D-brane realizations of runaway behavior and moduli stabilization

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Abstract: In this paper we find examples of moduli stabilization and runaway behavior which can be treated exactly. This is shown for supersymmetric field theories which can be realized on the world volume of D-branes. From a geometric point of view, these field theories lift moduli spaces of vacua by deforming lines of singularities where supersymmetric fractional branes can be located in the geometry without D-branes.

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1. Introduction

Supersymmetric string compatifications on Calabi-Yau manifolds are usually characterized by having continuous families of solutions that satisfy the string equations of motion, this family is called the moduli space of a compactification. One can move inside the family by exciting closed string fields, and since the total change of energy is zero, these fields are massless. These massless fields are scalars with respect to the four dimensional physics, and are called moduli.

It is believed that non-perturbative effects can cause an effective superpotential on the moduli space which lifts the degeneracy of these vacua [29], and leaves behind a finite number of supersymmetric vacua. Thus there is no true moduli space, but only an asymptotic region where some of the moduli fields can be considered to be very light. This is usually a region with runaway behavior, and moduli roll towards ten dimensional flat space.

Most of the full structure of moduli space is inaccessible to computations because the string dilaton is one of the moduli, and we have very little understanding of the theory at strong coupling to determine the structure of the moduli space and the superpotential on it.

In most circumstances all we understand is an expansion of the theory about a weak coupling point, and we are forced to look for solutions which do not stray too far from the weak coupling regime.
For most results however, one cannot sum the full set of non-perturbative corrections, and the effective superpotential on the moduli space is given roughly by

\[ W_{\text{eff}} = W_1 + W_2 + \text{Other uncontrolled non-perturbative corrections} \quad (1.1) \]

where one believes \( W_1 \) and \( W_2 \) dominate in some region of moduli space. Here, we explicitly write two contributions that are associated with distinct dependence on the closed string dilaton, so that one can balance the two effects and produce a finite vev for the dilaton, hopefully in a perturbative regime for the calculation of some quantities (this has been called a racetrack scheme. It was discussed originally in \cite{21}. For a more recent discussion see \cite{14}).

In this paper we will explore toy models for moduli stabilization in supersymmetric field theories. The main points of the paper are to exploit the recent advances in describing the structure of supersymmetric vacua by matrix models \cite{13}, and to geometrize the field theory behavior into aspects of the geometry of a system of D-branes, so that we can come into contact with the stabilization of moduli for more geometric setups. At the same time, retaining just a field theory calculation and decoupling gravity and the dilaton, because we are taking a non-compact Calabi-Yau geometry.

The main advantage of the setup described in this paper is that it can be argued to be exact, due to their relations to matrix models. In this sense it is now possible to make certain arguments on the whole moduli space of a theory, instead of a more usual procedure of taking limits in various regions where different manipulations give a tractable answer \cite{16}.

This program should be viewed as baby steps towards producing vacua as described in the work of Kachru et al. \cite{18}, where first one describes a supersymmetric compactification, and at the very end one adds anti-D3 branes to break supersymmetry on an F-theory geometry. this final step produces a De-Sitter like vacuum in string theory. It has been argued by Susskind \cite{23} based on ideas by Bousso and Polchinski \cite{6} that there is possibly a very large number of these models. Under these circumstances it is important to understand under what conditions can one trust the calculations that one is performing. See also the recent discussion by Douglas \cite{15}, where an attempt is made to count vacua.

The paper is organized as follows:

In section \ref{section1} we study the topology of moduli spaces and the conditions under which classical moduli spaces can be lifted by quantum corrections. We argue that there need to be singularities in codimension one on the classical moduli space for this to happen. In section \ref{section2} we give a D-brane realization of the Affleck-Dine-Seiberg system by putting a collection of fractional branes on a \( \mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2 \) singularity. We study the geometry of the system in detail and show that confining fractional branes remove the three lines of singularities when one computes the deformed geometry.
We also study a Seiberg-dual version of the system which allows for more easy generalizations. Next, in section 4, we study a variation of a racetrack scheme which allows for gaugino condensation in two gauge groups to stabilize the position of a brane. This example can be obtained by deformations of an $\mathcal{N} = 2$ theory softly broken to $\mathcal{N} = 1$. The theory has various vacua with very different properties. We give a qualitative analysis of the light spectrum of particles in some of the vacua which are interesting. We close the paper with some concluding remarks.

