Sensitive and stable vector magnetometer for operation in zero and finite fields

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Abstract: We report on an optically pumped magnetometer that uses multiple laser beams to pump and probe spin-polarized Cs atoms. The selected sensor geometry allows for operation in finite magnetic fields as well as close to zero field. In finite fields the magnetometer employs free spin precession signals to determine the field modulus and direction as described in a separate publication. This publication focuses on the magnetometer operation close to zero field, which is based on a ground state Hanle resonance. The four laser beams permit the simultaneous measurement of two orthogonal magnetic field components in a differential detection scheme that greatly suppresses technical laser power noise. Sensitivities better than 54 fT/Hz1/2 could be demonstrated simultaneously for both measurement channels in a well shielded environment. A minimum Allan deviation, limited by residual field fluctuations, of better than 40 fT was observed for integration times of 2s. The magnetometer achieves high sensitivity and stability in offset fields as well as close to zero field and is, thus, a universal tool for low frequency magnetic field measurements.

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1. Introduction

Optically pumped magnetometers (OPM) [1, 2] are among the most sensitive magnetic field sensors known today [3]. In such sensors, atomic spin polarization is created using optical pumping [4, 5] and its change due to the interaction with the magnetic field is monitored by optical means. Most commonly, the sensors monitor vector spin polarization (spin orientation), which is created with circularly polarized pumping light [6] and precesses in the magnetic field with the Larmor frequency \( \omega_L = \gamma |B| \). In this study we focus on \(^{133}\text{Cs}\) atoms in the ground state with the overall atomic spin \( F = 4 \). The precession of that spin is characterized by the gyromagnetic ratio \( \gamma \approx 3.5 \) Hz/nT and the effective transverse spin coherence time \( T_2 \) with its corresponding spin relaxation rate \( \Gamma = 1/T_2 \).

The large variety of OPM sensors can be classified according to the magnetic field strength \( |B| \) they are designed for. The first class are zero-field sensors which operate at field strengths \( |B| \ll \Gamma/\gamma \). Their Larmor precession is much slower than the spin relaxation and their operation is based on the ground state Hanle effect [7]. The very sensitive SERF magnetometers [3, 8] are a sub class of zero-field sensors. Zero-field sensors are usually sensitive to a single magnetic field component given by the geometry of the sensor. Using modulation techniques that periodically change one or several magnetic field components, it is possible to gain sensitivity to more than one component of the magnetic field vector [9].

A second class of optically pumped magnetometers is designed to operate in a finite magnetic fields \( |B| \gg \Gamma/\gamma [10, 11] \). Under those conditions the Larmor precession is much faster than the spin-relaxation rate, which means that many Larmor periods can be observed during the coherence time using magnetic resonance techniques. Magnetometers in this regime include the classical \( M_x \) and \( M_z \) [11] designs that use oscillating magnetic fields to manipulate the precessing...
spins as well as all-optical variants where all necessary interactions are mediated by light [12]. Such magnetometers primarily measure the modulus of the magnetic field and can in addition be sensitive to the direction of the field [12, 13].

The magnetometer setup used in this study is capable of delivering highly sensitive and stable readings in zero field as well as in finite fields. The performance in finite fields is published in [13], where we show that the magnetometer can be used for sensitive vector field measurements with 80 fT magnitude resolution and better than 8 µrad angular resolution for a few seconds integration time. In the work reported here we show that the same module can achieve similar performance in zero field when operated in Hanle mode.

2. Experimental setup

The experimental setup uses four laser beams, which all contribute to optical pumping and probing of the Cs spin. We describe the ensemble average $\langle F \rangle$ of the Cs spin as the magnetization $M \propto \langle F \rangle$ of the medium. Figure 1 shows the magnetometer module with the integrated optics that create the four laser beams. The magnetometer module was placed inside the Biomagnetic Shielded Room (BMSR-2) [14] at PTB Berlin. Inside the shield, a system of Helmholtz-coils with 1.40 m and 1.60 m diameter could provide homogeneous magnetic fields along the $x$, $y$, and $z$ axes, respectively. The active medium of the magnetometer consisted of Cs vapor at room temperature contained in an evacuated glass cell. The inside surface of the cell was coated with paraffin to prevent spin depolarization during wall collisions [15]. The light from a external-cavity diode laser, resonant with the $F=4$ to $F=3$ Cs $D_1$ transition (895 nm), was guided to the magnetometer head via a polarization-maintaining single-mode fiber. Close to the magnetometer module, a non-magnetic beam-splitting setup distributed the light to four multi-mode fibers with 0.4 mm core diameter and approx. 1 m length. The large core diameter was chosen because non-magnetic (plastic) setups do not permit coupling into single-mode fibers. The fibers were well fixed in order to minimize the conversion of mechanical vibrations to light intensity changes. Each of the four light beams in the magnetometer head was formed by an optical module (om in Fig. 1) that linearly polarizes and collimates the light from each multi-mode fiber. A second polarizer in the optical module ensured that each beam is circularly polarized with the same helicity when it
We start with the evolution of the magnetization vector $M$. Its time evolution is given by the Bloch equations

