The rock breaking capability analyses of sonic drilling

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Abstract
Sonic drilling technology uses the longitudinal vibration of a drill string to realize fast and effective drilling. By setting the top and bottom boundary conditions of the drill string during drilling, a dynamic model of flexible sonic string percussive drilling is established in this article. At a certain drilling depth, with the excitation frequencies as the control parameters, the maximum impact force and rock breaking energy utilization rate are used to evaluate the rock breaking capability of the sonic drilling system under the linear bit–rock model. A surface diagram of the maximum breaking force reached within the working frequencies and at varying drilling depths is obtained. The curve graph of the rock breaking energy utilization rate varying with drilling depth under the first six orders of resonance is also calculated. Analysing the influence of changing drilling parameters on the rock breaking capability of sonic drilling systems can provide theoretical guidance for the actual drilling process.

Keywords
sonic drilling, flexible percussion drilling, max impact force, rock breaking energy utilization rate

Highlights
- A physical model is established to study the interaction of the flexible drill string impacting rock
- The influences of the sonic vibration head and the isolation spring are considered in the mathematical model
- The maximum impact force reached in the working frequencies and drilling depths is calculated.
- The variation of system energy before and after bit impacting rock is revealed.

Introduction
The sonic technique is an advanced form of drilling that has the advantage of high efficiency in subsurface formations, exceptional power output and tooling penetration in a small, lightweight rig.¹ Sonic drilling can obtain high-quality samples without any drilling fluids, such as mud and air.² This feature makes sonic drilling the preferred method for environmental sampling.³ Many types of sonic rigs have been developed and widely used in the field. Boart Longyear Company completed a borehole by sonic drilling with a depth of 274.32 m at the Bingham Canyon Mine in 2012. High frequencies and resonant energies are generated inside the sonic vibrator to advance the drilling string into subsurface formations. The maximum drilling depth and drilling speed are greatly affected by vibration conditions and parameters. Currently, drillers still rely on experience to control the resonant energy to match the formation being encountered to achieve the maximum rate of penetration (ROP). Furthermore, in the process of shallow formation sampling, pure soil drilling cannot be completely realized, and rocks are occasionally encountered; to sample quickly and safely, the sonic drilling rig must have a certain rock breaking ability.

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The longitudinal vibration of the drilling string is not only adopted in sonic drilling rigs but also has been successfully applied to improve the drilling efficiency in deep rock breaking. A research project was conducted to develop resonance-enhanced drilling (RED) at the University of Aberdeen during the last decade. The main idea behind this technology is to apply an adjustable high-frequency dynamic stress (generated by axial oscillations) in combination with the rotary action to enhance the penetration rates by creating resonance conditions between the drill bit and hard rocks. Subsequently, a series of bit–rock interaction models considering a set of parameters were built to describe the main physical characteristics of rocks in a vibro-impact drilling system. The dynamic method for the stiffness identification of a rock via analysis of its corresponding impact duration was also developed to optimize the percussive drilling parameters. The rock breaking mechanism under harmonic vibro-impact is investigated based on the displacement response and energy response. These studies aim to superimpose axial drill bit vibrations upon normal rotary drilling to optimize the ROP for oil drilling in deep wells. These studies provide a highly valuable reference for drilling capability analyses of resonant sonic drilling on unexpected rocks.

The vibrations of the drilling strings dominate the ROP, drilling direction, core quality, and drilling tools. An understanding of the drilling string dynamics is crucial for the analysis of the drilling process. Jansen et al. modelled the drilling string as a mass–spring–damper system and developed lumped parameter models based on an ordinary differential equation (ODE). However, the distributed nature of a slim drilling string could not be reflected by this model. Consequently, a distributed parameter model was built to analyse the characterization of the vibration in infinite dimensions. This model was used by Tucker et al., Fridman et al., Saldivar et al. and Danciu et al. and is suitable for analysing the drilling string dynamics in oil drilling, geothermal drilling, and other deep drilling.

Recently, some attempts have been made to understand the resonant sonic drilling process to improve the drilling efficiency. In 2015, Latrach and Beji presented a vibration model as a hyperbolic partial differential equation with Neumann boundary conditions as well as a resonant sonic vibrator drill model in complex soils. These researchers used this model to define a reference trajectory and proposed a flatness-based control scheme guaranteeing efficient drilling. Mnafig and Beji presented a distributed parameter model of the drilling string vibrating in the axial direction. These investigators analytically quantified the system dynamics, the influence of the control variables, such as the resonant mode, and the weight on the bit. Ammari and Beji determined the wave peak amplitude from the resonant drilling string mode shape, and system’s stability was analysed around a practical frequency mode.

