Sparse Reconstruction Off-grid OFDM Time Delay Estimation Algorithm Based on Bayesian Automatic Relevance Determination

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Abstract. Aiming at the problem of single measurement vector (SMV) OFDM signal delay estimation in complex environment, a sparse reconstruction off-grid delay estimation algorithm based on Bayesian automatic correlation determination (OGBARD) is proposed. The algorithm adopts Bayesian framework, from the perspective of further mining useful information, introduces asymmetric autocorrelation to determine a priori parameters and off-grid parameters, and integrates them into the parameter estimation process, which improves the delay estimation accuracy under SMV and low signal-to-noise ratio (SNR) condition. Firstly, based on the channel frequency response (CFR) estimation value of the OFDM signal physical layer protocol data unit, the algorithm constructs a sparse representation model with off-grid parameters, and then introduces the probability hypothesis of noise vector, off-grid parameters and sparse coefficient vector in the model. Finally, based on Bayesian inference, the expectation maximization (EM) algorithm is used to estimate the hyperparameters. Simulation experiments show that the proposed algorithm has better estimation performance than the comparison algorithm and is closer to the Cramer–Rao Bound (CRB).

1. Introduction
Orthogonal frequency division multiplexing (OFDM) technology has been widely used in various digital transmission and communication systems [1]. A large number of applications of OFDM signals have also spawned the positioning requirements based on it. Therefore, many scholars have studied the positioning techniques based on it. In [2], the time delay estimation problem of OFDM multipath signals is studied. The received signal is modelled as a Gaussian autocorrelation model, and then the maximum likelihood estimation algorithm under the model is derived. However, this method needs to construct the covariance matrix described in the text offline according to the environment, which is more complex. In [3], a delay estimation algorithm of OFDM-WLAN system based on propagator method (PM) is proposed for the problem of high complexity of eigenvalue decomposition in multiple signal classification (MUSIC) algorithm. However, for finite samples, especially under single sample conditions, the estimated performance of this method in [3] is degraded. This is because the received signal covariance matrix is not full rank and cannot constitute a perfect orthogonal signal and a noise subspace. Although smoothing in the frequency domain can make the covariance matrix full rank, the effective bandwidth is lost and estimation accuracy is limited [4].

In many application scenarios, due to environmental and conditional limitations, it is impossible to repeat a large number of experiments to obtain enough observation samples, and only a small number of observation samples or even a single sample can be obtained. Under the condition of single
measurement vector (SMV) and low signal-to-noise ratio (SNR), the traditional time delay estimation algorithm has the disadvantages of lower precision and higher complexity. At the same time, sparse representation is a theory that has received much attention in recent years. In [5], the orthogonal matching pursuit (OMP) algorithm is applied to the impulse radio-ultra wideband (IR-UWB) delay estimation, and delay estimation of the single-path or multipath under known transmitted signal conditions are realized. However, because the greedy algorithm is too ‘greedy’, it is not able to evade the influence of the projection of the smaller energy atom in the residual on the selection of the atom with larger energy, so the estimation accuracy is limited.

In many application scenarios, for environmental and conditional limitations, it is not possible to repeat a large number of experiments to obtain sufficient observation samples, and even a small observation sample or single sample can be obtained. Under the condition of single measurement vector (SMV) and low signal to noise ratio (SNR), the traditional time delay estimation algorithm has the drawback of lower accuracy and higher complexity. Sparse representation is a theory that attracts attention in recent years. In [5], the orthogonal matching pursuit (OMP) algorithm is applied to impulse radio-ultra-wideband (IR-UWB) delay estimation to achieve single-path or multipath delay estimation under known transmit signal conditions. However, since the greedy algorithm is too “greedy”, the estimation accuracy is limited because the effect of the projection of the low energy atom of the residual on the selection of the high energy atom cannot be avoided. In [6], the compressive sampling matching pursuit (CoSaMP) algorithm is applied to the multipath delay estimation of OFDM signals, which has achieved good results under the SMV condition. Although the support set is reduced by the backtracking mechanism, the estimated performance decreases with the increase of the multipath number due to the characteristics of the greedy algorithm. In [7], the MFCOUSS algorithm is proposed to combine the matching pursuit (MP) and the focal underdetermined system solver (FOCUSS) algorithm, which provides good estimation performance under SMV conditions.

