Scaling of Nonlinear Longitudinal and Hall Resistivities near the Vortex Glass Transition

X. Hu, L. He, L. Yin, Z.H. Ning, H.Y. Xu, X.L. Xu, J.D. Guo, C.Y. Li and D.L. Yin*
Department of Physics, Peking University, Beijing 100871, China

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We show that the longitudinal current-voltage characteristics of superconductors in mixed state have the general form of extended power law. Isotherms simulated from this nonlinear equation fit the experimental I-V data of Strachan et al. [ Phys. Rev. Lett. 87, 067007 (2001)]. We determine the average pinning force in the flux creep and strong pinning regime and discuss both the puzzling scaling behavior $\rho_{xy} \propto \rho_{xx}^{\beta}$ and a recently found new scaling relationship of nonlinear Hall resistivity $\rho_{xy}(T)$.

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The vortex motion in the superfluid electrons has presented a persistent problem in the physics of high-temperature superconductors (HTS). One of the most puzzling phenomena is the sign change that has been observed in the Hall effect in the superconducting state of temperature superconductors (HTS). One of the most puzzling phenomena is the sign change that has been observed in the Hall effect in the superconducting state of most HTS and some conventional superconductors [1].

Another highly intriguing issue is the power law scaling relation between the Hall resistivity $\rho_{xy}$ and longitudinal resistivity $\rho_{xx}$ [2, 3]. Experiments show that the Hall effect at least depends on two factors: the doping level [4] and the vortex pinning [2, 3]. Based on the hypothesis of a vortex glass transition [1], Dorsey and Fisher [2] reasonably explained the puzzling scaling behavior $\rho_{xy} \propto \rho_{xx}^{\beta}$ with $\beta = 1.7 \pm 0.2$ observed by Luo et al. [2]. The Hall effect itself was attributed to a "particle-hole" asymmetry and the exponent $\beta$ was related to a specially chosen particle-hole asymmetry exponent $\lambda \approx 3$. They predicted "The nonlinear Hall electric field $E_y$ should exhibit universal scaling and, right at the transition, should vanish with a universal power of the current density $J_x$". This prediction has been verified by Wöltgens et al. [2] with an experimentally determined Hall-related exponent $\lambda = 3.4 \pm 0.3$. Furthermore, they found wide range scaling behavior $\rho_{xy} = A\rho_{xx}(J_x,T)^{2.0 \pm 0.2}$ in consistency with the model proposed by Vinokur, Geshkenbein, Feigel'man and Blatter (VGF) [2]. However, recently the widely accepted experimental evidence for vortex glass theory [1] has met serious doubt [11]. Strachan et al. show wide range accurate isothermal I-V measurements over 5 or 6 decades on a high quality 2200˚A thick YBa$_2$Cu$_3$O$_7-\delta$ film with $T_c \approx 91.5K$ and transition width about 0.5K [11] and find although the I-V isotherms measured in a magnetic field can be collapsed into scaling functions proposed by Fisher et al. [1] as is widely reported in the literature, these excellent data collapse can also be achieved for a wide range of exponents and glass temperatures $T_g$ as demonstrated in their Fig.2(a),2(b) and 2(c) [11]. Since the critical temperature $T_g$ cannot be determined uniquely, the correctness of the vortex glass picture has to be reinvestigated. In this work, we show that the longitudinal isothermal current-voltage characteristics have the general form of extended power law [12].

This remarkable feature naturally explains the insightful new findings of Strachan et al. [11] as well as the recently observed scaling relationship of $\rho_{xy}(T)$.

