Comparison of unsteady aerodynamics on wind turbine blades using methods of ranging fidelity

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Abstract. In this paper the unsteady aerodynamic loads for aeroelastic bend-twist coupling predicted by methods of various fidelity are compared. These are: a fast and simplistic strip theory model based on the analytical theories by Greenberg and Loewy, an Euler and a RANS unsteady CFD simulation. The test object is the DTU 10MW reference wind turbine. The comparison uses transfer functions with bending deflections and blade torsion as an input, aerodynamic thrust and pitching moment as an output. A structural feedback is not considered. The RANS simulation has the highest fidelity and is taken as a baseline for all other methods. The temporal discretization and simulation time of the CFD have to be chosen carefully as they influence the transfer behavior. As long as the flow separation is small, the Euler simulation is very accurate. The frequency response by the strip theory has a good agreement over a broad range of flow conditions in attached flow when a distributed torsion or a flap-wise deflection is applied. Severe deviations are found for an edge-wise deformation and in-plane forces.

1. Introduction
As the length and flexibility of modern wind turbine blades is increasing to reduce the cost of energy, aeroelastic effects gain more significance [11]. In the field of aeroelasticity, an accurate description of unsteady aerodynamic forces that respond to a structural oscillation is essential for stability analyses, but high fidelity CFD simulations are often not economically feasible in an industrial context. Therefore aerodynamic tools with a low to medium computational cost which have sufficient accuracy are used for aeroelastic analysis in early development stages. This paper focuses on the aerodynamic response, no structural interaction is considered.

While the use of Theodorsen’s model [20] is widely spread (e.g. [2], [7], [3]), only few studies (e.g. [15], [14], [17], [13]) include the returning wake or investigate unsteady aerodynamics in the frequency domain, which is beneficial for a stability analysis [8].

A strip theory method is compared to two unsteady 3D CFD (Euler and RANS) simulations, to identify its weaknesses and range of validity. The objective of this study is to identify differences between these methods and the implications in the context of dynamic aeroelasticity. The 2D strip theory makes various simplifications but it is formulated in the frequency domain, thus is very resource efficient and can be used directly to determine aeroelastic blade instabilities when coupled to a finite element beam model [9]. The unsteady RANS simulations are assumed
to reflect the actual flow most accurately and are treated as the baseline to determine the reduced accuracy of the Euler simulation.

This study compares the unsteady loads predicted by these different methods by means of transfer functions (TF) in the frequency domain for a unidirectional structural deformation. Therefore non-linear effects like dynamic stall cannot be taken into account. As the unsteady forces are regarded from an aeroelastic point of view, this paper focuses on forces related to bend-twist coupling. These are primarily the flap-wise bending force and twisting moment responding to a bending and torsional deformation.

The turbine utilized in this study is the DTU 10MW RWT. This is a generic three bladed horizontal axis upwind turbine. For all considerations the configuration without prebend and tilt is used. For a full description refer to Bak et al. [1].

2. Methods

2.1. Strip theory

The strip theory code (ST) has three degrees of freedom per element (two translations and one rotation) and is based on the works of Greenberg [4] and Loewy [12]. The blade is discretized as a set of individual airfoils with no interaction between them. This method is currently used by the in-house aeroelastic analysis tool FlatterRotor [5], developed at the EUROS GmbH. Greenberg’s theory describes the aerodynamic loads responding to a movement of the blade. After rearranging and reducing the original equations by Greenberg to only isolated deformation directions, the equations for lift and pitching moment with respect to the quarter chord are:

\[
L(k) = -\ddot{x} \pi \rho b^2 l \\
- \dot{y} \pi \rho b^2 l \alpha \\
+ \dot{\phi} \pi \rho b^3 l \cdot 1/2 \\
- \dot{x} \frac{L_{steady} C'(k)}{V \alpha_{eff}} \\
- \dot{y} \frac{L_{steady} (1 + C'(k))}{V} \\
+ \dot{\phi} \left( \frac{L_{steady} C'(k)b}{V \alpha_{eff}} + \pi \rho b^2 l V \right) \\
+ \dot{\phi} \frac{L_{steady} C'(k)}{\alpha_{eff}}
\]

\[
M_{1/4}(k) = \ddot{x} \pi \rho b^3 l \cdot 1/2 \\
+ \dot{y} \pi \rho b^3 l \alpha \cdot 1/2 \\
- \dot{\phi} \pi \rho b^3 l \cdot 3/8 \\
- \dot{\phi} \pi \rho V b^3 l.
\]

(1)

The orientation of the used coordinate system is: \(x\) towards the suction side, \(y\) towards the trailing edge and \(\phi\) as nose up around the blade axis. The origin is in the center of the root section. The nomenclature is defined as:

- \(\alpha\): Angle of attack
- \(\alpha_0\): Zero angle of attack
- \(\alpha_{eff}\): Effective angle of attack \((\alpha_{eff} = \alpha - \alpha_0)\)
- \(\rho\): Air density
- \(\Omega_R\): Rotational speed
- \(\omega\): Frequency
- \(b\): Half chord
- \(C'\): Lift deficiency function
- \(H_n^{(2)}\): Hankel function of the second kind
- \(J_n\): Bessel function of the first kind
- \(k\): Reduced frequency \((k = b \cdot \omega / V)\)
- \(L_{steady}\): Steady lift
- \(l\): Element length
- \(Q\): Number of blades
- \(V\): Flow velocity
The lift-curve slope of $2\pi$ is substituted with the effective lift curve slope according to the current steady lift at the respective blade station. Instead of the unsteady aerodynamics model by Theodorsen [20], Loewy’s formulation [12] is used as lift deficiency function to include the effect of the returning wake

$$C' \left( k \right) = \frac{H_1^{(2)} (k) + 2 J_1 (k) \cdot W (k)}{H_1^{(2)} (k) + i H_0^{(2)} (k) + 2 [J_1 (k) + i J_0 (k)] \cdot W (k)}.$$  \hspace{1cm} (2)

This theory is an extension of the Theodorsen model, with the additional terms weighted by $W$ as a function of the reduced frequency, wake spacing $h$ and frequency ratio $\omega / \Omega_R$,

$$W (k) = \frac{1}{e^h b \cdot e^{i 2 \omega / \Omega_R} - 1}.$$  \hspace{1cm} (3)

with the wake spacing

$$h = \frac{2 \pi \cdot V_D}{b \cdot Q \cdot \Omega_R}.$$  \hspace{1cm} (4)

The effective stream velocity at the rotor disk $V_D$ can be calculated from the steady stream velocity by a Blade Element Momentum (BEM) code. In the FlatterRotor implementation, a BEM algorithm according to Hansen [6] is used to calculate $V_D$ as well as all other steady aerodynamic information. Since the wake spacing describes the time independent intensity of the returning wake, see $h/b$ in Eqn. (3), a reference wake spacing is defined as

$$h_{ref} = \frac{2 \pi \cdot V_D}{b_{ref} \cdot Q \cdot \Omega_R}.$$  \hspace{1cm} (5)

The half chord at 75% rotor radius ($b_{ref} = 1.63 m$) is used. This method uses thin airfoil theory and assumes a 2D potential flow, a constant wake velocity, a pure periodic wake and no interaction between the elements or blades. The load calculation with a very fine frequency spacing takes a few seconds on a standard office computer.

2.2. CFD setup

For the CFD simulations, the fully compressible, unstructured, multigrid-accelerated, viscous Navier-Stokes solver TAU [16], developed at the German Aerospace Center (DLR), is used. For the RANS calculations, the turbulence is modeled with the Spalart-Allmaras one-equation model. Fully turbulent flow is assumed, i.e. no transition is modeled.

The computational grid consists of all three blades, the hub and the nacelle. It is an unstructured mesh of tetrahedra with prism layers built on the basis of a mostly structured surface mesh, which contains 415 cells in the radial direction and 100 cells along the chord on the suction and the pressure side. The mesh contains about 38 million cells in total. With the turbine diameter of $D = 178.3$ m, the whole domain has the shape of a cylinder with a radius of 6 D and an axial extension from 4 D upstream to 10 D downstream of the rotor. Whether a 120° azimuth model would be sufficient to model the dynamics of the wake is not investigated. Although it was not examined in this study, a full rotor model enables a non-uniform excitation, e.g. the deformation of a single blade.

A variation of the mesh dimensions and the resolution shows that the final mesh is large enough and reasonably resolved. For the pitch angle and the deformations, TAU’s internal grid deformation mechanism is used. All simulations are conducted with a uniform, constant, axial wind field. The Euler simulation is done with TAU as well, but does not use the high resolution prism layer since no boundary layer is modeled. The calculation time per RANS transfer function is approx. 3000 CPU hours, while the Euler TF only needs around 1200 CPU hours.
3. Test setup and execution
The different test conditions are listed in Table 1. Although the conditions overspeed and underspeed are not within the nominal operational range of the turbine, they are selected to cover a broader range of tip speed ratios (TSR) and a realistic overspeed. The pitched condition is chosen to investigate a similar wake spacing to underspeed while being within the operational range of the turbine and featuring a substantial pitch.

