The exact parity symmetric model and big bang nucleosynthesis

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Abstract

The assumption of exact, unbroken parity symmetry leads directly to a simple predictive resolution of the atmospheric and solar neutrino puzzles. This is because the existence of this symmetry implies the existence of a set of mirror neutrinos which must mix maximally with the known neutrinos if neutrinos have mass. The maximal mixing of the electron neutrino with the mirror electron neutrino with $3 \times 10^{-10} \text{eV}^2 \lesssim |\delta m^2| \lesssim 10^{-3} \text{eV}^2$ leads to a predicted reduction of the solar neutrino flux by a factor of 2, which is in quite good agreement with the experiments. The maximal mixing of the muon neutrino with the mirror muon neutrino with $|\delta m^2| \simeq 10^{-2} \text{eV}^2$ also solves the atmospheric neutrino puzzle. We show that there is a significant range of parameters where these solutions are not in conflict with standard Big Bang Nucleosynthesis when the creation of lepton asymmetry due to neutrino oscillations is taken into account.
It has been known for a long time\footnote{1} but not widely appreciated that it is possible to build a phenomenologically consistent gauge model which has a parity symmetry which need not be broken at all. In order to achieve \emph{unbroken} parity symmetry it is necessary to double the number of fermions and the gauge symmetry. However, while the number of particles is doubled the number of parameters is not significantly increased (only two additional parameters in the minimal model with massless neutrinos)\footnote{2}. If the neutrinos in the exact parity symmetric model have mass, and if mass mixing between ordinary and mirror neutrinos exists, then the mass eigenstate neutrinos must also be parity eigenstates. Because parity transformations simply interchange ordinary and mirror neutrinos, the mass eigenstate fields will be maximal combinations of ordinary and mirror weak eigenstates\footnote{2}. This result holds independently of the details of the origin of the neutrino masses.

The mirror neutrinos are essentially sterile as far as ordinary interactions are concerned. However unlike totally sterile neutrinos (such as right-handed neutrino gauge singlets) mirror neutrinos interact amongst themselves and with mirror particles with interactions of the same form and strength as ordinary neutrinos interact with ordinary particles. One advantage of mirror neutrinos over conventional sterile neutrinos is that the mirror gauge symmetry provides an excellent reason as to why they are not very heavy\footnote{2, 3}. If the parity symmetry connecting the ordinary and mirror worlds is unbroken, then the mirror neutrinos are set by the same scale as ordinary neutrinos\footnote{2}.

We will denote the three mirror neutrinos by \( \nu'_{e}, \nu'_{\mu}, \nu'_{\tau} \).

With small intergenerational mixing it follows from the unbroken parity symmetry of the model that \( \nu_{e} \) and \( \nu'_{e} \) will be approximately maximal mixtures of two mass eigenstates \( \nu_{1} \) and \( \nu_{2} \). Similarly, \( \nu_{\mu} \) and \( \nu'_{\mu} \) will each be approximately maximal mixtures of two mass eigenstates as will \( \nu_{\tau} \) and \( \nu'_{\tau} \). We will denote the \( \delta m^2 \) describing these maximal oscillations by \( \delta m^2_{e'e'} \), \( \delta m^2_{\mu'\mu'} \) and \( \delta m^2_{\tau'\tau'} \) respectively.

The maximal mixing of \( \nu_{e} \) and \( \nu'_{e} \) will solve the solar neutrino problem for the large range of parameters \footnote{4, 5, 6}$\begin{equation}
3 \times 10^{-10} \, eV^2 \lesssim |\delta m^2_{e'e'}| \lesssim 10^{-3} \, eV^2.
\end{equation}$

The maximal mixing of the electron neutrino with the mirror electron neutrino with \( \delta m^2_{e'e'} \) in the above range leads to a predicted reduction of the solar neutrino flux by a factor of 2 which is in quite good agreement with the experiments. (See Ref. \footnote{4} for a detailed comparison of the predictions of the exact parity model with the solar neutrino data\footnote{4}).

The deficit of atmospheric muon neutrinos can be explained if there are \( \nu_{\mu} - \nu'_{\mu} \) oscillations with \( \sin^2 2\theta_0 \gtrsim 0.5 \) and \( 10^{-3} \, eV^2 \lesssim |\delta m^2_{\mu'\mu'}| \lesssim 10^{-1} \, eV^2 \footnote{8, 9} \). The best fit\footnote{8} occurs for \( \sin^2 2\theta_0 \simeq 1 \) and

\begin{equation}
|\delta m^2_{\mu'\mu'}| \simeq 10^{-2} \, eV^2.
\end{equation}
The exact parity symmetric model is also compatible with the LSND signal\cite{10, 2}.

A potential problem with any model that has additional light degrees of freedom is that these extra states can contribute to the energy density of the early Universe and spoil the reasonably successful Big Bang Nucleosynthesis (BBN) predictions. This presents a problem for the exact parity symmetric model because it can potentially lead to a doubling of the energy density at the time of nucleosynthesis (which is equivalent to about 6 additional neutrinos). However, it is plausible that an initial macroscopic asymmetry between ordinary and mirror matter might exist, as can be arranged through the inflationary scenario proposed in Ref.\cite{11} (for example). Even if ordinary matter dominates mirror matter immediately after the Big Bang, the oscillations between the ordinary and mirror neutrinos might be expected to bring the mirror sector into equilibrium with the ordinary particles\cite{12}. For maximally mixed ordinary - mirror neutrinos, the following BBN bounds have been obtained\cite{13} assuming that the lepton number asymmetry could be neglected:

\begin{align}
|\delta m_{ee}^2| &\lesssim 10^{-8}\, eV^2, \\
|\delta m_{\mu\mu}^2|, |\delta m_{\tau\tau}^2| &\lesssim 10^{-6}\, eV^2.
\end{align}

(4)

With the parameter choices Eq.(2, 3), there is a potential conflict with the naive BBN bounds Eq.(4). However, these bounds do not hold if there is an appreciable lepton asymmetry in the early Universe for temperatures between 1 – 30 MeV\cite{14}. Remarkably, it turns out that ordinary-sterile neutrino oscillations can by themselves create lepton number\cite{13, 16, 17}. Recently, we have shown\cite{17} that the lepton number generated by ordinary - sterile neutrino oscillations can allow the bounds in Eq.(4) to be evaded by many orders of magnitude. Indeed, the bounds can be relaxed sufficiently so that the ordinary - sterile neutrino oscillation solutions to the solar and atmospheric neutrino anomalies do not significantly modify BBN.

