Entanglement and quantum interference

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In the history of quantum mechanics, much has been written about the double-slit experiment, and much debate as to its interpretation has ensued. Indeed, to explain the interference patterns for subatomic particles, explanations have been given not only in terms of the principle of complementarity and wave-particle duality but also in terms of quantum consciousness and parallel universes. In this paper, the topic will be discussed from the perspective of spin-coupling in the hope of further clarification. We will also suggest that this explanation allows for a realist interpretation of the Afshar Experiment.

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A. Description of the experiment

In itself, the double-slit experiment is easy to describe. Let us assume without loss of generality that we are using electrons to conduct the experiment. First a beam of electrons is fired through a single slit, and a normal diffraction pattern observed. Next the beam of electrons is fired at a pair of slits and an interference pattern is observed (Fig. 1). This interference pattern presents an immediate difficulty. If the electrons were truly particles then we would expect each electron to have passed through either one of the slits (but not both) according to the laws of independent probability.

Specifically, if the particle state is $|\psi\rangle = |\psi_1\rangle + |\psi_2\rangle$ and if the electron beam were composed of independent non-interacting particles then the probability distribution would be given by $|\psi_1|^2 + |\psi_2|^2$. Instead, a distribution of the form $|\psi_1 + \psi_2|^2$ is observed, consistent with a wave-front. This same interference pattern also emerges if electrons pass through the slits and hit the screen one at a time, thus excluding the possibility that they have interacted or interfered with each other. It would seem individual electrons interfere with themselves. Moreover, the matter is further complicated when we attempt to observe which route through the slits the individual electrons might have taken. To our surprise we will find that the distribution associated with particles now re-emerges. And so we might wonder, how did the electron know it was being observed and change back to behaving like a particle?

Some have tried to explain this in terms of quantum consciousness, whereby the electron is neither a particle nor a wave, but responds according to the type of question we ask, and the type of probing we do. Others would say that it is both a particle and a wave, and that the projection postulate forces it to choose one or the other state, according to how we measure it. Again others have attempted to explain the result in terms of parallel universes or from Cramer’s transactional viewpoint [1].

Before attempting to agree or disagree with any of the above interpretations, I would like to first re-examine the problem mathematically from the perspective of entanglement and rotational invariance, and see what this analysis suggests.

B. Rotationally-invariant states

Part of the difficulty with a strict particle interpretation of quantum interference is the assumption that particles independently pass through either one of the slits, with the slits serving merely as a filter or a conduit for the particles. In this paper, I would like to suggest that regardless of which path the electron takes, if the slits are sufficiently close together, then the acquired spin-state of an individual electron depends upon its interaction with both of the slits, unless inhibited from doing so. At the same time this does not mean that the particle has traveled through both slits, nor does it preclude that the particle has taken a definite route through the interferometer. However, it does require that we focus on the spin-state dependency as determined by the two apertures. Moreover, I think these results will be found not
only to be in full agreement with the quantum eraser effect obtained through parametric down conversion\textsuperscript{2,3}, but will also highlight the role of entanglement in interference phenomena, as well as clarify the role of conditional probability in physics.

1. A coupling principle:

It has been shown in previous work that isotropically spin-correlated states must occur in pairs\textsuperscript{1}. Specifically, if we denote spin-up and spin-down states by $|+\rangle$ and $|\rangle$, this means that there exist unique rotationally invariant pure states of the form

$$\psi = \frac{1}{\sqrt{2}}(|+\rangle |\rangle - |\rangle |+\rangle) \quad (1)$$

$$\psi = \frac{1}{\sqrt{2}}(|+\rangle |+\rangle + |\rangle |\rangle), \quad (2)$$

which correspond to the singlet state and its spinor conjugate respectively. These entangled states have the characteristic that if a spin-measurement is made in an arbitrary direction on one of the states, then the state of the other particle is known with certainty. For example, in the case of the singlet state, if a positive spin is detected along an arbitrary $z$-axis of particle one then a negative spin value will be detected along the corresponding $z$-axis of particle two. Indeed, I would suggest that it is precisely this spinor correlation which is the primary cause of the interference phenomena, and if this correlation is broken as often happens when we measure or try to detect a particle, then the interference phenomena disappears.

