Comparison of electric powers measured with digital devices relative to powers associated with distinctive physical phenomena

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Abstract. The article discusses the methods of measuring electrical powers in general-purpose digital devices. Based on the described algorithms, the measured values were classified and compared with the powers determined from the Currents’ Physical Component (CPC) power theory for single-phase and three-phase circuits. The results of calculations for an example measuring circuit were compared. The criticism of the measurement methods used today is presented. The method of measuring the CPC powers has been outlined.

1. Introduction

Modern power and energy meters include AD converters and microprocessor computing systems. Owing to this, reading, the remote transmission and automation of the measuring process are made easier. In addition to this, it is possible to use algorithms that implement the adopted power definitions.

The first microprocessor devices used to measure electrical power were very inaccurate, especially in the case of non-sinusoidal waveforms. To determine the electric power for a sinusoidal wave, it is enough to determine the rms current and voltage, and the phase shift between them. In this case, the active $P$ [W], reactive $Q$ [Var], and apparent $S$ [VA] power are described by energy properties.

Higher harmonics mean that the definitions of individual powers should be different than in the case of a sinusoidal waveform. It is important to think about the definitions of AC powers and try to answer the question: Which powers are measured by digital measuring systems and which powers should be used for such measurements?

The distribution on harmonics of current and voltage waveforms is the basis for power measurement.

$$u = u_1 + u_H, \quad i = i_1 + i_H,$$

where: $u_1$, $i_1$ – basic voltage and current components,
$u_H$, $i_H$ – harmonic components of voltage and current.

Active power, treated as the sum of active power of all the harmonics of current and voltage, is understood as the average value of instantaneous power:
\[ P = \frac{1}{nT} \int_{\tau}^{\tau+nT} u(t) \cdot i(t) \, dt, \]  

(2)

where: \( T \) – period, \( n \) – multiplicity periods to averaging.

Therefore, this power is a parameter determined indirectly from the measurements of instantaneous current and voltage.

2. Current and voltage measurement

Two analog-to-digital converters with a signal conditioner are used to measure current and voltage. The most common are transducers that measure single polarization waveforms. In this case, a fixed component is added to the measured signal, the value of which is adjusted to the amplitude of the signal measured. The use of operational amplifiers in the measuring channel can also introduce a fixed component. This means that the measurement of current and voltage is burdened with a measurement error, resulting from the appearance of a fixed current and voltage component. Multiplication of two sinusoidal waveforms, which contain constant components \( U_0 \) and \( I_0 \), leads to the following form:

\[
\{ U_m \cos(\omega t) + U_0 \} \cdot \{ I_m \cos(\omega t) + I_0 \} \\
= \frac{U_m I_m}{2} + U_0 I_0 + \left( U_0 I_m + I_0 U_m \right) \cos(\omega t) + \frac{U_m I_m}{2} \cos(2\omega t). 
\]  

(3)

The multiplication of two sine functions \( u(t) \) and \( i(t) \) gives the four members corresponding to three pulsations. Active power is hidden in the component suitable for \( \omega_0 \) pulsation. Therefore, the determination of active power requires the use of an LPF low-pass filter. Unfortunately, both components of Equation (3) are transferred through this filter.

Figure 1. Low-pass filter LPF passes both \( \omega_0 \) components

Figure 1 shows the influence of current and voltage constant components on the active power value.

3. Determination of active power using filters

Microprocessor power and energy measurement use an algorithm, the essence of which is based on the proper processing of digital current and voltage signals.

Figure 2. Structural diagram for measuring active power using filters.
The low-pass filter LPF removes the components of pulsations \( \omega_1 \) and \( \omega_2 \), while the HPF high-pass filter in the current channel removes the constant component \( U_0 \) from the signal. The HPF filter must be installed in one of the measuring channels. The impact of the HPF high-pass filter causes the phase shift of the signal in the current channel. To compensate for this error, a phase correction circuit is placed in this channel \( \Phi \) (figure 2). The algorithm for determining active power is also valid in the case of non-sinusoidal periodic current and voltage waveforms.

