Assessing Disk Galaxy Stability through Time

D. Valencia-Enríquez1,2, I. Puerari1, and I. Rodrigues3

1 Instituto Nacional de Astrofísica, Óptica y Electrónica, Calle Luis Enrique Erro 1, Santa María Tonantzintla, 72840 Puebla, Mexico
2 Corporación Universitaria Autónoma de Nariño, Carrera 28 No. 19-24, 52001 Pasto, Colombia
3 Instituto de Pesquisa e Desenvolvimento, Universidade do Vale do Paraíba, Av. Shishima Hifumi, 2911—Urbanova, São José dos Campos—SP, 12244-000, Brasil

Received 2018 September 18; revised 2019 February 21; accepted 2019 March 13; published 2019 April 16

Abstract

N-body simulations have shown that a bar in a galaxy can be triggered by two processes: (1) by its own instabilities in the disk, or (2) by interactions with other galaxies. Both mechanisms have been widely studied. However, the literature has not shown measurements of the critical limits of the disk stability parameters (DSPs). We show measurements of those parameters through the whole evolution in isolated disk models and find that the initial rotation configuration of those models stays in the stable or unstable regime from the initial to the final evolution. Then we perturbed the isolated models to study the evolution of DSPs under perturbation. We find that the critical limits of DSPs are not much affected in barred models, but when the bar is triggered by a perturbation, the disk falls into the unstable regime. We show in our models that a bar triggered by a light perturbation grows in two phases; first, the bar appears as a slow rotator, and then it evolves to be a fast rotator; second, when the perturbation is far from the target galaxy, the bar evolves from fast to slow rotator. When the bar is triggered by a heavy perturbation, it appears as a fast rotator and evolves to be a slow rotator, similar to classical bar models.

Key words: galaxies: evolution – galaxies: kinematics and dynamics – galaxies: structure – methods: numerical

1. Introduction

Evaluating the stability of a disk galaxy, as well as disentangling the formation and evolution of a barred galaxy from the observations, is a difficult task. Analytical works and N-body simulations are alternatives for studying its dynamics and bar evolution. Analyses of the stabilities are based on the determination of the dispersion relations and then investigation of unstable modes. If the size of the perturbation is much smaller than the size of the disk, the perturbation is cataloged as a local stability. The stellar Toomre stability criterion \( Q \) plays an important role in the formation of spiral arms fragmentation to local scale (Romeo & Mogotsi 2017). But, if the size of the perturbation is comparable with the size of the disk, the perturbation is classified as a global stability. However, it is very difficult to write down a universal dispersion relation stability criterion. In the cosmological context, where the halo is much larger than the disk, the bar perturbation cannot be classified as a global perturbation, but rather is a central characteristic of a full galaxy.

Bar formation in disk galaxies has been studied for several decades. Basically, a bar-like structure is triggered by two processes: (1) those that stem from internal causes, such as dynamical instabilities within individual galaxies, and (2) those that are produced by external (e.g., tidal) influences.

Lynden-Bell (1996) and Polyachenko & Polyachenko (2004) reviewed five different mechanisms for bar formation in isolated galaxies. (1) Because real bars have internal streaming motions, Freeman’s picture of bar formation tried to associate these motions with incompressible Jacoby and Riemann fluids. (2) Two density waves are reflected in the center of the galaxy and amplified via Toomre’s swing mechanism when the disk has no inner Lindblad resonance (ILR). (3) Contopoulos’ picture shows that the bar is a result of the distortion of circular orbits into eccentric orbits when the bar potential is already formed. (4) Kalsojs (1971, 1977) made a full stability analysis that led to an eigenvalue problem for normal modes of an axisymmetric stellar disk. The self-gravities of an ensemble of orbits (lobes) cooperate with one other to generate disk instability radial orbits, which result in the formation of slow Lynden-Bell bars. In practice, it has proved to be very difficult to find eigenvalues (Sellwood & Wilkinson 1993). Lynden-Bell (1979) suggested that bars may grow slowly through the gradual alignment of eccentric orbits. In addition, Polyachenko & Polyachenko (2004) discuss the formation of galactic structures viewed as low-frequency normal modes in disks consisting of precessing stellar orbits. They studied the properties of an integral equation via the Lynden-Bell derivative of the distribution function, which depends on the variation of angular momentum. They found that if such a derivate is positive, the bar mode can form; otherwise, the derivate is negative, and then spiral modes grow. The mechanism of bar formation constrains the angular velocity of the mode, which should be larger than the angular velocity of the orbital precession for fast bars or should be similar for slow bars (Polyachenko & Polyachenko 2003). Then the bar mode develops as a result of azimuthal tuning orbits in a massive disk. However, the wave decay in a light disk due to the wave mode meets the ILR. (5) A statistical focus examines bar formation from the rotating initial configurations when its spin parameter is lower than some critical limit.

These channels of bar formation are all connected by the change in angular momentum. It has been shown that the formation and evolution of bars in isolated disk galaxies depend on the angular momentum exchange between their resonances and components (halo, disk, or bulge; Athanassoula 2002 and references therein). Therefore, a study of the spin parameter \( \lambda_r \) (Equation (10)), which is a measure of specific angular momentum, and its critical limit \( \lambda_{cr} \) (Equation (14)) can assess the stability in situ of a disk model, and diagnose the growth of a bar instability as well. These parameters can be related to an empirical stability parameter of Efstathiou et al. (1982, hereafter EF82), which only depends on the rotation curve, disk mass, and disk radius scale, and it is relatively easy to obtain in real observed galaxies (Equation (13)). This parameter can

\[ Q = \frac{\sqrt{2m_0}}{c_0} \]

\[ \lambda_r = \frac{\sqrt{2m_0}}{c_0} \]

\[ \lambda_{cr} = \frac{\sqrt{2m_0}}{c_0} \]
evaluate roughly the stability of a disk because it is indirectly related to the exchange of angular momentum. In other words, the angular momentum exchange can be more efficient when the velocity dispersion is low and affects the mass distribution of the disk, halo, or bulge, which in turn affect the rotation curve and the scale radius of a disk. Therefore, our goal is to figure out the critical limit of the spin parameter, which relates the energy, mass distribution, and angular momentum of a disk galaxy by using \(N\)-body simulations to show the utility of \(\lambda_{\text{crit}}\) for diagnosing disk stability in \(N\)-body simulations. Furthermore, we claim the possibility of using \(c_m\) to give a rough estimate of the stability in a real galaxy, because getting the angular momentum or spin parameter from a real galaxy is a very difficult task.

The Mo et al. (1998, hereafter MO98) models let us manipulate the spin parameter directly and get a maximum and submaximum disk with the same \(M_D/M_H\) ratio. These models can be stable or unstable to bar formation, depending on the disk stability parameters (\(\lambda_N, \lambda_{\text{crit}}\), hereafter DSPs). Therefore, to accomplish our goal, we generate models with the same \(M_D/M_H\) ratio, but with different initial disk stability parameters to be stable or unstable to bar formation, and then we follow these parameters through time, because a follow-up of \(\lambda_{\text{crit}}\) over time has not been done. Likewise, we obtain measures of the parameter \(c_m\) through time to show and connect its behavior to the bar instability.

We know that galaxies in the universe are not isolated; in fact, they are interacting. Thompson (1981) showed that there is a large fraction of barred galaxies in the core of the Coma cluster, indicating that tidal interactions can trigger bar formation. Elmegreen et al. (1990) studied binary galaxy samples to search for possible correlations between the bar and the Hubble type, finding that binary systems have a factor of \(\sim 2\) excess of barred galaxies. Andersen (1996) made a study of the velocity distribution of disk galaxies in the Virgo cluster, with the result that only the barred spiral galaxies in the core of the Virgo Cluster may have been triggered by interactions. Marinova et al. (2011) investigated the properties of bright barred and unbarred galaxies in the Abell 901/902 cluster, explaining that a high-velocity dispersion in the core region benefits flyby interactions, which may increase the bar fraction while preserving intact the galaxy disk.

