1. Introduction

In the last couple of years the heavy-ion community put a lot of effort in understanding the mechanism present in heavy-ion collisions at ultrarelativistic energies in the experiments at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC). Recent experimental and theoretical results indicate that a new state of matter, the quark-gluon plasma (QGP), with a very short life time is created. Although the QGP is not available for direct observation, its properties can be deduced from the measurement of the produced hadrons in the final state.

The large values of the measured hadronic elliptic flow $v_2$ [1], which is the second coefficient of the Fourier series of the azimuthal particle multiplicity, suggests that equilibration of quarks and gluons occurs on a very short time scale $\leq 1$ fm/c. This also suggests that the shear viscosity over entropy density ratio $\eta/s$ of the QGP is very small, which means that the QGP behaves like a nearly perfect fluid. All these conclusions can be drawn from comparison of experimental results with hydrodynamic calculations. However, an understanding of the mechanism of fast thermalization can not be achieved in the scope of hydrodynamic models. The early pre-equilibrium dynamics of the QGP must be studied in the scope of the microscopic theory.

In contrast to the hydrodynamic approach, kinetic transport theory is a microscopic theory and thus allows to study processes of soft and hard processes simultaneously. This is in particular important for detailed understanding of further properties of the quark-gluonic medium, such as the suppression of jets and heavy-quarks. Suppression of jets, also known as jet quenching, is quantified by comparing the hadron multiplicities measured in heavy-ion collisions with appropriately scaled multiplicities from $p+p$ collisions.
collisions [2]. In addition, very exciting jet-associated particle correlations were observed [3], which might be the result of a conical emission off propagating shock waves in form of Mach Cones. These Mach Cones might be induced by high-energy partons traversing the expanding medium [4]. However, recent studies show that the most promising explanation for the appearing double-peak structure are fluctuating initial conditions [5]. Nevertheless, the issue about Mach cones is yet not clarified and demands a deeper investigation.

The kinetic transport model BAMPS (Boltzmann Approach to Multiparton Scatterings) [6] has been developed to provide a unified description of dynamics of the early QGP stage of heavy-ion collisions (HIC) including perturbative QCD based elastic and inelastic processes. BAMPS has been applied to provide explanations of fast thermalization on a very short time scale \( \leq 1 \text{ fm/c} \) [7] as well as a small value of \( \eta/s \approx 0.08 - 0.2 \) for \( \alpha_s = 0.6 - 0.3 \) [8, 9]. These results have been challenged in Ref. [10], which claims that a mis- or double-counting of symmetry factors when applying the Gunion-Bertsch (GB) matrix element to inelastic processes might lead to an overestimation of interaction rates in BAMPS by a factor of six. In Ref. [11] we addressed these issues in great detail by providing extensive numerical comparisons between the GB approximation and the exact leading order pQCD result for light partons. To this end, we proposed an improved version of the GB matrix element that cures some problems within the GB approximations and, thus, is valid in all regions of phase space. However, we want to emphasize that the deviations of rates computed in the original GB approximation compared to rates computed from the improved GB and the exact matrix element are caused deep within the approximations made by GB and are not given by simple symmetry factors as argued in Ref. [10]. The line of argumentation is outlined in Sec. 3. Recent BAMPS calculations with the original GB matrix element provide results on elliptic flow [12, 13] and jet quenching [14] at RHIC energies, which is for the first time done in a consistent and fully pQCD–based microscopic transport model. New calculations for various observables with the improved GB matrix element are underway. In addition, BAMPS has been used in certain works as a reference for hydrodynamic calculations. This opens the possibility to study hydrodynamic phenomena for arbitrary viscosity.

In these proceedings we demonstrate the application of BAMPS on several number of phenomena observed in the recent heavy-ion experiments. In Sec. 2 BAMPS results on elliptic flow and suppression of charm and bottom quarks for LHC energies are introduced. In the following Sec. 3 we show the comparison of the Gunion-Bertsch approximation to the exact result within leading order perturbative QCD. In Sec. 4 we introduce preliminary results on the application of jet reconstruction algorithms. Finally, in Sec. 5 we demonstrate the evolution of a jet inducing a shock wave in form of a Mach cone in relativistic heavy-ion collisions.

2. Elliptic flow and suppression of heavy quarks

Heavy quarks are a good probe to study the properties of the QGP. They are accurately calibrated due to their well known production process. Furthermore, heavy quarks are produced entirely in the early stage of the heavy ion collision due to their large mass [15] and are also tagged during hadronization due to flavor conservation. While heavy quarks at RHIC can only be measured indirectly via heavy flavor electrons, at LHC for the first time it is possible to reconstruct \( D \) mesons and, therefore, receive information only about charm quarks.

