Constraints on New Physics from $K \rightarrow \pi \nu \bar{\nu}$

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Abstract

We study constraints on new physics from the recent measurement of $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ by the E787 and E949 Collaborations. In our analysis we consider two models of new physics: (a) extra down type singlet quark model (EDSQ) and (b) $R$-parity violating Minimal Supersymmetric Standard Model (MSSM). We find that $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ along with other processes like $K_L \rightarrow \mu^+ \mu^-$, $\epsilon'/\epsilon$ provide useful bounds on the parameter $U_{sd}$, characterizing the off-diagonal $Z - d - \bar{s}$ coupling of model (a). The bounds on the Re($U_{sd}$) from $(K_L \rightarrow \mu^+ \mu^-)_\text{SD}$ and Im($U_{sd}$) from $\epsilon'/\epsilon$ are so tight that the branching ratio of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ can exceed the standard model value by at most a factor of two. For model b), we also obtain stringent bounds on certain combinations of product of two $\lambda'_{ijk}$ couplings originating from $L$ number violating operator $L_i Q_j D_k$ using $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \mu^+ \mu^-$ processes. Even with the stringent constraints on $U_{sd}$ (in model (a)) and on products of two $R$ couplings (model (b)), we find that the branching ratio $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ can be substantially different in both the above models from those predicted in the standard model.

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I. INTRODUCTION

Recently the experiment E949 at Brookhaven National Laboratory has detected an event for the rare decay $K^+ \rightarrow \pi^+\nu\bar{\nu}$. Combining previously reported two events by the experiment E787, a branching ratio $B(K^+ \rightarrow \pi^+\nu\bar{\nu}) = (1.47^{+1.30}_{-0.89}) \times 10^{-10}$ has been obtained. The central value of this branching ratio is about twice that of the standard model (SM) prediction $B(K^+ \rightarrow \pi^+\nu\bar{\nu}) = (7.2^{+2.1}_{-2.1}) \times 10^{-11}$. The SM prediction and the combined E787 and E949 results are consistent with each other within one standard deviation. Future improvement of the experimental sensitivity will verify whether any meaningful difference emerges. If the experimental result converges to the present central value, it would be an indication of new physics beyond the SM. In extensions of the SM there are new sources for flavor changing neutral currents which can affect the branching ratio for $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and reproduce the central value obtained by E787 and E949.

In this work we analyze the effect of the new E787 and E949 results on two class of models beyond the SM. The first one is the flavor changing neutral current (FCNC) mediated by Z boson in extra down-type quark singlet (EDQS) model. These extra down type singlet quarks appear naturally in each 27-plet fermion generation of $E_6$ Grand Unification Theories (GUTs). The mixing of these singlet quarks with the three SM down type quarks induces tree-level flavor changing neutral currents (FCNC) by Z exchange. These tree-level FCNC couplings can have significant effects on various kaon decay processes including $K \rightarrow \pi\nu\bar{\nu}$.

The second model, we consider is the $R$-parity violating minimal supersymmetric standard model (MSSM). The superpotential of the MSSM contains operators which violate lepton ($L$) and baryon ($B$) numbers. The simultaneous presence of both lepton and baryon number violating operators leads to rapid proton decay which contradicts the experimental bound on proton life-time. In order to keep the proton life-time within the experimental limit one has to impose certain additional symmetry in the model so that the baryon and lepton number violating interactions vanish. In most cases, a discrete multiplicative symmetry called $R$-parity is imposed, where $R = (-1)^{3B+L+2S}$, and $S$ is the spin of the particle. Under this new symmetry, all baryon and lepton number violating operators in the superpotential with mass dimension less or equal to four vanish. This not only forbids rapid proton decay but also predicts stable lightest supersymmetric particle (LSP), which
escapes the detection providing a unique signature of $R$-parity conserving MSSM. However, this symmetry is *ad hoc* in nature, with no strong theoretical arguments in support of it. There are many other discrete symmetries such as baryon parity and lepton parity, both of which can remove the unwanted operators from the superpotential thus preventing rapid proton decay. Since, there is no direct evidence supporting either $R$-parity conserving or $\bar{R}$ MSSM, it is interesting to probe the consequences of $\bar{R}$ model (in such a way that the either $B$ or $L$ number is violated but not both ) in light of some recent low energy data. Already extensive studies have been done to look for the direct as well as indirect evidence of $R$-parity violation from different processes and to put constraints on various $\bar{R}$ couplings [19, 20, 21, 22, 23, 24, 25, 26]. We consider constraints on the product of two $\bar{R}$ couplings that ensue from $K^+ \to \pi^+\nu\bar{\nu}$ decay as well as other kaon processes, such as $K_L \to \mu^+\mu^-$, $K^+ \to \pi^+\mu^+e^-$. In some cases, the bounds obtained in this analysis are stronger than the existing one. Interestingly even with such a stronger bounds, for some combinations of $\bar{R}$ couplings, the $\text{Br}(K_L \to \pi^0\nu\bar{\nu})$ is well above the standard model prediction.

The rest of the paper is organized in the following way. In Section II we discuss the tree level FCNC effects in the extra down type singlet quark model. We will then constrain this new FCNC parameter using the latest data on $\text{Br}(K^+ \to \pi^+\nu\bar{\nu})$, $\text{Br}(K_L \to \mu^+\mu^-)_{\text{SD}}$, and $\epsilon'/\epsilon$ in kaon decay into two pions. After constraining the FCNC parameter space we look for the prediction for the $CP$ violating processes: $K_L \to \pi^0\nu\bar{\nu}$ and $K_L \to \pi^0\epsilon^+\epsilon^-$ in the allowed range of parameter space. In Section III we obtain constraints on the product of two $\bar{R}$ couplings using the current data on $\text{Br}(K^+ \to \pi^+\nu\bar{\nu})$, and $\text{Br}(K_L \to \mu^+\mu^-)_{\text{SD}}$, followed by the prediction of $K_L \to \pi^0\nu\bar{\nu}$ and process in the allowed range of the $\bar{R}$ couplings. We summarize our results in the last section.