4. Active power as an average value of instantaneous power

The second method of determining active power is averaging the instantaneous power value. In this solution, powers are determined indirectly after the discretization of voltage and current waveforms. The average power value is equal to:

\[
P = \frac{1}{N} \sum_{n_0}^{n_0+N} p(n) \quad [\text{W}].
\]

where \( N \) – number of samples in period \( T \).

From the instantaneous values of voltage and current, rms values are determined:

\[
I = \sqrt{\frac{1}{N} \sum_{n_0}^{n_0+N} i^2(n)} \quad [\text{A}], \quad U = \sqrt{\frac{1}{N} \sum_{n_0}^{n_0+N} u^2(n)} \quad [\text{V}].
\]

The apparent power, reactive power and power factor are determined from the following equations:

\[
S = U \cdot I \quad [\text{VA}], \quad Q = \sqrt{S^2 - P^2} \quad [\text{Var}], \quad \cos \varphi = \frac{P}{S}
\]

The algorithm discussed is correct for sinusoidal waveforms. For non-sinusoidal periodic processes, the results obtained do not reflect electrical powers values.

5. Criticism of results obtained

The powers determined in both measurement methods are subject to a methodological error. In some cases, the results obtained do not reflect the physical quantities to which they were intended. Active power should be a power that has its physical reflection in energy transformation. The consumer should be charged with costs for with the power that they have actually consumed, for example, they have opted for mechanical power in the engine. The conversion of electricity into mechanical energy takes place for a given synchronous frequency of the source. The spinning magnetic flux is correlated only with the fundamental frequency. Supplying \( i_n \) harmonic current to the motor does not increase the useful mechanical power of the machine. This causes an increase in losses. Averaging the instantaneous power \( p(t) \) means that the measurement result contains all harmonics, even those that come from the power source and do not depend on the recipient.

By decomposing the current into physical components according to the theory of CPC [1,7,8,15,17,18], it can be noticed that there are powers that should not burden the recipient. In the case of non-linear receivers, the active power \( P \) is the sum of the active working power \( P_D \) and the active reflected power \( P_C \) [2,6,18]. The \( P_D \) component has a positive value - due to the energy flow from the source to the receiver, while the \( P_C \) takes a negative value - due to a change in the direction of current flowing through the receiver. This means that both algorithms will indicate incorrect values of active power. In this light, it is more beneficial to account only for the basic harmonics of the power.
Example
A single-phase nonlinear receiver in accordance with [18] has been connected to a real source as shown in the figure below.

Figure 3. A nonlinear receiver powered from a real source with several harmonics.

The non-linear receiver generates the third and fifth harmonics. The third harmonic occurs in both sources \( j_C \) and \( e_D \). From the CPC theory, the results depend on:

\[
P_1 = 6612.5 \text{ W}, \quad P_3 = -22.5 \text{ W}, \quad P_5 = -6.25 \text{ W}.
\]

Therefore:

\[
P_D = P_1, \quad P_C = -P_3 - P_5 = 28.75 \text{ W},
\]

\[
P = P_D - P_C = 6583.75 \text{ W}.
\]

The crms values of voltages and currents for subsequent harmonics are equal to:

\[
U(1) = 115 \text{ V}, \quad U(3) = -j15 \text{ V}, \quad U(5) = -(2.5 + j12.5) \text{ V}, \quad I(1) = (57.5 - j57.5) \text{ A}, \quad I(3) = (-4.5 - j1.5) \text{ A},
\]

\[
I(5) = -2.5 \text{ A}.
\]

This means that in the time domain:

\[
u = 115\sqrt{2} \sin \omega_1 t - 15\sqrt{2} \sin(3\omega_1 t + 1.57) - 12.75\sqrt{2} \sin(5\omega_1 t + 1.37) \text{ V},
\]

\[
i = 81.32\sqrt{2} \sin(\omega_1 t - 0.79) - 4.74\sqrt{2} \sin(3\omega_1 t + 0.32) - 2.5\sqrt{2} \sin(5\omega_1 t) \text{ A}.
\]

