LEPTON UNIVERSALITY

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Abstract

The Standard Model requires the three known leptonic families to have identical couplings to the gauge bosons. The present experimental tests on lepton universality are reviewed, both for the charged and neutral current sectors. Our knowledge about the Lorentz structure of the $l^− \rightarrow \nu_l l^− \bar{\nu}_l$ transition amplitudes is also discussed.

INTRODUCTION

The Standard Model (SM) is a gauge theory, based on the group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, which describes strong, weak and electromagnetic interactions, via the exchange of the corresponding spin–1 gauge fields: 8 massless gluons and 1 massless photon for the strong and electromagnetic interactions, respectively, and 3 massive bosons, $W^\pm$ and $Z$, for the weak interaction. The fermionic matter content is given by the known leptons and quarks, which are organized in a 3–fold family structure:

$$
\begin{bmatrix}
\nu_e & u \\
e^- & d
\end{bmatrix},
\begin{bmatrix}
\nu_\mu & c \\
\mu^- & s
\end{bmatrix},
\begin{bmatrix}
\nu_\tau & t \\
\tau^- & b
\end{bmatrix},
$$

(1)

where (each quark appears in 3 different colours)

$$
\begin{bmatrix}
\nu_l & q_u \\
l^- & q_d
\end{bmatrix} \equiv \begin{pmatrix}
\nu_l \\
l^-
\end{pmatrix}_L, \begin{pmatrix}
q_u \\
q_d
\end{pmatrix}_L, l^-_R, \ (q_u)_R, \ (q_d)_R,
$$

(2)
plus the corresponding antiparticles. Thus, the left-handed fields are $SU(2)_L$ doublets, while their right-handed partners transform as $SU(2)_L$ singlets. The 3 fermionic families in (1) appear to have identical properties (gauge interactions); they only differ by their mass and their flavour quantum number.

The gauge symmetry is broken by the vacuum, which triggers the Spontaneous Symmetry Breaking (SSB) of the electroweak group to the electromagnetic subgroup:

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{SSB} SU(3)_C \otimes U(1)_{QED}.$$ (3)

The SSB mechanism generates the masses of the weak gauge bosons, and gives rise to the appearance of a physical scalar particle in the model, the so-called Higgs. The fermion masses and mixings are also generated through the SSB mechanism.

The SM constitutes one of the most successful achievements in modern physics. It provides a very elegant theoretical framework, which is able to describe all known experimental facts in particle physics. A detailed description of the SM and its present phenomenological status can be found in Refs. [1] and [2], which discuss the electroweak and strong sectors, respectively.

In spite of its enormous phenomenological success, the SM leaves too many unanswered questions to be considered as a complete description of the fundamental forces. We do not understand yet why fermions are replicated in three (and only three) nearly identical copies? Why the pattern of masses and mixings is what it is? Are the masses the only difference among the three families? What is the origin of the SM flavour structure? Which dynamics is responsible for the observed CP violation?

The fermionic flavour is the main source of arbitrary free parameters in the SM: 9 fermion masses, 3 mixing angles and 1 complex phase (assuming the neutrinos to be massless). The problem of fermion–mass generation is deeply related with the mechanism responsible for the SSB. Thus, the origin of these parameters lies in the most obscure part of the SM Lagrangian: the scalar sector. Clearly, the dynamics of flavour appears to be “terra incognita” which deserves a careful investigation.

The flavour structure looks richer in the quark sector, where mixing phenomena among the different families occur (leptons would also mix if neutrino masses were non-vanishing). Since quarks are confined within hadrons, an accurate determination of their mixing parameters requires first a good understanding of hadronization effects in flavour–changing transitions. A rather exhaustive description of our present knowledge on the different quark couplings has been given in Ref. [3].

The leptonic sector is easier to analyze. The absence of a direct lepton–gluon vertex provides a much cleaner environment to study the structure of the weak currents and the universality of their couplings to the gauge bosons. In the pure leptonic transitions, strong interactions are only present through small higher–order corrections (vacuum polarization, . . . ). Thus, it is possible to obtain precise theoretical predictions which can be compared with the available data. Although hadronization is of course present in semileptonic decays, such as $\tau^- \rightarrow \nu_\tau \pi^-$, $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$, . . . , it only involves gluonic exchanges between the quarks of a single hadronic current. Taking appropriate ratios of different semileptonic transitions with identical hadronic components, the QCD effects cancel to a very good approximation. Therefore, semileptonic decays also provide
Table 1: Masses and lifetimes of the known leptons

|        | Mass                          | Lifetime                  |
|--------|-------------------------------|---------------------------|
| $e$    | $0.51099907 \pm 0.00000015$ MeV | $> 4.3 \times 10^{23}$ yr |
| $\mu$  | $105.658389 \pm 0.000034$ MeV  | $(2.19703 \pm 0.00004) \times 10^{-6}$ s |
| $\tau$ | $1777.00 ^{+0.30}_{-0.27}$ MeV | $(290.21 \pm 1.15) \times 10^{-15}$ s |
| $\nu_e$ | $< 10–15$ eV                  | $> 300 \text{ s} \times (m_{\nu_e}/\text{eV})$ (90% CL) |
| $\nu_\mu$ | $< 0.17$ MeV                | $> 15.4 \text{ s} \times (m_{\nu_\mu}/\text{eV})$ (90% CL) |
| $\nu_\tau$ | $< 18.2$ MeV              | Model dependent           |

accurate tests of the leptonic couplings.

The measured masses and lifetimes of the known leptons, shown in table 1, are very different. The mass spectrum indicates a hierarchy of the original Yukawa couplings, which increase from one generation to the other. A similar pattern occurs in the quark sector. The huge lifetime differences can be simply understood as a kinematic reflection of the different masses [see Eq. (18)]. How precisely we know that the underlying interactions are actually identical for the three lepton generations is the main question we want to address in the following.

**QED COUPLINGS**

A general description of the electromagnetic coupling of a spin–$\frac{1}{2}$ charged lepton to the virtual photon involves three different form factors:

$$T[l\bar{l}\gamma^*] = e \varepsilon_{\mu} (q) \not{l} \left[ F_1(q^2) \gamma^\mu + i \frac{F_2(q^2)}{2m_l} \sigma^{\mu\nu} q_\nu + \frac{F_3(q^2)}{2m_l} \sigma^{\mu\nu} \gamma_5 q_\nu \right] \not{l},$$

where $q^\mu$ is the photon momentum. Owing to the conservation of the electric charge, $F_1(0) = 1$. At $q^2 = 0$, the other two form factors reduce to the lepton magnetic dipole moment, $\mu_l \equiv (e/2m_l)(g_1/2) = e(1 + F_2(0))/2m_l$, and electric dipole moment $d_l = eF_3(0)/2m_l$.

