Ambiguity Analysis for Multitarget Estimation Using Random Permutated Frequency Diverse Arrays

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This work was supported by the National Natural Science Foundation of China under Grant 61871425, Grant 61631019, Grant 61861011, and Grant 61861008.

ABSTRACT For multitarget estimation using a frequency diverse array with a random permutated frequency increment (RP-FDA), ambiguity occurs when targets are located at specific locations and the frequency increment vector is not properly designed. To provide guidance for the design of a frequency increment vector to avoid ambiguous estimation, ambiguity analysis, including the principle of ambiguity, the relationship among ambiguity types, the relative target location, and the frequency increment vector, is necessary. In this paper, a relation matrix that describes the targets' relative location is established and analyzed to reveal the principle of ambiguity, that is, the rank deficiency of the relation matrix. Then, a law of relative target locations causing the rank deficiency of the relation matrix and the categories of the relative target locations are developed. Moreover, two cases of the relative target locations causing ambiguity are proposed, which are transformed based on the relative target locations causing rank deficiency. By employing the law of relative target locations causing ambiguity in a frequency increment vector pre-evaluation, more ambiguous frequency increment vectors can be identified compared with the identification using single-target criterion, which improve the unambiguous probability of a multitarget estimation. Numerical simulations verify the effectiveness of the proposed law of a relative target location causing ambiguity and of the rank deficiency examination to avoid ambiguity.

INDEX TERMS Frequency diverse array (FDA), random FDA, frequency increment vector identification, multitarget localization.

I. INTRODUCTION
Frequency diverse arrays (FDAs) have attracted increasing attention in recent years \cite{1}–\cite{3}. FDAs can produce a range-angle-dependent beampattern by employing a small linear frequency increment across antenna elements. This beampattern can be exploited for range-dependent interference suppression \cite{4}, range ambiguity resolution \cite{5}, performance enhancement in synthetic aperture radar \cite{6} and moving target indication \cite{7}. However, the beampattern of a conventional FDA is coupled in the range and angle dimensions, so that the range and angle of the target of interest cannot be estimated directly from the FDA radar output.

To decouple the range and angle in estimation, FDAs with random frequency increments (RFDAs), where the frequency increment of each array element obeys some kind of distribution, have been proposed to produce a thumbtack-like beampattern \cite{8}. For RFDAs, the strategy of frequency increments in a discrete uniform distribution is cost-effective because all the frequency increments across the array element are integral multiples of the unit frequency increment, which is easy to implement. Moreover, FDAs with frequency increments that are permutations of the linear frequency increments \cite{9}–\cite{13} are more efficient because they utilize the whole frequency increments. The permutations of the linear frequency increments are not unique, and the number of frequency increment permutations depends on the number of array elements. However, not all of the frequency increment permutations can decouple the range and angle of the target.
Some fake localizations occur when the frequency increment permutation is not chosen properly.

For a single target scenario, FDAs using Costas sequence modulated frequency increments [10] and random permuted FDAs (RP-FDAs) with frequency increment vectors satisfying the identifying criterion [9] have been introduced to properly choose the frequency increment vectors to avoid ambiguous estimation. However, for a multitarget scenario, the ambiguity depends not only on the frequency increment vectors but also on the relative location of the targets. Different from the phased array, the phenomenon that relative location of the targets affects the performance of estimation is unique for FDA, which needs to be studied and analyzed. Moreover, in a real application, it is possible that the targets are located at a specific relative location causing ambiguity, and then an incorrect localization is obtained. Furthermore, in electronic countermeasures, targets can be arranged at the specific relative target location by adversaries after frequency metrology, resulting in a detection with a false alarm. The ambiguity analysis for multitarget estimation using RP-FDAs is established to reveal the principle of ambiguity, that is, the rank of the matrix that describes the targets’ relative location is estimated to reveal the principle of ambiguity, that is, the rank of the matrix that describes the targets’ relative location is estimated.

In this paper, instead of null spectrum analysis, a relation matrix that describes the targets’ relative location is established to reveal the principle of ambiguity, that is, the rank deficiency of the relation matrix. Then a law for a relative target location causing ambiguity is sought. By employing the law of relative target locations causing ambiguity in a frequency increment vector pre-evaluation, more ambiguous frequency increment vectors can be identified compared with the identification using single-target criterion, which improves the unambiguous probability of a multitarget estimation. Certainly, the proposed ambiguity analysis method is also suitable for two-target scenario. The rest of this paper is organized as follows. In Section II, the signal model and ambiguity origin are briefly introduced. In Section III, the law of relative target locations causing ambiguity is derived and discussed in detail. Simulation results are presented to verify the effectiveness of the proposed law of a relative target location causing ambiguity and the significance of the ambiguity analysis in Section IV, while the conclusions are drawn in Section V.

