Illuminating hot Jupiters in caustic crossing

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ABSTRACT
In recent years a large number of hot Jupiters orbiting in a very close orbit around the parent stars have been explored with the transit and Doppler effect methods. Here in this paper we study the gravitational microlensing effect of a binary lens on a parent star with a hot Jupiter revolving around it. Caustic crossing of the planet makes enhancements on the light curve of the parent star in which the signature of the planet can be detected by high-precision photometric observations. We use the inverse ray shooting method with tree code algorithm to generate the combined light curve of the parent star and the planet. In order to investigate the probability of observing the planet signal, we do a Monte Carlo simulation and obtain the observational optical depth of \( \tau \sim 10^{-8} \). We show that about 10 yr of observations of Galactic bulge with a network of telescopes will enable us detecting about 10 hot Jupiters with this method. Finally we show that the observation of the microlensing event in the infrared band will increase the probability for detection of the exoplanets.

Key words: gravitational lensing: micro – planetary systems.

1 INTRODUCTION

Gravitational microlensing as one of the applications of general relativity is proposed by Paczyński (1986) for detecting the dark compact halo objects, the so-called MACHOs in the Galactic halo. While not enough MACHOs have been detected in the halo (Milsztajn & Lasserre 2001), still the microlensing technique has been used as an astrophysical tool for studying the atmosphere of the stars and exploring the exoplanets. In the standard method for exploring planets with microlensing, a star with the companion planets can play the role of lens and produce caustic lines where crossing the caustics by the source star produces a high magnification on the light curve (Mao & Paczynski 1991). In this case in addition to the standard microlensing light curve we can detect a short-duration spark due to the caustic crossing formed by the planet. A precise photometry of the event is essential to find out this short-duration signature of the planet. The advantage of this technique compared to the other methods of exoplanet detection is that it is sensitive to the observation of earth mass planets (Beaulieu et al. 2006) and also those planets located beyond the snow line (Gould et al. 2010). There are also other methods in gravitational microlensing such as planetary microlensing signals from the orbital motion of the source star around the common barycentre of source star–planet system (Rahvar & Dominik 2009).

In addition to the mentioned methods, Graff & Gaudi (2000) proposed caustic crossing of a close-in Jupiter-size planet, produced by a binary lens. In this case the planet’s light is magnified so much that it can be detected by a 10-m class telescope. Here we extend this work looking to the details of the light curves and study the most favourite passband for this observation. Since in a close-in Jupiter, the thermal emission due to the high temperature of the planet is more significant than the reflected light from the parent star, the observations in the infrared passband are more favourable than the visual passband. We also do a Monte Carlo simulation with a given observational strategy to obtain the number of observable events in terms of the parameters of the planet and the parent star. We emphasize that while the observations of the hot Jupiters are simpler in nearby stars via the eclipsing and Doppler methods, the microlensing method can detect distant systems and enable us to compare the statistics of the hot Jupiters with the nearby observations. One of the interesting features of the light curve for the planet caustic crossing is that the planet can cross the caustic more than that of parent star, as it traces effectively a longer path due to the revolving motion around the parent star.

The paper is organized as follows. In Section 2 we will introduce the caustic crossing of the parent star and planet system and generate the light curve with inverse ray shooting technique, introducing a new development in tree code algorithm. In Section 3 we study the characteristics of the light curve in terms of the orbital parameters of the planet and the parent star. In Section 4 we explain our Monte Carlo simulation for estimating the probability of illuminating hot Jupiters with this method. In Section 5 we give the conclusions.

