Stellar Tidal Processes Near Massive Black Holes

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Abstract

Close tidal interactions of stars with a central massive black hole (MBH) or with other stars in the high density cusp around it can affect a significant fraction of the stellar population within the MBH radius of influence. We consider three strong processes that have the potential of modifying stellar structure and evolution there. (1) Tidal spin-up by hyperbolic star-star encounters. (2) Tidal scattering of stars on the MBH. (3) Tidal heating of inspiraling stars—"squeezars"—that were tidally captured by the MBH. We discuss the implications for stellar populations near MBHs and for the growth of MBHs by tidal disruption of stars, and the possible observational signature of such processes near the MBH in the Galactic Center. We compare the event rates of prompt tidal encounters (tidal disruption and tidal scattering) and slow inspiral events (squeezars / tidal capture), and find that tidal capture is at least an order of magnitude less efficient than prompt disruption. This means that past studies, which assigned similar weights to prompt disruption and tidal capture, over-estimated the contribution of tidal disruption to the growth of the MBH by at least a factor of two.

1.1 Introduction

Strong tidal interactions involving stars are expected to occur frequently near a MBH in a galactic center.

First, the MBH is a mass sink, which drives a flow of stars from the MBH radius of influence $r_h$ to the center, to replace those it has destroyed. An inevitable consequence of this flow is that some stars are deflected into orbits whose periapse $r_p$ lies just outside the critical radius for destruction. We will focus here on the case where the MBH mass $m$ is small enough so that the tidal disruption radius $r_t = R_s(M/M_\odot)^{1/3}$, where $M_\odot$ and $R_\odot$ are the stellar mass and radius, lies outside the event horizon $r_h$, ($m \cdot 10^8 M_\odot$ for a solar type star). Such stars will suffer an extreme tidal impulse, but will not be destroyed, at least not on their first peri-passage. There are two possible outcomes: that the star is ultimately disrupted, or that it avoids subsequent encounters with the MBH. Both are considered in detail below.

Second, a variety of formation scenarios predict that MBHs should lie in the center of a high density stellar cusp (e.g. Bahcall & Wolf 1977; Young 1980). The diverging stellar density implies that there must be some volume around the MBH where close tidal encounters occur on timescales significantly shorter than the typical stellar lifetime. Such encounters will have a very different outcome from those that occur in globular clusters that do not
contain a MBH. In most cases the encounters will not lead to tidal capture. Instead the two stars will continue on their separate ways after experiencing a brief strong tidal impulse.

Extreme tidal interactions, which transfer energy and angular momentum from the orbit to the star, can affect its structure and subsequent evolution by heating it, spinning it up, mixing it, or ejecting some of its mass. This is interesting in view of the observed presence of unusual stellar populations near MBHs: the blue nuclear cluster in the inner 0.02 pc of the GC (Genzel et al. 1997), and around the MBH in M31 (Lauer et al. 1998); evidence for anomalously strong rotational dredge-up in an M supergiant near the MBH in the GC (Carr, Sellgren & Balachandran 2000), but not in a high density nuclear cluster without a MBH (Ramírez et al. 2000); the unusually high concentration of very rare extreme blue He supergiants around the Galactic MBH (Krabbe et al. 1991; Najarro et al. 1994).

The observational consequences of extreme tidal interactions cannot be predicted with certainty at this time, although some reasonable conjectures can be made (Alexander & Livio 2001; Alexander & Morris 2003). Irrespective of this uncertainty, it is clear, as shown below, that the amount of tidal energy deposited in the star can reach a significant fraction of its binding energy, and that the angular momentum extracted from the orbit can spin a star up to a significant fraction of its break-up velocity. It is therefore plausible to assume that the effects can be observationally interesting and proceed to explore the dynamical processes that give rise to such tidal interactions. Furthermore, tidal disruption and collisional stellar mass loss are important channels for supplying mass to a low-mass MBH (Murphy, Cohn & Durisen 1991), and so the consequences of extreme tidal processes may provide observable links between the properties of the stellar population near the MBH and its evolutionary history.

