Phase diffusion in stationary state of nonequilibrium Bose gas.

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Abstract

The low energy properties of the stationary state of a nonequilibrium Bose gas with the dynamical equilibrium between the escape and creation of particles are studied. The low energy spectrum of elementary excitations in such Bose gas is calculated at sufficiently low temperatures. Both in the nonequilibrium stationary state and in the thermodynamically equilibrium state the spectrum of the low energy excitations is formed by the long wavelength phase fluctuations of the complex Bose field. The spectrum of the phase fluctuations is found to have a diffusion behavior. Such behavior is in contrast to that in the thermodynamically equilibrium BEC systems in which the long wavelength phase fluctuations have the sound spectrum. The diffusion character of the phase fluctuations is due to noise generated by the escape and creation of particles. The diffusion character of the phase fluctuation spectrum results in the absence of coherency and, thus, in the absence of BEC and superfluidity as well.
I. INTRODUCTION.

The investigations of the coherent properties of a Bose gas of quasiparticles in the nonequilibrium systems attract large interest of experimentalists and theorists. The various efforts to realize the state with the Bose-Einstein condensate (BEC) in the nonequilibrium Bose gas of quasiparticles were carried out in different systems. The systems are, as follows, exciton gas in the double quantum wells in semiconductors [1]-[7], polariton systems, in particular, polariton systems in the two dimensional (2D) cavities located between two mirrors [8]-[15], and a gas of excited magnons [16]-[19]. All these systems are the candidates to observe BEC at relatively large temperatures. Some of them are interesting both for the fundamental and for the applied science. In all these systems quasiparticles are created by external forces and live during a finite time. In what follows, quasiparticles are called as particles for brevity.

The properties of the stationary state of a nonequilibrium Bose gas produced by the dynamic equilibrium between the escape and creation of particles due to external forces are considered in the present paper. In contrast to quantum atomic gases in traps in which BEC has been observed at ultralow temperatures and where the atoms have the infinite lifetime [1], [21], all the systems considered here have a finite lifetime for Bose particles. Provided the lifetime is long compared with the thermalization time, the system can be observed within the time interval small compared with the particle lifetime and large compared with the thermalization time. In this case the external pumping is unnecessary. However, if the lifetime is small compared with the thermalization time, a continuous external pumping of particles becomes necessary to reach a stationary state of the system. Below we analyze this case within the framework of the nonideal Bose gas model augmented with the processes of the continuous escape and creation of particles. Moreover, the latter processes are supposed to be noncoherent in accordance with many experiments, for example, [2]-[4], [10]-[12]. In particular, for polaritons in microcavities, a pumping produces particles with the large energies and then the particles relax to the low energy states. We assume that the processes of the escape and creation of particles are governed by random sources. In the stationary state the escape and creation terms balance each other in average but the noise generated by these two processes acts on the system continuously.

