Late-time acceleration and Phantom Divide Line Crossing with Non-minimal Coupling and Lorentz Invariance Violation

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Abstract

We consider two alternative dark energy models: a Lorentz invariance preserving model with a nonminimally coupled scalar field and a Lorentz invariance violating model with a minimally coupled scalar field. We study accelerated expansion and dynamics of equation of state parameter in these scenarios. While a minimally coupled scalar field has not the capability to be a successful dark energy candidate with cosmological constant line crossing, a nonminimally coupled scalar field in the presence of Lorentz invariance or a minimally coupled scalar field with Lorentz invariance violation have this capability. In the later case, accelerated expansion and phantom divide line crossing are the results of interactive nature of this Lorentz violating scenario.

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1 Introduction

Recent evidences from supernova searches data \([1,2]\), cosmic microwave background (CMB) results \([3-5]\) and also Wilkinson Microwave Anisotropy Probe (WMAP) data \([6,7]\), indicate an positively accelerating phase of the cosmological expansion today and this feature shows that the simple picture of universe consisting of pressureless fluid is not enough. In this regard, the universe may contain some sort of additional negative-pressure dark energy. Analysis of the three year WMAP data \([8-10]\) shows that there is no indication for any significant deviations from Gaussianity and adiabaticity of the CMB power spectrum and therefore suggests that the universe is spatially flat to within the limits of observational accuracy. Further, the combined analysis of the three-year WMAP data with the supernova Legacy survey (SNLS) \([8]\), constrains the equation of state \(w_{de}\), corresponding to almost 74\% contribution of dark energy in the currently accelerating universe, to be very close to that of the cosmological constant value. Moreover, observations appear to favor a dark energy equation of state, \(w_{de} < -1\) \([11]\). Therefore a viable cosmological model should admit a dynamical equation of state that might have crossed the value \(w_{de} = -1\), in the recent epoch of cosmological evolution. In fact, to explain positively accelerated expansion of the universe, there are two alternative approaches: incorporating an additional cosmological component or modifying gravity at cosmological scale. Multi-component dark energy with at least one non-canonical phantom field is a possible candidate of the first alternative. This viewpoint has been studied extensively in literature (see \([12]\) and references therein). Another alternative to explain current accelerated expansion of the universe is extension of general relativity to more general theories on cosmological scales. In this viewpoint, modified Einstein-Hilbert action resulting \(f(R)\)-gravity (see \([13]\) and references therein) and braneworld gravity \([14-16]\) are studied extensively. For instance, DGP (Dvali-Gabadadze-Porrati) braneworld scenario as an IR modification of general relativity explains accelerated expansion of the universe in its positive branch via leakage of gravity to extra dimension. In this model, equation of state parameter of dark energy never crosses \(\omega(z) = -1\) line, and universe eventually turns out to be de Sitter phase. But, in this setup if we use a single scalar field (ordinary or phantom) on the brane, we can show that equation of state parameter of dark energy can cross phantom divide line (PDL) \([17]\). Also quintessential behavior can be achieved in a geometrical way in higher order theories of gravity \([18]\).

From another viewpoint, impact of Lorentz invariance violation (LIV) on cosmology has been studied recently \([19,20]\). This issue has been studied in the context of scalar-vector-tensor theories \([19]\). It has been shown that Lorentz violating vector fields affect the
dynamics of the inflationary models. One of the interesting feature of this scenario is that the exact Lorentz violating inflationary solutions are related to the absence of the inflaton potential. In this case, the inflation is completely associated with the Lorentz invariance violation and depends on the value of the coupling parameters [20]. One important observation has been made in reference [21] which accelerated expansion and crossing of phantom divide line with one minimally coupled scalar field in the presence of a Lorentz invariance violating vector field has been shown. This model is essentially an interacting model which consists of interaction between scalar field and Lorentz violating vector field. One important consequence in quintessence model is the fact that a single minimally coupled scalar field has not the capability to explain crossing of phantom divide line, \( \omega_\phi = -1 \) [22]. However, a single but non-minimally coupled scalar field is enough to cross the phantom divide line by its equation of state parameter [12]. Currently, models of phantom divide line crossing are so important that they can realize that which model is better than the others to describe the nature of dark energy. In this respect, possible crossing of phantom divide line by equation of state parameter in model universes with a non-minimally coupled scalar field and Lorentz invariance violation is important.

With this preliminaries, the purpose of this paper is to study late time acceleration and phantom divide line crossing in two model universes: a model universe with a non-minimally coupled scalar field in the presence of Lorentz invariance symmetry and a model universe with a Lorentz invariance violating dark energy component with minimal coupling. In the former case we extend our study to moving domain wall picture of braneworld scenario. In this regard, we first study cosmological consequences of a non-minimally coupled scalar field. In this stage, with a numerical analysis of parameters space of the model, we show that accelerated expansion and crossing of phantom divide line are explainable in the Jordan frame. By transforming to Einstein’s frame, we show that this model cannot account for crossing of phantom divide line. Our strategy differs with existing literature in its special kind of numerical reasoning based on an appropriate ansatz. Then we extend this model to a braneworld setup. In this extension, brane is considered to be a moving domain wall in a background 5-dimensional anti de Sitter-Schwarzschild (AdSS\(_5\)) black hole bulk. In other words, we consider a static bulk configuration with two 5-dimensional anti de Sitter-Schwarzschild black hole spaces joined by a moving domain wall. Then we study dynamics of equation of state parameter of a non-minimally coupled scalar field in this setup. This model has also capability to explain accelerated expansion and phantom divide line crossing in a fascinating manner. Then we summarize cosmological equations of a Lorentz invariance violating model in the spirit of Scalar-Vector-Tensor theories. We find a relation between Lorentz Invariance violation
parameter and dynamics of scalar field. This relation explicitly shows the interactive nature of our Lorentz invariance violating model. This model, with a minimally coupled scalar field accounts for crossing of phantom divide line and also accelerated expansion. There are three important outcomes of our study: it is impossible to cross phantom divide line with a single and minimally coupled scalar field but non-minimal coupling of scalar field in the Jordan frame provides such an important feature. A non-minimally coupled scalar field on the moving domain wall is a good candidate for dark energy which explains both late-time acceleration and phantom divide line crossing. Also, a Lorentz invariance violation model with a minimally coupled scalar field accounts for late-time acceleration and phantom divide line crossing. It is important to note that non-minimal coupling of a scalar field and gravity may provides a basis for symmetry (such as Lorentz invariance) breaking as has been argued in reference [23]. To complete our study, based on recent observational data we obtain some important constraints on the parameters of the models in the favor of late-time accelerated expansion.