2 MICROLENSING LIGHT CURVE

A binary system deflects the light ray in a more complicated way than a single lens. Let us represent \( \xi \) as the position of the image in
the lens plane and $\eta$ the position of the source in the source plane. Then, the geometrical relation between these parameters is given by the lens equation as follows (Schneider & Wiess 1986):

$$\eta = \frac{D_\odot}{D_s} \frac{\xi - \theta_1}{\xi - \theta_2},$$

where $\alpha$ is the overall deflection angle due to a double lens and $D_s$, $D_\odot$, and $D_l$ are the distances of the source and lens from the observer and the distance between the lens and source, respectively. The deflection angle for the light ray is given as follows:

$$\alpha(\xi) = \frac{4GM_s}{c^2} \frac{\xi - \theta_1}{|\xi - \theta_1|^2} + \frac{4GM_\odot}{c^2} \frac{\xi - \theta_2}{|\xi - \theta_2|^2},$$

where $\theta_1$ and $\theta_2$ are the positions of the binary system in the lens plane and $M_\odot$ and $M_\odot$ are the masses of the lenses. Equation (1) as the lens equation is a fifth-order equation and in general the solution is not trivial. One of the possible methods for solving the lens equation is the inverse ray shooting method (Kaiser et al. 1986; Schneider & Wiess 1987). In this method we follow the position of the light ray that shoots from the observer to the lens plane. Knowing the position of the lenses, we can calculate the deflection angle. Substituting this in the lens equation results in the position of the source. We pixelize the source and the lens plane, and for each light ray passing from the lens plane and hitting the source plane, we count the number of hits inside each pixel. These numbers identify the magnification pattern in the source plane. We use the tree code method, as described later, for generating the image and the magnification of the source star.

In order to simplify our calculation we take the dimensionless parameters in the lens equation. Let us define the overall Einstein radius as follows:

$$R_E = \sqrt{\frac{4GM_\odot}{c^2} \frac{D_s D_\odot}{D_l}}.$$ (3)

We normalize the lens equation to this length-scale, which results in

$$x = r - \alpha(r),$$

where $x = D_s/D_l \times \eta/R_E$, $r = \xi/R_E$ and the deflection angle is given by

$$\alpha(r) = \alpha_1 \frac{r - r_1}{|r - r_1|^2} + \alpha_2 \frac{r - r_2}{|r - r_2|^2},$$

and $\alpha_1 = M_\odot/(M_1 + M_2)$, $r_1 = \theta_1/R_E$.

We take a straight line for the path of the centre of mass of the parent star and planet at the lens plane. Taking the mass of parent star larger than the mass of the planet, the parent star follows approximately a straight line as follows:

$$u_s = \left(-u_0 \sin \alpha + \frac{t - t_0}{t_\odot} \cos \alpha \right) \hat{i} + \left(u_0 \cos \alpha + \frac{t - t_0}{t_\odot} \sin \alpha \right) \hat{j},$$

where $u_0$ is defined as the minimum impact parameter of source star from the centre of the Cartesian coordinate system, normalized to the Einstein radius, $t_0$ is the time of impact parameter and $\alpha$ is the angle between $x$-axis and the trajectory of the source star. The rotation of the planet around the parent star makes a cycloid-like pattern on the source plane which is given by

$$u_p = u_s + \tilde{a}(\cos(\cot + \varphi) \cos(\alpha + \beta)$$

$$- \cos(\delta) \sin(\cot + \varphi) \sin(\alpha + \beta)) \hat{i}$$

$$+ \tilde{a}(\cos(\cot + \varphi) \sin(\alpha + \beta)$$

$$+ \cos(\delta) \sin(\cot + \varphi) \cos(\alpha + \beta)) \hat{j},$$

where $\tilde{a}$ is the projection of the planet orbit on the lens plane normalized to the Einstein radius, $\omega$ is the angular velocity of the planet around the source star which can be obtained from the Kepler’s third law:

$$\omega = \sqrt{\frac{G(M_p + M_\odot)}{a^3}}.$$

where $a$ is the orbital radius of the planet, and $\delta$ is the deviation of the normal vector to the orbital plane of the planet from our line of sight. For the hot Jupiters due to the tidal interaction of the planet and the parent star, we set the eccentricity to zero and one angle is sufficient for describing the orbital plane deviation. Here, $\beta$ is the angle between the trajectory of the source star and the projected semimajor axis of the planet and $\varphi$ is the initial phase of the planet.