It is well known that the possibility of BEC and superfluidity is closely connected with the behavior of the low energy part of the excitation spectrum. The low energy spectrum of
elementary excitations is found here for the stationary state of a nonequilibrium Bose gas at sufficiently low temperatures. In the nonequilibrium Bose gas in the stationary state as well as in the thermodynamically equilibrium gas with BEC the renormalized low energy spectrum of elementary excitations is governed by the long wave phase fluctuations of the complex Bose field. As is well known, the low energy spectrum of phase fluctuations and, correspondingly, the low energy part of the excitation spectrum in the thermodynamically equilibrium Bose gas with BEC has a sound-like behaviour [22]-[24]. We show here that, in contradiction with the thermodynamically equilibrium Bose systems, the spectrum of long wave phase fluctuations for the stationary state in the nonequilibrium Bose gas with the noncoherent particle creation and escape has a diffusion character at sufficiently small temperatures. The diffusion behaviour of the spectrum is due to the noise generated by the escape and creation processes. This noise exists in spite of the balance in average between the escape and creation terms. The frequency behavior for the different components of the self-energy part of the one-particle Green function [29], [30], [25] determines the influence of the noise on the system. The relation between the kinetic and imaginary parts of the retarded or advanced components in the self-energy part gives an effective temperature of the stationary state. The important difference between thermodynamically equilibrium Bose systems and nonequilibrium systems in the stationary state results from nonzero value of the zero-frequency Fourier component in the kinetic term of the one-particle self-energy part of nonequilibrium systems. This circumstance does lead to the diffusion character of the low energy spectrum of the phase fluctuations in the nonequilibrium stationary state of a Bose gas. At the same time, the fluctuations of the Bose field module has the same character as in the thermodynamically equilibrium state of Bose gas. The diffusion character of the spectrum of phase fluctuations results in the absence of coherency and, thus, in the absence of BEC and superfluidity. In particular, for the systems with phase diffusion, the interference experiments should not demonstrate the pronounced interference picture in contrast to similar experiments in the quantum atomic Bose gases in traps [20], [21]. Unlike thermodynamically equilibrium Bose gas with BEC, the classical Bose field for the stationary state of nonequilibrium Bose gas obeys the Fokker-Planck equation which is more complicated than the Gross-Pitaevskii one. This is completely due to an existence of the noise generated by the particle escape and creation. So, if we try to describe the system by the Gross-Pitaevskii equation, the noise term should be augmented to the right-hand side
of the Gross-Pitaevskii equation. As a result, the equation goes over to the Langevin-like equation.

At present, a majority of the systems used for the experimental BEC realization in nonequilibrium stationary state of quasiparticles have the quasi 2D geometrical configurations. For example, polaritons in microcavities are confined by two parallel mirror planes with a small spacing between them. In this case the bare spectrum of polaritons has a quadratic dispersion law in the region of sufficiently small momenta near the bottom of the spectrum. For the sufficiently small densities, such polariton systems can be described by the nonlinear Schroedinger Hamiltonian with a noncoherent pumping and escape of particles [8], [9]. The experiments on the BEC realization or laser effect in the polariton systems in microcavities are carried out actively at the present time [10]-[12].

In the present paper the coherent properties and the low energy spectrum of excitations in the stationary state of nonequilibrium Bose gas are studied in the assumption that the system considered can be described by the nonlinear Schroedinger action with a noncoherent pumping and escape of particles. As will be seen below, in such systems the renormalized low energy spectrum has a diffusion character both in two and in three dimensions. To be closer to experiments [10]-[12] in which the BEC of quasiparticles has been investigated, we consider the properties of a nonequilibrium Bose gas in the 2D geometrical configuration with a brief analysis for the typical properties of this dimensionality.

II. LOW ENERGY SPECTRUM OF A NONEQUILIBRIUM BOSE GAS IN THE STATIONARY STATE.

In the present paper we study the coherent properties of the stationary state in dilute 2D polariton gas in a microcavity under continuous noncoherent pumping and escape of particles at small effective temperatures. These properties are determined by the particles with the momenta near the bottom of the polariton spectrum. The polaritons are concentrated in the region near the bottom of the spectrum after relaxation from the excited states of particles created by the noncoherent pumping. In the region of sufficiently small momenta the bare polariton spectrum has a quadratic dependence on the momentum for polariton gas in a microcavity confined by two plane mirrors separated by sufficiently small spacing [8], [9]. The renormalized spectrum of elementary excitations in the region of small momenta is
determined by an effective interparticle interaction. In this region of momenta the polariton system of sufficiently small density can be considered as a dilute 2D Bose gas with the effective interparticle interaction independent of the momentum transfer. The Hamiltonian of such Bose gas with the omitted terms of the escape and pumping of particles can be taken in the form of the nonlinear Schroedinger Hamiltonian

$$\hat{H} = \int d^2r \left\{ \hat{\psi}^+ \left( -\frac{1}{2m} \vec{\nabla}^2 - \mu \right) \hat{\psi} + \frac{1}{2} g \left( \hat{\psi}^+ \hat{\psi} \right)^2 \right\}$$  \hspace{1cm} (1)$$

