Solitons and Black Holes in 4 and 5 Dimensions

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Abstract

Two lectures given in Paris in February 1985. Since the material as given has now appeared elsewhere [1,2] I have decided not to

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Contents

1 Introduction

2 Topology and the Initial Data

3 The Black Hole as Soliton

4 Solitons in 5-dimensions

5 Pyrgon-Monopole duality

1 Introduction

This is the written version of two lectures given in Paris in February 1985. Since the material as given has now appeared elsewhere [1,2] I have decided not to
repeat the lectures verbatim but rather to comment on the general problem of
solitons in gravity, in particular on the importance or otherwise of spatial and
spacetime topology contrasting the situation in 4 and in 5 spacetime dimensions.
My main point will be that while there are many similarities with the situation
in Yang-Mills-Higgs theory there are significant differences. In particular the
apparently inevitable occurrence of spacetime singularities and their conjectured
shielding by event horizons (Cosmic Censorship) means that one cannot assume
that the time evolution of initial data is continuous. This substantially alters
ones views of the importance of topology in the classical theory. It is highly
likely that the quantum theory - should it make mathematical sense - will be
similarly affected. The plan of the article is as follows: in section 2 I will discuss
some topological aspects of the initial value problem. In section 3 I will describe
why I don’t feel one can regard black holes as solitons except in the extreme
Reissner-Nordstrom case, and the relation of this to supergravity. In section 4
I will contrast the situation with that in 5-dimensions and I will argue that the
true analogue of magnetic monopoles in Yang-Mills theory are the multi-Taub
NUT solutions whose importance for Kaluza-Klein theory was first stressed by
Gross, Perry and Sorkin. Their relation to black holes will also be described.
In section 5 I will describe a duality conjecture analogous to that of Olive &
Montonen in the Yang-Mills case.

2 Topology and the Initial Data

It is an attractive idea that the way to study solitons and other topological
features in General Relativity is to start with an initial data set \((\Sigma, g_{ij}, K_{ij})\)
where \(\Sigma\) is a 3-dimensional manifold, \(g_{ij}\) a Riemannian metric and \(K_{ij}\) the
second fundamental form. The metric and second fundamental form should
satisfy certain constraints and be asymptotically flat. Indeed one could imagine
more than one asymptotic region, just as there is in the Schwarzschild vacuum
solution. The \(k\) asymptotic regions may be imagined to be compactified to give
a compact manifold \(\bar{\Sigma}\), \(\Sigma\) being diffeomorphic to \(\bar{\Sigma}\) with \(k\) points removed.
There is no complete topological classification of 3 manifolds but it is known
[3] that for orientable manifolds \(\Sigma\) may be expressed uniquely as the connected
sum of a number of ”prime manifolds” \(\Sigma_i\)

\[
\bar{\Sigma} = \Sigma_1 \# \Sigma_2 \ldots \# \Sigma_n \quad (1)
\]

A complete list of prime manifolds is not known but it is known that for instance
\(S^2 \times S^1\) and elliptic spaces \(S^3/\Gamma\) where \(\Gamma\) is a suitable discrete subgroup of \(SO(4)\)
with free action on \(S^3\) are prime. Initial data satisfying the constraints which
are orientable are, according to Schoen and Yau [4] probably limited to a sum
of \(S^2 \times S^1\)’s and elliptic spaces.

The existence of a unique factorization has led Witten [5] to argue that there
are no solitons in 4-dimensional gravity because if there were one would expect
an anti-soliton 3-metric such that one

\[
S^3 = \Sigma_s \# \Sigma_s \quad (2)
\]
where $\Sigma_s$ is the soliton 3-space topology and and $\bar{\Sigma}_s$ that of the anti-soliton. If $\Sigma_s$ is prime this is ruled out by the uniqueness. One now seems to have a problem with CPT since (2) implies that the soliton anti-soliton pair cannot have the quantum numbers of the vacuum. The way out of this particular difficulty would seem to be that topology is not a "good quantum number". This seems reasonable because it appears that any topologically non-trivial initial data set must evolve to give spacetime singularities in its future [6].

