Quantum Zeno effect
in the decay onto an unstable level*

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Abstract
Under certain assumptions it is shown that the decay of level 2 of a three-level system onto level 1 is slowed down because of the further decay of level 1 onto level 0. It is argued that this phenomenon may be interpreted as a consequence of the quantum Zeno effect. The reason why this may be possible is that the second decay (or accompanying photon radiation) may be considered as a sign of the transition $2 \rightarrow 1$ so that during the first transition the system is under continuous observation.

1 Introduction
The quantum Zeno effect (paradox) \cite{1} is the name for the phenomenon of freezing (or slowing down) the evolution of a continuously observed quantum system. Originally the effect has been discussed in the case of a spontaneously decaying system, and preventing (or slowing down) of the decay has been predicted. Later on it was argued \cite{2} that the Zeno effect cannot arise in the spontaneous decay. Instead, the Zeno effect has been thoroughly investigated and finally experimentally proved \cite{3} in a repeatedly measured two-level system otherwise undergoing Rabi oscillations. The possibility of the Zeno effect in the initial non-exponential stage of a spontaneous decay is yet under discussion \cite{4}. It was argued \cite{5} that the Zeno effect is observed in the real radioactive decay. A review on the subject can be found in \cite{6}.

*Published in Phys. Lett. A 257, 227-231 (1999)
We shall present below a simple model predicting slowing down of a spontaneous decay in the case if the final (after this decay) state of the system is also unstable. This situation could in principle be interpreted as the Zeno effect in the continuously observed spontaneous decay, the second decay serving as a mechanism for the observation of the first one.

We shall consider a 3-level system with level 2 spontaneously decaying onto level 1 and level 1 spontaneously decaying onto level 0. If the system is originally on level 2, then the decay $1 \rightarrow 0$ (practically, observation of a photon radiated simultaneously with this decay) is a sign that the system has already arrived at level 1 and therefore that the transition $2 \rightarrow 1$ occurred. Vice versa, the absence of the decay $1 \rightarrow 0$ means that the system is yet at level 2. Thus, the very possibility of the decay $1 \rightarrow 0$ means that the system prepared originally at level 2 is under permanent observation (measurement). Then, as a result of the Zeno effect, the system must be frozen at level 2 or at least the decay of this level must be essentially slowed down.

In Sect. 3 we shall confirm by a direct quantum-mechanical calculation that this is the case: the decay $2 \rightarrow 1$ is slowed down if level 1 is unstable, the greater instability of level 1, the less the rate of the decay $2 \rightarrow 1$. In Sect. 4 we shall return to the question whether this phenomenon can be interpreted as a result of the Zeno effect.

To make the calculation more clear, we shall consider in Sect. 2 the decay onto a stable level and then in Sect. 3 the model will be generalized to the case of interest.

# The decay onto a stable level

As the preliminary step, let us consider, by the method given in [7], a model of the decay $2 \rightarrow 1$ onto a stable level 1.

Let $H_0$ be a Hamiltonian of a multilevel system (atom) including also a continuous spectrum. The latter may originate from the interaction between the atom and the electromagnetic field (photons) which could be absorbed or radiated simultaneously with transitions of the atom. The nature of the continuous spectrum may be arbitrary, but for concreteness we shall speak of photons. The total Hamiltonian $H = H_0 + V$ will contain also a potential $V$ leading to the transition between levels accompanying by the photon number change.
Denote the state of the atom on level 2 by \( |2\rangle \). Suppose that there is no photons (more generally, no contribution from the continuous spectrum) in the state \( |2\rangle \). We wish to describe the decay of this state to the state \( |1E\rangle \) in which the atom is on level 1 and there are also some photons, so that the total energy of the atom and the electromagnetic field is \( E \). For simplicity we shall assume that the only non-zero matrix elements of the potential \( V \) are \( \langle 1E|V|2\rangle = \langle 2|V|1E\rangle \).

