QED cascade in a plane electromagnetic wave

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(Dated: September 18, 2018)

It is demonstrated by QED-PIC (particle-in-cell) simulations that self-sustained QED cascades can efficiently develop in a plane wave of extremely high intensity which is believed not suitable for cascading. In the simulations, the cascade starts in the light sail regime of laser-foil interaction. As a result, a constantly growing electron-positron pair plasma ‘cushion’ is formed in front of the foil. Then the cushion starts to absorb the laser energy and decouples the laser radiation from the moving foil thereby interrupting the ion acceleration. Nevertheless, the cascade continues to develop. The models describing the cushion front dynamics and the cushion electrodynamics are presented and their predictions are in a qualitative agreement with the numerical simulations.

With a fast progress in PW laser technology quantum electrodynamics (QED) cascades started to attract a great attention [1–5]. In a strong laser field QED cascade can be self-sustained and seeded by, for example, an electron at rest. In this case the seeded electron is accelerated in the laser field and emits high-energy photons, which, in turn, can create electron-positron \((e^+e^-)\) pairs as a result of Breit-Wheeler process [5]. The secondary particles are also involved in photon emission and pair photoproduction, thus, the cascade develops with avalanche-like production of \(e^+e^-\) plasma and gamma-quanta. The cascade development is very similar to another physical phenomenon, namely, an avalanche breakdown in a gas discharge [2]. Apart from the self-sustained QED cascades (or A (Avalanche)-type cascades [5]), where the laser field provides the energy for cascading, there are also ‘shower-like’ cascades (or S-type cascades [5]), where the cascade energy does not exceed the energy of the seed particles. The S-type cascades are well-known as an air showers produced by cosmic rays in atmosphere [9].

A number of papers are devoted to the search of optimal laser field configurations in order to facilitate experimental realization of QED cascading. Most of the proposed configurations are based on multiple colliding laser pulses [1, 10–16] or on tightly focused laser pulse [17] whose field structures are strongly different from a traveling plane wave. The QED cascade can also develop in the standing wave field structure formed by incident and reflected laser radiation interacting with solid targets [18]. However, this can be considered as a kind of a scheme with two counter-propagating laser pulses. The plane wave-like configurations are considered as not suitable for cascading because an electron initially at rest cannot be accelerated in this field in such way that it will be capable of emission of gamma-quanta with high enough energy [17, 19, 21].

The impact of QED effects on the laser-solid interaction are mainly explored in the hole boring regime [22–24], when the target thickness is much greater than the skin depth characterizing the laser penetration into the foil. Particularly, the production of the \(e^+e^-\) plasma cushions [21] is observed in numerical simulations [18–25]. The hole boring regime without QED effects is characterized by significant reflection of the incident laser radiation. This is not the case for the light sail (LS) regime when the target is a thin foil with the skin depth of the order of the target thickness [23–28]. In this case the foil can be continuously accelerated as whole and the laser reflection is negligible. The LS regime has gained substantial interest during the past years as one of the most efficient schemes for high-energy ion acceleration [29]. The regime is not yet explored at extremely high laser intensities when the QED effects play a key role.

In this paper we report on a new effect, namely on a development of a self-sustained QED cascade in a plane wave structure where it is generally believed that such cascades are suppressed. The effect is observed in quantum electrodynamics — particle-in-cell (QED-PIC) simulations of the extremely high intensity laser-foil interaction in the LS regime. It is demonstrated that the cascade yields a dense \(e^+e^-\) plasma and its front propagates towards the laser in the foil plasma reference frame that qualitatively resembles an ionization front propagating towards the microwave source during microwave breakdown in gases [30, 31]. An efficient production of the \(e^+e^-\) pairs in front of the moving foil forms an overdense \(e^+e^-\) plasma ‘cushion’ [24] which absorbs the laser radiation and decouples the radiation from the foil plasma thereby interrupting the ion acceleration in the LS regime. The cascade continues to develop even after the laser field is decoupled from the foil plasma, thus, the latter can be considered as a seed becoming negligible for the late stage of the cascade development.

The laser-foil interaction is simulated with 3D QED-PIC code QUILL [32], which enables modelling of QED effects via Monte-Carlo method. In the simulations the circularly polarized laser pulse with wavelength \(\lambda = 2\pi c/\omega_L = 1\mu m\) propagates along the \(x\)-axis and has a rectangular profile with smoothed edges along all axes and the dimensionless amplitude, \(a_0 = eE/m_e c \omega_L\), where \(\omega_L\) is the laser frequency, \(m_e\) and \(e > 0\) are the electron mass and charge value, respectively, \(c\) is the speed
of light. The transverse spatial size of the laser pulse is 10 $\mu m \times 10 \mu m$ and the pulse duration is 40 fs. The laser field structure is very close to a traveling plane wave. The foil plasma with the initial electron concentration $n_e$ has the thickness, $d$, and the same transverse size as the laser pulse. The simulation box size is $20\lambda \times 30\lambda \times 30\lambda$, the grid size is 2000 $\times$ 300 $\times$ 300. We conducted a series of simulations varying parameters $a_0$, $n_e$, and $d$, but satisfying condition $a_0 = \eta n_e d \lambda r_e$ corresponding to the LS regime $[23]$, where $r_e = e^2/m_e c^2$ is the classical electron radius, $\eta$ is the numeric coefficient of order of unity (in all simulations $\eta = 1.5$). The simulation results are shown in Figs. 1(a)–3.