According to the widely believed but still as yet unproven Cosmic Censorship Hypothesis [7] these singularities will be shielded inside event horizons. Furthermore it is also widely believed that the final state (in the classical theory) will consist of one or more time independent black holes. These black holes will have the metric of the Kerr solution. The consequences of this are rather disappointing as far as spatial topology is concerned. Suppose one started with for instance one of Sorkin’s non-orientable wormholes [8]. That is $\bar{\Sigma}_s = P$ the non-orientable $S^2$ bundle over $S^1$. It is not difficult to construct initial data with this topology [9]. This has a number of fascinating topological properties [3]. For instance, topologically:

$$P\#(S^2 \times S^1) \equiv P\#P$$

(3)

which one might interpret as saying that two non-orientable wormholes could turn into a non-orientable wormhole and a conventional orientable wormhole. All of this however will be invisible from infinity since presumably each, or maybe both, will be surrounded by event horizons and the fact that they are topologically non-trivial will play no role in the exterior dynamics; The final black hole solution will be a Schwarzschild or Kerr metric and no hint of the interior topology will show up in that.

Very much the same applies to the significance of the $\theta$ -vacuum structure of the initial data. One might view the configuration space $Q$ for gravity as the space of Riemannian metrics on $\bar{\Sigma}_s$ factored by the set of diffeomorphisms $\text{Diff}^+(\bar{\Sigma}_s)$ having a point on $\bar{\Sigma}_s$ (the point at infinity) and its tangent space invariant. If $\text{Diff}^+(\bar{\Sigma}_s)$ is not connected the configuration space $Q$ will not be simply connected and $\theta$ -vacuum analogous to those in Yang-Mills theory are possible [10]. A particular instance of this is the beautiful work of Sorkin and Friedman [11] on spin $\frac{1}{2}$ from gravity. Because $Q$ is not simply connected a rotation of the spacetime relative to infinity may result in one moving around a closed loop in $Q$ which is not homotopic to the constant path. The quantum wave function could in principle change sign under such a rotation. As an example consider as they do $\text{Diff}^+(\bar{\Sigma}_s)$ to be $S^3/\Gamma$ where $\Gamma$ is the 8 element group consisting of the unit quaternions and their negatives together with $\pm 1$. It is quite easy to construct time symmetric initial data corresponding to this space. The resulting space can be thought of as containing 7 black holes suitably identified [9]. Despite the exotic topology it seems rather likely that the end result will be just one large black hole. Again there will be no sign in the external metric of the initial exotic topology.

Finally as a final argument against the significance of 3-space topology let me remind the reader of the well known theorem of Serini, Einstein, Pauli and
Lichnerowicz which I like to paraphrase as "No solitons without horizons". The theorem states that there are no regular globally static solutions of the vacuum Einstein equations other than the flat one. The argument depends on the fact that if \( g_{00} = -V^2 \) with \( V > 0 \) on \( \Sigma \) and \( V \to 1 \) at infinity the field equations imply that

\[
\nabla_i \nabla^i V = 0 ,
\]

where \( \nabla_i \) is covariant differentiation with respect to the spatial metric \( g_{ij} \). The maximum principle immediately shows that \( V = 1 \). The remaining field equation now reads

\[
R_{ij} = 0
\]

where \( R_{ij} \) is the Ricci tensor of \( g_{ij} \) which in 3-dimensions shows that \( g_{ij} \) and hence the 4-dimensional metric must be flat.

3 The Black Hole as Soliton

The remarks in section 2 have been intended to convince the reader of the importance of the 4-dimensional dynamics of the theory as opposed to that of 3-dimensional initial data. This does not mean that one can necessarily regard black holes as solitons. Far from it. They have no fixed mass or angular momentum even in the classical theory. Indeed the non-decreasing property of the event horizon area is anything but solitonic. The situation is even worse in the quantum theory since we know from the work of Hawking [12] that black holes are unstable against thermal evaporation. We are still ignorant of the final outcome of this process which may not be calculable in Einstein theory but may require a consistent quantum theory of gravity. A plausible guess is that the hole simply disappears in a puff of radiation. If this is true the black hole should be regarded in the quantum theory as an unstable "intermediate state", rather than a stable particle-like state.

