Rotating black hole shadow in perfect fluid dark matter

Xian Hou, Zhaoyi Xu and Jiancheng Wang

Yunnan Observatories, Chinese Academy of Sciences, 396 Yangfangwang, Guandu District, Kunming 650216, P.R. China
Key Laboratory for the Structure and Evolution of Celestial Objects, Chinese Academy of Sciences, 396 Yangfangwang, Guandu District, Kunming 650216, P.R. China
Center for Astronomical Mega-Science, Chinese Academy of Sciences, 20A Datun Road, Chaoyang District, Beijing 100012, P.R. China
Key Laboratory of Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, P.R. China

E-mail: xianhou.astro@gmail.com, xuzy@ihep.ac.cn, jcwang@ynao.ac.cn

Received October 16, 2018
Revised December 4, 2018
Accepted December 11, 2018
Published December 21, 2018

Abstract. We study analytically the shadow cast by the rotating black hole in the perfect fluid dark matter. The apparent shape of the shadow depends upon the black hole spin $a$ and the perfect fluid dark matter intensity parameter $k$ ($k > 0$). In general, the shadow is a perfect circle in the non-rotating case ($a = 0$) and a deformed one in the rotating case ($a \neq 0$). The deformation gets more and more significant with the increasing $a$, similar to the Schwarzschild and Kerr black holes. In addition, there exists a reflection point $\tilde{k}_0$. When $k < \tilde{k}_0$, the size of the shadow decreases with the increasing $k$ and the distortion increases with the increasing $k$. When $k > \tilde{k}_0$, the size of the shadow increases with the increasing $k$ and the distortion decreases with the increasing $k$. Furthermore, the energy emission rate of the black hole in the perfect fluid dark matter increases with the increasing $k$ and the peak of the emission shifts to higher frequencies. Finally, we propose that to observe the effect of the black hole spin $a$ and the perfect fluid dark matter intensity $k$ on the shadow of the black hole Sgr $A^*$ at the center of the Milky Way, highly improved techniques would be necessary for the development of future astronomical instruments.

Keywords: GR black holes, astrophysical black holes, dark matter theory

ArXiv ePrint: 1810.06381
1 Introduction

Supermassive black holes are generally believed to reside at the heart of most galaxies including our own galaxy, the Milky Way, though direct detection of a black hole is still one of the most important unresolved problems in astronomy. Among the different methods employed to determine the nature of the black hole, i.e., mass and spin of the black hole, observing the shadow of the black hole remains probably the most exciting and interesting one. The black hole shadow is the optical appearance cast by the black hole when there is a bright distant source behind it. It appears as a two-dimensional dark zone for a distant observer, like us on Earth. The black hole shadow is a natural result of Einstein’s theory of General Relativity (GR), so it can not only provide us information on fundamental properties of the black hole, but also serves as a useful tool of testing GR. Attempts to observe the shadow of a black hole are ongoing. For example, the sub-millimeter “Event Horizon Telescope” (EHT)\(^1\) [1] based on the very-long baseline interferometry (VLBI) are expected to obtain the first images of the black hole Sgr A\(^*\) at the center of our own galaxy and of the black hole M87 in the Virgo A galaxy in the near future.

The boundary of the shadow of a non-rotating black hole, like the Schwarzschild black hole, is a perfect circle and was first studied by [2] and later by [3] who further considered the effect of a thin accretion disk on the shadow. The shadow of the rotating black hole, i.e., the regular Kerr black hole, is no longer circular but rather deformed [4, 5]. The apparent shape of the shadow depends upon the black hole space-time metric. Shadows in various black hole space-time have been examined intensively in the literature in the last decades [e.g., 6–26]. This topic has also been extended to black holes in modified GR [e.g., 27–30], to black holes with higher or extra dimensions [e.g., 26, 31–34] and black holes surrounded by plasma [e.g., 35, 36]. Multiple shadows of a single black hole or the shadow of multiple black holes have also been discussed recently [e.g., 19, 37–39]. Besides these analytical work on different types of black holes, there have been various analytical and simulation-based work dedicated to the black hole Sgr A\(^*\) by taking into account more realistic situations such as accretion flow and relativistic jets [e.g., 40–46]. Accretion models can thus be constrained by comparing with the EHT observations of Sgr A\(^*\). The possibility of testing theories of gravity basing

\(^1\)www.Eventhorizontelescope.org
on the shadow of Sgr A* has been explored equally in the literature [e.g., 47–53]. More interestingly, [54] suggested that the black hole shadow can be used to determine the matter category, such as dark matter, dust and radiation, around a black hole under the assumption of perfect fluid matter. See [55] for a recent review of the study of black hole shadows.