II. SIGNAL MODEL AND AMBIGUITY ORIGIN

A. SIGNAL MODEL

Consider a uniform linear array consisting of \( N \) elements and an interelement distance \( d \). The carrier frequency at the \( n \)th element is

\[
f_n = f_0 + m_n \Delta f, \quad n = 0, 1, \ldots, N - 1,
\]

where \( \Delta f \) is the frequency increment and \( f_0 \) is the reference carrier frequency. In addition, \( m_n \) is the \( n \)th element of the \( N \times 1 \) frequency increment vector \( \mathbf{m} \) containing a random permutation of the integers 0 to \( N - 1 \), where \( \mathbf{m} \) corresponds to the frequency increment permutation mentioned in Section I.

In the rest of the paper, the frequency increment vector \( \mathbf{m} \) is employed instead of the frequency increment permutation for convenient description.

A block diagram of the transmit and receive chain is shown in Fig. 1. The transmitted signal of the \( n \)th element can be expressed as \( \tilde{s}_n(t) = e^{j2\pi f_0 t} \) [8]. We consider the case in which the receiver is band-limited, with a narrow-band filter equipped on each element, and the \( n \)th element only receives/processes the signal with carrier frequency \( f_0 \) [15]. We choose the first element as the reference. Thus, the received echo of the \( n \)th element after filtering is [8]

\[
\tilde{y}_n(t) = \sum_{i=1}^{K} \beta_i(t - \tau_i)e^{j2\pi f_0 t - \frac{2\pi r_i n d \sin \theta_i}{\lambda}} + e_n(t)
\]

\[
= \left\{ \sum_{i=1}^{K} \beta_i(t - \tau_i)e^{j2\pi f_0 t + m_n \Delta f_{r_i}} e^{-j2\pi f_0 \frac{2\pi r_i n d \sin \theta_i}{\lambda}} \right\} + e_n(t),
\]

where \( \tau_i = \frac{2\pi r_i n d \sin \theta_i}{\lambda} \). Here, \( \beta_i(t), \theta_i \) and \( r_i \) represent the complex reflection amplitude, angle (measured from the normal direction to the target direction) and slant range of the \( i \)th target, respectively. The targets are considered stationary. \( K \) represents the number of targets, and \( e_n(t) \) is complex additive white Gaussian noise. To avoid grating lobes along the angle dimension of the transmit-receive beampattern, we consider \( d \leq \frac{\lambda}{4K} \) [8], [15]. To avoid grating lobes (periodical main lobe) along the range dimension of the transmit-receive beampattern, we consider \( \Delta f \leq \frac{c}{2R_{\max}} \), where \( R_{\max} \) denotes the maximum range of the area of interest. The term \( m_n \frac{2\Delta f_{r_i} \sin \theta_i}{c} \) is ignored because \( \Delta f \ll f_0 \) and \( d \sin \theta_i \ll r_i \). After coherent detecting \( e^{j2\pi f_0 t + m_n \Delta f_{r_i}} \), (2) becomes

\[
y_n(t) = \sum_{i=1}^{K} \xi_i(t)e^{j\psi_{n,i}} + e_n(t),
\]

where \( \xi_i(t) = \beta_i(t - \tau) e^{-j2\pi f_0 \frac{2\pi r_i}{\lambda}} \) and

\[
\psi_{n,i} = -2\pi m_n \frac{2\Delta f_{r_i}}{c} - n \frac{2\pi f_0 d \sin \theta_i}{c},
\]
Therefore, the received signal in vector form is
\[ y(t) = A(\psi) \xi(t) + e(t), \]
where \( y(t) = [y_0(t), \ldots, y_{N-1}(t)]^T \), \( \xi(t) = [\xi_0(t), \ldots, \xi_{N-1}(t)]^T \), \( A(\psi) = [a(\psi_1), \ldots, a(\psi_K)] \), and \( e(t) = [e_0(t), e_1(t), \ldots, e_{N-1}(t)]^T \). Signals in (5) can be used for target estimation either by matched filtering or subspace decomposition algorithms such as the multiple signal classification (MUSIC) algorithm.

