Heavy neutrino mixing and single production at Linear Collider

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Abstract

We study the single production of heavy neutrinos via the processes $e^-e^+ \rightarrow \nu N$ and $e^-\gamma \rightarrow W^- N$ at future linear colliders. As a base of our considerations we take a wide class of models, both with vanishing and non-vanishing left-handed Majorana neutrino mass matrix $m_L$. We perform a model independent analyses of the existing experimental data and find connections between the characteristic of heavy neutrinos (masses, mixings, CP eigenvalues) and the $m_L$ parameters. We show that with the present experimental constraints heavy neutrino masses almost up to the collision energy can be tested in the future experiments.

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1 Introduction

While the properties of charged fermions are tested with a very high accuracy the situation in the neutral fermion sector remains poorly understood. The issues of neutrino masses and mixings are still unsettled. In the Standard Model (SM) neutrinos are predicted to be massless. However, the solar and atmospheric neutrino deficits as well as the measurement of COBE satellite of the hot component of dark matter seems to indicate that neutrinos do have small but non-vanishing masses. All laboratory experiments so far have failed to measure these masses, having allowed one only to set upper limits on their values. If neutrinos are, indeed, massive there are two fundamental questions to be answered. Firstly, what is the nature of their masses i.e. whether neutrinos are Majorana or Dirac particles and, secondly, why neutrino masses are so tiny compared with the masses of charged leptons. Both of these questions might show in a new light if new neutrino species with a large mass were discovered.

The generic neutrino mass matrix allowed by gauge symmetry is of the form

$$M = \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix},$$

(1)

where $m_D$ is the submatrix of Dirac type masses and $m_L$ and $M_R$ are the submatrices of Majorana type masses for left- and right-handed neutrinos, respectively. The exact form of the mass matrix depends on specific model under consideration. As suggested by experimental data, there is a strong hierarchy among different types of masses. While the Dirac masses forming the matrix $m_D$ are naturally of the order of charged lepton masses, non-observation of right-handed neutrinos forces the mass scale of $M_R$ to be larger than the mass $M_Z$ of the neutral weak boson. In the SM supplemented with right-handed neutrino states the left-handed Majorana masses are zero at tree level (small radiative corrections of the order $m_L \sim O(1)$ eV can be expected). However, in models with left-handed triplet Higgs representations, e.g. in the left-right symmetric models (LRM), $m_L = h_M v_L$, where $h_M$ are unknown triplet Yukawa coupling constants and $v_L$ is the vev of the left-handed triplet field which, due to its contribution to the parameter $\rho = M_W^2/(M_Z^2 \cos^2 \theta_W)$, is constrained to be below 9 GeV. The bounds on $h_M$ are not particularly restrictive at the moment but will improve considerably in future experiments. At present we know that $m_L$ cannot exceed $O(1)$ GeV.

The physical neutrino states, including the ordinary light neutrinos $\nu_e$, $\nu_\mu$ and $\nu_\tau$, are found by diagonalizing the matrix. Observable variables emerging as a result of the diagonalization are the masses, mixings and CP parities of the mass eigenstate neutrinos. Particularly interesting are the mixings between the light and heavy neutrinos, for which there exist several constraints from various low-energy measurements, such as the universality of lepton couplings and the negative searches for neutrinoless double beta decay.

In this letter we study a single heavy neutrino production in future linear colliders (LC) in reactions

$$e^-e^+ \rightarrow \nu N,$$

(2)

$$e^-\gamma \rightarrow W^-N,$$

(3)
taking into account the existing constraints on the mixing between $\nu_e$ and the heavy neutrino $N$, assuming both $e^+e^-$ and $e^-\gamma$ collision options, and taking into account polarizations of the initial state particles. The importance of these reactions stems from the possibility to extend the kinematical discovery limit of heavy neutrinos almost by a factor of two when compared with the mass reach of the pair production processes. In the LC the photon beam can be obtained by scattering intensive laser pulses off the electron beam \cite{9}. In the case of linearly polarized laser light the energy spectrum of hard photons is strongly peaked at 84\% of the electron beam energy and its polarization rate is essentially the same as the electron beam one \cite{10}. We will take into account the polarization of the initial state particles.

