Bias-induced chiral current and geometrical blockade
in triangular triple quantum dot

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Abstract – We theoretically investigate the quantum transport properties of a triangular triple quantum dot (TTQD) ring connected to two reservoirs by an analytical derivation and an accurate hierarchical equations-of-motion calculation. We initially demonstrate a bias-induced chiral current under zero magnetic field caused by the coupling between the spin gauge field and spin current in a nonequilibrium TTQD that induces a scalar spin chirality, which lifts the chiral degeneracy and the time inversion symmetry. The chiral current oscillates with the bias within the Coulomb blockade regime, suggesting that the chiral spin qubit can be controlled by purely electrical manipulations. Then, the geometrical blockade of the transport current due to the localization of chiral states is elucidated by spectral function analysis.

In condensed matter physics, the geometric (Berry) phase coherence may strongly affect the charge dynamics. For example, the anomalous Hall effect in ferromagnetic metals exhibits a nonzero transverse resistivity even in the absence of an external magnetic field. The geometric phase of the Bloch wave functions plays a major role in this phenomenon [1]. The nontrivial spin texture in ferromagnetic metals produces a gauge flux that can be incorporated into transfer integrals by additional phase factors [2,3]. In the literature, this anomalous contribution has been attributed to the spin-orbit interaction and spin polarization of the conduction electrons. With respect to the strong Hund-coupling limit, the spin of the conduction electron aligns with the impurity spin. The resulting fictitious magnetic field produced by the noncoplanar spin configuration or spin chirality [1,4] implies the topological origin of the Hall conductivity. The fictitious magnetic field contains a uniform component generated by spin-orbit interactions [5–7].

Consider a minimal chiral spin model, as shown in fig. 1(a), with three local spins, $S_1$, $S_2$, and $S_3$, whose axes are tilted away from the overall magnetization axis. This triangular triple quantum dot (TTQD) system exhibits a nonmagnetic structure. However, a conduction electron moving in the background of these spins establishes a phase factor that can be given as $e^{i\Omega/2}$, where $\Omega$ is the solid angle subtended by three spins on the unit sphere. The chiral spin state is closely related to the chiral spin liquid and superconductivity states [8]. The geometric phase acts as a gauge field with two essential consequences: the anomalous Hall effect [1] and a chiral current $I_c \propto S_1 \cdot (S_2 \times S_3)$. Here, we obtain the chiral current using a nonmagnetic TTQD structure under zero magnetic field (see fig. 1(b), (c)), and show that this geometrical current induces a novel geometrical blockade effect.

TTQDs comprise three triangularly configured and coupled quantum dots connected to two or three reservoirs. As the smallest artificial molecules exhibiting topological properties, TTQDs have been extensively studied both experimentally [9–12] and theoretically [13–20]. They are expected to realize various quantum interference effects in the strong correlation regime [18,20]. Furthermore, as the qubits are encoded in chiral spin states,
they are promising candidates for application in quantum computing [14–16,21]. Each chiral qubit is embedded in a decoherence-free subspace, protecting it from collective noises [22] and random charge fluctuations [13].

However, the manipulation of chiral qubits or the chiral spin state is a challenging task. The existing proposals [13–16] split the degeneracy of the left- and right-hand chiral states in a perpendicular magnetic field. However, the application of external magnetic field to quantum computing would be impractical because it is incompatible with large-scale integrated circuits. Furthermore, localizing the required oscillating magnetic fields for quantum gate or qubit manipulation is extremely difficult.

In principle, the gauge field induced by the aforementioned geometric phase could behave as a magnetic field, negating the requirement for an applied field. Motivated by this insight, we propose that the chiral qubit and chiral spin state can be manipulated through the bias-voltage-induced chiral current. We validate our proposal using the Anderson triple-impurity model TTQD.

