Cluster structures of excited states in $^{14}$C

Tadahiro Suhara$^1$ and Yoshiko Kanada-En’yo$^2$

$^1$Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
$^2$Department of Physics, Kyoto University, Kyoto 606-8502, Japan

E-mail: suhara@yukawa.kyoto-u.ac.jp

Abstract. We investigate structures of excited states in $^{14}$C with a method of \( \beta \)-\( \gamma \) constraint antisymmetrized molecular dynamics in combination with the generator coordinate method. Various excited states with the developed 3\( \alpha \)-cluster core structures are suggested in positive- and negative-parity states. In the positive-parity states, triaxial deformed and linear-chain structures are found to construct excited bands. Interestingly, $^{10}$Be+$\alpha$ correlation is found in the cluster states above the $^{10}$Be+$\alpha$ threshold energy. By applying a generalized molecular orbital model to the linear-chain state of $^{14}$C, we confirm the $^{10}$Be+$\alpha$ correlation.

1. Introduction

Owing to the experimental progress of unstable nuclei, various exotic structures have been discovered in neutron-rich nuclei where excess neutrons play important roles. In light neutron-rich nuclei, cluster structure with excess neutrons is one of the hot subjects in experimental and theoretical studies, where searching for new cluster states in excited states has been performed.

In this point of view, neutron-rich C isotopes such as $^{14}$C are an interesting subject because they have excess neutrons compared with $^{12}$C, which is already well known to show various structures owing to the coexistence of shell-model and cluster features. The ground state of $^{12}$C has mainly a shell-model feature of the $p_{3/2}$-subshell closed configuration, whereas, in the excited states above the 3\( \alpha \) threshold energy, various configurations of 3\( \alpha \)-cluster structures were suggested in many theoretical works [1–11]. For example, the $\alpha$ condensation of weakly interacting three $\alpha$ clusters suggested in the $0^+_2$ and $2^+_2$ states [6–8], an equilateral-triangular structure of three $\alpha$ clusters in the $3^+_1$ state, and a linear-chainlike structure with an obtuse-angle-triangular 3\( \alpha \) configuration in the $0^+_3$ state [9,10] were suggested. Therefore, it is expected that $^{14}$C with two excess neutrons may also exhibit rich phenomena in the excited states. In particular, from an analogy of $^{12}$C, various cluster structures may appear in $^{14}$C as well as shell-model structures.

For the excited states of $^{14}$C, there are many experimental studies that indicate the appearance of cluster states such as $^{10}$Be+$\alpha$ cluster states [12–16]. Also on the theoretical side, cluster structures of $^{14}$C were suggested [17]. In the study of $^{14}$C with 3\( \alpha \)+2\( n \) cluster models, Itagaki et al. predicted that an equilateral-triangular structure of the well-developed three $\alpha$ clusters surrounded by excess neutrons is formed constructing $K^\pi = 0^+$ and $K^\pi = 3^-$ rotational bands in the excited states. Another interesting problem is whether a linear-chain 3\( \alpha \) structure with two excess neutrons exists in the excited states of $^{14}$C. Although there were discussions on the possibility of the linear-chain structure in neutron-rich C isotopes [12,18,19], there is no clear conclusion for stability of the linear-chain structure in $^{14}$C. Thus, the excited
states of $^{14}$C are attracting much interest recently, and therefore, systematic study of the ground and excited states is required.

Our aim is to investigate the ground and excited states of $^{14}$C while focusing on cluster features. In the present study, we apply a combination of the $\beta$-$\gamma$ constraint AMD and the generator coordinate method (GCM), which we call the $\beta$-$\gamma$ constraint AMD + GCM. This method has already been proved to be a powerful approach to describe various structures such as cluster and shell-model-like structures [20]. In particular, it is useful for the systematic study of cluster states in excited states owing to the superposition of basis AMD wave functions on the two-dimensional $\beta$-$\gamma$ plane. Therefore, it is suitable for the study of $^{14}$C.

2. Framework

In this section, we explain the $\beta$-$\gamma$ constraint AMD + GCM, briefly. For more details, the reader is referred to Ref. [20].

In the method of AMD, a basis wave function of an $A$-nucleon system $|\Phi\rangle$ is described by a Slater determinant of single-particle wave functions $|\varphi_i\rangle$ as $|\Phi\rangle = \frac{1}{\sqrt{A!}} \det \{ |\varphi_1\rangle, \cdots, |\varphi_A\rangle \}$. The $i$-th single-particle wave function $|\varphi_i\rangle$ consists of the spatial part $|\phi_i\rangle$, spin part $|\chi_i\rangle$, and isospin part $|\tau_i\rangle$ as

$$
|\varphi_i\rangle = |\phi_i\rangle|\chi_i\rangle|\tau_i\rangle, \tag{1}
$$

$$
\langle r|\phi_i\rangle = \left( \frac{2\nu}{\pi} \right)^{\frac{3}{2}} \exp \left[ -\nu \left( r - \frac{Z_i}{\sqrt{\nu}} \right)^2 + \frac{1}{2} Z_i^2 \right], \tag{2}
$$

$$
|\chi_i\rangle = \xi_{i\uparrow} |\uparrow\rangle + \xi_{i\downarrow} |\downarrow\rangle, \tag{3}
$$

where $\nu$ is the width parameter and is taken to be a common value for all the single-particle Gaussian wave functions in the present work. The spin orientation is given by the parameter $\xi_i$, while the isospin part $|\tau_i\rangle$ is fixed to be up (proton) or down (neutron). In a basis wave function $|\Phi\rangle$, $\{X\} = \{ Z, \xi \} = \{ Z_1, \xi_1, Z_2, \xi_2, \cdots, Z_A, \xi_A \}$ are complex variational parameters and they are determined by the energy optimization.

To describe various cluster and shell-model structures which may appear in the ground and excited states of $^{14}$C, we perform the energy variation with the constraints on the quadrupole deformation parameters, $\beta$ and $\gamma$.

In the calculations of energy levels, we superpose the parity and total-angular-momentum projected AMD wave functions $\hat{P}_MK^\pm|\Phi^\pm(\beta, \gamma)\rangle$. Thus, the final wave function for the $J^\pm$ state is given by a linear combination of the basis wave functions as $|\Phi^\pm\rangle = \sum_K \sum_n f_n(\beta, \gamma, K)\hat{P}_MK^\pm|\Phi^\pm(\beta, \gamma)\rangle$. The coefficients $f_n(\beta, \gamma, K)$ are determined using the Hill-Wheeler equation. In the limit of sufficient basis wave functions on the $\beta$-$\gamma$ plane, it corresponds to the GCM with the two-dimensional generator coordinates of the quadrupole deformation parameters, $\beta$ and $\gamma$.

3. Results

In this section, we explain the calculated results for the positive-parity states. For more details and results for the negative-parity states are described in Ref. [21].

We show the calculated energy levels for the positive-parity states of $^{14}$C in Fig. 1 as well as the experimental levels [13–16,22]. We also display the density distributions of the intrinsic wave functions of bands. Four columns on the left are the experimental data and six columns on the right are the theoretical results. States in the three columns from the left are the known positive-parity states. States which were discovered in the $^{10}$Be+$\alpha$ break-up reactions are displayed in the fourth column from the left. The lower three states are those with a strong population of $^{10}$Be($0^+_1$) and upper three states are $^{10}$Be($2^+_1$). We classified the theoretical states into five
groups by analyzing the intrinsic structures and $E2$ transition strengths. It is found that the ground band (1) has a shell-model-like structure, while three $\alpha$ clusters develop well in most of the excited states. The second and third column is the $K^\pi = 0^+$ and $K^\pi = 2^+$ side band (2), which has a triaxial structure. In these states, three $\alpha$ clusters have the isosceles-triangle configuration and excess neutrons occupy the $sd$-like orbitals. The fourth column is the $K^\pi = 0^+$ band (3), in which developed three $\alpha$ clusters have a linear-chainlike structure. It is interesting that the excess neutrons distribute around two of the three $\alpha$ clusters. This indicates the $^{10}\text{Be}$ correlation in the linear-chain states. For the excited states of $^{14}\text{C}$, there are many experimental studies that indicate the appearance of $^{10}\text{Be}+\alpha$ cluster states. Our results suggest that excited states with $^{10}\text{Be}+\alpha$ correlation are the candidates for these states.

Next, to confirm the $^{10}\text{Be}+\alpha$ in the linear-chain structure, we investigate the linear-chain structure in $^{14}\text{C}$ with a generalized molecular orbital model which can describe asymmetric
configurations of the linear-chain structure. In the previous calculation for $^{14}$C with AMD, there is a rotational band of linear-chain structure with $^{10}$Be+$\alpha$ correlation. However, there is a question whether $^{10}$Be+$\alpha$ correlation really exist. That is, in the $\beta$-$\gamma$ constraint AMD+GCM, detailed behavior of valence neutrons in the molecular orbitals which distribute whole system is not sufficiently described because a single particle wave function is approximated by a Gaussian wave packet. The calculated energy surface of $^{14}$C is shown as functions of the difference of the $\alpha$-cluster intervals $d_2 - d_1$ in Fig. 2. The intrinsic energy curve has the energy minimum at the symmetric configuration, $d_2 - d_1 = 0$ fm, while the positive-parity and the $0^+$ energy curves have energy minima at asymmetric configurations, $d_2 - d_1 > 0$ fm. At the energy minimum, $d_2 - d_1 = 1.7$ fm, the ratio of excess neutron distribution around left, middle, and right $\alpha$ clusters is $l : m : r = 1.0 : 4.3 : 0.6$. Valence neutrons gather around the middle $\alpha$ cluster. Since the middle $\alpha$ cluster is shifted to one side, the concentration of the amplitude at the middle $\alpha$ cluster means that valence neutron distribution leans toward the narrow side of two of three $\alpha$ clusters. That is, the asymmetric configuration of linear three $\alpha$ clusters and valence neutrons following the middle $\alpha$ cluster make $^{10}$Be+$\alpha$ correlation. More details are explained in Ref. [23].

4. Summary
We investigated $^{14}$C with $\beta$-$\gamma$ constraint AMD+GCM. Characteristic structures such as a triaxial structure and a linear-chain structure with the $^{10}$Be+$\alpha$ correlation appear in the excited bands. We confirmed the $^{10}$Be+$\alpha$ correlation in the linear-chain states of $^{14}$C by applying a generalized molecular orbital model to the system. As a future problem, we are estimating decay widths of $^{14}$C into $^{10}$Be+$\alpha$. We will report the detailed results in a future paper.

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