The sparse reconstruction algorithms are mainly grid based, and the higher the grid density, the higher the estimation accuracy, but the complexity of the algorithm increases. Related researchers have also started research to balance the relationship between the off grid effect and the complexity of the algorithm. In [8], a sparse reconstruction method based on twice meshing is proposed to estimate multipath delay of OFDM systems. Under the same estimation accuracy, the method can reduce the computational complexity compared to the one-time uniform meshing method. But the final result is still at grid points.

Although the sparse reconstruction algorithm mentioned above can improve the accuracy of time delay estimation under the condition of SMV, when the number of multipath increases, the accuracy of estimation decreases, and there exists the off-grid effect. Aimed at the problem of OFDM delay estimation under SMV condition, this paper proposes a sparse reconstruction off-grid time delay estimation algorithm based on Bayesian automatic relevance determination (OGBARD).

The rest of this article are summarized as follows: Section 2 introduces a signal model. The design of the proposed algorithm is described in Section 3. Section 4 is a simulation experiment. And the conclusion of this paper is in Section 5.

2. Signal Model

In the wireless positioning scenario, it is assumed that the radiation source and the receiving station are relatively stationary. Then a radio wave propagation channel under multipath conditions is modelled as a form of complex low-pass equivalent impulse response, which is usually given by (1)

$$h(t)=\sum_{i=1}^{L} a_i \delta(t-\tau_i),$$

where the number of concentrated propagation paths is $L$. $a_i = |a_i| e^{j\phi_i}$ is the complex fading factor of the $i^{th}$ path. $\tau_i$ is the propagation delay corresponding to $i^{th}$ path. It is assumed that the amplitude and delay of the complex fading factor are constant during the short observation period and that the
random phase is distributed uniformly in \((0, 2\pi)\).

In order to apply sparse reconstruction theory to delay estimation, it is necessary to process the model to construct the corresponding sparse representation model. First, the delay mesh formed by the set \(\{\tau_1, \tau_2, \cdots, \tau_N\}\) covers all possible multipath signal delay values in the scenario and defines the spacing between two adjacent delay values as the resolution \(r = |\tau_{i+1} - \tau_i|\). The set \(\{\tau_1, \tau_2, \cdots, \tau_L\}\) is a delay value corresponding to the actual multipath signal. Performing discrete Fourier transform (DFT) for (1), taking into account the additive white Gauss noise in the measurement process, then the channel frequency response can be expressed as

\[
\vec{h} = V \vec{\alpha} + \vec{\varepsilon},
\]

where \(\vec{h} = [\vec{h}_1, \vec{h}_2, \cdots, \vec{h}_N]^T \in \mathbb{C}^{M \times 1}\) is the CFR estimation vector and \(\vec{\varepsilon} = [\vec{e}_1, \vec{e}_2, \cdots, \vec{e}_u]^T \in \mathbb{C}^{M \times 1}\) is the additive Gaussian white noise vector. \(V = [v(\tau_1), v(\tau_2), \cdots, v(\tau_L)]^T \in \mathbb{C}^{M \times N}\) is a redundant DFT dictionary in which \(v(\tau_i) = [1, e^{-j2\pi\tau_1}, \cdots, e^{-j2\pi(M-1)\tau_1}]^T \in \mathbb{C}^{M \times 1}\). \(N\) represents the number of grids.

And in order to meet the requirements of sparsity, there should be \(NM \gg 1\).

