A Crucial Test for Color-octet Production Mechanism in $Z^0$ Decays

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Abstract

The direct production rates of $D$-wave charmonia in the decays of $Z^0$ is evaluated. The color-octet production processes $Z^0 \rightarrow ^3 D_J (c\bar{c})q\bar{q}$ are shown to have distinctively large branching ratios, the same order of magnitude as that of $J/\psi$ production, as compared with other $D$-wave charmonium production mechanisms. This may suggest a crucial channel to test the color-octet mechanism as well as to observe the $D$-wave charmonium states in $Z^0$ decays. In addition, a signal for the $^3 D_J$ charmonium as strong as $J/\psi$ or $\psi'$ with large transverse momentum at the Tevatron should also be observed.

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The systematic study of the heavy quark bound systems has played a very important role in obtaining information not only on the properties of heavy quarks themselves but also on Quantum Chromodynamics (QCD). The property of asymptotic freedom of QCD allows one to calculate the production and decays of heavy quark mesons perturbatively at high energy scale, while the nonperturbative part can be factored out as the wave functions or their derivatives at the origin.

Recent progress in the this area was stimulated by the experiment results of CDF detector at the Fermilab Tevatron. In the 1992-1993 run, the CDF data for the prompt production of $\psi$ and $\psi'$ at large transverse momentum region were observed to be orders of magnitude stronger than the lowest order perturbative calculations based on color-singlet model, which has ever gained some success in describing the production and decays of heavy quarkonia.

To resolve these discrepancies, Braaten and Yuan suggested that parton fragmentation represents the dominant source of prompt quarkonium production at high transverse momentum ($P_T \geq 6 \text{ GeV}$), though these processes are formally of high order in the strong coupling constant. Over the past few years, the fragmentation functions for S- and P-wave quarkonia have been calculated to lowest order, and they have subsequently been utilized to the issues of quarkonium production phenomenology in hadron collider. However, they still underestimate the rates of $\psi$ and $\psi'$ production more than an order of magnitude referring to the observed data at the Tevatron. These large discrepancies have called in question the simple color-singlet model description for quarkonium and stimulate ones to seek for new production mechanisms as well as new paradigms for treating heavy quark-antiquark bound systems that go beyond the color-singlet model.

To this end, a factorization formalism has recently been performed by Bodwin, Braaten, and Lepage in the context of nonrelativistic quantum chromodynamics (NRQCD), which provides a new framework to calculate the inclusive production and decay rates of quarkonia. In this approach, the calculations are organized in powers of $v$, the average velocity of the heavy quark (antiquark) in the meson rest frame, and in $\alpha_s$, the strong coupling constant.

The breakdown of color-singlet model stems from its overlook of the high Fock components contributions to quarkonium production cross sections. The color-octet term in the gluon fragmenting to $\psi(\psi')$ has been considered by Braaten and Fleming to explain the $\psi(\psi')$ surplus problem discovered by CDF. Taking $<O_{\psi}^{3S_1}>$ and $<O_{\psi'}^{3S_1}>$ as input
parameters, the CDF surplus problems for $\psi$ and $\psi'$ can be explained as the contributions of color-octet terms due to gluon fragmentation. By adjusting the values for matrix elements $<\mathcal{O}_8^{1/2}(3S_1)>$ and $<\mathcal{O}_8^{1/2}(3S_1)>$, the authors of Refs.\[12\][13] got self-consistent values of them in the contents of NRQCD.

Even though the color-octet mechanism has gained some successes in describing the production and decays of heavy quark bound systems, especially in explaining the transverse momentum spectrum of the quarkonium measured at the Tevatron\[14\], it still has a long way to go before finally setting its position and role in heavy quarkonium physics. Actually, by now the color-octet matrix elements are extracted only by fitting the data. Moreover, some recent studies\[15\][16][17] show that there are some inconsistencies about the normalization of the color-octet matrix elements in different processes. Therefore, the most urgent task among others needs to do is to confirm and identify the color-octet quarkonium signals.

While the first charmonium state, the $J/\psi$, has been found over twenty years, $D$-wave states, given the limited experimented data, have received less attention. However, this situation may be changed in both experimental and theoretical investigations. Experimentally, there are hopes of observing charmonium $D$-wave states in addition to $1^{--}(\psi(3770))$ in a high-statistic exclusive charmonium production experiment\[18\] and $b\bar{b}$ $D$-wave states in $\Upsilon$ radiative decays\[19\]. Theoretically, the analysis based on NRQCD shows that in $D$-wave quarkonium production the color-octet components play an even more important roles than in the $S$- and $P$-wave cases.

