Robustness of non-standard cosmologies solving the Hubble constant tension

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In this manuscript we reassess the potential of interacting dark matter-dark energy models in solving the Hubble constant tension. These models, mostly modifying late-time physics, have been proposed but also questioned as possible solutions to the $H_0$ problem. Here we examine several interacting scenarios against cosmological observations, focusing on the important role played by the calibration of Supernovae data. In order to reassess the ability of interacting dark matter-dark energy scenarios in easing the Hubble constant tension, we systematically confront their theoretical predictions for $H_0$ to SH0ES measurements of (a) the Hubble constant and (b) the intrinsic magnitude $M_B$, explicitly showing that the choice of prior is irrelevant, as a higher value of $H_0$ is always recovered within interacting scenarios. We also find that one of the interacting scenarios provides a better fit to the cosmological data than the $\Lambda$CDM model itself.

I. INTRODUCTION

A plethora of observations have led to confirm the standard $\Lambda$CDM framework as the most economical and successful model describing our current universe. This simple picture (pressureless dark matter, baryons and a cosmological constant representing the vacuum energy) has been shown to provide an excellent fit to cosmological data. However, there are a number of inconsistencies that persist and, instead of diluting with improved precision measurements, gain significance [1–10].

The most exciting (i.e. probably non due to systematics) and most statistically significant ($4 - 6\sigma$) tension in the literature is the so-called Hubble constant tension, which refers to the discrepancy between cosmological predictions and low redshift estimates of $H_0$ [11,13]. Within the $\Lambda$CDM scenario, Cosmic Microwave Background (CMB) measurements from the Planck satellite provide a value of $H_0 = 67.36 \pm 0.54$ km s$^{-1}$ Mpc$^{-1}$ at 68\% CL [13]. Near universe, local measurements of $H_0$, using the cosmic distance ladder calibration of Type Ia Supernovae with Cepheids, as those carried out by the SH0ES team, provide a measurement of the Hubble constant $H_0 = 73.2 \pm 1.3$ km s$^{-1}$ Mpc$^{-1}$ at 68\% CL [15]. This problematic $\sim 4\sigma$ discrepancy aggravates when considering other late-time estimates of $H_0$. For instance, measurements from the Megamaser Cosmology Project [16], or those exploiting Surface Brightness Fluctuations [17] only exacerbate this tension [4].

As previously mentioned, the SH0ES collaboration exploits the cosmic distance ladder calibration of Type Ia Supernovae, which means that these observations do not provide a direct extraction of the Hubble parameter. More concretely, the SH0ES team measures the absolute peak magnitude $M_B$ of Type Ia Supernovae standard candles and then translates these measurements into an estimate of $H_0$ by means of the magnitude-redshift relation of the Pantheon Type Ia Supernovae sample [20]. Therefore, strictly speaking, the SH0ES team does not directly extract the value of $H_0$, and there have been arguments in the literature aiming to translate the Hubble constant tension into a Type Ia Supernovae absolute magnitude tension $M_B$ [21,23]. In this regard, late-time exotic cosmologies have been questioned as possible solutions to the Hubble constant tension [22,24], since within these scenarios, it is possible that the supernova absolute magnitude $M_B$ used to derive the low redshift estimate of $H_0$ is no longer compatible with the $M_B$ needed to fit supernovae, BAO and CMB data.

A number of studies have prescribed to use in the statistical analyses a prior on the intrinsic magnitude rather than on the Hubble constant $H_0$ [7,23]. Following the very same logic of these previous analyses, we reassess here the potential of interacting dark matter-dark energy cosmology [21] in resolving the Hubble constant ([13,25–54] and references therein) and/or the intrinsic magnitude $M_B$ tension, by demonstrating explicitly from a full analysis that the results are completely independent of whether a prior on $M_B$ or $H_0$ is assumed (see also the recent [55]).