The purpose of this paper is to study the special case where the sterile neutrinos are mirror neutrinos\cite{2}. The mirror neutrinos are essentially sterile when probed by ordinary matter, however they do have significant self interactions. There are two main effects of the self interactions in the early Universe. First, the effective potential governing ordinary - mirror neutrino oscillations will gain a contribution from the interactions of the mirror neutrino with the background. Second, the mirror weak interactions can bring the mirror neutrinos into equilibrium with the other mirror particles. This effect is quite important because it will significantly modify the momentum distribution and the number density of mirror neutrinos compared with the case of sterile neutrinos. In Ref.\cite{17}, we showed in detail how lepton number creation can evade the naive bounds of Eq.(4) and we determined the required parameter space for strictly sterile neutrinos. In the present work we consider the case of mirror neutrinos. For this case it turns out that the effect of the mirror neutrino self interactions is to significantly enlarge the allowed region of parameter space compared to the case of strictly sterile neutrinos.

There are many independent $\delta m^2$, $\sin^2 2\theta_0$ parameters. We will need to make some assumptions otherwise we cannot say anything definite. We will assume that
intergeneration mixing is small and that
\[ m_{\nu_e}, m_{\nu'_e} < m_{\nu_\mu}, m_{\nu'_\mu} < m_{\nu_\tau}, m_{\nu'_\tau}. \] (5)

The assumption of small mixing between the generations is quite natural in our opinion in view of the situation with quarks. This assumption is also supported by the LSND experiment[10], which claims to have measured a small mixing between the muon and electron anti-neutrinos. The above assumption for the mass ranges of the neutrinos is also quite natural in view of the mass hierarchy between generations for the quarks and leptons. It is also compatible with the three experimental neutrino anomalies (solar, atmospheric and LSND). [Of course it is not the only possibility, but represents our best guess given the existing information].

For ordinary - sterile or ordinary - mirror neutrino two state mass mixing, the weak-eigenstates \((\nu_\alpha, \nu_s)\) will be linear combinations of two mass eigenstates \((\nu_a, \nu_b)\):

\[
\nu_\alpha = \cos \theta_0 \nu_a + \sin \theta_0 \nu_b, \quad \nu_s = -\sin \theta_0 \nu_a + \cos \theta_0 \nu_b. \] (6)

Note we will always define \(\theta_0\) in such a way so that \(\cos 2\theta_0 \geq 0\). We also adopt the convention that \(\delta m^2 \equiv m_b^2 - m_a^2\). Hence with this convention \(\delta m^2\) is positive (negative) provided that \(m_b > m_a\) (\(m_b < m_a\)).

Ordinary - sterile or ordinary - mirror neutrino oscillations can generate significant lepton number in the early Universe[15, 16, 17]. The origin of this phenomenon can be traced to the fact that the effective potential induced from the coherent forward scattering of neutrinos with the background is generally unequal to the effective potential for anti-neutrinos if the background is CP asymmetric[18, 19]. This means that the matter mixing angles for neutrinos are generally unequal to the matter mixing angles for anti-neutrinos, and thus the oscillation rates for neutrino oscillations need not be the same as the oscillation rates for anti-neutrino oscillations.

We begin by briefly reviewing the case of ordinary - sterile neutrino oscillations as developed in Ref.[17]. The evolution of lepton number \(L_{\nu_\alpha} \equiv (n_{\nu_\alpha} - n_{\bar{\nu}_\alpha})/n_\gamma\) (where the \(n_\text{'s}\) are the number densities and \(\alpha = e, \mu, \tau\)) in the early Universe generated by \(\nu_\alpha - \nu_s\) oscillations can be approximately described by the following equation[17],

\[
\frac{dL_{\nu_\alpha}}{dt} = \frac{\pi^2}{4\zeta(3)T^3} \int \frac{s^2 \Gamma_{\nu_\alpha} p \frac{c - b p}{x^p + (c - b p + a p)^2}[x^p + (c - b p - a p)^2]}{\[x^p + (c - b p + a p)^2\] + \Delta, \] \] (7)

where \(\Delta\) is a small correction term,

\[
\Delta \simeq \frac{-\pi^2}{8\zeta(3)T^3} \int \frac{s^2 \Gamma_{\nu_\alpha} p \frac{c - b p}{x^p + (a p)^2 + (b p - c)^2}[x^p + (c - b p - a p)^2]}{\[x^p + (c - b p + a p)^2\] + \Delta. \] (8)

In these equations, \(\zeta(3)\) is the Riemann zeta function of 3 (\(\zeta(3) \simeq 1.202\)), \(dn_{\nu_\alpha}^\pm \equiv dn_{\nu_\alpha} \pm dn_{\bar{\nu}_\alpha}\), and similarly for \(dn_{\bar{\nu}_\alpha}^\pm\). In the region where the lepton number is much less than 1, or equivalently, the chemical potential can be neglected,

\[
 dn_{\nu_\alpha} \simeq dn_{\bar{\nu}_\alpha} \simeq \frac{1}{2\pi^2} \frac{p^2 dp}{1 + e^{p/T}}. \] (9)
In Eq.(4), $c \equiv \cos 2\theta_0$, $s \equiv \sin 2\theta_0$, and the quantities, $b^p, a^p, x^p, \Gamma_{\nu_\alpha}^p$ are all functions of momentum of the form:

$$x^p = s^2 + \frac{\Gamma_{\nu_\alpha}^p}{4(\Delta_0^p)^2} \left( \frac{p}{\langle p \rangle} \right)^2, \quad \Gamma_{\nu_\alpha}^p = \Gamma_{\nu_\alpha} \frac{p}{\langle p \rangle},$$

$$a^p = -\sqrt{2G_F} v \alpha L^{(\alpha)}, \quad b^p = -\sqrt{2G_F} v \alpha A_e T^2 \frac{p}{\langle p \rangle},$$

where $n_\gamma = 2\zeta(3)T^3/\pi^2$, $\Delta_0^p = \delta m^2/2p$ and the thermally averaged collision frequencies $\Gamma_{\nu_\alpha}$ are

$$\Gamma_{\nu_\alpha} \simeq y_\alpha G_F^2 T^5,$$

where $y_\alpha \simeq 4.0, y_{\mu,\tau} \simeq 2.9$. In Eq.(10), $G_F$ is the Fermi constant, $M_W$ is the W-boson mass and $A_e \simeq 55.0$, $A_{\mu,\tau} \simeq 15.3$. The function $L^{(\alpha)}$ is given by

$$L^{(\alpha)} = L_{\nu_\alpha} + L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau} + \eta,$$

where $\eta$ is a small asymmetry term which arises from the asymmetries of baryons and electrons and is expected to be about $10^{-10}$. With the definitions Eq.(10) observe that Eq.(7) implies that significant lepton number can only be generated provided that $\delta m^2 < 0$ and in this case only for oscillations with $b^p < c$.

