X-RAY EMISSION FROM MAGNETICALLY TORQUED DISKS OF Oe/Be STARS

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ABSTRACT

The near-main-sequence B stars show a sharp dropoff in their X-ray-to-bolometric luminosity ratio in going from B1 to later spectral types. Here we focus attention on the subset of these stars that are also Oe/Be stars, to test the concept that the disks of these stars form by magnetic channeling of wind material toward the equator. Calculations are made of the X-rays expected from the magnetically torqued disk (MTD) model for Be stars discussed by Cassinelli et al., Maheswaran, and Brown et al. In this model, the wind outflow from Be stars is channeled and torqued by a magnetic field such that the flows from the upper and lower hemispheres of the star collide as they approach the equatorial zone. X-rays are produced by the material that enters the shocks above and below the disk region and radiatively cools and compresses while moving toward the MTD central plane. The model predictions are compared with ROSAT observations obtained for an O9.5 star, ζ Oph, by Berghöfer et al. and for seven Be stars from Cohen et al. Two types of fitting models are used to compare predictions with observations of X-ray luminosity versus spectral type. Extra consideration is also given here to the well-studied Oe star ζ Oph, for which we have Chandra observations of the X-ray line profiles of the triad of He-like lines from the ion Mg xx. Thus, the X-ray properties add to the list of observables that can be explained within the context of the MTD concept. This list already includes the Hα equivalent widths and white-light polarization of Be stars.

Subject headings: circumstellar matter — stars: early-type — stars: magnetic fields — stars: rotation — stars: winds, outflows — X-rays: stars

1. INTRODUCTION

In a survey of the X-ray emission of near-main-sequence B stars (or “B V” stars), Cassinelli et al. (1994) and Cohen et al. (1997) found a departure from the canonical “law” relating X-ray luminosity to the bolometric luminosity for hot-star X-rays: \( L_X/L_B = 10^{-7} \). This relation holds for stars throughout the O spectral range and extends to about B1 V. However, beyond that there is a sharp drop in the ratios in going to B3 V, by about 2 orders of magnitude. Cohen et al. (1997) investigated whether the sharp decrease could be explained merely by a reduction of the wind outflow from these stars and found that this could indeed explain the initial decrease in the X-ray luminosity, but that in going to even later B V stars another problem arose. The emission measures of the X-ray—producing material at spectral type B3 V and later become larger than the predicted wind emission measures for B stars for the smooth-wind case. Cohen et al. suggested that this could be the first indication that the late-B stars lie at the transition to the outer atmospheric structure of cool stars, for which surface magnetic fields control the X-ray properties. These ideas were based on a spherical radial outflow picture for B stars.

In fact, B stars are known to be rather rapid rotators. Bjorkman & Cassinelli (1993) developed the wind-compressed disk (WCD) model, which suggests that the wind from a rapidly rotating star will orbit toward the equatorial region, where it would shock and compress the incident gas. The model had success in explaining the polarization properties of emission-line Be stars (Wood et al. 1997). WCD was also supported, initially, by hydrodynamic simulations performed by Owocki et al. (1994). However, in a more detailed consideration of the flow-to-the-equator idea, Owocki et al. (1996) found that nonradial line forces in a rotating and distorted star tend to impede the flow to the equator and produce a bipolar flow instead. Hence, the cause of the disks that exist around Be stars, as opposed to polar plumes, has become a topic of much debate among theorists.

Observers also found problems with the WCD idea. Hanuschik et al. (1996) found in their observations of equator-on Be stars that the mass outflow speeds detectable in the disks were negligible compared with the steady but slow equatorial outflows predicted by the WCD model, although no one seems ever to have predicted whether in fact the WCD material is dense enough to be detectable in this way. Even more interesting was their observational conclusion that the azimuthal speed of the inner disk material was larger than the angular speed of the star from which the disk presumably originated (as it must be to remain in Keplerian orbit unless supported by other forces.) Specifically, observations of the equator-on Be star β Mon showed that the Fe ii line emission arising from the disk is broader than the \( v \sin i \) value derived from photospheric lines. Hanuschik et al. (1996) suggested that the equatorial disk material was in Keplerian motion about the star. In the context of any Keplerian paradigm, it is especially important to note that in order to form a Keplerian disk, there needs to be an increase in the specific angular momentum of the matter after it leaves the star. The mechanism for providing that additional angular momentum is currently a subject of debate (e.g., Baade & ud-Doula 2005; Brown & Cassinelli 2005).

