Relating Quantum Information to Charged Black Holes

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Abstract

Quantum non-cloning theorem and a thought experiment are discussed for charged black holes whose global structure exhibits an event and a Cauchy horizon. We take Reissner-Norström black holes and two-dimensional dilaton black holes as concrete examples. The results show that the quantum non-cloning theorem and the black hole complementarity are far from consistent inside the inner horizon. The relevance of this work to non-local measurements is briefly discussed.

Keywords: quantum non-cloning theorem, black hole information, black hole complementarity

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I. INTRODUCTION

The loss of information of a black hole has long been an interesting question in the theoretical physics\cite{1}. Although many physicists have devoted time searching for the answer to this question\cite{2-8}, the fundamental solution is still elusive. On the other hand, the development of quantum information theory \cite{9} draws attention to fundamental questions about what is physically possible and what is not. For instance, the quantum non-cloning theorem \cite{10}, which asserts, unknown pure states cannot be reproduced or copied by any physical means. Recently, there are growing interests in the quantum non-cloning theorem. The original proof of this theorem \cite{10} shows that the cloning machine violates the quantum superposition principle. The second version of the quantum non-cloning theorem states that a violation of unitarity makes cloning two non-orthogonal states impossible\cite{11,12}. However, the third version argues that if the unitarity of two non-orthogonal states is destroyed and only if they are linearly independent, then the states which are secretly chosen from a certain set can be probabilistically cloned \cite{13,14}. In this paper, we wish to discuss what the quantum non-cloning theorem might infer to a charged black hole.

We notice that, among the many efforts endeavoring to resolve the black hole information paradox, Susskind, Thorlacius and Uglum suggest the black hole complementarity principle \cite{3} which can be formulated as follows: i) From the point of view of an external observer the region just outside the horizon, stretched horizon, acts like a very hot membrane which absorbs, thermalizes and emits any information that falls to the black hole; ii) From the point of view of a freely falling observer there is nothing special at the horizon so a freely falling observer can cross the horizon in his way to the singularity.

As it is pointed out by Lowe et.al\cite{5}, this principle describes physical pictures which are apparently contradictory. According to this principle, an observer who remains outside the event horizon of a black hole can describe the black hole as a hot membrane, the stretched horizon, which absorbs and stores anything falling onto it. And there is no information loss: all the information stored in the membrane will eventually re-emitted in the form of Hawking radiation. On the other hand, an observer who falls freely into the black hole sees things very differently, no membrane, no stretched horizon, and nothing irregular at the event horizon. Moreover, from the point of view of the freely falling observer all the information entering into the black hole will never come back. Therefore, it seems that the
black hole can act as a cloning machine because the matter which has fallen past the event horizon and the Hawking radiation are not different objects. If the re-emitted quantum information has a chance to fall into the black hole, then once it crosses the event horizon there will be definitely two copies of the same quantum information. This violates the basic principle of the quantum mechanics. In relation to this conflict, Susskind et al show that duplicate information will be never detected in a Schwarzschild black hole because any measurement inside the black hole will require the energy far beyond the total energy of the black hole\[4,15\]. And if we make an assumption that quantum mechanics forbids information cloning as meaning that no real observer is ever allowed to detect duplicate information, then the quantum non-cloning theorem is preserved.

Nevertheless, we find that the relationship between the quantum non-cloning theorem and the black hole complementarity is still far from consistent for charged black holes whose global structure exhibits an event and a Cauchy horizon. In this paper, we wish to discuss the quantum non-cloning theorem for Reissner-Norström black holes(RN) and two-dimensional (2D) dilaton black holes as concrete examples, because the properties of RN black holes and 2D dilaton black holes are different very much from Schwarzschild black holes. This will be helpful for our thorough understanding of the quantum non-cloning theorem and the black hole complementarity.

