RARE two-point Dixon with dual bandwidths

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Purpose: To investigate the impact of dual readout bandwidths (dBW) in a dual echo fat/water acquisition and describe a dBW-rapid acquisition relaxation enhanced, or turbo spin echo sequence where the concept is used to improve SNR by removing dead times between refocusing pulses and avoiding redundant chemical-shift encoding.

Methods: Cramér-Rao bounds and Monte Carlo simulations were used to investigate a two-point fat/water model where the difference in bandwidths is incorporated. In vivo images were acquired at 1.5 and 3 T with the dBW-rapid acquisition relaxation enhanced, or turbo spin echo sequence. Typical bandwidth ratios were 1:2. SNR was compared with a single bandwidth sequence under identical scan parameters at 3T.

Results: Monte Carlo simulations and Cramér-Rao analysis demonstrate that number of signal averages can be improved with dual bandwidths compared to conventional single bandwidth acquisitions. The dBW-rapid acquisition relaxation enhanced, or turbo spin echo sequence can acquire images with high readout resolutions with well-conditioned sampling. An SNR improvement of 52% was measured, in line with the theoretical gain of 54%.

Conclusions: The proposed dBW-rapid acquisition relaxation enhanced, or turbo spin echo sequence is a highly SNR-efficient two-point rapid acquisition relaxation enhanced, or turbo spin echo sequence without dead times, and can acquire images at higher resolutions than current vendor-supplied alternatives.

KEYWORDS
chemical shift, Cramér-Rao bound, Dixon, Monte Carlo, NSA, RARE

INTRODUCTION

Fat/water separation using chemical shift encoded (CSE) echoes, often referred to as Dixon methods,1 are offered by most vendors today. In particular, rapid acquisition relaxation enhanced, or turbo spin echo (RARE)2 (FSE/TSE)-based CSE methods have been widely adopted in both research and clinical settings, such as head/neck, optic nerve, or even whole-body imaging.3

Where conventional fat saturation falls short at large fields of view, Dixon methods estimate field inhomogeneity and can account for off-resonance frequencies ranging beyond

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that of the RF pulse bandwidth, which is limited by the frequency offset due to chemical shift. Unlike STIR, which alters the image contrast and suffers from signal loss, fat/water separated sequences are an attractive alternative that have the added benefit of being compatible for cases in which a contrast agent has to be administered. In addition, recent publications have shown the diagnostic potential of fat images in differentiation of bone marrow metastases, even suggesting that the fat image from a $T_2$ weighted Dixon acquisition can replace the conventional T1 image.

While the elimination of a fat saturation or inversion pulse reduces the excitation duration, Dixon methods require multiple images acquired with different CSE which can result in a net increase of scan time. However, the additional echoes contribute to the SNR in the fat and water images. The relationship between the SNR, expressed in terms of effective number of signal averages (NSA), and the choice of CSE through dephasing times has been investigated for two and three echoes. The effective NSA equals the number of echoes when dephasing times are optimal, therefore other means of accelerating the sequence could in theory compensate for the multiple echo demand.

When acquiring CSE echoes in RARE imaging, CPMG conditions impose further demands on the sequence design. Since refocusing RF pulses must be temporally equidistant, spin echoes occur exactly midway between each pair, and as a consequence conventional spin echoes carry no CSE. To maintain CPMG while encoding chemical shift, readouts can be temporally shifted. In this case, not only is the RF echo spacing prolonged by twice the desired shift, but also dead times are introduced where no sampling occurs. For instance, given a readout duration of 4 ms, only 65% of the available acquisition time is spent sampling as 2.3 ms dead times are necessary at 3T to achieve opposed phase sampling.

An alternative is to apply an odd number of readout gradients with alternating polarity between each refocusing pulse, that is, a GRASE readout used for CSE. A major benefit of GRASE CSE is that no additional refocusing pulses are required and, therefore, no scan time penalty is incurred. This technique is currently offered as a clinical product sequence by several vendors. The case of three echoes where the second echo is in-phase is known as the fast spin-echo triple-echo Dixon (fTED) technique. While no dead times are present in the fTED sequence, other than non-sampled ramps, the use of multiple gradient lobes limits the spatial resolution due to gradient amplitude restrictions and peripheral nerve stimulation constraints. The sampling is also redundant as readouts mirrored around the spin echo share the same CSE, resulting in an effective NSA smaller than the number of echoes.

