Prospects for Constraining Neutrino Mass Using Planck and Lyman-Alpha Forest Data

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In this paper we investigate how well Planck and Lyman-Alpha forest data will be able to constrain the sum of the neutrino masses, and thus, in conjunction with flavour oscillation experiments, be able to determine the absolute masses of the neutrinos. It seems possible that Planck, together with a Lyman-α survey, will be able to put pressure on an inverted hierarchical model for the neutrino masses. However, even for optimistic assumptions of the precision of future Lyman-α datasets, it will not be possible to confirm a minimal-mass normal hierarchy.

I. INTRODUCTION

The determination of absolute neutrino masses is a key scientific goal for the coming decade. Neutrino flavour oscillation detections have shown that neutrinos do indeed have mass but unfortunately cannot determine their absolute masses. Particle physics experiments (e.g. tritium beta decay or neutrinoless double beta decay) offer the most direct probe of neutrino masses, but reaching limits of less than 1 eV is formidably challenging [1]. Sub-eV neutrino masses can also be probed indirectly via their effects on the energy density of the Universe and large scale structure. For this reason there has been considerable interest on constraints on neutrino masses from various current and future cosmological probes (see [2] for an excellent and comprehensive review).

In this paper we perform a detailed Markov Chain Monte Carlo analysis to assess the sensitivity of the Planck satellite [3] to neutrino masses. This problem is topical because Planck is scheduled for launch in late 2008 and should provide all-sky maps of the cosmic microwave background (CMB) of unprecedented precision. Planck will operate at nine frequencies from 30 to 857 GHz, seven of which will be sensitive to polarization. The sensitivity and frequency coverage of Planck should allow accurate subtraction of foregrounds resulting in an essentially complete reconstruction of the CMB temperature signal over a large area of the sky together with significant new information on its polarization. For a summary of the Planck instruments and its scientific programme, including constraints on neutrino masses, see [3]. For assessments of Planck constraints on neutrino masses from different perspectives and using a variety of complementary astrophysical data see [4, 5, 6, 7, 8].

Neutrino masses affect both the cosmic history and structure formation. One main effect is a reduction of power below a characteristic wavenumber corresponding to the Hubble scale when the neutrinos become non-relativistic 1

\[ k_\nu \sim 0.026 \left( \frac{m_\nu}{\text{eV}} \right)^{1/2} \Omega_m^{1/2} h \text{ Mpc}^{-1}, \]

(e.g. [6, 10, 11]). Planck will not directly probe the damping of fluctuations caused by massive eV-scale neutrinos (except possibly via weak lensing of the CMB). Nevertheless Planck will be vital since it will break parameter degeneracies that would otherwise exist in other cosmological datasets.

Observations of Lyman-α absorption in quasar spectra constrain the amplitude and slope of the cosmological matter power spectrum at redshifts between two and four on comoving megaparsec scales. Compared to galaxy redshift surveys Lyman-α surveys probe the matter power spectrum closer to the linear regime and sample smaller scales, giving a long lever arm when combined with observations of the CMB. Furthermore, the galaxy power spectrum is difficult to relate to the matter power spectrum on small scales where the fluctuations are highly non-linear and the damping effects of eV-mass neutrinos are most significant. Hence it is natural to consider combining Planck with a Lyman-α survey in an effort to assess what cosmology might contribute to the determination of neutrino masses over the next decade. Already, the tightest cosmological limits on neutrino masses come from combining Lyman-α datasets with CMB and other datasets (e.g. [12, 13]). In principle an ultra-large galaxy redshift survey, as anticipated from the Square Kilometer Array (SKA), might be able to measure the small effects of sub-eV scale neutrinos on the galaxy power spectrum on scales \( \gtrsim 0.02 h \text{ Mpc}^{-1} \) (see [8]). This is discussed further in Sections [X] and [X].

