A generalized self-veto probability for atmospheric neutrinos

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Abstract

Neutrino telescopes such as IceCube search for an excess of high energy neutrinos above the steeply falling atmospheric background as one approach to finding extraterrestrial neutrinos. For samples of events selected to start in the detector, the atmospheric background can be reduced to the extent that a neutrino interaction inside the fiducial volume is accompanied by a detectable muon from the same cosmic-ray cascade in which the neutrino was produced. Here we provide an approximate calculation of the veto probability as a function of neutrino energy and zenith angle.
INTRODUCTION

A downward atmospheric neutrino will be excluded from a sample of atmospheric neutrinos if the sample consists of events starting in the detector and if the neutrino has sufficiently high energy and sufficiently small zenith angle that a muon from the same event will enter the detector at the same time and be recognized. Such an event will be classified as an atmospheric muon and rejected. This strategy was originally suggested in [1] for the case of $\nu_\mu$ from the decays of charged pions and kaons, where the probability that the muon produced in the same decay reaches the detector can be calculated with an analytic approximation. A preliminary estimate of the more general case where the veto is provided by any muon from the same shower was used in evaluating the atmospheric neutrino background in the high energy starting event (HESE) analysis of IceCube [2].

In this paper we show how to calculate the more general case in which the neutrino can be accompanied by a muon produced in any branch of the same shower. Accounting for these extra muons increases the veto probability for $\nu_\mu$ only slightly. For $\nu_e$ however, such uncorrelated muons are the only source of accompanying muons since meson decays to electron neutrinos are accompanied by electrons rather than muons. Treating $\nu_e$ properly is especially important when considering neutrino fluxes from charmed mesons, which decay to $\nu_e$ and $\nu_\mu$ with nearly equal probability.

Calculating the probability that a neutrino is accompanied by a muon produced in any branch of the same shower requires a calculation that accounts for the correlation between the candidate neutrino and the entire shower structure. This is a straightforward Monte Carlo calculation that is limited only by the statistics of meson decay at high energy. The characteristic ratio of decay probability to interaction probability for a meson of type $\alpha$ is $\epsilon_\alpha/(E_\nu \cos \theta)$. At energies of 100 TeV and above, of current interest in IceCube, this ratio for kaons is significantly less than 1%. For prompt neutrinos from charm decay, the ratio is by definition large, but the production of charmed hadrons itself is rare and subject to large uncertainties. For these reasons it is useful to develop a numerical estimate of the probability that a neutrino is accompanied by an unrelated muon from the same event. If such an approximation can be shown to agree with the Monte Carlo result at low energy, it can be used to extend the veto calculation beyond the statistical limitations of the full Monte Carlo.
CALCULATIONS

The flux of atmospheric neutrinos can be obtained by integrating the production spectrum of neutrinos over atmospheric depth. The production spectrum is an integral over the parent spectrum of mesons that decay to produce neutrinos. The range of the integration is given by the maximum and minimum kinematically allowed values of $E_{\text{parent}}$ for a given $E_\nu$. For a power-law primary spectrum of nucleons, the integral over the neutrino production spectrum leads to the standard approximation [3] for the flux of $\nu_\mu + \bar{\nu}_\mu$ from decay of charged pions and kaons:

$$\phi_\nu(E_\nu) = \phi_N(E_\nu) \times \left\{ \frac{A_{\pi\nu}}{1 + B_{\pi\nu} \cos \theta E_\nu/\epsilon_{\pi}} + \frac{A_{K\nu}}{1 + B_{K\nu} \cos \theta E_\nu/\epsilon_{K}} \right\}.$$  

(1)

In the case of the two body decays of charged pions and charged kaons, the integral over the parent meson energy can be constrained to require

$$E_\mu + E_\nu > E_{\mu,\text{min}} + E_\nu,$$

(2)

where $E_{\mu,\text{min}}$ is the minimum muon energy needed to reach the depth of the detector and trigger it. This is the calculation of Ref. [1] which leads to a modified $\phi_\nu^*(E_\nu, E_{\mu,\text{min}})$, which is the flux of neutrinos accompanied by the muon from the same meson decay. Then the passing rate,

$$P(E_\nu, E_{\mu,\text{min}}) = \frac{\phi_\nu(E_\nu) - \phi_\nu^*(E_\nu, E_{\mu,\text{min}})}{\phi_\nu(E_\nu)},$$

(3)

gives the fraction of atmospheric $\nu_\mu$ that are not accompanied by the muon from the same decay in which the neutrino was produced. Our goal is to generalize the passing rate to atmospheric neutrinos of all flavors by including all muons produced in the same cosmic ray shower as the neutrino.

