Baryon Fluctuations and the QCD Phase Transition

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The dynamic separation into phases of high and low baryon density in a heavy ion collision can enhance fluctuations of the net rapidity density of baryons compared to model expectations. We demonstrate that event-by-event proton and antiproton measurements can be used to observe this phenomenon. We then perform real-time lattice simulations to show how these fluctuations arise and how they can survive through freeze out.

If the QCD phase transition is first order, matter at the appropriate temperatures and densities can form a mixed phase consisting of plasma droplets in equilibrium with a surrounding hadronic fluid. If formed in ion collisions, this mixed phase can produce large event-by-event fluctuations as the system hadronizes \( \Box \). In particular, extraordinary baryon number fluctuations \( \Box \) can accompany the first order transition expected at high baryon density \( \Box \).

In this paper we explore the dynamics of phase separation in nuclear collisions. The aims of this paper are twofold. First, we study the role of baryon number fluctuations as a probe of the order of the QCD transition. We focus on the high baryon density regime, where theory \( \Box \) and lattice simulations \( \Box \) suggests that the QCD phase transition is first order in a strict thermodynamic sense with baryon density as an order parameter. Our work may also apply to RHIC collisions, if the low baryon density systems produced at the highest energies approximate a first order transition \( \Box \). Second, we generalize techniques from condensed matter physics \( \Box \) to confront phase separation in the highly-nonequilibrium context of nuclear collisions. Our framework can be used to systematically address other probes as experimental information and theoretical understanding evolve.

To begin, we describe the character of mixed-phase baryon fluctuations and show how they can be measured. Measurement is not completely straightforward as, e.g., neutrons are not easily observed on an event-by-event basis. We then formulate a dissipative-hydrodynamic model of phase separation and perform numerical simulations for that model.

QCD with two massless flavors can exhibit a first order transition whose coexistence curve culminates in a tricritical point at temperature \( T_c \) and baryon chemical potential \( \mu_c \). For \( T > T_c \) and \( \mu < \mu_c \), a second order phase transition breaks/restores chiral symmetry. If the quark masses are sufficiently large, the second order transition is replaced by a smooth transformation (since chiral symmetry is explicitly broken). The first order line remains, however, with the tricritical point replaced by a critical point in the same universality class as a liquid–gas transition.

At RHIC, baryon density may also serve as an approximate order parameter for the nearly first order transition at small net baryon density. Lattice simulations \( \Box \) and general arguments \( \Box \) show that the baryon susceptibility \( \chi \) at \( \mu = 0 \) can increase suddenly as temperature is increased near \( T_h \sim 160 \text{ MeV} \), where the chiral order parameter and, e.g., the energy density change sharply. Jumps in the susceptibility commonly accompany first order transitions. For a liquid-gas transition, \( \chi = \partial \rho/\partial \mu \) is proportional to the compressibility: steam is much more compressible than water.

Large fluctuations in baryon number occur during phase separation in a first order transition. Figure 1b shows the phase diagram in the \( T - \rho \) plane \( \Box \) where \( \rho \) is the baryon density. A uniform system quenched into the outer parabolic region will separate into droplets at the high baryon density \( \rho_h \) surrounded by matter at density \( \rho_l \). The net baryon number \( N_B \) in a sub-volume of the system varies depending on the number of droplets in the sub-volume. The variance of the baryon number \( V_B = \langle N_B^2 \rangle - \langle N_B \rangle^2 \) can exceed the equilibrium expectation by an amount

\[
\Delta V_B \approx f(1-f)(\Delta N_B)^2,
\]

where \( f \) is the fraction of the high density phase in the sub-volume \( V \) and \( \Delta N_B = (\rho_h - \rho_l)V \). In contrast, an equilibrium system follows Poisson statistics, so that \( V_B = V + \langle V \rangle = \langle N + \overline{N} \rangle \), where \( N, V \) and \( \overline{N}, \overline{V} \) are the numbers and variances of baryons and antibaryons and \( N_B = N - \overline{N} \).

Experimenters can search for a “super-poissonian” variance such as \( \Box \) by measuring

\[
\Omega_p = \frac{V_{p-\overline{p}} - (N_p + N_{\overline{p}})}{(N_p + N_{\overline{p}})^2},
\]

where \( N_p \) and \( N_{\overline{p}} \) are the numbers of protons and antiprotons in a rapidity interval and \( V_{p-\overline{p}} \) is the variance of the net proton number \( N_p - N_{\overline{p}} \). This quantity vanishes in equilibrium and is related to the more familiar scaled variance \( \omega_p = (N_p + N_{\overline{p}})(1 + \Omega_p) \). Most importantly, \( \Omega_p \) is ideal for our application because of the property

\[
\Omega_p = \Omega_B \equiv \frac{V_B - \langle N + \overline{N} \rangle}{\langle N + \overline{N} \rangle^2}
\]
where \( N \) and \( \bar{N} \) are the numbers of baryons and antibaryons – including unseen neutrons and antineutrons (the proof follows). The conditions for which (3) holds are met by a range of thermal and Glauber models that respect isospin symmetry. Isospin fluctuations can alter \( q \) near the tricritical point or in the presence of a disorder-oriented chiral condensate, but those effects will be evident from pion measurements.