The $F_i(q^2)$ form factors are sensitive quantities to a possible lepton substructure. Moreover, $F_3(q^2)$ violates $T$ and $P$ invariance; thus, the electric dipole moments, which vanish in the SM, constitute a good probe of CP violation. Owing to their chiral changing structure, the magnetic and electric dipole moments may provide important insights on the mechanism responsible for mass generation. In general, one expects that a fermion of mass $m_f$ (generated by physics at some scale $M \gg m_f$) will have induced dipole moments proportional to some power of $m_f/M$.

The measurement of the $e^+e^- \rightarrow l^+l^-$ cross-section has been used to test the universality of the leptonic QED couplings. At low energies, where the $Z$ contribution is small, the deviations from the QED prediction are usually parameterized through:

$$\sigma(e^+e^- \rightarrow l^+l^-) = \sigma_{\text{QED}} \left( 1 \mp \frac{s}{s - \Lambda_\pm^2} \right)^2. \quad (5)$$

† A slightly different parameterization is adopted for $e^+e^- \rightarrow e^+e^-$, to account for the $t$–channel contribution.
The cut-off parameters $\Lambda_{\pm}$ characterize the validity of QED and measure the point-like nature of the leptons. From PEP and PETRA data, one finds:

\[ \Lambda_+ (e) > 435 \text{ GeV}, \Lambda_- (e) > 590 \text{ GeV}, \]
\[ \Lambda_+ (\mu) > 355 \text{ GeV}, \Lambda_- (\mu) > 265 \text{ GeV}, \Lambda_+ (\tau) > 285 \text{ GeV} \text{ and } \Lambda_- (\tau) > 246 \text{ GeV (95\% CL)}, \]

which correspond to upper limits on the lepton charge radii of about $10^{-3} \text{ fm}$.

The most stringent QED test comes of course from the high-precision measurements of the $e$ and $\mu$ anomalous magnetic moments:

\[ a_e \equiv (g_A - 2)/2 = \begin{cases} 
(115 965 214.0 \pm 2.8) \times 10^{-11} & \text{(Theory)} \\
(115 965 219.3 \pm 1.0) \times 10^{-11} & \text{(Experiment)} 
\end{cases} \]

\[ a_\mu \equiv (g_A - 2)/2 = \begin{cases} 
(1 165 917.1 \pm 1.0) \times 10^{-9} & \text{(Theory)} \\
(1 165 923.0 \pm 8.4) \times 10^{-9} & \text{(Experiment)} 
\end{cases} \]

Experimentally, very little is known about $a_\tau$ since the spin precession method used for the lighter leptons cannot be applied due to the very short lifetime of the $\tau$. The effect is however visible in the $e^+e^- \rightarrow \tau^+\tau^-$ cross-section. The limit $|a_\tau| < 0.023$ (95\% CL) has been derived from PEP and PETRA data. This limit actually probes the corresponding form factor $F_2(s)$ at $s \sim 35$ GeV. A more direct bound at $q^2 = 0$ has been extracted from the decay $Z \rightarrow \tau^+\tau^-\gamma$:

\[ |a_\tau| < 0.0104 \quad (95\% \text{ CL}) . \]

A slightly better, but more model-dependent, limit has been derived from the $Z \rightarrow \tau^+\tau^-$ decay width: $-0.004 < a_\tau < 0.006$.

In the SM the overall value of $a_\tau$ is dominated by the second order QED contribution:

\[ a_\tau \approx \alpha/2\pi. \] Including QED corrections up to $O(\alpha^3)$, hadronic vacuum polarization contributions and the corrections due to the weak interactions (which are a factor 380 larger than for the muon), the tau anomalous magnetic moment has been estimated to be:

\[ a_\tau|_{\text{th}} = (1.1773 \pm 0.0003) \times 10^{-3} . \]

So far, no evidence has been found for any CP-violation signature in the lepton sector. The present limits on the leptonic electric dipole moments are:

\[ d_e = (-0.3 \pm 0.8) \times 10^{-26} \text{ e cm}, \]
\[ d_\mu = (3.7 \pm 3.4) \times 10^{-19} \text{ e cm}, \]
\[ |d_\tau| < 5.8 \times 10^{-17} \text{ e cm}. \]

**CHARGED CURRENT UNIVERSALITY**

In the SM, the charged-current interactions are governed by an universal coupling $g$:

\[ \mathcal{L}_{\text{cc}} = \frac{g}{2\sqrt{2}} \left\{ W_\mu \left[ \sum_{ij} \bar{u}_i \gamma^\mu (1 - \gamma_5) V_{ij} d_j + \sum_l \bar{\nu}_l \gamma^\mu (1 - \gamma_5) l \right] + \text{h.c.} \right\} . \]
In the original basis of weak eigenstates quarks and leptons have identical interactions. The
diagonalization of the fermion masses gives rise to the unitary quark mixing matrix $V_{ij}$, which
couples any up–type quark with all down–type quarks. For massless neutrinos, the analogous
leptonic mixing matrix can be eliminated by a redefinition of the neutrino fields. The lepton
flavour is then conserved in the minimal SM without right–handed neutrinos.

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$

The simplest flavour–changing process is the leptonic decay of the muon, which proceeds through
the $W$–exchange diagram shown in Fig. 1. The momentum transfer carried by the intermediate
$W$ is very small compared to $M_W$. Therefore, the vector–boson propagator reduces to a contact
interaction,

$$-g_{\mu \nu} + g_{\mu \nu}/M_W^2 \rightarrow \frac{q^2 \ll M_W^2}{M_W^2} \frac{g_{\mu \nu}}{M_W^2}.$$  \hspace{1cm} (12)

The decay can then be described through an effective local 4–fermion Hamiltonian,

$$H_{\text{eff}} = G_F \sqrt{2} \left[ \bar{e} \gamma^\alpha (1 - \gamma_5) \nu_e \right] \left[ \bar{\nu}_\mu \gamma^\alpha (1 - \gamma_5) \mu \right],$$  \hspace{1cm} (13)

where

$$G_F = \frac{g^2}{8M_W^2}$$  \hspace{1cm} (14)

is called the Fermi coupling constant. $G_F$ is fixed by the total decay width,

$$\frac{1}{\tau_\mu} = \Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{G_F^2 m_\mu^5}{192 \pi^3} (1 + \delta_{RC}) f \left( \frac{m_\mu^2}{m_e^2} \right),$$  \hspace{1cm} (15)

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$, and

$$(1 + \delta_{RC}) = \left[ 1 + \frac{\alpha(m_\mu)}{2\pi} \left( \frac{25}{4} - \pi^2 \right) \right] \left[ 1 + \frac{3 m_\mu^2}{5 M_W^2} - \frac{2 m_e^2}{M_W^2} \right] = 0.9958$$  \hspace{1cm} (16)

takes into account the leading higher-order corrections. The measured lifetime, $\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6}$ s, implies the value

$$G_F = (1.16639 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2} \approx \frac{1}{(293 \text{ GeV})^2}.$$  \hspace{1cm} (17)

