Mission-Aware Medium Access Control in Random Access Networks

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Abstract

We study mission-critical networking in wireless communication networks, where network users are subject to critical events such as emergencies and crises. If a critical event occurs to a user, the user needs to send necessary information for help as early as possible. However, most existing medium access control (MAC) protocols are not adequate to meet the urgent need for information transmission by users in a critical situation. In this paper, we propose a novel class of MAC protocols that utilize available past information as well as current information. Our proposed protocols are mission-aware since they prescribe different transmission decision rules to users in different situations. We show that the proposed protocols perform well not only when the system faces a critical situation but also when there is no critical situation. By utilizing past information, the proposed protocols coordinate transmissions by users to achieve high throughput in the normal phase of operation and to let a user in a critical situation make successful transmissions while it is in the critical situation. Moreover, the proposed protocols require short memory and no message exchanges.

Index Terms — Mission-critical networking, MAC protocols, slotted Aloha, memory-based protocols.

1 Introduction

Network users may face critical situations where life or livelihood is at risk. Examples include a fire in a building, a natural disaster in a region, a heart attack of a patient, and a military attack by an enemy. When a network user detects a critical event, it is important for the user to inform relevant rescue parties of the event as early as possible so that they can take the necessary measures to mitigate the risk or help affected parties recover. This paper is concerned about delay in the transmission of information about critical events in mission-critical networking, which occurs between the detection of critical events by a network user and the response to them by a rescue party.

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We consider wireless communication networks in which users share a common channel and contend for access. We approach the problem of dealing with critical situations from a protocol designer’s perspective at the medium access control (MAC) layer. Since multiple packets transmitted at the same time result in a collision, MAC protocols are used to coordinate transmissions by users. Distributed coordination function (DCF), widely deployed in the IEEE 802.11a/b/g wireless local area network (WLAN) [1], does not differentiate users, and thus it is unable to coordinate the behavior of users in the event of critical situations so that a user in a critical situation uses the channel while others wait.

The enhanced version of DCF, called enhanced distributed channel access (EDCA), is deployed in IEEE 802.11e [2] and does differentiate users according to their access categories. EDCA specifies different contention window sizes and arbitration interframe spaces to different access categories, yielding a smaller medium access delay and more bandwidth for the higher-priority traffic categories [3]. However, EDCA is designed to support applications requiring quality-of-service, and as such it is not directly applicable to mission-critical networking in wireless networks. In particular, a user having highest-priority data shares the channel with other users. Although it obtains higher throughput than others, EDCA does not allow it to “capture” the channel until it finishes transmitting the highest-priority data.

In this paper, we discuss the problem of the protocol designer mainly in the context of a slotted Aloha system. The protocol designer cares about total throughput and fairness in the normal phase, in which there is no critical situation, while he is concerned about delay in the critical phase. Also, he takes the complexity of protocols into consideration in both phases. We show that the dual objective of the protocol designer — maximizing throughput and fairness in the normal phase while minimizing delay in the critical phase — can be achieved by a class of MAC protocols utilizing past information. The proposed protocols have the following desirable properties:

1. The system achieves high total throughput while yielding equal throughput to individual users in the normal phase of operation, when no user is in a critical situation.

2. Should a critical event occur, the user in a critical situation captures the channel after a short delay while other users wait until it transmits all the necessary information.

3. The protocols can be implemented without any message exchange. In particular, they do not require users to know whether other users are in a critical situation or not.

4. The protocols are based on short memory, thus requiring only a small memory space for each user.

Slotted Aloha was first introduced in [4]. Recently, the framework of game theory is used to analyze the noncooperative or cooperative behavior of users in slotted Aloha [5]–[9]. In [5], the strategy, or the decision rule, for a user is simply its transmission probability used over time to attain its desired throughput. In [6], the number of users contending for the channel varies over time, and users know the number of users currently in the system. The decision rule for a user used
in [6] is its transmission probability as a function of the number of users. Altman et al. [7] assume that information on the number of users in the system is unavailable to users and that newly arrived packets are always transmitted. The decision rule in their model is the transmission probability for backlogged packets. A correlation device is used in [8]. With the presence of a correlation device, the decision rule for a user considered in [8] is its transmission probability depending on random signals generated by the correlation device. Ma et al. [9] define two states for users, a free state and a backlogged state, and relax the assumption of [7] that newly arrived packets are always transmitted. The decision rule for a user in their model is two transmission probabilities used in each state.

In the game theoretic models above, the strategies are those in one-shot games even though interactions among users are repeated. That is, authors consider transmission strategies based only on current information (for example, the number of users, correlation signals, and the state of packets) in contrast to early work that considers transmission probabilities updated based on the histories of feedback information on the channel states (for example, [10] and [11]). We consider strategies as those in repeated games that depend not only on current information but also on past information. By opening up this possibility, we can design a simple distributed protocol that performs well both when there is a critical event and when there is none.

The rest of the paper is organized as follows. We describe the model in Section 2 and formulate the problem of the protocol designer in Section 3. In Section 4, we investigate the various trade-offs that the protocol designer faces and introduce our mission-aware MAC protocols. We extend the protocols to more general scenarios in Section 6. We conclude the paper in Section 7.

2 Model

We consider an idealized slotted Aloha system as in [12]. Users (pairs of transmitter-receiver nodes) share a communication channel though which they transmit packets. The total number of users is \( N \), and the set of users is denoted by \( \mathcal{N} = \{1, \ldots, N\} \). We assume that the number of users is fixed over time and known to users. In the case that users do not know the total number of users, they can estimate it by using techniques such as the one in [13], and the MAC protocols in this paper can be modified by replacing the actual number of users with an estimate.

Time is slotted, and slots are synchronized. We label slots by \( t = 1, 2, \ldots \). Packets are of the same size, and each packet requires one slot for transmission. A user always has a packet to transmit and makes a decision on whether to transmit or not in every slot [5] [9]. The action space of a user can be written as \( A = \{T, W\} \), where \( T \) stands for “transmit” and \( W \) for “wait.” We denote the action of user \( i \) by \( a_i \in A \) and an action profile or outcome by \( a = (a_1, \ldots, a_N) \). The set of outcomes is denoted by \( A \triangleq A^N \).

A packet is successfully transmitted if it is the only transmission in the slot. If there is more than one transmission, a collision occurs. If the transmission of a packet results in a collision, it

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1 Whether the current packet is new or backlogged is affected by past outcomes, but it contains very limited information about the past and can be considered as the “label” of the current packet.
is retransmitted in some later slot until it is successfully received. We assume that user $i$ senses whether the channel is idle (no transmission) or busy (at least one transmission) when it waits. We also assume that the receiver node sends an acknowledgement signal to the transmitter node when the transmission is successful. In this way, a user learns whether its transmission is successful (success) or not (failure). Hence, from a user’s point of view, there are four possible channel states, and we define the set of channel states by $S \triangleq \{\text{idle, busy, success, failure}\}$. We use $s_i \in S$ to denote the channel state of user $i$.

The system is subject to critical events such as emergencies and crises. When a critical event occurs, it assigns a user to carry out a mission of describing it to a rescue party. The amount of information required to describe the critical event depends on the nature of the particular event, and we model this feature by assuming that the number of packets required to complete a mission is determined by a random variable $X$. $X$ takes a value of a positive integer, and we use $x$ to denote the realized value of $X$. We call $x$ the length of a mission. We assume that $x$ is known only to the user to whom the mission is assigned. We say that a user is in a critical situation if it has a mission and in a normal situation otherwise. We denote the situation of user $i$ by $y_i \in Y$ where $Y \triangleq \{\text{normal, critical}\}$. We use $\mathbf{y} \triangleq (y_1, \ldots, y_N) \in \mathcal{Y} \triangleq Y^N$ to denote the entire situations of the system.

We say that the system is in the normal phase when every user is in a normal situation and in the critical phase when some user is in a critical situation. We assume that there can be at most one mission in the system at a time. We find this assumption realistic, considering typically a small number of users who share a wireless channel and the low frequency of critical events. Our mission-aware MAC protocols are developed based on this assumption, but we relax this assumption later in Section 5 and show that the protocols can be modified to deal with multiple missions at a time.

A user knows about its own past and current situations as well as its own past channel states. We define the history of user $i$ in slot $t$ as all information that user $i$ has at the beginning of slot $t$, which can be written as

$$H^t_i = (y_1^i, s_1^i; \ldots; y_{t-1}^i, s_{t-1}^i; y_t^i),$$

for $t = 1, 2, \ldots$. Let $\mathcal{H}_t \triangleq (Y \times S)^{t-1} \times Y$ be the set of all possible histories for a user in slot $t$. Then the set of all possible histories can be defined by $\mathcal{H} \triangleq \bigcup_{t=1}^{\infty} \mathcal{H}_t$.