Studying the black hole shadow in the presence of dark matter and dark energy would be of specific interests given that the Universe is dominated by dark mater (27%) and dark energy (68%), while the contribution of baryonic matter is minor (5% to the total mass-energy of the Universe), according to the Standard Model of Cosmology. Recently, the black hole shadow in quintessence has been discussed by [56] and [57], while [58] studied the shadow of the black hole Sgr A* in the Cold Dark Matter [CDM, 59–61] and Scalar Field Dark Matter [SFDM, e.g., 62–64] halos. The CDM model is the current leading dark matter model despite the fact that it is in tension with long-standing (and more recent) small-scale structure observations [65]. The SFDM model, although less popular, can provide good concordance with both large-scale and small-scale structure observations. Another alternative dark matter model is the phenomenological Perfect Fluid Dark Matter (PFDM) model [e.g., 66, 67], in which dark matter is described as a perfect fluid. Although simple, this model has the analytical form and the possibility of explaining the asymptotically flat rotation velocity in spiral galaxies [e.g., 68–70]. This work will investigate the shadow of rotating black hole in PFDM and discuss its applications to Sgr A*, complementary to the work mentioned above.

The paper is organized as follows. In section 2, we introduce the space-time metrics for the spherically symmetric and rotating black holes in PFDM. In section 3, we derive the complete null geodesic equations for a test particle moving around the rotating black hole in PFDM. In section 4, we study the apparent shapes of the shadow cast by the rotating black hole in PFDM. The energy emission rate of the rotating black hole in PFDM is investigated in section 5 and we discuss our results in section 6.

## 2 Black hole space-time in perfect fluid dark matter

### 2.1 Case of spherically symmetric black hole

The spherically symmetric black hole space-time metric in PFDM is [68, 70]

\[
\begin{align*}
\text{ds}^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2),
\end{align*}
\]

with

\[
\begin{align*}
f(r) &= 1 - \frac{2M}{r} + \frac{k}{r} \ln \left( \frac{r}{|k|} \right),
\end{align*}
\]

where \( M \) is the black hole mass and \( k \) is a parameter describing the intensity of the PFDM. If the PFDM is absent \((k = 0)\), the above space-time metric reduces to the Schwarzschild black hole.

### 2.2 Case of rotating black hole

The rotating black hole space-time metric in PFDM is [71]

\[
\begin{align*}
\text{ds}^2 &= -\left( 1 - \frac{2Mr - kr \ln \left( \frac{r}{|k|} \right)}{\Sigma^2} \right) dt^2 + \frac{\Delta}{\Sigma^2} dr^2 - \frac{2a \sin^2\theta}{\Sigma^2} \left( 2Mr - kr \ln \left( \frac{r}{|k|} \right) \right) d\phi dt \\
&\quad + \Sigma^2 d\theta^2 + \sin^2\theta \left( r^2 + a^2 + a^2 \sin^2\theta \frac{2Mr - kr \ln \left( \frac{r}{|k|} \right)}{\Sigma^2} \right) d\phi^2,
\end{align*}
\]
\[ \Sigma^2 = r^2 + a^2 \cos^2 \theta, \]  
(2.4)
\[ \Delta = r^2 - 2Mr + a^2 + kr \ln \left( \frac{r}{|k|} \right). \]  
(2.5)

If the PFDM is absent \((k = 0)\), the above space-time metric reduces to the Kerr black hole.

The value of \(k\) can be both positive and negative and is constrained theoretically as \(0 < k < 2M\) and \(-7.18M < k < 0\) [71]. Observational value could be obtained by fitting the rotation curves in spiral galaxies, similar to what have been done in [68] and [70], which shows that the order of magnitude is around \(10^{-6} \sim 10^{-7}\). Here we only consider the theoretical positive values of \(k\). The case of negative \(k\) can be studied in a similar way as presented in this work.

### 3 Null geodesics

The shape and size of the shadow depend upon completely on the geometry of the black hole. It is thus necessary to first study the geodesic structure of a test particle for the above space-time metric (eq. 2.3). To do this, we adopt the Hamilton-Jacobi equation and Carter constant separable method [72] to obtain the complete geodesic equations. The general form of the Hamilton-Jacobi equation can be expressed as

\[ \frac{\partial S}{\partial \sigma} = -\frac{1}{2} g^{\mu \nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu}, \]  
(3.1)

where \(\sigma\) is an affine parameter along the geodesics and \(S\) is the Jacobi action of which the separable solution is

\[ S = \frac{1}{2} m^2 \sigma - Et + S_r(r) + S_\theta(\theta). \]  
(3.2)

