High magnetic pulse by solenoids for intense ion beam transport

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Abstract. A high voltage pulser up to 50 kV, with a main pulse of 2 μs time duration, was realized using a fast high voltage capacitor of 150 nF (50 kV) connected to the ground by a homemade spark-gap. The circuit, contained in a metallic box to avoid irradiation, had short electric connections in order to reduce the inductances. The diagnostic was performed by two systems: a self-integrating Rogowski coil and an integrator connected to a current transformer, both suitable for pulses longer than 1 μs. To focus ion beams with energy reaching 40 keV, we made two solenoids having an inductance of 3.6 and 16 μH which were fed by the current generated by the pulser up to 12 kA, 1 μs at 50 kV of charging voltage of capacitor. The measurements determined a magnetic field at the solenoid center of 0.61 T for the more inductive solenoid and 1.14 T for the other one.

1. Introduction
In recent years, several techniques of laser-matter interactions have been used in order to produce ion beams. They have already found useful applications in many physical environments as well as out – of – physics applications (biology, medicine etc.). The problem we try to face in this paper is how to create devices able to focus ion beams using magnetic fields. Both quadrupole[1] and solenoidal[2] beam transport schemes are feasible [3], but we will focus on the latter one, attempting to find an optimized configuration to earn the maximum magnetic field strength.

2. Theory
In classical electrodynamics theory, it’s easy to show a variation along the radius of a solenoid gives rise to a force which can trap the particles in a magnetic field[4]. In a cylindrical symmetry, supposed the magnetic field \( \mathbf{B} \) is axisymmetric, that means in cylindrical coordinates \( B_\vartheta = 0 \) and \( \partial B/\partial \vartheta = 0 \), it’s possible to know from Maxwell’s equation \( \nabla \cdot \mathbf{B} = 0 \) written in cylindrical coordinates \((r, \vartheta, z)\) :

\[
B_r = -\frac{1}{2} r \left( \frac{\partial B_z}{\partial z} \right)_{r=0}
\]

assuming a negligible variation of the field with \( r \).

Since the expression for a magnetic field for \( r \to 0 \) in a finite solenoid is known, it’s possible to get the expression for the average force along the \( z \) axis as well as its radial component:
where \( v_{\perp} \) is the speed lying in the plane perpendicular to \( B \), depending on the particle \( q \) charge through the relation 
\[
:v_{\perp} = \frac{q r_L B}{m},
\]
where \( r_L \) is the Larmor radius of the particle trajectory.

The focusing of the ions is however not perfect. It depends on many factors, but our work can be simplified by the following considerations: a particle with \( v_{\perp} = 0 \) has no magnetic moment which is \( \mu = \frac{1}{2} \frac{m v_{\perp}^2}{B} \), and won’t feel any force along \( B \). A particle with small rate between speed perpendicular and parallel to \( B \) will also escape if the maximum value of the magnetic field \( B \) is not large enough. That’s why we can only focus on the maximum values reached by the magnetic field in the solenoid in order to check how good its focusing skills will be.

3. Materials and methods
We are going to test two different solenoids with different geometrical dimensions, comparing the inductance properties able to satisfy our needs. The first solenoid (denoted in the following with S1) has an inductance of \( 3.6 \mu H \) (11 rings, length 12 cm) and the second one (S2) an inductance of \( 16 \mu H \) (20 rings, length 20 cm). Both S1 and S2 have been realized by a copper wire with diameter 8 mm (S1) and 5 mm (S2). To generate \( B \), we connected the solenoids to a pulser with a (negative) high-voltage power supply, composed by a capacitor of 150 nF (100 kV), closed on the solenoids by a homemade fast spark gap. The sketch of the pulser is shown in Fig.1.

![Figure 1. Experimental setup. C: capacitor; SG: spark gap; RC: Rogowski coil, I: integrator; D: divider 20x.](image)

The power supply provided charge to the capacitor through the resistor \( R_c \) (200 k\( \Omega \)) and the resistor \( R_q \) (200 k\( \Omega \)). The discharge current in the capacitor is measured with two different diagnostic systems: a Rogowski coil and an integrator connected to a current transformer.

Our Rogowski coil [5] is composed of 140 rings realized by a conductive wire with diameter 0.8 mm in order to reduce the resistance. The diameter of the rings is 2 cm, while the one of the coil is 3.5 cm. Its inductance is \( 11 \mu H \) and its resistance 0.4 \( \Omega \). In order to get the properties of a self-integrator, the coil was closed on a load resistor of 0.5 \( \Omega \) (see Fig. 2a). In this conditions the integrating time \( L/R = 12 \mu s \). Therefore, the theory tells us the longest pulse the system can diagnose is less than 12 \( \mu s \), about 2 \( \mu s \).
The second diagnostic system, the integrator connected to a current transformer, consists of a coil with 38 rings connected to an integrator composed by a resistor of 331 Ω and an capacitor of 47 nF connected to ground. The relatively small number of rings allows to decrease the stray capacitance between the coil and the case, avoiding oscillations and allowing the application of the integrator circuit of high input impedance (See Fig. 2b). In this case the longest pulse the system can diagnose is also about 2 μs.

