A TRANSVERSAL FREDHOLM PROPERTY FOR THE
∂-NEUMANN PROBLEM ON G-BUNDLES

DEDICATED TO M.A. SHUBIN ON HIS 65TH BIRTHDAY

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Abstract. Let \( M \) be a strongly pseudoconvex complex \( G \)-manifold with compact quotient \( M/G \). We provide a simple condition on forms \( \alpha \) sufficient for the regular solvability of the equation \( \Box u = \alpha \) and other problems related to the \( \partial \)-Neumann problem on \( M \).

1. Introduction

Let \( M \) be a manifold which is the total space of a \( G \)-bundle

\[ G \longrightarrow M \longrightarrow X \]

with \( X \) compact. With respect to a \( G \)-invariant measure on \( M \), define the Hilbert space \( L^2(M) \). This decomposes as

\[ L^2(M) \cong L^2(G) \otimes L^2(X), \tag{1.1} \]

and if we assume that the action of \( G \) is from the right, then \( t \in G \) acts in \( L^2(M) \) by \( t \mapsto R_t \otimes 1 \). The von Neumann algebra of operators on \( L^2(G) \) commuting with right translations is denoted by \( \mathcal{L}_G \) and the corresponding algebra of bounded linear operators on \( L^2(M) \) that commute with the action of \( G \) is denoted by \( \mathcal{B}(L^2(M))^G \).

This has a decomposition itself as follows,

\[ \mathcal{B}(L^2(M))^G \cong \mathcal{B}(L^2(G) \otimes L^2(X))^G \cong \mathcal{L}_G \otimes \mathcal{B}(L^2(X)). \]

Definition 1.1. Let \( M \) be a \( G \)-manifold with quotient \( X = M/G \) and let \( \mathcal{H}_1, \mathcal{H}_2 \) be Hilbert spaces of sections of bundles over \( M \). A closed, densely defined, linear operator \( A : \mathcal{H}_1 \rightarrow \mathcal{H}_2 \) which commutes with the action of \( G \) is called transversally Fredholm if the following conditions are satisfied:

1. there exists a finite-rank projection \( P_{L^2(X)} \in \mathcal{B}(L^2(X)) \) such that \( \ker A \subset \text{im} (1_{L^2(G)} \otimes P_{L^2(X)}) \)
2. there exists a finite-rank projection \( P'_{L^2(X)} \in \mathcal{B}(L^2(X)) \) such that \( \text{im} A \subset \text{im} (1_{L^2(G)} \otimes P'_{L^2(X)})^\perp \).

This note will provide a simple example of this idea. Let \( M \) be a strongly pseudoconvex complex manifold which is also the total space of a \( G \)-bundle \( G \longrightarrow M \longrightarrow X \) with \( X \) compact. Furthermore, assume that \( G \) acts on \( M \) by holomorphic transformations. With respect to a \( G \)-invariant measure and Riemannian structure, define the Hilbert spaces of \((p,q)\)-forms \( L^2(M, \Lambda^{p,q}) \).

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On $M$, consider Kohn’s Laplacian, $\Box$ and its spectral decomposition, $\Box = \int_0^\infty \lambda dE_\lambda$ in $L^2(M, \Lambda^p, \gamma)$. If $q > 0$, it was shown in [P1] that if $\delta > 0$, then the Schwartz kernel of the spectral projection $P_\delta = \int_0^\infty dE_\lambda$ belongs to $C^\infty(M \times \overline{M})$. Choosing a piecewise smooth section $X \hookrightarrow M$, we may write points in $M$ as pairs $(t, x) \in G \times X$. The Schwartz kernel $K$ of $P_\delta$ then, almost everywhere, takes the form

$$K(t, x; s, y) = K(ts^{-1}, x; e, y) =: \kappa(ts^{-1}; x, y),$$

where we have used the $G$-invariance of $P_\delta$. It is also true that $\kappa$ has an expansion

$$\kappa(t; x, y) = \sum_{kl} \psi_k(x) h_{kl}(t) \bar{\psi}_l(y)$$

(1.2)

where $(\psi_k)_k$ is an orthonormal basis of $L^2(X)$. The functions $h_{kl}$ are smooth in $G$ with $\sum_{kl} \|h_{kl}\|^2_{L^2(G)} < \infty$, where $L^2_k(G)$ consists of the functions on $G$ that are square-integrable with respect to right-Haar measure (cf. proof of Lemma 6.2 in [P1]).

