Recent advances in semi-analytical scattering models for NDT simulation

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Abstract. For several years, CEA-LIST and partners have been developing ultrasonic simulation tools with the aim of modelling non-destructive evaluation. The existing ultrasonic modules allow us to simulate fully real ultrasonic inspection scenarios in a range of applications which requires the computation of the propagated beam, as well as its interaction with flaws. To fulfil requirements of an intensive use (for parametric studies), the choice has been made to adopt mainly analytical approximate or exact methods to model the scattering of ultrasound by flaws. The applied analytical theories (Kirchhoff and Born approximations, GTD, SOV...) were already described in previous GDR communication. Over the years, this "semi-analytical" approach has been enriched by adaptations and improvements of the existing models or by new models, in order to extend the applicability of the simulation tools. This paper is devoted to the following recent advances performed in the framework of this approach:
- The SOV method based on the exact analytical model for the scattering from a cylindrical cavity has been extended in 3D to account for field variations along the cylinder. This new 3D model leads to an improvement in simulation of small side-drilled holes.
- Concerning the geometrical theories of diffraction (GTD), subroutines for calculation of the 2D wedge diffraction coefficients (for bulk or Rayleigh incident waves) have been developed by the Waves and Fields Group and uniform corrections (UAT and UTD) are under investigation.
- Modelling of the contribution of the head wave and creeping wave to the echoes arising from a wedge.
Numerous experimental validations of the developed models are provided. New possibilities offered by these new developments are emphasized.

1. Introduction:
The CIVA software platform is developed at CEA-LIST and partners in the aim of simulating non-destructive evaluation [1]. Most of the developed models are based on semi-analytical methods since they are not heavy on computation time.

The ultrasonic simulation tools in CIVA allow one to fully predict the real ultrasonic inspection in a range of applications which requires the computation of the beam propagated, as well as its interaction with flaws. The transducer field is calculated using a pencil method derived from the Rayleigh integral for acoustic radiation [2]. Indeed, inside the coupling material, the Rayleigh integral
is computed directly by summing the contributions to the field of each source point of the discretized transducer surface. In the specimen the pencil method which is an extension of ray theory [3] is applied.

The beam to flaw interaction is dealt with using different modelling approaches depending upon the defect and the inspection characteristics. Three kinds of models for the scattering of ultrasound by flaws have been integrated: approximate analytical solutions, exact analytical solutions and numerical modelling methods.

The developed approximate analytical solutions are respectively:

- the Kirchhoff approximation [4] to model specular reflexions from volumetric voids (spherical or hemispherical holes, SDHs) and cracks (rectangular, CAD or elliptical planar, FBHs, multifaceted). This approximation is mostly valid if the observation direction is close to reflexion and is particularly suitable to simulate specular reflexion, corner effects, etc. The corresponding integrated model requires the meshing of the defect surface.

- the geometrical theory of diffraction (GTD) to treat scattering from crack edges [5,6]. This approximation is valid away from specular angles and forward paths. The corresponding integrated model requires the meshing of the flaw contour.

- the modified Born approximation to deal with solid inclusions. It provides an analytical solution for some flaw geometries (spherical, cylindrical and ellipsoidal) without any meshing of the flaw [7].

An exact analytical solution for the scattering from a cylindrical cavity, based on the Separation Of Variables (SOV) method, has been used to simulate the response of a side drilled hole in a 2D configuration [4]. This model is discussed later.

The main general assumptions applied to deal with the application of the semi-analytical models were described in [4].

These previous methods have been experimentally validated in the most commonly used configurations [8]. A first example of experimental validation is now proposed. It deals with an application of the Kirchhoff approximation.

