Field dependent nilpotent symmetry for gauge theories

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We construct the field dependent mixed BRST (combination of BRST and anti-BRST) transformations for pure gauge theories. These are shown to be an exact nilpotent symmetry of both the effective action as well as the generating functional for certain choices of the field dependent parameters. We show that the Jacobian contributions for path integral measure in the definition of generating functional arising from BRST and anti-BRST part compensate each other. The field dependent mixed BRST transformations are also considered in field/antifield formulation to show that the solutions of quantum master equation remain invariant under these. Our results are supported by several explicit examples.

I. INTRODUCTION

The BRST transformation plays a central role in the quantization, renormalizability, unitarity and other aspects of the gauge theories [1–5]. Such nilpotent transformation, characterized by an infinitesimal, global and anticommuting parameter, leaves the effective action including the gauge fixing term invariant. Similar to the BRST transformation, anti-BRST transformation is also a symmetry transformation where the role of ghost and anti-ghost fields are interchanged. The anti-BRST symmetry does not play as fundamental role as BRST symmetry itself but it is a useful tool in geometrical description of BRST/anti-BRST symmetric theories [6] and widely used in the investigation of perturbative renormalization of the gauge theories [7]. We consider the mixed BRST (MBRST)
transformation (defined as $\delta_m \phi = s_b \phi \delta \Lambda_1 + s_{ab} \phi \delta \Lambda_2$) with infinitesimal, anticommuting but global parameters $\delta \Lambda_1$ and $\delta \Lambda_2$ corresponding to the BRST variation ($s_b$) and anti-BRST variation ($s_{ab}$) of the generic fields $\phi$ respectively. However, the transformations $s_b$ and $s_{ab}$ defined in the MBRST transformation satisfy the absolute anticommuting relation $\{s_b, s_{ab}\} \phi = 0$. Such an infinitesimal mixed transformation is also a nilpotent symmetry transformation of the effective action as well as of the generating functional.

In the present work we construct the field dependent MBRST (FMBRST) transformation having finite and field dependent parameters. The usual finite field dependent BRST (FFBRST) and anti-BRST (FF-anti-BRST) transformations are the symmetry transformations of the effective action only but do not leave the generating functional invariant as the path integral measure in the definition of generating functional transforms in a non-trivial manner [8, 9]. The FFBRST and FF-anti-BRST transformations for the effective theory have many applications in gauge field theory [8–16]. Unlike the usual FFBRST and FF-anti-BRST transformations the FMBRST transformations are shown to be the symmetry of the both effective action as well as the generating functional of the theory. We construct the finite parameters in the FMBRST transformation in such a way that the Jacobian contribution due to FFBRST part compensates the same due to FF-anti-BRST part. Thus we are able to construct the finite nilpotent transformation which leaves the generating functional as well as the effective action of the theory invariant. We further show that the effect of FMBRST transformation is equivalent to the effect of successive operations of FFBRST and FF-anti-BRST transformations.

Our results are supported by several explicit examples. First of all we consider the gauge invariant model for single self-dual chiral boson in (1+1) dimensions [17–19], which is very useful in the study of certain string theoretic models [20, 21] and plays a very crucial role in the study of quantum Hall effect [22]. (3+1) dimensional Abelian as well as non-Abelian Yang-Mills (YM) theory in the Curci-Ferrari-Delbourgo-Jarvis (CFDJ) gauge [23–25] are also considered to demonstrate the above finite nilpotent symmetry.

The Lagrangian quantization of Batalin and Vilkovisky (BV) formulation [3, 4, 26, 27], which is also known as the field/antifield formulation, is considered to be one of the most
powerful and advanced technique of quantization of gauge theories involving the BRST symmetry \[3, 4, 28\]. To study the role of FMBRST transformation in field/antifield formulation we consider the same three simple models in BV formulation. We show that the FMBRST transformation does not change the generating functional written in terms of extended quantum action in BV formulation. Hence the FMBRST transformation leaves the different solutions of the quantum master equation in field/antifield formulation invariant.

The paper is organized as follows. In Sec. II, we discuss the infinitesimal MBRST transformation. Next the FMBRST transformation is constructed in Sec. III. The non-trivial Jacobian for such FMBRST transformation is evaluated in Sec. IV. Sec. V, is devoted to the explicit examples having FMBRST symmetry transformation. In Sec. VI, we study the field/antifield formulation in the context of FMBRST transformation. Sec. VII is reserved for concluding remarks.

II. THE INFINITESIMAL MBRST TRANSFORMATION

The generating functional for the Green’s function in an effective theory described by the effective action \( S_{\text{eff}}[\phi] \) is defined as

\[
Z = \int D\phi \ e^{i S_{\text{eff}}[\phi]},
\]

\[
S_{\text{eff}}[\phi] = S_0[\phi] + S_{gf}[\phi] + S_{gh}[\phi],
\]

where \( \phi \) is the generic notation for all fields involved in the effective theory. The infinitesimal BRST \( (\delta_b) \) and anti-BRST \( (\delta_{ab}) \) transformations are defined as

\[
\delta_b \phi = s_b \phi \delta \Lambda_1, \quad s_b^2 = 0
\]

\[
\delta_{ab} \phi = s_{ab} \phi \delta \Lambda_2, \quad s_{ab}^2 = 0,
\]

where \( \delta \Lambda_1 \) and \( \delta \Lambda_2 \) are infinitesimal, anticommuting but global parameters. Such transformations leave the generating functional as well as effective action invariant

\[
\delta_b Z = 0 = \delta_b S_{\text{eff}},
\]
\[ \delta_{ab} Z = 0 = \delta_{ab} S_{\text{eff}}. \]  

(2.6)

This implies that the effective action \( S_{\text{eff}}[\phi] \) and the generating functional are also invariant under the MBRST \( (\delta_m = \delta_b + \delta_{ab}) \) transformation

\[ \delta_m Z = 0 = \delta_m S_{\text{eff}}. \]  

(2.7)

Further such MBRST transformation is nilpotent because,

\[ \{s_b, s_{ab}\} = 0. \]  

(2.8)

Now, in the next section we construct the finite field dependent version of following infinitesimal MBRST symmetry transformation

\[ \delta_m \phi = s_b \phi \delta \Lambda_1 + s_{ab} \phi \delta \Lambda_2. \]  

(2.9)

III. CONSTRUCTION OF FMIBRST TRANSFORMATION

To construct the FMBRST transformation, we use to follow the similar method of constructing FFBRST transformation \[8\]. However, in this case unlike FFBRST transformation we have to deal with two parameters, one for the BRST transformation and the other for anti-BRST transformation. We introduce a numerical parameter \( \kappa(0 \leq \kappa \leq 1) \) and make all the fields \( (\phi(x, \kappa)) \) \( \kappa \)-dependent in such a way that \( \phi(x, \kappa = 0) \equiv \phi(x) \) and \( \phi(x, \kappa = 1) \equiv \phi'(x) \), the transformed field. Further, we make the infinitesimal parameters \( \delta \Lambda_1 \) and \( \delta \Lambda_2 \) field dependent as

\[ \delta \Lambda_1 = \Theta_1[\phi(x, \kappa)] d\kappa \]  

