Thin-shell mixing in radiative wind-shocks and the
$L_X \sim L_{bol}$ scaling of O-star X-rays

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ABSTRACT

X-ray satellites since Einstein have empirically established that the X-ray luminosity from single O-stars scales linearly with bolometric luminosity, $L_X \sim 10^{-7} L_{bol}$. But straightforward forms of the most favored model, in which X-rays arise from instability-generated shocks embedded in the stellar wind, predict a steeper scaling, either with mass loss rate $L_X \sim \dot{M} \sim L_{bol}^{1.7}$ if the shocks are radiative, or with $L_X \sim \dot{M}^2 \sim L_{bol}^{3.4}$ if they are adiabatic. This paper presents a generalized formalism that bridges these radiative vs. adiabatic limits in terms of the ratio of the shock cooling length to the local radius. Noting that the thin-shell instability of radiative shocks should lead to extensive mixing of hot and cool material, we propose that the associated softening and weakening of the X-ray emission can be parameterized as scaling with the cooling length ratio raised to a power $m$, the “mixing exponent.” For physically reasonable values $m \approx 0.4$, this leads to an X-ray luminosity $L_X \sim \dot{M}^{0.6} \sim L_{bol}$ that matches the empirical scaling. To fit observed X-ray line profiles, we find such radiative-shock-mixing models require the number of shocks to drop sharply above the initial shock onset radius. This in turn implies that the X-ray luminosity should saturate and even decrease for optically thick winds with very high mass-loss rates. In the opposite limit of adiabatic shocks in low-density winds (e.g., from B-stars), the X-ray luminosity should drop steeply with $\dot{M}^2$. Future numerical simulation studies will be needed to test the general thin-shell mixing ansatz for X-ray emission.

Key words: shock waves – stars: early-type – stars: winds – stars: mass loss – X-rays: stars

1 INTRODUCTION

Since the 1970’s X-ray satellite missions like Einstein, ROSAT, and most recently Chandra and XMM-Newton have found hot, luminous, O-type stars to be sources of soft ($\lesssim 1$ keV) X-rays, with a roughly linear scaling between the X-ray luminosity and the stellar bolometric luminosity, $L_X \sim 10^{-7} L_{bol}$. But straightforward forms of the most favored model, in which X-rays arise from instability-generated shocks embedded in the stellar wind, predict a steeper scaling, either with mass loss rate $L_X \sim \dot{M} \sim L_{bol}^{1.7}$ if the shocks are radiative, or with $L_X \sim \dot{M}^2 \sim L_{bol}^{3.4}$ if they are adiabatic. This paper presents a generalized formalism that bridges these radiative vs. adiabatic limits in terms of the ratio of the shock cooling length to the local radius. Noting that the thin-shell instability of radiative shocks should lead to extensive mixing of hot and cool material, we propose that the associated softening and weakening of the X-ray emission can be parameterized as scaling with the cooling length ratio raised to a power $m$, the “mixing exponent.” For physically reasonable values $m \approx 0.4$, this leads to an X-ray luminosity $L_X \sim \dot{M}^{0.6} \sim L_{bol}$ that matches the empirical scaling. To fit observed X-ray line profiles, we find such radiative-shock-mixing models require the number of shocks to drop sharply above the initial shock onset radius. This in turn implies that the X-ray luminosity should saturate and even decrease for optically thick winds with very high mass-loss rates. In the opposite limit of adiabatic shocks in low-density winds (e.g., from B-stars), the X-ray luminosity should drop steeply with $\dot{M}^2$. Future numerical simulation studies will be needed to test the general thin-shell mixing ansatz for X-ray emission.

Key words: shock waves – stars: early-type – stars: winds – stars: mass loss – X-rays: stars

Wade 2011). But in putatively single, non-magnetic O-stars, the most favored model is that the X-rays are emitted from embedded wind shocks that form from the strong, intrinsic instability (the “line-deshadowing instability” or LDI) associated with the driving of these winds by line-scattering of the star’s radiative flux (Owocki et al. 1988, Feldmeier et al. 1997, Dessart & Owocki 2003, Sundqvist & Owocki 2012).

This LDI can be simply viewed as causing some small ($\lesssim 10^{-3}$) fraction of the wind material to pass through an X-ray emitting shock, implying in the case that the full shock energy is suddenly radiated away that the X-ray luminosity should scale with the wind mass loss rate, $L_X \sim \dot{M}$. But within the standard Castor et al. 1975, hereafter CAK) model for such radiatively driven stellar winds, this mass loss...
with mass; see §2.4) that this filling factor should actually do indeed do indeed lead to an X-ray luminosity that scales linearly with bolometric luminosity, $L_x \sim L_{bol}$.

The analysis here examines instead the role of radiative cooling and, associated thin-shell instabilities (Vishniac 1994; Walder & Folini 1998; Schure et al. 2007; Parkin & Pittard 2010), in mixing shock-heated material with cooler gas, and thereby reducing and softening the overall X-ray emission. As summarized in figure 1, for a simple parameterization that this mixing reduction scales with a power (the “mixing exponent” $m$) of the cooling length, $\ell^m$, we find that the linear $L_x - L_{bol}$ law can be reproduced by assuming $m \approx 0.4$. The development below quantifies and extends a preliminary conference presentation of this thin-shell mixing ansatz (Owocki et al. 2011).

Specifically, to provide a quantitative basis for bridging the transition between radiative and adiabatic shock cooling, the next section (§2) first analyzes in detail the X-ray emission from a simplified model of a single, standing shock in steady, spherically expanding outflow. The following section (§3) then generalizes the resulting simple bridging law to account for thin-shell mixing, and applies this in a simple exospheric model to derive general scalings for X-ray luminosity from a wind with multiple, instability-generated shocks assumed to have a power-law number distribution in wind radius. A further application to computation of X-ray line profiles (§4) provides constraints on the mixing and shock-number-number-number-number exponents needed to match observed X-ray luminosity.

rate increases with luminosity\(^2\) as $\dot{M} \sim L_{bol}^{1/\alpha} \sim L_{bol}^{1.7}$, where the latter scaling uses a typical CAK power index $\alpha \approx 0.6$ (Puls et al. 2000). This then implies a super-linear scaling for X-ray to bolometric luminosity, $L_x \sim L_{bol}^{3.4}$, which is too steep to match the observed, roughly linear law.

In fact, the above sudden-emission scaling effectively assumes the shocks are radiative, with a cooling length that is much smaller than the local radius, $\ell \ll r$. In the opposite limit $\ell \gg r$, applicable to lower-density winds for which shocks cool by adiabatic expansion, the shock emission scales with the X-ray emission measure, $EM \sim \int \rho^2 dV$, leading then to an even steeper scaling of X-ray vs. bolometric luminosity, $L_x \sim \dot{M}^2 \sim L_{bol}^{3.4}$.

