Design of FIR Precoders and Equalizers for Broadband MIMO Wireless Channels with Power Constraints

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Abstract
This paper examines the optimum design of FIR precoders or equalizers for multiple-input multiple-output (MIMO) frequency-selective wireless channels. The matrix Bezout identity can be employed to design a FIR MIMO precoder/equalizer for a left/right coprime channel. Unfortunately, Bezout precoders/equalizers usually increase the transmit/noise power in order to compensate for the deep frequency fades in channels. To overcome this problem, we describe in this paper a convex optimization technique for the optimal synthesis of MIMO FIR precoders subject to transmit power constraints, and of MIMO FIR equalizers with output SNR constraints. The synthesis problem reduces to the minimization of a quadratic objective function under convex quadratic inequality constraints, so it can be solved by employing Lagrangian duality. Instead of solving the primal problem, we solve the lower dimensional dual problem for the Lagrange multipliers. The simulation results show that optimal precoders with power constraints outperform Bezout precoders.

1. INTRODUCTION

The increasing demand for high data rates communication combined with the lack of wireless spectrum have prompted the consideration in recent years of multi-antenna wireless communication systems that can support much higher data rates [2], [3], [4] than traditional single-input single-output wireless channels. However, for the case of frequency-selective MIMO wireless channels, the task of channel equalization becomes challenging, since the vector nature of the channel precludes the use of trellis-based equalization techniques due to the excessively large number of states required, and since MIMO decision-feedback equalizers [5], [6] have a high computational complexity. As a consequence, most broadband MIMO wireless system architectures address the equalization issue by purposefully trading off some amount of system performance against a lower implementation complexity. Usually, an orthogonal frequency division multiplexing (OFDM) modulation format [7], [8] or, single carrier frequency domain equalization [9], [10] can simplify the equalization problem but at the price of reducing data rate due to the overhead of the cyclic prefix. Similarly, the class of Bezout precoders or equalizers introduced in [1] for MIMO frequency-selective channels can be viewed as simplifying the equalization problem for a rectangular system. Specifically, for the case of a left coprime FIR channel, which arises generically when the number $p$ of transmit antennas is larger than the number $q$ of receive antennas, the matrix Bezout identity can be employed to design a FIR MIMO precoder that equalizes exactly the channel at the transmitter. Similarly, for a right coprime FIR channel, which arises generically when $p < q$, the Bezout identity yields a FIR zero-forcing MIMO equalizer. Unfortunately, when deep fades are present in the MIMO channel frequency response, Bezout precoders usually increase the transmit power, and Bezout equalizers tend to amplify the output noise power. To overcome this problem, we explore here the tradeoff existing between the power of the residual inter-symbol interference (ISI) and inter-channel interference (ICI) in the equalized channel and transmit power constraints or output SNR constraints. A convex optimization technique is proposed for the synthesis of MIMO FIR precoders (or equalizers) minimizing the power of the residual channel ICI and ISI subject to transmit power constraints (or output SNR constraints). The synthesis problem reduces to the minimization of a quadratic convex objective function under quadratic convex inequality constraints, so it can be solved by employing Lagrangian duality [11]. Instead of solving the primal problem, we solve the lower dimensional dual problem for the Lagrange multipliers.

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II. Optimal precoder design with power constraints

The system we consider has \( p \) transmit and \( q \) receive antennas. In this section, we assume that \( p > q \), and that the \( q \times p \) FIR channel \( H(z) = \sum_{n=0}^{d} H(n) z^{-n} \) with order \( d \) is known at the transmitter. For a left coprime channel, there exists \( p \times q \) FIR matrices \( F(z) = \sum_{n=0}^{d} F(n) z^{-n} \) obeying the matrix Bezout identity

\[ H(z)F(z) = E(z) = \text{diag}\{z^{-k_1}, \ldots, z^{-k_q}\}, 1 \leq i \leq q, \]  

