Leak-Diagnosis Approach for Water Distribution Networks based on a $k$-NN Classification Algorithm

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Abstract: This paper proposes an approach based on a $k$-Nearest Neighbour classification algorithm ($k$-NN) to identify regions in a water distribution network (WDN) that are affected under presence of leaks. The classification algorithm is trained with numerical data coming from a MATLAB simulator based on a dynamic model of the WDN that involve leaks in its formulation. Concretely, the training is done by using the numerical solutions of a dynamic model of the WDN under several leak cases. The dynamic model is formulated by taking into account typical assumptions of the rigid water column (RWC) theory and using the graph theory. The proposed approach was evaluated in a hydraulic pilot plant.

Keywords: Hydraulic network, leak diagnosis, pressure reduction, pipeline modeling, water distribution networks.

1. INTRODUCTION

Water distribution networks (WDN) are prone to different types of faults. The most common and harmful ones are leaks, which mainly occur in tanks, gaskets, joints, and accessories (Thornton et al., 2002; Bermúdez et al., 2018). Since pressure changes throughout a WDN when it is affected by a leak (Creaco and Pezzinga, 2018), most water companies have invested in developing different leak diagnosis algorithms based on pressure and flow measurements. For example, Valizadeh et al. (2009) developed a K-Nearest Neighbour classification algorithm (K-NN) to locate leaks by using only pressure and flow sensor data, without considering a dynamic hydraulic model.

However, detecting a fault is as important as reducing the effects of these on the WDN. In other words, it is imperative to reduce the amount of water that is lost due to leaks. For example, Abu-Mahfouz et al. (2019) proposes a control system for water demands at the nodes of a hydraulic network, using genetic algorithms and the dynamical network model. Brentan et al. (2018) propose a model of water demand predictions in the nodes of a hydraulic network using artificial neural networks (ANN), working together with the model of pumps and pressure reducing valves in the system. Authors in Bello et al. (2019) present an overview of pressure management in hydraulic networks and the techniques used to supply the demand for water through different approaches; based on data, dynamical system modeling, and optimization. Muhammetoglu et al. (2017) developed an analysis of pressure data through a hydraulic network, and implement a set of valves in a sector of the system to reduce overpressure. Laucelli et al. (2015) present the evolutionary regression modeling (EPR) technique to highlight possible problems in a WDN such as leaks and overpressures, through pressure/flow measurements of some system nodes. Nevertheless, despite the few reported papers in the literature, the problem of reducing water losses due to leaks still open.

This work proposes an approach based on a $k$-NN classification algorithm to locate the section of WDN affected by a leak. The classification algorithm is trained with simulations data, which are based on physical modeling of the hydraulic network. In particular, this is focused on the case study of a pilot WDN located at the Technological Institute of Tuxtla Gutierrez, which provides an example of topology and realistic physical parameters. Furthermore, the control valves are considered in the mathematical model of the WDN. Simulation results are presented to illustrate the applicability of the proposed method. The leak magnitude is reduced by activating control valves, which are located in critical nodes of the WND.

The paper continues as follows: the case study which is considered is presented in section 2, where the corresponding physical modeling is also given. The key ingredients of control valves and classification approach are then introduced in section 3, and application results are provided in section 4. Some conclusions are finally given in section 5.
2. CASE STUDY

The WDN considered in this work as a case-study corresponds to the experimental PVC pipeline (schedule 80) located at the Technological Institute of Tuxtla Gutierrez. The set-up is composed of two valves placed in strategic positions such that they allow the reconfiguration of the test apparatus: as a simple pipeline, or as a WDN with two branches. The P&I diagram is shown in Fig. 1. The instrumentation of the hydraulic system is composed of five valves to simulate leaks, two flow meters of Coriolis effect, two magnetic flow meters with enhanced double frequency excitation, which allows coping with the most severe applications. Eight industrial pressure sensors transmitters (Yokogawa EJA530E) located at the ends of the pipe, and a modular block data acquisition system (GM10).

3. MODELING APPROACH

In this contribution, the modeling of a WDN is based on the rigid water column (RWC) theory, which has been previously used by Shimada (1989); Kaltenbacher et al. (2017). Therefore, the following assumptions are considered:

(A1) The flow rate is supposed to be one-dimensional;
(A2) the cross-sectional area is constant along each pipeline;
(A3) the conduit walls of each pipeline are rigid, and the liquid fluid is incompressible;
(A4) convective changes in velocity are negligible;

In the following, the models of the elements conforming to a WDN are presented together with their constitutive laws. After this, the model of the overall WDN connecting the elements is derived by using graph theory.

3.1 Component models

A WDN consists of multiple elements (e.g., pipes, valves, leaks, reservoirs) that are characterized by dynamic and algebraic relationships between the flow $Q_j$ through the component $j$ and the pressure drop $\Delta H_j = H_i - H_{i+1}$ across that component, where subscripts $i$ and $i+1$ denote the two ends of component $j$ (De Persis and Kallesoe, 2011). The relationships for the elements considered in this contribution are introduced here below.