The elliptic flow \( v_2 \) and the nuclear modification factor \( R_{AA} \) are important observables for heavy quarks. Although those particles are rare probes, both observables are experimentally accessible for fragmentation and decay products of heavy quarks such as \( D \) mesons or heavy flavor electrons. The \( R_{AA} \) reflects how much energy heavy quarks loose in the QGP. The \( v_2 \) is large if heavy quarks interact often with the medium and pick up its collective flow.

All the calculations for heavy quarks in this section are done with a running coupling and an improved Debye screening. The latter means that the screening mass of the \( t \) channel of elastic scatterings is determined such that the energy loss matches the energy loss of a heavy quark calculated within the hard thermal loop approach.
Detailed studies in BAMPS [16, 17, 18, 19, 20, 21] show that elastic energy loss of heavy quarks alone is not compatible with the experimental data at RHIC and LHC. However, elastic energy loss explains a significant portion of the overall suppression. If we employ a running coupling and improved Debye screening the experimental data for both $v_2$ and $R_{AA}$ for both RHIC and LHC can be explained if the elastic cross section is multiplied with the artificial factor $K = 3.5$. This indicates that radiative energy loss should be about two times larger than the elastic energy loss. However, this must be checked in a forthcoming study, which will be carried out with the improved Gunion-Bertsch matrix element from Sec. 3 generalized to heavy quarks. On the left hand side of Fig. 1 the nuclear modification factor of $D$ mesons for very central events at LHC is depicted. Our prediction with BAMPS is slightly smaller than the experimental data points. This is in agreement with the observation that we also underestimate the $R_{AA}$ of non-prompt $J/\psi$, heavy flavor electrons, and muons [16].

The right hand side of Fig. 1 shows our predictions of the elliptic flow of $D$ mesons, non-prompt $J/\psi$, heavy flavor electrons, and muons. For $D$ mesons and electrons there is already data available which agrees well with our calculations.

3. Comparison of the Gunion-Bertsch cross section to the exact result

A commonly used approximation to the leading order perturbative QCD matrix element for partonic $2 \leftrightarrow 3$ processes is a result derived by GUNION and BERTSCH (GB) [25]. This approximation gives a comparatively simple expression for the gluon radiation amplitude in terms of the transverse momentum of the radiated gluon $k_\perp$ and the transverse exchanged momentum $q_\perp$.

When employing the matrix element to obtain rates or cross sections from phase space integration, the amplitude by GUNION and BERTSCH deviates from the exact result [26] in characteristic regions of the phase space [11], which leads to a sizeable deviation [10]. Therefore, we propose an improved version of the Gunion-Bertsch matrix element which agrees very well with the exact result in all phase space regions:

$$\left| \mathcal{M}_{qq' \rightarrow qq' g} \right|^2 \simeq 12g^2 \left| \mathcal{M}_{qq' \rightarrow qq' g} \right|_{sa}^2 \frac{q_\perp^2}{k_\perp^2 (q_\perp - k_\perp)^2}$$

where “sa” stands for small angle approximation, $g$ is the strong coupling and $\bar{x}$ the symmetrized longitudinal momentum fraction of the emitted gluon,

$$\bar{x} = \frac{k_\perp}{\sqrt{s}} |y|.$$
Figure 2. Left: Differential cross section $d\sigma/dy$ for the process $qq' \rightarrow qq'g$ calculated with the exact \cite{26}, original GB \cite{25}, GB with $(1-x)^2$, and improved GB with $(1-\bar{x})^2$ (Eq. (1)) matrix element. Right: Ratio of total cross section $\sigma_{2 \rightarrow 3}$ of the improved and original GB matrix elements to the exact one for different processes. For details see Ref. \cite{11}.

The derivation is explicitly shown in Ref. \cite{11}. The difference to the original GB matrix element consists of two parts, a) keeping a kinematic factor $(1-x)^2$, where $x$ is the fraction of light cone momentum carried by the radiated gluon; and b) respecting the symmetry of the process by explicitly combining results from the two gauge choices $A^+ = 0$ and $A^- = 0$, restricting the emission of gluons to the respective forward direction. The latter is necessary since the GB approximations are not applicable in the respective backward regions for both gauges \cite{11}. In contrast to the original GB matrix element, the improved result is not only valid at mid-rapidity, but also at forward and backward rapidity.

