The deconfinement transition in SU(N) gauge theories

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Abstract

We investigate the properties of the deconfinement transition in SU(4) and SU(6) gauge theories. We find that it is a 'normal' first order transition in both cases, from which we conclude that the transition is first order in the \( N \to \infty \) limit. Comparing our preliminary estimates of the continuum values of \( T_c/\sqrt{\sigma} \) with existing values for SU(2) and SU(3) demonstrates a weak dependence on \( N \) for all values of \( N \).
1 Introduction

The dependence of QCD and SU($N$) gauge theories on the number of colours, $N$, is of great interest [1, 2]. Through a number of recent lattice calculations [2, 3, 4, 5, 6, 7] we have learned a great deal about the mass spectrum, various string tensions and the topological properties of SU($N$) gauge theories at finite and infinite $N$. These calculations confirm the usual diagrammatic expectation that a smooth large-$N$ physics limit is achieved by keeping fixed the 't Hooft coupling $g^2N$. In addition there is some good evidence from very accurate calculations of the string tension, $\sigma$, in 2+1 dimensional gauge theories [8], that the leading corrections are $O(1/N^2)$ – again as expected. (Note however that there is room for a violation of this [2], but the evidence is equivocal as yet.)

An interesting question concerns the order of the deconfining phase transition at $N = \infty$. It has been argued [9] that the difference between the SU(3) gauge theory and QCD with light quarks (see [10] for recent reviews) can be reconciled with both being ‘close to’ large-$N$ if in fact the SU($N$) transition becomes second order at large-$N$. (See [11] for some more recent reflections on this.) This has prompted lattice studies [12, 13] of deconfinement in SU(4) which claim that the transition is first order there. However it is only in SU(6), or larger groups, that all possible escape routes close [9]. Thus we hope that the calculations in both SU(4) and SU(6) gauge theories [14] that we shall briefly report upon in this paper will serve to settle this question. (See also [15] for calculations at very large $N$, performed in a twisted Eguchi-Kawai single-site reduced model [16].)

In this letter we focus upon the order of the transition and upon our preliminary determination of how the critical temperature, $T_c$, varies with $N$. There are of course many other interesting aspects of the transition, some of which we are studying. We will mention some of these in the concluding section, and will present our detailed results in a longer publication [14]. There we will also discuss and compare several different methods for locating and analysing the deconfining phase transition. We will, in addition, apply these methods to the case of SU(2), where the transition is known to be 2nd order [17], and SU(3), where it is known to be weakly 1st order [18]. At the same time we will have higher statistics which should allow us to considerably improve upon the preliminary continuum values of $T_c/\sqrt{\sigma}$ that we present in this paper. However, we believe that we already have unambiguous evidence for the first order nature of the SU(4) and SU(6) phase transitions and this is what we wish to communicate in this letter.

2 Strategy and technical details

Our lattices are hyper-cubic with periodic boundary conditions and with lattice spacing $a$. We use the standard plaquette action

$$S = \beta \sum_{p} \{1 - \frac{1}{N} \text{ReTr} U_{p}\}$$

(1)
where the ordered product of the SU($N$) matrices around the boundary of the plaquette $p$ is represented by $U_p$. In the continuum limit this becomes the usual Yang-Mills action with

$$\beta = \frac{2N}{g^2}. \quad (2)$$

The simulations are performed with a combination of heat bath and over-relaxation updates, as described in [4, 3]. Our SU(4) and SU(6) updates involve 6 and 15 SU(2) subgroups respectively.

Consider a lattice whose size is $L^3L_t$ in lattice units. This may be regarded as a system at temperature

$$a(\beta)T = \frac{1}{L_t}. \quad (3)$$

We require $L \gg L_t$, so that it makes sense to discuss thermodynamics. We study the deconfining transition by fixing $L_t$ and varying $\beta$ so that $T$ passes through $T_c$.

