Large Interferometer For Exoplanets (LIFE):

VII. Practical implementation of a kernel-nulling beam combiner with a discussion on instrumental uncertainties and redundancy benefits

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ABSTRACT

Context. In the previous paper in this series, we identified that a pentagonal arrangement of five telescopes, using a kernel-nulling beam combiner, shows notable advantages for some important performance metrics for a space-based mid-infrared nulling interferometer over several other considered configurations for the detection of Earth-like exoplanets around solar-type stars.

Aims. We aim to produce a practical implementation of a kernel-nulling beam combiner for such a configuration, as well as a discussion of systematic and stochastic errors associated with the instrument.

Methods. We develop the mathematical framework around a beam-combiner based on a nulling combiner first suggested by Guyon et al. (2013), and then use it along with the simulator developed in the previous paper to identify the effects of systematic uncertainties.

Results. We find that errors in the beam combiner optics, systematic phase errors and the RMS fringe tracking errors result in instrument limited performance at ~4–7 µm, and zodiacal limited at ≥10 µm. Assuming a beam splitter reflectance error of |ΔR| = 5% and phase shift error of ∆φ = 3°, we find that the fringe tracking RMS should be kept to less than 3 nm in order to be photon limited, and the systematic piston error be less than 0.5 nm to be appropriately sensitive to planets with a contrast of 10⁻⁷ over a 4–19 µm bandpass. We also identify that the beam combiner design, with the inclusion of a well positioned shutter, provides an ability to produce robust kernel observables even if one or two collecting telescopes were to fail. The resulting four telescope combiner, when put into an X-array formation, results in a transmission map with an relative SNR equivalent to 80% of the fully functioning X-array combiner.

Conclusions. The advantage in sensitivity and planet yield of the Kernel-5 nulling architecture, along with an inbuilt contingency option for a failed collector telescope, leads us to recommend this architecture be adopted for further study for the LIFE interferometer.

Key words. Telescopes – Instrumentation: interferometers – Techniques: interferometric – Infrared: planetary systems – Planets and satellites: terrestrial planets

1. Introduction

Optical/mid-infrared nulling interferometry from space has been experiencing a resurgence of interest over the past few years, particularly with regards to detecting Earth-like exoplanets around solar-type stars. Such an idea is not new, having been first proposed by Bracewell (Bracewell 1978), and then through multiple studies resulting in two large missions: the European Space Agency’s Darwin (Léger et al. 1996) and NASA’s Terrestrial Planet Finder - Interferometer (TPF-I) (Beichman et al. 1999).

However, due to a myriad of reasons, not least concerning the lack of technological readiness, both missions were cancelled in the late 2000s.

Since then, various teams have continued to work towards the resurgence of nulling interferometry, leading to the formation of the Large Interferometer For Exoplanets (LIFE) initiative. This project is being considered as one of the large-class missions of the European Space Agency’s ‘Voyage 2050 program (Voyage 2050 Senior Committee 2021): a large space interferometer in the legacy of Darwin, working in the mid-infrared, with a goal to both detect and characterise Earth-like exoplanets that are difficult to access using other techniques such as single aperture coronography and transit spectroscopy. Significant work has already been done to characterise the planet yield of such a mission (Kammerer & Quanz 2018; Quanz et al. 2021), and the spectral requirements of the instrument (Konrad et al. 2021). A simulator tool to simulate observations and signal-to-noise requirements has also been developed (Dannert et al. 2022).

The renewal of interest in space interferometry also presents an opportune time to reanalyse the technology behind nulling interferometry. In particular, new technologies such as “kernel-nulling” (Martinache & Ireland 2018; Laugier et al. 2020), have opened up avenues to consider other telescope configurations away from the Emma X-array configuration decided upon in the Darwin/TPF-I era (Lay et al. 2005). The previous paper in this series (Hansen et al. 2022, hereby LIFE6) conducted a trade study between a number of different configurations, including the X-array, to determine whether other architectures would provide a higher yield and higher signal-to-noise. It was found that, in fact, an architecture consisting of five telescopes in a pentagonal shape, using a kernel-nulling beam combination scheme, outperformed the X-array in both detection and characterisation.

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In this paper, we propose a practical way of implementing the beam combination scheme discussed in the previous paper. We also discuss the systematic instrumental errors of such a beam combiner; how these change the dominant sources of photon noise, and how they impact the robustness and sensitivity of the kernel observable. Finally, we discuss a major advantage of this beam combination scheme - that even if a collector telescope is damaged or fails, the interferometer and beam combiner are still able to produce robust transmission maps with fewer telescopes.

2. Implementation of the Beam Combination Scheme

We devised an implementation of the Kernel-5 nuller beam combiner through the method of Guyon et al. [2013]. They posit that any predetermined unitary lossless transfer matrix \( M \) (denoted \( U \) in their notation) of \( n \) inputs, can be created through a series of \( n = \frac{(m+1)}{2} \) unequal beam splitters, with a phase shifting plate put in front of one of the inputs of each beam splitter. Such a design, for a five telescope combiner, can be seen in Figure [1].

This design also includes a set of \( m \) adaptive nullers (denoted AN) and \( m - 1 \) spatial filters (denoted SF) that are used to remove systematic amplitude and phase errors on input. The spatial filters are placed after first row of beam splitters (which provides the nulling) in order to precisely align the optics to get a deep null. These will be discussed further in Section [3].

