1. Introduction

Information signals are widely used in various fields, including their application in control systems. It should be noted that, given the powerful information resource of noise signals, the potential possibilities of their use have not been utilized in full when creating modern information technologies, developing the software and hardware information and control systems. This can be explained by the hypothesis that noise signals in radio-electronic systems are considered as interference, which, accordingly, underlies the methodology of research into reducing their intensity and the level of influence. Such a methodology is still relevant in the analysis and synthesis of technical systems for various purposes. This applies to issues related to the system noise immunity.

The wide range of tasks on processing the information signals under the action of interference is common knowledge. To solve them, classical and modern methods are used. The theoretical foundation of research into noise signals is the boundary theorems of the sum of independent random quantities, which were further advanced within the framework of the theory of linear and harmonized random processes and fields. The practical justification for such studies is the stochastic integrated physical mechanism of noise signals formation and their distribution in different mediums.
It should be noted that known methods for decreasing the signal/noise ratio are based on a limited series of theoretical ideas. An example would be the use of a reference model of the Gaussian random process of white noise as a noise interference [1]. Even though the continuous white noise has not been physically implemented, this model is widely used.

2. Literature review and problem statement

Paper [1] investigates the wireless information transfer systems using an adaptive filter based on the least-squares method (LSM) to reduce the impact of noise interference – the Gaussian white noise. However, the cited paper is generally applied to eliminate the attenuation within the wireless information transfer systems. The results of using an estimate of the signal/noise ratio (SNR) based on Kalman filtration are reported in [2]: the cognitive radio networks were investigated. It should be noted that the Kalman filtration algorithm is described to apply to the Gaussian white noise, for which the SNR estimate is projected, but no results for other types of noise interference were given.

The results of changing the dynamics of numerical series and textured objects in image research using a reference model of a random process of the Gaussian white noise, as a random process with uncorrelated (in a broad sense) or independent (in a narrow sense) values, were described in [3]. These results make it possible to evaluate the presence of a Gaussian Markov random field in the tasks on detecting a textured object in a noisy image. It would be expedient to see how these results could be applied to other subject areas. Work [4] examined the low levels of SNR in object detection systems and proposed the use of two new detectors under the action of non-Gaussian noise interference. However, it should be noted that the accuracy of measuring the parameters of a usable signal can be achieved for the case of a correlated non-Gaussian interference. Computer simulation of problems on the recognition of objects under conditions of indirect visibility (Non-Line-of-Sight) employs the methods of inverse correlography with the use of white noise implementations in line with a uniform distribution law [5]. This method is limited due to the use of only correlated non-Gaussian interference. According to the results reported in [6], the use of the phase-rotated spectral correlation detection method for spectral investigations provides for the enhancement of SNR versus the energy and single-cycle cyclo-stationary detector. The detectors, which operate on this principle, surpass energy detectors in terms of analytical results and model experiments but are quite complex in technical implementation. The results of the Xilinx Artix-7 FPGA chip operation in studying the algorithm for orthogonal mutual correlation under a strong exposure to the Gaussian white noise aimed at increasing the SNR are reported in [7]. To implement the algorithm of orthogonal reciprocal correlation of the investigated signals, an additional supporting signal is generated, the phase characteristics of which are shifted by $\pi/2$ relative to the investigated one. This is a known signal orthogonalization method, similar to the method for forming quadratic phases in signal systems. The basic provisions of the theory and practice of noise interference are given in [8]. The linear random processes with infinitely dividing distribution laws, including Gaussian and Poisson, have been used as a mathematical model of noise interference. In practice, noise interference studies that are most used in the study of radio-electronic systems are the white noises, including heat and fractional ones. A given work offers the use of the correlation orthogonal filters for processing the geophysical signals. However, the implementation of such an idea is not clearly formulated; no recommendations for its practical implementation are given. It is proposed, for the technical diagnostics tasks, to use a method for finding the piecewise-homogeneous intervals of noise signals [9]. The method reduces the effect of the interference component due to the use of singular spectral analysis and subsequent sliding processing of the noise sequence. However, when using such a method, it is necessary to clearly define the threshold values used for the sliding processing of the noise sequence, which is not described in practice.

It follows from our analysis that increasing the SNR has not fully used the possibility of decreasing the denominator values of this correlation. In a general statement, this is the issue related to increasing the noise immunity of technical systems.