2. Lifting moduli spaces

Given a classical moduli space of vacua, we can ask what properties of the moduli space are necessary to have a superpotential generated by quantum effects on the classical moduli space. The basic property of the effective superpotential is that it is given by a holomorphic (complex analytic) function on the moduli space of vacua. Traditional setups include a conserved $R$-charge which makes it possible to argue for the exact form of the superpotential. A review with many examples and guide to the literature can be found in [28]. The new matrix model ideas [13, 11, 9] can be argued to be exact, irrespective of the presence of these additional symmetries, and therefore one can now study many examples which were not possible in the past.

A very important point to remember is that the moduli spaces given by field theories are usually noncompact, with infinity being given by the region of large vevs for some fields in the SUSY field theory. Under good conditions, the infinity will be weakly coupled and therefore quantum corrections will be small. In effect, this gives us a compactification of the moduli space of vacua, and then the superpotential will be a complex analytic function (it could be multi-valued) on the compactified moduli space. If this function is non-constant, then because it is holomorphic it will necessarily have singularities somewhere in the middle of the moduli space. These are either monodromies or poles and should be associated to some massless particle being present at the singularity. Infinity can also have monodromies associated to it, so if one knows the structure of the singularities it is possible to guess the superpotential function inside the moduli space, by requiring a fixed type of behavior at each singularity. Necessarily all of these singularities are of complex codimension one in the moduli space. It is exactly this style of reasoning that produced the solution of $\mathcal{N} = 2$ field theories by Seiberg and Witten [27], except that the holomorphic object was the infrared gauge coupling on the moduli space, and the holomorphic map was to the upper half plane, and then modded out by the $SL(2, \mathbb{Z})$ S-duality group.

Indeed, these singularities associated to massless particles should be already present in the classical theory, so we find that the geometry of the moduli space requires classical singularities in codimension one. If these are not present, then the moduli space is not lifted, and the only other possibility for quantum effects on the moduli space is that it becomes deformed.
The analysis above can be done branch by branch on the moduli space, so it is possible to have theories where some branches of the moduli space are lifted and some others are not.

Obviously the above arguments can be further clarified with the help of some examples.

In the case of $\mathcal{N} = 4$ SYM, for gauge group $U(N)$ the classical moduli space is given by $\mathbb{C}^3/S_N$, the symmetric product of $N$ copies of $\mathbb{C}^3$, and it is described by a set of three commuting matrices of rank $N$, which can be diagonalized. The singularities of the moduli space occur at places where there is enhanced gauge symmetry. This is a set that requires us to fix three pairs of eigenvalues simultaneously. This phenomenon occurs in complex codimension three, and therefore the moduli space is not lifted by quantum corrections. Indeed, from the high amount of symmetry the moduli space is not deformed at all.

A second example consists of a field theory one of whose branches of moduli space is a (non-compact) Calabi-Yau geometry. In this case, the Calabi-Yau geometry can only have singularities in codimension two or higher, so again, the moduli space can not be lifted by quantum corrections. This example is relevant for a probe brane in the conifold geometry with fractional branes places at the conifold (the Klebanov-Strassler system [20]). In this case the geometry gets deformed.

Finally, we can consider the Affleck-Dine-Seiberg [1] field theory with gauge group $SU(N)$ and $N_F < N$ quarks $Q_i, \tilde{Q}_i$. One can argue that at generic points in the classical moduli space that the theory has an unbroken $SU(N - N_F)$ gauge group, which has a gaugino condensate.