A decision rule specifies a transmission probability following each history, and thus it can be represented by a mapping from $\mathcal{H}$ to $[0, 1]$. Let $\mathbb{N}_+ \triangleq \{0, 1, \ldots\}$ be the set of nonnegative integers. If a decision rule depends only on information obtained in the recent $m$ previous slots and the current slot, we say that it is based on $m$-period memory where $m \in \mathbb{N}_+$. Let $\mathcal{L}_m \triangleq (Y \times S)^m \times Y$ be the set of all $m$-period histories. Then a decision rule based on $m$-period memory can be written as

$$f_t : \mathcal{L}_m \rightarrow [0, 1].$$
\( f_t(L^t_i) \) gives the transmission probability for user \( i \) in slot \( t \) when the recent \( m \)-period history of user \( i \) is

\[
L^t_i = (y^{t-m}_i, s^{t-m}_i, \ldots, y^{t-1}_i, s^{t-1}_i, y^t_i),
\]

for \( t = 1, 2, \ldots \). We set \((y^{t'}_i, s^{t'}_i) = (\text{normal, idle})\) for \( t' \leq 0 \) as a default. A decision rule based on \( m \)-period memory is said to be stationary if it is independent of \( t \). Let \( F_m \) be the set of all stationary decision rules based on \( m \)-period memory. Then the set of all stationary decision rules based on finite memory is obtained by \( F = \bigcup_{m=0}^{\infty} F_m \). Given two nonnegative integers \( m_1 \) and \( m_2 \) with \( m_1 > m_2 \), we say that \( f \in F_{m_1} \) is equivalent to \( g \in F_{m_2} \) if \( f(L) = g(L') \) where \( L' \) is obtained by deleting information in the first \((m_1 - m_2)\) slots of \( L \), for all \( L \in L_{m_1} \). If \( f \in F_{m_1} \) is equivalent to \( g \in F_{m_2} \), then \( f \) can be implemented using only \( m_2 \)-period memory, and thus it can be rewritten as a decision rule based on \( m_2 \)-period memory.

We define a protocol as a profile of stationary decision rules based on finite memory \( f = (f_1, \ldots, f_N) \in F^N \). Given a protocol, we can derive four objects: 1) throughput, 2) short-term fairness, 3) expected average delay, and 4) complexity. We assume that the protocol designer cares about 1) and 2) in the normal phase and 3) in the critical phase. The definitions and the importance of these objects will be explained in the next section. The protocol designer is concerned about 4) overall. The complexity of a protocol can be defined as follows.

Given a protocol \( f \), we first define

\[
m_i = \min\{m \in \mathbb{N}_+ | \exists g \in F_m \text{ such that } f_i \text{ is equivalent to } g\},
\]

for each \( i \in \mathcal{N} \). Then \( m_i \) is the minimum length of memory required to implement the decision rule \( f_i \). We take the maximum of \( m_i \) across users to obtain

\[
m^*(f) = \max\{m_1, \ldots, m_N\}.
\]

Then \( m^*(f) \) is the minimum length of memory required to implement the protocol \( f \), and we say that the protocol \( f \) is based on \( m^* \)-period memory. Intuitively, a protocol is simpler when it is based on shorter memory. Thus, we call \( m^*(f) \) the complexity level of the protocol \( f \).

We assume that one of the objectives of the protocol designer is to prescribe a protocol with low complexity. In other words, the protocol designer is inclined to prescribe protocols based on short memory, for example, one-period memory. Considering the large memory spaces of computing devices, one may find that using decision rules based only on one-period memory is too restrictive. However, one-period memory-based decision rules are easy to follow and robust to variations on memory and computation constraints. Suppose that the protocol designer is uncertain about the memory and computation capacities of individual users. If a failure to follow the prescribed decision rule by a single user results in a total breakdown of the system, then the protocol designer wants to provide a simple protocol to ensure that every user can follow it. Moreover, analysis with decision
rules based on one-period memory is meaningful in that the performance of protocols based on one-period memory provides a lower bound on that of more complicated protocols based on longer memory.

3 Problem Formulation

We first consider the problem of the protocol designer separately in the normal phase and in the critical phase. After discussing the sub-problems in the two phases, we combine them to formulate the overall problem of the protocol designer.

3.1 Problem in the Normal Phase

We consider a time horizon during which there is no critical situation. In this case, \( y_i^t = \text{normal} \) for all \( i \in \mathcal{N} \) and \( t \) in the horizon. Since \( y_i \) is constant, we can reduce the domain of a stationary decision rule based on \( m \)-period memory from \( \mathcal{L}_m \) to \( S^m \). If a protocol \( f \) has a complexity level \( m^* \), then a Markov model can be constructed where the state space of the Markov chain is \( \mathcal{A}^{m^*} \). If \( f \) is chosen so that the induced Markov chain has only one ergodic class, then there exists a unique stationary distribution \( \pi \) on \( \mathcal{A}^{m^*} \) \([13]\). We define the throughput of user \( i \) by

\[
\tau_i(f) \triangleq \sum_{(a_1, \ldots, a_{m^*}) \in \mathcal{A}^{m^*}} \pi(a_1, \ldots, a_{m^*}) \left( \frac{1}{m^*} \sum_{m=1}^{m^*} I(a_m = a^i) \right),
\]

where \( m^* = m^*(f), a^i \in \mathcal{A} \) is the outcome in which only user \( i \) transmits, and \( I \) is the indicator function. That is, the throughput of user \( i \) is the frequency of its success in steady state. The total throughput of the system is defined by

\[
\tau(f) \triangleq \sum_{i=1}^{N} \tau_i(f),
\]

and the throughput profile by

\[
\tau^*(f) \triangleq (\tau_1(f), \ldots, \tau_N(f)).
\]

The protocol designer can evaluate the throughput profile at least in two aspects. First, he can measure the utilization of the channel by total throughput. Hence, considering the efficiency of protocols, he wants to obtain high total throughput. Second, he may have some preferences over the distributions of total throughput to users. This is related to QoS differentiation. In some cases, he may prefer to treat every user equally. In other cases, he may want to yield different throughput to different users in a certain proportion.

Given a protocol \( f \), we can compute the expected number of slots with consecutive successes of user \( i \) in steady state. Let \( \theta_i \) be the reciprocal of this expected value. Then \( \theta_i \in [0, 1] \), where \( \theta_i = 0 \)
means that the expected value is infinity. We take the minimum of \( \theta_i \) to obtain

\[ \theta^*(f) \triangleq \min\{\theta_1, \ldots, \theta_N\}, \tag{9} \]

and call \( \theta^*(f) \) the short-term fairness level of the protocol \( f \). As \( \theta^* \) gets larger, the expected duration of slots in which the channel is used by one user becomes shorter. Thus, the protocol designer prefers a protocol with a high short-term fairness level to guarantee periodic usage of the channel by users.

Summarizing the discussion so far, the protocol designer’s problem in the normal phase can be formulated as

\[ \text{(P-Norm)} \max_{f \in \mathcal{F}^N} U_N(\tau^*(f), \theta^*(f), m^*(f)), \tag{10} \]

where \( U_N \) is the utility function of the protocol designer in the normal phase, defined on \([0, 1]^N \times [0, 1] \times \mathbb{N}_+\). To make the utility function consistent with the preferences of the protocol designer, we assume that \( U_N \) is increasing in \( \tau_i(f) \), for each \( i \in \mathcal{N} \), and \( \theta^*(f) \) and decreasing in \( m^*(f) \).