3. The aim and objectives of the study

The aim of this study is to ensure an increase in SNR at the correlation system output by using the adaptive method of the orthogonal Laguerre filtration of noise interference. Such an approach would make it possible to reduce the value of the SNR denominator.

To accomplish the aim, the following tasks have been set:
- to analyze the operation of a correlation system with the input orthogonal Laguerre filters under the action of white noise with the infinitely dividing distribution laws as a process with the uncorrelated values for analog and digital variants;
- to design a functional structural–logical circuit of the correlation system with the complex orthogonal Laguerre filters, which would give an opportunity to use them in practice;
- to conduct a computer simulation experiment to substantiate the reduction of the influence of the interference at the output of the correlation system when using complex orthogonal Laguerre filters.

4. Analysis of the operation of the correlation system with the input orthogonal Laguerre filters

The responses from the $n$-channel linear system of the Laguerre filters $\xi(t)$ under the action of stationary white noise $\xi(t)$ (Fig. 1) are uncorrelated along the semi-axis of their lateral time-dependent shift as a sequence of the uncorrelated linear random processes.

The theoretical justification of the proposed assertion is based on the following results obtained using [9, 10].

For a continuous case, the reciprocal correlation function of the Laguerre filter response at $n > m$ is determined as follows:

$$R_{nn}(\tau) = \mathbb{M}\{\xi_n(t)\xi_n(t+\tau)\} = \begin{cases} (-1)^{n-m} \alpha^2 \sigma^2 \int_{-\infty}^{\infty} \exp(-2\alpha(t-\tau))dt, & \tau \in (0,\infty), \\ 0, & \tau \in (-\infty,0). \end{cases}$$ (1)
Typical continuous actual noise interference can be divided into two types:

a) the low-frequency broadband with a forming filter

\[ \phi_\alpha(t) = \alpha \exp(-\alpha t) U(t), \]

where \( U(t) \) is the Heaviside function; \( \alpha \) is the parameter of the pulse transition function of a filter (for example, RC filter);

b) the high-frequency narrowband with a forming filter

\[ \phi_\beta(t) = e^{-\beta t} \sin 2\pi ft, \]

where \( \beta \) and \( f \) are the parameters of the filter’s pulse transition function (for example, RLC filter). A series of publications [4, 8] use the following titles of actual noise interference, respectively:

- a) RC noise;
- b) RLC noise.

Given the known statistics on the implementations of model (3), the use of methods for statistical processing of time series and the LRP theory, the statistical estimates of the process characteristics \( \xi(\omega, \tau) \) and functions \( \xi(\omega, t) \) are determined unequivocally. A statistical estimation of the autocorrelation function is also determined (4). To solve practical problems, it is necessary to additionally introduce the following limitations, which are often when studying the operation of correlation systems. A statistical estimate \( r_\iota(s) \) is obtained under the condition that two ergodic hypotheses hold for model (3), namely, the mathematical expectation and correlation function. The need for the implementation of ergodic hypotheses for a stationary LRP is connected to the algorithm for computing a statistical estimation \( r_\iota(s) \) is implemented not by the operator of the mathematical expectation of model (3) but the correlation operator of the time-averaged stationary LRP implementations.

The adaptive method of the orthogonal Laguerre filtration is proposed to ensure that the correlation system with the input Laguerre filters produces a decrease in the interference effect at the output.

The main provisions of a given method are based on the following:

- using the orthogonal Laguerre filtration of actual noise interference based on the complex digital Laguerre filters (complex Laguerre filters) with corresponding weight coefficients;
- determining the autocorrelation function of an input noise interference to form, over the predefined sequence, a reduction of the values of this function over the corresponding time shift sequence;
- determining the weight coefficients of the components of a complex Laguerre filter depending on the predefined sequence of reduction of the value of the mutual correlation function of noise interference at the output of the system;
- deriving the reduced values of the mutual correlation function of actual noise interference at the output of the correlation system;
- evaluating the effectiveness of the proposed method.

To implement this method, we have developed a structural-logic scheme.

5. Development of the scheme of the correlation system with complex orthogonal Laguerre filters

The functional structural-logical scheme of the correlation system with complex Laguerre filters is shown in Fig. 2.
The filter itself is composed of a sequence of the Laguerre filters, interconnected in parallel (Fig. 1); and in each of them, the responses are independently formed under the interference influence.