It is clear that there is a quantization error in the above delay division scheme based on the grid. In order to compensate the grid error and not to increase the computational complexity, a first order Taylor expansion is proposed to calibrate the grid mismatch \([9]\). It is assumed that the quantization delay value \(\tau_n \in \bar{\tau}\) is the minimum error of the actual delay value of the delay grid. The vector \(\phi(\tau_i)\) corresponding to the actual multipath delay value is expanded by the first order Taylor expansion at the nearest quantization delay value \(\bar{\tau}_n\) to obtain the following equation

\[
v(\tau_i) \approx v(\bar{\tau}_n) + d(\bar{\tau}_n)(\tau_i - \bar{\tau}_n),
\]

where \(d(\bar{\tau}_n) = \frac{dv(\tau)}{d\tau}|_{\tau = \bar{\tau}_n}\) is the first derivative vector of \(v(\tau)\) to \(\tau\) in the following form

\[
d(\tau_i) = \frac{dv(\tau)}{d\tau} = [0 - j2\pi \Delta f e^{-j2\pi \Delta f \tau_1}, \cdots, -j2\pi \Delta f (M-1) e^{-j2\pi \Delta f (M-1) \tau_1}]^T
\]

Then the over-complete dictionary with off-grid parameter is

\[
\Phi(\phi) = V + \Psi \text{diag}(\phi),
\]

where the matrix \(\Psi = [d(\bar{\tau}_1), d(\bar{\tau}_2), \cdots, d(\bar{\tau}_N)]^T \in \mathbb{C}^{M \times N}\) is an over-complete matrix composed of the derivative vectors of the vectors \(v(\bar{\tau}_1)\) corresponding to each delay value in set \(\bar{\tau}\). \(\phi = [\phi_1, \phi_2, \cdots, \phi_N]^T\) is the off-grid parameter vector. And the model of (2) becomes

\[
\vec{h} = \Phi(\phi) \vec{\alpha} + \vec{\varepsilon}
\]

3. The Proposed Algorithm

3.1 The automatic relevance determination

3.1.1 Automatic relevance determination a priori hypothesis

We use an a priori distribution to make probability assumptions about unknowns of the signal model. The noise is white Gaussian noise, and the probability distribution is

\[
p(\vec{e} | \lambda) = \text{CN}(\vec{e} | \theta_{\lambda, \sigma^2}, \lambda^{-1} I_u),
\]

where \(\lambda = \sigma^{-2}\) is the inverse of the noise variance. Because the conjugate prior distribution of Gauss distribution is gamma distribution, we assume that the hyperparameters \(\lambda\) obeys the gamma distribution with parameters \(c\) and \(d\). Its probability density function is

\[
p(\lambda | c, d) = \text{Gamma}(\lambda | c, d)
\]
In order to make these priors non-informative, we set the parameters to very small values in the experiments.

For the sparse coefficient vector \( \mathbf{a} \in \mathbb{R}^{N \times 1} \), the different elements are not correlated, we assume that it satisfies the Gauss distribution.

\[
p(\mathbf{a} | \gamma) = \text{CN}(\mathbf{a} | \theta_{\mathbf{a} \gamma} , A)
\]

(9)

Where the matrix \( A = \text{diag}[\gamma] \in \mathbb{R}^{N \times N} \) is a diagonal matrix, and the non-negative hyperparameters in the vector \( \gamma = [\gamma_1, \gamma_2, \cdots, \gamma_N]^\top \) control the variance of each unknown sparse coefficient vector element.

Then it is assumed that the variances in complex Gauss distribution are independent of each other and obey the gamma distribution with parameters \( a \) and \( b \).

\[
p(\gamma | a, b) = \prod_{i=1}^{N} \text{Gamma}(\gamma_i | a, b)
\]

(10)

In the sparse extension of delay domain, the delay interval is evenly divided into a set of discrete points. Similar to uniformly partitioning grid points, we also assume that the off-grid parameters are uniformly distributed, and the probability distribution is as follows.

