Production and Decay of 750 Gev state of 6 top and 6 antitop quarks

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Abstract

Crude estimates are made of the branching ratios and pair production cross-section for our previously proposed bound state S of six top and six antitop quarks identified with the diphoton excess recently observed in ATLAS and CMS. We estimate the pair production cross section to be approximately 12 times that for fourth family quarks. Hence we predict $\sigma(pp \to SS + \text{anything}) \approx 2$ pb at 13 TeV and an increase by a factor 10 over the cross section at 8 TeV. Crude estimates of the main branching ratios relative to the diphoton decay give $\Gamma(S \to \gamma + \gamma) \propto 1$, $\Gamma(S \to t + \bar{t}) \propto 378$, $\Gamma(S \to \text{gluon + gluon}) \propto 117$, $\Gamma(S \to \text{Higgs + Higgs}) \propto 15$, $\Gamma(S \to W + W) \propto 30$ and $\Gamma(S \to Z + Z) = 15$. These estimates are consistent with the LHC bounds at 8 TeV within a factor 1.25. We expect the $S \to \gamma\gamma$ events to be produced together with another S resonance decaying typically into top-antitop or gluon-gluon jets.

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1 Introduction

We shall here explore the possibility that the diphoton excess in the inclusive $\gamma\gamma$ spectrum, recently found by the ATLAS and CMS collaborations [1, 2], with a mass of 750 GeV can be a bound state of particles already present in the Standard Model, namely a bound state of 6 top + 6 antitop quarks. Thus we would need no new fundamental particles, interactions or free parameters beyond the Standard Model to explain this peak, which otherwise looks like “new physics”!

For several years we have worked on the somewhat controversial idea [3, 4, 5, 6, 7, 8] that the exchange of Higgses and gluons between 6 top and 6 antitop quarks provides sufficiently strong attraction between these quarks for a very light (compared to the mass of 12 top quarks) bound state $S$ to be formed. The 6 tops + 6 antitops are all supposed to be in the 1s state in the atomic physics notation and, because of there being just 3 colors and 2 spin states for a top-quark, this is the maximum number allowed in the 1s shell.

Further speculations around this bound state were mostly built up under the assumption of a hoped for new principle – the multiple point principle [9, 10, 11] – from which we actually predicted the mass of the Higgs boson long before it was found [12]. This principle says that there shall be several phases of space (i.e. several vacua) with the same energy density. One of these should have a condensate of the bound states $S$. It was even speculated then that such a condensate – or new vacuum – could form the interior of balls, containing highly compressed ordinary matter, which make up the dark matter [13, 14, 15]. Thus the discovery, if confirmed, of the bound state $S$ could support a theory, in which dark matter could be incorporated into a pure Standard Model theory, only adding the multiple point principle, which predicts the values of coupling constants but otherwise without new physics.

It is the main goal of the present article to crudely estimate the relative decay rates for the important channels and the pair production rate of the particle $S$. The experimental cross sections $\sigma(p + p \to \gamma + \gamma + \text{anything})$ for the diphoton excess at 750 GeV are $(6 \pm 3) \text{ fb}$ at CMS and $(10 \pm 3) \text{ fb}$ at ATLAS for $\sqrt{s} = 13 \text{ TeV}$, while at 8 TeV they are respectively $(0.5 \pm 0.6) \text{ fb}$ and $(0.4 \pm 0.8) \text{ fb}$ [1, 2, 16, 17, 18]. For a singly produced resonance by gluon fusion, one expects a ratio $r = 5$ for the production cross section at 13 TeV compared to that at 8 TeV. For pair production, like in our model, one rather expects a ratio of $r = 10$, which fits the data better.

2 Decay Diagrams

Of course whatever the outgoing particles from the decay, they must somehow couple to the (anti)top quarks that are the constituents of the bound state $S$. Also with the large number of constituents it is needed that the majority of these constituents simply disappear in annihilations, in most cases into nothing – not even energy and momentum.

Let us consider the construction of a diagram for such a decay, by first ignoring the outgoing particles. So, we imagine an annihilation for each top quark having some spin and color with the antitop quark having just the opposite spin and color. This can be
represented by a blob symbolizing the bound state (Bethe Salpeter) wave function, with six loops attached to it representing the 6 top quarks annihilating with the 6 antitop quarks. A realistic decay diagram is now formed by attaching an outgoing line to one of the six loops for each of the final state particles. The important decay channels contain two final state particles, which can be attached to the same loop or to two different loops. The top + antitop decay mode has to be treated separately.

