Continuous-Variable Measurement-Device-Independent Quantum Key Distribution

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We propose a continuous-variable measurement-device-independent quantum key distribution (CV-MDI QKD) protocol, in which detection is conducted by an untrusted third party. Our protocol can defend all detector side channels, which seriously threaten the security of a practical CV QKD system. Its security analysis against arbitrary collective attacks is derived based on the fact that the entanglement-based scheme of CV-MDI QKD is equivalent to the conventional CV QKD with coherent states and heterodyne detection. We find that the maximal total transmission distance is achieved by setting the untrusted third party close to one of the legitimate users. Furthermore, an alternate detection scheme, a special application of CV-MDI QKD, is proposed to enhance the security of the standard CV QKD system.

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I. INTRODUCTION

Quantum key distribution (QKD)\textsuperscript{[1,2]} can establish a secure key between two legitimate partners (Alice and Bob) through insecure quantum and classical channels. In recent last decades research on QKD has evolved rapidly. Some commercial systems are available in the market now\textsuperscript{[3]}. QKD has two main approaches: one is discrete-variable (DV) QKD, and the alternative is continuous-variable (CV) QKD\textsuperscript{[4,5]}. CV QKD has the advantage of being compatible with standard telecommunication, especially no request on single photon detectors. A Gaussian modulated CV QKD protocol using coherent states\textsuperscript{[6,8]} has been proved to be secure against arbitrary collective attacks\textsuperscript{[9,10]}, which is optimal in both the asymptotic case\textsuperscript{[11]} and the finite size regime\textsuperscript{[12,13]}. A recent experiment has successfully distributed secure keys over an 80-km optical fiber\textsuperscript{[14]}, showing the potential of long distance communication using CV QKD protocols.

Generally speaking, the theoretical security analysis of QKD relies on some ideal theoretical models. However, the practical devices often have some imperfections and deviate from the theoretical models. Thus the mismatch between practical devices and their idealized models may open security loopholes, which make the practical systems vulnerable to attacks\textsuperscript{[15]}. In DV QKD systems, various types of attacks against imperfect devices were proposed, among which the attacks against the single photon detector are the most significant ones\textsuperscript{[16,18]}. Recently in CV QKD systems, several attack strategies against practical detectors were also proposed\textsuperscript{[19,24]}. For example, the calibration attack\textsuperscript{[20]} and local oscillator (LO) fluctuation attack\textsuperscript{[21]} take advantage of modifying LO to manipulate the measurement results, which will make Alice and Bob overestimate the secret key rate. The wavelength attack\textsuperscript{[22,23]} allows the eavesdropper to launch an intercept-resend attack because of the wavelength-dependent property of the fiber beamsplitter used in the heterodyne detector. The saturation attack\textsuperscript{[24]} can force Alice and Bob to underestimate the excess noise by saturating the homodyne detector, which can hide the presence of an intercept-resend attack.

A natural attempt to remove these attacks in a CV QKD system was to characterize the specific loophole and find a countermeasure. For instance, Jouguet\textit{et al.}\textsuperscript{[25]} proposed an efficient countermeasure against the calibration attack by monitoring the LO\textsuperscript{[20]}. Once an attack is known, prevention is usually simple. However, it is difficult to fully characterize real detectors and account for all loopholes. Therefore figuring out how to defend against general attacks on detectors in practical systems becomes critical in CV QKD.

Inspired by the novel detector-attack-immune protocols, i.e. measurement-device-independent (MDI) QKD protocols\textsuperscript{[25–27]}, which were well analyzed in theory\textsuperscript{[28–31]} and successfully demonstrated in experiments\textsuperscript{[32–33]}, here we propose a CV-MDI QKD protocol which can also defend all detector side channels. The main idea is that both Alice and Bob are senders and an untrusted third party, named Charlie, is introduced to realize the measurement. Such measurement results will be used by Alice and Bob in the post-processing step to generate.
By introducing the equivalent entanglement-based (EB) scheme of this protocol, we show the security analysis again arbitrary collective attacks, which is based on the fact that the entanglement-based scheme of CV-MDI QKD is equivalent to the CV QKD with coherent states and heterodyne detection \[8\]. A corresponding prepare-and-measure (PM) scheme is proposed for implementation. Moreover, the performance of our protocol against collective entangling cloner attack is presented via numerical simulations. When the distance between Alice and Charlie equals that between Bob and Charlie (symmetric case), the transmission distance is below 10 km. However, in the asymmetric case the transmission distance can be improved, reaching 80 km under certain conditions. This demonstrates the feasibility of our scheme.