The typical laser-foil interaction structure observed in the simulations is shown in Fig. 1(a). The time evolution of the particle density is demonstrated in Fig. 1(b) (see also video in the Supplemental Material [33]) for the simulation with parameters: $n_e = 5.9 \cdot 10^{23}$ $cm^{-3} \cong 530 m_e c^2$ ($n_{cr} = m_e \omega_L^2 / 4\pi e^2 \approx 10^{23} cm^{-3}$), $d = 1 \mu m$, $a_0 = 2500$. It follows from the simulations that for $ct/\lambda < 7$ the foil plasma is compressed into a thin layer reflecting small portion of the laser radiation while the ions are continuously accelerated. The number of the $e^+e^-$ pairs produced during this time period is insignificant. In the time period $7 < ct/\lambda < 14$ the non-uniform $e^+e^-$ plasma starts to build up. The laser energy is absorbed by the plasma and converted into gamma-quanta. The laser field is almost completely screened at the location of the foil particles, thus ion acceleration is suppressed [see Fig. 1(c)]. In the time period $14 < ct/\lambda < 28$ the QED cascade develops in the self-sustained regime despite the fact that the laser radiation is decoupled from the foil plasma. The $e^+e^-$ plasma cushion absorbs the laser energy without any reflection and the ions are no longer accelerated. The cushion expands towards the laser while the cascade front propagates with velocity $v_{cf} < c$ until all laser energy is absorbed by the produced $e^+e^-$ plasma.

We also performed simulations for different values of $a_0 = 1500$, 2000, 2500, 3000 at $d = 1 \mu m$ and for different values of $d = 0.5 \mu m$, 1 $\mu m$, 2 $\mu m$ at $a_0 = 2500$. It follows from Fig. 2 that the cascade front velocity depends little both on time and on the foil thickness while it strongly depends on the laser intensity and decreases with increase of $a_0$. It is interesting to note that the cushion plasma density at the late stage of the interaction is in several times higher than the relativistic critical density $a_0 m_e c^2$. In order to understand the role of the foil ions in the
QED cascade we simulate an interaction of the laser pulse with overdense $e^+e^-$ plasma layer initially at rest (without any ions). The laser pulse is the same as in the simulation for the foil. The layer parameters: $d = 1 \mu m$ and $n_e = 0.7 \alpha_0 n_{cr}$. It is observed in the simulation (see the Supplemental Material [34]) that the self-sustained cascade develops in similar way as in the case of the electron-ion plasma foil. Therefore the self-sustained QED cascade can develop in a plane electromagnetic wave if there is an appropriate seed and the wave is intense enough. For the parameters explored in the simulations the cascade efficiently develops at $a_0 > 1500$.

As we will see, the mechanism of the cascade front propagation is determined by the $e^+e^-$ pair photoproduction from the high-energy gamma-quanta counter-propagating to the laser radiation. These gamma-quanta are emitted by the relativistic electrons and positrons inside the cushion along their instantaneous velocity and have the greatest photoproduction probability. The QED cascade develops and expands as a result of coupling between the pair plasma produced by the gamma-quanta and the gamma-quanta emitted by the plasma and leaving the cascade volume [see Fig. 3(a)]. This leads to constant growing of the $e^+e^-$ plasma cushion towards the laser.

A simple phenomenological model based on discussed above mechanism can be constructed to describe propagation of the QED cascade front in the reference frame moving along the $x$-axis with the average positron velocity. We assume that the gamma-quantas are propagating in precisely opposite direction of the $x$-axis and produce the $e^+e^-$ pairs that are immobile in the average but have enough energy to produce gamma-quanta. In this case continuity equations can be written as follows:

$$\frac{\partial n_p}{\partial t} = W_p n_\gamma, \quad \frac{\partial n_\gamma}{\partial t} - e \frac{\partial n_p}{\partial x} = -W_p n_\gamma + 2W_r n_p \quad (1)$$

where $n_e, n_p = n_e$ and $n_\gamma$ are the densities of the electrons, positrons and the gamma-quantas, respectively, $W_p$ and $W_r$ are the mean probabilities of pair photoproduction and of photon emission, respectively. The probabilities are assumed to be constant. If the term with $\partial_x$ characterizing the spatial dispersion is neglected in Eqs. (1) then it is reduced to the equations describing QED cascade in the rotating electric field without spatial dynamics [13, 16].