Specifically, let the initial state

$$\psi = \frac{1}{\sqrt{2}}(|+\rangle |\rangle + |\rangle |\rangle) \quad (3)$$

represent the state as registered with respect to the normal at Q ($m=0$ in Fig. 1). In other words, from the perspective of Q, we not only assume that the incoming particle has come with equal probability from either aperture, but that its spinor component along the normal to the screen at Q has been induced by the apertures working in tandem, as captured in the tensor product of equation (3). Next, shift the normal at Q to P. In this case, the pure initial state will transform under rotation operators ($\mathcal{R}(\alpha), \mathcal{R}(\beta)$) into the mixed quantum state

$$\psi = \frac{1}{\sqrt{2}}\sqrt{1-\rho^2}(|+\rangle |\rangle - |\rangle |\rangle)$$

$$\rho^2 \leq 1, \quad \alpha, \beta$$

where $\rho^2 \leq 1$, and $\alpha$ and $\beta$ are the angles of incidence of the straight-line trajectories drawn from the apertures (Fig. 2). Note, this mixed state is composed of two isotropically spin-correlated pure states describing the spin state of a single particle arriving at P with probability $\rho^2$, where $\rho = \alpha - \beta$ is the angle subtended by the two rays connecting P to the two apertures (Fig. 2). This means that regardless of which aperture a particle may or may not take, the relative spin-states are correlated by the slits themselves, in such a way that the state ($|+\rangle |+\rangle - |\rangle |\rangle$) will be detected with probability $\rho^2$ and failed to be detected with probability $1 - \rho^2$. In fact, failure to detect means that the other state ($|+\rangle |\rangle - |\rangle |\rangle$) will have occurred. Also we note that intuitively, regardless of the particles trajectory, we can by analogy imagine the direction of the particle’s spin behaving like a compass needle, and pointing always in the direction of the normal to the screen.

Finally to conclude this section, an explicit derivation of equation (4) is given, using the SU(2) properties associated with rotational invariance. Indeed, by our initial assumption, if $\phi = 0$ then the state at $m = 0$ is of the form $|+\rangle |+\rangle - |\rangle |\rangle$ with intensity $I_0$. Applying the rotation operator $\mathcal{R}(\alpha)$ to the first component, gives the corresponding intensity at P, resulting from a particle moving along a ray making an angle of incidence $\alpha$ (Fig. 2) with the normal. It follows that

$$(\mathcal{R}(\alpha)|+\rangle |+\rangle + \mathcal{R}(\alpha)|\rangle |\rangle)$$

$$= \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$+ \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$-\sin(\alpha) \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] . \quad (5)$$

Similarly, if we multiply the first tensor component by the rotation operator $\mathcal{R}(\beta)$ and the second component by the operator $\mathcal{R}(\beta)$, and denote $\alpha - \beta = \phi$, we obtain:

$$(\mathcal{R}(\alpha), \mathcal{R}(\beta))(|+\rangle |+\rangle - |\rangle |\rangle) =$$

$$\begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} \cos(\beta) & \sin(\beta) \\ -\sin(\beta) & \cos(\beta) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$+ \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} \cos(\beta) & \sin(\beta) \\ -\sin(\beta) & \cos(\beta) \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Fig. 2
which on multiplying out gives
\[
\cos(\phi) \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] 
- \sin(\phi) \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] \left( \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \right].
\]

This is identical to equation (4) above, and further justifies our interpretation. Also note that \( \phi \) and \( \theta \) are related by the equation \( \phi = \frac{2\pi}{\lambda} \sin(\theta) \) (Fig. 1). [2]