where $g > 0$ is the two-particle scattering amplitude independent of the momentum transfer, $\hat{\psi}^+$, $\hat{\psi}$ are the creation and annihilation operators of Bose particles, $m$ is the effective mass, and $\mu$ is the chemical potential. We put here $\hbar = 1$. The particle density $n = <\hat{\psi}^+ \hat{\psi}>$ is supposed to be small. In the case of the low density Bose gas the chemical potential is $\mu = gn$. The consideration of low energy excitations gives the possibility to take the bare spectrum of particles in the form $\sqrt{p^2/2m}$, or in the coordinate representation it can be written as $(-\vec{\nabla}^2/2m)$. The effect of the escape and creation processes will be included into the description of the system via introducing the relaxation terms into the Gross-Pitaevskii action treated within the framework of the Keldysh-Schwinger technique [29], [30], [25] in the functional-integral formulation [31].

In the ”tetragonal” representation of the Keldysh-Schwinger technique the action of the system $S$ can be written in the terms of integrating over the double time contour with time reversion. The particle escape and creation processes can be included into action $S$ as random sources of the particle escape and creation. The action can be written as

$$S = \oint dt d^2r \left\{ \bar{\psi} \left( i\partial_t + \mu - \frac{1}{2m} \vec{\nabla}^2 \right) \psi - \frac{1}{2} g \left( \bar{\psi} \psi \right)^2 \right\} - \sum_{\alpha=1,2} \left( \bar{\psi} f_\alpha + \bar{f}_\alpha \psi \right) + \bar{f}_\alpha K_\alpha f_\alpha$$ \hspace{1cm} (2)$$

where $\psi$ and complex conjugate $\bar{\psi}$ are the Bose fields, $f_\alpha$, $\bar{f}_\alpha$ are random sources depending on the time and space coordinates, index $\alpha = 1$ corresponds to the particle escape and $\alpha = 2$ corresponds to the particle creation by the external forces, and $\bar{K}_\alpha$ is the white noise correlator of random sources $f_\alpha$. The corresponding generating functional $Z$ with the action $S$ (2) is written in the form of the functional integral over the fields $\psi$ and $f_\alpha$

$$Z = \int D\psi D\bar{\psi} \prod_{\alpha=1,2} Df_\alpha D\bar{f}_\alpha \exp \{ iS + i\delta S \}$$ \hspace{1cm} (3)$$
where the action $\delta S$ depends on the infinitesimal sources $\zeta$ as

$$
\delta S = \oint dt d^2 r (\bar{\psi} \zeta + \zeta \psi)
$$

(4)

The transition to the "triangular" representation can be performed by the substitution

$$
\Theta = \frac{1}{2} (\psi_+ + \psi_-)
$$

(5)

$$
\theta = \psi_+ - \psi_-
$$

where $\psi_+$ and $\psi_-$ are the Bose fields at the upper and lower branches of the time contour. The component $\Theta$ is the "classical" component of the field and $\theta$ is the component corresponding to quantum fluctuations. Note that the component $\Theta$ is nonzero for the case of the fields coinciding at the different branches of the time contour $\psi_+ (t, \vec{r}) = \psi_- (t, \vec{r})$, and the component $\theta$ vanishes in this case. The generating functional for low energy particles can be treated in the quasi classical approximation due to the large value of the low energy state occupation numbers. In this approximation the terms $\bar{\theta} \theta$ can be neglected as compared with the terms $\bar{\Theta} \Theta$. The transition to the fields $\Theta$, $\theta$ and the integration over the random sources $f_\alpha$ in the generating functional (3) with taking the inequality $\bar{\theta} \theta << \bar{\Theta} \Theta$ into account give the generating functional for the low energy fields. The corresponding action $S_{\text{QCl}} [\Theta, \theta]$ in the "triangular" representation [29], [30] reads

$$
Z = \int D\Theta D\bar{\Theta} D\theta D\bar{\theta} \exp \{ i S_{\text{QCl}} [\theta, \Theta] + i \delta S_{\text{QCl}} [J, j] \}
$$

