Shearlet-based compressed sensing with non-local similarity for MRI breast image reconstruction

Xiaotao Shao† | Caike Wei† | Yi Xie‡ | Zhongli Wang† | Yan Shen†

†School of Electronic and Information Engineering, Beijing Jiaotong University, Beijing, China
‡Beijing Xinghang Mechanical-Electrical Equipment Co., LTD, Beijing, China

Correspondence
Yan Shen, Beijing Jiaotong University, Beijing, 100044, China.
Email: sheny@bjtu.edu.cn

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Abstract
Magnetic resonance imaging (MRI) requires long detection time and makes patients uncomfortable. The proposed compressed sensing MRI compressed sensing with shearlet dictionary and non-local similarity model is established with shearlet dictionary and non-local similarity. The shearlet dictionary is adopted in MRI compressed sensing to represent breast tissues with sparser data in different scales and directions. The non-local similarity of an image is integrated to the model to preserve the lesion details of the reconstructed MRI images. The proposed model is solved by the split Bregman algorithm to obtain the optimized image iteratively. Experiments are performed on practical MRI breast images with sampling data of 13% and 10%. With the decrease of sampling data, the proposed method can reconstruct the image with better visual effect and higher peak signal-to-noise ratio (PSNR) and structural similarity index (SSIM) than traditional methods. There is an improvement of 13 dB of PSNR and 0.2 of SSIM under 10% data. The proposed method can reconstruct MRI images with less data and higher reconstruction quality compared with the traditional methods.

1 | INTRODUCTION

Magnetic resonance imaging (MRI) is an effective imaging modality in the diagnosis of breast diseases [1] because of its high-quality images with excellent soft tissue contrast and absence of emitted ionizing radiation. Apart from these advantages, the relation between the number of measured data sample and net scan time is hard to balance [2]. To obtain high spatial resolution, it is necessary to obtain as much data as possible. However, it will increase the scan time and may cause more artefacts. During the entire data acquisition process, patients stay in a relatively closed MRI space for a long time, which makes patients uncomfortable, even for the patients who need life support systems. Reducing measurement time can solve this problem, but will lead to the reduction of measurement data and introduces unpleasant artefacts in the reconstructed MRI images. To overcome these disadvantages, a novel MRI reconstruction method should be developed that allows for MRI reconstruction from the reduced measurement data. Compressed sensing (CS) offers the opportunity for a signal to be restored to reasonable resolution even when the sampling ratio is lower than that suggested by the Nyquist sampling theory [3]. CS has attracted more attention in applications ranging from image processing to medical imaging CS states that if images are sparse in a space domain or some transformation domain, they can be reconstructed from a small number of random linear measurements via solving a tractable convex optimization programme [4].

Researchers have studied CS applied in the MRI field. On the one hand, they seek new theories to improve the quality of reconstructed images; on the other hand, they seek sparser representation to reduce the data for reconstruction. Various theories are used to improve the reconstruction of MRI images. The penalized splitting approach and adaptive non-linear filtering strategy were used to reconstruct three-dimensional (3D) MRI images [1]. Adding extra regularization is also one of the strategies to improve the quality of image reconstruction. Duan et al. combined the norm and total variation (TV) regularization terms linearly and used the split Bregman iteration technique to reformulate the original constrained problem as a sequence of unconstrained problems [5]. Researchers have used variable splitting and alternating...
minimization of the augmented Lagrangian to solve the regularized reconstruction problem [6], or they have introduced the majorizing matrix in the range of the regularization matrix [2]. Abdullah applied the low-dimensional manifold model to exploit the patches’ similarity and redundancy, and used the dimension of the patch-manifold as a regularization term in a variational formulation [7]. Cheng adopted a first-order primal-dual framework to remove undersampling artefacts while keeping the features [8]. Meanwhile, some methods are considered in the reconstruction, such as low rank [9], patch-based method [10], and so on. These papers improve the quality of an image by using different reconstruction methods, or they combine other theories such as low rank and TV, and thus solve them as regularization terms. However, the quality improvement by these methods requires a higher sampling of MRI data. If the MRI sampling data is reduced, the reconstruction quality will be decreased.