$$
\begin{align*}
\dot{M}_x &= (M_x - y \times B_y + \Gamma (0 - M_x)), \\
\dot{M}_y &= (M_y - y \times B_x + \Gamma (0 - M_y)), \\
\dot{M}_z &= (M_z - B_z + \Gamma (0 - M_z)),
\end{align*}
$$

where $\Gamma$ is the effective relaxation rate, which includes the broadening due to optical pumping and assumes isotropic relaxation. The combined optical pumping of all four laser beams results in a source of magnetization along the $z$-direction, which is modeled as a relaxation towards $M = (0, 0, M_0)$. In the following expressions, we parametrize spin relaxation using the magnetic line width $\delta B = \Gamma/\gamma$. The atoms’ absorption coefficient for circularly polarized light contains a term proportional to the projection of the magnetization $M$ onto the $\mathbf{k}$ vector of the considered beam. Consequently, the four beams probe different projections of $M$, which for the steady-state solutions ($M = 0$) of Eq. (1) are given by

$$
\begin{align*}
\frac{M_{xz}}{M_0} &= \frac{B_z B_x + B_x B_z}{B_x^2 + B_y^2 + B_z^2 + \delta B^2}, \\
\frac{M_{yz}}{M_0} &= \frac{B_z (B_x \pm B_y) + \delta B (\delta B \pm B_x)}{B_x^2 + B_y^2 + B_z^2 + \delta B^2}.
\end{align*}
$$

Here $M_{xz}$ and $M_{-xz}$ are the projections of the magnetization probed by the laser beams along the $+xz$- and $-xz$-directions respectively. Correspondingly, $M_{+yz}$ and $M_{-yz}$ are the projections of the magnetization onto the $y-z$ plane.
In order to verify the validity of those line shapes, a slow ramp of the $x$ field component $B_x = B_{0x} + B_R$ was applied in the range from $-35.5$ nT $\leq B_R \leq 35.5$ nT, while the transmitted light power of the four laser beams was recorded. The top row of Fig. 2 shows the preamplifier voltages, which are proportional to the transmitted light power recorded for each of the four laser beams. These signals are in principle usable for sensitive magnetometry in the regions, where they show a significant slope $s = dM/dB$ with respect to the applied field. The most sensitive part of the slope for beams 3 and 4 are, however, not centered with respect to $B_R = 0$ which can be corrected using the lock-in technique introduced in Sec. 2. In this mode of operation, a modulation field $B_m = A_m \sin(\omega_m t)$ was applied along the $z$-direction. The oscillating field leads to a corresponding modulation of the detected signals with an amplitude that depends characteristically on the field components in $x$- and $y$-directions. The lock-in amplifiers are used to demodulate the signal and generate an output that is proportional to the signal component that oscillates in phase with the modulation. The top row of Fig. 2 shows the modulated signals at the input to the lock-in amplifiers, while the bottom row shows the output demodulated signals. The most sensitive part of the slope in the demodulated signals for beams 1 and 2 are now centered with respect to $B_R = 0$ and the two signals from the same plane have very similar shapes except for a different sign. Subtracting the two mirrored signals further increases the signal amplitude and suppresses correlated noise. This differential detection scheme is discussed later and leads to the data presented in Fig. 3.

