Fatigue crack growth under Mode I, II and III for plane-strain and plane-stress conditions

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Abstract

Fatigue crack growth (FCG) could be encountered in many mechanical components, which can be made from either thin or thick steel plates (or shells) and, therefore, be subjected to a plane-stress or a plane-strain condition, respectively. The loads applied in a solid body containing a narrow notch or a sharp crack will induce a yield zone near its tip with a dimension that will depend on the mechanical properties of the material, as well as on the thickness of the body, the crack length and the magnitude of the loads applied. Crack propagation can then occur under mode I, II, III or mixed-mode for general loading.

The paper presents $J_{II}$, $J_{III}$ and $J_{I}$ integral functions, which were correlated with the elastic stress intensity factors $K_{I}$, $K_{II}$ and $K_{III}$ for thin and thick CT specimens. The evaluation of J-Integral values was carried out for different crack lengths, along the crack front, and using the Finite Element Method (FEM), with collapsed nodes and midside nodes dislocated to $\frac{1}{4}$ of the edge’s length, in order to simulate the crack tip singularity. Interaction between in-plane, in-plane sliding and out-of-plane modes are also discussed in the paper. In addition, FCG rates under mode I, mode III and a mixed-mode (mode I+III) were experimentally determined, at room temperature, for a high-strength Cr-Mn austenitic stainless steel.

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Selection and peer-review under responsibility of the Politecnico di Milano, Dipartimento di Meccanica

Keywords: Fracture mechanics; fatigue crack growth; modes of cracking I, II, III; J-Integral; Compact Tension Specimen; Zencrack software.

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1. Introduction

Frequently fatigue crack propagation plays an important role in the fracture process and stress intensity factors at the crack tip are crucial to be known and correlated with fatigue crack growth rates, in order to define, for instance, the time-inspection periods if a Damage Tolerance Design was chosen to be applied during the in-service lifetime of a component. In this work, two geometries of Compact Tension Specimen (CTS) were analysed through a numerical method as if they were subjected to crack propagation under the modes of cracking I, II and III, either for plane-stress or plane-strain conditions.

Nomenclature

| Symbol | Description |
|--------|-------------|
| B      | Specimen Thickness |
| CTS    | Compact Tension Specimen |
| E      | Young’s Modulus |
| FCG    | Fatigue Crack Growth |
| FEM    | Finite Element Method |
| G      | Shear modulus |
| J      | J-Integral |
| J_I, J_{II}, J_{III} | J-Integral at the crack tip, corresponding to mode of cracking I, II or III, respectively |
| K_C    | Critical Stress Intensity Factor (Plane-Stress) |
| K_I, K_{II}, K_{III} | Stress Intensity Factors at the crack tip, corresponding to mode of cracking I, II or III, respectively |
| K_r    | Fracture Toughness (Plane-Strain) |
| RT     | Room temperature |
| SIF, K | Stress Intensity Factor |
| T      | Traction Vector |
| u      | Displacement vector |
| W      | Specimen Width |
| W      | Strain-Energy Density |
| Γ      | J-Integral closed contour |
| ν      | Poisson’s Coefficient |

2. The J-Integral

The introduction of the J-integral by Cherepanov [1] and Rice [2] as a fracture mechanics parameter presented a new method for evaluating the stress field close to a crack in an elastic or nonlinear elastic material. The J-integral is an energetic contour (Eq. 1) based on the energy-momentum tensor suggested by Eshelby [3]. It is defined around the crack tip, in the positive direction, counterclockwise, which starts on the lower flat face of the crack and continues along a well-defined path to its upper flat face. This energetic contour integral is path independent if the following conditions are met: quasi-static conditions (time independent processes, excluding dynamic effects associated with accelerations and kinetic-energy production), the absence of body forces, as well as thermal induced stresses and traction stress applied in the faces of the crack and a homogenous hyperelastic material, as mentioned by Brocks et al. [4] and Hellån [5]. In this case, J-integral value is assumed to have always the same value for all paths that enclose a hole or a crack, it is identical to the energy release rate for a plane crack extension [6], and the two-dimensional J-integral along the closed contour Γ is given by Eq.1 [2].