In terms of finding sparse representation to reduce the data for reconstruction, an image can be reconstructed with less data if it has sparse representation. TV and Bayesian dictionary learning were set as a sparse constraint to reconstruct an MRI image with a random sampling of 20% [11]. Datta applied interpolation to estimate the missing k-samples of undersampled multi-slice data to accelerate the scan time [12]. Liu et al. proposed a balanced sparse model that balances solutions between analysis and synthesis sparse models; it can reconstruct an image with 30% data [13]. Zhuang proposed joint priors capture global and local sparse nature of the MR image by jointing image and patch priors to promote image structures and suppress artefacts or noises [14]. The sparsity of the transformation domain was introduced into the reconstruction, such as partial Fourier transformation [15], gradient domain [16], wavelet domain [17, 18], and the joint sparsity in the wavelet and TV domains [19].

The quality of CS reconstruction depends on the sparsity of the image domain or its transformation domain and the reconstruction method. To reduce the sampling ratio of reconstruction, we should seek a sparser representation. In recent years, some transformations have been able to provide sparser representation for images, such as contourlet [20, 21], curvelet [22], and shearlet [23, 24] transformations, which are called redundant dictionaries. The wavelet and curvelet transformations were jointly set as the sparse regularization terms to reconstruct the images [25]. The TVCMRI model [26] used the sparsity of the wavelet domain and TV domain to solve the CS problem. Kida combined the optimum interpolation approximation and CS to improve the stability and accuracy of the reconstructed image [27]. Huang et al. used the TV model and transformation domain model alternatively to implement MRI CS reconstruction [28]. Panić et al. proposed a CS-MRI reconstruction algorithm with a Markov random field prior model based on the non-decimated shearlet transformation [29]. Contourlet and curvelet transformations will introduce various artefacts that affect the results of CS reconstruction. However, shearlet transformation has much sparser representation [24] than contourlet and curvelet transformations and overcomes the disadvantages of them. Therefore, we selected shearlet transformation in this paper to accomplish the CS reconstruction because of its prominent properties of sparsity, directional sensitivity, and efficient implementation.

The main contributions of our work include the following: (1) applying the shearlet redundant dictionary to MRI CS reconstruction with sparser representation; (2) the non-local similarity is adopted to preserve the details of the image, and combining shearlet transformation with non-local similarity to reduce the sampling ratio, by which only 10% sampling data is needed to reconstruct the image with high quality.

The organization of the paper is as follows. Section 2 introduces the shearlet transformation. Section 3 introduces the proposed MRI CS model based on shearlet transformation, which can effectively reconstruct an image with less data and meanwhile preserve the details with non-local similarity. Section 4 presents the experimental results and compares reconstructed results of the proposed method with previous CS reconstruction methods. Section 5 is the conclusion.

2 | THEORY

2.1 | Shearlet transformation

Shearlet transformation delivers optimally sparse approximations of images and allows for a unified treatment of the continuum and digital world [30]. Curvelet is implemented in frequency domain that will introduce various artefacts. Contourlet is implemented with tree-structured filter banks but it misses an associated continuum domain theory.

The digitization of the associated discrete shearlet transformation will be performed in the pseudo-polar Fourier transformation (PPFT) domain. The Fourier transformation of a continuous function, evaluated at frequencies expressed in polar coordinates, is an important conceptual tool for understanding physical continuum phenomena. An analogous tool, suitable for computations on discrete grids, could be very useful; however, no exact analogue exists in the discrete case. PPFT can solve this problem, which evaluates the discrete Fourier transformation at points on a trapezoidal grid in frequency space. PPFT is a 2D Fourier transformation of an image in a pseudo-polar coordinate system. The pseudo-polar grid consists of two basic subsets: the basic horizontal subsets \( C_{21} \) and \( C_{22} \) and the basic vertical subsets \( C_{11} \) and \( C_{12} \), represented by \( \Box \), \( \Box \), \( \Box \), and \( \Box \), respectively, as shown in Figure 1.