### 3.2 Problem in the Critical Phase

Now we consider a time horizon from the start to the end of a mission. Suppose that a mission is assigned to user \( i \) in slot \( t_0 \) and that user \( i \) completes its mission in slot \( t_1 \). Then for \( t = t_0, \ldots, t_1 \), \( y_i^t = \text{critical} \) and \( y_j^t = \text{normal} \) for \( j \neq i \). The number of slots needed to complete the mission is \( \hat{x}_i = t_1 - t_0 + 1 \). Once a protocol \( f \in \mathcal{F}^N \) and the \( m^* \)-period histories of users in slot \( t_0 \) \( L \triangleq (L_1, \ldots, L_N) \in \mathcal{L}_m^N \) are specified, where \( m^* = m^*(f) \), we can determine the probability distribution over the number of slots required for user \( i \) to complete the transmission of \( x \) packets. Thus, \( \hat{x}_i \) can be considered as a realization of a random variable, called \( \hat{X}_i \), whose probability distribution depends on \( x \), \( L \), and \( f \). We define \( \bar{X}_i(x, L, f) \) as the expected value of \( \hat{X}_i \) given \( x \), \( L \), and \( f \). We also define

\[ \bar{D}_i(x, L, f) \triangleq \bar{X}_i(x, L, f) - x, \tag{11} \]

which we call the expected delay in a mission of user \( i \). \( \bar{D}_i(x, L, f) \) represents the expected number of slots during a critical situation of user \( i \) that are not used for the successful transmission of user \( i \)’s packets when the length of the mission is \( x \), the \( m^* \)-period histories of users is \( L \), and the protocol is \( f \).

\( x \) follows the probability distribution of random variable \( X \), and \( f \) induces a stationary distribution on \( \mathcal{L}_m^N \) using a Markov model. Hence, we can calculate the expected value of \( \bar{D}_i \) given a protocol \( f \) to obtain

\[ D_i(f) \triangleq E_x[L \bar{D}_i(x, L, f)], \tag{12} \]
which can be considered as the average expected delay in a mission of user \( i \). Finally, we define the average expected delay of the protocol \( f \) by

\[
D^*(f) \triangleq \max\{D_1(f), \ldots, D_N(f)\}. \quad (13)
\]

The average expected delay measures the expected number of slots in which a user with a mission waits or experiences a collision during its mission. The party affected by a critical event can be rescued in a timely manner only when the mission is completed without delay. Therefore, the protocol designer prefers protocols that yield a small average expected delay. Note that \( \hat{x}_i \geq x \) for any realization of \( X \) and \( \hat{X}_i \), and thus \( D^*(f) \geq 0 \) for all \( f \in F^N \).

Suppose that the protocol designer has a utility function in the critical phase, \( U_C \), defined on \([0, +\infty) \times \mathbb{N}_+\). Then the protocol designer’s problem in the critical phase can be formulated as

\[
(P\text{-Crit}) \max_{f \in F^N} U_C(D^*(f), m^*(f)), \quad (14)
\]

where \( U_C \) is decreasing in \( D^*(f) \) and \( m^*(f) \).

### 3.3 Overall Problem

Depending on the arrival of critical events, the system is in the critical phase for some slots and in the normal phase for others. Hence, the protocol designer needs to find a protocol that performs well in both phases. There may exist a trade-off between the performance in the normal phase and that in the critical phase. When facing such a trade-off, the protocol designer needs to find a protocol that resolves the trade-off by solving the following overall problem:

\[
(\text{OP}) \max_{f \in F^N} U(\tau^*(f_N), \theta^*(f_N), m^*(f_N), D^*(f), m^*(f)), \quad (15)
\]

where \( f_N \) is the sub-protocol of \( f \) obtained by fixing \( y_{ti} = \text{normal} \) for all \( i \). \(^2 U \) denotes the overall utility function of the manager, defined on \([0,1]^N \times [0,1] \times \mathbb{N}_+ \times [0, +\infty) \times \mathbb{N}_+\), and it is increasing in the first two arguments and decreasing in the last three. In the formulation, the protocol designer may have different tolerance on the complexity in the two phases. For example, he may want to keep complexity low in the normal phase while allowing higher complexity in the critical phase.

### 4 Performance Analysis

This section investigates various trade-offs between the variables in the protocol manager’s problem. In the normal phase, we analyze the trade-off between total throughput and short-term fairness by imposing symmetry and fixing complexity. In the critical phase, we show the trade-off between the average expected delay and complexity. Finally, we illustrate the trade-off between short-term

\(^2\)Formally, \( f_{i,N} \) that constitutes \( f_N \) can be considered as a restriction of \( f_i \) to the subset of \( \mathcal{L}_m \), that contains \( y_i = \text{normal} \) only.
fairness and the average expected delay and between total throughput and the average expected
delay, which are variables of interest in different phases. The analysis in this section provides results
based on which the protocol designer can choose his optimal protocol once his utility function is
specified.

4.1 Performance in the Normal Phase

We analyze the performance in the normal phase using the constrained optimization approach to
(P-Norm). First, we impose a symmetry constraint which requires every user to follow the same
decision rule. This will be optimal when the protocol manager desires to yield the same throughput
to every user. Second, we fix the short-term fairness level and the complexity level. By varying
the short-term fairness level and finding optimal values of the constrained optimization problem,
we can trace the trade-off between total throughput and short-term fairness.

4.1.1 No Memory

For tractability, we consider stationary decision rules based on no memory and one-period memory.
We first consider the case where users do not use past information to determine their transmission
probabilities. In that case, a stationary decision rule is just a single transmission probability used
over time. Imposing the symmetry constraint, we denote the common transmission probability by
$p$. Then total throughput is given by

$$\tau(p) = Np(1 - p)^{N-1},$$

and the short-term fairness level is

$$\theta(p) = 1 - p(1 - p)^{N-1}. \tag{17}$$

Combining these two, we obtain

$$\theta + \frac{\tau}{N} = 1, \tag{18}$$

which illustrates a trade-off between total throughput and short-term fairness. Total throughput
is maximized at $p = 1/N$ while the short-term fairness level is maximized at $p = 0$ and $1$ where
total throughput is zero. Maximum total throughput $(1 - 1/N)^{N-1}$ converges to $1/e \approx 0.368$ as
$N \to \infty$. Note that this value is equal to the maximum achievable throughput of the stabilized
slotted Aloha system with an infinite set of nodes \[15\]. The short-term fairness level of the protocol
$p = 1/N$ converges to $1$ as $N \to \infty$.

When the protocol $f$ prescribes the same decision rule $f$ to every user, we use $f$ instead of $f$ as the argument of
functions whose original argument is a protocol.
4.1.2 One-period Memory

Now we consider stationary decision rules that utilize the channel states of the previous slot. A stationary decision rule for user \( i \) based on one-period memory in the normal phase can be expressed as \( f_i : S \rightarrow [0, 1] \). The reciprocal of the expected number of slots with consecutive successes of user \( i \) is given by

\[
\theta_i = 1 - f_i(\text{success}) \prod_{j \neq i} (1 - f_j(\text{busy})).
\]

We impose the symmetry constraint on the protocol and use \( f \) to denote the common stationary decision rule based on one-period memory. By setting the short-term fairness level at \( \theta \), we obtain a constrained version of (P-Norm):

\[
\begin{align*}
(P\text{-Norm1}) \\
\hat{\tau}(\theta) &= \max_{f \in F_1} \tau(f) \\
\text{subject to } f(\text{success})(1 - f(\text{busy}))^{N-1} &= 1 - \theta.
\end{align*}
\]

We first show that the protocol designer can achieve maximum total throughput 1 and the maximum short-term fairness level 1 at the same time with a symmetric stationary decision rule based on one-period memory when there are only two users.

**Proposition 1** With \( N = 2 \), \( \hat{\tau}(1) = 1 \).

**Proof**: Consider a decision rule \( \hat{f} \in F_1 \) defined by \( \hat{f}(\text{idle}) = \hat{f}(\text{failure}) = 1/2 \), \( \hat{f}(\text{busy}) = 1 \), and \( \hat{f}(\text{success}) = 0 \). Note that \( \hat{f} \) satisfies (21) with \( \theta = 1 \). The transition probability matrix on \( A = \{(W,W),(W,T),(T,W),(T,T)\} \) when both users use \( \hat{f} \) is given by

\[
P = \begin{bmatrix}
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}
\end{bmatrix}.
\]

From the structure of \( P \), we can see that \((W,W)\) and \((T,T)\) are transient states while \((W,T)\) and \((T,W)\) are ergodic states. Once an ergodic state is reached, \((W,T)\) and \((T,W)\) alternate. Thus, \( \tau_1(\hat{f}) = \tau_2(\hat{f}) = 1/2 \) and \( \tau(\hat{f}) = 1 \). Since \( \tau(f) \leq 1 \) for all \( f \in F \), \( \hat{f} \) attains the maximum of (P-Norm1).