II. Z MEDIATED FCNC WITH EXTRA DOWN TYPE SINGLET QUARK

FCNC mediated by Z boson contributing to $K^+ \to \pi^+\nu\bar{\nu}$ can be generated in many ways when going beyond the SM. A simple possibility arises from one new vector-like down type singlet quark in addition to the SM fermions. In the weak interaction basis, W and Z
interactions with quarks (in the current basis) can be written as

\[ \mathcal{L}_W = -\frac{g}{\sqrt{2}} \bar{U}_L^0 \gamma^\mu D_L^0 W^+, \]
\[ \mathcal{L}_Z = -\frac{g}{2c_W} [\bar{U}_L^0 \gamma^\mu U_L^0 - \bar{D}_L^0 \gamma^\mu D_L^0 - 2s_W^2 (Q_u \bar{U}_L^0 \gamma^\mu U^0 + Q_d \bar{D}_L^0 \gamma^\mu D^0 + Q_{d'} \bar{D'}_L^0 \gamma^\mu D'^0)] Z_\mu, \]

(1)

where \( U^0 = (u, c, t) \), and \( D^0 = (d, s, b) \) are the usual three generations of quarks in the SM, and \( D'^0 = d'^0 \) is the additional down type of quark singlet. One can easily generalize the model to include \( n \) generations of vector-like down-type quarks by using \( D_{d'}^0 = (d'_{1}, ..., d'_{n}) \).

In general \( D'^0 \) can mix with the ordinary quarks in \( D^0 \). The down quark mass matrix \( M_d \) is diagonalized by \( 4 \times 4 \) unitary matrices \( V^L \) and \( V^R \).

\[ \mathcal{L}_W = -\frac{g}{\sqrt{2}} \bar{U}_L^0 \gamma^\mu V D_L^0 W^+, \]
\[ \mathcal{L}_Z = -\frac{g}{2c_W} [\bar{U}_L^0 \gamma^\mu U_L^0 - \bar{D}_L^0 \gamma^\mu D_L^0 - 2s_W^2 (Q_u \bar{U}_L^0 \gamma^\mu U + Q_d \bar{D}_L^0 \gamma^\mu D) + Q_{d'} \bar{D'}_L^0 \gamma^\mu D'], \]

(2)

where \( Q_u, d \) are the electric charges of up and down quarks in unit of proton charge. \( U_{ij} = V^L_{di} V^L_{dj} \) is \( 4 \times 4 \) matrix, and \( V = V^L V^L \) which is a \( 3 \times 3 \) matrix different from the usual KM matrix \( V_{KM} \). We shall be concerned with only \( U_{sd} \) element of the \( U \) matrix in this paper.

In the absence of \( d' \), the theory reduces to the SM. The top-left \( 3 \times 3 \) block matrix in \( V \) corresponds to the usual \( V_{KM} \) matrix. The rest of the matrix elements in \( V \) are expected to be small since deviations away from the SM are constrained to be small from various experimental data.

### A. \( K^+ \to \pi^+ \nu \bar{\nu} \) process

The FCNC interactions as described by \( \mathcal{L}_Z \) in Equation 2 can contribute to \( K^+ \to \pi^+ \nu \bar{\nu} \) decay at the tree level by exchanging Z boson as shown in Figure 1. The relevant Lagrangian is given by

\[ \mathcal{L}^\nu_Z = \frac{g^2}{4c_W^2 m_Z^2} U_{sd} \bar{s}_L \gamma^\mu d_L \sum_{\ell=e,\mu,\tau} \bar{\nu}_{L_{\ell}} \gamma^\mu \nu_{L_{\ell}}. \]

(3)
FIG. 1: Feynman diagram for $Z$ exchange tree-level contribution to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ process

Using $<0|\bar{s}\gamma^\mu\gamma_5d|K^+> = i f_K p^\mu_K$ and including the SM contribution, one obtains

$$\frac{\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{\text{Br}(K^+ \rightarrow \pi^0 e^+ \nu)} = \frac{r_{K^+} \alpha^2}{2\pi^2 s_W|V_{us}|^2} \sum_{\ell=e,\mu,\tau} |\Delta_{K^+}^{SM} + \Delta_{K^+}^Z|^2;$$

$$\Delta_{K^+}^{SM} = \lambda_{sd}^c X_{NL}^\ell + \lambda_{sd}^t \eta_{\ell}^X X_0(x_t) \quad (4)$$

The factor $r_{K^+} = 0.901$ accounts for isospin breaking corrections, $\alpha = 1/128$, $s_W = \sin \theta_W$. $\Delta_{K^+}^{SM}$ is the SM contribution with the charm contributions at NLO found to be $X_{NL}^{e,\mu} = (10.6 \pm 1.5) \times 10^{-4}$, $X_{NL}^{\tau} = (7.1 \pm 1.4) \times 10^{-4}$ [4]. $\lambda_{sd}^t$ is defined as $V_{td}^* V_{ud}$. The top contribution is proportional to the term with $\lambda_{sd}^t$. $\eta_{\ell}^X$ is a QCD correction factor which is equal to 0.994. The function $X_0$ is given by:

$$X_0 = \frac{x}{8} \left[ \frac{x + 2}{x - 1} + \frac{3x - 6}{(x - 1)^2} \ln x \right]; \quad x = \frac{m_t^2}{m_W^2} \quad (5)$$

Using the current best fit values for the Wolfenstein parameters, $\lambda = 0.224$, $A = 0.839$, $\rho = 0.178$, $\eta = 0.341$, and experimental value of $\text{Br}(K^+ \rightarrow \pi^0 e^+ \nu) = 0.0487$, we obtain the SM value of $(7.28^{+0.13}_{-0.12}) \times 10^{-11}$ for $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. The theoretical error on the branching ratio is computed by allowing $\lambda_{sd}^t$ and the charm NLO corrections ($X_{NL}^{e,\mu}, X_{NL}^{\tau}$) to vary within $1\sigma$ from their central value. The central value of the SM branching ratio is a factor of 2 smaller than the experimental central value.

The term $\Delta_{K^+}^Z$ in Equation (4) characterizes new contribution from FCNC $Z$ interaction.
which is given by:

$$\Delta^Z_K = -\frac{\pi s_W^2}{\alpha} U_{sd}. \quad (6)$$

With this new contributions it is possible to reproduce the experimental central value, for example with \(\text{Re}(U_{sd}) = 0.339 \times 10^{-5}\) and \(\text{Im}(U_{sd}) = 0.055 \times 10^{-5}\), we obtain \(\text{Br}(K^+ \rightarrow \pi^+\bar{\nu}\nu) = 1.47 \times 10^{-10}\) using the central values of \(\lambda_c^{sd}, \lambda_t^{sd}\) and the charm NLO corrections.

One can also turn the argument around. By using the experimental data on \(\text{Br}(K^+ \rightarrow \pi^+\bar{\nu}\nu)\) one can constrain the new FCNC parameter \(U_{sd}\). In Figure 2 the grey shaded region represents 90% CL allowed region in the \(\text{Re}(U_{sd}) - \text{Im}(U_{sd})\) plane from \(\text{Br}^{\exp}(K^+ \rightarrow \pi^+\bar{\nu}\nu)\). The thickness of the band includes the 1\(\sigma\) theoretical error arising from the CKM parameters and the charm NLO contribution. From the Figure 2 one can see that the recent data on \(\text{Br}(K^+ \rightarrow \pi^+\bar{\nu}\nu)\) still allows a large part of the parameter space of \(U_{sd}\). However, in next two sub-sections we will show that the current information on the short distance contribution to \(K_L \rightarrow \mu^+\mu^-\) and the data on \(\epsilon'/\epsilon\) constrain the above allowed parameter space of \(U_{sd}\) quite severely.