(3.1)

\[ \delta \Lambda_2 = \Theta_2[\phi(x, \kappa)] d\kappa, \]  

(3.2)

where the prime denotes the derivative with respect to \( \kappa \) and \( \Theta_i[\phi(x, \kappa)](i = 1, 2) \) are infinitesimal field dependent parameters. The infinitesimal but field dependent MBRST transformations, thus can be written generically as

\[ \frac{d\phi(x, \kappa)}{d\kappa} = s_b \phi(x, \kappa) \Theta_1[\phi(x, \kappa)] + s_{ab} \phi(x, \kappa) \Theta_2[\phi(x, \kappa)]. \]  

(3.3)
Following the work in Ref. \[8\] it can be shown that the parameters $\Theta_i'[\phi(x, \kappa)] (i = 1, 2)$, contain the factors $\Theta_i'[\phi(x, 0)] (i = 1, 2)$, which are considered to be nilpotent. Thus $\kappa$ dependency from $\delta_b \phi(x, \kappa)$ and $\delta_{ab} \phi(x, \kappa)$ can be dropped. Then the Eq. (3.3) can be written as

$$\frac{d\phi(x, \kappa)}{d\kappa} = s_b \phi(x, 0) \Theta_1'[\phi(x, \kappa)] + s_{ab} \phi(x, 0) \Theta_2'[\phi(x, \kappa)].$$

(3.4)

The FMBRST transformations with the finite field dependent parameters then can be constructed by integrating such infinitesimal transformations from $\kappa = 0$ to $\kappa = 1$, such that

$$\phi' \equiv \phi(x, \kappa = 1) = \phi(x, \kappa = 0) + s_b \phi(x) \Theta_1[\phi(x)] + s_{ab} \phi(x) \Theta_2[\phi(x)],$$

(3.5)

where

$$\Theta_i[\phi(x)] = \int_0^1 d\kappa' \Theta_i'[\phi(x, \kappa')]\]$$

(3.6)

are the finite field dependent parameters with $i = 1, 2$.

Therefore, the FMBRST transformation corresponding MBRST transformation mentioned in Eq. (2.9) is given by

$$\delta_m \phi = s_b \phi \Theta_1 + s_{ab} \phi \Theta_2$$

(3.7)

It can be shown that above FMBRST transformation with some specific choices of the finite parameters $\Theta_1$ and $\Theta_2$ is the symmetry transformation of the both effective action and the generating functional as the path integral measure is generally invariant under such transformation.

IV. METHOD FOR EVALUATING THE JACOBIAN

For the symmetry of the generating functional we need to calculate the Jacobian of the path integral measure in the definition of generating functional. The Jacobian of the path integral measure for FMBRST transformation $J$ can be evaluated for some particular
choices of the finite field dependent parameters $\Theta_1[\phi(x)]$ and $\Theta_2[\phi(x)]$. We start with the definition,

$$D\phi = J(\kappa) \, D\phi(\kappa) = J(\kappa + d\kappa) \, D\phi(\kappa + d\kappa),$$

(4.1)

Now the transformation from $\phi(\kappa)$ to $\phi(\kappa + d\kappa)$ is infinitesimal in nature, thus the infinitesimal change in Jacobian can be calculated as

$$\frac{J(\kappa)}{J(\kappa + d\kappa)} = \Sigma_\phi \pm \frac{\delta\phi(x, \kappa)}{\delta\phi(x, \kappa + d\kappa)}$$

(4.2)

where $\Sigma_\phi$ sums over all fields involved in the path integral measure and $\pm$ sign refers to whether $\phi$ is a bosonic or a fermionic field. Using the Taylor expansion we calculate the above expression as

$$\frac{1}{J(\kappa)} \frac{dJ(\kappa)}{d\kappa} = - \int d^4x \left[ \Sigma_\phi(\pm) s_\phi(x, \kappa) \frac{\partial\Theta_1'[\phi(x, \kappa)]}{\partial\phi(x, \kappa)} + \Sigma_\phi(\pm) s_{ab}\phi(x, \kappa) \frac{\partial\Theta_2'[\phi(x, \kappa)]}{\partial\phi(x, \kappa)} \right].$$

(4.3)

The Jacobian, $J(\kappa)$, can be replaced (within the functional integral) as

$$J(\kappa) \to e^{i(S_1[\phi] + S_2[\phi])}$$

(4.4)

iff the following condition is satisfied

$$\int D\phi(x) \left[ \frac{1}{J} \frac{dJ}{d\kappa} - i \frac{dS_1[\phi(x, \kappa)]}{d\kappa} - i \frac{dS_2[\phi(x, \kappa)]}{d\kappa} \right] e^{i(S_{\text{eff}} + S_1 + S_2)} = 0,$$

(4.5)

where $S_1[\phi]$ and $S_2[\phi]$ are some local functionals of fields and satisfy the initial condition

$$S_i[\phi(\kappa = 0)] = 0, \ i = 1, 2.$$

(4.6)

The finite parameters $\Theta_1$ and $\Theta_2$ are arbitrary and we can construct them in such a way that the infinitesimal change in Jacobian $J$ (Eq. (4.3)) with respect to $\kappa$ vanishes

$$\frac{1}{J} \frac{dJ}{d\kappa} = 0.$$

(4.7)
Therefore, with the help of Eqs. \(4.5\) and \(4.7\), we see that
\[
\frac{dS_1[\phi(x, \kappa)]}{d\kappa} + \frac{S_2[\phi(x, \kappa)]}{d\kappa} = 0.
\]
It means the \(S_1 + S_2\) is independent of \(\kappa\) (fields) and must vanish to satisfy the initial condition given in Eq. \(4.6\) satisfied. Hence the generating functional is not effected by the Jacobian \(J\) as \(J = e^{i(S_1 + S_2)} = 1\). The nontrivial Jacobian arising from finite BRST parameter \(\Theta_1\) compensates the same arising due to finite anti-BRST parameter \(\Theta_2\). It is straight forward to see that the effective action \(S_{eff}\) is invariant under such FMBRST transformation.

V. EXAMPLES

To demonstrate the results obtained in the previous section we would like to consider several explicit examples in \((1+1)\) as well as in \((3+1)\) dimensions. In particular we consider the bosonized self-dual chiral model in \((1+1)\) dimensions, Maxwell’s theory in \((3+1)\) dimensions, and non-Abelian YM theory in \((3+1)\) dimensions. In all these cases we construct explicit finite parameters \(\Theta_1\) and \(\Theta_2\) of FMBRST transformation such that the generating functional remains invariant.