Thermally activated flux motion can be considered as the sequence of thermally activated jumps of the vortex segments or vortex bundles between the metastable states generated by disorder. Every elementary jump is viewed as the nucleation of a vortex loop, and the mean velocity of the vortex system is determined by the nucleation rate $\bar{f}_n [12]$

$$v \propto \exp \left(-\frac{\delta F}{kT}\right).$$

Here $\delta F$ is the free energy for the formation of the critical size loop or nucleus which can be found by means of the standard variational procedure from the free energy functional due to the in-plane displacement $u(z)$ of the moving vortex during loop formation

$$F_{\text{loop}}[u] = \int dz \left[\frac{1}{2} \eta \left(\frac{du(z)}{dz}\right)^2 + V_p(u(z)) - f_s \cdot u\right]$$

with

$$f_s = f_L + f_\eta = \frac{\Phi_0}{c} J_p \times e_z$$

and $J_p = J - J_f = J - \frac{E}{\rho_f}$,

$$f_\eta = -\eta v_{\text{vortex}},$$

where $f_L = \Phi_0 J \times e_z/c$ is the Lorentz force due to applied current $J$ and $f_\eta$ is the viscous force on vortex, $\eta$ is the Lorentz force due to applied current $J$ and $f_\eta$ is the viscous force on vortex, $\eta$ is the viscous drag coefficient $\eta \approx \Phi_0 \eta / \langle \rho_n \rangle$, $\rho_f$ is the flux flow resistance of a pinning free mixed state $\rho_f \approx \frac{\rho_n}{\langle \rho_n \rangle}$, as derived by Bardeen and Stephen [2].

Our equation (2) differs from the corresponding one used in previous vortex glass theories on that we considered both the Lorentz force $f_L$ and the viscous drag force $f_\eta$ on the vortex loop during its formation while in present vortex glass theories only bare Lorentz force $f_L$ has been taken into account [1, 13, 15].

As shown in [15], the barrier energy found by the conventional vortex glass model [7] has the general form with
that of collective pinning model and Boson glass model

\[ \delta F \equiv F_{\text{loop}}[L^*(J)] = U(J) \approx U_0 \left( \frac{J_0}{J} \right)^\mu, \quad (4) \]

with \( L^*(J) \) the critical size of vortex loop nucleation. Since the viscous drag force \( f_\nu \) has been taken into account in Eq. (2), now instead of Eq. (4) one must have the barrier energy

\[ \delta F = U(J_p) \approx U_0 \left( \frac{J_0}{J_p} \right)^\mu, \quad (5) \]

which implies a current-voltage characteristic of the form

\[ E(J) = \rho_f J \exp \left[ -\frac{U_0}{kT} \left( \frac{J_0}{J_p} \right) \right], \quad (6) \]

where \( U_0 \) is a temperature and field dependent characteristic pinning energy related to the stiffness coefficient and \( J_0 \) is a characteristic current density related to \( U_0 \), \( \mu \) is an numerical exponent.

Considering the real sample size effect we note that the barrier energy for vortex loop excitation cannot exceed the free energy value needed for generating a loop with real sample size \( D \) defined as

\[ \delta F = U(J) \leq U(J_D) \equiv F_{\text{loop}}[L^*(J = J_D) \equiv D]. \quad (7) \]

Therefore, the free energy barrier for vortex loop excitation never diverges even at vanishing applied current \( J \) as predicted by the conventional vortex glass theory [7].

Replacing \( J_p \) in Eq. (5) with the expression Eq. (4), and taking the logarithm of its both sides, one obtains

\[ J - J_f = \left( \frac{U_0}{kT} \right)^{1/\mu} J_0 \left[ \ln \left( \frac{J}{J_f} \right) / J_f \right]^{1/\mu} \]

\[ = \left( \frac{U_0}{kT} \right)^{1/\mu} J_0 (1 + h)^{-1} \ln \left( \frac{J_D}{J_{DF}} \right)^{-1/\mu}, \quad (8) \]

where \( J_{DF} = E(J_D)/\rho_f \) is much smaller than \( J_D \) and we define \( h = -\ln(J_D/J_{DF})/(\ln(J_D/J_{DF})^{1/\mu} \). Using the approximation \((1 + h)^{-1} \approx 1 - h \) for \(|h| < 1\), one finally obtains a general normalized form of the current-voltage characteristic as

\[ y = x \exp \left[ -\gamma (1 + y - x)^p \right], \quad (9) \]

where

\[ \gamma = \frac{U_0}{kT} \left( \frac{2J_0}{J_D} \right)^\mu, \quad x = \frac{J}{2J_D}, \quad y = \frac{E(J)}{2\rho_f J_D}, \quad p = \mu. \]