Recently, there is some clue for the $D$-wave $2^{--}$ charmonium state in $E705$ 300 GeV $\pi^\pm$- and proton- Li interaction experiment\[20\]. In this experiment there is an abnormal phenomenon that in the $J/\psi\pi^+\pi^-$ mass spectrum, two peaks at $\psi(3686)$ mass and at 3.836 GeV (given to be the $2^{--}$ state) are observed and they have almost the same height. Obviously, this situation is difficult to explain based upon the color-singlet model, because the production rate would be proportional to the squared second derivative of the wave function at the origin for the $D$-wave state, which is suppressed by $O(v^4)$, as compared to the squared wave function at the origin for the $S$-wave state. However, it might be explained with the NRQCD analysis. In NRQCD the Fock state expansion for $3D_J$ states is

$$|3D_J> = O(1)|Q\bar{Q}(3D_J, 1) + O(v)|Q\bar{Q}(3P_J, 8)g > + O(v^2)|Q\bar{Q}(3S_1, 8 or 1)gg > + \cdots.$$ (1)

In fact, considering the suppression due to the derivative of wave function at the origin, in production processes, contributions of the three terms written out in Eq.(1) have the
same order in \( v^2 \). So in the quark fragmentation, all of them are of the same order in both \( \alpha_s \) and \( v^2 \). However, in the gluon fragmentation for \( D \)-wave charmonium production processes, the \( S \)-wave color-octet \((^3S_1,8)\) production is \( O(1/\alpha_s^2) \) enhanced over the color-singlet \((^3D_J,1)\) and \((^3S_1,1)\) production in the short distance perturbative sector because the color singlet \((^3D_J,1)\) and \((^3S_1,1)\) have to couple to at least three gluons. The \( P \)-wave color-octet process is forbidden by charge conjugation invariance. By this argument, it might be easy to understand the \( E705 \) experiment data as long as the nonperturbative matrix element \( \langle O_8^{^3D_J}(^3S_1) \rangle \) is about the same order as \( \langle O_8^{\psi}(^3S_1) \rangle \) in magnitude, which is just expected in the framework of NRQCD.

Of course, this explanation requires the dominance of color-octet gluon fragmentation over other production mechanisms. At energies in fixed target experiments like \( E705 \), the color-octet gluon fragmentation dominance may or may not be the case. Moreover, the strong signal of \( J/\psi\pi^+\pi^- \) at 3.836GeV observed by \( E705 \) is now questioned by other experiments\(^{[21]}\). Nevertheless, if the \( E705 \) result is confirmed (even with a smaller rate, say, by a factor of 3, for the signal at 3.836GeV), the color-octet gluon fragmentation will perhaps provide a quite unique explanation for the \( D \)-wave charmonium production, since in all other mechanisms the \( D \)-wave production rate is expected to be much smaller than that for the \( S \)-wave states.

No matter what will be the final situation for the \( E705 \) result, it does remind us that in the NRQCD approach the production rate of \( D \)-wave heavy quarkonium states can be as large as that of \( S \)-wave states as long as the color-octet gluon production mechanism dominantes. This scenario can be tested in many processes, in particular, in the \( Z^0 \) decays.

Well over \( 10^6 \) \( Z^0 \) events have been accumulated at the CERN \( e^+e^- \) collider at \( \text{LEP} \) and further improvement are expected. This makes it possible to investigate rare decays of \( Z^0 \) and to precisely test QCD. Among others the production of charmonium states, in particular the \( ^3D_J \) states, in the \( Z^0 \) decays will be very interesting in testing the color-octet production mechanism.

