General Messenger Gauge Mediation

Thomas T. Dumitrescu,¹ Zohar Komargodski,² Nathan Seiberg² and David Shih²,³

¹Department of Physics, Princeton University, Princeton, NJ 08544, USA
²School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, USA
³Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08854, USA

We discuss theories of gauge mediation in which the hidden sector consists of two subsectors which are weakly coupled to each other. One sector is made up of messengers and the other breaks supersymmetry. Each sector by itself may be strongly coupled. We provide a unifying framework for such theories and discuss their predictions in different settings. We show how this framework incorporates all known models of messengers. In the case of weakly-coupled messengers interacting with spurions through the superpotential, we prove that the sfermion mass-squared is positive, and furthermore, that there is a lower bound on the ratio of the sfermion mass to the gaugino mass.
1. Introduction

Supersymmetry at the TeV scale is a leading candidate for physics beyond the Standard Model (SM). If it is realized in nature, it must be spontaneously broken in a hidden sector, and this breaking must then be mediated to the Supersymmetric Standard Model (SSM). In general, the mediation is highly constrained by precise experimental tests of flavor symmetry in the SM. Gauge mediation [1-9] solves this SUSY flavor problem by postulating that the hidden sector and the SSM only communicate via the SM gauge interactions. Since the gauge interactions are flavor blind, they automatically result in a flavor-universal SSM spectrum consistent with experiment.

Many different gauge mediation models have been constructed, giving rise to a wide variety of predictions. In [10], General Gauge Mediation (GGM) was formulated in order to incorporate these models into a uniform framework. The GGM setup consists of two sectors: a SUSY-breaking hidden sector with a weakly-gauged global symmetry that includes the SM gauge group, and a visible sector that includes the SSM. The defining assumption of GGM is that these two sectors completely decouple when the SM gauge couplings are taken to zero.¹ By treating the SM gauge interactions perturbatively, but allowing for complicated (in principle strongly-coupled) hidden-sector dynamics, it was possible to deduce the most general, model-independent features of gauge mediation. In particular, it was shown that the parameter space of GGM consists of three complex gaugino masses, and three real parameters which determine the sfermion masses.

In this paper we will focus on a subset of the models described by GGM. Many of the known gauge mediation models contain hidden-sector fields which are charged under the SM gauge group and acquire a non-supersymmetric spectrum, but do not themselves participate in the SUSY-breaking dynamics; such fields are called messengers. The prime example of this scenario is Minimal Gauge Mediation (MGM) [7-9], which serves as the foundation for many phenomenological studies of gauge mediation. In MGM, a pair of messengers is weakly coupled to a hidden-sector singlet X (a spurion) via the superpotential. SUSY-breaking is parameterized by the $F$-term vev of X, which is assumed to result from the dynamics of the hidden sector. The minimal setup has been extended by giving the messengers arbitrary supersymmetric masses and allowing the spurion to interact with the messengers through arbitrary Yukawa couplings [13]. Models of this type were used

¹ Given this definition, GGM does not address the $\mu$-problem of gauge mediation; it also does not allow for gauge messengers. See [11] and [12,13] for extensions of GGM in these directions.
in [15] to attain the correct number of GGM parameters, but without covering the full parameter space. The full parameter space was covered in [16] by also giving the messengers diagonal SUSY-breaking masses through a $D$-term spurion. This shows that the entire GGM parameter space is theoretically accessible, even just with models of messengers.

Our goal in this paper is to study general theories with a messenger sector. The motivation for GGM was to elucidate the most general predictions common to all gauge mediation models; here we would like to understand the most general consequences of having a messenger sector. The hidden sector of GGM now itself consists of two subsectors: a sector of SM-singlets in which SUSY is spontaneously broken at the scale $\sqrt{F}$ (this sector is denoted by subscript $h$), and a supersymmetric messenger sector characterized by the scale $M$ (this sector is denoted by subscript $m$), whose global symmetry contains the SM gauge group. We allow the most general interactions between the SUSY-breaking sector and the messenger sector, but we assume that these interactions are weak so that perturbation theory is applicable:

$$\delta \mathcal{L} = \frac{16\tilde{\lambda}}{\Lambda^{\Delta_h + \Delta_m - 2}} \int d^4 \theta \tilde{O}_h \tilde{O}_m + \frac{4\lambda}{\Lambda^{\Delta_h + \Delta_m - 3}} \int d^2 \theta O_h O_m + c.c. \ .$$  

We will refer to the two terms in (1.1) and their respective complex conjugates as Kähler potential and superpotential interactions. In general, the SUSY-breaking sector and the messenger sector are described by effective theories valid below some UV scale $\Lambda \gg M, \sqrt{F}$. The operators $O_h, \tilde{O}_h$ and $O_m, \tilde{O}_m$ belong to these sectors, respectively. $O_h, O_m$ are chiral superfields of dimensions $\Delta_h, \Delta_m$, while $\tilde{O}_h, \tilde{O}_m$ are unconstrained (in general complex) superfields of dimensions $\tilde{\Delta}_h, \tilde{\Delta}_m$. Finally, $\lambda, \tilde{\lambda}$ are dimensionless couplings; the numerical factors are for later convenience. We refer to this setup as General Messenger Gauge Mediation (GMGM).

Since GMGM is based on an effective Lagrangian valid below the scale $\Lambda$, we must clarify the UV-sensitivity of this setup. If the interactions (1.1) are renormalizable, their contributions to the soft masses are $\Lambda$-independent and can always be trusted. In general, there may be additional non-renormalizable interactions at the scale $\Lambda$ which could contribute to the soft masses. These operators are necessarily $\Lambda$-suppressed, and their contribution to the soft masses is subdominant. The situation is more subtle if the interactions (1.1) are not renormalizable. Now we can only trust our calculations if we know all contributing operators at the scale $\Lambda$, unless we can argue that the effects of certain such operators are subdominant. Throughout this paper, we will assume that one of these
conditions is satisfied. Note that in some cases, we are forced to include additional operators at the scale \( \Lambda \), which act as counterterms for divergences in the effective theory. Logarithmic divergences are an exception, since we can always trust the coefficient of the leading logarithm.

As we will see, the GMGM framework includes the aforementioned weakly-coupled spurion models with general \( F \)- and \( D \)-term SUSY-breaking parameters. It also includes models with hidden-sector gauge dynamics, such as those described in [17,18], the Semi-Direct Gauge Mediation models studied in [19-21], and messenger models with strong dynamics in the hidden sector [22-24]. In addition, the framework in principle makes it possible to study models in which the messengers themselves are strongly coupled.

The foundation for our calculations in GMGM is provided by the simple formulas derived in [16] for the gaugino mass and the sfermion mass-squared in GGM:

\[
M_{\tilde{g}} = \frac{g^2}{4} \int d^4x B(x),
\]
\[
m_{\tilde{f}}^2 = -\frac{g^4Y^2}{128\pi^2} \int d^4x A(x) \log(x^2M^2),
\]

where \( B(x) \) and \( A(x) \) are hidden-sector correlation functions defined as follows:

\[
B(x) = \langle Q^2(J(x)J(0)) \rangle, \\
A(x) = \langle Q^4(J(x)J(0)) \rangle.
\]

For simplicity, we take the SM gauge group to be \( U(1) \) throughout this paper. The group theory factors for the full case \( SU(3) \times SU(2) \times U(1) \) can be easily restored. As described in [10,16], \( J(x) \) is the bottom component of the \( U(1) \) current superfield through which the messengers couple to the SM gauge field, \( g \) is the gauge coupling, and \( Y \) is the charge of the sfermion. From (1.2), we see that the problem of determining the visible-sector soft masses reduces to calculating the correlators \( B(x) \) and \( A(x) \).

In section 2, we give a more detailed definition of GMGM. We expand in powers of the interactions (1.1) to obtain leading-order formulas for the \( B \)- and \( A \)-correlators. The formulas simplify because they factorize into products of correlators which are evaluated separately in the SUSY-breaking sector and the messenger sector. At leading order, the

\[ \text{footnote}^2 \text{ The GGM formalism is briefly reviewed in appendix A, where we also collect some new results on the GGM correlation functions. Note that our definition of } B(x) \text{ in (1.3) differs from the definition in [10,16] by a factor of 4.} \]
SUSY-breaking correlators are relatively simple, while the messenger correlators are always supersymmetric. As a general consequence of the formalism, we will show that the gaugino mass coming from Kähler potential interactions is typically suppressed relative to the corresponding sfermion mass. In particular, we will show that certain leading-order Kähler potential contributions to the gaugino mass vanish identically.

The GMGM formalism also leads to a controlled expansion in $\frac{F}{M^2} \ll 1$ when the SUSY-breaking splittings in the messenger sector are small. In this limit we recover the qualitative behavior of the soft masses in many known models. A different limit leads to the well-known spurion regime in which models of messengers are commonly studied.

Section 3 explains how some of the GMGM formulas of section 2 can be rewritten in terms of supersymmetric deformations of the messenger-sector Lagrangian. Using this technique, we show more conceptually why certain Kähler potential interactions do not generate gaugino masses at leading order.

In section 4, we apply the GMGM formalism to the much-studied case of weakly-coupled, renormalizable spurion models and calculate the leading-order soft masses in these models.

For the case of messengers coupling to spurions through a general superpotential, formulas for the soft masses were first obtained in [14] using the technique of wavefunction renormalization [25]. As we will show, these formulas imply that the sfermion mass-squared is always positive, and moreover that the ratio of the sfermion mass-squared to the gaugino mass-squared is bounded from below. For example, with $N$ messengers and SM gauge group $U(1)$, this ratio satisfies:

$$\frac{m_f^2}{M^2} \geq \frac{Y^2}{N}.$$

(1.4)

This inequality explains why models in which spurions only couple through the superpotential cannot cover the parameter space of GGM [14].

When the spurions couple to the messengers through Kähler potential interactions, the GMGM formulas imply that the leading-order gaugino mass always vanishes. In some cases, we reinterpret this vanishing as a consequence of the rescaling anomaly. We also use our results for spurion models to discuss a particular limit of Semi-Direct Gauge Mediation. In the regime we study, we find that without considerable fine-tuning, the sfermion mass is always much greater than the gaugino mass.
In appendix B, we reanalyze weakly-coupled spurion models in more detail. We use the techniques of section 3 to rederive the leading-order soft masses. By directly applying the formulas from [16], we also derive simple expressions for the full, all-orders soft masses. These were first obtained in [20], and we find complete agreement. Finally, we discuss the limitations of the wavefunction renormalization technique (even for small SUSY-breaking), and we explain why it happens to give correct answers for the soft masses in weakly-coupled spurion models.

2. General Messenger Gauge Mediation

In this section we give a more detailed definition of General Messenger Gauge Mediation (GMGM), and we show how the $B$- and $A$-correlators in (1.3) simplify in this framework. A general consequence of the formalism is that certain Kähler potential interactions do not generate a gaugino mass at leading order. Finally, we discuss two simplifying limits of the GMGM framework: the limit of small SUSY-breaking, and the spurion limit.

2.1. Definition of the Framework

Our definition of GMGM consists of the following three sectors:

1.) A visible sector consisting of the SSM with gauge group $G_{SM} = SU(3) \times SU(2) \times U(1)$ and characteristic scale $M_{\text{weak}} \sim 100 \text{ GeV}$.

