Supersymmetric Grand Unification with a Fourth Generation?

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Abstract

The possibility of incorporating a chiral fourth generation into a SUSY-GUT is investigated. Precision fits to electroweak observables require us to introduce light supersymmetric particles, with masses less than $M_Z$. These particles might also provide decay channels for the fourth generation quarks of mass $\sim 100$ GeV. We also require $\tan \beta$ to lie in the range $1.50 \lesssim \tan \beta \lesssim 1.75$ and obtain an upper limit on the lightest Higgs boson mass in the MSSM4 of 152 GeV.

1 Introduction

The Standard Model (SM) is, of course, phenomenologically very successful, while its supersymmetric extension stabilizes the gauge hierarchy problem and allows a grand unification of the SM interactions. However, there is no explanation of why there should be just three generations of quarks and leptons or their hierarchy of masses. In this paper we investigate the possibility of consistently incorporating a fourth generation into a supersymmetric grand unified theory.

We shall assume a structure akin to that of the minimal supersymmetric standard model (MSSM3), adding a complete chiral fourth generation and its associated supersymmetric partners (the so-called MSSM4). We note that at one-loop level the validity of gauge coupling unification is independent of the number of generations, and so this requirement does not discriminate between three and four generation models. However, the requirement that all the Yukawa couplings remain perturbative at energies up to the GUT

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scale places restrictive upper bounds on the fourth generation \( T, B, E \) and \( N \) masses and also constrains the supersymmetric parameter \( \tan \beta = \frac{v_U}{v_D} \) (\( v_U \) and \( v_D \) being the vacuum expectation values of the two Higgs doublets \( H_U \) and \( H_D \) that are present in the MSSM). In section 2 we investigate the masses and decay channels of the fourth generation particles that are consistent both with perturbative gauge coupling unification and the latest experimental bounds from direct searches. In section 3 we comment on the issues regarding fits to precision data in the MSSM4 before discussing the lightest Higgs boson mass in section 4.

## 2 Experimental Constraints and a Fourth Generation

All fourth generation models must adhere to certain experimental constraints, the first of which stems from precise measurements of the decay characteristics of the \( Z \)-boson performed at LEP. This has set a lower bound of \( M_F \geq \frac{M_Z^2}{2} \) on any non-SM particles that couple to the \( Z \)-boson. Ignoring the unnatural hierarchy emerging within the neutrino sector, we assume a Dirac mass \( M_N \approx \left( \frac{M_Z^2}{2} \right) \) for the heavy neutral lepton.

We begin our discussion by considering the leptonic sector where we assume \( M_E > M_N \). Under the assumption that the mixing between the fourth generation leptonic sector and the first three generations is negligible, the decay \( E \rightarrow NW^* \) will be dominant. Current experimental limits searching for \( E \rightarrow NW^* \) from \( e^+e^- \rightarrow E^+E^- \) production have been performed by the OPAL and L3 collaborations up to the kinematic limit \( M_E \sim 100 \text{ GeV} \) \[1\]. However, it turns out that to be consistent with perturbative unification we require \( M_E \sim \frac{M_Z}{2} \), so that the fourth generation charged lepton Yukawa coupling \( Y_E \) doesn’t become non-perturbative below the GUT scale. Therefore, in order to evade experimental bounds, the mass difference \( \Delta M_L \) must be less than \( \sim 5 \text{ GeV} \), which is in the region where the trigger efficiencies are significantly lowered and events are dominated by the two photon background \[1\]. We also note that with \( \Delta M_L \) the order of a few GeV the decay lifetime \( \tau (E \rightarrow NW^*) \) is too short for the heavy \( E \) lepton to leave a charged track in the electromagnetic calorimeter. Regarding the heavy neutrino, OPAL and L3 have set the bound \( M_N > 70 - 90 \text{ GeV} \) based on the search for \( N \rightarrow lW^* \) \((l = e, \mu \text{ or } \tau) \) provided the mixing matrix elements satisfy \( V_{Ne,\mu,\tau} > 10^{-6} \) \[1\].
However, if we assume that this mixing angle is negligible \( (V_{Ne,\mu,\tau} < 10^{-6}) \), then the neutrino is stable enough to leave the detector. In this case, the only relevant bound comes from the \( Z^0 \) decay width noted earlier. Based on the above discussion, in the rest of this paper we shall take:

