Radiative $B \rightarrow K^*\gamma$ transition in QCD

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ABSTRACT

We evaluate the branching ratios of the radiative decays $B \rightarrow K^*\gamma$, and $B \rightarrow K_1\gamma$ in the framework of three-point function QCD sum rules. We find: $\Gamma(B \rightarrow K^*\gamma)/\Gamma(b \rightarrow s\gamma) = 0.17 \pm 0.05$ and $\Gamma(B \rightarrow K_1\gamma)/\Gamma(b \rightarrow s\gamma) = 0.30 \pm 0.15$. We also study, in the infinite quark mass limit, the connection between the form factor of the radiative $B \rightarrow K^*\gamma$ transition and the weak hadronic form factors entering the semileptonic B-meson decays.

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Radiative $B$-meson decays of the type $B \to K_i^* \gamma$, with $K_i^* = K^*(892), K_1(1400)$, etc., are known to provide valuable information on the Standard Model (SM) at the quantum level [1]. In fact, these decays, induced by flavour changing $b \to s$ neutral currents, are controlled by the one-loop electromagnetic penguin operator which involves important SM parameters such as the top quark mass and the Cabibbo-Kobayashi-Maskawa matrix elements $V_{ts}$ and $V_{tb}$. At the quark level, it has been shown [2] that QCD corrections lift the Glashow-Iliopoulos-Maiani suppression leading to a sizeable enhancement of the branching ratio $BR(b \to s \gamma)$. Indeed, preliminary experimental results have been recently reported from the CLEO II Collaboration [3]:

$$BR(B \to K^*(892) \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}.$$  \ (1)

On the theoretical side, the calculation of the branching ratios of the exclusive processes induced by the $b \to s$ transition is plagued by the usual uncertainties involved in determining weak hadronic form factors. Estimates are available e.g. from quark models [4], and from the Heavy Quark Effective Theory in the two extreme cases where the chiral symmetry is assumed for the light quark sector [5], or where the strange quark is considered to be heavy [6]; the different estimates differ by up to an order of magnitude.

Among the various theoretical approaches, QCD sum rules [7] appear to be promising since they are, in principle, model independent. A calculation, in this framework, of $B \to K_s^* \gamma$ has been done some time ago [8] using the formalism of two-point functions. However, since $B_s^*$ cannot decay into $B K^*$ because of phase space, two-point function QCD sum rules provide information only on the coupling of the effective current to $B_s^*$. Vector Meson Dominance must then be invoked, and an independent estimate of the strong coupling constant $g_{B_s^*BK^*}$ is required. The latter lies outside the framework of two-point function QCD sum rules and, therefore, it is model dependent. A possible way out of this model dependency is offered by the formalism of QCD sum rules for three-point functions. In this case the hadronic form factors entering the matrix element of $B \to K_i^* \gamma$ can be determined provided the leptonic decay constants of the $B$ and $K_i^*$ mesons are known. The former can be estimated from two-point function QCD sum rules [9], as well as from lattice QCD [10], whereas the leptonic decay constant of $K_i^*$ can be extracted e.g. from the $\tau$-lepton decay data [11].

In this paper we pursue this three-point function QCD sum rule approach, which is similar in spirit to another calculation that can be found in the literature [12]; however,
as it will be shown below, the numerical results of [12] can be made more reliable, and a more accurate determination of the above mentioned decay channels can be obtained. In addition, we study here the connection which follows in the \( m_b \to \infty \) limit between the form factor of \( B \to K^* \gamma \) and the semileptonic B-meson form factors.

The electromagnetic penguin operator which governs the \( b \to s \gamma \) transition can be written, for \( m_s \ll m_b \), as follows:

\[
\mathcal{H}_{eff} = C m_b \epsilon^\mu \bar{s} \sigma_{\mu \nu} \frac{(1 + \gamma_5)}{2} q^\nu b ,
\]

where \( \epsilon \) and \( q \) are the photon polarization and momentum, respectively. The constant \( C \) contains the dependence on the Cabibbo-Kobayashi-Maskawa angles and on the heavy quark masses; neglecting \( m_c \) it reads:

\[
C = \frac{G_F e}{\sqrt{2} 4 \pi^2} V_{ts} V_{tb} F_2 \left( \frac{m_t^2}{m_W^2} \right) ,
\]

where the function \( F_2 \) is given by [2]:

\[
F_2(x) = x^{16/23} \left\{ \bar{F}_2(x) + \frac{116}{27} \left[ \frac{\eta^{10/23} - 1}{5} + \frac{\eta^{28/23} - 1}{14} \right] \right\} .
\]