The above sonic drilling studies mostly focus on soil drilling, except the works done by Sun et al. They provided an innovative analytical solution of a partial differential mathematical physical equation based on the modelling of the impact of the resonant flexible drilling string on a rock induced by the sonic vibrator. The maximum impact force of the sonic drill on the rock, the effects of the drilling string length, the rock stiffness and the damping ratio were studied systematically. As the rock stiffness and the order of the resonance increase, the magnitude of the impact peak increases, and the impact duration becomes shorter. As the drilling hole is extended, the drilling string becomes longer, and the largest impulse obviously decreases. However, the effects of the mass of the sonic vibrator and the isolation spring between the sonic vibrator and support component were not considered in the model.

This article investigates the rock breaking capability of sonic drilling and considers the influences of the drill string lengths and excitation frequencies on the maximum impact force and energy. It is organized as follows. Firstly, we introduce the principles and system model. We present a composite joint mass–spring model of the sonic vibrator and a distributed parameter model of the drilling string axial vibrations. Secondly, we analyse the drill bit force and energy for sonic drilling on the rock. Finally, the conclusion is presented.

**Mathematical modelling and solution**

In the sonic vibration head, as shown in Figure 1, the double hydraulic motor drives the two eccentric shaft systems to achieve high-speed movement. Through the synchronization system, the eccentric shaft rotation speed is the same but in the opposite direction; the horizontal vibration forces cancel each other out, and the vertical direction of the force is strengthened, resulting in a downward drilling force by drilling vibration. The sonic vibration head produces a vibration of 50–200 Hz to the drilling string. When the vibration overlaps with the natural resonance frequency of the drilling string, the system produces a resonance with a maximum drilling energy, and the soil around the drilling string generates local liquefaction, which reduces the damping of the drilling string and surrounding strata to achieve rapid drilling.

**Mathematical modelling**

The vibration characteristics of the sonic drilling system are closely related to the mass of the sonic vibration head, the stiffness of the isolation spring and the drilling parameters. In this article, the effects of the inertial force of the sonic
vibration head and the elastic force of the isolation spring are considered to establish a coupled viscous damping longitudinal vibration model for the sonic vibration head, drilling string and stratum.

During drilling in the soil strata, the constrained reaction force at the bottom of the drilling string is assumed to be zero, and the energy input to the system by the excitation force in a period is equal to the energy dissipated by viscous damping of the hole wall; then, the steady-state vibration model of the system is established.

When the rock is encountered, the steady state is broken, and the problem is transient. The kinetic energy and strain energy of the steady-state resonance system is converted into the potential energy acting on the rock (in view of the transient characteristics, the input energy of the excitation force can be ignored) to achieve effective rock breaking. Therefore, the transient vibration model of the system is established to analyse the reaction force acting on the constraint reaction at the bottom of the drilling string, that is, the reaction force acting on the impact force. After an impact occurs, the drilling bit rebounds immediately, separating from the rock until the next impact. Therefore, the contact duration in the half vibration cycle of sonic drilling is investigated for the transient problem, and we assume that the rock loading and unloading stiffness to the bit are the same. The nomenclature used in this article is shown in Table 1.

**Step 1: No contact**

When the sonic rig drills into the soil layer, that is, when the bit does not contact the rock, the surrounding soil layer interacts with the drilling string to form the main damping source in the sonic drilling. When the damping system is in steady-state vibration, it is assumed that the drilling string is a straight bar with equal section length $l$, and its longitudinal forced vibration governing equation is

$$\rho S \frac{\partial^2 u_1}{\partial t^2} + c \frac{\partial u_1}{\partial t} - ES \frac{\partial^2 u_1}{\partial x^2} = p(x, t) \delta(x) + M_0 \frac{\partial^2 u_1}{\partial t^2} \delta(x)$$

where $\rho$ is the density of the drilling string, $S$ is the cross-sectional area of the drilling string, $c$ is the viscous damping, $E$ is Young’s modulus of the drilling string, and the excitation force $p(x, t)$ acts on the top of the drilling string.
The influence of the vibration head mass $M_0$ is applied in the governing equation (1) in the form of inertial forces. The elastic force of the isolation spring is applied at the top of the drilling string

$$E_0 \frac{\partial^2 u_1(0,t)}{\partial x^2} = k_0 u_1(0,t)$$

where $k_0$ is the isolation spring stiffness. While ignoring the constraint reaction at the bottom of the drilling string, the free end of the drilling string is

$$\frac{\partial u_1(l,t)}{\partial x} = 0$$

Step 2: Loading and unloading contact

When the rock is suddenly drilled, the vibration mode of the steady longitudinal vibration drilling string will be seriously changed by the transient impact. If the energy input from the excitation force is ignored, then the governing equation is

$$\rho S \frac{\partial^2 u_2}{\partial t^2} + c \frac{\partial u_2}{\partial t} - ES \frac{\partial^2 u_2}{\partial x^2} = M_0 \frac{\partial^2 u_2}{\partial t^2} \delta(x)$$

Table 1. Nomenclature.

