HQET and Exclusive $B$ Decays

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Abstract

Exclusive semileptonic $B$ decays are discussed. The emphasis is on using semileptonic decays to determine $|V_{ub}|$ and $|V_{cb}|$. Recent progress in our understanding of $B$ semileptonic decays to excited charmed mesons is also reviewed.

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1 Introduction

In the area of exclusive $B$ decay most applications of heavy quark effective theory (HQET) have been to semileptonic decays, which is the topic I will concentrate on in this lecture. In Section II, I review the determination of $|V_{cb}|$ from exclusive $B \rightarrow D^* \ell \bar{\nu}_\ell$ decay. Section III reviews recent theoretical progress in our understanding of $B$ semileptonic decay to the doublet of excited charmed mesons with spin of the light degrees of freedom, $s_\ell = \frac{3}{2}$, and positive parity. Finally, in Section IV prospects for determining $|V_{ub}|$ from data on exclusive semileptonic $B$ (and $D$) decays are discussed. Other promising methods for determining $|V_{cb}|$ and $|V_{ub}|$ from $B$ decays involve using the operator product expansion (OPE) to predict inclusive decay rates and lattice QCD to predict exclusive decay matrix elements. However, these techniques will not be discussed in this lecture.

2 $|V_{cb}|$ and Exclusive $B \rightarrow D^* \ell \bar{\nu}_\ell$ Decay

In the near future much of the high energy experimental program will be devoted to testing whether the KM phase is responsible for the CP violation we observe in nature. This program revolves around using $B$-decays to determine the sides and angles of the unitarity triangle in as many ways as possible and checking for inconsistencies in the results. Since the deviations from the standard model may not be large, a precise determination of the angles and sides of the triangle is desirable.

At the present time CP nonconservation has only been observed in kaon decays. In the standard model it arises from second order weak $K^0 - \bar{K}^0$ mixing. The CKM elements that occur in the box diagram with a top quark in the loop are $(V_{td}^* V_{ts})^2 = (\rho - 1 - i\eta)^2|V_{cb}|^4|V_{us}|^2$. (With the usual conventions $\rho + i\eta$ are the coordinates in the complex plane of the vertex of the unitarity triangle that doesn’t lie on the real axis.) Therefore, $|V_{cb}|$ must be known very accurately if the measured value of the CP violation parameter $\varepsilon$ is to be compared with theory.
The semileptonic form factors that occur in $B \to D^{(*)}e\bar{\nu}_e$ decay are defined by
\begin{align}
\langle D(v')|\bar{c}\gamma^\mu b|B(v)\rangle &= h_+(w)(v + v')^\mu + h_-(w)(v - v')^\mu, \\
\langle D^*(v', \varepsilon)|\bar{c}\gamma^\mu b|B(v)\rangle &= ih_V(w)\varepsilon^\nu\varepsilon'^\alpha\varepsilon'_\mu v_\alpha v_\beta, \\
\langle D^*(v', \varepsilon)|\bar{c}\gamma^\mu\gamma_5 b|B(v)\rangle &= h_{A_1}(w)(w + 1)\varepsilon^\mu - h_{A_2}(w)(\varepsilon^* \cdot v)v^\mu - h_{A_3}(w)(\varepsilon^* \cdot v)v'^\mu.
\end{align}

In eqs. (2.1), (2.2) and (2.3) the $h_j$ are Lorentz scalar form factors that are functions of the dot product of the $B$ and $D^*$ four-velocities $w = v \cdot v'$. In the rest frame of the $B$ meson $w = E_{D^{(*)}}/m_{D^{(*)}}$ is the $\gamma$ factor for the recoiling $D^{(*)}$. At zero recoil $w = 1$.

Heavy quark spin symmetry implies that for $m_{c,b} \to \infty$ the form factors are given by [2]: $h_-(w) = h_{A_2}(w) = 0$ and $h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w)$. At the kinematic point $w = 1$ the QCD operator $\bar{c}\gamma_\mu b$ matches onto a generator of the flavor-spin symmetry in HQET [3]. Since the matrix elements of the symmetry generators are known, heavy quark symmetry implies the normalization condition [3, 4]
\begin{equation}
\xi(1) = 1.
\end{equation}

There are perturbative corrections to these predictions suppressed by $\alpha_s(m_{c,b})/\pi$ [5, 6, 7] and nonperturbative corrections suppressed by $\Lambda_{QCD}/m_{c,b}$ [8, 9]. The perturbative corrections are calculable and do not cause a loss of predictive power.