\(N\)-body simulations have shown that a bar in a disk galaxy can be triggered by interactions (Noguchi 1987; Gerin et al. 1990; Sundin et al. 1993; Miwa & Noguchi 1998, and reference therein). Gerin et al. (1990) and Sundin et al. (1993) showed that tidal effects can increase or decrease the strength of the bar, which depends on the mass implicated, the pericenter distance, and the relative phase between the bar and the companion. Miwa & Noguchi (1998) simulated close encounters, showing that bars generated by these encounters are confined to the ILR, producing slow bars, depending on the mass of the perturber. Recently, Moetazedian et al. (2017) showed that encounters with low-mass satellite galaxies may cause a delay in bar formation compared to the isolated case. Instead, they can cause an advance in the bar formation after a small bar is already formed in the center but its amplitude is still insignificant. They explain that the spiral wave created by the perturbation can interfere positively or negatively via swing amplification to form a bar. Likewise, Martinez-Valpuesta et al. (2017) showed that the evolution of the bar parameters (strength, length, and pattern speed) in disk galaxies, which form a bar-like structure in isolation, are not much affected, while such parameters triggered by a perturbation show some difference from their counterparts. The angular velocity of the bar that was triggered by a flyby is slower than in such a structure formed by a self-instability of the disk. They also showed that a slow flyby has a greater effect on the target galaxy.

Cosmological simulations show that bars form and then are destroyed in response to asymmetric halos and interactions with the substructure (Romano-Díaz et al. 2008). Lang et al. (2014) used \(N\)-body simulations to investigate the ability of galaxy flyby interactions to form bars, finding that the mass ratio between the main galaxy and the perturbation determines some properties of the bar in the target galaxy. This type of encounter can be as strong as a minor merger (Vesperini & Weinberg 2000) and therefore can change the properties of the disk as well as the disk stabilities, transforming the galaxy in a permanent way.

Therefore, flyby encounters appear in both simulations and observations, and such interactions may change the properties of the model and in turn the DSPs; then the disk might fall in the instability regime, which triggers the bar growth. Hence, another goal is to study the behavior of the DSPs and the experimental parameter \(c_m\) as the bar or spiral structures evolve in our models, now under perturbation, and prove they remain in the stability or instability regimes.

The growth of a barred galaxy has three main phases (Martinez-Valpuesta et al. 2006), which are characterized by three main observational parameters: length, strength, and pattern speed. In this paper, we also measure these parameters to study the growth of the bar. The first phase corresponds to bar formation and extends for \(\sim 2\) Gyr; the bar strength and bar length grow quickly. The second phase is the buckling of the bar, where the vertical symmetry in the bar is broken, weakening the bar. In this phase, the amplitude \(m = 2\) of the Fourier transform \(A_2\) reaches a maximum, saturating the bar (Martinez-Valpuesta et al. 2006).

The final phase of the bar is the secular evolution. Sellwood (1981) showed that the bar grows slowly by increasing its strength and length. Combes & Sanders (1981), on the other hand, reported that bars tend to weaken in the long term. The rate at which bar parameters change depends on the properties of the model. Debattista & Sellwood (1998) showed that the bar slows by dynamical friction in a dense dark matter (DM) halo, while Athanassoula et al. (2013) found that the higher the gas content of the disk, the slower is the growth of the bar. They also found that the halo triaxiality triggers bar formation earlier and leads to considerably less increase in bar strength. On the other hand, the pattern speed of the bar slows down during these phases (Weinberg 1985; Little & Carlberg 1991; Athanassoula 2003).

This paper is organized as follows. Section 2 presents a theoretical view, Section 3 shows a description of the \(N\)-body simulations and methods, and Sections 4 and 5 show the results for isolated and perturbed models, respectively. Section 6 presents the discussion and Section 7 the conclusions.

2. Theoretical Input

We have performed collisionless \(N\)-body simulations with the Gadget-2 code (Springel et al. 2001; Springel 2005). We present here 15 simulations of fully self-consistent models, all of them with a live exponential disk and live DM halo. The live halo ensures disk–halo angular momentum exchange, which
plays an important role in the formation and evolution of bars, as discussed by Athanassoula (2002). We simulated barred and unbarred models, aiming to monitor the disk stability parameters of the disk.

2.1. Models of Disk Galaxies

The initial conditions were set down following the methodology delineated by Springel & White (1999, hereafter SW99) and Springel et al. (2005), which is based on the analytic model of Mo et al. (1998, hereafter MO98).

The DM mass distribution was modeled with a Hernquist (1990) profile,

$$\rho_{\text{dm}} = \frac{M_{\text{dm}}}{2\pi r(r + a)^3},$$  \hspace{1cm} (1)

with cumulative mass profile $M(<r) = M_{\text{dm}}r^2/(r + a)^2$. This profile has the same DM as a Navarro–Frenk–White (NFW) profile (Navarro et al. 1996, 1997) within the $r_{200}$ radius ($r_{200}$ is the radius at which the mean enclosed DM density is 200 times the critical density, i.e., it contains the virial mass). The NFW profile is often given in terms of the concentration index $c$, defined as $c = r_{200}/r_d$, where $r_d$ is the scale length of the NFW halo. We then have the relation

$$a = \frac{r_{200}}{c} \sqrt{2[\ln(1 + c) - c/(1 + c)]}. \hspace{1cm} (2)$$

Furthermore, let us define

$$V_{200}^2 = \frac{GM_{200}}{r_{200}}$$ \hspace{1cm} (3)

to be the circular velocity at the virial radius.

The stellar component is modeled with an exponential surface density profile of scale length $r_d$:

$$\Sigma(r) = \Sigma_0 e^{-r/r_d}$$ \hspace{1cm} (4)

where $\Sigma_0 = M_d/(2\pi r_d)$. The vertical mass distribution is given by an isothermal sheet with a radially constant vertical scale length $z_0$. Therefore, three-dimensional stellar density in the disk is

$$\rho_s(r, z) = \Sigma(r) \left[ \frac{1}{2z_0} \text{sech}^2\left( \frac{z}{2z_0} \right) \right]. \hspace{1cm} (5)$$

A self-gravitating model is one in which the initial kinetic energy of the spherically symmetric halo may be computed by assuming that all particles move around the center on circular orbits, with speed equal to the circular velocity (SW99 and MO98), so that $E_{\text{kin}} = (GM_{200}/(2r_{200}))f_c$, where

$$f_c = \frac{c[1 - 1/(1 + c)^2 - 2\ln(1 + c)/(1 + c)]}{2[\ln(1 + c) - c/(1 + c)]^2}, \hspace{1cm} (6)$$

which comes from the change in the total energy resulting from the different density profile.

The total angular momentum of the halo $J_h$ with total energy $E_h$ is often characterized by the dimensionless spin parameter

$$\lambda_h = \frac{J_h|E_h|^{1/2}}{GM_h^{3/2}}. \hspace{1cm} (7)$$

The disk has a structure of a thin exponential disk, and it is cold and centrifugally supported. The mass disk $M_d$ is a fraction $m_d$ of $M_{200}$:

$$M_d = m_d M_{200}. \hspace{1cm} (8)$$

In a similar way, the angular momentum of the disk $J_d$ is a fraction $j_d$ of $J_h$:

$$J_d = j_d J_h. \hspace{1cm} (9)$$

Consequently, the spin parameter of the disk is

$$\lambda_d = \left( \frac{j_d}{m_d} \right) \lambda_h. \hspace{1cm} (10)$$

From this equation, we then determine the scale radius of the disk (MO98), given by

$$r_d = \frac{1}{\sqrt{2}} \lambda_d r_{200} f_c^{-1/2} f_r \hspace{1cm} (11)$$

where

$$f_r = 2 \left[ \int_0^{r_{200}} e^{-r/r_d} \frac{1}{r^3} \nu(r) \frac{d^2r}{\nu^2} \right]^{-1}. \hspace{1cm} (12)$$

Note that in practice the scale length $r_d$ in the initial disk is determined iteratively in order to satisfy Equations (13) and (17) from Springel & White (1999).

2.2. Criterion of Instability

Instabilities play a very important role in transforming and regulating the properties of disk galaxies. Local instabilities are affected by perturbations with lengths much smaller than the size of the disk. They can be transient and can regulate the evolution of a disk by driven features such as transient spiral structures and star formation due to the fragmentation and collapse of gas clouds. On the other hand, large disk instabilities (LDI), which are comparable to the size of the disk, can cause a significant transformation of the overall disk. Whenever a disk galaxy has LDI, it will evolve toward a new stable configuration, erasing information about the initial conditions under which the system was formed (Mo et al. 2010).