The exception to this would be if the hole carried a "central" charge. By central I mean completely conserved and not carried by any of the fundamental fields of the theory. For example in \( N=2 \) ungauged extended supergravity, [13] there is a Maxwell field. The fields of the \( N=2 \) supergravity multiplets are the graviton, the photon and the gravitino. These are all electrically neutral with respect to the Maxwell field - that is why the theory is "ungauged". It is quite possible for black holes to carry this charge - essentially because the lines of flux are "trapped in the topology" as people used to say in the days of "Geometrodynamics". The metric of such holes (if nonrotating) is that of Reissner and Nordstrom. It is parameterized by the mass \( M \) and charge \( Q \). Because of the duality, invariance of the theory of any magnetic charge may be rotated to zero by a suitable duality rotation. The singularity is clothed by an event horizon if

\[
M \geq \frac{|Q|}{\kappa}
\]

where \( \kappa^2 = 4\pi G \) and \( G \) is Newton’s constant. I have described in more detail elsewhere [1,2] how one may view (6) as a Bogomolny type inequality [see
The electric charge $Q$ is truly central in the sense of the super symmetry algebra and the inequality in (6) is saturated by extreme black holes which are "supersymmetric" in that they possess "Killing spinors". There exist a whole family of multiblack hole metrics [17] satisfying (6). These are the PapapetrouMajumdar metrics [18] which are included in the general class of Israel-Wilson metrics [19]. Tod [20] has shown that the Israel-Wilson metrics exhaust all the metrics with Killing spinors in N=2 supergravity. It has been known for some time that the throat of the extreme Reissner-Nordstrom metric has the geometry of the Robinson-Bertotti solution, i.e. the product metric on $S^2 \times AdS_2$ where $AdS_2$ is 2-dimensional anti-de Sitter space. The Robinson-Bertotti metric shares with flat space the property of being maximally supersymmetric - i.e. of having the largest possible number of Killing spinors; Thus the extreme Reissner-Nordstrom metrics spatially interpolate between the 2 possible "vacua" of $N=2$ ungauged supergravity. The possible relevance of this remark for spontaneous compactification is intriguing. For the present let me remark that this is typically soliton-like behaviour.

Since the charge is central it cannot be lost during Hawking evaporation and so a hole with an initial charge must settle down to the lowest mass state with that charge. This is the extreme (zero temperature) state. Thus extreme Reissner-Nordstrom holes seem to behave just like solitons. The hole with the opposite charge is clearly the anti-soliton and it seems extremely plausible that a soliton-anti-soliton might completely annihilate one another. They cannot do this classically if Cosmic Censorship holds since by Hawking's area theorem the final event horizon must have non-vanishing area but the resultant Schwarzschild black hole can then evaporate thermally. The main way in which the extreme holes differ from solitons is that there seems to be no way of fixing their mass or charge - i.e. no quantization rule. (6) Since the extreme holes (which need not all have the same mass), can remain in equilibrium it is reasonable to consider departures from equilibrium perturbatively. To lowest order they should move on geodesics on a suitable "moduli space", that is to lowest order the parameters specifying the solution should change slowly. This is the same approximation as has been used successfully in Yang-Mills theory [21,22]. In the present case the Papapetrou Majumdar solution (representing N black holes) is specified by giving the positions of N points in $\mathbb{R}^3$. In principal the points could coincide though I will argue in a short while that this doesn't happen. If the holes, having equal masses, were identical one would factor by the action of the permutation group $S_N$ on the N positions. Thus we know the moduli space. The metric is not known. However if one makes the approximation that one hole is very much smaller than all the others, one can anticipate that the motion of the small hole in the field of the others should be given by the standard equation for a charged geodesic (with charge = mass $\times \kappa$). In the slow motion limit this does indeed give non-relativistic geodesic motion in the metric

$$ds^2 = U^3d\mathbf{x}^2$$ (7)
where
\[ U = 1 + \sum_{i=1}^{N-1} \frac{GM_i}{|x - x_i|} \]  
(8)

This metric is complete on \( \mathbb{R}^3 - \{x_i\} \). In this approximation the holes would take an infinite time to merge or coalesce.

The quantum scattering of extreme holes could be studied in the non-relativistic limit by looking at the Schrödinger equation on the moduli space. This would presumably correspond to the scalar Laplacian with respect to the metric on the moduli space, though it is also possible that potential terms might appear due to one loop effects. In the case that the holes all had equal mass one should divide out by the permutation group. The moduli space would have fundamental group \( S_N \). The wave function could in principle then be even or odd under permutation. Thus one could imagine "fermionic" black holes! This is the analogue of the effect of Sorkin and Friedman I described above. It is possible to find extreme black holes in the N=4 ungauged extended supergravity theory as well [23]. They should also probably be thought of as solitons. Like the extreme holes in N=2 they also have no natural mass quantization - at least as far as classical or semi-classical considerations are concerned. To get a satisfactory quantization rule one seems forced to turn to Kaluza-Klein theory.

