Non-perturbative effects in WIMP scattering off nuclei in the NMSSM

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We explore a scenario in the Next-to-Minimal-Supersymmetric-Standard-Model (NMSSM) with both a light O(10) GeV neutralino and a CP-odd Higgs boson with significant coupling to down-type fermions, evading all current B physics, LEP and WMAP bounds. Motivated by a possible slight lepton universality breaking hinted in T decays, we consider the effect of the mixing of \(n_\eta\) resonances with the pseudoscalar Higgs on the spin-dependent scattering neutralino cross section off nucleons. We conclude that this mechanism could be relevant provided that non-perturbative effects enhance the effective \(n_\eta\)-nucleon coupling, taking over velocity/\(q^2\) suppression factors, perhaps giving a new insight into the current controversial situation concerning direct search experiments of dark matter.

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I. INTRODUCTION

Evidence has been accumulated both from astrophysics and cosmology that about 1/4 of the energy budget of the present universe consists of the so-called (cold) dark matter (DM), namely, a component which is non-relativistic and neither feels the electromagnetic nor the strong interaction. It is fair to say that the most popular DM candidate for a WIMP (Weakly Interacting Massive Particle) is the lightest supersymmetric particle (LSP) in supersymmetric models with R-parity conservation. Leaving aside the axion and the axino, the superpartners with the right properties for playing the role of a WIMP in the universe are the gravitino and the lightest neutralino (\(\chi\)) - by far, the most discussed case in the literature.

Although the LHC is running smoothly and collecting large amounts of data useful to look for physics beyond the Standard Model (SM), other complementary facilities are certainly needed, especially concerning DM detection. In fact, DAMA/LIBRA, CoGeNT, and more recently CRESST experiments have reported the observation of events in excess of the expected background, hinting at the existence of a light WIMP \cite{11,12}. However, exclusion limits set by other direct searches, such as Xenon10 \cite{4} and Xenon100 \cite{5}, are in tension with the above claims.

In the NMSSM, a light neutralino (as a DM candidate) can efficiently annihilate through the resonant s-channel via a light pseudoscalar Higgs mediator satisfying the requirements from the relic density \cite{6}. However, following a scan of the NMSSM parameter space, the authors of \cite{8} obtained upper limits on the spin-independent (SI) \(\chi\)-nucleon cross section which are substantially below the requirements of DAMA and CoGeNT. The spin-dependent (SD) cross section (via Z-exchange) was also found to be several orders of magnitude below current experimental bounds. On the other hand, the authors of \cite{8} were able to achieve a somewhat larger SI cross section in a similar scenario. Admittedly, such cross sections can be further enhanced by increasing the s-quark content of the nucleon, but the agreement with the low range of DAMA results turns out to be only marginally acceptable.

In this work we revisit SD \(\chi\)-nucleon scattering via pseudoscalar-exchange in the NMSSM, usually neglected in most analyses \cite{6}, which however might be enhanced due to a non-perturbative mechanism as later argued.

II. A LIGHT NMSSM CP-ODD HIGGS BOSON

The Higgs sector of the NMSSM contains six independent parameters: \(\lambda, \kappa, A_\lambda, A_\kappa, \tan \beta\) and \(\mu_{\text{eff}}\), whose definitions can be found elsewhere \cite{10}. Notice that \(\mu_{\text{eff}} = \lambda s\) is generated as the vev of the singlet field \(s \equiv \langle S \rangle\); it is also useful to define \(B_{\text{eff}} = A_\lambda + ks\).

Two physical pseudoscalar states appear in the spectrum of the NMSSM as superpositions of the MSSM-like state \(A_{\text{MSSM}}\) and the singlet-like state \(A_S\). In particular for the lightest CP-odd Higgs boson

\[
A_1 = \cos \theta_A A_{\text{MSSM}} + \sin \theta_A A_S,
\]

where \(\theta_A\) stands for the mixing angle \cite{10}. The \(A_1\) reduced coupling \(X_d\) to down-type quarks and leptons (normalized with respect to the coupling of the CP-even Higgs boson of the SM) reads

\[
X_d = \cos \theta_A \tan \beta \simeq -\frac{\lambda v (A_\lambda - 2ks)}{M_1^2 + 3k_A s} \tan \beta,
\]

where \(M_1^2 = 2\mu_{\text{eff}} B_{\text{eff}}/\sin 2\beta\).