The cross sections of the processes (2) and (3) are proportional to the light and heavy neutrino mixing angles. In the see-saw models \cite{11} these mixings are predicted to be very small. However, there are other models, e.g. based on $E_6$ and $SO(10)$ symmetry groups, where the light neutrinos are predicted to be massless at tree level by symmetry arguments \cite{12, 13}, in which case the neutrino mixing angles are not related to their masses as in the see-saw models and can be considerably larger allowing for observable effects. In order to cover all these possible scenarios we perform a model independent phenomenological study of the allowed space of parameters describing heavy neutrinos and relate them to the cross sections of the processes (2) and (3).

### 2 Experimental bounds on heavy neutrino mixings

The recent discussions of the bounds on the mixings between light and heavy neutrinos have based on simplified analyses as far as the Majorana mass submatrix $m_L$ is concerned. Either the conditions ensuing from $m_L$ have been neglected \cite{13} or the entire matrix have been taken zero \cite{14}. In the following analysis we will include the effects connected to the existence of a non-zero submatrix $m_L$.

Let us denote by $K$ the lepton analogy of the Kobayashi-Maskawa matrix. The experimental bounds on the elements of that matrix, $K_{\nu e}$ and $K_{Ne}$, describing the mixing of $\nu_e$ with light and heavy neutrinos, respectively, can be summarized as follows:

\[
\sum_{N(\text{heavy})} |K_{Ne}|^2 \leq \kappa^2 = 0.0054, \tag{4}
\]

\[
\sum_{\nu(\text{light})} K_{\nu e}^2 m_{\nu} \leq \kappa_{\text{light}}^2 = 0.65 \text{ eV}, \tag{5}
\]

\[
\sum_{N(\text{heavy})} K_{Ne}^2 \frac{1}{m_N} \leq \omega^2 = (2 - 2.8) \cdot 10^{-5} \text{ TeV}^{-1}. \tag{6}
\]

Here $m_{\nu}$ and $m_{N}$ denote the masses of light and heavy neutrinos, respectively. The first constraint comes from the LEP and low-energy measurements of lepton universality \cite{15}, and the constraints (5) and (6) from the lack of positive signal in neutrinoless double beta
decay mediated by the light \[^{[13]}\] and heavy neutrinos\[^{[14]}\], respectively. The constraint (4) is valid for both Dirac and Majorana neutrinos, while the conditions (5) and (6), which follows from neutrinoless double beta decay, give restriction only for Majorana neutrinos.

Diagonalization of the matrix (1) yields a relation

$$\sum_{\nu (\text{light})} K_{\nu e}^2 m_\nu + \sum_{N (\text{heavy})} K_{N e}^2 m_N = \langle m_L \rangle_{\nu e \nu e} \equiv \langle m_L \rangle, \quad (7)$$

which together with Eq. (3) gives the following constraint on the parameters of heavy neutrinos

$$\left| \langle m_L \rangle - \sum_{N} K_{N e}^2 m_N \right| < \kappa_{\text{light}}^2. \quad (8)$$

The inequalities (4), (6) and (8) establish the parameter space still allowed by the experimental data. This space depends on the values of $\langle m_L \rangle$, as well as on the number and CP properties of the right-handed neutrinos and their mass spectrum. Therefore, for the heavy neutrino masses $m_N$ testable at LC one can find the range of allowed values of $\langle m_L \rangle$. To shorten our notation let us denote the masses of heavy neutrinos by $M_1 \equiv M$, $M_2 = AM$, and $M_3 = BM$, with $A, B \geq 1$ (we will consider only the cases of at most three heavy neutrinos), and define the new parameters

$$\Delta = \frac{\langle m_L \rangle}{M}, \quad \delta = \frac{\kappa_{\text{light}}^2}{M}. \quad (9)$$

We will assume that CP is conserved and define unphysical CP phases of charged leptons in such a way that the neutrino mixing angles are purely real ($K_{N \nu e} = x_i$) if the CP parity of heavy neutrino is $\eta_{\text{CP}}(N_i) = +i$ and purely imaginary ($K_{N \nu e} = ix_i$) if $\eta_{\text{CP}}(N_i) = -i$. In our notation the CP-parity of the lightest heavy neutrino is always $+i$. This can be arranged without loss of generality by a proper redefinition of the CP eigenstates.