We demonstrate (3) by writing the joint probability for \( N_p \) and \( N_{\bar{p}} \) as \( \sum N \sum_{\bar{N}} p(N_p|N)p(N_{\bar{p}}|\bar{N})P(N, \bar{N}) \). The distribution \( P(N, \bar{N}) \), which determines \( \Omega_B \), is modified by phase separation; we make no assumptions about its form. We assume that the conditional probability \( p(N_p|N) \) for measuring \( N_p \) given \( N \) baryons is binomial, with \( q \) the chance that an individual baryon is a proton (see (4) for notation). We further take \( p(N_{\bar{p}}|\bar{N}) \) for antiprotons to be binomial with the same \( q \). These assumptions hold for most thermal and multiplescattering models. The average of the joint distribution is \( \langle N_p + N_{\bar{p}} \rangle = \sum N \sum_{\bar{N}} p(N_p|N)N_p \) and \( \bar{N} = \sum N \sum_{\bar{N}} p(N_{\bar{p}}|\bar{N})N_{\bar{p}} \) yield \( \langle N_p + N_{\bar{p}} \rangle = q(N + \bar{N}) \). The quantity \( \langle N + \bar{N} \rangle \) depends only on \( P(N, \bar{N}) \). Similarly, we find \( \langle (N_p - N_{\bar{p}})^2 \rangle = q^2(\langle N - \bar{N} \rangle^2) + q(1 - q)(N + \bar{N}) \). We combine these moments to obtain (4).

The antiproton contribution to (4) is large only at RHIC, where \( N_{\bar{p}}/N_p \sim 0.6 \) at \( \sqrt{s} = 130 \text{ A-GeV} \). At the top SPS energy, we estimate \( \bar{p} \) contributions to (4) to be at the few percent level in \( \text{Au+Au} \) at \( \sqrt{s} = 17.5 \text{ A-GeV} \), since \( N_{\bar{p}}/N_p \sim 6% \). The highest baryon density – and the greatest potential for observing a first order transition – is perhaps at lower energies.

We remark that Jeon and Koch and Asakawa et al. have proposed that hadronization may change the character of charge and baryon number fluctuations even in the absence of a phase transition (13). This effect is essentially poissonian, however, so it is not clear that it would cause \( \Omega_p \) to differ from zero, the equilibrium value, or that it could be tested without measuring neutrons. The effect on charge fluctuations is much more dramatic (13).

We now turn to describe the process of phase separation. To describe the state of the mixed phase, we follow the standard condensed matter practice (6) and write a Ginzburg-Landau free energy \( f = \kappa(\nabla \rho)^2/2 + f_0 \), where

\[
f_0 = -m^2(\rho - \rho_c)^2/2 + \lambda(\rho - \rho_c)^4/4 \tag{4}
\]
describes the excursions of the baryon density \( \rho \) from its equilibrium value in the uniform matter. For \( m^2 \sim T_c - T \) we find the correct liquid-gas critical exponents. The values \( \rho_h \) and \( \rho_q \) in fig. 1 correspond to the equilibrium densities at \( T < T_c \): \( \rho_h = \rho_c - \Delta \rho \) and \( \rho_q = \rho_c + \Delta \rho \), where \( \Delta \rho = \sqrt{m^2/\lambda} \). The \( \kappa \) term describes the droplet surface tension. For our \( f_0 \), we compute \( \sigma = (8\kappa m^3/9\lambda^2)^{1/2} \sim \kappa^{1/2} \) (14).

To describe the dynamics of the system, we must account for the fact that baryon number is conserved. Furthermore, it is crucial to include dissipation to describe this strongly fluctuating system. The simplest equations that meet these criteria are:

\[
\partial \rho / \partial t = M \nabla^2 \mu, \quad \mu = f_0' - \kappa \nabla^2 \rho; \tag{5}
\]

model B in (7). We illustrate that (6) describes diffusion in a stable liquid by considering fluctuations about the equilibrium density \( \rho = \rho_h + \delta \rho_k \exp(-ik \cdot x) \), where \( \delta \rho_k \ll \rho_h \). A system at this density is near the minimum of \( f_0 \), so that \( f_0' \approx f_0''(\rho_0)\delta \rho_k = 2m^2\delta \rho_k \). Therefore, (6) is standard diffusion equation at linear order in \( \delta \rho_k \).