$\tau$ Decay

The decays of the $\tau$ lepton proceed through the same $W$–exchange mechanism as the leptonic $\mu$
decay. The only difference is that several final states are kinematically allowed: $\tau^- \to \nu_\tau e^- \bar{\nu}_e$, $\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$, $\tau^- \to \nu_\tau d \bar{u}$ and $\tau^- \to \nu_\tau s \bar{u}$. Owing to the universality of the $W$–couplings,
all these decay modes have equal amplitudes (if final fermion masses and QCD interactions are
neglected), except for an additional $N_C|V_{ui}|^2$ factor ($i = d, s$) in the semileptonic channels, where $N_C = 3$ is the number of quark colours. Making trivial kinematical changes in Eq. (15), one easily gets the lowest–order prediction for the total $\tau$ decay width:

$$\frac{1}{\tau_\tau} \equiv \Gamma(\tau) \approx \Gamma(\mu) \left(\frac{m_\tau}{m_\mu}\right)^5 \left\{ 2 + N_C \left( |V_{ud}|^2 + |V_{us}|^2 \right) \right\} \approx \frac{5}{\tau_\mu} \left(\frac{m_\tau}{m_\mu}\right)^5,$$

(18)

where we have used the unitarity relation $|V_{ud}|^2 + |V_{us}|^2 = 1 - |V_{ub}|^2 \approx 1$. From the measured muon lifetime, one has then $\tau_\tau \approx 3.3 \times 10^{-13}$ s, to be compared with the experimental value $\tau_\tau^{\text{exp}} = (2.9021 \pm 0.0115) \times 10^{-13}$ s.

Table 2: Experimental values\footnote{1} of some basic $\tau$ decay branching fractions.

$$\begin{array}{lcc}
B_e & (17.786 \pm 0.072)\% \\
B_\mu & (17.317 \pm 0.078)\% \\
R_\tau^B & \equiv \left(1 - B_e - B_\mu\right)/B_e & 3.649 \pm 0.019 \\
\text{Br}(\tau^- \to \nu_\tau \pi^-) & (11.01 \pm 0.11)\% \\
\text{Br}(\tau^- \to \nu_\tau K^-) & (0.692 \pm 0.028)\% \\
\end{array}$$

The branching ratios into the different decay modes are predicted to be:

$$\text{Br}(\tau^- \to \nu_\tau l^- \bar{\nu}_l) \approx \frac{1}{5} = 20\%,$$

$$R_\tau \equiv \frac{\Gamma(\tau \to \nu_\tau + \text{Hadrons})}{\Gamma(\tau^- \to \nu_\tau \bar{\nu}_e \nu_e)} \approx N_C,$$

(19)

in good agreement with the measured numbers\footnote{1} given in table 2. Our naive predictions only deviate from the experimental results by about 20%. This is the expected size of the corrections induced by the strong interactions between the final quarks, that we have neglected. Notice that the measured $\tau$ hadronic width provides strong evidence for the colour degree of freedom.

The pure leptonic decays $\tau^- \to e^- \bar{\nu}_e \nu_\tau, \mu^- \bar{\nu}_\mu \nu_\tau$ are theoretically understood at the level of the electroweak radiative corrections\footnote{2}. The corresponding decay widths are given by Eqs. (15) and (16), making the appropriate changes for the masses of the initial and final leptons.
Using the value of $G_F$ measured in $\mu$ decay, Eq. (15) provides a relation between the $\tau$ lifetime and the leptonic branching ratios $B_l \equiv Br(\tau^- \rightarrow \nu_\tau l^- \bar{\nu}_l)$:

$$B_e = \frac{B_\mu}{0.972564 \pm 0.000010} = \frac{\tau_\tau}{(1.6321 \pm 0.0014) \times 10^{-12} \text{s}}.$$  \hspace{1cm} (20)

The errors reflect the present uncertainty of 0.3 MeV in the value of $m_\tau$.

![Figure 3: Relation between $B_e$ and $\tau_\tau$. The dotted band corresponds to the prediction in Eq. (20).](image)

The predicted $B_\mu/B_e$ ratio is in perfect agreement with the measured value $B_\mu/B_e = 0.974 \pm 0.006$. As shown in Fig. 3, the relation between $B_e$ and $\tau_\tau$ is also well satisfied by the present data. Notice, that this relation is very sensitive to the value of the $\tau$ mass $[\Gamma(\tau^- \rightarrow l^- \bar{\nu}_l \nu_\tau) \propto m_\tau^5]$. The most recent measurements of $\tau_\tau$, $B_e$ and $m_\tau$ have consistently moved the world averages in the correct direction, eliminating the previous ($\sim 2\sigma$) disagreement. The experimental precision (0.4\%) is already approaching the level where a possible non-zero $\nu_\tau$ mass could become relevant; the present bound $m_{\nu_\tau} < 18.2$ MeV (95\% CL) only guarantees that such effect is below 0.08\%.

**Semileptonic Decays**

Semileptonic decays such as $\tau^- \rightarrow \nu_\tau P^- \text{ or } P^- \rightarrow l^- \bar{\nu}_l \text{ [}P = \pi, K\text{]}$ can be predicted in a similar way. The effects of the strong interactions are contained in the so–called decay constants $f_P$, which parameterize the hadronic matrix element of the corresponding weak current:

$$\langle \pi^-(p)|\bar{d}\gamma^\mu\gamma_5 u|0\rangle \equiv -i\sqrt{2}f_\pi p^\mu,$$
$$\langle K^-(p)|\bar{s}\gamma^\mu\gamma_5 u|0\rangle \equiv -i\sqrt{2}f_K p^\mu.$$  \hspace{1cm} (21)

Taking appropriate ratios of different semileptonic decay widths involving the same meson $P$, the dependence on these decay constants factors out. Therefore, those ratios can be predicted.
rather accurately:

\[
R_{e/\mu} \equiv \frac{\Gamma(\pi^+ \to e^- \bar{\nu}_e)}{\Gamma(\pi^+ \to \mu^- \bar{\nu}_\mu)} = \frac{m_e^2(1 - m_e^2/m_\pi^2)^2}{m_\mu^2(1 - m_\mu^2/m_\pi^2)^2} \left(1 + \delta R_{e/\mu}\right) = (1.2351 \pm 0.0005) \times 10^{-4},
\]

\[
R_{\tau/\pi} \equiv \frac{\Gamma(\tau^- \to \nu_\tau \pi^-)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \frac{m_\tau^2}{2 m_\pi m_\mu^2} \left(1 - m_\tau^2/m_\pi^2\right)^2 \left(1 + \delta R_{\tau/\pi}\right) = 9774 \pm 15, \quad (22)
\]

\[
R_{\tau/K} \equiv \frac{\Gamma(\tau^- \to \nu_\tau K^-)}{\Gamma(K^- \to \mu^- \bar{\nu}_\mu)} = \frac{m_\tau^2}{2 m_K m_\mu^2} \left(1 - m_\tau^2/m_K^2\right)^2 \left(1 + \delta R_{\tau/K}\right) = 480.4 \pm 1.1,
\]

where \(\delta R_{e/\mu} = -(3.76 \pm 0.04)\%\), \(\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%\) and \(\delta R_{\tau/K} = (0.90 \pm 0.22)\%\) are the computed radiative corrections. These predictions are in excellent agreement with the measured ratios:

\(R_{e/\mu} = (1.2310 \pm 0.0037) \times 10^{-4}\), \(R_{\tau/\pi} = 9878 \pm 106\) and \(R_{\tau/K} = 465 \pm 19\).

