Product acceptance determination based on EWMA yield index using repetitive and MDS sampling schemes

M. Aslam\textsuperscript{a,}, M. Azam\textsuperscript{b}, R.A.K. Sherwani\textsuperscript{c}, C.-H. Yen\textsuperscript{d}, and C.-H. Jun\textsuperscript{e}

a. Department of Statistics, Faculty of Science, King Abdullah University, Jeddah 21551, Saudi Arabia.
b. Department of Statistics and Computer Sciences, University of Veterinary and Animals Sciences, Lahore 54000, Pakistan.
c. College of Statistical and Actuarial Sciences, University of the Punjab, Lahore, Pakistan.
d. Department of Industrial Engineering and Management Information, Huafan University, Taipei, Taiwan.
e. Department of Industrial and Management Engineering, POSTECH, Pohang 37673, Republic of Korea.

Received 19 February 2010; received in revised form 30 July 2010; accepted 3 January 2012

KEYWORDS
Repetitive group sampling; Multiple dependent state sampling; Exponentially weighted moving average; Lot sentencing; Non-linear optimization.

Abstract. This study presents a repetitive group sampling plan and a multiple dependent state sampling plan based on the EWMA (exponentially weighted moving average) yield index for product acceptance. The proposed plans utilize the current and previous information through EWMA statistic to reach a decision of lot sentencing. A non-linear optimization model is developed to determine the plan parameters of the proposed plans for various specified conditions. The performance of the proposed plans over several existing sampling plans is analyzed, showing that the proposed plans are efficient in reducing the sample size for lot sentencing. For industrial application, a real example is given to demonstrate the implementation of the proposed plans.

© 2022 Sharif University of Technology. All rights reserved.

1. Introduction

Acceptance sampling plans have been widely used in the manufacturing industry for lot sentencing, including inspection of raw material, semi-products, and final products. Customers would perform the inspection of goods when there is a need to verify the quality of goods submitted by suppliers. Mostly, the 100% inspection of goods is not feasible because of costs, time, destructive test, and so on. Therefore, the application of acceptance sampling plans is almost inevitable. A well-designed sampling plan should have high probability of acceptance for lots with good quality and low probability of acceptance for lots with bad quality. Generally, the parameters of a well-design sampling plan are determined by minimizing the sample size while satisfying the principle of two points on operating characteristic curve.

Attribute sampling plans and variable sampling plans have been developed for various situations in the literature. The former is used when data of interest is obtained for the counting process and the latter is used when data of interest is obtained for the measurement process. Attribute sampling plans are easier to apply than variable sampling plans, while variable sampling plans are more informative than attribute sampling plans. Therefore, both plans have been considered to be important for lot sentencing. Some researchers favor the attribute sampling plan, while others favor the variable sampling plan. More detailed information about the applications of two types of sampling plans can be seen in [1–27].

Most of variable acceptance sampling plans in the
literature use only the current information to make a decision about the submitted lot. This type of plans is called “memoryless” plan. The efficiency of variable sampling plans can be increased by utilizing the current and previous information about the lot. The EWMA statistic is very popular in designing the control chart as it gives more weight to the current data and decreasing weight to the past data. According to [28], the control charts based on EWMA can detect smaller shifts in the process. More details about the EWMA statistic can be seen in [29]. Due to the advantages of the EWMA statistic, researchers found it interesting to design a sampling inspection plan using the mentioned statistic. Recently, Aslam et al. [30] proposed a variable sampling plan using the EWMA statistic when the quality of interest follows normal distribution with known or unknown standard deviation. They compared the efficiency of the proposed plan with the existing plans. Yen et al. [31] designed the sampling plan for a yield index using the EWMA statistic.

It is important to note that the inspection cost is directly proportional to the sample size. Therefore, some other flexible sampling schemes such as a Repetitive Group Sampling Plan (RGSP) [32], Multiple Dependent State Sampling (MDSS) plan [33], and Multiple Dependent State Repetitive Plan (MDSRP) [34] are proposed. These sampling schemes provide a smaller sample size than the plan based on a single sampling scheme. By exploring the literature and to the best of authors’ knowledge, works on RGSP and MDSS with EWMA yield index have not been proposed yet. As mentioned, RGSP and MDSS schemes have been proved to be more economical than the single sampling plan. In addition, the EWMA yield index not only considers the quality of the current lot and the preceding lots, but also provides an exact measure on the process yield. For these motivations, the designing of MDSS and RGSP based on the EWMA yield index is proposed.

The rest of this paper is organized as follows. The yield index $S_{pk}$ is introduced in Section 2. In Sections 3 and 4, the designing of RGSP and MDSS is presented, respectively. In Section 5, advantages of the proposed plans are described. Section 6 provides an example to illustrate the proposed methodology. Finally, conclusions are made in Section 7.