Equation (9) can also be nearly equivalently expressed in an extended power law form

\[ \frac{y}{x} = e^{-\gamma(x - y + 1)^{n-1}}, \quad (10) \]

with \( n \) is a parameter determined by the condition that Eqs. (4) and (10) meet at the inflection points of \(ln y \sim \ln x \) curves for Eq. (9).

In Fig.1(a) we show the numerical solutions of Eq. (9) and Eq. (10) for comparison.

The complex longitudinal experimental current-voltage data are usually described phenomenologically with a form [15]

\[ E(J) = J\rho_{xx}(T, B, J) = J\rho_f e^{-U_{eff}(T, B, J)/kT}. \quad (11) \]

Equation (9) implies that effective barrier can be explicitly expressed as

\[ U_{eff}(T, B, J) = U_c(B, T)F[J/J_c(B, T)], \quad (12) \]

with \( J_c(B, T) = 2J_D(B, T) \).

Incorporating it into the commonly observed scaling behavior of magnetic hysteresis \( M(H) \) in superconductors, it can be shown that \( U_c(B, T) \) and \( J_c(B, T) \) in Eq. (12) must take the following forms [17]

\[ U_c(B, T) = \Psi(T)B^n \propto [T^*(B) - T]^\delta B^n, \]
\[ J_c(B, T) = \lambda(T)B^m \propto [T^*(B) - T]^\alpha B^m, \quad (13) \]

with \( T^*(B) \) being the irreversibility temperature where tilt modules and stiffness of vortex matter vanish. Considering \( \rho_f \propto T \), one finds from Eq. (12) the following temperature dependent relations for \( \gamma \) and \( I - V \)

\[ \gamma(T) = \gamma_0(T^* - T)^\delta/kT, \]
\[ I \propto xJ_D(T) \propto x(T^* - T)^\alpha, \]
\[ V \propto yJ_D(T)\rho_f \propto y(T^* - T)^\alpha T. \quad (14) \]

The combination of equations (9) and (14) enables us to reproduce the experimentally measured \( I - V \) isotherms of Strachan et al. [11]. In Fig.1(b) we show the comparison, where the parameters used are \( \alpha = 3.0, \delta = 0.5, p = 1.75, \gamma_0 = 3.21 \times 10^{-21} \) and the irreversibility temperature \( T^*(B = 4T) \) is assumed 88K similar to the Fig.1 in Ref. [11].

On the basis of nearly 100 simulated \( I - V \) isotherms from 68K to 88K with temperature intervals of 0.2K, we obtain three different scaling data collapse analysis with assumed critical temperature \( T_g \) of 81K,75K and 70K respectively. In Fig.2 we show the comparison of our result with the data collapses of measured \( I - V \) curves of Strachan et al. in Ref. [11] and find fair agreement.

