A standing problem of glassy systems with randomness and frustration is the possible instability of the glassy frozen states against infinitesimally weak perturbations such as an infinitesimal change of temperatures. A class of phenomenological scaling theories started first in the context of spin-glasses [1] suggests that the equilibrium glassy state dramatically changes beyond the so-called overlap length. Unfortunately, the validity of the prediction has not been proven explicitly except for some Migdal Kadanoff real space renormalization-group (MKRG) studies [2]. The problem remains very far from being resolved especially concerning the problem of temperature-chaos, i.e., the sensitivity to small temperature changes [3, 4].

The problem of temperature-chaos is now of great interest since it is considered as a possible mechanism of the rejuvenation (restart of aging) found in experiments such as spin-glasses [5] and polymer glasses [6]. Concerning the memory effect which is simultaneously observed in experiments, recently a novel dynamical memory was found in a coarsening model under explicit cycling of target equilibrium states [7].

We consider a much simpler model, the directed polymer in random media (DPRM) in 1+1 dimension [8], [9] which allows a much better access to the problem. It is known to be in the frozen phase at all finite temperatures in the sense that its scaling properties are always known to be in the frozen phase at all finite temperatures which allows a much better access to the problem. Indeed the anomalous sensitivity of the glassy systems to weak perturbations are predicted by scaling arguments which are partially supported by numerical studies [10, 11, 12, 13, 14, 15]. In the present paper, we present a unified analytical approach to deal with a variety of perturbations based on the concept of explicit replica symmetry breaking [16, 17] and confirm analytically the predictions of the scaling arguments for the first time.

DPRM belongs to the class of elastic manifolds in random media [18] so that its statistical mechanical properties are potentially relevant for a broad class of physical systems of much interest. The latter includes the domain walls of ferromagnets [19] with weak bond randomness and the flux lines in type-II superconductors with randomly distributed point like pinning centers [20], CDW and vortex lattice systems with weak random-periodic pinnings [21, 22].

We study the model described by the following Hamiltonian,

\[ H[V, h, \phi] = \int_0^L dz \left[ \frac{\kappa}{2} \left( \frac{d\phi(z)}{dz} \right)^2 + V(\phi(z), z) \right]. \]  

The scalar field \( \phi \) represents the displacement of the elastic object at point \( z \) in a 1-dimensional internal space of size \( L \). The 1st term in the Hamiltonian is the elastic energy, \( \kappa \) being the elastic constant. The random pinning medium is modeled as a quenched random potential \( V(\phi, z) \) with zero mean \( V(\phi, z) = 0 \) and short-ranged spatial correlation, \( \langle V(\phi, z) V(\phi', z') \rangle = 2D\delta(\phi-\phi')\delta(z-z') \). Here and in the following \( \langle \cdots \rangle \) stands for the average over different realizations of the random potential.

Our starting point is a system of two real replicas, say A and B, whose configurations \( \phi_A(z) \) and \( \phi_B(z) \) are subjected to exactly the same random potential and temperature. Second, we apply small perturbations to them. In the present letter, we consider

i) **Temperature difference** [11]: slightly different temperatures \( T_A = T + \delta T \ T_B = T - \delta T \) for A and B respectively with \( \delta T / T \ll 1 \)

ii) **Decorrelation of random potential** [11]: the random potential of B is made from that of A
as \( V_B = (V_A + \delta V')/\sqrt{1 + \delta^2} \) where \(|\delta| \ll 1\) and \( V'\) follows the same Gaussian distribution as \( V\). Then \( \bar{V}_G(\delta, z) = 0 \) and \( \bar{V}_G(\delta, z) = 2D_GG'\delta(\phi - \phi')(z - z') \) with \( D_{AA} = D_{BB} = D \) and \( D_{AB} = D/\sqrt{1 + \delta^2} < D \).

ii) Explicit short-ranged repulsive coupling \([2, 7]\): A and B replicas are subjected to explicit repulsive short-ranged interaction \( \epsilon \int_0^L dz \delta(\phi_A(z) - \phi_B(z)) \) with \( 0 < \epsilon \ll 1 \).

iii) Tilt field \([4]\): A and B replicas are subjected to a tilting field of opposite sign \(-h\phi_A(L) + h\phi_B(L)\) with \( h \ll 1 \).