An analysis of a particular region of the NMSSM parameter space where \(X_d\) can be relatively large at high \(\tan \beta\), together with a light CP-odd Higgs boson \((m_{\eta_\ell} \sim 10(10)\) GeV), was carried out in \cite{10} although without including the relic abundance constraint. Let us stress here again that this scenario is quite different from the PQ-symmetry-limit (\(\kappa \rightarrow 0\)) or R-symmetry-limit (\(A_\kappa, A_\lambda \rightarrow 0\)), where \(X_d\) remains moderate even in the large \(\tan \beta\) limit. Although not motivated by any symmetry as in the latter cases, we remark that \(\tan \beta \sim 1/|\mu_{\text{eff}} B_{\text{eff}}|\) for large values of \(\tan \beta\) \cite{11}, giving consistency to our scenario which implies (relatively) small values of \(|B_{\text{eff}}|\).

As pointed out in Refs. \cite{12,13} large \(X_d\) could induce a non-negligible mixing of the CP-odd state and \(\eta_\ell (nS)\) hadronic resonances. For the sake of simplicity, we only consider the \(A_1\) mixing with the nearest (in mass) pseudoscalar resonance, generically denoted hereafter as \(\eta_0\).
Thus, the $A_1$ and $\eta_b$ physical states can be written approximately as

\begin{align}
A_1 &= \cos \alpha \, A_{10} + \sin \alpha \, \eta_{b0} \quad (3) \\
\eta_b &= \cos \alpha \, \eta_{b0} - \sin \alpha \, A_{10} \quad (4)
\end{align}

where subindex zero refers to unmixed states throughout. (The dominant components may of course be reversed if $\alpha > \pi/4$.) In any event, one should keep in mind that the $A_{10}$ can mix (to a greater or lesser extent) with more than a single pseudoscalar hadronic state (see Ref. [13]).

The strength of the mixing is determined by the angle $\alpha$ given by [10, 14]

\[ \sin 2\alpha = \left[ 1 + \frac{(m_{A_{10}} - m_{\eta_{b0}})^2}{4 \delta m^4} \right]^{-1/2}, \]

where the imaginary part has been neglected and $\delta m^2$ can be computed by means of a non-relativistic quark potential model:

\[ \delta m^2 = \left( \frac{3m_{\eta_{b0}}^3}{4\pi v^2} \right)^{1/2} |R_{\eta_{b0}}(0)| \times X_d, \]

where $v = 246$ GeV and $R_{\eta_{b0}}(0)$ stands for the radial wave function at the origin of the corresponding $\eta_{b0}$ state (for more details see Ref. [14]).

As emphasized in Ref. [13], a substantial mixing of a $O(10)$ GeV CP-odd Higgs boson with $\eta_{b0}$ resonances can modify (hinder) a signal based on direct observation of a monochromatic peak in the photon spectrum of radiative $\Upsilon$ decays [16]. On the other hand, a light CP-odd Higgs could still show up as a slight breaking of lepton universality in the ratio $B_{\tau\tau}/B_{\ell\ell} \approx 1$, where $B_{\ell\ell}$ denotes the tauonic, and $B_{\ell\ell}$ the electronic ($\ell = e$) or muonic ($\ell = \mu$) branching ratios of the $\Upsilon$ resonance, respectively [17, 18].