**B. AMBIGUITY ORIGIN**

For the phased array, (4) becomes \( \psi_{n,i} = 2\pi(m_n 2\pi d \sin \theta_i) \). When \( \theta_i \neq \theta_j \), the steering vectors \( a(\psi_1), \ldots, a(\psi_K) \) are linearly independent. However, for an RP-FDA, the steering vector depends not only on \( \theta_i \) and \( n \) but also on \( m_n \) and \( r_i \). The steering vectors are not always linearly independent, even if \( r_i \neq r_j \) and \( \theta_i \neq \theta_j \). Therefore, the target locations (range and angle) need to be analyzed. By choosing one of the targets as reference \( Q_1(r_1, \theta_1) \), the location of the \( i \)th target is expressed as
\[
\begin{align*}
    r_i &= r_1 + \Delta r_i \\
    \sin \theta_i &= \sin \theta_1 + \Delta(\sin \theta_i),
\end{align*}
\]
where \( \Delta r_i \) represents the difference in the range between the \( i \)th target and the reference target, and \( \Delta(\sin \theta_i) \) represents the difference in the sine of the angle between the \( i \)th target and the reference target. Substituting (6) into the steering matrix in (5), then \( A(\psi) \) becomes
\[ A(\psi) = [a(\psi_1) \otimes 1] \circ J, \]
where \( 1 \) denotes the \( 1 \times K \) vector with all elements 1, \( \otimes \) denotes the Kronecker product and \( \circ \) denotes the Hadamard product.

\[ J = \\
\begin{bmatrix}
1 & e^{j\phi_{0,2}} & \ldots & e^{j\phi_{0,K}} \\
1 & e^{j\phi_{1,2}} & \ldots & e^{j\phi_{1,K}} \\
\vdots & \vdots & \ddots & \vdots \\
1 & e^{j\phi_{N-1,2}} & \ldots & e^{j\phi_{N-1,K}}
\end{bmatrix}, \]
\[ \phi_{n,i} = -4\pi \left[ m_n \frac{\Delta f}{c} \Delta r_i - \frac{f_{0d}}{c} \Delta(\sin \theta_i) \right]. \]

Since \( J \) implies the relationship among target locations, it is called the relation matrix.

Equation (7) shows the two aspects that can cause ambiguity in multitarget estimation using an RP-FDA. One is \( a(\psi_1) \), which depends only on the frequency increment vector \( m \). The ambiguities related to \( a(\psi_1) \) can be regarded as the ambiguities in single-target estimation. If a frequency increment vector \( m \) makes the single-target estimation ambiguous, ambiguity will undoubtedly arise in multitarget estimation. The principle and criteria to identify the unambiguous frequency increment vector \( m \) are described in [9] and are not the focus of this paper.

The other aspect is the relation matrix \( J \), which depends on both the relative target location and the frequency increment vector \( m \). If the relative target location and frequency increment vector \( m \) make the rank of relation matrix \( J \) deficient, the covariance matrix will consequently be rank deficient. Then, the dimension of the signal subspace will be less than the number of targets, resulting in the estimation being incorrect, with some false targets. In the rest of the paper, a law for a relative target location causing ambiguity is sought.

**III. AMBIGUITY ANALYSIS**

**A. RELATIVE TARGET LOCATIONS CAUSING RANK DEFICIENCY**

It is seen from (9) that periodicity of the phase \( \phi_{n,i} \) contributes to the linearly dependence of relation matrix \( J \). Since parameters \( m \) and \( n \) in (9) are both discrete integers, when each \( \Delta r_i \) and each \( \Delta(\sin \theta_i) \) are both integer multiples of reference value, it is possible that the relation matrix \( J \) is rank deficient. Therefore, a unit difference in range and a unit difference in angle should be defined. According to the signal model described in Section II-A, when \( d = \frac{\lambda_0}{4} \) and \( \Delta f = \frac{2K_{max}}{c} \) are selected, the whole detection area (range and angle) can be expressed as \( \Delta R = \frac{\pi c}{K} \) and \( \Delta(\sin \theta) = \frac{c}{2f_{0d}} \). Then, unit difference in range \( \Delta r \) and unit difference in angle \( \Delta(\sin \theta) \) are defined as
\[
\begin{align*}
    \Delta r &= \frac{\Delta R}{K} = \frac{c}{2K\Delta f}, \\
    \Delta(\sin \theta) &= \frac{\Delta(\sin \theta)}{K} = \frac{c}{2Kf_{0d}}.
\end{align*}
\]

(9) can be rewritten as
\[
\begin{align*}
    r_i &= r_1 + p_i \Delta r \\
    \sin \theta_i &= \sin \theta_1 + q_i \Delta(\sin \theta),
\end{align*}
\]
where \( i = 1, 2, \ldots, K \), \( p_i \in \{0, \pm 1, \pm 2, \ldots, \pm K - 1\} \), and \( q_i \in \{0, \pm 1, \pm 2, \ldots, \pm K - 1\} \). The diagram of the possible relative target locations is shown in Fig. 2. Therefore, we intend to find \( K \) locations from the \( K^2 \) locations.