2.) A messenger sector (denoted by subscript $m$) whose global symmetry group contains $G_{SM}$. All mass scales in this sector are of order $M$; there are no massless particles. The messenger sector may be strongly coupled.

3.) A SUSY-breaking sector (denoted by subscript $h$) consisting of $G_{SM}$-singlets. For simplicity we assume that the scale of all masses and correlation functions is set by the strength $\sqrt{F}$ of SUSY-breaking ($F^2$ is the total vacuum energy-density). Like the messenger sector, the SUSY-breaking sector may be strongly coupled.

Note that the messenger sector (2) and the SUSY-breaking sector (3) of GMGM together make up what is called the hidden sector in GGM. The visible sector and the messenger sector interact through the visible-sector gauge fields; they decouple when the visible-sector gauge couplings vanish. Since the SUSY-breaking sector is neutral under $G_{SM}$, the current multiplet which enters the GGM correlators in (1.2) and (1.3) only contains messenger-sector fields.
A key element of the GMGM framework is the assumption that the interactions between the SUSY-breaking sector and the messenger sector are \textit{weak} and can be treated in perturbation theory. This is what will allow us to simplify the $B$- and $A$-correlators in (1.3). With this assumption, the interactions between the SUSY-breaking sector and the messenger sector take the general form (1.1), which we repeat here for convenience:

\begin{equation}
\delta L = \frac{16 \tilde{\lambda}}{\Lambda \Delta_h + \Delta_m - 2} \int d^4 \theta \tilde{O}_h \tilde{O}_m + \frac{4 \lambda}{\Lambda \Delta_h + \Delta_m - 3} \int d^2 \theta O_h O_m + \text{c.c.} .
\end{equation}

The quantities appearing in (2.1) were defined below (1.1). We refer to the two terms in (2.1) together with their respective complex conjugates as Kähler potential and superpotential interactions. In general, the interaction Lagrangian $\delta \mathcal{L}$ might contain several such terms.

### 2.2. Soft Masses in GMGM

It is straightforward to expand in powers of the interactions (2.1) and compute the leading-order contributions to the $B$- and $A$-correlators in (1.3). We will organize the presentation in terms of the different types of operators that can couple the SUSY-breaking sector and the messenger sector. In each case, we will see how the $B$- and $A$-correlators factorize into separate correlators evaluated in these two sectors, and we will discuss the implications for the gaugino and sfermion masses.

**Superpotential Interactions**

\begin{equation}
\delta \mathcal{L} = \frac{4 \lambda}{\Lambda \Delta_h + \Delta_m - 3} \int d^2 \theta O_h O_m + \text{c.c.}
\end{equation}

\begin{equation}
= \frac{\lambda}{\Lambda \Delta_h + \Delta_m - 3} Q^2 (O_h O_m) + \text{c.c.} .
\end{equation}

Here we denote by $O$ the bottom component of the superfield $\mathcal{O}$. In the second line, we have traded the $\theta$-integral for the action of the supercharges $Q^2$. Note that this eliminates the extraneous numerical factors. At leading order, the interaction (2.2) gives the following contributions to the $B$- and $A$-correlators:

\begin{equation}
B(x) = \frac{\lambda \langle Q^2(O_h) \rangle_h}{\Lambda \Delta_h + \Delta_m - 3} \int d^4 y \langle Q^2(O_m(y)) J(x) J(0) \rangle_m ,
\end{equation}

\begin{equation}
A(x) = \frac{\lambda^2}{\Lambda^2 (\Delta_h + \Delta_m - 3)} \int d^4 y d^4 y' \left\langle Q^4 \left( O^\dagger_h(y) O_h(y') \right) \right\rangle_h \times \langle Q^4 \left( O^\dagger_m(y) O_m(y') \right) J(x) J(0) \rangle_m .
\end{equation}
Here we also use the subscripts $h$ and $m$ to highlight the factorization into SUSY-breaking correlators and messenger correlators.

We see that for superpotential interactions, the $B$-correlator is $\mathcal{O}(\lambda)$ while the $A$-correlator is $\mathcal{O}(\lambda^2)$. Thus, the gaugino and sfermion masses will be of the same order in the interaction. This is the desired behavior expected of gauge mediation spectra. Note that global symmetries which are unbroken in either the SUSY-breaking sector or the messenger sector can make the correlators in (2.3) vanish. In particular, an unbroken $R$-symmetry in the messenger sector under which $R(\tilde{O}_m) \neq 2$ results in the vanishing of the leading-order $B$-correlator and hence of the gaugino mass.

**General Kähler Potential Interactions**

\[
\delta \mathcal{L} = \frac{16\tilde{\lambda}}{\Lambda \Delta_h + \Delta_m - 2} \int d^4 \theta \tilde{O}_h \tilde{O}_m + \text{c.c.} \\
= \frac{\tilde{\lambda}}{\Lambda \Delta_h + \Delta_m - 2} Q^4 \left( \tilde{O}_h \tilde{O}_m \right) + \text{c.c.} + \text{(total derivative)}. \tag{2.4}
\]

Now the leading-order $B$- and $A$-correlators are given by:

\[
B(x) = \frac{\tilde{\lambda} \langle Q^4(\tilde{O}_h) \rangle_h}{\Lambda \Delta_h + \Delta_m - 2} \int d^4 y \left( \langle Q^2 \left( \tilde{O}_m(y) \right) J(x) J(0) \rangle_m \\
+ \left( \tilde{O}_{h,m} \to \tilde{O}^\dagger_{h,m} \right) + \text{(total x-derivative)} \right), \tag{2.5}
\]

\[
A(x) = \frac{\tilde{\lambda} \langle Q^4(\tilde{O}_h) \rangle_h}{\Lambda \Delta_h + \Delta_m - 2} \int d^4 y \left( \langle Q^4 \left( \tilde{O}_m(y) \right) J(x) J(0) \rangle_m + \text{c.c.} \right).
\]

As we will discuss in section 4, this formula for $A(x)$ includes the well-known supertrace contribution to the sfermion mass-squared in weakly-coupled models of messengers interacting with $D$-term spurions [27].

Although the SUSY-breaking sector operator $\tilde{O}_h$ generically acquires an $F$-term vev, this only leads to a total $x$-derivative in the $B$-correlator (2.3). The reason is that the contribution of such an $F$-term vev is proportional to the correlator

\[
\left( \tilde{O}_m(y) Q^4 (J(x) J(0)) \right)_m. \tag{2.6}
\]

Expanding $Q^4 (J(x) J(0))$ in components and using current conservation shows that this correlator is a total $x$-derivative. Consequently, it will not contribute to the gaugino
mass (1.2) upon integrating the $B$-correlator over $x$. In section 3, we will give another, more conceptual proof of this fact, which uses supersymmetric deformations and holomorphy. We conclude that only the $D$-term vev of $\tilde{O}_h$ contributes to the leading-order gaugino mass. This fact has already been observed in examples, such as [16,20].

Just like for the superpotential contribution, an unbroken $R$-symmetry in the messenger sector can lead to a vanishing $B$-correlator at leading order. However, in contrast to the superpotential contribution, the $B$- and the $A$-correlators are now both $\mathcal{O}(\tilde{\lambda})$. In general this will lead to a soft spectrum satisfying $m_{\tilde{f}} \gg M_{\tilde{g}}$, more fine-tuning in the SSM, and the phenomenology of split SUSY [28,29]. The hierarchy $m_{\tilde{f}} \gg M_{\tilde{g}}$ can potentially be avoided, if the $\mathcal{O}(\tilde{\lambda})$ contribution to the $A$-correlator vanishes (e.g. due to additional symmetries).

**Half-Chiral Kähler Potential Interactions**

These interactions are a subset of the previous case. They still have the form (2.4), except that now either $\tilde{O}_h$ or $\tilde{O}_m$ (but not both) is chiral. In these cases we see from (2.3) that the $\mathcal{O}(\tilde{\lambda})$ contribution to the $A$-correlator vanishes. The leading non-trivial contribution now arises at $\mathcal{O}(\tilde{\lambda}^2)$.

When $\tilde{O}_h$ is chiral, we obtain:

$$A(x) = \frac{\tilde{\lambda}^2}{\Lambda^2(\tilde{\Delta}_h+\tilde{\Delta}_m-2)} \int d^4y d^4y' \left\langle Q^4 \left( \tilde{O}_h(y)\tilde{O}_h(y') \right) \right\rangle_h \times \left\langle Q^4 \left( Q^2(\tilde{O}_m(y))\bar{Q}^2(\tilde{O}_m(y')) \right) J(x)J(0) \right\rangle_m.$$  \hspace{1cm} (2.7)

Note that by the discussion around (2.6), a chiral $\tilde{O}_h$ can never give a gaugino mass at $\mathcal{O}(\tilde{\lambda})$, since there is no $D$-term vev, and even a nonzero $F$-term vev does not contribute to the $B$-correlator at zero momentum. In a sense, this is the most dramatic manifestation of the vanishing gaugino mass discussed above. Now the leading-order gaugino mass is $\mathcal{O}(\tilde{\lambda}^2)$ and we again find a split-SUSY spectrum.

\footnote{It is straightforward to lower two powers of the interaction (2.4) and derive the $\mathcal{O}(\tilde{\lambda}^2)$ contribution to the $B$-correlator; we do not display the result here.}
When $\tilde{O}_m$ is chiral, the leading-order $B$- and $A$-correlators are given by:

\[
B(x) = \frac{\tilde{\lambda}}{\Lambda^{\Delta_h+\Delta_m-2}} \int d^4 y \left\langle Q^2(\tilde{O}_m(y)) J(x) J(0) \right\rangle_m ,
\]

\[
A(x) = \frac{\tilde{\lambda}^2}{\Lambda^{2(\Delta_h+\Delta_m-2)}} \int d^4 y d^4 y' \left\langle Q^4 \left( Q^2(\tilde{\tilde{O}}_h(y)) Q^2(\tilde{\tilde{O}}_h(y')) \right) \right\rangle_h \times \left\langle Q^4 \left( \tilde{O}_m(y) \tilde{O}_m(y') \right) J(x) J(0) \right\rangle_m .
\]

(2.8)

Note that in this case the $B$-correlator is $O(\tilde{\lambda})$, while the $A$-correlator is $O(\tilde{\lambda}^2)$, so that the gaugino and sfermion masses are of the same order in the interaction.

