Cosmic evolution during primordial black hole evaporation

Winfried Zimdahl
Fakultät für Physik, Universität Konstanz, PF 5560 M678, D-78457 Konstanz, Germany

Diego Pavón
Departamento de Física Universidad Autónoma de Barcelona, 08193 Bellaterra (Barcelona), Spain

(March 21, 2022)

PACS numbers 98.80Hw, 04.70Dy, 05.70.Ln

Primordial black holes with a narrow mass range are regarded as a nonrelativistic fluid component with an equation of state for dust. The impact of the black hole evaporation on the dynamics of the early universe is studied by resorting to a two-fluid model. We find periods of intense radiation reheating in the initial and final stages of the evaporation.

I. INTRODUCTION

The interest in primordial black holes (PBHs) dates back to Zel’dovich and Novikov [1], Hawking [2], and Carr and Hawking [3], who concluded they should be produced by density fluctuations in the primordial cosmic fluid, leading to a wide mass spectrum for these objects. The earlier their formation, the lower their mass. Since then there has been a manifold of proposals for the production of PBHs: Inhomogeneities triggered in the inflationary period of cosmic expansion [4], first order phase transitions [5,6], bubble wall collisions [7], the Gross-Perry-Yaffe mechanism of quantum gravitational tunneling from hot radiation [8] (see also [9] and [10]), quantum pair creation (based on the nonboundary proposal of the wave function of the universe) during inflation [11], quantum fluctuations in hybrid inflation [12] (see also [13]), and the decay of cosmic loops [14], to mention just a few.

This interest in PBHs is understandable as they may have had a notable impact on several areas of current interest, such as baryogenesis [15], dark matter [16], galactic nucleation [17], and the reionization of the universe [18]. In addition they can constrain the spectral index of primordial density perturbations [19].

A natural outcome of the copious production of black holes in some of the mentioned scenarios is that, sooner or later, a point will be reached where the energy density of the PBH component will significantly contribute to the energy density of the cosmic medium. For example, phase transitions based on grand unified theories [1] or bubble wall collisions in extended inflation [16] may lead to a PBH dominated universe. Also, in scenarios of hybrid inflation there may exist a regime in which the black holes will dominate the total energy density soon after the end of inflation [12]. This situation may last for some time before the contribution of the latter component to the total energy density becomes negligible because of its evaporation via Hawking’s process. (This stage has been termed “the binary phase” and studied in detail [21] for PBHs produced by the Gross-Perry-Yaffe mechanism.)

PBH formation processes from phase transitions, e.g. in connection with extended inflation, are characterized by a very narrow mass spectrum (see, e.g., [22] and references therein). In such a situation the assumption of only one mass $m_{(BH)}$ for all members of the population may be considered a reasonable approximation. On the other hand, a black hole of mass $m_{(BH)}$ may thermodynamically be characterized by a temperature $T_{(BH)} \propto m_{(BH)}^{-1}$ [23]. Consequently, with a single mass population of PBHs one may associate a single temperature as well. This suggests a picture of the PBH component in which $T_{(BH)}$ in some respect plays the role of an equilibrium fluid temperature.

On the basis of this interpretation it is the target of the present paper to study the cosmological dynamics during PBH evaporation within a model of two interacting and reacting fluids. The evaporation process in this model is pictured as a decay of the PBH “fluid” into a conventional fluid. We discuss the reheating of the latter as a consequence of the evaporation and show that one may describe this process in terms of a bulk pressure of the cosmic medium as a whole. The general formalism is then applied to the dynamics of a Friedmann-Lemaître-Robertson-Walker (FLRW) universe and the back reaction of the PBH evaporation on the cosmic scale factor is investigated.

To establish a transparent picture we introduce the following simplifying assumptions from the outset: (i) the PBHs can be thought of as particles of a perfect fluid with the equation of state for dust, (ii) both fluids share the same four-velocity, (iii) all the PBHs start evaporating at the same time. Once these assumptions are accepted one can apply the formalism of interacting and reacting
fluids to the mixture of both components (say radiation and the PBH fluid) [24]. Notice that if the PBHs did not evaporate, the evolution of the scale factor and the energy densities of both components would directly be given by the well-known results of Jacobs for a mixture of relativistic particles and dust [23]. However the fact that the PBHs radiate makes a big difference with respect to the rather traditional case of a mixture of radiation and dust with conserved fluid particle numbers.

A closely related work of Barrow et al. [24] considers the evaporation of PBHs with an initial power-law mass spectrum. The black holes in that paper do not constitute, however, a fluid in the sense discussed here (such kind of fluid picture looks less natural for a nonsingular distribution of masses). In general, the dynamical behavior of both configurations is different, especially with respect to the existence of solutions for which the energy density in PBHs is in equilibrium with radiation.

This paper is organized as follows. Section II recalls the general hydrodynamical formalism for two interacting and reacting perfect fluids [24,25] which previously has been applied to the reheating phase at the end of the inflationary expansion [26,27]. Section III specifies this framework to a mixture of thermal radiation plus primordial black holes and derives a general formula for the specific structure of the evaporation process in terms of an effective bulk viscous pressure and corresponding entropy production density, which is then used to discuss specific features of the evaporation process, especially a “reheating” of the radiation. The evolution equations for the energy densities of each fluid coupled to the Friedmann equation are solved numerically. Section IV introduces an effective one-component model of the cosmic medium and describes the PBH evaporation process in terms of an effective bulk viscous pressure of the system as a whole. We present a general formula for the latter, which is evaluated explicitly in section V for the initial stages of the evaporation process. Finally, section VI summarizes the results of this work. Units have been chosen so that \( c = k_B = \hbar = G = 1 \).

II. THE GENERAL TWO-FLUID MODEL

The energy-momentum tensor \( T^{ik} \) of the cosmic medium is assumed to split into two perfect fluid parts,

\[
T^{ik} = T^{ik}_{(1)} + T^{ik}_{(2)},
\]

where \( \rho_{(A)} \) denotes the energy density and \( p_{(A)} \) the equilibrium pressure of species-\( A \) particles. For simplicity we assume both components to share the same four-velocity \( u^i \), normalized so that \( u^i u_i = -1 \). The tensor \( h^{ik} = g^{ik} + u^i u^k \) projects any vectorial quantity on the hypersurface orthogonal to \( u^i \). The particle flow vector \( N^i_{(A)} \) of species \( A \) is defined as

\[
N^i_{(A)} = n_{(A)} u^i ,
\]

where \( n_{(A)} \) is the particle number density. We are interested in situations where neither the particle numbers nor the energy-momenta of the components are separately conserved, i.e., conversion of particles and exchange of energy and momentum between the components are admitted. The balance laws for the particle numbers are

\[
N^i_{(A)} = \dot{n}_{(A)} + \Theta n_{(A)} = n_{(A)} \Gamma_{(A)},
\]

where \( \Theta \equiv u^i \) is the fluid expansion and \( \Gamma_{(A)} \) is the rate of change of the number of particles of species \( A \). There is particle production for \( \Gamma_{(A)} > 0 \) and particle decay for \( \Gamma_{(A)} < 0 \), respectively. For \( \Gamma_{(A)} = 0 \) we have separate particle number conservation (“detailed balance”).

Interactions between the fluid components amount to the mutual exchange of energy and momentum. Consequently, there will be no local energy-momentum conservation for the subsystems separately. Only the energy-momentum tensor of the system as a whole is conserved.

Denoting the loss and source terms in the separate balances by \( t^i_{(A)} \), we write

\[
T^{ik}_{(A);k} = -t^i_{(A)},
\]

implying

\[
\dot{\rho}_{(A)} + \Theta (\rho_{(A)} + p_{(A)}) = u^a t^a_{(A)},
\]

and

\[
(\rho_{(A)} + p_{(A)}) \dot{a}^a + p_{(A)} h^{ak} = -\dot{h}^a t^i_{(A)}.
\]

All the considerations to follow will be independent of the specific structure of the \( t^i_{(A)} \). In other words, there are no limitations on the strength or the structure of the interaction.