To use 1D filter group to extract image direction information, it is necessary to adjust the horizontal subset and vertical subset of the pseudo-polar grid according to rectangular coordinate system, as shown in Figure 2. The horizontal and vertical subsets are combined successively and rearranged in the interval along the slope direction. In this way, a 1D filter set is used to divide the adjusted pseudo-polar grid and the frequency band. The 1D filter of the rectangular support in Cartesian coordinate corresponds to the wedge-shaped support domain in the pseudo-polar grid.
Consider the cone-adapted discrete shearlet transformation \( SH(\Phi, \Psi, \Psi, \Delta, \Lambda, \Lambda) = \Phi(\Phi, \Delta) \cup \Psi(\Psi, \Delta) \cup \Psi(\Psi, \Lambda, \Lambda), \)
where \( \Delta = \mathbb{Z}^2 \), which means the digital grid \( \mathbb{Z}^2 \) is projected onto the two symmetric pseudo-polar coordinates \( \Lambda \) and \( \Lambda \). It can be seen from Figure 1 that \( \Lambda \) consists of \( C_{11} \) and \( C_{12} \), and \( \Lambda \) consists of \( C_{21} \) and \( C_{22} \). This transformation includes three operations \( \Phi(\Phi, \Delta), \Psi(\Psi, \Lambda), \) and \( \Psi(\Psi, \Lambda, \Lambda) \). The operation \( \Phi(\Phi, \Delta) \) is defined by the scale function \( \Phi \), whose support is \( \Delta \). The shearlet operations \( \Psi(\Psi, \Lambda) \) and \( \Psi(\Psi, \Lambda, \Lambda) \) are defined by the shearlet function \( \Psi \) and \( \Psi \), whose support are \( \Lambda \) and \( \Lambda \), respectively. The black dots in Figure 1 indicate frequency points in different directions. Let \( \Psi \) be a classical shearlet function

\[
\psi(\xi) = \hat{\psi}(\xi_1, \xi_2) = \hat{\psi}_1(\xi_1)\hat{\psi}_2(\xi_2)
\]

(1)

where \( \hat{\psi} \) is the PPFT of \( \psi \). The operation \( \odot \) represents the PPFT, and \( (\xi_1, \xi_2) \in \mathbb{Z}^2 \). Equation (1) separates the classical shearlet function into two operations, \( \psi = \mathcal{L}(\mathcal{F}(\hat{\psi})) \) is a wavelet function with \( \hat{\psi} \in C^\infty(\mathbb{R}) \) and the support of \( \hat{\psi} \) is \([-4, -1/4] \cup [1/4, 4] \); \( \psi = \mathcal{L}(\mathcal{F}(\hat{\psi})) \) is a ‘bump’ function satisfying \( \hat{\psi} \in C^\infty(\mathbb{R}) \) and the support of \( \hat{\psi} \) is \([-1, 1] \).

Focusing on the cone \( C_{11} \) and \( C_{12} \), the discrete shearlet transformation of \( x \) is expressed as the following:

\[
x \mapsto x(2^{-r} \psi_1(4^{-j} \xi_1) \psi_2(4 \xi_2 + c))
\]

(2)

where \( x \) is the PPFT of \( x \). Other parameters are scale \( j \), orientation \( o \in \mathbb{Z}^2 \), position \( \tau \in \mathbb{R}^2 \), frequency point \( k \in \mathbb{R}^2 \), image size \( N \times N \), and \( h \in [-N/2, N/2] \) \( \cup [-N/2, N/2] \). The discrete shearlet transformation on \( C_{21} \) and \( C_{22} \) can be obtained by the same method of \( C_{11} \) and \( C_{12} \), but then \( \psi(o \xi_1, o \xi_2) = \psi(o \xi_1, o \xi_2) \) \( \in \mathbb{Z}^2 \). The scale parameter \( j \) represents the decomposition scale levels of MRI images, where \( j = 1, 2, \ldots, J \). As the decomposition level \( j \) increases, the image details are preserved, but the amount of shearlet coefficients is increased, which results in the increase of the computational complexity. Therefore, the two-level \( (j = 1, 2) \) decomposition of the shearlet transformation is adopted here for MRI images.

