Holographic dark energy interacting with dark matter in a Closed Universe

Norman Cruz\(^1\), Samuel Lepe\(^2\), Francisco Peña\(^3\) and Joel Saavedra \(^2\)

\(^1\)Departamento de Física, Facultad de Ciencia, Universidad de Santiago, Casilla 307, Santiago, Chile.

\(^2\)Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4950, Valparaíso, and

\(^3\)Departamento de Ciencias Físicas, Facultad de Ingeniería, Ciencias y Administración, Universidad de La Frontera, Avda. Francisco Salazar 01145, Casilla 54-D Temuco, Chile.

(Dated: July 24, 2008)

A cosmological model of an holographic dark energy interacting with dark matter throughout a decaying term of the form \(Q = 3(\lambda_1 \rho_{DE} + \lambda_2 \rho_m)H\) is investigated. General constraint on the parameters of the model are found when accelerated expansion is imposed and we found a phantom scenarios, without any reference to a specific equation of state for the dark energy. The behavior of equation of stated for dark energy is also discussed.

I. INTRODUCTION

In the recent years a number of observational facts from high redshift surveys of type Ia Supernovae, WMAP, CMB, etc, led us to believe that our universe is passing through an accelerating phase of expansion \([1]\). It is generally accepted that our universe might have also emerged from an accelerating phase in the past. Thus there might have two phases of acceleration of the universe, early inflation and late acceleration followed by a decelerating phase. In order to describe the present accelerating phase of the universe, it may be useful to consider dark energy in the theory. The observational data of the universe indicates that dark energy content of the universe is about 76% of the total energy budget of the universe. To accommodate such a huge energy various kinds of exotic matters are considered to identify possible candidate for the dark energy. Recently, holographic principle \([2], [3]\) is incorporated in cosmology \([4], [5], [6], [7]\) to constraint the dark energy content of the universe following
the work of Cohen et al. \cite{8}. The holographic principle, in simple words establish that all degrees of freedom of a region of space in are the same as that of a system of binary degrees of freedom distributed on the boundary of the region \cite{9}. This point of view, represents an approach from a consistent theory of quantum gravity (unfortunately not yet found) in order to clarify the nature of dark energy. The Holographic principle says that the number of degrees of freedom of a physical system should scale with its bounding area rather than with its volume. Along these lines the literature have been focused in explain the size of the dark energy density on the basis of holographic ideas, derived from the suggestions that in quantum field theory a short distance a cut-off is related to a long distance cut-off due to the limit set by the formation of a black hole \cite{8}. Interacting dark energy and dark matter approach has been extensively discussed in the literature, see Ref. \cite{10} and references therein.

One of the most used description for the interaction between dark energy and dark matter is describe through an interaction factor given by $Q \sim (\lambda_1 \rho_{DE} + \lambda_2 \rho_{DM}) H$, whose origin can be modelled from a phenomenological point of view \cite{10, 11, 12, 13} and front observational data \cite{14}. In this article we are going to consider dark energy from an holographic origin.

The plan of the paper is as follows: In Sec. II we described the interacting model, accelerated scenarios and phantom regimen. In Sec. III we discuss the behavior of the equation of state. Finally, we conclude in Sec. IV.

II. DARK ENERGY DECAYING TO DARK MATTER

In the following we modelled the universe assuming that is filled with dark matter (including both dark and barionic matter), with a density $\rho_m$, and a dark energy component, $\rho_{DE}$, which obey the holographic principle. We are going to assume that the dark matter component is interacting with the dark energy component through a source (loss) term $Q$ that enters the energy balance. The Friedmann’s field equation is given by

$$3H^2 + \frac{3k}{a_0^2} \chi^{-2} = \rho_{DE} + \rho_m,$$ (1)

The continuity equations for both fluids take the form

$$\dot{\rho}_{DE} + 3H (\rho_{DE} + p_{DE}) = -Q,$$ (2)

$$\dot{\rho}_m + 3H \rho_m = Q,$$ (3)
where $\chi = a/a_0$ and $8\pi G = 1$. Dark matter obey the equation of state corresponding to dust $p_m = 0$. In the following we shall use the interaction model given by

$$Q = 3(\lambda_1 \rho_{DE} + \lambda_2 \rho_m) \, H.$$  

(4)

where $\lambda_1$ and $\lambda_2$ are positive constants. This model are basic in a phenomenologically way from the interaction between dark energy and dark matter, and were extensively considered in the literature, see Refs. [10, 11, 12, 13] and references therein. Substituting Eq. (4) in Eq. (3), we obtain the corresponding coupled equation

$$\dot{\rho}_m + 3(1 - \lambda_2) \, H \rho_m = 3\lambda_1 \rho_{DE} H.$$  

(5)