2 Accelerated Expansion and PDL Crossing with a Ricci-Coupled Scalar Field

2.1 The Jordan Frame

For a model universe with a non-minimally coupled scalar field as matter content of the universe, the action in the absence of other matter sources in the Jordan frame can be written as follows

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{k_4^2} \alpha(\phi) R[g] - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right],$$  \hspace{1cm} (1)$$

where we have included an explicit and general non-minimal coupling of scalar field and gravity. For simplicity, from now on we set $k_4^2 \equiv 8\pi G_N = 1$. Variation of the action with respect to metric gives the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \alpha^{-1} T_{\mu\nu}. \hspace{1cm} (2)$$

$T_{\mu\nu}$, the energy-momentum tensor of the scalar field non-minimally coupled to gravity, is given by

$$T_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} (\nabla \phi)^2 - g_{\mu\nu} V(\phi) + g_{\mu\nu} \Box \alpha(\phi) - \nabla_\mu \nabla_\nu \alpha(\phi), \hspace{1cm} (3)$$
where □ shows 4-dimensional d’Alembertian. For FRW universe with line element defined as
\[ ds^2 = -dt^2 + a^2(t)d\Sigma_k^2, \] (4)
where \( d\Sigma_k^2 \) is the line element for a manifold of constant curvature \( k = +1, 0, -1 \), the equation of motion for scalar field \( \phi \) is
\[ \nabla^\mu\nabla_\mu \phi = V' - \alpha' R[g], \] (5)
where a prime denotes the derivative of any quantity with respect to \( \phi \). This equation can be rewritten as
\[ \ddot{\phi} + \frac{3\dot{a}}{a} \dot{\phi} + \frac{dV}{d\phi} = \alpha' R[g], \] (6)
where a dot denotes the derivative with respect to cosmic time \( t \) and Ricci scalar is given by
\[ R = 6\left( \dot{H} + 2H^2 + \frac{k}{a^2} \right). \] (7)
With this non-minimally coupled scalar field as matter content of the universe, cosmological dynamics are described by
\[ \frac{\dot{a}^2}{a^2} = -\frac{k}{a^2} + \frac{\rho}{3}, \] (8)
and
\[ \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p). \] (9)
The effect of non-minimal coupling of scalar field and gravity is hidden in the definition of \( \rho \) and \( p \). We assume that scalar field, \( \phi \), has only time dependence and using (3), we find
\[ \rho = \alpha^{-1}\left( \frac{1}{2}\dot{\phi}^2 + V(\phi) - 6\alpha' H \dot{\phi} \right), \] (10)
\[ p = \alpha^{-1}\left( \frac{1}{2}\dot{\phi}^2 - V(\phi) + 2\left( \alpha' \ddot{\phi} + 2H \alpha' \dot{\phi} + \alpha'' \dot{\phi}^2 \right) \right), \] (11)
where \( H = \frac{\dot{a}}{a} \) is Hubble parameter. Now, equation (9) takes the following form
\[ \frac{\ddot{a}}{a} = -\frac{1}{6}\alpha^{-1}\left( 2\dot{\phi}^2 - 2V(\phi) + 6\left( \alpha' \ddot{\phi} + H \alpha' \dot{\phi} + \alpha'' \dot{\phi}^2 \right) \right), \] (12)
and dynamics of equation of state parameter is given by
\[ w \equiv \frac{p}{\rho} = \frac{\dot{\phi}^2 - 2V(\phi) + 4\left( \alpha' \ddot{\phi} + 2H \alpha' \dot{\phi} + \alpha'' \dot{\phi}^2 \right)}{\dot{\phi}^2 + 2V(\phi) - 12\alpha' H \dot{\phi}}. \] (13)
From this equation, when \( \dot{\phi} = 0 \), we obtain \( p = -\rho \). In this case \( \rho \) is independent of \( a \) and \( V(\phi) \) plays the role of a cosmological constant. In the minimal case when \( \dot{\phi}^2 < V(\phi) \),
using (9) we obtain $p < -\frac{\rho}{3}$ which shows an accelerated expansion which is driven by cosmological constant. However, cosmological constant is not a good candidate for dark energy since its suffers from several conceptual problems such as its unknown origin and also need to huge amount of fine-tuning. In non-minimal case the cosmological dynamics depends on the value of non-minimal coupling. As a first goal in this paper we try to see whether late-time accelerated expansion and crossing of phantom divide line are explainable with this non-minimally coupled scalar field as candidate for dark energy or not. Although this issue is not new, our strategy for this purpose differs from existing approaches ( see for instance [12]). To have positively accelerated expansion we need $\rho + 3p < 0$ in equation (9). This is possible when the following relation holds

$$\alpha^{-1} \left[ 2\dot{\phi}^2 - 2V(\phi) + 6(\alpha'\dot{\phi} + H\alpha'\phi + \alpha''\dot{\phi}^2) \right] < 0.$$

