Study of the SU(3)_c ⊗ SU(3)_L ⊗ U(1)_X model with the minimal scalar sector

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A study of the three-family local gauge group SU(3)_c ⊗ SU(3)_L ⊗ U(1)_X with right-handed neutrinos is carried out. We use the minimal scalar sector able to break the symmetry in a proper way and produce, at the same time, masses for the fermion fields. We embed the structure into a simple gauge group and, by using experimental results from the CERN LEP, SLAC linear collider and atomic parity violation data, we also constrain relevant parameters for the new neutral and charged currents. We discuss the mass spectrum for the gauge boson sector and for the spin 1/2 particles. With the use of discrete symmetries and the introduction of extra scalar fields, a consistent mass spectrum could be constructed.

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I. INTRODUCTION

The number of fermion families in nature and the pattern of fermion masses and mixing angles are two of the most intriguing puzzles in modern particle physics. The 3-3-1 extension of the Standard Model (SM) of the strong and electroweak interactions, based on the local gauge group SU(3)_c ⊗ SU(3)_L ⊗ U(1)_X, provides an interesting attempt to answer the question on family replication. In fact, this extension has among its best features that several models can be constructed so that anomaly cancellation is achieved by an interplay between the families, all of them under the condition N_f = N_c = 3, where N_f is the number of families and N_c is the number of colors of SU(3)_c (three-family models)^1,2.

Two 3-3-1 three-family models have been studied extensively over the last decade. In one of them the three known left-handed lepton components for each family are associated to three SU(3)_L triplets as (ν_l, l^−_L, l^+_L), where l^+_L is related to the right-handed isospin singlet of the charged lepton l^+_L in the SM [1]. In the other model the three SU(3)_L lepton triplets are of the form (ν_l, l^−_L, ν_l^c)_L, where ν_l^c is related to the right-handed component of the neutrino field ν (a model with right-handed neutrinos)[2] . In the first model anomaly cancellation implies quarks with exotic electric charges −4/3 and 5/3, while in the second one the extra particles have only ordinary electric charges.

Our aim in this paper is to carry a phenomenology analysis of the 3-3-1 model in the version that includes right-handed neutrinos with a minimal content of Higgs scalars [3]. We are going to set updated constraints on several parameters of the model (including those related to the weak neutral currents), to check if the model can be embedded into a simple gauge group structure, and most important, to check if the model can reproduce realistic results as far as the fermion mass spectrum is concerned.

We already know, from the analysis presented in Refs. [1, 2], that models based on the 3-3-1 local gauge structure are suitable to describe some neutrino properties, because they include in a natural way most of the ingredients needed to explain the masses and mixing in the neutrino sector. In particular, Ref. [4] addresses this issue in the context of the model studied here. Even though we are going to concentrate mainly in other aspects of the model, we will show an alternative way to produce tiny neutrino masses with maximal mixing.

This paper is organized as follows: In Sec. II we review and present some novel features of the model. In Sec. III we study the fermion mass spectrum, and in Sec. IV we fix the mixing angles between the flavor diagonal neutral currents present in the model. In Sec. V we present our conclusions.

II. THE MODEL

Some of the formulas presented in this section are taken from Refs. [2] and [3]. Corrections to some minor printing mistakes in the original papers are included.

A. The gauge group

As it was stated above, the model we are interested in is based on the local gauge group SU(3)_c ⊗ SU(3)_L ⊗ U(1)_X which has 17 gauge bosons: one gauge field B^µ associated with U(1)_X, the 8 gluon fields G^µ associated with SU(3)_c which remain massless after breaking the symmetry, and another 8 gauge fields associated with SU(3)_L and that we write for convenience as

$$\frac{1}{2} \lambda_i A^\mu_i = \frac{1}{\sqrt{2}} \begin{pmatrix} D^\mu_1 & W^{+\mu} & K^{+\mu} \\ W^{-\mu} & D^\mu_2 & K^0 \delta_i^\mu \\ K^\delta_i & K^0 & D^\mu_3 \end{pmatrix},$$

(1)

where $D^\mu_1 = A^\mu_1 / \sqrt{2} + A^\mu_2 / \sqrt{6}$, $D^\mu_2 = -A^\mu_3 / \sqrt{2} + A^\mu_4 / \sqrt{6}$, and $D^\mu_3 = -2A^\mu_5 / \sqrt{6}$. λ_i, i = 1, 2, ..., 8, are the eight Gell-Mann matrices normalized as Tr(λ_i λ_j) = 2δ_ij.

The charge operator associated with the unbroken
where \( L_3 = \text{Diag}(1,1,1) \) (the diagonal \( 3 \times 3 \) unit matrix), and the \( X \) values are related to the \( U(1)_X \) hypercharge and are fixed by anomaly cancellation. The sine square of the electroweak mixing angle is given by \( \sin^2 \theta_W \) where \( g_1 \) and \( g_2 \) are the coupling constants of \( U(1)_X \) and \( SU(3)_L \) respectively, and the photon field is given by \( A_\mu^\alpha = S_W A_3^\mu + C_W \left[ \frac{T_W}{\sqrt{3}} A_8^\mu + \sqrt{1 - T_{W}^2 (3/3)} B_\mu \right], \) where \( C_W \) and \( T_W \) are the cosine and tangent of the electroweak mixing angle, respectively.

There are two weak neutral currents in the model associated with the two flavor diagonal neutral gauge weak bosons:

\[
Z_0^\mu = C_W A_3^\mu - S_W \left[ \frac{T_W}{\sqrt{3}} A_8^\mu + \sqrt{1 - T_{W}^2 (3/3)} B_\mu \right],
\]

\[
Z_\nu^\mu = - \sqrt{1 - T_{W}^2 (3/3)} A_8^\mu + \frac{T_W}{\sqrt{3}} B_\mu,
\]

and one current associated with the flavor non diagonal neutral gauge boson \( K^0 \nu \) which carries a kind of weak V-isospin charge. In the former expressions \( Z_0^\mu \) coincides with the weak neutral current of the SM. Using Eqs. (3) and (4) we can read that the gauge boson \( Y^\mu \) associated with the \( U(1)_Y \) hypercharge in the SM is:

\[
Y^\mu = \frac{T_W}{\sqrt{3}} A_8^\mu + \sqrt{1 - T_{W}^2 (3/3)} B_\mu.
\]

### B. The spin 1/2 particle content

The quark content for the three families is the following: \( Q_L^i \) is a \((u^i, d^i, D^i)_L \sim (3, 3, 0), i = 2, 3, \) for two families, where \( D^i_L \) are two extra quarks of electric charge \(-1/3\) (the numbers inside the parentheses stand for the \([SU(3)_c, SU(3)_L, U(1)_X]\) quantum numbers in that order); \( Q_L^1 = (d^1, u^1, U)_L \sim (3, 3^*, 1/3) \), where \( U_L \) is an extra quark of electric charge \(2/3\). The right handed quarks are \( u_{2L}^+ \sim (3^*, 1, -2/3) \), \( d_{2L}^+ \sim (3^*, 1, 1/3) \) with \( a = 1, 2, 3, \) a family index, \( D_{1L}^c \sim (3^*, 1, 1/3) \), \( i = 2, 3, \) and \( U_{1L}^c \sim (3^*, 1, -2/3) \).