\[
M_E \simeq 50 \text{ GeV} \quad ; \quad M_N \simeq 50 \text{ GeV}
\]

(1)

as representative masses of the fourth generation leptonic particles.

Direct searches for the fourth generation quarks is an ongoing process at the Fermilab Tevatron. Here we focus on the experimental restrictions for \( M_t > M_T > M_B \), so that the charged current (CC) decays \( B \to tW^- \) and \( B \to TW^- \) are kinematically forbidden. The leading CC decay mode will then be \( B \to cW^- \) which is doubly Cabibbo suppressed by the mixing matrix factor \( V_{cB}. \) In this situation, loop induced flavour changing neutral current (FCNC) decays can dominate provided \[2\]:

\[
\frac{|V_{cB}|}{|V_{tB}|} \lesssim \mathcal{O} \left( 10^{-2} - 10^{-3} \right)
\]

(2)

Several experiments have searched explicitly for \( B \) quarks decaying via FCNCs. The \( D\Phi \) collaboration \[3\] has excluded the range \( \frac{M_B^2}{2} < M_B < M_{Z^0} + M_b \) by a null search for both \( B \to b\gamma \) and \( B \to bg \). For masses \( M_t, M_T > M_B > M_{Z^0} + M_b \), the decay \( B \to bZ^0 \) is expected to dominate, except for \( B \to bh^0 \), if \( M_B > M_{h^0} + M_b \). The CDF collaboration \[4\] has performed a general search for long-lived particles that decay into a \( Z^0 \) gauge boson. This will encompass the FCNC decay of a fourth generation \( B \) quark decaying via \( B \to bZ^0 \), if the mixing matrix factor \( V_{tB} \simeq V_{TB} \) small enough to result in a long lifetime. By looking for \( Z^0 \to e^+e^- \) with a displaced vertex they are able to exclude a \( B \) quark mass up to \( M_B = 148 \text{ GeV} \) for \( c\tau_B = 1 \text{ cm} \), where \( \tau_B \) is the proper decay time of the \( B \) quark, and a branching ratio of \( Br \ (B \to bZ^0) = 100 \% \). However, this limit diminishes to \( M_B \sim 96 \text{ GeV} \) if \( c\tau_B > 22 \text{ cm} \). To date, this remains the only lower mass bound on quasi-stable \( B \) quarks as emphasized by Frampton et al. \[5\].

As regards the \( T \) quark, for \( M_T \gtrsim 100 \text{ GeV} \), there are two competing decay modes; \( T \to BW \) and \( T \to bW \). The \( BW \) decay will certainly dominate over the \( bW \) decay when the former is real, since the \( bW \) channel is suppressed by the mixing matrix factor \( V_{TB} \). However, when \( M_T - M_B < M_W \), the two body \( bW \) decay could be competitive with the three body \( BW^* \) decay. For large enough \( V_{TB} \) the \( T \to bW \) decay will be dominant. However, for \( M_T \lesssim M_t \),
such a dominance would lead to a large excess of $bW$ events relative to those already present from $t\bar{t}$ production and decay, and so this implies that $\frac{V_{tb}}{V_{TB}}$ must be small enough for the $T \rightarrow BW^*$ decay to dominate. The detection of the $T$ quark would then depend on the decay properties of the $B$ quark and the mass difference $M_T - M_B$. We refer to [6] and [7] for a discussion of this scenario, in particular Gunion et al. [6] finds the only way that $T \bar{T}$ events can evade being included in the CDF and $D\phi$ data sample is if $M_T - M_B$ is sufficiently small so that the $W^*$ in $T \rightarrow BW^*$ is virtual and the jets and leptons from the two $W^*$’s are soft. A further analysis on updated data is needed that takes into account the quasi-stable nature of the $B$ quark.