At a distance of 8 kpc (Reid 1993), the low mass 2 × 10^6 MBH in the Galactic Center (GC) (Ghez et al. 2000; Schödel et al. 2002) is the nearest and observationally most accessible MBH. Although it is heavily reddened (A_K ∼ 3", Blum et al. 1996), deep high resolution astrometric, photometric and spectroscopic IR observations of thousands of stars very close to the MBH provide information on their luminosity, effective temperature and orbits (e.g. Eckart, Ott & Genzel 1999; Figer et al. 2000; Gezari et al. 2002). Since the GC is the obvious first place to look for evidence of extreme tidal interactions, the results presented here will be applied to the GC, but it should be emphasized that these physical mechanisms are generally relevant for MBH in galactic nuclei.

1.2 Tidal spin-up by star-star encounters

Stars in the high Keplerian velocity field in a dense stellar cusp around a MBH will suffer numerous hyperbolic tidal encounters over their lifetimes. Although such encounters transfer some energy and angular momentum from the orbit to the colliding stars, they rarely remove enough energy for tidal capture. This is in marked contrast to the situation in the high density cores of globular clusters without a MBH, where the colliding stars are on nearly zero-energy orbits and close collisions lead to the formation of tidal binaries. The effects of hyperbolic encounters on the stars are mostly transient. The stellar dynamical and thermal timescales are very short compared to the mean time between collisions, and so apart from some mass-loss in very close collisions, the star is largely unaffected. It is however more difficult for the star to shed the excess angular momentum, since magnetic breaking operates on timescales of the order of the stellar lifetime (Gray 1992). High rotation is therefore the longest lasting dynamical after-effect of a close encounter. Over time, the stellar angular
momentum will grow in a random walk fashion due to successive, randomly oriented tidal encounters.

We consider the effect of the tides raised by an impactor star of mass $m$ on a target star of mass $M$ and radius $R$ as the impactor follows an unbound orbit with periapse $r_p$ from the target star. The tilde symbol will be used below to denote quantities expressed in units where $G = M = R = 1$. In these units, $e = 1$ is the centrifugal break-up angular frequency, $E_b = -1$ is the stellar binding energy, up to a factor of order unity, and $e_r = m^{1/3}$ is the MBH tidal radius.

The orbital energy $E$ and the orbital angular momentum $L$ that are transferred from the orbit to the star by an impulsive tidal encounter are related by $E = e_p L$, where $e_p$ is the relative angular velocity at periapse. We assume here for simplicity rigid body rotation. The change in the stellar angular velocity due to a single parabolic encounter is then given to the leading order in the linear multipole expansion by (Press & Teukolsky 1977)

$$e' = e_m^2 T_2(\frac{e_p}{e_r^6})$$

where $L$ is the stellar moment of inertia (assumed to remain constant), $T_2$ is the $^3_1$ tidal coupling coefficient (calculated numerically for a given stellar structure model), and $e = [e_r^6 = (1 + m)]^{1/2}$ is the dimensionless transit time of the encounter. The steep $e_r^{-6}$ dependence indicates that most of the contribution comes from very close encounters, where the linear expansion no longer holds. The formal divergence at small periapse must be truncated by non-linear effects, which have to be investigated numerically. Smoothed Particle Hydrodynamics (SPH) simulations (e.g. Fig. 1.2) of grazing and penetrating encounters show that as $e_p$ is decreased, $e$ first increases above the value predicted by linear theory, and then saturates at the onset of mass loss, as the ejecta carry away the excess angular momentum.

