Final State Interactions in $D^0 \to K^0 \bar{K}^0$

You-Shan Dai$^{a,d}$, Dong-Sheng Du$^{a,b}$, Xue-Qian Li$^{b,c}$, Zheng-Tao Wei$^a$ and Bing-Song Zou$^a$

$^a$ Institute of High Energy Physics, P.O.Box 918(4), Beijing 100039, China

$^b$ CCAST(World Laboratory), P.O.Box 8730, Beijing 100080, China

$^c$ Department of Physics, Nankai University, Tianjin 300071, China

$^d$ Department of Physics, Hangzhou University, Zhejiang, 310028 China

Abstract

It is believed that the production rate of $D^0 \to K^0 \bar{K}^0$ is almost solely determined by final state interactions (FSI) and hence provides an ideal place to test FSI models. Here we examine model calculations of the contributions from s-channel resonance $f_J(1710)$ and t-channel exchange to the FSI effects in $D^0 \to K^0 \bar{K}^0$. The contribution from s-channel $f_0(1710)$ is small. For the t-channel FSI evaluation, we employ the one-particle-exchange (OPE) model and Regge model respectively. The results from two methods are roughly consistent with each other and can reproduce the large rate of $D^0 \to K^0 \bar{K}^0$ reasonably well.

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I. Introduction

The importance of the final state interactions (FSI) in hadronic processes has been recognized for a long time. Recently its applications in D and B decays have attracted extensive interests and attentions of theorists. In many decay modes the FSI may play a crucial role.

Here the FSI refers to the soft rescattering processes at hadronic level [1]. Since all FSI processes are concerning non-perturbative QCD and cannot be reliably evaluated with any well-established theoretical framework, we have to rely on phenomenological models to analyze the FSI effects in certain reactions. The chiral lagrangian approach is proved to be reliable for evaluating hadronic processes, but there are too many free parameters which are determined by fitting data, so that its applications are much constrained. Therefore, we have tried to look for some simplified models which can give rise to reasonable estimation of FSI.

The decay $D^0 \to K^0 \bar{K}^0$ is a very interesting mode. Pham [2] and Lipkin [3] noticed the important role of FSI to this production long time ago. A direct $D^0 \to K^0 \bar{K}^0$ can only occur via a W-boson exchange based on the quark-diagram analysis [4], and moreover, since the CKM entries for $c \bar{u} \to d \bar{d}$ and $s \bar{s}$ have opposite signs, the reaction must be proportional to an SU(3) breaking, therefore according to the common knowledge obtained by studying B and D decays, direct $D^0 \to K^0 \bar{K}^0$ is much suppressed than $D^0 \to K^+ K^-$. However, the data show that $B(D^0 \to K^0 \bar{K}^0) \sim (6.5 \pm 1.8) \times 10^{-4}$ and $B(D^0 \to K^+ K^-) \sim (4.27 \pm 0.16) \times 10^{-3}$ [5]. Obviously, the $D^0 \to K^0 \bar{K}^0$ is realized through inelastic final state interactions. Namely, $K^0 \bar{K}^0$ is not a direct product of $D^0$ decay, but is secondary one from other hadrons (mesons, mainly) which have larger direct production rate in $D^0$ decays, via hadronic rescattering. Hence the decay $D^0 \to K^0 \bar{K}^0$ provides an ideal place to test model calculations of FSI.

Around $m_D = 1.86 GeV$ energy region, there is an abundant spectrum of resonances. The s-channel resonance contribution can be very important. However only those with appropriate quantum numbers ($J^{PC} = 0^{++}$) can contribute to the FSI for $D^0 \to K^0 \bar{K}^0$ mode. According to PDG Tables [6], there is only one $0^{++}$ resonance with mass sufficiently close to $m_D$, i.e., the $0^{++}$ component in $f_J(1710)$. In this work, we evaluate s-channel contribution by only accounting
$f_J(1710)$ and the rest is attributed to the t-channel exchange. For the t-channel exchange, we consider two approaches. One is the One-Particle-Exchange (OPE) model, concretely here it is the single-meson-exchange, while another is the Regge pole model. In fact, the Regge trajectories contain all non-perturbative QCD effects, but from another angle, its leading term is exactly the exchange of a meson with appropriate quantum numbers. The calculation with the single-meson-exchange scenario is obviously much simpler and straightforward. Moreover, some theoretical uncertainties are included in an off-shell form factor which modifies the effective vertices, therefore can compensate the residue effects which exist in a precise Regge pole model. This compensation can at least be of the same accuracy as the Regge pole model with several free parameters. One can trust that the results obtained in the two approaches should be qualitatively consistent, even not exactly equal. Our later numerical results confirm this allegation.

In Sec.II, we give the formulations for s- and t-channel FSI effects. For t-channel case, both the One-Particle-Exchange model and the Regge pole model are used. The numerical results and discussion are given in Sec.III.

