Precision adjustment of electromagnetic emitter of large-scale systems such as GLONASS

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Abstract. The multidimensional nonlinear regression-tensor model is investigated in the substantiation of necessary and sufficient conditions for the optimal multi-factorial process of precision calibration of the parameters of the electromagnetic radiation source in the geostationary orbit (including the large-scale GLONASS system). A robust-adaptive strategy of a posteriori formation of the target functional of electromagnetic observability of a weighted-distributed signal in an established complex of stationary ground points is proposed on the basis of observations of this signal, made with a tolerance.

1. Introduction

Complex electronic and mechanical processes are the cornerstone of modern orbital precision verification of astrotechnique [1–5], which actualizes the issues associated with the development of their mathematical models [6]. In this context, regression models are in demand [7], including an important class formed by regression-tensor systems [8]. These systems, on the one hand, encompass polynomial models, allowing analytic description on the basis of tensor calculus [9], strong differentiability of vector mappings [10], and the theory of extremal problems. On the other hand, these models do not just idly manifest themselves, but actively work [11], acquiring a prominent role in a posteriori multifactor nonlinear mathematical modeling of electron-mechanical [12] and optical-mechanical systems [13], providing (within the optimization of the target quality functional reception of an electromagnetic signal) adaptive tuning of parameters that lower the energy level of the side lobes of electromagnetic emitters [8, 11, 14].

In this connection, the main attention below is paid to the problems posed in the conclusions of the work [14]. Particularly, there is correction of the target functional of the intensity of the observed signal of the source of electromagnetic radiation (SER) in the geostationary orbit in the regression-tensor modeling of the alignment process of the spatial-geometric parameters of the SER. In this case, regression interpretations of multiply connected conditions imposed by complex constraints [15] permitting on the basis of matrix analysis [16] the construction of an optimal mode of adaptive adjustment of SER parameters (in particular, the geometry and orientation of its antenna) in terms of the mathematical input-output model are determined. The predictive model of the SER signal is constructed from the experimental data of orbital verification tests based on the two-criteria identification by the least-squares
method (LSM) of covariant tensors [9] of the nonlinear SER equation, as a multidimensional tensor regression with a minimum coordinate-matrix norm [17].

2. Motivation, Terminology and Formulation of Problems

Let \( R \) be the field of real numbers, \( R^n \) is \( n \)-dimensional vector space over \( R \) with the Euclidean norm \( \|y\|_R = \sqrt{\sum_{i=1}^{n} y_i^2} \) \( y = (y_1, \ldots, y_n) \in R^n \) be a column vector with elements \( y_1, \ldots, y_n \in R \) and let \( M_{n,m}(R) \) be the space of all \( n \times m \)-matrices with elements from \( R \). In addition, we assume that \( T^m_n \) is the space of all covariant tensors of \( k \)-valence, that is, real multi-linear forms \( f^{k,m}: R^n \times \cdots \times R^n \to R \) with the norm \( \|f^{k,m}\|_{T^m_n} = \left( \sum_{j} t_{j}^{2} \right)^{1/2} \), where \( \{t_{j}\} \) is the "matrix of coordinates" [9, p. 246] of the tensor \( f^{k,m} \) with respect to the canonical basis in \( R^n \).

Let, in the process of orbital alignment, \( \{b_i\}_{i=1}^{n} \) be a set of stationary points for ground-based reception of the signal of the SER located on a geostationary satellite (for example, GLONASS systems, i.e., radius vectors, connecting the satellite and points \( b_i \), are constant), some fixed (reference) vector of space-angular parameters of the antenna of the SER [11–15]. \( v \) is a purposeful variation of the physical and geometric predictors [7] in the process of precision adjustment. \( w(\omega + v) \in R^n \) is the vector of the predictive intensity of the calibration signal of the SER measured at the probing points.

In this formulation, for a compact description of the nonlinear adaptive process of parametric orbital adjustment of the SER, we select for consideration a multidimensional input-output prognostic system described by the vector-tensor \( k \)-valent equation of a multi-factorial regression of the form:

\[
w(\omega + v) = \text{col} \left( \sum_{j=0}^{m-1} f_{1}^{j,m}(v_1, \ldots, v), \ldots, \sum_{j=0}^{m-1} f_{n}^{j,m}(v_1, \ldots, v) \right) + \varepsilon(\omega, v).
\]

(1)

Here \( \varepsilon(\omega,:) : R^n \to R^n \) is not a parametrizable vector function of the class:

\[
\|\varepsilon(\omega,v)\|_R = o\left((v_1^2 + \ldots + v_m^2)^{1/2}\right).
\]

(2)

\( v = \text{col} \{v_1, \ldots, v_m\} \), \( f_{i}^{0,m} \) is a "rank 0" tensor representing the intensity of the signal \( w_i \), \( i = 1, n \) in the mode of the orbital electronic geometric tuning of the transmitting antenna of the SER.

