Dark Energy Accretion onto a Black Holes in an Expanding Universe

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Abstract

By using the solution describing a black hole embedded in the FLRW universe, we obtain the evolving equation of the black hole mass expressed in terms of the cosmological parameters. The evolving equation indicates that in the phantom dark energy universe the black hole mass becomes zero before the Big Rip is reached.

1 Introduction

In recent years, strong observational evidence shows that the current expansion of our universe is accelerating [1]. In the framework of the general relativity, this positive acceleration means that, at present, our universe is dominated by a mysterious component, the dark energy [2]. Usually, we may model the dark energy by a perfect fluid with a negative pressure \( p = w\rho < 0 \). The dark energy may be in the form of a vacuum energy with \( w = 1 \), or a quintessence model with \( w > -1 \). The observation gives the constraint [3]:

\[
w = -1.07^{+0.09}_{-0.08}(\text{stat} \ 1\sigma)\pm0.13(\text{sys}).
\]

Then there exists the possibility of the phantom models with \( w < -1 \). In this case, the universe undergoes the “supper accelerated” expansion. Finally, in the finite time, the Big Rip singularity is reached, and all bounded objects in the universe are torn apart [4].

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What would happen to black holes in an expanding universe filled with the phantom dark energy? In [5, 6], using the Schwarzschild metric, the authors have obtained the result

$$\dot{M} \equiv \frac{dM}{dt} = 4\pi G^2 A M^2 [p_\infty + \rho_\infty],$$

(1)

where $p_\infty$ and $\rho_\infty$ are respectively the pressure and energy density of the universe at the asymptotic limit $r \rightarrow +\infty$. And $A$ is a dimensionless constant. Then the author concluded that the masses of all black holes will tend to vanish as the universe approached the Big Rip. In [7], the authors obtained the same result (1), by using the non-static Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2GM(t)}{r}\right)dt^2 + \left(1 - \frac{2GM(t)}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

(2)

However, the metric is asymptotically flat and does not describe exactly the space-time of a black hole embedded in the FLRW universe. Actually, Eq.(1) is obtained by ignoring the backreaction of the phantom matter on the black hole. When the matter density of the background is low, the ignorance is reasonable. However, as the Big Rip is approached, the matter density becomes very large. The backreaction should not be neglected, and the metric (2) becomes unsuitable for describing the spacetime. Then, we should adopt some exact metric, in order to take into account of the backreaction.

Recently, new exact solutions describing black holes embedded in the FLRW universe have been suggested in [8, 9]. Then it is interesting for us to study the problem of the dark energy accretion on black holes by using the exact solutions. In [8], the accretion rate (Hawking-Hayward quasi-local mass added per unit time) is obtained. But the result is not suitable for the application in the cosmology. In order to study the evolution of the black hole mass as the universe expands, we should express the accretion rate in terms of the parameters at the asymptotic limit $r \rightarrow +\infty$, namely the cosmological parameters, $p_\infty$, $\rho_\infty$, et al..

In this paper we will try to obtain such an expression by using the solution with imperfect fluid and radial mass flow in [8]. Our result shows that the evolution of the black hole mass is governed by a second-order differential equation, which is different from Eq.(1).

The paper is organized as follows. In Section 2, we recall the results in [8]. In Section 3, we show the evolving equation of the black hole mass expressed in terms of the cosmological parameters. Section 4 contains a discussion and conclusions.
2 the Solution with Imperfect Fluid and Radial Mass Flow

In [8], the line element is written in the form

$$ds^2 = -\frac{B^2(t, r)}{A^2(t, r)} dt^2 + a^2(t) A^4(t, r) (dr^2 + r^2 d\Omega^2),$$

with $A(t, r) = 1 + \frac{Gm(t)}{2r}$ and $B(t, r) = 1 - \frac{Gm(t)}{2r}$. Here and after, we take $c = \hbar = 1$. Then, at the asymptotic limit $r \to +\infty$, the metric becomes the FLRW metric. In this space-time, the physically relevant mass is the Hawking-Hayward quasi-local mass [8, 10, 11]

$$m_H(t) = a(t) m.$$

In this paper, we study the solution with radial mass flow, $u(r, t)$. The cosmological matter is assumed to be described by the imperfect fluid energy-momentum tensor

$$T^{ab} = (p + \rho) u^a u^b + pg^{ab} + q^a u^b + q^b u^a,$$

with

$$u^a = \left( \frac{A}{B} \sqrt{1 + a^2 A^4 u^2}, u, 0, 0 \right), \quad q^a = (0, q, 0, 0).$$

