quantum privacy amplification for quantum secure direct communication

Fu-Guo Deng$^{1,3}$ and Gui Lu Long$^{1,2}$

$^1$Key Laboratory For Quantum Information and Measurements, and Department of Physics, Tsinghua University, Beijing 100084, P. R. China
$^2$Center for Atomic and Molecular NanoSciences, Tsinghua University, Beijing 100084, P. R. China
$^3$Key Laboratory of Beam Technology and Materials Modification of MOE, and Institute of Low Energy Nuclear Physics, Beijing Normal University, Beijing 100875, P. R. China

(Dated: November 24, 2018)

Using quantum mechanics, secure direct communication between distant parties can be performed. Over a noisy quantum channel, quantum privacy amplification is a necessary step to ensure the security of the message. In this paper, we present a quantum privacy amplification scheme for quantum secure direct communication using single photons. The quantum privacy amplification procedure contains two control-not gates and a Hadamard gate. After the unitary gate operations, a measurement is performed and one photon is retained. The retained photon carries the state information of the discarded photon, and hence reduces the information leakage. The procedure can be performed recursively so that the information leakage can be reduced to any arbitrarily low level.

PACS numbers: 03.67.Hk, 03.67.Dd, 03.67.-a

Quantum secure direct communication (QSDC) allows two distant parties to directly communicate securely. It has attracted attention recently. In contrast, quantum key distribution usually generates a string of random keys and, the secret message is transmitted later through a classical channel, usually using the Vernam one-time-pad cipher-system. Recently, a QSDC protocol using batch of single photons is proposed. It has two distinct features. First it uses single photons instead of entangled photon pairs. Secondly, the transmission is operated in a batch by batch manner. This feature is also reflected in the QSDC protocols in Refs. The secret message is encoded and released to the quantum channel in public only when the channel is assured secure so that no secret message is leaked even though an malicious eavesdropper may intercept the encoded qubits. Over a noisy quantum channel, the scheme is completely secure. In practice, channel noise is inevitable, error correction and privacy amplification must be used in order to reduce the information leakage below the desired level.

The basic steps in the QSDC protocol in Ref. contains 4 steps. First, Bob prepares a batch of N single photons randomly in one of four polarization states: $|0\rangle = |+\rangle$, $|1\rangle = |--\rangle$, $|x\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-x\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$. Bob sends this batch of photons to Alice. Alice stores most of the single photons and selects randomly a subset of single photons and performs measurement using either the $x$ or $y$ basis. Alice publishes the measuring-basis and the measured results of these single photons. Upon these information, Bob determines the error rate. If the error rate is higher than the threshold, the process is aborted, and if the error rate is below the threshold, the process continues and Alice encodes her message using two unitary operations on the stored single photons and then sends them back to Bob. Upon receiving these encoded photons, Bob makes appropriate measurement and reads out the message.

Classical privacy amplification (CPA) has been used for QKD, for instance the BB84 protocol. Quantum privacy amplification (QPA) has been used for QKD using entangled quantum system, for instance for the Ekert91 QKD scheme. Without privacy amplification, the error threshold in the proposed QSDC scheme in ref. will be very small. To allow the scheme to be operable over a noisy channel, quantum privacy amplification has to be used. In this paper, we present a quantum privacy amplification scheme for QSDC (QSDC-QPA). This QSDC-QPA can be used not only for the QSDC protocol, but also for the BB84 QKD protocol. It uses simple local gate operations and single particle measurement, and these operations could be implemented by technologies that is currently being developed.

The basic scientific problem for the proposed QSDC-QPA scheme can be expressed as follows. Suppose Bob sends Alice a batch of single photons, each photon is randomly prepared in one of the four quantum states $|0\rangle$, $|1\rangle$, $(|0\rangle + |1\rangle)/2$ and $(|0\rangle - |1\rangle)/2$, where $|0\rangle$ and $|1\rangle$ denote the horizontal and vertical polarization state of a photon. Due to channel noise and eavesdropping, an error bit rate $r$ is known for the photon batch ($r$ is four times of the error bit rate detected by Alice and Bob using random sampling, because eavesdropper’s interception causes only 25 percent of error). The QSDC-QPA task is to condense a portion of photons from the batch so that Eve’s information about the condensed photons is reduced to below a desired level.