II. A THOUGHT EXPERIMENT CONDUCTED ON AN RN BLACK HOLE

We first begin with the metric of general spherically symmetric space-time, which can be written as\[16,17\] ( Planck units is used: $\hbar = G = c = k = 1$ hereafter)

\[
ds^2 = e^{2U(r)} dt^2 - e^{-2U(r)} dr^2 - R^2(r) \left( d\theta^2 + \sin^2\theta d\varphi^2 \right).
\]

Following the idea of [16], we can concentrate on the "near horizon limit", and define $y = r - r_H$, where $r_H$ is the event horizon and $y \ll r_H$. The metric becomes

\[
ds^2 = e^{2U(y)} dt^2 - e^{-2U(y)} dy^2 - R^2(y) d\Omega^2.
\]

If we set

\[
\rho = \int e^{-U(y)} dy, \quad \omega = e^{U(y)} t/\int e^{-U(y)} dy,
\]

\[
(3)
\]
then the metric has the form

\[ ds^2 = \rho^2 d\omega^2 - d\rho^2 - R^2(y) d\Omega^2. \]  \hspace{1cm} (4)

We further define

\[ X^+ = \rho e^\omega, \quad X^- = -\rho e^{-\omega}, \]  \hspace{1cm} (5)

then the metric becomes

\[ ds^2 = dX^+ dX^- - R^2(y) d\Omega^2. \]  \hspace{1cm} (6)

In this way, the horizon is no longer singular. In the following, we would like to investigate the quantum non-cloning theorem for RN black holes. The line elements of a RN black hole are given by

\[ ds^2 = (1 - \frac{2M}{r} + \frac{Q^2}{r^2})dt^2 - (1 - \frac{2M}{r} + \frac{Q^2}{r^2})^{-1}dr^2 - r^2(d\theta + \sin\theta d\varphi)^2, \]  \hspace{1cm} (7)

where \( M, Q \) are the mass and the charge of the black hole. The horizon equation is

\[ r^2 - 2Mr + Q^2 = 0, \]  \hspace{1cm} (8)

and the solutions

\[ r_H = M + \sqrt{M^2 - Q^2}, \quad r_I = M - \sqrt{M^2 - Q^2} \]  \hspace{1cm} (9)

are the event horizon and the inner horizon respectively. Thus the space-time can be distinguished into three regions:

A: \( 0 < r < r_I \); B: \( r_I < r < r_H \); and C: \( r > r_H \).

We can find that, while the surface \( r = r_H \) is an event horizon in the same sense that \( r = 2M \) is an event horizon in the Schwarzschild space-time, the surface \( r = r_I \) is a horizon in a different sense. The original Schwarzschild singularity \( r = 0 \) here has become the inner horizon, while the RN singularity \( r = 0 \) corresponds to a negative value of \( r \) in the original Schwarzschild metric. In the following analysis, we still concentrate on the ”near horizon limit” and consider a small angular region near a point on the horizon. Define

\[ y = r - r_H; y \ll r_H, \]

\[ \rho = \frac{M + \sqrt{M^2 - Q^2}}{\sqrt{M^2 - Q^2}}(2y)^{-\frac{1}{2}}, \]

\[ \omega = \frac{\sqrt{M^2 - Q^2}}{(M + \sqrt{M^2 - Q^2})^2}t. \]  \hspace{1cm} (10)
FIG. 1: A thought experiment conducted on a RN black hole

and

$$X^+ = \rho e^\omega, \quad X^- = -\rho e^{-\omega}. \quad (11)$$

Thus the metric can finally be written as

$$ds^2 = dX^+dX^- - dx^idx^i \quad (12)$$
The lifetime of information stored in the stretched horizon is called the black hole information retention time[4]. According to the black hole complementarity, when a q-bit information is thrown into a black hole, an observer outside the event horizon can calculate how long it will be re-emitted in form of Hawking radiation. We can derive the information retention time of the RN black holes as follows. When a RN black hole radiates, the spectrum of particles is given by the Planck distribution[18]:

\[ dE_\omega = \frac{(\omega - e\phi)^3 d\omega}{e^{(\omega - e\phi)/T_H} - 1}, \]  

where \( dE_\omega \) is the radiation energy in the spectral range \( \omega \) to \( \omega + d\omega \). Integrating over \( \omega \) from \( e\phi \) to \( \infty \), one obtains the rate at which the black hole loses energy, i.e., mass

\[ \frac{dM}{dt} = -\sigma T_H^4 A. \]  

(14)

\( A = 4\pi r_H^2 \) is the area of the event horizon and \( \sigma \) the Stefan-Boltzman constant. Thus, for RN black holes with \( T_H = \frac{(r_H - r_I)}{4\pi r_H^2} \), it is given by

\[ \frac{dM}{dt} = -\frac{\sigma (r_H - r_I)^4}{(4\pi)^3 r_H^6}. \]  