4 Solitons in 5-dimensions

Much of the discussion about the relevance of topology in section 2 could be repeated here with 4 replacing 3. The details of the topological discussion would differ and we certainly don’t have detailed singularity theorems and black hole uniqueness theorems in higher dimensions - indeed we know very little about black holes in higher dimensions. However in higher dimensions gravity is even more attractive (having a force inversely as distance to the power of the dimension of spacetime minus 2) than in 4-dimensions. In 5-dimensions it depends on distance in the same way as the repulsive centrifugal force (which is inversely as distance cubed in all dimensions). In higher dimensions it rises even more rapidly than the centrifugal repulsion. This would seem to make gravitational collapse and spacetime singularities even more likely in higher dimensions. However there is an important difference. We are no longer obliged, nor would we wish, to confine ourselves to initial data which are asymptotically Euclidean. If we do so the argument that the vanishing of the Ricci tensor implies that the 4-space is flat still goes through according to Schoen and Yau’s Positive Action Theorem [25]. If we don’t require that the 4-metric be asymptotically Euclidean there are many complete Ricci flat 4-metrics, including one - that on the K3 surface - which is compact. Any gravitational instanton will give a static 5-metric with no horizons. Note that if we have no horizon we are still forced to have \( V = 1 \), that is the metric must be a product on \( \mathbb{R} \times M \), where \( M \) is the 4-manifold. In the older language the space time would be said to be "ultrastatic". Not all of these objects will be classically stable. The stability
will be governed by spectrum of the Lichnerowicz Laplacian acting on symmetric tensors on $\mathcal{M}$. If $\mathcal{M}$ has a self-dual metric this is known to be positive and hence the corresponding static lump will be classically stable. If $\mathcal{M}$ has a metric which is not self-dual the spectrum is not likely to have a positive spectrum and the corresponding lump will be unstable. Examples of this are the Euclidean Schwarzschild solution [26] and the "Taub-Bolt solution" [27].

The evolution of these objects in the full non-linear theory is unclear. The Euclidean Schwarzschild solution has the same asymptotics as the flat metric on $\mathbb{R}^3 \times S^1$ and so presumably it loses energy to gravitational radiation and attempts to settle down to the flat metric but it can’t do this without forming some sort of singularity since this would involve a spatial topology change. It seems likely that a black hole will be formed but this is not known. The same remarks apply to the Taub-Bolt metric which presumably tries to settle down to the Taub-NUT metric. Again black hole formation seems likely. It is possible that these black holes appear regular when viewed from a 4-dimensional standpoint in which case they should be included with those described in [24] and [28]. The Hawking effect may then cause these black holes to evolve to the flat or the Taub-NUT solution.

The boundary conditions of interest for Kaluza-Klein theory is that the metric be what has been called in this context asymptotically flat i.e. that it approach the flat product metric on $\mathbb{R} \times S^1$ at infinity or that it be asymptotically locally flat. The typical example of the latter is the self-dual Taub-NUT metric, or multi-Taub-NUT with $N$ centres. The topology at infinity in this case is $\mathbb{R} \times B_N$ where $B_n$ is the $S^1$ bundle over $S^2$ with Hopf invariant $N$ - i.e. the lens space $L(N,1)$.

Gross, Perry and Sorkin [29] have pointed out that the Taub-NUT solution plays the role of a magnetic monopole in Kaluza-Klein theory. Perry and myself [30] have shown that the monopole moment $P$ of any asymptotically locally flat solution should satisfy the Bogomolny type inequality

$$\frac{|P|}{2\kappa} \leq M$$

with equality in the supersymmetric self-dual case. It is interesting to note that the gravitational instanton solution of Atiyah & Hitchin [22] is self-dual but has a negative mass. This is presumably because it has the topology at infinity of $\mathbb{R} \times (S^3/\Gamma)$ where $\Gamma$ is the binary dihedral group. The crucial point here is whether or not suitable solutions of the Witten equation exist.