II. THEORETICAL FRAMEWORK

We adopt a flat cosmological model described by the Friedmann-Lemaître-Robertson-Walker metric. A possible parameterization of a dark matter-dark energy inter-
In the equations above, $T$ action is provided by the following expressions [56, 57]:

$$\nabla_{\mu}T^{\mu}_{\nu}(dm) = Q w_{\nu}^{(dm)}/a,$$  \hspace{0.5cm} (1)

$$\nabla_{\mu}T^{\mu}_{\nu}(de) = -Q w_{\nu}^{(dm)}/a.$$  \hspace{0.5cm} (2)

In the equations above, $T^{\mu}_{\nu}(dm)$ and $T^{\mu}_{\nu}(de)$ represent the energy-momentum tensors for the dark matter and dark energy components respectively, the function $Q$ is the interaction rate between the two dark components, and $w_{\nu}^{(dm)}$ represents the dark matter four-velocity. In what follows we shall restrict ourselves to the case in which the interaction rate is proportional to the dark energy density $\rho_{de}$ [56, 57]:

$$Q = \delta_{DMDE}H\rho_{de},$$  \hspace{0.5cm} (3)

where $\delta_{DMDE}$ is a dimensionless coupling parameter and $H = \dot{a}/a$. The background evolution equations in the coupled model considered here read [58]

$$\dot{\rho}_{dm} + 3H\rho_{dm} = \delta_{DMDE}H\rho_{de},$$  \hspace{0.5cm} (4)

$$\dot{\rho}_{de} + 3H(1 + w_{0,\delta d})\rho_{de} = -\delta_{DMDE}H\rho_{de}.$$  \hspace{0.5cm} (5)

The evolution of the dark matter and dark energy density perturbations and velocities divergence field are described in [26] and references therein.

It has been shown in the literature that this model is free of instabilities if the sign of the coupling $\delta_{DMDE}$ and the sign of $(1 + w_{0,\delta d})$ are opposite, where $w_{0,\delta d}$ refers to the dark energy equation of state [57, 59]. In order to satisfy such stability conditions, we explore three possible scenarios, all of them with a redshift-independent equation of state. In Model A, the equation of state $w_{0,\delta d}$ is fixed to $-0.999$. Consequently, since $(1 + w_{0,\delta d}) > 0$, in order to ensure a instability-free perturbation evolution, the dark matter-dark energy coupling $\delta_{DMDE}$ is allowed to vary in a negative range. In Model B, $w_{0,\delta d}$ is allowed to vary but we ensure that the condition $(1 + w_{0,\delta d}) > 0$ is always satisfied. Therefore, the coupling parameter $\delta_{DMDE}$ is also negative. In Model C, instead, the dark energy equation of state is phantom $(w_{0,\delta d} < -1)$, therefore the dark matter-dark energy coupling is taken as positive to avoid early-time instabilities. We shall present separately the cosmological constraints for these three models, together with those corresponding to the canonical ΛCDM.

| Model | Prior $w_{0,\delta d}$ | Prior $\delta_{DMDE}$ |
|-------|-----------------------|------------------------|
| A     | -0.999                | [-1.0, 0.0]            |
| B     | [-0.999, -0.333]      | [-1.0, 0.0]            |
| C     | [-3, -1.001]          | [0.0, 1.0]             |

TABLE I. Priors of $w_{0,\delta d}$, $\delta$ in models A, B, C.

III. DATASETS AND METHODOLOGY

In this Section, we present the data sets and methodology employed to obtain the observational constraints on the model parameters by performing Bayesian Monte Carlo Markov Chain (MCMC) analyses. In order to constrain the parameters, we use the following data sets:

- The Cosmic Microwave Background (CMB) temperature and polarization power spectra from the final release of Planck 2018, in particular we adopt the plikTTTEEE+lowl+lowlE likelihood [60, 61], plus the CMB lensing reconstruction from the four-point correlation function [62].

