Search for the $\Delta^{++}$ component in $^{12}$C ground state using $^{12}$C($\gamma, \pi^+p$) reaction

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Abstract

The differential cross section for the reaction $^{12}$C($\gamma, \pi^+p$) has been measured in the $\Delta$ resonance region at high recoil momenta of the residual nuclear system. The data are analysed under the assumption that the formation of $\pi^+p$ pairs may be interpreted as $\gamma\Delta^{++} \to \pi^+p$ process, proceeding on the $\Delta^{++}$ constituent in a target nucleus. As a result, the estimation $N_\Delta = 0.028 \pm 0.008$ deltas per nucleon in $^{12}$C was obtained.

PACS: 13.60 Le; 25.20 Lj;
Keywords: Pion photoproduction; $\Delta$ isobar configuration; coincidence measurement

The interaction of energetic projectile particles with nuclei at high momentum transfer implies probing the nuclear structure at short distances. A wide variety of developments in the short range nucleon-nucleon dynamics concerns the internal nucleon degrees of freedom. The nontrivial substructure of nucleon manifests itself in existence of the internally excited states of nucleon i.e. baryon resonances or isobar. Therefore, in modern nuclear models the conventional nuclear wave function consisting of nucleons only is supplemented by the exotic components, the so-called isobar configurations. The role of nuclear isobar configurations in nuclear reactions involving high momentum transfer is one of the most interesting questions of medium energy physics (see e.g. reviews [1, 2]).

It was found within the frame of different approaches that the strongest isobar admixture in nuclei stems from the $\Delta(1232)$-resonance. Most theoretical and experimental investigations of the $\Delta$-isobar configuration have been done for the deuteron, $^3$He and $^3$H and to a smaller degree for heavier nuclei. At the same time, the amount of virtual deltas for more massive nuclei with their higher density is expected to be more essential than in lighter nuclei. Here we would like to mention the explicit calculations of the $\Delta$ probability in $^4$He and $^{12}$C [3] that give estimates of about 4.5% and 3.2%, respectively. The examinations of the existence of virtual $\Delta$s had little success despite significant experimental efforts. As for the experimental search for the isobar admixture in nuclei with $A > 4$, the observation of $\Delta^{++}$ knock-out from $^9$Be in $p^9$Be collisions [4] and the recent double charge exchange experiment $A(\pi^+, \pi^-)$ on $^{12,13}$C, $^{90}$Zr, $^{208}$Pb [5] present the most interest. The corresponding data analysis point to the probable existence of the virtual $\Delta$s in p-shell nuclei at the level of $1 \div 3\%$. The photo- and electroproduction processes as a test of $\Delta$ admixture in nuclei were considered in Ref. [6]. Laget proposed the combined study of $(e, e'\Delta^{++})$ and $(e, e'\Delta^0)$.
reactions on $^3\text{He}$ \cite{7}. The corresponding investigations are performed now at MAMI and Jefferson Lab. \cite{8, 9}. It is noted in Ref. \cite{10}, that results for the reaction $^3\text{He}(e, e'\pi^\pm)$ obtained at MAMI do not contradict with the assumption of a preformed $\Delta^{++}$. As for the reactions with real photons, only one measurement of $^{12}\text{C}(\gamma, \pi^+p)$ reaction has been reported in Ref. \cite{11}. The estimation $\Delta$ admixture in $^{12}\text{C}$ from these data is difficult since the experimental cross section was averaged over the wide range of the angles of emitted particles.

