Effect of ambient pressure oscillation on the primary breakup of cylindrical liquid jet spray

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Abstract
The present simulation study investigates the effects of ambient pressure oscillation on cylindrical liquid jet sprays, using the volume of fluid method. The research is motivated by combustion instability in combustion engines, where strong harmonic pressure oscillation can damage internal structures. Oscillating pressure modulates not only the fuel mass flow rate but also the ambient gas density and liquid surface tension, and in liquid sprays, the ambient fluid density and surface tension can have substantial effects on spray breakup. In order to investigate the multiple property changes with ambient pressure oscillation, therefore, a new solver in OpenFOAM is developed. In the solver, liquid mass flow rate, ambient gas density, and liquid surface tension change simultaneously as a result of pressure oscillation. Simulations were conducted at a Reynolds number of 2000 and Weber number over 2000, conditions that are conducive to primary breakup in laminar flows. The simulations show that oscillations in ambient pressure significantly strengthen the surface instability of the liquid ligament, which depends on the surface tension–pressure coefficient, the mean pressure, and the amplitude of oscillation.

Keywords
Primary breakup, ambient pressure, oscillation, inlet velocity, surface tension, gas density

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1. Introduction
Combustion instability is a process in which unsteady pressure is coupled with unsteady heat release to amplify flow fluctuations. This phenomenon has long been a major concern in various engineering applications, including gas turbine combustors, afterburners, and industrial furnaces. Although the individual mechanisms of combustion instability are well understood and have been discussed in, for example, works by Toong et al.¹ Lieuwen and Zinn,² Acharya et al.,²,³ and Biswas,⁴ the complete pathway of the interplay of pressure and heat release remains a challenging problem.

As combustion instability is a system-wide problem, the contributions of each component of the system need to be thoroughly investigated. Among many components, fuel spray is considered to play an important role in combustion instability.⁵,⁶ While spray dynamics and breakup have been analyzed extensively,⁷–¹³ over a wide range of conditions, however, the physics of fuel sprays are as yet generally considered to be poorly understood. The common range of oscillation in combustor pressure is from 10% to 20% .¹⁴,¹⁵ In some cases, the amplitude can soar to 75% of the static chamber pressure for rocket combustors.¹⁶

Lieuwen and Zinn² investigated one of the mechanisms caused by interactions between combustor pressure oscillations and the fuel supply rates under lean conditions in a low NOX gas turbine, which produces large heat-release oscillations that could drive
instabilities. In their analysis, the onset of instability occurs in the following steps: (i) the pressure oscillation at the combustor travels upstream to the fuel injector at the speed of sound, (ii) the pressure oscillation at the fuel injector causes oscillation of the fuel supply rate, (iii) the oscillating fuel supply rate causes oscillation of the equivalence ratio at the fuel injector, (iv) the equivalence ratio fluctuation travels to the combustor at the flow velocity, and (v) finally, the equivalence ratio fluctuation causes oscillations in heat release. Hence, if the travel time of step (iii) is in phase with the pressure oscillation, instability arises. The key parameter is the travel time, as they demonstrated by changing the location of the fuel injector (effectively changing the travel time).

Chaparro and Cetegen\textsuperscript{17} focused on the effect of upstream flow modulation to understand the blow-off characteristics and stabilities of conical turbulent premixed flames. They concluded that the modulation of the liquid fuel is crucial to prevent combustion instabilities. Seo and Lee\textsuperscript{18} also studied combustion instability mechanisms in a lean premixed gas turbine combustor characterized by large pressure fluctuations and excessive heat transfer. They found that the pressure oscillations are strongly coupled with variations in flame structure.

Oscillating inlet velocity is also known to play a leading role in intensifying the spray breakup process. Chung et al.\textsuperscript{19} experimentally found that upstream acoustic modulation can affect liquid disintegration in a hollow-cone conical spray by shortening the breakup length. Chaves et al.\textsuperscript{20} and Geschner et al.\textsuperscript{21} subsequently investigated the effects of perturbations in a liquid jet modulated by well-defined periodic velocity fluctuation of finite amplitudes, and classified the different structures of jet spray observed in their experiments. They used a velocity-modulation atomizer driven by a piezoelectric transducer to produce flow rate oscillations, in a setup similar to that previously used by Takahashi et al.\textsuperscript{22} and Chung et al.\textsuperscript{23} Brenn et al.\textsuperscript{24} researched sheet and ligament formation processes utilizing the vibrational excitation of the liquid sheets that emerge from flat-fan pressure atomizers. In numerical simulations, Srinivasan et al.\textsuperscript{25} conducted computational analysis based on volume of fluid (VOF) method to investigate the effects of inlet velocity, liquid jet diameter and excitation parameters, modulation amplitude, and frequency, on the morphology of round laminar pulsed liquid jets. Salvador et al.\textsuperscript{26} simulated the primary atomization of jet spray at low computational cost, but still showing good agreement with experimental data. The results of all of these studies suggest that the upstream velocity fluctuation caused by pressure perturbation can improve the atomization of the spray.

In general, oscillation of ambient pressure leads to changes in ambient gas density and in the liquid surface tension and viscosity of fluids. The effects of ambient pressure and density have been studied extensively, but for stationary pressures and not dynamically changing pressures. Researchers have investigated the effect of pressure on surface tension.\textsuperscript{27-29} As shown in Figure 1, Rice\textsuperscript{27} indicated that the surface tension decreases linearly with pressure. Grosshans et al.\textsuperscript{30} applied the VOF approach using the direction-averaged curvature model to predict liquid atomization with several different ratios of liquid–gas density and viscosity, for given Reynolds and Weber numbers. Yousefifard et al.\textsuperscript{31} and Kim et al.\textsuperscript{32} numerically investigated the spray characteristics of several fuels under different chamber backpressures. Chen and Yang\textsuperscript{33} reported on the role of backpressure for a sharp-edged orifice, mainly discussing its effect on the discharge coefficient of jet flow.

Furthermore, many researchers have extended their investigations to other ambient parameters. Elbadawy et al.\textsuperscript{35} focused on the effects of the root mean square (RMS) turbulence produced by a fan on the spray characteristics of liquid fuel injected into a constant volume vessel and then used discrete phase model (DPM) modeling, achieving good agreement with experimental data.
Chakraborty\textsuperscript{36} performed a mathematical investigation of the breakup of viscous liquid droplets on surface tension time-harmonic modulation subjected to fluctuations in pressure, temperature, and concentration. Müller et al.\textsuperscript{37} conducted computational fluid dynamics simulations of the primary breakup of a high-viscosity liquid jet to predict the gas phase velocities, which govern atomization.