Finally, we also have included two shutters in the design (denoted S); these shutters can be used in the case of a collector telescope failure to reconfigure the beam combiner to produce robust observables with fewer telescopes. This will be discussed in more detail in Section [4]. The parameters for each beam splitter and phase shifter can be derived through working backwards from the predetermined matrix \( M \).

We start with the general case of a beam combiner with \( m \) inputs (labelled \( V_1 \) through \( V_m \)) and \( m \) outputs (\( W_1 \) through \( W_m \)) as depicted for \( m = 5 \) in Figure [1]. The phase shifting plate is put in front of the second input of each beam splitter (the beam entering from the left in the figure), imposing a phase shift of \( \phi_j \) for each \( j \)th plate. We define each beam splitter through a mixing angle \( \theta_j \), which is related to their reflectance \( R \) and transmittance \( T \) as follows:

\[
R = \sin \theta_j \quad T = \cos \theta_j
\]

Hence each phase shifter/beam splitter module can be described by a 2x2 matrix:

\[
C_j = \begin{bmatrix} \sin \theta_j & e^{i\phi_j} \cos \theta_j \\ -e^{i\phi_j} \sin \theta_j & \cos \theta_j \end{bmatrix}
\]

As we have \( m \) input beams, with only two interfering at any one time, the other \( m - 2 \) beams are represented by identity rows and columns. Each beam combining step can thus be represented as an \( m \times m \) block diagonal matrix \( A_j \), with the diagonals of the rows/columns corresponding to the combining beams being equal to \( C_j \) and having ones on the other diagonal terms. For example, for a beam splitter module combining beams two and three out of a five beam combiner, the matrix \( A_j \) is given by:

\[
A_j = \begin{bmatrix} 1 & 0^T & 0 & 0 \\ 0 & C_j & 0 & 0 \\ 0 & 0^T & 1 & 0 \\ 0 & 0^T & 0 & 1 \end{bmatrix}
\]

where \( 0 = [0,0] \) is a two element zero column vector.

The full beam combiner is hence described by a multiplication of the \( n \) matrices. We also note that each of the output electric fields of the beam combiner can have an arbitrary phase shift (\( \omega \)) relative to the first output, as we only measure the intensity of these beams. We represent this as the matrix \( B \), given by:

\[
B = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\omega} & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\omega} \end{bmatrix}
\]

Therefore, to create a beam combiner for any transfer matrix \( M \), we solve the following equation for parameters \( \theta_j, \phi_j (j = 1, \ldots, n) \) and \( \omega_j (k = 2, \ldots, m) \). This equation is built from the \( A_j \) matrices in ascending order, corresponding to the order in which light traverses the combiner from the top left corner to the bottom right:

\[
M = BA_nA_{n-1} \cdots A_2A_1
\]

For the Kernel-5 nuller, we have that \( m = 5 \) and \( n = 10 \), and we know the transfer matrix \( M \) from LIFE6:

\[
M = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & e^{i\frac{2\pi}{5}} & e^{i\frac{4\pi}{5}} & e^{i\frac{6\pi}{5}} & e^{i\frac{8\pi}{5}} \\ 1 & e^{i\frac{4\pi}{5}} & e^{i\frac{8\pi}{5}} & e^{i\frac{2\pi}{5}} & e^{-i\frac{2\pi}{5}} \\ 1 & e^{i\frac{8\pi}{5}} & e^{i\frac{2\pi}{5}} & e^{-i\frac{2\pi}{5}} & e^{i\frac{4\pi}{5}} \\ 1 & e^{i\frac{2\pi}{5}} & e^{i\frac{6\pi}{5}} & e^{-i\frac{2\pi}{5}} & e^{i\frac{4\pi}{5}} \end{bmatrix}
\]

In this investigation, we have swapped rows two and four from the matrix in LIFE6 so that the two kernel outputs are
formed from neighbouring pairs. Assuming the pentagonal formation described in LIFE6, this also allows us to form the deeper null, calculated from the difference in rows two and three, using fewer beam splitting modules; this null provides better performance in terms of stellar leakage. To be consistent between the naming schemes of the two papers, we define kernel 1 as the second order null \((|W_1|^2 - |W_2|^2)\) and kernel 2 as the fourth order null \((|W_2|^2 - |W_3|^2)\).

Working through element by element, we solve Equation 5 for this matrix \(M\), resulting in the parameters listed in Table 1. The output phase shifts \(\omega\) were all zero except for \(\omega_5\), which had a phase shift of \(\pi\).

We note here that this implementation of the beam combiner has some caveats:

1. The beam splitters and phase shifting plates are required to be stable over a large wavelength band (nominally \(4-19\mu m\) (Quanz et al. 2021)). This could be alleviated by increasing the number of beam trains, splitting the wavelength into a few coarse channels, and implementing multiple versions of the beam combiner. The downside to this method is the increase of optical components and space requirements; a pertinent problem for a space-based mission. We will return to this issue when we discuss phase chopping in Section 3.6.