In order to generate the light curve, we need the relative velocity of the binary lens with the parent star and companion planet. The relative transverse velocity of the source–observer line of sight with respect to the lens at the lens plane is given by (Kayser et al. 1986)

$$v_{rel} = x v_x - y v_y,$$

where $v_x$, $v_y$, and $v_\odot$ are the two-dimensional transverse velocities of the centre of mass of the lens system, source and observer with respect to the line of sight, respectively, and $x = D_s/D_l$ is the ratio of the distance of the lens to the source. The velocity of the observer is obtained from the local measurements of the Solar system in the Galactic frame. The velocity of the stars in the bulge is given by the dispersion velocity in this structure and the velocity of the stars in the disc is obtained from the combination of the dispersion and global velocities of the stars (Binney & Tremaine 1987).

Since the lens is a binary system, it will rotate around the centre of mass during the microlensing of the parent star and the companion planet. For simplifying our calculation, we fix the position of the binary system and obtain the relative motion of the source objects with respect to the rotating binary system. The situation is similar to the study of the motion of an object in rotating non-intertidal reference frame in classical mechanics. In the reference frame of the binary system, we do the following coordinate transformation for the position of the source object:

$$R(\Omega, \psi) = \left( \begin{array}{c} \cos \Omega \sin \psi t \\ \sin \Omega \sin \psi t \\ \cos \psi t \end{array} \right).$$

where $\Omega$ is the angular velocity of the binary lens, which is given by

$$\Omega = \sqrt{\frac{G(M_1 + M_2)}{d_\perp^2}} \cos \psi^3,$$

where $d_\perp$ is the apparent separation of the two lenses from each other and $\psi$ is the deviation of the normal vector to the binary plane from the line of sight.

For generating the light curve we should note that there are two source objects, namely the parent star and the planet. The total flux received by the observer then is the accumulation of the magnified flux of each component (Griest & Hu 1992; Han & Gould 1996):

$$A_{total} = F_A A_s + F_p A_p,$$

where $F_A$ and $F_p$ are the intrinsic flux of the parent star and the planet and $A_s$ and $A_p$ are the corresponding magnifications. Let us define the ratio of the intrinsic flux of the planet to the flux of the parent star by $\varepsilon = F_p/F_A$, which is much smaller than 1 (i.e. $\varepsilon \ll 1$). The total magnification can be written as

$$A_{total} = A_s + \varepsilon A_p.$$
The magnification of parent star (solid line) and the companion planet in the first peak, the planet crosses twice the caustic lines. In order to distinguish the caustic crossing of the planet in the first peak, we zoom in around the first peak of the parent star which is given in the right-hand corner of Fig. 2. We note that the recrossing of the caustic by the planet takes place with an interval time of about 1 h, and in order to detect this signal, we need a sampling rate less than this time-scale.

In the rest of this section we introduce the details in generation of the light curve in the numerical calculation. We should note that the tree code method used in our calculation is different than that introduced by Barnes & Hut (1986) and Wambégsanss (1999). In their method the lenses in the lens plane are treated corresponding to their distances to the light ray. In that approach the process of making cells on the lens plane depends on the distance of the lenses to the light ray and their distribution. For the weak gravitational regime where the deflection angle is obtained from the superposition of each lens, the closer lens enters into the calculation of the deflection angle with more accuracy than the distant ones. This method can be used for a large number of lenses as the passage of a quasar light ray through a galaxy.