In eq.(4) \( \eta = \alpha_s(\mu)/\alpha_s(m_W) \) where \( \mu \approx m_b \) is the typical scale of the process, and \( \bar{F}_2(x) \) reads [3]:

\[
\bar{F}_2(x) = \frac{x}{(x - 1)^3} \left[ \frac{2x^2}{3} + \frac{5x}{12} - \frac{7}{12} - \frac{3x^2 - 2x}{2(x - 1)} \ln(x) \right] .
\]

The function \( F_2(x) \) depends weakly on the top quark mass, e.g. in the range 90 GeV \( < m_t < 210 \) GeV, \( F_2(x) \) increases from 0.55 to 0.68. Using \( m_b = 4.6 \) GeV, \( \tau_B = 1.4 \) ps and \( m_t = 120 \) GeV, this leads to the prediction of the inclusive \( b \to s \gamma \) branching ratio: \( BR(b \to s \gamma) = 2.2 (|V_{ts}|/0.042)^2 \times 10^{-4} \).

Let us now consider the decay \( B \to K^*(892) \gamma \). According to eq.(3) the amplitude for \( B(p) \to K^*(p', \eta) \gamma(q, \epsilon) \) can be written as follows:

\[
\mathcal{A}(B \to K^* \gamma) = \langle K^*(p', \eta) \gamma(q, \epsilon) | J_{eff}^\mu | B(p) \rangle \epsilon_\mu ,
\]

where the effective current \( J_{eff}^\mu \) is given by:

\[
J_{eff}^\mu = C m_b \bar{s} \sigma^{\mu \nu} \frac{(1 + \gamma_5)}{2} q_\nu b .
\]
The two hadronic form factors $F_1$ and $G_2$ appearing in the matrix element in eq. (6), i.e.

$$
\langle K^* (p', \eta) | \bar{s} \sigma_{\mu \nu} \frac{(1 + \gamma_5)}{2} q^\nu b | B(p) \rangle = i \epsilon_{\mu \rho \sigma} \eta^{* \rho} p^\sigma F_1(q^2)
+ [\eta^*_\mu (m_B^2 - m_{K^*}^2) - (\eta^* \cdot q)(p + p')_\mu] G_2(q^2)
$$

(8)

($q = p - p'$) can be related using the identity $\sigma^{\mu \nu} \gamma_5 = -i 2 \epsilon^{\mu \nu \rho \sigma} \sigma_{\rho \sigma}$. In fact, $G_2 = F_1/2$, so that we need to compute only the form factor $F_1$ at the kinematical point $q^2 = 0$.

As usual in the QCD sum rules approach, we consider the three-point function correlator:

$$
T_{\alpha \mu \nu}(p, p', q) = (i)^2 \int dx \, dy \, e^{i(p' \cdot x - p \cdot y)} \langle 0 | T(J_{\alpha}(x) \hat{O}_{\mu \nu}(0) J_5^\dagger(y)) | 0 \rangle ,
$$

(9)

where the currents $J_\alpha$ and $J_5$ have the same quantum numbers of the $K^*$ and $B$ mesons, respectively, i.e.

$$
J_\alpha(x) = \bar{q}(x) \gamma_\alpha s(x) ,
J_5(y) = \bar{q}(y) i \gamma_5 b(y) ,
$$

(10)

whereas the operator $\hat{O}_{\mu \nu}(0)$ is given by

$$
\hat{O}_{\mu \nu}(0) = \bar{s}(0) \frac{1}{2} \sigma_{\mu \nu} b(0) ,
$$

(11)

($\sigma_{\mu \nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$). Using the decomposition

$$
T_{\alpha \mu \nu} q^\nu = i \epsilon_{\alpha \rho \sigma} p^\rho p'^\sigma T(p^2, p'^2, q^2)
$$

