Principals of the Continue model of Geomedia Construction

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Abstract. The problem of constructing a continuous model of a hierarchical-block geomedia is
considered. The basic structural element providing the geomedia structuring is determined, and
the transition from a hierarchically blocky to a hierarchically structured media is discussed. The
introduction of four structural levels is sufficient for the geomedia modeling within the Earth's
crust. To describe the fractured structures, a non-Euclidean model of a continuous medium is
applied, which considers the violation of the deformation compatibility conditions. An algorithm
for the transition to a continual description of such structures is presented, and nonsingular
solutions for a plane-stressed field are constructed. The performed analysis showed that this
approach requires applying the principle of hierarchical non-Euclidianity and a monolithic block
principle.

1. Introduction
The development of the geomedium continual model is one of the main tasks of geomechanics
throughout the history of its existence [1, 2]. This is due to the need to apply well-developed methods
of continuum mechanics to the problem of predicting the development of geomechanical processes both
within the rock mass and on the scale of the Earth's crust and geoid. Thus, the formation of Soil
Mechanics was accompanied by a fundamental dispute between P. Fillunger and K. Terzaghi about the
subject of the discipline itself [3]. Because of the insufficient development level of the geomechanics
theory of that time, general theoretical ideas of P. Fillunger were rejected and forgotten for a long time,
and soil mechanics was turned into a semi-empirical theory.

The formation of "Rock Mechanics" was also accompanied by a fundamental dispute between the
supporters of applying the classical principles of continuum mechanics to the rock mass and the direction
of taking into account the block structure of the rock mass, developed by L. Muller [4]. This determined
the further development of the theory of the geomedium in two directions: the mechanics of discrete and
continuous media. Both directions have taken fundamentally different positions, developed in many
respects in opposition to each other.

We should also mention Bucher's pulsation theory of the geoid formation and development [5]. It
was built based on the continuum concepts of the linear theory of elasticity without detailed elaboration
of the discrete structure of the mass. This gave in the solution the values of stresses in the center of the
Earth, orders of magnitude higher than the known ultimate strength of rocks.
In recent years, Sh. A. Mukhamediev attempted to develop a universal continual model of the geomedium, which would take into account its discrete nature [1]. The author has developed the fundamental provisions of such a model based on the general laws of the mechanics of effective media [6]. The use of the concept of an effective medium in inhomogeneous materials has shown a high degree of reliability, in particular, in the mechanics of continuous media with defects [7]. Starting with a proposal in 1958 by L.G. Kachanov [8] to introduce the concept of "effective stresses", methods for the transition to a continuous medium were developed on the principles of averaging the characteristics of a continuum with defects within the "representative elementary volume" (RVE).

Effective stresses are interpreted as stresses that lead to the same results as real stresses but acting already in the region of an "effective" continuous medium, where there are no defects. Similarly, a continuous medium with defects can be reduced to a continuous medium due to homogenization, as was considered in work mentioned above [1].

Note that all of the above approaches to developing a workable continual model of the geomedium, taking into account its block nature, have a drawback, which consists of the need to average the properties of the medium with fractured defects within the selected range of dimensions. A consequence of averaging is the impossibility of describing mesofracture structures at all levels of the block geomedium from the solutions of the mechanics' problems.

Meanwhile, it is the need to describe such structures as the zonal destruction of the mass around deep underground openings [9], studying the patterns of their formation and development, which is of primary interest in geomechanics. This prompts the search for a solution to creating a workable continual model of the geomedium.

At the same time, the developed principles for determining the effective characteristics of a medium with fracture defects can be successfully applied in the development of continual non-Euclidean models of a continuous medium with defects that specify the structural-hierarchical nature of the geomedium. Here the "monolithic block" principle constitutes one of the starting points of modeling, as shown below.

