Abstract
Leader-based protocols for consensus, i.e., atomic broadcast, allow some processes to unilaterally affect the final order of transactions. This has become a problem for blockchain networks and decentralized finance because it facilitates front-running and other attacks. To address this, order fairness for payload messages has been introduced recently as a new safety property for atomic broadcast complementing traditional agreement and liveness. We relate order fairness to the standard validity notions for consensus protocols and highlight some limitations with the existing formalization. Based on this, we introduce a new differential order fairness property that fixes these issues. We also present the quick order-fair atomic broadcast protocol that guarantees payload message delivery in a differentially fair order and is much more efficient than existing order-fair consensus protocols. It works for asynchronous and for eventually synchronous networks with optimal resilience, tolerating corruptions of up to one third of the processes. Previous solutions required there to be less than one fourth of faults. Furthermore, our protocol incurs only quadratic cost, in terms of amortized message complexity per delivered payload.

Keywords. Consensus, atomic broadcast, decentralized finance, front-running attacks, differential order fairness.

1 Introduction
The nascent field of decentralized finance (or simply DeFi) suffers from insider attacks: Malicious miners in permissionless blockchain networks or Byzantine leaders in permissioned atomic broadcast protocols have the power of selecting messages that go into the ledger and determining their final order. Selfish participants may also insert their own, fraudulent transactions and thereby extract value from the network and its innocent users. For instance, a decentralized exchange can be exploited by front-running, where a genuine message \( m \) carrying an exchange transaction is sandwiched between a message \( m_{\text{before}} \) and a message \( m_{\text{after}} \). If \( m \) buys a particular asset, the insider acquires it as well using \( m_{\text{before}} \) and sells it again with \( m_{\text{after}} \), typically at a higher price. Such front-running and other price-manipulation attacks represent a serious threat. They are prohibited in traditional finance systems with centralized oversight but must be prevented technically in DeFi. Daian et al. [7] have coined the term miner extractable value (MEV) for the profit that can be gained from such arbitrage opportunities.

The traditional properties of atomic broadcast, often somewhat imprecisely called consensus as well, guarantee a total order: that all correct parties obtain the same sequence of messages and that any message submitted to the network by a client is delivered in a reasonable lapse of time. However, these properties do not further constrain which order is chosen, and malicious parties in the protocol may therefore manipulate the order or insert their own messages to their benefit. Kelkar et al. [14] have recently introduced the new safety property of order fairness that addresses this in the Byzantine model.

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Kursawe [15] and Zhang et al. [20] have formalized this problem as well and found different ways to tackle it, relying on somewhat stronger assumptions.

Intuitively, order fairness aims at ensuring that messages received by “many” parties are scheduled and delivered earlier than messages received by “few” parties. The Condorcet paradox demonstrates, however, that such preference votes can easily lead to cycles even if the individual votes of majorities are not circular. The solution offered through order fairness [14] may therefore output multiple messages together as a set (or batch), such that there is no order among all messages in the same set. Kelkar et al. [14] name this property block-order fairness but calling such a set a “block” may easily lead to confusion with the low-level blocks in mining-based protocols.

In this paper, we investigate order fairness in networks with \( n \) processes of which \( f \) are faulty, for asynchronous and eventually synchronous atomic broadcast. This covers the vast majority of relevant applications, since timed protocols that assume synchronous clocks and permanently bounded message delays have largely been abandoned in this space.

We first revisit the notion of block-order fairness [14]. In our interpretation, this requires that when \( n \) correct processes broadcast two payload messages \( m \) and \( m' \), and \( \gamma n \) of them broadcast \( m \) before \( m' \) for some \( \gamma > \frac{1}{2} \), then \( m' \) is not delivered by the protocol before \( m \), although both messages may be output together. This guarantee is difficult to achieve in practice because Kelkar et al. [14] show that for the relevant values of \( \gamma \) approaching one half, the resilience of any protocol decreases. Tolerating only a small number of faulty parties seems prohibitive in realistic settings.

More importantly, we show that \( \gamma \) cannot be too close to \( \frac{1}{2} \) because \( \gamma \geq \frac{1}{2} + \frac{f}{n-f} \) is necessary for any protocol. This result follows from establishing a link to the differential validity notion of consensus, formalized by Fitzi and Garay [10]. Notice that block-order fairness is a relative measure. We are convinced that a differential notion is better suited to address the problem. We, therefore, overcome this inherent limitation of relative order fairness by introducing differential order fairness: When the number of correct processes that broadcast a message \( m \) before a message \( m' \) exceeds the number that broadcast \( m' \) before \( m \) by more than \( 2f + \kappa \), for some \( \kappa \geq 0 \), then the protocol must not deliver \( m' \) before \( m \) (but they may be delivered together). This notion takes into account existing results on differential validity for consensus [10]. In particular, when the difference between how many processes prefer one of \( m \) and \( m' \) over the other is smaller than \( 2f + \kappa \), then no protocol exists to deliver them in fair order.

Last but not least, we introduce a new protocol, called quick order-fair atomic broadcast, that implements differential order fairness and is much more efficient than the previously existing algorithms. In particular, it works with optimal resilience \( n > 3f \), requires \( O(n^2) \) messages to deliver one payload on average and needs \( O(n^2 L + n^3 \lambda) \) bits of communication, with payloads of up to \( L \) bits and cryptographic \( \lambda \)-bit signatures. This holds for any order-fairness parameter \( \kappa \). For comparison, the asynchronous Ae-equitas protocol [14] has resilience \( n > 4f \) or worse, depending on its order-fairness parameter, and needs \( O(n^3) \) messages.

To summarize, the contributions of this paper are as follows:

- It illustrates some limitations that are inherent in the notion of block-order fairness (Section 4.1).
- It introduces differential order fairness as a measure for defining fair order in atomic broadcast protocols (Section 4.2).
- It presents the quick order-fair atomic broadcast protocol for differentially order-fair Byzantine atomic broadcast with optimal resilience \( n > 3f \) (Section 5).
- It demonstrates that the quick order-fairness protocol has quadratic amortized message complexity, which is an \( n^2 \)-fold improvement compared to the most efficient previous protocol for the same task (Section 5.3).

The paper starts with a review of previous work (Section 2) and by describing our system model (Section 3).
2 Related work

Over the last decades, extensive research efforts have explored the state-machine replication problem. A large number of papers refer to this problem, but only a few of them consider fairness in the order of delivered payload messages. In this section, we review the related work on fairness.

Kelkar et al. [14] introduce a new property called transaction order-fairness which prevents adversarial manipulation of the ordering of transactions, i.e., payload messages. They investigate assumptions needed for achieving this property in a permissioned setting and formulate a new class of consensus protocols, called Aequitas, that satisfy order fairness. A subsequent paper by Kelkar et al. [12] extends this approach to a permissionless setting. Recently, Kelkar et al. [13] presented another permissioned Byzantine atomic-broadcast protocol called Themis. It introduces a new technique called deferred ordering, which overcomes a liveness problem of the Aequitas protocols.

Kursawe [15] and Zhang et al. [20] have independently postulated alternative definitions of order fairness, called timed order fairness and ordering linearizability, respectively. Both notions are strictly weaker than order fairness of transactions, however [12]. Timed order fairness assumes that all processes have access to synchronized local clocks; it can ensure that if all correct processes saw message \( m \) to be ordered before \( m' \), then \( m \) is scheduled and delivered before \( m' \). Similarly, ordering linearizability says that if the highest timestamp provided by any correct process for a message \( m \) is lower than the lowest timestamp provided by any correct process for a message \( m' \), then \( m \) will appear before \( m' \) in the output sequence. The implementation of ordering linearizability [20] uses a median computation, which can easily be manipulated by faulty processes [12].

The Hashgraph [3] consensus protocol also claims to achieve fairness. It uses gossip internally and all processes build a hash graph reflecting all of the gossip events. However, there is no formal definition of fairness and the presentation fails to recognize the impossibility of fair message-order resulting from the Condorcet paradox. Kelkar et al. [14] also show an attack that allows a malicious process to control the order of the messages delivered by Hashgraph.

A complementary measure to prevent message-reordering attacks relies on threshold cryptography [18, 5, 8]: clients encrypt their input (payload) messages under a key shared by the group of processes running the atomic broadcast protocol. They initially order the encrypted messages and subsequently collaborate for decrypting them. Hence, their contents become known only after the message order has been decided. For instance, the Helix protocol [2] implements this approach and additionally exploits in-protocol randomness for two additional goals: to elect the processes running the protocol from a larger group and to determine which messages among all available ones must be included by a process when proposing a block. This method provides resistance to censorship but still permits some order-manipulation attacks.

3 System model and preliminaries

3.1 System model

Processes. We model our system as a set of \( n \) processes \( P = \{ p_1, \ldots, p_n \} \), also called parties, that communicate with each other. Processes interact with each other by exchanging messages reliably in a network. A protocol for \( P \) consists of a collection of programs with instructions for all processes. Processes are computationally bounded and protocols may use cryptographic primitives, in particular, digital signature schemes.

