemical—contribute to the transition of the initial quasi-equilibrium system to the equilibrium state, and they can be considered as relaxation processes, and the characteristic transition time corresponding to each of them can be considered as relaxation times (transition processes to the equilibrium state are called relaxation). The relaxation time gives the time scale during which equilibrium is established for the process under consideration. The structure of the combustion zone depends on the relaxation processes occurring in it. The relaxation time can be defined as the residence time of the substance in the corresponding part of the combustion zone. In the simplest cases of burning mixed gases, the width of the combustion zone (in order of magnitude) is equal to \(\delta = a/u\), and the residence time of the substance in this zone is \(\tau = \delta/u = a/u^2\) [4]. Here \(a\) is the thermal diffusivity of the gas, and \(u\) is the flame propagation velocity. The value of \(\tau\) determines the relaxation time to establish equilibrium in chemical composition during the combustion of perfectly mixed gases.

The study of processes in the combustion zone is facilitated by the fact that often they are very different from each other in relaxation time. This allows us to introduce simplified combustion models in which some state parameters under certain conditions are considered unchanged (frozen), while other parameters (or the same parameters but in other conditions) are considered to be of equilibrium value. As an example, consider a detonation wave propagating in a gas mixture (e.g., in a...
The mixture of hydrogen with air, which initially has room temperature. The relaxation time of the chemical reaction in the case under consideration (i.e., the time of its transition to an equilibrium state in chemical composition into combustion products) is so long that the gas parameters in front of the wave can be considered unchanged. When an intense shock wave passes through the mixture, the temperature of the gas in the wave rises in a short time (of the order of the time of several collisions between molecules). Therefore, a shock wave can be considered as a discontinuity in which only an increase in temperature occurs with a constant chemical composition of the mixture. Behind the shock wave that heats the combustible mixture and high temperature, there is a fast chemical edition leading to the establishment of chemical equilibrium. In this case, the relaxation time is significantly larger than the shock compression time (of the order of $10^4$ molecular collisions), and a relaxation layer forms behind the shock wave. Diffusion and thermal conductivity, as relatively slow processes in the relaxation zone can be neglected. Such is the model of the detonation wave developed by Zeldovich [4].

The analysis of this model instead of the extremely simplified model of the detonation wave as a gas-dynamic discontinuity made it possible to significantly develop the theory of detonation. Using as a parameter, the relaxation time of processes in various regions of the combustion zone is especially effective in the analysis of unsteady multistage combustion of complex systems, for example, solid rocket fuels. The comparison of the relaxation time of processes in the condensed phase with the relaxation time of processes in the gas phase with a rapid change in external conditions, for example, the pressure in the combustion chamber, makes it possible to draw important qualitative conclusions about the occurrence of unsteady combustion for a wide range of technical systems with a chemical transformation mechanism that has not yet been studied.

3. Critical combustion conditions

The main features of combustion include the presence of critical phenomena—sudden changes in the mode of the process with a small change in temperature, pressure, etc. The conditions under which this abrupt regime change occurs are called critical conditions. The consideration of the theory of thermal ignition proposed by Semenov [5] makes it possible to quantify the critical conditions of ignition. From the condition of equal heat release and heat removal at the ignition boundary, we can obtain the ratio for determining the temperature of the reacting body (heating from chemical reactions):

$$T_1 - T_0 = \frac{RT_0^2}{E}.$$

It is enough that the temperature of the gas mixture rises due to a chemical reaction by several percent, and a transition to a quick chemical reaction occurs from an almost complete absence of a chemical reaction. Depending on the conditions under which the reaction proceeds, qualitatively different realizations can be obtained. The accumulation of heat in the reacting system causes a sharp transition from a slow, almost imperceptible reaction to a progressively accelerating reaction. Such sharp transitions are in fact natural for simple exothermic reactions. They are associated with the exponential dependence of the reaction rate on temperature according to the Arrhenius equation, with a powerful effect on the reaction process of “feedback”—the accumulation of heat in the reacting system, increasing the temperature of this system. The obtained critical ignition conditions separate the
region in which the quasistationary state of the system under consideration is possible from the region of the essentially unsteady process of burnout (“thermal explosion”). These critical conditions depend on the relationship between the heat release from the chemical reaction and the heat removal to the external environment, i.e., from the interaction of chemical and physical processes, which determine the impossibility of thermal equilibrium in the volume of a reacting combustible mixture with desired properties.