Wang et al recently presented a two-point single bandwidth sequence derived from fTED where partial Fourier is used to allow flexible RF echo-spacings while still acquiring in/opposed phase echoes. This sBW-RARE sequence enables higher spatial resolution, because more gradient time is available and partial Fourier allows smaller k-space coverage. However, dead times are present during the third gradient lobe that is used as a dephaser to maintain CPMG conditions.

The dual bandwidth technique has been used to partially remove dead times from an interleaved RARE sequence. In spin echo sequences, dead times have been eliminated by utilizing dual bandwidths. Here, we introduce a dual readout bandwidths (dBW-RARE) approach that aims to achieve high readout resolution, completely remove dead times and avoid redundant CSE while still complying with CPMG conditions. The problem of choosing optimal readout waveforms to minimize the noise in the estimates is also investigated, where a modified POCS technique was used to allow an additional degree of freedom. We demonstrate the technique for $T_2$ and PD weighted images on 1.5T and 3T systems.

This paper also investigates the noise properties of the inverse problem associated with fat/water imaging acquired with dual bandwidths. The use of two echoes for real-valued fat/water estimates with intrinsic off-resonance correction is explored. Closed-form Cramér-Rao bounds (CRB) and effective NSA are calculated and compared against Monte Carlo simulations. NSA under a single-species approximation is proposed as an option for selecting dephasing times where both fat and water images are equally weighted.

## Theory

### Dual bandwidths

Dual bandwidths can increase the scan efficiency by removing sequence dead times, but result in an uneven noise distribution in the echoes. This will affect the noise performance of the water and fat estimates, which is clearly illustrated in the following example: Consider two echoes acquired in and opposed phase, with weights $w_1$ and $w_2$, respectively. Let $w_1$ increase at the cost of $w_2$ to the point where no weight is applied to the second echo, that is, no time is spent sampling it. This results in a high SNR in the in-phase echo, while the opposed-phase echo contains no signal. As a result, the in-phase echo has high SNR, but separating water and fat is an ill-posed problem as there is no CSE and estimates will contain pure noise given an unbiased estimator. We use Cramér-Rao bound analysis for investigating in detail how the varying signal uncertainties from $w_1$ and $w_2$ propagate into the fat and water estimates.

#### 2.1 Cramér-Rao bounds

The two-point signal model acquired with dephasing times $t_1$ and $t_2$ is given by
where $\psi$ incorporates all nonchemical-shift sourced off-resonances, such as B0-inhomogeneities. The phase immediately following excitation is $\phi$, here modeled as equal for the fat ($F$) and water ($W$) species. The model matrix $A$ describes the phase accrual effect from chemical shift, where it is assumed that water is on resonance. The off-resonance due to chemical shift is described by $\omega$, and is known a priori. For simplicity, a single-peak model is used here but it is straightforward to perform this analysis with multiple fat resonances. Note that the dephasing times are relative to the spin echo, as described in Figure 1, and can be negative. To incorporate the effect of sampling the echoes with different weights, the diagonal matrix $A$ is introduced. Scaling the signal instead of the noise generates a model where the signal variance is equal in both echoes. Equation (1) contains four unknown real-valued parameters $W$, $F$, $\phi$, and $\psi$, composing an exactly determined system given the two acquired complex-valued echoes $S$.

The CRB express the minimum output variance in the estimates that can be achieved from an unbiased estimator. The Fisher information matrix $F$ is inversely related to the CRB in the sense that the diagonal elements of $F^{-1}$ are the CRB of the corresponding parameter. Under the assumption of complex Gaussian noise, $F$ can be derived using the Slepian-Bangs formula:

$$F = \frac{1}{\sigma^2} \sum_{n=1}^{2} \left( \text{Re} \left\{ \left( \frac{\partial S_n(\theta)}{\partial \theta} \right) \left( \frac{\partial S_n(\theta)}{\partial \theta} \right)^H \right\} \right),$$

where $\theta = [W, F, \phi, \psi]^T$ is the real-valued parameter vector and $\sigma^2$ is the variance of the measured signals. The partial derivatives of $S_n$ are

$$\frac{\partial S_n(\theta)}{\partial \theta} = W_n e^{i\psi_n + \phi} \left[ \begin{array}{c} 1 \\ -i (W + e^{i\omega t_n} F) \end{array} \right].$$