Failures. In our model, we distinguish two types of processes. Processes that follow the protocol as expected are called correct. Contrary, the processes that deviate from the protocol specification or may crash are called Byzantine.
Communication. We assume that there exists a low-level mechanism for sending messages over reliable and authenticated point-to-point links between processes. In our protocol implementation, we describe this as “sending a message” and “receiving a message”. Additionally, we assume first-in first-out (FIFO) ordering for the links. This ensures that messages broadcast by the same correct process are delivered in the order in which they were sent by a correct recipient.

Timing. This work considers two models, asynchrony and partial synchrony. Together they cover most scenarios used today in the context of secure distributed computing. In an asynchronous network, no physical clock is available to any process and the delivery of messages may be delayed arbitrarily. In such networks, it is only guaranteed that a message sent by a correct process will eventually arrive at its destination. One can define asynchronous time based on logical clocks. A partially synchronous network [9] operates asynchronously until some point in time (not known to the processes), after which it becomes stable. This means that processing times and message delays are bounded afterwards, but the maximal delays are not known to the protocol.

3.2 Byzantine FIFO Consistent Broadcast Channel

We are using a Byzantine FIFO consistent broadcast channel (BCCH) that allows the processes to deliver multiple payloads and ensures a notion of consistency despite Byzantine senders. The interface of such a channel provides two events involving payloads from a domain $\mathcal{M}$:

- A process invokes $bcch$-broadcast$(m)$ to broadcast a message $m \in \mathcal{M}$ to all processes.
- An event $bcch$-deliver$(p_j, l, m)$ delivers a message $m \in \mathcal{M}$ with label $l \in \{0, 1\}^*$ from a process $p_j$.

The label that comes with every delivered message is an arbitrary bit string generated by the channel. Intuitively, the channel ensures that if a message is delivered with some label, then the message itself is the same at all correct processes that deliver this label.

Definition 1 (Byzantine FIFO Consistent Broadcast Channel). A Byzantine FIFO consistent broadcast channel satisfies the following properties:

Validity: If a correct process broadcasts a message $m$, then every correct process eventually delivers $m$.

No duplication: For every process $p_j$ and label $l$, every correct process delivers at most one message with label $l$ and sender $p_j$.

Integrity: If some correct process delivers a message $m$ with sender $p_j$ and process $p_j$ is correct, then $m$ was previously broadcast by $p_j$.

Consistency: If some correct process delivers a message $m$ with label $l$ and sender $p_j$, and another correct process delivers a message $m'$ with label $l$ and sender $p_j$, then $m = m'$.

FIFO delivery: If a correct process broadcasts some message $m$ before it broadcasts a message $m'$, then no correct process delivers $m'$ unless it has already delivered $m$.

This primitive can be implemented by running, for every sender $p_i$, a sequence of standard consistent Byzantine broadcast instances [4, Sec. 3.10] such that exactly one instance in each sequence is active at every moment. Each consistent broadcast instance is identified by a per-sender sequence number. When an instance delivers a message, the protocol advances the sequence number and initializes the next instance. The sequence number serves as the label. Details of this protocol are described by Cachin et al. [4 Sec. 3.12.2]; notice that their protocol also ensures FIFO delivery, although this is not explicitly mentioned there.

In addition to the $bcch$-broadcast and $bcch$-deliver events, in our protocol we use the following methods to access the BCCH primitive: $bcch$-create-proof and $bcch$-verify-proof. Those methods ensure that
missing messages can be transferred in a verifiable way, and they are implemented as in the protocol for
verifiable consistent broadcast by Cachin et al. [5]. The input of bcch-create-proof is a list of messages
and it outputs a string s that contains a proof along with the list of messages to be sent. A process that
receives a message providing s can input this in bcch-verify-proof to verify the proof contained in s such
that it is impossible to forge a proof for a message that was not bcch-delivered.

Another two methods, bcch-get-length and bcch-get-messages, are used to get the number of sent
payload messages and to extract them.

3.3 Validated Byzantine Consensus

Validated Byzantine consensus [5] defines an external validity condition. It requires that the consensus
value is legal according to a global, efficiently computable predicate P, known to all processes. This
allows the protocol to recognize proposed values that are acceptable to an external application. Note
that it is not required that the decision value was proposed by a correct process, but all processes must
be able to verify the validity. A consensus primitive is accessed through the events vbc-propose(v) and
vbc-decide(v), where v ∈ V has a potentially large domain V and may contain a proof, which allows
processes to verify the validity of v.

Definition 2 (Validated Byzantine Consensus). A protocol solves validated Byzantine consensus with
validity predicate P if it satisfies the following conditions:

Termination: Every correct process eventually decides some value.

Integrity: No correct process decides twice.

Agreement: No two correct processes decide differently.

External validity: Every correct process only decides a value v such that P(v) = TRUE. Moreover, if
all processes are correct and propose v, then no correct process decides a value different from v.

We intend this notion to cover asynchronous protocols, which actually only terminate probabilisti-
cally, as well as eventually synchronous protocols. The difference is not essential to our use of them.

Originally, external validity has been defined for asynchronous multi-valued Byzantine consensus,
which requires randomized implementations [5]. But the property applies equally to consensus protocols
with partial synchrony.

Among the asynchronous protocols, recent work by Abraham et al. [1] improves the expected com-
munication (bit) complexity to O(Ln^2) from O(Ln^3) in the earlier work [5], where L is the maximal
length of a proposed value.

In Dumbo-MVBA [16] the communication complexity of this primitive is further reduced to O(Ln)
through erasure coding, where the input of each process is split into coded fragments, distributed to every
process, and recovered later.

Byzantine consensus protocols in the partial-synchrony model can easily be enhanced to provide ex-
ternal validity, when each process verifies P for every proposed value. For instance, the single-decision
versions of PBFT [6] and of HotStuff [19] achieve best-case complexities O(Ln^2) and O(Ln), respec-
tively; these values increase by a factor of n in the worst case.

3.4 Atomic Broadcast

Atomic broadcast ensures that all processes deliver the same messages and that all messages are output
in the same order. This is equivalent to the processes agreeing on one sequence of messages that they
deliver. Atomic broadcast is also called “total-order broadcast” or simply “consensus” in the context of
blockchains because it is equivalent to running a sequence of consensus instances. Processes may broad-
cast a message m by invoking a-broadcast(m), and the protocol outputs messages through a-deliver(m)
events.
Definition 3 (Atomic Broadcast). A protocol for atomic broadcast satisfies the following properties:

**Validity:** If a correct process a-broadcasts a message \( m \), then every correct process eventually a-delivers \( m \).

**No duplication:** No message is a-delivered more than once.

**Agreement:** If a message \( m \) is a-delivered by some correct process, then \( m \) is eventually a-delivered by every correct process.

**Total order:** Let \( m \) and \( m' \) be two messages such that \( p_i \) and \( p_j \) are correct processes that a-deliver \( m \) and \( m' \). If \( p_i \) a-delivers \( m \) before \( m' \), then \( p_j \) also a-delivers \( m \) before \( m' \).

## 4 Revisiting order fairness

In this section, we discuss the challenges of defining order fairness and highlight limitations of order fairness notions from previous works. We then introduce our refined notion of differential order-fair atomic broadcast.

### 4.1 Limitations

Defining a fair order for atomic broadcast in asynchronous networks is not straightforward since the processes might locally receive messages for broadcasting in different orders. We assume here that a correct process receives a payload to be broadcast (e.g., from a client) at the same time when it a-broadcasts it. If a process broadcasts a payload message \( m \) before a payload message \( m' \), according to its local order, we denote this by \( m \prec m' \). Furthermore, we abandon the validity property above in the context of atomic broadcast with order fairness and assume now that every payload message is a-broadcast by all correct processes. This corresponds to the implicit assumption made for deploying order-fair broadcast.

Even if all processes are correct, it can be impossible to define a fair order among all messages. This is shown by a result from social science, known as the Condorcet paradox, which states that there exist situations that lead to non-transitive collective voting preferences even if the individual preferences are transitive. Kelkar et al. [14] apply this to atomic broadcast and show that delivering messages in a fair order is not always possible. Their example considers three correct processes \( p_1, p_2, \) and \( p_3 \) that receive three payload messages \( m_a, m_b, \) and \( m_c \). While \( p_1 \) receives these payload messages in the order \( m_a \prec m_b \prec m_c \), process \( p_2 \) receives them as \( m_b \prec m_c \prec m_a \) and \( p_3 \) in the order \( m_c \prec m_a \prec m_b \). Obviously, a majority of the processes received \( m_a \) before \( m_b, m_b \) before \( m_c \), but also \( m_c \) before \( m_a \), leading to a cyclic order. Consequently, a fair order cannot be specified even with only correct processes.

One way to handle situations with such cycles in the order is presented by Kelkar et al. [14] with block-order fairness: their protocol delivers a “block” of payload messages at once. Typically, a block will contain those payloads that are involved in a cyclic order. Their notion requires that if sufficiently many processes receive a payload \( m \) before another payload \( m' \), then no correct process delivers \( m \) after \( m' \), but they may both appear in the same block. Even though the order among the messages within a block remains unspecified, the notion of block-order fairness respects a fair order up to this limit.

