Quantum teleportation is one of the important manifestations of quantum mechanics. In particular, quantum teleportation of continuous variable states has attracted a great deal of attention because of a high detection efficiency, handy manipulation of continuous variable states, and possibility of application to high-quality quantum communication. Two kinds of protocols have been suggested for continuous variable teleportation; one utilizes the entanglement between quadrature-phase variables and the other between the photon-number sum and the relative phase. Both the protocols employ a squeezed two-mode vacuum for the quantum channel. In this paper, we report how to optimize the quantum teleportation of quadrature-phase variables when the quantum channel and experimental conditions are not perfect.

There are a few obstacles which make the teleportation of quadrature-phase variables imperfect. The perfect quantum teleportation is possible only by a maximally-entangled quantum channel, i.e., by an infinitely squeezed state which is unphysical as it incurs the infinite energy. Moreover, when the quantum channel is exposed to the real world, it is influenced by the environment, which turns the pure squeezed state into a mixture and deteriorates the entanglement property. To maximize the channel entanglement, purification protocols for continuous variable states have been suggested by Parker et al. for partially-entangled pure states and by Duan et al. for mixed Gaussian states. However, the theoretical suggestions have not been realized by experiment. Further, there are other obstacles in experiment such as imperfect detection efficiency at the sending station and imperfect unitary transformation at the receiving station. We show that the imperfect conditions may be absorbed into the imperfect quantum channel while other apparatuses are treated perfect, and find the optimization condition for the teleportation under a given experimental condition. We show that blindly maximizing the initial entanglement of the quantum channel does not necessarily bring about the best teleportation.

We propose a realistic optimal strategy for continuous-variable teleportation in a realistic situation. We show that any imperfect quantum operation can be understood by a combination of an asymmetrically-decohered quantum channel and perfect apparatuses for other operations. For the asymmetrically-decohered quantum channel, we find some counter-intuitive results; teleportation does not necessarily get better as the channel is initially squeezed more. We show that decoherence-assisted measurement and transformation may enhance the fidelity for the asymmetrically mixed quantum channel.

Two modes $a$ and $b$ of the squeezed vacuum are distributed, respectively, to a sending and a receiving station. At the sending station, the original unknown state is entangled with the field mode $a$ of the quantum channel by a 50/50 beam splitter. Two conjugate quadrature variables are measured respectively for the two output fields of the beam splitter using homodyne detectors. Upon receiving the measurement results through the classical channel, the other mode $b$ of the squeezed vacuum is displaced accordingly at the receiving station. The quantum teleportation of quadrature-phase variables is well described by a phase-space distribution, in particular, the Wigner function that is the Fourier transform of its characteristic function $C(\eta) \equiv \text{Tr}[\rho \hat{D}(\eta)]$ for the state of the density operator $\rho$. $\hat{D}(\eta)$ is the displacement operator.

The quantum teleportation is completed by a unitary displacement operation at the receiving station. If a field state of its Wigner function $W(\alpha)$ is displaced by $\beta$, it is represented by the Wigner function $W(\alpha - \beta)$. In the experiment, the displacement operation is performed using a beam splitter of a high transmittance $T$. To displace a field state of the Wigner function $W(\alpha)$, it is injected into the beam splitter while a high intensity coherent state of amplitude $\beta/\sqrt{1-T}$ is injected to the other input port. The beam splitter operation results in the convolution between the two input states. Remembering that the synthesizing coherent state is the displaced vacuum, we find the Wigner function $W_d(\gamma - \beta)$ of the output field by

$$W_d(\gamma - \beta) = \frac{1}{1-T} \int d^2\alpha W(\alpha) W_{\text{vac}} \left( \frac{\gamma - \beta - \sqrt{T} \alpha}{\sqrt{1-T}} \right)$$

(1)

where $W_{\text{vac}}(\alpha)$ is the Wigner function for the vacuum. We can easily see that displacing a field by a beam splitter of its transmittance $T$ is equivalent to unitarily-displacing the field after it is mixed with the vacuum at a beam splitter of the same $T$. Note that mixing a field with the
vacuum at a beam splitter results in the same dynamics of the field influenced by the vacuum environment. Kim and Imoto found that assuming the coupling of the system with the environment $\kappa$ and the exposure time to the environment $\tau$, the normalized interaction time $R \equiv 1 - \exp(-\kappa \tau)$ is the same as $1 - T$.

