Simulation electromagnetic scattering on bodies through integral equation and neural networks methods

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Abstract. The paper deals with the issue of electromagnetic scattering on a perfectly conducting diffractive body of a complex shape. Performance calculation of the body scattering is carried out through the integral equation method. Fredholm equation of the second time was used for calculating electric current density. While solving the integral equation through the moments method, the authors have properly described the core singularity. The authors determined piecewise constant functions as basic functions. The chosen equation was solved through the moments method. Within the Kirchhoff integral approach it is possible to define the scattered electromagnetic field, in some way related to obtained electrical currents. The observation angles sector belongs to the area of the front hemisphere of the diffractive body. To improve characteristics of the diffractive body, the authors used a neural network. All the neurons contained a logsigmoid activation function and weighted sums as discriminant functions. The paper presents the matrix of weighting factors of the connectionist model, as well as the results of the optimized dimensions of the diffractive body. The paper also presents some basic steps in calculation technique of the diffractive bodies, based on the combination of integral equation and neural networks methods.

1. Introduction

Among the most significant tasks in the course of the technical units development, one should point out the issues of studying the processes of waves scattering on physical bodies of a complex shape for the radar frequency band length in the resonance area [1, 2].

The authors are going to examine the solution of the problem by means of the integral equation method.

2. Materials and methods

Let us make the recording of the Fredholm integral equation of the second time for the electrical current taking into account the boundary conditions for the surface of the perfectly conducting metal body [3, 4] (fig. 1)
Figure 1. Pattern of radio waves scattering on perfectly conducting metal body (side elevation)

\[ J_S(r) = 2n \times H^i(r) + \frac{1}{2\pi} n \times \int_s J_s(r) \times \nabla G ds', \quad (1) \]

where \( G = \exp(-jkr)/r \) presents a three-dimensional Green function which relates to the free space. It is characterized as a solution for the Helmholtz equation for the case of the \( \delta \)-shaped source; \( s \) describes the body’s surface; \( n \) is associated with the description of the outward normal to the body surface for the observation point;

\( J_S = [n \times H] \) – represents the surface density of the equivalent electrical current; \( H^i(r) = i^i_z + i^i_y + i^i_z \) is a vector of the plane incident electromagnetic wave, which is vertically polarized.

First it is necessary to determine unknown surface electrical currents \( J_s(r) \) flowing over the surface of the body. To do this, the authors have used the moments method to solve the integral equation (1) \([5-7]\). Having defined the system of basic functions, the authors have expanded the electrical currents over the surface of the body.

For calculation, the authors used piecewise constant functions. Afterwards this task was to define the system of testing functions. As testing functions the authors used Dirac \( \delta \)-functions. The use of such system in its physical interpretation demonstrates that providing boundary conditions for a magnetic field is not appropriate for the whole surface of body \( S \), but only for the digital number of its dots. The analysis of the simulation processes of electromagnetic scattering on the irregular-shaped bodies shows \([8, 9]\) that the exact accuracy of the solutions is achieved in case of choice of the required number of dots.

Having employed the moments method, the authors modified the integral equation (1) into the system of equations.

\[
\begin{bmatrix}
U_{xx} & U_{xy} & U_{xz} \\
U_{yx} & U_{yy} & U_{yz} \\
U_{zx} & U_{zy} & U_{zz}
\end{bmatrix}
\begin{bmatrix}
J_x \\
J_y \\
J_z
\end{bmatrix}
= \begin{bmatrix}
R_x \\
R_y \\
R_z
\end{bmatrix},
\quad (2)
\]
where $J_x, J_y, J_z$ represent the components of the electrical currents surface density.

To calculate matrix blocks in the system of equations (2), it is necessary to use the following expressions:

\[
(U_{xx})_{mn} = \frac{1}{2\pi} \int_I ((n_x)_m (\text{grad}_{x}^{'})_{mn} + (n_z)_m (\text{grad}_{z}^{'})_{mn}) ds_n - \delta_{mn};
\]

\[
(U_{xy})_{mn} = -\frac{1}{2\pi} \int_I (n_y)_m (\text{grad}_{x}^{'})_{mn} ds_n; (U_{xz})_{mn} = -\frac{1}{2\pi} \int_I (n_z)_m (\text{grad}_{x}^{'})_{mn} ds_n;
\]

\[
(U_{yx})_{mn} = -\frac{1}{2\pi} \int_I (n_x)_m (\text{grad}_{y}^{'})_{mn} ds_n; (U_{yz})_{mn} = -\frac{1}{2\pi} \int_I (n_z)_m (\text{grad}_{y}^{'})_{mn} ds_n;
\]

\[
(U_{yy})_{mn} = \frac{1}{2\pi} \int_I ((n_x)_m (\text{grad}_{y}^{'})_{mn} + (n_z)_m (\text{grad}_{y}^{'})_{mn}) ds_n - \delta_{mn};
\]

\[
(U_{zx})_{mn} = -\frac{1}{2\pi} \int_I (n_x)_m (\text{grad}_{z}^{'})_{mn} ds_n; (U_{zy})_{mn} = -\frac{1}{2\pi} \int_I (n_y)_m (\text{grad}_{z}^{'})_{mn} ds_n;
\]

\[
(U_{xz})_{mn} = \frac{1}{2\pi} \int_I ((n_x)_m (\text{grad}_{x}^{'})_{mn} + (n_y)_m (\text{grad}_{y}^{'})_{mn} ds_n - \delta_{mn},
\]

where $m, n = 1, \ldots, N$, where $N$ represents the number of dots needed for the diffractive body surface partition.

