Scattering of high-energy partons off ultrasoft
 gluon fluctuations in hot QCD plasma
 and energy losses

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Abstract

Within the framework of the effective theory for ultrasoft field modes a problem of inter-
acting an energetic color particle with ultrasoft fluctuations in hot gluon plasma is
considered. The procedure of calculation of certain effective current generating the pro-
cess of interaction is proposed, and the problem of its gauge independence is discussed.
The application of the theory developed to the problem of energy losses of the energetic
color particle propagating through hot QCD-medium is given. It is shown that the pertur-
bation approach for calculation of energy losses breaks down at the ultrasoft momentum
scale of plasma excitations.

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1 Introduction

In this work we study the problem of scattering of energetic color particles (quarks, gluons or more generally – partons) off plasma fluctuations at the ultrasoft momentum scale ($p_0 \sim g^4 T$, $|p| \sim g^2 T$, where $T$ is temperature of the system and $g$ is the coupling constant) in hot gluon plasma. The correct effective theory at the ultrasoft momentum scale is generated by the linearized Boltzmann-Langevin equation in the form first proposed by Bödeker [1]. Then this equation was obtained by a number of authors [2, 3, 4, 5] within the framework of different approaches. This transport equation includes the collision term for color relaxation and the Gaussian noise term that keeps the ultrasoft modes in thermal equilibrium.

The way for solving the problem stated is based on general ideas suggested in our previous work [6] dealing with the similar problem of scattering of high-energy particles off the plasma fluctuations at the soft momentum scale ($p_0 \sim gT$, $|p| \sim gT$). Following the idea of the work [6] we supplement the Boltzmann-Langevin equation by the Wong equation [7] describing the precession of the classical color charge $Q = (Q^a, a = 1, \ldots, N_c^2 - 1)$ of the energetic parton propagating through QCD-medium. The scattering process is generated by certain effective current. The calculation of this current is crucial step of the formalism under consideration. However in contrast to our previous work [6] the effective current here represents the expansion not in powers of the free gauge field $A_\mu^{(0)}(X)$ (that at this momentum scale is strongly damping by virtue of collisions), but in powers of the noise term $\nu(X, v)$ and also initial value of the color charge $Q_0$ of the energetic parton.

We work to lowest order in the color charge $Q_0$ of the parton, but to all orders in the noise term, since the external parton is a truly perturbative object, whereas the thermal color fluctuations at the ultrasoft scale $g^2 T$ are non-perturbative [8]. We apply the current approach to study of the energy losses of the energetic parton induced by scattering off the ultrasoft gluon fluctuations. We show that for weak gauge coupling this contribution to the total balance of the energy losses can be neglected, although here, there is some vagueness of the exact estimation of this type of energy loss.

This paper is organized as follows. In Section 2, the convention and notation used in this paper are summarized. In Section 3, the basic nonlinear integral equation for gauge potential is written out and the algorithm of the successive calculation of certain effective current generating the interaction process of the ultrasoft plasma fluctuations with the energetic color particle is proposed. In Section 4, we discuss the problem of gauge independence of matrix element for the simplest scattering process. In Section 5,
the energy losses caused by the scattering off ultrasoft gluon fluctuations in lower orders in powers of the noise term is analyzed.

2 Initial equations

We use the metric \( g^{\mu\nu} = \text{diag}(1, -1, -1, -1) \), choose units such that \( c = k_B = 1 \) and note \( X = (X_0, X) \), \( p = (p_0 \equiv \omega, p) \) etc.

At the space-time scale \( X \gg (gT)^{-1} \) the ultrasoft fluctuations of the gluon color density

\[ W(X, v) = W^a(X, v)T^a \quad ((T^a)^{bc} \equiv -if^{abc}) \]

satisfies the Boltzmann-Langevin equation

\[ [v \cdot D_X, W(X, v)] = -v \cdot E(X) - C[W](X, v) + \nu(X, v). \quad (2.1) \]

Here, \( D_\mu = \partial_\mu + igA_\mu(X) \) is the covariant derivative; \([ , ]\) denotes the commutator; \( E(X) = E^a(X)T^a \) is the chromoelectric field and \( C[W] \) is linearized collision term defined by the expression

\[ C[W](X, v) \equiv \hat{C}W(X, v) = \int \frac{d\Omega_{v'}}{4\pi} C(v, v')W(X, v') \quad (2.2) \]

with the collision kernel

\[ C(v, v') = \gamma \delta(S^2)(v - v') - m_D^2 g^2 N_c T^2 \Phi(v \cdot v'). \quad (2.3) \]

In the last expression \( \delta(S^2)(v - v') \) is the delta-function on the unit sphere,

\[ \Phi(v \cdot v') \simeq \frac{2}{\pi^2 m_D^2} \frac{(v \cdot v')^2}{\sqrt{1 - (v \cdot v')^2}} \ln \left( \frac{1}{g} \right), \quad m_D^2 = \frac{1}{3} g^2 N_c T^2 \]

within logarithmic accuracy and

\[ \gamma = m_D^2 \frac{g^2 N_c T^2}{2} \int \frac{d\Omega_{v'}}{4\pi} \Phi(v \cdot v') \]

is the damping rate for hard transverse gluon with velocity \( v \). Furthermore the function

\[ \nu(X, v) = \nu^a(X, v)T^a \]

is the noise term with the noise-noise correlation function

\[ \ll \nu^a(X, v)\nu^b(X', v') \gg = \frac{2T}{m_D^2} C(v, v')\delta^{ab}\delta^4(X - X'). \quad (2.4) \]

\[ \text{Here, we employ the parametrization} \ [5] \ \text{for deviation of the gluon density from the equilibrium} \]

\( \delta N(k, X) = -g W(X, v) dN(|k|)/d|k|, \) where \( N(|k|) \) is the Bose-Einstein distribution. More general expression for parametrization of off-equilibrium fluctuations is considered in Ref. \[9\].

\[ \text{Hereafter the square brackets denote functional dependence.} \]
The color current in the terms of the function $W(X, v)$ is
\[
 j_\mu(X) = m_D^2 \int \frac{d\Omega_v}{4\pi} v_\mu W(X, v). \tag{2.5}
\]

Following Blaizot and Iancu [10] we present the function $W(X, v)$ as the sum of two parts
\[
 W(X, v) = W^{\text{ind}}(X, v) + W(X, v),
\]
where $W^{\text{ind}}(X, v)$ is the solution of Eq. (2.1) in the absence of the noise $\nu(X, v)$
\[
 [v \cdot D_X, W^{\text{ind}}(X, v)] = - v \cdot E(X) - \hat{C} W^{\text{ind}}(X, v), \tag{2.6}
\]
and $W(X, v)$ is a fluctuating piece satisfying
\[
 [v \cdot D_X, W(X, v)] = - \hat{C} W(X, v) + \nu(X, v). \tag{2.7}
\]
Thus, $W(X, v)$ is proportional to $\nu$ and in general also depends on the gauge field $A_\mu(X)$.
The color current (2.5) is decomposed as
\[
 j_\mu(X) = m_D^2 \int \frac{d\Omega_v}{4\pi} v_\mu \left( W^{\text{ind}}(X, v) + W(X, v) \right) \equiv j_\mu^{\text{ind}}(X) + \zeta_\mu(X), \tag{2.8}
\]
where $\zeta_\mu(X)$ is the fluctuation current acting as the noise term in the Yang-Mills equation
\[
 [D^\nu, F_{\mu\nu}(X)] + \xi^{-1} \partial^\nu A_\nu(X) = j_\mu^{\text{ind}}(X) + \zeta_\mu(X). \tag{2.9}
\]
Here, $F_{\mu\nu} = F^{a}_{\mu\nu} t^a$ is the field strength tensor and $\xi$ is the gauge parameter fixing a covariant gauge.