The problem of nonlinear regression-tensor modeling of the multifactor optimal process of orbital calibration of the SER is presented and studied in detail in [11, 14] for the 2-valent model (1). With that, analytical solutions of three methodological positions of this problem were obtained:

(I) for a fixed vector-predictor \( \omega \in R^n \) and its open neighborhood \( V \subset R^n \), the analytic conditions are defined for which the vector function \( w(\cdot): V \to R^n \) of the indices of the intensity of the calibration signal of the SER at the probing points satisfies the regression system (1)–(2);

(II) a direct algorithm for identifying the coordinates of the tensors \( f_{i}^{j,m} \), \( i = 1, n; j = 0,2 \) in a 2-valent regression-tensor model (1) is constructed on the basis of the solution of a two-criterion LSM-Problem of the form:

\[
\begin{aligned}
&\min \left\{ \sum_{i=1}^{g} \left( \|w_i(t) - \text{col} \left( \sum_{j=0}^{m-1} f_{1}^{j,m}(v_{i(t)}, \ldots, v_{i(t)}), \ldots, \sum_{j=0}^{m-1} f_{n}^{j,m}(v_{i(t)}, \ldots, v_{i(t)}) \right) \right)^2 \right\}^{1/2} \\
&\min \left\{ \sum_{i=1}^{n} \sum_{j=0}^{m-1} \|f_{i}^{j,m}\|_{T^m_n}^2 \right\}^{1/2},
\end{aligned}
\]

(3)
where \( w_{(l)} \in \mathbb{R}^q, v_{(l)} \in \mathbb{R}^n, l = 1, q \), respectively, are the vectors of the experimental factor-predictors of the SER, i.e. \( w_{(l)} \) is a posteriori "reaction" to the target "variation" relative to the reference vector \( \omega \) coordinates under the condition \( \|v_{(l)}\|_{\mathbb{R}^n} < 1 \). This inequality is methodologically dictated by the condition (2). \( q \) is the number of orbital experiments performed (determined by the representativeness of the model (1)), taking into account the dynamic characteristics of the SER [18, 19].

(III) for the 2-valent regression-tensor model (1), with the given predictor \( \omega \in \mathbb{R}^n \) and the nominal condition \( \varepsilon(\omega, \cdot) \equiv 0 \), an analytical solution of the orbital calibration problem is obtained, as a non-linear "\( v \) -optimizing" of the variable (relative to the \( \omega \) vector) predictors of the adjustable physical and geometric parameters of the SER:

\[
\max_{v \in \mathbb{R}^n} F(v) := r_1 w_1(\omega+v) + \ldots + r_n w_n(\omega+v),
\]

where the vector function \( v \mapsto w(\omega + v) = \text{col}(w_1(\omega + v), \ldots, w_n(\omega + v)) \) has a coordinate representation according to the LSM-identified model (1)–(3), \( r_i > 0 \) is weights reflecting the priority of the ground-based probing points of the SER signal.

Statement of problems (according to the conclusions of [14]):

(i) to determine the necessary and sufficient conditions for the solvability of the optimization task (4) for the 3-valent model (1);

(ii) to construct an algorithm for correcting sufficient conditions for the extremum of the stationary point of task (i) on the basis of a \( r \)-parametric adjustment \( r \mapsto r^T w(\omega + v) \) of the functional:

\[
v \mapsto F(v) = r^T w(\omega + v).
\]

3. Optimization of the Process of Adjusting the SER-Parameters

Let us consider the task (i) – optimization of the process of adjustment of the parameters of the SER for the model (1) with \( k = 3 \). We note that the solution of the concomitant identification Problem (II) with \( k = 3 \) is a modification of assertion 2 of [14] (see also [20]).

