I. INTRODUCTION

Josephson junctions show phenomena caused by macroscopic-scale quantum coherence and non-linear dynamical properties. Their unique properties come from couplings between superconducting gauge-invariant phase differences and the electromagnetic field, leading to practical applications, as seen, e.g., in Ref. [1]. Macroscopic quantum tunneling (MQT) [2] is one of the characteristic phenomena of Josephson junctions. Superconducting-to-resistive switching events in current-biased Josephson junctions are related to MQT at low temperatures, where thermal excitations are negligible. A wide variety of junction systems (artificial niobium-based junctions [3,4], grain boundary junctions [5,6], and intrinsic Josephson junction stacks in single-crystalline high-$T_c$ cuprate superconductors [7-9]) show this tunneling phenomenon. The theoretical aspects have been studied well, depending on the types of Josephson junctions [10-21]. MQT in Josephson junctions plays an important role in Josephson phase qubits and the relevant superconducting quantum engineering [22-26]. Hence, studying MQT in Josephson junctions attracts a great deal of attention theoretically and experimentally, to find quantum characteristics of superconducting devices.

The discovery of superconducting materials, including magnesium diboride [27] and iron-based compounds [28], triggered the studies on multi-band superconductivity. These superconductors have intriguing properties, originating from the multiple superconducting gaps opening in different parts of the Fermi surfaces [29-33]. Notable Josephson effects are predicted in junctions with multi-band superconductors [21,34-36]. The characteristic behaviors in these systems originate from the presence of multiple gauge-invariant phase differences coupled by inter-band Josephson coupling [37-39]. Specifically, a phase mode induced by inter-band fluctuations, which is referred to as a Josephson-Leggett (JL) mode [40], can lead to singular behaviors. However, the JL-mode excitations are not coupled directly to the electric field, owing to their neutral-superfluid feature [41-43]. Instead, they interact with the Josephson plasma (JP) mode (i.e., in-phase motion of superfluids) [44-45]. Thus, a careful and systematic study of the interaction between the JP and the JL modes is desirable for exploring characteristic phenomena in Josephson junctions composed of multi-band superconductors.

In this paper, we construct a theory of MQT in a Josephson junction formed by a conventional single-band superconductor and a two-band superconductor (a hetero Josephson junction), to clarify the effects of the JL mode on low-temperature switching events in Josephson junctions. From theoretical considerations of the dynamics of gauge-invariant phase differences, we choose a tunneling path along the center-of-mass motion of the phase differences, and consider the inter-band fluctuations to be the environment for the center-of-mass motion. We evaluate the MQT escape rates, varying different junction parameters. We show that the inter-band fluctuations have both positive and negative effects on the MQT. Zero-point fluctuations of the JL mode enhance the MQT escape rate, whereas the quantum dissipation induced by the JL mode suppresses the quantum tunneling. The former was found by two of the authors (YO and MM) [21] focusing only on a specific junction parameter. Thus, the present approach successfully extends the previous results in Ref. [21] and reveals two distinct features of the JL mode, i.e., amplification and reduction. Moreover,
we show that the zero-point-fluctuation enhancement exceeds the quantum-dissipation suppression. Therefore, we find that the escape rate is significantly enhanced by the JL mode. In these junctions, the dissipation effect is marginal because there is only one dissipation channel corresponding to a monochromatic JL-mode. We also examine the dependence of the escape rate on the interband Josephson energy for junction parameters which are typical for BaFe$_2$As$_2$ and MgB$_2$. We find that the effects of inter-band fluctuations on MQT strongly depend on the nature of the JL mode characterized by the superconducting material parameters of the junction.

This paper is organized as follows. In Sec. II, we introduce a minimal model of multi-band Josephson junctions. In Sec. III, we describe a theory of MQT in this Josephson junction and derive the MQT escape-rate formula. In Sec. IV, we evaluate the escape rate for various junction parameters. Section V presents a summary.

II. MODEL

We study a minimal model of a multi-band Josephson junction, as seen in Fig. 1, to find an essential feature of the JP-JL coupling. The system is composed of a conventional single-gap superconductor and a two-gap superconductor. The superconducting electrodes are separated by an insulating layer with thickness $D$ and area $W$. The DC current $I_{ext}$ is applied to this junction. The right electrode has two gaps, $\Delta^{(1)}_R e^{i\phi_R}$ and $\Delta^{(2)}_R e^{i\phi_R}$, whereas the left electrode has a single gap $\Delta_L e^{i\phi_L}$. These superconducting phases are coupled to each other via the Josephson couplings $E_{11}$, $E_{12}$, and $E_{in}$. The standard Josephson energy associated with Cooper-pair tunneling between the superconducting electrodes is characterized by $E_{11}$ and $E_{12}$. The inter-band Josephson energy associated with the tunneling between the two bands is characterized by $E_{in}$.