**FIGURE 2. Diagram of the possible relative target locations.**

Substituting (10) to (12) into (9), \( \phi_{n,i} \) can be written as
\[ \phi_{n,i} = -2\pi m_n p_i - n q_i. \]

Because of the first order linear relationship among \( \phi_{n,i}, p_i \) and \( q_i \), when \( p_i \) and \( q_i \) are both linear with respect to \( i \) to first
order, the rank of $\mathbf{J}$ has a high probability of being less than $K$ as the vector $\mathbf{m}$ varies.

$$
\begin{align*}
    p_i &= C_1(i-1) - h_1K \\
    q_i &= C_2(i-1) - h_2K,
\end{align*}
$$

(14)

where $C_1 \in \{1, 2, \ldots, K - 1\}$, $C_2 \in \{1, 2, \ldots, K - 1\}$, $h \in \{0, 1, \ldots, 1\}$, $h_1$ is the value $h$ that satisfies $0 < r_1 + C_1(i - 1) - hK < \Delta R$ and $h_2$ is the value $h$ that satisfies $-\Delta R < \Delta \sin(\theta) < \frac{\Delta \sin(\theta)}{2}$. Suppose the basis of the signal subspace is $\mathbf{a}(\psi_1), \mathbf{a}(\psi_{K-1})$ and the other vector $\mathbf{a}(\psi_K)$ is the linear combination of the basis.

$$
\mathbf{a}(\psi_K) = l_1\mathbf{a}(\psi_1) + l_2\mathbf{a}(\psi_2) + \cdots + l_{K-1}\mathbf{a}(\psi_{K-1}).
$$

(15)

When estimating the locations of the target via projecting the searching vector $\mathbf{a}(\psi)$ to the noise subspace, the projection of $\mathbf{a}(\psi_K)$ is always 0 even if the $K$th target does not exist.

Equation (14) reveals that the relative target locations causing rank deficiency are uniformly distributed in the detection area in both the range and angle dimensions. This kind of target distribution is common in reality; examples include military formations and sensor layouts. Therefore, it is necessary to analyze the ambiguity in multitarget estimation using RP-FDAs to improve the probability of unambiguous estimation.

B. CATEGORIES OF RELATIVE TARGET LOCATIONS CAUSING RANK DEFICIENCY

According to (14), when the parameters $C_1$ and $C_2$ have different values, the relative target locations are different. The quantity of relative target locations is $(K - 1)^2$. However, because of the periodicity of (13), the relative target locations with different $C_1$ and $C_2$ may be in the same distribution, although they are of different orders. Fig. 3 shows the example of two different relative target locations in this case. The number of targets is $K = 5$, and the reference targets of these two relative locations are the same. The $i$th target $Q_i$ in Fig. 3(a) with $(C_1, C_2) = (1, 2)$ is located at a different location than the $i$th target $Q_i$ in Fig. 3(b) with $(C_1, C_2) = (3, 1)$, but all the targets are in the same distribution. The relative target locations in the same distribution are considered to be in the same category.

For each relative target location in a category, the corresponding frequency increment vectors causing rank deficiency are the same; therefore, we need only to consider one of the relative target locations for each category. By analyzing categories of relative target locations with different numbers of targets and different values of $C_1$ and $C_2$, and employing mathematical induction, the quantity of categories of the relative target locations is obtained. Let $H$ denote the quantity of categories of the relative target locations. It is a value related to the number of targets $K$, and can be expressed as

$$
H = \sum_{i \in \mathbb{U}} \left[ \frac{K}{i} - g(i) \right] - 1.
$$

(16)

where

$$
\begin{align*}
    U &= \{ \frac{j}{i} \text{ is an integer less than } K & (K/j) \text{ is an integer} \}, \\
    g(i) &= \begin{cases} 
    0, & \frac{K}{i} < \min_{\{i, (K/i)_cd\}} \{1\} \\
    \frac{K}{i} - \min_{\{i, (K/i)_cd\}} \{1\}, & \{i, (K/i)_cd\} \supseteq \{1\}. 
    \end{cases}
\end{align*}
$$

Here, $\{i, (K/i)_cd\}$ denotes the set consisting of the common division of $i$ and $K/i$.