II. The formulations

The direct decay amplitudes of $D^0 \rightarrow VV'$ and $D^0 \rightarrow PP'$ where $V(V')$ and $P(P')$ denote vector and pseudoscalar mesons, are given in many literatures and we will follow the conventions of [6].

(1) The s-channel resonance contribution.

Even though the spectrum is abundant at $m_D$ region, only the $0^{++}$ component of $f_J(1710)$ can make substantial contributions to the s-channel FSI. However the $0^{++}$ component of $f_J(1710)$ is still not well determined [4]. we use the data of $f_0(1710)$ for our later calculations. It is expected that the brought errors are within the error tolerance region of the present data.

To lowest order, the effective coupling of $f_0$ to $VV'$ and $PP'$ ($V,P$ are vector and pseudoscalar) which are concerned here, can be of forms

\[
L_I = \frac{g}{\sqrt{2}} \phi^+ \phi f \quad \text{PP'} f \quad (1)
\]

\[
L_I = \frac{g'}{\sqrt{2}} A_\mu A^\mu f \quad \text{VV'} f. \quad (2)
\]
With these lagrangians, the effective coupling constants \( g \) and \( g' \) are obtained by fitting the branching ratios of \( f_0 \) to \( VV' \) or \( PP' \).

The effective weak decay Hamiltonian for our process is:

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{us} V_{cs}^* [c_1 (\bar{s}c)_{V-A} (\bar{u}s)_{V-A} + c_2 (\bar{s}s)_{V-A} (\bar{u}c)_{V-A}] + V_{ud} V_{cd}^* [c_1 (\bar{d}c)_{V-A} (\bar{u}d)_{V-A} + c_2 (\bar{d}d)_{V-A} (\bar{u}c)_{V-A}] \right\}, \tag{3}
\]

where \( V_{us}, V_{cs}, V_{ud}, V_{cd} \) are the CKM matrix entries, \( V - A \) represents \( \gamma \mu (1 - \gamma_5) \).

The amplitude of the decay \( D^0 \to K^+ K^- \) is:

\[
A(D^0 \to K^+ K^-) = \frac{G_F}{\sqrt{2}} V_{us} V_{cs}^* a_1 < K^+ K^- | (\bar{s}c)_{V-A} (\bar{u}s)_{V-A} | D^0 >
= \frac{G_F}{\sqrt{2}} V_{us} V_{cs}^* a_1 (-f_{Kf_0} F_{DK}^D m_D^2) (m_D^2 - m_K^2), \tag{4}
\]

where \( a_1 = c_1 + \frac{1}{\sqrt{2}} c_2 \). The non-factorization effects are neglected here.

In terms of these effective couplings, the amplitude of \( D^0 \to K^0 \bar{K}^0 \) with the s-channel resonance \( f_0(1710) \) contribution can be written as

\[
A^{FSI} = \sum_{\text{all } MM'} \frac{1}{2} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \frac{i}{\delta^4(p_1 + p_2 - p_B) A(D^0 \to M + M')} \cdot g_{MM'} \cdot g_{KK} \frac{i}{s^2 - m_{f_0}^2 + i m_{f_0} \Gamma_{tot}} \times \kappa \tag{5}
\]

where \( \frac{i}{s^2 - m_{f_0}^2 + i m_{f_0} \Gamma_{tot}} \) is the relativistic Breit-Wigner resonance propagator for \( f_0(1710) \) and \( s \) is the total c.m. energy squared and \( \kappa \) is an isospin factor. The sum over the weak amplitudes \( A(D^0 \to MM') \) includes all possible states. The physical picture is shown in Fig.1. For other intermediate mesons other than \( f_0(1710) \), their propagators may provide a suppression factor \( 1/(m_D^2 - m^2) \) which would wash out their contributions.

Thus it is easy to derive

\[
A(D^0 \to K^+ K^- \to f_0(1710) \to K^0 \bar{K}^0)
= \frac{G_F^2 g_{KKf_0}^2}{\sqrt{2} \cdot 32 \pi} V_{us} V_{cs}^* a_1 f_{Kf_0} F_{DK}^D (m_D^2 - m_K^2)(1 - 4m_K^2/m_D^2)^{1/2} \times
\frac{(m_D^2 - m_f^2) - i \Gamma_f m_f}{(m_D^2 - m_f^2)^2 + \Gamma_f^2 m_f^2}. \tag{6}
\]
For the vector meson case, we have

\[ A(D^0 \rightarrow \rho^+ \rho^- \rightarrow f_0(1710) \rightarrow K^0 \bar{K}^0) = \frac{G_F g_{KKf} \cdot g_{\rho f}}{\sqrt{2}} V_{ud} V_{cs}^* f_K \sqrt{\frac{1}{3} \left( \frac{4}{3} \right) \left( 1 - \frac{4m_{\rho}^2}{m_D^2} \right)^{1/2}} \]

