The generation of the worm and wheel gears in a CAD soft

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Abstract. In this paper, we present the way to obtain in AutoCAD the solids that materialize the worm and wheel gears. The worm and wheel gears are practically used first of all due to their high transmission ratio. After the state of the art of the worm and wheel gears, we present the manufacturing possibilities of the wheels. Further on, one presents the way to obtain of the solids that materialize the cylindrical and globoid worms, using AutoLisp functions in AutoCAD. By Boolean operations, one extracts from the raw material that approximates the worm, the solid that materializes the tool. One thus generates cylindrical worms with 1, 2, and 3 starts and globoid worms with one start. The obtaining of the wheels is performed in a similar way, the tool being the already obtained worm.

1. Introduction

The teething of the gears using machine tools, by classical manufacturing procedures, is performed by copying or rolling. The procedures based on copying assume the obtaining of the teeth by milling with side-milling cutter or end-milling cutter, by shaping with pattern tool etc. The rolling procedure bases on the gearing between the tool and the raw material, which is manufactured. The tools have different shapes depending on the type of the machine tool, which generates the teeth. Usually, the shape of the tooth is an involute one, but there also exist having the shape of the teeth as cycloids, arcs of circle etc.

The worm and wheel gears are helical gears at which the angle between the axes is equal to 90°. The worm, which is also the driving element, has one or z1 starts (1 ... 4), number which also represents the number of teeth of the worm. The wheel, the driven element, has cylindrical shape with inclined toothing, with a number of z2 teeth. The transmission ratio of the gear \( i_{12} = \frac{z_2}{z_1} \) may have great values (10 ... 500), but, increasing the transmission ratio, the mechanical efficiency dramatically decreases; this fact is one of the main disadvantages of such gears [1, 2, 3].

The flank of the worm spire may be or may be not a ruled surface [4, 5, 6, 3]. The worm may have a cylindrical, conical or globoid shape [1, 2, 3, 7]. The toothing of the wheel depends on the shape of the worm with which it gears. The wheels are cylindrical with inclined teeth, globoid wheels, globoid conical gears etc.

The globoid worm and wheel gears have as main advantage the great contact ratio, all spires of the worm being in gear with the teeth of the wheel. Contrary to the cylindrical worm and wheel gears, where the calculation relations are obtained by solving problems of planar geometry, the calculation relations for the globoid worm and wheel gears are obtained by solving problems of spatial geometry
This aspect primarily affects the industrial manufacturing procedures for the accurate obtaining of the toothing.

The CAD softs may successfully simulate the generation of the gears on machine tools.

In Reference [1] authors present general aspects concerning the generation of the cylindrical gears in AutoCAD. They present a five steps working algorithm, in AutoCAD, where with AutoLisp functions one generates, by rolling without sliding, different gears. For the obtaining of the solid that materializes the gears one performs minimum 360 Boolean operations of extracting the tool from the raw material, which approximates the gear. Contrary to the classical manufacturing procedure, the stock left for machining is eliminated in one single rotation of the gear. One thus generates cylindrical gears with straight teeth, inclined teeth or conical gears with straight teeth.

Based on this algorithm, further on, we present the procedure to obtain the worm and wheel gears in a CAD soft.

2. The obtaining of the solids that materialize the cylindrical worms in AutoCAD

The cylindrical worms are classified based on the procedure of the obtaining the flank of the teeth. The Archimedes worm (ZA) has rectilinear flanks, with a trapezoidal planar section, similar to a screw with trapezoidal profile [1, 2, 3, 4, 6]. The involute worm (ZE) has, in frontal section, involute flanks, in transversal section the flanks being generated by straight lines tangent to the base cylinder [2, 3, 7]. The convolute worm (ZN) has the flanks generated after two straight lines situated in a plane perpendicular to the helix of the worm. In frontal section, the flanks look like elongated involute [3, 6, 7]. The worms obtained by milling with double conical side-milling cutter are named ZK1 worms, while the worms obtained with a conical end-milling cutter are called ZK2 worms [1, 3, 5]. From those presented above it also results the way of obtaining of worms by turning or milling.