When performing the sparse expansion of the delay domain, we divide the entire delay domain evenly to obtain a set of fixed discrete points. Without knowing the true delay of the multipath signal, the path may appear at any value of the time axis, so the off-grid delay parameter is assumed to be evenly distributed. Its prior probability distribution is

\[
p(\varphi) = \prod_{n=1}^{N} \text{U}(\varphi_n | [-\tau, \tau])
\]

(11)

3.1.2 Bayesian inference

Based on the above probability hypothesis and the Bayesian criterion, we find that the maximum posterior probability density of the sparse coefficient vector of (6) is

\[
p(\mathbf{a} | \mathbf{h}, \lambda, \varphi, \gamma) = \text{CN}(\mathbf{a} | \mu, \Sigma)
\]

(12)

Where \( \mu \) is the maximum posterior mean vector of \( \mathbf{a} \), \( \Sigma \) is the covariance matrix, and their expressions are respectively

\[
\mu = \lambda \Sigma \Phi^\dagger \mathbf{h}
\]

(13)

\[
\Sigma = (A^\dagger + \lambda \Phi^\dagger \Phi)^{-1}
\]

(14)

We consider \( \mathbf{a} \) as a hidden variable, and then we can use EM algorithm to solve the maximization problem of (15).

\[
(\hat{\lambda}, \hat{\varphi}, \hat{\gamma}) = \arg \max_{\lambda, \varphi, \gamma} E_{\mathbf{a} \sim \mathbf{a}, \varphi \sim \varphi} [\ln p(\mathbf{h}, \mathbf{a}, \lambda, \varphi, \gamma)]
\]

(15)

Where \( E_{\mathbf{a} \sim \mathbf{a}, \varphi \sim \varphi} [\bullet] \) represents the expectation of the posterior distribution of the sparse coefficient vector \( \mathbf{a} \), then we can deduce the formula for solving the hyperparameters.

For the parameter \( \lambda \), which determines the noise variance, the iteration renewal formula is

\[
\lambda \leftarrow \frac{M + c - 1}{\|\mathbf{h} - \Phi \mu\|^2 + \text{Tr}(\Phi^\dagger \Phi \Sigma) + d}
\]

(16)

For the parameter \( \gamma \), which determines the potential variance of different delay values after sparse representation, the iteration update formula is as follows.

\[
\gamma_i \leftarrow \frac{a - 2 + \sqrt{(2 - a)^2 + 4b(\Sigma_u + \mu_i)}}{2b}
\]

(17)

For the off-grid parameter vector \( \varphi \), the updated formula is

\[
\varphi \leftarrow \arg \min_{\varphi \in [-\tau, \tau]} [\|\varphi - \Pi \varphi - 2d\varphi\|]
\]

(18)

The expressions of matrix \( \Pi \) and vector \( \theta \) are respectively

\[
\Pi = \text{Re} \{\varphi^\dagger \varphi^\ast \odot (\mu \mu^\dagger + \Sigma)\}
\]

(19)
\[ \theta = \text{Re}\left\{\text{diag}(\mu^*)\Psi^*(\bar{H} - V\mu)\right\} - \text{Re}\left\{\text{diag}(\Psi^* V\Sigma)\right\} \]  

Equation (18) is derived and a solution can be obtained, i.e. \( \phi = \Pi^\top \theta \). However, it cannot be solved if \( \Pi \) is irreversible. Instead, the off grid parameters need to be solved iteratively. And the iteration formula is

\[ \hat{\phi}_i = \frac{\theta_i - (\Pi_u^\top \Psi)^\top \phi_i}{\Pi_u^\top \Psi}, \]  

where \( u_i \) denotes the vector formed after removing the \( i \)th element of the vector \( u \). Then the specific solution formula for the off-grid parameters can be obtained as

\[ \phi_i = \begin{cases} \hat{\phi}_i, & \text{if } \hat{\phi}_i \in [-\frac{\pi}{2}, \frac{\pi}{2}] \\ -\frac{\pi}{2}, & \text{if } \hat{\phi}_i < -\frac{\pi}{2} \\ \frac{\pi}{2}, & \text{otherwise} \end{cases} \]  

The EM algorithm can guarantee the local minimum solution of the algorithm convergence. For the convergence of the ARD algorithm, a more detailed analysis of global convergence is in [10].