This paper is organized as follows: In Sec. II, the detailed descriptions of both the EB and PM schemes of CV-MDI QKD are given. In Sec. III, we present the security analysis for the CV-MDI QKD protocol. In Sec. IV, we show the numerical simulation results of the secret key rate, and discuss the performance and potential applications.

II. CONTINUOUS-VARIABLE MEASUREMENT-DEVICE-INDEPENDENT QKD PROTOCOL

In the PM scheme of a standard QKD protocol such as Bennett-Brassard 1984 (BB84) \[30\], Alice randomly prepares an encoded quantum state and sends it to Bob for detection. This PM scheme can be formulated in an EB version of the protocol as follows: Alice first creates an entangled state such as a Bell state, and afterwards, she measures one mode of this entangled state in a certain basis, thereby producing the correct state for the other mode that is sent to Bob.

In practice, the PM scheme is usually easy to apply, while the equivalent EB scheme is convenient for security analysis. The EB scheme of DV-MDI QKD can be seen as a one-way protocol using entanglement swapping as an untrusted quantum relay \[25, 26\]. Here we use the same idea in our CV-MDI QKD protocol, which exploits the continuous variable entanglement swapping \[37, 38\]. The EB scheme of the CV-MDI QKD protocol shown in Fig. 1(a) is described as follows.

**Step 1.** Alice generates one two-mode squeezed (TMS) state and keeps mode \(A_1\) while sending the other mode, \(A_2\), to an untrusted third party (Charlie) through the channel with length \(L_{AC}\). Bob generates another TMS state and keeps the mode \(B_1\) while sending the other mode, \(B_2\), to Charlie through another channel with length \(L_{BC}\).

**Step 2.** Modes \(A'\) and \(B'\) received by Charlie interfere at a beam splitter (BS) with two output modes \(C\) and \(D\). Then both the \(x\) quadrature of \(C\) and \(p\) quadrature of \(D\) are measured by homodyne detections, and the measurement results \(\{X_C, P_D\}\) are publicly announced by Charlie.

**Step 3.** After receiving Charlie’s measurement results, Bob displaces mode \(B_1\) by operation \(\hat{D}(\beta)\) and gets \(\hat{\rho}_{B_1}' = \hat{D}(\beta) \hat{\rho}_{B_1} \hat{D}^\dagger(\beta)\), where \(\hat{\rho}_X\) represents the density matrix of mode \(X\), \(\beta = g (X_C + iP_D)\) and \(g\) represents the gain of the displacement. Then Bob measures mode \(B_1'\) to get the final data \(\{X_B, P_B\}\) using heterodyne detection. Alice measures mode \(A_1\) to get the final data.
\{X_A, P_A\} using heterodyne detection.

**Step 4.** Alice and Bob use an authenticated public channel to finish the parameter estimation, information reconciliation, and privacy amplification steps.

After Charlie’s measurements and Bob’s displacement, mode A₁ and mode B₁' become entangled. Therefore, after both Alice’s and Bob’s heterodyne detections, their final data are correlated. The equivalent PM scheme is shown in Fig. 1(b) which is described as follows.

**Step 1.** Alice randomly prepares a coherent state \(|x_A + ip_A\rangle\), where \(x_A\) and \(p_A\) are Gaussian distributed with variance \(V_A - 1\). Bob randomly prepares another coherent state \(|x_B + ip_B\rangle\), where \(x_B\) and \(p_B\) are Gaussian distributed with variance \(V_B - 1\). Both Alice and Bob send their coherent states to Charlie through two different channels.

**Step 2.** The two modes (A’ and B’) received by Charlie interfere at a BS with two output modes C and D. Then both the z quadrature of C and p quadrature of D are measured by homodyne detections, and the measurement results \(\{X_C, P_D\}\) are publicly announced by Charlie.

**Step 3.** When Alice and Bob receive Charlie’s measurement results, Bob modifies his data as \(X_B = x_B + kX_C, P_B = p_B - kP_D\), while Alice keeps hers unchanged, \(X_A = x_A, P_A = p_A\). \(k\) is the amplification coefficient related to channel loss (the relationship between \(k\) and \(g\) in the EB scheme is shown in Appendix A).

**Step 4.** Alice and Bob use an authenticated public channel to finish the parameter estimation, information reconciliation, and privacy amplification steps.

In the PM scheme, Alice and Bob prepare coherent states independently and do not require any measurements inside them. The third party, Charlie, is totally untrusted. That is why the protocol can be called measurement device independent. See Appendix A for the proof of equivalence between the EB and PM schemes.

