Fragmenting protostellar disks: properties and observational signatures

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ABSTRACT

Using numerical hydrodynamic simulations, we study the gravitational fragmentation of an unstable protostellar disc formed during the collapse of a pre-stellar core with a mass of 1.2\,M\odot. The forming fragments span a mass range from about a Jupiter mass to very-low-mass protostars and are located at distances from a few tens to a few thousand AU, with a dearth of objects at \(\lesssim 100\) AU. We explore the possibility of observational detection of the fragments in discs viewed through the outflow cavity at a distance of 250 pc. We demonstrate that one hour of integration time with the Atacama Larger Millimeter/sub-millimeter Array (ALMA) is sufficient to detect the fragments with masses as low as \(\approx 0.5\,M_{\text{Jup}}\) at orbital distances up to 800 AU from the protostar. The ALMA resolution sets the limit on the minimum orbital distance of detectable fragments. For the adopted resolution of our simulated ALMA images of 0.1''\,\,the fragments can be detected at distances down to 50 AU. At smaller distances, the fragments usually merge with the central density peak. The likelihood for detecting the fragments reduces significantly for a lower resolution of 0.5''. Some of the most massive fragments, regardless of their orbital distance, can produce characteristic peaks at \(\approx 5\mu\text{m}\) and hence their presence can be indirectly inferred from the observed spectral energy distributions of protostars.

Key words: stars: formation – instabilities – planetary systems: protoplanetary discs – accretion: accretion discs – hydrodynamics

1 INTRODUCTION

Observations of young protostellar discs and numerical modeling of disc formation and evolution both suggest that protostellar discs may be sufficiently massive to undergo gravitational fragmentation in the course of their evolution. Observations of young protostars embedded in parental cloud cores yield disc masses that sometimes exceed 0.1\,M\odot\,(e.g. Isella et al.\,2009; Jorgensen et al.\,2009). Numerical hydrodynamics simulations indicate that such massive discs are gravitationally unstable and may fragment if disc cooling is sufficiently fast (e.g. Johnson & Gammie\,2003; Stamatellos & Whitworth\,2009; Meru & Bate\,2011; Boss\,2012; Vorobyov\,2013).

The newly-formed fragments are in the sub-stellar mass regime and may be considered as embryos of giant planets and brown dwarfs, which may or may not evolve into finished objects. The majority of the fragments are torn up into the disc inner regions and probably onto the burgeoning protostar, triggering episodic accretion and luminosity bursts (Vorobyov & Basu\,2006; 2010a), a phenomenon supported by many indirect lines of evidence (see e.g. Dunham & Vorobyov\,2012). Some of the fragments may be ejected via many-body gravitational interaction with other fragments into the intracluster medium, forming upon contraction a population of freely-floating brown dwarfs (Basu & Vorobyov\,2013), or settle onto stable orbits providing a gateway for the formation wide-orbit gas giant and brown dwarf companions to low-mass stars (Vorobyov & Basu\,2010b; Boss\,2012; Vorobyov\,2013). Finally, fragments may loose their atmospheres via tidal stripping and reveal solid terrestrial or icy giant cores if the dust sedimentation timescale is sufficiently fast (Boley et al.\,2010; Navalkhit\,2010).

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Although the feasibility of disc fragmentation has been
confirmed by many theoretical and numerical studies, unambiguous observational detection of the fragments has been a challenging task. Recent observations using aperture masking interferometry have apparently been successful in identifying one such candidate (Kraus & Ireland 2012), although it might have already collapsed to a planetary-sized object as implied by the estimated age of the central star of 2 Myr. On the other hand, numerical hydrodynamic simulations seem to indicate that fragments with typical sizes of several tens of AU can be detected with the IRAM Plateau de Bure interferometer (Stamatellos & Whitworth 2011). In addition, the spiral structure in gravitationally unstable discs and fragmentation on scales of the order of thousands AU in starless and protostellar cores is likely to be detectable with the Atacama Large Millimeter/sub-millimeter Array (ALMA) (Cossins et al. 2010; Offner et al. 2012).

In this paper, we study numerically the formation and evolution of a protostellar disc subject to vigorous gravitational fragmentation in the embedded phase of star formation. We focus on the properties of the fragments and explore possibilities of their direct and indirect observational detection. In Section 2, an overview of the numerical model is given. Section 2.3 describes the adopted model for calculating the spectral energy distributions (SEDs). The main properties of the fragments are presented in Sections 3 and 4. The model SEDs are discussed in Section 5 and the synthetic ALMA images are presented in Section 6. The limitations of our model are discussed in Section 7 and main conclusions are given in Section 8.

2 MODEL DESCRIPTION AND BASIC EQUATIONS

2.1 Numerical hydrodynamics model

Our numerical hydrodynamics model for the formation and evolution of a star plus disc system is described in detail in Vorobyov & Basu (2010a) and is briefly reviewed below for the reader’s convenience. We start our numerical simulations from the gravitational collapse of a starless cloud core, continue into the embedded phase of star formation, during which a star, disc, and envelope are formed, and terminate our simulations in the T Tauri phase, when most of the envelope has accreted onto the forming star/disc system. In our numerical hydrodynamics simulation, the protostellar disc is not isolated but is exposed to intense mass loading from the envelope. In addition, the mass accretion rate onto the disc is not a free parameter of the model but is self-consistently determined by the gas dynamics in the envelope, which in turn is set by initial conditions in the core.

To avoid too small time steps, we introduce a “sink cell” at $r_{\text{sc}} = 7$ AU and impose a free inflow inner boundary condition and free outflow outer boundary condition so that that the matter is allowed to flow out of the computational domain but is prevented from flowing in. The sink cell is dynamically inactive; it contributes only to the total gravitational potential and secures a smooth behaviour of the gravity force down to the stellar surface. We monitor the gas surface density in the sink cell and when its value exceeds a critical value for the transition from isothermal to adiabatic evolution, we introduce a central point-mass object. In the subsequent evolution, 90% of the gas that crosses the inner boundary is assumed to land onto the central object plus the sink cell. The other 10% of the accreted gas is assumed to be carried away with protostellar jets. We assumed a 10% efficiency as the typical lower limit in the X-wind model of jet launching (Shu et al. 1994).

We make use of the thin-disc approximation to compute the gravitational collapse of rotating, gravitationally unstable cloud cores. This approximation is an excellent means to calculate the evolution for many orbital periods and many model parameters and its justification is discussed in Vorobyov & Basu (2010a). The basic equations of mass, momentum, and energy transport are

$$\frac{\partial \Sigma}{\partial t} = -\nabla_p \cdot (\Sigma v_p), \quad (1)$$

$$\frac{\partial}{\partial t} (\Sigma v_p) + [\nabla_p (\Sigma v_p \otimes v_p)]_p = -\nabla_p P + \Sigma g_p + \nabla \cdot (\Pi), \quad (2)$$

$$\frac{\partial}{\partial t} + \nabla_p \cdot (\nu v_p) = \nu (\nabla_p \cdot v_p) - \Lambda + \Gamma + \nabla' \cdot \Pi_{pp'}, \quad (3)$$

where subscripts $p$ and $p'$ refers to the planar components $(r, \phi)$ in polar coordinates, $\Sigma$ is the mass surface density, $\nu$ is the internal energy per surface area, $P$ is the vertically integrated gas pressure calculated via the ideal equation of state as $P = (\gamma - 1) c^2$ with $\gamma = 7/5$, $Z$ is the radially and azimuthally varying vertical scale height determined in each computational cell using an assumption of local hydrostatic equilibrium, $v_p = v_r \hat{r} + v_\phi \hat{\phi}$ is the velocity in the disc plane, $g_p = g_r \hat{r} + g_\phi \hat{\phi}$ is the gravitational acceleration in the disc plane, and $\nabla_p \cdot = \partial / \partial r + \hat{\phi} \partial / \partial \phi$ is the gradient along the planar coordinates of the disc.