Now, we show two key variables in this paper, the center-of-mass phase and the relative phase. Using the gauge-invariant phase differences $\theta^{(1)}$ and $\theta^{(2)}$ between the electrodes (See Fig. 1), the center-of-mass phase is

$$\theta = \frac{\alpha_2}{\alpha_1 + \alpha_2} \phi^{(1)} + \frac{\alpha_1}{\alpha_1 + \alpha_2} \phi^{(2)},$$

with a dimensionless constant $\alpha_i$, related to the density of states of the $i$th-band electron near the interface between the right electrode and the insulator. The relative phase is

$$\psi = \theta^{(1)} - \theta^{(2)}.$$  

When the voltage difference is $V$, the Josephson relation is

$$\frac{\partial \theta}{\partial t} = \frac{2eD}{\hbar} V,$$

with the electric charge $e$ and the Planck constant $\hbar$. The Josephson relation indicates that $\theta$ is directly coupled to the electric field, but $\psi$ does not. In this paper, we focus on a short Josephson junction, that is, $D$ is much smaller than the Josephson penetration depth. Hence, we ignore the spatial modulation of $\theta^{(1)}$ and $\theta^{(2)}$, and the influence of solitonic excitations shown in Refs. 17 and 18.

III. FORMULATION

We formulate the MQT escape rate, based on the semi-classical approximation with the imaginary-time path-integral method. Our discussion is divided into four steps. First, we show the Lagrange formalism of our junction, useful for the path-integral method. Second, from a physical point of view, we find a plausible tunneling path for low-temperature switching events in the present junction. Our approach is to choose a specific path and reduce the issue into an effective 1D tunneling problem. Third, we show that the relevant Euclidean (imaginary-time) Lagrangian can be mapped into the Caldeira-Leggett model. In this step, we mainly use a small $\psi$-expansion. Finally, we obtain the MQT escape rate, using a technique based on the influence-functional method.

A. Real-time Lagrangian

The real-time effective Lagrangian $\mathcal{L}$ for our junction is

$$\mathcal{L}(\theta, \psi) = \frac{1}{2} m_{cm} \left( \frac{d\theta}{dt} \right)^2 + \frac{1}{2} m_{tt} \left( \frac{d\psi}{dt} \right)^2 + E_{11} \cos \theta^{(1)} + E_{12} \cos \theta^{(2)} + E_{in} \cos \psi + E_{J1} \gamma \theta,$$

where $m_{cm}$ and $m_{tt}$ are the mass parameters, and $E_{J1}$ and $E_{in}$ are the Josephson coupling energies.
with $m_{cm} = \hbar^2/2E_c$ and $m_{rlt} = \hbar^2/2(\alpha_1 + \alpha_2)E_c$. The charging energy is denoted by $E_c$. The total Josephson energy coupling is $E_j = E_{j1} + E_{j2}$. The dimensionless bias current is $\gamma = I_{ext}/I_c$. The critical current $I_c$ is related to $E_j$.

In this paper, we take a positive value for the inter-band Josephson coupling $E_{in}^{\text{cm}}$. Thus, we focus on the case of a 0 phase shift in the two-band superconductor (i.e., $\phi_R^{(1)} - \phi_R^{(2)} \equiv 0 \mod 2\pi$). We can find that our results do not change qualitatively, when a $\pi$ phase shift (i.e., $\pm s$-wave) occurs.

B. Determining a tunneling path

We now seek a predominant tunneling path on $(\theta, \psi)$. In this paper, we choose an in-phase tunneling path along the $\theta$-axis. Here, we justify this choice based on the physical properties of $\theta$ and $\psi$. First, $\theta$ has a direct coupling to the electric field, whereas $\psi$ does not. Thus, the switching event in this junction is caused by the tunneling of $\theta$. This tunneling is strongly enhanced by the bias current $\gamma$ because the potential barrier height along the $\theta$ axis decreases with increase of $\gamma$. Second, $\psi$ tends to be fixed to $0$ or $\pi$, because the dynamics of the relative phase is subjected to a restoring force induced by the inter-band coupling (in this paper, $\psi$ tends to be $0$ because $E_{in} > 0$). Although $\psi$ fluctuates around these fixed values, the amplitude of the fluctuations is relatively small compared to the oscillation of $\theta$, as described below. Moreover, the fluctuations are not affected by the bias current $\gamma$, in contrast to the tunneling of $\theta$. Therefore, the switching event in a high-bias current condition $\gamma \approx 1$ may occur, via the tunneling along the $\theta$-axis. Thus, we can reduce our issue to an effective 1D tunneling problem along this in-phase path.