### Universality Tests

All these measurements can be used to test the universality of the W couplings to the leptonic charged currents. Allowing the coupling \(g\) in Eq. (11) to depend on the considered lepton flavour (i.e., \(g_e, g_\mu, g_\tau\)), the ratios \(B_\mu/B_e\) and \(R_{e/\mu}\) constrain \(|g_\mu/g_e|\), while \(B_e/\tau\) and \(R_{\tau/e}\) provide information on \(|g_\tau/g_\mu|\). The present results are shown in tables 3 and 4, together with the values obtained from the comparison of the \(\sigma \cdot B\) partial production cross-sections for the various \(W^- \to l^- \bar{\nu}_l\) decay modes at the \(p-\bar{p}\) colliders.

#### Table 3: Present constraints on \(|g_\mu/g_e|\).

| \(g_\mu/g_e\) | \(B_\mu/B_e\) | \(R_{\pi \to e/\mu}\) | \(\sigma \cdot B_{W \to \mu/e}\) |
|----------------|----------------|--------------------|---------------------|
| \(1.0005 \pm 0.0030\) | \(1.0017 \pm 0.0015\) | \(1.01 \pm 0.04\) |

#### Table 4: Present constraints on \(|g_\tau/g_\mu|\).

| \(g_\tau/g_\mu\) | \(B_\tau\mu/\tau_\tau\) | \(R_{\tau/\pi}\) | \(R_{\tau/K}\) | \(\sigma \cdot B_{W \to \tau/\mu}\) |
|-------------------|---------------------------|----------------|---------------|---------------------|
| \(1.0001 \pm 0.0029\) | \(1.005 \pm 0.005\) | \(0.984 \pm 0.020\) | \(0.99 \pm 0.05\) |

The present data verify the universality of the leptonic charged–current couplings to the 0.15% (\(\mu/e\)) and 0.30% (\(\tau/\mu\)) level. The precision of the most recent \(\tau\)-decay measurements...
is becoming competitive with the more accurate $\pi$--decay determination. It is important to realize the complementarity of the different universality tests. The pure leptonic decay modes probe the charged–current couplings of a transverse $W$. In contrast, the decays $\pi/K \rightarrow l\bar{\nu}$ and $\tau \rightarrow \nu_\tau \pi/K$ are only sensitive to the spin–0 piece of the charged current; thus, they could unveil the presence of possible scalar–exchange contributions with Yukawa–like couplings proportional to some power of the charged–lepton mass. One can easily imagine new physics scenarios which would modify differently the two types of leptonic couplings. For instance, in the usual two Higgs doublet model, charged–scalar exchange generates a correction to the ratio $B_\mu/B_e$, but $R_{\pi\rightarrow e/\mu}$ remains unaffected. Similarly, lepton mixing between the $\nu_\tau$ and an hypothetical heavy neutrino would not modify the ratios $B_\mu/B_e$ and $R_{\pi\rightarrow e/\mu}$, but would certainly correct the relation between $B_l$ and the $\tau$ lifetime.

**NEUTRAL CURRENT UNIVERSALITY**

In the SM, all leptons with equal electric charge have identical couplings to the $Z$ boson:

$$L_{NC}^Z = \frac{g}{2 \cos \theta_W} Z_\mu \sum_l \bar{l}_\gamma^\mu (v_l - a_l \gamma_5) l,$$

where

$$v_l = T^l_3 (1 - 4 |Q_l| \sin^2 \theta_W), \quad a_l = T^l_3.$$  

This has been tested at LEP and SLC, where the effective vector and axial–vector couplings of the three charged leptons have been determined.

For unpolarized $e^+$ and $e^-$ beams, the differential $e^+e^- \rightarrow \gamma, Z \rightarrow l^+l^-$ cross-section can be written, at lowest order, as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8s} \left\{ A (1 + \cos^2 \theta) + B \cos \theta - h_l \left[ C (1 + \cos^2 \theta) + D \cos \theta \right] \right\},$$

where $h_l (= \pm 1)$ is the $l^-$ helicity and $\theta$ is the scattering angle between $e^-$ and $l^-$. Here,

$$A = 1 + 2v_e v_l \Re(\chi) + (v_e^2 + a_e^2) (v_l^2 + a_l^2) |\chi|^2,$$
$$B = 4a_e a_l \Re(\chi) + 8v_e a_e v_l a_l |\chi|^2,$$
$$C = 2v_e a_l \Re(\chi) + 2 (v_e^2 + a_e^2) v_l a_l |\chi|^2,$$
$$D = 4a_e v_l \Re(\chi) + 4v_e a_e (v_l^2 + a_l^2) |\chi|^2,$$

and $\chi$ contains the $Z$ propagator

$$\chi = \frac{G_F M_Z^2}{2\sqrt{2} \pi \alpha} \frac{s}{s - M_Z^2 + i s \Gamma_Z / M_Z}. \tag{27}$$

The coefficients $A$, $B$, $C$ and $D$ can be experimentally determined, by measuring the total cross-section, the forward–backward asymmetry, the polarization asymmetry and the forward–backward polarization asymmetry, respectively:
\[ \sigma(s) = \frac{4\pi\alpha^2}{3s} A, \]
\[ \mathcal{A}_{FB}(s) \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3B}{8A}, \]
\[ \mathcal{A}_{Pol}(s) \equiv \frac{\sigma(h_l=+1) - \sigma(h_l=-1)}{\sigma(h_l=+1) + \sigma(h_l=-1)} = \frac{C}{A}, \]
\[ \mathcal{A}_{FB,Pol}(s) \equiv \frac{N_F^{(h_l=+1)} - N_F^{(h_l=-1)} - N_B^{(h_l=+1)} + N_B^{(h_l=-1)}}{N_F^{(h_l=+1)} + N_F^{(h_l=-1)} + N_B^{(h_l=+1)} + N_B^{(h_l=-1)}} = \frac{-3D}{8A}. \]
With polarized $e^+e^-$ beams, one can also study the left–right asymmetry between the cross-sections for initial left– and right–handed electrons. At the $Z$ peak, this asymmetry directly measures the average initial lepton polarization, $P_e$, without any need for final particle identification:

$$A^0_{LR} \equiv A_{LR}(M_Z^2) = \frac{\sigma_L(M_Z^2) - \sigma_R(M_Z^2)}{\sigma_L(M_Z^2) + \sigma_R(M_Z^2)} = -P_e.$$  \hspace{1cm} (32)

