Nonlinear 2D Spin Susceptibility in a Finite Magnetic Field: Spin-Polarization Dependence

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(Dated: April 16, 2018)

By theoretically calculating the interacting spin susceptibility of a two dimensional electron system in the presence of finite spin-polarization, we show that the extensively employed technique of measuring the 2D spin susceptibility by linear extrapolation to zero-field from the finite-field experimental data is theoretically unjustified due to the strong nonlinear magnetic field dependence of the interacting susceptibility. Our work compellingly establishes that much of the prevailing interpretation of the 2D susceptibility measurements is incorrect, and in general the 2D interacting susceptibility cannot be extracted from the critical magnetic field for full spin polarization, as is routinely done experimentally.

PACS numbers: 72.25.Dc; 75.40.Gb; 71.10.Ca; 72.25.Ba;

The spin susceptibility, also called the Pauli susceptibility for the non-interacting case, is a fundamental property of great significance in condensed matter physics. For example, its behavior (e.g. temperature dependence) could distinguish between Fermi and non-Fermi liquids. The electron interaction induced density dependent enhancement of spin susceptibility is a key signature of many body effects in interacting Fermi liquids, which has been extensively studied during the last fifty years [1, 2, 3]. In fact, the magnetic susceptibility of an itinerant electron system is one of the key (as well as most-studied) thermodynamic properties of metallic systems. In this Letter, we show theoretically that the metallic magnetic susceptibility could depend rather strongly (and non-trivially) on the spin polarization of the system, and such a nonlinear polarization (or equivalently magnetic field) dependent spin susceptibility could have profound effects on the interpretation of many recent experimental measurements [4, 5, 6, 7, 8, 9] of 2D magnetic susceptibility in confined semiconductor structures. In fact, we believe that our theoretical work invalidates most of the recent interpretations of the 2D spin susceptibility measurements, particularly at lower carrier densities and higher fields where the nonlinear effects are strong. We emphasize that the spin-polarization (or the nonlinear field) dependence of the magnetic susceptibility is purely an interaction effect – a strictly 2D noninteracting system has only the usual linear free electron Pauli spin susceptibility.

The key theoretical idea introduced in this work is the observation, almost obvious on hindsight (but routinely ignored in the extensive recent experimental literature on the 2D susceptibility measurement), that in a finite magnetic field B the net spin polarization of an interacting 2D system is manifestly nonlinear in B, unlike the corresponding linear noninteracting Pauli susceptibility situation. This nonlinearity makes the experimental extraction of the interacting 2D susceptibility from a linear extrapolation of the finite-field spin-polarized data to the zero-field limit, as is often done, theoretically unjustified.

The specific relevance of our theoretical nonlinear susceptibility to 2D electron systems in semiconductor structures arises from the particular experimental methods, involving the application of an external magnetic field to spin-polarize the 2D system, typically used to measure the 2D spin susceptibility [4, 5, 6, 7, 8, 9]. In one technique, a tilted magnetic field, with components both parallel and perpendicular to the 2D layer, is used, and the coincidence of the spin-split Zeeman levels with the orbitally quantized Landau levels as manifested in the SdH oscillations of the 2D magnetoresistance is used to obtain the Zeeman energy and hence the susceptibility. In the other method, only an applied parallel magnetic field is used to fully spin-polarize the 2D system, and the observed kink in the magnetoresistance as a function of the applied field is identified as the saturation field B_c to completely polarize the system, leading to the measured magnetic susceptibility. We find that the strong nonlinear dependence of the interacting 2D susceptibility on the applied magnetic field makes it essentially impossible to extract the susceptibility from a measurement of B_c, and some of the controversial conclusions in the literature about the low-density behavior of the 2D susceptibility may have arisen from B_c-based measurements. We note that both experimental techniques involve spin-polarizing the 2D system, and only when this spin-polarization is rather small in magnitude, the susceptibility measurement is sensible.

For absolute theoretical clarity, we consider only the strict 2D limit neglecting the quasi-2D layer thickness effect completely since the finite layer thickness brings in the nonessential complications of the parallel field induced magneto-orbital coupling [10, 11] already at the noninteracting level, leading to a rather complex variation of the 2D susceptibility (due to the parallel field-induced magneto-orbital coupling for motion perpendicular...
ular to the 2D layer) with the carrier density and the applied field, most particularly at low (high) 2D densities (magnetic fields) when the field-induced magnetic length is comparable to the finite layer thickness. Since this is a conceptually simple (but numerically intricate) one-electron band-structure effect, completely independent of the many-body nonlinear effect of interest to us, we leave this out, considering only the strict 2D theoretical limit where the magneto-orbital coupling is, by definition, absent. We neglect thermal effects also, concentrating on \( T = 0 \), in order to focus entirely on the nonlinearity in the susceptibility.