(15)

We integrate (15) to get

\[ \int_0^t dt = -\frac{(4\pi)^3}{\sigma} \int_{M,Q}^0 \frac{r_H^6 dM}{(r_H - r_I)^2}. \]  

(16)

Substituting (9) into (16) and assuming \( Q = \lambda M \), where \( 0 \leq \lambda < 1 \), we have

\[ t = \frac{(4\pi)^3}{3\sigma} \frac{(1 + \sqrt{1 - \lambda})^6}{16(1 - \lambda)^2} M^3. \]  

(17)

Hence, the information retention time should have the same order of magnitude, this is to say \( t_R \sim M^3 \). Now, let's consider a thought experiment which is originally considered in [4] and repeated later in the review article[15]. For simplicity, we just repeat the same experiment process of that in [4,15] for a RN black hole, for that is helpful for our understanding on the relationship between black hole complementarity and quantum non-cloning theorem. When a RN black hole is formed, a q-bit information is thrown in before the black hole has a chance to evaporate. Here the information must be quantum information and has the general form: \( | \varphi \rangle = a | 0 \rangle + b | 1 \rangle \), because only quantum information has the nature of non-cloning. According to the observer who falls with the q-bit, the information...
at a later time will be localized behind the horizon at a point (a); see figure 1. On the other hand, an observer outside the horizon eventually sees all of the energy returned in the form of Hawking radiation. Thus, according to the black hole complementarity, a measurement can be performed on the radiation and the original information can be determined. We assume there is an observer $O$ stationed outside the horizon to collects information stored in the infalling q-bit. At that time, the observer jumps into the black hole, carrying the information to point (c) behind the horizon. Now there are two copies of the q-bit behind the horizon one at (a) and one at (c). A signal from (a) to (c) can reveal that information has been duplicated and then the quantum non-cloning theorem is violated. The analysis as follows can show us how the quantum non-cloning theorem is preserved in region B and how it is violated in region A.

In the thought experiment, the point (c) must occur before the trajectory of $O$ intersects the inner horizon. On the other hand, $O$ may not cross the event horizon until the information retention time has elapsed. The implication of the two constraints is most easily seen using the following coordinates:

$$
X^+ = \rho e^{\omega},
$$
$$
X^- = -\rho e^{-\omega},
$$
$$
\omega = \frac{\sqrt{M^2 - Q^2}}{(M + \sqrt{M^2 - Q^2})^2} t = \frac{\sqrt{1 - \lambda t}}{(1 + \sqrt{1 - \lambda})^2 M}.
$$

(18)

An observer outside the horizon must wait a time $t \sim CM^3$ (the time which we have obtained in (17) and here $C$ is a positive constant), to collect a bit from the Hawking radiation. Thus, the observer may not jump into the black hole until $X^+ \sim e^{CM^2}$, which means that $O$ should be at a point satisfying

$$
X^- < e^{-CM^2}.
$$

(19)

This requires that the message sent between (a) and (c) must be sent within a time interval $\delta t$ of the same order of magnitude, an incredibly short time ($\delta t \sim e^{-CM^2}$).

The uncertainty principle of quantum mechanics requires that the quanta of the message have energy of order $(\delta t)^{-1}$, which can be written as

$$
E_{\text{signal}} \sim e^{CM^2}.
$$

(20)

Clearly, the energy required is much greater than the total energy or mass of the black hole. Therefore, it is impossible to detect duplicate information in the region between the event
horizon and the inner horizon of RN black holes. The physical process discussed in [4,15] also show that any communication between (a) and (c) need a super-planckian frequency. As it is pointed in [15], there must be something wrong with the usual ideas of local quantum field theory in black hole background because a theory should not predict things which are in principle unobservable.
In fact, when we turn to discuss the quantum non-cloning theorem within the inner horizon, the duplicate information is not undetectable any longer. RN black holes are different from Schwarzschild black holes: While Schwarzschild black holes have space-like singularities, the singularity of a RN black hole is time-like. As the inner horizon is not a mathematical singularity of the geometry, we could concentrate on the near horizon limit.