These results can be incorporated into the linear theory by simple prescriptions, making it possible to calculate the mean spin-up of a test star by averaging over all impact parameters, orbital energies and impactor masses for a given model of the nuclear stellar cluster (Alexander & Kumar 1999). Figure 1.1 shows the mean spin-up of a solar mass star after 10 Gyr near the MBH in the GC, as function of distance from the MBH, in a high density, $n_r/r_-^{-1.5}$ cusp of a continuously star forming population that is deduced to exist there (Alexander & Sternberg 1999; Alexander 1999). The mean spin-up reaches values as high as $e = 0.3$ within $0.03$ pc, ($\sim 60$ times higher than is typical in the field) and decreases to $10\%$ within $0.3$ pc (0.1 to 0.2 of $e$). The spin-up effect falls off only slowly with distance from the black hole because the increased tidal coupling in slower collisions at larger distances compensates for the decrease in the stellar density. Thus, long-lived main sequence stars with inefficient magnetic breaking are expected to rotate at a significant fraction of their centrifugal breakup velocity in a large volume of the dense stellar cusp around a MBH.

### 1.3 Tidal scattering by the central black hole

Tidal disruption is an important channel for feeding low-mass MBHs that accrete from a low density cusp where collisional mass-loss is low. Numeric models of the growth of a MBH in a central cluster suggest that the fraction of the MBH mass that is supplied by tidal disruption ranges between $f = 0.15$ (Murphy, Cohn & Durisen 1991) and $f = 0.65$ (Freitag & Benz 2002), depending on model assumptions. We will here $f = 0.25$ as a representative value.
Dynamical analyses of the 2-body scattering by which stars are deflected into “loss-cone” orbits that bring them within $e_\oplus$ of the MBH (Frank & Rees 1976; Frank 1978; Lightman & Shapiro 1977; Magorrian & Tremaine 1999; Syer & Ulmer 1999) show that tidally disrupted stars are typically on slightly unbound orbits relative to the MBH, and that they mostly originate from the MBH radius of influence, $e_\oplus = e^2$, where $e$ is the 1D velocity dispersion far from the MBH. The stellar mass enclosed within $e_\oplus$ is comparable to $m$. The cross-section for such stars to pass within $e_\oplus$ of the MBH scales as $\eta$ (Hills 1975; see also §1.5). It then follows that for every star on an orbit with $0 < e < 2e_\oplus$ that is promptly disrupted, there is a star that skirts the tidal disruption zone on an orbit with $e = 2e_\oplus$. Such “tidally scattered” stars narrowly escape disruption on their first peri-passage after being subjected to extreme tidal distortion, spin-up, mixing ad mass-loss (Fig. 1.2). We will now argue that there is also a high probability that these stars will avoid subsequent total disruption, either by being deflected off their orbit or by missing the MBH due to its Brownian motion (Alexander & Livio 2001).

First, the scattering timescale is shorter than the dynamical one, and so stars wander in and out of the loss-cone several times during one orbital period. After the first peri-passage,

This follows from the fact that the stars are on nearly parabolic orbits and that the scattering is isotropic. First, since the enclosed stellar mass within $e_\oplus$ roughly equals the MBH mass, a star on a plunging orbit will pass the MBH with a velocity slightly above the local escape velocity. Second, $e_\oplus$ is where the loss-cone replenishment efficiency peaks sharply, at the transition from the empty loss-cone (“diffusive”, or “small angle scattering”) regime and the full loss-cone (“pinhole”, or “large angle scattering”) regime. Deflection by large angles relative to the loss cone opening angle leads to an isotropic redistribution of the velocity.
the stars are on very eccentric orbits with apoastron \( 2e \), and so there is a considerable chance that they will be scattered again out of the loss-cone before their next close passage, and avoid eventual orbital decay and disruption.

Second, the survival probability is further increased by the Brownian motion of the BH relative to the dynamical center of the stellar system. A low mass MBH which evolves in an initially constant density core of radius \( \rho_0 \) is estimated to have Brownian fluctuations with an amplitude that is much larger than the tidal radius (Bahcall & Wolf 1976),

\[
\frac{h \rho_1}{\rho_0} \frac{e_0}{\rho_0^{5/6}} \sim 1
\]  

(1.2)

The Brownian motion proceeds on the dynamical timescale of the core, which is comparable to the orbital period of the tidally scattered stars. The orbits of these stars take them outside of \( e_0 \), where they are dynamically affected only by the center of mass of the nucleus, and not by the relative shift between the MBH and the stellar mass. Therefore, on re-entry into the volume of influence, their orbit will not bring them to the same peri-distance from the MBH.