The left panel of Fig. 2 compares the rapidity spectrum of the emitted gluon as given by different approximations of the matrix element by depicting the differential cross section $d\sigma/dy$ for the process $qq' \rightarrow qq'g$. Although the original GB matrix element agrees very well with the exact result at mid-rapidity (the region in which GUNION and Bertsch were mainly interested in) large deviations can be seen at forward and backward rapidity. Based on a detailed analytic investigation of the underlying approximations, our proposed improved version of the GB matrix element agrees very well in all phase space regions with the exact result. This remarkable agreement holds for virtually all center of mass energies $\sqrt{s}$ as well as for the other radiative processes $qg \rightarrow qgg$ and $gg \rightarrow ggg$, which is depicted on the right hand side of Fig. 2.

The implementation of the new improved GB matrix element in our transport model BAMPS is currently under way. As part of this study we will also implement the running coupling for elastic and radiative processes as well as radiative processes for heavy quarks in Sec. 2.

4. Jet reconstruction within BAMPS

Another method for studying the parton energy loss inside a heavy-ion medium is the reconstruction of jets. The initial hard scattering processes of the approaching nucleons lead to back-to-back parton pairs, which gain a high amount of virtuality during these scattering processes. In the subsequent evolution of the partons, they try to decrease their virtuality by splitting processes like $q \rightarrow qg$ or $g \rightarrow gg$, which can be described by the DGLAP evolution equation \cite{27, 28, 29}. These fragmentation processes lead to particle showers with a broad angle and momentum distribution. In order to provide a description of the energy loss mechanism inside the created medium, jet reconstruction methods \cite{30, 31} are used. They combine single shower particles to a common “full jet” based on their distances $\Delta R = \sqrt{(\Delta y)^2 + (\Delta \phi)^2}$ to the jet axis. The idea behind reconstructing jets within heavy-ion collisions is that when subtracting appropriately any background medium contribution out of the jets, jet reconstruction allows the direct
study of the in-medium energy loss of the initial high \( p_t \) partons.

In \( p + p \) collisions, where no medium creation is expected, these splitting processes already lead to an imbalance in the momenta of the reconstructed jets with the two highest transverse momenta. These jets are associated with the initial back-to-back parton pair and the momentum asymmetry is caused by stochastically distributed vacuum splitting processes out of the considered jet cone. Experimental results \([32, 33, 34]\) in \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \) Pb + Pb collisions at the LHC show a significant enhancement of this momentum imbalance in central HIC in comparison to \( p + p \)-collisions. As a measure of this enhancement the momentum imbalance \( A_J \),

\[
A_J = \frac{p_{t;\text{Leading}} - p_{t;\text{Subleading}}}{p_{t;\text{Leading}} + p_{t;\text{Subleading}}},
\]

is defined, where \( p_{t;\text{Leading}} \) (\( p_{t;\text{Subleading}} \)) is the reconstructed transverse momentum of the jet with the highest (second highest) transverse momentum. The suppression of balanced events in HIC is supposed to be the result of different in-medium energy loss of the two partons within the created bulk medium, which is a consequence of a non-central spatial production point of the initial di-jet pair.

In this section we present our preliminary results on the momentum imbalance \( A_J \) simulated within the transport model BAMPS with a constant coupling \( \alpha_s = 0.3 \). For the initial momentum spectra of the partons we use a distribution sampled according to a parametrized parton distribution \([35]\), starting at \( p_{t,0} = 100 \text{ GeV} \). Because BAMPS describes only scattering processes of particles on the mass-shell, it is necessary to model the initial splitting processes of the virtual partons properly for reproducing the findings in \( p + p \) collisions. Therefore the shower routines of the event generator PYTHIA \([36]\) are used to model the virtual splitting processes. Because the medium modification of the created parton showers is to be evaluated within the BAMPS framework, it is necessary to switch off any hadronization processes and terminate the splitting processes within PYTHIA prematurely. For that reason, the standard PYTHIA global termination criterion in the virtuality \( Q_0 = 1 \text{ GeV} \) is replaced by an energy-dependent minimum virtuality scale \( Q_0 = \sqrt{t_{\text{min}}} \) depending on the individual parton energy and a global shower time \( \tau \).

Throughout this section the shower time is assumed as \( \tau = 0.2 \text{ fm} \). Calculations within a static medium showed that the energy loss of the reconstructed jets is, for realistic values of \( \tau \), nearly independent of the used shower time. The initial spatial production points of the parton pairs are determined by a Glauber modeling of the initial nucleus-nucleus collisions based on a Woods-Saxon density profile.