2. This implementation is schematically drawn as bulk optics. While photonics provides multiple advantages in terms of spatial filtering and space requirements (in terms of both footprint and space compatibility), demonstrations of far mid-infrared achromatic directional couplers and phase shifts have not, to the authors’ knowledge, occurred. We note however that progress is ongoing in this area, particularly at the shorter end of the mid-infrared band (e.g. Kenchington Goldsmith et al. 2017; Gretzinger et al. 2019). In principle, all components could be photonic.

3. Systematic Instrumental Uncertainties

In this section, we analyse the systematic uncertainties associated with this beam combiner implementation, and the effects of phase fluctuations of the input beams more broadly, on the robustness and sensitivity of the kernel-nulling architecture.

3.1. Adaptive nullers and alignment procedure

In this architecture, systematic errors can come from a number of places: errors in the phase and amplitude of the input beams, as well as errors in the optical elements of each beam splitter module. We can, however, eliminate some of these errors immediately through a careful calibration process using an adaptive nuller.

Adaptive nullers, as described by Lay et al. (2003), are compensators that can adjust the phase and amplitude of an input beam of light through the use of a deformable mirror (DM). The light is spectrally dispersed onto a DM, adjusted to tune the wavelength dependent phase and amplitude, before being redispersed and recollimated for beam combination. Amplitude is tunable through phase tilts orthogonal to the dispersion direction, when combined with spatial filtering. This is an invaluable tool for correction of the beams on input, and has also been shown to provide stable achromatic phase shifts (Peters et al. 2010).

For our purposes, the adaptive nuller can also be used to eliminate any errors in the top nulling row of beam splitter modules (\(A_1\) through \(A_5\)) with a process similar to the following:

1. Modulate the adaptive nuller on input \(V_1\) such that it maximises amplitude for all wavelengths. This input is set as the global phase reference.
2. Modulate the adaptive nuller on input \(V_2\) so that, as well as entering the spatial filter and removing wavelength dependence, it forms a null output on output \(W_2\).
3. Repeat stage two with the other three inputs, forming nulls on outputs \(W_3, W_4\) and \(W_5\) respectively.
4. If necessary, reduce the amplitude of input \(V_1\) and repeat steps 2 & 3.

In this manner, the phases of the input beams can be managed to completely remove any phase errors in the top nulling beam splitter optics, and any errors in the amplitudes of the inputs will just result in less overall light; the inputs can be modified such that they all have the amplitude of the dimmest input beam. The spatial filters at the end of the first row are required to remove unwanted spatial modes of the light that would prevent deep nulls.

3.2. Beam combiner optical errors

While the adaptive nullers are able to negate the effects of optical errors in the top nulling row of beam splitters, the remaining six modules will still contribute to errors in the kernel-nulls. To simulate this, we apply random fluctuations to the optical parameters based on a predefined RMS error:

\[
\theta = \theta_0 + \frac{|\Delta R|}{\cos \theta_0} x \cos \theta_0 \\
\phi = \phi_0 + \Delta \phi x
\]

where \(x \in [-1, 1]\) is a random number, which is uniformly distributed to simulate the effect of a typical pass/fail optical specification. The parameters \(\theta_0\) and \(\phi_0\) are the true values as defined in Table 1. \(|\Delta R|\) denotes the error in the reflectance of the beam splitter, and \(\Delta \phi\) represents the error in the phase shifter. For the remainder of this analysis, we will consider three sets of these uncertainties and refer to the pair by their \(|\Delta R|\) amount. These uncertainties are chosen for realistic manufacturing tolerances from optics suppliers:

\[
|\Delta R| = 2\% \quad \Delta \phi = 1^\circ \\
|\Delta R| = 5\% \quad \Delta \phi = 3^\circ \\
|\Delta R| = 10\% \quad \Delta \phi = 6^\circ
\]

We ran a Monte-Carlo simulation to find the standard deviation of the kernel maps as a function of angular coordinate when these errors are applied. We assume a pentagonal arrangement of the telescopes, equivalent to Figure 7 in LIFE6. The two maps, assuming \(|\Delta R| = 5\%\), are shown in Figure 2. Here we see that the kernels show a maximum standard deviation of 8% of the total telescope flux, with an average standard deviation of 2.7% and 2.2% of kernel 1 and 2 respectively. We also find that the average standard deviations for \(|\Delta R| = 2\%\) are 1% and 0.9%, and for \(|\Delta R| = 10\%\) we have 5.7% and 4.6% respectively.

We also examine the effect of these uncertainties on the modulation efficiency (RMS azimuthal average) of the kernel map; this highlights the fluctuations that may influence the power of the planet signal as the array rotates. We show this in Figure 3 where we overlay the modulation efficiency as a function of radius with no error, normalised by flux per telescope, on top of the average of twenty random draws with \(|\Delta R| = 5\%\). What is apparent here is that with this amount of error, the modulation...
efficiency is not significantly affected, indicating the information in the signal does not significantly change with these optical errors even though the detailed map structure requires additional calibration or modelling. There is a small amount of variation around the null of the first kernel; we will address the effect on the null depth and stability in the following section.

