Research Article

A New Growth Curve for Predicting Production Performance of Water-Flooding Oilfields

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Oil production and water cut prediction is one of the most important research contents of reservoir production performance analysis. The growth curve method has the advantages of the general water drive curve method and the combined solution model method with fewer parameters and simple and fast calculation process and so it has been widely used in well production prediction. Based on the analysis of 4W and 4Y4 model growth curves, a new generalized growth curve of the well production performance is proposed. The new model can forecast cumulative oil production, annual oil production, and water cut at different oilfield development periods. A MATLAB program was developed to derive the parameters in the new model. The built model was applied to the production data of the Samattalol oilfield and Daqing oilfield. The predicted cumulative oil production, annual oil production, and water cut are all close to the actual production data, and satisfactory results are obtained, which demonstrates the practicability and reliability of the new model.

1. Introduction

Water flooding is the most common oilfield development technology around the world. With the oilfields gradually entering the stage of medium or high water cut, accurate prediction of reservoir production performance is very important for reservoir engineers to formulate reasonable measures to stabilize oil production and control water production and to adjust oilfield development plans [1–2]. Oil production and water cut are two major contents of reservoir production performance analysis. Generally, the main methods to predict the annual oil production and water cut of the reservoir include numerical reservoir simulation, water drive curve, and analytical model. The numerical reservoir simulation requires a complex geological model, and it is time-consuming and has low efficiency [3–8]. Since the water drive curve method was proposed in the 1980s, there have been about 50 kinds of water drive curves, but many water drive curves have certain limitations in calculating different types of reservoirs, and the calculation accuracy is not high and the calculation results sometimes differ greatly [9–14]. In contrast, the analytical method has better practical conditions and is suitable for the whole process of oilfield production and water cut changes [15, 16]. Ling et al. proposed an oil cut power function fitting method for the decline rate of the type A water drive curve. However, the method is only suitable for predicting the production of reservoirs with water cut greater than 70% which conforms to the characteristics of type A water drive [17]. Liu et al. established a new dynamic calculation method of displacement efficiency and volume sweep coefficient of water drive reservoirs. The new method overcomes the dependence of most methods on the relative permeability curves of oil and water phases [18]. Based on the correlation between the oil-water relative permeability ratio and water saturation, a new water-flooding type curve was created by combination with the conventional reservoir engineering methods [19–21].
Based on the previous studies about the growth curve model, combined with the 4W and 4Y4 types of growth curve, the new prediction model of annual oil production, cumulative oil production, and water cut are established. The reliability of these built models is verified by the actual production data of the Samottorol oilfield and Daqing oilfield.

2. Establishment of the New Model

Weibull, Gompertz, and logistic growth curves are used to predict the oilfield development performance. Yu [22] studied the growth curves extensively and summarized the following laws: when time \( t \) tends to 0, cumulative oil production \( N_P \) tends to 0; when \( t \) tends to infinity, \( N_P \) tends to \( N_{R_{max}} \). The study of growth curve includes the characteristics, application, recoverable reserves, and development index of the curve. It can be seen that the growth curve plays an important role in the analysis and prediction of reservoir production performance. Due to the complexity of production history, it is very difficult to describe the whole process of oilfield development quantitatively. The water cut variations of many oilfields do not follow the single 4W or 4Y4 water drive type curve but a type in-between these two water drive type curves. A simple and practical comprehensive water drive type curve is established. The new model expands the application range of the water drive curve, including both the 4W water drive type curve and the 4Y4 water drive type curve.

Yu [22] proposed the 4W growth curve for predicting oilfield development performance. The curve form is simple, and only has three coefficients \( \alpha, \beta, \) and \( c \). The growth curve of 4W type is given by

\[
N_P = N_{R_{max}} \left(1 - e^{-t^{\alpha}/\alpha}\right)^c.
\]  

(1)

Suppose that \( a = 1/\alpha \) and \( b = \beta \), the 4W growth curve equation can be rewritten as

\[
N_P = N_{R_{max}} \left(1 - e^{-t^{\beta}/\beta}\right)^c.
\]  

(2)

The growth curve of the 4Y4 type is shown as

\[
N_P = N_{R_{max}} \left(\frac{e^{atb} - 1}{e^{atb} + 1}\right)^c.
\]  

(3)

Equations (2) and (3) have similar mathematical expressions. According to the relationship between cumulative oil production and development time in equations (2) and (3), the new growth curve for predicting oilfield development performance is established:

\[
N_P = N_{R_{max}} \left(\frac{e^{atb} - 1}{e^{atb} + d}\right)^c.
\]  

(4)

Equation (4) is the new cumulative oil production prediction model.

