De Broglie wave in vacuum, matter and nanostructures

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Abstract. Properties of de Broglie waves and their differences from any other waves, for example, electromagnetic, are discussed. Their little-known properties are given: 1) the propagation velocity is greater than the speed of light \( V > c \); 2) dispersion even in vacuum (waves of different frequencies propagate at different speeds, which results in the spreading of any wave packets associated with the particles); 3) the connection of the de Broglie wave with another physical wave moving with a speed \( v \) less than the speed of light, but so always \( v \cdot V = c^2 \), and for the photons \( v = V = c \).

According to later views of de Broglie, this second wave is not the group wave. Possible mechanisms of generating the de Broglie waves based on spherical and cylindrical hollow quantum resonators, which can be specially created in matter using specific nanostructures are considered. Perhaps their nature itself realizes them for any micro-objects, regardless of their electrical, magnetic and gravitational properties. That is, this phenomenon is universal.

Applications and realizations of this idea are discussed in the part II of our article.

1. Introduction

Up to date, about 400 elementary particles are known and there is no doubt that several hundred more elementary particles will be discovered. Only three of them are stable (\( e, p, \nu \) - electron, proton, and neutrino), others are not elementary, and decay with different lifetimes, that is, they are all resonances (excited states of systems consisting of more fundamental particles, for example, \( e, p, \nu \)). Nevertheless, it is postulated that each of the 400 particles mentioned above is described by its wave function, which satisfies its first-order “corpuscular” equation. Thus, the overall picture does not seem very attractive.

The most advanced quantum theory currently is quantum field theory, which introduces particle creation and annihilation operators, the number of postulates in which is very large and the interpretation of its mathematical apparatus is difficult and often ambiguous. Microscopic systems in which the number of degrees of freedom does not change during evolution (there are no acts of birth and annihilation of particles) can be described by relativistic quantum mechanics, in which the wave function still remains a complex function and does not become an operator, as is the case in quantum field theory.

The basis of modern relativistic quantum mechanics are the following 5 axioms [1]:

1. The hypothesis of the existence of a complex wave function \( \psi(\vec{r}, t) \) completely (in the quantum sense) characterizing the physical system. This function belongs to one of the irreducible representations of the Lorentz group, which are divided into only 2 classes: tensor and spinor. In the probabilistic version of the interpretation (and there are several dozen of them [2]), the squared modulus of this function is the probability density.

2. De Broglie hypothesis, according to which the physical state of a free particle is described by a plane wave

\[
\psi = \psi_0 \exp \left\{ \frac{i}{\hbar} (Et - \vec{p} \cdot \vec{r}) \right\}.
\]  

3. Corpuscular-wave dualism according to which the wave function simultaneously satisfies:
The “corpuscular” equation, which is usually (but not necessarily) a linear equation of the first order. For an electron, this equation is the Dirac equation:

\[ \left( \gamma^\mu \frac{\partial}{\partial x^\mu} + m \right) \psi = 0. \]  

(2)

the wave equation, which is often called the Klein-Gordon-Fock equation:

\[ (\Box + m^2) \psi = 0. \]  

(3)

The class of solutions of this last equation is much richer than the solutions of linear first-order corpuscular equations (for example, Dirac equations), therefore, only those solutions that simultaneously satisfy the principle of wave-particle dualism, i.e., equations (2) and (3) at the same time, are chosen. Usually the dynamic equation of first-order evolution is chosen so that its solutions for a free particle automatically satisfy the wave equation. This is exactly what takes place [3] for most well-known equations (Schrödinger, Pauli, Dirac) and for other similar equations constructed on the Dirac-Clifford algebra

\[ \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 \delta^{\mu\nu} \]  

(4)

and Petiaux-Duffin-Kemmer algebra

\[ \beta^\mu \beta^\nu \beta^\alpha + \beta^\alpha \beta^\nu \beta^\mu = \delta^{\mu\nu} \beta^\alpha + \delta^{\alpha\nu} \beta^\mu. \]  

(5)

It becomes obvious that all the equations for the observed particles (and there are about 400 of them) can be written in a single form of the Dirac equation (2), but with different matrices \( \gamma \) and \( \beta \), of different ranks, depending on the spin value.

4. **The correspondence rule**, according to which each classical quantity (which in the general case is a function of coordinates, momenta and time) is associated with an operator whose eigenvalues are observable quantities. The last assumption imposes certain restrictions on the type of operators, namely, all operators that display physical quantities must be Hermitian, since only such operators have a spectrum of eigenvalues given by a set of real numbers.

5. **The principle of superposition**, according to which if the system can be in two states described by the wave functions \( \psi_1 \) and \( \psi_2 \), then it can also be in the third state at the same time, which is a linear combination of the first two:

\[ \psi_3 = c_1 \psi_1 + c_2 \psi_2. \]  

(6)

Of all the hypotheses described above, the most fundamental is the de Broglie hypothesis of a wave associated with a particle, regardless of the presence or absence of some particle properties (mass, charge, spin, etc.) in the particle.