In our tree code method we divide the lens plane into cells and choose a same-size pixel to surround the source at the source plane. The procedure starts with dividing the lens plane into four parts, and in the source plane we choose a pixel with the same size to surround the source object where the source is located at the centre of the cell. We shoot enough number of light rays to each cell in the lens plane to cover the cell’s area. Using the lens equation, if at least one of the rays collides with the source cell, we accept that cell for the next dividing step. Otherwise, we remove this cell from the lens plane. We continue this dividing procedure up to the stage where we have enough pixels to cover the source object having reasonable resolution as much smaller cells provide better resolution and less statistical fluctuation in the magnification calculation. Since this pixelizing is time consuming, specially in the Monte Carlo simulation, we stop the dividing procedure up to a desire resolution. For instance, for the parent star, we cover the source with a $5 \times 5$ grid. Fig. 3 depicts the pixelizing procedure in this algorithm. In the final stage of pixelizing, the remaining number of cells in the lens plane compared to that of cells in the source plane that covers the source star provides the magnification due to gravitational lensing. Also, the remaining pixels in the lens plane provide the shape of the images. We should note that due to the smaller size of the planet with respect to the source star, the magnification of the planet does not suffer as much as the parent star from the finite-size effect. The result of planet caustic crossing will be like a flashing; let us call it ‘planet flashing’. While this flashing in terms of magnification is much larger than that of the magnification of the source star (see Fig. 2), the intrinsic flux of the planet relative to the flux of the source star should be sufficient to be observed by a telescope. In the next section we will discuss this issue in detail.

3 CHARACTERISTICS OF THE LIGHT CURVE

In this section, we study the details of the light curve by looking into the physical specifications of the source star and the companion planet as well as the binary lens. One of the main factors in the observability of the planet by this method is the ratio of the planet’s flux as the signal to the flux of the parent star as the background. Using equation (13) we can describe the signal to the background in terms of the total and parent star magnifications:

$$\frac{\delta F}{F} = \frac{A_{\text{total}}}{A_{\star}} - 1.$$  \quad (14)

The overall flux received from a planet by the observer contains the radiation of the thermal energy due to the intrinsic temperature of the planet and the reflection of the parent star’s light. Assuming the thermal emission of the planet as a blackbody radiation (Lopez-Morales & Seager 2007), the temperature of the planet can be calculated by taking into account the absorption of the radiation received from the parent star and reradiating it through the Boltzman law. Hence the planet’s temperature is given by

$$T_p = T_\star \left( \frac{R_s}{a} \right)^{1/2} \left[ f(1 - A_R) \right]^{1/4}.$$  \quad (15)
where $R_p$ is the radius of the planet. On the other hand, the flux of the planet due to the reflection of parent star’s light is given by

$$ F_p^{(ref)} = F_p A_g \left( \frac{R_p}{a} \right)^2 g(\Phi), $$

(18)

where $A_g$ is the geometrical albedo, assuming the Lambert’s law, $A_g = 2/3 A_s$, $\Phi$ is the phase of the planet and $g(\Phi)$ is a function of the planet phase indicating a fraction of the lighted area of planet in front of the observer. Now the ratio of the overall planet’s flux which comprises the thermal and reflection terms to the star’s flux is given by

$$ \frac{F_p}{F_*} = g(\Phi) \left[ A_g \left( \frac{R_p}{a} \right)^2 + \left( \frac{R_p}{R_*} \right)^2 \left( \frac{T_p}{T_*} \right)^4 \right], $$

(19)

where eliminating the planet’s temperature from equation (15), the flux ratio can be obtained in the simpler form of

$$ \frac{F_p}{F_*} \approx \left( \frac{R_p}{a} \right)^2 [A_g + f (1 - A_g)] g(\Phi). $$

(20)

The important point in this equation is that it is independent of the temperature and the radius of the source star and depends only on the chemical composition of the planet atmosphere which changes $A_g$, the size and the distance of the planet from the parent star. During gravitational lensing we multiply the corresponding magnifications of the planet and the parent star to the intrinsic flux of the source objects and obtain the overall receiving flux.

