Non-coaxial Deformation Characteristics of Fine Sand under Principal Stress Rotation

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Abstract. A series of pure principal stress rotation tests with dynamic hollow cylinder apparatus were conducted on saturated fine sand. The elastic torsional shear strain was reported to account for a large proportion of the total torsional shear strain, which could not be ignored. On this basis, the non-coaxial angle is redefined and the non-coaxial deformation characteristics and influencing factors of saturated fine sand are studied. The results show that the principal stress rotation will cause obvious plastic deformation of the fine sand and that the initial shear stress has a more significant influence on the non-coaxial deformation characteristics of the sand compared to the intermediate principal stress coefficient.

1. Introduction

Most of the classic constitutive sand models adopt the traditional theory of plastic flow, which assumes that the direction of plastic strain increment is consistent with the direction of stress increment [1]. However, because the above rule is derived from isotropic metal materials, it does not fully apply to geotechnical anisotropic materials. A large number of laboratory tests [2-6] demonstrate that the direction of plastic strain increment is inconsistent with the direction of principal stress increment in the process of loading, which is called the non-coaxial characteristics of the soil.

Complex dynamic loads, such as traffic and waves, will cause the principal stress axis rotation of the soil, resulting in obvious plastic deformation of the soil. Yu et al. [7] and Yang et al. [8] have reported that the non-coaxial characteristics will significantly reduce soil strength and make the soil “soft” macroscopically. Tong et al. [9] once reported that if in actual engineering the effect of principal stress rotation is not considered, the design would be dangerous. Therefore, studying the non-coaxial characteristics of the soil and its influencing factors is of great importance for building a more accurate constitutive model and guiding engineering design.

At present, it is still controversial whether elastic strain in the calculation of a non-coaxial angle should be separated. Because it is difficult to separate the elastic strain from the total strain in actual test measurement and considering that the elastic strain represents a small proportion of the total strain [10], most researches [11-13] have replaced the plastic strain increment with the total strain increment approximately; moreover, they consider the non-coaxial angle as the angle between the total strain increment direction and the principal stress direction. However, Blanc et al. [14] performed a principal stress rotation test on dry sand and reported that the elastic strain cannot be ignored for analyzing non-coaxial characteristics.

Aiming at the abovementioned controversial point, a series of Wenhe fine sand tests were performing using a hollow cylinder apparatus, the stress–strain relationship of the soil and the influence factors of non-coaxial characteristics were analyzed. In particular, in the calculation of non-coaxial angles, whether the influence of elastic strain should be ignored is addressed.
2. Test equipment and scheme

2.1. Test equipment and test soil
The test instrument used in this article is TJ-5Hz hollow cylinder apparatus of the University Tongji. The instrument can independently apply axial force, torque, internal pressure, and external pressure to control the average principal stress \( p \), shear stress \( q \), intermediate principal stress coefficient \( b \), and the principal stress direction angle \( \alpha \). In this manner, the loading of complex stress paths can be realized. The computer data acquisition system named GDSLAB provides favorable conditions for accurate measurement of soil stress and strain parameters. Hight et al. [15] provided the calculation of stress–strain parameters of the hollow cylindrical specimen.

The soil used for the tests was fine sand from Wenhe, Shandong, with a maximum dry density of 1.89 g/cm\(^3\) and a minimum dry density of 1.42 g/cm\(^3\). A hollow cylinder sample of 200 mm × 100 mm × 60 mm was prepared using a layered wet tamping method and the relative density of the fine sand was controlled to \( D_r = 65\% \).

2.2. Test plan and stress path
First, the sample is saturated with a back pressure of 100 kPa such that the pore pressure coefficient \( B \) can reach 0.95 or more to obtain a saturated sample. The shear stress \( q \) is then increased to a predetermined value to start the principal stress rotation test. The test plan is shown in Table 1. The shear stress \( q \) and the intermediate principal stress coefficient \( b \) remain constant throughout the test, and only the direction angle of the principal stress \( \alpha \) changes. Figure 1 shows the stress path on the deviatoric stress space.

