Detecting gravitational radiation from neutron stars using a six-parameter adaptive MCMC method

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Abstract. We present a Markov chain Monte Carlo technique for detecting gravitational radiation from a neutron star in laser interferometer data. The algorithm can estimate up to six unknown parameters of the target, including the rotation frequency and frequency derivative, using reparametrization, delayed rejection and simulated annealing. We highlight how a simple extension of the method, distributed over multiple computer processors, will allow for a search over a narrow frequency band. The ultimate goal of this research is to search for sources at a known location, but uncertain spin parameters, such as may be found in SN1987A.

1. Introduction

Rapidly rotating neutron stars could be an important source of gravitational wave signals. Several mechanisms have been proposed that would cause them to emit quasi-periodic gravitational waves [5, 6].

Interferometric gravitational wave detectors are now operating in numerous locations around the world [1, 2, 3, 4], and much work has gone into the development of dedicated search algorithms for these signals. Radio observations can provide the sky location, rotation frequency and spin-down rate of known pulsars, and this knowledge simplifies the analysis. This was the case for the recent search for a signal from PSR J1939+2134 [7]. When the position and phase evolution of a source are not known, all-sky hierarchical strategies are required, and these have huge computational requirements [8, 9].

Here we concentrate on the search for a gravitational wave signal from a known location, but where spin parameters of the rotating neutron star are not well known (but within a narrow band). SN1987A is a good example of a poorly parameterised source for which the sky location is known, but where there are large uncertainties in the frequency and spin-down parameters of a putative neutron star [10]. In particular,
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we consider a search with six unknown parameters: the gravitational wave amplitude $h_0$, the polarization angle $\psi$ (which depends on the position angle of the spin axis in the plane of the sky), the phase of the signal at a fiducial time $\phi_0$, the inclination of the spin axis with respect to the line-of-sight $\iota$, the uncertainty in the absolute value of the signal frequency $\Delta f$, and the frequency derivative $\dot{\Delta} f$.

We use a Bayesian Markov chain Monte Carlo (MCMC) technique for this analysis as MCMC methods have been applied successfully to similar problems involving large numbers of parameters [11]. In a previous study [14], we used a Metropolis-Hastings (MH) algorithm [12, 13] for a similar search, but with only five parameters ($\Delta f$ being absent). When the frequency derivative $\Delta f$ is included in the basic MH routine of [14] the large correlation between $\Delta f$ and $\Delta \dot{f}$ makes the parameter search difficult, and the basic MH algorithm becomes inefficient. In order to adequately sample the parameter space we implemented a combination of three different strategies for accelerating convergence of Markov chains: reparameterisation, the delayed rejection method of Tierney and Mira [15] (which is an adaptive version of the MH algorithm), and simulated annealing [16] (which is a Monte Carlo approach to global optimization). The parameter $\Delta f$ is highly correlated with $\Delta \dot{f}$, and a strong correlation also exists between $h_0$ and $\cos \iota$. An initial transformation of these variables to near orthogonality yields a more tractable parameter space that is more effectively sampled.

The heterodyne manipulation of the data used in this study is identical to that presented (by two of us) in an end-to-end robust Bayesian method of searching for periodic signals in gravitational wave interferometer data [17], and is also described in [7]. A brief summary of this heterodyne technique is given in Sec. 2. Our delayed rejection method, as well as the reparameterisation strategy, is presented in Sec. 3. In Sec. 4 we present the results of this study, using synthesized signals, for this six parameter problem. A brief discussion of the long term goals for this work are presented in Sec. 5.

2. The gravitational wave signal

Gravitational waves from spinning neutron stars are expected to be weak at the Earth, therefore long integration periods are necessary to extract the signal. It is therefore important to take proper account of the antenna patterns of the detectors and the Doppler shift due to the motion of the Earth.