The lepton content is given by the three \([SU(3)_L, SU(3)_c, SU(3)_L] \) anti-triplets \( L_{aL} \) or \( (e^a, \nu^a, N^a_0)_L \sim (1, 3^*, -1/3), a = 1, 2, 3; e = e, \mu, \tau \) respectively, and the three singlets \( e_{1L} \sim (1, 1, 1), \) where \( \nu^a_0 \) is the neutrino field associated with the lepton \( e_a \) and \( N^a_0 \) plays the role of the right-handed neutrino field associated to the same flavor. Notice that this model does not contain exotic charged leptons, and universality for the known leptons in the three families is present at tree level in the weak basis.

With these quantum numbers it is just a matter of counting to check that the model is free of the following gauge anomalies \[3\]: \([SU(3)_c]^3\); (as in the SM \([SU(3)_c]^3\) is vectorlike); \([SU(3)_L]^3\) (six triplet and six anti-triplets), \([SU(3)]^2 U(1)_X\); \([SU(3)_L]^2 U(1)_X\); \([grav]^2 U(1)_X\) and \([U(1)_X]^3\), where \([grav]^2 U(1)_X\) stands for the gravitational anomaly \[4\].

### C. \( SU(6) \) as a covering group

The Lie algebra of \( SU(3) \otimes SU(3) \otimes U(1) \) is a maximal subalgebra of the simple algebra of \( SU(6) \). The five fundamental irreducible representations (irreps) of \( SU(6) \) are: \([6], [6^*], [15], [15^*]\) and \([20]\) which is real. The branching rules for these fundamental irreps into \( SU(3)_c \otimes SU(3)_L \otimes U(1)_X \) are \[15\]:

\[
[6] \rightarrow (3, 3, -1/3) \oplus (1, 3, 1/3),
\]

\[
[15] \rightarrow (3^*, 1, -2/3) \oplus (1, 3^*, 2/3) \oplus (3, 3, 0),
\]

\[
[20] \rightarrow (1, 1, 1) \oplus (1, 1, -1) \oplus (3, 3^*, 1/3)
\]

\[
\oplus (3^*, 3, -1/3),
\]

where we have normalized the \( U(1)_X \) hypercharge according to our needs.

From these branching rules and from the fermion structure presented above, it is clear that all the particles in this 3-3-1 model can be included in the following \( SU(6) \) reducible representation:

\[
5[6^*] + 3[20] + 4[15],
\]

which, besides the particles in the representations already stated in the previous section, includes new exotic particles, for example:

\[
(N^0, E^+, E^{+^*})_L \sim (1, 3^*, 2/3) \subset [15],
\]

\[
E^-_L \sim (1, 1, -1) \subset [20],
\]

\[
(D^{ec}, U^{ec}, U^{ee})_L \sim (3^*, 3, -1/3) \subset [20].
\]

The analysis shows that the reducible representation in Eq. (6) is anomalous. The simplest \( SU(6) \) reducible representation which is free of anomalies and includes the fields in Eq. (6) is given by \[15\]:

\[
8[6^*] + 3[20] + 4[15],
\]

which includes a good deal of new exotic particles (all of them with ordinary electric charges): four families of 3-3-1 up and down type quarks, four more exotic down-type quarks, plus eight families of 3-3-1 lepton triplets, among other particles.

Our analysis, even tough producing a messy spectrum, is full of physical content, and it seems to contradict the analysis presented in Ref. \[6\] where a three-family standard model of particles can be embedded only into a group of rank eight or larger (as for example \( SU(8) \)). The point here is that the extra fields in Eq. (7) are not
a vectorlike structure with respect to the 3-3-1 subgroup of $SU(6)$ (the structure in Eq. (1) violates the so called survival hypothesis and then the analysis presented in Refs. 8 does not follow).

D. The minimal scalar sector

For this model the minimal scalar sector able to properly break the symmetry, to provide with masses to the eight gauge bosons related to the eight broken generators in $SU(3)_L \otimes U(1)_X$ and, at the same time, able to provide with Yukawa terms for the fermion fields in the model, is given by 3: $\phi_0^2 = (\phi_1^2, \phi_1^0, \phi_0^0) \sim (1, 3^*, -1/3)$, and $\phi_2^2 = (\phi_2^0, \phi_2^0, \phi_2^0) \sim (1, 3^*, 2/3)$, with Vacuum Expectation Values (VEV) given by $\langle \phi_1 \rangle^T = (0, v_1, V)$ and $\langle \phi_2 \rangle^T = (v_2, 0, 0)$.

The usual analysis shows that this set of VEV breaks the symmetry in one single step

$$SU(3)_c \otimes SU(3)_L \otimes U(1)_X \rightarrow SU(3)_c \otimes U(1)_Q.$$  \(8\)

For the particular value $v_1 = 0$, the symmetry breaking chain becomes

$$3 - 3 - 1 \rightarrow V SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_Q,$$  \(9\)

which in turn allows for the matching conditions $g_2 = g_3$ and

$$g_2 = \frac{1}{g_1^2} + \frac{1}{3g_2^2},$$  \(10\)

where $g_2$ and $g'$ are the gauge coupling constants of the $SU(2)_L$ and $U(1)_Y$ gauge groups in the SM, respectively. Then, for $v_1 = 0$ the SM becomes a low energy theory of this particular 3-3-1 gauge structure.

We will see in the next section that this scalar structure properly breaks the symmetry and provides with masses for the gauge bosons, but it is not enough to produce a consistent mass spectrum for the quark sector, and at least one more Higgs triplet, which does not develop VEV, must be introduced.