The main result of the present paper is the fact that when $\kappa$ corresponds to $P_\delta$, the sum in equation (1.2) can be taken to be finite. This means that the spectral projections of $\Box$ are subordinate to simple projections of the form $P = 1_{L^2(G)} \otimes P_{L^2(X)}$ with $P_{L^2(X)}$ the projection onto the space spanned by the $\psi_k$ that appear in the sum. Since there are finitely many, we have that rank $P_{L^2(X)} < \infty$. Thus our main result in this note is

**Theorem 1.2.** Let $M$ be a strongly pseudoconvex complex manifold which is also the total space of a $G$-bundle $G \to M \to X$ with $X$ compact. Furthermore, assume that $G$ acts on $M$ by holomorphic transformations. It follows that for $q > 0$, the Laplacian $\Box$ in $L^2(M, \Lambda^p, \gamma)$ is transversally Fredholm.

We will also show that the $\bar{\partial}$-Neumann problem has regular solutions for $g \in \text{im} P^\perp$.

As well as sharpening the results in [P1], the results of this note will be useful in studying the $\bar{\partial}$-Neumann problem and its consequences for $G$-manifolds with nonunimodular structure group; in [P1], $G$ was always assumed unimodular. These $G$-manifolds, among others, occur naturally as complexifications of group actions, as shown in [HHK].

The present results, in addition to the amenability property introduced in [P2], will lead to a better understanding of two important exemplary nonunimodular $G$-manifolds discussed in [GHS]. One of these has a large space of $L^2$-holomorphic functions while the other has $L^2O = \{0\}$.

**Remark 1.3.** All the results in this note remain valid for weakly pseudoconvex $M$ satisfying a subelliptic estimate, and for the boundary Laplacian, $\Box_b$, [P3].

2. Invariant operators in $L^2(M)$

Here we briefly sketch the construction of the Schwartz kernel (1.2) of $P_\delta$. We will continue to simplify notation by suppressing the operators’ acting in bundles; some additional details are in [P1].

On the group alone, the projection $P_L$ onto a translation-invariant subspace $L \subset L^2(G)$ is a left-convolution operator with distributional kernel $\kappa$,

$$(P_L u)(t) = (\lambda_u)(t) = \int_G ds \kappa(ts^{-1})u(s), \quad (u \in L^2(G)),$$
where $ds$ is the right-invariant Haar measure.

Let us lift this definition to $L^2(M)$ by taking the decomposition $[2]$ a step further. Letting $(\psi_k)_k$ be an orthonormal basis for $L^2(X)$, we may write

$$L^2(M) \cong L^2(G) \otimes L^2(X) \cong \bigoplus_k L^2(G) \otimes \psi_k,$$

and with respect to this decomposition write matrix representations for operators in $L^2(M)$ as

$$B(L^2(M)) \ni P \mapsto [P]_{kl}, \quad P_{kl} \in B(L^2(G)).$$

When $P \in B(L^2(M))^G$ each of the $P_{kl}$ is an operator commuting with the right action and thus is a left convolution operator. Thus $P_{kl} = \lambda_{h_{kl}}$ for distributions $h_{kl}$ on $G$, as in the expansion $[2]$. When $P$ is a self-adjoint projection, we find that the matrix of convolutions $H = [\lambda_{h_{kl}}]_{kl}$ is an idempotent in that $\sum_k H_{jk} H_{kl} = H_{jk}$ and the matrix corresponding to $P^*$, has matrix representation $[\lambda_{h_{ik}}]_{kl}$.

3. Regularity of the $\bar{\partial}$-Neumann problem on $G$-manifolds

We provide a brief list of the properties of the $\bar{\partial}$-Neumann problem relevant to our work here and refer the reader to $[FK, GHS, P1]$ for more detail. With the invariant measure and Riemannian structure on $M$ define the Sobolev spaces $H^s(M, \Lambda^{p,q})$ of $(p,q)$-forms on $M$. Note that the $G$-invariance of the structures and the compactness of $X$ imply that any two such Sobolev spaces are equivalent. A word on notation: we will write $A \lesssim B$ to mean that there exists a $C > 0$ such that $|A(u)| \leq C|B(u)|$ uniformly for $u$ in a set that will be made clear in the context.