for arbitrary delays \( z^{-k_i} \). The delays \( k_i \) can be selected optimally such that the power of the precoder \( F(z) \) is minimized \([1]\). Precoders satisfying (2.1) form the class of Bezout precoders. In spite of their apparent simplicity, Bezout precoders have a significant defect. They may increase the transmit power significantly to compensate for deep fades in the singular values of \( H(e^{j\theta}) \). To overcome this defect, we propose an optimal precoder that achieves the optimal trade-off between channel equalization and transmit power constraints. To measure the mismatch between the concatenated system \( H(z)F(z) \) with the ideal zero-forcing channel \( E(z) \), we employ the objective function

\[
J(F) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \| E(e^{j\theta}) - H(e^{j\theta})F(e^{j\theta}) \|^2 d\theta, \tag{2.2}
\]

where \( \|M\|^2_2 = \text{tr}(MM^H) \) denotes the squared Frobenius norm. Consider the \( (d + r + 1)q \times (r + 1)p \) block Toeplitz matrix

\[
\Gamma(H) = \begin{bmatrix}
H(0) & 0 & \cdots & 0 \\
H(1) & H(0) & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
H(d) & \cdots & H(1) & H(0) \\
0 & 0 & \ldots & H(d)
\end{bmatrix}
\tag{2.3}
\]

and the block column matrices

\[
E = \begin{bmatrix}
E(0) \\
E(1) \\
\vdots \\
E(r + d)
\end{bmatrix}, \quad F = \begin{bmatrix}
F(0) \\
F(1) \\
\vdots \\
F(r)
\end{bmatrix}
\tag{2.4}
\]

of dimension \( (d + r + 1)q \times q \) and \( (r + 1)p \times q \), respectively. The criterion \( J(F) \) can be expressed in matrix form as

\[
J(F) = \text{tr}[(\Gamma(H)F - E)^H(\Gamma(H)F - E)]. \tag{2.5}
\]

Note that the MSE between the received vector signal \( y(n) \) with the channel noise variance \( \sigma_n^2 I_q \) and the input vector signal \( s(n) \) with covariance matrix \( \sigma_s^2 I_q \) can be expressed as

\[
\text{MSE} = E[\|y(n) - s(n)\|^2] = \sigma_n^2 J(F) + \sigma_s^2 \tag{2.6}
\]

so that minimizing \( J(F) \) is equivalent to minimizing the MSE. Since the Hessian \( \nabla^2 J(F) = \Gamma^H(H) \) is positive semi-definite, the objective function \( J \) is convex. In the following subsections, we construct precoders that minimize \( J \) under several types of transmit power constraints. In all cases, the constraints are convex, so that the resulting convex optimization problems can be solved by Lagrangian duality methods.

A. Power constraints on the precoder columns

We first consider the case where a power constraint is imposed on each column of the precoder. Let \( f_j(z) \) denote the \( j \)th column of the matrix filter \( F(z) \). We consider the case where a power constraint of the form

\[
\frac{1}{2\pi} \int_{-\pi}^{\pi} \| f_j(e^{j\theta}) \|^2 d\theta = t_j^2 f_j \leq P_j \tag{2.7}
\]

is imposed on \( f_j(z) \) for \( 1 \leq j \leq q \). This constraint can be viewed as modelling a situation where each user \( j \) is guaranteed a fixed quality of service (QoS) in the form of an allocated power \( P_j \). Since the objective function \( J(F) \) can be decomposed as

\[
J(F) = \sum_{j=1}^{q} J_j(f_j) \tag{2.8}
\]

with

\[
J_j(f_j) = \|\Gamma(H)f_j - e_j\|^2, \tag{2.9}
\]

where \( e_j \) is the \( j \)th column of the matrix \( E \), the minimization of \( J(F) \) under the constraints (2.7) for \( 1 \leq j \leq q \) is equivalent to the separate minimization for each \( J_j(f_j) \) under the constraint (2.7). Since \( J_j(f_j) \) and the constraint (2.7) are both convex, this optimization problem can be solved either in primal space or dual space \([11]\). Because of the large size of \( f_j \), the primal problem has a large dimension, whereas the dual problem reduces to a scalar optimization problem since there is only one constraint. Therefore we solve the dual form of the optimization problem.