Transfer of mass and torquing of the outflow could be produced by magnetic fields rooted in the star’s surface, as is well known from magnetic-rotator theory (Lamers & Cassinelli 1999). The existence of magnetic fields in hot stars is now well established...
(Donati et al. 2001, 2002). Thus, the magnetically torqued disk (MTD) model was proposed in Cassinelli et al. (2002) for the disks around Be stars. In this model, the star is pictured as having a co-aligned dipolar field that both channels and torques the wind from the star toward a disk. Given that the detailed structure of the magnetic fields of these stars has not yet been determined, it seems reasonable to use a pure dipole as the most conservative hypothesis. Minimal magnetic fields were derived from the need to torque the outflow and the much denser disk material to Keplerian speeds, and the required fields were compared with upper and lower limits for hot-star fields that had been derived by Maheswaran & Cassinelli (1988, 1992). For stars of spectral class B2 V (which corresponds to the most common class of Be stars), the field required to torque the dense disk is about 300 G, while that required to torque the wind is only around 10 G. This difference exists because the field needed to torque a wind is proportional to the square root of the density, and the density of the wind is about 3 orders of magnitude lower than the density at the equatorial plane in the disk. The fact that the higher figure is comparable to the fields that have now been derived from multiline Zeeman-effect measurements of the very slowly rotating star \( \beta \) Cep (Donati et al. 2001) shows that both the flow and the disk will in reality undergo magnetic torquing for this star. In other stars, the magnetic fields (whose strengths have not yet been measured) may lie in an intermediate regime in which the wind material would be torqued to high specific angular momentum while being channeled but have entirely Keplerian dynamics in the denser disk (Owocki et al. 2005). The MTD model was shown to produce the H\( \alpha \) emission observed in Be stars (Doazan et al. 1991), as well as the level of intrinsic polarization seen (Quirrenbach et al. 1997). It is important to note that only a fraction of B stars are Be stars, and those stars identified as Be stars only spend a fraction of their time in a state with identifiable Be star features. Therefore, it is not necessary for a theoretical paradigm to cause a magnetically torqued disk for all possible sets of stellar and magnetic parameters; it is only necessary for a disk model to encompass a wide enough range of parameter space so as to make it reasonable that some stars display disks some of the time. A detailed comparison with observations will only be possible when the “duty cycles” of Be stars and the Be star fraction have been better determined observationally.

In the original MTD model, the stellar wind mass flux and wind speed distribution were taken to be uniform over the stellar surface. However, in the case of a rapidly rotating star this assumption is invalid, since the rotation results in gravity darkening (von Zeipel 1924) and reduces the wind mass flux and the terminal speed in the equatorial region (Owocki et al. 1998). By incorporating gravity darkening into the MTD model (MTDGD), Brown et al. (2004) derived several important disk properties, such as the dependence of the disk’s mass density distribution on its extent, the total number of disk particles, and the functional dependencies of the emission measure and polarization on the rotation rate \( S_0 \) and wind velocity law \( (\beta) \). In contrast to what had been expected, they found that the critical rotation (or \( S_0 = 1 \)) is not optimal for creation of hot-star disks.

One important omission in the basic MTD formulation, as well as in MTDGD to date, is the erroneous neglect of gravity in the disk density structure (see Brown & Cassinelli 2005). Although \( g_0 \propto z \) is zero in the equatorial plane, its increase with \( z \) will enhance the density in the disk and cause it to grow over time. It will likely increase the H\( \alpha \) and polarization predictions. In turn, this will eventually result in radial escape of disk material, although whether this is slow and steady or episodic, as claimed by Owocki & ud-Doula (2003), is not yet clear.

Other computational and analytical attempts to model disks similar to those envisioned here have met with mixed results. Owocki & ud-Doula (2003) criticized the MTD idea because the model was found to be unstable in their MHD simulations. However, numerical simulations by Keppens & Goedbloed (1999, 2000) and Matt et al. (2000) demonstrated the existence of disks around some hot stars, such as post-asymptotic giant branch stars. Also, Maheswaran (2003) has studied this scenario using analytical MHD and found that the magnetically torqued disks are likely to be persistent, which casts doubt on the numerical simulations of Owocki & ud-Doula. Thus, further work is needed to test the basic ideas of the MTD and MTDGD models, either thorough numerical and analytical MHD calculations or the use of observational diagnostics from radio to X-ray wavelengths.

In this paper, however, we are primarily concerned with X-ray emission in the MTDGD models—an emission that occurs well upstream of the dense disk—and explaining the X-ray anomalies associated with Be stars. These will be little affected by the density in the disk itself, although our use of MTD to find the outer disk radius will make our X-ray estimates of the source emission measure a little too high. A successful model should be able to explain the dropoff at B2 Ve and the apparently excessive X-rays of late B V stars while using disk parameters consistent with theory and the entire set of observational data. This process can then be inverted to use the X-ray properties of a star to derive limits on its wind, disk, and rotational properties. In § 2, we describe how X-ray emission is produced in the model. The effects of model parameters on the X-ray emission are discussed in § 3. Comparisons of model predictions with both ROSAT and Chandra observations are presented in § 4. The discussion and conclusions are in § 5.

### 2. Model for X-ray Emission

The basic concept for X-ray production in MTDGD models is that there are shock-heated regions above and below an equatorial disk where the winds from the upper and lower hemispheres of the star collide. To establish the X-ray emission properties, we need to make model predictions of the density and temperature structure in the postshock regions. The disk and predisk densities are dependent on the mass and momentum flux from the corresponding latitude zones of the stellar surface, which are funneled by magnetic flow tubes to the disk. The temperature depends on the speed at which the matter collides with the shocks at the disk boundaries. We treat these aspects in turn and then discuss how we combine various parts of the heated disk to determine the resultant X-ray spectrum and total X-ray luminosity.

