Damage or change detection in a small scale model of steel bridge deck under static loading by extensometry

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Abstract. In order to monitor the health state of a bridge structure and to ensure that it still fulfills its functions, an identification method for possible changes or damages has been developed by their locations and the assessment of the damage or change rate. The method is based on a comparison between two structural responses (displacements or stresses) under static loadings. The first is obtained by numerical modeling of the healthy structure, where the second by experimental measurements on the real structure under its operational conditions using sensors placed on structural elements. The two responses are analyzed by an algorithm (IAFEM) based on the inverse analysis method by computing the structure’s eventual change or damage factors. The method was initially tested experimentally on a small scale model of a steel bridge deck, built in laboratory, and by analyzing the displacement and stress responses of the structure.

Keywords: bridge, damage identification, inverse analysis, static, sensors, extonsometry

1. Introduction
The health of civil infrastructures (bridges, buildings, etc.) is directly related to the citizen’s life security on the one hand and the economic development of the countries on the other hand, especially the old structures that are still operational. Considering the importance of the impact of a possible degradation, it is therefore essential to monitor them to ensure that they still fulfill their functions or require repair.

Structural health monitoring is the design of a process that will enable to evaluate the integrity of a structure and will define any changes (damage) that may affect it and thereby disrupt its normal functioning [1]. The performance of such a system is determined by the level of accessibility to the information collected on these changes (detection of the damage, its location, its quantification and the estimation of the damaged structure’s residual life [2].

According to Worden K and Dulieu-Barton JM [3] there are two complementary approaches for damage identification:

The first approach considers it to be a classification problem, in which pattern recognition algorithms are used to group in damage classes the different characteristics extracted from the measurement data of the system. The second approach regards damage identification as an inverse problem and generally adopts a reference model (that of healthy structure), and attempts to relate the changes obtained in the measurement data to those obtained by simulation to damage, so the gap is produced due to damage. The algorithms used for this approach are mainly based on linear algebra or
The main existing methods based on the exploitation of the dynamic characteristics according to the two approaches as well as their applications can be consulted in the work of Fritzen CP and Kraemer P[4]; Doebling SW and all [5] and Friswell MI [6].

We are interested in this work to the method mentioned in Boumechra N [7] which is based on an inverse analysis of the static displacement responses of one or more points of the structure subjected to a moving load in order to define the real stiffness and therefore a possible change of stiffness, compared to a reference numerical model developed in finite elements, of the healthy structure. The real stiffness determined could then be greater or less than an initial value assumed by reference to the numerical model. This will allow us first of all to check the behavior of the real structure as well as its composition with respect to this design model and in a second place to detect if there are damages in the case where this stiffness would be smaller.

The aim of our work is to evaluate by an inverse analysis the real stiffness of a steel structure composed of frame elements, realized in the laboratory, with respect to displacement and stress measurements using displacement sensors and strain gauges respectively.

2. Methodology

2.1. Method principle

The principle is the initial definition of a structure’s geometrical dimensions, physical properties, and supports conditions. Subjected to a movable static load, this structure is deformed so we can record displacements or stresses at several points (static response). From these results, a research to identify the real stiffness of each structural element will be done. The real stiffness thus determined may be greater or smaller than the initial or assumed value of this one. If this stiffness is smaller, then it is said that there is damage. The mathematical method developed is an inverse analysis of those displacement and / or stress responses of one or several points of the structure. This inverse analysis developed on structure’s finite element numerical model allows the evaluation of each finite element’s real stiffness Boumechra N [7].

The assumptions of the developed method are as follows: the geometrical and mechanical characteristics of the undamaged structure are assumed to be known, the structure is elastic and linear, the intensity of the moving load is known and the dynamic effect of the moving load is neglected.

The studied structure is discretized into \( n \) finite elements. For each stiffness \( [K_i] \) of the finite element \( i \) corresponds a correction coefficient \( \alpha_i \). If this coefficient is positive, it is said that there is an additional stiffness. If this coefficient is negative, it is said that there is damage on the finite element and is thus called reduction coefficient. From this, we develop the system’s static equilibrium equation which is defined in linear analysis as:

\[
[K]\{\delta(x_0)\} = \{F(x_0)\}
\]  

(1)

The structural stiffness matrix \([K]\) is the assembly of the stiffness matrices of each finite element. The load vector \( \{F(x_0)\} \) varies according to the position of the moving load \( x_0 \). So from the global displacement vector \( \{\delta(x_0)\} \), we can determine the movement of the section \( w_S(x_0) \) by the matrix \([N]\) defining the displacement of the studied section \( S \).