Turbulent viscosity is taken into account via the viscous stress tensor $\Pi$, the expression for which is provided in Vorobyov & Basu (2010a). We parameterize the magnitude of kinematic viscosity $\nu$ using a modified form of the $\alpha$-prescription

$$\nu = \alpha c_s Z F_\alpha(r), \quad (4)$$

where $c_s^2 = \gamma P / \Sigma$ is the square of effective sound speed calculated at each time step from the model’s known $P$ and $\Sigma$. The function $F_\alpha(r) = 2 \pi^{-1} \tan^{-1} [(r_d / r)^{10}]$ is a modification to the usual $\alpha$-prescription that guarantees that the turbulent viscosity operates only in the disc and quickly reduces to zero beyond the disc radius $r_d$. In this paper, we use a spatially and temporally uniform $\alpha$, with its value set to $5 \times 10^{-3}$. This choice is based on the work of Vorobyov & Basu (2009), who studied numerically the long-term evolution of viscous and self-gravitating disks. They found that the temporally and spatially averaged $\alpha$ in protostellar discs should lie in the $10^{-3}$ to $10^{-2}$ limits. Smaller values of $\alpha$ ($\lesssim 10^{-4}$) have little effect on the resultant mass accretion history (dominated in this case by gravitational torques), while larger values ($\alpha > 10^{-1}$) destroy circumstellar disks during less than 1.0 Myr of evolution and are thus inconsistent with mean disk lifetimes of the order of 2-3 Myr.

The radiative cooling $\Lambda$ in equation (3) is determined using the diffusion approximation of the vertical radiation transport in a one-zone model of the vertical disc structure (Johnson & Gamments 2003).
\[ \Lambda = F_e \sigma T_{mp}^4 \frac{r}{1 + r^2}, \]  
(5)

where \( \sigma \) is the Stefan-Boltzmann constant, \( T_{mp} = \mathcal{P} \mu / R \Sigma \) is the midplane temperature of gas, \( \mu = 2.33 \) is the mean molecular weight, \( R \) is the universal gas constant, and \( F_e = 2 + 20 \tan^{-1}(r)/(3r) \) is a function that ensures a correct transition between the optically thick and optically thin regimes. We use frequency-integrated opacities of [Bell & Lin (1994)]. The heating function is expressed as

\[ \Gamma = F_e \sigma T_{irr}^4 \frac{r}{1 + r^2}, \]  
(6)

where \( T_{irr} \) is the irradiation temperature at the disc surface determined by the stellar and background black-body irradiation as

\[ T_{irr}^4 = T_{bg}^4 + \frac{F_{irr}(r)}{\sigma}, \]  
(7)

where \( T_{bg} \) is the uniform background temperature (in our model set to the initial temperature of the natal cloud core) and \( F_{irr}(r) \) is the radiation flux (energy per unit time per unit surface area) absorbed by the disc surface at radial distance \( r \) from the central star. The latter quantity is calculated as

\[ F_{irr}(r) = \frac{L}{4\pi r^2} \cos \gamma_{irr}, \]  
(8)

where \( \gamma_{irr} \) is the incidence angle of radiation arriving at the disc surface at radial distance \( r \) calculated as [Vorobyov & Basu (2010a)]

\[ \cos \gamma_{irr} = \cos \alpha \cos \delta (\tan \alpha - \tan \delta), \]  
(9)

where

\[ \cos \alpha = \frac{dr}{\sqrt{dr^2 + dZ^2}}, \quad \cos \delta = \frac{r}{\sqrt{r^2 + Z^2}}, \]  
(10)

\[ \tan \delta = \frac{dZ}{dr}. \]  
(11)

The stellar luminosity \( L_* \) is the sum of the accretion luminosity \( L_{s,acc} = GM_* M/2R_* \), arising from the gravitational energy of accreted gas and the photospheric luminosity \( L_{s,ph} \) due to gravitational compression and deuterium burning in the star interior. The stellar mass \( M_* \) and accretion rate onto the star \( \dot{M} \) are determined self-consistently during numerical simulations via the amount of gas passing through the sink cell. The stellar radius \( R_* \) is calculated using an approximation formula of [Palla & Stahler (1991)], modified to take into account the formation of the first molecular core. The photospheric luminosity \( L_{s,ph} \) is taken from the pre-main sequence tracks for the low-mass stars and brown dwarfs calculated by [D’Antona & Mazitelli (1997)]. More details are given in [Vorobyov & Basu (2010a)].

Equations (1)–(3) are solved using the method of finite differences with a time explicit solution procedure in polar coordinates \((r, \phi)\) on a numerical grid with 512 \times 512 grid zones. The advection is treated using a third-order-accurate piecewise parabolic interpolation scheme of [Colella & Woodward (1984)]. The update of the internal energy per surface area \( \epsilon \) due to cooling \( \Lambda \) and heating \( \Gamma \) is done implicitly using the Newton-Raphson method of root finding, complemented by the bisection method where the Newton-Raphson iterations fail to converge. The viscous heating and force terms in equations (2) and (3) are implemented in the code using an explicit finite-difference scheme, which is found to be adequate for \( \alpha \lesssim 0.01 \). The radial points are logarithmically spaced. The innermost grid point is located at the position of the sink cell \( r_{sc} = 7 \) AU, and the size of the first adjacent cell varies in the 0.07–0.1 AU range depending on the cloud core size. This corresponds to the radial resolution of \( \Delta r = 1.1–1.6 \) AU at 100 AU.

### 2.2 Initial setup

For the initial distribution of the gas surface density \( \Sigma \) and angular velocity \( \Omega \) in the pre-stellar cores we take those from [Basu (1997)]

\[ \Sigma = \frac{r_0 \Sigma_0}{\sqrt{r^2 + r_0^2}}, \]  
(12)

\[ \Omega = 2\Omega_0 \left( \frac{r_0}{r} \right)^2 \left[ 1 + \left( \frac{r}{r_0} \right)^2 - 1 \right]. \]  
(13)

These radial profiles are typical of pre-stellar cores formed as a result of the slow expulsion of magnetic field due to ambipolar diffusion, with the angular momentum remaining constant during axially-symmetric core compression. Here, \( \Omega_0 \) and \( \Sigma_0 \) are the angular velocity and gas surface density at the disc centre and \( r_0 = \sqrt{\mathcal{A}_0^2 / \pi G \Sigma_0} \) is the radius of the central plateau, where \( \mathcal{A}_0 \) is the initial sound speed in the core. The gas surface density distribution described by equation (12) can be obtained (to within a factor of unity) by integrating the three-dimensional gas density distribution characteristic of Bonnor-Ebert spheres with a positive density-perturbation amplitude \( \mathcal{A} \) [Dapp & Basu (2003)]. In all models the value of \( A \) is set to 1.2. We note that the properties of protostellar discs formed as a result of the gravitational collapse of pre-stellar cores depend weekly on the form of the initial density and angular velocity distributions in the core as long as the initial core mass and ratio of rotational to gravitational energy \( \beta \) are the same [Vorobyov (2012)]. The initial temperature of pre-stellar cores is set to 10 K.

### 2.3 Constructing spectral energy distributions

To construct a spectral energy distribution (SED) of our model object, we split it into three constituent parts: the accreting protostar, the inner disc, and the outer dynamic disc. This partitioning is motivated by the specifics of our numerical model—we do not compute the dynamical evolution of the system in the inner several AU (inside the sink cell) due to severe Courant limitations on the hydrodynamical time step.

The basic assumption entering our calculation of the SEDs is that our model object is viewed through an outflow cavity. This allows us to neglect the reprocessing of

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1 This definition of the midplane temperature is accurate within a factor of unity [Zhu et al. (2012)].
light by the envelope and use surface densities and effective temperatures derived directly from numerical hydrodynamics modelling of the outer disc and analytic model of the inner disc. At the same time, this assumption imposes limitations on the viewing angle, which should not exceed the opening semi-angle of the outflow cavity $\theta_c$. Below, we explain in more detail the main ideas and equations involved in calculating the model SEDs.

2.3.1 The outer dynamic disc

Numerical hydrodynamics simulations provide us with gas surface densities $\Sigma$ and midplane temperatures $T_{\text{mp}}$ in each grid cell $(i, j)$ of the outer dynamic disc. We use a solution to the one-dimensional radiation transfer equation to calculate the radiative flux density $F_\nu$ leaving each grid cell of the disc surface

$$F_\nu^{\text{disc}} = \frac{S}{d^2} B_{\nu}(T_{\text{surf}}) \left( 1 - e^{-\Sigma \kappa_\nu \sec \gamma_{\text{incl}}} \right),$$

where $B_{\nu}(T_{\text{surf}})$ is the Plank function, $d$ is the distance to the considered object, set to 100 pc in the paper, $\gamma_{\text{incl}}$ is the inclination angle of the disc with respect to the observer (i.e., an angle between the disc rotation axis and the line of sight), and $S$ is the projected area of the disc surface seen by the observer. The inclination angle $\gamma_{\text{incl}}$ is set to zero in the present paper. The effect of non-zero inclination, but not exceeding the opening semi-angle of the outflow cavity, is discussed in Section 4 and will be considered in detail in a follow-up paper. The total flux is found by summing inputs from all individual grid cells of the disc, which amount to 512 × 512 zones for the typical numerical resolution of our hydro simulations.

