Partially coherent sources which produce the same far zone optical force as a laser beam

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On applying a theorem previously derived by Wolf and Collett, we demonstrate that partially coherent Gaussian Schell model fluctuating sources (GSMS) produce exactly the same optical forces as a fully coherent laser beam. We also show that this kind of sources helps to control the light-matter interaction in biological samples which are very sensitive to thermal heating induced by higher power intensities; and hence the invasiveness of the manipulation. This is a consequence of the fact that the same photonic force can be obtained with a low intensity GSMS as with a high intensity laser beam. This property is not necessary to produce a highly directional intensity with electric vector \( \mathbf{E}(\mathbf{r},\omega) \) on a dipolar particle in vacuum with permittivity \( \varepsilon_0 \), i.e., that whose scattering cross section can be fully expressed in terms of the two first Mie coefficients \( a_1 \) and \( b_1 \), can be written as a sum of a gradient (conservative) and a scattering plus curl (non-conservative) force [17–20],

\[
F_i(\mathbf{r},\omega) = F_i^{\text{cons}}(\mathbf{r},\omega) + F_i^{\text{nc}}(\mathbf{r},\omega)
\]

\[
= \frac{\varepsilon_0}{4} \text{Re} \omega \partial_\mathbf{r} \left\langle E_j^* (\mathbf{r},\omega) E_j (\mathbf{r},\omega) \right\rangle + \frac{\varepsilon_0}{2} \text{Im} \omega \left\langle E_j^* (\mathbf{r},\omega) \partial_\mathbf{r} E_j (\mathbf{r},\omega) \right\rangle,
\]

\[(1)\]

where \((i,j) = (x,y,z)\) and \(\alpha_e\) is the electric polarizability of the particle. The electric field can be written in terms of an angular spectrum of plane waves \( \mathbf{e}(ks_\perp,\omega) \) [14, 21, 22]

\[
\mathbf{E}(\mathbf{r},\omega) = \int \mathbf{e}(ks_\perp,\omega) e^{i k s_\perp \cdot d \mathbf{s}_\perp},
\]

(2)

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Addressing then a planar GSMS, its cross spectral density tensor \( W_{ij}^{(0)}(\rho_1, \rho_2, \omega) = \langle E_i^* (\rho_1) E_j (\rho_2) \rangle \) at the plane \( z = 0 \) of the source, is given by [14]

\[
W_{ij}^{(0)}(\rho_1, \rho_2, \omega) = \sqrt{S_i^{(0)}(\rho_1, \omega) S_j^{(0)}(\rho_2, \omega)} \mu_{ij}^{(0)}(\rho_2 - \rho_1, \omega)
\]

where \( S^{(0)} \) and \( \mu^{(0)} \) are the spectral density and the spectral degree of coherence of the source, respectively. For this type of sources these quantities are both Gaussian, i.e.,

\[
S_i(\rho, \omega) = A_i \exp[-\rho^2/(2\sigma^2_{s,i}(\omega))],
\]

\[
\mu_{ij}(\rho_2 - \rho_1, \omega) = B_{ij} \exp[-|\rho_2 - \rho_1|^2/(2\sigma^2_{g,ij}(\omega))].
\]

The parameter \( A_i \) is the maximum of the spectral intensity at \( \rho = 0 \), whereas \( B_{ij} \) is \( |B_{ij}| = 1 \) if \( i = j \) and \( |B_{ij}| \leq 1 \) if \( i \neq j \).

The widths \( \sigma(\omega)_{s,i} \) and \( \sigma(\omega)_{g,ij} \) are usually known as the spot size or the correlation or spatial coherence length of the source, respectively. Notice that these parameters cannot be chosen arbitrarily, they have to fulfill a series of conditions [15]. In this work, for simplicity, we restrict ourselves to the case in which the field is completely polarized, i.e., the degree of polarization is equal to 1 [23], or equivalently, the electric field only fluctuates in one direction (for example in the \( x \)-direction). At fixed frequency and in order to simplify the notation, in what follows we shall write \( \sigma_{s,i} \) and \( \sigma_{g,ij} \) without \( \omega \) dependence nor the Cartesian subindex, understanding that we are dealing with the \( x \)-component of the electric field.

Now we turn to analytically calculate the optical forces in the far-zone. To this end we need the angular correlation tensor at the plane of the source. This has been already calculated, thus the trace of the angular correlation tensor reads as [14]

\[
\text{Tr} A_{ij}(ks_{s,i}, ks_{s,j}) \propto A_{xx}(ks_{s,i}, ks_{s,j}) = \frac{A}{(4\pi)^2(a^2 - b^2)} e^{-(\alpha k^2 s^2_{s,i} + \alpha k^2 s^2_{s,j} - 2\beta s_{s,i}s_{s,j})},
\]

where \( a = 1/(4\sigma^2_s) + 1/(2\sigma^2_g), b = 1/(2\sigma^2_g), \alpha = a/4(a^2 - b^2) \) and \( \beta = b/4(a^2 - b^2) \).

