Study on the One-Proton Halo Structure in $^{23}$Al *

FANG De-Qing$^1$, $^1$ MA Chun-wang$^{1,2}$, MA Yu-Gang$^1$, CAI Xiang-Zhou$^1$, CHEN Jin-Gen$^{1,2}$, CHEN Jin-hui$^{1,2}$, GUO Wei$^1$, TIAN Wen-Dong$^1$, WANG Kun$^{1,2}$, WEI Yi-Bin$^{1,2}$, YAN Ting-Zhi$^{1,2}$, ZHONG Chen$^1$, ZUO Jia-xi$^{1,2}$, and SHEN Wen-Qing$^1$

$^1$Shanghai Institute of Applied Physics, Chinese Academy of Sciences, P. O. Box 800-204, Shanghai 201800 and $^2$Graduate school of Chinese Academy of Sciences

(Dated: November 10, 2018)

The Glauber theory has been used to investigate the reaction cross section of proton-rich nucleus $^{23}$Al. A core plus a proton structure is assumed for $^{23}$Al. HO-type density distribution is used for the core while the density distribution for the valence proton is calculated by solving the eigenvalue problem of Woods-Saxon potential. The transparency function in an analytical expression is obtained adopting multi-Gaussian expansion for the density distribution. Coulomb correction and finite-range interaction are introduced. This modified Glauber model is apt for halo nuclei. A dominate $s$-wave is suggested for the last proton in $^{23}$Al from our analysis which is possible in the RMF calculation.

PACS numbers: 25.60.Dz, 24.10.-i

---

Interest in reaction cross section ($\sigma_R$) has grown over the last decades with the development of radioactive ion beams (RIBs). $\sigma_R$ for light exotic nuclei has been studied extensively both theoretically and experimentally. Up to now much effort has been devoted to the study of neutron halo structure, such as the discovery of neutron halo nuclei $^6^3$He, $^{11}$Li, $^{11,14}$Be $^3$ and the recent work on $^{19}$C $^2$ $^8$ and $^{20}$Ne. At the same time, some work has been performed on proton halo nuclei, e.g. $^8$B and $^{17}$Ne. Due to Coulomb effect, the formation of proton halo is more difficult and complicated compared to neutron halo structure. And discrepancies were found in proton halo nuclei, especially on $^8$B $^3$. Recently the $\sigma_R$ was measured for the proton halo candidate $^{22}$Al, whose separation energy of the last proton is very small (0.125 MeV) $^5$. Enhancement in $\sigma_R$ was observed for $^{23}$Al compared to its neighbors $^3$ $^{10}$. According to the shell model, the last proton in $^{23}$Al is in the $1d_{5/2}$ orbit. But the RMF calculation shows that it is possible to have the inversion of the $1s_{1/2}$ and $1d_{5/2}$ orbit in $^{23}$Al $^1$. Thus it is very interesting and important to know which orbit the last proton occupies.

Several methods are available to study the total reaction cross section, i.e. the multi-step scattering theory of Glauber $^{12}$, the transport model method of Ma et al. $^{13}$ and the semi-empirical formula of Kex et al. $^{14}$ and Shen et al. $^{15}$ etc. For the first one, the optical limit approximation of the Glauber approach is the most common used method $^{12}$. In the high energy domain, the Glauber model is based on the individual nucleon-nucleon collisions along the classical straight-line trajectory in the overlap volume of the colliding nuclei $^{12,16}$. This model has been extended to low energies by taking into account the Coulomb distortion of the eikonal trajectory $^{17}$. Recently, it has been found that the Glauber model always underestimates $\sigma_R$ by 10-50% at intermediate energies, if one assumes HO-type nucleon density distributions and determines the width parameter by reproducing the interaction cross section at relativistic energies $^{18}$. To improve the calculation of the Glauber model at low energies, the finite-range interaction was considered and its energy dependence was investigated by fitting the $\sigma_R$ of $^{12}$C + $^{12}$C from low to high energies $^{19}$. And also the few-body Glauber model was widely used to study nuclei with halo structure, which assumes core plus one-(two-)nucleon structure $^{20}$. However, all these calculations are done by solving a multi-dimensional numerical integration or Monte-Carlo simulation. To simplify the calculation, the Glauber model in an analytical form has been obtained if we use Gaussian function for density distributions $^{17}$. But this method can only be used to describe stable nuclei. In order to study nuclear reaction involving exotic nuclei, the separation energy dependent diffuseness was introduce into this method $^{21}$. Here we present a more general analytical form by expanding the density distributions of projectile and target using multi-Gaussian functions, which can be used to calculate $\sigma_R$ for both stable nuclei and nuclei with halo structure.

