Top Mass Effects in Higgs Production at Hadron Colliders *

KEMAL J. OZEREN

Fachbereich C, Bergische Universität Wuppertal
42097 Wuppertal, Germany

We derive the first four terms of an expansion in $m_{tt}^2/m_t^2$ of the total Higgs cross section through gluon fusion. At NLO we demonstrate the excellent convergence of this series to the known result keeping the exact top mass dependence. At NNLO there is no known exact result, and our work represents a thorough quantitative investigation of the effects of finite top mass at this order. We discuss the applicability of our approach, and present numerical results for the LHC and Tevatron.

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1. Introduction

The forthcoming LHC experiments are expected to elucidate the mechanism of electroweak symmetry breaking. The most popular theoretical models of this phenomenon invoke a new scalar field called the Higgs boson, and the LHC has been designed with this firmly in mind. To discover the Higgs it is important to thoroughly understand its production and decay modes, as well as any relevant backgrounds. In this talk we describe recent work concerning the production of a Standard Model Higgs boson through gluon fusion. This process proceeds via a massive top quark, and as one can see from Fig. 1 the leading order diagrams are already one loop. At this order an exact analytical expression for the cross section is known, but at higher orders in $\alpha_s$ analytic formulae retaining the exact top mass dependence are only partially known $[1,2,3]$, although a numerical code known as HIGLU $[4]$ is available, which evaluates the NLO cross section exactly. Fortunately, one finds that the cross section is very well reproduced by working in an effective theory in which the top quark is integrated out, and then weighting with the leading order mass dependence. Comparisons with

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HI&LU show that this procedure works extremely well up to NLO \[5, 6, 7\]. This is usually taken as sufficient justification for also using the effective theory approach at higher orders; the NNLO contributions \[8, 9, 10\] are known only in the effective theory.

To test the heavy-top limit approximation, we work in the full theory, including the top quark, and evaluate the cross section at NNLO as an asymptotic series in \(1/M_t\). The technology to perform this expansion is well known, and automated in the q2e/exp framework \[11\]. We assume \(M_t\) heavier than all other scales in the problem, and express all Feynman integrals as convolutions of massive vacuum integrals with at most three loops, and massless 3/4/5-point functions through 2/1/0 loops, respectively. We note that the purely virtual contributions at NNLO have recently been calculated by two separate groups \[12, 13\].

We are able to directly evaluate the single real emission contribution in terms of hypergeometric functions, which we then expand in \(\epsilon\) with the Hyp-Exp \[14\] package. For the double real emission part we perform an expansion in powers of \((1-x)\) and then integrate term by term. To demonstrate the cancellation of infrared poles we must then of course similarly expand the single real part. After renormalising the coupling \(\alpha_s\) (for which we adopt the \(\overline{MS}\) scheme), top mass \(M_t\) and gluon wave function (both in the on-shell scheme) we are left with only \(1/\epsilon^2\) and \(1/\epsilon\) infrared poles. These are absorbed in the PDFs as per the usual mass factorisation procedure.

The description provided here is necessarily brief - we refer the reader to Ref. \[15\] for more details, full analytic results and comprehensive references.

### 2. Small-\(x\) Behaviour

The total cross section is a convolution of the partonic cross section with non-perturbative PDFs. Equivalently, we can write it as an integral over the luminosity,

\[
\sigma = \sum_{\alpha, \beta} \int_{m_t^2/S}^{1} dx \frac{E_{\alpha\beta}(x)}{x} \tilde{\sigma}_{\alpha\beta}(x),
\]

where \(x\) is related to the partonic centre of mass energy by \(x = M_h^2/\hat{s}\). The small \(x\) region of this integral therefore corresponds to high energy.
For the LHC the partonic cms energy can range up to 14 TeV, and so our assumption that the top mass is the heaviest scale in the process clearly breaks down here. Fortunately the behaviour of the cross section in the limit $x \to 0$ is known exactly, i.e. keeping the full top mass dependence [16]. We can therefore improve our results, which are valid for large $x$, by smoothly matching them onto the small-$x$ results of Ref. [16].