E. The gauge boson sector

After breaking the symmetry with $\langle \phi_i \rangle$, $i = 1, 2$, and using the covariant derivative for triplets $D^\mu = \partial^\mu - ig_3 \lambda_a A_\mu^a + \partial_0 \mu R, \mu I_3$, we get the following mass terms in the gauge boson sector.

1. Spectrum in the charged gauge boson sector

In the basis $(K_0^\pm, W_\mu^\pm)$, the square mass matrix produced by the VEV of the scalar fields is

$$M_\pm^2 = \frac{g_3^2}{2} \begin{pmatrix} (V_1^2 + V_2^2) & v_1 V_1 & v_1 V_2 \\ v_1 V_1 & (v_1^2 + v_2^2) & v_1 V_2 \\ v_1 V_2 & v_1 V_2 & (v_1^2 + v_2^2) \end{pmatrix},$$  \(11\)

a symmetric mass matrix having eigenvalues $M_{W^\pm}^2 = g_3^2 v_1^2/2$ and $M_{K_0}^2 = g_3^2 (v_1^2 + v_2^2 + V^2)/2$, related to the physical fields $W_\mu^\pm = \kappa (V W_\mu - v_1 K_\mu)$, and $K_0^\pm = \kappa (V K_0 + v_1 W_\mu)$, respectively. The first of them is associated with the known charged weak current $W_\mu^\pm$, and the second with a new one $K_0^\pm$ predicted by this model ($\kappa^2 = v_1^2 + V^2$ is a normalization factor). From the experimental value $M_{W^\pm} = 80.423 \pm 0.039$ GeV we obtain $v_2 \simeq 175$ GeV as in the SM, with $v_1$ being a free parameter as far as the $M_{W^\pm}$ mass value is concerned.

Notice that for $v_1 = 0$ there is no mixing between the two charged gauge bosons, and the mass for the known physical gauge boson $M_{W^\pm}$ is not altered. A crude value for $v_1$ can be estimated at this point in the following way: by using for the mass of $W_\mu^\pm = (\cos \eta W_\mu - \sin \eta K_\mu)$ the experimental value given above, and using the relationship

$$\tan^2 \eta = \frac{v_1^2}{V^2} = \frac{M_{W^\pm}^2 - M_{sm}^2}{M_{W^\pm}^2 - M_{sm}^2} \simeq \frac{M_{W^\pm}^2 - M_{sm}^2}{g_3^2 V^2}/2,$$  \(12\)

with $M_{sm} = 80.380 \pm 0.023$ (the $W$ mass calculated from the SM), and $V \sim 1$ TeV, we get $0 \leq \eta \leq 0.1$, which in turn implies $0 < v_1 < 100$ GeV, small than the electro-weak scale any way, with the largest value for $\eta$ coming from the large experimental and theoretical errors in the value for $M_{W^\pm}$. We are going to show below that $v_1$ must be very small since it is related to the mass scale of the neutrinos in the model.

2. Spectrum in the neutral gauge boson sector

For the five electrically neutral gauge bosons we get first, that the imaginary part of $K_0^\pm = (K_{0R}^\pm + i K_{0I}^\pm)/\sqrt{2}$ decouples from the other four electrically neutral gauge bosons, acquiring a mass $M_{K_0}^2 = g_3^2 (v_1^2 + V^2)/2$. Then, in the basis $(B^\mu, A_{1mun}, A_{2mun}, K_{0R}^\mu)$, a singular 4 \times 4 matrix is obtained with the eigenvector associated with the zero eigenvalue corresponding to the photon field $A_1^\mu$ in Eq. (13). The remaining $3 \times 3$ mass matrix, in the basis $(Z^\mu, Z'^\mu, K_{0I}^\mu)$, reduces to

$$\begin{pmatrix} \frac{v_1^2 + v_2^2}{s_\delta} & \frac{v_1^2 C_{2W} - v_2^2}{s_\delta} & \frac{-C_{W^\mu V}}{s_\delta} \\ \frac{v_1^2 C_{2W} - v_2^2}{s_\delta} & \frac{v_1^2 C_{2W} - v_2^2}{s_\delta} & \frac{-C_{W^\mu V}}{s_\delta} \\ \frac{-C_{W^\mu V}}{s_\delta} & \frac{-C_{W^\mu V}}{s_\delta} & \frac{v_1^2 C_{2W} - v_2^2}{s_\delta} \end{pmatrix},$$  \(13\)

times a coefficient $\delta^2 g_3^2 / 2 C_{2W}$, where $C_{2W} = C_{W^\mu} - S_{W^\mu}$ and $\delta^2 = (3 - 4 S_{W^\mu})$. The eigenvectors and eigenvalues of this matrix should correspond to the physical fields and their masses, respectively.

From this matrix we see that in the limit $v_1 = 0$, $K_{0I}^\mu$ does not mix with the fields $Z^\mu$ and $Z'^\mu$ and it picks up a mass value equal to the mass of $M_{K_0^\pm}$, which in turn implies that $K_{0I}^\mu$ and $K_{0I}^\mu$ (the antiparticle of $K_{0I}^\mu$) have equal masses, as they should in a well CPT behaved field theoretical framework.
Allowing for a $v_1 \neq 0$ and using the matrix entries in Eq. (13), we can read the mixing between $Z_0^\mu$ and $K_R^\mu$

$$\tan(2\psi) = \frac{2Vv_1C_W}{V^2C_W - v_1^2 - v_2^2S_W^2},$$

(14)

which goes to zero in the limit $v_1 = 0$ as it should be. In this limit $K_R^\mu$ decouples from $Z_0^\mu$, and also from $Z_0^\mu$. In what follows, even when $\psi \neq 0$, we will assume it takes a very small value and we will use $\cos \psi \approx 1$ and $\sin \psi \approx \psi$.

More relevant is the mixing between $Z_0^\mu$ and $Z_0^\mu$ which is given by

$$\tan(2\theta) = \frac{2\sqrt{(3 - 4S_W^2)(v_1^2 - v_2^2C_{2W})}}{4V^2C_W^4 - 2v_2^2C_{2W} - v_1^2(3 - 4S_W^2 - C_{2W}^2)}$$

$$\frac{v_1 \rightarrow 0}{v_1 = 0} \frac{v_2^2 - 3 - 4S_W^2}{2V^2C_W^4 - v_2^2C_{2W}}.$$  

(15)

The physical fields are

$$Z_1^\mu = Z_0^\mu \cos \theta \cos \psi - Z_0^\mu \sin \theta - K_R^\mu \cos \theta \sin \phi,$$

$$Z_2^\mu = Z_0^\mu \sin \theta \cos \psi + Z_0^\mu \sin \phi,$$

$$K_0^\mu = Z_0^\mu \sin \cos \phi + K_R^\mu \cos \phi,$$

where $\phi$ is the mixing angle between $Z_0^\mu$ and $K_R^\mu$ which is also very small and it is zero in the limit $v_1 = 0$. In this limit the physical fields are $K_0^\mu$ and

$$Z_1^\mu = Z_0^\mu \cos \theta - Z_0^\mu \sin \theta,$$

$$Z_2^\mu = Z_0^\mu \sin \theta + Z_0^\mu \cos \theta,$$

where the mixing angle $\theta$ is going to be bounded in Sec. [LV] using experimental constraints.