(12)

we can compute the invariant amplitude $T$ in QCD through the Operator Product Expansion for $p^2, p'^2$ large and spacelike [3], obtaining a perturbative term and non-perturbative power corrections parameterized in terms of quark and gluon condensates. Keeping the terms up to $D = 5$, and in the limit $m_s = 0$, we have:

$$
T^{QCD}(p^2, p'^2, q^2 = 0) = \frac{3m_b^4}{8\pi^2} \int ds \, ds' \, \frac{s'}{(s - s')^3} \frac{1}{(s - p^2)(s' - p'^2)}
- \frac{m_b}{2} < \bar{q}q > \frac{1}{(m_b^2 - p^2)(-p'^2)}
+ \frac{m_b}{2} g < \bar{q} \sigma G q > \left[ \frac{m_b^2}{2(m_b^2 - p^2)^3(-p'^2)} \right]
+ \frac{m_b^2}{3(m_b^2 - p^2)^2(-p'^2)^2} \frac{1}{2(m_b^2 - p^2)^2(-p'^2)} ,
$$

(13)

3The extrapolation from $q^2$ large and spacelike to $q^2 = 0$ is possible due to the large $b$-quark mass.
In eq.(13) we have not included the $O(\alpha_s)$ correction in the perturbative term. As it will become clear shortly, the one loop perturbative contribution turns out to be rather small, on account of the restricted integration range in the dispersion relations. Hence, it is reasonable to expect the two-loop contribution to be negligible. Also, the $D = 4$ non-perturbative term from the gluon condensate is not yet known. However, we do not expect this term to influence much the results. For instance, in other QCD sum rule applications to heavy-light quark systems, where the Wilson coefficient of the gluon condensate has been calculated, it has been proven to contribute very little to the sum rules. Current uncertainties in the values of the leptonic decay constants will dominate the final error on $F_1(0)$.

The invariant amplitude $T(p^2, p'^2, q^2)$ can be related, by means of a double dispersion relation, to a hadronic spectral density which gets contributions from the lowest lying resonances, the $B$ and $K^*$ mesons, plus higher resonances and a continuum:

$$T^H(p^2, p'^2, 0) = f_B \frac{m_B^2}{m_b} f_{K^*} m_{K^*} \frac{1}{(p^2 - m_B^2)(p'^2 - m_{K^*}^2)} F_1(0)$$

$$+ \text{higher resonances} + \text{continuum}. \quad (14)$$

In eq.(14), $f_{K^*}$ and $f_B$ are the leptonic decay constants of the $K^*$ and $B$ mesons, respectively, defined by: $\langle 0 | J_\mu | K^*(p', \eta) \rangle = f_{K^*} m_{K^*} \eta \mu$, and $\langle 0 | J_5 | B(p) \rangle = f_B m_B^2 / m_b$. According to duality, the higher resonance states and the continuum contribution to the hadronic spectral function can be modelled by perturbative QCD. A prediction for $F_1(0)$ can then be obtained by equating the hadronic and the QCD sides of the sum rule in the duality region

$$m_b^2 < s < s_0$$

$$0 < s' < \bar{s} = \min(s - m_b^2, s'_0), \quad (15)$$

where $s_0$ and $s'_0$ are effective thresholds.

The convergence of the series involving the power corrections, and the dominance of the lowest lying states, can be improved by performing a double Borel transform, with the result:

...
\[
\frac{f_B}{m_b} m_B^2 f_{K^*} m_{K^*} \exp \left[ -\frac{m_B^2}{M^2} - \frac{m_{K^*}^2}{M'^2} \right] F_1(0) = \\
= \frac{3m_b^4}{8\pi^2} \int_{m_B^2}^{s_0} ds \int_0^s ds' \frac{s'}{(s-s')^3} \exp \left[ -\frac{s}{M^2} - \frac{s'}{M'^2} \right] \\
= -\frac{m_b}{2} <\bar{q}q> \exp \left[ -\frac{m_B^2}{M^2} \right] \left[ 1 - m_0^2 \left( \frac{m_b^2}{4M^2} + \frac{m_B^2}{3M^2M'^2} - \frac{1}{2M^2} \right) \right],
\]

(16)

where we have adopted the notation \( g <\bar{q}\sigma Gq> = m_0^2 <\bar{q}q> \).

Equation (16) is the sum rule for \( F_1(0) \). We use the following values of the parameters: \( <\bar{q}q> = (-0.23 \text{ GeV})^3 \), and \( m_0^2 = 0.8 \text{ GeV}^2 \); the effective thresholds are varied in the range \( 33 - 36 \text{ GeV}^2 \) for \( s \) and \( 1.4 - 1.8 \text{ GeV}^2 \) for \( s' \). The value of \( f_{K^*} \) can be obtained from the decay \( \tau^- \rightarrow K^{*-}\nu_\tau \): \( f_{K^*} = 0.22 \pm 0.01 \text{ GeV} \); as for \( m_b \) and \( f_B \) we use \( m_b = 4.6 \text{ GeV} \) and, consequently, \( f_B = 0.18 \pm 0.01 \text{ GeV} \) as computed in [14].