To begin with, we describe the formulation of $\sigma_R$ in the framework of Glauber model in the optical limit approximation. If we distinguish neutrons and protons in the projectile and target, the reaction cross section can be expressed as

$$\sigma_R = 2\pi \int_0^\infty |1 - T(b)| b db$$

where $T(b)$ is the transparency function at impact parameter $b$

$$T(b) = \exp\left(-\int dr \int f(r - r') \sum_{i=n,p} \right)$$

---

*Supported by the Major State Basic Research Development Program in China Under Contract No. G2000774004, the National Natural Science Foundation of China (NSFPC) under Grant No 10405032, 10328259 and 10135030 and the Phosphor program in Shanghai under contract No. 03QA14066.

$^\dagger$Corresponding author. Email address: fangdq@sinr.ac.cn
\[
\sigma_{ij} = \sum_{j=n,p} \sigma_{ij}^2 \rho_{P_1}^2(r) \rho_{T_1}^2(r') \mathrm{d}r' \quad (2)
\]

where \( f(r) \) is the finite-range function
\[
f(r) = \frac{1}{\gamma_0^2} \exp(-\frac{r^2}{\gamma_0^2}) \quad (3)
\]

with \( \gamma_0 \) being the finite-range parameter. \( \sigma_{ij} \) is the nucleon-nucleon collision cross section. \( \rho_{P_1}^2 \) is the neutron (proton) density distribution in the projectile (target) with the z axis integrated [17]. With the input density distribution of the projectile and target, the reaction cross section can be obtained after the multi-dimensional numerical integration.

If we assume Gaussian distributions for the density of projectile and target, which is suitable for light stable nuclei. Then we can obtain an analytical transparency function [17]. To extend this method for both stable nuclei and halo nuclei with a long tail in the density distribution, we express the density distribution by N Gaussian functions
\[
\rho_{ij}(r) = \sum_{k=1}^{N} \rho_{ij}^k(0) e^{-(r/a_{ij}^k)^2} \quad (i=P,T; j=n,p) \quad (4)
\]

where \( \rho_{ij}^k(0) \) and \( a_{ij}^k \) are the amplitude and width parameters of the Gaussian functions. Then the transparency function can be written in an analytical form similarly [17, 21]

\[
T(b) = \exp(-\pi \sum_{i=n,p} \sum_{j=n,p} \sum_{k=1}^{N} \sum_{k'=1}^{N'} \sigma_{ij} \rho_{P_1}^k(0) \rho_{T_1}^{k'}(0) \rho_{P_1}^k(0) \rho_{T_1}^{k'}(0) \frac{(a_{P_1}^k)^3 (a_{T_1}^{k'})^3}{(a_{P_1}^k)^2 + (a_{P_1}^{k'})^2 + \gamma_0^2} \times \exp[-(\frac{b'}{a_{P_1}^k})^2 + (\frac{b'}{a_{P_1}^{k'}})^2 + \gamma_0^2]) \quad (5)
\]

where \( b' \) is the impact parameter after considering Coulomb correction
\[
b'^2 = \frac{b^2}{1 - V_c/E_{\text{CM}}} = \frac{b^2}{1 - 1.44 Z_P Z_T/(R_{\text{int}} E_{\text{CM}})} \quad (6)
\]

here \( Z_P (Z_T) \) refers to the charge number of the projectile (target), \( R_{\text{int}} \) is the interaction radius and \( E_{\text{CM}} \) is the incident energy in the center of mass frame [17].