Proposition 1 shows that channel sharing between two users can be achieved without communication when they use the decision rule \( \hat{f} \). Initially, they contend with each other with transmission probability 1/2. Once a user succeeds, they take a turn by alternating between \( T \) and \( W \). This perfect channel sharing scheme is no longer possible with three or more users. If three or more users use \( \hat{f} \), then a success can last only one slot because it will be followed by a collision for sure, and as a result the system will be in a collision state most of the time.
Let us partition the set of outcomes $A$ into $(N+1)$ sets according to the number of transmissions in outcomes. That is, we express $A = A_0 \cup \cdots \cup A_N$ where $A_k$ is the set of outcomes with $k$ transmissions, for $k = 0, 1, \ldots, N$. We obtain an approximate solution to (P-Norm1) by finding a decision rule $f$ in $F_1$ that maximizes one-step transition probabilities to $A_1$, in which a successful transmission occurs, when followed by every user.

First, suppose that the outcome in the previous slot is in $A_0$, i.e., the channel was idle. Then every user transmits with probability $f(\text{idle})$. If every user uses the same transmission probability, say $p$, then the probability of success is given by $Np(1-p)^{N-1}$, and this expression is maximized at $p = 1/N$. Hence, we set $f(\text{idle}) = 1/N$ to maximize the one-step transition probability from $A_0$ to $A_1$.

Next, suppose that the outcome in the previous slot is in $A_1$, i.e., there was a successful transmission. Then one user transmits with probability $f(\text{success})$ while $(N-1)$ users with $f(\text{busy})$. The probability of success in the current slot is given by

$$f(\text{success})(1-f(\text{busy}))^{N-1} + (N-1)f(\text{busy})(1-f(\text{busy}))^{N-2}(1-f(\text{success})).$$

(23)

The first term in (23) is fixed at $1 - \theta$ by (21). The second term is positive if $f(\text{success}) < 1$ and $f(\text{busy}) > 0$. If $\theta$ is small, however, the second term is near zero. So we ignore the effect of the second term.

We consider two combinations of $f(\text{success})$ and $f(\text{busy})$ that satisfy (21):

$$f(\text{success}) = 1 - \theta \quad \text{and} \quad f(\text{busy}) = 0,$$

(24)

and

$$f(\text{success}) = 1 \quad \text{and} \quad f(\text{busy}) = 1 - \frac{1}{N-\sqrt{1-\theta}}.$$  

(25)

Ma et al. [9] adopt (25) for their two-state protocol. The main difference between these two combinations is that with (24) a capture by a user ends when the user releases the channel whereas with (25) it ends when another user creates a collision. We choose (24) over (25) for the following two reasons. First, the probabilities in (24) are independent of the number of users while $f(\text{busy})$ in (25) depends on it. Thus, (24) will be more robust in achieving a desired duration of consecutive successes in an environment where the number of users is unknown. Second, (24) yields a more fair use of the channel than (25) in the following sense. With (24), when a capture ends, the channel goes to an idle state in which every user contends on an equal basis. Hence, a user who captures the channel next time is chosen equally likely among $N$ users. On the other hand, since $f(\text{busy}) \approx 0$ in (25) when $\theta$ is not large and $N$ is not small (for example, $f(\text{busy}) = 0.0543$ when $\theta = 0.2$ and $N = 5$), it is most likely that a capture ends by the transmission of one other user. Since $f(\text{busy}) \approx 0$, those who waited in the collision are likely to wait until the contention is resolved between the two users who collided. Hence, when a capture by a users ends, the same user will capture the channel again next time with probability near 1/2. This implies that there are fewer “changes of hands” with (25) than with (24).

Lastly, suppose that the outcome in the previous slot is in $A_2$ through $A_N$, i.e., there was a
collision. The transmission probability that has not been specified is \( f(\text{failure}) \). With transmission probabilities chosen so far, i.e., \( f(\text{idle}) = 1/N, f(\text{busy}) = 0, \) and \( f(\text{success}) = 1 - \theta \), a transition from a success state to a collision state is not possible, and from an idle state, \( A_2 \) is most likely among \( A_2 \) through \( A_N \). Hence, we choose \( f(\text{failure}) \) to maximize the one-step transition probability from \( A_2 \) to \( A_1 \). Since there are two users who transmit with \( f(\text{failure}) \) while others wait following an outcome in \( A_2 \), the one-step transition probability is maximized at \( f(\text{failure}) = 1/2 \).

The discussion so far provides an approximate solution to the problem of maximizing one-step transition probabilities to a success state, which we denote by \( \tilde{f} \) where \( \tilde{f}(\text{idle}) = 1/N, \tilde{f}(\text{busy}) = 0, \tilde{f}(\text{success}) = 1 - \theta \), and \( \tilde{f}(\text{failure}) = 1/2 \). The next proposition provides a lower bound on the maximum value of (P-Norm1) by deriving the expression for \( \tau(\tilde{f}) \).

**Proposition 2** Suppose \( \theta > 0 \) in (P-Norm1). Define \( q_k = C^N_k (1/N)^k (1 - 1/N)^{N-k} \) for \( k = 0, \ldots, N \). Define recursively from \( k = N \) down to 2 by \( J_k(k) = 1 \) and

\[
J_{k'}(k) = \frac{C_{k+1}}{2k+1} J_{k'}(k+1) + \frac{C_{k+2}}{2k+2} J_{k'}(k+2) + \cdots + \frac{C_{k'}}{2k'-1} J_{k'}(k')
\]

for \( k' = k + 1, \ldots, N \). Also, define

\[
G_k = \frac{2^k}{2^k - 1} \sum_{j=k}^{N} J_j(k) q_j
\]

for \( k = 2, \ldots, N \), and

\[
G_1(\theta) = \frac{1}{\theta} \left( 1 - q_0 - \sum_{k=2}^{N} \frac{G_k}{2^k} \right)
\]

Then

\[
\hat{\tau}(\theta) \geq \frac{G_1(\theta)}{1 + G_1(\theta) + G_2 + \cdots + G_N}.
\]

If \( \theta = 0 \), then \( \hat{\tau}(0) = 1 \).

**Proof:** The lower bound in (29) is total throughput attained at \( \tilde{f} \). Since every user uses the same decision rule, we can use \( \{A_0, \ldots, A_N\} \) as the set of Markov states instead of \( A \). Let \( P(k'|k) \) be
the transition probability from $A_k$ to $A_{k'}$ when $\tilde{f}$ is used. The transition probabilities are given by

\begin{align}
P(k'|0) &= q_{k'} \quad \text{for } k' = 0, \ldots, N, \\
P(k'|1) &= \begin{cases} 
\theta & \text{for } k' = 0 \\
1 - \theta & \text{for } k' = 1 \\
0 & \text{for } k' = 2, \ldots, N,
\end{cases} \\
P(k'|k) &= \begin{cases} 
\frac{C_{k'}^k}{2^k} & \text{for } k' = 1, \ldots, k \\
0 & \text{for } k' = k + 1, \ldots, N,
\end{cases} \quad \text{for } k = 2, \ldots, N.
\end{align}

(30) (31) (32)

If $\theta = 0$, then $A_1$ is the unique ergodic state, and thus $\tau(\tilde{f}) = 1$ implying $\hat{\tau}(0) = 1$. If $\theta > 0$, then every state of the Markov chain is positive-recurrent since $P(0|k) > 0$ for all $k = 0, \ldots, N$ and $P(k'|0) > 0$ for all $k' = 0, \ldots, N$. We denote the unique stationary distribution by $(\pi_k)_{k=0}^N$ where $\pi_k$ is the probability of $A_k$ in steady state. Using the stationarity condition $\pi_k' = \sum_{k=0}^N P(k'|k)\pi_k$ for $k = 0, \ldots, N$ (one of them redundant), we obtain $\pi_k = G_k \pi_0$ for $k = 1, \ldots, N$. Imposing the probability condition $\sum_{k=0}^N \pi_k = 1$, we get

$$\pi_1 = \frac{G_1(\theta)}{1 + G_1(\theta) + G_2 + \cdots + G_N},$$

(33)

which is total throughput at the approximate solution.

\[ G_2 \text{ through } G_N \text{ are independent of } \theta, \quad \text{and } G_1 \text{ is decreasing in } \theta. \] This implies that the lower bound is decreasing in the short-term fairness level $\theta$, leading to a trade-off between throughput and fairness. Since $G_1 \to \infty$ as $\theta \to 0$, total throughput can be made arbitrarily close to 1 by choosing $\theta$ sufficiently small, which sacrifices fairness. Figure 1 illustrates the trade-off between total throughput and the short-term fairness level at the optimal decision rule to (P-Norm1), which is computed using numerical methods, and at the approximate solution $\tilde{f}$ with $N = 10$. Figure 1 also shows feasible combinations of throughput and fairness with no memory.