The inefficient detection at the sending station is another factor which degrades the teleportation. When the two photomultipliers of a homodyne detector have the same efficiency $\eta$, the imperfect homodyne detector is described by a perfect homodyne detector with a beam splitter in front. A field passes through the beam splitter of the transmission $\eta$ and it is mixed with the vacuum which has been injected into the other input port. The inefficiency of the detection can also be passed to the quantum channel. We will discuss later that inefficiency at the sending station gives an effect not only to the quantum channel but also to the original unknown field to teleport.

We have seen that imperfect operations at the receiving and sending stations can be understood as a combination of the perfect operations with an imperfect mixed quantum channel. Imperfection at the displacement operation is absorbed by the field mode to the receiving station. The field mode to the sending station can absorb inefficiency in the homodyne detection. These considerations lead the quantum channel mixed asymmetrically due to a different condition for each mode of the quantum channel. The study of the teleportation using the asymmetrically-decohered quantum channel is important not only because it can explain the experimental situation but also because it gives novel features and deeper understanding of the nature of entanglement for the continuous variables. If quantum teleportation is used for a quantum communication, it is more likely that the two modes of the quantum channel will undergo different environmental conditions. To the best of our knowledge, the impact of the asymmetric channel on the quantum teleportation has not yet been seriously explored.

The quantum channel, which is initially in the two-mode squeezed vacuum of squeezing factor $s$, is influenced by the thermal environments. Assuming that two thermal modes are independently coupled with the quantum channel, the dynamics of the quantum channel is described by a Fokker-Planck equation in the interaction picture. Solving the equation, the time-dependent Wigner function is obtained as

$$W_{ab}(\alpha_a, \alpha_b) = \mathcal{N} \exp \left\{ -\frac{2}{m_a m_b - c_a c_b} \left[ m_a |\alpha_a|^2 + m_b |\alpha_b|^2 \right] + \sqrt{c_a c_b} (\alpha_a \alpha_b + \alpha^*_a \alpha_b) \right\},$$

where $m_i = R_i (1 + 2\bar{n}_i) + T_i \cosh 2s$ and $c_i = R_i \sinh 2s$, $(i = a, b)$; $\bar{n}_i$ is the average thermal photon number of the environment for the channel mode $i = a, b$. The normalized interaction time $R_i (\equiv 1 - T_i)$ is zero when the quantum channel is not subject to the environment and grows to unity when the channel completely assimilates the environment.

It has been shown that a two-mode Gaussian state is separable when a semi-positive well-defined $P$ function can be assigned to it after some local operations. The two-mode squeezed state subject to the thermal environment is separable when

$$(m_a - 1)(m_b - 1) \geq c_a c_b.$$ \hspace{1cm} (3)

As a special case, if the channel mode $b$ is influenced by the vacuum, i.e., $\bar{n}_b = 0$, the channel becomes separable when

$$R_a \geq \frac{1}{1 + \bar{n}_a}.$$ \hspace{1cm} (4)

For this case, the separation condition depends only on the average thermal photons influencing the channel mode $a$, even when the channel is minimally squeezed at the initial instance.

By the ideal teleportation, the original state is recovered at the receiving station. However, when the channel is not maximally entangled the teleportation is not ideal. The fidelity $F$ is defined as $F = \pi \int d^2\alpha W_o(\alpha) W_f(\alpha)$, where $W_o(\alpha)$ and $W_f(\alpha)$ are the Wigner functions, respectively, for the original and teleported states, to show how close the teleported state is to the original state. One of the important assumptions in teleportation is that the original state is unknown so that the teleportation protocol has to be selected to optimize the average fidelity over all the possible original states. However, the average of the fidelity defined above is zero for continuous variables because of the scope of the possible original states. We thus take a subset composed of all coherent states and find the strategy to optimize the teleportation. A coherent state is one of a few manifestly quantum and extremely useful states generated in laboratories. Because the coherent states are nonorthogonal it is impossible to discern them with certainty. Any state can be written as a weighted sum of coherent states.