In view of the greatly improved accuracy of the recent measurements of the leptonic BFs (and likely so in the forthcoming BaBar analysis of $\Upsilon(3S)$ decays [19]), it seems advisable to remove the dependence on the final-state lepton mass ($m_{\ell}$) dividing the branching fraction (BF) by $K(x_\ell) = (1 + 2x_\ell)(1 - 4x_\ell)^{1/2}$, which behaves as a (smoothly) decreasing function of $x_\ell = m_{\ell}^2/M_\Upsilon^2$: $B_{\ell\ell} = B_{\ell\ell}/K(x_\ell)$, with $\ell = e, \mu, \tau$. Therefore defining

\[ \mathcal{R}_{\ell\ell} = \frac{B_{\ell\ell} - B_{\ell\ell}}{B_{\ell\ell}} = \frac{\mathcal{B}_{\ell\ell} - 1}{\mathcal{B}_{\ell\ell}} = 1 \quad \ell = e, \mu, \]

the contribution of a pseudoscalar Higgs to the (inclusive) decay rate would imply an enhancement of the tauonic mode and therefore small but positive values of $\mathcal{R}_{\tau\ell}$ [17].

The experimental results obtained from the PDG listing [20] are shown in Table 1. The good agreement of the $\psi(2S)$ measurements with the SM expectations, together with the systematic disagreement of the $\Upsilon$ family, are consistent with a slight enhancement of the tauonic decay mode of $\Upsilon$ resonances versus the electronic and muonic decay modes, due to an extra contribution of a light pseudoscalar Higgs boson, which couples to down-type quarks (at large $\tan\beta$) but of negligible effect for up-type quarks in this limit.

Furthermore, unexpected values for the hyperfine splittings in the bottomonium spectrum: $\Delta E_{\text{hyp}}(nS) = m_{\Upsilon(nS)} - m_{\eta(nS)} (n = 1, 2, 3)$ can be induced by the mixing [11, 12]. In particular, $\Delta E_{\text{hyp}}(1S) = 69.3 \pm 2.8$ MeV obtained by BaBar and CLEO using hindered radiative $\Upsilon(3S)$ decays [20] appears to be significantly larger than expected from perturbative QCD, estimated to be $42 \pm 13$ MeV [21]. However, the recent Belle measurement of the $\eta_b(1S)$ mass based on the $h(1P) \to \eta_b(1S)\gamma$ decay [22] leads to $\Delta E_{\text{hyp}}(1S) = 59.3 \pm 3.1$ MeV, in very good agreement with lattice NRQCD calculations [23]. Note in passing that the discovery of the $\eta_b(2S)$ state and its mass determination would provide a crucial check when compared to the lattice prediction for $\Delta E_{\text{hyp}}(2S)$ [23].

### A. NMSSM scan including WMAP bounds

In order to assess the impact of the $A_{10} - \eta_{b0}$ mixing on DM phenomenology, we first have to review the present bounds from B physics, LEP and DM relic abundance. To this aim the latest version of NMSSMTools [24] was employed to scan the NMSSM parameter space with micrOMEGAs turned on in the main code. We focus on a narrow mass window for $A_1$, where current experimental constraints still permit large $X_d$ values [25] under examination in this paper.

The following conditions were required to be satisfied:

i) $10 \lesssim m_{A_1} \lesssim 10.58$ GeV (i.e. below $B\bar{B}$ threshold).

ii) Relatively large values of $X_d$, for large $\tan\beta$.

iii) A light neutralino of mass $O(10)$ GeV, as a WIMP candidate satisfying the WMAP bounds.

The selected ranges of NMSSM parameters correspond to a particular but motivated scenario [12, 13]. We

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### Table I: Phase-space corrected leptonic branching fractions, $\mathcal{B}(\Upsilon(nS) \to \ell\ell)$ (in %), and error bars (summed in quadrature) of $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ resonances [20]. Error bars of the ratios $R_{\ell\ell}(nS)$ are likely overestimated because of expected correlations between the numerator and denominator experimental uncertainties.