\[
w_S(x_0) = [N]\{\delta(x_0)\} = [N][K]^{-1}\{F(x_0)\}
\]

(2)

After the assembly of the structure’s global stiffness matrix, we have:

\[
[K] = [K_0] + \sum_{i,j} \alpha_i [K_i]
\]

(3)

The global stiffness matrix \([K_0]\) and the elementary stiffness matrices \([K_i]\) of the assumed healthy structure are defined from the initial information about the structure’s geometrical dimensions and material’s mechanical characteristics. Therefore, the matrix \([K]\) is function of variables \( \alpha_i \) that are
unknown and to be determined. Because of the importance of the system, Neumann series are used to
inverse the global stiffness matrix \([K]\):

\[
[K]^{-1} = \left( [K_0] + \sum_{i=1}^{n} \alpha_i [K_i] \right)^{-1} = \left( \sum_{q=0}^{T} \left( [K_0]^{-1} \left( \sum_{i=1}^{n} \alpha_i [K_i] \right) \right)^q \right) [K_0]^{-1}
\]

(4)

Where \( T \) is the maximum order of the Neumann series. It must be chosen so great enough to ensure
convergence of the series. So the inverse matrix of \([K]\) is a multivariable polynomial matrix \((\alpha_i, \ i = 1, n)\)
of a degree \( T \). It is a matrix whose elements are multivariate polynomials of degree \( T \) in \( n \) variables \( \alpha_i \).
The Neumann series converges to the following condition:

\[
\left( [K_0]^{-1} \left( \sum_{i=1}^{n} \alpha_i [K_i] \right) \right)^q [K_0]^{-1} < 1
\]

(5)

This indicates that the difference of stiffness, due to a reduction or an excess, must be small. After
substitution of Equations (2) and (4), the displacement \( w_s(x_0) \) is rewritten as follows:

\[
w_s(x_0) = [N] \left( \sum_{q=0}^{T} \left( [K_0]^{-1} \left( \sum_{i=1}^{n} \alpha_i [K_i] \right) \right)^q \right) [K_0]^{-1} \{F(x_0)\}
\]

(6)

Equation (6) defines an algebraic relationship with \( n \) unknowns \((\alpha_i)\). It is a multivariate polynomial
equation of \((\alpha_i, \ i = 1, n)\) variables and of \( T \) degree. A set of \( m \) loading vectors \( \{F_i\} = \{F(x_0)\} \) can be
developed corresponding to \( m \) positions of the moving load \( P \). For these \( m \) loading vectors correspond
\( m \) displacements of the selected point \((w_{s,i} = w_s(x_0))\). The data of the static displacement response thus
allows the development of a system of \( m \) nonlinear equations (polynomial) and \( n \) unknowns, which is as follows:

\[
f_d(\alpha_i, \ i = 1, m) = w_{s,j} - [N] \left( \sum_{q=0}^{T} \left( [K_0]^{-1} \left( \sum_{i=1}^{n} \alpha_i [K_i] \right) \right)^q \right) [K_0]^{-1} \{F_i\} = 0
\]

(7)

Normal Stress in a given section point or fiber defined by its local coordinates \( y_L \) and \( z_L \), is
computed from the vector of the local element forces defined by its two ends nodes \( i \) and \( j \).

\[
\sigma_f(x_0) = \frac{N}{A} + \frac{M_y}{I_y} \cdot z_L + \frac{M_z}{I_z} \cdot y_L
\]

(8)

The local solicitations vector is written as follows:

\[
\{soll\} = \left( N_i, V_{y,i}, V_{z,i}, M_{xi}, M_{yi}, M_{zi}, N_j, V_{y,j}, V_{z,j}, M_{xi}, M_{yi}, M_{zi} \right)
\]

(9)

\[
\{soll\} = [K_{L,i}] [\delta_{L,i}] = [K_{L,i}] [T_{ri}] [N_{L,i}] [\delta_G] = [K_{L,j}] [T_{ri}] [N_{L,i}] [K]^{-1} \{F_i\}
\]

(10)

\[
\{soll\} = [K_{L,i}] [T_{ri}] [N_{L,i}] \left( \sum_{q=0}^{T} \left( [K_0]^{-1} \left( \sum_{i=1}^{n} \alpha_i [K_i] \right) \right)^q \right) \{K_0\}^{-1} \{F_i\}
\]

(11)