Table 5: Measured values of $\Gamma_l \equiv \Gamma(Z \to l^+l^-)$ and the leptonic forward–backward asymmetries. The last column shows the combined result (for a massless lepton) assuming lepton universality.

| $l$   | $\Gamma_l$ (MeV) | $A^0_{FB}(\%)$ |
|-------|-----------------|----------------|
| $e$   | 83.96 $\pm$ 0.15 | 1.60 $\pm$ 0.24 |
| $\mu$ | 83.79 $\pm$ 0.22 | 1.62 $\pm$ 0.13 |
| $\tau$ | 83.72 $\pm$ 0.26 | 2.01 $\pm$ 0.18 |
| $l$   | 83.91 $\pm$ 0.11 | 1.74 $\pm$ 0.10 |

Table 6: Measured values of the different polarization asymmetries.

| $\mathcal{A}^{0,\tau}_{Pol}$ | $\mathcal{A}^{0,\tau}_{FB,Pol}$ | $-\mathcal{A}^0_{LR} = \mathcal{P}_e$ | $\mathcal{P}_l = \frac{-\mathcal{A}^{0,\tau}_{FB,Pol} + \mathcal{A}_{Pol}}{2}$ |
|-----------------------------|------------------------------|----------------------------|--------------------------------|
| $-0.1401 \pm 0.0067$       | $-0.1382 \pm 0.0076$        | $-0.1542 \pm 0.0037$        | $-0.1523 \pm 0.0044$        |

Tables 5 and 6 show the present experimental results for the leptonic $Z$–decay widths and asymmetries. The data are in excellent agreement with the SM predictions and confirm the universality of the leptonic neutral couplings. There is however a small ($\sim 2\sigma$) discrepancy between the $P_e$ values obtained from $\mathcal{A}^{0,\tau}_{FB,Pol}$ and $\mathcal{A}^0_{LR}$. Assuming lepton universality, the combined result from all leptonic asymmetries gives

$$\mathcal{P}_l = -0.1500 \pm 0.0025. \hspace{1cm} (33)$$

The measurement of $\mathcal{A}^{0,\tau}_{Pol}$ and $\mathcal{A}^{0,\tau}_{FB,Pol}$ assumes that the $\tau$ decay proceeds through the SM charged–current interaction. A more general analysis should take into account the fact that the $\tau$–decay width depends on the product $\xi \mathcal{P}_\tau$ (see the next section), where $\xi$ is the corresponding Michel parameter in leptonic decays, or the equivalent quantity $\xi_h (= h_{\nu_\tau})$ in the semileptonic modes. A separate measurement of $\xi$ and $\mathcal{P}_\tau$ has been performed by ALEPH ($\mathcal{P}_\tau = -0.139 \pm 0.040$) and L3 ($\mathcal{P}_\tau = -0.154 \pm 0.022$), using the correlated distribution of the $\tau^+\tau^-$ decays.

The combined analysis of all leptonic observables from LEP and SLD ($\mathcal{A}^0_{LR}$) results in the effective vector and axial–vector couplings given in table 7. The corresponding 68% probability contours in the $a_l$–$v_l$ plane are shown in Fig. 4. The measured ratios of the $e$, $\mu$ and $\tau$ couplings provide a test of charged–lepton universality in the neutral–current sector.

The neutrino couplings can be determined from the invisible $Z$–decay width, by assuming three identical neutrino generations with left–handed couplings (i.e., $v_{\nu_\tau} = a_{\nu_\tau}$), and fixing the

---

§ A small 0.2% difference between $\Gamma_\tau$ and $\Gamma_{e,\mu}$ is generated by the $m_\tau$ corrections.
Table 7: Effective vector and axial–vector lepton couplings derived from LEP and SLD data.

|                     | Without Lepton Universality | With Lepton Universality |
|---------------------|-----------------------------|---------------------------|
|                     | LEP                         | LEP + SLD                 |
| $v_e$               | $-0.0368 \pm 0.0015$        | $-0.03828 \pm 0.00079$    |
| $v_\mu$             | $-0.0372 \pm 0.0034$        | $-0.0358 \pm 0.0030$      |
| $v_\tau$            | $-0.0369 \pm 0.0016$        | $-0.0367 \pm 0.0016$      |
| $a_e$               | $-0.50130 \pm 0.00046$      | $-0.50119 \pm 0.00045$    |
| $a_\mu$             | $-0.50076 \pm 0.00069$      | $-0.50086 \pm 0.00068$    |
| $a_\tau$            | $-0.50116 \pm 0.00079$      | $-0.50117 \pm 0.00079$    |

|                     | $v_\mu/v_e$                 | $v_\tau/v_e$              |
| $v_\mu/v_e$         | $1.01 \pm 0.11$             | $0.935 \pm 0.085$         |
| $v_\tau/v_e$        | $1.001 \pm 0.062$           | $0.959 \pm 0.046$         |
| $a_\mu/a_e$         | $0.9989 \pm 0.0018$         | $0.9993 \pm 0.0017$       |
| $a_\tau/a_e$        | $0.9997 \pm 0.0019$         | $1.0000 \pm 0.0019$       |

The universality of the neutrino couplings has been tested with $\nu_\mu e$ scattering data, which fixes the $\nu_\mu$ coupling to the $Z$: $v_{\nu_\mu} = a_{\nu_\mu} = 0.502 \pm 0.017$.

The measured leptonic asymmetries can be used to obtain the effective electroweak mixing angle in the charged–lepton sector:

$$ \sin^2 \theta_{\text{lept}}^{\text{eff}} \equiv \frac{1}{4} \left( 1 - \frac{v_l}{a_l} \right) = 0.23114 \pm 0.00031$$

Including also the hadronic asymmetries, one gets $\sin^2 \theta_{\text{lept}}^{\text{eff}} = 0.23165 \pm 0.00024$ with a $\chi^2$/d.o.f. = 12.8/6.

**LORENTZ STRUCTURE OF THE CHARGED CURRENTS**

Let us consider the decay $l^- \to \nu_l l'^- \bar{\nu}_{l'}$, where the lepton pair $(l, l')$ may be $(\mu, e)$, $(\tau, e)$, or $(\tau, \mu)$. The most general, local, derivative–free, lepton–number conserving, four–lepton interaction
Figure 4: 68% probability contours in the $a_l - \nu_l$ plane from LEP measurements.\cite{lep} The solid contour assumes lepton universality. Also shown is the 1σ band resulting from the $A_{LR}^0$ measurement at SLD. The grid corresponds to the SM prediction.