- $a$: Wave propagation velocity (m/s)
- $C$: Damping of the drilling string vibration (Ns/m²)
- $d$: Drill string inner diameter (mm)
- $D$: Drill string outer diameter (mm)
- $S$: Drill string cross-sectional area (m²)
- $l$: Length of the drilling string (m)
- $E$: Elastic modulus of the drilling string (Pa)
- $\rho$: Drill string density (kg/m³)
- $2m_e$: Total static moment me of the sonic vibrator (kg m)
- $t$: Time (s)
- $t_0$: Time when the bit contacts rock in the resonance state (s)
- $t_0$: Time before the impact occurs (s)
- $k_0$: Stiffness of the isolation spring (N/m)
- $M_0$: Mass of the sonic vibration head (kg)
- $k_l$: Load stiffness of the rock (N/m)
- $k_u$: Unload stiffness of the rock (N/m)
- $\Phi(x)$: Longitudinal vibration mode of the drilling string
- $q(t)$: Modal coordinate of the drilling string
- $u_1$: Distance field of the drilling string before bit impacting rock
- $u_2$: Distance field of the drilling string after bit impacting rock
- $x$: Distance from the vertices at time $t$ (m)
- $\delta(x)$: $\delta(x) = 1$ when $x = 0$ and $\delta(x) = 0$ when $x \neq 0$
- $\xi_i$: $i$-th order damping ratio before bit impacting rock
- $\zeta_i$: $n$-th order damping ratio after bit impacting rock
- $\omega$: Angular frequency of the sonic frequency vibrator (rad/s)
- $\omega_n$: $i$-th order modal frequency of $\Phi(x)$ before bit impacting rock (rad/s)
- $\omega_n$: $n$-th order modal frequency of $\Phi(x)$ after bit impacting rock (rad/s)
- $m_{dn}$: $n$-th order damped natural frequency after bit impacting rock (rad/s)
- $E_{p0}$: Kinetic energy of the sonic vibration head (J)
- $E_{p0}$: Potential energy the of isolation spring (J)
- $E_k$: Kinetic energy of the drilling string (J)
- $E_{sk}$: Strain energy of the drilling string (J)
- $E_{ssk}$: Potential energy of the spring at rock (J)
- $\alpha$: Proportion of the maximum rock potential energy to total energy (%)
The rock drilled into is regarded as an elastic medium with a stiffness coefficient of $k$, and the upper and lower boundary conditions of the drilling string are

$$\text{ES} \frac{\partial u_2(0,t)}{\partial x} = k_0 u_2(0,t)$$  \hspace{1cm} (6)$$

and

$$\text{ES} \frac{\partial u_2(l,t)}{\partial x} = -k_0 u_2(l,t)$$  \hspace{1cm} (7)$$

The steady-state solution in no contact is the initial condition of the loading contact

$$u_2(x,0) = \sum_{i=1}^{N} \Phi_i(x) q_i(t_0) = f_1(x)$$  \hspace{1cm} (8)$$

and

$$\frac{\partial u_2(x,0)}{\partial t} = \sum_{i=1}^{N} \Phi_i(x) \dot{q}_i(t_0) = f_2(x)$$  \hspace{1cm} (9)$$

where $t_0$ is the time when the steady vibration drilling string bit is in the equilibrium position, and its determination is shown in (46).

The particle on the steady longitudinal vibration drilling string moves in a simple harmonic motion around its own equilibrium position, and the drill bit may encounter the rock at any position. While the bit is in its balance position, the end velocity is at its maximum. This article will examine the process in which the bit is just in balance before impact and then begins to hit and bounce off the rock.

**Solution method and natural frequency of the system**

Xiao et al.\textsuperscript{23} obtained the explicit expression of the modal frequency with respect to the vibrator mass and system structure parameters through mathematical analysis, but the vibrator mass was still placed in the top boundary condition. To consider the influence of the vibration head mass, it is placed in the governing equation by means of the inertial force, which leads to the elastic support boundary. The characteristic equations represented by the symmetric (off-diagonal) modal mass matrix and the diagonal modal stiffness matrix are obtained by modifying the traditional separation variable solution method, and the inherent vibration characteristics of the system are solved. Then, the vibration response is solved according to the Galerkin-type matrix equation. For the solution of the steady-state and transient vibration problems, refer to Appendix 1 and Appendix 2 for details.