As in previous MTD papers, the dimensionless rotation rate of the star, \( S_0 \), is defined as the Keplerian fraction by

\[
S_0 = \sqrt{\frac{\Omega^2 R^3}{GM}},
\]

for stellar angular velocity \( \Omega \). This determines the inner and outer boundaries of the disk and the effects of rotational gravity darkening on the disk. The ratio \( \gamma \), which, together with \( S_0 \), determines the effect of the magnetic field on the disk and wind, is defined as

\[
\gamma = \left( \frac{B^2}{8\pi GM \rho_0} \frac{2R}{v_w^2} \right)^{1/2},
\]

where \( \rho_0 = \left[ \frac{M(4\pi R^2 v_w^2)}{v_w} \right]^{1/2} \) is the characteristic density of the cool gas at the equatorial plane and \( M/(4\pi R^2 v_w^2) \) is the characteristic density for the wind. Therefore, \( \gamma \) is a measure of the
magnetic field energy density relative to the gravitational energy density of the wind material near the star. So, a unit value for $\gamma$ provides an indication of the minimal field needed to form a disk. The field then determines the latitude range of the stellar flow that forms a disk, and the inner and outer radii.

The mass flux from the base of the wind is given in MTDGD theory by

$$F_m(x) = \frac{\dot{M}}{4\pi R^2(1 - \frac{2}{3}S_0^2)} x^{-3} \left\{ 1 - S_0^2 \left[ 1 - \left( \frac{1}{x} \right)^2 \right] \right\}$$  \hspace{1cm} (3)

and the wind speed by

$$v_w(x) = v_\infty \left( 1 - \frac{1}{x} \right)^\beta \left\{ 1 - S_0^2 \left[ 1 - \left( 1 - \frac{1}{x} \right)^2 \right] \right\}^{1/2},$$  \hspace{1cm} (4)

where $x$ is the radial distance in the disk from the center of the star in units of the stellar radius (i.e., $r/R_s$) and $v_\infty$ is the terminal velocity of the wind flow if it were unimpeded to infinity. Thus, from $F_m$ the preshock mass density approaching the disk is

$$\rho_0 = \frac{F_m}{v_w} = \frac{\dot{M}}{4\pi R^2 v_\infty} x^{-3} \left( 1 - \frac{1}{x} \right)^{-\beta} \left\{ 1 - S_0^2 \left[ 1 - \left( 1 - \frac{1}{x} \right)^2 \right] \right\}^{1/2}.$$  \hspace{1cm} (5)

We assume the disk is formed by the shock compression above and below the equatorial plane. Note that, for simplicity, strong, normal shocks are assumed here, while the time-variable structures in the disks and the radiative overstability in the shocks (e.g., Pittard et al. 2005) are not taken into account in the model. Resultant shock temperatures greater than $10^6$ K will produce X-ray emission.

In terms of the jump conditions, at the top of the disk the shock density is 4 times the wind density, namely,

$$\rho_s = 4\rho_0.$$  \hspace{1cm} (6)

The temperature at the wind interface of the shocked disk is given by

$$T_s(x) = 1.44 \times 10^7 v_\infty^2(x),$$  \hspace{1cm} (7)

where $T_s$ is the shock temperature in kelvins and $v_\infty$ is the incident wind speed in units of $10^8$ cm s$^{-1}$.

According to the standard X-ray models by Hillier et al. (1993) and later by Feldmeier et al. (1997), the energy emitted per second per hertz from a volume $dV$ in all directions is given by

$$d\epsilon_\nu = n_p n_e \tilde{\Lambda}_\nu(T_s) dV,$$  \hspace{1cm} (8)

where $n_p$ is the proton density, $n_e$ is the electron density, and $T_s$ is the temperature reached at the wind-disk shock, located at $(r, \phi, z)$, using cylindrical geometry in which $r$ is the radial distance in the equatorial plane, $\phi$ is the azimuthal angle, and $z$ is the distance above the equatorial plane. $\tilde{\Lambda}_\nu$ in this equation is determined by averaging across the cooling length, as given by Feldmeier et al. (1997):

$$\tilde{\Lambda}_\nu(T_s) = \frac{1}{L_c} \int_{-L_c}^{L_c} \tilde{\Lambda}_\nu(T_s(z')) \delta(z') dz',$$  \hspace{1cm} (9)

where $\Lambda_\nu$ is the frequency-dependent cooling function of a hot plasma, $z$ is the location of the shock front, and $z'$ is the coordinate in the cooling layer, of extent $L_c$, which is related to the velocity and density of postshock gas and the chemical composition. The functions $\tilde{\Lambda}_\nu$ and $\delta(z')$ describe the normalized density and temperature stratification in the postshock region, respectively. Note that we neglect the plasma motions in the postshock flow, which can generate small-scale magnetic structure that may provide some magnetic support and thus reduce the postshock compression.

In our treatment, we use the functional forms of $\tilde{\Lambda}_\nu$ and $\delta(z')$ for the temperature and density stratifications, respectively, as defined by Feldmeier et al. (1997), who considered plane-parallel shock fronts. However, here the cooling layer above the disk of a Be star is split into many concentric rings (e.g., $\sim$200 rings) along the disk’s radial extent, and each ring is sliced into vertical sub-layers (e.g., $\sim$100 sub-layers), as illustrated in Figure 1. Thus, each sub-layer has a specific density and temperature, which are assumed to be constant throughout the sub-layer. With these, one may obtain the emission measure for each ring and sublayer from

$$\Delta(EM)_i = n_p n_e \Delta V_i = \frac{\rho_i^2}{m_p \mu_e \mu_p} \Delta V_i,$$  \hspace{1cm} (10)

where $\Delta V_i$ and $\rho_i$ are the volume and mass density of the $i$th sub-layer, respectively, and $\mu_e$ and $\mu_p$ are mean particle weights per electron and proton, respectively. Then, the X-ray emission from each ring and sublayer is given (with $\Delta V_i = 2\pi r_i \Delta z_i \Delta r_i$) by

$$\Delta L_{\nu,i} = \Delta(EM)_i \Lambda_\nu(T_s),$$  \hspace{1cm} (11)

where $\Lambda_\nu(T_s)$ is the cooling function at temperature $T_s$, frequency $\nu$, and chemical abundance (assumed solar), as given by the Astrophysical Plasma Emission Database (APED), described by Smith et al. (1998).