Panels (b) and (c) of fig. 1 show our TTQD structure in a quantum transport setup. Each QD exists in the local moment regime, where each QD is occupied by one electron and preserves 1/2 magnetic moment corresponding to the parameters of \( \varepsilon = 0 \) and a relatively small inter-dot coupling \( t \). QD2 and QD3 are coupled to the electronic reservoirs L and R, respectively. The total composite Hamiltonian, \( H_T = H_{\text{dots}} + H_{\text{res}} + H_{\text{coup}} \), is described by the Anderson impurity model, where

\[
H_{\text{dots}} = \sum_{j,k=1}^{3} \sum_{s=\uparrow,\downarrow} t_{jk}^s \hat{d}_{js}^\dagger \hat{a}_{ks} + \sum_{j=1}^{3} U_j \hat{n}_{j\uparrow} \hat{n}_{j\downarrow}. \tag{1}
\]

Here, \( \hat{n}_{js} \equiv \hat{d}_{js}^\dagger \hat{d}_{js} \) is the number operator of an electron that occupies the specified on-dot spin orbital. For clarity, we assume that the \( C_{3v} \) symmetry of the TTQD holds, where \( t_{12} = t_{23} = t_{31} = t \). The on-dot Coulomb repulsion and energy are given as \( U_j = U \) and \( \varepsilon_j \equiv t_{jj} = -U/2 \), respectively. The non-interacting Fermion reservoir is described as \( H_{\text{res}} = \sum_{\alpha \in \{L,R\}} \sum_{ks} (\varepsilon_{\alpha ks} + \mu_\alpha) \hat{c}_{\alpha ks}^\dagger \hat{c}_{\alpha ks} \), with \( \mu_L = eV/2 = -\mu_R \). The TTQD system-reservoir coupling is described as \( H_{\text{coup}} = \sum_{ks} (t_{12} \hat{c}_{L2s}^\dagger \hat{d}_{2s} + t_{31} \hat{c}_{R3s}^\dagger \hat{d}_{3s} + \text{H.c.}) \).

In the following analysis, we initially investigate the chiral current induced by a magnetic field applied to an isolated TTQD, which unambiguously identifies the chiral current operator \( \hat{I}_c \) (cf. eq. (3)). Using this current operator, we also evaluate the chiral current induced by the bias voltage. Let us begin with a pristine TTQD in the absence of a magnetic field and reservoirs. The ground state of the isolated TTQD is four-fold degenerate, with each QD occupied by one spin without chirality. Thus, the three spins are coplanar with degenerate chiral states. Under a perpendicular magnetic field, the ring-like TTQD structure is threaded with degenerate chiral states. Under a perpendicular magnetic field, the ring-like TTQD structure is threaded with degenerate chiral states. Under a perpendicular magnetic field, the ring-like TTQD structure is threaded with degenerate chiral states. Under a perpendicular magnetic field, the ring-like TTQD structure is threaded with degenerate chiral states.

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where \( n \) denotes the average electron occupation number on each dot. In the half-filling situation, \( n = 1 \); thus, the \( t \)-term vanishes. The \( J \)-term denotes the usual Heisenberg exchange interaction with \( J = 4t^2/U \). The final term is the chiral term with \( \chi = 24t^3\sin(2\pi\phi/\phi_0)/U^2 \), where \( \phi \) is the magnetic flux enclosed by the TTQD and \( \phi_0 = hc/e \) is the quantum flux unit. The chiral operator of the TTQD is given as \( \hat{S}_1 \cdot (\hat{S}_2 \times \hat{S}_3) = -4i\sum_{\alpha \nu} d_{\alpha \nu}^d d_{\nu \alpha}^c - d_{\alpha \nu}^c d_{\nu \alpha}^d d_{\nu \alpha}^d d_{\nu \alpha}^c d_{\nu \alpha}^d d_{\nu \alpha}^c \). This operator splits the four-fold degenerate ground state of the total spin-\( \frac{1}{2} \) subspace into two chiral-state pairs. The first pair is the minority spin pair circling clockwise (+) and anticlockwise (−), expressed as \( |q_{\pm}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\downarrow\rangle + e^{\pm \frac{2\pi}{3}} |\downarrow\uparrow\downarrow\rangle + e^{\pm \frac{4\pi}{3}} |\downarrow\uparrow\downarrow\rangle) \). An analogous pair, with all flipped spins, exists for \( S_z = -1/2 \). Considering the degeneracy in \( S_z \), we simplify \( |q_{\pm}\rangle \) to \( |q_{\pm}\rangle \) and assume the same \( |q_+\rangle \) values for \( S_z = -1/2 \) and \( S_z = 1/2 \). These states are the eigenstates of the TTQD chiral operator constituting the final term of eq. (2), which describes the circular transfer difference between the chiral and anticlockwise directions of the electrons. From the Hellman-Feynman theorem, the chiral current is defined as \( I_c \equiv -\frac{\hbar}{\pi e} \frac{\partial F_{\text{dot}}}{\partial \phi} = -\frac{\hbar}{\pi e} \frac{\partial F_{\text{dot}}}{\partial \phi} \), where \( F_{\text{dot}} \) is the free energy induced by the magnetic field [25]. Substituting the effective Hamiltonian eq. (2) into the definition, we obtain