![Figure 1. Inspection of a plane specimen including several backwall breaking flaws of various heights.](image)

In the given example, a plane specimen including several backwall breaking flaws of various heights (from 2mm to 20mm) is inspected using a transducer generating both longitudinal and transverse waves. In figure 1 are shown the experimental and the simulated (using the Kirchhoff approximation).
approximation) obtained B-Scans when scanning the specimen. In the experimental B-Scans are observed notably the following echoes:
- the P wave corner echo (a) including a backwall reflexion without mode conversion;
- In a similar way, the shear wave corner echo (c);
- A mixed corner echo (b) due to mode conversion at the backwall;
All these echoes are quantitatively and qualitatively reproduced in simulation. See [8] for details.

The following parts of the paper will be devoted to the advances performed at CEA-LIST in analytical diffraction models.

2. The Separation of Variables method (scattering from a cylindrical cavity).

2.1. Interest and principle of this method
The semi-analytical Kirchhoff approximation, in a wide range of situations, leads to satisfactory quantitative predictions with a low computational overhead. Nonetheless, this model may cause some discrepancies in the prediction of small flaw responses, since it is a high frequency approximation. Particularly, experimental validations carried out in a pulse-echo configuration on small side drilled holes highlighted some differences between Kirchhoff simulation and the measured shear waves. These discrepancies could be explained by the theoretical principle of Kirchhoff approximation: it takes into account specular reflection on a cavity but not the creeping waves. These waves (called Franz waves [9]) propagate around the cavity circumference and may have a significant influence especially at low $ka$ (product of the wave number $k$ by the defect radius $a$). They are the analogues on curved surfaces of Stoneley waves (surface waves excited by grazing incidence) on flat surfaces. The creeping waves differ from Stoneley waves in two respects: they are attenuated because energy is radiated tangentially away from the cylinder; there are many circumferential modes $n$ of different speeds but the damping increases with $n$.

That is why the separation of variables (SOV) method has been investigated. This method [10] is used to obtain an exact analytical model for the scattering from a cavity. The principle of the SOV method has been described in detail in [4]: it enables one to obtain series developments of the scattered waves. Terms of these series expressed are products of Hankel and complex exponential functions. A parametric study has been performed to optimize the number of series terms to compute so as to ensure the series convergence. The SOV diffraction coefficients have been theoretically validated by comparison with the literature [11].

Two models of echo simulation based on the SOV method have been developed. The first model developed (presented in GDR 2008) is a 2D one: it supposes that the fields radiated by both the emitter and the receiver are invariant along the cylinder length. To relax the previous hypothesis, the development of a "2.5D" SOV model has been carried out. This model takes into account the field variation along the flash line which is the first cylinder line insonified by the beam assumed to be a plane incident wave. It supposes an invariant diffraction coefficient along the flash line. This SOV model in CIVA is developed for a cylinder.

2.2. Experimental validations
Numerous experimental validations of the SOV model have been performed [8,12] including comparisons with the Kirchhoff model. An example of experimental validation is presented in figure 2 below: it deals with inspection at 5MHz of two small side drilled holes (SDH) of respectively 0,5 and 1mm diameter (corresponding respectively to $ka=4.9$ and $ka=2.4$) with a SW45° contact transducer. The corner echo measured on a slot is taken as the amplitude reference. Given in the figure are comparisons of both maximum amplitudes and A-Scans corresponding to echoes obtained with several simulations (Kirchhoff, SOV and a finite element method FEM detailed in [4]) and with measure. The Kirchhoff model is unable to predict the creeping circumferential wave (chronologically the second and less intense observed echo) and leads to a discrepancy with the measurement because the creeping wave affects the maximum amplitude of the first echo (specular reflection on the cylinder). SOV and FEM give rise to a good agreement in amplitude with the measured result. The SOV model provides a
more satisfactory prediction of the creeping wave behavior in term of both time of flight and relative amplitude compared to the specular echo.