2. The detection process:

We now ask what happens when we try to detect the particle route? Mathematically, the answer is quite straightforward and can be handled using projection operators. Specifically, the transmitted rotationally invariant state, given by \( |\psi\rangle = 1/\sqrt{2}(|\uparrow\rangle |\uparrow\rangle + |\downarrow\rangle |\downarrow\rangle) \), is reduced by the detector (placed at slits 1 and/or 2) to the state
\[
P_i |\psi\rangle = 1/\sqrt{2} P_i(|\uparrow\rangle |\uparrow\rangle + |\downarrow\rangle |\downarrow\rangle)
= 1/\sqrt{2} (|\uparrow\rangle + |\downarrow\rangle)
\]
where \( i = 1 \) or \( 2 \) represents the aperture from which the particle is emitted. In other words, from the perspective of this paper, the measuring process breaks the entanglement, causing the interference to disappear. The spins are no longer correlated and the probability distribution is now associated with two independent random variables representing the proportion of particles emerging respectively from each aperture. In neither case, does it follow that the particle has entered both slits simultaneously and interacted with itself. Rather, the double slit experiment seems to be telling us that regardless of the slit from which the particle leaves, both slits have contributed to inducing an isotropically-polarized state relative to the slits themselves, which is then manifested in an interference effect, unless interrupted by the detection process. Moreover, this perspective is not only compatible with the Afshar experiment, but also offers a realist interpretation of his results [2].

Finally, it should be noted that if the above interpretation is correct, then it should also be possible to design an experiment to destroy the correlation using Stern-Gerlach magnets, without ever obtaining “which way” information; thereby demonstrating that it is not our knowledge of the situation that determines the final quantum state, but rather the way in which the states are correlated and transformed from coherent states into non-coherent states. An experiment similar to this is described by Qureshi and Rehman in [3]. However, in their case they first use a magnet to obtain “which way” information as the particles enter the slit and then erase this information with another magnet after the particles exit the slits, to produce interference. What is being proposed here is the opposite. First particles enter the slits undetected and form an interference pattern on the screen. Then a Stern-Garlach magnet field, placed orthogonal to the screen and its normal, is turned on as the particles emerge from the slits. If the above theory is correct then the interference pattern will disappear by strategically positioning the magnet.

![Fig. 3](image-url)

3. Multiple slits:

In this section, we briefly analyze the interference phenomena caused by three or more slits. At first this might seem a formidable problem to resolve, given the many ways that the slits may influence the polarization. However, because of the coupling principle, we need only worry about paired states.

For example, in the case of three slits lying on a line, the possible paired states are of the form, \( |\psi_{12}\rangle, |\psi_{23}\rangle \) or \( |\psi_{13}\rangle \), where each \( |\psi_{ij}\rangle \) equals
\[
\cos(\phi_{ij}) (|\uparrow\rangle |\uparrow\rangle + |\downarrow\rangle |\downarrow\rangle) + \sin(\phi_{ij}) (|\uparrow\rangle |\downarrow\rangle + |\downarrow\rangle |\uparrow\rangle)
\]
with \( \phi_{ij} \) being the angle subtended by rays drawn from \( a_i \) and \( a_j \) and meeting at \( P \) (Fig. 3). Moreover, it is not difficult to show that once we have the correct correlation written down for any pair of slits, then it is easy to deduce the correlation associated with any other pair of slits. Indeed, it seems remarkable that multiple slit correlations can be reduced to paired correlations. To show this explicitly it is sufficient to consider an interferometer made of three apertures:

Let \( u = \frac{1}{\sqrt{2}} (|\uparrow\rangle |\uparrow\rangle + |\downarrow\rangle |\downarrow\rangle) \) and \( v = \frac{1}{\sqrt{2}} (|\downarrow\rangle |\downarrow\rangle - |\uparrow\rangle |\uparrow\rangle) \),