### 3.3. Null depth

From the plots in Figure 3, the most concerning trend induced by errors in the optical elements is the effect on the null; whether the null no longer reaches the desired depths and thus greatly increasing stellar leakage. Using the simulation machinery detailed in LIFE6, we calculated the base-10 logarithm of the ratio of the stellar leakage noise and zodiacal background light as a function of wavelength, assuming a 2 m aperture size. The wavelength range chosen is between 4 and 19 µm, to align with that of Quanz et al. (2021) and LIFE6. We calculated these plots for two stars located at 5 pc: a M5V dwarf based on Proxima Centauri, and a G2V dwarf based on the Sun. The latter was chosen based on the closest stars of F/G stellar type: there are three stars within 6 pc (σ Ceti, e Eri and η Cas) and can be considered on
average to be roughly a solar-type star at 5 pc. Hence this can be used as an extreme scenario on stellar leakage. This simulation was then repeated for $|\Delta R|$ values of 2%, 5% and 10%, to see the effect of optical errors on stellar leakage. The plots are shown in Figure 4.

We can see from these plots that kernel 2 is more sensitive to these optical errors; as kernel 1 is only a second-order null, it is more dominated by stellar leakage, and as such the noise floor is above that induced by these beam combiner optical errors. The fourth-order null of kernel 2, however, produces a much smaller stellar leakage and as such, a reduction in the null depth arising from optical errors is more apparent. We find that an error of 2% results in a 6 fold increase in stellar leakage for the solar type star, with a similar increase at 10% for the M-dwarf.

While this error results in quite a shift, particularly for the solar type star, we remind the reader that these stars are extreme scenarios. Most stars in the LIFE catalogue (Quanz et al., 2021) are further than 5 pc away, and the amount by which stellar leakage dominates at the short wavelengths decreases with distance. There are also few non M-dwarfs within 5 pc, and a 2% error on an M-dwarf measurement does not result in a large change in the leakage to zodiacal ratio. Nevertheless, this does indicate that optical errors in the beam combiner are important to minimise, particularly in suppressing stellar leakage at the short wavelengths. Conversely, optical beam combiner errors do not matter beyond approximately 8 µm as measurements will be strongly zodiacal background dominated.

### 3.4. Null stability

In LIFE6, we provided a simple approximation of the RMS fringe tracking requirements to remain limited by photon background noise, rather than fluctuations in the null. We found that the interferometer should aim for $<9$ nm RMS when looking at M-dwarfs, and around 2 nm for G-dwarfs, both at about 5 pc. We now look further into this, including the impact of optical beam combiner errors on the fringe tracking requirements and null stability.

From equation 50 in LIFE6, recall that the minimum fringe tracking error needed to remain photon noise limited can be estimated from this equation:

$$\langle \phi^2 \rangle F_{\text{star}} < \max \left[ \frac{P_{\text{zodiacal}}}{A}, F_{\text{leakage}} \right].$$

where $F_{\text{star}}$ is the stellar flux, $P_{\text{zodiacal}}$ is the zodiacal light (in photons/s), $F_{\text{leakage}}$ is the stellar leakage flux and $A$ is the single telescope aperture.

We simulate a random erroneous phase of all five telescopes, adding a term:

$$\phi_i = \frac{2\pi}{A} X_i$$

where $i$ represents the index of the telescope, $\lambda$ the wavelength in nanometers and $X \sim N(0, \sigma)$ is a random variable pulled from a normal distribution with zero mean and a standard deviation of $\sigma$, the RMS fringe tracking error in nanometers. We then calculate the mean square response over a large number of random phases, $\langle \phi^2 \rangle$ and multiply by the stellar flux to obtain the noise due to null fluctuations. In Figure 5, we plot as a function of wavelength the base-10 logarithm of the ratio of the null fluctuations against the maximum background of that given wavelength. This was plotted for the same two stars and optical beam combiner errors as for Figure 4. An RMS fringe tracking error of $\delta =5$ nm was assumed in this plot.

Firstly, we note that as in Figure 4, kernel 1 is never dominated by the null fluctuations, again due to it not providing as sensitive a null. We can also see the areas for which stellar leakage and zodiacal light dominate as a function of wavelength: dominant stellar leakage is represented by a flat line following the same functional form as the stellar flux, and the downwards curve represents being zodiacal light dominated. As to be expected from the previous discussion, we see that with a greater beam combiner optical error, the stellar leakage dominates for a larger part of the spectrum. The stronger leakage also washes out the effect of null fluctuations: with a strong enough optical error, these terms dominate and as such null stability becomes less important.

In Table 2, we plot the minimum fringe tracking RMS needed to remain dominated by photon noise at all wavelengths for stars of type G, K and M at 5 pc, as well as for differing amounts of beam combiner optical error. Interestingly, the RMS fringe tracking requirement is stricter than the previous calculation, being less than 1 nm when looking at G-dwarfs with no optical errors and about 6 nm for an M-dwarf. A balance needs to be struck with regards to acceptable stellar leakage and the achievable fringe tracking uncertainties. With $|\Delta R| = 5\%$, the additional stellar leakage is not increased too much, and the

