Adaptive Submodular Influence Maximization with Myopic Feedback

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Abstract
In this paper, we study the problem of adaptive influence maximization in social networks. It has been proved that if the problem satisfies the adaptive submodularity property, then an adaptive greedy policy is guaranteed to provide an \((1 - 1/e)\)-approximation of the optimal policy. Nevertheless, so far such a property has only been verified in the case of the full-adoption feedback model. As adaptive decision making is time-critical, we consider a more realistic feedback model, called myopic. In this direction, we introduce an alternative utility function proving that it is adaptive monotonic and adaptive submodular. It allows us to demonstrate that our myopic adaptive greedy policy provides theoretical guarantees as it retains the \((1 - 1/e)\)-approximation ratio. Furthermore, we show that the adaptive submodularity property does not hold if we allow the nodes to be deactivated randomly over time. Empirical analysis on real-world networks reveals the benefits of the myopic adaptive greedy strategy to the influence maximization problem.

1. Introduction
Graphs are useful models for specifying relationships within a collection of objects. Numerous real-life situations could be represented as nodes linked by edges, including social, biological or computer networks. Discovering the most influential nodes in such networks has been the objective of considerable research in AI community. One of the most practical applications is that of product placement or viral marketing. Consider a directed social network in which nodes correspond to potential customers. If a customer owns a product then can recommend it to his friends, according to a given diffusion model that simulates the word-of-mouth effect. Given a fixed budget, our objective is to select a set of customers to give a product for free, in order to maximize the spread of influence through the network, i.e., to maximize the number of peoples that will finally buy this product.

Influence maximization (IM) in social networks was first studied by Domingos & Richardson (2001). Kempe et al. (2003) reformulated influence maximization as a discrete optimization problem by introducing two of the most popular diffusion models: Independent Cascade (IC) and Linear Threshold (LT) model. They demonstrated that finding an optimal set of at most \(k\) seed nodes, with \(k\) to represent our budget, that maximizes influence in the network is NP-hard under both models. Nevertheless, they proved that the utility function to maximize, which is the expected number of influenced nodes, is monotone and submodular. These properties in conjunction with the results of Nemhauser et al. (1978) imply that the greedy algorithm is guaranteed to provide an \((1 - 1/e)\)-approximation of the optimal set.

Feige (1998) highlighted that this result is tight if \(P \neq NP\), and considered as near-optimal (Nemhauser & Wolsey, 1978; Vondrak, 2010). The aforementioned seminal works have inspired a large part of other research works, either to provide alternative frameworks (Wang et al., 2010; Lu et al., 2013; Aslay et al., 2014; He & Kempe, 2016; Tang & Yuan, 2016), or to speed up the greedy algorithm via heuristics providing theoretical results (Chen et al., 2009; Goyal et al., 2011; Borgs et al., 2014) or scalability guarantees (Leskovec et al., 2007; Jung et al., 2012; Kim et al., 2013).

Most of the works on influence maximization are restricted to the non-adaptive setting, where all seed nodes must be selected in advance. The main drawback of this assumption is that the particular choice of seed nodes is completely driven by the diffusion model and the edge probability assignment. Apparently, it may leads to a severe overestimation of the actual spread resulting from the chosen seed nodes (Goyal et al., 2011). Under this prism, we focus on the adaptive setting of the influence maximization problem. Instead of selecting a number of seed nodes in advance, we select one (or more) node at a time, then we observe how its activation propagates through the network and, based on the observations made so far, we adaptively select the next seed node(s). Actually, it constitutes a se-
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2. Problem Statement and Preliminaries

A social network is typically modeled as a directed graph \( G = (V, E) \) with each node \( u \in V \) to represent a person, and the edges \( E \) to reflect the relationships among them. Similar to Golovin & Krause (2011), we consider from now on the IC model to simulate the diffusion process in a social network. In this model, only the seed nodes are initially active. Afterwards, each time where a node \( u \) first becomes active, it has a single chance to activate/influence each of its inactive neighbors \( v \), succeeding with known influence probability \( p_{uv} \). The diffusion process continues until no further activations are possible.

We consider that each edge \( e \in E \) is associated with a particular state \( o \in O \), with \( O \) to be a set of possible states (describing whether an edge is live or dead). Let us denote by \( \phi : E \rightarrow O \) a particular realization of the influence function, indicating the status of edges in a particular state of the world. It is also assumed that the realization \( \Phi \) is a random variable with known probability distribution, \( p(\phi) = P[\Phi = \phi] \).

In our sequential decision making problem, after choosing a seed node \( u \in V \), we get a partial observation of the ground truth influence graph \( \phi \). More specifically, after each step, our knowledge so far will be represented as a partial realization \( \psi \subseteq E \times O \), which is a function from a subset of \( E \) to their states. We use the notation \( \text{dom}(\psi) \), called as domain of \( \psi \), to refer to the set of nodes that are observed to be active through \( \psi \). More clearly, we say that a partial realization observes an edge \( e \), if some node \( u \in \text{dom}(\psi) \) has revealed its status. A partial realization \( \psi \) is said to be consistent with \( \phi \), denoted by \( \phi \sim \psi \), if the state of all edges which are observed by \( \psi \) are the same in \( \phi \). Also, we say that \( \psi \) is a subrealization of \( \psi' \), if both of them are consistent with some \( \phi \), and \( \text{dom}(\psi) \subseteq \text{dom}(\psi') \).