Let $f_{\text{norm}1}$ be the solution to (P-Norm1). We study the structure of $f_{\text{norm}1}$ fixing $\theta = 0.1$ and compare it with $\tilde{f}$. Again, we rely on numerical methods to compute $f_{\text{norm}1}$. Table 1 and Figure 2 show optimal decision rules. $f_{\text{norm}1}(\text{idle})$ and $f_{\text{norm}1}(\text{failure})$ are close to those in approximate solution. As the second term of (23) is accounted in the optimal solution, $f_{\text{norm}1}(\text{busy})$ and $f_{\text{norm}1}(\text{success})$ take intermediate values between (21) and (25). We can see that the approximate solution is quite close to the optimal solution. As a result, the lower bounds found in Proposition 1 are close to maximum total throughput as shown in Table 1 and Figure 3. Table 2 and Figure 3 make a comparison of total throughput under four different decision rules in the normal phase. A two-state protocol is proposed in [9] where users use different transmission probabilities depending on whether they are in a free state or in a backlogged state. Total throughput under $\eta$-short-term fairness is given in equation (6) of [9]. We set $\eta = 1/\theta = 10$ so that the expected numbers of slots with consecutive successes are the same under (21) and under $\eta$-short-term fairness. The total throughput of the two-state protocol can be obtained by a stationary decision rule based on one-period mem-
ory \( f_{\text{two}} \) where \( f_{\text{two}}(\text{success}) = 1 \) and \( f_{\text{two}}(\text{idle}) = f_{\text{two}}(\text{busy}) = f_{\text{two}}(\text{failure}) = 1 - \frac{1}{N} \). Since \( f_{\text{two}} \) does not fully utilize information from the previous slot, there is a reduction in obtained total throughput compared to that obtained using \( f_{\text{norm}} \). \( f_{\text{one}} = \frac{1}{N} \) is the optimal decision rule based on no memory. Again, utilizing no information decreases maximum attainable throughput. Note that \( f_{\text{norm}} \), \( \tilde{f} \), and \( f_{\text{two}} \) have the same short-term fairness level 0.1 while that of \( f_{\text{one}} \) is \( 1 - \frac{1}{N} \). If users do not use past information, it is not very likely that a user succeeds for two or more consecutive slots. As a result, the short-term fairness level of decision rules based on no memory is very high. For example, \( \theta^*(f_{\text{one}}) = 0.9613 \) when \( N = 10 \). If we solve (P-Norm1) at \( \theta = 1 - \frac{1}{N} \), decision rules based on one-period memory yield no higher total throughput than those based on no memory as illustrated in Figure 1. This implies that the key feature of decision rules based on one-period memory is their ability to correlate between successful users in the current slot and in the future slots. The degree of correlation is determined by \( \theta \). When \( \theta \) is close to 1, this correlation does not exist, and thus utilizing information from the previous slot does not help to increase throughput.

Finally, we analyze the performance of stationary decision rules based on one-period memory in an environment of IEEE 802.11 DCF considered in [16]. Now, the duration of a slot depends on the state of the channel. Let \( \sigma_0 \), \( \sigma_1 \), and \( \sigma_2 \) be the duration of a slot when the channel state is idle, success, and collision, respectively. Then total throughput is expressed as

\[
\tau = \frac{P_1 E[P]}{P_0 \sigma_0 + P_1 \sigma_1 + P_2 \sigma_2}
\]

where \( E[P] \) is the average packet payload size and \( P_0 \), \( P_1 \), and \( P_2 \) are the probabilities of idle, success, and collision states, respectively. Note that in the idealized slotted Aloha model, we assume the size of each packet equal to the slot duration and ignore overhead so that \( \sigma_0 = \sigma_1 = \sigma_2 = E[P] \), and thus the expression for total throughput is reduced to \( P_1 \), the probability of success. With stationary decision rules based on one-period memory, the probabilities can be calculated as \( P_0 = \pi(A_0) \), \( P_1 = \pi(A_1) \), and \( P_2 = \sum_{k=2}^{N} \pi(A_k) \) where \( \pi(B) \) is the probability of outcomes in \( B \subset A \) in the stationary distribution.

To obtain numerical results, we use parameters specified by IEEE 802.11a PHY mode-8 [17], which are tabulated in Table 1. Based on the parameters, we obtain \( E[P] = 18432 \), \( \sigma_0 = 486 \), \( \sigma_1 = 22656 \), and \( \sigma_2 = 21626 \) in bits. We set up a new problem called (P-Norm2) by replacing the objective function in (P-Norm1) with (34). We call the optimal solution to (P-Norm2) \( f_{\text{norm}} \). Table 2 lists the optimal decision rules for (P-Norm2) with \( \theta = 0.1 \). Compared to \( f_{\text{norm}} \), \( f_{\text{norm}} \) prescribes lower transmission probabilities. Since an idle slot is a lot shorter than a slot in success or collision states, reaching an idle state is not very costly compared to reaching a collision state. Hence, \( f_{\text{norm}}(\text{busy}) \) and \( f_{\text{norm}}(\text{success}) \) have the structure of (24), and \( f_{\text{norm}}(\text{idle}) \) and \( f_{\text{norm}}(\text{failure}) \) are chosen lower than corresponding values in \( f_{\text{norm}} \) to avoid collision states.

Figure 1 compares total throughput in this scenario under three different decision rules. \( f_{\text{one}} \) uses the single transmission probability that maximizes (34) whereas \( f_{\text{DCF}} \) uses the single trans-
mission probability that corresponds to the contention window-based exponential backoff (EB) protocol with $CW_{\text{min}} = 16$ and $CW_{\text{max}} = 1024$, which can be calculated using equations (7) and (9) of [16]. We find that the transmission probabilities derived from DCF are suboptimal as the number of users increases and that there is a significant performance improvement by utilizing information obtained in the previous slot in this environment too.

Figure 5 illustrates the trade-off between throughput and fairness in the DCF environment with $N = 10$. As in the slotted Aloha system, total throughput reduces as the short-term fairness level increases. The point corresponding to the operation of DCF is not on the boundary as it operates suboptimally. Again, the gain from utilizing past information comes from serial correlation among successful users, which is possible when $\theta$ is not large. Since there is overhead in DCF, total throughput does not converge to one as $\theta$ goes to zero.

4.2 Performance in the Critical Phase

We now consider slots in which some user is in a critical situation. As a benchmark case, suppose that the entire situations of the system is known to all users. Then $y_i^t$ in the histories of user $i$ is replaced by $y^t$, and users can adjust their transmission probabilities depending on others’ situations as well as on their own situations. With the public knowledge of $y$, the lower bound for $D^*$ can be attained with a protocol based on no memory. Define a decision rule $f_0$ in the critical phase by $p_i^t = 1$ if $y_i^t = \text{critical}$, $p_i^t = 0$ if $y_j^t = \text{critical}$ for some $j \neq i$. $f_0$ uses current information only. Suppose that a mission arrives to user $i$ in slot $t_0$. If every user follows $f_0$, then user $i$ captures the channel for $x$ slots starting from slot $t_0$. Then $t_1 = t_0 + x - 1$, and we have $\hat{x}_i = x$ for any value of $x$, which lead to $D^*(f_0) = 0$.

However, the assumption that every user knows the situations of others is unrealistic considering the distributed nature of wireless networks. Hence, it is more natural to assume that each user $i$ knows only about its situation, $y_i$. In this scenario, $f_0$ cannot be used since users do not know whether there is another user who is in a critical situation. Suppose that users use $f_{\text{norm}} \in F_1$ when they are in a normal situation and $f_{\text{crit}} \equiv 1$ in a critical situation. We impose an important constraint on $f_{\text{norm}}$:

$$f_{\text{norm}}(\text{busy}) = 0. \quad (35)$$

Then the remaining transmission probabilities $f_{\text{norm}}(\text{idle}), f_{\text{norm}}(\text{success}),$ and $f_{\text{norm}}(\text{failure})$ determine both total throughput and the average expected delay while $f_{\text{norm}}(\text{success})$ also determines the short-term fairness level by $\theta = 1 - f_{\text{norm}}(\text{success})$ given (35). By varying these three transmission probabilities, we can obtain the feasible combinations of total throughput, short-term fairness, and the average expected delay. In Table 5 we describe the structure of $f_{\text{norm}}$ and compare it against the persistence probability-based EB protocol described in [18].