Using an adaptive method in practice implies the stage of training.

At the training stage, the autocorrelation function of the input noise component \( r_{s}(s_j) \) is determined, which is described by a stationary LRP in form (3). Laguerre filters under the effect of input noise interference, noise interference in the form from the system as a mutual correlation function of the input signals, taking into consideration the expressions (4) and (5), a response of Laguerre filters is determined as follows:

\[
\psi^{(r)}(t_j) = \sum_{i=1}^{n} \alpha_{i} \psi^{(1)}(t_j),
\]

\[
\psi^{(s)}(t_j) = \sum_{i=1}^{n} \beta_{i} \psi^{(2)}(t_j),
\]

(5)

where

\[
\{ \psi^{(r)}(t_j), \psi^{(s)}(t_j), k \in [-m], j \in [-n] \}
\]

is the system of orthonormal Laguerre functions of the discrete argument, \( \{ \alpha_{i}, \beta_{i}, k \in [-m], j \in [-n] \} \) – the valid numerical weight coefficients, in this case,

\[
\alpha_{i} \psi^{(r)}(t_j) \neq 0; \quad \beta_{i} \psi^{(s)}(t_j) \neq 0.
\]

We obtain, at the output of the correlation system, taking into consideration the expressions (4) and (3), a response from the system as a mutual correlation function of the input noise interference in the form

\[
r_{aw}(s_j) = \sigma_{r}^{2} \sum_{j=-\infty}^{\infty} \sum_{r=-\infty}^{m} \sum_{k=-\infty}^{n} \alpha_{i} \beta_{k} h_{i}^{(2)}(\tau) r_{s}(s_j - \tau),
\]

(6)

where a mutual correlation transformation of the complex Laguerre filters is determined as follows:

\[
h_{i}^{(2)}(\tau) = \sum_{j=-\infty}^{\infty} \sum_{\tau=-\infty}^{\infty} \psi^{(r)}(t_j) \psi^{(s)}(t_j + \tau),
\]

(7)

In general, expression (6) is the algorithm for the functioning of a correlation system with the input complex Laguerre filters under the effect of input noise interference, which is described by a stationary LRP in form (3).

In order to reduce the impact of noise interference, a sequence of values for the mutual correlation function \( r_{aw}(s_j) \) is taken to be 0 over the predefined set of time-shift points \( s_j \in [-K, K] \).

Next, it is necessary to determine the weight coefficients \( \alpha_{i} \) and \( \beta_{i} \) in expression (5), based on the condition \( K \leq m(n-1) \). To this end, it is necessary to solve the system of equations (6) relative to unknown \( \alpha_{i} \) and \( \beta_{i} \), by selecting \( m \) and \( p \) so that the number of equations equals the number of unknowns. Such a correlation system with the orthogonal filters makes it possible to ensure, for the input noise interference over the set of shifts \( s_j \in [-K, K] \), the reduced values \( r_{aw}(s_j) \).

To estimate the efficiency of the adaptive method for reducing the effect of noise interference, we shall use the expressions to compute the following numerical efficiency coefficients with the use of values for the mutual correlation function \( r_{aw}(s_j) \), over the interval of time shifts \( s_j \in [-P, P] \).

\[
A_{1} = 10 \cdot \lg \left( \frac{1}{2P} \sum_{j=-P}^{P} \left| r_{aw}(s_j) \right| \right),
\]

(8)

\[
A_{2} = 10 \cdot \lg \left( \frac{1}{2P} \sum_{j=-P}^{P} \left| r_{aw}(s_j) \right| \right),
\]

The use of expression (8) has been theoretically substantiated to a greater degree.

The correlation system with the input complex Laguerre filters is adaptive to:

- the type and value of the intensity of the input noise stationary interference, which is taken into consideration by its autocorrelation function;
- the frequency range of the examined interference, which is determined by the choice of the \( \lambda \) parameter for the complex Laguerre filters;
- the number of channels for two complex Laguerre filters, which enables obtaining \( 2m(n-1) \) values of a sequence of reducing the correlation function at the output of the system.

The implementation of such a functional structural-logical scheme makes it possible to practically use the complex orthogonal Laguerre filters in order to reduce the signal/noise ratio.

Fig. 2. The structural-logical scheme of a correlation system to implement an adaptive method.