In the adaptive version of the influence maximization problem, we have to design a policy \( \pi \), determining sequentially which node(s) must be selected as seed(s) at each time step, given \( \psi \). We call as \( E(\pi, \Phi) \subseteq V \) the seed nodes that have been selected following the policy \( \pi \) under realization \( \phi \). Moreover, we are given a utility function \( f(S, \phi) \triangleq |\sigma(S, \phi)| \), with \( \sigma(S, \phi) \) to be the set of the influenced nodes at the end of the process under realization \( \phi \) and given the seed set \( S \). Actually, what we want is to discover a policy \( \pi^* \) that maximizes the expected utility given by \( f_{\text{avg}}(\pi) \triangleq \mathbb{E}_\Phi[f(E(\pi, \Phi), \Phi)] \). This optimization problem can be written more concretely as:

\[
\pi^* \in \arg \max_{\pi} f_{\text{avg}}(\pi) \quad \text{s.t.} \quad |E(\pi, \phi)| \leq k, \forall \phi.
\]

To provide generalizations of monotonicity and submodularity in such an adaptive setting, Golovin & Krause (2011) adopt the expected marginal gain notion.

In this paper, we introduce an alternative utility function that instead of the total number of the active nodes at the end of the diffusion, returns the cumulative number of active nodes through the time. In order to present our theoretical analysis in a strict way, we resort to a layered graph representation, similar to the one presented by Kempe et al. (2003), with each one of the graph’s layers to illustrate the diffusion in network at a particular time stamp. Additionally, we propose a new diffusion model, which constitutes a modified version of the so-called Myopic feedback model. In the modified myopic feedback model, an active node has more than one opportunities to activate each one of its neighbors. The main contribution of this work is that we achieve to prove rigorously that the considered objective function is adaptive monotone and adaptive submodular under both the standard and the modified myopic feedback model. Therefore, the proposed myopic adaptive greedy policy is theoretically guaranteed to reach an \( (1 - 1/e) \)-approximation ratio in terms of the expected utility of the optimal adaptive policy. We also prove that our assumption under which the active nodes cannot be deactivated through the time constitutes a necessary condition to verify the adaptive submodularity property of the proposed utility function in both myopic feedback models. Finally, the superiority of the myopic adaptive greedy strategy over other adaptive heuristic strategies and a non-adaptive greedy strategy to the IM problem has been demonstrated on real-life social networks.
**Definition 1.** The conditional expected marginal benefit of \( v \in V \), conditioned on partial realization \( \psi \), is given as:

\[
\Delta(v|\psi) \triangleq \mathbb{E}_\Phi \left[ f(\text{dom}(\psi) \cup \{v\}, \Phi) - f(\text{dom}(\psi), \Phi)|\Phi \sim \psi \right].
\]

This leads us to the following definitions of adaptive monotonicity and adaptive submodularity, defined w.r.t. to the distribution \( p(\phi) \) over realizations.

**Definition 2.** The utility function \( f \) is adaptive monotone iff, for all \( \psi \) such that \( \mathbb{P}(\Phi \sim \psi) > 0 \), we have \( \Delta(v|\psi) \geq 0 \).

**Definition 3.** \( f \) is adaptive submodular iff, for all \( \psi \leq \psi' \) and for all \( v \in V \setminus \text{dom}(\psi') \), we have \( \Delta(v|\psi) \geq \Delta(v|\psi') \).

Adaptive submodular functions share many interesting properties. For instance, adaptive submodularity is preserved by nonnegative linear combinations and truncations. In the rest of the paper we focus on the next key result, which is a theoretical guarantee for the adaptive greedy policy.

**Theorem 1 (Golovin & Krause (2011)).** Let \( \pi^\psi \) be the adaptive greedy policy selecting \( k \) seed nodes. If the utility function \( f \) is adaptive monotone and adaptive submodular w.r.t. \( p(\phi) \), then \( f_{avg}(\pi^\psi) \geq (1 - 1/e)f_{avg}(\pi^\ast) \).

Actually, Theorem 1 is a direct extension of the non-adaptive bound, which was proved to be near-optimal. In the adaptive influence maximization problem, the following two concrete feedbacks can be considered:

- **Full-adoption feedback:** after selecting a seed node at time \( t \), we observe the entire propagation (cascade) in graph at \( t + 1 \), and then we select the next seed node;
- **Myopic feedback:** we only observe the status (active or not) of the neighbors of the seed nodes at \( t + 1 \).

Therefore, in the myopic feedback model, selecting a node at time \( t \) has an effect at time \( t + 2 \), \( t + 3 \), and so on. Nevertheless, Golovin & Krause (2011) have shown that the utility function \( f \) holds its adaptive submodular property only under the full-adoption feedback model. Thus, Theorem 1 cannot be applied directly in the case of myopic feedback model (simple counterexamples reported by Golovin & Krause (2011); Vaswani & Lakshmanan (2016)).

### 3. Myopic Feedback through Layered Graphs

The limitations of the full-adoption feedback (in most applications the propagation in the network is not instantaneous) motivates us to migrate on the myopic feedback model, which seems to fit better on the real world. However, Theorem 1 is not applicable as \( f \) is not adaptive submodular anymore under the IC model with myopic feedback. To deal with this situation, we propose the following modifications. Instead of considering the total number of active nodes at the end of the process as the utility function, we employ the cumulative number of active nodes over time. More concretely, given a finite horizon \( T \), the proposed utility function is defined as \( \tilde{f} \triangleq \sum_{t=1}^{T} |\sigma_t(S, \phi)| \), where \( \sigma_t(S, \phi) \) represents the set of active nodes at time \( t \) if the seed set \( S \) has been selected under realization \( \phi \).