The decomposition of the shearlet transformation is shown in Figure 3. The input image \( x \), \( x \in \mathcal{L}_2(\mathbb{R}^2) \), is transformed into the PPFT domain, and the frequency domain coefficient \( X \) is obtained. Next, the scale filters, consisting of a low-pass (LP) filter and band-pass (BP) filter, are used to divide the spectrum into LP frequency part \( a_j \) and BP frequency part \( b_j \). Then BP frequency part \( b_j \) is divided into multiple directions by convolving with the ‘shear filter’, as shown in the blue dotted shadow block. It can be seen from Figure 3 that if \( j = 2 \), then the frequency domain coefficient \( X \) is decomposed into the LP frequency part \( a_2 \) by convolving with the LP filter \( L_2 \) and into the BP frequency part \( b_2 \) by convolving with the BP filter \( B_2 \). Then, BP frequency part \( b_2 \) is decomposed in multiple directions by convolving with the shear filter. There are eight directions in total in the BP frequency part, as shown in the blue dotted shadow block \( c_1 \) of Figure 3. Next, the LP frequency part \( a_1 \) is decomposed into the LP frequency part \( a_2 \) by convolving with the LP filter \( L_1 \) and into the BP frequency part \( b_1 \) by convolving with the BP filter \( B_1 \). The BP frequency part \( b_2 \) is decomposed into multiple directions \( c_2 \). Finally, an inverse Fourier transformation of pseudo-polar coordinates (IPPT) is performed to obtain the shearlet coefficients \( \text{coef} \). As shown in Figure 3, \( a_l \) is the LP frequency part of \( j \)-level, \( a_l = X \ast L_{p1} \ast L_{p2} \ast \cdots \ast L_{pj} \); \( b_l \) is the BP frequency part of \( j \)-level, \( b_l = X \ast L_{p1} \ast L_{p2} \ast \cdots \ast L_{pj} \ast B_{p1} \ast S_{p} \); \( \ast \) is the
convolution operator, \( L_P \), \( B_P \), and \( S \) represent LP filter, BP filter, and shear filter of \( j \)-level, respectively.

The two different LP filters \( L_1 \) and \( L_2 \) can be shown, respectively, in Figure 4a after the above operation in Figure 3. Figure 4b shows the five shear filters of different directions selected from \( c_1 \) of Figure 3. The number and direction of different direction filters can be selected according to specific requirements, so as to extract frequency information in the required directions and achieve better reconstruction results.

### 2.2 Analysis of sparse representation

To find the dictionary, which represents the images sparser and can preserve more information in different scales and directions, we have conducted experiments on the CS reconstruction based on curvelet, contourlet and shearlet dictionaries. The CS reconstruction model is \( x = \text{arg} \min_x \| \Psi x \|_1 \) s.t. \( y = \Phi x \), where \( \Phi \) is the measurement matrix; \( \Psi \) is the redundant dictionary including curvelet, contourlet, and shearlet transformations; \( y \) is the measured vector; \( x \) is the reconstructed vector and is sparse on the redundant dictionary \( \Psi \). The classical image ‘Lena’ is used in the experiment and the optimal dual l1-analysis method is adopted to solve this model [31].

Figure 5 shows the PSNR comparison of contourlet, curvelet, and shearlet redundant dictionaries based on CS. It can be observed from Figure 5 that the CS reconstruction with shearlet has the highest PSNR. Even at the sampling ratio point of 20\%, PSNR of shearlet can reach 15 dB, but PSNRs of curvelet and contourlet are about 3 dB. When the sampling ratio is around 50\%, PSNR of shearlet can reach to 22 dB, PSNRs of curvelet and contourlet are under 10 dB. Therefore, shearlet is superior over curvelet and contourlet based on CS.

Figure 6 is the comparison of structural similarity (SSIM) among the three redundant dictionaries. The structural similarity is defined as the joint function of the luminance, contrast, and structure of the image. It can be seen from Figure 6 that SSIM is raising up with the increasing of the sampling ratio. SSIM of the CS reconstructed image with shearlet dictionary is much higher than that with curvelet and contourlet. Especially at the sampling ratio point of 20\%, SSIM with shearlet can reach 0.6 and SSIM with other two dictionaries are only around 0.1. Therefore, the image reconstructed based on shearlet has higher similarity of the original image.

### 3 THE PROPOSED METHOD

MRI collects data about breasts in different magnetic fields. The common model of data acquisition with incomplete measurement for MRI CS is given by the following equation:

\[
y = \Phi A x
\]

where \( x \) represents the MRI image, \( A \) is the discrete Fourier transformation, \( \Phi \) is the measurement matrix, and \( y \) is the sampled data.