2.3. Simplifying Limits and their Phenomenology

In formula (2.3), the leading-order $A$-correlator due to superpotential interactions does not completely factorize into separate contributions from the SUSY-breaking sector and the messenger sector, because of the momentum dependence of the SUSY-breaking correlator

\[
\left\langle Q^4 \left( O^\dagger_h(y) O_h(y') \right) \right\rangle_h .
\]

(2.9)

In this subsection, we will discuss two scenarios in which this momentum dependence becomes trivial and the $A$-correlator truly factorizes. In these cases we will also be able to say more about the soft spectrum. The two simplifying limits are:

- **Small SUSY-Breaking:** $F \ll M^2$. Since the messenger correlator in (2.3) decays exponentially at long distance, we only need to consider the SUSY-breaking correlator (2.3) at scales $|y - y'| \lesssim \frac{1}{M} \ll \frac{1}{\sqrt{F}}$. The SUSY-breaking sector at these scales is close to a fixed point. Thus (2.3) can be approximated by the OPE of $O^\dagger_h$ and $O_h$ in this fixed-point CFT. The OPE translates into an expansion in $F/M^2$ and we can therefore focus on the leading operator $O_\Delta$ in the OPE which acquires a SUSY-breaking $D$-term vev $\left\langle Q^4(O_\Delta) \right\rangle_h \neq 0$. Now formula (2.3) for the $A$-correlator simplifies:

\[
A(x) = \frac{\lambda^2 \left\langle Q^4(O_\Delta) \right\rangle_h}{\Lambda^{2(\Delta_h+\Delta_m-3)}} \int d^4 y d^4 y' (y - y')^{\Delta - 2\Delta_h} \left\langle Q^4 \left( O^\dagger_m(y) O_m(y') \right) J(x) J(0) \right\rangle_m .
\]

(2.10)

An analogous discussion applies to the hidden-sector correlators in (2.7) and (2.8).
Comparing (2.10) with the $B$-correlator in (2.3), we see that the ratio of the sfermion mass-squared to the gaugino mass-squared is given by:

$$\frac{m_f^2}{M^2} \sim \left( \frac{\sqrt{F}}{M} \right)^{\Delta - 2\Delta_h}.$$  \hspace{1cm} (2.11)

There are three cases:

1.) If $\Delta < 2\Delta_h$, the sfermion mass is much larger than the gaugino mass. The little hierarchy problem of the SSM is exacerbated, and the phenomenology is that of split SUSY [28,29].

2.) If $\Delta > 2\Delta_h$, the sfermions are very light compared to the gauginos. This is the interesting regime for the mechanism of hidden-sector renormalization to operate. Indeed, our result (2.11) agrees with [22,23].

3.) The boundary case $\Delta = 2\Delta_h$ will typically produce comparable sfermion and gaugino masses, just like ordinary gauge mediation.

In a general CFT any of these options can in principle be realized. If the SUSY-breaking sector is asymptotically free, we can say more: since the product $O_h^\dagger O_h$ is a good operator in the free CFT, we know that $\Delta \leq 2\Delta_h$. Thus, we can either get sfermions which are much heavier than the gauginos (1), or a spectrum of comparable sfermions and gauginos (3). If $O_h$ is composite in the free UV regime, it is guaranteed that $\Delta < 2\Delta_h$ and we end up with split SUSY. If $O_h$ is a fundamental singlet we get $\Delta = 2\Delta_h = 2$ and obtain a spectrum of comparable sfermions and gauginos. This property of singlets is one reason they have played an important role in model building (see e.g. [30]).

- **Spurion Limit.** At sufficiently low energies, the dynamics of the SUSY-breaking sector becomes trivial, essentially containing only the vacuum energy and the Goldstino. Although we expect to find new states at the scale $\sqrt{F}$ (for instance, the scalar superpartner of the massless Goldstino), the dynamics of the theory could remain very weakly coupled and essentially classical through some much larger scale $\Lambda \gg \sqrt{F}$. This can happen in non-trivial examples, such as SUSY QCD with massive flavors below the strong-coupling scale [34]; another example will be discussed in subsection 4.3. Below the scale $\Lambda$, the only contribution to (2.9) comes from picking the $F$-terms of the two operators; the correlator factorizes because the theory is classical:

$$\langle Q^4 \left( O_h^\dagger(y)O_h(y') \right) \rangle_h = \left| \langle Q^2(O_h) \rangle_h \right|^2.$$  \hspace{1cm} (2.12)
Again, formula (2.3) for the $A$-correlator simplifies:

$$A(x) = \frac{\lambda^2}{\Lambda^2(\Delta_h + \Delta_m - 3)} \int d^4 y d^4 y' \left\langle Q^2 (O_m(y)) Q^2 (O_m(y')) J(x) J(0) \right\rangle_m . \quad (2.13)$$

At scales larger than $\Lambda$, the dynamics of the SUSY-breaking sector becomes important and factorization no longer occurs. Since we cut off our effective theory at the scale $\Lambda$, this does not affect the result in (2.13).

This situation precisely corresponds to the well-known spurion regime in which models of messengers are commonly studied. Comparing (2.13) to the $B$-correlator in (2.3), we see that in this regime the sfermion and gaugino masses are of the same order in SUSY-breaking, so that we have

$$\frac{m_f^2}{M_y^2} \sim 1 . \quad (2.14)$$

This is expected from our experience with spurion models of messengers.

Note that if $F \ll M^2$ the spurion limit may overlap with the limit of small SUSY-breaking. In the spurion limit the dynamics of the SUSY-breaking sector is trivial up to the scale $\Lambda$. For small SUSY-breaking this always corresponds to case (3) discussed in the previous bullet point, and results in comparable sfermion and gaugino masses, consistent with (2.14). However, the spurion limit may also apply in some cases where $F \gtrsim M^2$.

### 3. Simplifying the Messenger Correlators

The GMGM expressions for the $B$- and $A$-correlators derived in section 2 all contain integrated products of SUSY-breaking correlators $\langle \ldots \rangle_h$ and messenger correlators $\langle \ldots \rangle_m$. In this section, we show how some of the supersymmetric messenger correlators can be rewritten in terms of certain deformations of the original messenger Lagrangian. This is particularly useful if the correlator coming from the SUSY-breaking sector has trivial momentum dependence and factors out of the integral. In this case, the $B$- and $A$-correlators can essentially be calculated in a supersymmetric theory. We will use these results to elucidate certain aspects of the GMGM framework. They also lead to a very compact treatment of weakly-coupled spurion models (see section 4 and appendix B).

Consider the following supersymmetric deformation of the messenger Lagrangian:

$$\delta L_m = \frac{4 \epsilon}{\Lambda^{\Delta_m - 3}} \int d^2 \theta O_m + \frac{16 \beta}{\Lambda^{\Delta_m - 2}} \int d^4 \theta \bar{O}_m + \text{c.c.}$$

$$= \frac{\epsilon}{\Lambda^{\Delta_m - 3}} Q^2 (O_m) + \frac{\beta}{\Lambda^{\Delta_m - 2}} Q^4 \left( \bar{O}_m \right) + \text{c.c.} + \text{(total derivative)} . \quad (3.1)$$

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As before, both $O_m$ and $\tilde{O}_m$ are in general complex and appear in (3.1) together with their complex conjugates. We also define the supersymmetric messenger correlator

$$C_{\text{SUSY}}(x; \epsilon, \tilde{\epsilon}) \equiv \langle J(x)J(0) \rangle_m,$$

where the arguments $\epsilon, \tilde{\epsilon}$ indicate that this correlator is to be evaluated in the messenger theory deformed by (3.1).

Derivatives of $C_{\text{SUSY}}$ with respect to $\epsilon, \tilde{\epsilon}$ insert the operators appearing in (3.1) into the correlator on the right-hand side of (3.2). As discussed above, this can be used to rewrite various contributions to the leading-order $B$- and $A$-correlators, as follows:

1.) Differentiating $C_{\text{SUSY}}$ with respect to $\epsilon$ yields (2.3), the leading-order superpotential contribution to the $B$-correlator:

$$B(x) = \frac{\lambda \langle Q^2(O_h) \rangle_h}{\Lambda^\Delta_h} \frac{\partial}{\partial \epsilon} C_{\text{SUSY}}(x; \epsilon, \tilde{\epsilon}) \bigg|_{\epsilon=\tilde{\epsilon}=0}.$$

Formulas similar to (3.3) were obtained in [32,33] for examples of this type.

2.) Differentiating $C_{\text{SUSY}}$ with respect to $\tilde{\epsilon}$ yields (2.5), the leading-order Kähler potential contribution to the $A$-correlator:

$$A(x) = \frac{\tilde{\lambda} \langle Q^4(\tilde{O}_h) \rangle_h}{\Lambda^\tilde{\Delta}_h} \frac{\partial}{\partial \tilde{\epsilon}} C_{\text{SUSY}}(x; \epsilon, \tilde{\epsilon}) \bigg|_{\epsilon=\tilde{\epsilon}=0} + \text{c.c.}.$$

3.) A second derivative of $C_{\text{SUSY}}$ with respect to $\epsilon$ yields (2.13), the leading-order superpotential contribution to the $A$-correlator in the spurion limit:

$$A(x) = \frac{\lambda^2 \langle Q^2(O_h) \rangle_h^2}{\Lambda^{2\Delta_h}} \frac{\partial^2}{\partial \epsilon \partial \epsilon^*} C_{\text{SUSY}}(x; \epsilon, \tilde{\epsilon}) \bigg|_{\epsilon=\tilde{\epsilon}=0}.$$

This only holds in the spurion limit. Generically, the superpotential contribution to the $A$-correlator cannot be written in terms of a supersymmetric deformation, because of the non-trivial momentum dependence of the SUSY-breaking two-point function $\langle Q^4 (O_h(y)O_h(y')) \rangle_h$ which appears in (2.3).

Note that the leading-order Kähler potential contribution (2.5) to the $B$-correlator or the half-chiral contributions (2.7) and (2.8) to the $A$-correlator cannot be written as supersymmetric deformations of the messenger Lagrangian.

---

5 Note that $C_{\text{SUSY}}$ is the common supersymmetric limit of the functions $C_{\alpha}$ defined in [10] (see appendix A).
We now give an alternate, more conceptual proof that the $F$-term vevs of hidden-sector operators in the Kähler potential do not generate a leading-order gaugino mass.\footnote{An argument along these lines has been conjectured in \cite{34}. The fact that $F$-term vevs in the Kähler potential cannot generate leading-order gaugino masses may allow the discussion in \cite{34} to be generalized further.} Such a contribution would be proportional to the supersymmetric messenger correlator

$$\left\langle Q^4(\tilde{O}_m(y)) J(x) J(0) \right\rangle_m \sim \left. \frac{\partial}{\partial \epsilon} C_{\text{SUSY}}(x; \epsilon, \tilde{\epsilon}) \right|_{\epsilon = \tilde{\epsilon} = 0}, \quad (3.6)$$

evaluated at zero momentum. However, $C_{\text{SUSY}}$ at zero momentum is (by definition) the wavefunction renormalization of the gauge multiplet at one-loop in the visible gauge coupling. In other words, the zero-momentum effective action for the gauge multiplet is given by

$$\frac{1}{4} \int d^2 \theta (1 + g^2 \mathcal{C}) W^2_\alpha + \text{c.c.}, \quad (3.7)$$

so that

$$\tilde{C}_{\text{SUSY}}(p = 0) = \mathcal{C} + \overline{\mathcal{C}}. \quad (3.8)$$

Here $\tilde{C}_{\text{SUSY}}(p)$ is the Fourier transform of $C_{\text{SUSY}}(x)$. Thus, $\tilde{C}_{\text{SUSY}}(p = 0)$ is the sum of a holomorphic function $\mathcal{C}$ of the microscopic hidden-sector couplings and an anti-holomorphic function $\overline{\mathcal{C}}$ of these couplings. This splitting into holomorphic plus anti-holomorphic functions is only true at zero momentum. Since $\tilde{\epsilon}$ is not a holomorphic parameter, $\tilde{C}_{\text{SUSY}}(p = 0)$ cannot depend on it, and hence the $\tilde{\epsilon}$ derivative in (3.6) must vanish at zero momentum.