Each component is governed by a separate Gibbs equation:

\[
T_{(A)} ds_{(A)} = d \frac{\rho_{(A)}}{n_{(A)}} + \frac{p_{(A)}}{n_{(A)}} \frac{1}{n_{(A)}},
\]

where \( T_{(A)} \) and \( s_{(A)} \) are the temperature and entropy per particle of species \( A \), respectively. Using the balances (1) and (3), one finds for the time behavior of the entropy per particle

\[
n_{(A)} T_{(A)} \dot{s}_{(A)} = u^a t^a_{(A)} - (\rho_{(A)} + p_{(A)}) \Gamma_{(A)},
\]

With nonvanishing source terms in the balances for \( n_{(A)} \) and \( \rho_{(A)} \), the change in the entropy per particle is different from zero in general.

The equations of state of the fluid components are assumed to have the general form

\[
p_{(A)} = p_{(A)} (n_{(A)}, T_{(A)}), \quad \rho_{(A)} = \rho_{(A)} (n_{(A)}, T_{(A)}),
\]
i.e., particle number densities $n_{(A)}$ and temperatures $T_{(A)}$ are regarded as the basic thermodynamical variables. The temperatures of the fluids are different in general.

Differentiating the last of the relations (11), using the balances (4) and (6) as well as the general relation

$$\frac{\partial \rho_{(A)}}{\partial n_{(A)}} = \frac{\rho_{(A)} + \rho_{(A)}}{n_{(A)}} - \frac{T_{(A)}}{n_{(A)}} \frac{\partial \rho_{(A)}}{\partial T_{(A)}}$$

(11)

that follows from the requirement that the entropy is a state function, we find the following expression for the temperature behavior [30]:

$$\dot{T}_{(A)} = -T_{(A)} (\Theta - \Gamma_{(A)}) \frac{\partial \rho_{(A)}}{\partial T_{(A)}} + \frac{u_{a} t_{a}^{(A)} - \Gamma_{(A)} (\rho_{(A)} + p_{(A)})}{\partial \rho_{(A)} / \partial T_{(A)}}$$

(12)

Here we have used the abbreviations

$$\frac{\partial \rho_{(A)}}{\partial T_{(A)}} \equiv \frac{\partial \rho_{(A)}}{\partial T_{(A)}} \, \frac{\partial T_{(A)}}{\partial n_{(A)}}$$

$$\frac{\partial \rho_{(A)}}{\partial T_{(A)}} \equiv \left( \frac{\partial \rho_{(A)}}{\partial T_{(A)}} \right)_{n_{(A)}}$$

etc.

Both the source terms $\Gamma_{(A)}$ and $u_{a} t_{a}^{(A)}$ in Eqs. (11) and (12) back react on the temperature. The numerator of the second term on the right-hand side of Eq. (14) coincides with the right-hand side of Eq. (13), i.e., the corresponding terms disappear in the special case $\dot{s}_{(A)} = 0$.

For $\Gamma_{(A)} = u_{a} t_{a}^{(A)} = 0$ and with $\Theta = 3a/a$, where $a$ is a characteristic length scale (which in the homogeneous and isotropic case coincides with the scale factor of the Robertson-Walker metric), the equations of state $p_{(2)} = n_{(2)} T_{(2)}$, $\rho_{(2)} = 3n_{(2)} T_{(2)}$ for radiation (subindex (2)) reproduce the well-known $T_{(2)} \sim a^{-1}$ behavior. With $p_{(1)} = n_{(1)} T_{(1)}$, $\rho_{(1)} = n_{(1)} m + \frac{2}{3} n_{(1)} T_{(1)}$ one recovers $T_{(1)} \sim a^{-2}$ for matter (subindex (1)).

The entropy flow vector $S_{(A)}^{a}$ is defined by

$$S_{(A)}^{a} = n_{(A)} s_{(A)} u_{(A)}$$

(13)

and the contribution of component $A$ to the entropy production density becomes

$$S_{(A):a}^{a} = n_{(A)} s_{(A)} u_{(A)} \dot{\rho}_{(A)} + n_{(A)} \dot{\rho}_{(A)}$$

$$= \left( \frac{\rho_{(A)} + p_{(A)}}{n_{(A)} T_{(A)}} \right) n_{(A)} \dot{\rho}_{(A)} + \frac{u_{a} t_{a}^{(A)}}{T_{(A)}}$$

(14)

where relation (11) has been used.

According to the balances (11) the condition of energy-momentum conservation for the system as a whole,

$$\left( T_{(1)} + T_{(2)} \right)_{:k} = 0$$

implies

$$\dot{t}_{(1)} = -\dot{t}_{(2)}$$

(15)

There is no corresponding condition, however, for the particle number balance as a whole. Defining the integral particle number density $n$ as $n = n_{(1)} + n_{(2)}$, we have

$$\dot{n} + \Theta n = n_{(1)} \dot{\rho}_{(1)} + n_{(2)} \dot{\rho}_{(2)}$$

(16)

$\Gamma$ is the rate by which the total particle number $n$ changes. We do not require $\Gamma$ to be zero since total particle number conservation is only a very special case.

The entropy per particle is

$$s_{(A)} = \frac{\rho_{(A)} + p_{(A)}}{n_{(A)} T_{(A)}} - \frac{\mu_{(A)}}{T_{(A)}}$$

(18)

where $\mu_{(A)}$ is the chemical potential of species $A$. Introducing the expression (18) into Eq. (14) yields

$$S_{(A):a}^{a} = \frac{\mu_{(A)}}{T_{(A)}} n_{(A)} \dot{\rho}_{(A)} + \frac{u_{a} t_{a}^{(A)}}{T_{(A)}}$$

(19)

For the total entropy production density $S_{a}^{a} = S_{(1):a}^{a} + S_{(2):a}^{a}$ we obtain

$$S_{a}^{a} = \frac{\mu_{(2)}}{T_{(2)}} n_{(2)} - \frac{\mu_{(1)}}{T_{(1)}} n_{(1)} \dot{\rho}_{(1)}$$

$$+ \left( \frac{1}{T_{(1)}} - \frac{1}{T_{(2)}} \right) u_{a} t_{a}^{(1)}$$

(20)

The condition $S_{a}^{a} = 0$ requires the well-known equilibrium conditions (see, e.g., [32] (chapter 5))

$$\mu_{(1)} = \mu_{(2)} ; T_{(1)} = T_{(2)}$$

(21)

as well as $\Gamma = 0$.

III. BASIC RELATIONS FOR AN EVAPORATING BLACK HOLE COMPONENT

In this section we apply the formalism outlined above to a mixture of massless radiation (component 2) and evaporating PBHs (component 1). We assume all the black holes of the same mass $m_{(BH)}$, and that they evaporate solely into radiation particles, though this is admittedly a rough approximation [33]. The PBH component is treated as a nonrelativistic fluid with

$$p_{(BH)} = 0$$

(22)

and the energy density
The number $N_{(BH)}$ of PBHs in a comoving volume $a^3$, $N_{(BH)} = n_{(BH)} a^3$, is not preserved and, according to Eq. (4), we may write down a balance equation for the corresponding PBH number flow vector $N'_{(BH)} = n_{(BH)} u^i$. 

$$N'_{(BH);i} = \dot{n}_{(BH)} + \Theta n_{(BH)} = n_{(BH)} \Gamma_{(BH)} .$$

(24)

From Eq. (23) the black hole energy balance becomes [cf. Eq. (3)]

$$\dot{\rho}_{(BH)} + \Theta \rho_{(BH)} = u_a t^{a}_{(BH)}$$

(25)

with

$$u_a t^{a}_{(BH)} = \rho_{(BH)} \left[ \Gamma_{(BH)} + \frac{\dot{n}_{(BH)}}{m_{(BH)}} \right] .$$

(26)

The entropy per black hole is

$$s_{(BH)} = 4\pi m_{(BH)}^2 ,$$

(27)

while the black hole temperature is related to its mass by the well-known formula [23]

$$T_{(BH)} = \frac{1}{8\pi m_{(BH)}} .$$

(28)

The temperature $T_{(BH)}$ is attributed to each PBH individually, i.e., it is not a conventional fluid temperature. With this basic conceptional difference in mind it is nevertheless possible to associate standard fluid-type quantities to the single-mass PBH component. Specifying the general relation (18) to the case at hand, a chemical potential may be associated with the black hole component by

$$\mu_{(BH)} = \frac{\rho_{(BH)}}{n_{(BH)}} - T_{(BH)} s_{(BH)} = \frac{1}{2} m_{(BH)} .$$

(29)