Use of the non-Euclidean models has shown its high efficiency in physical mesomechanics [10] in connection with the completeness of describing the physical and mechanical properties of materials, which has become especially important in recent years. Many publications have appeared in which various characteristics of the internal structure of materials were introduced, including scalar and tensor characteristics of "damage," "defectiveness," etc. On the other hand, the use of electron microscopy to study the fine structure of crystals in the 1950s led to the introduction of such concepts as dislocations, disclinations, vacancies, and other characteristics of crystal structure defects into the materials science literature.

For a macroscopic level of consideration of the plastic deformation of solids in the form of a two-stage process: elastic and plastic, the modern theory of defects in crystals makes it possible to qualitatively explain many laws governing the behavior of materials under various loading conditions. However, experimental studies of their plastic deformation show [10] that processes of structural rearrangement occur at this stage, accompanied by the development of deformation at intermediate - between microscopic and macroscopic - scales. The separation of such scales in a crystal is determined by the fact that a new qualitative state is formed in it, characterized by the existence of so-called structural levels of deformation.

The need to formulate a new paradigm for describing a deformable solid in the hierarchy of the spectrum of structural-scale levels was analyzed in [11]. It is shown that it is possible to offer an abbreviated description of the entire spectrum of defects based on their shared property, which is associated with the development of local structural transformations that form in zones of stress concentrators of various scales. Then the process of structural transformations in the deformed crystal develops over the entire spectrum of scale levels. The problems arising here lie at the junction of physics and mechanics of a deformable solid and have become the subject of research in physical mesomechanics.

The emergence of "Physical mesomechanics" as a scientific discipline that studies and simulates the mesolevel of deformable bodies with defects has forced to revise the approaches to constructing mathematical models of the geomedium. The application of the mesomechanics principles in geomechanics, when modeling the geomedium and forecasting geodynamic phenomena, was reflected
in the works [12, 13]. At the same time, the authors of this article simultaneously and independently developed an approach for modeling the hierarchy of a block geomedium based on non-Euclidean models [14, references therein].

2. The general property of non-Euclidean structures
Apparently, for the first time in the physical literature, the necessity of introducing the transition from the classical (Euclidean) theory to the non-classical (non-Euclidean) theory was apparently pointed out in the works of K. Kondo and W. Bilby [15, 16]. Many authors have proposed variants of generalization of the classical theory of elasticity within the framework of various non-Euclidean models of a continuous medium, and an analysis of this problem can be found in [17, 18]. Common to all approaches is the use of non-Euclidean geometric objects: the torsion, non-metricity, and curvature tensor as variables characterizing the geometric structure of the internal interactions of particles of a continuous medium with each other. The idea of a generalization of the classical theory was analyzed in the most precise form by S.K. Godunov [19], who pointed out that the identification of changes in the internal metric of a material, which determines the change in its internal energy, and the corresponding change in the shape of a body in the Euclidean metric of an external observer, is an additional hypothesis postulated in the classical theory.

The need to introduce non-Euclidean objects was implicitly emphasized in analyzing various engineering problems [17, p. 2213]. Thus, the assumptions about the incompatibility of deformations used in [20] when creating equipment for measuring the level of internal stresses in welded structures are clearly related to the nonzero curvature tensor in the space inside the material.

From a physical perspective, the geometric non-Euclidean characteristics of spaces are internal variables and cannot be measured directly. Although defects are the physical reason for the existence of internal nonzero stresses in the material, at the macroscale level, at a certain grain size, a distribution of dislocations is possible, which does not lead to macroscopic stress fields [21]. In this case, when interpreting the results of an experimental study of the actual material behavior, a significant role is played by the resolution level in measurements, which depends on the scale of the processes under study. At the same time, at a given - macroscopic - scale level of consideration of the materials' behavior, it does not matter which physical defects determine the resulting incompatibility of deformations.

In this case, the shared kinematic property for all defects is the incompatibility of the medium elements, which should be considered a fundamental element in studying the internal structure of materials. Then, on the selected spatial scale of the material, to describe its behavior, one should introduce as an additional variable the incompatibility parameter (one or several), which determines the degree of the material structural heterogeneity. By definition, the parameter of incompatibility has a kinematic origin, and the violation of the homogeneity of the material occurs when certain force conditions are met.