4. Volumetric combustion in a gas mixture stream

Combustion in a moving mixture usually occurs in a combustion wave propagating through the mixture and is characterized by the presence of inhomogeneities and transport processes. But, if such an intensive mixing of the initial mixture with combustion products is ensured in the flowing combustion chamber so that the parameters of the mixture are the same at all points of the chamber, then combustion in such a complete mixing chamber will proceed according to the volumetric mechanism.

Consider the possible stationary modes of the process in the chamber of complete mixing. Note that in the case of thermal ignition, the critical ignition conditions corresponded to the transition from the region of a quasistationary process to the region of nonstationary process of progressive burnout—“thermal explosion.” In the case of a full mixing chamber, critical conditions determine the transition from one stationary level to another stationary level, and the duration of the existence of stationary states of the system is not fundamentally unlimited. We assume that at the entrance to the chamber, there is a gas with a concentration of $C_0$ and a temperature of $T_a$. In the chamber, due to complete mixing, the temperature $T$ and gas concentration $C$ remain unchanged; the same temperature and composition have a gas exiting the combustion chamber. Next, $c$ is the relative concentration of the reacting substance (the mass of a given substance per unit mass of the mixture), which is associated with the volume concentration $C$ by the ratio

$$c = \frac{C}{\rho},$$

where $\rho$ is the density of the substance.

Loss of heat through the walls into the environment will not be taken into account.

To find possible stationary modes of the process in the combustion chamber, we study the equation of its heat balance. This equation, given that all the heat released as a result of a chemical reaction, goes to increase the thermal part of the enthalpy of combustion products (neglecting the kinetic energy of the latter due to its smallness), we write in the form

$$V_k WH = \dot{m}c_p(T - T_0),$$

where $H$ is the heat of combustion of the mixture, $V_k$ is the volume of the combustion chamber, $W$ is the reaction rate, $m$ is the second mass of gas passing through the combustion chamber, and $c_p$ is the heat capacity of the gas (we consider it constant).

Chemical reaction rate $W = k(c\rho)^n \exp \left(-\frac{E}{RT}\right)$. For simplicity, we assume that the reaction is first order ($n = 1$). Then the reaction rate, expressed through the dimensionless concentration, can be represented as
\[
W' = \frac{W}{\rho} = kc \exp \left( -\frac{E}{RT} \right).
\]

From the conditions of constancy of the total enthalpy, i.e., the sum of the thermal part of the enthalpy and chemical energy (combustion in the chamber proceeds at constant pressure and without heat exchange with the environment), we find the relationship between concentration and temperature in the form
\[
C = c_0 \frac{T_2 - T}{T_2 - T_0}.
\]
Thus, as the temperature rises, the concentration of the reactant decreases (burnout).

Now the reaction rate equation can be written as
\[
W' = k_1 (T_2 - T) \exp \left( -\frac{E}{RT} \right),
\]
where \(k_1 = \frac{c_0}{T_2 - T_0} k\).

With a large value of the thermal effect of the reaction and, correspondingly, a larger increase in the temperature of the reacting gas, the prevailing effect on the reaction rate is temperature; a decrease in the concentration is much weaker. Therefore, the curve in the case under consideration has an exponential character and reaches a maximum only after 80–90% of the starting material is consumed.

These features of the temperature dependence of the reaction rate are the cause of sharp changes in the combustion mode. With low heat release, the effect of a decrease in concentration on the reaction rate is more significant than the effect of temperature. In this case, the transition from one combustion mode to another can occur gradually, smoothly. We rewrite Eq. (1) in the form
\[
HW' = \frac{\dot{m} \cdot c_p (T - T_0)}{V_k \rho},
\]
where \(HW'\) is the rate of heat release of the chemical reaction; \(\frac{\dot{m} \cdot c_p (T - T_0)}{V_k \rho}\) the rate of heat removal from the combustion chamber by the flow of matter.