Together with relation (28) the expression (23) for the PBH energy density fits into the general structure given in Eqs. (10). The corresponding partial derivatives are

$$\frac{\partial \rho_{(BH)}}{\partial T_{(BH)}} = -\frac{\rho_{(BH)}}{T_{(BH)}} , \quad \frac{\partial \rho_{(BH)}}{\partial n_{(BH)}} = m_{(BH)} .$$

(30)

Furthermore, the relationship

$$\frac{\partial \rho_{(BH)}}{\partial T_{(BH)}} = -\frac{1}{2} \frac{m_{(BH)}}{T_{(BH)}^2} ,$$

(31)

holds. It is essential that the general temperature law (23) is applicable as well and specifies [cf. Eqs. (23) and (24)] to

$$\dot{T}_{(BH)} = \frac{u_a t^{a}_{(BH)} - \Gamma_{(BH)} \rho_{(BH)}}{\partial \rho_{(BH)} / \partial T_{(BH)}} = -T_{(BH)} \frac{\dot{n}_{(BH)}}{m_{(BH)}} .$$

(32)

which is consistent with formula (28). The BH temperature behavior $T_{(BH)} \propto m_{(BH)}^{-1}$ may be regarded as a special case of the general fluid temperature law (13). This provides the main motivation for our fluid approach to PBHs. On the other hand, this approach relies on the possibility of treating the black holes as an ensemble of noninteracting particles. The validity of the latter assumption can be shown by a simple Newtonian argument [21]. The relative velocity of neighboring black holes due to the Hubble expansion is

$$v_{exp} = \frac{d}{dt} \left( n_{(BH)}^{-1/3} \right) .$$

With the help of Eq. (24) we obtain

$$v_{exp} = \frac{\Theta}{3} n_{(BH)}^{-1/3} \left( 1 - \frac{\Gamma_{(BH)}}{\Theta} \right)$$

for this quantity. Since for a spatially flat FLRW universe

$$\frac{\Theta}{3} = \sqrt{\frac{8\pi}{3} \left( n_{(BH)} m_{(BH)} + \rho_{(2)} \right)}$$

is valid, we find

$$v_{exp} = n_{(BH)}^{-1/3} \sqrt{\frac{8\pi}{3} \left( n_{(BH)} m_{(BH)} + \rho_{(2)} \right)} \left( 1 - \frac{\Gamma_{(BH)}}{\Theta} \right) .$$

The velocity required for a BH to escape the (Newtonian) gravitational pull of its nearest neighbor is

$$v_{esc} = \sqrt{2 m_{(BH)} n_{(BH)}^{-1/3}} = n_{(BH)}^{-1/3} \sqrt{2 n_{(BH)} m_{(BH)}} .$$

Comparing the expressions for $v_{exp}$ and $v_{esc}$, we find that $v_{exp} > v_{esc}$, provided

$$\sqrt{\frac{4\pi}{3} \left( 1 + \frac{\rho_{(2)}}{n_{(BH)} m_{(BH)}} \right) \left( 1 - \frac{\Gamma_{(BH)}}{\Theta} \right)} > 1 .$$

The latter relation is always satisfied for $\Gamma_{(BH)} \leq 0$, the case of main interest here, but also for a certain range of values $\Gamma_{(BH)} > 0$. We conclude that the behavior of the individual BHs is primarily determined by the expansion of the universe and not by their mutual interaction. This justifies their treatment as a noninteracting particle species.

Let us assume from now on that the source terms on the right-hand side of Eq. (8) cancel among themselves, i.e., that the entropy per particle of component 2 is preserved. This isentropy condition amounts to the assumption that the fluid-2 particles at any stage are amenable to a perfect fluid description. With $\dot{s}_{(2)} = 0$, via Eq. (8) equivalent to

$$u_a t^{a}_{(2)} = \left( \rho_{(2)} + p_{(2)} \right) \Gamma_{(2)} ,$$

(33)

the expression (23) for the temperature behavior simplifies considerably:
\[ \dot{s}^{(2)} = 0 \Rightarrow \frac{T^{(2)}}{\Gamma^{(2)}} = -\left( \Theta - \Gamma^{(2)} \right) \frac{\partial p^{(2)}}{\partial \rho^{(2)}}. \]  

(34)

Combining Eqs. (10) and (23), one has

\[ u_a^{a(BH)} = -u_a^{a(BH)} = -\left( \rho^{(2)} + p^{(2)} \right) \Gamma^{(2)}, \]  

(35)

and we find the expression

\[ \Gamma^{(2)} = -\rho^{(2)} / \rho^{(2)} = 2 \left( m^{(BH)} / m^{(BH)} \right) \]  

(36)

for the rate of change of the fluid-2 particle number. Negative rates \( \Gamma^{(BH)} \) and \( m^{(BH)} / m^{(BH)} \) (evaporation) imply a positive \( \Gamma^{(2)} \) and vice versa.

The contribution of the black hole component to the entropy production density is [cf. Eqs. (13) and (24)]

\[ S^{a(BH)} : a = n^{a(BH)} s^{a(BH)} \left[ \Gamma^{(BH)} + \frac{\dot{s}^{a(BH)}}{s^{a(BH)}} \right] \]  

(37)

with

\[ \frac{\dot{s}^{a(BH)}}{s^{a(BH)}} = 2 m^{a(BH)} / m^{a(BH)}, \]  

(38)

according to Eq. (27). The fluid-2 component contributes with

\[ S^{a} : a = n^{a} s^{a} \Gamma^{a} \]  

(39)

Assuming now the second component to be radiation with \( p^{(2)} = \rho^{(2)}/3 \) and \( \mu^{(2)} = 0 \), the overall entropy production density (20) becomes

\[ S^{a} = \rho^{(BH)} \Gamma^{(BH)} \left[ 2T^{(BH)} - \frac{1}{T^{(2)}} \right] + \rho^{(BH)} \frac{m^{(BH)}}{m^{(BH)}} \left[ \frac{1}{T^{(BH)}} - \frac{1}{T^{(2)}} \right], \]  

(40)

where we have used relations (23) and (33). This formula for the entropy production density is still completely general and, within the fluid picture, holds for PBH formation (\( \Gamma^{(BH)} > 0 \)) and mass accretion (\( \dot{m}^{(BH)} > 0 \)) as well as for the evaporation process (\( \Gamma^{(BH)} < 0 \) and \( \dot{m}^{(BH)} < 0 \)). For \( T^{(2)} > 2T^{(BH)} \) the second law favors the formation of PBHs (\( \Gamma^{(BH)} > 0 \)) and mass accretion (\( \dot{m}^{(BH)} > 0 \)), while for \( T^{(2)} < T^{(BH)} \) the reverse processes dominate.

Let us assume that at some initial time \( t_0 \) the black hole temperature coincides with the radiation temperature, i.e., \( T^{(BH)}(t_0) = T^{(2)}(t_0) \). Under this condition the second term in Eq. (10) vanishes and the entropy production density reduces to

\[ S^{a} = -\frac{1}{2} \rho^{(BH)} \Gamma^{(BH)} = -n^{a(BH)} s^{a(BH)} \Gamma^{(BH)} \]  

(41)

at \( t = t_0 \). To obtain the second relation (11) we have used Eqs. (27) and (28). Obviously, this case requires \( \Gamma^{(BH)} < 0 \) in order to satisfy \( S^{a} > 0 \), i.e., decay (\( \Gamma^{(BH)} < 0 \)) of the PBH component. In other words, the formation of PBHs (\( \Gamma^{(BH)} > 0 \)) is thermodynamically forbidden if the temperature of the PBHs coincides with the temperature of the ambient radiation, i.e., for \( T^{(BH)} = T^{(2)} \).