One of the most important characteristics of these GSMS sources is that the behavior of the emitted field can be beam-like. To ensure this in the far-zone, the following necessary and sufficient conditions have to be fulfilled [14]

\[
\frac{1}{(2\sigma_s^2)} + \frac{1}{\sigma_g^2} \ll \frac{2\pi^2}{\lambda^2}.
\]

Next, in order to obtain the force in SI units, we redefine the parameter \( A \) as \( A/\varepsilon_0 c \), where \( A \) is the peak intensity of the source in \( W/m^2 \). Substituting Eq. (8) into Eqs. (3)-(4), approximating \( s_z \simeq 1 - 1/2s^2_z \), and after a long tedious but straightforward calculation, one derives the different components of the force. Then, performing the \( s_\perp \) and \( s'_\perp \) integrations, the conservative components finally are

\[
F_{x,y}^{\text{cons}} = -\text{Re} \alpha A \frac{1}{\varepsilon_0 c} \frac{1}{4\sigma_s^2 \Delta(z)^4} e^{-\frac{2\sigma_s^2}{(2\sigma_s \Delta(z))^2}} (x, y)
\]

and

\[
F_z^{\text{cons}} = \text{Re} \alpha A \frac{A_z}{4k^2 \sigma_s^4 \delta^2 \Delta(z)^2} \frac{1}{\varepsilon_0 c} (\rho^2 - 2\sigma_s^2 \Delta(z)^2) \frac{e^{-\frac{2\rho^2}{(2\sigma_s \Delta(z))^2}}}{(x, y)}
\]

On the other hand the non-conservative forces read

\[
F_{x,y}^{\text{nc}} = \text{Im} \alpha A \frac{1}{\varepsilon_0 c} \frac{1}{2k^2 \sigma_s^2 \delta^2 \Delta(z)^2} \frac{1}{\varepsilon_0 c} (x, y)
\]

and

\[
F_z^{\text{nc}} = \text{Im} \alpha A \frac{A_z}{4k^2 \sigma_s^4 \delta^2 \Delta(z)^2} \frac{1}{\varepsilon_0 c} \left[ \frac{1}{2} k^4 \sigma_s^4 \delta^2 \Delta(z)^2 - a k^6 \sigma_s^2 \delta^2 \Delta(z)^2 \right] + \left( \frac{k^4}{2\delta^4} - \frac{k^2}{4z^2} \right) \frac{e^{-\frac{2\rho^2}{(2\sigma_s \Delta(z))^2}}}{(x, y)}
\]

where \( 1/\delta^2 = 1/(2\sigma_s) + 1/\sigma_g^2 \) and \( \Delta(z) = [1 + (z/k\sigma_\delta)^2]^{1/2} \).

Eqs. (10)-(13) express the force exerted on a dipolar particle in the far zone by the field emitted from a GSMS of any state of coherence.

Fig. 1. (Color online). Spectral density (left) and spectral degree of coherence (right) at \( z = 0 \) for different source parameters which generate the same radiant intensity in the far-zone.

Now we address the ET for GSMS [16], and its experimental confirmation [24]. This theorem establishes that any GSMS will generate the same radiant intensity \( J(\theta) = r^2 \text{Tr} W_{ij}(r, r, \omega), (\theta = \rho/z), \) as a laser whose spectral density at the plane \( z = 0 \) is \( S_i(\rho, \omega) = A_i \exp[-\rho^2/(2\sigma_\rho)] \), if the following conditions are fulfilled

\[
\frac{1}{\sigma_g^2} + \frac{1}{(2\sigma_s^2) = \frac{1}{(2\sigma_\rho)^2}, A = \left( \frac{\sigma_g}{\sigma_s} \right)^2 A_i.
\]

Fig. (1) shows the spectral density and the spectral degree of coherence for the same parameters as in Ref. [16].
perform the calculations, we consider a dipolar latex-

forces from such a partially coherent light, it is natural

same radiant intensity. Then, in the context of optical

appropriately spatially coherent source, like a laser beam, as the

partially coherent Gaussian-Schell model source. As seen on comparing Figs. 1 and 2, a source with small

parameters, one can minimize the optical peak intensity

trapping constitutes a new principle for optical nanomanip-

fluctuating sources and a high power laser beam, (or in
general between different GSMS), as regards the depth
and width of the potential well created by the photonic
trap, constitutes a new principle for optical nanomanip-

In biophysical experiments, where there is sample dam-
age produced by high values of the power intensity of
the incident beam, this issue acquires vital importance,
this equivalence of low peak intensity partially coherent
fluctuating sources and a high power laser beam, (or in
general between different GSMS), as regards the depth
and width of the potential well created by the photonic
trap, constitutes a new principle for optical nanomanip-

The potential well is usually improved on focusing the
emitted light through a thin lens or ABCD system [26].
We have checked this phenomenon when the light is fo-
cused by a thin lens, and we get results with the partially
coherent source which are quite similar to those obtained
with the coherent beam.

In summary, we have studied the different components
of the optical force from a Gaussian Schell model source.
it has been demonstrated that in the far-zone such a par-
tially coherent source can produce a force equal to that
exerted by a laser beam; such a force being larger as the
beam-condition is more strongly fulfilled. In opposition
to this, in the near-field the maximum force corresponds
to a minimum force in the far-field. We believe that these
results should trigger a renewed interest, as well as ex-
periments, in optical manipulation.

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