In such a mathematical formulation, the non-linear prognostic equation (1) can be presented in the following vector-matrix-tensor form:

\[
w(\omega + v) = c + A v + \text{col}(v^T B_1 v + f_1^3 m(v, v, v), \ldots, v^T B_n v + f_n^3 m(v, v, v)) + \varepsilon(\omega, v),
\]

where \( c \in \mathbb{R}^n, A \in M_{m,m}(R), B_i \in M_{m,m}(R), i = 1, n \). Without loss of generality, we assume that each matrix \( B_i \) has an upper triangular structure; this simplifies the implementation of the LSM-algorithm (3).

According to (1) with \( k = 3 \), the functional of electromagnetic observability (5) is twice continuously differentiable, which guarantees the equality of mixed derivatives:

\[
\frac{\partial^2 F(v_1, \ldots, v_m)}{\partial v_g \partial v_p} = \frac{\partial^2 F(v_1, \ldots, v_m)}{\partial v_p \partial v_g}, \quad \forall g, p = 1, m.
\]

Therefore, in the solution of the optimization problem (4) for the 3-valent model (6), we may assume the following Proposition 1 below being the main result, according to (7), Theorem 3 [10, p. 505] and Theorem 7.2.5 of [16]. But first let us preliminary assume that \( B_i^T := (B_i + B_i^T) \in M_{m,m}(R), i = 1, n \), where each \( B_i \) is the matrix of the system (6) (the matrix of the tensor \( f_i^{3,m} \) in the formulation, when it is not considered symmetric in the system (1)). Moreover, we consider the vector function:

\[
v \mapsto \Phi(v) := (r_1 B_1^T + \ldots + r_n B_n^T)^{-1} (A^T + [\nabla f_1^3 m(v, v, v), \ldots, \nabla f_n^3 m(v, v, v)]^T) r,
\]

where \( \nabla f_i^3 m(v, v, v) \) is the gradient of the functional \( v \mapsto f_i^3 m(v, v, v) \).

**Proposition 1.** The stationary points \( v^* \in \mathbb{R}^n \) of task (i) are the solutions of equation
\[ v^* + \Phi(v^*) = 0. \]  

A sufficient condition \( F(v^*) = \max \left| F(v) : v \in \mathbb{R}^n \right| \) is the requirement that \( v^* \), as a stationary point of the functional (5), be of elliptic type. In other words, at the point \( v^* \) for the Hessian \( G(v, r) \) of the functional (5), inequalities:

\[ \det \left[ b_{ij} \right]_p < 0, \quad p = 1, m, \]

must be performed, where \( \left[ b_{ij} \right]_p \in M_{n, p}(R) \), \( p = 1, m \) are the main Hessian sub-matrices

\[
G(v^*, r) = r_1 \left( B_1^* + \left[ \partial^2 f_n^3_m(v, v, v) / \partial v_p \partial v_p \right]_{v^*} \right) + \ldots \\
+ r_n \left( B_n^* + \left[ \partial^2 f_n^3_m(v, v, v) / \partial v_p \partial v_p \right]_{v^*} \right) \in M_{m, m}(R),
\]

which is equivalent – the characteristic numbers \( \lambda_p(v^*, r) \) of the matrix \( G(v^*, r) \) satisfy:

\[ \lambda_p(v^*, r) < 0, \quad p = 1, m. \]

**Corollary 1.** For \( k = 2 \), the Hessian of the functional (5) and the conditions (9), (10) are invariant to the position of the stationary point \( v^* \), while the Hessian is equal to \( G(r) = r_1 B_1^* + \ldots + r_n B_n^* \), which results in a linear dependence of the numbers \( \lambda_p(r) \), \( p = 1, m \) on the normalization of the vector \( r \).

If \( \text{rank} \, G(r) = m \), then the solution of equation (8) is unique and has the form \( v^* = -G^{-1}(r) A^T r \), which makes the position of the point \( v^* \) invariant to the normalization of the vector \( r \).

One of the factors, affecting the geometry of the stationary point \( v^* \) of the Proposition 1, is the digital adaptive parametric adjustment \( r \mapsto G(v^*, r) \) that results in the performance of elliptical conditions (9) or (10), which is the research subject of the next section.