A. Bosonized chiral model

We start with the generating functional for the bosonized self-dual chiral model as
\[
Z_{CB} = \int D\phi \ e^{iS_{CB}},
\]
where \(D\phi\) is the path integral measure in generic notation. The effective action \(S_{CB}\) in \((1+1)\) dimensions is given as
\[
S_{CB} = \int d^2x [\pi_\phi \dot{\phi} + \pi_{\dot{\phi}} \dot{\phi} + p_u \dot{u} - \frac{1}{2} \pi_\phi^2 + \frac{1}{2} \pi_{\dot{\phi}}^2 + \pi_{\dot{\phi}} (\dot{\varphi}' - \dot{\vartheta}' + \lambda) + \pi_\phi \lambda + \frac{1}{2} B^2]
\]
\[ + B(\dot{\lambda} - \varphi - \vartheta) + \dot{\bar{c}}\bar{c} - 2\bar{c}c], \quad (5.2) \]

where the fields \( \varphi, \vartheta, u, B, c \) and \( \bar{c} \) are the self-dual field, Wess-Zumino field, multiplier field, auxiliary field, ghost field and anti-ghost field respectively. The nilpotent BRST and anti-BRST transformations for this theory are

**BRST:**

\[
\begin{align*}
\delta_b \varphi &= c \delta \Lambda_1, \quad \delta_b \lambda = -\dot{\bar{c}} \delta \Lambda_1, \quad \delta_b \vartheta = c \delta \Lambda_1, \\
\delta_b \pi \varphi &= 0, \quad \delta_b u = 0, \quad \delta_b \pi \vartheta = 0, \quad \delta_b \bar{c} = B \delta \Lambda_1, \\
\delta_b B &= 0, \quad \delta_b c = 0, \quad \delta_b p_u = 0,
\end{align*}
\]

**anti-BRST:**

\[
\begin{align*}
\delta_{ab} \varphi &= -\bar{c} \delta \Lambda_2, \quad \delta_{ab} \lambda = \dot{\bar{c}} \delta \Lambda_2, \quad \delta_{ab} \vartheta = -\dot{\bar{c}} \delta \Lambda_2, \\
\delta_{ab} \pi \varphi &= 0, \quad \delta_{ab} u = 0, \quad \delta_{ab} \pi \vartheta = 0, \quad \delta_{ab} c = B \delta \Lambda_2, \\
\delta_{ab} B &= 0, \quad \delta_{ab} \bar{c} = 0, \quad \delta_{ab} p_u = 0,
\end{align*}
\]

where \( \delta \Lambda_1 \) and \( \delta \Lambda_2 \) are infinitesimal, anticommuting and global parameters. Note that \( s_b \) and \( s_{ab} \) are absolutely anticommuting i.e. \((s_b s_{ab} + s_{ab} s_b) \phi = 0\). In this case the MBRST symmetry transformation \((\delta_m \equiv \delta_b + \delta_{ab})\), as constructed in section II, reads as

\[
\begin{align*}
\delta_m \varphi &= c \delta \Lambda_1 - \bar{c} \delta \Lambda_2, \quad \delta_m \lambda = -\dot{\bar{c}} \delta \Lambda_1 + \dot{\bar{c}} \delta \Lambda_2, \\
\delta_m \vartheta &= c \delta \Lambda_1 - \bar{c} \delta \Lambda_2, \quad \delta_m \pi \varphi = 0, \quad \delta_m u = 0, \\
\delta_m \pi \vartheta &= 0, \quad \delta_m \bar{c} = B \delta \Lambda_1, \quad \delta_m B = 0, \quad \delta_m c = B \delta \Lambda_2, \\
\delta_m p_u &= 0.
\end{align*}
\]

The FMBRST transformations corresponding to the above MBRST transformations are constructed as

\[
\begin{align*}
\delta_m \varphi &= c \Theta_1 - \bar{c} \Theta_2, \quad \delta_m \lambda = -\dot{\bar{c}} \Theta_1 + \dot{\bar{c}} \Theta_2, \\
\delta_m \vartheta &= c \Theta_1 - \bar{c} \Theta_2, \quad \delta_m \pi \varphi = 0, \quad \delta_m u = 0, \\
\delta_m \pi \vartheta &= 0, \quad \delta_m \bar{c} = B \Theta_1, \quad \delta_m B = 0, \\
\delta_m c &= B \Theta_2, \quad \delta_m p_u = 0,
\end{align*}
\]
where Θ₁ and Θ₂ are finite field dependent parameters and are still anticommuting in nature. We construct the finite parameters Θ₁ and Θ₂ as

\[ \Theta_1 = \int \Theta_1' d\kappa = \gamma \int d\kappa \int d^2x [\bar{c}(\dot{\lambda} - \varphi - \vartheta)], \tag{5.7} \]

\[ \Theta_2 = \int \Theta_2' d\kappa = -\gamma \int d\kappa \int d^2x [c(\dot{\lambda} - \varphi - \vartheta)], \tag{5.8} \]

where γ is an arbitrary parameter.

Using Eq. (4.3), the infinitesimal change in Jacobian for the FMBRST transformation given in Eq. (5.6) can be calculated as

\[ \frac{1}{J} \frac{dJ}{d\kappa} = 0. \tag{5.9} \]

The contribution from second and third terms in the R.H.S. of Eq. (4.3) cancels each other. This implies that the Jacobian for path integral measure is unit under FMBRST transformation. Hence the generating functional as well as the effective action are invariant under FMBRST transformation

\[ Z_{CB} \left( \int D\phi \ e^{iS_{CB}} \right) \xrightarrow{FMBRST} Z_{CB}. \tag{5.10} \]

Now, we would like to consider of the effect of FFBRST transformation with finite parameter Θ₁ and FF-anti-BRST transformation with finite parameter Θ₂ independently. The infinitesimal change in Jacobian \( J_1 \) for the FFBRST transformation with the parameter Θ₁ is calculated as

\[ \frac{1}{J_1} \frac{dJ_1}{d\kappa} = \gamma \int d^4x \left[ B(\dot{\lambda} - \varphi - \vartheta) + \dot{\bar{c}} c - 2\bar{c}c \right]. \tag{5.11} \]

To write the Jacobian \( J_1 \) as \( e^{iS_1} \) in case of BRST transformation, we make following ansatz for \( S_1 \) as

\[ S_1 = i \int d^4x \left[ \xi_1(\kappa) \ B(\dot{\lambda} - \varphi - \vartheta) + \xi_2(\kappa) \ \dot{\bar{c}}c \right. \]

\[ \left. + \xi_3(\kappa) \ \bar{c}c \right], \tag{5.12} \]