2. The yield index $S_{pk}$

According to [31], “process yield has been a standard criterion used in the manufacturing industry as a common measure on process performance”. The process yield index is defined as the proportion of products manufactured within the given specification limits. The product manufactured beyond the specification limits incurs the extra cost of rework. As mentioned by [35], “traditionally, products manufactured within the specification limits are considered to be equally conforming and those outside the specification limits are considered to be equally nonconforming”. The process yield is defined mathematically as:

$$Y = \int_{L_{SL}}^{U_{SL}} f(x) dx,$$

where $L_{SL}$ is the Lower Specification Limit, $U_{SL}$ the Upper Specification Limit, and $f(x)$ the probability density function (pdf) of the quality variable of interest $X$. The disadvantage of Eq. (1) is that it is unable to differentiate the products within the specification limit and beyond the specification limits [35]. Boyle [36] developed the capability index, called yield index $S_{pk}$ which is helpful to provide the exact measure of process yield. This index is defined as follows:

$$S_{pk} = \frac{1}{3} \Phi^{-1} \left\{ \frac{1}{2} \Phi \left( \frac{U_{SL} - \mu}{\sigma} \right) + \frac{1}{2} \Phi \left( \frac{\mu - L_{SL}}{\sigma} \right) \right\}$$

where $\Phi(.)$ is the cumulative distribution function (cdf) of the standard normal distribution, $\Phi^{-1}$ the inverse function of the standard normal distribution, $\mu$ the mean, and $\sigma$ the standard deviation of the process. This index is more efficient than other capability indices since it provides a one-to-one relationship to the process yield [31]. Yen et al. [31] reported the process yield and non-conformities in Parts Per Million (PPM), when $S_{pk} = 1.0(0.1)2.0$ and popular performance requirements are 1.00, 1.33, 1.50, 1.67, and 2.00.

In practice, $\mu$ and $\sigma$ are unknown and they are estimated from the data. The best linear unbiased estimator of $\mu$ is $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$ and that of $\sigma^2$ is $S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1}$. The estimator $\hat{S}_{pk}$ is expressed as follows:

$$\hat{S}_{pk} = \frac{1}{3} \hat{\Phi}^{-1} \left\{ \frac{1}{2} \hat{\Phi} \left( \frac{U_{SL} - \bar{X}}{S} \right) + \frac{1}{2} \hat{\Phi} \left( \frac{\bar{X} - L_{SL}}{S} \right) \right\}$$

As the exact distribution of this estimator is analytically intractable, Lee et al. [37] derived the approximately normal distribution of $\hat{S}_{pk}$ as given below:

$$\hat{S}_{pk} \sim N \left( S_{pk}, \frac{a^2 + b^2}{36n \sigma^2 (3S_{pk})} \right),$$

where $\phi(.)$ is the pdf of the standard normal distribution, and

$$a = \frac{1}{\sqrt{2}} \left\{ 3C_p(2 - C_a)\phi(3C_p(2 - C_a)) + 3C_pC_a\phi(3C_pC_a) \right\},$$
\[ b = \phi(3C_p(2 - C_a)) - \phi(3C_p C_a), \]
\[ d = (USS - LSL)/2, \]
\[ C_p = (USS - LSL)/6\sigma, \]
\[ C_a = 1 - |\mu - M|/d. \]

3. Designing of the proposed plan using repetitive group sampling

The Repetitive Group Sampling Plan (RGSP) attracted the researchers due to its simplicity as compared to the double sampling plan or the sequential sampling plan. The RGSP is proven to be more efficient than the single and double sampling schemes [8, 19, 21, 38, 39, 40]. The proposed RGSP based on the EWMA version of yield index is stated as follows:

**Step 1.** Take a random sample of size from a submitted lot at time \( t \). Compute sample mean \( \overline{X} = \sum_{i=1}^{n} X_i/n \) and sample standard deviation \( S = \sqrt{\sum_{i=1}^{n} (X_i - \overline{X})^2/(n-1)} \). Calculate the yield index \( \hat{S}_{plk} \) as in Eq. (3) for the current lot. Choose the smoothing constant \( \lambda (0 < \lambda < 1) \) and compute the following EWMA yield index:

\[ \hat{S}_{plk}^{EWM_A \lambda} = \lambda \hat{S}_{plk} + (1 - \lambda) \hat{S}_{plk}^{EWM_A \lambda -1}. \]

**Step 2.** The decision about the submitted lot is stated as follows:

(a) Accept the lot if \( \hat{S}_{plk}^{EWM_A \lambda} \geq k_a; \)

(b) Reject the lot if \( \hat{S}_{plk}^{EWM_A \lambda} < k_r; \)

(c) If \( k_r \leq \hat{S}_{plk}^{EWM_A \lambda} < k_a \), repeat Step 1.

The proposed RGSP is based on three parameters: sample size \( n \), acceptance value \( k_a \), and rejection value \( k_r. \) The proposed RGSP is the extension of the plan designed by [31] because it reduces to the latter when \( k_a = k_r \). The Operating Characteristic (OC) function of the proposed RGSP is derived as follows. The lot acceptance probability based on a single sample is given as:

\[ P\left( \hat{S}_{pk}^{EWM_A \lambda} \geq k_a \right) = 1 - \Phi \left( \frac{k_a - S_{pk}}{\sqrt{\lambda(2-\lambda)/(\sigma^2 + \sigma^2)(36n\phi^2(3S_{pk}))^{-1}}} \right). \]  

(5)

The probability of repetitive sampling for a submitted lot is given as follows:

\[ P\left\{ k_r \leq \hat{S}_{plk}^{EWM_A \lambda} < k_a \right\} = \Phi \left( \frac{k_a - S_{pk}}{\sqrt{\lambda(2-\lambda)/(\sigma^2 + \sigma^2)(36n\phi^2(3S_{pk}))^{-1}}} \right) - \Phi \left( \frac{k_r - S_{pk}}{\sqrt{\lambda(2-\lambda)/(\sigma^2 + \sigma^2)(36n\phi^2(3S_{pk}))^{-1}}} \right). \]  

(6)

Therefore, the OC function is given by Eqs. (7) and (8) as shown in Box I. The Average Sample Number (ASN) of the proposed RGSP is given by Eq. (9) as shown in Box II.

