RESONANT REPULSION OF KEPLER PLANET PAIRS

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Abstract

Planetary systems discovered by the Kepler space telescope exhibit an intriguing feature. While the period ratios of adjacent low-mass planets appear largely random, there is a significant excess of pairs that lie just wide of resonances and a deficit on the near side. We demonstrate that this feature naturally arises when two near-resonant planets interact in the presence of weak dissipation that damps eccentricities. The two planets repel each other as orbital energy is lost to heat. This moves near-resonant pairs just beyond resonance, by a distance that reflects the integrated dissipation they experienced over their lifetimes. We find that the observed distances may be explained by tides if tidal dissipation is unexpectedly efficient (tidal quality factor $\sim 10$). Once the effect of resonant repulsion is accounted for, the initial orbits of these low-mass planets show little preference for resonances. This could constrain their origin.

Key word: planets and satellites: dynamical evolution and stability

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1. INTRODUCTION

NASA’s Kepler mission is revolutionizing our knowledge of planetary systems. It has already discovered thousands of transiting planetary candidates, including hundreds of systems with two or more planets (Batalha et al. 2012). Most of these are Neptune- or Earth-sized planets. To date, one of the most intriguing Kepler discoveries is that, while the spacing between planets appears to be roughly random, there is a distinct excess of planetary pairs just wide of certain resonances, and a nearly empty gap just narrow of them (Lissauer et al. 2011; Fabrycky et al. 2012). These features are particularly prominent near the 3:2 and 2:1 resonances, and affects planets that fall within a few percent of resonances (Fabrycky et al. 2012).

Is this resonance asymmetry a feature planetary systems are born with, or one they acquire much later on? Many studies have reported that planets become trapped into first-order resonances when they migrate in protoplanetary disks (e.g., Lee & Peale 2002; Snellgrove et al. 2001; Papaloizou & Szusztkiewicz 2005). In fact, the presence of resonances among giant planets detected by radial velocity has been regarded as strong evidence for disk migration (e.g., Marcy et al. 2001; Tinney et al. 2006). However, Kepler’s low-mass planets appear to be less influenced by resonances, and the pile-ups just outside resonances are partly counterbalanced by the gaps inside them.

In this paper, we identify a process that can modify the pair separation and give rise to the observed resonance asymmetry. But first, let us consider a commonly invoked mechanism, tidal circularization. If the inner planet is eccentric, tides raised on it would damp its eccentricity, decrease its semimajor axis, and hence increase the period ratio of the pair (Novak et al. 2003; Terquem & Papaloizou 2007).\textsuperscript{4} Adopting the equilibrium tide expression from Hut (1981), the damping rate for a pseudo-synchronized planet is

$$\gamma_e = \frac{1}{e} \frac{1}{dt} = \frac{9}{2} \frac{k_2}{T_i} q (1 + q) \left( \frac{R_i}{a} \right)^8,$$  

where $q = M_* / m_1$ is the mass ratio of the star to planet, $k_2$ is the tidal love number, $R_1$ is the inner planet’s radius, and $\alpha$ is its orbital separation. In this tidal model, $T_i = R_i^3 / (G M_1 m_1)$ where $\tau_1$ is the assumed constant tidal lag time which we take to be $\tau_1 = \tau_1 / (2 Q_1)$, with $Q_1$ being the inner planet’s tidal quality factor (Goldreich & Soter 1966) and $P_1$ its orbital period.

The orbital decay rate is $\dot{a} / a = 2 \dot{e} e$ because orbital angular momentum is largely conserved (assuming that $e \ll 1$). So, tidal evolution could potentially circularized orbits inward of $\sim 10$ days. As it does so, it moves the inner planet inward by $\dot{a} / a \sim \dot{e}$. This increases the period ratio for a planet pair by a fractional amount of $3 \dot{e}^2 / 2 = 1.5 \%(e_1 / 0.1)^2$. However, tidal circularization alone cannot reproduce the observed asymmetry: assuming all near-resonant pairs were initially uniformly distributed in their period ratios, all systems march to larger period ratios by a comparable amount. This produces neither gap nor peak.

A more selective mechanism is required. In this paper, we show that for a pair of planets that happen to lie near a mean-motion resonance, dissipation causes the planets to repel each other. The rate of repulsion is greatest at exact resonance and falls off steeply away from resonance. Planets that are initially slightly closer than resonance are pushed wide of the resonance; those that are initially wider are pushed even farther apart. And planet pairs far away from the resonance are not affected.