One approach to a numerical evaluation of the passing rate is to use an approximate form for the yield of muons per primary nucleus. We use an approximation based on parameterization of simulations that is sometimes referred to as the Elbert formula [4]. This approximation gives a good description of the average properties of muon bundles generated by primary cosmic ray nuclei of mass $A$ and total energy $E$ [5]. (See also Ref. [6].) We find, using the simulations described in the results section, that the Elbert formula can be
generalized to describe fluxes of $\nu_\mu$ and $\nu_e$ by adding one additional parameter. In integral form, the approximation is

$$N_l(> E_l, A, E, \theta) = K_l \frac{A}{E_l \cos \theta} x^{-p_1} (1 - x^{p_2}),$$

with $x \equiv AE_l/E$ and the constants $K$, $p_1$, $p_2$, and $p_3$ given for different leptons $l$ in Table I. The approximation is valid above a few TeV, where pions and kaons are more likely to interact than decay in flight. The decay probability is proportional to $1/E_l \cos \theta$ [7]. The same form can be made to describe leptons from the decays of charmed mesons like the $D^\pm$ that decay promptly before they can re-interact by removing the decay-probability factor:

$$N_l(> E_l, A, E, \theta) = K_l A x^{-p_1} (1 - x^{p_3})^{p_2}.$$  

Fig. I shows the approximate lepton yields as a function of lepton energy, primary energy, and zenith angle.

The response function gives the distribution of primary energy of nuclei of mass $A$ that produce leptons of a given energy $E_l$ as

$$R_l(A, E, E_l, \theta) = \phi_N(A, E) \times \frac{dN_l(> E_l, A, E, \theta)}{dE_l}.$$  

Then the flux of leptons is

$$\phi_l(E_l, \theta) = \Sigma_A \int dE R_l(A, E, E_l, \theta).$$

To estimate the passing rate of neutrinos we evaluate

$$P(\nu, E, \theta) = \frac{\Sigma_A \int dE R_\nu(N_\mu = 0)}{\Sigma_A \int dE R_\nu},$$

where $P(N_\mu = 0|A, E, E_{\mu,\text{min}}, \theta)$ is the probability that no muons from a shower initiated by a cosmic ray of the given mass, energy, and zenith angle penetrate to the depth of the detector without dropping below the detection threshold $E_{\mu,\text{min}}$. This can be approximated as the Poisson probability

$$P(N_\mu = 0|E, E_{\mu,\text{min}}, \theta) = e^{-N_\mu(E, \tilde{E}_{\mu,\text{min}}(\theta), \theta)},$$

where $\tilde{E}_{\mu,\text{min}}(\theta)$ is the surface energy required to reach the detector with $E_{\mu,\text{min}}$ 50% of the time and $N_\mu$ is the cumulative muon yield evaluated at that energy. A Python implementation of this calculation is included with the online supplemental materials for this article.
The central idea of the estimate is to weight the probability of zero muons according to the weights that give rise to the flux of neutrinos of a given $E_\nu$. It should be noted that the same idea can be applied to estimate the atmospheric neutrino veto efficiency of a surface detector like IceTop [8] by replacing $P(N_\mu = 0|A, E, E_{\mu,\text{min}}, \theta)$ with $1 - \epsilon(A, E, \theta)$ where $\epsilon$ is the surface detector’s trigger efficiency for showers initiated by a cosmic rays of the given mass, energy, and zenith angle.