Let us more closely examine the dynamical behaviors of $\theta$ and $\psi$ around the in-phase tunneling path. The amplitudes of $\theta$-oscillations and $\psi$-fluctuations are characterized by, respectively, $m_{cm}^{-1}$ and $m_{rlt}^{-1}$. Since the dimensionless constants $\alpha_1$ and $\alpha_2$ are small ($\alpha_1 < 1$), we have small fluctuations of $\psi$. Hence, we examine the switching event in this Josephson junction, using the in-phase tunneling path with small relative-phase fluctuations. Along this tunneling path, the dynamics of $\theta$ is expressed by a particle under the so-called washboard potential. Furthermore, the dynamics of $\psi$ can be expressed by a simple harmonic oscillator, with angular frequency

$$\omega_L = \frac{1}{\hbar} \sqrt{2(\alpha_1 + \alpha_2)E_c E_{in}}.$$  

In other words, $\psi$ is regarded as a bosonic environment for the center-of-mass phase. We will show these points, via the derivation of the Euclidean Lagrangian with the expansion of $\mathcal{L}$ around the in-phase tunneling path.

C. Euclidean Lagrangian around an in-phase tunneling path

Now we derive the Euclidean Lagrangian, with three steps. Throughout this paper, we denote the imaginary time as $\tau (= it)$. First, we take a small $\psi$-expansion, up to second order. We obtain the Euclidean Lagrangian

$$\mathcal{L}^E = \mathcal{L}^E_{\text{cm}} + \mathcal{L}^E_{\text{rlt}} + \mathcal{L}^E_{\text{int}},$$

with

$$\mathcal{L}^E_{\text{cm}} = \frac{m_{cm}}{2} \left( \frac{d\theta}{d\tau} \right)^2 - E_1 (\cos \theta + \gamma \theta),$$  

$$\mathcal{L}^E_{\text{rlt}} = \frac{m_{rlt}}{2} \left( \frac{d\psi}{d\tau} \right)^2 + \frac{1}{2} m_{rlt} \omega_L^2 \psi^2,$$  

$$\mathcal{L}^E_{\text{int}} = g_1 E_j \psi^2 \cos \theta - g_2 E_j \psi \sin \theta.$$  

The first term ($\mathcal{L}^E_{\text{cm}}$) is the Lagrangian for the center-of-mass $\theta$, the second term ($\mathcal{L}^E_{\text{rlt}}$) is for the relative phase $\psi$, and the third term describes the interaction between

FIG. 2. (Color online) Schematic diagrams of two effects of the Josephson-Leggett (JL) mode on a macroscopic quantum tunneling (MQT) process. (a) MQT-enhancement by the decrease of the barrier height. The curves indicate the potential energy with (blue) and without (red) JL mode. (b) MQT-suppression by dissipation (quantum dissipation). The wiggling arrow schematically shows the energy dissipation from $\theta$ to $\psi$ via linear coupling $C_{\text{int}} \theta \psi$. 

\(\theta\) and \(\psi\). The coupling constants \(g_+\) and \(g_-\) are

\[
g_+ = \frac{1}{2E_1} \left[ \frac{\alpha_1^2}{(\alpha_1 + \alpha_2)^2} E_{11} + \frac{\alpha_2^2}{(\alpha_1 + \alpha_2)^2} E_{12} \right],
\]
\[
g_- = \frac{1}{E_3} \left[ -\frac{\alpha_1}{(\alpha_1 + \alpha_2)^2} E_{11} + \frac{\alpha_2}{(\alpha_1 + \alpha_2)^2} E_{12} \right].
\]

We find that \(g_+\) is positive, whereas \(g_-\) vanishes when the parameters of the respective gaps are equivalent: \(\alpha_1 = \alpha_2\) and \(E_{11} = E_{12}\).