When parameter \( d = 1 \), equation (4) is transformed into equation (2), and when \( d = 1 \), equation (4) is transformed into equation (3). Therefore, both equations (2) and (3) are special cases of equation (4).

Taking the derivation with respect to the time \( t \) on both sides of equation (4), the corresponding new prediction model of annual oil production can be obtained as follows:

\[
Q = \frac{dN_P}{dt} = N_{R_{max}} \left(\frac{abce^{atb}t^{b-1}(d+1)}{(e^{atb}+d)^2}\right) \left(\frac{e^{atb} - 1}{e^{atb} + d}\right)^c - 1.
\]  

(5)

According to the law of growth curve, when \( t \) tends to 0, \( N_P \) tends to 0 and then water cut \( f_w \) tends to 0; when \( t \) tends to infinity, \( N_P \) tends to \( N_{R_{max}} \) and then \( f_w \) tends to 1 (taking the limit water cut as 1). Therefore, equation (4) can be rewritten as follows:

\[
f_w = \left(1 - e^{-t^{\beta}/\beta}\right)^c.
\]  

(6)

Equation (6) is the new established water cut prediction model. The parameter \( d \) in equations (4)–(6) is unknown, which overcomes the limitation of the original model and expands the application range of the growth curve in reservoir production performance analysis.

When \( d = 0 \), the corresponding water cut prediction model of the 4W growth curve is

\[
f_w = \left(1 - e^{-t^{\beta}/\beta}\right)^c.
\]  

(7)

When \( d = 1 \), the corresponding water cut prediction model of the 4Y4 growth curve is given as follows:

\[
f_w = \left(1 - \frac{e^{atb} - 1}{e^{atb} + 1}\right)^c.
\]  

(8)

3. Solution of the Model

There are two methods to solve cumulative oil production in equation (4) and water cut model in equation (6). One is to apply the MATLAB programming method to obtain the values of parameters \( a, b, c, d, \) and \( N_{R_{max}} \) through multiple regression. The other method is to rewrite equation (4) into

\[
\ln \left[ \ln \left(\frac{1 + d \times \left(N_P/N_{R_{max}}\right)^{1/c}}{1 - \left(N_P/N_{R_{max}}\right)^{1/c}}\right) \right] = \ln \left( a + b \ln t \right).
\]  

(9)

The left and right sides of equation (9) are double logarithmic linear relations. Through the linear trial and error method, when the linear correlation is the best, the values of \( a, b, c, d, \) and \( N_{R_{max}} \) are obtained and the variation of cumulative oil production with time can be calculated by equation (4). Due to the complexity of equation (5), it is difficult to calculate the relevant parameters by the above two methods. The annual oil production can be calculated by the following formula:

\[
Q(t) = N_P(t) - N_P(t-1).
\]  

(10)
The water cut equation parameters $a$, $b$, $c$, and $d$ can also be calculated by the MATLAB programming method or the linear trial and error method, and the change of water cut with time can be calculated by substituting the above parameters into equation (6). Different $c$ and $d$ values are selected and tried to determine whether the two sides of equation (11) are linear. If $c$ and $d$ are appropriate values, the left and right sides of equation (11) show the linear relationship. Generally, this attempt needs to be done many times. However, a MATLAB program is developed in the calculation, and the program can quickly calculate the best $c$ and $d$ values by using the least square method. The MATLAB program gives more accurate results than the linear trial and error method. The water cut double logarithmic linear equation is

$$\ln \left( \frac{1 + d \times f_{\text{wc}}^{1/c}}{1 - f_{\text{wc}}^{1/c}} \right) = a + b \ln t. \quad (11)$$

### 4. Model Validation

Using the new prediction model, the production performances of the Samattalol oilfield and Daqing oilfield, which are relatively advanced in water-flooding development, are predicted. The predicted results are compared with actual production data to verify the accuracy and reliability of the new model.