The differential decay rate for $B \to D^*e\bar{\nu}_e$ is
\begin{equation}
\frac{d\Gamma}{dw}(B \to D^*\ell\bar{\nu}_e) = \frac{G_F^2m_B^5r^3(1 - r)^2(w^2 - 1)^{1/2}(w + 1)^2}{192\pi^3} \\
\times \left[1 + \frac{4w}{w + 1} - \frac{1 - 2wr + r^2}{(1 - r)^2}\right] |V_{cb}|^2|\mathcal{F}_{B \to D^*}(w)|^2.
\end{equation}

where $r = m_{D^*}/m_B$. $\mathcal{F}_{B \to D^*}(w)$ can be expressed in terms of the form factors $h_j(w)$ and in the $m_{c,b} \to \infty$ limit $\mathcal{F}_{B \to D^*}(w) = \xi(w)$. The known normalization at $w = 1$ allows an extraction of $|V_{cb}|$ from an extrapolation of data on this decay to the zero recoil kinematic point.

The structure of the symmetry breaking corrections to $\mathcal{F}_{B \to D^*}(1)$ is
\begin{equation}
\mathcal{F}_{B \to D^*}(1) = 1 + \delta\eta_A(\alpha_s) + 0 + \delta_1/m_{c,b} + \ldots = 0.91 \pm 0.05.
\end{equation}

In the above $\delta\eta_A(\alpha_s)$ is the perturbative QCD correction. It is known to order $\alpha_s^2$ and has the value $-0.04$ [10]. There is no correction of order $\Lambda_{QCD}/m_{c,b}$ and hence the zero for the
third term in eq. (2.6). This result is known as Luke’s theorem [9]. The terms of order \( (\Lambda_{\text{QCD}}/m_{c,b})^2 \) and higher are estimated by models [11, 12, 13] to have the value \(-0.05\). Since this is a model dependent contribution I assign it a 100% theoretical uncertainty and this is the source of the theoretical error on the right hand side of eq. (2.6). The experimental data gives [14, 15] \(|V_{cb}| = (35.2 \pm 1.4) \times 10^{-3}\) which when combined with eq. (2.6) yields

\[ |V_{cb}| = (38.6 \pm 2.3_{\text{exp}} \pm 2_{\text{th}}) \times 10^{-3}. \] (2.7)

Part of the nonperturbative corrections to \(F_{B \rightarrow D^*}(1)\) are calculable in a model independent way. This part has a nonanalytic dependence on the light quark masses and takes the form [12]

\[ \delta_{1/m_c^2} + \ldots = \frac{g^2\Delta^2}{(4\pi f_\pi)^2} Y(\Delta/m_\pi), \] (2.8)

where \(Y\) is a known function of \(\Delta = m_{D^*} - m_D\) divided by the pion mass. (For simplicity of presentation in eq. (2.8) I have taken the limit \(m_b \to \infty\) and only kept nonperturbative corrections suppressed by powers of \(1/m_c\).) In eq. (2.8) \(g\) is the \(D^*D\pi\) coupling and \(f_\pi\) is the pion decay constant. The coupling \(g\) also occurs in the \(D^*\) decay width

\[ \Gamma(D^{*+} \rightarrow D^{0}\pi^+) = \frac{1}{6\pi} \frac{g^2}{f_\pi^2} |\vec{p}_\pi|^3 \simeq 0.2g^2 \text{ MeV}. \] (2.9)

The branching ratio for \(D^{*+} \rightarrow D^{0}\pi^+\) is 68.3%. A measurement of the \(D^*\) width would fix \(g\) and reduce somewhat the theoretical uncertainty in eq. (2.6). Our expectation, based on the chiral quark model [18] is that \(g^2 \sim 1/2\), but the uncertainty in this estimate is very large.

Part of the experimental error in the determination of \(|V_{cb}|\) arises from the extrapolation to zero recoil. This can be reduced by using a parametrization for the \(w\) dependence of \(F_{B \rightarrow D^*}(w)\) that is constrained by dispersion relations and perturbative QCD [16, 17].

