Mesons as Open Strings in a Holographic Dual of QCD

Toshiya IMOTO,1,* Tadakatsu SAKAI2,** and Shigeki SUGIMOTO3,***

1Daiwa Securities Capital Markets Co. Ltd.,
Grand Tokyo North Tower, Tokyo 100-6753, Japan
2Department of Physics, Nagoya University, Nagoya 464-8602, Japan
3Institute for the Physics and Mathematics of the Universe,
The University of Tokyo, Kashiwa 277-8568, Japan

(Received May 26, 2010; Revised July 8, 2010)

We study meson spectrum obtained from massive open string modes in a holographic dual of QCD constructed on the basis of a D4/D8-brane configuration in type IIA string theory. The spectrum includes mesons with spin higher than one. Taking into account the effect of curved background perturbatively, we obtain a mass formula for these mesons. The mass formula exhibits the linear Regge behavior at the leading order with subleading non-linear corrections. We argue that the string spectrum captures some features of the observed meson spectrum. For example, \( a_2(1320), b_1(1235), \pi(1300), a_0(1450), \) etc., are identified as the first excited massive open string states and \( \rho_3(1690), \pi_2(1670), \) etc., are identified as the second excited states.

Subject Index: 129, 168

§1. Introduction

String theory was originally born as a theory of hadrons in the late 1960s.1)–3) One of the nice features of string theory is that the spectrum exhibits the linear Regge behavior, which is one of the mysterious properties found in the observed hadron spectrum. That is, the spin \( J \) and mass squared \( M^2 \) of the hadrons lie on the linear trajectory as

\[
J = \alpha_0 + \alpha' M^2 ,
\]

where \( \alpha_0 \) and \( \alpha' \) are parameters called Regge intercept and Regge slope, respectively. However, there were some difficulties in regarding string theory as the fundamental theory of hadrons at that time. For example, the space-time dimension of string theory is higher than four, and the Regge intercepts turned out to be inconsistent with the observed hadron spectrum. In fact, the Regge intercepts for the leading trajectories are \( \alpha_0 = 1 \) and \( \alpha_0 = 2 \) for open and closed strings, respectively. This implies that there are massless particles with \( J = 1 \) and \( J = 2 \) in the open and closed string spectra, respectively. The massless spin-two state in the closed string spectrum was later interpreted as graviton,4), 5) and string theory has become a promising candidate of a unified theory that includes quantum gravity. Instead, QCD is now established as the fundamental theory of hadrons.

*1) E-mail: imotoggl@gmail.com
**1) E-mail: tsakai@eken.phys.nagoya-u.ac.jp
***1) E-mail: shigeki.sugimoto@ipmu.jp
The situation has drastically changed since the discovery of the gauge/string duality. (For a review, see Ref. 6.) It is conjectured that string theory in a certain ten-dimensional curved space-time can be equivalent to a four-dimensional gauge theory. Many examples of this type of duality have been found. Since the space-time dimension in string theory description is higher than four, it is often called a holographic description of the four-dimensional gauge theory. In some examples, it can be shown that massless particles with \( J = 1, 2 \) in ten-dimensional string theory correspond to massive particles in the four-dimensional gauge theory. Therefore, the gauge/string duality has a potential to solve the old problems in string theory as a theory of hadrons.

A holographic description of a gauge theory that realizes four-dimensional \( SU(N_c) \) QCD with \( N_f \) massless flavors at low energy was proposed in Ref. 7). It is constructed from a D4/D8-brane configuration in type IIA string theory. The closed string sector is the same as that considered by Witten in Ref. 8), in which a supergravity solution corresponding to \( N_c \) D4-branes wrapped on an \( S^1 \) with supersymmetry breaking boundary condition is used to obtain a holographic description of four-dimensional pure \( SU(N_c) \) Yang-Mills theory. This configuration has been used in the study of glueball spectrum. (See, for example, Refs. 9)–12)). As mentioned above, although the gravitons are massless in the ten-dimensional curved space-time, they yield massive glueballs in the four-dimensional world. In addition, to incorporate \( N_f \) flavors of quarks in the system, \( N_f \) D8-branes are placed in the Witten’s D4-brane background as probes. It was argued in Refs. 7) and 13) that the open strings attached on the D8-branes are interpreted as mesons and the low energy effective theory is written as a five-dimensional \( U(N_f) \) Yang-Mills - Chern-Simons (YM-CS) theory. It turned out that this five-dimensional YM-CS theory reproduces various phenomenological models constructed to describe the properties of hadrons. Furthermore, many quantities such as masses and couplings calculated in the holographic description are roughly in good agreement with the experimental results. This five-dimensional gauge field originates from the massless modes in the open string spectrum. It produces a massless pseudo-scalar meson (pion) together with an infinite tower of massive vector and axial-vector mesons with \( J^{PC} = 1^- \) and \( 1^+ \), where \( P \) and \( C \) are parity and charge conjugation parity, respectively. Clearly, it can only cover a part of the whole meson spectrum observed in the experiments. In particular, the mesons with \( J \geq 2 \) cannot be obtained from the five-dimensional gauge field. This is, however, not a serious problem of the model. As suggested from the original idea of string theory, it is natural to expect that higher spin mesons are obtained from the massive excited states in the open string spectrum. The purpose of this paper is to explore this direction. For related works, see Refs. 14)–32).

In this paper, we study the meson spectrum obtained from the massive open string states. Although it is difficult to quantize strings in the curved background with Ramond-Ramond (RR) flux, we are able to estimate the masses of these mesons assuming that the ’t Hooft coupling \( \lambda \) is large. The leading term in the mass formula is just that obtained in the flat space-time, which gives the linear Regge behavior (1.1). Taking into account the effect of the curved background perturbatively, we obtain the next-to-leading terms that give corrections to the formula (1.1). We
also determine the quantum numbers $J^{PC}$ for these mesons and try to identify the meson spectrum obtained from string theory with that observed in the experiment. We argue that it is plausible to identify $a_2(1320)$, $b_1(1235)$, $\pi(1300)$, $a_0(1450)$, etc., as those obtained from the first excited massive open string states, and $\rho_3(1690)$, $\pi_2(1670)$, etc., as the second excited states.

The organization of this paper is as follows. First, we briefly review the model in §2. In §3, we analyze the meson spectrum that is obtained from the massive open string modes. The results are compared with the experimental data in §4. In §5, we summarize our results and discuss possible future directions. In Appendix A, we analyze the spectrum of the second excited open string states that could be identified with excited mesons including those with $J = 3$. In Appendix B, we classify $\mathbb{Z}_2$ symmetries of the system we work in. This plays an important role in identifying the open string states with the mesons found in the experiments. Appendix C is devoted to a study of the effect of RR flux into the meson mass formula.

§2. Brief review of the model

In this section, we provide a brief review of the holographic QCD proposed in Ref. 7) with the aim of fixing our notation and convention. Here, we only describe the necessary ingredients of the model for this paper. See Refs. 7) and 13) for more details.

The model is constructed using a system with D4/D8/D8-branes in type IIA string theory compactified on an $S^1$. To break the supersymmetry completely, we impose the anti-periodic boundary condition on all the fermions of the system along the $S^1$. The radius of the $S^1$ is denoted as $M_{KK}^{-1}$, although we mainly employ the unit with $M_{KK} = 1$ in the following. The $M_{KK}$ dependence can easily be recovered from the dimensional analysis. To realize four-dimensional $SU(N_c)$ Yang-Mills theory, we consider $N_c$ D4-branes wrapped on the $S^1$, and $N_f$ D8-D8 pairs are added to obtain $N_f$ flavors of massless quarks. The D8-branes and D8-branes are placed at the antipodal points on the $S^1$ and extended along the other nine directions.