The technical parameters of the sonic vibrator and drilling string are shown in Table 2. The natural frequency of the steady and transient vibration of the drilling string can be solved through Appendix 1 and Appendix 2. With an increase in the matrix order N, the values of the first six natural frequencies tend to be stable. In the actual sonic drilling operation, the excitation frequency provided by the top motor is approximately 0–200 Hz. With changes in drill string lengths and working conditions, drivers often cannot find the appropriate frequency to achieve high-efficiency resonant drilling but can only rely on experience, which greatly reduces the drilling efficiency, so it is necessary to obtain the natural frequency of the system changing with the length of the drill string. The first six natural frequencies of the sonic system before and after the bit impacting rock with varying lengths of the drill string are shown in Figure 2.

As seen from Figure 2, before and after impacting rock, the natural frequencies of the system are inversely proportional to the drilling depths; especially at higher orders, the natural frequencies decrease more obviously as the drilling depths deepen. In addition, it can be seen that the first six natural frequencies of the system after the impact are slightly higher than those before the impact. This is due to the introduction of elastic rocks at the end of the drill string in the model, which leads to an increase in the stiffness of the system. The general working frequencies of sonic drilling are 0–200 Hz. Figure 2 can provide guidance for the actual drilling process.

**Analysis of the drilling string bottom force and energy**

When the drill bit hits the rock layer, the vibration response of the system changes instantaneously, and the bottom of the drilling string exerts a force on the rock. This section discusses the influencing factors of this force and then analyses the rock breaking energy.
Analysis of the drilling string bottom force

According to (28), the natural frequency of the system is calculated. The vibration mode (19) satisfies the boundary conditions (6) and (7), the modal coordinates (44) and (45) are determined by the initial conditions (8) and (9) and the elastic support displacement at the bottom of the drilling string is obtained

\[ u_2(l,t) = \sum_{n=1}^{N} \Phi_n(l) q_n(t) \]

(10)

Assuming that the initial contact time and position are \( t_0 \) and \( u_1(l,t_0) \), respectively, the relative displacement \( \Delta u = u_2(l,t) - u_1(l,t_0) \) of the spring (the rock deformation is simplified as the spring) at a certain moment \( t \) after contact, and then the bottom of the drilling string exerts a force on the rock force

\[ F(t,t_0) = k_i \Delta u(t,t_0) \]

(11)

The maximum force exerted on the rock at the bottom of the drilling string can be found by the above formula

\[ F_{max} = k_i \Delta u(t_1,t_0) \]

(12)

where \( t_1 \) is the moment when the particle velocity at the bottom of the drilling string is zero after impact and \( t_0 \) is the moment when the particle velocity at the bottom of the drilling string reaches its maximum (see (46) for the time \( t_0 \) determination).

Without considering the influence of the sonic vibration head mass and isolation spring on the top, reference 19 indicates that with increasing drilling string length, the peak value of the impact force on the rock clearly decreases, but the impact duration increases. When the damping coefficient increases, the peak value of the impact force obviously decreases but has little influence on the duration of the impact. In view of the complex structure of the sonic vibration head on the top of the sonic rig, the mass of the sonic vibration head on the top is relatively large, which consumes part of the system energy. The top isolation spring also stores a large amount of energy. It is necessary to systematically study the rock breaking capability of sonic drilling considering the influence of the top sonic vibration head mass and top isolation spring stiffness.
In sonic drilling systems, with increasing drilling depth, the excitation frequencies are the main parameter controlling the drilling efficiency, and in the process of impacting rocks, we often focus on the maximum impact force. Figure 3 shows that the maximum impact forces reached at excitation frequencies of 0–200 Hz and drilling depths of 20–100 m.

Figure 3 clearly shows that at shallower drilling depths, there are fewer local peak points of the maximum impact force at working frequencies of 0–200 Hz, and the higher the resonance frequency is, the higher the peak point will be. With increasing drilling depth, the peak points gradually decrease, but more local peak points appear in the working frequency range, reaching 8 at a drilling depth of 100 m. It can also be found that the peak point is relatively higher at a higher resonance frequency. In summary, when sonic drilling is in a resonance working environment, it can achieve a greater impact force and then may reach the strength of the rock, thus breaking the rock.

Analysis of the drilling string energy

Energy calculation

The variation mechanism of the energy response is crucial for tracking the drilling process. Methods of modelling of energy response of rock with and without damping under harmonic vibro-impacting are undertaken and the corresponding equations of energy dissipation are presented in this study. During the sonic drilling process, energy conversion and storage mainly include the sonic vibration head kinetic energy $E_{M_0}$, the isolation spring potential energy $E_{k_0}$, the drilling string kinetic energy $E_{sk}$, the drilling string strain energy $E_{ss}$, and the rock deformation potential energy $E_{kl}$, which are calculated as follows.