The number $n_R$ of heavy neutrino species varies in different models. Let us consider the bounds for the $\Delta$ and the $K_{N \nu e}$ mixing parameters separately in the cases where this number is $n_R = 1, 2$ or $3$.

- $n_R = 1$

From the inequalities (4), (6) and (8) we obtain

$$-\delta < \Delta < \min(\kappa_{\text{light}}^2, \omega^2 M) + \delta, \quad (10)$$

implying that relatively small values of $\langle m_L \rangle$, e.g., $|\langle m_L \rangle| \leq 2 \times 10^{-4}$ GeV for $M = 100$ GeV, are tolerated by data. In this case the largest possible value of $K_{N \nu e}$ is restricted to be

$$(K_{N \nu e})_{\text{max}}^2 = \min(\Delta + \delta, \omega^2 M, \kappa_{\text{light}}^2). \quad (11)$$

\(^1\)To find these values, complicated analyses of nuclear matrix elements must be performed. It has been argued in Ref. \[^{[17]}\] that the bounds on $\omega^2$ can be more than one order of magnitude less restrictive than we present here. However, our considerations will not be affected by this change. It is important for our discussion that such a bound can be derived from the experimental data \[^{[18]}\].
• $n_R = 2$

The light-heavy mixing angles are restricted by the inequalities which depend on the CP eigenvalues of the heavy neutrinos as

\[
\begin{align*}
x_1^2 + x_2^2 &\leq \kappa^2, \\
|\Delta - x_1^2 \mp Ax_2^2| &\leq \delta, \\
-|x_1^2 \mp \frac{x_2^2}{A}| &\leq \omega^2 M,
\end{align*}
\]

where the signs $-(+)$ stand for the same (different) values of $\eta_{CP}$ of the neutrinos. From these inequalities we obtain bounds on acceptable values of $\Delta$ and $x_1^2$ as follows.

(i) If $\eta_{CP}$'s of both neutrinos are the same then

\[
-\delta \leq \Delta \leq \min(\kappa^2, A^2 \omega^2 M) + \delta, \tag{13}
\]

and the largest allowed value of $K_{N_1e}$ can be found from the expression

\[
(x_1^2)_{\text{max}} = \min\left(\Delta + \delta, \frac{A^2 \omega^2 M - \Delta + \delta}{A^2 - 1}, \frac{\kappa^2 - \Delta + \delta}{A - 1}\right) < \min(\omega^2 M, \kappa^2). \tag{14}
\]

(ii) If $\eta_{CP}(N_1) = -\eta_{CP}(N_2) = +i$ the allowed values of $\Delta$ are in the range

\[
-\min(A^2, (A - 1)\kappa^2 + A\omega^2 M) - \delta \leq \Delta \leq \min(\omega^2 M, \kappa^2) + \delta, \tag{15}
\]

and the maximal value of $K_{N_1e}$ is given by

\[
(x_1^2)_{\text{max}} = \min\left[\frac{\kappa^2 + A\omega^2 M - \Delta}{A + 1}, \frac{\kappa^2 - \Delta + \delta}{A - 1}\right] \leq \min\left[\frac{\kappa^2 + A\omega^2 M}{A + 1}, \kappa^2\right]. \tag{16}
\]

• $n_R = 3$

In this case the inequalities take a form

\[
\begin{align*}
x_1^2 + x_2^2 + x_3^2 &\leq \kappa^2, \\
|\Delta - x_1^2 \mp Ax_2^2 \mp Bx_3^2| &\leq \delta \\
-|x_2^2 \mp \frac{x_3^2}{A} \mp \frac{x_1^3}{B}| &\leq \omega^2 M.
\end{align*}
\]

As $\Delta$ can be positive or negative and $A > B$ or $A < B$, there are only three independent CP configurations: CP eigenvalues of all neutrinos are the same, $\eta_{CP}(N_1) = \eta_{CP}(N_2) = -\eta_{CP}(N_3)$ or $\eta_{CP}(N_1) = -\eta_{CP}(N_2) = -\eta_{CP}(N_3)$.