The analytical extended power law equations (9), (10) and (12) enable one to resolve the nontrivial problem of determining the positive scaling function \( \Gamma(v) \) between the average pinning force \( \langle F_p \rangle_t \) and vortex velocity \( v_L \) used in the VGFB model [4] as well as by Wang, Dong and Ting (WDT) [17] in the form \( \langle F_p \rangle_t = -\Gamma(v_L)\mathbf{v}_L \). Thus, the analytical equation for the nonlinear Hall re-
The conductance can be derived as

\[ \rho_{xy} = \frac{\beta_0 \rho_{xx}^2}{\Phi_0 B} \left[ \eta(1 - C) - 2\mathcal{C}(v_L) \right] \]

or

\[ \rho_{xy} = \mu m \rho_n B \left\{ (1 + C) \frac{\rho_{xx}^2}{\rho_f} - 2C \frac{\rho_{xx}}{\rho_f} \right\}, \quad (15) \]

with the longitudinal nonlinear resistivity defined in Eqs. (11) and (12), \( \rho_{xx} \approx \rho_f e^{-U_c/kT} \). The parameter \( C \) is defined as \( C \equiv C(1 - \Phi/H_{c2}) \) with \( \Phi \) the average magnetic field over the vortex core and \( C \) is describing the contact force on the surface of vortex core \( 17 \). \( \beta_0 = \mu m H_{c2} \) with \( \mu m = \tau e/m \) the mobility of the charge carrier and \( H_{c2} = \Phi_0/(2\pi \xi^2) \) the upper critical magnetic field.

This equation is based on the conventional theories of Bardeen and Stephen \( 14 \) as well as Nozières and Vinen \( 18 \) for pinning free motion of vortex which predict that the Hall effect stems from quasinormal core and hence has the same sign as in the normal state. However, the microscopic theories of the vortex dynamics in clean superconductors \( 19 \) predict that even in the absence of pinning, the vortex contribution to Hall current may have the sign different from the sign of Hall effect in the normal state. In latter case, increasing pinning can remove the Hall anomaly and even result in a second sign reversal of \( \rho_{xy} \) observed in some strongly anisotropic materials \( 6, 21, 22 \).

With consideration of this factor, Hall resistivity equation \( 15 \) should be modified to the form

\[ \rho_{xy}(T) = \mu_m \rho_n B \cdot \frac{A_s(T)}{A_n(T)} \left\{ \left(1 + C\right) \frac{\rho_{xx}^2}{\rho_f} - 2C \frac{\rho_{xx}}{\rho_f} \right\}, \quad (16) \]

where \( A_s(T) \) and \( A_n(T) \) are the Hall force coefficients of
the sample in pinning free superconducting and normal states respectively 19.

Besides the well-known puzzling scaling behavior \( \rho_{xy} \approx \rho_{xx} \), recently we find a new universal scaling relation of the form

\[
\log_{10} \left| \frac{\rho_{xy}}{\rho_m} \right| \approx \Phi \left( \frac{T - T_0}{T_m - T_0} \right), \tag{17}
\]

where \( \rho_m \) is the first extreme value of Hall resistivity from low temperature side and \( T_m \) is the temperature corresponding to \( \rho_m \). \( T_0 \) corresponds to the temperature where the Hall resistivity become first measurable at the low temperature side.

This scaling relationship is consistent with the general Hall resistivity equation 10. In the case of YBCO (Fig.3(a) 1, 20), the factor \( A_s(T)/A_n(T) > 0 \), we see the Hall anomaly at temperatures around \( T_m \), on the other hand, for the strongly anisotropic materials HBCO(Fig.3(c) 8) and TBCCO(Fig.3(d) 22), we see the double sign reversal of \( \rho_{xy}(T) \), perhaps owing to that \( A_s(T)/A_n(T) < 0 \), as mentioned by Kopnin and Vinokur 8.

In conclusion, we demonstrate that the longitudinal \( E(J) \) response have the general form of extended power law. This nonlinear behavior naturally explains the scaling of longitudinal and Hall resistivities near the vortex glass transition.