| Condition     | underspeed | rated | overspeed | pitched |
|---------------|------------|-------|-----------|---------|
| $V_{wind}$ [m/s] | 11.4       | 11.4  | 11.4      | 16.2    |
| $\Omega_R$ [1/min] | 6         | 9.6   | 12        | 9.6     |
| Pitch [$^\circ$] | 0         | 0     | 0         | 13      |
| $h_{ref}$ [-]       | 17.8      | 9.46  | 6.78      | 17.96   |
| TSR [-]           | 4.91      | 7.86  | 9.83      | 5.53    |
| $dt$ [s]           | 0.0278    | 0.0174| 0.0139    | 0.0174  |
| $f_{Nyquist}$ [Hz]     | 17.9      | 28.7  | 35.9      | 28.7    |
| $\Delta f$(RANS) [Hz] | 0.05     | 0.08  | 0.10      | 0.08    |
| $\Delta f$(Euler) [Hz] | 0.033    | 0.053 | 0.067     | 0.053   |
| $\Delta f$(ST) [Hz]    | 0.008     | 0.008 | 0.008     | 0.008   |

With the time step dependent on the rotor speed, the absolute time step and Nyquist frequency varies. The CFD time data is transformed into the frequency domain with a fast Fourier transformation (FFT). The transfer functions are compared up to 4 Hz, which is high enough to cover the first five modal frequencies while guaranteeing a high enough resolution in the time domain for the FFT. The margin from 4 Hz to the Nyquist frequency is the lowest in the underspeed condition, but sufficiently high for all other conditions. The frequency spacing $\Delta f$ depends on the simulation time and is shown in Table 1 as well. When using Euler, three rotations are modeled, while RANS simulates only two. The size of the time step is set to 1° azimuth angle. For the strip theory, the frequency spacing is set to 0.008 Hz.

Since the strip theory needs a steady BEM solution, a comparison between the methods for the steady state of rated and pitched is carried out. Besides the CFD method, results from the AVATAR Project [18], a Bladed V4 calculation and a QLLT simulation with QBlade, are used for a plausibility check. The deviations are pronounced on the root (<20% rotor radius) but within about 13% on the mid-board between RANS and BEM. It is assumed that the baseline flow condition is comparable between the methods, but inherent deviations of the unsteady response close to the tip and root cannot be ruled out. As an example the comparison for the steady state is shown in Figure 1.

3.1. Excitation signal
Four deformation types are considered in this study: distributed flap-wise and edge-wise bending, distributed twist (torsion) and constant twist (pitch). The focus is on the flap-wise and torsional type of input since these are the most relevant for classical flutter. The distribution shape is purely quadratic. The radial dimension is held constant across all methods. The signal amplitude is very small since the comparison requires all methods to behave linearly and avoid hysteresis effects like dynamic stall. For flap-wise and edge-wise deformations the amplitude at the tip is set to 0.01 m. The tip twist for a torsional deformation is set to 0.1° and the blade pitch deformation to 0.01°. Through the structural twist of the blade, edge and flap deformations are slightly coupled, depending on the global pitch and the blade station.
3.2. Calculation of transfer function
A transfer function is calculated as output over input. The output is the aerodynamic load at the local section and the input is the tip deformation amplitude. The global TF is the sum or integration of local TFs over the length of the blade. In the radial direction, 25 blade elements are used. The first 19 m of the blade are not included, because the section close to the root consists of very thick profiles where the flow is more comparable to the flow around blunt bodies rather than airfoils and strong deviations in the steady state are observed. Local TFs are plotted with the respective reduced frequency. A global coordinate system is introduced to compare the results. This right-hand system is defined with the origin of the z axis in the center of the circular root section and x in the axial downstream direction. The resulting y direction is tangential to the rotor axis and within the rotor plane. The out-of-plane thrust force $F_x$ and pitching moment about the z axis $M_z$ are compared.

For the strip theory, a pure harmonic input and output are considered to derive the transfer function in the frequency domain.

Since the CFD operates in the time domain, the data has been corrected to only show the influence of the excitation. This is done by calculating the difference between the time data from a simulation with and without a deformation. All unsteady CFD simulations use a steady precursor simulation with a fully developed wake system as a starting condition to reduce the required simulation time. No window function is used for the FFT, as the TF is derived with the derivative of the time data. This deformation is applied to all blades after five time steps as a step function in the global coordinate system. The global CFD TF is calculated as the average of all three blade TFs. No substantial deviations in the results between different blades are found.