For $T < T_c$ we are in the confining phase, while for $T > T_c$ we will be in one of $N$ equivalent deconfined phases. To distinguish all these phases we define the Polyakov loop

$$l_p(\vec{x}) = \text{Tr} \prod_t U_t(\vec{x}, t) \quad (4)$$

and denote by $\bar{l}_p$ its average over $\vec{x}$ for a given field configuration. For $T < T_c$ we will have $\bar{l}_p \approx 0$ while for $T > T_c$ the $N$ deconfined vacua will be characterised by values $\bar{l}_p \approx cz_n$ where $z_n$ is one of the $N$'th roots of unity and $c > 0$ will depend on $\beta$, $L_t$, $N$ etc. For $L$ large enough the fluctuations of $\bar{l}_p$ become small enough that a field configuration can be unambiguously categorised into one of the phases by its value of $\bar{l}_p$. Near $T_c$ there may be tunnelling between these phases. This process will involve field configurations that contain a mix of phases and will have intermediate values of $\bar{l}_p$. Note that the usual characteristic of first order lattice transitions, a clear double peak structure when one plots a histogram of the number of fields against the action (energy) is not so useful here. This is because we are dealing with a physical rather than a bulk transition. Any difference in the action will therefore be $O(a^4)$ and this is extremely small compared to the $O(1/\beta)$ value of the action. It is only on the very largest lattices that the fluctuations in the latter, which are also $O(1/\sqrt{L^3})$, become small enough for the beginnings of a double peak structure to appear. By contrast, clear multiple peak structures in histograms of $\bar{l}_p$ already appear on small lattices.

In a finite spatial volume $V$ the phase transition is smeared over a finite range of $T$. One needs to choose a value for $T_c(V)$ in this range and then to extrapolate this to $V = \infty$ to obtain the value of $T_c$ in the thermodynamic limit. Different criteria will differ at finite $V$ but should extrapolate to the same value of $T_c$ at $V = \infty$. Good criteria will be ones with finite-$V$ corrections that are simple, small and have a known functional form. We have performed calculations with a number of criteria which produce consistent results. In this paper we present values of $\beta_c$ obtained from the value of $\beta$ at which the Polyakov loop susceptibility has a peak. To obtain the susceptibility as a continuous function of $\beta$ we use a standard reweighting technique [19] applied to those calculations that lie close enough to $T_c$. This will be described in some more detail in the next Section.
Tunnelling between the confined and deconfined vacua is a first sign that the phase transition is first order. However its observation at $T \neq T_c$ and/or on small volumes may be misleading. Equally a lack of tunnelling on small volumes may be misleading; for example, the transition may be weakly first order. It is therefore crucial to perform a finite volume study to identify what volumes are large enough to be useful. Given that near $T_c$ the important length scale is $aL_t = 1/T_c$ one would expect that one needs at least $L \simeq 3L_t$ in order that the periodic spatial volume can accommodate two phases and some intervening surface in a realistic fashion. In a finite volume study a first order transition would be characterised, at large $V$, by a latent heat that stays non-zero, a mass gap that does not vanish (i.e. a correlation length that does not diverge), an appropriately behaved susceptibility, and clear tunnelling that becomes exponentially rare as $V$ grows.

The computational cost will clearly be minimised by using the smallest possible values of $L_t$. It is however well known that with the standard plaquette lattice action there is a cross-over in the strong-to-weak coupling transition region which becomes a strong first-order phase transition at larger $N$. (See e.g. [4] for a discussion.) For SU(4) this appears to be smooth cross-over [4] but by SU(6) it is certainly a strongly first order bulk phase transition [5]. This presents some problems. In particular any calculations that are going to be useful for continuum extrapolations will need to be made on the weak-coupling side of this transition. For SU(4) deconfinement this means that we have to use $L_t \geq 5$. For SU(6) the finite-T transition for $L_t = 5$ is quite close to the strongly first order bulk phase transition and so this would not be a good place to address the delicate question of whether the former is also first order or not. For that we will use $L_t = 6$. However the computational expense of SU(6) calculations at $L_t = 6$ is such that we will not be able to perform the kind of finite size study that distinguishes most unambiguously between 1st and 2nd order phase transitions. Fortunately the phase transition turns out to be reasonably strong first order, rather than being weak first order as in SU(3). This makes it relatively easy to study, even within our tight constraints.