Possible solutions to the heat balance equation, where \(q_{xp} = HW'\) curvilinear dependence and.
\[
q = \frac{\dot{m} c_p (T - T_0)}{V_k \rho},
\]
straight lines allow you to set stationary temperature levels of combustion due to various conditions. One can find out the effect on the combustion regime of changes in various parameters: \(m, V_k, T_0, H, etc\). To do this, one should consider how the combustion regime will change in the case of a high thermal effect of the reaction with changing parameters.

\(\frac{\dot{m}}{V_k \rho}\) and other conditions unchanged. It is easy to see that the expression \(\frac{\dot{m}}{V_k \rho} = \frac{1}{\tau_0}\) is an important parameter of combustion analysis. Here \(\tau_0\) is the gas residence time in the combustion chamber.

5. Unsteady burning

Unsteady combustion processes include ignition, extinction, flame propagation limits, flame front instability (here we mean intrinsic, spontaneous instability), the transition of combustion to detonation, and oscillations during combustion. The study of these phenomena is of great practical interest. New tasks and problems in
the field of combustion processes that have arisen as a result of the rapid development of aircraft engines, their thrust growth and, accordingly, their burning power, and the use of high-energy components relate mainly to issues of unsteady combustion, especially combustion oscillations. Unfortunately, the theory of unsteady combustion is not fully developed.

6. Flame front instability

The instability of the laminar flame front can be caused by changes in the flame structure due to the difference in the diffusion coefficients of the components of the combustible mixture, leading to a partial change in the composition of the mixture (enrichment of the mixture with a faster diffusing component) immediately before the flame front. Stratification of the mixture under the influence of selective diffusion causes local changes (increase and decrease) in the burning rate, as a result of which the smooth surface of the flame is bent, the flame breaks up into separate cells—a cellular structure of the flame is formed, and polyhedral flames appear on the burners. The flame stability is also influenced by the ratio between the diffusion coefficients $D$ and the thermal diffusivity of the component in the mixture in deficiency: if $D > a$, and then from physical considerations, we can expect an increase in the flame velocity in areas that are convex towards the initial mixture and decrease in concave ones, which leads to instability flame, if $D < a$ flame will be stable. The instability of the flame depends not only on the structure of the flame and the diffusion-thermal processes occurring in it but also on the interaction between the flame and the gas flow in the immediate vicinity of it, i.e., from hydrodynamic processes [6, 7].

7. Unsteady phenomena in combustion chambers

In the combustion chambers of power plants, in rocket and aircraft engines, and in other plants with nonequilibrium processes accompanied by intense heat generation (nuclear reactors, chemical reactors with exothermic processes, etc.), there are always random pressure fluctuations with different frequencies, the amplitude of which can achieve significant values (in rocket engines up to 3% of the average pressure in the combustion chamber of the Russian Federation). This kind of unsteadiness in combustion chambers—sound noise—is usually associated with hydrodynamic causes of the same nature as in ordinary turbulent gas flows. In fact, the nature of sound noise and the mechanism of its generation in combustion chambers are almost not studied.