If there is a moving energetic color charged parton (quark or gluon) in the hot gluon plasma, then on the right-hand side of field equation (2.9) it is necessary to add appropriate color current $j_Q^{\mu}(X)$. One expects the world line of this parton to obey the classical trajectory in the manner of Wong [7]. In the leading order in the coupling constant the color current of the energetic color parton has the form [6]
\[
 j_Q^{a\mu} = g \tilde{v}^\mu U^{ab}(t, t_0) Q_0^b \delta^{(3)}(x - \tilde{v}t), \quad \tilde{v}^\mu = (1, \tilde{v}), \tag{2.10}
\]
where
\[
 U(t, t_0) = T \exp \{-i g \int_{t_0}^{t} (\tilde{v} \cdot A^a(\tau, \tilde{v} \tau)) T^a d\tau \}
\]
is the evolution operator taking into account the color precession along the parton trajectory, $\tilde{v}$ is the velocity of the energetic parton and $Q_0^a$ is initial value of its color charge.
3 Construction of effective current

Now we are in position to construct the effective theory of nonlinear interaction of the energetic color particle with ultrasoft gluon fluctuations in spirit of the work [6]. As was mentioned in Introduction, the main point here, is deriving certain effective current generating this process of nonlinear interaction. For this purpose, at first we represent the solutions \( W^{\text{ind}}(x, v) \) and \( W(x, v) \) of dynamical equations (2.6), (2.7) as a formal expansion in powers of interaction field \( A_\mu(X) \). Rewriting the Yang-Mills equation (2.9) in the momentum space\(^3\) and taking into account above-mentioned expansions, we obtain the basic nonlinear integral equation for the gauge potential \( A_\mu(p) \) at the ultrasoft momentum scale \( p_0 \sim g^4 T, |p| \sim g^2 T \)

\[
^{*}D^{-1}_{\mu\nu}(p) A^{\mu\nu}(p) = -j^{(\text{tot})a}_{\mu}[A, \nu](p),
\]

where an expression for total current in representation of the interaction field reads

\[
j^{(\text{tot})a}_{\mu}[\nu, A](p) = j^{\text{ind}}_{\mu}[A](p) + j^{(3)}_{a\mu}[A](p) + \zeta^{a}_{\mu}[A, \nu](p).
\]

Note that in deriving Eq. (3.1) the part linear in \( A_\mu(p) \) of the induced current (that is, the one involving the polarization tensor \( \Pi_{\mu\nu}(p) \)) has been extracted out from \( j^{(\text{tot})a}_{\mu} \) and included in the effective inverse propagator on the left-hand side of (3.1). Conversely, the tree-level 3-gluon and 4-gluon vertices which were originally presented on the left-hand side of Eq. (2.9) have been now reabsorbed into the induced current on the right-hand side of Eq. (3.1). Inverse gluon propagator on the left-hand side of Eq. (3.1) is defined by expression

\[
^{*}D_{\mu\nu}(p) = -P_{\mu\nu}(p) \ast \Delta^I(p) - Q_{\mu\nu}(p) \ast \Delta^I(p) + \xi D_{\mu\nu}(p) \Delta^0(p),
\]

where \( \Delta^0(p) = 1/p^2; \ast \Delta^I(p) = 1/(p^2 - \Pi^I(p)), \Pi^I(p) = 1/2 \Pi^{\mu\nu}(p) P_{\mu\nu}(p), \) and \( \Pi^I(p) = \Pi^{\mu\nu}(p) Q_{\mu\nu}(p) \) with ultrasoft gluon self-energy \( \Pi_{\mu\nu}(p) \) \[11\]

\[
\Pi_{\mu\nu}(p) = m_D^2 \left\{-g_{\mu\nu}g_{\rho\sigma} + \omega \int \frac{d\Omega_\nu}{4\pi} \int \frac{d\Omega_\nu'}{4\pi} \left\langle \frac{1}{v \cdot p + i\hat{C}} v \right| v' \right\}
\]

The explicit form of the Lorentz matrices \( P_{\mu\nu}, Q_{\mu\nu} \) and \( D_{\mu\nu} \) is given in Appendix. The symbols \( \langle v| \) and \( |v'\rangle \) in the definition of \( \Pi_{\mu\nu}(p) \) possess the following properties

\[
\left\langle v \right| v' \rangle = \delta^{(2)}(v - v'), \quad \left\langle v \left| \hat{C} \right| v' \right\rangle = \left\langle v' \left| \hat{C} \right| v \right\rangle = C(v, v'), \quad v_\mu |v'\rangle = v'_\mu |v'\rangle
\]

\(^3\)We add the current of the hard color charged parton (2.10) to the right-hand side of Eq. (2.9)
and satisfy the “completeness relation”

$$\int \frac{d\Omega}{4\pi} |\nu\rangle \langle \nu| = 1.$$  

In the works [12, 13] explicit analytical expressions for functions \(\Pi^i(p)\) and \(\Pi^j(p)\) in the terms of the continued fractions are given.

The induced color current on the right-hand side of Eq. (3.2) by virtue of the Boltzmann equation (2.6) is expressed as

$$j_{\mu}^{\text{ind}}[A](p) = \sum_{s=2}^{\infty} j_{\mu}^{(s)a}(A, \ldots, A),$$  

where

$$j_{\mu}^{(s)a}(A, \ldots, A) = \frac{1}{s!} g^{s-1} \int \Gamma_{\mu_1 \ldots \mu_s}^{a_1 \ldots a_s}(p, -p_1, \ldots, -p_s) A^{a_1 \mu_1}(p_1) A^{a_2 \mu_2}(p_2) \ldots A^{a_s \mu_s}(p_s)$$

$$\times \delta^{(4)}(p - \sum_{i=1}^{s} p_i) \prod_{i=1}^{s} dp_i.$$  

(3.5)

In the last expression the coefficient functions \(\Gamma_{\mu_1 \ldots \mu_s}^{a_1 \ldots a_s}\) are one-particle-irreducible amplitudes for the ultrasoft fields in the thermal equilibrium or more concisely—ultrasoft amplitudes (USA) introduced by Blaizot and Iancu [11]. USA represent the generalization of usual HTL-amplitudes [14] by including the effects of the collisions among the thermal particles, in this case among the hard transverse gluons.