However it is important to take into account that several of the decay products carry global conserved quantum numbers which, in the two loop case, need to be transferred from one top antitop annihilation loop to the next one. For instance the Higgs carries weak $SU(2)$ charge (also weak hypercharge). Only the photon, or rather its and $Z^0$'s component coupling to the weak hypercharge, is the exception and carries no global conserved charge (apart from charge conjugation). In order to achieve such a transfer of the global charge from one loop attached to a final state particle carrying the charge to the loop attached to the other final state particle, another particle (e.g. a Higgs particle) has to be exchanged between the loops.

We shall postpone the discussion of the particles having a global conserved charge until section 3.2 and consider here the main factors, in the diphoton decay, suppressing the amplitude for both photons coming via the same loop $T_1$ and via 2 different loops $T_2$ respectively.

- **Suppression factor $T_1$ for one loop giving the final state**

In order that a pair of (anti)top quarks – with the compensating color and spin of course – can disappear into zero four momentum and zero spin, they should be sufficiently close, i.e. at the same point, or at least within the distance of a top quark Compton wavelength. Thus the amplitude for such total annihilation has to be proportional to the amplitude for finding them at the same point. We may get an idea of this amplitude by a dimensional argument considering the wave function for the position of a top quark relative to its compensating antitop quark $\Psi(\vec{r})$. In a Gaussian wave function ansatz, we can write such a wave function as the product of one associated with each of the three dimensions $\Psi(\vec{r}) = \psi(x)\psi(y)\psi(z)$. For these wave functions we need an estimate of the radius $b/m_t$ of the bound state. In our earlier work \[5\] we estimated $b \sim 1$ from the requirement that the bound state should bind to have a small mass compared to the sum of the constituent quark masses. This estimate was refined a bit further in \[19\] to give $b = 2.3$. The definition of the radius $b/m_t$ which we use is that

$$<x^2> = \frac{<r^2>}{3} = \frac{b^2}{m_t^2} \tag{1}$$

where this $x$-fluctuation should be that of a single (anti)top relative to the whole center of mass. So the normalized wave function along the x-axis for one constituent becomes

$$\psi(x) = \sqrt{\frac{m_t}{\sqrt{2\pi}b}} \exp\left(-\frac{m_t^2}{4b^2}x^2\right). \tag{2}$$
The relative wave function for two (say compensating) constituents is given by the convolution in x-space of two such wave functions:

$$\psi_{\text{rel}}(x) = \sqrt{\frac{m_t}{2\sqrt{\pi}b}} \exp\left(-\frac{m_t^2}{8b^2}x^2\right). \quad (3)$$

The amplitude for finding the two (anti)top quarks with the same x coordinate is thus $$\psi_{\text{rel}}(0) = \sqrt{\frac{m_t}{2\sqrt{\pi}b}}$$. Assuming, on dimensional grounds, that the top mass gives the relevant scale, we extract the typical suppression factor

$$\xi_x = \psi_{\text{rel}}(0)/\sqrt{m_t} = \sqrt{\frac{1}{2\sqrt{\pi}b}}. \quad (4)$$

for the annihilation of the pair of (anti)top quarks having this relative wave function. The amplitude for the decay of the whole bound state gets suppressed by such a factor for each totally annihilating pair and for each dimension of space. Thus the amplitude of suppression from one annihilating pair becomes

$$\xi = \xi_x^3 \xi_y \xi_z = \left(\sqrt{\frac{1}{2\sqrt{\pi}b}}\right)^3 = (4\pi b^2)^{-3/4} = 0.043. \quad (5)$$

The one loop decay has one more pair annihilating into the vacuum than the two loop decay and is thus suppressed by an extra factor of $$\xi$$.

When the various (anti)top pairs annihilate, they can deliver their full energy to one loop or another from which the final state particles arise. In the one loop case all the 6 pairs deliver their energy to the surviving pair and this has a combinatorial chance of $$(1/6)^6$$. On the other hand in the case of two loops, each surviving pair having got contributions of energy from 3 pairs, we would have the probability

$$\frac{1}{10} = (1/6)^6 * \left(\frac{6}{3}\right) / 2 = (1/6)^6 * 10. \quad (6)$$

Thus there is a combinatorial suppression by a factor of 1/10 of the one loop decay relative to the two loop decay.