Pipe and pipe section: The equation of motion for each pipe (or pipe section) of a WDN is given as

$$\dot{Q}_j = \beta_j (H_i - H_{i+1}) - \alpha_j Q_j |Q_j| - \beta_j \Delta H v_j, \quad (1)$$

where $Q_j$ is the flow rate ($m^3/s$) through pipe $j$, $H_i$ is the piezometric head (mH2O) at the inlet of pipe $j$, $H_{i+1}$ is the piezometric head (mH2O) at the outlet of pipe $j$, $\Delta H v_j$ is the pressure drop across an in-line device (e.g., a valve, a pump), $\beta_j = g A_{r,j}/L_j$ is the inertial term associated to pipe $j$, $g$ is the acceleration of gravity, $A_{r,j}$ is the cross-sectional area of pipe $j$, $L_j$ is the length of pipe $j$, $\alpha_j = f_j(Q_j)/2\phi_j A_{r,j}$, $\phi_j$ is the inner pipe diameter, and $f(Q_j)$ is the friction factor, which is calculated according to the flow regime (laminar, transitional or turbulent).

Leak node: The continuity equation for a leak is given by the following equation:

$$\dot{Q}_i = \frac{1}{A_{\ell,i}} (Q_j - Q_{j+1} - Q_{\ell,i}). \quad (2)$$

where $A_{\ell,i}$ is the leak area.

The discharge of the leak can be calculated with the Torricelli’s equation given by:

$$Q_{\ell,i} = C_{\ell,i} A_{\ell,i} (2gH_i)^{-1/2}, \quad (3)$$

where $C_{\ell,i}$ is the discharge coefficient.

The motion equations for a pipeline with a leak are given as follows

$$\dot{Q}_j = \beta_j (H_{i-1} - H_i) - \alpha_j Q_j |Q_j|, \quad (4)$$

$$Q_{j+1} = \beta_{j+1} (H_i - H_{i+1}) - \alpha_{j+1} Q_{j+1} |Q_{j+1}|, \quad (5)$$

where all the variables with sub-index $j$ are associated to the flow through the section $j$, which is before the leak node $i$, and the variables with sub-index $j+1$ are associated to the flow trough section $j+1$, which is after the leak node $i$.

Branch node: The continuity equation for a branch can be written as

$$\dot{H}_i = \frac{1}{A_{n_i}} (Q_j - Q_{j+1} - Q_k), \quad \forall k \neq j, j + 1 \quad (6)$$

where $Q_k$ is the flow rate through the branch $k$ connected to the node and $A_{n_i}$ is the node surface.

In-line valve: The equation for the pressure drop across a check valve or gate valve is

$$\Delta H v_j = H_i - H_{i+1} = \frac{Q_j |Q_j|}{(v_j^2 E_j)^2}, \quad (7)$$
where \( Q_j \) is the flow rate through the valve, \( r_j \) is the nondimensional effective gate opening, \( E_j \) is a valve parameter determined by the energy dissipation potential of the valve. If the valve is fully open then \( r_j = 1 \). If the valve is fully closed then \( r_j = 0 \). When the valve is partially open the value of \( r_j \) is defined by

\[
r_j = \frac{Cv_j'Av_j'}{Cv_jAv_j} \tag{8}
\]

where \( Cv_j \) and \( Cv_j' \) are the coefficients of discharge representing losses through a fully open valve and partially open valve, respectively. \( Av_j \) and \( Av_j' \) are the cross-sectional area of the valve orifice when fully and partially open, respectively. The valve size parameter is defined as

\[
E_j = \frac{Q_j}{r_j} \tag{9}
\]

If (7) is substituted into (1) then the continuity equation for a pipeline with a valve can be written as

\[
\dot{Q}_j = \beta_j (H_i - H_{i+1}) - \alpha_j Q_j |Q_j| - \beta_j \frac{Q_j |Q_j|}{(r_j E_j)^2}, \tag{10}
\]

