Stabilization of the p-wave superfluid state in an optical lattice

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It is hard to stabilize the p-wave superfluid state of cold atomic gas in free space due to inelastic collisional losses. We consider the p-wave Feshbach resonance in an optical lattice, and show that it is possible to have a stable p-wave superfluid state where the multi-atom collisional loss is suppressed through the quantum Zeno effect. We derive the effective Hamiltonian for this system, and calculate its phase diagram in a one-dimensional optical lattice. The results show rich phase transitions between the p-wave superfluid state and different types of insulator states induced either by interaction or by dissipation.

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The observation of the s-wave superfluid state in a Fermionic atomic gas represents a remarkable breakthrough in the study of many-body physics with ultracold atoms [1]. The Feshbach resonance plays an important role in those experiments, enhancing the interatomic interaction so that the superfluid phase can be entered at a temperature that is experimentally achievable. It is of great interest to realize the superfluid state with other pairing symmetries as well. The p-wave superfluid state is the next candidate for observation, and has attracted the interest of many experimental and theoretical groups [2, 3]. The p-wave Feshbach resonance has been recently observed in experiments [2], and can push the single-component Fermi gas to the strongly interacting region and open a door towards observation of the p-wave superfluid state in this system. However, compared with the s-wave Feshbach resonance, a key difficulty with the p-wave resonance is that the inelastic collisional loss in this system is typically large [2, 4, 5], which forbids thermalization of the gas within the system lifetime [6].

In this paper, we discuss how a dissipation-induced blockade mechanism can stabilize the p-wave superfluid state in an optical lattice in the strongly interacting region. Such a dissipation-induced blockade has been reported in recent experiments to realize the Tonks gas [4] or for simulation of effective three-body interactions with cold bosonic atoms. We apply this mechanism to stabilize the single-component Fermion system in an optical lattice in the presence of the p-wave Feshbach resonance. The p-wave Feshbach resonance has been considered recently on single sites in a very deep optical lattice [2, 8]. Here, instead, we focus on the many-body physics by deriving an effective Hamiltonian for this system, taking into account the atomic hopping in the reduced Hilbert space caused by the dissipation-induced blockade. This effective Hamiltonian provides a starting point to understand the quantum phases. We compute the phase diagram of the system explicitly with well-controlled numerical methods for an anisotropic lattice where the atom tunnelling is dominantly along one dimension. The results show rich phase transitions between the p-wave superfluid state, a dissipation-induced insulator state, the Mott insulator state, and different kinds of metallic states. Although these results are obtained from one-dimensional calculations, we expect these phases to correspond also to similar phases in higher dimensions.

We consider a single-component Fermi gas near a p-wave Feshbach resonance. If this strongly interacting gas is loaded into an optical lattice, many different Bloch bands can be populated, in particular when the resonance is broad (as it is the case for some recent experiments [2]). However, we can derive an effective single-band model for this system that is independent of the interaction details. Following a strategy similar to the s-wave Feshbach resonance case [9], we first analyze the local Hilbert space structure on a single lattice site. When we have zero or one atom on the site \(i\), the states are simply denoted by \(|0\rangle\_i\) and \(|a\rangle\_i = a\_i^\dagger |0\rangle\_i\), respectively. For the case of two atoms on a single site, the exact two-body physics has recently been calculated [8], and there are several two-body energy levels separated by an energy difference of the order of the band gap. If we assume that the system temperature is significantly below the band gap, only the lowest two-body state is relevant. We refer to this state as a dressed molecule level, and denote it by \(|b\rangle\_i = b\_i^\dagger |0\rangle\_i\). Note that the wave function of \(|b\rangle\_i\) in general includes contributions from many of the original atomic orbitals [3]. It has a p-wave symmetry in space and is antisymmetric under exchange of the two atoms in this dressed molecule.