Hardin et al. [16] reported that at the initial stage of the soil loading and unloading curve, the tangent modulus can be considered as the elastic modulus of the soil. To obtain an elastic modulus and a shear modulus of the soil sample for calculating the elastic strain, this study first conducts an RC test, which changes the soil between compression, torsion, and pure shear.

| Test number | Stress path       | \( p \) /kPa | \( q \) /kPa | \( b \) | \( \alpha \) |
|-------------|------------------|-------------|-------------|--------|------------|
| RC          | A-B-A (4 times)  | 100         | 25          | 0.0    | 0°–45°–0°  |
| R1          | A-B-C-D-A        | 100         | 25          | 0.0    | 0°–180°    |
| R2          | A-B-C-D-A        | 100         | 25          | 0.5    | 0°–180°    |
| R3          | A-B-C-D-A        | 100         | 25          | 1.0    | 0°–180°    |
| R4          | A-B-C-D-A        | 100         | 50          | 0.0    | 0°–180°    |
| R5          | A-B-C-D-A        | 100         | 50          | 0.5    | 0°–180°    |
| R6          | A-B-C-D-A        | 100         | 50          | 1.0    | 0°–180°    |
3. Modulus calculation
Figure 2 shows the stress–strain curve of the RC test. From this, the axial elastic modulus $E = 64.50 \text{MPa}$ can be obtained. Considering that the sample is prepared using a wet tamping method, the soil sample obtained can be regarded as elastic and isotropic, and the elastic modulus of the soil sample is $E = 64.50 \text{MPa}$; moreover, the shear modulus is calculated as $G = 25.94 \text{MPa}$.

4. Strain analysis
Figure 3 shows the relationships between the torsional shear strain and the principal stress direction angle in the R series of tests. Although the final change in the elastic torsional shear strain $\gamma_{z\theta}^e$ is 0, it fluctuates greatly during the rotation of the principal stress. Moreover, its amplitude accounts for a large proportion of the total torsional shear strain $\gamma_{z\theta}$ and cannot be ignored. The change law of plastic torsional shear strain $\gamma_{z\theta}^p$ with the principal stress direction angle $\alpha$ is the same. Unlike the elastic torsional shear strain $\gamma_{z\theta}^e$ development discipline, the plastic torsional shear strain $\gamma_{z\theta}^p$ will continue to increase in varying degrees in the later stage of the test, and this part of the deformation cannot be recovered. Moreover, it can be found that the higher the initial shear stress $q$, the greater the amplitude

Figure 1. Stress path in deviatoric stress space

Figure 2. Stress–strain curves of RC

(a) Axial  (b) Tangential

Figure 3. Strain analysis
of the elastic strain $\gamma'_{z\theta}$. Furthermore, under the same shear stress condition, the change in elastic torsional shear strain $\gamma'_{z\theta}$ is the same regardless of the value of the intermediate principal stress coefficient $b$. This indicates that the initial shear stress $q$ has a greater effect on the elastic torsional shear strain $\gamma'_{z\theta}$; however, the intermediate principal stress coefficient $b$ does not affect $\gamma'_{z\theta}$. For the plastic strain, the larger the intermediate principal stress coefficient $b$, the larger the initial shear stress $q$, the greater the absolute value of $\gamma'_{z\theta}$, and the more significant the accumulation of the plastic.