Suppose that a mission arrives to user $i$ in slot $t_0$. We examine the decisions of users using the decision rule that prescribes $f_{\text{norm}}$ in case of a normal situation and $f_{\text{crit}}$ in case of a critical
situation, depending on the outcome in slot $t_0 - 1$. First, we consider the case where user $i$ succeeded in slot $t_0 - 1$, i.e., $a^t_{t_0 - 1} = a^i$. User $i$ transmits its packet while others wait in slot $t_0$ because user $i$ uses $f_{\text{crit}}$ and user $j \neq i$ uses $f_{\text{norm}}$ which prescribes the transmission probability $p^t_{j_{t_0 - 1}} = f_{\text{norm}}(s^t_{j_{t_0 - 1}}) = f_{\text{norm}}(\text{busy}) = 0$ by (35). These decisions remain unchanged until user $i$ completes its mission in slot $t_0 + x - 1$. When user $i$ switches to $f_{\text{norm}}$ in slot $t_0 + x$, it is expected to capture the channel for additional $1/\theta$ slots. To prevent this and reset the system, we require that a user in a critical situation should release the channel when it returns to a normal situation.

The mission-aware protocol described so far is summarized in Table 6 and named as Protocol 1. Note that Protocol 1 is based on one-period memory. We denote Protocol 1 by $\mathbf{f}_1$. In the case where $L$ contains $a^i$ as the most recent outcome, we have $\hat{x}_i = x$ and thus

$$\bar{D}_i(x, L, \mathbf{f}_1) = 0$$

for all $x \in \text{supp}(X)$ where $\text{supp}(X)$ denotes the support of the random variable $X$.

Second, we consider the case where some user $j \neq i$ succeeded in slot $t_0 - 1$, i.e., $a^t_{t_0 - 1} = a^j$. Then user $i$ transmits in slot $t_0$ because it uses $f_{\text{crit}}$, but user $j$ transmits with probability $1 - \theta$ because $f_{\text{norm}}(\text{success}) = 1 - \theta$. Hence, with probability $\theta$ user $i$ starts transmitting its packets from slot $t_0$, and with probability $1 - \theta$ a collision between the packets of user $i$ and $j$ occurs in slot $t_0$. If a collision occurs, then the two users contend for the channel with $p^t_i = 1$ and $p^t_j = f_{\text{norm}}(\text{failure})$ from slot $t_0 + 1$ until user $i$ captures the channel. The number of slots until the first success of user $i$ follows a geometric distribution with parameter $1 - f_{\text{norm}}(\text{failure})$. Hence, we obtain

$$\bar{D}_i(x, L, \mathbf{f}_1) = \frac{1 - \theta}{1 - f_{\text{norm}}(\text{failure})}$$

for all $x \in \text{supp}(X)$ and $L$ with $a^j$ as the most recent outcome. Note, however, that user $j$ learns that there is a user in a critical situation when encountering a failure after a success because it cannot happen when every user uses $f_{\text{norm}}$. Again, (35) is crucial for this observation. Then user $j$ can back off in slot $t_0 + 1$ instead of contending with the user in a critical situation. This enhancement is incorporated in Protocol 2 of Table 7, which we denote by $\mathbf{f}_2$. Note that Protocol 2 is based on two-period memory. Under Protocol 2, user $i$ starts transmitting in slot $t_0$ with probability $\theta$ and in slot $t_0 + 1$ with probability $1 - \theta$. Therefore, the expected delay is

$$\bar{D}_i(x, L, \mathbf{f}_2) = 1 - \theta$$

for all $x \in \text{supp}(X)$ and $L$ with $a^j$ as the most recent outcome. Comparing (37) and (38), we can see that the higher short-term fairness level reduces the expected delay for a user if a different user succeeded in the previous slot. This is true because as $\theta$ is larger, the probability of yielding gets higher.

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4$L$ having $a$ as the most recent outcome means that $L_i$ has the channel state for user $i$ in the most recent slot that corresponds to $a$.  

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Third, we consider the case where a collision occurred in slot $t_0 - 1$. Let $k'$ be the number of users who transmitted in slot $t_0 - 1$ among users other than user $i$. Then according to Protocols 1 and 2, user $i$ transmits with probability 1, $k'$ users transmit with probability $f_{\text{norm}}(\text{failure})$, and the remaining users wait in slot $t_0$. Note that unlike in the previous case, an inference about the existence of a critical situation cannot be made because another collision following a collision is not a zero-probability event under $f_{\text{norm}}$. The collision state will last until user $i$ succeeds. The number of users contending for the channel remains the same or decreases during collisions, and fixing the number of contenders at $k'$ will provide an upper bound for the expected delay. This leads us to

$$
\tilde{D}_i(x, L, f) \leq \frac{1}{(1 - f_{\text{norm}}(\text{failure}))^{k'} - 1},
$$

for all $x \in \text{supp}(X)$, for $L$ such that $k'$ users among users other than user $i$ transmitted in the most recent outcome, and for $f = f_1, f_2$. Consider an outcome with $k$ transmitters, i.e., $a \in A_k$. When users follow the same decision rule, the probability that $k' = k$, i.e., user $i$ is not one of the $k'$ transmitters, is $C_k^{N-1}/C_k^N = (N - k)/N$ and the probability that $k' = k - 1$, i.e., user $i$ is one of the $k$ transmitters, is $C_{k-1}^{N-1}/C_k^N = k/N$. Hence, we have

$$
\tilde{D}_i(x, L, f) \leq \frac{N - k}{N} \cdot \frac{1}{(1 - f_{\text{norm}}(\text{failure}))^k} + \frac{k}{N} \cdot \frac{1}{(1 - f_{\text{norm}}(\text{failure}))^{k-1} - 1}
$$

$$
= \frac{N - k f_{\text{norm}}(\text{failure})}{N(1 - f_{\text{norm}}(\text{failure}))^k} - 1,
$$

for all $x \in \text{supp}(X)$, $L$ with $a^0 \in A_k$, $k = 2, \ldots, N$, and $f = f_1, f_2$.

Lastly, we consider the case where the channel was idle in slot $t_0 - 1$. Then according to Protocols 1 and 2, user $i$ transmits with probability 1 while other users transmit with probability $f_{\text{norm}}(\text{idle})$. As in the previous case, no inference about the existence of a critical situation based on the channel state in slot $t_0$ can be made because any outcome can be reached following an idle state under $f_{\text{norm}}$. In slot $t_0$, user $i$ succeeds with probability $(1 - f_{\text{norm}}(\text{idle}))^{N-1}$, and a collision in which one of the transmitters is user $i$ occurs with probability $1 - (1 - f_{\text{norm}}(\text{idle}))^{N-1}$. Hence, we have

$$
\tilde{D}_i(x, L, f) \leq \sum_{k'=1}^{N-1} \frac{C_k^{N-1} f_{\text{norm}}(W, \text{idle})^{k'} (1 - f_{\text{norm}}(W, \text{idle}))^{N-k'-1}}{(1 - f_{\text{norm}}(T, \text{failure}))^{k'}},
$$

for all $x \in \text{supp}(X)$, $L$ with $a^0 \in A_0$, and $f = f_1, f_2$. Since $f_{\text{norm}}$ induces a stationary distribution on $\mathcal{A}$, we can compute upper bounds on the average expected delays of Protocols 1 and 2 using the definition given in [13] and the results so far.

**Proposition 3** Let $\pi$ be the stationary distribution over $\mathcal{A}$ under $f_{\text{norm}}$. Then for any probability
distribution for $X$, the average expected delays of Protocols 1 and 2 satisfy

$$D^*(f_1) \leq \pi(A_0) \sum_{k'=1}^{N-1} \frac{C_k^{N-1} f_{\text{norm}}(\text{idle})^{k'} (1 - f_{\text{norm}}(\text{idle}))^{N-k'-1}}{(1 - f_{\text{norm}}(\text{failure}))^{k'}} + \pi(A_1) \frac{N - 1}{N} \frac{1 - \theta}{1 - f_{\text{norm}}(\text{failure})} + \sum_{k=2}^{N} \pi(A_k) \left[ \frac{N - kf_{\text{norm}}(\text{failure})}{N (1 - f_{\text{norm}}(\text{failure}))^k} - 1 \right]$$

and

$$D^*(f_2) \leq \pi(A_0) \sum_{k'=1}^{N-1} \frac{C_k^{N-1} f_{\text{norm}}(\text{idle})^{k'} (1 - f_{\text{norm}}(\text{idle}))^{N-k'-1}}{(1 - f_{\text{norm}}(\text{failure}))^{k'}} + \pi(A_1) \frac{N - 1}{N} (1 - \theta) + \sum_{k=2}^{N} \pi(A_k) \left[ \frac{N - kf_{\text{norm}}(\text{failure})}{N (1 - f_{\text{norm}}(\text{failure}))^k} - 1 \right].$$

\textbf{Proof}: Since every user uses the same decision rule under $f_1$ and $f_2$, $D_t(f)$ is the same across users for $f = f_1, f_2$. Forming a weighted average of (39), (37), (40), and (41) where the weights are given by $\pi(A_1)/N, (N - 1)\pi(A_1)/N, \pi(A_k)$, and $\pi(A_0)$, respectively, we obtain the upper bound on the average expected delay of Protocol 1 given in (42). Using (38) instead of (37), we obtain the upper bound on the average expected delay of Protocol 2 given in (43).