The sampling plan, which provides protection to producer and consumer, is considered as efficient. Let \( \alpha \) be the producer’s risk and \( \beta \) be the consumer’s risk. The producer desires that the lot acceptance probability be larger than \( 1 - \alpha \) when the quality level of lot is at the acceptance quality level (AQL = \( S_{AQL} \)), while the consumer desires the lot acceptance probability to be smaller than \( \beta \) when quality level of lot is at the limiting quality level (LQL = \( S_{LQL} \)). The three plan parameters of the proposed plan are obtained using the following non-linear optimization model:

\[ \text{MIN}_{n, k_a, k_r} \text{ASN}(S_{AQL}). \]  

(10a)

Subject to:

\[ \pi_A(S_{AQL}) \geq 1 - \alpha, \]  

(10b)
\[
\text{ASN}(S_{pk}) = \frac{n}{1 - \left( \Phi \left( \frac{k_e - S_{pk}}{\sqrt{\lambda/(2-\lambda)(\lambda^2 + \phi^2)(\lambda^3 + \phi^2)^2}} \right) \right)^n - \Phi \left( \frac{k_e - S_{pk}}{\sqrt{\lambda/(2-\lambda)(\lambda^2 + \phi^2)(\lambda^3 + \phi^2)^2}} \right) - 1}
\]

Box II

\[\pi_A(S_{LQL}) \leq \beta, \quad 0 < k_r < k_e.\]  

The plan parameters of the proposed RGSP at different values of \(\lambda, \alpha, \) and \(\beta\) are presented for \((S_{AQI}, S_{LQL}) = (1.33, 1.0)\), as shown in Table 1. In Table 1, the plan parameters are given for \((S_{AQI}, S_{LQL}) = (1.50, 1.33)\). The smoothing constant lambda is a weight on the current information as compared to past information. Thus, a small value of lambda (around 0.1 \(- 0.3\)) is recommended as long as the process is stable. A larger value of lambda may be used if the user seeks to reflect more weight on the current lot. In fact, in our tables, we compare the parameter values for various values of lambda, which is the same as the sensitivity analysis.

From Tables 1 and 2, we find that the plan parameters have the following trends:

1. At fixed values of \(\alpha\) and \(\lambda\), ASN decreases as \(\beta\) increases;
2. At fixed values of \(\alpha\) and \(\beta\), ASN increases as \(\lambda\) increases;
3. At fixed values of \(\beta\) and \(\lambda\), ASN decreases as \(\alpha\) increases;
4. The acceptance constant and rejection constant are both larger than 1 given the quality levels (\(S_{AQI}, S_{LQL}) = (1.33, 1.0)\) and \((S_{AQI}, S_{LQL}) = (1.50, 1.33)\).

It is noted that the EWMA statistic considers the effect of accumulating the past sample sizes and the current one simultaneously. Therefore, the normal approximation even for a very small sample size can be justified.

### Table 1. Parametric values of the proposed RGSP and various \(\lambda\) under \((S_{AQI}, S_{LQL}) = (1.33, 1.0)\).

| \(\lambda\) | \(\alpha\) | \(\beta\) | \(n\) | \(k_l\) | \(k_u\) | ASN | \(\lambda\) | \(\alpha\) | \(\beta\) | \(n\) | \(k_l\) | \(k_u\) | ASN |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.10 | 0.010 | 0.025 | 0.050 | 0.075 | 0.100 | 0.100 | 0.200 | 0.250 | 0.250 | 0.250 | 0.250 | 0.100 | 0.100 |
| 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 |
| 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 |
| 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 |
| 1.30 | 1.30 | 1.30 | 1.30 | 1.30 | 1.30 | 1.30 | 1.30 | 1.30 | 1.30 | 1.30 | 1.30 | 1.30 | 1.30 |
| 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 |
| 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 |
| 0.050 | 0.050 | 0.050 | 0.050 | 0.050 | 0.050 | 0.050 | 0.050 | 0.050 | 0.050 | 0.050 | 0.050 | 0.050 | 0.050 |
| 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 |
| 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 |
| 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 |
| 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 |
| 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 |
| 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 |
| 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
Table 2. Parametric values of the proposed RGSP and various λ under \((S_{QL}, S_{LTP}) = (1.50, 1.33)\).

| \(\alpha\) | \(\beta\) | \(n\) | \(k_a\) | \(k_r\) | \(ASN\) | \(\lambda = 0.1\) | \(\lambda = 0.3\) | \(\lambda = 0.5\) | \(\lambda = 1.0\) |
|---|---|---|---|---|---|---|---|---|---|
| 0.010 | 18 | 1.4266 | 1.2634 | 23.90 | 40 | 1.4718 | 1.3446 | 80.55 | 40 | 1.5574 | 1.2508 | 285.07 | – | – | – |
| 0.025 | 16 | 1.4466 | 1.3550 | 23.03 | 48 | 1.4534 | 1.3450 | 76.73 | 50 | 1.5245 | 1.2710 | 222.80 | – | – | – |
| 0.050 | 14 | 1.4440 | 1.3456 | 22.21 | 40 | 1.4384 | 1.3441 | 73.37 | 50 | 1.5017 | 1.2791 | 189.28 | – | – | – |
| 0.075 | 13 | 1.4314 | 1.3408 | 21.11 | 40 | 1.4262 | 1.3452 | 70.56 | 40 | 1.4880 | 1.2801 | 176.18 | 50 | 1.5058 | 0.9986 | 770.81 |
| 0.100 | 15 | 1.4144 | 1.3532 | 20.67 | 48 | 1.4150 | 1.3500 | 68.14 | 50 | 1.4733 | 1.2803 | 159.00 | 50 | 1.4756 | 1.0165 | 5901.13 |

Note: (×) shows that plan parameters do not exist.

