Testing the Gravitational Redshift with Atomic Gravimeters?

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Abstract—Atom interferometers allow the measurement of the acceleration of freely falling atoms with respect to an experimental platform at rest on Earth’s surface. Such experiments have been used to test the universality of free fall by comparing the acceleration of the atoms to that of a classical freely falling object. In a recent paper, Müller, Peters and Chu [Nature 463, 926-929 (2010)] argued that atom interferometers also provide a very accurate test of the gravitational redshift (or universality of clock rates). Considering the atom as a clock operating at the Compton frequency associated with the rest mass, they claimed that the interferometer measures the gravitational redshift between the atom-clocks in the two paths of the interferometer at different values of gravitational potentials. In the present paper we analyze this claim in the frame of general relativity and of different alternative theories, and conclude that the interpretation of atom interferometers as testing the gravitational redshift at the Compton frequency is unsound. The present work is a summary of our extensive paper [Wolf et al., arXiv:1012.1194] and its frequency compared to a similar clock

on ground. This yielded a test of the gravitational redshift with about 10⁻⁴ accuracy. The European Space Agency will fly in 2013 the Atomic Clock Ensemble in Space (ACES) [3], including a highly stable laser-cooled atomic clock, which will test (in addition to many other applications in fundamental physics and metrology) the gravitational redshift to a precision of about 10⁻⁶. The gravitational redshift is also tested in null redshift experiments in which the rates of different clocks (based on different physical processes or different atoms) are compared to each other [6].

Quite generally in a modern context, tests of GR measure the difference between the predictions of GR and of some generalized alternative theory or theoretical framework (see [7] for a review). It has to be kept in mind that the classification and inter-comparison of different tests have then to be defined with respect to the framework used.

Atom interferometers have reached high sensitivities in the measurement of the gravitational acceleration [8, 9]. This yields very important tests of the weak equivalence principle (WEP) or universality of free fall (UFF) when comparing the free fall of atoms with that of classical macroscopic matter (in practice a nearby freely falling corner cube whose trajectory is monitored by lasers). The relative precision of such tests of the UFF is currently 7 × 10⁻¹⁵, using Cs [8, 10] or Rb [9] atoms. Although it remains less sensitive than tests using macroscopic bodies of different composition [2, 11] which have reached a precision of 2 × 10⁻¹³, this UFF test is interesting as it is the most sensitive one comparing the free fall of quantum objects (namely Cesium atoms) with that of a classical test mass (the corner cube). In this contribution, which summarizes the extensive paper [11], we investigate whether such experiments can also be interpreted as tests of the gravitational redshift.

II. OVERVIEW OF OUR EXTENSIVE PAPER [11]

In a recent paper, Müller, Peters and Chu [10] (hereafter abbreviated as MPC) proposed a new interpretation of atom interferometry experiments as testing the gravitational redshift, that is also the universality of clock rates (UCR), with a precision 7 × 10⁻¹⁵, which is several orders of magnitude better than the best present [4] and near future [5] clock tests.
A. Analogy with clock experiments

The main argument of MPC (see also the more detailed papers [12], [13]) is based on an analogy between atom interferometry experiments and classical clock experiments. The idea of clock experiments is to synchronize a pair of clocks when they are located closely to one another, and move them to different elevations in a gravitational field. The gravitational redshift will decrease the oscillation frequency of the lower clock relative to the higher one, yielding a measurable phase shift between them. There are two methods for measuring the effect. Either we bring the clocks back together and compare the number of elapsed oscillations, or we measure the redshift by means of continuous exchanges of electromagnetic signals between the two clocks. In both methods one has to monitor carefully the trajectories of the two clocks. For example, in the second method one has to remove the Doppler shifts necessarily appearing in the exchanges of electromagnetic signals.

In the first method, the phase difference between the two clocks when they are recombined together, can be written as a difference of integrals over proper time,

$$\Delta \varphi_{\text{clock}} = \omega \left[ \int_{I} d\tau - \int_{II} d\tau \right] \equiv \omega \oint d\tau.$$  \hspace{1cm} (1)

The two clocks have identical proper frequency $\omega$. We denote by $I$ and $II$ the two paths (with say $I$ being at a higher altitude, i.e. a lower gravitational potential) and use the notation $\oint d\tau$ to mean the difference of proper times between the two paths, assumed to form a close contour. The integrals in (1) are evaluated along the proper times of the clocks, and we may use the Schwarzschild metric to obtain an explicit expression of the measured phase shift in the gravitational field of the Earth.