Note that the phasor $e^{i(\psi_n + \phi)}$ is cancelled by the Hermitian transpose in Equation (2). Using Cramer’s rule, we can find the element at row $r$ and column $c$ of $F^{-1}$:

$$F^{-1}_{r,c} = \frac{M_{r,c}}{\det(F)},$$

where $M_{r,c}$ is the minor of $F^T$, that is, the determinant of the matrix formed by removing row $c$ and column $r$ after transposing $F$. We seek the diagonal elements of $F^{-1}$. Thus, only the principal minors ($r = c$) need consideration. The suffix will instead indicate the model parameter, that is, $\text{CRB}_W = \frac{M_{r,c}}{\det(F)}$. $\text{CRB}_F$ can be calculated from $\text{CRB}_W$ by swapping $W$ and $F$, showing the conjugate nature of $\text{CRB}_F$ and $\text{CRB}_W$ with respect to the fat:water ratio, as previously described.

### 2.2 Single species approximation

Equation (3) reveals that $F$ is dependent on the relative fat and water content. Indeed, expanding Equation (4) results in
a lengthy quartic polynomial of $F$ and $W$. For clarity, we explicitly add the relative water and fat content as arguments to the CRB equations, that is, $\text{CRB}_{W}(W, F)$. By assuming that a voxel contains either water or fat, calculations of the lower bounds for water are simplified to

$$\text{CRB}_{W}(1, 0) = \sigma^2 w^2 \cos^2 (\omega t_1) + w^2 \cos^2 (\omega t_2)$$

and

$$\text{CRB}_{W}(0, 1) = \sigma^2 \frac{w^2 + w^2}{w^2_1 w^2_2 (\cos (\omega t_1) - \cos (\omega t_2))^2}$$

The effective number of signal averages, NSA, is defined as the variance of the measured signal, $\sigma^2$, divided by the variance of the estimate. As previously stated, $\text{CRB}_{W}(W, F) = \text{CRB}_{F}(F, W)$. The average NSA under the single species approximation is, therefore, equal in the fat and water image and the subscript can be dropped:

$$\text{NSA}_{ss} = \frac{\sigma^2}{2 \times \text{CRB}(1,0)} + \frac{\sigma^2}{2 \times \text{CRB}(0,1)} = \frac{w^2_1 w^2_2 (\cos (\omega t_1) - \cos (\omega t_2))^2 \left( w^2_1 \cos^2 (\omega t_1) + w^2_2 \cos^2 (\omega t_2) + 2 \right)}{2 \left( w^2_1 + w^2_2 \right) \left( w^2_1 \cos^2 (\omega t_1) + w^2_2 \cos^2 (\omega t_2) \right)}$$

In the conventional single-bandwidth case were the two echoes are acquired in separate acquisitions, NSA_{ss} \in [0, 2]. \text{NSA}_{ss} = 1$. Equation (7) simplifies to

$$\text{NSA}_{ss,\text{BW}} = \frac{(\cos (\omega t_1) - \cos (\omega t_2))^2 (\cos^2 (\omega t_1) + \cos^2 (\omega t_2) + 2)}{4 (\cos^2 (\omega t_1) + \cos^2 (\omega t_2))}.$$  

(8)

3 | METHODS

3.1 | Monte Carlo simulations

Monte Carlo simulations were carried out to verify the results from the Cramér-Rao calculations above. The forward model BAAX was applied once and 1000 samples were generated for each dephasing time pair $(t_1, t_2)$ and set of weights by adding complex noise. Noise was added to the real and imaginary channels, drawn from a normal distribution of zero mean and standard deviation $\sigma = 0.01$. Non chemical-shift off-resonances were estimated by minimizing the cost function $J$ (as described in Ref. 24) for 50 candidates, placed on a 1 Hz grid with the center point around the ground truth of 40 Hz:

$$J(\psi) = \| \text{Se}^{-i\phi} - \text{B}(\psi) \text{LRe} \left( \text{L}^{-1} \text{B}(\psi)^H \text{Se}^{-i\phi} \right) \|_2^2,$$

(9)

where $L = \Delta A$ and the $H$ superscript denotes the conjugate transpose operator. The initial phase $\phi$ was calculated for each field map candidate $\psi$ according to Bydder et al.

$$\phi = \frac{1}{2} \angle (L^H B(\psi)^H S) \text{Re} (L^H L)^{-1} (L^H B(\psi)^H)$$

(10)