The total X-ray emission for the entire shocked disk is found by summing the emission over all the rings and sublayers and multiplying by 2 to account for the upper and lower shock fronts:

$$L_\nu = 2 \sum_i \Delta L_{\nu,i},$$  \hspace{1cm} (12)
Integrating over the frequency range concerned, we obtain the X-ray luminosity,

\[ L_X = \int_{\nu_1}^{\nu_2} L_{\nu} \, d\nu, \]  

(13)

where \( \nu_1 \) and \( \nu_2 \) are the lower and upper X-ray frequencies in the energy band of the instrument, 0.1 and 2.4 keV in the case of ROSAT, for example. Calculations were carried out with sufficient numbers of rings and sublayers so that the results were no longer dependent on the specific number of rings and sublayers.

### 3. EFFECTS OF MODEL PARAMETERS ON X-RAY EMISSION

The MTDGD models have been worked out for main-sequence stars with given effective temperatures, mass-loss rates, and terminal velocities. Our interest is in the X-ray properties at each spectral type. Our interest is in the X-ray properties at each spectral type, as in previous papers dealing with the MTD model.

Each of the three model parameters (\( \beta, S_0, \) and \( \gamma \)) affects the X-ray emission measure and the ratio \( L_X/L_B \) in different ways. To explain these various effects, we discuss the results from our modeling of the star \( \zeta \) Oph, for which we find the following:

1. Changing the velocity law's \( \beta \)-value can significantly affect \( L_X/L_B \). Changing \( \beta \) has a small effect on the disk extent, but it significantly affects the X-ray source, as a result of the dependence of the cooling length on the wind velocity as parameterized by \( \beta \). The cooling length depends on velocity as \( L_c \propto v_4^4/\rho_4 \propto v_5^4 \) (see Feldmeier et al. 1997). Therefore, the overall effect on the emission measure is \( EM \propto \rho_2^2 L_c \propto \rho_3^2 v_4^4/\rho_4 \propto v_5^4 \). If, for example, \( \beta \) is increased, meaning that a more slowly accelerating wind is incident upon the disk, then the emission measure from shock regions becomes lower and in turn \( L_X/L_B \) is lowered, as shown in Figures 2 and 3 for various values of \( \beta \).

2. Changing \( \gamma \) also gives rise to a change in \( L_X/L_B \). Increasing \( \gamma \) leads to an increase in the radial extent of the disk, and the extension occurs primarily outwardly, away from the star. Thus, there is a greater radial range of emitting material channeled to the disk. In Figure 2, we show the dependence of \( L_X/L_B \) on the magnetic parameter \( \gamma \).

3. Increasing the rotation rate parameter \( S_0 \) affects \( L_X/L_B \) in three ways: (a) When the rotation rate is smaller than some turnover value (e.g., for given values of \( \beta = 1 \) and \( \gamma = 1.5 \), this is \( S_0 \sim 0.5 \)), the gravity darkening is negligible, so increasing rotation helps to form the disk. Hence, the greater the rotation rate, the larger the amount of matter channeled to the disk. (b) As the rotation rate increases further, the disk’s inner radius gets closer to the star, and therefore a slower and denser (and so cooler) wind reaches the disk. Hence, we find a situation similar to that discussed above regarding the \( \beta \)-value, and \( L_X/L_B \) actually decreases. (c) If \( S_0 \) is increased even further, beyond the turnover value, the gravity-darkening effect plays an important role in significantly reducing the mass flow to the disk from equatorial regions on the star, and of course \( L_X/L_B \) decreases significantly. Figures 2 and 3 also show this trend for given \( \gamma \) and \( \beta \).

In the tentative test, we show that when determining \( L_X/L_B \) from the model, we have to fix two of the three free parameters to some values and then find how \( L_X/L_B \) varies with the remaining
one and whether the value of the remaining one is appropriate given the observed value of \( L_X / L_B \). Figures 2 and 3 show the varying trends and imply the probable values for these three free parameters. Strictly, the ranges of the parameters can only really be determined by using a search to define the surface in three-dimensional \((S_0, \gamma, \beta)\)-space in which fits are acceptable. Such a search will be carried out in the future, and it may be more tightly constrained by fitting not only the X-ray luminosity but also the X-ray spectral hardness. From the above results, we may derive for our program stars, such as \( \zeta \) Oph, a least-squares formula that looks like \( L_X = k S_0^\alpha \gamma^\beta \), where the constant \( k \) is a certain fraction of the wind kinetic energy converted to X-rays and the precise amount depends on the spin rate \( S_0 \), on \( B \), and on \( \beta \). By doing a numerical partial derivative for each power to fit our results, we might find the ranges of the three powers to be \( p \sim 1-4 \), \( q \sim 5-8 \), and \( w \sim 2-5 \), which would show that \( L_X \) seems to be highly dependent on the value of \( \gamma \). This may explain why we are able to fit all the stars so exactly with our model.