The multi Taub-NUT solutions are specified by giving $N$ non-coincident points in $\mathbb{R}^3$. Permutating the points gives the same metric so the moduli space is the well known configuration space $((\mathbb{R}^3)^N - \Delta)/S_N$ where $\Delta$ is the set of points in $((\mathbb{R}^3)^N$ where two or more points coincide and $S_N$ is as before the permutation group on $N$ symbols. The metric on the moduli space is under study. Again the quantum mechanics offers the possibility of multivalued wave functions though whether these monopoles can really be thought of as fermions remains at present unclear. An important property of the Taub-NUT solutions
is that the magnetic charge $P$ satisfies the Dirac quantization condition:

$$eP = 2\pi$$  \hspace{1cm} (10)

where $e$ is the basic unit of charge in Kaluza-Klein theory. This in turn implies that (using the equality in (9)) the mass $M$ is quantized:

$$M = \frac{1}{4\pi \kappa e}$$  \hspace{1cm} (11)

Given their stability and the quantization of the mass and magnetic charge it seems reasonable to regard the Taub-NUT solutions as representing solitons though this does require, as in section 2, that some of the topological numbers associated with the object are not conserved. In the present case two such numbers are of interest. The Hirzebruch signature and the Euler number. The multiple monopole has non-vanishing Hirzebruch signature. Roughly it corresponds to magnetic charge. Since this can be read off from the asymptotic boundary conditions one might expect this to be conserved. The Euler number is a different matter however. This cannot be determined from infinity and given the likely occurrence of singularities there seems to be no good reason for it to be conserved. Another argument, due to Hawking, is that the Euclidean action in General Relativity is not scale invariant. This means that it may cost arbitrarily little action to pass from one topological configuration to another. This is unlike the case in Yang-Mills theory in 4-dimensions where the action is scale invariant and typically topologically different configurations differ by an amount $\frac{8\pi^2}{g^2}$ where $g$ is the coupling constant. If one does accept them as solitons one sees a number of striking resemblances with the massive modes of the Kaluza-Klein theory. This is the subject of the next section.

5 Pyrgon-Monopole duality

The physical content of the 5-dimensional Kaluza-Klein theory when viewed from the point of view of 4-dimensions

- 1) A set of massless states, the graviton, graviphoton and dilaton
- 2) A tower of massive states of spin 0, 1 and 2 each with mass $m$ and charge $e$ given by

$$m = n\frac{|e|}{2\kappa}$$  \hspace{1cm} (12)

where $n = 1, 2, 3, \ldots$

At the linearized level all the massive states are trivially stable. When one takes into account interactions one might expect the higher mass states to decay into lower mass states but a charged state cannot decay into a neutral state. Thus the lowest mass states, $n = 1$, should be absolutely stable except against annihilation with their antiparticle states. These stable lowest mass states have
been called Pyrgons [31]. Thus the perturbative physical Hilbert space consists of massless states, pyrgons and antipyrgons. In a supersymmetric theory the pyrgons fit into massive supermultiplets with central charge. In N=8 for example the relation (12) corresponds to the maximal central charge allowed. This is necessary to avoid states with spin greater than 2.

Now the G-P-S monopoles possess in the N=8 supergravity model of Cremmer [32] the maximum permitted number of Killing spinors and hence supersymmetries. As shown in [30] they fit into supermultiplets when the zero modes are taken into account. There is a rather close analogy, indeed one is tempted to say a duality, between the monopoles of Kaluza-Klein theory and the pyrgons. This suggested duality is analogous to that which has been suggested in Yang-Mills theory [33]. In the present case we suggest that there might exist in the full quantum theory operators which create and annihilate monopole states. In addition there will be operators which create the massless states. If these satisfy an effective field theory it is essentially unique - it must be the original field theory of the pyrgons. This is essentially because of the supermultiplets structure. Thus we have the conjectured dualities:

\[
\begin{align*}
\text{monopole} & \leftrightarrow \text{pyrgon} \\
\text{massless fields} & \leftrightarrow \text{massless fields} \\
\text{antimonopole} & \leftrightarrow \text{anti-pyrgon}
\end{align*}
\]

It is difficult to see with present day techniques how such a conjecture could be verified. In the Yang-Mills case some partial evidence has come from a study of magnetic and electric dipole moments. It has been verified that the gyromagnetic ratio of the ordinary Yang-Mills particles equals the gyroelectric ratio of the monopoles plus fermionic zero-modes [34]. It is known that the gyromagnetic ratios of the pyrgons are anomalous and equal unity, rather than the Dirac value of 2 [35]. It would be interesting to calculate the electric dipole moments of G-P-S monopoles with their fermionic zero-modes. Further insight into this conjectured duality might come from a study of monopole-pyrgon interactions. A number of authors [36] have pointed out that there is no "Callan-Rubakov" effect [37] which would catalyze the decay of pyrgons. This is most easily seen from the fact that scalar modes on Taub-NUT are well defined and using the covariantly constant spinor fields on Taub-NUT one can obtain all solutions of the Dirac equation. Thus if \( \phi \) is a covariantly constant spinor on Taub-NUT and a solution of the wave equation with energy \( \phi_\omega \) then:

\[\psi_\pm = (\phi_\omega \pm \frac{1}{\omega}(\partial \phi_\omega)) \epsilon\]

are solutions of the Dirac equation with the same energy.