| $\Upsilon(nS)$ | $\mathcal{B}(\ell^+\ell^-)$ | $\mathcal{B}(\mu^+\mu^-)$ | $\mathcal{B}(\tau^+\tau^-)$ | $R_{\tau/\ell}(nS)$ | $R_{\tau/\ell}(nS)$ |
|---------------|----------------|----------------|----------------|----------------|----------------|
| $\Upsilon(1S)$ | $2.48 \pm 0.07$ | $2.48 \pm 0.05$ | $2.62 \pm 0.10$ | $0.057 \pm 0.050$ | $0.057 \pm 0.046$ |
| $\Upsilon(2S)$ | $1.91 \pm 0.16$ | $1.93 \pm 0.17$ | $2.01 \pm 0.21$ | $0.052 \pm 0.141$ | $0.041 \pm 0.141$ |
| $\Upsilon(3S)$ | $2.18 \pm 0.21$ | $2.18 \pm 0.21$ | $2.30 \pm 0.30$ | $0.056 \pm 0.171$ | $0.056 \pm 0.171$ |
| $\psi(2S)$ | $0.773 \pm 0.017$ | $0.77 \pm 0.08$ | $0.772 \pm 0.100$ | $-0.001 \pm 0.100$ | $0.002 \pm 0.100$ |
set $\mu_{\text{eff}} \simeq 200$ GeV and $\tan \beta = 45$ throughout our analysis, while $\lambda$ and $\kappa$ run over the range $[0.3, 0.5]$, with $M_A \in [400, 550]$ GeV. In order to get the highest possible $X_d$ values compatible with the bounds, we set $A_\kappa \in [-20, -30]$ GeV for $m_\chi \simeq 2 - 3$ GeV, dropping to $A_\kappa \simeq -10$ GeV for $m_\chi \simeq 10 - 12$ GeV. Note that decreasing $|A_\kappa|$ implies lowering $X_d$ as can be inferred from Eq. (2) by keeping $M_A$ fixed within the above interval.

In Fig. 1 we plot $X_d$ versus $m_{A_1}$ for different values of the neutralino mass, namely $m_\chi = 3, 3.5, 4, 10$ and 12 GeV. For $m_\chi$ close to 5 GeV the values of $X_d$ are very small as expected on the grounds of a too efficient annihilation rate of dark matter due to the resonant condition: $2m_\chi \simeq m_{A_1}$. Notice that values of $X_d \gtrsim 8$ are possible for $m_\chi \gtrsim 10$ GeV, being particularly large ($X_d \gtrsim 10$) for $m_\chi \simeq 4$ GeV. It is also worth noting that if condition (iii) is removed from the scan, the allowed values of $X_d$ become higher, notably about $m_\chi = 5$ since the aforementioned resonant condition does not apply anymore.

Let us also remark that the current limit from B factories [20] on $B [Y(3S) \to \gamma + \text{invisible}] < (0.7 - 30) \times 10^{-6}$, for $s_{\text{inv}}^{1/2} < 7.8$ GeV where $s_{\text{inv}}$ denotes the missing invariant-mass squared, actually does not impose any bound on the $\chi$ mass (as $m_{A_1} \gtrsim 10$ GeV).

### III. NEUTRALINO SCATTERING OFF NUCLEI

Next we address neutralino elastic scattering off nuclei based on our analysis of the NMSSM parameter space. As is well-known, the SI scattering cross section is enhanced by a coherent factor proportional to the atomic number squared $A^2$. For SD interactions, the cross section depends on the total spin of the nucleus and is typically a factor $A^2$ smaller.

The total WIMP-nucleus cross section has contributions from both the SI and SD interactions, though one contribution is expected to dominate the other depending on the target nucleus (e.g. according to the even/odd number of protons and neutrons) and the detection technique employed in the experiment. The contributions to the SI cross section arise in the interaction Lagrangian of the WIMP with quarks and gluons of the nucleon from scalar and vector couplings whereas the SD part is attributed to the axial-vector couplings. Pseudoscalar interaction is usually neglected because of a strong velocity and/or momentum transfer suppression.