3. Hierarchy of structural levels of the block geomedium
Turning to the analysis of the hierarchical-block geomedium from the position of mesomechanics, it should be pointed out that the block structure of the rock mass was initially established in geological studies [22]. At the same time, a classification of the hierarchy of blocks appeared, in which the corresponding levels were distinguished, based on the ability of rocks to destroy specifically [23]. The block structure has, as a rule, the form of "masonry" and is formed as a result of geological processes, and the sizes of blocks are considered from 0.1 mm to the first hundreds of kilometers. The phenomenological model of the geomedium hierarchical-block structure was considered in [34] using the "fundamental canonical series" idea at the quantitative level. In [15], the hierarchical-block nature of the geomedium was noted, which, when passing to the mesoscale, presupposes shear and rotation as the main types of deformations.

However, it is well known in geomechanics that the main structural element of the geomedium is a tension joint that develops during shear [25]. Note that the problem of the origin of tension joint in a rock mass under conditions of all-round compression was discussed in geological publications long before the advent of geomechanics, but no unambiguous explanation was found [22]. At present, the
theory of shear-tension destructions at compression has been developed quite fully [6, 25]. Moreover, the block character of the geomedium plays a decisive role in this.

At the first level of the hierarchical-block geomedium, represented by rock samples, blocks are mineral grains that have clearly distinguished boundaries in the experiment, which set the heterogeneity of the rock's strength properties. At the boundaries of mineral grains, as the most weakened areas of the geomaterial, during compression, shear mesodefects appear, generating tension sites in the end areas (see Fig. 1). With an increase in the load, the shear-tension mesocracks merge, and a tension macrocrack develops in the rock sample. The direction of its growth coincides with the direction of the maximum stresses during the uniaxial compression of the sample.

![Figure 1. Formation of shear-tension mesocrack [25]](image)

The theory of shear-tension fracture of rocks presented in [25] describes all the basic experimental facts known from tests of rock samples and makes it possible to determine the maximum length of a stable shear-tension macrocrack. The size of the structural blocks (the diameter of the mineral grain $d_{зерн} = 0.1 \ .. 5 \ \text{мм}$) to the sample size (diameter 3–5 cm, height 6–10 cm) averages 1:100. The sizes of microcracks for a mineral are on the order of the size of a mineral grain. The sizes of tension mesocracks, arising during compression of a rock sample, vary from 1 to 5–10 grain diameters [25]:

$$2l_{обр}^{макс–мезо} = (5 \div 10)d_{зерн} = (0.5 \div 5)\text{мм}$$ (1)

The critical length of a steady macrocrack of tension under uniaxial compression $2l_{обр}^{макс–макро}$ (the limiting length of a steady macrodefect of a rock sample) is 6–10 cm for most rocks [25].

When moving to the next upper structural level in the rock blocks hierarchy of the geomedium for openings with a diameter of 3–5 m (which corresponds to the conditions of most single mine openings of the coal and ore industry), the ratio of 1:100 between the size of the rock sample and the diameter of the openings also remains. If we assume that the principle of geometric similarity [19] between the block structure of the rock sample and the mass at the hierarchically second level of the block geomedium is valid, then the size of the mass blocks is equal to $d_{мас} = 100d_{зерн} = (1 \div 5)\text{см}$, and the ratios (2) determine the corresponding sizes of mesodefects:

$$2l_{мезо–макро}^{мин} = d_{макро} = (1 \div 5)\text{см}$, $2l_{мезо–макро}^{макс} = (5 \div 10)d_{мас}$$ (2)