In forced combustion chambers with high pressure values, in some cases, regular pressure fluctuations occur with a frequency close to the natural (acoustic) frequency of gas oscillations in the reaction volume and with an amplitude increasing in time (which can stabilize at some level). This type of unsteadiness—acoustic combustion instability—is associated with the excitation and amplification of acoustic (sound) waves in the combustion chamber. The interaction of acoustic waves with the combustion process causes fluctuations in the burning rate and, accordingly, fluctuations in the rate of heat generation, which under the conditions determined by the Rayleigh criterion [2, 3, 7–10] leads to the conversion of the heat of combustion into the mechanical energy of acoustic waves. The physically clear interpretation of acoustic combustion instability as a self-oscillating process with acoustic feedback is applicable to a wide range of problems of excitation and amplification of sound waves in heat-generating systems, in various types of
combustion chambers. Sound noise in the combustion chambers is also a self-oscillating process in which the energy source is the combustion process and the feedback is through the influence of sound waves on combustion. In this case, undamped oscillations of a stochastic nature arise, having a wide frequency band and random phases. Excitation and amplification of sound waves in combustion chambers as a result of their interaction with combustion processes lead to the generation of sound noise and to acoustic combustion instability, accompanied by pressure fluctuations. The boundary conditions for wave disturbances are determined by the geometry of the path and the parameters of the regime. For example, the ratio of the length and diameter of the cylindrical combustion chamber, the configuration of the nozzle on one side and the nozzle head on the other, determine the conditions for the excitation of a certain mode of acoustic vibrations at some fixed frequencies corresponding to a certain vibration form (tone). The number of natural frequencies of the camera (path) is unlimited. The most essential for practice are the lower harmonics. In the study of their forms and frequencies of combustion chambers, its analogy with acoustic resonators is often used. However, the combustion processes of two-phase flows are neglected, or they are significantly simplified, the mixture formation provided by the specific nozzle head and the processes of the flow of combustion products in the subsonic part of de Laval nozzle are not taken into account. All this distinguishes wave phenomena in combustion chambers from processes in a pipe closed at one end. At the same time, the simplified task allows one to attract the well-known acoustics ratios for a closed cylindrical pipe with rigid walls [8–11], also the classic Fundamentals of Acoustics by E. Skuchik and many other works. In the framework of the linear theory, one can quite accurately take into account the influence of the geometry of the chamber and nozzle on the shape and frequency of natural acoustic vibrations. To evaluate the effect on the oscillatory process of the spatial distribution of combustion, as well as to obtain analytical relationships when acoustic absorbers are installed in the chamber, the amplitude of acoustic pulsations can theoretically be determined only on the basis of nonlinear theory. Linear approaches are convenient for analyzing the conditions for the excitation of oscillations and determining relations for the instability boundary. At the same time, nonlinear models describe the processes of growth of disturbances and transition to the state of limit cycles.

At present, there is no theory that takes into account all aspects of the combustion instability problem in engines; methods for calculating combustion stability at the engine design stage have not been developed. At the same time, the results of a theoretical analysis of simplified models of combustion instability in engines are successfully used in practical studies to predict the effect of changes in design and operating parameters on instability, to search for means to increase stability, and to conduct targeted experimental work when developing engines with respect to stability.

In the analysis of a number of problems associated with combustion instability in engines, a simplified description of the dynamics of the combustion process is successfully applied by introducing the delay time between the moment the fuel enters the combustion chamber and the moment it is converted into final combustion products. The model of the delay time is based on the replacement of the real burnout curve by a step function characterized by a single quantitative indicator—the delay time or relaxation time of the combustion process. With this approach, the complex and almost unstudied mechanism of unsteady combustion in engines is excluded from consideration. For the first time, the model of delay time was proposed by Natanzon in 1949 in solving the problem of low-frequency instability in short-threaded rocket engines [9]. Crocco and Chen [10] developed a model of pressure-dependent delay time (pressure-sensitive delay time model) and applied to the analysis of low-frequency and high-frequency (acoustic) combustion
instabilities in liquid-propellant rocket (LPR). When assessing the stability of combustion in energy engines, the influx and loss of acoustic energy in the combustion chamber are determined. In the case of solid fuel engines, the determination of the influx of acoustic energy is simplified, since the combustion zone can be considered combined with the burning surface of the charge. The developed experimental methods using special model chambers (T-shaped chambers) make it possible to determine the acoustic conductivity of the burning surface for a given fuel and then calculate the influx of acoustic energy, taking into account the distribution of the amplitude of the oscillations for the considered acoustic mode over the burning surface [11]. The loss of acoustic energy in the combustion chamber can be approximately calculated from known acoustic relationships and determined experimentally on a model of the combustion chamber.