\[
I_c = -\frac{2e\hbar}{\pi} t^3 \bar{\Omega} \hat{S}_1 \cdot (\hat{S}_2 \times \hat{S}_3). \tag{3}
\]

The TTQD constitutes the shortest loop when each dot lies in its local moment region. The coefficient \( t^3/U^2 \) denotes the lowest-order nonvanishing contribution to the circling current. Figure 1(d) plots the chiral current as a function of flux in the equilibrium state. The period of chiral current is double the flux period. Thus, the chiral operator breaks the symmetry of TTQD from \( C_{3v} \) to \( C_3 \) and induces chiral current. Note that the chiral current in fig. 1(d) deviates from the sinusoidal pattern of the chiral-state pairs. The first pair is the minority spin pair \( \sim \frac{1}{2} \) subspace as well as the TTQD are set to \( \Gamma_L = \Gamma_R = \Gamma = 0.025 \text{meV} \). The temperature, set to \( T = 0.6 \text{K} \), is considerably greater than the Kondo temperature (approximately \( T_K \sim 3.6 \times 10^{-4} \text{mK} \)). Figure 1(e) plots the calculated chiral current \( I_c \) as a function of the bias voltage \( V \). The \( V < U/2 \) region, where the resulting \( I_c(V) \) oscillates, is the Coulomb blockade regime in this study. Further increasing the bias \( (V > U/2) \) will push the system out of the Coulomb blockade regime, whereby \( I_c \) will gradually decrease to become zero when the transport current \( I_c \) increases. Alternatively, when the transport current is small, the chiral current is approximately equal to the bond current between dots. The comparison between chiral current and bond current is illustrated in the SM. Note that in the calculation of fig. 1(e) the dot-electrode coupling \( \Gamma \) is small compared to \( t (\Gamma/t = 0.1) \), for which the magnitude of the chiral current is remarkable. For larger dot-electrode coupling, the amplitude of chiral current is narrowed, with a greater transport current. Nevertheless the qualitative behavior of the chiral current is the same as that of the small \( \Gamma \). For even larger \( \Gamma \), compared to the chiral current, the transport current dominates the electron transport and leads to a indistinguishable chiral current.