where \( \xi_i(i = 1, 2, 3) \) are arbitrary \( \kappa \)-dependent constants and satisfy the initial conditions \( \xi_i(\kappa = 0) = 0 \).
The essential condition in Eq. (4.5) satisfies with Eqs. (5.11) and (5.12) iff
\[
\int d^4x \left[ -B(\dot{\lambda} - \phi - \vartheta)(\xi_1' + \gamma) - \dot{c}\dot{c}(\xi_2' + \gamma) \\
- \dot{c}\dot{c}(2\gamma - \xi_3') + B\dot{c}\Theta'_1(\xi_1 - \xi_2) + Bc\Theta'_1(2\xi_1 + \xi_3) \right] \\
= 0,
\]
(5.13)
where prime denotes the derivative with respect to \( \kappa \). Equating the both sides of the above equation, we get the following equations
\[
\xi_1' + \gamma = 0, \quad \xi_2' + \gamma = 0, \quad \xi_3' - 2\gamma = 0, \\
\xi_1 - \xi_2 = 0 = 2\xi_1 + \xi_3.
\]
(5.14)
The solution of above equations satisfying the initial conditions is
\[
\xi_1 = -\gamma\kappa, \quad \xi_2 = -\gamma\kappa, \quad \xi_3 = 2\gamma\kappa.
\]
(5.15)
Then the expression for \( S_1 \) in terms of \( \kappa \) becomes
\[
S_1 = i \int d^4x \left[ -\gamma\kappa B(\dot{\lambda} - \phi - \vartheta) - \gamma\kappa\dot{c}\ddot{c} + 2\gamma\kappa\dddot{c} \right].
\]
(5.16)
On the other hand the infinitesimal change in Jacobian \( J_2 \) for the FF-anti-BRST parameter \( \Theta_2 \) is calculated as
\[
\frac{1}{J_2} \frac{dJ_2}{d\kappa} = -\gamma \int d^4x \left[ B(\dot{\lambda} - \phi - \vartheta) + \dot{c}\ddot{c} - 2\dot{c}\dddot{c} \right].
\]
(5.17)
Similarly, to write the Jacobian \( J_2 \) as \( e^{iS_2} \) in the anti-BRST case, we make ansatz for \( S_2 \) as
\[
S_2 = i \int d^4x \left[ \xi_4(\kappa) B(\dot{\lambda} - \phi - \vartheta) + \xi_5(\kappa) \dot{c}\ddot{c} \\
+ \xi_6(\kappa) \dddot{c} \right],
\]
(5.18)
where arbitrary \( \kappa \)-dependent constants \( \xi_i(i = 4, 5, 6) \) have to be calculated.

The essential condition in Eq. (4.5) for the above Jacobian \( J_2 \) and functional \( S_2 \) provides
\[
\int d^4x \left[ B(\dot{\lambda} - \phi - \vartheta)(\xi_1' - \gamma) + \dot{c}\dot{c}(\xi_2' - \gamma) - \dddot{c}(2\gamma + \xi_3') \right. \\
+ \left. B\dddot{c}\Theta'_2(\xi_4 - \xi_5) + Bc\Theta'_2(2\xi_4 + \xi_6) \right] = 0.
\]
(5.19)
Comparing the L.H.S. and R.H.S. of the above equation, we get following equations

\[ \xi'_4 - \gamma = 0, \quad \xi'_5 - \gamma = 0, \quad \xi'_6 + 2\gamma = 0, \]
\[ \xi_4 - \xi_5 = 0 = 2\xi_4 + \xi_6. \]  
(5.20)

Solving the above equations, we get the following values for \( \xi_i \)'s

\[ \xi_4 = \gamma\kappa, \quad \xi_5 = \gamma\kappa, \quad \xi_6 = -2\gamma\kappa. \]  
(5.21)

Putting these values in expression of \( S_2 \), we get

\[ S_2 = i \int d^4x \left[ \gamma\kappa B(\lambda - \varphi - \vartheta) + \gamma\kappa\dot{\bar{c}}\dot{c} - 2\gamma\kappa\bar{c}c \right]. \]  
(5.22)

Thus under successive FFBRST and FF-anti-BRST transformations the generating functional transformed as

\[ Z_{CB} \left( \int D\phi \ e^{iS_{CB}} \right) \xrightarrow{(FFBRST)(FF-anti-BRST)} Z_{CB} \left( \int D\phi \ e^{iS_{CB} + S_1 + S_2} \right), \]  
(5.23)

Note for the particular choices of \( \Theta_1 \) and \( \Theta_2 \), the \( S_1 \) and \( S_2 \) cancel each other. Hence \( Z_{CB} \) remains invariant under successive FFBRST and FF-anti-BRST transformations. It is interesting to note that the effect of FMBRST transformation is equivalent to successive operation of FFBRST and FF-anti-BRST transformations.

**B. Maxwell’s theory**

The generating functional for Maxwell theory, using Nakanishi Lautrup type auxiliary field \( (B) \), can be given as

\[ Z_M = \int D\phi \ e^{iS_M^{\text{eff}}}, \]  
(5.24)

where the effective action in covariant (Lorentz) gauge with the ghost term is

\[ S_{eff}^M = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \lambda B^2 - B\partial_{\mu} A^\mu - \bar{c}\partial_{\mu} \partial^\mu c \right]. \]  
(5.25)
The infinitesimal off-shell nilpotent BRST and anti-BRST transformations under which the effective action $S_{eff}^M$ as well as generating functional $Z_M$ remain invariant, are given as

**BRST:**
\[
\begin{align*}
\delta_b A_\mu &= \partial_\mu c \delta\Lambda_1, \quad \delta_b c = 0 \\
\delta_b \bar{c} &= B \delta\Lambda_1, \quad \delta_b B = 0.
\end{align*}
\] (5.26)

**Anti-BRST:**
\[
\begin{align*}
\delta_{ab} A_\mu &= \partial_\mu \bar{c} \delta\Lambda_2, \quad \delta_{ab} \bar{c} = 0, \\
\delta_{ab} c &= -B \delta\Lambda_2, \quad \delta_{ab} B = 0.
\end{align*}
\] (5.27)

The nilpotent BRST transformation ($s_b$) and anti-BRST transformation ($s_{ab}$) mentioned above are absolutely anticommuting in nature i.e. \( \{s_b, s_{ab}\} \equiv s_b s_{ab} + s_{ab} s_b = 0 \). Therefore the sum of these two transformations ($s_b$ and $s_{ab}$) is also a nilpotent symmetry transformation. Let us define MBRST transformation ($\delta_m \equiv \delta_b + \delta_{ab}$) in this case, which is characterized by two infinitesimal parameters $\delta\Lambda_1$ and $\delta\Lambda_2$, as
\[
\begin{align*}
\delta_m A_\mu &= \partial_\mu c \delta\Lambda_1 + \partial_\mu \bar{c} \delta\Lambda_2, \\
\delta_m c &= -B \delta\Lambda_2, \\
\delta_m \bar{c} &= B \delta\Lambda_1, \\
\delta_m B &= 0.
\end{align*}
\] (5.28)

The nilpotent FMBRST symmetry transformation for this theory is then constructed as
\[
\begin{align*}
\delta_m A_\mu &= \partial_\mu c \Theta_1 + \partial_\mu \bar{c} \Theta_2, \\
\delta_m c &= -B \Theta_2, \\
\delta_m \bar{c} &= B \Theta_1, \\
\delta_m B &= 0.
\end{align*}
\] (5.29)

where $\Theta_1$ and $\Theta_2$ are finite, field dependent and anticommuting parameters. We choose particular $\Theta_1$ and $\Theta_2$ in this case as
\[
\Theta_1 = \int \Theta_1' dk = \gamma \int dk \int d^4x [\bar{c} \partial_\mu A^\mu],
\] (5.30)
\[ \Theta_2 = \int \Theta_2' d\kappa = \gamma \int d\kappa \int d^4x [c\partial_\mu A^\mu], \]  

(5.31)

where \( \gamma \) is an arbitrary parameter. The infinitesimal change in Jacobian using Eq. (4.3) for the FMBRST transformation with the above finite parameters vanishes. It means that the path integral measure and hence the generating functional is invariant under FMBRST transformation.