![Diagram showing relative amplitude comparison](image)

**Figure 2.** Inspection of a plane specimen including several backwall breaking flaws of various heights.

| Relative Amplitude (reference : slot) | Kirchhoff | SOV | FEM | Measure |
|--------------------------------------|-----------|-----|-----|---------|
| SDH 1mm                              | - 9.5dB   | - 11dB | - 13dB | - 13dB |
| SDH 0.5mm                            | - 10.9dB  | - 16.3dB | - 15.7dB | - 16.5dB |

3. Models derived from the Geometrical Theory of Diffraction (GTD)

3.1. Introduction

The Geometrical Theory of Diffraction (GTD) is commonly used to model scattering from flaw edges or wedges.

In the case of embedded cracks, two different strategies [6] have been investigated at CEA-LIST concerning the calculation of the GTD coefficients: a so-called “projected 2D” option, which is based on the projection of the incoming and scattered wave vectors over the plane normal to the flaw edge, and a “pure” 3D GTD code, using developments carried out by Larissa Fradkin and co-workers at London South Bank University (LSBU).

In addition to embedded cracks, the modelling of the scattering from wedges is under investigation at LSBU which is studying the performances of two different 2D codes for the wedge 2D GTD diffraction coefficient [13].
3.2. Simulation of edges diffraction from embedded cracks

The Kirchhoff approximation deals with specular reflection contrary to GTD but is not always accurate for diffracted echoes. Indeed, the Kirchhoff approximation lies in the fact that the near field scattered by a crack is approximated by the reflected field as stated by geometrical acoustics. On the other hand, the Geometrical Theory of Diffraction (GTD) is commonly used to deal with the prediction of diffraction phenomena. GTD is also an alternative model which relies on a ray theory [5]: contrary to geometrical acoustics, it is notably able to predict a non zero contribution to the diffracted field by a rectangular flaw in the shadow area.

Nevertheless GTD fails at predicting accurate diffracted amplitudes notably in the case the of direction of observation close to :

- the direction of specular reflection;
- the shadow boundary;
- the critical longitudinal angle for shear waves: in that case, the GTD coefficient is not continuous.

In the two previous directions, the GTD coefficient diverges and it is even invalid in a wide area in the vicinity of these directions (represented in the Figure below)

![Figure 3. Description of the geometrical reflected and shadow areas of a rectangular flaw.](image)

To improve the GTD predictions in these cases, our first step was to include a modification of GTD coefficients around these special observation directions.

3.2.1. Modification of GTD coefficients around special observation directions.

The developed GTD models are applied successfully in the majority of the inspection configurations and notably in TOFD inspections. But, they can provide problematic results in some complex cases notably for 3D configurations. Indeed the divergences or discontinuities of the GTD diffraction coefficients can lead to the presence of artefacts in the simulated results. Let us consider for instance a pulse-echo configuration of inspection of an elliptical crack (see Figure 5.a) which corresponds to an incidence with an angle quite critical to the normal to the flaw plane. Such a flaw can be detected thanks to the echoes scattered by the top and the bottom edges which can be distinguished in a true B-Scan (as those represented on Figure 5.b) superimposed on the flaw location.

In Figure 4.a and 4.b are respectively shown, versus the observation angle, the modulus and the phase of the 2D GTD coefficient for an incident angle corresponding to the previous configuration. In that pulse echo inspection, the observation angle is near the critical angle ($\theta = 57^\circ$) and consequently the GTD coefficient suffers from a discontinuity in the neighbourhood (in term both of module and phase). Besides, we observe a divergence of the coefficient at the specular direction ($\theta = 120^\circ$) and at the forward direction ($\theta = 240^\circ$) which is accompanied by a sign change ($\pi$ phase shift).

In order to deal with such problematic configurations with the GTD models, we have decided to include simple treatments to avoid the presence of artefacts in the simulated results. In the following figure are thus presented the simulated C-Scans obtained without and with the treatment. The B-Scan with the treatment is represented too (superimposition of the true reconstructed B-Scan on the flaw location) and shows that the elliptical crack could be detected thanks to the echoes scattered respectively by the top and bottom edges. These edges are easily observed in the treated C-Scan with two distinguishable lobes whereas the initial C-Scan highlights an artefact which prevents a satisfactory detection of the flaw.
Consequently this treatment provides a qualitative improvement of GTD in some complex configurations. Nevertheless specific experimental validations have to be carried out in these problematic configurations.