Second, in order to remove the non-linearity with respect to \(\psi\) in the interaction Lagrangian, we use the mean-field approximation. The expectation values of \(\psi\) and \(\psi^2\) for the ground state (i.e., zero-temperature limit) of \(L'_{\text{int}}\) are

\[
\langle \psi \rangle_\psi = 0, \quad \langle \psi^2 \rangle_\psi = \frac{\hbar}{2m_{\text{int}}\omega_L} \equiv \Omega^2,
\]

where the symbol \(\langle \cdot \rangle_\psi\) indicates the expectation value with respect to \(\psi\). Using these values, we rewrite \(L'_{\text{int}}\) as a summation of the expectation values and the deviation from them, \(L'_{\text{int}} = \langle L'_{\text{int}} \rangle_\psi + \delta L'_{\text{int}}\). Omitting higher-order fluctuations, we obtain the linearized interaction Lagrangian,

\[
L_{\text{int}}^E = g_+ \omega^2 \cos \theta - g_- \psi \sin \theta.
\]

We then derive the effective Euclidean Lagrangian with the mean-field approximation,

\[
L^E(\theta, \psi) = \frac{m_{\text{cm}}}{2} \left( \frac{d\theta}{d\tau} \right)^2 + V_{\text{eff}}(\theta) + L_{\text{int}}^E - g_- \psi \sin \theta,
\]

with

\[
V_{\text{eff}}(\theta) = -E_1 [1 - \varepsilon] \cos \theta + \gamma \theta,
\]

where \(\varepsilon = g_+ \Omega^2\).

Third, we expand Eq. (13) around the local minimum of \(V_{\text{eff}}\), denoted by \(\theta_0 = \arcsin[\gamma/(1 - \varepsilon)]\). In this paper, we focus on the case \(\gamma \approx 1\); this is typical for MQT experiments. After performing a constant phase-shift transformation, which does not change the path-integral measure, we obtain

\[
L^E(\theta, \psi) \approx \frac{m_{\text{cm}}}{2} \left( \frac{d\theta}{d\tau} \right)^2 + \bar{V}_{\text{eff}}(\theta) + L_{\text{int}}^E - C_{\text{int}} \theta \psi + \delta V,
\]

with

\[
\bar{V}_{\text{eff}}(\theta) = \frac{\hbar^2 \omega^2_{\text{eff}}(\gamma)}{4E_c} \left( \theta^2 - \frac{\theta^3}{\theta_1} \right),
\]

where \(\theta_1 = \cot \theta_0\) and \(C_{\text{int}} = E_3 g_- \cos \theta_0\). We have dropped constants irrelevant to \(\theta\) and \(\psi\). The current-dependent Josephson-plasma frequency is

\[
\omega_{\text{eff}}(\gamma) = \sqrt{2E_c E_1 (1 - \varepsilon)} \left[ 1 - \left( \frac{\gamma}{1 - \varepsilon} \right)^2 \right]^{1/4}.
\]

We stress that the effect of the JP-JL coupling explicitly appears in this formula. Furthermore, we find that the interaction term \(C_{\text{int}} \theta \psi\) is essentially the same as the system-bath interaction in the Caldeira-Leggett model. The last term in eq. (15) is the counterterm \(\delta V = (C_{\text{int}}^2/2m_{\text{int}}\omega_L^2)^2\), which is added for reproducing Hooke’s law between \(\theta\) and \(\psi\) [i.e., \((\theta - \psi)^2\)].

### D. Escape rate formula

We now show the formula for the MQT escape rate \(\Gamma\). At the low-temperature limit, the escape rate is \(\Gamma = (2/\hbar \beta) \text{Im} \langle K(\beta) \rangle\), with the inverse temperature \(\beta\) and

\[
K(\beta) = \int_{-\infty}^{\infty} d\psi \int_{\theta(0) = 0, \psi(0) = \psi} d\theta \langle D(h) \rangle \langle D(\psi) \rangle \exp \left\{ -\frac{1}{\hbar} \int_0^{\beta} d\tau \bar{L}^E(\theta(\tau), \psi(\tau)) \right\}.
\]

One of the authors (SK) \(\text{developed a method to evaluate the MQT escape rate, for this class of Lagrangian. This approach is essentially the same as the influence-functional method.}\) Thus, we find that when \(\beta \rightarrow \infty\) (i.e., zero-temperature limit) for the case \(\gamma \approx 1\)

\[
\Gamma = \omega_{\text{eff}} \sqrt{\frac{30S_B}{\pi \hbar} \left( 1 + \frac{S_D}{2S_B} \right)} \exp \left[ -\frac{1}{\hbar} (S_B + S_D) \right],
\]

with

\[
S_B = \frac{8}{15} \frac{\hbar^2 \omega_{\text{eff}}^2 \theta_1^4}{2E_c},
\]

\[
S_D = \frac{8 \pi C_{\text{int}}^2 \theta_1^2}{m_{\text{int}} \omega_{\text{eff}}^2} \int_0^\infty g(z) dz,
\]

where \(g(z)\) means the effects of the memory kernel in terms of the influence functional method,

\[
g(z) = \frac{z^4}{(\omega_L/\omega_{\text{eff}}^2 + z^2)^{3/2}} \text{sinh}^2(\pi z).
\]