The moduli space is parametrized by the $N_F \times N_F$ meson matrix $M_{ij} = Q_i \tilde{Q}_j$. Generic points in moduli space are characterized by $M$ having maximal rank $N_F$. The singularities in the moduli space are characterized by $M$ having smaller rank. The order parameter that determines this property is whether the single equation on moduli space $\det(M) = 0$ is true or not. Thus, these singularities occur in codimension one, and are associated to the field theory having a point of enhanced symmetry $SU(N - N_F + 1)$. In this example, the classical moduli space has an effective superpotential given by [1]

$$W_{\text{eff}} \sim \left[ \frac{\Lambda^{3N-N_f}}{\det M} \right]^{(N-N_f)-1}$$

(2.1)

Obviously this effective superpotential is singular exactly at the classical singularities in moduli space. Here one does not get a pole at the singularities unless $N_f = N - 1$. This is the same condition required for the superpotential to be generated by instantons. In the other cases there are monodromies at the singularities, which can be associated to motion between the $(N - N_F)$ vacua of the pure $SU(N - N_F)$ theory.
3. Affleck-Dine-Seiberg with D-branes.

Now we want to use the results of the past section to start building D-brane field theories which have runaway behavior or moduli stabilization.

As we saw, we require that the moduli space have singularities in codimension one. The simplest such moduli space would be a one complex dimensional manifold. A D-brane with such a moduli space is usually a fractional brane at a curve of singularities, e.g. a D5 brane wrapped on a holomorphic two cycle which has shrunk to zero size.

The natural place to find such geometries is in orbifold with fixed lines of singularities. For example, let us take take $\mathbb{C}^3/\mathbb{Z}_2$. Here, the fixed point set of the group action is the singular locus, and it gives rise to an $\mathcal{N} = 2$ SYM theory for a fractional brane. However, this theory has too much supersymmetry, and there are no singularities in the moduli space of a single D-brane. To remedy this situation, we can introduce a marked point on the moduli space, by performing an additional orbifold, to obtain the $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold.

This theory is described by the following quiver diagram

\[
\begin{array}{ccccc}
A & & B & \\
& & & & \\
& & & & \\
C & & D & \\
\end{array}
\]

where we have labeled the nodes of the quiver with capital letters $A, B, C, D$. The quiver is not-chiral, and all edges correspond to two chiral multiplets with opposite quantum numbers under the gauge group. We will label these as $\phi_{XY}$ where $X$ and $Y$ indicate the two gauge groups under which it is charged.

The geometry of the orbifold is given by one equation in four variables

\[uvw = t^2\]  

(3.1)

and it contains three lines of singularities meeting at the origin. These lines of singularities correspond to the locus $u, v = 0$, $v, w = 0$ and $w, u = 0$. A single brane in the bulk has brane content $A + B + C + D$, and for this brane the variables $u, v, w, t$ can be identified as follows

\[u = \phi_{AB}\phi_{BA}, v = \phi_{AC}\phi_{CA}, w = \phi_{AD}\phi_{DA}, t = \phi_{AB}\phi_{BD}\phi_{DA}\]  

(3.2)

A straightforward manipulation of the F-terms show that these variables satisfy equation \[3.1\].
Now, the fractional branes at the singularities are constructed from combinations of two different fractional branes like $A + B$. This brane has a one dimensional moduli space characterized by the vev of the gauge invariant field $\phi_{AB}\phi_{BA} = u$ which gives us a brane which spans the $u$-line of singularities. Notice that we have a marked point at the origin where the gauge group is enhanced to $U(1) \times U(1)$.

This configuration does not get its moduli space lifted however, since it can be argued to be a configuration which can be obtained by orbifolding an $\mathcal{N} = 2$ theory without adding extra $\mathcal{N} = 1$ fractional branes.

However, we can consider the following configuration of branes $NA + NF B$. This configuration for $N_A \neq N_B$ confines either the gauge group $U(N_A)$ or the gauge group $U(N_B)$, and is exactly the gauge theory that one would obtain from the Affleck-Dine-Seiberg system if one gauged the vector like $U(N_F)$ flavor symmetry. We will now take $N > N_F$, and we know that this particular configuration has runaway behavior.