Furthermore the classical parton current \(j_{Q\mu}^{a}\) is given by expression [6]

$$j_{Q\mu}^{a}[A](p) = j_{Q\mu}^{(0)a}(p) + \sum_{s=1}^{\infty} j_{Q\mu}^{(s)a}(A, \ldots, A),$$  

(3.6)

where \(j_{Q\mu}^{(0)a}(p) = g/(2\pi)^3 Q^a_0 \bar{\nu}_\mu \delta(\bar{\nu} \cdot p) \) is the initial current of the energetic parton, and

$$j_{Q\mu}^{(s)a}(A, \ldots, A) = \bar{\nu}_\mu \sum_{s=1}^{\infty} \frac{g^{s+1}}{(2\pi)^3} \int \frac{1}{(\bar{\nu} \cdot (p_1 + \ldots + p_s))(\bar{\nu} \cdot (p_2 + \ldots + p_s)) \ldots (\bar{\nu} \cdot p_s)}$$

$$\times (\bar{\nu} \cdot A^{a_1}(p_1)) \ldots (\bar{\nu} \cdot A^{a_s}(p_s)) \delta\left(\bar{\nu} \cdot \left(p - \sum_{i=1}^{s} p_i\right)\right) \prod_{i=1}^{s} dp_i (T^{a_1} \ldots T^{a_s})_{ab} Q^b_0.$$  

(3.7)

Finally, the fluctuation current \(\zeta_{\mu}^{a}\) functionally depending on both the gauge field and the noise term (by virtue of equation for fluctuating piece (2.7)) has a following structure

$$\zeta_{\mu}^{a}[A, \nu](p) = \zeta_{\mu}^{(0)a}[\nu](p) + \sum_{s=1}^{\infty} \zeta_{\mu}^{(s)a}[A, \nu](p).$$  

(3.8)
Here,

\[ \zeta^{(0)a}_\mu(p) \equiv \zeta^{(0)a}_\mu(p) = im^2_D \int \frac{d\Omega_v}{4\pi} \int \frac{d\Omega_{v_1}}{4\pi} \nu_\mu \langle v \rangle \left| \frac{1}{v \cdot p + i\xi} \right| v_1 \nu^a(p, v) \]  
(3.9)
is the fluctuation color current generated only by Langevin source on the right-hand side of the Boltzmann equation (2.3) and

\[ \zeta^{(s)a}(A, \nu)(p) \equiv \zeta^{(s)a}_\mu(p) = im^2_D g^s \int \frac{d\Omega_v}{4\pi} \int \prod_{i=1}^{s+1} \frac{d\Omega_{v_i}}{4\pi} \times v_\mu \langle v \rangle \left| \frac{1}{v \cdot p + i\xi} \right| v_1 \nu^{a_1} \cdots \nu^{a_{s+1}}(p_{s+1}, v_{s+1}) \]  
(3.10)

The next step is a derivation of a formal solution of basic field equation (3.1) by the approximation scheme method. Discarding the nonlinear terms in \( A_\mu \) and product \( A_{\mu\nu} \) on the right-hand side of Eq. (3.1), we obtain in the first approximation

\[ \ast D_{\mu\nu}^{-1}(p) A^{a\nu}(p) = -j_Q^{(0)a} \mu(p) - \zeta^{(0)a}_\mu(p), \]

where \( \zeta^{(0)a}_\mu(p) \) is defined by Eq. (3.9). The general solution of this equation is

\[ A^{a}_\mu(p) = A^{(0)a}_\mu(p) - \ast D_{\mu\nu}(p) \left\{ j_Q^{(0)a}(\nu)(p) + \zeta^{(0)a}(\nu)(p) \right\}. \]

Here, \( A^{(0)a}_\mu(p) \) is a solution of homogeneous equation (3.11) (free field), and the last term on the right-hand side represents the field induced by a high-energy color particle in medium and the noise term. From the physical point of view it is evident that at the ultrasoft momentum scale, where the effects of the collisions among the hard thermal particles are essential, free plasma waves in the system are to be absent. In other words, the dispersion equation \( \ast D_{\mu\nu}^{-1}(p) = 0 \) with inverse propagator (3.3) at the ultrasoft momentum scale defines a very significant damping rate of plasma waves preventing their existence and therefore one can set \( A^{(0)a}_\mu(p) \equiv 0 \). Thus the interaction field \( A_\mu(p) \) defined as a solution of Eq. (3.1) with current (3.2) will be represented in the form of a functional expansion in powers of the noise term \( \nu(p, \nu) \) (Guerin, [15]) and color charge \( Q_0^a \).

We can write out a more general structure of the interacting gauge field as follows:

\[ A^{a}_\mu(p) = -\ast D_{\mu\nu}(p) J^{(tot)\nu}(\nu, Q_0)(p), \]

(3.11)
where the total effective current is

$$J^{\text{tot}}_{\mu} [\nu, Q_0](p) = \sum_{n=0}^{\infty} J^{(n)a}_{\mu} [\nu](p) + \sum_{n=1}^{\infty} J^{(n)a}_{\mu} [\nu](p). \quad (3.12)$$

Here, the terms $J^{(n)a}_{\mu} [\nu](p)$ and $J^{(n)a}_{\mu} [\nu](p)$ are proportional to the $n$th power of $\nu$. $J^{(0)a}_{\mu} (p)$ is initial current of the energetic parton and $J^{(1)}_{\mu} [\nu](p) \equiv \zeta^{(0)a}_{\mu} [\nu](p)$ is initial fluctuation current. The last sum on the right-hand side of Eq. (3.12) is independent of color charge $Q_0$ and describes the processes of self-interaction of ultrasoft plasma fluctuations. The first sum contains relevant information on the interaction of the energetic parton with ultrasoft fluctuations. Below we will calculate the first correction $J^{(1)a}_{\mu}$ to the initial current $J^{(0)a}_{\mu} = g/(2\pi)^3 Q_0^2 \delta \nu \delta (\bar{v} \cdot p)$. For this purpose we need the relations

$$A^a_{\mu}(p)|_{\nu=Q_0=0} = 0, \quad \frac{\delta A^{\mu}(p)}{\delta Q_b^0}|_{\nu=Q_0=0} = -\frac{g}{(2\pi)^3} \delta^{ab} \ast D^{\mu
u}(p) \delta \nu \delta (\bar{v} \cdot p), \quad (3.13)$$

following from general formula (3.11).