To model the output demodulated signals of the lock-in amplifier, we substitute $B_z \rightarrow B_z + B_m$ in (Eq. 2), expand the expressions into a Taylor series and select the terms with $\sin(\omega_m t)$ up to the third order. We then take the difference between the expressions obtained for $M_{xz}$ and $M_{-xz}$ (beam 1 and beam 2); and $M_{yz}$ and $M_{-yz}$ (beam 3 and beam 4), respectively. This results in the line shapes of the differential output signals $f_{xz}$ and $f_{yz}$, measured by beams in $xz$- and $yz$-planes:

$$f_{xz} = -2A_M \frac{B_x(D - 2B_z^2) + 2B_yB_z \delta B}{D^2}$$

$$+ 3A^3_M \frac{B_x(8B_z^4 - 8B_z^2D + D^2)}{2D^4}$$

$$+ 3A^3_M \frac{B_y(8B_z^4 - 8B_z^2D + D^2) - 4B_yB_z \delta B(D - 2B_z^2)}{2D^4}.$$

$$f_{yz} = 2A_M \frac{B_y(D - 2B_z^2) - 2B_xB_z \delta B}{D^2}$$

$$- 3A^3_M \frac{B_y(8B_z^4 - 8B_z^2D + D^2) - 4B_yB_z \delta B(D - 2B_z^2)}{2D^4}.$$
Fig. 3. Differential lock-in output signals from beams in \(xz\)-plane - sensitive to \(B_x\) component (blue dashed line), in \(yz\)-plane - sensitive to \(B_y\) component (yellow dashed line), together with global fit (red solid line) according to Eq. (5). The residual field \(B_{0x}\) causes the small offset between the zero crossing and \(B_R = 0\).

Here \(D = B_x^2 + B_y^2 + B_z^2 + \delta B^2\) is the typical resonance denominator. The terms proportional to \(A^3_M\) disappear if the Taylor series is expanded only to linear order. Close to zero field (\(|B| \ll \delta B\)), the magnetometer signals simplify to linear zero crossings

\[
fxz = -\frac{2A_M}{\delta B^2} B_x, \quad fyz = \frac{2A_M}{\delta B^2} B_y.
\]  
(4)

This approximation shows that the \(fxz\)-signal is mostly sensitive to the \(B_x\) component of the field, and the \(fyz\) signal to the \(B_y\) component. To first order both signals are not sensitive to \(B_z\).

In order to compensate for possible misalignments between the laser beams that define the \(xz\)- and \(yz\)-planes and the coils producing the ramping field, the anticipated line shapes of the measured signals are parameterized by

\[
f = \begin{cases} 
A_1 \cdot fxz + A_2 \cdot fyz & (xz\text{-signal}) \\
A_3 \cdot fxz + A_4 \cdot fyz & (yz\text{-signal}) 
\end{cases}
\]
(5)

where the amplitudes \(A_1\), \(A_2\), \(A_3\) and \(A_4\) take into account mutual contributions of the differential signals. Since the functions \(f\) are dimensionless, the amplitudes contain the overall laser power and scaling factors such as amplifier gains. They also permit to model differences in laser power in the different beams.

The experimental line shapes are shown in Fig. 3 along with a simultaneous fit of the two functions from Eq. (5). One can conclude, that experimental data are well reproduced by the model predictions. In this fit, the quantities \(A_1\ldots A_4\), \(B_{0x}\ldots B_{0y}\), and \(\delta B\) were set as free parameters. This permits to determine values of the transverse components of the residual magnetic field \(B_{0x}\) and \(B_{0y}\), which yields: \(B_{0x} = 0.329(32)\) nT, \(B_{0y} = -0.366(24)\) nT, \(\delta B = 3.483(43)\) nT. The given uncertainties are statistical 1\(\sigma\) errors determined by the fitting algorithm.

The field component \(B_x = B_{0x} + B_R\) was ramped very slowly (60 s ramping time) in order to record the data shown in Fig. 3, since the magnetometer has a limited bandwidth. The effective bandwidth is given by the time constant of the low-pass filter in the lock-in amplifier (\(T_c = 100\) ms...
Fig. 4. Time series of the magnetic field measured during the night from December 14 to 15, 2016. The black line shows the evolution of $B_x = 62 \text{ pT}$ and the blue line of $B_y = 30 \text{ pT}$. The offsets were chosen to fit the two time series in the same graph. Both time series were smoothed using a moving average with a window length of 10 min in order to highlight the long term evolution. For each averaging window a noise density spectrum was computed and the average noise density for a frequency band from 0.5 to 1.2 Hz is displayed as the width of the gray bands. In order to be visible, the noise density was scaled by a factor of $50 \text{ pT} / (\text{pT/Hz}^{1/2})$. The time with the best noise density observed during this night is indicated as a scale reference. The analysis shown in Figs. 5 and 6 is based on the indicated periods.