\[ \left[ m_{\rho} (m_D + m_{\rho}) f_\rho A_1 \left( 2 + \frac{(m_D^2 - 2m_{\rho}^2)}{4m_{\rho}^4} \right) - \frac{2m_{\rho} f_\rho A_2 (m_D^2 - m_{\rho}^2)^2}{2m_{\rho}^2} \right. \]

\[ + \left. \frac{m_D^4 (m_D^2 - 2m_{\rho}^2)}{8m_{\rho}^4} \right] \times \frac{(m_D^2 - m_f^2)^2 - i \Gamma f m_f}{(m_D^2 - m_f^2)^2 + \Gamma_f^2 m_f^2}, \]  

where all \( F^{DK}_0, a_1, A_1, A_2 \) etc. are defined according to the conventions in [6].

For \( A(D^0 \rightarrow \pi^+ \pi^- \rightarrow f_0(1710) \rightarrow K^0 \bar{K}^0) \), we need to replace \( g_{KKf}^2, V_{us} V_{cs}^* m_K^2 \) in the expression by \( g_{KKf}^2 g_{\rho f}, V_{ud} V_{cd}^* m_{\pi}^2 \). For other intermediate states such as \( K^{*+} K^{*-} \) and \( \rho^+ \rho^- \), we have no data about their branching ratios of \( f_0(1710) \), so we do not consider their s-channel contribution at present.

(2) t-channel contribution: The OPE model.

In the OPE model, a single t-channel (the same as u-channel) virtual particle is exchanged, (see Fig.2) and it is natural to assume that the lightest particle with proper quantum number dominates.

The exchange scenario has been studied in \( D \rightarrow VP \) and \( B \rightarrow \pi K \) cases [7, 8]. The effective vertices of strong interaction for the rescattering process, such as \( g_{KK\pi}, g_{\rho KK} \) etc. are gained from data provided the flavor SU(3) symmetry holds. However, since the t-channel exchanged particles P and V are off their mass shell, so a phenomenological form factor \( F(\Lambda) = \frac{\Lambda^2 - m^2}{\Lambda^2 - m^2} \) is introduced to compensate the off-shell effect at the vertices [9]. It is noted that \( \Lambda \) is a parameter which takes value between 1.2~2.0 GeV in normal sense. As pointed out in last section, the parameter \( \Lambda \)-value would smear the errors caused by assuming the dominates of one-particle exchange.

Obviously, for \( D^0 \rightarrow K^0 \bar{K}^0 \) final state, PV intermediate state is forbidden, meanwhile we also ignore contributions from the intermediate states with more than two mesons or baryons, which are definitely much smaller.

There are two key aspects to make the concerned processes substantial. First the direct production amplitude of \( D \rightarrow PP \) or \( VV \) must be large enough, and the second, the scattering
The amplitude of PP (or VV) → $K^0\bar{K}^0$ is not small. It depends on the effective couplings and how far the propagating meson deviates from its mass shell. Since the scattering PP (or VV) → $K^0\bar{K}^0$ are, in general, inelastic processes, the absolute values of the amplitudes are smaller than unity.

First, let us study which channels of D → PP or VV are substantially large. Here let us just make some order estimations of the amplitudes before doing concrete calculations.

Based on the quark diagrams, definitely $D^0 \rightarrow K^+ K^−, K^{*+} K^{*-}, \pi^+ \pi^−, \rho^+ \rho^−$ have larger amplitudes because they are realized via the so-called external W-emission \cite{4} which are much larger than other mechanisms.

For $D^0 \rightarrow \pi^0 \pi^0$ or $\rho^0 \rho^0$, even though they can happen via the internal W-emission, the amplitudes are about 3 times smaller than the external W-emission as
\[
\sqrt{\frac{\Gamma(D^0 \rightarrow \pi^0 \pi^0)}{\Gamma(D^0 \rightarrow \pi^+ \pi^-)}} \sim 0.3 \sim \sqrt{\frac{\Gamma(D^0 \rightarrow \rho^0 \rho^0)}{\Gamma(D^0 \rightarrow \rho^+ \rho^-)}}.
\]

Therefore, in our later calculations, we neglect contributions from such intermediate states.

(i) The $D^0 \rightarrow PP \rightarrow K^0\bar{K}^0$ case

Here we present the formulae for $D^0 \rightarrow K^+ K^− → K^0\bar{K}^0$ as an example and a similar expression can be written down for $D^0 \rightarrow \pi^+ \pi^− → K^0\bar{K}^0$. In this case the exchanged meson is $\rho^\pm$.