Kinetic energy of the sonic vibration head is

$$E_{M_0} = \frac{1}{2} M_0 \left(\frac{\partial u_0}{\partial t}\right)^2 = \frac{1}{2} M_0 \sum_{n=1}^{N} \left| \Phi_n(0) \dot{q}_n(t) \right|^2$$

(13)

Potential energy of the isolation spring is

$$E_{k_0} = \frac{1}{2} k_0 u_0^2(0, t)^2 = \frac{1}{2} k_0 \sum_{n=1}^{N} \left| \Phi_n(0) q_n(t) \right|^2$$

(14)

Kinetic energy of the drilling string is
Strain energy of the drilling string is

\[ E_{sk} = \frac{1}{2} \rho S \left( \frac{\partial u_s(x,t)}{\partial t} \right)^2 \text{dx} = \frac{1}{2} \rho S \sum_{n=1}^{N} \left[ \Phi_n(x) q_n(t) \right]^2 \text{dx} \]  

(15)

Potential energy of the spring at rock is

\[ E_k = \frac{1}{2} k u_s^2(l,t) = \frac{1}{2} k \sum_{n=1}^{N} \left[ \Phi_n(l) q_n(t) \right]^2 \]  

(17)

**Energy distribution**

Before drilling into the rock, the system is in a steady-state resonance condition, while after drilling into the rock, the system shows a transient non-resonance response, and the energy stored in each part will change, as shown in Figure 4. Before drilling into the rock, the energy in the drilling string system changes over time in a relatively stable manner, and the kinetic energy and potential energy of the system alternately change. When drilling into the rock, the steady-state form is destroyed, the mechanical energy in the drilling string decreases, while the potential energy expressed by equation (17) in the rock increases. As time advances, there will be a maximum rock potential energy under certain impact conditions. It is necessary to study how much of system’s energy can be transferred to the rock at the hole bottom, which determines its ability to break rock. The proportion of the maximum rock potential energy to total energy is defined as \( \alpha \).

Before drilling into the rock, the eccentric motor keeps the sonic drilling string system working, the resonance state ensures high drilling ability and the viscous damping keeps dissipating energy. After impacting the rock, the bottom boundary conditions will change, the vibration mode and natural frequency of the drilling string will also change accordingly and the drilling string system will not be in a resonance state. It can be found from Figure 3 that the peak point of the maximum impact force only appears at the resonant frequency and is significantly higher than those achieved at the surrounding non-resonant frequencies. Therefore, it is necessary to analyse the proportion of rock breaking energy in the total energy of the system under the working conditions of resonance. The curve of rock breaking energy utilization with the drilling depths under resonance is shown in Figure 5.

Figure 5 shows that in the case of the first sixth-order resonance, with increasing drilling depth, the rock breaking energy utilization rate of the sonic drilling system shows different rules. When the system works at the lower resonance of orders 1 and 2, with the deepening of the drilling depths, the rock breaking energy utilization rate shows a downward trend. However, in the case of higher resonance, with the deepening of drilling depths, the rock breaking energy utilization rate...
will first increase and then decrease abruptly, and with the increase of resonance order, the location of this sudden change will be delayed correspondingly. In addition, the rock breaking energy utilization rate is relatively high in the lower resonance working environment. Therefore, in combination with Figure 3, under specific formation conditions, choosing the appropriate frequency can not only achieve a certain impact force but also obtain a higher rock breaking energy utilization rate.

Conclusions

The vibration characteristics of the sonic drilling system are closely related to the sonic vibration head mass, isolation spring stiffness and drilling parameters. The influences of the inertia force of the sonic vibration head and the elastic force of the isolation spring are considered to establish a coupled viscous damping longitudinal vibration model with a top excitation system to achieve the best drilling effect on a linear rock model.

Within the working frequency range, as the drilling depths deepen, the resonance frequency becomes denser, and the maximum impact force decreases significantly. When the system works at a lower resonance, the rock breaking energy utilization rate decreases with increasing drilling depth. However, in the case of higher resonance, the rock breaking energy utilization rate will increase first and then decrease abruptly, and with the increase in resonance order, the position of this sudden change will be delayed correspondingly. The analyses of the energy distribution indicate that the mechanical energy of the drill string provides the main energy for rock breaking, and only a small part of the energy can be transferred to the hole bottom in the deep drilling case. Therefore, simple sonic drilling is not suitable for deep hole rock breaking.

To sample quickly and safely, the sonic drilling rig should have a certain ability to break unexpected rocks. It is necessary to reasonably design each part, including the sonic vibration head structure and mass and the isolation spring mechanical properties. Additionally, the excitation frequency also needs to be tuned with the resonance order and drilling depth. This article provides a solid foundation for exploring the more complex flexible impact theory.

