Glueballs as gravitons

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We study the implications of the graviton solutions in the glueball spectrum in the bottom-up approach of the AdS/CFT correspondence.

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I. INTRODUCTION

Quantum Chromodynamics (QCD), the theory of the strong interactions, has eluded an analytical solution since its formulation 1. One of the aspects of QCD which has attracted much attention since its formulation is the glueball spectrum 2, 3. In an attempt to understand this theory a procedure to extend the AdS/CFT correspondence breaking conformal invariance and supersymmetry was proposed 4–6. In this so called top-down approach the glueball spectrum has been studied 7–11. The AdS geometry of the dual theory is an AdS-black-hole geometry where the horizon plays the role of an infrared (IR) brane. One relevant feature found in ref.10 is that the graviton, not the dilaton, corresponds to the lightest scalar glueball. This feature is in good agreement with the lattice spectrum were the lightest scalar glueball has a much lower mass than its immediate excitation which is almost degenerate with the tensor glueball 26–31.

A different strategy, the so-called bottom-up approach starts from QCD and attempts to construct a five-dimensional holographic dual. One implements duality in nearly conformal conditions defining QCD on the four dimensional boundary and introducing a bulk space which is a slice of AdS5 whose size is related to $\Lambda_{QCD}$ 24–28. This is the so called hard wall approximation. Later on, in order to reproduce the Regge trajectories, the so called soft wall approximation was introduced 24–25. Within the bottom-up strategy and in both, hard wall and the soft wall approximations, glueballs arising from the correspondence of fields in AdS5 have been studied 26–31.

The analysis in the top-down approach led to the conclusion that scalar and tensor glueballs arise from the correspondence with the graviton 9, 11. The purpose of this investigation is to find the role of the AdS graviton in the correspondence in the bottom-up approach. We study the structure of the scalar and tensor components of the AdS5 graviton and compare with the conventional spectra of the bottom-up approach to search for their interpretation 26–51. Next we discuss the spectrum in the hard wall approximation and later we shall proceed with the discussion in the soft wall approximation. We end up by drawing some conclusions.

II. GLEUCEBALLS AS HARD WALL GRAVITONS

According to AdS/CFT correspondence massless scalar string states are dual to boundary scalar glueball operators 20. The glueball operators are massless and respecting conformal invariance. Once we introduce a size in the AdS space there is an infrared cut off in the boundary which is proportional $1/\Lambda_{QCD}$ breaking conformal invariance. The presence of the slice implies an infinite tower of discrete modes for the bulk states. These bulk discrete modes are related to the masses of the non-conformal glueball operators.

In the bottom-up approach supergravity fields in the AdS5 slice times a compact $S_5$ space are considered an approximation for a string dual to QCD. The metric of this space can be written as

$$ds^2 = \frac{R^2}{z^2} (dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu) + R^2 d\Omega_5,$$

where $\eta_{\mu\nu}$ is the Minkowski metric and the size of the slice in the holographic coordinate $0 < z < z_{max}$ is related to the scale of QCD $z_{max} = \frac{1}{\Lambda_{QCD}}$. The field equation for the dual of the scalar glueballs is

$$\partial_x^2 \Phi - \frac{3}{z} \partial_z \Phi + \eta_{\mu\nu} \partial_x^\mu \partial_x^\nu \Phi = 0.$$  \hspace{1cm} (2)

Considering plane wave solutions

$$\Phi(x, z) = C_k e^{-iP_x z^2} J_2(u_k z),$$  \hspace{1cm} (3)

the corresponding boundary conditions determine the spectrum by the following equations 26–28.

$$\text{Dirichlet} \quad J_2(\chi_k) = 0, \quad \text{Neumann} \quad J_1(\xi_k) = 0.$$  \hspace{1cm} (4)

Here $J_2$ and $J_1$ are Bessel functions, for the Dirichlet modes $u_k = \chi_k \Lambda_{QCD}$, while for the Neumann modes $u_k = \xi_k \Lambda_{QCD}$ and $k$ labels the energy modes. The corresponding solutions for the mass of the glueballs in dimensional units $\Lambda_{QCD}$ are given by the zeros of the corresponding Bessel functions.
We have reproduced conventional wisdom [26, 27]. In the next section we show that this approach is in relative good agreement with the lattice spectrum.