In (16), $U$ represents the set of values that $C_1$ can take, in which $K/i$ need to be an integer. If $K/i$ is an integer, there will be $i$ target/targets in the same range with the reference target. Otherwise, there will only be 1 target in each range. Then, the relative target locations will be in the same distribution as one of the scenario when $C_1 = 1$. $K/i$ is the maximum quantity of value that $C_2$ can take when $C_1$ is determined. Among the $K/i$ values, some of them will generate an incorrect relative target distribution with two or more targets in the same location, which should excluded from the category quantity. The quantity of incorrect distributions is represented by $g(i)$. Note that when $\{i, (K/i)_cd\}$ has more than 3 elements, it is hard to calculate $g(i)$. The minimum value of $g(i)$ is given; therefore, the quantity of categories $H$ calculated from (16) is larger than the true value. However, the redundancy ensures the reliability of the unambiguous frequency increment vectors. Table 1 illustrates the quantity of categories with different numbers of targets.

C. RELATIVE TARGET LOCATIONS CAUSING AMBIGUITY

Though the relative target locations that satisfy (14) can cause the rank deficiency of the relation matrix $\mathbf{J}$ and further cause the rank deficiency of the covariance matrix, the estimation of the target location remains correct. However, once the relative target location transforms slightly on the basis of (14), ambiguous estimation appears. Two types of transformation are possible:

Case 1: The range and angle of the detection area are evenly divided into $K + 1$. Randomly select one target from the $K + 1$ targets that satisfy (14), and then discard it.

Case 2: The range and angle of the detection area are evenly divided into $K$. Randomly select one target from the $K$ targets that satisfy (14), and move it to another location beside the other target locations.
### TABLE 1. Number of categories with different numbers of targets.

| $K$ | $U$   | $(\{i, K/i\})$ | $(\{i, K/i\})_{\text{odd}}$ | $g(i)$ | $(C_1, C_2)$ | $H$ |
|-----|-------|----------------|--------------------------|--------|-------------|-----|
| 5   | $\{1\}$ | $(1, 5)$ | $(\{1\})$ | 0 | $(1, 1)$ | 4 |
| 6   | $\{1, 2, 3\}$ | $(1, 6), (2, 3), (3, 2)$ | $(\{1\}, (\{1\}, (\{1\})$ | 0, 0, 0 | $(1, 1), (2, 1), (3, 1)$ | 10 |
| 8   | $\{1, 2, 4\}$ | $(1, 8), (2, 4), (4, 2)$ | $(\{1\}, (\{1, 2\}), (\{1, 2\})$ | 0, 2, 1 | $(1, 1), (2, 1), (4, 1)$ | 10 |
| 9   | $\{1, 3\}$ | $(1, 9), (3, 3)$ | $(\{1\}, (\{1, 3\})$ | 0, 1 | $(1, 1), (1, 2), (3, 1)$ | 10 |
| 36  | $\{1, 2, 3, 4, 6, 9, 12, 18\}$ | $(1, 36), (2, 18), (3, 12), (4, 9)$ | $(\{1\}, (\{1, 2\}), (\{1, 3\}), (\{1\})$ | 0, 9, 4, 0 | $(1, 1)$ | $< 71$ |

Fig. 4 shows examples of relative target locations causing ambiguity with $K = 5$. Fig. 4(a) shows the relative target location in Case 1 when the 4th target $Q_4$ is discarded. Fig. 4(b) shows the relative target location in Case 2 when the 4th target $Q_4$ is discarded and a substitute target $Q'_4$ is placed. Note that whether a frequency increment vector is ambiguous is not affected by the target being discarded. These two kinds of relative target locations are also common in real life; examples include military formations with one target located in an erroneous location and sensor layouts with one sensor node inoperable.

**FIGURE 4.** Examples of relative target locations causing ambiguity with $K = 5$.

### D. AMBIGUITY PRE-EVALUATION USING RANK EXAMINATION

Though the relative target locations causing ambiguity is transformed from the relative target locations causing rank deficiency, the rank of the relation matrix $J$ and the rank of the covariance matrix after transformation are both equal to the number of targets $K$. When executing the location estimation, the ambiguity cannot be eliminated by traditional rank compensation. Therefore, the ambiguity needs to be identified by examining the rank of the relation matrix $J$ composed of the target locations causing rank deficiency. When the number of targets is known a priori, both cases causing ambiguity and all categories of relative target locations causing rank deficiency must be considered one by one when pre-evaluating the frequency increment vector. The pre-evaluation steps are illustrated as follows.

**Step 1:** Select a frequency increment vector, and initialize the number of cases and the quantity of categories of relative target locations causing rank deficiency.