A striking fact about the scalar modes on the Taub-NUT background is that the massive scalar pyrgon wave equation separates in 2 different coordinate systems. One system is the standard radial variables in which the metric is

\[ds^2 = (1 + \frac{2N}{\rho})^{-1}4N^2(d\psi + \cos \theta d\phi)^2 + (1 + \frac{2N}{\rho})(d\rho^2 + \rho^2(\sin^2 \phi d\phi^2)) \quad (13)\]
where \(0 \leq \psi \leq 4\pi\). Thus \(8\pi N = 2\pi R_K\), where \(R_K\) is the radius of the Kaluza-Klein circle. The scalar field has the form

\[
\phi = e^{-i\omega t} e^{i\frac{\psi}{2}} Y_{lm}(\theta) e^{im\phi} f_n(\rho)
\]

(14)

where \(Y_{lm}(\theta)e^{im\phi}\) is a spin weighted spherical harmonic and where \(f_n(\rho)\) is a non-relativistic Coulomb wave function with angular momentum \(l\) but where the Coulomb potential is energy dependent, i.e. depends upon \(\omega\), that is \(f\) satisfies

\[
\frac{1}{\rho^2} \frac{d}{d\rho} \left( \rho^2 \frac{df}{d\rho} \right) - \frac{l(l+1)f}{\rho^2} + (2N\omega^2 - \frac{n^2}{N}) f + (\omega^2 - \frac{n^2}{4N^2}) f = 0
\]

(15)

There are no bound states, just scattering states. Since the radial equation (15) is a Coulomb one one might anticipate that scattering is better described using parabolic coordinates, defined by

\[
\xi = \rho(1 + \cos \theta) \quad \eta = \rho(1 - \cos \theta)
\]

This is in fact true. The wave equation also separates in the \(t,\phi,\xi,\eta\) coordinates. Using them one can give a simple description of the scattering. The classical orbits are especially simple being conic sections. They are, when projected into the 3-space spanned by \(\rho,\theta,\phi\), the intersection of a cone centred at \(\rho = O\) with a plane, the intersection being a hyperbola in general.

The existence of 2 different coordinate systems in which the wave equation separates is often taken as the indication of hidden symmetries and indeed of a "spectrum generating algebra". The precise nature of this algebra in the present case has not been worked out. It is tempting to speculate that it may be related to the known existence of Kac-Moody algebras in Kaluza-Klein theory [38].

Another tempting speculation is that these ideas will find their full expression in string theory. Michael Green [39] has remarked that if one considers 10-dimensional string theory where 10-D of the spacelike dimensions form a torus, each of whose radii equals \(R\) one obtains string states with masses satisfying

\[
(mass)^2 = \sum_{i=0}^{\infty} \left( \frac{M_i^2}{R_i^2} + \frac{R_i^2 N_i^2}{\alpha'^2} \right) + \frac{2}{\alpha'} (N_0 + \bar{N}_0)
\]

(16)

\(N_0\) and \(\bar{N}_0\) are occupation numbers for higher string states. The integers \(\{M_i\}\) are Kaluza-Klein charges resulting from the periodicity in the 10-D compact dimensions. The integers \(\{N_i\}\) are topological charges associated with the number round the \(i\)'th compact dimension. Consider the limit

\[
R \to 0 \quad \text{and} \quad \frac{\alpha'}{R} \quad \text{constant} = \lambda
\]

(17)

The resulting D-dimensional field theory has an infinite number of massive spin 2 supermultiplets whose masses are determined by \(\lambda\). This theory is apparently identical to the theory obtained by starting with 10-dimensional N=2
supergravity and compactifying on a hypertorus with (10-D) dimensions having finite radii $R = \lambda$.

Now set $D = 5$. The reduction of N=2 d=10 to 5 dimensions gives Cremmers’s N=8 D=5 model, with its pyrgon states. On the other hand from (17) we see that the states corresponding to zero Kaluza-Klein charge but non-vanishing topological winding numbers will survive in this limit. These presumably correspond to the magnetic monopole states.

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