The isospin dependence of the nuclear force and its impact on the many-body system

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Abstract. A major goal of contemporary nuclear physics is to improve our knowledge of the nuclear matter equation of state. In particular, the equation of state of isospin-asymmetric matter (that is, with unequal concentrations of protons and neutrons), is not well understood, mostly due to our limited knowledge of the symmetry energy. The latter reduces the binding energy in a nucleus with unequal number of protons and neutrons, and is crucial for understanding nuclear stability. We review experimental, phenomenological, and theoretical facts about the symmetry energy. We emphasize the importance of adopting a microscopic approach towards a better understanding of this important quantity.

1. Introduction
Nearly eight decades ago, Bethe and Weizäcker [1, 2] proposed the “liquid drop model” for the nuclear binding energies which, in spite of its simplicity, turned out to be remarkably successful. The model is essentially a mass formula which provides the total binding energy per nucleon as the sum of several contributions,

\[ \frac{B}{A} = a_V - \frac{a_s}{A^{1/3}} - \frac{a_C Z(Z-1)}{A^{4/3}} - \frac{a_{sym}(A-2Z)^2}{A^2} - \frac{\Delta E}{A}. \]  

The first four contributions are known as the volume, the surface, the Coulomb, and the symmetry term, respectively. The last term contains smaller contributions, originating from effects such as pairing. The (attractive) volume term is the largest contribution and represents the energy per nucleon of an infinite, charge-neutral, isospin-symmetric system. Such idealized system is known as symmetric nuclear matter, and the value of the \( a_V \) coefficient is the binding energy per nucleon of infinite symmetric matter, approximately 15 MeV. Clearly, the (repulsive) Coulomb term favors a nucleus with a large neutron excess. On the other hand, the distabilizing effect of the symmetry term increases with larger values of \( N - Z = A - 2Z \). Fits to empirical nuclear binding energies determine the coefficients of the mass formula, resulting in a value of about 23 MeV for \( a_{sym} \).

Although this simple model captures the main features of the nuclear binding energies, a deep understanding of nuclear structure and nuclear stability, all the way to the (yet unknown) drip lines, requires a microscopic approach to the interaction of hadrons in nuclei. This is still a very challenging problem that goes to the very core of nuclear physics. In fact, our present knowledge of the nuclear force in free space is, in itself, the result of decades of struggle [3]. The nature of the nuclear force in the medium is of course a much more complex problem, as
it involves aspects of the force that cannot be constrained through free-space nucleon-nucleon (NN) scattering. Predictions of nuclear properties are the ultimate test for many-body theories.

Nuclear matter, which was defined above, is an alternative and convenient theoretical laboratory for many-body theories. Being an infinite (and thus translationally invariant) system of nucleons subjected only to their mutual strong interactions, it is theoretically simpler than an actual nucleus.

Adopting what is known as “local density approximation,” one can relate the energy per particle in nuclear matter to the one in finite systems, through an appropriate energy-density functional. Vice versa, empirical information on nuclei can be extrapolated to an infinite system to provide constraints for theoretical models of nuclear matter.

For the case of (cold) isospin-asymmetric nuclear matter, where proton and neutron densities are different, the energy per particle is a function of both the total density and the relative concentrations of protons and neutrons. Naturally, the latter dependence is closely related to the symmetry term in the Bethe-Weizäcker formula, Eq. (1).

In the \textit{ab initio} approach to the many-body problem, one starts with fundamental few-nucleon forces. Recently, chiral effective theories of the nuclear force, originally proposed by Weinberg [4], have become popular as a mean to respect the symmetries of Quantum Chromo Dynamics (QCD) while retaining the basic degrees of freedom typical of low-energy nuclear physics, pions and nucleons. Chiral effective theories provide a well-defined scheme to determine the appropriate many-body diagrams to be included at each order of the perturbation. However, being based on a low-momentum expansion, interactions derived from chiral perturbation theory are not suitable for applications in dense nuclear/neutron matter, where high Fermi momenta are involved. We will discuss this issue further in Sec. 3.

Although predictions of symmetric nuclear matter properties from different models are reasonably convergent, predictions of the symmetry energy can vary dramatically, particularly so for its density dependence. It is the purpose of this article to explore and discuss the present status of our knowledge of the symmetry energy and, more broadly, the equation of state (EoS) of asymmetric matter. One of our main goals is to demonstrate the importance of pursuing a microscopic approach towards a better understanding of the properties of dense many-body systems.