Figure 6 plots the upper bounds on the average expected delays of Protocols 1 and 2 found in Proposition 3 when $f_{\text{norm}}(\text{idle})$ and $f_{\text{norm}}(\text{failure})$ are chosen to maximize total throughput given the constraints $f_{\text{norm}}(\text{busy}) = 0$ and $f_{\text{norm}}(\text{success}) = 0.9$. As the number of users increases, the average expected delay gets longer. Since a critical event occurs most likely following a success state ($\pi(A_1) \approx 0.8$ under $f_{\text{norm}}$ with $\theta = 0.1$), the second terms in the right-hand sides of (42) and (43) dominate the other terms. As a result, the overestimation used in (39) will not have a large impact on the values of the upper bounds in Proposition 3, and the upper bounds will be close to the actual average expected delays. Figure 6 also shows the trade-off between the average expected delay and complexity. The protocol designer can reduce the average expected delay by increasing the complexity level from 1 to 2.

So far, we have used the average expected delay to measure the performance of a protocol in the critical phase. Suppose that the protocol designer is also interested in the worst-case delay as well as in the average expected delay of a protocol. Both Protocols 1 and 2 have a sequence of outcomes with a positive probability that a user in a critical situation has to wait for an arbitrary large number of slots before it starts to transmit, although the probability of such a sequence of outcomes is close to zero when the number of waiting slots is large. The protocol designer can bound realized delays by $m$ with a protocol based on $m$-period memory. The idea is to make users in a normal situation back off after experiencing $m$ consecutive collisions so that a user in a critical situation, if any, can capture the channel. When user $i$ is in a critical situation, the possible outcomes under Protocols 1 and 2 are either user $i$’s success or a collision. Since the delay can go infinitely long through consecutive collisions, user $i$ is guaranteed to start its transmission after $m$
slots at latest if such modification is applied. Protocol 3 is proposed in Table 8 to introduce this modification. Note that this modification will have almost no impact on total throughput in the normal phase because it is very unlikely to have \( m \) consecutive collisions in either phase when \( m \) is moderately large, and as a result it can be thought of as a safety device which is rarely used.

4.3 Accounting for Both Phases

We have seen that it is crucial to set \( f_{\text{norm}}(\text{busy}) = 0 \) to allow a user in a critical situation to capture the channel during its mission without others knowing about the presence of the mission. The specification of the remaining transmission probabilities determines the total throughput, the short-term fairness level, and the average expected delay of Protocols 1 and 2. By varying the remaining probabilities, the protocol designer can find attainable combinations of total throughput, short-term fairness, and the average expected delay, and then he can choose the most preferred one among them.

We first investigate the relationship between fairness and delay. Figure 7 depicts the combinations of short-term fairness levels and upper bounds on the average expected delay. We fix \( N = 10 \) and choose \( f_{\text{norm}}(\text{idle}) \) and \( f_{\text{norm}}(\text{failure}) \) to maximize total throughput given the constraints \( f_{\text{norm}}(\text{busy}) = 0 \) and \( f_{\text{norm}}(\text{success}) = 1 - \theta \). There are two counteracting effects when the short-term fairness level increases. First, the system stays in idle and collision states more often as illustrated in Figure 1, and in these states the expected delay is higher than in success states. Second, the expected delay decreases when a user other than the one with a mission was successful in the previous slot, as reflected in the second terms in the right-hand sides of (42) and (43). The difference between (42) and (43) is that \((1 - \theta)\) is multiplied by \(1/(1 - f_{\text{norm}}(\text{failure})) \approx 2 \) in (42) while it is not in (43). Thus, the second effect is stronger in (42) than in (43). Figure 7 shows that the second effect is dominant in (42) while the first in (43). The upper bound on the average expected delay gets lower as fairness increases with Protocol 1 whereas it gets higher with Protocol 2.

Figure 8 illustrates the trade-off between throughput and delay with \( \theta = 0.1 \) and \( N = 10 \). Given the transmission probabilities \( f_{\text{norm}}(\text{idle}) \) and \( f_{\text{norm}}(\text{failure}) \) that maximize total throughput fixing \( f_{\text{norm}}(\text{busy}) = 0 \) and \( f_{\text{norm}}(\text{success}) = 1 - \theta \), there is no need to consider larger transmission probabilities for \( f_{\text{norm}}(\text{idle}) \) and \( f_{\text{norm}}(\text{failure}) \) because it will decrease total throughput and increase the average expected delay. Hence, we use values for \( f_{\text{norm}}(\text{idle}) \) between 0 and 0.11 and for \( f_{\text{norm}}(\text{failure}) \) between 0 and 0.5, and some feasible combinations are shown in Figure 8. The protocol designer can choose the values for \( f_{\text{norm}}(\text{idle}) \) and \( f_{\text{norm}}(\text{failure}) \) to yield the most preferred combination of throughput and delay.

5 Extension to Concurrent Missions

So far, we have considered a system in which there can be at most one mission in the system at a time. In this section, we describe how the proposed protocols can be modified in the presence of
multiple missions.

We first assume that $y$ is publicly known. Alternatively, we may assume that $y_i$ is known only to user $i$ but every user knows the number of missions in the system. We denote $f_{\text{norm}1}$ with $N \geq 3$ users by $f_{\text{norm}1}(N)$ and define $f_{\text{norm}1}(1) \equiv 1$ and $f_{\text{norm}1}(2) \triangleq \hat{f}$ where $\hat{f}$ is the two-user alternating scheme introduced in Proposition 1. Let $n^i_t$ be the number of critical situations in $y^i_t$. With the public knowledge of $y$, we can consider the following protocols.

1. **First-come first-served protocol**
   Users in a critical situation conduct their missions in the same order as their missions arrive. That is, if there are users in a critical situation when a mission arrives to a user, it waits until all the missions that arrived earlier are completed. If multiple missions arrive at the same time, the users with these missions contend with each other with an equal transmission probability to determine the turn. (Let $n^*$ be the number of missions that arrived at the same time. Then $n^*$ users transmit with probability $1/n^*$ until some user succeeds. After the successful user finishes its mission, the remaining $(n^* - 1)$ users contend with transmission probability $1/(n^* - 1)$ to determine the second user who uses the channel. This process is repeated until the last user finishes its mission.)

2. **Sharing protocol**
   Users in a critical situation use $f_{\text{norm}1}(n^i_t)$ to share the channel equally while users in a normal situation wait in the critical phase. Note that there are slots in idle or collision states when $n^i \geq 3$, which is not the case with the first-come first-served protocol unless multiple missions arrive at the same time.

Now we consider the case where each user knows only about its situation. For the moment, we assume that the system can have at most two critical situations at a time. We discuss how Protocol 2 can be modified in such a scenario. Suppose that the second mission arrives to user $j$ in slot $t_0$ while user $i$ is in a critical situation. Then $a^{t_0-1} = a^i$, but user $j$ does not know whether the successful user is in a normal situation or in a critical situation. User $j$ transmits in slot $t_0$. The transmission by user $j$ informs user $i$ that there exists another user who is also in a critical situation. If user $i$ were in a normal situation, it would respond by waiting in slot $t_0 + 1$ according to Protocol 2 so that user $j$ could capture the channel. However, since user $i$ is in a critical situation, it responds by transmitting in slot $t_0 + 1$ to inform user $j$ of its critical situation. Then the presence of two missions becomes a common knowledge between the two users after two slots. From slot $t_0 + 2$ on, users $i$ and $j$ use $\hat{f}$ to share the channel with the following modification. In the transient period until one user succeeds, they always transmit following an idle slot to prevent other users who are unaware of the missions from taking the channel. Once one user succeeds, they alternate between $(a_i, a_j) = (T, W)$ and $(a_i, a_j) = (W, T)$ until one of the missions ends. After one of the missions ends, an idle slot occurs, and the situation becomes the same as the one with one mission arriving following an idle slot. We can decrease the expected delay by requiring the user who completed its mission earlier than the other to wait in the next slot.
If three or more missions can occur at the same time, then the dispersion of information on the number of critical situations through changes in transmission probabilities becomes more complicated and takes long if possible. Thus, if critical events occur frequently to multiple users at the same time, the broadcast by users to signal their critical situations to others will be valuable in mission-critical networking.