**Lemma 3.1.** Suppose that $M$ is strongly pseudoconvex and $U$ is an open subset of $M$ with compact closure. Assume also that $\zeta_1, \zeta_2 \in C^\infty_c(U)$ for which $\zeta_2|_{\text{supp}(\zeta_1)} = 1$. If $q > 0$ and $\alpha|_U \in H^s(U, \Lambda^{p,q})$, then $\nabla(\Box + 1)^{-1} \alpha \in H^{s+1}(\bar{M}, \Lambda^{p,q})$ and

$$\|\zeta_2(\Box + 1)^{-1} \alpha\|_{s+1}^2 \lesssim \|\zeta_1 \alpha\|^2_{s} + \|\alpha\|^2_0.$$

**Proof.** This is Prop. 3.1.1 from $[FK]$ extended to the noncompact case in $[P1]$. \qed

It follows easily (Corollary 4.3, $[P1]$) that the image of the Laplacian’s spectral projection $P_\delta$ is contained in $C^\infty(M, \Lambda^{p,q})$.

In order to derive properties of the Schwartz kernel of $P_\delta$, we will need global Sobolev estimates strengthening the previous result. The following assertion (Theorem 4.5 of $[P1]$) provides global a priori Sobolev estimates on $M$ and is a generalization of Prop. 3.1.11, $[FK]$ to the noncompact case. Note that this crucially uses the uniformity on $M$ guaranteed by the $G$-action and the compactness of $X$.

**Lemma 3.2.** Let $q > 0$. For every integer $s \geq 0$, the following estimate holds uniformly,

$$\|u\|^2_{s+1} \lesssim \|\Box u\|^2_s + \|u\|^2_0, \quad (u \in \text{dom}(\Box) \cap C^\infty(\bar{M}, \Lambda^{p,q})).$$

The previous two lemmata give

**Corollary 3.3.** For $q > 0$, let $\Box = \int_0^\infty \lambda dE_\lambda$ be the spectral decomposition of the Laplacian $\Box$ and for $\delta \geq 0$, define $P_\delta = \int_0^\delta dE_\lambda$. Then $\text{im } P_\delta \subset H^\infty(M)$. 

Proof. The assertion follows from lemmata 3.1, 3.2 and the fact that im $P_b \subset \text{dom} \Box^k$ for all $k = 1, 2, \ldots$. Thus the estimates
\[
\|\Box^{k-s} u\|_{s+1} \lesssim \|\Box^{k-s+1} u\|_{s} + \|\Box^{k-s} u\|_0, \quad (s = 1, 2, \ldots, k)
\]
hold for $u \in \text{im } P_b$. These can be reduced to the result. \hfill \Box

Remark 3.4. By results in [E, P3], these regularity properties essentially hold true for $G$-manifolds $M$ that are weakly pseudoconvex but satisfy a subelliptic estimate. Similar results hold for the boundary Laplacian $\Box_b$ as indicated in [P1].

4. The finiteness result

In this section, we modify an ingenious lemma from [GHS]. In the original setting, this lemma asserts that on a regular covering space $\Gamma \to M \to X$, it is true that any closed, invariant subspace $L \subset L^2(M)$ that belongs to some $H^s(M)$ ($s > 0$) has the following property. There exists an $N < \infty$ and a $\Gamma$-equivariant injection $P_N$ such that
\[
L \xrightarrow{P_N} L^2(\Gamma) \otimes \mathbb{C}^N.
\]
This result has analogues in [A] and Theorem 8.10, [LL], gotten by different methods.

Here, we will use essentially the same proof as in [GHS] to obtain a similar result for $G$-bundles. We will need the following

Definition 4.1. For any positive integer $s$, let $H^{0,s}(G \times X) = L^2(G) \otimes H^s(X)$ be the completion of $C_c^\infty(G \times X)$ in the norm defined by
\[
\|u\|^2_{H^{0,s}(G \times X)} = \int_G dt \|u(t, \cdot)\|^2_{H^s(X)}.
\]
Clearly $\|\cdot\|_{H^{0,s}(G \times X)} \leq \|\cdot\|_{H^s(M)}$ and so $H^s(M) \subset H^{0,s}(G \times X)$.