In Ref.[17], we showed that the distribution of sterile states is governed approximately by the equations

$$\frac{dz}{dt} = \frac{(1 - z)}{4} \frac{\Gamma_{\nu_\alpha}^p s^2}{x^p + (c - b^p + a^p)^2}, \quad \frac{d\bar{z}}{dt} = \frac{(1 - \bar{z})}{4} \frac{\Gamma_{\nu_\alpha}^p s^2}{x^p + (c - b^p - a^p)^2},$$

where

$$z \equiv \frac{dn_{\nu_\alpha}/dp}{dn_{\nu_\alpha}/dp}, \quad \bar{z} \equiv \frac{dn_{\nu_\alpha}/dp}{dn_{\nu_\alpha}/dp}.$$

Thus, the equation governing the evolution of $L_{\nu_\alpha}$ has the approximate form,

$$\frac{dL_{\nu_\alpha}}{dt} = \frac{1}{4\zeta(3)T^3} \int_0^\infty \frac{s^2 \Gamma_{\nu_\alpha}^p a^p(c - b^p)}{[x^p + (c - b^p + a^p)^2][x^p + (c - b^p - a^p)^2]} \frac{(1 - z^+) p^2 dp}{(1 + e^{p/T})} + \Delta,$$

where $\Delta$ is a small correction term,

$$\Delta \simeq \frac{1}{8\zeta(3)T^3} \int_0^\infty \frac{s^2 \Gamma_{\nu_\alpha}^p [x^p + (a^p)^2 + (b^p - c)^2]}{[x^p + (c - b^p + a^p)^2][x^p + (c - b^p - a^p)^2]} \frac{z^- p^2 dp}{(1 + e^{p/T})},$$

with $z^\pm \equiv (z \pm \bar{z})/2$ and we have neglected a small term proportional to $L_{\nu_\alpha}$.

In the case of ordinary - mirror neutrino oscillations, there are two important modifications. First, the effective potential of the mirror neutrinos is generally non-negligible. Second, the mirror weak interactions can bring the mirror neutrino into equilibrium with other light mirror particles. We now discuss these points in more detail.

In the case of $\nu_\alpha - \nu_\beta'$ oscillations, the dynamics depend on the difference of the effective potentials,

$$V = V_\alpha - V_\beta',$$
where \( V_\alpha (V'_\beta) \) is the effective potential experienced by a pure weak (mirror) eigenstate. These effective potentials can be expressed in terms of the parameters \( a^p, b^p \) \((a'^p, b'^p)\) as follows,

\[
V_\alpha = (-a^p + b^p)\Delta_0^p, \quad V'_\beta = (-a'^p + b'^p)\Delta_0^p. \quad (18)
\]

If the number of mirror neutrinos is much less than the number of ordinary neutrinos then \( b'^p \simeq 0 \). [Note that the \( b \)-part of the effective potential is proportional to the number densities of the background particles. This dependence is not given explicitly in Eq. (10) since for this equation the number densities were set equal to their equilibrium values]. The parameter \( a^p \) has the form

\[
a^p \equiv -\sqrt{2}G_F n_\gamma L^{(\beta)}, \quad (19)
\]

where \( L^{(\beta)} \) is given by

\[
L^{(\beta)} = L_{\nu'_\beta} + L_{\nu'_e} + L_{\nu'_\mu} + L_{\nu'_\tau} + \eta', \quad (20)
\]

and \( L_{\nu'_\beta} \) are the mirror lepton numbers, which are defined by \( L_{\nu'_\beta} \equiv (n_{\nu'_\beta} - n_{\bar{\nu}'_\beta})/n_\gamma \) (note that \( n_\gamma \) is the number density of ordinary photons) and \( \eta' \) is a function of the mirror baryon/electron number asymmetries [which is defined analogously to \( \eta \)]. We will assume that \( \eta' \) is small and can be approximately neglected. Thus, assuming that the number density of mirror particles is much less than the number density of ordinary particles, the modification of the effective potential due to the mirror interactions of \( \nu'_\beta \) can be approximately taken into account by simply replacing \( L^{(\alpha)} \) in the definition of \( a^p \) by \( L^{\sim \alpha\beta'} \equiv L^{(\alpha)} - L^{(\beta)} \). Since ordinary + mirror lepton number is conserved (and we will assume that it is zero), it follows that

\[
L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau} + L_{\nu'_e} + L_{\nu'_\mu} + L_{\nu'_\tau} = 0. \quad (21)
\]

For example, if we consider \( \nu_\alpha - \nu'_\beta \) oscillations in isolation then \( L_{\nu'_\beta} = -L_{\nu_\alpha} \) and \( \sim L^{\sim 2L^{(\alpha)}} \).