The Lagrangian associated with the minimization of \( J_j(f_j) \) under the constraint (2.7) takes the form

\[
L_j(f_j, \lambda_j) = J_j(f_j) + \lambda_j (t_j^2 f_j - P_j) = t_j^2 (M + \lambda_j I) f_j - t_j^2 \Gamma(H) H e_j - e_j^H \Gamma(H) f_j - \lambda_j P_j + 1, \tag{2.10}
\]
where \( M = \Gamma(H)^H \Gamma(H) \). In order to find a saddle point \((f^opt, \lambda^opt)\), we first fix \( \lambda_j \) and minimize the Lagrangian over \( f_j \), yielding

\[
f^opt_j(\lambda_j) = (M + \lambda_j I)^{-1} \Gamma(H)^H e_j.
\] (2.11)

The dual function is then given by

\[
G_j(\lambda_j) = L_j(f^opt_j(\lambda_j), \lambda_j)
\]

\[
= -e_j^H \Gamma(H) (M + \lambda_j I)^{-1} \Gamma(H)^H e_j + \lambda_j - \lambda_j P_j
\] (2.12)

over the domain \( D = \{ \lambda_j : \lambda_j \geq 0, M + \lambda_j I > 0 \} \). The unique solution \( \lambda^opt_j \) of the dual problem is obtained by maximizing the concave function \( G_j(\lambda_j) \) over \( D \). This can be accomplished by using Newton techniques. Then, substituting \( \lambda^opt_j \) in (2.11) gives the optimal solution

\[
f^opt_j = (M + \lambda^{opt}_j I)^{-1} \Gamma(H)^H e_j.
\] (2.13)

B. Total power constraint

Next, consider the case where a constraint is imposed on the total power used by all transmit antennas. This constraint can be expressed as

\[
tr\left( \frac{1}{2\pi} \int_{-\pi}^{\pi} F^H(e^{j\theta})F(e^{j\theta})d\theta \right) = tr(F^H F) \leq P_T.
\] (2.14)

The Lagrangian corresponding to the minimization of \( J(F) \) under the constraint (2.14) can be expressed as

\[
L(F, \lambda) = J(F) + \lambda (tr(F^H F) - P_T).
\] (2.15)

Minimizing the convex function (2.15) gives

\[
F^{opt}(\lambda) = (M + \lambda I)^{-1} \Gamma(H)^H E.
\] (2.16)

The dual function is therefore given by

\[
G(\lambda) = L(F^{opt}(\lambda), \lambda)
\]

\[
= tr\left[ -E^H \Gamma(H) (M + \lambda I)^{-1} \Gamma(H)^H E \right] + q - \lambda P_T
\] (2.17)

over the domain

\[
D = \{ \lambda : \lambda \geq 0, M + \lambda I > 0 \}.
\] (2.18)

Proceeding as in the previous subsection, the optimal Lagrangian multiplier \( \lambda^{opt} \) is obtained numerically by maximizing \( G(\lambda) \) over \( D \) with Newton techniques. Substitution of \( \lambda^{opt} \) inside (2.16) then yields the optimal precoder.

C. Power constraint on each transmit antenna

In practical situations the power amplifier for each transmit antenna has a region of linear operation. It is more realistic to impose a power constraint on each transmit antenna, or equivalently on each row of the precoder. Let \( f_i(z) \) denote the \( i \)th row of the precoder, with \( 1 \leq i \leq p \). Then the power constraint on antenna \( i \) can be expressed as

\[
\frac{1}{2\pi} \int_{-\pi}^{\pi} |f_i(e^{j\theta})|^2 d\theta = tr(F^H C_i F) \leq P_i
\] (2.19)

with

\[
C_i = I_{r+1} \otimes \langle u_i u_i^H \rangle,
\] (2.20)

