Double charm states in QCD sum rules

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Abstract.
In this note we revisit and improve the calculation of the $T_{cc}$ mass using the double ratio of QCD sum rules (QCDSR), assuming that this state is described by a molecular current. In a previous work we used a tetraquark current. We conclude that with both currents we arrive at nearly the same results for the $T_{cc}$ mass, which turns out to be close to the mass of the $X(3872)$.

1. Introduction
Recently we have learned that there are hadronic states whose quantum numbers and main properties cannot be explained by a simple quark-antiquark or three-quark configuration. These states are called exotic states. The most well studied state is the $X(3872)$ resonance (assumed to be an $1^{++}$ axial vector meson), which was discovered by BELLE in $B$-decays and confirmed by BABAR, CDF and D0 (for a recent review see [1]). Its most popular picture consists of a molecular configuration, $DD^* + DD^*$. Here we focus on another exotic meson, which has similar structure, similar quark content and similar mass: the four-quark state ($QQ\bar{u}\bar{d}$) with quantum numbers $I = 0$, $J = 1$ and $P = +1$ which, following Ref.[2], we call $T_{QQ}$, where $Q$ stands for $c$ or $b$. As already noted previously [2, 3], the $T_{bb}$ and $T_{cc}$ states with $J^P = 1^+$ cannot split into a pair of two $B$ or two $D$ mesons. If their masses are below the $BB^*$ or $DD\pi$ thresholds, these decays are also forbidden. As a result, $T_{QQ}$ becomes stable with respect to the strong interaction, and must decay radiatively, or even weakly if the mass becomes lower than the threshold made of two pseudoscalar mesons.

In constituent models with a flavor-independent central potential, the stability of tetraquark ($QQ\bar{u}\bar{d}$) configurations comes from a favorable effect when the charge-conjugation symmetry is broken. This is the same mechanism by which, in QED, the loosely bound positronium molecule evolves into the very stable hydrogen molecule. It has been observed that in the large $m_Q$ limit, the light degrees of freedom cannot resolve the closely bound $QQ$ system. This results in bound states similar to the $\Lambda_Q$ states, with $QQ$ playing the role of the heavy antiquark [4]. The ($QQ\bar{q}\bar{q}$) states have been studied using a variety of potential models [3, 5, 6]. The corresponding four-body problem is very delicate, as discussed in [7]. All authors agree that such states become bound when the quark over the antiquark mass ratio becomes sufficiently large. Detailed four-body calculations, using a pairwise central potential supplemented by a chromomagnetic interaction, indicate that $T_{bb}$ is rather well bound, and $T_{cc}$ possibly bound by a few MeV below $DD^*$. For instance, the prediction of Ref. [2] is, in units of MeV:

$$M_{T_{cc}} = 3876 \sim 3905\, ,\quad M_{T_{bb}} = 10519 \sim 10651\, .$$

(1)
A non-pairwise confinement has also been considered [8], inspired by the large coupling regime of QCD, where it is shown [9] that it is more favorable to build stable tetraquarks. In this improved quark model, as well as in conventional quark models, it is found that \((QQq\bar{q})\) has an energy lower than \((Q\bar{Q}q\bar{q})\). Another variant was considered in [10], with a chiral potential model, which includes meson-exchange forces between quarks, instead of the chromomagnetic interaction.

The existence of a \(D\bar{D}^* + D\bar{D}^*\) molecule was predicted in Ref. [11] on the basis of the pion-exchange dynamics. A comprehensive discussion on meson molecules can be found in [12]. Here, the pion is exchanged between the hadrons, as in the Yukawa theory of nuclear forces. The \(DD^*\) and \(D\bar{D}^*\) vertices are identical, as well as the \(D^*D^*\) and \(D\bar{D}^*\) ones. There is only an overall change of sign, due to the G-parity of the pion. Therefore, if the pion-exchange dynamics is able to bind the \(D\bar{D}^* + D\bar{D}^*\) molecule, the same is true for the \(DD^*\) molecule. The difference between these two states can only come from the short-range part of the interaction, which is accurately described by QCDSR.

In what follows we discuss the mass determination of the \(T_{QQ}\) and the ways to improve it in the framework of QCDSR. We explore the connection between the \(X(3872)\) and the \(T_{QQ}\) and argue that the existence of the first implies the existence of the second.