The standard numerical analysis of eq.(16) consists in looking for a region in the \((M^2, M'^2)\) space where the value of \( F_1(0) \) is reasonably independent of the Borel variables \( M^2, M'^2 \), and of the thresholds \( s_0, s'_0 \). In addition, one may look for a hierarchy in the series of power corrections, although there are a few known examples where this hierarchy is absent, without affecting the reliability of the results. It turns out that, in the present case, the perturbative contribution in (16) is smaller than the \( D = 3 \) non-perturbative term, due to the tiny integration region which corresponds to reasonable choices of the thresholds. For this reason we are forced to relax the criterion of the hierarchy among all the terms in the power series expansion. In this way, as shown in fig.(1), we find a stability region in correspondence to the value \( F_1(0) = 0.35 \pm 0.05 \) (the error is obtained by varying the parameters in their allowed intervals). This result allows us to estimate the fraction of inclusive \( b \rightarrow s\gamma \) decays represented by the exclusive channel \( B \rightarrow K^{*}\gamma \):

\[
\frac{\Gamma(B \rightarrow K^{*}\gamma)}{\Gamma(b \rightarrow s\gamma)} = \left( \frac{m_B}{m_b} \right)^3 \left( 1 - \frac{m_{K^*}^2}{m_B^2} \right)^3 |F_1(0)|^2 = 0.17 \pm 0.05 .
\]

(17)

Using the computed value of the inclusive branching ratio we find \( BR(B \rightarrow K^{*}\gamma) = (4 \pm 1) \times 10^{-5} \), to be compared with eq.(1).

For a recent calculation of the beauty quark mass see [15].
The result in eq. (17) is smaller than that obtained in [12] by roughly a factor of 2. First, there is a technical difference between our determination of \( F_1(0) \) and that of [12]. While we start from the three-point function (9), which leads to a one-step determination of \( F_1(0) \) from the sum rule (16), the authors of [12] use the equations of motion to rewrite \( F_1(0) \) as a linear combination of two form factors which satisfy two different QCD sum rules. We find that this procedure is unnecessary, and that it may introduce artificial uncertainties in the results for \( F_1(0) \). Second, the authors in [12] use \( f_B \simeq 130 \) MeV, a value which is not confirmed by recent lattice and QCD sum rules analyses both for finite and infinite heavy quark masses [9, 10].

A calculation similar to the one described above can be done for the transition \( B \to K_1 \gamma \). Since the electromagnetic penguin is a spin-flip operator, the \( K_1 \) meson is the \( ^3P_1 \), \( 1^+ \) orbital excitation in the kaon system and, in the framework of QCD sum rules, it can be interpolated by the current \( J_\alpha = \bar{q} \gamma_\alpha \gamma_5 s \).

As it is well known, there are two \( 1^+ \) states in the strange particle spectrum, i.e. \( K_1(1270) \) and \( K_1(1400) \), whose full widths are: \( 90 \pm 20 \) MeV and \( 174 \pm 13 \) MeV, respectively [16]. The leptonic decay constants of these states can be estimated using experimental information on the hadronic decays of the \( \tau \)-lepton. From the measured branching ratio [11] \( \text{BR}(\tau \to (K_1(1270) + K_1(1400)) \nu_\tau) = 1.14 \pm 0.5 \times 10^{-2} \) and using [11] for the fraction of the \( \tau \) decay into \( K_1(1270) \) and \( K_1(1400) \), the values: \( 0.48 \pm 0.38 \) and \( 0.66 \pm 0.35 \), respectively, we find: \( f_{K_1(1270)} = 180 \pm 81 \) MeV and \( f_{K_1(1400)} = 258 \pm 89 \) MeV. These values are in agreement, within errors, with a previous estimate of the leptonic decay constant of the \( ^3P_1 \) octet [17], based on two-point function QCD sum rules.