\section*{F. Currents}

\subsection*{1. Charged currents}

The Hamiltonian for the currents, charged under the generators of the SU(3)$_L$ group, is $H^{\text{C}} = g_3(W^\mu J_{W}^\mu + K_\mu J_{K}^\mu + K_0^\mu J_{K_0}^\mu)/\sqrt{2} + h.c.$, with

$$J_{W}^\mu = \frac{3}{a} \sum_{a=1}^{3} \bar{u}_L^\gamma \gamma^\mu d_L^\gamma - \bar{d}_L^\gamma \gamma^\mu u_L^\gamma - \bar{d}_L^\gamma \gamma^\mu e_a L,$$

$$J_{K}^\mu = \frac{3}{a} \sum_{a=1}^{3} \bar{u}_L^\gamma \gamma^\mu d_L^\gamma - \bar{d}_L^\gamma \gamma^\mu e_{\alpha L},$$

$$J_{K_0}^\mu = \frac{3}{a} \sum_{a=1}^{3} \bar{d}_L^\gamma \gamma^\mu d_L^\gamma - \bar{d}_L^\gamma \gamma^\mu u_L^\gamma - \bar{d}_L^\gamma \gamma^\mu e_{\alpha L},$$

where $K_0^\mu$ is electrically neutral but carries weak V-isospin, besides it is flavor non diagonal.

\subsection*{2. Neutral currents}

The neutral currents $J_\mu(EM), J_\mu(Z)$ and $J_\mu(Z')$, associated with the Hamiltonian $H^0 = eA^\mu J_\mu(EM) + (g_3/C_W)Z^\mu J_\mu(Z) + (g_1/\sqrt{3})Z'^\mu J_\mu(Z')$, are

$$J_\mu(EM) = \frac{2}{3} \sum_{a=1}^{3} \bar{u}_L^\gamma \gamma^\mu u_a + \bar{\nu}_L^\gamma \gamma^\mu \nu_a U,$$

$$-\frac{1}{3} \sum_{a=1}^{3} \bar{d}_L^\gamma \gamma^\mu d_a + \sum_{i=1}^{2} \bar{D}_L^\gamma \gamma^\mu D^i,$$

$$- \sum_{a=e,\mu,\tau} \bar{e}_a^\gamma \gamma^\mu e_a,$$

$$= \sum_f q_f \bar{\gamma}_\mu f,$$

$$J_\mu(Z) = J_{\mu,L}(Z) = -S^\delta_W J_\mu(EM),$$

$$J_\mu(Z') = -J_{\mu,L}(Z') + T_W J_\mu(EM),$$

where $e = g_3S_W^2 = g_1C_W \sqrt{(1 - T_W^2)/3}$ is the electric charge, $q_f$ is the electric charge of the fermion $f$ in units of $e$, and $J_\mu(EM)$ is the electromagnetic current.

The left-handed currents are

$$J_{\mu,L}(Z) = \frac{1}{2} \sum_{a=1}^{3} \left( \bar{u}_L^\gamma \gamma^\mu u_a - \bar{d}_L^\gamma \gamma^\mu d_a - \bar{d}_L^\gamma \gamma^\mu u_a L \right)$$

$$= \sum_F F_L T_3^\mu \gamma^\mu F_L,$$

(16)

$$J_{\mu,L}(Z') = S^{-1}_{2W} \left[ \bar{u}_L^\gamma \gamma^\mu u_a 2L + \bar{d}_L^\gamma \gamma^\mu d_a 2L - \bar{d}_L^\gamma \gamma^\mu d_1 L \right]$$

$$- \sum_a \left( \bar{u}_L^\gamma \gamma^\mu u_a L - \bar{d}_L^\gamma \gamma^\mu d_a L \right),$$

$$+ T_{2W}^{-1} \left[ \bar{d}_L^\gamma \gamma^\mu u_a 2L + \bar{d}_L^\gamma \gamma^\mu u_a 3L \right]$$

$$- \bar{d}_L^\gamma \gamma^\mu u_1 L - \sum_a \left( \bar{u}_L^\gamma \gamma^\mu u_a L + \bar{d}_L^\gamma \gamma^\mu d_a L \right),$$

$$+ T_{-1} \left[ \bar{d}_L^\gamma \gamma^\mu d_2 L + \bar{d}_L^\gamma \gamma^\mu d_3 L \right]$$

$$- \bar{d}_L^\gamma \gamma^\mu u_1 L - \sum_a \left( \bar{u}_L^\gamma \gamma^\mu u_a L \right),$$

$$= \sum_F F_L T_3^\mu \gamma^\mu F_L,$$

(17)

where $S_{2W} = 2S_W C_W, T_{2W} = S_{2W}/C_{2W}, T_{3f} = Dg(1/2, -1/2, 0)$ is the third component of the weak isospin, $T_{3f} = Dg(S^{-1}_{2W}, T_{2W}, -T_{-1})$ is a convenient 3 x 3 diagonal matrix, acting both of them on the representation 3 of SU(3)$_L$ (the negative value when acting on the representation 3*, which is also true for the matrix $T_{3f}$) and $F$ is a generic symbol for the representations 3 and 3* of SU(3)$_L$. Notice that $J_\mu(Z)$ is the neutral current of the SM (with the extra fields included in $J_\mu(EM)$).
This allows us to identify $Z_u$ as the neutral gauge boson of the SM, which is consistent with Eqs. (1) and (1).