## III. SECURITY ANALYSIS

It is well known that the security of a PM scheme is equivalent to that of the corresponding EB scheme for a QKD protocol. In the EB scheme in Fig. 1(a), if one further assumes that both Bob’s initial TM state and the displacement operation inside himself are also untrusted, then the protocol could be seen as the well-known one-way CV QKD protocol using coherent states and heterodyne detection. The equivalent one-way model is shown in Fig. 1(b). Thus the EB scheme of CV-MDI QKD is just one specific case of the equivalent one-way model with more constraints on Eve. Therefore, the secret key rate of the equivalent one-way model should be a lower bound of the EB scheme. Suppose the secret key rates (with reverse reconciliation) of the EB scheme of CV-MDI QKD and the equivalent one-way model are, respectively, \(K^R_2\) and \(K^R_1\); then \(K^R_2 \leq K^R_1\). Although \(K^R_1\) is not very tight, it is easy to calculate [10, 11]

\[
K^R_2 = \beta R I(X_A, P_A : X_B, P_B) - \chi_2(X_B, P_B : E),
\]

where \(I(X_A, P_A : X_B, P_B)\) is the classical mutual information between Alice and Bob, \(\beta\) is the reconciliation efficiency, and \(\chi_2(X_B, P_B : E)\) is the Holevo bound of the mutual information between Bob and Eve. Also \(\chi_2(X_B, P_B : E) = S(\hat{\rho}_E) - S(\hat{\rho}_E|X_B, P_B)\), where \(S(\hat{\rho}_E)\) is the von Neumann entropy of the quantum state \(\hat{\rho}_E\).

Based on the theorem of the optimality of Gaussian collective attacks [43, 44], the upper bound of \(\chi_2(X_B, P_B : A_1B_1')\) is only determined by the covariance matrix \(\gamma_{A_1B_1'}\) of the quantum state \(\hat{\rho}_{A_1B_1'}\). In a practical experiment, \(\gamma_{A_1B_1'}\) can be calculated through the parameter estimation step.

From the analysis above, the secret key rate \(K^R_2\) is based on the assumption that Eve controls Charlie, and it’s actually calculable in a practical experiment. Therefore, the CV-MDI QKD protocol using \(K^R_2\) as the secret key rate is immune to all collective attacks against detectors.

## IV. SIMULATION RESULTS AND DISCUSSION

### A. Numerical simulation

As discussed above, in experiment, Alice and Bob can get the covariance matrix \(\gamma_{A_1B_1'}\) through the parameter estimation step. Then they can calculate the secret key rate \(K^R_2\). In numerical simulation, a model to simulate

![Diagram](image-url)
the CV-MDI QKD protocol is provided, including what the channels are and what Charlie does. The model is shown in Fig. 8

We assume the channels from Alice to Charlie and Bob to Charlie are under two independent entangling cloner [39] attacks, and Charlie performs a standard CV entanglement-swapping process as the EB scheme requires. All the simulations in this paper are under this model.

We should point out that Eve’s attack described here is not the optimal one. The entangling cloner attack is usually used to model a Gaussian channel affected by the environment (Eve) and is analyzed to get a sense of a protocol’s performance in experiment. In Fig. 3, the channel parameters transmittance and excess noise on Alice’s side (Bob’s side) are $\eta_A$ ($\eta_B$) and $\varepsilon_A$ ($\varepsilon_B$). We assume that both channel losses are $\alpha = 0.2$ dB/km, and thus $\eta_A = 10^{-\epsilon L_{AC}/10}$ and $\eta_B = 10^{-\epsilon L_{BC}/10}$. The quadratures’ relations are shown in Appendix B. The covariance matrix $\gamma_{A;B'}$ is

$$\gamma_{A;B'} = \begin{pmatrix} V_{A} & I_{2} \varepsilon_{z} \sigma_{z} & \sqrt{T (V_{A}^{2} - 1) \sigma_{z}} & T (V_{A} - 1) + 1 + T \varepsilon_{z}^{2} \end{pmatrix} I_{2}$$  \tag{2}$$

where

$$I_{2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_{z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad T = \frac{\eta_{A}}{2} g^2,$$  \tag{3}$$

and

$$\varepsilon' = 1 + \frac{1}{\eta_{A}} [\eta_{B} (\chi_{B} - 1) + \eta_{A} \chi_{A}]$$

$$+ \frac{1}{\eta_{A}} \left( \frac{\sqrt{2}}{g} \sqrt{V_{B} - 1} - \sqrt{\eta_{B}} \sqrt{V_{B} + 1} \right)^{2}.$$  \tag{4}$$

$$\chi_{A} = \frac{1 - \eta_{A}}{\eta_{A}} + \varepsilon_{A}, \quad \chi_{B} = \frac{1 - \eta_{B}}{\eta_{B}} + \varepsilon_{B},$$

and $g$ is the gain of displacement.