The created parton showers are subsequently evolved within an offline recorded BAMPS background event. At every timestep the shower particles may interact with medium particles which afterwards become shower particles by their own. Only the so defined shower particles are used for reconstructing jets by the “anti-k_t”-algorithm \([37]\) with resolution parameter \( R = 0.3 \) as implemented in the common “FastJet”-package \([38]\). As a first step for considering background effects, transverse momentum cuts \( p_{t;\text{cut}} \) are used to separate shower particles from the underlying background medium. Therefore the in-medium scatterings processes of each shower particle are stopped individually when reaching \( p_{t;\text{cut}} \).

Figure 3 (left) shows the calculated \( A_J \) distribution for central \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \) Pb + Pb collisions (0-10%, which corresponds to a mean impact parameter \( b = 3.4 \text{ fm} \)) in comparison with data by CMS for different \( p_{t;\text{cut}} \) values. Experimental trigger conditions by CMS (\( p_{t;\text{Leading}} > 120 \text{ GeV}, p_{t;\text{Subleading}} > 30 \text{ GeV}, \Delta \phi > \frac{\pi}{2} \) and \( |\eta_{\text{jet}}| < 2 \)) are employed and an effective modeling of the experimental jet energy resolution is used. For that an independent Gaussian smearing of the reconstructed jet momenta based on a comparison between PYTHIA simulations and \( p + p \) data by CMS \([34]\) is applied.

The calculations show that the shape of the \( A_J \) distribution is highly sensitive to the momentum cut in the shower evolution. Simulations with an earlier termination of the scattering processes lead to a momentum imbalance which is peaked around \( A_J \approx 0.6 \) whose value is limited by the employed trigger conditions. For lower values of \( p_{t;\text{cut}} \) and therefore a later cut in the shower evolution, the distribution gets shifted to more balanced events.

The used cut parameter \( p_{t;\text{cut}} \) is obviously not sufficient to fulfill the assumed separation between the shower particles and the underlying medium particles. With a lower \( p_{t;\text{cut}} \) parameter the further
in-medium scattering processes lead to a higher contamination of medium momentum within the reconstructed jet momenta. This momentum contamination is similar to both the leading and subleading jet and at the same time greater than the in-medium momentum loss out of the reconstructed jets. Hence, the $A_J$ distribution is shifted to events with balanced leading jet momenta.

To subtract the background contamination in the reconstructed jets more effectively, we have studied the sampling of appropriate background events in which the scattered medium particles are removed to prevent a double-counting of particles simultaneously in the shower and the medium list. These pruned events can afterwards be superposed with the simulated shower particles to get events with all shower and background particles. After reconstructing jets based on these events, common experimental subtraction algorithms [39, 34] can be used to subtract the background contamination within the reconstructed jets. However, the sampling of background events with removed scattered medium particles is not trivial: Because of the used test-particle method within BAMPS, the background medium events have different test-particle numbers than the overlayed shower events. Therefore we employ a local sampling method based on a grid in the $\eta - \phi$-plane. Detailed studies have shown that this sampling procedure leads to an uncertainty of $\approx 10\%$ in the transverse momentum of jets reconstructed in background events without additional shower particles.

Figure 3 (right) shows the calculated $A_J$ distribution with the already described parameters and trigger conditions but with reconstructed jets based on shower as well as medium particles which are subtracted by a “CMS noise/pedestal”[34]-like background subtraction method. The distribution of $A_J$ is not longer dependent on the used cut $p_{t;\text{cut}}$ in the shower evolution: It shows a strong peak structure at high $A_J$ values independent from $p_{t;\text{cut}}$. This means, that the medium $p_t$ contamination is successfully removed by the employed background subtraction. However, the enhancement and thereby the relative jet momentum loss within BAMPS is too strong in comparison with data. This is in qualitative agreement with former studies of the nuclear modification factor $R_{AA}$ both at RHIC and LHC [20].

We have presented a procedure in which it is possible to simulate the evolution of the parton shower within a partonic medium while considering any recoil effects and further in-medium scattering processes. While considering these further in-medium scattering processes it is insufficient to use a trivial momentum cut in the in-medium shower evolution to separate between the shower and medium regime. Furthermore, we have shown that when studying reconstructed jets based on events with shower and medium particles together with a background subtraction, the momentum imbalance is independent from the employed cut in the shower evolution. Similar to the single inclusive hadron suppression,
the momentum imbalance of reconstructed jet pairs within BAMPS is too strong in comparison with experimental data. However, recent studies [11] propose an improvement of the Gunion-Bertsch matrix element, which is employed for the radiative $2 \to 3$ processes within BAMPS. The implementation of the improved Gunion-Bertsch matrix element into BAMPS is currently under way and with it we expect much less partonic energy loss. Furthermore, it is planned to realize a stochastic modeling of the Landau-Pomeranchuk-Migdal effect for radiative processes of hard probes and to investigate its implications for the magnitude and shape of high-pt and jet observables.