We adopt frequency-dependent opacities $\kappa_\nu$ from Ossenkopf & Henning (1994). These opacities were derived for three cases of a) thick ice mantles, b) thin ice mantles, and c) no ice mantles. In this work, we adopt opacities derived for the case of thin ices, commonly referred to as “OH5” opacities in the literature. The dependence of our results on the adopted opacity is discussed in more detail in Section 4.

To calculate the Plank function $B_{\nu}$ in equation (14), we need to know the disc surface temperature $T_{\text{surf}}$. Stellar/background irradiation of the disc surface, viscous/shock heating of the disc interiors, and blackbody cooling from the disc surface determine the temperature balance of the disc in our model. Therefore, we use the following equation to calculate the surface temperature of the disc in our model

$$T_{\text{surf}}^4 = \frac{1}{2} F_\nu T_{\text{mp}}^4 + \frac{1}{1 + \tau} + \frac{F_{\text{irr}}(r)}{\sigma},$$

where the factor 1/2 is introduced because only the fluxes reaching/leaving one side of the disc are considered. We also note that the effect of disc opacity is now taken into account via the factor $1/(1 + \tau)$ rather than via $\tau/\Gamma_0 (1 + \tau)$. The latter expression yields too low temperatures in the optically thin limit $\tau \ll 1.0$ in cases when the heating sources are absent.

2.3.2 The central accreting protostar

The central protostar provides a significant contribution to the SED at shorter wavelengths. The radiative flux density from the protostar $F_\nu^{\star}$ is calculated assuming a black-body radiation spectrum with effective temperature $T_{\text{eff}}$

$$F_\nu^{\star} = \frac{\pi R_\star^2}{d^2} B_{\nu}(T_{\text{eff}}),$$

where $R_\star$ is the radius of the protostar and $T_{\text{eff}}$ is defined by both the accretion and photospheric luminosities ($L_{\text{accr}}$ and $L_{\text{ph}}$, respectively) as

$$T_{\text{eff}} = \left( \frac{L_{\text{accr}} + L_{\text{ph}}}{4\pi R_\star^2 \sigma} \right)^{1/4},$$

where $\sigma$ is the Stefan-Boltzmann constant. The accretion luminosity is calculated from the model’s known accretion rate $\dot{M}$ onto the protostar, while the photospheric luminosity is taken from the pre-main sequence tracks of D’Antona & Mazzitelli (1997) for low-mass stars and brown dwarfs.

2.3.3 The inner disc

The properties of the disc in the inner several AU cannot be calculated self-consistently because of the sink cell, which is introduced for numerical convenience to avoid too small timesteps near the origin where the grid lines converge. Nevertheless, the inner disc may contribute significantly to the total SED. We reconstruct the structure of the inner disc using the azimuthally-averaged gas surface density profile in the outer disc and adopting a simple model for the surface temperature of the inner disc.

The reconstruction of the physical properties of the inner disc proceeds in three steps. First, we calculate the inner disc truncation radius $R_{\text{in}}$ where the gas temperature exceeds the dust sublimation threshold $T_{\text{d.s.}} = 1500$ K (Dullemond et al. 2001). The effect of varying $T_{\text{d.s.}}$ is discussed in Section 4. The calculation is based on the assumption of energy balance between the energy absorbed by spherical, blackbody dust grains $\pi a^2 (L_{\text{accr}} + L_{\text{ph}})/(4\pi r^2)$ and energy emitted by dust grains $\sigma T_{\text{d}}^4 4\pi a^2$, where $a$ is the radius of dust grains Dunham et al. 2010). The resulting expression for the sublimation radius is

$$R_{\text{in}} = \sqrt{\frac{L_{\text{accr}} + L_{\text{ph}}}{16\pi \sigma T_{\text{d.s.}}^4}}.$$  

We assume that the inner rim of the disc at $R_{\text{in}}$ is heated to the temperature of dust sublimation $T_{\text{d.s.}}$.

Second, we calculate the gas surface density distribution in the inner disc using the following relation

$$\Sigma_{\text{in}}(r) = \Sigma_{\text{in,0}} \left( \frac{r}{r_{\text{in}}} \right)^{-p},$$

where $r_{\text{in}}$ is the outer radius of the inner disc and $\Sigma_{\text{in,0}}$ is the corresponding gas surface density. To make a smooth transition between the inner and outer discs, we perform the least-squares fitting to the azimuthally averaged gas surface density of the latter. Based on this fitting, we choose a value of $\Sigma_{\text{in,0}}$ at $r_{\text{in}}$ such that the two discs are smoothly joined. The exponent $p$ varies between 1.5 for gravitationally unstable discs and approximately 1.0 for viscosity-dominated discs (e.g. Vorobyov & Basu 2004). Since the inner disc is likely to be gravitationally stable, we chose $p = 1.0$. We note however that variations in the density profile of the inner disc...
insignificantly influence the resulting SED due to the high optical depth of the inner disc.

Finally, the surface temperature of the inner disc is found adopting a simple model of a viscous disc in Keplerian rotation. Viscosity generates dissipation of energy in the vertical column of the disc at a rate \( \dot{E} \):

\[
\dot{E} = \frac{9}{4} \nu \Sigma \Omega^2,
\]

where \( \Omega = (GM_*/r^3)^{1/2} \).

Assuming further that the inner disc radiates from its surface as a black-body, one can write the disc surface temperature as

\[
T_{\text{surf}} = \frac{9}{8\sigma} \nu \Sigma \Omega^2 + T_{\text{bg}}^4 + \frac{F_{\text{irr}}}{\sigma},
\]

where \( T_{\text{bg}} \) is a constant background temperature. The inner disc scale height \( Z_{\text{in}} \), needed to calculate the incident radiation flux \( F_{\text{irr}} \), is found by assuming the disc aspect ratio from Dunham & Vorobyov (2012)

\[
Z_{\text{in}}/r = C \left( \frac{r}{r_\text{in}} \right)^{0.25},
\]

where the scaling factor \( C \) is found by applying the disc scale height of the outer dynamic disc at \( r_\text{in} \). This ensures a continuous transition between the inner and outer disc vertical scale heights. For a steady-state disc, the dynamic viscosity \( \nu \Sigma \) can be expressed in terms of the mass accretion rate \( \dot{M} \) as (Pringle 1981)

\[
\nu \Sigma = \frac{\dot{M}}{3\pi} \left( 1 - \left( \frac{R_\text{in}}{r} \right)^{1/2} \right).
\]

For the mass accretion rate \( \dot{M} \) in the inner disc we adopted the value calculated at the inner inflow boundary of the outer dynamic disc, \( r_\text{in} = 7 \) AU. Once the surface temperature and density in the inner disc have been calculated, the input to the total SED is found using equation (13). The implementation of a more accurate, time-dependent model of the inner disc, subject to mass loading from the outer dynamic disc, remains to be a future improvement of our numerical model.

### 3 DISC INSTABILITY AND FRAGMENTATION

Theoretical and numerical studies of the evolution of protostellar discs indicate that disc fragmentation is a complicated phenomenon, which can be influenced by both the internal disc physics and external environment. The latter may influence the disc susceptibility to fragmentation directly through, e.g., stellar and background irradiation, or indirectly by setting the initial conditions in cloud cores that favor or disfavor fragmentation in subsequently formed discs due to, e.g., higher/lower infall rates onto the disc.

In this section, we describe the time evolution of protostellar discs formed as a result of the gravitational collapse of pre-stellar cores with different initial mass and angular momentum. We consider two prototype cases: model 1 with initial core mass \( M_\text{c} = 1.22 M_\odot \) and ratio of rotational to gravitational energy \( \beta = 8.8 \times 10^{-3} \), and also model 2 with \( M_\text{c} = 0.74 M_\odot \) and \( \beta = 2.75 \times 10^{-3} \). We have chosen the two models based on our previous numerical simulations of Vorobyov & Basu (2010a) so as to obtain discs with vigorous and occasional gravitational fragmentation.