Given the estimated off-resonance $\hat{\psi}$ and $\hat{\phi}$ for each sample, real-valued estimates of $X$ were calculated using weighted least-squares:

$$\hat{X} = \begin{bmatrix} L & L^* \end{bmatrix}^* \left[ \text{SB}(\hat{\psi}) e^{-i\hat{\phi}} \right]$$

(11)

where the + and * superscript denotes the pseudoinverse and conjugate operator, respectively. The NSA of the species is calculated from the signal variance divided by the variance of the estimate $\rho \in [W, F]$ of interest:

$$\text{NSA}_{\rho} = \frac{\sigma^2}{\text{Var} (\hat{\rho})}.$$  

(12)

3.2 | dBW-RARE pulse sequence

The single readout trapezoid in conventional RARE imaging is replaced with two trapezoids, centered around the spin echo event. The coupled readout trapezoids have equal area but differ in amplitude and duration. Data are sampled on the plateau of the low amplitude gradient. The high amplitude gradient is sampled when the amplitude is equal to or larger than the low amplitude gradient in order to avoid ramp compensation blips. Since an even number of bipolar readout gradients are used, the net magnetic moment of the readout block is zero. The refocusing pulse following each readout will mirror the k-space trajectory around the DC point, and the sign of the readout block would need to be alternated in order to achieve Cartesian sampling. In our implementation, readout dephaser and phasers instead surround the coupled readout trapezoids. This reduces eddy current effects and associated stimulated echo pathways while still complying with CPMG conditions. An illustration of the sequence is shown in Figure 1, where the redundant sampling associated with a naive single bandwidth approach is shown together with sBW-RARE. Also seen in the figure is the improved sampling by allowing a partial Fourier acquisition. A more detailed description of the sequences is available as Supporting Information, showing all boards and gradient waveforms together with magnetic moments and slew rate plots.
3.2.1 Optimally coupled readouts

A plot of Equation (8) is shown in Figure 2 where the CSE phase is used as a surrogate for dephasing times $t_1$ and $t_2$, avoiding the field strength dependent angular frequency $\omega$ of fat, with equal weights. Overlaying the NSA are the dephasing times of each sample pair, visualized as lines with symbols marking the k-space center for the sampling strategies shown in Figure 1. The line length is equal to the available acquisition time $t_a$ for all but sBW-RARE. A naive single bandwidth acquisition is restricted to the ill-conditioned diagonal, a result of the redundant sampling. sBW-RARE can acquire well-condition samples, but at the cost of shorter sampling duration as indicated by the line length. The sampling line slope is changed when the bandwidth differs, improving the CSE compared to the naive case. The k-space center point can be translated along the line to further improve the sampling efficiency with partial Fourier acquisitions.

It is commonly stated that three-point Dixon techniques have a maximum NSA of 3, while a double echo acquisition has a capped NSA of 2. This is suitable for multiple acquisitions or multi-echo gradient-echo sequences since additional readout windows increase the total sampling duration. With the idea of replacing a single readout trapezoid with two bipolar gradients, maximum NSA should be 1. We then use the user supplied bandwidth $\tau_p$ and readout matrix size $N_t$ as a proxy for determining the available acquisition time $t_a = N_t / \tau_p$ (Figure 1). Once $t_a$ is known, the question of selecting the relative gradient amplitude arises. We seek to qualitatively separate the fat and water signal in high resolution applications, where it is reasonable to assume that most voxels contain either water or fat, hence Equation (7) is suitable as the objective function for determining the optimally coupled readouts. These are found using a brute-force search where the partial Fourier factor and the fraction of the first readout duration $f = w_1 / (w_1 + w_2)$ are varied. NSA_{ss} is to be interpreted as the sampling efficiency compared to a conventional single readout trapezoid acquiring a non-CSE echo during the same $t_a$ that is, NSA_{ss} \in [0,1]. For this to hold, weights must be chosen such that the combined signals have unit variance, ie, $w_1^2 + w_2^2 = 1$, and can be expressed in terms of $f$:

$$w_1 = \frac{f}{\sqrt{(1-f)^2 + f^2}}$$
$$w_2 = \frac{1-f}{\sqrt{(1-f)^2 + f^2}}$$

Since sampling is switched off to some extent during gradient switching in dBW-RARE, we add an additional penalty term to Equation (7) where the non-sampled fraction of $t_a$ is included.