Nevertheless, momentum-dependent interactions have been put forward [25–29] in order to alleviate the tension between the DAMA signal and the null results from other experiments. In this work we propose that a significant $A_{10} - \eta_0$ mixing could dramatically modify the SD $\chi$-nucleon cross section, in analogy with the well-known vector-meson-dominance model (VMD) for electron (or real photon) scattering off nuclei [30], where the virtual (or real) photon interacts with nucleons via one of its hadronic components. In Fig. 2 we depict two graphs illustrating how the mixing of a pseudoscalar (left), or a photonic mediator (right), with hadronic resonances can modify the effective coupling to the nucleon for WIMP and electron scattering, respectively.

Likewise, a similar mechanism could be envisaged for the mixing of scalar resonances (e.g. $\chi_0$ states) and a light enough CP-even Higgs boson. However, it is widely accepted that, contrary to the CP-odd Higgs, present bounds exclude a $O(10)$ GeV scalar boson with relatively large couplings to quarks and leptons (see however [31]). In fact, the lightest scalar Higgs state in our NMSSM scan has a mass $\gtrsim 110$ GeV, whereby the mixing with hadronic states would be negligible.

The axial-vector and pseudoscalar $\eta_0 NN$ couplings defined via

\[
\mathcal{L}_{\eta_0 NN} = \frac{g_{\eta_0 NN}}{2M_N} \bar{N} \gamma_\tau N \eta_0 - i g_{\eta_0 NN} \bar{N} \gamma_5 N \eta_0 \quad (8)
\]

lead to a $q^2$-suppression factor at the rate level, where $q$ is the momentum transfer of the neutralino to the nucleon. An additional $q^2$ factor stems from the $A_{10} \chi \chi$ vertex, yielding altogether a $(q^2/M_N^2)^2$ suppression factor in the scattering cross section, where $M_N$ stands for the target and neutralino mass, respectively.

The effective coupling to the nucleon of either the mixed $A_1$ or $\eta_0$ state via its hadronic component reads

\[
g_{A_1 NN}^{\text{eff}} = \sin \alpha \times g_{\eta_0 NN} \quad , \quad g_{\eta_0 NN}^{\text{eff}} = \cos \alpha \times g_{\eta_0 NN} \quad . \quad (9)
\]

respectively, where the mixing angle $\alpha$ is given by Eq. (4).

In turn, the effective coupling of either the mixed $A_1$ or $\eta_0$ state to the neutralino reads

\[
g_{A_1 \chi \chi}^{\text{eff}} = \cos \alpha \times g_{A_{10} \chi \chi} \quad , \quad g_{\eta_0 \chi \chi}^{\text{eff}} = - \sin \alpha \times g_{A_{10} \chi \chi} \quad (10)
\]

respectively, as a result of its Higgs component.

Let us remark that both $A_1$ and $\eta_0$ are considered as physical states in our model, i.e. mass eigenvalues of the full Hamiltonian without transitions among their constituents apart from interactions with external particles. Note, however, that the WIMP scattering off nuclei takes place at very low momentum transfer, so that the pseudo-scalar mediator in Fig. 2 is quite off-shell. To the extent
that the interaction amplitude is not too sensitive to the energy difference between on-shell and off-shell states, should our model make sense.

On the other hand, let us note that the \( Z \)-boson coupling to the neutralino (\( g_{Z\chi\chi} \)) should be of the same order as the \( A_{10} \) coupling, but the \( A_{10} \)-coupling to strange quarks would be tiny (especially for a dominantly singlet-like \( A_{10} \)) as compared to the \( Z \)-boson coupling to nucleons (\( g_{ZNN} \)). However, the effective coupling \( \eta_{\chi} \) of mixed \( A_{1}/\eta_{\chi} \) states to the latter could be significantly enhanced via a non-perturbative effect. Indeed, the experimental value \( B[\eta_{\chi} \to pp] \simeq 10^{-3} \) turns out to be unexpectedly large in view of the helicity suppression resulting in a perturbative framework. Even after including finite mass effects, a discrepancy of three or four orders of magnitude still remains with respect to theoretical calculations.\( \eta_{\chi} \). We shall return later to this point of paramount importance in our work, after examining the spin-dependent \( \chi \)-nucleon cross section mediated by a pseudoscalar Higgs versus a \( Z \)-boson.