We identify the baryon diffusion coefficient at \( \rho_0 \), as \( D = 2m^2M \). In general, diffusion drives the system towards homogeneity at all density for which \( f_0''(\rho) > 0 \).

![FIG. 1. Free energy (a) and phase diagram (b) vs. baryon density for (6).](image)
We remark that the large magnitude of $D$ suggested by [17] is consistent with our assumption in [6] that baryon diffusion is the dominant transport mode for baryons at high density. Our model superficially suggests a slower onset of the instability for a substantially smaller value of $D$. However, if $D$ were truly small then it would be necessary to include transport mechanisms involving convection and viscosity. In fact, viscosity must dominate near $\mu = 0$, where diffusion processes are strictly irrelevant [23].

To describe nuclear collisions, we extend [5] to include drift due to Bjorken longitudinal flow:

$$\frac{\partial \rho}{\partial \tau} + \rho/\tau = M \nabla^2 \mu, \quad (8)$$

where $\tau$ is the proper time and $\mu$ is given by [3,6]. The new drift term forces the average density to decrease as $\langle \rho \rangle \propto \tau^{-1}$, driving the system through the phase coexistence region. Fluctuations grow when densities approach $\rho_c$ (see fig. 1). To derive the drift term, observe that [5] follows from baryon current conservation, which more generally implies $\partial_\mu j^\mu = 0$. The current is $j^\mu = pu^\mu + j^\mu_d$, where $u^\mu$ is a fluid velocity that includes a contribution from the meson flow, and $j^\mu_d$ is the diffusion current, $\propto \nabla \mu$ when $u = (1,0,0,0)$. The left and right sides of [5] respectively follow from $\partial_\mu (pu^\mu)$ and $\partial_\mu j^\mu_d$ for Bjorken flow.

For times $t \gg \tau_R$, the system undergoes a nonlinear evolution in which droplets merge, reducing their surface energy. To study this regime, we write the evolution equation (5) in the dimensionless form:

$$\frac{\partial \psi}{\partial \hat{\tau}} + \frac{\epsilon + \psi}{\hat{\tau}} = -\frac{1}{2} \nabla^2 (\psi - \psi^3 + \nabla^2 \psi) \quad (9)$$

where we use the dimensionless coordinates $\hat{\tau} = 8\tau/\tau_R$ and $\hat{\psi} = \psi \xi/\epsilon$. The dimensionless order parameter $\hat{\psi} \equiv (\rho - \rho_c)/\Delta \rho$ equals $\pm 1$ when $\rho = \rho_{c,0}$. The only remaining parameter is $\epsilon = \rho_c/\Delta \rho$, which controls the strength of the first order transition. Here, we take $\epsilon = 1$ corresponding to a strongly first order transition. Observe that [5] depends on the temperature and density scale only through $\epsilon$. This is an artifact of our very simplistic quadratic $f_0$; we will introduce a more realistic free energy density in later work to study the role of temperature in the evolution.

We solve (5) numerically on a 2+1 dimensional lattice following Grant et al. [12]. We use a forward Euler method to evolve the system in time for a time step $\Delta \tau = 0.05$. We study the evolution in the transverse plane and in the rapidity $\eta$-$x_T$ plane, where $x_T$ is a cartesian transverse coordinate. The laplacian in the $\eta$-$x_T$ case is

$$\nabla^2 = \frac{1}{\tau^2} \frac{\partial^2}{\partial \eta^2} + \nabla^2_{x_T}. \quad (10)$$

To treat the higher spatial derivatives we extend the next-nearest-neighbor algorithm developed by Oono and Puri [16] and used in [15] to account for the asymmetric $\eta$-$x_T$ lattice. We write

$$\hat{\nabla}^2 \psi = \frac{1}{2(\Delta x)^2} \left( \sum_{NN} \psi + \frac{1}{4} \sum_{NNNN} \psi - \frac{5}{2} \psi \right) \quad (11)$$

where the first sum runs over the four nearest neighbors (NN) and the second over the four adjacent next-next-nearest neighbors (NNNN). Oono and Puri use the diagonal next-nearest-neighbors instead in (11) – a formulation that requires a symmetric lattice. We take $\Delta x = 1$. We find that our results are practically indistinguishable from NNN results [13] for this spacing on a symmetric lattice. To study longitudinal expansion, it suffices to replace one coordinate $\Delta x$ for $\hat{\tau} \Delta \eta$.