Motivated by the work of ref. [10] in the top-down approach, we study the contribution of the scalar component of the graviton field in the sliced AdS$_5$ geometry. The equation for the scalar component of the graviton field in this AdS$_5$ geometry with an appropriate choice of gauge is exactly Eq. (2). Thus in this case the spectrum of the scalar field and the graviton coincide if we use for the latter the same boundary conditions.

Let us recall that the energy modes of the scalar component of the graviton and the scalar field obtained by solving Eq. (2) with Dirichlet and Neumann boundary conditions lead to Eqs. (3) which we have shown in Table I.

In the case of particles with spin, in particular we address those of even spin, in the conventional bottom-up approach the spectrum of the glueballs is determined by equations of motion of scalar fields with mass in AdS$_5$,

$$\partial^2 \Phi - \frac{3}{z} \partial_z \Phi + (\eta_{\mu \nu} \partial_\mu \partial_\nu - \frac{(\mu R)^2}{z^2}) \Phi = 0.$$  (5)

where $(\mu R)^2 = J(J+4)$, $J$ being the spin, see refs. [25, 27].

Considering again plane wave solutions the spectrum is given by,

Dirichlet $J_n(\chi_{n,k}) = 0$,

Neumann $(2-n)J_n(\xi_{n,k}) + \xi_{n,k} J_{n-1}(\xi_{n,k}) = 0$,  (6)

where $J_n$ are Bessel functions, $n = 2 + J$ and $k$ labels the modes.

The tensor component of the graviton is degenerate with the scalar field unless a AdS$_5$ mass term is added. This must be done in order to have the adequate conformal dimension on the boundary, which is given by the mass. Once the mass term is included the tensor graviton spectrum is determined by Eq. (4). In Table II we show the modes of the tensor field.

Summarizing, in AdS$_5$ in the hard wall approximation, the graviton spectrum determines the same equations as obtained previously by introducing scalar and tensor fields [26, 27].

### III. A HARD WALL GLUEBALL SPECTRUM

We proceed to compare the just described AdS$_5$ spectrum to the quenched lattice glueball spectrum [13, 18].

| $k$ | 1 | 2 | 3 | 4 | 5 | ...
|-----|---|---|---|---|---|---|
| Dirichlet | 5.136 | 8.417 | 11.620 | 14.796 | 17.960 | ...
| Neumann | 3.832 | 7.016 | 10.173 | 13.324 | 16.471 | ...

TABLE I: Energy modes for the scalar glueball with Dirichlet and Neumann boundary conditions in units of $\Lambda_{QCD}$

| $k$ | 1 | 2 | 3 | 4 | 5 | ...
|-----|---|---|---|---|---|---|
| D tensor | 7.588 | 11.068 | 14.375 | 17.616 | 20.827 | ...
| N tensor | 5.981 | 9.537 | 12.854 | 16.096 | 19.304 | ...

TABLE II: Energy modes for Dirichlet (D) and Neumann (N) boundary conditions both for the tensor field.

In order to perform the comparison we need to fix the scale of the AdS$_5$ calculation. For that purpose we fix here the lightest glueball mass. D and N stand for the boundary conditions Dirichlet and Neumann respectively. Given the crudeness of the model the fit does quite a good job in reproducing the spectrum.

| $k$ | 0$^+$ | 2$^+$ | 0$^{++}$ | 2$^{++}$ | 0$^{+\prime}$ |
|-----|-------|-------|--------|--------|--------|
| MP  | 1.73  | 2.40  | 2.67   |        |        |
| YC  | 1.71  | 2.39  |        |        |        |
| LTW | 1.47  | 2.15  | 2.75   | 3.37   | 3.39   |
| Lattice | 1.64 | 2.31 | 2.74 | 3.37 | 3.39 |
| D   | 1.64  | 2.42  | 2.69   | 3.53   | 3.71   |
| N   | 1.64  | 2.56  | 3.00   | 4.08   | 4.34   |

TABLE III: Glueball masses (GeV) from lattice calculations MP [13], YC [19] and LTW [18] and Lattice (average) compared with AdS$_5$ model calculations. D stands for Dirichlet, N for Neumann boundary conditions.