A. Pseudoscalar Higgs versus \( Z \)-boson exchange

The \( \chi \)-nucleon scattering cross section with \( A_{1}/\eta_{h} \) mixing (\( \sigma_{SD}^{m} \)) can be estimated from the ratio \( r = \sigma_{SD}^{m}/\sigma_{SD}^{Z} \), where \( \sigma_{SD}^{Z} \) denotes the \( Z \)-exchange SD cross section, according to:

\[
r \sim \sin^{2}(2\alpha) \left[ \frac{g_{A_{1}XX}}{g_{ZXX}} \right]^{2} \left[ \frac{g_{\eta_{h}NN}}{g_{ZNN}} \right]^{2} \left[ \frac{90(\text{GeV})}{m_{A_{1}}} \right]^{4} \times \left[ \frac{q^{2}}{M_{S}^{2}} \right]^{2} F^{2}[m_{A_{1}}, m_{\eta_{h}}, q^{2}] \quad (11)
\]

where \( F[m_{A_{1}}, m_{\eta_{h}}, q^{2}] \) takes into account the interference effect due to the sign difference in Eqs. (9, 10). At vanishing \( q^{2} \) and the range of masses considered in this work, one gets \( F[m_{A_{1}}, m_{\eta_{h}}, 0] \simeq |m_{A_{1}} - m_{\eta_{h}}|/m_{\eta_{h}} \).

Actually a summation over all possible mixed states should be understood in (11), albeit with variable weight depending on both their mixing strength and the effective \( \eta_{\alpha}(nS) \)-nucleon coupling.

Let us first examine the ratio

\[
\left[ \frac{g_{A_{1}XX}}{g_{ZXX}} \right]^{2} \simeq \frac{N_{11}^{2} N_{13}^{2}}{N_{13}^{2} - N_{14}^{2}} \frac{4 g_{1}^{2} \cos^{2} \theta_{W}}{g_{2}^{2}} \cos^{2} \theta_{A} \quad (12)
\]

with \( N_{1i} \) denoting the different (bino, higgsino ...) components of the lightest neutralino, and the couplings \( g_{1} \) and \( g_{2} \) satisfy: \( g_{1}^{2}/g_{2}^{2} = \tan^{2} \theta_{W} \), where \( \theta_{W} \) is the Weinberg angle. Bounds from the invisible decay width of the \( Z \)-boson imply that \( |N_{11}^{2} - N_{14}^{2}| < 0.11 \). Moreover, in our scan we always find \( N_{11}^{2} \) close to unity (at large tan \( \beta \) where also \( N_{13}^{2} >> N_{14}^{2} \)), and \( \cos^{2}(\theta_{A}) \in [10^{-2} - 10^{-1}] \); hence we can write

\[
\left[ \frac{g_{A_{1}XX}}{g_{ZXX}} \right]^{2} \simeq \frac{4 N_{11}^{2} \sin^{2} \theta_{W} \cos^{2} \theta_{A}}{N_{13}^{2}} \sim \mathcal{O}(10^{-1} - 1) \quad (13)
\]

Setting \( m_{A_{1}} = 10 \text{ GeV}, \sin^{2}(2\alpha) \simeq 10^{-1} \) (typically expected from (10)) and \( q^{2} \simeq (100 \text{ MeV})^{2} \) as reference values, we are led to

\[
\frac{g_{\eta_{h}NN}}{g_{ZNN}} \gtrsim \mathcal{O}(10^{3}) \quad (14)
\]

so that the \( A_{1}/\eta_{h} \) exchange channel would become comparable numerically to the \( Z \)-exchange, i.e. \( r \sim 1 \).