4. Weakly-Coupled Examples

In this section we explore a subset of the models we presented in section 2: weakly-coupled spurion models of messengers.

4.1. SUSY-Breaking in the Superpotential

The most general renormalizable messenger theory with $N$ pairs $\Phi_i, \tilde{\Phi}_i$ of chiral messengers which couple to a spurion $X$ through the superpotential is given by:

$$\mathcal{L} = \int d^4 \theta \left( \Phi_i^\dagger \Phi_i + \tilde{\Phi}_i^\dagger \tilde{\Phi}_i \right) + \int d^2 \theta (X \lambda_{ij} + m_{ij}) \Phi_i \tilde{\Phi}_j + \text{c.c.}. \quad (4.1)$$
Here $\Phi_i, \tilde{\Phi}_i$ have $U(1)$ charges $+1$ and $-1$ respectively. The spurion $X$ acquires an $F$-term vev $X|_{\theta^2} = f$ ($f$ real). This scenario was dubbed (Extra)Ordinary Gauge Mediation (EOGM) in [14], where the phenomenology of such models was studied in detail (especially for the case where the theory possesses an $R$-symmetry).

In the language of section 2 we have (with an obvious generalization to multiple superpotential interactions):

$$O_h = X, \quad (O_m)_{ij} = \Phi_i \tilde{\Phi}_j,$$

with $\Delta_h = 1$ and $\Delta_m = 2$. The couplings between $O_h$ and $O_m$ are $\sim \lambda_{ij}$. Since this model is renormalizable, the UV-cutoff $\Lambda$ does not appear in the interaction Lagrangian and the theory is fully calculable. For this model to be phenomenologically viable, we need to impose messenger parity (a symmetry that exchanges $\Phi$ and $\tilde{\Phi}$ under which the global $U(1)$ current is odd) and CP invariance. To satisfy these requirements, we assume that there exists a basis in which $m$ is real and diagonal with eigenvalues $m_i$, and that in this basis $\lambda$ is real and symmetric.

The GMGM framework tells us how to compute the leading-order contributions to the gaugino and sfermion masses: use the general formula (2.3) for superpotential interactions, with the operators (4.2). The result for the gaugino mass is:

$$M_{\tilde{g}} = -\frac{g^2 f}{8\pi^2} \sum_{i=1}^N \frac{\lambda_{ii}}{m_i},$$

where $\lambda_{ii}$ are the diagonal entries of $\lambda$ in the basis where $m$ is real and diagonal. For the sfermion mass-squared, we find:

$$m^2 \tilde{f} = \frac{g^4 Y^2 f^2}{64\pi^4} \sum_{i=1}^N \left( \frac{\lambda_{ii}^2}{m_i^2} + \sum_{j=1}^N \frac{\lambda_{ij}^2}{m_i^2 - m_j^2} \log \frac{m_i^2}{m_j^2} \right).$$

These formulas are rederived in appendix B using the results of section 3.

Both (4.3) and (4.4) were obtained in [14] using the wavefunction renormalization technique [25]. As we explain in appendix B, this technique is in general not applicable (even for small SUSY breaking), but several peculiarities of the EOGM Lagrangian (4.1) render it valid in this particular case.

An important property of the expression in (4.4) is that it is strictly greater than zero. We have therefore shown that the leading-order sfermion mass-squared in the most general
renormalizable messenger model with $F$-term breaking is positive. In fact, we can prove a bound for the ratio of the sfermion mass-squared to the gaugino mass-squared. Since the second term on the right-hand side of (4.4) above is manifestly positive, we have

$$m_{\tilde{f}}^2 \geq \frac{g^4 Y^2 f^2}{64\pi^4} \sum_{i=1}^{N} \frac{\lambda_{ii}^2}{m_i^2}.$$  \hspace{1cm} (4.5)

Using the fact that

$$\sum_{i=1}^{N} \frac{\lambda_{ii}^2}{m_i^2} \geq \frac{1}{N} \left( \sum_{i=1}^{N} \frac{\lambda_{ii}}{m_i} \right)^2,$$  \hspace{1cm} (4.6)

we derive the inequality

$$\frac{m_{\tilde{f}}^2}{M_\tilde{g}^2} \geq \frac{Y^2}{N}.$$  \hspace{1cm} (4.7)

It is clear that this inequality is saturated when all off-diagonal elements of $\lambda_{ij}$ vanish and the ratio $\frac{\lambda_{ii}}{m_i}$ is the same for each messenger. For the simple case of a $U(1)$ visible gauge group considered here, this is the usual definition of Minimal Gauge Mediation (MGM) with $N$ messengers. Indeed, the ratio of the sfermion mass-squared to the gaugino mass-squared in such an MGM model is precisely $\frac{Y^2}{N}$. The inequality thus states that renormalizable spurion models of weakly-coupled messengers with $F$-term breaking can never give rise to scalars which are lighter than the ones we get in MGM. This explains why such models are not sufficient to cover the entire GGM parameter space [15].

Although we have only considered the case where the visible gauge group is $U(1)$, it is clear that inequalities such as (4.7) can be obtained for messengers in arbitrary vectorlike representations of any gauge group. Our discussion above is limited to leading order in SUSY-breaking. It would be interesting to study the corrections to the inequality which arise at higher orders in $f$ (in many examples these are known to be small). Other corrections come from renormalization group running in the visible sector; we have not analyzed these effects in detail.

4.2. SUSY-Breaking in the Kähler Potential

We now consider the effect of coupling the messengers to a SUSY-breaking $D$-term spurion $V|_{\theta^4} = D$ ($D$ real) through the Kähler potential. We start from the Lagrangian

$$L = \int d^4 \theta \Phi_i^\dagger (\delta_{ij} + V\bar{\lambda}_{ij}) \Phi_j + \bar{\Phi}_i^\dagger (\delta_{ij} + V\bar{\lambda}_{ij}) \bar{\Phi}_j + \int d^2 \theta m_{ij} \Phi_i \bar{\Phi}_j + c.c..$$  \hspace{1cm} (4.8)
The matrix $\tilde{\lambda}$ must be Hermitian and messenger parity requires us to choose the same $\tilde{\lambda}$ for the $\Phi$ and the $\tilde{\Phi}$. In the language of section 2 (with an obvious generalization to multiple operators):

$$\tilde{\mathcal{O}}_h = V, \quad (\tilde{\mathcal{O}}_m)_{ij} = \Phi_i^\dagger \Phi_j + \tilde{\Phi}_i^\dagger \tilde{\Phi}_j,$$

with couplings $\sim \tilde{\lambda}_{ij}$. Such terms do not by themselves contribute to the gaugino mass at any order in $D$ because of $R$-symmetry in the messenger sector. We will now analyze the sfermion mass-squared at leading order in $D$. After substituting (4.9) into (2.5), we obtain:

$$m^2_{\tilde{f}} = \frac{g^4 Y^2 D}{32\pi^4} \sum_{i=1}^N \tilde{\lambda}_{ii} \left( \log \frac{\Lambda^2}{m_i^2} - 2 \right). \quad (4.10)$$

Here $\tilde{\lambda}_{ii}$ are the diagonal elements of $\tilde{\lambda}$ in the basis in which $m$ is real and diagonal. The logarithmic divergence is a result of the fact that the SUSY-breaking $D$-term has introduced a non-vanishing supertrace $\sim \text{Tr} \tilde{\lambda}$ into the messenger sector [27]. To render the model calculable we assume that $\text{Tr} \tilde{\lambda} = 0$ so that the supertrace vanishes. This gives

$$m^2_{\tilde{f}} = -\frac{g^4 Y^2 D}{32\pi^4} \sum_{i=1}^N \tilde{\lambda}_{ii} \log m_i^2. \quad (4.11)$$

Note that as opposed to the $F$-term contribution (4.4), this does not have definite sign. Formula (4.11) is rederived in appendix B using the results of section 3.

One could also have studied half-chiral terms in the Kähler potential, such as

$$\frac{1}{\Lambda} \int d^4 \theta X \Phi^\dagger \Phi + \text{c.c.}, \quad (4.12)$$

where $X$ acquires an $F$-term vev $X|_{\theta^2} = f$, as in the previous subsection. For simplicity, we now discuss a single pair of messengers ($N = 1$) of mass $m$. We have shown in section 2, and again in section 3, that such terms do not contribute to the gaugino mass at leading order. In the language of this section, this can be viewed as a consequence of the rescaling anomaly. Redefining

$$\Phi \to \left(1 - \frac{X}{\Lambda}\right) \Phi,$$

we are assuming that there are no additional operators at the scale $\Lambda$ which can generate a sfermion mass.

The fact that (4.12) violates messenger parity is irrelevant for this discussion.

The appearance of the rescaling anomaly in this example was already observed in [16]. Our discussion here serves to clarify its role in light of the general results of sections 2 and 3.
the SUSY-breaking terms change according to

\[ \frac{1}{\Lambda} \int d^4 \theta X \Phi \Phi^\dagger + \text{c.c.} \to - \frac{1}{\Lambda^2} \int d^4 \theta X^\dagger X \Phi \Phi^\dagger \Phi - \frac{m}{\Lambda} \int d^2 \theta X \Phi \Phi^\dagger + \text{c.c.} , \]  

(4.14)

where the second term on the right-hand side arises from the mass term \( m\Phi \Phi^\dagger \) in the superpotential. The terms in (4.14) are a combination of \( F \)- and \( D \)-term spurions. Naively applying (4.13), the \( F \)-term leads to a gaugino mass

\[ M_{\tilde{g}} = \frac{g^2}{8\pi^2} \frac{f}{\Lambda} . \]  

(4.15)

However, the rescaling (4.13) is anomalous and shifts

\[ W_\alpha^2 \to \left( 1 + \frac{g^2}{8\pi^2} \log \left( 1 - \frac{X}{\Lambda} \right) \right) W_\alpha^2 . \]  

(4.16)

This generates a contribution to the gaugino mass which exactly cancels (4.15). The role of the anomaly in this example is further elucidated in appendix B. The anomaly does not affect the sfermion mass, which can be obtained from (4.14) using the formulas derived above for \( F \)- and \( D \)-term spurions.

4.3. Semi-Direct Gauge Mediation

Semi-Direct Gauge Mediation models [19-21] are concrete examples of weakly-coupled, completely calculable gauge mediation. They contain a SUSY-breaking sector and a messenger sector; both sectors and the interactions which couple them can be treated in perturbation theory. In this sense, they are described by the GMGM framework. However, this framework is most useful when the dynamics of the SUSY-breaking sector and the messenger sector do not depend on the weak interaction between them. We then expand to a given power in this interaction, but can in principle treat the factorized SUSY-breaking and messenger correlators exactly. This is not the case for semi-direct models, where the small coupling between the SUSY-breaking sector and the messenger sector also plays a crucial role in the dynamics of the SUSY-breaking sector itself (hence the name “Semi-Direct Gauge Mediation”). In this case one must resort to ordinary perturbation theory to also expand the SUSY-breaking correlator to the desired order.

\footnote{For earlier work along similar lines, see [17].}
Note that once the SUSY-breaking vacuum has been fixed, these issues do not affect the first-order formulas derived in section 2 for general Kähler potential interactions. However, determining the higher-order contributions in general requires a full loop-calculation. These contributions are particularly important if the first-order contribution vanishes. We will see below that even in such cases, there is a parameter regime of Semi-Direct Gauge Mediation which can be analyzed using the results of the previous two subsections and appendix B.