These order of magnitude arguments are verified by more detailed analysis (see §1.5), which shows that the survival probability of tidally scattered stars is \( P_s \approx 0.8-0.9 \). It then follows that the mass fraction of surviving tidally scattered stars within \( e_0 \), which passed once within \( e_0 < 2e_0 \) of the MBH, is comparable to the mass fraction of the MBH supplied by tidal disruption. Depending on the definition of what constitutes an extreme tidal interaction, there exists a maximal periapse, parameterized by \( b_e \), that corresponds to sufficiently strong tidal interactions. For example, for a solar type star, \( b_e = 1.25 \) corresponds to a minimal tidal energy deposition of \( \mathcal{E} = 0 \Omega_2 \) (Eq. 1.4). Since the angular velocity at periapse is \( \Omega_p = (2-b_e)^{1/2} \Omega_1 = 1 \Omega_1 \) and the solar moment of inertia \( \mathcal{I} = 0 \Omega_7 \), this corresponds also to a minimal angular spin deposition of \( \mathcal{F} = 0 \Omega_2 \) and a minimal spin-up of \( \Omega = 0.28 \). Over time, the mass fraction of tidally scattered stars within \( \rho_1 \) will rise to \( f(b_e-1)P_s \sim 0.05 \) (for \( f = 0.25, b_e = 1.25 \) and \( P_s = 0.8 \)). Tidally scattered stars thus constitute a non-negligible fraction of the stellar population in the MBH radius of influence, and will remain there as relics of the early stages of the MBH evolution even after its mass grows above the tidal disruption limit, possibly detectable by correlations between unusual spectral properties and highly eccentric orbits.

### 1.4 Squeezars: Tidally powered stars

A small fraction of the stars that are deflected into orbits with \( \rho_0 < 1 \) will be tidally caught by the MBH and spiral into an ever tighter orbit as the tides gradually extract orbital energy each peri-passage. The orbital energy that a star has to lose to circularize from an \( \mathcal{E} = 0 \) orbit exceeds its own binding energy by orders of magnitude,

\[
\mathcal{E}_c = \frac{mb^{2}}{2b} \sim 1
\]  

(1.3)

where the periapse is parametrized by \( \rho_1 = b_e \). A tidally heated star—a “squeezar”—will ultimately be disrupted by expanding beyond its Roche lobe or by radiating above its Eddington luminosity.

The orbital and internal evolution of a tidally heated star in the course of the inspiral depends on its initial structure and is coupled to the changes in its mechanical and thermal...
properties in response to the tidal heating. One approach to the challenging problem of modeling squeezar evolution is to consider two simplified cases that likely bracket the range of possible responses (Alexander & Morris 1993): (1) Surface heating and radiative cooling ("hot squeezar"), where the tidal oscillations dissipate in a very thin surface layer that expands moderately and radiates at a significantly increased effective temperature (McMillan, McDermott & Taam 1987). (2) Bulk heating and adiabatic expansion ("cold squeezar"), where the tidal oscillations dissipate in the stellar bulk and cause a large, quasi self-similar expansion at a constant effective temperature (Podsiadlowski 1996).

Given these prescriptions, the evolution of the squeezar orbit, size, luminosity and temperature can be derived from the tidal energy deposition equation

$$E = \frac{T_2(b^{3/2})R^5}{b^3} \quad \text{for } (\mathbf{r} \cdot \mathbf{v}) \neq 1; \quad (1.4)$$
where $R$ is the expanded stellar radius in terms of the original radius, and the orbital equations for the semi-major axis $a$, the period $P$ and eccentricity $e$,

\[ a = -m = 2E; \quad P = 2 \sqrt{a^3/(1 + m)}; \quad e = 1 - e_p = a; \]  

(1.5)

where $E$ is the orbital energy, and Keplerian orbits near the MBH are assumed.