As in previous studies [7, 14, 17] we consider the signal expected from a non-precessing triaxial neutron star. The gravitational wave signal from such an object is at twice its rotation frequency, $f_s = 2f_r$, and we characterise the amplitudes of each polarization with overall strain factor, $h_0$. The measured gravitational wave signal will also depend on the antenna patterns of the detector for the ‘cross’ and ‘plus’ polarisations, $F_{\times,+}$, giving a signal

$$s(t) = \frac{1}{2}F_{+}(t; \psi)h_0(1 + \cos^2 \iota) \cos \Psi(t) + F_{\times}(t; \psi)h_0 \cos \iota \sin \Psi(t),$$

A simple slowdown model provides the phase evolution of the signal as

$$\Psi(t) = \phi_0 + 2\pi \left[f_s(T - T_0) + \frac{1}{2}\dot{f}_s(T - T_0)^2\right],$$

where

$$T = t + \delta t = t + \frac{\vec{r} \cdot \vec{n}}{c} + \Delta T.$$
Here, \( T \) is the time of arrival of the signal at the solar system barycenter, \( \phi_0 \) is the phase of the signal at a fiducial time \( T_0 \), \( \vec{r} \) is the position of the detector with respect to the solar system barycenter, \( \vec{n} \) is a unit vector in the direction of the neutron star, \( c \) is the speed of light, and \( \Delta T \) contains the relativistic corrections to the arrival time [18].

If \( f_s \) and \( \dot{f}_s \) are known from (for example) radio observations, the signal can be heterodyned by multiplying the data by \( \exp[-i \Psi(t)] \), low-pass filtered and resampled, so that the only time varying quantity remaining is the antenna pattern of the interferometer. We are left with a simple model with four unknown parameters \( h_0, \psi, \phi_0 \), and \( \epsilon \). If there is an uncertainty in the frequency and frequency derivative then we have two additional parameters, the differences between the signal and heterodyne frequency and frequency derivatives, \( \Delta f \) and \( \Delta \dot{f} \), giving a total of six unknown parameters.

A detailed description of the heterodyning procedure is presented elsewhere [7, 17]. Here we just provide a brief summary of this standard technique. The raw signal, \( s(t) \), is centered near twice the rotation frequency of the neutron star, but is Doppler modulated due to the motion of the Earth and the orbit of the neutron star if it is in a binary system. The modulation bandwidth is typically \( 10^4 \) times less than the detector bandwidth, so one can greatly reduce the effective data rate by extracting this band and shifting it to zero frequency. In its standard form the result is one binned data point, \( B_k \), every minute, containing all the relevant information from the original time series but at only \( 2 \times 10^{-6} \) the original data rate. If the phase evolution has been correctly accounted for at this heterodyning stage then the only time-varying component left in the signal will be the effect of the antenna pattern of the interferometer, as its geometry with respect to the neutron star varies with Earth rotation. Any small error, \( \Delta f \), in the heterodyne frequency will cause the signal to oscillate at \( \Delta f \) (plus the residual Doppler shift). We estimate the noise variance, \( \sigma_k^2 \), in the bin values, \( B_k \), from the sample variance of the contributing data. It is assumed that the noise is stationary over the 60 s of data contributing to each bin.

### 3. The adaptive Metropolis-Hastings algorithm

After heterodyning, the signal on which we wish to carry out our MCMC analysis has the form [17]

\[
y(t_k; \mathbf{a}) = \frac{1}{4} F_x(t_k; \psi) h_0 (1 + \cos^2 \epsilon) e^{i \Delta \Psi(t)} - \frac{i}{2} F_x(t_k; \psi) h_0 \cos \epsilon e^{i \Delta \Psi(t)},
\]  

where \( t_k \) is the time of the \( k \)th bin \( B_k \) and \( \mathbf{a} = (h_0, \psi, \phi_0, \cos \epsilon, \Delta f, \Delta \dot{f}) \) is a vector of our unknown parameters. \( \Delta \Psi(t) \) represents the residual phase evolution of the signal, equalling \( \phi_0 + 2\pi[\Delta f (T - T_0) + \Delta \dot{f} (T - T_0)^2 / 2] \). The objective is to fit this model to the antenna output data

\[
B_k = y(t_k; \mathbf{a}) + \epsilon_k,
\]

where \( \epsilon_k \) is assumed to be normally distributed noise with a mean of zero and known variance \( \sigma_k^2 \). Assuming exchangeability of the binned data points, \( B_k \), the joint likelihood that these data \( \mathbf{d} = \{B_k\} \) arise from a model with a certain parameter vector \( \mathbf{a} \) is [17]

\[
p(\mathbf{d}|\mathbf{a}) \propto \prod_k \exp \left[ -\frac{1}{2} \frac{|B_k - y(t_k; \mathbf{a})|^2}{\sigma_k} \right] = \exp \left[ -\frac{\chi^2(\mathbf{a})}{2} \right],
\]
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where

$$\chi^2(a) = \sum_k \left| \frac{B_k - y(t_k; a)}{\sigma_k} \right|^2.$$  

(7)