Our strategy will therefore be as follows. We start with SU(4) where we perform a detailed finite size study of the deconfining transition for lattices with $L_t = 5$. We shall establish that it is first order and will quantify its dependence on the spatial volume. We then perform a study of the SU(6) phase transition on a $16^36$ lattice, using our earlier finite-size study to extrapolate it to the thermodynamic limit. We also have begun calculations in SU(4) for $L_t = 6$ and in SU(6) for $L_t = 5$; these enable us to make some preliminary estimates of $T_c/\sqrt{\sigma}$ in the continuum limit. At the same time the $L_t = 6$ SU(4) calculations provide for a direct comparison with the SU(6) calculations at $L_t = 6$.

3 $T_c$ in SU(4)

Our $L_t = 5$ study has been performed on $L = 12, 14, 16, 18, 20$ lattices, with between 200000 to 300000 sweeps at each value of $\beta$ and $L$. For $\beta$ in a narrow region around $\beta = 10.635$ we find very clear tunnelling transitions on all lattice sizes. These become more rare as $L \uparrow$, just as one would expect for a first order transition. In Fig. we illustrate this with the time
histories of $|\bar{l}_p|$ on the $L = 14$ and $L = 20$ lattices at $\beta = 10.635$.

We define a normalised Polyakov loop susceptibility

$$\frac{\chi_l}{V} = \frac{\langle |\bar{l}_p|^2 \rangle - \langle |\bar{l}_p| \rangle^2}{V}$$

(5)

where we expect

$$\lim_{T \to T_c} \chi_l \xrightarrow{V \to \infty} \begin{cases} V, & \text{if 1'\text{st order}} \\ V^\gamma, & \text{if 2'\text{nd order}} \end{cases}$$

(6)

with $\gamma$ a constant that depends on both the space-time dimension and the critical exponents of the transition. For each value of $L$ we calculate $\chi_l$ as a function of $\beta$, determine the value at which it has a maximum and use that as our estimate of $\beta_c$, and hence $T_c$. This is a good, although not unique, criterion irrespective of whether the transition is first or second order. To identify the maximum we interpolate between the values of $\beta$ at which we have performed calculations (usually there are three that have sufficient tunnellings to be useful) using a standard reweighting technique [19]. Fig. 2 shows a histogram of the plaquette, averaged over the lattice volume, for the $20^35$ lattices reweighted to the critical beta-value. It clearly shows the double peaking that is characteristic of a first order transition. In Fig. 3 we plot our values of $\beta_c$ against the inverse volume expressed in units of $T$. We note that if the transition is first order, as our’s clearly is, then there exist arguments [18] that with this way of determining $\beta_c$ the leading finite-$V$ correction should be

$$\beta_c(V) = \beta_c(\infty) - \frac{h}{VT^3}.$$  

(7)

Here $h$ is independent of the lattice action and of the lattice spacing $a$ up to lattice corrections that will be small if $a$ is small. We observe from Fig. 3 that all our values of $\beta_c$ are consistent with eqn(7). Moreover the value of $h$ we thus extract

$$h = 0.09 \pm 0.02$$

(8)

is close to the SU(3) value, $h \simeq 0.1$ [18]. We take this to imply that $h$ depends weakly on $N$ and will use the value in eqn(8) to extrapolate to $V = \infty$ in the SU(6) case where we do not perform an explicit finite volume study. We remark that if $h$ is indeed independent of $N$, then this implies that the volume dependence of $T_c(V)/T_c(\infty)$ is $O(1/N^2)$, i.e. there is no volume dependence in the $N \to \infty$ limit.