Both these scalings ignore the effect of bound-free absorption of X-rays by the cool, unshocked material that represents the bulk of the stellar wind. Owocki & Cohen (1994) (hereafter OC99) showed that accounting for wind absorption can lead to an observed X-ray luminosity that scales linearly with $L_{bol}$, but this requires specifying ad hoc a fixed radial decline in the volume filling factor for X-ray emitting gas. We show below (§2) that this filling factor should actually be strongly affected by the level of radiative cooling. Moreover, while modern observations of spectrally resolved X-ray emission profiles by Chandra and XMM-Newton do indeed show the expected broadening from shocks embedded in the expanding stellar wind, the relatively weak blue-red asymmetry indicates that absorption effects are modest in even the densest OB-star winds (Cohen et al. 2010, 2011). Since many stars following the $L_x$-$L_{bol}$ empirical law have weaker winds that are largely optically thin to X-rays (Cohen et al. 1997, Nazé et al. 2011), it now seems clear that absorption cannot explain this broad $L_x$ scaling.

\(L_x \sim \dot{M}^{1/\alpha} \sim L_{bol}^{1.7}\) if $m \approx 1-\alpha \approx 0.4$.

\[L_x \sim k_x \dot{M} \Delta V^2\]
\[k_x \sim r/\ell \ll 1\]
\[L_x \sim \dot{M}^2 \sim L_{bol}^{3.4}\]
\[L_x \sim M \sim L_{bol}^{1.7}\]
\[L_x \sim \dot{M} \sim L_{bol}^{1-m} \sim L_{bol}^{(1-m)/\alpha}\]
\[L_x \sim L_{bol}^{(1-m)/\alpha} \sim L_{bol}^{1.7}\]
Where general scaling laws in §3, the final section (§6) concludes with a brief summary and outlook for future work.

2 STANDING SHOCK MODEL

2.1 Energy Balance

Most previous analyses of X-rays from massive stars (e.g., Wojdowski & Schulz 2003; Cohen et al. 2011) have been cast in terms of a density-squared emission measure from some fixed volume filling factor for X-ray emitting gas (see however Krolik & Raymond 1985; Hillier et al. 1993; Feldmeier et al. 1997; Antokhin et al. 2004). So let us begin by showing explicitly how that picture must be modified (see however Krolik & Raymond 1985; Hillier et al. 1993; Feldmeier et al. 1997; Antokhin et al. 2004). By focusing on the simple example of a steady, standing wind shock, it is possible to carry out an analytic analysis that derives, more or less from first principles, a simple bridging law between the scalings for adiabatic vs. radiative shocks.

Specifically, let us consider an idealized model in which a spherically symmetric, steady-state stellar wind with mass loss rate \( \dot{M} \) and constant, highly supersonic flow speed \( V_\infty \) undergoes a strong, standing shock at some fixed radius \( r = r_\ast \). Relative to an unshocked wind with density \( \rho_w = \dot{M}/4\pi V_\infty r^2 \), the post-shock flow at \( r > r_\ast \) has a density that is a factor \( \rho/\rho_w \) higher, with a post-shock speed that is a factor \( v/V_\infty = 1/4 \) lower. The reduction in flow kinetic energy results in a high, immediate post-shock temperature,

\[
T_s = \frac{3 \mu V_\infty^2}{16 k} = 14 \text{MK} \left( \frac{V_\infty}{1000 \text{km s}^{-1}} \right)^2,
\]

where \( k \) is Boltzmann’s constant, and the latter evaluation assumes a standard molecular weight \( \mu = 0.62 m_p \), with \( m_p \) the proton mass.

But following this sudden shock increase, the combined effects of adiabatic expansion and radiative cooling cause the flow temperature \( T \) to decrease outward. For pressure \( P = \rho k T/\mu \) and internal energy density \( e = (3/2) P \), the steady-state energy balance for a general vector velocity \( \mathbf{v} \) is

\[
\nabla \cdot (e \mathbf{v}) = -P \nabla \cdot \mathbf{v} - \rho \Lambda_m(T),
\]

where \( \Lambda_m \equiv \Lambda/\langle \mu_e \mu_p \rangle \), with \( \Lambda(T) \) the optically thin cooling function (e.g., Smith et al. 2001), and \( \mu_e \) and \( \mu_p \) respectively the mean mass per electron and per proton.

In general, we should also include a detailed momentum equation to account for possible acceleration of the post-shock flow, for example from the inward pull of stellar gravity, or the outward push of the gas pressure gradient. But the analysis here is greatly simplified if we make the reasonable assumption that these two countervailing accelerations roughly cancel, and so give a constant speed \( v = V_\infty/4 \) for all \( r > r_\ast \). For this case of a steady, spherical, constant-speed post-shock outflow, the vector energy equation (2) reduces to a simple differential equation for the decline in temperature with radius \( r \),

\[
\frac{dT}{dr} = \frac{4}{3} \frac{T}{r} - \frac{2 \mu}{3 k v} \rho \Lambda_m(T),
\]

wherein the first and second terms on the right-hand-side respectively represent the effects of adiabatic expansion and radiative cooling. This can alternatively be cast in terms of a temperature scale length,

\[
\frac{1}{H_T} \equiv -\frac{1}{T} \frac{dT}{dr} = \frac{4}{3 r} + \frac{\kappa_c \rho}{3},
\]

where

\[
\kappa_c(T) \equiv \frac{8 \mu \Lambda_m(T)}{k T V_\infty}
\]

is a mass cooling coefficient (with CGS units \( \text{cm}^2 \text{g}^{-1} \)), defined as the inverse of a characteristic cooling column mass. The corresponding cooling length is given by \( \ell = 4/\kappa_c \rho \), defined such that the radiative and adiabatic cooling terms are equal \( \ell = r \).