(i) The case of equal CP eigenvalues of all heavy neutrinos is qualitatively the same as for $n_R = 2$ and we obtain

\[
-\delta \leq \Delta \leq \max\left\{\min(A\kappa^2, A^2 \omega^2 M), \min(B\kappa^2, B^2 \omega^2 M)\right\} + \delta, \tag{18}
\]

and

\[
(x_1^2)_{\text{max}} = \min\left\{\Delta + \delta, \max\left[\min\left(\frac{B^2 \omega^2 M - \Delta + \delta}{B^2 - 1}, \frac{B\kappa^2 - \Delta + \delta}{B - 1}\right), \min\left(\frac{A^2 \omega^2 M - \Delta + \delta}{A^2 - 1}, \frac{A\kappa^2 - \Delta + \delta}{A - 1}\right)\right]\right\} \leq \min(\omega^2 M, \kappa^2). \tag{19}
\]
(ii) The allowed region of the neutrino mixing angles in the case $\eta_{\text{CP}}(N_1) = \eta_{\text{CP}}(N_2) = -\eta_{\text{CP}}(N_3) = i$ is sketched in Fig. 1. The solutions belong to $\Omega_2$ plane which lay within $\Omega_1$ and $\Omega_3$ planes. Its situation depends on the value of $\Delta$. The constraints on $\Delta$ are different if $A > B$ or $A \leq B$. For $A > B$ we obtain

$$- \min \left\{ B\kappa^2, \max \{ B^2\omega^2M, (B-1)\kappa^2 + B\omega^2M \} \right\} - \delta \leq \Delta \leq \min \{ A\kappa^2, (A-B)\kappa^2 + AB\omega^2M \} + \delta,$$

and for $A \leq B$

$$- \min \left\{ B\kappa^2, (B-1)\kappa^2 + B\omega^2M \right\} - \delta \leq \Delta \leq \min \{ A\kappa^2, A^2\omega^2M \} + \delta.$$

 Independently of the relation between $A$ and $B$ the largest possible value of $K_{N_{1e}}^2$ is given by

$$(x_1^2)_{\text{max}} = \min \left[ \frac{B\kappa^2 + \Delta - \delta}{1+B}, \max \left[ \frac{B^2\omega^2 - \Delta + \delta}{B^2 - 1}, \frac{(A-B)\kappa^2 + A\omega^2M - \Delta + \delta}{(A-1)(B+1)} \right] \right] \leq \min \left[ \frac{\kappa^2 + BM\omega^2}{B+1}, \kappa^2 \right].$$

(iii) The case $\eta_{\text{CP}}(N_1) = -\eta_{\text{CP}}(N_2) = -\eta_{\text{CP}}(N_3) = i$ is similar to $n_R = 2$ (ii) one. The region of allowed values of $\Delta$ is given by

$$- \max \left\{ \min \{ A\kappa^2, (A-1)\kappa^2 + A\omega^2M \}, \min \{ B\kappa^2, (B-1)\kappa^2 + B\omega^2M \} \right\} - \delta \leq \Delta \leq \min \{ \omega^2M, \kappa^2 \} + \delta,$$

and the maximally allowed mixing $K_{N_{1e}}^2$ can be found from

$$(x_1^2)_{\text{max}} = \max \left\{ \min \left[ \frac{A^2\omega^2M - \Delta + \delta}{A^2 - 1}, \frac{A\kappa^2 + \Delta - \delta}{A + 1} \right], \min \left[ \frac{B^2\omega^2M - \Delta + \delta}{B^2 - 1}, \frac{B\kappa^2 + \Delta - \delta}{B + 1} \right] \right\} \leq \max \left\{ \frac{\kappa^2 + A\omega^2M}{A + 1}, \frac{\kappa^2 + B\omega^2M}{B + 1} \right\}.$$

All the cases considered above showed that the regions of allowed values of $\Delta$ depend on the number of heavy neutrinos, their CP properties and masses in a rather complicated way. In general, $\Delta$ can be both positive and negative and in magnitude as large as several times of $\kappa^2$ depending on the values of $A$ and $B$. Letting $\Delta$, $A$, $B$ and neutrino masses to vary one can obtain large neutrino mixing angles which provide observable effects in the LC experiments.