The couplings of the flavor diagonal mass eigenstates $Z_{1u}^u$ and $Z_{2u}^u$ are given by

$$H^{NC}_{1u} = \frac{g_3}{2C_W} \sum_{i=1}^{2} Z_{iu}^u \sum_{f} \{ f \gamma_\mu [a_{IL}(f)(1-\gamma_5) + a_{IR}(f)(1+\gamma_5)] f \}$$

$$= \frac{g_3}{2C_W} \sum_{i=1}^{2} Z_{iu}^u \sum_{f} \{ f \gamma_\mu [g(f)_{iV} - g(f)_{iA} \gamma_5] f \},$$

with

$$a_{IL}(f) = \cos \theta \cos \psi (T_{3f} - q_f S_{W}^2) + \Theta \sin \theta T_{1f} - \Theta \sin \theta q_f T_{W},$$

$$a_{IR}(f) = -q_f \{ \cos \theta \cos \psi S_{W}^2 + \Theta \sin \theta T_{W} \},$$

$$a_{2L}(f) = S_A (T_{3f} - q_f S_{W}^2) - \Theta \cos \theta \cos \phi (T_{3f} - q_f T_{W}),$$

$$a_{2R}(f) = -q_f \{ S_A S_{W}^2 - \Theta \cos \theta \cos \phi T_{W} \},$$

where $S_A = (\sin \theta \cos \phi \cos \psi - \sin \phi \sin \psi)$, and $\Theta = S_W C_W / \sqrt{(3-4S_W^2)}$. From these coefficients we can read

$$g(f)_{1V} = \cos \theta \cos \psi (T_{3f} - 2q_f S_{W}^2) + \Theta \sin \theta T_{1f} - \Theta \sin \theta q_f T_{W},$$

$$g(f)_{2V} = S_A (T_{3f} - 2q_f S_{W}^2) - \Theta \cos \theta \cos \phi (T_{3f} - 2q_f T_{W}),$$

$$g(f)_{1A} = \cos \theta \cos \psi T_{3f} + \Theta \sin \theta T_{1f},$$

$$g(f)_{2A} = S_A T_{3f} - \Theta \cos \theta \cos \phi T_{3f}. \tag{19}$$

The values of $g_{1V}$ and $g_{1A}$, with $i = 1, 2$, are listed in Tables I and II.

As we can see, in the limit $\theta = 0$ the couplings of $Z_{1u}^u$ to the ordinary leptons and quarks are the same as in the SM; due to this property we can test the new physics beyond the SM predicted by this particular model.

### III. FERMION MASSES

The Higgs scalars introduced in Sec. II break the symmetry in an appropriate way and, at the same time, are the only scalar representations able to produce mass terms for the fermion fields via Yukawa interactions.

#### A. The up quark sector

The Yukawa terms for the up quark sector are

$$L_Y^u = Q_L^u \phi_1^* C(h^U_{1L} c + \sum_{a=1}^{3} h_a^u w_a^e c) + \sum_{i=2}^{3} Q_L^U \phi_2 C(h_a^u u_a^L c + h_i^U U_i^L) + h.c., \tag{20}$$

where the $h$'s are Yukawa couplings and $C$ is the charge conjugation operator. By introducing a symmetry which forbids the term proportional to $h_a^u$ and $h_i^U$ (see below) and in the basis $(u^1, u^2, u^3, U^u)$ we get, from Eq. (20), the following tree-level mass matrix

$$M_u = \begin{pmatrix}
0 & h_{21}^u & h_{31}^u & v_2^u \\
h_{21}^u & 0 & h_{32}^u & 0 \\
h_{31}^u & h_{32}^u & 0 & 0 \\
v_2^u & 0 & 0 & h^U V
\end{pmatrix}, \tag{21}$$

The matrix $M_u^\dagger M_u$ has one eigenvalue equal to zero associated to the quark $u^1$ which we identify as the $u$ quark in the first family. The other three eigenvalues are: one at the scale $V^2$ with eigenvector $U^u$ and two at the electroweak scale $v_2^u$, one of them suppressed by differences of Yukawa couplings; let us see:

First, if we set the 8 Yukawa couplings equal to a common real value $h^u$, then two zero eigenvalues are obtained, one corresponding to $u^1$ and the other one associated to $(u^2 - u^3)/\sqrt{2}$. The two eigenvalues different from zero are: $(h^U V)^2$ related to the exotic $U$ quark and $(2h^u v_2^u)^2$ related to the eigenvector $(u^2 + u^3)/\sqrt{2}$ which we may identify as the top quark $t$, with a proper mass if we set $h^u = 0.5$.

Next, let us set $h_{21}^u = h_{23}^u = h^U = h^u$, $h_{22}^u = h_{32}^u = h_a$ and $h_{23}^u = h_{32}^u = h_b$, then we have now only one zero eigenvalue related with the $u^1$ quark in the first family, one eigenvalue equal to $(h_a V)^2$ associated to the exotic $U$ quark, a third eigenvalue $(h_a^2 + h_b^2) v_2^u$ with eigenvector $(u^2 - u^3)/\sqrt{2}$ that we may identify with the top quark $t$, and a fourth eigenvalue $(h_a^2 - h_b^2) v_2^u$ with eigenvector $(u^2 - u^3)/\sqrt{2}$ that we may identify with the charm quark $c$.

This result is quite interesting by itself because it allows us to identify, for the up quark sector, the mass eigenstates as a function of the weak eigenstates, and the model becomes a realistic one (at least for the up quark sector) as far as we can identify a mechanism able to generate afterwards a mass for the $u$ quark in the first family.

Clearly there is not an ingredient able to generate this mass in the model presented so far. What we propose here is to introduce a third scalar $\phi_3^u = (\phi_3^u, \phi_3^u, \phi_3^u) \sim (1, 3^*, -1/3)$ which does not develop a VEV, but that couples to the up quark sector via a Yukawa term of the form $Q_L^u \phi_3^u C \sum_a h_a^u w_a^e c$. Then the up quark picks up a radiative mass via the diagram depicted in Fig. 4 where the mixing in the neutral sector is due to a totally antisymmetric term in the scalar potential of the form $\lambda \phi_1^2 \phi_2^* \phi_3$, where the $SU(3)_C$ indexes are understood.