It is likely that the two observed states are a mixture of the \( ^1P_1 \) and \( ^3P_1 \) states induced e.g. by the \( K^*\pi \) channel. Since the mixing angle is not well known, we assume that the \( ^3P_1 \) state is an intermediate state with mass \( 1300 \) MeV and leptonic decay constant \( f_{K_1} = 200 \) MeV. Using the threshold \( s_0' = 3 - 4 \) GeV\(^2\) we obtain the value \( F_1(0) = 0.5 \pm 0.1 \), which corresponds to

\[
\frac{\Gamma(B \to K_1 \gamma)}{\Gamma(b \to s \gamma)} = 0.30 \pm 0.15 .
\]  

This result indicates that this transition could be dominant with respect to the \( B \to K^*\gamma \) decay. It is rather amusing to recall that while both the \( ^3P_1 \) and the \( ^1P_1 \) states are produced in \( \tau \)-lepton decays, as well as in hadronic reactions, the latter is forbidden in
the radiative decay of the B meson. Hence, the observation of the above decay channel could shed light on the mixing between the two states $K_1(1270)$ and $K_1(1400)$.

Our analysis of the $B \to K^*\gamma$ transition can be concluded by discussing the limit $m_b \to \infty$. Although this does not affect our estimates of the exclusive radiative decays, it is important in itself as it can provide information on the form factors entering the semileptonic decays of the B-meson. In fact, it is known that in the limit of an infinitely heavy $b$ quark one can relate the hadronic matrix element of the effective operator responsible for the transition $b \to s\gamma$ to the matrix elements of the weak currents between a heavy and a light meson [18], by using the flavour and spin symmetries of the Heavy Quark Effective Theory (HQET) [19]. Indeed, one obtains the relation:

$$F_1(q^2) = \left\{ \frac{q^2 + m_B^2 - m_{K^*}^2}{2m_B} \frac{V(q^2)}{m_B + m_{K^*}} - \frac{m_B + m_{K^*}}{2m_B} A_1(q^2) \right\}$$

(19)

among the form factor $F_1$ responsible for the transition $B \to K^*\gamma$ and the semileptonic form factors $V(q^2)$ and $A_1(q^2)$ defined by:

$$\langle K^*(p',\eta) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B(p) \rangle = \frac{2V(q^2)}{m_B + m_{K^*}} \epsilon_{\mu\nu\lambda\sigma} \eta^\nu p^\lambda p'^\sigma$$

$$+ i(m_B + m_{K^*}) A_1(q^2) \eta^\mu + ...$$

(20)

($B = B^- \text{ or } B^0$); in (20) the ellipses denote terms proportional to $(p+p')_\mu$ or $q_\mu$. In using HQET to relate $F_1$ to $V$ and $A_1$ one meets the following problem. The relation (19) is supposed to hold for $q^2 \approx q^2_{max} = (m_B - m_{K^*})^2$, i.e. at zero recoil where the predictions of HQET are in general reliable, whereas in $B \to K^*\gamma$ we need $F_1(q^2)$ at $q^2 = 0$. Moreover at $q^2 \approx q^2_{max}$ we have the following scaling behaviour of the form factors:

$$F_1(q^2_{max}) \approx \sqrt{m_b}$$
$$V(q^2_{max}) \approx \sqrt{m_b}$$
$$A_1(q^2_{max}) \approx \frac{1}{\sqrt{m_b}}.$$ 

(21)

Therefore, the contribution from $A_1$ in (19) should, in principle, be neglected in the infinite quark mass limit, as it is next-to-leading. However, $V(q^2)$ and $A_1(q^2)$ might have a different $q^2$ behaviour and, hence, one cannot exclude that $A_1(0)$ is comparable to $V(0)$ in the $m_b \to \infty$ limit. QCD sum rules can shed some light on this issue.
The limit $m_b \to \infty$ in the sum rule for $F_1(0)$, eq. (16), can be taken after defining new low-energy variables $y$, $y_0$ and $E$ as follows:

\[
\begin{align*}
    s &= m_b^2 + 2m_b y \\
    s_0 &= m_b^2 + 2m_b y_0 \\
    M^2 &= 2m_b E ;
\end{align*}
\] (22)