Let us consider the teleportation using the quantum channel. Before any action at the receiving and sending stations, the total state is a product of the original unknown state and the quantum channel. Setting homodyne detectors at the two output ports of the beam splitter, quadrature-phase variables $p_1$ and $q_2$ are measured at the respective output ports. Upon receiving each pair of measurement results $g \equiv \sqrt{2} (q_2 - ip_1)$, the quadrature variables of the channel field $b$ is displaced by $g'(g)$ accordingly. For a coherent original state $|\alpha\rangle$, we can calculate the measurement-conditioned Wigner function for the teleported state. Using the definition of the fidelity given above, $F(\alpha, g, g')$ is calculated. The average fidelity for the total set of coherent states is found:

$$\bar{F} = \int d^2\alpha \int d^2g F(\alpha, g, g') = \mathcal{F}_o \int d^2g \exp[-E |g - g'|^2]$$ \hspace{1cm} (5)
where $E$ is a channel-dependent factor and

$$F_o = \frac{1}{1 + \frac{1}{2}(m_a + m_b) - \sqrt{m_a m_b}}$$

Eq. (6) readily shows that only when the displacement $g'$ is the same as $g$, the average fidelity may have a finite value and $\bar{F} = F_o$ is the optimum average fidelity in this case. Braunstein and Kimble found that the teleportation is optimized when $g = g'$ for the pure quantum channel and the fidelity $\bar{F}$ is greater than $1/2$ when the pure channel is entangled [2]. Using Eqs. (3) and (6), we find that for the symmetrically decohered mixed channel, i.e., $m_a = m_b$, the fidelity is greater than $1/2$ as far as the channel is entangled. However, when the channel is asymmetrically decohered, the value $1/2$ is not necessarily the critical fidelity for the standard teleportation scheme described above. Another important fact is found that the teleportation does not necessarily get better as the quantum channel is initially squeezed more. To illustrate more clearly, assume $\bar{n}_a = 0$. For $R_o = 0$ and $R_a = 1$, the quantum channel is inseparable as shown in Eq. (3) but Eq. (6) gives the fidelity $F_o = 1/(2 + \sinh^2 s)$, which means that the more initially squeezed the quantum channel, the smaller the fidelity is.

In Fig. 1, the fidelity $F_o$ is plotted against initial squeezing $s$ for the asymmetric quantum channel where the channel mode $b$ is influenced by the vacuum environment for the normalized interaction time $R_b = 0.01$ and 0.05 while the channel mode $a$ is not influenced by an environment. The teleportation via the quantum channel, which has been decohered by the vacuum with interaction time $R_b = 0.01$, in fact, corresponds to the teleportation with the pure squeezed quantum channel and imperfect displacement using the beam splitter of transmittance $T = 99\%$. It is clearly seen that even for the quantum channel of seemingly-negligible asymmetry, if the channel is initially squeezed more than a certain degree, the teleportation becomes worse. By the first-derivative of $F_o$ with regard to $s$, we find that the teleportation is optimized when the squeezing is $e^{-2s} = |t_a - t_b|/(t_a + t_b)$ for a fixed channel condition $t_a$ and $t_b$, where $t_i = \sqrt{T_i}$ ($i = a, b$). Note that this result does not depend on the temperature of the environment.

For a mixed quantum channel, the standard teleportation of the orthogonal measurement and the unitary transformation the best strategy? The straightforward answer is beyond the scope of this paper but the following argument gives some hint. Let us consider the asymmetrically mixed channel by taking $\bar{n}_b = 0$ and $\bar{n}_a \neq 0$. For given initial squeezing and $T_o$, the fidelity is maximized to $F_o = 1/[1 + (1 + \bar{n}_a)(1 - T_o)]$ only when $T_b = (\sinh 2s/2 \sinh^2 s)^2 T_o$. We find that the critical fidelity $1/2$ is recovered to coincide with the separation condition under this condition. This shows that some further decoherence may enhance the fidelity [5]. The transformation accompanied by decoherence is no longer unitary which implies that a general transformation may optimize the fidelity. With the same argument, we find that a general measurement may optimize the fidelity for asymmetrically mixed quantum channel.