To proceed further, we assume a conformal coupling of scalar field and gravity as $\alpha(\phi) = \frac{1}{2}(1 - \xi \phi^2)$. In fact in general relativity, and in all other metric theories of gravity in which the scalar field is not part of the gravitational sector, such a conformal coupling with $\xi = \frac{1}{6}$ is necessary. Then we obtain

$$(1 - 3\xi)\dot{\phi}^2 - V(\phi) + 3\xi^2 \phi^2 + 6\xi H\phi\dot{\phi} + 3\xi \phi \frac{dV}{d\phi} < 0.$$

By imposing the weak energy condition and restricting study to the case with $\xi \leq 1/6$, one finds [24]

$$-2V + 3\xi \phi \frac{dV}{d\phi} < 0$$

and a necessary condition for cosmic acceleration is therefore

$$V - \frac{3\xi}{2} \phi \frac{dV}{d\phi} > 0, \quad \xi \leq \frac{1}{6}.$$

In this case to have cosmic acceleration with $\xi > 0$, the potential $V(\phi)$ should vary with $\phi$ slower than power-law potential $V_c(\phi) = V_0 \left( \frac{\phi}{\phi_0} \right)^{2/3}$. However, when $\xi < 0$, the necessary condition for cosmic acceleration requires that $V$ grow faster than $V_c$ as $\phi$ increases [24]. As a specific example to show how this model works, if we set $V(\phi) = \lambda \phi^n$, condition (17) gives $\lambda \left( 1 - \frac{2n\xi}{2} \right) > 0$ which yields $\xi \leq 2/3n$. Figure 1 gives a qualitative description of required relation between potential and non-minimal coupling to have accelerated expansion. As this figure shows, for positive $\xi$, only for $0 < \xi < 0.026$, this non-minimal model has the capability to explain accelerated expansion. For a more general consideration, we try a reliable ansatz so that $\phi(t) \approx \frac{A}{t^2}$ ( we assume a decreasing
power law ansatz for scalar field with $\beta > 1$) and $a(t) \approx Bt^\nu$. Accelerated expansion requires $\nu > 1$. Now equation (12) can be rewritten as follows

$$\nu(\nu-1)t^{-2} = -\frac{1}{3}(1-\xi t^{-2\beta})^{-1}\left\{2\beta^2t^{-2\beta-2} - 2t^{-3\beta} - 6[\beta(\beta+1)\xi t^{-2\beta-2} + \beta\nu\xi t^{-2\beta-2} - \xi\beta^2t^{-2\beta-2}]\right\}$$

(18)

Considering terms of order $t^{-2\beta-2}$, we find

$$3\nu(\nu-1)\xi = 6[\beta(\beta+1)\xi + \beta\nu\xi - \xi\beta^2] - 2\beta^2.$$  

(19)

On the other hand, equation (6) for spatially flat FRW geometry gives

$$\beta(\beta + 1)t^{-\beta-2} - 3\nu\beta t^{-\beta-2} + nt^{-\beta n+\beta} = 6\xi\nu t^{-\beta-2} - 12\xi\nu^2 t^{-\beta-2}.$$  

(20)

By considering terms of order $t^{-\beta-2}$, we find

$$\beta(\beta + 1) - 3\nu\beta = 6\xi\nu - 12\xi\nu^2.$$  

(21)

Now we have two equations (19) and (21) for three parameters $\xi$, $\beta$ and $\nu$. We first solve equation (19) for $\nu$ to obtain

$$\nu = \frac{3\xi - 6\beta\xi \pm \sqrt{(3\xi - 6\beta\xi)^2 + 12\xi[6[\beta(\beta+1)\xi - \xi\beta^2] - 2\beta^2]}}{6\xi}.$$  

(22)
A numerical analysis shows that reality of $\nu$ is preserved if we choose positive $\xi$ with $1 \leq \beta < 1.9$ (note that this condition is supported by equation (21) which gives $\beta^2 - 2\beta + 1 \geq 0$). With this requirements and taking positive sign in (22) we obtain possible values of $\nu$ in this ansatz. The result is shown in figure 2. This figure shows that with nonminimally coupled scalar field one can explain accelerated expansion, that is $\nu > 1$, naturally. Thus non-minimal coupling of scalar field and gravity in the Jordan frame provides a suitable framework for explanation of late-time accelerated expansion. Note that with negative sign in equation (22) it is impossible to find a positive $\nu$.

\[\omega = \frac{\beta^2 t^{-2\beta - 2} + 2t^{-\beta n} + 4[-\xi \beta (\beta + 1)t^{-2\beta - 2} + 2\nu \xi \beta t^{-2\beta - 2} - \xi \beta^2 t^{-2\beta - 2}]}{\beta^2 t^{-2\beta - 2} + 2t^{-\beta n} - 12\beta \xi \nu t^{-2\beta - 2}}. \quad (23)\]

Figure 3 shows the crossing of phantom divide line with equation of state parameter of this non-minimally coupled scalar field. On the other hand, as figure 4 shows, in the case of $\xi = 0$, that is a single minimally coupled scalar field, there is no crossing of phantom divide barrier, as has been emphasized by other literature [22].
Figure 3: A non-minimally coupled scalar field in Jordan frame has the capability to have crossing of phantom divide line by its equation of state parameter in a suitable domain of parameter space.