Further on, we will present the obtaining of the solids that materialize cylindrical worms in AutoCAD, by copying the manufacturing procedure by planing, the obtaining by turning being described at the globoid worms.

The main geometric elements of the gear are in figure 1 [1]. The geometric elements of the worm and wheel gear are in function of the axial modulus \( m_x \) (which has standard values). The angle \( \alpha_{0x} \) is usually equal to 20°.

![Figure 1. Worm and wheel gear.](image)

The diameter of the work is given in function of the diametric coefficient \( q \) with the aid of expression (1). This coefficient, depending on the number of starts and the transmission ratio has values between 7 and 16,

\[
q = \frac{d_1}{m_x}. \tag{1}
\]

The value of angle \( \gamma \) of the reference helix is given by the relation
\[ \text{tg } \gamma = \frac{\pi m_z z_1}{2 \pi q} = \frac{z_1}{q}. \]  

(2)

For the construction of the solid that materializes the tool, which will generate the tooth, the following relations are necessary

\[ h_{x_i} = \frac{m_z}{2} (q + 2), \quad h_{x_f} = 1.25m_z, \]  

(3)

while for the construction of the solid that materializes the solid, which approximates the worm we need the relations

\[ r_{x_i} = \frac{m_z}{2} (q + 2), \quad r_{x_f} = \frac{m_z}{2} (q - 2.5), \]  

(4)

\[ L = \begin{cases} 
(11 + 0.06z_2)m_z \text{ for } z_4 = 1, 2, \\
(12.5 + 0.09z_2)m_z \text{ for } z_4 = 3, 4,
\end{cases} \]  

(5)

\[ a = 0.5(r_{x_f} + r_{x_i}) = 0.5m_z(q + z_2). \]  

(6)

Comparing to the classical manufacturing process, in AutoCAD, at one step, one will completely eliminate the machining allowance. The tool, which materializes a facing mill, is a circular sector with lateral faces dull at an angle \( \alpha \). If the radius of the mill is very large, then one may use a parallelepipedic solid with the edges dull with the angle \( \alpha \). In both cases, the tool is inclined in the tangential plane by an angle \( \gamma \) given by relation (2). The solid will generate the empty spaces between the teeth of the worm. We used this solution due to its simple explanation.

The raw material that approximates the worm consists in to coaxial cylinders. The first cylinder has the diameter smaller that \( d_{f_i} \), on it being assembled the bearings of the worm. The second cylinder has the external diameter \( d_{f_i} \) and the length \( L_{\text{worm}} = L + 2t \), where \( t \) marks the values of the bevels at the two ends of the worm. In this way, the bevels will not affect the total length \( L \) of the worm, given by relation (5).

We wrote two AutoLisp Functions called Tool_and Raw_material in order to generate the solids described below.

During the generation of the toothing of worm, the solid that approximates the worm rotates and translates, the tool remaining fixed. We chose this solution for a smaller number of positioning operations of solids. We may also use the case in which the raw material rotates and the tool translates.

If \( \Delta \phi \) is the rotational step, then the displacement \( \Delta s \) is given by relation \( \Delta s = \frac{\Delta \phi \cdot \pi}{360^\circ} \cdot p z_1 \). The rotation angle of the worm is \( \phi = \phi + \Delta \phi \), while its displacement reads \( s = s + \Delta s \).

At a rotation of \( \phi = 360^\circ \) of the raw material, it displaces with the distance \( s = p \). For the complete cut surface, the raw material will have to perform \( n \) rotations, \( n = \frac{L_{\text{worm}}}{pz_1} \).

The Boolean operations of extracting the solid that materializes the tool from the solid that materializes the raw material are also performed with an AutoLisp function. It calls the functions Tool and Raw_material that generate the described above solids and name them Tool and Raw_material.

In a while loop in which one assigns values to the angle \( \phi \) in interval \( \phi \in \left[ 0 \div \frac{L_{\text{worm}}}{pq_1} \right] \), with constant step \( \Delta \phi = 1^\circ \) one makes the following operations in AutoCAD:
– one rotates the raw material above the axis \( Ox \) with the angle \( \Delta \phi \).
– one moves the raw material along the axis \( Ox \) with the distance \( \Delta s = \frac{\Delta \phi \circ \rho z_1}{360^\circ} \).
– one copies the tool and name the copy as Tool1,
– one positions the copy of tool at the point \( (L_{worm} + 1.5m, -r_f, 0) \).
– one extracts (the AutoCAD command Subtract) the solid Tool1 from the solid Raw_material,
– one passes to the next angle \( \phi = \phi + \Delta \phi \).