4. Designing the proposed plan using multiple dependent state sampling

Most available sampling plans only consider the present state of a lot, that is, accepting or rejecting a lot is based on the present lot quality. To overcome this issue, Wortham [33] first introduced the concept of multiple dependent state sampling plan, which accepts or rejects a lot based on not only the quality of current lot but also the quality of preceding lots. This sampling plan can be used in cases when lots are submitted serially. In this section, a Multiple Dependent State Sampling (MDSS) plan is proposed based on yield index by referring to [10]. The proposed procedure is given as below:

Step 1. Take a random size from the submitted lot at time \(t\). Compute the sample mean \(\overline{X}\) and the sample standard deviation \(S\). Calculate the yield index \(\hat{S}_{pk}\). Choose the smoothing constant \(0 < \lambda < 1\) and compute the following EWMA yield index:

\[
\hat{S}^{EWMA}_{pk} = \lambda \hat{S}_{pk} + (1 - \lambda)S_{pk}^{EWMA}_{pk-1}.
\]

Step 2. The decision about the submitted lot is made as follows:

(a) Accept the lot if \(\hat{S}^{EWMA}_{pk} \geq k_a\);

(b) Reject the lot if \(\hat{S}^{EWMA}_{pk} < k_r\);

(c) If \(k_r \leq \hat{S}^{EWMA}_{pk} < k_a\), then accept the lot when \(n\) preceding lots have been accepted under the condition of \(\hat{S}^{EWMA}_{pk} \geq k_a\). Otherwise, reject the lot.

This proposed plan is based on four plan parameters: sample size \(n\), acceptance value \(k_a\), rejection value \(k_r\), and the number of preceding lots accepted \(i\). This plan is a generalization of [31]. It reduces to [31] when \(i = 1\).

The OC function \(\pi_A^{MDSS}(S_{pk})\) of the proposed plan is derived as follows:

\[
\pi_A^{MDSS}(S_{pk}) = P\{\hat{S}^{EWMA}_{pk} \geq k_a\} + P\{k_r \leq \hat{S}^{EWMA}_{pk} < k_a\} + [P\{\hat{S}^{EWMA}_{pk} \geq k_a\}]^i.
\]

Based on Eqs. (5) and (6), Eq. (11) can be rewritten as follows:
Table 3. Parametric values of the proposed MDSS with \( i = 2 \) and various \( \lambda \) under \((S_{AQL}, S_{LTDP}) = (1.33, 1.0)\).

| \( \alpha \) | \( \beta \) | \( n \) | \( k_a \) | \( k_r \) | \( \lambda = 0.1 \) | \( \lambda = 0.3 \) | \( \lambda = 0.5 \) | \( \lambda = 1.0 \) |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.010 | 0.010 | 7 | 1.1408 | 0.1070 | 24 | 1.1425 | 0.1038 | 44 | 1.1413 | 0.8150 | 118 | 1.0896 | 0.2163 |
| 0.025 | 0.050 | 6 | 1.1732 | 0.4290 | 20 | 1.0921 | 0.3687 | 40 | 1.1303 | 1.1074 | 106 | 1.0718 | 0.3919 |
| 0.050 | 0.100 | 5 | 1.1057 | 0.0677 | 18 | 1.1143 | 0.3131 | 34 | 1.1140 | 0.3586 | 97 | 1.0740 | 0.6982 |
| 0.075 | 0.075 | 5 | 1.1083 | 0.3859 | 15 | 1.0686 | 0.4479 | 24 | 1.0433 | 0.5578 | 83 | 1.0652 | 0.6948 |

\[
\pi^M_{DSS}(S_{pk}) = \\
\left( 1 - \Phi \left( \frac{k_a - S_{pk}}{\sqrt{\frac{\lambda}{(2 - \lambda)} \left( (a^2 + b^2) \{36n \phi^2(3S_{pk}) \}^{-1} \} \right)} \right) \right) + \left\{ \Phi \left( \frac{k_a - S_{pk}}{\sqrt{\frac{\lambda}{(2 - \lambda)} \left( (a^2 + b^2) \{36n \phi^2(3S_{pk}) \}^{-1} \} \right)} \right) \right\} - \Phi \left( \frac{k_r - S_{pk}}{\sqrt{\frac{\lambda}{(2 - \lambda)} \left( (a^2 + b^2) \{36n \phi^2(3S_{pk}) \}^{-1} \} \right)} \right) \left[ 1 - \Phi \left( \frac{k_a - S_{pk}}{\sqrt{\frac{\lambda}{(2 - \lambda)} \left( (a^2 + b^2) \{36n \phi^2(3S_{pk}) \}^{-1} \} \right)} \right) \right]^{i}.
\]

For specified values of \( i \), the three plan parameters of the proposed plan can be obtained using the following non-linear optimization model:

\[
MIN_{n, k_a, k_r} = n, \ 
\] (13a)

Subject to:

\[
\pi^M_{DSS}(S_{AQL}) \geq 1 - \alpha, \ 
\pi^M_{DSS}(S_{LTDP}) \leq \alpha, \ 
\]

Based on Eqs. (13a) to (13c), the plan parameters of the proposed plan are given in Tables 3 and 4. It is noted the behavior of plan parameters in Tables 3 and 4 is the same as those in Tables 1 and 2. The codes are shown in a supplementary file.