4. MODEL CALCULATIONS AND COMPARISONS WITH OBSERVATIONS

The physical quantities of the eight program stars are listed in Table 1. We ran several models and compared the results with observations.

4.1. Comparisons with ROSAT Observations

For all program stars, we took MTDGD models to compute \( L_X \) in three ROSAT energy bands—soft (0.1–0.4 keV), hard (0.5–2.0 keV), and total (0.1–2.4 keV)—and determined the hardness ratio \( HR = (H - S)/(H + S) \), the emission measure, and the ratio of X-ray to bolometric luminosity, \( \log (L_X / L_B) \), which can be compared with the ROSAT observations. In these calculations, we can choose various values of the parameters \( \beta, S_0 \), and \( \gamma \). But for simplicity, we always use the minimal magnetic field for \( \gamma \) as given by Cassinelli et al. (2002) and then fix either \( \beta \) or \( S_0 \) and let the other one adjust. Note, in fact, that what we did is to adjust the parameters \( S_0 \) and \( \beta \) until the fit is perfect. Hence, we are assuming the model and inferring a range of \( S_0 \) and \( \beta \) that is acceptable.

From the calculations with the MTDGD model, we achieve the observed X-ray \( L_X / L_B \) ratio, within the allowed range of adjustments of the model’s free parameters. In fitting the observed \( \log (L_X / L_B) \), we have adopted the following two different approaches.

In the first, we use the observed projected velocity \( v \sin i \) and take the average value of \( \langle \sin i \rangle \) for a random set of inclination angles to estimate a surface rotation speed. This allows us to estimate the rotation rate \( S_0 \) of each star. This sort of approach has been used by Chandrasekhar & Münch (1950) and later by many authors, such as Porter (1996), to analyze the rotation of Be stars. Also, in this first approach we use the threshold (i.e., minimal) magnetic field given by Cassinelli et al. (2002) as our tentative field strength. Thus, for this case we have one adjustable parameter, the velocity-law index \( \beta \), to provide a fit to the observations. As seen in Table 2, with the exception of just one star, the value of \( \beta \) needed for the program stars is larger than unity. These values correspond to slowly accelerating winds, as compared with estimates of \( \beta \) for spherical winds. The calculation results are also shown in Table 2, from which one can see that the model results for the emission measure and \( L_X / L_B \) fit the observations quite well.

### Table 1

| Star   | Type  | \( T_e \) (K) | \( R_e \) (R\(_{\odot}\)) | \( M_e \) (M\(_{\odot}\)) | \( M \) (M\(_{\odot}\) yr\(^{-1}\)) | \( v_{\infty} \) (km s\(^{-1}\)) | Distance (pc) | \( v \sin i \) (km s\(^{-1}\)) |
|--------|-------|---------------|-----------------|-------------------|-----------------|-----------------|---------------|-----------------|
| \( \zeta \) Oph | O9.5 Vn | 5.04 | 31,600 | 8.00 | 25.0 | 4.0 \times 10^{-8} | 1500 | 154 | 385 |
| \( \kappa \) CMa | B1.5 IVe | 4.21 | 24,690 | 6.84 | 12.9 | 4.2 \times 10^{-9} | 1560 | 308 | 200 |
| \( \eta \) Cen | B1.5 Ve | 3.99 | 24,690 | 5.31 | 11.2 | 1.5 \times 10^{-9} | 1660 | 110 | 345 |
| \( \delta \) Cen | B2 IVe | 4.01 | 23,010 | 6.31 | 9.8 | 2.0 \times 10^{-9} | 1310 | 138 | 155 |
| \( \mu \) Cen | B2 IVe | 3.89 | 23,010 | 5.50 | 10.4 | 8.5 \times 10^{-10} | 1470 | 163 | 180 |
| \( \alpha \) Ara | B2 IVe | 3.77 | 23,010 | 4.79 | 9.8 | 4.5 \times 10^{-10} | 1540 | 122 | 315 |
| \( \alpha \) Eri | B3 Ve | 3.33 | 19,320 | 4.07 | 6.9 | 4.2 \times 10^{-11} | 1330 | 27 | 250 |
| \( \alpha \) Col | B7 I Ve | 2.45 | 12,790 | 3.39 | 3.7 | 3.0 \times 10^{-12} | 1250 | 44 | 210 |

