Effect Summaries for Thread-Modular Analysis
Sound Analysis despite an Unsound Heuristic

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Abstract We propose a novel guess-and-check principle to increase the efficiency of thread-modular verification of lock-free data structures. We build on a heuristic that guesses candidates for stateless effect summaries of programs by searching the code for instances of a copy-and-check programming idiom common in lock-free data structures. These candidate summaries are used to compute the interference among threads in linear time. Since a candidate summary need not be a sound effect summary, we show how to fully automatically check whether the precision of candidate summaries is sufficient. We can thus perform sound verification despite relying on an unsound heuristic. We have implemented our approach and found it up to two orders of magnitude faster than existing ones.

1 Introduction

Verification of concurrent, lock-free data structures has recently received considerable attention [2312526]. Such structures are both of high practical relevance and, at the same time, difficult to write. A common correctness notion in this context is linearity [13], which requires that every concurrent execution can be linearized to an execution that could also occur sequentially. For many data structures, linearity reduces to checking control-flow reachability in a variant of the data structure that is augmented with observer automata [2]. This control-flow reachability problem, in turn, is often solved by means of thread-modular analysis [416]. Our contribution is on improving thread-modular analyses for verifying linearity of lock-free data structures.

Thread-modular analyses compute the least solution to a recursive equation

\[ X = X \cup \text{seq}(X) \cup \text{interfere}(X). \]

The domain of \( X \) is sets of views, partial configurations reflecting the perception of a single thread about the shared heap. Crucially, thread-modular analyses abstract away from the correlation among the views of different threads. Function \( \text{seq}(X) \) computes a sequential step, the views obtained from \( X \) by letting each thread execute a command on its own views. This function, however, does not reflect the fact that a thread may change a part of the shared heap seen by others. Such interference steps are computed by \( \text{interfere}(X) \). It is this function that we improve on. Before turning to the contribution, we recall the existing approaches and motivate the need for more work.

In the merge-and-project approach to interference (e.g., [491619]), a merge operation is applied on every two views in \( X \) to determine all merged views consistent with the given ones. On each of the consistent views, some thread performs a sequential step, and
the result is projected to what is seen by the other threads. The approach has problems with efficiency. The number of merge operations is exactly the square of the number of views in the fixed point. In addition, every merge of two views is expensive. It has to consider all consistent views whose number can be exponential in the size of the views.

The learning approach to interference \cite{31,21} derives, via symbolic execution, a symbolic update pattern for the shared heap. The learning process is integrated into the fixed-point computation, which incurs an overhead. Moreover, the number of update patterns to be learned is bounded by the number of reachable views only. An interference step applies the learned update patterns to all views, which again is quadratic in the number of views. Moreover, although update patterns abstract away from thread-local information, computing each application still requires a potentially expensive matching. There are, however, fragments of separation logic with efficient entailment \cite{5}.

What is missing is an efficient approach to computing interferences among threads.

Main ideas of the contribution. We propose to compute \textit{interfere}(X) by means of so-called effect summaries. An effect summary for a method \(M\) is a stateless program \(Q_M\) which over-approximates the effects that \(M\) has on the shared heap. With such summaries at hand, the interference step can be computed in linear time by executing the method summaries \(Q_M\) for all methods \(M\) on the views in the current set \(X\). This is a substantial improvement in efficiency over merge-and-project and learning techniques, which require time roughly quadratic in the size of the fixed-point approximant, \(X\), and possibly exponential in the size of views.

Technically, statelessness is defined as atomicity and absence of persistent local state. We found both requirements typically satisfied by methods of lock-free data structures. For our approach, this means stateless summaries are likely to exist (which is confirmed by our experiments). The reason why the atomicity requirement holds is that the methods have to preserve the integrity of the data structure under interleavings. The absence of persistent state holds since interference by other threads may invalidate local state at any time.

We propose a heuristic to compute, from a method \(M\), a stateless program \(Q_M\) which is a candidate for being an effect summary of \(M\). Whether or not this candidate is indeed a summary of \(M\) is checked on top of the actual analysis, as discussed below. Our heuristic is based on looking for occurrences of a programming idiom common in lock-free data structures which we call \textit{copy-and-check blocks}. Such a block is a piece of code that, despite lock-free execution, appears to be executed atomically. Roughly, we identify each such block and turn it into an atomic program.

Programmers achieve the above mentioned atomicity of copy-and-check blocks by first creating a local copy of a shared variable, performing some computation over it, checking whether the copy is still up-to-date and, if so, publishing the results of the computation to the shared heap. A classic implementation of such blocks is based on \textit{compare-and-swap} (CAS) instructions. In this case, for a local variable \(t\) and a shared variable \(T\), the copy-and-check block typically starts with an assignment \(t=T\) and finishes with executing \texttt{CAS}(T, t, x) which atomically checks whether \(t==T\) holds and, if so, changes the value of \(T\) to \(x\). Hereafter, we will denote such blocks as \textit{CAS blocks}, and we will concentrate on them since they are rather common in practice. However, we note
that the same principle can be used to handle other kinds of copy-and-check blocks, e.g., those based on the load-link/store-conditional (LL/SC) mechanism.

The idea of program analysis exploiting the intended atomicity of CAS blocks by treating these blocks as atomic is quite natural. The reason why it is not common practice is that this approach is not sound in general. The atomicity may be introduced too coarsely, and, as a result, an interfere\((X)\) implementation based on the guessed candidate summaries may miss interleavings present in the actual program. For our analysis, this means that its soundness is conditional upon the fact that the candidate summaries used are indeed proper effect summaries. It must be checked that they are stateless and that they cover all effects on the shared heap. We propose a fully automatic and efficient way of performing those checks. To the best of our knowledge, we are the first to propose such checks.

To check whether candidate summaries indeed cover the effects of the methods for which they were constructed, the idea is to let the methods execute under any number of interferences with the candidate summaries and see if some effect not covered by the candidate summaries can be obtained. Formally, we use the program \(Q = \bigoplus_i Q_{M_i}\), which executes a non-deterministically chosen candidate summary \(Q_{M_i}\) of a method \(M_i\), execute the Kleene iteration \(Q^*\) in parallel with each method \(M\), and check whether the following inclusion holds:

\[
\text{Effects}(M \parallel Q^*) \subseteq \text{Effects}(Q^*).
\]

If this inclusion holds, \(Q^*\) covers the actual interference all methods may cause. Hence, our novel implementation of interfere\((X)\) explores all possible interleavings. The cost of the inclusion test is asymptotically covered by that of computing the fixed point, and practically negligible. It can be checked in linear time (in the size of the fixed point) by performing, for every view in \(X\), a sequential step and testing whether the effect of the step can be mimicked by the candidate summaries. It is worth pointing out the cyclic nature of our reasoning: we use the candidate summaries to prove their own correctness.

Statelessness is an important aspect in the above process. It guarantees that the sequential iteration of \(Q^*\) explores the overall interference the methods of the data structure cause. As we are interested in parametric verification, the overall interference is, in fact, the one produced by an unbounded number of concurrent method invocations. Hence, computing this interference using candidate summaries requires us to analyse the program \(\prod\infty Q\), which is a parallel composition of arbitrarily many \(Q\) instances. However, statelessness guarantees that each of these instances executes atomically without retaining any local state. While, the atomicity ensures that the concurrent \(Q\) instances cannot overlap, the absence of local state ensures that \(Q\) instances cannot influence each other, even if executed consecutively by the same thread. Hence, we can use a single thread executing the iteration \(Q^*\) in order to explore the interference caused by \(\prod\infty Q\). This justifies the usage of \(Q^*\) for the effect coverage above. The check for statelessness is similar to the one of effect coverage. If both tests succeed, the analysis information is guaranteed to be sound.

**Overview of the approach, its advantages, and experimental evaluation.** Overall, our thread-modular analysis proceeds as follows. We employ the CAS block heuristic to compute candidate summaries. We use these candidates to determine the interferences in the fixed-point computation. Once the fixed point has been obtained, we check whether
the candidates are valid summaries. If so, the fixed point contains sound information, and can be used for verification (or, an on-the-fly computed verification result can be used). Otherwise, verification fails. Currently, we do not have a refinement loop because it was not needed in our experiments.

Our method overcomes the limitations in the previous approaches as follows. The summary program, $Q$, is quadratic in the syntactic size of the program—not in the size of the fixed point. The interference step executes the summary on all views in the current set $X$, which means an effort linear rather than quadratic in the fixed-point approximant. Moreover, $Q$ is often acyclic and hence needs linear time to execute, as opposed to the worst case exponential merge or match. In our benchmarks, we needed at most 5 very short summaries, usually around 3–5 lines of code each. The computation of candidate summaries (based on cheap and standard static analyses) and their check for validity are separated from the fixed point, and the cost of both operations is negligible.

We implemented our thread-modular analysis with effect summaries on top of our state-of-the-art tool [12] based on thread-modal reasoning with merge-and-project. We applied the implementation to verify linearizability in a number of concurrent list implementations. Compared to [12], we obtain a speed-up of two orders of magnitude. Moreover, we managed to infer stateless effect summaries for all our case studies except the DGLM queue [6] under explicit memory management (where one needs to go beyond statelessness). However, we are not aware of any automatic approach that would be able to handle this algorithm.

2 Effect Summaries on an Example

The main complication for writing lock-free algorithms is to guarantee robustness under interleavings. The key idea to tackle this issue is to use a specific update pattern, namely the CAS-blocks discussed in Section 1. We now show how CAS blocks are employed in the Treiber’s lock-free stack implementation under garbage collection, the code of which is given in Listing 1. The push method implements a CAS block as follows: (1) copying the top of stack pointer, top=ToS, (2) linking the node to be inserted to the current top of stack, node.next=top, and (3) making node the new top of stack in case no other thread changed the shared state, CAS(ToS,top,node).

Similarly, pop proceeds by: (1) copying the top of stack pointer, top=ToS, (2) querying its successor, next=top.next, and (3) swinging ToS to that successor in case the stack did not change, CAS(ToS,top,next).

Following the CAS-block idiom, the only statements modifying the shared heap in the Treiber’s stack are the CAS operations. Hence, we identify three types of effects on the shared heap. First, a successful CAS in push makes ToS point to a newly allocated cell that, in turn, points to the previous value of ToS. Second, a successful CAS in pop moves ToS to its successor ToS.next. Since we assume garbage collection, the removed element is not freed but remains in the shared heap until collected. Third, the effect of any other statement on the shared heap is the identity.