5. Advantages of proposed plans

In this section, we will compare the proposed plans with the existing plans in terms of ASN. To justify the performance of the proposed plans, we will set the same values of specified parameters for all sampling plans. It is noted that a sampling plan which provides the smaller values of ASN is considered to be more efficient.

5.1. The proposed RGSP vs [31] plan

First, we compare the efficiency of the proposed RGSP with the plan developed by Yen et al. [31]. For
Table 4. Parametric values of the proposed MDSS with $i = 2$ and various $\lambda$ under $(S_{AQI}, S_{LQL}) = (1.50, 1.33)$.

| $\alpha$ | $\beta$ | $n$ | $k_a$ | $k_r$ | $\lambda = 0.1$ | $\lambda = 0.3$ | $\lambda = 0.5$ | $\lambda = 1.0$ |
|---|---|---|---|---|---|---|---|---|
| 0.010 | 35 | 13.910 | 0.6271 | 130 | 1.0499 | 0.5094 | 210 | 1.3750 | 0.4431 | - | - | - |
| 0.050 | 27 | 13.714 | 0.2596 | 97 | 1.3939 | 0.3900 | 156 | 1.3636 | 0.7519 | 419 | 1.3660 | 0.6833 |
| 0.075 | 27 | 13.714 | 0.4755 | 88 | 1.3902 | 0.7449 | 131 | 1.3608 | 1.1404 | 414 | 1.3635 | 0.6965 |
| 0.100 | 25 | 13.856 | 0.3119 | 82 | 1.3854 | 1.1077 | 131 | 1.3622 | 0.7275 | 384 | 1.3549 | 0.3745 |
| 0.010 | 33 | 1.174 | 0.7303 | 107 | 1.3947 | 0.6782 | 190 | 1.3815 | 0.6351 | 505 | 1.3941 | 0.5732 |
| 0.025 | 28 | 14.010 | 1.0627 | 93 | 1.4103 | 0.7763 | 160 | 1.3772 | 1.1674 | 452 | 1.3742 | 0.6754 |
| 0.050 | 21 | 13.770 | 1.1337 | 79 | 1.4025 | 0.8973 | 140 | 1.3688 | 0.3110 | 365 | 1.3745 | 0.5507 |
| 0.075 | 19 | 1.3612 | 0.6365 | 71 | 1.3972 | 0.3064 | 135 | 1.3794 | 0.5162 | 365 | 1.3699 | 1.0139 |
| 0.100 | 18 | 1.3571 | 0.4443 | 65 | 1.3925 | 0.5371 | 122 | 1.3925 | 0.6613 | 288 | 1.3572 | 0.7418 |
| 0.010 | 28 | 1.4128 | 1.1738 | 76 | 1.3913 | 0.7866 | 176 | 1.4244 | 1.3866 | 436 | 1.3045 | 0.6401 |
| 0.025 | 24 | 1.4167 | 0.6921 | 75 | 1.3924 | 0.4058 | 135 | 1.3852 | 0.5877 | 337 | 1.3870 | 0.6285 |
| 0.050 | 20 | 1.4101 | 0.9152 | 65 | 1.4099 | 0.5746 | 121 | 1.3876 | 1.0446 | 299 | 1.3782 | 0.4950 |
| 0.075 | 18 | 1.4027 | 0.3582 | 58 | 1.4043 | 1.0083 | 102 | 1.3793 | 0.5640 | 283 | 1.3661 | 0.9708 |
| 0.100 | 16 | 1.3991 | 0.7180 | 53 | 1.3997 | 0.7612 | 100 | 1.3995 | 1.1916 | 254 | 1.3603 | 0.3830 |
| 0.010 | 25 | 1.4042 | 1.0600 | 75 | 1.4015 | 0.8104 | 142 | 1.3983 | 0.8159 | 364 | 1.3947 | 1.1042 |
| 0.025 | 21 | 1.4223 | 0.2300 | 67 | 1.3972 | 0.9966 | 111 | 1.3978 | 0.2840 | 356 | 1.3942 | 0.2600 |
| 0.050 | 17 | 1.4157 | 0.4025 | 57 | 1.4153 | 0.5212 | 97 | 1.3809 | 0.5452 | 285 | 1.3834 | 0.2833 |
| 0.075 | 15 | 1.4099 | 1.1529 | 50 | 1.4103 | 0.7559 | 92 | 1.3821 | 0.5867 | 232 | 1.3675 | 0.7466 |
| 0.100 | 14 | 1.3657 | 0.6306 | 45 | 1.4049 | 0.7363 | 86 | 1.4058 | 0.9944 | 193 | 1.3628 | 1.1439 |

Note: (-) shows that plan parameters do not exist.

comparison, we selected $\lambda = 0.1, 0.3$ and $(S_{AQI}, S_{LQL}) = (1.33, 1.0)$. The values of ASN for both sampling plans are shown in Table 5.