To study the consequence of perturbations, we analyze disorder average of partition function \( \bar{Z}_{A+B}^n(L) \) in which both real replicas \( G = A, B \) are further replicated into \( n\)-replicas \( \alpha = 1, \ldots, n \). As noticed by Kardar \([2]\), if an analytical continuation for \( n \to 0 \) is possible, such a replicated partition function can be identified as a generator of cumulant correlation functions of sample-to-sample fluctuations of free-energies \([2]\).

\[
\ln \bar{Z}_{A+B}^n(L) = n[-\beta A F_A(L) - \beta B F_B(L)] + \frac{n^2}{2} [-\beta A F_A(L) - \beta B F_B(L)]^2 + \ldots
\]  

where \([\ldots]_c^n\) stands for \( p\)-th cumulant correlation functions of the total free-energies \(-\beta A F_A(L) - \beta B F_B(L)\) with \( F_G(L) \) and \( \beta_G \) being free-energy and inverse temperature of each subsystem \( G = A, B \).

Then if there is a complete change of the free-energy landscape due to infinitesimally weak perturbations of strength \( \delta \ll 1 \), the statistical correlations between \( A \) and \( B \) should be lost asymptotically in the triple limits \( n \to 0 \) followed by \( L \to \infty \) and finally \( \delta \to 0 \). Here the order of the limits is crucial. Then the total partition function \( \bar{Z}_{A+B}^n(\delta, L) \) under perturbation should reduce to a product of the partition functions of the sub-systems \( \bar{Z}_A^m(L) \) and \( \bar{Z}_B^n(L) \) for \( A \) and \( B \) respectively,

\[
\lim_{\delta \to 0} \lim_{L \to \infty} \lim_{n \to 0} \bar{Z}_{A+B}^n(\delta, L) = \bar{Z}_A^m(L) \times \bar{Z}_B^n(L)
\]

We use this factorization as the definition of the decorrelation of the free-energy landscape (chaos).

To be specific, we consider that one end of all replicas is fixed at \( x = 0 \) and the other end fixed at \( \{x_G, \alpha\} \). The partition function of the total system can be expressed by a path integral over all possible configurations of \( 2 \times n \) replicas,

\[
\bar{Z}_{A+B}^n\{x_G, \alpha\}, L) = \int \prod_{G=A,B, \alpha=1, \ldots, n} \mathcal{D}\phi_G, \alpha \ e^{-S_{A+B}[\phi_G, \alpha]} (4)
\]

with a certain effective action \( S_{A+B}[\phi_G, \alpha] \). Then in turn, the partition function can be generated as a path integral of a quantum system in imaginary time with a Schrödinger equation,

\[
-\frac{d}{dL} \mathcal{Z}_{A+B}^n\{x_G, \alpha\}, L) = \mathcal{H}_{A+B}\{x_G, \alpha\}, L). (5)
\]

with the Schrödinger operator

\[
\mathcal{H}_A = -\sum_{G, \alpha} \frac{k_B T_G}{2k} \frac{\partial^2}{\partial x_{G, \alpha}^2} + \sum_{(G, \alpha), (G', \beta)} \frac{D_{GG'}}{(k_B T_G)(k_B T_{G'})} \delta(\phi_G, \alpha(z) - \phi_{G', \beta}(z)) + \epsilon \sum_{\alpha} \delta(\phi_A, \alpha(z) - \phi_B, \alpha(z)) - \frac{h}{k_B T} \sum_{\alpha} \frac{\partial}{\partial x_{A, \alpha}} - \frac{h}{k_B T} \sum_{\alpha} \frac{\partial}{\partial x_{B, \alpha}}. (6)
\]

Here we have two different kinds of bosons for \( A \) and \( B \) each of which has \( n \) particles. The 1st term represents the kinetic energy. The 2nd term stands for attractive short-ranged interactions between the bosons where the sum is taken over all possible pairs of bosons. The 3rd term is the explicit repulsive coupling. The last term is due to the tilt field applied to \( A \) and \( B \) subsets in the opposite directions.

The Schrödinger operator can be decomposed as

\[
\mathcal{H}_{A+B} = \mathcal{H}_0 + \delta \mathcal{H} (7)
\]

where

\[
\mathcal{H}_0 = -\sum_{G, \alpha} \frac{k_B T_G}{2k} \frac{\partial^2}{\partial x_{G, \alpha}^2} - \frac{D}{(k_B T)^2} \sum_{(G, \alpha), (G', \beta)} \delta(x_G, \alpha - x_{G', \beta}). (8)
\]

is the Schrödinger operator in the absence of perturbations and \( \delta \mathcal{H} \) is a perturbation term representing one of the specific perturbations i)-iv). Note that the original operator \( \mathcal{H}_0 \) has a high replica (permutation) symmetry: it is symmetric under all possible permutations of bosons not only among \( A \) and \( B \) subsets but among the whole set.