In the holographic description, the D4-branes are replaced with the corresponding curved background considered by Witten in Ref. 8). The D8-branes are treated as probes, assuming $N_f \ll N_c$. Then, the D8-branes and D8-branes are smoothly connected, and the system becomes $N_f$ D8-branes embedded in the Witten’s D4-brane background. The topology of the background geometry is $\mathbb{R}^{1,3} \times \mathbb{R}^2 \times S^4$ and we parametrize the $\mathbb{R}^{1,3}$ and $\mathbb{R}^2$ by $x^\mu$ ($\mu = 0, 1, 2, 3$) and $(z, y)$, respectively. The metric and dilaton configurations can be written as

$$ds^2 = \frac{4}{27} \lambda_s^2 d\tilde{s}^2,$$

$$d\tilde{s}^2 = K(r)^{1/2} \eta_{\mu\nu} dx^\mu dx^\nu + K(r)^{-5/6} dr^2 + K(r)^{-1/2} r^2 d\theta^2 + \frac{9}{4} K(r)^{1/6} d\Omega_4^2, \quad (2.1)$$

$$e^\phi = \frac{\lambda^{3/2}}{3\sqrt{3}\pi N_c} K(r)^{1/4}, \quad (2.2)$$
where \((r, \theta)\) is the polar coordinate of the \(\mathbb{R}^2\) related to \((z, y)\) by
\[
z = r \sin \theta, \quad y = r \cos \theta, \tag{2.3}
\]
\(d\Omega_4^2\) is the line element on the unit \(S^4\) and \(K(r) = 1 + r^2.\) The parameters \(l_s\) and \(\lambda\) correspond to the string length and ‘t Hooft coupling in QCD, respectively. In addition, the RR 4-form field strength \(F_4\) is proportional to the volume form of the \(S^4\) and satisfies
\[
\frac{1}{2\pi} \int_{S^4} F_4 = N_c. \tag{2.4}
\]

It is important to note that the string length \(l_s\) only appears in the overall factor in the metric. When we consider a string with tension \(T = 1/(2\pi l_s^2)\) embedded in this background, the \(l_s\) dependence in the string world-sheet action cancels out and the system is equivalent to a string with tension
\[
\tilde{T} \equiv \frac{2\lambda}{27\pi}, \tag{2.5}
\]
embedded in the background, whose metric is given by \(ds^2\) in (2.1). Therefore, the masses of the massive string modes remain finite in the decoupling limit \(l_s \to 0,\) implying that the holographic dual description of QCD involves the whole massive states of a string theory rather than just its massless sector. In the following, we consider strings with tension \(\tilde{T}\) embedded in the background with metric given by \(ds^2.\) This is equivalent to setting
\[
\alpha' \equiv l_s^2 = \frac{27}{4} \lambda^{-1}, \tag{2.6}
\]
which is allowed because the \(l_s\) dependence cancels out anyway.** In this convention, we can omit the tilde in \(\tilde{T}\) and \(ds^2.\) From (2.2) and (2.6), we see that the loop correction and \(\alpha'\) correction in string theory correspond to \(\lambda^{3/2}/N_c\) and \(1/\lambda\) corrections, respectively. We assume that these corrections are small, which is justified when \(1 \ll \lambda^{3/2} \ll N_c.\)

In the D4/D8/D8-brane configuration (before replacing the \(N_c\) D4-branes with the corresponding supergravity background), gluons are obtained as the massless modes in the Kaluza-Klein (KK) decomposition of the gauge field on the D4-brane wrapped on \(S^1.\) The massive KK modes in the \(SU(N_c)\) gauge field as well as fermions and scalar fields on the D4-brane world-volume are artifacts of the model and do not have any counterparts in QCD. They all have masses of order \(M_{KK},\) and hence, our model deviates from QCD when the energy is higher than this scale. In principle, we should take a limit \(M_{KK} \to \infty\) and \(\lambda \to 0\) with the QCD scale (e.g., the rho meson mass) kept fixed at the experimental value to send the cutoff to infinity. This is analogous to taking the continuum limit in lattice QCD. The QCD scale \(\Lambda_{QCD}\) is related to \(M_{KK}\) by \(\Lambda_{QCD} = f(\lambda)M_{KK}\) with an unknown function \(f(\lambda).\) To take

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\*\* \(r\) is related to \(U/U_{KK}\) used in Ref. 7) by \((U/U_{KK})^3 = 1 + r^2.\)

\*\*\* This convention differs from that in Ref. 13) by a factor of 2/3 on the right-hand side.
the $M_{KK} \to \infty$ limit, we have to know how the function $f(\lambda)$ behaves in the small $\lambda$ region. However, since our calculation can only be trusted when $\lambda$ is large, this procedure is beyond the scope of this paper. Instead, we use the experimental values of two quantities as inputs to fix $M_{KK}$ and $\lambda$, and expect that the predictions will become more accurate by taking into account the $1/\lambda$ and $1/N_c$ corrections. In Refs. 7) and 13), the experimental values of $\rho$ meson mass $m_\rho|_{\text{exp}} \simeq 776$ MeV and pion decay constant $f_\pi|_{\text{exp}} \simeq 92.4$ MeV are used to fix the values of the parameters $M_{KK}$ and $\lambda$ as

$$M_{KK} \simeq 949 \text{ MeV}, \quad \lambda \simeq 16.6.$$  \hspace{1cm} (2.7)

With this parameter choice, various properties of hadrons predicted in the model turned out to be in reasonably good agreement with the experimental results. Although the cutoff scale $M_{KK}$ is rather low, we can expect that the effect of the cutoff is much milder than that of lattice QCD. In the energy scale of a few GeV, this system can be regarded as a holographic dual of massless QCD with extra massive fields whose masses are of order $M_{KK}$. For many of the low energy quantities of hadrons we are interested in, the main contributions are from the massless gluons and quarks, and the effects of the extra degrees of freedom should not be large enough to alter the order of magnitude. The fact that our formulation does not suffer from the finite volume effects and keeps exact Poincaré symmetry as well as chiral symmetry also helps to reduce the error.

In this paper, we analyze the open strings attached on the D8-branes that are interpreted as mesons in QCD. The D8-branes are placed at $y = 0$, and extended along $x^\mu$, $z$ and $S^4$ directions. The system is invariant under the $SO(5)$ isometry that rotates the $S^4$. Since the quarks and gluons are invariant under this $SO(5)$, only the $SO(5)$ invariant states can be identified with the mesons in QCD. For this reason, we will focus on the $SO(5)$ invariant field configurations. The $SO(5)$ non-invariant states are the artifacts of the model. Such states cannot be the bound states made only of the quarks and gluons, but necessarily involve massive $SO(5)$ non-invariant constituents, and hence, they are expected to be decoupled if we were able to send the cutoff to infinity, as explained above. In §3.1, we will show that there is a $\mathbb{Z}_2$ symmetry that can also be used to clean up the spectrum.

In Refs. 7) and 13), the massless sector of the open string is analyzed. It was shown there that the effective theory on the D8-brane world-volume is given by a five-dimensional YM-CS theory after dimensional reduction on the $S^4$. Expanding the five-dimensional gauge theory with respect to a complete set of normalizable functions of $z$, we obtain an infinite tower of four-dimensional meson fields, including those interpreted as pion, $\rho$-meson, $a_1$-meson, etc. The main goal of this paper is to extend this analysis to the massive open string states.

One of the important quantities used in this paper is the string tension. If we use the values (2.7) in (2.6) and (2.5), we obtain $\alpha' \simeq 0.45$ GeV$^{-2}$ and $T \simeq (0.59 \text{ GeV})^2$. (See also Refs. 33), 12), and 6).) Unfortunately, this value of string tension $T$ is considerably larger than the value estimated in lattice gauge theory $T|_{\text{lattice}} \simeq (0.44 \text{ GeV})^2$. (For a review, see, for example, Ref. 34.) We should not be too serious about this discrepancy, because our analysis is not accurate enough.
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It may be cured by taking into account the $1/\lambda$ and $1/N_c$ corrections.\(^{\ast}\) Another possible correction may be due to the effect of quark masses. Note that the pion decay constant is sensitive to the quark mass, and its value in the chiral limit is about $5 \sim 15\%$ smaller than the experimental value. If we use this value to fix $\lambda$, the value of $\lambda$ will become smaller and the predicted values of $\alpha' \text{ and } T$ will become closer to the expected ones. We will revisit this issue in \S 4.

\section{3. Massive string modes in the model}

In this section, we consider massive open string states in our model. Since it is not easy to quantize strings in the curved space-time described in the previous section, we restrict our analysis to the cases with $\lambda \gg 1$, in which the effect of the non-trivial background can be considered perturbatively.

As explained in \S 2, we have to pick up states that are invariant under $\text{SO}(5)$ isometry. Then, the open string states can be regarded as particles in the five-dimensional space-time parametrized by $x_\mu (\mu = 0, 1, 2, 3)$ and $z$. We will soon show that the states that can be interpreted as mesons in QCD should also be invariant under a $\mathbb{Z}_2$ symmetry. First, we classify the states that are invariant under $\text{SO}(5)$ and $\mathbb{Z}_2$ in \S 3.1, and then identify the parity and charge conjugation quantum numbers in \S 3.3. We demonstrate these procedures by considering the first excited massive string states, which include a spin-2 meson. The results for the second excited states are given in Appendix A.