\(m, E\) and \(L\) are, respectively, the test particle’s mass, energy and angular momentum, with respect to the rotation axis. \(S_r(r)\) and \(S_\theta(\theta)\) are, respectively, functions of \(r\) and \(\theta\). Combining eq. (3.1) and eq. (3.2) and applying the variable separable method, we get the null geodesic equations for a test particle around the rotating black hole in PFDM as

\[ \Sigma \frac{dt}{d\sigma} = \frac{r^2 + a^2}{\Delta} [E(r^2 + a^2) - aL] - a(E \sin^2 \theta - L), \]  
(3.3)
\[ \Sigma \frac{dr}{d\sigma} = \sqrt{\mathcal{R}}, \]  
(3.4)
\[ \Sigma \frac{d\theta}{d\sigma} = \sqrt{\Theta}, \]  
(3.5)
\[ \Sigma \frac{d\phi}{d\sigma} = \frac{a}{\Delta} [E(r^2 + a^2) - aL] - \left( aE - \frac{L}{\sin^2 \theta} \right), \]  
(3.6)

where \(\mathcal{R}(r)\) and \(\Theta(\theta)\) read as

\[ \mathcal{R}(r) = [E(r^2 + a^2) - aL]^2 - \Delta [m^2 r^2 + (aE - L)^2 + \mathcal{K}], \]  
(3.7)
\[ \Theta(\theta) = \mathcal{K} - \left( \frac{L^2}{\sin^2 \theta} - a^2 E^2 \right) \cos^2 \theta, \]  
(3.8)
with $\mathcal{K}$ the Carter constant. The dynamics of the test particle around the rotating black hole in PFDM are fully described by the geodesic equations (3.3)–(3.6). The boundary of the shadow is completely determined by the unstable circular orbit which satisfies the condition

$$\mathcal{R} = \frac{\partial \mathcal{R}}{\partial r} = 0. \quad (3.9)$$

Introducing two impact parameters $\xi$ and $\eta$ as

$$\xi = L/E, \quad \eta = \mathcal{K}/E^2, \quad (3.10)$$

and considering the case of photons ($m = 0$), for an observer at the infinity (photons will arrive near the equatorial plane with $\theta = \pi/2$), we obtain

$$(r^2 + a^2 - a\xi)^2 - [\eta + (\xi - a)^2](r^2f(r) + a^2) = 0, \quad (3.11)$$

$$4r(r^2 + a^2 - a\xi) - [\eta + (\xi - a)^2](2rf(r) + r^2f'(r)) = 0. \quad (3.12)$$

From eqs. (3.11) and (3.12), we can get the expressions of $\xi$ and $\eta$ as

$$\xi = \frac{(r^2 + a^2)(rf'(r) + 2f(r)) - 4(r^2f(r) + a^2)}{a(rf'(r) + 2f(r))}, \quad (3.13)$$

$$\eta = \frac{r^3[8a^2f'(r) - r(rf'(r) - 2f(r))^2]}{a^2(rf'(r) + 2f(r))^2}. \quad (3.14)$$

Furthermore, we have

$$\xi^2 + \eta = 2r^2 + a^2 + \frac{16(r^2f(r) + a^2)}{(rf'(r) + 2f(r))^2} - \frac{8(r^2f(r) + a^2)}{rf'(r) + 2f(r)} \quad (3.15)$$

$$= 2r^2 + a^2 + \frac{8\Delta[2 - (rf'(r) + 2f(r))^2]}{(rf'(r) + 2f(r))^2}, \quad (3.16)$$

where

$$f'(r) = \frac{2M + k}{r^2} - \frac{k}{r^2} \ln \left( \frac{r}{|k|} \right). \quad (3.17)$$

4 Black hole shadow

In order to study the shape of the black hole shadow, we introduce the celestial coordinates $\alpha$ and $\beta$ as

$$\alpha = \lim_{r_o \to \infty} \left( -r_o^2 \sin \theta_o \frac{d\phi}{dr} \right), \quad (4.1)$$

$$\beta = \lim_{r_o \to \infty} \left( r_o^2 \frac{d\theta}{dr} \right). \quad (4.2)$$

Here we assume the observer is at infinity. $r_o$ is the distance between the observer and the black hole, and $\theta_o$ is the inclination angle. $\alpha$ and $\beta$ are the apparent perpendicular distances of the shadow as seen from, respectively, the axis of symmetry, and its projection on the equatorial plane.
Using the null geodesic equations (3.3)—(3.6), we get the relations between celestial coordinates and impact parameters as

\[ \alpha = -\frac{\xi}{\sin \theta}, \]  
(4.3)

\[ \beta = \pm \sqrt{\eta + a^2 \cos^2 \theta - \xi^2 \cot^2 \theta}. \]  
(4.4)

In the equatorial plane \( (\theta = \pi/2) \), \( \alpha \) and \( \beta \) reduce to

\[ \alpha = -\xi, \]  
(4.5)

\[ \beta = \pm \sqrt{\eta}. \]  
(4.6)