Since the six (anti)top pairs in the bound state are bosons, in states that only differ by color and spin, the interference terms between the decay amplitudes arising from various permutations of these pairs should get the same phase. The resulting constructive interference means that the combinatorial factor, 1/10, suppressing the one loop case relative to the two loop case should apply to the decay amplitude.

Collecting together the above two suppression factors, we get an overall suppression factor for the one loop case of

$$T_1 = \xi/10 = 0.0043. \quad (7)$$
• **Suppression factor** \( T_2 \) **for two loops giving the final state**

If both final state particles originate from the *same* (anti)top quark annihilation pair the large spatial momentum of these particles is achieved by the exchange of a fundamental particle (a top quark) in the loop between them, while their collected three momentum is just zero. So, in the one loop case, the bound state wave function causes no severe suppression of the emission amplitude.\(^1\) If, however, the two final state particles arise from two *different* (anti)top quark annihilations, the final large spatial momentum of the order of \( m_S/2 \) for each of the decay particles has to result from the momentum of the antitop and top quarks annihilating into that final state particle. This means that, in momentum representation, the emission amplitude is proportional to the momentum wave function at the momentum being say \( \vec{p} = (m_S/2, 0, 0) \). We must now estimate the width of the momentum distribution for the two (anti)top pairs, which emit the final state particles, after the other 4 pairs have annihilated into the vacuum. So we imagine that three of the (anti)top pairs give their momenta to one final pair, while the other three give their momenta to the other final pair. This would mean that the momentum distribution of one of the final pairs, or equivalently of one final state particle, is given by the convolution of the momentum distributions for 3 pairs, meaning 6 (anti)top quarks. Fourier transforming the single constituent wave function \((2)\), we obtain

\[
\tilde{\psi}(p_x) \propto \exp \left( -\frac{p_x^2 b^2}{m_t^2} \right). \tag{8}
\]

The distribution of the final state momentum, prior to its emission, is like that of the sum of 6 single constituent momenta \( \vec{p}_{fin} = \sum_{j=1}^{6} \vec{p}_j \). Thus we obtain the dependence of the final state particle wave function on \( p_{x\ fin} \):

\[
\tilde{\psi}_{outgoing}(p_{x\ fin}) \propto \exp \left( -\frac{p_{x\ fin}^2 b^2}{6m_t^2} \right). \tag{9}
\]

Substituting \( p_{x\ fin} = m_S/2 \) into \((9)\), we obtain the wave function suppression amplitude \( T_2 \) for the case of the two final state particles (each with its own wave function) coming from two *different* annihilating pairs to be:

\[
T_2 = \left[ \exp \left( -\frac{(m_S/2)^2 b^2}{6m_t^2} \right) \right]^2 = [\exp(-4.1)]^2 = 0.00025 \tag{10}
\]

Now, however, the Gaussian wave function form is *not trustable* for such a large momentum. Rather we should use a more realistic form of the wave function like the one from say an atomic s-wave. The Fourier transform of such a more realistic wave function has the form

\[
\tilde{\psi}(\vec{p}) \propto \left( \frac{1}{1 + \frac{1}{2} \ast \frac{(\vec{p}^2 b^2)}{6m_t^2}} \right)^2. \tag{11}
\]

\(^1\)The effects of a residual suppression could be incorporated by reducing the value of \( T_1 \) and hence \( \epsilon \) in eq. \((13)\) which, in any case, has a large uncertainty.
Inserting the momentum value $|\vec{p}| = m_S/2$ for the decay we obtain, for the atom-like wave function form, the suppression factor

$$T_2 = \left(1 + \frac{1}{2} \times \frac{(m_S/2)^2 b^2}{6m_t^2}\right)^{-4} = 0.106^2 \approx 0.011.$$  

(12)

Thus the ratio of the diphoton decay amplitude via one loop to the photon pair decay amplitude (or better the component of the photon coupling to the weak hypercharge) via two loops becomes:

$$\epsilon = \frac{T_1}{T_2} = \frac{0.0043}{0.011} = 0.39.$$  

(13)