All other hypotheses have certain disadvantages. So, for example, the used rules of correspondence “physical quantity” ↔ “operator” turned out to be ambiguous. Moreover, for some physical quantities (for example, mass, velocity) we do not know the operators. For a scalar particle (spin \( s = 0 \)), usually described by a Klein-Gordon second-order equation, the density of associated (corresponding) current turned out to be not positively defined (can take negative values). If the scalar particle is described by the first-order equation of Petio-Duffin-Kemmer, then its wave function turns out to be 5-component and consists of a scalar and 4-vector \((\psi = (\varphi, A_\mu))\), that is, it belongs to the reducible representation of the Lorentz group contrary the axiom No.1.

2. **Materials and methods of research**

2.1 **Part I. De Broglie waves in vacuum**

The concept of energy quanta was introduced by Planck in 1900 according to the formula:

\[ E = h \omega. \]  

(7)
Five years later in 1905, Einstein established a universal relationship between energy and mass:

$$E = mc^2.$$  \hspace{1cm} (8)

And only after 17 years [4] Louis de Broglie was the first to equate these formulas to each other “by virtue of some Great Law of Nature” [5]

$$m_0c^2 = \hbar \omega_0.$$  \hspace{1cm} (9)

He suggested that inside any particle at rest with mass $m_0$ there is an oscillatory process described by the quantity $e^{i\omega_0 t}$ (this is not yet a wave). He understood that formula (9) describes the oscillatory process only in its own frame of reference. In any other reference system, it is violated, since for a moving electron its mass increases as $m_0/\sqrt{1 - \beta^2}$, and the frequency of the clock decreases as $\omega = \omega_0 \cdot \sqrt{1 - \beta^2}$. Here $\beta = \frac{v}{c}$. To eliminate this contradiction, de Broglie suggested that internal oscillations with frequency $\omega_0$ instantly generate oscillations at any point in the Universe with the same frequency $\omega_0$, that is, all points of the Universe oscillate in time with the particle oscillations. We can say that the speed of synchronization of the internal clock of a stationary particle with the clock at any point in the Universe equals infinity ($V = \infty$), since it occurs instantly. So far, we have only oscillations, but not waves. However, taking into account the Lorentz transformations for time in a reference frame moving with speed $v$

$$t \rightarrow \frac{t - vx/c^2}{\sqrt{1 - \beta^2}}, \quad \beta = \frac{v}{c},$$  \hspace{1cm} (10)

the expression for the oscillations turns into the expression for the de Broglie wave with the velocity $V > c$:

$$e^{i\omega_0 t} \rightarrow e^{i\frac{\omega_0}{\sqrt{1 - \beta^2}}(t - \frac{x}{V})} = e^{i\frac{\omega_0}{\sqrt{1 - \beta^2}}(\frac{x}{V})}.$$  \hspace{1cm} (11)

Since, taking into account (10) - (11), the left and right sides of (9) are already transformed by the same way, you can “erase the zeros” in (9) and write the relativistically covariant expression that is valid in any reference frame:

$$mc^2 = \hbar \omega.$$  \hspace{1cm} (12)

From (11) it follows that the velocity $V$ of the phase (according to de Broglie [5]) wave is always equal or greater to than the speed of light, so that always

$$v \cdot V = c^2.$$  \hspace{1cm} (13)

Hence, for the particle at rest, we have the uncertainty $0 \cdot \infty = c^2$, and for the photon we have the identity $c \cdot c = c^2$. The de Broglie wave has (by definition) a wavelength:

$$\lambda = \frac{2\pi V}{\omega} = \frac{2\pi \hbar \sqrt{1 - \beta^2}}{m_0 v} = \frac{2\pi \hbar}{p} = \frac{h}{p} = \lambda_B.$$  \hspace{1cm} (14)

Here $p = m_0 v/\sqrt{1 - \beta^2}$ is the particle momentum, $\hbar = h/2\pi$. In the nonrelativistic case, formula (14) takes a more well-known form

$$\lambda_B = \frac{h}{mv}.$$  \hspace{1cm} (15)

In his dissertation [5], Louis de Broglie introduced his wave associated with a particle that gave the meaning of a phase wave propagating with a velocity $V$ greater than the speed of light. According to
modern concepts, it is a tachyon, the existence of which is not prohibited by the special theory of relativity. However, it does not carry energy. De Broglie did not discuss in detail the physical meaning of the wave he introduced, confining himself to the phrase: “Explaining this meaning will be a difficult task for the extended theory of electromagnetism” [6]. This meaning is still not fully understood at the present time, as well as the interpretation of quantum mechanics itself [2]. De Broglie identified a physical wave with a speed of a lower speed of light \( v \leq c \) with the group velocity of the wave packet, which, in turn, can be identified with the real physical speed of the particle itself. However, wave packets formed from de Broglie waves spread even in a vacuum (we discuss this a bit later in this paper). Apparently, knowing this, he later showed that a physical wave with a velocity \( v < c \) admits another wider interpretation “without involving the concept of a group of waves” [7].

