Spectral Correlation Between the Non-uniform Input Ground Motion and the Dynamic Response of the Surrounding Rocks of a Long Tunnel

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**Abstract:** To generate several different time-history curves of ground motion, which have the ability to reflect local site effects, such as the traveling-wave effect and attenuation effect, for long circular tunnels, a random method for the synthesis of ground motion is employed. The dynamic response of the surrounding rocks of a long tunnel under the uniform and non-uniform input ground motions is analyzed using the vertical input method. Based on this, the spectral correlation between the input ground motion and the dynamic response of the surrounding rocks is investigated through the coherence function. Unlike the results under the uniform input ground motion, the transmission coefficient of the surrounding rock monitoring points of each section along the axial direction of the tunnel under the non-uniform input ground motion varies in the main frequency band. In addition, at the same frequency, the transmission coefficient of the monitoring points gradually increases along the tunnel’s axial direction, but the increasing magnitude is variable. The correlation function of each monitoring point along the tunnel axis under the non-uniform input ground motion generally decreases with frequency. Moreover, the further the distance from the tunnel entrance, the faster the attenuation of correlation. The maximum coherence coefficient and the axial distance of the tunnel are basically in conformity with the quadratic polynomial relationship. However, the maximum coherence coefficient of the non-uniform input ground motion is considerably greater than that of the uniform input ground motion. The gap increases with the distance, and the maximum gap is about 0.15.
1. Introduction

In traffic engineering, highways and railways will expand deeper in western China. Owing to the criticality of the terrain and location, long tunnels that are several kilometers or even several tens of kilometers in length have become the main structure and throat of lifeline projects, including highways and railways. However, in western China, most of the long tunnels are approaching or crossing the seismically active fault zone, with the basic seismic intensity of the project site being mostly VII or above. Over the past century, a series of large earthquakes that have occurred worldwide caused the cracking, collapse, and destruction of underground engineering structures [1–4]. It is found that earthquake damage along the tunnel has many different types and that the damage mode of one earthquake is extremely complicated. These unique phenomena of earthquake damage are caused by non-uniform ground motion [5].

The input ground motion for the dynamic response of tunnels includes the uniform and non-uniform conditions. Currently, scholars have conducted a systematic research on the surrounding rock deformation and the mechanical characteristics of the structure under the uniform input ground motion. Zheng et al. [6] conducted analysis on tunnel stability using two methods, namely, dynamic finite element static analysis and complete dynamic analysis, under the action of earthquake. It is pointed out that local tensile failure first occurs at the top of the tunnel and then at the sides. Qu et al. [7] and Bao et al. [8] analyzed the displacement, stress, and plastic zone of the diversion tunnel cross section under the action of earthquake. Gao et al. [9] conducted shaking table tests on tunnel models with different types of seismic waves, seismic intensity, and different burial depths. The results reveal that the changing law of the tunnel lining stress with burial depth is similar to that with different types of seismic waves and different peak values of seismic acceleration. With regard to the dynamic response of a tunnel under the non-uniform input ground motion, oblique incidence is selected as the main input mode. However, oblique incidence only considers the traveling-wave effect of ground motion, not the attenuation effect and coherence effect. Therefore, a few researches were conducted on the dynamic response of a tunnel under the non-uniform input ground motion based on the stochastic process theory. Earlier, Li et al. [10] employed the non-uniform input ground motion to study the dynamic response of underground caverns. Based on this, Zhao et al. [11] studied the dynamic nonlinear response of underground rock caverns under the excitation of strong seismic traveling waves. In addition, they pointed out that when the amplitude of local vibration is greater than 0.2 g, the traveling-wave effect of ground motion significantly increases the damage of the underground cavern. Li and Song [12] created a numerical model of the longitudinal dynamic response of the foundation-tunnel system and investigated the longitudinal traveling-wave effect of the tunnel under the action of non-uniform earthquakes.

It can be concluded that the researches still focus on the dynamic response of the tunnel and the destruction characteristics of the structure. Research analyzing the spectral correlation between the non-uniform input ground motion and the dynamic response of the surrounding rocks of the long tunnel is scarce. Therefore, in this paper, a long and large water conveyance tunnel in the southwest China is used as the research object, and the acceleration time-history curves of the non-uniform input ground motion are generated in accordance with the stochastic engineering theory. The dynamic response of the surrounding rocks of the long tunnel under the uniform and non-uniform input ground motion is analyzed and compared.
motions is analyzed using the vertical input method. On this basis, the spectral correlation between the input ground motion and the dynamic response of the surrounding rocks is analyzed through the coherence function.