To describe the transition \( |2\rangle \rightarrow |1E\rangle \), consider the general state of the system in the form

\[
|\psi\rangle = a_2(t) |2\rangle e^{-iE_2t} + \sum_E a_{1E}(t) |1E\rangle e^{-iEt}
\]

(1)

where the natural units (\( \hbar = 1 \)) are used and the integration in energy is denoted as a sum. To return to the usual units, we have to replace \( t \) by \( t/\hbar \). To return to the genuinely continuous spectrum, we have to replace a sum over \( E \) by integration over \( E \) with the weight \( \rho_1(E) \) presenting the local density of states \( |1E\rangle \).

Substituting this form for the state in the Schrödinger equation, we have the following equations for the coefficients \( a_2, a_{1E} \):

\[
\dot{a}_2 e^{-iE_2t} = -i \sum_E \langle 2|V|1E\rangle a_{1E} e^{-iEt},
\]

(2)

\[
\dot{a}_{1E} e^{-iEt} = -i \langle 1E|V|2\rangle a_2 e^{-iE_2t}.
\]

(3)

To solve these equations, let us accept the anzatz \( a_2(t) = \exp(-\gamma_2 t) \) corresponding to the exponential law of the decay of level 2 (this law is valid for not too small times). Then Eq. (4) will take the form

\[
i \sum_E a_{1E}(2|V|1E) e^{-i(E-E_2)t} = \gamma_2 e^{-\gamma_2 t}
\]

(4)

while Eq. (3) may be explicitly solved to give

\[
a_{1E}(t) = \frac{\langle 1E|V|2\rangle}{E - E_2 + i\gamma_2} \left[ 1 - e^{i(E-E_2 + i\gamma_2)t} \right].
\]

(5)

The initial condition \( a_{1E}(0) = 0 \) is used to describe the system being initially on level 2.
Now we have to substitute the expression (3) for the function \( a_{1E}(t) \)
in Eq. (4). Evaluating the sum (integral) on energies in Eq. (4), we shall
assume that the weight function \( \rho_1(E) \) and the matrix element \( \langle 1E|V|2 \rangle \) are
slow functions of energy and can be replaced by the constants equal to the
values of these functions at \( E = E_2 \) (the energy of level 2, the point where the
denominator in Eq. (3) has minimum). Under this assumption the energy
integral can be evaluated. Eq. (4) may be shown to be satisfied provided
that
\[
\gamma_2 = \pi \rho_1(E_2) \langle 1E_2|V|2 \rangle^2.
\]
This is nothing else than the “Fermi’s golden rule” for the decay of an
unstable level.

3 The decay onto a decaying level

Let us apply an analogous consideration to the three-level system of interest: level 2 may decay to level 1, and level 1 in turn may decay to level 0. The
general state of the system (again containing a continuous spectrum, photons) may be presented in the form
\[
|\psi\rangle = a_2(t) |2\rangle e^{-iE_2t} + \sum_E (a_{0E}(t) |0E\rangle + \sum_E a_{1E}(t) |1E\rangle) e^{-iEt}. \tag{7}
\]
Here \( |1E\rangle \) denotes the state of the atom at level 1 and the general energy of
the system (atom plus photons) \( E \), \( |0E\rangle \) is an analogous state but with the
atom at level 0. The sums over energies will be later replaced by the integrals
with the corresponding weights: \( \rho_1(E) \) for the states \( |1E\rangle \) and \( \rho_0(E) \) for \( |0E\rangle \).
The Hamiltonian of the system will be taken in the form \( H = H_0 + V \) with the
following non-zero matrix elements of \( V \): \( \langle 1E|V|2 \rangle \) and \( \langle 0E|V|1E' \rangle \). Then
the Schrödinger equation gives the following equations for the coefficients:
\[
\dot{a}_2 = -i \sum_E \langle 2|V|1E \rangle a_{1E} e^{-i(E-E_2)t}, \tag{8}
\]
\[
\dot{a}_{1E} = -i \langle 1E|V|2 \rangle a_2 e^{-i(E_2-E)t} - i \sum_{E'} \langle 1E|V|0E' \rangle a_{0E'} e^{-i(E'-E)t}, \tag{9}
\]
\[
\dot{a}_{0E} = -i \sum_{E'} \langle 0E|V|1E' \rangle a_{1E'} e^{-i(E'-E)t}. \tag{10}
\]
To solve this set of equations, we shall present them in the vector form

\[
\begin{align*}
\dot{a}_2 &= -i V_{21} a_1 \\ 
\dot{a}_1 &= -i V_{12} a_2 - i V_{10} a_0 \\ 
\dot{a}_0 &= -i V_{01} a_1 
\end{align*}
\]  