So, the combined action of resonant interaction and damping naturally gives rise to the observed resonance asymmetry. This effect, which we term “resonant repulsion,” was investigated by Greenberg (1981) for the Galilean satellites, by Lithwick & Wu (2008) to possibly account for the orbits of Pluto’s minor moons, and by Papaloizou (2011) for multiple planet systems. Batygin & Morbidelli (2012) independently arrived at many of
the results presented in this paper; their paper was posted to arxiv.org at the same time as this one.

2. RESONANT REPULSION

We consider the evolution of two planets orbiting a star and assume that the interaction between the planets is predominantly due to the 2:1 resonance. We will also include weak external eccentricity-damping forces. The energy (or Hamiltonian) of the two planets is, to leading order in eccentricity,

\[ H = - \frac{GM_* m_1}{2a_1} - \frac{GM_* m_2}{2a_2} - \frac{G m_1 m_2}{a_2} \times (f_1 e_1 \cos (2\lambda_2 - \lambda_1 - \varpi_1) + f_2 e_2 \cos (2\lambda_2 - \lambda_1 - \varpi_2)), \]

where we follow standard notation (e.g., Murray & Dermott 2000), with the orbital parameters for the inner planet denoted by \(a_1, e_1, \lambda_1, \varpi_1\), and those for the outer planet subscripted by 2. The mass of the star and planets are \(M_*\), \(m_1\), \(m_2\), and the Laplace coefficients are \(f_1 = -(2 + \alpha D/2) b_{1/2}^2\) and \(f_2 = (3/2 + \alpha D/2) b_{1/2}^2 - 2\alpha\) (Murray & Dermott 2000). Near 2:1 resonance (\(\alpha = 2^{-2/3}\)), the Laplace coefficients are \(f_1 = -1.19\) and \(f_2 = 0.428\).

We choose units such that

\[ GM_* = 1, \]

and assume that the eccentricities are small. In terms of the complex eccentricity

\[ z_j \equiv e_j e^{i m_j}, \]

the equations of motion for planet \(j\) are (e.g., Murray & Dermott 2000; Lithwick & Wu 2008)

\[ \frac{d\lambda_j}{dt} = \frac{m_j}{a_j} \frac{\partial H}{\partial \lambda_j}, \]

\[ \frac{dz_j}{dt} = -\frac{i}{m_j \sqrt{a_j}} \frac{\partial H}{\partial z_j}, \]

\[ \frac{da_j}{dt} = -\frac{2 \sqrt{a_j}}{m_j} \frac{\partial H}{\partial a_j}. \]

To leading order in \(m_j / M_*\), the semimajor axes are constant, and the equations for \(\lambda_j\) are

\[ \frac{d\lambda_j}{dt} = n_j, \]

where

\[ n_j \equiv a_j^{-3/2}. \]

Hence

\[ \lambda_j \approx n_j t. \]

The eccentricity equations become, after adding damping terms,

\[ \frac{dz_1}{dt} = i \mu_2 n_2 \left( \frac{a_2}{a_1} f_1 e^{i \phi} - \gamma_1 z_1 \right), \]

\[ \frac{dz_2}{dt} = i \mu_1 n_2 f_2 e^{i \phi} - \gamma_2 z_2, \]

where

\[ \mu_j \equiv m_j / M_* \]

\[ \phi \equiv 2\lambda_2 - \lambda_1 \approx -2 \Delta \cdot n_2 t. \]

Here,

\[ \Delta \equiv \frac{n_1 - 2n_2}{2n_2} \]

is the fractional distance to nominal resonance. When \(\Delta < 0\), the pair is on the near side of resonance, otherwise it is on the far side.\(^5\) The \(\gamma_{ej}\) in Equations (12) and (13) denote the eccentricity damping rates on each of the two planets due to some external force (e.g., tides or a dissipative disk). We assume that \(\gamma_{ej} \ll |\Delta n_2|\).