We aim at reconstructing image \( x \) from sampled data \( y \) because \( x \) has sparser representation in shearlet domain than in contourlet and curvelet domains. We set the shearlet transformation as the redundant dictionary \( \Psi \), and \( \Psi \in \mathbb{R}^2 \) \((m < n)\), \( x \) can be expressed as \( x = \Psi^T s \), \( s \) is the shearlet coefficients of image \( x \).

According to CS theory, MRI CS claims to reconstruct an MRI image from sampled data by enforcing the image sparsity; that is, \( x \) can be accurately reconstructed by solving the following minimization problem:

\[
x = \arg \min_x \{ \| \Psi x \|_1 \} \text{ s.t. } y = \Phi A x
\]

The \( N \times N \) matrix \( A \) is the projection matrix, namely, discrete Fourier transformation.

To reduce the distortion of the reconstructed image, we improve the model in Equation (4) with the non-local similarity properties [32, 33] of the image, which are used as a regularization term to optimize the details of the MRI image, as shown in Equation (5). The proposed model in Equation (5) is called CS with shearlet dictionary and non-local similarity (CSSN):

\[
x = \arg \min_x \left\{ \| \Psi x \|_1 + \lambda \sum_p \| \nabla_\omega x \|_1 \right\} \text{ s.t. } y = \Phi A x
\]
The term $\|\nabla_{\omega}x\|_1$ is the non-local similarity term and $\nabla_{\omega}x$ is the non-local gradient operation; $\lambda$ is the weight of the non-local similarity term. The non-local similarity term characterizes the textures or structures of MRI images within non-local areas, and thus it preserves the sharp edges effectively to maintain image’s non-local consistency.

The $\nabla_{\omega}x$ of the non-local similarity term can be written as follows:

$$\nabla_{\omega}x = \sqrt{\sum_{p,q \in \Omega_q} (x(p) - x(q))^2 \omega(p, q)}$$  \hspace{1cm} (6)

and

$$\omega(p, q) = \exp\left(-\frac{\|R_p(x) - R_q(x)\|_2^2}{\mu^2}\right)$$ \hspace{1cm} (7)

where $R_p(x)$ is the image block centred at pixel $p$ of size $N_p \times N_p$ ($N_p = 5$); $R_q(x)$ is centred at the neighbouring pixel $q$ with the same size of $R_p(x)$ and moves in a larger searching window $\Omega_q$ of $N_q \times N_q$ ($N_q = 21$) around $R_p(x)$. The weight function $\omega(p, q)$ used here is to approximate the similarity between the two adjacent patches $R_p(x)$ and $R_q(x)$ [33].

The non-local similarity term can be obtained in the following steps. First, divide the image $x$ into blocks $R_p(x)$ of size $N_p$. Second, for each block $R_p(x)$, we search blocks that are most similar to it within the searching window $\Omega_q$. Finally, weight it by the weight function $\omega(p, q)$, and compute the Euclidean distance between different blocks. Therefore, the method of non-local similarity can preserve more textures and fine details.

Equation (5) is non-smooth and non-separable. We apply the split Bregman algorithm to the constrained minimization problem (5) and change it into the unconstrained problem. Let $d = \Psi x$ and $w = \nabla_{\omega}x$. Introduce the parameters of constraint term: $\mu_1, \mu_2$, and $\mu_3$. The following equations are obtained from Equation (5):

$$x = \arg\min_{x} \left\{ \|d\|_1 + \lambda\|w\|_1 + \frac{\mu_1}{2}\|y - \Phi Ax\|_2^2 \right\}$$  \hspace{1cm} (8)

$$+ \frac{\mu_2}{2}\|d - \Psi x + b\|_2^2 + \frac{\mu_3}{2}\|w - \nabla_{\omega}x + c\|_2^2 \right\}$$

$$b^{k+1} = b^k + d^{k+1} - \Psi x^{k+1}$$  \hspace{1cm} (9)
and

\[ c^{k+1} = c^k + w^{k+1} - \nabla_w x^{k+1} \quad (10) \]