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[1] A brief review of the literature is given in S. J. Hagen, A. W. Smith, M. Rajeswari, J. L. Peng, Z. Y. Li, R. L. Greene, S. N. Mao, X. X. Xi, S. Bhattacharya, Qi Li, and C. J. Lobib, Phys. Rev. B 47, 1064 (1993).
[2] J. Luo, T. P. Orlando, J. M. Graybeal, X. D. Wu, and R. Muenchhausen, Phys. Rev. Lett. 68, 690 (1992).
[3] W. N. Kang, B. W. Kang, Q. Y. Chen, J. Z. Wu, S. H. Yun, A. Gapud, J. Z. Wu, K. Chu, D. K. Christen, R. Kerchner, and C. W. Chu, Phys. Rev. B 59, R9031 (1999) and the references cited therein.
[4] T. Nagaoka, Y. Matsuda, H. Obara, A. Sawa, T. Terashima, I. Chong, M. Takano, and M. Suzuki, Phys. Rev. Lett. 80, 3594 (1998).
[5] P. J. M. Wöltgens, C. Dekker, and H. W. de Wijn, Phys. Rev. Lett. 71, 3858 (1993).
[6] N.B. Kopnin and V.M. Vinokur, Phys. Rev. Lett. 83, 4864 (1999).
[7] M. P. A. Fisher, Phys. Rev. Lett. 62, 1415 (1989); see also D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B 43, 130 (1990).
[8] A. T. Dorsey and M. P. A. Fisher, Phys. Rev. Lett. 68, 694 (1992).
[9] V. M. Vinokur, V. B. Geshkenbein, M. V. Feigel’man, and G. Blatter, Phys. Rev. Lett. 71, 1242 (1993).
[10] R. H. Koch, V. Foglietti, W. J. Gallagher, G. Koren, A. Gupta, and M. P. A. Fisher, Phys. Rev. Lett. 63, 1511 (1989).
[11] D. R. Strachan, M. C. Sullivan, P. Fournier, S. P. Pai, T. Venkatesan, and C. J. Lobb, Phys. Rev. Lett. 87, 067007 (2001).
[12] Z. H. Ning et al., cond-mat/0407234.
[13] D. R. Nelson and V. M. Vinokur, Phys. Rev. B 48, 13060 (1993); V. M. Vinokur, Physica A 200, 384 (1993); U. C. Täuber and D. R. Nelson, Phys. Rev. B 52, 16106 (1995).
[14] J. Bardeen and M. J. Stephen, Phys. Rev. 140, A1197 (1965).
[15] For a review, see G. Blatter et al., Rev. Mod. Phys. 66, 1125 (1994); E. H. Brandt, Rep. Prog. Phys. 58, 1465 (1995); Y. Yeshurun, A. P. Malozemoff and A. Shaulov, Rev. Mod. Phys. 68, 911 (1996); L. F. Cohen and H. J. Jensen, Rep. Prog. Phys. 60, 1581 (1997); M. E. Fisher, Rev. Mod. Phys. 70, 653 (1998).
[16] For example, see M. Tinkham, Introduction to Superconductivity, para. 9.6, 2nd ed., NY: McGraw-Hill (1996).
[17] Z. D. Wang, Jinming Dong, and C. S. Ting, Phys. Rev. Lett. 72, 3875 (1994).
[18] P. Nozières and W.F. Vinen, Philos. Mag. 14, 667 (1966).
[19] N. B. Kopnin and A. V. Lopatin, Phys. Rev. B 51, 15291 (1995); M. V. Feigel’man et al., JETP Lett. 62, 834 (1995); A. van Otterlo et al., Phys. Rev. Lett. 75, 3736 (1995); G. Blatter et al., Phys. Rev. Lett. 77, 566 (1996); D. I. Khomskii and A. Freimuth, Phys. Rev. Lett. 75, 1384 (1995).
[20] H. Y. Xu et al., Phys. Rev. B 66, 054513 (2002).
[21] H. C. Ri, R. Gross, F. Gollnik, A. Beck, and R. P.
Huebener, Phys. Rev. B 50, 3312 (1994).

[22] S. J. Hagen, C. J. Lobb, R. L. Greene, and M. Eddy, Phys. Rev. B 43, 6246 (1991).