In recent years, some advanced experimental and numerical studies have been applied to offer detailed insight into the phenomena of the primary breakup processes of liquid jets. Warncke et al.\textsuperscript{38} conducted highly resolved direct numerical simulation (DNS) embedded in a coarser large eddy simulation (LES) of pre-filming airblast atomization, in order to get more accurate liquid ligament and droplet structures, corresponding to the data obtained by high-speed shadowgraphy experiments. Bravo et al.\textsuperscript{39} performed simulations using the VOF interface tracking method coupled to the Lagrangian particle method to capture the breakup instabilities of jets and the resulting droplets, compared to experimental measurements using X-ray radiography. Tachibana et al.\textsuperscript{40} applied LES of spray combustion to the investigation of the thermo-acoustic instability mechanisms and droplet breakup process in practical aeroengine combustors, considering dynamic pressure with a peak frequency of approximately 500 Hz and an amplitude of more than 40 kPa. They compared the results with experimental data obtained from optical measurements, such as planar laser induced fluorescence (OH-PLIF) and OH/chemiluminescence.

The dependency of viscosity on pressure change is different for gases and liquids. Compared with gas, liquid viscosity is much less dependent on pressure. The viscosity of water, for example, changes by only 0.67% within the pressure range of 0.1–5 MPa.\textsuperscript{41} For gases, especially ideal gases, the change in viscosity can be considered pressure independent at low and medium pressures below 5 MPa.\textsuperscript{42,43}

In this study, we first discuss the effect of pressure oscillation on flow properties: inlet velocity, ambient density, and surface tension. We conducted numerical simulations employing the VOF method to simulate the primary spray behavior and compared these results in different cases with imposed continuous variable oscillations. All the simulations were conducted with a 2D laminar model, at forcing Reynolds number below 2000 and Weber number over 2000, with liquid properties selected to maintain laminar flow.

2. Methodology

The primary breakup of a liquid jet surrounded by ambient gas is modeled using the VOF method. Deshpande et al.\textsuperscript{44} published a detailed description of the interFoam-code in OpenFOAM, and showed its capabilities in interface capturing. InterFoam uses fixed values for thermodynamic properties, however, so it was necessary to modify the code for this research. The new solver is able to synchronously impose sinusoidal oscillation on the inlet velocity, gas density, and liquid surface tension driven by the oscillating ambient pressure. In the process of solving the pimple loop in interFoam, the gas density and liquid surface tension are updated every iteration to match the relationships given by equations (10) and (11). As the pressure change is global, the gas density and the liquid surface tension are updated globally as well. The modification is encoded on the U-equation subroutine and the P-equation subroutine. This solver treats the two-phase flow as a single incompressible continuum with variable gas density and changing liquid surface tension.

2.1. Flow parameters changed by ambient pressure

In the present configuration (see Figure 2), the pressure oscillation propagates upstream into the inlet duct and produces standing oscillations at the fuel injector. The pressure oscillation affects the fuel spray dynamics by influencing injector inlet velocity, gas density near the injector, and transport properties such as surface tension. The changes in the parameters are formulated as follows. To simplify the analysis, relationships are linearized for inclusion in the solver.

The inlet velocity has the following relationship

\[
U_{jet} = K\sqrt{P_{res} - P_{am}}
\]

(1)

where \(K\) is an orifice discharge coefficient, \(P_{res}\) is the pressure of the reservoir, and \(P_{am}\) is the ambient pressure.\textsuperscript{45} Assuming \(P_{res}\) does not change, the inlet velocity change as expressed in the quasi-steady orifice equation is linearized as follows

\[
U' = \frac{K}{2\sqrt{P_{res} - P_{am}}} P'
\]

(2)

where \(U'\) and \(P'\) represent the fluctuating terms of the inlet velocity and the ambient pressure. If the equation...
is divided by the averaged equation (1), it can be rewritten as

$$\frac{U'}{U_{\text{jet}}} = -\frac{1}{2(P_{\text{res}}/P_{\text{am}} - 1)P_{\text{am}}}$$  (3)

This indicates that the relative magnitude of the inlet velocity oscillation depends on the pressure ratio ($P_{\text{res}}/P_{\text{am}}$) and the relative magnitude of pressure oscillation. For example, if the reservoir pressure is twice the ambient pressure, the relative amplitude of inlet velocity oscillation is half the relative amplitude of ambient pressure. In general, the velocity oscillation would not change linearly with pressure at high-pressure oscillations. Still, the difference between the linearization (equation (3)) and the square-root dependence (equation (1)) is not drastic at high pressures: for example, the difference is less than 10% at 40% of ambient pressure. In general, the velocity oscillation is half the relative amplitude of inlet pressure oscillations. Also, as the motivation is understanding the onset of combustion instability where a narrow band response is dominant, we focused on a single harmonic oscillation. Hence, it was our preferred choice to have a single harmonic response in velocity.

The change in density of the ambient gas follows the ideal gas equation and is determined by the ambient pressure under constant temperature as

$$PM = \rho RT$$  (4)

where $R$ is the universal gas constant (=8.314 m$^3$Pa-mol$^{-1}$-K$^{-1}$), $T$ is the temperature, and $M$ is the molar mass of the gas. By linearizing the above equation, the density fluctuation is related as follows

$$\rho' = P'M/RT$$  (5)

The pressure dependence of liquid surface tension is rather complicated and depends on the combination of gases and liquids, according to their different solubilities, as described in the work of Rice$^{27}$ and Sachs and Meyn.$^{28}$ Vavruch$^{46}$ developed a method to evaluate the surface tension–pressure coefficient for pure liquids, as follows

$$\sigma = \sigma_0 - \frac{E_i}{P_l}(P_{\text{am}} - P_0)$$  (6)

where $E_i$ is the internal energy per unit surface energy and $P_l$ is the liquid pressure. With a parameter $\Gamma = E_i/P_l$, the linearized form of the above equation can be described as

$$\sigma' = -\Gamma \cdot P'$$  (7)

The coefficient $\Gamma$ (the ratio of $E_i$ and $P_l$) is approximately constant for any given gas–liquid group, especially within the pressure range up to 5 MPa,$^{29,47}$ but $\Gamma$ may vary substantially for different combinations of gases and liquids. For example, the decrement of surface tension is around 4 dyne/cm, which equals 5.6% of the initial value in the methane (gas) and water (liquid) group, for pressure changes from 0.01 MPa to 3 MPa. For the CO$_2$ (gas) and water (liquid) group, the change reaches almost 30% under the same conditions. When the coefficient $\Gamma$ is larger, the amplitude of oscillating surface tension rises with the ambient pressure oscillation (i.e., it is more sensitive to pressure change). Although $\Gamma$ is a specific value for a given fluid, various values are used to understand its dependency on the primary breakup of jet spray.

In summary, the flow parameters change harmonically given the oscillation of ambient pressure, as follows

$$P_{\text{am}} = \bar{P} + P' \cdot \sin(2\pi ft)$$  (8)

$$U_{\text{inlet}} = \bar{U} + \frac{K}{2\sqrt{P_{\text{res}} - P_{\text{am}}}}P' \cdot \sin(2\pi ft)$$  (9)

$$\rho_{\text{am}} = \bar{\rho} + \frac{1}{RT}P' \cdot \sin(2\pi ft)$$  (10)

$$\sigma = \bar{\sigma} - \Gamma P' \cdot \sin(2\pi ft)$$  (11)

With these relationships incorporated into the solver, these varying factors can be applied to the jet spray.