Here, the purely spatial vector field $q^a$ describes a radial energy flow. The condition

$$\lim_{r \to +\infty} u(t, r) = \lim_{r \to +\infty} q(t, r) = 0,$$

should be imposed, in order for the momentum-energy tensor [5] to become the perfect fluid momentum-energy tensor in cosmology at the asymptotic infinity $r \to +\infty$,

$$T_{\infty}^{ab} = (p_{\infty} + \rho_{\infty}) u^a u^b + p_{\infty} g^{ab}. $$

The Einstein equations give four independent equations [8],

$$\dot{m}_H = -\frac{1}{2} a B^2 A \sqrt{1 + a^2 A^4 u^2} (p + \rho) u,$$

$$8\pi G p = - \left( \frac{A}{B} \right)^2 \left[ 3C^2 + 2(\dot{C} + \frac{GmC}{r AB}) \right],$$

$$8\pi G \rho = \left( \frac{A}{B} \right)^2 \left[ 3C^2 + (\dot{C} + \frac{GmC}{r AB}) \frac{2a^2 A^4 u^2}{1 + a^2 A^4 u^2} \right],$$

$$q = -(p + \rho) \frac{u}{2},$$
with $A \equiv \int \int d\theta d\phi \sqrt{g_{\Sigma}} = 4\pi r^2 a^2 A^4$ representing the area of a spherical surface with isotropic radius $r$, and $C \equiv \frac{\dot{a}}{a} + \frac{2\dot{A}}{A}$. Here and after, the dot denotes the derivative with respect to the time.

This is the solution describing a black hole embedded in the FLR W universe [8]. As $r \to \infty$, Eq.(9) becomes the Friedmann equation

$$8\pi G \rho = 3\frac{\dot{a}^2}{a^2}.$$ 

Then Eq.(9) may be consider as a generalized Friedmann equation. Eq.(8) and (11) indicate a generalized conservation law

$$\dot{\rho} (1 + a^2 A^4 u^2) + \dot{p} (a^2 A^4 u^2) + 3\left(\frac{\dot{a}}{a} + 2\frac{\dot{A}}{A}\right)(p + \rho)(1 + a^2 A^4 u^2)$$

$$+ (p + \rho)2a^2 A^4 u^2\left(\frac{\dot{a}}{a} + 2\frac{\dot{A}}{A} + \frac{\dot{u}}{u}\right) = 0,$$

which, as $r \to +\infty$, becomes the conservation law in the FLR W universe

$$\dot{\rho} + 3\frac{\dot{a}}{a}(p + \rho) = 0.$$

Additionally, the radial component of the energy-momentum conservation law, $T^\mu_{\nu 1} = 0$, gives another conservation law

$$0 = \frac{a^2 A^5}{2B\sqrt{1 + a^2 A^4 u^2}} \{5\left(\frac{\dot{a}}{a} + 2\frac{\dot{A}}{A}\right)(1 + a^2 A^4 u^2)(p + \rho)u + (p + \rho)a^2 A^4 u^3\left(\frac{\dot{a}}{a} + 2\frac{\dot{A}}{A} + \frac{\dot{u}}{u}\right)$$

$$+ (1 + a^2 A^4 u^2)\left[(\dot{\rho} + \dot{p})u + (p + \rho)\dot{u}\right] + p' + \left(\frac{B'}{B} - \frac{A'}{A}\right)(p + \rho)(1 + a^2 A^4 u^2)\}

$$+ (1 + a^2 A^4 u^2)\left[(\dot{\rho} + \dot{p})u + (p + \rho)\dot{u}\right] + p' + \left(\frac{B'}{B} - \frac{A'}{A}\right)(p + \rho)(1 + a^2 A^4 u^2)\}

(12)

Here, we note that the time component of the energy-momentum conservation law, $T^\nu_{\nu 0} = 0$, does not give an independent equation, which can be derived by using the equations (7) and (11).

3 the Evolving Equation in terms of Cosmological Parameters

Eq.(7) gives the accretion rate. Since $u < 0$, the mass $m_H$ increases if $p + \rho > 0$, stays constant in a de-Sitter background, and decreases for the phantom dark energy $p + \rho < 0$. However, this equation is not convenient for us to study the evolution of the black hole mass as the universe expands. In
cosmology, what we need is an evolving equation of \( m_H \) expressed in terms of the parameters at the asymptotic limit \( r \rightarrow +\infty \), namely the cosmological parameters.

To obtain such an evolving equation, we may take the limit of Eq.(7) as \( r \rightarrow +\infty \). Then we have

\[
\dot{m}_H = -2\pi a^3 (p_\infty + \rho_\infty) \lim_{r \rightarrow +\infty} (ur^2),
\]

where \( p_\infty(t) \) and \( \rho_\infty(t) \) are respectively the limit of \( p(t, r) \) and \( \rho(t, r) \) as \( r \rightarrow +\infty \). This result indicates that, as \( r \rightarrow +\infty \),

\[
u(t, r) \simeq \frac{u_\infty(t)}{r^2} + O(r^{-3}).
\]

Similarly, as \( r \rightarrow \infty \), we may expect

\[
\begin{align*}
p(t, r) &\simeq p_\infty(t) + \frac{p_1(t)}{r} + O(r^{-2}), \\
\rho(t, r) &\simeq \rho_\infty(t) + \frac{\rho_1(t)}{r} + O(r^{-2}).
\end{align*}
\]

Then, up to the order \( r^{-2} \), Eq.(12) gives

\[
\frac{d}{dt} \left[ a^5 (p_\infty + \rho_\infty) u_\infty \right] = 2a^3 [p_1 - Gm(p_\infty + \rho_\infty)]
\]

At the same time, up to the order \( r^{-1} \), Eq.(11) indicates two equations

\[
\begin{align*}
\dot{\rho}_\infty + 3 \frac{\dot{a}}{a} (p_\infty + \rho_\infty) &= 0, \\
\dot{\rho}_1 + 3 \frac{\dot{a}}{a} (p_1 + \rho_1) + 3G \dot{m}(p_\infty + \rho_\infty) &= 0.
\end{align*}
\]

Here, naturally, we can impose the equation of state

\[
p_\infty(t) = w \rho_\infty(t),
\]

where \( w \) is a dimensionless constant. Further, we assume the same equation of state for \( p_1(t) \) and \( \rho_1(t) \)

\[
p_1(t) = w \rho_1(t).
\]

Together the two equations and Eq.(18), we can solve Eq.(19), and obtain

\[
\rho_1(t) = 3G(m_0 - m)(p_\infty + \rho_\infty),
\]
where \( m_0 \) is an integral constant with the dimension of mass. Now, Eq.(17) can be rewritten as

\[
\frac{d}{dt}[a^5(p_\infty + \rho_\infty)u_\infty] = 2Ga^3[3wm_0 - (3w + 1)m](p_\infty + \rho_\infty).
\] (23)

Using this equation and Eq.(13), finally we can obtain the evolving equation of \( m_H \)

\[
\ddot{m}_H + 2\frac{\dot{a}}{a}\dot{m}_H - 4\pi G[(3w + 1)m_H - 3wm_0a](p_\infty + \rho_\infty) = 0,
\] (24)

where \( \dddot{m}_H \equiv \frac{d^2m_H}{dt^2} \). This is a second-order differential equation as we expect from the second-order differential equations (7)-(9).

4 Discussion and Conclusion

In the last section, we obtain the evolving equation of the mass of a black hole embedded in the FLRW universe. Both Eq.(1) and our result, Eq.(24), imply that the black hole mass decrease in the phantom dark energy universe, but the qualitatively evolving behaviors of the black hole mass indicated by the two equation are different. Due to Eq.(1), it has been shown in [5, 12] that the mass of a black hole tends to zero in the phantom dark energy universe approaching the Big Rip. However, by using our result Eq.(24), we find that the black hole mass in the dark energy universe has been zero before the Big Rip singularity is reached.

In order to show this, firstly let us rewrite Eq.(24) in terms of the derivative with respect to the scale factor \( a(t) \),

\[
\frac{d^2m_H}{da^2} + \frac{3(1-w)}{2}a^{-1}\frac{dm_H}{da} = \frac{3}{2}(3w + 1)(1 + w)a^{-2}m_H - \frac{9}{2}w(1 + w)m_0a^{-1}.
\] (25)

Here, we have used the equations

\[
\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_\infty, \quad \rho_\infty \propto a^{-3(1+w)}.
\]

The general solution of Eq.(25) is

\[
m_H(t) = C_1[a(t)]^{1+3w} - C_2[a(t)]^{-3(1+w)/2} + \frac{3(1+w)}{3w+5}m_0a(t),
\] (26)

where \( C_1 \) and \( C_2 \) are the integral constants. Use \( t_0 \) to denote the initial moment. The two constants are determined by the initial mass, \( m_H(t_0) \), and the initial accretion rate \( \dot{m}_H(t_0) \).
In a phantom dark energy scenario, \( w \) is close to \(-1\) and \( w < -1 \). Then we may have

\[
1 + 3w < 0, \quad 1 > -3(1 + w)/2 > 0, \quad \frac{3(1 + w)}{3w + 5} < 0.
\]

Using \( m_H(t_0) > 0 \) and \( \dot{m}_H(t_0) < 0 \), generally we may have

\[
C_1 > 0, \quad C_2 > 0.
\]

Then Eq.(26) indicates that, before the Big Rip singularity is reached, the black mass has been zero at the moment \( t_e \) which is determined approximately by the equation

\[
C_1[a(t_e)]^{1+3w} = C_2[a(t_e)]^{-3(1+w)/2} - \frac{3(1 + w)}{3w + 5} m_0 a(t_e).
\]

Physically, this result seems to be better, since the Big Rip singularity becomes unnecessary for the disappearance of the black holes. Of course, the analysis above may be not rigid when the black hole mass is at the order of the Planck mass, since the effect of the Hawking radiation might dominate the evolution of the black hole with the Planck mass.

In [7], the author have shown that Eq.(1) can be generalized to the non-static metric (2). Then we might expect that Eq.(1) should be reduced by taking \( a(t) \) to be constant, \( \dot{a} = 0 \), in Eq.(24). However, it is not the case. The reason is that Eq.(1) in [7] is obtained under the assumption

\[
\lim_{r \to \infty} (ur^2) = AG^2M^2.
\]

From Eq.(17), we know the assumption for the solution used in this paper is unreasonable. Then it becomes natural that Eq.(1) can not be reduced from Eq.(24) with \( \dot{a} = 0 \).

Even, we should note that the previously known solution for the perfect fluid, Eq.(4), cannot be reduced from our result Eq.(24) by taking \( q^a = 0 \) in Eq.(5). In fact, \( q^a \neq 0 \) is necessary. It has been shown in [8] that, for the perfect fluid described by taking \( q^a = 0 \) in Eq.(5), the Einstein equations indicate

\[
p + \rho = 0.
\]

This can be also learned from Eq.(10). This implies that only the de-Sitter equation of state \( p = -\rho \) is allowed. Eq.(7) accordingly yields \( \dot{m}_H = 0 \). So no accretion happens for a single perfect fluid. Then the imperfect fluid (5) with \( q^a \neq 0 \) is necessary for the accretion to happen when the metric (3) is
used. This indicates that the previously known accretion rate (1) is not the limit of our result (24) as $q^a \to 0$.

However, we do not think our result (24) is better than the result (1). The reason is that the imperfect fluid solution which our result (24) is based on has the unphysical behavior \cite{8}—the superluminal motion of the fluid as $r \to m/2$. For this reason our result cannot be used as a "better approximation" as compared with the result (1). We may take Eq.(24) as an interesting try.

Summarily, in the paper, by using the exact solution describing a black hole embedded in the FLRW universe \cite{8} and the equation of state (20), we obtain the evolving equation of the black hole mass expressed in terms of the cosmological parameters, which is convenient for the application in cosmology. Our result, a second-order differential equation (24), is different from the result (1) in \cite{5}. Our result indicates that the black mass in the phantom dark energy universe has been zero at the moment $t_e$ before the Big Rip moment $t_B$, while, due to Eq.(1), the mass of a black hole tends to zero in the phantom dark energy universe approaching the Big Rip. Here, we emphasize again that our result (24) is not better than the result (1) because of the unphysical behavior of the imperfect fluid solution which our result (24) is based on.

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