THERMAL HADRON PRODUCTION
IN Si-Au COLLISIONS

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Abstract

The most abundantly produced hadron species in Si–Au collisions at the BNL-AGS (nucleons, pions, kaons, antikaons and hyperons) are shown to be in accord with emission from a thermal resonance gas source of temperature $T \approx 110$ MeV and baryochemical potential $\mu_B \approx 540$ MeV, corresponding to about 1/3 standard nuclear density. Our analysis takes the isospin asymmetry of the initial state fully into account.
1 Introduction

The basic aim of high energy nuclear collisions is the production of strongly interacting matter. Following the primary collision, multiple scattering and production processes are expected to provide the rapid increase in entropy needed for local equilibration. Hence the first question to be addressed by nuclear collision experiments is whether they indeed produce matter, i.e., large-scale strongly interacting systems in local equilibrium. QCD thermodynamics predicts that at sufficiently high density, such matter will be a plasma of deconfined quarks and gluons (QGP). Since this prediction presupposes equilibrium, experimental evidence for thermalisation is a prerequisite to establishing QGP production. It would be ideal to test thermalisation at each stage in the evolution of the produced systems. However, unambiguous tests exist so far only for the final hadronic stage. In particular, we can check if the abundance of hadrons emitted in nuclear collisions is governed by thermal composition laws \[1, 2\]. The existence of such a “chemical equilibrium” means that the presence of different hadron species at freeze-out is specified by the freeze-out temperature \(T\) and the freeze-out baryochemical potential \(\mu_B\) determining the baryon density; hence the two parameters \(T\) and \(\mu_B\) fix the production ratios of all emitted hadrons, in much the same way the abundances of light nuclei are determined by the conditions of the early universe at the end of the strong interaction era.\[1\] If the initial state contains more neutrons than protons, it is necessary to assure overall charge conservation, which leads, e.g., to a \(\pi^-/\pi^+\) ratio different from unity. The analysis presented below takes such deviations from an isospin-symmetric initial state fully into account. Since the neutron to proton and the \(\pi^-/\pi^+\) ratios deviate from one by about ten to twenty percent, the results from this analysis will generally differ from the isospin-symmetric case by this order of magnitude. Estimates of the effects of a charge chemical potential have been presented previously \[4, 5\].

The confirmation of chemical equilibrium among the emitted hadrons at freeze-out of course does not imply that the previous stages were hadronic. It is also not connected to any specific expansion pattern, and in particular, it does not imply isotropic emission from a single stationary “fireball”. Starting from a uniform medium, hydrodynamic flow preserves chemical equilibration, but for anisotropic boundary conditions, it leads to anisotropic momentum distributions. Hence the momentum spectra of thermally emitted hadrons and the source radii obtained from hadron-hadron interferometry are needed to determine the nature of the expansion. Hadron production according to thermal composition laws is thus perfectly compatible with the existence of a QGP at an earlier point in the evolution. It in fact provides a suitable basis for such a possibility: if the final hadrons were emitted from an equilibrated source, it is quite possible that this source was locally thermalised already in its earlier stages and thus could have consisted of deconfined matter.

The experimental study of thermalisation is therefore of fundamental importance. A recent comprehensive analysis of BNL-AGS data from \(Si-Au\) collisions \[6, 7\] provides first evidence that the most abundantly observed hadrons were indeed emitted from a thermal

\[^{1}\]It has been noted \[8\] that hadrons of a given species could be in local thermal equilibrium even though chemical equilibrium between the different species is not attained.
source. If supported by further results from $Au-Au$ collisions at the AGS and at higher energies from the CERN-SPS, this conclusion would be the decisive step in showing that strongly interacting matter can be produced by nuclear collisions. In view of the importance of such a result, we find it useful to analyse the present AGS data independently in a self-contained hadrosynthesis approach. Our analysis will therefore be based solely on particle ratios, without any source volume or flow features, and it will, as mentioned, take into account the initial state isospin asymmetry.