The restrictions on the Yukawa couplings in the up quark sector can be realized by requiring invariance under an anomaly free discrete $Z_2$ symmetry [15], with the
TABLE I: The $Z'_2 \rightarrow \bar{f} f$ couplings.

| $f$ | $g(f)_{1V}$ | $g(f)_{1A}$ |
|-----|-------------|-------------|
| $u^2$, $u^3$ | $(\frac{1}{2} - \frac{4S_W^2}{3}) \cos \theta \cos \psi + \Theta(S_{1W}^1 - \frac{2T_W^1}{3}) \sin \theta$ | $\frac{1}{2} \cos \theta \cos \psi + \Theta S_{1W}^1 \sin \theta$ |
| $u^1$ | $(\frac{1}{2} - \frac{4S_W^2}{3}) \cos \theta \cos \psi - \Theta(T_{1W}^1 + \frac{2T_W^1}{3}) \sin \theta$ | $\frac{1}{2} \cos \theta \cos \psi - \Theta T_{1W}^1 \sin \theta$ |
| $d^2$, $d^3$ | $-\frac{1}{2} + \frac{2S_W^2}{3} \cos \theta \cos \psi + \Theta(T_{1W}^1 + \frac{2T_W^1}{3}) \sin \theta$ | $-\frac{1}{2} \cos \theta \cos \psi + \Theta T_{1W}^1 \sin \theta$ |
| $d^1$ | $-\frac{1}{2} + \frac{2S_W^2}{3} \cos \theta \cos \psi - \Theta(S_{1W}^1 - \frac{2T_W^1}{3}) \sin \theta$ | $-\frac{1}{2} \cos \theta \cos \psi - \Theta S_{1W}^1 \sin \theta$ |
| $U$ | $\frac{2S_W^2}{3} \cos \theta \cos \psi - \Theta(T_{1W}^1 + \frac{2T_W^1}{3}) \sin \theta$ | $\Theta T_{1W}^1 \sin \theta$ |
| $D_{1,2}$ | $\frac{2S_W^2}{3} \cos \theta \cos \psi + \Theta(T_{1W}^1 + \frac{2T_W^1}{3}) \sin \theta$ | $-\Theta T_{1W}^1 \sin \theta$ |
| $\varepsilon_{1,2,3}$ | $-\frac{1}{2} + \frac{2S_W^2}{3} \cos \theta \cos \psi - \Theta(S_{1W}^1 - 2T_W) \sin \theta$ | $-\frac{1}{2} \cos \theta \cos \psi - \Theta S_{1W}^1 \sin \theta$ |
| $\nu_{1,2,3}$ | $\frac{1}{2} \cos \theta \cos \psi - \Theta T_{1W}^1 \sin \theta$ | $\frac{1}{2} \cos \theta \cos \psi - \Theta T_{1W}^1 \sin \theta$ |
| $N^1_{1,2,3}$ | $-\Theta T_{1W}^1 \sin \theta$ | $-\Theta T_{1W}^1 \sin \theta$ |

TABLE II: The $Z'_2 \rightarrow \bar{f} f$ couplings.

| $f$ | $g(f)_{2V}$ | $g(f)_{2A}$ |
|-----|-------------|-------------|
| $u^2$, $u^3$ | $(\frac{1}{2} - \frac{4S_W^2}{3})S_A - \Theta(S_{1W}^1 - \frac{2T_W^1}{3}) \cos \theta \cos \phi$ | $\frac{1}{2}S_A - \Theta S_{1W}^1 \cos \theta \cos \phi$ |
| $u^1$ | $(\frac{1}{2} - \frac{4S_W^2}{3})S_A + \Theta(T_{1W}^1 + \frac{2T_W^1}{3}) \cos \theta \cos \phi$ | $\frac{1}{2}S_A + \Theta T_{1W}^1 \cos \theta \cos \phi$ |
| $d^2$, $d^3$ | $\frac{1}{2} + \frac{2S_W^2}{3}S_A - \Theta(T_{1W}^1 + \frac{2T_W^1}{3}) \cos \theta \cos \phi$ | $\frac{1}{2}S_A - \Theta T_{1W}^1 \cos \theta \cos \phi$ |
| $d^1$ | $\frac{1}{2} + \frac{2S_W^2}{3}S_A + \Theta(S_{1W}^1 - \frac{2T_W^1}{3}) \cos \theta \cos \phi$ | $\frac{1}{2}S_A + \Theta S_{1W}^1 \cos \theta \cos \phi$ |
| $U$ | $\frac{2S_W^2}{3}S_A + \Theta(T_{1W}^1 + \frac{2T_W^1}{3}) \cos \theta \cos \phi$ | $\Theta T_{1W}^1 \cos \theta \cos \phi$ |
| $D_{2,3}$ | $\frac{2S_W^2}{3}S_A - \Theta(T_{1W}^1 + \frac{2T_W^1}{3}) \cos \theta \cos \phi$ | $-\Theta T_{1W}^1 \cos \theta \cos \phi$ |
| $\varepsilon_{1,2,3}$ | $-\frac{1}{2} + \frac{2S_W^2}{3}S_A + \Theta(S_{1W}^1 - \frac{2T_W^1}{3}) \cos \theta \cos \phi$ | $-\frac{1}{2}S_A + \Theta S_{1W}^1 \cos \theta \cos \phi$ |
| $\nu_{1,2,3}$ | $\frac{1}{2}S_A + \Theta T_{1W}^1 \cos \theta \cos \phi$ | $\frac{1}{2}S_A + \Theta T_{1W}^1 \cos \theta \cos \phi$ |
| $N^1_{1,2,3}$ | $\Theta T_{1W}^1 \cos \theta \cos \phi$ | $\Theta T_{1W}^1 \cos \theta \cos \phi$ |

FIG. 1: One loop diagram contributing to the radiative generation of the up quark mass.

The following assignments of $Z_2$ charge $q$

$$q(Q^a_L, U^c_L, D^c_L, \varepsilon^c_L, \phi_1) = 0,$$

$$q(u^a_L, d^c_L, l^c_L, \phi_2, \phi_3) = 1,$$

for $a = 1, 2, 3$ and $i = 1, 2$; where we have included the down quark and lepton sectors, anticipating the analysis which follows.

As can be seen, we have avoided a hierarchy of the Yukawa couplings in the up quark sector just by introducing the extra Higgs field $\phi_3$, obtaining in this way a neat mass spectrum in this sector.