**B. Constraints from other kaon processes**

The same flavor changing \(Z\) interaction in Equation (2) will also have effects on other flavor changing kaon processes. The \(\Delta S = 1\) and \(\Delta S = 2\) effective Lagrangian relevant for our discussions are given by

$$L^{\Delta S=1} = \frac{g^2}{4\alpha s_W^2 m_Z^2} U_{sd} \bar{s}_L \gamma^\mu d_L \tilde{f} \gamma_\mu(2I_3 L - 2Q_f s_W^2) f,$$

$$L^{\Delta S=2} = \frac{G_F}{\sqrt{2}} \left[ U_{sd}^2 - \frac{8}{\pi^2 s_W^2} U_{sd} \sum_{\alpha=c,t} \lambda_{\alpha,5}^s Y_5(x) \right] \bar{s}_L \gamma^\mu d_L \bar{s}_L \gamma_\mu d_L \quad (7)$$

where, \(Y_5(x) = (x/8) [(x-4)/(x-1) + 3x \log x/(x-1)^2]\). The second term in \(L^{\Delta S=2}\) is due to one loop effect which is important for gauge invariance, but is numerically small [11]. Taking \(f = \nu, e, \mu\), \(L^{\Delta S=1}\) can contribute to processes like \(K^+ \rightarrow \pi^+\bar{\nu}\nu\), \(K_L \rightarrow \pi^0\bar{\nu}\nu\), \(K_L \rightarrow \mu^+\mu^-\) and \(K_L \rightarrow \pi^0 e^+ e^-\). Taking \(f = u, d\), \(L^{\Delta S=1}\) can contribute to \(\epsilon'/\epsilon\). While \(L^{\Delta S=2}\) can contribute to \(\epsilon_K\) and \(\Delta M_K\).

**Constraint from \(K_L \rightarrow \mu^+\mu^-\)**
FIG. 2: The shaded region is 90% CL allowed parameter space in \(\text{Re}(U_{sd}) - \text{Im}(U_{sd})\) plane from \(\text{Br}(K^+ \rightarrow \pi^+ \nu\bar{\nu})\) data. The area enclosed by the two solid horizontal and two vertical dotted lines is allowed by \(\text{Br}(K_L \rightarrow \mu^+ \mu^-)_{SD}\) and \(\epsilon'/\epsilon\) respectively at 90% CL. Both \(\text{Re}(U_{sd})\) and \(\text{Im}(U_{sd})\) are measured in units of \(10^{-5}\).

An interesting limit on the FCNC parameter \(U_{sd}\) comes from the short-distance contribution to the decay \(K_L \rightarrow \mu^+ \mu^-\). The \(K_L \rightarrow \mu^+ \mu^-\) branching ratio can be decomposed into dispersive part (\(\text{Re} A\)) and the absorptive part (\(\text{Im} A\)). The absorptive can be determined very accurately from the branching ratio \(\text{Br}(K_L \rightarrow \gamma\gamma)\) and the resulting \(|\text{Im} A|^2\) alone almost saturates the branching ratio \(\text{Br}(K_L \rightarrow \mu^+ \mu^-) = (7.07 \pm 0.18) \times 10^{-9}\) \cite{11}, leaving a very small room for the dispersive contribution \(\text{Re} A\). \(\text{Re} A\) can be further decomposed in \(\text{Re} A_{LD} + \text{Re} A_{SD}\). Combining results from Refs. \cite{7, 28}, we have 90%CL bound on \(\text{Br}(K_L \rightarrow \mu^+ \mu^-)_{SD} \leq 2.5 \times 10^{-9}\), with \(|\text{Re} A_{SD}|^2 \equiv \text{Br}(K_L \rightarrow \mu^+ \mu^-)_{SD}\). In our analysis we use this value of \(\text{Br}(K_L \rightarrow \mu^+ \mu^-)_{SD}\) to constrain the FCNC parameter \(\text{Re}(U_{sd})\).
expression for $\text{Br}(K_L \to \mu^+ \mu^-)\text{SD}$ is given by [11]:

$$
\frac{\text{Br}(K_L \to \mu^+ \mu^-)_{\text{SD}}}{\text{Br}(K^+ \to \mu^+ \nu)} = \frac{\tau_{K_L}}{\tau_{K^+}} \frac{\alpha^2}{\pi^2 s_W^4 |V_{us}|^2} \times \left[T_{\text{SM}} + \text{Re}(\Delta Z_K)^2\right],
$$

(8)

where,

$$
T_{\text{SM}} = Y_{NL} \text{Re} (\lambda_{sd}^e) + \eta_t^Y Y_0(x_t) \text{Re} (\lambda_{sd}^t)
$$

(9)

At 90%CL we get following bound on $\text{Re}(U_{sd})$:

$$
-1.12 \times 10^{-5} \leq \text{Re}(U_{sd}) \leq 3.45 \times 10^{-6}
$$

(10)

The lower bound is obtained by taking $\text{Re}(\lambda_{sd}^e) = (-0.2204 - 0.0023)$, $\text{Re}(\lambda_{sd}^t) = (-3.04 - 0.31) \times 10^{-4}$ and the charm NLO contribution $Y_{NL} = (2.94 + 0.28) \times 10^{-4}$. The upper bound is obtained by using $\text{Re}(\lambda_{sd}^e) = (-0.2204 + 0.0022)$, $\text{Re}(\lambda_{sd}^t) = (-3.04 + 0.32) \times 10^{-4}$ and the charm NLO contribution $Y_{NL} = (2.94 - 0.28) \times 10^{-4}$. In Figure 2 the above bound is shown by the area enclosed by vertical dotted lines. This shows that a substantial part of the parameter space which was allowed by the $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$ is ruled out by the $\text{Br}(K_L \to \mu^+ \mu^-)_{\text{SD}}$.

**Constraint from $\epsilon'/\epsilon$**

Following the notation of Ref. [11], we write the down the expression for $\epsilon'/\epsilon$:

$$
\epsilon'/\epsilon = F_{\epsilon'}(x_t)\text{Im}(\lambda_{sd}^t) + \Delta_{\epsilon'}
$$

(11)

where, $F_{\epsilon'}(x_t)$ and can be found in the Ref. [11] and $\Delta_{\epsilon'} = -\frac{\pi s_W^2}{\alpha} (P_X + P_Y + P_Z)\text{Im}(U_{sd})$, with $P_{X,Y,Z}$ are given in Ref. [11].