2.1.1 Dispersion of de Broglie waves in a vacuum

As a result of transformation into a moving reference frame with the velocity \( v < c \), the frequency of de Broglie waves according to (11) increases:

\[
\omega = \frac{\omega_0}{\sqrt{1 - \beta^2}}.
\] (16)

Here \( \omega_0 \) is a frequency in the reference frame at rest, \( \beta = \frac{v}{c} \).

This shows that waves with different frequencies propagate in vacuum at different speeds. This phenomenon is called dispersion. We find the explicit form of the dependence \( v = f(\omega) \). From (16) we obtain:

\[
v = c \sqrt{1 - \frac{\omega_0^2}{\omega^2}},
\] (17)

or taking into account (13) for de Broglie wave we get

\[
V = \frac{c}{\sqrt{1 - \frac{\omega_0^2}{\omega^2}}}.
\] (18)

For the particle at rest, \( \omega_0 = \omega \) and we get \( V = \infty \), that is, we come to the previously considered case of instantaneous synchronization of all clocks, when there are in-phase oscillations at all points of the Universe with the actual absence of a propagating wave. With increasing particle energy (its physical velocity \( v \)), the de Broglie phase wave velocity \( V \) decreases and tends to the speed of light \( c \).

2.1.2 The difference between de Broglie waves and electromagnetic waves

The most well-studied waves are electromagnetic waves described by the vectors \( \vec{E} \) and \( \vec{H} \). For example, in a vacuum for a free photon we have a solution in the form of a plane wave:

\[
\vec{E}(\vec{r}, t) = \vec{E}(\vec{r}) e^{i(\omega t - \vec{p} \cdot \vec{r} + \varphi)},
\] (19)

where \( \vec{E}(\vec{r}) \) is the amplitude, \( E \) is energy (frequency \( \omega = E/\hbar \)), \( \vec{p} \) is the momentum (wave vector \( \vec{k} = \vec{p}/\hbar \)), \( \varphi \) is the phase, polarization (helicity) \( s \) is inside in the amplitude \( \vec{E}(\vec{r}) \). A similar formula holds for the magnetic field vector. It follows from (19) that an electromagnetic wave is characterized by 5 parameters: amplitude, energy (frequency), momentum (wave vector \( \vec{k} = \vec{p}/\hbar \)), phase \( \varphi \) and polarization \( s \), which is hidden in the vector properties of the amplitude \( \vec{E} \). However, only 4 parameters are independent (for example, amplitude, frequency, phase, and polarization) due to the dispersion relation between energy and momentum \( E^2 = p^2 c^2 + m^2 c^4 \) (or \( \omega^2 = c^2(\omega_0^2 + k_0^2) \)), for electromagnetic waves \( m = 0 \). All these 4 parameters are used in modern technologies. For example, amplitude, frequency, phase, and polarization modulations are used to transmit information.

Unlike electromagnetic waves for the de Broglie plane waves

\[
\psi = A_0 e^{i(\omega t - \vec{k} \cdot \vec{r} + \varphi)}
\] (20)
in the probabilistic interpretation of quantum mechanics, everything looks like completely different.

Since waves with small and large amplitudes are normalized to “1”, the constant $A_0$ does not carry useful information. Formula (20) can be rewritten identically in the form of the product of two exponentials:

$$\psi = A_0 e^{i(\omega t - k \vec{r})} e^{i\varphi} .$$

(21)

Since the phase $\varphi$ is a constant, when normalized, it disappears. In addition, the de Broglie wave does not have polarization. Thus, there remains only one parameter for control (modulation) - frequency (energy). However, even this single parameter is unsatisfactory due to the dispersion of de Broglie waves even in vacuum.

Obviously, the reason for the dispersion of de Broglie waves in vacuum is the possibility of their propagation at different speeds (from the speed of light $V = c$ for photons to the infinite speed $V = \infty$ for a particle at rest) due to the presence of a rest mass of the particle associated with the de Broglie wave. However, the dispersion of de Broglie waves in a vacuum disappears for particles with zero rest mass (photons, and, possibly, neutrinos). For a photon, both waves attributed to the particle have the same velocities ($v = V = c$) and a unique opportunity arises (maybe the only one) to effectively control the quantum properties of the electromagnetic wave using the four parameters discussed above (amplitude, frequency, phase and polarization) which is difficult make for particles with mass due to the presence of dispersion phenomena even in vacuum.

Many physical phenomena occurring at nonrelativistic velocities ($v \ll c$) can be described by simpler nonrelativistic equations, which, in addition to mathematical simplifications, also solve complex problems of interpretation of quantum mechanics. So, for example, Schrödinger was the first to obtain the relativistic equation for an electron without taking into account spin, which is now often called the scalar Klein-Gordon-Fock equation, or the wave equation (3). However, he himself discovered that associated density with this equation can take negative values and this complicates the interpretation of solutions to this equation. From the point of view of Schrödinger, the nonrelativistic equation, now bearing his name

$$i\hbar \frac{\partial \psi}{\partial t} = H \psi$$

(22)

more acceptable in nonrelativistic quantum mechanics, which confirmed its further development. Moreover, it always means that all solutions of equation (22) with the first time derivative still satisfy the second-order wave equation (3), thereby demonstrating the phenomenon of particle-wave dualism in nonrelativistic physics.