Figure 1 presents the gas surface density images for model 1 in the inner 2000 × 2000 AU box at various times since the formation of the central protostar. The entire computational box is almost 10 times larger. The disc forms at \( t = 0.01 \) Myr and becomes gravitationally unstable at \( t = 0.04 \) Myr. A first set of fragments appear in the disc at \( t = 0.1 \) Myr. The subsequent evolution is characterized by vigorous gravitational instability and fragmentation leading sometimes to a nearly complete breakup of the disc into massive fragments and dense filamentary arms (i.e., at \( t = 0.13 \) Myr). Some of the fragments possess mini-discs of their own, complicating even more the dynamics of the entire disc-like structure.

When studying the susceptibility of a protostellar disc to gravitational fragmentation, the following two criteria are usually investigated (e.g., Rafikov 2003; Vorobyov & Basu 2010a).

(i) The ratio of the local cooling time \( t_\text{c} = c/\Lambda \) to the local dynamical time \( \Omega^{-1} \) should be smaller than a few (Gammie 2001; Rice et al. 2003; Meru & Bate 2012), with the actual value depending on the physical conditions in the disc. Following Vorobyov & Basu (2010a), we will refer to the dimensionless quantity \( t_\text{c} \Omega^{-1} \) as the \( G \)-parameter and adopt \( G = 1 \) as a fiducial (and conservative) critical value.

(ii) The Toomre criterion \( Q = c_\pi \Omega/(\pi G \Sigma) \) for a Keplerian disc is smaller than some critical value \( Q_{\text{cr}} \), usually taken to be unity (Toomre 1964). The value of \( Q_{\text{cr}} \) may depend on the physical conditions and may vary by a factor of unity (Polyachenko et al. 1997). We have chosen a conservative value of \( Q_{\text{cr}} = 1.0 \).

We applied these two criteria to our model at \( t = 0.13 \) Myr in a fashion similar to figures 3 and 4 in Vorobyov & Basu (2010a). We found that the fragments obey both criteria. At the same time, there are a few regions in the disc that are characterized by \( Q < 1.0 \) but with no obvious fragmentation taking place in there. The lack of fragmentation in these regions is explained by slow cooling. We also note that both criteria for disc fragmentation are violated in the inner several tens of AU, in agreement with previous studies of fragmenting discs.

We note that the total mass of the disc (including fragments and the sink cell) varies between 0.19 \( M_\odot \) at \( t = 0.09 \) Myr and 0.25 \( M_\odot \) at \( t = 0.6 \) Myr, with the maximum value of 0.31 \( M_\odot \) reached at \( t = 0.25 \) Myr. The mass of the central star grows from 0.32 \( M_\odot \) at \( t = 0.09 \) Myr to 0.76 \( M_\odot \) at \( t = 0.6 \) Myr. For the disc mass estimates, we used an algorithm described in Dunham & Vorobyov (2012) and a critical surface density for the disc to envelope transition of 0.1 g cm\(^{-2}\).

Finally, in Figure 2 we present an example of a gravitationally unstable disc showing only occasional fragmentation. The panels show the gas surface density maps in model 2 for the inner 800 × 800 AU box. The time elapsed since the formation of the central protostar is indicated in each panel. The disc in this model forms at \( t = 0.035 \) Myr, appreciably later than in model 1 due to smaller angular momentum of the parental pre-stellar core. The resulting disc
mass is smaller by almost a factor of two than in model 1. The disc is gravitationally unstable and exhibits a flocculent spiral structure but is mainly stable to fragmentation, in contrast to model 1. The only distinct episode of disc fragmentation takes place around $t = 0.13$ Myr, but the fragment has quickly migrated through the inner sink cell during just several orbital periods. In the subsequent evolution, the disc shows little signs of fragmentation. We note that the disc age in each panel is approximately equal to that in the corresponding panel in Figure 1.
Properties of fragmenting protostellar discs

4 PROPERTIES OF THE FRAGMENTS

In this section, we explore the properties of the fragments in model 1 with the purpose of determining the feasibility of using observations to indirectly or directly detect their presence. Figure 3 shows the number of fragments \( N_f \) calculated every 0.01 Myr after the formation of the protostar using a fragment tracking algorithm described later in this section. Evidently, \( N_f \) varies significantly with time, reaching local peak values every \( \approx 0.1 \) Myr. An increase in the number of fragments shows recent fragmentation, and a decrease shows recent destruction/accretion of the fragments.

It is interesting to note that the time intervals between recurrent peaks is of the same duration as the characteristic time of mass infall onto the disc

\[
t_{\text{infall}} = \frac{M_d}{\dot{M}_d},
\]

where \( M_d \) is the disc mass and \( \dot{M}_d \propto c_s^3/G \) is the mass infall rate onto the disc. Indeed, for the disc mass of 0.2–0.25 \( M_\odot \) and mass infall rates of \((2 - 3) \times 10^{-6} M_\odot \) typical for our model, the characteristic infall time ranges between 0.07 Myr and 0.12 Myr.

The approximate match between periodicity of the peaks in Figure 3 and \( t_{\text{infall}} \) suggests that the mass infall may cause periodic bursts of disc fragmentation, at least in the embedded phase of disc evolution which lasts for 0.4-0.45 Myr in this model. The important implication of this phenomenon is that the fragments can be present in the disc for a significantly longer time period than expected for just one burst of disc fragmentation in numerical hydrodynamics simulations without disc infall (e.g. Stamatellos et al. 2011), significantly increasing the probability for observing the fragments.

The two bottom panels in Figure 4 present vertically integrated pressures \( P_c \), surface densities \( \Sigma_c \), midplane temperatures \( T_{\text{mp,c}} \) and surface temperatures \( T_{\text{surf,c}} \) derived at the geometrical centres of the fragments in model 1. As before, these data were calculated by searching for and identifying the fragments in the inner 5000 AU of the computational mesh every 0.01 Myr starting from \( t = 0.03 \) Myr and ending at \( t = 0.6 \) Myr. The obtained masses and radial positions of the fragments were then split into 20 logarithmically spaced bins. One can identify two local maxima in the mass distribution function, one belonging to the planetary-mass regime at \( \approx 5 M_{\text{Jup}} \) and the other lying in the upper brown-dwarf-mass regime at \( \approx 60 - 70 M_{\text{Jup}} \). The whole mass spectrum extends from about a Jupiter mass to very-low-mass stars. The radial positions of the fragments have a well-defined maximum at \( \approx 400 - 500 \) AU. The number of the fragments gradually declines at smaller distances and no fragments are seen at \( r < 30 \) AU. The dearth of fragments at \( r < 100 \) AU is most likely explained by slow cooling that impedes fragmentation in the inner several tens AU (Johnston & Gammie 2003; Meru & Bate 2011) and fast inward radial migration of the fragments. Numerical simulations show that once migration has started it proceeds very fast and fragments spiral into the inner few AU (and probably onto the star) during just a few orbital periods (Vorobyov & Basu 2006, 2010b; Machida et al. 2011, Cha & Navarkshin 2011), making it difficult to observe such fragments. We note that we do not follow the fate of the fragments once they have passed through the sink cell at 7 AU.

