Neutrino tri-bi-maximal mixing from $\Delta(27)$

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Abstract. The observed neutrino mixing, having a near maximal atmospheric neutrino mixing angle and a large solar mixing angle, is close to tri-bi-maximal, putting leptonic mixing in contrast with the small mixing of the quark sector. We discuss a model in which $\Delta(27)$ (a subgroup of $SU(3)$) is the family symmetry, and tri-bi-maximal mixing directly follows from the vacuum structure enforced by the discrete symmetry. The model accounts for the observed quark and lepton masses and the CKM matrix, as well as being consistent with an underlying stage of Grand Unification.

Proceedings entry for SUSY 2006 $^1$, based on [1].

INTRODUCTION

The observed neutrino oscillation parameters are consistent with a tri-bi-maximal structure $^2$:

$$U_{PMNS} \propto \begin{pmatrix}
-\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\
\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\
\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}
\end{pmatrix}$$ (1)

This simple form of leptonic mixing contrasts with the CKM matrix. Family symmetries have been used to justify this structure, including some based on underlying $SU(3)$ family symmetry $^3$.

$\Delta(27)$ is the semi-direct product group $Z_3 \ltimes Z_3'$ $^4$. This discrete group is interesting as a family symmetry because it is the smallest subgroup of $SU(3)$ that has the useful feature of having distinct triplets and anti-triplets (3 dimensional irreducible representations).

| Field | $Z_3$ | $Z_3'$ |
|-------|-------|-------|
| $\phi_1$ | $\phi_1$ | $\phi_2$ |
| $\phi_2$ | $\alpha\phi_2$ | $\phi_3$ |
| $\phi_3$ | $(\alpha)^2\phi_3$ | $\phi_1$ |

The triplets $\phi_i$ transform as shown in TABLE $^1$. $\Delta(27)$ allows all the $SU(3)$ invariants (being its subgroup) plus some additional invariants. Unlike smaller subgroups like

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$^1$ http://susy06.physics.uci.edu/talks/3/varzielas.pdf
Δ(12) [4], it forbids invariants constructed solely from 2 triplets (e.g. $\phi \phi \equiv \phi_1 \phi_1 + \phi_2 \phi_2 + \phi_3 \phi_3$ is not invariant).

We can thus have left-handed fermions $\psi_i$ and their charge conjugates $\psi_i^c$ share the same transformation properties under the family symmetry, which allows straightforward embedding in GUTs (a desirable feature).

In fact, one can construct models having the fermions (and their conjugates) transforming e.g. as triplets and the flavons responsible for breaking down the family symmetry transforming as anti-triplets. The only allowed terms that are quadratic in the fermions (triplets) arise when flavons (anti-triplets) are included to make the appropriate family invariant contractions, and these become mass terms for the fermions when the flavons acquire non-vanishing vacuum expectation values (vevs).

THE MODEL

The model aims to reproduce fermion masses and mixings, with particular emphasis on reproducing neutrino tri-bi-maximal mixing. We now discuss what is required in order to obtain successful mass structures.

The model relies on the seesaw mechanism in order to obtain the effective neutrino masses - so the heavy Majorana neutrinos mass matrix plays a role in determining the effective neutrino mass matrix (influencing the leptonic mixing).

A particularly interesting structure for the Majorana mass matrix is one where there is strong hierarchy - Sequential Dominance (SD) scenarios (see [3] and references therein).

Within SD, one can readily construct Yukawa structures that are phenomenologically viable for all the fermions: the difference between the contrasting leptonic and quark mixing is caused by the seesaw mediated intervention of the strongly hierarchical Majorana masses, that can for example transform the otherwise hierarchical dominance of the 3rd family contribution characteristic of charged fermions, into a negligible effect for the effective neutrino mass matrix.

The aim is to obtain viable quark structure as in [5] and tri-bi-maximal mixing for the neutrinos after seesaw has taken place. The charged leptons will introduce small corrections to the neutrino mixing angles and yield a near tri-bi-maximal PMNS matrix.