2.2 Part II. Localized de Broglie waves in matter and nanostructures

Below we will consider the amazing quantum effects caused by the de Broglie waves, which for the photon turned out to be as if it is built into the electromagnetic wave, which leads to unusual phenomena in localized nanostructures and extended continuous media.

Nanostructured carbon materials raise interest due to their unique properties. Semiconductor and nanomaterial technologies have a number of theoretical problems related to the analytical description and numerical simulation of processes of spatial unlimited cumulation of de Broglie waves ($\psi_n$ -functions) of electrons at the focuses of spherically and cylindrically symmetric quantum resonators (dots, lines, and wells) for these particles. These phenomena are similar to those in conventional acoustic resonators (see Fig. 1) and resonators for the accumulation of electromagnetic waves (lasers, waveguides, optical fibers, etc.). It turns out that in the nanoworld it is possible to form metastable (partially open) cumulatively dissipative quasistationary convective structures caused by quantum effects and the reflection of previously captured dissipative convective electron flows back to the center of the hollow quantum resonator by polarizing spherical or cylindrical-symmetric “mirrors”(potentials) [8-10]. These structures, for example, metastable endoanions with endoelectrons inside, are formed as a result of polarization capture (cumulation to the center — focus or focal axis) of electrons by hollow fullerene molecules or polarized nanotubes [8-10] (Fig. 2). In the internal cavity of supramolecules, the focusing of the electron occurs as a result of the action of polarization forces on it (Coulomb fields from the polarized molecule differently charged with differently charged surfaces). In this paper, a historical and analytical study of the unlimited cumulation of the $\psi_e^{\infty}$ electron function in quantum spherical (0D-points) and cylindrically symmetric (1D-wires) resonators is carried out. As a result of the analytical studies of convective cumulative dynamic processes inside quantum
spherically and cylindrically symmetric resonators and comparing the analytical results with the results of existing experiments, we formulated the basics of cumulative quantum mechanics (CQM) and classified quantum resonators and their corresponding quantum dots, wires and wells for any (plane [11], cylindrical [8-10] and spherical [8-10]) symmetry with electrons localized in them, limited $\psi_n$ and unbounded $\psi_{n-1/2}$ -functions and their own energy spectrum $E_n > 0$ and $E_n < 0$ and $E_{n-1/2} > 0$ and $E_{n-1/2} < 0$, respectively. (An electron with an energy greater than zero is localized in the region of a hollow molecule by a polarization barrier [8-10].) As an illustration, in this work, the CQM is used to analytically describe the quantum size effect of the Vysikailo type I, studied in [8-10] and caused by polarization capture of electrons into the internal cavity of allotropic hollow forms of carbon: fullerenes and nanotubes [8-10]. The CQM in [12] is used to describe analytically the concentration-quantum-size effect of the Vysikailo type II, as well as to analytically identify and study the parameters that control the macroproperties of nanostructured materials. Analytical calculations are compared in details with the experimental observations available in the literature and specially organized experimental studies at the TISNCM in Troitsk Moscow RF. For planar symmetry of light wave phenomena, two bright phenomena are known - these are Fresnel diffraction (respectively, interference) and Fraunhofer diffraction (interference). Our generalized Fresnel and Fraunhofer interference of de Broglie waves of electrons with $E_n > 0$ and $E_{n-1/2} > 0$, which are polarized quantum resonators cumulating to the center for these waves, are studied in this paper in the framework of a simple model - "quantum particle in a box, cylinder or ball ". As a result of regularization of unbounded solutions for three types of symmetry (planar - $k = 0$, spherical - $k = 1$ and cylindrical - $k = 0.5$), a general expression is obtained for the solutions ($E_{n0}$, $\psi_n(r)$, $E_{n-1/2}$, $\psi_{n-1/2}(r)$, $W_n(r)$, $W_{n-1/2}(r)$ are the probabilities densities of the particle in the dr layer) and the normalization cumulation coefficients of $\psi_n$ -functions - $A_n$ and $B_n$ for the corresponding $\psi_n(r)$ and $\psi_{n-1/2}(r)$ depending on the parameter $k$ determined by the symmetry type of the resonator. After a series of simplifications, the simple model that we used to describe quantum resonances in electron capture was reduced to the well-known model of G.A. Gamow, proposed by him to describe the penetration of an $\alpha$-particle through the potential barrier of the atomic nucleus. The Gamow model was used to analytically calculate the decay probability of radioactive nuclei observed in experiments, and to determine the possibility of decay of atomic nuclei with the emission of one or two protons (V.I. Goldanskii, 1960, 1965). In historical terms, the value of the theory is especially great because Gamow theory was the first successful application of quantum mechanics to the atomic nucleus, that is, to femtoworld objects (with dimensions of $\sim 10^{-15}$ m). Apparently, in [8-10], for the first time, a model with a polarization barrier, similar to the Gamow model, was used to describe the processes of polarization capture of electrons by hollow polarizing carbon molecules — C60,70 in the nanoworld (with sizes of $\sim 10^{-9}$ m). Our regularization of unbounded solutions or CQM can be used to describe the cumulative phenomena of de Broglie waves of particles in atomic nuclei and the phenomena of cumulation in classical physics.