Thus we can generate a reasonable glueball spectrum from the graviton scalar and tensor components. We proceed next to look into more complicated models to learn about the possible roles of the graviton components.

### IV. GLUEBALLS AS SOFT WALL GRAVITONS

Another procedure to determine the spectrum of QCD from AdS$_5$ has been a mechanism for a gravitational background which cut-offs smoothly in the holographic coordinate. The mechanism introduced some time ago for that purpose which was defined to reproduce the Regge behavior of the mesons consists in incorporating a dilaton field $\Phi$ and a metric $g_{MN}$ with characteristic properties [24]. In this formalism the glueballs are described by 5d fields propagating on this background with the action given by

$$I = \int d^5 x \sqrt{-g} e^{-\Phi} \mathcal{L},$$  (7)

where $\mathcal{L}$ is the field lagrangian density, $\Phi$ the dilaton field and $g = \det g_{MN}$. Since our aim is to find the glueballs associated with the AdS graviton we generalize the metrics studied previously without changing the results for the conventionals. We propose the metric

$$g_{MN}(z) = e^{-\alpha^2(z^2/R^2)} \frac{R^2}{z^2}(-1,1,1,1,1)$$  (8)
and the dilaton field

$$\Phi(z) = \beta^2 z^2.$$  \hfill (9)

This is a slight modification of the simplest metric studied in ref. \[28\] with the aim of studying the behavior of the $AdS$ graviton. In order to implement the condition of Regge trajectories used in previous calculation \[24, 28\] we must impose

$$\frac{3\alpha^2}{2} + \beta^2 = 1.$$  \hfill (10)

It is important to note that the change of metric affects the lagrangian term in the action leading to an additional multiplicative factor $e^{\alpha^2 z^2}$ in the integral which is the reason for the factor 3/2 in Eq. (10).

Given these restriction the spectra for the glueballs arising from the scalar and tensor fields, using $R = 1$ in Eq. (8), are those of ref. \[28\]

$$M_0^2 = 4n + 8,$$

$$M_2^2 = 4n + 12.$$  

respectively.

Our next step is to find the solution for the scalar component of the graviton. The corresponding equation is

$$-\Psi''(z) + V_{GS}(z)\Psi(z) = M^2\Psi(z),$$  \hfill (11)

where

$$V_{GS}(z) = \frac{8e^{-\alpha^2 z^2}}{z^2} - \frac{17}{4z^2} - 7\alpha^2 - \frac{15\alpha^4 z^2}{4}.$$  \hfill (12)

Let us perform the change of variables $\tau = \alpha z/\sqrt{2}$, Eq. (11) becomes

$$-\frac{d^2\Psi}{d\tau^2}(\tau) + V_{GSS}(\tau)\Psi(\tau) = \Lambda^2\Psi(\tau),$$  \hfill (13)

where

$$\Lambda^2 = \frac{2M^2}{\alpha^2}$$  \hfill (14)

and

$$V_{GSS}(\tau) = \frac{8e^{-2\tau^2}}{\tau^2} - \frac{17}{4\tau^2} - 14 - 15\tau^2.$$  \hfill (15)

In order to study the boundary conditions at the origin we Taylor expand the exponential to second order, leading to potential of the form

$$V_{low}(\tau) \sim \frac{15}{4\tau^2}$$  \hfill (16)

at small $\tau$, which leads to the low $\tau$ behavior for the field function

$$\Psi(\tau) \sim \tau^{5/2}.$$  \hfill (17)

Note that $\alpha$ is just a scale in the mass equation Eq. (14), and that the mode solutions $\Lambda_k$ are independent of this scale.

For $\alpha^2 > 0$ no mode solutions are found. The solutions are damped oscillations as shown in Fig. [1].