Here, \(S_B\) is the bounce action of the tunneling particle \(\theta\) along the extremal path on the potential \(\bar{V}_{\text{eff}}(\theta)\), whereas \(S_D\) is the dissipative action, which corresponds to the energy dissipation from \(\theta\) to the environment \(\psi\).

Before closing this subsection, let us summarize the role of the JL mode on MQT based on our theory described above. On the one hand, the zero-point fluctuations give a positive non-zero \(\varepsilon\). This quantity effectively reduces the tunneling barrier height, as seen in Eq. (14). As a result, the zero-point fluctuations enhance the MQT escape rate. On the other hand, the linear interaction \(C_{\text{int}} \theta \psi\) in Eq. (15) causes energy dissipation from \(\theta\) to the environment \(\psi\). The effect of this quantum dissipation appears as the dissipative action \(S_D\) in Eq. (19), then it suppresses the MQT escape rate. In Figs. 2 (a) and (b), we show the schematics of these two major roles of the JL mode.
FIG. 3. (Color online) Macroscopic quantum tunneling escape rate, versus the density-of-states difference $\delta \alpha$ and the Josephson-energy difference $\delta E_J$ between the two tunneling channels shown in Fig.1. $\omega_p = \sqrt{2E_cE_J}/\hbar$ is the Josephson-plasma frequency. The black mesh surface ($\Gamma$) shows the escape rate with the inter-band fluctuations. In contrast, the purple mesh surface ($\Gamma_0$) indicates the escape rate without the inter-band fluctuations.

FIG. 4. (Color online) Ratio of the dissipative action to the bounce action, $S_D/S_B$. The open square and triangle correspond to the parameters for the iron-based superconductor BaFe$_2$As$_2$ and MgB$_2$, respectively.

IV. ESCAPE RATES WITH DIFFERENT JUNCTION PARAMETERS

We now numerically evaluate the MQT escape rate \[ \Gamma \] in order to discuss the MQT in general 2-band Josephson junctions, we perform the calculations for different junction parameter sets ($\alpha_i$, $E_{Ji}$), with fixed $E_c/E_J (=0.002)$, $\gamma (=0.9)$, and $\alpha_1 + \alpha_2 (=0.1)$. We use two parameters characterizing the differences between the two tunneling channels,

$\delta \alpha = \frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2}, \quad \delta E_J = \frac{E_{J1} - E_{J2}}{E_J}. \quad (23)$

The former characterizes the density-of-states difference in the vicinity of the interface, while the latter is the normalized Josephson-energy difference. We examine $\Gamma$ in the parameter space ($\delta \alpha$, $\delta E_J$).

Figure 3 shows the MQT escape rate as a function of $\delta \alpha$ and $\delta E_J$ for $E_{in}/E_J = 0.1$. $\omega_p = \sqrt{2E_cE_J}/\hbar$ is the Josephson-plasma frequency. The black mesh indicates the escape rate with the JL mode, while the purple surface indicates the bare escape rate $\Gamma_0$, namely $\Gamma$ without the JL mode (i.e. $C_{int} = \varepsilon = 0$). At the origin of the ($\delta \alpha$, $\delta E_J$) space in which all band parameters are equivalent, $C_{int}$ becomes zero. Therefore, the MQT suppression by quantum dissipation does not appear at this
The MQT in this ideal condition was studied by two of the authors (YO and MM). Figure 3 indicates that the MQT escape rate is enhanced by the JL mode for various hetero Josephson junctions with two-band superconductors, whereas the effect of MQT suppression is marginal.

The results in Fig. 3 indicates that the energy dissipation is relatively small, compared to the energy of the bounce motion of \( \theta \). To clarify this point, we calculate the ratio of the dissipative action to the bounce action. Figure 4 shows the contour map of \( S_D/S_B \), with different junction parameters \( (\delta \alpha, \delta E_J) \). The open square and the triangle in Fig. 4 indicate the parameter sets \( (\delta \alpha, \delta E_J) = (-0.126, 0.251) \) and \( (-0.236, 0.447) \), respectively. The former is evaluated by typical material parameters for \( \text{BaFe}_2\text{As}_2 \) while the latter for \( \text{MgB}_2 \). We find that \( S_B \) is much smaller than \( S_D \). It is noteworthy that the environment \( \psi \) oscillates with single angular frequency \( \omega_L \). Thus, there is only one dissipation channel in our system. This fact would lead to a small energy dissipation.