**Figure 1.** The scheme of the acoustic resonator for unlimited cumulation and accumulation of energy of fluid oscillations, manifested in the formation of a cavity in the fluid in the center of the resonator. 1 - source of ultrasound; 2 - surface of an acoustic spherically symmetric resonator; 3 - cavity (bubble) in the liquid, due to the accumulation of energy in a standing cos-wave in the center of the resonator.
Figure 2. Scheme [8-10] of cumulative discharge of an electron with an energy of 0.24 to 20 eV into the cavity of $C_{60,70}$ molecules and the resonant formation of the endoion $e_k@C_n$ (a pulsating soliton or localized de Broglie standing electron wave in the $C_n$ region, where $n = 20, 60, 70, 98$). A sequential decrease in the de Broglie wavelength of an electron incident on the same polarized $C_n$ molecule is presented. The radius of the endion (the location of the reflecting polarization potential - the reflecting "mirror") $R_p = R_{C_{60}} + r_{ind}$.

2.2.1 Sources, paradoxes and fundamentals of cumulative quantum mechanics
Quantum resonators for electrons in the case of plane symmetry have been studied both experimentally and theoretically [11]. Such resonators are called quantum 0D points (boxes), 1D wires, and 2D wells or 2D baths (with complex profiling of the well potential). The probability of finding electrons is maximum in the center of planar resonators in the case of symmetric $\psi_{n-1/2}$ - eigenfunctions of electrons relative to the center of the resonator (cos-waves) and is equal to zero at their boundaries (Fig. 2). For asymmetric $\psi_n$ - functions of electrons (or for sin-waves) in the center of the quantum resonator and at its boundaries $\psi_n = 0$. Thus, in planar resonators, in the region of their center, there is limited cumulation of the eigen $\psi_{n-1/2}$-functions of electrons only for cos-waves (see Fig. 2 on p. 110 in [11] and Fig. 2 in this article). As a result of the self-organization of a standing wave, two types of waves can be realized - these are cos-waves (fundamental tone) and sin-waves (overtones) with the corresponding energy spectra $E_{n-1/2} \sim (n-1/2)^2$ and $E_n \sim n^2$.

2.2.2 General concept of cumulation
In geometric optics, the energy flux is conserved, and as the rays converge to the focus of the lens, its intensity increases inversely with the square of the distance to the focus - $1/r^2$ - in the case of spherical symmetry and $1/r$ - in the case of cylindrical symmetry (generalized Gauss theorem on the conservation of flux). In the framework of wave optics, the phenomena of geometric cumulation with spherical or cylindrical symmetry are supplemented by such phenomena as interference and Fresnel diffraction (the wave node in the center of the resonator) and Fraunhofer interference and diffraction (antinode in the center of the resonator). Definition of cumulation was given by Ya.B. Zeldovich: “Cumulation, that is, the concentration in a small volume ($\Delta V$) of force, energy or other physical quantity ($W$), is a most important natural phenomenon”, fully defines the phenomena of focusing and self-focusing of energy-mass-pulse flows in any including extreme, natural phenomena in any environment. Cumulation is called unlimited if $W \to \infty$ as $\Delta V \to 0$. Cumulation is the most important phenomenon in nature, however, it is still ignored in the framework of the classical new quantum mechanics, in particular, the requirement is used that the $\psi_n$ - function of a localized particle must be bounded everywhere. When particles are cumulated by quantum spherically symmetric or cylindrically symmetric "mirrors" (Fig. 2), the $\psi_{n-1/2}$-functions must also cumulate (grow) unlimitedly to the focusing center, as well as the intensity density of the electromagnetic beam converging to the focus, or energy density of standing cos waves in spherical or cylindrical symmetric acoustic resonators. The effective cumulation of $\psi_{n-1/2}$-functions of particles (to the center - focus) is carried out by spherically symmetric $\psi_{n-1/2} \sim 1/r$. It would seem that cumulation (to the axis) is less effective for cylindrical symmetry of a quantum resonator, since $\psi_n \sim 1/r^{1/2}$. We should not forget about the polarization forces arising along the axis of the cylindrical structure (along $z$ and directed along $r$). We will not consider these 2D effects. We direct the reader to [11] (the case of plane symmetry). Spherically symmetric quantum resonators are called spherically symmetric 0D quantum dots (see, for example, [10]). (This classification is
also relevant within the framework of CQM. It fully corresponds to the classification of quantum dots in the form of a box with plane symmetry [11]. Similarly, cylindrically symmetric quantum resonators are called 1D wires (quantum lines, etc.) [10], which corresponds to names with planar symmetry [11]. The cylindrically symmetric quantum wires include polarizable carbon nanotubes of constant diameter.