The limitations of the geometrical theory of diffraction show the interest of a more generic theory of diffraction. Our future step will be as a perspective to study and develop a model based on an uniform geometrical theory of diffraction [14] (UGTD) which merges the complementary theories of Kirchhoff and GTD in order to eliminate their drawbacks.

### 3.2.2. Theoretical principle of UGTD in the acoustic scalar case

The UGTD principle has already been studied for cracks embedded in a acoustic medium by LSBU. It consists, for a semi-infinite planar crack, in decomposing the Kirchhoff integral field $u^{KA}$ into a geometrical contribution $u^{KA,GE}$ and into an edge diffraction contribution. Outside shadow and reflected areas, the geometrical contribution $u^{KA,GE}$ models the sum of the reflected and incident fields and the edge diffraction contribution has the form in 2D of a cylindrical scattered wave weighted by a specific Kirchhoff diffraction coefficient $D^{Kirchhoff}$. Inside shadow and reflected areas, the expressions
of these two Kirchhoff contributions are modified thanks to a uniform asymptotic evaluation of the Kirchhoff integral [15]. Then the main hypothesis of the developed UGTD is to express everywhere the UGTD field as the sum of the geometrical Kirchhoff contribution and the GTD field:

$$u_{UGTD} = u_{KA,GE} + u_{GTD} = u_{KA} + (D_{GTD} - D_{Kirchoff}) \sum \frac{i}{2\pi} \frac{e^{ikr}}{\sqrt{kr}}$$

(1)

where r is the distance between the edge and the observation point and k the wave number. As a consequence, the UGTD method consists in adding the total Kirchhoff field and a modified GTD field (a cylindrical scattered wave in 2D) characterized by a diffraction coefficient which is the difference $D_{UGTD} = D_{GTD} - D_{Kirchoff}$. The diffraction coefficients can be found analytically and depend of the incident $\phi_0$ and observation $\phi$ angles versus the normal to the edge in the flaw plane:

$$D_{Kirchoff} = -\frac{1}{2} \left( \tan \frac{\phi - \phi_0}{2} + \tan \frac{\phi + \phi_0}{2} \right), \quad D_{GTD} = -\frac{1}{2} \left( \frac{1}{\cos \frac{\phi - \phi_0}{2}} + \frac{1}{\cos \frac{\phi + \phi_0}{2}} \right)$$

(2)

The UGTD diffraction coefficient $D_{UGTD} = D_{GTD} - D_{Kirchoff}$ has no singularities at shadow boundaries where the GTD coefficients diverge (for $\phi \pm \phi_0 = \pi$). The UGTD is able to model both reflection and diffraction phenomena and relaxes the numerical failures of the initial GTD at shadow boundaries. This approach could be applied in elastodynamics too.

An application of this theory will be studied with the aim of modelling the echoes due to immersed targets.

4. Modelling of creeping and head waves.

4.1. Modelling of the creeping wave reflection

4.1.1. Observation of discrepancies between simulated and experimental results for critical incidence angle at the backwall

The contribution of a head wave is usually small compared to the compressional and shear bulk waves in corner echo inspection, as mostly geometrical (specular or quasi specular) echoes are concerned. However, in some configurations, especially near critical angles (at backwall reflection, for instance), its contribution to echo response may be significant. This point is illustrated here by an experimental validation of corner echo prediction. In this study a stainless steel component composed of two different bottom slopes of $0^\circ$ and $10^\circ$ is considered. For each slope, the specimen contains a 10 mm height by 20 mm length rectangular backwall breaking flaw. The pulse-echo response of these defects is measured by a 45° refracted SV-waves planar contact transducer. Simulation of interaction between incident wave and flaw is performed with CIVA software (therefore, only using bulk waves contribution modelled thanks to the Kirchhoff model).