To obtain the allowed masses of the fourth generation quarks that are consistent with perturbative gauge coupling unification we perform a renormalization group study of the MSSM4. Specifically, this enables us to place upper limits on the masses of the $T$ and $B$ quarks by ensuring their Yukawa couplings $Y_T$ and $Y_B$ run perturbatively to the GUT scale $M_{GUT}$:

$$Y_{T,B}^2 (\mu) \leq 4\pi \quad \text{for} \quad M_{Z^0} \leq \mu \leq M_{GUT} \quad (3)$$

where $M_{GUT}$ is defined to be the scale where $\alpha_1(\mu) = \alpha_2(\mu)$. In our analysis we have neglected all the Yukawa couplings from the first three generations, except that of the third generation $t$ quark whose mass we take to be $M_t = 175$ GeV. As is typical with four generation models, we also require small values of $\tan \beta$ so as to avoid $Y_B(M_{Z^0}) \geq O(\sqrt{4\pi})$. Further details of the procedure for running the renormalization group equations in the MSSM4 can be found in [8]. In Figure 1 we have plotted the maximum $T$ quark mass ($M_T^{max}$) versus $M_B$, given that perturbative unification must occur. We plot for $\tan \beta = 1.60, 1.65$ and $1.70$, and we fix $M_t = 175$ GeV. We have taken $M_B \geq 96$ GeV, which corresponds to the absolute experimental lower bound on a quasi-stable $B$ quark as discussed earlier. We can see from Figure 1 that higher $\tan \beta$ values allow for a larger $M_T^{max}$, though at the expense of restricting $M_B$, and as such tends to favour the hierarchy:

$$M_T > M_B \quad (4)$$

This mass hierarchy is necessary if we consider the $T$ and $B$ quarks to have the standard model decay channels $T \rightarrow BW^*$ and $B \rightarrow bZ^0$, as discussed earlier in this section. As we decrease $\tan \beta$ then the hierarchy in Eq. 4

\footnote{If $M_T < M_B$ then we would expect $T \rightarrow bW$ to be the dominant decay channel of the $T$ quark, which is excluded experimentally based on searches for the third generation top quark.}
Figure 1: Plot of $M_T^{max}$ versus $M_B$ for $\tan \beta = 1.60$, 1.65 and 1.70, as obtained from the requirement of perturbative gauge coupling unification. The third generation $t$ quark mass is taken as $M_t = 175$ GeV. The fourth generation leptonic masses are taken as $M_E = M_N = 50$ GeV.

Figure 2: Same as Figure 1 but with $M_t = 170$ GeV.
becomes harder to maintain. Overall, we find that $\tan \beta$ can take on the values $1.50 < \tan \beta < 1.75$ without the Yukawa couplings becoming non-perturbative below the GUT scale. In Figure 2 we also plot $M_T^{\text{max}}$ versus $M_B$, but this time using $M_t = 170$ GeV. We can see that this value of $M_t$ increases $M_T^{\text{max}}$ by a few GeV for a given $M_B$ and $\tan \beta$. For $M_t = 180$ GeV the allowed ranges of $M_T$ and $M_B$ are very constrained, in fact we only find solutions for $\tan \beta \geq 1.66$ and $M_T, M_B < 100$ GeV.