According to \( \tilde{f} \), if a node is active for three time steps, it will yield a reward equal to 3 instead of 1 as in case of \( f \). The proposed utility function is consistent with many real life situations. Consider, for instance, the case of platforms with a monthly subscription, like Netflix or Amazon. Those services charge each active user every month on the date he signed up. Thus, the companies’ profit increase as the users be active for longer periods. Thus, the value of an active node is additive over time.

To represent the evolution of the network over time, we resort to a layered graph representation, denoted as \( \mathcal{G}^L \). A graph’s layer corresponds to the representation of the original graph at a particular time step, with \( \mathcal{L}_t \) to denote the set of nodes on layer \( t \). Consider for example the original graph illustrated at Fig. 1(a) and its evolution over three successive time steps, with green nodes being active nodes. We retrieve the same amount of information as in the case of the layered graph, Fig. 1(b). Indeed, node \( v \) is active at time \( t \) if and only if \( u_t \) is active in the layered graph. Then, it influences its neighbor \( v \) at time \( t + 1 \) with probability \( p_{uv} \). Thus, there is a possibly live edge from \( u_t \) to \( v_{t+1} \). For the sake of simplicity, in the rest of the paper we use the indexing \( f_{\mathcal{G}} \) or \( f_{\tilde{\mathcal{G}}} \) to explicitly declare that function \( f \) or \( \tilde{f} \) is estimated on graph \( \mathcal{G} \).

In the layered graph representation, if \( u_t \) is active then \( u_{t+1} \) will also be active with probability \( 1 \), which reflects that the nodes can only switch from being inactive to being active. This assumption will be relaxed in Section 4. Another assumption is that an active node has multiple opportunities to influence its neighbors. This is a slight modification from the standard model, which is still consistent with most applications. In Section 4, we prove that the proposed utility function is adaptive submodular under both the modified and the standard IC model with myopic feedback.

![Figure 1. Influence propagation representation over (a) Original graph \( \mathcal{G} \) and (b) Layered graph \( \mathcal{G}^L \).](image-url)
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It can be easily verified that the two networks, the original and the layered one, are closely linked. The following lemma highlights the fact that computing \( f'_{G^L} \) is equivalent to computing \( f \) on the layered graph, i.e. \( f'_{G^L} \).

**Lemma 1.** For each set \( S \) of seed nodes (with time indices) and realization \( \phi \), it holds that \( f'_{G^L}(S, \phi) = f_{G^L}(S, \phi) \).

**Proof.** It suffices to remark that the number of active nodes on layer \( L_t \) is equal to the number of active nodes on \( G \) at time \( t \). Summing up the active nodes of each layer \( L_t \) is the same by applying \( f \) on \( G^L \), which is equivalent to summing up the number of active nodes on \( G \) at each timestep.

In our model, the time dependency is even stronger compared to previous models. Partial realizations \( \psi \) should now indicate the status of observed nodes and edges as well as the corresponding timesteps, as nodes can be activated over multiple timesteps and edges can be crossed multiple times. Actually, we need to know up to which time step, the \( \psi \) contains observations. This leads us to the next definition.

**Definition 4.** Let \( \Psi \) be the set of all possible partial realizations. The time function \( T : \Psi \rightarrow \{1, ..., T\} \) returns, for a particular \( \psi \), the largest time index from observed nodes and edges, and 1 if \( \psi = \emptyset \).

In a nutshell, choosing \( u \) as a seed node having observed \( \psi \) with \( T(\psi) = t \leq T \), is the same as choosing \( u_t \) as a seed node in the layered graph, since the process is now at time \( t \). In this point, let us provide a last definition.

**Definition 5.** The marginal gain of choosing \( u \) as a seed node, having observed \( \psi \) with \( T(\psi) = t \), and for the ground truth realization \( \phi \) of the network, is defined as:

\[
\delta_{\phi}(u|\psi) \triangleq f'_{G^L}(\text{dom}(\psi) \cup \{u_t\}, \phi) - f'_{G^L}(\text{dom}(\psi), \phi).
\]

Definition 5 is useful for the analysis of the following lemmas. First, Lemma 2 is a markovian result on layers. It shows that, to evaluate \( \delta_{\phi}(u|\psi) \), we only need information from the current layer, \( T(\psi) \). Information from past layers have no impact on the marginal gain of adding \( u \) to seed nodes at time \( T(\psi) \). Then, Lemma 3 is a submodularity result on \( \delta_{\phi}(\cdot|\psi) \), that will be central in proofs of Section 4.

**Lemma 2.** The marginal gain of choosing \( u \) as a seed node on \( G^L \), under partial realization \( \psi \) with \( T(\psi) = t \), is:

\[
\delta_{\phi}(u|\psi) = f_{G^L}([L_t \cap \text{dom}(\psi)] \cup \{u_t\}, \phi) - f_{G^L}(L_t \cap \text{dom}(\psi), \phi).
\]

**Proof.** Based on Def. 5 and Lemma 1, we get that \( \delta_{\phi}(u|\psi) = f_{G^L}(\text{dom}(\psi) \cup \{u_t\}, \phi) - f_{G^L}(\text{dom}(\psi), \phi) \).