Remarkably, our results show that the chiral spin qubit can be controlled by electrical manipulations alone, negating the requirement for a magnetic field. In the Coulomb blockade regime \( (V < U/2) \), the lead-dressed ground state under bias comprises the aforementioned chiral pairs \( \{|q_\pm\rangle\} \); see fig. 1(b) and (c). In particular, compared to the magnetic-field counterpart of fig. 1(d), the bias-dressed states at \( V = 0.05 \) and \( 0.25 \text{mV} \) are associated with the maximal clockwise and anticlockwise chiral currents, respectively. Figure 1(f) displays the reduced density matrix diagonal elements of the two chiral-state populations, \( \rho_{q_+,q_+} \) and \( \rho_{q_-,q_-} \), as functions of bias. The sign and magnitude of the difference, \( \rho_{q_+,q_+} - \rho_{q_-,q_-} \), correlate well with the observed direction and magnitude of the chiral current in fig. 1(e). At \( V = 0.15 \text{meV} \), the chiral ground states are degenerate, \( i.e., \rho_{q_+,q_+} = \rho_{q_-,q_-} \); therefore, \( \Gamma = 0 \), and the effective magnetic flux is \( \phi_{\text{eff}} = \phi_0/4 \). This state physically differs from \( V > U/2 \) beyond the Coulomb blockade regime, where \( \phi_{\text{eff}} \approx 0, \rho_{q_+,q_+} \approx \rho_{q_-,q_-} \), and \( I_c \approx 0 \).

*Spin gauge field coupling analysis*: The aforementioned observations can be analyzed and understood as follows. Bias breaks the inversion symmetry of the TTQD, which is required for the coupling between the spin gauge field and the spin current. To decouple the on-site Coulomb interaction in our Anderson impurity model, we applied the Hubbard-Stratonovich approach that will be detailed in the SM. To maintain the spin rotation invariance, we introduce a unitary transformation \( \Psi_j = R_j \Psi_j \) in the spin space. Here, \( R_j \) is a site- and time-dependent \( SU(2) \) rotation matrix that satisfies \( \sigma \Psi_j = R_j \sigma R_j^\dagger \). The electron operators are given in the spinor form \( \Psi_{\uparrow} = (d_{j^+}^d, d_{j^+}^c) \). By expressing the dot spin in polar

\[ \text{See SM for the details of derivations of the coupling action.} \]
form $S = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)^T$, the matrix $R = e^{\frac{i}{2} \phi} e^{-\frac{i}{2} \theta} e^{\frac{i}{2} \phi} e^{-\frac{i}{2} \theta}$ rotates the spin-up state $|\uparrow\rangle$ to the direction of dot spin as $|S⟩ = R|\uparrow⟩$. Consequently, the kinetic term of $H_{\text{dot}}$ exhibits a covariant form because $t_{jk} \Psi_{j,k}^\dagger \rightarrow t_{jk} \Psi_{j,k}^\dagger (R_k - R_j) \Psi_{j,k}$ and $\Psi_{j,k}^\dagger \partial_\tau \Psi_{j,k} \rightarrow \Phi_{jk}^\dagger (\partial_\tau + R_j \partial_\tau R_j) \Phi_{jk}$. The $SU(2)$ spin gauge field is defined as $A_{jk}(r) = -iR_k^\tau (R_k - R_j)$. In terms of the Pauli matrices, these expressions are exact as $A_{jk} = \sum_r A_{jk} \sigma_r$, where $r = x, y, z$ is the direction in spin space. The gauge field is minimally coupled to the spin current with the coupling energy given as

$$H_A = \sum_{jk} j_{jk}^c A_{jk} + \sum_{jr} s_{jr} A_{jr}^c, \tag{4}$$

where $s_{jr}^c = \Phi_{jr}^\dagger \sigma_r \Phi_{jr}$ and $j_{jk}^c = -\frac{e}{\hbar} (\Phi_{jr}^\dagger \sigma_r \Phi_{jr} - \Phi_{jr} \sigma_r \Phi_{jr})$ are the spin-density and spin-current operators, respectively. These operators are coupled to $A_{jr}^c$ and $A_{jr}^c$, i.e., the temporal and spatial components of the spin gauge field, respectively. Thus, the localized spins are expected to reorient themselves to minimize the coupling energy.

