Vertical vibration of elevator compensating sheave due to brake activation of traction machine

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Abstract. This paper shows an elevator dynamic model that calculates the compensating sheave motion during a brake activation of the traction machine. A simplified mathematical formulation is derived to evaluate the vertical static displacement of the compensating sheave. The vertical vibration induced by the brake of the traction machine is also evaluated and it is concluded that the maximum vibration occurs when the vertical vibration by the car stop timing synchronizes the phase of the vibration by the brake activation. The maximum vibration is also evaluated by a mathematical formulation and it shows that the vertical vibration is proportional to the building height. The derived equations contribute to the elevator’s optimal design.

1. Introduction
Most elevators applied to tall buildings include compensating ropes to satisfy the balanced rope tension between the car and the counter weight. The compensating ropes receive tension by the compensating sheave, which is installed at the bottom space of the elevator shaft [1, 2]. The compensating sheave is only suspended by the compensating ropes, therefore, the sheave can move vertically during the car traveling. It is important to evaluate the vertical displacement of the compensating sheave, because the displacement is one of the key factors to determine the pit depth of the elevator shaft.

This paper shows the static displacement and the vertical vibration of the compensating sheave. Firstly, an elevator system model is proposed to evaluate the vertical motion of each component. The derived simulation model indicates that the vertical static displacement depends on the car position and the car loading condition. Based on the simulation results, we can produce a simplified mathematical formulation to evaluate the static displacement.

Secondary, the vertical vibration induced by the brake of the traction machine is evaluated numerically. As the simulation results correspond with the experimental ones, we can investigate the worst condition of the vibration by changing the elevator’s system parameters. It is concluded that the maximum vibration occurs when the vertical vibration induced by the car stop timing synchronizes the phase of the vibration induced by the brake activation. By the result, we can also introduce a mathematical formulation to evaluate the maximum vibration.

Finally, we evaluate the relation between the vertical vibration and the building height. By the derived equation, we can conclude that the amplitude of the vertical vibration induced by the brake is proportional to the building height.
In the end, the derived simulation model and mathematical formulations contribute to the elevator’s optimal design, especially for the pit depth evaluation.

2. Multi-body dynamic model
Figure 1 shows the multi-body dynamic model of our elevator system. Since the suspension and compensating ropes are treated as continuous bodies, they are modeled by alternating the intensive mass and spring [3, 4].

The multi-body dynamic model in figure 1 is represented by the following equation of motion:

\[ M \ddot{x} + C \dot{x} + Kx = F, \]

where \( M, C, K \) and \( F \) are the system’s inertia matrix, damping matrix, stiffness matrix and generalized force vector respectively. \( x \) is the vector of state variables, which includes the translational and rotational values. Equation (1) includes models of rope slip on the sheaves, tension loss, and the transient brake force of the traction machine [5].

When the brake is activated at the traction machine due to abnormal behaviors in the elevator system, the car and the counter-weight receive a large vertical vibration at the same time. The vibration also induces the vertical motion against the compensating sheave (figure 2).

3. Simulation results
3.1. Static displacement of the compensating sheave
The vertical displacement of the compensating sheave \( x_s \) in figure 1 is evaluated in each car position. Figure 3 shows the compensating sheave’s vertical displacement when the car moves downward from the top floor to the bottom floor. During the acceleration and deceleration time, the compensating sheave moves downward. The simulation result of equation (1) shows the same time response against the experimental one.

When the car stops at the top floor, the stiffness of the suspension rope at the car side is high enough. Therefore the car loading condition doesn’t affect the compensating sheave’s vertical motion at the top floor. The vertical position of the compensating sheave is determined by the stretch of the suspension rope at the counter-weight side. On the other hand, when the car stops at the bottom floor without any load, the sheave moves upward compared to the top floor’s stopping condition as shown in figure 3 and figure 4(real line). This is because the stretch
of the suspension rope at the car side is shorter due to lighter weight of the car against the counter-weight as shown in figure 5. If the car has a rated load, the sheave moves downward (dashed line in figure 4) due to heavier weight of the car against the counter-weight.