2 Hadronic Ratios.

The measured hadron production ratios listed in Table 1 form the basis of our analysis. If the hadrons are emitted from a source in full chemical equilibrium, each measured ratio determines a range of $T-\mu_B$ values with which it is compatible. This range can be calculated on the basis of an ideal gas of all observed hadrons and hadronic resonances, requiring overall charge and strangeness conservation and correct resonance decays [1].

Let us briefly sketch the method. If at freeze-out we have an ideal gas of hadrons and hadronic resonances, then the system is described by the partition function

$$\ln Z(T, \mu_B, \mu_S, \mu_Q) = \sum_i \lambda_B^{S_i} \lambda_S^{S_i} \lambda_Q^{Q_i} W_i. \tag{1}$$

Here $W_i$ is the phase space factor for hadrons of species $i$ (mesons, baryons and their antiparticles), with $S_i$, $B_i$ and $Q_i$ denoting the strangeness, baryon number and charge of the hadron in question. The baryonic fugacity is defined as $\lambda_B = \exp(\mu_B/T)$, that for the strangeness as $\lambda_S = \exp(\mu_S/T)$, and that for charge as $\lambda_Q = \exp(\mu_Q/T)$. The phase space factors are given by

$$W_i = \frac{d_i m_i^2 V T}{2\pi^2} K_2 \left(\frac{m_i}{T}\right), \tag{2}$$

with $d_i$ denoting the spin degeneracy, $m_i$ the mass of hadron species $i$; $V$ is the volume of the system. We include in the sums in eq. (1) all hadrons listed in the latest Particle Data Compilation [8], excluding charm and bottom resonances; decays are taken into account with their experimental branching ratios, with an educated guess being made whenever the information on decays is not complete. The resulting table contains 479 entries. The partition function (1) then determines all thermal properties of the system in terms of the four parameters $T$, $\mu_B$, $\mu_S$ and $\mu_Q$. The chemical potential for the strangeness, $\mu_S$, can be fixed by requiring the overall strangeness of the system to vanish. Similarly, the chemical potential for the charge, $\mu_Q$, is fixed by requiring the charge/baryon ratio of the final state to be equal to that of the initial state. If there is a one-stage freeze-out of all thermal hadrons, then all production ratios ($\pi/p$, $K/\pi$, $K/\bar{K}$, $\rho/\pi$, $\phi/\pi$, $Y/p$, ...) are given in terms of the two remaining parameters $T$ and $\mu_B$. For example, we can use the measured $\pi^+/p$ and $K^+/\pi^+$ ratios to fix the values of $T$ and $\mu_B$; all other ratios are then predicted, if the different particle species are indeed present according to their equilibrium weights. To fix the overall charge/baryon, we thus have to estimate the number of interacting
protons and neutrons in the initial state. For a central $A - B$ collision ($A \ll B$), the number $N_{\text{part}}$ of participant nucleons is the sum of the nucleons in $A$ and those in a tube of radius $R_A$ through nucleus $B$,

$$N_{\text{part}} \simeq [A + (\pi R_A^2)n_0 2R_B] \simeq [A + 1.5 A^{2/3} B^{1/3}].$$

(3)

Here $n_0 = 0.17$ fm$^{-3}$ is standard nuclear density; $R_A = 1.12 A^{1/3}$, and similarly for $B$. For $Si - Au$ collisions, we thus obtain $N_{\text{part}} \simeq 108$. With $Z/A = 0.5$ for $Si$ and $Z/A = 0.4$ for $Au$, this is made up of $N_p \simeq 46$ protons and $N_n \simeq 62$ neutrons. We thus have to fix the overall charge/baryon of the final state at $46/108$ by suitably adjusting the charge potential $\mu_Q$ at the temperature $T$ and baryochemical potential $\mu_B$ obtained from particle ratios. As noted, the strangeness potential $\mu_S$ is fixed to give the final state a vanishing overall strangeness.