Another important effect of the mirror interactions is that the momentum distribution of the mirror neutrinos will be approximately Fermi-Dirac distributions, and the other mirror particles will be excited until

\[
T_{\nu'_e} = T_{\nu'_\mu} = T_{\nu'_\tau} = T_{e'} = T_{\gamma'} \equiv T', \quad (22)
\]

(we consider the \( T' \sim 100 \text{ MeV} \) region only where we can approximately neglect the excitation of mirror muons and other heavier mirror particles). Of course the light mirror particles will only be excited provided that there are sufficient mirror neutrinos around so that the interaction rates will be faster than the expansion rate, that is,

\[
hG_F^2 T'^5 \gtrsim 5.5T^2/M_P, \quad (23)
\]
where $M_P$ is the Planck mass and $h$ is a numerical parameter which depends on the particular interaction (see Table 1 of Ref. [20] for a list of the reaction rates). Typically, $h \sim 1/8$. Solving Eq. (23), we obtain the condition

$$T' \gtrsim 2 \left( \frac{T}{\text{MeV}} \right)^{\frac{5}{2}} \text{MeV}. \quad (24)$$

Assuming that the above condition is approximately satisfied, the system of ordinary and mirror particles form two weakly coupled thermodynamic systems; the system comprising the ordinary particles at a temperature $T$, and the system comprising the mirror particles which has a distinct temperature $T'$. Let us determine the equation governing the evolution of $T'$. Initially, we will assume that $T' = 0$. Ordinary - mirror neutrino oscillations can then generate a mirror neutrino $\nu_\beta'$ say. The rate at which mirror neutrinos are created/destroyed by $\nu_\alpha - \nu_\beta'$ oscillations is governed by the rate equation,

$$\frac{d}{dt} \left[ n_{\nu_\alpha'} + n_{\bar{\nu}_\alpha'} \right] = \frac{1}{n_{\nu_\alpha} + n_{\bar{\nu}_\alpha}} \int \Gamma(\nu_\alpha \rightarrow \nu_\beta')(dn_{\nu_\alpha} - dn_{\nu_\beta'}) + \Gamma(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta')(dn_{\bar{\nu}_\alpha} - dn_{\bar{\nu}_\beta'}). \quad (25)$$

The reaction rates are given by [17, 22],

$$\Gamma(\nu_\alpha \rightarrow \nu_\beta') = \frac{1}{2} \Gamma_{\nu_\alpha} \sin^2 2\theta_m \left( \sin^2 \frac{\tau}{2L_{\text{osc}}} \right) = \frac{1}{4} \left[ \frac{\Gamma_{\nu_\alpha} s^2}{x^p + (b^p + a^p - c)^2} \right]. \quad (26)$$

The rate for for the process $\Gamma(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta')$ can be obtained by replacing $a^p \rightarrow -a^p$ in the above equation. If Eq. (24) is satisfied, then the momentum distribution of sterile neutrinos has the form

$$dn_{\nu_\beta'} \simeq dn_{\bar{\nu}_\beta'} \simeq \frac{1}{2\pi^2} \frac{p^2 dp}{1 + e^{p/T'}}, \quad (27)$$

where we have neglected the mirror neutrino chemical potential which is small provided that the lepton number is sufficiently small. Thus using Eqs. (26) and Eq. (27), it is straightforward to show that Eq. (25) can be expressed as follows,

$$\frac{d}{dt} \left[ n_{\nu_\beta'} + n_{\bar{\nu}_\beta'} \right] \simeq \frac{1}{6\zeta(3)T^3} \int_{0}^{\infty} \frac{s^2 \Gamma_{\nu_\alpha}(b^p - c)^2 + (a^p)^2 + x^p)p^2 F(p, T, T') dp}{[x^p + (c - b^p + a^p)^2][x^p + (c - b^p - a^p)^2]}, \quad (28)$$

where

$$F(p, T, T') \equiv \frac{1}{1 + e^{p/T}} - \frac{1}{1 + e^{p/T'}}. \quad (29)$$

During the process whereby the mirror interactions excite the other mirror particles and thermalize the momentum distributions, the number density of sterile neutrinos is generally not conserved. Thus, we cannot directly use Eq. (28) to determine the evolution of $T'$. However, the mirror interactions must conserve energy. If the energy density of the mirror particles is much less than the energy density of the ordinary
particles then the process whereby the mirror sector comes into equilibrium with itself should not significantly affect the expansion rate of the Universe. For this reason, and because of the conservation of energy, it follows that the energy density of the mirror particles normalized to the energy density of the ordinary particles should to a good approximation not change due to the expansion of the Universe. We will denote this quantity by \( \gamma \equiv \rho'/\rho \). In the region where 1 MeV \( \lesssim T \lesssim 100 \) MeV, and \( 1 \) MeV \( \lesssim T' \lesssim 100 \) MeV, then \( \gamma \simeq T^4/T^4 \) (assuming that we are in a region where Eq.(24) is valid). Using Eq.(28), it is straightforward to show that the evolution of \( \gamma \), which is entirely due to ordinary - mirror transitions, satisfies the following equation (where we are assuming that \( \gamma \ll 1 \)),

\[
\frac{d\gamma}{dt} \simeq \frac{1}{19\zeta(3)N T^4} \int_0^\infty \frac{s^2 \Gamma_{\nu\alpha}(b^p - c)^2 + (a^p)^2 + x^p p^3 F(p, T, T') dp}{[x^p + (c - b^p + a^p)^2][x^p + (c - b^p - a^p)^2]},
\]

(30)

where \( N = \rho'/\rho' \). Using \( \rho' = 3\rho' + \rho'_\nu + \rho'_\nu = 6\frac{1}{2}\rho' \), it follows that \( N \simeq 6.14 \).

In the region where Eq.(24) holds, Eq.(7) and Eq.(27) imply that the evolution of lepton number obeys the following equation,

\[
\frac{dL_{\nu\alpha}}{dt} = \frac{1}{4\zeta(3)T^3} \int_0^\infty \frac{s^2 \Gamma_{\nu\alpha} a^p(c - b^p)(a^p) F(p, T, T') dp}{[x^p + (c - b^p + a^p)^2][x^p + (c - b^p - a^p)^2]} + \Delta,
\]

(31)

where

\[
\Delta \simeq \frac{-3L_{\nu\alpha}}{2\pi^2} \int_0^\infty \frac{s^2 \Gamma_{\nu\alpha} [x^p + (a^p)^2 + (b^p - c)^2] G(p, T, T') dp}{[x^p + (c - b^p + a^p)^2][x^p + (c - b^p - a^p)^2]},
\]

(32)

and

\[
G(p, T, T') \equiv \frac{1}{T^3 (1 + e^{p/T})^2} - \frac{1}{T'^3 (1 + e^{p/T'})^2}.
\]

(33)

Equations (30) and (31) are coupled differential equations that can be solved for \( L_{\nu\alpha}, T' \).