The delay estimate of the path can be obtained according to the one-to-one correspondence between the position of the non-zero element in the hyperparameters \( \gamma \) and the potential arrival path delay. The corresponding index value of the path in the time delay grid are

\[ \text{inde} = \text{arg } \text{nonzero}(\gamma_i) \]  

Then the multipath delay estimation with off-grid correction are given by

\[ \hat{\tau}_i = \tau_{\text{inde}} + \phi_{\text{inde}} \]  

### 3.2 Algorithm steps

Based on the above analysis, the complete process of the proposed method is summarized in Algorithm 1.

**Algorithm 1**

**Input:** The observation data: \( \bar{H} \), the over-complete matrix \( V \) and \( \Psi \);

**Initialization:** Initialize the parameters: \( \gamma^{(0)}, \lambda^{(0)} \) and \( \phi^{(0)} \), set a small positive number \( \epsilon > 0 \), the maximum number of iterations \( \text{maxiter} \) and set \( i = 0 \);

**Steps:**

1. According to the hyperparameters \( \gamma \) and \( \phi \) to construct the over-complete dictionary \( \Phi(\phi) \) and diagonal matrix \( A = \text{diag}(\gamma) \);
2. Via (13) and (14) to calculate \( \mu \) and \( \Sigma \);
3. Update the hyperparameters \( \gamma^{(i)} \) according to (17), update \( \lambda^{(i)} \) via (16) and update \( \phi \) via (21) and (22);
4. if \( \|\gamma^{(i+1)} - \gamma^{(i)}\| / \|\gamma^{(i)}\| < \epsilon \) or the number of iterations reaches the pre-set value, the iteration stops; If not, set \( t = t + 1 \), jump to step 1;
5. Via (23) and (24) to obtain the estimated value of the delay.

**Output:** The estimated value of the delay \( \hat{\tau} \).

### 4. Simulation analysis

In this paper, OFDM delay estimation algorithm is studied under the wireless positioning system model. To verify the practicality and robustness of the proposed algorithm, we use the Monte Carlo experiment to compare the proposed algorithm with the PM algorithm in [3] (using smoothing to make the covariance matrix a full rank), the CoSaMP algorithm in [6], the MFCOUSS algorithm in [7] and the CRB. According to IEEE 802.11a protocol [11], the OFDM system parameters set in simulations are shown in Table 1.
Table 1. System parameters

| Simulation Parameters          | Value        |
|-------------------------------|--------------|
| FFT cycle (μs)                | 3.2          |
| System bandwidth (MHz)        | 20           |
| Number of subcarriers         | 64           |
| Carrier frequency (GHz)       | 2.4          |
| Number of channel paths       | 3            |
| Delay of the paths (ns)       | [62,152,236] |
| Power of the paths (dB)       | [0, -1, -2]  |

We define the total root mean square error (RMSE) and RMSE for a single path of delay to measure the performance of the algorithm

\[
RMSE_{total} = \sqrt{\frac{1}{KL} \sum_{i=1}^{K} \sum_{k=1}^{L} (\hat{\tau}_{ik} - \tau_{ik})^2}
\]  
\[
RMSE_{i} = \sqrt{\frac{1}{K} \sum_{k=1}^{K} (\hat{\tau}_{ik} - \tau_{ik})^2}, i = 1, 2, \cdots, L
\]

The number of Monte Carlo simulations is \(K\). \(\hat{\tau}_{ik}\) is the estimated value of the \(i\)th Monte Carlo experimental delay of the \(k\)th path. And \(\tau_{ik}\) is the actual delay value of the \(i\)th path. In the proposed algorithm, we set \(a = 1\), \(b = 0.01\) and \(c = d = 1\times10^{-4}\). The equipment used in all simulation experiments is a Win7 system, Intel Xeon CPU (4-core 3.3 GHz) computer.