We discard the free solutions to Equations (12) and (13) because they decay to zero at the rates \(\gamma_{ej}\), much faster than the rate of semimajor axis evolution, as we shall see below. The forced eccentricities are, to first order in \(\gamma_{ej} / (\Delta n_2) \ll 1\):

\[ z_1 = -\frac{\mu_2}{2\Delta} f_1 \sqrt{\frac{a_2}{a_1}} e^{i \phi} \left( 1 - i \frac{\gamma_1}{2\Delta n_2} \right), \]

\[ z_2 = -\frac{\mu_1}{2\Delta} f_2 e^{i \phi} \left( 1 - i \frac{\gamma_2}{2\Delta n_2} \right). \]

The small phase shift, \(O(\gamma_{ej})\), relative to the undamped forced eccentricities plays a crucial role in resonant repulsion.

Inserting the above forced eccentricities into the semimajor axis equations yields, as in Lithwick & Wu (2008),

\[ \frac{d \ln a_1}{dt} = -\frac{\beta}{2 \Delta^2} (\gamma_{ej} f_1^2 \beta + \gamma_{ej} f_2^2) - \gamma_{al} |z_1|^2, \]

\[ \frac{d \ln a_2}{dt} = \frac{\mu_1^2}{\Delta^2} (\gamma_{ej} f_1^2 \beta + \gamma_{ej} f_2^2) - \gamma_{al} |z_2|^2, \]

where

\[ \beta \equiv \frac{\mu_2 \sqrt{a_2}}{\mu_1 \sqrt{a_1}} \]

and we have included additional damping terms with rates \(\gamma_{ej} f_j^2\).

This form for the damping rate is applicable for any process that conserves angular momentum, such as tides (see below). By contrast, if the planet is migrated in a disk, or pushed by tides raised on the central body (as for Jupiter’s moons), the induced rate of change of \(a\) would be independent of eccentricity. We shall not consider those kinds of forces.

We conclude that the distance to resonance changes at the rate

\[ \frac{d \Delta}{dt} = \frac{3}{4} \frac{\mu_1^2}{\Delta^3} \Gamma, \]

where

\[ \Gamma \equiv (2 + \beta) (\gamma_{ej} f_1^2 \beta + \gamma_{ej} f_2^2) + \frac{\gamma_{al} f_1^2 \beta^2 - \gamma_{al} f_2^2}{2}. \]

\(^5\) For brevity, we often refer to nominal resonance (\(\Delta = 0\)) as simply resonance. This should not be confused with a pair being locked in resonance, i.e., in a state where the resonant angles librate.
The parameters are chosen such that resonant repulsion on an initially flat distribution of pairs at three later times. The three red curves show the effect of 3:2 resonances (Batalha et al. 2012). The three red curves show the effect of tides are the source of damping, then that make $|\Delta| \ll 1$. We verify this rate with an N-body simulation below.

As long as $\Gamma > 0$, as we shall argue is the case, then $\Delta$ always increases, independent of the sign of $\Delta$. A pair of planets that is initially spaced closer than nominal resonance ($\Delta < 0$) will tend to be pushed outside of resonance, i.e., to $\Delta > 0$. A pair initially outside of resonance will be pushed even farther apart. We term this effect resonant repulsion. Furthermore, since the speed of migration is slowest far from resonance, the region near nominal resonance ($\Delta = 0$) should be unoccupied, and resonant pairs should evacuate the resonance region and pile up outside. This will lead to an asymmetry, with more planets outside of nominal resonance than inside.

Two planets that initially have $\Delta = \Delta_0$ repel each other to $\Delta > \Delta_0$, and at time $t$ they migrate to

$$\Delta(t) = (\Delta_{mig}^3 + \Delta_0^3)^{1/3},$$

where

$$\Delta_{mig}(t) = \left(\frac{9}{4} \mu_i^2 \Gamma t\right)^{1/3}.$$  \hspace{1cm} (24)

Figure 1 illustrates the effect on the distribution of period ratios.

The sign of $\Gamma$ is always positive due to eccentricity damping alone, i.e., to the $\gamma_{ej}$ terms in Equation (23). Furthermore, if tides are the source of damping, then $\gamma_{ai} = 2\gamma_{ej}$ by angular momentum conservation, leaving $\Gamma > 0$; this is also true for any form of damping that conserves angular momentum. Other forms of damping could in principle result in values of $\gamma_{ai}$ that make $\Gamma$ negative. However, the fact that Kepler pairs are piled up outside of resonances argue that this did not happen.