In order to draw any inference on the unknown parameter vector $a$ we need the (posterior) probability of $a$ given $d$ which can be obtained from the likelihood via an application of Bayes’ theorem. The unnormalized posterior density

$$p(a|d) \propto p(a)p(d|a)$$

(8)

is the product of the prior density of $a$, $p(a)$, and the joint likelihood. Accordingly, appropriate priors have to be chosen for the particular parameters. In this study we use uniform priors with prior ranges $[-\pi, \pi]$, $[-\pi/4, \pi/4]$ and $[-1, 1]$ for the angle parameters $\phi_0$, $\psi$ and $\cos \iota$ respectively. For $h_0$ we also specify a uniform prior with boundary $[0, 1000]$ in units of the rms noise. For the frequency and spindown uncertainty we use suitable uniform priors with ranges of $[-160, 160]$ Hz and $[-10^{-9}, 10^{-9}]$ Hz s$^{-1}$ for $\Delta f$ and $\Delta \dot{f}$, respectively.

The normalized posterior density $p(a|d) = p(a)p(d|a)/p(d)$ cannot be evaluated analytically, so we use Monte Carlo methods to explore $p(a|d)$. If we can simulate from $p(a|d)$, we can estimate all interesting quantities, including the posterior means of all parameters from the corresponding sample means, to any desired accuracy by increasing the sample size.

However, drawing independent samples in a six-dimensional parameter space is not feasible. It has already been shown that MCMC methods can be used to parameterise gravitational wave signals of low signal-to-noise ratio with four unknown parameters. These simulate a Markov chain, constructed so that its stationary distribution coincides with the posterior distribution and the sample path averages converge to the expectations. A minimal requirement for this is the irreducibility of the chain and hence the ability of the chain to reach all parts of the state space. A specific MCMC technique is the MH algorithm which does not require the normalization constant, only the unnormalized posterior density of Eq. (8). We employed the MH algorithm for the four and five parameter pulsar detection problems. The efficiency of the MH algorithm depends heavily on the choice of the proposal density. Intuition suggests that the closer the proposal distribution is to the target, the faster convergence to stationarity is achieved. Default choices such as a Gaussian proposal or a random walk result in very slow mixing for this 6-parameter problem. To increase the speed of convergence, we employed an adaptive technique, adaptive in the sense that it allows the choice of proposal distribution to depend upon information gained from the already sampled states as well as the proposed but rejected states. The idea behind the delayed rejection algorithm specified by is that persistent rejection, perhaps in particular parts of the state space, may indicate that locally the proposal distribution is badly calibrated to the target. Therefore, the MH algorithm is modified so that on rejection, a second attempt to move is made with a proposal distribution that depends on the previously rejected state. This adaptive Monte-Carlo method was generalized for the variable dimension case and renamed the ‘delayed rejection method’. Since we have a fixed dimension problem here we implemented the original version, and also the generalization that uses the reversible jump method. It turned out that the delayed rejection with the reversible jump method was not that beneficial for this particular problem and thus we will explain the original delayed rejection algorithm here.
For the Metropolis-Hastings algorithm a new state in a Markov chain is chosen first by sampling a candidate \( a' \) from a certain proposal distribution \( q_1(a'|a_n) \) usually depending on the current state \( a_n \) and then accepting or rejecting it with a probability \( \alpha_1(a'|a_n) \) depending on the distribution of interest. This rejection is essential for the convergence of the chain to the intended target distribution. The choice of a good proposal distribution is important to avoid persistent rejections in order to achieve good convergence of a chain. However in different parts of the state space different proposals are required. When a proposed MH move is rejected, a second candidate \( a'' \) can be sampled with a different proposal distribution \( q_2(a''|a',a_n) \) that can depend on the previously rejected proposal. Since a rejection suggests a bad fit of the first proposal, a different form of proposal can be advantageous in the second stage. To preserve reversibility of the Markov chain and thus to comply with the detailed balance condition, the acceptance probabilities for both the first and the second stage are given by