Figure 1.3 shows the evolution of a $1M_\odot$ hot squeezar, with the tidal coupling coefficient calculated for a model of the Sun (Alexander & Kumar 2001). In particular, the squeezar evolutionary model relates the inspiral time $t_i$ to the initial orbital period $P_0$ and the periapse $b$. The mean number of squeezars at any given time in the GC is given by $n = t_i$, where $t_i$ is the inspiral event rate (estimated in §1.5). As shown below, $\pi \sim 0.1$ in the GC if the loss-cone is replenished by two body scattering in a spherically symmetric system.

The observational implications of having on average $\pi$ squeezars near the MBH can be expressed by considering the properties of the “leading squeezar” (the one with the shortest period). The leading squeezar has, on average, completed $\pi = t_i = (\pi + 1)$ of its inspiral. Figure 1.3 also presents the results in terms of the average properties of the leading squeezar, as function of $\pi$. It is evident that the effects of tidal heating on the leading squeezar can be quite pronounced even if $\pi$ is small, and that the properties of the leading squeezar can be much more extreme if $\pi$ was under-estimated by the neglect of non-spherical effects.

### 1.5 Prompt disruption vs. slow inspiral

The orbital decay of a tidally heated star is just one case of a dissipative interaction that can lead to orbital inspiral. Other possibilities include gravitational wave (GW) emission (Hils & Bender 1995; Sigurdsson & Rees 1997; Freitag 2001, 2003) or drag against a massive accretion disk (Ostriker 1983; Syer, Clarke & Rees 1991; Viklovskij & Czerny 2002). Unlike prompt disruption or tidal scattering, where the star reaches the MBH directly in less than the initial orbital period $P_0$, slow inspiral proceeds gradually over a timescale $t_i \approx P_0$, which is typically a steeply rising function of the periapse. For the extracted orbital energy to power a high luminosity of gravity waves, tidal heat, or mechanical energy in the disk, as the case may be, the star has first to decay into a short period orbit. The time available for inspiral is limited by two-body collisions similar to those that deflected the star into its eccentric orbit in the first place, since they can deflect it again to a wider orbit where the dissipation is inefficient. Because $t_i \sim P_0$, this poses a much more severe constraint for an inspiraling star than for a promptly disrupted star.

Novikov et al. (1992) estimated that tidal capture by a MBH occurs for orbits with $b < b_c$ 3, which means that stars are scattered into tidal capture orbits and subsequently disrupted at a rate that is $b_c - 1$ 2 times faster than the rate at which they are scattered into prompt tidal disruption orbits. These considerations led to the suggestion (Frank & Rees 1976; Novikov et al. 1992; Magorrian & Tremaine 1999) that slow tidal inspiral may be at least as important as prompt disruption for feeding the MBH and for producing observable tidal flares (Frank & Rees 1976). This implies that the already large contribution of prompt tidally disrupted stars to the mass budget of a low-mass MBH (§1.3) should be further scaled by the ratio of

Based on the requirement that the orbital energy extracted by the first peri-passage should decrease the apoapse to $2e - e_0$, where $e_0$ is the distance from which the star was scattered into the loss-cone. This tidal capture criterion does not include timescale considerations.
Fig. 1.3. The evolution of a hot squeezar in the GC (Alexander & Morris 2003). The 1$M\odot$ star is on an orbit with $b = 1.5$ and $P_0 = 1.4 \times 10^4$ yr. The star is disrupted when $R = b$ at $t_{\text{disrupt}} = 3.7 \times 10^5$ yr. At that point the tidal luminosity exceeds the intrinsic luminosity by a factor of 640, but the orbit is still almost parabolic with eccentricity $e = 1 - 2 \times 10^{-4}$. The average of properties of the leading squeezar as a function of the mean number of squeezars can be read off the top axis.