The most relevant Yukawa terms in the superpotential are as follows:

\[
P_Y \sim \frac{1}{M^2} \bar{\phi}_i^j \psi_i \bar{\phi}_j \psi_j H 
+ \frac{1}{M^3} \bar{\phi}_i^{23} \psi_i \bar{\phi}_j^{23} \psi_j^{c} \text{HH}_{45} 
+ \frac{1}{M^2} \bar{\phi}_i^{23} \psi_i \bar{\phi}_1^{23} \psi_j^c H 
+ \frac{1}{M^2} \bar{\phi}_i^{123} \psi_i \bar{\phi}_2^{123} \psi_j^c H 
\]

$H$ stands for the usual SM or SUSY Higgs, and $H_{45}$ is an additional scalar field whose role is discussed in [1].
The flavon vevs that yield the desired masses are:

\[ \langle \bar{\phi}_3 \rangle = (0, 0, a) \quad (6) \]

\[ \langle \bar{\phi}_{23} \rangle = (0, -b, b) \quad (7) \]

\[ \langle \bar{\phi}_{123} \rangle = (c, c, c) \quad (8) \]

The combination of the \( P_Y \) terms with these \( \bar{\phi} \) vevs leads to similar textures to those in [5] for quarks, and a charged lepton mass matrix very similar to that of the down quarks (i.e. with small mixing).

It is important to discuss how the vevs of eqs. (6) to (8) arise. One way of aligning the vevs relies solely on soft terms. The key there is that the symmetry is discrete and thus breaks the continuum of vacuum states. A simple example of how that can arise is through soft quartic terms proportional to \( m^2 \bar{\phi}^i \bar{\phi}^i \bar{\phi}^j \bar{\phi}^j \bar{\phi}^k \bar{\phi}^k \). This type of term is not allowed by \( SU(3) \), but is allowed by \( \Delta(27) \). If \( \bar{\phi} \) acquires a non-zero vev, the minimisation of the quartic term determines the direction of the vev to be that of eq.(6) - when the coefficient of the quartic term is negative; or eq.(8) - when the coefficient of the quartic term is positive. Relative alignment as in eq.(7) comes from mixed terms involving e.g. \( \bar{\phi}_{23} \bar{\phi}_{123} \). The complete alignment is discussed in detail in [1].

The \( \Delta(27) \) family symmetry isn’t enough in order to ensure that only the desired \( P_Y \) terms are allowed. We introduce additional symmetries to further constrain terms that can spoil the mass structures.

### TABLE 2.

| Field | \( \Delta(27) \) | \( U(1) \) | \( R \) symmetry |
|-------|-----------------|-------------|-----------------|
| \( \psi \) | 3               | 0           | 1               |
| \( \psi^c \) | 3               | 0           | 1               |
| \( H \) | 1               | 0           | 0               |
| \( H_{45} \) | 1               | 2           | 0               |
| \( \bar{\phi}_3 \) | 3               | 0           | 0               |
| \( \bar{\phi}_{23} \) | 3               | -1          | 0               |
| \( \bar{\phi}_{123} \) | 3               | 1           | 0               |

The charge assignments in TABLE 2 give rise to the superpotential in eqs.(2) to (5), forbidding unwanted terms.

The symmetries may be extended to the sector giving rise to Majorana masses as described in [1]. They fulfill SD and combine with the specific structure of the Yukawa terms (namely eq.(4) and eq.(5)) to yield a tri-bi-maximally mixed effective neutrino mass matrix.

### CONCLUSION

After obtaining the tri-bi-maximal neutrino mixing, it’s necessary to take into account the charged lepton mixing in order to get the PMNS angles [6]. The charged lepton mass
matrix was also obtained from the model, and so we conclude that the model predicts the following leptonic mixing angles \[1\]:

\[
\sin^2 \theta_{12} = \frac{1}{3} \pm 0.052 \pm 0.052
\]

\[
\sin^2 \theta_{23} = \frac{1}{2} \pm 0.061 \pm 0.058
\]

\[
\sin^2 \theta_{13} = 0.0028
\]

In conclusion, the model is phenomenologically viable, and is consistent with underlying Grand Unification.

The model relies on the seesaw mechanism under a SD scenario, and also on misalignment of vevs (in this case through relatively simple soft terms allowed by the family symmetry). The tri-bi-maximal mixing is, in that sense, directly related to the discrete group \(\Delta(27)\).

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