RECOILLESS RESONANCE ABSORPTION OF TRITIUM ANTINEUTRINOS AND TIME-ENERGY UNCERTAINTY RELATION

S. M. Bilenky

Joint Institute for Nuclear Research, Dubna, R-141980, Russia

Abstract

We discuss neutrino oscillations in an experiment with Mössbauer recoilless resonance absorption of tritium antineutrinos, proposed recently by Raghavan. We demonstrate that small energy uncertainty of antineutrinos which ensures a large resonance absorption cross section is in a conflict with the energy uncertainty which, according to the time-energy uncertainty relation, is necessary for neutrino oscillations to happen. The search for neutrino oscillations in the Mössbauer neutrino experiment would be an important test of the applicability of the time-energy uncertainty relation to a newly discovered interference phenomenon.

1 Introduction

Uncertainty relations play an important role in the quantum theory. They are based on fundamental general properties of the theory and manifest the nature of it. There are two different types of the uncertainty relations in the quantum theory: Heisenberg uncertainty relations and time-energy uncertainty relations.

The Heisenberg uncertainty relations are based on commutation relations for hermitian operators, which correspond to physical quantities. Let us consider two hermitian operators $A$ and $B$. From the Cauchy-Schwarz inequality we have (see, for example, [1])

$$\Delta_a A \Delta_a B \geq \frac{1}{2} |\langle a|[A, B]|a\rangle| .$$

(1)

Here $|a\rangle$ is some state and

$$\Delta_a A = \sqrt{\langle a|(A - \langle a|A|a\rangle)^2|a\rangle}$$

(2)
is the standard deviation of $A$ in the state $|a\rangle$. If operators $A$ and $B$ satisfy the commutation relation $[A, B] = iC$, where $C$ is a hermitian operator, than we have the uncertainty relation

$$\Delta_a A \Delta_a B \geq \frac{1}{2} |\langle a|C|a\rangle|.$$  

(3)

For canonically conjugated quantities the right-handed parts of the Heisenberg uncertainty relations do not depend on the state $|a\rangle$. For example, for operators of momentum $p$ and coordinate $q$, which satisfy the commutation relation $[p, q] = 1/i$, from (1) we obtain the standard uncertainty relation

$$\Delta_a p \Delta_a q \geq \frac{1}{2}.$$  

(4)

The Heisenberg uncertainty relations are applicable to the states at the fixed time $t$. They mean in particular that physical quantities whose operators do not commute can not have simultaneously definite values.

Time-energy uncertainty relation have a completely different character. It was a subject of intensive discussions and controversy from the early years of the quantum theory. In the literature exist different time-energy uncertainty relations with different meaning of quantities which enter into them (see, for example, review [2]).

The time energy uncertainty relation is based on the fact that dynamics of a quantum system is determined by the Hamiltonian. The most direct and general derivation of the time-energy uncertainty relation was given by Mandelstam and Tamm [3].

Let us consider the evolution equation for any operator $O(t)$ in the Heisenberg representation. We have

$$-i \frac{\partial}{\partial t} O(t) = [H, O(t)],$$  

(5)

where $H$ is the total Hamiltonian (which does not depend on time). From (1) and (5) we find

$$\Delta_a E \Delta_a O(t) \geq \frac{1}{2} |\frac{\partial}{\partial t} \langle a|O(t)|a\rangle|.$$  

(6)

We can rewrite the inequality (6) in the form of the time-energy uncertainty relation

$$\Delta_a E \Delta_a t \geq \frac{1}{2}.$$  

(7)
Here

$$\Delta a t = \frac{\Delta a O(t)}{\left| \frac{\partial}{\partial t} \langle a | O(t) | a \rangle \right|}. \quad (8)$$

We assumed that the derivative $\frac{\partial}{\partial t} \langle a | O(t) | a \rangle$ is different from zero ($|a\rangle$ is a non stationary state). The quantity $\Delta a t$ has a dimension of time. It depends on the state $|a\rangle$ and operator $O(t)$. Different systems were considered in [3, 2].

It follows from (8) that $\Delta a t$ is the time interval which is necessary for the average value $\langle a | O(t) | a \rangle$ to be changed by one standard deviation $\Delta a O(t)$. In other words $\Delta a t$ characterizes the time interval during which the state of the system significantly varies.

Neutrino oscillations is a non stationary phenomenon. This was demonstrated by the recent accelerator K2K [4] and MINOS [5] neutrino oscillation experiments in which time of neutrino production and neutrino detection was measured. The only parameter which characterizes evolution of a neutrino state in the case of neutrino oscillations is period of oscillations (or oscillation length). In the simplest case of two neutrinos, the period of oscillations is given by the expression

$$t_{osc} = \frac{4\pi E}{\Delta m^2}, \quad (9)$$

where $E$ is the neutrino energy and $\Delta m^2 = m_2^2 - m_1^2$.