The cross-sections of tidal capture and prompt disruption, $b_c$ = 1. If the relative contribution of inspiraling stars were indeed so high, the implications would be far-reaching: stars could supply most or even all of the MBH mass, thereby establishing a direct link between $m$ and stellar dynamics on a scale of $e_r$. However, a small initial periapse does not in itself guarantee inspiral and ultimate disruption. The star must also have enough time to complete its orbital decay.

This time constraint can be taken into account correctly by considering only stars that are scattered from a volume that is close enough to the MBH so that the inspiral can be completed in time (Alexander & Hopman 2003). The inspiral time $t_0$ increases with $e_p$. Therefore, the volume from which a scattered star can inspiral faster than the time it takes for two body relaxation to significantly deflect it, decreases with $e_p$. The maximal possible periapse corresponds to the point where the available volume shrinks to zero (The truncation of the cusp near the MBH by destructive stellar collisions also limits the available volume).

Table 1.1 lists the inspiral event rate, the mean number of inspiraling stars, and the maximal periapse for inspiral in the GC for three processes: hot and cold 1$M\odot$ squeezars and for gravitational wave inspiral by 0.6$M\odot$ white dwarfs (WD) that comprise 10% of an old stellar population. The rates were estimated for a spherical single-mass Keplerian stellar cusp, $n_1 / r^{-1.8}$, normalized to contain a total stellar mass $2 \times 10^6 M\odot$ ($= m$) within $r_h = 1.8$
Table 1.1. *Inspiral in the Galactic Center.*

| Process               | $i$ (yr$^{-1}$) | $\pi$ | max $r_p$ |
|-----------------------|-----------------|-------|-----------|
| Hot squeezar          | $3(-6)$         | 0.2   | $2.4r_s$  |
| Cold squeezar         | $4(-6)$         | 0.2   | $2.8r_s$  |
| WD gravity waves      | $2(-7)$         | 0.64  | $25r_s$   |

pc (after Schödel et al. 2002). The predicted prompt disruption rate for this simple model is

\[ p = 9 \times 10^5 \text{yr}^{-1}, \]

in general agreement with independent estimates from previous studies,

\[ p = 5 \times 10^5 \text{yr}^{-1} \] (Syer & Ulmer 1999) and \[ p \text{ few } 10^6 \text{yr}^{-1} \] (Alexander 1999). The rate of WD inspiral derived here, \[ i = 2 \times 10^7 \text{yr}^{-1} \], is also consistent with the estimates of Sigurdsson & Rees (1997) and Freitag (2003). The survival probability of tidally scattered stars can also be calculated by the formalism of Alexander & Hopman (2003), and it is found to be close to unity, \[ P_s = 0.8-0.9, \] as anticipated by general arguments (§1.3).

However, we find that the tidal inspiral rate is only \[ 0.05 \] of the prompt disruption rate, and not the factor 2–3 enhancement due to tidal capture that was assumed by previous studies. We conclude that the contribution of tidal capture to the MBH mass budget and to the tidal flaring rate from galactic nuclei is negligible compared to prompt disruption. Past studies, which assigned similar weights to prompt disruption and tidal capture, over-estimated the contribution of tidal disruption to the growth of the MBH by at least a factor of two.

1.6 Summary

We have shown that strong tidal interactions of stars with a MBH or with other stars in the high density cusp around a MBH can deposit large amounts of orbital energy and angular momentum in a significant fraction of the stellar population within a large volume around the MBH. We propose that such interactions can alter the evolution and appearance of stars in galactic centers and thereby probe the evolution of the MBH and the stellar system around it. We explored tidal spin-up by star-star encounters, tidal scattering by a MBH and tidal inspiral into a MBH. We showed that tidal capture is inefficient in the presence of two body scattering.

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