### 2.2. Governing equation

The governing mass and momentum conservation equations are presented as$^{48,49}$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0$$  (12)

$$\frac{\partial \rho U}{\partial t} + \nabla \cdot (\rho U U) = -\nabla \cdot P + \nabla \cdot \tau + [\sigma_0 - \Gamma \cdot (P_{\text{am}} - P_0)] \cdot \kappa \cdot \hat{n}$$  (13)

where the last term, $[\sigma_0 - \Gamma \cdot (P_{\text{am}} - P_0)] \cdot \kappa \cdot \hat{n}$ in equation (13) describes the surface tension force transformed into a volumetric force, considering its change driven by pressure. $\kappa$ represents the curvature of the interface and $\hat{n}$ means the interface unit normal.
vector on the interface, using the continuous surface force model.\textsuperscript{50}

In terms of interface, the volume fraction, \( \alpha \), as

\[
\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) = 0
\] (14)

is used to describe the interface between the liquid and the air and the effective local density and viscosity in the cells can be estimated as follows

\[
\rho = \rho_l \alpha + \rho_g (1 - \alpha)
\] (15)

\[
\mu = \mu_l \alpha + \mu_g (1 - \alpha)
\] (16)

where the subscripts \( l \) and \( g \) denote liquid and gas, respectively.

### 2.3. Physical model, boundary condition, and fluid properties assumptions

An axial symmetric domain is used for the simulations, as shown in Figure 2. The diameter of the injector orifice \( (D) \) is 5 mm, and the spray domain is 90 mm \( \times \) 25 mm \( (18D \times 5D) \). The ambient pressure is set to oscillate sinusoidally in the simulation domain. The inlet velocity, ambient density, and liquid surface tension oscillate following equations (8) to (10). For reference cases, fixed density and fixed liquid surface tension simulations are also calculated. The frequency of oscillation is fixed at 100 Hz, and various oscillation amplitudes of ambient pressure are considered, as summarized in Table 1. The mean inlet velocity is 1 m/s and the mean ambient pressures are 0.5, 1, 2, and 3 MPa. The oscillation amplitudes of inlet velocity increase with ambient pressure. The amplitude of pressure oscillation ranges from 5\% to 60\%. The upper limit of the amplitude is made higher to exemplify the perturbation and breakage. All 131 cases were simulated; the detailed list is shown in Table 3.

The basic characteristic flow numbers influencing the spray development process are the Reynolds number, the Weber number, and the Strouhal number, which relate the critical nature of inertia, the surface forces, and the ratio of inertial forces influencing jet breakup. These dimensionless numbers are defined as follows

\[
Re = \frac{\rho_l U D}{\mu_l}
\] (17)

\[
We = \frac{\rho_l U^2 D}{\sigma}
\] (18)

\[
St = \frac{\pi D f}{U}
\] (19)

Of course, the ambient gas properties may also affect the jet spray. In order to be conducive to primary breakup in laminar flows, all fluid properties are determined as shown in Table 2. The resulting Reynolds number is 2000, the Weber number ranges from 2008 to 20,032 and the Strouhal number is 1.57.

### 2.4. Validation

1) **Mesh independence.** The mesh is on a rectangular coordinate system with different levels of refinement in different regions. The mesh is most refined near the shear layer, where gas and liquid interact, starting from the injector outlet with an oblong size of 60 mm \( \times \) 10 mm. The most refined area includes approximately half of all cells, to capture the deformation of wavy ligaments and the separation of droplets in this area. The rest of the grid becomes gradually coarser, as shown in Figure 2(a). As shown in Figure 3(b), a wedge

| Table 1. Conditions. |
|-----------------------|
| Mean velocity (m/s) | Mean ambient pressure (MPa) | Amplitude of ambient pressure oscillation (%) | Oscillating frequency (Hz) | Amplitude of velocity oscillation (%) | Density | Surface tension |
| 1 | 0.5, 1, 2, 3 | 5, 10, 20, 30, 40, 50, 60 | 100 | 2.5, 5, 10, 15, 20, 25, 30 | Fixed or following equation (2) | Fixed or following equation (3) |

| Table 2. Fluid properties at normal pressure. |
|------------------|
| Liquid density | Liquid viscosity | Surface tension coefficient | Gas density | Gas viscosity | Surface tension-pressure coefficient |
| 1000 kg/m\(^3\) | \(2.5 \times 10^{-6}\) m\(^3\)/s | 0.0025 N/m | 1 kg/m\(^3\) | \(1.48 \times 10^{-5}\) m\(^3\)/s | \(2.58 \times 10^{-11}\) m\(\cdot\)7.76 \times 10^{-10}\) m |
domain with 5° was used to simulate a cylinder jet in a 2D axial-symmetry model.

To test the grid sensitivity of the simulation, Case 1 (Table 3) was solved on four grid resolutions of 360 K, 720 K, 950 K, and 1.68 M. Figure 4 shows a comparison of the axial velocity of the liquid flow along the x-axis for the four grids. For reference, the corresponding spray shape is drawn on the bottom of the graph in red. As shown in the figure, the simulation with the 950 K grid shows invariant results on further grid refinement, so the 950K grid was used for the current study.

(2) Comparison with reference results. To further validate the simulation method, the current method was compared with two sets of reference data. Figure 5 shows a comparison between our simulated data and a simulation of a modulated jet flow with only inlet velocity oscillation by Srinivasan et al.25 For this comparison study, the following conditions were set same as the reference: Strouhal number, Reynolds number, Weber number, and the relative velocity oscillation magnitude. As the reference data do not specify the time of capture, the time was estimated to best of our effort. As the flow is modulated with a single harmonic oscillation, the error for the time of capture is less likely to be significant. The two sets of results are superimposed, with the red color representing the current solver, and the blue showing the reference. For quantitative comparison, total surface areas are calculated: 6.12 mm² for the reference data and 6.53 mm² for our simulation. The difference is around 6.7%, indicating a good agreement was made with the reference data.

Another comparison was made with an experimental data from Geschner et al.21 In the experiment, the jet flow was driven by a piezoceramic atomizer to generate a bell-shaped structure of ligament and the image was captured by a CCD camera. Our simulation used the same Strouhal number, Reynolds number, Weber number, as the experiment. The reference data did not specify the time of capture or the magnitude of velocity oscillation. Hence, we conducted simulations with a range of velocity oscillations and estimated the time of capture to best of our effort. With the best of our estimation, our simulation data agree well with the experimental data, as shown in Figure 6. Again, red represents the current simulation, and grey represents the data of Geschner et al.21 Quantitative comparison such as the surface area comparison is not possible in this case as the inner structure cannot be captured from the reference data.

The comparison with these two previous works, one computational and the other experimental, shows that our simulation setup is suitable for use in the analysis of the characteristics of primary breakup under oscillating conditions using VOF methodology.