According to the general approach to computation of effective currents \[6, 16\] the expression for the first correction $J^{(1)a}_{\mu}$ in the leading order in the coupling constant (i.e. a linear part over color charge $Q_0$) is defined by the following expression:

$$J^{(1)a}_{\mu} [\nu](p) = Q^0_0 \int dp' \int \frac{d\Omega_{\nu'}}{4\pi} v^b(p', \nu') \left[ \frac{\delta^2 J^{\text{ind}(2)a}_{\mu} [A](p)}{\delta v^b(p', \nu') \delta Q_0^0} + \frac{\delta^2 J^{(1)a}_{\mu} [A](p)}{\delta v^b(p', \nu') \delta Q_0^0} + \frac{\delta^2 \zeta^{(1)a}_{\mu} [A, \nu](p)}{\delta v^b(p', \nu') \delta Q_0^0} \right]|_{\nu=Q_0=0}. \quad (3.14)$$

By using explicit expressions for the currents $J^{\text{ind}(2)a}_{\mu}$, $J^{(1)a}_{\mu}$ and $\zeta^{(1)a}_{\mu}$ (Eqs. (3.5), (3.7) and (3.10)) and relations (3.13), after simple calculations we arrive at the expression for the desired current correction

$$J^{(1)a}_{\mu} [\nu](p) = -i(T^{\text{ab}})^{\nu}_{\text{c}} Q^0_0 \left( \frac{\alpha_s}{2\pi^2} \right) m^2_D \int dp' \int \frac{d\Omega_{\nu'}}{4\pi} R_{\mu}(p, p'; \bar{v}, \nu') i

\delta (\bar{v} \cdot (p - p')) \delta (\bar{v} \cdot p) \delta \nu \delta \nu', \quad (3.14)$$

where $\alpha_s = g^2/4\pi$, and

$$R_{\mu}(p, p'; \bar{v}, \nu') = -\frac{\bar{v}_\mu}{(\bar{v} \cdot p') \nu'} \int \frac{d\Omega_{\nu'}}{4\pi} \left( \bar{v}_\nu \ast T^{\nu\nu'}(p') \nu' \right) \langle v \mid \frac{1}{v \cdot p' + i\lambda} \mid v' \rangle \quad (3.15)$$

$$-i \Gamma_{\mu\nu\lambda}(p, -p', -p + p') \ast D^{\nu\nu'}(p') \int \frac{d\Omega_{\nu'}}{4\pi} \nu' \langle v \mid \frac{1}{v \cdot p' + i\lambda} \mid v' \rangle \ast D^{\lambda\nu'}(p' - p') \bar{v}_\nu.$$
\[
+ \int \frac{d\Omega_v}{4\pi} \int \frac{d\Omega_{v_1}}{4\pi} v_\mu \langle v' | v \rangle \frac{1}{v \cdot p + iC} | v_1 \rangle v_{1\nu} \langle v_1 | \frac{1}{v_1 \cdot p' + iC} | v' \rangle \ast D^{\nu\nu'} (p - p') \delta_{\nu'}. 
\]

Remind that the three-gluon vertex \( \Gamma_{\mu_1\mu_2} \) on the right-hand side of Eq. (3.15) is ultrasoft amplitude. Its explicit form is defined in Ref. [13]. The diagrammatic interpretation of different terms on the right-hand side of Eq. (3.15) is presented in Fig. 1.

Figure 1: The process of the scattering of the energetic parton off ultrasoft plasma fluctuations in the linear approximation over both noise \( \nu(p', \nu') \) and initial color charge of parton \( Q_0 \). The blob on the waveline stands for the ultrasoft-gluon propagator and 3-gluon ultrasoft vertex. The double line denotes energetic parton with velocity \( \tilde{v} \). Triple line denotes collision-resummed propagator \( \langle v | \frac{1}{v \cdot p' + iC} | v' \rangle \), and cross denotes the noise source.

We note also that the first two terms on the right-hand side of Eq. (3.15) can be presented in a more compact form

\[
- K^{(1)}_{\mu\nu} (\tilde{v}, p, -p') \ast D^{\nu\nu'} (p') \int \frac{d\Omega_v}{4\pi} v_{\nu'} \langle v | \frac{1}{v \cdot p' + iC} | v' \rangle, 
\]

where the function

\[
K^{(1)}_{\mu\nu} (\tilde{v}, p, -p') = \frac{\tilde{v}_\mu \tilde{v}_\nu}{\tilde{v} \cdot p'} + \Gamma_{\mu\nu\lambda} (p, -p', -p + p') \ast D^{\lambda\lambda'} (p - p') \delta_{\nu'}. 
\]

was introduced in work [6]. This function defines a matrix element of the nonlinear damping process of soft plasma excitations. In the next section we consider the convolution \( \tilde{u}_\mu(p) R^\mu (p, p'; \tilde{v}, v') \), where \( \tilde{u}_\mu(p) \) is the longitudinal projector in the covariant gauge (see Appendix). Such a convolution appears in the problem of the energy losses (Section 5). We prove that this convolution is gauge-invariant at least in the classes of temporal and covariant gauges in a weak sense, in spirit of Bödeker [11].
The general expression for a current correction of second order over $\nu$ in linear approximation over the color charge $Q_0$ has the following form:

$$J_{Q_0}^{(2)\nu}(p) = Q_0 g^4 \left( \frac{\alpha_s}{2\pi^2} \right) m_D^4 \int dp' \int dp'' \int \frac{d\Omega_{p'}}{4\pi} \int \frac{d\Omega_{p''}}{4\pi} \mathcal{R}_{\mu}^{abcd}(p', p''; \tilde{v}, v', v'') \times \left\{ \nu^b(p', v') \nu^c(p'', v'') - \left\langle \nu^b(p', v') \nu^c(p'', v'') \right\rangle \delta(\tilde{v} \cdot (p - p' - p'')) \right\}, \quad (3.17)$$

where

$$\mathcal{R}_{\mu}^{abcd}(p, p', p''; \tilde{v}, v', v'') = f^{abe} f^{ced} \mathcal{R}_{\mu}(p, p', p''; \tilde{v}, v', v'') + f^{ace} f^{bed} \mathcal{R}_{\mu}(p, p'', p'; \tilde{v}, v'', v'). \quad (3.18)$$

We will not write out an explicit expression for function $\mathcal{R}_{\mu}(p, p', p''; \tilde{v}, v', v'')$ by virtue of its excessive awkwardness. On the right-hand side of Eq. (3.17) we insert a term with $\left\langle \nu^2 \right\rangle$ to maintain stochasticity of the gauge field $A_{\mu}$ (condition $\left\langle A_{\mu} \right\rangle = 0$).