2. The Symmetry Energy

2.1. Some phenomenological features

Isospin-asymmetric nuclear matter is characterized by the neutron density, $\rho_n$, and the proton density, $\rho_p$. It is more convenient to refer to the total density $\rho = \rho_n + \rho_p$ and the asymmetry (or neutron excess) parameter $\alpha = \frac{\rho_n - \rho_p}{\rho}$. Clearly, $\alpha=0$ corresponds to symmetric matter and $\alpha=1$ to neutron matter.

Expanding the energy/particle in isospin asymmetric matter with respect to the asymmetry parameter yields

$$e(\rho, \alpha) = e_0(\rho) + \frac{1}{2} \left( \frac{\partial^2 e(\rho, \alpha)}{\partial \alpha^2} \right)_{\alpha=0} \alpha^2 + O(\alpha^4).$$

(2)

To a very good degree of approximation, the energy per particle in isospin asymmetric matter can be written as

$$e(\rho, \alpha) \approx e_0(\rho) + e_{sym}(\rho) \alpha^2,$$

(3)

which is to be compared with the sum of the first and fourth terms on the right-hand side of Eq. (1), with $\alpha$ replacing $\frac{N-Z}{A}$. A typical value for $e_{sym}$ at nuclear matter saturation density ($\rho_0$) is 30 MeV, with theoretical predictions spreading approximately between 26 and 35 MeV.
A measure for the density dependence of the symmetry energy is the parameter defined as

\[ L = 3\rho_0 \left( \frac{\partial e_{\text{sym}}(\rho)}{\partial \rho} \right)_{\rho_0} \approx 3\rho_0 \left( \frac{\partial e_{\text{n.m.}}(\rho)}{\partial \rho} \right)_{\rho_0}, \]  

(4)

where we have used Eq. (3) with \( \alpha = 1 \). Thus, \( L \) is sensitive to the gradient of the energy per particle in neutron matter \( (e_{\text{n.m.}}) \), that is, the neutron matter pressure. As to be expected on physical grounds, the neutron skin,

\[ S = \sqrt{<r_n^2>} - \sqrt{<r_p^2>}, \]

(5)

where \( \sqrt{<r_n^2>} \) and \( \sqrt{<r_p^2>} \) are the r.m.s. radii of the neutron and proton distributions, is highly sensitive to the same energy gradient.

Predictions of \( L \) by phenomenological models show a very large spreading. Values ranging from -50 to +100 MeV are found from the numerous parametrizations of Skyrme interactions [5], all chosen to fit the binding energies and the charge radii of a large number of nuclei.

The definition of \( L \) in Eq. (4) originates from an expansion of the symmetry energy in powers of the density (relative to saturation density):

\[ e_{\text{sym}}(\rho) \approx e_{\text{sym}}(\rho_0) + \frac{L}{3} \left( \frac{\rho - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2, \]

where

\[ K_{\text{sym}} = 9\rho_0^2 \left( \frac{\partial^2 e_{\text{sym}}(\rho)}{\partial \rho^2} \right)_{\rho = \rho_0}. \]

(7)

2.2. Experimental constraints on the symmetry energy

In this section we review some recent measurements to constrain the symmetry energy. We refer the reader to Ref. [6] and references therein for more details.

Several types of experiment can be exploited to constrain the density dependence of the symmetry energy. They include:

- Heavy ion collisions.
- Nuclear binding energies.

The EoS is an important part of the input of transport models describing heavy ion collisions, and thus can be constrained through measurements of selected observables in ion-ion scattering. Sensitivity to the EoS can be attained by varying the number of protons and neutrons of the colliding nuclei. Emission of particles from neutron-rich or proton-rich systems is sensitive to the symmetry potential, which is repulsive on neutrons and attractive on protons (in a neutron-rich environment). Neutron-proton spectral ratios as well as neutron, hydrogen, and fragment flows are sensitive to the density dependence of the symmetry energy and have been used for that purpose. Also, in a collision where target and projectile have different values of \( N/Z \), the system tends towards a state of constant value of the asymmetry parameter \( \alpha \), a phenomenon known as “isospin diffusion”, the rate of which is controlled by the strength of the symmetry potential. Neutron and proton spectra from central collisions of \( ^{124}\text{Sn} + ^{124}\text{Sn} \) and \( ^{112}\text{Sn} + ^{112}\text{Sn} \) at 50 MeV per nucleon have been measured, as well as transverse collective flows of hydrogen, helium isotopes, and other fragments at 35 MeV per nucleon in \( ^{70}\text{Zn} + ^{70}\text{Zn}, ^{64}\text{Zn} + ^{64}\text{Zn}, \) and \( ^{64}\text{Ni} + ^{64}\text{Ni} \) reactions.