![Figure 3. Sheave’s vertical displacement $x_s$ during car downward motion (no loading)](image1)

**Figure 3.** Sheave’s vertical displacement $x_s$ during car downward motion (no loading) [3]

![Figure 4. Displacement difference between no loading and rated loading](image2)

**Figure 4.** Displacement difference between no loading and rated loading

![Figure 5. Relation between sheave’s vertical displacement and car position](image3)

**Figure 5.** Relation between sheave’s vertical displacement and car position

The equivalent rope stiffness $K$ in figure 5 is expressed by the serial spring connection between the suspension rope and the shackle spring at the rope end.

$$K = n \times \frac{K_s K_r}{K_s + K_r}, \quad (2)$$

where $K_s$ and $K_r$ are the stiffness of the suspension rope and the shackle spring respectively. $n$ is the number of rope in the elevator system.

By using the equivalent rope stiffness $K$, the relative displacement of the compensating sheave $x_s$ is represented by the following simple equation:

$$x_s = \frac{x_d}{2}, \quad x_d = \frac{M_w G}{(M_c + M_L) G} = \frac{(M_w - M_c - M_L) \times G}{K}, \quad (3)$$
where \( G, M_w, M_c \), and \( M_L \) are the gravitational acceleration, the counter-weight mass, the car mass and the car loading mass respectively. \( x_d \) is the relative stretch of the suspension rope between the car side and the counter-weight side. As the compensating sheave is the movable pulley, the vertical displacement \( x_s \) is half of \( x_d \).

### 3.2 Vertical vibration during the brake activation

When the brake is activated during the car running, the compensating sheave receives a large vibration as shown in figure 6. The simulation result of equation (1) shows the same time response against the experimental one. The vibration behavior can be described by figure 7.

![Figure 6. Sheave displacement during brake activation](image)

![Figure 7. Relationship between sheave vibration and brake timing](image)

The car and the counter-weight receive a step force by the brake. The force induces a vertical vibration and the vibration continues while the car slows down its speed. When the car stops, the compensating sheave receives another step force and then it induces another vibration.

If the first vibration and the second vibration are synchronized at the car stopping time, the second vibration is magnified and a larger vibration occurs after the car stopping time as shown in figure 7.

In this section, we evaluate the above large vibration at the car stopping time. At first, we introduce a generalized mass \( M \), which represents the car weight or the counter-weight one.

\[
M = \begin{cases} 
M_c + M_s/2 + M_r + M_L & \text{(bottom floor condition)} \\
M_w + M_s/2 + M_r & \text{(top floor condition)} 
\end{cases}, 
\tag{4}
\]

where \( M_c \) and \( M_r \) are the weight of compensating sheave and suspension rope respectively. The rope weight \( M_r \) is expressed as the following equivalent mass.

\[
M_r = n \times \rho \times L/3, 
\tag{5}
\]

where \( \rho \) and \( L \) are the linear density of the suspension rope and the rope length respectively.

As the generalized mass \( M \) is suspended by the rope, the mass causes vertical vibration by the brake force of the traction machine. In the actual system, the vertical vibration decreases gradually due to the rope damping effect. However we ignore the damping effect to evaluate the worst condition.

The car velocity pattern also affects the vibration. In figure 7, the brake is activated under the constant speed condition. On the other hand, figure 8 shows the vibration behavior under the brake occurrence at the car acceleration time.
When the car is accelerating at $\beta G$, the brake is activated and the car is decelerated to $-\alpha G$ by the brake force.