Now, the infinitesimal change in Jacobian for the FFBRST transformation with the parameter \( \Theta_1 \) is calculated as

\[ \frac{1}{J_1} \frac{dJ_1}{d\kappa} = \gamma \int d^4x \left[ B\partial_\mu A^\mu + \bar{c}\partial_\mu \partial^\mu c \right]. \]  

(5.32)

To write the Jacobian \( J_1 \) as \( e^{iS_1} \) in the BRST case, we make following ansatz for \( S_1 \) as

\[ S_1 = i \int d^4x \left[ \xi_1 B\partial_\mu A^\mu + \xi_2 \bar{c}\partial_\mu \partial^\mu c \right]. \]  

(5.33)

The essential condition in Eq. (4.5) is satisfied subjected to

\[ \int d^4x \left[ B\partial_\mu A^\mu (\xi'_1 + \gamma) + \bar{c}\partial_\mu \partial^\mu c (\xi'_2 + \gamma) - B\partial_\mu \partial^\mu c \Theta'_1 (\xi_1 - \xi_2) \right] = 0, \]  

(5.34)

where prime denotes the derivative with respect to \( \kappa \). Equating the both sides of the above equation, we get the following Eqs.

\[ \xi'_1 + \gamma = 0, \quad \xi'_2 + \gamma = 0, \quad \xi_1 - \xi_2 = 0. \]  

(5.35)

The solution of above equations satisfying the initial conditions \( \xi_i = 0, (i = 1, 2) \) is

\[ \xi_1 = -\gamma \kappa, \quad \xi_2 = -\gamma \kappa. \]  

(5.36)

Putting these value in the Eq. (5.33), the expression of \( S_1 \) becomes

\[ S_1 = -i\gamma \kappa \int d^4x \left[ B\partial_\mu A^\mu + \bar{c}\partial_\mu \partial^\mu c \right]. \]  

(5.37)

However, the infinitesimal change in Jacobian \( J_2 \) for the FF-anti-BRST transformation with the parameter \( \Theta_2 \) is calculated as

\[ \frac{1}{J_2} \frac{dJ_2}{d\kappa} = -\gamma \int d^4x \left[ B\partial_\mu A^\mu + \bar{c}\partial_\mu \partial^\mu c \right]. \]  

(5.38)
Similarly, to write the Jacobian \( J_2 \) as \( e^{iS_2} \) in the anti-BRST case, we make the following ansatz for \( S_2 \) as

\[
S_2 = i \int d^4x \left[ \xi_3 B \partial_\mu A^\mu + \xi_4 \bar{c} \partial_\mu \partial^\mu c \right]. \tag{5.39}
\]

The essential condition in Eq. (4.5) for Eqs. (5.38) and (5.39) provides

\[
\int d^4x \left[ B \partial_\mu A^\mu (\xi'_3 - \gamma) + \bar{c} \partial_\mu \partial^\mu c (\xi'_4 - \gamma) \right. \\
- B \partial_\mu \partial^\mu \bar{c} \Theta'_2 (\xi_3 - \xi_4) \left. \right] = 0. \tag{5.40}
\]

Comparing the L.H.S. and R.H.S. of the above equation we get following equations

\[
\xi'_3 - \gamma = 0, \quad \xi'_4 - \gamma = 0, \quad \xi_3 - \xi_4 = 0. \tag{5.41}
\]

Solving the above equations, we get the following values for \( \xi \)'s

\[
\xi_3 = \gamma \kappa, \quad \xi_4 = \gamma \kappa. \tag{5.42}
\]

Plugging back these value of \( \xi_i (i = 3, 4) \) in Eq. (5.39), we obtain

\[
S_2 = i \gamma \kappa \int d^4x \left[ B \partial_\mu A^\mu + \bar{c} \partial_\mu \partial^\mu c \right]. \tag{5.43}
\]

From Eqs. (5.37) and (5.43), one can easily see that \( S_1 + S_2 = 0 \). Therefore, under successive FFBRST and FF-anti-BRST transformations with these particular finite parameters \( \Theta_1 \) and \( \Theta_2 \) respectively, the generating functional transformed as

\[
Z_M \left( \int D\phi \ e^{iS_{\text{eff}}^M} \right) ^{(\text{FFBRST})(\text{FF-anti-BRST})} \rightarrow Z_M \left( \int D\phi \ e^{iS_{\text{eff}}^M + S_1 + S_2} \right). \tag{5.44}
\]

Hence the successive operation of FFBRST and FF-anti-BRST transformation also leaves the generating functional \( Z_M \). This reconfirms that FMBRST transformation has same effect on both the effective action and the generating functional as the successive FFBRST and FF-anti-BRST transformations.
C. Non-Abelian YM theory in CFDJ gauge

The generating functional for non-Abelian YM theory in CFDJ gauge can be written as

$$Z_{YM}^{CF} = \int D\phi e^{iS_{YM}^{CF}[\phi]},$$

(5.45)

where $\phi$ is generic notation for all the fields in the effective action $S_{YM}^{CF}$

$$S_{YM}^{CF} = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \frac{\xi}{2} (h^a)^2 + \imath h^a \partial_\mu A^{\mu a} - \frac{1}{2} \partial_\mu c^a (D^\mu c)^a + \frac{1}{2} (D_\mu c)^a \partial^\mu c^a - \frac{\xi g^2}{8} (f^{abc} \bar{c}_b c^c)^2 \right],$$

(5.46)

with the field strength tensor $F_{\mu\nu}^a = \partial_\mu A^{\mu a} - \partial_\nu A^\nu_a + g f^{abc} A^\mu_b A^\nu_c$ and $h^a$ is the Nakanishi Lautrup type auxiliary field. The effective action as well as the generating functional are invariant under following infinitesimal BRST and anti-BRST transformations

**BRST:**

$$\delta_b A^{\mu a}_\mu = - (D_\mu c)^a \delta \Lambda_1, \quad \delta_b c^a = - \frac{g}{2} f^{abc} \bar{c}_b c^c \delta \Lambda_1$$

$$\delta_b (ih^a) = - \frac{g}{2} f^{abc} \left( \imath h^b \bar{c}_d + \frac{g}{4} f^{cde} \bar{c}_b c^d \right) \delta \Lambda_1,$$

(5.47)

**Anti-BRST:**

$$\delta_{ab} A^{\mu a}_\mu = - (D_\mu c)^a \delta \Lambda_2, \quad \delta_{ab} c^a = - \frac{g}{2} f^{abc} \bar{c}_b c^c \delta \Lambda_2$$

$$\delta_{ab} (ih^a) = - \frac{g}{2} f^{abc} \left( \imath h^b \bar{c}_d + \frac{g}{4} f^{cde} \bar{c}_b c^d \right) \delta \Lambda_2,$$