**RESULTS**

In Fig. 2 we compare the veto passing fraction (fraction of neutrinos that arrive at a depth of 1950 m in ice with no muons above 1 TeV) in two cases: once considering only muons produced in the same decay as the $\nu_\mu$ [1], and once considering uncorrelated muons from other branches of the shower (Eq. 8). As might be expected, the uncorrelated veto by itself is not as efficient as the same-decay veto, but it applies to $\nu_e$ as well as $\nu_\mu$.

Fig. 3 shows a comparison of the analytic calculations of the passing rate with a full Monte Carlo calculation. We simulated showers with CORSIKA [9] and SIBYLL 2.1 [10] hadronic interactions, weighting the showers to the H3a spectrum of Ref. [11]. We then used PROPOSAL [12] to propagate the muons in each shower through ice to a vertical depth of 1950 m, and tabulated the fraction of neutrinos where no muons reached depth with more than 1 TeV as a function of neutrino energy, flavor, and zenith angle. Since $\nu_\mu$ may be vetoed either by a muon from the same vertex or from the rest of the shower, we approximate the passing rate as

$$P_{\text{total}} \approx P_{\text{correlated}} \times P_{\text{uncorrelated}},$$

where the first factor is the passing rate from Ref. [1] and the second is from Eq. 8. While this approximation accounts for the correlated muon more than once, it nonetheless describes the full Monte Carlo calculation quite well. For $\nu_e$ there is no partner muon, and the passing rate is described well by Eq. 8 alone.

Fig. 4 shows a similar comparison, but only considers neutrinos from the decays of charmed mesons simulated with DPMJET 2.55 [13]. Here, the passing rate for $\nu_\mu$ is nearly the same as in the conventional case, while for $\nu_e$ it is slightly higher. This happens because neutrinos from charm decay tend to carry a larger fraction of the shower energy than conventional neutrinos, and so come from a population of showers with less energy and fewer
muons on average.

Finally, Fig. 5 shows the effective neutrino fluxes that can be observed in IceCube if all neutrino events with accompanying muons above 1 TeV are removed from the sample. The most notable feature is that the up-down symmetry of the atmospheric neutrino flux is distorted for zenith angles smaller than $75^\circ$. The effect is especially stark for the prompt neutrino flux, which would otherwise be completely isotropic, mimicking a diffuse flux of extragalactic neutrinos.

Self-veto provides a powerful tool for disentangling astrophysical neutrinos from an otherwise irreducible atmospheric neutrino background, and vice versa. The generalized calculation presented here can be used to estimate passing rates for conventional and prompt neutrinos of all flavors.

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[1] S. Schönert, T. K. Gaisser, E. Resconi, and O. Schulz, Phys. Rev. D 79 (2009), 10.1103/PhysRevD.79.043009

[2] M. Aartsen et al. (IceCube Collaboration), Science 342 (2013), 10.1126/science.1242856.

[3] T. K. Gaisser, Cosmic rays and particle physics (Cambridge University Press, Cambridge and New York, 1990).

[4] J. W. Elbert, in Proceedings of the DUMAND Summer Workshop (Scripps Institution of Oceanography, La Jolla, CA) (1979).

[5] T. Gaisser and T. Stanev, Nucl. Instrum. Meth. A 235 (1985), 10.1016/0168-9002(85)90260-8

[6] P. Lipari, Astroparticle Physics 1 (1993), 10.1016/0927-6505(93)90006-Y

[7] $\cos^* \theta$ is $\cos \theta$ evaluated at the average altitude of the first interaction parameterized as in Ref. [14]. The correction is important for zenith angles larger than $70^\circ$. 
TABLE I. Parameters of the modified Elbert formula for different lepton flavors and production processes.

| Parameterization | K   | \(p_1\) | \(p_2\) | \(p_3\) | Equation |
|------------------|-----|---------|---------|---------|---------|
| Elbert \(\mu\)   | 14.5 | 0.757   | 5.25    | 1       | (4)     |
| Conventional \(\mu\) | 49.5 | 0.626   | 4.94    | 0.580   | (4)     |
| Conventional \(\nu_\mu\) | 79.9 | 0.463   | 4.37    | 0.316   | (4)     |
| Conventional \(\nu_e\) | 0.805 | 0.619   | 9.78    | 0.651   | (4)     |
| Charm \(\nu_\mu\) and \(\nu_e\) | 0.000780 | 0.604   | 7.34    | 0.767   | (5)     |