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Figure 5. The curve of system’s rock breaking energy utilization rate changes with the drilling depth under resonance conditions.
References

1. Reece R. Good vibes: sonic drilling excels in tailings applications. *Eng Min J* 2010; 211: 73.
2. Wang Y, Liu B, Zhou Q, et al. Design of a sonic drill based on virtual prototype technology. *T Can Soc Mech Eng* 2013; 37: 185–196. DOI: 10.1139/tsmme-2013-0011.
3. Burlingame MJ, Egin D, and Armstrong WB. Unit weight determination of landfill waste using sonic drilling methods. *J Geotechnical Geoenvironmental Eng* 2007; 133: 609–612.
4. Wiercigroch M. Resonance enhanced drilling: method and apparatus. Patent No. WO2007141550, 2007.
5. Pavlovskaia E, Hendry DC, and Wiercigroch M. Modelling of high frequency vibro-impact drilling. *Int J Mech Sci* 2015; 91: 110–119. DOI: 10.1016/j.ijmecsci.2015.08.009.
6. Liao M, Liu Y, Páez Chávez J, et al. Dynamics of vibro-impact drilling with linear and nonlinear rock models. *Int J Mech Sci* 2018; 146–147: 200–210. DOI: 10.1016/j.ijmecsci.2018.07.039.
7. Kovalyshen Y. Self-excited axial vibrations of a drilling assembly: modeling and experimental investigation. In: Paper presented at the 47th U.S. rock mechanics/geomechanics symposium, 23–26 June 2013, San Francisco, California.
8. Li S, Tian S, Li W, et al. Rock breaking mechanism and drilling performance of harmonic vibro-impact drilling. *JOP Conf Ser Earth Environ Sci*, 2020; 570:022036. DOI:10.1088/1755-1315/570/2/022036.
9. Jansen JD and van den Steen L. Active damping of self-excited torsional vibrations in oil well drillstrings. *J Sound Vibration* 1995; 179: 647–668. DOI: 10.1016/j.jsvi.1995.0042.
10. Tucker WR and Wang C. An integrated model for drill-string dynamics. *J Sound Vibration* 1999; 224: 123–165. DOI: 10.1006/jsvi.1999.2169.
11. Fridman E, Mondié S, and Saldivar B. Bounds on the response of a drilling pipe model. *IMA J Math Control Inf* 2010; 27: 513–526. DOI: 10.1093/imamci/dnq024.
12. Saldivar B, Knüppel T, Woittennek F, et al. Flatness-based control of torsional-axial coupled drilling vibrations. *IFAC Proc* 2014; 47: 7324–7329. DOI: 10.3182/20140824-6-ZA-1003.02205.
13. Danciu D, Islam B, and Stng F. Computational modeling and oscillations damping of axial vibrations in a drilling system. In: 22nd International conference on system theory, control and computing, Sinaia, Romania, 2018, pp. 105–110, DOI: 10.1109/ICSTCC.2018.8540738.
14. Latrach K and Lotfi B. Axial vibrations tracking control in resonant sonic tunnel drilling system. In: 54th IEEE conference on decision and control, Osaka, Japan, 2015, pp. 2495–2500. DOI: 10.1109/CDC.2015.7402583.
15. Latrach K and Beji L. Analysis and control of axial vibrations in tunnel drilling system. In: 2nd IFAC workshop on automatic control in offshore oil and gas production, Florianópolis, Brazil, 27–29 May, 2015.
16. Zoghlami N, Latrach K, and Beji L. Bottomhole pressure stabilizing observer-based controller in tunnel drilling system. In: 3rd International conference on control. engineering & information technology (CEIT), Tlemcen, Algeria, pp. 1–6. 2015, DOI: 10.1109/CEIT.2015.7233182.
17. Ammari K and Beji L. Reconstructed drill-bit motion for sonic drillstring dynamics. In: 24th Mediterranean conference on control and automation (MED), Athens, Greece, pp. 814–819. 2016. DOI: 10.1109/MED.2016.7535944.
18. Sun L, Bu C, Hu P, et al. The transient impact of the resonant flexible drill string of a sonic drill on rock. *Int J Mech Sci* 2017; 122: 29–36. DOI: 10.1016/j.ijmecsci.2017.01.014.
19. Drivdahl KS, Able RE, Nevenner TA, et al. Methods of preloading a sonic drill head and methods of drilling using the same. Patent No. 8356677US, 2013.
20. Roussy R. The development of sonic drilling technology. *GeoDrilling Int* 2002; 10: 12–14.
21. Xiao J, Bu C, Hu Y, et al. Influence of sonic vibrator mass on the modal frequency of the drill string. *T Can Soc Mech Eng* 2020; 44: 65–71. DOI: 10.1139/tsmme-2019-0032.
22. Li S, Yan L, Li W, et al. Research on energy response characteristics of rock under harmonic vibro-impact drilling. *J Vib Eng Technol* 2019; 7: 487–496. DOI: 10.1007/s42417-019-00146-9.
23. Hamdan MN and Jabran BA. Free and forced vibrations of a restrained uniform beam carrying an intermediate lumped mass and a rotary inertia. *J Sound Vib* 1991; 150: 203–216. DOI: 10.1016/0022-460X(91)90616-R.
Appendix 1. No contact vibration response