We now assume that the holographic dark energy takes the form

$$\rho_{DE} = 3c^2 H^2,$$  

(6)

different cases with $c^2 \lesssim 1$ were well described from theoretical point of view in Ref. [15] and from the observational view in Ref. [16]. Substituting the expression Eq. (6) in Eq. (5), and using Eq. (1), we obtain the following solution for $\rho_m$

$$\rho_m (\chi) = \left[ C_1 \chi^\Delta + k \frac{9c^2\lambda_1}{(1 - c^2) a_0^2 \Delta} \right] \chi^{-2},$$  

(7)

where $C_1$ is a positive constant of integration, and $\Delta$ is a constant defined by

$$\Delta \equiv 3 \left( \lambda_2 + \frac{c^2}{1 - c^2} \lambda_1 \right) - 1.$$  

(8)

The expression given by Eq. (7) reproduces in the limit with no interaction the usual behavior for dust, i.e., $\rho_m \sim \chi^{-3}$. Nevertheless, it is straightforward to see that Eq. (5) is equivalent to the continuity equation of a fluid with an equation of state $p_{eff} = -(\lambda_1/r + \lambda_2) \rho_m$, where $r$ is the coincidence parameter $r = \rho_m/\rho_{DE}$. In other words, due to the interaction with dark energy the dark matter behaves like a fluid with negative pressure.

In order to obtain an expression for the Hubble parameter, we introduce in Eq. (1) the holographic dark energy for $\rho_{DE}$, and from Eq. (7), yields

$$H^2 (\chi) = \frac{1}{3(1 - c^2)} \left[ C_1 \chi^\Delta - k \delta \right] \chi^{-2}.$$  

(9)

$^1 a_0$ is the present day value of the scale factor.
As we shall see below, a closed universe \((k = 1)\) implies a non constant coincidence parameter with the cosmic time. Since \(H^2 > 0\) we choose \(\delta < 0\). Taking the derivative with respect to the cosmic time of the above equation yields

\[
\dot{H}(\chi) = \frac{1}{6(1 - c^2)} \left[ C_1 (\Delta - 2) \chi^{\Delta} + 2k\delta \right] \chi^{-2},
\]

where \(\delta\) is a constant defined by

\[
\delta \equiv \frac{3}{a_0^2} \left( 1 - \frac{3c^2}{(1 - c^2) \Delta} \right) \lambda_1.
\]

From Eqs. (9) and (10) and since \(H^2(\chi) + \dot{H}(\chi) = \ddot{\chi}\), we obtain the expression for the acceleration

\[
\ddot{\chi} = \frac{C_1}{6(1 - c^2)} \Delta \chi^{\Delta - 1}.
\]

The deceleration parameter \(q\) is given by

\[
q = -\frac{a}{aH^2} = -\left( 1 + \frac{\dot{H}}{H^2} \right),
\]

and from Eqs. (10) and (9) we obtain that

\[
q(\chi) = -\left( 1 + \frac{1}{2} \frac{\left[ C_1 (\Delta - 2) \chi^{\Delta} + 2k\delta \right]}{C_1 \chi^{\Delta} - k\delta} \right),
\]

\[A. \text{ An accelerated universe}\]

We investigate first the conditions to have an accelerating universe, imposing that \(\ddot{\chi} > 0\). Eq. (12) tell us that \(\Delta\) must be positive. Now we are going to analyze the quantity \(\Delta\) according to the constraints \(0 < \Delta < 2\) and \(\Delta \geq 2\) for \(\delta < 0\). It is straightforward to obtain the following constraints for \(\lambda_1\) and \(\lambda_2\). In the former case

\[
\frac{1 - c^2}{c^2} \left( \frac{1}{3} - \lambda_2 \right) < \lambda_1 < \frac{1 - c^2}{c^2} (1 - \lambda_2),
\]

\[0 < \lambda_2 < \frac{1}{3},
\]

and when \(\Delta \geq 2\),

\[
\lambda_1 > \frac{2}{3} \left( \frac{1 - c^2}{c^2} \right),
\]

\[0 < \lambda_2 < \frac{1}{3}.
\]
In these universes with accelerated expansion the dark matter component which, as we have seen before (see Eq. (7)), has a negative pressure, behaves like the sum of two fluids both with decreasing energy density when the scale factor grows (if $0 < \Delta < 2$). On the other hand, if $\Delta > 2$ both dark components diverge when $\chi$ grows.