3. Results and discussion

The effects of different oscillating flow properties on spray breakup are compared and discussed in this section. The investigated effects are the oscillation amplitude of pressure, mean pressure, variable density, and variable surface tension, and the interaction of variable density and variable surface tension. In the first subsection, the effects are compared qualitatively and quantitatively first by visual comparison, the most intuitive means of comparison, and then based on total surface area and radial ejection velocity. In the second subsection, detailed droplet formation mechanisms are explained.

The total surface area of liquid flow, denoted as $S_{\text{spray}}$, is obtained by integrating all areas of the spray jet interface. For fair comparison, because the penetration length is different for different conditions, the total area is normalized by a reference area. Here, the reference area is chosen as the surface area of a cylinder with one closed end and with the diameter of the injector ($D$) and the penetration length of the spray ($L$)

$$S_{\text{ref}} = \pi DL + \pi D^2 / 4$$

Radial ejection velocity refers to the radial velocity of the liquid on the imaginary interface of the spray trunk. For consistency, the radial location 0.05D is chosen for the imaginary interface as shown in Figure 7. The velocity reflects the stretching ability of the spray jet in the radial direction.

3.1. Effect of the different oscillating parameters

(1) Effect of oscillation amplitude of pressure. Figure 8 shows visual differences between spray flows under different amplitudes of pressure oscillation. As the oscillation amplitude of ambient pressure increases, the inlet velocity oscillation increases, according to equation (3). Note that the density and the surface tensions are fixed for these cases.
Table 3. List of parameters for numerical experiments.

| Case | Mean velocity (m/s) | Mean ambient pressure (Mpa) | Amplitude of ambient pressure oscillation (%) | Oscillating frequency (Hz) | Amplitude of velocity oscillation (%) | Density | Coefficient of surface tension to pressure (m) |
|------|--------------------|-----------------------------|---------------------------------------------|---------------------------|---------------------------------------|---------|-----------------------------------------------|
| 1    | 1                  | 0.5                         | 60                                          | 100                       | 30                                    | Fixed   | Fixed                                         |
| 2    | 2                  | 1                           | 60                                          | 100                       | 30                                    | Fixed   | Fixed                                         |
| 3    | 3                  | 2                           | 60                                          | 100                       | 30                                    | Fixed   | Fixed                                         |
| 4    | 4                  | 3                           | 60                                          | 100                       | 30                                    | Fixed   | Fixed                                         |
| 5    | 5                  | 0.5                         | 60                                          | 100                       | 30                                    | Oscillating | Fixed                                      |
| 6    | 6                  | 1                           | 60                                          | 100                       | 30                                    | Oscillating | Fixed                                      |
| 7    | 7                  | 2                           | 60                                          | 100                       | 30                                    | Oscillating | Fixed                                      |
| 8    | 8                  | 3                           | 60                                          | 100                       | 30                                    | Oscillating | Fixed                                      |
| 9    | 9                  | 0.5                         | 50                                          | 100                       | 25                                    | Fixed   | Fixed                                         |
| 10   | 10                 | 1                           | 50                                          | 100                       | 25                                    | Fixed   | Fixed                                         |
| 11   | 11                 | 2                           | 50                                          | 100                       | 25                                    | Fixed   | Fixed                                         |
| 12   | 12                 | 3                           | 50                                          | 100                       | 25                                    | Fixed   | Fixed                                         |
| 13   | 13                 | 0.5                         | 50                                          | 100                       | 25                                    | Oscillating | Fixed                                      |
| 14   | 14                 | 1                           | 50                                          | 100                       | 25                                    | Oscillating | Fixed                                      |
| 15   | 15                 | 2                           | 50                                          | 100                       | 25                                    | Oscillating | Fixed                                      |
| 16   | 16                 | 3                           | 50                                          | 100                       | 25                                    | Oscillating | Fixed                                      |
| 17   | 17                 | 0.5                         | 40                                          | 100                       | 20                                    | Fixed   | Fixed                                         |
| 18   | 18                 | 1                           | 40                                          | 100                       | 20                                    | Fixed   | Fixed                                         |
| 19   | 19                 | 2                           | 40                                          | 100                       | 20                                    | Fixed   | Fixed                                         |
| 20   | 20                 | 3                           | 40                                          | 100                       | 20                                    | Fixed   | Fixed                                         |
| 21   | 21                 | 0.5                         | 40                                          | 100                       | 20                                    | Oscillating | Fixed                                      |
| 22   | 22                 | 1                           | 40                                          | 100                       | 20                                    | Oscillating | Fixed                                      |
| 23   | 23                 | 2                           | 40                                          | 100                       | 20                                    | Oscillating | Fixed                                      |
| 24   | 24                 | 3                           | 40                                          | 100                       | 20                                    | Oscillating | Fixed                                      |
| 25   | 25                 | 0.5                         | 30                                          | 100                       | 15                                    | Fixed   | Fixed                                         |
| 26   | 26                 | 1                           | 30                                          | 100                       | 15                                    | Fixed   | Fixed                                         |
| 27   | 27                 | 2                           | 30                                          | 100                       | 15                                    | Fixed   | Fixed                                         |
| 28   | 28                 | 3                           | 30                                          | 100                       | 15                                    | Fixed   | Fixed                                         |
| 29   | 29                 | 0.5                         | 30                                          | 100                       | 15                                    | Oscillating | Fixed                                      |
| 30   | 30                 | 1                           | 30                                          | 100                       | 15                                    | Oscillating | Fixed                                      |
| 31   | 31                 | 2                           | 30                                          | 100                       | 15                                    | Oscillating | Fixed                                      |
| 32   | 32                 | 3                           | 30                                          | 100                       | 15                                    | Oscillating | Fixed                                      |
| 33   | 33                 | 0.5                         | 20                                          | 100                       | 10                                    | Fixed   | Fixed                                         |
| 34   | 34                 | 1                           | 20                                          | 100                       | 10                                    | Fixed   | Fixed                                         |
| 35   | 35                 | 2                           | 20                                          | 100                       | 10                                    | Fixed   | Fixed                                         |
| 36   | 36                 | 3                           | 20                                          | 100                       | 10                                    | Fixed   | Fixed                                         |
| 37   | 37                 | 0.5                         | 20                                          | 100                       | 10                                    | Oscillating | Fixed                                      |
| 38   | 38                 | 1                           | 20                                          | 100                       | 10                                    | Oscillating | Fixed                                      |
| 39   | 39                 | 2                           | 20                                          | 100                       | 10                                    | Oscillating | Fixed                                      |
| 40   | 40                 | 3                           | 20                                          | 100                       | 10                                    | Oscillating | Fixed                                      |
| 41   | 41                 | 0.5                         | 10                                          | 100                       | 5                                     | Oscillating | Fixed                                      |
| 42   | 42                 | 1                           | 10                                          | 100                       | 5                                     | Oscillating | Fixed                                      |
| 43   | 43                 | 2                           | 10                                          | 100                       | 5                                     | Oscillating | Fixed                                      |
| 44   | 44                 | 3                           | 10                                          | 100                       | 5                                     | Oscillating | Fixed                                      |
| 45   | 45                 | 0.5                         | 10                                          | 100                       | 5                                     | Fixed   | Fixed                                         |
| 46   | 46                 | 1                           | 10                                          | 100                       | 5                                     | Fixed   | Fixed                                         |
| 47   | 47                 | 2                           | 10                                          | 100                       | 5                                     | Fixed   | Fixed                                         |
| 48   | 48                 | 3                           | 10                                          | 100                       | 5                                     | Fixed   | Fixed                                         |
| 49   | 49                 | 0.5                         | 5                                           | 100                       | 2.5                                   | Oscillating | Fixed                                      |
| 50   | 50                 | 1                           | 5                                           | 100                       | 2.5                                   | Oscillating | Fixed                                      |
| 51   | 51                 | 2                           | 5                                           | 100                       | 2.5                                   | Oscillating | Fixed                                      |
| 52   | 52                 | 3                           | 5                                           | 100                       | 2.5                                   | Oscillating | Fixed                                      |