We first consider constraints on neutrino masses from the current Sloan Digital Sky Survey Lyman-α data (denoted SDSSlya below). We then consider the improvement in these constraints arising from a hypothetical percent-level determination of the power spectrum from

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1 Where \( m_\nu \) is the neutrino mass, \( \Omega_m \) is the mass density in units of the critical density and \( h \) is the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\).
a future Lyman-α survey. Neutrino flavour oscillation experiments are consistent with one of two minimal values for the sum of neutrino masses, either 0.056 eV or 0.095 eV. The former value comes from assuming the neutrinos fall into a normal hierarchy, with the mass of the intermediate-mass neutrino closer to that of the lightest neutrino than that of the heaviest one. The latter value comes from assuming the neutrinos fall into an inverted hierarchy, with the mass of the intermediate-mass neutrino closer to that of the heaviest neutrino than that of the lightest one (see e.g. [2]). For a total mass significantly greater than these values, the mass splittings are much smaller than the individual masses and the neutrinos are said to be degenerate. These observations suggest two clear goals for a cosmological determination of neutrino mass: firstly to see whether neutrino masses are degenerate or not and secondly to differentiate between a normal and an inverted hierarchy. In addition, it may be possible to put useful constraints on physics beyond the standard model that impinges on neutrino mass; e.g. the study of thermal leptogenesis in [14] that gives an upper bound on the mass of a light neutrino of 0.1 eV.

II. COSMOLOGICAL IMPLICATIONS OF NEUTRINO MASS

The evolution of both the background universe and perturbations within it are sensitive to the neutrino mass spectrum. For reviews see [2, 15]. One main effect of massive neutrinos is a suppression of power on small scales (Eq. 3) that is roughly proportional to the neutrino fraction of the matter content of the universe. However, the effect can be partially degenerate with changes in other cosmological parameters, so for robust results uncertainties in other parameters should be marginalized out.

III. MASSIVE NEUTRINOS IN CAMB AND COSMOMC

Our analysis makes use of the software packages CAMB2 [16] and CosmoMC3 [17]. CAMB calculates the linear-theory CMB power spectrum and optionally multiple matter power spectra for a given cosmological model. CosmoMC uses Markov Chain Monte Carlo to sample from the posterior distribution of cosmological parameters from a given likelihood function. The likelihood function uses theoretical calculations from CAMB in combination with real (or mock) datasets. The standard set of six parameters commonly used is \(\{\Omega_b h^2, \Omega_{\text{cdm}} h^2, \theta, \tau, n_s, \ln(10^{10} A_s)\}\), where \(\Omega_b\) is the baryon density divided by the critical density, \(\Omega_{\text{cdm}}\) is the dark matter density (including potential massive neutrinos) divided by the critical density, \(h\) is the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\), \(\theta\) is the acoustic horizon angular scale, \(\tau\) is the optical depth from reionization, \(A_s\) and \(n_s\) are the amplitude and spectral index (at a fiducial wavenumber of 0.05 Mpc\(^{-1}\)) of the primordial adiabatic scalar curvature perturbation power spectrum \(P(k)\). We follow this usage here, and use the shorthand \(\{6\}\) to denote these six parameters. In this paper, the dark energy is assumed to be a cosmological constant. Massive neutrinos are introduced via the parameter \(f_\nu \equiv \Omega_\nu / \Omega_{\text{cdm}}\), where \(\Omega_\nu\) is the massive neutrino density divided by the critical density today. The sum \(\sum m_\nu\) of the neutrino masses is related to \(f_\nu\) by

\[
\sum m_\nu \approx 93.12 \Omega_{\text{cdm}} h^2 f_\nu \text{ eV}.
\]

(2)

Throughout this paper we assume only the usual three neutrino species. We either take them all to have the same mass \(\sum m_\nu/3\), or take two of them to be massless and one to be massive with mass \(m = \sum m_\nu\) in this case. In the first case we denote the standard six parameters along with \(\sum m_\nu\) by \(\{6 + \Sigma\}\) and in the second case we write \(\{6 + m\}\). We shall also sometimes consider a running scalar spectral index, with running \(\eta_{\text{run}} \equiv (d/d\ln k)^2 \ln P(k)\) and assumed constant in \(k\).