(5.48)

where $\delta \Lambda_1$ and $\delta \Lambda_2$ are infinitesimal, anticommuting and global parameters. The infinitesimal MBRST symmetry transformation ($\delta_m = \delta_b + \delta_{ab}$) in this case is written as:

$$\delta_m A^{\mu a}_\mu = - D_\mu c^a \delta \Lambda_1 - D_\mu c^a \delta \Lambda_2,$$

$$\delta_m c^a = - \frac{g}{2} f^{abc} \bar{c}_b c^c \delta \Lambda_1 - \left( \imath h^a + \frac{g}{2} f^{abc} \bar{c}_b c^c \right) \delta \Lambda_2,$$

$$\delta_m (ih^a) = - \frac{g}{2} f^{abc} \left( \imath h^b \bar{c}_d + \frac{g}{4} f^{cde} \bar{c}_b c^d \right) \delta \Lambda_2,$$

(5.50)
\[
\delta_m \bar{c}^a = \left( i h^a - \frac{g}{2} f^{abc} \bar{c}^b c^c \right) \delta \Lambda_1 - \frac{g}{2} f^{abc} \bar{c}^b c^c \delta \Lambda_2,
\]
\[
\delta_m (ih^a) = - \frac{g}{2} f^{abc} \left( i h^b c^c + \frac{g}{4} f^{cde} \bar{c}^b c^d c^e \right) \delta \Lambda_1 - \frac{g}{2} f^{abc} \left( i h^b c^c + \frac{g}{4} f^{cde} \bar{c}^b c^d c^e \right) \delta \Lambda_2.
\]
(5.49)

Corresponding FMBRST symmetry transformation is constructed as:
\[
\delta_m A^a_\mu = - D_\mu \bar{c}^a \Theta_1 - D_\mu \bar{c}^a \Theta_2,
\]
\[
\delta_m c^a = - \frac{g}{2} f^{abc} \bar{c}^b c^c \Theta_1 - \left( i h^a + \frac{g}{2} f^{abc} \bar{c}^b c^c \right) \Theta_2,
\]
\[
\delta_m \bar{c}^a = \left( i h^a - \frac{g}{2} f^{abc} \bar{c}^b c^c \right) \Theta_1 - \frac{g}{2} f^{abc} \bar{c}^b c^c \Theta_2,
\]
\[
\delta_m (ih^a) = - \frac{g}{2} f^{abc} \left( i h^b c^c + \frac{g}{4} f^{cde} \bar{c}^b c^d c^e \right) \Theta_1 - \frac{g}{2} f^{abc} \left( i h^b c^c + \frac{g}{4} f^{cde} \bar{c}^b c^d c^e \right) \Theta_2,
\]
(5.50)

with two arbitrary finite field dependent parameters \( \Theta_1 \) and \( \Theta_2 \). The generating functional \( Z_{YM}^{CF} \) is made invariant under the above FMBRST transformation by constructing appropriate finite parameters \( \Theta_1 \) and \( \Theta_2 \). We construct the finite nilpotent parameters \( \Theta_1 \) and \( \Theta_2 \) as
\[
\Theta_1 = \int \Theta_1 d\kappa = \gamma \int d\kappa \int d^4x [\bar{c}^a \partial_\mu A^{\mu a}],
\]
(5.51)
\[
\Theta_2 = \int \Theta_2 d\kappa = \gamma \int d\kappa \int d^4x [\bar{c}^a \partial_\mu A^{\mu a}],
\]
(5.52)

where \( \gamma \) is an arbitrary parameter. Following the same method elaborated in previous two examples, we show that the Jacobian for path integral measure due to FMBRST transformation given in Eq. (5.50) with finite parameters \( \Theta_1 \) and \( \Theta_2 \) becomes unit. It means that under such FMBRST transformation the generating functional as well as effective action remain invariant. The Jacobian contribution for path integral measure due to FFBRST transformation with parameter \( \Theta_1 \) compensates the same due to FF-anti-BRST transformation with parameter \( \Theta_2 \). Therefore, under the successive FFBRST and FF-anti-BRST transformations the generating functional remains invariant as
\[
Z_{YM}^{CF} (\text{FFBRST})(\text{FF-anti-BRST}) \rightarrow Z_{YM}^{CF}.
\]
(5.53)

Again we see the equivalence between FMBRST and successive operation of FFBRST and FF-anti-BRST transformations.
We end up the section with conclusion that in all the three cases the FMBRST transformation with appropriate finite parameters is the finite nilpotent symmetry of the effective action as well as the generating functional of the effective theories. Here we also note that the successive operations of FFBRST and FF-anti-BRST also leave the generating functional as well as effective action invariant and hence equivalent to FMBRST transformation.

VI. FMBRST SYMMETRY IN FIELD/ANTIFIELD FORMULATION

In this section we consider the field/antifield formulation using MBRST transformation. Unlike BV formulation using either BRST or anti-BRST transformations, we need two sets of antifields in BV formulation for MBRST transformation. We construct FMBRST transformation in this context. The change in Jacobian under FFBRST transformation in the path integral measure in the definition of generating functional is used to adjust with the change in the gauge-fixing fermion $\Psi_1$ [28]. Hence the FFBRST transformation is used to connect the generating functionals of different solutions of quantum master equation [10, 13]. However in case of BV formulation for FMBRST transformation we need to introduce two gauge-fixing fermions $\Psi_1$ and $\Psi_2$. We construct the finite parameters in FMBRST transformation in such a way that contributions from $\Psi_1$ and $\Psi_2$ adjust each other to leave the extended action invariant. This implies that we can construct appropriate parameters in FMBRST transformation such that generating functionals corresponding to different solutions of quantum master equations remain invariant under such transformation. These results can be demonstrated with the help of explicit examples. We would like to consider the same examples of previous section for this purpose.
A. Bosonized chiral model in BV formulation

We recast the generating functional in Eq. (5.1) for (1+1) dimensional bosonized chiral model using both BRST and anti-BRST exact terms as

\[ Z_{CB} = \int D\phi \, e^{iS_{CB}}, \]

\[ = \int D\phi \, \exp \left[ i \int d^2x \left\{ \pi_\phi \dot{\phi} + \pi_\vartheta \dot{\vartheta} + p_u \dot{u} - \frac{1}{2} \pi_\phi^2 \right. \right. \]
\[ \left. \left. + \frac{1}{2} \pi_\vartheta^2 + \pi_\vartheta (\varphi' - \vartheta' + \lambda) + \frac{1}{2} \pi_\phi \lambda + \frac{1}{2} s_b \Psi_1 \right. \right. \]
\[ \left. \left. + \frac{1}{2} s_{ab} \Psi_2 \right\} \right], \] (6.1)

here Lagrange multiplier field \( u \) is considered as dynamical variable and expression for gauge-fixing fermions for BRST symmetry (\( \Psi_1 \)) and anti-BRST symmetry (\( \Psi_2 \)) respectively are

\[ \Psi_1 = \int d^2x \, c(\dot{\lambda} - \varphi - \vartheta + \frac{1}{2} B). \] (6.2)
\[ \Psi_2 = \int d^2x \, c(\dot{\lambda} - \varphi - \vartheta + \frac{1}{2} B). \] (6.3)