8. Acoustic combustion instability

Intracameral self-oscillations can be modeled by a system of two dynamic links: a combustion link (a system of equations describing the conversion processes) and an acoustic link closed by positive feedback. The input coordinate of the combustion unit is the pressure perturbation $\delta p$ and the gas flow velocity perturbation $\delta u$ and the output perturbation of the flow rate of the combustion products $\delta G$. The dimensionless perturbations of the parameters appear to be related to the stationary mean values, for example [12]:

$$\delta p = \frac{p'}{p_0}, \delta \rho = \frac{1}{\gamma} \delta p', \delta u = \frac{u'}{u_0}. \quad (3)$$

Here $\gamma$ is the adiabatic exponent, $\delta \rho$ is the perturbations of gas density, and $\delta u$ is the pulsations of the gas flow rate.

Flow perturbations are associated with perturbations of the density and gas flow rate, such as

$$\delta G = \delta \rho + \delta u. \quad (4)$$

The perturbations arising in the combustion zone of the axial components of the mass flow density lead to additional pressure pulsations that propagate to the nozzle and are reflected from the surface of its subsonic part. Thus, the dynamic properties of the acoustic link are characterized by its frequency response $\theta$, which has the form

$$\delta p = \theta(\delta G(t)). \quad (5)$$

The combustion unit can be represented by a nonlinear dependence

$$\delta G(t) = H(\delta p(t)), \quad (6)$$

where $H$ is the frequency response of the combustion unit.

For high-frequency acoustic vibrations arising in the considered dynamic system, which constructively constructs a cylindrical combustion chamber with a short subsonic part of the nozzle, the values of the vibration frequencies correspond to the resonant maxima of the frequency characteristic of the acoustic unit. In turn, the vibration modes (modes) are close to the vibration modes of a conservative system similar to the link under consideration (Table 1).
Following the works [2] also the classic Fundamentals of Acoustics by E. Skuchik and many other works, we present the solution of the wave equation in cylindrical coordinates ($x, r, \phi$) in the following form:

$$\delta p = \sum_{mnk} [\delta p]_{mnk} J_m \left( \nu_{mn} \frac{r}{R} \right) \cos m\phi \cos (\omega_{mnk} t + \psi_{mnk}), \quad (7)$$

where $\sum_{mnk} [\delta p]_{mnk}$ is the amplitude of the mode ($m, n, k$) in the transverse and axial directions, $\omega_{mnk}$ is the frequency of the vibration mode ($m, n, k$), $\psi_{mnk}$ is the phase shift of the modes ($m, n, k$), $R$ is the radius of the combustion chamber, $J_m(z)(z = \nu_{mn} \frac{r}{R})$ is a Bessel function of the first kind satisfying the Bessel equation, and $\nu_{mn}$ are the roots of the equation $\frac{d}{dz} J_m \left( \nu_{mn} \frac{r}{R} \right) = 0, (m = 0, 1, 2...).$

The acoustic link is almost linear [12]; therefore, the relationship of the input and output signal for it is determined by the ratio

$$\delta p_{mnk} = \bar{\theta} (i\omega_{mnk}) \delta G_{mnk}, \quad (8)$$

where $\bar{\theta}$ is the acoustic frequency response of the camera for the mode ($m, n, k$).

In a linear consideration of the instability problem of a given dynamic system, i.e., completing the harmonic linearization of formulas (6) and (7), we obtain the relation for the frequency response of the combustion unit

$$\bar{H}(i\omega_{mnk}, |\delta p_{mnk}|) = \frac{\delta G_{mnk}}{\delta p_{mnk}}, \quad (9)$$

where the pressure perturbations have the form

$$\delta p = |\delta p|_{mnk} e^{i\omega_{mnk} t} J_m \left( \nu_{mn} r \right) \cos m\phi \cos (\omega_{mnk} t + \psi_{mnk}), \quad (10)$$

or the same in complex form

$$\delta p = \delta p_{mnk} e^{i\omega_{mnk} t} J_m \left( \nu_{mn} r \right) \cos m\phi, \quad (11)$$

for each individual mode, the oscillation amplitude is $\delta p_{mnk} = \frac{1}{2} |\delta p|_{mnk} e^{i\omega_{mnk}}$.