2.2 X-ray luminosity

For any local post-shock temperature \( T \), let \( f_x(T) \) represent the fraction of radiation emitted in an X-ray bandpass of interest. Neglecting for now any wind absorption, the total X-ray luminosity from this single standing shock is then given by radial integration of the associated X-ray emission,

\[
L_{xs} = 4\pi \int_{r_\ast}^{\infty} \rho^2 \Lambda_m(T) f_x(T) r^2 dr.
\]

Using (1), this can be recast as an integral over temperature,

\[
L_{xs} = 12\pi \int_0^{T_s} T \frac{\rho^2 \Lambda_m(T)}{4 + \kappa_c(T) \rho} f_x(T) \frac{dT}{T},
\]

\[
\approx 48\pi r_\ast^3 \rho_{\infty}^2 \Lambda_m(T_s) \frac{f_x(T_s) \delta T_s}{2T_s},
\]

where the latter approximation\(^4\) uses single-point trapezoidal integration, assuming that \( f_x \) declines from its post-shock value \( f_x(T_s) \) to zero over a temperature range \( \delta T_s \) from the initial post-shock temperature \( T_s = T(r_\ast) \). Here \( \rho_{\infty} \equiv \dot{M}/(4\pi V_\infty r_\ast^2) \) is the wind density just before the shock,

\[^4\text{ The ratio } \ell/r \text{ differs only by an order-unity factor from the ratio of cooling to escape time, } \chi \equiv t_{\text{cool}}/t_{\text{esc}}, \text{ defined by Stevens et al. (1992) to characterize the transition from radiative to adiabatic shocks in colliding stellar winds.}\]

\[^5\text{ Aside from } f_x(T), \text{ the combination of other factors in the integrand for } \frac{dT}{T} \text{ becomes constant in } T \text{ in the radiative limit } \kappa_c \rho > 1, \text{ and scales as } T^{-3/4} \text{ in the opposite, adiabatic limit. For } f_x(T) \text{ that declines roughly linearly with temperature, the approximate trapezoidal integration } \frac{dT}{T} \text{ thus becomes nearly exact in the radiative limit, while mildly underestimating the actual value in the adiabatic limit, for example by about 15% for } \delta T_s = T_s/2.\]
and

\[ \kappa_{cs} \equiv \kappa_c(T_s) = \frac{128 \Lambda_m(T_s)}{3 \sqrt{2}} \]

(8a)

\[ = 2 \sqrt{3} \Lambda_m(T_s) \left( \frac{kT}{\mu} \right)^{-3/2} \]

(8b)

\[ \approx 1000 \frac{\text{cm}^2}{g} T_s^{-2} \approx 750 \frac{\text{cm}^2}{g} T_s^{-2} \text{cm}^2, \]  

(8c)

with \( T_\gamma \equiv T_s/10^7 \text{K} \) and \( T_{\text{kev}} \equiv kT_s/\text{keV} \). The numerical evaluation in \( \kappa_{cs} \) assumes an approximate fit to the cooling function, \( \Lambda(T_s) \approx 4.4 \times 10^{-31}/\sqrt{T_\gamma} \) erg cm\(^2\) s\(^{-1}\), over the relevant range of shock temperatures, \( 10^{6.5} K < T_s < 10^{7.5} K \) (Schure et al. 2009). Recall that the shock cooling length is set by \( \ell_s = 1/\kappa_{cs} \rho_{vs} \).

For context, the mass absorption coefficient for bound-free absorption of X-rays when smoothed over bound-free edges, also roughly follows an inverse-square scaling with energy. Over the relevant energy range 0.5–2 keV, we can use the the opacity curves of, e.g., Cohen et al. (2010), Leutenegger et al. (2010), or Hervé et al. (2012) to write an approximate scaling form,

\[ \kappa_{bf} \approx 30 \frac{\text{cm}^2}{g} E_{\text{keV}}^{-2}, \]

(9)

where \( E_{\text{keV}} \) is now the X-ray photon energy in keV.

By casting the cooling strength in an opacity form normally used to describe absorption, we are thus able to make direct comparisons between cooling and absorption, and so characterize their respective domains of importance. This is further facilitated by the quite fortunate coincidence that both have similar inverse-square scalings with their associated energy.

Since these respective energies are usually roughly comparable, \( E_{\text{keV}} \approx T_{\text{kev}} \), the fact that the numerical factor for \( \kappa_{cs} \) is about 25 times greater than for \( \kappa_{bf} \) means that cooling can become important even in winds that are too low density to have significant absorption. As such, winds with adiabatic shocks are always optically thin, whereas shocks in optically thick winds are always radiative. Moreover, as detailed in \( \kappa_{cs} \) even in the bulk of O-star winds for which X-ray absorption is weak or marginal, the structure of X-ray emission associated wind shocks should be dominated by radiative cooling.

2.3 Bridging law between radiative and adiabatic limits

Noting that the pre-shock kinetic energy luminosity of the wind is

\[ L_w = 2 \pi r_s^2 \rho_{vs} V_\infty^3 = \frac{M V_\infty^2}{2}, \]

(10)

we can use \( \kappa_{cs} \) to eliminate the cooling function \( \Lambda_m \) from \( \kappa_{bf} \), recasting the X-ray luminosity scaling as a “bridging law” between the radiative and adiabatic shock limits,

\[ L_{xs} = f_{xs} \frac{9}{16} \frac{L_w}{1 + \ell_s/\ell_{rad}}, \]

(11)

where \( f_{xs} \equiv f_s(T_s) \delta T_s/2T_s \) is now a cooling-integrated shock X-ray fraction.

Note here that the combination of factors multiplying \( L_w \) on the right-hand-side of \( L_{xs} \) is just the kinetic energy conversion factor introduced in the summary figure

\[ k_x \equiv \frac{9}{16} \frac{f_{xs}}{1 + \ell_s/\ell_{rad}}. \]

(12)

For high-density, radiative shocks with \( \ell_s \ll r_s \),

\[ L_{xs, rad} = f_{xs} \frac{9}{16} L_w = f_{xs} \frac{9}{32} \frac{M V_\infty^2}{2}. \]

(13)

As a physical interpretation, \( 9/16 \) is just the fraction of wind kinetic energy that is converted to post-shock heat, which is radiated away before any losses to adiabatic expansion, with the fraction \( f_{xs} \) emitted in the X-ray bandpass of interest. Note that this scales linearly with density and thus mass loss rate, showing that a standard density-squared emission measure does not represent an appropriate scaling for emission from radiative shocks.

For lower-density, adiabatic shocks with \( \ell_s \gg r_s \), this X-ray emission is reduced by the ratio \( r_s/\ell_s \), giving the scalings,

\[ L_{xs, ad} = f_{xs} \frac{9}{16} \frac{r_s}{\ell_s} L_w \]

(14a)

\[ = f_{xs} \frac{9}{8} \frac{3}{\pi} V_\infty^3 \kappa_{cs} \rho_{vs}^2 \]

(14b)

\[ = f_{xs} \frac{48}{\pi} \Lambda_m(T_s) \rho_{vs}^2 \]

(14c)

\[ = f_{xs} \frac{3}{\pi} \frac{V_\infty^3}{V_\infty^3} \left( \frac{M}{V_\infty^2} \right)^2, \]

(14d)

which thus recovers the density-squared scaling for X-ray luminosity, showing that emission measure does provide an appropriate scaling for emission from adiabatic shocks.

