A simplified algorithm for 3D mesh model considering the influence of edge features

Rumeng Lv¹, Xiaobing Chen¹,²* and Bingying Zhang³

¹ Huaiyin Institute of Technology, Jiangsu Province, China
² Huaiyin Institute of Technology, Jiangsu Province, China
³ Huaiyin Institute of Technology, Jiangsu Province, China
*Corresponding author’s e-mail: chenxiaobing@hyit.edu.cn

Abstract. Aiming at the problem that most of the existing grid simplification algorithms for 3D models can not deal with a large number of boundary or non-popular grid models, this paper proposes a grid simplification algorithm for 3D models based on traditional algorithms. The algorithm mainly studies the geometric features of the model, considering the calculation methods and characteristics of edge shrinkage, and introduces the edge feature factors on the basis of the traditional algorithm, that is, the triangular area and side length factors of local area are introduced in the calculation of folding cost. In addition, the gaussian curvature characteristics of the 3D model are also included. Experimental results show that the proposed algorithm can keep the detail features of the mesh model well, and greatly reflect the quality and effect of mesh simplification after simplification.

1. Introduction

In the process of 3D mesh model simplification, under certain constraints, relatively detailed mesh model is sometimes unnecessary to be generated in specific scenes. In order to improve the efficiency of mesh model loading and reduce the simplified mesh model of vertices and faces, a model with fewer faces is used to replace the original one. In recent years, many mesh simplification algorithms are mostly based on the more important algorithm -- edge folding algorithm, and most mesh simplification algorithms are derived and developed based on edge folding algorithm.

In order to simplify the mesh, the algorithm takes the square sum of the distance between the new vertex and the associated plane of the two vertices to be folded as the quadratic error measure, so as to generate the simplified mesh model. The edge folding algorithm proposed by Garland and Heckbert[1][2] in 1997 and 1999 has good quality after simplification. However, the characteristics of model key are not clarified. Zhang Shixue[3] et al. classified vertices and edges of the 3D model and allocated different coefficients, which effectively reduced the generation of long and narrow triangles, but reduced the efficiency of grid simplification. Meng Jun[4] proposed a fast mesh simplification algorithm, which used vertex weights to indicate the importance of vertices and used priority queues storing vertex weights to control the order of edge contraction. Compared with Zhang Shixue's algorithm, the simplification speed of the algorithm was improved, but the feature problems of the model were ignored. Therefore, Li Long[5] et al. introduced absolute curvature of the vertices of the grid model when calculating the folding cost, and the algorithm can effectively preserve the geometric features of the model, but the simplification quality of the algorithm needs to be improved. Hou[6] et al. redesigned the weight calculation method of QEM algorithm, and introduced the square of the maximum deviation found in the triangle adjacent to
the vertex as the importance of the vertex into the error measure. The contour information of the model
generated by the algorithm was well preserved, but the details of the model were not obvious. Wang
Jun[7] introduced the concept of approximate measure of edge curvature, and the algorithm ran efficiently
and effectively retained the details of the grid, but the simplified grid model was of low quality.

On the theoretical basis of the above research, aiming at the problems of insufficient detail feature
retention and insufficient grid optimization after simplification, this paper proposes a grid simplification
algorithm, which utilizes the influence of geometric features of the model on quadratic error to simplify
relatively flat regions. At the same time, the detailed parts and key feature areas in the model are better
reserved. Experimental results show that the proposed method retains more features of the model than
previous methods.

2. Measure of quadratic error

Quadratic error measurement is proposed by Garland and Heckbert. This algorithm is mainly divided
into two parts: one is initialization of grid vertices, that is, assigning a quadratic error matrix \( Q \) to each
vertex in the grid; the other is the iterative process of opposite edge contraction operation. Using the
sum of squares of the distance between the new vertex generated by edge folding operation and the
associated plane of the two vertices of the folded edge as the error measure, the folding cost of the model
is calculated as \( \Delta v = v^T Q v \). For vertices in three dimensions \( v = [v_x, v_y, v_z, 1]^T \), the set of planes associated
with it is \( \text{plane}(v) \). Let the plane equation \( P \) in three-dimensional space be \( ax + by + cz + d = 0 \). Where
\( a^2 + b^2 + c^2 = 1 \), \( d \) is a constant. This gives us the distance between our new vertex \( v' \) and the plane \( P \)

\[
d^2 = (p^T v')^2 = v^T (pp^T) v = v^T K_p v \tag{1}
\]