Let us now consider the second term in Eq. (10). This contribution, which vanishes for \( T^{(BH)}(t_0) = T^{(2)}(t_0) \), represents the entropy production density in case the number of PBHs is constant, i.e., for \( \Gamma^{(BH)} = 0 \). Restricting ourselves momentarily to a fixed BH number, there are two possibilities for the subsequent evolution of the system. For \( T^{(BH)} < T^{(2)} \) immediately after \( t = t_0 \) a positive entropy production requires \( \dot{m}^{(BH)} / m^{(BH)} < 0 \), i.e., PBH evaporation. This process will be studied in some detail below. For \( T^{(BH)} < T^{(2)} \) on the other hand, the condition \( S^{a} > 0 \) is only satisfied for \( \dot{m}^{(BH)} / m^{(BH)} > 0 \), i.e., for mass accretion. Becoming larger, the BHs also become colder [cf. Eq. (28)] and their growth seems to continue indefinitely, i.e., until they have eaten up the entire universe. However, such a kind of scenario is thermodynamically impossible as can be seen by the following argument. The essential point is that the BH mass accretion back reacts on the temperature of the ambient radiation. For an accretion rate \( \dot{m}^{(BH)} / m^{(BH)} > 0 \) the rate (30) is negative (recall that \( \Gamma^{(BH)} = 0 \) in the present discussion). The cooling rate (24) of the radiation temperature then becomes

\[ \frac{T^{(2)}}{T^{(2)}} = \frac{1}{3} \left( \Theta + 3 \rho^{(BH)} \frac{\dot{m}^{(BH)}}{4 \rho^{(BH)} m^{(BH)}} \right). \]  

(42)

It is obvious that from some time on the temperature \( T^{(2)} \) will cool off faster than \( T^{(BH)} \) [cf. Eq. (24)]. Consequently, \( T^{(2)} \) will approach \( T^{(BH)} \). As soon as \( T^{(2)} \) has fallen below \( T^{(BH)} \), mass accretion stops since for \( T^{(2)} < T^{(BH)} \) the rate \( \dot{m}^{(BH)} / m^{(BH)} \) has to be negative in order to guarantee a positive entropy production, i.e., the process now proceeds in the reverse direction. We conclude that the second law of thermodynamics forbids a catastrophic growth of the PBHs. These considerations may also be regarded as a justification of the “initial” condition \( T^{(BH)}(t_0) = T^{(2)}(t_0) \) for the evaporation process.

Now we continue discussing the case with \( \Gamma^{BH} \) different from zero. The situation \( S^{a} = 0 \) generally corresponds to a “detailed balance” at \( t = t_0 \), i.e., both reactions, the formation of PBHs and their evaporation, proceed at the same rate such that there is no change in the net numbers \( N^{(BH)} \) and \( N^{(2)} \). This is equivalent to \( \Gamma^{(BH)}(t_0) = \dot{m}^{(BH)}(t_0) / m^{(BH)}(t_0) = \Gamma^{(2)}(t_0) = 0 \). For \( t < t_0 \) the formation of PBHs and accretion, i.e., \( \Gamma^{(BH)} > 0 \), \( \dot{m}^{(BH)} / m^{(BH)} > 0 \) and \( \Gamma^{(2)} < 0 \) are thermodynamically favored. For \( t > t_0 \) the PBH evaporation with \( \Gamma^{(BH)} < 0 \), \( \dot{m}^{(BH)} / m^{(BH)} < 0 \) and \( \Gamma^{(2)} > 0 \) dominates.
We are interested here in the second part of this process, i.e., in the evolution of the universe for \( t \geq t_0 \). We will not discuss issues of PBH formation (see the introduction for possible mechanisms) but simply assume that at some early time a considerable amount of cosmic matter was in the form of PBHs of the same mass and that this component subsequently (e.g., until the epoch of nucleosynthesis) decayed. We will investigate the thermodynamic aspects of this evaporation process and its implications for the cosmological dynamics.

According to Eq. (33) negative values of the rates \( \Gamma_{(BH)} \) and \( \dot{m}_{(BH)}/m_{(BH)} \) imply a positive \( \Gamma_{(2)} \). The production rate \( \Gamma_{(2)} \) may either be larger or smaller than the expansion rate \( \Theta \). For \( \Gamma_{(2)} < \Theta \) the fluid temperature decreases according to Eq. (34), while the BH temperature increases according to Eq. (2). It follows that \( T_{(BH)} > T_{(2)} \) at \( t > t_0 \). The evaporation process will continue since \( T_{(2)} < T_{(BH)} \) requires \( \Gamma_{(BH)} < 0 \) and \( \dot{m}_{(BH)}/m_{(BH)} < 0 \) to guarantee \( S^a > 0 \) in Eq. (40).

For \( \Gamma_{(2)} > \Theta \), however, hypothetically realized e.g. by a large initial ratio \( \rho_{(BH)}/\rho_{(2)} \), the fluid temperature increases. If this increase is smaller than the increase in \( T_{(BH)} \) we have again \( T_{(2)} < T_{(BH)} \) and the PBH evaporation goes on since it remains thermodynamically favorable (\( S^a > 0 \)). But an increase in \( t_{(BH)} \) stronger than that in \( T_{(BH)} \), results in a fluid temperature which is higher than \( T_{(BH)} \). For \( T_{(2)} > 2T_{(BH)} \) a positive entropy production (41) requires \( \Gamma_{(BH)} > 0 \) and \( \dot{m}_{(BH)}/m_{(BH)} > 0 \), implying a quick transition to a negative \( \Gamma_{(2)} \), i.e., the process confines itself. A strong “reheating” of the fluid will stop the evaporation and reverse the process. Now, the second law requires PBHs to be formed out of the radiation and accrete mass. A negative \( \Gamma_{(2)} \) on the other hand, will make \( T_{(2)} \) subsequently decrease [cf. Eq. (2)]. If \( T_{(2)} \) has fallen below \( T_{(BH)} \), the evaporation process may continue (see the discussion following Eq. (41)).

We conclude that the PBH evaporation is a self-confining process and, consequently, the accompanying “reheating” of the radiation is limited on general thermodynamical grounds.

To study the corresponding dynamics of the two-component system in more detail we resort to the decay law [33]

\[
\dot{m}_{(BH)} = -\frac{A}{m_{(BH)}^2}
\]

where \( A \) may be taken as a positive-definite constant to be determined by the condition \( m_{(BH)}(t_0 + \tau) = 0 \), with \( \tau \) the lifetime of a Schwarzschild black hole that obeys a law of the type [12] from its formation up to its final disappearance, leaving no remnant behind. We assume that all PBHs start evaporating at the same time and that accretion of the ambience matter on the black holes can be ignored [21]. In reality Eq. (42) is an approximation to Hawking’s law since \( A \) is not rigorously constant over the entire radiation period, but depends on the number of particle species emitted by the black hole, whence it slowly increases with the inverse of the black hole mass (see [33] and [34]). For the rate \( \dot{m}_{(BH)}/m_{(BH)} \) one finds

\[
\frac{\dot{m}_{(BH)}}{m_{(BH)}} = -\frac{1}{3\tau} \left( \frac{m_{(BH)}}{m_{(BH)}}(t_0) \right)^3.
\]

Integration of this equation yields

\[
m_{(BH)}(t) = m_{(BH)}(t_0) \left[ 1 - \frac{t - t_0}{\tau} \right]^{1/3}.
\]

The last expression allows one to write the evaporation rate (43) as

\[
\frac{\dot{m}_{(BH)}}{m_{(BH)}} = -\frac{1}{3\tau} \frac{1}{1 - \frac{t - t_0}{\tau}}.
\]

The number decay rate of the PBHs is determined by their inverse lifetime, i.e.,

\[
\Gamma_{(BH)} = -\frac{1}{\tau}.
\]

Formulas (45) and (46) hold if the PBHs are purely decaying with the reverse process entirely suppressed.

In order to describe the process of formation and subsequent evaporation of PBHs in full detail, one needs explicit expressions for the rates \( \Gamma_{(BH)} \) and \( \dot{m}_{(BH)}/m_{(BH)} \) through the transition period from predominant PBH formation to the evaporation phase, where \( \Gamma_{(BH)} \) and \( \dot{m}_{(BH)}/m_{(BH)} \) switch from positive to negative values. While our general framework up to Eq. (40) is able to cover this most general case, the simple expressions (45) and (46) for \( \dot{m}_{(BH)}/m_{(BH)} \) and \( \Gamma_{(BH)} \), respectively, are valid only for pure evaporation.

To simplify our description we will assume the interval between the time at which \( \Gamma_{(BH)} \) changes its sign and the time at which the expressions (45) and (46) are valid to be negligibly small, so that it may be justified to approximately identify both times. Under this condition the simple expressions (45) and (46) may be used for the whole range \( t \geq t_0 \). As a consequence, we have to deal with a nonzero decay rate already at \( t = t_0 \) and the “initial” entropy production rate is positive, according to Eq. (11).