A quantity analogous to $\chi_l$, but with plaquettes replacing the Polyakov loop, is the specific heat $C(\beta)$:

$$\frac{1}{\beta^2}C(\beta) = \frac{\partial}{\partial \beta} \langle \bar{u}_p \rangle = N_p \langle \bar{u}_p^2 \rangle - N_p \langle \bar{u}_p \rangle^2$$

(9)

where $\bar{u}_p$ is the average value for a given lattice field of the plaquette $u_p \equiv \text{ReTr}U_p/N$, and $N_p = 6L^3 L_t$ is the total number of plaquettes. In Fig. 4 we plot $C(\beta_c)/V$ against $1/V$. We observe that it is certainly consistent with going to a finite value as $V \to \infty$, as one would expect for a first order transition. We show in Fig. 4 a fit with a leading linear correction in $1/V$ of the kind that one would expect irrespective of whether the transition was first or
second order. For a first order transition, as we appear to have here, we can obtain the latent
heat, \( L_h \), from the intercept
\[
\lim_{V \to \infty} \frac{1}{\beta^2 N_p} C(\beta) = \frac{1}{4}(\langle \bar{u}_{p,c} \rangle - \langle \bar{u}_{p,d} \rangle)^2 = \frac{1}{4} L_h^2
\]  
(10)
where \( \bar{u}_{p,c} \) and \( \bar{u}_{p,d} \) are the average plaquette values at \( \beta = \beta_c \) in the confined and deconfined
phases respectively. From the fit in Fig.4 we obtain
\[
L_h = 0.00197(5).
\]  
(11)
We note that if we take the \( L_t = 4 \) SU(3) latent heat in \[21\] and naively scale it to \( L_t = 5 \), we
obtain \( L_h(N = 3) \approx 0.0013 \) which is substantially smaller than our above SU(4) value. This
shows explicitly that the SU(4) transition is more strongly first-order than the SU(3) one.

We have seen in Fig.4 that the value of \( \bar{p} \) allows us to label the phase of a field configuration
almost unambiguously. So for values of \( \beta \) close to \( \beta_c \), where all phases are sampled, we can
calculate the average plaquette in each phase separately and hence the difference \( \Delta s \) between
the confined and deconfined phases. Calculated at \( \beta_c \) this is the finite-\( V \) latent heat of the
transition. There is some uncertainty in separating configurations in a single phase from those
that are tunnelling. This is a systematic error that we can estimate by varying the cuts on \( \bar{p} \), and we include it in our final error estimates. In Fig.5 we show how \( \Delta s \) varies with the spatial
size, \( L \), for those of our calculations that are closest to \( \beta_c \) (see Fig.3). We see that the latent
heat is at most weakly dependent on the spatial volume and appears to have a finite \( V = \infty \)
limit, confirming once again that this is a first order phase transition. The value is pleasingly
close to that in eqn(11), which has been obtained under very different assumptions.

The value of \( \Delta s \) provides a measure of how strong is the transition. To express it in
physical units we note that if one separates the plaquette into an ultraviolet perturbative
piece and the gluon condensate \[21\], then the lattice renormalisation factor multiplying the
latter is \( Z \approx 1 \). Thus we can express \( \Delta s \equiv a^4 m_s^4 \) where \( m_s \) is a physical scale. Noting that
for \( \beta \approx \beta_c \) we have \( \Delta s \approx 0.002 \) and that \( a T_c = 1/L_t = 0.2 \) we see that \( m_s \approx T_c \). That is
to say, a typical dynamical scale indicating that the strength of this first order transition is
quite ordinary. We remark that our preliminary calculations with \( L_t = 6 \) indicate that lattice
corrections are small and that this statement holds in the continuum limit.

From the fit in Fig.3 we obtain the \( V = \infty \) critical value \( \beta_c = 10.63709(72) \). Interpolating
previously calculated values of the string tension \[11\] to \( \beta_c \), and using \( T_c = 1/5a(\beta_c) \), we obtain
\[
\frac{T_c}{\sqrt{\sigma}} = 0.6024 \pm 0.0045 \quad \text{at } a = 1/5T_c
\]  
(12)
where the bulk of the error comes from the string tension. If lattice corrections are very small,
as is found to be the case in SU(3) \[18\], then eqn(12) should provide a good estimate of the value in the continuum limit. In fact our very preliminary \( L_t = 6 \) calculations give a \( V = \infty \)
value \( \beta_c = 10.780(10) \) which translates to \( T_c/\sqrt{\sigma} = 0.597(8) \), where the major part of the
error comes from the uncertainty in \( \beta_c \). This, taken together with the value in eqn(11), gives
us the extrapolated continuum value

\[ \lim_{a \to 0} \frac{T_c}{\sqrt{\sigma}} = 0.584 \pm 0.030. \]  

(13)

In making such an extrapolation we need to assume the dominance of the leading \( O(a^2\sigma) \) lattice correction, and this is justified by the observed dominance of this correction for other physical quantities \[4, 3]\.