6 Conclusion

We have studied the issue of delay in mission-critical networking. In the context of wireless communication networks, we have proposed a novel class of MAC protocols that utilize not only current information but also past information. This allows users to coordinate their behavior without explicit message exchanges. In the normal phase, the system can attain high throughput by allowing a successful user to capture the channel for a period. In the critical phase, the proposed protocols make a user in a critical situation capture the channel after a short delay without any message passing about its critical situation. The proposed protocols fulfill the objective of the protocol designer in both phases while maintaining low complexity.

For analytic tractability, we mainly focused on decision rules based on one-period memory. It will be interesting to investigate the properties of optimal decision rules based on longer memory such as two-period memory and how the trade-off between throughput and fairness changes when longer memory is utilized in the normal phase. Another potential advantage from utilizing longer memory is the transmission of more information through the change in transmission probabilities. One of the reasons that the proposed protocols work well in a distributed setting is that users can communicate implicitly through their choices of transmission probabilities. When the set of possible decision rules expands as longer memory is used, there are potentially more “codes” that can be conveyed through transmission decisions.

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Table 1: Optimal decision rules for (P-Norm1), $f_{\text{norm}1}$, with $\theta = 0.1$

| $N$ | $f_{\text{norm}1}(\text{idle})$ | $f_{\text{norm}1}(\text{busy})$ | $f_{\text{norm}1}(\text{success})$ | $f_{\text{norm}1}(\text{failure})$ |
|-----|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 3   | 0.338                           | 0.034                           | 0.964                           | 0.493                           |
| 4   | 0.255                           | 0.025                           | 0.971                           | 0.490                           |
| 5   | 0.205                           | 0.020                           | 0.975                           | 0.488                           |
| 10  | 0.103                           | 0.010                           | 0.982                           | 0.485                           |
| 15  | 0.069                           | 0.006                           | 0.984                           | 0.485                           |
| 20  | 0.052                           | 0.005                           | 0.985                           | 0.484                           |

Table 2: Comparison of total throughput under different decision rules in the normal phase

| $N$ | $f_{\text{norm}1}$ | $f$   | $f_{\text{two}}$ | $f_{\text{one}}$ |
|-----|--------------------|-------|------------------|------------------|
| 3   | 0.8275             | 0.8199| 0.5808           | 0.4444           |
| 4   | 0.8235             | 0.8139| 0.5541           | 0.4219           |
| 5   | 0.8214             | 0.8104| 0.5391           | 0.4096           |
| 10  | 0.8175             | 0.8038| 0.5116           | 0.3874           |
| 15  | 0.8163             | 0.8017| 0.5030           | 0.3806           |
| 20  | 0.8157             | 0.8007| 0.4988           | 0.3774           |

Table 3: IEEE 802.11a PHY mode-8 parameters

| Parameters               | Values       |
|--------------------------|--------------|
| Packet payload           | 2304 octets  |
| MAC header               | 28 octets    |
| ACK frame size           | 14 octets    |
| Data rate                | 54 Mbps      |
| Propagation delay        | 1 $\mu$s     |
| Slot time                | 9 $\mu$s     |
| PHY header time          | 20 $\mu$s    |
| SIFS                     | 16 $\mu$s    |
| DIFS                     | 34 $\mu$s    |

Table 4: Optimal decision rules for (P-Norm2), $f_{\text{norm}2}$, with $\theta = 0.1$

| $N$ | $f_{\text{norm}2}(\text{idle})$ | $f_{\text{norm}2}(\text{busy})$ | $f_{\text{norm}2}(\text{success})$ | $f_{\text{norm}2}(\text{failure})$ |
|-----|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 3   | 0.077                           | 0                               | 0.9                             | 0.136                           |
| 4   | 0.056                           | 0                               | 0.9                             | 0.146                           |
| 5   | 0.043                           | 0                               | 0.9                             | 0.143                           |
| 10  | 0.021                           | 0                               | 0.9                             | 0.151                           |
| 15  | 0.014                           | 0                               | 0.9                             | 0.153                           |
| 20  | 0.010                           | 0                               | 0.9                             | 0.156                           |
Table 5: Description of the decision rule used in the normal phase, $f_{norm}$, and the persistence probability-based EB protocol

| $s_{i-1}$ | $f_{norm}$ | EB protocol |
|---|---|---|
| idle | $p^{t}_{i} \approx 1/N$ | $p^{t}_{i} = p^{t-1}_{i}$ |
| busy | $p^{t}_{i} = 0$ | $p^{t}_{i} = p^{\text{max}}_{i}$ |
| success | $p^{t}_{i} = 1 - \theta$ | $p^{t}_{i} = \max\{\beta_{i} p^{t-1}_{i}, p^{\text{min}}_{i}\}$ (0 < $\beta_{i}$ < 1) |
| failure | $p^{t}_{i} \approx 0.5$ | $p^{t}_{i}$ |

Table 6: [Protocol 1] Mission-aware MAC protocol based on one-period memory

**Decision rule for user $i$**

1. Set $p^{t}_{i} = 1$ if $y^{t}_{i} = \text{critical}$.
2. Set $p^{t}_{i} = 0$ if $y^{t-1}_{i} = \text{critical}$ and $y^{t}_{i} = \text{normal}$.
3. Set $p^{t}_{i} = f_{norm}(s^{t-1}_{i})$ if $y^{t-1}_{i} = \text{normal}$ and $y^{t}_{i} = \text{normal}$.

(As in Section 2, we set $y^{t}_{i} = \text{normal}$ and $s^{t}_{i} = \text{idle}$ for $t' \leq 0$ in all protocols.)

Table 7: [Protocol 2] Mission-aware MAC protocol based on two-period memory

**Decision rule for user $i$**

1. Set $p^{t}_{i} = 1$ if $y^{t}_{i} = \text{critical}$.
2. Set $p^{t}_{i} = 0$ if $y^{t-1}_{i} = \text{critical}$ and $y^{t}_{i} = \text{normal}$.
3. Set $p^{t}_{i} = 0$ if $s^{t-2}_{i} = \text{success}$ and $s^{t-1}_{i} = \text{failure}$.
4. Set $p^{t}_{i} = f_{norm}(s^{t-1}_{i})$ if $y^{t-1}_{i} = \text{normal}$ and $y^{t}_{i} = \text{normal}$ except for 3.

Table 8: [Protocol 3] Mission-aware MAC protocol based on $m$-period memory

**Decision rule for user $i$**

1. Set $p^{t}_{i} = 1$ if $y^{t}_{i} = \text{critical}$.
2. Set $p^{t}_{i} = 0$ if $y^{t-1}_{i} = \text{critical}$ and $y^{t}_{i} = \text{normal}$.
3. Set $p^{t}_{i} = 0$ if $s^{t-2}_{i} = \text{success}$ and $s^{t-1}_{i} = \text{failure}$.
4. Set $p^{t}_{i} = 0$ if $s^{t-m}_{i} = \cdots = s^{t-1}_{i} = \text{failure}$ and $y^{t}_{i} = \text{normal}$.
5. Set $p^{t}_{i} = f_{norm}(s^{t-1}_{i})$ if $y^{t-1}_{i} = \text{normal}$ and $y^{t}_{i} = \text{normal}$ except for 3 and 4.
Figure 1: Trade-off between throughput and fairness with $N = 10$

Figure 2: Optimal decision rules for (P-Norm1) with $\theta = 0.1$
Figure 3: Total throughput under different decision rules in the normal phase \((f_{\text{approx}} = \tilde{f})\)

Figure 4: Total throughput under different decision rules in the DCF environment
Figure 5: Trade-off between throughput and fairness in the DCF environment with $N = 10$

Figure 6: Upper bounds on the average expected delays of Protocols 1 and 2
Figure 7: Relationship between fairness and delay under Protocols 1 and 2 with $N = 10$

Figure 8: Relationship between throughput and delay under Protocols 1 and 2 with $N = 10$