In the region where

\[
T' \ll 2 \left( \frac{T}{\text{MeV}} \right)^{\frac{3}{2}} \text{ MeV},
\]

(34)

the number of mirror particles are insufficient to enable the mirror sector to come into equilibrium with itself. In the case of Eq.(34), the momentum distribution and number density of mirror neutrinos are not significantly affected by the mirror interactions. This means that the momentum distribution of mirror neutrinos should be the same as with the case of sterile neutrinos. Thus the momentum distribution and number density of mirror neutrinos should be governed approximately by Eq.(13) with the evolution of lepton number governed approximately by Eq.(15). Of course even in the case of Eq.(34), the effects of the mirror interactions on the effective potential must still be taken into account, that is, we must replace \( L^{(\alpha)} \) by \( \tilde{L}^{\alpha\beta} \).
There are several ways in which the creation of lepton number(s) can prevent sterile/mirror neutrinos from coming into equilibrium. One way is that one set of oscillations $\nu_\alpha - \nu_s$ creates $L_{\nu_\alpha}$. The lepton number $L_{\nu_\alpha}$ can then suppress other, independent oscillations such as $\nu_\beta - \nu_s$ oscillations (with $\beta \neq \alpha$) for example. A more direct, but less dramatic way in which the creation of lepton number can help prevent the sterile/mirror neutrinos from coming into equilibrium, is that the lepton number generated from say $\nu_\alpha - \nu_s$ oscillations itself suppresses the $\nu_\alpha - \nu_s$ oscillations [24]. We will examine the latter effect here (the former effect will be studied in a moment). Previous work [13] obtained the BBN bound on ordinary - sterile neutrino oscillations with large $|\delta m^2| \sim 10^{-4} \ eV^2$ (with $\delta m^2 < 0$) and small $\sin^2 2\theta_0 \sim 10^{-2}$. This bound can be approximately parametrized as follows

$$\sin^2 2\theta_0 \sim 3 \times 10^{-5} \left( \frac{eV^2}{|\delta m^2|} \right)^{1/2}.$$  \hspace{1cm} (35)

This bound arises by assuming that the $\nu_\alpha - \nu_s$ oscillations do not bring the sterile $\nu_s$ state into equilibrium. Note that this bound neglected the creation of lepton number and it also did not include the effects of the distribution of neutrino momenta. However, in the realistic case, the creation of $L_{\nu_\alpha}$ (after it occurs), will suppress the $\nu_\alpha - \nu_s$ oscillations and the actual bound might be expected to be somewhat less stringent than Eq. (35). Nevertheless, for the case of truly sterile neutrinos we found [17] that Eq. (35) turned out to be a reasonable approximation in the realistic case where the momentum distribution and the creation of lepton number was taken into account. This is largely due to the fact that the creation of sterile neutrinos suppresses the lepton number creating oscillations and consequently delays the point where significant lepton number can be created [17]. In the case of mirror neutrinos, there will be much less of this type of effect because the number density of the mirror neutrinos is kept low due to the excitation of the other mirror particles. Also, the thermalization of the neutrino momentum distributions means that not all of the mirror neutrinos will have low momentum. Thus, we would expect that the bound Eq. (35) should be weakened somewhat.

For the case of ordinary - mirror neutrino oscillations, the BBN bound for large $|\delta m^2| \sim 10^{-4} \ eV^2$ (with $\delta m^2 < 0$) and small $\sin^2 2\theta_0 \sim 10^{-2}$ can be obtained by solving the coupled differential equations, Eq. (30) and Eq. (31). Doing this numerical exercise, we obtain the following bounds assuming that $\delta N_{eff} = \rho' / \rho_{\nu_\alpha} < 0.6$ (1.5) [21] (where $N_{eff}$ is the effective number of neutrinos present during nucleosynthesis),

$$\sin^2 2\theta_0 \sim 3(6) \times 10^{-4} \left( \frac{eV^2}{|\delta m^2|} \right)^{1/2} \quad \text{for } \nu_{\mu,\tau} - \nu_\beta' \text{ oscillations},$$

$$\sin^2 2\theta_0 \sim 1(2) \times 10^{-4} \left( \frac{eV^2}{|\delta m^2|} \right)^{1/2} \quad \text{for } \nu_e - \nu_\beta' \text{ oscillations}.$$ \hspace{1cm} (36)

Comparing these bounds with Eq. (35), we see that the BBN bound on $\sin^2 2\theta_0$ for ordinary - mirror neutrino oscillations is about an order of magnitude weaker than the corresponding bound for ordinary - sterile neutrino oscillations.
We now identify the region of parameter space for which the maximal ordinary
mirror neutrino oscillations can solve the solar and atmospheric neutrino anomalies
without leading to a significant modification of BBN.

We first study the maximal ordinary - mirror neutrino oscillation solution to the
atmospheric neutrino problem. We will assume that the various oscillations can be
approximately broken up into the pairwise oscillations $\nu_\alpha - \nu_\beta$. We will denote the
various oscillation parameters in a self-evident notation,

$$b_{\alpha \beta'}, a_{\alpha \beta'}^p$$ for $\nu_\alpha - \nu_\beta'$ oscillations,

where $\alpha, \beta = e, \mu, \tau$. We will denote the mixing parameters, $\delta m^2, \sin^2 2\theta_0$ appropriate
for $\nu_\alpha - \nu_\beta'$ oscillations by $\delta m^2_{\alpha \beta'}, \sin^2 2\theta_{0}^\alpha$. Note that lepton number cannot be
created by $\nu_\alpha - \nu_\beta'$ oscillations until $\langle b_{\alpha \beta'}^p \rangle \sim \cos 2\theta_0^\alpha$. Recall that the $b^p$ parameter
is inversely proportional to $\delta m^2$ [see Eq. (21)]. Thus, the earliest point during the evo-

du-4lution of the Universe where lepton number can be created due to ordinary - mirror
neutrino oscillations occurs for oscillations which have the largest $|\delta m^2|$. Note that
these oscillations should satisfy the BBN bound Eq. (36) if we demand that the sterile
nu-3-1e-trino oscillations occurs for oscillations which have the largest $|\delta m^2|$. Note that
these oscillations should satisfy the BBN bound Eq. (36) if we demand that the sterile
neutrino should not significantly modify BBN. Note that the $\nu_\mu - \nu_\mu'$ oscillations have
quite small $|\delta m^2_{\mu\mu'}| \sim 10^{-2}$ eV$^2$, and $\cos 2\theta_{0}^{\mu\mu'} \sim 0$ (assuming maximal or near max-

ima-limit mixing), and thus these oscillations themselves cannot produce significant lepton
number. However, the $|\delta m^2|$ for $\nu_\tau - \nu_\mu'$ (or $\nu_\tau - \nu_\mu'$) oscillations can be much larger.
Also note that $\delta m^2 < 0$ if $m_{\nu_\tau} > m_{\nu_\mu'}$ (or $m_{\nu_\tau} > m_{\nu_\mu}$).