\(^2\)Of course, one could follow a different approach: take some theoretical model for $m_L$ as a starting point and then use the experimental data to constrain the possible spectrum of heavy neutrinos and their CP parities.
3 Single heavy neutrino production at LC

The processes (2), (3) occur entirely due to the light-heavy neutrino mixings and, therefore, their cross sections are proportional to the mixing factors $K_{N_1\nu}$. The situation is simplest when there exists just one heavy neutrino with which the electron neutrino mixes, i.e. $n_R = 1$, or when all the heavy neutrinos have the same CP parities since the experimental bound (11) constrains $K_{2N_1\nu}$ to be very small (see Eqs. (11), (14), (19)). In these cases the cross sections are very small, e.g., for the process $e^- e^+ \rightarrow \nu N$ $\sigma = 0(1) \text{ fb}$ ($M_N = 500 \text{ GeV}$ and $\sqrt{s} = 1(2) \text{ TeV}$) and with the anticipated integrated luminosities of $L=10-100 \text{ fb}^{-1}$ it will be practically impossible to detect these processes. Therefore, we have to study the cases of several heavy neutrinos with differing CP eigenvalues.

In order to obtain large mixing angle $(x_1^2)_{\text{max}}$ and to illustrate its dependence on $\Delta$ we shall consider the most natural situation $n_R = 3$ for two cases $\Delta = 0$ and $\Delta \neq 0$. In the latter case the appropriate range of $|\Delta|$ can be obtained by choosing, e.g., $|\langle m_L \rangle| = \mathcal{O}(1)$ GeV and varying heavy neutrino mass in the range testable at the LC, $M = 0.1 - 1 \text{ TeV}$. Let us first study the case (ii) with $A > B$. If $\Delta = 0$ then it follows from Eq. (22) that for quite large range of $A$ and $B$ mixing angles close to the absolute maximum are allowed. In particular, if $B = 1$ (i.e., two Majorana neutrinos are degenerate and form one Dirac neutrino) then the mixing angle can be as large as $\kappa^2/2 = 0.0027$ independently of the value of $A$. If we allow for non-zero $\Delta$ according to Eq. (20) we see that its large positive values yield small mixing angles. However, choosing $\Delta = -(B - 1)\kappa^2$ we obtain in the limit $\delta \ll \kappa^2$

$$
(K_{N_1\nu})_{\text{max}} = \frac{\kappa^2}{B + 1}.
$$

As we see, in this case there is a continuous range of relatively large possible mixing angles $K_{N_1\nu}$.

If $B > A$ the situation is somewhat different. According to Eq. (22), if $\Delta = 0$ the only possibility to have sizable mixing angles is to have $B = 1$ which yields $(x_1^2)_{\text{max}} = \kappa^2/2$. If $B \neq 1$ the maximum mixing angle decreases very quickly to a value proportional to $\omega^2$. However, for $\Delta \neq 0$ one can have a continuous range of large mixing angles, described by Eq. (25) for the minimal $\Delta$. The results for the case (iii) are very similar to the latter ones (here $A$ and $B$ are symmetric).

Note that the experimental bounds (4)-(6) correlate the allowed values of $\Delta$ with neutrino masses and mixings. While for the previously given representative values of $\langle m_L \rangle$ and $M$ one can have any value of $A$ and $B$ then for the larger $\langle m_L \rangle$ which give, e.g., $|\Delta| = 0.1$ small values of $A, B$ are excluded. Consequently, in this case the maximally allowed mixing angles are suppressed by large $A, B$. However, since $M$ is expected to be large, it would be difficult accommodate so large values of $\Delta$ with the existing phenomenology.

The discussion above can be summarized as shown in Fig. 2 where we plot properly normalized $(x_1^2)_{\text{max}}$ against $B$. The maximal mixing angles for $\Delta = 0$ in the cases (ii) $B > A$ and (iii), which can be close to maximum only if $B \approx 1$, are described by short-dashed line. For the other cases, i.e. $\Delta \neq 0$ or case (ii) $A > B$, one has a variety of possibilities to obtain mixings close to the global maximum which is presented in figure...
by solid line. In particular, an extreme situation with $\Delta = -0.1$ is depicted with long-dashed line.