Where \( K_p \) is a 4*4 symmetric matrix:

\[
K_p = pp^T = \begin{bmatrix}
a^2 & ab & ac & ad \\
ab & b^2 & bc & bd \\
ac & bc & c^2 & cd \\
ad & bd & cd & d^2
\end{bmatrix}
\tag{2}
\]

Let \( Q(v) = \sum_{p \in \text{plane}(v)} K_p \) be the quadratic error matrix of vertex \( v \), in the edge folding operation, the
folding cost is \( \Delta(v') = v^T (Q + Q) v \). use \( Q + Q \) to represent the quadratic error matrix of new vertex \( v' \),
where \( \Delta(v') \) is the error cost of new vertex \( v \).

3. Simplified algorithm description

The mesh density of the simplified model generated by QEM algorithm is relatively uniform, and the
feature retention effect of the large-scale simplified model generated is general, which cannot be well
applied in the field with obvious model details. Therefore, this paper considers factors affecting model
features to achieve the purpose of feature retention.

3.1. The area of a triangle's local area

For example, the literature [9] takes edge curvature as a new quadratic error measure, which can improve
the simplification quality of the model, but cannot well reflect the detailed features of the model.
Therefore, the shape of the triangle in the grid model is determined by using the average area of the local
area of the vertex. The larger the average area of the local area of the vertex is, the flatter the feature of
the local area will be; otherwise, the triangle shape around the vertex is more dense. In this paper, the
local average area of vertex triangle is used to measure the distribution of mesh density in the model, so
as to reflect the change of local surface.

There are many ways to calculate the triangular area, such as the cross product of triangular vectors
or the formula of triangular apex points is \( S(Tr) \). In this paper, the coordinates of three vertices of a
triangle are set by using the first calculation method are \(v_0 \left( x_0, y_0, z_0 \right), v_1 \left( x_1, y_1, z_1 \right), v_2 \left( x_2, y_2, z_2 \right)\). Let the product of the faces of the triangle be \(S(Tr) = \frac{1}{2} |v_0 v_2 \times v_0 v_1|\). Considering the influence of the area size of the adjacent triangle at a point in the local area and the mesh density on the model feature loss, the average area of the adjacent triangle at vertex IV is calculated as follows:

\[
\sum_{i=1}^{n} S(Tr_i) = \overline{S}(v_i) \tag{3}
\]

Where \(n\) is the number of adjacent triangles of vertex \(v_i\) and \(S(Tr)\) is the area of the \(i\)-th triangle in the adjacent triangles of vertex \(v_i\).

3.2. side length

On vertex characteristics were measured after the local area, single use vertex local average size will remain simplified model part of the detailed features, but only triangular mesh morphological characteristics described in the model, will lead to a decline in the quality of grid model, they may also make the uneven distribution of grid, makes the model simplified general effect. In addition, in the calculation of Angle method, if the internal angles of a triangle are equal, the shape of the triangle tends to 1. If the inner Angle of a triangle is close to 0, the triangle shape ranges from 0 to 1, but the calculation is relatively complicated. Based on the above considerations, this paper chooses to calculate the length of the connection between two vertices in the grid model, and the length of side length determines the importance of model features.

The Euclidean distance method is used to calculate the length of the connection between the two vertices in the model. The folded edge length between edges \(v_i, v_j\) is calculated as follows:

\[
l(v_i, v_j) = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2} \tag{4}
\]

Where the coordinates of \(v_i\) and \(v_j\) are \(v_i = (x_i, y_i, z_i)\) and \(v_j = (x_j, y_j, z_j)\).

