THE STOCHASTIC UNIVERSE:

Professor A.M. Mathai’s 75th Birthday

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Abstract. A brief description and results of A.M. Mathai’s research programme on statistics and probability, initiated in the 1970s, and its relation to physics is given.

Introduction

Professor A.M. Mathai, Director of the Centre for Mathematical Sciences India and Professor Emeritus of the Department of Mathematics and Statistics at McGill University Canada, initiated a research programme on statistics and probability for physics in the 1970s with the publication of three research monographs (see figures below) focusing on information theory and entropic functional (Jaynes, Shannon, Boltzmann-Gibbs), the normal distribution as emerging from random processes in statistics and physics (Gauss, Maxwell-Boltzmann), and the statistical characterization of random variables in terms of generalized hypergeometric functions (Meijer, Fox).

In past centuries, there have been remarkable relations between mathematics and physics (Manin 1981, Rompe and Treder 1979). Mathematics influenced the development of theoretical physics or problems arising from physics and vice versa theory and experiment in physics encouraged developments in mathematics and statistics. The history of special functions of mathematical physics is a testament for such developments.

Mathematics and Statistics

The classical subdivisions of mathematics are geometry, algebra, and analysis. Geometry studies a space made up of points with various sorts of structure. Analysis considers a function, possibly describing some physical quantity evolving in time, and its derivatives. Algebra handles a set of things with a law of composition. There is a fourth branch to the logic of mathematics still under hot debate how it is part of mathematics: Probability theory and statistical inference used frequently as
foundations for scientific models (Uffink 2007). Carl Friedrich von Weizsäcker (2006), in his seminal study The Structure of Physics, elaborates that physics formulates general laws of nature in at least four forms emanating from the subdivisions of mathematics: (i) a family of functions, (ii) a differential equation, (iii) an extremum principle, and (iv) a symmetry group. Such general laws have been discovered and formulated in special and general relativity (Greene 2000) and quantum (field) theory (Veltman 2003), but not in statistical physics which developed not yet a set of generally accepted formal axioms, particularly for statistical thermodynamics and stochastic theory of non-equilibrium systems (Ebeling and Sokolov 2005). This is where Mathai’s research programme is making contributions in one way or another: (i) standard and fractional differential equations for reaction, diffusion, reaction-diffusion, (ii) entropic, distributional, and differential pathways to beta-1, beta-2, and gamma families of distributions, and (iii) connection between fractional calculus and statistical distribution theory.

Physics

Concerning the interaction between physics and mathematics, twentieth century examples are Riemannian geometry in general relativity and the impact of quantum mechanics on the development of functional analysis. Einstein finalized general relativity in 1915, while quantum field theory has been an open frontier since its foundation in 1927 by Dirac based on Heisenberg’s and Schroedinger’s fundamental discoveries for quantum mechanics. This was 80-85 years ago. For the next fifty years there was not much interaction between theoretical physics and mathematics. Mathematics turned to more abstract accomplishments, while quantum field theory was formulated in a rather formal way. This changed in the mid-1970s when nonabelian gauge theories emerged as quantum field theories (Greene 2000).

The generally agreed perception is that there are two fundamental theories in twentieth century physics: general relativity and quantum field theory. General relativity describes gravitational forces on the scale of the macrocosmos (Greene 2000), while quantum field theory describes the interaction of elementary particles, electromagnetism, strong, and weak forces at the scale of the microcosmos (Veltman 2003). There remains to exist an inconsistency between the two theories. The formal quantization of general relativity leads to infinite formulas. Einstein invented general relativity to resolve an inconsistency between special relativity and Newtonian gravity (Pais 1982). Quantum field theory was invented to reconcile Maxwell’s electromagnetism and special relativity with nonrelativistic quantum mechanics (Hoffmann 2010). But there were two basically different approaches. In Einstein’s “thought experiments”, which led to the discovery of general relativity, the logical framework came first. Then in Riemannian geometry of general relativity, the correct mathematical framework was found. In the development of quantum field theory on the other hand, there was no a priori conceptual basis; experimental clues played an important role, but there was no mathematical model. To date string theory is progressing in the pursuit to a formal quantization of general relativity: if accomplished it might be called a revolution in physics in the twenty first century.

A second revolution in the twenty first century may be the development of statistical mechanics beyond Maxwell, Boltzmann, and Gibbs, taking into account the need for a physical theory for stochastic theory of non-equilibrium systems (Prigogine 1980, Ebeling and Sokolov 2005). Indications are here that the mathematics for such a theory is given in fractional calculus with the H-function playing a central role in the solutions of stochastic fractional differential equations of the type of Liouville, master, Fokker-Planck, and reaction-diffusion (, Mandelbrot 1981, Klages et al. 2008, Honerkamp 1994, Tsallis 2009). In this regard, Mathai made contributions in his research programme. However, the physical interpretation of fractional time and space derivatives and integrals, applied to physical phenomena of reaction and diffusion, has not been discovered yet and a prospective physical
theory based on a “Schroedinger equation for thermodynamics”, as referred to by Prigogine, even if he surely had in mind a type of master equation, fully incorporating the concept of entropic functionals, has not been discovered. Surely the H-function and fractional calculus seem to fit equations already known that govern spatio-temporal pattern formation which are central to a fully developed mathematical and physical theory for non-equilibrium systems.

A review of a large number of specific results that emanated in Mathai’s long teaching and research carrier in mathematics, statistics, and natural sciences is contained in two recently published books that have been prepared under his leading guidance:
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