To investigate the \( \sigma_R \) of \(^{23}\text{Al}\), we assume a core \(^{22}\text{Mg}\) plus one proton structure since the weak bind between the last proton and the core. HO-type distribution is used for the core density. The density distribution for the valence proton was calculated by solving the eigenvalue problem of Woods-Saxon potential [22]
\[
\frac{d^2R(r)}{dr^2} + \frac{2n^2}{r^2}[E - U(r) - \frac{l(l+1)h^2}{2m}]R(r) = 0 \quad (7)
\]

\[
U(r) = -V_0 f(r) + V_{ins}(1 \cdot s) r_0^3 \frac{d}{dr} f(r) + V_{\text{Coul}}
\]

where \( f(r) = [1 + \exp(-r/R)]^{-1} \) with \( R = r_0 A_i^{1/3} \) \( (V_{ins} = 17\text{MeV}) \). \( V_0 \) is the depth of potential, \( V_{\text{Coul}} \) is the Coulomb potential. In the calculation the diffuseness \( (a) \) and radii parameter \( (r_0) \) were chosen to be 0.67 fm and 1.27 fm [23].

First we assume HO-type distribution for the \(^{12}\text{C} \) target and determine the HO width parameter by reproducing the interaction cross section of \(^{12}\text{C} + ^{12}\text{C} \) at relativistic energy. Then the \( \sigma_R \) for \(^{12}\text{C} + ^{12}\text{C} \) at different energies are calculated and compared them with experimental data. In the calculation, the finite-range parameter was taken from Ref. [13].

\[
\begin{align*}
\gamma_0^2 &= 2\beta^2 \\
\beta &= 0.996 \exp\left(-\frac{E}{1000\text{MeV}}\right) + 0.089
\end{align*}
\quad (8)
\]

where \( E \) is the incident energy in the laboratory frame in MeV. With the consideration of the energy dependent finite-range interaction, the underestimation at low energies in the zero-range Glauber model was removed as described in Ref. [10].

**TABLE I:** The reaction cross sections for \(^{22}\text{Mg} \) and \(^{23}\text{Al} \) with carbon target.

| Nuclei energy (AMeV) | \( \sigma_R \) (mb) reference |
|----------------------|-----------------------------|
| \(^{22}\text{Mg} \)    | 33.4 1531±125 [19]          |
| \(^{23}\text{Al} \)    | 35.9 1892±145 [9]           |

Then we assume HO distribution for the \(^{22}\text{Mg} \) core, and adjust the HO width parameter to fit the measured \( \sigma_R \) at 33.4A MeV as given in Table. [4]

![FIG. 1: Density distributions of \(^{22}\text{Mg} \) and \(^{23}\text{Al} \). The dotted line is the density distribution of \(^{22}\text{Mg} \). The solid and dashed line are the density distribution of \(^{23}\text{Al} \) in s- and d-wave, respectively. For comparison, the density distribution of \(^{23}\text{Al} \) in the s-wave calculated by multi-Gaussian expansion (Eq.(4)) is also plotted as the dotted-dashed line. It is almost overlapped with the original density distribution.](image-url)
Since spin parity assignment for the ground state of $^{23}$Al is unknown experimentally, we took the assumed value (1/2+) from Ref. [10]. The mixing configuration for the last proton was considered as following

$$
\phi \propto \sqrt{\mathcal{F}[^{22}\text{Mg}(0^+) \otimes \nu_{1/2}, J=1/2^+]} 
+ \sqrt{\mathcal{F}[^{22}\text{Mg}(2^+) \otimes \nu_{5/2}, J=1/2^+]}
$$