3.3. Gaussian curvature

Literature [1] proposed a second error algorithm in surface has a good effect on simplified, but there is no good to keep the details of the simplified model, this is because the models in the process of simplifying the details are ignored, and can reflect the characteristics of the model surface curvature, to smooth the degree of curvature of curve, the greater the curvature, the greater the degree of curvature of curve. Moreover, the surface of the grid model is a second-order non-differentiable surface, which cannot be calculated by the traditional curvature calculation method. Therefore, gaussian curvature of the model surface is adopted to increase the calculation cost, so that the edges of the non-boundary vertices are processed later.

The vertex curvature is calculated by the sum of the angles and areas of triangles in the grid. Let \(\alpha_i\) be the Angle of the \(i\)-th triangle in the vertex neighborhood, and approximate by gauss-Bonnet theorem:

\[
\oint_{\partial D} GdS + \sum_{i=1}^{n} \alpha_i = 2\pi \tag{5}
\]

The Gaussian curvature \(G\) is solved by formula (6). Assuming that \(G\) is constant in the local region of the vertex, it can be approximated as the surrounding center of gravity region, where \(D\) is the sum of angles in the local neighborhood of the vertex, \(DSN\) is the area element of the surface, and \(D\) is the area of the \(i\)-th triangular surface in the neighborhood \(n\) of the vertex center of gravity, as shown in Figure 1:
3.4. Error measure and new point position

The purpose of this paper is to reduce the error between the original model and the simplified model and retain the details of the model under the given simplification rate. Since the calculation of error measure and the position of new points are both important factors affecting the simplification effect of the model, the simplified error measure method has a large amount of calculation of geometric measure, so it is not suitable for the grid simplification application with high efficiency requirements, so this paper calculates a new error measure based on the quadratic error measure method of Garland.

In the previous algorithm, the second error calculation is fold after vertex peace in the distance, and price for folding, in this study, the introduction of local regional average size, length and gaussian curvature calculation, the triangle vertex area, the smaller the average area of $\overline{S_v}$ vertex gaussian curvature $G_v$ change more intense, thus establishing relation, Denoted by $\phi$, the boundary treatment and vertex characteristic factors of the model are considered when calculating the error, and the distance between the vertex and the relevant plane is calculated. The following formula is used to calculate the quadratic error:

$$
\phi = l(v_i, v_j) G(v_i, v_j) (\overline{S_v} + \overline{S_v})^{-1}
$$

Add $\phi$ as a weight to the edge cost calculation. When the edge $(v_i, v_j)$ shrinks to the new vertex $v = [x \ y \ z]^T$, let $G(v_i, v_j) = |G_v| + |G_v|$, and map $G(v_i, v_j)$ to $[0, 1]$, the total error matrix of the contracted edge is as follows:

$$
Q(v) = l(v_i, v_j) G(v_i, v_j) (\overline{S_v} + \overline{S_v})^{-1} (Q(v_i) + Q(v_j))
$$

In 3d mesh model, QEM algorithm is generally used to determine the position of new vertices on triangular edges of mesh. In this paper, mesh simplification is carried out based on edge folding method, and the schematic diagram of boundary vertices is shown in Figure 2:
For the edge shrink operation \((v_i, v_j)\rightarrow v\), the position of the new vertex \(v\) needs to be determined. It is known from Section 3.1 that the edge contraction cost function equation is a quadric surface equation. The new vertex position can be obtained by using the reversibility of the obtained matrix. That is, during each edge contraction, the error is derived and minimized, so that the calculated mesh is more likely to be close to the original mesh model. As a quadratic error, the optimal position of the new vertex \(x, y\) and \(Z\) coefficients can be easily found through solution.