It is believed \cite{7, 8} that a single particle exchange in the t-channel would make dominant contributions to the FSI. For $D^0 \rightarrow K^+ K^− → K^0\bar{K}^0$ process shown in Fig.2, $K^+, K^−$, and the t-channel exchanged $\rho$ form a triangle diagram. As a matter of fact, for a pure FSI process, we only need to evaluate the absorptive part of the triangle. Definitely, the dispersive part of this loop can be calculated in terms of the dispersion relation \cite{9}, and generally it is expected to be of the same order as the absorptive part of the loop.

According to the Cutkosky rule, we make cuts to let $K^+, K^−$ be on shell, leaving $\rho^\pm$ to be off shell. At the $K^+ \rho^\pm K^0$ vertex the effective Hamiltonian is $g_{\rho KK} \epsilon_\mu (p_{K^+} + p_{K^0})^\nu$.

The amplitude of $D^0 \rightarrow K^+ K^− → K^0\bar{K}^0$ is:
\[
A^{FSI} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{d^3p_3}{(2\pi)^3} \frac{d^3p_4}{(2\pi)^3} \delta^4(p_1 + p_2 - p_3 - p_4) A(D^0 \rightarrow K^+ K^-) \cdot g_{\rho KK}^2 (p_1 + p_3)^\mu (p_2 + p_4)\nu (-g_\mu\nu + \frac{k_\mu k_\nu}{m_\rho^2}) \frac{i}{k^2 - m_\rho^2} F(k^2)
\]
\[ H = -(p_1 \cdot p_2 + p_1 \cdot p_4 + p_2 \cdot p_3 + p_3 \cdot p_4) \text{ and } A(D^0 \rightarrow K^+K^-) \text{ is the direct weak production amplitude.} \]

\[ F(k^2) = \frac{\Lambda^2 - m^2}{\Lambda^2 - p_{\rho}^2} \text{ as suggested in Ref. [7],} \]

(ii) \( D^0 \rightarrow VV \rightarrow K^0 \bar{K}^0 \) case

The case for intermediate states of two vector mesons (VV) has been studied in \( B \rightarrow pK^* \rightarrow \pi K \) processes [8]. It is shown that the VV intermediate states give a significant contribution to final state interactions. Here we take \( D^0 \rightarrow K^{*+}K^{*-} \rightarrow K^0 \bar{K}^0 \) as an example, while the expression for \( D^0 \rightarrow \rho^+\rho^- \rightarrow K^0 \bar{K}^0 \) is in close analog.

The amplitude for \( D^0 \rightarrow K^{*+}K^{*-} \) decay is:

\[ A(D^0 \rightarrow K^{*+}K^{*-}) = \frac{G_F}{\sqrt{2}} V_{us} V_{cs}^* a_1 \cdot M^{K^{*+}K^{*-}}, \]  

where

\[ M^{K^{*+}K^{*-}} = <K^{*+}|(\bar{u}s)_{V-A}|0> <K^{*-}|(\bar{s}c)_{V-A}|D^0> \]

\[ = m_{K^*}(m_D + m_{K^*})f_{K^*}A_{K^*}D_{K^*}(m_{K^*})^2(\epsilon_{K^{*+}} \cdot \epsilon_{K^{*-}}) \]

\[ - \frac{2m_{K^*}}{m_D + m_{K^*}} f_{K^*}A_{K^*}D_{K^*}(m_{K^*})^2(\epsilon_{K^{*+}} \cdot p_D)(\epsilon_{K^{*-}} \cdot p_D) \]

\[ - \frac{2m_{K^*}}{m_D + m_{K^*}} f_{K^*}V_{K^*}D_{K^*}(m_{K^*})^2 \epsilon_{\mu\nu\rho} \epsilon_{\mu\nu\rho} \epsilon_{K^{*+}}^{\mu} c_{K^{*-}}^{\nu} p_{K^{*+}}^\rho p_{K^{*-}}^{\nu}. \]

Unlike the \( D^0 \rightarrow K^+K^- \rightarrow K^0 \bar{K}^0 \), the t-channel exchanged particle in \( D^0 \rightarrow K^{*+}K^{*-} \rightarrow K^0 \bar{K}^0 \) is \( \pi^\pm \). Since it is the lightest meson of right quantum number, it should give rise to the largest contribution. The amplitude for the final state interaction of the process \( D^0 \rightarrow K^{*+}K^{*-} \rightarrow K^0 \bar{K}^0 \) is:

\[ A^{FSI} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^32E_1} \frac{d^3p_2}{(2\pi)^32E_2} (2\pi)^4 \delta^4(p_1 + p_2 - p_B) \]
In most literatures, the energy region for s-channel resonance is [10]:

\[ \theta = \frac{\lambda}{s} \text{ where } S \text{ is the S-matrix of strong interaction, } \theta \text{ is the angle between } \vec{p}_1 \text{ and } \vec{p}_3, \text{ and} \]