The non-trivial $r$ dependence of the metric (2.1) is analyzed perturbatively in \S 3.2, where we show that it makes the five-dimensional particles stay around $z = 0$ and they behave as four-dimensional mesons.

\subsection{3.1. Classification of open string states}

When $\lambda$ is very large, the string length is much shorter than the scale of the curvature radii of the background, and it is possible to approximate the space-time around a string by the flat space-time $\mathbb{R}^{1,3} \times \mathbb{R}^2 \times \mathbb{R}^4$, where the $\mathbb{R}^4$ factor denotes a local patch around a point in $S^4$. We parametrize the flat ten-dimensional space-time by $x^M (M = 0, 1, \ldots, 9)$, and also use the notation $(z, y) = (x^4, x^5)$ and $\theta^a = x^a (a = 6, 7, 8, 9)$.

We are interested in the $\text{SO}(5)$ invariant states created by open strings attached on the D8-branes placed at $y = 0$. When we approximate the background by the flat space-time, the $\text{SO}(5)$ isometry of $S^4$ becomes $\text{SO}(4)_{6-9}$ rotational and translational symmetry acting on $\mathbb{R}^4$ parametrized by $\theta^a$. Here, $\text{SO}(n)_{A_1 \sim A_n}$ denotes the orthogonal group acting on the $n$-dimensional space parametrized by $(x^{A_1}, \ldots, x^{A_n})$. Therefore, we ought to find $\text{SO}(4)_{6-9}$ invariant states with the wave function constant along $\theta^a$ directions. As a result, the system is reduced to a five-dimensional space-time parametrized by $x^\mu (\mu = 0, 1, 2, 3)$ and $z$.

To obtain the open string spectrum, it is convenient to consider D9-branes that are related to the D8-branes by T-duality along the $y$-direction.\(^{\ast\ast}\) We first compact-

\(^{\ast}\) See Ref. 19) for the calculation of the $1/\lambda$ correction to the string tension.

\(^{\ast\ast}\) Here, we use the D9-brane picture just to learn the open string spectrum on the D8-branes,
ify the $y$-direction to $S^1$, but restrict our attention to the sector with zero winding number. This system is T-dual to $N_f$ D9-branes with zero momentum along the $S^1$ direction. Here, we use the notation $\tilde{y} = \tilde{x}^5$ to parametrize the T-dualized $S^1$-direction. The background RR 4-form field strength is mapped to an RR 5-form field strength proportional to $d\tilde{y} \wedge d\theta^6 \wedge d\theta^7 \wedge d\theta^8 \wedge d\theta^9$. Then, the $SO(5)$ invariant states on the D8-branes are in one-to-one correspondence with the open string states on the D9-branes that are invariant under the $SO(4)_{6\sim 9}$ and the translational symmetry along the five-dimensional space parametrized by $(\tilde{y}, \theta^a)$. There is an additional constraint we have to impose to obtain the states corresponding to the mesons in QCD. In fact, as we will show in Appendix B, a $Z_2$ action generated by

$$
(\tilde{y}, x^9) \rightarrow (-\tilde{y}, -x^9)
$$

(3.1)

is a symmetry of the whole system, under which quarks and gluons are invariant. Therefore, the mesons in QCD should correspond to the open string states that are also invariant under this $Z_2$. We call this $Z_2$ symmetry “$\tau$-parity”, following Ref. 11.

Massive particles created by the strings attached on the D9-brane can be classified by representations of $SO(9)_{1\sim 9}$ that is the little group for the ten-dimensional massive particle. We first classify the massive string modes by the representations of $SO(9)_{1\sim 9}$, decompose them by the representations of $SO(5)_{1\sim 5} \times SO(4)_{6\sim 9}$ subgroup, and extract the $SO(4)_{6\sim 9}$ invariant components. Then, we pick up the components that are invariant under the $\tau$-parity (3.1).

Let us examine the first excited states explicitly following the procedure explained above. We use the light-cone gauge quantization in the NSR formalism to quantize the open strings on the D9-brane in the T-dualized picture. (See, for example, Ref. 35.) We take $x^0 \pm x^1$ as the light-cone directions. Since the states in R-sector cannot be invariant under $SO(4)_{6\sim 9}$, we consider only the NS-sector. The physical states are constructed by acting with an arbitrary number of the bosonic creation operators $\alpha^I_{-n}$ ($I = 2 \sim 9$; $n = 1, 2, 3, \ldots$) and an odd number of fermionic creation operators $\psi^J_{-r}$ ($J = 2 \sim 9$; $r = 1/2, 3/2, 5/2, \ldots$) on the NS-vacuum $|p\rangle$ as

$$
\alpha^{I_1}_{-n_1} \cdots \alpha^{I_k}_{-n_k} \psi^{J_1}_{-r_1} \cdots \psi^{J_{2l+1}}_{-r_{2l+1}} |p\rangle
$$

(3.2)

with non-negative integers $k$ and $l$. Here, the NS-vacuum $|p\rangle$ is an eigenstate of the momentum operator with the momentum $p$ satisfying the mass shell condition

$$
-p^2 = \frac{N}{\alpha^2} = \frac{4}{27} \lambda N \equiv m_0^2,
$$

(3.3)

where we have used (2.6) and

$$
N \equiv \sum_{i=1}^k n_i + \sum_{i=1}^{2l+1} r_i - \frac{1}{2}
$$

(3.4)

and do not care about the RR tadpole cancellation.

* This $Z_2$ is T-dual of the “$\tau$-parity” used in Ref. 11) to classify the closed string states corresponding to glueballs in QCD. The invariance under this $Z_2$ implies that the massless scalar field on the D8-brane world-volume considered in §4.2 of Ref. 7) cannot be interpreted as scalar mesons in QCD. See Appendix B for further comments.
Here, the momentum $p$ can take non-zero values only in the five-dimensional components $p = (p^\mu, p^z)$ ($\mu = 0, \cdots, 3$), because we impose the translational invariance along $(y, \theta^a)$.

The first excited state with $N = 1$ is

$$\psi_{-3/2}^J |p\rangle, \quad \alpha_{-1/2}^J \psi_{-1/2}^J |p\rangle, \quad \psi_{-1/2}^J \psi_{-3/2}^J |p\rangle,$$

which gives vector representation ($\Box_8$), rank-2 tensor representation ($\Box_8 \otimes \Box_8$), and rank-3 anti-symmetric tensor representation ($\Box_{56}$) of $SO(8)_{2\sim 9}$, respectively. Here, the subscript of each Young tableau represents the dimension of the representation. The rank-2 tensor representation is reducible and it can be decomposed to singlet ($\mathbb{1}$), symmetric traceless representation ($\Box_{35}$), and anti-symmetric representation ($\Box_{28}$).

Although the $SO(9)_{1\sim 9}$ symmetry is not manifest in the light-cone gauge, it is not difficult to see that these multiplets are obtained from $\Box_{84} \oplus \Box_{14}$ of $SO(9)_{1\sim 9}$. The decomposition of these representations with respect to the representation of $SO(5)_{1\sim 5} \times SO(4)_{6\sim 9} \subset SO(9)_{1\sim 9}$ is given by

$$\Box_{14} = (\Box_{14}, \mathbb{1}) \oplus (\mathbb{1}, \Box_9) \oplus (\Box_5, \mathbb{1}) \oplus (\mathbb{1}, \mathbb{1}),$$

$$\Box_{84} = (\Box_{10}, \mathbb{1}) \oplus (\mathbb{1}, \Box_4) \oplus (\Box_5, \mathbb{1}) \oplus (\mathbb{1}, \mathbb{1}).$$

Therefore, the $SO(4)_{6\sim 9}$ invariant states are

$$\mathbb{1} \oplus \Box_{10} \oplus \Box_{14}$$

of $SO(5)_{1\sim 5}$.

Thus far, we have considered the first excited massive states for the open strings attached on the D9-brane, which is T-dual of the D8-brane. The field content for the D8-brane is obtained from the dimensional reduction of that for the D9-brane. Then, the five-dimensional fields corresponding to the first excited states on the D8-brane can be obtained by the dimensional reduction of the six-dimensional fields listed in (3.7), that is, the scalar field $\varphi$, rank-3 anti-symmetric tensor field $A_{\alpha \beta \gamma}$, and traceless symmetric tensor field $h_{\alpha \beta}$ ($\alpha, \beta, \gamma = 1, \ldots, 5$). The components that are invariant under the $\tau$-parity (3.1) are

$$\varphi, \quad A_{MNP}, \quad h_{MN}, \quad h_{yy}$$

with $M, N, P = 1, 2, 3, z$.