In practice we do not integrate over all the wavelengths, as the detector of the telescope may be sensitive to a specific passband. To find the optimal wavelength for the detection, we maximize the ratio of the intensity of the planet to that of the parent star, i.e. $\delta I(v)/I(v)$. Using equations (16) and (18), assuming a constant reflection index for all the wavelengths (i.e. $\frac{dA_g}{dv} \simeq \text{const}$), this ratio is obtained as

$$ \frac{\delta I(v)}{I(v)} = \left[ \frac{R_p}{a} \right]^2 \frac{dA_g}{dv} \left[ 1 + \left( \frac{R_p}{R_*} \right)^2 \right] g(\Phi). $$

(21)

We should note that the reflection flux is mainly important in the optical bands, where the thermal flux dominates in the infrared and submillimeter wavelengths.

Hot Jupiters have small albedo of $A_g \lesssim 0.2$ (Rowe et al. 2008), so most of the radiation of the parent star is absorbed by the planet’s atmosphere and is reradiated in the infrared band. The result is a small share of luminosity of the planet in the reflected flux. The exception in the hot Jupiters happens in the cases with the period of motion smaller than 3 d. This class of hot Jupiters, so-called Veryhot Jupiters, has a surface temperature larger than 2500 K in which they reradiate considerable amount of thermal flux in the optical passband. However, the peak of the spectrum is in the infrared band (Lopez-Morales & Seager 2007). Out of this exception, the most hot Jupiters have a considerable flux in infrared due to the thermal emission (Deming & Seager 2009). We plot equation (21) in Fig. 4 as an evidence that in the infrared passband, we have the most contrast between the flux of the planet and the parent star. Here we compare the planet luminosity to the parent star luminosity as a function of the wavelength. In order to see the sensitivity of this fraction to the mass of the parent star, we plot this function for four masses of the parent star. Choosing the parent star as a main-sequence star, the radius of the star can be eliminated in favour of the mass, using

$$ R_* = M_*^{1.2}, $$

(22)
In the upper panel, the solid line represents the luminosity of the planet with $T_p = 1500$ K and the dashed line is given for the parent star with $T_\star = 5778$ K. The radius of the planet and the source star is given in megameter. The lower panel shows the ratio of the luminosity for the planet to the parent star.

where the corresponding parameters are normalized to that of the Sun’s values. Fig. 5 shows that M-dwarf stars are more favourable for the hot Jupiter detection in the wavelengths longer than 15 μm. However, we should note that for the M-dwarf star, the abundance of the hot Neptunes is more than the hot Jupiters (Ida & Lin 2005).

4 MONTE CARLO SIMULATION

In this section we do a Monte Carlo simulation to obtain quantitatively the sensitivity of the planet detection to the parameters of the binary lens and the planetary system. Finally we provide the probability for detecting desired events.

First of all we need to generate the parameters of the lens and the source according to the physical distribution of the parameters. For better analysis, we divide the parameter space into the lens and the source parameters. For the lens parameters we take $q = M_\ell / M_\star$, the ratio of the masses in the binary system to change uniformly in the range of $q \in [0, 1]$, the mass of one of the lenses is taken from the Salpeter mass function (Salpeter 1995) in the range of $M_\ell \in [0.1, 3] M_\odot$ and the distance between the lenses is taken uniformly in the range of $d \in [0.1, 5 \text{ au}]$. The location of the lens from the observer is calculated from the probability function of microlensing detection $\mathrm{d}t_\ell / \mathrm{d}x \propto p(x) \sqrt{\chi(1-x)}$, where $x$ changes in the range of $x \in [0, 1]$. The velocity of the lens is taken to be the combination of the global (Rahal et al. 2009) and the dispersion velocity (Binney & Tremaine 1987) of the disc and bulge. We take a thin disc for modelling the disc and take our line of sight towards the Galactic bulge with the latitude angle in the range of $b \in [1, 2]$.