We will first consider the system comprising $\nu_\tau, \nu_\mu$ and $\nu_\mu'$ (and their anti-particles).
The effects of the other neutrinos will be discussed in a moment. We will assume for
definitiveness that $m_{\nu_\tau} > m_{\nu_\mu}$ so that $|\delta m^2_{\mu\mu'}| > |\delta m^2_{\mu\mu'}|$ and the $\nu_\tau - \nu_\mu'$ oscillations create $L_{\nu_\tau}$ first (with $L_{\nu_\tau}, L_{\nu_\mu}$ assumed to be initially negligible).

Recall that in the case of $\nu_\alpha - \nu_\beta'$ oscillations, the effect of the mirror interactions
on the effective potential can be taken into account by replacing $L^{(\alpha)}$ in the definition
of $a^p$ by $L^{(\alpha)} = L^{(\alpha)} - L^{(\beta)}$. For the $\nu_\tau, \nu_\mu, \nu_\mu'$ system,

$$L^{(\mu\mu')} \simeq 2L_{\nu_\mu} + L_{\nu_\tau} - 2L_{\nu_\mu'} \simeq 4L_{\nu_\mu} + 3L_{\nu_\tau},$$

$$L^{(\tau\mu')} \simeq 2L_{\nu_\tau} + L_{\nu_\mu} - 2L_{\nu_\mu'} \simeq 4L_{\nu_\tau} + 3L_{\nu_\mu},$$

where we have used Eq. (21) with $L_{\nu_\mu} \simeq L_{\nu_\mu} \simeq L_{\nu_\tau} \simeq 0$. Thus, the creation of $L_{\nu_\tau}$ by $\nu_\tau - \nu_\mu'$ oscillations also implies the creation of the quantity $L^{(\mu\mu')}$. Because $a^p_{\mu\mu'}$ is
directly proportional to $L^{(\mu\mu')}$ it follows that the creation of a large $L^{(\mu\mu')}$ will make $a^p_{\mu\mu'}$
large (i.e. significantly greater than 1), which thereby suppresses the oscillations (note that $\sin^2 2\theta_{0}^{\mu\mu'} \sim \sin^2 2\theta_{0}^{\mu\mu'}/a_{\mu\mu'}^2$ if $a_{\mu\mu'} \gg 1$). However, note that $\nu_\mu - \nu_\mu'$ oscillations
can potentially destroy $L^{(\mu\mu')}$ (because for these oscillations $L^{(\mu\mu')} \simeq 0$ is an approximately
stable fixed point for the temperature range of interest). Thus, we must obtain the
region of parameter space where $\sim L^{\mu\mu}$ created by $\nu_\tau - \nu'_\mu$ oscillations does not get subsequently destroyed by $\nu_\mu - \nu'_\mu$ oscillations. We will determine this region of parameter space by numerically solving the coupled differential equations governing the evolution of $L_{\nu_\tau}, L_{\nu_\mu}$ and $T'$. [An approximate analytic computation could be done following similar reasoning to the case of sterile neutrinos[17]. Note however that the approximations used in this analytic derivation are not valid if the point where the destruction of $\sim L^{\mu\mu}$ due to $\nu_\mu - \nu'_\mu$ oscillations reaches a maximum can occur during the phase where the creation of $\sim L^{\mu\mu}$ due to $\nu_\tau - \nu'_\mu$ oscillations is still growing exponentially].

The rate of change of $L_{\nu_\mu}$ and $L_{\nu_\tau}$ due to the $\nu_\tau - \nu'_\mu, \nu_\mu - \nu'_\mu$ oscillations can be obtained from Eq.(31). This leads to the following coupled differential equations,

$$
\frac{dL_{\nu_\mu}}{dt} \simeq \frac{1}{4\pi^2} \int_0^\infty \frac{\sin^2 2\theta^\mu_{\nu'\mu}}{p} \int_0^\pi F(p,T,T') dp \int_0^{\pi/2} \sin p \cos p [\frac{1}{2\pi} \int_0^\infty d\nu \sin^2 2\theta^\mu_{\nu'\mu} \int_0^\infty \sin \nu' \nu' \nu' \nu' + \pi] F(p,T,T') dp,
$$

(39)

where the equation governing the evolution of $T'$ can be obtained from Eq.(34), and it is

$$
\frac{d\tau}{dt} \simeq \frac{1}{10\pi^2} \int_0^\infty \int_0^\pi F(p,T,T') dp \int_0^{\pi/2} \sin p \cos p F(p,T,T') dp \int_0^\infty \sin^2 2\theta^\mu_{\nu'\mu} \int_0^\infty \sin \nu' \nu' \nu' \nu' + \pi F(p,T,T') dp,
$$

(40)

Note that Eq.(34) and Eq.(39) are coupled differential equations which must be solved simultaneously. In our numerical integration of Eqs.(39, 40), we will assume for definiteness that $\sin^2 2\theta^\mu_{\nu'\mu}$ and that $\delta m^2_{\mu\nu}$ are given by the best fit for the atmospheric neutrino data, i.e. $\sin^2 2\theta^\mu_{\nu'\mu} \simeq 1$ and $|\delta m^2_{\mu\nu}| \simeq 10^{-2} eV^2$. The results of this exercise are shown in Figure 1[23, 26]. In the region above the solid line, the $\sim L^{\mu\mu}$ created by $\nu_\tau - \nu'_\mu$ oscillations does not get subsequently destroyed by $\nu_\mu - \nu'_\mu$ oscillations. The numerical work also demonstrates that the lepton number is generated early enough and is large enough to suppress the $\nu_\mu - \nu'_\mu$ oscillations sufficiently so that these oscillations do not create a significant number of $\nu'_\mu$ states. The requirement that $\nu_\tau - \nu'_\mu$ oscillations do not produce too many sterile states implies an upper limit on $\sin^2 2\theta^\mu_{\nu'\mu}$ [see Eq.(34)]. This upper limit has been shown in the Figure assuming for definiteness that $\rho'/\rho_\nu < 0.6$ (dashed-dotted line). Also shown in Figure 1 (dashed line) is the cosmological energy density bound $|\delta m^2_{\nu\nu}| \simeq 1600 eV^2[27].
Comparing the allowed region of parameter space shown in Figure 1 with the analogous case for sterile neutrinos, it is apparent that the corresponding allowed region for mirror neutrinos is somewhat larger than the allowed region for sterile neutrinos. The increase of parameter space (which is about an order of magnitude larger for $\sin^2 2\theta_{\mu'\mu}'$) in the case of mirror neutrinos is primarily due to the result that the bound, Eq. (36), is considerably less stringent than the bound, Eq. (35).