We consider a small angular region near a point on the horizon and define:

\[ y' = r - r_I, \quad y' \ll r_I \]
\[ \rho' = \sqrt{2y'} \frac{M - \sqrt{M^2 - Q^2}}{\sqrt{M^2 - Q^2}} \]
\[ \omega' = \frac{\sqrt{M^2 - Q^2}}{(M - \sqrt{M^2 - Q^2})^2} t, \]

and

\[ X'^+ = \rho' e^{-\omega'}, \]
\[ X'^- = \rho' e^{\omega'}, \]

where \( X'^+ \) should correspond to \( X^- \) for consistency. Then the metric near the inner horizon can be finally written as

\[ ds^2 = dX^+ dX^-. \]

For an observer inside the inner horizon, the two copies of q-bit must appear at a time interval no less than the so called black hole information retention time (see figure2), which is to say

\[ X'^- > \rho' e^{\omega'}. \]

Therefore, according to the quantum uncertainty principle, the energy required to send a message to detect duplicate information inside the inner horizon, reads

\[ E_{\text{signal}} \sim e^{-CM^2}, \]

which is apparently permitted by the quantum mechanics. Indeed, once the q-bit information and the observer have a chance to cross the inner horizon, the quantum non-cloning theorem seems unavoidable violated. We can easily see from (7) that

\[ g_{00} < 0, g_{11} > 0, g_{22} > 0, g_{33} > 0, (r > r_H) \]
\[ g_{00} > 0, g_{11} < 0, g_{22} > 0, g_{33} > 0, (r_H > r > r_I) \]
\[ g_{00} < 0, g_{11} > 0, g_{22} > 0, g_{33} > 0, (r < r_I). \]
The region between the event horizon and the inner horizon is a region which \( r \) is the coordinate of time, and \( t \) is the coordinate of space. However, for regions \( r < r_I \) and \( r > r_H \), space and time are not interchanged. An observer who crossed the inner horizon can act freely and the singularity is avoidable for the observer. This is a main difference between a RN black hole and a Schwarzschild black hole. Thus, the black hole complementarity and the quantum non-cloning theorem become incompatible in region A.

Moreover, what discussed in [5,15] about the quantum Xerox principle is almost entirely cannot be applied to extremal RN black holes. For extremal RN black holes, the two horizons coincide \( r_H = r_I \). The event horizon becomes degenerate, with nonzero area but vanishing temperature. Theoretically, there exists two possibilities for the black hole complementarity and the quantum non-cloning theorem: one is that black hole complementarity or quantum non-cloning theorem is wrong; the other is that they are both correct but there is some physics in RN black holes still unknown. As to this puzzle, we want to say that, instability of the inner horizon is not discussed here. Small external perturbations to the inner horizon are not strong enough to change the metric but if we consider quantum electrodynamics process in region B, the metric can be modified and it can be proved that the true space-time singularity is indeed created and no inner horizon formed then [17].

Now, we still lack evidence to prove that the experiment conducted on a RN black holes can generate enough perturbations to change the nature of the inner horizon. And also, when we talk about q-bit information here, we talk about it by only making local measurements. Quantum information theory is not a local theory. The essence of quantum information theory is entanglement and non-locality. In quantum theory one talks about communication between distant entangled states, where each state separately sees only random events and it is through entanglement which quantum information is transmitted. Local measurements cannot be an effective way to tell what is true and what is not true in the above thought experiments. However, we do not mean to talk about quantum information theory and the instability of the inner horizon in detail. To answer the above question we should first know the nature of entanglement in the background of curved space-time. This is a rather difficult problem because quantum gravity is needed.
III. QUANTUM NON-CLONING THEOREM IN TWO-DIMENSIONAL DILATON BLACK HOLES

It is interesting to extend our discussion to 2D dilaton black holes. The properties of 2D dilaton black holes have been widely investigated in the past several years and it was found that many basic results of standard 4D black hole physics find their counterpart in the 2D case[19]. The metric of the 2D dilaton black hole adopted is as follows:

\[ dS^2 = N^2 dt^2 - (N^2)^{-1} dr^2, \]  

(27)

where

\[ N^2 = 1 - \frac{2M}{\Lambda}e^{-\Lambda r} + \frac{Q^2}{\Lambda^2}e^{-2\Lambda r}. \]  