\([K_{L,i}], [T_{ri}] \) et \([N_{L,i}]\) are respectively, local stiffness matrix, transformation matrix from the global
coordinate system to the local coordinate system of the corresponding finite element and the nodes’
degrees of freedom extraction matrix of the corresponding finite element.
Thus, in the same way as equation (7), we can develop m nonlinear relations of corresponding stresses of the section (\( \sigma_{S,i} = \sigma_{S}(x_0) \)) and n unknowns, which is the following:

\[
f_\sigma (\alpha_i)_{i=1,n} = \sigma_{S,i} - \sigma_{S}(x_0) = 0
\]  

(12)

The correction coefficients resolve requires the solving of nonlinear equations system (7) or (12). It consists of an optimization problem. The Levenberg-Marquardt algorithm is the most efficient technique for solving this type of non-linear functional.

From this mathematical development, a Matlab code has been written for the inverse analysis of those displacement and/or stress responses on a finite element model: frame element with 6 degrees of freedom per node, and thus the computing of the correction coefficients (\( \alpha_i \)). These coefficients allow the detection of the eventual damage rate or excess of stiffness in the structure’s elements.

2.2. Experimental process and structure description

The structure to be studied is a small scale model of a steel bridge deck, composed of two chords (L40x40 x 4 mm²), 7 floor beams (L40x40 x 4 mm²), and two stringers (40 x 3 mm²) (figure 1a), resting on two hinged supports and two movable supports, with a length of 2.4m, an width of 0.5m, gussets of 2.5 mm of thickness with several shapes and dimensions (in plan) are fastened by welding to the chords, the connections between chords and floor beams are bolted and those between floor beams and stringers are welded.

A tensile test was performed to measure the elasticity modulus of the system’s steel material. It is equal to 210000 N / mm² with a Poisson coefficient of 0.30. A variable loading of 2x1kN was applied on 5 different longitudinal positions at the stringers–floor beams intersections.

Displacement measurements were performed at the ends of the floor beams by HBM inductive displacement sensors (WA-50mm-T) (figure 1b) [8]. The stress measurements were carried out by HBM strain gauges (LY11-10 / 120 ohms) on three points chosen on the chords near the mid-span (figure 1c) [9]. It was mounted in a half bridge circuit and using the multiple wires technology (5 wires) in order to increase the sensitivity of the signal and compensate for the effects of connection cables (voltage drop, zero drift) and temperature variation. A 4800 HZ carrier frequency power supply, shielded and properly grounded wiring were used to reduce electromagnetic noise and thermoelectric voltages according to Keil S [11] and Hoffmann K [12]. The data acquisition was done using the Quantum MX410 amplifier [10], the visualization and computer data processing by Catman software (HBM).

2.3. Structure modeling (Numerical reference model)

The finite element model is built by the frame finite elements with 6 degrees of freedom per node. It is performed by a Matlab code: 92 nodes, 81 frame finite elements, and 552 degrees of freedom (figure2).The model have two hinged supports (65; 86), two movable supports (71; 92) which are leaning on the brick walls (figure 1a). 28 Rigid links, are assigned to connection points to take into account the elements superposition like those between stringers and transvers beams or between main beams and transvers beams. In this numerical model, the existing gussets plates are neglected.

2.4. Static displacement and stress responses

Two parallel moving static loads of 1kN are applied at stringers–floor beams intersections. The loading step is 0.40m. Thus the displacement responses at the nodes (66; 67; 68; 69; 70; 87; 88; 89; 90 and 91) and stress responses at the nodes (15; 22 and 38) according to the different positions of the moving load (5 positions) have been recorded using displacement sensors and strain gauges: the total of 50 data from vertical displacements and 15 from normal stresses. The inverse analysis concerns a numerical model composed of 81 finite elements from which 81x4 = 324 unknowns or correction coefficients. The stiffness correction coefficients are presented with respect to cross section A and moments of inertia Ix, Iy and Iz.
3. Results and discussion
The inverse analysis to evaluate the real stiffness of the metallic system was made in three steps: that of displacements, stresses then displacements + stresses.

The results of the inverse analysis of the displacement responses recorded for different positions of the point loads, are illustrated in (figure 3). The axial stiffness with respect to the cross section A and the torsional stiffness with respect to torsional moment of inertia are variable between +0.2 and -0.23 in the chords (main beams), negligible on the stringers and quite variable between +0.00 and -0.23 for floor beams. For the flexural stiffness relative to the moment of inertia I_y, the correction coefficients of the main beams are close to 0.80 except finite elements close to the supports while the flexural stiffness corresponding to the moment of inertia I_z, their participations are insignificant. This is true considering the structural system’s geometry which is horizontal and the loads are perpendicular thereto.
Figure 3. Inverse analysis results corresponding to the displacements, with respect to the cross section $A$, the torsional moment of inertia $I_x$ and the flexural moments of inertia $I_y$, $I_z$ respectively.