Recently a study shows\(^{[22]}\) that the leading order color-octet process in \( \alpha_s \), say \( Z^0 \to \psi g \), has a relatively small branching ratio because of the large momentum transfer, and this is also the case for the \( D \)-wave charmonium production. The dominant color-octet processes for \( Z^0 \to ^3D_J q\bar{q} \) as well as \( Z^0 \to \psi q\bar{q} \) begin at order \( \alpha_s^2 \) as shown in Fig.1. Here \( q \) represents
From Ref. [22] we readily have
\[
\Gamma(Z \rightarrow ^3D_{Jq\bar{q}}) = \Gamma(Z \rightarrow q\bar{q}) \frac{\alpha_s^2(2m_c)}{36} \frac{n_8^3(3S_1)}{m_c^3} \{5(1 - \xi^2) - 2\xi \ln \xi - [2Li_2(\frac{\xi}{1 + \xi}) - 2Li_2(\frac{1}{1 + \xi}) - 2 \ln(1 + \xi) \ln \xi + 3 \ln \xi + \ln^2 \xi(1 + \xi)^2}\} \tag{2}
\]
in the limit \(m_q = 0\), where \(Li_2(x) = -\int_0^x dt \ln(1 - t)/t\) is the Spence function.

From Eq. (2) we can get the branching ratios of \(Z^0 \rightarrow ^3D_{Jq\bar{q}}\). In the numerical calculation, we take [22][13]
\[
m_c = 1.5 GeV, \quad M_{3D} \approx 2 m_c, \quad \alpha_s(2m_c) = 0.253 \tag{3}
\]
and
\[
< O_8^{3D_1}(3S_1) > \approx < O_8^{\psi'}(3S_1) > = 4.6 \times 10^{-3} GeV^3. \tag{4}
\]
Here, in NRQCD \(< O_8^{3D_2}(3S_1) >\) should be of the same order as \(< O_8^{J/\psi}(3S_1) >\) or \(< O_8^{\psi'}(3S_1) >\), and we just take [11] as a tentative value for it, where the value of \(< O_8^{\psi'}(3S_1) >\) was determined by fitting the CDF data for surplus production of \(\psi'\) at the Tevatron [13].

From approximate heavy-quark spin symmetry relations, we have
\[
< O_8^{3D_1}(3S_1) > \approx \frac{3}{5} < O_8^{3D_2}(3S_1) > ,
\]
\[
< O_8^{3D_2}(3S_1) > \approx \frac{7}{5} < O_8^{3D_2}(3S_1) > . \tag{5}
\]
Summing over all the quark flavors \((q = u, d, s, c, b)\), one may obtain the decay widths
\[
\sum_q \Gamma(Z^0 \rightarrow ^3D_1q\bar{q}) \approx 0.7 \times 10^{-4} GeV, \tag{6}
\]
\[
\sum_q \Gamma(Z^0 \rightarrow ^3D_2q\bar{q}) \approx 1.2 \times 10^{-4} GeV, \tag{6}
\]
\[
\sum_q \Gamma(Z^0 \rightarrow ^3D_3q\bar{q}) \approx 1.7 \times 10^{-4} GeV, \tag{6}
\]
and the fraction ratios
\[
\frac{\Gamma(Z^0 \rightarrow ^3D_1q\bar{q})}{\Gamma(Z^0 \rightarrow q\bar{q})} = 2.0 \times 10^{-4}, \tag{7}
\]
\[
\frac{\Gamma(Z^0 \rightarrow ^3D_2q\bar{q})}{\Gamma(Z^0 \rightarrow q\bar{q})} = 3.4 \times 10^{-4}, \tag{7}
\]
\[
\frac{\Gamma(Z^0 \rightarrow ^3D_3q\bar{q})}{\Gamma(Z^0 \rightarrow q\bar{q})} = 4.8 \times 10^{-4}. \tag{7}
\]
The dominant color-singlet processes occur as shown in Fig.2 and Fig.3. Corresponding to the quark fragmentation in Fig.2, the branching ratios of $^3D_J$ production in color-singlet processes are $2.3 \times 10^{-6}$, $3.6 \times 10^{-6}$, and $1.7 \times 10^{-6}$ for $J = 1, 2, 3$, respectively, which are obtained from the universal fragmentation calculations\cite{24}. There should also be color-octet processes through quark fragmentation as in Fig.2. However, indirect evidence indicates that they are not dominant relative to the color-singlet processes\cite{24}.