2. Numerical model and acceleration time-history curves of the non-uniform input ground motion

2.1 Numerical model
The water conveyance tunnel project has a length of 63.426 km and a diameter of 8.4 m. It is a typical long tunnel in a complex seismic geological environment. To analyze the dynamic response of the water conveyance tunnel under the uniform and non-uniform input ground motions, assuming that the tunnel length is 1000 m and the burial depth is more than 100 m, a three-dimensional numerical model is constructed, which is presented in Figure 1(a). The center of the tunnel entrance is indicated as the origin and the axial direction of the tunnel as the y-direction. The direction perpendicular to the axial direction is indicated as the x-direction and the vertical direction as the z-direction. The monitoring scheme, which is presented in Figure 1, is arranged to facilitate the analysis of the dynamic response of the surrounding rocks of the tunnel. Figure 1(b) presents the layout of the surrounding rock monitoring points of the tunnel, and Figure 1(c) presents the layout of the surrounding rock monitoring points distributed along the elevation.

Figure 1. Numerical model and layout of the surrounding rock monitoring points
2.2 Acceleration time-history curves of the non-uniform input ground motion

Because the tunnel is 1000 m long, it is assumed that an acceleration time-history curve is generated every 50 m. Based on the stochastic process theory and displacement reference verification [10], a total of 21 acceleration time-history curves are generated, as presented in Figure 2. It can be observed that the waveforms of the acceleration time-history of the excitation points are the same. With the distance increase, the ground motion time-history exhibits a certain delay effect, and the peak acceleration value gradually decreases. Thus, the acceleration time-history curves of all the generated excitation points reflect well both the traveling-wave effect and attenuation effect, indicating the feasibility of the acceleration time-history curve of the non-uniform input ground motion. To analyze the effect of the non-uniform input ground motion on long tunnels, the uniform input seismic motion is chosen as the comparison condition. The acceleration time-history curve of the 1# excitation point is used for the acceleration time-history curve of the uniform input ground motion. The input ground motion is horizontal vibration in the y-direction, and the mode is vertical input.

![Figure 2. Acceleration time-history curves of the non-uniform input ground motion](image)

3. Frequency spectrum analysis of the surrounding rock

3.1 Uniform input ground motion

The results indicate the consistency of the acceleration time-history curves of the surrounding rock monitoring points at the same position in each section along the axial direction of the tunnel under the uniform input ground motion. Thus, the Fourier spectrum and transmission function of the monitoring points at the y = 600 m section under the uniform input ground motion are analyzed, as presented in Figure 3. Moreover, it can be found that the Fourier spectrum of each monitoring point of the tunnel section is similar to that of the input ground motion. In the main frequency band, the change in the transmission coefficient with frequency generally exhibits a trend of linear decrease with increasing frequency. The transmission coefficient changes from 1.6 to 0.8, and the transmission coefficients of the different monitoring points are also different. The general order is top > right = left > bottom, as presented in Figure 3(b).
3.2 Non-uniform input ground motion

Due to the difference in the acceleration time-history curves of the surrounding rock monitoring points, which are at the same position in each section along the axial direction of the tunnel under the non-uniform input ground motion, the Fourier spectrum and transmission function of the right monitoring point at different sections are analyzed, as presented in Figure 4. Similar to the results under the uniform input ground motion, the Fourier spectrum of the surrounding rocks is the same as that of the input ground motion. However, the transmission coefficient of the monitoring point of each section in the main frequency band is different from that under the uniform input ground motion, as presented in Figure 4(d). In the y = 0 m and y = 200 m sections, the transmission coefficient increases in the frequency range 0.5–0.7 Hz. Subsequently, the transmission coefficient decreases linearly with frequency. In the y = 400 m, y = 600 m, and y = 800 m sections, the transmission coefficient increases in the frequency range 0.5–0.7 Hz, thus exhibiting a similar process to sine wave change. Afterward, although the transmission coefficient remains the same, and vibration change with frequency is repeated, the vibration change amplitude increases along the axial direction of the tunnel. In the frequency range 0.5–0.7 Hz, the change in the transmission coefficient of the surrounding rock monitoring point of the y = 1000 m section is similar to that of the y = 800 m section. However, at a frequency range 0.7–1.5 Hz, the transmission coefficient of the surrounding rock monitoring point of this section gradually decreases with repeated vibration of the frequency. Besides, the transmission coefficient of the monitoring point at the same frequency gradually increases along the axial direction of the tunnel, but the increase magnitude is different.
4. Correlation of seismic dynamic response of the surrounding rocks

The correlation between two earthquake series can be expressed by the coherence function, which is

$$
\gamma_{ij}(\omega, d) = \frac{S_{ij}(\omega)}{\sqrt{S_{ii}(\omega)S_{jj}(\omega)}} \tag{1}
$$

where $S_{ii}(\omega)$ and $S_{jj}(\omega)$ denote the self-power spectral density of the ground motion at points i and j, respectively. $S_{ij}(\omega)$ denotes the cross-power spectral density of the ground motion at points i and j.