5. Transition from ideal to viscous Mach cones in BAMPS

Highly energetic partons propagating through the hot and dense QGP rapidly lose their energy and momentum as the energy is deposited in the medium. Measurements of two- and three-particle correlations in heavy-ion collisions show a complete suppression of the away-side jet, whereas for lower $p_T$ a double peak structure is observed in the two-particle correlation function [3]. One possible origin of these structures were assumed to be the interaction of fast partons with the soft matter which generates collective motion of the medium in form of Mach cones. [4, 40, 41, 42, 43, 44]. Although today the most promising explanation for this phenomenon is due to fluctuating initial conditions[5], we address here the question whether the jet-induced Mach cones can still be considered as possible candidates.

In a previous work [45] we demonstrated in BAMPS the transition of Mach cones from perfect fluid limit to the highly viscous regime by adjusting the shear viscosity over entropy density ratio $\eta/s$. For this purpose we used a simplified setup of a static box in order to investigate the pure evolution of a Mach cone. Using two different source terms with an infinite energy reservoir we have shown, that the double peak structure in extracted two-particle correlations appears only, if the source deposits only energy, but no momentum. However, this only holds if in addition the energy deposition is large enough. Furthermore, a finite shear viscosity over entropy density ratio $\eta/s$ can destroy this double peak structure.

Although this source term with only energy but no momentum deposition was able to explain under some circumstances the double peak structure in two-particle correlations, the physical motivation for this source term is weak, since its properties does not fit in the usual picture of a jet assumed in heavy-ion collisions. Therefore, the origin of such a double peak structure observed in heavy-ion collision is most probably not the source alone, but maybe medium effect, such as the expansion of the medium. Furthermore, due to the infinite energy in our previous setups, the source never stopped, which also may have an effect of the final results.

In this proceeding we demonstrate first preliminary results using a more realistic setup in order to describe the evolution of jet-induced Mach cones in relativistic collisions of heavy-ions. For this purpose we use smoothed initial conditions for the initial setup and consider only central collisions, $b = 0$, in order to avoid effects coming from elliptic flow $v_2$. For the initial single-particle distribution function we use the following parametrization:

$$f(x, p) = K \frac{1}{E} \left( \frac{Q^n}{Q^n + p_T^n} \right)^m e^{-(y_{rap}/\sigma_y^2)} e^{-(z^2/\sigma_z^2)} T_A (x_T - b/2) T_B (x_T + b/2)$$  \hspace{1cm} (4)

where $T_A$ and $T_B$ are the nuclear thickness functions and $y_{rap}$ is the longitudinal momentum rapidity. The parameters we use are $Q = 1.3$ GeV, $n = 4$, $m = 1.5$, $\sigma_y = 1$ and $\sigma_z = 0.13$ fm. For the normalization we use $K = 0.0135$. The parameters are chosen in such a way to fit the final spectra of recent heavy-ion collisions at RHIC. The shear viscosity over entropy density ratio is kept constant $\eta/s = 0.08$ in the whole simulation. We use only binary collisions with an isotropic angle distribution. The jet is initialized at midrapidity with $E = 50$ GeV and looses its energy only via binary scatterings. Due to the test particle method [6] the number of jets in BAMPS has to be scaled with the number of test particles, $N_{test}$. This is required in order to preserve the correct relation between jet and medium.

On the left hand side of Fig. 4 we show a snapshot at $t = 8$ fm/c at midrapidity of the simulation in BAMPS. In order to demonstrate the induced shock wave of the jet, we set the initial position of the jet...
to $x = -4$ fm and $y = 0$ fm. The jet initially propagates in positive $x$-direction. As expected, due to the sufficient small viscosity we observe the development of a strongly curved shock wave propagating through the medium. The strongly curved profile is a result of the loosing energy of the jet as well as the fact that the medium expands very fast. On the right hand side of Fig. 4 we show the corresponding two-particle correlations of the simulation at $t = 12$ fm/$c$. We clearly observe only a peak at $\Delta \phi = 0$, and not a double peak. This indicates, that we either have to use special momentum cuts in order to observe a double peak structure, or the expansion of the medium as well as the more realistic jet scenario are still not enough to obtain a double peak structure. Further investigations on this topic using the transport model BAMPS are currently under way.

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