The processes in Fig.3 are more complicated. For in the most important kinematic region the virtual gluon is nearly on its massshell, $^3D_J$ production in the gluon fragmentation color-singlet process may be separated to be $Z^0 \rightarrow q\bar{q}g^*$ with $g^* \rightarrow^3D_Jgg$. We can estimate the partial width following the way in Ref.\cite{25}, and the differential decay rate of $Z^0 \rightarrow q\bar{q}g^*$ may then be obtained. With the definition

$$\Gamma(g^* \rightarrow AX) = \pi \mu^3 P(g^* \rightarrow AX),$$  \hspace{1cm} (8)

the calculation of decay distribution $P(g^* \rightarrow^3D_Jgg)$ for the gluon of virtuality $\mu$ is very complicated and lengthy (the detailed calculation will be given elsewhere\cite{25}), and in the nonrelativistic limit it is proportional to the second derivative of the radial wave function at the origin. As in the cases of $P$-wave charmonium production, $g^* \rightarrow^3D_Jgg$ processes also have the infrared divergences involved, which are associated with the soft gluon in the final state. Strictly speaking, the divergences can be cancelled in the framework of NRQCD, but here we simply deal with it following the way of\cite{7} by imposing a lower cutoff $\Lambda$ on the energy of the outgoing gluon in the quarkonium rest frame. As discussed in Ref.\cite{7}, the cutoff $\Lambda$ can be set to be $m_c$ to avoid large logarithms in the divergent terms.

The decay widths of $Z^0$ to color-singlet charmonium state $^3D_J$ by gluon fragmentation can be evaluated via

$$\Gamma(Z^0 \rightarrow q\bar{q}g^*; g^* \rightarrow^3D_Jgg) = \int \frac{d\mu^2}{\mu^2_{\text{min}}} \Gamma(Z^0 \rightarrow q\bar{q}g^*)P(g^* \rightarrow^3D_Jgg),$$  \hspace{1cm} (9)

where the cutoff $\Lambda = m_c$ is transformed into a lower limit on $\mu^2_{\text{min}} = 12m_c^2$.

In the numerical calculation, taking\cite{23,27}\hspace{1cm} $\alpha_s(2m_c) = 0.253, \hspace{0.5cm} m_c = 1.5GeV, \hspace{0.5cm} |R_D^{\prime}(0)|^2 = 0.015GeV^7,$

and summing over all the flavors $q \hspace{0.5cm} (q = u, d, s, c, b)$, we obtain

$$\frac{\Gamma(Z^0 \rightarrow q\bar{q}g^*; g^* \rightarrow^3D_JX)}{\Gamma(Z^0 \rightarrow q\bar{q})} = 4.3 \times 10^{-7},$$
\[
\frac{\Gamma(Z^0 \to q\bar{q}^*; g^* \to ^3D_2 X)}{\Gamma(Z^0 \to q\bar{q})} = 2.1 \times 10^{-6}, \\
\frac{\Gamma(Z^0 \to q\bar{q}^*; g^* \to ^3D_3 X)}{\Gamma(Z^0 \to q\bar{q})} = 1.2 \times 10^{-6}.
\] (11)

Among the three triplet states of D-wave charmonium, \(^3D_2\) is the most prominent candidate to discover firstly. Its mass falls in the range of 3.810 ~ 3.840 \(GeV\) in the potential model calculation\(^{28}\)[29], that is above the \(D\bar{D}\) threshold but below the \(D\bar{D}^*\) threshold. However the parity conservation forbids it decaying into \(D\bar{D}\). It, therefore, is a narrow resonance. Its main decay modes are expected to be,

\[^3D_2 \to J/\psi\pi\pi, \quad ^3D_2 \to ^3P_J\gamma(J = 1, 2), \quad ^3D_2 \to 3g.\] (12)

We can estimate the hadronic transition rate of \(^3D_2 \to J/\psi\pi^+\pi^-\) from the Mark III data for \(\psi(3770) \to J/\psi\pi^+\pi^-\)\(^{30}\) and the QCD multipole expansion theory\(^{31}\)[32]. The Mark III data give\(^{30}\) \(\Gamma(\psi(3770) \to J/\psi\pi^+\pi^-) = (37 \pm 17 \pm 8) KeV\) or \((55 \pm 23 \pm 11) KeV\) (see also Ref.\(^{32}\)). Because the \(S - D\) mixing angle for \(\psi(3770)\) and \(\psi(3686)\) is expected to be small (say, \(-10^\circ\), see Ref.\(^{33}\) for the reasoning), the observed \(\psi(3770) \to J/\psi\pi^+\pi^-\) transition should dominantly come from the \(^3D_1 \to J/\psi\pi^+\pi^-\) transition, which is also compatible with the multipole expansion estimate\(^{32}\). Then using the relation\(^{31}\)

\[d\Gamma(^3D_2 \to ^3S_1 2\pi) = d\Gamma(^3D_1 \to ^3S_1 2\pi)\]

and taking the average value of the \(\Gamma(\psi(3770) \to J/\psi\pi^+\pi^-)\) from the Mark III data, we may have

\[\Gamma(^3D_2 \to J/\psi\pi^+\pi^-) = \Gamma(^3D_1 \to J/\psi\pi^+\pi^-) \approx 46 KeV.\] (13)