B. The down quark sector

The most general Yukawa terms for the down quark sector, using just $\phi_1$ and $\phi_2$, are

$$L_Y^d = Q^1_L \phi_2 C \left( \sum_i h^d_i D^c_L + \sum_a h^d_a d^c_L \right) + \sum_i Q^1_L \phi_1 C \left( \sum_a h^d_a d^c_L + \sum_i h^d_i D^c_L \right) + h.c.$$  \hspace{1cm} (22)

In the basis $(d^1, d^2, d^3, D^2, D^3)$ and with the $Z_2$ symmetry introduced above, this lagrangian produces a treelevel down quark mass matrix of the form

$$M_d = \begin{pmatrix} h^d_1 v_2 & 0 & 0 & 0 & 0 \\ h^d_2 v_2 & 0 & 0 & 0 & 0 \\ h^d_3 v_2 & 0 & 0 & 0 & 0 \\ 0 & h^d_{21} v_1 & h^d_{32} v_1 & h^d_{22} V & h^d_{32} V \\ 0 & h^d_{23} v_1 & h^d_{33} v_1 & h^d_{23} V & h^d_{33} V \end{pmatrix}.$$  \hspace{1cm} (23)

The matrix $M^0_d M_d$ has two eigenvalues equal to zero (even for $v_1 \neq 0$). For $h^d_1 = h^d_2 = h^d_3 \equiv h^d$, and $h^d_{ij}$ of order one, the nonzero eigenvalues are: two of the order of $V$ that we may identify with the masses of the heavy
exotic quarks $D^1$ and $D^2$, and one of the order of $3h^dv_2$ that we may identify with the mass of the ordinary down quark $d^1$, with a hierarchy $h^d/h^u \sim 10^{-4}$ (this hierarchy can be eliminated by introducing more scalars). The two states with zero eigenvalues are linear combinations of $d^2$ and $d^3$ that we may identify with the bottom and strange quarks.

The ingredient that generates radiative masses for the two tree-level massless states is just the scalar field $\phi_3$ used in the up quark sector, via a Yukawa term of the form $Q_L^\dagger \phi_3 C \sum_a h^d_a d^a_L$. The diagram in Fig. 2 shows how to generate these masses at the one loop level, one of them enhanced by sum of Yukawa couplings and the other one depleted by difference of Yukawa couplings.

Notice that the mass eigenstates are related to the weak eigenstates (up to small mixing of the order of $v_1$ with the two exotic down quarks) in the following way: for the bottom quark $b \approx (d^3 + d^1)/\sqrt{2}$, for the strange quark $s \approx (d^3 - d^1)/\sqrt{2}$, and for the down quark $d \approx d^1$ plus mixing via radiative loops with the other four down type quarks.

Our findings here are that the spectrum in the down quark sector is not as neat as it is in the up quark sector, and that the up-down hierarchy is unavoidable in the context of the analysis presented.

C. The charged lepton sector

The Yukawa term for the three charged leptons allowed by the discrete symmetry is

$$\mathcal{L}_Y^l = \sum_{a,b=1}^3 h^e_{a\beta} L_{aL} \phi_2^c e_{bL} + h.c.,$$

which, in the basis $(e^1, e^2, e^3)$, produces a mass matrix of the form

$$M_e = v_2 \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}. \quad (25)$$

With all the Yukawa couplings equal, this is a democratic type mass matrix which, as we know, is a good starting point to generate consistent lepton mass matrices. The disadvantage here is again the strong hierarchy to be imposed between the Yukawa couplings in the up, down and leptonic sectors.

We can avoid again the hierarchy by using a discrete symmetry in order to eliminate the tree-level mass term, and introducing Higgses (leptoquarks) able to produce radiative masses for all the charged leptons.

D. The neutral lepton sector

The discrete $Z_2$ symmetry avoids the only possible Yukawa term leading to tree-level neutrino masses \[4\], which is of the form $\bar{\nu}_a L_i \nu_b L_j \phi_2$. However, the neutrinos can get radiative Dirac type masses using the mixing between the two charged gauge bosons as depicted in Fig. 3. The mass matrix generated in this way is of the democratic type with large mixing among the several flavors. Notice also that the value of $v_1$ must be very small in order to have a small neutrino mass scale, in agreement with the analysis in Sec. II (if $v_1 = 0$ the mechanism in Ref. \[4\] is still available).

Notice the particular way how the neutrinos mix and get masses in this model. They acquire only Dirac type masses via a one loop radiative correction, which is different to the well known ways of generating neutrino masses, namely, the Zee mechanism \[12\] and the see-saw mechanism.

IV. CONSTRAINTS ON THE GAUGE BOSONS MIXING ANGLES

After the identification of the mass eigenstates, we can properly bound $\sin \theta$ and $M_{Z_2}$ by using parameters measured at the $Z$ pole from CERN $e^+e^-$ collider (LEP), SLAC Linear Collider (SLC), and atomic parity violation constraints which are given in Table IV.

The expression for the partial decay width for $Z_1^\mu \to f\bar{f}$ is

$$\Gamma(Z_1^\mu \to f\bar{f}) = \frac{N_C G_F M_{Z_1}^3}{6\pi\sqrt{2}} \rho \left( \frac{3\beta - \beta^3}{2} \right)^2 |g(f)_{V}|^2$$
\[+ \beta^3 [g(i)_{1A}]^2 \{1 + \delta f\} R_{EW} R_{QCD},(26)\]

where \(f\) is an ordinary SM fermion, \(Z_i^f\) is the physical gauge boson observed at LEP, \(N_C = 1\) for leptons while for quarks \(N_C = 3(1 + \alpha_S/\pi + 1.405\alpha^2/\pi^2 - 12.77\alpha^3/\pi^3)\), where the 3 is due to color and the factor in parentheses represents the universal part of the QCD corrections for massless quarks (for fermion mass effects and further QCD corrections which are different for vector and axial-vector partial widths, see Ref. [13]); \(R_{EW}\) are the electroweak corrections which include the leading order QED corrections given by \(R_{QED} = 1 + 3\alpha/(4\pi)\). \(R_{QCD}\) are further QCD corrections (for a comprehensive review see Ref. [14] and references therein), and

\[\beta = \sqrt{1 - 4m_f^2/M_Z^2}\]

is a kinematic factor which can be taken equal to 1 for all the SM fermions except for the bottom quark. The factor \(\delta f\) contains the one loop vertex contribution which is negligible for all fermion fields except for the bottom quark, for which the contribution coming from the top quark, at the one loop vertex level, is parameterized as \(\delta_b \approx 10^{-2}[m_b^2/(2M_Z^2)] + 1/5\) [15]. The \(\rho\) parameter can be expanded as \(\rho = 1 + \delta_\rho + \delta_\nu\) where the oblique correction \(\delta_\rho\) is given by \(\delta_\rho \approx 3G_F m_f^2/(8\pi^2\sqrt{2})\), and \(\delta_\nu\) is the tree level contribution due to the \((Z' - Z)\) mixing which can be parameterized as \(\delta_\nu \approx (M_Z^2/M_{Z_1}^2 - 1)\sin^2\theta\). Finally, \(g(f)_{1V}\) and \(g(f)_{1A}\) are the coupling constants of the physical \(Z_i^f\) field with ordinary fermions which, for this model, are listed in Table 4.