![Figure 4: Base-10 logarithm of the ratio of the stellar leakage to zodiacal light against wavelength for two different stars and varying amounts of beam combiner optical error. The black dashed line divides the upper region where the combiner is dominated by stellar leakage, and the lower region where the instrument is zodiacal limited.](image)
Fig. 5: Base-10 logarithm of the ratio of the null fluctuation noise to background noise against wavelength for a fringe tracking RMS of $\delta = 5$ nm. The background noise is chosen to be the maximum of stellar leakage and zodiacal light for that given wavelength. Plotted for two different stars and varying amounts of beam combiner optical error. The black dashed line divides the maximum of stellar leakage and zodiacal light for that given wavelength. The contrast for an Earth-like planet and that systematic errors will not show up as false signals. In the mid-infrared, the contrast for a Earth-like planet is $1 \times 10^{-7}$ (Defrère et al. 2018), and so we need to ensure the error in the kernel lies below this amount. This is shown on the plots as a black dotted line. From this, it is apparent that systematic piston error needs to be kept as low as possible, especially at short wavelengths and with large optical errors. At $\lambda = 4 \mu m$, an optical error of $|\Delta R| = 2\%$ requires a systematic piston error $< 0.75 \text{ nm}$, whereas $|\Delta R| = 10\%$ requires a very stringent 0.3 nm or less. The longer wavelength plot is more lenient, suggesting a systematic error of $< 1.8 \text{ nm}$ at $|\Delta R| = 2\%$ and 0.7 nm for $|\Delta R| = 10\%$. This finding emphasises a result that has been consistent across all of these investigations: the shorter wavelengths are much more affected by systematic errors.

We also fitted a quadratic to each of the curves in Figure 6b and plotted the coefficient against the respective error amount $|\Delta R|$. This is shown in Figure 7. We identify that the coefficient of the quadratic curve scales linearly with beam combiner error, and hence show that the kernel error is overall third order in systematic errors.

3.6. Phase chopping

While kernel-nulling allows us to remove on-axis symmetric photon noise sources, as well as being resistant to second-order errors in piston (Martinache & Ireland 2015), there are still residual instrumental noise sources that could be removed through phase chopping, namely detector noise. This technique involves rapidly swapping the rows two and three, and four and five, with each other. In doing so, the kernel output remains the same but the signals are being measured on different detectors. Hence we
can remove any slowly variable detector bias or gain effects in the system.

Phase chopping with our beam combiner design is theoretically not difficult; all that is required is for each of the phase shifts in front of beam splitter modules $A_i$ through $A_{10}$ to have the sign changed. Mathematically:

$$\phi_i \rightarrow -\phi_i \quad i \in [5, 10] \quad (11)$$

This could be made to happen, for example, by putting phase shifting optics on piezo stages and rapidly moving them by a fraction of a wavelength to induce a rapid phase shift sign flip (and hence the "chop").

Of course, this induces another error term - as we are working over a large wavelength range and these changing phase shifts will only work at specific wavelength, elsewhere in the bandpass will incur a degree of chromatic phase error. This can be minimised by reducing the size of the bandpass (such as the use of multiple beam trains as described in Section [2]), by making the reference wavelength for phase chopping in the centre of the bandpass, and by designing the beamsplitter to have a wavelength independent 0 or $\pi$ phase shift. This last point allows us to halve the amount of phase shift error that would otherwise occur, as well as ensure that both outputs on either side of the phase chop have symmetrical errors.

To model this error, we split our nominal wavelength range into three, evenly spaced with regard to the amount of phase error induced at the end of the sub-bandpasses. These ranges became 4.6-7.0µm, 6.7-11.2µm and 11.2-19.0µm. The amount of error induced by the phase chop is given by:

$$\delta_i = \min(2\phi_i, 2\pi - 2\phi_i) \quad i \in [5, 10] \quad (12)$$

$$\sigma_{\phi_i} = \frac{\delta_i}{2} \left( \frac{\lambda}{\lambda_c} - 1 \right) \quad (13)$$

where $\lambda_i$ is the central (reference) wavelength of the relevant sub-bandpass, $\delta$ is the minimum change in phase required to flip the sign of the beam splitter phase shift, and $\sigma_{\phi_i}$ is the error induced by this phase chop. For the Kernel-5 beam combiner, these phase changes and the maximum error associated with them at the edge of the sub-bandpass (in radians) are:

$$\delta_5 = 2.17 \quad |\sigma_{\phi_5}| = 0.28$$

$$\delta_6 = 0.60 \quad |\sigma_{\phi_6}| = 0.08$$

$$\delta_7 = 1.62 \quad |\sigma_{\phi_7}| = 0.21$$

$$\delta_8 = 0.29 \quad |\sigma_{\phi_8}| = 0.04$$

$$\delta_9 = 0.99 \quad |\sigma_{\phi_9}| = 0.13$$

$$\delta_{10} = \pi \quad |\sigma_{\phi_{10}}| = 0.40$$

To see the effect that this chromatic phase chop error would have on the measurements, we performed the same simulation as in Section [12] except adding the relevant phase chop error to the $\Delta \phi$ of each beam splitter. This is shown in Figure 8.

We can see in these plots that the added chromatic error indeed makes the stellar leakage considerably worse for kernel 2, with kernel 1 being less affected for the same reasons as in Section [13]. We also see the effects of chromaticity - the leakage is at a minimum in the centre of each sub-bandpass (where there should be no added error) and increases to a local maximum/inflection point at the edges. The effect is quite strong at the shortest wavelengths, reducing the effect of the null by an order of magnitude. However, the second kernel is still zodiacal dominated beyond 8µm; hence this will only be a problem for the shortest wavelengths around the closest stars.