The error estimate associated with theory in eq. (2.7) is rather adhoc. Of course there is not really a correct way to assign an error from theory. What one needs is another method for determining \(|V_{cb}|\). The consistency between it and the exclusive method then provides a measure of the theoretical uncertainties. Fortunately such a method exists which uses the inclusive \(B\) semileptonic decay rate [14]. At the present time this approach has a somewhat smaller experimental uncertainty. The theoretical uncertainty associated with the inclusive technique is also expected to be small. However, uncertainties from possible violations of
quark-hadron duality that are not apparent at low orders in the OPE are very difficult to estimate. The inclusive way of determining $|V_{cb}|$ yields a value close to that in eq. (2.7) indicating that theoretical uncertainties in both of these methods are indeed less than 5% .

3 $B$ Semileptonic decay to Excited Charm Mesons

The members of the excited $s_i^7 = \frac{3}{2}^+$ doublet have been observed. They are the $D_1(2420)$ and $D_2^*(2460)$. Recently there have been measurements of the $B$ branching ratio to $D_1 e \bar{\nu}_e$ and a limit on the branching ratio to the $D_2^* e \bar{\nu}_e$ final state \cite{19, 20},

$$Br(B^+ \to D_1^0 e^- \bar{\nu}_e) = (0.56 \pm 0.14)\%, \quad (3.1)$$
$$Br(B^+ \to D_2^* e^- \bar{\nu}_e) \leq 0.8\%. \quad (3.2)$$

The branching ratios to the ground state doublet, $Br(B \to D^0 e \bar{\nu}_e) = 1.8 \pm 0.4\%$ and $Br(B \to D^* e \bar{\nu}_e) = 4.6 \pm 0.3\%$, indicate that about 35% of $B$ semileptonic decays are to excited mesons and nonresonant final states.

In terms of Lorentz scalar form factors the matrix elements of the weak vector and axial vector form factors are

$$\langle D_1 (v', \varepsilon)|\bar{c}\gamma^\mu b|B(v)\rangle = f_V \varepsilon^* \mu + (f_{V_2} \varepsilon^* \mu + f_{V_3} \varepsilon^* \nu)(\varepsilon^* \cdot v), \quad (3.3)$$
$$\langle D_1 (v', \varepsilon)|\bar{c}\gamma^\mu \gamma^5 b|B(v)\rangle = i f_{A_1} \varepsilon^* \alpha \gamma \varepsilon^* \beta \gamma', \quad (3.4)$$
$$\langle D_2^* (v', \varepsilon)|\bar{c}\gamma^\mu \gamma^5 b|B(v)\rangle = k_{A_1} \varepsilon^* \alpha \gamma \varepsilon^* \beta \gamma' + (k_{A_2} \varepsilon^* \alpha \gamma + k_{A_3} \varepsilon^* \beta \gamma) \varepsilon^* \alpha \gamma \varepsilon^* \beta \gamma', \quad (3.5)$$
$$\langle D_2^* (v', \varepsilon)|\bar{c}\gamma^\mu \gamma^5 b|B(v)\rangle = i k_V \varepsilon^* \alpha \gamma \varepsilon^* \beta \gamma \varepsilon^* \alpha \gamma \varepsilon^* \beta \gamma'. \quad (3.6)$$

The form factors $f_j$ and $k_j$ are functions of $w$. Note that the $B \to D_2^*$ zero recoil matrix elements of the vector and axial vector currents vanish by Lorentz invariance, independent of the values of $k_j(1)$. However, the $B \to D_1$ zero recoil matrix element of the vector current is nonzero if $f_{V_1}(1) \neq 0$. Heavy quark symmetry implies that in the $m_{c,b} \to \infty$ limit, $f_{V_1}(1) = 0$. Since most of the phase space is near zero recoil, $1 < w < 1.3$, the $\Lambda_{QCD}/m_{c,b}$ corrections which cause the zero recoil $D_1$ matrix element not to vanish are very important.

