Optimal organization of multi-profile production

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Abstract. The effectiveness of the use of modern production equipment to a large extent depends on its workload. However, it is impossible to manufacture only one type of product; therefore, production becomes multi-profile, the role of coordinating it with the market increases. Both the market and the production process are stochastic, so the probabilistic agreement of these two processes on the basis of the Kolmogorov equation in the canonical form is considered. The level of coordination is estimated by the difference in the probability density that the market price for a given product exceeds the level of its production cost using the technology implemented in multi-production. The latter probability density assesses the readiness of technologies for implementation, including the availability of raw materials, materials, energy, serviceable equipment and devices — all of this consumes control resources. The optimal allocation of these control resources in the preparation of technologies can significantly improve production efficiency. Of practical significance is the revealed fact that the dependence of control actions on time is exponential: the value of organizational control resources decreases as it moves away from the moment when demand for this type of product arises. Dependence on demand is logarithmic and the value of organizational control resources increases with increasing demand for this type of product.

1. Introduction

Multi-profile production, in general, is intended for the implementation of a number of manufacturing techniques for products of the selected range. It is usually managed according to a predetermined schedule [1,2]. For the formation of prices and sales, the company monitors very carefully [3-5]. Obviously, both the pricing process on the market and the volume of production at the enterprise due to the influence of a large number of random factors are stochastic in nature. On the other hand, the technologies implemented by production should be maintained at a given level of readiness, which is also stochastically evaluated. The readiness of technology for implementation, in addition to the readiness of the control program, provides for the availability of raw materials, materials, energy, serviceable equipment and devices — all of this consumes control resources. It is clear that the even distribution of these resources across all the technologies used is not the best solution — the funds should be invested cumulatively, in accordance with the market demand. Naturally, in order to increase efficiency, multi-profile production it is necessary to produce exactly the products that are in demand on the market at a given time and in the required volumes. However, if the market requires more than the enterprise can produce, the task of choosing the most profitable products arises. In this connection, we investigate the problem of optimal control of probabilistic matching of two processes — market and production, with the aim of maximizing the economic efficiency of multi-profile production.
2. Theory

Probabilistic description of market and production processes is found in the literature quite often [6-10]. In this paper, the managerial task is posed: to increase the efficiency of multi-profile production, primarily by increasing the equipment load factor [11,12] by making the products in demand on the market, based on the probabilistic agreement of the two above processes.

Let the probability of exceeding the market price for the i-th type of production cost of its production using this technology is of Markov nature, and is described by the Kolmogorov equation [13], presented here in canonical form:

\[
\frac{\partial \omega_{1i}}{\partial t} = b \frac{\partial^2 \omega_{1i}}{\partial x^2}
\]  

(1)

where \( \omega_{1i} \) is the density of the probability described above, depending on the market demand for products \( x \) and time \( t \); \( b \) is the diffusion coefficient, which determines here the rate of withdrawal of products from the zone of exceeding the price of products, respectively, according to its cost. This equation can be solved using the source function [14].

The readiness of the \( i \)-th technology that creates these products should be addressed by an organizational control action similar to (1). Let \( u_i(x,t) \) be the number of shares of control resources directed to the organization of readiness of the \( i \)-th technology. Denoting the probability density of the availability of \( i \)-th technology for the production of \( \omega_{2i} \), we have:

\[
\frac{\partial \omega_{2i}}{\partial t} = b \frac{\partial^2 \omega_{2i}}{\partial x^2} + u_i(x,t)
\]  

(2)

The task of managing the readiness of technology is to ensure that the possibility of producing products as closely as possible corresponding to a favorable period of exceeding market demand over the cost of manufacturing products using this technology. This approach dominates the choice of various management strategies [15, 16].

Subtract from (1) -th equation (2) – th and get:

\[
\frac{\partial(\omega_{1i} - \omega_{2i})}{\partial t} = b \frac{\partial^2(\omega_{1i} - \omega_{2i})}{\partial x^2} - u_i
\]  

(3)

We introduce the designation of probable unavailability of the \( i \)-th technology

\[ s_i = \omega_{1i} - \omega_{2i} \]  

(4)

Then (3) is converted as follows.

\[
\frac{\partial s_i}{\partial t} = b \frac{\partial^2 s_i}{\partial x^2} - u_i
\]  

(5)