Higher excited states can be constructed in a similar manner. We present the results for the second excited states in Appendix A. One of the important properties worth mentioning here is that, as it has been well-known from the early days of string theory, the spectrum exhibits linear Regge behavior. Namely, the highest spin $J$ for each mass level is $J = N + 1 = 1 + \alpha' m_0^2$. In §3.2, we show that the masses of four-dimensional mesons are modified from this linear behavior.

3.2. Mass spectrum

To obtain four-dimensional meson fields from the five-dimensional fields obtained in §3.1, we expand the five-dimensional fields using a complete set of normalizable
functions of $z$ that is chosen to extract mass eigenstates. The idea is the same as
the usual Kaluza-Klein (KK) decomposition. Although the space in the $z$ direction
is non-compact, the non-trivial $z$ dependence of the metric induces a potential that
prevents particles from moving away to infinity and the system effectively becomes
four-dimensional.

For example, the five-dimensional scalar field $\varphi$ is expanded as

$$\varphi(x^\mu, z) = \sum_{n=0}^{\infty} \varphi^{(n)}(x^\mu) \phi_n(z),$$

(3.9)

where $\{\phi_n(z)\}_{n \geq 0}$ is a complete set of normalizable functions of $z$ to be determined
and $\varphi^{(n)}(x^\mu)$ denote the four-dimensional meson fields. The field equation corre-
sponding to the mass shell condition (3.3) in the flat space-time limit is

$$\left( \eta^{\mu\nu} \partial_\mu \partial_\nu + \partial_z^2 - m_0^2 \right) \varphi(x^\mu, z) = 0,$$

(3.10)

where $\mu, \nu = 0, \ldots, 3$ and $m_0^2 = N/\alpha'$ as defined in (3.3). Here, we ignore the
interaction terms assuming that the string coupling $g_s \simeq \lambda^{3/2}/N_c$ (see (2.2)) is
small. In the following, we claim that the leading correction to this equation in
the curved space-time can be included by just replacing $\eta^{\mu\nu}$ in (3.10) by the curved
metric $g^{\mu\nu}(z) = K(z)^{-1/2} \eta^{\mu\nu}$ in (2.1) as

$$\left( g^{\mu\nu}(z) \partial_\mu \partial_\nu + \partial_z^2 - m_0^2 \right) \varphi(x^\mu, z) = 0$$

(3.11)

for large $\lambda$. More precisely, for the warped geometry with $g_{\mu\nu}(z) = f(z) \eta_{\mu\nu}$ and
$f(z) = 1 + c z^2 + O(z^4)$ (in our case, $f(z) = K(z)^{1/2}$ and $c = 1/2$), the terms we
should keep in (3.11) $\times f(z)$ are

$$\left( \eta^{\mu\nu} \partial_\mu \partial_\nu + \partial_z^2 - m_0^2 (1 + c z^2) \right) \varphi(x^\mu, z) = 0,$$

(3.12)

and all the other terms are sub-leading for large $\lambda$. This statement holds not only
for the scalar fields but also for any tensor fields.

To see that the other possible terms in the equations of motion (3.12) can be
neglected, it is convenient to rescale $z$ as

$$w = \lambda^{1/4} z$$

(3.13)

and treat $w$ as an $O(1)$ variable. The motivation for this rescaling is that, as we will
soon show, the typical width of the wave functions in the $z$-direction turns out to
be of $O(1)$ in terms of the rescaled coordinate $w$. Recall that $m_0^2$ defined in (3.3) is
proportional to $\lambda$, and hence, $\eta^{\mu\nu} \partial_\mu \partial_\nu$ in (3.12) should be considered to be of order $\lambda$.
Because of the rescaling (3.13), $\partial_z^2 = \lambda^{1/2} \partial_w^2$ and $m_0^2 z^2 = (m_0^2 \lambda^{-1/2}) w^2$ in (3.12)
are of order $\lambda^{1/2}$. These terms give the leading corrections to the flat space-time
limit. We keep up to $O(\lambda^{1/2})$ terms and neglect the smaller contributions in the
field equation assuming $\lambda$ is large. It is easy to see that the contributions from
possible $O(z^2)$ terms in front of $\partial_z^2$ and $O(z^4)$ terms in $m_0^2 f(z)$ in (3.12) are at most
of $O(1)$ and can be neglected. Since (3.10) is exact in the flat space-time limit,
all the other corrections should involve the derivatives of the background metric, dilaton, and RR fields. The metric and dilaton configurations in (2.1) and (2.2) are independent of $x^\mu$, and the $z$-derivatives of metric and dilaton ($\partial_z g_{MN}$ and $\partial_z \phi$) are of $\mathcal{O}(\lambda^{-1/4})$. Therefore, the terms with derivatives of metric and dilaton field cannot contribute to the $\mathcal{O}(\lambda^{1/2})$ terms. This implies that, for example, we do not have to replace the derivatives in (3.11) with the covariant derivatives, since the terms with Christoffel symbols are sub-leading. The corrections involving the curvature of the space-time are also negligible. One might think that the higher derivative terms may have larger contributions, since $\partial_z \sim \mathcal{O}(\lambda^{1/4})$. However, since each derivative is accompanied by $l_s \sim \lambda^{-1/2}$, the higher derivative terms can be neglected as well.

The effect of the RR field is trickier. We show in Appendix C that the corrections to the equations of motion (3.12) due to the background RR field (2.4) can also be neglected within our approximation.

From (3.12), the masses of the four-dimensional meson fields are obtained as the eigenvalues of the eigenequation

$$(-\partial_z^2 + m_0^2 (1 + c z^2)) \phi_n(z) = M_n^2 \phi_n(z) \, .$$

The eigenfunctions $\{\phi_n(z)\}_{n \geq 0}$ form a complete set, and are used to expand the five-dimensional field as in (3.9), with the eigenvalue $M_n^2$ interpreted as the mass squared of the $n$th meson field $\varphi^{(n)}(x^\mu)$. The eigenequation (3.14) is the same as the Schrödinger equation of the harmonic oscillator so that

$$M_n^2 = m_0^2 + 2m_0 \sqrt{c} \left(n + \frac{1}{2}\right) = \frac{N}{\alpha'} + \sqrt{\frac{2N}{\alpha'}} \left(n + \frac{1}{2}\right) M_{KK}$$

with $n = 0, 1, \ldots$, where we have used $c = 1/2$ and (3.3), and recovered $M_{KK}$ by the dimensional analysis. The eigenfunctions are $\phi_n(z) \propto H_n(\xi) \exp(-\xi^2/2)$, where $\xi = (m_0^2 c)^{1/4} z$ and $H_n(\xi)$ is the $n$th Hermite polynomial, and they satisfy $\phi_n(-z) = (-1)^n \phi_n(z)$.

It would be instructive to derive the formula (3.15) in a more direct way. The action for a particle of mass $m_0$ placed on the D8-brane is

$$S = -m_0 \int dt \sqrt{-g_{tt}} = -m_0 \int dt K(z)^{1/4} \, ,$$

which implies that this particle is trapped around $z = 0$ with a potential

$$V(z) \equiv m_0 K(z)^{1/4} \approx m_0 \left(1 + \frac{z^2}{4}\right) + \mathcal{O}(z^4) \, .$$

Approximating the potential $V(z)$ by a harmonic oscillator, the energy eigenvalues are obtained as

$$M_n \approx m_0 + \frac{1}{\sqrt{2}} \left(n + \frac{1}{2}\right) M_{KK} \, .$$

This result consistently reproduces (3.15) up to $\mathcal{O}(\lambda^{1/2})$ terms.
3.3. Parity and charge conjugation

In Ref. 7), it was shown that the parity transformation in QCD corresponds to flipping the sign of the spatial coordinates in the five-dimensional space-time:

\[(x^0, x^1, x^2, x^3, z) \rightarrow (x^0, -x^1, -x^2, -x^3, -z)\],

and the charge conjugation is a \(\mathbb{Z}_2\) transformation acting on the massless five-dimensional field \(A_M(x^\mu, z)\) \((M = 0, 1, 2, 3, z; \mu = 0, 1, 2, 3)\) on the D8-brane as

\[A_\mu(x^\mu, z) \rightarrow -A_\mu^T(x^\mu, -z), \quad A_z(x^\mu, z) \rightarrow A_z^T(x^\mu, -z)\].