We first focus on disk instabilities that can trigger a bar in an isolated disk galaxy. For that purpose, the most relevant studies are those of EF82 and MO98. EF82 used N-body techniques to investigate disk instabilities of exponential disks embedded in a variety of halos and found that the bar instability for a stellar disk is characterized by the parameter $\epsilon_m$:

$$\epsilon_m = \frac{V_{\text{max}}}{\sqrt{GM_d/r_d}}. \hspace{1cm} (13)$$

They found that if $0.7 \leq \epsilon_m \leq 1.2$, then the disk is unstable to bar formation. If $\epsilon_m > 1.2$, then the disk is stable, but this parameter seems not to work well (Athanassoula 2008; Saha & Naab 2013). However, MO98 and SW99 show that disk stability is characterized by a lower limit of disk spin parameter $\lambda_d$; then they obtain a relation of this critical limit $\lambda_{\text{crit}}$ with the help of $\epsilon_m$, but they use an approximation of $V_{\text{max}}$ in their models; we use the $V_{\text{max}}$ obtained directly from the N-body simulation, and we deduce a relation similar to that of MO98 for $\lambda_{\text{crit}}$. Thus, following the same methodology as MO98, we use the $\epsilon_m$ formula together with Equation (11) and establish $\epsilon_{m, \text{crit}} \approx 1$ for the disk stability; then, we obtain a relation of a
lower limit of the disk spin parameter, as follows:

\[
\lambda_{\text{crit}} = \left( \frac{\varepsilon_{\text{crit}}}{V_{\text{max}}} \right)^2 \frac{GM_d \sqrt{2f_r}}{r_{200} f_r} = \frac{GM_d \sqrt{2f_r}}{V_{\text{max}}^2 r_{200} f_r} \quad (14)
\]

where \( V_{\text{max}} \) is the maximum circular velocity, \( M_d \) is the mass of the disk, \( f_r \) is an integral that depends on \( r_{200} \), \( v_c(r) \) is the circular velocity, \( V_{200} \) is the circular velocity given by Equation (3), and \( f_c \) is a function that depends on the concentration of the halo. Thus, we can find a disk stable against bar formation if \( \lambda_d > \lambda_{\text{crit}} \), but otherwise it is unstable, as follows:

\[
\begin{align*}
&\lambda_d > \lambda_{\text{crit}}, \quad \text{stable against bar formation} \\
&\lambda_d \leq \lambda_{\text{crit}}, \quad \text{unstable to bar formation.}
\end{align*}
\]

This parameter has the advantage that it depends on the angular momentum exchange and the mass distribution of the components, concentration, and scale radius. In addition, it also depends on the velocity dispersion \( \sigma \) because for higher \( \sigma \) the particles have less time to exchange energy and angular momentum, and therefore \( \lambda_d \) varies less. To prove this criterion, we set down four isolated disk models, and these models were subjected to 11 other interactions.

3. Methodology

This section includes a description of the initial conditions of the models, which consist of a disk and a DM halo, the interactions, and the tools we develop to analyze the models.

3.1. Isolated Model Setup

The initial conditions are generated following the methodology delineated in SW99 and MO98 as described before. Our models have been evolved from 0 to 6 Gyr. In the four models we present here, only one initial parameter was modified: the spin parameter \( \lambda \equiv \lambda_d \). Most of the structural parameters are given in Figure 1. It shows models from disk-dominated to halo-dominated ones. We remark that these models were generated with the same \( M_p/M_d \) ratio, changing then the disk radial scale length, central surface density, and the Toomre parameter \( Q \). The Toomre parameter \( Q \) increases according to the spin parameter in the initial conditions (Figure 2).

The isolated models (Table 1) have \( 7 \times 10^6 \) particles: \( 2 \times 10^6 \) to simulate the disk and \( 5 \times 10^6 \) to simulate the halo. We ensured that the mass of the halo particles is not larger than eight times the mass of the disk particles. Additionally, we repeat the simulation \( \Lambda \lambda_05 \), doubling its number of particles to \( 14 \times 10^5 \), \( 4 \times 10^6 \) in the disk and \( 10 \times 10^6 \) in the halo, and also we reduce to half the softening parameters, keeping all of the other physical parameters unchanged. As the analysis shows (Section 4), the main characteristics of the model remain unchanged: the spin parameter \( \lambda_d \) and the stability experimental parameter \( \varepsilon_{\text{crit}} \) time evolution do not depend on these numerical parameters.

The simulations were performed with the Gadget-2 code (Springel 2005) with its default units, where the velocity unit is equal to \( 1 \text{ km s}^{-1} \), the length unit is equal to \( 1 \text{ kpc} \), and the mass unit is equal to \( 1 \times 10^{10}M_\odot \). The gravitational forces were computed with a hierarchical multipole expansion, in which short-range forces are computed with the “tree” method, while long-range forces are determined with Fourier techniques with a tolerance parameter \( \theta_{\text{tol}} = 0.5 \). The softening length for the disk particles is \( \epsilon = 0.01 \), and for the halo ones it is \( \epsilon = 0.1 \). They are chosen so that the maximum interparticle force shall not exceed the typical mean-field strength (Dehnen & Read 2011). Thus, we ensure that two-body relaxation will not artificially induce chaotic orbits. Time integration is based on a quasi-symplectic scheme where long-range and short-range forces can be integrated with different time steps given by \( \Delta t = \sqrt{2\eta\epsilon/|a|} \), where \( \eta = 0.01 \) and \( a \) is the acceleration of the last time step. With these parameters, we ensure that the energy conservation was better than \( 10^{-3} \). We assessed the numerical robustness by experimenting with fewer particles and a bigger softening according to Dehnen & Read (2011), getting similar results.

3.2. Setting Up the Encounters

We use the models set up in Section 3.1 to subject them to a perturbation. In 3.1, we modeled four disk galaxies where we only change the spin parameter to get two classical models: two models where the disk dominates, another one where the halo dominates, and another one where the disk and the halo have the same contribution in the inner region of the rotation curve.

We use a Hernquist profile to build the stellar and halo components of the perturbation using the methodology established by Springel & White (1999) and Springel et al. (2005). We set up three groups of encounters where we change the mass of the perturbed to have different tidal forces at the pericenter; Table 2 lists such interactions. The first group (Pw) has interactions where the total mass of the perturber is approximately half of the total mass of the target model, the second group (Pm) has interactions where the total mass of the perturber is similar to the total mass of the target model, and finally, the last one (group Ps) has interactions where the total mass of the perturber is almost two times the total mass of the target model.

Later in the article, we will show that isolated models form a bar at different times, for example, in model A03 after the first gigayear, in model A04 after the second gigayear, and in model A05 at the end of the simulation, while model A06 does not form a bar-like structure. We set up the interactions from the second gigayear in these isolated models, and we calculate the passage at around the third gigayear to analyze the tidal perturbation at different stages of the bar as well as the growth of the bar in the stable model.

The interactions follow a coplanar, prograde, and hyperbolic orbit around the disk, reaching the pericenter at the first gigayear of its evolution (third gigayear in our interactions). We set the pericenter distance for all interactions approximately to \( r_p = 3 r_{75} = 19.4 \text{ kpc} \), where \( r_{75} \) is the radius containing 75% of the disk mass of model A03.

Based on our experience, we use the mass contained up to the radius \( r_p \) of target galaxy \( M_t(r_p) \) and perturbation galaxy \( M_p(r_p) \) as a point mass to calculate an approximation of the orbit that the perturbation will trace two like bodies, where

\[
M_t(r_p) = M_{D_t}(r_p) + M_{H_t}(r_p)
\]

and

\[
M_p(r_p) = M_{D_p}(r_p) + M_{H_p}(r_p),
\]

where the subscripts \( H, D, \) and \( S \) mean the halo, disk, and spherical components.

The DM mass distribution was modeled with a Hernquist (1990) profile (Equation (1)); this profile has the same DM to
The NFW profile is often given in terms of the concentration index \( c \), defined as \( c = r_{200}/r_s \), where \( r_s \) is the scale length of the NFW halo, and the characteristic overdensity is

\[
\delta_0 = \frac{200 \left( 1 + c \right)^{3/2}}{3 \ln(1 + c) - c/(1 + c)}. \tag{19}
\]

Then, the mass within radius \( r_p \) for the halos \( M_{Ht} \) and \( M_{Hp} \) is

\[
M_{Ht}(r_p) = 4\pi \rho_{\text{crit}} \delta_0 r_s^3 \left[ 1 + \ln(1 + cx) \right], \tag{20}
\]

where \( x = r_p/r_{200} \).