From (1), (2), it can be seen that taking into account the "masonry" of the block structure of the mass in the order of size $2l_{мезо–макро}^{макс} = 2l_{обр}^{макро–макро}$, which can be taken as an estimate when carrying out uniaxial compression tests. This allows us to consider a macrodefect at the hierarchically first level of the geomedium (rock sample) as a mesodefect of minimum length at the hierarchically second (upper) level of the geomedium - a mine opening. In this case, the maximum length of the mass mesofracture will be $2l_{мезо–макро}^{макс} \approx 10 – 100 \ \text{см}$, which corresponds to the parameters of most fracture systems in the rock mass.
Thus, a steady maximum length macrocrack of the neighboring lower hierarchical level sample will serve as a meso-fracture of the adjacent upper hierarchical level of the underground opening. The ratio 1: 100 gives the scale of the rock mass, within which the conditions of geometric similarity are preserved, and while maintaining the mechanisms of rock destruction in the rock mass by tension under compression for openings with a diameter of up to 3–5 m, the requirements of physical similarity are met. The latter means that the mesofracture process in the mass around the openings occurs according to the same laws characteristic of rock samples: after the stage of chaotic development, the mesofracture process is localized, leading to zonal deformation of the enclosing rocks [27]. Further development of such a process leads to the formation of tear-off macrocracks under compression conditions associated with zonal destruction (disintegration) [27]. The subsequent transfer of the obtained results to all levels of the hierarchical block array within the crust makes it possible to estimate the maximum sizes of steady macrofractures. The final results are shown in Table.

The proposed approach expands the ideas of physical mesomechanics to a hierarchical-block geomedium: the maximum size of a steady shear-tension macrocrack of the lower neighboring hierarchical level determines the minimum size of an upper-level block and limits its level in the block hierarchy from below. In this case, interblock shear-tension cracks are considered as mesocracks, and intrablock ones – as microcracks.

After determining the scale within which the requirements for averaging the mesovolumes properties [13] are fulfilled, the construction of a continuous medium model with shear-type fracture defects at all levels of the hierarchical-block geomedium is directed along a constructive path. In this case, the shear in the process of shear-tension failure under conditions of all-round compression [25] remains decisive in the formation of a macrocrack, and the tension part provides, in turn, the rotational mobility of a block medium of the corresponding mesolevel. This corresponds to the condition of work [11] to satisfy the requirement to preserve the shear-rotational mechanism of the mesofracture process at each structural level of the geomedium. Therefore, it seems reasonable to apply non-Euclidean models [14] to the geomedium at all levels of its block hierarchy, subject to certain principles [31].

| Table The structural block sizes at different structural levels of the block geomedium |
| Structural level | The structural block size | Length of steady macrodefect | Actual range of the structural block sizes | Citation |
|------------------|---------------------------|-------------------------------|------------------------------------------|---------|
| Rock Sample      | 0.5 mm                     | 10 cm                         | 0.1-0.5 mm                               | [25]    |
| Rock Mass (mining opening) | 5 cm                      | 10 m                          | 10-100 cm                                | [26]    |
| Rock Mass (clearing opening) | 5 m                       | 1000 m                        | 2-5 m                                    | [28]    |
| Rock Mass (crust) | 500 m                      | 100 km                        | 20-180 km                                | [29, 30]|

4. Non-Euclidean model of the geomedium at various structural levels

In the classical model of a continuous medium, when describing the stress field, a kinematic constraint associated with the fulfillment of the compatibility condition (Saint-Venant condition) is used, and dynamic relations for the elements of the medium. When extending the classical theory, an incompatibility function is introduced [17, 23], and this, in turn, leads to the rejection of the Saint-Venant compatibility condition. The thermodynamic substantiation of the relations of the non-Euclidean model is given in [32].