In this paper we present the experimental results for the differential cross section of the reaction

$$\gamma + ^{12}\text{C} \rightarrow \pi^+p + X.$$ \hspace{1cm} (1)

The measurements are performed in the kinematical region of high momenta transferred to the residual nuclear system $X$. Our investigations are designed to determine the admixture of $\Delta(1232)$-component in the $^{12}\text{C}$ ground state. We analyse the data under the assumption that $\pi^+p$ pairs are the product of the $\gamma\Delta^{++} \rightarrow \pi^+p$ process which proceeds on the $\Delta^{++}$-constituent preexisting in a target nucleus. The method of using reactions of the type (1) also examined in our previous paper \cite{12} has two main advantages. First, since the $\Delta^{++}$ cannot be excited by photons on a single nucleon the interpretation of measurements is free from the difficulty to distinguish the knocked-out $\Delta$s from those created in the reaction. Second, since virtual $\Delta$s are produced in the NN collisions, it is reasonable to expect that the probability for finding $\Delta$ in a nucleus is quadratic in nuclear density. Therefore, the search for the $\Delta$-components in electromagnetic reactions is preferred since the photon beam is able to probe the whole nuclear volume up to the region of highest densities. As a consequence, more direct test of isobar configuration may be carried out. This distinguishes reactions (1) from those using strongly interacting probes, which undergo multiple scattering before reaching central-nuclear densities.

Measurements of the differential cross section of the reaction $^{12}\text{C}(\gamma, \pi^+p)$ were performed at the Tomsk synchrotron. The bremsstrahlung photon beam have been produced by electrons with energy 500 and 420 MeV. The experimental setup is shown in Fig. 1. It includes two channels for detecting the charged pion and the proton in coincidence.

The pions with 181 Mev/c mean momentum were detected by a strongly focusing magnetic spectrometer at the angle 54° with respect to the photon beam. The angular and momentum acceptances were $3 \cdot 10^{-3}$ sr and 24%, respectively. It was determined by the telescope that includes two scintillation counters. The momentum resolution of 1% was available by use of the scintillation hodoscope \cite{13} located in the focal plane of the spectrometer. Proton channel includes $(\Delta E, E)$ - system of the scintillation counters on the polystyrene base and two auxiliary scintillation counters with absorbers intended for the energy calibration and for monitoring the stability of the $(\Delta E, E)$ - system. The solid angle of the proton channel defined by the $\Delta E$-counter was equal to 0.26 sr. The mean polar angle $\theta_p$ with respect to the photon beam and the angular coverage of the proton channel were equal to 75° and ±19°, respectively. The $E$-detector includes three scintillation counters of dimensions $10 \times 10 \times 50$ cm$^3$, which were disposed on the top of one another. In the region of the proton energy $T_p = 40 \div 120$ MeV the analysis of the impulse amplitudes from the $\Delta E$- and $E$-counters allows us to determine the polar angle and the energy of the registered proton with accuracies better than $\sigma_\theta = 3^\circ$ and $\sigma_E = 4$ MeV, respectively. The accuracy of the azimuthal angle measurement determined by the counter dimensions was $\sigma_\phi \sim 2^\circ$. The auxiliary scintillation counters chose narrow-direct beam of the particles. The pions
and protons of the beam with the minimal energy were used for the energy calibration of the proton channel. This minimal energy was determined by the material thickness of the proton channel. We carried out the calibration and the major run simultaneously.

To reduce the cosmic background, the pion channel detectors were covered with the big-area scintillation counter Sa functioning in the anticoincidence condition. With the similar purpose the final trigger was formed only during the accelerator radiation impulse.

The target was a carbon slab of the natural isotope composition $4.35 \cdot 10^{22}$ nuclei/cm$^2$ in thickness. The total energy of the photon beam was measured by the Gauss-quantameter $^{14}$.

In the present experiment 45 events of the $\pi^+p$-coincidences have been yielded. Figure 2 shows the distribution of events as a function of the proton emission angle and the proton energy for two accelerator electron energy.

The events are concentrated in the vicinity of two proton energies 50 MeV and 85 MeV. This effect can readily be seen on the lower panel corresponding to the electron energy $E_e = 420$ MeV. At this energy two $\pi^+p$ production mechanisms may be recognized. The first mechanism is caused by the binary process $\gamma \Delta^{++} \rightarrow \pi^+p$, being the subject of the present investigation. The second one results from the $\pi^o p$- and $\pi^+n$-pairs production with following charge-exchange rescattering of neutral pions and neutrons. It is reasonably to expect, that the events stemming from the $\pi^o p$- and $\pi^+n$-pairs production are mainly grouped in the region of small proton energies. This is due to the fact that the processes of quasifree $\pi^+n$ and $\pi^o p$ production dominate in the region of small momenta of the residual nuclear system. In the kinematical conditions considered in our experiment, this region corresponds to the small proton energies. Therefore, we assume that for the energies $T_p \leq 55$ MeV both mechanisms mentioned above are significant. At the same time, in the region $T_p \geq 70$ MeV where the intensity of the quasifree pion photoproduction decreases in approximately $10^2$ times, the background effects are of minor importance and the registered events must be due mainly to the direct $\pi^+p$ formation on preexisting deltas.