The bound on $\text{Im}(U_{sd})$ from $\epsilon'/\epsilon$ depends upon the sign of $\text{Im}(U_{sd})$. For $\text{Im}(U_{sd}) > 0$, the upper bound looks like :

$$
\text{Im}(U_{sd}) \leq \frac{(\epsilon'/\epsilon)^\text{exp} - (\epsilon'/\epsilon)^\text{SM}}{-\frac{\pi s_W^2}{\alpha} (P_X + P_Y + P_Z)}
$$

(12)

The experimental value for $\epsilon'/\epsilon$ is $(1.8 \pm 0.4) \times 10^{-3}$. To maximize the $\text{Im}(U_{sd})$, one should take maximum and minimum values for $(\epsilon'/\epsilon)^\text{exp}$ and $(\epsilon'/\epsilon)^\text{SM}$ respectively. The minimum value for the $(\epsilon'/\epsilon)^\text{SM}$ can be obtained by taking the lowest allowed values of $\text{Im}(\lambda_{sd}^t)$, $B_6$ and $B_8$ (defined in Ref. [11]). With these choices of input parameters we have found

$$
\text{Im}(U_{sd}) \leq 3.72 \times 10^{-6}
$$

(13)
Now, for \( \text{Im}(U_{sd}) < 0 \), we have

\[
- \text{Im}(U_{sd}) \leq \frac{(\epsilon'/\epsilon)_{\text{SM}} - (\epsilon'/\epsilon)_{\text{exp}}}{-\frac{\pi s_W}{\alpha}(P_X + P_Y + P_Z)} \tag{14}
\]

In this case we take \((\epsilon'/\epsilon)_{\text{max}}\) and \((\epsilon'/\epsilon)_{\text{min}}\). This can be achieved by taking largest allowed values of \(\text{Im}(\lambda_{sd}^t_\alpha)\), \(B_6\) and \(B_8\). This gives us

\[
- \text{Im}(U_{sd}) \leq 1.91 \times 10^{-6} \tag{15}
\]

Clearly the bounds on the FCNC parameter \(\text{Im}(U_{sd})\) depends upon the experimental values of \((\epsilon'/\epsilon)_{\text{max}}\) and \((\epsilon'/\epsilon)_{\text{min}}\). Finally combining Equation 13 and Equation 15 we get a bound on \(\text{Im}(U_{sd})\) at 90\%CL :

\[
- 1.91 \times 10^{-6} \leq \text{Im}(U_{sd}) \leq 3.72 \times 10^{-6} \tag{16}
\]

In Figure 2 this bound is depicted by area enclosed by two parallel solid lines. This bound further constrain the parameter space of \(U_{sd}\) which was otherwise allowed by the \(\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})\) and \(\text{Br}(K_L \to \mu^+ \mu^-)_{\text{SD}}\).

**Remark on \(\Delta M_K\) and \(\epsilon_K\)**

The contribution from the new Z flavor changing neutral current to \(\Delta M_K^Z\) and \(\epsilon_K^Z\) are given by

\[
M_{12}^K = \frac{G_F^2 M_W^2 f_K^2 B_K M_K}{12\pi^2} \left[-8U_{ds}(\lambda_{ds}^c \eta_c Y_0(x_c) + \lambda_{ds}^t \eta_t Y_0(x_t)) + \frac{4\pi s_W^2}{\alpha} \eta_e U_{sd}^2\right],
\]

\[
\Delta M_K^Z = 2\text{Re}(M_{12}^K), \quad \epsilon_K^Z = e^{i\pi/4} \frac{\text{Im}(M_{12}^K)}{\sqrt{2}\Delta M_K} \tag{17}
\]

We observe that \(\Delta M_K\) and \(\epsilon_K\) evaluated from the above are close to the standard model predictions after constraints from \(K^+ \to \pi^+ \nu \bar{\nu}\) and \(K_L \to \mu^+ \mu^-\) is taken into account. Uncertainties in long distance contributions to \(\Delta M_K\) make these process unsuitable for obtaining strong constraints.

**Predictions for \(K^+ \to \pi^+ \nu \bar{\nu}, K_L \to \pi^0 \nu \bar{\nu}\) and \(K_L \to \pi^0 e^+ e^-\)**

In the previous discussions we have shown how recent data on \(\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})\), \(\text{Br}(K_L \to \mu^+ \mu^-)_{\text{SD}}\) and \(\epsilon'/\epsilon\) could be used to constrain the tree level FCNC parameter \(U_{sd}\) in extra down type singlet quark model. In view of the tight constraints on \(U_{sd}\) obtained, we now examine the range of allowed branching ratios for \(K^+ \to \pi^+ \nu \bar{\nu}, K_L \to \pi^0 \nu \bar{\nu}\) and \(K_L \to \pi^0 e^+ e^-\) within the allowed parameter space of \(U_{sd}\).
FIG. 3: Variation of \( \text{Br} (K^+ \to \pi^+ \nu \bar{\nu}) \) (in units of \( 10^{-10} \)) in the allowed parameter space of the \( \text{Re} (U_{sd}) \) and \( \text{Im} (U_{sd}) \) (both in units of \( 10^{-5} \)) as shown by the rectangular box in Figure 2.

In Figure 3 we show the variation of \( \text{Br}(K^+ \to \pi^+ \nu \bar{\nu}) \) (in units of \( 10^{-10} \)) in the allowed \( \text{Re}(U_{sd}) - \text{Im}(U_{sd}) \) plane. It turns out that the branching ratio can reach up to \( 1.5 \times 10^{-10} \) at the edge of the allowed parameter \( \text{Re} (U_{ds}) \) and \( \text{Im} (U_{ds}) \). Another point to note is that the branching ratio depends very weakly on the FCNC parameter \( \text{Im} (U_{ds}) \). In the computation of this branching ratio we consider only the central values for all theoretical input parameters.