Given a set \( S \) of seed nodes on the \( G^L \), the utility function \( f \) is given by \( f_{G^L}(S, \phi) = \sum_{t'=1}^{T} \sigma(S_t, \phi) \cap L_{t'} \). Then we get that:

\[
\delta_{\phi}(u|\psi) = \sum_{t'=1}^{T} \sigma(\text{dom}(\psi) \cup \{u_t\}, \phi) \cap L_{t'} - \sum_{t'=1}^{T} \sigma(\text{dom}(\psi), \phi) \cap L_{t'} = \sum_{t'=1}^{T} \sigma(\text{dom}(\psi) \cup \{u_t\}, \phi) \cap L_{t'} - \sum_{t'=1}^{T} \sigma(\text{dom}(\psi), \phi) \cap L_{t'}
\]

**Algorithm 1** Myopic Adaptive greedy policy

**Input:** \( G, T \)

1. \( \psi \leftarrow \emptyset, \ S \leftarrow \emptyset \)
2. for \( t = 1 \) to \( T \) do
3. Compute \( \Delta(v|\psi), \forall v \in L_t \setminus S \)
4. Select \( v^* \in \arg \max \Delta(v|\psi) \)
5. \( S \leftarrow S \cup \{v^*\} \)
6. Update \( \psi \) observing (one-step) myopic feedback
7. \( S \leftarrow S \cup \text{dom}(\psi) \)
8. end for
9. return \( S \) (final set of influenced nodes)

**Lemma 3.** For partial realizations \( \psi \subseteq \psi' \) with \( T(\psi) = T(\psi') = t \) and any \( v \in V \), we get \( \delta_{\phi}(v|\psi) \geq \delta_{\phi}(v|\psi') \).

**Proof.** Let \( R(v_t, \phi) \) denotes the set of nodes that can be reached from node \( v_t \) via a path consisting of live edges, under realization \( \phi \). For any \( A \subseteq L_t \) (layer \( t \) of \( G^L \)), we have \( f_{G^L}(A, \phi) = \|U \cup A \cap R(u, \phi) \). Let us now consider the quantity \( f_{G^L}(A \cup \{v_t\}, \phi) - f_{G^L}(A, \phi) \) to be equal to the number of elements of \( R(v_t, \phi) \) that are not already contained in \( U \cup A \cap R(u, \phi) \). Clearly, this quantity is larger or equal to the number of elements of \( R(v_t, \phi) \) that are not contained in the bigger set \( U \cup B \cap R(u, \phi) \). Setting \( A = L_t \cap \text{dom}(\psi) \) and \( B = L_t \cap \text{dom}(\psi') \) we can conclude that \( \delta_{\phi}(v|\psi) \geq \delta_{\phi}(v|\psi') \).

**4. Theoretical Guarantees for the Myopic Adaptive Greedy Strategy**

We are now ready to formally state our main result, which is an approximation guarantee for the proposed myopic adaptive greedy policy. As already mentioned, the policy starts with an empty set \( S_0 = \emptyset \), and repeatedly chooses as seed node the node that gives the maximum expected marginal gain under partial realization \( \psi \). An overview of the myopic adaptive greedy policy is presented in Alg. 1.

**Theorem 2.** The adaptive greedy policy \( \pi^* \) obtains at least \((1 - 1/e) \) of the value of the best policy for the adaptive influence maximization problem under the modified IC model with myopic feedback and \( f \) as utility function. In other words, if \( \tilde{f}_{avg}(\pi^*) \triangleq \mathbb{E}_{\Phi}[f_{G}(E(\pi, \Phi), \phi)] \), we get that:

\[
\tilde{f}_{avg}(\pi^*) \geq (1 - 1/e) \tilde{f}_{avg}(\pi^*)
\]
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Proof. Actually, we want to prove that the utility function \( f_G \) is adaptive monotonic and adaptive submodular w.r.t. \( p(\cdot) \). This will allow us to directly apply Theorem 1 to complete the proof. Adaptive monotonicity can be easily verified, since \( f_G(\cdot, \phi) \) is itself monotonic for each \( \phi \).

At this point, let us choose two subrealizations \( \psi, \psi' \) with \( \psi \subseteq \psi' \) and \( u \notin \text{dom}(\psi') \). To prove adaptive submodularity in a rigorous way, i.e. to prove that \( \Delta(u|\psi) \geq \Delta(u|\psi') \), we adopt here the methodology of Golovin & Krause (2011). They define a coupled distribution \( \mu \) over pairs of realizations \( \phi \sim \psi \) and \( \phi' \sim \psi' \). Let \( X \triangleq \{X_{uv} : (u, v) \in E\} \) denote the Bernoulli r.v. of the model, and let us explicit the realizations’ dependency over these r.v.: we indeed have \( \phi(E) = \phi(E(X)) \) that we denote by \( \phi(X) \) for conciseness. Using this formalism, we define \( \mu \) implicitly in terms of a joint distribution \( \mu \) on \( X \times X' \) where \( \phi = \phi(X) \) and \( \phi' = \phi'(X') \) (the realizations generated from these sets of random edge statuses), so \( \mu(\phi(X), \phi'(X')) = \hat{\mu}(X, X') \). Because \( \psi \subseteq \psi' \), we derive that all edges observed by \( \psi \) have the same state - 0 or 1 - in \( \phi \) and \( \phi' \), since \( \phi \sim \psi \) and \( \phi' \sim \psi' \). All \( (X, X') \in \text{support}(\hat{\mu}) \) have these properties.