Let us consider a generic point in the moduli space. This will be characterized by the meson matrix $M = \phi_{BA}\phi_{AB}$. With the $U(N_F)$ symmetry we can diagonalize it, and we obtain $N_F$ branes at generic points in the $u$ curve of singularities. Also, we get $N - N_F$ branes stuck at the origin.

These branes at the origin in the low energy effective field theory are pure $U(N - N_F)$ and the $SU(N - N_F)$ confines.

Confinement in geometric situations usually leads to a geometric transition: a deformation of the complex structure due to exchanging even cycles where branes can wrap by fluxes [17, 20]. The shape of the deformation in this case can be argued by holomorphy [24, 25, 26].

In the field theory there is a non-anomalous $U(1)^3$ symmetry which is the remnant of the R-symmetry of the unorbifolded gauge theory. $u, v, w$ are charged under these global symmetries and one can not deform the cubic term in the equation without breaking these symmetries. The only allowed deformation can be put in the form

$$uvw = t^2 + c$$

where $c$ is a constant. We will leave a matrix model derivation of this effect for the appendix.

Given that this is the form of the deformation, we can readily understand why the moduli space for fractional branes is lifted. Clearly the above geometry is regular for $c \neq 0$, all of the fractional branes get their moduli space lifted because there are no singularities left over where we can support a supersymmetric fractional brane. Again, the result of the appendix shows that the moduli space of a brane in the bulk is not lifted.

Also, one can derive the Affleck-Dine-Seiberg superpotential directly from matrix models (see for example [10, 2, 22]), and the result is given exactly by

$$W_{eff} = (N - N_F)(S \log S - S) + \tau S + S \log \det(M)$$

(3.4)
where $S$ is the gaugino condensate for the unbroken gauge group. So long as $S \neq 0$, one can see that there is no saddle point for $M$.

Surprisingly, this has implication for the conformal field theory associated to the singularity. One can consider points in the moduli space where the branes are split according to configurations

$$2NA + N(B + C) + N(C + D) + N(B + D)$$

In these configurations at generic points in moduli space the gauge group $U(2N)_{A}$ reduces to pure gauge theory in the deep infrared, and it confines. The geometric transition described before still takes place, and the branch of moduli space is lifted. In essence one can show that any branch which leaves confining branes at the origin is lifted.

This is a slightly surprising result, as we are used to thinking of orbifolds of $\mathcal{N} = 4$ gauge theory as being essentially classical objects, and the classical moduli space as being exact. This is only true when the moduli correspond exclusively to branes in the bulk (the discussion of codimension of singularities in the previous section gives that result).

Now, let us explore a related theory where we take a formal Seiberg duality on the group $U(N)$ on the configuration $NA + N_{F}B$. This gives us the following quiver diagram

![Quiver Diagram](image)

Where the new gauge group is given by $(N_{F} - N)A' + N_{F}B'$. We can now take $N < N_{F}$, or even negative. The advantage is that now we can consider the moduli space as being given by the adjoint field $\phi_{B'B'}$, and we also have the superpotential

$$\text{tr} \left( \phi_{B'B'} \phi_{B'A'} \phi_{A'B'} \right)$$

If we ignore the $A'$ branes, then we have an $\mathcal{N} = 2$ gauge theory on the volume of the branes $B'$, and therefore the moduli space is not lifted. However, once we include the branes $A'$ it is possible to integrate the quarks for the gauge group $U(N_{A'})$ at generic points in the moduli space for the field $\phi_{B'B'}$, which is identified with the meson matrix $M$.

The result of the integration gives an effective potential for the field $\phi$ from a disk diagram computation

$$W_{\text{eff}} = N_{A'}(S \log S - S) + \tau_{A'}S - S \log \det(M)$$

Notice that the only difference between 3.4 and 3.7 is the sign of the term that contains the determinant of the meson field. This can be argued because in the first
case one obtains the term from integrating ghosts (a gauge fixing procedure), while in the other case they are obtained from integrating matter.