From Table 5, we see that the proposed RGSP provides smaller ASN than the plan in [31]. For example, given $\alpha = 0.010, \beta = 0.010$, and $\lambda = 0.1$, ASN of the proposed RGSP is 4.25, while that of the existing plan is 7. Thus, the proposed plan is more efficient than the sampling plan in [31].

5.2. The proposed RGSP vs traditional RGSP

Now, we will compare the efficiency of the proposed RGSP with memoryless (traditional) sampling plan. The proposed plan reduces to traditional RGSP when $\lambda = 1$. We selected the same values of all the parameters for this comparison. For comparison, we selected $\lambda = 0.1, 0.3$ and $(S_{AQI}, S_{LQL}) = (1.33, 1.0)$. The values of the ASN for both plans are shown in Table 6.

From Table 6, we see that the proposed RGSP provides smaller ASN than the traditional RGSP plan. For example, given $\alpha = 0.010, \beta = 0.010$, and $\lambda = 0.1$, ASN of the proposed RGSP is 4.25, while that of the existing plan is 81.74. Thus, the proposed plan is more efficient than the traditional RGSP plan.

5.3. The proposed MDSS plan vs [31]

To compare the efficiency of the proposed MDSS plan
with that of the existing plan [31], we again set the same values of all plan parameters for all sampling plans. The plan parameters of both the proposed sampling plans are shown in Table 7. For comparison, we selected \( \lambda = 0.1, 0.3 \) and \((S_{AQL}, S_{LQL}) = (1.50, 1.33)\).

For example, given \( \alpha = 0.010, \beta = 0.025 \), and \( \lambda = 0.5 \), ASN of the proposed MDSS plan is 178, while that of the existing plan is 210. Thus, the proposed MDSS plan is more efficient than the existing plan.

### 6. Applications in industry

We illustrate the application of the proposed plan with the help of color STN displays data where each pixel is divided into \( R, G, \) and \( B \) sub-pixels [31]. Suppose that the membrane thickness of each pixel is variable of interest. The engineer wants to inspect the quality of color STN displays using the proposed plans. The target value \( T \), specification limits USL, and LSL of membrane thickness are set as \( T = 12000 \), \( USL = 12500 \), and \( LSL = 11500 \), respectively. In the contract, suppose \( \alpha = 0.05, \beta = 0.10, \lambda = 1.0 \), and \((S_{AQL}, S_{LQL}) = (1.33, 1.0)\). Based on the predetermined values, if the proposed MDSS plan with two accepted preceding lots is applied, we find that the corresponding parameters \((n, k_0, k_r)\) of the sampling plan are \((55, 1.1207, 0.7415)\) from Table 3. Hence, the 55 inspected samples will be taken from one lot randomly to make lot sentencing. The sample data used are the same as those in [31].

Yen et al. [31] demonstrated that the data were well fitted to a normal distribution with the help of a normal probability plot. From the sample data in [31], we can obtain \( \bar{X} = 11715.2, S = 49.21 \) and the calculated value of \( S_{pk} \) is 1.5072. Assume that \( \hat{S}_{EWM A_{-1}} = 1.1052; \) then, \( \hat{S}_{EWM A_{+1}} \) is calculated as 1.5072 with \( \lambda = 1.0 \).

Therefore, the lot will be accepted by the consumer since the value of \( \hat{S}_{EWM A_{+1}} = 1.5072 \), is larger than the critical acceptance value 1.1207 significantly.

### 7. Conclusion remarks

This study designed two sampling plans for the quality characteristic of interest with normal distribution. The necessary measures for both sampling plans were pre-
presented. A detailed study was provided to demonstrate the advantages of the proposed plans. From a comparative study, we concluded that the proposed RGSP plan outperformed other plans using single sampling, repetitive sampling, and the proposed MDSS plans. To illustrate the application of the proposed plan, color STN displays data of a real case were presented. The proposed plans could be applied to circumstances with expensive inspections. For scope of application in industries, they can be applied in the mobile industry, automobile industry, electronic industry, and so on. For the direction of future research, the proposed plan using a cost model can be considered. Also, this study can be extended for the quality characteristic of interest with non-normal distribution.

Acknowledgement

The authors are deeply thankful to editor and reviewers for their valuable suggestions to improve the quality of manuscript. This article was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah. The author, Muhammad Aslam, therefore, acknowledge with thanks DSR technical and financial support. The work by Chi-Hyuck Jun was supported by Korea Institute for Advancement of Technology (KIAT) grant funded by the Korea Government (MOTIE) (P0008691, HRD Program for Industrial Innovation).

Supplementary data is available at:

file:///C:/Users/SHAMILA/AppData/Local/Temp/Supplementary%20file.pdf

References

1. Collani, E.V. “A note on acceptance sampling for variables”, *Metrika*, 38(1), pp. 19–36 (1981).

2. Seidel, W. “Is sampling by variables worse than sampling by attributes? A decision theoretic analysis and a new mixed strategy for inspecting individual lots”, *Sankhyā: The Indian Journal of Statistics, Series B*, pp. 96–107 (1997).

3. Govindraju, K. and Balamurali, S. “Chain sampling plan for variables inspection”, *Journal of Applied Statistics*, 25(1), pp. 103–109 (1998).