In the limit $m_{c,b} \to \infty$ heavy quark spin symmetry implies that the form factors $f_j$ and
$k_j$ can be expressed in terms of a single function of $w$ \[21\].

\[
\begin{align*}
\sqrt{6} f_{A}(w) &= -(w + 1)\tau(w) & k_{V}(w) &= -\tau(w) \\
\sqrt{6} f_{V1}(w) &= (1 - w^2)\tau(w) & k_{A1}(w) &= -(1 + w)\tau(w) \\
\sqrt{6} f_{V2}(w) &= -3\tau(w) & k_{A2}(w) &= 0 \\
\sqrt{6} f_{V3}(w) &= (w - 2)\tau(w) & k_{A3}(w) &= \tau(w)
\end{align*}
\] (3.7)

Note that unlike the $B \to D^{(*)} e\bar{\nu}_e$ case $\tau(1)$ is not fixed by heavy quark symmetry. In the infinite mass limit $f_{V1}(1) = 0$ because of the factor of $(1 - w^2)$ in eq. (3.7).

An analysis of the $\Lambda_{QCD}/m_{c,b}$ corrections gives \[22\]

\[
\sqrt{6} f_{V1} = -4(\bar{\Lambda}^* - \bar{\Lambda})\tau(1)/m_c, \tag{3.8}
\]

where $\bar{\Lambda}$ is the mass of the light degrees of freedom in a member of the $s^\pi_L = \frac{1}{2}^-$ doublet and $\bar{\Lambda}^*$ is the mass of the light degrees of freedom in a member of the $s^\pi_L = \frac{3}{2}^+$ doublet. The difference $\bar{\Lambda}^* - \bar{\Lambda}$ can be expressed in terms of known hadron masses yielding $\bar{\Lambda}^* - \Lambda \simeq 0.39$GeV. This is the most important $\Lambda_{QCD}/m_{c,b}$ correction because it is the only one at zero recoil.

Away from zero recoil other $\Lambda_{QCD}/m_{c,b}$ corrections arise and some model dependence occurs. In the infinite mass limit $R = Br(B \to D_s^* e\bar{\nu}_e)/Br(B \to D_1 e\bar{\nu}_e) = 1.65$ and the measured value of the $B \to D_1 e\bar{\nu}_e$ decay rate implies $|\tau(1)| = 1.27$. Including the $\Lambda_{QCD}/m_{c,b}$ corrections changes these results to \[22\] (see also \[23\]) $R \simeq 0.6$ and $|\tau(1)| \simeq 0.7$. (For these predictions $\tau'(1)/\tau(1) = -1.5$ and $\tau(w) = \tau(1) + \tau'(1)(w - 1)$ were used.) The $\Lambda_{QCD}/m_{c,b}$ corrections lead to the expectation that $R < 1$, which is opposite from what the infinite mass limit gives.

4 \hspace{1cm} |V_{ub}| \hspace{1cm} From Exclusive $B$ Decay

Recently branching ratios for $B \to \pi e\bar{\nu}_e$ and $B \to \rho e\bar{\nu}_e$ have been measured \[24\]. One of the original applications of heavy quark symmetry was to take the measured $D \to K^* e\bar{\nu}_e$ form factors and use $SU(3)$ (light quark) flavor symmetry and heavy quark symmetry to get $B \to \rho e\bar{\nu}_e$ form factors \[25\]. For such decays the form factors are defined by

\[
\begin{align*}
\langle V(p', \varepsilon) | \bar{q} \gamma_{\mu} Q | H(p) \rangle &= i g^{(H \to V)} \varepsilon_{\mu \lambda \sigma} \varepsilon^{* \nu}(p + p')^\lambda(p - p')^\sigma, \\
\langle V(p', \varepsilon) | \bar{q} \gamma_{\mu} \gamma_5 Q | H(p) \rangle &= f^{(H \to V)} \varepsilon^{* \mu} + a_+^{(H \to V)}(\varepsilon^* \cdot p)(p + p')_\mu + a_-^{(H \to V)}(\varepsilon^* \cdot p)(p - p')_\mu.
\end{align*}
\] (4.1)

It is convenient to view the form factors $f$, $g$ and $a_{\pm}$ as functions of $y = v \cdot v'$ where $p = m_H v$ and $p' = m_V v'$. Then $q^2 = m^2_H + m^2_V - 2m_H m_V y$. In this section $y$ is used for $v \cdot v'$ (instead
of \( w \) as a reminder that the light up down and strange quarks are not treated as heavy. Assuming pole dominance for the form factors the \( D \to K^* \bar{v}_e \) data gives \[26\]

\[
\begin{align*}
    f^{(D\to K^*)}(y) &= \frac{(1.9 \pm 0.1)\text{GeV}}{1 + 0.63(y - 1)}, \\
    a_+^{(D\to K^*)}(y) &= -\frac{(0.18 \pm 0.03)\text{GeV}^{-1}}{1 + 0.63(y - 1)}, \\
    g^{(D\to K^*)}(y) &= -\frac{(0.49 \pm 0.04)\text{GeV}^{-1}}{1 + 0.96(y - 1)}.
\end{align*}
\]