The phase shift measured by an atom interferometer contains a contribution which is similar to the clock phase shift (1), so it is tempting to draw an analogy with clock experiments. In this analogy, the role of the clock’s proper frequency is played by the atom’s (de Broglie-)Compton frequency $\omega_C = mc^2/\hbar$, where $m$ denotes the rest mass of the atom. However the phase shift includes also another contribution $\Delta \varphi_{\ell}$ coming from the interaction of the laser light used in the beam-splitting process with the atoms. Thus,

$$\Delta \varphi = \omega_C \oint d\tau + \Delta \varphi_{\ell}.$$  \hspace{1cm} (2)

Here we are assuming that the two paths close up at the entry and exit of the interferometer; otherwise, additional terms have to be added to (2). The schematic view of the atom interferometer showing the two interferometer paths $I$ and $II$ is given by Fig. 1. The first term in (2) is proportional to the atom’s mass through the Compton frequency and represents the difference of Compton phases along the two classical paths. In contrast, the second term $\Delta \varphi_{\ell}$ does not depend on the mass of the atoms.

At first sight, the first term in (2) could be used for a test of the gravitational redshift, in analogy with classical clock experiments, and the precision of the test could be very good, because the Compton frequency of the Cäsium atom is very high, $\omega_C \approx 2\pi \times 3.0 \times 10^{25}$ Hz. However, as shown in a previous brief comment [14] and the detailed paper [11], this re-interpretation of the atom interferometer as testing the UCR is fundamentally incorrect.

At this point, we want to mention other crucial differences between atom interferometry and clock experiments, which reinforce our conclusions. In clock experiments the trajectories of the clocks are continuously controlled for instance by continuous exchange of electromagnetic signals. In atom interferometry in contrast, the trajectories of the atoms are not measured independently but theoretically derived from the Lagrangian and initial conditions. Furthermore, we expect that it is impossible to determine independently the trajectories of the wave packets without destroying the interference pattern at the exit of the interferometer.

Another important difference lies in the very notion of a clock. Atomic clocks use the extremely stable energy difference between two internal states. By varying the frequency of an interrogation signal (e.g. microwave or optical), one obtains a resonant signal when the frequency is tuned to the frequency of the atomic transition. In their re-interpretation, MPC view the entire atom as a clock ticking at the Compton frequency associated with its rest mass. But the “atom-clock” is not a real clock in the previous sense, since it does not deliver a physical signal at Compton frequency, as also recently emphasized in the same context in [15].

Note also that the phase shift (1) for clocks is valid in any gravitational field, with any gravity gradients, since it is simply the proper time elapsed along the trajectories of the clocks in

1See also the reply of MPC [15] to our brief comment. After completion of our work [14], [11], several independent analysis have appeared [16], [17] supporting our views and consistent with our conclusions.
a gravitational field. By contrast, the phase shift $\varphi$ is known only for quadratic Lagrangians and cannot be applied in a gravitational field with large gravity gradients, or more generally with any Lagrangian that is of higher order.

\section*{B. Analysis in general relativity}

The clear-cut argument showing that the atom interferometer does not measure the redshift is that the “atom-clock” contribution, i.e. the first term in (2), is in fact zero for a closed total path \cite{[13], [19]. As a specific example, let us consider the prediction from GR, which has been extensively treated in \cite{[20]. Here we only present a very basic analysis sufficient for our purposes. The appropriate Lagrangian is given by the proper time $d\tau = (\gamma g_{\mu\nu}dx^\mu dx^\nu/c^2)^{1/2}$, i.e. $L_{\text{GR}} = -mc^2 d\tau/dt$, and is derived to sufficient accuracy using the Schwarzschild metric generated by the Earth,

\begin{equation}
L_{\text{GR}}(z, \dot{z}) = -mc^2 + \frac{GM}{r_\oplus} - mgz + \frac{1}{2}m\dot{z}^2,
\end{equation}

where $r_\oplus$ is the Earth’s radius, $g = GM/r_\oplus^2$ is the Newtonian gravitational acceleration, $G$ is Newton’s gravitational constant, $M$ is the mass of the Earth, $m$ the mass of the atom, $c$ the speed of light in vacuum, $z$ is defined by $r = r_\oplus + z$ with $r$ the radial coordinate, and we neglect the post-Newtonian corrections. For simplicity we restrict ourselves to only radial motion, which is sufficient for this argument in this paper.