3. RESONANT REPULSION BY TIDES

In this section, we focus on the case when the dissipation is provided by tidal damping. The rate of eccentricity damping $\gamma_{ei}$ is given by Equation (2). In addition, $\gamma_{ai} = 2\gamma_{ej}$ by angular momentum conservation, and we may ignore tides on the outer planet ($\gamma_{a2} = \gamma_{a2} = 0$) because tidal damping rates are steep functions of orbital period. Therefore, Equation (25) becomes

$$\Delta_{mig} \approx 0.006 \left(\frac{Q_1}{10}\right)^{-1/3} \left(\frac{k_2}{0.1}\right)^{1/3} \left(\frac{m_1}{10 M_\odot}\right)^{1/3} \left(\frac{R_1}{2 R_\oplus}\right)^{5/3} \frac{M_\odot}{M_\odot}^{−8/3} \left(\frac{P_1}{5 \text{ day}}\right)^{−13/9} \left(\frac{t}{5 \text{ Gyr}}\right)^{1/3} \times (2\beta + 2\beta^2)^{1/3}. \hspace{1cm} (26)\]  

Figure 2 shows an N-body simulation with tides of two planets initially on the near side of resonance. Resonant repulsion pushes them to the far side, in agreement with the analytic solution (Equations (24) and (26)). There is modest disagreement when the pair crosses through nominal resonance when the expansion in small $e$ becomes invalid. The free eccentricities damp away after a brief initial period ($\leq 2 \times 10^4$ yr). On crossing nominal resonance, they are regenerated, but then quickly damp away again. Damping locks the system into libration (of both resonant angles), but this has little dynamical significance, as it is merely a consequence of the eccentricities taking on their purely forced values.

Figure 3 shows the “resonant repulsion time” ($\tau_{RR}$) for all reported Kepler pairs. This is the timescale over which resonant repulsion by tides moves a pair toward or away from the nearest first-order resonance. Mathematically, $\tau_{RR} \equiv |\Delta/\Delta|$, where $\Delta$ is the observed fractional distance and $\Delta$ is the rate predicted by resonant repulsion (Equation (22)) assuming tidal damping is operating with $Q_1 = 10$, $k_2 = 0.1$, and using the observed planet and stellar parameters. On this plot, systems that have $\tau_{RR}$ longer than their age have not experienced significant resonant
systems with very small $t_{RR}$ reveal many are related to three-body effects: the turquoise circles indicate pairs where one or both planets are engaged in at least two resonances simultaneously (defined as $|\Delta| < 5\%$). Our simple picture of resonant repulsion may break down in these cases. “Uncertain systems” refer to those where the nominal total mass $\geq 1000 M_{\oplus}$ (assuming Earth density), and we discard them from consideration for fear of contamination.

(A color version of this figure is available in the online journal.)

**Figure 3.** Timescale for resonant repulsion to move the period ratio of Kepler planet pairs by a distance $|\Delta|$, where $\Delta$ is the observed fractional distance to the closest first order resonance. We adopt KIC system parameters, with updated values for KOI-961 (red dots) from Muirhead et al. (2012). For tidal dissipation, $Q_1 = 10$ and $k_2 = 0.1$. The lower panel zooms in to the resonant region. If $t_{RR} \gg$ system age (the horizontal line is the age of the Sun), the period ratios should have evolved little since birth, while for $t_{RR} \ll$ age, we do not expect the systems to linger at the observed ratios. The fact that most pairs lie at or above the horizontal line is consistent with resonant repulsion by tides. Close inspection of systems with very small $t_{RR}$ reveal many are related to three-body effects: the turquoise circles indicate pairs where one or both planets are engaged in at least two resonances simultaneously (defined as $|\Delta| < 5\%$). Our simple picture of resonant repulsion may break down in these cases. “Uncertain systems” refer to those where the nominal total mass $\geq 1000 M_{\oplus}$ (assuming Earth density), and we discard them from consideration for fear of contamination.

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repulsion, while all those with shorter $t_{RR}$ should have moved to the right.

A number of inferences may be drawn. First, most systems far from resonances ($|\Delta| \geq 10\%$) have experienced negligible resonant repulsion and were most likely born with the period ratio they have today.

Second, systems within 1%–10% of resonance exhibit $t_{RR}$ that are as long as, or longer than, the typical age of systems (a few Gyr). This is consistent with resonant repulsion by tides: systems with shorter $t_{RR}$ would have been moved to the right until $t_{RR}$ was comparable to the age of the system. Near the 2:1 resonance, it appears that pairs as far left as 1.8 and as far right as 2.2 could have been affected by the repulsion.