For concreteness, we specialize our discussion to the theory discussed in [20]. Here the SUSY-breaking sector is taken to be the 3-2 model of [35]. The parameters of the 3-2 model are chosen to satisfy $h \ll g_2, g_3 \ll 1$; we will denote $\alpha_2 = \frac{g_2^2}{4\pi}$. There are $2N_f$ messengers $\ell_i$ ($i = 1, \ldots, 2N_f$), which transform as $SU(2)$-doublets. The messengers have a supersymmetric mass term given by

$$W = m\ell^2 ,$$

(4.18)

and couple to the SUSY-breaking sector through the $SU(2)$ gauge fields.

To compute the soft masses at first order in the interaction, we apply the general formulas (2.5) from section 2. At this order, we see that the gaugino mass vanishes due to an $R$-symmetry in the messenger sector, while the sfermion mass vanishes because the $SU(2)$ generators are traceless. This requires us to compute beyond first order, which confronts us with all the difficulties discussed above.

There is, however, a parameter regime of Semi-Direct Gauge Mediation in which the dynamics of the SUSY-breaking sector becomes essentially trivial and the model reduces to a theory of weakly-coupled messengers interacting with spurions, like the theories studied in the previous two subsections and in appendix B. The SUSY-breaking sector and

\[ W_{\text{eff}} = \frac{\Lambda_3^7}{\det(QQ)} + hQ\overline{Q}L , \quad (4.17) \]

where $Q = (\overline{U}, \overline{D})$. 

\[ 11 \text{ The 3-2 model consists of an } SU(3) \times SU(2) \text{ gauge theory with matter content resembling a single generation of the standard model: } Q \in (3, 2); L \in (1, 2); U, D \in (3, 1). \text{ The numbers in parentheses label the } SU(3) \text{ and } SU(2) \text{ representations respectively of the matter fields. Assuming that } \Lambda_3 \gg \Lambda_2 \text{ (here } \Lambda_2, \Lambda_3 \text{ are the strong-coupling scales of the } SU(3), SU(2) \text{ gauge groups), the } SU(3) \text{ dynamics dominates and the theory is described by an effective superpotential} \]
the messenger sector only interact through the $SU(2)$ gauge fields, which are completely Higgsed at the scale

$$\Lambda \sim \Lambda_3/h^{1/7}. \tag{4.19}$$

Thus, the SUSY-breaking sector essentially becomes trivial below this scale as far as the messengers are concerned. If we assume that $m \ll \Lambda$, the model reduces to an effective theory of messengers interacting with spurions and endowed with a natural UV-cutoff $\Lambda$. These spurions are gauge invariant operators of the 3-2 model, and the messengers interact with them through an effective Kähler potential (see section 3 of [20]). We can thus analyze this theory using the language of the previous two subsections, with spurions giving rise to the following contributions to the messenger spectrum [20]:

1.) At zeroth order in $\alpha_2$, the SUSY-breaking sector gives the messengers diagonal mass-splittings $M_{\text{mess}}^2 \sim m^2 \pm m_d^2$ through a $D$-term spurion as in (4.8):

$$V|_{\theta^i} = D \sim m_d^2 \sim h^2 \Lambda^2. \tag{4.20}$$

At this order the supertrace vanishes. The requirement that the messengers not be tachyonic restricts $m^2 \gtrsim h^2 \Lambda^2$, so that the allowed parameter range for $m$ is given by

$$h\Lambda \lesssim m \ll \Lambda. \tag{4.21}$$

2.) At $\mathcal{O}(\alpha_2)$, the 3-2 model generates a half-chiral Kähler potential interaction of the type displayed in (4.12) with:

$$X|_{\theta^2} = f \sim \frac{\Lambda m_{od}^2}{m} \sim \frac{\alpha_2}{4\pi} h \Lambda^2. \tag{4.22}$$

Here and below we will keep track of loop factors such as $\frac{\alpha_2}{4\pi}$, but drop all other numerical $\mathcal{O}(1)$ factors. As explained at the end of the previous section, $f$ gives the messengers off-diagonal masses $m_{od}^2 \sim \frac{\alpha_2}{4\pi} h \Lambda m$. However, due to the rescaling anomaly, no gaugino mass is generated at $\mathcal{O}(f)$.

3.) A non-zero, negative supertrace for the messengers is also generated at $\mathcal{O}(\alpha_2)$:

$$\text{Str } M_{\text{mess}}^2 \sim -N_f \alpha_2 h^2 \Lambda^2. \tag{4.23}$$

We now discuss the leading non-trivial contributions of these three items to the sfermion mass-squared. The $D$-term generates a negative sfermion mass-squared at $\mathcal{O}(D^4)$, 

while the $F$-term gives a positive contribution at $\mathcal{O}(f^2)$. The supertrace (4.23) gives rise to a logarithmically divergent term $\sim -\text{Str}M^2_{\text{mess}}\log \Lambda^2/m^2$; this always dominates the $\mathcal{O}(f^2)$ contribution from the $F$-term, which we consequently drop. We thus obtain for the sfermion mass-squared:

$$m^2_{\tilde{f}} \sim Y^2 \left( \frac{\alpha_g}{4\pi} \right)^2 N_f m^2 \left( \frac{h\Lambda}{m} \right)^2 \left( \alpha_2 \log \frac{\Lambda^2}{m^2} - C_{\tilde{f}} \left( \frac{h\Lambda}{m} \right)^6 \right), \quad (4.24)$$

where $C_{\tilde{f}}$ is a positive $\mathcal{O}(1)$ constant and $\alpha_g = \frac{g^2}{4\pi}$.

Because the supertrace leads to a log-divergent contribution which is cut off at the scale $\Lambda$ at which we defined our spurion model, the sfermion mass is UV-sensitive. While this leading logarithmic piece is universal and can be trusted, there are finite threshold corrections at the scale $\Lambda$ which cannot be calculated in the spurion limit we are considering. These corrections can be estimated to be of the same order as the coefficient of the leading logarithm, i.e. they are $\mathcal{O}(\alpha_2)$. Since we limit ourselves to the large-logarithm limit, these unknown threshold corrections can safely be ignored. Note that in a certain regime, the threshold corrections may be comparable to (or even dominate) the finite $D$-term contribution (the second term in (4.24)). In that case, the latter can also be ignored relative to the large logarithm. In other regimes, this negative $D$-term contribution can dominate the threshold corrections; in that case, we must ensure that the log-enhanced term is sufficiently large to avoid a tachyonic sfermion mass.

It is important to note that the UV-sensitivity discussed in the previous paragraph is an artifact of truncating the full theory to a spurion model. In the example we are considering, the full theory is renormalizable and leads to finite, calculable soft masses. Here the general discussion from the introduction applies: the more information we have about the structure of the theory at the cutoff scale $\Lambda$, the more reliable statements we can make about the soft masses.

Unlike the sfermion mass, the gaugino mass does not suffer from a UV-ambiguity. As explained above, the $D$-terms by themselves never generate a gaugino mass, and the $\mathcal{O}(f)$ contribution vanishes due to the rescaling anomaly. Using the formulas from appendix B, we see that the leading non-trivial contributions to the gaugino mass arise at $\mathcal{O}(fD^2) + \mathcal{O}(f^3)$:

$$M_{\tilde{g}} \sim \left( \frac{\alpha_g}{4\pi} \right) \left( \frac{\alpha_2}{4\pi} \right) N_f m \left( \frac{h\Lambda}{m} \right)^3 \left( \left( \frac{h\Lambda}{m} \right)^2 + C_{\tilde{g}} \left( \frac{\alpha_2}{4\pi} \right)^2 \right), \quad (4.25)$$
where $C_\tilde{g}$ is a positive $O(1)$ constant. Parametrically either the first or the second term dominates, depending on the relative size of $\frac{\alpha_2^2}{4\pi}$ and $\frac{h\Lambda}{m}$.

We now compare the sfermion mass-squared to the gaugino mass-squared:

\[
\frac{m_{\tilde{f}}^2}{M_g^2} \sim \frac{4\pi Y^2 N_f \log \frac{\Lambda^2}{m^2}}{(\frac{\alpha_2}{4\pi})^4 \left( \frac{h\Lambda}{m} \right)^2 \left( \frac{h\Lambda}{m} \right)^2 + C_\tilde{g} \left( \frac{\alpha_2}{4\pi} \right)^2}.
\] (4.26)

For the parameter range (4.21) and $\alpha_2 \ll 1$, the sfermion mass is always much greater than the gaugino mass. (This can be avoided by fine-tuning the hidden-sector parameters.) Thus, the phenomenology is that of split SUSY, and the fine-tuning problem of the SSM is exacerbated. This does not rule out the possibility of a more desirable phenomenology for the case $m \gtrsim \Lambda$, where the spurion treatment breaks down and a genuine loop_calculation is required.

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Appendix A. Remarks on General Gauge Mediation

In this appendix we briefly review the formalism of General Gauge Mediation (GGM) and collect some results on the GGM correlation functions which have not yet been discussed in the literature.
A.1. Review of GGM

The GGM formalism applies to theories which decouple into a SUSY-breaking hidden sector and a supersymmetric visible sector when the visible-sector gauge couplings vanish. For simplicity, we will take the visible sector to consist of a $U(1)$ gauge theory with coupling $g$ and a single flavor $f$ of charge $Y$. Of central importance are the correlation functions of the hidden-sector global $U(1)$ current multiplet. A global symmetry current $j_\mu$ is embedded in a real superfield $J$ satisfying $D^2 J = 0$. In components:

\[ J = J + i \theta j - i \bar{\theta} j - \theta \sigma^\mu \partial_\mu j + \frac{1}{2} \theta^2 \bar{\theta} \sigma^\mu \partial_\mu j - \frac{1}{2} \bar{\theta}^2 \theta \sigma^\mu \partial_\mu j - \frac{1}{4} \theta^2 \bar{\theta}^2 \partial^2 J, \]

(A.1)

with $\partial_\mu j^\mu = 0$.

We define the functions $C_a(x)$ ($a = 0, 1/2, 1$) and $B(x)$ through:

\[ \langle J(x) J(0) \rangle = C_0(x) \]
\[ \langle j_\alpha(x) j_{\dot{\alpha}}(0) \rangle = -i \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu C_{1/2}(x) \]
\[ \langle j_\mu(x) j_\nu(0) \rangle = (\eta_{\mu\nu} \partial^2 - \partial_\mu \partial_\nu) C_1(x) \]
\[ \langle j_\alpha(x) j_\beta(0) \rangle = \frac{1}{4} \epsilon_{\alpha\beta} B(x) . \]

(A.2)

Note that our normalization of $B$ differs from the conventions in [10,16] by a factor of 4. The Fourier transforms of the functions $C_a, B$ will be denoted by $\tilde{C}_a, \tilde{B}$. When SUSY is unbroken, all the $C_a(x)$ are equal, and we denote their common limit by $C_{\text{SUSY}}(x)$. The SUSY-breaking gaugino mass and sfermion mass-squared are then given by:

\[ M_{\tilde{g}} = \frac{g^2}{4} \tilde{B}(p = 0) , \]

(A.3)

\[ m_{\tilde{f}}^2 = -g^4 Y^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \left( \tilde{C}_0(p) - 4\tilde{C}_{1/2}(p) + 3\tilde{C}_1(p) \right) . \]

(A.4)

Finally, recall from [16] that we can write

\[ B(x) = \langle Q^2 (J(x) J(0)) \rangle , \]

(A.5)

\[ A(x) \equiv -8\partial^2 \left( C_0(x) - 4C_{1/2}(x) + 3C_1(x) \right) = \langle Q^4 (J(x) J(0)) \rangle , \]

(A.6)

where $Q^2(\cdots) = \{ Q_\alpha, [Q_\alpha, \cdots] \}$ and $Q^4(\cdots) = \{ Q_\alpha, [Q_\alpha, [Q_\alpha, \cdots]] \}$. The order of the supercharges in (A.3) and (A.6) is inconsequential because of translational invariance. Using (A.6) and integrating by parts, formula (A.4) can be put into the form (1.2).