Due to the time response of the acceleration in figure 8, the vibration of $M$ induced by the brake activation is derived by a simple 1-DOF(degree of freedom) vibration model as shown in figure 5.

$$x_i(t) = \frac{M \times G}{K} \{-\beta + (\alpha + \beta)(1 - \cos \omega t)\}, \quad (6)$$

where $\omega$ is the natural frequency of the generalized mass $M$ suspended by the equivalent rope stiffness $K$.

$$\omega = \sqrt{\frac{K}{M}}. \quad (7)$$

Equation (6) is derived by the following initial condition.

$$x_i(0) = -\frac{M \times \beta G}{K}, \quad x_i(0) = 0. \quad (8)$$

By the step function of the brake force $(\alpha+\beta)G$, equation (6) shows the sinusoidal wave response. In figure 8, $D_a$ and $D_b$ correspond to the following equation.

$$D_a = \frac{\alpha MG}{K}, \quad D_b = \frac{\beta MG}{K}. \quad (9)$$

From the second term of equation (6), the vibration amplitude $C_1$ during deceleration due to the traction brake is given by

$$C_1 = D_a + D_b = \frac{(\alpha + \beta) MG}{K}. \quad (10)$$

The residual vibration after the car stopping is also described by a sinusoidal wave of the same frequency in equation (7). The following equation is derived by a simple 1-DOF vibration model in the same manner as equation (6).

$$z(t) = A \cos \omega(t - t_i) + B \sin \omega(t - t_i), \quad (11)$$
where $t_i$ is the time of car stopping and the arbitrary values $A, B$ are determined by the following initial condition.

\[ z(t_i) = x_i(t_i), \quad \dot{z}(t_i) = \dot{x}_i(t_i). \] (12)

Then equation (11) is expressed by

\[ z(t) = x_i(t_i) \cos \omega(t - t_i) + \frac{\dot{x}_i(t_i)}{\omega} \sin \omega(t - t_i) = \sqrt{x_i^2(t_i) + \left(\frac{\dot{x}_i(t_i)}{\omega}\right)^2} \sin(\omega t + \phi), \] (13)

where $\phi$ is the constant of phase shift.

The maximum amplitude of equation (13) is derived by the following condition.

\[ \cos \omega t_i = -1 \quad \rightarrow \quad t_i = \frac{\pi(2N - 1)}{\omega}, \quad (N = 1, 2, \cdots). \] (14)

In this case, the maximum amplitude is given by

\[ Z = \max\{z(t)\} = \frac{M \times G}{K} (2\alpha + \beta). \] (15)

In figure 8, the maximum amplitude $C_2$ as the residual vibration is given by the following equation.

\[ C_2 = D_a + C_1 = 2D_a + D_b = \frac{(2\alpha + \beta)MG}{K} \equiv Z. \] (16)

The compensating sheave’s vibration is half of the generalized mass $M$, because it moves by the mechanism of movable pulley. Therefore the maximum vibration of the compensating sheave is expressed as follows.

\[ x_s = \frac{Z}{2} = \frac{M \times G}{2K} (2\alpha + \beta). \] (17)

### 3.3. Influence of building height

Equation (17) is the function of the equivalent rope stiffness $K$. If the rope is long enough, equation (2) is approximated by

\[ K \approx n \times K_r = n \times \frac{E_r A_r}{L}, \] (18)

where $E_r$ and $A_r$ are the rope’s Young modulus and the cross section area respectively. The rope length $L$ can be approximated by the building height.

Substituting equation (18) into equation (17), it is assumed that the maximum vibration is proportional to the rope length $L$, or the building height.

\[ x_s \approx \frac{MG}{2nE_r A_r} (2\alpha + \beta)L. \] (19)

To evaluate the above assumption, the maximum vibration is calculated in each building height. Figure 9 shows the several simulation results of the maximum vibration in the actual elevator specifications.

The result of equation (17) which is marked by a circle matches the simulation result marked by a square. As both results are proportional to the building height, the above assumption is acceptable.
4. Conclusions
We evaluated the vertical displacement and vibration of the compensating sheave. Our formula suggests that the static displacement depends on the car position and the car loading condition. We also get another conclusion that the maximum vertical vibration induced by the brake of the traction machine is proportional to the building height. As the maximum vertical vibration and the static displacement are expressed by simple formulas, we can use them for the elevator’s optimal design.

References
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