4. Sensitivity and long term stability

In order to determine the sensitivity and stability of the magnetometer, we recorded the lock-in amplifier output for long periods of time (overnight). We prepared the measurement by carefully degaussing the BMSR-2 and tried to keep the field inside the room as constant as possible. The signal corresponded to the point $B_R = 0$ in Fig. 3, where small changes of the field are linearly translated to signal changes according to Eq. (4). The resulting time series of magnetic field values for the $x$ and $y$ direction are depicted in Fig. 4 and show a typical behavior for the BMSR-2. The field in the $x$-direction is mainly influenced by the door and needs two hours after the door was closed to stabilize. Apart from that, field changes are dominated by the magnetic disturbances caused by the Berlin subway lines. Usually, the subway stops operating for a few
Fig. 5. Noise density spectrum $\rho(f)$ of the recorded time-series for $B_x$ (black) and $B_y$ (blue). Due to the logarithmic scaling of the frequency axis the frequency bins at higher frequencies are very closely spaced. Here several bins were averaged in order to reduce the scatter. The red curve is computed using a segment of the $B_x$ time series that shows particularly low noise (see text). The gray curve shows the original spectrum $\rho(B_x)$ before low-pass filtering.

hours in the night and the best noise performance is obtained during that time. Unfortunately, construction works in the subway tunnels were carried out during our data taking, which caused disturbances almost constantly throughout the night. As indicated in Fig. 4, the following noise and stability analysis is based on a time series that covers the time from 11:42 pm till 08:02 am on December 15, 2016, which does not include the two hour period needed to let the door settle.

Figure 5 shows the noise density spectrum of the measured magnetic field components $B_x$ and $B_y$. In addition to the two spectra for the entire time series, the spectrum of a 3.4 h period starting at 1:09 am (red curve) of the $B_x$ time series is shown, which was selected for particularly low noise probably due to reduced subway activity during that time. All spectra show a white noise region extending down to $\approx 0.1$ Hz. The selected segment shows the lowest white noise density averaged over a frequency span from 0.5 Hz to 1.1 Hz of 35 fT/Hz$^{1/2}$. The noise shown in the time series that cover the whole night is slightly higher with $\rho(B_x) = 44$ fT/Hz$^{1/2}$ and $\rho(B_y) = 54$ fT/Hz$^{1/2}$. The difference between the two channels is due to slightly different probe powers and the non-uniformity of the noise in BMSR-2.

For frequencies above 2 Hz, all original spectra showed an increase in noise due to vibrational resonances in the shielding room and an imperfect suppression of noise in the lock-in amplifier (see gray curve in Fig. 5). The imperfection was due to a reduced lock-in time constant of $T_c = 30$ ms for this measurement which was chosen in order to maximize the usable bandwidth. In order to suppress that noise, all time series were filtered with a steep FIR low-pass filter [17] with a flat pass-band from DC to 1.9 Hz and an equivalent noise bandwidth of 2.3 Hz. The long time stability of the readings for $B_x$ and $B_y$ was characterized using Allan deviations (ADEV) of the same times series as in the spectra. Figure 6 shows the dependence of the ADEV on the integration time ($\sigma - \tau$ plot). All ADEV-plots were calculated with overlapping samples (see Sec. 5.2.2 in [18]) in order to reduce the scatter. The use of the low-pass filter induces
correlations in the samples which lead to a rising ADEV at small integration times $\tau < 0.2$ s.

The white noise region for the measurements of $B_x$ (black and red curves in Fig. 5) are slightly sloped, while the one for $B_y$ is almost flat. As a consequence, only the ADEV for $B_y$ shows a segment between $\tau = 0.2$ s and $\tau = 0.5$ s that follows the expected $\tau^{-1/2}$ scaling. The ADEV for longer integration times is dominated by the instability of the magnetic field in the BMSR-2 and shows typical values for this shield. We expect that the curves would follow the statistical scaling for longer integration times if the experiment were repeated in a more stable field. Under the given conditions we find a minimum ADEV of 32 fT for $\tau = 1$ s for the selected data set and slightly worse values of about 40 fT for the entire time series.

5. Conclusions

In summary, the results from our experiment demonstrate highly sensitive simultaneous measurements of the magnetic field vector components along two orthogonal directions. The measurements are based on a ground-state Hanle resonance, that is well described by the derived algebraic expressions for the line shapes. We find that the sensor geometry with four laser beams allows for efficient magnetometer operation in offset fields [13] as well in zero-field conditions. The use of two laser beams in one plane permits a differential detection scheme, which greatly suppresses the influence of common mode power fluctuations. This is particularly useful in the zero-field case, where the detection is based on signal components with frequencies of about 10 Hz which can be strongly affected by technical noise of the laser power and frequency. A lock-in detection scheme further suppresses instabilities in the zero-field case by shifting the detection frequency away from DC. We found a high stability of the readings with an almost flat part of the noise spectrum extending down to 0.1 Hz. In contrast to most SERF magnetometers, the four beam magnetometer is thus particularly well suited to detect low-frequency magnetic signals. The instability that leads to a rising Allan deviation (Fig. 6) for integration times $\tau > 1$ s is typical for
BMSR-2 and thus probably not induced by the magnetometer.