The bottom panel of Figure 4 shows the geometrical centres of the fragments in model 1. As before, these data were calculated by searching for and identifying the fragments in the inner 5000 AU every 0.01 Myr starting from \( t = 0.03 \) Myr and ending at \( t = 0.6 \) Myr. The two bottom panels in Figure 4 present vertically integrated pressures \( P_c \), surface densities \( \Sigma_c \), midplane temperatures \( T_{\text{mp,c}} \) and surface temperatures \( T_{\text{surf,c}} \) derived at the geometrical centres of the fragments in model 1. As before, these data were calculated by searching for and identifying the fragments in the inner 5000 AU every 0.01 Myr starting from \( t = 0.03 \) Myr and ending at \( t = 0.6 \) Myr. The obtained masses and radial positions of the fragments were then split into 20 logarithmically spaced bins. One can identify two local maxima in the mass distribution function, one belonging to the planetary-mass regime at \( \approx 5 M_{\text{Jup}} \) and the other lying in the upper brown-dwarf-mass regime at \( \approx 60 - 70 M_{\text{Jup}} \). The whole mass spectrum extends from about a Jupiter mass to very-low-mass stars. The radial positions of the fragments have a well-defined maximum at \( \approx 400 - 500 \) AU. The number of the fragments gradually declines at smaller distances and no fragments are seen at \( r < 30 \) AU. The dearth of fragments at \( r < 100 \) AU is most likely explained by slow cooling that impedes fragmentation in the inner several tens AU (Johnston & Gammie 2003; Meru & Bate 2011) and fast inward radial migration of the fragments. Numerical simulations show that once migration has started it proceeds very fast and fragments spiral into the inner few AU (and probably onto the star) during just a few orbital periods (Vorobyov & Basu 2006, 2010b; Machida et al. 2011, Cha & Navarkshin 2011), making it difficult to observe such fragments. We note that we do not follow the fate of the fragments once they have passed through the sink cell at 7 AU.

\[
\frac{\partial \Phi}{\partial r} + \frac{1}{r} \frac{\partial \Phi}{\partial \phi} > 0,
\]
and surface temperatures $T_{\text{surf}}$ which have not yet dissociated molecular hydrogen. Implying that all the fragments are the first hydrostatic cores vs. gas surface density $\Sigma_{c}$.

Panels of Figure 4 show surface temperatures vs. surface density. The surface densities greater than $100 \, \text{g cm}^{-2}$, where most fragments are located, do not, as a rule, exceed $100 \, \text{g cm}^{-2}$ (solid line) and $\Sigma_{c} \geq 10^{3} \, \text{g cm}^{-2}$ (dashed line). The corresponding exponents are 2.0 and 1.5.

In Figure 5 we present surface density images, zoomed-in on the four fragments in model 1 with midplane temperature exceeding $10^{3} \, \text{K}$. Black curves outline the fragments as found by our fragment tracking algorithm. Yellow circles mark the corresponding Hill radii. Time elapsed since the formation of the central protostar is indicated in each panel. The scale bar is in $\text{g cm}^{-2}$.

$$R_{\text{H}} = r_{l} \left( \frac{M_{f}}{3 \, M_{\ast} + M_{f}} \right)^{1/3},$$

where $r_{l}$ is the radial distance from the protostar to the fragment and $M_{f}$ is the mass of the fragment confined within the black curve.

The properties of the four fragments are listed in Table 1. In particular, the columns from left to right present: 1) the number of the fragment, 2) the time at which the fragment is seen in the disc, 3) the mass of the fragment, 4) the mass confined within the Hill radius $M_{H}$, 5) the radial distance to the fragment, 6) the radius of the fragment $R_{f}$, 7) the Hill radius of the fragment, 8) the midplane temperature of the fragment, 9) the surface temperature of the fragment, 10) the surface density of the fragment, and 11) the optical depth to the midplane $\tau$. The last four quantities were calculated at the geometrical centers of the fragments. The radius of the fragment is found by calculating the surface area occupied by the fragment and assuming that the fragment has a circular form.

Evidently, all four fragments with $T_{\text{mp,c}} \geq 10^{3} \, \text{K}$ are in the brown-dwarf-mass regime. In fact, we do not find any planetary-mass objects with midplane temperatures exceeding $400 \, \text{K}$. Fragments that are closer to the protostar are generally smaller in size, probably due to tidal stripping.

We do not see any significant correlation between $T_{\text{mp,c}}$ and $r_{l}$ for the entire sample of the fragments in model 1, implying that hot fragments may be found at any distances and not only at $r < 170 \, \text{AU}$ as in the current model. The last column in Table 1 illustrates a strong dependence of opacity on midplane temperature. For fragment F2 with $T_{\text{mp,c}} = 1180 \, \text{K}$ the opacity is $\tau = 256$, while for fragment F1
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5 MODEL SPECTRAL ENERGY DISTRIBUTIONS

In this section, we calculate model SEDs using the method laid out in Section 2. Our object is split into three constituent parts: the protostar, the inner disc, and the outer dynamic disc (plus envelope if present). Figure 6 shows the azimuthally averaged gas surface density (Σ; left panels) and surface temperature (Tsurf; right panels) profiles in model 1 at the four times when the most massive fragments are present in the disc. The inner and outer discs are joined together at r1 = 20 AU. By assumption, the radial profiles of Σ and Tsurf in the inner disc are axisymmetric, but they are not in the outer disc—the averaging procedure could not smooth out azimuthal inhomogeneities caused by spiral arms and fragments. The most massive fragments manifest themselves by clear-cut peaks in Σ at r ≈ 50 AU (t = 0.09 Myr), r ≈ 110 AU (t = 0.1 Myr), and r ≈ 160 AU (t = 0.11 Myr) and t = 0.13 Myr). There are also a number of secondary peaks corresponding to less massive fragments. Tsurf exceeds 10^4 K near the dust sublimation radius (Rν = 0.1 – 0.15 AU, depending on the total luminosity) and declines gradually to 10 K in the envelope. There are also local maxima seen in Tsurf at the positions of the fragments but their magnitudes are considerably smaller than those of Σ due to substantial input of isotropic stellar irradiation into the thermal balance of the outer disc.

Figure 7 presents the resulting SEDs in model 1. In particular, individual contributions from the central protostar, inner disc, and outer disc are shown by the dotted, dashed, and dash-dotted lines, respectively. The solid line is the total SED. A visual inspection of Figure 7 reveals that hot and massive fragments may contribute significantly to the total radiative flux. In particular, the contribution from the outer disc has a characteristic double- peaked structure, with the peak at shorter wavelengths produced by the most massive and hot fragments in the disc. The input from the fragments to the total flux may sometimes exceed that from the inner disc (dotted lines). As a result, the total flux may have a characteristic secondary peak at λ ≈ 5 μm comparable in magnitude to that produced by the protostar at λ ≈ 0.5 μm. This feature is clearly visible at t = 0.09 Myr and t = 0.13 Myr when fragments F1 and F4 have mid-plane temperatures exceeding 1300 K. At these high temperatures dust grains start to evaporate, which leads to a sharp drop in opacity and the corresponding increase in the surface temperature. As Table 1 demonstrates, Tsurf exceeds 700 K for fragment F1 at t = 0.09 Myr and fragment F4 at t = 0.13 Myr. In comparison, fragment F2 at t = 0.1 Myr and fragment F3 at t = 0.11 Myr are characterized by mid-plane temperatures that are below the dust sublimation temperature of ≈ 1200 K. Their surface temperatures are below 450 K, their input to the total SEDs is comparable to that of the inner disc, and the secondary peak at λ ≈ 5 μm is less evident.

To highlight the predicted SED differences between

Figure 6. Azimuthally averaged gas surface density (left-hand column) and effective temperature (right-hand column) distributions in our model at four times after the formation of the central protostar.

Figure 7. Model spectral energy distributions in model 1 at four times after the formation of the central star. Individual contributions from the central protostar, inner disc, and outer disc are shown by the dotted, dashed, and dash-dotted lines, respectively. The solid line is the total SED.
Table 1. Characteristics of the fragments

| fragment | time (Myr) | \( M_f \) \((M_{\text{Jup}})\) | \( M_{\text{H}} \) \((M_{\text{Jup}})\) | \( r_f \) (AU) | \( R_f \) (AU) | \( R_{\text{H}} \) (AU) | \( T_{\text{mp}} \) (K) | \( T_{\text{surf}} \) (K) | \( \Sigma \) (g cm\(^{-2}\)) | \( \tau \) |
|----------|-----------|----------------------------|-----------------|-------------|--------------|-------------------|-----------------|-------------------|----------------|---|
| F1       | 0.09      | 55                         | 68              | 48          | 8            | 17               | 1660            | 1020              | \( 8.7 \times 10^4 \) | 18.5   |
| F2       | 0.1       | 32                         | 47              | 108         | 9            | 32               | 1180            | 380               | \( 2.0 \times 10^4 \) | 256    |
| F3       | 0.11      | 52                         | 63              | 163         | 16           | 56               | 1190            | 445               | \( 1.8 \times 10^4 \) | 139    |
| F4       | 0.13      | 64                         | 84              | 158         | 21           | 55               | 1375            | 740               | \( 3.5 \times 10^4 \) | 32     |

\( M_f \)–the mass of the fragment, \( M_{\text{H}} \)–the mass confined within the Hill radius, \( r_f \)–the radial distance to the fragment, \( R_f \)–the radius of the fragment, \( R_{\text{H}} \)–the Hill radius of the fragment, \( T_{\text{mp}} \)–the midplane temperature of the fragment, \( T_{\text{surf}} \)–the surface temperature of the fragment, \( \Sigma \)–the surface density of the fragment, and \( \tau \)–the optical depth to the midplane of the fragment.