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\textsuperscript{12} This differs from the notation in [10,16] by some powers of $x$. We adopt this convention so that we can consistently denote by $\tilde{f}(p) = \int d^4 x f(x) e^{-ipx}$ the Fourier transform of a function $f(x)$, while matching the conventions of [10,16] in momentum space.
A.2. Goldstinos in GGM

Supersymmetric theories without FI-terms or non-trivial target-space geometry in the UV have a Ferrara-Zumino multiplet \([36,38]\) containing the supercurrent and the energy-momentum tensor. It is organized in terms of a real vector superfield \(J_\alpha \dot{\alpha}\) satisfying
\[
\overline{D}^\dot{\alpha} J_\alpha = D_\alpha X, \quad \overline{D}_\alpha X = 0.
\]
(A.7)

We see that there must be a chiral superfield \(X\) which is well-defined in all supersymmetric theories we discuss. In \([39]\), it was shown that at low energies the operator \(X\) flows to an operator \(X_{NL}\) as follows:
\[
X \rightarrow \frac{8F}{3} X_{NL},
\]
\[
X_{NL} = \frac{G^2}{2F_X} + \sqrt{2}\theta G + \theta^2 F_X.
\]
(A.8)

Here \(G\) is the massless Goldstino fermion; \(X_{NL}\) satisfies \(X_{NL}^2 = 0\). At very low energies, \(F_X\) can be replaced by its expectation value \(F\), where \(F^2\) is the vacuum energy-density.

Consider a general superfield \(O\), which is well-defined in the UV. Suppose that the expectation values of its \(\theta^2, \theta^4\) components are \(F_O, G_O, D_O\), respectively. At very low energies this superfield must flow to
\[
O \rightarrow \frac{F_O}{F} X_{NL} + \frac{G_O}{F} X_{NL}^\dagger + \frac{D_O}{F^2} X_{NL} X_{NL} + \cdots,
\]
(A.9)

where the dots stand for corrections with more fermions or more derivatives. If there are other massless particles, they could mix into (A.9) as well, but this will not change our final answer.

We can use the decomposition (A.9) to extract the \(F, G, D\)-components of the superfield \(O\) by projecting onto combinations of \(X_{NL}, X_{NL}^\dagger\). Equivalently, we can consider correlation functions of \(O\) and combinations of \(X, X^\dagger\) at very long distances as follows:
\[
F_O = \frac{3}{2} \pi^4 F^2 \lim_{y \to \infty} y^6 \langle X^\dagger(y)O(0) \rangle,
\]
\[
G_O = \frac{3}{2} \pi^4 F^2 \lim_{y \to \infty} y^6 \langle X(y)O(0) \rangle,
\]
(A.10)
\[
D_O = \frac{9}{4} \pi^8 F^4 \lim_{y \to \infty} y^{12} \langle X^\dagger X(y)O(0) \rangle.
\]

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Here $X$ denotes the bottom component of the superfield $X$. The last equation in (A.10) contains no self-contractions between the $X$ operators. This is evident from the expansion (A.9).

These observations allow us to rewrite the $B$- and $A$-correlators (A.5) and (A.6) as

$$B(x) = \langle Q^2(J(x)J(0)) \rangle = 6\pi^2 F^2 \lim_{y \to \infty} y^6 \langle X^\dagger(y)J(x)J(0) \rangle,$$

$$A(x) = \langle Q^4(J(x)J(0)) \rangle = 36\pi^4 F^4 \lim_{y \to \infty} y^{12} \langle X^\dagger X(y)J(x)J(0) \rangle.$$  \hspace{1cm} (A.11)

This converts the GGM formulas to correlation functions of bottom components of well-defined superfields, without the need to perform SUSY variations. Physically the first formula in (A.11) describes two Goldstinos $G^2$ propagating from infinity at essentially zero momentum and contracting with $J(x)J(0)$. Similarly the second formula describes four zero-momentum Goldstinos $G^2G^2$ coming from infinity.

Appendix B. More on Weakly-Coupled Messengers

In this appendix, we give a more detailed treatment of the weakly-coupled spurion models discussed in section 4.

B.1. SUSY-Breaking in the Superpotential

The method of section 3 tells us how to compute the visible soft masses for the messenger Lagrangian

$$\mathcal{L} = \int d^4\theta \left( \Phi_i^\dagger \Phi_i + \tilde{\Phi}_i^\dagger \tilde{\Phi}_i \right) + \int d^2\theta \left( X\lambda_{ij} + m_{ij} \right) \Phi_i \tilde{\Phi}_j + c.c. , \hspace{1cm} (B.1)$$

with $X|_{\theta^2} = f$ (see subsection 4.1), by studying the supersymmetric Lagrangian

$$\mathcal{L}_{SUSY} = \int d^4\theta \left( \Phi_i^\dagger \Phi_i + \tilde{\Phi}_i^\dagger \tilde{\Phi}_i \right) + \int d^2\theta \mathcal{M}_{ij} \Phi_i \tilde{\Phi}_j + c.c. . \hspace{1cm} (B.2)$$

Here $\mathcal{M}$ is a general complex supersymmetric mass matrix that should be distinguished from $m_{ij}$.

Because of $SU(N) \times SU(N)$ invariance, all physical observables depend only on the eigenvalues $\mu_i^2$ of $\mathcal{M}$. Thus, the function $\tilde{C}_{SUSY}(p)$ takes the simple form

$$\tilde{C}_{SUSY}(p) = \frac{1}{8\pi^2} \sum_{i=1}^N \left( \log \frac{\Lambda^2}{\mu_i^2} + g \left( \frac{p^2}{\mu_i^2} \right) \right), \hspace{1cm} (B.3)$$
where \( \Lambda \) is a UV-cutoff. The numerical coefficient and the functional form of \( g \) are fixed by a one-loop calculation. We will only need the asymptotic behavior

\[
g(p^2 \to 0) = -1 + \mathcal{O}(p^2),
\]

\[
g(p^2 \to \infty) = -\log \frac{p^2}{\mu_i^2} + 1 + \mathcal{O}(1/p^2).
\]

Substituting this into (B.3), we see that

\[
\tilde{C}_{\text{SUSY}}(p = 0) = \frac{N}{8\pi^2} \left( \log \Lambda^2 - 1 \right) - \frac{1}{8\pi^2} \left( \text{Tr} \log \mathcal{M} + \text{Tr} \log \mathcal{M}^\dagger \right).
\]

Thus, at zero momentum the answer breaks up into a holomorphic and an anti-holomorphic part, as was explained in section 3.

It is now straightforward to calculate the soft masses. In the language of sections 3 and 4, we consider the deformation \( \mathcal{M} = m + \epsilon \lambda \) in (B.3) and differentiate as in (3.3) to obtain the gaugino mass:

\[
M_\tilde{g} = -g^2 f \left( \frac{8\pi^2}{\partial} \text{Tr} \log (m + \epsilon \lambda) \right) = -g^2 f \left( \frac{8\pi^2}{\partial} \text{Tr}^{-1} \lambda = -g^2 f \sum_{i=1}^{N} \frac{\lambda_{ii}}{m_i},
\]

where \( m_i \) are the eigenvalues of \( m \) and \( \lambda_{ii} \) are the diagonal entries of \( \lambda \) in the basis where \( m \) is real and diagonal.

To calculate the sfermion mass-squared, we need to calculate a second derivative of \( \tilde{C}_{\text{SUSY}}(p) \) with respect to \( \epsilon \) and \( \epsilon^* \), as in (3.5). The result then needs to be integrated as in (1.2) (or equivalently (A.4)). For this class of theories, there is a trick to perform the integral.\(^\text{13}\) Because of the simple form of \( \tilde{C}_{\text{SUSY}}(p) \) in (B.3), the \( \epsilon \)-derivatives can be converted to momentum derivatives using the chain rule:

\[
\frac{\partial^2}{\partial \epsilon \partial \epsilon^*} \tilde{C}_{\text{SUSY}}(p) = -\frac{p^2}{8\pi^2} \sum_{i=1}^{N} \left( \frac{\partial^2 \log \mu_i^2}{\partial \epsilon \partial \epsilon^*} \frac{\partial g}{\partial p^2} - \left| \frac{\partial \log \mu_i^2}{\partial \epsilon} \right|^2 \frac{\partial}{\partial p^2} \left( p^2 \frac{\partial g}{\partial p^2} \right) \right).
\]

It is now straightforward to integrate, picking up the boundary values according to (B.4). We obtain for the sfermion mass-squared:

\[
m_f^2 = g^4 Y_f^2 \frac{f^2}{64\pi^4} \sum_{i=1}^{N} \left( \frac{\partial^2 \log \mu_i^2}{\partial \epsilon \partial \epsilon^*} \log \mu_i^2 + \left| \frac{\partial \log \mu_i^2}{\partial \epsilon} \right|^2 \right)
\]

\[
= g^4 Y_f^2 \frac{f^2}{128\pi^4} \frac{\partial^2}{\partial \epsilon \partial \epsilon^*} \text{Tr} \log^2 \mathcal{M}^\dagger \mathcal{M}
\]

\[
= g^4 Y_f^2 \frac{f^2}{64\pi^4} \sum_{i=1}^{N} \left( \frac{\lambda_{ii}^2}{m_i^2} + \sum_{j=1, j \neq i}^{N} \frac{\lambda_{ij}^2}{m_i^2 - m_j^2} \log \frac{m_i^2}{m_j^2} \right).
\]

\(^{13}\) In some special cases this has been discussed in \([32,33]\).
In (B.6) and (B.8) we find perfect agreement with the answers quoted in subsection 4.1. These results were first obtained in [14] using the wavefunction renormalization technique [25]. In the next subsection, we will reexamine this technique in more detail, comment on its limitations (even for small SUSY-breaking), and explain why it happens to give correct answers for the soft masses in weakly-coupled spurion models.