Applying the simple rates (45) and (46) from \( t_0 \) on will provide us with a transparent picture of the evaporation process, although it implies the assumption of an equality of \( T_{(BH)} \) and \( T_{(2)} \) at a time \( t_0 \) somewhat later than the “real” beginning \( \Gamma_{(BH)} = \dot{m}_{(BH)}/m_{(BH)} = 0 \) of the evaporation process.

Assuming \( \rho_{(2)} = \rho_{(BH)}/3 \) in Eq. (36), the production rate \( \Gamma_{(2)} \) at \( t = t_0 \) is

\[\Gamma_{(2)}(t_0) = \frac{\beta}{\tau}, \quad \beta = \frac{\rho_{(BH)}(t_0)}{\rho_{(2)}(t_0)}, \tag{47}\]

where \( \beta \) is the initial ratio of the PBH energy density to the radiation energy density. Given the PBH lifetime
\[ \tau, \text{ the initial production rate is determined by the ratio of the energy densities. According to Eq. (34) this rate} \]

\[ \text{fixes the subsequent behaviour of } T_{(2)} \text{. It is obvious from Eq. (34), that for a rate } \Gamma_{(2)} = 3H - 3m_{(BH)} / m_{(BH)}, \]

\[ \text{where } H \equiv \frac{\dot{a}}{a} \text{ is the Hubble parameter, the fluid temperature } T_{(2)} \text{ increases at the same rate as } T_{(BH)} \text{ [cf. Eq. (23)]. Since evaporation is only possible for } T_{(2)} < T_{(BH)}, \]

\[ \text{this is the highest rate for which there is evaporation. A still higher rate interrupts the evaporation and thermodynamically favors a temporary PBH formation phase until } T_{(2)} \approx T_{(BH)} \text{ again (see the discussion following Eq. (11)). Combining this maximum rate at } t = t_0 \text{ with the rate (45) at } t = t_0, \text{ provides us with the condition for evaporation,} \]

\[ \beta \leq 3H_0 + 1, \quad (48) \]

\[ \text{where } H_0 \text{ is the Hubble parameter at } t_0. \text{ This condition limits the initial ratio } \beta. \text{ For } \tau \leq H_0^{-1} \text{ the PBHs evaporate within one Hubble time and } \beta \text{ is restricted to } \beta \leq 4. \text{ If the BH lifetime is much larger than the Hubble time, i.e., } \tau \gg H_0^{-1}, \text{ the initial PBH abundance may be larger, i.e. } \rho_{(BH)} \gg \rho_{(2)} \text{ corresponding to } \Gamma_{(2)} \gg \Gamma_{(BH)}, \text{ is possible at } t = t_0. \]

\[ \text{Notice that these are restrictions following from the thermodynamics of the decay process. We did not take into account here any limits on the fractional abundance from PBH formation mechanisms which might be more restrictive than the present ones.} \]

\[ \text{We may define a “reheating” temperature } T_{(2)}^{rech} \text{ as the maximum temperature for the radiation fraction by } T_{(2)}^{rech} = 0. \text{ According to Eq. (14) the corresponding condition is } \Gamma_{(2)} = 0. \text{ With } \rho_{(2)} = 3n_{(2)}T_{(2)} \text{ and Eq. (17), i.e., assuming the reheating to proceed in the initial phase of the evaporation process, we find} \]

\[ T_{(2)}^{rech} \approx \frac{1}{3} \frac{n_{(BH)}}{n_{(2)}} \frac{m_{(BH)}}{\Theta \tau}. \quad (49) \]

\[ \text{The energy balance (25) with the “sink” (26), applied to the evaporation process, becomes} \]

\[ \dot{\rho}_{(BH)} + \Theta \rho_{(BH)} = \rho_{(BH)} \left[ \Gamma_{(BH)} + \frac{m_{(BH)}}{m_{(BH)}} \right]. \quad (50) \]

\[ \text{with the rates (46) and (45). For the radiation component we have} \]

\[ \dot{\rho}_{(2)} + \frac{4}{3} \Theta \rho_{(2)} = -\rho_{(2)} \left[ \Gamma_{(BH)} + \frac{m_{(BH)}}{m_{(BH)}} \right]. \quad (51) \]

\[ \text{With respect to “equilibrium” solutions found by Barrow \textit{et al.} (21), in which the ratio of the energy density in the PBH component to the energy density of the ambient radiation remains constant, we address the question whether a corresponding behavior is possible in the present context. To this purpose we have to study whether a relation } \rho_{(BH)} = \tilde{\beta} \rho_{(2)} \text{ with } \tilde{\beta} = \text{ const is compatible with the equations (10) and (11). Introducing} \]

\[ \rho_{(BH)} = \tilde{\beta} \rho_{(2)} \text{ in either Eq. (54) or Eq. (51) provides us with a relation} \]

\[ \Gamma_{(2)} = \frac{1}{4} \frac{\tilde{\beta}}{\beta + 1} \Theta \quad (52) \]

\[ \text{with } \Gamma_{(2)} = -\frac{3}{4} \tilde{\beta} \left[ \Gamma_{(BH)} + m_{(BH)} / m_{(BH)} \right] \text{ [cf. Eq. (16)]. This is obviously from PBH formation mechanisms which might be more restrictive than the present ones.} \]

\[ \text{Integration of equations (51) and (54) with the rates (46) and (45) provides us with the expressions} \]

\[ \rho_{(BH)} (t) = \rho_{(BH)} (t_0) \frac{a_0^3}{a_3} \left[ 1 - \frac{t - t_0}{\tau} \right]^{1/3} \exp \left[ \frac{t - t_0}{\tau} \right], \quad (53) \]

\[ \text{for the energy density of the black hole component, and} \]

\[ \rho_{(2)} (t) = \rho_{(2)} (t_0) \frac{a_0^3}{a_4} + \frac{1}{a_4} \rho_{(BH)} (t_0) \frac{a_0^3}{a_3} \times \]

\[ \times \int_{t_0}^{t} dt a(t) \frac{4 - \frac{t - t_0}{\tau}}{\left[ 1 - \frac{t - t_0}{\tau} \right]^{2/3}} \exp \left[ \frac{t - t_0}{\tau} \right], \quad (54) \]

\[ \text{for the radiation energy density, respectively. The dynamics of the entire system is determined by the energy densities (33) and (54) together with the field equations for } a(t). \text{ In the case of a spatially flat FLRW universe the latter reduce to the Friedmann equation} \]

\[ 3 \frac{\dot{a}^2}{a^2} = 8\pi \left( \rho_{(BH)} + \rho_{(2)} \right). \quad (55) \]

\[ \text{We have numerically integrated the integro-differential set of equations (53), (33) and (53) and shown the results for } \rho_{(BH)} \text{ and } \rho_{(2)} \text{ in figures 1 and 2, respectively. We have included the possibility of no black hole at all (} \beta = 0 \text{) for the sake of completeness and comparison with the evolution of a uncontaminated FLRW radiation universe. The impact of the evaporation process on the expansion rate may be understood in terms of an effective bulk viscous pressure of the cosmic fluid as a whole as will be discussed in the following sections. It is well known that a bulk pressure tends to increase the expansion rate (38).} \]

\[ \text{Denoting the number of relativistic particles by } N_{(2)} = n_{(2)} a^3, \text{ the following general relations hold for the radiation component through the whole process:} \]
\[ \rho_{(2)} \propto \frac{N^{4/3}}{a^4}, \quad T_{(2)} \propto \frac{N^{1/3}}{a} = n^{1/3}_{(2)}, \]  

(56)

implying \( \rho_{(2)} \propto T^{4}_{(2)}. \)

Toward the end of the evaporation, i.e., for \( t \to t_0 + \tau, \) the rates \( \dot{m}_{(BH)}/m_{(BH)} \) and \( \Gamma_{(2)} \) diverge, leading to explosive particle emission in the final stages of the evaporation process. As a result we have an increase in the radiation energy density, equivalent to a second phase of intense reheating, whereas the energy density in the PBH component falls down sharply. (We mention that there have been suggestions in the literature according to which the PBHs do not evaporate completely but leave behind Planck-sized remnants [22, 37]. If this were the case, the final reheating would be reduced.) By direct inspection we confirm that there is no phase with a constant ratio between both energies densities (see the discussion preceding and following Eq. [22]).