Kelkar et al. [14] specify “sufficiently many” as a \( \gamma \)-fraction of all processes, where \( \gamma \) represents an order-fairness parameter such that \( \frac{1}{2} < \gamma \leq 1 \). More precisely, block-order fairness considers a number of processes \( \eta \) that all receive (and broadcast) two payload messages \( m \) and \( m' \). Block-order fairness for atomic broadcast requires that whenever there are at least \( \gamma \eta \) processes that receive \( m \) before \( m' \), then no correct process delivers \( m \) after \( m' \) (but they may deliver \( m \) and \( m' \) in the same block).

Kelkar et al. [14] explicitly count faulty processes for their definition. Notice that this immediately leads to problems: If \( \gamma \eta < 2f \), for instance, the notion relies on a majority of faulty processes, but no guarantees are possible in this case. Therefore, we only count on events occurring at correct processes here and define a block-order fairness parameter \( \tau \) to denote the fraction of correct processes that receive one message before the other.
Moreover, we assume w.l.o.g. that all correct processes eventually broadcast every payload, even if this is initially input by a single process only. This simplifies the treatment compared to original block-order fairness, which considers only processes that broadcast both payload messages, \( m \) and \( m' \) [13]. Our simplification means that a correct process that has received only one payload will receive the other payload as well later. This process should eventually include also the second payload for establishing a fair order. It corresponds to how atomic broadcast is used in practice; hence, we set \( \eta = n - f \).

In asynchronous networks, furthermore, one has to respect delayed. Their absence reduces the strength of the formal notion of block-order fairness in asynchronous networks even more.

In the following, we discuss the range of achievable values for \( \tau \). Since we focus on models that allow asynchrony, we assume \( n > 3f \) throughout this work. Fundamental results on validity notions for Byzantine consensus in asynchronous networks have been obtained by Fitzi and Garay [10]. Recall that a consensus protocol satisfies termination, integrity, and agreement according to Definition 2. Standard consensus additionally satisfies:

**Validity:** If all correct processes propose \( v \), then all correct processes decide \( v \).

Notice that this leaves the decision value completely open if only one correct process proposes something different. In their notion of strong consensus, however, the values proposed by correct processes must be better respected, under more circumstances:

**Strong validity:** If a correct process decides \( v \), then some correct process has proposed \( v \).

Unfortunately, strong consensus is not suitable for practical purposes because Fitzi and Garay [10, Thm. 8] also show that if the proposal values are taken from a domain \( \mathcal{V} \), then the resilience depends on \( |\mathcal{V}| \). In particular, strong consensus is only possible if \( n > |\mathcal{V}|f \).

Related to this, they also introduce \( \delta \)-differential consensus, which respects how many times a value is proposed by the correct processes. This notion ensures, in short, that the decision value has been proposed by “sufficiently many” correct processes compared to how many processes proposed some different value. More precisely, for an execution of consensus and any value \( v \in \mathcal{V} \), let \( c(v) \) denote the number of correct processes that propose \( v \):

**\( \delta \)-differential validity:** If a correct process decides \( v \), then every other value \( w \) proposed by some correct process satisfies \( c(w) \leq c(v) + \delta \).

To summarize, whereas the standard notion of Byzantine consensus requires that all correct processes start with the same value in order to decide on one of the correct processes’ input, strong consensus achieves this in any case. It requires that the decision value has been proposed by some correct process. However, it does not connect the decision value to how many correct processes have proposed it. Consequently, strong consensus may decide a value proposed by just one correct process. Differential consensus, finally, makes the initial plurality of the decision value explicit. For \( \delta = 0 \), in particular, the decision value must be one of the proposed values that is most common among the correct processes. More importantly, differential validity can be achieved under the usual assumption that \( n > 3f \).

We now give another characterization of \( \delta \)-differential validity. For a particular execution of some (asynchronous) Byzantine consensus protocol, let \( v^* \) be (one of) the value(s) proposed most often by correct processes, i.e.,

\[
v^* = \arg \max_v c(v).
\]

**Lemma 1.** A Byzantine consensus protocol satisfies \( \delta \)-differential validity if and only if in every one of its executions, it never decides a value \( w \) with \( c(w) < c(v^*) - \delta \).

**Proof.** Assume first that the protocol satisfies \( \delta \)-differential validity and a correct process decides any value \( v \) in the domain. Then every other value \( w \) proposed by a correct process satisfies \( c(w) \leq c(v) + \delta \). In particular, this implies \( c(v^*) \leq c(v) + \delta \), which is equivalent to, \( c(v) \geq c(v^*) - \delta \). Hence, the protocol never decides a value \( x \) with \( c(x) < c(v^*) - \delta \).
To show the reverse direction, suppose \( x \) is such that \( c(x) < c(v^*) - \delta \) and a correct process decides \( x \). This does not satisfy \( \delta \)-differential validity because also \( v^* \) has been proposed by a correct process but \( c(v^*) > c(x) + \delta \).

For consensus with a binary domain \( V = \{0, 1\} \), this means that a consensus protocol satisfies \( \delta \)-differential validity if and only if in every one of its executions with, say, \( c(0) > c(1) + \delta \), every correct process decides 0.

No asynchronous consensus algorithm for agreeing on the value proposed by a simple majority of correct processes exists, however. Fitzi and Garay [10] Thm. 11 prove that \( \delta \)-differential consensus in asynchronous networks is not possible for \( \delta < 2f \):

**Theorem 2 ([10]).** In an asynchronous network, \( \delta \)-differential consensus is achievable only if \( \delta \geq 2f \).

The above discussion already hints at issues with achieving fair order in asynchronous systems. Recall that Kelkar et al. [14] present atomic broadcast protocols with block-order fairness for the asynchronous setting with order-fairness parameter \( \gamma \) (whose definition includes faulty processes). The corruption bound is stated as

\[
 n > \frac{4f}{2\gamma - 1}.
\]

For \( \gamma = 1 \), which ensures fairness only in the most clear cases, there are \( n > 4f \) processes required. For values of \( \gamma \) close to \( \frac{1}{2} \), the condition becomes prohibitive for practical solutions.

In fact, even when using our interpretation, \( \gamma \) cannot be too close to \( \frac{1}{2} \), as the following result shows. It rules out the existence of \( \gamma \)-block-order-fair atomic broadcast in asynchronous or eventually synchronous networks for \( \gamma < \frac{1}{2} + \frac{f}{n-\gamma} \).

**Theorem 3.** In an asynchronous network with \( n \) processes and \( f \) faults, implementing atomic broadcast with \( \gamma \)-fair block order is not possible unless \( \gamma \geq \frac{1}{2} + \frac{f}{n-\gamma} \).

**Proof.** Towards a contradiction, suppose there is an atomic broadcast protocol ensuring \( \gamma \)-fair block order with \( \frac{1}{2} < \gamma < \frac{1}{2} + \frac{f}{n-\gamma} \). We will transform this into a differential consensus protocol that violates Theorem 2.

The consensus protocol works like this. All processes initialize the atomic broadcast protocol. Upon `propose(v)` with some value \( v \), a process simply `a-broadcasts` \( v \). When the first value \( v' \) is `a-delivered` by atomic broadcast to a process, the process executes `decide(v')` and terminates.

Consider any execution of this protocol such that all correct processes propose one of two values, \( m \) or \( m' \). Suppose w.l.o.g. that \( c(m) = \gamma(n - f) \) and \( c(m') = (1 - \gamma)(n - f) \), i.e., \( m \) is proposed \( c(m) \) times by correct processes and more often than \( m' \), since \( \gamma > \frac{1}{2} \). It follows that \( \gamma(n - f) \) correct processes `a-broadcast` \( m \) before \( m' \) and \( (1 - \gamma)(n - f) \) correct processes `a-broadcast` \( m' \) before \( m \).

According to the properties of atomic broadcast all correct processes `a-deliver` the same value first in every execution. Moreover, the atomic broadcast protocol `a-delivers` \( m \) before \( m' \) by the \( \gamma \)-fair block order property. This implies that the consensus protocol decides \( m \) in every execution and never \( m' \). Since no further restrictions are placed on \( m \) and on \( m' \), this consensus protocol actually ensures \( \delta \)-differential validity for some \( \delta < c(m) - c(m') \) by Lemma 1.

However, the \( c(m) \) and \( c(m') \) satisfy, respectively,

\[
 c(m) = \gamma(n - f) < \left( \frac{1}{2} + \frac{f}{n-\gamma} \right)(n - f) = \frac{n+f}{2} \\
 c(m') = (1 - \gamma)(n - f) > \left( 1 - \frac{1}{2} - \frac{f}{n-\gamma} \right)(n - f) = \frac{n-3f}{2}
\]

and, therefore, \( \delta < c(m) - c(m') < \frac{n+f}{2} - \frac{n-3f}{2} = 2f \). But \( \delta \)-differential asynchronous consensus is only possible when \( \delta \geq 2f \), a contradiction.
4.2 Differential Order-Fairness

The limitations discussed above have an influence on order fairness. The condition on \( \delta \) to achieve \( \delta \)-
differential consensus directly impacts any measure of fairness. It becomes clear that a relative notion for block-order fairness, defined through a fraction like \( \tau \), may not be expressive enough.