This fits very well with continuing all the results for negative values of $N$, and considering this as a calculation that was performed in the brane category. As discussed in [5], Seiberg dualities correspond to basis changes for fractional branes, and the natural basis are determined by terms in the Kahler potential. In the above example $A' \sim -A$ corresponds to the class of the antibrane of $A$ in a different region of the Kahler moduli where the collection of mutually BPS branes is different.

For this case the Affleck-Dine-Seiberg superpotential looks as follows, once we have integrated out $S$

$$W_{\text{eff}} \sim (\det M)^{1/N_{A'}}$$

(3.8)

notice that this superpotential grows at infinity in the moduli space. However, when we analyze it one eigenvalue of $M$ at a time with all others fixed, it grows slower than a polynomial. If we scale the meson variables as $M \rightarrow tM$, $W_{\text{eff}} \sim t^{N_F/N_{A'}}$ which grows slower than $t$ for $N_F < N_{A'}$. In this case the effective potential at infinity (for canonical fields) is given by

$$|\partial W/\partial \phi|^2 \sim t^{2N_F/N_{A'}-2}$$

(3.9)

goes to zero, and one still has runaway behavior, as the total energy will decrease going to infinity.

Notice that in all of these cases there is monodromy of the superpotential at infinity, so $W_{\text{eff}}$ still has to have singularities at finite values, because one needs a place where the cut of $W_{\text{eff}}$ originating at infinity ends. For the most part in this situation the discussion in the previous section about the topological features of moduli space goes unchanged: $\partial W/\partial \phi$ still vanishes at the boundary and one has monodromies for $\partial W/\partial \phi$ inside the moduli space, but it does not need to vanish in the interior.

Let us now compare this result with the literature. This type of example has been discussed in [16], where it was argued that a quantum deformed moduli space was incompatible with the F-terms which are produced from adding sources which only appear linearly. In the case above, this is the field $\phi_{B'B'}$, and the quantum moduli space would correspond to the bilinears $\phi_{B'A'}\phi_{A'B'}$, under special circumstances. Also it was argued in various limits that gaugino condensation would produce an effective superpotential on the moduli space for the $\phi$. The difference now is that we are not forced to take limits. The matrix model technology lets us do a complete analysis on the whole moduli space, independent of the couplings.

4. An example of moduli stabilization

Let us consider the results of the previous section, particularly the last part, where
we had a Seiberg-dual version of the ADS superpotential. It is interesting to write models where one can produce moduli stabilization and not just runaway behavior. The simplest such setup is to setup a racetrack scheme: two non-perturbative effects compete with each other to stabilize the vacua[23].

The simplest such setup in the considerations we have been making is to take a one complex dimensional space with two singularities. At each of the singularities place fixed branes which are not allowed to move, but that repel the brane with the moduli space from the singularity.

This can be done by introducing two confining gauge groups in the following quiver diagram

\[
\begin{array}{c}
\text{B1} \\
\text{B2} \\
\end{array}
\quad Q \quad \tilde{Q} \quad A \quad q \quad \tilde{q}
\]

With a superpotential given by

\[\phi_A Q \tilde{Q} + (\phi_A + m)q \tilde{q}\] (4.1)

At generic points in moduli space for \(\phi_A\), both \(Q, \tilde{Q}\) and \(q, \tilde{q}\) are massive and can be integrated out. The disc diagram computations produce an effective potential for \(\phi\) and the gaugino condensates for branes \(B_1, B_2\) given by

\[W_{\text{eff}} = N_1(S_1 \log S_1/\Lambda^3 - S_1) + \tau_1 S_1 + N_2(S_2 \log S_2/\Lambda^3 - S_2)\] (4.2)

\[+ \tau_2 S_2 - S_1 \log(\det(\phi/\Lambda)) - S_2 \log(\det((\phi + m)/\Lambda))\] (4.3)

Again, we can go to the eigenvalue basis for \(\phi\) by gauge transformations, and we obtain the effective superpotential for each eigenvalue \(\lambda\) as given by

\[-S_1 \log(\lambda/m) - S_2 \log(\lambda/m + 1)\] (4.4)

which produces a supersymmetric (semiclassical) vacuum at

\[\lambda = -\frac{S_1}{S_1 + S_2} m\] (4.5)

Once the branes are stabilized we see that the vacuum has \(U(N_A)\) pure gauge symmetry in the IR, and the \(SU(N_A)\) will confine producing extra non-perturbative effects.