The equations of motion are deduced from (3) using the principle of least action or the Euler-Lagrange equations, and readily $\ddot{z} = -g$. Then, when integrating (3) along the resulting paths, and calculating the difference of action integrals $\Delta \varphi_S$ as defined by the first term in (2), one finds

\begin{equation}
\Delta \varphi_S = 0.
\end{equation}

The key point about the result (4) is a consistent calculation of the two paths in the atom interferometer and of the phases along these paths, both derived from the same classical action, using in a standard way the principle of least action. At the deepest level, the principle of least action and its use in atom interferometry comes from the Feynman path integral formulation of quantum mechanics or equivalently the Schrödinger equation \cite{[18]. Thus only the second term $\Delta \varphi_t$ remains in (2). The final phase shift,

\begin{equation}
\Delta \varphi = \Delta \varphi_t = k g T^2,
\end{equation}

depends on the wavevector $k$ of the lasers, on the interrogation time $T$ and on the local gravity $g$. This shows that the atom interferometer is a gravimeter or accelerometer. The phase shift (5) arises entirely from the interactions with the lasers and the fact that the atoms are falling with respect to the laboratory in which the experiment is performed. The atom’s Compton frequency is irrelevant.

\section*{C. Analysis in the modified Lagrangian formalism}

This framework, which we call the “modified Lagrangian formalism”, is an adaptation for our purpose of a powerful formalism for analyzing tests of the EEP and its various facets: the WEP, the local Lorentz invariance (LLI) and the local position invariance (LPI) \cite{[21], [22], [7]. This formalism allows deviations from general relativity and metric theories of gravity, with violations of the UFF and UCR, and permits a coherent analysis of atom interferometry experiments.

The formalism is defined by a single Lagrangian, that is however different from the GR Lagrangian. For our purposes it is sufficient to use a strongly simplified “toy” Lagrangian chosen as a particular case within the “energy conservation formalism” of Nordtvedt and Haugan \cite{[21], [22] (see \cite{[7] for a review). To keep in line with MPC we choose an expression similar to the Lagrangian of GR given by (3), namely

\begin{equation}
L_{\text{modified}} = -m_0 c^2 + \frac{GMm_0}{r_\oplus} - (1 + \beta_X^{(a)}) m_0 g z + \frac{1}{2}m_0 \dot{z}^2,
\end{equation}

where $\beta_X^{(a)}$ denotes a dimensionless parameter characterizing the violation of LPI, and depending on the particular type $X$ of mass-energy or interaction under consideration; e.g. $\beta_X^{(a)}$ would be different for the electromagnetic or the nuclear interactions, with possible variations as a function of spin or the other internal properties of the atom, here labelled by the superscript $(a)$. Thus $\beta_X^{(a)}$ would depend not only on the type of internal energy $X$ but also on the type of atom $(a)$.

The Lagrangian (6) describes the Newtonian limit of a large class of non-metric theories, in a way consistent with Schiff’s conjecture and fundamental principles of quantum mechanics. Most alternative theories commonly considered belong to this class which encompasses a large number of models and frameworks (see \cite{[7] and references therein), like most non-metric theories (e.g. the Belinfante-Swihart theory \cite{[23]), some models motivated by string theory \cite{[24]), some general parameterized frameworks like the energy conservation formalism \cite{[21], [22], the TH\textsubscript{EM} formalism \cite{[25], and the Lorentz violating standard model extension (SME) \cite{[26], [27].

Within this general formalism there is no fundamental distinction between UFF and UCR tests, as violation of one implies violation of the other, thus testing one implies testing the other. Depending on the theory used, different experiments test different parameters or parameter combinations at differing accuracies, so they may be complementary or redundant depending on the context. By varying (6) we obtain the equations of motion of the atom as

\begin{equation}
\ddot{z} = -(1 + \beta_X^{(a)}) g,
\end{equation}

which shows that the trajectory of the atom is affected by the violation of LPI and is not universal. In fact we see that $\beta_X^{(a)}$ measures the non-universality of the ratio between the atom’s passive gravitational mass and inertial mass. Thus, in the modified Lagrangian framework the violation of LPI implies a violation of WEP and the UFF, and $\beta_X^{(a)}$ appears to be the UFF-violating parameter. This is a classic example \cite{[21], [22] of the validity of Schiff’s conjecture, namely that it is impossible in any consistent theory of gravity to violate LPI (or LLI) without also violating WEP.