Finally, note that the general bridging law \( L_{xs} \) can alternatively be written as a modification of either the radiative or adiabatic scaling,

\[ L_{xs} = \frac{L_{xm, rad}}{1 + \ell_s/\ell_{rad}} \]

(15a)

\[ = \frac{L_{xm, ad}}{1 + r_s/\ell_{rad}}. \]

(15b)

This shows that the X-ray luminosity is always limited to be somewhat below the smaller of the radiative or adiabatic luminosities, i.e. \( L_{xs} \approx \min(L_{xm, rad}, L_{xm, ad}) \).

2.4 Local X-ray emissivity

To facilitate application of these single standing-shock scalings to the more complex case of multiple embedded wind shocks generated from the line-deshadowing instability, we next recast these results in terms of the local X-ray emission from an individual shock. Since X-ray emission arises from collision of ions and electrons, it is common to write the X-ray emissivity (per unit volume and solid angle) as scaling with the square of the local density,

\[ \eta = C_s f_v \rho^2, \]

(16)

where \( C_s \) is a constant that depends on the shock strength and atomic physics, and \( f_v \) represents a local volume filling factor for shocked gas that is sufficiently hot to emit X-rays. If each individual shock has an associated filling factor \( f_{vs} \) proportional to its post-shock cooling length, then the total filling factor from an ensemble of shocks can be written,

\[ f_s(r) = f_{vs} \frac{dN_s}{d ln r} = f_{xs} n_s(r), \]

(17)
where \( N_s(r) \) is the cumulative number of shocks up to radius \( r \), and \( n_s \) measures the local differential number of new, emerging shocks. This formalism emphasizes the importance of the number of shocks and their spatial distribution; compared to the traditional emission measure approach, it should provide more physically motivated constraints for wind-shock X-ray production in massive stars.

In particular, the analysis below of multiple, instability-generated shocks assumes a power-law scaling for \( n_s \) (see eq. 25); but for the above single-shock model, this just takes the form of a Dirac delta-function, \( n_s = \delta(r - r_s) \). The associated X-ray luminosity is then given by integration of the emissivity \( \eta_s \) over solid angle and volume,

\[
L_x = 16\pi^2 \int C_s f_{vs} r \delta(r - r_s) \rho^2 r^2 \, dr .
\]

Upon trivial evaluation over the delta function, comparison with the scalings (14c) and (15b) yields the identifications,

\[
C_s = \frac{3}{\pi} \Lambda_{\infty}(T_s) f_{xx} \quad \text{and} \quad f_{vs} = \frac{1}{1 + r_a/n_s} .
\]

This thus now sets a bridging law at the level of an individual shock, with \( f_{xx} \) characterizing the fraction of single-shock emission measure that actually contributes to radiative emission. The adiabatic limit \( r_a \gg r_s \) gives \( f_{vs} \approx 1 \) and so a density-squared scaling for the emissivity (16), the radiative limit reduces this by the small factor \( r_a/r_s \ll 1 \), thus giving this emissivity a linear-density scaling set by the total kinetic energy flux through the shock.

Note that the factor \( C_s \) depends only on the shock strength (through the post-shock temperature \( T_s \)), while the shock filling factor \( f_{vs} \) depends on the local shock radius and cooling length, as set by the pre-shock wind density. Thus, although these scalings are derived from a simple model of a single, standing shock, they should also be generally applicable to more complex, moving shock structures, if specified in terms of the pre-shock wind density and the relative velocity jump that sets the shock strength and temperature.

### 3 X-RAYS FROM INSTABILITY-GENERATED EMBEDDED WIND SHOCKS

#### 3.1 Exospheric scaling for \( L_x \)

To model the X-ray emission from multiple, embedded wind shocks, let us next write the X-ray luminosity in a fully general, 3-D form that accounts for possible directional dependencies in emission and absorption,

\[
L_x = \int d^3r \int d\Omega \eta_s(r, n) e^{-\tau(r, n)} ,
\]

where the optical depth \( \tau(r, n) \) accounts for bound-free absorption of X-rays emitted at location \( r \) in the observer direction \( n \).

In principle, instability-generated, X-ray emitting shocks will be associated with a complex, 3D, stochastic wind structure (Dessart & Owocki 2003). But upon averaging over small scales, this can be reasonably well described by a globally spherical wind emission model (OC99). In modeling line emission, accounting for the observed Doppler shift from wind expansion still requires including a directional dependence of the emissivity and optical depth; see (4) and Owocki & Cohen (2001, hereafter OC01). But to derive the general scalings for the X-ray luminosity, one can again take the total X-ray emissivity to be an isotropic function of the local radius, \( \eta_s(r) \). Moreover, given the weak to isotropic importance of absorption in all but the densest winds, its overall role in scaling relations can be roughly taken into account through a simple exospheric approximation (OC99, Leutenegger et al. 2010), for which the integrations over solid angle and volume in (21) reduce to just a single integration in radius,

\[
L_x \approx 16\pi^2 \int_{R_1}^{\infty} \eta_s(r) r^2 \, dr.
\]

This integral lower bound is taken from the larger of the X-ray onset radius and the radius for unit radial optical depth, i.e. \( R_1 \equiv \max[R_a, R_s] \). For a wind with mass loss rate \( \dot{M} \) and flow speed \( V_\infty \), this radius for transition from optically thick to thin X-ray emission is given by

\[
R_1 \equiv \tau_s R_s = \frac{\kappa_{\text{opt}} \dot{M}}{4\pi V_\infty^2} \approx 25 R_\odot \frac{M_{-6}}{E_{\text{kev}} V_\infty^2} ,
\]

where \( \tau_s \) is a characteristic wind optical depth to the stellar surface radius \( R_s \), and the latter equalities use the scaling (4) for the bound-free X-ray opacity \( \kappa_{\text{opt}} \) with \( M_{-6} \equiv M/(10^{-6} M_\odot \text{yr}^{-1}) \) and \( V_\infty \equiv V_\infty/1000 \text{km s}^{-1} \).

In direct analogy, we can similarly define a characteristic adiabatic radius for transition from radiative to adiabatic cooling of the associated wind shocks,

\[
R_a \equiv \frac{\kappa_{\text{cs}} \dot{M}}{4\pi V_\infty^2} \approx 625 R_\odot \frac{M_{-6}}{T_{\text{kev}}^2 V_\infty} ,
\]

where \( T_{\text{kev}} = k T_s/\text{keV} \). For X-rays emitted with photon energy comparable to the shock energy, \( E_{\text{kev}} \approx T_{\text{kev}} \), we find \( R_a/R_1 \approx \kappa_{cs}/\kappa_{\text{opt}} \approx 25 \). As detailed below, this implies there can be extensive wind regions in which the density is high enough for shocks to be radiative, but too low for much wind absorption of their emitted X-rays. In particular, independent of the mass loss rate, any shocks formed near the radius \( R_1 \), where X-rays have near unit optical depth, should always be strongly radiative.