Looking at Figures 1 and 2 it is clear that we could arrange $M_T$ and $M_B$ so that the standard model decay channels:

$$T \rightarrow BW^* \quad B \rightarrow bZ^0$$  \hspace{1cm} (5)

are kinematically accessible, and dominate on the basis that the mixing angles $V_{Bc}$ and $V_{Tb}$ are suppressed. An updated analysis based on RUNII data at Fermilab is essential to exclude this scenario. However, we also note that there are other decay channels for the $T$ and $B$ quarks that become possible in the MSSM4. For instance, we can consider the possibility of light supersymmetric particles providing decay channels for the $T$ and $B$ quarks. In this situation one can constrain the masses of the light (i.e. $< M_{Z^0}$) neutralino ($\tilde{\chi}_1^0$) and chargino ($\tilde{\chi}_1^\pm$) pair which, as we will see in section 3, is already required by fits to precision data, in order to allow the following two body decays:

$$T \rightarrow \tilde{B}_1\tilde{\chi}_1^+ \quad B \rightarrow \tilde{B}_1\tilde{\chi}_1^0$$  \hspace{1cm} (6)

where $\tilde{B}_1$ is the lightest mass eigenvalue in the fourth generation sbottom sector. It has already been shown by Gunion et. al. [6] that $\tilde{B}_1$ is typically the lightest squark in the MSSM4. Ensuring both of the decay channels in Eq. 6 are kinematically accessible, combined with the constraints from perturbative unification and precision data fits, places severe restrictions on the allowed spectrum. We discuss this in more detail in section 3.

3 Precision Measurements and a Fourth Generation

It is difficult to provide bounds from precision data without a fully consistent study taking into account exact particle masses, any light supersymmetric particle spectra present and mixings between different flavours. However, Maltoni et. al. [9] has pointed out that a highly degenerate neutralino and
The chargino pair can provide the necessary contributions to the precision parameters that are needed to cancel that of the fourth generation, whilst at the same time being consistent with LEP bounds [10]. Specifically they require:

\[ M_{\pi^+} \lesssim \Delta M_\chi \lesssim 3 \text{ GeV} \quad \text{and} \quad M_\chi \sim 60 \text{ GeV} \quad (7) \]

where we define the notation \( \Delta M_\chi = M_{\tilde{\chi}^+_1} - M_{\tilde{\chi}^0_1} \) and \( M_\chi = M_{\tilde{\chi}^+_1} \gtrsim M_{\tilde{\chi}^0_1} \).

Looking at their results, we see that the magnitude of the contribution to the fitted parameters from this sector is highly dependent on \( M_{\tilde{\chi}^+_1} \). Deviations from \( M_{\tilde{\chi}^+_1} \) larger than \(+30 \text{ GeV}\) or \(-5 \text{ GeV}\) are ruled out at the 2\( \sigma \) level. We now discuss the influence of these precision data fits on the scenario where the \( T \) and \( B \) quarks decay into supersymmetric particles.

If allowed, the \( B \to \tilde{B}_1 \tilde{\chi}^0_1 \) decay in Eq. (6) will certainly dominate over the one-loop FCNC decay \( B \to bZ^0 \) and two-generation decays \( B \to cW^- \) that traditional searches have looked for. Typical (pole) masses required for this scenario to be viable are:

\[ M_B (\simeq M_T) \gtrsim 100 \text{ GeV} \]
\[ M_{\tilde{B}_i} \lesssim 50 \text{ GeV} \]
\[ 55 \text{ GeV} \leq M_\chi \leq M_B - M_{\tilde{B}_i} \quad (8) \]

We assume that \( \theta_{\tilde{B}} \simeq 1.17 \text{ rad} \), where \( \theta_{\tilde{B}} \) is the mixing angle that relates the \( \tilde{B}_L \) and \( \tilde{B}_R \) weak eigenstates to the mass eigenstates \( \tilde{B}_1 \) and \( \tilde{B}_2 \) through the matrix:

\[ \begin{pmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{\tilde{B}} & \sin \theta_{\tilde{B}} \\ -\sin \theta_{\tilde{B}} & \cos \theta_{\tilde{B}} \end{pmatrix} \begin{pmatrix} \tilde{B}_L \\ \tilde{B}_R \end{pmatrix} \quad (9) \]