Obviously, the perturbation term \( \delta \mathcal{H} \) breaks the replica (bosonic) symmetry explicitly. Then the interesting question is whether such a term can break the symmetry in the thermodynamic limit even if it is infinitesimally small. If a decorrelation of the free-energy (chaos) takes place, it should manifest itself as reduction of this high symmetry, i.e. replica symmetry breaking, such that replica symmetry remains at most only within each subset \( A \) or \( B \). Our problem can be considered as a particular example of the problem of explicit replica symmetry breaking proposed by Parisi and Virasoro \([4]\), who tried to give a prescription to define replica symmetry breaking in a sound thermodynamic sense by introducing infinitesimally weak replica symmetry breaking terms. Indeed our approach is an extension of the work by Parisi \([4]\).
who implemented the idea of explicit replica symmetry breaking in the context of DPRM.

The ground state of the Schrödinger operator \( \mathcal{S} \) can be obtained via the Bethe ansatz \([23]\). It is a bound state involving all particles described by the wavefunction

\[
\langle \Psi_{RS}| \{ x_{G,\alpha} \} \rangle \sim \exp \left( -\lambda \sum_{((G,\alpha),(G',\beta))} |x_{\alpha,G} - x_{\beta,G'}| \right)
\]

with \( \lambda = \kappa D/(k_B T)^3 \). Since it is completely replica symmetric, we call this as RS (replica symmetric) state. Following Parisi \([17]\), we consider also an excited state

\[
\langle \Psi_{RSB}| = < \Psi_{RS}^A | < \Psi_{RS}^B | < \Psi_{RS}^A | \Psi_{RS}^B | = 0.
\]

where \( < \Psi_{RS}^A | \) and \( < \Psi_{RS}^B | \) are similar bound states described by the wavefunction like \([11]\) but constituted by only A and B particles. Assuming that the centers of mass of the two bound states are infinitely far from each other so that their overlap is zero, such a state becomes also an eigenstate. Since replica symmetry is reduced here, we refer to the state as RSB (replica symmetry broken) state.

Let us consider the difference (gap) of the eigen value of the RS ground state and the RSB excited state under the perturbation

\[
\Gamma(n) = \Gamma_0(n) + \delta \Gamma(n)
\]

where \( \Gamma_0(n) = k_B T/\kappa \lambda^2 n^3 \) is the original 'energy gap' and \( \delta \Gamma(n) \) is the correction due to the perturbation. Assuming that the amplitudes of the perturbations are small enough, one may compute the correction term at 1st order of perturbation as,

\[
\delta \Gamma(n) = \frac{\langle \Psi_{RSB}|\delta \mathcal{H}|\Psi_{RSB} \rangle - \langle \Psi_{RS}|\delta \mathcal{H}|\Psi_{RS} \rangle}{\langle \Psi_{RSB}|\Psi_{RSB} \rangle - \langle \Psi_{RS}|\Psi_{RS} \rangle}.
\]

As we outline later, the correction terms can be computed for the cases that we consider and we find a generic form for the leading term

\[
-\delta \Gamma(n) \sim \Delta n^p \quad 1 \leq p < 3
\]

with \( \Delta > 0 \) being a constant which represents the strength of perturbations and \( p \) being a positive integer smaller than 3. We call the exponent \( p \) as order of perturbation. Here 'leading' term means a term with smallest power of \( n \) which becomes most important in the \( n \to 0 \) limit that we consider. Let us note here that we have chosen the detailed form of the perturbations i)-iv) such that this gap \([13]\) remains invariant under exchange \( A \leftrightarrow B \). Combining the results one finds,

\[
\Gamma(n) = \Gamma_0(n) \left[ 1 - \left( \frac{n}{n^*(\Delta)} \right)^{(3-p)} \right]
\]

with

\[
n^*(\Delta) = \left( \frac{\Delta}{\lambda^2 k_B T/\kappa} \right)^{1/(3-p)}.
\]

Let us recall that we have to take the \( n \to 0 \) limit before \( L \to \infty \). Since \( p < 3 \) holds for all the cases we consider, an arbitrarily small perturbation \( \Delta \) will induce a level crossing at \( n^*(\Delta) \) below which the contribution of RSB excited state to the replicated partition function becomes larger than that of the original ground state (RS), i. e. there is replica symmetry breaking. The result \([14]\) strongly suggests that the factorization of the partition function \([6]\), i.e. complete change of the free-energy landscape (or chaos), takes place in the thermodynamic limit even with infinitesimally small (but non-zero) perturbation of strength \( \Delta \ll 1 \),

\[
\lim_\Delta \lim_{L \to \infty} \lim_{n/n^* \to 0} \frac{Z_{RSB}^n(\Delta, L)}{Z_{RS}^n(L) \times Z_{B}^n(L)} = \begin{cases} 1 & (p < 3) \\
\end{cases}
\]

Now let us further exploit from the above result to find a more physical picture. In the absence of perturbations, the logarithm of the replicated partition function is known to have a functional form \([24, 25]\) \( \ln Z_{RS}^n(\beta F) \) \( \sim \ln L^\beta \) \( \propto L^\beta \) with \( \beta \) being the exact stiffness exponent \( \beta = 1/3 \). On the other hand, \([4]\) implies \( n/n^* \) is another natural variable of the replicated partition function \([3]\).