Figure 2 plots estimates of \( R_a/R_0 \) vs. O-star spectral subtype, for shocks with the temperature \( T_{\text{kev}} = 0.5 \), as expected for wind-shocks generated by the line-deshadowing instability (Runacres & Owocki 2002, Dessart & Owocki 2003), and estimated from the observed, relatively soft X-ray spectra (Wojdowski & Schulz 2005, Zhelov & Palli 2007, Nazé et al. 2012). The large \( R_a \) values confirm that wind-shocks of this energy should be radiative.

This plot can also be readily used to estimate expected unit optical depth radii. In particular, for an emission line at \( E_{\text{kev}} = 1 \), the corresponding \( R_1 \) would be a factor 100
smaller than the $R_a$ plotted in Figure 2. Since this means most O-stars would have $R_1/R_o < 1$, we see that absorption effects should be weak to moderate throughout the O-star domain, even while the shocks are generally radiative. See Figure 3 for further discussion and illustration of these scalings.

### 3.2 Bridging law with thin-shell mixing

Previous analyses using the density-squared emissivity (16) (e.g., OC99, OC01) have directly parameterized the X-ray filling factor $f_s$ as following some specified radial function, e.g., a power-law. But instead let us now parameterize the shock number distribution in (17) by an analogous power-law,

$$n_s(r) \equiv n_{so}\left(\frac{R_o}{r}\right)^{\beta} ; \quad r > R_o,$$

with $n_s = 0$ for $r < R_o$. Both instability simulations (Owocki & Puls 1999; Runacres & Owocki 2002; Dessart & Owocki 2003) and X-ray profile fitting and He-like $f/\ell$ ratios (Leutenegger et al. 2002; Cohen et al. 2003, 2011, 2014) suggest an initial onset for shock formation around $R_o \approx 1.5R_*.$

Applying (29) in (17), the analysis in (24) provides a more physical model of shock X-ray emission that accounts explicitly for the effects of radiative cooling. However, it still does not account for any thin-shell mixing. The inherent thinness ($\ell \ll r$) of radiative shock cooling zones makes them subject to various thin-shell instabilities (Vishniac 1994), which in numerical simulations lead to highly complex, turbulent shock structure (e.g., Walder & Folini 1998). Parkin & Pittard (2010) discuss how the inherently limited spatial resolution of radiative shock simulations leads to a “numerical conduction” that transports heat from high to low temperature gas, resulting in a severe, but difficult-to-quantify reduction in the X-ray emission.

While the specific mechanisms within hydrodynamical simulations may indeed depend on such numerical artifacts, the perspective advocated here is that such an overall reduction in X-ray emission is likely a natural consequence of the turbulent cascade induced by the thin-shell instability; this should lead to substantial physical mixing between cool and hot material, with the softer and more efficient radiation of the cooler gas effectively reducing the emission in the X-ray bandpass.

Pending further simulation studies to quantify such mixing and X-ray reduction, we make here the plausible ansatz that the reduction should, for shocks in the radiative limit $\ell/r \ll 1$, scale as some power $m$ of the cooling length ratio, $(\ell/r)^m$. To ensure that the mixing becomes ineffective in the adiabatic limit – for which the cooling layer is too extended to be subject to thin-shell instability – we recast the bridging law (28) for the shock volume filling factor $f_{vs}$ in the generalized form,

$$f_{vs} \approx \frac{1}{(1 + r/\ell)^{1+m}} = \frac{1}{(1 + \kappa_c\rho r)^{1+m}} ,$$

with the level of mixing now controlled by a positive value of the mixing exponent $m$. To simplify notation in the analysis to follow, we have dropped here the subscripts (“s” or “w”) for the quantities (e.g., $\ell$, $r$, $\kappa_c$, $\rho$) on the right-hand-side. Recalling that the cooling coefficient $\kappa_c$ takes the scalings given in equations (38)-(40), we can use the adiabatic radius $R_a$ from (24) to characterize the asymptotic regimes for (29).

If $R_a < R_o$, then the shocks are adiabatic throughout the wind, and we recover the standard density-squared emissivity (16) with a specified volume filling factor set by $f_s = n_s$.

If $R_a > R_o$, this adiabatic scaling still applies in the outer wind, $r > R_a$; but in the inner regions $R_o < r < R_a$, the cooling term in the denominator of (29) dominates, giving the emissivity (16) now a reduced dependence on density,

$$\eta_c \approx C_s n_{so} \rho^{-1-m} \frac{(\kappa_c r)^{1+m}}{R_o} ; \quad R_o < r < R_a .$$

Without mixing ($m = 0$), the density dependence is thus linear, but with mixing ($m > 0$), it becomes sub-linear.

As noted above, absorption is a modest effect in even dense O-star winds, with $\tau_a$ at most of order unity, implying then that $R_1 \approx R_a$ (Cohen et al. 2010). But the stronger coefficient $(\kappa_c/\kappa_{bd}) = R_a/R_1 \approx 25$ for radiative cooling means that such winds typically have $R_a > R_o$, implying that most shocks remain radiative well above the wind acceleration region where they are generated.

### 3.3 $L_x$ scalings with $M/V_\infty$

Let us now turn to the scalings for the overall X-ray luminosity. Applying the emissivity (16) and filling factor (28) to the exospheric model (22) with the power-law shock distribution (29), we find

$$L_x \approx 16\pi^2 C_p \int_{R_1}^{\infty} \frac{n_{so}^2}{(1 + \kappa_c\rho r)^{1+m}} \, r^2 \, dr = C_p \left[ \frac{M}{V_\infty} \right] \int_{R_1}^{\infty} \frac{dr}{[\rho r (\rho r)^{1-m}(\rho r + R_a)]^{1+m}} ,$$

where $C_p \equiv C_s n_{so} R_o^m$, and the latter equality uses mass conservation to cast the integral in terms of the scaled wind
velocity, \(w(r) \equiv v(r)/V_\infty\). This is generally taken to have a ‘beta-law’ form \(w(r) = (1 - R_s/r)^\beta\), with the canonical case \(\beta = 1\) giving \(rw = r - R_s\), and the constant-speed case \(\beta = 0\) giving \(rw = r\). For these special cases, \(\beta\) gives some results for full integrations of (25).

But even for a generic velocity law \(w(r)\), we can readily glean the essential scalings of the \(L_x\) from (25) in key asymptotic limits of adiabatic vs. radiative shocks.