(6)

$$
S_{\text{QCl}} [\theta, \Theta] = \int dt d^2 r \left\{ \bar{\Theta} \left( i \partial_t + \mu - \frac{\vec{\rho}^2}{2m} + i \hat{\Gamma}_R (\tilde{\omega}) \right) \Theta + 
+ \Theta \left( i \partial_t + \mu - \frac{\vec{\rho}^2}{2m} + i \hat{\Gamma}_A (\tilde{\omega}) \right) \theta -
- g \bar{\Theta} \Theta (\bar{\Theta} \Theta + \bar{\theta} \theta) + i \bar{\theta} \Gamma_K (\tilde{\omega}) \theta \right\}
$$

(7)

The frequencies and gradients of the fields describing low energy excitations and producing the action $S_{\text{QCl}} [\theta, \Theta]$ are supposed to be small on the scale of $\mu = g < \bar{\Theta} \Theta > = gn$, where $\mu$ is the chemical potential, $\hat{\rho} = -i \vec{\nabla}$, $\tilde{\omega} = i \partial_t$ are the momentum and frequency operators in the space-time representation. The frequency-dependent terms $\hat{\Gamma}_R (\omega)$, $\hat{\Gamma}_A (\omega)$, $\hat{\Gamma}_K (\omega)$ are the imaginary parts of the corresponding self-energy part components of one-particle
Green function. The indices $R$, $A$, $K$ denote the retarded, advanced and kinetic components, respectively. These terms can be expressed via the relaxation parts corresponding to the escape $\hat{\Gamma}^{(\text{out})}(\omega)$ and creation $\hat{\Gamma}^{(\text{in})}(\omega)$ of particles

\[
\hat{\Gamma}_R(\omega) = \hat{\Gamma}^{(\text{out})}_R(\omega) - \hat{\Gamma}^{(\text{in})}_R(\omega) \\
\hat{\Gamma}_A(\omega) = \hat{\Gamma}^{(\text{out})}_A(\omega) - \hat{\Gamma}^{(\text{in})}_A(\omega) \\
\hat{\Gamma}_K(\omega) = \hat{\Gamma}^{(\text{out})}_K(\omega) + \hat{\Gamma}^{(\text{in})}_K(\omega)
\] (8)

For the stationary state, the relaxations $\hat{\Gamma}_R(\omega)$, $\hat{\Gamma}_A(\omega)$ vanish at zero frequency $\hat{\Gamma}_R(0) = \hat{\Gamma}_A(0) = 0$. In contrast, the term $\hat{\Gamma}_K(0) = \hat{\Gamma}^{(\text{out})}_K(0) + \hat{\Gamma}^{(\text{in})}_K(0)$ is positive and nonzero since this term is a sum of two positive nonzero terms $\hat{\Gamma}^{(\text{out})}_K(0)$ and $\hat{\Gamma}^{(\text{in})}_K(0)$ \[29\]. The kinetic component of the self-energy part $\hat{\Gamma}_K$ describes the noise generated by the particle escape and creation. For small frequencies $\omega$, or in the time representation for small time derivative $\hat{\omega} = i\partial_t$, the relaxation terms can be expanded in $\omega$. The zero frequency components $\hat{\Gamma}_R(0)$ and $\hat{\Gamma}_A(0)$ have zero value in the stationary state due to the dynamic balance of the particle escape and creation processes. For this reason, in the limit of small frequencies the relaxations $\hat{\Gamma}_R(\omega)$, $\hat{\Gamma}_A(\omega)$ can be expanded into a series in $\omega$. The first term of the expansion has the form

\[
\hat{\Gamma}_R(\omega) = -\hat{\Gamma}_A(\omega) = \kappa \omega = \kappa i\partial_t
\] (9)

where the constant $\kappa$ can be written as a ratio of $\Gamma_K(0)$ to some constant $T^*$, which can be interpreted as an effective temperature of the stationary state of the system

\[
\kappa = \frac{\Gamma_K}{4T^*}
\] (10)