The AutoLisp function, which realizes the operations presented above is named Worm.

\[
\text{(Defun C:Worm()}
\]
\[
\text{(Command "Erase" "All" "Osnap" "OFF" "Ortho" "OFF")}
\]
\[
\text{(Tool) (Raw_material)}
\]
\[
\text{(Command "Copy" Tool "" (List 0 0) (List 0 0)) (Setq Tool1(EntLast))}
\]
\[
\text{(Command "Move" Tool1 "" (List 0 0) (List (+ Lworm (* 1.5 m)) (/ df1 -2) 0))}
\]
\[
\text{(Command "Subtract" Raw_material "" Tool1 "")}
\]
\[
\text{(Setq Raw_material(EntLast))}
\]
\[
\text{(Setq phi 0 Angle(* 360 (/ Length_worm p z1)))}
\]
\[
\text{(While (< phi angle)}
\]
\[
\text{(Command "Rotate3D" Raw_material "" "x" "" ang_step)}
\]
\[
\text{(Setq displacement(/ (* p ang_step z1) 360))}
\]
\[
\text{(Command "Move" Raw_material "" 0,0" (list displacement 0))}
\]
\[
\text{(Command "Copy" Tool "" (List 0 0) (List 0 0)) (Setq Tool1(EntLast))}
\]
\[
\text{(Command "Move" Tool1 "" (List 0 0) (List (+ Lworm (* 1.5 m)) (/ df1 -2) 0))}
\]
\[
\text{(Command "Subtract" Raw_material "" Tool1 "")}
\]
\[
\text{(Setq phi(+ phi ang_step))}
\]
\[
\text{)}
\]

In figure 2 we represented the raw material and the tool at the beginning of the Boolean operations and at the end when we obtained the solid that materializes the worm with one start.

![Figure 2](image1.png)

**Figure 2.** Worm with one start and the tool at the beginning and after the cutting.

For the obtaining of the worms with many starts one uses the same function Worm which completes with many copies of the tool called Tool2, Tool3, Tool4, positioned at the right of the Tool1 with the distances \( p, 2p, 3p \). In figures 3 and 4 are the photographic representations of the worms with two and four starts, obtained with the AutoLisp function Worm, based on the previous algorithm.

![Figure 3](image2.png)

**Figure 3.** Worm with two starts.

![Figure 4](image3.png)

**Figure 4.** Worm with four starts.
3. The obtaining of the solids that materialize the globoid worms

For the obtaining of the solid that materializes the globoid worm we have chosen for turning with a profiled utter. While the worm rotates about its own axis (point $O_1$ in figure 1), the profiled cutter describes a rotational motion, the rotational center being point $O_2$ (figure 1). The standards recommend the shape of the raw material from which one obtains the worm. In figure 5, keeping into account these recommendations, we represented the solid, which materializes the raw material. It was generated with the AutoLisp function called Pinion:

```lisp
(Defun Pinion() (Command "Cylinder" (List 0 0 ) "D" d0 Lworm) (Command "Rotate3D" "L" "" Y" (List 0 0 0 ) "90") (Command "CHAmfer" (List 0 0 ) "" bevel bevel "L" (List 0 0 ) "") (Command "CHAmfer" (L worm 0 ) "" bevel bevel "L" (List L worm 0 ) "") (setq dist(/ ( - L_P Lworm) 2.0)) (Command "Cylinder" (List (* -1 dist) 0 0 ) "D" dia_rul L_P) (Command "Rotate3D" "L" "" Y" (List (* -1 dist) 0 0 ) "90") (Command "Union" "All" "") (Setq Pinion(EntLast)) (Command "Move" Pinion "" (List (* 0.5 Lworm) 0) (List 0 a))
)
```