(11)

(12)

(13)

where the following vectors and matrices are introduced:

\[
\begin{align*}
(a_1)_E &= a_{1E}, \quad (a_0)_E = a_{0E}, \\
(V_{21})_E &= \langle 2|V|1E \rangle e^{-i(E-E_2)t}, \quad (V_{12})_E = \langle 1E|V|2 \rangle e^{-i(E_2-E)t}, \\
(V_{10})_{EE'} &= \langle 1E|V|0E' \rangle e^{-i(E-E')t}, \quad (V_{01})_{EE'} = \langle 0E|V|1E' \rangle e^{-i(E'-E)t}.
\end{align*}
\]

(14)

Let us introduce also the integral operations acting on the time-dependent vectors:

\[
I_{kl} a = -i \int_0^t V_{kl} a \, dt, \quad J = I_{10} I_{01}.
\]

(15)

Then Eqs. (12, 13) may be replaced by the integral equations

\[
\begin{align*}
a_1 &= I_{12} a_2 + I_{10} a_0, \\
a_0 &= I_{01} a_1
\end{align*}
\]

(16)

having the solution

\[
a_1 = (1 - J)^{-1} I_{12} a_2 = \sum_{n=0}^{\infty} J^n I_{12} a_2.
\]

(17)

Making use of the anzatz \( a_2 = e^{-\Gamma_2 t} \) in the right-hand side of this equation and substituting the resulting expression for \( a_1 \) in Eq. (16), we have the following equation for \( \Gamma_2 \):

\[
-i \sum_{n=0}^{\infty} V_{21} J^n I_{12} e^{-\Gamma_2 t} = -\Gamma_2 e^{-\Gamma_2 t}.
\]

(18)

We can evaluate each term in the sum. For calculating sums (integrals) over energies, we shall use the same approximation as in the preceding section considering all matrix elements of \( V \) and the weight functions \( \rho_1(E) \) for states \( |1E\rangle \) and \( \rho_0(E) \) for states \( |0E\rangle \) slow functions of energies. Then each term in the left-hand side of Eq. (18) can be evaluated.
It turns out that the terms corresponding to the given \( n \) differ from the term corresponding to \( n - 1 \) only by the numerical factor \((-N)\) where
\[
N = \pi^2 \rho_0(E_2)|\langle 0E_2|V|1E_2 \rangle|^2 \rho_1(E_2).
\] (19)
This gives
\[
\Gamma_2 = \frac{\gamma_2}{1 + N}
\] (20)
where \( \gamma_2 \) is defined by Eq. (6). Eq. (20) is proved in the assumption that \( N < 1 \), however this does not exclude that it may be valid also in a wider region. The assumptions about the behavior of matrix elements of \( V \) and functions \( \rho_1 \) and \( \rho_2 \) taken above are essential.

The formula (20) leads to the main conclusion. The entity \( N \) in its denominator is proportional to the rate of the decay of level 1 in the situation when the system starts in the state \( |1E_2 \rangle \). In other words, \( N \) is a measure of instability of level 1 (under the condition that there are also photons so that the total energy of the system is \( E_2 \)). We see therefore that the rate of the decay of level 2 decreases because of instability of the target level 1. The more instability of level 1, the less the rate of the decay \( 2 \rightarrow 1 \).

The last claim may be made more concrete if we (roughly) estimate the rate \( \Gamma_1 \) of the decay of level 1. It depends on the energy band of the decaying states \( |1E \rangle \). Since these states result in the decay of the state \( |2 \rangle \), the energy \( E \) should be of the order of \( E_2 \) and the width \( \Delta E \) of the energy band is of the order of \( \Gamma_2 \). The rate of the decay of the level 1 may be obtained (as a rough estimate) by multiplication of the number (19) by \( \Delta E \) giving \( \Gamma_1 \sim \Gamma_2 N \). According to Eq. (20), the slowing down of the decay \( 2 \rightarrow 1 \) is essential if \( \Gamma_1 \) is larger than or of the order of \( \Gamma_2 \).