### Table 2

| Star | \( \beta \) | \( S_0 \) (G) | \( B \) (G) | \( T_e \) (K) | \( R_e \) (R\(_{\odot}\)) | \( X_i \) (R\(_{\odot}\)) | \( X_o \) (R\(_{\odot}\)) | \( \log EM_X \) | \( \log (L_X / L_B) \) | \( HR \) | \( \log EM_{\text{th}} \) | \( \log (L_X / L_B) \) |
|------|-------------|----------------|---------|---------------|-----------------|-----------------|-----------------|-----------------|-----------------|------|-----------------|-----------------|
| \( \zeta \) Oph | 1.29 | 0.64 | 2072 | 1.60 | 1.35 | 1.86 | 53.91 | -7.47 | 0.630 | 54.30 | -7.47 |
| \( \kappa \) CMa | 2.43 | 0.42 | 917 | 2.09 | 1.78 | 2.47 | 52.65 | -7.94 | 0.521 | 52.60 | -7.94 |
| \( \eta \) Cen | 1.53 | 0.69 | 514 | 1.39 | 1.28 | 1.81 | 52.99 | -8.43 | 0.397 | 51.90 | -8.42 |
| \( \delta \) Cen | 2.81 | 0.36 | 684 | 2.42 | 1.98 | 2.77 | 51.90 | -8.57 | 0.264 | 51.77 | -8.57 |
| \( \mu \) Cen | 1.34 | 0.38 | 685 | 2.77 | 1.91 | 2.55 | 53.07 | -7.01 | 0.913 | 52.83 | -7.01 |
| \( \alpha \) Ara | 1.61 | 0.64 | 326 | 1.48 | 1.35 | 1.88 | 51.42 | -8.78 | 0.358 | 51.33 | -8.78 |
| \( \alpha \) Eri | 1.45 | 0.56 | 130 | 1.78 | 1.47 | 2.01 | 50.80 | -8.89 | 0.558 | 50.77 | -8.89 |
| \( \alpha \) Col | 0.77 | 0.59 | 44 | 1.95 | 1.42 | 1.89 | 50.41 | -8.28 | 0.838 | 50.50 | -8.28 |

Notes.—We use the average rotation rate and threshold magnetic field as fixed parameters and then fit the X-rays by adjusting \( \beta \) through the simulations. In the table, \( X_i \) and \( X_o \) represent the inner and outer extent of the disk with stellar radius as the unit, respectively. The subscripts \( i \) and \( o \) on \( EM, L_X / L_B \), and \( HR \) denote “theoretical” and “observational,” respectively.
well for a given set of free parameters. We plot $L_X/L_B$ versus spectral type in Figure 4 and versus magnetic field $B$ and $S_0$ in Figure 5 in terms of the model results in Table 2. We also list the derived hardness ratio of X-rays with regard to certain disk properties and find it to be marginally in agreement with observations. The hardness ratio of ζ Oph (HR$_0$ = 0.630) fits the observation (HR$_0$ = 0.908) marginally well. Unfortunately, the observed hardness ratios for other program stars are not available from ROSAT, but we still list the model results in the table for future comparison.

As our second approach, we follow the radiatively driven wind theory and simply assume $\beta = 1$; then we can fit the observed $L_X/L_B$ by adjusting the remaining parameter, $S_0$. Note that the threshold field used here is determined by $\beta$ and $S_0$ for a given star as in Cassinelli et al. (2002). The calculation results are shown in Table 3, in which the range of $S_0$ for the various stars is from 0.49 to 0.88. This range is consistent with traditional values of the rotation rate parameter $\Omega_0$ or $v$ sin $i$ at the surface. Thus, we can use the observed line width, derived from Table 1 and assuming $\beta = 1$, to derive $S_0$ and thus the angular speed at the disk at a determined radius where temperatures reach values on the order of 5 MK, at which Mg xi can form. We find that this is at about 1.8$R_*$. The model that is used to calculate the line profile assumes a ring of emission at 1.8$R_*$ and uses the postshock density and temperature at that location. For comparison, the MTDGD radial location is about 22% larger than the source location obtained by Gagné et al. (2005; 1.2$R_*$ to 1.4$R_*$) in their model of the young magnetic O star θ1 Ori C. From the model predictions in Table 2, we know the value of $S_0$ and thus the angular speed at the Mg xi formation region. From Table 1, we know the value of $v$ sin $i$ at the surface. Thus, we can use the observed line width, assumed to be from the orbital velocity of the source region, to derive the inclination factor $\sin i$. This corresponds to an angle $i = 53^\circ$. The predicted X-ray source temperature of the ring of emission is determined from equation (7) using the incident wind speed determined by $v_w$ from Table 1 and assuming $\beta = 1.29$. This yields $T = 4.94$ MK, which is near the temperature of 5 MK at which the Mg xi ion has its maximum ion fraction.

A key feature of our model line calculations is that we include “real” temperature- and density-dependent line emissivities, which means that there is only “one” normalization applied to the total line-complex calculation (i.e., we do not apply individual line X-ray sources. The $f/i$ line ratios obtained from He-like ion $f$ (forbidden, intercombination, resonance) emission lines provide a diagnostic of the radial locations of the shock source regions for these ions (e.g., Kahn et al. 2001; Waldron & Cassinelli 2001, 2007). Here we use the MTDGD model for ζ Oph to find the source region associated with the ion Mg xi and compare the line profile that would form in this region against observations of the line from the Chandra High Energy Transmission Grating Spectrometer (HETGS).

The primary goal of this exercise is to demonstrate that the MTDGD concept can reproduce the general properties of the Mg xi $f$ lines without adjusting the fit parameters. MTDGD predicts that the Mg xi $f$-emitting region is an annulus above the disk at a determined radius where temperatures reach values on the order of 5 MK, at which Mg xi can form. We find that this is at about 1.8$R_*$. The model that is used to calculate the line profile assumes a ring of emission at 1.8$R_*$ and uses the postshock density and temperature at that location. For comparison, the MTDGD radial location is about 22% larger than the source location obtained by Gagné et al. (2005; 1.2$R_*$ to 1.4$R_*$) in their model of the young magnetic O star θ1 Ori C. From the model predictions in Table 2, we know the value of $S_0$ and thus the angular speed at the Mg xi formation region. From Table 1, we know the value of $v$ sin $i$ at the surface. Thus, we can use the observed line width, assumed to be from the orbital velocity of the source region, to derive the inclination factor $\sin i$. This corresponds to an angle $i = 53^\circ$. The predicted X-ray source temperature of the ring of emission is determined from equation (7) using the incident wind speed determined by $v_w$ from Table 1 and assuming $\beta = 1.29$. This yields $T = 4.94$ MK, which is near the temperature of 5 MK at which the Mg xi ion has its maximum ion fraction.