The channel \( K_L \to \pi^0 \nu \bar{\nu} \) in the SM and in the model under consideration is dominantly \( CP \) violating. In the standard model this decay mode is mostly determined by the intermediate top quark state and the uncertainty due to the charm contribution is negligible.
FIG. 4: Variation of $\text{Br} \left( K_L \rightarrow \pi^0 \nu \bar{\nu} \right)$ (in units of $10^{-10}$) in the allowed range of $\text{Im} \left( U_{sd} \right)$ (in units of $10^{-5}$).

compared to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ channel. We have [11]

$$\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{Br}(K^+ \rightarrow \pi^0 e^+ \nu)} = r_{K_L} \frac{\tau_{K_L}}{\tau_{K^+}} \frac{\alpha^2}{2\pi^2 s_W^4 |V_{us}|^2} \sum_{\ell=e,\mu,\tau} |\text{Im} \left( \Delta_{K}^{SM} + \Delta_{K}^{Z} \right)|^2. \tag{18}$$

where, $\Delta_{K}^{SM}$ and $\Delta_{K}^{Z}$ are defined in Equation (4) and (6) respectively. The standard model branching fraction for the process $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is $2.93^{+0.84}_{-0.67} \times 10^{-11}$ in the same ballpark as other estimates [7, 29, 30]. The errors correspond to the 1σ error in the CKM elements.

In Figure 4 we show the variation of $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ (in units of $10^{-10}$) in the allowed $\text{Re}(U_{sd}) - \text{Im}(U_{sd})$ parameter space. Being a $CP$ violating process, the branching ratio is solely dependent upon the $\text{Im} \left( U_{sd} \right)$ part of the FCNC parameter $U_{sd}$ and it can reach as high as $10 \times 10^{-11}$ at the edge of the allowed region. Presently there is an upper bound on the branching ratio of $5.9 \times 10^{-7}$ at 90% CL [17].

We now discuss the $CP$ violating mode $K_L \rightarrow \pi^0 e^+ e^-$. It has recently been shown [31, 32, 33] that there are two main contributions to this process. The amplitude arising from the short distance physics, and a mixing contribution arising from conversion of $K_L$ to
$K_S$ and subsequent decay of $K_S$ into $\pi^0 e^+ e^-$. Using the experimental input on $\text{Br}(K_S \rightarrow \pi^0 e^+ e^-)$, it has been shown that this mixing contribution dominates over the short distance contribution in the rate by a factor of $5 \sim 6$. As a result of this the new physics contribution to the short distance amplitude through $Z$ exchange, using constraints on the parameter $U_{sd}$ in Figure 2, hardly affects the total rate. The experimental bound on $K_L \rightarrow \pi^0 e^+ e^-$ is larger than theory prediction by an order of magnitude, so no stronger bounds on $U_{sd}$ can be obtained.

III. FCNC FROM $R$ PARITY VIOLATING INTERACTIONS IN MSSM

The most general superpotential of the minimal supersymmetric standard model (MSSM) can contain $R$-parity violating interaction terms:

$$
W_R = \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}^c_k + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}^c_k + \lambda''_{ijk} \hat{U}^c_i \hat{D}^c_j \hat{D}^c_k
$$

(19)

where, $\hat{E}^c_i$, $\hat{U}^c_i$ and $\hat{D}^c_i$ are $i$-th type singlet lepton, up-type and down-type quark superfields, $\hat{L}_i$ and $\hat{Q}_i$ are $SU(2)$ doublet lepton and quark superfields. The symmetry of the superpotential requires $\lambda_{ijk} = -\lambda_{jik}$ and $\lambda''_{ijk} = -\lambda''_{ikj}$. It is clear from the Equation 19 that the first two terms violate lepton number, whereas, the last one violate baryon number. Simultaneous presence of both lepton and baryon number violating terms in the superpotential will lead to rapid proton decay. To prevent this we can have either lepton or baryon number violating terms but not both in the superpotential. The $R$ interactions, in general can have $27\lambda'$-type and 9 each of $\lambda$ and $\lambda''$-type of new couplings, which in general can be complex. The phase of single coupling can be absorbed in the definition of the sfermion field, but the product of couplings can have a nontrivial phase. In our analysis we shall assume that only $\lambda_{ijk}$ type of lepton number violating couplings are present. Furthermore, we will constrain product of two $\lambda'$ type of couplings at a time and assume that there are no accidental cancellations and that only one product dominates at a time.

From the $R$ superpotential (Equation 19) one can write down the effective Lagrangian.
relevant for our purpose generated by the exchanging of different sfermions.

\[ \mathcal{L}_R = \frac{\lambda'_{ijk} \lambda_{j'k}}{2m_{d_i}^2} \left[ \bar{d}_L^i \gamma^\mu d_L^i \gamma_\mu d_L^j + e_L^i \gamma^\mu e_L^i \bar{u}_L^i \gamma_\mu u_L^j - \nu_{L}^i \gamma^\mu \nu_{L}^i \bar{d}_L^i \gamma_\mu d_L^j - \bar{e}_L^i \gamma^\mu \nu_{L}^i \bar{u}_L^i \gamma_\mu d_L^j \right] \]

\[ \lambda'_{ijk} \lambda_{j'k} \bar{u}_L^i \gamma_\mu u_L^j \bar{d}_R^k \gamma_\mu d_R^{k'} \]

\[ \lambda'_{ijk} \lambda_{j'k} \bar{d}_R^k \gamma_\mu d_R^{k'} \bar{u}_L^i \gamma_\mu u_L^j \]

\[ \frac{\lambda'_{ijk} \lambda_{j'k}}{2m_{d_i}^2} \bar{u}_L^i \gamma_\mu u_L^j \bar{d}_R^k \gamma_\mu d_R^{k'} \]

\[ \alpha, \beta \text{ are color indices.} \]

\section{Constraints from $K^+ \to \pi^+ \nu\bar{\nu}$ process}

In the $R$ MSSM, $K^+ \to \pi^+ \nu\bar{\nu}$ process receives two non zero contributions from the exchange of $\tilde{d}_R^i$ and $\tilde{d}_L^j$ squarks. Keeping the SM contribution one obtains:

\[ \frac{Br(K^+ \to \pi^+ \nu\bar{\nu})}{Br(K^+ \to \pi^0 e^+ \nu)} = \frac{r_{K^+ \to \pi^+ \nu\bar{\nu}}}{2\pi^2 s_W^2 |V_{us}|^2} \left[ \sum_{\ell=e,\mu,\tau} \left| \Delta_{SM}^{\ell} \right|^2 + \sum_{i \neq j} \left| \Delta_{Kii'} \right|^2 \right], \quad (21) \]

where, the contribution from the lepton number violating operator $\tilde{L}_i \tilde{Q}_j \tilde{D}_k^c$ is given by:

\[ \Delta_{Kii'}^{R} = \frac{\pi s_W^2}{\sqrt{2} G_F} \left[ \frac{\lambda'_{ij2} \lambda_{ij1}^{*}}{2m_{d_i}^2} - \frac{\lambda'_{ik1} \lambda_{ik2}^{*}}{2m_{d_k}^2} \right]. \quad (22) \]

There are two contributions arising from the product of two types of $\lambda'$ couplings, in one case ($\lambda'_{ij2} \lambda_{ij1}^{*}$), the propagator squark coming from left handed doublet quark superfield $\tilde{Q}_j$, whereas in the other case ($\lambda'_{ik1} \lambda_{ik2}^{*}$), the propagator squark coming from right handed singlet quark superfield $\tilde{D}_k^c$. In our analysis we take one combination of couplings to be nonzero at a time by setting all others to zero. The limits on $R$ couplings are usually quoted for $m_f = 100$ GeV. Following this general practise, through out our analysis we assume sfermion masses to be degenerate with $m_f = 100$ GeV, and limits for higher $m_f$ can be obtained easily by scaling.