![Fig. 1: Typical solution for the graviton equation for $\alpha^2 > 0$.](image)

The well behaved modes appear for $\alpha^2 < 0$. In this case we use the variable $t = i\tau, t > 0$ and $a = i\alpha, a^2 > 0$. Eqs. (13), (14) and (20) now become

$$-\frac{d^2\Psi}{dt^2}(t) + V_{GSS}(t)\Psi(\tau) = \Lambda^2\Psi(t),$$  \hfill (18)

where

$$\Lambda^2 = \frac{2M^2}{a^2}$$  \hfill (19)

and

$$V_{GSS}(t) = \frac{8e^{2t^2}}{t^2} - \frac{17}{4t^2} + 14 - 15t^2.$$  \hfill (20)

It is important to note the change of sign in the exponential but also in the constant term which lead to quantized modes solutions. We show in Fig. [2] the solutions for the first three modes with the low correct $t$ behavior, $\Psi(t) \sim t^{5/2}$. The values of the modes are given in Table [IV].

The equations for the mode energies are now given by Eq. (19),
FIG. 2: Typical solution for the equation of the scalar component of the graviton for $a^2 > 0$. The solid curve shows the wave function for the lowest (k=0) mode, the dashed curve that of the first mode (k=1) and the dotted curve that of the second mode (k=2).

TABLE IV: The scalar modes of the graviton equation before "a"-scaling.

| k  | 0  | 1  | 2  | 4  | ... |
|----|----|----|----|----|-----|
| scalar graviton | 7.341 | 9.065 | 10.818 | 12.568 | ... |

The tensor component of the graviton field leads to the same equation as the scalar component. By adding a mass term as before the potential changes to

$$V_{GSS}(t) = \frac{8e^{2t^2}}{t^2} + \frac{4J(J+4) - 17}{4t^2} + 14 - 15t^2,$$

which for the tensor graviton $J = 2$, leads to a short range potential

$$V_{low}(t) \sim \frac{63}{4t^2},$$

at small $t$, which leads to a different low $t$ behavior for the field function

$$\Psi(t) \sim t^4.$$  

The corresponding $\Lambda_k$ modes obtained by solving the differential equation with this low $t$ behavior, as was done before for the scalar graviton components, are shown in Table[V] In Fig. 3 we show the mode solutions.

In Table[V] we show the corresponding mass modes $M_k$ for scalar and tensor fields and scalar and tensor graviton components.

With these results we now proceed to analyze the glueball spectrum in the next section.

V. A SOFT WALL GLUEBALL SPECTRUM

In order to fit the modes to the glueball masses we need to fit two scales: one associated with the conventional fields and the other with the graviton modes. Our calculation at this point is naive and corrections should be applied [28]. However the results we are going to present suffice for the purposes of the present investigation. They are shown in Table[VII].

The first two lines represent the graviton spectrum adjusting the single scale to the lattice data. The first fit is based on the low glueball spectrum obtained by adjusting the lowest glueball mass to the second scalar mode to keep the $0^{++}, 2^{++}, 0^{++}, 2^{++}$ ordering thereafter. Since the graviton has two scalar components below its tensor component, the lowest scalar has a mass below the lowest lattice scalar mass, in correspondence with some sum rule calculations [32]. This spectrum leads to very low excitations. The second shown spectrum fits the 2.71 GeV resonance leading to a spectrum with an extra scalar,
which has not been seen in lattice calculations, and the gap between the higher lying states is too small. We conclude that the graviton modes alone cannot give rise to a reasonable spectrum.

We next analyze the conventional field spectrum considering fundamental the doubling of the $2^{++},0^{++}$. We assume these states to be approximately degenerate, by fitting initially the two to the 2.15 GeV resonance and then to the 2.71 GeV. The fit produces the low lying spectrum but fails badly in the highest pair. The latter on the contrary does a good job in the high lying states but misses greatly the lowest glueball mass.

Lastly we choose a mixed scenario. The graviton is used for the low lying modes and the field spectrum for the high lying modes as happens naturally in AdS$_5$. We show two fits. In the first we fit the second mode to 1.64 GeV to fix the metric scale, and the first scalar field mode to 2.71 GeV. This fit leads again to a low first graviton scalar mode, maybe a remnant of the scale dilaton. The second fit is based on fitting the first graviton tensor component to the 2.15 GeV resonance, keeping the field fits as before. The fit is excellent if it were not for the additional scalar at 2.00 GeV. One can see moreover that in this case the gap in the graviton sector is small and therefore three additional scalars appear before the 2.71 GeV resonance as shown in Table VII.