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A key feature of our model line calculations is that we include “real” temperature- and density-dependent line emissivities, which means that there is only “one” normalization applied to the total line-complex calculation (i.e., we do not apply individual line...
normalizations). Our emissivities are determined by the MEKAL plasma emission code (Mewe et al. 1995). The main advantage of this code is that it allows one to explore density-sensitive lines, which is an important aspect in studies of the He-like $\text{f}r$ line formation process in early-type stars. It is well established that the observed behavior of the He-like $\text{f}r$ lines, in particular the relative strengths of the $i$- and $f$-lines (i.e., the $f/i$ line ratio), is not due to density effects but instead depends on the strength of the extreme-UV (EUV) and UV photospheric flux (e.g., Kahn et al. 2001; Waldron & Cassinelli 2001, 2007). Although current emissivity codes do not provide a means for determining line emission dependencies on EUV/UV flux (as first demonstrated by Blumenthal et al. 1972), one of us (W. L. W.) has developed a special algorithm to simulate the dependence of $f/i$ on EUV/UV flux. Since we know how the $f/i$ ratio depends separately on density and on EUV/UV flux, by equating these two relationships we can determine what we call an “effective EUV density” for any radial wind location for a given input photospheric EUV/UV flux. This effective EUV density is then used in the MEKAL code to determine the relative strengths of the $i$- and $f$-lines. We point out that the actual value of this effective EUV density is “not” an actual physical density, but only a parameter that is used to simulate the effects of the EUV/UV flux on the $f/i$ ratio.

The model Mg $\text{X}_i\text{f}r$ emission lines are calculated by a simple integration over $\phi$, since the radial position of the emission zone is fixed. The emission is attenuated by the radial and azimuthally dependent line-of-sight cool stellar wind X-ray continuum optical depth through the disk, including the effects of stellar occultation. The model X-ray emission includes all emissivities in a given wavelength region, for example, the $\text{f}r$ lines, their satellite lines, any other lines that may be present, and the continuum. Once we specify the location and temperature of the disk-distributed X-ray plasma (as determined by the MTDGD model), along with the effective EUV density as determined from a known photospheric flux, there is basically only a single free parameter, the normalization factor obtained from the fit, which gives the total emission measure of the integrated disk-distributed X-ray sources (i.e., a measure of $n_e^2$ times the volume of the X-ray-emitting plasma). The predicted HETGS MEG +1/i−1 counts are compared with the observed counts in the top panel of Figure 6 (using a bin size of 0.01 Å). The bottom panel shows the predicted input normalized model flux used to generate the model MEG +1/i−1 first-order counts from the extracted ancillary response file (ARF) and redistribution matrix file (RMF) appropriate for the $\zeta$ Oph data. The most obvious feature seen in the $r$- and $i$-lines is the characteristic double-peaked structure as expected from viewing a collection of disk sources seen at a large inclination angle. Since there is no radial velocity component in our model, the blue and red peaks should have the same strength. However, the red peak of the $r$-line is slightly larger than the blue peak, which we attribute to the presence of several weaker satellite lines redward of the $r$-line. The most notable effect of other lines is seen in the vicinity of the $f$-line, where the double-peaked characteristic is masked by these other lines. There is also a slight emission excess on the blue side of the $f$-line profile, which suggests that the disk-confined X-ray sources may have an outward radial velocity component.

### Table 3

| Star     | $\beta$ | $S_0$ | $B$  | $\gamma$ | $X_i$ | $X_f$ | $\log EM_r$ | $\log (L_{X;}/L_\odot)$ | $HR_r$ | $\log EM_r$ | $\log (L_{X;}/L_\odot)$ |
|----------|---------|-------|------|----------|-------|-------|-------------|--------------------------|-------|-------------|--------------------------|
| $\zeta$ Oph | 1.0     | 0.752 | 1922 | 1.49     | 1.21  | 1.72  | 53.91       | -7.47                    | 0.636 | 54.30       | -7.47                    |
| $\kappa$ CMa | 1.0     | 0.821 | 604  | 1.38     | 1.14  | 1.68  | 52.65       | -7.94                    | 0.526 | 52.60       | -7.94                    |
| $\eta$ Cen | 1.0     | 0.879 | 477  | 1.30     | 1.09  | 1.66  | 51.98       | -8.42                    | 0.440 | 51.90       | -8.42                    |
| $\delta$ Cen | 1.0    | 0.815 | 392  | 1.38     | 1.15  | 1.68  | 51.90       | -8.57                    | 0.250 | 51.77       | -8.57                    |
| $\mu$ Cen | 1.0     | 0.495 | 546  | 2.21     | 1.60  | 2.11  | 53.08       | -7.01                    | 0.390 | 51.33       | -7.01                    |
| $\alpha$ Ara | 1.0    | 0.858 | 291  | 1.32     | 1.11  | 1.67  | 51.41       | -8.78                    | 0.390 | 51.33       | -8.78                    |
| $\alpha$ Eri | 1.0    | 0.725 | 112  | 1.54     | 1.24  | 1.74  | 50.81       | -8.89                    | 0.561 | 50.77       | -8.89                    |
| $\alpha$ Col | 1.0    | 0.488 | 50   | 2.24     | 1.61  | 2.13  | 50.41       | -8.28                    | 0.850 | 50.50       | -8.28                    |