This value of \( \theta_{\tilde{B}} \) is chosen so that \( \tilde{B}_1 \) decouples from the \( Z^0 \)-boson, thereby maintaining consistency with the measured total \( Z^0 \) width if \( M_{\tilde{B}_1} < \frac{M_{Z^0}}{2} \). It turns out that \( \theta_{\tilde{B}} \simeq 1.17 \text{ rad} \) is also required in order to evade the experimental direct searches for the light \( \tilde{B}_1 \) squark. We discuss this in more detail later in this section.

The decay rates for \( T \to \tilde{B}_1 \tilde{\chi}^+_1 \) and \( B \to \tilde{B}_1 \tilde{\chi}^0_1 \) depend crucially on the amount of \( \tilde{B}_L - \tilde{B}_R \) mixing. It is especially important to consider the effect of fixing \( \theta_{\tilde{B}} \simeq 1.17 \text{ rad} \) on the \( T \to \tilde{B}_1 \tilde{\chi}^+_1 \) decay. The Lagrangian for the \( \tilde{B}_1 - T - \tilde{\chi}^+_1 \) interaction is given by [11]:

\[ \mathcal{L}_{TB_1\tilde{\chi}^+_1} = g\overline{T}P_R \left( -U_{11} \cos \theta_{\tilde{B}} + U_{12} Y_B \sin \theta_{\tilde{B}} \right) \tilde{\chi}^+_1 \tilde{B}_1 \\
+ g\overline{T}P_L \left( Y_T V_{12} \cos \theta_{\tilde{B}} \right) \tilde{\chi}^+_1 \tilde{B}_1 + h.c. \quad (10) \]
where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ and $U_{ij}$ and $V_{ij}$ ($i, j = 1, 2$) are the $2 \times 2$ unitary matrices diagonalizing the charged gaugino-higgsino matrix:

$$U^* \left( \begin{array}{cc} M_2 & \sqrt{2} \sin \beta M_W \\ \sqrt{2} \cos \beta M_W & \mu_H \end{array} \right) V^{-1} = \left( \begin{array}{cc} M_{\tilde{\chi}_1^\pm} & 0 \\ 0 & M_{\tilde{\chi}_2^\pm} \end{array} \right)$$

(11)

We take $\mu_H$ and $M_2$ to be in the parameter regions where we expect a light degenerate chargino-neutralino pair, required in the MSSM4 by fits to precision data [9]. If we take $M_2 \ll |\mu_H|$ then the $\tilde{\chi}_1^0$ and $\tilde{\chi}_1^\pm$ are both gaugino-like, with $M_{\tilde{\chi}_1^\pm} \simeq M_2$. In this case the mass of the lightest neutralino is given by $M_{\tilde{\chi}_1^0} \simeq \min(M_1, M_2)$, where $M_1$ is the $U(1)$ gaugino soft mass. Therefore, small mass splittings only occur if $M_2 < M_1$. The mixing matrix elements in the chargino sector for the gaugino-like case are given by:

$$V_{11} \sim U_{11} \sim 1 \quad V_{12} \sim U_{12} \sim 0$$

(12)

On the other hand, if $|\mu_H| \ll M_2$ then $\tilde{\chi}_1^0$ and $\tilde{\chi}_1^\pm$ are both higgsino-like with degenerate masses $M_{\tilde{\chi}_1^\pm} \simeq M_{\tilde{\chi}_1^0} \simeq |\mu_H|$. In this case, the mixing matrix elements in the chargino sector obey:

$$V_{11} \sim U_{11} \sim 0 \quad V_{12} \sim \text{sgn}(\mu_H) \quad U_{12} \sim 1$$

(13)

We can see from Eq. (10) that the decay rate for $T \rightarrow \tilde{B}_1 \tilde{\chi}_1^+$ will be suppressed if the light chargino $\tilde{\chi}_1^+$ is gaugino-like, since $V_{12} \sim U_{12} \sim 0$ and $\cos \theta_{\tilde{B}}$ is fixed at 0.39. Therefore, in order to ensure that $T \rightarrow \tilde{B}_1 \tilde{\chi}_1^+$ dominates over $T \rightarrow bW^+$, without appealing to a suppression of the CKM mixing matrix element $V_{tb}$, we would expect the light chargino $\tilde{\chi}_1^+$ to be higgsino-like.