Different shapes of the shadow can be obtained by plotting \( \beta \) against \( \alpha \). Through calculations, we find \( f(r) \) has a reflection point at \( k_0 = 2/(1 + e) \). When \( k < k_0 \), \( f(r) \) monotonically decreases with \( k \), while when \( k > k_0 \), \( f(r) \) monotonically increases with \( k \). Such reflection point of \( f(r) \) results in a corresponding reflection point of the shadow at \( \tilde{k}_0 \sim 0.8 \), as was shown in figure 1. When \( k < \tilde{k}_0 \), the shadow is a perfect circle and the size decreases with the increasing \( k \) in the non-rotating case \((a = 0)\), while in the rotating case \((a \neq 0)\), the shadow gets more and more distorted with the increasing \( a \), and the size decreases with the increasing \( k \), similar to the case of \( a = 0 \). When \( k > k_0 \), the structure of the shadow is similar to what we find for \( k < \tilde{k}_0 \) except that the size of the shadow instead increases with the increasing \( k \).

This interesting behavior may be understood as a natural result of the metric and/or the PFDM mass distribution. Since the PFDM density \( \rho \sim r^{-3} \), it could be considered as a point mass distribution, which is similar to the mass distribution of the Schwarzschild black hole. In this case, we could imagine that the system is composed of two parts: the original black hole with mass \( M \) and the black hole corresponding to the PFDM with mass \( M_1 \). The horizon of the system would be determined by the original black hole horizon \( 2M \) and the PFDM black hole horizon \( 2M_1 \). The PFDM black hole grows with the intensity parameter \( k \). When \( k < \tilde{k}_0 \), the PFDM black hole horizon is less than \( 2M \) and has an inhibitory effect on the original black hole horizon [70]. The larger the \( k \) is, the smaller the horizon of the system is, thus the smaller the shadow is. When \( k > \tilde{k}_0 \), the PFDM black hole horizon is larger than \( 2M \). Since the horizon of the original black hole is unchanged, PFDM would dominate the system. Therefore, the larger the \( k \) is, the larger the horizon is, and the larger the shadow is.

To extract detailed information from the shadow and connect to astronomical observations, we adopt the two observables defined in [5]: the radius of the shadow \( R_s \) which approximately describes the size of the shadow and \( \delta_s \) which measures its deformation [Figure 3, 58]. \( R_s \) is defined as the radius of a reference circle passing through three points: \( A(\alpha_r, 0), B(\alpha_t, \beta_t) \) and \( D(\alpha_b, \beta_b) \). From the geometry of the shadow, \( R_s \) can be calculated as

\[ R_s = \frac{(\alpha_t - \alpha_r)^2 + \beta_t^2}{2|\alpha_r - \alpha_t|}, \]  
(4.7)

and \( \delta_s \) can be expressed as

\[ \delta_s = \frac{d_s}{R_s} = \frac{|\alpha_p - \tilde{\alpha}_p|}{R_s}, \]  
(4.8)

where \( d_s \) is the distance between the most left position of the shadow and of the reference circle.
For non-rotating black holes \((a = 0)\), the shadow is a perfect circle with radius of \(R_s\). So
\[
\alpha^2 + \beta^2 = \xi^2 + \eta = R_s^2.
\] (4.9)

We show in figure 2 the evolution of the radius \(R_s\) and the distortion parameter \(\delta_s\) with the parameters \(a\) and \(k\) on both sides of the reflection point \(\tilde{k}_0\). We find that when \(k < \tilde{k}_0\), \(R_s\) decreases with the increasing \(k\), but almost does not vary with \(a\) (a constant has been added to visualize the trend of \(R_s\) for different \(a\)), while \(\delta_s\) increases monotonically with the increasing \(k\) and \(a\). When \(k > \tilde{k}_0\), the evolution pattern is contrary to that of \(k < \tilde{k}_0\). This result is consistent with what can be inferred from figure 1.

## 5 Energy emission rate

We assume that, for an observer located at infinity, the black hole shadow approaches to the high energy absorption cross section of the black hole. For a spherically symmetric black hole, the high energy absorption cross section oscillates around a limiting constant value \(\sigma_{lim}\) which is approximately equal to the geometrical cross section of the photon sphere \([73, 74]\) and can be expressed as \([9]\)
\[
\sigma_{lim} \approx \pi R_s^2,
\] (5.1)

where \(R_s\) is the radius of the black hole shadow. We extend this result to the rotating black hole considered in this work, given that the shadow resembles a standard circle even for extreme values of \(a\) and \(k\) (figure 1). The high energy emission rate of the black hole reads as
\[
\frac{d^2 E(\omega)}{d\omega d\tau} = \frac{2\pi^2 \sigma_{lim}}{e^{\omega/T} - 1} \omega^3,
\] (5.2)
where $\omega$ is the frequency of photon and $T$ is the Hawking temperature for the outer event horizon ($r_+$) which can be expressed as

$$T = \frac{r_+^2 f'(r_+)(r_+^2 + a^2) + 2a^2 r_+(f(r_+)-1)}{4\pi(r_+^2 + a^2)^2}. \quad (5.3)$$

If the PFDM is absent ($k = 0$), eq. (5.3) reduces to the regular Kerr black hole

$$T_{\text{Kerr}} = \frac{r_+^2 - a^2}{4\pi r_+(r_+^2 + a^2)}, \quad (5.4)$$

where $r_+ = M + \sqrt{M^2 - a^2}$.