A naive quasi-particle picture to determine the spin-polarization \( \zeta = (n_\uparrow - n_\downarrow)/n \) (where \( n_{\uparrow(\downarrow)} \) is the spin up (down) electron density and \( n = n_\uparrow + n_\downarrow \) is the total electron density) of the 2D electron system in an applied magnetic field \( B \), is to separate the spin-up quasiparticles and spin-down quasiparticles, and to use a simple relation \( E_{F_\uparrow} = -\mu_B B + \mu_B B \), where \( E_{F_\uparrow} \) is the renormalized Fermi energy for the spin up (down) quasiparticles, which is dependent on the up (down) Fermi wavevector \( k_{F_\uparrow} = \sqrt{k^2 + \zeta} \) ( \( k_{F_\downarrow} = k_F \sqrt{1 - \zeta} \) with \( k_F \) being the Fermi wavevector in the unpolarized state. Through this relation one can determine \( \zeta \), and then obtain the susceptibility. This naive picture is suitable for deriving the zero-field susceptibility in the limit \( \zeta \) (or \( B \)) \( \to 0 \), and also for all fields in the noninteracting electron model, but for the interacting system and at finite fields, this simple relation does not hold. A more complete theoretical treatment is then needed in considering the finite field situation when eventually at some density dependent critical field \( B_c(n) \), the 2D system will undergo a first order transition to a fully spin-polarized system. (At finite temperature, this first order transition will be rounded, but the basic physics remains the same.)

We study the magnetization by calculating the total energy per particle of the 2D system as a function of density, spin-polarization, and magnetic field within the ring diagram approximation \( [12, 13] \) which is exact at high density. In an applied magnetic field \( B \), the polarization \( \zeta \) which minimizes the energy then corresponds to the magnetization of the system. The total energy per particle of the system can be written as \( E(r_s, \zeta, B) = E_K(r_s, \zeta) + E_Z(B) + E_C(r_s, \zeta) \) where \( E_K \) is the kinetic energy, \( E_Z \) is the Zeeman energy due to the finite magnetic field, and \( E_C \) is the interaction (Coulomb) energy calculated within the many-body ring diagram approximation. It is useful to mention here that the 2D spin polarization properties (but not the nonlinear aspects of importance in our work) have been theoretically studied with numerical quantum Monte Carlo techniques \( [14] \) which are in principle more sophisticated than our analytic many-body approximation, but the essential qualitative features (i.e. the nonlinearity in the magnetic field) that are relevant for the present purpose are already present in our ring-diagram calculation which becomes exact in the high-density limit. We have used the notation of the interaction parameter \( r_s \), the so-called Wigner-Seitz radius, which is the dimensionless interparticle separation measured in the units of the effective Bohr radius \( a_B: r_s = (\pi n)^{-1/2}/a_B \). It is easy to obtain

\[
E_K = \frac{1}{2} \left( \frac{k_{F_\uparrow}^2}{2m} n_\uparrow + \frac{k_{F_\downarrow}^2}{2m} n_\downarrow \right) = \frac{1 + \zeta^2}{4\alpha^2 r_s^2} (ma_B^2)^{-1},
\]

\[
E_Z = -\mu_B B \frac{n_\uparrow}{n} + \mu_B B \frac{n_\downarrow}{n} = -\mu_B B \zeta,
\]

where \( m \) is the electron mass, \( \alpha = \sqrt{1/\mu} \) is the electron magnetic moment (i.e. the Bohr magneton). The Coulomb energy can be written as \( E_C = E_{ex} + (E_C - E_{ex}) \) where \( E_{ex} \), the exchange energy, can be written as

\[
E_{ex} = -\frac{2}{3\pi a r_s} [(1 + \zeta)^{3/2} + (1 - \zeta)^{3/2}] (ma_B^2)^{-1}.
\]

The rest, the correlation energy, is then

\[
E_C - E_{ex} = \int \frac{d^2q d\omega}{2n(2\pi)^3} [\ln(\epsilon(q, i\omega)) - \epsilon(q, i\omega) + 1] = \frac{2(ma_B^2)^{-1}}{\alpha^4 \pi r_s^2} \int_0^\infty x dx dz [\ln(\epsilon(x, iz) - \epsilon(x, iz) + 1] \quad (3)
\]

where \( \epsilon(q, i\omega) \) is the dynamic dielectric function \( (13) \).

![FIG. 1](Color online.) Calculated energy \( E \) (in arbitrary units) per particle as a function of spin polarization \( \zeta \) in an applied magnetic field \( B \) ranging from 0 to \( B_c \) with steps 0.2\,B_c for \( r_s = 5 \) 2D electron system. (Note that \( B_c \) is a function of \( r_s \).) Inset: the corresponding \( r_s = 1 \) results.