As with most phenomenological studies of supersymmetric models with $R$-parity, we must ensure that the lightest supersymmetric particle (LSP) is neutral for cosmological reasons. This is usually taken to be the neutralino $\tilde{\chi}_1^0$ in the MSSM3. The masses in Eq. (8) contradict this requirement. One possible solution presents itself, however, if we assume that the LSP is in fact the fourth generation sneutrino $\tilde{N}_1$ which becomes stable due to $R$-parity [3]. Since $M_{\tilde{N}_1} < \frac{M_{\tilde{\tau}_1}}{2}$ we must arrange the mixing angle $\theta_{\tilde{N}}$ in the

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2 $\mu_H$ is the bilinear term that couples the two Higgs doublets in the superpotential, whilst $M_2$ is the $SU(2)$ gaugino soft mass that appears in the supersymmetric breaking Lagrangian.

3 Previous studies of the MSSM4 have shown that for some of the parameter space it is natural to assume that $N_1$ is the LSP [4].
fourth generation sneutrino mass matrix such that $\tilde{N}_1$ decouples from the $Z^0$-boson. A right-handed LSP $\tilde{N}_1$ does indeed not couple to the $Z^0$-boson. The light fourth generation bottom squark will then decay via the semi-leptonic channel $\tilde{B}_1 \rightarrow c l \tilde{N}_1^*$, where $l = e, \mu, \tau$. Such a decay involves the factor $|V_{Be}V_{Nl}|$, leading to a long lifetime. Experiments looking for hadronizing sbottom squarks currently only exclude the range $5 \text{ GeV} \leq M_{\tilde{B}_1} \leq 38 \text{ GeV}$, if the mixing angle $\theta_{\tilde{B}} \simeq 1.17 \text{ rad}$ such that $\tilde{B}_1$ decouples from the $Z^0$-boson \cite{12}. Searches for long lived charged particles in $p\bar{p}$ collisions at CDF have only excluded particles in the mass range $50 \text{ GeV} - 270 \text{ GeV}$ \cite{13}.

4 The Higgs Sector of the MSSM4

At tree-level the lightest Higgs boson mass $M_{h^0}$ in the MSSM4 is bounded from above by the relation:

$$M_{h^0} \leq M |\cos 2\beta| \leq M_{Z^0}$$

$$M \equiv \text{min} (M_{Z^0}, M_{A^0})$$

(14)

This has already been ruled out from experimental data taken at LEP \cite{14}. It is therefore of vital importance to check that the radiative corrections in the MSSM4 can provide a large enough contribution to raise the mass of the lightest Higgs boson to be above its experimental lower bound.

To take the radiative corrections into account in the MSSM4, we use the one-loop effective potential, given by:

$$V_{\text{eff}} = V_0 + V_1 + \ldots$$

(15)

where $V_0$ is the tree-level potential, $V_1$ contains the one-loop contributions, and the ellipses represent the higher-loop corrections which we shall ignore. The one-loop radiative correction to the effective potential in the MSSM4 is given by:

$$V_1 = \sum_k \frac{1}{64\pi^2} (-1)^{2J_k}(2J_k + 1)C_k m_k^4 \left( \ln \frac{m_k^2}{Q^2} - \frac{3}{2} \right)$$

