Ghost imaging based on pulsed radiation of terahertz spectrum range modulated by random phase screen

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Abstract. We present mathematical and computer modelling of ghost imaging technique based on broadband pulsed radiation of terahertz spectrum range, modulated by a random phase screen. The modelling is performed in both frequency and time domains. The quality of the reconstructed images is discussed and estimated.

1. Introduction
Ghost imaging for terahertz (THz) radiation is a popular subject in the field of optical imaging. This is due to the wide range of applications of the THz technology. As THz range contains the vibration eigen frequencies of many molecules, it represents very significant value for biology and medicine applications. Many of materials are transparent for THz radiation, that makes it very promising in applications such as security, medicine, and quality control [1]. These applications, as many of others related to THz technology, need imaging systems operating at THz frequencies, making the investigation of ghost imaging for THz radiation highly-demand topic.

In this paper we propose mathematical model of ghost imaging for terahertz pulsed radiation, modulated by random phase screen. Modelling is performed both in frequency and time domains. It is well known that classical pseudo-thermal light ghost imaging requires random or structured intensity modulation in laser beams. For this purpose, spatial light modulators (SLM) are frequently used in visible spectral range. However, due to the complexity of the implementation of SLM for THz spectral range, in this work a random phase screen is employed to obtain random modulation of the intensity in THz pulses.

2. Mathematical model
The scheme of ghost imaging which is used in our model is a classical pseudo thermal light ghost imaging setup shown in figure 1, in which THz pulses are incident onto the rotating plate with random rough surface on its back side, representing a random phase screen. The random phase screen introduces a phase shift which subsequently leads to the appearance of random intensity distributions (speckle patterns). After that beam is divided into two parts by beam splitter, the first beam interacts with the object and propagates to the single pixel detector. The second beam is recorded by multi pixel detector without interacting with the object. Then cross correlation function between speckle patterns registered
by multi pixel detector and overall intensity of speckle patterns transmitted through the object is calculated.

![Figure 1. Model of ghost imaging for pulsed THz radiation](image)

2.1. Model of random phase screen

In order to provide a modulation of THz pulses, it was used a model of Gaussian random rough surface [2], which is shown in figure 2. This surface has the following two parameters: root mean square deviation of the surface height $h_{RMS}$ and correlation length $Cl$.

![Figure 2. Samples of Gaussian random rough surface representing the random phase screen](image)

2.2. Construction of ghost imaging in the frequency domain

In the frequency domain we consider using Gaussian beams with the field distribution $E_{in}$ which can be described by:

$$E_{in}(x, y, z) = e^{ikz}E_0 \frac{-ik\alpha^2}{z-ik\alpha^2} e^{\frac{ik}{2}(x^2+y^2-z^2-ik\alpha^2)}, \quad (1)$$

where $x, y$ are the spatial coordinates, $\alpha$ is the spatial parameter of Gaussian beam, $E_0$ is the initial amplitude of beam. Let us consider Gaussian beam that is incident to the random phase screen at $z = 0$. Using the phase shift introduced by random phase screen $h(x, y)$, the field distribution of beam right after the random phase screen is evaluated in the following way:

$$E_r(x, y, z = 0) = E_{in}(x, y, z = 0) \cdot e^{i\frac{2\pi}{\lambda}(n-1)h(x,y)}, \quad (2)$$

where $\lambda$ is a wavelength, $n$ is the refraction index of material of plate with random rough surface on its back side, $h(x, y)$ is the function of heights of random phase screen.

Propagation of modulated beam along $z$ axis over the distance $\Delta z = d_2$ is calculated using Fresnel propagator described by:

$$E_r(x_2, y_2, z = d_2) = \frac{e^{ikz}}{i\lambda\Delta z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_r(x_1, y_1) e^{i\frac{\pi}{\lambda\Delta z}(x_2-x_1)^2+(y_2-y_1)^2} dx_1 dy_1, \quad (3)$$

where $x_1, y_1$ are the source plane coordinates, $x_2, y_2$ are the observation plane coordinates, $\Delta z$ is the distance between source and observation planes (distance of beam propagation), $E_r(x_1, y_1)$ and $E_r(x_2, y_2)$ is the field distribution in the source and observation planes respectively.
Propagating along the z axis, the beam acquires a random intensity distribution (speckle pattern) which is used in the ghost image construction (Figure 3).