4 \( T_c \) in SU(6)

Our high statistics SU(6) calculations have been performed on 16\(^3\)6 lattices at values of \( \beta \) close to \( \beta = 24.845 \). In physical units this is like a 14\(^3\) lattice with \( L_t = 5 \) and although this is not large, we have seen in our SU(4) calculations (see Figs.1-4) that the physics of such volumes is quite close to the thermodynamic limit.

In Fig.6 we plot the values of \(|\bar{l}_p|\) for the sequence of configurations that we obtain at \( \beta = 24.845 \). We see well defined tunnelling between confined and deconfined phases, characteristic of a first order transition. This is further illustrated in Fig.7 which displays the very clear separation between the phases – this time at \( \beta = 24.85 \). Using the reweighting technique we extract a critical value \( \beta_c = 24.850(3) \). We extrapolate this to \( V = \infty \) using eqn(8) with the value of \( h \) in eqn(8). Recall that this is motivated by the fact that one finds \( h \) to have the same value in SU(3) and SU(4). Doing so we obtain \( \lim_{V \to \infty} \beta_c = 24.855 \pm 0.003 \). This can be translated into a value of \( T_c \) just as in the case of SU(4), using the values of the SU(6) string tension calculated in [5], to give

\[ \frac{T_c}{\sqrt{\sigma}} = 0.588 \pm 0.002 \quad \text{at } a = 1/6T_c. \]  

(14)

Just as in SU(4), we would expect this to provide a good estimate of the continuum value. We have in addition performed some preliminary calculations on 16\(^3\)5 lattices, increasing and decreasing \( \beta \) while crossing the deconfining phase transition. From the average plaquette values it is clear that these calculations are all on the weak coupling side of the nearby bulk transition and can therefore be used for a continuum extrapolation. (Whether this remains so in a much higher statistics calculation and for other volumes is something we are in the process of investigating [14].) These preliminary \( L_t = 5 \) calculations provide, after extrapolation to \( V = \infty \), an estimate \( \beta_c = 24.519(33) \) which translates to \( T_c/\sqrt{\sigma} = 0.580(12) \). Taken together with with the value in eqn(14) this gives us the continuum value

\[ \lim_{a \to 0} \frac{T_c}{\sqrt{\sigma}} = 0.605 \pm 0.026. \]  

(15)

As in the case of SU(4) we need to assume the dominance of the leading \( O(a^2\sigma) \) lattice correction, and the justification is the same as it was there [4].

As for the strength of the first order transition, we remark that our value of \( \Delta s \) is larger than our (preliminary) value on a directly comparable 16\(^3\)6 SU(4) calculation, showing that the transition is certainly not weakening as \( N \to \infty \).
5 Discussion

Our careful finite size study in SU(4) for $aT = 1/5$ reveals quite clearly that the deconfining phase transition is first order with a latent heat that is not particularly small. This confirms previous work [12, 13]. Our SU(6) calculations do not involve a finite size study, but have been performed on a spatial volume that, on the basis of our SU(4) calculations, should be large enough for our purposes. The calculations were performed closer to the continuum limit, $aT = 1/6$, in order to avoid any possible confusion with a nearby strongly first order bulk transition. On our $16^3 \times 6$ lattices we found very clear, if rare, tunnelling between the confining and deconfining phases, characteristic of a typical first order transition. The extrapolation to the thermodynamic limit was made using the dependence on $V$ found in SU(4). This can be justified by the fact that the SU(4) volume dependence is the same as found in SU(3), suggesting that it depends at most weakly on $N$. The SU(6) transition appears to be at least as strong as the SU(4) one, which tells us that the SU($N = \infty$) transition is first order and not particularly weak.