For small $\omega$, the dependence of $\Gamma_K(\omega)$ on frequency $\omega$ can be neglected as the zero frequency component of this term is nonzero $\Gamma_K(0) = \Gamma_K \neq 0$. The effective temperature $T^*$ is supposed to be much smaller than the Berezinskii-Kosterlitz-Thouless temperature $T_c$ \[26\], \[27\], \[28\]. Moreover, we suppose that $T^* << \mu$. The infinitesimal term $\delta S_{QCI}^{(\text{pol})}[J,j]$ is proportional to the infinitesimal sources $J$ and $j$ obtained from the infinitesimal sources $\zeta$ by the transformation \[3\]. In this case the sources $\zeta_+$, $\zeta_-$ play a role of the fields $\psi_+, \psi_-$

6
\[ \delta S_{QCI} [J, j] = \int dt d^2 r \left[ (\overline{\theta} J + \overline{J} \theta) + (\overline{\Theta} j + \overline{J} \Theta) \right] \]  \hfill (11)

Using notations (9) and (10), we can rewrite the action \( S_{QCI} [\theta, \Theta] \) as

\[
S_{QCI} [\theta, \Theta] = \int dt d^2 r \left\{ \overline{\theta} \left[ i (1 + i \kappa) \partial_t \Theta - F \right] + \left[ -i (1 - i \kappa) \partial_t \overline{\Theta} - \overline{F} \right] \theta + i \overline{\theta} \Gamma_K \theta \right\} \hfill (12)

where the functions \( F \) and \( \overline{F} \) are

\[
F = \left( \frac{\overline{p}^2}{2m} + g \overline{\Theta} \Theta - \mu \right) \Theta \hfill (13)
\]

\[
\overline{F} = \left( \frac{\overline{p}^2}{2m} + g \overline{\Theta} \Theta - \mu \right) \overline{\Theta}
\]

One can see that the functions \( F \) and \( \overline{F} \) can be represented in the form of the functional derivatives

\[
F = \delta H_0; \quad \overline{F} = \overline{\delta} H_0 \hfill (14)
\]

where \( \delta = \delta/\delta \Theta \) and \( \overline{\delta} = \delta/\delta \overline{\Theta} \) are the functional derivatives over the fields \( \Theta \) and \( \overline{\Theta} \), respectively. Thus, the functional \( H_0 \) has the form of the energy

\[
H_0 = \int d^2 r \left( \frac{\overline{\theta}^2}{2m} - \mu \right) \Theta \hfill (15)
\]

The integral over the fields \( \theta \) and \( \overline{\theta} \) in the generating functional \( Z \) (6) has the Gauss form and can easily be calculated

\[
Z = \int D\Theta D\overline{\Theta} \exp (-S [\Theta] + i \delta S [j]) \hfill (16)
\]

The action \( S [\Theta] \) can be written as

\[
S [\Theta] = \frac{1}{\Gamma_K} \int dt d^2 r \left\{ \left[ i (1 + i \kappa) \partial_t \Theta - F \right] \left[ -i (1 - i \kappa) \partial_t \overline{\Theta} - \overline{F} \right] \right\} \hfill (17)
\]

Note that, in the case \( \Gamma_K \to 0 \) due to the form of action (17), the contribution to the functional integral (16) is given by the configurations of the field \( \Theta \) obeying the standard
Gross-Pitaevskii equation. As is well known, the solution of this equation in the assumption of the small magnitudes of momenta compared with the inverse correlation length $\xi_0^{-1} \sim \sqrt{m\mu}$ yields the sound spectrum of low energy excitations.