4. Results
In this paper, only selected transfer functions are shown, for a more comprehensive comparison see [10]. The units of the magnitude illustrate the units of output and input.
4.1. Level of confidence in the CFD results

While the strip theory is formulated in the frequency domain, for the CFD method the discretization in the time domain can have a major influence on the results. Other distorting effects (see [19]) are caused by the averaging in the RANS simulation and the spectral leakage by the FFT. To determine a possible influence of the temporal discretization, the condition overspeed is computed with RANS for a pitch input with 0.5° azimuth angle per time step. The resulting global transfer function is shown in Figure 2 as magnitude (Mag) and phase for a 1° pitch input.

![Figure 2. Global RANS pitch-Fx TFs at overspeed.](image)

At the frequency of multiples of three revolutions (3P = 0.6 Hz at overspeed) the influence of the returning wake is visible as a reduction in magnitude. In this paper, this is referred to as a Loewy drop, while the maximum between those drops is referred to as a Loewy peak. With rising frequency the intensity of the Loewy drops and peaks decays in the form of a $1/e$-function. Comparing the two RANS TFs, in the low frequency range up to 2 Hz there is no change in the frequency response, but small differences are noticeable for higher frequencies. These deviations are small, but the position of the Loewy drops changes. Due to the frequency resolution and possible leakage from the FFT, the exact Loewy drop position cannot be determined with a high certainty and is not subject of this investigation. The general trend of the frequency response is largely independent from the time step in this range. This applies to $M_z$ TFs as well. Whether a larger time step without deviations up to 2 Hz would be possible is not investigated.

Since only 2-3 revolutions are simulated, the effect of the simulation time is shown with the example of an Euler torsion-$F_x$ TF with varying simulation time in Figure 3. Calculating the TF with fewer revolutions increases the frequency spacing. However, mainly the drops and peaks seem to be affected by the simulation time. The discrepancy at the first Loewy drop is quite large. With rising frequency the TF with one revolution quickly approaches the TFs with two or three revolutions, but at least two revolutions seem to be necessary. This behavior is less pronounced with the not shown $M_z$ TF. This effect is present at RANS simulations as well.

The deformation amplitude (0.01° for pitch, 0.1° for torsion) does not affect the shape of the frequency response, which indicates that non-linear effects have an insignificant impact. The difference in magnitude and phase is caused by the distribution of the reduced frequency when summing up the local TFs to global TFs.
4.2. Torsional deformation in rated condition
As a direct comparison between the methods, the local TFs are analyzed at the reference blade station of 75% rotor radius for the rated condition. The airfoil at this station is the FFA-W3-241 airfoil with a relative thickness of 24.1%. A reduced frequency of 0.6 corresponds to 3.95 Hz. The TFs are shown in Figure 4 for $F_x$ and Figure 5 for $M_z$. The magnitude shows the section load response per $1^\circ$ of tip torsion.

For the local torsion-$F_x$ TF, the general trend of the amplitude is reproduced by the strip theory, but a persistent offset of 15% - 30% is present. Regarding the phase, after 6P ($k = 0.15$) the ST phase does not follow the CFD phase as well, but the differences are small. For frequencies lower than the first Loewy peak the strip theory predicts a different behavior than the CFD simulation. While the amplitude starts with a level high altitude according to the strip theory, the CFD simulation shows a drop towards the zero frequency. This behavior is present for all forces and conditions for rotational deformations.

In Figure 5 the torsion-$M_z$ TF shows an even better agreement for the amplitude between the strip theory and RANS over a wide frequency range. Differences increase for frequencies towards the lower and the higher end of the 4 Hz spectrum.
The differences between the RANS and Euler results indicate the importance of viscous drag. With a different algorithm and simulation time, slight variations are expected, especially close to Loewy drops or peaks. After about 15P (k = 0.35) a slight amplitude offset to the RANS calculation is present in the $F_x$ TF and growing with rising frequency. The course of the phase already starts to separate at 9P (k = 0.22). For $M_z$ TFs an additional magnitude deviation for lower frequencies is present. Even though the assumptions of the strip theory do not apply for the thicker airfoil at 50% rotor radius with a relative thickness of 26%, the deviations between the local TFs are comparable to the shown TFs at 75% rotor radius.

4.3. Flap-wise bending deformation in rated condition
In figure 6 the TFs for a flap-wise bending motion are shown. The magnitude of a TF for a bending deflection rises with the reduced frequency and starts with an magnitude close to zero. To remove this trend, the TF is determined for an applied deflection velocity instead of a deformation. In the frequency domain this is equivalent to dividing the TF by $i\omega$, which reduces the phase by 90° and the CFD magnitude rises proportionately for low frequencies. The ST TF starts with a level magnitude, similar to the TFs from a torsional deformation (Figure 4), but for CFD TFs this is not the case. This tendency can be judged as an indication that for frequencies lower than 3P, the quality of the CFD TF is not as high as for the higher frequencies. The amplitude of the flap-$F_x$ TF is very similar to the torsional input (Figure 4), because the heave velocity of an airfoil is related to a change in the angle of attack. The offset for magnitude and phase between CFD and the ST TFs is similar to one observed at torsion-$F_x$ TFs.