The Cs medium used is optimized for room-temperature operation with a narrow linewidth of about 3 nT. Using a SERF medium with the four beam pumping and readout scheme would result in a larger detection bandwidth and potentially higher sensitivity. However, the magnetometer cannot measure in an offset field if a medium in the SERF regime is used. The ability to switch from zero-field to finite-field operation without need to change anything in the sensor head allows for a very flexible use of the sensor. The device and methods are thus very useful as a general-purpose magnetometer and especially in applications that require operation in both magnetic field regimes. A possible application of this sensor is its use in precision experiment like the search for an electric dipole moment (EDM) of the neutron [19, 20]. In normal operation such experiment use a coil producing a field of 1 µT is powered inside a strong magnetic shield. Without the need to change the magnetometer and open the shield the magnetometer may now be used to investigate with the coil powered off the properties of the shield, i.e. characterizing the quality of degaussing [16].

The magnetometer is well suited for all applications that require detecting small changes in the magnetic field. In precision experiments magnetic field changes that correlate with changes in the experimental conditions are often a source of systematic uncertainties. In EDM searches such uncertainties, for example, can be caused by magnetic field changes correlated with the reversal of the electric field (see Sec. 8B in [20]). The four beam magnetometer permits to study such correlations with or without offset field and thus gains additional information about the possible cause of the correlation. The noise spectrum in Fig. 5 shows that the highest sensitivity is obtained for detection frequencies between 1 Hz and 2 Hz. This fact is also reflected by the part of the ADEV in Fig. 6 that scales approximately statistically e.g. proportional to $\tau^{-1/2}$. A detection frequency of 1 Hz corresponds to averaging the time series over segments of $\tau = 0.5$ s. The curve $\rho(B_{\text{selected}})$ in Fig. 6, which is least effected by external noise, shows that the magnetometer can compare one segment of length $\tau = 0.5$ s to the next one with a statistical uncertainty of 35 fT.

The corresponding statistical uncertainty for $\tau = 0.5$ s in a finite magnetic field is 110 fT according to the recordings presented in [13]. This measurement was taken in a pulsed mode of operation that is intrinsically not as sensitive as the continuous recording used in zero field. Continuous magnetic resonance techniques [11] can be adapted to the four beam setup and should further improve the performance in finite field mode.

In both modes of operation the ADEV curves demonstrate a high stability which permits to compare events that are separated in time by hundreds of seconds with a statistical uncertainty of better than 300 fT. This is important, for example, for studying correlations due to experimental parameters that cannot be quickly modulated like the reversal of a strong electric field.