Let the production have an assortment of \( n \) types of products and, accordingly, technologies and for each technology \( u_i \), shares of control resources are spent to prepare them for the production of a given product. The general probability of technology availability for production in the first approximation can be defined as the product of the partial probabilities of the readiness of each technology, considering them independent events. The unwillingness of the \( i \)-th technology is reduced after each metered organizational management impact on the value of \( q_i \). Considering the managerial impact independent, the overall change in the readiness of technology for production will be obtained in the form of a work or an exponential function \( q_i^{u_i} \). Then the total probability describing the readiness for the production of products in general will be expressed as

\[
P = \prod_{i=1}^{n} (1 - q_i^{u_i})
\]  

(6)

To simplify the expression (6), we take \( P_i = 1 - q_i \to 1 \). Making the multiplication and getting rid of the values of the second and higher orders of smallness, we obtain the general unpreparedness for production:

\[
Q(\bar{u}) = \sum_{i=1}^{n} q_i^{u_i}
\]  

(7)

where \( \bar{u} = \{ u_1, u_2, ..., u_n \} \) is the vector of shares of control actions.
Similar reasoning leads to a similar formula for the probable unavailability of \( i-th \) technology to market requirements

\[
S(\bar{s}) = \sum_{i=1}^{n} p_i^{s_i}
\]  
(8)

Accordingly, the probabilistic unavailability of \( i-th \) technology to meet the market demand is reduced by \( p_i^{s_i} \), and \( \bar{s} = \{ s_1, s_2, ..., s_n \} \).

The total costs of organizational measures related to improving production readiness are expressed as a linear relationship:

\[
C = C(\bar{u}) = \sum_{i=1}^{n} c_i u_i
\]  
(9)

where \( c_i \) is the cost of a single organizational measure that increases the availability of the \( i-th \) technology.

The control task is set as follows: to find the optimal distribution of shares of control actions \( u_i \) to ensure minimum costs at a given level of unreadiness for market requirements.

To solve the optimal control problem by the Euler – Lagrange method, we compose the Lagrangian:

\[
F(\bar{u}) = \sum_{i=1}^{n} c_i u_i + \sum_{i=1}^{n} \alpha_i p_i^{s_i} + \sum_{i=1}^{n} \psi_i \left( \frac{\partial s_i}{\partial t} - b \frac{\partial^2 s_i}{\partial x^2} + u_i \right) + \sum_{i=1}^{n} \varepsilon_i (\alpha_i p_i^{s_i} - q_i^{u_i})
\]  
(10)

Here, the first two terms make up the integrand function in the functional, the third one requires the satisfaction of the equations of the control object, and the last one fulfills the stated constraints.

To ensure the extremum \( F(\bar{u}) \) we make up the Euler equations for all variables:

\[
\begin{cases}
\frac{\partial F(\bar{u})}{\partial u_i} = c_i - \psi_i c_i - \varepsilon_i q_i^{u_i} lnq_i = 0 \\
\frac{\partial s_i}{\partial t} - b \frac{\partial^2 s_i}{\partial x^2} + u_i = 0 \\
\alpha_i p_i^{s_i} lnq_i - \frac{d \psi_i}{\partial t} = 0 \\
\alpha_i p_i^{s_i} - q_i^{u_i} = 0, \quad i = 1, ..., n
\end{cases}
\]  
(11)

Logarithm of the fourth equation of the system, we get

\[
ln\alpha_i + s_i lnq_i - u_i lnq_i = 0
\]  
(12)

Express from here \( s_i \)

\[
s_i = \frac{lnq_i}{lnp_i} u_i - \frac{ln\alpha_i}{lnp_i}
\]  
(13)

and substitute into the second equation of system (11), we get

\[
\frac{lnq_i}{lnp_i} \frac{\partial u_i}{\partial t} - \frac{b \cdot lnq_i}{lnp_i} \frac{\partial^2 u_i}{\partial x^2} + u_i = 0
\]  
(14)

Apply to the solution of this equation the method of separation of variables

Let be

\[
u = v(t) \cdot w(x)
\]  
(15)

Substitute in (14) and omit the indexes for simplicity

\[
\frac{w(x) \cdot lnq_i}{lnp_i} \frac{\partial v_i}{\partial t} - \frac{b \cdot v(t) \cdot lnq_i}{lnp_i} \frac{\partial^2 w}{\partial x^2} + v(t) \cdot w(x) = 0
\]  
(16)

We divide both sides of the equation by the product (13)

\[
\frac{lnq_i}{v(t) \cdot lnp_i} \frac{\partial v_i}{\partial t} - \frac{b \cdot lnq_i}{w(x) \cdot lnp_i} \frac{\partial^2 w}{\partial x^2} + 1 = 0
\]  
(17)