The publically available version of CAMB has recently been upgraded by one of us (AL) to handle arbitrary mass splittings. Fig. 1 shows how the matter power spectrum is sensitive to assumptions about the neutrino masses. For this work we have also modified CosmoMC to allow two neutrinos to be massless and one to be massive. In all cases CosmoMC uses \(f_\nu\) as its base parameter, with \(m\) or \(\sum m_\nu\) as derived parameters.

IV. WMAP3 AND MASSIVE NEUTRINOS

In Ref. [12] the WMAP team present constraints on neutrino properties based on the 3-year WMAP data [18, 19] either with or without other datasets. Using WMAP data alone, they find \(\sum m_\nu < 1.8\) eV at 95% confidence. Along with either SDSS or 2dFGRS galaxy redshift data, they find \(\sum m_\nu < 1.3\) eV or 0.9 eV at 95% confidence respectively. We ran seven-parameter \(\{6 + \Sigma\}\) chains against WMAP and found \(\sum m_\nu < 1.7\) eV at 95% confidence (using version 2 of the WMAP team likelihood code) in good agreement with [12].

V. WMAP3 AND SDSSLYA

A recent paper obtains an impressive 95% upper bound for \(\sum m_\nu\) of only 0.17 eV, using a combination of CMB, galaxy and Lyman-α data [13]. Their routine to calculate the likelihood for a model in light of the SDSSLYA data has been made publicly available at [20]. The CMB and
The power spectra of three models are illustrated relative to the power spectrum for three equal-mass neutrinos with \( \sum m_\nu = 0.059 \) eV. The upper solid blue curve corresponds to massless neutrinos, the dotted curve corresponds to the minimal normal hierarchy, and the lower curve corresponds to the minimal inverted hierarchy. Also shown is a model with \( \sum m_\nu = 0.13 \) eV and other cosmological parameters changed so that the model is nearly degenerate in likelihood with the fiducial one against future Planck and Lyman-\( \alpha \) datasets.

FIG. 1: A plot illustrating the dependence of the matter power spectrum on the neutrino mass spectrum. The power spectra of three models are illustrated relative to the power spectrum for three equal-mass neutrinos with \( \sum m_\nu = 0.059 \) eV. The upper solid blue curve corresponds to massless neutrinos, the dotted curve corresponds to the minimal normal hierarchy, and the lower curve corresponds to the minimal inverted hierarchy. Also shown is a model with \( \sum m_\nu = 0.13 \) eV and other cosmological parameters changed so that the model is nearly degenerate in likelihood with the fiducial one against future Planck and Lyman-\( \alpha \) datasets.

other data are somewhat in tension, the small-scale data favouring a higher overall normalization of the power spectrum than the CMB. Since adding neutrino mass only lowers the power on small scales, the combined datasets prefer no neutrino mass at all. The tension between the datasets could be merely a statistical fluctuation: the two datasets happen to give a tighter mass constraint than expected from most a priori possible realizations of the data. Alternatively it could indicate that there is some inconsistency in the datasets, for example one, or both, having some unaccounted-for systematic error. Or it could indicate that our modelling is too simplified, e.g. the primordial power spectrum varies strongly with wavenumber.

We performed a seven-parameter \( \{6 + \Sigma\} \) joint analysis of the WMAP and SDSS/LYA data, and obtained a 95% upper bound on \( \sum m_\nu \) of 0.39 eV (v2 WMAP code) or 0.35 eV (v1 WMAP code), intermediate between that of \( \{13\} \) and those of the WMAP team using galaxy survey data mentioned above. Note that we used the same WMAP and Lyman-\( \alpha \) forest data and code as \( \{13\} \). However we did not use the additional CMB galaxy survey and supernovae data used by Ref. \( \{13\} \): including these datasets gives them a significantly tighter constraint than from just Lyman-\( \alpha \) and WMAP alone.