Hamiltonian, consistent with locality and Lorentz invariance\cite{59,60,61,62,63}

$$\mathcal{H} = 4 \frac{G_{\nu l}}{\sqrt{2}} \sum_{n, \epsilon, \omega} g_{n \epsilon \omega} \left[ \bar{\nu}_l \Gamma^n (\nu_l) \right] \left[ \bar{\nu}_l \lambda \Gamma_{n \epsilon \omega} \right],$$

contains ten complex coupling constants or, since a common phase is arbitrary, nineteen independent real parameters which could be different for each leptonic decay. The subindices $\epsilon, \omega, \sigma, \lambda$ label the chiralities (left–handed, right–handed) of the corresponding fermions, and $n$ the type of interaction: scalar ($I$), vector ($\gamma \mu$), tensor ($\sigma \mu\nu / \sqrt{2}$). For given $n, \epsilon, \omega$, the neutrino chiralities $\sigma$ and $\lambda$ are uniquely determined.

Taking out a common factor $G_{\nu l}$, which is determined by the total decay rate, the coupling constants $g_{n \epsilon \omega}$ are normalized to\cite{64}

$$1 = \frac{1}{4} \left( |g_{RR}^S|^2 + |g_{RL}^S|^2 + |g_{LR}^S|^2 + |g_{LL}^S|^2 \right) + 3 \left( |g_{RL}^T|^2 + |g_{LR}^T|^2 \right) + \left( |g_{RR}^V|^2 + |g_{RL}^V|^2 + |g_{LR}^V|^2 + |g_{LL}^V|^2 \right).$$

In the SM, $g_{LL}^V = 1$ and all other $g_{n \epsilon \omega}^\alpha = 0$. 

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For an initial lepton polarization $P_l$, the final charged–lepton distribution in the decaying–lepton rest frame is usually parameterized in the form

$$\frac{d^2\Gamma}{dx \, d\cos \theta} = \frac{m_l \omega^4}{2\pi^3} G_{\nu l}^2 \sqrt{x^2 - x_0^2} \left\{ F(x) - \frac{\xi}{3} P_l \sqrt{x^2 - x_0^2} \cos \theta A(x) \right\},$$

(38)

where $\theta$ is the angle between the $l^-$ spin and the final charged–lepton momentum, $\omega \equiv (m_l^2 + m_{\nu l}^2)/2m_l$ is the maximum $l^-$ energy for massless neutrinos, $x \equiv E_{\nu l}/\omega$ is the reduced energy, $x_0 \equiv m_{\nu l}/\omega$ and

$$F(x) = x(1 - x) + \frac{2}{9} \rho \left(4x^2 - 3x - x_0^2\right) + \eta x_0(1 - x),$$

$$A(x) = 1 - x + \frac{2}{3} \delta \left(4x - 4 + \sqrt{1 - x_0^2}\right).$$

(39)

For unpolarized $l^-$s, the distribution is characterized by the so-called Michel parameter $\rho$ and the low–energy parameter $\eta$. Two more parameters, $\xi$ and $\delta$, can be determined when the initial lepton polarization is known. If the polarization of the final charged lepton is also measured, 5 additional independent parameters ($\xi', \xi'', \eta', \alpha', \beta'$) appear.

For massless neutrinos, the total decay rate is given by

$$\Gamma = \frac{m_l^5 \tilde{G}_{\nu l}^2}{192\pi^3} f \left(\frac{m_{\nu l}^2}{m_l^2}\right) (1 + \delta_{RC}),$$

(40)

where

$$\tilde{G}_{\nu l} \equiv G_{\nu l} \sqrt{1 + 4\eta \frac{m_{\nu l} g(m_{\nu l}^2/m_l^2)}{f(m_{\nu l}^2/m_l^2)}},$$

(41)

and $g(z) = 1 + 9z - 9z^2 - z^3 + 6z(1 + z) \ln z$. Thus, $\tilde{G}_{e\mu}$ corresponds to the Fermi coupling $G_F$, measured in $\mu$ decay. The $B_\mu/B_e$ and $B_\tau/B_\mu$ universality tests, discussed in the previous section, actually prove the ratios $|\tilde{G}_{\mu\tau}/\tilde{G}_{e\tau}|$ and $|\tilde{G}_{e\tau}/\tilde{G}_{e\mu}|$, respectively. An important point, emphatically stressed by Fetscher and Gerber, concerns the extraction of $G_{e\mu}$, whose uncertainty is dominated by the uncertainty in $\eta_{\mu\rightarrow e}$.

In terms of the $g_{e\omega}^u$ couplings, the shape parameters in Eqs. (38) and (39) are:

$$\rho = \frac{3}{4}(\beta^+ + \beta^-) + (\gamma^+ + \gamma^-),$$

$$\xi = 3(\alpha^- - \alpha^+) + (\beta^- - \beta^+) + \frac{7}{3}(\gamma^+ - \gamma^-),$$

$$\xi\delta = \frac{3}{4}(\beta^- - \beta^+) + (\gamma^+ - \gamma^-),$$

$$\eta = \frac{1}{2} \text{Re} \left[ g_{LL}^V g_{RR}^{S_u} + g_{RR}^V g_{LL}^{S_u} + g_{LR}^V \left(g_{RL}^{S_u} + 6g_{RL}^T\right) + g_{RL}^V \left(g_{LR}^{S_u} + 6g_{LR}^T\right)\right].$$

(42)
where\(^{14}\)

\[
\begin{align*}
\alpha^+ & \equiv |g_{RL}^V|^2 + \frac{1}{16} |g_{RL}^S + 6g_{RL}^T|^2, \\
\beta^+ & \equiv |g_{RR}^V|^2 + \frac{1}{4} |g_{RR}^S|^2, \\
\gamma^+ & \equiv \frac{3}{16} |g_{RL}^S - 2g_{RL}^T|^2,
\end{align*}
\]

\[
\begin{align*}
\alpha^- & \equiv |g_{LR}^V|^2 + \frac{1}{16} |g_{LR}^S + 6g_{LR}^T|^2, \\
\beta^- & \equiv |g_{LL}^V|^2 + \frac{1}{4} |g_{LL}^S|^2, \\
\gamma^- & \equiv \frac{3}{16} |g_{LR}^S - 2g_{LR}^T|^2,
\end{align*}
\]

are positive–definite combinations of decay constants, corresponding to a final right– (\(\alpha^+, \beta^+, \gamma^+\)) or left– (\(\alpha^-, \beta^-, \gamma^-\)) handed lepton. In the SM, \(\rho = \delta = 3/4, \eta = \eta'' = \alpha' = \beta' = 0\) and \(\xi = \xi' = \xi'' = 1\).