The stellar component of the perturbation \( M_{Sp} \) was also modeled with a Hernquist profile; therefore the mass contained at radius \( r_p \) is

\[
M_{Sp}(r_p) = M_{Sp0} \left( \frac{r_p}{r_p + a} \right)^3, \tag{21}
\]

where \( M_{Sp0} \) is the total mass of the stellar component for the perturbation galaxy, and the mass within radius \( r_p \) of the exponential disk is then

\[
M_{Dp}(r_p) = \frac{M_D}{1 - \left( 1 + \frac{r_p}{r_d} \right) e^{r_p/r_d}}. \tag{22}
\]

Then we get the position and initial velocity of the perturber to get that orbit from the motion equations of two bodies. Obviously,
determine the scale radius of the halo than the pericenter calculated by the theoretical orbit. Figure 3 compares the theoretical and simulated orbits. Orbits traced by the perturbation from the disk density center shows that the evolution of the disk to the exponential disk also was used to calculate the instantaneous angular velocity of the bar Ωb, and then it was used to fix the bar reference frame. From the bar reference frame, we calculate the bar axis lengths by providing a density threshold to define the bar limits. In order to measure the size of the bar, first we obtain a profile of the bar with both the major and minor axes, dividing them into cells and then calculating their surface density; the limits of the bar were calculated using the density threshold provided. The next step was to change iteratively the size or number of cells until reaching a convergence of 10⁻² on the length of the bar axes.

4. Results for Isolated Models

In this section, we describe the growth of the bar, the calculations of the disk instabilities through time, and the results from the Fourier transform analysis.

4.1. Stability Criterion through Time

Figure 4 shows the face-on logarithm surface density maps for all of our models, including the Aλ05_M14 model, at times 0, 1, 2, 3, 4, 5, and 6 Gyr. Model Aλ03 in the top row has the lowest value of ε0 and λc, being λd < λc (see Figure 5); this model maintains λd < λc for the full evolution. This model forms a bar very quickly, and it is maintained during the whole evolution. Model Aλ04 is shown in the second row; it also has λd < λc and 0.7 < ε0 < 1 (see Figure 6), and it forms the bar at around 2 to 3 Gyr, and the bar is also maintained during the entire evolution. The third row shows the model Aλ05; this model has λd ≈ λc and ε0 a little less than unity (see Table 1). It forms a weak bar perturbation around the fourth gigayear; the Aλ05_M14 model has a very similar nature. We notice that the numerical resolution affects modestly the angular velocity of the bar structure, as we can see in Figure 4, where the position of the bar is different between these models at the same snapshot; however, we observe that the DSP does not have a high dependence on the resolution, as shown in Figures 5 and 6. The last row presents the model Aλ06; this model has λd > λc and ε0 > 1. This model is stable and shows some weak spiral waves. On the other hand, we observe that extended disk can generate longer bars than concentrate models, in agreement with the results of Cervantes Sodi & Sánchez García (2017); it was found that the length of the bars depends on the surface brightness: high surface brightness galaxies host longer bars than their low surface brightness counterparts. Cervantes Sodi & Sánchez García (2017) attribute this result to an anticorrelation of the surface brightness with the spin.

Figure 5 shows the evolution of the DSPs (λd and λc) for our simulations. Model Aλ03, in the upper left panel of Figure 5, begins with λd < λc (see upper right panel of Figure 5). The Aλ05 (black lines) and Aλ05_M14 models (cyan lines), in the bottom left panel, start with λd ≈ λc, but once the bar begins to form, λd becomes smaller than λc and remains like that to the end of the simulation. Model Aλ06, in the bottom right panel, shows that the spin parameter is larger than the critical spin parameter during the whole evolution.

Figure 3. Orbits traced by the perturbation from the disk density center (the bar) of the target galaxy. The color lines are represented in the figure, and the dashed lines show the theoretical orbits. We can see that the pericenter is less than the pericenter calculated by the theoretical orbit.

this theoretical orbit is slightly different from the simulated orbit. Figure 3 compares the theoretical and simulated orbits.

The target models are Aλ03, Aλ04, Aλ05, and Aλ06, and the perturbation model has 1 × 10⁶ particles: 1 × 10⁴ to simulate the stellar component (it is a Hernquist profile) and 9.9 × 10⁵ to simulate the halo. The simulations were performed with the Gadget-2 code (Springel 2005) where the tolerance parameter is θtol = 0.5 and the softening length is ε = 0.01. In the same form as we mention in Section 3.1, we perform again the interaction PwAλ06 while doubling the number of particles in the perturbation (2 × 10⁶ particles: 2 × 10⁴ to simulate the stellar component, and 1.98 × 10⁶ to simulate the halo, PwAλ06_2M); also, we reduce to half the softening parameters, keeping all other physical parameters unchanged. The analysis of all characteristics of the perturbed disk (Section 5) shows that the evolution of the disk is very similar to the case with a perturbation of few N particles. Likewise, we assessed the numerical robustness by experiment with fewer particles and bigger softening according to Dehnen & Read (2011), getting similar results.

3.3. Measurement of Parameters

We determine the spin parameters λc using Equations (7) and (10) from the phase space of the simulation. The λc was calculated using Equation (14). First, we calculate the potential as a function of radius, then we determine the derivative of this curve to get the rotation curve vₗ(r) for a snapshot, and so we obtain the Vmax. To calculate fₑ, we fit the halo profile to the NFW profile to determine the scale radius of the halo rₜ and the radius r₂₀₀, then we calculate the halo concentration using Equation (2). Finally, we fit the profile of the disk to the exponential disk (Equation (4)) to obtain the scale length radius of the disk, then we calculate the integral fₑ (Equation 12). Also, we use these parameters to calculate the experimental stability parameter εₑ. It should be mentioned that we calculate all of these parameters using the phase space for each saved snapshot of the simulations.

As we said before, the bars are characterized by three main observational parameters: length, strength, and pattern speed. In order to measure these parameters, we compute Fourier coefficients for modes from m = 1 to m = 10 in the disk particles and monitor their amplitudes and phase variations across the disk as a function of time. We use the amplitude m = 2 to measure the strength and growth rate of the bar that is being formed in the disk. The phase m = 2 of the Fourier coefficients was also used to calculate the instantaneous angular velocity of the bar Ωb, and then it was used to fix the bar reference frame. From the bar reference frame, we calculate the bar axis lengths by providing a density threshold to define the bar limits. In order to measure the size of the bar, first we obtain a profile of the bar with both the major and minor axes, dividing them into cells and then calculating their surface density; the limits of the bar were calculated using the density threshold provided. The next step was to change iteratively the size or number of cells until reaching a convergence of 10⁻² on the length of the bar axes.

We determine the spin parameters λc using Equations (7) and (10) from the phase space of the simulation. The λc was calculated using Equation (14). First, we calculate the potential as a function of radius, then we determine the derivative of this curve to get the rotation curve vₗ(r) for a snapshot, and so we obtain the Vmax. To calculate fₑ, we fit the halo profile to the NFW profile to determine the scale radius of the halo rₜ and the radius r₂₀₀, then we calculate the halo concentration using Equation (2). Finally, we fit the profile of the disk to the exponential disk (Equation (4)) to obtain the scale length radius of the disk, then we calculate the integral fₑ (Equation 12). Also, we use these parameters to calculate the experimental stability parameter εₑ. It should be mentioned that we calculate all of these parameters using the phase space for each saved snapshot of the simulations.

As we said before, the bars are characterized by three main observational parameters: length, strength, and pattern speed. In order to measure these parameters, we compute Fourier
The experimental stability parameter, $\epsilon_m$, is plotted in Figure 6. This figure shows the evolution of $\epsilon_m$ for all of the models. The black line depicts the A\(\lambda\)03 model, the red line depicts the A\(\lambda\)04 model, the green line shows the A\(\lambda\)05 model, the cyan line depicts the A\(\lambda\)05_M14 model, and blue line depicts the model A\(\lambda\)06. While the evolution of $\epsilon_m$ shows values in the range from 0.7 to 1 for barred models, the evolution of $\epsilon_m$ for the A\(\lambda\)06 model displays values larger than one. Furthermore, the $\epsilon_m$ parameter for models A\(\lambda\)03 and A\(\lambda\)04 shows an increase during the evolution, but not exceeding unity, while $\epsilon_m$ for models A\(\lambda\)05 and A\(\lambda\)05_M14 fluctuates around unity at the end of the simulation. This last model evolves very similarly to the A\(\lambda\)05 model, which means that the DSPs and the $\epsilon_m$ parameter do not depend on the number of particles in our models.

