Numerical study on characteristics of sound and wake generated by flow past triangular cylinder at various incident angles

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Abstract. Characteristics of sound and wake generated by flow past a triangular cylinder with Mach number 0.3 at various incident angles are studied by numerically solving the compressible Navier-Stokes equations using OpenFOAM. The Reynolds number in this study is varied from 140 to 180. Vortex streets behind the cylinder are observed to have different patterns as the Reynolds number and the angle of incidence are varied. The mean lift coefficient has zero value when the cylinder is symmetric and maximum value at incident angle approximately 30°. The mean drag coefficient is observed varying when incident angle increases. The Strouhal number observed has the value in the range of 0.15 – 0.21. Sound is generated mainly from a fluctuation in the lift and drag forces due to the shedding of vortices. The frequency of sound in far field is observed shifting when the incident angles is increased because of the different behavior of the fluctuating lift and drag forces.

1. Introduction
Aerodynamic noise is the sound generated by the fluid motion itself [1]. Curle [2] predicted that if solid boundaries are present in the flow field, the fluctuating force on the boundaries can generate sound, and for a flow at sufficiently low Mach numbers the sound generated by the fluctuating force on the boundaries may dominate sounds generated from other sources. Sound generated by flow past a circular cylinder has been studied both experimentally [3, 4] and computationally [5-7]. Inoue and Hatakeyama [5] studied the sound generated by a flow past a circular cylinder at low Mach numbers and low Reynolds number by solving the two-dimensional compressible Navier-Stokes equations using a finite difference method. They found that the lift and drag coefficients are not so affected by increasing the Mach number. The Stouhal number has the same value for all Mach numbers in their study. The sound field is dominated by the lift dipole, i.e., the fluctuating lift force acting on the cylinder produces sound that propagates perpendicularly to the incoming uniform flow. The acoustic field is well predicted by the Curle’s analogy after the Doppler effect has been taken into account.

Sound generated by flow past a rectangular cylinder has been studied by, for example, Liow et al. [8], Rokugou et al. [9] and Inoue et al. [10]. Inasawa et al. [11] studied the sound generated by the flow past a rectangular cylinder at different aspect ratios at Mach numbers lower than 0.3 and Reynolds number of 150. Their results show that the Strouhal number varies as the aspect ratios of the cylinder is increased. The predominant sound is generated by the lift dipole with the same frequency as the vortex shedding frequency. The sound with twice frequency of
the vortex shedding frequency are generated by the drag dipole, and the fluctuating mass in the source region near the cylinder. The sound with the same frequency as the secondary vortex street developed in the downstream region for some values of aspect ratio of the cylinder can be observed. Their results also show that the sound generated from the fluctuating mass is strengthened as increasing the Mach number.

The flow past the triangular cylinder has received even less attention than the rectangular cylinder. Most of the purposes of previous studies are to study fluid properties [12-14], and heat transfer problems [15, 16]. Ali et al. [17] studied a sound generated by a flow past a bluff body with different shapes of bluff body including triangular cylinder by using the Curle’s solution. However, the sound from flow past triangular cylinder was studied only at one incident angle. Moreover, by using the Curle’s solution, the other sounds generated cannot be predicted except the sound generated from fluctuating force. This leads to the purpose of this study which is to study the sound generated by flow past the triangular cylinder at different incident angles by numerically solving the compressible Navier-Stokes equations using a finite volume method. The lift and drag coefficients are examined as a function of incident angles and the Reynolds number. The patterns of the vortex street are compared with Ng et al. [12] whether or not the compressible and incompressible flows give the same pattern of vortex street. The sound in the far field is observed varying as the incident angle is increased because of the changing of how lift and drag coefficients fluctuate.

2. Method

The 2D simulation of flow over a triangular cylinder is simulated using OpenFOAM. The geometry model and meshing processes are prepared in Gmsh software. Then, they are imported to OpenFOAM for solving by the rhoPimpleFoam solver using a finite volume scheme. The rhoPimpleFoam solver is the transient solver for flow of compressible fluids. The compressible Navier-Stokes equations are solved with the ideal gas equation, the Newton’s law of viscosity, and the Fourier’s law of heat conduction as follows.

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \\
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p - \nabla \cdot \tau, \\
\frac{\partial (\rho E)}{\partial t} + \nabla \cdot (\rho \mathbf{u} E) = -\nabla \cdot (\rho \mathbf{u} p) + \nabla \cdot (\tau \cdot \mathbf{u}) - \nabla \cdot \mathbf{j},
\]

where \( \rho \) is the density, \( \mathbf{u} \) the velocity, \( p \) the pressure, \( \tau \) the viscous stress tensor, \( E \) the total energy, \( \mathbf{j} \) the heat flux, \( R \) the specific gas constant, \( T \) the temperature, \( \mu \) the dynamic viscosity, and \( k \) the thermal conductivity. The Prandtl number is assumed to be 0.713. The lift coefficient, drag coefficient, pressure field calculated in OpenFOAM are imported and calculated for the mean value, root mean square, frequency spectrum of lift and drag coefficients and pressure field in MATLAB. ParaView is used for visualization of the vorticity field.