Figure 4: Equation of state parameter of a single minimally coupled scalar field (with $\xi = 0$), cannot explain crossing of phantom divide line [22].
2.2 The Einstein frame

Now we study the situation in Einstein frame by a conformal transformation. The action (1) in Jordan frame can be rewritten as follows

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \hat{R} - \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \xi R \phi^2 - V(\phi) \right] \]  \hspace{1cm} (24)

where we assumed \( k_4^2 = 1 \) and \( \alpha(\phi) = \frac{1}{2}(1 - \xi \phi^2) \) and \( \xi \) is a non-minimal coupling. The metric signature convention is chosen to be (+ − − −) with spatially flat Robertson-Walker metric as follows

\[ ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j. \]  \hspace{1cm} (25)

To obtain the fundamental background equations in Einstein frame, we perform the following conformal transformation

\[ \hat{g}_{\mu \nu} = \Omega g_{\mu \nu}, \quad \Omega = 1 - \xi \phi^2. \]  \hspace{1cm} (26)

Here we use a hat on a variable defined in the Einstein frame. The conformal transformation gives

\[ S = \int d^4x \sqrt{-\hat{g}} \left[ \frac{1}{2} \hat{R} - \frac{1}{2} F^2(\phi) \hat{g}^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \hat{V}(\phi) \right], \]  \hspace{1cm} (27)

where by definition

\[ F^2(\phi) \equiv \frac{1 - \xi \phi^2(1 - 6\xi)}{(1 - \xi \phi^2)^2} \]  \hspace{1cm} (28)

and

\[ \hat{V}(\phi) \equiv \frac{V(\phi)}{(1 - \xi \phi^2)^2}. \]  \hspace{1cm} (29)

Therefore, one may redefine the scalar field as follows

\[ \frac{d\hat{\phi}}{d\phi} = F(\phi) = \frac{\sqrt{1 - \xi \phi^2(1 - 6\xi)}}{1 - \xi \phi^2}. \]  \hspace{1cm} (30)

When we investigate the dynamics of universe in the Einstein frame, we should transform our coordinates system to make the metric in the Robertson-Walker form

\[ \dot{a} = \sqrt{\Omega} a, \quad \dot{\Omega} = \sqrt{\Omega} dt, \]  \hspace{1cm} (31)
and we obtain
\[ ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j. \] (32)

Note that the physical quantities in Einstein frame should be defined in this coordinate system. Now the field equations can be written as follows
\[ \dot{H}^2 = \frac{1}{3} \left[ \frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 + \dot{V}(\phi) \right] = \frac{\rho}{3}, \] (33)
\[ \frac{d^2 \phi}{dt^2} + 3H \frac{d\phi}{dt} + \frac{d\dot{V}}{d\phi} = 0 \] (34)

where \( \dot{H} = \frac{\dot{a}}{a} \). We assume that scalar field \( \phi \) has only time dependence and we find dynamics of equation of state as follows
\[ \dot{\omega}_{\phi} = \frac{\dot{p}}{\rho} = \frac{1}{4} \left( \frac{d\phi}{dt} \right)^2 - \ddot{V}(\phi) + \frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 + \dot{\phi}, \] (35)

This is an interesting result: it shows that a non-minimally coupled scalar field in Einstein frame cannot support the phantom phase. In fact, conformal transformation from Jordan frame to Einstein frame transforms the equation of state parameter to its minimal form but with a redefined scalar field and in this case it is impossible to achieve phantom phase (and therefore no crossing of phantom divide line).

3 Braneworld Considerations

In this section we show that a minimally coupled scalar field localized on the brane provides even more suitable candidate for explanation of accelerated expansion and phantom divide line crossing. With this motivation, in which follows, along with studies in [25-28], we consider a moving domain wall picture of braneworld to discuss the issues of late-time acceleration and phantom divide line crossing of equation of state parameter with a non-minimally coupled scalar field localized on the brane. Following [26], we consider a static bulk configuration with two 5-dimensional anti de Sitter-Schwarzschild (AdSS\(_5\)) black hole spaces joined by a moving domain wall. To embed this moving domain wall into 5-dimensional bulk, it is then necessary to specify normal and tangent vectors to this domain wall with careful determination of normal direction to the brane. We assume
that domain wall is located at coordinate \( r = a(\tau) \) where \( a(\tau) \) is determined by Israel junction conditions \([29]\). In this model, observers on the moving domain wall interprets their motion through the static 5-dimensional bulk background as cosmological expansion or contraction. Now consider the following line element \([26]\)

\[
dS_{5\pm}^2 = -\left(k - \frac{\eta_\pm}{r^2} + \frac{r^2}{\ell^2}\right)dt^2 + \frac{1}{k - \frac{\eta_\pm}{r^2} + \frac{r^2}{\ell^2}}dr^2 + r^2\gamma_{ij}dx^i dx^j,
\]

where \( \pm \) stands for left(\(-\)) and right(\(+\)) side of the moving domain wall, \( \ell \) is curvature radius of AdS\(_5\) manifold and \( \gamma_{ij} \) is the horizon metric of a constant curvature manifold with \( k = -1, 0, 1 \) for open, flat and closed horizon geometry respectively and \( \eta_\pm \neq 0 \) generates the electric part of the Weyl tensor on each side. This line element shows a topological anti de Sitter black hole geometry in each side. Using Israel junction conditions \([29]\) and Gauss-Codazzi equations we find the following generalization of the Friedmann and acceleration equations

\[
\frac{\ddot{a}}{a^2} + \frac{k}{a^2} = \frac{\rho}{3} + \frac{\eta}{a^4} + \frac{\ell^2}{36}\rho^2, \quad (37)
\]

\[
\frac{\ddot{a}}{a} = -\frac{\rho}{6}(1 + 3w) - \frac{\eta}{a^4} - \frac{\ell^2}{36}\rho^2(2 + 3w), \quad (38)
\]

where we have adapted a \( Z_2 \)-symmetry with \( \eta_+ = \eta_- \equiv \eta \) and \( \omega \) is defined as \( \omega = \frac{\eta}{\rho} \). Assuming that brane is tensionless, in which follows we discuss two cases with \( \eta = 0 \) and \( \eta \neq 0 \) separately. Note that \( \eta \) is the coefficient of a term which is called holographic matter term. For \( \eta = 0 \), each sub-manifolds of bulk spacetime are exact AdS\(_5\) spacetimes.