To estimate the uncertainty in the ratio $\epsilon$ we take the parameter $\xi_x$, giving $\xi = \xi_x^3$, which is estimated only by dimensional arguments, to have an uncertainty of the order of a factor 2. The counting of the combinatorics for getting the energy collected on the one or two pairs we take to have an uncertainty by a factor $10/3 = 3.3$ in the amplitude (allowing $\left(\begin{array}{c}6 \\ 3 \end{array}\right)$ to be replaced by $\left(\begin{array}{c}4 \\ 2 \end{array}\right)$ in eq. (6). The wave function(s) we take to be uncertain by a factor 2 in the exponent, meaning by a factor $\exp(4.1/2)$. Adding the squares of the logarithms of these uncertainties leads to the total uncertainty for our estimate of the ratio $\epsilon$

$$\Delta(\ln \epsilon) = \sqrt{(\ln 8)^2 + (\ln 3.3)^2 + (4.1/2)^2} = \sqrt{4.3 + 1.4 + 4.2} = \sqrt{9.9} = 3.1.$$  

(14)

This means that that this ratio $\epsilon$ is uncertain by a factor $\exp(3.1) = 20$.

### 3 Branching Ratios

We shall now estimate the decay rates of the bound state $S$ into the important 2 body channels relative to the diphoton decay rate. Firstly we consider the one loop decay mechanism, which will be dominant for large values of $\epsilon = T_1/T_2$ and then the two loop decay mechanism which will be dominant for small values of $\epsilon$.

#### 3.1 One Loop Case

The main decay channels, apart from the $S \rightarrow t \bar{t}$ decay, correspond to the same diagram structure differing only in the couplings of the final state particles to the annihilating top antitop loop. So their relative decay rates should essentially be given by the ratios of the coupling strengths involved. We shall now introduce effective couplings for the various final state particles, analogous to the fine structure constant $\alpha = e^2/4\pi$ in QED. These effective couplings are used in Table [1] to roughly estimate the relative decay rates of the bound state $S$, via the one loop mechanism, into the dominant 2 body final states.

**Photon and transverse Z.** The electric charge of the top quark is $q = 2e/3$ and the effective coupling for of the photon to the $t \bar{t}$ loop is $4\alpha/9$. 


Table 1: Assuming dominance of one top antitop pair giving the final state, relative predictions are given for the partial decay widths of $S$ and for the branching ratios relative to the diphoton decay width compared to the experimental upper bounds from ref. [18]. In our model $r \approx 10$.

The corresponding effective coupling of $Z$ to the $t\bar{t}$ loop is

$$\frac{\alpha}{2\sin^2 \theta_W \cos^2 \theta_W} \left[ \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right)^2 + \left( - \frac{2}{3} \sin^2 \theta_W \right)^2 \right] = \frac{4\alpha}{9} \cdot 0.92. \quad (15)$$

We take $\alpha = 1/129$ and the Weinberg angle to be given by $\sin^2 \theta_W = 0.23$.

**Gluon.** The vertex for a gluon of color $i$ coupling to a top quark is $g_s \lambda^i/2$. Averaging over the colors of the top quark, the effective coupling of the gluon to the $t\bar{t}$ loop becomes

$$\frac{\alpha_s}{3} T_R \left( \frac{\lambda^i}{2} \right)^2 = \frac{\alpha_s}{6}. \quad (16)$$

We take $\alpha_s = 0.1$ and then sum over the 8 color states of the gluon.

**Higgs and longitudinal $W^\pm$ and $Z^0$.** According to the Goldstone Boson Equivalence Theorem [20], in the high energy limit the couplings of the longitudinal $W^\pm$ and $Z^0$ become equal to those of the corresponding eaten Higgs fields. The Higgs field coupling to the $t\bar{t}$ loop is

$$\alpha_h = \frac{g_t^2/2}{4\pi} = 0.035 \quad (17)$$

where $g_t$ is the top quark Yukawa coupling constant. Also the Higgs field has only one helicity state unlike the transverse gauge bosons and the top quark.

**Transverse $W^\pm$.** The $W^\pm$ gauge fields are formed from two real fields, $W_1$ and $W_2$, lying in the adjoint representation of SU(2). So their effective coupling to the $t\bar{t}$ loop is

$$\frac{1}{2} \frac{\alpha}{\sin^2 \theta_W} \left( \left( \frac{\sigma^i}{2} \right)^2 \right)_{t_L t_L} = \frac{\alpha}{8 \sin^2 \theta_W} = 0.54\alpha, \quad (18)$$