### 4.2. Growth of the Bar

The evolution of the bar and the spiral structure is shown in Figure 7, in which we plot the Fourier amplitude for the $m = 2$ mode, $A_2(t, R)$. For model A\(\lambda\)03 (top panel), we observe that the rapid growth of the bar is followed by spiral waves until $A_2$ reaches a maximum; after that, the disk increases its velocity dispersion, maintaining the bar perturbation, and some weak spiral waves are driven transiently by the bar (Athanassoula 1980; Salo et al. 2010). In the second panel, model A\(\lambda\)04, the bar grows at around 2–3 Gyr, and the spiral structures generated here are stronger than those generated in model A\(\lambda\)03. Models A\(\lambda\)05 (third panel) and A\(\lambda\)05_M14 (fourth panel) show that the bar grows at around 4–5 Gyr and the saturation at around 6 Gyr; these panels show the bar growth accompanied by some strong bisymmetrical structures at larger radii. Finally, model A\(\lambda\)06 (bottom panel) only shows weak and transient waves in the $m = 2$ mode. The white lines in these panels represent the radius of the bar, which is half the length of the bar (second panel of Figure 10).

Figure 8 shows the amplitude for the mode $m = 2$ Fourier coefficient for different radii as a function of time. As we show in Valencia-Enríquez et al. (2017), these plots can be understood as growing curves of the structures that are being assembled. This means they represent the strength of the structures that are developing in the disk. In these curves, we can identify the three main phases of bar growth by the amplitude curves for the inner radii; the first phase, the growth of the bar, corresponds to an exponential rise of its amplitude, from which we can get the growth rate $\omega$ of the bar (see Table 3); the second phase corresponds to saturation of its amplitude (around the maximum amplitude); and the final
asymptotically toward unity, while the stable model keeps \( \omega = 1 \).

Figure 5. The spin parameter of the disk \( \lambda_d \) (Equation (10)) is depicted with a dashed line, while its critical spin parameter \( \lambda_{\text{crit}} \) (Equation (14)) is depicted with a continuous line as a function of time. The model name is given at the bottom right corner. The bottom left panel provides a comparison between the A\(\lambda_05\) (black lines) and A\(\lambda05\_M14\) (cyan lines) models. We can observe that models forming a bar exhibit \( \lambda_d < \lambda_{\text{crit}} \) and the model stable against bar formation exhibits \( \lambda_d > \lambda_{\text{crit}} \).

Figure 6. Evolution of the stability experimental parameter \( \epsilon_m \) defined in EF82. We observe how the bar models begin in the range \( 0.7 < \epsilon < 1 \) and evolve asymptotically toward unity, while the stable model keeps \( \epsilon_m \) beyond the unity. The A\(\lambda_05\) and A\(\lambda05\_M14\) models show a very similar behavior.

Table 1

| Models   | \( \lambda_{\text{d,init}} \) | \( \lambda_{\text{crit}} \) | \( \epsilon_m \) | Final Status     |
|----------|-------------------------------|----------------------|-----------------|------------------|
| A\(\lambda_03\) | 0.03                          | 0.05                 | 0.76            | Strong bar       |
| A\(\lambda_04\) | 0.04                          | 0.055                | 0.86            | Barred          |
| A\(\lambda_05\) | 0.05                          | 0.056                | 0.95            | Weak bar        |
| A\(\lambda05\_M14\) | 0.05                          | 0.056                | 0.95            | Weak bar        |
| A\(\lambda_06\) | 0.06                          | 0.057                | 1.03            | Unbarred        |

We observe how the bar models begin in the range \( 0.7 < \epsilon < 1 \) and evolve asymptotically toward unity, while the stable model keeps \( \epsilon_m \) beyond the unity. The A\(\lambda_05\) and A\(\lambda05\_M14\) models show a very similar behavior.

phase, the secular evolution, corresponds to flattening of its curve (bar saturation).

In Figure 8, model A\(\lambda_03\), in the upper left panel, shows the fastest growth rate of the bar, \( \omega = 3.82 \text{ km s}^{-1} \text{kpc}^{-1} \); the maximum amplitude is reached around 0.5–1.5 Gyr, and after the second gigayear the bar saturates. Model A\(\lambda_04\), in the upper right panel, presents a growth rate of \( \omega = 0.84 \text{ km s}^{-1} \text{kpc}^{-1} \); the second phase is around 2–3 Gyr, and the bar saturates after the fourth gigayear. The A\(\lambda_05\) (black lines) and A\(\lambda05\_M14\) (cyan lines) models, in the bottom left panel, reach the maximum amplitude after the fourth gigayear, and the growth rate is around \( \omega = 0.39 \text{ km s}^{-1} \text{kpc}^{-1} \). Finally, model A\(\lambda_06\), in the bottom right panel, shows weak bisymmetrical structures that evolve with a very slow growth rate of \( \omega = 0.14 \text{ km s}^{-1} \text{kpc}^{-1} \). We have found a relation between the measured growth rate and the initial \( \lambda \) in the form growth rate \( \propto \lambda^{-4.65} \) (Figure 9).

Some other measurements were made to characterize the bar. In Figure 10 we present the instantaneous angular velocity \( \Omega_B \), length of the bar \( l \), axial ratio \( b/a \), and ratio \( R = R_{\text{CR}}/a_b \), where \( R_{\text{CR}} \) and \( a_b \) are the corotation radius and bar length, respectively. This parameterization permits a classification of bars into “fast” \( (1.0 < R < 1.4) \) or “slow” \( (R > 1.4) \); Athanassoula 1992; Debattista & Sellwood 2000). Model A\(\lambda_03\), depicted with a black line, shows a constant decrease in the angular velocity approximately from 30 to 20 km s\(^{-1}\) kpc\(^{-1}\). For model A\(\lambda_04\) (red line), this decrease starts from 20 to 12 km s\(^{-1}\) kpc\(^{-1}\). The length of the bars is kept almost constant for both models, as well as the ratio between their axes. Furthermore, in their evolution, the bars change from almost fast \( (R \sim 1.4) \) to slow \( (R > 1.4) \), as we observe in the rightmost panel of Figure 10. The A\(\lambda_05\) and A\(\lambda05\_M14\) models, depicted with green and cyan lines, respectively, show an angular velocity of around 12 km s\(^{-1}\) kpc\(^{-1}\). For these models, the bar grows in size until it achieves the longest length of approximately 11 kpc. Afterward, the bar shrinks to 7 kpc at the maximum amplitude phase (around \( T = 5 \text{ Gyr} \)). However, the bar seems to be destroyed after that, and the measurements of bar parameters are more difficult and less precise.
5. Results for Interactions

Figures 11–13 show the face-on logarithm surface density maps for all of our interacting models at times of 2, 3, 4, 5, and 6 Gyr. The simulations start from the second gigayear; therefore the pericenter passages in all encounters are around the third gigayear (second column of these figures). We have to bear in mind that all encounters pass at around 40 km s$^{-1}$ kpc$^{-1}$, so the duration of interaction is similar in all encounters. We can observe that those tidal interactions produce well-defined, strong spiral arms and extended tidal features, such as bridge and tail, that are all transient but distinct in nature (Toomre & Toomre 1972; Oh et al. 2008). The models in which the bar is already formed show strong spiral structures, but the bar seems not to be affected, while models where the bar is not yet created show a thin and durable oval triggered by the strong tidal pull. Figure 11 shows the interactions of group Pw; the tidal pull is the lightest, so these interactions generate wide spirals in all models and a wide oval in simulations where the bar is not formed yet. We add to the last row the PwA.06 model to compare it to the PwA.06 model. We notice that they are apparently very similar in nature, which means that the perturbation resolution does not affect the model evolution as long as they have the same mass distribution. Figure 12 shows the interactions of group Pm, where the tidal pull is relatively strong; not only do these interactions generate narrow spirals in all models, but they also generate a thin central oval in the disk, where the bar is not formed yet. Finally, Figure 13 shows the interactions of group Ps. Here the tidal pull is the strongest, generating the thinnest spirals and ovals of all interactions. Observing these interactions, we can notice that the stronger the interaction, the thinner the spiral structure in the target model.