The effective action \( S_{CB} \) is invariant under combined BRST and anti-BRST transformations given in Eq. (5.5). The generating functional \( Z_{CB} \) can be written in terms of antifields \( \phi_1^* \) and \( \phi_2^* \) corresponding to all fields \( \phi \) as

\[ Z_{CB} = \int D\phi \, \exp \left[ i \int d^2x \left\{ \pi_\phi \dot{\phi} + \pi_\vartheta \dot{\vartheta} + p_u \dot{u} - \frac{1}{2} \pi_\phi^2 \right. \right. \]
\[ \left. \left. + \frac{1}{2} \pi_\vartheta^2 + \pi_\vartheta (\varphi' - \vartheta' + \lambda) + \frac{1}{2} \pi_\phi \lambda + \frac{1}{2} \phi_i^* \dot{c} - \frac{1}{2} \phi_2^* \dot{c} \right. \right. \]
\[ \left. \left. + \frac{1}{2} \phi_i^* \dot{c} - \frac{1}{2} \phi_2^* \dot{c} + \frac{1}{2} \phi_1^* B + \frac{1}{2} \phi_2^* B - \frac{1}{2} \phi_i^* \dot{c} \right) \right], \] (6.4)

where \( \phi_i^* (i = 1, 2) \) is a generic notation for antifields arising from gauge-fixing fermions \( \Psi_i \). The above relation can further be written in compact form as

\[ Z_{CB} = \int D\phi \, e^{iW_{\Psi_1 + \Psi_2}[\phi, \phi_i^*]}, \] (6.5)
where $W_{\Psi_1+\Psi_2}[\phi, \phi^*_i]$ is an extended action for the theory of self-dual chiral boson corresponding the gauge-fixing fermions $\Psi_1$ and $\Psi_2$.

This extended quantum action, $W_{\Psi_1+\Psi_2}[\phi, \phi^*_i]$ satisfies certain rich mathematical relations commonly known as quantum master equation \[4\], given by

\[
\Delta e^{iW_{\Psi_1+\Psi_2}[\phi, \phi^*_i]} = 0 \quad \text{with} \quad \Delta \equiv \frac{\partial_r}{\partial \phi} \frac{\partial_r}{\partial \phi^*_i}(-1)^{\epsilon+1}.
\] (6.6)

The generating functional does not depend on the choice of gauge-fixing fermions \[3\] and therefore extended quantum action $W_{\Psi_i}$ with all possible $\Psi_i$ are the different solutions of quantum master equation. The antifields $\phi^*_i$ corresponding to each field $\phi$ for this particular theory can be obtained from the gauge-fixed fermion $\Psi_1$ as

\[
\begin{align*}
\varphi_1^* &= \frac{\delta \Psi_1}{\delta \varphi} = -\bar{c}, \quad \vartheta_1^* = \frac{\delta \Psi_1}{\delta \vartheta} = -\bar{c}, \quad c_1^* = \frac{\delta \Psi_1}{\delta c} = 0, \\
\bar{c}_1^* &= \frac{\delta \Psi_1}{\delta \bar{c}} = -\frac{1}{2}(\dot{\lambda} - \varphi - \vartheta + \frac{1}{2}B), \\
B_1^* &= \frac{\delta \Psi_1}{\delta B} = \frac{1}{2}\bar{c}, \quad \lambda_1^* = -\frac{\delta \Psi_1}{\delta \lambda} = -\dot{c}.
\end{align*}
\] (6.7)

Similarly, the antifields $\phi_2^*$ can be calculated from the gauge-fixing fermion $\Psi_2$ as

\[
\begin{align*}
\varphi_2^* &= \frac{\delta \Psi_2}{\delta \varphi} = -c, \quad \vartheta_2^* = \frac{\delta \Psi_2}{\delta \vartheta} = -c, \\
c_2^* &= \frac{\delta \Psi_2}{\delta c} = (\dot{\lambda} - \varphi - \vartheta + \frac{1}{2}B), \quad \bar{c}_2^* = \frac{\delta \Psi_2}{\delta \bar{c}} = 0, \\
B_2^* &= \frac{\delta \Psi_2}{\delta B} = \frac{1}{2}c, \quad \lambda_2^* = -\frac{\delta \Psi_2}{\delta \lambda} = -\dot{c}.
\end{align*}
\] (6.8)

Now we apply the FMBRST transformation given in Eq. (5.6) with the finite parameters written in Eqs. (5.7) and (5.8) to this generating functional. We see that the path integral measure in Eq. (6.5) remains invariant under this FMBRST transformation as the Jacobian for path integral measure is 1. Therefore,

\[
Z_{CB} \left( \int D\phi \ e^{iW_{\Psi_1+\Psi_2}} \right) \underbrace{\text{(FFBRST)(FF-anti-BRST)}}_{\cdots} \rightarrow Z_{CB}.
\] (6.9)

Thus the solutions of quantum master equation in this model remain invariant under FMBRST transformation as well as under consecutive operations of FFBRST and FF-anti-BRST transformations. However the FFBRST (FF-anti-BRST) transformation connects...
the generating functionals corresponding to the different solutions of the quantum master equation \[10, 14\].

**B. Maxwell’s theory in BV formulation**

The generating functional for Maxwell’s theory given in Eq. (5.24) can be recast using BRST and anti-BRST exact terms as

\[
Z_M = \int D\phi \, e^{i \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} s_b \Psi_1 + \frac{i}{2} s_a \Psi_2 \right]}, \tag{6.10}
\]

where the expressions for gauge-fixing fermions \( \Psi_1 \) and \( \Psi_2 \) are

\[
\Psi_1 = \int d^4x \, \bar{c}(\lambda B - \partial \cdot A), \tag{6.11}
\]

\[
\Psi_2 = -\int d^4x \, c(\lambda B - \partial \cdot A). \tag{6.12}
\]

The generating functional for such theory can further be expressed in fields/antifields formulation as

\[
Z_M = \int D\phi \, \exp \left[ i \int d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} A_{\mu 1}^* \partial^\mu \bar{c} + \frac{1}{2} A_{\mu 2}^* \partial^\mu c + \frac{1}{2} \bar{c}_1 \lambda B - \frac{1}{2} \bar{c}_2 \lambda B \right) \right]. \tag{6.13}
\]

In the compact form above generating functional is written as

\[
Z_M = \int D\phi e^{i W_{\Psi_1 + \Psi_2} [\phi, \phi^*]}, \tag{6.14}
\]

where \( W_{\Psi_1 + \Psi_2} [\phi, \phi^*] \) is an extended action for the Maxwell’s theory corresponding to the gauge-fixing fermions \( \Psi_1 \) and \( \Psi_2 \).