Nuclear binding energies.
This method relies on the liquid droplet model we referred to in Section 1. Because the symmetry energy contribution to the total binding energy is small, a reliable determination of both the symmetry energy and its slope can be difficult. All other contributions must be known with very high accuracy. With improved droplet models which allow to reproduce binding energies within
0.1\%, it was possible to determine the symmetry energy within 1 MeV and its slope within 30 MeV [6].

Neutron skin measurements with hadronic or electroweak probes.

The relation between the thickness of the neutron skin and the slope of the symmetry energy can be understood in terms of the pressure gradient (due to the symmetry potential) that pushes neutrons outwards (and acts in a similar manner in the interior of neutron stars). Experiments aimed at extracting the neutron skin as a measure of the slope of the symmetry energy use either electroweak or hadronic probes. Particularly for the latter, theoretical uncertainties in the models used to analyse the data are hard to quantify.

Parity-violating electron scattering experiments are now a realistic option to determine neutron distributions with unprecedented accuracy. The neutron radius of \(^{208}\text{Pb}\) is expected to be re-measured at the Jefferson Laboratory in the PREXII experiment planned for the near future. Parity-violating electron scattering at low momentum transfer is especially suitable to probe neutron densities, as the \(Z^0\) boson couples primarily to neutrons. A much higher level of accuracy can be achieved with electroweak probes than with hadronic scattering. With the success of this program, reliable empirical information on neutron skins will be able to provide more stringent constraints on the density dependence of the symmetry energy.

Measurements of the neutron skin of \(^{48}\text{Ca}\) have also been proposed and are expected to take place in the near future with the CREX experiment [7]. Being much lighter than \(^{208}\text{Pb}\), \(^{48}\text{Ca}\) is considerably more sensitive to surface and shell effects, which makes it possible to extract useful structure information. Furthermore, \textit{ab initio} calculations are feasible for Calcium isotopes, thus CREX experiments have the potential to test models of three-nucleon forces and density functional theories.

Brief summary of present constraints.

In summary, the authors of Ref. [6] conclude that consistent constraints can be obtained from structure measurements and heavy-ion collisions. Their analyses, which include densities between \(0.3\rho_0\) and \(\rho_0\), yield a constraint centered at approximately \((32.5, 70)\) MeV for the symmetry energy and the \(L\) parameter, respectively. The extracted value for the neutron skin of \(^{208}\text{Pb}\) is \(0.18 \pm 0.027\) fm, which is found to be consistent with the symmetry energy constraints.

3. Our microscopic approach to isospin-asymmetric nuclear matter

After the development of QCD and the understanding of its symmetries, chiral effective theories [4] were developed as a way to respect the symmetries of QCD while keeping the degrees of freedom (nucleons and pions) typical of low-energy nuclear physics. However, chiral perturbation theory (ChPT) has definite limitations as far as the range of allowed momenta is concerned. For the purpose of applications in dense matter, where higher and higher momenta become involved with increasing Fermi momentum, NN potentials based on ChPT are unsuitable. On the other hand, relativistic meson theory is an appropriate framework to deal with the high momenta encountered in dense matter. In particular, the one-boson-exchange (OBE) model has proven very successful in describing NN elastic data in free space up to high energies, and has a good theoretical foundation. We seek a momentum-space potential developed within a relativistic scattering equation. Furthermore, we require a potential that uses the pseudovector coupling for the interaction of nucleons with pseudoscalar mesons. With these constraints in mind, as well as the requirement of a good description of the NN data, Bonn B [3] is a reasonable choice.