In the case $\Gamma_K \neq 0$ the fluctuations of the Bose field $\Theta$ should be taken into account in the integral for the generating functional (16). Multiplying brackets in the action $S[\Theta]$ (17), we obtain

$$S[\Theta] = \frac{1}{\Gamma_K} \int dtd^2r \left\{ (1 + \kappa^2) \left( \frac{\partial_t \Theta}{\partial_r \Theta} \right) + FF + i \left( \frac{\partial_t \Theta}{\Gamma} F - F \frac{\partial_t \Theta}{\partial_r \Theta} \right) \right\}$$

(18)

The complex fields $\Theta$ and $\bar{\Theta}$ can be rewritten in the module-phase representation $\Theta = \rho \exp(i\varphi)$, $\bar{\Theta} = \rho \exp(-i\varphi)$. In this representation the corresponding terms of the action (18) take the form

$$\frac{\partial_t \Theta}{\partial_r \Theta} = \rho^2 (\partial_t \varphi)^2 + (\partial_t \rho)^2$$

(19)

$$FF = f^2 \rho^2 - \frac{1}{m} f \rho \left( \nabla^2 \rho - \rho \left( \nabla \varphi \right)^2 \right) +$$

$$+ \left( \frac{1}{2m} \right)^2 \left[ \left( \nabla^2 \rho \right)^2 - 2 \rho \left( \nabla^2 \rho \right) \left( \nabla \varphi \right)^2 + \rho^2 \left( \nabla^2 \varphi \right)^2 + \rho^2 \left( \nabla^2 \varphi \right) + 4 \rho \left( \nabla \rho \right) \left( \nabla \varphi \right) \left( \nabla^2 \varphi \right) \right]$$

(20)

where $f$ is

$$f = g \rho^2 - \mu$$

(21)

and

$$i \left[ \left( \frac{\partial_t \Theta}{\partial_r \Theta} \right) F - F \left( \frac{\partial_t \Theta}{\partial_r \Theta} \right) \right]$$

$$= 2f \rho^2 \partial_t \varphi + \frac{1}{m} (\partial_t \rho) \left[ 2 \left( \nabla \rho \right) \left( \nabla \varphi \right) + \rho \nabla^2 \varphi \right] -$$

$$- \frac{1}{m} (\rho \partial_t \varphi) \left[ \nabla^2 \rho - \rho \left( \nabla \varphi \right)^2 \right]$$

(22)

The module $\rho$ of the field $\Theta$ can be represented as a sum of the average value and fluctuation of module $\rho = \rho_0 + \delta \rho$, where $\rho_0$ is the bare value of the average module. [28]
\( \rho_0 = \sqrt{\mu / g} \), and \( \delta \rho \) is the module fluctuation. For small frequencies \( \omega \ll \mu \) and momenta \( | \vec{p} | \ll \sqrt{m \mu} \) the fluctuations \( \delta \rho \) are small compared with the average module \( \delta \rho \ll \rho_0 \). The fluctuations of the phase in the 2D case can be large \([26],[27],[28]\). For small module fluctuations \( \delta \rho \ll \rho_0 \), the function \( f \) takes the form

\[
f = 2g\rho_0 \delta \rho = 2\mu \delta \rho
\]

(23)

The expansion over the module fluctuations \( \delta \rho \) for the small values of the time and space derivatives \( \left( \frac{1}{2m} \nabla^2 \delta \rho \right), (\partial_t \delta \rho) \ll \mu \rho_0 \) leads to

\[
\mathcal{F}F = 4\mu^2 \delta \rho^2 + 2\mu \rho_0 \delta \rho \left( \frac{1}{m} \left( \nabla \varphi \right)^2 \right) + \rho_0^2 \left( \frac{1}{2m} \left( \nabla \varphi \right)^2 \right)^2 + \rho_0^2 \left( \frac{1}{2m} \nabla^2 \varphi \right)^2
\]

(24)

\[
i((\partial_t \Theta F) - \mathcal{F} (\partial_t \Theta)) = 4\mu \rho_0 \delta \rho (\partial_t \varphi) + 2\rho_0^2 (\partial_t \varphi) \left( \frac{1}{2m_{pol}} \right) \left( \nabla \varphi \right)^2
\]

(25)

\[
(\partial_t \Theta) (\partial_t \Theta) = \rho_0^2 (\partial_t \varphi)^2 + (\partial_t \delta \rho)^2
\]