\[
\alpha_1(a'|a_n) = \min\left(1, \frac{p(a')p(d|a')q_1(a_n|a')}{p(a_n)p(d|a_n)q_1(a'|a_n)}\right)
\]

and

\[
\alpha_2(a''|a_n) = \min\left(1, \frac{p(a'')p(d|a'')q_1(a'|a'')q_2(a_n|a',a'')[1 - \alpha_1(a'|a'')]}{p(a_n)p(d|a_n)q_1(a'|a_n)q_2(a''|a_n,a')[1 - \alpha_1(a'|a_n)]}\right)
\]

respectively. Fig. 1 illustrates the idea of delayed rejection. When the second stage proposal step is applied due to rejection of the first, the chain has, in order to preserve the reversibility, to imply a return path which comprises a fictive stationary Markov chain consisting of a fictive stage 1 proposal step from \( a'' \) to \( a' \) which is rejected followed by an accepted fictive second stage move to \( a_n \). Although

![Figure 1. The delayed rejection method. In case of rejection of the first, bold step a second, more timid move is proposed. In order to maintain the reversibility of the Markov Chain the acceptance probability has to consider a fictive return path.](image-url)
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The coherence between $\Delta f$ and $\Delta \dot{f}$ is obvious since the data is sampled from time $t_{\text{start}}$ to $t_{\text{end}}$, where the heterodyned signal traverses a frequency from $f_{\text{start}} = \Delta f + \frac{1}{2} \Delta \dot{f} \cdot t_{\text{start}}$ to $f_{\text{end}} = \Delta f + \frac{1}{2} \Delta \dot{f} \cdot t_{\text{end}}$, time $t = 0$ is an epoch time when $f = \Delta f$. Hence it is much more natural to work with these frequencies as new parameters and vary them with a certain correlation which influences $\Delta \dot{f}$ indirectly. The original parameters are then obtained by the simple linear transformation

$$\Delta f = f_{\text{start}} - \frac{1}{2} \Delta \dot{f} \cdot t_{\text{start}}$$

and

$$\Delta \dot{f} = 2 \cdot \frac{f_{\text{end}} - f_{\text{start}}}{t_{\text{end}} - t_{\text{start}}}.$$  

Since the Jacobian of this transformation is constant the prior distributions for the new parameters $f_{\text{start}}$ and $f_{\text{end}}$ are flat as well.

Another cross-correlation can be observed between the parameters $h_0$ and $\cos \iota$ that arises from the fact that $h_0$ can be seen as a scaling factor and $\cos \iota$ as a non-linear weighting between the plus and cross polarisation part of the model. As seen in Eq. 4, the plus part is multiplied by the factor $a_1 = \frac{1}{4} h_0 (1 + \cos^2 \iota)$ while the cross part encloses the term $a_2 = \frac{1}{2} h_0 \cos \iota$. The original parameters can be derived from

$$h_0 = 2 \left( a_1 + \sqrt{a_1^2 - a_2^2} \right),$$

and

$$\cos \iota = \frac{2a_2}{h_0}.$$  

As mentioned above, the prior distribution of the parameters $h_0$ and $c = \cos \iota$ are chosen uniform with joint probability density function

$$f(h_0, c) = \begin{cases} (2h_0)^{-1}, & \text{if } 0 \leq h_0 < h_0^*, \quad -1 \leq c \leq 1, \\ 0, & \text{otherwise}, \end{cases}$$

where for this study $l_{h_0} = 1000$ in units of the rms noise. This implies a joint prior distribution for the parameters $a_1$ and $a_2$ of the form

$$g(a_1, a_2) = \begin{cases} (2l_{h_0})^{-1}, & \text{if } |a_2| \leq a_1 \leq \frac{4a_1^2 + l_{h_0}^2}{4l_{h_0}^2} \leq \frac{l_{h_0}}{2}, \\ 0, & \text{otherwise} \end{cases} \end{equation}$$

with Jacobian

$$\det J = \frac{2}{\sqrt{a_1^2 - a_2^2}}.$$  

Since the Jacobian is positive for the above restrictions we can write

$$g(a_1, a_2) = \begin{cases} \frac{1}{l_{h_0} \sqrt{a_1^2 - a_2^2}}, & \text{if } |a_2| \leq a_1 \leq \frac{4a_1^2 + l_{h_0}^2}{4l_{h_0}^2} \leq \frac{l_{h_0}}{2}, \\ 0, & \text{otherwise} \end{cases}.$$  