Eqs. (1) can be solved with the one-sided Fourier transform [35], i.e. by the expansion of their solution as a series of complex exponents with real $k$ values and complex $\omega$ values. If the initial distribution of the pair plasma density and the gamma-quantas density are $n_p (0, x)$ and $n_\gamma (0, x)$, respectively, then the solution for the pair plasma density is

$$n_p (t, x) = \int_{-\infty + i\sigma}^{\infty + i\sigma} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \int dx' \frac{e^{ik(x-x')-i\omega t}}{\omega - \omega + \sigma} \times \frac{W_p n_p (0, x') + \sigma (\omega + \sigma k) n_\gamma (0, x')}{\Delta (\omega, k)} , \quad (2)$$

where $\Delta (\omega, k) = \omega^2 + \omega (iW_p + \sigma k) + 2W_p W_r$, $\sigma$ is a real number so that the contour path of integration is in the region of convergence. The initial distribution perturbations propagates along characteristics determined by the dispersion relation $\Delta (\omega, k) = 0$ with the following solution: $\omega = -ck + iW_p \pm \sqrt{ck + iW_p}^2 - 8W_r^2 / 2$. The group velocity of the perturbations then can be found $v_{gr} = \partial \text{Re}[\omega] / \partial k$. In the absence of the laser field the QED processes are suppressed, $W_r = W_p = 0$. In this case the plasma and the gamma-quantas are decoupled from each other and the dispersion relation yields $\omega = -k$ for the gamma-quantas and $\omega = 0$ for the plasma, which correspond to $v_{gr} = -c$ and $v_{gr} = 0$, respectively.

It follows from the analysis of the dispersion relation that (i) in the case of large wave numbers $k$ $\gg$ $W_r / c > W_p / c$ the perturbations are damped since $\text{Im} [\omega] = -W_p / 2 < 0$; (ii) the perturbations with small wavenumbers, $k$ $\ll$ $W_p / c$, are the most unstable with the growth rate $\Gamma \equiv \text{Im} [\omega] = (W_p / 2) (\sqrt{1 + 8\gamma} - 1)$ (where $r = W_r / W_p$) that coincides with the QED cascade growth rate derived in Refs. [13, 16] for the field configuration corresponding to the rotating electric field. In this case the dispersion relation for the unstable perturbations is $\omega \approx \pm \sqrt{8\gamma} (s-1) / 2 + i\Gamma$, where $s = (1 + 8\gamma)^{-1 / 2}$. The parameter $s$ tend to zero in the classical limit ($a_0 \to 0$ hence $r = W_r / W_p \to \infty$) and peaks in the strong QED limit $a_0 \to \infty$, $r \approx 4$ and $s \approx 0.17$ [36]. Therefore, in the case of small $k$ the group velocity of the unstable perturbations is $v_{gr} \approx -0.41 c$. These result coincide well with the results of numerical solution of Eqs. (1) for various shapes of the initial seed of pairs and gamma-quantas.

In the reference frame moving along with the pair plasma, the cascade front velocity $v_{cf}$ should coincide with the above-mentioned group velocity, $v_{gr} \approx -0.41 c$. In the laboratory reference frame the relation between the cascade front velocity and the mean cushion plasma velocity along the $x$-axis, $\tilde{v}_x$, relates by the Lorentz transform $v_{cf} = (\tilde{v}_x + v_{gr}) / (1 + \tilde{v}_x v_{gr} / c^2)$ from that $\tilde{v}_x$ can be found. The model predicts $\tilde{v}_x = 0.61 c$ for $v_{cf} = 0.27 c$ [$a_0 = 2500$, see Fig. 3(b)] that is reasonably close to value of the averaged positron velocity $\tilde{v}_x \approx 0.75 c$ retrieved from the simulation.

To characterize the cascade in more detail the EM field distribution and the particle dynamics inside the $e^+e^-$ plasma cushion are studied. It follows from Fig. 3(b) that the field structure is close to the circularly polarized wave with $\mathbf{E} \perp \mathbf{B}$ that declines in the plasma within several laser wavelengths, where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic field respectively.
magnetic fields, respectively. The key feature of the EM field is the magnetic field predominance $B > E$ inside the $e^+e^-$ plasma. In such field the electrons and positrons do not gain energy [see the line $\gamma/a_0$ in Fig. 3(c)], thus the self-sustained cascading is suppressed deep inside the cushion. Although the electrons and positrons outrun the cascade front on average, for some time their velocity along the $x$-axis is less than the front velocity [see Fig. 3(c)] for the positron distribution in $x - v_x$ space and for $v_{ef}$ and $\bar{v}_x$. Therefore, they can emit gamma-quanta which can get to the vacuum region and produce new $e^+e^-$ pairs in the laser field. This field then accelerates the created electrons and positrons towards the cushion, thus the self-sustained cascade develops on the interface of the vacuum region and the cushion, as shown schematically in Fig. 3(a). Note that the magnetic field dominance and probable signatures of a cascade in a plane wave are also observed in the simulations for the case of a linear polarization [14].