To investigate the possible inconsistency of the WMAP and SDSS/LYA data, we constructed a fake WMAP dataset, keeping the noise the same as for the real data but replacing the \( C_l \)'s themselves by those of the 6-parameter model that is the best fit to the WMAP and SDSS/LYA datasets taken together. With \( \{6 + \Sigma\} \) chains, faked WMAP alone gives a 95% upper limit to \( \sum m_\nu \) of 1.5 eV, comparable to that from real WMAP. However, faked WMAP with SDSS/LYA gives a 95% upper limit to \( \sum m_\nu \) of 0.70 eV, double that of real WMAP with SDSS/LYA; see Fig. 2. A possible interpretation of this result is that the WMAP + SDSS/LYA limit on the neutrino mass is spuriously low, by a factor of about two, because of some unidentified systematic error in one or both datasets. However, as mentioned above, the tension between the two datasets seen in Fig. 2 could be a statistical fluke or an inadequacy in the theoretical model.

VI. MOCK PLANCK DATA FOR COSMOMC

In this section we describe how we construct very simplified mock Planck data for Cosmomc, used for forecasting future constraints on the neutrino masses. We consider only the linear-theory CMB power spectrum. If the CMB lensing signal (via the power-spectrum of the weak lensing deflection field) can also be used, significantly better constraints might be obtainable than by using the CMB power spectrum alone. This is discussed by Lesgourgues et al. in \( \{7\} \), who find that including the weak lensing deflection field from an idealised Planck experiment (unlikely to be realised in practice) leads to limits on the neutrino masses comparable to those discussed below from combining Planck with Ly\( \alpha \) data.

First of all we assume an underlying cosmology, taking parameters from the best-fit six-parameter models coming from either WMAP alone or WMAP and SDSS/LYA. Next we run the CAMB software to generate a theoreti-
cal “prior” power spectrum distribution with mean $C_{l}^{\nu}$ based on the input cosmological parameters. For comparison purposes we sometimes include one non-zero neutrino mass 0.06 eV when generating this power spectrum.

The log-likelihood for some model with power spectrum $C_{l}$ averaged over sky realizations turns out, up to an irrelevant constant, just to be the log-likelihood of those $C_{l}$‘s evaluated taking the sky power spectrum to be its ensemble average $C_{l}^{\nu}$

Hence we do not need to make a specific realization of the sky or numerically average over many of them for our forecasting; we just imagine the data to have exactly the ensemble average power spectrum. This procedure gives error bars consistent with those obtained from most actual realizations, but has the advantage that the maximum likelihood parameters should be at their true values rather than moving around between different realizations. (See [21, 22] for related discussions.)

We then create the mock dataset for use with cosmoMC [22]. The sky power spectrum $C_{l}$ is set to be the prior spectrum described in the previous paragraph. We work on the full sky and convolve with a beam window function and add noise (see e.g. [24] for the procedure). The beam is assumed to be Gaussian and symmetric, and the noise is assumed to be white and uniform across the sky (we neglect any foregrounds). The beam width and pixel noise are chosen to approximate those of the Planck 143 GHz High Frequency Instrument channel [3].

We label the prior power spectra derived from current data as follows: $C^{W}$ from the six-parameter fit to WMAP alone assuming massless neutrinos, $C^{WS}$ from the six-parameter fit to WMAP and SDSSLYA assuming massless neutrinos, $C_{0.06}^{W}$ from the six-parameter fit to WMAP assuming one neutrino mass of 0.06 eV and $C_{0.06}^{WS}$ from the six-parameter fit to WMAP and SDSSLYA with one neutrino mass of 0.06 eV.

We also ran eight-parameter $\{6 + \Sigma + n_{\text{run}}\}$ chains with both neutrino mass $\sum m_{\nu}$ and scalar spectral index running $n_{\text{run}}$. Despite thoughts that both might affect the power spectrum on small scales in a qualitatively similar way, we found that these two parameters were not in fact degenerate and that the 95% upper limit on $\sum m_{\nu}$ was only moderately weakened to 0.87 eV.

VIII. PLANCK AND SDSSLYA

We now consider how well Planck might do in conjunction with SDSSLYA data. Running $\{6 + \Sigma\}$ chains against $C^{W}$ and SDSSLYA, we obtain the tight constraint $\sum m_{\nu} < 0.10$ eV at 95% confidence, three times tighter than WMAP and SDSSLYA suggesting that Planck data together with existing Lyman-\(\alpha\) data may be capable of placing severe pressure on an inverted hierarchy. The same constraint on $\sum m_{\nu}$ is obtained using $C_{0.06}^{W}$ in place of $C^{W}$.

