Planar Electric Trap for Neutral Particles

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A new geometry to trap neutral particles with an ac electric field using a simple electrode structure is described. In this geometry, all electrodes are placed on a single chip plane, while particles are levitated above the chip. This enables easy construction of the trap and good optical access to the trap.

1. Introduction

To investigate the properties of particles such as atoms and molecules, it is desirable to hold them in an isolated environment. For this purpose, various types of traps have been developed. Among them, magneto-optical traps (MOTs) and magnetic traps (MTs) are successfully used for a variety of applications.1) However, they require the particles to have either a closed transition (MOT) or a magnetic moment (MT), which restricts the range of particles that can be trapped. Electric traps (i.e., traps that use an electric field), on the other hand, have the advantage that they can trap almost any type of neutral particles. It is easily shown that it is impossible to trap neutral particles using a static electric field,2) and an ac electric trap was proposed to overcome this difficulty.3–5) The disadvantage of such an electric trap is its shallow trap depth (potential depth for sodium atoms is 190 µK for an electric field of 1 kV mm−1), and it is needed to make a small trap to increase the curvature of the potential and hold particles against gravity. However, ac electric traps proposed to date have solid structures, and thus, the fabrication of small traps has been difficult. For such technical reasons, ac electric traps have not been realized until recently.6–9)

In this paper, we propose a new geometry of an ac electric trap in which all electrodes are placed on a single plane, while particles are trapped above that plane. Such geometry enables one to fabricate the trap structure on a chip with high precision, which raises the possibility of fabricating micro-structured atom and molecule chips that benefit from the advantages of long decoherence time,10) low electric power consumption, and scalability of electric traps.

2. Principle of the Dynamical Trapping

We start from the trap configuration described in ref. 5, schematically shown in Fig. 1, in which all electrodes are located on a single plane (xy-plane). Two pairs of electrodes are placed on x- and y-axes symmetrically [i.e., at (±s0, 0, 0) and (0, ±s0, 0)]. There are two phases in the operation of this trap: In phase A (phase B), a voltage is applied to one of the pairs of electrodes on the xy-plane symmetrically [i.e., at (±s0, 0, 0) and (0, ±s0, 0)]. The Stark potential that a neutral particle experiences in the electric field E(r) is

\[ V(r) = -\frac{1}{2} \alpha |E(r)|^2, \quad (1) \]

where \( \alpha \) is the polarizability of the particle. \( \alpha \) is always positive for atoms or molecules in a stable state. In phase A or B, the potential \( V(r) \) has the form

\[ V_A(r) = \frac{1}{2} m \omega_0^2 (r_1^2 + r_2^2 + \xi_0 \xi_2^2) + V_0 \]

\[ V_B(r) = \frac{1}{2} m \omega_0^2 (r_1^2 + \xi_0 \xi_2^2) + V_0, \quad (2) \]

up to the 2nd order in \( r \). Here, \( m \) is the mass of the particle and \( \omega_0 \) and \( \xi_0 \) are the angular frequency of oscillation in the y-direction in phase A (x-direction in phase B). For spherical electrodes, \( \eta_0 = 2 \) and \( \xi_0 = 1 \).11) By switching alternately between phases A and B for time \( T_A \) and \( T_B \) respectively (usually \( T_A = T_B \)), particles are trapped around the origin (0, 0, 0): They are dynamically captured in the xy-plane (similar to an RF ion trap11), whereas the confinement in the z-direction is static. Larger \( \xi_0 \) gives stronger confinement in the z-direction, while the stable region for the driving frequency \( \Omega = 2\pi/T \) is reduced with increasing \( \eta_0 \) (\( T = T_A + T_B \) is the period of the applied voltage).

3. Lifting the Trapping Point off the Chip Plane

Particles are trapped at the saddle point of \( |E(r)|^2 \). To lift up this trapping point off the xy-plane, we place additional electrodes in between the existing electrodes to bend the field lines and create a saddle point other than the origin (Fig. 2). We maintain the electrode configuration symmetric under \( x \leftrightarrow -x \), \( y \leftrightarrow -y \), and \( x \leftrightarrow y \). Now, eq. (2) becomes

\[ V_A(r) = \frac{1}{2} m \omega_0^2 [-\eta x^2 + y^2 + \xi(z-h)^2] + V \]

\[ V_B(r) = \frac{1}{2} m \omega_0^2 [x^2 - \eta y^2 + \xi(z-h)^2] + V, \quad (3) \]