From Equations (21) and (22) we see that only in the case of same flavor neutrinos in the final state ($i = i'$), the $R$ contributions will interfere with the standard model one. In Figure 3 (a), the area within the circle represent the 90% CL allowed region in the $Re(\lambda'_{ij2} \lambda_{ij1}^{*}) - Im(\lambda'_{ij2} \lambda_{ij1}^{*})$ (first combination of $R$ couplings in Equation 22) plane. Similarly, the 90% CL allowed regions for the second combination of $R$ couplings is shown in the Figure 3 (b) in
the Re($\lambda'_{i,k}^\nu \lambda'^*_{2,k}$) - Im($\lambda'_{i,k}^\nu \lambda'^*_{2,k}$) plane. The solid contour represent $\pi^+\nu_e\nu_e$ and $\pi^+\nu_\mu\nu_\mu$ (with $i = 1, 2$) final state, while the dashed one correspond to $\pi^+\nu_\tau\nu_\tau$ (with $i = 3$) final state. The product of $R$ couplings are in units of $10^{-5}$. The marginal difference between the solid and dashed contours arises due the difference in the charm contribution at NLO for $e, \mu$ and $\tau$.

The relative shifts of the bounds between Figure 5 (a) and (b) can traced to the relative sign difference between two combinations of $R$ couplings in Equation (22).

In the scenario, with $i \neq i'$, the $R$ operators contribute to the amplitude $\mathcal{M}(K^+ \rightarrow \pi^+\nu\bar{\nu})$ incoherently, without interfering with the standard model. Moreover, both combination of $R$ couplings in Equation (22) will have same contribution to $K^+ \rightarrow \pi^+\nu\bar{\nu}$ process. From the experimentally observed branching ratio of $K^+ \rightarrow \pi^+\nu\bar{\nu}$ process, we obtains $|\lambda'_{i,j2}^\nu \lambda'^*_{j,1}| \leq 0.89 \times 10^{-5}$ at 90% CL for $m_{\bar{u}_L} = 100$ GeV. The same bound will also apply for the other combination of $R$ couplings $\lambda'_{i',k1}^\nu \lambda'^*_{k,2}$ in Equation (22).

**Constraints from $K_L \rightarrow \mu^+\mu^-$ process**

The $K_L \rightarrow \mu^+\mu^-$ process bounds the real part of the product $\lambda'_{2j1}^\nu \lambda'^*_{2j2}$. The expression of the branching ratio Br($K_L \rightarrow \mu^+\mu^-$)$_{SD}$ has been given in Equation (8). The $R$ contribution can be easily included by replacing $\Delta^Z_{K_L}$ in Equation (8) by $-(\pi s_W^2/\sqrt{2}G_F\alpha)\lambda'_{2j1}^\nu \lambda'^*_{2j2}/2m_{\bar{u}_L}^2$. Using the 90% CL upper bound on the Br($K_L \rightarrow \mu^+\mu^-$)$_{SD} \leq 2.5 \times 10^{-9}$ (as mentioned before), we obtain

$$-3.719 \times 10^{-6} < \text{Re}(\lambda'_{2j1}^\nu \lambda'^*_{2j2}) < 1.14 \times 10^{-6}.$$  \hspace{1cm} (23)

for $m_{\bar{u}_L} = 100$ GeV.

**Constraints from $K^+ \rightarrow \pi^+\mu^-e^-$ process**

This lepton number violating process has no contribution from the standard model. However, in $R$ model, $\lambda'_{1j1}^\nu \lambda'^*_{2j2}$ coupling can induce such decay process mediated by virtual $\bar{u}_L$.

The branching ratio is given by

$$\frac{\text{Br}(K^+ \rightarrow \pi^+\mu^-e^-)}{\text{Br}(K^+ \rightarrow \pi^0e^+\bar{\nu})} = \frac{r_{K^+} \alpha^2}{2\pi s_W^4 |V_{us}|^2} |\Delta^R_{K^+}|^2,$$  \hspace{1cm} (24)

$$\Delta^R_{K^+} = \frac{\pi s_W^2}{\sqrt{2}G_F\alpha} \left( \frac{\lambda'_{1j1} \lambda'^*_{2j2}}{2m_{\bar{u}_L}^2} \right).$$  \hspace{1cm} (25)

Experimentally we only have an upper bound on the branching ratio, Br($K^+ \rightarrow \pi^+\mu^-e^-) < 2.8 \times 10^{-11}$ at 90% CL. Using this bound we obtain at 90% CL