The normalization constraint (37) is equivalent to \(\alpha^+ + \alpha^- + \beta^+ + \beta^- + \gamma^+ + \gamma^- = 1\). It is convenient to introduce the probabilities \(Q_{\omega}\) for the decay of an \(\omega\)–handed \(l^-\) into an \(\epsilon\)–handed daughter lepton,

\[
\begin{align*}
Q_{LL} & = \frac{1}{4} |g_{LL}^S|^2 + |g_{LL}^V|^2 = \frac{1}{4} \left( -3 + \frac{16}{3} \rho - \frac{1}{3} \xi + \frac{16}{9} \xi \delta + \xi' + \xi'' \right), \\
Q_{RR} & = \frac{1}{4} |g_{RR}^S|^2 + |g_{RR}^V|^2 = \frac{1}{4} \left( -3 + \frac{16}{3} \rho + \frac{1}{3} \xi - \frac{16}{9} \xi \delta - \xi' + \xi'' \right), \\
Q_{LR} & = \frac{1}{4} |g_{LR}^S|^2 + |g_{LR}^V|^2 + 3 |g_{LR}^T|^2 = \frac{1}{4} \left( 5 - \frac{16}{3} \rho + \frac{1}{3} \xi - \frac{16}{9} \xi \delta + \xi' - \xi'' \right), \\
Q_{RL} & = \frac{1}{4} |g_{RL}^S|^2 + |g_{RL}^V|^2 + 3 |g_{RL}^T|^2 = \frac{1}{4} \left( 5 - \frac{16}{3} \rho - \frac{1}{3} \xi + \frac{16}{9} \xi \delta - \xi' - \xi'' \right).
\end{align*}
\]

Upper bounds on any of these (positive–semidefinite) probabilities translate into corresponding limits for all couplings with the given chiralities.

For \(\mu\) decay, where precise measurements of the polarizations of both \(\mu\) and \(e\) have been performed, there exist\(^{14}\) upper bounds on \(Q_{RR}, Q_{LR}\) and \(Q_{RL}\), and a lower bound on \(Q_{LL}\). They imply corresponding upper bounds on the 8 couplings \(|g_{RR}^n|^2, |g_{LR}^n|^2\) and \(|g_{RL}^n|^2\). The measurements of the \(\mu^-\) and the \(e^-\) do not allow to determine \(|g_{LL}^S|\) and \(|g_{LL}^V|\) separately\(^{14}\). Nevertheless, since the helicity of the \(\nu_\mu\) in pion decay is experimentally known\(^{14}\) to be \(-1\), a lower limit on \(|g_{LL}^V|\) is obtained from the inverse muon decay \(\nu_\mu e^- \rightarrow \mu^- \nu_e\). The present (90\% CL) bounds\(^{14}\) on the \(\mu\)–decay couplings are shown in Fig. 5. These limits show nicely that the bulk of the \(\mu\)–decay transition amplitude is indeed of the predicted \(V-A\) type.

The experimental analysis of the \(\tau\)–decay parameters is necessarily different from the one applied to the muon, because of the much shorter \(\tau\) lifetime. The measurement of the \(\tau\) polarization and the parameters \(\xi\) and \(\delta\) is possible due to the fact that the spins of the \(\tau^+\tau^-\) pair produced in \(e^+e^-\) annihilation are strongly correlated\(^{10,12,13,14}\). Another possibility is to use the beam polarization, as done by SLD. However, the polarization of the charged lepton emitted in the \(\tau\) decay has never been measured. In principle, this could be done for the decay \(\tau^- \rightarrow \mu^- \nu_\mu \bar{\nu}_e\) by stopping the muons and detecting their decay products\(^{14}\). An alternative method would be\(^{23}\) to use the radiative decays \(\tau \rightarrow l^- \bar{\nu}_l \nu_\tau \gamma\) (\(l = e, \mu\)), since the distribution of the photons emitted by the daughter lepton is sensitive to the lepton spin. The measurement of the inverse decay \(\nu_l l^- \rightarrow \tau^- \nu_\tau\) looks far out of reach.
Figure 5: 90% CL experimental limits for the normalized $\mu$–decay couplings $g_{\mu}^n \equiv g_{\mu\omega}^n / N^n$, where $N^n \equiv \max(\vert g_{\mu\omega}^n \vert) = 2, 1, 1/\sqrt{3}$ for $n = S, V, T$. (Taken from Ref. 54).

Figure 6: 90% CL experimental limits for the normalized $\tau$–decay couplings $g_{\tau\omega}^n \equiv g_{\tau\omega}^n / N^n$, assuming $e/\mu$ universality.

The present experimental status on the $\tau$–decay Michel parameters is shown in table 8. For comparison, the values measured in $\mu$ decay are also given. The improved accuracy of the most recent experimental analyses has brought an enhanced sensitivity to the different shape parameters, allowing the first measurements of $\eta_{\tau \rightarrow \mu}, \xi_{\tau \rightarrow e}, (\xi\delta)_{\tau \rightarrow e}$ and $(\xi\delta)_{\tau \rightarrow \mu}$ without any $e/\mu$ universality assumption.

The determination of the $\tau$ polarization parameters allows us to bound the total probability for the decay of a right–handed $\tau$: \[ Q_{\tau_R} \equiv Q_{RR} + Q_{LR} = \frac{1}{2} \left[ 1 + \frac{\xi}{3} - \frac{16}{9} (\xi\delta) \right]. \] (45)

One finds (ignoring possible correlations among the measurements):

\[ Q_{\tau_R}^{-\mu} = 0.05 \pm 0.10 < 0.20 \quad (90\% \text{ CL}), \]
\[ Q_{\tau_R}^{-e} = -0.03 \pm 0.16 < 0.25 \quad (90\% \text{ CL}), \]
\[ Q_{\tau_R}^{-l} = 0.02 \pm 0.06 < 0.12 \quad (90\% \text{ CL}), \]

where the last value refers to the $\tau$ decay into either $l = e$ or $\mu$, assuming identical $e/\mu$ couplings. Since these probabilities are positive semidefinite quantities, they imply corresponding limits on all $\vert g_{RR}^n \vert$ and $\vert g_{LR}^n \vert$ couplings.
Table 8: World average Michel parameters. The last column ($\tau \to l$) assumes identical couplings for $l = e, \mu$. $\xi_{\mu \to e}$ refers to the product $\xi_{\mu \to e} P_{\mu}$, where $P_{\mu} \approx 1$ is the longitudinal polarization of the $\mu$ from $\pi$ decay.

|        | $\mu \to e$         | $\tau \to \mu$  | $\tau \to e$  | $\tau \to l$ |
|--------|---------------------|------------------|--------------|-------------|
| $\rho$ | $0.7518 \pm 0.0026$ | $0.733 \pm 0.031$ | $0.734 \pm 0.016$ | $0.741 \pm 0.014$ |
| $\eta$ | $-0.007 \pm 0.013$  | $-0.04 \pm 0.20$ | —            | $0.047 \pm 0.076$ |
| $\xi$  | $1.0027 \pm 0.0085$ | $1.19 \pm 0.18$  | $1.09 \pm 0.16$ | $1.04 \pm 0.09$ |
| $\xi\delta$ | $0.7506 \pm 0.0074$ | $0.73 \pm 0.11$  | $0.80 \pm 0.18$ | $0.73 \pm 0.07$ |

A measurement of the final lepton polarization could be even more efficient, since the total probability for the decay into a right–handed lepton depends on a single Michel parameter:

$$Q_{lR}' \equiv Q_{RR} + Q_{RL} = \frac{1}{2}(1 - \xi') .$$

(47)

Thus, a single polarization measurement could bound the five RR and RL complex couplings. Another useful positive–definite quantity is

$$\rho - \xi\delta = \frac{3}{2}\beta^+ + 2\gamma^- ,$$

(48)

which provides direct bounds on $|g_{RR}^V|$ and $|g_{RR}^S|$. A rather weak upper limit on $\gamma^+$ is obtained from the parameter $\rho$. More stringent is the bound on $\alpha^+$ obtained from $(1 - \rho)$, which is also positive–definite; it implies a corresponding limit on $|g_{RL}^V|$. Table 9 gives the resulting (90% CL) bounds on the $\tau$–decay couplings. The relevance of these limits can be better appreciated in Fig. 3, where $e/\mu$ universality has been assumed.