If this were deemed to be too great an error to propagate uncorrected, we could add a thin wedge of glass to a second piezo stage in front of each of the effected beam splitters that could act as a corrector for this chromatic effect. The downside for this correction is that this doubles the number of piezos and would considerably increase the beam combiner’s complexity.
4. Redundancy for Failed Telescopes

4.1. Kernel-5 nuller

One significant advantage of the “Guyon”-type beam combiner design for the Kernel-5 nuller described in Section 2, on top of the planet yield advantages discussed in LIFE6, is the ability for it to continue producing robust observable measurements even if a collecting spacecraft fails. In other words, the Kernel-5 nuller will still be able to function with only four telescopes. This is not applicable to the traditional X-array beam combiner - if one of the telescopes of that design fails, the main mission objectives for detecting Earth-like exoplanets is severely compromised.

This safeguard against a damaged telescope can be implemented through the use of a well placed shutter in the midst of the beam splitters, shown as S in Figure 1. If a collector telescope fails, all that is required is for the four operating telescopes to move into input positions two through five (that is, the failed telescope is corresponds to input V₁), and the shutter to close. We can emulate this in matrix notation through blocking beam one at the start of the relay (representing the failed telescope; F) and then blocking beam two in between beam splitting modules C₄ and C₇ (representing the shutter; S). Inserting these into equation [5]

\[
\begin{align*}
F &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix} \\
S &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \end{bmatrix}
\end{align*}
\]

\[
\tilde{M} = BA_{10}A_{9}A_{8}A_{7}A_{6}A_{5}SA_{4}A_{3}A_{2}A_{1}F
\]

(14)

\[
\tilde{M} = \frac{1}{\sqrt{5}} \begin{bmatrix} 0 & -\sqrt{5} & 0 & 0 & \sqrt{5} \\ -\sqrt{5} & 0 & 0 & \sqrt{5} & 0 \\ 0 & 0 & 1 & \sqrt{5} & -\sqrt{5} \\ \sqrt{5} & -\sqrt{5} & -\sqrt{5} & 0 & 0 \\ -\sqrt{5} & \sqrt{5} & \sqrt{5} & -\sqrt{5} & 0 \\ \sqrt{5} & -\sqrt{5} & \sqrt{5} & -\sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} & \sqrt{5} & -\sqrt{5} & -\sqrt{5} \\ \sqrt{5} & -\sqrt{5} & \sqrt{5} & -\sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} & \sqrt{5} & -\sqrt{5} & -\sqrt{5} \\ \end{bmatrix}
\]

(15)

where

\[
a = \sqrt{\frac{5}{2(5 + \sqrt{5})}}
\]

\[
b = \frac{1}{2} \sqrt{\frac{1}{2}(5 + \sqrt{5})}
\]

It is apparent from this system that output \( W₁ \) is again the bright output. What is less apparent is that outputs \( W₂ \) and \( W₃ \), and \( W₄ \) and \( W₅ \) form enantiomorphic pairs; an attribute that allows them to form a kernel-null \cite{Laugier2020}. To demonstrate this, we perform a relative phase shift at the output (that is, change \( \omega \)) so that the contribution of input 2 is always real; this should result in the kernel-null pairs becoming complex conjugates of each other. We plot the pairs of outputs in a “Complex Matrix Plot”, akin to Laugier et al. (2020), in Figure 9, where it is easily seen that the pairs are mirror images of each other. Thus, even with one telescope no longer working, the system is able to produce two kernel-null outputs.

We show the output kernel maps in Figure 10, where we have assumed that the remaining four telescopes have changed configuration into a 6:1 X-array formation as in LIFE6 and Lay (2006). We find that one kernel produces a maximum transmission of 0.65 single telescope fluxes, and the other producing 2.75 telescope fluxes (together producing an efficiency of 85% compared to the X-array, or 68% with respect to the original 5 telescopes). However, due to the cold shutter, the nullled outputs contain 0.8 times the normal background per mode. If we consider an SNR metric defined as the maximum transmission over the background per telescope, noting that the background is multiplied by a factor of \( \sqrt{2} \) due to each kernel being the difference of two outputs, we find that the total SNR of this “shuttered” beam combiner is:

\[
SNR = \sqrt{\left(\frac{2.75}{\sqrt{2}\times0.8}\right)^2 + \left(\frac{0.65}{\sqrt{2}\times0.8}\right)^2} = 2.23
\]

(17)

For the non-damaged X-array configuration, the maximum transmission is four telescope fluxes out of one output; the same SNR metric is thus 2.83. The relative SNR of the damaged array is therefore 80% of the equivalent non-damaged four telescope array architecture. This infers that this configuration, made out of necessity due to a collector telescope failure, results in only a
Angular position (\(B/B\))

20% SNR reduction compared to the default X-array beam combination architecture with 100% of its telescopes functioning. Hence this beam combiner design offers good protection against collector telescope malfunctioning. We note that the reason kernel 2 specifically contains most of the transmission is determined solely by the arrangement of telescopes in the array; a different arrangement would result in kernel 1 having the maximum transmission.