We begin with the most abundantly observed hadron species, $p$, $\pi^{\pm}$, $K^{\pm}$ and $\Lambda$, since these are most likely to be thermalised. What we denote by $\Lambda$ will always include the $\Sigma^0$, since the two are experimentally not separable. ¿From the production rates of these six hadron species, we get five independent ratios. As seen in Fig. 1, four of these in fact cross in a common region in the plane of temperature and baryochemical potential, so that pions, nucleons, kaons, antikaons and lambdas are observed according to their thermal weights. The freeze-out parameters thus obtained are

$$T_F = 110 \pm 5 \text{ MeV} \quad \mu_B^F = 540 \pm 20 \text{ MeV}. \quad (4)$$

The fifth ratio, which can be taken either as the $K^-/\pi^-$ or the $\pi^-/\pi^+$ ratio, is also in agreement with the above values.

In Fig. 2 we show that when charge conservation is not enforced (i.e. the isospin symmetric case, with $\mu_Q = 0$), there is never a consistency between the $\pi^+/p$ and $K^+/K^-$ experimental ratios. The additional constraint of charge conservation, normalized by the initial state participating nucleons, provides an overlap region for these experimental ratios, allowing a consistent set of thermodynamic parameters to be determined.

We have also checked that the finite volume corrections discussed in [6] do not modify our results. In Table 1, we list all thermal ratios as determined by these values, together with the measured ratios. It is evident that the emission rates of nucleons, pions, kaons, antikaons and hyperons are correctly described by thermal composition. The corresponding baryon density is $n_B = 0.055 \pm 0.025$ fm$^{-3}$ and hence about $1/3$ standard nuclear density, indicating considerable expansion before freeze-out. The freeze-out values (4) agree with those determined in [3]; our temperature is slightly lower. In [3], the temperature range for freeze-out was fixed by a study of $\Delta$ production. However, the determination of the $\Delta$ yield is quite delicate, based on pion transverse momentum distributions at small $p_T$ and/or reconstruction over a partially known background [9]. To avoid the uncertainties this leads to, we use the directly measured hadron ratios. For the freeze-out parameters (3), we get an overall thermal ratio $\Delta/N = 0.25 \pm 0.02$.

Given the initial state baryon number of 108, we can extract from the ratios of Table 1 the thermal abundances of the different species; they are listed in Table 2. Also shown there are the experimental abundances, obtained in the same way from the experimental ratios.
Next we turn to less copiously produced hadron species. In Table 1, we also list their measured ratios together with the corresponding predictions from thermal emission at the freeze-out parameters (4). We see that the value for $\phi$ production agrees well 0.1 $\phi$’s per event. Multiply strange baryons ($\Xi$) seem experimentally somewhat more abundant than thermally predicted. A definite disagreement with the thermal predictions is found for antibaryons ($\bar{p}$ and $\bar{\Lambda}$): these are produced much more copiously than their thermal predictions. Before discussing the possible origin of this, we consider the onset of equilibrium for the different species.

3 Thermalisation.

Thermalisation in the AGS energy range can lead to an increase or to a decrease of hadron abundances relative to those measured in $p - p$ or $p - A$ interactions. Let us note three examples in some detail.

A well documented case is the $K^+ / \pi^+$ ratio: it grows from a $p - p$ value near 0.05 to a four times larger thermal value, above 0.2, in central $Si - Au$ and $Au - Au$ collisions (11, 12). Similarly, the $\phi / \pi^+$ ratio increases by more than a factor two from its $p - p$ value to the thermal result found in $Si - Au$ data (13).

In contrast, the number of pions produced per participating nucleon decreases at AGS energy towards its thermal value. In Fig. 3, we show the $\pi / N_{\text{part}}$ ratio in $p - A$ collisions at 14.6 GeV beam momentum (14). To obtain these values, we approximate the pion multiplicity by $3\langle h_- \rangle$, where $\langle h_- \rangle$ is the average number of negative hadrons; $\pi / N_{\text{part}}$ is defined as the average number of participants in a $p - A$ collision,

$$N_{\text{part}} = 1 + n_0\pi r_0^2(3/4)(2R_A),$$

(5)