It is natural to expect that in the case of the neutrino oscillations $\Delta t$ in the time-energy uncertainty relation (7) is given by the period of oscillations

$$\Delta t \simeq t_{osc}. \quad (10)$$

As we will discuss in the next section accelerator neutrino oscillation experiments confirm this expectation.

## 2 Neutrino oscillations is non stationary phenomenon

One of the most important recent discovery in the particle physics was the discovery of neutrino oscillations [7, 8, 9, 10, 11, 12, 13, 4, 5]. All existing at

\[\text{[Footnote]}\]

This expression follows from the standard theory of neutrino oscillations (see, for example, [6]). Up to the factor $4\pi$ it can be obtained, however, from general considerations. In fact, $E$ in the numerator is determined by the Lorenz boost and $\Delta m^2$ in the denominator follows from dimensional reasons and the requirement: $t_{osc} \rightarrow \infty$ at $m_2 \rightarrow m_1$. 

\[\text{[End of Footnote]}\]
present data are in agreement with the assumption that the fields of the flavor neutrinos \( \nu_{LL}(x) \) \((l = e, \mu, \tau)\) are mixtures of the left-handed components of the three massive neutrino fields (see reviews \([6, 14]\))

\[
\nu_{LL}(x) = \sum_{i=1}^{3} U_{li} \nu_{iL}(x) .
\] (11)

Here \( U \) is the PMNS \([15, 16]\) neutrino mixing matrix and \( \nu_{iL}(x) \) is the field of neutrino with mass \( m_{i} \).

In the case of the three-neutrino mixing the probabilities of the transition between different flavor neutrinos depend on six parameters: two mass-squared differences \( \Delta m_{12}^{2} \) and \( \Delta m_{23}^{2} \), three mixing angles \( \theta_{12}, \theta_{23} \) and \( \theta_{13} \) and CP-phase \( \delta \). However, two parameters are small: \( \Delta m_{12}^{2} \Delta m_{23}^{2} \equiv 3 \cdot 10^{-2} \) and \( \sin^{2} 2\theta_{13} \leq 5 \cdot 10^{-2} \). If we neglect contribution of the small parameters to the transition probabilities, two-neutrino \( \nu_{\mu} \Leftrightarrow \nu_{\tau} \) oscillations take place in the atmospheric-LBL region of the values of the parameter \( \frac{L}{E} \) (\( L \) is the distance between neutrino production and detection points and \( E \) is the neutrino energy). For the probability of \( \nu_{\mu} (\bar{\nu}_{\mu}) \) to survive we have in this case (see review \([6]\))

\[
P(\nu_{\mu} \rightarrow \nu_{\mu}) = P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\mu}) \simeq 1 - \frac{1}{2} \sin^{2} 2\theta_{23} (1 - \cos \Delta m_{23}^{2} \frac{L}{2E}) .
\] (12)

In the reactor KamLAND region \( \bar{\nu}_{e} \leftrightarrow \bar{\nu}_{\mu,\tau} \) oscillations take place. The \( \bar{\nu}_{e} \) survival probability has two-neutrino form

\[
P(\bar{\nu}_{e} \rightarrow \bar{\nu}_{e}) = 1 - \frac{1}{2} \sin^{2} 2\theta_{12} (1 - \cos \Delta m_{12}^{2} \frac{L}{2E}) .
\] (13)

Finally, the probability of the solar \( \nu_{e} \) to survive is given by the standard two-neutrino matter expression which depends on \( \Delta m_{12}^{2}, \sin^{2} \theta_{12} \) and the density of electrons in the sun.

From the analysis of the data of the Super-Kamiokande atmospheric neutrino experiment it was found \([7]\)

\[
1.5 \cdot 10^{-3} \leq \Delta m_{23}^{2} \leq 3.4 \cdot 10^{-3} \text{ eV}^{2}, \quad \sin^{2} 2\theta_{23} > 0.92 .
\] (14)

From the global analysis of the data of the KamLAND and solar neutrino experiments the following values of the parameters \( \Delta m_{12}^{2} \) and \( \tan^{2} \theta_{12} \) were obtained \([9]\)

\[
\Delta m_{12}^{2} = 7.9^{+0.6}_{-0.5} \cdot 10^{-5} \text{ eV}^{2}, \quad \tan^{2} \theta_{12} = 0.40^{+0.10}_{-0.07} .
\] (15)
Finally, from analysis of the data of the reactor CHOOZ experiment\(^\text{[17]}\) for the parameter $\sin^2 \theta_{13}$ the following upper bound was found

$$\sin^2 \theta_{13} \leq 5 \cdot 10^{-2}.$$  \hspace{1cm} (16)