(continued)
| Case | Mean velocity (m/s) | Mean ambient pressure (Mpa) | Amplitude of ambient pressure oscillation (%) | Oscillating frequency (Hz) | Amplitude of velocity oscillation (%) | Density | Coefficient of surface tension to pressure (m) |
|------|----------------------|-----------------------------|---------------------------------------------|-------------------------|--------------------------------------|---------|-----------------------------------------------|
| 53   | 1                    | 0.5                         | 5                                           | 100                     | 2.5                                  | Fixed   | Fixed                                         |
| 54   | 1                    | 1                           | 5                                           | 100                     | 2.5                                  | Fixed   | Fixed                                         |
| 55   | 1                    | 2                           | 5                                           | 100                     | 2.5                                  | Fixed   | Fixed                                         |
| 56   | 1                    | 3                           | 5                                           | 100                     | 2.5                                  | Fixed   | Fixed                                         |
| 57   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Fixed   | $2.58 \times 10^{-11}$                        |
| 58   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Fixed   | $8.62 \times 10^{-11}$                       |
| 59   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Fixed   | $5.172 \times 10^{-10}$                      |
| 60   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Fixed   | $7.76 \times 10^{-10}$                       |
| 61   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Fixed   | $5.172 \times 10^{-10}$                      |
| 62   | 1                    | 2                           | 40                                          | 100                     | 20                                   | Fixed   | $2.58 \times 10^{-11}$                       |
| 63   | 1                    | 2                           | 40                                          | 100                     | 20                                   | Fixed   | $8.62 \times 10^{-11}$                       |
| 64   | 1                    | 2                           | 40                                          | 100                     | 20                                   | Fixed   | $2.58 \times 10^{-10}$                       |
| 65   | 1                    | 2                           | 40                                          | 100                     | 20                                   | Fixed   | $5.172 \times 10^{-10}$                      |
| 66   | 1                    | 2                           | 40                                          | 100                     | 20                                   | Fixed   | $7.76 \times 10^{-10}$                       |
| 67   | 1                    | 2                           | 20                                          | 100                     | 10                                   | Fixed   | $2.58 \times 10^{-11}$                       |
| 68   | 1                    | 2                           | 20                                          | 100                     | 10                                   | Fixed   | $8.62 \times 10^{-11}$                       |
| 69   | 1                    | 2                           | 20                                          | 100                     | 10                                   | Fixed   | $2.58 \times 10^{-10}$                       |
| 70   | 1                    | 2                           | 20                                          | 100                     | 10                                   | Fixed   | $5.172 \times 10^{-10}$                      |
| 71   | 1                    | 2                           | 20                                          | 100                     | 10                                   | Fixed   | $7.76 \times 10^{-10}$                       |
| 72   | 1                    | 2                           | 10                                          | 100                     | 5                                    | Fixed   | $2.58 \times 10^{-11}$                       |
| 73   | 1                    | 2                           | 10                                          | 100                     | 5                                    | Fixed   | $8.62 \times 10^{-11}$                       |
| 74   | 1                    | 2                           | 10                                          | 100                     | 5                                    | Fixed   | $2.58 \times 10^{-10}$                       |
| 75   | 1                    | 2                           | 10                                          | 100                     | 5                                    | Fixed   | $5.172 \times 10^{-10}$                      |
| 76   | 1                    | 2                           | 10                                          | 100                     | 5                                    | Fixed   | $7.76 \times 10^{-10}$                       |
| 77   | 1                    | 2                           | 5                                           | 100                     | 2.5                                  | Fixed   | $2.58 \times 10^{-11}$                       |
| 78   | 1                    | 2                           | 5                                           | 100                     | 2.5                                  | Fixed   | $8.62 \times 10^{-11}$                       |
| 79   | 1                    | 2                           | 5                                           | 100                     | 2.5                                  | Fixed   | $2.58 \times 10^{-10}$                       |
| 80   | 1                    | 2                           | 5                                           | 100                     | 2.5                                  | Fixed   | $5.172 \times 10^{-10}$                      |
| 81   | 1                    | 2                           | 5                                           | 100                     | 2.5                                  | Fixed   | $7.76 \times 10^{-10}$                       |
| 82   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Oscillating | $2.58 \times 10^{-11}$                       |
| 83   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Oscillating | $8.62 \times 10^{-11}$                       |
| 84   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Oscillating | $2.58 \times 10^{-10}$                       |
| 85   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Oscillating | $5.172 \times 10^{-10}$                      |
| 86   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Oscillating | $7.76 \times 10^{-10}$                       |
| 87   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Oscillating | $2.58 \times 10^{-11}$                       |
| 88   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Oscillating | $8.62 \times 10^{-11}$                       |
| 89   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Oscillating | $2.58 \times 10^{-10}$                       |
| 90   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Oscillating | $5.172 \times 10^{-10}$                      |
| 91   | 1                    | 2                           | 60                                          | 100                     | 30                                   | Oscillating | $7.76 \times 10^{-10}$                       |
| 92   | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $2.58 \times 10^{-11}$                       |
| 93   | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $8.62 \times 10^{-11}$                       |
| 94   | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $2.58 \times 10^{-10}$                       |
| 95   | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $5.172 \times 10^{-10}$                      |
| 96   | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $7.76 \times 10^{-10}$                       |
| 97   | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $2.58 \times 10^{-11}$                       |
| 98   | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $8.62 \times 10^{-11}$                       |
| 99   | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $2.58 \times 10^{-10}$                       |
| 100  | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $5.172 \times 10^{-10}$                      |
| 101  | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $7.76 \times 10^{-10}$                       |
| 102  | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $2.58 \times 10^{-11}$                       |
| 103  | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $8.62 \times 10^{-11}$                       |
| 104  | 1                    | 2                           | 70                                          | 100                     | 20                                   | Oscillating | $2.58 \times 10^{-10}$                       |

(continued)
As the velocity oscillation increases, the flow forms a larger bell-shaped structure.\textsuperscript{17} With the formation of longer ligaments and the separation of more droplets, the liquid core flow tends to become thinner. Compared with the jet in Figure 8(a), the thickness of the liquid core in Figures 8(b), 8(c) and 8(d) decreases.