With the effects of the Treiber’s stack identified, we can turn towards finding an approximation. For that, consider the program fragments from Listing 1. S1 covers the effects of the CAS in push, S2 covers the effects of the CAS in pop, and, lastly, S3 produces the identity-effect covering all remaining statements. Then, the summary program is $Q = S1 @ S2 @ S3$. 
struct Node { data_t data; Node next; }
shared Node ToS;
void push(data_t in) {
    Node node = new Node(in);
    while (true) {
        Node top = ToS;
        node.next = top;
        if(CAS(ToS, top, node)){
            return;
        }
    }
}
S1: atomic {
    /* push */
    Node node = new Node(*);
    node.next = ToS;
    ToS = node;
}

bool pop(data_t& out) {
    while (true) {
        Node top = ToS;
        if(top == NULL){
            return false;
        }
        Node next = top.next;
        if(CAS(ToS, top, next)){
            out = top.data;
            return true;
        }
    }
}
S2: atomic {
    /* pop */
    assume(ToS != NULL);
    ToS = ToS.next;
}
S3: atomic { /* skip */
Listing 1. Pseudo code of the Treiber's lock-free stack [28] and its effect summaries.

To obtain the non-trivial summaries S1 and S2, it suffices to concentrate on the block of code between the top=ToS assignment and the subsequent CAS(ToS, top, _) statement. Without going into details (which will be provided in Section 5), the summaries result from considering the code between the two statements atomic, performing simplification of the code under this atomicity assumption, and including some purely local initialization and finalization code (such as the allocation in the push method).

3 Programming Model

A concurrent program P is a parallel composition of threads T. The threads are while-programs formed using sequential composition, non-deterministic choice, loops, atomic blocks, skip, and primitive commands. The syntax is as follows:

$$P ::= T | P || P \quad T ::= T_1; T_2 | T_1 \oplus T_2 | T^* | \text{atomic } T | \text{skip} | C.$$  

We use \text{Thrd} for the set of all threads. We also write \text{P}^* to mean a program P with the Kleene star applied to all threads. The syntax and semantics of the commands in \text{C} are orthogonal to our development. We comment on the assumptions we need in a moment.

We assume programs whose threads implement methods from the interface of the lock-free data structure whose implementation is to be verified. The fact that, at runtime, we may find an arbitrary (finite) number of instances of each of the threads corresponds to an arbitrary number of the methods invoked concurrently. The verification task is then formulated as proving a designated shared heap unreachable in all instantiations of the program. Since thread-modular analyses simultaneously reason over all instantiations of the program, we refrain from making this parameterization more explicit. Instead, we consider program instances simply as programs with more copies of the same threads.

We model heaps as partial and finite functions \text{h}: \text{Var} \cup \mathbb{N} \rightarrow \mathbb{N}. Hence, we do not distinguish between the stack and the heap, and let the heap provide valuations for both the program variables from \text{Var} and the memory cells from \mathbb{N}. We use \mathbb{H} for the set of all heaps. Initially, the heap is empty, denoted by \text{emp} with \text{dom(emp)} = \emptyset. We write \mathbb{L} if a partial function is undefined for an argument: \text{h}(e) = \mathbb{L} if \text{e} \notin \text{dom(h)}.

We assume each thread has an identifier from \text{Tid} \subseteq \mathbb{N}. A program state is a pair \text{(s, ef)} where \text{s} \in \mathbb{H} is the shared heap and ef: Tid \rightarrow \text{Thrd} \times \mathbb{H} maps the thread
identifiers to thread configurations. A thread configuration is of the form \((T, o)\) with \(T \in \text{Thrd}\) and \(o \in \mathbb{H}\) being a heap owned by \(T\). If \(cf = \{i \rightarrow (T, o)\}\) contains a single mapping, we write simply \((s, (T, o))\).

Our development crucially relies on having a notion of separation between the shared heap \(s\) and the owned heap \(o\) of a thread \(T\). However, the actual definitions of what is owned and what shared are a parameter to our development. We just require the separation to respect disjointness of the shared and owned heaps and to be defined such that it is preserved across execution of program statements. The latter is formalized below in Assumption 1. To render disjointness formally, we say that a state \((s, cf)\) is separated, denoted by \(\text{separate}(s, cf)\), if, for every \(i_1, i_2 \in \text{dom}(cf)\) with \(cf(i_j) = (T_{i_j}, o_{i_j})\) and \(i_1 \neq i_2\), we have \(\text{dom}(s) \cap \text{dom}(o_{i_j}) \cap \mathbb{N} = \emptyset\) and \(\text{dom}(o_{i_1}) \cap \text{dom}(o_{i_2}) \cap \mathbb{N} = \emptyset\).

Note that, in order to allow for having thread-local variables, the heaps need to be disjoint only on memory cells (but not on variables), thus the additional intersection with \(\mathbb{N}\).

We use \(\rightarrow\) to denote program steps. The sequential semantics of threads is as expected for sequential composition, choice, loops, and skip. An atomic block \(\text{atomic } T\) summarizes a computation of the underlying thread \(T\) into a single program step. The semantics of primitive commands depends on the actual set \(C\). We do not make it precise but require it to preserve separation in the following sense.

**Assumption 1.** For every step \((s, (T, o)) \rightarrow (s', (T', o'))\) with \(\text{separate}(s, (T, o))\), we have \(\text{separate}(s', (T', o'))\).

The semantics of a concurrent program incorporates the requirement for separation into its transition rule. A thread may only update the shared heap and those parts of the heap it owns. No other parts can be modified. Therefore, we let threads execute in isolation and ensure that the combined resulting state is separated:

\[
\begin{align*}
(s, cf(i)) \rightarrow (s', cf') & \quad cf' = cf[i \rightarrow cf'] \quad \text{separate}(s', cf') \\
(s, cf) \rightarrow (s', cf') & \quad \text{(PAR)}
\end{align*}
\]

Despite a precise notion of separation is not needed for the development of our approach in Section 4, we give, for illustration, the notion we use in our implementation and experiments. In the case of garbage collection (like in Java), the owned heap of a thread includes, as usual, its local variables and cells accessible from these variables, which were allocated by the thread, but never made accessible through the shared variables. The shared heap then contains the shared variables, all cells that were once made accessible from them, as well as cells waiting for garbage collection. For the case with explicit memory management, we need a more complicated mechanism of ownership transfer where a shared cell can become owned again. We propose such a mechanism in Section 6.

We assume the computation of the programs under scrutiny to start from an initial state \(\text{init}_P = (s_{\text{init}}, cf_{\text{init}, P})\) where \(s_{\text{init}}\) is the result of an initialization procedure. The initial thread configurations, denoted by \(cf_{\text{init}, T}\), are of the form \((T, emp)\). The initialization procedure is assumed to be part of the input program. We are interested in the shared heaps reachable by program \(P\) from its initial state:

\[
SH(P) := \{s | \exists cf. \text{init}_P \rightarrow^* (s, cf)\}.
\]
4 Interference via Summaries

We now present our new approach to computing the effect of thread interference steps on the shared heap (corresponding to evaluating the expression $\text{interfere}(X)$ from Section 1 for a set of views $X$) in a way which is suitable for concurrency libraries. In particular, we introduce a notion of a stateless effect summary $Q$: a program whose repeated execution is able to produce all the effects on the shared heap that the program under scrutiny, $P$, can produce. With a stateless effect summary $Q$ at hand, one can compute $\text{interfere}(X)$ by repeatedly applying $Q$ on the views in $X$ until a fixed point is reached. Here, statelessness assures that $Q$ is applicable repeatedly without any need to track its local state.

Later, in Section 5, we provide a heuristic for deriving candidates for stateless effect summaries. Though our experiments show that the heuristic we propose is very effective in practice, the candidate summary that it produces is not guaranteed to be an effect summary, i.e., it is not guaranteed to produce all the effects on the shared heap that $P$ can produce. A candidate summary which is not an effect summary is called unsound.

To guarantee soundness of our approach even when the obtained candidate summary is unsound, we provide a test of soundness of candidate summaries. Interestingly, as we prove, it is the case that even (potentially) unsound candidate summaries can be used to check their own soundness—although this step appears to be cyclic reasoning.

4.1 Stateless Effect Summaries

We start by formalizing the notion of statelessness. Intuitively, a thread is stateless if it terminates after a single step and disposes its local heap. Formally, we say that a thread $T$ of a program $Q$ is stateless if, for all reachable shared heaps $s \in \text{SH}(Q^*)$ and all transitions $(s, cf \xrightarrow{\text{init}, T} (s', cf))$, we have $cf = (\text{skip}, \text{emp})$. A program $Q$ is stateless if so are all its threads. Note that statelessness should hold from all reachable shared heaps rather than from just all heaps. While an atomic execution to \text{skip} would be easy to achieve from all heaps, a clean-up yielding \text{emp} can only be achieved if we have control over the thread-local heap. Also note that statelessness basically requires a thread to consist of a top-level atomic block to ensure termination in a single step.

Next, we define the effects of a program $P$, denoted by $\text{EF}(P) \subseteq \mathbb{H} \times \mathbb{H}$, to be the set $\text{EF}(P) = \{(s, s') | \text{init} \xrightarrow{P} (s, cf) \xrightarrow{*} (s', cf')\}$. This set generalizes the reachable shared heaps, $\text{SH}(P)$: it contains all atomic (single-step) updates $P$ performs on the heaps from $\text{SH}(P)$.

Altogether, a program $Q$ is a (stateless) effect summary of $P$ if it is stateless and $\text{EF}(T \parallel Q^*) \subseteq \text{EF}(Q^*)$ holds for all threads $T \in P$. We refer to this inclusion as the effect inclusion. Intuitively, it states that $Q^*$ subsumes all the effects $T$ may have under interference with $Q^*$. The below lemma, proved in the appendix, shows that the effect inclusion can be used to check whether a candidate summary is indeed an effect summary. Moreover, the check can deal with the different threads separately.

**Lemma 1.** If $Q$ is stateless and $\text{EF}(T \parallel Q^*) \subseteq \text{EF}(Q^*)$ holds for all $T \in P$, then we have $\text{EF}(P) \subseteq \text{EF}(P \parallel Q^*) \subseteq \text{EF}(Q^*)$.

In what follows, we describe our novel thread-modular analysis based on effect summaries. We assume that, in addition to the program $P$ under scrutiny, we have
a program $Q$ which is a candidate for being a summary of $P$ (obtained, e.g., by the heuristic that we provide in Section 5). In Section 4.2, we first provide a fixed point computation where the interference step is implemented by a repeated application of the candidate summary $Q$. We show that if the candidate summary $Q$ is an effect summary, then the fixed point we compute is a conservative over-approximation of the reachable shared heaps of $P$. Next, in Section 4.3, we show that the fact whether or not $Q$ is indeed an effect summary of $P$ can be checked efficiently on top of the computed fixed point (even though the fixed point need not over-approximate the reachable shared heaps of $P$).

In the case that the test of Section 4.3 fails, $Q$ is not an effect summary of $P$, and our verification fails with no definite answer. As a future work, one could think of proposing some way of patching the summaries based on feedback from the failed test. However, in our experiments, using the heuristic computation of candidate summaries proposed in Section 5, this situation has not happened for any program where a stateless effect summary exists. In the only experiment where our approach failed (the DGLM queue under explicit memory management, which has not been verified by any other fully automatic tool), the notion of stateless effect summaries itself is not strong enough. Hence, a perhaps more interesting question for future work is how to further generalize the notion of effect summaries.