In closing this section we give also an expression for current $J_{\mu}^{(2)\nu}(p)$ in the second sum of expansion of the effective current $\mathcal{J}_{\mu}^{(2)\nu}(p)$. This current defines the simplest process of the nonlinear interaction of the thermal ultrasoft fluctuations produced by the noise term $\nu$ among themselves. Computing a second order variation in $\nu^2(p, v)$ of initial expression for the total current $\mathcal{J}_{\mu}^{(2)\nu}(p)$ and taking into account (3.18) we obtain

$$J_{\mu}^{(2)\nu}(p) = -g m_D^4 (T^v)^{\alpha_1 \alpha_2} \int dp_1 \int dp_2 \int \frac{d\Omega_{p_1}}{4\pi} \int \frac{d\Omega_{p_2}}{4\pi} T_{\mu}(p, -p_1, -p_2; v_1, v_2) \times \nu^{\alpha_1}(p_1, v_1) \nu^{\alpha_2}(p_2, v_2) \delta(p - p_1 - p_2),$$

where

$$T_{\mu}(p, -p_1, -p_2; v_1, v_2) \equiv \Gamma_{\mu\nu_1\nu_2}(p, -p_1, -p_2) * \mathcal{D}^{\nu_1\nu_2}(p_1) \left\{ \int \frac{d\Omega_v}{4\pi} \nu^{\nu_1} \left\langle v \right| \frac{1}{v \cdot p_1 + iC} \left| v_1 \right\rangle \right\} \times * \mathcal{D}^{\nu_1\nu_2}(p_2) \left\{ \int \frac{d\Omega_{v'}}{4\pi} v''^{\nu_2} \left\langle v' \right| \frac{1}{v' \cdot p_2 + iC} \left| v_2 \right\rangle \right\} + \int \frac{d\Omega_v}{4\pi} \int \frac{d\Omega_{v'}}{4\pi} \int \frac{d\Omega_{v''}}{4\pi} \nu^{\nu_1} \left\langle v \right| \frac{1}{v \cdot p + iC} \left| v' \right\rangle \times \left[ v''^{\nu_1} \left\langle v'' \right| \frac{1}{v'' \cdot p_1 + iC} \left| v_1 \right\rangle - v''^{\nu_2} \left\langle v'' \right| \frac{1}{v'' \cdot p_2 + iC} \left| v_2 \right\rangle \right].$$

The diagrammatic interpretation of different contributions determining the process of nonlinear self-interaction is presented on Fig.2.
Figure 2: The simplest process of nonlinear self-interaction of ultrasoft gluon fluctuations.

4 Gauge invariance of $\bar{u}_\mu(p)R^\mu(p, p'; \vec{v}, \nu')$

By using an explicit expression \((3.15)\) for the function $R^\mu(p, p'; \vec{v}, \nu')$ and the effective Ward identity for the ultrasoft three-gluon amplitude \([11, 13]\), we can represent the convolution $\bar{u}_\mu R^\mu$ in the following form

$$\bar{u}_\mu(p)R^\mu(p, p'; \vec{v}, \nu') = p^2R^0(p, p'; \vec{v}, \nu') + \omega \left[ \int \frac{d\Omega_\nu}{4\pi} \left( \bar{v}_\nu * D^{\nu\nu'}(p') \nu_{\nu'} \right) \left( \frac{1}{v \cdot p' + \i C} \right) \right]$$

$$+ \delta_{\lambda'}^{\nu'} \left( \frac{p'' - p' \lambda'}{p'^2} \right) * D^{\lambda\lambda'}(p - p') \bar{v}_{\lambda'} - \left( \delta_{\nu'}^{\nu'} - \frac{(p - p')^{\lambda'}(p - p')_\nu}{(p - p')^2} \right) \bar{v}_{\lambda'} * D^{\nu\nu'}(p') \right) \times$$

$$\times \int \frac{d\Omega_{\nu'}}{4\pi} v_{\nu'} \left( \frac{1}{v \cdot p' + \i C} \right) \nu_{\nu'} \left( \frac{1}{v \cdot p' + \i C} \right)$$

$$\left[ \int \frac{d\Omega_{v_1}}{4\pi} \left( v \cdot p \right) \nu_{v_1} \left( \frac{1}{v \cdot p + \i C} \right) v_{1\nu} \left( \frac{1}{v_1 \cdot p' + \i C} \right) \nu'_{1\nu'} \right].$$

In deriving \((4.1)\) we consider that $\bar{v} \cdot (p - p') = 0$ by virtue of $\delta$-function in integrand of Eq. \((3.14)\). Below we will show that on the right-hand side of Eq. \((4.1)\) all terms proportional to $\omega$ either equal to zero or mutually cancel out.

In projector

$$\left( \delta_{\nu'}^{\nu'} - \frac{(p - p')^{\lambda'}(p - p')_\nu}{(p - p')^2} \right)$$

in convolution with $\bar{v}_{\nu'}$, the last term vanishes by virtue of the above-mentioned constraint $\bar{v} \cdot (p - p') = 0$, and contribution of $\delta_{\nu'}^{\nu'}$ cancels with the second term on the right-hand side of Eq. \((4.1)\).
Furthermore we transform the last term on the right-hand side of Eq. (4.1). Here, we use a relation
\[
\int \frac{d\Omega_{\nu}}{4\pi} (v \cdot p) \left< v \left| \frac{1}{v \cdot p + i\hat{C}} \right| v_1 \right> = \int \frac{d\Omega_{\nu}}{4\pi} \delta^{(S^2)}(v - v_1) = 1,
\]
which is true by virtue of the property:
\[
\int \frac{d\Omega_{\nu}}{4\pi} v^4 \pi \left< v \left| \hat{C} \right| v_1 \right> = \int \frac{d\Omega_{\nu}}{4\pi} C(v, v_1) = 0.
\]
Taking into account the above-mentioned relation for the last term in Eq. (4.1) after replacement \( v_1 \to v \), we can present this term as follows:
\[
\int \frac{d\Omega_{\nu}}{4\pi} v^4 \pi v_{\nu} \left< v \left| \hat{C} \right| v' \right> \equiv -p_{\lambda}' D_{\lambda\lambda}' (p - p') \hat{v}_\lambda',
\]
that exactly cancels out with the first term from a projector
\[
\left( \delta_{\nu}' - \frac{p_{\nu}' p_{\lambda}'}{p^2} \right).
\]
Thus on the right-hand side of Eq. (4.1), besides a first term \( p^2 R^0 \) different from zero, the following term is left
\[
- \frac{p_{\lambda}'}{p^2} \cdot D^{\lambda\lambda'} (p - p') \hat{v}_\lambda \int \frac{d\Omega_{\nu}}{4\pi} (v \cdot p') \left< v \left| \frac{1}{v \cdot p' + i\hat{C}} \right| v' \right> \equiv - \frac{p_{\lambda}'}{p^2} \cdot D^{\lambda\lambda'} (p - p') \hat{v}_\lambda.
\]
Here, on the right-hand side we take into account relation (4.2). Thus this term is independent of unit vector \( v' \). The consequence of this important property will be the fact that in the expression for current \( J_{Q\mu}^{(1)}[v](p) \) (3.14) the solid integration \( \int d\Omega_{\nu'}/4\pi \) appears only in combination
\[
\int \frac{d\Omega_{\nu'}}{4\pi} \nu^b(p', \nu').
\]
Furthermore in subsequent discussion we will follow reasoning of Bödeker [1].