5.1. Disk Instabilities through Time

Figure 14, first column, plots 14(a) and (d), depicts the interactions with the lightest companion (Pw). Including the PwA.06 model (purple line), the second column depicts the interactions with the companion that has a mass similar to the target galaxy (Pm). The last column depicts the interactions with the heaviest perturbation (Ps). The measurements of the DSPs, Figures 14(a) to (c), show similar behavior in all interactions. In these figures, the spin parameters $\lambda_{\text{crit}}$ are drawn as solid lines, while $\lambda_p$ is depicted as dashed lines. The experimental stability parameter $\epsilon_m$ is displayed in Figures 14(d) to (f).

In general, at the time of encounter, the impulse on particles of the target galaxy given by the perturbation makes the spin parameter $\lambda_p$ increase and the critical spin parameter $\lambda_{\text{crit}}$ decrease, and the stability parameter $\epsilon_m$ decreases as well. After the perturbation passes and is far from the studied galaxy, $\lambda_p$...
The change in DSP depends on the mass, distance, and velocity around the pericenter, which means that the growth of spirals and bar properties evolves differently (Noguchi 1987; Gerin et al. 1990; Sundin & Sundelius 1991; Miwa & Noguchi 1998; Oh et al. 2008; Martinez-Valpuesta et al. 2017; Moetazedian et al. 2017). For our purpose, we only experiment with the mass of the perturbation, setting almost constant the pericenter distance and the angular velocity of the perturbation.

5.2. Evolution of Bar Parameters

Since the bar and tidal pull produce strong features in the mode \( m = 2 \) of the Fourier component, we measure such amplitude and display it in Figure 15 for all our interactions. Figures 15(a)–(c) show the amplitude in color scale, and Figures 16(a)–(c) show curves of that amplitude for different radii. The passing of the perturbation causes the amplitude to increase strongly and transiently from the outer to the middle region of the disk. From there, substantial amplitudes move toward the inner part of the disk in all interactions.

Models PwA\( \lambda \)03, PmA\( \lambda \)03, and PsA\( \lambda \)03 (\( ^*\)A\( \lambda \)03 \( ^*\)) represent the top panels of Figures 15(a)–(c), respectively, show the evolution of a bar in the inner region of the disk, which is almost constant through time, and it appears not to be affected by the interaction (see also upper left panels of Figures 16(a)–(c)). In the outer part of the disk, the “growing curves” (Figures 16(a)–(c)) show that the amplitude of transient tidal spirals is higher than the one in the bar at the interaction time. Then, after the perturbation is far from the target galaxy, the change in DSP depends on the mass, distance, and velocity around the pericenter, which means that the growth of spirals and bar properties evolves differently (Noguchi 1987; Gerin et al. 1990; Sundin & Sundelius 1991; Miwa & Noguchi 1998; Oh et al. 2008; Martinez-Valpuesta et al. 2017; Moetazedian et al. 2017). For our purpose, we only experiment with the mass of the perturbation, setting almost constant the pericenter distance and the angular velocity of the perturbation.

The change in DSP depends on the mass, distance, and velocity around the pericenter, which means that the growth of spirals and bar properties evolves differently (Noguchi 1987; Gerin et al. 1990; Sundin & Sundelius 1991; Miwa & Noguchi 1998; Oh et al. 2008; Martinez-Valpuesta et al. 2017; Moetazedian et al. 2017). For our purpose, we only experiment with the mass of the perturbation, setting almost constant the pericenter distance and the angular velocity of the perturbation.
some spirals can survive transiently, while other weaker spiral waves may be driven temporarily by the bar.

In models PwA04, PmA04, and PsA04 (+A04), the perturbation passes when the bar reaches a maximum amplitude for mode $m = 2$ (MA phase), which causes the MA phase to finish quickly, saturating the inner part of the disk (see second panels of Figures 15(a)–(c) and upper right panels of Figures 16(a)–(c)), and strong tidal spirals can survive longer than those of models +A05.

Before the perturbation passes, models PwA05, PmA05, and PsA05 (+A05) in both the third panels of Figures 15(a)–(c) and the bottom left panels of Figures 16(a)–(c), respectively, do not form a bar yet. Thus, the interaction happens before the MA phase, causing high amplitudes of mode $m = 2$ that grow first in the outer region of the disk, and then less strong amplitudes grow in the inner region of the disk, then accelerating the formation of a larger, narrow bar. Tidal spirals can survive for more time, and it seems to be connected to the bar transiently. In particular, interaction PsA05, which has the highest perturbation, shows this behavior more conspicuously than the others.

Models PwA06, PmA06, and PsA06 (+A06), shown in both the last panels of Figures 15(a)–(c) and the bottom right panels of Figures 16(a)–(c), respectively, show behavior similar to model +A05. As we showed before, model A06 is stable against bar formation; however, when it is subjected to a perturbation, the DSPs fall below the stability limits (see Figures 14(a)–(c)), causing bar formation (see the snapshots and measurements of the Fourier transform). Likewise, the PwA06_M2 model, which is shown in the last panel of Figures 15(d) and 16(a), presents a nature akin to its fiducial model.

Additionally, from the “growing curves,” we calculated the growth rate of spirals and bar, which is depicted with a straight red line in Figures 16(a)–(c). For models +A03 and +A04, the bar is already formed, so we calculated the growth rate of strong spirals that were triggered by the interaction. For the other models, we calculated the growth rate of the bar. We summarize in Table 4 the growth rate of tidal spirals (bold numbers) and the growth rate of the bar.

Figure 17 shows the growth of the bar that is characterized by the observational parameters. From the top row to the last one, we display the evolution of the bar for models of groups Pw, Pm, and Ps, respectively. We can observe in all encounters that at the time of interaction, the measurements of $\Omega_B$ and $l$ show a bump due to the impact given by the perturbation.

The evolution of a bar for models +A03 (black lines) and +A04 (red lines) is not much affected by the perturbation. They evolve similarly to their isolated counterpart (see Figure 10). However, there is a slight difference in the bar axis length. For example, while model A03 shows an increase of around 1 kpc during the interval from two to six, perturbed models +A03 do not present such an increase, and after the perturbation passes, the bar seems to maintain the same size throughout the evolution. On the other hand, the slowdown of the bar for model A04 falls at a constant rate from 21 to 12 km s$^{-1}$ kpc$^{-1}$, while such a slowdown seems to stop after the perturbation overly, decreasing it and keeping it around 12 km s$^{-1}$ kpc$^{-1}$. Particularly, the $\Omega_B$ for model PsA04 still oscillates between 10 and 16 km s$^{-1}$ kpc$^{-1}$ during the rest of the simulation.

The interactions in +A05 (green lines), the third row of Figure 17, cause bar formation to start earlier than in the isolated model A05. While the bar angular velocity of the A05 model is around 12 km s$^{-1}$ kpc$^{-1}$, this, for perturbed models, changes from 12 km s$^{-1}$ kpc$^{-1}$ at the beginning to 9 km s$^{-1}$ kpc$^{-1}$ when the perturbation is far away. Moreover, the bar reaches its maximum length at the MA phase: for example, for model A05 it is around 11 kpc at 4–5 Gyr, for model PwA05 it is around 10 kpc at 3–5 Gyr, for models PmA05 and PsA05 it is around 11 kpc at 3 Gyr. These variations cause the $R$ parameter to also have large changes: for example, the bars of models A05 and +A05 appear as slow rotators and tend to become fast rotators.

We can observe clearly the bar formation of models +A06 (blue lines) and PwA06_M2 (purple lines), which was triggered by the perturbation. After the perturbation passes, the bar appears with the lowest angular velocity, which is around 9 km s$^{-1}$ kpc$^{-1}$, and it stays at this speed throughout the evolution. In addition, the bar length in these models reaches the largest radius in both the MA phase and after that event as well, and the bars in these models are the most narrow (see the ratio between the axes of the bar, blue lines in the third column of Figure 17). Particularly, the bar in the PwA06 and PwA06_M2 models, which have the lightest interaction, appears as a slow rotator and evolves toward a fast rotator; then when the perturbation is far away, the bar evolves from fast to slow rotator. In contrast, in heavier interactions (PmA06 and PsA06), the bar appears as fast and evolves toward a slow rotator.