In our model it is easy to check that the coincidence parameter is a decreasing function of the scale between two extreme values. The expression for $r$, evaluated from Eqs. (6) and (7), yields

$$r(z) = \frac{1}{c^2} \left[ 1 + \frac{3k}{a_0^2 C_1} \frac{1}{(1+z)^{\Delta - k\delta}} \right],$$

(17)

where $1 + z = \chi^{-1}$. Deriving the parameter $r$ with respect to the variable $\chi$, we obtain the following expression in terms of the acceleration

$$\frac{dr(\chi)}{d\chi} = -\frac{18k(1-c^2)^2}{c^2a_0^2 (C_1\chi^{\Delta - k\delta})^2} \frac{\ddot{\chi}}{(C_1\chi^{\Delta - k\delta})^2}.$$  

(18)

Note that if we have imposed an accelerated expansion ($\ddot{\chi} > 0$) a decreasing $r$ is obtained only for $k > 0$. So in our approach open universe leads to not desirable physical results. Evaluating the limits of $r$ for $r(z \to \infty)$ ($\chi \to 0$) and $r(z \to -1)$ ($\chi \to \infty$) we obtain

$$r(z \to \infty) \to \frac{\lambda_1}{1/3 - \lambda_2}; \quad r(z \to -1) \to \frac{1-c^2}{c^2}.$$  

(19)

For flat universes $r(z)$ is always constant. In the case of a closed universe $r$ goes to a constant in the future cosmic evolution.

In order to analyze the behavior of the interaction term $Q$ we rewrite Eq. (11) in the form

$$Q = 3(\lambda_1 + \lambda_2 r(z)) \rho_{DE} H(z),$$

(20)

and explicitly for our holographic model the interaction takes the form

$$Q(z) = 9c^2 (\lambda_1 + \lambda_2 r(z)) H^3(z).$$

(21)

If the only condition on $\Delta$ is to be positive (with $\delta < 0$) we obtain a reasonable physical result since the behavior of $Q$ as the cosmic time evolves is a decreasing function. In the limit $z \to -1$, we obtain $Q(z \to -1) \to 0$, since $H(z \to -1) \to 0$. Meanwhile, the dark energy density (see Eq. (7)) is also a decreasing function of the cosmic time, which justify a decreasing interaction between the two fluids considered. This scenario occurs if $0 < \Delta \leq 2$. If $\Delta > 2$ we have divergences in the interaction term and in the dark components.
B. The phantom case

It is interesting to note that phantom scenarios are allowed by Eq. (12) since admit solutions of the type \( \chi(t) = (t_s - t)^{-\beta} \) with \( \beta > 0 \). In the phantom scenarios where \( \Delta > 2 \), we have a divergence for the scale factor \( \chi \) and in the Hubble parameter, since \( H(z \to -1) \to (1 + z)^{1-\Delta/2} \). The singularity of the scale factor occurs at \( t = t_s \), which can be evaluated integrating twice Eq. (12). We obtain that the cosmic time is given by

\[
t + A_2 = \int \frac{d\chi}{\sqrt{\frac{C_1}{3(1-c^2)}} + A_1},
\]

Choosing for simplicity, \( A_1 = 0 \) and fixing \( A_2 \) in order to have \( \chi(t_0) = 1 \), we obtain

\[
\chi^{\Delta/2-1} = \frac{1}{\sqrt{\frac{C_1}{3(1-c^2)}} (\frac{\Delta}{2} - 1)} (t_s - t)^{-1}.
\]

The expression for \( t_s \) is then given by

\[
t_s = t_0 + \frac{1}{\sqrt{\frac{C_1}{3(1-c^2)}} (\frac{\Delta}{2} - 1)}.
\]

At this future time we obtain the usual singularities that characterize a big rip solution. We note that for the current time from Eq. (14) it is straightforward to obtain

\[
q(0) > -\frac{1}{2} \Delta,
\]

in accord to the current observational data \[17\].