Comparing the covariance matrix (2) with the one-way protocol, we can find that $\varepsilon'$ represents the equivalent excess noise of the equivalent one-way model of CV-MDI QKD. Here we choose $g = \sqrt{\frac{2}{\eta_{B}} \sqrt{V_{B} - 1} / \sqrt{V_{B} + 1}}$ to minimize the equivalent excess noise; thus we have

$$\varepsilon' = \varepsilon_{A} + \frac{1}{\eta_{A}} [\eta_{B} (\varepsilon_{B} - 2) + 2].$$  \tag{5}$$

We first consider the perfect reconciliation efficiency case $\beta_{R} = 1$. The simulation result in Fig. 4 is the secret key rate of the symmetric case, which means $L_{AC} = L_{BC}$. The red dashed line is under the conditions that variance $V_{A} = V_{B} = 10^{5}$ and excess noise $\varepsilon_{A} = \varepsilon_{B} = 0$, and the black solid line is under the con-
for the red dashed line is \( \beta \) distance step. (b) The left panel is the secret key rate vs transmission for one-way CV QKD, which contains two steps; the first is \( L_{AB} \approx 3 \) km, around 7 km, referring to a 1.4 dB loss. If we will discuss how to extend the transmission distance.

In short-distance communications. In the next section, symmetric case. Hence the symmetric case can be useful if the transmission distance gets longer. That’s why we This is a very large value, and it will increase quickly even if there is no excess noise in either channel, the case cannot result in an optimal system performance.

Figure 5(a) shows the secure key rate in an asymmetric case \( \left( L_{AC} \neq L_{BC} \right) \). When \( L_{BC} \) increases, the maximal \( L_{AC} \) decrease dramatically. If Charlie’s position can be close to Bob, the total transmission distance \( L_{AB} \) can be a relatively longer distance, up to 80 km in theory [the top black solid line in Fig. 5(b)]. Thus, the asymmetric case of CV-MDI QKD indicates the detection scheme that will be discussed below.

The detection scheme discussed here is the same as the EB scheme of CV-MDI QKD except that Bob takes over Charlie’s operations in Fig. 1(a) with \( L_{BC} = 0 \) km. Figure 6(a) shows the PM model of this detection scheme, which contains two steps, i.e., the measurement step and the data-processing step. The measurement step is a modified heterodyne detection in which the vacuum state introduced by the BS in the standard heterodyne detection is replaced by a Gaussian-modulated coherent state \( |\alpha_B\rangle \). The data-processing step uses the measurement results \( \{ X_C, P_D \} \) and the means \( \{ x_B, p_B \} \) of the quadratures of \( |\alpha_B\rangle \) to construct Bob’s final data \( \{ X_B = x_B + kX_C, P_B = p_B - kP_D \} \), where \( k \) is the amplification coefficient and will be traversed to find an optimal value \( k_{opt} \) which makes the secret key rate the highest. The new detection scheme has two advantages. First, it can defend against all collective attacks against detectors, which is the most important feature. Second, the traversing \( k \) can be done classically.

In a practical experiment, the reconciliation efficiency is not 100%. Figure 6(b) shows the results for a practical reconciliation efficiency \( \beta_R = 0.95 \) [14], the maximal transmission distance using our detection scheme can be 40 km.

In the above analysis, almost perfect detection efficiency was assumed in our simulation. Next, we will consider a practical homodyne detector which has imperfections such as finite efficiency and electronic noise [46]. These imperfections will increase the “equivalent excess noise,” i.e., \( \varepsilon_{imD} = \varepsilon_A + \eta_B \left( \varepsilon_B - 2 \right) + 2 + 2 \chi_{Det} / \eta_A = \varepsilon' + 2 \chi_{Det} / \eta_A \), which holds when traversing the displacement gain \( g \) to minimize it under two independent entangling cloner attacks. Here \( \chi_{Det} = \left( 1 - \eta_D \right) / \eta_D + \varepsilon_{Det} / \eta_D \), \( \eta_D \) is the detector’s efficiency, and \( \varepsilon_{Det} \) is the variance of the electronic noise. Thus the secret key rate will decrease when the detector is imperfect. For instance, if the detector’s efficiency reduces to 90%, even though the electronic noise is zero, the transmission distance will be less than 10 km. Actually, to get a nonzero transmission distance the detector’s efficiency has to be above 85.5% [17]. However, these imperfections can be compensated for by optical preamplifiers [48, 50].