Finally, for clarifying the dependence of \( \Gamma \) on the inter-band Josephson energy, we calculate \( \Gamma \), with different \( E_{in}/E_J \). Figure 5(a) shows \( \Gamma \) as a function of \( E_{in}/E_J \). In this calculation, we use again the typical material parameters for \( \text{BaFe}_2\text{As}_2 \) and \( \text{MgB}_2 \), as seen in Fig. 4. We find that \( \Gamma \) sharply increases with decreasing \( E_{in}/E_J \). In order to understand this behavior, we plot \( \varepsilon \) and \( C_{int} \) as functions of \( E_{in}/E_J \) in Figs. 5(b) and (c). The magnitude of the zero-point fluctuations of the JL mode \( \varepsilon \) is large, with decreasing \( E_{in}/E_J \). This behavior corresponds to the fact that the JL angular frequency \( \omega_L \) decreases when \( E_{in} \) decreases. Thus, the zero-point fluctuations \( \Omega^2 \) increase, for small \( E_{in} \) [See Eq. (11), as well]. Therefore, the reduction of the tunneling barrier height is marked for small \( E_{in} \). We also find that the coupling strength of the quantum dissipation \( C_{int} \) for the \( \text{MgB}_2 \) parameter set is larger than the \( \text{BaFe}_2\text{As}_2 \) parameter set, as shown in Fig. 5(c). In contrast, we find little difference in \( \varepsilon \) for these two parameter sets. Hence, the energy dissipation of \( \text{MgB}_2 \) is remarkable, compared to \( \text{BaFe}_2\text{As}_2 \). As a result, the enhancement of the MQT rate for \( \text{MgB}_2 \) is smaller than that of \( \text{BaFe}_2\text{As}_2 \).

Let us now summarize our results. The MQT in hetero Josephson junctions with two-band superconductors is strongly affected by the presence of the JP-JL coupling. In other words, the MQT escape rate reflects the nature of the JL mode characterized by the superconducting material parameters, i.e., the density of states near the interfaces and the inter-band Josephson energy.

V. CONCLUSION

We constructed a theory of macroscopic quantum tunneling (MQT) in Josephson junctions consisting of multi-band superconductors, and clarified the effect of inter-band phase fluctuations, namely, the Josephson Leggett (JL) mode on the MQT. In order to discuss the essential effect of the JL mode, we employed a minimal model of the multi-band Josephson junction: a hetero Josephson junction consisting of a conventional single-gap superconductor and a two-gap superconductor. We focused on the in-phase tunneling path along the center-of-mass motion of the phase differences, which is directly related to low-temperature switching events. In the tunneling process along the in-phase path, the effect of the JL mode is caused by the interaction between the JP mode and the JL mode. We derived a Lagrangian which explicitly includes the JL-JP coupling by using a mean-field approximation. The derived Lagrangian is similar to that of the Caldeira-Leggett model for dissipative quantum tunneling. Based on the imaginary-time path-integral method, we derived a formula for the escape rate from this Lagrangian.

In our junction, the JL mode plays two major roles which are opposite to each other: (i) the enhancement of quantum tunneling by lowering the tunneling barrier height, and (ii) the suppression of quantum tunneling by quantum dissipation. We calculated the MQT escape rate, systematically varying the junction parameters. We clarified that the enhancement effect is dominant, and that the MQT escape rate is significantly enhanced by the JL mode. The amount of the MQT enhancement depends on the properties of the JL mode characterized by the superconducting material parameters, such as the inter-band Josephson energy and the density of states near the interfaces of the junctions. Therefore, a precise analysis of the MQT would provide valuable information for the JL mode in multi-band superconductors.

ACKNOWLEDGEMENTS

We wish to thank for I. Kakeya and S. Kashiwaya for valuable discussions and comments. This work is partially supported by a Grant-in-Aid for JSPS Fellows, a Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture of Japan (Grant No. 24510146). FN is partially supported by RIKEN iTHERES Project, MURI Center for Dynamic Magneto-Optics, JSPS-RFBR Contract No. 12-02-92100, and a Grant-in-Aid for Scientific Research (S).