4. Adaptation of the Target Functional on the r-Parameter Family of its Hessians

Let us consider the formulation (ii): for a stationary point of the optimization task (i), construct a numerical procedure for correcting the weight coefficients \( r \in \mathbb{R}^n \), starting from the fulfillment of the spectral conditions (10); ensuring the elliptic nature of the stationary point \( v^* \) of Proposition 1. This formulation is relevant in the problem of orbital calibration of the parameters of the SER, when at some ground points \( b_i \) it is necessary to weaken (i.e. \( r_j < 0 \)) the reception of the SER signal.

Let an initial vector \( r_0 \in \mathbb{R}^n \) of weight coefficients from the formulation (ii) be given. For example, a heuristic choice \( r_0 \) can be made based on the equality of its coordinates \( r_{0i}, i = 1, n \) to the values of some functions \( \Psi_j : R \to R \) that depend on the values of the functionals \( J_j(v) := w_j(\omega + v), i = 1, n \) from the auxiliary problems of optimal prediction of the quality of orbital alignment of the SER for individual target indicators \( w_j \). We denote by \( v^0 \in \mathbb{R}^n \) some stationary point of the functional (5) in case when the \( r \)-priority of the probing points is equal to \( r_0 \). Correspondingly, by \( G_0 \in M_{m, m}(R) \) we denote the Hessian of the given functional calculated for the pair \((r_0, v^0)\) and let \( G_i := B_i^* + \left[ \partial^2 f_n^3_m(v, v, v) / \partial v_p \partial v_p \right]_{v^0}, \quad i = 1, n. \)

Then, for an admissible linear variation \( \Delta r \) of the coordinates of the vector \( r_0 = \text{col}(r_{01}, \ldots, r_{0n}) \) given (by virtue of the comments to formula (4)), the region of this variation \( W \subset \mathbb{R}^n \) of the form:

\[ \Delta r := \text{col}(\Delta r_1, \ldots, \Delta r_n) \in W, \quad r_i = r_{0i} + \Delta r_i > 0, \quad i = 1, n, \]
\( \Delta r \)-parametric family of linear variations of the Hessian \( G(v^0, r_0 + \Delta r) \) is determined by a matrix \( m \times m \)-manifold of the form:

\[
G_0 + \sum_{i=1}^{\Delta r} G_i, \quad \Delta r \in W. \tag{11}
\]

**Proposition 2.** Let \( r = r_0 + \Delta r, \{ (\lambda_p(r_0), x_p), p = 1, m \} \subset R \times R^m \) be the set of eigen-pairs of Hessian \( G_0 \), i.e. \( \lambda_p(r_0)x_p = G_0x_p, p = 1, m \), and let, starting from the implementation of the manifold (11), be given the numbers \( g_{pi} = x_p^T G_r x_p / x_p^T x_p \), then the eigenvalues \( \lambda_p(v^0, r_0 + \Delta r), p = 1, m \) of the Hessian \( G(v^0, r_0 + \Delta r) \) have the form:

\[
\lambda_1(v^0, r_0 + \Delta r) = \lambda_1(r_0) + \sum_{i=1}^{\Delta r} g_{1i} \Delta r_i + o\left(\|\Delta r\|^2_{R^r}\right),
\]

\[
\lambda_m(v^0, r_0 + \Delta r) = \lambda_m(r_0) + \sum_{i=1}^{\Delta r} g_{mi} \Delta r_i + o\left(\|\Delta r\|^2_{R^r}\right).
\]

**Corollary 2.** Let \( k = 2, n = m, \Lambda(r_0) := \{ \lambda_1(r_0), \cdots, \lambda_m(r_0) \} \) be the vector of characteristic numbers of the matrix \( (r_0 b_1^T + \cdots + r_m b_m^T) \) and \( \{ x_p \}_{p=1}^{m} \) be the eigenvectors corresponding to them. In addition, let \( \Lambda^* := \{ \lambda_1^*, \cdots, \lambda_m^* \} \) be a certain vector of characteristic numbers, "standard/exemplary" by criterion (10), and \( B := [b_{pi}] \) is the \( m \times m \) matrix with elements \( b_{pi} = x_p^T b_i^T x_p / x_p^T x_p \).

Then for \( r_0 + \Delta r \), where the variation vector has a representation \( \Delta r = B^{-1}(\Lambda^* - \Lambda(r_0)) \), the eigenvalues of the Hessian \( G(r_0 + \Delta r) \) will be \( o\left(\|\Delta r\|^2_{R^r}\right) \) – close to the reference values \( \{ \lambda_p^* \}_{p=1}^{m} \).

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