Known work on the study of the probability of finding an electron in a spherical-symmetric quantum dots — 0D resonators, for example, [10, 13]. For these resonators, the authors choose the boundary conditions both at the cavity boundary and the additional boundary condition at its center [10]. Such a statement of the boundary condition in the center of the resonator can be carried out directly by demanding that the $\psi_n$ -function be equal to zero or by requiring the regularity of the $\psi_n$-function at zero (in the center of the resonator.) (Despite the fact that the boundary conditions are called boundary because they are chosen only at the cavity boundary, and in the center, the solution organizes itself as a result of resonance.) An additional requirement on the regularity of solutions leads to: 1) the exclusion of the cumulation phenomenon and 2) the exclusion of a number of solutions in the case of a spherical and cylindrical symmetry from the full range of solutions available in the case of plane symmetry. As a result of such requirements, phenomena with planar symmetry become richer than phenomena with spherical and cylindrical symmetry, although there are no physical grounds for this. In the formal inclusion of solutions irregular at zero in the case of spherical symmetry, Brazilian physicists were especially successful [13]. They are honest on a computer, but formally, in essence, they calculated the probability $(W_n = 4\pi r^2 |\psi_n|^2)$ of an electron in a $dr$ layer inside a spherically symmetric quantum 0D resonator with an infinite potential only at its outer boundary (sphere) and published these calculations profiles of Bessel functions and their corresponding $W_d(r)$ in [13]. The solutions presented in their work on the numerical modeling of the $W_d(r)$ profiles in a well at the boundary with infinite potential (according to the analytical solutions [10] $W_{\circ} \sim \sin^2(k_0 r)$ and $W_{\circ^*} \sim \cos^2(k_0 r)$) have as nodes in the center of the resonator (sin-waves) and antinodes (cos-waves - *), which fully corresponds to the resonance solutions known by Helmholtz for acoustic spherical resonators with cumulation of the energy density in the center of such resonators by cos-waves [10]. In this case, the authors of [13] did not notice (ignore) the problem of the unlimited growth of the eigenfunctions $\psi_n$ of electrons $\psi_d(r) \sim \cos(k_0 r)/r \rightarrow \infty$ as $r \rightarrow 0$ in the case of solutions with antinodes in the center of a spherically symmetric quantum 0D- resonator (or spherically symmetric quantum dot). The solutions $\psi_d(r) \sim \cos(k_0 r)/r$ in a number of works on quantum dots [14] and in atomic physics textbooks are discarded due to the irregularity of the eigen $\psi_n$ -functions in the center of a spherical or cylindrical resonator (at zero). The lack of attention to this problem in [13] is due to the fact that the weight coefficient $4\pi r^2$ cuts the unlimited growth of $|\psi_n|^2 \sim \cos^2(k_0 r)/r^2$ in the region of the center of the spherical quantum dot, and therefore the problem of unlimited growth of $\psi_d(r) \sim \cos(k_0 r)/r$ to the center of the spherical resonator was successfully ignored by the authors of [13] (not noticed). The unlimited cumulation of eigenfunctions $\psi_n$ -functions at spherically symmetric quantum dots in [13] for cos-waves was left without discussion. The problem of unlimited cumulation of the eigenfunctions of electrons for cos-waves in a spherical quantum 0D point has not yet been adequately investigated by the scientific community to modify the postulate of the boundedness everywhere of any eigen $\psi_n$ –functions of particles in nano- and femtoworlds.

3. Results and discussion

3.1 Regularization of solutions unlimited in the center of the resonator for cos-waves

The paradoxes in the description in the framework of ordinary quantum mechanics are two obvious inconsistencies. The first paradox is that cos-waves (with eigenfunctions $\psi_{n=1/2}$-symmetric with respect to the center of the resonator) in classical quantum mechanics have the right to exist only in the case of plane symmetry [11] (see Figs. 3 and 2 on p. 110 in [11]), and none of the researchers object against this. (In this case, it is not discussed what the electron energy is at infinity behind the real barrier [11].) As shown in [10], with polarization capture in the cavity, electrons with resonance energy $E_n > 0$ are localized, which is a complete analogy to the capture of acoustic or electromagnetic waves, the energy of which is always greater than zero, therefore all these states are metastable and after the dissipation of energy, an empty resonator remains. Electrons in atoms have an energy less than zero, and these wave phenomena are stationary. In the case of spherical ($k = 1$) or cylindrical ($k = 0.5$) symmetry, according to the postulate on the boundedness of $\psi_n$ -functions, the eigen $\psi_n$ -functions of cos-waves with $\psi_{n=0} \sim \cos(k_0 r)/r^k$ [10, 15] are not considered separate researchers taking place in real life [14], that is, for $k \neq 0$, some of the solutions are forbidden by this, as the author believes, by an erroneous postulate requiring regularity of $\psi_{n=0}$-functions in the center of the resonator. The solution to this paradox should be sought while taking into account the energy spectrum of cos waves and the normalization coefficient $2^k \pi^{3/2} r^k$, which regularizes the $\psi_{n=0}$ - function ~
$2^{2k} \pi^{1/2} r \cos(k r) r^k = 2^{2k} \pi^{1/2} \cos(k r)$ in the center of the resonator with the corresponding type of symmetry. Indeed, $\psi_{\text{ncos}}$ - the function in all manipulations over it is used simultaneously with the normalization coefficient determined by the type of symmetry. And only with planar symmetry ($k = 0$) this coefficient is equal to 1. For other types of symmetry, this coefficient is different from 1 and regularizes $\psi_{\text{ncos}}$-functions. A random solution to this paradox was performed in [13] when calculating the possible profiles of the probability of an electron being in a spherically symmetric quantum resonator depending on the distance to the center of the resonator for different kinetic energies. Therefore, in [13], regular probability profiles of the presence of electrons at spherically symmetric quantum dots and for quantum states forbidden by ordinary quantum mechanics with their own $\psi_{\text{ncos}}$-functions are given.