In Figure 5, the coherence function between the two points along the tunnel axis and the entrance of the tunnel under the non-uniform input ground motion is presented. From the figure, it can be seen that the coherence function between the monitoring points along the tunnel axis and the entrance of the tunnel under the non-uniform input ground motion increases in the frequency range 0–1 Hz, which is different from that under the uniform input ground motion. However, the overall trend is attenuating with frequency; the further the distance, the faster the attenuation. The changes in the coherence coefficient of the monitoring points of the tunnel with frequency are the same; however, the values slightly differ. Figure 5(d) demonstrates the change in the maximum coherence coefficient of each monitoring point along the axial direction of the tunnel. From the figure, it can be seen that the maximum coherence coefficients of the monitoring points of the tunnel are approximately the same. Moreover, the change in the maximum coherence coefficient along the axial direction of the tunnel is consistent with that under input ground motion. Although a certain gap exists, it is less than 0.05, which indicates that the numerical simulation results are reasonable in coherence test.

Figure 6 presents the coherence function of input ground motion and the seismic response at the monitoring points along the elevation of the $y = 400$ m and $y = 800$ m sections under the non-uniform input ground motion. For each monitoring point at the same section, the trend of the coherence function with frequency is similar to that under the uniform input ground motion. However, in the $y = 400$ m and $y = 800$ m sections, the trend is different. For example, when the frequency is less than 8 Hz, the coherence function of monitoring point 2# in the $y = 400$ m section decreases linearly with frequency; in the $y = 800$ m section, the coherence function of monitoring point 2# also decreases with frequency, but there is a step in the frequency range 3–5 Hz. Similarly, the maximum value, rate of
change, range of change, number of changes, and frequency change range of the coherence coefficient of other monitoring points at different elevations in the two sections are also different. Figure 6(c) presents the maximum value of the coherence function of the monitoring points along the elevation of these two sections. It decreases with increasing distance, and the relationship between the maximum coherence coefficient and the distance also conforms to the quadratic polynomial relationship. However, the maximum coherence coefficient of the non-uniform input ground motion is significantly greater than that of the uniform input ground motion. The greater the distance, the greater the gap. The maximum gap is about 0.15, and the maximum coherence coefficient of the non-uniform input ground motion also exhibits a different trend at different sections. The maximum gap does not exceed 0.06.

![Figure 5. Coherence function of the monitoring points along the tunnel axis under the non-uniform input ground motion](image)

(a)  
(b)  
(c)  
(d)
Figure 6. Coherence function of each monitoring point along the elevation of $y = 400$ m and $y = 800$ m sections under the non-uniform input ground motion

5. Conclusion

In this paper, the dynamic response of the surrounding rocks of the long tunnel under the uniform and non-uniform input ground motions is analyzed. The spectral correlation between the input ground motion and the dynamic response of the surrounding rocks through the coherence function is also studied. The conclusion is obtained as follows:

(1) In the main frequency band, the change in the transmission coefficient with frequency generally exhibits a trend of linear decrease with increasing frequency under the uniform input ground motion. The order of the transmission coefficient of the different monitoring points is top > right = left > bottom.

(2) The transmission coefficient of the surrounding rock monitoring points of each section along the axial direction of the tunnel under the non-uniform input ground motion is different from that in the main frequency band. In addition, the transmission coefficient of the monitoring points at the same frequency gradually increases along the axial direction of the tunnel.

(3) The correlation function of each monitoring point along the tunnel axis under the non-uniform input ground motion generally decreases with frequency, and the further the distance from the tunnel entrance, the faster the attenuation of correlation.

(4) The maximum coherence coefficient and the distance of the tunnel meets the quadratic polynomial relationship under the uniform and non-uniform input ground motions. However, the maximum coherence coefficient of the non-uniform input ground motion is significantly greater than that of the uniform input ground motion. Further, the maximum coherence coefficient of the non-uniform input ground motion also exhibits a different trend at different sections.

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