The next two statements in this section follow [GHS] closely. Lemma 4.2 is taken verbatim and Theorem 4.3 is a small variation on Prop. 1.5 of that article.

Lemma 4.2. Let $X$ be a compact Riemannian manifold, possibly with boundary and let $(\psi_k)_k$ be any complete orthonormal basis of $L^2(X)$. Then, for all $s > 0$ and $\delta > 0$ there exists an integer $N > 0$ such that for all $u \in H^s(X)$ in the $L^2$-orthogonal complement of $(\psi_k)_k$ we have the uniform estimate
\[
\|u\|_{L^2(X)} \leq \delta \|u\|_{H^s(X)}, \quad (u \in H^s(X), \ u \perp \psi_k, \ k = 1, 2, \ldots, N).
\]

Proof. Assuming the contrary, there exist $s > 0$ and $\delta > 0$ so that for each $N > 0$ there is an $u_N \in H^s(X)$ with $\langle u_N, \psi_k \rangle = 0$ for $k = 1, 2, \ldots, N$ and $\|u_N\|_s < 1/\delta \|u_N\|_0$. Without loss of generality we may rescale the $u_N$ to unit length. By Sobolev’s compactness theorem, the sequence $(u_N)_N$ is a compact subset of $L^2(X)$. By the requirement that each $u_N$ be orthogonal to $\psi_k$ for $k = 1, 2, \ldots, N$, the sequence converges weakly to zero. This contradicts the choice of normalization. \hfill \Box

Theorem 4.3. Assume that $G$ is a Lie group and $G \to M \to X$ is a $G$-bundle with compact quotient, $X$. Let $L$ be an $L^2$-closed, $G$-invariant subspace in $H^\infty(M)$, such that for $s \in \mathbb{N}$ sufficiently large, $L \subset H^s(M)$ and
\[
\|u\|_{H^s(M)} \lesssim \|u\|_{L^2(M)}
\]
holds uniformly for $u \in L$. Then $L \subset \text{im } (1_{L^2(G)} \otimes P_{L^2(X)})$ where $P_{L^2(X)}$ is a finite-rank projection in $L^2(X)$. 

Proof. First, assume that \( M \cong G \times X \) is a trivial bundle. For each fixed \( t \in G \), define the slice at \( t \), \( S_t = \{(t, x) \in M \mid x \in X\} \), and note that by the trace theorem, the restrictions of functions in \( L \) to these slices are in \( H^\infty(S_t) \). Note also that the invariance of \( L \) implies that all the restrictions \( L|_{S_t} \) are identical. At the identity \( e \in G \), choose an orthonormal basis \((\psi_j)_j\) for \( L^2(S_e) \cong L^2(X) \). Let \( L \) satisfy the assumptions of the theorem and define a map \( P_N : L \to L^2(G) \otimes \mathbb{C}^N \) by

\[
(P_N u)(t) = (u_1(t), u_2(t), \ldots, u_N(t)),
\]

where

\[
u_j(t) = \langle u|_{S_t}, \psi_j \rangle_{L^2(X)}, \quad j = 1, 2, \ldots, N.
\]

We will show that \( P_N \) is injective for large \( N \). Assume that \( u \in L \) and \( P_N u = 0 \). The smoothness of all the structures implies that \((P_N u)(t) = 0\) identically. Lemma \( 4.2 \) and invariance imply that there is a \( \delta_N > 0 \) such that

\[
\|u\|_{L^2(S_t)}^2 \leq \delta_N^2 \|u|_{S_t}\|_{H^\infty(S_t)}^2, \quad (t \in G).
\]

Integrating over \( t \in G \) we obtain

\[
\|u\|_{L^2(M)}^2 \leq \delta_N^2 \|u\|_{H^\infty(G \times X)}^2 \leq \delta_N^2 \|u\|_{H^\infty(M)}^2.
\]

If this were possible for any \( N \), this would contradict the estimate \( 4.1 \) unless \( u = 0 \), since \( \delta_N \to 0 \) as \( N \to \infty \). To obtain the result for a trivial bundle, let \( N \) be the least integer for which \( P_N \) is injective and choose \( N \) elements \( v_1, v_2, \ldots, v_N \in L \) whose restrictions to \( S_e \) are linearly independent. The result for a general bundle follows by a trivialization argument. \( \square \)

Remark 4.4. We should note here that the assumptions are redundant. For \( L \) to be \( L^2 \)-closed and in \( H^\infty(M) \) implies the validity of an estimate \( 4.1 \) for any \( s \).