We now start to define our notion of order-fair atomic broadcast; it has almost the same interface as regular atomic broadcast. The primitive is accessed with of-broadcast(m) for broadcasting a payload message \( m \) and it outputs payload messages through of-deliver(M) events, where \( M \) is a set of payloads delivered at the same time; \( M \) corresponds the block of block-order fairness. We want to count the number of correct processes that of-broadcast a message \( m \) before another message \( m' \) and introduce a function

\[
b : M \times M \rightarrow \mathbb{N}
\]

for all \( m \) and \( m' \) that were ever of-broadcast by correct processes. The value \( b(m, m') \) denotes the number of correct processes that of-broadcast \( m \) before \( m' \) in an execution. As above we assume w.l.o.g. that a correct process will of-broadcast \( m \) and \( m' \) eventually and that, therefore, \( b(m, m') + b(m', m) = n - f \).

Can we achieve that if \( b(m, m') > b(m', m) \), i.e., when there are more correct processes that of-broadcast message \( m \) before \( m' \) than correct processes that of-broadcast \( m' \) before \( m \), then no correct process will of-deliver \( m' \) before \( m \)? Using a reduction from \( \delta \)-differential consensus, as in the previous result, we can show that this condition is too weak.

**Theorem 4.** Consider an atomic broadcast protocol that satisfies the following notion of order fairness for some \( \mu \geq 0 \):

**Weak differential order fairness:** For any \( m \) and \( m' \), if \( b(m, m') > b(m', m) + \mu \), then no correct process a-delivers \( m' \) before \( m \).

Then it must hold \( \mu \geq 2f \).

**Proof.** Towards a contradiction, suppose there is an atomic broadcast protocol, which ensures that for all payload messages \( m \) and \( m' \) with \( b(m, m') > b(m', m) + \mu \) and \( \mu \geq 0 \), no correct process a-delivers \( m' \) before \( m \) and that \( \mu < 2f \). We will transform this into a differential consensus protocol that violates Theorem 2.

The consensus protocol works like this. All processes initialize the order-fair atomic broadcast protocol. Upon propose(v) with some value v, a process simply of-broadcasts v. When the first value \( v' \) is of-delivered to a process, the process executes decide(\( v' \)) and terminates.

Consider any execution of this protocol such that all correct processes propose one of two values, \( m \) or \( m' \). Suppose w.l.o.g. that \( m \) is proposed \( c(m) \) times by correct processes and more often than \( m' \), which is proposed \( c(m') \) times, with \( c(m) + c(m') = n - f \) and that \( c(m) > c(m') + \mu \). It follows that \( b(m, m') = c(m) \) correct processes of-broadcast \( m \) before \( m' \) and \( b(m', m) = c(m') \) correct processes of-broadcast \( m' \) before \( m \), hence, \( b(m, m') > b(m', m) + \mu \).

According to the properties of atomic broadcast, all processes of-deliver the same value first in every execution. Moreover, the protocol of-delivers \( m \) before \( m' \) because \( b(m, m') > b(m', m) + \mu \). This implies that the consensus protocol decides \( m \) in every such execution. Since no further restrictions are placed on \( m \) and \( m' \) and since \( c(m) - c(m') > \mu \), this consensus protocol actually implements \( \mu \)-differential consensus by Lemma 1. However, achieving \( \mu \)-differential asynchronous consensus requires that \( \mu \geq 2f \) according to Theorem 3. But \( \mu < 2f \) by the above assumption. This is a contradiction.

On the basis of this result, we now formulate our notion of \( \kappa \)-differentially order-fair atomic broadcast, using a fairness parameter \( \kappa \geq 0 \) to express the strength of the fairness. Smaller values of \( \kappa \) ensure stronger fairness in the sense of how large the majority of processes that of-broadcast some \( m \) before \( m' \) must be to ensure that \( m \) will be of-delivered before \( m' \) and in a fair order.

Recall that throughout this work, we assume that if one correct process of-broadcasts some payload \( m \), then every correct process eventually also of-broadcasts \( m \). For reasons that are discussed later
(in the remarks after the protocol description in Section 5.2), we use a weaker formal notion of validity, which considers executions with only correct processes.

**Definition 4 (κ-Differentially Order-Fair Atomic Broadcast).** A protocol for κ-differentially order-fair atomic broadcast satisfies the properties no duplication, agreement and total order of atomic broadcast and additionally:

**Weak validity:** If all processes are correct and of-broadcast a finite number of messages, then every correct process eventually of-delivers all of these of-broadcast messages.

**κ-differential order fairness:** If \( b(m, m') > b(m', m) + 2f + \kappa \), then no correct process of-delivers \( m' \) before \( m \).

Compared to the above notion of weak differential order fairness, we have \( \kappa = \mu - 2f \). We show in the next section how to implement κ-differentially order-fair atomic broadcast.

## 5 Quick order-fair atomic broadcast protocol

This section presents first an overview of our quick order-fair atomic broadcast algorithm in Section 5.1. A detailed description of the implementation follows in Section 5.2, along with the pseudocode in Algorithm 1-2. Finally, the complexity of the algorithm is discussed in Section 5.3.

### 5.1 Overview

The protocol concurrently runs a Byzantine FIFO consistent broadcast channel (BCCH) and proceeds in rounds of consensus. BCCH allows processes to deliver multiple messages consistently. An incoming of-broadcast event with a payload message \( m \) triggers BCCH and bcch-broadcasts \( m \) to the network. Additionally, every process keeps a local vector clock that counts the payloads that have been bcch-delivered from each sending process. Every process also maintains an array of lists \( msgs \) such that \( msgs[i] \) records all bcch-delivered payloads from \( p_i \).

When a process bcch-delivers the payload message \( m \), it increments the corresponding vector-clock entry and appends \( m \) to the appropriate list in \( msgs \). As soon as sufficiently many new payloads are found in \( msgs \), a new round starts. Each process signs its vector clock and sends it to all others. The received vector clocks are collected in a matrix, and once \( n - f \) valid vector clocks are recorded, a new validated Byzantine consensus (VBC) instance is triggered. The process proposes the matrix and the signatures for consensus, and VBC decides on a common matrix with valid signatures. This matrix defines a cut, which is a vector of indices, with one index per process, such that the index for \( p_j \) determines an entry in \( msgs[j] \) up to which payload messages are considered for creating the fair order in the round. It may be that the index points to messages that a process \( p_i \) does not store in \( msgs[j] \) because they have not been bcch-delivered yet. When the process detects such a missing payload, it asks all other processes to send the missing payload directly and in a verifiable way, such that every process will store all payloads up to the cut in \( msgs \).

Once all processes received the payloads up to the cut, the algorithm starts to build a graph that represents the dependencies among messages that must be respected for a fair order. This graph resembles the one used in Aequitas [14], but its semantics and implementation differ. The vertices in the graph here are all new payload messages defined by the cut and an edge \((m, m')\) indicates that \( m \) should at most be of-delivered before \( m' \).

The graph results from two steps. In the first step, the process creates a vertex for every payload message that appears in a distinct lists in \( msgs \) and it is not yet of-delivered. In the second step, the algorithm builds a matrix \( M \) such that \( M[m][m'] \) counts how many times \( m \) appears before \( m' \) in \( msgs \) (up to the cut). \( M[m][m'] \) can be interpreted as votes, counting how many processes want to order \( m \) before \( m' \). Notice that entries of \( M \) exist only for \( m \) and \( m' \) where at least one of \( M[m][m'] \) and \( M[m'][m] \) is non-zero.
If the difference between entries $M[m][m']$ and $M[m'][m]$ is large enough, then the protocol adds a directed edge $(m, m')$ to the graph. The edge indicates that $m'$ must not be of-delivered before $m$. More precisely, assuming that messages $m$ and $m'$ have been observed by at least $n - f$ processes, such an edge is added for all $m$ and $m'$ with $M[m][m'] > M[m'][m] - f + \kappa$. The condition is explained through the following result.

**Lemma 5.** If $b(m, m') > b(m', m) + 2f + \kappa$, then $M[m][m'] > M[m'][m] - f + \kappa$.

**Proof.** At least $M[m][m'] - f$ correct processes have of-broadcast $m$ before $m'$ because $M[m][m']$ may include reports about $m$ and $m'$ in $msgs$ from up to $f$ incorrect processes. In other words,

$$b(m, m') \geq M[m][m'] - f \iff M[m'][m] \leq b(m, m') + f$$

At most $M[m][m'] + 2f$ correct processes have of-broadcast $m$ before $m'$ because $M[m][m']$ may include reports about $m$ or $m'$ in $msgs$ from up to $f$ incorrect processes, and there may be up to $2f$ correct processes whose arrays were not considered in this number. That is,

$$b(m, m') \leq M[m][m'] + 2f \iff M[m][m'] \geq b(m, m') - 2f$$

Suppose $b(m, m') > b(m', m) + 2f + \kappa$. The above implies

$$M[m'][m] \geq b(m, m') - 2f$$

$$> b(m', m) + 2f + \kappa - 2f$$

$$= b(m', m) + \kappa$$

$$\geq M[m'][m] - f + \kappa.$$ 

Thus, whenever $M[m][m'] > M[m'][m] - f + \kappa$, we need to prevent that the protocol of-delivers $m'$ before $m$. We do this by adding an edge from $m$ to $m'$ to the graph; as shown later, this ensures that $m'$ is not of-delivered before $m$. \hfill \square