The energy emission rate evolution with the photon frequency $\omega$ is shown in figure 3 for different values of the parameters $a$ and $k$ on both sides of the reflection point $\tilde{k}_0$. It is clear that the peak of the emission increases with the increasing $k$ and shifts to higher frequencies regardless of the range of $k$ specified.

6 Discussion

In this work, we study the rotating black hole shadow in the perfect fluid dark matter by investigating how the shadow varies with the black hole spin $a$ and the perfect fluid dark
Figure 3. Evolution of the black hole energy emission rate with the frequency $\omega$ for different values of the parameters $a$ and $k$ ($k > 0$), with $\tilde{k}_0$ the reflection point. Upper panel: $k < \tilde{k}_0$; lower panel: $k > \tilde{k}_0$.

| $a$  | 0.3   | 0.6   | 0.9   |
|------|-------|-------|-------|
| $k$  | 0.001 | 0.01  | 0.07  |
| $R_s$| 5.175 | 5.049 | 4.536 |
| $\delta_s$ (%)| 0.011 | 0.011 | 0.013 |
| $\theta_s$| 26.4662| 25.8184| 23.1978|

| $a$  | 0.3   | 0.6   | 0.9   |
|------|-------|-------|-------|
| $k$  | 0.8   | 1.5   | 1.8   |
| $R_s$| 3.248 | 3.510 | 3.708 |
| $\delta_s$ (%)| 0.007 | 0.004 | 0.003 |
| $\theta_s$| 16.6099| 17.9493| 18.9604|

Table 1. The observables $R_s$, $\delta_s$ and the angular radius $\theta_s$ for the supermassive black hole Sgr A* at the center of the Milky Way, for different values of perfect fluid dark matter parameter $k$ and black hole spin $a$. $\tilde{k}_0$ is the reflection point. Note that not all decimal points are shown, for purpose of clarity.

matter intensity $k$ ($k > 0$). We show that for an observer located at an infinite distance and in the equatorial plane of the black hole, the shadow is a perfect circle in the non-rotating case ($a = 0$) and a deformed one in the rotating case ($a \neq 0$). The evolution of the shadow size $R_s$ and the distortion parameter $\delta_s$ is dependant of the range of $k$ specified. When $k$
is smaller than the reflection point $\tilde{k}_0$, $R_s$ decreases with the increasing $k$ and $\delta_s$ increases with the increasing $k$, while when $k > \tilde{k}_0$, instead, $R_s$ increases with the increasing $k$ and $\delta_s$ decreases with the increasing $k$. On the other hand, the shadow gets more and more distorted with the increasing $a$ while the size does not vary significantly. Under the assumption that the black hole shadow equals to the high energy absorption cross section, we further calculate the energy emission rate of the black hole in the perfect fluid dark matter. Independent of the range of $k$ chosen, the emission rate increases with the increasing $k$ and the peak shifts to higher frequencies.

In addition, we can estimate the angular radius of the shadow as $\theta_s = R_s M / D$, where $M$ is the black hole mass and $D$ is the distance between the black hole and the observer. [12] further proposed to estimate the angular radius as $\theta_s = 9.87098 \times 10^{-6} R_s (M / M_\odot) (1 \text{kpc} / D) \mu$ as with $M_\odot$ the solar mass. Taking the supermassive black hole Sgr A* at the center of the Milky Way as example, $M = 4.3 \times 10^6 M_\odot$ and $D = 8.3$ kpc. The calculation result is summarized in table 1. We find that to extract the information of the perfect fluid dark matter intensity parameter $k$, an angular resolution of $1 \mu$as will be enough, while for the black hole spin $a$, a resolution of much less than $1 \mu$as will be needed. Nevertheless, both resolutions required are out of the capacity of the current astronomical instruments. Currently, EHT has a resolution of $\sim 60 \mu$as at 230 GHz and is expected to achieve $15 \mu$as by observing at a higher frequency of 345 GHz and adding more VLBI telescopes. The space-based VLBI RadioAstron [75]$^2$ will obtain a resolution of $\sim 1 - 10 \mu$ as in the future. We conclude that future observations with highly improved techniques would be able to achieve the resolution required to observe the perfect fluid dark matter influence on the shadow of the black hole Sgr A*.

When submitting the paper, we were aware of a similar work by [76], in which the complete black hole solution in PFDM with a cosmological constant reported by [71] were explored. We notice that when the cosmological constant is not considered, their result on the black hole shadow is consistent with ours. They in addition studied the effects of the PFDM parameter and the cosmological constant on the deflection angle of light, while we investigated in detail the evolution of the shadow as well as the black hole emission rate.