Now we consider a localized non-minimally coupled scalar field on the brane and discuss its cosmological implications especially on late-time dynamics. In this case we use energy density and pressure of scalar field defined in equations \((10)\) and \((11)\) as the only matter source on the brane. In this case, equation \((38)\) takes the following form

\[
\frac{\ddot{a}}{a} = -\frac{1}{6\alpha}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) - 6\alpha' H\dot{\phi}\right)\left(1 + 3\frac{\dot{\phi}^2 - 2V(\phi) + 4(\alpha'\ddot{\phi} + 2H\alpha'\dot{\phi} + \alpha''\dot{\phi}^2)}{\dot{\phi}^2 + 2V(\phi) - 12\alpha' H\dot{\phi}}\right)
\]

\[-\frac{\ell^2}{36\alpha^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) - 6\alpha' H\dot{\phi}\right)^2 \left(2 + 3\frac{\dot{\phi}^2 - 2V(\phi) + 4(\alpha'\ddot{\phi} + 2H\alpha'\dot{\phi} + \alpha''\dot{\phi}^2)}{\dot{\phi}^2 + 2V(\phi) - 12\alpha' H\dot{\phi}}\right), \quad (39)
\]

where \( H = \frac{\dot{a}}{a} \) is Hubble parameter on the moving domain wall. This is a complicate relation and to explain its cosmological implications, we have to consider either some limiting cases or specify \( \alpha(\phi), V(\phi) \) and \( \phi \). One can apply the ansatz introduced in the last section with conformal coupling of scalar field and Ricci scalar on the brane to investigate late time behavior of this equation. But, due to existence of several fine-tunable parameters and a combination of plus and minus signs in this relation, essentially
it is possible to find a domain of parameters space that satisfies the condition $\ddot{a} > 0$ in the favor of positively accelerated expansion. For instance, if we set $\phi = \phi_0 e^{-\kappa t}$ with $\kappa > 0$, $a = a_0 t^\nu$, $V = \lambda \phi^n$, and $A = \frac{\ddot{a}}{a}$, then equation (39) gives

$$A = -\frac{E_1 (1 + 3 E_2)}{3 - 3 \xi \phi} - \frac{\ell^2 E_1^2 (2 + 3 E_2)}{9 - 9 \xi \phi}$$

(40)

where

$$E_1 \equiv 0.5 \kappa e^{-2\kappa t} + \lambda e^{-\kappa nt} - 6 \xi e^{-2\kappa t} \nu \kappa$$

and

$$E_2 \equiv \left(\kappa^2 e^{-2\kappa t} - 2 \lambda e^{-\kappa nt} + 8 \xi e^{-2\kappa t} \nu \kappa \right) \left(\kappa^2 e^{-2\kappa t} + 2 \lambda e^{-\kappa nt} - 12 \xi e^{-2\kappa t} \nu \kappa \right)^{-1}.$$

Figure 5 shows the possibility of accelerated expansion ($A > 0$ for $\nu > 1$ in some appropriate domain of parameter space (for example with $\lambda = \kappa = \ell = 1$ and $\xi = -0.1$)). The case with $\eta \neq 0$ accounts for accelerated expansion in even more simpler manner due to its wider parameter space. In this braneworld setup, equation of state parameter with above ansatz (defined before equation (40)) has the following form

$$\omega(t) = \frac{\kappa^2 e^{-2\kappa t} - 2 \lambda e^{-\kappa nt} + 8 \nu \xi \kappa e^{-2\kappa t}}{\kappa^2 e^{-2\kappa t} + 2 \lambda e^{-\kappa nt} - 12 \nu \xi \kappa e^{-2\kappa t}}$$

(41)

Figure 6 shows the dynamics of equation of state parameter in this case with above mentioned ansatz. As this figure shows, equation of state parameter crosses the phantom divide line $\omega = -1$. On the other hand, crossing of phantom divide line with $\eta \neq 0$ is easily achieved due to wider parameter space in this case. As we have emphasized in introduction, models of phantom divide line crossing are so important that they can realize that which model is better than the others to describe the nature of dark energy. In this sense a non-minimally coupled scalar field on the brane provides a good candidate for explaining accelerated expansion and crossing of phantom divide line as a reliable candidate for dark energy.
Figure 5: Accelerated expansion with a nonminimally coupled scalar field on the brane.

Figure 6: Crossing of phantom divide line with non-minimally coupled scalar field on the brane.
4 Lorentz Invariance Violating Cosmology

The purpose of this section is to study the effect of an explicit violation of Lorentz invariance via incorporating a vector field in the action. Following references [19,20], we summarize the cosmological dynamics of a Lorentz invariance violating scenario with a minimally coupled scalar field. Our goal is to find a relation between Lorentz Invariance violation parameter and dynamics of scalar field. This relation will affect the equation of state of minimally coupled scalar field which is the central object of subsequent discussions. In fact, this relation reflects the interactive nature of this model.