2. \(T_{QQ}\) in QCD sum rules

The first study of tetraquarks with two heavy quarks within QCD sum rules was done in [13], where these states were represented by diquark-antidiquark currents. This study was revisited and improved in [14]. Here we compare in detail the \((QQq\bar{q})\) and \((Q\bar{Q}q\bar{q})\) configurations. Such a comparison was also performed in Ref. [15], where the authors have studied heavy tetraquarks using a crude color-magnetic interaction, with flavor symmetry breaking corrections. In [13] it was found that (in units of GeV):

\[
M_{T_{cc}} = 4.0 \pm 0.2, \quad M_{T_{bb}} = 10.2 \pm 0.3.
\]

The short-range part of the interaction can be tested by the QCD sum rules approach. Therefore, we can study the ratio of the masses of the \(T_{cc}\) and \(X(3872)\) states, by using the double ratios of sum rules (DRSR), which is widely applied for accurate determinations of the ratios of couplings and masses and form factors. This accuracy is reached due to partial cancellations of the systematics of the method and of the QCD corrections in the DRSR. More recently, the DRSR was used to study different possible currents for the \(X(3872)\) [16]. It was found that (within the accuracy of the method) the different structures \((3 - 3\) and \(6 - 6\) tetraquarks and \(D\bar{D}^* + D\bar{D}^*\) molecule) lead to the same prediction for the mass. The two-point functions of the \(X(3872)\) (assumed to be an \(1^{++}\) axial vector meson) and the \(T_{cc}\) (assumed to be a \(J^P = 1^+\) state) is defined as:

\[
\Pi_i^{\mu\nu}(q) = i \int d^4x \, e^{iqx} \langle 0 | T[j_i^{\mu}(x)j_i^{\nu}(0)] | 0 \rangle
\]

\[
= -\Pi_{i1}(q^2)(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) + \Pi_{i0}(q^2) \frac{q^\mu q^\nu}{q^2},
\]

(3)

where \(i = X, \ T_{cc}\). The two invariants, \(\Pi_1\) and \(\Pi_0\), appearing in Eq. (3) are independent and have respectively the quantum numbers of the spin 1 and 0 mesons.

We assume that the \(X(3872)\) and \(T_{cc}\) states are described by the molecular currents:

\[
j_X^\mu(x) = \bar{q}_a(x)\gamma^\mu c_a(x)\bar{c}_b(x)\gamma_5q_b(x) - \bar{q}_a(x)\gamma^\mu c_a(x)\bar{c}_b(x)\gamma_5q_b(x),
\]

(4)

and

\[
j_{T_{cc}}^\mu(x) = \bar{q}_a(x)\gamma^\mu c_a(x)\bar{c}_b(x)\gamma^\mu c_b(x)
\]

(5)
where $a$ and $b$ are color indices. The correlation function, $\Pi_{1i}$ in Eq. (3), can be written in terms of a dispersion relation:

$$
\Pi_{1i}(q^2) = \int_{4m_i^2}^{\infty} ds \, \frac{\rho_i(s)}{s - q^2} + \cdots ,
$$

where $\pi\rho_i(s) \equiv \text{Im}[\Pi_{1i}(s)]$ is the spectral function.

The correlation function in Eq. (3) is evaluated in two ways: using the operator product expansion (OPE) and using the information from hadronic phenomenology. In the OPE side we work at leading order of perturbation theory in $\alpha_s$, and we consider the contributions from condensates up to dimension six. In the phenomenological side, the correlation function is estimated by inserting intermediate states for the $X$ and $T_{cc}$ states via their couplings $\lambda_i$ to the molecular currents, $\langle 0 | j_{\mu}^i | M_i \rangle = \lambda_i \epsilon_{\mu}$, where $M_i \equiv X, T_{cc}$, $j_{\mu}^i$ are the currents in Eqs. (4) and (5).

Using the Ansatz: “one resonance plus QCD continuum”, where the QCD continuum comes from the discontinuity of the QCD diagrams from a continuum threshold $t_c$, the phenomenological side of Eq. (3) can be written as:

$$
\Pi_{\mu\nu}^{\text{phen}}(q^2) = \frac{\lambda_i^2}{M_i^2 - q^2} \left(-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{M_i^2}\right) + \cdots ,
$$

where the Lorentz structure $g_{\mu\nu}$ projects out the $1^+$ state. The dots denote higher axial-vector resonance contributions that will be parametrized, as usual, by the QCD continuum. After making a Borel transform on both sides, and transferring the continuum contribution to the QCD side, the moment sum rule and its ratio read:

$$
\mathcal{F}_i(\tau) \equiv \lambda_i^2 e^{-M_i^2\tau} = \int_{4m_i^2}^{t_c} ds \, e^{-s\tau} \rho_i(s)
$$

$$
\mathcal{R}_i(\tau) \equiv -\frac{d}{d\tau} \log \mathcal{F}_i(\tau) \simeq M_i^2 ,
$$

where $\tau \equiv 1/M^2$ is the sum rule variable with $M$ being the Borel mass. In the following, we shall work with the DRSR:

$$
\tau_{T_{cc}/X} \equiv \sqrt{\mathcal{R}_{T_{cc}}/\mathcal{R}_X} \simeq \frac{M_{T_{cc}}}{M_X}.
$$

Using QCD sum rules, one can usually estimate the mass of the $X$-meson, from the ratio $\mathcal{R}_X$ in Eq. (8), which is related to the spectral densities obtained from the current (4). A tetraquark current for the $X(3872)$ was used in Ref. [17] and its mass was estimated to be:

$$
M_X \simeq \sqrt{\mathcal{R}_X} = (3925 \pm 127) \text{ MeV} .
$$

Although the uncertainty in Eq. (10) is still large, considering the fact that this result was obtained in a Borel region where there is pole dominance and OPE convergence, one can say that the QCD sum rules support the existence of such a state and that the value obtained for $M_X$ is in reasonable agreement with the experimental value $M_X = (3872.2 \pm 0.8) \text{ MeV}$.