For the most part we can ignore this possibility if we take just \(N_A = 1\). Even if we have \(N_A \neq 1\) one can argue that this can happen at very small scales compared to any other scale by judiciously choosing the relations between the couplings \(\tau_A, \tau_1, \tau_2\) at a very high scale (the string scale for example). Since we can arrange for branes \(A\) to become infrared free for \(Q\) and \(q\) massless, the coupling will run in the IR so that it
is very weakly coupled at the scale \( m \), while at the same time we can arrange for the couplings \( \tau_1, \tau_2 \) to be of order one at the scale \( m \), because the matter content is not sufficient to stop the running coupling constant from becoming large. This can be arranged even if all of the coupling constants \( \tau_i \) are perturbative and approximately equal at the UV scale.

There are various interesting aspects of the above field theory. It can be produced by taking branes on a deformed \( A_3 \) singularity. This begins with an \( \mathcal{N} = 2 \) SYM theory. One can softly break it to \( \mathcal{N} = 1 \) by adding mass terms for some of the adjoint of the fields, and still keep the geometric engineering geometry, reducing the singularity to an \( A_1 \) singularity almost everywhere. These type of geometric constructions have been discussed in [7, 8], where they choose to generically lift the moduli space for fractional branes completely.

The \( A_3 \) singularity has four nodes, and the fourth node is not present in the above discussion, so it can be used to make the deformation geometrical. This produces an example of moduli stabilization in a system of D-branes with softly broken \( \mathcal{N} = 2 \) SUSY.

The above example can be phrased also as moduli stabilization by fluxes. Once the gauginos of the branes \( B_1, B_2 \) condense, they give rise to a geometric transition and are replaced by fluxes. The fractional brane with a moduli space gets localized because there is a flux induced superpotential on the brane.

The interesting questions to analyze in this setup are the conditions under which we can make \( \phi/m \) very large, so that the theory for the group of branes \( A \) is perturbative so that the fields \( Q, \tilde{Q}, q, \tilde{q} \) have a large mass, and to ask how much fine tuning is involved in accomplishing this condition.

The simplest case to set this up is to take the limit \( m \to 0 \). Then one has the two singularities in the complex \( \phi \) plane coalescing.

The effective superpotential for the field \( \phi \) once \( S_1, S_2 \) are integrated out is given by

\[
W_{\text{eff}} = -(N_1 S_1 + N_2 S_2) = \Lambda^3 (A(\phi/\Lambda)^{1/N_1} + B(\phi/\Lambda)^{1/N_2})
\]

and this expression has a saddle point at a non-zero value of \( \phi \) so long as \( N_2 \neq N_1 \). At the minimum of the potential \( S_1 = -S_2 \), and \( |\phi| \sim \Lambda \exp^{(N_2 \tau_1 - N_1 \tau_2)/(N_2 - N_1)} \). Which can be even very large compared to \( \Lambda \), without much fine tuning on the couplings \( \tau_i \).

One can ask what the value of \( \phi \) is for a weakly coupled setup for \( \tau_1 = \tau_2 = \tau \) and large (lets say at the string scale). Then one easily sees that \( |\phi| \sim \Lambda |\exp \tau| \).