The antifields for gauge-fixed fermion \( \Psi_1 \) are calculated as

\[
A_{\mu 1}^* = \frac{\delta \Psi_1}{\delta A^\mu} = \partial_\mu \bar{c}, \quad \bar{c}_1^* = \frac{\delta \Psi_1}{\delta \bar{c}} = (\lambda B - \partial \cdot A),
\]

\[
c_1^* = \frac{\delta \Psi_1}{\delta c} = 0, \quad B_1^* = \frac{\delta \Psi_1}{\delta B} = \lambda \bar{c}. \tag{6.15}
\]
The antifields $\phi^*_2$ can be calculated from the gauge-fixed fermion $\Psi_2$ as
\[
A^*_{\mu 2} = \frac{\delta \Psi_2}{\delta A^\mu} = -\partial^\mu c, \quad c^*_{2} = \frac{\delta \Psi_2}{\delta c} = 0, \\
c^*_{2} = \frac{\delta \Psi_2}{\delta \bar{c}} = -(\lambda B - \partial \cdot A), \quad B^*_{2} = \frac{\delta \Psi_2}{\delta B} = -\lambda c.
\] (6.16)

Now implementing the FMBRST transformation mentioned in Eq. (5.29) with parameters given in Eqs. (5.30) and (5.31) to this generating functional we see that the Jacobian for path integral measure for such transformation becomes unit. Hence, the FMBRST transformation given in Eq. (5.29) is a finite symmetry of the solutions of quantum master equation for Maxwell’s theory.

Now, we focus on the contributions arising from the Jacobian due to independent applications of FFBRST and FF-anti-BRST transformations. We construct the finite parameters of FFBRST and FF-anti-BRST transformations in such a way that Jacobian remains invariant. Therefore, the generating functional remains invariant under consecutive operation of FFBRST and FF-anti-BRST transformations with appropriate parameters as
\[
Z_M \xrightarrow{(FFBRST)(FF-\text{anti-}BRST)} Z_M.
\] (6.17)

It also implies that the effect of consecutive FFBRST and FF-anti-BRST transformations is same as the effect of FMBRST transformation on $Z_M$.

C. Non-Abelian YM theory in BV formulation

The generating functional for this theory can be written in both BRST and anti-BRST exact terms as
\[
Z_{YM}^{CF} = \int D\phi \ e^{i\int d^4x \left[ -\frac{1}{4} F^a_{\mu \nu} F^{a \mu \nu} + \frac{1}{2} s_a \Psi_1 + \frac{1}{2} s_{ab} \Psi_2 \right]},
\] (6.18)

with the expressions of gauge-fixing fermions $\Psi_1$ and $\Psi_2$ as
\[
\Psi_1 = -\int d^4x \ \bar{c}^a (i\frac{\xi}{2} h^a - \partial \cdot A^a),
\] (6.19)
\[ \Psi_2 = \int d^4 x \ e^a \left( i \frac{\xi}{2} h^a - \partial \cdot A^a \right). \] (6.20)

We re-write the generating functional given in Eq. (5.45) using field/antifield formulation as,

\[ Z_{YM}^{CF} = \int D\phi \exp \left[ i \int d^4 x \left\{ -\frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} - \frac{1}{2} A^a_{\mu 2} D^\mu c^a \\
- \frac{1}{2} A^a_{\mu 2} D^\mu c^a + \frac{1}{2} c_1^a \left( i h^a - \frac{g}{2} f^{abc} \bar{c}^b c^c \right) \\
- \frac{1}{2} c_2^a \left( i h^a + \frac{g}{2} f^{abc} \bar{c}^b c^c \right) - \frac{1}{2} h_1^a \left( \frac{g}{2} f^{abc} h^b c^c \right) \\
- \frac{i g^2}{8} f^{abc} f^{cde} \bar{c}^b c^c c^e \right) \right] \] (6.21)

This generating functional \( Z_{YM}^{CF} \) can be written compactly as

\[ Z_{YM}^{CF} = \int D\phi \ e^{i W_{\Psi_1 + \Psi_2} [\phi, \phi^*]}, \] (6.22)

where \( W_{\Psi_1 + \Psi_2} [\phi, \phi^*] \) is an extended quantum action for the non-Abelian YM theory in CFDJ gauge.

The antifields are calculated with the help of gauge-fixed fermion \( \Psi_1 \) as

\[ A_\mu^a = \frac{\delta \Psi_1}{\delta A^a_{\mu}} = -\partial_\mu c^a, \quad c_1^a = \frac{\delta \Psi_1}{\delta c^a} = 0, \]

\[ c_1^{\alpha *} = \frac{\delta \bar{\Psi}_1}{\delta \bar{c}^a} = -(i \frac{\xi}{2} h^a - \partial \cdot A^a), \]

\[ h_1^a = \frac{\delta \bar{\Psi}_1}{\delta \bar{h}^a} = -i \frac{\xi}{2} c^a. \] (6.23)

The explicit value of antifields can be calculated with \( \Psi_2 \) as

\[ A_\mu^a = \frac{\delta \Psi_2}{\delta A^a_{\mu}} = \partial_\mu c^a, \quad c_2^a = \frac{\delta \Psi_2}{\delta c^a} = 0, \]

\[ c_2^{\alpha *} = \frac{\delta \bar{\Psi}_2}{\delta \bar{c}^a} = (i \frac{\xi}{2} h^a - \partial \cdot A^a), \]

\[ h_2^a = \frac{\delta \bar{\Psi}_2}{\delta \bar{h}^a} = i \frac{\xi}{2} c^a. \] (6.24)

We observe here again that the Jacobian for path integral measure in the expression of generating functional \( Z_{YM}^{CF} \) arising due to FMBRST transformation and due to successive
operation of FFBRST and FF-anti-BRST transformations remains unit for appropriate choice of finite parameters. Thus, the consequence of FMBRST transformation given in Eq. (5.50) with the finite parameters given in Eqs. (5.51) and (5.52) is equivalent to the subsequent operations of FFBRST and FF-anti-BRST transformations with same finite parameters.

VII. CONCLUDING REMARKS

FFBRST and FF-anti-BRST transformations are nilpotent symmetries of the effective action. However these transformations do not leave the generating functional invariant as the path integral measure changes in a nontrivial way under these transformations. We have constructed infinitesimal MBRST transformation which is the combination of infinitesimal BRST and anti-BRST transformations. Even though infinitesimal MBRST transformation does not play much significant role, its field dependent version has very important consequences. We have shown that it is possible to construct the field dependent MBRST (FMBRST) transformation which leaves the effective action as well as the generating functional invariant. The finite parameters in the FMBRST transformation have been chosen in such a way that the Jacobian contribution from the FFBRST part compensates the same arising from FF-anti-BRST part. We have considered several explicit examples with diverse character in both gauge theories as well as in field/antifield formulation to show these results. It is interesting to point out that the effect of FMBRST transformation is equivalent to successive operations of FFBRST and FF-anti-BRST transformations. We have further shown that the generating functionals corresponding to different solutions of quantum master equation remain invariant under such FMBRST transformation whereas the independent FFBRST and FF-anti-BRST transformations connect the generating functionals corresponding to the different solutions of the quantum master equation. It will be interesting to see whether these FMBRST transformations put further restrictions on the relation of different Green’s function of the theory to simplify the renormalization program. In particular, such FMBRST trans-
formations may be helpful for the theories where BRST and anti-BRST transformations play independent role.

Acknowledgments

We thankfully acknowledge the financial support from the Department of Science and Technology (DST), Government of India, under the SERC project sanction grant No. SR/S2/HEP-29/2007.

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