Our soft model calculation is just a first step in the analysis since probably additional terms might be needed to reproduce the correct ordering of the spectrum. However it comes out of our analysis that the graviton modes do a good job for the lowest modes and the field modes for the higher modes as in AdS$_7$. In the first fit the lowest scalar mode can be physically understood as a smaller mass fundamental state in a more precise calculation and could be related to the scale dilaton appearing in sum rule and 1/N calculations. As in our previous analysis of AdS$_7$, we also conclude that a very low mass glueball requires an almost doubling of the 1.64 GeV resonance and a $2^{++}$ below the 2.71 GeV resonance as seen in Table VII. The second fit requires from an additional $0^{++}$ below the first $2^{++}$ resonance. If we compare with the present lattice data the gap of the graviton spectrum is too small in this model as shown in Table VIII. The consequence of this result is that the metric will have to be modified to eliminate these excitations from the spectrum. On the other hand, the degeneracy of the field, modes which we have considered relevant for the fitting of the higher lying spectrum, is only approximate and may be eliminated by modifying the dilaton field and/or the metric.

### Table VII: Glueball masses (GeV) from lattice calculations

|       | $0^{++}$ | $0^{++}$ | $2^{++}$ | $0^{++}$ | $2^{++}$ | $0^{++}$ |
|-------|----------|----------|----------|----------|----------|----------|
| MP    | 1.73     | 2.40     | 2.67     |          |          |          |
| YC    | 1.71     | 2.39     |          |          |          |          |
| LTW   | 1.47     | 2.15     | 2.75     | 3.37     | 3.39     |          |
| Lattice | 1.64 | 2.31 | 2.71 | 3.37 | 3.39 |          |
| graviton | 1.32 | 1.64 | 1.76 | 1.96 | 2.09 | 2.27 |
| graviton | 1.83 | 2.27 | 2.43 | 2.71 | 2.90 | 3.14 |
| fields | 1.75  | 2.15 | 2.15 | 2.48 | 2.48 |          |
| fields | 2.20  | 2.71 | 2.71 | 3.13 | 3.13 |          |
| mixed | 1.32 | 1.64 | 1.76 | 2.71 | 3.32 | 3.32 |
| mixed | 1.62 | 2.00 | 2.15 | 2.71 | 3.32 | 3.32 |

### Table VIII: Average glueball masses (GeV) from lattice calculations

|       | $0^{++}$ | $0^{++}$ | $2^{++}$ | $0^{++}$ | $0^{++}$ | $2^{++}$ |
|-------|----------|----------|----------|----------|----------|----------|
| Lattice | 1.64 | 2.31 | – | – | 2.71 | 3.37 |
| mixed | 1.32 | 1.64 | 1.76 | 1.96 | 2.09 | 2.71 |
| mixed | 1.62 | 2.00 | 2.15 | 2.40 | 2.56 | 2.71 |

### VI. CONCLUSIONS

We have discussed the spectrum of the scalar and tensor glueballs under the assumption that in an AdS$_5$ approach scalar and tensor components of the graviton might play a significant role corresponding to the lowest lying glueballs. We have studied the problem in a hard and soft wall approaches. In the former the graviton reproduces exactly the same equations as the field approach. By fitting an energy scale the results of the model reproduce reasonably well the lattice results. In the soft wall approach we reproduce for the fields the dilaton approach. For the graviton we find new solutions which depend on a metric parameter $a^2 > 0$. The metric grows as $e^{a^2 z^2}$ which implies that it grows at short distances becoming pure AdS$_5$ at infinity leading to a potential which is able to bind. We have solved numerically the equations for the scalar and the tensor components, adding to the latter a mass term $\sim J(J+4)/z^2$. The graviton components give good fits for the low mass spectrum, while the field solutions reproduce correctly the gap of the upper modes. We find two solutions, one which reproduces the lattice spectrum with additional scalar modes and a second which has a very low mass scalar. The latter is characterized by an almost doubling of the $0^{++}$ (1.64) mode and the $2^{++}$ (1.76) tensor and some additional scalars, a result which also appeared in the top-down approach. These additional scalars might be an artifact of the soft model which leads to a small gap in the graviton modes.

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