### 3.3.1 Adiabatic shocks in optically thin wind

First for low-density winds with optically thin emission from adiabatic shocks, \(R_1 < R_a < R_o\), we can effectively drop the \(R_a\) term in the denominator, and set the lower bound of the integral to the fixed onset radius, \(R_i = R_o\), yielding

\[
L_x \approx C_p \left[ \frac{M}{V_\infty} \right]^2 \int_{R_o}^{\infty} \frac{dr}{w^2(r)^{\alpha+1}} ; \quad R_1 < R_a < R_o .
\]

Since the resulting integral then is just a fixed constant that is independent of \(M\), the X-ray luminosity recovers the standard adiabatic scaling \(L_x \sim (M/V_\infty)^2\).

### 3.3.2 Radiative shocks in optically thin or thick wind

For high-density winds with radiative shocks and so \(R_o < R_a < R_s\), the \(R_0\) now dominates its term, and so can be pulled outside the integral. Rescaling the remaining integrand in terms of the initial radius \(R_i\), we find

\[
L_x = C_p \left[ \frac{M}{V_\infty} \right]^2 \frac{R_i^{m-p}}{R_o^{m+1}} \int_1^{R_1} \frac{dr}{w^2(r)^{1-m}} ; \quad R_o < R_a ,
\]

where again the integral is now essentially independent of \(M/V_\infty\).

For the intermediate-density case in which the radiative shock emission is optically thin, \(R_1 < R_s < R_a\), the integration lower limit is fixed at the onset radius, \(R_1 = R_o\), which is independent of \(M/V_\infty\). But since \(R_s \sim M/V_\infty\), the overall scaling is \(L_x \sim (M/V_\infty)^{1-m}\).

For the case of very dense, optically thick winds with radiative shocks, \(R_o < R_s < R_a\), the lower boundary at \(R_1 = R_s\) gives the residual integral an additional dependence on \(R_1^{m-p}\); since \(R_1\) too scales with mass loss rate, the dependence on the mixing index \(\alpha\) cancels. At the radius \(R_1\) the cooling length ratio \(\ell/\rho\) is always the same, implying that in optically thick winds the observed radiative shock emission is likewise fixed for any mass loss rate. The luminosity scaling thus becomes independent of mixing, scaling just with shock number index as \(L_x \sim (M/V_\infty)^{1-p}\).

This is in fact the same scaling found in OC99 for optically thick winds, but with \textit{adiabatic} shocks. Indeed, OC99 argued that assuming a filling factor \(f_0 \sim r^{-0.4}\) could give a sub-linear dependence on mass loss rate, \(L_x \sim M^{0.6}\), and so possibly reproduce the \(L_x \sim L_{bol}\) relation. Subsequent analysis of X-ray line profiles observed from Chandra and XMM-Newton have shown, however, that optical depth effects are quite moderate even for O-stars like ζ Puppis with quite dense winds (Cohen et al. 2011). The bulk of O-star winds are simply too low-density for this optical thickness scaling to apply, and so this cannot be the explanation for the \(L_x \sim L_{bol}\) relation.

### 3.3.3 Summary of asymptotic scalings

To summarize, power-law shock-number models with thin-shell mixing have the asymptotic scalings,

\[
L_x \sim \left[ \frac{M}{V_\infty} \right]^2 ; \quad R_1 < R_a < R_o ; \quad \text{adiabatic, thin} \quad (31a)
\]

\[
L_x \sim \left[ \frac{M}{V_\infty} \right]^{1-m} ; \quad R_1 < R_o < R_a ; \quad \text{radiative, thin} \quad (31b)
\]

\[
L_x \sim \left[ \frac{M}{V_\infty} \right]^{1-p} ; \quad R_o < R_1 < R_a ; \quad \text{radiative, thick} \quad (31c)
\]

where the progression represents a trend of increasing \(M/V_\infty\). The first applies for weak winds, for example from early B main-sequence stars, which typically show weak X-ray emission, with \(L_x\) well below \(10^{-7} L_{bol}\) (Cohen et al. 1997). The middle scaling for intermediate-density winds is the most relevant for the bulk of O-type stars found to follow the \(L_x \sim L_{bol}\) relation. The last applies only to the strongest winds, e.g. very early O supergiants like HD93129A, for which analysis of X-ray line profiles show moderate absorption effects, with \(\tau_s \gtrsim 1\) (Cohen et al. 2011).

### 3.4 Link between \(M\) and \(L_{bol}\) scaling

As noted in the introduction, and summarized in figure 1, straightforward application of CAK wind theory implies a direct dependence of mass loss rate on luminosity that scales as \(M \sim L_{bol}^{1/\alpha}\), where \(\alpha \approx 0.6\) is the CAK power index; for the bulk of O-type stars with intermediate-density winds and thus \(L_x \sim M^{1-m}\), reproducing the observed \(L_x \sim L_{bol}\) relation thus simply requires a mixing exponent \(m \approx 1 - \alpha \approx 0.4\).

More generally, let us now consider how this requirement is affected if one accounts also for a secondary dependence on stellar mass \(M\), which in turn can give a further indirect dependence on \(L_{bol}\). Specifically, within CAK wind theory, \(M \sim M^{3/2}\) and \(V_\infty \sim M^{1/2}\), and so if we in turn assume from stellar structure a mass-luminosity dependence \(M \sim L_{bol}\), where \(s \approx 1/3\) (e.g., Maeder 2009, p. 360), we find

\[
\log \left( \frac{M}{V_\infty} \right) \sim \frac{2 - s(2 - \alpha)}{2\alpha} \log(L_{bol}) \sim \frac{\log(L_x)}{1 - m} ,
\]

where the latter relation makes use of the scaling (31b).

Reproducing the empirical \(L_x \sim L_{bol}\) relation thus now requires

\[
m = \frac{2(1 - \alpha) - s(2 - \alpha)}{2 - s(2 - \alpha)}.\]

Accounting for a stellar structure scaling \(s \approx 1/3\) with a CAK index \(\alpha \approx 0.6\) thus now requires a mixing exponent

\[7\]

\[8\]

Indeed, this star could be viewed as a transitional object to the WNH type Wolf-Rayet stars, for which absorption effects should strongly attenuate any X-rays from instabilities in the wind acceleration region; see. Of course, the terminal speed also depends on stellar radius as \(V_\infty \sim 1/\sqrt{R}\), but the diverse luminosity classes of X-ray emitting O-stars makes it difficult to identify any systematic dependence on \(L_{bol}\) that might influence the overall \(L_x - L_{bol}\) relation.
of radiative cooling affects X-ray line profiles. Following Owocki & Cohen (2001, hereafter OC01), the directional Doppler-shift of the X-ray line emission within the expanding wind is modeled through a line emissivity \( \eta_\lambda(r, \mu) \) at an observer's wavelength \( \lambda \) along direction cosine \( \mu \) from a radius \( r \). The resulting X-ray luminosity spectrum \( L_\lambda \) is computed from integrals of the emission over direction and radius, attenuated by bound-free absorption within the wind (cf. OC01 equation 1),

\[
L_\lambda = 8\pi^2 \int_{-1}^{1} d\mu \int_{R_s}^{\infty} drr^2 \eta_\lambda(\mu, r) e^{-\tau(\mu, r)}.
\]  

The absorption optical depth \( \tau(\mu, r) \) is evaluated by converting to ray coordinates and then integrating for each ray with a fixed impact parameter from the local position to the observer. For the standard \( \beta = 1 \) velocity law, the integrals are analytic, with overall scaling in proportion to \( \tau \equiv R_1/R_s \).