In Fig. 1, we present the energy per particle \( E \) as a function of spin polarization \( \zeta \) in different applied magnetic field \( B \). As we can see from Fig. 1 for small enough \( r_s \) (\( r_s < r_s^* \approx 5.5 \)), the value of which is obvious from Fig. 2, the system prefers zero spin polarization at \( B = 0 \). As \( B \) increases, the energy curve shifts down while the minimum energy corresponds to a non-zero spin polarization \( \zeta^* \). When \( B \) increases to \( B_c \), there exist two \( \zeta^* \) values which minimize the energy. For example, in \( r_s = 5 \) case as shown in Fig. 1 when \( B = B_c \) one energy minimum corresponds to \( \zeta^* = 0.15 \) and the other corresponds to
discontinuity is smoothened somewhat.

![Fig. 2](image-url)  
**FIG. 2:** (Color online.) Calculated full polarization critical magnetic field \( B_c \) as a function of \( r_s \) in units of the corresponding non-interacting value \( B_{c,0} \). Inset: the discontinuous jump of spin polarization \( \zeta^* \) at \( B_c \).

The ground state energy per particle as a function of \( B \) and \( r_s \) is an important result, from which other physical quantities can be derived. For example, the critical polarization magnetic field \( B_c \), which is a function of \( r_s \), can be determined through the above procedure for each \( r_s \) value. Using the polarization magnetic field for non-interacting 2D electron system \( B_{c,0} = E_F/\mu_B \) as the unit, we plot the \( B_c \) for the interacting 2D electron system as a function of \( r_s \) in Fig. 2. From this figure we see that \( B_c \) decreases monotonically as \( r_s \) increases, and that at \( r_s = r_s^*(\sim 5.5) \), \( B_c \) decreases to zero, and the system is spontaneously spin-polarized. This result confirms those of previous theoretical calculations \(^{12,13}\) in the ring diagram approximation. In the inset of Fig. 2 we show the discrete jump of the spin polarization at \( B = B_c \) as a function of \( r_s \). We emphasize that the exact value of \( r_s^*(\sim 5.5) \) here depends on the model and the approximation scheme, and is much larger \(^{13}\) for realistic quasi-2D systems. Also at finite \( T \), the abrupt discontinuity is smoothened somewhat.

![Fig. 3](image-url)  
**FIG. 3:** (Color online.) Calculated spin polarization as a function of magnetic field \( B \) for \( r_s = 5 \). Inset: the corresponding \( r_s = 1 \) results. The relevance of O, A, C, D in defining various susceptibility are discussed in the text.

From the ground state energy we are able to determine the magnetization curve \( \zeta^*(B) \) (Fig. 3), from which we notice that the magnetization increases as a convex function of \( B \) (the convexity is seen clearly in the increasing of the susceptibility shown in Fig. 4), and experiences a discrete jump at \( B = B_c \). For \( B > B_c \), the system remains fully polarized (\( \zeta^* = 1 \)). As mentioned, the magnetization jump in small \( r_s \) system is less pronounced.

![Fig. 4](image-url)  
**FIG. 4:** (Color online.) Calculated spin susceptibility \( \chi^* \) (red solid curves) and semi-linear spin susceptibility \( \chi_5^* \) (blue dashed curves) as a function of magnetic field \( B \) for \( r_s = 1 \) 2D electron system. (The tilted field measurements essentially obtain \( \chi_5^* \).) Inset: the corresponding \( r_s = 5 \) results.

The nonlinear spin susceptibility \( \chi^* = n(d\zeta^*/dB) \) can be derived from magnetization \( \zeta^* \) shown in Fig. 3. Since the magnetization curve has a jump at \( B = B_c \), the spin susceptibility \( \chi^* \) is only meaningful for magnetic field within the range of \( 0 \leq B < B_c \). In Fig. 4 we present calculated spin susceptibility (using the non-interacting Pauli susceptibility \( \chi \) as the unit) as a function of \( B \) for two different \( r_s \) values: \( r_s = 1 \) and \( 5 \). It is worth
mentioning that $\chi^*$ always increases with increasing $B$, i.e. the nonlinearity of the interacting 2D susceptibility is a monotonically increasing function of $B$ in the $0 < B < B_c$ range. The quantitative behavior of nonlinear $\chi^*(r_s; B/B_c)$ is also a strong function of $r_s$, as one can see by comparing the main figure and the inset in Fig. 4. The susceptibility remains finite for all $B$ up to $B_c$, after which $\chi^*$ is not well-defined. (In Fig. 4 we also show the result for, what we call, the semi-linear spin susceptibility $\chi^*_S$, which is related to experimental studies of the susceptibility and is defined below.)