We now study the DRSR of the $T_{cc}/X$ defined in Eq. (9). In Fig. 1, we show the $\tau$-dependence of the ratio for $\sqrt{t_c} = 4.15$ GeV and for two values of $m_c = 1.23$ GeV and 1.47 GeV. From Fig. 1 one can see that there is a $\tau$-stability around $\tau \simeq 0.4$ GeV$^{-2}$ and for this value of $\tau$, we get:

$$
\tau_{T_{cc}/X} = 1.00 \pm 0.01 .
$$
Figure 1. The double ratio $r_{Tcc/X}$ defined in Eq. (9) as a function of $\tau$ for $\sqrt{t_c} = 4.15$ GeV and for two values of $m_c = 1.23$ (solid line) and 1.47 GeV (dashed line).

Figure 2. The double ratio $r_{Tcc/X}$ as a function of $t_c$ for $\tau = 0.4$ GeV$^{-2}$ and for two values of $m_c = 1.23$ (solid line) and 1.47 GeV (dashed line).

In Fig. 2, we show the $t_c$-dependence of the ratio for $\tau = 0.4$ GeV$^{-2}$ and for two values of $m_c = 1.23$ GeV and 1.47 GeV. From this figure one can see that the ratio increases with $t_c$. However, considering the large range of $t_c$ presented in the figure, the ratio does not differ more than 3% from 1.

Our analysis has shown that the $D\bar{D}^* + c.c.$ and $DD^*$ currents lead to the same mass predictions within the accuracy of the approach. The accuracy of the DRSR is bigger than the normal QCDSR because the DRSR are less sensitive to the exact value and definition of the heavy quark mass and to the QCD continuum contributions. As mentioned before, this accuracy is reached due to partial cancellations of the systematics of the method and of the QCD corrections in the DRSR. Therefore, if the observed $X(3872)$ is a molecular $D\bar{D}^* + c.c.$ state its molecular cousin $DD^*$ should also be a bound state. Its mass can be obtained by using the experimental mass for the $X(3872)$ in Eq. (11):

$$M_{Tcc} = (3872.2 \pm 39.5) \text{ MeV}.$$ (12)
Using the same interpolating field in Eqs. (4) and (5) with the charm quark replaced by the bottom one, we can analyze the DRSR:

\[ \frac{r_{T_{bb}}/X_b}{R_{X_b}} \equiv \sqrt{\frac{R_{T_{bb}}}{R_{X_b}}} \approx \frac{M_{T_{bb}}}{M_{X_b}}. \]  

(13)

We obtain:

\[ r_{T_{bb}}/X_b = 1.00. \]  

(14)

Therefore, we can predict the degeneracy between the masses of the \( T_{bb} \) and of the \( X_b \).

3. Conclusions

In this note we have presented a refinement of the determination of the \( T_{cc} \) mass. In [13], using a tetraquark current we arrived at \( M_{T_{cc}} = 4.0 \pm 0.2 \text{ GeV} \). Here, using a molecular current and the DRSR method we arrive at \( M_{T_{cc}} = 3.87 \pm 0.04 \text{ GeV} \). The DRSR method relates two particles with the same quantum numbers and, due to the cancellations mentioned above, allows for a more precise determination of the mass. In the present case the associated particle is the well-established \( X(3872) \). Within the uncertainties the new determination of the \( T_{cc} \) mass agrees with the previous one. The sum rules for the \( X(3872) \) and for the \( T_{cc} \) have the same (good) quality. This suggests that the existence of the former implies the existence of the latter.

From the microscopic point of view, QCDSR describes accurately short range interactions. If both the \( T_{cc} \) and the \( X(3872) \) are meson molecules we should also include long range interactions, i.e., meson exchanges. If the dominant long range process is the pion exchange between the corresponding mesons, in the present case it would be exactly the same for the two molecules in question. We can then conclude that if the observed \( X(3872) \) meson is a \( DD^*+ \) molecule, then the \( DD^* \) molecule, associated to the \( T_{cc} \), should also exist with approximately the same mass. An extension of the analysis to the \( b \)-quark case leads to the same conclusion. This question will certainly be experimentally studied at the LHC in the near future.

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