4.2 Summaries in the Fixed-Point Computation

To explore the reachable shared heaps of program $P$, we suggest a thread-modular analysis which explores the reachable states of threads $T \in P$ in isolation. To account for the possible thread interleavings of the original program, we apply interference steps to the threads $T$ by executing the provided summary $Q$. Conceptually, this process corresponds to exploring the state space of the two-thread programs $T \parallel Q^*$ for all syntactically different $T \in P$. Technically, we collect the reachable states of those programs in the following least fixed point:

$$
X_0 = \{(s_{\text{init}}, (T, \text{emp})) | T \in P\}
$$

$$
X_{i+1} = X_i \cup \text{seq}(X_i) \cup \text{interfere}(X_i).
$$

Since $Q^*$ has no internal state, the analysis only keeps the thread-local configurations of $T$. Functions $\text{seq}(\cdot)$ and $\text{interfere}(\cdot)$ compute sequential steps (steps of $T$) and interference steps (steps of $Q^*$), respectively, as follows:

$$
\text{seq}(X_i) = \{(s', cf') | \exists (s, cf) \in X_i, (s, cf) \rightarrow (s', cf')\}
$$

$$
\text{interfere}(X_i) = \{(s', cf') | \exists s, cf'. \text{ separate}(s', cf) \land s \neq s'.\}
$$

$$
(s, cf) \in X_i \land (s, cf_{\text{inst}, Q}) \rightarrow (s', cf')\}.
$$

Function $\text{seq}(X_i)$ is standard. For $\text{interfere}(X_i)$ we apply $Q$ to each configuration $(s, cf) \in X_i$ by letting it start from the shared heap $s$ and its initial thread-local configuration $cf_{\text{inst}, Q}$. Then we extract the updated shared heap, $s'$, resulting in the post configuration $(s', cf)$. Altogether, this procedure applies to the views in $X_i$ the shared heap updates dictated by $Q$. The thread-local configurations, $cf$, of threads $T$ are not changed by interference. This locality follows from statelessness.
The following lemma, which is proven in the appendix, states that the set of shared heaps collected from the above fixed point is indeed the set of reachable shared heaps of all $T \parallel Q^*$. Let $X_k$ be the fixed point and define $\mathcal{R} = \{ s \mid \exists cf. (s, cf) \in X_k \}$.  

**Lemma 2.** If $Q$ is a summary of $P$, then $\mathcal{R} = \bigcup_{T \in P} SH(T \parallel Q^*)$.

With the state space exploration in place, we can turn towards a soundness result of our method: given an appropriate summary $Q$, the fixed point computation over-approximates the reachable shared heaps of $P$.

**Theorem 1.** If $Q$ is a summary of $P$, then we have $SH(P) \subseteq SH(Q^*) = \mathcal{R}$.

The rational behind the theorem is as follows. Relying on $Q$ being a summary of $P$ provides the effect inclusion. So, Lemma 1 yields $EF(P \parallel Q^*) \subseteq EF(Q^*)$. From the definition of effects we can then conclude $SH(P \parallel Q^*) \subseteq SH(Q^*)$. Thus, we have $SH(P) \subseteq SH(Q^*)$ because $SH(P) \subseteq SH(P \parallel Q^*)$ is always true. This shows the first inclusion. Similarly, the effect inclusion gives $SH(T \parallel Q^*) \subseteq SH(Q^*)$ by the definition of reachability. Hence, we conclude using Lemma 2.

### 4.3 Soundness of Summarization

Soundness of our method, as stated by Theorem 1 above, is conditioned by $Q$ being a summary of $P$. In our framework, $Q$ is heuristically constructed and there is no guarantee that it really summarizes $P$. Hence, for our analysis to be sound, we have to check summarization; we have to establish (1) the effect inclusion, and (2) statelessness of $Q$.

To that end, we check that (1) every update $T$ performs on the shared heap in the system $T \parallel Q^*$ can be mimicked by $Q$, and that (2) every execution of $Q$ terminates in a single step and does not retain persistent local state. We implement those checks on top of the fixed point, $X_k$, as follows:

\[
\forall (s, cf) \in X_k \ \forall s', cf', i \ \exists cf''.
\]

\[
(s, cf) \rightarrow (s', cf') \implies (s, cf_{\text{init},Q}) \rightarrow (s', cf'') \land \quad \text{(CHK-MIMIC)}
\]

\[
(s, cf_{\text{init},Q}(i)) \rightarrow (s', cf') \implies cf' = (\text{skip, emp}) \quad \text{(CHK-STATELESS)}
\]

The above properties indeed capture our intuition. The former, (CHK-MIMIC), states that, for every explored $T$-step of the form $(s, cf) \rightarrow (s', cf')$, the effect $(s', s')$ is also an effect of $Q$. That is, executing $Q$ starting from $s$ yields $s'$. This establishes the effect inclusion as required by Lemma 1. The latter check, (CHK-STATELESS), states that every thread of $Q$ must terminate in a single step and dispose its owned heap. This constraint is relaxed to those shared heaps which have been explored during the fixed-point computation. That is, it ensures statelessness of $Q$ on all heaps from $\mathcal{R}$. The key aspect is to guarantee that $\mathcal{R}$ includes $SH(Q^*)$ as required by the definition. We show that this inclusion follows from the check.

The above checks rely on the fixed point, which, in turn, is computed using the candidate summary $Q$. That is, we use $Q$ to prove its own correctness. Nevertheless, our development results in a sound analysis as stated by the following theorem, the proof of which can be found in the appendix.

**Theorem 2.** The fixed point $X_k$ satisfies (CHK-MIMIC) and (CHK-STATELESS) if and only if $Q$ is a summary of $P$. 

5 Computing Effect Summaries

We now provide our heuristic for computing effect summaries. It is based on CAS blocks between an assignment \( t = T \), denoted as checked assignment, and a CAS statement \( \text{CAS}(T, t, x) \), denoted as checking CAS below. Since we compute a summary for each such block, the number of summaries is at most quadratic in the size of the input.

In what follows, consider some method \( M \) given by its control-flow graph (CFG) \( G = (V, E, v_{\text{init}}, v_{\text{final}}) \). The CFG has a unique initial and a unique final state, which we will use in our construction. Return commands are assumed to lead to the final state. As we are only interested in the effect on the shared heap, we drop return values from return commands. Likewise, we skip assignments to output parameters unless they are important for the flow of control in \( M \). We assume the summaries to execute with non-deterministic input values, and so we replace every input parameter with a symbolic value \( * \). Conditionals, loops, and CAS commands are represented by two edges, for the successful and failing execution, respectively. Let \( e_{\text{asgn}} := (v_{\text{asgn}}, t = T, v_{\text{asgnt}}) \) be the CFG edge of the checked assignment, and let the successful branch of the checking CAS be \( e_{\text{cas}} := (v_{\text{cas}}, \text{CAS}(T, t, x), v_{\text{cassuc}}) \). Next, let \( e_{\text{asgn}}' := (v_{\text{asgn}}, t = T, v_{\text{asgnt}}') \) be a copy of the checked assignment to be used as the beginning of the CAS block, and let \( e_{\text{cas}}' := (v_{\text{cas}}, \text{CAS}(T, t, x), v_{\text{cassuc}}') \) be a copy of the checking CAS to be used as the end of the CAS block. Here, \( v_{\text{asgnt}}' \) and \( v_{\text{cassuc}}' \) are fresh nodes.

To give a concise description of effect summaries, the following shortcuts will be helpful. We write \( \text{rand}(G) \) for the CFG obtained from \( G \) by replacing each occurrence of a shared variable by a non-deterministic value \( * \). By \( G - S \), we mean the CFG obtained from \( G \) by dropping all edges carrying commands from the set \( S \). Given nodes \( v_1 \) and \( v_2 \), we denote by \( G(v_1, v_2) \) the CFG obtained from \( G \) by making \( v_1 / v_2 \) the initial/final node, respectively. Given two CFGs \( G \) and \( G' \), we define \( G; G' \) as their disjoint union where the single final state of \( G \) is merged with the single initial state of \( G' \). Finally, we allow compositions \( e; G \) and \( G; e \) of a CFG \( G \) with a single edge \( e \), by viewing \( e \) as a CFG consisting of a single edge with the initial/final nodes being the initial/final nodes of \( e \), respectively.

The construction of the summary proceeds in two steps. First we identify the CAS block and create the control-flow structure, then we clean it up using data flow analysis and generate the final code of the summary. Note that the clean-up step is optional but generates a concise form beneficial for verification.

**Step 1: Control-flow structure.** A summary consists of an initialization phase, followed by the CAS block, and a finalization phase. The first step results in the CFG

\[ G_{\text{init}}; G_{\text{block}}; G_{\text{final}}. \]

The guiding theme of the construction is to preserve all sequences of commands that may lead through the CAS block.

In the initialization phase, which is intended for purely local initialization, the method is assumed to be interrupted by other threads in the sense that the values of shared pointers may spontaneously change. Therefore, we replace all dependencies on shared variables by non-deterministic assignments. Moreover, all return commands are removed since we have not yet passed the CAS block. Eventually, when arriving at the \( v_{\text{asgn}} \) location,
the summary non-deterministically guesses that the CAS block should begin, and so the control is transferred to it via the $e_{\text{asgn'}}$ edge. Hence, the initialization is:

$$G_{\text{init}} := (\text{rand}(G) - \{\text{return}\})(v_{\text{init}}, v_{\text{asgns}}).$$

The CAS block begins with the $e_{\text{asgn'}}$ edge, i.e., with the checked assignment, and ends with the $e_{\text{cas'}}$ edge, i.e., the checking CAS statement. From the CAS block, we remove all control-flow edges with assignments $t = \top$ as we fixed the checked assignment when entering the CAS block (other assignments of the form $t = \top$, if present, will give rise to other CAS blocks; and a repeated execution of the same checked assignment then corresponds to a repeated execution of the summary). We also remove the return commands as the finalization potentially still has to free owned heap. Failing executions of the checking CAS do not leave the CAS block (and typically get stuck due to the removed checked assignments). Successful executions may leave the CAS block, but do not have to. Eventually, the summary guesses the last successful execution of the checking CAS and enters the finalization phase. Hence, we get the following code:

$$G_{\text{block}} := e_{\text{asgn'}}; ((G - \{\text{return}, t = \top\})(v_{\text{asgnt}}, v_{\text{cas}}); e_{\text{cas'}}).$$

Sometimes, the checked assignment can use local variables assigned prior to the checked assignment. In such a case, we add copies of edges with these assignments before the $e_{\text{asgn'}}$ edge. This happens, e.g., in the enqueue procedure of Michael & Scott’s lock-free queue where the sequence $\text{tail} = \text{Tail}; \text{next} = \text{tail}.next$; is used. If $\text{next} = \text{tail}.next$ is the checked assignment, we start $G_{\text{block}}$ with edges containing $\text{tail} = \text{Tail}$ and $\text{next} = \text{tail}.next$.