(9)

where $f(0 \leq f \leq 1)$ denotes the relative s-wave spectroscopic factor. $\nu_{1/2}(r)$ and $\nu_{5/2}$ refer to the wavefunction of s- and d-wave. The density distribution of the valence proton can be calculated by $\phi^2(r)$. The wavefunction for the valence proton in $2s_{1/2}$ or $1d_{5/2}$ orbit was calculated by adjusted the depth of Woods-Saxon potential to reproduce the separation energy of the last proton in $^{23}$Al. The density distribution of $^{23}$Al was obtained by adding that of the core and the valence proton. For simplicity, we used the same density distributions for both the ground($0^+$) and excited($2^+$) states of $^{22}$Mg. Comparison of $^{23}$Al’s density distributions in s- and d-wave with the core was shown in Fig. 1. From the figure, the density distributions of $^{23}$Al are larger than that of $^{22}$Mg. The tail of s-wave is much longer than d-wave. In the expansion, we used 12 Gaussian functions for the projectile and 4 Gaussian functions for the target density distribution. In the fitting, $\chi^2$ is less than $10^{-9}$. As we can see in Fig. 1, good agreement between the original and fitted density was obtained.

![FIG. 2: Transparency function (T(b)) for $^{23}$Al in the s-wave on $^{12}$C target at 35.9A MeV. The dotted line is calculated by the Glauber model in the optical limit approximation. The dashed line is the results with finite-range interaction. And the solid line is calculation by the Glauber model with both finite-range interaction and Coulomb correction.](image)

With the Gaussian parameters, we can directly calculate the transparency function (T(b)) using Eq.(5). The effect of finite-range and Coulomb correction on T(b) was compared in Fig. 2. As we can see, T(b) is saturated to unity as the impact parameter increase. Compared with the Glauber model in the optical limit approximation, the finite-range effect makes T(b) decrease slower as the decrease of the impact parameter which leads to larger reaction cross section. But the Coulomb correction makes T(b) decrease faster as the decrease of the impact parameter.

Dependence of the reaction cross section on the s-wave spectroscopic factor was calculated and shown in Fig. 3. The $\sigma_R$ of $^{23}$Al was also plotted [4]. It is found that within the uncertainties our analysis suggests dominate s-wave for the last proton in $^{23}$Al. The lower limit of the s-wave spectroscopic factor is around 0.65. But it should be pointed out that the error of $\sigma_R$ for $^{23}$Al is comparable with the difference in $\sigma_R$ between s- and d-wave. And even the s-wave calculation still underestimates the experimental $\sigma_R$ data. Zhao et al. suggested an enlarged core in $^{23}$Al which lead to larger reaction cross section [4]. That maybe one of the reasons for the underestimation in our calculations. In order to obtain more confirmative conclusion, much more precise measurement and theoretical study of $\sigma_R$ for $^{23}$Al is required. And the measurement of momentum distribution for the residue after one-proton remove is expected which is much easy to distinguish s- and d-wave.

For comparison, we also assume HO core plus Yukawa-square tail for the density distribution of $^{23}$Al and determine the parameters by reproducing the experimental $\sigma_R$. The Yukawa-square tail is known to be a good approximation to the shape of a single-particle density at the outer region of a core with centrifugal and Coulomb barriers [4]. The assumed density is written as

$$
\rho(r) = \begin{cases} 
\text{HO-type} & (r < r_c) \\
Y \exp(-\lambda r)/r^2 & (r \geq r_c)
\end{cases}
$$

(10)

![FIG. 3: Dependence of the reaction cross section on the s-wave spectroscopic factor. The shaded area comes from the experimental error of $^{22}$Mg and $^{23}$Al’s $\sigma_R$ data. The dot is the $\sigma_R$ of $^{23}$Al.](image)
FIG. 4: Density distributions of $^{23}\text{Al}$. The solid and dashed line are the density distribution of $^{23}\text{Al}$ in $s$- and $d$-wave, respectively. The thick line is the extracted density distribution assuming HO plus Yukawa distribution. The shaded area is the error from the experimental $\sigma_R$ data.

where $r_c$ is the crossing point of these two functions, the factor $Y$ is to keep the equality of the two distributions at $r_c$. The width parameter of the HO-type core is fitted to the $\sigma_R$ of $^{22}\text{Mg}$. Then the parameters $\lambda$ and $r_c$ are fitted to the $\sigma_R$ of $^{23}\text{Al}$ given in Table I with an additional normalization condition $\int \rho(r)d^3r = Z$ and $Z$ is the charge number of $^{23}\text{Al}$.