This joint prior density has the shape shown in Fig. 2. These reparameterisations result in a fast mixing Markov chain but still, the choice of a suitable proposal distribution is essential. Usually, a multivariate Normal distribution is utilized for the proposal distributions $q_1(\mathbf{a}^*|\mathbf{a}_n)$ and $q_2(\mathbf{a}^*|\mathbf{a}, \mathbf{a}_n)$, with means equal to the current state and
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Figure 2. Joint prior density of $a_1$ and $a_2$ for a given boundary $b_{h_0}$ for the parameter $h_0$.

different variances depending on the stage. Larger variances are chosen for the ‘bold’ first stage steps, while smaller variances are more beneficial for the ‘timid’ second stage candidates. The covariance matrix has to comprise the correlation between the parameters $f_{\text{start}}$ and $f_{\text{end}}$ since this correlation indirectly controls the parameter $\Delta \dot{f}$ as mentioned above. Hence choosing proposals for $f_{\text{start}}$ and $f_{\text{end}}$ with a correlation of 1 would imply no change of $\Delta \dot{f}$ because both parameters are changed in the same way while a correlation of 0 would have a great impact on $\Delta \dot{f}$ since $f_{\text{start}}$ and $f_{\text{end}}$ are changed completely uncorrelated. Thus the correlation between $f_{\text{start}}$ and $f_{\text{end}}$ has to be treated randomly in order to control $\Delta \dot{f}$. Best results are obtained when a correlation of 0 is chosen with probability 0.5 for the bold moves of $\Delta \dot{f}$ and a correlation of almost 1 otherwise for timid moves of $\Delta \dot{f}$. The proposals for the parameters $a_1$ and $a_2$ are sampled independently since they represent scaling factors for the plus and cross polarisation part, respectively. Finally we have to consider the correlation between the original parameters $\psi$ and $\phi_0$ which are not reparameterised. Pilot runs show that they are highly correlated. Hence the proposal distribution is adapted accordingly.

Unfortunately, the posterior distribution features very narrow modes in a large parameter space that has to be scanned. Thus a simple Normal distribution is not suitable for a proposal distribution as pilot runs have revealed. Instead, a proposal distribution with long tails and strong narrow mode is required. This can easily be achieved by generating a random sample between two boundaries $b_l$ and $b_h$ for the standard deviation of the proposal by generating a random weight for the weighted geometric mean of these two boundaries. Hence we sample standard deviations according to

$$\sigma = b_h^w b_l^{-w} \text{ where } w \sim \beta(a, b)$$

is Beta-distributed with parameters $a$ and $b$. The resulting proposal distribution is symmetric with very long tails and a strong narrow mode. In order to obtain higher standard deviations for the first stage the choice of $w \sim \beta(2, 1)$ (with mean $\frac{2}{3}$) is adequate while for the second stage $w \sim \beta(1, 2)$ (with mean $1/3$) samples smaller standard deviations.

The implementation of the ideas outlined above leads to reasonable acceptance rates and hence to a much better convergence of the Markov chain. While during the burn-in period it is mainly the stage 1 candidates that are accepted, the Markov chain
is driven mainly by stage 2 candidates after the burn-in. But still, the stationary
distribution features many distinct modes that carry the risk of trapping the Markov
chain. Therefore, we regard the posterior as a canonical distribution

\[
p(a|d) \propto p(a)p(d|a) \propto p(a) \exp \left[ -\frac{\chi^2(a)}{2} \right] \\
\quad \times \exp \left[ -\frac{\chi^2(a) - 2 \log [p(a)]}{2} \right] \\
\quad \times \exp \left[ -\beta \left( \chi^2(a) - 2 \log [p(a)] \right) \right].
\] (19)