3.2.2 Data acquisition

Informed consent was obtained from all three volunteers in accordance with the institutional review board policy.

All 3T images were acquired on a Signa Premier (GE Healthcare). Images of the optic nerve were also acquired on a 1.5T Optima 450w (GE Healthcare) using a 16-channel head/neck Rx coil. PD-weighted MSK images were acquired with dedicated coils from GE Healthcare: 8 channel Rx foot/ankle, 18 channel Rx shoulder, 18 channel Tx/Rx knee. Acquisition parameters are listed in Table 1.

3.2.3 SNR measurement

The same scan parameters were used for both sBW- and dBW-RARE, resulting in different dephasing times $t_1$ and $t_2$ as seen in Table 1. For both sequences, each phase encoding was acquired twice in an interleaved manner, giving two fully sampled images with a temporal resolution of TR. This method
was chosen over the alternative of doubling the sampling rate\textsuperscript{26} as the reference method had aliasing in the frequency encoding direction due to the high readout amplitude. As a result, inter-TR motion such as flow affects the measurements. The two images were fat/water separated independently from each other, and the resulting estimates were subtracted. The difference images were squared, followed by a division of 2 to account for the overestimation from subtracting two measurements. Noise maps were calculated by subsequent smoothing with a 6 mm circular filter. SNR maps were formed by dividing the fat/water estimates from the first measurement with the corresponding noise map. SNR was extracted from ROIs placed in the orbital fat and gray matter, avoiding regions with flow. The same ROIs were used for sBW- and dBW-RARE. A theoretical SNR gain was calculated from the square root of the ratio between sampling durations.

### RESULTS

#### 4.1 Noise analysis

The NSA of the water and fat estimates from the Monte Carlo simulations were compared visually with the corresponding analytical NSA presented in Figure 3, showing excellent agreements with the simulations. The Monte Carlo simulation results are available as supplementary material.

Figure 3A,B shows NSA\textsubscript{W} at 0% and 100% fat ratio in the single bandwidth case, that is, $w_1 = w_2 = 1/\sqrt{2}$ for dephasing time pairs equivalent to a phase shift between 0 and $2\pi$ radians. Note that the NSA\textsubscript{W} = NSA\textsubscript{F} if the fat and water content are swapped, hence not shown. The single-species approximated NSA\textsubscript{SS} is also shown.

The bottom row of Figure 3 shows how NSA\textsubscript{W} is dependent on $f$ and $t_2$ with the first echo being acquired in-phase. This is evaluated at 0 and 100% fat ratio, as well as with the single-species approximation. Results show that the effective NSA of water can be improved with dual band-widths if the second echo is not acquired exactly opposed phase.

#### 4.2 dBW-RARE

##### 4.2.1 Optimally coupled readouts

The objective function in Equation (7) is shown in Figure 4 for three different $t_{ar}$ simulated with infinite slewrate.
Ill-conditioning is seen at $f = 1/2$. Higher $\text{NSA}_{ss}$ can be achieved when $t_2$ is increased, with two optima mirrored around $f = 1/2$. Partial Fourier strongly increases $\text{NSA}_{ss}$. An interactive version of Figure 4 is available at the online repository, where the unweighted version can be seen for comparison.

### 4.2.2 In vivo experiments

Fat and water were successfully separated in all slices in all datasets. The acquired echoes shown in the figures have been data synthesized using the POCS routine. First and second echo images of the 1.5 T examination of the orbits are shown in Figure 5, together with the reconstructed water and fat images. As expected, the noise level is larger in the second echo due to its short readout duration. Also shown is the set of dephasing times for the two echoes, and shortening of $t_2 - t_1$ from partial Fourier. High resolution PD-weighted knee, ankle, and shoulder images are presented in Figures 6-8, respectively. The knee dataset was also reconstructed using a single-peak fat model, available as supplementary material.

### 4.2.3 SNR comparison

Images from the SNR measurements and the ROIs are shown in Figure 9. The extraocular muscles and the optic nerve are well separated from their surrounding fat tissue. SNR in the fat tissue is increased by 51% with dBW-RARE compared to SBW-RARE. In gray matter, a similar increase of 52% is seen in dBW-RARE. This is in line with the theoretical gain of 54% from the increased sample duration in dBW-RARE.