Corollary 4.5. Let \( \Box = \int_0^\infty \lambda dE_\lambda \) be the spectral resolution of the Laplacian and for \( \delta > 0 \) let \( P_\delta = \int_0^\delta dE_\lambda \) be a spectral projection. Also choose a piecewise smooth section \( x : X \hookrightarrow M \). It follows that \( P_\delta \) has a representation

\[
(P_\delta u)(t, x) = \sum_{k=1}^N \int_{G \times X} ds dy \psi_k(x)h_{kl}(st^{-1})\psi_l(y)u(s, y),
\]

where \((\psi_k)_k\) are an orthonormal basis of \( L^2(X) \) and \( H = [h_{kl}]_{kl} \) is a self-adjoint, idempotent convolution operator in \( \bigoplus_1^N L^2(G) \) with \( h_{kl} \in C^\infty(G) \). Also,

\[
\sum_{k=1}^N \|h_{kl}\|_{L^2(G)}^2 = \sum_{k=1}^N h_{kk}(e) < \infty.
\]

Proof. By Corollary \( 4.3 \) the theorem applies. Apply the Gram-Schmidt procedure to the \((\psi_k)_k^N \) above, obtaining the \((\psi_k)_k^N \). The decomposition is described in \( [P2] \). \( \square \)

Remark 4.6. In the case that \( G \) is unimodular, \( \sum_{kl} \|h_{kl}\|_{L^2(G)}^2 < \infty \) is the same as saying that \( P_\delta \) is in the \( G \)-trace class, which we established in \( [P1] \) in the setting in which \( M \) is strongly pseudoconvex and in \( [P3] \) where \( M \) satisfies a subelliptic estimate. The new content of Corollary \( 4.5 \) is the finiteness of the sum \( (4.4) \), etc. This transverse dimension gives a meaningful (though much rougher) measure of the spectral subspaces of \( \Box \) (and \( \Box_\delta \)) than the \( G \)-dimension when \( G \) is unimodular, but is also defined when the group is not assumed unimodular as, for example, in \( [HHK] \) and in important examples in \( [GHS] \). We should note that \( [HHK] \) also
deals with the situation in which the $G$-action is only proper, rather than free as we assume here.

5. Applications

We will give a version of the solution of the $\bar{\partial}$-Neumann problem, for our noncompact $M$. The version valid for $M$ compact, e.g. Prop. 3.1.15 of [FK], is unlikely to remain valid in our setting because the Neumann operator on a noncompact space is usually unbounded.

Let $\Box = \int_0^\infty \lambda dE_\lambda$ be the spectral decomposition of the Laplacian on $M$ and for $\delta > 0$ put

$$L_\delta = \text{im} \int_\delta^\infty dE_\lambda \quad \text{and} \quad P_\delta = \int_0^\delta dE_\lambda.$$  

In this section we will show that $\Box u = g$, and the $\bar{\partial}$-Neumann problem have regular solutions for $g \in L_\delta$.

**Lemma 5.1.** If $g \in L_\delta \cap C^\infty(\bar{M})$, then the solution $u$ of $\Box u = g$ is smooth.

**Proof.** Let $g \in L_\delta \cap C^\infty(\bar{M})$ and solve $\Box u = g$ in $L^2(M)$. Note that $\|u\|_{L^2(M)} \leq (1/\delta)\|g\|_{L^2(M)}$. Adding $u$ to both sides of the equation, $(\Box + 1)u = g + u$, we obtain that $(\Box + 1)u = \Box u + u = g + u$. Applying $(\Box + 1)^{-1}$, the real estimate, Lemma 3.1 provides that

$$\|\zeta u\|_{s+1} \lesssim \|\zeta_1 (g + u)\|_{s} + \|g + u\|_{0} \leq \|\zeta_1 g\|_{s} + \|\zeta_1 u\|_{s} + \|g + u\|_{0}.$$  