B.2. Comments on Wavefunction Renormalization

We begin by reviewing the wavefunction renormalization technique [25] for the sfermion mass in Minimal Gauge Mediation (MGM). Consider a single pair of messengers $\Phi, \tilde{\Phi}$ with superpotential

$$W = X\tilde{\Phi}\Phi. \quad (B.9)$$

For now $X$ is a background chiral superfield and SUSY is unbroken. To obtain the sfermion mass, we need to calculate the $X$-dependent supersymmetric effective action for the sfermion fields $Q, \tilde{Q}$. There are two types of operators which can appear in this effective action: operators which contain the UV-cutoff $\Lambda$ and operators which are $\Lambda$-independent. The only place where $\Lambda$ can appear is inside perturbation theory logarithms; the first non-trivial such operator appears at two-loop order and contributes to the anomalous dimension of $Q, \tilde{Q}$. In MGM it is given by:

$$\delta K \sim \log^2 \frac{X^\dagger X}{\Lambda^2} \left( Q^\dagger Q + \tilde{Q}^\dagger \tilde{Q} \right). \quad (B.10)$$

Now consider $\Lambda$-independent terms. We organize the effective action as an expansion in the number of supercovariant derivatives acting on $X, X^\dagger$. Since $X$ is the only mass scale, there is no operator without covariant derivatives. There are, however, many operators with covariant derivatives, such as

$$\int d^4\theta \frac{D^2 X^\dagger D^2 X}{(X^\dagger X)^2} \left( Q^\dagger Q + \tilde{Q}^\dagger \tilde{Q} \right). \quad (B.11)$$

To introduce SUSY-breaking, we give the $F$-term of $X$ a vev $X|_{\theta^2} = f$. The contribution of (B.10) and (B.11) to the sfermion mass-squared is (up to coefficients) given by:

$$m^2_f \sim \frac{f^2}{|X|^2} + \frac{f^4}{|X|^6}. \quad (B.12)$$

In this equation $X$ denotes the bottom-component vev of the background superfield $X$. The wavefunction renormalization technique correctly captures the $O(f^2)$ term in (B.12)
because the only operator (B.10) which can be written at this order is the cutoff-dependent anomalous dimension. The higher orders are not captured and are more difficult to calculate.

This discussion suggests that the wavefunction renormalization technique may not even capture the leading SUSY-breaking contribution in theories with more than one mass scale. Consider, for example, an O’Raifeartaigh-like model with superpotential

$$W = M(X)_{ij} \Phi_i \tilde{\Phi}_j + Y_{ij} \Phi_i \tilde{\Phi}_j,$$  \hfill (B.13)

where $M(X)$ is a general matrix function of the background chiral superfield $X$ and the Kähler potential is canonical. Suppose we want to calculate the one-loop effective potential for $Y$ when $X$ acquires a SUSY-breaking vev $X|_{\theta^2} = f$. It is easy to check that the wavefunction renormalization technique fails to give the correct answer even at leading order in $f$. The reason is that using the matrices $M(X)$ and $\rho$ we can construct many cutoff-independent operators without covariant derivatives. To see this, consider the effective Kähler potential generated for $Y$ when the chiral fields $\Phi, \tilde{\Phi}$ are integrated out at one-loop; it is given by [31]:

$$K_{\text{eff}} \sim \text{Tr} \left( (M + \rho Y)^\dagger (M + \rho Y) \log \frac{(M + \rho Y)^\dagger (M + \rho Y)}{\Lambda^2} \right).$$  \hfill (B.14)

Expanding the logarithm, we obtain many $\Lambda$-independent operators without covariant derivatives (for instance, a tadpole for $Y$). Therefore, the wavefunction renormalization technique does not correctly capture the leading-order SUSY-breaking effect. We generally expect this to be the case, unless the theory has essentially only one scale set by a single superfield.

The example of MGM discussed above exactly falls into this class of trivial theories. More generally, consider a free theory of $N$ messengers of the type discussed in subsections 4.1 and B.1, with superpotential

$$W = (X \lambda_{ij} + m_{ij}) \Phi_i \tilde{\Phi}_j = M_{ij}(X) \Phi_i \tilde{\Phi}_j.$$  \hfill (B.15)

The effective action for this theory can only depend on the eigenvalues $\mu^2_i$ of the matrix $M(X)$. Since the messengers are decoupled and the gauge interactions are flavor blind, ratios of different eigenvalues cannot appear. Thus, the same argument as for MGM shows that there are no cutoff-independent operators at leading order in $f$, and at that order the result is correctly captured by the wavefunction renormalization technique. Note that the assumption that the messengers are free is crucial for this argument.
Proceeding as in subsection B.1, we now compute the sfermion mass-squared for the messenger Lagrangian
\[ \mathcal{L} = \int d^4 \theta \Phi_i^\dagger (\delta_{ij} + V \tilde{\lambda}_{ij}) \Phi_j + \tilde{\Phi}_i^\dagger (\delta_{ij} + V \tilde{\lambda}_{ij}) \tilde{\Phi}_j + \int d^2 \theta m_{ij} \Phi_i \tilde{\Phi}_j + \text{c.c.}, \] (B.16)
with \( V|_{\theta^i} = D \) and \( \tilde{\lambda} \) Hermitian, by analyzing the SUSY theory given by
\[ \mathcal{L}_{\text{SUSY}} = \int d^4 \theta \Omega_{ij} (\Phi_i^\dagger \Phi_j + \tilde{\Phi}_i^\dagger \tilde{\Phi}_j) + \int d^2 \theta m_{ij} \Phi_i \tilde{\Phi}_j + \text{c.c.}. \] (B.17)
Here \( \Omega \) is Hermitian positive-definite. As discussed in subsection 4.2, the Lagrangian (B.16) by itself never gives rise to gaugino masses because of \( R \)-symmetry.

We want to compute \( \tilde{C}_{\text{SUSY}}(p) \), as a function of the deformation \( \Omega = 1 + \tilde{\epsilon} \tilde{\lambda} \), and then take a derivative with respect to \( \tilde{\epsilon} \) as in (3.4). To calculate \( \tilde{C}_{\text{SUSY}}(p) \), we first diagonalize \( \Omega \) through a unitary transformation:
\[ U^\dagger \Omega U = \text{diag}(|\omega_i|^2), \] (B.18)
where the \( \omega_i \) are unique up to a phase. This \( SU(N) \) transformation is not anomalous, and we can rewrite the Lagrangian as
\[ \mathcal{L}_{\text{SUSY}} = \sum_{i=1}^N \int d^4 \theta |\omega_i|^2 \left( \Phi_i^\dagger \Phi_i + \tilde{\Phi}_i^\dagger \tilde{\Phi}_i \right) + \int d^2 \theta m_i' \Phi_i \tilde{\Phi}_i + \text{c.c.}, \] (B.19)
where \( m' = U^T m U \). We now rescale \( \Phi_i \rightarrow \omega_i^{-1} \Phi_i, \tilde{\Phi}_i \rightarrow \omega_i^{-1} \tilde{\Phi}_i \) and obtain
\[ \mathcal{L}_{\text{SUSY}} = \sum_{i=1}^N \int d^4 \theta \left( \Phi_i^\dagger \Phi_i + \tilde{\Phi}_i^\dagger \tilde{\Phi}_i \right) + \int d^2 \theta m_i'' \Phi_i \tilde{\Phi}_i + \text{c.c.}, \] (B.20)
where \( m'' = \text{diag}(\omega_i^{-1}) m' \text{diag}(\omega_i^{-1}) \). However, this rescaling is anomalous and we pick up a correction of the form
\[ W^2_{\alpha} \rightarrow \left( 1 - \frac{g^2}{4\pi^2} \sum_{i=1}^N \log \omega_i \right) W^2_{\alpha}. \] (B.21)

Note that the eigenvalues \( \mu_i^2 \) of \( m''m'' \) are the physical masses of the messengers in this supersymmetric theory; they depend on the matrix \( \Omega \) and its eigenvalues \( |\omega_i|^2 \) in a
complicated way. Since we have analyzed models of the type (B.20) in subsection B.1, we only need to add the contribution of the anomaly to obtain

\[
\tilde{C}_{\text{SUSY}}(p) = \frac{1}{8\pi^2} \sum_{i=1}^{N} \left( \log \frac{A_i^2}{\mu_i^2} + g \left( \frac{p^2}{\mu_i^2} \right) - 2 \log |\omega_i|^2 \right).
\]

(B.22)

Here \( g \) is the same function as in (B.3). It is easy to check that at zero momentum (B.22) breaks up into the sum of a holomorphic and an anti-holomorphic part, in accordance with the discussion in section 3. In particular, \( \tilde{C}_{\text{SUSY}}(p = 0) \) is \( \Omega \)-independent.

We now set \( \Omega = 1 + \tilde{\epsilon} \tilde{\lambda} \) and differentiate with respect to \( \tilde{\epsilon} \). Since \( \tilde{C}_{\text{SUSY}} \) is independent of \( \Omega \) at zero momentum, only \( g \) can contribute to this derivative. Converting \( \tilde{\epsilon} \)-derivatives to momentum derivatives as before, we obtain:

\[
\frac{\partial}{\partial \tilde{\epsilon}} \tilde{C}_{\text{SUSY}}(p) = -\frac{p^2}{8\pi^2} \sum_{i=1}^{N} \frac{\partial \log \mu_i^2}{\partial \tilde{\epsilon}} \frac{\partial g}{\partial p^2}.
\]

(B.23)

Performing the integral is again trivial, and we get:

\[
m_i^2 = \frac{g^4 Y^2 D}{32\pi^4} \sum_{i=1}^{N} \tilde{\lambda}_{ii} \left( \log \frac{A_i^2}{m_i^2} - 2 \right).
\]

(B.24)

Here \( \tilde{\lambda}_{ii} \) are the diagonal elements of \( \tilde{\lambda} \) in the basis in which \( m \) is real and diagonal. As discussed in subsection 4.2, we assume that \( \text{Tr} \tilde{\lambda} = 0 \) to ensure a vanishing supertrace and render the model calculable. This finally gives

\[
m_i^2 = -\frac{g^4 Y^2 D}{32\pi^4} \sum_{i=1}^{N} \tilde{\lambda}_{ii} \log m_i^2,
\]

(B.25)

in accordance with our result from subsection 4.2.

**B.4. Comments on Vanishing Gaugino Masses**

In this subsection we elaborate on the discussion at the end of subsection 4.2 and elucidate why chiral spurions in the Kähler potential cannot generate leading-order gaugino masses.

Consider a single free chiral superfield \( \Phi \). The theory has a global \( U(1) \) symmetry with a conserved current \( J = \Phi^\dagger \Phi \). The anomaly is the statement that while we can satisfy

\[
D_i^2 \langle J(x_1, \theta_1, \bar{\theta}_1) J(x_2, \theta_2, \bar{\theta}_2) J(x_3, \theta_3, \bar{\theta}_3) \rangle = 0 \quad (i = 1, 2, 3)
\]

(B.26)
at separated points, there is no set of contact terms which makes \((B.26)\) true at coincident points. One way to cancel this anomaly is to introduce an additional chiral superfield \(\tilde{\Phi}\) with its own current \(\tilde{J} = \tilde{\Phi}^\dagger \tilde{\Phi}\). This makes it possible to choose contact terms such that the current \(\tilde{J} = J - \tilde{J}\) satisfies \(D^2 \tilde{J} = 0\) in all correlation functions – even at coincident points.

Perhaps the most dramatic consequence of these contact terms is that even though the fields \(\Phi\) and \(\tilde{\Phi}\) are decoupled, we must have

\[
\langle J J J \rangle \neq \langle J \tilde{J} \tilde{J} \rangle .
\]

The two correlators in \((B.27)\) differ by contact terms at coincident points. No Feynman diagram with intermediate \(\Phi\) or \(\tilde{\Phi}\) fields leads to such contact terms. However, if we add heavy regulator fields, then the contact terms are generated by diagrams with intermediate regulator particles. This phenomenon is very common in the presence of anomalies.