The parity transformation (3.19) works for the massive sector in the same manner. However, the charge conjugation (3.20) is given only for the massless gauge field and we need to know how it acts on the massive string modes. Because the charge conjugation interchanges the left-handed and right-handed components of the quark field, we expect that the coordinate \(z\) should be mapped to \(-z\). In addition, it relates a field with its transpose as in (3.20), implying that the orientation of the open string should be flipped by the charge conjugation. Therefore, the charge conjugation corresponds to an orientifold action that involved the reflection of \(z\). An orientifold action consistent with our brane configuration was found in Ref. 36), in which O6-planes are added to the system to obtain a holographic description of \(O(N_c)\) and \(USp(N_c)\) QCD. It is shown that the \(\mathbb{Z}_2\) symmetry corresponding to the charge conjugation is the orientifold action associated with an O6\(^+\)-plane, which is defined by

\[(z, x^8, x^9) \rightarrow (-z, -x^8, -x^9)\]

(3.21)

together with world-sheet parity transformation.*)

Let \(C\) be the generator of this \(\mathbb{Z}_2\) represented on the open string states. The action of \(C\) on states in the light-cone gauge (3.2) can be read from the relations:

\[C\alpha^\parallel_{-n}C^{-1} = (-1)^n\alpha^\parallel_{-n}, \quad C\alpha^\perp_{-n}C^{-1} = -(-1)^n\alpha^\perp_{-n},\]

\[C\psi^\parallel_{-r}C^{-1} = e^{i\pi\psi^\parallel_{-r}}, \quad C\psi^\perp_{-r}C^{-1} = -e^{i\pi\psi^\perp_{-r}},\]

\[C|0; A, B\rangle = i|0; B, A\rangle\],

(3.22)

where \(\parallel = 2, 3, 6, 7; \perp = z, y, 8, 9\), and \(|0; A, B\rangle\) is the NS-vacuum with Chan-Paton indices \(A, B\). Since we are interested in the \(SO(4)_{6-9}\) invariant states, we can also use \(\mathbb{Z}_2\) defined by (3.22) with \(\parallel = 2, 3, 6, 7, 8, 9\) and \(\perp = z, y\) to examine the charge conjugation parity. It acts on the \(SO(4)_{6-9}\) invariant states of the form (3.2) as

\[(-1)^{N + n_y + n_z + 1}\]

where \(N\) is the excitation level defined in (3.4), \(n_y\) and \(n_z\) are the numbers of \(y\) and \(z\) indices, respectively. Furthermore, it is accompanied by interchanging the Chan-Paton indices and mapping \(z \rightarrow -z\) in the argument of the wave function.

*) The orientifold action associated with an O6-plane is defined by \(I\Omega(-1)^{F_L}\) for closed strings, where \(I\) is an involution that flips three spatial coordinates, \(\Omega\) is the world-sheet parity transformation, and \(F_L\) is the left-moving space-time fermion number.
It is easy to check that the charge conjugation defined in (3.22) yields (3.20) for the massless states $\psi_{-1/2}^M |p\rangle$. For the first excited states obtained in (3.8), the charge conjugation $C$ acts as

\[
A_{ijk}(x^\mu, z) \rightarrow A^T_{ijk}(x^\mu, -z), \quad h_{ij}(x^\mu, z) \rightarrow h^T_{ij}(x^\mu, -z), \quad h_{i}(x^\mu, z) \rightarrow -h^T_{i}(x^\mu, -z), \quad h_{zz}(x^\mu, z) \rightarrow h^T_{zz}(x^\mu, -z), \quad h_{yy}(x^\mu, z) \rightarrow h^T_{yy}(x^\mu, -z), \quad \varphi(x^\mu, z) \rightarrow \varphi^T(x^\mu, -z),
\]

where $i, j, k = 1, 2, 3$.

Four-dimensional meson fields are obtained by expanding the five-dimensional fields as in (3.9). The parity and charge conjugation properties can be read from (3.19) and (3.23), together with the fact that the mode functions satisfy $\phi_n(-z) = (-1)^n \phi_n(z)$. The spin $J$, parity $P$, and charge conjugation parity $C$ for the lightest mesons with $n = 0$ in the first excited massive string states (3.8) are as follows:

\[
\begin{array}{ccccccc}
 & h_{ij}^{(0)} & h_{iz}^{(0)} & h_{zz}^{(0)} & h_{yy}^{(0)} & A_{ijk}^{(0)} & A_{ijz}^{(0)} & \varphi^{(0)} \\
 J^{PC} & 2^{++} & 1^{--} & 0^{++} & 0^{++} & 0^{--} & 0^{++} & \end{array}
\]

(3.24)

For the second lightest modes with $n = 1$, $P$ and $C$ are all flipped, compared with those with $n = 0$.

§4. Comparison with the experimental data

Here, we try to compare our results with the experimental data. Since our model only contains massless quarks, we set $N_f = 2$ and focus on the light unflavored mesons. In particular, we consider the isovector mesons, because isoscalar mesons could be mixed with glueballs. The isovector mesons seen in the meson summary table in Ref. 37) are listed in Table I. A plot of mass squared against spin for these mesons is shown in Fig. 1.

From Table I and Fig. 1, we observe that the mass squared $M^2$ against spin $J$ of $\rho(770)$, $a_2(1320)$, $\rho_3(1690)$, $a_4(2040)$, $\rho_5(2350)$, and $a_6(2450)$ lies on a linear trajectory satisfying

\[
J = \alpha_0 + \alpha' M^2
\]

(4.1)

with $|\alpha_0|_{\text{exp}} \simeq 0.53$ and $|\alpha'|_{\text{exp}} \simeq 0.88 \text{ GeV}^{-2}$. The mesons in this $\rho$-meson trajectory are the lightest mesons with a given spin, and hence, they should correspond to the states with $N = J - 1$ and $n = 0$ in (3.15). The mass formula (3.15) implies

\[
J \simeq 1 + \alpha' M^2_{n=0} - \frac{\alpha'}{\sqrt{2}} M_{KK} M_{n=0} + \mathcal{O}(1/\lambda)
\]

(4.2)

for these mesons except for $\rho(770)$. The lightest vector meson $\rho(770)$ is obtained from the massless sector with $N = 0$, for which we cannot use the mass formula (3.15). As mentioned in §3.1, the leading terms in (4.2) reproduce the linear trajectory (4.1).
Table I. Isovector mesons from meson summary table in Ref. 37). The left row denotes $J^P$ and each three- or four-digit number on the table represents the approximate mass used as a part of the name of the meson in units of MeV. The superscript $\Delta$ is assigned for the mesons that are not regarded as being established.

| $J^P$ | Name | Mass (MeV) |
|-------|------|------------|
| $0^{-+} (\pi)$ | 135 | 1300 | 1800 |
| $0^{++} (a_0)$ | 980 | 1450 |
| $1^{--} (\rho)$ | 770 | 1450 | 1570$\Delta$ | 1700 | 1900$\Delta$ | 2150$\Delta$ |
| $1^{++} (a_1)$ | 1260 | 1640$\Delta$ |
| $1^{-+} (b_1)$ | 1235 |
| $1^{-+} (\pi_1)$ | 1400 | 1600 |
| $2^{++} (a_2)$ | 1320 | 1700$\Delta$ |
| $2^{-+} (\pi_2)$ | 1670 | 1880 | 2100$\Delta$ |
| $3^{--} (\rho_3)$ | 1690 | 1990$\Delta$ |
| $4^{++} (a_4)$ | 2040 |
| $5^{--} (\rho_5)$ | 2350$\Delta$ |
| $6^{++} (a_6)$ | 2450$\Delta$ |

Fig. 1. Mass squared against spin of the isovector mesons in Ref. 37).

with $\alpha_0 = 1$. The subleading $O(\lambda^{-1/2})$ term in (4.2) gives the deviation from it. Note that a similar non-linear behavior is found in the analysis of spinning strings in Refs. 15) and 19). Unfortunately, as discussed in §2, the value of $\alpha'$ evaluated from (2.6) and (2.7) is not good enough to fit the experimental data. Given the fact
that the correction to $\alpha'$ in (2.6) and (2.7) should be large, it may be interesting to see if the observed meson masses can be obtained by adjusting the value of $\alpha'$. In fact, if we set $\alpha' \simeq 1 \text{ GeV}^{-2}$, the masses predicted by the formula (4.2) are in very good agreement with those of $a_2(1320)$, $\rho_3(1690)$, $a_4(2040)$, $\rho_5(2350)$, and $a_6(2450)$. (See Fig. 2.) Here, the $O(\lambda^{-1/2})$ term in (4.2) plays a crucial role, because the linear Regge trajectory with $\alpha_0 = 1$ can never give a good fit to the experimental data.