The nonadiabatic term in $H_A$ is a vector product of the dot spins. This term identifies the Dzyaloshinskii-Moriya (DM) interaction. While preserving the spin states, an adiabatic process produces a spin Berry phase. The effective inter-site transfer integral is $t_{jk}^{\text{eff}} = t \langle n_j | n_k \rangle \cos \theta_{jk}$, where $\theta_{jk}$ is the angle between two on-site spins and $\varphi_{jk}$ is the solid angle spanned by $n_j$, $n_k$, and $z$ [35,36]. Furthermore, the polarization energy $U/2$ splits the spin degeneracy in the rotating frame. The ground-state spinor field is reduced to the simple form $\Phi_{jk}^c \equiv d_{jk}^c$; similarly, the spin current is reduced to $j_{jk} = -it(\Phi_{jr}^\dagger \sigma_r \Phi_{jr} - \Phi_{jr} \sigma_r \Phi_{jr})$. For simplicity, consider a continuous situation with almost identical directions being observed for the two spins. In the steady state, i.e., $\nabla \cdot j = 0$, the current can be expressed as $j = \nabla \times f$, where the vector field $f = f \hat{z}$ is perpendicular to the planar direction. Integrating by parts, we obtain $j \cdot A^{\text{ad}} \sim -f \cdot \nabla \times A^{\text{ad}}$. The current source $f$ couples to the curvature of the adiabatic gauge field, which acts as the effective magnetic field

$$B^{\text{eff}} \equiv (\nabla \times A^{\text{ad}})_z = -\frac{\hbar}{4} [S \cdot (\nabla \times S) \times \nabla \cdot S]. \tag{5}$$

Given $S_j \sim S_k + (\delta_{jk} \nabla) S_k$ and replacing the differentials by $S_j$, we obtain the discrete form, namely, the chiral interaction, $S_j \cdot (S_k \times S_k)$, among triple dots.

By $C_{3v}$ symmetry, the above derivations show that the coupling energy $H_A$ of the isolated TTQD is zero. An open TTQD system with finite bias breaks the inversion symmetry with a bond current that minimizes $H_A$. This indicates a charge current associated with the scalar chirality $S_1 \cdot (S_2 \times S_3)$ of the three spins. The chirality is the solid angle spanned by the three spins, and the current is plotted in fig. 1(a).

**Geometrical blockade:** Figure 1 implies the blockade behavior of the transport current. We now demonstrate this effect concretely. Each dot preserves the local spin with $\epsilon = -U/2 = -0.5$ mV, and the chiral ground states degenerate at $V = 0.15$ meV. The plots of $I_c$, $I_t$, and $\rho_{q\pi}$ in fig. 1(e) and (f) can be naturally divided into three regions of bias: (a) the blockade region ($0 < V < 0.15$ mV), (b) the coexistence region ($0.15 < V < 0.45$ mV), and (c) the conduction region ($0.45 < V < 0.7$ mV). Panels (a), (b), and (c) of fig. 2 present the corresponding nonequilibrium spectral functions $\{A_i(\omega)\}$ in these three regions. As shown in fig. 2(a), $A_i(\omega)$ of each QD peaks near the Fermi level ($\epsilon_F = 0$). Intuitively, a resonance transport current $I_t$ should be induced even under a small bias value. However, $I_t \sim 0$ at $V = 0.15$ mV (see fig. 1(e)). In the same region, $I_c$ has finished its semi-period with a maximum of $|I_c| \sim 4.3$ nA. Although the observed transport current is $I_t \sim 0$, the significant $I_c$ presents a type of geometrical blockade because of the formation of the geometrical chiral state (cf. fig. 1(f)). The resulting localization of the chiral states does not contribute to the transport current $I_t$. Within a specific bias zone, we describe the geometrical blockade effect as $\gamma \equiv \int_{V_{c1}}^{V_{c2}} |I_c| dV / \int_{V_{c1}}^{V_{c2}} I_c dV$. Not surprisingly, this parameter is large ($\gamma = 25$) in the blockade region of $0 < V < 0.15$ mV.