| Final state $f$ | Bound | Relative prediction | $\Gamma(S \rightarrow f)/\Gamma(S \rightarrow \gamma\gamma)$ | Comment |
|-----------------|-------|---------------------|--------------------------------------------------|---------|
| $\gamma\gamma$ | $< 0.8(r/5)$ | $(4\alpha/9)^2 = 1.2 \times 10^{-6}$ | 1 | |
| gluon + gluon   | $< 1300(r/5)$ | $8(\alpha_{s}/6)^2 = 2.3 \times 10^{-3}$ | 190 | |
| Higgs + Higgs   | $< 20(r/5)$ | $\alpha_{h}^2/4 = 3 \times 10^{-4}$ | 25 | Higgs-particles |
| ZZ              | $< 6(r/5)$ | $\alpha_{h}^2/2 = 6 \times 10^{-4}$ | 50 | longitudinal |
| WW              | $< 20(r/5)$ | $\alpha_{h}^2/4 = 3 \times 10^{-4}$ | 25 | longitudinal |
| $Z\gamma$       | $< 2(r/5)$ | $(4\alpha/9)^2 \times 0.92 = 2.2 \times 10^{-5}$ | 1.8 | |
| ZZ              | $< 6(r/5)$ | $(4\alpha/9)^2 \times (0.92)^2 = 1.0 \times 10^{-5}$ | 0.8 | transverse |
| WW              | $< 20(r/5)$ | $2(0.54\alpha)^2 = 3.5 \times 10^{-5}$ | 3 | transverse |
| top + antitop   | $< 300(r/5)$ | $3\alpha_{h}^2 = 5.9 \times 10^{-3}$ | 494 | |
| $\Gamma_{total}(S)/\Gamma(S \rightarrow \gamma\gamma)$ | | | 791 | |
where the extra factor of 1/2 is due to $W^\pm$ only interacting with left-handed top quarks. The final sum over $i = 1, 2$ gives a factor of 2 in the decay rate.

**Top antitop.** In principle one top and one antitop quark can simply escape, while the other 5 pairs annihilate into the vacuum. However this would happen with an amplitude suppressed by the square of the wave function for finding each of the escaping quarks having spatial momentum $m_S/2$, i.e. suppressed by the factor $T_2$ of eq. [12] If the top and antitop exchange a gluon between themselves, as they escape, they can get most of the kinematically needed spatial momentum exchanged through that gluon propagator and thus the emission could occur at first from where the wave function is not so suppressed. We assume that, crudely, having such an exchange of some fundamental particle essentially eliminates the suppression due to the wave function, as we do for the other one loop processes (see footnote 1). Thus in amplitude we expect that, in compensation for the $\alpha_s$ factor due to the gluon exchange, the wave function suppression factor $T_2$ can be dropped. So, for the top antitop decay we consider a one loop diagram consisting of a gluon exchange between the quarks. We therefore assume that the ratio of the top antitop decay rate to the other one \( t\bar{t}\) loop decays is well estimated by taking the ratio of the coupling strengths involved. The gluon exchange diagram involves the quadratic Casimir of the quark representation (rather than the index involved in the $S \rightarrow gg$ decay) and the effective coupling for the production of a quark of color $a$ becomes

$$\alpha_{t\bar{t}} = \frac{\alpha_s}{3} \sum_i \left( \left( \frac{\lambda^i}{2} \right)^2 \right)_{aa} = \frac{4\alpha_s}{9}. \quad (19)$$

Finally we sum over the 3 color states of the quark.

### 3.2 Two Loop case

We now consider the case where the two final state particles are emitted by two different quark pairs. The gluon-gluon decay channel is special, because the gluons can be emitted from two “crossed” loops, in which the two (anti)top quark pairs swap color indices. For the other channels (apart from $S \rightarrow \ell t$ decay) the two (anti)top pairs separately annihilate into a colorless final state particle. However, for particles with a global conserved quantum number (i.e. all the other relevant final state particles except for the hypercharge coupling part of the photon or of the $Z^0$) the emission from two different pairs is forbidden, unless a particle carrying the global quantum number is exchanged between the two pairs. This means that a diagram for e.g. $S \rightarrow Higgs + Higgs$ will, in addition to the $6 - 2 = 4$ pairs annihilating into the vacuum, have two (anti)top pairs coming out of the bound state vertex, exchanging a Higgs particle and producing the 2 final state Higgs particles.