If more that two atoms coming to a single site, different from the s-wave case, the state will not be stable due to big three-particle inelastic collision loss [4]. At first sight, this seems to mean that the system will become unstable. However, in a lattice, there is a subtle dissipation induced blockade mechanism [6] which forbids population of the unstable three-particle state and thus stabilizes the whole system. The basic idea is illustrated in Fig. 1.
The three-particle state $|3\rangle$ has a large bandwidth characterized by its inelastic collision rate $\gamma$ and an energy shift characterized by the on-site atom-dressed-molecule interaction energy $\Delta_3$. If a single atom tunnel through a barrier with a hopping rate $t$ to form this state $|3\rangle$, the probability of getting $|3\rangle$ is given by $t^2/\left(\gamma^2 + \Delta_3^2\right)$ (similar to a two-level transition with a detuning $\Delta_3$ and a bandwidth $\gamma$). So the net collisional loss of the system is bounded by $\gamma_{\text{eff}} = \gamma t^2/\left(\gamma^2 + \Delta_3^2\right) \leq t^2/\left(2\Delta_3\right)$ no matter how large the inelastic collision rate $\gamma$ is. Near the Feshbach resonance, the atom-dressed-molecule interaction energy $\Delta_3$ is comparable with the lattice band gap (thus much larger than $t$), the net collisional loss $\gamma_{\text{eff}}$ is therefore small compared with the atomic hopping required to thermalize the system. The reduction of population in the noisy state $|3\rangle$ is called the dissipation-induced blockade (or interpreted as the quantum Zeno effect); the blockade is actually induced by both dissipation and interaction when $\Delta_3$ and $\gamma$ are comparable. Due to this mechanism one can achieve many-body thermal equilibrium in a lattice even if there is big inelastic collision loss.

Due to the dissipation-induced blockade discussed above, on each site we have only three relevant levels: $|0\rangle_i$, $|a\rangle_i$, and $|b\rangle_i$ as the low energy configurations. The energy difference between the states $|a\rangle_i$ and $|b\rangle_i$ can be tuned with the external magnetic field via the Feshbach resonance. We then take into account the atomic hopping, which changes the level configurations of the neighboring sites under the constraint that the atom number be conserved. Thus, there are only three possible processes, as illustrated in Fig. 2. Corresponding to these configuration changes, the general Hamiltonian for this lattice system then takes the form

\begin{equation}
H = \sum_i \left[ (\Delta b_i^\dagger b_i - \mu(a_i^\dagger a_i + 2b_i^\dagger b_i)) - \sum_{\langle i,j \rangle} P \left[ t_1 a_i^\dagger a_j + t_2 (b_i^\dagger - b_j^\dagger) a_i^\dagger a_j + t_3 b_i^\dagger b_j a_i^\dagger a_i + H.c. \right] P \right], \tag{1}
\end{equation}
thermodynamic limit. The algorithm has been shown to work with high precision compared to known results for the Hubbard model [15]. For simplicity, we take the hopping rates $t_1 = t_2 = t_3 = t$, and use $t$ as the energy unit. Then we have effectively only two parameters, $\Delta$ and $\mu$ (in units of $t$) in the Hamiltonian (1). To figure out the complete phase diagram with respect to these two parameters, we calculate $\partial \langle H \rangle / \partial \Delta$ and $\partial \langle H \rangle / \partial \mu$ as functions of $\Delta$ or $\mu$ for the ground state of $H$, and use the characteristics of these curves to identify the phase transition points.