This idea can be extended further to three telescopes (that is, two telescopes failing) by implementing a second shutter between \(C_3\) and \(C_6\). This results in a transfer matrix of:

\[
\mathbf{M} = \frac{1}{\sqrt{5}} \begin{pmatrix}
0 & 0 & 1 & 1 & 1 \\
0 & \frac{\sqrt{5}}{5} + ai & \frac{\sqrt{5}}{5} - ci & -\frac{\sqrt{5}}{5} + ei \\
0 & \frac{\sqrt{5}}{5} - ai & \frac{\sqrt{5}}{5} + ci & -\frac{\sqrt{5}}{5} - ei \\
0 & -\frac{\sqrt{5}}{5} - bi & -\frac{\sqrt{5}}{5} - di & \frac{\sqrt{5}}{5} - fi \\
0 & -\frac{\sqrt{5}}{5} + bi & -\frac{\sqrt{5}}{5} + di & \frac{\sqrt{5}}{5} + fi
\end{pmatrix}
\]

where numerically

\[
\begin{align*}
a = 0.7551280988643292 & & d = 0.2707664135274219 \\
b = 0.9048040910575242 & & e = 0.3918568348616487 \\
c = 1.1469849337259779 & & f = 0.6340376775301025
\end{align*}
\]

Again, we find that there are two sets of enantiomorphic pairs, shown in a CMP in Figure 11. The resultant maps have a maximum transmission of 0.91 and 1.47 telescope fluxes, and hence an array efficiency of 79% compared to the Kernel-3 design, or 48% compared to the original Kernel-5 design. The shutters also effectively reduce the background of the outputs by a factor of 0.6, which results in an effective SNR 1.58. This is 74% of the SNR (2.12) of a three telescope combiner with all telescopes functioning. The modified combiner would therefore be adequate to continue the mission after a failure of two spacecraft.

4.2. Modified X-array

While we stated that the X-array design does not allow for this redundancy advantage, this is only the case for the traditional beam combiner design consisting of two combiners with a \(\pi\) phase shift along the nulled baseline, and then a \(\frac{\pi}{6}\) phase chop of these nulled outputs. The X-array/Bracewell design could in fact be implemented in the same “Guyon”-type beam combiner as described in Section 2.

Consider a combiner with \(m = 4\) inputs and \(n = 6\) beam splitter modules, shown in Figure 12 along with the phase shifts and reflectance parameters found in Table 3. Other than the parameters and number of inputs/outputs, this design is identical to that of the Kernel-5 nuller described in Section 2. When the parameters are inserted into equation [5] we obtain the following transfer matrix:

\[
\mathbf{M} = \frac{1}{\sqrt{4}} \begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & i & -i & -1 \\
i & 1 & -1 & -i \\
-1 & 1 & 1 & -1
\end{pmatrix}
\]

The middle two nulled rows of this matrix is equivalent to the middle two rows of the transfer matrix of the traditional X-array beam combiner found in equation 6 of LIFE6, with a different...
Table 3: Optical parameters for the beam combiner design of the X-array, discussed in Section 4.2 and displayed in Figure 12

| Mixing Angle ($\theta$) | Reflectance coefficient $|R|$ | Phase Shift ($\phi$) |
|-------------------------|-------------------------------|-----------------------|
| $C_1$ $-\frac{\pi}{4} \approx -0.785$ | $\frac{1}{\sqrt{2}} \approx 0.707$ | $\pi$ |
| $C_2$ $\arcsin\left(\frac{1}{\sqrt{3}}\right) \approx 0.615$ | $\frac{1}{\sqrt{3}} \approx 0.577$ | $\pi$ |
| $C_3$ $\frac{\pi}{6} \approx 0.618$ | $\frac{1}{2}$ | $\pi$ |
| $C_4$ $\pi - \arcsin\left(\frac{2}{\sqrt{5}}\right) \approx 2.230$ | $\sqrt{\frac{2}{5}} \approx 0.791$ | $\arctan(2) \approx 1.107$ |
| $C_5$ $- \arcsin\left(\frac{1}{\sqrt{3}}\right) \approx -0.615$ | $\frac{1}{\sqrt{3}} \approx 0.577$ | $\arctan(3) \approx 1.249$ |
| $C_6$ $-\frac{\pi}{4} \approx -0.785$ | $\frac{1}{\sqrt{2}} \approx 0.707$ | $\frac{3\pi}{4} \approx 2.356$ |

Fig. 11: Complex Matrix Plot of the “damaged” Kernel-5 beam combiner with three telescope inputs.

Thus, if a telescope was to fail in this variant of the X-array, the remaining telescopes could move into a triangular position (like in the Kernel-3 nuller of LIFE6) and the beam combiner could still produce a robust observable. Such a map is shown in Figure 14.

This map has a maximum transmission of 1.73 telescope fluxes, an efficiency of 58% compared to a fully functioning three telescope combiner, or 43% compared to the undamaged X-array. As before, the shutter will reduce the background in the nulled outputs, this time by a factor of 0.75. This results in an effective SNR of 1.41, 66% of the fully functioning Kernel-3 array. While this is substantially less than the 100% efficiency of the X-array with four telescopes, nonetheless this modified combiner would be adequate to continue on the mission in the event of a collector telescope failure.