In the expressions mentioned above (3), the authors introduced labeling $\delta_{mn}$, Kronecker’s delta:

\[
\text{grad} G_{mn} = -\hat{r}_{mn} \frac{1+ jkr_{mn}}{r_{mn}^2} \exp(-jkr_{mn}) = \hat{r} \text{grad}_{x}^{'}, (\text{grad}_{y}^{'})_{mn}, \text{grad}_{z}^{'},
\]

where $\hat{r}_{mn} = \frac{r_{mn}}{|r_{mn}|}$, which is calculated as a unitary vector, coming from the destination point to the observation point [10].

In the system of equations, the column-vector of the absolute terms is determined by the following expressions:

\[
(R_x)_m = 2((n_y)_m (H_y^i)_m - (n_z)_m (H_x^i)_m);
\]

\[
(R_y)_m = -2((n_x)_m (H_x^i)_m - (n_z)_m (H_y^i)_m);
\]

\[
(R_z)_m = 2((n_x)_m (H_y^i)_m - (n_y)_m (H_x^i)_m).
\]

After solving the indicated system of equations (2), it is necessary to calculate the scattered electromagnetic field corresponding to the far-field region. There is a link between that field and calculated electrical current $J_\delta(r)$ based on the following expression:
\[
H_{sc}^{\text{sc}}(r) = \frac{j\alpha}{4\pi} \sqrt{\mu_0 \varepsilon_0} \exp(-jk r) \int_s \mathbf{J}_s(r') \times \hat{r} \exp(jkr \cdot r') \, ds',
\]

where \( k \) – a wavelength constant;
\( \mu_0 = 4 \cdot \pi \cdot 10^{-7} \, \text{H} / \text{m} \), \( \varepsilon_0 = 8.85 \cdot 10^{-12} \, \text{F} / \text{m} \) is a magnetic and electrical constant;
\( r' \) – a radius-vector of the destination point;
\( r \) – a radius-vector of the observation point, corresponding to the far-field region.

After that let us calculate the effective scattering pattern of the examined diffractive body on the basis of the following formula [8-10]:

\[
\sigma = \lim_{r \to \infty} 4\pi r^2 \left| \frac{H_{sc}^{\text{sc}}(r)}{H^1(r)} \right|^2.
\]

3. Results and Discussion

Difference between the prescribed value of the effective scattering pattern for the given observation angle and the value of the effective scattering pattern relating to the synthesized body was considered as an objective function. The authors also examined a two-level feedforward neural network, synthesized through the base of the evidences of the estimated search space point and containing 5 neurons in the first layer and 1 neuron in the second layer. All the neurons contained a logsigmoid activation function and weighting factors as discriminant functions.

The neural networks can also be used for solving the system of equations that is coming from discretizing the integral equation of electromagnetic scattering. But, the neural networks size can be intractable because of the large number of the moment matrix elements. When the problem is considered, the process of selection for the elements in the moment matrix is very crucial.

For the examined bodies given in figure 1, the authors obtained the following results: for observation angle \( \theta = 40^\circ \) and the level of the effective scattering pattern of 15 dB: a) \( a=1.7\lambda, \ b=0.25\lambda \); b) \( d=0.71\lambda, \ f=3.6\lambda, \ g=0.45\lambda \), in the direction perpendicular to the figure, the body dimension was 1.6\( \lambda \).

The basic stages in the technique of the calculating diffractive body of an irregular shape may be enumerated as the following:

1. Determining the initial dimensions of the diffractive structure.
2. Calculating scattering characteristics through the method of integral equations.
3. Optimizing characteristics of the diffractive body through the method neural networks.
4. Results output.
Table 1. Matrix of the weighting factors in a connectionist model

| Layer number | Neuron number in the layer | Number of the neuron penetration |
|--------------|----------------------------|---------------------------------|
|              | 1                          | 2                               |
| 1            | 11.2                       | -27.4                           |
|              | 2                          | -28.3                           |
|              | 3                          | 17.5                            |
|              | 4                          | 55.4                            |
|              | 5                          | 15.2                            |
| 2            | -498                       | -564                            |

4. Conclusion
In the paper, the problem electromagnetic wave scattering on the body with complex shape is considered. The moment method approach is used as a powerful tool for solving the integral equation. For optimization scattering characteristics, the neural nets method is used. The examined method can be employed to the wide range of the electrodynamic bodies.

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