Our preliminary calculations at other values of $a$ suggest that the values of $T_c$ in eqns(11,14) are indeed close to the continuum values. This is not surprising given the very small lattice corrections observed in SU(3) calculations with the same lattice action [15]. If we take these SU(3) values with older SU(2) ones [7] and plot them against $1/N^2$ together with our SU(4) and SU(6) values, as in Fig.8, we see that we can describe the values for all $N$ with just a leading large-$N$ correction

$$\frac{T_c}{\sqrt{\sigma}} = 0.582(15) + 0.43(13) \frac{1}{N^2}, \quad (16)$$

even though the SU(2) transition is second order, the SU(3) is weakly first order and the SU(4) and SU(6) transitions are ‘normal’ first order transitions. Our ongoing calculations will significantly reduce the errors; both by providing accurate values of $\beta_c$ at $L_t = 5,6$ for SU(6) and SU(4) respectively, and because we intend to obtain more accurate values of the string tension at the critical values of $\beta$ corresponding to $L_t = 5$, where the error on our string tension extrapolation/interpolation is particularly large. In addition our SU(3) calculations will use the same methods, and so a comparison with larger $N$ should provide direct evidence for the strengthening of the transition as $N$ increases.

There are of course many other interesting properties of the phase transition, some of which we have been studying. We will report on this work elsewhere [14]. For example, topological fluctuations are suppressed across $T_c$ and we find that this effect becomes dramatically stronger at larger $N$. Small instantons are explicitly suppressed as $N \uparrow$ while larger instantons are suppressed as $T \uparrow$ in the deconfined phase. It appears that $T_c$ is large enough for these effects to ‘cross’ at $T_c$ at large $N$ so that there is essentially no topology left for $T > T_c$. We also calculate the mass gap in the neighbourhood of the transition, separately for the confined and deconfined phases, and explicitly confirm that it does not vanish at $T_c$. We calculate the spatial string tensions of $k$-strings near $T_c$ and confirm approximate Casimir scaling [3] in both phases, although the string tensions are significantly different in the two phases (at the same value of $\beta$). Comparing $k = 1$ and $k = 2$ Polyakov loops we see that deconfinement appears to occur for both at the same value of $T_c$. And our finite volume study in SU(6)
should determine whether our claim in this paper, that finite volume effects disappear as $1/N^2$, is indeed correct. Finally our extra calculations at other other values of $\alpha$ will enable us to improve upon the calculations reported herein.

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Figure 1: The modulus of the average value of the Polyakov loop for a sequence of $14^35$ (top) and $20^35$ (bottom) field configurations at $\beta = 10.635$ in SU(4).
Figure 2: A histogram of the values of the average plaquette, for the $20^35$ lattice at $\beta_c$ in SU(4).
Figure 3: The critical value of $\beta$ plotted against the inverse spatial volume, $V$, expressed in units of the temperature, $T$. On $L^35$ lattices in SU(4). The straight lines are the extrapolations to infinite volume, one excluding the $L = 12$ value. The corresponding extrapolated values are shown slightly to the left of $1/V = 0$. 
Figure 4: The specific heat at the critical value of $\beta$, $C(\beta_c, V)$, normalised to the spatial volume $V$ and plotted against $1/V$, with $V$ in units of $T$. The intercept ($\bullet$) at $V = \infty$ provides a measure of the latent heat. For SU(4) and $L_t = 5$. 
Figure 5: The difference of the average plaquette in the confining and deconfining phases on $L^3$ SU(4) lattices plotted versus $L$. For $\beta = 10.635$ (o) and $\beta = 10.637$ (●).
Figure 6: The modulus of the average value of the Polyakov loop for a sequence of $16^3 6$ SU(6) field configurations at $\beta = 24.845$. 
Figure 7: A histogram of the values the modulus of the Polyakov loop, $\bar{l}_{\mu}$, for the SU(6) calculation at $\beta = 24.85$. 
Figure 8: The deconfining temperature, $T_c$, in units of the string tension, $\sigma$ plotted versus $1/N^2$. The straight line is the large-$N$ extrapolation with a leading $O(1/N^2)$ correction.