In principle, this integrated optical depth can be affected by the “porosity” associated with optically thick clumps or anisotropic “pancakes” (Feldmeier et al. 2003), with potential consequences for interpreting the asymmetry of X-ray line profiles in terms of the wind mass loss rate Oskinova et al. (2006). But for the optically thin (\( \tau \ll 1 \)) or marginally optically thick (\( \tau \sim 1 \)) lines considered here, individual clumps should be optically thin (Owocki & Cohen 2002; Sundqvist et al. 2012), and so such effects are not important for the discussion below, which focuses on the overall profile width.

4.2 Scaling analysis

As noted, applications of this OC01 formalism assuming a density-squared emission show that a spatially constant X-ray volume filling factor \( f_v \) gives generally quite good fits to observed X-ray lines. To examine the effect of radiative cooling, let us now apply the more general bridging-law scalings of (16) and (26), assuming the simple power-law form for \( \eta_\lambda(r, \mu) \) and velocity \( v \). We can again infer basic scaling results by inspection of this integrand in the limits of adiabatic vs. radiative shocks. For the adiabatic case, this follows the scaling in (29), with radial dependence \( 1/w^2p^{p+1} \). To match observed profiles, such adiabatic emission models require constant \( f_v = n_s \), with the zero power-law exponent \( p = 0 \) implying a \( 1/(w^2) \) variation of the integrand.

By contrast, the radiative limit follows the scaling in (30), with direct radius dependence \( 1/p^{p+1-m} \) and velocity dependence \( 1/w^{1-m} \) that is weaker than the \( 1/w^2 \) of the adiabatic model. Focusing first just on the former, we see that reproducing the \( 1/r^2 \) integrand needed to fit observed profiles would now require \( p = 1 + m \). In practice, to compensate for the weaker inverse-speed dependence, fitting the observed profile width requires a somewhat steeper shock-number decline, as we now quantify.
4.3 X-ray line profiles from radiative shocks

For a $\beta = 1$ velocity law with $R_o = 1.5R_a$ and various specified values of the exponents $p$ and $m$, figure 3 plots normalized X-ray line profiles for optically thin ($\tau_x \ll 1$; left) and marginally optically thick ($\tau_x = 1$; right) lines. In both cases, the black curve represents the adiabatic, constant-filling-factor ($p = 0$) model that gives generally good fits to observed profiles.

The other curves show results for radiative shocks. Without mixing, the red curve for radiative shocks with constant shock number ($p = 0$) is far too broad; fitting the favored black profile now requires a $p = 1.5$ (blue curve), which is even steeper than the predicted $p = m + 1 = 1$ needed to compensate for the weaker direct radial scaling. With mixing exponent of $m = 0.4$, we find a $p = 1.5$ shock-number model gives profiles (not shown) that are still somewhat too broad. But with a somewhat steeper number exponent $p = 2$, the blue curve again nearly reproduces the black curve.

An overall conclusion is thus that, for radiative shock models with mixing at a level needed to reproduce the $L_x \sim L_{bol}$ relation, matching observed X-ray emission requires a steep radial decline ($p \approx 2$) in shock number above the onset radius.

4.4 Decline of $L_x$ in optically thick winds

Such a steep radial drop off in shock number has important implications for the scaling of X-ray luminosity for the highest density stars that become optically thick to bound-free absorption, i.e. with $\tau_x > 1$ and thus $R_1 > R_o$. Namely, from the scaling given by (31), we see that taking $p \approx 2$ implies that the X-ray luminosity for such optically thick winds should now decline inversely with mass loss rate, $L_x \sim M^{-1} \sim 1/M$. If the X-ray emission is concentrated near an onset radius within the wind acceleration zone, the bound-free absorption by the overlying, optically thick wind significantly attenuates the net X-rays seen by an external observer. For early O-type supergiants with dense winds, this can lead to a reduced X-ray luminosity, but because the overall decline of bound-free opacity with X-ray energy, the observed spectrum can be hardened. The recent analysis of X-rays from the O2If star HD93129A provides a potential example approaching this limit (Cohen et al. 2011).

5 SCALING RESULTS FOR FULL EVALUATION OF $L_x$ INTEGRAL

5.1 Constant-speed wind with $\beta = 0$

As a supplement to the asymptotic $L_x$ scalings given in (28), let us finally consider full solutions for the general integral (28). For a constant speed model with $\beta = 0$ and thus $w(r) = 1$, general analytic integration is possible in terms of the incomplete beta function. But the typical properties can be more simply gleaned by examining the special case $p = 1$, for which the integral in (28) takes the simpler analytic form,

$$L_x = \frac{C_p\rho_k}{16\pi^2m(1-m)} \left[ \frac{1 + mR_o/R_a}{(1 + R_o/R_a)m} - 1 \right],$$

wherein the square-bracket factor sets the scalings with $M/V_\infty$, with the preceding terms just fixing the overall normalization. For low-density, optically thin winds, the initial radius is fixed to the onset radius, so that $R_a/R_1 \sim R_a/R_0 \sim M/V_\infty$. For $R_o/R_0 \ll 1$, expansion of the square-bracket term recovers the adiabatic scaling $L_x \sim (M/V_\infty)^2$ of (33a), while for $R_o/R_0 \gg 1$, it becomes proportional to $R_o^{-1-m}$ and so recovers the radiative, optically thin scaling (31b). For high-density, optically thick winds, the ratio $R_o/R_1 = R_a/R_1$ becomes independent of mass loss rate, and so the square-bracket factor and thus $L_x$ approach a constant value, in accord with (31c) for this case with $p = 1$.

5.2 Standard wind with $\beta = 1$

For a standard $\beta = 1$ velocity law with $rw = r - R_\star$, analytic integration of (28) can be cast in terms of the Appell hypergeometric function; but in practice it is more straightforward just to carry out the integration numerically.