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First, we consider electrodes as point charges and place them near the new saddle point \((0, 0, h)\). Let \(\phi(\mathbf{r})\) be the scalar potential. In phase A (and similarly in phase B), because \(\phi(x, y, z) = -\phi(-x, y, z)\) and \(\phi(x, -y, z) = \phi(x, -y, z)\),

\[
\begin{align*}
E_x(x, y, z) &= +E_x(-x, y, z) \\
E_y(x, y, z) &= -E_y(-x, y, z) \\
E_z(x, y, z) &= -E_z(-x, y, z) \\
E_x(x, y, z) &= +E_x(x, -y, z) \\
E_y(x, y, z) &= -E_y(x, -y, z) \\
E_z(x, y, z) &= +E_z(x, -y, z),
\end{align*}
\]

such that \(\partial_x E_x|_{(0,0,0)} = \partial_y E_y|_{(0,0,0)} = \partial_z E_z|_{(0,0,0)} = \partial_z E_z|_{(0,0,0)} = 0\). Also, \(\partial_x E_x|_{(0,0,0)} = \partial_y E_y|_{(0,0,0)} = 0\) from \(\partial_x |E|^2|_{(0,0,0)} = 0\) and \(\nabla \times \mathbf{E} = 0\). From these equalities, \(\nabla^2 |E|^2|_{(0,0,0)} = 2 \sum_{j=x,y,z} (\partial_j |E|_{(0,0,0)})^2 = 0\), and thus, \(\eta - \xi = 1\) is still satisfied.

### 3.1 Design procedure: step 1

First, we consider electrodes as point charges and place them on the \(xy\)-plane, as shown in Fig. 3. Four outer point charges placed at \(s_{\text{out}} = (\pm s_0, 0)\), \((0, \pm s_0)\) correspond to the four electrodes of the original configuration (Fig. 1) and we place charges of \(\pm q_0\) or 0 at the appropriate phase. Four inner point charges at \(s_{i} = (\pm s, \pm s), (\pm s, \mp s)\) are added to the original configuration with charges \(\pm q\) depending on the phase (see figure). In Fig. 4, we plot parameters \(h/s_0\), \(\omega/\omega_0\), and \(\xi\) that appear in eq. (3) as functions of \(q/q_0\) for several \(s/s_0\) values (\(\eta\) is calculated from \(\xi\) as \(\eta = \xi + 1\)). Here, \(\omega_0\) is \(\omega\) for the original configuration, i.e.,

\[
\omega_0 = \sqrt{\frac{1}{m} \left| \frac{d^2 \phi}{d \mathbf{r}^2} \right|_{s=0, q=0}} = \frac{2}{s_0^2} \left( \frac{3\epsilon}{4\pi \epsilon_0 s_0} \right) \frac{q_0}{m}.
\]

Normalized potential

\[
V_{\text{norm}}(\mathbf{r}) = \frac{2\xi^2}{\alpha} \left( \frac{4\pi \epsilon_0 s_0}{q_0} \right)^2 V(\mathbf{r})
\]

is plotted along the \(z\)-axis for different values of \(q\) in Fig. 5, for different values of \(s\) in Fig. 6, and along the \(x\)- (y-)
direction in Fig. 7. We choose \( s = 0.3 s_0 \) and \( q = 0.06 q_0 \) to develop the design procedure further. Relevant parameters for \( s = 0.3 s_0 \) and \( q = 0.06 q_0 \) are compared with those for the original configuration in Table I (see §5 for \( \chi \)).

### 3.2 Design procedure: step 2

We then replace the point charges with electrodes of finite size. For this, we first calculate the equipotential surface of the electric field. Figure 8(a) shows the plot of the normalized scalar potential

\[
\phi_{\text{norm}}(\mathbf{r}) = \frac{4\pi \epsilon_0 s_0}{q_0} \phi(\mathbf{r})
\]

in the chip plane \((z = 0)\). Equipotential surfaces of \( \phi_{\text{norm}} = \pm \alpha \) \((\alpha > 0)\) around the outer point charges at \( s_{\text{out}} \) are nearly spherical for large \( \alpha \) and small \( q/q_0 \), and can be well imitated by disc-shaped electrodes of radius \( r_{\text{out}} = 2 s_0/(2\alpha + 1) \) centered at \( s_{\text{out}} \). However, smaller \( \alpha \) gives higher electric field assuming that the voltage applied to the electrodes is fixed. In Fig. 8(b), we choose \( \alpha = 2.0 \) \((r_{\text{out}} = 0.4 s_0)\). We apply a voltage of \( \pm v_0 \) or 0 at an appropriate phase to these outer electrodes:

\[
a = \frac{4\pi \epsilon_0 s_0}{q_0} v_0,
\]

i.e.,

\[
V(\mathbf{r}) = \frac{\alpha}{2 \alpha^2} \left( \frac{v_0}{s_0} \right)^2 V_{\text{norm}}(\mathbf{r}).
\]