The finalization phase, again, cannot rely on shared variables. However, here, we preserve the return statements to terminate the execution:

$$G_{\text{final}} := \text{rand}(G)(v_{\text{cassuc}}, v_{\text{final}}).$$

Figure 1 illustrates the construction on the pop method in Treiber’s stack. Instead of a CFG, we give the source code. STOP represents deleted edges and the fact that we cannot move from one phase to another not using the new edges.

Step 2: Cleaning-up and summary generation. We perform copy propagation using a must analysis that propagates an assignment $y = x$ to subsequent assignments $z = y$, resulting in $z = x$. That it is a must analysis means the propagation is done only if $z = y$ definitely has to use the value of $y$ that stems from the assignment $y = x$. Moreover, we perform the copy propagation assuming that the entire summary executes atomically. For the initialization phase, the result is that the non-deterministic values for shared variables propagate through the code. Similarly, for the CAS block, the shared variables themselves propagate through the code. For the finalization phase, non-deterministic values propagate only in the case when a local variable does not receive its value from the CAS block. As a result, after the copy propagation, the CAS and the finalization block may contain conditionals that are constantly true or constantly false. We replace those that evaluate to true by skip and remove the edges that evaluate to false. The result of the copy propagation is illustrated in Figure 2.
Subsequently, we perform a live variables analysis. A variable is live if it may occur in a subsequent conditional or on the right-hand side of a subsequent assignment. Otherwise, it is dead. We remove all assignments to dead variables including output parameters. In our running example, all assignments to local variables as well as to the output parameter can be removed.

Next, we remove code that is unreachable, dead, or useless. Unreachable code can appear due to modifications of the CFG. Dead code does not lead to the final location. Useless code does not have any impact on the values of the variables used, which can concern even (possibly infinite) useless loops.

Finally, in the resulting code, conditionals are replaced by assume statements, and the entire code is wrapped into an atomic block. For the pop method in Treiber’s stack, we get the summary given in Listing 1.

6 Generalization to Explicit Memory Management

We now generalize our approach to explicit memory management. The problem is that the separation between the shared and owned heap is difficult to define and establish in this
Ownership as understood in garbage collection, where no other thread can access a cell that was allocated by another thread but not made shared, does not exist any more. Memory can be freed and reallocated, with other threads still holding (dangling) pointers to it. These threads can read and modify that memory, hence the allocating thread does not have strong guarantees of exclusivity. However, programmers usually try to prevent effects of accidental reallocations: threads are designed to respect ownership. That is, a thread should be allowed to execute as if it had exclusive access to the memory it owns.

Our development is parameterized by a notion of separation between the shared and owned heap. To generalize the results, we provide a new notion of ownership suitable for explicit memory management. However, the new notion is not guaranteed to be preserved by the semantics. Instead, we include into our fixed-point computation a check that the program respects this ownership, and give up the analysis if the check fails.

To understand how the heap separation is influenced by basic pointer manipulations, we consider the following set of commands $C$:

$$x = \text{malloc}, \ x = \text{free}, \ x = y, \ x = y . \text{sel}_i, \ x . \text{sel}_i = y .$$

Here, $x, y$ are program variables and $\text{sel}_0, \ldots, \text{sel}_n$ are selectors, from which the first, say $m$, are pointer selectors and the rest are data selectors. Command $x = \text{malloc}$ allocates a record, a free block of addresses $a + 0, \ldots, a + n$, and sets $h(x)$ to $a$. Command $x = \text{free}$ frees the record $h(x) + 0, \ldots, h(x) + n$. Selectors correspond to field accesses: $x . \text{sel}_i$ refers to the content of $a + i$ if $x$ points to $a$. The remaining commands have the expected meaning.

### 6.1 Heap Separation

We work with a three-way partitioning of the heap into shared, owned, and free addresses. Free are all addresses that are fresh or have been freed and not reallocated. Shared is every address that is reachable from the shared variables and not free. The reachability predicate, however, requires care. First, we must obviously generalize reachability from the first memory cell of a record to the whole record. Second, we must not use undefined pointers for reachability. A pointer is undefined if it was propagated from uninitialized or uncontrolled memory. Letting the shared heap propagate through such values would make it possible for the entire allocated heap to be shared (since undefined pointers can have an arbitrary value). Then, owned is the memory which is not shared nor free. The owned memory is partitioned into disjoint blocks that are owned by individual threads. A thread gains ownership by moving memory into the owned part, and loses it when the memory is removed from the owned part. The actions by which a thread can gain ownership are (1) allocation and (2) breaking reachability from shared variables by an update of a pointer or a shared variable (ownership transfer). An ownership violation is then a modification of a thread’s owned memory by another thread. This can in particular be (1) freeing or publishing the owned memory or (2) an update of a pointer therein. A program respects ownership if it cannot reach an ownership violation.

Let us discuss these concepts formally. We use $\bot$ to identify free cells. That is, in a heap $h$ address $a$ is free if $h(a) = \bot$ (also written $a \notin \text{dom}(h)$). A record is free if so are all its cells. Consequently, the $\text{free}$ command sets all cells of a record to $\bot$. The shared heap is identified by reachability through defined pointers starting from the shared
variables. For undefined pointers we use the symbolic value $\text{udef}$. Initially, all variables are undefined. Moreover, we let allocations initialize the selectors of records to $\text{udef}$. Initially, all variables are undefined. Moreover, we let allocations initialize the selectors of records to $\text{udef}$.

Let $\text{Ptrs}$ be the shared pointer variables. Then, the addresses of the shared records in a heap $h$, denoted by $\text{records}(h) \subseteq \mathbb{N} \cup \{\bot\}$, are collected by the following fixed point (where the address of a record is its lowest address):

$$S_0 = \{a | \exists x \in \text{Ptrs}. h(x) = a \neq \text{udef}\}$$

$$S_{i+1} = \{b | \exists a \in S_i. \exists k. a \neq \bot \land 0 \leq k < m \land h(a + k) = b \neq \text{udef}\}$$

All addresses within the shared records are then shared. The remaining cells, i.e., those that are neither free nor shared, are owned. This definition establishes a sufficient separation for Assumption 1. It is automatically lifted to the concurrent setting by Rule (PAR) following the intuition from above.

It remains to detect ownership violations, which occur whenever a thread modifies cells owned by other threads. Due to the separation integrated into Rule (PAR), threads execute with only the shared heap and their owned heap being visible. The remainder of the heap is cut away. By choice of $\bot$ to identify free cells, the cut away part appears free to the acting thread. In particular, the parts owned by other threads appear free. Hence, in order to avoid ownership violations, a thread must not modify free cells. To that end, an ownership violation occurs if (A) a free cell is freed again, (B) a free cell is written to, or (C) a free cell is published to the shared heap. For (A) and (B) we extend the semantics of commands to raise an ownership violation if a free cell is manipulated. For (C) we analyse all program steps of threads and check whether the step results in a shared heap where $\bot$ is made reachable.

Formally, we have the following rules.

$$\exists \text{sel}. (s \uplus o)(x). \text{sel} \notin \text{dom}(s \uplus o) \quad \Rightarrow \quad (s, (x = \text{free}, o)) \rightarrow \text{violation} \quad \text{(A)}$$

$$\forall \text{sel}. (s \uplus o)(x). \text{sel} \notin \text{dom}(s \uplus o) \quad \Rightarrow \quad (s, (x.\text{sel} = y, o)) \rightarrow \text{violation} \quad \text{(B)}$$

$$\quad \Rightarrow \quad (s, cf) \rightarrow (s', cf') \quad \bot \in \text{records}(s') \quad \text{(C)}$$

Note that reading out free cells is allowed by the above rules. This is necessary because lock-free algorithms typically perform speculating reads and check only later whether the result of the read is safe to use. Moreover, note that our detection of ownership violations can yield false-positives. A cell may not be owned, yet an ownership violation is raised because it appears free to the thread. We argue that such false-positives are desired as they access truly free memory. Put differently: an ownership violation detected by the above rules is either indeed an ownership violation or an unsafe access of free memory, that is, a bug.

### 6.2 Ownership Transfer

The above separation is different from the one used under garbage collection in the earlier sections. When an address becomes unreachable from the shared variables, it
is transferred into the acting thread’s owned heap (although other threads may still have pointers to it). We introduce this ownership transfer to simplify the construction of summaries. The idea is best understood on an example.

Under explicit memory management, threads free cells that they made unreachable from the shared variables to avoid memory leaks. Consider, for example, the method pop in Treiber’s stack (Listing 1). There, a thread updates the ToS variable making the former top of stack, say a, unreachable from the shared heap. In the version for explicit memory management, a is then freed before returning. If ownership was not transferred and address a stayed shared, then two summaries would be needed: one for the update of ToS and one for freeing a. However, a stateless version of the latter summary could not learn which address to free since it starts with the empty local heap and with a unreachable from the shared heap. If, on the other hand, ownership of a is transferred to the acting thread, then the former summary can include freeing a (which does not change the shared heap). Moreover, it is even forced to free a in order to remain stateless since a would otherwise persist in its owned heap.

We stress that our framework can be instantiated with other notions of separation, like an analogue of the one for garbage collection or the one of [12], which both do not have ownership transfer. This would complicate the reasoning in Section 4, but could lead to a more robust analysis (ownership transfer is prone to ownership violations).

6.3 ABA Prevention

Additional synchronization mechanisms can be incorporated into our approach. For instance, lock-free data structures may use version counters to prevent the ABA problem [20]: a variable leaves and returns to the same address, and an observer incorrectly concludes that the variable has never changed. A well-known scenario of this type causes stack corruption in a naive extension of Treiber’s stack to explicit memory management [20]. To give the observer a means of detecting that a variable has been changed, pointers are associated with a counter that increases with every update.

In our analysis, such version counters must be persistent in the shared memory. Since this is an exception from the above definition of separation, a presence of version counters must be indicated by the user (e.g., the user specifies that the version counter of a pointer a is always stored at address a + 1). The semantics is then adapted in such a way that (1) version counters remain in the shared heap upon freeing, (2) are retained in case of reallocations, and (3) are never transferred to a thread’s owned heap. The modifications can be easily implemented, and are detailed in Appendix B.3. Last, the thread-modular abstraction has to be adjusted since keeping all counters ever allocated in every thread view is not feasible. One solution is to remember only the values of those counters that are attached to the allocated shared and the thread’s own heap.

7 Experiments and Discussion

To substantiate our claim for practical benefits of the proposed method, we implemented the techniques from Sections 4. Therefore, we modified our previous lineariz...
ability checker \cite{12} to perform our novel fixed-point computation. The modifications were straightforward and implemented along the lines of Section 4.