**Step 2:** Consider one of the cases causing ambiguity. Calculate the quantity of categories according to the considered case.

**Step 3:** Select a target’s relative location causing rank deficiency from one of the categories.

**Step 4:** Substitute the frequency increment vector and the relative target location into the relation matrix $J$. Test the rank of $J$; if the rank is less than the number of targets, the frequency increment vector is ambiguous and returns to Step 1; if the rank is equal to the number of targets, continue.

**Step 5:** Check whether all the categories are examined; if not, return to Step 3; otherwise, continue.

**Step 6:** Check whether both cases are examined; if not, return to Step 2; otherwise, the frequency increment vector is unambiguous.

### IV. NUMERICAL RESULTS

To show the effectiveness of the proposed law of relative target locations causing ambiguity and the results of the rank deficiency examination, several simulations based on MATLAB are illustrated in this section. Consider a uniform linear array with a reference carrier frequency $f_0 = 3$ GHz and frequency increment $\Delta f = 15$ KHz. The element spacing is $d = 0.025$ m, the number of array element is $N = 11$. The estimation of the range and angle for multitargets using the MUSIC algorithm is executed. All the estimates are computed based on 100 snapshots [16]. The number of targets is estimated via the minimum description length (MDL) criterion [17].
A. EXAMPLE ESTIMATION RESULTS OF TARGETS WITH LOCATIONS CAUSING AMBIGUITY

Fig. 5 shows an example of target locations causing ambiguity. Fig. 5(a) illustrates a kind of target location causing ambiguity, which is the same as the target location in Fig. 4(b). As a comparison, a kind of target location that does not satisfy the proposed law is considered and is shown in Fig. 5(b). The number of targets is $K = 5$, and the target locations of these two scenarios are listed in Table 2. Fig. 6(a) and Fig. 6(c) demonstrate the MUSIC spectra of the targets in Fig. 5(a) with frequency increment vectors $m_1 = [5, 9, 4, 7, 2, 10, 8, 3, 0, 1, 6]$ and $m_2 = [5, 7, 9, 10, 1, 0, 6, 2, 3, 8, 4]$, respectively. The MUSIC spectra of targets in Fig. 5(b) with frequency increment vectors $m_1$ and $m_2$ are shown in Fig. 6(b) and Fig. 6(d), respectively. The signal-to-noise ratio (SNR) is 20 dB. For the targets whose relative location does not satisfy the proposed law, the estimation with frequency increment vectors $m_1$ and $m_2$ are both unambiguous, where the number and location of the spectrum peaks are both in agreement with the true targets. However, for the targets whose relative location satisfies the proposed law causing ambiguity, the estimation with the frequency increment vector $m_2$ is ambiguous because a false target appears in Fig. 6(c). This verifies the effectiveness of the proposed law of relative target locations that cause ambiguity.

TABLE 2. Target locations in Fig. 5.

| Target | Case 2, $(C_1, C_2) = (1, 2)$ | Irregular target location |
|--------|-------------------------------|---------------------------|
| 1      | $(1000m, -23.58^\circ)$      | $(2500m, -30^\circ)$      |
| 2      | $(3000m, 23.58^\circ)$        | $(6000m, -20^\circ)$      |
| 3      | $(5000m, -53.13^\circ)$       | $(5020m, -10^\circ)$      |
| 4      | $(7000m, -23.58^\circ)$       | $(3000m, 23.58^\circ)$    |
| 5      | $(9000m, 53.13^\circ)$        | $(9000m, 50^\circ)$       |

B. NOISE ANALYSIS

In this subsection, we examine how ambiguity pre-evaluation is affected by noise. We take an unambiguous estimation as an example, where the scenario is the same as in Fig. 6(a). Fig. 7 shows the root mean square error (RMSE) performance of the range and the angle estimations versus the SNR. The RMSE performance is evaluated by Monte Carlo simulation where the number of Monte Carlo trials is 500 [18]. When the SNR is less than $-1$ dB, the RMSE of the range and angle both rise sharply. This is because the source enumeration is incorrect when the noise power is greater than the signal power. However, when the SNR is over 0 dB, satisfactory localization performance of the range and angle can be achieved, which is similar to the localization performance of the traditional MUSIC algorithm. Therefore, in the rest of the paper, we always assume that the SNR is 20 dB.
C. PERCENTAGE OF AMBIGUITY WITH DIFFERENT TARGET LOCATIONS

To characterize the proportion of the frequency increment vectors that should be excluded from executing an estimation, the percentage of ambiguous frequency increment vectors is defined as

\[ \beta = \frac{N_{\text{Ambiguous}}}{N_{\text{Test}}} \times 100\% \]  