Get
\[
\frac{\ln q_i}{v(t) \cdot \ln p_i} \frac{\partial v_i}{\partial t} - \frac{b \cdot \ln q_i}{w(x) \cdot \ln p_i} \frac{\partial^2 w}{\partial x^2} - 1 = M
\]

This chained equality is transformed into a system of two equations
\[
\begin{align*}
\frac{\partial v(t)}{\partial t} - \frac{M \cdot \ln p_i}{\ln q_i} v(t) &= 0 \\
\frac{\partial^2 w(x)}{\partial x^2} - \frac{(M + 1) \ln p_i}{b \cdot \ln q_i} w(x) &= 0
\end{align*}
\]

The solution of the equations obtained is found in the reference book [17]
\[
\begin{align*}
v(t) &= A_4 e^{\frac{M \cdot \ln p_i}{\ln q_i}} \\
w(x) &= A_2 \cosh \left( x \sqrt{\frac{(M + 1) \ln p_i}{b \cdot \ln q_i}} \right) + A_3 \sinh \left( x \sqrt{\frac{(M + 1) \ln p_i}{b \cdot \ln q_i}} \right)
\end{align*}
\]

Since at \( x = 0 \) (zero demand for products) there is no point in wasting control resources, that is, \( u = 0 \), in the second equation only the second term remains
\[
u(x, t) = A_4 e^{\frac{M \cdot \ln p_i}{\ln q_i}} \sinh \left( x \sqrt{\frac{(M + 1) \ln p_i}{b \cdot \ln q_i}} \right)
\]

When finding the integration constant and the constant \( M \), we take into account the fact that the control starts at a certain demand that has already arisen \( x_0 \), then
\[
u(x_0, 0) = u_0
\]

Or
\[
u_0 = \sinh \left( x_0 \sqrt{\frac{(M + 1) \ln p_i}{b \cdot \ln q_i}} \right)
\]

From here
\[
M = \frac{b \cdot \ln q_i}{\ln p_i} \left( \text{arsh}(u_0) \right)^2 - 1
\]

3. Data and Method

For a visual representation of the dependence of the control action that optimizes the distribution of resources directed at improving the availability of technology, we will construct a schedule. From the above theory, we have
\[
u(x, t) = A_4 e^{\frac{M \cdot \ln p_i}{\ln q_i}} \sinh \left( x \frac{\text{arsh}(u_0)}{x_0} \right)
\]

The specified values of the coefficients for the construction of graphs are summarized in Table 1. Here, simple values of quantities are chosen in order to bring to the fore the dependence on time and on the magnitude of the demand for this type of product.

| Reduction share of probable unavailability of \( i \)-th technology, \( q_i \) | Reduction share of probable unavailability of \( i \)-th technology, \( p_i' \) | Probability diffusion coefficient, \( b \) | Initial value of control resources, \( u_0 \) | Initial demand for products, \( x_0 \) | Constant integration, \( A_4 \) |
|---|---|---|---|---|---|
| 0.01 | 0.01 | 1 | 1 | 1 | 6.2 |
4. Results and discussion

Figure 1 shows the graph of the control actions separately: an exponential graph (first the abscissa axis is upper in figure 1) versus time and a logarithmic graph (the lower one is first abscissa axis in figure 1).

![Graph showing exponential and logarithmic graphs](image)

**Figure 1.** Change of control components (green - time dependence, red - demand level dependence).

Here, the control actions are deposited on the y-axis in fractions (dimensionless), on the abscissa - time in days and demand for in the quantity of the product (piece). As we see, the dependence of control actions on time is exponential: the value of organizational control resources decreases as the distance from the moment when demand for this type of product arises. Dependence on demand is logarithmic and the value of organizational control resources increases with increasing demand for products.

On the basis of the revealed dependence, a method of optimal redistribution of control resources is easily developed, which should remove resources in time exponentially from one product profile and transfer them to another profile for which demand has arisen at the moment, and coordinate the value of these resources with a logarithm of the amount of demand.

5. Conclusion

Thus, the optimal distribution of organizational control resources in the preparation of technologies implemented by multi-profile production in accordance with the requirements of the market can significantly improve its efficiency. Of practical significance is the fact that the dependence of control actions on time is exponential: the value of organizational control resources decreases as the distance from the moment when demand for this type of product occurs. Dependence on demand is logarithmic and the value of organizational control resources increases with increasing demand for this product profile.

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