Equipotential surfaces of \( \phi_{\text{norm}} = 0 \) around the inner point charges at \( s_{\text{in}} \) are again nearly spherical for \( q/q_0 \ll s/s_0 \), and the inner point charges can be replaced by disc-shaped electrodes of radius \( r_{\text{in}} = q/q_0 \left( (s_0^2 - 2 s_0 x + 2 x^2)^{-1/2} - (s_0^2 + 2 s_0 x + 2 x^2)^{-1/2} \right) \) \((r_{\text{in}} = 0.11 s_0 \text{ for } q = 0.06 q_0, \ s = 0.3 s_0)\) centered at \( s_{\text{in}} \), which are always connected to the ground (0 voltage).

### 4. Trapping Particles Against Gravity of the Earth

In this section, we give some practical values to trap neutral atoms and dielectric spheres on the earth. We orient the \( z \)-axis along the gravity direction. We set a condition that the gravitational sag should be smaller than \( \sigma s_0 \) \((\text{with } \sigma \ll 1)\): \( g/(\xi \omega^2) \leq \sigma s_0 \). This gives

\[
\omega_0 \geq \frac{g}{\sigma s_0 \xi \left( \frac{\omega}{\omega_0} \right)^2}.
\]

On the other hand,
\[ \omega_0 = \frac{2}{s_0^2} \sqrt{\frac{3\alpha v_0}{m}}. \]

We set the trap design as \( s = 0.3s_0, q = 0.06q_0, \) and \( a = 2.0, \) which gives \( \omega = 0.85\omega_0 \) and \( \xi = 0.94. \) In addition, we set \( \sigma = 0.1 \) and \( g = 9.81 \text{ m s}^{-2}. \)

First, we discuss trapping neutral atoms. We choose sodium atoms as an example: \( \alpha = 2.68 \times 10^{-10} \text{ F m}^{-2}, \) \( m = 3.82 \times 10^{-26} \text{ kg}, \) and thus, \( \alpha/m = 7.0 \times 10^{-14} \text{ F m}^{-2} \text{ kg}^{-1}. \) If we set \( s_0 = 0.5 \text{ mm}, \) then the condition of the gravitational sag requires \( \omega_0 \geq 2\pi \times 85 \text{ Hz}. \) This is satisfied with an applied voltage of \( v_0 = 290 \text{ V}. \) Setting \( v_0 = 290 \text{ V} \) gives \( \omega = 2\pi \times 73 \text{ Hz}, \) and using Fig. A-2, the trap should be stable for \( 7.2 < T < 8.8 \text{ ms} \) with \( T_A = T_B = T/2. \)

Next, we discuss trapping of a dielectric material. The polarizability of a dielectric sphere of radius \( a \) and dielectric constant \( \epsilon \) is

\[ \alpha = \frac{4\pi\epsilon_0}{\epsilon + 2\epsilon_0} a^3 \]

[mass is \( m = (4\pi/3)\rho a^3 \), where \( \rho \) is the density], such that

\[ \frac{\alpha}{m} = \frac{3\epsilon_0}{\rho} \frac{\epsilon - \epsilon_0}{\epsilon + 2\epsilon_0} \]

is independent of the radius \( a. \) Polystyrene (\( \epsilon = 2.5\epsilon_0, \) \( \rho = 1.05 \times 10^3 \text{ kg m}^{-3} \)), for example, has \( \alpha/m = 8.4 \times 10^{-15} \text{ F m}^2 \text{ kg}^{-1} \), which is roughly one order of magnitude smaller than that of sodium atoms.

5. Trapping of Polar Particles

In the case of polar particles of permanent dipole moment \( \mu, \) the force under the electric field can be written as follows, assuming that the direction of the dipole moment is always oriented parallel to the electric field (see Appendix C):

\[
F_i(r) = \sum_j \mu_j \nabla E_j(r)
= \sum_j \mu_j \frac{E_j(r)}{E(r)} \nabla E_j(r)
= \mu \sum_j \frac{E_j(r)}{E(r)} \nabla E_j(r)
= \mu \frac{E(r)}{2E(r)} \nabla \left[ (E(r))^2 \right]
= \mu \nabla E(r).
\]

Thus, the force is derived from the potential

\[
V(r) = -\mu E(r).
\]