However, there is a concern that this constraint might be artificially tight simply because of the possible discrepancy between the WMAP and SDSSLYA datasets discussed in Section [V]. To address this, we ran $\{6 + \Sigma\}$ chains against $C^{WS}$ and SDSSLYA, rather than $C^{W}$ and SDSSLYA. Although the SDSSLYA data effectively enters twice, this procedure should give an indication of what might happen if the tension between the WMAP and SDSSLYA data is caused by systematic errors. This yields $\sum m_{\nu} < 0.27$ eV, a significantly weaker constraint. Thus we see that that bound on the neutrino mass is highly sensitive to the assumed input model. The ability of Planck and SDSSLYA to constrain an inverted hierarchy therefore depends on which of these input models is closer to the truth.

IX. PLANCK AND PERCENT LEVEL MEASURES OF THE MATTER POWER SPECTRUM

In this Section we analyse what might be learned from Planck and new Lyman-\(\alpha\) surveys of greater statistical power than SDSSLYA. For example, [26] investigates how an extended Ly\(\alpha\) survey might perform in Constraining dark energy and curvature, assuming an experimental configuration with characteristics similar to that proposed for galaxy baryonic acoustic oscillation surveys. Here we take a very simple approach and consider a survey that would be able to measure the matter power spectrum at one or more effective redshifts and at one or more scales to better than five percent accuracy.

Since Lyman-\(\alpha\) surveys effectively measure distances in velocity units (see e.g. [27]), we choose our scales likewise. We consider the following hypothetical datasets:

1. $P_{\alpha 1}^{101}$, consisting of a single data point at an effective redshift of 3 and wavenumber 0.009 s/km, with a 1% fractional error,
2. \( P^{3@1}_{1\%} \), consisting of three data points at an effective redshift of 3 at wavenumbers 0.002 s/km, 0.009 s/km and 0.02 s/km, with 1% fractional errors,

3. \( P^{3@3}_{1\%} \), consisting of three data points at each of three redshifts of 2, 3 and 3.5 at wavenumbers of 0.002 s/km, 0.009 s/km and 0.02 s/km, with 1% fractional errors, and

4. \( P^{3@5\%} \), consisting of three data points at each of three redshifts of 2, 3 and 3.5 at wavenumbers of 0.002 s/km, 0.009 s/km and 0.02 s/km, with 5% fractional errors.

The datasets are constructed by evolving the \( z = 0 \) matter power spectrum output from CAMB at the appropriate scale back to the appropriate redshift using the standard formula for the growth of linear inhomogeneities with the appropriate parameters for the assumed background model. This “data” is fed into a version of the lyaf90 module of CosmoMC in order to perform the likelihood calculation for models.

Running \( \{6+m+n_{\text{run}}\} \)-parameter chains against \( C_{0.06}^W \) and faked Lyman-\( \alpha \) datasets yields the limits shown in Table I for \( \sum m_\nu \) at 95% confidence.

Corresponding one-dimensional likelihood plots are shown in Fig. 3. Note that all curves peak in the vicinity of the added neutrino mass of 0.06 eV. Fig. 4 shows how adding future Lyman-\( \alpha \) data to Planck data breaks degeneracies and thus substantially improves the limits shown in Fig. 2. However, none of the curves in Fig. 3 tend to zero as the neutrino mass tends to zero and thus none of the dataset combinations is capable of unambiguously detecting neutrino mass in the minimal normal hierarchy model.

In this paper we have focussed on combining CMB and Lyman-\( \alpha \) data for the reasons given in the introduction. In light of the above results we also considered the additional degeneracy-breaking effects that a future galaxy survey might provide. Combined with our most optimistic \( P^{3@3\%}_{1\%} \) Lyman-\( \alpha \) dataset along with Planck, such a galaxy survey would have to effectively measure \( \Omega_m h \) to better than 2% in order to yield a 95% confidence positive detection of neutrino mass for the minimal hierarchy (assuming the improved constraint comes from degeneracy-breaking alone). For comparison, the SDSS and 2dF galaxy surveys constrain \( \Omega_m h \) to an accuracy of about 10% [28, 29] and so substantially larger redshift surveys would be required to constrain the shape of the matter power spectrum to the level required to constrain a minimal hierarchy. At present, the best prospect seems to be a large-scale galaxy survey of \( \sim 10^9 \) galaxies detected with the SKA over the redshift range \( 0 - 1.5 \) [8].