3. Experimental setting

The flow past the equilateral triangular cylinder of side length \( d \) is simulated in the square computation domain of size \( 620d \times 620d \) shown in figure 1a. The fluid flows from left to right.
Figure 1. The geometry of the simulation. (a) the triangular cylinder of side length \(d\) is centered in the square computational domain of size \(620d \times 620d\) (not to scale), black dots represent points where the pressure is collected over time. (b) The triangular cylinder of side length \(d\) with the inclination \(\alpha\). The frontal height \(h\) is used for calculating the Reynolds number as characteristic length.

sides of the domain. The cylinder is located at the middle of the domain. The Reynolds number used in this study is defined as \(Re = U_\infty h/\nu\) where \(h\) is the frontal height of the cylinder related to \(d\) by the relation \(h/d = \sin(60^\circ - |\alpha - 30^\circ|) + \sin(|\alpha - 30^\circ|)\) and \(\alpha\) is the angle measured counterclockwise with the incoming uniform flow with speed \(U_\infty\) shown in figure 1b. The angle \(\alpha\) is in the range of \(0^\circ - 60^\circ\). The Mach number is defined as \(M = U_\infty/c_\infty\), where \(c_\infty\) is the speed of sound in the fluid. In this study, \(M = 0.3\). The boundary conditions are given as follows. At the inlet, \(u_x = U_\infty, u_y = 0, \partial p/\partial x = 0,\) and \(T = T_\infty\). At the outlet, the wave transmissive boundary condition in OpenFOAM is used. \(\partial u_x/\partial x = 0, \partial u_y/\partial x = 0, Dp/Dt = 0,\) and \(\partial T/\partial x = 0.\) On the cylinder surface, the adiabatic no-slip boundary condition is applied. \(u_x = 0, u_y = 0, \partial p/\partial n = 0,\) and \(\partial T/\partial n = 0.\) Pressure fields at the distance \(75d\) away from the center of cylinder are recorded over time for the frequency analysis. The polar coordinate \((r, \theta)\) is used for analyzing the pressure signal.

4. Results and discussion

4.1. The lift and drag coefficients

Figure 2 shows the profiles of drag \((C_D)\) and lift coefficients \((C_L)\) at \(\alpha = 0^\circ\) and \(\alpha = 12^\circ\) with \(Re = 160\). The profiles of \(C_D\) are different because the behavior of vortex shedding depends on \(\alpha\) and \(Re\). On the other hand, the profile of \(C_L\) looks the same except for the mean value.

Figure 3 shows the mean of drag \((C_{D,mean})\) and lift coefficients \((C_{L,mean})\) at each \(\alpha\) where \(Re\) are varied. The results show the same trend as those reported by Ng et al. [12] for incompressible flow. The plot of \(C_{L,mean}\) shows the same trend for all \(Re\) in this study. The plot shows that \(C_{D,mean}\) vanishes at \(\alpha = 0^\circ\) and \(\alpha = 60^\circ\) due to the symmetry of the cylinder and has its minimum at \(\alpha \approx 30^\circ\) for all \(Re\). The plot shows \(C_{L,mean}\) looks symmetric about its minimum. The plot of \(C_{D,mean}\) also shows the similar trend as \(Re\) is increased. From the figure, \(C_{D,mean}\) has its minimum at \(\alpha \approx 24^\circ\) but is not symmetric about its minimum. At \(\alpha = 0^\circ\), \(C_{D,mean}\) has a smaller value than that at \(\alpha = 60^\circ\). Because when \(\alpha = 60^\circ\), the larger wake are developed behind the cylinder thus gives rise in \(C_{D,mean}\) as mentioned by Ng et al. [12].

Figure 4 shows the plot of root mean square of drag \((C_{D,rms})\) and lift coefficients \((C_{L,rms})\). From the plots, \(C_{L,rms}\) increases as \(\alpha\) increases. While \(C_{D,rms}\) increases for \(0^\circ \leq \alpha \leq 36^\circ\) then decreases. According the Curle’s analogy, the sound generated by the fluctuation of lift force
Figure 2. The profile of (a) lift and (b) drag coefficients at Re = 160 plotted against non-dimensional time $tU_\infty/d$.

Figure 3. Plots of (a) mean lift coefficient and (b) mean drag coefficient at different incident angles for three values of Reynolds number.

gets louder when $\alpha$ is increased. While the sound generated by drag force is strongest at $\alpha \approx 36^\circ$. Increasing Re results in higher fluctuation both in lift and drag coefficients.

Figure 5 depicts the power spectral density (PSD) of the lift coefficient on the cylinder. The results show that the fluctuating lift force can have multiple frequencies depending on $\alpha$ and Re. The single frequency peak of lift coefficient is observed when $\alpha = 0^\circ$ shown in figure 5a. The plot shows the example of the cases of $\alpha$ that give multiple frequency components. Changing Re does not significantly affect on the spectrum as shown in figure 5b. The sound generated from lift force fluctuation can then have many frequencies depending on $\alpha$.