\[
\begin{align*}
H_1 & = 4g_{K^*K^0}^2 [(p_3 \cdot p_4) - \frac{(p_2 \cdot p_3)(p_2 \cdot p_4)}{m_2^2}] - \frac{(p_1 \cdot p_3)(p_1 \cdot p_4)}{m_1^2} + \frac{(p_1 \cdot p_2)(p_2 \cdot p_3)(p_2 \cdot p_4)}{m_1^2 m_2^2} \\
H_2 & = 4g_{K^*K^0}^2 [(p_3^0 m_4^0) - \frac{(p_2^0 p_3^0)(p_2 \cdot p_4)}{m_2^2}] - \frac{(p_1^0 p_4^0)(p_1 \cdot p_3)}{m_1^2} + \frac{(p_1^0 p_2^0)(p_2 \cdot p_3)(p_2 \cdot p_4)}{m_1^2 m_2^2}.
\end{align*}
\]

As mentioned above, the expression for \( A^{FSI} (D^0 \to \rho^+ \rho^- \to K^0 \bar{K}^0) \) is similar, the only distinction is that for \( D^0 \to \rho^+ \rho^- \to K^0 \bar{K}^0 \), the exchanged particle is \( K^\pm \) instead.

(3) t-channel contribution: The Regge pole model

The principles of Regge theory are [10]: (i) The scattering amplitudes are analytic functions of the angular momentum \( J \); (ii) A particle of mass \( m \) and spin \( \sigma \) will be on a Regge trajectory \( \alpha(t) \) (where \( t \) is the Mandelstam invariant parameters) and \( \sigma = \alpha(m^2) \); (iii) The partial wave amplitude has a pole of the form \( \frac{1}{J - \alpha(t)} \). It is suggested that the Regge theory provides a very simple and economical description of total cross section at high energy region [11].

The invariant amplitude for the scattering of particles with helicities \( \lambda_i \) from the Regge phenomenology is [10]:

\[
\mathcal{M}_{i \to f}^{\lambda_1 \lambda_2 ; \lambda_3 \lambda_4} = -\left( \frac{-t}{s_0} \right)^{\lambda/2} e^{-i\pi \alpha(t)} + \frac{J}{2 \sin \pi \alpha(t)} \gamma_{\lambda_1 \lambda_2} \left( \frac{s}{s_0} \right)^{\alpha(t)}
\]

where \( s \) and \( t \) are Mandelstam invariants, and \( \lambda = |\lambda_3 - \lambda_1| + |\lambda_4 - \lambda_2| \); \( J \) is the signature for Regge trajectory. For Pomeron and \( \pi \) trajectory, \( J = +1 \); For \( \rho \) and \( K^* \) trajectory, \( J = -1 \).

This expression corresponds to an asymptotic behavior when \( s \gg s_0 \) and \( s_0 \) is a scale parameter.

In most literatures, \( s_0 \) is taken as 1 GeV\(^2\). This Regge asymptotic behavior works very well in the energy region \( \sqrt{s} \geq 5 GeV \). We extend the energy region to \( \sqrt{s} = m_D \). The legitimacy is likely because we have accounted the s-channel resonance \( f_0(1710) \) contribution separately,
while contributions from the rest resonances can be treated as a smooth function of $s$ which is determined by the crossed t-channel exchange \[\text{[12]}\] and it is the fundamental of the Regge pole theory. Thus we can assume that there would not be a large deviation from the Regge asymptotic behavior. $\gamma(t)$ is a residue function. The linear Regge trajectory as an approximation is taken for our calculations $\alpha(t) = \alpha_0 + \alpha't$. $\alpha'$ is nearly a universal parameter for all Regge trajectories (except for Pomeron), $\alpha' \approx 0.9$. $\alpha_0 = 0.5$ for $\rho$ and $\omega$ trajectories; $\alpha_0 = 0.3$ for $K^*$ trajectory; $\alpha_0 = 0$ for $\pi$ trajectories; $\alpha_0 = -0.3$ for $K$ trajectories. But in our calculation we have adopt approximation $\alpha_0 = 0.5$ for $\rho$ and $K^*$ trajectories; $\alpha_0 = 0$ for $\pi$ and $K$ trajectories in order to carry out dispersion integration analytically.

We take the $D^0 \to K^{*+}K^{*-} \to K^0\bar{K}^0$ as an example and for the other intermediate states expressions are similar.