- Type Ia Supernovae distance moduli measurements from the Pantheon sample [20]. These measurements constrain the uncalibrated luminosity distance $H_0d_L(z)$, or in other words the slope of the late-time expansion rate (which in turn constrains the current matter energy density, $\Omega_{0,m}$). We refer to this dataset as SN.

- Baryon Acoustic Oscillations (BAO) distance and expansion rate measurements from the 6dFGS [63], SDSS-DR7 MGS [64], BOSS DR12 [65] galaxy surveys, as well as from the eBOSS DR14 Lyman-α (Lyα) absorption [66] and Lyα-quasars cross-correlation [67]. These consist of isotropic BAO measurements of $D_V(z)/r_d$ (with $D_V(z)$ and $r_d$ the spherically averaged volume distance and sound horizon at baryon drag, respectively) for 6dFGS and MGS, and anisotropic BAO measurements of $D_M(z)/r_d$ and $D_H(z)/r_d$ (with $D_M(z)$ the comoving angular diameter distance and $D_H(z) = c/H(z)$ the radial distance) for BOSS DR12, eBOSS DR14 Lyα, and eBOSS DR14 Lyα-quasars cross-correlation.

- A gaussian prior on $M_B = -19.244 \pm 0.037$ mag [23], corresponding to the SN measurements from SH0ES.

- A gaussian prior on the Hubble constant $H_0 = 73.2 \pm 1.3$ km s$^{-1}$ Mpc$^{-1}$ in agreement with the measurement obtained by the SH0ES collaboration in [15].

For the sake of brevity, data combinations are indicated as CMB+SN+BAO (CSB), CMB+SN+BAO+$H_0$ (CSBH) and CMB+SN+BAO+$M_B$ (CSBM).

Cosmological observables are computed with CLASS [68, 69]. In order to derive bounds on the proposed scenarios, we modify the efficient and well-known cosmological package MontePython [70], supporting the Planck 2018 likelihood [71]. We make use of CalPriorSNIa, a module for MontePython, publicly available at https://github.com/valerio-marra/CalPriorSNIa that implements an effective calibration prior on the absolute magnitude of Type Ia Supernovae [29, 72].
TABLE II. Mean values and 68% CL errors on $\omega_{cdm} \equiv \Omega_{cdm} h^2$, the current dark energy density $\Omega_{0, \text{fd}}$, the current matter energy density $\Omega_{0, m}$, the Supernovae Ia intrinsic magnitude $M_B$, the Hubble constant $H_0$ and the clustering parameter $\sigma_8$ within the standard ΛCDM paradigm. We also report the minimum value of the $\chi^2$ function obtained for each of the data combinations.

| Parameter | CSB | CSBH | CSBM |
|-----------|-----|------|------|
| $\omega_{cdm}$ | $0.1193 \pm 0.0010$ | $0.1183 \pm 0.0009$ | $0.1183^{+0.0006}_{-0.0005}$ |
| $\Omega_{0, \text{fd}}$ | $0.6889^{+0.0057}_{-0.0061}$ | $0.6958^{+0.0056}_{-0.0050}$ | $0.6956^{+0.0057}_{-0.0049}$ |
| $\Omega_{0, m}$ | $0.3111^{+0.0057}_{-0.0061}$ | $0.3042^{+0.0056}_{-0.0050}$ | $0.3044^{+0.0057}_{-0.0049}$ |
| $M_B$ | $-19.42 \pm 0.01$ | $-19.40 \pm 0.01$ | $-19.40 \pm 0.01$ |
| $H_0$ | $67.68^{+0.41}_{-0.46}$ | $62.81^{+0.42}_{-0.41}$ | $62.80^{+0.41}_{-0.40}$ |
| $\sigma_8$ | $0.8108^{+0.0061}_{-0.0058}$ | $0.8092^{+0.0060}_{-0.0065}$ | $0.8090^{+0.0064}_{-0.0059}$ |
| minimum $\chi^2$ | $3819.46$ | $3836.50$ | $3840.44$ |