### Table 3. Continued

| Case | Mean velocity (m/s) | Mean ambient pressure (Mpa) | Amplitude of ambient pressure oscillation (%) | Oscillating frequency (Hz) | Amplitude of velocity oscillation (%) | Density | Coefficient of surface tension to pressure (m) |
|------|---------------------|----------------------------|-----------------------------------------------|---------------------------|--------------------------------------|---------|---------------------------------------------|
| 105  | 1                   | 0.5                        | 20                                            | 100                       | 10                                   | Oscillating | $5.172 \times 10^{-10}$ |
| 106  | 1                   | 0.5                        | 20                                            | 100                       | 10                                   | Oscillating | $7.76 \times 10^{-10}$ |
| 107  | 1                   | 2                          | 20                                            | 100                       | 10                                   | Oscillating | $2.58 \times 10^{-11}$ |
| 108  | 1                   | 2                          | 20                                            | 100                       | 10                                   | Oscillating | $8.62 \times 10^{-11}$ |
| 109  | 1                   | 2                          | 20                                            | 100                       | 10                                   | Oscillating | $2.586 \times 10^{-10}$ |
| 110  | 1                   | 2                          | 20                                            | 100                       | 10                                   | Oscillating | $5.172 \times 10^{-10}$ |
| 111  | 1                   | 2                          | 20                                            | 100                       | 10                                   | Oscillating | $7.76 \times 10^{-10}$ |
| 112  | 1                   | 0.5                        | 10                                            | 100                       | 5                                    | Oscillating | $2.58 \times 10^{-11}$ |
| 113  | 1                   | 0.5                        | 10                                            | 100                       | 5                                    | Oscillating | $8.62 \times 10^{-11}$ |
| 114  | 1                   | 0.5                        | 10                                            | 100                       | 5                                    | Oscillating | $2.586 \times 10^{-10}$ |
| 115  | 1                   | 0.5                        | 10                                            | 100                       | 5                                    | Oscillating | $5.172 \times 10^{-10}$ |
| 116  | 1                   | 0.5                        | 10                                            | 100                       | 5                                    | Oscillating | $7.76 \times 10^{-10}$ |
| 117  | 1                   | 2                          | 10                                            | 100                       | 5                                    | Oscillating | $2.58 \times 10^{-11}$ |
| 118  | 1                   | 2                          | 10                                            | 100                       | 5                                    | Oscillating | $8.62 \times 10^{-11}$ |
| 119  | 1                   | 2                          | 10                                            | 100                       | 5                                    | Oscillating | $2.586 \times 10^{-10}$ |
| 120  | 1                   | 2                          | 10                                            | 100                       | 5                                    | Oscillating | $5.172 \times 10^{-10}$ |
| 121  | 1                   | 2                          | 10                                            | 100                       | 5                                    | Oscillating | $7.76 \times 10^{-10}$ |
| 122  | 1                   | 0.5                        | 5                                             | 100                       | 2.5                                  | Oscillating | $2.58 \times 10^{-11}$ |
| 123  | 1                   | 0.5                        | 5                                             | 100                       | 2.5                                  | Oscillating | $8.62 \times 10^{-11}$ |
| 124  | 1                   | 0.5                        | 5                                             | 100                       | 2.5                                  | Oscillating | $2.586 \times 10^{-10}$ |
| 125  | 1                   | 0.5                        | 5                                             | 100                       | 2.5                                  | Oscillating | $5.172 \times 10^{-10}$ |
| 126  | 1                   | 0.5                        | 5                                             | 100                       | 2.5                                  | Oscillating | $7.76 \times 10^{-10}$ |
| 127  | 1                   | 2                          | 5                                             | 100                       | 2.5                                  | Oscillating | $2.58 \times 10^{-11}$ |
| 128  | 1                   | 2                          | 5                                             | 100                       | 2.5                                  | Oscillating | $8.62 \times 10^{-11}$ |
| 129  | 1                   | 2                          | 5                                             | 100                       | 2.5                                  | Oscillating | $2.586 \times 10^{-10}$ |
| 130  | 1                   | 2                          | 5                                             | 100                       | 2.5                                  | Oscillating | $5.172 \times 10^{-10}$ |
| 131  | 1                   | 2                          | 5                                             | 100                       | 2.5                                  | Oscillating | $7.76 \times 10^{-10}$ |

**Figure 4.** Axial velocity distribution of liquid flow along the $x$-axis at different grid resolutions.

As the velocity oscillation increases, the flow forms a larger bell-shaped structure.\textsuperscript{17} With the formation of longer ligaments and the separation of more droplets, the liquid core flow tends to become thinner. Compared with the jet in Figure 8(a), the thickness of the liquid core in Figures 8(b), 8(c) and 8(d) decreases.

**Figure 5.** Overlay of a simulation result of Srinivasan et al.\textsuperscript{25} and our simulation result.

**Figure 6.** Present results and experimental image. Gray: Geschner et al.\textsuperscript{21} and red: present simulation.
by 22%, 39%, and 48%, respectively. That is, more of the liquid holds more expanding energy in the radial direction and this amplifies the development of atomization. Increased ligament stretching in the radial direction is the leading stimulus of the primary breakup of jet flow in these cases. Furthermore, as the oscillation amplitude is enhanced, the attenuation of the axial velocity also slows down, to produce a longer penetration length through the decreased air resistance of the narrower liquid core flow.

Figure 9 shows the effect of oscillation amplitude as demonstrated by surface area ratio. The oscillation amplitude strongly correlates with the development of the spray and its breakup. The correlation is linear up to 2 MPa mean ambient pressure and exhibits saturation at 3 MPa. As the pressure oscillation increases from 5% to 60%, the surface area ratio increases up to 2.9 times at 0.5 MPa and 2.6 times at 3 MPa, respectively.

In Figure 10, the distribution of radial ejection velocity near the liquid core is shown along the axial direction at different oscillation amplitudes. Large radial ejection velocity of the liquid implies higher potential for expansion in the radial direction, which causes easier and earlier primary breakup. As the amplitude of oscillation increases, the radial ejection velocity also increases, making more liquid expand radially to interact with the incoming ambient gas flow. However, changes in ambient mean pressure seem to have less effect on the radial ejection velocity. This will be considered in detail in Section 3.2.

(2) Effect of ambient mean pressure. Figure 11 shows the effect of mean ambient pressure for constant amplitude of liquid pressure oscillation over a range of mean pressures. Note that the density of ambient air and the liquid surface tension are fixed.