4.4. Influence of flow conditions
The influence of the flow condition on the frequency response is investigated using global TFs. Depending on the wake spacing, the intensity of the returning wake effect varies. With a high reference wake spacing the intensity of the Loewy drops decreases and the effect fully fades after a frequency of 6P - 9P, see Figure 7. The condition has little impact on the phase, while the rotational speed increases the magnitude level. The constant offset detected on the local level is present as well. Although the deviations vary along the rotor radius, on a global level the general shape is represented with good accuracy relative to the overall behavior.

The position up to which the flow is attached changes with the flow condition. The intensity of the flow separation in the CFD simulation is not analyzed, but from the comparison of the steady angle of attack to the stall angle according to the profile polar data, the stall region is
approximated. At rated and pitched the angle of attack is with a margin of about 5° below the stall angle for the considered sections of the blade. For the overspeed condition, the station at 19 m is close to stall, but from 22 m upwards, the margin towards stall is over 5°. The whole blade operates at the stall onset in the underspeed condition, up to 35 m a strong flow separation is presumed. A comparison of local TFs at 22% rotor radius shows, that in this region differences between all methods are significant in this condition.

Figure 8 shows flap-$F_x$ TFs for the underspeed and pitched condition. The influence of the flow condition on the transfer function is similar to that of a torsional input. Deviations between RANS and Euler TFs are small, except for the underspeed condition. If the blade operates on the onset of stall, viscous effects are not negligible. On local TFs close to the root, the RANS TFs feature irregular peaks for low frequencies, which are assumed to be caused by unsteady vortex shedding. These local effects are for the underspeed condition strong enough to influence the global frequency response.
5. Conclusion
In this paper the transfer functions between various rotor blade deformations and the corresponding unsteady aerodynamic loads are compared between an analytical model and CFD simulations. It is shown that the transfer functions by Euler CFD simulations are very accurate considering the reduced calculation time. This is likely caused by the low influence of viscous drag in nominal operating conditions. There is the possibility that Euler and RANS TFs deviate, especially for low frequencies, if the blade operates close to stall. Depending on the time scale of the effects to be investigated and the intensity of the flow separation, it could be advantageous to use Euler to allow for a longer simulation time or shorter time step.

With the RANS solution as a baseline, results indicate that forces related to bend-twist coupling (torsional and flap-wise bending excitation to thrust and pitching moment) are well represented by the strip theory. When comparing the global TFs over all conditions, the differences vary and are the most intense for the underspeed condition, however the general trend stays the same, and the effect of various tip speed ratios, rotational speeds and pitching angles can be represented. Other than by a stall in the underspeed condition, no significant deviations are observed on a local level because of thick airfoils or caused by the vortex in the tip region, where the Mach number reaches up to 0.33 in the overspeed condition. The missing interaction between the elements and the assumption of a 2D flow is no substantial limitation and the Loewy theory is able to model the behavior of the returning wake with good accuracy.

6. Future work
While edge-wise deformations are not shown, significant deviations are found in magnitude (up to a factor of 2) and phase (up to 90°). For all input deformations, the TFs to the driving torque $M_z$ feature stronger deviations in magnitude and phase than are observed with $F_x$ and $M_z$ TFs. The influence of the different flow conditions cannot be fully represented by the analytical model either, especially for edge TFs. Among other simplifications, the absence of drag in the ST model is assumed to be a reason for this, but it is possible that a different model (e.g. [21]) would yield better results for the excitation of the edge-wise degree of freedom.

As seen in Figure 7, the CFD TF position of the Loewy drop does not always occur at multiples of 3P. Whether this is a numerical or physical effect needs to be investigated with a finer frequency spacing and time step. To explore the causes to the observed differences, the CFD results would need to be analyzed in the time domain as well to investigate the wake interaction.
behavior.

Since unsteady RANS simulations are very resource intensive, a method of medium fidelity, such as a lifting line free vortex wake code, could be favorable to investigate an edge-wise excitation and in-plane forces with a high temporal resolution.

Acknowledgments
The authors would like to thank the DLR Institute of Aeroelasticity for providing the necessary computational resources for the simulations, as well as Jens Nitzsche and Prof. Wolf Krüger for the technical expertise and enabling the cooperation.

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