(28)

\( M \) and \( Q \) are, respectively, the mass and charge of a black hole, and \( \Lambda \) is the cosmological constant. The horizons of the black hole are

\[ r_{\pm} = \Lambda^{-1} \ln[\Lambda^{-1}(M \pm \sqrt{M^2 - Q^2})]. \]  

(29)

And it is assumed that \( Q < M \). The space-time can also be separated into three regions: A: \( 0 < r < r_-; \) B: \( r_- < r < r_+; \) and C: \( r > r_+ \). The Hawking temperature of the black hole is

\[ T_+ = \frac{\Lambda(M^2 - Q^2)^{1/2}}{2\pi(M + (M^2 - Q^2)^{1/2})} \]  

(30)

We can further define

\[ y = r - r_+; \quad y \leq r_+ \]

\[ \rho = \frac{\sqrt{2}(e^{\Lambda y} - 1)^{1/2}}{\sqrt{(M^2 - Q^2)^{1/2}[M + (M^2 - Q^2)^{1/2}]}} \]

\[ \omega = \frac{(M^2 - Q^2)^{1/2}[M + (M^2 - Q^2)^{1/2}]}{\Lambda} t, \]  

(31)

and

\[ X^+ = \rho e^{-\omega}, X^- = \rho e^{\omega} \]  

(32)

Then, the metric can be rewritten as

\[ dS^2 = dX^+ dX^- \]  

(33)
The black hole information retention time can be obtained by using the one-dimensional Stefan-Boltzman formula

\[
\frac{dM}{dt} = \frac{\pi^2 T^2 k_B^2 A}{6},
\]

where \( k_B \) is the Boltzman constant and \( A \) is the "area" of the 2D dilaton black hole. After integration of Eq.(34), we can express the information retention time in a simple way, say

\[
t_R \sim C M,
\]

where \( C \) is a constant and \( Q = \lambda M \ (0 \leq \lambda < 1) \) is assumed in the integration. Following the same process in section II, we find that the message sent between the two observers in region B requires the quanta of energy \( E_{\text{signal}} \sim e^{CM^3} \), which is different from Eq.(24), but still greater than the total energy of the black hole. While inside the inner horizon, the duplicate information is no longer undetectable. After the similar calculation to section II, we obtain the energy required to send a message to detect duplicate information inside the inner horizon, which goes as \( E_{\text{signal}} \sim e^{-CM^3} \). The results also show that the non-cloning theorem can be violated for the 2D dilaton black holes.

IV. CONCLUSION

In summary, we have discussed the quantum non-cloning theorem in RN black holes and 2D dilaton black holes. We find that the quantum non-cloning theorem can well established in the region between the inner horizon and event horizon, but to be violated inside the inner horizon. This seems to indicate that the black hole complementarity principle or the quantum non-cloning theorem is confront with a challenge and we may need some new physics to resolve the dilemma. Quantum information theory and the instability of the inner horizon are proposed as possible directions. We would like to investigate them in our future work. In addition, what we discussed about the RN black holes is also applicable to other charged black holes such as Garfinkle-Horne dilaton black holes and Gibbons-Maeda dilaton black holes[20].

Recently, Horowitz and Maldacena (HM) have proposed an alternative model of black hole evaporation by imposing a final state boundary condition at black hole singularities, to resolve the apparent contradiction between string theory and semiclassical arguments over whether black hole evaporation is unitary [21]. This model requires a specific final state at
black hole singularity which is perfectly entangled between the collapsing matter and the incoming Hawking radiation. Thus, information in the black hole can be "teleported" out in the outgoing Hawking radiation and the quantum non-cloning theorem can be well preserved in the whole process. The proposal might shed light on the problems suffered by the black hole complementarity. However, Gottsman, Preskill, and later Yurtsever and Hockney argued that the proposed constraint must lead to nonlinear evolution of the initial quantum state, and one cannot ensure the black hole final state to be maximally entangled[22,23,24]. Giddings and Lippert pointed out that the HM scenario depends on assuming inside and outside Hilbert spaces and may conflict with the expectations for observations made by the inside observers[25]. In [26], we extend HM’s proposal to Dirac fields and find that if annihilation of the infalling positrons and the collapsed electrons inside the horizon is considered, then the nonlinear evolution of collapsing quantum state can be avoided.