The measured signals provided by the two detectors, allow us to know the current flowing into the solenoid and consequently the magnetic field inside. As the theory predicts, the output waveform signal is a damped sinusoid.

The calibration of both the systems was performed by means of a sinusoidal pulse very close to the one obtained during the experiment. It’s obtained by a pulser having a 150 nF capacitor connected by a fast switch to a 50 Ω coaxial structure. Additionally, during the calibration process we used an additional resistor in the circuit of 4.5 Ω to measure the current on the resistor, see Fig. 3.

From the displayed signal, we could calculate the input/output ratio. It’s a time-dependent quantity, so the computation of its value was performed in coincidence of the first peak of the signal (≈2 μs). Adopting this method the extracted values are 335 A/V for the Rogowski coil and 519 A/V for the integrator connected to the current transformer at an applied voltage of 20 kV.
4. Magnetic field mapping

The high current flowing into the solenoid and the presence of high voltage don’t allow to measure the magnetic field. What we do to overcome this obstacle is, in a first place, to take a look at the theory: in both a finite and an infinite solenoid, the expression of the magnetic field keeps its linear dependence on the current, although its space dependence drastically changes. In addition to this, as we showed in theory section, the knowledge of the magnetic field \( B_z \) as a function of \( z \) on the solenoid’s axis \((r=0)\) can provide us a value for the radial magnetic field and allows us to calculate the average force exerted on the particle along \( z \).

We show a table with the geometrical features of the examined solenoids:

| Feature     | S1        | S2        |
|-------------|-----------|-----------|
| Radius \( a \) | 30 mm    | 45 mm    |
| Coils number \( N \) | 11        | 20        |
| Length \( L \) | 12 cm    | 20 cm    |

The solenoids described above have been mapped in a low – current regime (10 A) by means of a Hirst GM07 gaussmeter. The resulting plots are shown here compared with the theoretical behavior:

It’s easy to notice the better agreement of the theory with the magnetic mapping of S2, probably due to a more uniform coil distribution per length unit.
Figure 7. Magnetic field mapping in S2.

The plot takes the origin of the $z$ axis in the middle of the solenoids, so that we can stress how close to the theoretical symmetric behavior the solenoids are. The analytical expression provided by the theory for the magnetic field is [6]:

$$B_z(z) \cong \frac{\mu_0 NI}{2L} \left[ \frac{z+\frac{L}{2}}{\sqrt{(z+\frac{L}{2})^2+a^2}} - \frac{z-\frac{L}{2}}{\sqrt{(z-\frac{L}{2})^2+a^2}} \right]$$

for $r \to 0$.

Now it’s possible to calculate the maximum value of the magnetic field. Using the procedure described above, denoting the low - current regime with the subscript L and the high - current one with H, we can get an expression for the ratio of the magnetic fields in the same position, hence the magnetic field we’re interested to:

$$\frac{B_H}{B_L} = \frac{I_H}{I_L} \Rightarrow B_H = \frac{B_L}{I_L} I_L.$$

This provides us the following values of the magnetic field strength, reached with the maximum applied voltage, 50 kV. We denote with the subscripts 1 and 2, S1 and S2 respectively:

$$B_1 = 1.14T$$
$$B_2 = 0.61T$$

while the flowing currents resulted of 11.7 kA and 5.4 kA, respectively. The magnetic field mapping helps us, using the previous formula, to calculate the magnetic field in the boundaries of the solenoid, obtaining at 50 kV, in $z$ direction:

$$B_1 \cong 0.41T$$
$$B_2 \cong 0.28T.$$

Using the relation between $z$ and $r$ components, it’s possible to get the radial component in the boundaries ($r = a, z = -L/2$) for S1 and S2:

$$B_{1r} = 0.31T$$
$$B_{2r} = 0.09T.$$

useful to know the conditions of the particles when they enter the solenoids.

Typical waveform of the magnetic field (current) is showed in Fig. 8 recorded by the Rogowski coil for S1.
As predicted by the theory, the output signal is a damped sinusoid having a frequency of 215 kHz. The corresponding signal for S2 was 103 kHz.

5. Conclusions
In this paper the design of a fast high voltage circuit to feed focusing solenoids has been proposed. The experimental setup has been optimized so that the maximum intensity of the current can be achieved in the solenoid and as a consequence also high magnetic field strengths inside the solenoids able to focus ion beams\cite{7, 8}. A rapid calculation \cite{9} based on the solenoids dimensions and our maximum magnetic field intensities can estimate a maximum beam energy for a proton beam about 40 keV in a non-relativistic regime.

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