IV. THE EFFECTIVE ONE-COMPONENT MODEL

In the present section we try to find an effective one-component description for the system of black holes and radiation. Since our considerations are based on the assumption that at some time \( t_0 \) the temperatures \( T_{(BH)} \) and \( T_{(2)} \) coincide, it seems tempting to introduce an equilibrium temperature for the system as a whole, following the lines of [22]. The problem here is different, however, compared with a mixture of two conventional fluids. Once the evaporation has started, an approximate equilibrium is no longer maintained, since there are no interactions like collisional events between the components. With their shrinking the black holes become hotter and hotter, while the radiation temperature decreases with the expansion of the universe, apart from phases of reheating. Therefore, the concept of an equilibrium temperature of the system as a whole intuitively might appear rather formal, except perhaps in the initial stage of the process. With the help of this quantity, however, a unified description of the cosmic matter will be possible which comprises both conventional fluid aspects and properties characterizing the BH component.

In a general two-fluid model close to equilibrium the Gibbs equation is

\[ T ds = d\rho \frac{1}{n} - (\mu_{(1)} - \mu_{(2)}) \frac{d n_{(1)}}{n}, \]  

(57)

where \( s \) is the entropy per particle. In the case of interest here the temperature \( T \) is the equilibrium temperature of the system of a conventional fluid and a PBH component. The temperatures \( T_{(1)} \) and \( T_{(2)} \) of the previous sections do not appear as variables in the present effective one-temperature description. An (approximate) equilibrium for the entire system is usually assumed to be established through the interactions between the subsystems on the right-hand sides of Eqs. (1) and (7). This reasoning does not hold in the case in which one of the components are evaporating black holes. Here, it is the circumstance that radiation is evaporated at the BH temperature which may constitute a situation which is similar to a thermal equilibrium between two fluids. We assume that analogously to relations (10) the cosmic medium as a whole is characterized by equations of state

\[ p = p(n, n_{(1)}, T), \quad \rho = \rho(n, n_{(1)}, T). \]  

(58)

If the first component is a black hole component, the first of the equation of state (58) specifies to

\[ p = p_2(n_2, T) \]  

(59)

because of Eq. (22).

If in the expressions for the entropies per particle the temperatures \( T_{(1)} \) and \( T_{(2)} \) are identified among themselves and with \( T \), and \( \rho(T) = \rho_{(1)}(T) + \rho_{(2)}(T) \) as well as \( p(T) = p_{(1)}(T) + p_{(2)}(T) \) are used, the description based on Eq. (57) is consistent with the one relying on Eq. (8) for \( n s(T) = n_{(1)} s_{(1)}(T) + n_{(2)} s_{(2)}(T). \)

The equilibrium temperature \( T \) is defined by [28, 39]

\[ \rho_{(1)}(n_{(1)}, T_{(1)}) + \rho_{(2)}(n_{(2)}, T_{(2)}) = \rho(n_{(1)}, T). \]  

(60)

As was shown by one of the authors [24], nonvanishing source terms \( \Gamma_{(A)} \) in the particle number balances [4] (deviations from “detailed balance”) give rise to a new type of effective bulk pressure for the cosmic medium as a whole (“reactive” bulk pressure), i.e. to enlarged entropy production. Here we try to apply this concept to the case in which the process responsible for the deviations from “detailed balance” is PBH evaporation.

In order to find an explicit description we write the energy-momentum tensor of the system as a whole tentatively as

\[ T^{ik} = \rho u^i u^k + (p + \Pi) h^{ik}, \]  

(61)

i.e., we try to map essential features of the evaporation process on an effective bulk pressure \( \Pi \) which we determine below by consistency requirements. The relations \( T_{;k}^{ik} = 0 \) imply the energy balance

\[ \dot{\rho} + \Theta \left[ \rho + p + \Pi \right] = 0 \]  

(62)

for the effective one-temperature description.

\[ \dot{\rho} \] From the Gibbs-equation (57) one finds for the change in the entropy per particle

\[ n \dot{s} = -\frac{\Theta}{T} \Pi - \frac{\dot{\rho} + p}{T} \Gamma - \frac{n_{(1)} n_{(2)}}{n} \left( \frac{\mu_{(1)} - \mu_{(2)}}{T} \right) \left[ \Gamma_{(1)} - \Gamma_{(2)} \right]. \]  

(63)

The expression for the entropy production density becomes
\[ S^a_{\alpha} = n s \Gamma + n s \dot{\mu} . \]  

Introducing here Eq. (63) and the effective one-component chemical potential
\[ \mu = \frac{\rho + p}{n} - T s , \] we find
\[ S^a_{\alpha} = -\frac{\Theta}{T} \Pi - \frac{n \mu}{T} \Gamma 
- \frac{n \Gamma_1 n_2}{n} \left( \frac{\mu_1 - \mu_2}{T} \right) \left[ \Gamma_1 - \Gamma_2 \right] . \]  

Identifying again the first fluid with the black hole component, using Eqs. (17), (20) and (24) as well as the decomposition
\[ n \mu = n_{(BH)} \mu_{(BH)} + n_{(2)} \mu_{(2)} = \frac{1}{2} \rho_{(BH)} , \] (recall that \( \mu_{(2)} = 0 \)) one obtains
\[ S^a_{\alpha} = -\frac{\Theta}{T} \Pi - \frac{1}{2} \rho_{(BH)} T \Gamma_{(BH)} . \]  

The last expression for the entropy production density has to be consistent with Eq. (44) of the two-component description. This requirement provides us with the following expression for the effective viscous pressure,
\[ -\frac{\Theta}{T} \Pi = \frac{1}{2} \rho_{(BH)} \left( \Gamma_{(BH)} + 2 \frac{\dot{n}_{(BH)}}{n_{(BH)}} \right) \left[ \frac{1}{T_{(BH)}} - \frac{1}{T_{(2)}} \right] 
+ \frac{1}{2} \rho_{(BH)} \Gamma_{(BH)} \left[ \frac{1}{T} - \frac{1}{T_{(2)}} \right] . \]  

The last relation shows that the evaporation process may be described in terms of an effective viscous pressure \( \Pi \). In other words, PBH evaporation, if regarded as a specific case of interfluid reactions, gives rise to a viscous pressure of the cosmic medium as a whole. The viscous pressure vanishes for \( \Gamma_{(BH)} = \dot{n}_{(BH)}/m_{(BH)} = \Gamma_{(2)} = 0 \), i.e., no net change in the fluid particle numbers (“detailed balance”). This is also the limiting case of noninteracting fluids. We emphasize again that the quantity \( \Pi \) in Eq. (69) is exclusively a consequence of the evaporation process. “Conventional” bulk viscous fluid pressures, e.g., due to scattering of radiation particles by the BHs, have been neglected here for simplicity.

By the field equations for a viscous cosmic medium with the energy-momentum tensor \( \Pi \) the quantity \( \Pi \) influences the cosmic dynamics. Restricting ourselves again to the homogeneous, isotropic and spatially flat case we have
\[ 3 \frac{a^2}{a^2} = 8 \pi \rho , \quad \left( \frac{\dot{a}}{a} \right)^2 = -4 \pi (\rho + p + \Pi) . \]  

While physically the present one-component picture is equivalent to the two-component description of the previous section, the mapping of certain features of the evaporation process onto an effective bulk pressure \( \Pi \) may provide a more transparent understanding of how the PBH evaporation modifies the cosmological dynamics.