In the expression for the energy losses of energetic color parton (Eq. (5.1)) the noise \( \nu^b(p', \nu') \) enters only in the form of the 2-point function: \( \ll \nu \nu' \gg \). However the correlation function of \( \nu^a(p, v) \) and \( \int d\Omega_{\nu'}/4\pi \nu^b(p', \nu') \) vanishes
\[
\ll \nu^a(p, v) \int d\Omega_{\nu'}/4\pi \nu^b(p', \nu') \gg = 0
\]
due to (2.3) and (4.3). Thus we can set \( \int d\Omega_{\nu'}/4\pi \nu^b(p', \nu') \equiv 0 \), where the symbol ‘w’ denotes equality in a weak sense, so that the term (4.4) can be set equal to zero in a weak sense.
Taking into account the above-mentioned, we can write the convolution (4.1) in the form
\[ \bar{u}_\mu(p) R^\mu(p, p'; \hat{v}, v') \equiv p^2 R^0(p, p'; \hat{v}, v'). \] (4.5)
Furthermore we consider the convolution \( \bar{u}_\mu R^\mu \) in the temporal gauge. For this purpose we perform replacements (see Appendix)
\[ \bar{u}_\mu(p) \rightarrow \tilde{u}_\mu(p) = \frac{p^2}{(p \cdot u)}(p_\mu - u_\mu(p \cdot u)), \]
\[ *D_{\mu\nu}(p) \rightarrow *\tilde{D}_{\mu\nu}(p) = -P_{\mu\nu}(p) *\Delta^t(p) - \tilde{Q}_{\mu\nu}(p) *\Delta^l(p) + \xi_0 \frac{p^2}{(p \cdot u)^2} D_{\mu\nu}(p), \]
where \( \xi_0 \) is a gauge parameter fixing the temporal gauge. Then after analogous computations we come to the expression
\[ \tilde{u}_\mu(p) R^\mu(p, p'; \hat{v}, v') \equiv p^2 R^0(p, p'; \hat{v}, v'). \] (4.6)
All the terms on the right-hand side of Eq. (4.6) (as well as Eq. (4.5)) including gauge parameter vanish. Thus as follows from equalities (4.5) and (4.6), at least in the classes of the temporal and covariant gauges the function \( \bar{u}_\mu R^\mu \) is gauge-invariant in a weak sense.

5 Energy losses of energetic parton induced by scattering off ultrasoft gluon fluctuations

As an application of the formalism developed in previous sections we consider the problem of calculation of the energy losses of the energetic parton induced by scattering process off ultrasoft boson excitations of the medium. As initial expression for the energy loss a classical expression for parton energy loss per unit length (Eq. (7.1) in [6]) with replacement of the expectation value \( \langle \cdot \rangle \) over statistical ensemble by averaging \( \langle \langle \cdot \rangle \rangle \) over the random white noise) is used
\[ \left( -\frac{dE}{dx} \right)_{\text{ul,soft}} = \frac{1}{|V|} \lim_{\tau \to \infty} \frac{1}{\tau^{3/2}} \int d\mathbf{x} dt \int dQ_0 \Re \langle J^a_\mu(x, t) \cdot E^a_\mu(x, t) \rangle \] (5.1)
\[ = -\frac{1}{|V|} \lim_{\tau \to \infty} \frac{(2\pi)^4}{\tau} \int d\mathbf{p} d\omega \int dQ_0 \omega \Im \langle J^a_\mu(p) *\tilde{D}^{\mu\nu}(p) J^a_{\nu}(p) \rangle. \]
Here, the color integration is
\[ \int dQ_0 = \prod_{a=1}^{d_A} dQ^a_0 \delta(Q^a_0 Q^a_0 - C_2), \quad d_A = N^2_c - 1, \]
where $C_2$ is the second order Casimir ($C_2 = C_A$ for energetic gluon and $C_2 = C_F$ for energetic quark). In the last line of Eq. (5.1) $J_{Q_0}(p) \equiv J_{Q_0}^0 (\nu, Q_0 | p) \approx \sum_{n=0}^{\infty} J_{Q_0}^{(n)0}(\nu | p) Q_0$ is the effective color current of hard parton and $*\tilde{D}^{\mu\nu}(p)$ is a gluon propagator (in the temporal gauge) at the ultrasoft momentum scale

\[ *\tilde{D}^{ij}(p) = \left( \frac{p^2}{\omega^2} \right) \frac{p_i^j}{p^2} *\Delta^l(p) + \left( \delta^{ij} - \frac{p_i^j}{p^2} \right) *\Delta^t(p), \quad *\tilde{D}^{0i}(p) = *\tilde{D}^{00}(p) = 0. \quad (5.2) \]

First of all, we write out the expression for the energy loss connected with initial current $J_{Q_0}^0 | \nu, Q_0 | p \equiv g/(2\pi)^3 \delta^{ab} \tilde{v}_\mu \delta(\tilde{v} \cdot p) Q_0^a$. Substituting this current into (5.1), taking into account the structure of propagator (5.2) and rules

\[ \int dQ_0 \; Q_0^a Q_0^b = \frac{C_2}{d_A} \delta^{ab}, \quad \left[ \delta(v \cdot p) \right]^2 = \frac{1}{2\pi} \tau \delta(v \cdot p), \]

we obtain

\[ \left( - \frac{dE^{(0)}}{dx} \right)_{\text{ul,soft}} = - \frac{1}{|\tilde{v}|} \left( \frac{N_c \alpha_s}{2\pi^2} \right) \left( \frac{C_2}{N_c} \right) \int dp \; \frac{\omega}{p^2} \left( \text{Im} \left( p^2 \Delta^l(p) \right) + (\tilde{v} \times p)^2 \text{Im} \left( *\Delta^t(p) \right) \right) \delta(\tilde{v} \cdot p). \quad (5.3) \]

This expression defines the polarization losses of energetic parton connected with large distance collisions [17]. However unlike [17] here integrand (and in particular scalar propagators $*\Delta^t(p) = 1/(p^2 - \Pi^t(p))$) is defined at the ultrasoft momentum scale.

Now we define the expression for the energy losses produced by the following term in the expansion of the effective current: $J_{Q_0}^{(1)0} (\nu, Q_0 | p) \approx J_{Q_0}^{(1)ab} (\nu | p) Q_0^b$ (Eq. (3.14), (3.15)). For this purpose we substitute the current (3.14) into general expression for the energy losses (5.1). For the sake of simplification we keep only a longitudinal part in propagator $*\tilde{D}^{\mu\nu}(p)$

\[ *\tilde{D}^{\mu\nu}(p) \rightarrow - \tilde{Q}^{\mu\nu}(p) *\Delta^l(p) \equiv - \frac{\tilde{u}^{\mu}(p) \tilde{u}^{\nu}(p)}{\tilde{u}^2(p)} *\Delta^l(p). \quad (5.4) \]