In the previous section, we have shown that the most massive and hot fragments can leave characteristic signatures in the SEDs of young protostellar discs observed through outflow cavities. In this section, we explore the possibility of detecting the fragments directly using interferometric observations and non-fragmenting discs, we show in Figure 6 the spectral energy distributions for the non-fragmenting model 2. Four ages of the disc that approximately match those of model 1 in Figure 7 were considered. Evidently, the SED of the outer disk (dash-dotted lines) has a single-peaked form with a maximum at \( \approx 100\mu \text{m} \). The outer disk contributes a negligible fraction to the total flux at \( \lambda \approx 5\mu \text{m} \), unlike the fragmenting model 1 showing a secondary peak at this wavelength. Nevertheless, there is a small feature at \( \lambda \approx 5\mu \text{m} \), which is not related to the outer disc and is likely caused by simplistic modelling of the inner disc due to the limitations of our numerical hydrodynamics scheme.

Several recent studies have presented observations and models for targets in Taurus. No triple-peaked SEDs like those seen at 0.09, 0.11, or 0.13 Myr in Figure 7 are seen in these studies, suggesting that such massive fragments are not present most of the time. Indeed, such massive fragments should only last at most a few thousand years before migrating onto the star, approximately 1% of the embedded stage duration of a few \( \times 10^5 \) yr (Evans et al. 2009; Vorobyov 2010a; Dunham & Vorobyov 2012). Even if several fragments form and migrate inward during the embedded stage, the total time when such features are observable is likely less than 10%, especially considering the necessity of favorable viewing geometry. Thus it is not surprising that no triple-peaked SEDs like those shown in Figure 6 have been detected given that studies like those listed above generally only contain a few dozen embedded sources. Should such a feature be detected in the future, it would be a strong indication of a massive disk fragment.

6 SYNTHETIC ALMA IMAGES OF FRAGMENTING DISCS

In the previous section, we have shown that the most massive and hot fragments can leave characteristic signatures in the SEDs of young protostellar discs observed through outflow cavities. In this section, we explore the possibility of detecting the fragments directly using interferometric observations with the Atacama Large Millimeter/sub-millimeter Array (ALMA).

To investigate this possibility, we first generated synthetic images following the same procedures described above for producing SEDs except, instead of summing over all spatial positions, we calculated the intensity in each pixel in a cartesian coordinate grid with a grid spacing of 5 AU (0.025") at 345 GHz. We then used the simobserve and simanalyzer tasks in the CASA software package3 to simulate 1 hour of ALMA integration at both 230 and 345 GHz, where these particular frequencies were chosen as the best compromise between resolution, sensitivity, and availability of suitable weather for such observations. The antenna configuration most closely corresponding to a synthesized beam of 0.1" (25 AU at the assumed distance of 250 pc) was selected for each frequency, since such a resolution is the minimum necessary to adequately resolve the disc into its individual fragments. With these simulated observing parameters, the 3\( \sigma \) detection limits of the ALMA images are approximately 0.2 mJy beam\(^{-1}\) at 230 GHz and 0.8 mJy beam\(^{-1}\) at 345 GHz.

Figure 8 presents the gas surface densities in model 1

3 CASA, the Combined Astronomy Software Applications, is the ALMA data reduction and analysis package; see http://casa.nrao.edu for details.
Properties of fragmenting protostellar discs

![Image of gas surface density maps and simulated ALMA images at 230 GHz (middle) and 345 GHz (bottom). The beam size is indicated in white in the bottom right of each panel. The images are displayed on a logarithmic scaling to capture the full dynamic range of the central density peaks, surrounding fragments, and spiral disc structure.]

**Figure 9.** Top row: Gas surface density maps of the inner 2000 × 2000 AU at five times since the formation of the central protostar. The black arrows point to the most massive fragments F1–F4. Middle and bottom rows: The corresponding synthetic ALMA images at 230 GHz (middle) and 345 GHz (bottom). The beam size is indicated in white in the bottom right of each panel. The images are displayed on a logarithmic scaling to capture the full dynamic range of the central density peaks, surrounding fragments, and spiral disc structure.

For instance, fragments F2–F4 at a radial distance of about 100–150 AU are clearly seen in the image and their peak fluxes are well above the detection limits of 0.2 (0.8) mJy beam$^{-1}$ at 230 (345) GHz. On the other hand, fragments that are located at radial distances $r \lesssim 50$ AU start to merge with the central flux peak and are more difficult to detect with our chosen resolution of 0.1″. For instance, fragment F1 (at $r_f = 48$ AU) is beginning to merge with the central peak. This is not surprising considering that the corresponding linear resolution is 25 AU for the adopted distance of 250 pc. However, we note that, once ALMA beings full-science array operations, angular resolutions better than 0.1″ will be achievable, enabling even very closely spaced fragments to be separated (although longer integration times than the 1 hour considered here will be required to reach comparable sensitivities). We also note that the spiral structure is clearly evident in some images.
While the log scaling is useful to emphasize the spiral structure, it unfortunately also enhances the noise. Images with a linear scaling chosen to oversaturate the central density peaks highlight the weak, distant fragments, thus Figure 10 presents such images for the same five times as in Figure 9. The maximum image intensities exceed 20 mJy beam$^{-1}$ but we have set the maximum of the image scales to values in the approximate range of 1–4 mJy beam$^{-1}$, thus oversaturating the images by factors of three or more. Evidently, this helps to resolve weaker and more distant fragments at the cost of losing fragments at $r_f \lesssim 100$ AU, which are now almost completely merged with the central density peaks.

The yellow arrows in the middle and bottom rows of Figure 10 indicate the least massive fragments that can be detected with ALMA. These fragments have masses of $3.5 \, M_{\text{Jup}}$, $2.5 \, M_{\text{Jup}}$, $2.0 \, M_{\text{Jup}}$, $1.5 \, M_{\text{Jup}}$, and $1.0 \, M_{\text{Jup}}$ at $t = 0.09$ Myr, $t = 0.10$ Myr, $t = 0.11$ Myr, $t = 0.13$ Myr, and $t = 0.3$ Myr respectively. Obviously, ALMA can detect fragments down to the planetary mass regime. The largest orbital distance of detectable fragments is about 850 AU.

To illustrate the effect of reduced resolution, we generated the simulated ALMA images in model 1 at the same five times as in Figure 10 but for a resolution of 0.5″. The corresponding images are shown in Figure 11 using the same oversaturated scaling as in Figure 10. Evidently, fragments located at radial distances from the protostar $r \lesssim 200$ AU have completely merged with the central peak. In addition, close-separation fragments now appear as one fragment (e.g., at $t = 0.09$ Myr and 0.3 Myr). Consequently, the total number of identifiable fragments has reduced considerably. We conclude that a resolution of 0.5″ can be used for discs located at distances $\lesssim 150$ pc.

7 MODEL CAVEATS

Uncertainties in dust opacity. In this study, we used the
frequency-integrated opacities of Bell & Lin (1994) to calculate the disc thermal balance and also the frequency-dependent opacities of Ossenkopf & Henning (1994) for the case of thin ices to calculate the flux. Uncertainties in the former may affect the calculated surface temperatures and hence the shape of the model SEDs. In particular, if dust grains evaporate at higher temperatures than assumed in Bell & Lin (1994), than the characteristic secondary peak at $\lambda \approx 5 \mu m$ (see Fig. 7) will be less pronounced. Numerical simulations with other opacity tables are needed to assess the effect.

On the other hand, uncertainties in the adopted frequency-dependent opacities have little effect on the resulting SEDs at $\lambda \lesssim 100 \mu m$ due to high optical depth at these frequencies but may have some effect at longer wavelengths. To evaluate the magnitude of this effect, we calculated the SEDs with frequency-dependent opacities of Ossenkopf & Henning (1994) for the three cases of thin ice, thick ice, and no ice. The resulting radiative fluxes at 230 GHz and 345 GHz differ by a factor of two at most. Finally, we note that opacity is varying vertically, an effect that cannot be captured in our 2D approach.