Last but not least, Golovin & Krause (2011) define \( \hat{\mu} \) s.t. the status of all edges not observed by \( \psi \) and \( \psi' \) are the same in \( X \) and \( X' \), i.e. \( X_{uv} = X'_{uv} \) for all edges \( (v, w), \) or \( \hat{\mu}(X, X') = 0 \). Here, we have randomness for all edges unobserved by \( \psi \). Selecting them independently with \( \mathbb{E}[X_{uv}] = p_{uv} \) according to \( p(\cdot) \), we end up with

\[
\hat{\mu}(X, X') = \prod_{(v, w) \text{ unobserved by } \psi} p_{uv}^{X_{uv}}(1 - p_{uv})^{1 - X_{uv}},
\]

for \( (X, X') \) satisfying previous constraints, and \( \hat{\mu}(X, X') = 0 \) otherwise (see Golovin & Krause, 2011) for details).

Then, let us remark that \( \delta_\phi(u|\psi) \geq \delta_\phi(u|\psi') \) for all \( (\phi, \phi') \in \text{support}(\mu) \). There are three different situations:

i) The first one is \( \psi \subseteq \psi' \) with \( T(\psi) = t < T(\psi') \): consider w.l.o.g. that \( T(\psi) = t + 1 \). Node \( u_t \) is therefore activated in \( G^L_t \), after observing \( \psi \). Let \( \psi_+ \) denotes the partial realizations combining \( \psi \) and observing one more step of the process - from layer \( t \) to layer \( t + 1 \) - without adding any seed node, w.r.t. \( \phi \). Also, let \( A \) denotes the set of activated nodes of layer \( t + 1 \), observed by \( \psi_+ \) and that would not have been activated if \( u_t \) was not selected as seed node, except \( u_{t+1} \). We have: \( \delta_\phi(u|\psi) = (1 + \delta_\phi(u \cup A|\psi_+)) \geq (1 + \delta_\phi(u \cup A|\psi_+)) = (2) \delta_\phi(u \cup A|\psi_+)) \geq (3) \delta_\phi(u|\psi') \). (1) comes from the fact that \( G^L \) is feedforward, therefore activating \( u \) brings a reward of 1 at time \( t \), plus the reward from the future. (2) is verified since \( (\phi, \phi') \in \text{support}(\mu) \) and quantities depend only on layers from \( t + 1 \) to \( T \) (Lemma 2), i.e. on edges unobserved by \( \psi_+ \) and \( \psi' \). (3) is due to the monotonicity of the set function \( \delta_\phi(\cdot|\psi_+) \). (4) is a direct application of Lemma 3 since \( \psi_+ \subseteq \psi' \) is always true under the given realizations.

This result is still valid for \( T(\psi') = t + x, \) with \( x > 1 \). Indeed, one can easily verify that we still get that \( \delta_\phi(u|\psi) \geq \delta_\phi(u \cup A|\psi_+) \) with \( A \) to be defined as before and \( \psi_+ \) to denote the partial realizations combining \( \psi \) and observing \( x \) more step of the process without adding any seed node, w.r.t. \( \phi \). Then, the remaining steps of the proof are identical with those of the case where \( x = 1 \).

ii) Then, if \( T(\psi) = T(\psi') = t \), a direct application of Lemmas 2 and 3 leads us to \( \delta_\phi(u|\psi) \geq \delta_\phi(u|\psi') \).

iii) The third case \( T(\psi) > T(\psi') \) is impossible, since it would violate the \( \psi \subseteq \psi' \) hypothesis.

Having proved that \( \delta_\phi(u|\psi) \geq \delta_\phi(u|\psi') \) for all \( (\phi, \phi') \in \text{support}(\mu) \), we use the trick of (Golovin & Krause, 2011) to conclude. Since \( \delta_\phi(u|\psi) - \delta_\phi(u|\psi') \triangleq f_G(\text{dom}(\psi')) - f_G(\text{dom}(\psi')) \) we derive that \( \Delta(u|\phi) = \sum_{(\phi, \phi')} \mu(\phi, \phi') \delta_\phi(u|\psi') \leq \sum_{(\phi, \phi')} \mu(\phi, \phi') \delta_\phi(u|\psi) = \Delta(u|\phi) \) proving that \( f_G \) is adaptive submodular.

4.1. Standard Myopic Feedback Model Analysis

Our analysis so far has been based on the assumption that an active node has multiple opportunities to influence its neighbors. In this section, we prove that the same theoretical guarantees are hold in the case of the standard IC diffusion model with myopic feedback presented by Golovin & Krause (2011). From now on let us call this model as standard myopic feedback model. In contrast to the modified myopic model, the nodes in the standard version have a single chance to influence/activate their neighbors.

The main difficulty now is that the influence weights among two nodes in the layered graph representation \( G^L \) are not the same along the time anymore. More specifically, it is no longer true that the influence probabilities between \( u_t \) and \( u_{t+1} \) with \( t \in \{1, \ldots, T-1\} \) in the \( G^L \) are equal to \( p_{uv} \in [0, 1] \). Let us define the function \( \mathbb{I}_\psi(u_t) = \mathbb{I}\{u_t \notin \text{dom}(\psi) \}, \forall i \in \{1, \ldots, t-1\} \) that indicates whether the node \( u \) has been activated before time \( t \) (0) or not (1). Therefore, the influence probabilities on the edges between \( u_t \) to \( u_{t+1} \) \( (\forall t \in \{1, \ldots, T-1\} \) will now be equal to \( p_{uv} \mathbb{I}_\psi(u_t) \). Actually, it is like adding randomness into the edges’ probabilities that is strongly related to the decisions taken by the following policy.