Naively one would expect the above ratio to be of order \( \alpha_{s}/\sqrt{\alpha_{em}} \) (where \( \alpha_{s} \) and \( \alpha_{em} \) denote the strong and electromagnetic coupling strengths, respectively) which hardly can yield such a large factor. Yet a big enhancement of (14) could be plausible if a non-perturbative mechanism contributes to the \( \eta_{\alpha} \)-nucleon coupling, as it likely happens in the \( \eta_{h}(1S) \) decay into a \( pp \) pair (though at quite larger \( r^{2} \)).

B. Non-perturbative (instanton-induced) effects

Indeed, an explanation of the large observed \( \eta_{h}(1S) \to pp \) decay rate seems to require a fundamental modification of the perturbative approach to account for this decay mode. Different proposals have been put forward in terms of a non-perturbative mechanism: mixing of the resonance and gluonium states, \( \xi_{\alpha} \), \( \alpha \)-meson effects, \( \chi_{\alpha} \), intermediate \( Z \)-boson contribution or higher Fock components.\( \eta_{\alpha}(nS) \). Despite many uncertainties, it is conceivable that a long-distance contribution also affects \( \eta_{\alpha}(nS) \) resonances.

Let us focus hereafter on instanton effects, which play a fundamental role in understanding the QCD vacuum and many other topics related to hadronic physics, especially concerning light hadrons (see e.g. [77] for a review). Nevertheless, instanton effects can still be relevant in heavy quark systems, e.g. for the non-perturbative gluon condensate in charmonium [38].

In particular, the authors of Ref. [39] studied the instanton contribution to non-perturbative chiral symmetry breaking in proton-proton scattering at high energy. The same authors later considered this interaction contributing to the decay of the \( \eta_{h} \) resonance into a \( pp \) pair [33, 40]. The idea is that the meson resonance annihilates into two gluons (perturbative part) that are absorbed by instantons, which couple to a baryon pair (non-perturbative part).
In the following we envisage whether such an instanton-induced interaction could still affect the $\eta_b$ resonance coupling to baryons for a momentum transfer from $q = m_{\eta_b}(1S)$ down to $q = 100$ MeV. If so, the WIMP scattering off nuclei might then bear an unexpected resemblance to $p-p$ elastic scattering (at small $-t$).

Instanton effects are usually assumed to depend linearly on the instanton density given by $34,41$:

$$\frac{dn(\rho)}{d\rho} \sim \frac{1}{\rho^2} \left[\alpha_s(\rho^{-1})\right]^{-6} \exp\left[-\frac{2\pi}{\alpha_s(\rho^{-1})}\right], \quad (15)$$

where $\rho$ denotes the instanton size (such that $\rho \lesssim 1/q$). Quite in general, instanton effects are expected to be weighted by the Euclidean instanton action of the exponential factor in Eq. $(15)$, becoming more relevant at smaller $q^2$ (hence larger $\alpha_s$).

Now, in order to obtain an estimate of the $\eta_b(1S) \rightarrow p\bar{p}$ decay rate within this framework, we first rescale the perturbative part of the calculation in $34$ to the bottomonium system according to:

$$\frac{|R_{\eta_b}(0)|^2}{|R_{\eta_b}(0)|^2} \times \frac{\alpha_s^2(m_{\eta_b})}{\alpha_s^2(m_{\eta_b})} \times \frac{m_{b_1}^2}{m_{b_0}^2} \approx 0.3, \quad (16)$$

where the wave function of the spin-singlet state can be approximated by the corresponding spin-triplet one $43$.

Turning now to the non-perturbative part of the calculation, we will assume (as usual) that the instanton-induced coupling to nucleons depends on the number of instantons $n(1/q)$ relevant in the process, obtained by integration of the instanton density $(15)$ over $\rho \leq 1/q$. Note that the resulting value should be controlled by the exponential dependence on $\rho$ around the inverse mass of the heavy resonance. Varying the momentum transfer to baryons, from $q = m_{\eta_b}$ to $q = m_{\eta_b}, n(1/q)$ decreases roughly by two orders of magnitude, hence lowering the rate (which depends on $n(1/q)^2$) by about four orders of magnitude.