The split Bregman algorithm is applied here to solve this problem. The reconstructed image \( x \) will be obtained after the following iteration process. Then, Equation (8) can be decomposed as three subproblems by iteratively minimizing \( x, d, \) and \( w \), respectively, as shown in Equations (11)–(13):

\[ x^{k+1} = \arg \min_x \left\{ \frac{\mu_1}{2} ||y - \Phi Ax||_2^2 + \frac{\mu_2}{2} ||d - \nabla_x x^k||_2^2 + \frac{\mu_3}{2} ||w - \nabla_w x^k + c^k||_2^2 \right\} \quad (11) \]

\[ d^{k+1} = \arg \min_d \left\{ ||d||_1 + \frac{\mu_2}{2} ||d - \nabla_x x^{k+1} + b^k||_2^2 \right\} \quad (12) \]

\[ w^{k+1} = \arg \min_w \left\{ \lambda ||w||_1 + \frac{\mu_3}{2} ||w - \nabla_w x^{k+1} - c^k||_2^2 \right\} \quad (13) \]

Combining these steps (Equations (11)–(13)) with the update of \( b^k \) and \( c^k \) (Equations (9) and (10)), we can solve the subproblems involved in (8) with appropriate iterations.

Given \( x^k \) and \( w^k \), the subproblem of (11) is a strictly convex quadratic function that can be solved by the gradient descent method. On setting the gradient of the objective function (11) to zero, the solution for (11) is achieved:

\[ x^k = - \left( A^T \Phi^T \Phi A + \mu_2 I \right)^{-1} \left( \mu_1 W \left( d^k + b^k \right) + \mu_3 \left(w^k + c^k - \nabla_w x^k\right) + A^T \Phi^T y \right) \quad (14) \]

The subproblem of (12) can be solved by threshold shrinking method. The coefficients \( d^k \) in the shearlet domain are subjected to hard threshold shrinkage, expressed as follows:

\[ d^{k+1} = \text{shrink} \left( d^k \right) = \begin{cases} 0, & d < T; \\ d, & d > T; \end{cases} \quad (15) \]

where \( T \) is the threshold, which can be adjusted according to the different images.

The MRI CS reconstruction for the proposed CSSN algorithm can be described as follows:

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**Algorithm 1**

**Input:** original image: \( x \); projection matrix: \( A \); measurement matrix: \( \Phi \)

**Output:** \( x^* \) : recovered image.

**Initialization:**

Set \( k = 1; b^0 = 0; c' = 0; x^1 \) is obtained by an initial reconstruction method based wavelet transformation, \( x^1 = WTCS (x) \).

Obtain sampling data: \( y = \Phi x \), where \( x \) is the projection data.

For \( k = 1: k_{\text{max}} \)

- Analyze nonlocal similarity, compute \( w^k \)

\[ w^k = \arg \min_w \left\{ \lambda ||w||_1 + \frac{\mu_2}{2} ||w - \nabla_w x^k - c^k||_2^2 \right\} \]

- Solve the following equation, obtain \( x^{k+1} \)

\[ d^{k+1} = \text{shrink} \left( d^k \right) \]

\[ w^{k+1} = x^{k+1} - c^k \]

If \( ||x^{k+1} - x^k||_2^2 / ||x^k||_2^2 < \text{tol} \)

- Break;

- Update \( b^{k+1}, c^{k+1} \)

\[ b^{k+1} = b^k + d^{k+1} - x^{k+1} \]

\[ c^{k+1} = c^k + w^{k+1} + x^{k+1} \]

- \( k = k+1 \)

**End**

\( x^* = x^{k+1} \)

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The reconstruction process of our method can be explained as follows: First, set the initial value of the parameters and obtain the measurement data \( y \). Second, solve the problem (5) about the measurement \( y \) by using the split Bregman algorithm, the problem (5) is decomposed as three subproblems (11)–(13), which can be solved iteratively instead of the problem (5). In the iterative process, the non-local similarity of the image \( w^k \) is first computed; then \( x^{k+1} \) is obtained by solving problem (14). Next, the threshold shrinkage processing is performed on the shearlet coefficients of the \( d^{k+1} \), which removes some of the high frequency coefficients and can effectively suppress the noise; then, the previously calculated variables are used to update \( b^{k+1} \) and \( c^{k+1} \). Finally, the iteration stops when \( ||x^{k+1} - x^k||_2^2 / ||x^k||_2^2 < \text{tol} \), where tolerance \( \text{tol} \) is set as \( 10^{-3} \) here; \( x^{k+1} \) and \( x^k \) are the reconstructed image of the \( k+1 \)th and \( k \)th iterations.