As a consequence, it becomes difficult to get results for all realizations \( \phi \) of the ground truth graphs as in Lemmas 2 and 3. To overcome this problem, we assume w.l.o.g. that the status of the edges from node \( u \) to \( v \) in the \( G^L \) are revealed right after the first and last attempt of the \( u \) to activate node \( v \). Actually, all edges have the same status (live or dead) that depends on whether this attempt was success-
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ful or not. It can be easily verified that this assumption has no impact on the diffusion process. For instance, let us consider a particular state of the world under which we observe that the first and single chance of node \( u \) to influence \( v \) is successful. Then, assuming that statuses of all edges between \( u_t \) and \( v_{t+1} \) with \( t \in \{1, \ldots, T-1\} \) in \( G^L \) are all equal to 1 (live) will not modify the final spread. Indeed, the edges prior to the one actually caused to influence \( v \) has no longer impact on the spread as the network \( G^L \) is feedforward. On the other hand, the status of the edges that belong on subsequent layers has no impact at the diffusion process. For instance, let us forcing active nodes to remain active through-time that such inequalities are demonstrated on the adaptive setting under myopic feedback model. Using the generalization of Theorem 1, we also retrieve the \((1-e^{-\ell/\alpha k})\) bound for any \( \alpha \)-approximate (\( \ell \)-truncated) greedy policies. Moreover, it can be easily verified that the bounds (Theorems 2 and 3) are still valid even if we select \( n > 1 \) seed nodes at each time step.

5. Non-Progressive Adaptive Submodular IM

So far, we have been relied on the hypothesis that active nodes can not be deactivated randomly. Nevertheless, the application itself can determine if this hypothesis is realistic or not. In the case of our layered graph representation, we can easily relax this assumption, by simply replacing the “1” with a random probability over the edges between the same nodes (see Fig. 2(a) for example). Our model along with the main notations are still well defined under this relaxation. However, it appears that it destroys the reasoning of the proof of Theorem 2. Indeed, as we demonstrate below, the utility function \( f_G \) is no longer adaptive submodular. Additionally, we show that the adaptive submodularity property is also violated even in the case of the full-adoption feedback by using the utility function \( f_G \).

Lemma 4. Forcing active nodes to remain active throughout the process constitutes a necessary condition to verify the adaptive submodularity property of:

i) \( f_G \) in the modified Myopic feedback framework;
ii) \( f_G \) in the standard Myopic feedback framework;
iii) \( f_G \) in the Full-adoption feedback framework.

Proof. i) In the case of the modified myopic feedback model, consider the (upper) layered graph of Fig 2(b) that consists of six random edges. There are \( 2^6 = 64 \) ground truth graphs, each of them being obtained with probability \( 1/64 \) since edges are independent Bernoulli r.v., \( B(1/2) \). We want to add \( v \) to the set of seed nodes. Now, consider \( \psi \) where we only know that \( u \) is activated at \( t = 1 \) (\( T(\psi) = 1 \)), and \( \psi' \) where we also observed that \( (u_1, u_2) \)
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and \((u_1, v_2)\) are dead edges \(\left( T(\psi') = 2 \right)\). Clearly, \(\psi \subseteq \psi'\).

A simple decomposition of all possible ground truth graphs leads to \(\Delta(v|\psi') = 1 + \left[ \frac{2}{5} \times \frac{1}{2} + 1 \times \frac{1}{10} + 0 \times \frac{6}{25} \right] = \frac{11}{10}\): a reward of \(1\) for activating \(v_1\) and possibly a marginal gain of adding \(v_2\) and \(v_3\) \((0 \text{ in } 48 \text{ ground truth realizations}, 1 \text{ in } 10 \text{ of them}, 2 \text{ in } 6 \text{ of them})\). We also get that \(\Delta(v|\psi') = 1 + \frac{1}{5} = 1.5\): \(v_2\) is active (seed) while \(v_3\) is active with probability \(1/2\). Therefore, \(\Delta(v|\psi') > \Delta(v|\psi)\) that violates the adaptive submodularity property.

ii) We consider again the same scenario as previously, with \(\psi \subseteq \psi'\). Similarly, a simple decomposition of all possible ground truth graphs gives \(\Delta(v|\psi) = 1 + \frac{1}{5} \times \left[ \frac{1}{2} \times \frac{1}{5} + \frac{1}{2} \times \frac{4}{5} \right] = \frac{11}{10}\). On the other hand, we get that \(\Delta(v|\psi') = 1 + \frac{1}{5} = 1.5\) as \(v_2\) is a seed node and \(v_3\) remains active with probability \(1/2\). Once again, we conclude that the adaptive submodularity property does not hold since \(\Delta(v|\psi') > \Delta(v|\psi)\).

iii) Let us consider the (bottom) graph of Fig. 2(b), where active nodes have a probability of \(1/2\) to be deactivated at each time step. Recall that our utility function is now the number of activated nodes at the end of the process, and let \(T = 2\). Suppose also that we want to choose \(a\) as seed node. Consider two policies: choose \(a\) at \(t = 1\) and \(d\) at \(t = 2\) (so \(\psi = \emptyset\)) or choose \(d\) at \(t = 1\) and then \(a\) at \(t = 2\) (so \(\psi'\) only contains the information that \(d\) is activated). We get that \(\Delta(a|\psi) < 3\) \((a, b, c\) are active at \(t = 1\), but they have a non-null probability to deactivate at \(t = 2\)) and \(\Delta(a|\psi') = 3\) (the process ends after nodes \(a, b\) and \(c\) are activated via choosing \(a\) as seed node). Since \(\psi \subseteq \psi'\), adaptive submodularity is once again violated.