In the discussion so far, we have assumed that the two messages $m$ and $m'$ were received by at least $n - f$ processes. Observe that every process can only contribute with 1 to either $M[m][m']$ or to $M[m'][m]$, but not to both. However, it may occur that only a few processes receive $m$ and $m'$ before the cut, which implies that $M[m][m']$ may be very small, for example. But then that count might actually grow later and take on values up to $n - f - M[m'][m]$. For this reason, we extend the condition derived from Lemma 5 in the algorithm as follows: if $n - f - M[m'][m] > M[m'][m] - f + \kappa$ (which implies that $M[m'][m]$ is small, i.e., $M[m'][m] < \frac{n - 2f}{2}$), we also add an edge between $m$ and $m'$. In summary, then, the algorithm adds an edge from $m$ to $m'$ whenever

$$\max \{ M[m][m'], n - f - M[m'][m] \} > M[m'][m] - f + \kappa.$$ 

Creating the graph in this manner leads to a directed graph that represents constraints to be respected by a fair order. Notice that two messages may be connected by edges in both directions when the difference is small and $\kappa < f$, i.e., there may be a cycle $(m, m')$ and $(m', m)$. This means that the difference between the number of processes voting for one or the other order is too small to decide on a fair order. Longer cycles may also exist. All payload messages with circular dependencies among them will be of-delivered together as a set. For deriving this information, the algorithm repeatedly detects all strongly connected components in the graph and collapses them to a vertex. In other words, any two vertices $m$ and $m'$ are merged when there exists a path from $m$ to $m'$ and a path from $m'$ to $m$. This technique also handles cases like those derived from the Condorcet paradox.

Finally, with the help of the collapsed graph, all payload messages defined by the cut are of-delivered in a fair order: First, all vertices without any incoming edges are selected. Secondly, these vertices are sorted in a deterministic way and the corresponding payloads are of-delivered one after the other. Then the processed vertices are removed from the graph and another iteration through the graph starts. As
Figure 1. The execution of Example 1 in which three correct processes $p_1, p_2, p_3$ of-broadcast messages that form a cycle, which makes it impossible to sort them in a fair order. After $m_a$, and after the protocol has computed cut $c$, an unbounded number of additional payloads might follow (see text).

soon as there are no vertices left, i.e., all payload messages are of-delivered, the protocol proceeds to the next round.

Note that cycles may also extend beyond the cut, as shown by Kelkar et al. [13]. Therefore, the algorithm holds back payload messages and does not of-deliver them while they may still become part of a longer cycle. This is ensured by counting how many times a message appears in $msgs$ up to the cut. In particular, let $C[m]$ count this number for a message $m$. We require that any message is only of-delivered when $C[m] \geq \frac{n+f-\kappa}{2}$, i.e., after $m$ appears in $msgs$ often enough such that it cannot become part of a cycle later or already be in a cycle that will grow later, e.g., through payloads that arrive after the cut.

Example 1. Let us consider a system of $n = 4$ processes, of which three ($p_1, p_2, p_3$) are correct and one ($p_4$) is faulty ($f = 1$). We fix the order-fairness parameter $\kappa = 0$, the notion is trivially satisfied for higher values of $\kappa$. Every correct process of-broadcasts three messages $m_a, m_b,$ and $m_c$, in an order that forms a Condorcet cycle. The Byzantine process $p_4$ does not of-broadcast. Suppose all messages have been bcch-delivered in round $r$ to all correct processes, as shown in Figure 1. Then the protocol obtains the cut $c = [3, 2, 1, 0]$.

From Algorithm 1-2 (L46), the matrix $M$ and the corresponding graph (L48) are

\[
M = \begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 1 \\
2 & 1 & 0
\end{bmatrix}
\]

Notice that arbitrarily many payload messages that are of-broadcast immediately after $m_a$ by $p_1$–$p_3$ might follow and arrive only in a future round, after cut $c$. The protocol cannot know this yet and must therefore postpone of-delivery of $m_a$. As captured by the condition that $C[m_b] = 1 < \frac{n+f-\kappa}{2}$, no payload message is of-delivered in this round. The protocol continues with another round $r'$ obtaining a cut $c'$, cf. Figure 1. Then the matrix $M$ and the graph become

\[
M = \begin{bmatrix}
0 & 2 & 1 \\
1 & 0 & 2 \\
2 & 1 & 0
\end{bmatrix}
\]
At this point, the protocol of-delivers \( \{m_a, m_b, m_c\} \) together, from a collapsed vertex, because now \( C[m_i] = 3 \geq \frac{n-f}{2} \) for \( i \in \{a, b, c\} \).

5.2 Implementation

Algorithm 1–2 shows the quick order-fair atomic broadcast protocol for a process \( p_i \). The protocol proceeds in rounds, maintains a round counter \( r \) (L1), and uses a boolean variable \( \text{inround} \), which indicates whether the consensus phase of a round is executing (L2).

Every process maintains two hash maps: \( \text{msgs} \) (L3) and \( \text{vc} \) (L4), in which process identifiers serve as keys. Hash map \( \text{msgs} \) contains ordered lists of \( \text{bcch-delivered} \) payload from each process in the system. Variable \( \text{vc} \) is a vector clock counting how many payload messages were \( \text{bcch-delivered} \) from each process.

**Rounds.** In each round, a matrix \( L \) (L5) and a list \( \Sigma \) (L6) are constructed as inputs for consensus. The matrix \( L \) will consist of vector clocks from the processes and \( \Sigma \) will contain the signatures of the processes. Additionally, every process maintains a list of integers called \( \text{cut} \) (L7) that are calculated in every round. This cut represents an index for every list in \( \text{msgs} \) to determine the payload to be used for creating the fair order. Initially, all values are zero. Finally, all of-delivered payload messages are included in a set \( \text{delivered} \) (L1), to prevent a repeated delivery in future rounds.

The protocol starts when a client submits a payload message \( m \) using an of-broadcast\( (m) \) event. BCCH then broadcasts \( m \) to all processes in the network (L11). When \( m \) with label \( l \) from process \( p_j \) is \( \text{bcch-delivered} \) (L12), the vector clock \( \text{vc} \) for process \( p_j \) is incremented. The attached label \( l \) is not used by the algorithm and only serves to define that all correct processes \( \text{bcch-deliver} \) the same payload following Definition 1. Additionally, payload \( m \) is appended to the list \( \text{msgs}[j] \) using an operation \( \text{append}(m) \) (L14).

When the length of \( p_j \)'s list in \( \text{msgs} \) exceeds the \( \text{cut} \) value for \( p_j \), new payloads may have arrived that should be ordered (L15). This tells the protocol to initiate a new round. A new round could also be triggered later, as described in the remarks at the end of this section.

The first step of round \( r \) is to set the flag \( \text{inround} \). Secondly, the protocol digitally signs the vector clock \( \text{vc} \) and obtains a signature \( \sigma \). The values \( r \), \( \sigma \), and \( \text{vc} \) are then sent in a \( \text{STATUS} \) message to all processes (L16–L18). When process \( p_i \) receives a \( \text{STATUS} \) message from \( p_j \), it validates the contained signature \( \sigma' \) using \( \text{verify}(j, \text{vc}', \sigma') \) (L19). An additional security check is made by comparing the locally stored round number \( r \) with the round number \( r' \) from the message. If both conditions hold, the vector clock \( \text{vc}' \) is stored as row \( j \) in matrix \( L \) (L20) and \( \sigma' \) is stored in list \( \Sigma \) at index \( j \) (L21).

**Defining a cut.** As soon as \( p_j \) has received \( n - f \) valid \( \text{STATUS} \)-messages (L22), it invokes consensus (VBC, L23) for the round through \( \text{vbc-propose} \) with proposal \( (L, \Sigma) \). The predicate of VBC checks that a proposal consists of a matrix \( L \) and a vector \( \Sigma \) such that for at least \( n - f \) values \( j \), the entry \( \Sigma[j] \) is a valid signature on row \( j \) of \( L \). When the VBC protocol subsequently decides, it outputs a common matrix \( L' \) of vector clocks and a list \( \Sigma' \) of signatures (L25). The process then uses \( L' \) to calculate the cut, where \( \text{cut}[j] \) is the largest value \( s \) such that at least \( f + 1 \) elements in column \( j \) in \( L' \) are bigger or equal than \( s \) (L28). In other words, \( \text{cut}[j] \) represents how many payload messages from \( p_j \) were \( \text{bcch-delivered} \) by enough processes. This value is used as index into \( \text{msgs}[j] \) to determine the payloads that will be considered for creating the order in this round.

The algorithm then makes sure that all processes will hold at least all those payloads in \( \text{msgs} \) that are defined by \( \text{cut} \). Each process detects missing payload messages from sender \( p_j \) from any difference between \( \text{vc}[j] \) and \( \text{cut}[j] \) (L30). If there are any, the process broadcasts a \( \text{MISSING} \)-message to all others. When another process receives such a request from \( p_j \) and already has the requested payloads in \( \text{msgs} \), it extracts them into a variable \( \text{resend} \) (L34). More precisely, it extracts a proof from the BCCH primitive with which any other process can verify that the payload from this particular sender is genuine. This
is done by invoking \texttt{bcch-create-proof(resend)} (L35); the messages and the proof are then sent in a \texttt{RESEND}-message to the requesting process \(p_j\) (L36).