We start with the following action for a typical scalar-vector-tensor theory which admits Lorentz invariance violation

\[ S = S_g + S_u + S_\phi, \] (42)

where the actions for the tensor field \( S_g \), the vector field \( S_u \), and the scalar field \( S_\phi \) are defined as follows

\[ S_g = \int d^4x \sqrt{-g} \frac{1}{16\pi G} R, \] (43)
\[ S_u = \int d^4x \sqrt{-g} \left[ -\beta_1 \nabla^\mu u^\nu \nabla_\mu u_\nu - \beta_2 \nabla^\mu u^\nu \nabla_\nu u_\mu - \beta_3 (\nabla_\mu u^\mu)^2 - \beta_4 u^\mu u_\nu \nabla_\mu u_\alpha + \lambda (u^\mu u_\mu + 1) \right], \] (44)
\[ S_\phi = \int d^4x \sqrt{-g} \mathcal{L}_\phi. \] (45)

Action (42) is allowed to contain any non-gravitational degrees of freedom in the framework of Lorentz violating scalar-tensor-vector theory of gravity. As usual, we assume \( u^\mu u_\mu = -1 \) and that the expectation value of vector field \( u^\mu \) is \( <0|u^\mu u_\mu|0>=-1 \) [30]. \( \beta_i(\phi) \) \( (i = 1, 2, 3, 4) \) are arbitrary parameters with dimension of mass squared and \( \mathcal{L}_\phi \) is the Lagrangian density for scalar field. Also, \( \sqrt{\beta_i} \) are mass scale of Lorentz symmetry breakdown [19,30]. The detailed cosmological consequences of this action are studied in reference [19]. Assuming a homogeneous and isotropic universe, we describe the universe with the following metric

\[ ds^2 = -N^2(t)dt^2 + e^{2\alpha(t)} \delta_{ij}dx^i dx^j, \] (46)

where \( N \) is a lapse function and the scale of the universe is determined by \( \alpha \). By variation of the action with respect to metric and choosing a suitable gauge, one obtains the following field equations

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi GT_{\mu\nu}, \] (47)
where $T_{\mu\nu} = T_{\mu\nu}^{(u)} + T_{\mu\nu}^{(\phi)}$ is the total energy-momentum tensor, $T_{\mu\nu}^{(u)}$ and $T_{\mu\nu}^{(\phi)}$ are the energy-momentum tensors of vector and scalar fields, respectively. The time and space components of the total energy-momentum tensor are given by [20]

$$T_0^0 = -\rho_u - \rho_\phi, \quad T_i^i = p_u + p_\phi,$$

(48)

where the energy density and pressure of the vector field are calculated as follows

$$\rho_u = -3\beta H^2,$$

(49)

$$p_u = \left(3 + 2\frac{H'}{H} + 2\frac{\beta'}{\beta}\right)\beta H^2,$$

(50)

$$\beta \equiv \beta_1 + 3\beta_2 + \beta_3,$$

(51)

a prime denotes the derivative of any quantity with respect to $\alpha$ and $H \equiv d\alpha/dt = \dot{\alpha}$ is the Hubble parameter. One can see that $\beta_4$ does not contribute to the background dynamics [19,20]. The energy equations for the vector field $u$ and scalar field, $\phi$ are as follows

$$\rho'_u + 3(\rho_u + p_u) = +3H^2\beta',$$

(52)

$$\rho'_\phi + 3(\rho_\phi + p_\phi) = -3H^2\beta',$$

(53)

respectively. There is a non-conservation scheme in this setup due to energy-momentum transfer between scalar and vector fields. This is very similar to the case studied by Zimdahl et al [35]. As they have shown, a coupling between a quintessence scalar field and a cold dark matter (CDM) fluid leads to a stable, constant ratio for the energy densities of both component compatible with a power law accelerated cosmic expansion. In fact this coupling is responsible for accelerated expansion and possible crossing of PDL line. In our Lorentz invariance violating scenario this coupling is present between scalar field and vector field as is manifested from equations (52) and (53) corresponding to equations (4) and (5) of Ref. [35] with $\delta \equiv -3H^2\beta \Pi_u = 3H^2\beta \Pi_u$, where $\Pi_u$ and $\Pi_u$ are effective pressure of vector and scalar component. Nevertheless, the total energy equation in the presence of both the vector and the scalar fields reads

$$\rho' + 3(\rho + p) = 0, \quad (\rho = \rho_u + \rho_\phi),$$

(54)

which shows the conservation of total energy density. With these preliminaries, dynamics of the model is described by the following Friedmann equations

$$\left(1 + \frac{1}{8\pi G\beta}\right)H^2 = \frac{1}{3\beta}\rho_\phi,$$

(55)

$$\left(1 + \frac{1}{8\pi G\beta}\right)(HH' + H^2) = -\frac{1}{6}\left(\frac{p_\phi}{\beta} + \frac{3p_\phi}{\beta}\right) - H^2\frac{\beta'}{\beta}.$$  

(56)
In the absence of vector field, that is, when all $\beta_i = 0$, one recovers the standard equations of dynamics. For the scalar sector of our model we assume the following Lagrangian

$$L_\phi = -\frac{\varepsilon}{2} (\nabla \phi)^2 - V(\phi),$$

(57)

where $(\nabla \phi)^2 = g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi$. Ordinary scalar fields are correspond to $\varepsilon = 1$ while $\varepsilon = -1$ describes phantom fields. For the homogeneous scalar field, the density $\rho_\phi$ and pressure $p_\phi$ are given as follows

$$\rho_\phi = \frac{\varepsilon}{2} H^2 \phi'^2 + V(\phi),$$

(58)

$$p_\phi = \frac{\varepsilon}{2} H^2 \phi'^2 - V(\phi).$$

(59)

The corresponding equation of state parameter is

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = -\frac{1 - \varepsilon H^2 \phi'^2 / 2V}{1 + \varepsilon H^2 \phi'^2 / 2V}. $$

(60)

Now the Friedmann equation takes the following form [20]

$$H^2 = \frac{1}{3\bar{\beta}} \left[ \frac{\varepsilon}{2} H^2 \phi'^2 + V(\phi) \right],$$

(61)

where $\bar{\beta} = \beta + \frac{4}{8\pi G}$. Using this equation we can show that

$$\phi' = -2\varepsilon \bar{\beta} \left( \frac{H \phi}{H} + \frac{\bar{\beta} \phi}{\beta} \right).$$