(17)

where \( N_{\text{Test}} \) is the number of frequency increment vectors taken as the tested samples, which is randomly selected from all frequency increment vectors and \( N_{\text{Ambiguous}} \) is the number of frequency increment vectors that generate an ambiguous estimation among all the tested frequency increment vectors. In the rest of the paper, we use the percentage of ambiguity instead of the percentage of ambiguous frequency increment vectors for simplicity. Fig. 8 demonstrates the percentage of ambiguity with different target locations when \( K = 5 \). Here, we randomly select 500 frequency increment vectors as the tested frequency increment vectors. Five categories of target locations are examined, including an irregular target location and four target locations causing ambiguity. The irregular target locations are the same as the target locations in Fig. 5(b). The other target locations are Case 1 with \((C_1, C_2) = (1, 1)\), Case 1 with \((C_1, C_2) = (1, 2)\), Case 2 with \((C_1, C_2) = (1, 1)\) and Case 2 with \((C_1, C_2) = (1, 2)\). Moreover, to show more examples, numbers of array elements from \( N = 6 \) to \( N = 15 \) are all considered.

![Figure 8. Percentage of ambiguous frequency increment vectors with different target locations.](image)

The percentages of ambiguity for the 4 target locations causing ambiguity are much higher than that for the irregular target locations, especially when the number of array elements is not much greater than the number of targets. This verifies the necessity of identifying the target locations causing ambiguity and filtering the corresponding frequency increment vectors.

D. PERCENTAGE OF AMBIGUITY USING RANK EXAMINATION VERSUS MUSIC ESTIMATION

Fig. 9 illustrates the comparison between the percentage of ambiguity using rank examination and the results obtained from MUSIC estimation. The tested frequency increment vectors are the same for these two methods. Here, two categories of relative target locations are considered: Case 1 with \((C_1, C_2) = (1, 1)\) and Case 2 with \((C_1, C_2) = (1, 1)\). The number of targets is \( K = 5 \). The percentage of ambiguity using rank examination is almost the same as the percentage obtained by MUSIC estimation, especially when the number of elements is larger than 11. The reason for the difference when \( N < 11 \) includes two aspects. One is the error in the ambiguous MUSIC estimation, and the other is the exceptional frequency increment vectors causing ambiguity in addition to the one causing rank deficiency. In Fig. 9, the percentage of ambiguity considering all the categories of relative target locations using rank examination is given as well. The percentage of ambiguity with total categories using rank examination is less than 100\% until the number of elements \( N > 12 \), where the percentage of ambiguity is the same as the one using MUSIC estimation for each category. As a consequence, the percentage of ambiguity with the total categories using rank examination can be regarded as the same as that using MUSIC estimation. This verifies the effectiveness of the proposed rank deficiency examination to avoid ambiguity.

![Figure 9. Percentage of ambiguous frequency increment vectors with rank examination vs MUSIC estimation.](image)

E. PERCENTAGE OF AMBIGUITY USING DIFFERENT CRITERION

Fig.10 shows the percentage of ambiguity using one-target criterion, two-target criterion and five-target criterion. It is seen that compared with the percentage of ambiguity using one-target criterion in [9], the percentage of ambiguity using two-target criterion and five-target criterion are greater, especially when the number of array elements is small, which means that the multitarget criterion can identify more ambiguous frequency increment vectors to guarantee the reliability of estimation. For the five-target scenario, the identification probability can be improved by up to 99.7\%. Moreover, the percentage of ambiguity rises sharply as the number of targets increase, which verify the necessary and effectiveness of the ambiguity analysis of multitarget using RP-FDA.
V. CONCLUSION
In this paper, the ambiguity in multitarget estimation using an RP-FDA is analyzed. Different from single-target estimation, the ambiguity also depends on the relative location of the targets. The relationship among the ambiguity, the relative target location and the frequency increment vector is depicted by the relation matrix. By analyzing the rank of the relation matrix, a law of relative target locations causing ambiguity is proposed, and the rank deficiency examination to avoid ambiguity is presented. When the targets are distributed uniformly and regularly in the detection area and one or more targets work unusually, ambiguity occurs. The ambiguity analysis can provide guidance for the frequency increment vector design to ensure unambiguous estimation. This is of practical value in the estimation of military formations, sensor layouts and so on. Moreover, in addition to the relative target locations proposed in this paper, there exist other target locations that can cause ambiguity in estimation. However, the associated influence on the percentage of ambiguity is very small; therefore, this aspect is not considered in this paper.