Figure 6 presents the geometry of the problem and the ray path of SV-mode corner echo for $0^\circ$ and $10^\circ$ configuration and the simulated and experimental B-Scan image, as well as the superposition of both echodynamic (amplitude versus scanning) curves. Results were normalized using a side drilled hole used as a reference. A very good agreement is observed for the $0^\circ$ configuration, while some important differences of amplitude and overall behaviours are observed between these two results for the $10^\circ$ slope configuration. For instance, quite a strong amplitude response is experimentally observed for such a configuration, while the probe is still “far” from the flaw, which means that an interaction occurs before the shear wave beam intersects the flaw. As the presented simulation only takes into account interactions of bulk waves with the flaw, it becomes obvious that for this
configuration it is necessary to simulate head and creeping waves interaction (the shear waves beam hits the backwall with a nominal incidence angle of 35°, close to the critical angle, about 33° in stainless steel).

![Diagram](image)

**Figure 6.** Experiment and Simulated B-Scan images and superimposed echodynamic curves of planar (left) and 10° backwall slope mock-ups with one vertical flaw inspected with a contact 45° shear waves probe.

### 4.1.2. Simulation of the creeping wave reflection

The contributions related to interactions of head and creeping waves with the flaw were not taken into account in the simulation until now. A time of flight analysis based on geometrical ray tracing has highlighted a potential interaction between a creeping wave and the flaw to explain the observed differences between simulated and experimental results in the 10° slope configuration. Geometrical paths related to this contribution are presented in figure 8.a). This interaction (a) corresponds to the reflection of the creeping wave on the flaw bottom. Indeed, at an incident critical angle, a creeping wave is generated at the backwall and is reflected on the flaw tip into another creeping wave. During its propagation along the backwall, this creeping wave will then generate a head wave propagating with an angle to the backwall normal equal to the critical angle.
In previous articles [16, 17], we have described the modelling of the creeping wave reflection (Figure 9.a) at a tip of a wedge made up of a backwall surface and a backwall breaking flaw. This model is based on a ray theory [3].

4.1.3. Experimental validation of the creeping wave reflection simulation

Modelling of the creeping wave reflection is now applied to simulate the echo response for the 10° slope configuration. To the initial simulation which models only interactions of the bulk wave with the flaw is added the contribution of the head wave radiated by a reflected creeping wave on the flaw bottom. The B-scan and the spatial repartition obtained with simulation is presented and compared to experiment in Figure 7. We can see by comparing figures (a) and (b) a good agreement between experimental and calculated B-scans. On figure (c) is shown first the initial calculation which models only bulk wave interactions. We observe then that the amplitude calculated with the addition of head wave improves modelling compared to experimental results (black line).

![Figure 7](image)

**Figure 7.** Measured (a) and calculated the addition of a head wave (b) B-scans of corner echo, and spatial distribution of corner echo (c) obtained with experiment (black line), finite element calculation (dotted grey line), CIVA calculation (dashed grey line) and CIVA calculation with add of head wave (grey line), according to the 10 degrees slope bottom of the specimen.

On this figure the result obtained with a finite element method FEM detailed in [4] is presented too (dotted grey line). We notice that it also provides a good prediction of experimental results.

4.2. Other contributions of creeping waves and head waves

Henceforth, the developed tools have been extended to simulate the reflection of a creeping wave which is generated along the flaw surface (Figure 8.b). Further, it is possible to take into account a backwall reflection before the generation of the creeping wave so that the contributions c) and d) (Figure 8) can be modelled.
4.3. Account of a creeping wave after a backwall reflection