The violation of LPI is best reflected in classical redshift experiments with clocks which can be analysed using a cyclic
gedanken experiment based on energy conservation [21], [22], [2]. The result for the frequency shift in a Pound-Rebka type experiment is

$$Z = \left(1 + \alpha_X^{(a)}\right) \frac{g \Delta z}{c^2},$$

where the redshift violating (or UCR violating) parameter $\alpha_X^{(a)}$ is again non-universal. The important point, proven in [21], [22], [7], is that the UCR-violating parameter $\alpha_X^{(a)}$ is related in a precise way to the UFF-violating parameter $\beta_X^{(a)}$, namely

$$\beta_X^{(a)} = \alpha_X^{(a)} \frac{E_X}{m c^2} - \frac{\alpha_X^{(a)}}{E_X},$$

where $E_X$ is the internal energy responsible for the violation of LPI, and $m$ is the sum of the rest masses of the particles constituting the atom. Therefore we can compare the different qualitative meaning of tests of UCR and UFF. For a given set of UFF and UCR tests their relative merit is given by [9] and is dependent on the model used, i.e. the type of anomalous energy $E_X$ and its dependence on the used materials or atoms.

We now consider the application to atom interferometry. In the experiment of [8], [10] the “atom-clock” that accumulates a phase is of identical composition to the falling object (the same atom), hence one has to consistently use the same value of $\beta_X^{(a)}$ when calculating the trajectories and the phase difference using the Lagrangian [6]. It is then easy to show that $\Delta \varphi_S = 0$ with the above Lagrangian [18], [20], [28]. The vanishing of $\Delta \varphi_S$ in this case is a general property of all quadratic Lagrangians and comes from consistently using the same Lagrangian for the calculation of the trajectories and the phase shift. It is related to the cancellation between the kinetic term and the gravitational potential energy term in the Lagrangian [6].

Then the total phase shift of the atom interferometer is again given by the light interactions only, which are obtained from the phases at the interaction points evaluated using the trajectory given by [6]. One then obtains

$$\Delta \varphi = \Delta \varphi_r = \left(1 + \beta_X^{(a)}\right) k g T^2.$$  

We first note that in this class of theories the Compton frequency plays no role, as $\Delta \varphi_S = 0$. Second, we note that although $\beta_X^{(a)}$ appears in the final phase shift, this is entirely related to the light phase shift coming from the trajectory of the atoms, and thus is a measurement of the effective free fall acceleration $(1 + \beta_X^{(a)})g$ of the atoms, which is given by [7]. In [8], [10] the resulting phase shift is compared to $k g T^2$ where $\hat{g}$ is the measured free fall acceleration of a falling macroscopic corner cube, i.e. $\hat{g} = \left[1 + \beta_X^{(a)(\text{corner cube})}\right]g$, also deduced from [7]. In this class of theories the experiment is thus a test of the UFF, as it measures the differential gravitational acceleration of two test masses (Cesium atom and corner cube) of different internal composition, with precision

$$\left|\beta_X^{(a)(\text{Cs})} - \beta_X^{(a)(\text{corner cube})}\right| \lesssim 7 \times 10^{-9}.$$  

Note that this expression is equivalent to the one obtained by MPC in their recent paper [13] (p. 4), but different from the one obtained in the same paper (p. 3) for UCR tests.[7]

D. Analysis in the multiple Lagrangian formalism

In the second framework, which we call the “multiple Lagrangian formalism”, the motion of test particles (atoms or macroscopic bodies) obeys the standard GR Lagrangian in a gravitational field, whereas the phase of the corresponding matter waves obeys a different Lagrangian. The MPC analysis in [10] belongs to this framework, which raises extremely difficult problems.

The WEP is assumed to be valid for the motion of massive classical particles which thus obeys the standard Lagrangian of general relativity, $L_{\text{particle}} = L_{\text{GR}}$, i.e. to first order

$$L_{\text{particle}} = -m c^2 + \frac{G M m}{r} - m g z + \frac{1}{2} m \dot{z}^2.$$  

On the other hand the action integral to be used for the computation of the phase shift of the quantum matter wave is assumed to be calculated from the different Lagrangian

$$L_{\text{wave}} = -m c^2 + \frac{G M m}{r} - (1 + \beta) m g z + \frac{1}{2} m \dot{z}^2,$$

where the parameter $\beta$ that measures the deviation from GR (i.e. $\beta = 0$ in GR) enters as a correction in the atom’s gravitational potential energy. Integrating the Lagrangian along the paths given by the Lagrangian [12] shows that $\Delta \varphi_S = \beta k g T^2$, where $\beta$ takes the meaning of a redshift-violation parameter. In particular we notice that $\beta$ could be “universal”, in contrast with the parameter $\beta_X^{(a)}$ in [6] which depends on the type of atom (a) and on some internal energy $X$ violating LPI. In the multiple Lagrangian formalism, because WEP is valid we can always test the value of $\beta$ by comparing $g$ as obtained from the free-fall of test bodies, to the phase difference cumulated by matter waves (as proposed by MPC [10]).