These form factors are measured only over the kinematic region \( 1 < y < 1.3 \). Over this range \( f^{(D\to K^*)} \) changes by less than 20%. However, the full kinematic range for \( B \to \rho \bar{v}_e \) is much larger, \( 1 < y < 3.5 \).

The differential decay rate for \( B \to \rho \bar{v}_e \) is

\[
\frac{d\Gamma(B \to \rho \bar{v}_e)}{dy} = \frac{G_F^2 |V_{ub}|^2}{48\pi^3 m_B m_B^2} S^{(B \to \rho)}(y),
\]

where

\[
S^{(H\to V)}(y) = \sqrt{y^2 - 1} \left[ f^{(H\to V)}(y) \right]^2 (2 + y^2 - 6yr + 3r^2) \\
+ 4 \text{Re} \left[ a_+^{(H\to V)}(y) f^{(H\to V)^*}(y) \right] m_H^2 r (y - r)(y^2 - 1) \\
+ 4 \left| a_+^{(H\to V)}(y) \right|^2 m_H^4 r^2 (y^2 - 1)^2 + 8 \left| g^{(H\to V)}(y) \right|^2 m_H^4 r^2 (1 + r^2 - 2yr)(y^2 - 1) \right]
= \sqrt{y^2 - 1} \left| f^{(H\to V)}(y) \right|^2 (2 + y^2 - 6yr + 3r^2) [1 + \delta^{(H\to V)}(y)],
\]

with \( r = m_V / m_H \). The function \( \delta^{(H\to V)} \) depends on the ratios of form factors \( a_+^{(H\to V)} / f^{(H\to V)} \) and \( g^{(H\to V)} / f^{(H\to V)} \). \( S^{(B \to \rho)}(y) \) can be estimated using combinations of \( SU(3) \) flavor symmetry and heavy quark symmetry. \( SU(3) \) symmetry implies that the \( B^0 \to \rho^+ \) form factors are equal to the \( B \to K^* \) form factors and the \( B^- \to \rho^0 \) form factors are equal to \( 1 / \sqrt{2} \) times the \( B \to K^* \) form factors. Heavy quark symmetry implies the relations

\[
\begin{align*}
    f^{(B\to K^*)}(y) &= \left( \frac{m_B}{m_D} \right)^{1/2} f^{(D\to K^*)}(y), \\
    a_+^{(B\to K^*)}(y) &= \left( \frac{m_D}{m_B} \right)^{1/2} a_+^{(D\to K^*)}(y), \\
    g^{(B\to K^*)}(y) &= \left( \frac{m_D}{m_B} \right)^{1/2} g^{(D\to K^*)}(y). \tag{4.5}
\end{align*}
\]

The second relation is obtained using \( a_-^{(D\to K^*)} = -a_+^{(D\to K^*)} \), valid in the large \( m_c \) limit.
Using eq. (4.5) and SU(3) to get the $B^0 \to \rho^+ \ell \bar{\nu}_\ell$ form factors (in the region $1 < y < 1.5$) from those for $D \to K^* \ell \nu_\ell$ given in eq. (19) yields $S^{(B\to\rho)}(y)$ plotted in Fig. 1 of Ref. [27]. The numerical values in eq. (19) differ slightly from those used in Ref. [27]. This makes only a small difference in $S^{(B\to\rho)}$, but changes $\delta^{(B\to\rho)}$ more significantly. In the region $1 < y < 1.5$, $|\delta^{(B\to\rho)}(y)|$ defined in eq. (4.4) is less than 0.06, indicating that $a_+^{(B\to\rho)}$ and $g^{(B\to\rho)}$ make a small contribution to the differential rate in this region.

This prediction for $S^{(B\to\rho)}$ can be used to determine $|V_{ub}|$ from a measurement of the $B \to \rho \ell \bar{\nu}_\ell$ semileptonic decay rate in the region $1 < y < 1.5$. This method is model independent, but cannot be expected to yield a very accurate value of $|V_{ub}|$. Typical SU(3) violations are at the $10 - 20\%$ level and one expects similar violations of heavy quark symmetry.