To illustrate the results of applying the non-Euclidean model to the first structural levels of the geomedium (Table), let us consider the case of a plane-deformed state of the medium in a stationary
setting. Then the inconsistency function is scalar, i.e., the non-Euclidean model has an additional parameter \( R \) that is unique in comparison with the classical model - this distinguishes the two-dimensional case from the three-dimensional case:

\[
R = \frac{\partial^2 \varepsilon_{11}}{\partial x_2 \partial x_1} + \frac{\partial^2 \varepsilon_{22}}{\partial x_1 \partial x_1} - 2 \frac{\partial^2 \varepsilon_{12}}{\partial x_1 \partial x_2},
\]

where \( \varepsilon_{ij} \) are the components of the strain tensor. The equations for the stress field in the absence of mass forces coincide with the Cauchy equilibrium equations

\[
\frac{\partial \sigma_{ij}}{\partial x_j} = 0.
\]

The non-Euclidean model presented above was applied to describe the severe-deformed state of a highly compressed rock sample [32]. The constructed solutions make it possible to identify the focal area of the preparation of macrofracture in the sample in the pre-fracture stage, to establish the periodic nature of the stress distribution both along the sample perimeter and along its height.

An experimental study of the nature of rock sample deformation in the pre-fracture stage of loading was carried out while developing a multi-point method for determining the deformations of a highly compressed rock sample. On samples of cylindrical dacite, deformations were recorded using strain gauges based on 10 mm and using the UIU2002 deformation measuring station as recording equipment. The coincidence between the results of experimental and theoretical studies is shown in Fig. 2, which shows their satisfactory qualitative agreement.

![Figure 2](image)

**Figure 2.** Research result of a rock sample deformations in the middle part (in height) in the pre-fracture loading area: (A) - experiment, (B) - comparison of theory and experiment.

Formulated above the Non-Euclidian model of the continuous media was used at the second structural level of geomedia (see Table) to analysis of various effects of zone deformation and destruction of a rock mass round cylindrical deep mining opening [27, 32]. From the decision of a plane problem of mechanics the equations for the description of periodic character of a stresses field round opening have been received.

Using the fracture criterion [25], it was shown that the characteristics of the zones are weakly dependent on the values of the elastic modulus of rocks and Poisson's ratio. The model efficiency has been demonstrated for a wide range of rock mass properties in strong rocks \( \sigma_c = 150 \) MPa, where \( \sigma_c \) is the strength of the rock) and for weak rocks \( \sigma_c = 15 \) MPa). Algorithms and programs were developed to calculate the values of stresses and criterion functions [32].
The correlation between theory and experimental study was assessed by comparing the measured radial displacements near openings at great depths (Nikolaevsky mine, Dalnegorsk, Russia) with the results predicted by the model (Fig. 3). It was found that the quantitative differences between the predicted and measured research results (excluding the contour zone) do not exceed 47%, with complete qualitative satisfaction at a distance of up to four zones from the contour. Studies of weak and strong rocks show that the main factor affecting the zonal parameters of the fracture structure is the value of the stress level in the rock mass (opening depth). With increasing depth, an increase in the number of destruction zones and their radial extent is observed.

Figure 3. Measuring station around the opening (A). Theoretical and experimental radial deformations (B).

Thus, the continual non-Euclidean geomedium models confirm their efficiency when describing mesostructures of the zonal and source type at various structural levels of the geomedium [32].

5. Conclusion

Geomechanics of highly compressed rocks and masses is a combined theory where the geomechanically determined hierarchical-block structure is replaced by a hierarchy of structural levels of the geomedium. The main structural element here is a shear-tension crack, which develops at the boundaries of the blocks. Each structural level, in turn, is modeled by a continual non-Euclidean model that meets all the requirements of physical mesomechanics.

Replacing the geomedium structural levels with a system of non-Euclidean models differing from each other only in the values of the parameters used in them corresponds to the principle of non-Euclidean hierarchy. Another essential principle is the principle of "monolithic block," where within each structural level, real blocks of a rock mass are modeled by an effective continuous medium, the properties of which are determined as a result of standard micromechanical procedures [6, 9].

As a result, the hierarchy of structural levels in the mass was identified at its various scales, and the block geomedium is presented in a graded form. With this approach, the study of the geomedium critical states in the process of its structural transformations under rock mass strong compression becomes the subject of geomechanics.

Acknowledgments

The work of author 1 was partially supported by the Russian Science Foundation (project no. 19-19-00408).

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