#### 4.1. Samattalol Oilfield

The Samattalol oilfield was put into operation in 1969 and has been developed since more than 50 years. The production data of the oilfield from 1969 to 1990 are shown in Table 1 [23]. Through the application of the MATLAB program, the calculated parameters are $a = 3.0405$, $b = 0.3374$, $c = 2.4246$, and $d = 611.5469$ and recoverable reserves are $N_{\text{Rmax}} = 26050.1809 \times 10^4$. The equation of cumulative oil production in the Samattalol oilfield can be expressed as follows:

$$N_p = 26050.1809 \left( \frac{e^{3.0405 \times 0.3374} - 1}{e^{3.0405 \times 0.3374} + 611.5469} \right)^{2.4246}. \quad (12)$$

If the cumulative oil production is substituted into equation (10), the annual oil production of the Samattalol oilfield can be calculated. Based on the data of water cut in Table 1, using the program, the calculated parameters are $a = 1.2522$, $b = 0.65536$, $c = 1.0645$, and $d = 1423.7735$. The equation of water cut in the Samattalol oilfield is

$$f_w = \left( 1 - \frac{1424.7355}{e^{1.2522t} + 1423.7735} \right)^{1.0645}. \quad (13)$$

#### 4.2. Daqing Oilfield

The Daqing oilfield was put into operation in 1965 and has been developed since more than 50 years, too. The production data of the oilfield from 1965 to 1987 are shown in Table 2 [24]. Substituting the cumulative oil production data and time into equation (4), the calculated parameters are $a = 1.0804$, $b = 0.64711$, $c = 0.59278$, and $d = 654.1989$ and recoverable reserves are $N_{\text{Rmax}} = 2953.0503 \times 10^4$. The equation of cumulative oil production in the Daqing oilfield can be expressed as follows:

$$N_p = 2953.0503 \left( \frac{e^{1.0804 \times 0.64711} - 1}{e^{1.0804 \times 0.64711} + 654.1989} \right)^{0.59278}. \quad (14)$$