The profile of the tool, which will generate the teeth of the worm is that represented in figure 6. The solid that materializes the tool is realized with the AutoLisp function Tool:

```lisp
(Defun Tool() (setq xsmall(* 2 m (/ (Sin Alpha1_rad) (Cos Alpha1_rad))) asmall(- (/ p 2) xsmall) bsmall(+ (/ p 2) xsmall) CD(/ (* 2 m) (Cos Alpha1_rad))) (Command "Pline" (List (* bsmall -0.5) (* m -3.0)) (List (* bsmall 0.5) (* m -3.0)) (List (* bsmall 0.5) (* m -2.0)) (List (* asmall 0.5) 0) (List (* asmall -0.5) 0) (List (* bsmall -0.5) (* m-2.0)) "C"); (Command "Extrude" "L" "" 1 "") (Setq Tool(EntLast))
)
```

**Figure 5.** Raw material for globoid worm.  
**Figure 6.** Profile of the tool.

The obtaining of the worm is also performed with an AutoLisp function. In a while loop, in the angular interval $0^0 + \left(\frac{\pi}{9} + 1\right)360^0$ one makes the following operations:

- one rotates the raw material (Pinion) with the constant angle $\varphi = 1^0$ about the rotational axis;
- one copies the tool (Tool) and assigns to it a name (e1);
- one positions the copy of the tool, by a translation, from the point $(0,0)$ to the point $\left(0, \frac{d_{w2}}{2} + m\right)$ and a rotation about the point $(0,0)$ of angle $-\frac{z_0}{2} \varphi_2 + \Delta \varphi_2$, where by $z_0$ we denoted the number of
teeth in gear with the worm, \( z_0 = \frac{z_2}{2} + 1 \), and by \( \varphi_2 \) we denoted the angular step of the wheel

\[
\varphi_2 = \frac{360}{z_2};
\]

at one step the tool rotates with the angle \( \Delta \varphi_2 = \frac{\varphi_2}{360} \).

– from the solid that materializes the raw material (Pinion) one extracts the solid that materializes the tool;

– one passes to the next step.

The function that realizes these operations is named Globoid_worm:

\[
\text{(Defun C:Globoid_worm () (arxload "geom3d.arx") (Command "Erase" "All" "" "" "" "" "" "") (Data) (Tool) (Raw_material) (Setq Nr_steps_Rotation (/ 360 ang_step)) (Setq Angle_Max_Pinion (* z0 360)) (Setq Nr_Total_steps(* z0 Nr_steps_Rotation)) (Setq Ang_step_Tool(/ phi_grad Nr_steps_Rotation)) (Setq phi 0 rot (* -0.5 z0 phi_grad)) (While (< phi Angle_Max_Pinion) (Command "Rotate3D" Pinion "x" (List 0 a) ang_step) (Setq rot(+ rot Ang_step_Tool)) (Command "Copy" Tool "" (0,0,0) (0,0,0)"" (List 0 0 0) (List 0 (+ (/ dw2 2) m) 0)) (Command "Rotate" e1 "" (List 0 0) rot) (Command "Subtract" Pinion "" e1 "") (Setq phi(+ phi ang_step)) )).
\]

The function calls the following Data function.

\[
\text{(Defun Data()) (Setq ang_step 1 z1 1 z2 30.0 m 2.0 p(* m Pi) q 12.0 dw2(* m z2) dw1(* m q) a(+ (/ dw1 2) (/ dw2 2.0)) phi_grad(/ 360 z2) phi_rad(* phi_grad/ Pi 180)) z0(+ (/ z2 9) 1.0) lphi1_rad(* 0.5 (- z0 1) phi_rad) Alphal1_grad(* Alphal1_rad (/ 180 Pi)) d0(+ dw1 m m) L(- d0 (* 0.03 a) bevel(/ d0 2) Lworm(* L (* 2 bevel)) dia_rul(- dw1 (* 3 m)) L_P (* 2 Lworm)) .}
\]

The function also calls the functions Tool and Raw_material previously defined. In figures 7 and 8, we present the photographic representations of the globoid worms with one start. In figure 7 the wheel has \( z_2 = 40 \) teeth, and in figure 8 it has \( z_2 = 30 \) teeth.