where $r_0 \simeq 0.85$ fm is the proton radius. Eq. (5) thus counts the number of nucleons contained in a tube of radius $r_0$ through the nucleus; the factor $(3/4)$ comes from averaging over the impact parameter. The ratio $\pi / N_{\text{part}}$ is seen to be $A$-independent; it remains constant at $1.5 \pm 0.2$ over the whole range from $p - N$ to $p - Au$. To obtain the corresponding experimental value of $\pi / N_{\text{part}}$ for $Si - Au$ collisions, we multiply the measured $\pi^+ / p$ ratio by $3 \times 1.14$ (where the factor 1.14 accounts for the observed $\pi^- / \pi^+$ ratio (15)) and divide it by 2.35 in accord with 46 participating protons and 62 neutrons. We thus find a significantly lower value, $1.15 \pm 0.12$, which is in accord with the thermal value $133/108 \approx 1.23$. This lower value is also supported by data from $Si - Al$ collisions (14). The ratio $\pi / N_{\text{part}}$ thus decreases from its $p - A$ value of about 1.5 to a somewhat lower thermal value.

Because of enhanced annihilation chances, the ratio of antibaryon to baryon production is also expected to decrease in a dense baryonic medium. In support of this, the $\bar{p} / p$ ratio appears to decrease in going from $p - p$ to $A - B$ (16).

There is also no particular reason why all ratios should approach thermalisation by the same mechanism or at the same rate. Pion-nucleon and pion-pion interactions result in more abundant kaon production than nucleon-nucleon interactions; thus an environment
with more such collisions will drive the kaon to pion ratio up. In nucleus-nucleus collisions, some of the available energy is needed to thermalise the participating nucleons and therefore is removed from meson production \[17\]; hence at the AGS, with very limited energy, \(\pi/N\) decreases to its thermal limit. As mentioned, antinucleons are more readily absorbed in nuclear matter, so that the \(\bar{p}/p\) and \(\bar{\Lambda}/\Lambda\) ratios in nuclear collisions are expected to be smaller than in \(p-p\) interactions. And in general one would expect the approach to be slower for less copiously produced particles. The onset of thermalisation will therefore be quite specific for different species, making a final uniform thermal distribution all the more striking.

In view of the general evolution towards thermalisation, it seems misleading to single out enhanced kaon/pion or hyperon/nucleon ratios as “strangeness enhancement”. There are fewer pions per nucleon in nucleus-nucleus collisions than in \(p-p\) or \(p-A\) interactions: this alone would drive the \(K/\pi\) ratio up, even for constant kaon production. Moreover, there are strange hadron species (e.g., antikaons) whose thermal production rate is smaller than that in hadron-hadron collisions, and there are non-strange species (e.g., \(\Delta\)’s) with enhanced thermal production.

We return now to the discrepancy between experimental and thermal antibaryon production. The thermal rates are determined by the dependence on the baryochemical potential \(\mu_B\): increasing \(\mu_B\) at fixed temperature decreases the ratio \(\bar{p}/p\) as \(\lambda_B^2 = \exp(-2\mu_B/T)\). However, such a suppression presupposes that the antibaryons can really experience the thermal medium, and that appears not clear. The fate of antiprotons in nuclear matter has been studied in detail in low energy \(p-A\) interactions \[18\]. Here one also expects enhanced annihilation, particularly in the production of very slow \(\bar{p}\)’s, which spend a long time in the medium. In contrast, one finds that over a large momentum range down to 0.5 GeV/c, the antipions apparently do not interact with the medium. This has been interpreted in terms of a very long formation time for baryon-antibaryon pairs in nuclear collisions, so that antiprotons emerge as well-defined particles only after leaving the nucleus \[13\]. It has also been considered as the effect of a specific screening of antiprotons in nuclear matter \[20\]. In any case, until the considerable transparency of nuclear matter for antiprotons is understood, it is not clear what role they will play in the medium produced in nucleus-nucleus collisions. It thus seems safest to exclude them from thermal considerations of AGS data, which are in the energy range studied in the mentioned \(p-A\) collisions \[18\].