For the E1 transition \(^3D_2 \to ^3P_J\gamma(J = 1, 2)\), using the potential model with relativistic effects being considered\(^{34}\), we find

\[\Gamma(^3D_2 \to \chi_{c1}\gamma) = 250 KeV, \quad \Gamma(^3D_2 \to \chi_{c2}\gamma) = 60 KeV,\] (14)

where the mass of \(^3D_2\) is set to be 3.840\(GeV\). As for the \(^3D_2 \to 3g\) annihilation decay, an estimate gives\(^{35}\)

\[\Gamma(^3D_2 \to 3g) = 12 KeV\] (15)

From (13), (14), and (15), we find

\[\Gamma_{tot}(^3D_2) \approx \Gamma(^3D_2 \to J/\psi\pi\pi) + \Gamma(^3D_2 \to \chi_{c1}\gamma) + \Gamma(^3D_2 \to \chi_{c2}\gamma) + \Gamma(^3D_2 \to 3g) \approx 390 KeV,\] (16)
and

\[ B(3D_2 \to J/\psi \pi^+ \pi^-) \approx 0.12. \]  \hspace{1cm} (17)

Compared with \( B(\psi' \to J/\psi \pi^+ \pi^-) = 0.324 \pm 0.026 \), the branching ratio of \( 3D_2 \to J/\psi \pi^+ \pi^- \) is only smaller by a factor of 3, and therefore the decay mode of \( 3D_2 \to J/\psi \pi^+ \pi^- \) is observable, if the production rate of \( 3D_2 \) is of the same order as \( \psi' \).

The other two states of the triplet, \( 3D_1 \) and \( 3D_3 \), are above the open channels threshold and are not narrow, and therefore are difficult to detect. It might be interesting to note that the **OPAL** Collaboration at **LEP** has analysed the \( J/\psi \pi^+ \pi^- \) spectrum in \( Z^0 \) decays[36], and there seems to be some events above the background around 3.77\(GEV \), whether these events are associated with the \( D \)-wave \( 1^{--} \) charmonium state \( \psi(3770) \) might remain interesting.

In conclusion, from the calculations and discussions above one can clearly see that the branching ratios of gluon fragmenting to color-octet \( 3D_J \) states are two to three orders larger than the dominant color-singlet processes. This large divergences are much helpful in distinguishing the two production mechanisms in experiment. On the other hand, because the production rates of \( \psi' \) and \( 3D_2(c\bar{c}) \) in color-octet mechanism are of the same amount of magnitude, the \( 2^{--} \) charmonium production in \( Z^0 \) decay may provide a crucial channel to test color-octet production mechanism at **LEP** with present luminosity. As for the \( b\bar{b} \) system,, the production of \( D \)-wave bottomonium states by color-singlet and color-octet mechanisms are similar to that of charmonium, but the disparity of the branching ratios between these two mechanisms is not as evident as that of charmonium production.

Finally, we would like to point out that the above discussion also applies to the \( 3D_J \) production at the Tevatron. As in the case of \( J/\psi \) and \( \psi' \) production at large momentum transverse, the \( 3D_J \) production will also be dominated by the color-octet gluon fragmenta-
tion. This implies that a signal for \( 3D_J \) states, which can be as strong as \( J/\psi \) or \( \psi' \), should be observed at the Tevatron. This will also be a crucial test of the color-octet mechanism. Detailed analysis will be given elsewhere.

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Figure Captions

Fig.1. One of the contributing Feynman diagrams of color-octet mechanism in $Z^0 \rightarrow 3D_Jq\bar{q}$ processes.

Fig.2. One of the Feynman diagrams corresponding to quark fragmentation processes in $Z^0$ decays.

Fig.3. Diagrams for $3D_J$ production from gluon jet in color-singlet mechanisms. (a) virtual gluon production in $Z^0$ decays (b) $3D_J$ production in gluon fragmentation.
Fig. 1
Fig. 3