We now apply these statements to study the gaugino mass for a single messenger pair \(\Phi, \tilde{\Phi}\) interacting with a chiral spurion \(X\) through the Kähler potential. For now, we assume that the messengers are massless, so that in the notation of this subsection the Lagrangian is given by

\[
\mathcal{L} = \int d^4\theta \left( \tilde{J} + \left( 1 + \frac{X}{A} + \frac{X^\dagger}{A} \right) J \right) .
\]

To leading order in \(X|\theta_2 = f\), the gaugino mass is determined by integrating the correlator

\[
b(x_1, x_2, x_3) \sim \langle Q^4(J(x_1))\tilde{J}(x_2)\tilde{J}(x_3) \rangle .
\]

Note that \(D^2 J = 0\) implies that \(Q^4 J = 0\) and therefore \(b\) vanishes at separated points. However, it does not vanish identically: there are finite contact terms at coincident points. Consequently, the massless theory based on \((B.28)\) leads to a gaugino mass. This mechanism is identical to the one discussed in \([40]\).

If we add a supersymmetric mass term

\[
W = m\Phi\tilde{\Phi} ,
\]

with arbitrary nonzero \(m\), then there is an additional contribution to the correlation function \(b\). The reason is that the current \(J\) is no longer conserved, and thus \(b\) no longer
vanishes at separated points. However, when integrated, the contribution from separated points exactly cancels the contact-term contribution from coincident points. As was already discussed at the end of subsection 4.2, this can be directly seen by rescaling $\Phi \rightarrow (1 - \Lambda) \Phi$. Classically, this leads (at leading order in $X$) to canonical kinetic terms and an $X$-dependent superpotential, which would seem to generate a gaugino mass. However, quantum mechanically the rescaling is anomalous, and the effect of the anomaly precisely cancels the superpotential contribution to the gaugino mass. This is consistent with our arguments from sections 2 and 3 based on the explicit evaluation of correlation functions and holomorphy.

In summary, we see that for $m = 0$ the anomaly leads to a nonzero gaugino mass even though there is no obvious diagram in the low-energy theory which generates such a mass. For nonzero $m$ the gaugino mass vanishes. There is a diagram in the low-energy theory which exactly cancels the contribution in the $m = 0$ theory. Therefore, in interesting models of messengers, Kähler potential operators of the type considered in this subsection do not generate leading-order gaugino masses.

**B.5. Spurion Models Beyond Leading Order**

In this subsection we will display the full, all-orders gaugino mass and sfermion mass-squared for the weakly-coupled spurion models that we considered in section 4 and the previous subsections of this appendix. The full theory is defined by

$$\mathcal{L} = \int d^4 \theta \Phi_i \Phi_j \delta_{ij} + V \Phi_j \Phi_j + \Phi_i \Phi_j + \int d^2 \theta (X \phi_i + m_{ij}) + \Phi_i \Phi_j + c.c. \quad (B.31)$$

We will directly work in a basis in which $m$ is diagonal with real eigenvalues $m_i$. In this basis messenger parity and CP conservation require that $\lambda$ be real and symmetric; $\tilde{\lambda}$ must always be Hermitian. The $F$- and $D$-term spurions $X$ and $V$ acquire expectation values $X|_{\theta^2} = f$ and $V|_{\theta^4} = D$ respectively, which we take to be real without loss of generality.

This model contains $N$ Dirac fermion pairs $\psi_i, \tilde{\psi}_i$ with masses $m_i$, and $2N$ complex scalars $(\phi_i, \tilde{\phi}_i)$ with mass matrix

$$\mathcal{M} = \begin{pmatrix} m_i^2 - D\tilde{\lambda} & -f\lambda \\ -f\lambda & m_i^2 - D\lambda \end{pmatrix}. \quad (B.32)$$

By a unitary transformation we can bring $\mathcal{M}$ to block-diagonal form:

$$\mathcal{M} \rightarrow \begin{pmatrix} m_i^2 + f\lambda - D\tilde{\lambda} & 0 \\ 0 & m_i^2 - f\lambda - D\tilde{\lambda} \end{pmatrix} \equiv \begin{pmatrix} \mathcal{M}_+ & 0 \\ 0 & \mathcal{M}_- \end{pmatrix} \quad (B.33)$$
Since the matrices $\mathcal{M}_\pm$ are Hermitian, there are unitary matrices $U_\pm$ such that $U_\pm^\dagger M_\pm U_\pm = \text{diag}(m_{\pm 1}^2, m_{\pm 2}^2, \ldots, m_{\pm N}^2)$. To avoid tachyonic messengers, we need to ensure that $m_{\pm i}^2 > 0$. In practice, this means choosing $f$ and $D$ sufficiently small compared to the supersymmetric messenger masses $m_i$. The supertrace is given by

$$\text{Tr} \, \mathcal{M}_+ + \text{Tr} \, \mathcal{M}_- - 2 \text{Tr} \, m_i^2 = -2D \text{Tr} \, \tilde{\lambda},$$  \hspace{1cm} (B.34)$$

and we need to assume $\text{Tr} \, \tilde{\lambda} = 0$ to render the model calculable.

Using the formulas in appendix B of [16], we immediately obtain for the gaugino mass:

$$M_\tilde{g} = -\frac{g^2}{8\pi^2} \sum_{i,j=1}^N (\pm)(U_\pm^\dagger)_{ij}(U_\pm^\dagger)_{ji} \frac{m_{\pm i}^2 \log \frac{m_{\pm i}^2}{m_{\pm j}^2}}{m_{\pm i}^2 - m_{\pm j}^2}. \hspace{1cm} (B.35)$$

To calculate the sfermion mass-squared, we first compute the GGM correlation functions defined in (A.2). From (B.4) in [16] we get:

$$\tilde{C}_0(p) = \sum_{i,j=1}^N (U_\pm^\dagger U_\mp)_{ij}(U_\pm^\dagger U_\mp)_{ji}I(p, m_{\pm i}, m_{\mp j}) ,$$

$$\tilde{C}_{1/2}(p) = \frac{1}{p^2} \sum_{i=1}^N \sum_{i,j=1}^N \left(J(m_{\pm i}) - J(m_i)\right)$$

$$+ \frac{1}{p^2} \sum_{i,j=1}^N (U_\pm^\dagger)_{ij}(U_\pm^\dagger)_{ji} \left(p^2 + m_{\pm i}^2 - m_j^2\right)I(p, m_{\pm i}, m_j) , \hspace{1cm} (B.36)$$

$$\tilde{C}_1(p) = \frac{1}{3p^2} \left(\sum_{i=1}^N \sum_{i=1}^N \left(p^2 + 4m_{\pm i}^2\right)I(p, m_{\pm i}, m_{\pm i}) + 4J(m_{\pm i})\right)$$

$$+ 4 \sum_{i=1}^N \left(p^2 - 2m_i^2\right)I(p, m_i, m_i) - 2J(m_i) \right) .$$

The Euclidian loop integrals $I(p, m_1, m_2)$ and $J(m)$ have been defined in (B.5) of [16] as follows:

$$I(p, m_1, m_2) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{((p+q)^2 + m_1^2)((q^2 + m_2)^2)},$$

$$J(m) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 + m^2}, \hspace{1cm} (B.37)$$

where a sharp momentum cutoff $q^2 \leq \Lambda^2$ has been imposed. To obtain the sfermion mass-squared (1.2), we substitute the $\tilde{C}_a(p)$ of (B.36) into (A.2) and do the momentum integral.
Note that as \( p^2 \to \infty \), the functions \( \tilde{C}_a(p) \) in (B.36) only differ at \( \mathcal{O}(1/p^4) \), so that this integral is guaranteed to be UV-convergent\(^{14}\). To compute the relevant two-loop integrals we follow Martin \^[41\]. The final answer is completely finite and can be simplified by using various properties of dilogarithms. After the dust settles, we obtain for the sfermion mass-squared:

\[
m^2_f \tilde{f} = \frac{g^4 Y^2}{64\pi^4} \sum_{\pm} \left( \sum_{i=1}^N \left( m_{\pm i}^2 \log m_{\pm i}^2 - m_i^2 \log m_i^2 \right) \right) \\
+ \sum_{i,j=1}^N \left( \frac{1}{2} (U_\pm^\dagger U_\pm)_{ij} (U_\pm^\dagger U_\pm)_{ji} m_{\pm i}^2 \text{Li}_2 \left( 1 - \frac{m_{\pm j}^2}{m_{\pm i}^2} \right) - 2 (U_\pm^\dagger)_{ij} (U_\pm)_{ji} m_{\pm i}^2 \text{Li}_2 \left( 1 - \frac{m_{\pm j}^2}{m_{\pm i}^2} \right) \right) .
\]

(B.41)

It is straightforward to expand these expressions to leading order in \( D \) and \( f \), in which case they exactly reduce to the formulas derived in section 4 and subsections B.1, B.2.

Formulas for the all-order soft masses in these models were first obtained in \^[26\], and we find complete agreement with (B.35) and (B.41). Special cases of these formulas have been considered in \^[41,42\].

\(^{14}\) This is a general result which holds in any renormalizable theory: the difference of any two \( \tilde{C}_a(p) \) vanishes at least as rapidly as \( 1/p^4 \) as \( p^2 \to \infty \). To prove this, we act on components of the current multiplet \( J \) in (A.3) with the supercharges to obtain the following two relations:

\[
\sigma_{\dot{\alpha} \alpha} \langle Q_\alpha \overline{Q}_{\dot{\alpha}} (j^\mu (x) J(0)) \rangle = 6 \partial^2 \left( C_0(x) - 2 C_{1/2}(x) + C_1(x) \right), \quad (B.38)
\]

\[
\langle Q_\alpha \overline{Q}_{\dot{\alpha}} (j^\alpha (x) \overline{J}(0)) \rangle = -2 \partial^2 \left( C_0(x) + 2 C_{1/2}(x) - 3 C_1(x) \right) . \quad (B.39)
\]

Consider the OPE of \( j^\mu (x) J(0) \) as \( x^\mu \to 0 \). Since the current superfield \( J \) has dimension 2, we have

\[
j^\mu (x) J(0) \sim \frac{O^\mu}{x^{-\Delta_O+6}} + \frac{V^\mu}{x^{-\Delta_V+5}} + \cdots , \quad (B.40)
\]

where \( O \) and \( V^\mu \) are scalar and vector operators of dimension \( \Delta_O \) and \( \Delta_V \) respectively, and the dots denote less singular terms. By Lorentz invariance, only \( V^\mu \) can contribute to (B.38), and moreover it cannot be a descendent. Hence \( V^\mu \) must be a primary. The unitarity bound for a primary vector operator is \( \Delta_V \geq 3 \), which is saturated by a conserved current. Fourier transforming (B.38), we conclude that the combination of the \( \tilde{C}_a(p) \) on the right-hand side vanishes at least as rapidly as \( 1/p^4 \) as \( p^2 \to \infty \). The argument is completely analogous for the OPE of \( j_\alpha (x) \overline{J}_{\dot{\alpha}}(0) \), which contains a potentially different primary vector operator \( V'^\mu \) with \( \Delta_{V'} \geq 3 \). This allows us to conclude that the result also holds for the difference of any two \( \tilde{C}_a(p) \).

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