On the other hand, the detection scheme discussed here is very effective against the local oscillator fluctuation attack [21] and the calibration attack [20]. The main idea of these two kinds of attacks is that Eve can control the scale of the measurement result by manipulating the LO. If the correct measurement result is denoted by \( X_O \), then Eve can force Bob to get a fake measurement result \( X'_O = \sqrt{\eta} X_O \), referring to Eq. (3) in [21]. When using the detection scheme, the only measurement results are conditions that variance \( V_A = V_B = 40 \) and excess noises \( \varepsilon_A = \varepsilon_B = 0.002 \), which is reasonable according to experiment [14].

From Fig. 4, we can see that in both cases, the maximal total transmission distance \( L_{AB} = L_{AC} + L_{BC} \) is around 7 km, referring to a 1.4 dB loss. If \( L_{AC} = 3.5 \) km, even if there is no excess noise in either channel, the equivalent excess noise in Eq. (3) is still around 0.35. This is a very large value, and it will increase quickly if the transmission distance gets longer. That’s why we cannot extract the secret key at a longer distance for the symmetric case. Hence the symmetric case can be useful in short-distance communications. In the next section, we will discuss how to extend the transmission distance.

**B. Discussion and application**

Equation (4) indicates that \( \varepsilon' \) is not symmetric if permitting \( \eta_A \) and \( \eta_D \) because the postprocessing step is not symmetric because only Bob modifies his data while Alice keeps hers unchanged. This means that the symmetric case cannot result in an optimal system performance.

Figure 5(a) shows the secure key rate in an asymmetric performance. The simulation model of this detection scheme, which contains two steps, i.e., the measurement step and the data-processing step. The measurement step is a modified heterodyne detection in which the vacuum state introduced by the BS in the standard heterodyne detection is replaced by a Gaussian-modulated coherent state \( |\alpha_B\rangle \). The data-processing step uses the measurement results \( \{ X_C, P_D \} \) and the means \( \{ x_B, p_B \} \) of the quadratures of \( |\alpha_B\rangle \) to construct Bob’s final data \( \{ X_B = x_B + kX_C, P_B = p_B - kP_D \} \), where \( k \) is the amplification coefficient and will be traversed to find an optimal value \( k_{opt} \) which makes the secret key rate the highest. The new detection scheme has two advantages. First, it can defend against all collective attacks against detectors, which is the most important feature. Second, the traversing \( k \) can be done classically.

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On the other hand, the detection scheme discussed here is very effective against the local oscillator fluctuation attack [21] and the calibration attack [20]. The main idea of these two kinds of attacks is that Eve can control the scale of the measurement result by manipulating the LO. If the correct measurement result is denoted by \( X_O \), then Eve can force Bob to get a fake measurement result \( X'_O = \sqrt{\eta} X_O \), referring to Eq. (3) in [21]. When using the detection scheme, the only measurement results are...
When Eve employs those two attacks, Bob will get fake results \(\{\sqrt{\pi} X_C, \sqrt{\pi} P_D\}\) and amplify them by the amplification coefficient \(k\) in the data-processing step. Because \(k\) will be traversed for all possible values to get the highest secret key rate, premultiplying a proportional coefficient \(\sqrt{\eta}\) for the measurement results by Eve will only lead to a different optimal value of \(k\) while the highest secret key rate is still the same. This can be proven easily because traversing \(k\) equals traversing \(k\sqrt{\eta}\) when \(\eta\) is a constant. Therefore, by using our detection scheme, the final secret key rate under these two kinds of attacks not only is never overestimated but also remains the same as in the nonattack case.

\[\{X_C, P_D\}\].

V. CONCLUSION

In this paper, we proposed the continuous-variable measurement-device-independent quantum key distribution protocol, which is immune to all collective attacks against detectors. The numerical simulation under the entangling cloner attack indicates that the transmission distance between Alice and Bob is limited in the symmetric case \((L_{AC} = L_{BC})\). However, when Charlie is close to Bob, the maximal total transmission distance can still reach 80 km for \(\beta_R = 1\) and 40 km for \(\beta = 0.95\). A corresponding detection scheme for the one-way protocol was proposed which is immune to all collective attacks against detectors. This detection scheme only requires slight revisions to the existing CV-QKD systems and thus shows its feasibility.