4. Cassady, C.R. and Nachlas, J.A. “Evaluating and implementing 3-level acceptance sampling plans”, *Quality Engineering*, 15(3), pp. 361–369 (2003).

5. Lin, H.C. and Sheen, G.J. “Practical implementation of the capability index C pk based on the control chart data”, *Quality Engineering*, 17(3), pp. 371–390 (2005).

6. Gao, Y. and Tang, L.C. “Chain sampling scheme under constant inspection errors”, *Quality and Reliability Engineering International*, 22(8), pp. 889–903 (2006).

7. Pearn, W.L. and Wu, C.W. “Critical acceptance values and sample sizes of a variables sampling plan for very low fraction of defectives”, *Omega*, 34(1), pp. 90–101 (2006).

8. Balamurali, S. and Jun, C.H. “Repetitive group sampling procedure for variables inspection”, *Journal of Applied Statistics*, 33(3), pp. 327–338 (2006).

9. Wu, C.W. and Pearn, W.L. “A variables sampling plan based on Cpk for product acceptance determination”, *European Journal of Operational Research*, 184(2), pp. 549–560 (2008).

10. Balamurali, S. and Jun, C.H. “Multiple dependent state sampling plans for lot acceptance based on measurement data”, *European Journal of Operational Research*, 180(3), pp. 1221–1230 (2007).

11. Morita, M., Arizono, I., Nakase, L., et al. “Economical operation of the C pm control chart for monitoring process capability index”, *The International Journal of Advanced Manufacturing Technology*, 43(3-4), pp. 301–311 (2009).

12. Tsai, T.R. and Lin, C.W. “Acceptance sampling plans under progressive interval censoring with likelihood ratio”, *Statistical Papers*, 51(2), pp. 259–271 (2010).

13. Klufa, J. “Exact calculation of the Dodge-Romig LTPD single sampling plans for inspection by variables”, *Statistical Papers*, 51(2), pp. 297–305 (2010).

14. Vangjeli, E. “ASN-minimax double sampling plans by variables for two-sided specification limits when the standard deviation is known”, *Statistical Papers*, 53(1), pp. 229–238 (2012).

15. Aslam, M., Jun, C.H., and Rasool, M. “Regular plans: A reliability sampling plan based on progressive interval censoring under Pareto distribution of second kind”, *IEMS*, 10(2), pp. 154–160 (2011).

16. Aslam, M., Balamurali, S., Jun, C.H., et al. “Optimal designing of an SkSp-V skip-lot sampling plan with double-sampling plan as the reference plan”, *The International Journal of Advanced Manufacturing Technology*, 60(5-8), pp. 733–740 (2012).

17. Wang, C.H. and Tsai, W. “Determining the production lot size with a heuristic inspection policy for controlling the quality of input materials and products”, *International Journal of Systems Science*, 43(11), pp. 2030–2039 (2012).

18. Wu, C.W., Aslam, M., and Jun, C.H. “Variables sampling inspection scheme for resubmitted lots based on the process capability index Cpk”, *European Journal of Operational Research*, 217(3), pp. 560–566 (2012).

19. Aslam, M., Lio, Y., and Jun, C.H. “Repetitive acceptance sampling plans for Burr type XII percentiles”, *The International Journal of Advanced Manufacturing Technology*, 68(1-4), pp. 495–507 (2013).

20. Nezhad, M.S.F. and Nasab, H.H. “Designing a single stage acceptance sampling plan based on the control threshold policy”, *International Journal of Industrial Engineering*, 22(3), pp. 143–150 (2011).
21. Asham, M., Yen, C.H., and Jun, C.H. “Variable repetitive group sampling plans with process loss consideration”, *Journal of Statistical Computation and Simulation*, 81(11), pp. 1417–1432 (2011).

22. Negrin, I., Pamet, Y., and Schechtman, E. “Developing a sampling plan based on Cpk-unknown variance”, *Quality and Reliability Engineering International*, 27(1), pp. 3–14 (2011).

23. Wu, C.W., Liao, M.Y., and Chen, J.C. “An improved approach for constructing lower confidence bound on process yield”, *European Journal of Industrial Engineering*, 6(3), pp. 369–390 (2012).

24. Yen, C.H., Chang, C.H., and Asham, M. “Repetitive variable acceptance sampling plan for one-sided specification”, *Journal of Statistical Computation and Simulation*, 85(6), pp. 1102–1116 (2015).

25. Asham, M., Azam, M., and Jun, C.H. “Multiple dependent state sampling plan based on process capability index”, *Journal of Testing and Evaluation*, 41(2), pp. 340–346 (2013).

26. Liu, S.W. and Wu, C.W. “Design and construction of a variables repetitive group sampling plan for unilateral specification limit”, *Communications in Statistics-Simulation and Computation*, 43(8), pp. 1866–1878 (2014).

27. Asham, M., Yen, C.H., Chang, C.H., et al. “Multiple dependent state variable sampling plans with process loss consideration”, *The International Journal of Advanced Manufacturing Technology*, 71(5-8), pp. 1337–1343 (2014).

28. Montgomery, D.C., *Introduction to Statistical Quality Control*, John Wiley & Sons (2007).

29. Čišar, P. and Čišar, S.M. “Optimization methods of EWMA statistics”, *Acta Polytechnica Hungarica*, 8(5), pp. 73–87 (2011).