Integrating over initial color charge, averaging with respect to the noise term and taking into account weak equality (4.6), we obtain the following (after (5.3)) contribution to the energy loss of the energetic parton

\[ \left( - \frac{dE^{(1)l}}{dx} \right)_{\text{ul,soft}} = -(2\pi)^7 \frac{2T m_D^2}{|\tilde{v}|} \left( \frac{N_c \alpha_s}{2\pi^2} \right)^2 \left( \frac{C_2}{N_c} \right) \int dp \int dp' \left( \frac{\omega}{p^2 p'^2} \right) \text{Im} \left( p^2 \Delta^l(p) \right) \delta(\tilde{v} \cdot (p - p')) \]

\[ \times \int \frac{d\Omega_v}{4\pi} \int \frac{d\Omega_{v'}}{4\pi} \left[ \mathcal{R}_0(p, p'; \tilde{v}, v) \mathcal{C}(v, v') \mathcal{R}_0^*(p, p'; \tilde{v}, v') \right]. \quad (5.5) \]
Here, the function $R_0(p, p'; \bar{v}, v')$ is defined by Eq. (3.15), and $C(v, v')$ is a collision kernel defined by Eq. (2.3). The contribution to the energy loss (5.5) of the term (3.16) can be presented in the following form:

$$
- \frac{(2\pi)^{7}}{|\bar{v}|} m^2_D \left( \frac{N_c \alpha_s}{2\pi^2} \right)^2 \left( \frac{C_2}{N_c} \right) \int dp \int dp' \left( \frac{\omega}{p^2} \right) \mathrm{Im} \left( p^2 \Delta^l(p) \right) \delta(\bar{v} \cdot (p - p')) \tag{5.6}
$$

$$
\times \left[ K_{0\nu}^{(1)}(\bar{v} | p, -p') \ast \bar{D}^{\nu\nu'}(p') \left( -\frac{2T}{\omega'} \right) \mathrm{Im} \Pi_{\nu'\lambda}(p') K_{0\lambda}^{(1)}(\bar{v} | p, -p') \ast \bar{D}^{\lambda\lambda'}(p') \right],
$$

where the imaginary part of the retarded polarization tensor is $\Pi_R^{(1)}(p) = -\omega m^2_D \int d\Omega_{\nu} \int d\Omega_{\nu'} \int d\Omega_{\nu_1} \int d\Omega_{\nu_2} v_{\mu} v_{\nu} \langle v_1 \rangle C(v_1, v_2) \langle v' \rangle \frac{1}{v \cdot p - i \hat{C} | v_2 \rangle}$.

Furthermore, if in propagators $\ast \bar{D}^{\nu\nu'}(p')$ and $\ast \bar{D}^{\lambda\lambda'}(p')$ we keep only longitudinal part (5.4), then expression (5.6) is written in the compact form (cp. with Eq. (7.11) in [6])

$$
- \frac{(2\pi)^{7}}{|\bar{v}|} m^2_D \left( \frac{N_c \alpha_s}{2\pi^2} \right)^2 \left( \frac{C_2}{N_c} \right) \int dp \int dp' \left( \frac{\omega}{p^2} \right) \mathrm{Im} \left( p^2 \Delta^l(p) \right) \left( \frac{\omega'}{p'^2} \right) \mathrm{Im} \left( p^2 \Delta^l(p') \right) \tag{5.7}
$$

$$
\times \frac{1}{\omega'} \left| K_{0\nu}^{(1)}(\bar{v} | p, -p') \bar{u}^{\nu}(p') \right|^2 \delta(\bar{v} \cdot (p - p')).
$$

We note that the imaginary part of the polarization tensor in Eq. (5.6) appears in the combination

$$
\left( -\frac{2T}{\omega'} \right) \mathrm{Im} \Pi_R^{(1)}(p').
$$

This function is the simplest example of (2-point) function of a new set of n-point amplitudes (additional to the ultrasoft amplitudes $\Gamma_{\mu_1 \ldots \mu_n}^{a_1 \ldots a_n}$) proposed by Guerin [15, 18] needed to fulfill the Kubo-Martin-Schwinger constraints. The existence of these amplitudes is related to the presence of the damping caused by the collision operator $\hat{C}$.

Unfortunately the third term on the right-hand side of Eq. (3.16) generated by the fluctuation current $\zeta_\mu$ gives no possibility to write out a complete expression for the energy loss $(-dE^{(1)}(dE)_{ul,soft}$ in a simpler and clear form similar to equation (5.7). In general case we can proceed as follows. We expand the collision kernel (2.3) in spherical harmonics (normalized by unit)

$$
C(v, v') = 4\pi \gamma \sum_{l,m} \sum_{l',m'} c_{lm, lm'} Y_l^m(v) Y_{l'}^{m'}(v').
$$

By virtue of the fact that the spherical harmonics are eigenvectors of the collision kernel, i.e., $\gamma^{-1}\hat{C} Y_l^m(v) = \lambda_l Y_l^m(v)$, the coefficients $c_{lm, lm'}$ in the expansion (5.8) are simply expressed in terms of eigenvalues $\lambda_l$ of the collision operator

$$
c_{lm, lm'} = (-1)^{l'-m'} \lambda_l \delta_{m,-m'} \delta_{l,l'}.
$$

15
where
\[ \lambda_l = 1 - \frac{2}{\pi} \int_{-1}^{+1} dz \frac{z^2}{\sqrt{1-z^2}} P_l(z) \]
and \( P_l(z) \) are Legendre polynomials. The explicit expressions for \( \lambda_l \) are [12]
\[ \lambda_{2n} = 1 - 2 \left[ \frac{(2n)!}{2^{2n}(n!)^2} \right]^2 \left( 1 + \frac{1}{(2n)^2 + 2n - 2} \right) \geq 0, \quad (5.10) \]
\[ \lambda_{2n+1} = 1, \quad n = 0, 1, 2, \ldots . \]
Substituting the expansion (5.8) in (5.5), taking into account (5.9) and property \( Y_{l}^{*m}(\nu) = (-1)^{l-m}Y_{l}^{-m}(\nu) \), we can write the expression for the energy loss (5.5) in the form of an expansion in modules squared of “partial amplitudes”
\[ \left( -\frac{dE^{(1)}_l}{dx} \right)_{ul,soft} = -\frac{(2\pi)^6}{|\vec{v}|} \left[ 2^m D_l \left( \frac{N_c \alpha_s}{2\pi^2} \right)^4 \left( \frac{C_2}{\alpha_s} \right) \int dp \int dp' \left( \frac{\vec{p}}{p^2} \right) \text{Im} \left( p^2 \Delta_l^l(p) \right) \delta(\vec{v} \cdot (p - p')) \times \left[ \sum_{n=0}^{\infty} \sum_{m=-(2n+1)}^{\infty} |R_{2n+1}^{(m)}(p, p'; \vec{v})|^2 + \sum_{n=1}^{\infty} \lambda_{2n} \sum_{m=-2n}^{m=2n} |R_{2n}^{(m)}(p, p'; \vec{v})|^2 \right] \right], \quad (5.11) \]
where
\[ R_{l}^{(m)}(p, p'; \vec{v}) = \int d\Omega_{\nu'} R_{0}(p, p'; \vec{v}, \nu') Y_{l}^{m}(\nu') \]
are coefficient functions in the expansion of \( R_{0} \) in spherical harmonics with respect to unit vector \( \nu' \). By virtue of (5.10) the expression in square brackets in Eq. (5.11) is positively definite and therefore the right-hand side of (5.11) has a positive sign under condition \( \text{Im} \left( p^2 \Delta_l^l(p) \right) \leq 0 \).