First, we rewrite the helicity amplitude of $D^0 \to VV$ decay in a convenient form \[\text{[13]}\]:

\[
A_{\lambda_1\lambda_2} = <V_1(k_1, \lambda_1)V_2(k_2, \lambda_2)|H_w|D^0(p)>
= \epsilon^*_\mu(k_1, \lambda_1)\epsilon^*_\nu(k_2, \lambda_2)[a\gamma^{\mu\nu} + \frac{b}{m_1m_2}p^\mu p^\nu + ic\frac{\epsilon^{\mu\nu\alpha\beta}k_{1\alpha}p_{\beta}}{m_1m_2}]
\] (14)

where $\lambda_1$, $\lambda_2$ are the helicity of $V_1$, $V_2$, and $\epsilon_\mu$, $\epsilon_\nu$ are the polarization vector of $V_1$, $V_2$. From Eq.(10), the above factors for decay $D^0 \to K^{*+}K^{*-}$ are:

\[
a = \frac{G_F}{\sqrt{2}}V^{*}_{us}V_{cs}a_1(m_D + m_{K^*})m_{K^*}f_{K^*}A^{DK^*_1}(m_{K^*}^2)
\]

\[
b = \frac{G_F}{\sqrt{2}}V^{*}_{us}V_{cs}a_1\frac{2m_{K^*}^3}{(m_D + m_{K^*})}f_{K^*}A^{DK^*_2}(m_{K^*}^2)
\]

\[
c = \frac{G_F}{\sqrt{2}}V^{*}_{us}V_{cs}a_1\frac{2m_{K^*}^3}{(m_D + m_{K^*})}f_{K^*}V^{DK^*}(m_{K^*}^2)
\]

In the rest frame of the $D^0$, $K^{*+}$ and $K^{*-}$ have the same helicity. According to \[\text{[13]}\], there are three independent helicity amplitudes:

\[
A_{++} = -a + \sqrt{x^2 - 1}c
\]

\[
A_{--} = -a - \sqrt{x^2 - 1}c
\]

\[
A_{00} = -xa - (x^2 - 1)b
\]

where $x \equiv \frac{k_1k_2}{m_{K^*}^2} = \frac{m_D^2 - 2m_{K^*}^2}{2m_{K^*}^2}$. 


The discontinuity of amplitude for the final state interaction of \( D^0 \rightarrow K^{*+}K^{*-} \rightarrow K^0\bar{K}^0 \) is:

\[
\text{Disc}A^{FSI} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^32E_1} \frac{d^3p_2}{(2\pi)^32E_2} (2\pi)^4 \delta^4(p_1 + p_2 - p_B) \cdot A(D^0 \rightarrow K^{*+}K^{*-})_{\lambda\lambda} M^{\lambda\lambda;00}(K^{*+}K^{*-} \rightarrow K^0\bar{K}^0),
\]

where \( \lambda \) is the helicity of the intermediate state \( K^* \). The discontinuity of this amplitude precisely corresponds to the absorptive part of the hadronic triangle (see Fig. 2) for the one-particle-exchange case where \( K^{*+}, K^{*-} \) are on-shell. For the rescattering of \( K^{*+}K^{*-} \rightarrow K^0\bar{K}^0 \), the exchange trajectory is \( \pi \). The helicity amplitude \( A_{++}, A_{--}, A_{00} \) all contribute to the same helicity state \( \{00\} \) of \( K^0\bar{K}^0 \). 

\[
\text{Disc}A^{FSI} = \sqrt{1 - \frac{4m_{K^*}^2}{m_D^2}} \frac{1}{16\pi s} \int_{t_{\text{min}}}^{t_{\text{max}}} dt (A_{++}M^{++;00} + A_{--}M^{--;00} + A_{00}M^{00;00})
\]

\[
= \epsilon_{\pi} \left( \frac{s}{s_0} \right)^{\alpha_0 - 1} (\epsilon_{\pi} s_0)^{\alpha_0 - 1}
\]

where \( \epsilon_{\pi} \) represents the value except the factor \( (\frac{s}{s_0})^{\alpha_0 - 1} \) and is calculated numerically.

The full amplitude of the final state interaction of \( D^0 \rightarrow K^{*+}K^{*-} \rightarrow K^0\bar{K}^0 \) can be obtained by using the dispersion relation.

\[
A^{FSI} = \frac{\epsilon_{\pi}}{\pi} \int_{4m_{K^*}^2}^{\infty} ds \left( \frac{s}{s_0} \right)^{\alpha_0 - 1} = \frac{\epsilon_{\pi}}{\pi m_D^2} \ln \left( 1 - \frac{m_D^2}{4m_{K^*}^2} \right)
\]

For the process \( D^0 \rightarrow \rho^+\rho^- \rightarrow K^0\bar{K}^0 \), the leading trajectory of rescattering is \( K \). For the process \( D^0 \rightarrow K^+K^- \rightarrow K^0\bar{K}^0 \) and \( D^0 \rightarrow \pi^+\pi^- \rightarrow K^0\bar{K}^0 \), the leading trajectory of rescattering are \( \rho \) and \( K^* \) respectively.