When process \(p_j\) receives a \texttt{RESEND}-message with a missing payload from \(p_k\), it first verifies the provided proof \(s'\) from the message by invoking \texttt{bcch-verify-proof(s')} function (L38). If the proof is valid, \(p_j\) extracts (L40) the payload messages through \texttt{bcch-get-messages(s')}, appends them to \(msgs[k]\), and increments \(vc[k]\) accordingly. The process repeats this until \(msgs\) contains all payloads included in the cut.

**Ordering messages.** At this point, every process stores all payloads \(msgs\) that have been \texttt{bcch-delivered} up to the cut. The remaining operations of the round are deterministic and executed by all processes independently. The next step is to construct the directed dependency graph \(G\) that expresses the constraints on the fair order of the payload messages. Vertices \(V\) in \(G\) represent payload messages that may be \texttt{of-delivered} and edges \((E)\) in \(G\) express constraints on the order among these payloads. First, all messages within the cut that are not yet delivered are added as vertices to the set \(V\) (L42).

Then, for each pair of messages \(m\) and \(m'\) in \(V\), the algorithm constructs \(M\) (L46) such that \(M[m][m']\) counts how many times a payload \(m\) appears before payload \(m'\) in the cut. In the same loop, the algorithm counts how many times message \(m\) appears within the cut and stores this result in array \(C\) (L47). Finally, all entries \(M[m][m']\) and \(M[m'][m]\) are compared and if condition \(max\{M[m][m'], n−f − M[m'][m]\} > M[m'][m] − f + \kappa\) holds, then a directed edge from \(m\) to \(m'\) is added (L48). This edge indicates that \(m\) must not be ordered after \(m'\), i.e., that \(m\) is \texttt{of-delivered} before \(m'\) or together with \(m'\).

Any payloads that cannot be ordered with respect to each other now correspond to strongly connected components of \(G\). A strongly connected component is a subgraph, which for each pair of vertices \(m\) and \(m'\) contains a path from \(m\) to \(m'\) and one from \(m'\) to \(m\). In the next step, a graph \(H = (W,F)\) is created and all strongly connected components in \(H\) are repeatedly collapsed until \(H\) contains no more cycles. This is done by contracting the edges in each connected component and merging all its vertices (L49–L51).

The algorithm further considers all vertices \(w\) without incoming edges and which satisfy condition \(C[m] \geq \frac{n + f + \kappa}{2}\), checked in function \texttt{stable(w)} (L60). All such \(w\) will be sorted in a deterministic way (L53). Notice that \(w\) may correspond to a message from \(M\) or a recursive set of sets of messages. Therefore function \texttt{flatten(w)} (L62) is used to extract payload messages and \texttt{of-deliver} them (L54). All \texttt{of-delivered} payload messages are added to \texttt{delivered} (L55) to prevent a repeated processing. Finally, \(w\) is removed from \(H\) (L56), and a next pass of extracting vertices with no incoming edge follows. This is repeated until all vertices have been processed and \texttt{of-delivered}.

The algorithm then initializes \(L\), sets \texttt{inround} to \texttt{FALSE}, increments the round number \(r\), and starts the next round (L57–L59).

**Remarks.** The condition for starting a round in L15 only waits until one single payload exists in \(msgs\) that was not considered before. This is necessary for liveness but not very efficient. This number can be increased such that a new round starts only after \(K = \Theta(n)\) new payload messages have arrived. Note that this threshold affects the amortized message and bit complexities that are considered in Section 5.3.

Recall that our model assumes that every correct process \texttt{of-broadcasts} all payload messages. For simplicity, though, our validity property has been formulated only for executions without faulty processes. It could be strengthened so that it holds for all executions, in which the processes do not \texttt{of-broadcast} an unbounded number of them that form a Condorcet cycle.

The protocol can also be changed to satisfy the even stronger liveness property of Kelkar et al. [13], which the Themis protocol satisfies. To deal with Condorcet cycles of unbounded length, one would modify the interface of order-fair broadcast so that it additionally outputs \texttt{of-startblock} and \texttt{of-endblock} events that carry no parameters. Furthermore, \texttt{of-deliver} would only output single payload messages from \(M\). An output “block” then consists of all payloads that are \texttt{of-delivered} between a \texttt{of-startblock}
Algorithm 1 Quick order-fair atomic broadcast (code for $p_i$).

State
1: $r \leftarrow 1$: current round
2: $\text{inround} \leftarrow \text{FALSE}$
3: $\text{msgs} \leftarrow [\cdot]: \text{HashMap}[\{1, \ldots, n\} \rightarrow [\cdot]]$: array of ordered lists of $bcch$-delivered messages
4: $\text{vc} \leftarrow [\cdot]: \text{HashMap}[\{1, \ldots, n\} \rightarrow \mathbb{N}]$: array of counters for $bcch$-delivered messages
5: $L \leftarrow [0]_{\times n}^n$: matrix of logical timestamps, constructed from $n$ vector clocks
6: $\Sigma \leftarrow [\cdot]^n$: list of signatures from STATUS messages
7: $\text{cut} \leftarrow [0]^n$: the cut decided for the round
8: $\text{delivered} \leftarrow \emptyset$: set of delivered messages

Initialization
9: Byzantine FIFO consistent broadcast channel ($bcch$)

10: $\textbf{upon}$ of-broadcast($m$) do
11: \hspace*{1em} $bcch$-broadcast($m$)

12: $\textbf{upon}$ bcch-deliver($p_j$, $l$, $m$) do
13: \hspace*{1em} $\text{vc}[j] \leftarrow \text{vc}[j] + 1$
14: \hspace*{1em} $\text{msgs}[j].\text{append}(m)$

15: $\textbf{upon}$ exists $j$ such that $\text{len}(\text{msgs}[j]) > \text{cut}[j] \land \neg\text{inround}$ do \hspace*{1em} // perhaps waiting longer
16: \hspace*{1em} $\text{inround} \leftarrow \text{TRUE}$
17: \hspace*{1em} $\sigma \leftarrow \text{sign}(i, \text{vc})$
18: \hspace*{1em} send message [STATUS, $r$, $vc$, $\sigma$] to all $p_j \in \mathcal{P}$

19: $\textbf{upon}$ receiving message [STATUS, $r'$, $vc'$, $\sigma'$] from $p_j$ such that $r' = r \land \text{verify}(j, vc', \sigma')$ do
20: \hspace*{1em} $L[j] \leftarrow vc'$
21: \hspace*{1em} $\Sigma[j] \leftarrow \sigma'$

22: $\textbf{upon}$ $\{|p_j \in \mathcal{P} \mid \Sigma[j] \neq \bot\} \geq n - f$ do
23: \hspace*{1em} vbc-propose((L, $\Sigma$)) for validated Byzantine consensus in round $r$
24: \hspace*{1em} $\Sigma \leftarrow [\cdot]^n$

25: $\textbf{upon}$ vbc-decide((L', $\Sigma'$)) in round $r$ do \hspace*{1em} // calculate the cut
26: \hspace*{1em} for $j \in \{1, \ldots, n\}$ do \hspace*{1em} // for each row in $L'$
27: \hspace*{1em} \hspace*{1em} $\text{cut}[j]$ is the largest $s$ such that at least $f + 1$ elements in column $j$ in $L'$ are at least $s$
28: \hspace*{1em} \hspace*{1em} $\text{cut}[j] \leftarrow \max\{s \mid \{L'[k][j] \geq s\} > f\}$
29: \hspace*{1em} \hspace*{1em} for $j \in \{1, \ldots, n\}$ do \hspace*{1em} // check for missing messages
30: \hspace*{1em} \hspace*{1em} $\textbf{if} \text{vc}[j] < \text{cut}[j] \text{then}$ \hspace*{1em} // some messages that are bcch-broadcast by $p_j$ are missing
31: \hspace*{1em} \hspace*{1em} \hspace*{1em} send message [MISSING, $r$, $j$, $vc[j]$] to all $p_k \in \mathcal{P}$

32: $\textbf{upon}$ receiving message [MISSING, $r'$, $k$, $index$] from $p_j$ such that $r' = r$ do
33: \hspace*{1em} $\textbf{if} \text{vc}[k] \geq \text{cut}[k] \text{then}$
34: \hspace*{1em} \hspace*{1em} $\text{resend} \leftarrow \text{msgs}[k][\text{index} \ldots \text{cut}[k]]$ \hspace*{1em} // copy messages from $p_k$
35: \hspace*{1em} \hspace*{1em} $s \leftarrow \text{bcch-create-proof}(\text{resend})$
36: \hspace*{1em} \hspace*{1em} send message [RESEND, $r$, $k$, $s$] to $p_j$ \hspace*{1em} // send missing messages to $p_j$

37: $\textbf{upon}$ receiving message [RESEND, $r'$, $k'$, $s'$] from $p_j$ such that $r' = r \land \text{len}(\text{msgs}[k]) < \text{cut}[k]$ do
38: \hspace*{1em} $\textbf{if} \text{bcch-verify-proof}(s') \text{then}$
39: \hspace*{1em} \hspace*{1em} $\text{vc}[k] \leftarrow \text{vc}[k] + \text{bcch-get-length}(s')$
40: \hspace*{1em} $\text{msgs}[k].\text{append}(\text{bcch-get-messages}(s'))$
Algorithm 2 Quick order-fair atomic broadcast (code for $p_i$).