(26)

The substitution of the above expressions into action (18) gives the generating functional and the corresponding action

\[
Z = \int D\delta \rho D \varphi \exp \{ iS[\delta \rho, \varphi] + i\delta S_{\delta \rho, \varphi} \} \tag{27}
\]

\[
S[\delta \rho, \varphi] = \frac{1}{\Gamma_K} \int dt d^2 r \left\{ \frac{1}{m} \left( \nabla \varphi \right)^2 \right. + 4\mu^2 \delta \rho^2 + 4\mu \rho_0 \delta \rho \left( \frac{1}{2m} \left( \nabla \varphi \right)^2 \right) + \rho_0^2 \left( \frac{1}{2m} \left( \nabla \varphi \right)^2 \right)^2 + \rho_0^2 \left( \frac{1}{2m} \nabla^2 \varphi \right)^2 + 4\mu \rho_0 \delta \rho \partial_t \varphi + 2 \left( \rho_0^2 \partial_t \varphi \right) \frac{1}{2m} \left( \nabla \varphi \right)^2 \left. \right\} \tag{28}
\]

Integrating the generating functional \( Z \) (27) over the module fluctuations \( \delta \rho \), we obtain the generating functional depending on the phase field \( \varphi \) alone

\[
Z = \int D \varphi \exp \{ iS[\varphi] + i\delta S_{\varphi} [j] \} \tag{29}
\]

Here the effective action \( S[\varphi] \) for the small values of \( \partial_t \varphi \) and \( \frac{1}{2m} \nabla^2 \varphi \) compared with \( \kappa \mu \) if \( \kappa < 1 \) or \( \mu \) if \( \kappa > 1 \) takes the form
\[ S[\varphi] = \frac{1}{\Gamma_K} \int dt d^2r \left\{ \varphi^2 \rho_0^2 (\partial_t \varphi)^2 + \rho_0^2 \left( \frac{1}{2m} \nabla^2 \varphi \right)^2 \right\} \]  

(30)

and

\[ \delta S_\varphi = \int dt d^2r \left[ \bar{\varphi} \rho_0 \exp(\text{in}) + j \rho_0 \exp(\text{in}) \right] \]

The action (30) describes a diffusion. It can be rewritten as

\[ S[\varphi] = \frac{\varphi^2 \rho_0^2}{\Gamma_K} \int dt d^2r \left\{ (\partial_t \varphi)^2 + \left( D_0 \nabla^2 \varphi \right)^2 \right\} \]  

(31)

where \( D_0 \) is the diffusion coefficient

\[ D_0 = \frac{1}{2m \varphi} \]  

(32)

The action (31) gives the following expression for the phase correlator

\[ <\varphi_{\mathbf{k} \omega} \varphi_{-\mathbf{k}' \omega'}> = \frac{\left( \Gamma_K / \varphi^2 \rho_0^2 \right)}{\omega^2 + \left( D_0 k^2 \right)^2} \delta \left( \mathbf{k} - \mathbf{k}' \right) \delta \left( \omega - \omega' \right) \]  

(33)

Near the pole this correlator can be rewritten as

\[ <\varphi_{\mathbf{k} \omega} \varphi_{-\mathbf{k}' \omega'}> = \frac{\left( \Gamma_K / \varphi^2 \rho_0^2 \right)}{2iD_0 k^2 \left( \omega + iD_0 k^2 \right)} \delta \left( \mathbf{k} - \mathbf{k}' \right) \delta \left( \omega - \omega' \right) \]  

(34)