At the energy $E_e = 500$ MeV, one can see the higher density of events corresponding to the proton energies less than 70 MeV. Here the events due to the $\pi^+p$-pairs formation
in the reaction $^{12}$C($\gamma, \pi^+\pi^- p$)$^{11}$B are expected to give some contribution. The estimation of this background has been done in the framework of the quasifree approximation. The corresponding formalism is presented in detail in Ref. [15]. The calculation shows that in the region $T_p \approx 55 \div 70$ MeV the experimental yield is due practically to the two-pion production mechanism, while for the proton energies $T_p \geq 75$ MeV the net effect from this process is estimated to be less than 1%. The above mentioned grounds led us to select for the analysis only the events, which lay within the interval $T_p = 70 \div 110$ MeV.

We write the differential cross section as

$$\frac{d^3\sigma}{dE_p d\Omega_p d\Omega_\pi} = \frac{N_{\pi p} E_{\gamma_{\text{max}}}}{\Delta E_\pi \Delta \Omega_\pi \Delta E_p \Delta \Omega_p W_\gamma t} \int f(E_\gamma) \left| \frac{\partial E_\gamma}{\partial E_\pi} \right| \, dE_\gamma,$$

where $E_\gamma, E_\pi,$ and $E_p$ are the energies of the photon, pion, and proton, respectively; $N_{\pi p}$ is the number of events in the phase space defined by the intervals $\Delta E_\pi, \Delta E_p, \Delta \Omega_\pi, \Delta \Omega_p; W_\gamma$ is the total energy of the photon beam; $t$ is the target thickness; $E_{\gamma_{\text{max}}}$ is the endpoint energy of the bremsstrahlung spectrum $f(E_\gamma)$, which is normalized as

$$\int_0^{E_{\gamma_{\text{max}}}} f(E_\gamma) E_\gamma \, dE_\gamma = E_{\gamma_{\text{max}}}.$$

The kinematical quantities not measured in the reaction were determined by solving the set of kinematical equations under the assumption that the residual nucleus $^{11}$Be is in its ground state.

The cross section was averaged in the intervals $E_\pi = 211 \div 246$ MeV, $\theta_p = 56 \div 94^\circ$, $E_p = 70 \div 110$ MeV. The mean photon energy for the events laying in the kinematical region under consideration was 345 MeV. Within the procedure described above we obtain the following result.
Here letters $\gamma$, the elementary process $\gamma$ respectively. The total energies and momenta of the participating particles are denoted by $E$ where the measurement error is statistical.

Now we recall briefly the method that was used in our calculation. As has been noted, the elementary process $\gamma^{\Delta++} \rightarrow \pi^+ p$ was assumed to be the only reaction mechanism. We employ the impulse approximation and use the closure relation when summing over the states of the residual undetected nuclear system. This gives the differential cross section of the reaction $^{12}\text{C}(\gamma, \pi^+ p)$ in lab system as

$$\frac{d^3\sigma}{dE_p d\Omega_p d\Omega_\pi} = 8.5 \pm 2.5 \frac{nb}{MeV sr^2}, \quad (2)$$

Here letters $\gamma$, $\pi$, $p$, $\Delta$ and $f$ stand for the photon, pion, proton, $\Delta$ and final nucleus respectively. The total energies and momenta of the participating particles are denoted by $E_i$ and $p_i$, $f_\pi$ and $f_\rho$ are the attenuation factors taking into account the absorption of the produced pions and protons in nuclear medium (see e.g. [16]) ; $\lambda$ is a index of the photon polarization; $t_\lambda$ is the elementary $\gamma^{\Delta++} \rightarrow \pi^+ p$ amplitude; the function $\rho_{\Delta++}(\mathbf{p}_\Delta)$ has the meaning of delta momentum distribution in the ground state of initial nucleus.