The effects of the vibration head mass and isolation spring on the intrinsic characteristics and vibration response of the drilling string system under excitation are considered. The influence of the vibration head mass is in the governing equation (1) in the form of the inertial force, and the elastic force of the isolation spring is the boundary condition (3) of the upper end of the drilling string. The displacement of the drilling string is expanded into an infinite series of regular modes by means of mode superposition

\[ u_1(x,t) = \sum_{i=1}^{N} \Phi_i(x)q_i(t) \]  

(18)

where \( q_i(t) \) is the modal coordinate.

Natural vibration of the undamped drilling string

If the damping and inertia force of the vibration head are ignored, then the natural mode of the longitudinal free vibration of the undamped drilling string is

\[ \Phi_i(x) = A_i \sin \frac{\omega_i}{a} x + B_i \cos \frac{\omega_i}{a} x \quad (i = 1, 2, \ldots) \]  

(19)

where \( \omega_i(i = 1, 2, \ldots) \) is the modal frequency of \( \Phi_i(x) \) and \( A_i \) and \( B_i \) are undetermined constants. Combined with the boundary conditions (3) and (4), there is

\[ A_i = \frac{k_0}{\rho S a \omega_i} B_i \]  

(20)

The equation for determining \( \omega_i \) is as follows

\[ \tan \frac{\omega_i}{a} l = \frac{k_0}{\rho S a \omega_i} \]  

(21)

The inherent vibration characteristics of the system with vibration head mass

According to the equation and boundary conditions satisfied by the \( i \)-th natural mode \( \Phi_i(x) \) of the drilling string, the modal mass is obtained as

\[ M = \begin{bmatrix} M_{11} & M_{12} & \cdots & M_{1N} \\ M_{21} & M_{22} & \cdots & M_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ M_{N1} & M_{N2} & \cdots & M_{NN} \end{bmatrix} \]  

(22)

Modal stiffness is

\[ K = \begin{bmatrix} K_{11} & 0 & \cdots & 0 \\ 0 & K_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K_{NN} \end{bmatrix} \]  

(23)

where \( \odot \) when \( i = j \)

\[ M_d = \int_0^l \rho S \Phi_i(x) \Phi_i(x) dx + M_0 \Phi_i(0) \Phi_i(0) \]  

(24)

\[ K_d = \int_0^l E S \Phi_i'(x) \Phi_i'(x) dx + K_0 \Phi_i(0) \Phi_i(0) \]  

(25)
when \( i \neq j \)

\[
M_{ij} = M_0 \Phi_i(0) \Phi_j(0) \\
K_{ij} = 0
\]  

(26)  

(27)

In summary, it can be seen that the modal mass and modal stiffness are symmetric matrices, which are substituted into the undamped eigenvalue equation

\[
[K - \omega^2 M] \psi = 0
\]  

(28)

According to (28), the natural frequency \( \omega_1, \omega_2, \ldots \) and regularized eigenvector \( \psi \) of the system are solved.

**Viscous damped vibration response of the system with vibration head mass under excitation**

The modal coordinates \( q_i(t) \) satisfy the following equation

\[
M \ddot{q}(t) + C \dot{q}(t) + Kq(t) = Q(t)
\]  

(29)

where modal damping

\[
C = \begin{bmatrix}
C_{11} & 0 & \cdots & 0 \\
0 & C_{22} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & C_{NN}
\end{bmatrix}
\]  

(30)

the modal force

\[
Q = \begin{bmatrix}
Q_{11} \\
Q_{21} \\
\vdots \\
Q_{N1}
\end{bmatrix}
\]  

(31)

among them

\[
C_{ii} = \int_0^1 c \Phi_i(x) \Phi_i(x) \, dx
\]  

(32)

and

\[
Q_{ii} = 2meo^2 \sin \omega_0 \Phi_i(0)
\]  

(33)

where \( \psi^T M \psi = E \) is the identity matrix, and \( \psi^T K \psi = \Lambda \) is the diagonal matrix, where

\[
\Lambda = \text{diag}(\omega_1^2, \omega_2^2, \ldots, \omega_N^2)
\]  

(34)

make

\[
q = \psi \eta
\]  

(35)

Then, (29) can be expressed as

\[
\ddot{\eta}(t) + C_N \dot{\eta}(t) + \Lambda \eta(t) = Q_N(t)
\]  

(36)

where \( \psi^T Q = Q_N \) and \( \psi^T C \psi = C_N \) is the coupling matrix (the off-diagonal elements may not be zero). To simplify the calculation, the off-diagonal elements are set to zero, and the influence of the off-diagonal elements is ignored.