For our purposes it is better to keep \( m \) in the discussion since we can also find moduli stabilization in the case \( N_1 = N_2 \). With just one fractional brane, the saddle point equations for \( S_1, S_2 \) give the following result

\[
S_1 = \Lambda^3 e^{-\tau_1/N_1 + 2\pi i k_1/N_1} (\phi/\Lambda)^{1/N_1}
\]

\[
S_2 = \Lambda^3 e^{-\tau_2/N_2 + 2\pi i k_2/N_2} ((\phi + m)/\Lambda)^{1/N_2}
\]
where $k_1, k_2$ are integers which choose the vacua of the two $U(N_i)$ gauge groups. Since generically $S_1, S_2$ have different dependence on $\phi$ it is possible to give vevs to $\phi$ so that $S_1 \sim -S_2$ and if $\phi$ is large with respect to $m$ then it will look like the previous calculation. It is still difficult to solve for $\phi/m = -S_1/(S_1 + S_2)$. In general this will give rise to a polynomial problem for $\phi$ of very high degree ($N_1N_2$ in most cases).

Notice that one can also take phases where $S_1, S_2$ are not oriented opposite to each other, and these give vevs to $\phi$ which are of order $m$. If $m \ll \Lambda$ one can not trust the Kahler potential for the field $\phi$ and one can not estimate the mass for the field $\phi$ reliably.

When $\phi/m$ is large we can ignore the $m$ dependence on the $S_i$, and the theory reduces to $m \sim 0$, so we can trust the effective lagrangian for $\phi$, as it ends up with a large vev compared to $\Lambda$. From here one can estimate the mass of the field $\phi$ from the effective action

$$S_{eff} \sim \int d^4 \theta \frac{1}{g^2} \phi^4 \phi + \int d^2 \theta W_{eff}(\phi)$$

and where $g$ could be the associated $\mathcal{N} = 2$ coupling. The mass for $\phi$ is then of order

$$g^2 \frac{\partial^2 W_{eff}}{\partial \phi^2} \sim g^2 \Lambda^3 / \phi^2$$

which can be very small if $g$ is small and perturbative. Having $\phi$ perturbative is more or less equivalent to requiring that the dilaton is fixed at a point where perturbation theory is valid in other setups.

Notice that the mechanism that chooses the large vev of $\phi$ depends on choosing the right vacuum among $N_1N_2$ of them. Without any fine tuning it was possible to find vacua with large vevs of $\phi$. This is a discrete choice that has the same structure as the one described in [6], where there is one choice among a large set of discrete fluxes that can balance the cosmological constant (this was called the discretuum of vacua: among the very many possible solutions with generic behavior, there is a good chance of finding one good vacuum).

Also for this one vacuum the vev of $W_{eff}$ is suppressed with respect to other vacua, as it is given by $-N_1S_1 - N_2S_2$. If this were a situation where gravity has not been decoupled because of the infinite volume CY, then at these values of $S_i$ the contributions of $S_1$ and $S_2$ would work against each other making the cosmological constant small (but negative) at the same time.

In this example, on top of guaranteeing that we have one light particle that sits at a perturbative value, we also have at the same time tuned the vev of the superpotential to be relatively small compared to other vacua, without requiring any additional tuning. In this sense we have found an example where two effects which are desirable for model building are correlated.
5. Conclusion

In this paper we have presented various constructions where D-brane moduli spaces are lifted by non-perturbative quantum corrections, sometimes producing supersymmetric vacua where the moduli fields are light and have a large vev.

These moduli spaces are usually for a D-brane which is located at a curve of singularities in a Calabi-Yau geometry. Because we studied non-compact Calabi-Yau geometries it was possible to reduce the problem to ordinary supersymmetric field theory, and one could treat the moduli stabilization mechanism exactly by using matrix model techniques. These situations are fairly simple to engineer and are a variation on racetrack schemes to stabilize the moduli fields.

It would be interesting if these models could be extended to a setup where gravity is dynamical in four dimension and where one can also understand supersymmetry breaking in a controllable manner.