(62)

Substituting this equation into the Friedmann equation, the potential of the scalar field can be written as

$$V = 3\bar{\beta} H^2 \left[ 1 - \frac{2}{3} \varepsilon \bar{\beta} \left( \frac{\bar{\beta} \phi}{\beta} + \frac{H \phi}{H} \right)^2 \right].$$

(63)

Note that in the above equations the Hubble parameter $H$ has been expressed as a function of $\phi$, $H = H(\phi(t))$. One can show that the equation of state has the following form

$$\omega_\phi = -1 + \frac{4}{3} \varepsilon \bar{\beta} \left( \frac{H \phi}{H} + \frac{\bar{\beta} \phi}{\beta} \right)^2$$

$$= -1 + \frac{1}{3} \varepsilon \frac{\phi'^2}{\bar{\beta}}.$$

(64)

Equations (62) and (64) are essential equations in forthcoming arguments. We stress here that violation of the Lorentz invariance which has been introduced by existence of
a vector field in the action, now has incorporated in the dynamics of scalar field and equation of state via existence of $\bar{\beta}$ reflecting interactive nature of the model. This interesting feature allows us to study crossing of phantom divide line by equation of state parameter of minimally coupled scalar field and late-time accelerated expansion as a result of interaction in the context of Lorentz invariance violation. We need to solve these two equations, (62) and (64), to find dynamics of scalar field $\phi$ and the equation of state $\omega_\phi$. This will be achieved only if the Hubble parameter $H(\phi)$ and the vector field coupling, $\bar{\beta}(\phi)$ are known. In which follows, our strategy is to choose some different cases of the Hubble parameter $H(\phi)$ and the vector field coupling $\bar{\beta}(\phi)$ and then investigating possible crossing of phantom divide barrier and late-time acceleration in this context. We obtain suitable domains of parameter space which have the capability to explain late-time acceleration and crossing of phantom divide line by equation of state parameter.

4.1 Late-Time Acceleration

In reference [21] we have studied late-time acceleration and phantom divide line crossing with Lorentz invariance violating fields for several interesting cases. Here we extend that study for a more general case. The condition for positive acceleration of the universe, that is, $\dot{a} > 0$ can be rewritten as $H'/H > -1$ in this Lorentz invariance violating scenario. We consider a general case where both the vector field coupling and the Hubble parameter are functions of scalar field $\phi$ defined as follows (see [21] for motivations behind choosing this ansatz)

$$H = H_0 \phi^\zeta, \quad \bar{\beta}(\phi) = m \phi^n, \quad n > 2$$

(65)

In which follows, we consider just a quintessence scalar field with $\varepsilon = 1$. Using equation (62), for this case we obtain

$$\phi(t) = \left[ H_0(t - t_0)(-4 \zeta m + 4 \zeta mn + 2 \zeta^2 m - 4 mn + 2 mn^2) + \phi_0 \right]^{-\frac{1}{n+1}}$$

(66)

and using equation (64) we find

$$\omega_\phi(t) = -1 + \frac{4}{3} m \phi^{n-2}(t)(\zeta + n)^2$$

(67)

Now we obtain a condition for positively accelerated expansion, $H'/H > -1$. We use equation (64), (65) and (67) to find

$$m^2 < \frac{1}{4(-1)^n \phi^{n-2}(t)(\zeta + n)^2}, \quad n > 2,$$

(68)
This relation can be used to constraint parameters of this model in order to have late-time acceleration by confrontation with observational data. Now we use relations (65) and (66) to obtain dynamics of scale factor

\[ a(t) = a_0(t_0) e^\Upsilon \]  

(69)

where

\[ \Upsilon \equiv \frac{2\phi_0^{(n+\zeta-2)} m(n+\zeta)(n+\zeta-2)H_0(t-t_0) + \frac{1}{2}\phi_0}{2\phi_0^{(n+\zeta-2)}(n-2)m(n+\zeta)} e^\Omega - \phi_0 \]

and

\[ \Omega \equiv -\zeta \left( \ln \left[ \frac{2m(n+\zeta)(n+\zeta-2)H_0(t-t_0) + \phi_0}{n+\zeta-2} \right] \right) \]

The functional form of scale factor in this case is very complicated. To find an understandable relation, we expand relation (69) in Taylor series. Choosing \( n = 3, \zeta = -2 \) and \( m = -0.1 \), we find

\[ a(t) = 0.286504 + 0.447663 t + 0.237821 t^2 + 0.0352651 t^3 + O(t^4) \]  

(70)

This relation shows that a Lorentz invariance dark energy model explicitly accounts for cosmic accelerated expansion as a result of interactive nature of the model. Figure 7 shows the variation of scale factor with time. Evidently, it has positive second derivation and so accounts for accelerated expansion.

### 4.2 Crossing the Phantom Divide Line

We can obtain dynamics of equation of state parameter for minimally coupled scalar field in this Lorentz invariance violating model. With \( \phi \) defined as (66), the equation of state takes the following form

\[ \omega(t) = -1 + \frac{4}{3} m \left[ \frac{(\zeta+n)^2}{H_0(t-t_0)(-4\zeta m + 4\zeta mn + 2\zeta^2m - 4mn + 2mn^2) + \phi_0} \right]^{(n-2)/(n+\zeta-2)} \]  

(71)

which explicitly has a dynamical behavior. This model allows us to choose a suitable parameter space to explain crossing of phantom divide barrier by equation of state parameter. This parameter space should be checked by observational data in order to have a reasonable cosmological model.