It is evident that differences in the ambient pressure have at least two significant effects on the primary breakup. First, as the pressure increases, the gas density increases, lowering the liquid–gas density ratio. This low-density ratio causes the jet to break up faster, generating smaller droplets. The aerodynamic drag force acting on the ligaments and droplets is higher, due to the high gas density. Second, at higher pressure the liquid expansion is suppressed in the radial ejection direction, decreasing the spreading angle. As a result, under high gas pressure, the liquid ligament is compressed to tighten toward the center without a noticeable change in the liquid core thickness. That is, unlike the effect of pressure oscillation amplitude, the influence of high mean pressure on the development of primary breakup is to enhance the dynamic interaction between the produced ligaments and the incoming ambient gas flow. This trend is reflected in Figure 12, which shows the influence of the ambient mean pressure on the jet flow. As the mean pressure increases, the ligaments become longer and are prone to breakage.

The change in ambient mean pressure has a negligible effect on radial ejection velocity, as shown
Figure 10. Radial ejection velocity ratio under different oscillation amplitudes with fixed density and surface tension. (a) $P_{am}=0.5$ MPa; (b) $P_{am}=3$ MPa.

Figure 11. Jet spray under various ambient mean pressures: $P_{am}$ equals (a) 0.5 MPa, (b) 1 MPa, (c) 2 MPa, and (d) 3 MPa. 40% $P_0$, 20% $U_0$ with fixed density and fixed surface tension.

Figure 12. Surface area ratio under various ambient mean pressures with fixed density and fixed surface tension for various oscillation amplitudes.

in Figure 13, although at higher mean pressures the oscillation of the radial velocity may increase slightly without significant change in amplitude.

(3) Effect of variable ambient density. The effect of variable ambient density is investigated, considering cases with fixed and with variable (equation (5)) gas density. Note that the liquid surface tension is fixed here, to isolate any additional effects. Figure 14 shows visual comparisons of the density effects at three pressure oscillation amplitudes: 10% amplitude in Figure 14(a) and 14(b), 20% amplitude in Figure 14(c) and 14(d), and 60% amplitude in Figure 14(e) and 14(f), respectively. At low oscillation magnitudes such as $P'=10\%$ or $20\%$, the effect of density is negligible: the jet structure is

Figure 13. Radial ejection velocity ratios at different mean pressures for $P'=60\%$ with fixed density and fixed surface tension.
maintained, the ligament breakup is similar, the liquid core thickness is almost identical, the penetration length only differs by 0.16%.

In contrast, when the oscillation magnitude is large ($P' = 60\%$), jet breakup occurs earlier with variable gas density, and more droplets are produced. The liquid core thickness becomes thinner, by around 10%, and the penetration length of the jet at a given time increases accordingly by 0.5%. This indicates that variable ambient density makes both ligaments and liquid core thinner and longer, and these effects clearly make the breakup occur sooner and more intensely. The fact that the change becomes more prominent at high oscillating magnitude indicates that the ambient density effect is a secondary effect, not the primary effect. Still, the oscillating gas density strengthens the aerodynamic interaction between the ligaments and the liquid core, enhancing the liquid breakup.

Figure 15 shows a quantitative comparison of surface area ratios for various cases. If the pressure oscillation magnitude is small (10% and 20% $P'$), the effect of the variable gas density is quite small; the two lines of surface area ratio at different mean pressures almost overlap in Figure 15. However, the changing density of the ambient gas gradually starts to play a role when the pressure oscillation amplitude reaches a certain level. Because the ligament becomes thinner, stronger aerodynamic interaction caused by the density oscillation starts to accelerate the separation of the droplet on the wavy surface of the ligament.

Figure 16 shows the trend of the development of jet breakup with change of pressure oscillation amplitude at different mean pressures. As the oscillation amplitude increases, the difference between fixed density case and variable density case gradually increases. The difference increases more rapidly under higher mean pressure.

Changing density can slightly promote the rapid expansion of the spray flow, as shown in Figure 17. As compared with the effect of the oscillation...
amplitude in Figure 9, the influence of varying density on the radial ejection velocity is not only smaller, but also begins to lag noticeably by 0.08 m downstream of inlet (seventh peak). This shows that oscillating density greatly strengthens the interaction between the liquid ligaments and ambient gas, but has relatively little effect on the liquid expansion.

(4) Effect of variable surface tension. To investigate the effect of surface tension, multiple values of surface tension-pressure coefficient $\Gamma$ in equation (7) are used for simulations. Figure 18 shows that the oscillating surface tension only affects the stretching and thinning of ligaments, without any influence on the liquid core. As the surface tension sensitivity increases, the enhanced oscillation of surface tension weakens its ability to hinder the breakup of liquid flow, and the liquid ligaments stretch longer and thinner. This is beneficial to the development of primary breakup and

![Figure 16. Surface area ratio for different pressure oscillation amplitudes with fixed and variable gas density at $P_{am} = 0.5$ MPa and 2 MPa.](image)

![Figure 17. Radial ejection velocity ratio at 60% $P^r$ with different densities and surface tensions at $P_{am} = 2$ MPa.](image)

![Figure 18. Spray breakup for various coefficients of surface tension: $\Gamma$ equals (a) 0 (fixed), (b) $2.58 \times 10^{-11}$ m, (c) $2.586 \times 10^{-10}$ m, and (d) $7.76 \times 10^{-10}$ m. Fixed density, $P = 60\%$ and $P_{am} = 2$ MPa.](image)
further atomization. However, the effect of the oscillating surface tension is limited to large oscillation, becoming noticeable only when the initial ligament is sufficiently thin.

The surface area ratio is also calculated for a range of sensitivities of surface tension to pressure, as shown in Figure 19. From this perspective, this higher sensitivity will cause a higher amplitude of oscillating surface tension with the ambient pressure oscillation. It is well known that high surface tension can prevent the collapse of ligaments and delay breakup by creating a force against droplet surface perturbations. However, the high surface tension is weakened by increased oscillation, allowing droplets to form more quickly.

As shown in Figure 19, the extension of ligaments and the formation of droplets are enhanced with increasing coefficient of surface tension to pressure. The maximum surface area ratio increases proportionally with the pressure oscillation magnitudes. When oscillation magnitudes are small such as 5% and 10% $P_0$, the maximum surface area ratio reaches up to 9% or 11% within the investigated range. As the oscillation magnitude is 20% $P_0$, the maximum surface area ratio is 27%. As for the largest oscillation at 60% $P_0$, the maximum surface area ratio goes up to nearly 40%.

Figure 20 shows that the radial ejection velocity is independent of the surface tension sensitivity. Hence, the strengthening of the spray breakage, shown in Figure 16, is mainly caused by the decrease of liquid stability of the ligament.

(5) Effect of variable density and surface tension. As the density and the surface tension oscillate, the interaction between the liquid filament and the droplet is further enhanced, while the thickness of the liquid core changes little, as shown in Figure 21. Figure 21(a) shows the case with fixed surface tension ($\Gamma = 0$), while Figure 21(b) and 21(c) shows oscillating surface tension with increasing $\Gamma$. With the increase of the pressure-surface tension coefficient, the ligament tends to grow longer and be more twisted, which leads to the formation of smaller droplets.