When the bias is increased to $0.15 < V < 0.45$ mV, $I_c$ entered its second semi-period with a smaller maximum $|I_c| \sim 2.5$ nA (see fig. 1(e)). Meanwhile, a noticeable $I_t$ can be observed to increase with $V$. As shown in fig. 2(b), $I_t$ was induced by excitation at $\omega \sim \pm 0.25$ meV. This new peak of $A(\omega)$, which was absent in the blockade region, grew with $I_t$ but the chiral state remained localized. The channel near $\epsilon_F$ remained blocked and contributed little to $I_t$. In the coexistence region, characterized by the simultaneous appearance of $I_c$ and $I_t$, the geometrical blockade parameter reduced to $\gamma = 4$.

The observed interplay between $I_t$ and $I_c$ can be understood as follows. Once an electron becomes an $I_t$ carrier from the left-to-the-right reservoir, it will not contribute to the circular current $I_c$. Physically, this behavior is related to the decreased occupation of electrons on the chiral states as $I_t$ increases (see fig. 1(e) vs. fig. 1(f)). In particular, we refer to $0.45 < V < 0.7$ mV as the conduction region. The corresponding spectral functions, $\{A_i(\omega)\}$, are shown in fig. 2(c). At $\omega \sim \pm 0.25$ meV, the transport excitation peak became sufficiently high and $I_t$ enlarged to the nA order. Although both the left and right chiral states remained near $\epsilon_F$, they were almost degenerate and
Bias-induced chiral current and geometrical blockade in triangular triple quantum dot

Fig. 2: Nonequilibrium spectral functions $A_i(\omega) = A_s(\omega)$ when $i = 1, 2, 3$ and $s = \uparrow, \downarrow$ in the (a) blockade region ($0 \leq V \leq 0.15 \text{mV}$), (b) coexistence region ($0.15 < V \leq 0.45 \text{mV}$), and (c) conduction region ($0.45 < V \leq 0.7 \text{mV}$). At the upper right of each plot is the calculated value of $\gamma \equiv \int_{V=0}^{V_s} |I_\omega|dV / \int_{V=0}^{V_s} dV$ in each region.

marginally occupied; accordingly, $I_c$ reduced to the order of pA. The geometrical blockade was lifted, and $\gamma$ was 0.04 in this region.

Finally, we elaborate the importance and the range of the Coulomb interaction. In an open quantum metal ring without electron-electron correlations ($U = 0$), the circling current can induce a finite magnetic moment [37]. However, this current is always accompanied by a transport current can induce a finite magnetic moment [37]. How-
chiral current can be measured and controlled. For ex-
by Coulomb and geometrical blockades, denoting that the
marginaly occupied; accordingly,

The geometrical nature. The bias-induced chiral current may lead to innovative applications of TTQD, ranging from magnetoelectric devices to chiral quantum computation.

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REFERENCES

[1] Taguchi Y., Oohara Y., Yoshizawa H., Nagaosa N. and Tokura Y., Science, 291 (2001) 2573.
[2] Anderson P. W., Phys. Rev., 115 (1959) 2.
[3] Nagaosa N. and Lee P. A., Phys. Rev. Lett., 64 (1990) 2450.
[4] Ohgushi K., Murakami S. and Nagaosa N., Phys. Rev. B, 62 (2000) R6065.
[5] Ye J., Kim Y. B., Millis A. J., Shraiman B. I., Majumdar P. and Tešanović Z., Phys. Rev. Lett., 83 (1999) 3737.
[6] Tatarà G. and Kohno H., Phys. Rev. B, 67 (2003) 113316.
[7] Tatarà G. and Garcia N., Phys. Rev. Lett., 91 (2003) 076806.
[8] Wen X. G., Wilczek F. and Zee A., Phys. Rev. B, 39 (1989) 11413.
[9] Rogge M. C. and Haug R. J., Phys. Rev. B, 77 (2008) 193306.
[10] Kotzian M., Gallego-Marcos F., Platero G. and Haug R. J., Phys. Rev. B, 94 (2016) 035442.
[11] Noiri A., Kawasaki K., Otsuka T., Nakajima T., Ye J., Kim Y. B., Millis A. J., Shraiman B. I., Majumdar P. and Tešanović Z., Phys. Rev. Lett., 83 (1999) 3737.
[12] Hong C., Yoo G., Park J., Cho M.-K., Chung Y., Sim H.-S., Kim D., Choi H., Umański V. and Mahalu D., Phys. Rev. B, 97 (2018) 241115(R).
[13] Hsieh C.-Y. and Hawrylak P., *Phys. Rev. B*, **82** (2010) 205311.