In the following, we argue the possible identification of the open string states in (3.24) and (A.3) with the mesons listed in Table I. Since there is an ambiguity in the value of $\alpha'$, we do not use the values of meson masses predicted using (3.15) explicitly for our purpose. Instead, we rely on the property of the mass formula (3.15) that the mesons identified with the open string states with the same $(N,n)$ should be nearly degenerate. Noting the fact that the mesons $a_2(1320)$, $\rho_3(1690)$, $a_4(2040)$, $\rho_5(2350)$, and $a_6(2450)$ are naturally identified with string modes with $N = 1, 2, 3, 4, 5$ and $n = 0$, we expect that the mesons identified with open string states in (3.24) and (A.3) should have masses close to that of $a_2(1320)$ ($m_{a_2} \simeq 1318 \text{ MeV}$) and $\rho_3(1690)$ ($m_{\rho_3} \simeq 1689 \text{ MeV}$), respectively.

Let us begin by reconsidering the identification of the massless $(N = 0)$ modes studied in Refs. 7) and 13). According to Refs. 7) and 13), the massless five-dimensional gauge field on the D8-brane produces a pseudo-scalar meson ($J^{PC} = 0^{-+}$), vector mesons ($J^{PC} = 1^{--}$), and axial-vector mesons ($J^{PC} = 1^{++}$). The predicted masses of the low-lying states are listed in Table I. The massless pseudo-scalar meson in Table II is interpreted as pion. It appears as a massless Nambu-Goldstone particle associated with the spontaneous chiral symmetry breaking, as shown in Ref. 7). We could not reproduce the mass of the pion, simply because our model corresponds to QCD with massless quarks. It would be worth emphasizing that the vector and axial-vector mesons obtained from the massless five-dimensional

![Fig. 2. A plot of (4.2) with $\alpha' = 1.1 \text{ GeV}^{-2}$. The dots represent mesons in the $\rho$-meson trajectory.](https://academic.oup.com/ptp/article-abstract/124/2/263/1829170)
Table II. Mesons obtained from the massless sector in Refs. 7) and 13). Here, we have used the rho meson mass $m_\rho \simeq 776$ MeV as an input to fix the value of $M_{KK}$ as in (2.7).

| $J^{PC}$  | $0^-$($\pi$) | 0   | 2435 |
|-----------|--------------|-----|------|
| 1$^{--}$($\rho$) | [776]        | 1607| 2435 |
| 1$^{++}(a_1)$ | 1189         | 2024| 2849 |

gauge field are all massive, despite the fact that the Regge intercept for the Regge trajectory obtained in the flat space-time limit is $\alpha_0 = 1$. In Refs. 7) and 13), the lightest vector and axial-vector mesons in Table II are interpreted as $\rho(770)$ and $a_1(1260)$, respectively. Since we do not see any axial-vector mesons with $J^{PC} = 1^{++}$ in (3.24), it is indeed plausible to identify $a_1(1260)$ as a meson obtained from the massless string modes. The predicted mass (1189 MeV) is very close to the observed value (1230 ± 40 MeV) for $a_1(1260)$. The interpretation of the heavier modes in Table II is less clear. Among the vector mesons in Table I, $\rho(1570)$ has the closest mass to the second lightest vector meson in Table II. However, since our approximation is not good enough to make a definite identification, $\rho(1450)$ and $\rho(1700)$ are also good candidates.

Our result for the first excited massive string modes in (3.24) predicts that there should be mesons with $J^{PC} = 2^{++}, 1^{--}, 1^{+-}, 0^{++},$ and $0^{-+}$ with approximately the same masses. The candidates for the mesons with $J^{PC} = 2^{++}, 1^{+-}, 0^{++},$ and $0^{-+}$ are $a_2(1320), b_1(1235), a_0(1450),$ and $\pi(1300)$, respectively. These mesons cannot be interpreted as those from the massless open string modes in Table II and it is nice to have candidates of these mesons in the first excited massive string states. The masses of these mesons are reasonably close to each other. Note that $a_0(980)$, which is the lightest meson with $J^{PC} = 0^{++}$ in Table I, is considered to be a four-quark state or two-meson resonance. (See the section of “Non-$q\bar{q}$ Candidates” in Ref. 37.) Since the interaction among $q\bar{q}$ mesons vanishes in the large $N_c$ limit, the four-quark states cannot appear as stable bound states in large $N_c$ QCD. Therefore, we do not interpret $a_0(980)$ as one of the $J^{PC} = 0^{++}$ states in (3.24). According to (3.24), two more $0^{++}$ states are predicted. We are not sure how to interpret these modes. The most plausible candidate for the $J^{PC} = 1^{--}$ state in (3.24) is $\rho(1450)$. However, since we also have $J^{PC} = 1^{--}$ states in the $N = 0$ sector as discussed above, this identification is rather ambiguous.

The spectrum of second excited massive open string modes with $N = 2$ is analyzed in Appendix A. Among those summarized in (A.3), the states with $J^{PC} = 3^{--}$ and $2^{--}$ do not appear in the spectrum of the massless ($N = 0$) and the first excited ($N = 1$) states. Therefore, they should be interpreted as the lightest mesons with $J^{PC} = 3^{--}$ and $2^{--}$ in Table I, that is, $\rho_3(1690)$ and $\pi_2(1670)$, respectively. It is encouraging to note that the masses of these two mesons are very close to each other. $\pi_2(1880)$ could also be a candidate for the $2^{--}$ meson in (A.3). Although the experimental evidence for the existence is insufficient, the second lightest mesons with $J^{PC} = 1^{++}$ and $2^{++}$ in Table I ($a_1(1640)$ and $a_2(1700)$) have approximately the same mass as $\rho_3(1690)$, indicating that these mesons could be interpreted as
those in (A.3). In (A.3), we find six $J^{PC} = 0^{-+}$ states. Although the interpretation of the degeneracy is unclear, a natural candidate for one of these states is $\pi(1800)$, since we have interpreted $\pi(1300)$ as one of the $N=1$ states above. There are also a lot of $J^{PC} = 1^{--}$ states in (A.3), which could be interpreted as the $1^{--}$ mesons in Table I. For example, $\rho(1700)$ is nearly degenerate with $\rho_3(1690)$ and it is tempting to interpret it as one of the $N=2$ states, although we cannot exclude the possibility that $\rho(1700)$ is one of the $1^{--}$ states found in the spectrum of $N=0$ and $N=1$ states as discussed above. The mesons with $J^{PC} = 1^{-+}$ can be obtained from the states with $(N,n) = (1,1)$ and $(N,n) = (2,0)$. $\pi_1(1400)$ and $\pi_1(1600)$ are the candidates in Table I. However, the mass of $\pi_1(1400)$ (1351 ± 30 MeV) is rather close to those of the $(N,n) = (1,0)$ states considered above, and it seems unclear if $\pi_1(1400)$ should be interpreted as one of the $(N,n) = (1,1)$ states or $(N,n) = (2,0)$ states. In fact, it is known that mesons with $J^{PC} = 1^{-+}$ are exotic states that cannot be obtained as $qq$ bound states. $\pi_1(1400)$ and $\pi_1(1600)$ are thus regarded as four-quark states or hybrid states ($q\bar{q}$ pairs bound by excited gluons). There are some works suggesting that $\pi_1(1400)$ is a four-quark state and $\pi_1(1600)$ is a candidate of a hybrid meson.\textsuperscript{38,39} If this is the case, $\pi_1(1400)$ is not interpreted as the open string states considered in this paper. On the other hand, the hybrid mesons exist as narrow resonances in large $N_c$ QCD\textsuperscript{40} and should appear as open string states in our analysis. The mass of $\pi_1(1600)$ (1662\textsuperscript{+15}_{-11}$ MeV) is also close to that of $\rho_3(1690)$ and it is plausible to interpret it as the $J^{PC} = 1^{--}$ state with $(N,n) = (2,0)$ in (A.3).

It might look strange that there are no clear candidates in Table I to be identified with the open string massive states with $n \geq 1$. For example, the states with $N = n = 1$ include those with $J^{PC} = 0^{-+}$, $0^{+-}$, and $2^{--}$, which are not found in the experiments. This fact may suggest that the states with $n \geq 1$ are relatively heavy or correspond to wide resonances. This may again indicate that $\pi_1(1600)$ corresponds to a state with $(N,n) = (2,0)$ rather than that with $(N,n) = (1,1)$. If we naively use our mass formula (3.18) with the value of $M_{KK}$ in (2.7), we obtain $M_{n+1} - M_n \simeq 671$ MeV, which predicts that the masses of the states with $(N,n) = (1,1)$ are approximately 2000 MeV. It is, however, unclear to what extent we can trust this value, since the contribution from the second term in (2.7) is comparable to or larger than that from the first term for $n \geq 1$.