III. THE BEHAVIOR OF THE EQUATION OF STATE

In this section we investigate the equation of state of the dark energy, which has been only restricted by the holographic criteria and by its interaction with dark matter. In doing so, we reduce the two fluid components to an equivalent one fluid with an equation of state \( p = \omega \rho \) and pressure \( p \). We make this by taking the equations of state \( p_{DE} = \omega_{DE} \rho_{DE} \) and \( p_m = 0 \), and replacing the Eq. (2) and Eq. (3). The sum of these equations yields

\[
\dot{\rho} + 3H(1 + \omega)\rho = 0
\]

where \( \rho = \rho_{DE} + \rho_m \) and \( \omega = \omega_{DE} \rho_{DE} \).
The Friedmann equation becomes

\[ 3H^2 = \rho - \frac{3k}{a_0^2} \chi^{-2}. \]  

(27)

It is straightforward to obtain the following expression for \( \omega \)

\[ 1 + \omega = -\frac{2}{3} \left[ \frac{\dot{H} - \frac{k}{a_0^2} \chi^{-2}}{H^2 + \frac{k}{a_0^2} \chi^{-2}} \right]. \]  

(28)

Using the expression for \( \rho_{DE} \) and \( Q \) yields

\[ 1 + \omega = -\frac{1}{3} \left[ \frac{C_1(\Delta - 2)\chi^\Delta + 2k\beta}{C_1\chi^\Delta - k\beta} \right], \]  

(29)

with \( \beta = \delta - \frac{3(1-c^2)}{a_0^2} \frac{3}{a_0^2} \left[ 1 - \frac{3\chi^\Delta}{\Delta} \right] \). From the definition of \( \omega \) we obtain

\[ \omega(1 + r) = \omega_{DE}. \]  

(30)

Using in the above equation the expression for \( r \) given by Eq.(17) we obtain the equation of state for the dark energy fluid

\[ \omega_{DE} = -\frac{1}{3c^2} \left[ 1 + \frac{\Delta}{C_1\chi^\Delta - k\beta} \right]. \]  

(31)

Without an explicit identification of a particular model for the dark energy, it is of interest to evaluate the limit \( \omega(z \to -1) \) and evaluate \( \omega(z \to \infty) \), in order to know the future and early behavior, respectively, of the effective fluid of the universe. For the future we obtain

\[ \omega(z \to -1) \to -1 - \frac{1}{3}(\Delta - 2), \]  

(32)

where, as we mentioned before, \( \Delta > 2 \) implies phantom behavior. The cosmological constant is obtained for \( \Delta = 2 \) and for early times yields

\[ \omega(z \to \infty) \to -\frac{1}{3}. \]  

(33)

From Eq.(31) we obtain the expression for \( \omega_{DE}(z) \), which is given by

\[ \omega_{DE}(z) = -\frac{1}{3c^2} \left[ 1 + \frac{\Delta}{1 - k\beta C_1^{-1}(1 + z)} \right]. \]  

(34)

As the cosmic time evolves from now, the equation of state of the dark energy behaves like

\[ \omega_{DE}(z \to -1) \to -\frac{1 + \Delta}{3c^2}, \]  

(35)

and at early times the dark energy becomes

\[ \omega_{DE}(z \to \infty) \to -\frac{1}{3c^2}. \]  

(36)

Those results show that the dark energy evolves from a near behaviors like as stringy gas to a phantom behavior.
IV. DISCUSSION