In what follows we are going to use the experimental values [4]: \(M_{Z_1} = 91.188\) GeV, \(m_t = 174.3\) GeV, \(a_s(m_Z) = 0.1192\), \(\alpha(m_Z)^{-1} = 127.938\), and \(\sin^2\theta_W = 0.2333\). These values are introduced using the definitions \(R_i = \Gamma_{Z}(q_i)/(\Gamma_{Z}(hadrons))\) for \(q = e, \mu, \tau, b, c, s, u, d\).

As a first result, notice from Table 4 that this model predicts \(R_e = R_\mu = R_\tau\) in agreement with the experimental results in Table III independent of any flavor mixing at the tree-level.

The effective weak charge in atomic parity violation, \(Q_W\), can be expressed as a function of the number of protons \((Z)\) and the number of neutrons \((N)\) in the atomic nucleus in the form

\[Q_W = -2[(2Z + N)c_{1u} + (Z + 2N)c_{1d}], (27)\]

where \(c_{1q} = 2g(e)_{1A} g(q)_{1V}\). The theoretical value for \(Q_W\) for the cesium atom is given by [16] \(Q_W(\frac{1331C}{S}) = -73.19 \pm 0.13 + \Delta Q_W\), where the contribution of new physics is included in \(\Delta Q_W\) which can be written as [17]

\[\Delta Q_W = \left[1 + 4\frac{S_W^4}{1 - 2S_W^2}\right] Z - N \delta_\nu + \Delta Q_W'. (28)\]

The term \(\Delta Q_W'\) is model dependent and it can be obtained for our model by using \(g(e)_{1A}\) and \(g(q)_{1V}\), \(i = 1, 2, 3\), from Tables IV and V. The value we obtain is

**TABLE III: Experimental data and SM values for some parameters related with neutral currents.**

| \(\Delta Q_W'\) | \(\Delta Q_W = (3.75Z + 2.56N)\sin\theta + (1.22Z + 0.41N)\frac{M_Z^2}{M_{Z_1}^2}\) |
|-----------------|---------------------------------|
| \(\exp\)        | \(Q_{W}^{SM} - Q_{W}^{SM} = 0.45 \pm 0.48\) |

which is 1.1 \(\sigma\) away from the SM predictions.

The discrepancy between the SM and the experimental data for \(\Delta Q_W\) is given by [16]

\[\Delta Q_W = Q_{W}^{ex} - Q_{W}^{SM} = 0.45 \pm 0.48, (30)\]

As we can see, the mass of the new neutral gauge boson is compatible with the bound obtained in \(p\bar{p}\) collisions at the Fermilab Tevatron [18]. From our analysis we can also see that for \(\theta = 0\), \(M_{Z_2}\) peaks at a finite value larger than 100 TeV which still copes with the experimental constraints on the \(\rho\) parameter.

V. CONCLUSIONS

During the last decade several 3-3-1 models for one and three families have been analyzed in the literature, the most popular one being the Pletz-Frampton model [1] which is certainly not the simplest construction. Other two different three-family models, more appealing but not so popular in the literature, were presented in Refs. [2] and [10]. The first one, studied in this paper, contains right-handed neutrinos, and the second one without right-handed neutrinos but with an extra exotic electron per family. The systematic analysis presented in
Refs. [21] and [3] shows that there are an infinite number of models based on the 3-3-1 gauge structure, most of them including particles with exotic electric charges; but the number of models with particles without exotic electric charges are just a few. Other two 3-3-1 models for one family and only with ordinary electric charges are analyzed for example in Refs. [21].

For the model presented in this paper we have studied the mixing of the two charged gauge bosons and its implications. Also we analyzed the mixing of the three electrically neutral gauge bosons and set limits for the mixing angles using precision measurements of the electroweak sector.

There are in this model three mass scales: the 3-3-1 scale $V \sim 1$ TeV, the electroweak scale $v_2 \approx 175$ GeV, and a new very small scale $v_1$ proportional to the neutrino mass scale.

By the use of a discrete $Z_2$ symmetry we have constructed an appealing mass spectrum for the fermions in this model; in particular we have carried a program with a minimum set of Higgs scalars (three) and VEV in which: the exotic fermions get heavy masses at the TeV scale; the quarks $t, c$ and $d$ get tree-level masses, with the ordinary up quark sector getting masses at the $v_2$ scale and with a hierarchy between the up and down sectors; the mass for the $c$ quark is suppressed by differences of Yukawa couplings, and the other ordinary quarks, $b, s$ and $u$, get radiative masses, with the $Z_2$ symmetry responsible of the weak isospin breaking.

The quark mass spectrum constructed in this way is just one example of how to proceed. Different discrete symmetries and extra scalar Higgs fields with and without VEV may be used in order to avoid hierarchies and produce a more realistic mass spectrum [22].

The neutrino masses and oscillations have been analyzed for this 3-3-1 model in Ref. [4] using a different approach from the one sketched here. In particular these authors use four $SU(3)_L$ scalar triplets instead of three, one of them with zero VEV as in this model, plus one charged scalar, three more $SU(3)_L$ singlets of neutral spin 1/2 particles (one per family), and the extra assumption of a diagonal mass matrix for the charged leptons.

In the main text we have proposed an alternative way of generating tiny neutrino masses with large mixing, but a detailed analysis of our proposal is beyond the aims of this paper.

In summary, in this paper we have presented original results completing previous analysis of the 3-3-1 model with right-handed neutrinos. First, the bounds on the mixing parameters of the neutral currents have been updated. But most important, our Higgs sector and VEV are different from the ones introduced in previous papers. They imply quite different mass matrices for gauge bosons and fermion fields, and a quite different phenomenology. The most important fact about our Higgs sector is that it allows for an acceptable fermion mass spectrum. Then we identify, for the first time for this model, the quark mass eigenstates; this allows us to do a consistent phenomenological analysis and to set reliable bounds on new physics coming from heavy neutral currents. It is worth noticing that the same scalar sector and VEV structure we have considered, have been used in Ref. [23] in order to show that, by allowing for a nonzero and small VEV $v_1$, the spontaneous breaking of
the lepton number can be implemented in the model. A new results to remark is also the embedding of this 3-3-1 structure into $SU(6)$.

The maybe unpleasant hierarchies present in the model can be avoided by introducing more Higgs scalar fields and new VEV, but again, the analysis is beyond the reach of this paper [22].

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