with inverse temperature \(\beta\). During the burn-in period this inverse temperature can
pass through values starting at a low value (thus high temperature) and ending up
at \(\beta = \frac{1}{T}\) which coincides with the posterior distribution. This simulated annealing
technique was introduced by Metropolis et al. [12] and allows scanning of the whole
parameter space by permitting larger steps. For the annealing schedule an exponential
temperature curve is applied. For a certain number of iterations \(t_s\), it starts with an
inverse temperature \(\beta_0\) until it reaches \(\beta = \frac{1}{T}\). The inverse temperature follows the function

\[
\beta(t) = \begin{cases} 
\beta_0 \exp \left[ \frac{t}{t_s} \log \left( \frac{a}{\beta_0} \right) \right], & \text{if } 0 \leq t \leq t_s, \\
\frac{1}{T}, & \text{if } t > t_s,
\end{cases}
\] (20)
depending on the current iteration \(t\). Since the starting temperature is dependent on
the data set which is influenced by the amplitude \(h_0\) of the signal it has to be adapted
accordingly.

4. Results with simulated signals

We have synthesized fictitious data, and passed it through our six parameter MCMC
routine. The presentation of results here is similar to that of the four and five
parameter study of [14]. The artificial signals were embedded within white and
normally distributed noise. The ability of the MCMC algorithm to successfully find
the signal and estimate the six parameters was demonstrated, and is presented below.
The artificial signals \(s(t)\) were synthesized assuming a source at \(RA = 4^h 41^m 54^s\) and
dec = 18° 23′ 32″, as would be seen by the LIGO-Hanford interferometer. The signals
were then added to noise; we assumed a signal at 300 Hz and a corresponding noise
spectral density of at that frequency of \(h(f) = 8 \times 10^{-23} \text{Hz}^{-1/2}\). The amplitude of
the signal used in our test runs was varied in the range \(h_0 = 4.0 \times 10^{-24} \text{ to } 1.5 \times 10^{-22}\).
The length of the data set corresponded to 14 400 samples or 10 days of data at a rate
of one sample per minute (which was the rate used for the LIGO/GEO S1 analysis
described in [13]).

In Fig. 8 we display the MCMC generated posterior probability distribution functions (pdfs) for an example signal. The real parameters for this signal were:
\(h_0 = 1.5 \times 10^{-22}\), \(\psi = 0.4\), \(\phi_0 = 1.0\) (both in radians), \(\cos \iota = 0.878\), \(\Delta f = 7.0 \times 10^{-3} \text{ Hz}\) and \(\Delta \dot{f} = -2.5 \times 10^{-10} \text{ Hz s}^{-1}\). For this example the program ran for \(10^6\) iterations.
For a signal this large only about \(2.5 \times 10^4\) iterations were needed for the burn-in,
and this data is discarded from the analysis. Short-term correlations in the chain were
eliminated by ‘thinning’ the remaining terms; we kept every 250th item in the chain. In
this example the MCMC yielded median values and 95% posterior probability intervals
of \(h_0 = 14.91 \times 10^{-23}\) (13.41 \(\times 10^{-23}\) to 15.84 \(\times 10^{-23}\)), \(\psi = 0.439\) (−0.552 to 0.707),
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\[ \phi_0 = 0.964 \ (0.696 \text{ to } 1.958), \cos \iota = 0.884 \ (0.828 \text{ to } 0.988), \Delta f = 6.99999772 \times 10^{-3} \text{Hz} \]

\[ (6.99999217 \times 10^{-3} \text{Hz to } 7.0000314 \times 10^{-3} \text{Hz}) \text{ and } \Delta f = -2.4999541 \times 10^{-10} \text{Hz s}^{-1} \]

\[ (-2.5000767 \times 10^{-10} \text{Hz s}^{-1} \text{ to } -2.4998272 \times 10^{-10} \text{Hz s}^{-1}). \]

The 95\% posterior probability interval is specified by the 2.5\% and 97.5\% quartiles of \[ p(a_i|d). \]

**Figure 3.** MCMC estimates of the posterior pdf (kernel density) for the six parameters \[ h_0, \psi, \phi_0, \cos \iota, \Delta f \text{ and } \Delta \dot{f}. \] This synthesized signal had real parameters of:

\[ h_0 = 15 \times 10^{-23}, \psi = 0.4, \phi_0 = 1.0, \cos \iota = 0.878, \Delta f = 7.0 \times 10^{-3} \text{Hz} \text{ and } \Delta \dot{f} = -2.5 \times 10^{-10}. \]