Near the trap center \( r_c, \)

\[
E(r) = \frac{E(r)^2 + E(r_c)^2}{2E(r_c)},
\]

such that the dynamics of the polar particles can be treated in the same way as that of the nonpolar particles by replacing \( \alpha \) with \( \mu / E(r_c). \) We define a dimensionless parameter \( \chi \equiv E(r_c)/E_0 \) where

\[
E_0 = E(0)|_{q=0} = \frac{q_0}{4\pi\epsilon_0s_0} |\nabla \Phi_{\text{norm}}(0)|_{q=0} = \frac{2v_0}{as_0}.
\]

Now, \( \omega_0 = \omega/\sqrt{\mu/E(r_c)} = \omega_0/\sqrt{\mu/E_0} \), and dielectric spheres, and polar particles. The parameters for this trap were comparable to those indicated in ref. 6. In the final step of the design, we used disc-shaped electrodes to imitate the equipotential surface of the electric field. To analyze its effect, we are now calculating the electric field using numerical methods. The result is still preliminary, but it shows that the field near the trapping point is relatively independent of the shape of the inner electrodes. Thus, as an application for our trap, a particle conveyor such as that shown in Fig. 10 should be possible.

6. Conclusions and Outlook

We presented a new design of an ac electric trap for neutral particles and demonstrated that it is possible to establish a trapping point above the chip surface with a planar electrode structure. We also discussed the feasibility of this design by presenting typical parameters for trapping neutral atoms, dielectric spheres, and polar particles. The parameters for this trap were comparable to those indicated in ref. 6. In the final step of the design, we used disc-shaped electrodes to imitate the equipotential surface of the electric field.4 To analyze its effect, we are now calculating the electric field using numerical methods. The result is still preliminary, but it shows that the field near the trapping point is relatively independent of the shape of the inner electrodes. Thus, as an application for our trap, a particle conveyor such as that shown in Fig. 10 should be possible.

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Appendix A: Stability

In this section, we investigate the stable condition of the dynamical confinement. The equations of motion are separated in \( x-, y-, \) and \( z- \) directions so that we explore the motion in the \( x- \) direction only. We follow the procedure described in ref. 4 and define a state vector \( X(t) \) as

\[
\omega_0 = \frac{2}{s_0^2} \sqrt{\frac{3\mu v_0}{m\chi\alpha s_0}} = \frac{6\mu v_0}{\chi\alpha s_0^2}.
\]

A plot of \( \chi \) is given in Fig. 9. For a particle of bulk material of permanent polarization \( P \) and density \( \rho, \mu/m = P/\rho \) such that \( \omega_0 = \sqrt{6Pv_0/(\chi\alpha s_0^2)} \) is independent of the shape and size of the particle. To give practical parameters, we again use the trap design \( s = 0.3s_0, q = 0.06q_0, \) and \( a = 2.0, \) and consider trapping of BaTiO\(_3\) microparticles (\( P = 0.26 \text{ C m}^{-2} \) and \( \rho = 5.5 \times 10^3 \text{ kg m}^{-3} \)). If we set \( s_0 = 3.0 \text{ mm}, \) then the condition on the gravitational sag with \( \sigma = 0.1 \) is satisfied for \( \omega_0 \geq 2\pi \times 35 \text{ Hz}, \) which implies that \( v_0 \geq 5.9 \text{ V}. \) At \( v_0 = 5.9 \text{ V}, \) \( \omega = 2\pi \times 29 \text{ Hz}, \) and, from Fig. A-2, the trap should be stable for \( 18 < T < 21 \text{ ms}. \)
trapping points levitated off the chip surface. Electrodes shown in black are connected to the ground.

Fig. 10. Possible design of a particle conveyor. “+” denotes the trapping points levitated off the chip surface. Electrodes shown in black are connected to the ground.

\[
X(t) = \begin{pmatrix} x(t) \\ v(t) \end{pmatrix}, \quad (A·1)
\]

where \(x(t)\) (\(v(t)\)) is the position (velocity) of the particle at time \(t\). The time evolution of \(X(t)\) is described by a time evolution matrix \(U(t)\) as \(X(t) = U(t)X(0)\):

\[
U(t) = \begin{pmatrix} x^{(1)}(t) & x^{(2)}(t) \\ v^{(1)}(t) & v^{(2)}(t) \end{pmatrix}, \quad (A·2)
\]

with \(U(0) = I\) (\(I\) is the unit matrix). We assume that phase \(A\) starts at \(t = 0\) (Fig. A·1) so that \(U(nT + t) = U(t)U(T)^n\) with \(U(T) = U_B U_A\), where \(U_A\) (\(U_B\)) is the time evolution matrix for phase A (B):