41: upon $\text{len}(\text{msgs}[j]) \geq \text{cut}[j]$ for all $j \in \{1, \ldots, n\}$ do
42: \hspace{1em} $V \leftarrow \left( \bigcup_{j \in \{1, \ldots, n\}} \text{msgs}[j][1 \ldots \text{cut}[k]] \right) \setminus \text{delivered}$
43: \hspace{1em} $M \leftarrow [] : \text{HashMap} [\mathcal{M} \times \mathcal{M} \rightarrow \mathbb{N}]$ \hspace{1em} // counts in how many msgs arrays $m$ occurs before $m'$
44: \hspace{1em} $C \leftarrow [] : \text{HashMap} [\mathcal{M} \rightarrow \mathbb{N}]$ \hspace{1em} // counts how many times $m$ appears in msgs arrays
45: \hspace{1em} for $m, m' \in V$ do
46: \hspace{2em} $M[m][m'] \leftarrow \{ j \in \{1, \ldots, n\} \mid m \text{ appears before } m' \text{ in } \text{msgs}[j][1 \ldots \text{cut}[k]] \}$
47: \hspace{2em} $C[m] \leftarrow \{ p_j \mid m \in \text{msgs}[j][1 \ldots \text{cut}[k]] \}$
48: \hspace{1em} $E \leftarrow \{ (m, m') \mid \max \{ M[m][m'], n - f - M[m'][m] \} > M[m'][m] - f + \kappa \}$ \hspace{1em} // add all edges
49: \hspace{1em} $H \leftarrow (V, E)$ \hspace{1em} // $(V, E) = G$
50: \hspace{1em} while $H$ contains some strongly connected subgraph $\overline{H} = (\overline{W}, \overline{F}) \subseteq H$ do
51: \hspace{2em} $H \leftarrow H / \overline{F}$ \hspace{1em} // collapse vertices in $\overline{W}$ into a single vertex $\overline{w}$ via edge contraction
52: \hspace{1em} while $\exists w \in \text{sort}(W) : \text{indegree}(w) = 0 \land \text{stable}(w)$ do \hspace{1em} // in deterministic order
53: \hspace{2em} $\text{of-deliver}(\text{flatten}(w))$ \hspace{1em} // $w$ may be a message or a (recursive) set of sets of messages
54: \hspace{2em} $\text{delivered} \leftarrow \text{delivered} \cup \text{flatten}(w)$ \hspace{1em} // keep track of delivered messages
55: \hspace{2em} $W \leftarrow W \setminus \{ w \}$
56: \hspace{1em} $L \leftarrow [0]^{n \times n}$
57: \hspace{1em} $\text{inround} \leftarrow \text{FALSE}$
58: \hspace{1em} $r \leftarrow r + 1$ \hspace{1em} // move to the next round

60: function $\text{stable}(w)$ \hspace{1em} // $w$ may be a message from $\mathcal{M}$ or a (recursive) set of sets of messages
61: \hspace{1em} return $(w \in \mathcal{M} \land C[w] \geq \frac{n + f - \kappa}{2}) \lor \bigwedge_{w' \in w \cup w': w \in \mathcal{M}} \text{stable}(w')$

62: function $\text{flatten}(w)$ \hspace{1em} // $w$ may be a message from $\mathcal{M}$ or a (recursive) set of sets of messages
63: \hspace{1em} return $\{ m \in w \mid m \in \mathcal{M} \} \cup \bigcup_{w' \in w \cup w': w \in \mathcal{M}} \text{flatten}(w')$
event and the subsequent of-endblock event. However, long cycles occur very infrequently in realistic scenarios, as shown by Kelkar et al. [13].

If consensus is not “black box” and treated in a modular way, more efficient variations of this protocol become possible. In particular, the ordering rounds may be integrated with a leader-based Byzantine consensus protocol [4]. This implies that multiple leaders in successive consensus rounds (or “epochs”) may be needed to agree on the cut of one ordering round. The Themis protocol [13] adopts this pattern.

The protocol satisfies another natural property, which has not been made explicit before in the literature, but is achieved by several existing protocols [14, 20, 13], not only by quick order-fair broadcast. Consider an execution in which the correct processes may be needed to agree on the cut of one ordering round. The Themis protocol [13] adopts this pattern.

5.3 Complexity

In this section, we analyze the complexity of the quick order-fair atomic broadcast protocol. We use two measures: message complexity and communication (bit) complexity. Moreover, we compare our results with existing algorithms from the literature.

Message complexity. If the Byzantine FIFO consistent broadcast channel (BCCH) is implemented using “echo broadcast” [17], it takes $O(n)$ protocol messages per payload message. Since more than $f$ processes of-broadcast each payload message and $f$ is proportional to $n$, the overall message complexity of BCCH is $O(n^2)$. Under high load, batching could be used to reduce the number of messages incurred by BCCH. In the protocol itself, every process sends $O(n)$ STATUS, MISSING, and RESEND messages, which also amounts to $O(n^2)$ messages.

The cost of validated Byzantine consensus (VBC) depends on the assumptions used for implementing it. In the asynchronous model, optimal protocols [14] achieve $O(n^2)$ messages on average. Assuming that $K$ new payload messages are delivered in each round, this becomes $O(Kn)$ per payload. Choosing $K = \Omega(n)$ reduces the amortized cost of consensus to $O(n)$ messages per payload message. Note that when using an implementation of VBC with complexity $O(n^3)$, as the algorithm of Cachin et al. [5], we can choose $K$ proportional to $n$ and may again obtain expected amortized message complexity $O(n^2)$.

With a partially synchronous consensus protocol according to Section 3.3, VBC uses $O(n)$ messages in the best case and $O(n^2)$ messages in the worst case. The total amortized cost of quick order-fair atomic broadcast per payload, therefore, is also $O(n^2)$ messages in this implementation.

Communication (bit) complexity. If digital signatures are of length $\lambda$ and payload messages are at most $L$ bits, the bit complexity of BCCH for one sender is $O(nL + n^2\lambda)$, and since we assume that $O(n)$ processes broadcast each message, this becomes $O(n^2L + n^3\lambda)$. Optimal asynchronous VBC protocols [14] have $O(nL + n^2\lambda)$ expected communication cost, for their payload length $L$. Since the proposals for VBC are $n \times n$ matrices, the bit complexity of this phase is $O(n^3 + n^2\lambda)$. Assuming that $K$ is proportional to $n$, the amortized bit complexity of VBC per payload is $O(n^2 + n\lambda)$. From this, it follows that the amortized bit complexity of the protocol per payload message is $O(n^3L + n^3\lambda)$.

Discussion. Table 1 gives an overview of message complexities of algorithms with different notions for fair payload message ordering. We compare our quick order-fair atomic broadcast with the algorithms introduced by Kelkar et al. [14] and Zhang et al. [20]. We leave out from the overview the protocol by Kursawe [15] since it has a completely different approach for solving fair payload message ordering.

The asynchronous Aequitas protocol [14] Sec. 7) provides fair order using a FIFO Broadcast primitive, implemented by OARcast of Ho et al. [11]. The implementation of OARcast described there uses $n$ ARcasts [11] for each payload, where one ARcast causes $\Theta(n^2)$ network messages. Since Aequitas
requires that every correct process broadcasts each payload, the total complexity increases by another factor of \( n \). Thus, each payload message incurs a cost of \( \Theta(n^4) \) messages in the gossip phase. Moreover, one instance of set agreement is executed for each payload, and each one of them calls \( n \) binary consensus protocols. Therefore Aequitas uses \( \Omega(n^4) \) messages for delivering one payload, which exceeds the cost of quick order-fair broadcast at least by the factor \( n^2 \).

Ordering linearizability \([20]\) is defined using a logical order of events observed on each process. Its implementation in the Pompé protocol, however, appears to require synchronized clocks in the sense of knowing bounds on differences between local clocks. Hence, the complexity of Pompé cannot be compared to that of asynchronous protocols for order fairness. Irrespective of this difference, its cost is \( O(n^2) \) messages and one instance of Byzantine consensus per payload message. The communication complexity of this protocol is \( O(n^3L) \) since each process broadcasts a \( \text{SEQUENCE} \)-message to all others with contents of length \( O(nL) \).

Themis \([13]\) relies strongly on a leader \( p_\ell \) to construct a fair order. If \( p_\ell \) does not perform its task timely, the protocol may switch to another leader, similarly to existing leader-based protocols. For assessing the complexity of Themis here, we consider the optimistic case, but note that the complexities stated for some other protocols, in particular for the quick order-fair broadcast, do not depend on timely leaders.