4.2. Strouhal number

Because the predominant sound has the same frequency as vortex shedding frequency, figure 6 presents the Strouhal number variation at different $\alpha$ and Re. The results show the same trend as reported by Tu et al. [14] for incompressible flow. The Strouhal number gradually increases for $0^\circ < \alpha < 20^\circ$, and rapidly decreases for $\alpha > 20^\circ$. Changing Re of the flow does not have an
Effect on the trend of the Stouhal number.

4.3. The generated sound

Figure 7 shows the fluctuation pressure at two locations \((r, \theta) = (75d, 90^\circ)\) and \((75d, 180^\circ)\) of the flow with \(\alpha = 0^\circ\) and Re = 160. The spectrum of the sound is presented on the right showing the frequencies of the generated sound. From the results, the sound at \(\theta = 90^\circ\) gives a louder sound due to the higher fluctuation in lift coefficient. The first peak of the spectrum is associated with the vortex shedding frequency. The harmonics of the first peak are also observed in the spectrum. The sound at \(\theta = 180^\circ\) has the first peak at twice of the vortex shedding frequency equal to the frequency of drag fluctuation. Figure 8 shows the fluctuation pressure for the case \(\alpha = 36^\circ\) and Re = 180 at the same location as in figure 7. In this case, the sound at \(\theta = 180^\circ\) has the profile almost like at \(\theta = 90^\circ\) except for the smaller amplitude of fluctuation. The spectrum of the sound at two locations are almost the same. The frequency peak associated with the vortex shedding frequency can be observed at \(\theta = 180^\circ\).
Figure 6. Plot of Strouhal number versus incident angle at different Reynolds number.

Figure 7. (a) Time histories of fluctuation pressure at $r = 75d$ of the case $\alpha = 0^\circ$ and $Re = 160$ at different $\theta$. (b) Spectrum of the sound pressure.

4.4. The vorticity

The vorticity of the wake behind the cylinder is presented in figure 9 for selected cases. Most of the wake characteristics pattern behind the cylinder are found to be bi-layered wake arrangement or having the secondary vortex street as described by Ng et al. [12]. For most of the parameter in this study, the downstream region near the cylinder show the Bénard-von Kármán vortex street. The vortex soon rearrange into two layers of vortex with the same sign convecting downstream called bi-layered wake arrangement. At some point, the bi-layered wake reform into bigger vortex street called secondary vortex street. The form of the secondary vortex can be different as shown in figure 9. For instance when $\alpha = 18^\circ$ and $Re = 180$, the secondary vortex street looks like the Bénard-von Kármán vortex street with bigger scale. That is the vortices with opposite sign are alternately generated in a shedding cycles. On the other hand, the secondary vortex street when $\alpha = 42^\circ$ and $Re = 180$ looks like 2P-like wake. That is two pairs of vortices with opposite sign are shed from each side of the cylinder.
Figure 8. (a) Time histories of fluctuation pressure at $r = 75d$ of the case $\alpha = 36^\circ$ and $Re = 180$ at different $\theta$. (b) Spectrum of the sound pressure.

Figure 9. Vorticity of flow past triangular cylinder at different incident angles for $Re = 180$. Red, and blue colors represent positive and negative signs of vorticity, respectively.
5. Conclusions

Flow past a triangular cylinder is numerically studied by solving the two-dimensional compressible Navier-Stokes equations with selected incident angles and the Reynolds numbers at low Mach number. Results show that the mean drag coefficient decreases when $0^\circ < \alpha < 25^\circ$ then increases for $\alpha > 25^\circ$. The highest mean drag coefficient is observed at $\alpha = 60^\circ$ due to the larger wake developed behind the cylinder. The mean lift coefficient has zero value when the cylinder is symmetric that is $\alpha = 0^\circ$ and $60^\circ$. The maximum mean lift coefficient is observed at $\alpha \approx 30^\circ$.

Varying Re does not affect the trends of mean lift and drag coefficients, root mean square of lift and drag coefficients, and Strouhal number significantly for Re in this study. Spectrum of lift coefficient can show up from one to many frequency peaks depending on $\alpha$ and Re. The Strouhal number is found gradually increasing for $0^\circ < \alpha < 10^\circ$, then suddenly decreasing for $\alpha > 10^\circ$. The fluctuating pressure at $(r,\theta) = (75d,90^\circ)$ has a higher amplitude than that at $(75d,180^\circ)$ as expected due to the higher fluctuation in the lift coefficient. The vorticity of the wake is observed to have patterns from the Bénard-von Kármán vortex street, bi-layered wake arrangement, and secondary vortex street. The Bénard-von Kármán vortex street is normally observed in downstream region near the cylinder. The wake is then developed to have either bi-layered wake arrangement or secondary vortex street or both depending on $\alpha$ and Re.

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