In this case, it is possible to analyze the dynamic properties of the combustion zone for each individual vibration mode, independently of the others.

### Table 1.
The values of the roots $\nu_{mn}$ for the case of transverse vibrations in a cylindrical combustion chamber.

| Fashion fluctuations | $m$ | $n$ | $\nu_{mn}$ | Fashion fluctuations | $m$ | $n$ | $\nu_{mn}$ |
|----------------------|-----|-----|------------|----------------------|-----|-----|------------|
| First tangential     | 1   | 0   | 1.841      | First radial         | 0   | 1   | 3.883      |
| Second tangential    | 2   | 0   | 3.054      | Second radial        | 0   | 2   | 7.16       |
| Third tangential     | 3   | 0   | 4.2        | 3-я Радиальная       | 0   | 3   | 10.174     |
| Combined             | 1   | 1   | 5.332      | Combined             | 2   | 1   | 6.708      |
| Combined             | 3   | 1   | 8.015      | Combined             | 1   | 2   | 8.527      |
| Combined             | 1   | 3   | 11.707     | Combined             | 2   | 2   | 9.97       |
| Combined             | 2   | 3   | 13.171     | Combined             | 3   | 2   | 11.346     |
| Combined             | 3   | 3   | 14.588     |                      |     |     |            |

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The condition for the excitation of self-oscillations in the combustion chamber is represented by the complex equation:

$$\bar{H} \cdot \bar{\theta} = 1.$$  \hspace{1cm} (12)

The frequency characteristics of the acoustic link and the combustion zone are uniquely associated with conductivity $\alpha = \frac{\sigma_0}{\rho_0}$.

$$\bar{H} = \frac{1 + \alpha_{\text{top}}}{\gamma}$$ \hspace{1cm} (13)

$$\frac{1}{\bar{\theta}} = \frac{1 + \alpha_{\text{acoustic}}}{\gamma}.$$

Here $\alpha_{\text{top}} = \alpha_{\text{top}}^+(\omega) + i \alpha_{\text{top}}^-(\omega)$ is the conductivity of the combustion unit and $\alpha_{\text{acoustic}} = \alpha_{\text{acoustic}}^+(\omega) + i \alpha_{\text{acoustic}}^-(\omega)$ is the conductivity of the acoustic link.

Eqs. (12) and (13) imply the equality for the real and imaginary parts of the conductivity of the system links

$$\left\{ \begin{array}{l}
\alpha_{\text{top}}^+(\omega) = \alpha_{\text{acoustic}}^+(\omega), \\
\alpha_{\text{top}}^-(\omega) = \alpha_{\text{acoustic}}^-(\omega).
\end{array} \right.$$  \hspace{1cm} (14)

Particular solutions of Eq. (11) corresponding to fixed values of the indices $(m, n, k)$ correspond to different vibration modes. Longitudinal vibrations ($m = 0; n = 0; k \neq 0$) propagate along the chamber in the axial direction. In this case, the tapering part and the critical section of the nozzle are considered as boundaries from which pressure waves are reflected. The geometric characteristics of a combustion chamber with a supersonic nozzle include the length, diameter of the cylindrical part, the shape and dimensions of the subsonic part of the nozzle, and the shape and design of the nozzle head. The geometry of the combustion chamber determines the values of the natural frequencies of self-oscillations and the height of the resonance maxima of the amplitude-phase frequency response of the acoustic link. At the resonant frequency, the imaginary part of the conductivity for the acoustic link is equal to zero $\alpha_{\text{acoustic}}^-(\omega) = 0$, and the real one is characterized by the ratio $\alpha_{\text{acoustic}}^+(\omega) = \frac{\alpha_{\text{acoustic}}^+(\omega)}{\alpha_{\text{acoustic}}^-} > 0$. It follows that the flow of acoustic energy at the exit of the acoustic link is equal to the flux emitted through the nozzle. The maximum value of the modulus of the frequency response of the acoustic link is achieved at frequencies equal to the values of the intrinsic resonances:

$$|\bar{\theta}|_{\text{max}} = \frac{\gamma}{1 + \alpha_{\text{acoustic}}^+(\omega)}.$$ \hspace{1cm} (15)