Acknowledgments

We acknowledge the anonymous referee for a constructive report that has significantly improved this paper. We acknowledge the financial support from the National Natural Science Foundation of China under grants No. 11503078, 11573060 and 11661161010.

References

[1] S. Doeleman et al., *Event-horizon-scale structure in the supermassive black hole candidate at the Galactic Centre*, Nature **455** (2008) 78 [arXiv:0809.2442] [ssSPIRE].

[2] J.L. Synge, *The escape of photons from gravitationally intense stars*, Mon. Not. Roy. Astron. Soc. **131** (1966) 463.

[3] J.P. Luminet, *Image of a spherical black hole with thin accretion disk*, Astron. Astrophys. **75** (1979) 228

[4] J.M. Bardeen, *Timelike and null geodesics in the Kerr metric*, in *Black holes (Les astres oculus)*, C. Dewitt and B.S. Dewitt eds., Gordon and Breach, New York, U.S.A. (1973).

\[http://www.asc.rssi.ru/radioastron/index.html\]
K. Hioki and K.I. Maeda, Measurement of the Kerr spin parameter by observation of a compact object’s shadow, Phys. Rev. D 80 (2009) 024042 [arXiv:0904.3575] [inSPIRE].

A. de Vries, The apparent shape of a rotating charged black hole, closed photon orbits and the bifurcation set $A_4$, Class. Quant. Grav. 17 (2000) 123.

R. Takahashi, Black hole shadows of charged spinning black holes, Publ. Astron. Soc. Jap. 57 (2005) 273 [astro-ph/0505316] [inSPIRE].

N. Tsukamoto, Black hole shadow in an asymptotically-flat, stationary and axisymmetric spacetime: the Kerr-Newman and rotating regular black holes, Publ. Astron. Soc. Jap. 57 (2005) 273 [astro-ph/0505316] [inSPIRE].

S.-W. Wei and Y.-X. Liu, Observing the shadow of Einstein-Maxwell-Dilaton-Axion black hole, JCAP 11 (2013) 063 [arXiv:1311.4251] [inSPIRE].

A. Abdujabbarov et al., Shadow of Kerr-Taub-NUT black hole, Astrophys. Space Sci. 344 (2013) 429 [arXiv:1212.4949] [inSPIRE].

J. Schee and Z. Stuchlík, Optical phenomena in the field of braneworld Kerr black holes, Int. J. Mod. Phys. D 18 (2009) 983 [arXiv:0810.4445] [inSPIRE].

L. Amarilla and E.F. Eiroa, Shadow of a rotating braneworld black hole, Phys. Rev. D 85 (2012) 064019 [arXiv:1112.6349] [inSPIRE].

L. Amarilla and E.F. Eiroa, Shadow of a Kaluza-Klein rotating dilaton black hole, Phys. Rev. D 87 (2013) 044057 [arXiv:1301.0532] [inSPIRE].

C. Bambi, F. Caravelli and L. Modesto, Direct imaging rapidly-rotating non-Kerr black holes, Phys. Lett. B 711 (2012) 10 [arXiv:1110.2768] [inSPIRE].

F. Atamurotov, A. Abdujabbarov and B. Ahmedov, Shadow of rotating non-Kerr black hole, Phys. Rev. D 88 (2013) 064004 [inSPIRE].

M. Wang, S. Chen and J. Jing, Shadow casted by a Konoplya-Zhidenko rotating non-Kerr black hole, JCAP 10 (2017) 051 [arXiv:1701.09451] [inSPIRE].

C. Bambi and N. Yoshida, Shape and position of the shadow in the $\delta = 2$ Tomimatsu-Sato space-time, Class. Quant. Grav. 27 (2010) 205006 [arXiv:1004.3149] [inSPIRE].

Z. Younsi et al., New method for shadow calculations: application to parametrized axisymmetric black holes, Phys. Rev. D 94 (2016) 084025 [arXiv:1607.05767] [inSPIRE].

P.V.P. Cunha et al., Shadows of Einstein–dilaton–Gauss–Bonnet black holes, Phys. Lett. B 768 (2017) 373 [arXiv:1701.00079] [inSPIRE].

S. Dastan, R. Saffari and S. Sorounshfar, Shadow of a Kerr–Sen dilaton-axion black hole, arXiv:1610.09477 [inSPIRE].

A. Abdujabbarov, M. Amir, B. Ahmedov and S.G. Ghosh, Shadow of rotating regular black holes, Phys. Rev. D 93 (2016) 104004 [arXiv:1604.03809] [inSPIRE].

M. Amir and S.G. Ghosh, Shapes of rotating nonsingular black hole shadows, Phys. Rev. D 94 (2016) 024054 [arXiv:1603.06382] [inSPIRE].