### 4.3 Supplementary material

In the spirit of reproducible research, we provide a repository on Github at henricryden.github.io/dbwRARE. The repository hosts data from all MR acquisitions, source code in Python for generation of the provided figures, as well as Monte Carlo simulations. Details from the SNR calculations are provided. A full plot of dBW and sBW-RARE sequence, and an interactive plot of Figure 4 showing the
associated gradient pairs and fat dephasing of the search space is also available. The in vivo knee dataset reconstructed with single- and multi-peak fat models is also presented.

**Discussion**

Pineda et al discovered in their three-point Cramér-Rao analysis that an NSA is maximized by equally spacing the
fat/water dephasing in the unit circle around a phase shift of $\pi/2$. This discovery of asymmetric encoding was proven optimal for all fat:water ratios.\textsuperscript{11} In two-point models, encoding is known to be optimal in an in-phase and opposed-phase acquisition.\textsuperscript{8-10} The Cramér-Rao bound analysis presented here, shown in Figure 3, states that in- and opposed phase images should be acquired with identical weights. However, for every pair of dephasing times other than in- and opposed phase, dual bandwidths improve the effective NSA. The effect is particularly strong in voxels containing the single species of interest. In other words, NSA in water-only regions in the water image can be increased by acquiring signals with unequal weights for some dephasing time pairs. A special case of this is seen when the second echo is acquired with a phase shift of $\pi/2$. From Figure 3D, it is evident that the NSA peaks when the quadrature echo is acquired with a dominant weight. The opposite case where more time is spent on the in-phase signal is not efficient.

The assumption of voxels containing either water or fat holds for high resolution imaging for MSK and optic nerve imaging shown here. However, this approximation does not hold in general. For instance, liver diseases cause a moderate...
fat infiltration\textsuperscript{29} and Equation (7) might not result in optimal echo times. It should be noted though, that in-phase and opposed-phase sampling is the most efficient for any given fat fraction.\textsuperscript{10} We believe that the single species approximation is a pragmatic option when selecting dephasing times where both fat and water images are equally optimized. Choosing dephasing times automatically was necessary in this application, but can be applied to other sequences in order to simplify workflow and guaranteeing optimal encoding given other sequence parameters. While Equation (7) is equally applicable to gradient echo imaging, it remains to be investigated how these sequences might benefit from dual bandwidth sampling.

Cramér-Rao bound analysis yields lower bounds of the variance in fat and water from an unbiased estimator. Since most fat/water separation methods incorporate spatial smoothing to resolve $\psi$ and $\phi$ maps, some bias is introduced. As evident from Equation (3), the NSA of fat and water are not dependent of the relative content if the nonlinear terms are known. Moreover with a linear model, optimal encoding is achieved when $t_1 - t_2$ corresponds to a phase shift of $\pi$, regardless of $t_1$.\textsuperscript{9} While Cramér-Rao analysis gives powerful insights of the inverse problem, this bias should be kept in mind when selecting image parameters.

Since the fTED technique fixes the gradient echo spacing to $\pi$ with the second echo in phase, the NSA from CSE is constantly 8/9.\textsuperscript{11} Estimating the field map from the three bipolar images is not trivial, and the suggested approach is to separate the problem. Two estimations of $\psi$ are performed separately: One on the first and second echo, and another on the second and third echo.\textsuperscript{16} The two field maps are subsequently combined. While successful, the field map estimation is biased and Cramér-Rao analysis is not entirely applicable.

The objective functions shown in Figure 4 have several interesting properties. The plots are symmetric around $f = 1/2$, which is expected since the model does not

**FIGURE 7**  Sagittal dBW-RARE PD-weighted images of the ankle at 3T. The cervical ligament in the sinus tarsi (white arrows) as well as the plantar calcaneocuboid ligament (green arrows) are clearly depicted. Images were acquired with $t_o = 4.0$ ms

**FIGURE 8**  Axial dBW-RARE PD-weighted images of the shoulder at 3T. The fat/water images are free from swaps, even in the challenging low signal region near the lungs. The articular cartilage at the glenoid is visible in the water image (arrow). Images were acquired with $t_o = 4.5$ ms
include transverse decay, hence $t_1$ and $t_2$ can be swapped in Equation (7). The dephasing time difference $t_2 - t_1$ is solely dependent on the partial Fourier fraction, which corresponds to $\pi$ radians dephasing at dashed lines. Gradient couples at red lines carry identical CSE, posing an ill-conditioned inverse problem. At a partial Fourier fraction of 0.5, this is expected since the k-space center acquisition of the first and second trapezoid effectively coincides. When $t_a$ is large enough to cover more than 1 period of fat/water dephasing, there are more than two bandwidth ratios available to choose with equal dephasing times. Incorporating the penalty of dual bandwidths, it is clear that the two gradient pairs closest to $f = 1/2$ are much better conditioned and the only pairs that need consideration.