4 | NUMERICAL EXPERIMENTS

We evaluate the proposed method on the images in the website https://radiopaedia.org shown in Figure 7. Figure 7a is the middle part of a right breast and Figure 7b is the upper-left part of a left breast. Breast masses with burrs on the edge, marked by the white boxes, are important indications for diagnosis. Therefore, the reconstructed lesion details should be preserved as far as possible. We use the random mask, with the sampling ratio defined in Equation (16), where \( m \) is the number of nonzero values and \( N \times N \) is the size of the image:
The experimental results of the proposed CSSN method are compared with the classical methods including SparseMRI [34], FCSA [35], RecPF [36], TVCMRI [26], WaTMRI [37], and SeSCI [38]. We use the PSNR and SSIM as the evaluation criterion.

The different sampling ratio $\beta$ is adopted in this paper to validate that the proposed method can reconstruct the MRI images with less data (lower $\beta$) and higher reconstruction image quality than other methods. The reconstructed images of Figure 7 with the proposed method and others methods are shown in Figures 8–10. Figure 8 shows the reconstructed MRI images with the sampling ratio $\beta$ of 10%; Figures 9 and 10 show the reconstructed MRI images with the sampling ratio $\beta$ of 13%. With the decrease of $\beta$ from 13% to 10%, all the CS methods obtain lower reconstructed image quality, but the proposed method still gets much better reconstruction quality than other methods.

Figure 8 shows the reconstructed results with the sampling ratio $\beta$ of 10%, from left to right, using RecPF, SparseMRI, TVCMRI, WaTMRI, FCSA, SeSCI, and the proposed CSSN methods. From a visual perspective, the proposed CSSN method in this paper is superior to other methods in the case of a sampling ratio of 10%. CSSN method in the last column of Figure 8 can reconstruct the lesion area with more detailed information (such as burrs, marked by the arrow) than other methods in the first to fifth columns of Figure 8. As shown in the first row of Figure 8, the traditional methods shown in Figure 8 (columns 1 to 5) cannot reconstruct the lesion information with clear details, the dark areas of images (marked by white boxes) are blurred, and their contrasts are low, which may affect a doctor’s diagnosis. The reconstructed results of the second row of the first to fifth columns of Figure 8 are blurred, the mass information in the image cannot be resolved, especially for the second column of Figure 8, the detailed information of the mass is illegible. It is critical to preserve the lesion details for diagnosis, but artefacts are introduced in the reconstruction results of the RecPF, SparseMRI, TVCMRI, WaTMRI, FCSA, and SeSCI methods. The proposed CSSN method can reconstruct the images by using 10% data with high quality. Therefore, the lesion features are preserved effectively.

The comparison of PSNR and SSIM of the reconstructed images with 10% data by different methods are shown in Table 1. It can be seen from Table 1 that the PSNR and SSIM of the proposed method are higher than those of other CS methods. The PSNR value of the proposed method has an increment of at least 13 dB compared with other methods. The SSIM value of the proposed method has an increment of 0.2 compared with other methods. Therefore, the CSSN method can reconstruct an image with less data (10%) while preserving the details of the image.

The above experiments are carried out with the sampling ratio of 10%. If we increase the sampling ratio, all CS methods can reconstruct images with higher quality. Figure 9 shows the reconstructed results at the sampling ratio of 13% by using all the methods. It can be seen from the last column of Figure 9 that the proposed CSSN method still has better reconstruction quality than other methods with clearer edges of lesions (marked by white boxes) under the sampling ratio of 13%. To compare the details of reconstructed images with different methods, the areas marked by white boxes in Figure 9 are magnified in Figure 10, which shows the burrs around the masses, one of the most important features of cancer diagnosis. The reconstructed results of the proposed method shown in

\[
\beta = \frac{m}{N \times N} \quad (16)
\]

**Figure 7** MRI images of infiltrating ductal carcinoma in a breast downloaded from website: (a) the middle part of the right breast and (b) the upper left part of the left breast

**Figure 8** From left to right are the compressed-sensing-reconstructed MRI images with 10% data using RecPF, SparseMRI, TVCMRI, WaTMRI, FCSA, SeSCI, and the proposed CSSN
FIGURE 9 From left to right are the compressed-sensing-reconstructed MRI images with 13% data using the RecPF, SparseMRI, TVCMRI, WaTMRI, FCSA, SeSCI, and the proposed CSSN methods.