The mean of the \[ h_0 \] distribution here is \[ 14.84 \times 10^{-23}. \]

With the noise level used, \[ h(f) = 8 \times 10^{-23}, \] we were able to successfully detect signals with amplitudes of \[ h_0 \geq 4.0 \times 10^{-24} \] with 10 days of data. This should be compared with the results presented in [14] where with just four parameters (\( h_0, \psi, \phi_0 \) and \( \cos \iota \)), we were able to confidently detect signals with an amplitude four times smaller. The addition of the new frequency parameters has the disadvantage of complicating the search due to the corresponding increase in the size of the parameter.
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For our study, we let the initial burn-in of the Markov chain last for as long as $3.5 \times 10^5$ iterations, and if the signal was not found by this time the search was terminated. It may be possible to find smaller signals with a longer burn-in.

Fig. 4 shows the MCMC estimated posterior for the smallest value of the parameter $h_0$ that we were able to identify with the MCMC code. The true parameter values for this run were $h_0 = 4.0 \times 10^{-24}$, $\psi = 0.4$, $\phi_0 = 1.0$, $\nu = 0.5$ (cos $\iota = 0.878$), $\Delta f = 7.0 \times 10^{-3}$ Hz and $\Delta \dot{f} = -2.5 \times 10^{-10}$ Hz s$^{-1}$. In this run the MCMC yielded a mean value and 95% posterior probability interval of $h_0 = 4.8 \times 10^{-24}$ (0.34 $\times 10^{-24}$ to 0.74 $\times 10^{-24}$). Fig. 5 displays the MCMC estimated posterior for the parameters $\Delta f$ and $\Delta \dot{f}$, which provides mean values and 95% posterior probability intervals of $\Delta f = 7.0 \times 10^{-3}$ Hz (6.9998 $\times 10^{-3}$ Hz to 7.0002 $\times 10^{-3}$ Hz), and $\Delta \dot{f} = -2.500 \times 10^{-10}$ Hz s$^{-1}$ ($-2.505 \times 10^{-10}$ Hz s$^{-1}$ to $-2.496 \times 10^{-10}$ Hz s$^{-1}$).

As can be seen from Figs. 4 and 5, even with small signal level it is still possible to extract the most astrophysically important parameters. For this MCMC run there were a total of $10^6$ iterations, with the first $3.5 \times 10^5$ as the burn-in.

Figure 4. MCMC estimate of the posterior pdf (kernel density) for the parameter $h_0$ from a six parameter search using synthesized data. The real parameter value for this signal was $h_0 = 4.0 \times 10^{-24}$. This was the smallest signal detectable by the MCMC method for the noise level used.

5. Discussion and conclusions

In the simplest application, the method demonstrated here could complement searches for signals from known pulsars [7, 17]; our method could be used to verify the frequency and frequency derivative values. The real advantage of the technique would come about in a search for a signal at a known location, but where the frequency information pertaining to the neutron star is not well known; a search for a signal from SN1987A [10] would be a possible application. In the demonstration here the heterodyning process provides a band of 1/60 Hz. It would be straightforward to expand this search to a bandwidth of 5 Hz by running the code on 300 processors, a task easily accomplished on a cluster of computers. For 10 days of data it takes a single 2.8 GHz personal computer approximately an hour to conduct about $3.3 \times 10^4$ iterations of our MCMC code. There are more iterations done per time interval at the beginning of a run because at that time more stage-1 steps are accepted. We believe that these MCMC methods offer great potential benefits for gravitational radiation.
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Figure 5. MCMC estimate of the posterior pdfs (kernel densities) for the parameters $\Delta f$ and $\Delta \dot{f}$ from a six parameter search using synthesized data with the smallest detectable signal $h_0 = 4.0 \times 10^{-24}$. The real parameters for this signal were: $\Delta f = 7.0 \times 10^{-3}$ Hz and $\Delta \dot{f} = 2.5 \times 10^{-10}$ Hz s$^{-1}$.

searches where the signals depend on a large number of parameters.

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