In Fig.4 the extracted density distribution was compared with the calculated density for $^{23}\text{Al}$ in $s$- and $d$-wave. The extracted density is closer to $s$-wave than $d$-wave. It demonstrates a long tail in the density distribution and supports the conclusion that the $s$-wave is dominate for the last proton in $^{23}\text{Al}$.

In summary, the Glauber theory has been used to investigate the reaction cross section. Adopting multi-Gaussian expansion for the density distribution of projectile and target, an analytical form for the transparency function was deduced. The energy dependent finite-range interaction and Coulomb correction were considered. It has been pointed out that this modified Glauber model is suitable for studying the $\sigma_R$ of both stable nuclei and nuclei with exotic structure. We used this model to study the reaction cross section of proton-rich nucleus $^{23}\text{Al}$. A core plus proton structure was assumed for $^{23}\text{Al}$. HO-type density distribution was used for the core while the density distribution for the valence proton was calculated by solving the eigenvalue problem of Woods-Saxon potential. The $s$- and $d$-wave mixing configuration was studied. Density distribution was extracted for $^{23}\text{Al}$ assuming HO plus Yukawa-square tail. A dominate $s$-wave was suggested in our analysis which indicates a proton halo structure in $^{23}\text{Al}$.

[1] Tanihata I et al. 1985 Phys. Rev. Lett. 55 2676; Tanihata I et al. 1985 Phys. Lett. B 160 380; Tanihata I et al. 1992 Phys. Lett. B 287 307
[2] Bazin D et al. 1995 Phys. Rev. Lett. 74 3569; Bazin D et al. 1998 Phys. Rev. C 57 2156
[3] Ozawa A et al. 2002 Eur. Phys. J. A 13 163
[4] Warner R E et al. 1995 Phys. Rev. C 52 R1166
[5] Negoita F, Borcea C and Carstoiu F 1996 Phys. Rev. C 54 1787
[6] Fukuda M et al. 1999 Nucl. Phys. A 656 209
[7] Obut I M et al. 1996 Nucl. Phys. A 609 74
[8] Audi G and Wapstra A H 1993 Nucl. Phys. A 565 66
[9] Cai X Z et al. 2002 Phys. Rev. C 65 024610
[10] Zhang H Y et al. 2002 Nucl. Phys. A 707 303
[11] Chen J G et al. 2004 Chin. Phys. Lett. 21 2140
[12] Glauber R J 1959 Lectures in theoretical physics. Vol. 1 (Interscience, New York, 1959)
[13] Ma Y G et al. 1993 Phys. Lett. B 302 386; Ma Y G et al. 1993 Phys. Rev. C 49 850.
[14] Kox S et al. 1987 Phys. Rev. C 35 1678.
[15] Shen W Q et al. 1989 Nucl. Phys. A 491 130.
[16] Karol P J 1975 Phys. Rev. C 11 1203
[17] Charagi S K and Gupta S K 1990 Phys. Rev. C 41 1610
[18] Ozawa A et al. 1996 Nucl. Phys. A 608 63
[19] Zheng T et al. 2002 Nucl. Phys. A 709 103
[20] Ogawa Y, Suzuki Y and Yabana K 1994 Nucl. Phys. A 571 784
[21] Feng J et al. 1993 Phys. Lett. B 305 9
[22] Speth J et al. 1977 Phys. Rep. 33 127
[23] Yamaguchi T et al. 2003 Nucl. Phys. A 724 3
[24] Zhao Y L, Ma Z Y, Chen B Q and Shen W Q 2003 Chin. Phys. Lett. 20 53