While we have focussed on the $\sim \mu'\mu'$ generated by $\nu_\tau - \nu'_e$ oscillations, similar results will also hold for $\sim \mu'\mu'$ generated by $\nu_\tau - \nu'_e$ oscillations. Note that in this case,

$$
\begin{align*}
\sim \mu'\mu' & \simeq 2L_{\nu_\mu} + L_{\nu_\tau} - 2L_{\nu'_\mu} - L_{\nu'_\tau} \simeq 4L_{\nu_\mu} + 2L_{\nu_\tau}, \\
\sim \tau e' & \simeq 2L_{\nu_\tau} + L_{\nu'_\mu} - 2L_{\nu'_e} - L_{\nu'_\nu} \simeq 4L_{\nu_\tau} + 2L_{\nu_\mu}.
\end{align*}
$$

(41)

In this case, solving the appropriate coupled differential equations, we find that the $\sim \mu'\mu'$ generated by $\nu_\tau - \nu'_e$ oscillations does not get destroyed by $\nu_\mu - \nu'_\mu$ oscillations for the range of parameter space shown in Figure 2. Of course we only require that the oscillation parameters are in the allowed region shown in Figure 1 or Figure 2.

Note that our results should only be considered as approximate because we have only included the $\nu_\tau - \nu'_e$ and $\nu_\mu - \nu'_\mu$ oscillations (where $\beta = \mu$ for Figure 1 and $\beta = e$ for Figure 2). In the most general case, the system is considerably more complicated. In general one should also include the effects of the oscillations with $\delta m^2 > 0$ as well as ordinary - ordinary and mirror - mirror neutrino oscillations. The oscillations with $\delta m^2 > 0$, include oscillations such as $\nu_\mu - \nu'_e$ [given the assumption Eq. (5)]. Focusing on this example, these oscillations generate $L_{\nu_\mu}, L_{\nu'_e}$ such that $L(\mu) - L(\tau') \rightarrow 0$. They cannot prevent $\nu_\tau - \nu'_e$ or $\nu_\tau - \nu'_e$ oscillations from generating $L_{\nu_\mu}$ and hence $\sim \mu'\mu'$. For this reason these oscillations cannot change anything qualitatively and for simplicity we have neglected them. The effect of ordinary - ordinary neutrino oscillations, such as $\nu_\tau - \nu_\mu$ oscillations is to generate $L_{\nu_\tau}, L_{\nu_\mu}$ such that $L_{\nu_\mu} - L_{\nu_\tau} \rightarrow 0$. These oscillations also cannot prevent $\sim \mu'\mu'$ being generated by $\nu_\tau - \nu'_\mu$ (or $\nu_\tau - \nu'_\mu$) oscillations. Also the rate of change lepton number due to these oscillations is typically suppressed compared to the rate of change of lepton number due to ordinary - sterile (or mirror) neutrino oscillations. Finally, the effect of mirror - mirror neutrino oscillations may also be important, although the precise effect of these oscillations is less clear.

We turn to a brief discussion of the maximal oscillation solution to the solar neutrino problem. Recall that maximal $\nu_e - \nu'_e$ oscillations can lead to a simple predictive solution to the solar neutrino problem for the large range of parameters, Eq. (15). Note that much of this parameter space is naively in conflict with BBN [see Eq. (16)]. However, $\nu_\tau - \nu'_e$ or $\nu_\tau - \nu'_\mu$ oscillations will generate $\sim \tau e'$ which can suppress the $\nu_e - \nu'_e$ oscillations provided that $\nu_e - \nu'_e$ oscillations do not destroy the $\sim \tau e'$ asymmetry. The situation is completely analogous to the case involving the $\nu_\mu - \nu'_\mu$ oscillations that we have been
studying above. For such large $|\delta m^2_{\tau\mu}|$ or $|\delta m^2_{\tau e}|$ identified in the Figures, it turns out that essentially the entire parameter space Eq.(4) does not lead to any significant modification of BBN (which is largely due to the fact that $|\delta m^2_{ee}| \gtrsim |\delta m^2_{\mu\mu'}|$).

In summary, the solutions of the atmospheric and solar neutrino problems suggested by the exact parity model are not in conflict with BBN for a significant range of parameters. Consistency with BBN does require relatively large values of the parameters,

$$|\delta m^2_{\tau\mu'}| \text{ (or } |\delta m^2_{\tau e'}|) \gtrsim 25 \text{ eV}^2 \text{ (50 eV}^2).$$

This suggests that $m_{\nu_{\tau}} \gtrsim 5 \text{ eV}$. Such values of the tau mass can also be motivated independently by the evidence for dark matter. Thus, demanding that the exact parity model solution of the atmospheric neutrino anomaly be consistent with BBN implies that the tau neutrino mass should be in a cosmologically interesting range. Also note that this tau neutrino mass range will be probed by the NOMAD and CHORUS experiments[28].

Figure Captions

Figure 1. Region of parameter space in the $\sin^2 2\theta^\mu\mu'$, $-\delta m^2_{\tau\mu'}$ plane where the $L^\mu\mu'$ created by $\nu_{\tau} - \nu'_{\mu'}$ oscillations does not get destroyed by $\nu_{\mu} - \nu'_{\mu}$ oscillations. This region which in the Figure is denoted by the “Allowed Region”, includes all of the parameter space above the solid line. We have assumed that $\sin^2 2\theta^\mu\mu' \simeq 1$ and $|\delta m^2_{\mu\mu'}| = 10^{-2} \text{ eV}^2$ (which is the best fit to the atmospheric neutrino data). Also shown (dashed line) is the cosmology bound $|\delta m^2_{\tau\mu}| \gtrsim 1600 \text{ eV}^2$. The dashed-dotted line is the nucleosynthesis bound Eq.(36) assuming that $N_{\text{eff}} = \rho'/\rho_{\nu_{\tau}} \approx 0.6$.