[8] R. Abbasi et al. (IceCube Collaboration), Nucl. Instrum. Meth. A 700 (2013), 10.1016/j.nima.2012.10.067
[9] D. Heck, J. Knapp, J. Capdevielle, G. Schatz, and T. Thouw, CORSIKA: A Monte Carlo Code to Simulate Extensive Air Showers, Tech. Rep. FZKA 6019 (Forschungszentrum Karlsruhe, 1998).
[10] E.-J. Ahn, R. Engel, T. K. Gaisser, P. Lipari, and T. Stanev, Phys. Rev. D 80 (2009), 10.1103/PhysRevD.80.094003
[11] T. K. Gaisser, Astroparticle Physics 35 (2012), 10.1016/j.astropartphys.2012.02.010
[12] J.-H. Koehne, K. Frantzen, M. Schmitz, T. Fuchs, W. Rhode, D. Chirkin, and J. B. Tjus, Computer Physics Communications 184 (2013), 10.1016/j.cpc.2013.04.001
[13] P. Berghaus, T. Montaruli, and J. Ranft, Journal of Cosmology and Astroparticle Physics 2008, 003 (2008)
[14] D. Chirkin, (2004), arXiv:hep-ph/0407078 [hep-ph]
[15] M. Honda, T. Kajita, K. Kasahara, S. Midorikawa, and T. Sanuki, Phys. Rev. D 75 (2007), 10.1103/PhysRevD.75.043006
[16] R. Enberg, M. H. Reno, and I. Sarcevic, Phys. Rev. D 78 (2008), 10.1103/PhysRevD.78.043005
(a) Cumulative lepton yields for vertical, 1 PeV proton showers.

(b) Zenith angle dependence of the cumulative conventional $\nu_e$ yield evaluated at 10% of the primary energy for different primary energies.

 FIG. 1. (Color online) Lepton yields from the modified Elbert formula. Each curve in (a) shows Eq. (4) evaluated with one of the parameter sets from Table I. The points in (b) show yields from CORSIKA simulation, the dotted lines a $1/\cos \theta$ dependence, and the solid lines a $1/\cos^* \theta$ dependence \cite{14}. All conventional lepton yields have the same zenith dependence.
FIG. 2. (Color online) Solid lines show the passing rate as a function of neutrino energy from the analytic calculation of Ref. [1] for various $\cos \theta$. Dashed lines show the calculation of Eq. 8. The passing rate in both calculations increases rapidly with the depth of the detector.

FIG. 3. (Color online) Comparison of approximations (solid lines) with Monte Carlo (crosses) for conventional neutrinos at 3 values of $\cos \theta$. Top panel: $\nu_\mu$ with solid lines showing the passing rate from the analytic calculation of Ref. [1]. Bottom panel: $\nu_e$ with solid lines showing the approximate calculation of Eq. 8.
FIG. 4. (Color online) Comparison of approximations (solid lines) with Monte Carlo (crosses) for neutrinos from charmed meson decay at 3 values of $\cos \theta$. Top panel: $\nu_\mu$ with solid lines showing the passing rate from the analytic calculation of Ref. [1]. While this calculation only applies strictly to 2-body decays of pions and kaons, it provides an adequate description of the muon/neutrino correlation in 3-body decays of D mesons as well. Bottom panel: $\nu_e$ with solid lines showing the approximate calculation of Eq. 8.
FIG. 5. (Color online) Effective atmospheric neutrino flux obtained by applying the passing-rate calculation presented here to the conventional flux calculation of Ref. [15] and the prompt flux calculation of Ref. [16]. The dotted lines in each panel show the total neutrino flux as a function of zenith angle for different energies, while the solid (dashed) lines show the portion of the $\nu_\mu$ ($\nu_e$) flux that can reach IceCube with no accompanying muons above 1 TeV. Above 100 TeV the up-going neutrino flux is suppressed by absorption in the Earth; this effect is not shown.