5. Conclusion

In this work, we have provided a practical method to implement a Kernel-5 beam combiner, using a collection of adaptive nullers,
Angular position (B/B)

\[
\begin{align*}
\text{Output 1} & : & & 0 & & 0.2 & & 0.4 & & 0.6 & & 0.8 & & 1.0 & & 1.2 & & 1.4 & & 1.6 & & 1.8 & & 2.0 \\
\text{Output 2} & : & & 0 & & 0.2 & & 0.4 & & 0.6 & & 0.8 & & 1.0 & & 1.2 & & 1.4 & & 1.6 & & 1.8 & & 2.0 \\
\text{Output 3} & : & & 0 & & 0.2 & & 0.4 & & 0.6 & & 0.8 & & 1.0 & & 1.2 & & 1.4 & & 1.6 & & 1.8 & & 2.0
\end{align*}
\]

Fig. 13: Complex Matrix Plot of the “damaged” X-array type beam combiner with three telescope inputs.

Angular position (B/B)

\[
\begin{align*}
\text{Output 1} & : & & 0 & & 0.2 & & 0.4 & & 0.6 & & 0.8 & & 1.0 & & 1.2 & & 1.4 & & 1.6 & & 1.8 & & 2.0 \\
\text{Output 2} & : & & 0 & & 0.2 & & 0.4 & & 0.6 & & 0.8 & & 1.0 & & 1.2 & & 1.4 & & 1.6 & & 1.8 & & 2.0 \\
\text{Output 3} & : & & 0 & & 0.2 & & 0.4 & & 0.6 & & 0.8 & & 1.0 & & 1.2 & & 1.4 & & 1.6 & & 1.8 & & 2.0
\end{align*}
\]

Fig. 14: Kernel map of the “damaged” X-array type beam combiner with three telescope inputs.

spatial filters, beam splitters and phase shifting plates. Adaptive nullers can be used to negate any phase errors induced by imperfections in four of the beam splitting modules, leaving optical errors in the remaining six modules to contribute to errors in the remaining system, including null depth, null stability and kernel sensitivity. These also influence requirements in systematic phase offset errors of the interferometer, as well as RMS fringe tracking errors.

Taken with a beam splitter reflectance error of |ΔR| = 5%, and associated phase shift error of Δφ = 3°, we find that in order to be photon limited and not limited by null fluctuations, we require a fringe tracking error less than 3 nm RMS. Furthermore, in order for the kernels to be appropriately sensitive to planets with a contrast of 1×10^{-7} over a bandpass from 4 to 19μm, we find that the systematic phase error must be less than 0.5 nm.

We do note, however, that these limits are strongly dominated by the shorter wavelengths, and that at longer wavelengths the requirements lessen substantially. Obtaining high signal in the shorter wavelength regions (around 4 μm) will therefore prove to be harder than at longer wavelengths beyond approximately 8 μm.

We have also shown a major benefit of the described beam combiner implementation: in introducing a well placed shutter between a coupler of the beam splitter modules, the Kernel-5 combiner can function as a four telescope combiner. This is a critical advantage if a collecting telescope were to fail or go offline. If these four telescopes were then placed into an X-array configuration, this modified combiner would produce an identical map to the original X-array architecture, albeit with a total throughput penalty of 15%, split over the two kernel outputs. This is offset by a reduction in the background due to the shutter, resulting in an SNR per telescope equal to 80% of the fully functional X-array. A further telescope could also be removed with the addition of a second shutter, leading to a Kernel-3 type map with a relative SNR 74% of the equivalent Kernel-3 beam combiner. Finally, we note that the beam combiner of the X-array itself could be designed in a similar way, and providing the same benefits as the Kernel-5 nuller. If one of the X-array telescopes were to fail, a Kernel-3 type map could be created with an efficiency of 58% and relative SNR 66% compared to the Kernel-3 combiner - lower than the equivalent Kernel-5 design, but nonetheless adequate to continue scientific observations.

The next step forward would be to investigate physically constructing such a beam combiner in a laboratory, to test the assumptions about errors and uncertainties in this paper. Furthermore, a more detailed study at the opto-mechanics of injection into a beam combiner like the one described would need to be addressed, for example how four telescopes in a rectangular formation could inject into the combiner designed for five in a pentagonal formation.

The advantage of telescope redundancy, along with the sensitivity advantages as discussed in LIFE6, further adds credence to the Kernel-5 beam combiner, with five telescopes in a pentagonal configuration, as the ideal architecture for the LIFE mission. We therefore suggest that future studies consider adopting this architecture in their analysis of future science and technological requirements for space-based mid-infrared nulling interferometry.

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Data Availability

Data is available upon request to the author.

References

Beichman, C. A., Woolf, N. J., & Lindensmith, C. A. 1999, The Terrestrial Planet Finder (TPF) : a NASA Origins Program to search for habitable planets, Tech. rep., Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California

Bracewell, R. N. 1978, Nature, 274, 780

Dannert, F., Ottiger, M., Quanz, S. P., et al. 2022, arXiv e-prints, arXiv:2203.00471

Defrère, D., Abis, O., & Beichman, C. A. 2018, in Handbook of Exoplanets, ed. B. Defrère, O. Absil, & C. A. Beichman, 10.1007/978-3-319-73038-4_106, 1007-1062

Gretzinger, T., Gross, S., Arriola, A., & Withford, M. J. 2019, Optics Express, 27, 8626

Guyon, O., Mennesson, B., Serabyn, E., & Martin, S. 2013, PASP, 125, 951

Hansen, J. T., Ireland, M. J., & the LIFE Collaboration. 2022, arXiv e-prints, arXiv:2201.04891

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