In this paper, the model grid vertices are classified, namely in the border of vertex to the border, the border edges on the vertex is a border, the border edge is only one adjacent surface, while the other side there are two adjacent surface, grid vertex classification, and adopt different methods to determine the location of the new vertex \(v\), can save computation efficiency, into and keep the model boundary, The non-boundary edge is treated as follows:

If the matrix is irreversible, then \(v = [x \ y \ z]^{T}\) is the new vertex of the folded edge; If the matrix \(r\) is reversible, since the last line of the \(r\) matrix is empty, the unique solution of the homogeneous vector \(v\) is obtained:

\[
\begin{bmatrix}
q_{11} & q_{12} & q_{13} & q_{14} \\
q_{21} & q_{22} & q_{23} & q_{24} \\
q_{31} & q_{32} & q_{33} & q_{34} \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \\
0 \\
1 \end{bmatrix}
\]

(8)

The traditional quadratic error measurement method based on feature degree weighting does not take into account the boundary problem of the model. In this paper, the boundary vertices or edges are not simplified, but reserved and restricted. However, if the simplified folding operation of boundary edge is restricted blindly, the simplification can only simplify other edges outside the boundary edge, but the effect of model simplification will be reduced. The key to solve the boundary edge problem is to set an appropriate weight for the boundary edge to increase the folding error of the edge, so as to preserve the boundary characteristics of the grid.

3.5. Algorithm steps
Based on the QEM algorithm, this algorithm introduces the local area, side length and Gaussian curvature of the triangle. According to the relationship between the three, the error cost function is obtained, and the new vertex positions are calculated for different feature vertices of the mesh model, which effectively deals with the problem of preserving the model boundary edges and features in the process of mesh simplification. The algorithm execution process is as follows:

- The user input iteration times and simplified targets were read, topological relationships of vertices in the grid were established, and different vertices in the model were classified, including boundary vertices.
- The quadratic error matrix, neighborhood triangular mean area and gaussian curvature of each vertex are calculated.
- The edge length, edge shrinkage error matrix, position of new vertices and edge shrinkage error of each edge are calculated.
- Sort the calculated edge shrinkage error.
- Each time the smallest error edge is selected from the edge shrinking error to perform the edge shrinking operation.
- The feature factors affected by edge contraction are updated, such as vertex position and edge contraction error.
- If the simplification conditions are not met, skip to Step 6; otherwise, the simplification ends.

4. Experimental results and analysis
In order to evaluate the effectiveness of the method, we will prepare several simplified models for verification, which are mainly compared with Garland method, triangular area weighting and literature [5]. Microsoft Visual Studio 2017 platform is mainly used in computers with Windows 10(64-bit) system,
Intel(R) Core(TM) I5-9300H CPU @ 2.40ghz processor and 8G RAM. OpenGL rendering engine is used to render and display the results.

Using the proposed method in this paper to the simplification of 3 d model, and under the condition of the same simplified rate of ratio, in figure 3, the original is given for 3 k simplify the 3 d model to 1 k, in the fish's eye and tail with the local characteristics of the present a good details, and the 3 d model is not obvious characteristics of the local grid density is small. In the case of high simplification rate, the important features of the model are maintained, as shown in the figure below:

![Figure 3. 90% effect of fish model simplification](image)

(a) Original model     (b) literature [1] method
(c) document [5]method      (d) Paper method

In figure 4, the original simplified to 1 k to 2 k of 3 d model, and use different methods to simplify to 90%, in the cow's stomach, head, etc with the local characteristics of the present a good details, and the 3 d model rate larger piece of characteristics, performance than other methods to keep the method presented in this paper the important features of the model, the diagram below:

![Figure 4. 90% effect of cow model simplification](image)

(a) Original model                             (b) literature [1] method
According to the experimental results, in the case of unweighted 3D model with high simplification rate, there are more inferior triangular shapes, and the details of the model cannot be maintained. In addition, by considering the edge feature factors, the visual effect of the model is increased, the overall feature retention is improved, and more details of the model are retained.

5. Conclusion
In this paper, a new mesh simplification algorithm based on QEM simplification algorithm is proposed, which weights edge feature factors to folding cost. Experiments show that the folding cost can not only maintain the overall feature variation of the triangular mesh model, but also retain more details, thus improving the overall visual effect of the triangular mesh model. However, the simplification effect of the algorithm is not obvious for areas with low sharpness. Therefore, more factors affecting the simplification effect are considered in the following work to improve the universality of the application of the model.

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