Simulation 1: Performance comparison of different algorithms

Set the delay interval to 1ns. The SNR is changed from -10 dB to 20dB. 200 Monte Carlo experiments are performed in several algorithms. As shown in Figure 1, the RMSE curve versus SNR of these algorithms are plotted and compared to CRB. When the SNR is high, the performance of the proposed algorithm is better than the other algorithm and is closer to the CRB. This is because the algorithm uses the ARD to produce the sparse solution for each iteration. An efficient global conflict between all possible values of the sparse coefficient vector is generated. This effectively avoids the floor effect caused by excessive "greedy" greedy algorithms.

![Figure 1](image1.png)

Figure 1. The relationship between RMSE of TOA and SNR of different algorithms
(a) First path; (b) Second path; (c) Third path

Simulation 2: Comparison of estimated performance under different multipath number conditions

In order to compare the estimation performance of each algorithm for the direct path under different multipath number conditions. The multipath number is set from 1 to 3 and the multipath delay and complex fading coefficient are the same as those of simulation 1. And the SNR is from -10dB to 20dB. 200 Monte Carlo experiments are performed on these algorithms. As shown in Figure 2, the RMSE curves versus SNR of these algorithms are plotted. It can be seen from Figure 2 that when the SNR is high, the RMSE curves of the proposed algorithm are close to each other under different
multipath number conditions, while the RMSE curves of other algorithms under different multipath number conditions are far apart. It indicates that the proposed algorithm is not sensitive to multipath numbers. Under the condition of single-path and high SNR, all algorithms show better estimation performance. The proposed algorithm, CoSaMP algorithm and the PM algorithm RMSE curve almost coincide. This is because the signal subspace is orthogonal to the noise subspace under single-path and SMV conditions.

Figure 2. Delay estimation RMSE of the first path under different multipath numbers

Simulation 3: The grid interval size impact on performance

Sparse reconstruction algorithms are mostly grid-based. Here, we compare the RMSE of CoSaMP algorithm, MFCOUSS algorithm and the proposed algorithm at different grid intervals. The value of grid interval is $r=[0.5,1,2,4,8]$ns, the number of Monte Carlo experiments is 200 and the SNR is 15dB. In each trial, the time delays of three paths are generated uniformly from the delays interval $[55,65]$ns, $[135,145]$ns and $[195,205]$ns respectively. The curves of total RMSE versus different grid interval are obtained as shown in Figure 3. It can be seen from the figure that the performance of the proposed algorithm is better under different grid interval because of the modification of the off-grid parameters.

Sparse reconstruction algorithms are mostly grid-based. Here we compare the RMSE of CoSaMP algorithm, the MFCOUSS algorithm and the proposed algorithm at different grid intervals. The grid interval value are $r=[0.5,1,2,4,8]$ns respectively. The SNR is set to 15dB and the number of Monte Carlo is 200. In each trial, the time delay of the three paths are uniformly generated from the delay interval $[55,65]$ns, $[135,145]$ns and $[195,205]$ns respectively. As shown in Figure 3, the curve of RMSE versus different grid interval are obtained. From the figure, we can see that the performance of the proposed algorithm is better under different grid interval.

Figure 3. The RMSE of different algorithms versus grid interval

5. Conclusion

In the OFDM delay estimation, the sparse reconstruction method can improve the estimation performance under the single sample condition. However, with the increase of the number of paths, the estimation accuracy of the existing sparse reconstruction delay estimation algorithms decreases,
and there exists the off-grid effect. In response to this problem, this paper is based on the Bayesian method, which eliminates the possible values of redundancy by using the parameterized, data-dependent prior distribution specification to solve the space. The first-order Taylor expansion is also used to construct the off-grid parameters to modify the time delay estimation value. It can be seen from the simulation experiments that the proposed algorithm has better estimation performance than the existing delay estimation algorithms under SMV or different multipath conditions.

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