The higher the conductivity of the nozzle, the lower the amplitude maximum of the frequency response of the oscillatory system. Changing the shape of the tapering part of the chamber leads to a change in the dispersion of the energy of acoustic vibrations, because energy losses are determined by the shape of the oscillating gas volume. The influence of the shape of the subsonic part of the nozzle on the chamber acoustics was studied in detail in [8–10, 12]. In the case of the implementation of transverse oscillation modes, the indices of Eqs. (7) and (8) take the following values: for radial ($m = 0; n \neq 0; k = 0$) and for tangential oscillations ($m \neq 0; n = 0; k = 0$) (Figure 1).
In longitudinal plane waves, particle paths are parallel to the direction of flow. When transverse modes of HF vibrations are excited, the particles of the medium oscillate along trajectories perpendicular to the direction of gas flow. Standing waves can be formed by the interaction of direct and reflected waves having the same frequency. Aggregations in a standing wave are characterized by a cross section with a maximum value of the amplitude of pressure oscillations and a zero amplitude of velocity oscillations, and the nodes of the waves are points with a zero pressure amplitude and a maximum velocity amplitude. The distance between successive nodes (or antinodes) is equal to half the wavelength. Radial vibrations are axisymmetric and directed from the axis to the side wall of the chamber. The pressure antinode points in the radial mode of oscillation are located in the center of the chamber and on its walls. Therefore, when radial acoustic vibrations are excited, the displacement of the combustion products leads to an increase in the gas pressure in the center of the combustion chamber and a decrease near its walls. After a time equal to half the oscillation period, the pressure in the center of the chamber, where it was high, becomes low and back against the wall. For tangential modes, two waveforms are distinguished. One of them is standing, having a wave form that is invariant in space but with a varying amplitude. Another form is a traveling wave in which the entire mass of gas, including a line of nodes, rotates. In the cylindrical part of relatively short combustion chambers, it is most often the tangential modes of acoustic vibrations that are excited. Of the two tangential modes, running or standing, in a chamber without partitions, a running mode is more common. The consequence of their excitement is, as a rule, significant damage to the material part. A sharp increase in the rate of heat transfer is associated with transverse instability modes, which often causes burnout of the firing base, nozzle nozzles, or walls of the combustion chamber for a very short period of time, for example, 100–150 ms.

Heat flows during unstable combustion can increase up to 10 or more times in comparison with a stable combustion mode. The fight against acoustic instability is one of the main problems in the development of high-power aircraft and rocket engines, including during boosting the regime. Modern experimental methods for assessing stability in engines are based on the creation of artificial disturbances in the combustion chambers (using an explosive sample inside the chamber or an explosion product stream outside the combustion chamber, etc.) with subsequent registration and analysis of the resulting oscillatory processes [2, 3, 13]. The creation of artificial perturbations of pressure or flow (equal to speed) in the combustion chambers, and the subsequent response of the combustion processes recorded during the experiment, as well as the registration of natural “noise” and the subsequent analysis of the results of the statistical processing of the measured signals allow us to
obtain quantitative estimates of the stability margins of the working process compared with other known approaches [3, 13]. The first method is similar to that used in mathematics to study the stability of systems of equations: an artificial perturbation is introduced into a working combustion chamber, in the other case, natural perturbations of the working process—“noise during combustion”—are used. For this, appropriate criteria were developed that quantitatively characterize the stability margins, which guarantee the engine’s operability under all the required modes and external conditions stipulated by the terms of reference.

The analysis of natural “noise” provides a large amount of information about the stability margins of the working process and, in addition, allows for the serial production of engines to control production stability in terms of maintaining the stability margin.