An important step in the study of the neutrino oscillations was the confirmation of the results of the SK atmospheric neutrino experiment\(^\text{[7]}\) by the long baseline K2K\(^\text{[4]}\) and MINOS\(^\text{[5]}\) accelerator neutrino oscillation experiments. From the analysis of data of the MINOS experiment for the parameters $\Delta m^2_{23}$ and $\sin^2 2\theta_{23}$ the following values were obtained\(^\text{[5]}\)

$$\Delta m^2_{23} = 2.74^{+0.44}_{-0.26} \times 10^{-3} \text{eV}^2, \quad \sin^2 2\theta_{23} > 0.87.$$ \hspace{1cm} (17)

The values (17) are in agreement with (14).

The experiments K2K and MINOS are also important from the point of view of the understanding of the origin of the neutrino oscillations (see\(^\text{[18],[19]}\)): in these experiments the time of neutrino production and neutrino detection was measured for the first time.

Let us consider as an example the K2K experiment. In this experiment neutrinos are produced in 1.1 $\mu s$ spills. Protons are extracted from the accelerator every 2.2 s. Let us denote the time of the neutrino production at the KEK accelerator $t_{KEK}$ and the time of the neutrino detection in SK detector $t_{SK}$. Neutrino events which satisfy the criteria

$$-0.2 \leq ((t_{SK} - t_{KEK}) - L/c) \leq 1.3 \mu s,$$ \hspace{1cm} (18)

where selected in the experiment.

In the K2K experiment the effect of neutrino oscillations was observed. This means that during the time interval $\Delta t = t_{SK} - t_{KEK}$ neutrino state is significantly changed (the initial $\nu_\mu$-state is transferred into a superposition of $\nu_\mu$ and $\nu_\tau$ states). The distance $L$ in the experiment is about 250 km and $\Delta t \simeq 0.8 \cdot 10^3 \mu s$. This time is comparable with the period of oscillations driven by $\Delta m^2_{23}$ which in the K2K experiment is approximately equal to $3.3 \cdot 10^3 \mu s$. Thus, in the case of the neutrino oscillations $\Delta t$ in the time-energy uncertainty relation (7) is of the order of the period of oscillations.
3 Recoilless creation and resonance absorption of tritium antineutrinos

In [20] it was proposed to detect the tritium $\bar{\nu}_e$ with energy $\simeq 18.6$ KeV in the recoilless Mössbauer transitions

$$^3\text{H} \rightarrow ^3\text{He} + \bar{\nu}_e, \quad \bar{\nu}_e + ^3\text{He} = ^3\text{H}. \quad (19)$$

It was estimated in [20] that the relative uncertainty of the energy of the antineutrinos produced in (19) is of the order

$$\frac{\Delta E}{E} \simeq 4.5 \cdot 10^{-16}. \quad (20)$$

With such an uncertainty it was estimated that the cross section of the recoilless resonance absorption of antineutrinos in the process $\bar{\nu}_e + ^3\text{He} = ^3\text{H}$ is equal to

$$\sigma_R \simeq 5 \cdot 10^{-32}\text{cm}^2 \quad (21)$$

Such a value is about nine orders of the magnitude larger than the normal neutrino cross section.

For the tritium antineutrino with the energy $\simeq 18.6$ KeV the length of the oscillations driven by $\Delta m^2_{23}$ is given by

$$L^{(23)}_{osc} \simeq 2.5 \frac{E(\text{MeV})}{\Delta m^2_{23}(\text{eV}^2)} m \simeq 18.6 \text{ m} \quad (22)$$

It was proposed in [20] to search for neutrino oscillations in the Mössbauer neutrino experiment. Such measurement would allow to determine the parameter $\sin^2 \theta_{13}$ (or to improve CHOOZ bound [16]). From (22) follows that the baseline of such an experiment is about 10 m.

Let us discuss possibilities of neutrino oscillations in the Mössbauer neutrino experiment from the point of view of the time-energy uncertainty relation. In order that neutrino oscillations driven by the ”large” atmospheric $\Delta m^2_{23}$ take place the following condition must be satisfied

$$\frac{\Delta E}{E} \gtrsim \frac{1}{4\pi} \frac{\Delta m^2_{23}}{E^2} \simeq 5.8 \cdot 10^{-13}. \quad (23)$$

From (23) we conclude that neutrino oscillations driven by $\Delta m^2_{23}$ can not be observed in the neutrino experiment with energy uncertainty given by (20).
We will discuss now neutrino oscillations driven by the small solar-KamLAND neutrino mass-squared difference $\Delta m_{12}^2$ given by (15) in the Mössbauer neutrino experiment. The oscillation length for the tritium neutrinos will be in this case about 30 times larger than $L_{\text{osc}}^{(23)}$. Thus, the baseline of the experiment must be about 300 meters. This make such an experiment very difficult. Let us see, however, whether the experiment is possible from the point of view of the time-energy uncertainty relation. The energy uncertainty in this case must satisfy the inequality

$$\Delta E \geq \frac{1}{4\pi} \frac{\Delta m_{12}^2}{E^2} \simeq 1.9 \cdot 10^{-14},$$

(24)

Thus, in the case if $\Delta E$ is given by (20) neutrino oscillations driven by the small $\Delta m_{12}^2$ are also impossible.