Nesting the supports of cutoff functions, concatenating and reducing these estimates for $s = 0, 1, \ldots$, we obtain that for each positive integer $s$ we have

$$\|\zeta u\|_{s+1} \lesssim \|\zeta_1 g\|_{s} + \|g + u\|_{0} \leq \|\zeta_1 g\|_{s} + (1 + 1/\delta)\|g\|_{0}.$$  

Thus $u \in C^\infty(M)$ by the Sobolev embedding theorem. \qed

**Corollary 5.2.** In $L_\delta$, the Laplacian satisfies the genuine estimate

$$\|u\|_{s+1} \lesssim \|\Box u\|_{s} + \|u\|_{0}, \quad (u \in L_\delta).$$

**Proof.** Let $(g_k)_k \subset L_\delta \cap H^\infty$ and $g_k \to g$ in $H^s(M)$. The previous lemma implies that there exists a sequence $(u_k)_k \subset C^\infty$ solving $\Box u_k = g_k$. Lemma 5.1 implies that $\|u_k\|_{s+1} \lesssim \|\Box u_k\|_{s} + \|u_k\|_{0}$ uniformly in $k$, so $(u_k)_k$ is Cauchy in the $H^{s+1}$ norm. \qed

**Lemma 5.3.** Suppose that $q > 0$, $\alpha \in L^2(M, N^{p,q})$, $\bar{\partial}\alpha = 0$, and $\alpha \in L_\delta$. Then there is a unique solution $\phi$ of $\bar{\partial}\phi = \alpha$ with $\phi \perp \ker(\bar{\partial})$. If $\alpha \in H^s(M, N^{p,q})$, then $\phi \in H^s(M, N^{p,q-1})$ and $\|\phi\|_{s} \lesssim \|\alpha\|_{s}$ for each $s$.

**Proof.** Taking $\alpha \in L_\delta$, there is a unique solution to $\Box u = \alpha$ orthogonal to the kernel of $\Box$; in fact $u \in L_\delta \subset (\ker \Box)^\perp$. Since $\bar{\partial}\alpha = 0$, applying $\bar{\partial}$ to

$$\Box u = \bar{\partial}^* \bar{\partial} u + \bar{\partial}\bar{\partial}^* u = \alpha$$  

gives that $\bar{\partial}\bar{\partial}^* \bar{\partial} u = 0$. This implies that $\langle \bar{\partial}\bar{\partial}^* \bar{\partial} u, \bar{\partial} u \rangle = 0$ which is equivalent to $\|\bar{\partial}\bar{\partial}^* \bar{\partial} u\|^2 = 0$. Thus $\bar{\partial}\bar{\partial}^* u = \alpha$ and we may take $\phi = \bar{\partial}^* u \in \text{im} \bar{\partial}^*$. But $\text{im} \bar{\partial}^* \subset (\ker \bar{\partial})^\perp$. The regularity claim follows immediately from Corollary 5.2 and the order of $\bar{\partial}^*$. \qed

Putting all these results together, we obtain
Corollary 5.4. Let $M$ be a complex manifold on which a subelliptic estimate holds. Assume also that $M$ is the total space of a bundle $G \to M \to X$ with $G$ a Lie group acting by holomorphic transformations with compact quotient $X = M/G$. With respect to a piecewise smooth section $X \hookrightarrow M$, define the slices $S_t$. Then there exists a finite-dimensional subspace $L|_{S_t} \subset L^2(X)$, such that the equation $\Box u = \alpha$ has solutions $u \in L^2(M)$ with uniform estimates on the space of $\alpha$ satisfying $\alpha|_{S_t} \perp L|_{S_t}$ for all $t \in G$.

Proof. Choose $\delta > 0$. Corollary 3.3 and Theorem 4.3 imply that there exists a finite rank projection $P_{L^2(X)} \in B(L^2(X))$ such that $P_\delta < 1_{L^2(G)} \otimes P_{L^2(X)}$. The orthogonal complement of the latter projection is $1_{L^2(G)} \otimes P_{L^2(X)}^\perp$, which contains $L_\delta$, on which the $\bar{\partial}$-Neumann problem is regular by the results of this section. Putting $L|_{S_t} = \text{im} P_{L^2(X)}$, we have the result.

Remark 5.5. A similar result holds for the $\bar{\partial}$-equation by Lemma 5.3.

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