We have also calculated the zero-field susceptibility $(n(d\chi^*/dB)|_{B=0})$, finding precise agreement with our earlier results [1]. We emphasize, however, that the experimental measurements [2,3,7,8,9] do not typically measure the nonlinear susceptibility shown in Fig. 4 or the zero-field susceptibility although most experimental interpretations automatically (and as we show in this Letter, incorrectly) assume that the experimentally measured susceptibility is the usual zero-field linear susceptibility.

One experimental way to study the spin susceptibility is to obtain the polarization field $B_c$ through magnetoresistance measurements [2,3,7,8,9], and then obtain the “spin susceptibility” from $B_c$ using the noninteracting formula. In fact this is not really the spin susceptibility $\chi^* = n(d\chi/dB)|_{B=0}$, but a different quantity which we call the linear spin susceptibility $\chi^*_L = n/B_c$. In Fig. 4 the susceptibility $\chi^*$ is represented by the derivative of the curve at point ‘O’, while the linear spin susceptibility $\chi^*_L$ is represented by the slope of line ‘OD’. These two quantities $\chi^*(B = 0)$ and $\chi^*_L$ (measured experimentally from the slope of the line ‘OD’ in Fig. 4) are certainly very different from each other, especially at larger $r_s$ values. We also note that the real critical field $B_c(D)$ corresponding to the point ‘D’ is much smaller than the extrapolated line ‘OC’ would indicate! In particular, $\chi^*_L$ would always be much larger than $\chi^*(B \to 0)$, and the experimental conclusion based on the measurement of $B_c$ is simply incorrect. It should be noted in this context that the semi-linear susceptibility $\chi^*_S$ (shown in Fig. 4 and discussed below) is always smaller in magnitude than $\chi^*$, and therefore in general, $\chi^*_L > \chi^*_S$.

Another experimental method (the tilted field method) to study the susceptibility is by matching Landau levels and Zeeman energy levels [3]. The experimental detail boils down to measuring, what we call, the semi-linear spin susceptibility $\chi^*_S(B) = n\chi^*(B)/B_c$ shown in Fig. 4. The easiest way to describe this quantity is by examining Fig. 5. The semi-linear spin susceptibility $\chi^*_S(B)$ at point $A$ is represented by the slope of line ‘OA’, while the susceptibility $\chi^*(B)$ is represented by the derivative of the magnetization curve at point ‘A’. Of course these two quantities are different, especially in a large magnetic field, as shown in Fig. 4. However, the experimental measurement of this semi-linear spin susceptibility $\chi^*_S$ is still reasonably meaningful in the following ways. One is that for $B = 0$, $\chi^*_S$ and $\chi^*$ coincide with each other as shown in Fig. 4 and therefore theoretically speaking, this measurement $\chi^*_S$ should be able to capture the true behavior of the zero-field susceptibility. Another meaningful aspect of this experiment is that the measurement $\chi^*_S$ shows that as $B$ increases, $\chi^*_S$ also increases $\chi^*$, which suggests that the magnetization curve is convex even though $\chi^*_S$ and $\chi^*$ are different. This observation agrees with our theoretical findings. We therefore conclude that the tilted field measurement leading to $\chi^*_S$ is reasonable (but still far from perfect) for measuring the 2D susceptibility for $B < B_c$, whereas the susceptibility $\chi^*_L$ (extracted from the measurement of $B_c$) is not particularly meaningful.

In conclusion, we have calculated the nonlinear magnetization and spin susceptibility as a function of magnetic field and density for 2D electron systems with long-ranged Coulomb interaction in an applied magnetic field. We find that most measurements of 2D spin susceptibility are incorrect because they do not incorporate the magnetic field-induced nonlinearity. Because of our neglect of sample details (e.g. finite width effects), our general theory is not directly comparable to the existing experimental data in any particular system, but our work establishes that any experiment in a finite magnetic field, cannot provide a meaningful measurement of the 2D susceptibility, except at the lowest fields and highest densities (i.e. for $B \ll B_c$) where our predicted nonlinear effects are quantitatively small. In particular, we show convincingly that an experimental measurement of $B_c$ (e.g. the parallel field magneto-transport data) most certainly does not provide a value for the zero-field interacting 2D susceptibility as has been uncritically assumed in most earlier works whereas the tilted field measurements, particularly in thin 2D samples at low magnetic fields, provide an approximate measurement of the susceptibility. Finally, we note that finite temperature effects would smoothen the discontinuity (at $B_c$) in the magnetization since there will be some finite thermal population of both spin up/down bands, but the same physics will apply qualitatively at low temperatures.

This work is supported by ONR, NSF, and LPS.

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