According to (36), to solve the particular solution \( \eta \)
\[ \eta_i(t) = \frac{Q_n(i,1) \sin(\omega t - \theta_i)}{\sqrt{(\Lambda(i,i) - \omega^2)^2 + (2\xi_i\omega \sqrt{\Lambda(i,i)})^2}} \]  

(37)

where \( \xi_i = C_N(i,i)/2\sqrt{\Lambda(i,i)} \) and \( \theta_i = \arctan(2\xi_i\omega \sqrt{\Lambda(i,i)}/\Lambda(i,i) - \omega^2) \). Then

\[ q_i(t) = \psi_i \eta_i \]  

(38)

Therefore, the forced vibration response \( u_i(x,t) \) of the system can be obtained by substituting (38) and (19) into (18).

The system response obtained by the above method satisfies the boundary condition of the elastic support on the top under any excitation frequency action. The numerical results converge to the stable value with an increase in the matrix order \( N \).

**Appendix 2. Loading and unloading contact vibration response**

**Natural vibration characteristics and response**

When a rock is suddenly drilled, the boundary conditions at the bottom of the drilling string change, and then, the vibration mode and natural frequency of the drilling string change. According to the boundary conditions (6) and (7), the mean coefficients \( A_n \) and \( B_n \) of the vibration mode (19) are satisfied

\[ A_n = \frac{k_0}{\rho S a_0 \omega_n} B_n \]  

(39)

The modal frequency \( \omega_n \) of \( \Phi_n(x) \) satisfies

\[ \frac{k_0}{\rho S a_0 \omega_n} = \frac{E S a_0 \sin(\omega_n/a)l - k_a \cos(\omega_n/a)l}{E S a_0 \cos(\omega_n/a)l + k_a \sin(\omega_n/a)l} \]  

(40)

Considering the initial conditions (8) and (9), the modal coordinates of the transient vibration problem can be further solved according to the same idea in Appendix 1

\[ q_n(t) = \exp\left(-\zeta_n \sqrt{\Lambda(n,n)} t\right) \left( q_{n0} \cos \varpi_{dn} t + \frac{\dot{q}_{n0} + \zeta_n \sqrt{\Lambda(n,n)} q_{n0}}{\varpi_{dn}} \sin \varpi_{dn} t \right) \]  

(41)

where

\[ \zeta_n = \frac{C_n(n,n)}{2\sqrt{\Lambda(n,n)}} \]  

(42)

\[ \varpi_{dn} = \sqrt{\Lambda(n \cdot n)} \sqrt{1 - \zeta_n^2} \]  

(43)

\( q_{n0} \) and \( \dot{q}_{n0} \) are determined by the initial conditions (8) and (9)

\[ q_{n0} = \int_0^l ESf_i'(x)\Phi_n'(x)dx + k_0 f_i(0) \Phi_n(0) + k_f(l) \Phi_n(l) \frac{1}{\Lambda(n,n)} \]  

(44)

\[ \dot{q}_{n0} = \int_0^l ESf_i''(x)\Phi_n''(x)dx + k_0 f_i(0) \Phi_n(0) + k_f(l) \Phi_n(l) \frac{1}{\Lambda(n,n)} \]  

(45)

Then, the transient response of the system is obtained.
**Initial impact time**

Before drilling into the rock, the drilling string is under a steady vibration condition, and the bit is in its balance position to perform harmonic motion. Assuming that the bit impacts the equilibrium position, the impact time $t_b$ is determined by the moment corresponding to the maximum velocity of the bit at the steady state. Time $t_b$ to meet

\[
v(l,t_b) = \sum_{i=1}^{N} \left( A_i \sin \frac{\omega_i}{a} l + B_i \cos \frac{\omega_i}{a} l \right) \psi \frac{Q_N(i,1) \omega \cos(\omega t_b - \theta)}{\sqrt{(\Lambda(i,i) - \omega^2)^2 + (2\zeta\omega \sqrt{\Lambda(i,i)})^2}} \]

thus, $t_0 = t_b + n(2\pi/\omega)$ is the initial impact time, where $n$ is a positive integer.