The elementary $\gamma^{\Delta++} \rightarrow \pi^+ p$ amplitude $t_\lambda$ was obtained within the diagramatic approach. The corresponding formulas are presented in our previous work [12]. Different terms of the amplitude are shown in Fig. 3 (a - e).

We include in the matrix element the new term (3.b) which was missed in the previous analysis [12]. As the calculation shows, this term is of minor importance when embedding the $\gamma^{\Delta++} \rightarrow \pi^+ p$ transition operator into the nuclear process and does not visibly influence our previous results. In the nonrelativistic limit up to the order $(p/M)^2$ the elementary amplitude may be written as

$$t_\lambda = i \sqrt{2} \frac{f_{\pi N \Delta}}{m_\pi} \epsilon \left\{ \frac{2 \mathbf{p}_\Delta \cdot \mathbf{e}_\lambda + i \frac{\mu_{\Delta++}}{3} \mathbf{\sigma}_\Delta \cdot [\mathbf{p}_\gamma \times \mathbf{e}_\lambda]}{E_\Delta + E_\gamma - E_\Delta + i \Gamma_\Delta/2} \right\}$$

$$+ i \frac{f_{\pi N \Delta}}{f_{\pi N \Delta}} \frac{3 \mu_{N \Delta}}{E_\Delta - E_\pi - E_\pi'} \mathbf{S} \cdot [\mathbf{p}_\gamma \times \mathbf{e}_\lambda] \mathbf{\sigma}_\Delta \cdot \mathbf{p}_\pi + 2 \frac{\mathbf{p}_p \cdot \mathbf{e}_\lambda + i \frac{\mu_p}{2} \mathbf{\sigma} \cdot [\mathbf{p}_\gamma \times \mathbf{e}_\lambda]}{E_\Delta - E_\pi - E_\pi'} \mathbf{S} \cdot \mathbf{p}_\pi$$

$$- \frac{4 M_p}{t - m_\pi^2} \mathbf{S} \cdot (\mathbf{p}_\pi - \mathbf{p}_\gamma) \mathbf{p}_\pi \cdot \mathbf{e}_\lambda - 2 M_p \mathbf{S} \cdot \mathbf{e}_\lambda \right\}$$

For the hadronic coupling constants we employ $f_{\pi N \Delta}^2/4\pi = 0.37$ from decay $\Delta \rightarrow \pi N$ and $f_{\pi \Delta} = 4/5 f_{\pi NN}$, as predicted by the trivial quark model. The magnetic moments used in the calculation are $\mu_p = 2.79$, $\mu_{\Delta++} = 4.3$ and $\mu_{N \Delta} = 3.24$ in terms of nuclear magnetons. The variables $E'_\rho = M_p + (\mathbf{p}_\Delta - \mathbf{p}_\pi)^2/2M_p$, $E'_\Delta = M_\Delta + (\mathbf{p}_\Delta - \mathbf{p}_\pi)^2/2M_\Delta$ and $E'_\Delta = M_\Delta + (\mathbf{p}_\Delta + \mathbf{p}_\pi)^2/2M_\Delta$ are the energies of the intermediate nucleon and deltas; $\mathbf{p}_{\pi \rho} = (\mathbf{p}_\pi M_p - \mathbf{p}_\rho E_\pi)/(M_p + E_\pi)$ and $\mathbf{p}_{\pi \Delta} = (\mathbf{p}_\gamma M_\Delta - (\mathbf{p}_\Delta - \mathbf{p}_\pi) E_\gamma)/(M_\Delta + E_\gamma)$ are the relative momenta in the $\pi p$ and $\gamma \Delta$ systems; $\mathbf{S}$ is transition operator between the states with spin.
Figure 3: Diagrams for the $\gamma \Delta^{++} \to \pi^+ p$ amplitude used in the present calculation: (a) s-channel term, (b), (c) u-channel terms, (d) pion pole term, (e) seagull term.