Figure 4. Inverse analysis results corresponding to stresses, with respect to the cross section $A$, the torsional moment of inertia $I_x$ and the flexural moments of inertia $I_y$ and $I_z$ respectively.
Inverse analysis of the stress responses which is illustrated in, (figure 4) is roughly similar to (figure 3). The axial stiffness with respect to the cross section A and these relative to torsional moment of inertia are generally variable between 0.00 and -0.21 in the main beams, negligible on stringers and quite variable for the floor beams. For the corresponding flexural stiffness at the moment of inertia Iy, the correction coefficients are variable between 0.00 and +0.40 and lower when approaching the supports. For the flexional stiffness corresponding to the moment of inertia Iz, their participations are negligible.

When the displacement and stress measurements were taken together in the inverse analysis, the main remark is that the flexural stiffness /Iy of the longitudinal or principal beams are generally increased from + 0% to + 80% except those near the supports (figure 5). For axial and torsional stiffness, they vary from -23% to + 20%. The participation of the stringers is negligible and also the flexural stiffness /Iz.

Figure 5. Inverse analysis results corresponding to the displacements + stresses, with respect to the cross section A, the torsional moment of inertia Ix and the flexural moments of inertia Iy, Iz.

It is worth noting that these results were based on the processing, by the IAFEM code, of 50, 15 and 65 pieces of information respectively to determine 324 unknowns or correction coefficients. It has been shown, in Boumechra's work [7], that it is necessary to have a greater number of information in order to have a better convergence.

Considering the design of the reduced model of the bridge deck studied in the laboratory and the results obtained by the developed inverse analysis code, it is noted that the addition of gussets plate on the main beams increased the flexural stiffness of this one. This result can be supported by a more approximate numerical model using plate or shell finite element (figure 6) and also frame finite element model with only the flexural stiffness correction in accordance with the results of the inverse
analysis. The (Table 1) confirms that the method developed makes it possible to evaluate the real stiffness of any structural system.

The difference found between the numerical and experimental values of node 22's stress is due to the position of the strain gauge: very close to the bolt (4 cm from the bolt) and therefore the effects of the stress concentration around the bolt’s hole.

![Figure 6. Plate finite elements model of the small scale steel bridge deck (Sap 2000).](image)

**Table 1.** Comparison between the different experimental and numerical models (Mid-span loading case).

|                  | Experimental model | Plate finite element model | Frame finite element model (+80% chords stiffness) | Frame finite element model (without stiffness change) |
|------------------|--------------------|----------------------------|---------------------------------------------------|-------------------------------------------------------|
| $\sigma$ (node 15) | 57.99 N/mm²   | 57.73 N/mm² (-0.5%)          | 60.8 N/mm² (+5%)                                   | 62.8 N/mm² (+8%)                                     |
| $\sigma$ (node 22) | 96.75 N/mm²   | 117.15 N/mm² (+21%)          | 108.4 N/mm² (+12%)                                 | 119.7 N/mm² (+23%)                                   |
| $\sigma$ (node 38) | 33.7 N/mm²    | 31.202 N/mm² (-7%)           | 34.7 N/mm² (+3%)                                   | 37.2 N/mm² (+10%)                                   |
| $\delta_v$ (node 66) | 9.5 mm        | 9.73 mm (+3%)                | 10.0 mm (+2%)                                     | 14.4 mm (+47%)                                      |
| $\delta_v$ (node 67) | 16.27 mm      | 16.76 mm (+3%)               | 17.4 mm (+7%)                                     | 25.5 mm (+47%)                                      |
| $\delta_v$ (node 68) | 19.31 mm      | 19.91 mm (+3%)               | 20.1 mm (+5%)                                     | 30.0 mm (+56%)                                      |

4. Conclusion
Structure and its components’ stiffness assessment is a method that may interest engineers to verify the conformity of any structural systems between those conceived in a design office and those determined by its real behavior or its response under different loading. It is also interesting to check numerical models or computation numerical approaches. The evaluation of stiffness also allows the engineer to detect possible damage on his structure. It will be more interesting to check this method on real bridges and under dynamic loads in the near future.
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