### X. CONCLUSIONS

In this paper we have studied how Planck, in combination with a Lyman-\( \alpha \) based measure of power on

| \( C_{0.06}^W \) | \( P^{1@1\%}_{1\%} \) | \( P^{3@1\%}_{1\%} \) | \( P^{3@3\%}_{1\%} \) | \( P^{3@5\%}_{1\%} \) |
|----------------|----------------|----------------|----------------|----------------|
| 0.13 eV         | 0.12 eV         | 0.11 eV         | 0.14 eV         |                    |

FIG. 3: A plot of the marginalized likelihoods for a single neutrino of mass \( m \) with assumed future datasets as discussed in the text. All curves use the \( C_{0.06}^W \) Planck dataset. As for the Lyman-\( \alpha \) dataset used, black (solid) corresponds to \( P^{3@5\%}_{1\%} \), red (dot-dash) to \( P^{1@1\%}_{1\%} \), green (short-dash) to \( P^{3@1\%}_{1\%} \) and blue (long-dash) to \( P^{3@3\%}_{1\%} \).

FIG. 4: A 2D contour plot indicating how a partial parameter degeneracy using only Planck data is lifted when Lyman-\( \alpha \) data is added. 68% and 95% confidence intervals are illustrated for the following three datasets (from broadest to tightest): blue, Planck alone; green, Planck with \( P^{3@3\%}_{1\%} \); red, Planck with \( P^{3@1\%}_{1\%} \).
megaparsec scales, might perform in constraining neutrino masses.

We find that Planck, in combination with existing Lyman-\(\alpha\) data, should be able to put significant pressure on the inverted hierarchy model. Some of the allowed parameter space for thermal leptogenesis models should also be constrained. These limits can be tightened by using more powerful Lyman-\(\alpha\) data. However, even for the ambitious Lyman-\(\alpha\) datasets assumed in Section 19 we conclude that it is unlikely that Planck will be able to positively detect a minimal-mass normal hierarchy.

Other cosmological data, in particular lensing of the the CMB, may improve the neutrino mass constraints, though extracting an accurate lensing signal from realistic Planck data is likely to be challenging and needs further investigation. For idealized Planck data,\(^7\) conclude that the weak lensing deflection field can improve the neutrino mass limits from Planck alone to a 1\(\sigma\) limit of 0.13 eV, comparable to our forecasts for Planck combined with SDSSLyA. In principal, a sufficiently large galaxy redshift survey, such as envisaged for the SKA, in combination with Planck could probe a minimal-mass normal hierarchy. This has been considered in some detail in\(^8\). Apart from the long timescale involved for such a survey (probably well beyond the next decade) it may prove difficult to relate the galaxy power spectrum to the underlying matter power spectrum to the required accuracy. (See\(^9\) for empirical evidence of scale dependent bias in the galaxy distribution over the wavenumber range 0.01 < \(k\) < 0.15 h Mpc\(^{-1}\).) Whether the precision envisaged by\(^8\) can be achieved remains to be seen.

The absolute values of the neutrino masses would offer important insights into physics beyond the standard model. There is a widespread hope that cosmological probes will be able to constrain neutrino masses to a precision better than the normal hierarchy characteristic mass of 0.06 eV. However, the detailed calculations presented here suggest that we should be more sanguine. The cosmological detection of 0.06 eV neutrinos would require extremely large cosmological datasets, free of systematic errors, in addition to Planck. Furthermore, the cosmological limits are dependent on physical assumptions (e.g. featureless varying power spectrum, and fixed dark energy) that may be difficult to justify experimentally. A convincing detection of a neutrinos mass \(\lesssim 0.1\) eV will require, at the very least, consistency between a number of independent cosmological datasets.

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