In Fig. 3, we show $\pi_b = \langle b_i^\dagger b_i \rangle$ and $\pi_a = \langle a_i^\dagger a_i \rangle$ as functions of $\Delta$ and $\mu$. One can clearly see several quantum phase transitions from this figure. First, with a fixed chemical potential $\mu = -0.5t$, one has $\pi_b = 1$ and $\pi_a = 0$ with a large negative detuning $\Delta$ (corresponding to strong attractive atomic interaction). In this case, each site is doubly occupied with two atoms. More than two atoms cannot move to the same site because of the dissipation-induced blockade. So this is an insulator phase stabilized by the dissipation. As one increases $\Delta$ with $\Delta > -2.15t$, the number of atoms on each site begins to fluctuate. If one looks at the pair correlation $\langle b_i^\dagger b_j \rangle$, it shows quasi-long-range behavior with slow algebraic decay. In Fig. 4(a), we show this correlation in the $k$-space, defined as $P_k = (1/N) \sum_{\rho=0}^N \langle b_k^\dagger b_{k+\rho} \rangle e^{ik\rho}$. The correlation $P_k$ is peaked sharply at $k = 0$. This corresponds to the $p$-wave superfluid phase. The $p$-wave character is inherited from the $p$-wave symmetry of the dressed molecule in space $b_i$ (or the atomic pair on the same site). The $p$-wave nature of the pairing is also manifested in the atomic pair wavefunction at different sites $\langle a_i^\dagger a_j \rangle$, which is obviously antisymmetric under exchange of the sites. In the one-dimensional case, the $p$-wave superfluid state is characterized by a quasi-condensate of the atomic pairs with a diverging pairing susceptibility. If one allows weak coupling between the one-dimensional tubes in the optical lattice, the $p$-wave quasi-condensate can easily be stabilized into a real condensate with a true long range pairing order.

If one further increases $\Delta$ in Fig. 3(a), one enters a phase where the total particle number per site $n = 2\pi_b + \pi_a$ is fixed at 1 (although the double occupation probability $\langle b_i^\dagger b_i \rangle$ varies with $\Delta$). This is a Mott insulator state with a finite gap to charge excitations. This can be seen more clearly in Fig. 3(b), where we fix the detuning $\Delta$, and show $\pi_b$ and $\pi_a$ as functions of the chemical
potential $\mu$. For this phase, the number density does not vary with $\mu$, so the system is incompressible as one expects for a Mott insulator phase. The correlation functions for this phase is shown in Fig. 4, where both the pair correlation and the charge density wave correlations are of short range. As one further moves to the right side in Fig. 3(a), there are two other phases: the normal mixture (NM) and the normal free gas (NFG). Both of these two phases are of a metallic nature with a finite compressibility (see Fig. 3(b)). The difference is that in the normal mixture phase, some sites are doubly occupied (with a finite $\langle b_i^\dagger b_i \rangle$). Several kinds of correlation functions for the normal mixture phase are shown in Fig. 4, and all of them decay rapidly with distance. In the NFG phase, the double occupation probability $\langle b_i^\dagger b_i \rangle$ reduces to zero, and one has a conventional single component free Fermion gas.

We have calculated the phase transition points for all $\Delta$ and $\mu$, and the result is summarized in Fig. 5 to give a complete two-parameter phase diagram. The five different phases are marked there. Under real experimental conditions, there is typically a weak global harmonic trap in addition to the optical lattice potential. As one moves from the trap center to the edge, the effective chemical potential $\mu$ gradually decreases under the local density approximation. So with a fixed interaction parameter $\Delta$, a line in the phase diagram of Fig. 5 across different $\mu$ gives the phase separation pattern of the Fermi gas in a harmonic trap. With a large positive $\Delta$, one has a Mott insulator state in the middle, surrounded by a normal Fermi gas. The phase transitions are most rich for small $|\Delta|$, where one can cross all the five different phases from the trap center to the edge. For large negative $\Delta$, the region of the $p$-wave superfluid state increases, but the Mott insulator and the normal mixture states eventually disappear when $\Delta < -3t$. As the $p$-wave superfluid state has a large stability region in the phase diagram, such a phase can be prepared experimentally by adiabatically ramping the Hamiltonian parameters following certain trajectories that suppress the three-particle occupation.

In summary, we suggest that the $p$-wave superfluid state near a Feshbach resonance can be stabilized in an optical lattice through a dissipation-induced blockade mechanism. We have derived the effective Hamiltonian to describe this strongly interacting system, taking into account the restriction of the Hilbert space due to this blockade mechanism. We solve the Hamiltonian in the anisotropic one-dimensional lattice through exact numerical calculations, and the result suggests rich phase transitions between the $p$-wave superfluid state and several kinds of insulator or metallic phases.

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