The most important problem of this formalism is that it is inconsistent to use a different Lagrangian (or metric) for the calculation of the trajectories and for the phases of the atoms or clocks. More precisely, it supposes that the fundamental Feynman path integral formulation of quantum mechanics, which is at the basis of the derivation of the phase shift in an atom interferometer [18], has to be altered in the presence of a gravitational field or could be wrong. Physically it amounts to making the distinction between the atoms when calculating their trajectories and the same atoms when calculating their phases, which is inconsistent as it

In an atom interferometer tests set limits on the same parameter combination as classical UFF tests, namely $\beta_1 + \xi^{\text{bind}} - \beta_2 - \xi^{\text{bind}}$ (13), p. 4, where the subscripts refer to test masses 1 and 2 (e.g. 1 = Cs and 2 = falling corner cube in [8] whilst 1 = Ti and 2 = Be in [11], and $\xi^{\text{bind}}$ refers to the nuclear binding energy of the test masses. On the other hand UCR tests in [13] set limits on either $\beta_1 - \xi^{\text{trans}}$ or $\xi^{\text{trans}} - \xi^{\text{trans}}$ (13), p. 3, where $\xi^{\text{trans}}$ is related to the atomic transition of the atom used in the clock and not to its nuclear binding energy.

3See “Methods” in [10], where they consider two scenarios. The first one clearly states that the trajectories are not modified whilst the atomic phases are. The second uses $g^\prime$ for the trajectories and $g(1 + \beta)$ for the phases, which again corresponds to two different Lagrangians.
is the same fundamental matter field in both cases. Even more, the basis for the derivation of the atom interferometry phase shifts is unjustified because the Feynman formalism is violated. To remain coherent some alternative formalism for the atomic phase shift calculations (presumably modifying quantum mechanics) should be developed and used. More generally the multiple Lagrangian formalism supposes that the duality between particles and waves in quantum mechanics gets somehow violated in a gravitational field.

The above violation of the “particle-wave duality”, implied by the multiple Lagrangian formalism, is very different from a violation of the equivalence principle in the ordinary sense. In this formalism a single physical object, the atom, is assumed to be described by two different Lagrangians, \( L \) applying to its phase shift, and \( L_{\text{particle}} \) applying to its trajectory. By contrast, in tests of the equivalence principle, one looks for the modification of the free fall trajectories or clock rates as a function of composition or clock type. Thus, different bodies or clocks are described by different Lagrangians, but of course for any single type of body we always have \( L_{\text{wave}} = L_{\text{particle}} \). The pertinent test-bed for equivalence principle violations is which does not imply any particle-wave duality violation.

The second problem of theories in the multiple Lagrangian formalism and of the interpretation of MPC is the violation of Schiff’s conjecture \[29\]. Indeed, we have assumed in this Section that the LPI aspect of the equivalence principle is violated but that for instance the WEP aspect remains satisfied. In a complete and self-consistent theory of gravitation one expects that the three aspects of the Einstein equivalence principle (WEP, LLI and LPI) are sufficiently entangled together by the mathematical formalism of the theory that it is impossible to violate one without violating all of them. The Schiff conjecture has been proved using general arguments based upon the assumption of energy conservation \[21\], \[22\]; it is satisfied, for instance, in the \( \Theta \varepsilon \mu \) formalism \[25\]. A contrario, we expect that violating the conjecture leads to some breakdown of energy conservation. In the present case this translates into postulating two Lagrangians \[12\] and \[13\] for the same physical object. Explicit theories that are logically and mathematically consistent but still violate Schiff’s conjecture are very uncommon (see \[7\] and references therein).

We conclude that the general statement of MPC according to which the atom interferometer measures or tests the gravitational redshift at the Compton frequency is incorrect. Instead, the interpretation of the experiment needs to be considered in the light of alternative theories or frameworks. Although one can consider particular alternative frameworks in which the statement of MPC could make sense, such frameworks raise unacceptable conceptual problems which are not at the moment treated in a satisfactory manner. In particular, they break the fundamental principles of quantum mechanics which are used for calculating matter wave phases. In most common and plausible theoretical frameworks the atom interferometry experiment tests the universality of free fall with the Compton frequency being irrelevant.

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