Even though phase space favours the $\eta_b \rightarrow p\bar{p}$ decay rate with respect to the $\eta_c$ by a factor $\approx 1.3$, we finally conclude, taking into account both perturbative and non-perturbative parts, that the partial width $\Gamma[\eta_b \rightarrow p\bar{p}]$ induced by instanton effects should be about four orders of magnitude (though with large uncertainties) smaller than $\Gamma[\eta_c \rightarrow p\bar{p}]$. The corresponding BF can be obtained making use of the central value for the $\eta_b(1S)$ full width $\Gamma_{\eta_b(1S)} = 12.4$ MeV, recently found by Belle $22$.

In sum, our order-of-magnitude estimate based on instanton-induced interaction reads:

$$B[\eta_b \rightarrow p\bar{p}] \approx 10^{-7} - 10^{-6} \quad (17)$$

Yet the $\eta_b(1S)$ can decay into a $p\bar{p}$ pair at an observable rate at B (Super) factories $42$. The experimental determination of $17$ thus becomes relevant to uncover possible non-perturbative effects (the present upper limit being $5 \times 10^{-4}$ $20$) associated to the $\eta_b$ state, permitting a reliable comparison with perturbative QCD predictions because of the heavier bottom mass.

On the other hand, as already commented, instanton-induced effects should become quite more important at low momentum transfer because of the exponential in Eq.$(15)$. Thus, a sizable non-perturbative effect in $\eta_b$ decays into baryons occurring at $q^2 \approx m_{\eta_b}^2$ should be enhanced (actually less suppressed) at the much lower energy scale set by the small momentum transfer in WIMP scattering off nuclei, $q \approx 100$ MeV. However, extrapolation to such low $q$ value from the $\eta_c$ or $\eta_b$ mass requires a further and detailed examination $43$.

Furthermore, our previous caveat concerning the off-shellness of the pseudoscalar mediator of Fig.2 is in order as indeed $q^2 << m_{\eta_b}^2$. In particular, its hadronic component could display in the nucleon-nucleon vertex a different behaviour than an on-shell $\eta_b$ resonance. Similarly, in a non-covariant language, one can invoke the time-energy uncertainty principle implying that the time for a virtual $bb$ pair is likely too short for the $\eta_b$ bound state to be formed. Therefore the above calculation of the perturbative part cannot be straightforwardly applied. Nonetheless, quantum numbers of the virtual $bb$ pair should still correspond to a pseudoscalar state thereby permitting two gluons to be emitted, ultimately leading to instanton-induced (spin-dependent) effective interaction of WIMPs with nuclei.

Conversely, such a mechanism should not significantly affect neutralino annihilation into SM particles via a $s$-channel exchange of a CP-odd Higgs boson, for the energy scale would be again of order $O(10)$ GeV, thereby avoiding an extra tension with indirect detection limits, such as cosmic-ray antiprotons $14,16$.

IV. SUMMARY

In this paper, we have considered a particular scenario within the NMSSM with both a light neutralino and a light CP-odd Higgs boson, the latter sizably mixing with pseudoscalar $\eta_b$ resonances. Implicit in our work is the idea that non-perturbative effects (e.g. instanton-induced interaction) may lead to a non-negligible pseudoscalar contribution to the $\chi$-nucleus scattering, thereby introducing a momentum-dependent form factor in the cross section which might be helpful (see e.g. Ref. $47$) to interpret the results of direct DM search experiments, with variable sensitivity along the nuclear recoil energy range.

To conclude we stress that an accurate experimental test of lepton universality in $Y$ decays, the discovery of the $\eta_b(2S)$ resonance together with the measurements of $B[\eta_b(nS) \rightarrow p\bar{p}]$ at a (Super) B factory $42$ could be relevant for a better understanding of DM searches and related astrophysical questions.

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