Notes added: Recently, we become aware of an independent work on the same subject [51]. Those authors use a different security analysis method based on conditional scenarios, which requires a relatively complex postprocessing technique. They also propose a model to describe Eve’s general attack and find the most powerful attack, which is very helpful for understanding Eve’s attack strategy.

VI. ACKNOWLEDGMENTS

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Appendix A: Equivalence between PM scheme and EB scheme

In CV QKD, the generation of a Gaussian-distributed coherent state is usually modeled by measuring one mode of a two-mode squeezed state using heterodyne detection and projecting another mode onto a coherent state. Therefore if we modify the PM scheme by replacing the state preparation step by two other steps, of which the first is Alice and Bob independently generate two TMS states \(\hat{\rho}_{A_1A_2}\) and \(\hat{\rho}_{B_1B_2}\) and the second is they measure modes \(A_1\) and \(B_1\) by heterodyne detections, then the modified protocol is equivalent to the original PM scheme. Next, we will show the equivalence between the modified protocol and the EB scheme.

Suppose Alice’s initial state is \(\hat{\rho}_{A_0A_1A_2}\), where mode \(A_0\) is the vacuum state introduced by the heterodyne detection. Bob’s initial state is \(\hat{\rho}_{B_0B_1B_2}\), where mode \(B_0\) is the vacuum state introduced by the heterodyne detection. The initial density matrix of whole system including Eve can be written as \(\hat{\rho}_0 = \hat{\rho}_{A_0A_1A_2} \otimes \hat{\rho}_E \otimes \hat{\rho}_{B_0B_1B_2}\). \(\hat{\rho}_E\) may contain multimodes.

For the modified protocol, suppose Alice’s initial measurement results are \(\{x_A, p_A\}\) and Bob’s are \(\{x'_B, p'_B\}\), then the state Eve gets is

\[
\rho_{A_2B_2E}^{(x_{A_{PA}}, x'_{B_{PB}})} = \frac{\langle x_A, p_A, x'_B, p'_B | U_{B_1A_0}^{B_1} U_{A_1A_0}^{A_1} \hat{\rho}_0 U_{A_1A_0}^{A_1} U_{B_1B_0}^{B_1} | x_A, p_A, x'_B, p'_B \rangle}{p(x_A, p_A, x'_B, p'_B)}
\] (A1)

where \(p(x_A, p_A, x'_B, p'_B)\) (\(p_{AB}\) for short) is the probability of getting measurement results \(\{x_A, p_A, x'_B, p'_B\}\). Eve will get the measurement results \(X_C\) and \(P_D\) according to a positive operator-valued measurement \(\{X_C, P_D\}\) on two certain modes, after applying a unitary operation \(U_{A_2B_2E}\). Thus the joint probability of \(\{x_A, p_A, x'_B, p'_B, X_C, P_D\}\) is

\[
p_{ABE} = Pr(x_A, p_A, x'_B, p'_B, X_C, P_D) = p_{AB} \langle X_C, P_D | U_{A_2B_2E} \hat{\rho}_{A_2B_2E} | X_C, P_D \rangle
\]

\[
= \langle x_A, p_A, x'_B, p'_B, X_C, P_D | U_{A_1A_0}^{B_1} U_{B_1B_0}^{B_1} U_{A_1A_0}^{A_1} U_{A_1A_0}^{A_1} U_{B_1B_0}^{B_1} | x_A, p_A, x'_B, p'_B, X_C, P_D \rangle
\]

(A2)

Because in the last step Bob modifies his data by \(X_B = x'_B + k_1 X_C, P_B = p'_B - k_1 P_D\), the joint probability of final
data \{X_A, P_A, X_B, P_B, X_C, P_D\} is

\[
p'_{ABE} = \Pr (X_A, P_A, X_B, P_B, X_C, P_D | D) = \Pr (X_A, P_A, X_B - k_1 X_C, P_B + k_1 P_D, X_C, P_D)_{M}
\]

\[
= \langle X_A, P_A, X_B - k_1 X_C, P_B + k_1 P_D, X_C, P_D | U^{BS}_{A_1, A_0} U^{BS}_{B_1, B_0} U^{BS}_{A_2, B_2} \hat{\rho}_0 U^{BS}_{A_3, A_0} \rangle \hat{D}_{B_0} (-ik_1 P_D) U^{BS}_{A_1, A_0} U^{BS}_{B_1, B_0} U^{BS}_{A_2, B_2} \hat{\rho}_0 U^{BS}_{A_3, A_0} \hat{D}_{B_0} (-ik_1 P_D) |X_A, P_A, X_B, P_B, X_C, P_D\rangle.
\]

(A3)