and we recall that the scaling law for $f_B$ is given by: $f_B = \hat{f}/\sqrt{m_b}$, modulo logs. Before taking the infinite mass limit we observe that the term arising from the $D = 5$ condensates in eq. (16) seems to diverge. However, as discussed in [20, 21, 22], this divergence is only apparent. In fact the contributions from the $D = 3$ and $D = 5$ condensates should be resummed since they arise from non local operators: they are only the first two terms in a power series of the non local operator in the expansion parameter $m_b^2$. However, the exact form of the non local operator is unknown, and the resummation procedure introduces an ambiguity. In [20, 21, 22] an exponential form $\exp(-m_b^2 A)$ was adopted, but a behaviour such as $(1 + m_b^2 A/n)^{-n}$ ($n = 1, 2, \ldots$) is also acceptable since the power series of both forms starts with the same term: $1 - m_b^2 A$. We observe, though, that for $B \to K^*\gamma$, and with $m_b \approx 4.6$ GeV (no doubt a large mass), eq. (16) shows the feature that the non-perturbative contribution dominates over the perturbative one. Should we resum the non perturbative contributions in the exponential form, we would obtain the unlikely result that, when $m_b \to \infty$, the dominant contribution to the sum rule would be provided by the perturbative term, contrary to the expectation based on the rather large value $m_b \approx 4.6$ GeV. Therefore, we shall assume the form

\[
(1 + m_b^2 A/n)^{-n} .
\] (23)

Moreover, if one adopts the reasonable criterion that when $m_b \to \infty$, the quark condensate contribution still dominates the sum rule, one gets $n = 1$ and therefore the sum rule takes the form:

\[
\begin{align*}
    f_{K^*m_K^*}\hat{f}\exp\left(-\frac{m_{K^*}^2}{M^2}\right)F_1(0) &= \frac{3}{4\pi^2 m_b^{3/2}} \int_0^{y_0} dy \int_0^{+\infty} ds's' \exp\left[-\frac{y}{E} - \frac{s'}{M^2}\right] \\
    &- \frac{\sqrt{m_b}}{2} \langle \bar{q}q \rangle \left(1 + \frac{m_0^2 m_b}{6EM^2}\right)^{-1}
\end{align*}
\] (24)

which shows that $F_1(q^2)$ behaves near the origin as follows:

\[
F_1(0) \approx \frac{1}{\sqrt{m_b}} .
\] (25)
The consequence is that the behaviour of $F_1(q^2)$ between $q^2 \approx 0$, eq.(24), and $q^2 \approx q_{\text{max}}^2$, eq.(21), can be interpolated by the simple pole formula suggested by a dispersion relation dominated by the nearest $1^-$ resonance:

$$F_1(q^2) = \frac{F_1(0)}{1 - q^2/m_{B^*}^2}. \quad (26)$$

Let us now examine the validity of eq.(19) at $q^2 = 0$. We can follow the same method discussed above to derive the behaviour of the sum rules for $V(0)$ and $A_1(0)$ in the $m_b \to \infty$ limit. We obtain:

$$f_K\cdot m_K\cdot \hat{f} \exp\left(-\frac{m_{K^*}^2}{M^2}\right)V(0) = -\frac{3}{4\pi^2 m_b^{3/2}} \int_{y_0}^{y_1} dy \int_0^{+\infty} ds' s' \exp\left[-\frac{y}{E} - \frac{s'}{M^2}\right] - \frac{\sqrt{m_b}}{2} \langle \bar{q}q \rangle \left(1 + \frac{m_0^2 m_b}{6EM'^2}\right)^{-1}, \quad (27)$$

$$f_K\cdot m_K\cdot \hat{f} \exp\left(-\frac{m_{K^*}^2}{M^2}\right)A_1(0) = -\frac{3}{4\pi^2 m_b^{3/2}} \int_{y_0}^{y_1} dy \int_0^{+\infty} ds' s' \exp\left[-\frac{y}{E} - \frac{s'}{M^2}\right] + \frac{\sqrt{m_b}}{2} \langle \bar{q}q \rangle \left(1 + \frac{m_0^2 m_b}{6EM'^2}\right)^{-1}. \quad (28)$$

The behaviour in eqs.(27)-(28) is compatible with the scaling laws in eq.(21) only if the form factors $V$ and $A_1$ have different $q^2$ dependence, for example $V$ is a simple pole analogous to (26) whereas $A_1$ is nearly constant in $q^2$. This is in agreement with the observation made in [23], where the $q^2$ dependence of the form factors of the semileptonic $B \to \rho$ transition has been explicitly investigated.

We notice, in closing, that using eqs.(24,27,28), the relation (19) is verified at $q^2 = 0$. This is a direct confirmation of the conjecture put forward in [18] and in [24], that in heavy-to-light transitions the heavy quark stays almost on its mass shell, and that the relations obtained at the zero recoil point remain applicable over the full $q^2$ region.

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Fig. 1. $F_1(0)$ versus the Borel parameters $M^2, M'^2$ for $s_0 = 36 \, GeV^2$, $s'_0 = 1.8 \, GeV^2$. 