| Time | Annual oil production ($\times 10^4t$) | Cumulative oil production ($\times 10^4t$) | Water cut (%) |
|------|--------------------------------------|------------------------------------------|---------------|
|      | Actual value | Prediction value | Relative error (%) | Actual value | Prediction value | Relative error (%) | Actual value | Prediction value | Relative error (%) |
| 1969 | 130         | 5.9             | 95.5                | 130.0        | 5.9             | 95.5                | 0.0          | 0.1              | 0.0                |
| 1970 | 430         | 343.1           | 20.2                | 560.0        | 349.0           | 37.7                | 0.0          | 0.3              | 0.0                |
| 1971 | 1000        | 1019.4          | -1.9                | 1560.0       | 1368.4          | 12.3                | 0.0          | 0.6              | 0.0                |
| 1972 | 2110        | 2227.2          | -5.6                | 3670.0       | 3595.6          | 2.0                 | 0.5          | 1.1              | -139.2             |
| 1973 | 3900        | 3996.2          | -2.5                | 7570.0       | 7591.8          | -0.3                | 2.0          | 1.9              | 5.0                |
| 1974 | 6120        | 6208.8          | -1.5                | 13690.0      | 13800.6         | -0.8                | 3.9          | 3.1              | 21.2               |
| 1975 | 8710        | 8621.9          | 1.0                 | 22400.0      | 22422.6         | -0.1                | 5.5          | 4.8              | 13.0               |
| 1976 | 11020       | 10940.1         | 0.7                 | 33420.0      | 33362.7         | 0.2                 | 9.2          | 7.2              | 20.8               |
| 1977 | 13000       | 12899.3         | 0.8                 | 46420.0      | 46262.0         | 0.3                 | 12.4         | 10.6             | 14.8               |
| 1978 | 14320       | 14326.3         | 0.0                 | 60740.0      | 60588.3         | 0.2                 | 16.1         | 14.9             | 7.3                |
| 1979 | 15080       | 15155.9         | -0.5                | 75820.0      | 75744.2         | 0.1                 | 17.7         | 20.4             | -15.6              |
| 1980 | 15480       | 15415.2         | 0.4                 | 91300.0      | 91159.3         | 0.2                 | 24.1         | 27.0             | -12.3              |
| 1981 | 15030       | 15191.3         | -1.1                | 106330.0     | 106350.7        | 0.0                 | 32.4         | 34.6             | -6.7               |
| 1982 | 14380       | 14598.3         | -1.5                | 120710.0     | 120949.0        | -0.2                | 42.7         | 42.7             | 0.0                |
| 1983 | 14000       | 13751.4         | 1.8                 | 134710.0     | 134700.4        | 0.0                 | 54.0         | 51.0             | 5.6                |
| 1984 | 13060       | 12752.5         | 2.4                 | 147770.0     | 147452.9        | 0.2                 | 61.0         | 59.0             | 3.2                |
| 1985 | 11090       | 11683.0         | -5.3                | 158860.0     | 159135.9        | -0.2                | 67.6         | 66.4             | 1.8                |
| 1986 | 10980       | 10603.5         | 3.4                 | 169840.0     | 169735.9        | 0.1                 | 72.1         | 72.9             | -1.1               |
| 1987 | 9880        | 9555.7          | 3.3                 | 179720.0     | 179295.2        | 0.2                 | 77.7         | 78.4             | -1.0               |
| 1988 | 8270        | 8566.1          | -3.6                | 187990.0     | 187862.1        | 0.1                 | 83.8         | 83.0             | 0.8                |
| 1989 | 7720        | 7649.5          | 0.9                 | 195710.0     | 195510.7        | 0.1                 | 85.4         | 86.8             | -1.6               |
| 1990 | 6400        | 6812.8          | -6.5                | 202110.0     | 202323.6        | -0.1                | 88.2         | 89.7             | -1.7               |
When the cumulative oil production is substituted into equation (10), the annual oil production of the Daqing oilfield can be calculated, too. Based on the water cut data in Table 2, the calculated parameters are $a = 0.99417$, $b = 0.69874$, $c = 0.74682$, and $d = 867.513$. The equation of water cut in Daqing oilfield is

$$f_w = \left(1 - \frac{867.513}{0.99417 \cdot 10^{0.69874t} + 867.513} \right)^{0.74682}. \quad (15)$$

Through the comparison of the predicted and actual data in Figures 1 and 2 and Tables 1 and 2, it can be seen that the cumulative oil production, annual oil production, and water cut calculated by the new prediction model are all close to the actual values and most of the prediction relative errors are less than 4.0%, which is generally in line with the development trend of the production performance data of the above two oilfields over time. There are 3 abnormal parameters in the oil well, and the error between the predicted value of the model and the actual value is large. Because there were only a few oil wells at the beginning of the production, the oil well production and water cut were unstable. The newly added wells can be responsible for the production and water cut, and the oilfield production enters a stable stage with the development of time. The results show that the new model has a good application effect in the Samattalol oilfield and Daqing oilfield, which provides reference for similar applications.