There are some other states in (A.3) that do not have plausible candidates in Table I. For example, our result (A.3) predicts that there are mesons with $J^{PC} = 0^{++}, 1^{+-},$ and $2^{--}$ that have masses close to $\rho_3(1690)$. It would be interesting if such mesons are found in future experiments.

§5. Summary and discussion

In this paper, we have studied the higher excited meson spectrum using a holographic description of QCD proposed in Ref. 7. One of the motivations of this investigation is to discuss whether or not the gauge/string duality holds beyond the massless sector in the holographic description. In fact, the higher excited mesons originate from the massive modes of open strings that end on probe D8-branes. We
derived a mass formula for these mesons by quantizing the open string perturbatively. It was found that the mass formula exhibits the linear Regge trajectory at the leading order in $1/\lambda$ expansions, with a non-linear term added as a subleading correction. By analyzing the discrete symmetries of our brane configuration, we read off the parity and charge conjugation quantum number of the open string states. It looks quite non-trivial that many of the open string states found in our analysis can be identified with the mesons found in the experiments. This result can be regarded as an evidence of the validity of the gauge/string duality including the massive excited string states. Thorough investigations toward this direction would be worthwhile. For example, it would be nice to work out the interactions that involve such excited meson from string theory. Similar analysis in the closed string sector would also be interesting.

There remain several important problems to clarify. First, as mentioned in §§2 and 4, the value of $\alpha'$ is not in good agreement with the expected value if we use (2.6) and (2.7). It would be interesting to see if it is improved by taking into account possible corrections. It is also important to calculate corrections to our mass formula (3.15) to see how much our results are affected by them. In particular, since most of the mesons we have been discussing are of the same order or heavier than the mass scale $\sim M_{KK}$ of the artifacts of the model, one might think that the deviation from realistic QCD would be significant. However, we know from many other nice results in holographic QCD that the effect of these artifacts seems to be much smaller than what one would naively expect. It is probable that our results described in §4 will not be changed much by taking into account the corrections. The situation may be analogous to the fact that the quenched approximation works very well in lattice QCD. Anyway, computation of the corrections should be done to justify all these.

Another related problem is the existence of the open string states that cannot be identified with the mesons in QCD. In §3.1, we restricted our attention to the states that are invariant under the $SO(5)$ and $\mathbb{Z}_2$ symmetry to exclude such artifacts from our consideration as many as we can. However, one should be aware that although this is a necessary condition to obtain mesons in QCD, it may not be a sufficient condition, and some of the states obtained in this paper might contain the artifacts of the model. To completely get rid of all the artifacts, we need to take a limit analogous to the continuum limit in lattice QCD, as discussed in §2. To this end, we have to extrapolate our analysis to small $\lambda$ region, in which supergravity approximation breaks down.

As pointed out in §4, our model predicts the states that have no candidate in Table I. Among them are the second excited massive modes with $J^{PC} = 0^{++}$, $1^{+-}$, and $2^{--}$, whose masses are expected to be approximately 1700 MeV. This fact does not lead to the conclusion that our model contradicts with the results of experiments immediately, since these states may correspond to wide resonances that are difficult to observe. If so, it would be interesting to compare our results with the predictions from the quark model. (For a review, see the section of “Quark model” in Ref. 37.) In fact, it is known that the mesons with $J^{PC} = 0^{++}$, $1^{+-}$, and $2^{--}$ can arise from the quark model. The lightest $q\bar{q}$ states with $J^{PC} = 0^{++}$ and $1^{+-}$ are those labeled as $n^{2S+1}L_J = 1^3P_0$ and $1^1P_1$, and are identified with $a_0(1450)$ and $b_1(1235)$, respectively.
respectively. Here, $S$, $L$, $J$, and $\hat{n}$ are the total spin carried by the constituent quark and anti-quark, orbital angular momentum, spin of the meson, and the principal quantum number corresponding to the radial excitation, respectively. Then, it is natural to identify the second lightest $J^{PC} = 0^{++}, 1^{+-}$ states and the lightest $J^{PC} = 2^{--}$ state in the quark model with the $J^{PC} = 1^{++}, 1^{--}$ states found as the open string states with $(N, n) = (2, 0)$ in (A.3). They are labeled as $\hat{n}^{2S+1}L_J = 2^3P_0$, $2^1P_1$, and $1^3D_2$, respectively. The masses of these states estimated in the quark model are indeed all close to 1700 MeV. (See, for example, Refs. 41 and 42.)

Acknowledgements

We would like to thank our colleagues at the Institute for the Physics and Mathematics of the Universe (IPMU) and the particle theory group at Nagoya University for helpful discussions. S.S. is especially grateful to Z. Bajnok, K. Hori, R. A. Janik, and T. Onogi for valuable discussions. T.S. thanks H. Fukaya, M. Harada, and M. Tanabashi for valuable discussions. The work of S.S. is supported in part by a Grant-in-Aid for Young Scientists (B), the Ministry of Education, Culture, Sports, Science and Technology (MEXT), Japan, a JSPS Grant-in-Aid for Creative Scientific Research (No. 19GS0219), and also by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. We are also grateful to the organizers of the workshop “Summer Institute 2008”, where part of this work was done.

Appendix A

Higher Excited States

The second excited states with $N = 2$ belong to $\Box_9 \otimes \Box_{84} \oplus \Box_9 \otimes \Box_{44}$ of the little group $SO(9)_{1-9}$. (See section 5.3 of Ref. 35)) The fields corresponding to the particles in $\Box_9 \otimes \Box_{84}$ and $\Box_9 \otimes \Box_{44}$ are denoted as $B_{A_1[A_2A_3A_4]}$ and $C_{A_1(A_2A_3)}$ with $\{A = 1, \ldots, 9\}$, respectively. Here, $[A_1, \ldots, A_n]$ and $(A_1, \ldots, A_n)$ denote antisymmetrization and symmetrization of the indices $A_1, \ldots, A_n$, respectively.

The $SO(4)_{6-9}$ invariant states are

\[ B_{\alpha_1[\alpha_2\alpha_3\alpha_4]} \quad B_{\alpha[a\alpha]} \quad \epsilon^{a_1a_2a_3a_4} B_{a_1[a_2a_3a_4]} \quad C_{\alpha_1(\alpha_2\alpha_3)} \quad C_{\alpha(\alpha\alpha)} \quad (A.1) \]

where $\alpha, \beta = 1, \ldots, 5$ and $a = 6, \ldots, 9$. The $\tau$-parity invariant components are

\[ B_{M_1[M_2M_3M_4]} \quad B_{M_1} \equiv B_{a[aMN]} \quad B_{M_2} \equiv B_{y[yMN]} \quad C_{M_1(M_2M_3)} \quad C_{M} \equiv C_{a(aM)} \quad C_{M} \equiv C_{a(aa)} \quad (A.2) \]

where $M = 1, \ldots, 4$. The lightest meson fields obtained from these fields together with their $J^{PC}$ are listed as follows:

\[ B_{j[i_1i_2i_3]}^{(0)} (1^{+-}) \quad B_{z[i_1i_2i_3]}^{(0)} (0^{++}) \quad B_{l[i_1i_2z]}^{(0)} (2^{++}, 1^{++}, 0^{++}) \quad B_{z[i_1i_2z]}^{(0)} (1^{+-}) \]

\[ B_{ij}^{[1,2](0)} (1^{+-}) \quad B_{iz}^{[1,2](0)} (1^{+-}) \quad C_{i(i_1i_2i_3)}^{(0)} (3^{--}, 2^{--}, 1^{--}) \]
\( C_{z(ij)}^{(0)} (2^{-+}) \), \( C_{z(ij)}^{(0)} (2^{-+}, 1^{-+}, 0^{-+}) \), \( C_{z(zz)}^{(0)} (1^{--}) \), \( C_{z(zi)}^{(0)} (1^{--}) \), \( C_{z(zj)}^{(0)} (1^{--}) \), \( C_{z(zj)}^{(0)} (0^{++}) \), \( C_{(1^{--})}^{[1-4](0)} (1^{--}) \), \( C_{(2^{--})}^{[1-4](0)} (0^{++}) \), (A.3)

where \( i, j = 1, 2, 3 \). Therefore, we have 3\(--\), 2\;+++, two 2\(--\), 2\(--\), seven 1\(--\), four 1\;+++, three 1\;+++, 1\;+++, two 0\;+++, six 0\(--\), as the lightest modes.