The correlator describes the properties of low energy excitations. For the thermodynamically equilibrium Bose system, the phase correlator has the real pole of the sound type for small momenta and frequencies. In contrast for \( \Gamma_K \neq 0 \), the phase correlator (34) has the diffusion pole at the small momenta and frequencies. This form of the phase correlator means the absence of the phase coherence and, as a result, the lack of BEC and superfluidity in the stationary state of nonequilibrium Bose gas with the dynamic balance between the particle escape and creation processes. For example, in the case of the stationary state of nonequilibrium Bose gas the quantum interference experiments, like those in trapped quantum atomic Bose gases on the interference of two independent Bose condensates [20, 21], cannot give the pronounced interference picture. Note that the derivation of the low energy phase correlator (33) does not use the D=2 dimensionality of the system and can be used without any change for nonequilibrium D=3 Bose gas in the stationary state.
The phase correlator at the equal time moments can easily be obtained from Eq. (33)

\[
\langle \varphi_{k,t} \varphi_{-k,t} \rangle = \left( \frac{\Gamma_K}{k^2} \right) \frac{1}{2D_0} \frac{1}{k^2} = \frac{\Gamma_K m}{2k^2} \frac{1}{k^2} \tag{35}
\]

The substitution of Eq. (10) into Eq. (35) gives the phase correlator in the form

\[
\langle \varphi_{k,t} \varphi_{-k,t} \rangle = \frac{4T^*}{m} \frac{1}{k^2} \tag{36}
\]

This form of the phase correlator is typical for the interacting Bose gas at low temperature. However, it results from the diffusion behavior of the excitation spectrum and not from the sound behavior of the spectrum.

The transition to the space coordinates in the 2D case leads to the typical expression for the Bose field correlator at large distances and time moments [26, 27]

\[
\langle \Theta_{r,t} \Theta_{r',t'} \rangle = \rho_0^2 \exp \left( - \frac{4T^* m}{\rho_0^2} \ln \left( \frac{s}{\xi_0} \right) \right) \tag{37}
\]

where \( \xi_0^{-1} = \sqrt{mg\rho_0^2}, s = \sqrt{|r - r'|^2 + D_0 |(t - t')|} \) and \( \xi_0 \) is the effective correlation length.

It should be noted that the behavior of the system in the stationary state differs from the behavior of the equilibrium Bose gas in the Berezinskii-Kosterlitz-Thouless state. In the equilibrium Bose gas there is a local condensate with the coherent phase in the regions which size is much smaller than the average distance between vortices and larger than \( \xi_0 \). The local condensate with the coherent phase in the stationary state of the nonequilibrium Bose system is absent. In this case the phase correlator has a diffusion character on all scales larger than \( \xi_0 \).

### III. CONCLUSION.

In the present paper the low energy properties of the stationary state of nonequilibrium Bose gas have been considered at sufficiently low temperatures \( T^* \ll \mu \). In the stationary state the particle escape and creation compensate each other in average, i.e., the stationary state requires zero value of relaxations \( \Gamma_R (\omega) \) and \( \Gamma_A (\omega) \) for zero frequency \( \omega = 0 \), where \( \Gamma_R (\omega) = \Gamma_R^{(out)} (\omega) - \Gamma_R^{(in)} (\omega) \). However, the quantum noise of the Bose field at zero frequency does not vanish and is characterized by \( \Gamma_K = \Gamma_K^{(out)} (0) + \Gamma_K^{(in)} (0) \), where the
kinetic component of the self-energy part $\Gamma_K$ at zero frequency is a sum of positive values $\Gamma_K^{(out)}(0)$ and $\Gamma_K^{(in)}(0)$. Due to this noise the low energy spectrum of elementary excitations, which is governed by the phase fluctuations of the Bose field, has the diffusion character \cite{34}. The diffusion character of the phase fluctuations means the absence of the phase coherency of the classical Bose field and, therefore, the absence of BEC in the stationary state of nonequilibrium Bose system. The diffusion character of the low energy spectrum of elementary excitations results from nonzero value of the zero frequency component of $\Gamma_K(\omega)$.

Note that the behavior of the module of the Bose field in the stationary nonequilibrium state at small effective temperatures is the same as in the thermodynamically equilibrium state, i.e., module of the Bose field has homogeneous nonzero average value and the field fluctuations are small compared with the average value. Moreover, the momentum behavior of the correlator of the phase fluctuations in the nonequilibrium stationary state is analogous to that in the thermodynamically equilibrium state with the Bose condensate.

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