![Fig. 2. The structural-logical scheme of a correlation system to implement an adaptive method.](image-url)
6. Results of computer simulation of the adaptive method of orthogonal Laguerre filtration of noise interference

The algorithm of the adaptive filtration has been implemented based on the complex Laguerre filters. The computer simulation experiment was conducted employing the Visual C++ programming environment.

To conduct the experiment, the following actions were performed:
1. One of the random process implementations has been received. A function for the normal distribution of pseudo-random values was used for this purpose.

Initial data:
Sample volume: N = 10,000.
Mathematical expectation \( M \xi = 0 \).
Variance \( D \xi = 1 \).

The chart of the obtained implementation \( r(i) \) is shown in Fig. 3.

![Fig. 3. The implementation of a discrete random process (first 1,000 values)](image)

The estimates of mathematical expectation and variances have been computed, \( M \xi = 0.0004 \) and \( D \xi = 0.9726 \), respectively.

The chart of the obtained implementation \( r(i) \) is shown in Fig. 3.

![Fig. 3. The implementation of a discrete random process (first 1,000 values)](image)

The estimation chart of the correlation function of one of the implementations of white noise \( R(s) \), where \( s \in (0,10) \), is shown in Fig. 4.

![Fig. 4. The estimation chart of the correlation function of one of the implementations of white noise \( R(s) \)](image)

The derived statistical characteristics of the white noise implementation confirm the expediency of using such a generator to obtain a sampling of the values with the Gaussian distribution. Further research is based on the received implementation.

2. Based on a sampling of the values with the Gaussian distribution law, the implementation of RLC-noise (Fig. 5) was formed to train the filter.

Initial data:
Implementation of the basic white noise.
Sample volume: \( N = 10,000 \).

Pulse transition function of the RLC filter:
\[ \phi_{RLC}(t) = e^{-\alpha t} \sin(2\pi ft), \quad i \in [0, N] \]

A forming filter parameter: \( \alpha = 0.1 \).
Frequency: \( f = 0.17 \).

![Fig. 5. The implementation of RLC noise (first 100 values)](image)

The estimation chart of the autocorrelation function of the implementation of RLC noise \( R_0(s) \), where \( s \in (0,50) \), is shown in Fig. 6.

![Fig. 6. The chart of the statistical estimation of the normalized autocorrelation function of the input noise interference at the stage of training](image)

The resulting chart shows that the autocorrelation function of the noise interference accepts rather large values, which affects the signal/noise ratio at the output of the system. In order to reduce the impact of interference, the experiment involved the following actions.

3. The estimates of the weight coefficients \( \beta \), were calculated based on formula (6).

Initial data:
Estimation of the values of the correlation function of RLC-noise \( \mathcal{R}_L(s) \).

The number of zeroing points \( = 15 \).
The complex Laguerre filter parameter \( \lambda = 0.1 \);
\( m = 1 \);
\( n = 16 \);
\( \alpha_1 = \alpha_2 = 1 \);
\( r \in [2, 16] \).

The value of the mutual correlation function at the stage training must be equated to \( r_{m}(s) = 0 \) over the range \( s \in (0,15) \).

We determined the matrix of coefficients at unknown \( \beta \) in the system of 15 linear equations (Fig. 7).

By using the obtained coefficients, the values of the coefficients \( \beta \) were determined (Fig. 8).
at points are not equal to 0 because it is...performing. To find a mutual correlation function, the following was performed. To experimentally compute the values of the same type, one can use the calculated coefficients of the complex orthogonal Laguerre filter. In the future, for the noise interference, the complex orthogonal filter fully eliminates the interference as

$$\mathcal{R}_{nn}(s) = 0$$

respectively.

Thus, the stage of training was performed for a given system performance. For the set

$$\beta = -5.89984 \text{ dB},$$

$$\beta = -5.90125 \text{ dB},$$

and for the set

$$\beta = -15.15,$$

$$\beta = -16.47063 \text{ dB},$$

$$\beta = -16.48631 \text{ dB}.$$
7. Discussion of results of using the adaptive method of orthogonal Laguerre filtration

The justification of the correlation system using the complex Laguerre filters has been given. To this end, the theoretical information on the analog and digital Laguerre filtration was provided. It should be noted that up to now the complex Laguerre filters have not been used in practice. Because of this, we have proposed an adaptive method of the orthogonal Laguerre filtration and outlined its main provisions. The use of the adaptive method of the orthogonal Laguerre filtration makes it possible to form the uncorrelated linear stationary random processes along the semi-axis of their mutual time shift, which describe a wide range of actual noise interference. In this, a given method differs significantly from previously known methods.