Ref. [27] proposed a method for getting a value of $S^{(B\to\rho)}(y)$ with small theoretical uncertainty. They noted that the “Grinstein-type” [28] double ratio

$$R(y) = \left[ \frac{f^{(B\to\rho)}(y)}{f^{(B\to K^*)}(y)} \right] / \left[ \frac{f^{(D\to\rho)}(y)}{f^{(D\to K^*)}(y)} \right]$$

(4.6)

is unity in the limit of SU(3) symmetry or in the limit of heavy quark symmetry. Corrections to the prediction $R(y) = 1$ are therefore suppressed by $m_s/m_{c,b}$ ($m_{u,d} \ll m_s$) instead of $m_s/\Lambda_{QCD}$ or $\Lambda_{QCD}/m_{c,b}$. A recent estimate of $R(1)$ using chiral perturbation theory gives [29] $R(1) = 1 - 0.035gg_2$ where $g_2$ is the $\rho \omega \pi$ coupling. Experimental information on $\tau \to \omega \pi \nu_\tau$ decay yields $g_2 \simeq 0.6$ [30].

Since $R(y)$ is very close to unity, the relation

$$S^{(B\to\rho)}(y) = S^{(B\to K^*)}(y) \left| \frac{f^{(D\to\rho)}(y)}{f^{(D\to K^*)}(y)} \right|^2 \left( \frac{m_B - m_{\rho}}{m_B - m_{K^*}} \right)^2$$

(4.7)

together with measurements of $|f^{(D\to K^*)}|$, $|f^{(D\to\rho)}|$, and $S^{(B\to K^*)}$ will determine $S^{(B\to\rho)}$ with small theoretical uncertainty. The last term on the right-hand-side makes eq. (4.7) equivalent to eq. (4.6) in the $y \to 1$ limit. The ratio of the $(2 + y^2 - 6yr + 3r^2) [1 + \delta^{(B\to V)}(y)]$ terms makes only a small and almost $y$-independent contribution to $S^{(B\to\rho)}/S^{(B\to K^*)}$ in the range $1 < y < 1.5$. Therefore, corrections to eq. (4.7) are at most a few percent larger than the deviation of $R(y)$ from unity.

$|f^{(D\to K^*)}|$ has already been determined. $|f^{(D\to\rho)}|$ may be obtainable in the future, for example from experiments at $B$ factories, where improvements in particle identification help reduce the background from the Cabibbo allowed decay. The measurement [31] $Br(D \to \rho^0 \ell \bar{\nu}_\ell)/Br(D \to \bar{K}^{*0} \ell \bar{\nu}_\ell) = 0.047 \pm 0.013$ already suggests that $|f^{(D\to\rho)}/f^{(D\to K^*)}|$ is close
to unity. Assuming $SU(3)$ symmetry for the form factors, but keeping the explicit $m_V$-dependence in $S^{(D\rightarrow V)}(y)$ and in the limits of the $y$ integration, the measured form factors in eq. (19) imply $Br(D \rightarrow \rho^0 \ell \nu_\ell)/Br(D \rightarrow \bar{K}^0 \ell \nu_\ell) = 0.044$.

$S^{(B\rightarrow K^*)}$ is obtainable from experimental data on $B \rightarrow K^* \nu_\ell \bar{\nu}_\ell$ or $B \rightarrow K^* \ell \bar{\ell}$. While the former process is very clean theoretically, it is very difficult experimentally. A more realistic goal is to use $B \rightarrow K^* \ell \bar{\ell}$, since CDF expects to observe $400 - 1100$ events in the Tevatron run II (if the branching ratio is in the standard model range). There are some uncertainties associated with long distance nonperturbative strong interaction physics in this extraction of $S^{(B\rightarrow K^*)}(y)$. But on average over $1 < y < 1.5$ this is probably less than a 10% effect [29]. Consequently a determination of $|V_{ub}|$ using this method with a theoretical uncertainty less than 10% seems feasible. Like the situation with $|V_{ub}|$ other methods, for example from the inclusive hadronic mass distribution [32] or from predictions for the form factors from lattice QCD [33] will be necessary to have confidence that the theoretical uncertainty is indeed this small.

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