FIGURE 10 Magnified area of Figure 9. From left to right are the magnified reconstructed MRI images with 13% data using the RecPF, SparseMRI, TVCMRI, WaTMRI, FCSA, SeSCI, and the proposed CSSN.

| Images | PSNR | SSIM |
|--------|------|------|
| MRI1   | 24.33| 0.745|
| MRI2   | 19.87| 0.421|
| MRI3   | 12.24| 0.335|
| MRI4   | 21.81| 0.265|
| Abbreviations: CS, compressed sensing; MRI, magnetic resonance imaging; PSNR, peak signal-to-noise ratio; SSIM, structural similarity index. |

Table 1 Comparison of PSNR and SSIM of CS-reconstructed images using 10% data, MRI1–MRI4 are chosen from 50 MRI images.

the first column of Figure 10 can preserve the micro burrs with clear edges, but other methods shown in Figure 10 (columns 1–5) have lost the information of micro burrs and cannot reconstruct them accurately (identified by the white arrows). In the second row of Figure 9, two large burrs can be seen around the mass area. The proposed method (the last column of Figure 9) can reconstruct the burrs with more preserved information, and a detailed comparison can be seen in the magnified images shown in the second column of Figure 10 where the white arrow is marked. Therefore, the proposed method of the last column of Figure 10 has sharper edges of the burrs than those of other methods.
It can be seen from Table 2 that the values of PSNR and SSIM of the proposed method are also higher than those of the other methods. The PSNR of the proposed method is at least 0.5 dB higher than other methods. The SSIM of the proposed method is 0.1 higher than the other methods.

Table 3 shows the running time for different algorithms. We use the image size of 256 × 256 under the sampling ratio of 13%. The time consumed was the average values over 50 MRI images.

We also have conducted experiments using all of the mentioned CS reconstruction methods with the 50 MRI images under different CS sampling ratios. Each curve in Figure 11 is the average of 50 PSNRs of the CS-reconstructed images, and each curve in Figure 12 is the average of 50 SSIMs of the CS-reconstructed images. The horizontal axes of Figures 11 and 12 are the sampling ratio; with the increase of the sampling ratio, more image data are used to reconstruct an image, which leads to quality improvement of reconstructed images and gradually decreases the gap between the proposed CSSN method and other methods. Therefore, the curves of PSNR and SSIM slope up with the increase of the sampling ratio, and thus the PSNRs and SSIMs of the FCSA, WaTMRI, and SeSCI methods approximate to that of the proposed method. However, in the case of a low sampling ratio (<20%), the PSNR and SSIM of the proposed CSSN method are much higher than that of the other methods, PSNR even increased to 13 dB at the sampling ratio of 10%. Therefore, the advantages of the proposed method is that it can reconstruct MRI images with less data (<10%) and preserve the details of lesions with higher PSNR and SSIM.
5 | CONCLUSION

In this paper, we propose a CSSN CS method that combines the shearlet dictionary and non-local similarity to reconstruct MRI images with less data and high reconstruction quality. The shearlet dictionary can provide a sparser representation of images and reduce the reconstruction data. Meanwhile, non-local similarity is used as one of the regularization terms of the proposed model to better preserve the image details. Therefore, the proposed method can preserve the lesion structures that have potential clinical significance to help doctors diagnose correctly. Experimental results on practical MRI images show that our proposed CSSN method can reconstruct an image with a 10% sampling ratio and obtain much higher PSNR than other methods. The proposed CSSN method can preserve the detailed edges of MRI images with a more pleasing visual effect and achieve higher SSIM than other methods. Therefore, the proposed CSSN method outperforms the state-of-the-art methods in both visual effects, PSNR, and SSIM.

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CONFLICT OF INTEREST

The authors declared that they have no conflicts of interest to this work.

ORCID

Xiaotao Shao https://orcid.org/0000-0003-0758-518X
Zhongli Wang https://orcid.org/0000-0002-3236-8219
Yan Shen https://orcid.org/0000-0001-9287-1206

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