To achieve a good conditioning of the inverse problem, in- and near opposed phase echoes are desired for two-point acquisitions. However, an opposed phase image contains high spatial frequency information in its phase, breaking the underlying conjugate mirror property that conventional partial Fourier methods are based on. A modified POCS method was used in this work, where the real valued property is instead enforced on the estimated fat and water signals. As $\phi$ and $\psi$ are estimated from the symmetrically sampled k-space, sufficient spatial resolution is required for them to be adequately resolved. We chose to limit the search space to partial Fourier fractions above 0.7 in this work as loss in image sharpness and Gibbs ringing artifacts appeared otherwise.

Since dBW-RARE already uses partial Fourier sampling along the frequency encoding direction, partial Fourier in the phase encoding directions cannot be used for acceleration. Additionally, $t_a$ needs to be sufficiently long to acquire two echoes with well-conditioned CSE. This might be limiting when applied to single-shot or 3D applications. While dBW-RARE is a Cartesian sequence, the proposed dual bandwidth readout is equally applicable to PROPELLER acquisitions. The added flexibility of different readout bandwidths can also be beneficial in radial bipolar sequences to achieve better CSE.

We pushed the limits of the dBW-RARE technique by acquiring MSK images with in-plane resolutions of 0.5 mm at 3T with short $t_a$ around 4 ms, seen in Figures 6-8. The synovial fluid and articular cartilage in Figure 6 are difficult to distinguish in the echoes, but appear well delineated in the water image. The epiphysis and growth plate are clearly distinguishable in the fat image. In low-signal regions near the lungs in Figure 8 where off-resonances are often large, the fat is correctly separated. The ankle images in Figure 7 show high anatomical details, with clear depiction of ligaments. A loss of image sharpness, particularly in the growth plate, was seen when using a single-peak fat model and is available in the supplementary material.

The 51–52% SNR gain from dBW-RARE compared to sBW-RARE is in agreement with the theoretical improvement of 54% from the sampling efficiency following the removal of dead times. sBW-RARE was chosen over fTED as the comparison with dBW-RARE can be performed with identical scan parameters without introducing reconstruction dependent bias.

The available acquisition time $t_a$ does not cover the entire duration available for gradients, as evident in Figure 1. The prephaser and dephaser surrounding the readout do not introduce any dead times however, as they are played out simultaneously with the crushers and phase encoding gradients. This holds as long as the prescribed pixels are square or the required crusher moment is sufficiently large.

6 | CONCLUSION

This paper has expanded the two-point Dixon model to incorporate the effects from acquiring images with different bandwidths. We have described the noise performance of the model when the introduced sampling weights are varied, and reported that an increase in NSA can be achieved when in-phase and opposed-phase echoes are not acquired.
Additionally, closed-form expressions of the NSA under a proposed single-species approximation, derived from Cramér-Rao analysis, has been presented and verified with Monte Carlo simulations.

The proposed dBW-RARE sequence is a two-point RARE sequence without dead times, where well-conditioned sampling is achieved through the use of dual bandwidths, further improved with partial Fourier. The choice of coupled trapezoids is automatically selected using the derived NSA expressions, guaranteeing optimal encoding given other sequence parameters without any user input. dBW-RARE can successfully acquire images with clinically relevant sequence parameters without additional scan time, at higher resolutions compared to fTED, and more efficiently compared to sBW-RARE.

CONFLICT OF INTEREST
Tim Sprenger is employed by GE Healthcare. Stefan Skare receives research support from GE Healthcare.

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SUPPORTING INFORMATION

Additional Supporting Information may be found online in the Supporting Information section.

dBW-RARE proton density weighted fat and water images reconstructed with a multi-peak (A), and single-peak (B) fat model. The image intensity inside the green box has been amplified to better visualize the reduced residual fat in the multi-peak reconstructed water image. The single-peak water image contains artifacts not present in the multi-peak reconstructed images, as indicated by the white arrows, and the growth plate is better delineated in a (blue arrow). Images were acquired with $t_a = 4.2$ ms

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