If lepton universality is assumed, the leptonic decay ratios $B_\mu/B_e$ and $B_e/\tau_\mu/\tau_\tau$ provide limits on the low–energy parameter $\eta$. The best sensitivity comes from $\hat{G}_{\mu\tau}$, where the term proportional to $\eta$ is not suppressed by the small $m_e/m_l$ factor. The measured $B_\mu/B_e$ ratio implies then:

$$\eta_{\tau \to l} = 0.005 \pm 0.027 .$$

(49)

This determination is more accurate that the one in table III, obtained from the shape of the energy distribution, and is comparable to the value measured in $\mu$ decay.

A non-zero value of $\eta$ would show that there are at least two different couplings with opposite chiralities for the charged leptons. Assuming the V–A coupling $g_{LL}^V$ to be dominant, the second one would be a Higgs–type coupling $g_{RR}^S$. To first order in new physics contributions, $\eta \approx \text{Re}(g_{RR}^S)/2$; Eq. (49) puts then the (90% CL) bound: $-0.08 < \text{Re}(g_{RR}^S) < 0.10$.

High–precision measurements of the $\tau$ decay parameters have the potential to find signals for new phenomena. The accuracy of the present data is still not good enough to provide strong constraints; nevertheless, it shows that the SM gives indeed the dominant contribution to the...
Table 9: 90% CL limits for the $g^\alpha_{\omega}$ couplings.

| decay amplitude | $\mu \to e$ | $\tau \to \mu$ | $\tau \to e$ | $\tau \to l$ |
|-----------------|------------|---------------|---------------|---------------|
| $|g^S_{RR}|$     | < 0.66    | < 0.71       | < 0.83        | < 0.57        |
| $|g^S_{LR}|$     | < 0.125   | < 0.90       | < 1.00        | < 0.70        |
| $|g^S_{RL}|$     | < 0.424   | $\leq 2$     | $\leq 2$      | $\leq 2$      |
| $|g^S_{LL}|$     | < 0.55    | $\leq 2$     | $\leq 2$      | $\leq 2$      |
| $|g^V_{RR}|$     | < 0.033   | < 0.36       | < 0.42        | < 0.29        |
| $|g^V_{LR}|$     | < 0.060   | < 0.45       | < 0.50        | < 0.35        |
| $|g^V_{RL}|$     | < 0.110   | < 0.56       | < 0.54        | < 0.53        |
| $|g^V_{LL}|$     | > 0.96    | $\leq 1$     | $\leq 1$      | $\leq 1$      |
| $|g^T_{RR}|$     | < 0.036   | < 0.26       | $\leq 0.29$   | < 0.20        |
| $|g^T_{LR}|$     | < 0.122   | $\leq 1/\sqrt{3}$ $\leq 1/\sqrt{3}$ $\leq 1/\sqrt{3}$ |

Future experiments should then look for small deviations of the SM predictions and find out the possible source of any detected discrepancy.

In a first analysis, it seems natural to assume that new physics effects would be dominated by the exchange of a single intermediate boson, coupling to two leptonic currents. Table 10 summarizes the expected changes on the measurable shape parameters in different new physics scenarios. The four general cases studied correspond to adding a single intermediate boson exchange, $V^+, S^+, V^0, S^0$ (charged/neutral, vector/scalar), to the SM contribution (a non-standard $W$ would be a particular case of the SM + $V^+$ scenario).

Table 10: Changes in the Michel parameters induced by the addition of a single intermediate boson exchange ($V^+, S^+, V^0, S^0$) to the SM contribution.

| $V^+$ | $S^+$ | $V^0$ | $S^0$ |
|-------|-------|-------|-------|
| $\rho - 3/4$ | < 0 | 0 | 0 | < 0 |
| $\xi - 1$ | $\pm$ | < 0 | < 0 | $\pm$ |
| $\delta \xi - 3/4$ | < 0 | < 0 | < 0 | < 0 |
| $\eta$ | 0 | $\pm$ | $\pm$ | $\pm$ |

SUMMARY

The flavour structure of the SM is one of the main pending questions in our understanding of weak interactions. Although we do not know the reason of the observed family replication, we have learned experimentally that the number of SM fermion generations is just three (and no
more). Therefore, we must study as precisely as possible the few existing flavours to get some hints on the dynamics responsible for their observed structure.

The lepton sector provides a clean environment to test the universality and Lorentz structure of the electroweak couplings. We want to investigate whether the mass is the only difference among the three fermion families. Naively, one would expect the $\tau$ to be much more sensitive than the $e$ or the $\mu$ to new physics related to the flavour and mass–generation problems. While many precision measurements of the electron and muon properties have been done in the past, it is only recently that $\tau$ experiments have achieved a comparable accuracy.

Lepton universality has been tested quite precisely, both in the charged and neutral current sectors. The leptonic couplings to the charged $W$ have been verified to be universal at the 0.15\% ($g_\mu/g_e$) and 0.30\% ($g_\tau/g_\mu$) level. The axial couplings of the $Z$ boson to the charged leptons have been measured with a comparable accuracy; universality is satisfied to the 0.17\% ($a_\mu/a_e$) and 0.19\% ($a_\tau/a_e$) level. The experimental precision is worse for the $Z$ vector couplings, which are known to be the same for the three charged leptons to 9\% ($v_\mu/v_e$) and 5\% ($v_\tau/v_e$) accuracy.

The Lorentz structure of the $l^- \rightarrow \nu_l l^- \bar{\nu}_l$ decay amplitudes has been investigated by many experiments. The present data nicely show that the bulk of the $\mu$–decay transition amplitude in indeed of the predicted V–A type. The available information on the leptonic $\tau$ decays, is still not good enough to determine the underlying dynamics; nevertheless, useful limits on possible new physics contributions start to emerge.

At present, all experimental results are consistent with the SM. There is, however, large room for improvements. Future experiments will probe the SM to a much deeper level of sensitivity and will explore the frontier of its possible extensions.

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