Our findings are listed in Table 1. Experiments were conducted on an Intel Xeon E5-2670 running at 2.60GHz. The table includes running times (averaged over ten runs) and the number of explored views (size of the computed fixed point). Our benchmarks include well-known data structures such as Treiber's lock-free stack \cite{28}, Michael&Scott's lock-free queue \cite{20}, and the lock-free DGLM queue \cite{6}. We do not include lock-free set implementations due to limitations of the tool in handling data—not due to limitations of our approach. We ran each benchmark under garbage collection (GC) and explicit memory management (MM). Additionally, we include for each benchmark a comparison between our novel fixed point using summaries and the optimized version of the classical thread-modular fixed point from \cite{12}.

Our experiments show that relying on summaries provides a significant performance boost compared to classical interference. This holds true for both garbage collection and explicit memory management. For garbage collection, we experience a speed-up of one order of magnitude throughout the entire test suite. Although comparisons among different implementations are inherently unfair, we note that our tool compares favorably to competitors \cite{2,3,30,31}. Under explicit memory management, the same speed-up is present for simple algorithms, like Treiber's stack. For slightly more complex implementations, like Michael&Scott's queue, we observe a more eminent speed-up of over two orders of magnitude. This speed-up is present even though the analysis explores a way larger search space than its classical counterpart. This confirms that our approach of reducing the complexity of interference steps rather than reducing the search space is beneficial for verification.

Unfortunately, we could not establish correctness of the DGLM queue under explicit memory management with neither of the fixed points. For the classical one, the reason was imprecision in the underlying shape analysis which resulted in spurious unsafe memory accesses. For our novel fixed point, the tool detected an ownership violation according to Section 6. While being correct, the DGLM queue indeed features such a violation. The update pattern in the \texttt{deque} method can result in freeing nodes that were made unreachable by other threads. The problematic scenario only occurs when the head

**Table 1.** Experimental results: a speed-up of up to two orders of magnitude.
of the queue overtakes the tail. Despite the similarity, this behavior is not present in Michael & Scott’s queue which is why it does not suffer from such a violation.

As hinted in [Section 6], one could generalize our theory in such a way that no ownership transfer is required. Without ownership transfer, however, freeing cells becomes an effect of the shared heap which cannot be mimicked: a stateless summary cannot acquire a pointer to an unreachable cell and thus not mimic the free. Consequently, one has to relax the assumption of statelessness. This inflicts major changes on the fixed point from [Section 4]. Besides program threads, it would need to include threads executing stateful summaries. Moreover, one would need to reintroduce interference steps. However, we argue that only such interference steps are required where stateful summaries appear as the interfering thread. Hence, the number of interference steps is expected to be significantly lower than for ordinary interference. We consider a proper investigation of these issues an interesting subject for future work.

8 Related Work

We already commented on the two approaches of computing interference steps. The merge-and-project approach [4, 9, 16, 19] suffers from low scalability and precision due to computing too many merge-compatible heaps. To improve precision of interference, works like [10, 27, 31] track additional thread correlations; ownership, for instance. However, keeping more information within thread states usually has a negative impact on scalability. Moreover, for the programs of our interest, those techniques were not applicable in the case of explicitly managed memory which does not provide exclusivity guarantees. Instead, [42] proposed to maintain views of two threads, allowing one to infer the context in which the views occur. Since this again jeopardizes scalability, [12] tailored ownership towards explicit memory management. Still, computing interference remained quadratic in the size of the fixed point. Our approach improves dramatically on the efficiency of [12] while keeping its precision.

The learning approach in [29, 31, 32] and [21, 22, 23] performs a variant of rely/guarantee reasoning [15] paired with symbolic execution and abstract interpretation, respectively. In a fixed point, the interference produced by a thread is recorded and applied to other threads in consecutive iterations. This computes a symbolic representation of the interference which is as precise as the underlying abstract domain (although the precision may be relaxed by further abstraction and hand-crafted joins). Our method improves on this in various aspects. First, we never compute the most precise interference information. Our summaries can be understood as a form of interpolant between the most precise approximation and the complement of the bad states. Second, our summaries are syntactic objects (program code) which are independent of the actual verification procedure and thus reusable. The learned interference may be reused only in the same abstract domain it was computed in. Third, we show how to lift our approach to explicit memory management what has not been done before. Fourth, our results are independent of the actual program semantics relying only on a small core language. Our development required to formulate the principles that libraries rely on (statelessness) which have not been made explicit elsewhere.

Another approach to make the verification of low-level implementations tractable is atomicity abstraction [18, 7, 17, 25, 24]. The core idea is to translate a given program into
its specification by introducing and enlarging atomic blocks. The code transformations must be provably sound, with the soundness arguments oftentimes crafted for a particular semantics only. While generating summaries is closely related to making the program under scrutiny more atomic, we pursue a different approach. Our rewriting rules (i.e. the computation of summaries) do not need to be, and indeed are not, provably sound, which allows for much more freedom. Nevertheless, we guarantee a sound analysis. Our sanity checks can be understood as an efficient, fully automatic procedure to check whether or not the applied atomicity abstraction was sound. Additionally, we do not rely on a particular memory semantics.

Simulation relations are widely used for linearizability proofs \cite{linearizability,verified-compilation} and verified compilation \cite{verified-compilation,verified-compilation}. There, one establishes a simulation relation between a low-level program and a high-level program stating that the latter preserves the behaviors of the former. Verifying properties of the low-level program then reduces to verifying the same property for the high-level program. Establishing simulation relations, however, suffers from the same shortcomings as atomicity abstraction.

Finally, \cite{grace-periods} introduces grace periods, an idiom similar to CAS blocks. It reflects the protocol used by a program to prohibit data corruption. During a grace period, it is guaranteed that a thread’s memory is not freed. However, no method for checking conformance to such periods is given. That is, soundness cannot be checked when relying on grace periods whereas our sanity checks can efficiently detect unsound verification results.

9 Conclusion

We proposed a new approach for verifying lock-free data structures. The approach builds on the so-called CAS blocks (or, more generally, copy-and-check code blocks) which are commonly used when implementing lock-free data structures. We proposed a heuristic that builds stateless program summaries from such blocks. By avoiding many expensive merge-and-project operations, the approach can greatly increase the efficiency of thread-modular verification. This was confirmed by our experimental results showing that the implementation of our approach compares favorably with other competing tools. Moreover, our approach naturally combines with recently proposed reasoning about ownership to improve the precision of thread-modular reasoning, which allowed us to handle complex lock-free code efficiently even under explicit memory management. Of course, our heuristically computed stateless summaries can miss some reachable shared heaps, but, as a major part of our contribution, we proved that one can check whether this is the case on the generated state space. Hence, we can perform sound verification using a potentially unsound abstraction.

In the future, we would like to investigate CEGAR to include missing effects into our summaries. The main question here is how to refine the program code of a summary using an abstract representation of the missing effects. Further, it may be necessary to introduce stateful summaries in order to include certain effects, as revealed by the DGLM queue under explicit memory management. Moreover, in theory, our approach could increase not only efficiency but also precision compared with other approaches. This is due to the atomicity of the CAS blocks that could rule out interleavings that other approaches would explore. We have not found this confirmed in our experiments. Nevertheless, we find it worth investigating the theoretical and practical aspects of this matter in the future.
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A Data Structure Implementations and Summaries

To complement our benchmarks from Section 7, we provide the considered data structure implementations and the corresponding summaries. In the code, we mark with bold font the extensions needed for explicit memory management, but not for garbage collection. For explicit memory management we use version counters to avoid the ABA problem.

Figure 3 gives an overview of structures and functions common to all algorithms. The coarse stack can be found in Figure 4, its summary in Figure 5. The coarse queue can be found in Figure 6, its summary in Figure 7. Treiber’s stack can be found in Figure 8, its summary in Figure 9. Michael & Scott’s queue can be found in Figure 10, its summary in Figure 11. The DGLM queue can be found in Figure 12, its summary in Figure 13. Note that the DGLM queue does not have a stateless summary under explicit memory management as discussed in Section 7.

```c
struct ptr_t {
    uint age;
    Node* ptr;
}

struct Node {
    data_t data;
    ptr_t next;
}

ptr_t create(data_t data) {
    ptr_t res;
    res.ptr = new Node();
    res.ptr->data = data;
    res.ptr->next.ptr = NULL;
    // version counters are not initialized intentionally
    return res;
}

atomic boolean CAS(ptr_t& dst, ptr_t cmp, ptr_t swp) {
    if (dst.age == cmp.age && dst.ptr == cmp.ptr) {
        dst.age = cmp.age + 1;
        dst.ptr = swp.ptr;
        return true;
    } else {
        return false;
    }
}

shared ptr_t ToS;
void init() {
    ToS.ptr = NULL;
}

void push(data_t in) {
    ptr_t node = create(in);
    Atomic {
        node.ptr->next = ToS;
        ToS = node;
    }
    return;
}

bool pop(data_t& out) {
    Atomic {
        if (ToS.ptr == NULL) {
            return false;
        } else {
            out = ToS.ptr->data;
            ptr_t top = ToS;
            ToS = ToS.ptr->next;
            free(top.ptr);
            return true;
        }
    }
    return;
}
```

Figure 3. Common functionality.

Figure 4. Coarse stack.
S1: atomic {
    /* push */
    ptr_t node = create(*);
    node.ptr->next = ToS;
    ToS = node;
}
S2: atomic { /* skip */ }
S3: atomic {
    /* pop */
    assume(ToS.ptr != NULL);
    ptr_t old = ToS;
    ToS.ptr = ToS.ptr->next.ptr;
    free(old.ptr);
}

Figure 5. Summaries for coarse stack.

shared ptr_t Head, Tail;
void init() {
    Head = create(*);
    Tail = Head;
}
void enq(data_t in) {
    ptr_t node = create(in);
    Tail.ptr->next = node;
    Tail = node;
}
return;

Figure 6. Coarse queue.