Non-zero inclination. In this paper, we have constructed SEDs and synthetic ALMA images using a methodology of Chiang & Goldreich (1997) and assuming zero inclination, i.e., for objects viewed pole-on. Our method is best applied to a later evolutionary stage when most of the parental core has dissipated, thus neglecting possible photon scattering or reprocessing by the envelope. To better compare with other studies that present synthetic SEDs of protostars based on global simulation data (e.g., Kurowsawa et al. 2004; Offner et al. 2012), we ran simple Monte Carlo radiative transfer models of a star+disk system embedded within a core. For this purpose, we used the dust radiative transfer package RADMC (Dullemond & Dominik 2004). The SEDs of the net star+disk system shown in Figure 7 were adopted as the input SEDs. A flattened, rotating density structure following the Terebev et al. (1984) solution, with an outflow cavity with an opening semi-angle of 30° was assumed for the core, and the adopted dust opacities were taken from OH5 (Ossenkopf & Henning 1994).

Figure 11. Same as Figure 10 except for a reduced resolution of 0.5′′.
modified to include both absorption and isotropic scattering opacities (see Dunham et al. 2010; Dunham & Vorobyov 2012) for details). The results are shown in Figure 12. Reprocessing by the dust in the core becomes significant for high inclinations but is negligible for inclinations smaller than the semi-opening angle of the outflow cavity, and the excess emission at long wavelengths from dust in the core does not hinder the ability to detect the triple-peaked SED indicative of massive disk fragments. Furthermore, our models do not suggest that scattering will significantly affect the observed mid-infrared SED to the point of hindering the detection of massive disk fragments, but we acknowledge that our models likely underestimate this effect since we are unable to properly treat scattering as anisotropic.

We also note that for large values of $\gamma_{\text{incl}}$, the position angle of the fragment in the disc becomes important. The fragment may be enhanced/suppressed if it is located on the farthest/nearest side of the disc with respect to the observer.

**Background irradiation.** The temperature of background irradiation is set to 10 K in the present study. A higher temperature of background irradiation makes the overall disc warmer. The net result is that the Jeans length is increasing and the fragments become more massive, which may facilitate their detection. At the same time, the disc becomes less prone to fragmentation and, as numerical hydrodynamics simulations of Vorobyov & Basu (2008) demonstrated, fragmentation ceases at $T_{\text{bg}} \gtrsim 50$ K.

**Dust sublimation temperature.** In this study, we adopted a dust sublimation temperature of 1500 K to calculate the sublimation radius in equation (13). We found that varying $T_{\text{d,s}}$ in the 1000–2000 K limits influences our results insignificantly. In particular, for $T_{\text{d,s}} = 1000$ K the maximum decrease in the flux by 18% (with respect to that calculated for $T_{\text{d,s}} = 1500$ K) occurs at $\lambda = 1.2$ $\mu$m, while for $T_{\text{d,s}} = 2000$ K, the corresponding increase in the flux by 46% occurs at $\lambda = 1.75$ $\mu$m.

**Numerical resolution.** The Truelove criterion (Truelove et al. 1998) states that the Jeans length has to be resolved by at least four grid zones in order to correctly study disc fragmentation. The Jeans length $R_J$ of a thin, fragmenting disc can be expressed as $R_J = \frac{2RT_d/\mu}{\pi G\Sigma_d}$.

\[
R_J = \left(\frac{\langle \nu^2 \rangle}{\pi G \Sigma_d}\right)
\]  

where $\langle \nu^2 \rangle = 2RT_d/\mu$ is the velocity dispersion of a thin disc, $R$ is the universal gas constant, $T_d$ is the gas mid-plane temperature in the fragment and $\Sigma_d$ is the gas surface density at the fragment-disc interface. Fragments usually condense out of densest sections of spiral arms. The typical surface densities and temperatures in spiral arms do not exceed 100 g cm$^{-2}$ and 100 K (Vorobyov 2011). Adopting these values for $\Sigma_d$ and $T_d$, the corresponding Jeans length is $R_J \approx 20$ AU.

In our model, the radial and azimuthal grid resolution at $r = 100$ AU is $\approx 1.0$ AU and the Jeans length is resolved by roughly 20 grid zones in each coordinate direction, sufficient to fulfil the Truelove criterion. The resolution deteriorates with increasing distance from the central star and the Truelove criterion is expected to break at $r \gtrsim 500$ AU, where the spatial resolution on our logarithmically spaced grid starts to exceed 5.0 AU and the Jeans length is resolved by less than four grid zones in each coordinate direction. However, fragmentation takes place mostly at radial distances from a few tens to a few hundreds AU. Fragments that are seen in our model at larger distances are most likely scattered from the inner disc regions due to gravitational interaction with other fragments. Higher resolution simulations are planned for the near future to confirm our results.

**Contraction timescale.** The grid resolution imposes limitations on the minimum size of a fragment of the order of 1.0 AU. This means that we cannot follow the contraction of fragments to planetary- or sub-stellar-sized objects and hence we may overestimate the feasibility of using (sub-)millimeter observations to detect the presence of fragments (fully formed planetary or sub-stellar objects will become essentially invisible for the ALMA). The contraction timescale of a fragment to a fully formed object strongly depends on its initial mass. A one-Jupiter-mass fragment will contract for about a few $\times 10^5$ yr, while a ten times more massive fragment will collapse in $< 10^4$ yr (Tohline 2002; Helled & Bodenheimer 2010; Boley et al. 2010). These timescales are comparable to or shorter than the timescale between successive bursts of disc fragmentation, $(7 - 12) \times 10^7$ yr (see Figure 3), which periodically regenerate the population of fragments in the disc. This implies that the probability of detecting fragments increases with decreasing mass of the fragment and it may be quite difficult to "catch" a massive proto-brown dwarf in the process of formation.

**The effect of environment and magnetic fields.** In the present study, we considered the formation of protostellar discs from isolated non-magnetized cores. Numerical hydrodynamics simulations of star cluster formation indicate that radiation feedback from young stars and tidal torques can significantly reduce the disk propensity to fragment (e.g.

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4 In the most extreme cases, some of the fragments may even be ejected from the disc into the intracluster medium (Basu & Vorobyov 2012).
Our model takes into account disc irradiation by the central protostar and background blackbody radiation with a temperature appropriate for isolated cores, \( T_{bb} = 10 \) K, but does not include tidal stripping of the disc. Our results are thus more applicable to isolated star formation in a quiescent environment. In addition, magnetic braking can significantly reduce the disc mass and size in the early Class 0 phase (sometimes even preventing disc formation), also diminishing the disc propensity to fragment (Li et al. 2011; Joos et al. 2012). The effect of magnetic fields can thus reduce the likelihood of detecting the fragments.

8 CONCLUSIONS

Using numerical hydrodynamics simulations in the thin-disc limit, we studied the formation and evolution of a gravitationally unstable protostellar disc with a particular emphasis on the properties of fragments formed via disc gravitational fragmentation. We constructed the spectral energy distribution of our model object viewed pole-on, including the flux from the central protostar, the inner unresolved disc and the outer disc plus envelope system. We also used the ALMA software and generated synthetic images of our model disc at two frequencies best fitted for the fragment detection. We found the following.

(i) The disc experiences multiple episodes of fragmentation in the embedded phase of star formation, with the time between recurrent fragmentation episodes approximately equal to the characteristic time of mass infall onto the disc \( M_d/M \). The fragments can therefore be present in the disc for a significantly longer time period than expected for just one burst of disc fragmentation in numerical hydrodynamics simulations without disc infall.

(ii) The mass distribution function of the fragments have two maxima, one in the planetary-mass regime around \( 5 \) \( M_{\text{Jup}} \) and the other in the upper brown-dwarf-mass regime around \( 60-70 \) \( M_{\text{Jup}} \). The whole mass spectrum extends from about a Jupiter mass to very-low-mass stars. The radial positions of the fragments have a maximum at \( 400-500 \) AU, with a dearth of fragments at \( r < 100 \) AU caused presumably by fast inward migration at these distances.

(iii) The majority of the fragments are characterized by the presence of massive \( (\geq 50 \) \( M_{\text{Jup}} \) ) and hot fragments in protostellar discs. Non-fragmenting discs lack notable peaks around 5 \( \mu \)m.