The measurement applied on mode B1 or B0 is homodyne detection (measuring the x quadrature of B1 or the p quadrature of B0); thus an extra displacement of the p quadrature on B1 or x quadrature on B0 will not change the joint probability \(p'_{ABE}\). Therefore

\[
p'_{ABE} = \langle X_A, P_A, X_B, P_B, X_C, P_D | \hat{D}_{B_1} (k_1 (X_C + i P_D)) \hat{D}_{B_0} (-k_1 (X_C + i P_D)) U^{BS}_{A_1, A_0} U^{BS}_{B_1, B_0} U^{BS}_{A_2, B_2} \hat{\rho}_0 \rangle \hat{D}_{B_1} (k_1 (X_C + i P_D)) \hat{D}_{B_0} (-k_1 (X_C + i P_D)) |X_A, P_A, X_B, P_B, X_C, P_D\rangle.
\]

(A4)

Suppose the quadratures of mode B0 and B1 are \(\hat{x}_{B_0}, \hat{p}_{B_0}\) and \(\hat{x}_{B_1}, \hat{p}_{B_1}\). After passing through the 50:50 BS first and then two displacements \(\hat{D}_{B_1} (k_1 (X_C + i P_D)), \hat{D}_{B_0} (-k_1 (X_C + i P_D))\), these quadratures become \(\hat{x}'_{A_0}, \hat{p}'_{A_0}\) and \(\hat{x}'_{A_1}, \hat{p}'_{A_1}\). Then

\[
\begin{align*}
\hat{x}'_{B_0} &= \sqrt{2} \left(\hat{x}_{B_0} + \hat{x}_{B_1}\right) + k_1 X_C = \frac{1}{\sqrt{2}} \left(\hat{x}_{B_0} + (\hat{p}_{B_0} + \sqrt{2} k_1 X_C)\right), \\
\hat{p}'_{B_0} &= \frac{1}{\sqrt{2}} \left(\hat{p}_{B_0} + \hat{p}_{B_1}\right) + k_1 P_D = \frac{1}{\sqrt{2}} \left(\hat{p}_{B_0} + (\hat{p}_{B_1} + \sqrt{2} k_1 P_D)\right), \\
\hat{x}'_{B_1} &= \sqrt{2} \left(\hat{x}_{B_0} - \hat{x}_{B_1}\right) - k_1 X_C = \frac{1}{\sqrt{2}} \left(\hat{x}_{B_0} - (\hat{p}_{B_0} + \sqrt{2} k_1 X_C)\right), \\
\hat{p}'_{B_1} &= \frac{1}{\sqrt{2}} \left(\hat{p}_{B_0} - \hat{p}_{B_1}\right) - k_1 P_D = \frac{1}{\sqrt{2}} \left(\hat{p}_{B_0} - (\hat{p}_{B_1} + \sqrt{2} k_1 P_D)\right).
\end{align*}
\]

(A5)

They are the same as firstly displacing \(\hat{D}_{B_1} \left(\sqrt{2} k_1 (X_C + i P_D)\right)\) on mode B1 and then passing through the BS. Thus, we have

\[
p'_{ABE} = \langle X_A, P_A, X_B, P_B, X_C, P_D | U^{BS}_{B_1, B_0} \hat{D}_{B_1} \left(\sqrt{2} k_1 (X_C + i P_D)\right) U^{BS}_{A_1, A_0} U^{BS}_{B_1, B_0} \hat{\rho}_0 \rangle \hat{D}_{B_1} \left(\sqrt{2} k_1 (X_C + i P_D)\right) U^{BS}_{B_1, B_0} |X_A, P_A, X_B, P_B, X_C, P_D\rangle.
\]

(A6)

For the EB scheme, Eve does the measurement first. We make the same assumption as above; then after Eve gets the measurement results \{X_C, P_D\}, the state left for Alice and Bob will be

\[
\hat{\rho}(X_C, P_D) = \frac{\langle X_C, P_D | U^{BS}_{B_2, E} U^{BS}_{A_2, A_0} U^{BS}_{B_1, B_0} |X_C, P_D\rangle}{p(X_C, P_D)},
\]

(A7)

where \(p(X_C, P_D)\) is the probability of getting measurement results \{X_C, P_D\}. Then Bob will displace mode B1 first by \(\hat{D}_{B_1} \left(g (X_C + i P_D)\right)\) and measure it using a heterodyne detector. The probability of getting the final data \{X_A, P_A, X_B, P_B\} given \{X_C, P_D\} is

\[
p_{AB|CD} = \Pr (X_A, P_A, X_B, P_B | X_C, P_D) = \langle X_A, P_A, X_B, P_B | U^{BS}_{B_1, B_0} \hat{D}_{B_1} \left(g (X_C + i P_D)\right) U^{BS}_{A_1, A_0} \rangle \hat{D}_{B_1} \left(g (X_C + i P_D)\right) U^{BS}_{B_1, B_0} |X_A, P_A, X_B, P_B\rangle.
\]