As an important point, we should be careful to choose the appropriate equation of state for components that are used to describe the universe energy-momentum content.
As we have emphasized earlier, a suitable coupling between a quintessence scalar field and other matter content can lead to a constant ratio of the energy densities of both components which is compatible with an accelerated expansion of the Universe or Crossing of phantom divide line (for more details see [35] and reference therein). In this respect and for instance, the holographic dark energy models studied in Ref. [22] have the phantom phase by adopting a native equation of state, whereas the authors in [36] have found accelerating phase only using the effective equation of state. Based on these arguments, we should explain what kind of equation of state is used for observing the nature of mixed fluids here. In our model, we have three sources of energy-momentum: 1- standard ordinary matter, 2- Scalar Field as a candidate of Dark Energy and 3- energy-momentum content depended on Lorentz violating vector field. Here we assume that standard matter has negligible contribution on the total energy-momentum content of the universe and we can consider a constant linear isothermal equation of state as $p_m = (\gamma - 1)\rho_m$ that $1 \leq \gamma \leq 2$ for it. For other two energy-momentum contents, it is possible to use the "trigger mechanism" to explain dynamical equation of state. This means that we assume scalar- vector-tensor theory containing Lorentz invarian ce violation which acts like the hybrid inflation models. In this situation, vector and scaler field play the roles of inflaton and the "waterfall" field respectively. In this regard, we can fine-tune parameter $m$ in equation (68) to obtain best fit model using the observational data. Of course, an attractor solutions and fine-tuning in Lorentz violation model for suitable inflation phase has been

Figure 7: Variation of scale factor $a(t)$ for different values of $t$ with $n = 3$, $\zeta = -2$ and $m = -0.1$. This figure shows accelerated expansion in this Lorentz invariance violating setup. The values of $\zeta$ are determined by relation (56).
studied in Ref. [20]. Therefore it is reasonable to expect that one of them will eventually dominate to explain inflation or accelerating phase and crossing of phantom divided line.

Figure 8 shows the dynamics of $\omega(t)$ in this setup, it crosses phantom divide line explicitly. The most important aspect of the present model is the fact that, Lorentz invariance violation provides a situation that one scalar field and another vector field, in an interactive picture, describe the phantom divide line crossing and universe late time acceleration. Figure 8 may be used also to explain why we are living in an epoch of $\omega < -1$ since in late time we see that $\omega < -1$. This is the second cosmological coincidence problem. Remember that $\bar{\beta}(t)$ plays the role of Lorentz invariance violation in this setup. Equation of dynamics for $\bar{\beta}(t)$ implicitly has an important meaning: by a suitable fine tuning one can construct a Lorentz violating cosmology consistent with observational data. In another words, this setup provides an important basis for testing LIV in cosmological context. Although many different models can also lead to phantom divided barrier crossing, our model is special in this respect since it contains only one scalar field and the presence of Lorentz violating vector field and interactive nature of this model control the crossing. In this sense, fine tuning of parameters space based on observational data restricts the value that $\bar{\beta}(t)$ can attain. Any non-vanishing value of $\bar{\beta}$ in our model shows violation of Lorentz symmetry in this cosmological setup. Lorentz
invariance violating inflation models constraint by WMAP and other observational data may provide other tests of LIV in cosmological setup. To see possible detection of Lorentz-violating fields in cosmology see [31,32,33]. In the presence of LIV, just one scalar field is enough to achieve phantom divide barrier crossing and existence of vector field controls the situation. Two important point should be stressed here: firstly, as figure 8 shows, there are some sudden jumps of the equation of state. In many existing models whose equation of state can cross the phantom divide line, \( \omega \) undulates around \(-1\) randomly ([34] and references therein). These jumps are actually a manifestation of this undulation which may be a signature of chaotic behavior of equation of state during its evolution. Secondly, as these figures show, crossing of the phantom divide line can occur at late-time. This fact, as second cosmological coincidence, needs additional fine-tuning in model parameters and trigger mechanism, for instance, can be used to alleviate this coincidence.

5 Summary

Light-curves analysis of several hundreds type Ia supernovae, WMAP observations of the cosmic microwave background radiation and other CMB-based experiments have shown that our universe is currently in a period of accelerated expansion. In this respect, construction of theoretical frameworks with potential to describe positively accelerated expansion and crossing of the phantom divide line by equation of state parameter, itself is an interesting challenge. According to existing literature on dark energy models, a minimally coupled scalar field is not a good candidate for dark energy model with equation of state parameter crossing the phantom divide line. On the other hand, a scalar field non-minimally coupled to gravity in the Jordan frame has the capability to be a suitable candidate for dark energy which provides this facilities. Although this issue has been studied in literature, our study here is different in its different approach based on numerical analysis of parameter space. We have extended this study to a braneworld setup where brane has been considered as a moving domain wall in a static bulk background. In this braneworld setup, non-minimally coupled scalar field provides even more reliable candidate for dark energy. Then we have extended our study to the Lorentz invariance violating dark energy model. We have shown that a minimally coupled scalar field in the presence of a Lorentz violating vector field provides a good candidate for dark energy with capability of describing late-time acceleration and phantom divide line crossing. One important observation here is the fact that this model achieve an interactive nature which this interaction is responsible for late-time acceleration and phantom divide line crossing. As some details of our analysis, we emphasize that due to complication of dynamical
equations, we have restricted our study to some specific form of non-minimal coupling and scalar field potentials and also we have considered some special form of time evolution for scale factor and scalar field. These choices, though especial, are natural and motivated from powerful grounds based on recent observational data. Crossing of phantom divide barrier by a single scalar field in the presence of a Lorentz violating vector field and with suitable fine tuning of model parameters in an interactive picture, is an important outcome in this context. This feature is more considerable were we emphasize that in the absence of Lorentz invariance violating vector field, it is impossible to cross phantom divide line just by one scalar field minimally coupled to gravity.

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