In Eq. (23) one has to deal with three generally different temperatures: The temperatures \( T_{(BH)} \) and \( T_{(2)} \) and the equilibrium temperature \( T \) of the system as a whole, defined by Eq. (20). The dynamics of \( T_{(BH)} \) and \( T_{(2)} \) is given by the laws (32) and (34), respectively. But we have still to find a corresponding relationship for \( T \). Such a law may be obtained via similar steps that led us to Eq. (12). There exists, however, the following complication. Because of the additional dependence of \( \rho \) on \( n_{(1)} \) one has now three partial derivatives of \( \rho \): \((\partial \rho / \partial T)_{n_{(1)}, n_{(2)}}, (\partial \rho / \partial n)_{T, n_{(1)}} \) and \((\partial \rho / \partial n)_{T, n_{(2)}} \). The requirement that \( s \) is a state function now leads to (40, 41)

\[ \frac{\partial \rho}{\partial n} = \frac{\rho + p}{n} - \frac{T}{n} \frac{\partial \rho}{\partial T} \left( \frac{\mu_{(1)} - \mu_{(2)}}{n} - T \frac{\partial}{\partial T} \left( \mu_{(1)} - \mu_{(2)} \right) \right) , \]  

generalizing Eq. (11), and the additional relation

\[ \frac{\partial \rho}{\partial n_{(1)}} = \mu_{(1)} - \mu_{(2)} - T \frac{\partial}{\partial T} \left( \mu_{(1)} - \mu_{(2)} \right) . \]  

Using the Gibbs-Duhem relations

\[ d \rho_{(A)} = n_{(A)} s_{(A)} d T_{(A)} + n_{(A)} d \rho_{(A)} \]  

for \( T_{(A)} = T \) together with Eq. (18), one finds

\[ \mu_{(A)} - T \frac{\partial \mu_{(A)}}{\partial T} = \frac{\rho_{(A)}}{n_{(A)}} . \]  

Consequently, the relations (71) and (72) may be written as

\[ \frac{\partial \rho}{\partial n} = \frac{\rho + p}{n} - \frac{T}{n} \frac{\partial \rho}{\partial T} - \frac{n_{(1)}}{n_{(2)}} \left[ \frac{\rho_{(1)}}{n_{(1)}} - \frac{\rho_{(2)}}{n_{(2)}} \right] \]  

and

\[ \frac{\partial \rho}{\partial n_{(1)}} = \frac{\rho_{(1)}}{n_{(1)}} - \frac{\rho_{(2)}}{n_{(2)}} . \]  

respectively, with \( \rho_{(1)} = \rho_{(1)}(T) \) and \( \rho_{(2)} = \rho_{(2)}(T) \), since we are within the one-temperature description.

Differentiating the second of the relations (88) and applying Eqs. (44, 17), (22, 74), and (76), we obtain

\[ \frac{\partial \rho}{\partial T} = - (\Theta - \Gamma) T \frac{\partial \rho}{\partial T} + \left[ T \Pi (\rho + p) \right] 
- \frac{n_{(1)} n_{(2)}}{n} \left[ \frac{\rho_{(1)}}{n_{(1)}} - \frac{\rho_{(2)}}{n_{(2)}} \right] \left[ \Gamma_{(1)} - \Gamma_{(2)} \right] . \]
Using here \( \rho = \rho_{(BH)} + \rho_{(a)} \) and the second relation of Eqs. (17), we find the evolution law for the equilibrium temperature of a system of a conventional fluid and a “fluid” of primordial single-mass black holes,

\[
T \frac{\partial \rho}{\partial T} = -T \left( \Theta - \Gamma_{(2)} \right) \frac{\partial \rho}{\partial T} + \rho_{(BH)} \frac{\dot{n}_{(BH)}}{m_{(BH)}} - \Theta \Pi + \frac{n_{(BH)}}{n} \left( T \frac{\partial p}{\partial T} - p \right) \left( \Gamma_{(BH)} - \Gamma_{(2)} \right).
\]  

(78)

The temperature law (78) comprises both the law (74) for the fluid temperature and the BH temperature law (72). This unifying feature may be considered as a justification of the equilibrium temperature concept for the cosmic medium. For \( \rho_{(BH)} = 0 \), Eq. (78) reduces to to the expression (14) for radiation while for \( \rho_{(a)} = \rho_{(a)} = \Pi = 0 \), equivalent to \( \rho = \rho_{(BH)} \) and \( T = T_{(BH)} \), it coincides with the behavior (32), equivalent to Eq. (28).

Inserting here the expressions (15) at \( t = t_0 \), (16) and (17) and taking into account \( \Pi (t_0) = 0 \) for \( T_{(BH)} (t_0) = T_{(BH)} (t_0) \) [cf. Eq. (33)], Eq. (78) at \( t = t_0 \) reduces to the perfect fluid temperature law

\[
T = -\frac{\partial p}{\partial \rho} \Theta, \quad (t = t_0).
\]

(79)

This result again proves the consistency of the concept of an equilibrium temperature \( T \) of the system as a whole.

V. ENTROPY PRODUCTION AND VISCOUS PRESSURE AT THE BEGINNING OF THE EVAPORATION PHASE

The temperature laws (32), (34) and (78) allow us to calculate the entropy production (18) and the viscous pressure (19) at the beginning of the black hole evaporation explicitly. For the temperatures we have in linear order:

\[
T (t) \approx T (t_0) - (t - t_0) \Theta T \frac{\partial \rho}{\partial T},
\]

(80)

\[
T_{(2)} (t) \approx T_{(2)} (t_0) - (t - t_0) \left( \Theta - \Gamma_{(2)} \right) T \frac{\partial \rho_{(2)}}{\partial \rho_{(2)}},
\]

(81)

and

\[
T_{(BH)} (t) \approx T_{(BH)} (t_0) - (t - t_0) T_{(BH)} \frac{\dot{n}_{(BH)}}{m_{(BH)}}.
\]

(82)

Assuming

\[
T (t_0) = T_{(2)} (t_0) = T_{(BH)} (t_0),
\]

(83)

the following relations hold at \( t = t_0 \):

\[
\frac{\partial \rho}{\partial T} = \frac{\rho_{(2)} - \rho_{(BH)}}{T}, \quad \frac{\partial p}{\partial \rho} = \frac{1}{3} \frac{\rho_{(2)}}{\rho_{(2)} - \rho_{(BH)}},
\]

\[
\frac{\partial \rho_{(BH)}}{\partial \rho} = \frac{T}{\rho_{(2)} - \rho_{(BH)}},
\]

(84)

and

\[
\frac{\partial p_{(2)}}{\partial \rho} = \frac{1}{3} \frac{\rho_{(BH)}}{\rho_{(2)} - \rho_{(BH)}}.
\]

(85)

Using the rates (15) at \( t = t_0 \), (16) and (17), we obtain

\[
T \approx \left[ 1 - \frac{t - t_0}{3 \tau} \frac{1}{1 - \beta \Theta} \right] T (t_0),
\]

(86)

\[
T_{(2)} \approx \left[ 1 - \frac{t - t_0}{3 \tau} \left( \Theta - \beta \right) \right] T (t_0),
\]

(87)

and

\[
T_{(BH)} \approx \left[ 1 + \frac{t - t_0}{3 \tau} \right] T (t_0).
\]

(88)

Either from Eq. (16) or Eqs. (18) and (19) we find for the entropy production density

\[
S_{\rho} \approx \frac{1}{2 \tau} \frac{\rho_{(BH)}}{T} \left[ 1 - \frac{t - t_0}{9 \tau} \left( 7 + 8 \beta + 3 H_0 \tau \right) \right].
\]

(89)

The back reaction of the evaporation process on the cosmological dynamics is determined by the viscous pressure \( \Pi \) which becomes in lowest order,

\[
\Pi = -\frac{5 - 8 \beta}{1 - \beta} \left[ 1 - \frac{1}{6 H_0 \tau} \right] \frac{t - t_0}{18 \tau} \rho_{(BH)}.
\]

(90)

Except in the range \( 5/8 < \beta < 1 \), the quantity \( \Pi \) has a negative sign because of relation (18). Obviously, \( \beta \approx 1 \) corresponding to \( \rho_{(a)} (t_0) \approx \rho_{(BH)} (t_0) \) is not a reasonable initial condition since \( \Pi \) diverges. The linear approximation breaks down for \( \rho_{(a)} (t_0) \approx \rho_{(BH)} (t_0) \). As follows from the first relation (34), the quantity \( \partial p / \partial T \) changes its sign for \( \rho_{(a)} (t_0) = \rho_{(BH)} (t_0) \). A reasonable initial condition may be \( \beta = 1/2 \), i.e., \( \rho_{(a)} (t_0) = 2 \rho_{(BH)} (t_0) \). In this case we find

\[
\Pi = -\frac{1}{9} \left[ 1 + \frac{1}{6 H_0 \tau} \right] \frac{t - t_0}{\tau} \rho_{(BH)} \left( \beta = \frac{1}{2} \right).
\]

(91)

Since we restricted ourselves to the initial stage of the evaporation, \( t - t_0 < \tau \) is valid. For \( \beta = 2 \) equivalent to \( \rho_{(BH)} (t_0) = 2 \rho_{(a)} (t_0) \) one obtains

\[
\Pi = -\frac{11}{18} \left[ 1 - \frac{1}{3 H_0 \tau} \right] \frac{t - t_0}{\tau} \rho_{(BH)} \left( \beta = 2 \right).
\]

(92)

If the PBH component makes up almost all of the cosmic matter at \( t = t_0 \) we have \( \beta \gg 1 \) and
\[ \Pi \approx \frac{4}{9} \left[ 1 - \frac{\beta}{3H_0\tau} \right] \frac{t - t_0}{\tau} \rho_{(BH)}, \quad (\beta \gg 1). \quad (93) \]

In the opposite limit \( \beta \ll 1 \) the viscous pressure is

\[ \Pi \approx -\frac{5}{18} \left[ 1 + \frac{1}{3H_0\tau} \right] \frac{t - t_0}{\tau} \rho_{(BH)}, \quad (\beta \ll 1). \quad (94) \]

Although these explicit expressions for \( \Pi \) are only valid in linear approximation, i.e., in the initial stage of the expansion rate of the universe. Its net effect is a higher expansion rate of the universe.

\[ \gamma \to \gamma \left[ 1 + \frac{\Pi}{\rho + p} \right] \quad (95) \]

initially, i.e., \( \gamma \) is effectively reduced. Via the field equations, a negative viscous pressure tends to increase the expansion rate of the universe.