$$|\lambda'_{1j1} \lambda'^*_{2j2}| \leq 2.684 \times 10^{-6}.$$  \hspace{1cm} (26)
| Couplings | bounds | source |
|-----------|---------|--------|
| $\lambda'_{i1k}\lambda'^*_{12k}$ | $-1.168 \times 10^{-5} \leq \text{Re}(\lambda'_{i1k}\lambda'^*_{12k}) \leq 0.67 \times 10^{-5}$ | $K^+ \rightarrow \pi^+\nu\bar{\nu}$ |
| | $-0.85 \times 10^{-5} \leq \text{Im}(\lambda'_{i1k}\lambda'^*_{12k}) \leq 1.0 \times 10^{-5}$ | $K^+ \rightarrow \pi^+\nu\bar{\nu}$ |
| $|\lambda'_{112}\lambda'_{111}|$ | $4.8 \times 10^{-7}$ | $\Delta M_K$ [34] |
| $|\lambda'_{122}\lambda'_{121}|$ | $4.6 \times 10^{-7}$ | $\Delta M_K$ [34] |
| $\lambda'_{132}\lambda'^*_{131}$ | $-0.67 \times 10^{-5} \leq \text{Re}(\lambda'_{132}\lambda'^*_{131}) \leq 1.17 \times 10^{-5}$ | $K^+ \rightarrow \pi^+\nu\bar{\nu}$ |
| | $-1.0 \times 10^{-5} \leq \text{Im}(\lambda'_{132}\lambda'^*_{131}) \leq 0.85 \times 10^{-5}$ | $K^+ \rightarrow \pi^+\nu\bar{\nu}$ |
| $|\lambda'_{212}\lambda'_{211}|$ | $4.8 \times 10^{-7}$ | $\Delta M_K$ [34] |
| $|\lambda'_{222}\lambda'_{221}|$ | $4.6 \times 10^{-7}$ | $\Delta M_K$ [34] |
| $\lambda'_{232}\lambda'^*_{231}$ | $-3.719 \times 10^{-6} \leq \text{Re}(\lambda'_{232}\lambda'^*_{231}) \leq 1.14 \times 10^{-6}$ | $K_L \rightarrow \mu^+\mu^-$ |
| | $-1.0 \times 10^{-5} \leq \text{Im}(\lambda'_{232}\lambda'^*_{231}) \leq 0.85 \times 10^{-5}$ | $K^+ \rightarrow \pi^+\nu\bar{\nu}$ |
| $|\lambda'_{312}\lambda'_{311}|$ | $4.8 \times 10^{-7}$ | $\Delta M_K$ [34] |
| $\lambda'_{32j}\lambda'^*_{3j1}$ | $-0.67 \times 10^{-5} \leq \text{Re}(\lambda'_{32j}\lambda'^*_{3j1}) \leq 1.168 \times 10^{-5}$ | $K^+ \rightarrow \pi^+\nu\bar{\nu}$ |
| $(j = 2, 3)$ | $-1.0 \times 10^{-5} \leq \text{Im}(\lambda'_{32j}\lambda'^*_{3j1}) \leq 0.85 \times 10^{-5}$ | $K^+ \rightarrow \pi^+\nu\bar{\nu}$ |
| $|\lambda'_{11k}\lambda'_{22k}|$ | $4.0 \times 10^{-7}$ | $\mu Ti \rightarrow e Ti$ [35] |
| $|\lambda'_{21k}\lambda'_{12k}|$ | $4.3 \times 10^{-7}$ | $\mu Ti \rightarrow e Ti$ [35] |
| $|\lambda'_{11k}\lambda'_{32k}|$ | $0.89 \times 10^{-5}$ | $K^+ \rightarrow \pi^+\nu\bar{\nu}$ |
| $|\lambda'_{21k}\lambda'_{32k}|$ | $0.89 \times 10^{-5}$ | $K^+ \rightarrow \pi^+\nu\bar{\nu}$ |
| $|\lambda'_{31k}\lambda'_{12k}|$ | $0.89 \times 10^{-5}$ | $K^+ \rightarrow \pi^+\nu\bar{\nu}$ |
| $|\lambda'_{31k}\lambda'_{22k}|$ | $0.89 \times 10^{-5}$ | $K^+ \rightarrow \pi^+\nu\bar{\nu}$ |
| $|\lambda'_{ij2}\lambda'_{ij1}|$ | $0.89 \times 10^{-5}$ | $K^+ \rightarrow \pi^+\nu\bar{\nu}$ |

TABLE I: Current relevant upper bounds on the values of products of two $\mathcal{R}$ couplings. Bounds corresponding to Ref. [34] and Ref. [35] for certain combinations are stronger than the one obtained in here. $\blacklozenge$ $K_L \rightarrow \mu e$ process put stronger limits ($8 \times 10^{-7}$) on the combinations: $|\lambda'_{212}\lambda'_{111}|$, $|\lambda'_{222}\lambda'_{121}|$ and $|\lambda'_{232}\lambda'_{131}|$ [36].

For $m_{\tilde{u}_L} = 100$ GeV. However, in Ref. [36] authors have obtained stronger bound of $|\lambda'_{ij1}\lambda'_{2j2}| \leq 8.0 \times 10^{-7}$ from $K_L \rightarrow \mu e$ process.

We now display in Table I the best current upper bounds on the product of two $\mathcal{R}$ couplings with the processes which provides the limit. We present bounds, obtained
FIG. 5: 90% C.L. allowed regions for the product of $R$ couplings (with $i = i'$)(in units of $10^{-5}$), which interfere with the SM from Br $(K^+ \rightarrow \pi^+ \nu\bar{\nu})$ data. The solid and dashed contours correspond to $(i = e, \mu)$ and $(i = \tau)$ case respectively. Figure (a) and (b) correspond to the first and second combination of $R$ couplings in Equation 22.

by us from $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ as well as some earlier bounds from other processes like $\Delta M_K$ and $\mu \rightarrow e$ conversion in the nuclei which are some times stronger.

From our analysis, it is clear that the bounds obtained on the different combinations of $R$ couplings from $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ process is of the order of $10^{-5}$. In the case where the pair of $R$ couplings which interfere with the standard model we obtain bounds on real and imaginary part of the pair separately from Figure 5 while in the non-interfering case, we obtain bound on the magnitude of the pair of $R$ couplings involved. Note that earlier bounds on the above combination were $\sim \mathcal{O}(10^{-3})$ allowing significant room for enhancement of the process $K^+ \rightarrow \pi^+ \nu\bar{\nu}$. The other process $K_L \rightarrow \mu^+ \mu^-$ puts a limit on Re($\lambda_{232}^{'}\lambda_{231}^{''}$), which is of the order of $10^{-6}$. The $\Delta M_K$ and $\mu \rightarrow e$ conversion in the nuclei provide bounds of the order of $10^{-7}$ on some combinations of $R$ couplings which are otherwise weakly constrained.
by our analysis. 

**Prediction for** $K_L \to \pi^0 \nu \bar{\nu}$ **process**

In the presence of $L$ number violating $\lambda_{ijk} L_i Q_j D^c_k$ operator, several products of two $R$ couplings can contribute to the $CP$ violating process $K_L \to \pi^0 \nu \bar{\nu}$. We have

$$\frac{\text{Br}(K_L \to \pi^0 \nu \bar{\nu})}{\text{Br}(K^+ \to \pi^0 e^+ \nu)} = \kappa \left[ \sum_{\ell=e,\mu,\tau} |\text{Im}(\Delta_{SM}^R + \Delta_{K,\ell \ell}^R)|^2 + \sum_{i\neq i'} |\Delta_{K,ii'}^R|^2 \right]$$

$$\kappa = \frac{r_{K_L}}{r_{K^+}} \frac{\tau_{K_L}}{\tau_{K^+}} \frac{\alpha^2}{2 \pi^2 s_W^4 |V_{us}|^2}$$

(27)

$$\Delta_{K,ii'}^R = \frac{\pi s_W^2}{2 G_F \alpha} \left[ \frac{\lambda_{ij1}^* \lambda_{ij2}^*}{2 m_{ii}^2} - \frac{\lambda_{ij2k}^* \lambda_{ij1k}^*}{2 m_{ik}^2} + \frac{\lambda_{ij1k}^* \lambda_{ij2k}^*}{2 m_{ik}^2} \right].$$

(28)

One notes that for $i = i'$, the decays are $CP$ violating, but for $i \neq i'$, the decays are not necessarily $CP$ violating which is very different from the SM.