The special feature of applying the method of the adaptive orthogonal Laguerre filtration when using the digital variant of processing the investigated signals is that it makes it possible to avoid solving a series of hardware tasks related to the creation of the appropriate analog modules of the correlation system.

The advantages of the method are as follows:

1. The theoretical justification, based on the results, of the theory of orthogonal functions and random processes, including LRPs.
2. The solution of the applied tasks using:
   - known results of the theory and practice of processing stationary noise interference because the signals and interference create the non-stationary combinations and this largely restricts the range of research methods while the stationary noise interference is typical of the most applied problems;
   - the digital variant of the method as an algorithm for operating a correlation system and the implementation of the concept model–algorithm–software in a computer simulation;
   - the reduction of impact exerted by noise interference described by a stationary LRP for SNR assessment;
   - the uniqueness of the orthogonal Laguerre filtration of a stationary white noise process – to form a sequence of uncorrelated noise processes over a significant time interval of mutual shift;
   - the potential capabilities to undertake, based on an LRP model, noise interference research not only within the framework of the correlation theory but with the use of higher moments;
   - the pulse transition functions of linear forming filters.

Restrictions on the use of the adaptive method include:

1. The Laguerre filters are filters of lower frequencies, which narrows the frequency range of the examined noise interference.
2. In order to define SNR, additional research should be performed taking into consideration the transformation of signals in the links of a correlation system with the input Laguerre filters.
3. For the practical use of the method, it is necessary to conduct a training phase to the input noise interference.
4. In each case, it is necessary to assess the effectiveness of the method and decide on its application taking into consideration:
   - an infinite interval of the orthogonality of Laguerre polynomials replaced with a finite one;
   - changes in the values of the Laguerre discrete argument polynomials at their quantization;
   - unbiasedness and effectiveness of the obtained statistical estimates of the mutual correlation function $r_{mn}(s)$.

The subject areas of method application are statistical hydroacoustics, geophysics, and radiophysics. The method could be used for vibration diagnostics and non-destructive control of technical systems, as well as in noise-reduction, which is a relevant field in the theory and practice of modern information technologies of measurements.

A computerized modeling experiment was implemented to further the expediency of the adaptive method of orthogonal Laguerre filtration. As a result of an example of calculating the parameters of pulse transitional characteristics with the orthogonal Laguerre filters, it is possible to draw the following conclusion. If one uses a predefined interference, it is possible to calculate the system of orthonormal functions of the Laguerre discrete argument. The values of these functions make it possible to build a filter that enables determining the characteristics of the correlation conversion of a usable signal over a certain offset range.

Thus, based on the use of the adaptive method of the orthogonal Laguerre filtration, we can create correlation systems to reduce the influence of stationary noise interference.

7. Conclusions

1. The operation of a correlation system with the input orthogonal Laguerre filters under the action of the white noise process with the infinitely dividing distribution laws as a process with the uncorrelated values for analog and digital variants has been analyzed. At the known statistics of the model implementations, the use of methods for the statistical processing of time series, and the LRP theory, the statistical evaluations of the process characteristics and functions are determined unequivocally; the statistical evaluation of the autocorrelation function is also determined. The adaptive method of the orthogonal Laguerre filtration has been proposed to ensure that the correlation system with the input Laguerre filters produces, at the output, a decrease in the interference impact.

2. Based on the proposed adaptive method, the functional structural-logical scheme of a correlation system with the complex orthogonal Laguerre filters has been developed, in which the filter is composed of a sequence of the Laguerre filters, which are interconnected in parallel, each of them independently forms the feedback under the action of interference. The proposed structural-logical scheme would make it possible to use them in practice to reduce SNR at the output of the system.

3. A computer simulation experiment has been conducted to substantiate the reduction of an interference influence at the output of the correlation system when using complex orthogonal Laguerre filters. As a result of the experiment, for an actual noise interference belonging to the RLC class of noise, the values of performance coefficients were obtained, $-5$ dB and $-16$ dB, depending on the range of values. Such results indicate a significant reduction in the impact of interference over the selected ranges.
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