![Image](image_url)

(a) Plastic strain  
(b) Elastic strain

**Figure 3.** Variations of torsional shear strain with $\alpha$ in R

5. **Definition of non-coaxial angle**

In plastic theory, the inconsistency between the direction of plastic strain increment and the direction of principal stress increment is defined as the non-coaxial characteristics. In the measurement process, the elastic strain is difficult to separate from the total strain, and the proportion of elastic strain in the total strain is small [10]. Therefore, many researchers [11-13] approximately replace the plastic strain increment with the total strain increment and consider the non-coaxial angle to be the angle between the total strain increment direction $\beta_{de}$ and the principal stress direction $\alpha$, which is recorded as $\beta$. The schematic of $\beta$ in deviatoric stress space is shown in Figure 4, and the specific calculation is as follows:

$$\beta = \beta_{de} - \alpha$$  \hspace{1cm} (1)

$$\beta_{de} = \frac{1}{2} \arctan \left( \frac{d\gamma_{z\theta}}{d\varepsilon_z - d\varepsilon_{\theta}} \right)$$  \hspace{1cm} (2)

$$\alpha = \frac{1}{2} \arctan \left( \frac{2\tau_{\theta z}}{\sigma_z - \sigma_0} \right)$$  \hspace{1cm} (3)
Yan et al. [17] reported that the non-coaxial properties are greatly affected by the torsional shear strain. The above experiments showed that the elastic torsional shear strain $\gamma_{\theta}^\varepsilon$ accounts for a larger proportion of the total torsional shear strain $\gamma_{\theta}$ . Moreover, the elastic torsional shear strain affects the direction of the total strain by affecting the elastic strain. If the elastic strain is directly ignored in the calculation of the non-coaxial angle, and the increment direction of the total strain is used rather than the increment direction of the plastic strain, the non-coaxial characteristics of soil cannot be accurately reflected, as shown in figure 4. Therefore, the effect of elastic strain on non-coaxial is considered in this study. The non-coaxial angle is defined as the angle between the increment direction $\beta_{de}^\varepsilon$ of the plastic strain and the principal stress direction $\alpha$ , which is denoted as $\beta^\varepsilon$ . Moreover, the calculation $\beta^\varepsilon$ is as follows:

$$\beta^\varepsilon = \beta_{de}^\varepsilon - \alpha$$  \hspace{1cm} (4)

$$\beta_{de}^\varepsilon = \frac{1}{2} \arctan \left( \frac{d\gamma_{\theta}^\varepsilon}{d\varepsilon_{\theta}^\varepsilon - d\varepsilon_{\theta}^p} \right)$$  \hspace{1cm} (5)

6. Research on non-coaxial characteristics

Figure 4 shows the relationship between the non-coaxial angle $\beta^\varepsilon$ and the direction angle of the principal stress $\alpha$ . The fine sand shows non-coaxial characteristics during the pure principal stress rotation, and the development law of the non-coaxial angle $\beta^\varepsilon$ is the same under different shear stress $q$ and different intermediate principal stress coefficients $b$ . Under the same initial shear stress
condition, the non-coaxial angle $\beta^p$ will slightly decrease with an increase in the intermediate principal stress coefficient $b$. Moreover, it can be seen that when $q = 50$ kPa, the non-coaxial angle of sand is reduced significantly compared to when $q = 25$ kPa. This shows that the effect of the initial shear stress $q$ on the non-coaxial deformation characteristics of saturated fine sand is more significant than the intermediate principal stress coefficient $b$.

\[ \text{Figure 5. Variations of } \beta^p \text{ with } \alpha \text{ in R} \]

7. Conclusions
(1)During the rotation of the principal stress, the elastic torsional shear strain accounts for a large proportion of the total torsional shear strain, thus ignoring its influence and approximately replacing the direction of the plastic strain increment with the direction of the total strain increment for non-coaxial analysis will result in inaccurate results. When defining the non-coaxial angle, the effect of the elastic strain should be considered.

(2)Pure principal stress rotation will cause significant plastic strains. The larger the intermediate principal stress coefficient $b$, the higher the shear stress $q$, and the more obvious the plastic accumulation.

(3)The lower the shear stress $q$ and the smaller the intermediate principal stress coefficient $b$, the more obvious the non-coaxial characteristics of the sand. Moreover, the initial shear stress $q$ influences the non-coaxial deformation characteristics of saturated fine sand more significantly than the intermediate principal stress coefficient $b$.

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