To explain the predominance of the magnetic field in the cushion let us consider a high-intensity circularly polarized plane wave travelling in a homogeneous $e^+e^-$ plasma. We use the asymptotic theory of the electron motion [37, 38] to calculate the current density in the Maxwell’s equations. Accordingly to Fig. 3(b) we suppose that the electric and magnetic fields are almost perpendicular to each other, however the Lorentz invariant $\mathbf{E} \cdot \mathbf{B} < 0$ [in Fig. 3(b) $\varphi$ is slightly greater than $\pi/2$]. Thus, there is a reference frame $K'$ moving along the $x$-axis with the speed $(\mathbf{E} \times \mathbf{B})_x/B^2 \approx E/B$ so that the component of the electric field perpendicular to the magnetic field vanishes and the electric field is directed exactly opposite to the magnetic one. In the $K'$-frame the electric field induces the current along itself, giving in the laboratory reference frame the current density $J = -2en_p\bar{v}_1\mathbf{B}/B$, where $n_p$ is the positron density (half of the overall plasma density) and $\bar{v}_1$ is the average component of the positron velocity perpendicular to the $x$-axis.

Noting that for a circularly polarized wave the vectors $\mathbf{B}$, $\partial_t \mathbf{E}$ and $\nabla \times \mathbf{B}$ are parallel to each other (as well as $\mathbf{E}$, $\partial_t \mathbf{B}$ and $\nabla \times \mathbf{E}$), and $\mathbf{B}$ rotates counterclockwise in the $yz$-plane with the increase of $x$ [see Fig. 3(b)]. Thus, the Maxwell’s equations yield:

$$kE = \frac{\omega}{c}B, \quad kB = \frac{\omega}{c}E - 8\pi en_p\bar{v}_1/c,$$

where $\omega$ is the frequency of the wave and $k$ is its wavenumber. We suppose that $\bar{v}_1$ relates with the mean particle velocity along the $x$-axis, $v_x$, as follows: $\bar{v}_1 = \nu(c^2 - \bar{v}_x^2)^{1/2}$, where $\nu < 1$ is a numerical coefficient introduced because of the large spread of the particle velocity distribution in the cushion. The simulations show that independently on the laser amplitude and time $\nu \approx 1/6$. Then $\bar{v}_x$ can be calculated from Eq. (3):

$$\frac{\bar{v}_x}{c} = \left(\frac{\sqrt{1 + 4S^2} - 1}{2S^2}\right)^{1/2}, \quad S = \frac{2n_p \nu}{n_e a},$$

where $a = eE/mc\omega$ and $n_e = m\omega^2/4\pi e^2$.

Although the presented solution of the Maxwell’s equations does not take into account plasma inhomogeneity and the field absorption, $\bar{v}_x$ calculated from Eq. (4) for $n_p$ and $E$ retrieved from the simulations [shown in Fig. 3(c) with triangles] coincides well with the drift velocity $E/B$ (crosses) and the mean positron velocity obtained in the simulation (solid line). In QED plasmas the parameter...
In conclusion, it follows from our results that (i) a QED cascade is an imminent process for most of high-field phenomena and can develop in a plane electromagnetic wave, or in other words, in wider range of the field configurations than it is supposed; (ii) the LS regime of the ion acceleration becomes inefficient at extremely high intensities when the overdense electron-positron plasma cushion is produced in front of the foil. It is shown that the cushion expands towards the laser because of intensive backward emission of the gamma-quanta with subsequent pair photoproduction. The models describing the cushion front propagation and the electrodynamic of the cushion plasma are proposed and are in a qualitative agreement with the numerical simulations. However further investigations are needed to develop a self-consistent theory of laser-plasma interaction in QED regime.

The work is supported in parts by the Russian Science Foundation by Grant No. 18-11-00210 (numerical simulations), by the Presidium of the Russian Academy of Sciences and by the Grants Council under the President of the Russian Federation through Grant No. MK-2218.2017.2 (analytical model).

S \propto n_p/a_0 typically increases with the increase of a_0 because of efficient avalanche-like pair production and sharp dependence of the cascade growth rate on the a_0. Thus, according to Eq. [4], the higher a_0, the lower both \bar{v}_x and v_{cf}. This is in agreement with the simulation results (see the upper row in Fig. [2]).