Themis lets all processes send their local orderings to \( p_\ell \) first. Suppose these consist of approximately \( K = \Theta(n) \) payload messages each. Then \( p_\ell \) constructs a graph \( G \) on these and sends \( G \) and some justification information to all processes. They maintain local graphs, update them in response, and potentially output some payload messages. This incurs a cost of \( O(n) \) messages. Since \( G \) contains \( K \) nodes and, in general, \( O(K^2) \) edges, the average communication complexity is \( O(n^2 + nL) \) in the best case.

| Notion                  | Algorithm            | Avg. messages | Avg. communication |
|------------------------|----------------------|---------------|--------------------|
| Block-Order-Fairness \([14]\) | Async. Aequitas \([14]\) | \( O(n^4) \)  | \( O(n^4L) \)      |
| Ordering Linearizability \([20]\) | Pompé \([20]\)        | \( O(n^2) \)  | \( O(n^3L) \)      |
| Block-Order-Fairness \([13]\) | Themis \([13]\)       | \( O(n) \)    | \( O(n^2 + nL) \)  |
| Differential Order Fairness | Quick o.-f. broadcast | \( O(n^2) \)  | \( O(n^2L + n^3\lambda) \) |

Table 1. Overview of different notions for fair message ordering and corresponding algorithms, with their expected message and communication complexities. The summary assumes \( L \geq \lambda \). (* The Pompé protocol requires synchronized clocks.)

### 6 Analysis

In this section, we show that the quick order-fair atomic broadcast protocol in Algorithm \([1,2]\) implements \( \kappa \)-differentially order-fair atomic broadcast. The properties to be satisfied are (Definition \([4]\): no duplication, agreement, total order, strong validity and \( \kappa \)-differential order fairness.

**Lemma 6.** No message is of-delivered more than once in Algorithm \([1,2]\).

**Proof.** The check in \([1,2]\) of the protocol implementation ensures that no payload message is of-delivered more than once. In the step when the protocol creates graph vertices, payload messages that are already contained in variable \( \text{delivered} \) are filtered out. Those messages will not be included in the graph and cannot be of-delivered again. Note that even in the case when a payload message \( m \) is bcch-delivered multiple times, because of filtering in \([1,2]\) it is not possible that \( m \) is of-delivered more than once. \( \square \)

**Lemma 7.** In Algorithm \([1,2]\) if a message \( m \) is of-delivered by some correct process, then \( m \) is eventually of-delivered by every correct process.

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Proof. Suppose that a payload message $m$ is of-delivered by some correct process $p_i$ in round $r$. Following the protocol steps, in round $r$ all correct processes decide on the same $L'$. This is guaranteed by the agreement property of the validated Byzantine consensus because no two correct processes decide differently. Since the matrix $L'$ is used to construct the cut deterministically, all correct processes construct the same cut (Lemma 8).

We can then distinguish two cases: In the first case, all correct processes have already bcch-delivered $m$ and store it in $msgs$. In the second case, there are some correct processes that have never heard of $m$ simply because of some delays in the network. Then these correct processes send a MISSING-message to all processes, requesting the delivery of their missing payloads. Since every message included in the cut was announced by $f + 1$ processes, and therefore also by at least one correct process, some process will respond with a RESEND-message containing $m$. Once all these messages are delivered, all correct processes store $m$ in $msgs$.

In the next step, every correct process builds graph $G$. Each vertex in the graph is constructed deterministically from the same information by every process, concretely, from the payload messages in $msgs$ and excluding those that are already in the delivered set (Lemma 9). Following the protocol, every correct process will eventually construct the same $G$ and output the same sequence of payload messages, also including $m$.

Lemma 8. Let $m$ and $m'$ be two messages such that $p_i$ and $p_j$ are correct processes that of-deliver $m$ and $m'$. In Algorithm 1–2, if $p_i$ of-delivers $m$ before $m'$, then $p_j$ also of-delivers $m$ before $m'$.

Proof. Consider two distinct payload messages $m$ and $m'$ and let $p_i$ and $p_j$ be any two correct processes that of-deliver both messages. Assume that $p_i$ of-delivers $m$ before $m'$. If $p_i$ of-delivers $m$ and $m'$ in round $r$, then both messages were included in the cut for $p_i$. Due to the argument used to establish the agreement property in Lemma 8, it must be that $m$ and $m'$ were also included in the cut for process $p_j$ in round $r$. The rest of the protocol, i.e., building a graph and of-delivering messages is deterministic. Therefore, $p_j$ delivers these two messages in round $r$ and also of-delivers $m$ before $m'$. Extending this argument over all rounds of the protocol, it follows that every correct process of-delivers the same sequence of payload messages.

Lemma 9. If all processes are correct and of-broadcast a finite number of messages in Algorithm 1–2 then every correct process eventually of-delivers these messages.

Proof. Let $p_i$ be some correct process that of-broadcasts a payload message $m$. Due to the validity property of the underlying Byzantine FIFO consistent broadcast channel, every correct process eventually bcch-delivers $m$. According to the algorithm, in every round $r$ a process $p_i$ waits for $n − f$ processes to receive signed vector clocks to propose a matrix of logical timestamps $L$ for validated Byzantine consensus (Lemma 8). The termination property of validated Byzantine consensus guarantees that every correct process eventually decides some value and according to the agreement property, no two correct processes decide differently. The resulting common $L'$ allows then each process to determine if $m$ is considered in the current round $r$. A message $m$ is considered if at least $f + 1$ processes have bcch-delivered $m$ and reported it in their vector clock (Lemma 8). Additionally, if $m$ is considered in the current round but some process $p_i$ has not bcch-delivered $m$ yet, $p_i$ will request that other processes send it the missing payload message (Lemma 8). Further, all messages in $msgs$ are added as vertices to the graph $G$. Moreover, because every process of-broadcasts a finite number of messages, every possible graph that is created will be finite. Since $m$ was of-broadcast by a correct process $p_i$, all processes are correct and of-broadcast a finite number of messages, $m$ will eventually be of-delivered.

Lemma 10. In Algorithm 1–2 if $b(m, m') > b(m', m) + 2f + \kappa$, then no correct process of-delivers $m'$ before $m$.

Proof. Recall that $b(m, m')$ is the number of correct processes that receive and of-broadcast $m$ before $m'$. Consider any two payload messages $m$ and $m'$ such that $b(m, m') > b(m', m) + 2f + \kappa$. 

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Suppose $m$ and $m'$ are both included in the cut of some round and none of them has been of-delivered yet. The protocol defines a threshold based on $M$ for creating an edge between two vertices. Lemma 5 shows that the condition for differential order fairness ensures that $M[m][m'] > M[m'][m] - f + \kappa$ in the protocol, where $M[m][m']$ counts how many times $m$ appears before $m'$ in $msgs$. Moreover, as explained in connection with Lemma 5, the algorithm extends this condition for adding an edge $(m, m')$ to

$$\max \{ M[m][m'], n - f - M[m'][m] \} > M[m'][m] - f + \kappa,$$

in order to cope with particularly small values of $M[m'][m]$. This may be the case when the full relative ordering information about $m$ and $m'$, in the sense that $M[m][m'] + M[m'][m] \geq n - f$, is not yet available with the cut. The implementation then adds an edge from $m$ to $m'$ to the graph (L48). This implies that $m'$ will not be of-delivered before $m$ because the algorithm respects this order by traversing the graph starting with vertices that have no incoming edges. Therefore, $m$ is either of-delivered before $m'$ or both messages are delivered together, within the same set. Moreover, observe that (2) ensures that graph generated by the protocol is connected.

Consider now the case that $m'$ is not included at all in the cut of the current round $r$. We want to show that for all $m \in V$ of the graph $G$, if $m$ is of-delivered in round $r$, there cannot be such an $m'$, for which an edge $(m', m)$ would be added at a later round and which might therefore violate $\kappa$-order fairness. Recall that an edge $(m', m)$ is added to $G$ in a round whenever (2) holds.

To be more precise, we show that the condition in L53 and the properties of stable() ensure $\kappa$-differential order fairness for such $m$ and $m'$. Let $\tilde{w}$ be a node of the graph as in L53. If every $m$ in flatten($\tilde{w}$) appears at least $\frac{n + f - \kappa}{2}$ times in $msgs$ up to the cut, i.e., satisfies stable($m$), it means that no $m'$ (not in the cut) can be ordered before the messages in $G$ of subsequent rounds. In fact, let $m \in flatten(\tilde{w})$ be such that $\text{stable}(m) = \text{TRUE}$ and $m'$ not be in the cut. Since $\text{stable}(m)$ holds, $C[m] \geq \frac{n + f - \kappa}{2}$ also means that $M[m][m'] \geq \frac{n + f - \kappa}{2}$. Thus,

$$M[m'][m] \leq n - M[m][m'] \leq n - \frac{n + f - \kappa}{2} = \frac{n - f + \kappa}{2}$$

in any future round as well. But this implies $M[m'][m] - M[m][m'] \leq -f + \kappa$, and thus, no edge $(m', m)$ is added according to (2). The argument given earlier then shows that order fairness is maintained. Notice that this takes care of scenarios as in Example 1 that include some message $\tilde{m} \in flatten(\tilde{w})$ with $\neg\text{stable}(\tilde{m})$. There may exist a further message $m'$ not included in the cut such that $m'$ must be ordered not after $\tilde{m}$.

Lemmas 6-10 directly imply the following theorem, which concludes the analysis of the protocol.

**Theorem 11.** Algorithm 1-2 implements $\kappa$-differentially order-fair atomic broadcast.

### 7 Conclusion

The quick order-fair atomic broadcast protocol guarantees payload message delivery in a differentially fair order. It works both for asynchronous and eventually synchronous networks with optimal resilience, tolerating corruptions of up to one third of the processes. Compared to existing order-fair atomic broadcast protocols, our protocol is considerably more efficient and incurs only quadratic cost in terms of amortized message complexity per delivered payload.

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