To simulate the creeping wave reflection (Figure 9.a) at a wedge tip, we previously modelled [16, 17] the head wave amplitude. Indeed the displacement amplitude of the head wave received at a point \( S_j \) on the probe and due to the reflection of the creeping pressure wave at the bottom of the flaw is:

\[
\mathbf{u}_s(S_j, \omega) = -T_{cw} R_{w-c} v_0(\omega) e^{i(\omega t - \tau)} \frac{1}{2\pi V_w} \frac{R_s' R_{ls}' V_l}{r^{\frac{3}{2}} l^{\frac{1}{2}}} \tan(\theta') \Delta S_{rw} \mathbf{n}_w ,
\]

with \( \Delta S_{rw} \) the surface which surrounds the point source \( S_j \), \( v_0 \) the particular velocity normal to probe surface, \( T_{cw} \) the transmission coefficient from wedge to component (incoming longitudinal wave in the wedge to shear wave radiated inside the material), \( T_{cw} \) the transmission coefficient from component to wedge. Further, \( \mathbf{n}_w \) is the polarization direction of the wave propagating in the wedge with the velocity \( V_w \), and \( V_l \) is the longitudinal velocity in the component. The coefficients \( R_s' \) and \( R_{ls}' \) are respectively normalized reflection coefficient [3] from shear wave to longitudinal wave and from longitudinal wave to shear wave at the critical angle \( \theta' \). Distances \( r = r + r_j \) and \( l = l_j + l_s \) are represented in the following figure 9.a.
To take into account a backwall reflection before the generation of the creeping wave, we extend the model developed to deal with the configuration without backwall reflection (figure 9.a). In that goal, we employ a mirror strategy to model the backwall reflection. The principle of this mirror effect is described on figure 9.b, for an incident path from the source to the wedge tip. A reception path from the wedge tip to a point $S_j$ on the crystal surface can be involved similarly. The point $S'_i$ is the image of the source point $S_i$ by symmetry with respect to the backwall surface and then $S'_i$, is the projection of $S'_i$, on the entry surface where the creeping wave propagates (in green on figure 9.b). In the case of a backwall reflection, the entry surface is the analogue of the backwall surface for the case without backwall reflection. Consequently, in case of no mode conversion at the backwall, the distance $r_i$ is calculated along the entry surface between $S'_i$ and the wedge tip. Finally, the amplitude of the head wave at a point $S_j$ on the probe is modelled using (3) by defining distances $r = r_i + r_j$ and $l = l_i + l_j$ thanks to the mirror strategy and by adding a plane wave reflection coefficient on the backwall.

Experimental measurements have to be carried out on both surface and backwall breaking flaws to validate the modelling of the creeping wave reflection along flaws with or without a backwall reflection.

5. Conclusion
This paper was devoted to the description of recent advances performed in the modelling of diffraction by cracks in the context of NDE. Three " semi-analytical" approaches, which have been enriched or extended in order to extend the applicability of the simulation tools, have been presented. Numerous experimental validations have been carried out for each developed model so as to highlight their accuracy.

The first invoked method is an exact analytical model based on Separation Of Variables (SOV): it has been developed for a cylindrical cavity without flaw disorientation (2,5D model). This SOV model provides a better prediction than the Kirchhoff approximation for SV waves and small flaws. It could be extended to a 3D configuration and to a spherical void.

We focused then on models derived from the Geometrical Theory of Diffraction (GTD) which is commonly used to deal with diffraction phenomena. A modification of the GTD coefficients has been included around special observation directions to improve the GTD simulation of the scattering from cracks edges. In future, progress towards uniform corrections of GTD in a view of a generic model (able to treat both reflection and diffraction effects) are envisaged. New GTD developments could enable also the treatment of flaws in anisotropic media (GTD coefficients for transversely-isotropic
media) and the modelling of the scattering from wedges. The study of the interaction of a flaw with a Rayleigh surface wave is in progress.

As to the modelling of creeping and head waves, the simulation of the reflection of a creeping wave on a wedge tip is henceforth available if this wave propagates along a specimen or a flaw surface even after one backwall reflection. Experimental validations have confirmed that the account of the head wave reflection must be included in the simulation.

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