When other authors [14] choose an additional condition in the center of the resonator ($\psi_n(0) = 0$ or regularity requirements at the zero of the $\psi_n$ function), all cumulative processes to the focus are discarded, that is, the entire spectrum of solutions with antinodes in the center of the resonator (cos-waves and their energy spectra) is excluded for both spherically and cylindrically symmetric ordinary and quantum resonators (quantum dots or wires). And this happens only on the basis of the postulate about the boundedness everywhere of the $\psi_n$ - eigenfunctions for any particles, erroneously acting so far in classical quantum mechanics. Based on this postulate, almost everywhere in works on quantum mechanics, the authors require regularity at zero of the $\psi_n$ - eigenfunction for all particles. However, as you know, the physical meaning does not have its own $\psi_n$-function, but the probability of a particle in the volume $dV$.

In cases of spherical and cylindrical symmetries of quantum dots at zero, a feature arises for the $\psi_{\text{ncos}}$-function of the electron, but, as indicated, this feature is not for the probability of finding the particle at zero, since the weight coefficient is $4\pi^{3/2}$ for the case of spherical symmetry and the weight coefficient $2\pi$ in the case of cylindrical symmetry regulates at zero the probability of finding a particle with an infinite value $\psi_{\psi\cos(0)} \sim \cos(k r) r^{1/2} \rightarrow \infty$ as $r \rightarrow 0$. The physical meaning and, therefore, the requirement for the regularity of the $\psi_n$ function should be related to the $\psi_n$ function normalized to the root of the weight coefficient (for spherical symmetry, $r \psi_{n=1/2} \sim \cos(k r) r^{1/2} \rightarrow 1$ as $r \rightarrow 0$). As a result of the removal of the regularity requirement for the eigen $\psi_{n=1/2}$ - cos-functions at zero in spherical and cylindrical-symmetric cases, a whole class of resonant solutions with cos-waves corresponding to solutions with plane symmetry appears. Solutions with cos-waves should be discarded if in the center ($r = 0$) there is a real object (for example, an atomic nucleus) through which an electron cannot penetrate, and an electron, like de Broglie's sin-wave, is reflected from this object. But there are no physical grounds to reject cos-waves in hollow quantum resonators due to the requirement of regularity of the eigenfunction of the $\psi_{n,1/2}$ function in the center of the resonator. This is proved in this paper analytically and in comparison with experiments. The second paradox is that, within the framework of classical wave mechanics (acoustics and electrodynamics), cumulative solutions of the type $\sim \cos(k r) r^k \neq 0$ are known since Helmholtz [15] and were investigated when solving the problems of solonluminescence and cavitation. The non-recognition of cos waves (or the Fraunhofer interference generalized by us) for hollow quantum resonators for de Broglie waves with spherical and cylindrical symmetries is a big mistake and we draw on it the attention of the whole scientific community. Such non-recognition is equivalent to non-recognition of cumulative phenomena in quantum mechanics and the restriction of the de Broglie hypothesis to quantum phenomena and the exclusion of spherical and cylindrically symmetric quantum 0D and 1D resonators from the center of resonance (and diffraction at the center of the resonator) of Vysikailo-de Broglie-Fraunhofer for de Broglie waves of electrons and other particles in their respective quantum resonators. And this is why de Broglie-Fresnel interference (and diffraction at their center) for de Broglie waves of electrons has been studied both theoretically (mathematically) and experimentally. These phenomena have long been studied for classical acoustic and electromagnetic waves and their resonators both theoretically (mathematically) [15] and experimentally. In [8-10], we solved the paradoxes and problems that exist in classical quantum mechanics with respect to acoustics and physics of electromagnetic waves, where such problems have long been unnoticed. The solution to these paradoxes is solved by comparing the experimentally measured resonance energies of electron capture by hollow $C_{60}$ and $C_{70}$ molecules with analytical calculations carried out within the framework of CQM. It should be noted that a large number of paradoxes (mismatch of phenomena with mathematical models) is generated by untimely (hasty) taking of mathematical limits. If the limits are taken in accordance with physical phenomena (in particular, with the laws of cumulation), then the paradoxes disappear (are solved) by themselves (become obvious), and new solutions (cos-waves) appear in a number of problems.