It was stressed, however, in [21] that due to impurities, lattice defects and other effects, which were not taken into account in [20], the real value for $\frac{\Delta E}{E}$ can be about two order of magnitude larger than (20). In this case inequality (24) could be satisfied. However, resonance cross section will be about four order of magnitude smaller than (21).

In conclusion, the very small energy uncertainty of antineutrinos produced in the two-body recoilless tritium decay $^3\text{H} \rightarrow ^3\text{He} + \bar{\nu}_e$ which provides a very large resonance cross section of the antineutrino absorbtion in the recoilless transition $\bar{\nu}_e + ^3\text{He} = ^3\text{H}$ is in a conflict with the energy uncertainty which, according to the time-energy uncertainty relation, is necessary for neutrino oscillations to happen. Thus, the Mössbauer neutrino oscillation experiment could be an important tool for the test of the fundamental time-energy uncertainty relation in a newly discovered interference phenomenon. Such test can not be performed in usual neutrino oscillation experiments (see [19]).

It is a pleasure for me to thank W. Potzel, F. von Feilitzsch, H.C.S.Lam and Wei Liao for interesting discussions. I acknowledge ILIAS program for support and the TRIUMF theory department for the hospitality.

References

[1] A. Messiah, Quantum mechanics vol.I North Holland (1970).

[2] P. Busch, arXiv:quant-ph/0105049.

[3] L. Mandelstam and I.E. Tamm, J. Phys.(USSR) 9 (1945) 249.
[4] K2K Collaboration, M.H. Alm et al., Phys. Rev. Lett. 90 (2003) 041801.

[5] MINOS Collaboration, D. G. Michael et al. arXiv:hep-ex/0607088.

[6] S.M. Bilenky, C. Giunti and W. Grimus. Prog. Part. Nucl. Phys. 43 (1999) 1.

[7] Super-Kamiokande Collaboration, Y. Ashie et al., Phys. Rev. Lett. 93 (2004) 101801; Phys. Rev. D71 (2005) 11205.

[8] SNO Collaboration, S.N. Ahmed et al., Phys. Rev. Lett. 87 (2001) 071301; 89 (2002) 0111301; 89 (2002) 011302; 92 (2004) 181301.

[9] KamLAND Collaboration, T. Araki et al., Phys. Rev. Lett. 94 (2005) 081801; hep-ex/0406035.

[10] B. T. Cleveland et al., Astrophys. J. 496 (1998) 505.

[11] GALLEX Collaboration, W. Hampel et al., Phys. Lett. B 447 (1999) 127; GNO Collaboration, M. Altmann et al. Phys. Lett. B 616 (2005) 174.

[12] SAGE Collaboration, J. N. Abdurashitov et al., Nucl. Phys. Proc. Suppl. 118 (2003) 39.

[13] Super-Kamiokande Collaboration, S. Fukuda et al., Phys. Rev. Lett. 86 (2001) 5651.

[14] M.C. Gonzalez-Garcia and M. Maltone arXiv:0704.1800.

[15] B. Pontecorvo, J. Exptl. Theoret. Phys. 33 (1957) 549. [Sov. Phys. JETP 6 (1958) 429 ]; J. Exptl. Theoret. Phys. 34 (1958) 247 [Sov. Phys. JETP 7 (1958) 172 ].

[16] Z. Maki, M. Nakagava and S. Sakata, Prog. Theor. Phys. 28 (1962) 870.

[17] CHOOZ Collaboration, M. Apollonio et al., Phys. Lett. B 466 (1999) 415; M. Apollonio et al., Eur. Phys. J. C 27 (2003) 331; hep-ex/0301017.

[18] S.M. Bilenky and M.D. Mateev, Phys.Part.Nucl.38 (2007)117, hep-ph/0604044.
[19] S.M. Bilenky, F. von Feilitzsch, W. Potzel, J.Phys.G34 (2007)987, hep-ph/0611285

[20] R.S. Raghavan hep-ph/0601079

[21] W. Potzel, Phys. Scr. T127 (2006) 85.