The small-$x$ results are,

$$\hat{\sigma}^{(1)}_{gg}(x) = 3 \sigma_0 C^{(1)} + O(x), \quad \hat{\sigma}^{(2)}_{gg}(x) = -9 \sigma_0 C^{(2)} \ln x + c + O(x),$$

for the NLO and NNLO cross sections respectively, where the coefficients $C^{(1)}$ and $C^{(2)}$ are given in Ref. [16] in the form of a numerical table, out of which we construct simple interpolating functions. The constant $c$ was not determined, and we set it to zero. Our strategy for matching these expressions with our results is to construct functions which have the correct behaviour in each of the limits $x \to 0$ and $x \to 1$, up to some order $N$ in an expansion in powers of $(1-x)$. We write

$$\hat{\sigma}^{(1)}_{gg}(x) = \hat{\sigma}^{(1)}_{gg,N}(x) + (1-x)^{N+1} \left[ 3 \sigma_0 C^{(1)} - \hat{\sigma}^{(1)}_{gg,N}(0) \right],$$

$$\hat{\sigma}^{(2)}_{gg}(x) = \hat{\sigma}^{(2)}_{gg,N}(x) - 9 \sigma_0 C^{(2)} \left[ \ln x + \sum_{k=1}^{N} \frac{1}{k} (1-x)^k \right],$$

where $\hat{\sigma}^{(n)}_{gg,N}$ denotes the expansion of the partonic cross section around $x = 1$ through $O((1-x)^N)$.

### 3. Numerical Results

In order to strictly test the heavy-top limit, we apply a consistent $1/M_t$ expansion to the partonic cross section, without factoring the LO mass dependence into the higher order terms. At NLO we therefore define,

$$\hat{\sigma}^{\text{NLO}}_{\alpha\beta}(M_t^n) = \sigma_0 \delta_{\alpha g} \delta_{\beta g} \delta(1-x) + \frac{\alpha_s}{\pi} \hat{\sigma}^{(1)}_{\alpha\beta}(M_t^n).$$

In Fig. 2 we compare the $gg$-channel NLO cross section, evaluated according to Eq. (4), including successive terms in the $1/M_t$ expansion, with the exact result from the numerical code HIGLU. We observe excellent convergence of the series at both the LHC and Tevatron. Unfortunately the low-$x$ behaviour of the other subprocesses ($qg$, $q\bar{q}$ and also the NNLO channels $qq$ and $qq'$) is not known. However, their numerical contribution at NLO is small, at the level of a few percent in the case of $qg$ and at the permille level for $q\bar{q}$. 

Fig. 2. Ratio of the $gg$ induced component of the NLO hadronic cross section as obtained from Eq. (3) to the value obtained from HIGLU \cite{4}, when keeping successively higher orders in $1/M_t$ (decreasing dash-length corresponds to increasing order); the dotted line is the result obtained from the pure soft expansion $\hat{\sigma}_{gg}^{(1)N}$ through order $1/M_t^{10}$ without the matching of Eq. (3).

Fig. 3. Ratio of the $gg$ induced component of the NNLO hadronic cross section as obtained from Eq. (3) to the heavy-top result of Eq. (1) (decreasing dash-length corresponds to increasing order in $1/M_t$); the dotted line is the result obtained from the pure soft expansion $\hat{\sigma}_{gg}^{(2)N}$ through order $1/M_t^6$ without the matching of Eq. (3).

The success of our approach at NLO means we can confidently apply it at higher orders. In Fig. 3 we compare the NNLO total cross section in our $1/M_t$ expansion approach with that obtained in the heavy top effective theory. The LO mass dependence is factored in only up to NLO, so that we can
examine the truly NNLO mass effects. We observe very good convergence towards the heavy-top results, which assures us of the high quality of the latter. The two results differ by less that 0.5%, which clearly justifies the use of the heavy-top effective theory so far in the literature, and also in future studies.

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