The diagrams for the emission of colorless final state particles with global charges all look very similar and even similar to the diagram for $S \rightarrow t + \bar{t}$ decay, in which an inner top quark loop exchanges a gluon with each of the outgoing $t$ and $\bar{t}$ quarks. They namely all have three loops in addition to the four (anti)top loops annihilating into nothing. Thus we assume that these various diagrams give decay rates essentially in the ratios of the coupling strengths involved. We note that the wave function suppression factor $T_2$ is
Table 2: Assuming dominance of two top antitop pairs giving the final state, relative predictions are given for the partial decay widths of $S$ and for the branching ratios relative to the diphoton decay width compared to the experimental upper bounds from ref. [18]. In our model $r \simeq 10$.

| Final state $f$ | Bound | Relative prediction | $\Gamma(S\to f) / \Gamma(S\to \gamma\gamma)$ | Comment |
|----------------|-------|---------------------|-----------------------------------------|---------|
| $\gamma\gamma$ | $< 0.8(r/5)$ | $(0.236\alpha)^2 = 3.35 \times 10^{-6}$ | 1 | |
| gluon + gluon   | $< 1300(r/5)$ | $8(\alpha_s/18)^2 = 2.5 \times 10^{-4}$ | 74 | |
| Higgs + Higgs   | $< 20(r/5)$ | $\alpha_h^4/(4T_2) = 3.4 \times 10^{-5}$ | 10 | Higgs-particles |
| ZZ              | $< 6(r/5)$ | $\alpha_h^4/(4T_2) = 3.4 \times 10^{-5}$ | 10 | longitudinal |
| WW              | $< 20(r/5)$ | $\alpha_h^4/(2T_2) = 6.8 \times 10^{-5}$ | 20 | longitudinal |
| $Z\gamma$      | $< 2(r/5)$ | $2(0.236\alpha)^2 \tan^2 \theta_W = 2.0 \times 10^{-6}$ | 0.6 | |
| ZZ              | $< 6(r/5)$ | $(0.236\alpha)^2 \tan^2 \theta_W = 3.0 \times 10^{-7}$ | 0.09 | transverse |
| WW              | $< 20(r/5)$ | $2(0.54\alpha)^4/T_2 = 6 \times 10^{-8}$ | 0.02 | transverse |
| top + antitop   | $< 300(r/5)$ | $3\alpha_h^4/T_2 = 1.06 \times 10^{-3}$ | 316 | |

$\Gamma_{total}(S)/\Gamma(S\to \gamma\gamma): = 432$

Partially relaxed by the additional particle exchanged between the two loops. However the emission vertices for the two final state particles are connected by 3 propagators in series rather than by only one propagator as in the one loop case. The transfer of the final state momentum through these 3 propagators between the vertices is less effective and thus the relevant momenta for the bound state wave function are larger than in the one loop case. We assume that, crudely, the wave function suppression factor $T_2$ is thereby relaxed to a factor of $\sqrt{T_2}$ and not replaced by unity as was done in the one loop case. On the other hand, the diphoton and gluon-gluon decay amplitudes retain the full wave function suppression factor $T_2$.

The diagrams for the emission of the weak hypercharge coupled components of the photon and $Z^0$ and the “crossed” loop diagram for gluon decay are a bit simpler, with only two loops in addition to the 4 annihilation into nothing loops. So we have to adopt a rule to compare the two loop diagrams with the three loop diagrams. We shall assume that the addition of a loop to a diagram roughly leads to an extra fine structure constant factor $\alpha_x$ appropriate to the exchanged particle times the old diagram. We could of course have used another expansion parameter instead of $\alpha_x$; for example $\alpha_x/\pi$ which would have led to a reduction in the Higgs + Higgs, ZZ, WW and top + antitop decay branching ratios in Table 2 by an order of magnitude. The gluon + gluon branching ratio does not suffer from this uncertainty.

Let us now consider the effective couplings to be used in Table 2 to roughly estimate the relative decay rates of the bound state $S$, via the two loop mechanism, into the dominant two body final states.

**Photon and transverse Z.** The hypercharge coupled superposition of the photon and $Z^0$ is described by the field $B_\mu = \cos \theta_W A_\mu - \sin \theta_W Z_\mu$. It couples with an average squared charge $[(2/3)^2 + (1/6)^2]/2 = 0.236$ to a top quark. The two loop diphoton decay is dominated by the production of this $B_\mu$ component and so the effective coupling for the photon is $0.236\alpha$. 

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| Final state $f$ | Bound $\Gamma(S\rightarrow f)$ | $\Gamma(S\rightarrow \gamma\gamma)$ | Comment |
|-----------------|-----------------------------|-----------------|---------|
| $\gamma\gamma$  | $<0.8(r/5)$                 | 1               |         |
| gluon + gluon   | $<1300(r/5)$                | 117             |         |
| Higgs + Higgs   | $<20(r/5)$                  | 15              | Higgs-particles |
| $ZZ$            | $<6(r/5)$                   | 15              | longitudinal |
| $WW$            | $<20(r/5)$                  | 30              | longitudinal |
| $Z\gamma$       | $<2(r/5)$                   | 1.0             |         |
| $ZZ$            | $<6(r/5)$                   | 0.3             | transverse |
| $WW$            | $<20(r/5)$                  | 1.1             | transverse |
| top + anti top  | $<300(r/5)$                 | 378             |         |

$\Gamma_{total}(S)/\Gamma(S\rightarrow \gamma\gamma):$ 558

Table 3: Benchmark model with $\epsilon^2 = 0.15$. Predictions are given for the decay branching ratios of $S$ relative to the diphoton decay width and compared to the experimental upper bounds from ref. [18]. In our model $r \simeq 10$.

The corresponding effective coupling of $Z$ is $0.236\alpha\tan\theta_W$.

**Gluon.** Averaging over the colors of the two (anti)top pairs, the effective coupling of a gluon of color $i$ for the “crossed” diagram is

$$\frac{\alpha_s}{g} Tr \left( \frac{\lambda_i}{2} \right)^2 = \frac{\alpha_s}{18}.$$  

(20)

**Higgs, longitudinal $Z^0$, $W^\pm$ and top antitop.** We use the same effective couplings as in the one loop case.

### 3.3 Benchmark Model

We estimated the ratio of the photon pair decay amplitude via one loop to the hypercharged component of the photon pair decay amplitude via two loops in eq. (13) to be $\epsilon = 0.39$, but with an uncertainty of order 20. So it is not clear which mechanism dominates. We take, as a benchmark model, the combination of the two loop decay rates from Table 2 plus $\epsilon^2 = 0.15$ times the one loop decay rates from Table 1, neglecting possible interference terms between the two decay mechanisms, and calculate the corresponding relative decay branching ratios given in Table 3.

### 4 The Production Cross Section

Our bound state particle $S$ would be produced in pairs by the gluon fusion process $gg \rightarrow SS$. (We think that producing just one $S$ would be very suppressed by essentially our $\xi$ factors.) We shall estimate the cross section for this process by using an impulse approximation, according to which the two gluons interact with just one quark inside the bound state.
Let us first consider the scattering process related to $gg \rightarrow SS$ by crossing symmetry: the scattering of a gluon by the bound state $S$ in the impulse approximation [21]. That is to say we take it, that a gluon scatters on just one of the constituents. The scattering amplitude is then written as a matrix element of the constituent density in the bound state $S$ multiplied by the amplitude for the gluon scattering on one constituent. The matrix element of the density of constituents, which we call the form factor $F(p^2)$, is the Fourier transform of the spatial density of constituents in $S$. Since there are twelve constituent (anti)top quarks we have $F(0) = 12$. Here the four momentum $p$ – which in the $gS \rightarrow gS$ process is purely space-like – is that delivered from the gluon to the $S$. Next we invoke crossing symmetry and analyticity to continue the amplitude for this $gS \rightarrow gS$ process into the process of two gluons pair creating two $S$’s: $gg \rightarrow SS$. The important four momentum $p$ for the form factor $F(p^2)$ thereby gets continued from its spatial value for the $gS \rightarrow gS$ scattering to a time-like value for the $gg \rightarrow SS$ pair production process. In the space-like region for $p$ the form factor $F(p^2)$ will typically fall off as $p^2$ becomes numerically bigger, since the density is typically somewhat smooth and its Fourier transform falls for large (space-like) four momenta. A naive analytical continuation therefore suggests that going into the opposite direction in the variable $p^2$ – namely the time-like direction – the form factor would grow with larger and larger numerical time-like $p$ until it meets a singularity. This is what one sees in cases like annihilation of say $e^+ + e^-$ into two pions [22] or into proton antiproton [23]. In the case of pions the form factor $F(p^2)$ actually grows with increasing $p^2$ even above the threshold for producing the two pions; it first meets its effective singularity at the $\rho$-resonance, after which the form factor turns down for further increase in $p^2$. For proton antiproton the singularities are met already below the threshold, so that above the threshold the form factor is already decreasing with numerically raising $p^2$. The $S$ states have rather strong Higgs fields around them and hence we expect significant (possibly resonant) dispersive contributions to the $S$ form factor from the SS channel above threshold. So we expect the form factor for $S$ in the region above the threshold for pair production to be within a factor of 10 or so from its value $F(0) = 12$ at $p^2 = 0$. However we have to take into account that the twelve different quarks and antiquarks have rather different colors. So certain combinations of the gluon color states will not interact with certain quarks and some couplings will get alternating signs. It follows that the interference terms between amplitude contributions from the gluons attaching to different (anti)quarks have complicated signs. Hence we have decided to ignore the interference terms. So the square of the sum of the 12 amplitudes from each of the constituent quarks gets approximated by the sum of the numerical squares of the 12 terms individually.

The actual estimation of the final cross section is done by making use of the already calculated production cross section for a fourth family (vectorlike) quark [24]. Thus we predict the production cross section of our bound state pair should be 12 times the pair production cross section for a fourth family quark having a mass of 750 GeV, but with an order of magnitude uncertainty. The mass of 750 GeV is needed, rather than the top quark mass, in order to give the correct kinematics for the $gg \rightarrow SS$ process. Using the quark pair production cross sections calculated in [24], we predict the cross section $\sigma(pp \rightarrow SS + anything)$ to be $12 \times 0.02 \sim 0.2$ pb at $\sqrt{s} = 8$ TeV and $12 \times 0.2 \sim 2$ pb at $\sqrt{s} = 14$ TeV. The total decay width relative to the diphoton decay width $\Gamma_{total}(S)/\Gamma(S \rightarrow \gamma\gamma)$
differs by less than a factor of 2 between the one loop and two loop mechanisms. Taking the benchmark value of $\Gamma_{\text{total}}(S)/\Gamma(S \to \gamma\gamma) = 558$, we predict the production cross section for $S$ decaying to diphotons at $\sqrt{s} = 13$ TeV to be $2/558 \text{ pb} \simeq 4 \text{ fb}$, consistent with the cross sections observed by ATLAS and CMS.

5 Conclusion

We have argued that the recently observed excess of diphoton events with a mass of 750 GeV can be identified with the decay of our proposed bound state $S$ of 6 top quarks and 6 antitop quarks [3, 4, 5], without violating any experimental bounds for alternative decays. In our model the resonance $S$ is produced in pairs by two colliding gluons. We estimate the pair production cross section, using the impulse approximation, to be roughly a factor of 12 – the number of (anti)top constituents – times bigger than the production cross section for a pair of fourth family (vectorlike) quarks. This leads to an $S$ production cross section of 2 pb at 13 TeV. Using the benchmark model of Table 3 for the decay branching ratios, a diphoton production cross section of order 4 fb is obtained in agreement with ATLAS and CMS. We also expect the diphoton excess events to be accompanied by another $S$ resonance decaying typically into top + antitop or gluon + gluon jets. This of course applies to the other decay modes as well as the diphoton decay.

We have considered two decay mechanisms: one loop decay in which one annihilating top antitop loop emits both final state particles and two loop decay in which the two final state particles come from different top antitop annihilating pairs. The decay by a simple escape of a top and an antitop, each carrying spatial momentum $m_S/2$, is suppressed by the bound state wave function. This suppression is ameliorated by gluon exchange and the top + antitop decay mode has the largest branching ratio in both cases. The one loop decay rate relative to the two loop decay rate is determined by the parameter $\epsilon^2$, which we estimated for our benchmark model to be 0.15 but with a huge uncertainty factor of order 400.

We note that the Higgs + Higgs and the longitudinal WW and ZZ decay modes are close to their experimental bounds for both the one loop and two loop decay mechanisms. It should also be remarked that the total decay amplitude for our bound state $S$ is suppressed by 4 or 5 factors of $\xi \sim 0.04$ (one for each (anti)top pair annihilating into nothing) and thus a decay rate suppressed by a factor of $\xi^8$ or $\xi^{10}$. Hence, in our model, the resonance $S$ would be very narrow with a width much less than the energy resolution at LHC.

It must be admitted that our estimates of the branching ratios and cross section are indeed very crude and subject to large uncertainties, but it is quite difficult to do much better. Nevertheless it is in principle possible in our scheme to calculate everything concerning the new particle, because our model is fundamentally nothing but the Standard Model!
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