6. Discussion

6.1. Discussion of Isolated Models

In the setup of the isolated models, we only changed the spin parameter $\lambda$ to generate disk-dominated and halo-dominated
models, and we study the DSPs and the properties of the bar through time.

Disk-dominated models form a bar relatively quickly, while halo-dominated ones do not form such a structure. It has been shown that the growth of a bar in a disk galaxy is more efficient when the rotation curve is dominated by the disk, due to the exchange of angular momentum between, for example, halo and disk (Athanassoula 2013). Thus, the rate at which bar parameters change depends on the properties of the model, as well as the initial $Q$ parameter. The local stability, the $Q$ Toomre parameter, remains almost constant in model A$\lambda$06, but this parameter increases with the growth of the bar in the other models (see Figure 2). Although this parameter is a good indicator to know whether a model is susceptible to bar formation at the beginning of its evolution, the growth of the bar makes this parameter increase, leaving the stability limits unknown. Besides, Romeo & Mogotsi (2018) showed that the Romeo-Falstad $Q_N$ parameter (Romeo & Falstad 2013), which is related with the Toomre $Q$ parameter, hardly correlates with the angular momentum $\Lambda$. However, MO98 and SW99 showed

Figure 11. Face-on surface logarithm density maps for interactions of group Pw. The color scale is at the top left, and time increases from the left to the right in units of Gyr. The top row shows the snapshots of model PwA$\lambda$03, the second row shows snapshots of model PwA$\lambda$04, and the third row shows snapshots of PwA$\lambda$05. The last two rows show snapshots of the PwA$\lambda$06 and PwA$\lambda$06 M2 models; notice that their evolution is very similar.
that disk stability possesses a lower limit on its spin parameter to get a stable disk, but it remains unclear what this limit is and how these disk stabilities behave during the evolution of a disk. In other words, are the DSPs maintained \(\lambda_d > \lambda_{\text{crit}}\) or \(\lambda_d < \lambda_{\text{crit}}\), or do they change like the \(Q\) parameter?

We get a stable disk against bar formation when the model begins with a spin parameter greater than its critical spin parameter; the model A\(\lambda_06\) starts with that configuration \((\lambda_d > \lambda_{\text{crit}})\), which is kept during the whole simulation, and the model does not form a bar (see bottom right panel of Figure 5). On the other hand, we obtain a disk unstable to bar formation when its spin parameter is less than its critical spin parameter. Models A\(\lambda_03\) and A\(\lambda_04\) start with that configuration \((\lambda_d < \lambda_{\text{crit}})\); although the models change \(\lambda_d\) and \(\lambda_{\text{crit}}\) during the evolution of the simulation, they conserve their configuration (see upper panels of Figure 5).

Additionally, we show that the time evolution of the DSPs does not depend on the number of particles. The A\(\lambda_05\) and A\(\lambda_05\_M14\) models, which have a different number of particles but are equal in nature, start with stability parameters very close to the lower stability limit. They begin to form an oval structure from 3 to 4 Gyr when the critical spin parameter \(\lambda_{\text{crit}}\) increases to be larger than the spin parameter \(\lambda_d\). These models conserve that configuration until the end of simulation.

The growth of the bar for all barred models shows some differences that are due to the central properties of the model. For the A\(\lambda_03\) model, the growth rate of the bar is higher, its spirals are weaker (Figures 7 and 8), the instantaneous angular velocity of the bar is higher, and its length is shorter (see Figure 10) than the same parameters for model A\(\lambda_04\). We also noticed that the \(R\) parameter evolves from slow to fast for both models A\(\lambda_03\) and A\(\lambda_04\) (see last panel of Figure 10), similar to classical bars shown in other works. The observational parameters of the bar for model A\(\lambda_05\) are very diffuse because the bar is just forming at the end of the simulation. In fact, at the beginning, this model seems to be stable, but after the fourth gigayear, the model starts to form a bar structure when \(\lambda_{\text{crit}}\) becomes higher than \(\lambda_d\), reaching the second phase of its growth at the end of the simulation.

Unlike in Saha & Naab (2013) and Athanassoula (2008), the parameter \(\epsilon_m\) saves the conditions established by EF82 in our models. The model A\(\lambda_06\) presents \(\epsilon_m > 1\), which is stable against bar formation, while the other models show \(0.7 < \epsilon_m < 1\), which are unstable to bar formation. This
parameter seems to work well in our models, which has a larger halo with an NFW profile.

6.2. Discussion of the Interactions

We measured the evolution of the disk stability parameters on isolated models to characterize the properties of a galactic disk as stable or unstable to bar formation. We showed that the DSP configuration of an initial disk susceptible to bar formation keeps such configuration below the stability limits through the entire evolution; in contrast, a stable disk holds such DSPs above the stability limits. The growth rate of the bar depends on how close the DSPs are to the stability limits, showing that if the DSPs are below and far from the stability limits, the growth rate of the bar is higher.

In this work, we subjected the isolated models to different perturbations in coplanar hyperbolic orbits to examine the evolution of their DSPs and how this affects the growth of bars in models stable and unstable to bar formation; we do not take care of the evolution of the perturbation. The perturbations were modeled by an extended live spherical halo and stellar components with a Hernquist profile, respectively, so that the interactions are more realistic. Using a spherical galaxy as a perturbation permits that the pull given by this one is smoother than if the perturbation were a point of mass.

We explore the interactions by only changing the mass of the perturbation where the total mass of the perturbation is half (Pw), similar (Pm), and two times (Ps) the total mass of the target galaxy. Therefore, the interaction force at the pericenter is stronger with heavier perturbations (see Table 2). The different perturbations affect similarly the DSPs and the bar parameters; however, the stronger the interactions, the more noticeable are the changes in the measurements of the parameters.

The flyby of the perturbation causes a bump in the spin parameter $\lambda_d$, and then when the perturbation is far from the target galaxy, it tends to return to similar values. Conversely, the critical spin parameter $\lambda_{crit}$ decreases slightly at the time of interaction, and then it increases, overtaking during the simulation the previous values that it had before the interaction. The increment on $\lambda_{crit}$ is due to the scale radius of the halo $r_s$ shrinking a larger percentage of distance than the radius $r_{200}$; thus the halo becomes more concentrated. Likewise, the $\epsilon_{sm}$ parameter decreases abruptly. This fall occurs because the
The passing of the perturbation causes a vigorous exchange of angular momentum between resonances and components (halo and disk), and then the disk also changes the radial scale $r_d$. Nevertheless, the small decrease in $r_d$ does not affect much the parameter $\lambda_{\text{crit}}$. We notice that the DSPs of perturbed models have the same behavior as that of isolated models, as well as the experimental stability parameter. For example, the parameter $\lambda_{\text{crit}}$ for interactions $\lambda_06$, which is stable to bar formation in isolation, overtakes the spin parameter $\lambda_d$ after the moment of interaction, and the $\epsilon_m$ parameter is set to the range $0.7 < \epsilon_m < 1$; then the model is identified as unstable to bar formation.
Although the \( m \) parameter is a simple comparison of the rotation curve and the circular velocity of a hypothetical particle subject to a point-mass potential that has a mass equal to that of the disk, it seems to work well in our interacting models. Therefore, it could be a good indicator for assessing the stability of a disk at least approximately, and also it could be used in real galaxies to assess and restrict some parameters of a disk galaxy.

Toomre & Toomre (1972) demonstrated that tidal perturbations distort extended portions of a disk to produce elongated and narrow features, phenomenologically called the bridge and tail. The bridge is built on the near side of the disk toward the perturber, while the tidal tail, or counterstream, forms on the far side (Oh et al. 2008). Together, these two features generate high amplitudes in the Fourier transform of mode \( m = 2 \). Such an amplitude illustrates the dynamical responses of disks to a tidal perturbation.

Figure 15 shows that the tidal pull evolves from the outer to the inner region of the disk, generating a vigorous spiral wave that excites the epicycle orbits of individual particles. Therefore, this triggers rapid bar formation in a disk without a bar. However, a bar already developed in a disk is marginally affected; for example, we observe that the bar becomes slightly oval, but the axis ratio is almost constant during the evolution of the simulation (see Figure 17). After the perturbation is far away, the tidal tail and bridge dissipate quickly, but some spirals grow transiently. These spirals are stronger and longer in models without a bar before the perturbation passes.