![FIG. 2. Order parameter in the transverse plane in the absence of expansion. Droplets tend to merge.](image-url)

Figures 2 and 3 show 2+1 dimensional numerical simulations of (5) in the transverse plane. Only longitudinal expansion is considered so the coordinates are cartesian with periodic boundary conditions. For comparison, fig. 2 shows results in which expansion is neglected by omitting the term $(\epsilon + \psi)/\hat{\tau}$ in (5). Expansion shown in fig. 3 prevents droplets from merging as in fig. 2. The expanding system reaches $\rho_e$ at $\tau_0 = 5$ fm. Because this is a dissipative system, we must apply thermal noise at each lattice site at $\tau_0$ to seed phase separation (noise at earlier times is dampened). The memory of the initial conditions is essentially lost for $\tau - \tau_0 > \tau_R$.

We now study the rapidity dependence of baryon number fluctuations. Figure 3 shows the computed variance for two different initial times and for two rapidity intervals. The variance is computed from a sample of 5000 simulated events, each unique due to the thermal noise. We see that the super-poissonian fluctuations grow appreciably by $\tau \sim 2\tau_0$. This variance drops as the rapidity interval is increased. We find that variance is governed by the ratio $\tau_0/\tau_R$, which compares the expansion and droplet-growth time scales.
estimates are reliable. It is therefore important that the literature, so it is not clear whether these benchmark central Au+Au collisions yield $\Omega$.

However, we have been unable to find experimental results on net-proton fluctuations in $pp$ or $pA$ collisions. If it turns out that light and heavy ion fluctuations are similar, it may be necessary to correlate baryon measurements with other signals to extract phase transition information, as in [17].

Nevertheless, we stress that it is unlikely that superpoissonian fluctuations in nucleon-nucleon ($NN$) collisions – if present – result in significant fluctuations in $AA$ interactions unless there is a major source of coherence or collectivity. If we treat the $AA$ collision as a superposition of $NN$ subcollisions, then $\Omega_p(AA) = \Omega_p(NN)/N(b)$, where $N(b)$ is the number of participant nucleons. To obtain a rough upper bound on $\Omega_p(NN)$, we take the total charge fluctuations measured to be $\sim 0.6$ in 200 GeV $pp$ collisions from Whitmore’s review [19]. For $Au+Au$ collisions at $b < 10$ fm, we estimate $\Omega_p(Au + Au) = \Omega_p(NN)/N(0) < 0.01$, where we use the wounded nucleon model to compute $N(b) \approx 59$ for $b = 10$ fm and 372 for $b = 0$. RQMD $Au+Au$ simulations for impact parameters fall below this bound [18].

We expect $\Omega_p$ to dramatically increase in heavy ion systems compared to light ones. In central S+S we expect the $NN$ contribution to $\Omega_p$ to be below 1%, as implied by our wounded nucleon model estimate. Since there is no evidence of a phase transition in such light systems at AGS or SPS, the appearance of fluctuations at the level of fig. 4 in $Au+Au$ would be impressive. But is there any source of coherence or collectivity other than a phase transition? Gluon junction effects [22] can lead to correlated baryon production in $pp$, $pA$ and $AA$ collisions. This effect is only partially included in RQMD [23]. We are currently studying how gluon junctions can effect $\Omega_p$ [18].

In summary, we have studied the phenomenological impact of baryon density, a proposed order parameter of the putative first order QCD phase transition at high baryon density [6]. We have shown that phase separation in the nonequilibrium heavy ion system can lead to large baryon fluctuations. These fluctuations are superpoissonian and, consequently, can be extracted by measuring protons alone. For [b] with $\langle p \rangle \propto \tau^{-1}$, the system is unstable only for $\tau < 2.3 \tau_0$. We extend the calculations to much longer times to demonstrate that the fluctuations in rapidity survive well past the freeze out time, of order 10–30 fm, in accord with [6].

For sufficiently large $\tau_0$, final state fluctuations can be substantial. However, we have seen that a more rapid expansion corresponding to smaller $\tau_0$ leads to an “inflation” that prevents the fluctuations from having a large impact on the final state. If experiments find that the non-poissionian component of fluctuations is small, we must use information from flow signals to ascertain the degree of this inflation.

We emphasize that these calculations include diffu-
sion, which dampens the fluctuations once the system becomes stable. While diffusion is the primary mechanism for dampening fluctuations at high density, viscosity becomes more important at small net baryon density. Several key questions remain: At what energy do heavy ion collisions reach a baryon density where the phase transition is strongly first order? Is there a residual modification of fluctuations due to the near transition at zero baryon density? To what extent does cooling, convection, viscosity and collision-geometry alter $\Omega_p$ compared to our estimates? Finally, we note that our mixed-phase effect may be compensated to some extent by the effect due to the difference between fluctuations in a plasma compared to a hadron gas. Nevertheless, the strength of the signal in our exploratory calculations invites further work.

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