For the source objects, the corresponding angles in the trajectory of the source system, $\alpha, \beta$ and $\gamma$ in equation (7), are taken uniformly. The minimum impact parameter is in the range of $u_0 \in [0, 1]$ and we take the mass of the parent star from the Salpeter mass function in the range of $M_\star \in [0.1, 3 M_\odot]$. Since the parent star is assumed to belong to the main sequence, the radius of the star is taken from equation (22). On the other hand, the luminosity of the parent star can be obtained from (Eddington 1926)

$$L_\star = L_\odot \left( \frac{M_\star}{M_\odot} \right)^{3.5}. \quad (23)$$

For the parameters of the planet, the inclination angle $\delta$ of the planetary plane to the line of sight is taken uniformly in the range of $\delta \in [-\pi/2, \pi/2]$. We take the mass of the planet in the range of $M_p \in [0.1, 10 M_\oplus]$ and distance of the planet from the parent star in the range of $a \in [0.01, 0.1 \text{ au}]$. For the close-in hot Jupiters at a distance less than ~ 0.05 au, which are of concern to us, the radius not only is a function of the planet’s mass, but also depends on the distance of the planet from the parent star (Fortney et al. 2007). Close-in planets are heated by the parent star and their atmosphere inflates. For the hot planets, in contrast to the conventional relation between the mass and the size, the small-mass planets inflate more than the massive ones. We take the mass–radius–distance relation of hot planets from Fortney et al. (2007).

Before performing Monte Carlo simulation to count the number of events with the desired signal to the background flux, we do a first-order estimation, just counting geometrically the number of planets whose trajectories cross the caustic lines. In our simulation we follow the path of the planet in the magnification pattern that has already been generated by the inverse ray shooting and assign the magnification of the source object along its trajectory. We calculate the deviation of the flux along the path to identify any sharp peak in the light curve. This signature indicates a caustic crossing. Our simulation shows that in microlensing events with binary lens and for the condition $u_0 \sim 1$, almost 36 per cent of planets can cross the caustic lines. We obtain almost the same amount of caustic crossing for theparent stars. We call this fraction of events with the caustic crossing as the geometrical criterion for the hot Jupiter detection.

In reality we should measure the flux of source star and the enhancements due to the hot Jupiter on the background light curve. First, we assume a telescope with 1 per cent photometric precision. We can change this criterion according to the size of telescope and a limiting magnitude on the brightness of the source star. With new techniques such as defocusing of the telescope, we can achieve dispersions of the order of 5 mmag with a medium-size telescope in terms of the relative flux dispersion to the background which is of the order of $10^{-4}$ (Southworth et al. 2009).

In our Monte Carlo simulation we look for the maximum magnification of the planet when it crosses the caustic line and obtain the flux ratio of the planet to the source star at that point. Here we do not take into account the sampling rate of the observations of the event. A typical duration for the magnification of the planet
Table 1. Detection efficiency of hot Jupiter for various optical, infrared and far-infrared passbands. The first column describes the atmospheric model of the planet, the second and the third columns represent the efficiency for the optical and red bands, the fourth to the tenth columns show the detection efficiency for the infrared and far-infrared passbands. The 11th column stands for the submilimeter and the last column shows the average detection efficiency over all the wavelengths, weighted to the spectrum of the planet.

| Band Wavelength (μm) | Optical | R | J | H | K | L | M | N | Q | Submilimeter | Overall wavelength |
|----------------------|---------|---|---|---|---|---|---|---|---|--------------|-------------------|
|                      | 0.55    | 0.825 | 1.25 | 1.65 | 2.2 | 3.45 | 4.7 | 10 | 20 | 450          |                   |
| $A_B = 0; f = \frac{1}{4}$ | 0.01     | 0.04 | 0.13 | 0.23 | 0.36 | 0.67 | 0.94 | 2.62 | 5.33 | 10.74 | 0.01          |
| $A_B = 0; f = \frac{1}{2}$ | 0.09     | 0.21 | 0.39 | 0.57 | 0.85 | 1.50 | 2.38 | 5.85 | 9.33 | 14.35 | 0.11          |
| $A_B = 0.3; f = \frac{1}{4}$ | 0.03     | 0.06 | 0.13 | 0.21 | 0.32 | 0.57 | 0.80 | 2.06 | 4.45 | 9.81 | 0.03          |
| $A_B = 0.3; f = \frac{1}{2}$ | 0.08     | 0.18 | 0.34 | 0.49 | 0.70 | 1.19 | 1.80 | 4.72 | 7.90 | 13.18 | 0.11          |
| $A_B = 0.5; f = \frac{1}{4}$ | 0.05     | 0.07 | 0.13 | 0.20 | 0.29 | 0.51 | 0.70 | 1.60 | 3.66 | 9.0 | 0.06          |
| $A_B = 0.5; f = \frac{1}{2}$ | 0.09     | 0.16 | 0.29 | 0.41 | 0.58 | 0.99 | 1.40 | 3.77 | 6.69 | 12.08 | 0.11          |