REFERENCES

[1] W.-Q. Wang, H. C. So, and A. Farina, “An overview on Time/Frequency modulated array processing,” IEEE J. Sel. Topics Signal Process., vol. 11, no. 2, pp. 228–246, Mar. 2017.

[2] S. L. Wang, Z.-H. Xu, X. Liu, W. Dong, and G. Wang, “Subarray-based frequency diverse array for target range-angle localization with monopulse processing,” IEEE Sensors J., vol. 18, no. 14, pp. 5937–5947, Jul. 2018.

[3] S. Y. Nusenu, S. Huaizong, P. Ye, W. Xuehan, and A. Basit, “Dual-function radar-communication system design via sidelobe manipulation based on FDA butler matrix,” IEEE Antennas Wireless Propag. Lett., vol. 18, no. 3, pp. 452–456, Mar. 2019.

[4] J. Xu, G. Liao, S. Zhu, and H. C. So, “Deceptive jamming suppression with frequency diverse MIMO radar,” Signal Process., vol. 113, pp. 9–17, Aug. 2015.

[5] C. Wang, J. Xu, G. Liao, X. Xu, and Y. Zhang, “A range ambiguity resolution approach for high-resolution and wide-swath SAR imaging using frequency diverse array,” IEEE J. Sel. Topics Signal Process., vol. 11, no. 2, pp. 336–346, Mar. 2017.

[6] D. Ran and X. Jia, “Enhanced back projection algorithm for linear frequency diverse array synthetic aperture radar imaging,” in Proc. 11th Eur. Conf. Synth. Aperture Radar EURAS. Jun. 2016, pp. 1–5.

[7] P. Baizert, T. B. Hale, M. A. Temple, and M. C. Wicks, “Forward-looking radar GMTI benefits using a linear frequency diverse array,” Electron. Lett., vol. 42, no. 22, p. 1311, 2006.

[8] Y. Liu, H. Ruan, L. Wang, and A. Nehorai, “The random frequency diverse array: A new antenna structure for uncoupled direction-range indication in active sensing,” IEEE J. Sel. Topics Signal Process., vol. 11, no. 2, pp. 295–308, Mar. 2017.

[9] J. Li, H. Li, and S. Ouyang, “Identifying unambiguous frequency pattern for target localisation using frequency diverse array,” Electron. Lett., vol. 53, no. 19, pp. 1331–1333, Sep. 2017.

[10] S. Y. Nusenu, Z. Wang, and W.-Q. Wang, “FDA radar using costas sequence modulated frequency increments,” in Proc. CIE Int. Conf. Radar (Radar), Oct. 2016, pp. 1–4.

[11] W. Wu and F. Xi, “Target localization for FDA-MIMO radar with random frequency increment via atomic norm minimization,” in IEEE MTT-S Int. Microw. Symp. Dig., May 2019, pp. 1–4.

[12] A. Basit, I. M. Qureshi, W. Khan, A. N. Malik, and B. Shoaib, “Beam pattern synthesis for a cognitive frequency diverse array radar to localize multiple targets with same direction but different ranges,” in Proc. 13th Int. Bhurban Conf. Appl. Sci. Technol. (IBCAST), Jan. 2016, pp. 682–688.

[13] X. Wang and Q. Long, “Uncoupled FDA beampattern synthesis by discrete element position and frequency offsets pairing,” in Proc. 16th Eur. Radar Conf. (EurRAD), Oct. 2019, pp. 57–60.

[14] J. Li, S. Ouyang, K. Liao, and X. Sun, “Identifying unambiguous frequency patterns for two-target localization using frequency diverse array,” Signal Process., vol. 171, Jun. 2020, Art. no. 107452.

[15] A. M. Jones, “Frequency diverse array receiver architectures,” M.S. thesis, Wright State Univ., Dayton, OH, USA, 2011.

[16] Y. Wang, W.-Q. Wang, and H. Chen, “Linear frequency diverse array manifold geometry and ambiguity analysis,” IEEE Sensors J., vol. 15, no. 2, pp. 984–993, Feb. 2015.

[17] F. Haddadi, M. Malek-Mohammadi, M. M. Nayebi, and M. R. Aref, “Statistical performance analysis of MLD source enumeration in array processing,” IEEE Trans. Signal Process., vol. 58, no. 1, pp. 452–457, Jan. 2010.

[18] K. Gao, H. Shao, H. Chen, J. Cai, and W.-Q. Wang, “Impact of frequency increment errors on frequency diverse array MIMO in adaptive beamforming and target localization,” Digit. Signal Process., vol. 44, pp. 58–67, Sep. 2015.

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