**III. Numerical results and discussion**

To reproduce the experimental data \( B(D^0 \rightarrow K^0\bar{K}^0) \sim 6.5 \times 10^{-4} \), we need the amplitude to be \( |A(D^0 \rightarrow K^0\bar{K}^0)| \sim 3.35 \times 10^{-7} GeV \). Now we examine numerical results of various FSI amplitudes.

For the s-channel contribution, we take the experimental data as inputs: \( m_f = 1710 \text{ MeV} \); \( \Gamma_{\text{tot}} = 133 \pm 14 \text{ MeV} \); \( B(K\bar{K}) = \Gamma_{K\bar{K}}/\Gamma_{\text{tot}} = 0.38 \); \( B(\pi\pi) = \Gamma_{\pi\pi}/\Gamma_{K\bar{K}} = 0.39 \). Since other channels of \( f_0(1710) \) decays have not been measured yet, we do not include their contribution
in this numerical estimation. We expect that they will give similar contribution as \(K^+K^-\) and \(\pi^+\pi^-\) modes. The numerical results of the s-channel \(f_0(1710)\) contributions are tabulated in Table.1.

Table 1: FSI amplitudes from s-channel contribution of \(f_0(1710)\).

| Decay Mode | \(A^{FSI}(\text{GeV})\) |
|------------|-----------------------|
| \(D^0 \rightarrow K^+K^- \rightarrow K^0\bar{K}^0\) | \((-0.24 - i0.53) \times 10^{-7}\) |
| \(D^0 \rightarrow \pi^+\pi^- \rightarrow K^0\bar{K}^0\) | \((0.13 + i0.32) \times 10^{-7}\) |
| total | \((-0.11 - i0.21) \times 10^{-7}\) |

One can notice that contributions from the \(K^+K^-\) and \(\pi^+\pi^-\) intermediate states interfere destructively, because \(V_{cd} \approx -V_{us}\). The sum of two contributions is small compared with what experimental data needs. However if parameters of \(f_0(1710)\) change\[14\], the s-channel contributions may become more important. For more precise estimation, we shall wait for further experimental information on \(0^{++}\) resonances near \(M_D\).

For the OPE model and the Regge pole model, we take \(c_1 = 1.26, c_2 = -0.51\); decay constants \[5, 15\]: \(f_\pi = 0.13 GeV\), \(f_K = 0.16 GeV\), \(f_\rho = 0.221 GeV\), \(f_{K^*} = 0.221 GeV\); and form factors \[6, 15\]:

\[
    F_0^{D\pi}(0) = 0.692, \quad A_1^{D\rho}(0) = 0.775, \quad A_2^{D\rho}(0) = 0.923,
\]

\[
    F_0^{DK}(0) = 0.762, \quad A_1^{DK^*}(0) = 0.880, \quad A_2^{DK^*}(0) = 1.147.
\]

For the OPE model, the effective strong coupling constants are given in \[5\]: \(g_{K^*K\pi} = 5.8\) and \(g_{\rho\pi\pi} = \sqrt{2}g_{\rho KK} = 6.1\). The parameter \(\Lambda\) in the off-shell form factor \(F(k^2)\) varies in a range of 1.2 to 2.0 GeV\[9\]. In Table 2, we tabulate the results corresponding to three cases: \(\Lambda = 1.2\) GeV, \(\Lambda = 1.6\) GeV, \(\Lambda = 2.0\) GeV.

Here three points are worthy of note. (1) The process \(D^0 \rightarrow K^{++}K^{*-} \rightarrow K^0\bar{K}^0\) has the largest contribution. The reason is because the exchanged particle is the lightest meson, the pion. This conclusion is the same as in \[5, 8\]. (2) The predicted amplitude of process \(D^0 \rightarrow K^{*+}K^{*-} \rightarrow K^0\bar{K}^0\) is not very sensitive to the choice of the parameter \(\Lambda\). By contrary, for
Table 2: FSI amplitudes from t-channel contributions in the OPE model.

| Decay Mode | \( A^{FSI}(\text{GeV}) \) |
|------------|----------------------------|
| \( D^0 \to K^+ K^- \to K^0 \bar{K}^0 \) | \( \Lambda = 1.2\text{GeV} \) | \( \Lambda = 1.6\text{GeV} \) | \( \Lambda = 2.0\text{GeV} \) |
| \( D^0 \to \pi^+ \pi^- \to K^0 \bar{K}^0 \) | \( i1.02 \times 10^{-7} \) | \( i3.11 \times 10^{-7} \) | \( i4.89 \times 10^{-7} \) |
| \( D^0 \to K^{**} K^{*-} \to K^0 \bar{K}^0 \) | \( i4.37 \times 10^{-7} \) | \( i5.89 \times 10^{-7} \) | \( i6.91 \times 10^{-7} \) |
| \( D^0 \to \rho^+ \rho^- \to K^0 \bar{K}^0 \) | \( -i1.79 \times 10^{-7} \) | \( -i3.02 \times 10^{-7} \) | \( -i3.98 \times 10^{-7} \) |
| total | \( i2.08 \times 10^{-7} \) | \( i2.75 \times 10^{-7} \) | \( i3.30 \times 10^{-7} \) |