Imagine the response of the combustion process in the chamber to an artificially created pressure impulse in it in the form of a sum of expressions [2]:

$$ y = Ay_0 \exp \left( -\int_0^t \delta dt \right) \cdot \sin \left( \int_0^t \omega dt + \varphi_0 \right), \quad (16) $$

where:

- $Ay_0$ is the initial amplitude of the deviation.
- $\delta = \delta (t)$ and $\omega = \omega (t)$ are the attenuation coefficient and oscillation frequency.
- $\varphi_0$ is the initial phase of the oscillations.

The damping coefficient of oscillations has the following physical meaning:

$$ \delta = \frac{(E_2 - E_1)}{2E_\Sigma}, \quad (17) $$

where $E_2$ is the flow of vibrational energy (per unit time) dissipated by the system, $E_1$ is the flow of vibrational energy entering the system, and $E_\Sigma$ is the total vibrational energy in the system.

It is convenient to replace the attenuation coefficient by a dimensionless quantity obtained by multiplying by a period, i.e., decrement of oscillations $\delta T$. Oscillations experimentally obtained in the combustion chambers of engines can be analyzed by presenting them in the form (16). The values of $\delta$ and $\omega$ fully characterize the oscillations; this allows us to use them as experimental indicators of stability.

To determine the experimental values of the decrement of pressure fluctuations, including noise, the following methods can be applied: spectral, correlation, amplitude, and the instantaneous method [13].

The identification of the spectrum of the measured vibration frequencies is carried out in order to isolate the natural frequencies of the acoustic path under study, on which vibration amplification (resonance) is realized during the development of the product. The main method is to compare the experimental frequencies with the calculated values for the natural modes of oscillation of the gas volume of the tract. Information on the values of various other parameters is necessary for statistics and clarification of their limit values, as well as, if necessary, changes to the design of the combustion chamber affecting stability.

To create algorithms for the quantitative assessment of the stability margin of the combustion chambers of rocket engines, a number of tasks were required:

- Selection of optimal indicators of process stability.
- Development of equipment for registration and processing of source information.
• Study of the relationship of stability indicators with structural and operational parameters of engines.

• Determination of the required values of indicators to ensure sufficient reserves of stability of the working process of engines of various schemes. The margin of stability with respect to the hard excitation of self-oscillations in the combustion chambers is sufficient if, after the introduction of a pulse perturbation, the pressure fluctuations decay quickly enough, and a certain relationship between the initial peak and the gas pressure in the chamber before its creation is satisfied. However, the selection of the optimal ratios of the parameters of the working process necessary to assess the sufficient stability margins with respect to the acoustic vibration modes required the processing of a large amount of statistical data for a number of engines [3].

The minimum impulse from artificial disturbance is selected taking into account the fact that it:

• Must exceed a value below which probabilistic excitation of the instability of the working process may occur.

• Should significantly exceed the level of ripple “noise” to ensure the calculation of stability parameters with sufficient accuracy. The maximum value of the initial pressure pulse should be limited due to the minimum interference in the working process of the chamber under study.

It was established that in order to assess the stability of the working process to finite disturbances, it is necessary [2, 3] to introduce a pressure pulse within $15 A_{sv} < A_m < 25 A_{sv}$ and determine the relaxation time $t_p$.

where:

- $A_m$ is the average value of the absolute maximum of the pulse.
- $A_{sv}$ is the average rectified value of the pulsations of natural noise.
- $t_p = t_1 + t_e$ is the total relaxation time of the working process.
- $t_1$ is the time of exposure to the working process of the first wave of perturbation.
- $t_e$ is the time of decreasing the amplitude of the pressure fluctuations $e$ times.

The combustion chamber is considered resistant to finite disturbances if $t_p < 15$ ms.

When testing full-scale engines, it is necessary to evaluate the decrements and spectra of pressure oscillations before the disturbance is introduced and after the oscillations are damped. The decrements of pressure fluctuations and spectra should not differ within the accuracy of measurements. Their significant difference will mean the instability of the work process.
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