(A8)

The joint probability of all data \{X_A, P_A, X_B, P_B, X_C, P_D\} is

\[
p''_{ABE} = p_{AB|CD} p(X_C, P_D) = \langle X_A, P_A, X_B, P_B, X_C, P_D | U^{BS}_{B_1, B_0} \hat{D}_{B_1} \left(g (X_C + i P_D)\right) U^{BS}_{A_1, A_0} \rangle U^{BS}_{B_1, B_0} |X_A, P_A, X_B, P_B, X_C, P_D\rangle.
\]

(A9)

It is easy to check that the two joint probabilities \(p'_{ABE}\) and \(p''_{ABE}\) are the same once we let \(g = \sqrt{2} k_1\). Therefore the EB scheme is equal to the modified protocol, and because of the equivalence between the modified protocol and the PM scheme, the EB scheme further equals the PM scheme.

In the modified protocol, if Bob’s initial measurement results are \{\hat{x}'_{B_1}, \hat{p}'_{B_1}\}, then the coherent state sent out from him is \(|x_A + ip_A\rangle\), where \(x_A = \sqrt{\frac{2}{1 + V_0}} x'_{A_1}\) and \(p_A = \sqrt{\frac{2}{1 + V_0}} p'_{A_1}\). So the amplification coefficient \(k\) in the PM scheme is described as

\[
k = \frac{V_0}{\sqrt{1 + V_0}} = \frac{g}{\sqrt{1 + V_0}}.
\]
Appendix B: Relationship of quadratures used in numerical simulation

After passing through the channels,

\[
\begin{align*}
    \hat{A}' &= \sqrt{\eta_A} \hat{A}_2 + \sqrt{1 - \eta_A} \hat{E}_2 \\
    \hat{B}' &= \sqrt{\eta_B} \hat{B}_2 + \sqrt{1 - \eta_B} \hat{E}_5
\end{align*}
\]

(B1)

Modes \(A'\) and \(B'\) interfere on the 50:50 BS, then modes \(C\) and \(D\) are

\[
\begin{align*}
    \hat{C} &= \frac{1}{\sqrt{2}} \left( \hat{A}' - \hat{B}' \right) = \frac{1}{\sqrt{2}} \left( \sqrt{\eta_A} \hat{A}_2 - \sqrt{\eta_B} \hat{B}_2 \right) + \frac{1}{\sqrt{2}} \left( \sqrt{1 - \eta_A} \hat{E}_2 - \sqrt{1 - \eta_B} \hat{E}_5 \right) \\
    \hat{D} &= \frac{1}{\sqrt{2}} \left( \hat{A}' + \hat{B}' \right) = \frac{1}{\sqrt{2}} \left( \sqrt{\eta_A} \hat{A}_2 + \sqrt{\eta_B} \hat{B}_2 \right) + \frac{1}{\sqrt{2}} \left( \sqrt{1 - \eta_A} \hat{E}_2 + \sqrt{1 - \eta_B} \hat{E}_5 \right)
\end{align*}
\]

(B2)

After the displacement, mode \(B'_1\) becomes

\[
\begin{align*}
    \hat{B}'_{1x} &= \hat{B}_{1x} + g\hat{C}_x = \left( \hat{B}_{1x} - g\frac{\sqrt{\eta_A}}{\sqrt{2}} \hat{B}_{2x} \right) + g\frac{\sqrt{\eta_A}}{\sqrt{2}} \hat{A}_{2x} + \frac{1}{\sqrt{2}} \left( \sqrt{1 - \eta_A} \hat{E}_{2x} - \sqrt{1 - \eta_B} \hat{E}_{5x} \right) \\
    \hat{B}'_{1p} &= \hat{B}_{1p} + g\hat{D}_p = \left( \hat{B}_{1p} + g\frac{\sqrt{\eta_A}}{\sqrt{2}} \hat{B}_{2p} \right) + g\frac{\sqrt{\eta_A}}{\sqrt{2}} \hat{A}_{2p} + \frac{1}{\sqrt{2}} \left( \sqrt{1 - \eta_A} \hat{E}_{2p} + \sqrt{1 - \eta_B} \hat{E}_{5p} \right)
\end{align*}
\]

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