![Figure 3 (a, b). Intensity distribution of beam (a) before the random phase screen, (b) propagated over distance z = 15 cm](image)

At the next stage of ghost image formation, the cross-correlation function (see [3]) between the speckle pattern \( I_r(x, y, v) \) and the overall intensity transmitted through the object \( B_r \), is evaluated by means of:

\[
I_r(x, y, v) = |E_r(x_2, y_2, z = d_2)|^2, \quad (4)
\]

\[
B_r = \int dx dy I_r(x, y, z = d_2) T(x, y), \quad (5)
\]

where \( T(x, y) \) is the transmission function of the object. This provides the reconstructed image \( G(x, y, v) \).

\[
G(x, y, v) \equiv \frac{1}{N} \sum_{r=1}^{N} (B_r - \langle B \rangle) I_r(x, y, v) = \langle B \cdot I(x, y, v) \rangle - \langle B \rangle \langle I(x, y, v) \rangle, \quad (6)
\]

where \( v \) is the radiation frequency, \( \langle \cdot \rangle \equiv \frac{1}{N} \sum_{r} \) denotes an ensemble average over \( N \) realizations of ghost imaging algorithm.

### 2.3. Construction of ghost imaging in the time domain

Considering ghost imaging in the time domain, we provide a method allowing to describe a ghost image reconstruction using THz pulses. In order to take into account all spectral components containing in THz pulse spectrum \( S(v) \), we take an inverse Fourier transform of field distribution \( E_r(x, y, z = d_2, 2\pi v/c) \) multiplied with \( S(v) \) as shown in equation:

\[
E_r(x, y, z = d_2, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i2\pi v t} S(v) E_r(x, y, z = d_2, 2\pi v/c) dv. \quad (7)
\]

After getting time-dependent field distribution \( E_r(x, y, z, t) \), it is possible to calculate the overall speckle pattern from the pulse by integrating \( E_r(x, y, z, t) \) over the pulse duration:

\[
I_r(x, y) = \int |E_r(x, y, z, t)|^2 dt = \int |S(2\pi v) E_r(x, y, z, 2\pi v/c)|^2 dv. \quad (8)
\]

Then the image reconstruction is obtained by substitution of \( I_r(x, y) \) into the cross-correlation function, described by equation (6).

### 2.4. Signal-to-noise ratio determination

For calculating the signal-to-noise ratio (SNR) the formula 3 from [4]:

\[
SNR = \frac{\langle I_f \rangle - \langle I_b \rangle}{\left( \sigma_f + \sigma_b \right)/2}, \quad (9)
\]

where \( \langle I_f \rangle \) is the average intensity of the image feature (part of image corresponding to object transmission area) and \( \langle I_b \rangle \) is the average intensity of the image background (which does not correspond to object transmission area). Values \( \sigma_f \) and \( \sigma_b \) are the standard deviations of the intensities in the feature and the background areas, respectively.
3. Numerical results

In this section we present numerical simulation results for both frequency and time domains. As a sample object it was used a ring, shown in figure 4 (a), with inner and outer radii \( R_{in} = 6 \, mm \), \( R_{out} = 9 \, mm \) respectively. Numerical results for the frequency domain are obtained for frequency \( \nu = 0.7 \, THz \) and shown in the figure 4 (b, c). Parameters of random phase screen are as follows: \( h_{RMS} = 0.8 \, mm \), \( Cl = 1.5 \, mm \), \( n = 1.46 \). The distance between the random phase screen and registration plane is \( \Delta z = 15 \, cm \).

Figure 4(a, b, c). (a) Object to be reconstructed, images reconstructed in the frequency domain for different numbers of realizations of ghost imaging algorithm (b) \( N = 10000 \); (c) \( N = 20000 \)

The increase of number of realizations of ghost imaging algorithm expectedly leads to the growth of quality of reconstructed image which is also proven by the increasing of SNR.

In the time domain the modelling was performed considering THz pulses with duration of 1 ps and spectral profile with central frequency \( \nu_0 = 0.5 \, THz \) as shown in the figure 5 (a). Reconstructed images for different number of realizations of ghost imaging algorithm are presented in figure 5 (b, c).

Figure 5(a, b, c). (a) Pulse spectrum, images reconstructed in the time domain for different numbers of realizations of ghost imaging algorithm (b) \( N = 10000 \); (c) \( N = 20000 \)

In case of image reconstruction with pulsed THz radiation the values of SNR of obtained images are about 20\% lower comparing with the frequency domain results. It can be explained by the fact that speckle patterns from different spectral components overlap each other. This leads to a decrease of the sharpness of reconstructed images.

4. Conclusion

We presented a mathematical and computer model of ghost imaging algorithm based on pulsed THz radiation modulated by random phase screen. The possibility of using a model of Gaussian random rough surface was shown for modulation of THz pulses. An approach of generating the speckle patterns which is capable of reconstructing ghost images even in case of working with broadband THz pulses was demonstrated.

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