To investigate the viability of a single heavy neutrino production at LC we present the cross sections of the processes $e^+ e^- \rightarrow \nu N$ and $e^- \gamma \rightarrow W^- N$ in Fig. 3 and Fig. 4, respectively, for various collision energies of the LC. For the mixing angle $|K_{N_1 e}|^2$ we choose the maximal allowed value of $\kappa^2/2 = 0.0027$. Since the cross sections are proportional to $|K_{N_1 e}|^2$ one can easily scale their values by an appropriate factor if the experimental constraints become stronger. As can be seen in the figures, the typical cross sections are of the order of 100 fb which with the anticipated luminosities would mean a few thousand events per year. Therefore, studies of the processes (4) and (3) would allow to extend the heavy neutrino mass range testable in the LC experiments almost up to the kinematical threshold.

4 Conclusions

We have studied the viability of a single heavy neutrino production at LC via the processes $e^- e^+ \rightarrow \nu N$ and $e^- \gamma \rightarrow W^- N$. Unlike in the previous works we have taken into account effects of non-vanishing left-handed Majorana neutrino mass matrix $\langle m_L \rangle$. The result depends in a crucial way on the number of heavy neutrinos, their mass spectrum, CP eigenvalues and the value of the left-handed mass matrix parameter $\langle m_L \rangle$. If there is only one heavy neutrino or if CP eigenvalues of all heavy neutrinos are the same then the constraints coming from the negative search of neutrinoless double beta decay constrain the cross sections much below the observable limit. In the other cases the neutrino mixing angle can be large, up to the maximally allowed $\kappa^2/2$. In particular, if $\langle m_L \rangle \neq 0$ there is quite large continuous parameter space that allows for observable effects in the LC experiments. With the present experimental constraints on neutrino mixings heavy neutrinos masses almost up to the kinematical threshold will be testable in future colliders.

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Figure captions

Fig.1. Sketch of allowed mixing angles $x_{1}^{2}$, $x_{2}^{2}$, $x_{3}^{2}$ for the case $n_{R} = 3$ with $\eta_{CP}(N_{1}) = -\eta_{CP}(N_{2}) = -\eta_{CP}(N_{3})$. Solutions belong to the $\Omega_{2}$ plane which is situated between $\Omega_{1}$ and $\Omega_{3}$ ones. Maximal $x_{1}^{2}$ is realized when the most protrude point S of the $\Omega_{2}$ plane approaches the point $S'$. 

Fig.2. Dependence of the maximal mixing angle $K_{N_{1}e}^{2}$ which is normalized to $\kappa^{2}/2$ on $B$. Solid line represents the global maximum as a function of $B$. Short-dashed line describes the behaviour of maximal mixing angle in the case $\eta_{CP}(N_{1}) = -\eta_{CP}(N_{2}) = -\eta_{CP}(N_{3})$, $\Delta = 0$ while long-dashed line shows a case with very large $\Delta$. 

Fig.3. Maximally allowed cross sections for the process $e^{-}e^{+} \rightarrow \nu N$ for different CM energies as functions of the lightest heavy neutrino mass. The mixing angle is taken to be $K_{N_{1}e}^{2} = \kappa^{2}/2 = 0.0027$.

Fig.4. Maximally allowed cross sections for the process $e^{-}\gamma \rightarrow NW^{-}$ for different CM energies as functions of heavy neutrino masses. Electron beam is assumed to be left-handedly and photon beam $\tau = -1$ linearly polarized. Curves denoted by $a$ and $b$ present cross sections of the lightest heavy neutrino production for $B = 1$ and $B = 5$, respectively.
\[ x_1^2 + x_2^2 + x_3^2 - \kappa^2 = 0 \]

\[ \Omega_1: \quad Bx_1^2 + \frac{B}{A} x_2^2 - x_3^2 + BM\omega^2 = 0 \]

\[ \Omega_2: \quad x_1^2 + Ax_2^2 - Bx_3^2 - \Delta = 0 \]

\[ \Omega_3: \quad Bx_1^2 + \frac{B}{A} x_2^2 - x_3^2 - BM\omega^2 = 0 \]

\[ (x_1^2)_{\text{max}} = \frac{\kappa^2 + BM\omega^2}{B + 1} \]

Figure 1:
Figure 2:
Figure 3:

\[ e^- e^+ \rightarrow \nu N \]
Figure 4: