On minimal $Z'$ explanations of the $B \to K^* \mu^+ \mu^-$ anomaly

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Recently LHCb has announced a discrepancy of 3.7σ in one of the theoretically clean observables accessible through studies of angular correlations in $B \to K^* \mu^+ \mu^-$. We point out that in the most minimal $Z'$ setup that can address this anomaly there is a model-independent triple-correlation between new physics (NP) in $B \to K^* \mu^+ \mu^-$, $B_s^-\bar{B}_s$ mixing, and non-unitarity of the quark-mixing matrix. This triple-correlation can be cast into a simple analytic formula that relates the NP contribution $\Delta C_9$ to the Wilson coefficient of the semileptonic vector operator to a shift in the mass difference $\Delta M_{B_s}$ and a violation of $|V_{us}|^2 + |V_{ub}|^2 + |V_{ub}|^2 = 1$. In contrast to the individual observables the found relation depends only logarithmically on the $Z'$ mass. We show that that our findings allow for useful future tests of the pattern of NP suggested by the $B \to K^* \mu^+ \mu^-$ anomaly.

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Introduction

The latest LHCb measurements of the decay distributions in $B \to K^* \mu^+ \mu^-$ display several deviations from the standard model (SM) predictions $\textsuperscript{11}$ $\textsuperscript{12}$. With 3.7σ the most significant discrepancy arises in the variable $P_5'$ (the analogue of $S_5$ in $\textsuperscript{13}$), which combines theoretical and experimental benefits, while retaining a high sensitivity to NP effects in $b \to s \gamma, t\ell^-\ell^+$. Further LHCb studies combined with a critical assessment of theoretical errors (see in particular $\textsuperscript{14}$) will be necessary to clarify whether the observed deviations are a real sign of NP or simply flukes.

Shortly after the LHCb announcement, the new results have been combined with existing data on other rare and radiative $b \to s$ modes into global fits $\textsuperscript{15}$ $\textsuperscript{17}$. The most surprising outcome of the analysis $\textsuperscript{16}$ is that the whole pattern of deviations seen by LHCb can be explained by adding a single effective interaction of the form $\textsuperscript{1}$

$$\mathcal{L}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{4\pi} V_{ts}V_{tb} C_9 (\bar{s}\gamma_\mu P_L b) (\bar{\mu}\gamma^\alpha_\mu) + \text{h.c.},$$

(1)

to the SM Lagrangian if the NP effects $\Delta C_9$ in the Wilson coefficient $C_9 = C_9^{\text{SM}} + \Delta C_9 \approx 4.1 + \Delta C_9$ $\textsuperscript{18}$ are large and destructive $\textsuperscript{1}$

$$\Delta C_9 \sim -1.5.$$ \hspace{1cm} (2)

This solution is intriguing not only because it is pure and simple but also because it is highly non-standard and cannot be obtained – at least to our knowledge – in the most common NP models such as supersymmetry, extra dimensions or partial compositeness (the study $\textsuperscript{19}$ confirms this naive expectation).

An obvious though ad hoc way to obtain $\textsuperscript{2}$ is to postulate the existence of a new neutral gauge boson (a $Z'$) with TeV-scale mass $M_{Z'}$ and rather particular couplings to fermions $\textsuperscript{20}$; to avoid disastrous CP-violating contributions to $B_s^-\bar{B}_s$ mixing the $Z'$ should couple proportionally to the combination $V_{ts}V_{tb}$ of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements to the left-handed $sb$ current; since the $B \to K^* \mu^+ \mu^-$ data seem to prefer a vector rather than an axial-vector coupling to the $\mu\mu$ bi-linear, the $Z'$ should furthermore feature left-handed and right-handed muon couplings of close to equal strength. The explicit construction of a realistic $Z'$ model with these properties and its rich phenomenology will be presented elsewhere $\textsuperscript{21}$.

In this letter we study a phenomenological model that has the above features built in by assumption. We point out that in such a simplified theory there is a model-independent triple-correlation between $\Delta C_9$, the relative shift $\Delta B_s$ in the mass difference of the $B_s$-meson system and first-row unitarity violation of the CKM matrix parametrised by $\Delta_{\text{CKM}}$. While the correlation between the variables $\Delta C_9$ and $\Delta B_s$ is well known their connection to $\Delta_{\text{CKM}}$ has, as far as we are aware, not been discussed in the literature. We show that by means of the triple-correlation it is possible to write the NP contribution $\Delta C_9$ as a simple analytic function of the shifts $\Delta_{B_s}$ and $\Delta_{\text{CKM}}$. In this way the quadratic sensitivity of $\Delta C_9$ on the inverse of the mass $M_{Z'}$ is turned into a logarithmic dependence. This implies not only that an observation of $\Delta C_9 \neq 0$ necessarily leads to a deviation in $B_s^-\bar{B}_s$ mixing ($\Delta_{B_s} \neq 0$) and a violation of CKM unitarity ($\Delta_{\text{CKM}} \neq 0$), but also that the pattern of the modifications depends weakly on the NP scale.

In the case of minimal $Z'$ models, the consistency of $\textsuperscript{2}$ can
hence be tested in a simple manner against theoretically clean quark-flavour observables. We discuss the present status and future prospects of these tests as well as other cross-checks that can be performed at SuperKEKB.

**Toy model**

Assuming lepton-flavour universality the $Z'$ interactions described in the introduction take the following form

$$
\mathcal{L}_{Z'} \supset \left( V_{tb} V_{tb} g_{bL}^{b*} \bar{s} Z' P_L b + h.c. \right) + \frac{g_{L}^{W}}{2} \sum_{\ell = e, \mu, \tau} (\bar{\ell} Z' \ell + \bar{\nu}_\ell Z' P_L \nu_\ell),
$$

(3)

where the coupling constants $g_{bL}^{b*}$ and $g_{L}^{W}$ are real and $P_L = (1 - \gamma_5)/2$ projects out left-handed fields. Note that $g_{bL}^{b*} = g_{bL}^{b*} + g_{L}^{W} = 2g_\pi^{b*}$, $g_A = g_{bL}^{b*} - g_{L}^{W} = 0$ and $g_{L}^{s} = g_{L}^{W}/2$, where the final relation is a consequence of $SU(2)_L$ invariance. Throughout our work we will assume that (3) encodes all relevant $Z'$-fermion interactions.

**Triple-correlation**

The new-physics correction $\Delta C_9$ to the Wilson coefficient of the semileptonic vector operator in (1) is found by calculating the tree-level $Z'$-exchange contribution to the partonic $b \to s u^+ \mu^-$ process. We obtain

$$
\Delta C_9 = -\frac{1}{16} \frac{\pi}{\alpha} \left[ g_{bL}^{b*} g_{L}^{W} \right],
$$

(4)

where $G_F \simeq 1.17 \times 10^{-5}$ GeV$^{-2}$ denotes the Fermi constant and $\alpha \simeq 1/128$.

Tree-level $Z'$ exchange also affects the mass difference $\Delta M_{B_s}$. We find

$$
\Delta M_{B_s} = \frac{\Delta M_{B_s}^{\text{SM}}}{\alpha M_{Z'}^2},
$$

(5)

where $s_w^2 \simeq 0.23$ denotes the sine of the weak mixing angle and $S \simeq 2.3$ [10] is the leading-order SM box contribution. The parameter $\tilde{r}$ encodes renormalisation group effects and is given by [11]

$$
\tilde{r} \simeq 0.985 \left[ 1 - 0.029 \ln \left( \frac{M_{Z'}^2}{1 \text{ TeV}} \right) \right].
$$

(6)

A violation of first-row CKM unitarity is a classic probe of additional neutral gauge bosons [12]. The amount of CKM unitarity violation is determined from the difference of the one-loop $Z'$ corrections to quark $\beta$-decay amplitudes from which the CKM elements are extracted as well as muon decay which normalises those amplitudes. Examples of Feynman diagrams relevant in our toy model [3] are shown in Fig. 1. Notice that the contributions to $b \to u e^- \bar{\nu}_e$ ($s \to u e^- \bar{\nu}_e$) are suppressed relative to $\mu^- \to e^- \nu_\mu \bar{\nu}_e$ by $V_{tb} V_{tb} V_{us} = O(\lambda^3) = O(1\%)$ ($V_{tb} V_{tb} V_{us} = O(\lambda^5) = O(1\%)$) with $\lambda \simeq 0.23$ denoting the Cabibbo angle. The flavour-changing contributions to CKM unitarity violation are hence for all practical purposes negligible, and one obtains [12]

$$
\Delta_{\text{CKM}} = \sum_{q = d, s, b} |V_{uq}|^2 - 1,
$$

(7)

$\Delta_{\text{CKM}} = 3 \times 16 \sqrt{3} G_F \pi s_w^2 \mu^2 / M_{Z'}^2 - 1 - \frac{M_W^2}{M_{Z'}^2},$

with $M_W \simeq 80.4$ GeV denoting the $W$-boson mass.

The relations [5] and [7] can now be used to eliminate the factor $g_{L}^{W} M_{Z'}^2$, entering (4) in favor of $\Delta_{B_s}$ and $\Delta_{\text{CKM}}$. Keeping only the leading-logarithmic term in the Taylor expansion of (7) around $M_{B_s}^2 / M_{Z'}^2 = 0$ (which is an excellent approximation for $M_{Z'} = \mathcal{O}(1 \text{ TeV})$), we get

$$
\Delta C_9 = -\frac{2\pi}{\sqrt{3} \tilde{r}} \left( S |\Delta_{B_s}| |\Delta_{\text{CKM}}| \right)^{1/2}.
$$

(8)

This is the simple formula advertised in the abstract and the introduction.

By means of (8) we can now check the consistency of the preferred fit solution [9] against the non-observation of NP in the mass difference $\Delta M_{B_s}$, [13] and the absence of CKM unitarity violation. At present one has [6]

$$
\Delta C_9 = [-1.9, -1.3],
$$

(9)

at 68% confidence level (CL), while [14, 15]

$$
|\Delta_{B_s}| < 20\%, \quad |\Delta_{\text{CKM}}| < 1.2\%,
$$

(10)

at 95% CL. The resulting constraints are shown in the upper panel of Fig. 2. The yellow, orange and red regions in the plot corresponds to a $Z'$ mass of 1.3 and 10 TeV, respectively, while the gray area indicates values

![FIG. 1. Examples of one-loop box corrections to muon (left) and bottom-quark (right) decays involving $W$ and $Z'$ bosons. For the case of the strange-quark decay the roles of $b$ and $s$ in the right diagram are interchanged.](image-url)
of $|\Delta_{B_s}|$ and/or $|\Delta_{\text{CKM}}|$ inconsistent with (10). We see that a large and negative shift $\Delta C_9$, as suggested by the $B \to K^\ast \mu^+ \mu^-$ anomaly, is consistent in our toy model (3) only if $|\Delta_{B_s}|$ and $|\Delta_{\text{CKM}}|$ are both non-zero and obey $\sqrt{|\Delta_{B_s}|/|\Delta_{\text{CKM}}|} = \text{const.}$ (for fixed $\Delta C_9$ and $M_{Z'}$). Interestingly, explaining the anomaly in $B \to K^\ast \mu^+ \mu^-$ with heavier $Z'$ bosons, one faces stronger constraints from $\Delta_{B_s}$ and $\Delta_{\text{CKM}}$ due to the appearance of the logarithm $\ln(M_{Z'}^2/M_{B_s}^2)$ in the denominator of (9).

To illustrate the potential of the triple-correlation as a cross-check, we also present results of a possible, though hypothetical, future scenario. We assume that the $B \to K^\ast \mu^+ \mu^-$ anomaly is confirmed with higher significance leading to

$$\Delta C_9 = [-1.7, -1.5],$$

and that the theoretical understanding of $\Delta_{B_s}$ and $\Delta_{\text{CKM}}$ is improved by a factor of 2, implying

$$|\Delta_{B_s}| < 10\%, \quad |\Delta_{\text{CKM}}| < 0.6\%.$$  \hfill (12)

In this case we obtain the results shown in the lower panel of Fig. 2. It is evident from the plot that in such a futuristic scenario an explanation of (11) by a minimal $Z'$ with mass of $O(5 \text{ TeV})$ would become testable. This is interesting because such heavy $Z'$ bosons would very likely escape direct detection at the LHC even at 14 TeV and high luminosity. Future improvements in lattice-QCD determinations of the $B_s$-meson decay constant $f_{B_s}$, the hadronic parameter $\bar{B}_{B_s}$ as well as $V_{cb}$, which represent the dominant sources of error in $\Delta M^\text{SM}_{B_s}$ [16], could hence also have a vital impact on $B \to K^\ast \mu^+ \mu^-$ studies. A similar statement applies to improved tests of CKM unitarity that call for better determinations of $|V_{ud}|$ and $|V_{us}|$. In the long run, extractions of $|V_{ud}|$ by future neutron decay studies [17] could play a key role here, since they are, unlike nuclear beta decay, not limited by the theoretical knowledge of nuclear corrections.

**Other implications**

Constraints on the structure of $\bar{s}Z'b$ and $\bar{\mu}Z'\mu$ interactions arise also from the measurements of $B_s \to \mu^+ \mu^-$ by LHCb [18] and CMS [19]. In this context it is important to realise that our phenomenological model predicts $\text{Br}(B_s \to \mu^+ \mu^-) = \text{Br}(B_s \to \mu^+ \mu^-)_{\text{SM}}$, since the axial-vector coupling between the $Z'$ and muons is set to zero by hand. Finding a notable deviation from the SM in $B_s \to \mu^+ \mu^-$ would thus imply that the structure of (3) has to be extended by allowing for $g_{A}^{\mu} \neq 0$. Such a modification will unavoidably lead to a correlated effect in $B \to K^\ast \mu^+ \mu^-$ that can be probed and constrained by further LHCb data. Accurate measurements of the variable $P_4'$ [3] (or $S_4$ in the notation of [4]) will play a key role in this context [20].

Since $SU(2)_L$ invariance requires $g_{L'}^{\mu} = g_{L}^{\mu}$, a deviation like (2) will also affect the $b \to s\nu\bar{\nu}$ transitions. In the case of the minimal model (3), one arrives at the prediction ($F = K^\ast, K, X_s$)

$$\Delta_{\nu\bar{\nu}} = \frac{\text{Br}(B \to F\nu\bar{\nu})}{\text{Br}(B \to F\nu\bar{\nu})_{\text{SM}}} - 1 \hfill (13)$$

$$= -\frac{s_w^2}{X} \Delta C_9 + \mathcal{O}(M_{W}^4/M_{Z'}^2),$$

with $X \simeq 1.47$ [21] denoting the SM loop contributions to $b \to s\nu\bar{\nu}$ from the $Z$-penguin and electroweak-box

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**FIG. 2.** Upper panel: parameter space in the $|\Delta_{B_s}|$-$|\Delta_{\text{CKM}}|$ plane favoured by the $B \to K^\ast \mu^+ \mu^-$ anomaly. The gray regions indicate the parameter ranges that are presently disfavoured at 95% CL. Lower panel: a possible future projection assuming a confirmation of the $B \to K^\ast \mu^+ \mu^-$ anomaly with improved statistics as well as a reduction of the theory uncertainty in $|\Delta_{B_s}|$ and $|\Delta_{\text{CKM}}|$. The yellow, orange and red regions in both plots correspond to $M_{Z'} = 1.3$ and 10 TeV, respectively. Consult the text for further explanations.
diagrams. Inserting (9) into (13) hence implies that a future measurement of $\Delta_{\nu\nu}$, which is not fully unrealistic at SuperKEKB, would show an excess of 20 – 30% (see also [7]), if the $B \to K^{*}\mu^{+}\mu^{-}$ anomaly and its implications survive further scrutiny. Notice that (13) is a rather model-independent result, because it only assumes lepton-flavour universality and the absence (or smallness) of right-handed currents.

**Conclusions and outlook**

We have pointed out that in minimal $Z'$ models that aim at explaining the pattern of NP suggested by the 3.7$\sigma$ anomaly recently observed in $B \to K^{*}\mu^{+}\mu^{-}$, there exists a triple-correlation that connects NP in the Wilson coefficient of the semileptonic vector operator ($\Delta C_9$) to a shift in the mass difference of $B_s-B_d$ mixing ($\Delta B_s$) and a violation of first-row CKM unitarity ($\Delta_{\text{CKM}}$). As a result, precision determinations of $\Delta_{\nu\nu}$ can in principle be used to probe and over-constrain the $B \to K^{*}\mu^{+}\mu^{-}$ anomaly in minimal $Z'$ scenarios. These tests become more powerful for heavier $Z'$ bosons, since $\Delta_{\text{CKM}}$ being a one-loop effect introduces a logarithm $\ln(M_Z^2/M_W^2)$ that suppresses $\Delta C_9$ when written in terms of $\Delta_{B_s}$ and $\Delta_{\text{CKM}}$. We emphasised that in models with purely vector couplings between the $Z'$ and muons, the $B \to K^{*}\mu^{+}\mu^{-}$ anomaly would leave no imprint in $B_s \to \mu^{+}\mu^{-}$, which is only sensitive to the axial-vector part of the $Z'$-muon coupling. Due to $SU(2)_L$ invariance, effects in the $b \to s\nu\nu$ transitions are on the other hand inevitable, and amount to rate enhancements of around 25%, in the case of theories with lepton-flavour universality and purely left-handed currents.

In this note we have studied a toy model that contains only the $Z'$-fermion couplings that are necessary to generate a large, negative contribution $\Delta C_9$. Clearly, such a model is not realistic in the sense that in explicit $Z'$ scenarios neither the axial-vector $Z'$-muon coupling nor the flavour-diagonal $Z'$-quark couplings will be exactly zero. Interestingly, CKM unitarity can be shown to remain a stringent constraint on the structure of complete $Z'$ models that can accommodate large shifts in $\Delta C_9$. Further powerful constraints on such scenarios also arise from precision measurements of atomic parity violation and electron-electron Møller scattering, which are sensitive to the axial-vector $Z'$-electron coupling. A detailed discussion of the possible phenomenological implications of the $B \to K^{*}\mu^{+}\mu^{-}$ anomaly will be presented in [9]. In fully realistic $Z'$ models the triple-correlation found in our work will therefore not appear in its pure form. On general grounds, certain correlations between the quark-flavour-changing $b \to s\ell^{+}\ell^{-},\nu\bar{\nu}$ transitions and modifications in $\mu^{-} \to e^{-}\nu\bar{\nu}\bar{\nu}$, as well as parity-violating $e^{-} \to e^{-}$ observables are however expected to remain, if a new neutral gauge boson should be responsible for the deviations in $B \to K^{*}\mu^{+}\mu^{-}$ as seen by LHCb.

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