4 Summary

We have here addressed the question of chemical equilibrium at freeze-out in a self-contained fashion, including the isospin asymmetry of the initial state. Our conclusion, in full agreement with that of \[1\], is that in \(Si-Au\) collisions at the AGS all copiously produced hadron species are emitted in accord with their thermal weights, as calculated for an ideal gas of hadrons and hadron resonances with the freeze-out parameters given by eq. (4). Some less frequently produced species also agree with this. Our conclusion is thus supported by six or seven independent and directly measured hadron production ratios,
so that it seems quite well-founded. In contrast, the production of the (relatively rare) antibaryons is not suppressed as much as expected in a baryon-rich environment. This agrees with results from $\bar{p}$ production in $p - A$ studies. The freeze-out temperature determined here can in principle be counterchecked by the measured transverse momentum spectra, provided we know the expansion pattern. In the absence of any collective flow effects, we would have for light hadrons ($m_\pi \simeq 0$)

$$\langle |p_T| \rangle \simeq 2T \simeq 210 \text{ MeV},$$

which is definitely too small. As noted in [6], this could be taken as an indication of predicted flow effects [3]. However, to establish conclusively the presence of such effects requires a more complete analysis, comparing in particular the change in transverse momentum spectra in going from $p - A$ to $A - B$ collisions. Although of great interest, that is beyond the scope of the present note. Stimulating discussions with M. Gazdzicki, D. Kharzeev, R. Stock and in particular with J. Stachel are gratefully acknowledged.

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Table 1: Ratios of Hadron Species in Si-Au Collisions at the AGS (Thermal parameters: \( T = 110 \pm 5 \text{ MeV}, \mu_B = 540 \pm 20 \text{ MeV} \))

| Ratio       | Experimental | Thermal     |
|-------------|--------------|-------------|
| \( \pi^+ / p \) | 0.80 ± 0.08  | 0.87 ± 0.15 |
| \( K^+ / \pi^+ \) | 0.19 ± 0.02  | 0.21 ± 0.02 |
| \( K^+ / K^- \) | 4.40 ± 0.40  | 4.51 ± 0.62 |
| \( \Lambda / p \) | 0.20 ± 0.04  | 0.16 ± 0.02 |
| \( K^- / \pi^- \) | 0.035 ± 0.005 | 0.038 ± 0.006 |
| \( \Xi^- / \Lambda \) | \((1.2 \pm 0.2) \times 10^{-1}\) | \((4.9 \pm 0.5) \times 10^{-2}\) |
| \( \phi / \pi^+ \) | \((4.5 \pm 1.2) \times 10^{-3}\) | \((4.6 \pm 1.3) \times 10^{-3}\) |
| \( \bar{p} / p \) | \((4.5 \pm 0.4) \times 10^{-4}\) | \((7.2 \pm 6.3) \times 10^{-5}\) |
| \( \bar{\Lambda} / \Lambda \) | \((2.0 \pm 0.8) \times 10^{-3}\) | \((3.4 \pm 3.0) \times 10^{-4}\) |

Table 2: Abundances of Hadron Species in Si-Au Collisions at the AGS (Thermal parameters: \( T = 110 \pm 5 \text{ MeV}, \mu_B = 540 \pm 20 \text{ MeV} \))

| Species      | Experimental | Thermal     |
|--------------|--------------|-------------|
| nucleons     | 94           | 94          |
| pions        | 120          | 133         |
| kaons        | 14           | 17          |
| hyperons     | 14           | 12          |
| antikaons    | 3            | 4           |
| \( \Xi^- \)'s | 2 \( \times 10^{-1}\) | 2 \( \times 10^{-1}\) |
| \( \phi \)'s | \(2 \times 10^{-2}\) | 6 \( \times 10^{-3}\) |
| antinucleons | \(4 \times 10^{-2}\) | \(6 \times 10^{-3}\) |
| antihyperons | \(3 \times 10^{-2}\) | \(4 \times 10^{-3}\) |
Figure Captions:

- Fig. 1: The $T - \mu_B$ regions determined by the indicated particle ratios (including experimental errors). The charge is kept fixed at 46, corresponding to 46 participant protons and 62 participant neutrons.

- Fig. 2: The $T - \mu_B$ regions determined by the indicated particle ratios (including experimental errors). The charge chemical potential is taken to be zero, thus neglecting the isospin asymmetry of the initial state.

- Fig. 3: The ratio $\pi/N_{\text{part}}$ as function of the number of participating nucleons $N_{\text{part}}$ in $p - A$ and in $Si - Al$ and $Si - Au$ collisions at the AGS.