**VI. CONCLUSIONS**

Based on the remarkable fact that the BH temperature law \( T_{(BH)} \propto m_{(BH)}^{-1} \), naturally fits into the general formula for the temperature of a fluid with variable particle number, we set up a scheme in which the evaporation of PBHs is interpreted as a deviation from “detailed balance” in an interacting and reacting two-fluid system. A single-mass PBH configuration shares essential features with a pressureless, nonrelativistic fluid. We modelled the evaporation process as a decay of such kind of “fluid” into a conventional relativistic fluid (radiation). We investigated the thermodynamics of this system, especially the entropy production and, with the help of the second law, derived general limits on the reheating of the radiation due to the evaporation. We found that intense reheating, i.e., an increase in the radiation energy density, may occur both in the initial and final stages of the process. For our single-mass PBH ensemble there are no “equilibrium” solutions, i.e., solutions for which the ratio of the energy densities of the PBHs and the radiation remain constant, as obtained in [24] for a PBH configuration with a power-law mass spectrum. The impact of the PBH evaporation on the cosmological dynamics may be described in terms of an effective bulk pressure of the cosmic substratum as a whole which we evaluated explicitly for the initial phase of the process. Its net effect is a higher expansion rate of the universe.

**Acknowledgement**

This paper was supported by the Deutsche Forschungsgemeinschaft, the Spanish Ministry of Education (grant PB94-0718) and NATO (grant CRG 940598). We are grateful to Vicenç Méndez for computational assistance.

[1] Ya. B. Zel’dovich and I. D. Novikov, Sov. Astron. 10, 602 (1967).
[2] S. W. Hawking, Mon. Not. R. Astr. Soc. 152, 75 (1971).
[3] B. J. Carr and S. W. Hawking, Mon. Not. R. Astr. Soc. 168, 399 (1974).
[4] B. J. Carr and J. Lidsey, Phys. Rev. D 48, 543 (1993); 50, 853 (1994).
[5] M. Y. Khlopov and A. Polnarev, Phys. Lett. B 97, 383 (1980).
[6] H. Kodama, M. Sasaki, and K. Sato, Progr. Theor. Phys. 68, 1979 (1982).
[7] J.D. Barrow, E.J. Copeland, E.W. Kolb, and A.R. Liddle, Phys. Rev. D 43, 984 (1991).
[8] D. Gross, M. J. Perry, and L. G. Yaffe, Phys. Rev. D 25, 330 (1982); 36, 1603 (1987).
[9] J. I. Kapusta, Phys. Rev. D 30, 831, (1984).
[10] T. Piran and R. M. Wald, Phys. Lett. 90 A, 20 (1982).
[11] R. Bouss and S. W. Hawking, Helv. Phys. Acta 69, 316 (1996); Phys. Rev. D 54, 6312 (1996).
[12] J. García-Bellido, A. Linde, and D. Wands, Phys. Rev. D 54, 6040 (1996).
[13] P. Ivanov Non-linear metric perturbations and production of primordial black holes, preprint astro-ph/9708224 (1997).
[14] A. Polnarev and R. Zembroricz, Phys. Rev. D 43, 1106 (1988).
[15] J. D. Barrow, Mon. Not. R. Astr. Soc. 192 427 (1980); D. Lindley, Mon. Not. R. Astr. Soc. 196 317 (1981); L. M. Krauss, Phys. Rev. Lett 49, 1459 (1982).
[16] B. J. Carr, Helv. Phys. Acta 69, 434 (1996).
[17] B. J. Carr, Astrophys. J. 201, 1 (1975).
[18] M. Gibilisco, Int. J. Mod. Phys. A 11, 5541 (1996).
[19] A. M. Green and A. R. Liddle, Phys. Rev. D 56, 6166 (1997).
[20] A. M. Green, A. R. Liddle, and A. Riotto, Phys. Rev. D 56, 7559 (1997).
[21] G. Hayward and D. Pavón, Phys. Rev. D. 40, 1748 (1989).
[22] J.D. Barrow, E.J. Copeland, and A.R. Liddle, Phys. Rev. D 46, 645 (1992).
[23] S. W. Hawking, Commun. Math. Phys. 43, 199 (1975).
[24] W. Zimdahl, Mon. Not. R. Astr. Soc. 288, 665 (1997).
[25] K. C. Jacobs, Nature 215, 1156 (1967).
[26] J. D. Barrow, E. J. Copeland and A. R. Liddle, Mon. Not. R. Astr. Soc., 253, 675 (1991).
[27] W. Zimdahl and D. Pavón, Gen. Relativ. Grav. 26, 1259
(1994).
[28] W. Zimdahl and D. Pavón, Mon. Not. R. Astr. Soc. 266, 872 (1994).
[29] W. Zimdahl, D. Pavón, and R. Maartens, Phys. Rev. D 55, 4681 (1997).
[30] M. O. Calvão, J. A. S. Lima and I. Waga, Phys. Lett. A 162, 223 (1992); J. A. S. Lima and A. S. M. Germano, Phys. Lett. A 170, 373 (1992); W. Zimdahl and D. Pavón, Phys. Lett. A 176, 57 (1993).
[31] S.R. de Groot, W. A. van Leeuwen, and Ch. G. van Weert, Relativistic kinetic theory (North-Holland, Amsterdam, 1980).
[32] J. Bernstein, Kinetic theory in the expanding universe (Cambridge University Press, Cambridge, 1988).
[33] D. N. Page, Phys. Rev. D 13, 198 (1976).
[34] K. S. Thorne, W. H. Zurek, and R. H. Price, “The thermal atmosphere of a black hole” in Black holes: The membrane paradigm, edited by K. S. Thorne, R. H. Price, and D. A. Macdonald (Yale University Press, New Haven, 1986).
[35] W. Zimdahl, J. Triginer, and D. Pavón, Phys. Rev. D 54, 6101 (1996).
[36] W. Zimdahl, Phys. Rev. D 57, 2245 (1998).
[37] B.J. Carr, J. Gilbert, and J. Lidsey, Phys. Rev. D 50, 4853 (1994).
[38] N. Udey and W. Israel, Mon. Not. R. Astr. Soc. 199, 1137 (1982).
[39] W. Zimdahl, Mon. Not. R. Astr. Soc. 280, 1239 (1996).
[40] W. Zimdahl and H. Sponholz, Physica A 163, 895 (1990).
[41] W. Zimdahl, Class. Quantum Grav. 8, 677 (1991).

List of captions for figures

Figure 1.- Evolution of the energy density of the PBH fluid during the evaporation process.

Figure 2.- Evolution of the energy density of the radiation fluid during the evaporation process.
\[
\frac{\rho_{\text{BH}}(t)}{\rho_{\text{BH}}(t_0)}
\]

\[\frac{(t-t_0)}{\tau}\]
\[ \frac{\rho_2(t)}{\rho_2(t_0)} \]

Graph showing the function \( \frac{\rho_2(t)}{\rho_2(t_0)} \) against \( \frac{(t-t_0)}{\tau} \) for different values of \( \beta \):

- \( \beta = 0 \)
- \( \beta = 1/2 \)
- \( \beta = 2 \)
- \( \beta = 1 \)