[14] Hsieh C.-Y., Rene A. and Hawrylak P., *Phys. Rev. B*, **86** (2012) 115312.

[15] Luczak J. and Bulka B. R., *Phys. Rev. B*, **90** (2014) 165427.

[16] Luczak J. and Bulka B. R., *Quantum Inf. Process.*, **16** (2016) 10.

[17] Hsieh C.-Y., Shim Y.-P., Korkusinski M. and Hawrylak P., *Rep. Prog. Phys.*, **75** (2012) 114501.

[18] Weymann I., Bulka B. R. and Barnaš J., *Phys. Rev. B*, **83** (2011) 195302.

[19] Wrześniowski K. and Weymann I., *Phys. Rev. B*, **92** (2015) 045407.

[20] Niklas M., Trottmann A., Donarini A. and Grifoni M., *Phys. Rev. B*, **95** (2017) 115133.

[21] Gimenez I. P., Hsieh C.-Y., Korkusinski M. and Hawrylak P., *Phys. Rev. B*, **79** (2009) 205311.

[22] Viola L., Fortunato E. M., Pravia M. A., Knill E., Laflamme R. and Cory D. G., *Science*, **293** (2001) 2059.

[23] Scarola V. W. and Das Sarma S., *Phys. Rev. A*, **71** (2005) 032340.

[24] MacDonald A. H., Girvin S. M. and Yoshioka D., *Phys. Rev. B*, **37** (1988) 9753.

[25] Byers N. and Yang C. N., *Phys. Rev. Lett.*, **7** (1961) 46.

[26] Lai C.-Y., Ventra M. D., Scheibner M. and Chien C.-C., *EPL*, **123** (2018) 47002.

[27] Jin J. S., Zheng X. and Yan Y. J., *J. Chem. Phys.*, **128** (2008) 234703.

[28] Hu J., Luo M., Jiang F., Xu R. X. and Yan Y. J., *J. Chem. Phys.*, **134** (2011) 244106.

[29] Li Z. H., Tong N. H., Zheng X., Hou D., Wei J. H., Hu J. and Yan Y. J., *Phys. Rev. Lett.*, **109** (2012) 266403.

[30] Ye L. Z., Wang X. L., Hou D., Xu R. X., Zheng X. and Yan Y. J., *WIREs Comput. Mol. Sci.*, **6** (2016) 608.

[31] Hou W. J., Wang Y. D., Wei J. H., Zhu Z. G. and Yan Y. J., *Sci. Rep.*, **7** (2017) 2486.

[32] Kiruchi T., Koretsune T., Arita R. and Tatara G., *Phys. Rev. Lett.*, **116** (2016) 247201.

[33] Tatara G., *Physica E*, **106** (2019) 208.

[34] Roßler U., Bogdanov A. and Pfleiderer C., *Nature*, **442** (2006) 797.

[35] Woo S., Litzius K., Krüger B., Im M.-Y., Caretta L., Richter K., Mann M., Krone A., Reeve R. M., Weigand M. et al., *Nat. Mater.*, **15** (2016) 501.

[36] Nagaosa N., *J. Phys. Soc. Jpn.*, **75** (2006) 042001.

[37] Cini M., Perfetto E. and Stefanucci G., *Phys. Rev. B*, **81** (2010) 165202.

[38] Rogge M. C. and Haug R. J., *Phys. Rev. B*, **78** (2008) 153310.

[39] Gaudreau L., Sachrajda A. S., Studenikin S., Kam A., Delgado F., Shim Y. P., Korkusinski M. and Hawrylak P., *Phys. Rev. B*, **80** (2009) 075415.