where \( u_i \) denotes the unit column vector of length \( p \) with the \( i \)th entry being 1, and \( \otimes \) is the Kronecker product. The Lagrangian associated to the minimization of \( J(F) \) under the constraints (2.19) for \( 1 \leq i \leq p \) can be written as

\[
L(F, \lambda) = J(F) + \lambda^T c(F),
\] (2.21)

with

\[
c(F) = \begin{bmatrix} c_1(F) & c_2(F) & \ldots & c_p(F) \end{bmatrix}^T,
\]

\[
\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \ldots & \lambda_p \end{bmatrix}^T
\] (2.22)

where \( c_i(F) = tr(F^H C_i F) - P_i \). The computation of the optimum precoder \( F^{opt} \) then proceeds along the same lines as in the previous two subsections, except that \( \lambda \) represents now a Lagrange multiplier vector, which gives

\[
F^{opt} = (M + \sum_{i=1}^{p} \lambda_i^{opt} C_i)^{-1} \Gamma(H)^H E.
\] (2.23)

III. Optimal equalizer design with SNR constraints

When \( p < q \), provided the channel is right coprime, a Bezout equalizer can be used to recover the transmitted vector signal [1]. But, like all zero-forcing equalizers, Bezout equalizers tend to amplify the received noise. Therefore, we consider below the design of equalizers that minimize the power of the remaining ICI and ISI component at their output while obeying SNR constraints. This problem is of course dual to the optimal precoder design with transmit power constraints considered earlier. The power of the ISI and ICI component can be expressed as

\[
P_{ISI} = E\left[ ||G(n) * H(n) - E(n) * x(n)||^2 \right] = \sigma_n^2 J(G)
\] (3.1)
where

\[ J(G) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \| G(e^{j\theta}) - \bar{G}(e^{j\theta}) \bar{H}(e^{j\theta}) \|_F^2 d\theta \]

\[ = \| G \bar{H}^H \|_F^2 - \| E \|_F^2. \]  

(3.2)

and

\[ G = [ G(0), G(1), \ldots, G(r) ] \]

\[ E = [ E(0), E(1), \ldots, E(d + r) ]. \]  

(3.3)

(3.4)

The SNR between the desired signal and the noise at the equalizer output can be expressed as

\[ \text{SNR} = \frac{E[|E(n) \ast x(n)|^2]}{E[|G(n) \ast v(n)|^2]} = \frac{\sigma_n^2}{\sigma^2 \text{tr}(GG^H)}. \]  

(3.5)

From (3.1), we see that minimizing the ISI and ICI power is equivalent to minimizing \( J(G) \). Also, a constraint of the form \( \text{SNR} \geq S \) is equivalent to the power constraint

\[ \frac{1}{2\pi} \int_{-\pi}^{\pi} G(e^{j\theta}) G^H(e^{j\theta}) d\theta = \text{tr}(GG^H) \]

\[ \leq P_T = \frac{\sigma_n^2}{\sigma^2 S}. \]  

(3.6)

This reduces the equalizer design to a problem of the same form as considered in subsection II.B for the design of a precoder with a total power constraint.

Instead of imposing an aggregate SNR constraint, it may be preferable to require that each component of the vector signal at the equalizer output obeys a separate SNR constraint. Let \( g_i(z) \) and \( e_i(z) \) denote the \( i \)-th rows of the FIR matrices \( G(z) \) and \( E(z) \), with \( 1 \leq i \leq p \). Then the SNR for the \( i \)-th component of the equalizer output can be expressed as

\[ \text{SNR}_i = \frac{E[|e_i(n) \ast x(n)|^2]}{E[|g_i(n) \ast v(n)|^2]} = \frac{\sigma^2}{\sigma^2_i |g_i|^2}. \]  

(3.7)

where \( g_i \) denotes the \( i \)-th row of the matrix \( G \) specified by (3.3). Then, minimizing the ISI and ICI power under the SNR constraints \( \text{SNR}_i \geq S_i \) for \( 1 \leq i \leq p \) is equivalent to minimizing

\[ J(G) = \sum_{i=1}^{p} J_i(g_i) \]  

with

\[ J_i(g_i) = \| g_i \bar{H}^H - e_i \|_F^2, \]

(3.8)

(3.9)

where \( e_i \) denotes the \( i \)-th row of \( E \), under the constraints

\[ \| g_i \|^2 \leq P_i = \sigma^2_i / (\sigma^2 S_i). \]  

(3.10)

This problem is clearly equivalent to the separate minimization of \( J_i(g_i) \) under the constraint (3.10), which is a problem of the same type as was considered in subsection II.A.