As a result of the multiplication of these functions, the value is obtained:

\[
p = u \cdot i = 6641.25 + 8377.50 \sin(2\omega_1 t - 2.30) + 1676.40 \sin(4\omega_1 t + 1.67) + 1277.62 \sin(6\omega_1 t + 1.98) + 97.50 \sin(8\omega_1 t + 0.08) - 31.87 \sin(10\omega_1 t + 2.94) \text{ VA}.
\]

In the situation presented in the example, the active power in both algorithms is equal to the average value from \( p(t) \) and it amounts to \( \overline{p} = 6641.25 \text{ W} \). The difference between this value and the effective value of active power \( \overline{p}(i) \) is equal to the active reflected power \( \overline{p} - \overline{p}(i) = 28.75 \text{ W} = P_C \). This means that the digital watt meter has exceeded its measurement by 28.75 W, which is 0.44% of the indication.

The reactive power measured in both algorithms is the Budeanu's reactive power, and it is right only for a sinusoidal waveforms. The criticism of this power is widely known in the literature. According to the CPC theory, reactive power \( Q \) should be interpreted as a parameter dependent only on the phase shift between current and voltage for all harmonics. Only this interpretation of reactive power makes it possible to perform reactive power compensation.

Power measurement with three-phase meters is based on the summation of the values measured in each phase individually. In the case of an unbalanced receiver, line currents are asymmetrical. The result is the appearance of a nonzero value of the unbalanced current. This component increases transmission and production losses in the source, which is not observed in the measurement of power in each phase individually.

Apparent power determined from (6) is a value dependent on the rms value of current and voltage. In the three-phase measurement algorithm, the result of the sum of apparent powers in each phase is the arithmetic apparent power. In addition to the arithmetic apparent power, the definitions of
geometric apparent power and the Buchholz apparent power are known. The results obtained from these definitions are identical only in the case of source symmetry, for a three-phase balanced receiver, with sinusoidal current and voltage waveforms. The apparent power \( S \) and the power factor \( \lambda \) are virtual quantities that cannot be directly justified by physical features. Nevertheless, the power factor is closely related to energy losses in the energy system. An increase in transmission losses results in a reduction of the power factor values. The selection of the definition of apparent power should be made paying special attention to the assessment of these losses. In [15], Czarnecki analyzed the value of power factors for individual definitions of apparent power in an unbalanced three-phase circuit. The conclusion from the analyzes conducted is the statement that these losses are correctly estimated for the power factor, calculated from the Buchholz apparent power.

Currently, the power measurement algorithms are compliant with the guidelines given in [3]. According to the CPC theory, in the case of asymmetry, unbalanced current \( i_a \) appears, which with the following components: active \( i_a \), reactive \( i_q \), and scattering \( i_s \) - affects the power grid. A fair share in the costs of using the power grid is possible when the power definitions given in the CPC theory are accepted for settlement. Among the several powers determined in this theory, the most important is the active power for the basic harmonic \( P_1 \). The remaining components of the apparent power \( S \) should be treated as inactive power. The consumer, when they know the individual components of the inactive power, will know what actions they must take to reduce the costs of energy purchase. Figure 4 shows the diagram of the algorithm for measuring three-phase power in a four-wire system. The corresponding equations are presented in [7] and [8].

![Diagram](image)

**Figure 4.** The structure of the power measurement algorithm according to the CPC theory.

If the CPC power theory is taken into account for electricity bills, it would be fairer. An electricity consumer would know what action he must do to make electricity charges lower.

### 6. Conclusions

The adoption of erroneous algorithms in digital power and energy meters results in incorrect billing for electricity. Irregularities in the measurement of power and energy affect both: individual consumers and energy companies participating in the internal settlement process. The algorithms implemented determine quantities that do not reflect the energy status of the distribution network. It is a consequence of accepting incorrect power definitions. When the power measurement algorithm is used in accordance with the CPC power theory, it will force recipients to balance three-phase receivers and to compensate for reactive power and higher harmonics. A change in the settlement method will take place only after the power definition has been accepted according to the CPC theory by the entire
scientific community. By observing the degree of advancement of works on CPC power theory, it can be assumed that in a few years, there may be a change in the ways of defining power - on a system precisely accounting for the individual components of the current.

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