A cosmological model of an holographic dark energy interacting with dark matter throughout a decaying term of the form $Q = 3(\lambda_1 \rho_{DE} + \lambda_2 \rho_m) H$ in a closed universe is investigated. General constraint on the parameters of the model are found when accelerated expansion is imposed. A dynamical coincidence parameter was found with dynamical decaying in the late times, that it is consistent with the constraint $c^2 < 1$. The dynamical parameters $H, Q, \rho_{DE}$ and $\rho_{DM}$ have the adequate limits at early and late times. We found that a phantom solutions is allowed, because Eq. (12) admit solutions of the type $\chi(t) = (t_s - t)^{-\beta}$ with $\beta > 0$. Here we obtain a deceleration parameter given by $q(0) > -\frac{1}{2} \Delta$ with $\Delta > 2$ and this constraint on $\Delta$ is consistent with the observational data, without any reference to a specific equation of state for the dark energy. At this point, we investigated the behavior of equation of state of the dark energy, which was only restricted by the holographic criteria and by its interaction with dark matter. We found, that the early limit is described by $\omega_{DE}(z \rightarrow -1) \rightarrow \frac{1+\Delta}{3c^2}$ and the late times is given by $\omega_{DE}(z \rightarrow \infty) \rightarrow -\frac{1}{3c^2}$. Those results show that the dark energy evolves from a near behaviors like as stringy gas to a phantom behavior.

V. ACKNOWLEDGEMENTS

NC, SL and JS acknowledge the hospitality of the Physics Department of Universidad de La Frontera where part of this work was done. SL acknowledges the hospitality of the Physics Department of Universidad de Santiago de Chile. FP acknowledges the hospitality of the Physics Institute of Pontificia Universidad Católica de Valparaíso. We acknowledge the partial support to this research by CONICYT through grant N° 11060515 (JS) and by Dirección de Estudios Avanzados PUCV. It also was supported from DICYT 040831 CM, Universidad de Santiago de Chile (NC), DIUFRO DI08-0041, of Dirección de Investigación y Desarrollo, Universidad de La Frontera (FP) and DI-PUCV, Grants 123.701/08 (SL) and 123.789 (JS), Pontificia Universidad Católica de Valparaíso.
References

[1] A. G. Riess et al., Astrophys. J. 607, 665 (20034); S. Perlmutter et al., Nature 51, 391 (1998); S. Perlmutter et al., Astrophys. J 517, 565 (1999); P. de Bernardis et al., Nature 404, 955 (2000); M. Tegmark et al., Phys. Rev. D 69, 103501 (2004); D. N. Spergel et al., astro-ph/0603449.

[2] W. Fischler and L. Susskind, [hep-th/9806039], L. Susskind, [hep-th/9901079], D. Bigatti and L. Susskind, TASI Lectures on the Holographic Principle, [hep-th/0002044].

[3] R. Boussou, JHEP 9907, 004 (1999), JHEP 9906, 028 (1999); Class. Quantum Grav. 17, 997 (2000).

[4] S. D. Hsu, Phys. Lett. B 594, 13 (2004).

[5] M. Li, Phys. Lett. B 603, 1 (2004).

[6] J. Zhang, X. Zhang and H. Liu, Phys. Lett. B 651, 84 (2007); X. Zhang, Phys. Lett. B 648, 1 (2007); M. R. Setare, arXive: 0705.3517; B. Chen, M. Li and Y. Wang, Nucl. Phys. B 774, 256 (2007); M. R. Setare, [gr-qc/0610008], [hep-th/0609069]; S. Nojiri and S. D. Odintsov, Gen. Rel. Grav. 38,1285 (2006); Y. Gong and Y. Z. Zhang, Class. Quantum Grav. 22, 4895 (2005); D. Pavón and W. Zimdahl, [hep-th/0511053].

[7] Q. G. Huang and M. Li, JCAP 8, 13 (2004).

[8] A. G. Cohen, D. B. Kaplan and A. E. Nelson, 1999 Phys. Rev. Lett. 82, 4971.

[9] L. Susskind, 1995 J. Math. Phys. 36, 6377.

[10] J. H. He and B. Wang, JCAP 0806, 010 (2008).

[11] B. Wang, J. Zang, C. Y. Lin, E. Abdalla and S. Micheletti, Nucl. Phys. B 778, 69 (2007).

[12] B. Wang, C. Y. Lin and E. Abdalla, Phys. Lett. B 637, 357 (2006).

[13] B. Wang, Y. Gong and E. Abdalla, Phys. Rev. D 74, 083520 (2006).

[14] M. Quartin, M. O. Calvao, S. E. Joras, R. R. R. Reis and I. Waga, JCAP 0805, 007 (2008).

[15] Q. G. Huang and M. Li, JCAP 0408, 013 (2004).

[16] Q. G. Huang and Y. G. Gong, JCAP 0408, 006 (2004).

[17] U. Seljak, A. Slosar and P. McDonald, JCAP 0610, 014 (2006).