S1: atomic {
    /* push */
    ptr_t node = create(*);
    Tail.ptr->next = node;
    Tail = node;
}
S2: atomic { /* skip */ }
S3: atomic {
    /* pop */
    assume(Head.ptr->next != NULL);
    ptr_t old = Head;
    Head = Head.ptr->next;
    free(old.ptr);
}

Figure 7. Summaries for coarse queue.

shared ptr_t ToS;
void init() {
    ToS.ptr = NULL;
}
void push(data_t in) {
    ptr_t node = create(in);
    while (true) {
        ptr_t top = ToS;
        node.ptr->next = top;
        if(CAS(ToS, top, node)){
            return;
        }
    }
}
bool pop(data_t& out) {
    while (true) {
        ptr_t top = ToS;
        if(top.ptr == NULL){
            return false;
        }
        ptr_t next = top.ptr->next;
        if(CAS(ToS, top, next)){
            out = top.ptr->data;
            free(top.ptr);
            return true;
        }
    }
}

Figure 8. Treiber’s lock-free stack [28].
Figure 9. Summaries for Treiber’s lock-free stack.

```c
shared ptr_t Head, Tail;

void init() {
    Head = create(*);
    Tail = Head;
}

void enq(data_t in) {
    ptr_t node = create(in);
    while (true) {
        ptr_t tail = Tail;
        ptr_t next = tail.ptr->next;
        if (tail.age != Tail.age) continue;
        if (tail.ptr != Tail.ptr) continue;
        if (next.ptr == NULL) {
            if (CAS(tail.next, next, node)) {
                CAS(Tail, tail, next);
                return;
            }
        } else {
            CAS(Tail, tail, next);
        }
    }
}

bool deq(data_t& out) {
    while (true) {
        ptr_t head = Head;
        ptr_t tail = Tail;
        ptr_t next = head.ptr->next;
        if (head.age != Head.age) continue;
        if (head.ptr != Head.ptr) continue;
        if (head.ptr == tail.ptr) {
            if (next.ptr == NULL) return false;
            else CAS(Tail, tail, next);}
        else {
            out = next.ptr->data;
            if (CAS(Head, head, next)) {
                free(head.ptr);
                return true;
            }
        }
    }
}
```

Figure 10. Michael&Scott’s non-blocking queue [20].

```c
S1: atomic {
    /* enq (add new node) */
    assume(Tail.ptr->next.ptr == NULL);
    ptr_t node = create(*);
    Tail.ptr->next.ptr = node.ptr;
    Tail.ptr->next.age++;
    Tail.age++;
}

S2: atomic {
    /* enq (swing Tail) */
    assume(Tail.ptr->next.ptr != NULL);
    Tail.ptr = Tail.ptr->next.ptr;
    Tail.age++;
}

S3: atomic {
    /* pop */
    ptr_t old = ToS;
    ToS.ptr = ToS.ptr->next.ptr;
    ToS.age++;
    free(old.ptr);
}

S4: atomic {
    /* deq (swing Tail) */
    assume(Head.ptr == Tail.ptr);
    assume(Head.ptr->next.ptr != NULL);
    Tail.ptr = Tail.ptr->next.ptr;
    Tail.age++;
}

S5: atomic {
    /* deq */
    assume(Head.ptr != Tail.ptr);
    ptr_t old = Head;
    Head.ptr = Head.ptr->next.ptr;
    Head.age++;
    free(old.ptr);
}
```

Figure 11. Summary for Michael&Scott’s non-blocking queue.
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shared ptr_t Head, Tail;

void init() {
    Head = create(*);
    Tail = Head;
}

void enq(data_t in) {
    ptr_t node = create(in);
    while (true) {
        ptr_t tail = Tail;
        ptr_t next = tail.ptr->next;
        if (tail.age != Tail.age) continue;
        if (tail.ptr != Tail.ptr) continue;
        if (next.ptr == NULL) {
            if (CAS(tail.next, next, node)) {
                CAS(Tail, tail, next);
                return;
            }
        } else {
            CAS(Tail, tail, next);
        }
    }
}

bool deq(data_t& out) {
    while (true) {
        ptr_t head = Head;
        ptr_t next = head.ptr->next;
        if (head.age != Head.age) continue;
        if (head.ptr != Head.ptr) continue;
        if (next.ptr == NULL) {
            return false;
        } else {
            out = next.ptr->data;
            if (CAS(Head, head, next)) {
                ptr_t tail = Tail;
                if (head == tail) {
                    CAS(Tail, tail, next);
                }
                free(head.ptr);
                return true;
            }
        }
    }
}

Figure 12. DGLM lock-free queue [6].

S1: atomic {
    /* enq (add new node) */
   assume(Tail.ptr->next.ptr == NULL);
    ptr_t node = create(*);
    Tail.ptr->next = node;
}

S2: atomic {
    /* enq (swing Tail) */
    /* deq (swing Tail) */
   assume(Tail.ptr->next.ptr != NULL);
    Tail.ptr = Tail.ptr->next.ptr;
}

S3: atomic {
    /* deq */
   assume(Head.ptr->next.ptr != NULL);
    Head.ptr = Head.ptr->next.ptr;
}

S4: atomic {
    /* skip */
}

Figure 13. Summary for the DGLM lock-free queue under garbage collection. The algorithm has no summary under explicit memory management.
B Semantics

We provide the detailed semantics for the core language from Section 3, as well as semantics for garbage collection and explicit memory management satisfying the requirements discussed in Sections 3 and 6.

B.1 Core Sequential Semantics

Section 3 introduced the core command language used throughout the paper. Its semantics is made precise in Figure 14.

\[
\begin{align*}
(s, (T_1, o)) & \rightarrow (s', (T_1', o')) \\
(s, (T_1; T_2, o)) & \rightarrow (s', (T_1'; T_2, o')) \\
(s, (\text{skip}; T, o)) & \rightarrow (s, (T, o)) \\
(s, (T_1, o)) & \rightarrow (s', (T_1', o')) \\
(s, (T_1 \oplus T_2, o)) & \rightarrow (s', (T_1', o')) \\
(s, (T^*, o)) & \rightarrow (s, (T; T^*, o)) \\
(s, (T^*, o)) & \rightarrow (s, (\text{skip}, o)) \\
\end{align*}
\]

Figure 14. Semantics of the command language core from Section 3.

B.2 Garbage Collection Semantics

A possible semantics for garbage collection follows.

\textbf{Commands.} In order to present how pointer primitives behave under garbage collection, we consider the following instantiation of \( C \):

\[
C ::= x = \text{malloc} \mid x = y \mid x = y.\text{sel}_i \mid x.\text{sel}_i = y.
\]

We assume a typed language where non-pointer values cannot be used or stored as pointers. Therefore, let \( PSel \subseteq \{sel_0, \ldots, sel_n\} \) be the selectors of pointer type, and \( PSel = \{sel_0, \ldots, sel_n\} \setminus PSel \) the ones of non-pointer type.

\textbf{Separation.} The heap is separated into three partitions: free, owned and shared memory. A discussion of the individual parts is in order.

\textbf{Free cells, free records.} We identify free addresses by \( \bot \). That is, a cell located at address \( a \) is free in a heap \( h \) if \( h(a) = \bot \). Recall that we may equivalently write \( a \notin \text{dom}(h) \). A record is considered free if so are all its cells. Formally, we use the predicate \( \text{isfree}_h(a) \) to denote that at address \( a \) in heap \( h \) sufficiently many cells are available for an object:

\[
\text{isfree}_h(a) : \iff \{a, \ldots, a + n\} \cap \text{dom}(h) = \emptyset.
\]

Note that, due to the semantics of Rule (PAR) and the usage of \( \bot \) here, freed memory and memory owned by other threads is indistinguishable.
Owned cells. Ownership is granted upon allocation. After a record is allocated the allocating thread owns all the record’s cells. Ownership is lost when the record is made reachable through the shared variables. Once lost, ownership cannot be reclaimed.

Shared cells. Shared are all cells that are neither owned nor free. A cell becomes shared if it is published, i.e., if it is made reachable from the shared variables. If a cell is made unreachable from the shared variables it remains shared nevertheless.

Let $\mathit{Ptrs}$ be the shared pointer variables. Then, the records reachable from the shared (pointer) variables in a heap $h$, denoted by $\mathit{records}(h)$, are those records collected by the following fixed point:

$$
\begin{align*}
S_0 &= \{a \mid \exists x \in \mathit{Ptrs}. h(x) = a\} \\
S_{i+1} &= \{b \mid \exists a \in S_i \exists \mathit{sel} \in \mathit{PSel}. h(a.\mathit{sel}) = b\}.
\end{align*}
$$

Note that we layout the semantics in such a way that $\bot$ is never reachable from the shared variables. To do so, it suffices to initialize pointer selectors upon allocation to null. This is the case because once a thread acquires a pointer to a cell, that cell remains accessible: if the cell is shared it will remain shared, if the cell is owned then it is owned by the thread holding the reference and it may either remain owned or become and remain shared. (Hence, ownership violations as discussed for explicit memory management in Section 6 cannot occur.)

Separating heaps. During execution, the boundaries between the shared and owned heap may be adjusted due to, e.g., assignments. In order to refer to the consequences of such updates, we introduce a function $\mathit{readjust}(s, o, h')$ which, for a heap $s \uplus o$, partitioned into a shared part $s$ and an owned part $o$, and an entire updated heap $h'$, provides a partitioning $h' = s' \uplus o'$ into a shared part $s'$ and an owned part $o'$ according to the above intuition of how the shared and owned parts behave. Formally, $\mathit{readjust}(s, o, h')$ sets the domains of $s'$ and $o'$ to

$$
\begin{align*}
dom(s') &= \dom(s) \cup \mathit{Pub} \quad \text{and} \quad \dom(o') = \dom(o) \setminus \mathit{Pub} \\
\text{with} \quad \mathit{Pub} &:= \{a + 0, \ldots, a + n \mid a \in \mathit{records}(h')\}.
\end{align*}
$$

This has the effect of moving the published addresses, $\mathit{Pub}$, from the owned heap to the shared heap. Note that this definition requires $\dom(s \uplus o) = \dom(h')$. As we will see later, the only exception to this usage is the transition rule for $\mathit{malloc}$. We will handle this corner case in the transition rule in favor of a more lightweight $\mathit{readjust}(-)$.

Semantics. Next up are the transition rules for the individual statements of the commands from $C$ as instantiated above.

Errors. For simplicity we assume that there is a dedicated shared variable $\mathit{fail}$ to indicate execution failures. We assume that execution is aborted as soon this variable is set.

$$
(s, (\mathit{err}, o)) \rightarrow (s[\mathit{fail} \rightarrow 1], (\mathit{skip}, o)) \quad \text{(ERR)}
$$
Effect Summaries for Thread-Modular Analysis

Allocation. A `malloc` allocates new memory for an object. The memory has to be free prior to the allocation. The selectors are default-initialized to `null`, i.e., $0 \in \mathbb{N}$.

\[
\begin{align*}
    a &\in \mathbb{N} \quad \text{isfree}_\mathbb{N}(a) \quad x \in \text{Ptrs} \\
    s' &= s[x \to a, a + 0 \to 0, \ldots, a + n \to 0] \\
    (s, (x = \text{malloc}, o)) &\to (s', (\text{skip}, o)) \quad (\text{MALLOCSHARED}) \\
    a &\in \mathbb{N} \quad \text{isfree}_\mathbb{N}(a) \quad x \notin \text{Ptrs} \\
    o' &= s[x \to a, a + 0 \to 0, \ldots, a + n \to 0] \\
    (s, (x = \text{malloc}, o)) &\to (s, (\text{skip}, o')) \quad (\text{MALLOCOWNED})
\end{align*}
\]

Note that the allocated memory could be in use by another thread. Such allocations are prevented in concurrent executions by the semantics of Rule (PAR) from Section 3 as the resulting concurrent state would not be separated. That is, we do not need care for such spurious allocations here in the sequential semantics.