TABLE III. Mean values and 68% CL errors on $\omega_{cdm} \equiv \Omega_{cdm} h^2$, the current dark energy density $\Omega_{0, \text{fd}}$, the current matter energy density $\Omega_{0, m}$, the dimensionless dark matter-dark energy coupling $\delta_{\text{DMDE}}$, the Supernovae Ia intrinsic magnitude $M_B$, the Hubble constant $H_0$ and the clustering parameter $\sigma_8$ within the interacting model A, see Tab. I. We also report the minimum value of the $\chi^2$ function obtained for each of the data combinations.

| Parameter | CSB | CSBH | CSBM |
|-----------|-----|------|------|
| $\omega_{cdm}$ | $0.109^{+0.011}_{-0.005}$ | $0.09 \pm 0.01$ | $0.096^{+0.011}_{-0.009}$ |
| $\Omega_{0, \text{fd}}$ | $0.724^{+0.017}_{-0.028}$ | $0.758^{+0.026}_{-0.024}$ | $0.754^{+0.026}_{-0.028}$ |
| $\Omega_{0, m}$ | $0.276^{+0.017}_{-0.028}$ | $0.242^{+0.026}_{-0.024}$ | $0.245^{+0.026}_{-0.024}$ |
| $M_B$ | $-0.116^{+0.044}_{-0.100}$ | $-0.219^{+0.033}_{-0.086}$ | $-0.203^{+0.034}_{-0.087}$ |
| $H_0$ | $68.59^{+0.65}_{-0.70}$ | $69.73^{+0.71}_{-0.72}$ | $69.67^{+0.75}_{-0.85}$ |
| $\sigma_8$ | $0.90^{+0.04}_{-0.08}$ | $1.01^{+0.06}_{-0.11}$ | $1.00^{+0.07}_{-0.12}$ |
| minimum $\chi^2$ | $3819.86$ | $3831.90$ | $3833.86$ |

IV. MAIN RESULTS AND DISCUSSION

We start by discussing the results obtained within the canonical ΛCDM scenario. Table I presents the mean values and the 1σ errors on a number of different cosmological parameters. Namely, we show the constraints on $\omega_{cdm} \equiv \Omega_{cdm} h^2$, the current dark energy density $\Omega_{0, \text{fd}}$, the current matter energy density $\Omega_{0, m}$, the Supernovae Ia intrinsic magnitude $M_B$, the Hubble constant $H_0$ and the clustering parameter $\sigma_8$ arising from three possible data combinations considered here and above described: CMB+SN+BAO (CSB), CMB+SN+BAO+$H_0$ (CSBH), CMB+SN+BAO+$M_B$ (CSBM). Interestingly, all the parameters experience the very same shift regardless the prior is adopted on the Hubble constant or on the intrinsic Supernovae Ia magnitude $M_B$. The mean value of $H_0$ coincides for both the CSBH and the CSBM data combinations, as one can clearly see from the dashed and dotted black lines in Fig. 1. Figure 2 presents the two-dimensional allowed contours and the one-dimensional posterior probabilities on the parameters shown in Tab. I. Notice that all the parameters are equally shifted when adding the prior on $H_0$ or on $M_B$, except for $\sigma_8$ which remains almost unchanged. Notice also that the value of the current matter density, $\Omega_{0, m}$, is smaller when a prior from SN measurements is considered: due to the larger $H_0$ value that these measurements imply, in order to keep the CMB peaks structure unaltered, the value of $\Omega_{0, m}$ should be smaller to ensure that the product $\omega m h^2$ is barely shifted.

We focus now on Model A, which refers to an interacting cosmology with $w_{0, \text{fd}} = -0.999$ and $\delta_{\text{DMDE}} < 0$. Table II presents the mean values and the 1σ errors on the same cosmological parameters listed above, with the addition of the coupling parameter $\delta_{\text{DMDE}}$, for the same three data combination already discussed.