A. Saha, S.M. Modumudi and S. Gangopadhyay, Shadow of a noncommutative geometry inspired Ayón Beato García black hole, Gen. Rel. Grav. 50 (2018) 103 [arXiv:1802.03276] [inSPIRE].

A. Grenzebach, V. Perlick and C. Lämmerzahl, Photon Regions and Shadows of Kerr-Newman-NUT Black Holes with a Cosmological Constant, Phys. Rev. D 89 (2014) 124004 [arXiv:1403.5234] [inSPIRE].
[25] V. Perlick, O.Yu. Tsupko and G.S. Bismovatyi-Kogan, Black hole shadow in an expanding universe with a cosmological constant, Phys. Rev. D 97 (2018) 104062 [arXiv:1804.04898] [nSPIRE].
[26] E.F. Eiroa and C.M. Sendra, Shadow cast by rotating braneworld black holes with a cosmological constant, Eur. Phys. J. C 78 (2018) 91 [arXiv:1711.08380] [nSPIRE].
[27] L. Amarilla, E.F. Eiroa and G. Giribet, Null geodesics and shadow of a rotating black hole in extended Chern-Simons modified gravity, Phys. Rev. D 81 (2010) 124045 [arXiv:1005.0607] [nSPIRE].
[28] R. Kumar, B.P. Singh, M.S. Ali and S.G. Ghosh, Rotating black hole shadow in Rastall theory, arXiv:1712.09793 [nSPIRE].
[29] J.R. Mureika and G.U. Varieschi, Black hole shadows in fourth-order conformal Weyl gravity, Can. J. Phys. 95 (2017) 1299 [arXiv:1611.00399] [nSPIRE].
[30] T. Vetsov, G. Gyulchev and S. Yazadjiev, Shadows of black holes in vector-tensor galileons modified gravity, arXiv:1801.04592 [nSPIRE].
[31] U. Papnoi, F. Atamurotov, S.G. Ghosh and B. Ahmedov, Shadow of five-dimensional rotating Myers-Perry black hole, Phys. Rev. D 90 (2014) 024073 [arXiv:1407.0834] [nSPIRE].
[32] A. Abdujabbarov et al., Energetics and optical properties of 6-dimensional rotating black hole in pure Gauss-Bonnet gravity, Eur. Phys. J. C 75 (2015) 399 [arXiv:1508.00331] [nSPIRE].
[33] M. Amir, B.P. Singh and S.G. Ghosh, Shadows of rotating five-dimensional charged EMCS black holes, Eur. Phys. J. C 78 (2018) 399 [arXiv:1707.09521] [nSPIRE].
[34] B.P. Singh and S.G. Ghosh, Shadow of Schwarzschild–Tangherlini black holes, Annals Phys. 395 (2018) 127 [arXiv:1707.07125] [nSPIRE].
[35] F. Atamurotov and B. Ahmedov, Optical properties of black hole in the presence of plasma: shadow, Phys. Rev. D 92 (2015) 084005 [arXiv:1507.08131] [nSPIRE].
[36] V. Perlick, O.Yu. Tsupko and G.S. Bismovatyi-Kogan, Influence of a plasma on the shadow of a spherically symmetric black hole, Phys. Rev. D 92 (2015) 104031 [arXiv:1507.04217] [nSPIRE].
[37] P.V.P. Cunha, C.A.R. Herdeiro, E. Radu and H.F. Runarsson, Shadows of Kerr black holes with scalar hair, Phys. Rev. Lett. 115 (2015) 211102 [arXiv:1509.00021] [nSPIRE].
[38] J. Grover et al., Multiple shadows from distorted static black holes, Phys. Rev. D 97 (2018) 084024 [arXiv:1802.03062] [nSPIRE].
[39] A. Yumoto, D. Nitta, T. Chiba and N. Sugiyama, Shadows of multi-black holes: analytic exploration, Phys. Rev. D 86 (2012) 103001 [arXiv:1208.0635] [nSPIRE].
[40] H. Falcke, F. Melia and E. Agol, Viewing the shadow of the black hole at the galactic center, Astrophys. J. 528 (2000) L13 [astro-ph/9912263] [nSPIRE].
[41] S.C. Noble, P.K. Leung, C.F. Gammie and L.G. Book, Simulating the emission and outflows from accretion disks, Class. Quant. Grav. 24 (2007) S259 [astro-ph/0701778] [nSPIRE].
[42] J. Dexter, E. Agol, P.C. Fragile and J.C. McKinney, The submillimeter bump in Sgr A* from relativistic MHD simulations, Astrophys. J. 717 (2010) 1092 [arXiv:1005.4062] [nSPIRE].
[43] M. Moscibrodzka, H. Falcke, H. Shiozawa and C.F. Gammie, Observational appearance of inefficient accretion flows and jets in 3D GRMHD simulations: application to Sagittarius A*, Astron. Astrophys. 570 (2014) A7 [arXiv:1408.4743] [nSPIRE].
[44] C.-K. Chan et al., The power of imaging: constraining the plasma properties of GRMHD simulations using EHT observations of Sgr A*, Astrophys. J. 799 (2015) 1 [arXiv:1410.3492] [nSPIRE].
A.E. Broderick et al., *Modeling seven years of event horizon telescope observations with radiatively inefficient accretion flow models*, Astrophys. J. **820** (2016) 137 [arXiv:1602.07701] [inSPIRE].