Assignment 1. Assignments of the form $x = y$ simply copy the valuation of $y$ to $x$. The boundary between the shared and owned heap may be readjusted.

\[
\begin{align*}
    (s', o') &= \text{readjust}(s, o, (s \uplus o)[x \to (s \uplus o)(y)]) \\
    (s, (x = y, o)) &\to (s', (\text{skip}, o')) \quad (\text{ASSIGN1})
\end{align*}
\]

Assignment 2 Assignments of the form $x = y \cdot \text{sel}_i$ dereference pointer $y$. For the assignment to succeed $y$ must be non-null. As before the shared heap may need readjustment.

\[
\begin{align*}
    (s \uplus o)(y) &= 0 \\
    (s, (x = y \cdot \text{sel}_i, o)) &\to (s, (\text{err}, o)) \quad (\text{ASSIGN2-NUL})
\end{align*}
\]

\[
\begin{align*}
    (s \uplus o)(y) &= a \neq 0 \\
    (s', o') &= \text{readjust}(s, o, (s \uplus o)[x \to (s \uplus o)(a \cdot \text{sel}_i)]) \\
    (s, (x = y \cdot \text{sel}_i, o)) &\to (s', (\text{skip}, o')) \quad (\text{ASSIGN2})
\end{align*}
\]

Assignment 3 Assignments of the form $x.\text{sel}_i = y$ follow the same rules as the previous assignments.

\[
\begin{align*}
    (s \uplus o)(x) &= 0 \\
    (s, (x.\text{sel}_i = y, o)) &\to (s, (\text{err}, o)) \quad (\text{ASSIGN3-NUL})
\end{align*}
\]

\[
\begin{align*}
    (s \uplus o)(x) &= a \neq 0 \\
    (s', o') &= \text{readjust}(s, o, (s \uplus o)[a.\text{sel}_i \to (s \uplus o)(y)]) \\
    (s, (x.\text{sel}_i = y, o)) &\to (s', (\text{skip}, o')) \quad (\text{ASSIGN3})
\end{align*}
\]

B.3 Explicit Memory Management Semantics

A possible semantics for explicit memory management follows which detects ownership violations as discussed in Section 6.
Commands. We consider the following pointer manipulating primitives:

\[
C ::= \text{err} \mid x = \text{malloc} \mid x = \text{free} \mid x = y \mid x.y \cdot x.y = y.
\]

As for the garbage collection case, we use pointer selectors, PSel, non-pointer selectors, \(\overline{\text{PSel}}\), and assume that non-pointer types are not cast to pointer types.

Separation. As for garbage collection, the heap consists of free, owned and shared memory. The separation is detailed in Section 6 and thus not repeated here.

Separating heaps. To separate a heap into a shared and an owned part we have to compute the records reachable from the shared variables, \(\text{Ptrs}\), following defined pointers. We use the symbolic value \(\text{undef}\) to denote undefined values of any type. Then, the addresses of the shared records in a heap \(h\) are given by the least fixed point to the following equation:

\[
S_0 = \{ a \mid \exists x \in \text{Ptrs}, h(x) = a \neq \text{undef} \}
S_i+1 = S_i \cup \{ b \mid \exists a \in S_i, \exists \text{sel} \in \text{PSel}. a \neq \bot \land h(a.\text{sel}) = b \neq \text{undef} \}.
\]

Let \(S_{\text{lfp}}\) be the least fixed point. Then, the function \(\text{readjust}(h)\) returns a pair \((s', o')\) which resembles \(h\) split into a shared heap \(s'\) and an owned heap \(o'\) according to the computed shared records. Formally, we have \(h = s' \uplus o'\) and

\[
\begin{align*}
\text{dom}(s') &= \text{dom}(h) \cap \bigcup_{a \in S_{\text{lfp}}} \{ a, \ldots, a+n \} \quad \text{and} \\
\text{dom}(o') &= \text{dom}(h) \setminus \text{dom}(s').
\end{align*}
\]

Ownership violations. Recall from Section 6 that ownership violations occur by modifying, freeing, or publishing memory owned by other threads. We handle the former two violations when dealing with assignments and freeing, respectively. Publishing, however, can happen whenever the boundary between the shared and owned heaps are adjusted. In such a case a cell owned by another thread is made reachable from the shared variables. Since those cells appear freed as discussed above, we can recognize an ownership violation due to publishing by checking whether \(\bot\) is contained in \(S_{\text{lfp}}\). For a concise presentation, we let \(\text{readjust}(h)\) return \text{err} in such occasions.

To make all possible occurrences of ownership violations visible, we include their detection in the semantics (unlike presented in Section 6 where ownership violations due to publishing where detected on top of program steps).

Semantics The detailed semantics are in order.

Errors. As for garbage collection, we assume a dedicated shared variable \(\text{fail}\) to indicate execution failures.

\[
(s, (\text{err}, o)) \rightarrow (s[\text{fail} \rightarrow 1], (\text{skip}, o)) \quad (\text{ERR})
\]
Effect Summaries for Thread-Modular Analysis

Allocation. A malloc allocates new memory for an object. The memory has to be free prior to the allocation. The newly allocated memory contains uncontrolled values, i.e., udef. We have:

\[
\begin{align*}
\text{malloc}(a) & \implies \text{isfree}_{\text{local}}(a) \implies (s', o') = \text{readjust}(h') \\
(s, (x = \text{malloc}, o)) & \implies (s', (\text{skip}, o')) \\
\end{align*}
\]

Here, readjust(·) cannot fail unless an ownership violation occurred in the preceding execution. Hence, we do not include a rule for such cases as we assume that we would have aborted the execution already.

Deallocation. A free marks a cell available for reallocation. As discussed for isfree(·) above we do so by setting the memory content to ⊥. A free removes an entire object from the heap. The object must not already be free, i.e., we explicitly forbid double free (in particular because it could lead to an ownership violation). Also, no shared record is allowed to be freed as for otherwise an allocation is not guaranteed to provide ownership (the allocated cell could still be referenced by a shared object).

\[
\begin{align*}
x \in \{\bot, \text{undef}\} & \implies (s, (x = \text{free}, o)) \rightarrow (s, (\text{err}, o)) \\
\text{FREE-FAIL} \\
x \in \text{dom}(s \uplus o) \quad \exists sel \in PSel. (s \uplus o)(x, sel) \notin \text{dom}(s \uplus o) & \implies (s, (x = \text{free}, o)) \rightarrow (s, (\text{err}, o)) \\
\text{FREE-DUPLICATE} \\
x \in \text{dom}(s \uplus o) \quad \neg \text{isfree}_{\text{local}}((s \uplus o)(x)) & \implies \text{err} = \text{readjust}((s \uplus o[a \rightarrow \bot, \ldots, a + n \rightarrow \bot]) \\
(s, (x = \text{free}, o)) & \rightarrow (s, (\text{err}, o)) \\
\text{FREE-SHARED} \\
x \in \text{dom}(s \uplus o) \quad \neg \text{isfree}_{\text{local}}((s \uplus o)(x)) & \implies (s', o') = \text{readjust}((s \uplus o[a \rightarrow \bot, \ldots, a + n \rightarrow \bot]) \\
(s, (x = \text{free}, o)) & \rightarrow (s', (\text{skip}, o')) \\
\text{FREE} \\
\end{align*}
\]

Assignment 1. Assignments of the form \(x = y\) simply copy the valuation of \(y\) to \(x\). The boundary between the shared and owned heap may be readjusted. Consequently, they can fail if an ownership violation due to publishing occurs.

\[
\begin{align*}
\text{ASSIGN1-OV} \\
\text{ASSIGN1} \\
\end{align*}
\]
Assignment 2 Assignments of the form \( x = y.\text{sel}_i \) dereference pointer \( y \). For the assignment to succeed \( y \) must be a defined pointer, that is, have a valuation other than \( \text{undef} \) and \( \bot \). Otherwise, an unsafe operation is performed which may lead to a segmentation fault. As before the shared heap may need readjustment.

\[
\begin{align*}
\text{ASSIGN2-SEG} & : y \in \{\bot, \text{undef}\} \\
& \quad (s, (x = y.\text{sel}_i, o)) \rightarrow (s, (\text{err}, o))
\end{align*}
\]

\[
\begin{align*}
\text{ASSIGN2-OV} & : y \in \text{dom}(s \uplus o) \\
& \quad (s \uplus o)(y) = a \\
& \quad \text{err} = \text{readjust} \left( (s \uplus o)[x \rightarrow (s \uplus o)(a.\text{sel}_i)] \right) \\
& \quad (s, (x = y.\text{sel}_i, o)) \rightarrow (s, (\text{err}, o))
\end{align*}
\]

Assignment 3 Assignments of the form \( x.\text{sel}_i = y \) dereference pointer \( x \). As above, we require this access to be safe. More importantly, however, we have to prevent ownership violations. The previous assignments updated variables. As such, they could only modify the shared or the owned heap. Now, we are updating a memory cell. We have to prevent updates of cells that belong to others. Since such cells appear freed, we must prevent updates to freed cells.

\[
\begin{align*}
\text{ASSIGN3-SEG} & : x \in \{\bot, \text{undef}\} \\
& \quad (s, (x.\text{sel}_i = y, o)) \rightarrow (s, (\text{err}, o))
\end{align*}
\]

\[
\begin{align*}
\text{ASSIGN3-OV-MOD} & : x \in \text{dom}(s \uplus o) \\
& \quad (s \uplus o)(x.\text{sel}_i) \notin \text{dom}(s \uplus o) \\
& \quad (s, (x.\text{sel}_i = y, o)) \rightarrow (s, (\text{err}, o))
\end{align*}
\]

\[
\begin{align*}
\text{ASSIGN3-OV-PUB} & : x \in \text{dom}(s \uplus o) \\
& \quad (s \uplus o)(x) = a \quad a.\text{sel}_i \in \text{dom}(s \uplus o) \\
& \quad \text{err} = \text{readjust} \left( (s \uplus o)[a.\text{sel}_i \rightarrow (s \uplus o)(y)] \right) \\
& \quad (s, (x.\text{sel}_i = y, o)) \rightarrow (s, (\text{err}, o))
\end{align*}
\]

\[
\begin{align*}
\text{ASSIGN3} & : x \in \text{dom}(s \uplus o) \\
& \quad (s \uplus o)(x) = \neg \text{isfree}_{\text{avf}}(a) \\
& \quad (s', o') = \text{readjust} \left( (s \uplus o)[a.\text{sel}_i \rightarrow (s \uplus o)(y)] \right) \\
& \quad (s, (x.\text{sel}_i = y, o)) \rightarrow (s', (\text{skip}, o'))
\end{align*}
\]
Version counters. To make the above semantics practical for the usage of version counters we have to ensure that threads can access these counters at any time. In particular, it must be possible to access version counters of records owned by other threads. Intuitively, we make version counters shared no matter if the record they are contained in is shared.