Notice again that all the parameters are equally shifted to either smaller or larger values, regardless the prior is adopted on either $H_0$ or $M_B$. In this case the shift on the Hubble parameter is larger than that observed within the ΛCDM model, as one can notice from the blue curves depicted in Fig. 1. Interestingly, we observe a 2σ indication in favor of a non-zero value of the coupling $\delta_{\text{DMDE}}$ when considering the CSBH and the CSBM data combinations. Indeed, while the value of the minimum $\chi^2$ is almost equal to that obtained in the ΛCDM framework for the CSB data analyses, when adding either a prior on $H_0$ or on $M_B$, the minimum $\chi^2$ value is smaller than that obtained for the standard cosmological picture: therefore, the addition of a coupling improves the overall fit. Figure 3 presents the two-dimensional allowed contours and the one-dimensional posterior probabilities obtained within Model A. It can be noticed that the prior on the Hubble constant and on the intrinsic magnitude lead to the very same shift, and the main conclusion is therefore prior-independent: there is a $\sim 2\sigma$ indication for a non-zero dark matter-dark energy coupling when considering either $H_0$ or $M_B$ measurements, and the value of the Hubble constant is considerably larger, alleviating the $H_0$ tension.

Focusing now on Model B, which assumes a negative coupling $\delta_{\text{DMDE}}$ and a constant, but freely varying, dark energy equation of state $w_{0, \text{fd}}$ within the $w_{0, \text{fd}} > -1$ region, we notice again the same shift on the cosmological parameters, regardless the prior is introduced in the Hubble parameter ($H_0$) or in the Supernovae Ia intrinsic magnitude ($M_B$), as can be noticed from Tab. III. As in Model A, the value of $H_0$ in this interacting cosmology is larger than within the ΛCDM framework (see the red curves in Fig. 1), albeit slightly smaller than in Model A, due to the strong anti-correlation between $w_{0, \text{fd}}$ and $H_0$ [56, 73]. Consequently, a larger value of $w_{0, \text{fd}} > -1$ implies a lower value of $H_0$. Nevertheless, a 2σ preference for a non-zero value of the dark matter-dark energy coupling is present also in this case, and also when the CSB dataset is considered: for the three data combinations presented here, there is always a preference for a non-zero dark matter-dark energy coupling. Notice that

\[ \delta_{\text{DMDE}} = \begin{cases} 0 & \text{for } \Lambda \text{CDM} \\ -0.999 & \text{for Model A} \\ -0.999 \text{ or } -1.000 & \text{for Model B} \end{cases} \]
stated, we considered as numerical fluctuations. Since, as previously
A and Model B, however, are small enough to be con-
Hermite polynomial basis functions. The differences between the minimum
χ^2 value of the interacting model B, see Tab. I. We also report the minimum
TABLE IV. Mean values and 68% CL errors on ω_{cdm} as given in Tab. I. We show constraint obtained within model A (green), model B (red) and model C (blue) for the CMB+SN+BAO data combination (solid lines), CMB+SN+BAO+H_0 (dashed lines) and CMB+SN+BAO+M_B (dotted lines).

| Parameter | CSB | CSBH | CSBM |
|-----------|-----|------|------|
| ω_{cdm}  | 0.077^{+0.036}_{-0.014} | 0.061^{+0.044}_{-0.019} | 0.065^{+0.036}_{-0.017} |
| Ω_0, fld | 0.785^{+0.034}_{-0.034} | 0.825^{+0.045}_{-0.044} | 0.818^{+0.034}_{-0.041} |
| Ω_0, m  | 0.215^{+0.081}_{-0.090} | 0.174^{+0.069}_{-0.082} | 0.182^{+0.075}_{-0.081} |
| w_0, fld | -0.909^{+0.026}_{-0.090} | -0.917^{+0.026}_{-0.082} | -0.918^{+0.026}_{-0.081} |
| δ_{DMDE} | -0.3^{+0.26}_{-0.14} | -0.45^{+0.22}_{-0.16} | -0.43^{+0.24}_{-0.15} |
| M_B     | -19.41 ± 0.02 | -19.38 ± 0.02 | -19.38 ± 0.02 |
| H_0     | 68.28^{+0.79}_{-0.85} | 69.68^{+0.75}_{-0.76} | 69.57^{+0.75}_{-0.76} |
| σ_s     | 1.30^{+0.01}_{-0.51} | 1.60^{+0.06}_{-0.76} | 1.53^{+0.03}_{-0.71} |
| Minimum χ^2 | 3819.96 | 3832.28 | 3836.24 |