These equations can be used to increase or decrease the flow rate between nodes of the hydraulic network (in-line), however it is also possible to consider the Haze-Williams Equation, which is a head-flow expression between nodes \( i, i+1 \) where are located (Dai and Li, 2016):

\[
Q_{V_j} = \frac{\alpha C_{HW} D_{j}^{2.63} (H_i - H_{i+1}) |H_i - H_{i+1}|^{0.54}}{L_j^{0.54}}, \tag{11}
\]

where \( \alpha \) is a system-dependent constant (SI 0.2787), \( Q_{V_j} \) is the flow that passes through the valves in \([m^3/s]\), \( D_j \) indicates the pvc pipeline diameter \( C_{HW} = 150, L_j \) is the length corresponding to \( H_i - H_{i+1} \), which represents the pressure drop across the valve. However, some parameters can be compacted in a \( R_j \) coefficient by reducing the previous Equation to (12):

\[
Q_{v_j} = V(t) R_j \Delta H_j |\Delta H_j|^{0.54} \tag{12}
\]

where \( \Delta H_j = H_i - H_{i+1} = V/Av_j \). Therefore, to have a variable flow control, is added the parameter \( V(t) \) to the equation (12), which allows to adjust the diaphragm to throttle the flow, leaving:

\[
Q_{v_j} = V(t) R_j \Delta H_j |\Delta H_j|^{0.54} \tag{13}
\]

where \( R_j \) take values from 0 to \( \infty \) (Ulanicki et al., 2008).

3.2 Network model

A WDN can be represented as a graph in order to organize the continuity and momentum equations in a graph-theoretical framework and facilitate the numerical implementation of the network model. Each pipe, which may include a valve, is called “a link”, with a direction arbitrarily defined in the graph. The two ends of a link are called “nodes,” where a branch or a leak may exist or pipes may be connected. The number of links and nodes are \( l \) and \( n \), respectively.

The global model of the WDN can expressed as follows

\[
\dot{x} = \left( \begin{array}{c}
-K \Psi A \\
-\Pi A^T \\
0
\end{array} \right) x + \left( \begin{array}{c}
\Psi B C_1 \\
\Pi B C_2
\end{array} \right) \tag{14}
\]

where \( x \in \mathbb{R}^n \) is the state vector given as \( x = [Q \ H]^T \), where \( Q \in \mathbb{R}^l \) is a vector comprising the flow rates through the links and \( H \in \mathbb{R}^n \) is the vector comprising the pressures at the nodes. \( A \in \mathbb{R}^{l \times n} \) is the node-link incidence matrix that describes the topology of a WDN and can be obtained as follows:

\[
A_{ij} = \begin{cases} 
+S_j & \text{if link } j \text{ starts at node } i \\
0 & \text{if link } j \text{ is not incident to node } i \\
-S_j & \text{if link } j \text{ leaves at node } i 
\end{cases} \tag{15}
\]

where \( S_j = \text{status of link } j \) given by

\[
S_j = \begin{cases} 
1 & \text{if link } j \text{ is open} \\
0 & \text{if link } j \text{ is closed} 
\end{cases} \tag{16}
\]

\( B C_1 \in \mathbb{R}^l \) and \( B C_2 \in \mathbb{R}^n \) are vectors that contains the boundary conditions (known variables) associated to the momentum and continuity equations, respectively, or in other words, the boundary conditions associated to the links and nodes, respectively. \( K \in \mathbb{R}^{l \times l} \) is a diagonal matrix composed of dissipation terms \( K_j \) in its diagonal, which are defined as

\[
K_j = -\alpha_j (Q_j) |Q_j| |Q_j| - \beta_j \frac{Q_j |Q_j|}{(r_j E_j)^2}. \tag{17}
\]

\( \Psi \in \mathbb{R}^{l \times l} \) is a diagonal matrix composed of the inertial terms \( \beta_j \). \( \Pi \in \mathbb{R}^{n \times n} \) is a diagonal matrix composed of the inverses of areas. \( 0 \in \mathbb{R}^{l \times n} \).

3.3 Model of the WDN pilot plant

The pilot plant has \( n = 11 \) nodes and \( l = 10 \) links. The nodes 1, 3, 5, 6 and 7 are leak nodes. The nodes \( H_{BC1}, H_{BC2}, H_{BC1}^2, H_{BC}^2, H_{BC}^2 \) are boundary nodes with measured piezometric heads. The nodes 2 and 4 are branch nodes. The links 1, 3, 5, 7 and 9 involve in-line valves. The links 2, 4, 6, 8 and 10 do not involve any in-line device. According to this, the dynamic of the pilot plant can be described by the following set of equations

\[
\begin{align*}
\dot{Q}_1 &= \beta_1 (H_{BC1}^2 - H_1) - \alpha_1 (Q_1) |Q_1| Q_1 |Q_1| - \beta_1 \frac{Q_1 |Q_1|}{(r_1 E_1)^2} \\
\dot{H}_1 &= \frac{1}{A_{E_1}} (Q_1 - Q_2 - Q_{E_1}) + Q_{e_1} \\
\dot{Q}_2 &= \beta_2 (H_1 - H_2) - \alpha_2 (Q_2) Q_2 |Q_2| \\
\dot{H}_2 &= \frac{1}{A_{E_2}} (Q_2 - Q_3 - Q_7) \\
\dot{Q}_3 &= \beta_3 (H_2 - H_3) - \alpha_3 (Q_3) Q_3 |Q_3| - \beta_3 \frac{Q_3 |Q_3|}{(r_3 E_3)^2} \\
\dot{H}_3 &= \frac{1}{A_{E_3}} (Q_3 - Q_4 - Q_{E_3}) + Q_{e_2}
\end{align*} \tag{18}
\]
In order to identify the section where the leak occurs, the valves $v_{L1}$, $v_{L4}$ and $v_{L5}$ are considered, representing a leak in each section of the hydraulic network, which are classified by the $k$-NN algorithm. The principle of $k$-NN algorithm is that the most similar samples belonging to the same class have high probability (Zhang et al., 2018), and is based on a simple learning model, which is presented in Fig. 3 (Cambroner and Moreno, 2006).