Figure 6. The detection efficiency for different parameters of the lens for three different observational strategies with $10^{-2}$ (dotted line), $10^{-3}$ (dashed line) and $10^{-4}$ (solid line) photometric precisions.

Figure 7. The detection efficiency for different parameters of the planet and the parent star for three different observational strategies with $10^{-2}$ (dotted line), $10^{-3}$ (dashed line) and $10^{-4}$ (solid line) photometric precisions.

during the caustic crossing is of the order of 1 h and we assume to have a network of telescopes to cover the event. For the photometric precision of 1 per cent, in Table 1 we show the detection efficiency for various passbands and atmospheric models of the hot Jupiters. Longer wavelengths are more favourable for detection of the planet than the shorter ones. Also, planets with high reradiating property (i.e. $f = 2/3$ in this simulation), meaning less advection in the atmosphere, are more favourable for the detection.

In order to study the sensitivity of the planet detection on the parameters of the model, we plot the detection efficiency in terms of the relevant parameters of the lens, source and planet in Figs 6 and 7 for three cases of photometric precisions of $10^{-2}$, $10^{-3}$ and $10^{-4}$. We ignore the irrelevant parameters that do not enter in the efficiency function. The detection efficiency function in terms of the lens and the source–planet parameters is given as follows.

(i) The first parameter is the distance between two lenses. Here the detection efficiency rises with increasing distance between the two lenses, and after a peak around $2R_\text{E}$, it decreases. The physical interpretation of this feature is due to the topological configuration of the caustic lines. It is shown in Schneider & Wiess (1986) that for the case of $q = 1$ and distance between the lenses in the range of $[2/\sqrt{8}, 2R_\text{E}]$, the caustic lines are topologically connected whereas beyond this range the caustic lines detach from each other. Having continuous caustic lines increases the probability of caustic crossing both by the parent star and the planet.

(ii) The second parameter is $x = D_\text{l}/D_\star$, the relative distance of the lens to the source star. In the detection efficiency diagram, the efficiency reaches to the maximum value around $\sim 0.5$ where the Einstein radius has the maximum size. A larger Einstein radius results in a longer duration for the microlensing event and a higher probability for the caustic crossing.

(iii) The third parameter is the impact parameter. Decreasing the impact parameter increases the detection efficiency. A smaller impact parameter from the centre of the lens configuration increases the probability of the caustic crossing. On the other hand,
statistically, the impact factor may also affect the magnification of the light curve. We test this hypothesis amongst the simulated events showing that the small impact parameters result in a higher signal to the background flux.

(iv) The ratio of the lens masses, \( q \). It seems that changing \( q \) has a geometrical effect on the shape of the caustic lines, where increasing it towards the symmetric shape, \( q = 1 \), maximizes the detection efficiency.

(v) The semimajor axis of the planet, \( a \), shown in Fig. 7. The hot Jupiters reside in the range of \([0.01, 0.1]\) au. For the smaller \( a \), there is a larger detection efficiency than the larger \( a \). The dependence of the detection efficiency on the semimajor axis results from the reflection of the flux of the parent star from the planet, proportional to the inverse square of the distance. The other effect is due to the intrinsic thermal luminosity of the planet, closer to the parent star, thus making the temperature of planet higher.