Therefore, the theoretical results presented in our paper and those of (Golovin & Krause, 2011) are not directly applicable in the case where the active nodes can be deactivated. However, this hypothesis of active nodes deactivation may be consistent with many applications, including some versions of the product placement problem (e.g. customers could reject the product after a while). In this direction, we are still able to prove a weaker inequality at each time step. We still consider the previous framework, but now active nodes are allowed to be deactivated randomly. At each time step \(t\), we choose \(k_t\) seed nodes from layer \(L_t\) of \(G^L\), in order to maximize the expected spread in the future having observed which nodes are currently active, i.e. active nodes on \(L_t\). Then, we have the following theorem.

**Theorem 4.** Let \(t \in \{1, \ldots, T\}\), let \(S_t \subseteq L_t\) the (observed) set of active nodes at time \(t\) and consider the following problem: \(A^* \in \arg\max_{A \subseteq \bigcup_{t \in \{1, \ldots, T\}} \{A \subseteq L_t \cup S_t, \Phi\}} \sum_{t=1}^{T} \Phi[f_{L_t \cup \ldots \cup L_T}(A \cup S_t, \Phi)]\). Then, a greedily constructed set \(A^* \subseteq \bigcup_{t \in \{1, \ldots, T\}} \{A \subseteq L_t \setminus S_t, \Phi\}\) is guaranteed to achieve a \((1 - 1/e)\)-approximation of the optimal set: \(\sum_{t=1}^{T} \Phi[f_{L_t \cup \ldots \cup L_T}(A^* \cup S_t, \Phi)] \geq (1 - 1/e) \sum_{t=1}^{T} \Phi[f_{L_t \cup \ldots \cup L_T}(A^* \cup S_t, \Phi)]\).

**Proof.** We easily derive from the proof of Lemma 3 that \(f_G\) is submodular, for any layered graph \(G^L\) (i.e., also for layered graphs \(L_t \cup \ldots \cup L_T\)). Indeed, we proved that for any \(\phi\) and \(A \subseteq B \subseteq L_t, f_G(A) + f_G(B) - f_G(A \cup \phi) \geq f_G(B) - f_G(A, \phi)\). Moreover, submodularity being preserved under nonnegative linear combinations, then the objective function of Theorem 2 is also itself submodular. Indeed, the expectation is a weighted sum of submodular functions, weights being probabilities, according to \(p(\phi)\). Therefore, we conclude by applying the classical result of (Nemhauser et al., 1978).

This result is weaker than those of Theorems 2 and 3, since it is simply a “step-by-step” inequality on each seeding, but not anymore on the entire policy. However, it is free from the assumption that active nodes should remain active.

**6. Empirical Analysis**

We conducted experiments on three networks from the Stanford’s SNAP database\(^1\). The first one is a rather small directed ego network from Twitter \(|V| = 228, |E| = 9,938\). We also study two medium-size (undirected) real networks, a social network from Facebook \(|V| = 4,039, |E| = 88,234\) and a collaboration network from Arxiv General Relativity and Quantum Cosmology section \(|V| = 5,242, |E| = 28,980\) (see appendix for more details).

In our analysis, we have consider both the standard and the modified IC diffusion model with myopic feedback, where we want to adaptively select \(k\) seed nodes, one at each time. The time horizon is defined as \(T = k + 1\), i.e., the diffusion process stops one step right after selecting the last seed node. As it is not possible to actually compute the optimal set of influential nodes, we compare the performance of the adaptive greedy strategy w.r.t. three alternative heuristics to identify influential seed nodes. These heuristics adaptively choose: (i) the node with highest betweenness centrality; (ii) the node with highest degree; and (iii) a random node among inactive nodes. Comparisons have been also made with a non-adaptive version of the greedy strategy, which chooses the \(k\) seed nodes (one at each step) in advance.

Similar to (Kempe et al., 2003; Gotovos et al., 2015), we set an identical influence probability at each edge, \(p = 0.1\). Nevertheless, different influence probabilities have also been considered (see appendix), leading to similar conclusions. The expected marginal gains are approximated via Monte Carlo sampling (1,000 simulations).

Figure 5 illustrates the empirical means of the expected utility \(f\) as well as the ±1 standard deviation deviations over 100 runs in the modified and the standard IC models. As expected, the adaptive greedy strategy outper-

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\(^1\)https://snap.stanford.edu/data/
forms the other strategies in all cases. Actually, in the case of the modified myopic feedback (Fig. 5(a)) the adaptivity is more profitable as we gradually gain more knowledge about the ground truth influence graph. This happens due to the fact that each node in the modified IC model has more than one chance to influence its neighbors. Therefore, the ground truth influence graph is revealed in a much faster rate than in the case of the original IC model. Moreover, the improvement on the performance that the adaptive greedy strategy achieves compared to that of the non-adaptive greedy depends on the network’s properties. For instance, in the case of the modified myopic feedback model (Fig. 5(a)) the adaptivity is more profitable as we gradually gain more knowledge about the ground truth influence graph. Actually, the performance of the non-adaptive greedy strategy is worse even when it is compared with that of the simple adaptive degree or centrality strategies under the modified myopic feedback. On the other hand, it seems that adaptivity doesn’t improve the performance a lot in the case of where we apply the standard myopic feedback in a dense graph, like Facebook (Fig. 5(b)). Summing up, it should be noticed that our analysis validates our initial claim that the performance of the proposed myopic adaptive greedy policy will be least as good as that of the non-adaptive greedy policy.