Combining the two, we conjecture the following scaling ansatz,

\[
\ln \frac{Z_{RSB}^n(\Delta, L)}{Z_{RS}^n(L) \times Z_{B}^n(L)} = g(2n L^{1/3})
\]

where \( \bar{T} \) is the average free-energy per unit length. Here we introduced a characteristic length \( L^* \) defined as

\[
L^* \sim (n^*)^{-3} \sim \Delta^{-3/(3-p)}
\]

The \( n \to 0 \) limit induces the thermodynamic limit \( L \to \infty \) if the variable \( n L^{1/3} = x \) is fixed. Then for a fixed \( x \), two limiting behaviors are expected. First the small length scale regime \( L/L^* \ll 1 \) corresponds to \( n/n^* \gg 1 \) so that the effect of perturbation will be small and the partition function is essentially the same as that of the unperturbed system of \( 2 \times n \)-replicas \( g(x, L/L^* \to 0) \equiv g(2 x) \). Second the large length regime \( L/L^* \gg 1 \) corresponds to \( n/n^* \ll 1 \). Thus the decoupling \([16]\) implies \( g(x, L/L^* \to \infty) \equiv 2 x \).

The above result implies the crossover length \( L^* \) should correspond to the overlap length \( L^* \) anticipated by the scaling argument. In the following we present the outline of our results for the specific perturbations. Details will be presented elsewhere \([28]\).
Temperature and potential change
In the case of i) small temperature difference, one finds by the perturbation calculation that the correction to the gap \((\Delta)\) is characterized by order \(p = 2\) with amplitude \(\Delta \propto (\delta T/T)^2\). Then using \((3)\) and \((8)\), we find the overlap length \(L^* \sim (\delta T/T)^{-1}\). Similarly, the case of ii) small decorrelation of random potential also amounts to the same order \(p = 2\) with \(\Delta \sim \delta^2\). Then one finds the overlap length \(L^* \sim \delta^{-6}\). Quite remarkably, the overlap-length obtained here is in agreement with the prediction of the scaling arguments \([11, 15]\). Furthermore, it is interesting to note that the two perturbations which look very different at a first sight, turn out to be the same in the replica space.

Explicit repulsive coupling
The case of iii) explicit short-ranged repulsive coupling has been considered by Parisi \([17]\) and the result gives order \(p = 1\) and amplitude \(\Delta \sim \epsilon\). Thus one finds the overlap-length \(L^* \sim \epsilon^{-3/2}\) which agrees with the prediction \([12]\) based on a scaling argument.

Tilt field
Finally we consider the case of iv) tilt field. In this case, the 1st order correction term \((13)\) vanishes. Fortunately, we can proceed as the following. The Schrödinger operator with the tilt field can be rewritten into the fully symmetric form as the unperturbed one \((5)\) by shifting the momenta, \(\partial/\partial x^x_{A,\alpha} = \partial/\partial x_{A,\alpha} - h/k_B T\) and \(\partial/\partial x^x_{B,\alpha} = \partial/\partial x_{B,\alpha} + h/k_B T\). Therefore the exact ground state under the tilt field can be obtained from the Bethe’s wave-function \((3)\) by undoing the previous shifting of moments. On the other hand, the unperturbed RSB wavefunction \((10)\) remains as a valid eigenstate under the tilt field. Combining these results, we obtain the correction to the gap due to the tilt field as \(\delta \Gamma(n) = nh^2/k_B T\) from which we read off order \(p = 1\) and amplitude \(\Delta \sim h^2\), which yields the overlap length \(L^* \sim h^{-3}\). The latter agrees with the prediction \([12]\) based on a scaling argument.

To summarize we have presented a unified analytical approach to the problem of the sensitivity to perturbations of the equilibrium glassy states of 1+1 dimensional DPRM based on the idea of explicit replica symmetry breaking. Our result in replica space suggests that perturbations can be classified according to their order \(p\) and the symmetries that are preserved. Therefore, apparently different perturbations can bring about the same effects. The universal features of the crossover from the weakly perturbed regime \(L/L^* \ll 1\) to the strongly perturbed regime \(L/L^* \gg 1\) can be examined numerically \([28]\). The prediction of the scaling arguments is supported for 4 different kinds of perturbations. It may be promising to apply the approach of the present paper in the studies of other glassy systems including spin-glasses.

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