3.2 General provisions on boundary conditions
The boundary conditions are called boundary conditions because they are placed on the boundary, and not in the center of the resonator. If a certain object is located in the center of the resonator, for example, an atomic nucleus or other impenetrable ball or point, then an additional boundary condition should be put on this object. So, P. Dirac introduced a δ-function to describe the behavior of free electrons and justified an additional boundary condition ($\psi_n(0) = 0$) for electrons stable with respect to their $K$-capture by the atomic nucleus. But if the resonator is empty, then the boundary conditions should be set only at its boundaries [10, 15] and, accordingly, all possible resonances should be investigated for both acoustic and electromagnetic, and for quantum resonators [10, 15]. Therefore, for hollow quantum resonators, one should study both generalized interference (diffraction at the center of the resonator) of Fresnel (sin-wave) and generalized by the author interference (diffraction at the center of the resonator) of Fraunhofer (cos-wave) [10]. We note again that the additional condition on the regularity of the eigenfunctions $\psi_n$-functions in the center of a hollow (absolutely empty) resonator, according to the author, is an error leading to the exclusion of a number of solutions in the case of quantum hollow spherically and cylindrically symmetric resonators. Researchers do not make such mistakes in the case of plane symmetry [11]. But with spherical or cylindrical symmetry, due to the requirement in classical quantum mechanics about the boundedness of the $\psi_n$ function (or the regularity of the eigenfunctions $\psi_n$ in the center of the resonator) in the whole space, one has to throw out a number of solutions responsible for the unlimited cumulation of $\psi_{n\text{cos}}$-functions, thereby violating de Broglie's postulate on the equivalence of wave phenomena in classical physics and quantum mechanics. Authors of this article believe that it is necessary to modify the requirement that the $\psi_n$ – eigenfunctions must be limited and thereby reformulate the foundations of cumulative quantum mechanics. The noted problems, paradoxes and their solutions affect the basics of describing the ballistic conductivity of nanotubes, their heat transfer, etc. and are very important for 4D problems of nanotechnology [10, 11].

According to the de Broglie principle, these problems should be solved by using an analogy with acoustic and electromagnetic phenomena in classical and quantum physics [10].

4. Conclusions

An analytical model of polarization resonance electron captures (dynamic localization due to self-formation of a potential barrier that cumulates this electron inside a hollow molecule) is proposed within the CQM [8-10]. For polarization capture by allotropic hollow forms of carbon: fullerenes and nanotubes, the electron energy $E_n > 0$. The problem of the polarization effect Vysikailo of the first type (or the problem of polarization cumulation of de Broglie waves of electrons with a characteristic size of $\sim 1$ nm) is reduced to the Gamow idea of a quantum particle in a box with a potential barrier at its border. The spectrum of energy states localized by the barrier $E_n > 0$ (metastable FQ particle — partially open quantum dot, line or well), is determined by the effective internal dimensions box ($R r_{\text{rad}}$) with polarizing forces, effectively acting at a distance $r_{\text{rad}}$ from the polarizing molecule. CQM allows for $E_{n, n+1/2} > 0$ to describe both the bounded cumulation of $\psi_n(r)$-functions in generalized de Broglie-Fresnel interference and the unbounded cumulation of $\psi_{n+1/2}(r)$-functions to the center of the quantum resonator in the generalized Vysical girl de Broglie-Fraunhofer interference in hollow polarized spherical or cylindrical symmetric quantum resonators for de Broglie waves of electrons. Within the framework of CQM, the eigenquantum pairs are analytically calculated: $\psi_n(r)$ -functions, respectively, the stratified probability profiles of the particle in the cavity — $W_n(r)$, and $E_n > 0$ are the eigen energies of electrons localized in the quantum resonator (C60 and 70, etc.) by polarization forces. It is proved that along with the classical energy spectrum for asymmetric $\psi_n$ – functions (sin-waves) with $E_n \sim n^2$ for hollow quantum resonators, quantum resonances for symmetric $\psi_n$ – functions (cos-waves) with $E_n \sim (n-1/2)^2$ (n-1) exist and are realized in experiments.

We have created nanostructured materials [12], including materials with increased strength and wear resistance, heterogeneous at the nanoscale, physically doped with nanostructures - quantum traps for free electrons. The solution of this problem will give the possibility to create new nanostructured materials and to investigate their various physical parameters with high accuracy, to design, to manufacture and to operate devices with new technical and functional capabilities, including for the nuclear industry and rocket science. It has been established that the presence of nanographite in the composite significantly improves the toughness and expands the range of possible applications compared to pure carbides.

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