The excited states with \( N \geq 3 \) can be analyzed in a similar way. The classification of the excited states with respect to the \( SO(9) \) little group can be obtained using the techniques used in Refs. 43) and 44).

### Appendix B

#### Discrete Symmetry

In this appendix, we classify the \( \mathbb{Z}_2 \) symmetries of the Witten’s D4-brane background with probe D8-branes. Type IIA string theory has \( \mathbb{Z}_2 \) symmetries generated by \( I_{\text{even}}, I_{\text{odd}} \), and \( (-1)^F_L \), where \( I_{\text{even}} \) and \( I_{\text{odd}} \) are spatial involutions that flip the sign of even and odd numbers of coordinates, respectively, \( \Omega \) is the world-sheet parity transformation, and \( F_L \) is the left moving space-time fermion number.\(^1\)

\( (-1)^F_L \) acts on the fields in R-R sector and R-NS sector as multiplication by \(-1\), and hence, it does not keep the RR 4-form field strength of the background \( (2 \cdot 4) \) invariant. Instead, \( P_\tau \equiv I_{\theta}(-1)^F_L \) is a symmetry of the background, as it keeps the \( F_4 \) flux as well as the metric invariant. Here, \( I_{1i2\ldots in} \) denotes the involution \( (x^{i1}, x^{i2}, \ldots, x^{in}) \rightarrow (-x^{i1}, -x^{i2}, \ldots, -x^{in}) \). In §3.3, we have learned two other \( \mathbb{Z}_2 \) symmetries corresponding to parity and charge conjugation symmetries generated by \( P \equiv I_{123} \), and \( C \equiv I_{289}(\Omega)(-1)^F_L \), respectively. The spatial involutions \( I_{\text{even}} \) that generate \( \mathbb{Z}_2 \) subgroups of \( SO(1, 3) \times SO(5) \) isometry are of course symmetries of the system. On the other hand, the spatial involutions \( I_{\text{even}} \) that involve \( y \rightarrow -y \) are not symmetries of the system, since they map the D8-branes placed at \( y = 0 \) to \( \mathbf{D}^{\overline{8}} \)-branes. However, if we combine it with \( (-1)^F_L \), which maps \( \mathbf{D}^{8} \)-branes back to D8-branes, the D8-branes are kept invariant. Therefore, \( P_\tau \) is a symmetry of the system. It is then easy to show that the \( \mathbb{Z}_2 \) symmetries of the system we should consider are those generated by the combinations of \( P_\tau \), \( P \), \( C \), and elements of a \( \mathbb{Z}_2 \) subgroup of \( SO(1, 3) \times SO(5) \) isometry.

\( P_\tau \) defined above acts in the same way as “\( \tau \)-parity” considered in Ref. 11). Let us show that the quarks and gluons are invariant under this \( \tau \)-parity. To this end, we turn to the D4/D8/\( \mathbf{D}^{\overline{8}} \)-brane configuration. As briefly reviewed in §2, QCD is realized on the D4-brane world-volume in the D4/D8/\( \mathbf{D}^{\overline{8}} \)-brane system embedded in a flat ten-dimensional space-time \( \mathbb{R}^{1,3} \times S^1 \times \mathbb{R}^5 \). The coordinates of the \( \mathbb{R}^{1,3} \) and \( S^1 \) factors correspond to the coordinates \( x^\mu \) and \( \theta \) used in the background \( (2 \cdot 1) \), respectively, and the radial and angular directions of the \( \mathbb{R}^5 \) factor correspond to \( r \) and \( S^4 \), respectively. In this appendix, we parametrize the \( \mathbb{R}^5 \) factor by \((x^5, \ldots, x^9)\). Although these coordinates are not the same as those used in §3.1, the involution \( x^9 \rightarrow -x^9 \) acts in the same way. In this picture, the D4-branes are extended along

\(^1\) We do not care much about \( (-1)^{F_s} \), where \( F_s \) is the space-time fermion number, since its action is trivial on the mesons.
$\mathbb{R}^{1,3}$ and $S^1$ directions and placed at the origin of $\mathbb{R}^5$, while the D8-branes and $\overline{D8}$-branes are extended along $\mathbb{R}^{1,3} \times \mathbb{R}^5$ directions and placed at $\theta = \pi/2$ and $\theta = -\pi/2$, respectively. The gluons are created by $4$-$4$ strings and the quarks are created by $4$-$8$ and $4$-$\overline{8}$ strings, where a $p$-$p'$ string is an open string stretched between D$p$-brane and D$p'$-brane.

The involution $y \rightarrow -y$ corresponds to the involution acting on the $S^1$ as $\tau \rightarrow -\tau$, where we have defined $\tau \equiv \theta - \pi/2$. Therefore, $P_\tau$ acts as $I_{\tau \bar{\theta}}(-1)^{F_L}$ in this D4/D8/$\overline{D8}$ system. To know how $P_\tau$ acts on gluons and quarks, it is convenient to T-dualize the system along the $S^1$ direction and consider a D3/D9/$\overline{D9}$ system. By parametrizing the T-dualized $S^1$ by $\bar{\tau}$, the $\tau$-parity $P_\tau$ can be interpreted as a 180-degree rotation in the $\bar{\tau}$-$x^9$ plane around the D3-brane. Since the gauge field on the D3-brane and the massless fermions in the spectrum of the 3-9 and 3-$\bar{9}$ strings are invariant under the rotation in the $\bar{\tau}$-$x^9$ plane, we conclude that the gluons and quarks are invariant under the $\tau$-parity $P_\tau$.

**Appendix C**

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**Contribution of RR Field to the Mass Formula**

Since the radius of the $S^4$ is proportional to $M_{KK}^{-1}$, the background RR 4-form field strength $F_4$ (2.4) in our convention is proportional to $N_c M_{KK}^{-1}$.** Then, a possible mass term for open string modes $\Phi$ and $\Phi'$ induced by the RR background on the D8-brane world-volume will be schematically written as

$$\int d^9 x \alpha' g_s F_4 \text{Tr}(\Phi \Phi') \sim \int d^9 x \lambda^{1/2} M_{KK}^2 \text{Tr}(\Phi \Phi') .$$

(C.1)

Here, we assume that the kinetic terms of $\Phi$ and $\Phi'$ are canonically normalized, and the Lorentz indices of $F_4$, $\Phi$, and $\Phi'$ are properly contracted. We only consider the isovector mesons, for which there are no mixing terms with closed string states. In our convention, the $g_s$ dependence of the effective action agrees with the string loop expansion if it is written in terms of $F_4' = g_s F_4$. Since the dimensionless combination $\alpha'^2 F_4'$ is of $O(\lambda^{-1/2})$, the higher order terms with respect to the background RR 4-form field strength can be neglected. The mass matrix element obtained from (C.1) is of $O(\lambda^{1/2})$, which is of the same order as $O(\lambda^{1/2})$ term in (3.15). Thus, one might think that the $O(\lambda^{1/2})$ term in our mass formula (3.15) would be modified by taking into account the effect of the background RR field.

However, it can be shown that the terms like (C.1) can only be possible when $\Phi$ and $\Phi'$ are not in the same excitation level. Then, by diagonalizing the mass matrix, it is easy to see that the contribution of the mixing term (C.1) to the mass squared of mass eigenstates is at most of $O(1)$. Therefore, the contribution of $F_4$ in the background in the equation of motion (3-12) can be neglected.

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**1) Because we impose the anti-periodic boundary condition for the fermions along the $S^1$, the T-duality along this $S^1$ yields Type 0B string theory. This boundary condition is not essential in our argument, since we are only interested in the transformation property of the open strings under $P_\tau$.**

**2) See also Appendix A of Ref. 7) for our convention of the RR fields.**
To see that $\Phi$ and $\Phi'$ cannot be in the same excitation level, it is convenient to T-dualize the system along the $y$ direction as we did in §3.1. Then, the D8-branes are mapped to D9-branes and the RR 4-form field strength $F_4$ is mapped to an RR 5-form field strength $F_5$. We use the same notation $\Phi$ and $\Phi'$ for the open string modes on the D9-brane. Recall that type IIB string theory with D9-branes is invariant under the world-sheet parity transformation $\Omega$. Then, because $F_5$ is odd under $\Omega$, $\text{Tr}(\Phi \Phi')$ should also be odd under $\Omega$ for allowing an interaction term as $F_5 \text{Tr}(\Phi \Phi')$. This implies $(-1)^{N+N'} = -1$, where $N$ and $N'$ are the excitation levels of $\Phi$ and $\Phi'$, respectively, and hence, $\Phi$ and $\Phi'$ cannot be in the same excitation level.

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