| Time | Annual oil production ($\times 10^4$ t) | Cumulative oil production ($\times 10^4$ t) | Water cut (%) |
|------|---------------------------------------|-------------------------------------------|--------------|
|      | Actual value | Prediction value | Relative error (%) | Actual value | Prediction value | Relative error (%) | Actual value | Prediction value | Relative error (%) |
| 1965 | 106.1        | 93.6           | 11.8              | 106.1        | 93.6           | 11.8              | 6.8          | 0.9           | 86.0              |
| 1966 | 63.4         | 58.5           | 7.7               | 169.5        | 152.1          | 10.2              | 11.9         | 1.8           | 84.9              |
| 1967 | 48.4         | 63.5           | −31.1             | 217.9        | 215.6          | 1.1               | 19.6         | 2.9           | 85.4              |
| 1968 | 68.6         | 72.1           | −5.1              | 286.5        | 287.7          | −0.4              | 2.4          | 4.2           | 75.9              |
| 1969 | 81.4         | 82.7           | −1.6              | 367.9        | 370.4          | −0.7              | 4.5          | 6.0           | 32.4              |
| 1970 | 92.1         | 94.7           | −2.9              | 460.0        | 465.1          | −1.1              | 6.7          | 8.2           | 21.7              |
| 1971 | 109.7        | 107.8          | 1.8               | 569.7        | 572.9          | −0.6              | 10.8         | 10.9          | −0.9              |
| 1972 | 123.5        | 121.3          | 1.8               | 693.2        | 694.2          | −0.1              | 15.1         | 14.3          | 5.5               |
| 1973 | 121.7        | 134.8          | −10.7             | 814.9        | 829.0          | −1.7              | 16.4         | 18.4          | −11.9             |
| 1974 | 146.5        | 147.3          | −0.6              | 961.4        | 976.3          | −1.5              | 19.8         | 23.2          | −17.0             |
| 1975 | 173.2        | 158.0          | 8.8               | 1134.6       | 1134.3         | 0.0               | 24.9         | 28.7          | −15.4             |
| 1976 | 180.1        | 166.0          | 7.8               | 1314.7       | 1300.3         | 1.1               | 34.2         | 34.9          | −2.2              |
| 1977 | 171.2        | 170.3          | 0.5               | 1485.9       | 1470.6         | 1.0               | 43.7         | 41.7          | 4.6               |
| 1978 | 163.2        | 170.4          | −4.4              | 1649.1       | 1640.9         | 0.5               | 51.8         | 48.7          | 6.0               |
| 1979 | 161.1        | 166.1          | −3.1              | 1810.2       | 1807.0         | 0.2               | 56.5         | 55.7          | 1.4               |
| 1980 | 151.5        | 157.8          | −4.1              | 1961.7       | 1964.8         | −0.2              | 64.4         | 62.5          | 3.0               |
| 1981 | 145.5        | 146.1          | −0.4              | 2107.2       | 2110.8         | −0.2              | 70.5         | 68.8          | 2.4               |
| 1982 | 130.2        | 132.1          | −1.4              | 2237.4       | 2242.9         | −0.2              | 75.1         | 74.4          | 0.9               |
| 1983 | 116.3        | 116.8          | −0.5              | 2353.7       | 2359.7         | −0.3              | 80.0         | 79.3          | 0.8               |
| 1984 | 99.2         | 101.4          | −2.2              | 2452.9       | 2461.1         | −0.3              | 83.1         | 83.5          | −0.5              |
| 1985 | 89.9         | 86.6           | 3.7               | 2542.8       | 2547.7         | −0.2              | 84.0         | 86.9          | −3.5              |
| 1986 | 79.9         | 72.9           | 8.7               | 2622.7       | 2620.7         | 0.1               | 85.5         | 89.7          | −4.9              |
| 1987 | 72           | 60.8           | 15.6              | 2694.7       | 2681.4         | 0.5               | 87.2         | 91.9          | −5.4              |
oilfields to apply the new model to predict production performance.

5. Conclusion

(1) According to the growth curves of 4W and 4Y4, the new generalized growth curve for predicting development performance of water-flooding oilfields is established. The new model overcomes the limitation of the original parameters and has wider applicability and flexibility. The new model can predict the changes of cumulative oil production, annual oil production, and water cut with time.

(2) Through the development of a MATLAB program, the actual values of parameters in the new model can be easily and quickly solved and the performance prediction equation in line with the actual oilfield production can be obtained. The cumulative oil production can be obtained by the growth curve, and the expression of annual oil production can be calculated by the cumulative oil production. The water cut prediction model is suitable for the whole stage of oilfield development.

(3) The predicted value of the new model is close to the actual value, and most of the prediction relative errors are less than 4.0%. The established model is reliable and practical.

Nomenclature

\( a, b, c, \) and \( d \): Model parameters, constants
\( N_{R_{\text{max}}} \): Recoverable reserves, 10^4t
\( N_j \): Cumulative oil production, 10^4t
\( Q \): Annual oil production, 10^4t
\( t \): Time, year
\( f_w \): Water cut, f.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.
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