Concerning our approach to nuclear matter, we adopt the Dirac-Brueckner-Hartree-Fock (DBHF) method. The main strength of the DBHF approach is its inherent ability to account for important three-body forces through its density dependence. More specifically, we refer to three-body forces arising from excitation of a virtual nucleon-antinucleon pair. For a detailed review of our application of the DBHF method to symmetric and asymmetric matter, we refer the reader to Ref. [8].
4. The symmetry energy and the role of the isovector mesons

Recently we explored the role of all isovector channels for the symmetry energy from the point of view of an ab initio model [9]. The main point of the ab initio approach is that mesons are tightly constrained by the free-space data and their parameters are never readjusted in the medium (this is what we mean by “parameter-free”). Furthermore, the contributions from the various mesons are fully iterated, thus giving rise to correlation effects. The corresponding predictions can be dramatically different than those which may be produced in first-order calculations.

In Fig. 1 we show the density dependence of the symmetry energy obtained with the Bonn A, B, and C potentials. The potential model dependence comes almost entirely from differences among predictions of the energy in symmetric nuclear matter. With the three sets of predictions, we mean to estimate the uncertainty to be expected when using different parametrizations for the isovector mesons, while describing the free-space NN data quantitatively.

In contrast, we show in Fig. 2 predictions of the symmetry energy with the numerous parametrizations of the Skyrme model [5]. Clearly, the constraint from the free-space data reduces dramatically the spreading among theoretical predictions.

![Figure 1](attachment:image1.png)

**Figure 1.** The symmetry energy as predicted with Bonn A, B, and C.

![Figure 2](attachment:image2.png)

**Figure 2.** Predictions of the symmetry energy vs. density for various parametrizations of the Skyrme model.

We have examined the effect of the isovector mesons ($\pi$, $\rho$, and $\delta$) on the difference between the potential energies of pure neutron matter and symmetric matter [9]. Our findings are easily understood in terms of the contributions of each meson to the appropriate component of the nuclear force and the isospin dependence naturally generated by isovector mesons. We find that the pion gives the largest contribution to this difference. The contribution of the pion is often overlooked, possibly because this meson is missing from some mean field models, which are popular among users of equations of state. It is our opinion that conclusions regarding the interplay of $\rho$ and $\delta$ in phenomenological models [10] should be taken with caution.

We emphasize the fundamental differences between our approach and the one of mean field models, particularly pionless QHD theories. First, these differences are of conceptual relevance, since free-space NN scattering and the NN bound state are essentially determined by pion physics. Furthermore, they can impact in a considerable way conclusions with regard to isospin dependent systems/phenomena. In order to have a fundamental basis, a microscopic theory of the nuclear many-body problem has to start from the bare NN interaction with all its components.
5. Conclusions and outlook

The symmetry energy plays a crucial role for the binding energies of nuclei and thus nuclear stability. In the idealized system known as infinite nuclear matter, it represents the reduction of binding energy per particle in the presence of isospin asymmetry.

The details of the density dependence of the symmetry energy [i.e. its derivative(s)] are still quite controversial. And yet, important observables depend crucially on those details, the most egregious example being the neutron skin thickness. Furthermore, the potential energy part of the symmetry energy controls the isospin-dependent dynamics in heavy-ion reactions. We have reviewed the rich and diverse effort which is presently going on to improve the available constraints on the symmetry energy and, more generally, the energy and pressure in asymmetric matter.

In reviewing our theoretical approach to symmetric and asymmetric matter, we argued that relativistic nuclear physics is a valid paradigm, particularly when high momenta (and/or high Fermi momenta) are involved. The ability of the DBHF approach to effectively incorporate important three-body effects is what makes this method attractive and convenient.

We then concentrated on what builds the symmetry energy in a relativistic meson-theoretic model. On simple physical grounds, one can expect that the isovector mesons would be the important building blocks. This is in fact the case. Of those, we identified the pion as by far the most important. This brings the tensor force into the spotlight, not only as a powerful saturation mechanism, but also as the main nuclear force component for the symmetry energy. We observed that the spreading of symmetry energy predictions obtained with different NN potentials (constrained by free-space data) is moderate, in contrast with phenomenological models.

In closing, we point out that our microscopic effective interactions (that is, the Dirac-Brueckner-Hartree-Fock G-matrix) can be useful for applications to nuclear reactions [11, 12]. Most recent work of our group includes the solution of the Bethe-Goldstone equation in three-dimensional space (so that standard angle-average approximations could be removed from the Pauli blocking operator) [13], and the development of microscopic in-medium NN cross sections suitable for applications in nucleus-nucleus collisions [12].

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