\[
U_A = \begin{pmatrix} \cosh \sqrt{\eta_0 T_A} & \frac{1}{\sqrt{\eta_0}} \sinh \sqrt{\eta_0 T_A} \\ \sqrt{\eta_0} \sinh \sqrt{\eta_0 T_A} & \cosh \sqrt{\eta_0 T_A} \end{pmatrix}, \quad (A·3)
\]

\[
U_B = \begin{pmatrix} \cos \omega T_B & \frac{1}{\omega} \sin \omega T_B \\ -\omega \sin \omega T_B & \cos \omega T_B \end{pmatrix}, \quad (A·4)
\]

with \(T_A = T_B = T/2\). Let \(\lambda\) be the eigenvalue of \(U(T)\):

\[
\lambda^2 - 2\beta \lambda + 1 = 0, \quad (A·5)
\]

Then, the time evolution of a general state vector becomes \(X(t) = U(t)X(0) + W(t)\).

\[
W(2T) = U(T)W(T) + W(T)
\]

\[
W(3T) = U(T)\left[U(T)W(T) + W(T)\right] + W(T)
\]

\[
W(nT) = \left[U(T)^{n-1} + U(T)^{n-2} + \cdots + U(T) + 1\right]W(T) = \frac{1 - U(T)^n}{1 - U(T)} W(T) \quad (B·1)
\]
To evaluate the time average of $X(t)$, we set $t = nT + \tau$ with $n = 0, 1, 2, \ldots$ and $0 \leq \tau < T$, and take the average over $n$ and $\tau$. Because $\bar{U}(T) = 0$ inside the stable region, we obtain

$$
\bar{X}(nT + \tau) = \frac{1}{1 - \bar{U}(T)} W(T) + \bar{W}(\tau).
$$

After a lengthy calculation, eq. (B·4) yields

$$
\bar{X}(nT + \tau) = x_0 \left[ \frac{-T_A + \eta T_B}{\eta T} + \frac{(\cosh \sqrt{\eta \omega T_A} - 1) \sin \omega T_B - \sqrt{\eta} \cos \omega T_B}{(1 - \beta \omega T) \eta T_A} \right]^{1/2}
$$

and $\bar{v}(nT + \tau) = 0$, where $x_0 = F_0/(m \omega^2)$. In the limit $\omega \tau \to 0$, $\bar{v}(nT + \tau) = x_0 T/(T_B - \eta T_A)$ as expected. The effective trap frequency is calculated as

$$
\omega_{\text{eff}} = \sqrt{x_0 / \bar{v}(nT + \tau) \omega}.
$$

In Fig. B·1, we plot $\omega_{\text{eff}}/\omega$ for $T_A = T_B = T/2$ as a function of $\omega T$ and $\eta$. From this, $\omega_{\text{eff}}/\omega = 0.25\ldots-0.33$ is obtained for $\eta = 1.5\ldots-2.0$.

**Appendix C: Dynamics of the Polarization Orientation of a Polar Particle**

Consider a polar particle of size $L$ and volume $u (u \sim L^3)$ that is made of a bulk material under an electric field $E$. Let $\theta$ be the angle between the electric field $E$ and the polarization $P$ of the material. Then,

$$
I \dot{\theta} = N = -uPE \sin \theta,
$$

where $I$ is the moment of inertia and $N$ is the torque. $I \sim \rho L^2$, where $\rho$ is the density of the material [$I = (\pi/60)\rho L^5$ for a sphere of diameter $L$]. Assuming $\theta$ is small, $\theta \sim -[PE/(\rho L^2)]\theta$ (sphere: $\theta \sim -[10PE/(\rho L^2)]\theta$). The angular frequency of oscillation of the polarization direction is thus $\omega_{\text{osc}} \sim \sqrt{PE/\rho}/L$ (sphere: $\omega_{\text{osc}} = \sqrt{10PE/\rho}/L$). If the switching time between phases A and B is longer than $\omega_{\text{osc}}^{-1}$, then the direction of the polarization adiabatically follows that of the electric field. For BaTiO$_3$ (see §5) with $L = 1\mu$m and $E = 1\text{V/mm}$, $\omega_{\text{osc}} \sim 2 \times 10^5\text{s}^{-1}$ (sphere: $\omega_{\text{osc}} = 7.8 \times 10^5\text{s}^{-1}$).

Fig. B·1. Plot of $\omega_{\text{eff}}/\omega$ as a function of $\omega T$ and $\eta (T_A = T_B = T/2)$.

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