(iv) The majority of the fragments in our model can be detected with ALMA using just one hour of integration time and a resolution of 0.1", provided that the disc is favourably inclined. The detection limit on the mass of the fragments is \( 1.5 \) \( M_{\text{Jup}} \) at a distance of 250 pc. It is advisable to use a log scaling to resolve the spiral structure and fragments at small distances \( \lesssim 100 \) AU, while the oversaturated linear scaling is best fitted to detect distant and low-mass fragments. The likelihood of detecting the fragments reduces significantly for a lower resolution of 0.5" but can probably be used for objects located at distances of the order of 150 pc and less.

In this paper, we have explored only one fragmentation model. More numerical simulations exploring a wide parameter space of initial core masses, the effects of different inclinations, and environments are further needed.

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APPENDIX A: TESTING THE SED ALGORITHM

To test the spectral energy distribution algorithm described in \( \S \) 2.4, we tried to reproduce the model of Chiang & Goldreich (1997) for a flared disc heated by stellar irradiation. In this model, the disc surface temperature is calculated as

\[
T_{\text{surf}} = \left( \frac{\alpha}{2} \right)^{1/4} \left( \frac{R_*}{r} \right)^{1/2} T_* \tag{A1}
\]

where the grazing angle is \( \alpha = 0.005r_*^{-1} + 0.05r_*^{2/7} \) and \( r_* \) is the radial distance in AU. We used this temperature distribution and applied the inner and outer disc cutoff radii 0.07 AU and 200 AU, respectively. The star was modeled as a spherical black body with temperature \( T_* = 4000 \) K and radius \( R_* = 2.5 \) \( R_\odot \), as in Chiang & Goldreich (1997).

The comparison of the resulting SED with that presented in Chiang & Goldreich (1997) is shown in Figure A1. Our SED is shown by the solid line, with the individual contributions of the disc and the star plotted by dashed and dotted lines, respectively. The SED of Chiang & Goldreich (1997) is indicated with filled circles (stellar input) and asterisks (disc input), respectively. Figure A1 indicates that our algorithm can reasonably well reproduce the results of Chiang & Goldreich (1997).

REFERENCES

Basu S., 1997, ApJ, 485, 240
Basu, S., Vorobyov, E. I. 2012, ApJ, 750, 30
Bate, M. R. 2009, MNRAS, 392, 590

\footnote{We note that Chiang & Goldreich (1997) used a value of 270 AU for the outer disc radius.}
**Figure A1.** Comparison of our test SED with that of Chiang & Goldreich (1997). Individual contributions to the test SED from the central star and the disc are shown with dotted and dashed lines, respectively; the solid line is the total flux. The SED of Chiang & Goldreich are plotted with asterisks (disc input) and filled circles (stellar input).

Beckwith, S. V. W., Sargent, A. I., Chini, R. S., Güsten, R. 1990, AJ, 99, 924
Bell, K. R., Lin, D. N. C., 1994, ApJ, 427, 987
Boss, A. P. 2011, ApJ, 731, 74
Boss, A. P. 2012, MNRAS, 419, 1930
Boley, A. C., Hayfield, T., Mayer, L., Durisen, R. H. 2010, Icarus, 207, 509
Cha, S.-H., Nayakshin, S. 2011, MNRAS, 415, 3319
Chiang, E. I., Goldreich P. 1997, ApJ, 490, 368
Colella, R., Woodward, P. R. 1984, J. Comput. Phys., 54, 174
Cossins, P., Lodato, G., Testi, L. 2010, MNRAS, 407, 181
D'Alessio, P., Calvet, N., Hartmann, L., Lizano, S., & Canto, J. 1999, ApJ, 527, 893
D'Antona, F., & Mazzitelli. I. 1997, Memorie della Societa Astronomica Italiana, 68, 807
Dapp, W. B., & Basu, S. 2009, MNRAS, 395, 1092
Dullemond, C. P., Dominik, C., Natta, A. 2001, ApJ, 560, 957
Dullemond, C. P., & Dominik, C. 2004, A&A, 417, 159
Dunham, M. M., Evans, N. J., II, Terebey, S., Dullemond, C. P., & Young, C. H. 2010b, ApJ, 710, 470
Dunham, M. M., Vorobyov, E. I. 2012, ApJ, 747, 52
Evans, N. J., II, Dunham, M. M., Jørgensen, J. K., et al. 2009, ApJS, 181, 321
Furlan, E., McClure, M., Calvet, N., et al. 2008, ApJS, 176, 184
Furlan, E., Luhman, K. L., Espaillat, C., et al. 2011, ApJS, 195, 3
Gammie, C. F., 2001, ApJ, 553, 174
Helled, R., Bodenheimer, P. 2010, Icarus, 207, 503
Isella, A., Carpenter, J. M., Sargent, A. I. 2009, 701, 260
Johnson, B. M., & Gammie C. F. 2003, ApJ, 597, 131
Joos, M., Hennebelle, P., & Sargent, A. I. 2012, A&A, 543, 128
Jørgensen, J. K., van Dishoeck, E. F., Visser, R., Bourke, T. L., Wilner, D. J., Lommen, D., Hogerheijde, M. R., Myers, P. C. 2009, A&A, 507, 861
Kratter, K. M., Matzner, Ch. D., Krumholz, M. R. 2008, ApJ, 681, 375
Kraus, A. L., Irland, M. J. 2012, ApJ, 745, 5
Kurosawa, R., Harries, T. J., Bate, M. R., & Symington, N. H. 2004, MNRAS, 351, 1134
Li, Z.-Y., Krassnepolsky, R., & Shang, H. 2011, 738, 180
Machida, M. N., Inutsuka, S., Matsumoto, T. 2011, ApJ, 729, 42
Meru, F., Bate, M. R. 2011, MNRAS, 410, 559
Meru, F., Bate, M. R. 2012, arXiv:1209.1107
Nayakshin, S. 2010a, MNRAS, 408, L36
Offner, S. S. R., Klein, R. I., McKee, C. F., & Krumholz, M. R. 2009, ApJ, 703, 131
Offner, S. S. R., Capodilupo, J., Schnee, S., Goodman, A. A. 2012, MNRAS, 420, L53
Offner, S. S. R., Robitaille, T. P., Hansen, C. E., McKee, C. F., & Klein, R. I. 2012, ApJ, 753, 98
Ossenkopf, V. & Henning, Th. 1994, A&A, 291, 943
Palla, F., & Stahler, S. W. 1991, ApJ, 375, 288
Polyachenko V. L., Polyachenko E. V., Strel’nikov A. V., 1997, Astronomy Letters 23, 525
Pringle, J. E., 1981, ARA&A, 19, 137
Rafikov, R. R. 2005, ApJ, 621, 69
Rice, W. K. M., Armitage, P. J., Bate, M. R., & Bonnell, I. A. 2003, MNRAS, 339, 1025
Robitaille, T. P., Whitney, B. A., Indebetouw, R., & Wood, K. 2007, ApJS, 169, 328
Shu, F. H., Najita, J. R., Ostriker, E., et al. 1994, ApJ, 429, 78
Stamatellos, D. Whitworth, A. P. 2009, MNRAS, 392, 413
Stamatellos, D. Whitworth, A. P., & Hubber, D. A. 2011a, ApJ, 730, 32
Stamatellos, D., Maury, A., Whitworth, A., André, P. 2011b, MNRAS, 413, 1787
Terebey, S., Shu, F. H., & Cassen, P. 1984, ApJ, 286, 529
Tohline, J.E. 2002, ARA&A, 40, 349
Toomre, A. 1964, ApJ, 139, 1217
Truelove, J. K. et al. 1998, ApJ, 495, 821
Vorobyov, E. I., Basu, S., 2006, ApJ, 650, 956
Vorobyov, E. I., & Basu, S. 2009, MNRAS, 393, 822
Vorobyov, E. I., 2010a, ApJ, 713, 1059
Vorobyov, E. I., 2010b, ApJ, 723, 1294
Vorobyov, E. I., & Basu, S. 2010a, ApJ, 719, 1896
Vorobyov, E. I., & Basu, S. 2010b, ApJL, 714, 133
Vorobyov, E. I. 2011, ApJ, 729, 146
Vorobyov, E. I. 2012, Astron. Reports, 56, 179
Vorobyov, E. I., 2013, A&A, in revision
Zhu, Z., Hartmann, L., Nelson, R. P., Gammie, C. F. 2012, ApJ, 746, 110