Why does it happen? One may intuitively expect that the more the quantum channel is squeezed, the better teleportation is. It is true when the channel remains pure and not influenced by an environment. However, when the channel is asymmetrically mixed, the conjecture may be wrong. There are many parameters which influence the teleportation. To make the analysis simple without losing the interesting features, we assume for the rest of the paper that the channel is exposed to low-temperature environments only for short periods of time, i.e., $R_i \ll T_i/\bar{n}_i$. The separation condition (3) shows that the quantum channel remains entangled longer as it is initially squeezed more. However, as we have seen in Fig. 1, the fidelity of teleportation can be worse with increasing the initial squeezing. For the short interaction time with low-temperature environments, we write the Wigner function [2] to highlight the EPR correlation [15] of the quantum channel as follows

$$W_{ab}(\alpha_a, \alpha_b) \approx N \exp \left(\frac{-2}{m_a m_b - c_a c_b} \left\{ (t_b q_a + t_a q_b)^2 + t_b p_a - t_a p_b \right\} \right. \right.$$

$$\left. \left. + e^{-2s\{(t_b q_a - t_a q_b)^2 + (t_b p_a + t_a p_b)^2}\right\}}\right).$$

As the initial squeezing $s \to \infty$, the Wigner function $W_{ab} \to C \delta(t_b q_a + t_a q_b) \delta(t_b p_a - t_a p_b)$. It is clear that the asymmetric channel has the EPR correlation between the scaled quadrature variables; positions $q_a$ and $q_b', (t_a/t_b)q_b$, and momenta $p_a$ and $p_b', (t_a/t_b)p_b$. This is somewhat similar to the relation between an original phase space and amplified or dissipated phase space depending on the scale factor $t_a/t_b$. When the channel is strongly squeezed the channel entanglement teleports the original photon to the scaled space, which brings about large noise in the teleported state. For a large squeezing, even though the quantum channel is strongly entangled, the strong entanglement results in inefficient teleportation because the entanglement is between the scaled spaces.

The fact that the asymmetrically mixed quantum channel shows quantum correlation in differently-scaled spaces is not inherent in the two-dimensional qubit system where maximally correlated observables remain unchanged under the asymmetric decoherence. Consider a bipartite system in a spin-singlet state which is asymmetrically decohered in the phase-insensitive and isotropic environment as for the continuous variable system we discuss in this paper. Its correlation function for the spin variables along the directions $\alpha$ and $\beta$ of two subsystems is given by

$$(U(a) \sigma_z U^\dagger(a) \otimes U(b) \sigma_z U^\dagger(b)) \propto -\cos \theta_{ab},$$

where $U$ is a unitary operator and $\theta_{ab}$ the relative angle of the two vectors $a$ and $b$. We clearly see that the maximal correlation between anti-parallel is preserved even after the asymmetric decoherence.

An imperfect detection efficiency at the receiver station can also be analyzed as a combination of perfect
detection with a beam splitter in front. At the beam splitter, not only the channel state but also the unknown original field are mixed with the vacuum. This is why the average fidelity is maximized to \( F = 1/\left[ e^{-2s} + 1/T_a \right] \) for the displacement factor \( g' = g/t_a \) assuming the channel mode \( b \) is not subject to the environment. This shows that the teleportation is more efficient when the channel is initially squeezed more, which is in agreement with the earlier result [2].

We have shown that the important experimental errors can be absorbed by an imperfect mixed quantum channel while the experimental operations are assumed to be perfect. Because the experimental error does not occur symmetrically between the receiving and sending stations, it is important to study the influence of the asymmetrically-decohered quantum channel on the teleportation. For the asymmetrically-decohered quantum channel we found that the strong initial squeezing does not always optimize the teleportation because the asymmetric quantum channel has the EPR correlation between the differently-scaled phase spaces. We found that a measurement and transformation accompanied by decoherence optimizes the teleportation.

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**FIG. 1.** The fidelity \( F \) is plotted against the degree of initial squeezing \( s \). The channel mode \( a \) is not subject to the environment, i.e., \( R_a = 0 \) and the channel mode \( b \) is subjected to the vacuum environment with the normalized interaction time \( R_b = 0.01 \) (solid line) and 0.05 (dotted line). The channel is thus inseparable. The decohered quantum channel with \( R_b = 0.01 \) corresponds to the pure channel for the imperfect displacement with the transmittance \( T = 99\% \).