Fig. 3. Classifier Diagram

This algorithm uses the Euclidean distance function to calculate the similarity or difference between classes. The class consists of a set of measurements that represent features of the system. Therefore, if the classifying algorithm is trained with a set of points $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)$, and the new values of the system are the points $\psi = (\psi_1, \psi_2, \ldots, \psi_n)$, the algorithm defines the class using the distance equation, for a two-dimensional space:

$$D(\zeta, \psi) = \sqrt{\sum_{i=1}^{n} (\zeta_i - \psi_i)^2}$$  

(20)
For test data, the output of the \( k \)-NN classifier is the class with the highest frequency among the \( k \)-nearest neighbors. In more complex network topologies than Fig. 1, and considering noisy measurements, non-Euclidean distance metrics can be selected and the selection of the optimal number of neighbors is important (Santos-Ruiz et al., 2019). For the work described in this paper, the Euclidean distance with 10 neighbors \((k = 10)\) was sufficient to obtain the required data separability.

In this work, the \( k \)-NN classification algorithm was trained with only 4 pressure sensors \((H_{BC}^1, H_{BC}^2, H_{BC}^3, H_{BC}^4)\), and four classes are considered. These classes correspond to: nominal operation without leak; leak in the main pipeline; leak in one branch; leak in the other branch. Only pressure measurements are considered due to the fact that in real hydraulic networks, they are the most commonly available tools. It is important to mention that these experiments were carried out in four different operating points of the hydraulic pump, corresponding to 35 Hz, 40 Hz, 45 Hz, and 50 Hz. Then for each leak scenario, or class, four experiments were done. The classification regions obtained are presented in Fig. 4. As it can be seen, in the presence of leaks, the data presents some separability, which facilitates the identification of the class. Note that this approach only identifies the section where the leak occurs, which is very important to apply the water loss reduction method presented below.

![Fig. 4. k-NN classification for leak detection.](image)

5. DETECTION AND REDUCTION OF THE LEAK IN THE HYDRAULIC NETWORK

5.1 Use of control valves

The simulation of the hydraulic network was performed in Matlab with the system parameters presented in Table 1. Three different leak scenarios are considered. First, a leak is between nodes 1 and 2, which corresponds to Section 1, and the leak is simulated at 40 seconds. Due to the separability of the data, the classification algorithm identifies the section where the leak occurs, and then, the control valve in that section is activated 100 seconds later. As it can be seen in Fig. 5, under the leak period, the flow rate is reduced in the demanding nodes. But after activation of the control valve, the flow rate changes drastically, and the water loss is reduced in the demanding nodes \(Q_6\), \(Q_8\), and \(Q_{10}\). The global water loss reduction is around 3%. Note that even if the classification algorithm identifies the class immediately, the control valve is activated only after 100[s], in order to better illustrate its impact on the water loss reduction.

![Fig. 5. Reduction of water losses due to the control valve in section 1.](image)

For the second scenario, a leak is introduced in section 2 at time at 40 [s], and Fig. 6 shows its transitory effect of the flows \(Q_7, Q_8\). The section is again well identified by the classification algorithm, and the leak again reduced when the control valve is activated at time 120 seconds.

For the third case, a leak is simulated in section 3 at 40 seconds, and the system classifier indeed finds it. In this case, Fig. 7 shows the leak transient effect on flow rates \(Q_9, Q_{10}\), and again its reduction after activation of the control valve at time 100 seconds.

6. CONCLUSIONS

In this paper, a method was proposed to locate leaking regions in a water distribution network. This method is based on a classification algorithm and the dynamic model of a water distribution network. Note that better training of the classification algorithm can be done since leaks
can be simulated at different coordinates of each pipe. The proposed method was implemented in a real system by considering a pre-tuned mathematical model, together with EPANET simulations. In a more realistic scenario, it could be necessary to identify or compute the model parameters such as the roughness, the friction factor, etc. In addition, the proposed methodology to obtain the mathematical model, which includes the valve and leakage model, can be used to implement control techniques to reduce water losses and guarantee water supply to users despite the presence of leaks. Future work will be done to consider an integrated leak tolerant control method by considering optimization control techniques.

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