(vi) Period of motion of planet, \( T \). The dependence of the efficiency on the period is, in the same way as the semimajor axis, due to the Kepler's law. A shorter duration for the period of the planets resembles that of the smaller semimajor axis.

(vii) Radius of the planet, \( R_p \). The intrinsic flux of the planet both in reflection of the parent star's light and thermal emission is proportional to the square of radius of the planet. Hence larger planets can be detected easily than the smaller ones.

(viii) Radius of the source star, \( R_s \). The effect of the radius of the source star on the relative intensity of the planet to the parent star is shown in equation (21). Increasing the radius of the star decreases this ratio.

Finally we come to conclude on the possibility of detection of the hot Jupiters from this method. The comparison of the OGLE data with the model constructed from the Hipparcos data indicates that about \( f_p \sim 0.5 \times 10^{-2} \) fraction of the stars have hot Jupiters and very hot Jupiters (Gould et al. 2006). On the other hand, the radial velocity observations of the planets indicate that about \( f_p \sim 10^{-2} \) fraction of the solar type stars have hot Jupiters (Marcy et al. 2005). From the Monte Carlo simulation we obtain the average detection efficiency \( \langle \epsilon_p \rangle \), for the planet detection with three photometric precisions of \( 10^{-2} \), \( 10^{-3} \) and \( 10^{-4} \) as 0.03, 0.30 and 0.35, respectively. The optical depth for the planet detection can be obtained by

\[
\tau_p = \langle \epsilon_p \rangle f_p \tau, \tag{24}
\]

where \( \tau \) is the optical depth for the microlensing events towards a given direction. For the direction of the Galactic bulge, \( \tau = 4.48 \times 10^{-6} \) (Sumi et al. 2006), and hence the optical depth for the planet detection is about \( \tau_p \sim 10^{-8} \). We can obtain the number of events for \( N_{bg} \) background stars with \( T \) exposure time as

\[
N_p = \frac{\pi T N_{bg}}{2 (\epsilon_h)} \times \tau_p, \tag{25}
\]

where the average Einstein crossing time for the Galactic bulge events is about \( \langle \epsilon_h \rangle = 28 \) d. In Table 2 we provide the numerical values for the optical depth and number of events for each photometric case.

### Table 2

| Photometric precision | \( 10^{-2} \) | \( 10^{-3} \) | \( 10^{-4} \) |
|-----------------------|-------------|-------------|-------------|
| \( \tau_p \)        | \( 1.34 \times 10^{-9} \) | \( 1.34 \times 10^{-8} \) | \( 1.57 \times 10^{-8} \) |
| \( N_p \)            | 3           | 28          | 32          |

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**5 CONCLUSION**

In this paper we examined the possibility of hot Jupiter detection through caustic crossing of a binary lens by a planet as the source object. The effect of this caustic crossing due to the small size of the planet is like an illumination on the microlensing light curve of the parent star. Taking the flux of the parent star as the background light and the illumination of the planet as the signal, we studied the physical characteristics of the planet such as the orbital size, the atmospheric property and the size of the planet on one hand and the characteristics of the parent star and lens on the other hand on the observability of planet.

In the next step, we did a Monte Carlo simulation to obtain the detection efficiency of the planet with this method. We showed that taking just geometrical caustic crossing of the planets, 36 per cent of the population can be illuminated. However, in reality, due to the photometric error, the peak generated by the planet may not be detected. We used three different photometric precisions in the observation and obtained the detection efficiency, assuming that we use a network of telescopes to have enough sampling of data much less than 1 h, a typical time of the caustic crossing of the planet. We showed that for longer passbands, the detection efficiency increases due to the more relative emissivity of planet compared to the parent star.

Finally we estimate the number of hot Jupiters that can be observed with this method towards the Galactic bulge. With a 10-yr monitoring of \( 10^7 \) stars towards the Galactic bulge, we can detect of the order of 10 hot Jupiters with this method. This observation may be done by the next generation of the microlensing surveys towards the Galactic bulge.
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