(16)

where the sum is taken over all particles in the loop; $C_k = 6(2)$ for coloured (uncoloured) fermions; $J_k$ are the spins and $m_k$ are the field dependent masses.
Figure 3: Plot of the lightest Higgs boson mass $M_{h^0}$ versus $M_S$. The upper and lower solid curves correspond to $M_T = M_B = 100$ GeV; $M_E = M_N = 50$ GeV and $\tan\beta = 1.65$ for the maximal-mixing and no-mixing cases, respectively. The dotted curves correspond to the uncertainty in $M_{h^0}$ from allowing $M_T$, $M_B$ and $\tan\beta$ to take on any value as long as perturbative unification occurs.

of the particles in the loops at the renormalization scale $Q$. We choose to minimize the potential at the scale $Q = \max(M_t, M_T)$.

In this analysis we will consider contributions to $V_1$ that arise from the $t$, $T$, $B$, $E$ and $N$ fermions and their corresponding superpartners. Since we are at low $\tan\beta$ we can ignore contributions to $V_1$ from the third generation (s)bottom (s)quark. To calculate the lightest Higgs boson mass we make the approximations:

$$M_{\tilde{l}}, M_{A^0}, M_{\tilde{q}} = \mathcal{O}(M_S) \quad ; \quad l = E, N \quad ; \quad q = t, T, B \quad (17)$$

where $M_S$ represents the scale of the soft supersymmetric breaking terms. Further details of the procedure can be found in [8].

In Figure 3 we plot the mass of the lightest Higgs boson mass $M_{h^0}$ versus $M_S$ for $M_t = 175$ GeV, and values of $M_T$, $M_B$ and $\tan\beta$ that results in perturbative unification. The bold curves correspond to taking $M_T = M_B = 100$ GeV, $M_E = M_N = 50$ GeV and $\tan\beta = 1.65$. We have also determined the allowed $(M_T, M_B, \tan\beta)$ parameter space that results in perturbative
unification, and from these sets of values we calculate the lightest Higgs boson mass. For a given $M_S$, we retain the maximum and minimum values of $M_{h^0}$ returned, which are represented by the dotted curves. We have also plotted for the maximal-mixing and minimal-mixing scenarios, as discussed in detail by Espinosa [15]. Overall, we can see that the maximum Higgs boson mass $M^\text{upper}_{h^0}$ consistent with perturbative unification has a value of $M^\text{upper}_{h^0} \simeq 152$ GeV. This is safely above the LEP lower bound.

5 Conclusions

We have seen that it is possible to incorporate a fourth generation into a supersymmetric GUT model, requiring the existence of a light, degenerate neutralino and chargino pair in order to provide the necessary cancellations in precision data fits. The fourth generation masses are then tightly constrained with typical values of $M_T \simeq M_B \simeq 100$ GeV and $M_E \simeq M_N \simeq 50$ GeV. To retain perturbative consistency to the unification scale we constrain $\tan \beta$ to lie in the interval $1.50 \lesssim \tan \beta \lesssim 1.75$. In order to provide decay channels to the fourth generation quarks, it might be that the LSP is the sneutrino $\tilde{N}_1$ with a mass $M_{\tilde{N}_1} \lesssim 40$ GeV. We would also expect a light $\tilde{B}_1$ with $M_{\tilde{B}_1} \simeq 40–50$ GeV along with a degenerate neutralino and chargino pair $M_{\tilde{\chi}} \simeq 55–60$ GeV. Such a light $\tilde{B}_1$ squark is chosen to decouple from the $Z^0$-boson, but would be copiously produced at the Fermilab Tevatron and should be searched for. The upper limit on the lightest Higgs boson mass is $M^\text{upper}_{h^0} \simeq 152$ GeV. The supersymmetric spectrum needed to satisfy all these constraints cannot be obtained from MSUGRA scenarios with universal parameters at the unification scale. We should also remark that a large degree of fine-tuning of parameters is involved in such a model.

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