Last, many systems very near resonances ($|\Delta| \leq 1\%$) exhibit such short $t_{RR}$ that they should have migrated to much larger $\Delta$ values. At first sight, their presence is troubling. However, an inspection of the *Kepler* catalog reveals that many of these are in triples or higher multiple systems, and these planets are engaged simultaneously in two or more two-body resonances. The worst-off cases are in simultaneous resonances, reminiscent of the Laplace resonance of Jupiter’s moons (Yoder & Peale 1981). Moreover, the fraction of multiples is much higher among systems with $t_{RR}$ falling below the solar age line than for other random pairs. Our simple picture of resonant repulsion fails when the planet is subject to two or more resonances. In this case, exact resonance may be maintained for a much longer time because the planets form a heavy ladder with an effectively large inertia. The prevalence of simultaneous resonances in these short $t_{RR}$ systems spurs us to hypothesize that all pairs with short $t_{RR}$ in Figure 3 are results of three-body effects, and that these resonances are not primordial, but a combined effect of resonant repulsion and three-body effects.

Removing the colored circles in Figure 3, we see a relatively clear picture that most pairs stay where they were born with, while pairs very close to resonances experience repulsion and are shifted by a few percent to larger period ratios.

**4. DISCUSSION**

In this work, we investigate the peculiar fact that there is an excess of *Kepler* planet pairs just wide of resonance, and a deficit just inward of resonance. We propose that dissipation is responsible for this asymmetry. Two nearly resonant planets whose eccentricities are weakly damped repel each other (Greenberg 1981; Lithwick & Wu 2008; Papaloizou 2011). This is because dissipation damps away the planets’ free eccentricities, but the eccentricities that are forced by the resonance persist despite dissipation. Planets are typically repelled when dissipation acts on these forced eccentricities. As such, resonant interaction allows dissipation to continuously extract energy from the orbits. Resonant repulsion pushes pairs from the near side to the far side of resonance, and naturally explains the *Kepler* result. Pairs accumulate at a fractional distance $\Delta_{\text{mag}}$ wide of each resonance, with $\Delta_{\text{mag}} \sim (\mu^2/t_{\text{damp}})^{1/3}$, where $t_{\text{damp}}$ is the typical eccentricity damping time and $t$ is the system age (Equations (24)–(25)).

For the source of dissipation, we focused on tidal damping in the inner planet. The typical distance planets can repel each other is of order a few percent or less for *Kepler* parameters if the tidal damping is efficient. The deficit of pairs immediately inward of resonance may be explained by this repulsion, and the distances outward of resonance where planet pairs are found are consistent with the theoretically estimated repulsion distance.

However, a number of inconsistencies between theory and data require further investigation. For instance, many pairs remain very close to resonance despite a short resonant repulsion time. These are often found in systems with more than two planets where the planet pairs are engaged simultaneously in more than one resonance. We therefore speculate that in fact all systems with short resonant repulsion time are consequences of three-body effects. This may be confirmed using transit-timing variation or other tools.

If resonant repulsion is the reason behind the resonance asymmetry, its signature should be observable in future studies. The planets should currently have nearly zero free eccentricities, and as a result both of the resonant angles should be locked at their center-of-resonance values, with very small libration amplitude. This can be tested with radial velocity measurements or with transit-time variations (Lithwick et al. 2012). Furthermore, if tidal damping is the dominant dissipation mechanism, we expect that the resonance asymmetry should vanish for planets at orbital periods greater than 10–20 days. Long-term *Kepler* monitoring will decide between tides or alternative damping mechanisms, e.g., damping by a gaseous or planetesimal disk.

Our study suggests that the initial period distribution of *Kepler* planets was relatively flat, without major pile-ups at or near resonances.6 This is in contrast to Jovian mass planets and

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6 The chain of resonances observed in systems like KOI-500, KOI-730 (Lissauer et al. 2011), KOI-2038 (Fabrycky et al. 2012) might also not be primordial, but a combined result of resonant repulsion (a two-body effect) and three-body interactions.
could help constrain the origin of these low-mass planets. If disk migration is responsible for their current location, it must somehow have avoided pushing the planets into resonances, perhaps because the migration rate was very fast—faster than the resonant libration rate. Alternatively, planets may be formed in situ (Hansen & Murray 2012) and have therefore avoided convergent migration.

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