On the other hand, the perturbation does not much affect the observational parameters in barred models. In contrast, when the perturbation triggers the bar, the observational parameters appear with less angular velocity and longer and shorter major and minor axes of the bar, respectively, and when the perturbation is heavy (e.g., \( Pm \) and \( Ps \) groups), the bar rotates from fast to slow, but when the perturbation is light, the bar grows in two phases: first it appears as a slow rotator and evolves toward a fast rotator, then it evolves from fast to a slow rotator (see models \( PwA \lambda 06 \) and \( PwA \lambda 06_M2 \)).

Finally, different from other results, like that of Martinez-Valpuesta et al. (2017), we find that the bar growth, which is triggered by a light interaction, develops in two phases, in which the bar grows from slow to fast rotator. However, when the value of \( R \) is close to one, the growth changes from fast to slow rotator.
7. Conclusions

In this work, we have followed for the first time in N-body simulations the critical spin parameter $\lambda_{\text{crit}}$ and the experimental stability parameter $\varpi_m$ to characterize the stability of a disk galaxy model. We get an isolated model unstable to bar formation by setting $\lambda_d < \lambda_{\text{crit}}$ (its rotation curve is dominated by the disk), while a model stable against bar formation is achieved by setting $\lambda_d > \lambda_{\text{crit}}$ (its rotation curve is dominated by the halo). Moreover, we show that the configuration of stability, for example, $\lambda_d > \lambda_{\text{crit}}$ or $\lambda_d < \lambda_{\text{crit}}$, is saved for a long time no matter what structure is forming in the disk. Following the same line of research, we perturbed the isolated models to understand the nature of the formation and evolution of a bar in disk galaxies. There, we illustrate how the DSPs are affected when the models are subjected to a perturbation. We reported one of the most important conclusions of the whole work, which in general is as follows: “The bars in our disk galaxy models are formed below the stability limits in both isolated and perturbed disks, and this depends on how close the parameters are to their critical values.”

The growth rate of the bar in our models depends on the configuration of the DSPs ($\lambda_d$, $\lambda_{\text{crit}}$). We have found that the growth rate is high if the spin parameter of the disk $\lambda_d$ is lower than the stability limit $\lambda_{\text{crit}}$, as well as if $\varpi_m$ is far from unity. The model Aλ05 (Aλ05_M14) also shows that the bar starts to evolve when the stability limit $\lambda_{\text{crit}}$ overtakes the spin parameter $\lambda_d$, that is, even if the initial configuration sets a model as stable, its own evolution with exchanges of angular momentum between the disk and halo can place it in the unstable regime. The stability parameter $Q$ increases during the evolution; in contrast, we have found that the models maintain their initial configuration on DSP (unstable or stable regime), except when the initial DSPs are close to the critical stability limits, such as in model Aλ05.

With respect to observational parameters of the bar, we show that the evolution of those parameters mainly depends on the central properties of the model. A more concentrated mass distribution in a disk generates a shorter bar that rotates faster than a more extended mass distribution model in both isolated and perturbed models.

D.V.E. and I.P. thank the Mexican Foundation Conacyt for grants that support this research. I.R. thanks the Brazilian agency CNPq (Project 311920/2015-2). Part of the numerical work was developed using the Hipercubo Cluster (FINEP 01.10.0661-00, FAPESP 2011/13250-0 and FAPESP 2013/17247-9) at IP&D–UNIVAP.

ORCID iDs

D. Valencia-Enríquez https://orcid.org/0000-0002-2184-3387

References

Andersen, V. 1996, AJ, 111, 1805
Athanassoula, E. 1980, A&A, 88, 184
Athanassoula, E. 1992, MNRAS, 259, 345
Athanassoula, E. 2002, ApJL, 569, L83
Athanassoula, E. 2003, MNRAS, 341, 1179
Athanassoula, E. 2008, MNRAS, 390, L69
Athanassoula, E. 2013, in Secular Evolution of Galaxies, ed. J. Falcón-Barroso & J. H. Knapp (Cambridge: Cambridge Univ. Press), 305
Athanassoula, E., Machado, R. E. G., & Rodionov, S. A. 2013, MNRAS, 429, 1949
Cervantes Sodi, B., & Sánchez García, O. 2017, ApJ, 847, 37
Combes, F., & Sanders, R. H. 1981, A&A, 96, 164
Debattista, V. P., & Sellwood, J. A. 1998, ApJL, 493, L5
Debattista, V. P., & Sellwood, J. A. 2000, ApJ, 543, 704
Dehnen, W., & Read, J. I. 2011, EPJP, 126, 55
Elstathioiu, G., Lake, G., & Negroponte, J. 1982, MNRAS, 199, 1069
Elmegreen, D. M., Elmegreen, B. G., & Bellin, A. D. 1990, ApJ, 364, 415
Gerin, M., Combes, F., & Athanassoula, E. 1990, A&A, 230, 37
Hernquist, L. 1990, ApJ, 356, 359
Kalnajs, A. J. 1971, ApJ, 166, 275
Kalnajs, A. J. 1977, ApJ, 212, 637
Lang, M., Holley-Bockelmann, K., & Sinha, M. 2014, ApJL, 790, L33
Little, B., & Carlberg, R. G. 1991, MNRAS, 250, 161
Lynden-Bell, D. 1979, MNRAS, 187, 101
Lynden-Bell, D. 1996, LNP, 474, 7
Marinova, I., Jogee, S., Heideman, A., et al. 2011, MSAIS, 18, 61
Martinez-Valpuesta, I., Aguerri, J. A. L., González-García, A. C., Dalla Vecchia, C., & Stringer, M. 2017, MNRAS, 464, 1502
Martinez-Valpuesta, I., Shlosman, I., & Heller, C. 2006, ApJ, 637, 214
Miwa, T., & Noguchi, M. 1998, ApJ, 499, 149
Mo, H. J., Mao, S., & White, S. D. M. 1998, MNRAS, 295, 319
Moetazedian, R., Polyachenko, E. V., Berczik, P., & Just, A. 2017, A&A, 604, A75
Mo, H. J., Mao, S., & White, S. D. M. 1999, MNRAS, 295, 319
Mo, H. J., Mao, S., & White, S. D. M. 1999, ApJ, 545, 740
Mo, H. J., Mao, S., & White, S. D. M. 1999, ApJ, 563, 1009
Mo, H. J., Mao, S., & White, S. D. M. 1999, ApJ, 563, 1009
Mo, H. J., Mao, S., & White, S. D. M. 2000, ApJ, 539, 682
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1996, ApJ, 462, 563
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Noguchi, M. 1987, MNRAS, 228, 635
Oh, S. H., Kim, W.-T., Lee, H. M., & Kim, J. 2008, ApJ, 683, 94
Polyachenko, V. L., & Polyachenko, E. V. 2003, ApJL, 589, 547
Polyachenko, V. L., & Polyachenko, E. V. 2004, ARep, 48, 877
Romano-Díaz, E., Shlosman, I., Heller, C., & Hoffman, Y. 2008, ApJL, 687, L13
Romeo, A. B., & Falstad, N. 2013, MNRAS, 433, 1389
Romeo, A. B., & Mogotsi, K. M. 2017, MNRAS, 469, 286
Romeo, A. B., & Mogotsi, K. M. 2018, MNRAS, 480, L23
Saha, K., & Naab, T. 2013, MNRAS, 434, 1287
Salo, H., Laurikainen, E., Buta, R., & Knapen, J. H. 2010, ApJL, 715, L56
Sellwood, J. A. 1981, A&A, 99, 362
Sellwood, J. A., & Wilkinson, A. 1993, RPPh, 56, 173
Springel, V. 2005, MNRAS, 364, 1105
Springel, V., Di Matteo, T., & Hernquist, L. 2005, MNRAS, 361, 776
Springel, V., & White, S. D. M. 1999, MNRAS, 307, 162
Springel, V., Yoshida, N., & White, S. D. M. 2001, NewA, 6, 79
Sundin, M., Donner, K. J., & Sundelius, B. 1993, A&A, 280, 105
Sundin, M., & Sundelius, B. 1991, A&A, 245, L5
Thompson, L. A. 1981, ApJL, 244, L43
Toomre, A., & Toomre, J. 1972, ApJ, 178, 623
Valencia-Enriquez, D., Puerari, I., & Chaves-Velasquez, L. 2017, RMxAA, 53, 257
Vesperini, E., & Weinberg, M. D. 2000, ApJ, 534, 598
Weinberg, M. D. 1985, MNRAS, 213, 451