As we see from the Equation 28 that several combinations of two $R$ couplings are involved in this case. In our numerical calculation we consider each combination of $R$ couplings one by one. For $i = i'$ these $R$ contributions interfere with the standard model one, while for $i \neq i'$, these new operators create neutrino pair which are not $CP$ eigenstate. We will treat these two cases separately. First we assume $i = i'$, in this case the branching ratio $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ can reach as large as $2.0 \times 10^{-9}$ for the allowed values of $\lambda_{131}^* \lambda_{132}^*, \lambda_{232}^* \lambda_{231}^*, \lambda_{332}^* \lambda_{331}^*$ couplings. In fact, we find that for all the relevant pairs of $R$ couplings (with $i = i'$), whose bound is $\sim \mathcal{O}(10^{-5})$, the maximum value of the branching ratio is $2.0 \times 10^{-9}$, which is almost two order of magnitude larger than the standard model prediction: $2.93^{+0.84}_{-0.67} \times 10^{-11}$. Experimentally we only have an upper limit for this branching ratio, which is $5.9 \times 10^{-7}$ at 90% CL. In the second scenario, where $i \neq i'$, $K_L \to \mu e$ and $\mu Ti \to e Ti$ set bounds of the order of $\mathcal{O}(10^{-7})$ on the magnitude of following combination of $R$ couplings: $| \lambda_{11k}^* \lambda_{22k}^* |$, $| \lambda_{21k}^* \lambda_{12k}^* |$, $| \lambda_{212}^* \lambda_{111}^* |$, $| \lambda_{222}^* \lambda_{121}^* |$, and $| \lambda_{232}^* \lambda_{131}^* |$. As can be seen from the Table I that the bound on the magnitude of other combinations of $R$ couplings are $0.89 \times 10^{-5}$ obtained from $K^+ \to \pi^+ \nu \bar{\nu}$. We find that the branching ratio can reach up to $1.39 \times 10^{-9}$ for the pair of $R$ couplings in Equation 28 whose magnitude satisfy the above mentioned limit.

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Note that we do not use $\epsilon'/\epsilon$ to put bounds on products of $\lambda'$ couplings because the theoretical expression for this quantity involves $R$ scalar couplings, involving more model dependence.
IV. CONCLUSIONS

In this paper we have examined the effects of new physics originated from two different kind of models on several rare flavor changing processes involving $K$ meson: $K^+ \rightarrow \pi^+ \nu \bar{\nu}, K_L \rightarrow \mu^+ \mu^-$, $e'/e$, $K_L \rightarrow \pi^0 e^+e^-$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$. In the first model, in addition to the SM quarks we also have one extra down-type singlet quark. The presence of such an additional singlet quark leads to a new off-diagonal $Z$ mediated FCNC coupling $U_{ij}$, between the SM quarks of flavor $i$ and $j$. In this paper we have considered off-diagonal $Z$ coupling between first two generation down type SM quarks, denoted by a complex parameter $U_{sd}$. We then obtained 90% C.L. bound on this mixing parameter using known experimental data on $K^+ \rightarrow \pi^+ \nu \bar{\nu}, K_L \rightarrow \mu^+ \mu^-$ and $e'/e$. It turned out that the allowed parameter space of $U_{sd}$ from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ process is severely constrained from $e'/e$ (imaginary part of $U_{sd}$): $-1.91 \times 10^{-6} \leq \text{Im}(U_{sd}) \leq 3.72 \times 10^{-6}$, while the $(K_L \rightarrow \mu^+ \mu^-)$SD constrains the real part of $U_{sd}$: $-1.12 \times 10^{-5} \leq \text{Re}(U_{sd}) \leq 3.45 \times 10^{-6}$. We did not find any significant deviation from the SM prediction of $K^0 - \bar{K}^0$ oscillation. The value of $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ can reach up to $1.5 \times 10^{-10}$ at the edge of the allowed parameter space of $U_{sd}$. Moreover, we can also reproduce the central value of experimentally measured $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. We have also studied other $CP$ violating kaon processes, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 e^+e^-$ in the allowed parameter space of $U_{sd}$. The value of $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ can reach as high as $10 \times 10^{-11}$ at the edge of the allowed parameter space. At present from experiment we have an upper limit on this branching ratio $5.9 \times 10^{-7}$ at 90% CL [17]. We have found that the process $K_L \rightarrow \pi^0 e^+e^-$ is very weakly dependent on the new physics parameter $U_{sd}$, because of the fact that the dominant contribution to this decay amplitude arises from the mixing of $K_L$ and $K_S$, followed by $K_S \rightarrow \pi^0 e^+e^-$ decay.

The second model, we have considered is the $\mathbb{R}$ MSSM. We have computed the bounds on the product of two $\mathbb{R}$ couplings of the type $\lambda'\lambda'$ using $K^+ \rightarrow \pi^+ \nu \bar{\nu}, K_L \rightarrow \mu^+ \mu^-$ and $K^+ \rightarrow \pi^+ \mu^+ e^-$ processes. We have assumed that the product of two $\mathbb{R}$ couplings are in general complex and all the sfermion masses are degenerate with mass of 100 GeV in order to compare with earlier bounds obtained in literature. One can obtain bounds for any other sfermion mass by scaling. In deriving the bounds full standard model amplitudes have been taken into account. One should note that in several cases, our bounds are complement to the bounds obtained from other processes like $\Delta M_K, K_L \rightarrow \mu e$ and $\mu Ti \rightarrow e Ti$. We
have found that processes like $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ $K_L \rightarrow \pi^0 \nu \bar{\nu}$ can be significantly enhanced compared to their standard model predictions. The constraints on the product of $R$ couplings $\lambda_{131}^\prime \lambda_{132}^\star$ from the decay mode $K_L \rightarrow \pi^0 e^+ e^-$ is $\sim \mathcal{O}(10^{-4})$, which is one order of magnitude weaker than the bound obtained from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. As we have explained before, the dominant contribution to the $K_L \rightarrow \pi^0 e^+ e^-$ process arises from the mixing between $K_L$ and $K_S$. After taking into account this mixing contribution, the standard model prediction for the $\text{Br}(K_L \rightarrow \pi^0 e^+ e^-) = (3.2^{+1.2}_{-0.8}) \times 10^{-11}$ [34], whereas the experimental upper bound in $5.1 \times 10^{-10}$ at 90$\%$CL, leaving a very small room for the new physics contribution.

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