$\gamma(E_\gamma, p_\gamma) \quad \pi^+(E_\pi, p_\pi) \quad \Delta^{++}(E_\Delta, p_\Delta) \quad p(E_p, p_p)$

3/2 and 1/2; $\sigma_\Delta$ is the analog of the Pauli spin matrix for the spin $3/2$ object (concrete representations of $S$ and $\sigma_\Delta$ are given e.g. in [2]).

The delta momentum distribution $\rho_{\Delta^{++}}(p)$ obeys the following normalization condition

$$\int \rho_{\Delta^{++}}(p) \frac{dp}{(2\pi)^3} = \frac{AN_\Delta^S}{4}, \quad (3)$$

where $N_\Delta^S$ is the number of deltas per nucleon in the ground state of the nucleus $^{12}\text{C}$ and $A = 12$ is the mass number. This function was calculated as

$$\rho_{\Delta^{++}}(p) = 4 \cdot \frac{4}{3} \pi R^3 \ n_\Delta^S(p), \quad (4)$$

where $R = 3.2$ fm is the square-well radius and $n_\Delta^S(p)$ is the $\Delta$’s occupation number in $^{12}\text{C}$. For lack of available analysis for the function $n_\Delta^S(p)$ (or $\rho_{\Delta^{++}}(p)$) for p-shell nuclei, we use the results of Ref. [17] where the $\Delta$’s occupation number inside the nuclear matter $n_\Delta^m(p)$ is presented. In order to relate $n_\Delta^S(p)$ to $n_\Delta^m(p)$, we assume that the momentum distribution
\( \rho_{\Delta^+}(p) \) of deltas in \(^{12}\)C is proportional to that for the nuclear matter. This leads to

\[
n^\Delta_c(p) = \frac{N^\Delta_c}{N^\Delta_m} \left( \frac{r^m_0}{r^c_0} \right)^3 n^m_\Delta(p). \tag{5}
\]

In the actual calculation we use the following radial parameters \((r_0 = RA^{-1/3})\) \[18\]

\[
r^m_0 = 1.12 \text{ fm}, \quad r^c_0 = 1.4 \text{ fm}. \tag{6}
\]

Treating \(N^\Delta_\Delta\) as a free parameter we determine its value by fitting our calculation to the experimental result \([2]\). Taking \(N^m_\Delta = 0.07\) from \([17]\) this approach gives

\[
N^\Delta_\Delta = 0.028 \pm 0.008 \text{ deltas per nucleon}, \tag{7}
\]

where indicated uncertainty is statistical only. This value is in rough agreement with the results obtained by other authors for p-shell nuclei \([4, 5]\).

The differential cross section as function of the energy of the proton \(E_p\) received from data for \(E_e = 500 \text{ MeV}\) and \(E_e = 420 \text{ MeV}\) as mean values in the intervals \(\Delta \theta_p = 56^0 \div 94^0\), \(\Delta E_\pi = 211 \div 246 \text{ MeV}\) are given in Fig.4. The differential cross section calculated in our model with \(N^{^{12}C}_\Delta = 0.028\) are also shown.

![Figure 4](image)

Figure 4: The differential cross section of the the \(^{12}C(\gamma, \pi^+p)\) reaction at mean \(E_\gamma = 340 \text{ MeV}\) as a function of the kinetic energy \(T_p\) for the intervals \(\Theta_p = 56^0 \div 94^0\), \(E_\pi = 211 \div 246 \text{ MeV}\). The solid curve is the result of the our the calculations.

It must be kept in mind that our quantitative conclusion depends strongly on the model adopted for the \(\Delta\) dynamics in nuclei. This question is also briefly considered in \([12]\). Thus it is highly desirable to have more realistic model for the delta momentum distribution in \(^{12}\)C.

This work was supported by the Russian Foundation for Basic Research under the Contracts No. 96-02-16742, No. 97-02-17765, and No. 99-02-16964.
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