Supporting Information

Evaluation of statistical methods for quantifying fractal scaling in water quality time series with irregular sampling

Qian Zhang\textsuperscript{1}, Ciaran J. Harman\textsuperscript{2}, James W. Kirchner\textsuperscript{3,4,5}

\textsuperscript{1}University of Maryland Center for Environmental Science at the US Environmental Protection Agency Chesapeake Bay Program Office, 410 Severn Avenue, Suite 112, Annapolis, MD 21403 (formerly, Department of Geography and Environmental Engineering, Johns Hopkins University, 3400 North Charles Street, Baltimore, Maryland 21218)
\textsuperscript{2}Department of Environmental Health and Engineering, Johns Hopkins University, 3400 North Charles Street, Baltimore, Maryland 21218
\textsuperscript{3}Department of Environmental System Sciences, ETH Zurich, Universitätstrasse 16, CH-8092 Zurich, Switzerland
\textsuperscript{4}Swiss Federal Research Institute WSL, Zürcherstrasse 111, CH-8903 Birmensdorf, Switzerland
\textsuperscript{5}Department of Earth and Planetary Science, University of California, Berkeley, California 94720

Correspondence to: Qian Zhang (qzhang@chesapeakebay.net)
Figure S1. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed $\beta$ values, series length of 9125, and NB ($\lambda = 0.01, \mu = 1$) distributed gap intervals. The blue dashed lines indicate the true $\beta$ values.
Figure S2. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed $\beta$ values, series length of 9125, and NB ($\lambda = 0.1, \mu = 1$) distributed gap intervals. The blue dashed lines indicate the true $\beta$ values.
Figure S3. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed $\beta$ values, series length of 9125, and NB ($\lambda = 1, \mu = 1$) distributed gap intervals. The blue dashed lines indicate the true $\beta$ values.
Figure S4. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed $\beta$ values, series length of 9125, and NB ($\lambda = 10$, $\mu = 1$) distributed gap intervals. The blue dashed lines indicate the true $\beta$ values.
Figure S5. Comparison of standard deviation in estimated spectral slope in irregular data that are simulated with varying prescribed $\beta$ values (30 replicates), series length of 9125, and mean gap interval of 2 (i.e., $\mu = 1$).
Figure S6. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed $\beta$ values, series length of 9125, and NB ($\lambda = 0.01$, $\mu = 14$) distributed gap intervals. The blue dashed lines indicate the true $\beta$ values.
Figure S7. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed $\beta$ values, series length of 9125, and NB ($\lambda = 0.1, \mu = 14$) distributed gap intervals. The blue dashed lines indicate the true $\beta$ values.
Figure S8. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed $\beta$ values, series length of 9125, and NB ($\lambda = 1, \mu = 14$) distributed gap intervals. The blue dashed lines indicate the true $\beta$ values.
Figure S9. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed $\beta$ values, series length of 9125, and NB ($\lambda = 10, \mu = 14$) distributed gap intervals. The blue dashed lines indicate the true $\beta$ values.
Figure S10. Comparison of standard deviation in estimated spectral slope in irregular data that are simulated with varying prescribed $\beta$ values (30 replicates), series length of 9125, and mean gap interval of 15 (i.e., $\mu = 14$).
Figure S11. Histogram of concentration residuals from the WRTDS method, expressed in natural log concentration units, for total nitrogen (TN) at the nine Chesapeake Bay monitoring sites. See Table 1 for site and data details.
Figure S12. Histogram of concentration residuals from the WRTDS method, expressed in natural log concentration units, for total phosphorus (TP) at the nine Chesapeake Bay monitoring sites. See Table 1 for site and data details.
Figure S13. Histogram of concentration residuals from the WRTDS method, expressed in natural log concentration units, for nitrate-plus-nitrite (NO$_x$) at the six Lake Erie and Ohio River monitoring sites. See Table 1 for site and data details.
Figure S14. Histogram of concentration residuals from the WRTDS method, expressed in natural log concentration units, for total phosphorus (TP) at the six Lake Erie and Ohio River monitoring sites. See Table 1 for site and data details.