the other three processes, the amplitudes are more sensitive to the choice. As well-known, the OPE model is more applicable when the virtual exchanged particle is close to its mass shell. In fact, the heavier the particle is, or the further it is off its mass shell, then the more sensitive the amplitude is to the parameter \( \Lambda \). (3) We only calculate the absorptive part which gives imaginary part of the FSI amplitudes only. It gives the correct order of magnitude of the FSI effects. The dispersive real part of the FSI amplitudes can be calculated in terms of the dispersion relation[9] with additional parameters, and generally it is of the same order of magnitude of the absorptive part. So the OPE model can reproduce the FSI effects rather well.

For the Regge pole model, two different treatment of the residue function \( \gamma(t) \) are assumed. Model I assumes constant residue functions \( \gamma[16] \): \( \gamma_{\pi\pi\rho}^2 = \sqrt{2} \gamma_{KK\rho} = \sqrt{\gamma_{0} \cdot \frac{2\gamma_{0}^2}{\gamma_{pp}}} \approx 72 \); Model II takes \( \gamma(t) \) to make Eq.(13) to be the same as in the OPE model for \( t \) near the mass squared of the leading exchanged particle. The numerical results are listed in Table 3.

Comparing the imaginary part in Regge pole models with the OPE results in Table 2, the biggest difference is for \( D \to VV \to PP \) modes in Model I. Model I is the conventional approximation of Regge model for high energies. It is obviously not a good approximation for the \( M_D \) energy region. We found that the main problem is: the high energy approximation for the t-dependent couplings, \( \frac{-l}{s_0}^{1/2} \) in Eq.(13), is not good for \( VV \to PP \) at the \( M_D \) energy. If we replace the \( \frac{-l}{s_0}^{1/2} \gamma(t) \) in Eq.(13) by the corresponding effective couplings in the OPE
Table 3: FSI amplitudes in Regge pole models.

| Decay Mode | $A^{FSI}$(GeV) | Model I | Model II |
|------------|---------------|---------|----------|
| $D^0 \rightarrow K^+K^- \rightarrow K^0\bar{K}^0$ | $(-0.31 - i2.61) \times 10^{-7}$ | $(-1.06 - i2.18) \times 10^{-7}$ |
| $D^0 \rightarrow \pi^+\pi^- \rightarrow K^0\bar{K}^0$ | $(0.38 + i3.17) \times 10^{-7}$ | $(-1.47 + i2.38) \times 10^{-7}$ |
| $D^0 \rightarrow K^{*-}K^{*-} \rightarrow K^0\bar{K}^0$ | $(-1.13 + i0.07) \times 10^{-7}$ | $(-4.08 + i2.75) \times 10^{-7}$ |
| $D^0 \rightarrow \rho^+\rho^- \rightarrow K^0\bar{K}^0$ | $(1.0 - i0.67) \times 10^{-7}$ | $(2.80 - i1.12) \times 10^{-7}$ |
| total | $(-0.06 - i0.04) \times 10^{-7}$ | $(-3.81 + i1.83) \times 10^{-7}$ |

model, then we get Model II which gives results roughly consistent with OPE results.

In summary, for the t-channel FSI contributions to $D^0 \rightarrow K^0\bar{K}^0$, the OPE model and Regge pole model with a proper treatment of $\gamma(t)$ (Model II) are roughly consistent with each other and can reproduce the experimental data reasonably well. They may be used to estimate t-channel FSI effects for other D decay channels. The Regge pole model assuming a constant $\gamma(t)$ (Model I) is not suitable for $M_D$ energy region. The s-channel FSI contribution from known $0^{++}$ resonances is small.

The situation of FSI for B meson decays should be different. There is an s-dependent suppression factor $\left(\frac{s}{s_0}\right)^{\alpha(t)}$ in Regge pole model. The discontinuity of the final state interaction amplitude is proportional to $\left(\frac{s}{s_0}\right)^{\alpha_0-1}$. For inelastic rescattering which the exchange trajectory $\alpha_0 < 1$, the discontinuity of the final state interaction amplitude decreases as the energy increases. This predicts that the final state interaction will be small in high energy region. There is no such s-dependent suppression factor in OPE model. At high energies, the t-channel exchange of heavier particles will become more important. The s-independent off-shell form factors in OPE model may be not enough to compensate these effects. We will continue our study in $B-$region and the results will be published elsewhere.
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Figure Captions

Fig. 1. The s-channel resonance FSI contribution. $j$ represents the intermediate states.
Fig. 2. Final-state interactions in $D^0 \rightarrow K^0 \bar{K}^0$ due to one particle exchange.