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A. Derivation of the deformation of the $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ geometry

The quiver diagram for the $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ geometry is given below.

\begin{center}
\begin{tikzpicture}
\node (A) at (0,0) {A};
\node (B) at (2,0) {B};
\node (C) at (0,-2) {C};
\node (D) at (2,-2) {D};
\draw (A) -- (B);
\draw (A) -- (C);
\draw (A) -- (D);
\draw (B) -- (C);
\draw (B) -- (D);
\draw (C) -- (D);
\end{tikzpicture}
\end{center}

The superpotential of the theory is given by the following expression

\[ W = \text{tr} \left( \phi_{AB}\phi_{BD}\phi_{DA} - \phi_{AB}\phi_{BC}\phi_{CA} \\
+ \phi_{BA}\phi_{AC}\phi_{CB} - \phi_{BA}\phi_{AD}\phi_{DB} \\
+ \phi_{CD}\phi_{DB}\phi_{BC} - \phi_{CD}\phi_{DA}\phi_{AC} \\
+ \phi_{DC}\phi_{CA}\phi_{AD} - \phi_{DC}\phi_{CB}\phi_{BD} \right) \]

(A.1)

We want to choose the theory with brane content $NA + (A + B + C + D)$, this is, one brane in the bulk in the presence of fractional branes at the singularity.
To calculate the moduli space of the brane in the bulk, and therefore the complex structure of the deformation, we need to split the $U(N+1)$ indices of $\phi_{XA}$ and $\phi_{AX}$ into the $U(N)$ singlet and the part in the fundamental. The singlet is going to be part of the moduli space of vacua, so it will not be integrated out, while the matter in the fundamental will be massive and can be integrated out. This manner of calculating has been described in detail in the papers [3, 4].

Here we will just include the partial gaugino condensate $S$ for the $U(N)$ unbroken gauge group. The masses for the fields which are charged under the $U(N)$ gauge field are given by the $3 \times 3$ matrix

$$M = \begin{pmatrix} 0 & -\phi_{BC} & \phi_{BD} \\ \phi_{CB} & 0 & -\phi_{CD} \\ -\phi_{DB} & \phi_{CD} & 0 \end{pmatrix} \quad (A.2)$$

Before we just include the determinant of the mass term, we need to use a gauge fixing procedure to get the $U(N)$ gauge group inside the $U(N+1)$ group, this gives a contribution from ghosts in the matrix model [12]. In situations with adjoints this is the contribution from the Vandermonde determinant. Here, we get instead, by gauge fixing $\phi_{AB}$ the following effective superpotential:

$$W_{eff} = -S \log(\phi_{AB}\phi_{BA}) \quad (A.3)$$

which makes us take the color component of $\phi_{AB}$ and set it to zero. The contribution from the mass term of the field $\phi_{AC}, \phi_{AD}, \phi_{CA}, \phi_{DA}$ then gives us

$$W_{eff2} = S \log(\phi_{CD}\phi_{DC}) \quad (A.4)$$

The total effective superpotential for the moduli fields is the sum of equations [A.3] and [A.4].

Now, let us consider the gauge invariant variables $u = \phi_{BA}\phi_{AB}, v = \phi_{BC}\phi_{CB}, w = \phi_{BD}\phi_{DB}$ and $t = \phi_{BC}\phi_{CA}\phi_{AB}$. If we square $t$ we obtain

$$t^2 = \phi_{BC}\phi_{CA}\phi_{AB}\phi_{BC}\phi_{CA}\phi_{AB} \quad (A.5)$$

Now we can use the deformed (by $S$) F-term equations of motion for the moduli fields to change the subscripts as follows

$$t^2 = \phi_{BC}\phi_{CA}\phi_{AD}\phi_{DC}\phi_{CA}\phi_{AB} \quad (A.6) = \phi_{BC}[\phi_{CB}\phi_{BD} - S/\phi_{DC}]\phi_{DC}\phi_{CA}\phi_{AB} \quad (A.7) = S\phi_{BC}\phi_{CA}\phi_{AB} + \phi_{BC}\phi_{CB}\phi_{BD}\phi_{DB}\phi_{BA}\phi_{AB} \quad (A.8) = St + uvw \quad (A.9)$$

After a linear change of variables in $t$ we can turn this expression into the form

$$uvw = t^2 + c \quad (A.10)$$

which is exactly what was expected due to holomorphy arguments in equation [3.3].
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