Let us write out the expression for the energy losses following from the current correction of the second order in \( \nu \): \( J_{Q_{ul}}^{(2)}[\nu](p) \) (Eqs. (3.17), (3.18)). The reasoning analogous above-mentioned leads to the following formula
\[ \left( -\frac{dE^{(2)}_l}{dx} \right)_{ul,soft} = -\frac{2}{|\vec{v}|} \left[ (2\pi)^6 T_{\gamma} \right] \left[ 2^m D_l \left( \frac{N_c \alpha_s}{2\pi^2} \right)^4 \left( \frac{C_2}{\alpha_s} \right) \int dp \int dp' \int dp'' \left( \frac{\vec{p}}{p^2} \right) \text{Im} \left( p^2 \Delta_l^l(p) \right) \right] \sum_{l, m, \tilde{m}} \lambda_l \lambda_{\tilde{m}} \left[ |R_{l, l}^{(m, \tilde{m})}(p, p', p''; \vec{v})|^2 + |R_{\tilde{l}, l}^{(\tilde{m}, m)}(p, p''', \vec{v})|^2 \right] + \text{Re} \left( R_{l, l}^{(m, \tilde{m})}(p, p', \vec{v}) R_{l, l}^{(\tilde{m}, m)}(p, p'', \vec{v}) \right) \delta(\vec{v} \cdot (p - p' - p'')), \quad (5.12) \]
where \( \sum_{l, m} = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \) and
\[ R_{l, l}^{(m, \tilde{m})}(p, p', p''; \vec{v}) = \int d\Omega_{\nu'} \int d\Omega_{\nu''} R_{0}(p, p', \vec{v}, \nu', \nu'') Y_{l}^{m}(\nu') Y_{l}^{\tilde{m}}(\nu''). \]
In deriving (5.12) we consider that the function \( R(\mu, p, p', p''; \bar{v}, v', v'') \) satisfies relation similar to (4.6). Note that if we introduce a new functions

\[
R(M, \tilde{M})(p, p', p''; \tilde{v}) \equiv \frac{1}{2} \left( R(M, \tilde{M})(p, p', p''; \tilde{v}) + R(M, \tilde{M})(p, p', p''; \tilde{v}) \right),
\]

\[
R^{(A)}(\mu, \tilde{M})(p, p', p''; \tilde{v}) \equiv \frac{1}{2} \left( R(M, \tilde{M})(p, p', p''; \tilde{v}) - R(M, \tilde{M})(p, p', p''; \tilde{v}) \right),
\]

then an expression within parentheses in integrand of Eq. (5.12) is rewritten in the following form (for the sake of brevity we suppress arguments of functions)

\[
|R(M, \tilde{M})(p, p', p''; \tilde{v})|^2 + |R^{(A)}(\mu, \tilde{M})(p, p', p''; \tilde{v})|^2 + \text{Re} \left( R(M, \tilde{M})(p, p', p''; \tilde{v}) R^{*}(M, \tilde{M})(p, p', p''; \tilde{v}) \right) = 3 |R^{(S)}(\mu, \tilde{M})(p, p', p''; \tilde{v})|^2 + |R^{(A)}(\mu, \tilde{M})(p, p', p''; \tilde{v})|^2.
\]

The right-hand side of the last expression is explicitly positive definite.

By using the explicit expressions for the energy losses, one can roughly estimate their order in the coupling constant. For values \( \omega \sim g^4 T, |p| \sim g^2 T \) from Eqs. (5.3), (5.11) and (5.12) the following estimations can be obtained:

\[
\left( -\frac{dE}{dx} \right)_{\text{ul,soft}} \sim \alpha_s^4 T^2,
\]

\[
\left( -\frac{dE}{dx} \right)_{\text{ul,soft}} \sim \alpha_s^4 T^2,
\]

\[
\left( -\frac{dE}{dx} \right)_{\text{ul,soft}} \sim \alpha_s^4 T^2
\]

up to the possible logarithmic factor \( \ln(1/g) \). It is seen that at the ultrasoft momentum scale the energy losses are parametrically strongly suppressed for \( \alpha_s \ll 1 \) as compared with the losses induced by scattering of the energetic parton off plasma excitations at the soft momentum scale [6]. However the estimations (5.13) suggest that not only the first three terms, but all terms of higher orders will be values of the same order in the coupling constant, i.e., \( (dE^{(n)})/dx_{\text{ul,soft}} \sim \alpha_s^4 T^2, n > 2 \). Thus from formal point of view for the estimation of real value of the energy losses we must take into account the whole series \( \sum_{n=0}^{\infty} J^{(n)ab}[\nu](p) Q^0_0 \) in initial expression for the energy losses (5.1).

6 Conclusion

In this work the procedure of calculation of the effective current generating the processes of the nonlinear interaction of the energetic color particle with the ultrasoft gluon fluctuations was proposed. The energy losses associated with these processes are also taken into account. We have established that the contribution of each term in the expansion of the effective current in powers of the noise term \( \nu(X, \mathbf{v}) \) to the energy losses is strongly suppressed in the coupling\(^5\) and all these contributions are of the same order in \( g \). By

\(^5\)This means that probably, the mechanism is irrelevant for phenomenology at RHIC.
virtue of the last circumstance perturbation approach for calculation of the energy losses breaks down at the ultrasoft momentum scale. The complete calculation of the energy losses \( (dE/dx)_{\text{ul,soft}} \) turns out to be non-perturbative and thus it requires an all-order resummation that cannot be performed via the methods developed here.

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Appendix

Here we present an explicit form of projection operators \( P_{\mu\nu}(p) \) and \( Q_{\mu\nu}(p) \) in two gauges:

- in the **covariant gauge**

\[
P_{\mu\nu}(p) = g_{\mu\nu} - D_{\mu\nu}(p) - Q_{\mu\nu}(p), \quad Q_{\mu\nu}(p) = \frac{\tilde{u}_{\mu}(p)\tilde{u}_{\nu}(p)}{\tilde{u}^2(p)} , \quad D_{\mu\nu}(p) = \frac{p_{\mu}p_{\nu}}{p^2} ,
\]

\[
\tilde{u}_{\mu} = p^2 u_{\mu} - p_{\mu} (p \cdot u),
\]

- in the **temporal gauge**

\[
P_{\mu\nu}(p) = g_{\mu\nu} - u_{\mu} u_{\nu} - \frac{(p \cdot u)^2}{p^2} \tilde{Q}_{\mu\nu}(p) , \quad \tilde{Q}_{\mu\nu}(p) = \frac{\tilde{u}_{\mu}(p)\tilde{u}_{\nu}(p)}{\tilde{u}^2(p)} ,
\]

\[
\tilde{u}_{\mu}(p) = \frac{p^2}{(p \cdot u)} (p_{\mu} - u_{\mu} (p \cdot u)),
\]

where \( u_{\mu} \) is global 4-velocity of the non-Abelian plasma. One assume that we are in a rest frame of a heat bath, so that \( u_{\mu} = (1,0,0,0) \).
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