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**References**

1. D. Budker and D. F. J. Kimball, eds., Optical Magnetometry (Cambridge University Press, 2013).
2. D. Budker and M. Romalis, “Optical magnetometry,” Nat. Phys. 3, 227–234 (2007).
3. I. K. Kominis, T. W. Kornack, J. C. Allred, and M. V. Romalis, “A subfemtotesla multichannel atomic magnetometer,” Nature 422, 596–599 (2003).
4. C. Cohen-Tannoudji and A. Kastler, “Optical pumping,” in *Progress in Optics*, vol. 5 E. Wolf, ed. (North-Holland, 1966).
5. W. Happer and B. S. Mathur, “Effective operator formalism in optical pumping,” Phys. Rev. 163, 12–25 (1967).
6. Y. Shi, T. Scholtes, Z. D. Grujić, V. Lebedev, V. Dolgovskiy, and A. Weis, “Quantitative study of optical pumping in the presence of spin-exchange relaxation,” Phys. Rev. A 97, 013419 (2018).
7. N. Castagna and A. Weis, “Measurement of longitudinal and transverse spin relaxation rates using the ground-state Hanle effect,” Phys. Rev. A 84, 053421 (2011).
8. W. C. Griffith, S. Knappe, and J. Kitching, “Femtotesla atomic magnetometry in a microfabricated vapor cell,” Opt. Express 18, 27167–27172 (2010).
9. S. J. Seltzer and M. V. Romalis, “Unshielded three-axis vector operation of a spin-exchange-relaxation-free atomic magnetometer,” Appl. Phys. Lett. 85, 4804–4806 (2004).
10. E. B. Alexandrov and V. A. Bonch-Bruevich, “Optically pumped atomic magnetometers after three decades,” Opt. Eng. 31, 711–717 (1992).
11. A. Weis, G. Bison, and Z. D. Grujić, “Magnetic resonance based atomic magnetometers,” in *High Sensitivity Magnetometers. Smart Sensors, Measurement and Instrumentation*, A. Grosz, M. J. Haji-Sheikh, and S. C. Mukhopadhyay, eds. (Springer, Cham, 2017).
12. B. Patton, E. Zhivun, D. C. Hovde, and D. Budker, “All-optical vector atomic magnetometer,” Phys. Rev. Lett. 113, 013001 (2014).
13. N. Castagna, G. Bison, K. Bodek, Z. Chowdhuri, L. Hayen, V. Héline, M. Kasprzak, K. Kirch, P. Knowles, H.-C. Koch, S. Komposch, A. Kozela, J. Krempel, B. Lauss, T. Lefort, Y. Lemièreme, A. Michedlishvili, O. Naviliat-Cuncic, F. Piegsa, P. Prashanth, G. Quéméner, M. Rawlik, D. Ries, S. Roccia, D. Rozpedzik, P. Schmidt-Wellenburg, N. Severjins, A. Weis, E. Wursten, G. Zsigmond, “Highly stable atomic vector magnetometer based on free spin precession,” Opt. Express 23, 22108–22115 (2015).
14. J. Bork, H.-D. Hahlbohm, R. Klein, and A. Schnabel, “The 8-layered magnetically shielded room of the PTB: design and construction,” in *Proceedings of the 12th International Conference on Biomagnetism, BIOMAG 2000*, J. Nenonen, R. J. Ilmoniemi, and T. Katila, eds. (Helsinki University of Technology, 2001), pp. 970–973.
15. N. Castagna, G. Bison, G. di Domenico, A. Hofer, P. Knowles, C. Macchione, H. Saudit, and A. Weis, “A large sample study of spin relaxation and magnetometric sensitivity of paraffin-coated Cs vapor cells,” Appl. Phys. B 96, 763–772 (2009).
16. F. Thiel, A. Schnabel, S. Knappe-Grueneberg, D. Stolfuss, and M. Burghoff, “Demagnetization of magnetically shielded rooms,” Rev. Sci. Instrum. 78 (2007).
17. R. E. Crochiere and L. R. Rabiner, “Interpolation and decimation of digital signals – a tutorial review,” Proc. IEEE 69, 300–331 (1981).
18. W. J. Riley, *Handbook of Frequency Stability Analysis*, NIST special publication (U.S. Department of Commerce, National Institute of Standards and Technology, 2008).
19. C. Baker, Y. Chibane, M. Chouker, P. Geltenbort, K. Green, P. Harris, B. Heckel, P. Iaydjiev, S. Ivanov, I. Kilvington, S. Lamoreaux, D. May, J. Pendlebury, J. Richardson, D. Shiers, K. Smith, and M. van der Grinten, “Apparatus for measurement of the electric dipole moment of the neutron using a cohabiting atomic-mercury magnetometer,” Nucl. Instruments Methods Phys. Res. A 736, 184 – 203 (2014).
20. J. M. Pendlebury, S. Afach, N. J. Ayres, C. A. Baker, G. Ban, G. Bison, K. Bodek, M. Burghoff, P. Geltenbort, K. Green, W. C. Griffith, M. van der Grinten, Z. D. Grujić, P. G. Harris, V. Héline, P. Iaydjiev, S. N. Ivanov, M. Kasprzak, Y. Kermaidie, K. Kirch, H.-C. Koch, S. Komposch, A. Kozela, J. Krempel, B. Lauss, T. Lefort, Y. Lemièreme, D. J. R. May, M. Musgrave, O. Naviliat-Cuncic, F. M. Piegsa, G. Pignol, P. N. Prashanth, G. Quéméner, M. Rawlik, D. Rebreyend, J. D. Richardson, D. Ries, S. Roccia, D. Rozpedzik, A. Schnabel, P. Schmidt-Wellenburg, N. Severjins, D. Shiers, J. A. Thorne, A. Weis, O. J. Winston, E. Wursten, J. Zejma, and G. Zsigmond, “Revised experimental upper limit on the electric dipole moment of the neutron,” Phys. Rev. D 92, 092003 (2015).