Figure 4 plots the resulting X-ray luminosity $L_x$ (scaled by the dimensional factor outside the square brackets in (33)) vs. adiabatic radius $R_\star$ (scaled by the shock-onset radius $R_o$), as computed from numerical integration of (28) for the selected, labelled values of the mixing exponent $m$ and shock number exponent $p$. As seen from (24), plotting vs. $R_\star$ can be viewed as a proxy for plotting vs. the wind density parameter $M/V_\infty$.

The left vertical line at $R_o/R_1 = 1$ represents the transition from low-density adiabatic shocks, for which $L_x \sim M^2$, to intermediate-density radiative shocks, for which $L_x \sim M^{1-m}$. The right vertical line represents the transition from optically thin to thick winds, implemented here through a sudden change in the integration lower bound, $R_1 = \max(R_1, R_o)$, with the shock onset radius fixed at $R_0 = 1.5R_a$, and $R_1$ the radius for unit optical depth. Like $R_a$, $R_1$ scales with $M/V_\infty$, with values that are a small, fixed fraction $R_1/R_o \equiv f_{ia}$ of the adiabatic radius. Since observed X-rays are typically a modest factor higher energy than the characteristic wind shock (e.g., $E_{kev} \approx 1 \approx 2T_{kev}$), the curves plotted in figure 4 assume, following (28) and (21), $f_{ia} = R_1/R_o = \kappa_{ef}/\kappa_c \approx 1/100$. For very large $R_o > R_o/f_{ia} \approx 100R_o$, we thus have $R_1 > R_o$, leading to a declining $L_x$, as predicted by the optically thick wind scaling $L_x \sim R_o^{-1-m}$ from (31c).

But for moderately dense winds, with $R_o < R_\star < R_o/f_{ia}$ (between the vertical lines in the figure), the increasing $L_x$ approaches the power-law variation $R_o^{-1-m}$ predicted by (31b). In particular, the black curve with $p = 2$ and $m = 0.4$ represents the preferred model with sub-linear scaling in $R_\star$, and thus in $M/V_\infty$, implying a nearly linear scaling of $L_x$ with $L_{bol}$.

Specifically, for typical values for stellar radius ($R_\star \approx 10 - 20R_\odot$) and wind terminal speed ($V_\infty \approx 2000$ km $s^{-1}$), this intermediate-density regime with $L_x \sim L_{bol}$ applies to wind mass loss rates that range from below $10^{-7}$ $M_\odot$ yr$^{-1}$ to a few times $10^{-6}$ $M_\odot$ yr$^{-1}$; this essentially encompasses the
entire O-star spectral range for which the $L_x \sim 10^{-7} L_{bol}$ relation is found to hold.

6 CONCLUDING SUMMARY

The central result of this paper is that, in the common case of moderately dense O-star winds with radiative shocks ($R_a > R_o$), thin-shell mixing can lead to this sub-linear scaling of the X-ray luminosity with the mass-loss rate, $L_x \sim (M/V_{\infty})^{1-m}$. Depending on the secondary scalings of wind density with bolometric luminosity, one finds that $m \approx 0.2 - 0.4$ can give roughly the linear $L_x \sim L_{bol}$ law that is empirically observed for O-star X-rays. Further simulation work will be needed to see if such mixing exponent values are appropriate, and indeed to test the validity of the basic mixing exponent ansatz.

But in the course of exploring this idea of thin shell mixing, the analysis here has lead to several interesting secondary results with validity and implications that are largely independent of mixing or any specific model for it. A summary list includes:

(i) In contrast to previous analyses that invoked a density-squared emission measure description for shock production of X-rays, we derive here a more general bridging law showing how the density-squared scaling of adiabatic shocks transitions to a single-density scaling for radiative shocks.

(ii) For radiative shocks, the X-ray volume filling factor is not fixed (as is commonly assumed), but is reduced by the narrow extent of the shock layer, $f_\xi \sim \ell/r$.

(iii) For nearly all O-stars, the large radiative-adiabatic transition radius, $R_a > R_o \approx 1.5 R_*$, implies that instability-generated shocks in the wind acceleration region should follow the radiative scaling, giving the X-ray luminosity a linear scaling with mass loss rate, $L_x \sim \dot{M}/V_{\infty}$.

(iv) For low-density winds of lower luminosity (early B) stars, shocks should indeed become adiabatic, implying then a steep ($L_x \sim \dot{M}^2$) decline of X-ray luminosity, as is in fact generally found for single, non-magnetic, early B-type stars, for which the inferred X-ray emission measure often approaches that of the full wind (Cohen et al. 1997, 2008).

(v) Matching observed X-ray emission lines with such models of radiative shocks with or without thin-shell mixing requires the shock number to have a moderately steep decline above the X-ray onset radius, with power-law exponent $p \approx 1.5 - 2$.

(vi) This in turn implies that the scaling of X-ray luminosity for dense, optically thick winds should saturate and even decline with increasing mass loss rate.
This last result on X-ray absorption is perhaps not too relevant for most of the O-stars following the $L_x - L_{bol}$ relation, for which optical depth effects are weak to marginal. But it can become important for the dense, moderately optically thick winds of extreme, early O-stars like HD93129A, which can be viewed as a transitional object to the WNH type Wolf-Rayet stars. More generally, the high-density of Wolf-Rayet winds implies that absorption should strongly attenuate X-rays from any instability-generated shocks in their wind acceleration region. As such, the observed hard X-rays seen from Wolf-Rayet stars like WR6 (EZ CMa) seem unlikely to be explained by this standard model of LDI shocks (Oskinova et al. 2012). This also has potential implications for interpreting observed X-rays from very massive stars that have $L_x \sim 10^{-7} L_{bol}$ despite having very high wind optical depths (Crowther et al. 2010), and whether these might instead originate from wind-wind collisions of close, undetected binary companions.

Indeed, the mixing ansatz in this paper could also be applied to model X-ray emission from colliding wind binaries, and their $L_x$ scaling with orbital separation. Wide binaries with adiabatic shocks should still follow the usual inverse distance scaling, as directly confirmed by observations of multi-year-period elliptical systems like WR140 and η Carinae (Corcoran 2011). But in close, short (day to week) period binaries with radiative shocks (Antokhin et al. 2004), mixing could reduce and limit the effective X-ray emission from the wind collision (Parkin & Pittard 2010), and thus help clarify why such systems often hardly exceed the $L_x \approx 10^{-7} L_{bol}$ scaling found for single stars (Oskinova 2002, Corcoran 2011, Gagné 2011).

Finally, in addition to exploring such effects in colliding wind binaries, a top priority for future work should be to carry out detailed simulations of the general effect of thin-shell mixing on X-ray emission, and specifically to examine the validity of this mixing-exponent ansatz for modeling the resulting scalings for X-ray luminosity.

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