7. Conclusion

We proposed the myopic adaptive greedy strategy for the adaptive influence maximization task. It is the first time where a policy like this one offers provable approximation guarantees under both the standard IC model and a modified version of it with myopic feedback. This is achieved by proposing an alternative utility function that returns the cumulative number of active nodes over time, which corresponds to the number of active nodes on a layered graph representation. Nevertheless, several interesting directions for future work remain. So far, we assumed that the influence graph was fully known, which may be a strong assumption in practice. In future work we intend to relax this assumption, studying alternative problems where influence probabilities must be adaptively learned in order to maximize influence. How to efficiently perform such dual task
in an adaptive way is still an open research question.

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Appendix

The purpose of this section is to provide more details about the materials that have been presented in the our paper. First of all, we provide a simple toy example in order justify our claim (Section 1) that the adaptive setting leads to higher spreads compared to the non-adaptive. Secondly, we analyze the three social networks used through our experiments, and provide information about the alternative heuristics used for comparisons. Finally, we report additional results exported by our empirical analysis.

A. Adaptive setting leads to higher spreads

To defend our claim, we give a simple example. Consider the network shown in Figure. 4(a) with influence probabilities \( p_{ab} = 0.9 \) and \( p_{ac} = 0.1 \). Let \( k = 2 \) (seed nodes - our budget). The non-adaptive greedy algorithm will select as seed nodes the \( a \) (at \( t = 1 \)) and \( c \) (at \( t = 2 \)). Nevertheless, based on the true world (see Fig. 4(b)), we observe that nodes \( a \) and \( b \) are active at time \( t = 2 \). Hence, we will infer that the edges \((a, b)\) and \((a, c)\) are live and dead, respectively. Therefore, the non-adaptive setting will lead to a reward equal to 2, as only nodes \( a \) and \( c \) will be activated finally, but not \( b \). Roughly speaking, we are going to make an offer at an already influenced user. On the other hand, the adaptive myopic strategy will first choose the node \( a \) and then will observe the status of the outgoing edges of node \( a \). In other words, he will observe that \( a \) managed to influence \( c \) but not \( b \). Hence, he will choose node \( b \) as the second seed node, since it is the only one which is not activated at this point. This leads to a reward of 3, which is higher than the non-adaptive setting, since all nodes are finally activated.

![Figure 4](https://snap.stanford.edu/data/)

B. Experimental Networks Description

As noticed in the main part of the paper, we conducted experiments on three networks from the Stanford’s SNAP database. The first one is a rather small graph, which represents an ego network from Twitter. The dataset is a subset - a “circle” - from the list of social circles from Twitter crawled from public sources. The graph consists of 228 nodes and 9,938 edges. The second one is a social network extracted from Facebook. Data were anonymously collected from survey participants using the Facebook app. The graph is undirected, and has 4,039 nodes and 88,234 edges.

The third graph is that of the Arxiv General Relativity and Quantum Cosmology collaboration network. In this graph, we have an undirected edge from \( i \) to \( j \), if author \( i \) co-authored a ArXiv paper with author \( j \) (between 1993 and 2003). This graph has 5,242 nodes and 28,980 edges.

A number of useful statistics about these graphs are provided in Table 1. Mean degree is the mean number of edges exiting nodes. A.P.L. stands for Average Path Length, which is the average number of nodes in the shortest path between two nodes of the graph. Moreover, the diameter of a graph is the length of the longest shortest path between two nodes.

C. Adaptive Greedy and Alternative Heuristics

In case of the modified myopic feedback model, we chose to implement the improved accelerated version of the adaptive greedy strategy (Golovin & Krause, 2011), for computational reasons. The algorithm is based on so-called lazy evaluations, i.e. on a clever use of the adaptive submodularity inequality to significantly reduce running times in practice by diminishing the number of nodes on which Monte Carlo simulations should be performed. The pseudo-code and the justification of this improved adaptive greedy algorithm is reported in (Golovin & Krause, 2011).

We compared the performance of the adaptive greedy strategy with the performance of alternative adaptive heuristics to select influential nodes. These heuristics adaptively choose, as seed node:

- the node with the highest centrality measure. In our experiments we use the betweenness centrality measure, which is equal to the number of shortest paths from all nodes to all others that pass through a node;
- the node with the highest degree, i.e. the node with the highest number of outgoing edges;
- a third baseline approach adaptively chooses seed nodes at random, from inactive nodes.

Finally, we also implemented a non-adaptive version of the greedy strategy which chooses in advance the \( k \) seed nodes, one for each time step.

D. Additional Results

In this section, we provide complementary results from our experimental study for the case of the modified myopic feedback model. Fig. 5 illustrates the empirical means of the expected utility \( f \) as well as the \( \pm 1 \) standard deviation intervals over 100 runs. These results correspond to
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| Network     | Nodes | Edges    | Mean degree | Max. degree | A.P.L. | Diameter | Type  |
|-------------|-------|----------|-------------|-------------|--------|----------|-------|
| Twitter     | 228   | 9,938    | 43.6        | 125         | 2.1    | 6        | Directed |
| ArXiv GR-QC | 5,242 | 28,980   | 11.1        | 162         | 6.1    | 17       | Undirected |
| Facebook    | 4,039 | 88,234   | 43.7        | 1,045       | 3.7    | 8        | Undirected |

Table 1. Statistics of the real-world networks used through experimental analysis

![Graphs](image)

Figure 5. Expected cumulative number of active nodes ($\tilde{f}_G$) vs. number of seeds for real world networks.

the case where the influence probabilities are equal to $1/4$ ($p = 1/4$) at each edge. The settings and the conclusions are similar to them of the main paper.