The basic operation of QSDC-QPA is shown in Fig. for two qubits. It consists of two control not (CNOT)
The state of the joint system of single photon 1 and 2 is changed to state of the second qubit measurement is |0⟩. ϕ₁ and ϕ₂ are the original states of the control and target qubit, respectively.

### Table I: The state of the output qubit when the result of the second qubit measurement is |0⟩.

| ϕ₂   | +z   | −z   | +x   | −x   |
|-------|------|------|------|------|
| +z    | 0    | 1    | 0    | 1    |
| −z    | 1    | 0    | 1    | 0    |
| +x    | 0    | 1    | 0    | 1    |
| −x    | 0    | 0    | 0    | 0    |

where

\[ |\psi⟩_{\text{out}} = \frac{1}{\sqrt{2}} \left( (a_1 b_2 + b_1 a_2) |0⟩_1 + (a_1 b_2 - b_1 a_2) |1⟩_1 \right) |0⟩_2 \]

After the two CNOT gates and H-gate operations, the state of the joint system of single photon 1 and 2 is changed to

\[ |\psi⟩_{\text{out}} = \frac{1}{\sqrt{2}} \left( (a_1 b_2 + b_1 a_2) |0⟩_1 + (a_1 b_2 - b_1 a_2) |1⟩_1 \right) |0⟩_2 \]

That is, Eve has no knowledge at all about the output state. But for Bob who has prepared the original states of the two qubits, he will know completely the output state when Alice tells him the σ₂,z measurement result.

If it happens that Eve has complete information about both qubits, she will know the output state exactly just like Bob. However the probability this can happen is only

\[ P_2 = r^2. \]

We can use the output qubit again as a control qubit and choose a third qubit from the batch as the target qubit and perform QPA operation on them. In this way, as more qubits are used in the QPA process, Eve’s information is reduced exponentially

\[ P_m = r^m, \]

where \( m \) is the number of qubits that have been used in the GPA. In this way, Alice can condense a portion of single photons from a batch of \( N \) photons with negligibly small information leakage. Then Alice can perform unitary operations to encode her secret message.

In summary, quantum privacy amplification on a batch of polarized single photons can be done with quantum mechanics for secure direct communication.

This work is supported the National Fundamental Research Program Grant No. 001CB309308, China National Natural Science Foundation Grant No. 60073009, 10325521, the Hang-Tian Science Fund, the SRFDP program of Education Ministry of China.

---

[1] A. Beige, B.-G. Englert, C. Kurtsiefer, and H Weinfurter, Acta Phys. Pol. A 101, 357 (2002); J. Phys. A: Math. Gen. 35, L407 (2002).
[2] K. Boström and T. Felbinger, Phys. Rev. Lett. 89, 187902 (2002).
[3] F.G. Deng, G.L. Long, and X.S. Liu, Phys. Rev. A 68, 042317 (2003).
[4] F.G. Deng and G.L. Long, Phys. Rev. A 69, 052319 (2004).
[5] F.L. Yan and X. Zhang, e-print quant-ph/0311132.
[6] Z.J. Zhang and Z.X. Man, e-print quant-ph/0403218.
[7] Q.Y. Cai and B.W. Li, Chin. Phys. Lett. 21, 601 (2004).
[8] Q.Y. Cai and B.W. Li, Phys. Rev. A 69, 054301 (2004).
[9] C.H. Bennett and G. Brassard, in Proceedings of IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India (IEEE, New Yoir, 1984), pp.175-179.
[10] A.K. Ekert, Phys. Rev. Lett. 67, 661 (1991).
[11] Vernam, Cipher printing telegraph systems for secret wire and radio telegraphic communications, J. Amer. Inst. Elec. Eng., Vol. 55 (1926), 109-115.
[12] C.H. Bennett, G. Brassard, and J.M. Robert, SIAM J. Comput. 17, 210 (1988); C.H. Bennett, G. Brassard, C. Crépeau, and U.M. Maurer, IEEE Trans. Inf. Theory 41, 1915 (1995).
[13] C.H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. Smolin, and W.K. Wootters, Phys. Rev. Lett. 76, 722 (1996).
[14] D. Deutsch, A. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, and A. Sanpera, Phys. Rev. Lett. 77, 2818 (1996).

FIG. 1: Quantum privacy amplification operation for two qubits.