Thus, for channels with more receive than transmit antennas, the design of ISI and ICI minimizing equalizers under output SNR constraints gives rise to problems identical to those considered earlier for the dual case of more transmit than receive antennas. This is not a surprise since for MIMO channels that operate in time division duplex, the channel is the same in both directions, so that a precoder designed for transmission in one direction becomes automatically an equalizer when the transmission direction is reversed.

IV. SIMULATION RESULTS

Since the same design technique is applicable to both equalizers and precoders, we only show results for the precoder case. In our simulations, a 4-input-2 output single-carrier frequency selective wireless channel with length 5 is considered. The transmit signal uses an uncoded QPSK constellation format and the combined response of the transmit and receive filters has a raised cosine spectrum with a roll-off factor of 0.2. The stationary broadband transmission channel from input \( k \) to output \( m \) is described by a five-path fading model. We assume that the transmit antennas are independent and are located in the far field of the receiver antennas, and receive antennas are uniform linear arrays, so that for each propagation ray emanating from a fixed transmit antenna the receive antennas have the same fading amplitudes, arriving angles, and multipath delays. The channel is stationary during the transmission of one block but changes independently from one block to another. The simulation results shown below represent an average over 1000 random channels. The channel noise is simulated by adding independent complex circular white Gaussian noise sequences which zero-mean and variance \( \sigma^2_n \) to each receiver antenna signal. In all plots, the SNR (in dB) is defined as

\[ \text{SNR} = \frac{\text{tr}(FF^H)}{\sigma^2} \]  

where \( \sigma^2 \) is the variance of the noise at each receiver antenna signal. In all plots, the SNR (in dB) is defined as

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bound, although for a fixed order, both the optimal and Bezout precoders appear to suffer from a BER floor effect.

Fig. 2 shows the BER performance for optimal precoders designed with different power constraints. The power constraint applied to each column of the precoder is $P_2 = P_T/q$, where $q = 2$ denotes the number of receive antennas and thus of users. In other words, the total power is divided evenly among users. Similarly, the power constraint applied to each transmit antenna is $P_T/p$ where $p = 4$. Thus in this case, the total transmit power $P_T = 2$ is divided evenly across all 4 transmit antennas. In all cases, the precoder has order 5. Fig. 2 shows that all three types of power constrained precoders exhibit a similar BER performance. Although the difference in performance among power constrained precoders is relatively small, it appears that the fewer the constraints, the better the performance.

V. Conclusion

In this paper, to overcome a limitation of Bezout precoders or equalizers for frequency-selective MIMO channels, a technique has been presented for the optimal synthesis of precoders subject to transmit power constraints, or equalizers subject to output SNR constraints. This design technique is applicable to unbalanced channels with either $p > q$ or $q > p$, and assumes full channel knowledge at either the transmitter or the receiver. It formulates the design of precoders or equalizers minimizing the ISI and ICI power under transmit power or SNR constraints as a positive semi-definite quadratic minimization problem with convex positive definite quadratic constraints. Lagrangian duality is employed to solve the lower dimensional dual form of this problem. Due to the reciprocity principle of wave propagation, the design of optimal precoders and equalizers have an identical form.

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Fig. 1. Comparison of the BER performance of Bezout precoders and optimal precoders with a total power constraint for orders 3, 5 and 7.

Fig. 2. BER performance comparison for optimal precoders designed with a total power constraint, or with power constraints on the precoder columns or rows.