In Fig. 4 we depict the two-dimensional allowed contours and the one-dimensional posterior probabilities obtained for Model B. From a comparison to Fig. 2 and also confronting the mean values of Tab. IV to those shown in Tab. III (and, to a minor extent, to those in Tab. III), one can notice that the value of Ω_{0, fld} is much larger. The reason for this is related to the lower value for the present matter energy density Ω_{0, m} (the values are also shown in the tables), which is required within the interacting cosmologies when the dark matter-dark energy coupling is negative. In the context of a universe with a negative dark coupling, indeed, there is an energy flow from dark matter to dark energy. Consequently, the (dark) matter content in the past is higher than in the standard ΛCDM scenario and the amount of intrinsic (dark) matter needed today is lower, because of the extra contribution from the dark energy sector. In a flat universe, this translates into a much higher value of Ω_{0, fld}. On the other hand, a lower value of Ω_{m, 0} requires a larger value of the clustering parameter σ_s to be able to satisfy the overall normalization of the matter power spectrum. In any case, we find again that the addition of a prior on either H_0 or M_B leads to exactly the same shift for all the cosmological parameters. Therefore, Model B also provides an excellent solution to the Hubble constant tension, although at the expense of a very large σ_s.

Finally, Tab. V shows the mean values and the 1σ er-
errors on the usual cosmological parameters explored along this study, for Model C. Notice that this model benefits from both its interacting nature and from the fact that $w_0,\text{fld} < -1$ and $\delta_{\text{DMDE}} > 0$. Both features of the dark energy sector have been shown to be excellent solutions to the Hubble constant problem. As in the previous cases, the shift in the cosmological parameters induced by the addition of a prior is independent of its nature, i.e. it is independent on whether a prior on $H_0$ or $M_B$ is adopted. Within this model, the value of the Hubble constant is naturally larger than within the $\Lambda$CDM model (see the blue lines in Fig. 1), regardless of the data sets assumed in the analyses. Despite its phantom nature, as in this particular case $w_0,\text{fld} < -1$ to ensure an instability-free evolution of perturbations, Model C provides the best-fits to any of the data combinations explored here, perform-
FIG. 3. 68% CL and 95% CL allowed contours and one-dimensional posterior probabilities on a selection of cosmological parameters within model A, considering three data combinations: CMB+SN+BAO (red), CMB+SN+BAO+\(H_0\) (blue) and CMB+SN+BAO+\(M_B\) (green).

\[ \text{ing even better than the minimal } \Lambda \text{CDM picture, as one can clearly notice from the last row of Tab. V. This fact makes Model C a very attractive cosmological scenario which can provide a solution for the long-standing } H_0 \text{ tension. We must remember that model C, however, has two degrees of freedom more than the standard } \Lambda \text{CDM paradigm. Figure 5 illustrates the two-dimensional allowed contours and the one-dimensional posterior probabilities obtained within Model C. Notice that here the situation is just the opposite one of Model B: the value of } \Omega_{0,\text{fld}} \text{ is much smaller than in standard scenarios, due to the larger value required for the present matter energy density } \Omega_{0,\text{m}} \text{ when the dark matter-dark energy coupling } \delta_{\text{DMDE}} > 0 \text{ and } w_{0,\text{fld}} < -1. \text{ This larger value of the present matter energy density also implies a lower value for the clustering parameter } \sigma_8, \text{ in contrast to what was} \]
FIG. 4. 68% CL and 95% CL allowed contours and one-dimensional posterior probabilities on a selection of cosmological parameters within model B, considering three data combinations: CMB+SN+BAO (red), CMB+SN+BAO+$H_0$ (blue) and CMB+SN+BAO+$M_B$ (green).