4 Discussion

We showed, under certain assumptions, that the rate of the decay is slowed down by instability of the target level. It has been argued in Introduction that this may be interpreted as a consequence of the quantum Zeno effect. However, this must be compared with the arguments against the Zeno effect in spontaneous decays.

Some authors argued [2] that the quantum Zeno effect is impossible in spontaneous decay because of its exponential law (contrary to the quadratic
small-time asymptotic of Rabi oscillations). One more doubt may be based on the following. A spontaneously decaying atom located in plasma is subject to repeated scattering of electrons on the atom. These events of scattering may be thought of as repeated measurements of the atom discriminating its levels. In this situation the Zeno effect, if existing, could slow down the decay. In reality the rate of the decay changes insignificantly [8] due to scattering of electrons.

This gives an additional argument against the Zeno effect in spontaneous decay. The phenomenon discussed in the preceding section may then be interpreted as a Zeno-like but not genuinely Zeno effect.

In our opinion, the conclusion should not be as radical as this. Instead, one may consider the possibility of the Zeno effect for different types of continuous measurements. In the situation considered in the present paper the transition $1 \rightarrow 0$ is an evidence that the system has already arrived at level 1, but after this evidence has been obtained the system is no more at level 1. If we consider a 2-level system with both levels 2 and 1 as our measured system, then the measurement leads to destruction of this system. On the contrary, scattering electrons gives information about the level the atom is on and leaves it at the same level. The measurements described by von Neumann’s projections (discussed in most papers on the Zeno effect) act analogously.

Therefore, the results obtained in the present may point out that 1) the quantum Zeno effect does not arise in a spontaneous decay if the measurement is “minimally disturbing” (described by projectors), but 2) the effect takes place if the measurement is “destructive” i.e. leads to the disappearance of the measured state.

Some other remarks must be added. The conclusion about slowing down the decay $2 \rightarrow 1$ due to the decay $1 \rightarrow 0$ has been proved above under certain assumptions. The most important of them are that the phase volumes of the transitions may be correctly accounted by the weights $\rho_1$, $\rho_0$ and these weights are slow functions of energy $E$. Under other conditions the conclusion about slowing down of the decay would be different. This may be considered as one more argument against the Zeno interpretation of slowing down, but instead this may point out on necessity of more accurate treatment of the concept of “observation”.

Observation of the decay is characterized by the time of observation and energy of the decay products. Simple statements that the decay “is observed”
or “is not observed” are hardly quite adequate. One needs quantitative characteristics of the observations. The degree of slowing down the decay must depend on these characteristics. The assumption accepted in the present paper that the functions $\rho_1(E), \rho_0(E)$ are slowly varying, means that the products of the decays are efficiently observed in a wide energy band. In this condition the Zeno effect may be expected. If the functions $\rho_1(E), \rho_0(E)$ have the shape of narrow peaks, the Zeno effect may be absent because of inefficient observation.

It may be remarked in this connection that the very existence of the decay products may naively be considered as an evidence of the decay. If one accept this point of view, he is forced to conclude that the decay is always under continuous observation and therefore is always subject to the Zeno effect. This is however invalid (see the paper of A.Peres in [II]) because the decay products have to be considered as a part of the system necessary for the description of the decay itself.

The secondary decay in a three-level system analyzed in the present paper may be in most cases considered as external to the primary decay. Hence, this secondary decay may be treated as an observation and must lead to slowing down of the primary decay. Even in this case much more accurate and detailed analysis is necessary to have complete and reliable description of the Zeno effect. This analysis has to include all temporal and energy characteristics of the process. As a limiting case, it cannot be excluded that in some conditions (for certain characteristics of the system and its environment) the secondary decay cannot be considered as being external in respect to the primary decay.

Summing up, we suggest that the complete analysis of the Zeno effect in decay requires more detailed definition of the concept of observation and of the Zeno effect itself. The results of the present paper show that the development of this sort must be fruitful.

**ACKNOWLEDGEMENT**

The author acknowledges the fruitful discussions with V. Namiot and A. Panov concerning interpretation of the results. The work was supported in part by the Russian Foundation of Basic Research, grant 98-01-00161.
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