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[1] S. W. Hawking, Comm. Math. Phys. 43, 199 (1975) ; Phys. Rev.D 14 2460 (1976).
[2] G. ’t Hooft, Nucl. Phys B 335, 138 (1990).
[3] L. Susskind, L. Thorlacius and J. Uglum, Phys. Rev. D 48, 3743 (1993); arXiv:hep-th/9306069.
[4] L. Susskind and L. Thorlacius, Phys. Rev. D 49, 966 (1994); arXiv:hep-th/9308100.
[5] D. A. Lowe, J. Plchinski and L. Susskind, Phys. Rev. D 48, 3743 (1993); arXiv:hep-th/9506138.
[6] D. N. Page, Phys. Rev. Lett. 71, 1291 (1993).
[7] Y. Kiem, E. Verlinde and H. Verlinde, Phys. Rev. D 52, 4668 (1995); arXiv:hep-th/9502074.
[8] C. R. Stephens, G. ’t Hooft and B. F. Whiting, Class. Quan. Grav.11, 621 (1994).
[9] C.H. Bennet, Phys. Today 48, No.10, 24 (1995).
[10] W. K. Wotters and W. H. Zurek, Nature 299, 802 (1982).
[11] H. P. Yuen, Phys. Lett. A 113, 405 (1986).
[12] G.M. D’Ariano and H. P. Yuen, Phys. Rev. Lett. 76 2832 (1996); H.Barnum, G. M. Caves, C.A. Fuchs, R. Jozsa and B. Schumacher, Phys. Rev. Lett. 76 2818 (1996).
[13] L. M. Duan and G. C. Guo, Phys. Lett. A 243, 261 (1998).
[14] L. M. Duan and G. C. Guo, Phys. Rev. Lett. 80, 4999 (1998).
[15] D. Bigatti and L. Susskind, arXiv:hep-th/0002044
[16] G. W. Gibbons and R. E. Kallosh, Phys. Rev. D 51, 2839 (1995).
[17] I.D. Novikov and V. P. Frolov, Physics of Black Holes, Dordrecht: Kluwer Academic Publishers (1989).
[18] L. D. Landau and E. M. Lifshitz, Statistical Physics, Oxford: Reed Educational and Professional Publishing Ltd (1980).
[19] S. Nojiri and S. D. Odintsov, Int.J.Mod.Phys. A16, 1015 (2000); D. Grumiller, W. Kummer and D.V. Vassilevich, Phys.Rept.369, 327 (2002); D. Grumiller, arXiv:gr-qc/0105078 Y. Kiem, C.Y. Lee and D.Park, Class.Quant.Grav. 15 2973 (1998); Y. G. Shen, D.M. Chen and T. J. Zhang, Phys. Rev. D 56 6698 (1997); T. Dereli and R.W. Tucker, Class.Quant.Grav. 11 2575 (1994); S. Mignemi, Annals Phys. 245 23 (1996); Y. Kazama, Y. Satoh and A. Tsuchiya, Phys.Rev.D 51 4265 (1995); O. Lechtenfeld and C. Nappi, Phys.Lett.B 288 72 (1992)
[20] D. Garfinkle, G.T. Horowitz and A. Strominger, Phys.Rev.D 43 3140 (1991); J. Horne and G.T. Horowitz Phys.Rev.D 46 1340 (1992); G.W. Gibbons and K. Maeda, Nucl.Phys.B 298 741 (1988); X.H. Ge and Y.G. Shen, Class.Quant.Grav.21 1941 (2004); X.H. Ge and Y.G. Shen, Chin. Phys. Lett.21 1413, (2004).
[21] G.T. Horowitz and J. Maldacena , JHEP 0402 008,(2004)
[22] D. Gottsman and J. Preskill, JHEP 0403 026 (2004) arXiv:hep-th/0311269
[23] U. Yurtsever and G. Hockney, arXiv:gr-qc/0409112
[24] J. G. Russo, arXiv:hep-th/0501132
[25] S. B Giddings and M. Lippert, Phys.Rev. D69 124019, (2004)
[26] X.H. Ge and Y.G. Shen, Phys. Lett. B (in press) arXiv:hep-th/0501131