Note.—The same as Table 2, but here we use $\beta = 1$ and the threshold magnetic field as fixed parameters instead and then fit the X-rays by adjusting $S_0$ in the simulations.
of a few hundred kilometers per second, or perhaps that the wind region above the disk has standard radiation-driven wind shocks that contribute to the overall emission. Nevertheless, we conclude that the MTDGD model can reproduce the observed Mg\(\text{xi}\) line shapes and line strengths, and, more importantly, the model predicts an \(f/i\) ratio that is very good agreement with the observations, although this model fit to the HETGS data predicts a log EM of 54.52, which is approximately 40% larger than the observed emission measure derived from ROSAT observations, as listed in Table 2. An explanation for this small difference may be that \(\zeta\) Oph is a variable X-ray source (Waldron 2005) and at the time of the Chandra observation the X-ray flux was about 40% larger when the star was observed with ROSAT.

5. DISCUSSION AND CONCLUSIONS

The magnetically torqued disk model including gravity-darkening effects, or MTDGD, has been tested to see if it can explain the basic X-ray properties of Oe/Be stars, for which the model was developed. It has already been found in the original papers on the MTD concept by Cassinelli et al. (2002) and Brown et al. (2004) that the idea of mass in Be disks being channeled by magnetic fields is consistent with the H\(\alpha\) luminosities of Be stars and that the mass in the disks as derived from polarization observations is also explainable, although we again note that inclusion of \(g_s\) will likely increase these in the model.

We have used a model that explains how matter can enter a disk with sufficient angular momentum to explain the quasi-Keplerian disks of Be stars, and the model uses field strengths that are comparable to those being found for other B stars. However, an essential property of the MTDGD model is that it requires the X-rays to be produced as a result of the abrupt braking of the wind at the shock fronts. In summary, we have arrived at several interesting results. (1) We have tested the prediction that X-rays should be produced by the impact of channeled winds onto a disk. These X-rays would probably not be predicted from other current Be star models, such as those in which the disk is produced by an extraction of angular momentum from the surface of a critically rotating star. (2) The model is based on the assumption that Be stars are rotating at their traditional values of about 70% critical, and these rotation rates were found to be sufficient to explain the X-ray emission within the context of the MTD picture. (3) The model is found to require fields on the order of \(10^7\) G for Be stars, and our results show that these are adequate for the broadband X-ray production. (4) Broadband ROSAT X-ray fluxes can be produced from B1 to B8 with MTDGD model parameters. (5) Fits are achievable even for the late B8 stars without invoking the presence of a dwarf M companion. (6) The model predicts that the helium-like ion Mg\(\text{xi}\) has an \(R_{\text{vir}}\) of about 1.8 stellar radii, which agrees well with where that line emission is expected to arise in \(\zeta\) Oph from Chandra observations; that is, the \(f/i\) lines are predicted to occur there and to be broad, with a half-width of about 400 km s\(^{-1}\), and the temperature structure is consistent with the formation of the Mg\(\text{xi}\) line.

Our discussion thus far has dealt with understanding the fundamental properties of Be stars as revealed by the ROSAT and Chandra observations. We have raised several questions during this paper that can now be addressed. (1) For our latest star, B7IVe, we found that the observed ROSAT level of X-rays could be produced if the velocity law had a value \(\beta = 0.77\). This small value means that the channeled wind is colliding with the disk at a larger fraction of the terminal wind speed than is the case for our other stars, which have \(\beta\) values ranging from 1.3 to 2.8. The other required parameters for this star seem plausible: \(B \sim 50\) G and \(S_0 \sim 0.6\) (in the first of our two fitting procedures). Cohen et al. (1997) suggested that the emission measures needed to explain X-rays from late-B stars were excessive, and that perhaps X-rays from a magnetically confined region at the base of the wind are needed. So from our model of this star, it appears that a magnetically confined X-ray formation region at the base is not required. A bipolar magnetic field could instead just be torquing and channeling the wind toward the disk through X-ray-emitting shocks. (2) The sharp dropoff of the X-ray luminosity beyond about B2 V also seems to be explainable with a plausible range of our \(\beta\) and \(S_0\) and magnetic field values of about 685–130 G for our B2 and B3 stars.

It appears that the MTDGD model has the ability to answer two of the more difficult questions concerning existing X-ray observations of Be stars, with plausible parameters. The model can explain the X-ray luminosity across the B spectral band, and it can explain reasonably well the observed line profile results from Chandra. Finally, it is important to note that our model results are not dependent on the nature of the high-density regions on the equatorial plane, for which there is controversy regarding field wrapping and magnetic breakouts. This is because our models are in effect providing information only about the X-ray formation regions at the boundaries of the disk, and the flow through these boundaries occurs before the matter reaches the cooled, compressed region near the equatorial plane. We do not require that the gas be controlled by a strong field all the way to the equatorial plane, and in fact we think that the gas is no longer dominated by the field in that region and is free to acquire a quasi-Keplerian orbital motion. The needed angular momentum is transferred to the gas in the preshock magnetically torqued and channeled MTD flow.

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