then equation (4) reduces in this notation to the compact form:
\[
[R(\alpha), R(\beta)]u = \cos(\beta - \alpha)u - \sin(\beta - \alpha)v
= [I, R(\beta - \alpha)]u. \tag{6}
\]
Similarly, we can show by direct calculation that
\[
[R(\alpha), R(\beta)]v = \sin(\beta - \alpha)u + \cos(\beta - \alpha)v
\]
\[
= [I, R(\beta - \alpha)]v.
\]
(7)
Combining these two results and recalling that \(|\psi_{12}\rangle = [I, R(\beta - \alpha)]u\) allows us to write
\[
[R(\beta), R(\gamma)]|\psi_{12}\rangle = [I, R(\gamma - \beta)]|\psi_{12}\rangle
\]
\[
= [I, R(\gamma - \beta)][I, R(\beta - \alpha)]u
\]
\[
= [I, R(\gamma - \beta + \beta - \alpha)]u
\]
\[
= [I, R(\gamma - \alpha)]u
\]
\[
= \cos(\gamma - \alpha)u - \sin(\gamma - \alpha)v
\]
\[
= \cos(\phi_{13})u - \sin(\phi_{13})v
\]
\[
= |\psi_{13}\rangle
\]
(8)

There is a certain beauty in this result. It shows that if at any point \(P\) we can write down the state function for any pair of apertures, then by means of rotation operators we can also transform it into the corresponding state function for any other pair. To do so, it is sufficient to know the angle a ray from the aperture makes with the normal to point \(P\). For example, if we were to modify the above and work with rotations \((R(\alpha), R(\beta))\) and \((R(\alpha), R(\gamma))\) we could have obtained the state function \(|\psi_{23}\rangle\).

Note: (1) In the case of spin 0 particles like mesons the above results would also apply, provided some type of isotropic polarized states occur. A photon may be used as an example of this type of polarization.

(2) In this paper we have assumed that \(|+\rangle |+\rangle |+\rangle |--\rangle\) is transmitted and the state \(|+\rangle |--\rangle |--\rangle |+\rangle\) is absorbed. However, the theory is completely symmetrical and we could equally have assumed the opposite without any loss of generality. Indeed, in the case of the quantum eraser effect, it could be argued that in some cases the state \(|+\rangle |--\rangle |--\rangle |+\rangle\) is registered on the detector, while the, \(|+\rangle |+\rangle |+\rangle |--\rangle\) is absorbed and not detected.

C. Conclusion

By associating quantum interference with rotationally-invariant states induced by the slits, we have succeeded in showing that it is possible for an object such as an electron or photon to exhibit both particle and wave properties in a non-mutually exclusive way. Furthermore, from the perspective of this paper, there is no contradiction in considering these objects to be particles at all times, in that they take a definite route through the interferometer, while simultaneously exhibiting wave characteristics associated with the isotropically-spin correlated states and induced by the apertures themselves, in accordance with the laws of conditional probability. Indeed, to the extent that we consider the object to be a particle then it will always be a particle. However to the extent that the spin (or polarization) states are defined and correlated relative to the apertures, they will also exhibit wave characteristics. Nevertheless, to capture these properties simultaneously is very difficult in that the act of path detection is also responsible for undoing the spin-correlation, which also means to undo the wave characteristics. Wave properties need to be detected prior to detecting the particle properties if both properties are to be exhibited. The two observables do not commute, but at the same time, it seems to me that the Afshar experiment has succeeded in demonstrating that these two properties can co-exist simultaneously, in a way that is compatible with a realist interpretation of complementarity.

Finally, it is worth emphasizing that if the above formulation were to be regarded as correct, then in my opinion the focus of interferometer research should move away from particle detection to trying to better understand how the correlated states are induced.

[1] J. Cramer, Rev. Mod. Phys., 58, 647(1986)
[2] S. Walborn et al., Phys. Rev. A, 65, 033818, 2002
[3] L. Orozco, http://grad.physics.sunysb.edu/~amarch
[4] P. O’Hara, Poun. Phy., 33(9), 1349(2003)
[5] Halliday and Resnick, Fund. of Physics 3rd ed., 904-910
[6] S. Afshar, Proc. SPIE 5866, 229(2005).
[7] T. Qureshi and Z. Rehman, arXiv quant-ph/0501010v1