The change of surface area ratio by the surface tension coefficient is plotted in Figure 22 at different conditions. The effect of the surface tension coefficient is small at low $P'$, while the effect is meaningful at high $P'$: for example, the surface area ratio increases by 5% at 20% $P'$, while 25% at 60% $P'$. The effect of variable density shows similar trends: the difference is small at low $P'$ but large at high $P'$. When comparing between variable density and fixed density cases, the differences between the cases are almost negligible at $P' = 5\%$ and 20%, while the differences are 15% and 22% at $P' = 40\%$ and 60%, respectively.
Figure 23 shows that the simultaneous oscillation of ambient density and surface tension slightly change the radial ejection velocity of the liquid. The difference in the radial ejection velocities occurs near the inlet, but is not very significant.

### 3.2 Droplet formation mechanism

This subsection details droplet formation mechanisms for cases in which density and surface tension oscillate with the oscillating pressure. The key underlying mechanism of droplet formation is Kelvin–Helmholtz (KH) instability. The KH instability produces waves on the surface of stratified two-fluid flows if the relative velocity difference is larger than a critical value. The waves grow unstably and form ligaments. As the ligaments radiate into the surrounding gas, the incoming airflow forces the liquid–gas interface to deform. Combining with the effect of the thinning process on the wavy surface of the liquid ligament, a pinch-off droplet formation mechanism is identified, as shown in Figure 24.

In Figure 24, the vortices' structures are apparent near the end of each ligament. These vortices are the main force enhancing the instabilities of the ligaments and the ligaments become longer and thinner. When a small section of ligament, as circled in Figure 24, is stretched enough, the presence of wavy liquid–gas interface dynamics begins to play an important role to promote breakup and collapse into droplets. At higher mean pressure of the ambient gas, more surrounding gas interacts with the liquid jet, leading to better break up. See also Section 3.1.3 for the mean pressure effect.

In the case of constant density, the velocity of the incoming gas stream changes little over time, as shown in Figure 25(a). The simulation with variable density gives a different result, as shown in Figure 25(b), the magnitude of gas velocity changes periodically. When the velocity of the incoming gas flow rises very quickly, the ligament surface is further reshaped; the velocity instantaneously tears the thinner ligament apart and forms several sections of separated ligaments. Furthermore, the rapidly changing velocity over a
short time can also generate new vortices, which further destroy liquid ligaments. The ligament sections continuously break up, due to localized wave instability, when the ligaments are sufficiently thin and unstable. Therefore, the varying gas velocity and more vortices lead to faster primary breakup of droplets. As an example, the ligament starts to collapse at 52 ms in the case of variable gas density (Figure 25(b)), while there is no sign of breakage for the case of fixed gas density (Figure 25(a)). By the time the ligament under fixed gas density begins to break (at 58 ms), the ligament under variable gas density is completely broken.
Figure 26 shows a comparison between fixed surface tension and variable surface tension simulations. The effect of oscillating surface tension is relatively weaker than that of oscillating density. For example, the ligament shapes are already different at 47 ms for the density effect comparison (see Figure 25), while the ligament shapes are still similar at 65 ms for the surface tension comparison (see Figure 26). Given the similar thin ligaments, the variable surface tension weakens the stability of the liquid itself, leading to more liquid droplets, as shown in Figure 26. When the liquid droplets are detached at 75 ms, the ligament is torn into strips for the fixed surface tension case, while the ligament forms complete spherical liquid droplets for the variable surface tension case.

A different droplet formation process with oscillating density and surface tension is demonstrated in Figure 27. Similar to Figure 25(b), the oscillating gas flow velocity and new vortices in the ligament stretching space enhance the detachment of droplets. However, surface tension oscillation further twists the shape of the ligaments. It takes longer for the ligament to be distorted, and the droplet generation process is slower than in the case with only changing density. Still, the subsequent collapse of these ligaments can produce more and smaller droplets in the primary breakup process.

4. Conclusion

Primary breakup of liquid jets under different oscillating parameters was numerically investigated, motivated by combustion instability problems. In particular, the research focuses on the effects of oscillating flow properties induced by oscillating ambient pressure. As the ambient pressure oscillates, the flow properties, inlet velocity, density of ambient gas, and liquid surface tension also oscillate accordingly. A new solver was developed based on the interFoam code, using the VOF method, in order to consider the effect of changes in those flow properties. All simulations were conducted for Reynolds number around 2000 to maintain laminar flow in the simulation, and the Weber number was set to be higher than 2000 to make the early primary breakup available in the calculation domain. Parametric studies were conducted to investigate the effects of mean ambient pressure, magnitude of ambient pressure oscillation, variable ambient density, and variable liquid surface tension.

Among the investigated parameters, the amplitude of pressure oscillation has the most significant effect on the primary breakup. As the amplitude increases, the formation of ligaments and droplets strengthens, due to increased radial ejection velocity and higher flow resistance. In contrast, the mean ambient pressure has negligible impact on the radial expansion of jet flow, but influences liquid–gas interfacial dynamics in the wavy ligaments and accelerates generation of smaller droplets by thinning the ligaments through localized wave instability.

Variable density has slight effects on the mass fraction of the liquid core flow, but mainly influences the formation of liquid ligaments and droplets by enhancing localized wave instability. When the surface tension–pressure coefficient increases and the oscillation amplitude reaches a certain level, the stability of ligaments is notably weakened and the breakage and disintegration of droplets are significantly strengthened. The effect of variable surface tension is not evident, however, when the oscillation amplitude is small.

Droplet formation mechanisms under different oscillating conditions were also investigated. The jet flow with constant density and surface tension follows the pinch-off type breakup in the thinning process of ligaments. As the density oscillates periodically, vortices are generated, instantly impacting the surface of ligaments to enhance primary breakup. Variable surface tension has a negligible effect in generating
ligaments, but once thin ligaments are formed, the shape and number of droplets are highly dependent on surface tension. If both the density and the surface tension oscillate, ligaments tend to become more distorted, forming more and smaller droplets in the breakup process.

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**Appendix**

**Notation**

| Symbol | Description |
|--------|-------------|
| $E_s$ | internal energy per unit surface energy |
| $D$ | orifice diameter |
| $f$ | frequency |
| $\dot{m}, \dot{M}$ | mass flow rate |
| $\hat{n}$ | normal vector |
| $P,P$ | pressure |
| $R$ | ideal gas constant |
| $Re$ | Reynolds number |
| $T$ | temperature |
| $t$ | time |
| $St$ | Strouhal number |
| $U, \vec{U}$ | axial velocity |
| $V$ | radial ejection velocity |
| $We$ | Weber number |
\( \alpha \) volume fraction
\( \rho \) density
\( \sigma, \tau \) surface tension
\( \Gamma \) surface tension-pressure coefficient
\( \tau \) shear stress
\( \kappa \) curvature

**Superscript**

\( \prime \) oscillating quantity

**Subscripts**

- \( am \) ambient
- \( g \) gas
- \( inlet \) inlet
- \( jet \) jet
- \( l \) liquid
- \( ref \) reference
- \( res \) reservoir
- \( spray \) spray
- \( 0 \) the state at normal pressure and temperature