R. Gold, J.C. McKinney, M.D. Johnson and S.S. Doeleman, *Probing the magnetic field structure in Sgr A* on black hole horizon scales with polarized radiative transfer simulations, Astrophys. J. **837** (2017) 180 [arXiv:1601.05550] [inSPIRE].

A. Broderick and A. Loeb, *Testing general relativity with high-resolution imaging of Sgr A*, J. Phys. Conf. Ser. **54** (2006) 448 [astro-ph/0607279] [inSPIRE].

C. Bambi and K. Freese, *Apparent shape of super-spinning black holes*, Phys. Rev. D **79** (2009) 043002 [arXiv:0812.1328] [inSPIRE].

A.E. Broderick, T. Johannsen, A. Loeb and D. Psaltis, *Testing the no-hair theorem with event horizon telescope observations of Sagittarius A*, Astrophys. J. **784** (2014) 7 [arXiv:1311.5564] [inSPIRE].

D. Psaltis, F. Ozel, C.-K. Chan and D.P. Marrone, *A general relativistic null hypothesis test with event horizon telescope observations of the black-hole shadow in Sgr A*, Astrophys. J. **814** (2015) 115 [arXiv:1411.1454] [inSPIRE].

A.A. Abdujabbarov, L. Rezzolla and B.J. Ahmedov, *A coordinate-independent characterization of a black hole shadow*, Mon. Not. Roy. Astron. Soc. **454** (2015) 2423 [arXiv:1503.09054] [inSPIRE].

T. Johannsen et al., *Testing general relativity with the shadow size of Sgr A*, Phys. Rev. Lett. **116** (2016) 031101 [arXiv:1512.02640] [inSPIRE].

D. Psaltis, F. Ozel, C.-K. Chan and D.P. Marrone, *A general relativistic null hypothesis test with event horizon telescope observations of the black-hole shadow in Sgr A*, Astrophys. J. **814** (2015) 115 [arXiv:1411.1454] [inSPIRE].

T. Harko, *Bose-Einstein condensation of dark matter solves the core/cusp problem*, JCAP **05** (2011) 022 [arXiv:1105.2996] [inSPIRE].
[65] S. Tulin and H.-B. Yu, Dark matter self-interactions and small scale structure, Phys. Rept. 730 (2018) 1 [arXiv:1705.02358] [inSPIRE].

[66] V.V. Kiselev, Quintessence and black holes, Class. Quant. Grav. 20 (2003) 1187 [gr-qc/0210040] [inSPIRE].

[67] F. Rahaman et al., Perfect fluid dark matter, Phys. Lett. B 694 (2011) 10 [arXiv:1009.3572] [inSPIRE].

[68] V.V. Kiselev, Quintessential solution of dark matter rotation curves and its simulation by extra dimensions, gr-qc/0303031 [inSPIRE].

[69] V.V. Kiselev, Vector field and rotational curves in dark galactic halos, Class. Quant. Grav. 22 (2005) 541 [gr-qc/0404042] [inSPIRE].

[70] M.-H. Li and K.-C. Yang, Galactic dark matter in the phantom field, Phys. Rev. D 86 (2012) 123015 [arXiv:1204.3178] [inSPIRE].

[71] Z. Xu, J. Wang and X. Hou, Kerr–Anti-de Sitter/de Sitter black hole in perfect fluid dark matter background, Class. Quant. Grav. 35 (2018) 115003 [arXiv:1711.04538] [inSPIRE].

[72] B. Carter, Global structure of the Kerr family of gravitational fields, Physic. Rev. 174 (1968) 1559.

[73] B. Mashhoon, Scattering of electromagnetic radiation from a black hole, Phys. Rev. D 7 (1973) 2807 [inSPIRE].

[74] C.W. Misner, K.S. Thorne and J.A. Wheeler, Gravitation, W.H. Freeman and Company, U.S.A. (1973).

[75] RadioAstron collaboration, N.S. Kardashev and V.V. Khartov, RadioAstron — A telescope with a size of 300 000 km: main parameters and first observational results, Astronomy Reports 57 (2013) 153 [arXiv:1303.5013] [inSPIRE].

[76] S. Haroon et al., Shadow and deflection angle of rotating black holes in perfect fluid dark matter with a cosmological constant, arXiv:1810.04103.