Figure 2. Region of parameter space in the $\sin^2 2\theta^e\mu'$, $-\delta m^2_{\tau e'}$ plane where the $L^\mu\mu'$ created by $\nu_{\tau} - \nu'_{e'}$ oscillations does not get destroyed by $\nu_{\mu} - \nu'_{\mu}$ oscillations. As in Figure 1, we have assumed that $\sin^2 2\theta^\mu\mu' \simeq 1$ and $|\delta m^2_{\mu\mu'}| = 10^{-2} \text{ eV}^2$. Also shown (dashed line) is the cosmology bound $|\delta m^2_{\tau\mu}| \approx 1600 \text{ eV}^2$. The dashed-dotted line is the nucleosynthesis bound Eq.(36) assuming that $\delta N_{\text{eff}} = \rho'/\rho_{\nu_{\tau}} \approx 0.6$.

Acknowledgements

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The BBN bound on the number of effective neutrino degrees of freedom, $N_{\text{eff}}$, has been the subject of some discussion lately. N. Hata et. al, Phys. Rev. Lett. 75, 3977 (1995), argue that the bound on $N_{\text{eff}}$ is $N_{\text{eff}} \sim 2.7$. However several other authors have disputed this conclusion. For example, the bounds $N_{\text{eff}} \sim 3.9$, 4.5, 4.0 have been respectively obtained by C. J. Copi, D. N. Schramm and M. S. Turner, Phys. Rev. Lett. 75, 3981 (1995); P. J. Kernan and S. Sarkar, Phys. Rev. D54, 3681 (1996); K. A. Olive and D. Thomas, University of Minnesota Preprint UMN-TH-1515/96, hep-ph/9610319.
Note that in terms of $a^p, b^p$, the matter mixing angle $\sin^2 2\theta_m$ and the function $\langle \sin^2 \frac{\tau}{2L_m} \rangle$ (the brackets $\langle \ldots \rangle$ denote the average over collision times, $\tau$) are given by\footnote{In deriving the equation for $\Delta$, we have used $dn_{\nu^\alpha} = n_\gamma L_{\nu^\alpha} \frac{6}{\pi^2 T^3} \frac{p^2 e^{p/T} dp}{(1 + e^{p/T})^2} + \mathcal{O}(L_{\nu^\alpha}^2)$, which can easily be obtained by expanding $dn_{\nu^\alpha}$ in a power series assuming that the chemical potential is small (see e.g. \cite{17}). Note that $dn_{\tilde{\nu}^\beta}$ can be obtained by replacing $T \rightarrow T'$ in the above equation.

Recently, D. P. Kirilova and M. V. Chizhov, \texttt{hep-ph/9608270} have examined the BBN bounds on $\nu_e - \nu_s$ oscillations for $|\delta m^2| \lesssim 10^{-7} \text{eV}^2$, including the effects of the neutrino asymmetry and momentum distribution.

In our numerical work we found that Eq.(24) was approximately valid for the allowed region with $\sin^2 2\theta_0 \lesssim 10^{-6}$. For $\sin^2 2\theta_0 \mu^\prime \lesssim 10^{-6}$, we found that there were few enough mirror neutrinos produced so that it did not matter significantly whether the rate of change of lepton number was obtained from Eq.(39,40) or Eqs.(15,13) (with $L^{(\alpha)} \rightarrow L^{\alpha\beta'}$).

Note that Eq.(7) [and hence Eq.(39)] is only valid provided that the evolution of the lepton number is sufficiently smooth (see Ref.\cite{17} for a detailed discussion about the region of validity of the static approximation). In Ref.\cite{17} we showed that a necessary condition for Eq.(7) to be valid is that $|\partial L_{\nu^\alpha}/\partial T| \lesssim 4 \times 10^{-11} \left( \frac{T}{\text{MeV}} \right)^4 \frac{1}{\text{MeV}}$.

In our numerical work we found that the above equation was approximately valid in the allowed region of Figures 1,2 provided that $\sin^2 2\theta_0 \lesssim 10^{-5}$. It would be a useful exercise to check our analysis (especially in the region where $\sin^2 2\theta_0 \sim 10^{-5}$) by performing a more accurate study using the density matrix equations (including the neutrino momentum distribution).}
If neutrinos are approximately stable (which would be expected unless some new interactions exist) then there is a stringent cosmology bound on the mass of the neutrinos which implies that $m_{\nu_e} \lesssim 40 \text{ eV}$ (see e.g. the review by G. Gelmini and E. Roulet, in Ref.[19]). Note that $m_{\nu_e} \lesssim 40 \text{ eV}$ implies that $|\delta m_{\tau\beta'}^2| \lesssim 1600 \text{ eV}^2$.

The NOMAD and CHORUS experiments are $\nu_\mu \rightarrow \nu_\tau$ appearance experiments. These experiments can potentially measure $|\delta m_{\tau\mu}^2|$. Note that within the context of the exact parity symmetric model we expect that $|\delta m_{\tau\mu}^2| \approx |\delta m_{\tau\mu'}^2|$. To see this, observe that $\delta m_{\tau\mu}^2$ is related to $\delta m_{\tau\mu'}^2$, $\delta m_{\mu\mu'}^2$ by the relation, $\delta m_{\tau\mu}^2 = \delta m_{\tau\mu'}^2 - \delta m_{\mu\mu'}^2$. Thus, for large $|\delta m_{\tau\mu}^2| \approx 1 \text{ eV}^2$, $\delta m_{\tau\mu}^2 \approx \delta m_{\tau\mu'}^2$ assuming that $|\delta m_{\mu\mu'}^2| \approx 10^{-1} \text{ eV}^2$ (as suggested by the atmospheric neutrino anomaly).
Figure 1

$-\delta m^2_{\mu\nu'} \text{ (eV}^2\text{)}$

$\sin^2 \theta_{\mu\nu'}$

Allowed Region
Figure 2

Allowed Region

$-\delta m^2_{\tau_e'}$ (eV$^2$)

$\sin^2 \theta$ $\tau_{e'}$

$10^{-9}$ $10^{-8}$ $10^{-7}$ $10^{-6}$ $10^{-5}$ $10^{-4}$

$10^1$ $10^2$ $10^3$