Features of using linear graphs in developing mathematical model of metal melting process in induction crucible furnace

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Abstract. The use of linear graphs in the development of a mathematical model of metal melting in an induction crucible furnace allows to determine the metal melting temperature and control the excess temperature in the main parts of the furnace. In addition, if it is necessary to optimize the thermal mode of operation of the furnace according to the presented graph conversion method, a graph model of any part of the furnace with the desired thermal parameters can be obtained.

Modern induction heating plants are complex high-performance, energy-intensive units with a capacity of tens of megawatts. High efficiency of the heating process is equivalent to energy saving and automatically leads to economical technology. In turn, the development and creation of efficient control systems for induction heating plants with high technical and economic indicators and high efficiency is one of the most pressing tasks in induction heating technology.[1]

Due to the difficulty of calculating the process of heating the molten metal and the main structural parts of induction crucible furnaces from the point of view of engineering practice, it is of great interest to develop a mathematical model based on linear graphs used in the analysis of linear electric circuits.[4]

The induction crucible furnace can be represented as a thermal circuit consisting of four interconnected thermal bodies.[1] The system of thermal balance equations for the steady state of such a scheme is as follows:

\[
\begin{align*}
\Lambda_{11}\theta_1 - \Lambda_{12}\theta_2 - \Lambda_{13}\theta_3 = P_1, \\
-\Lambda_{21}\theta_1 + \Lambda_{22}\theta_2 - \Lambda_{23}\theta_3 - \Lambda_{24}\theta_4 = P_2, \\
-\Lambda_{31}\theta_1 - \Lambda_{32}\theta_2 + \Lambda_{33}\theta_3 - \Lambda_{34}\theta_4 = P_3, \\
-\Lambda_{42}\theta_2 - \Lambda_{43}\theta_3 + \Lambda_{44}\theta_4 = P_4,
\end{align*}
\]

(1)

where \(\theta_1, \theta_2, \theta_3, \theta_4\) – excess temperature of molten metal, inductor, lining and crucible induction furnace housing, °C; \(P_1, P_2, P_3, P_4\) – power loss of corresponding parts of crucible induction furnace, kW; \(\Lambda_{12}, \Lambda_{13}, \Lambda_{14}, \Lambda_{23}, \Lambda_{24}, \Lambda_{33}, \Lambda_{34}, \Lambda_{43}, \Lambda_{44}\) – thermal conductivity between the parts of the crucible induction furnace under consideration; \(\Lambda_1\) – heat dissipation of molten metal through open arch of crucible; \(\Lambda_2\) – heat transfer of furnace housing; \(\Lambda_{11} = \Lambda_{12} + \Lambda_{13} + \Lambda_4\) – total thermal conductivity of the molten metal; \(\Lambda_{22} = \Lambda_{21} + \Lambda_{23} + \Lambda_{24}\) – total thermal conductivity of inductor; \(\Lambda_{33} = \Lambda_{31} + \Lambda_{32} + \Lambda_{34}\) – total thermal conductivity of lining; \(\Lambda_{44} = \Lambda_{22} + \Lambda_{43} + \Lambda_4\) – total thermal conductivity of the housing.

The system of thermal balance equations (1) can be represented as directional graphs according to the methodology presented in [2, 3]:

Fig.1. System of thermal balance equations (1) in the form of directed graphs

Inverse the graph shown in Fig.1 (Fig. 2).

Fig.2. The inversion of the graph shown in Fig.1
We exclude the node \( \theta_4 \) from the graph presented in Fig. 2 and get two loops with the following transmissions \( A_{34}A_{43}/A_{33}A_{44} \) and \( A_{24}A_{42}/A_{22}A_{44} \) (Fig 3).

![Graph 3](image3.png)

Further, from the graph shown in Fig. 3, we exclude the loop with transmissions \( A_{34}A_{43}/A_{33}A_{44} \) and \( A_{24}A_{42}/A_{22}A_{44} \) (Fig 4).

![Graph 4](image4.png)

After removing the node \( \theta_4 \), the graph model of the thermal process has the form with three unknowns (Fig 4). We will exclude the node \( \theta_3 \) and get two loops with the following transfers \( A_{13}A_{31}/A_{11}A_{33} \) and \( A_{23}A_{32}/A_{22}A_{33} \) (Fig 5).

![Graph 5](image5.png)

Excluding transmission loops \( A_{13}A_{31}/A_{11}A_{33} \) and \( A_{23}A_{32}/A_{22}A_{33} \) from the graph shown in Fig. 5 and excluding the node \( \theta_2 \), we obtain a graph model with formation of a loop with transmission \( A_{12}A_{21}/A_{11}A_{22} \) thermal process with two desired thermal parameters (Fig 6).

![Graph 6](image6.png)

Excluding the node \( \theta_2 \) we get a thermal model with one unknown thermal parameter (Fig 7).

![Graph 7](image7.png)
To implement the algorithm for solving the problem, a program was compiled. Thus, the presented mathematical graph model has a non-complex algorithm and easily programmed on personal computers that allows to optimize process of receiving liquid metal in the induction crucible furnace on the basis of control and management power ($P_1$, $P_2$, $P_3$, $P_4$) and thermal (for example, $\alpha_1$, etc.) parameters. In addition, according to the presented graph conversion method, it is possible to obtain a graph model of an inductor with the desired thermal parameter $\theta_2$ if it is necessary to optimize the thermal mode of operation of the inductor.

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