**Figure 7.** Globoid worm (\( z_2 = 40 \) teeth). **Figure 8.** Globoid worm (\( z_2 = 30 \) teeth).
4. The obtaining of the wheels which materialize the wheels

Due to the toroidal shape of the wheel, the teeth cannot be defined by using a reference rack. It is defined relative to a reference worm. Keeping into account the notations in figure1, the main geometric elements for the raw material, which materializes the wheel, are:

\[ r_2 = \frac{m_z z_2}{2}, \quad r_{a1} = \frac{m_z}{2}(z_2 + 2), \quad r_{f1} = \frac{m_z}{2}(z_2 - 2.5), \quad (7) \]

\[ b = \begin{cases} 
0.75d & \text{if } z_1 \leq 3, \\
0.67d_{a1} & \text{if } z_1 = 4.
\end{cases} \quad (8) \]

For the obtaining of the solid that materializes the wheel one uses as tool the already obtained worm. In order to diminish the dimension of the file that contains the wheel, we will tooth only one sector containing a tooth and an empty space, the obtained solid being multiplied in a polar way, about the rotational axis, with \( z_2 \) copies.

For the obtaining of the sector to be toothed, one uses the AutoLisp function Sector:

(Defun Sector() (Command "Pline" "0,0" (Polar (List 0 0) (- (/ Pi 2) Phi_rad) (/ de 2.0)) "A" "CE" (List 0 0) (List 0 (/ de 2.0)) "L" "C")
(Command "Extrude" "L" "" b "")
(Command "Move" "L" "" (List 0 0) (List 0 (/ b -2.0))) (Setq Tooth(EntLast))
)

Figure 9. Sector that will be toothed.  
Figure 10. Globoid worm and wheel gear.

In figure 9 we represented the solid that materializes the sector of the wheel generated by the previous AutoLisp function, the wheel being in contact with the already obtained worm. At a complete rotation of the worm one obtains a tooth of the wheel. Similar to the previous case, in a while loop,
one performs 360 steps, at each step extracting a rotated copy of the worm from the sector. When the worm rotates with one degree, the sector rotates with the angular step $\Delta \phi_2 = \frac{\phi_2}{360}$. The rotational angle of the worm is contrary oriented relative to the sense of rotation of the sector. After the obtaining of the solid that materializes $1/z_2$ of the wheel, it is multiplied in polar way, about the rotational axis, and the $z_2$ copies are united with the AutoCAD command Union. In figure 10 is presented the photographic representation of the worm and wheel gear obtained with the previously described procedure.

5. Conclusions
The worm and wheel gears are preferred when the transmission ratios have great values. These gears assure, in certain conditions, the transmission of great torques with relative great mechanical efficiency. In industry, they are also preferred due to the reduced noise during the exploitation. In some cases, the single-sense transmission of the motion is an advantage. In most cases the manufacturing of the worms is difficult and requires some wear-in operations for the worm and wheel gear. In this paper we obtained solids that materialize the worm and wheel gears, using AutoLisp functions in AutoCAD. We generated cylindrical and globoid worm and wheel gears with the aid of some algorithms inspired from the classical manufacturing procedures. By repetitive Boolean operations, one extracts from the raw material that approximates the worm, the solid that materializes the tool. One thus generates cylindrical worms with 1, 2, and 3 starts and globoid worms with one start. The obtaining of the solids that materialize the wheels is performed in a similar way.

The realized AutoLisp functions are considered to be used at the generation of the worm and wheel gears no matter the number of starts or the shape of the worm. For the generation of the solid that materializes the wheel one uses as the tool, the already obtained worm. In order to diminish the dimension of the file that contains the wheel, one is toothing only one circular sector that contains a tooth and an empty space, the obtained solid being then multiplied, in a polar way, about the rotational axis, the number of copies being given by the number of the teeth of the wheel. In this paper, we did not consider the profile displacement. The relations can be easily modified in order to consider this aspect too. The procedure used to obtain the worms is the same, the only difference being the position of the tool.

In a future paper we will study the possibility to obtain the gear with solids in AutoCAD considering the clearance during the functioning. With minimum modifications, the presented AutoLisp functions can be also used for worm and wheel gears at which the angle between the axes is not equal to $90^0$, or the wheel is a rack.

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