\[ J = \int_{\Gamma} \left( Wdy - T \frac{\partial u}{\partial x} ds \right) \] (1)
Where $\Gamma$ is a closed contour surrounding the crack tip, $W$ represents the strain energy density or work of deformation per unit of volume, $T$ is the traction vector, which includes the Cartesian components of stress tensor and is defined to the outward normal along $\Gamma$ ($T_i = \sigma_{ij} n_j$) and $u$ is the displacement vector. The stress intensity factors (SIF) and the J-integral are two approaches for evaluating the stress field at the vicinity of a crack tip and for a linear elastic material the stress intensity factors can be related to the J-integral, in combined modes, using Eq.2 and Eq.3 for plane-strain and plane stress conditions, respectively [4,7].

$$J = J_I + J_{II} + J_{III} = \frac{1-\nu^2}{E} \left( K_I^2 + K_{II}^2 \right) + \frac{1+\nu}{E} K_{III}^2$$  \hspace{1cm} (2) \\
$$J = J_I + J_{II} + J_{III} = \frac{1}{E} \left( K_I^2 + K_{II}^2 \right) + \frac{1}{2G} K_{III}^2; G = \frac{E}{2(1+\nu)}$$  \hspace{1cm} (3)

Crack propagation will then occur when the strain energy release rate criterion is met, that is when the stress intensity factor at the crack tip reaches a critical value [8]. In addition, variables such as the crack length, or the specimen thickness, are also important to define the extent of yielding at the crack tip, but also to influence crack propagation due to mixed mode. In fact, crack growth within a finite thickness is never restricted to mode I but instead is a mixture of modes I and III [5]. Therefore, mode I or mode III could be dominant and if mode II load is also included, the crack motion could occur out of its original tangent plane violating the assumption of a codirectional growth [5]. Mode I is the most studied crack propagation mode. On the contrary, modes II and III are not so extensively studied and this work aims to study them from the numerical point of view, but also to study mode III and mixed mode (I+III) from the experimental point of view.

3. Fracture Mechanics - Finite Element Analyses

3.1. Plane-Strain and Plane-Stress Models

In order to calculate fracture mechanics parameters such as $K$ and $J$, a CT specimen was designed according ASTM E399 and ASTM E647 standards [9, 10] and numerical simulations were carried out (Fig.1). Two finite element models were designed in ANSYS Mechanical software and the width (W) and thickness (B) of the specimens were intended to induce plane-strain or plane-stress conditions (W=64mm, B=32mm and W=26mm, B=2.5mm). The 3D numerical models were designed with four layers along the thickness of the specimen, using finite element type SOLID186, in order to calculate $J$-values along the crack front. Each model was submitted to three types of loading, namely Modes I, II and III, assuming crack lengths equal to $a/W = 0.45, a/W = 0.50, a/W = 0.55$, as illustrated in Fig.1.

![Fig.1. CT specimens modeled with different through-thickness crack lengths, (a/W), namely: 0.45, 0.50 and 0.55, for plane-strain and plane-stress conditions.](image)

The ZENCRACK software was used to introduce a crack-block named s03_t23x1, with 3 contours per each single node, for extracting the numerical values of J and K along the crack front. As four finite element layers were introduced along the model’s thickness, the numerical values were calculated in five nodes at the crack front. Some results are presented in Fig.2.
4. FCG rate experimental tests

Fatigue crack growth rate tests were carried out in an Instron bi-axial servo-hydraulic machine (Fig. 3), at RT, using CT specimens in the plane-stress condition, which were made of a high-strength austenitic Cr-Mn stainless steel (Fig. 3). The loads applied induced either a mode I, a mode III or a combined mode (I+III) of cracking.

![Chemical composition and mechanical properties at RT of the high-strength stainless steel tested.](image)

Yield Strength, $\sigma_y$: 480 MPa; Tensile Strength, $\sigma_R$: 800 MPa; Ductility: $> 40\%$

5. Conclusions

Stress intensity factors were calculated on CT specimens for Mode I, II and III, for both plane-strain and plane-stress conditions. Numerical $K_i$ values are in good agreement with values calculated from standards ASTM E399 and ASTM E647. FCG rate tests were carried out in mode I, mode III and combined mode (I+III).

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