30. Asham, M., Azam, M., and Jun, C.H. “A new lot inspection procedure based on exponentially weighted moving average”, *International Journal of Systems Science*, pp. 1–9 (2013)(ahead-of-print).

31. Yen, C.H., Asham, M., and Jun, C.H. “A lot inspection sampling plan based on EWMA yield index”, *The International Journal of Advanced Manufacturing Technology*, pp. 1–8 (2014).  

32. Sherman, R.E. “Design and evaluation of a repetitive group sampling plan”, *Technometrics*, 7(1), pp. 11–21 (1965).

33. Wortham, A. and Baker, R. “Multiple deferred state sampling inspection”, *The International Journal of Production Research*, 14(6), pp. 719–731 (1976).

34. Asham, M., Wu, C.W., Jun, C.H., et al. “Developing a variables repetitive group sampling plan based on process capability index C pk with unknown mean and variance”, *Journal of Statistical Computation and Simulation*, 83(8), pp. 1507–1517 (2013).

35. Yen, C.H. and Chang, C.H. “Designing variables sampling plans with process loss consideration”, *Communications in Statistics-Simulation and Computation*, 38(8), pp. 1579–1591 (2009).

36. Boyles, R.A. “Process capability with asymmetric tolerances”, *Communications in Statistics-Simulation and Computation*, 23(3), pp. 615–635 (1994).

37. Lee, J.C., Hung, H.N., Pearn, W.L., et al. “On the distribution of the estimated process yield index Spk”, *Quality and Reliability Engineering International*, 18(2), pp. 111–116 (2002).

38. Asham, M., Wu, C.W., Azam, M., et al. “Variable sampling inspection for resubmitted lots based on process capability index Cpk for normally distributed items”, *Applied Mathematical Modelling*, 37(3), pp. 667–675 (2013).

39. Asham, M., Khan, N., Azam, M., et al. “Designing of a new monitoring t-chart using repetitive sampling”, *Information Sciences*, 269, pp. 210–216 (2014).

40. Asham, M., Yen, C.H., Chang, C.H., et al. “Two-stage variables acceptance sampling plans using process loss functions”, *Communications in Statistics-Theory and Methods*, 41(20), pp. 3633–3647 (2012).

Biographies

Muhammad Asham introduced the area of Neutrosophic Statistical Quality Control (NSQC) for the first time. He is the founder of Neutrosophic Inferential Statistics (NIS) and NSQC. His contribution is the development of neutrosophic statistics theory for the inspection, inference, and process control. He originated the theory in these areas under Neutrosophic Statistics.

Muhammad Azam holds his Master’s degree in Statistics from Islamia University Bahawalpur in 1996 with distinction (Gold Medalist). He completed his MPhil from QAU, Islamabad in 2006 and PhD from University of Innsbruck Austria in 2010. He has been involved in teaching for various institutes for the last 21 years. He started his career as a lecturer in Statistics from Punjab Education Department and served there for 13 years. In 2010, he joined the Forman Christian College University Lahore as an Assistant Professor and served there for five years. In 2015, he joined as an Associate Professor and the Chairman of the Department of Statistics and Computer Science, UVAS, Lahore. On January 04, 2018, Dr. Azam was selected as a Professor of Statistics and he also continued working as the Chairman of the Department till March 13, 2018. On March 14, 2018, he was assigned the responsibility as the Dean of Faculty of Life Sciences Business Management (FLSBM). He has published more than 80 research articles mostly published in impact factor international journals. He has attended a number of national and international conferences/workshops. His research interests include survey sampling, statistical quality control, decision trees, and ensemble classifiers. He has produced 25
MPhil students. Currently, 4 PhD and 4 MPhil research students are working under his supervision.

Rehan Ahmad Khan Sherwani was born in Lahore, Pakistan in 1981. He received the Master’s degree in Statistics and the PhD degree from the University of the Punjab, Pakistan. He is currently working as an Assistant Professor at the College of Statistics, University of the Punjab, Lahore. He has numerous publications in peer-reviewed national and international research journals. His areas of specialization include regression analysis, multilevel models, structural equation models, mixed models, and their applications. He is a member of Boards of Studies of University of the Punjab and GC University Faisalabad, Pakistan. He has also worked as a member of Punjab Technical Committee for Census 2017; a member of Dean of the Faculty of Science, Purchase Committee; a Focal Person of QEC Faculty of Science, University of the Punjab; a Coordinator of MSc Biostatistics Programme, and a Coordinator of MSc Business Statistics and Management Programme. He is also a member of National Curriculum Review Committee by HEC for the subject of Statistics.

Ching-Ho Yen received the MS degree in Statistics from National Tsing Hua University in 2000, and the PhD degree in Industrial Engineering and Management from National Yang Ming Chiao Tung University in 2007. He is currently a Professor at the Department of Industrial Engineering and Management Information, Huaian University, Taipei, Taiwan. He is interested in statistics, quality control, and data mining.

Chi-Hyuck Jun received the BS degree in Mineral and Petroleum Engineering from Seoul National University, the MS degree in Industrial Engineering from the Korea Advanced Institute of Science and Technology, and the PhD degree in Operations Research from the University of California at Berkeley. Since 1987, he has been with the Department of Industrial and Management Engineering, Pohang University of Science and Technology (POSTECH), where he is currently a Professor Emeritus at the Department of Industrial and Management Engineering. He is interested in data mining and reliability/quality.