V. FINAL REMARKS

In this study we have tried to reassess the ability of interacting dark matter-dark energy cosmologies in alleviating the long-standing and highly significant Hubble constant tension. Despite the fact that in the past these models have been shown to provide an excellent solution to the discrepancy between local measurements and high redshift, Cosmic Microwave Background estimates of $H_0$, there have been recent works in the literature questioning their effectiveness, related to a misinterpretation of SH0ES data, which indeed does not directly extract the value of $H_0$. We have therefore computed the ability of
interacting cosmologies of reducing the Hubble tension by means of two possible different priors in the cosmological analyses: a prior on the Hubble constant and, separately, a prior on Type Ia Supernova absolute magnitude. We combine these priors with Cosmic Microwave Background (CMB), Type Ia Supernovae (SN) and Baryon Acoustic Oscillation (BAO) measurements, showing that the constraints on the cosmological parameters are independent of the choice of prior, and that the Hubble constant tension is always alleviated. This last statement is also prior-independent. Furthermore, one of the possible interacting cosmologies considered here, with a phantom nature, provides a better fit than the canonical ΛCDM framework for all the considered data combinations, but with two extra degrees of freedom. We therefore conclude that interacting dark-matter dark-energy cosmolo-
TABLE V. Mean values and 68% CL errors on $\omega_{\text{adm}} \equiv \Omega_0 h^2$, the current dark energy density $\Omega_{0, \text{fid}}$, the current matter energy density $\Omega_{0, m}$, the dark energy equation of state $w_{0, \text{fid}}$, the dimensionless dark matter-dark energy coupling $\delta_{DMDE}$, the supernovae Ia intrinsic magnitude $M_B$, the Hubble constant $H_0$ and the clustering parameter $\sigma_8$ within the interacting model C, see Tab. I. We also report the minimum value of the $\chi^2$ function obtained for each of the data combinations.

| Parameter       | CSB               | CSBH              | CSBM              |
|-----------------|-------------------|-------------------|-------------------|
| $\omega_{\text{adm}}$ | $0.138^{+0.008}_{-0.012}$ | $0.137^{+0.007}_{-0.016}$ | $0.135^{+0.008}_{-0.012}$ |
| $\Omega_{0, \text{fid}}$ | $0.655^{+0.032}_{-0.021}$ | $0.671^{+0.031}_{-0.021}$ | $0.675^{+0.027}_{-0.018}$ |
| $\Omega_{0, m}$ | $0.345^{+0.032}_{-0.021}$ | $0.329^{+0.031}_{-0.031}$ | $0.325^{+0.027}_{-0.018}$ |
| $w_{0, \text{fid}}$ | $-1.087^{+0.042}_{-0.042}$ | $-1.131^{+0.041}_{-0.041}$ | $-1.117^{+0.044}_{-0.044}$ |
| $\delta_{DMDE}$ | $0.183^{+0.063}_{-0.180}$ | $0.173^{+0.051}_{-0.170}$ | $0.150^{+0.051}_{-0.150}$ |
| $M_B$ | $-19.41 \pm 0.02$ | $-19.38 \pm 0.02$ | $-19.37 \pm 0.02$ |
| $H_0$ | $68.29^{+0.66}_{-0.91}$ | $69.74^{+0.73}_{-0.73}$ | $69.67^{+0.78}_{-0.77}$ |
| $\sigma_8$ | $0.735^{+0.045}_{-0.057}$ | $0.748^{+0.068}_{-0.034}$ | $0.755^{+0.051}_{-0.047}$ |
| minimum $\chi^2$ | 3818.24 | 3830.56 | 3835.10 |

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