Hereafter, let $VSel \subseteq PSel$ be the of selectors which carry version counters and $\overline{VSel} = (PSel \cup PSel) \setminus VSel$ the selectors that do not. Moreover, let $Adr \subseteq \mathbb{N}$ be the addresses that conform to the alignment restrictions required by version counters.

The following adaptions to the above semantics are required.

Free cells. When freeing, the version counters remain valid. We have to adapt $isfree_h(a)$ to account for this.

$$isfree_h(a) : \iff \forall sel \in VSel. a.sel \notin dom(h).$$

Separating heaps. Version counters remain always shared. The set of all possible cells carrying version counters is: $M = \{a.sel \mid a \in Adr \wedge sel \in VSel\}$. Then, we modify the definition of $readjust(h) = (s', o')$ to retain version counters in the shared heap:

$$dom(s') = dom(h) \cap \left( M \cup \bigcup_{a \in S_i} \{a, \ldots, a+n\} \right)$$
$$dom(o') = dom(h) \setminus dom(s').$$

Allocation. The values of version counters persist allocations. If a fresh cell is allocated, then they are initialized to a non-deterministic value. The following replaces (MALLOC):

$$a \in Adr \quad isfree_{x\leftarrow o}(a) \quad (s', o') = readjust(h')$$
$$d_0, \ldots, d_n \in \mathbb{N} \quad \forall sel_i \in VSel. a.sel_i \in dom(s) \implies d_i = s(a.sel_i)$$
$$h' = (s \cup o)[x \rightarrow a, \forall sel_i \in VSel. a.sel_i \rightarrow udef, \forall sel_i \in \overline{PSel}. a.sel_i \rightarrow d_i]$$
$$\quad (s, (x = malloc, o)) \rightarrow (s'', (skip, o'))$$

Deallocation. Tags are not deleted upon deallocation. The following replaces (FREE):

$$x \in dom(s \cup o) \quad \neg isfree_{x\leftarrow o}(s \cup o)(x)$$
$$\quad (s', o') = readjust((s \cup o)[\forall sel \in VSel. a.sel \rightarrow \bot])$$
$$\quad (s, (x = free, o)) \rightarrow (s', (skip, o'))$$

\footnote{For version counters to work one has to prevent arbitrary reuse of cells. In particular, certain cells are designated version counters and may not be used for other pointer or data payload. This requires allocations to be aligned, e.g., by the size of records.}
C Missing Proofs

This section gives detailed proofs for the presented theory. For simplicity we avoid remapping thread identifiers among executions of different programs. Therefore, without loss of generality, we assume during the proofs throughout this section that the following holds:

\[ \text{cf}_{\text{init}, P \parallel Q}^* = \text{cf}_{\text{init}, P} \cup \text{cf}_{\text{init}, Q}^* \]
and
\[ \text{cf}_{\text{init}, T \parallel Q}^* = \{ 0 \mapsto (T, \text{emp}) \} \cup \text{cf}_{\text{init}, Q}^* . \]

C.1 Auxiliaries

Fact 1. Let \( Q \) be a program and \( T \in Q \) one of its threads. Then, for every step \( (s, (T, \text{emp})) \rightarrow (s', (\text{skip}, \text{emp})) \) of \( T \), we have \( (s, \text{cf}_{\text{init}, Q}^*) \rightarrow^* (s', \text{cf}_{\text{init}, Q}^*) \).

Lemma 3. Let \( Q \) be a stateless program and \( T \in Q \) one of its threads. Then, for every \( (s, s') \in \text{EF}(T \parallel Q^*) \), there is some \( T' \in Q \) with \( (s, (T', \text{emp})) \rightarrow (s', (\text{skip}, \text{emp})) \).

Corollary 1. Let \( Q \) be a stateless program. Then, for every \( (s, s') \in \text{EF}(Q^*) \), we have \( (s, \text{cf}_{\text{init}, Q}^*) \rightarrow^* (s', \text{cf}_{\text{init}, Q}^*) \).

Corollary 2. Let \( Q \) be a stateless program and let \( T \in Q \) be one of its threads. Then, \( \text{EF}(T \parallel Q^*) \subseteq \text{EF}(Q^*) \).

Corollary 3. Let \( Q \) be a stateless program. Then, for every \( (s, s') \in \text{EF}(Q^*) \), there is some \( T \in Q \) with \( (s, (T, \text{emp})) \rightarrow (s', (\text{skip}, \text{emp})) \).

Proof (Fact 1). Let \( Q \) be a program and \( T \in Q \) one of its threads. Furthermore, let \( (s, (T, \text{emp})) \rightarrow (s', (\text{skip}, \text{emp})) \) be a valid \( T \)-step. Then, the semantics give:

\[ (s, (T^*, \text{emp})) \rightarrow (s, (T; T^*, \text{emp})) \rightarrow (s', (\text{skip}; T^*, \text{emp})) \rightarrow (s', (T^*, \text{emp})). \]

Now, let \( i \in \text{Tid} \) be such that \( \text{cf}_{\text{init}, Q}^*(i) = (T^*, \text{emp}) \). Such \( i \) must exist because we have \( T \in Q \). Then, using the above sequence of \( T^* \) steps gives a valid sequence:

\[ (s, \text{cf}_{\text{init}, Q}^*) = (s, \text{cf}_{\text{init}, Q}^*[i \mapsto (T^*, \text{emp})]) \rightarrow^* (s', \text{cf}_{\text{init}, Q}^*[i \mapsto (T^*, \text{emp})]) = (s', \text{cf}_{\text{init}, Q}^*). \]

Note that this sequence is valid by the semantics, because at any time the local heaps of all threads are \( \text{emp} \). Hence, concurrent steps are possible. This concludes the claim.

Proof (Lemma 3). Let \( Q \) be a stateless program and \( T \in Q \) one of its threads. We now show that the executions of \( T \parallel Q^* \) are of of a particular form.
**IB:** The empty execution of $T \parallel Q^*$ reaches only its initial state, $(s_{\text{init}}, cf_{\text{init},T \parallel Q^*}).$ The initial state of $Q^*$ is $(s_{\text{init}}, cf_{\text{init},Q^*}).$ Hence, $Q^*$ reaches $s_{\text{init}}$ in zero steps: $(s_{\text{init}}, cf_{\text{init},Q^*}) \xrightarrow{\emptyset} (s_{\text{init}}, cf_{\text{init},Q^*}).$ Moreover, the initial configurations are of the form $(s_{\text{init}}, cf_{\text{init},T \parallel Q^*}) = (T, \emptyset)$ and $(s_{\text{init}}, cf_{\text{init},T \parallel Q^*}(i) = (T_i, \emptyset))$ for every $i \in \text{dom}(cf_{\text{init},Q^*})$ and some thread $T_i \in T.$

**II:** For every $\text{init}_{T \parallel Q^*} \rightarrow^* (s, cf)$ we have

(a) $\text{init}_{Q^*} \rightarrow^* (s, cf_{\text{init},Q^*})$

(b) $cf(0) \in \{T, \emptyset\}, \{(\text{skip}, \emptyset)\}$

(c) $cf(i) \in \{\{(\text{skip}, \emptyset), (\text{skip}, T^*_i, \emptyset), (T, \emptyset), (T_0; T^*_i, \emptyset)\} \text{ for every } i, T_i \text{ with } cf_{\text{init},Q^*}(i) = (T^*_i, \emptyset)}$

**IS:** Consider now $\text{init}_{T \parallel Q^*} \rightarrow^* (s, cf) \rightarrow_{i} (s', cf')$ for some $i.$ The semantics give $cf(j) = cf'(j)$ for all $j \neq i.$ Hence, for proof obligations (b) and (c) it suffices to show that $cf'(i)$ has the desired form. We do a case distinction on $i.$

**Case $i = 0.$** By induction (b) we must have $cf(0) = (T, \emptyset)$ with $T \neq \text{skip}$ as for otherwise the last step would not be possible. The induction hypothesis (a), furthermore, provides $s \in SH(Q^*).$ Then, we have $cf'(0) = (\text{skip}, \emptyset)$ because $T$ is stateless as $Q.$ Hence, $cf'$ has the appropriate form and the step $(s, (T, \emptyset)) \rightarrow (s', (\text{skip}, \emptyset))$ is valid. Now, Fact 1 together with induction hypothesis (a) yields: $\text{init}_{Q^*} \rightarrow^* (s, cf_{\text{init},Q^*}) \rightarrow^* (s', cf_{\text{init},Q^*}).$ This concludes the case.

**Case $i \neq 0.$** We cannot have $cf(i) = (\text{skip}, \emptyset)$ as the last step would not be possible otherwise. For $cf(i) \in \{\{(\text{skip}, T^*_i, \emptyset), (T^*_i, \emptyset)\} \text{ we immediately get } s = s' \text{ and thus } \text{init}_{Q^*} \rightarrow^* (s', cf_{\text{init},Q^*}) \text{ by induction. Moreover, the semantics immediately give the desired form of } cf'. \text{ So it remains to consider } cf(i) = (T_i; T^*_i, \emptyset) \text{ with } T_i \neq \text{skip. Due to } s \in SH(Q^*) \text{ by induction (a) and statelessness of } T_i \in Q \text{ we have } (s, (T_i, \emptyset)) \rightarrow (s', (\text{skip}, \emptyset)). \text{ Hence, we get the desired } cf'(i) = (\text{skip}, T^*_i, \emptyset). \text{ Now, Fact 1 together with induction hypothesis (a) yields: } \text{init}_{Q^*} \rightarrow^* (s, cf_{\text{init},Q^*}) \rightarrow^* (s', cf_{\text{init},Q^*}). \text{ This concludes the case.}

Let $(s, s') \in EF(T \parallel Q^*)$ be some effect. It stems from an execution of the form $\text{init}_{T \parallel Q^*} \rightarrow^* (s, cf) \rightarrow_{i} (s', cf')$ for some $i.$ Analogous to the induction step from above, we conclude that there is some thread $T'$, namely the one with identifier $i$, that can perform the valid step $(s, (T', \emptyset)) \rightarrow (s, (\text{skip}, \emptyset)).$

**Proof (Corollary 1).** Let $Q$ be a stateless program and $(s, s') \in EF(Q^*)$ one of its effects. We invoke Lemma 3 to get some $T \in Q$ with $(s, (T, \emptyset)) \rightarrow (s', (\text{skip}, \emptyset)).$ Then, we conclude applying Fact 1 onto this step.

**Proof (Corollary 2).** Let $Q$ be a stateless program and $T \in Q$ one of its threads. Let $(s, s') \in EF(T \parallel Q^*)$ be some effect. The effect stems from an execution of the form $\text{init}_{T \parallel Q^*} \rightarrow^* (s, cf) \rightarrow (s', cf').$ According to the induction from the Proof of Lemma 3 we have $\text{init}_{Q^*} \rightarrow^* (s, cf_{\text{init},Q^*}).$ Invoking Lemma 3 on the effect $(s, s')$ yields some $T' \in Q$ with $(s, (T', \emptyset)) \rightarrow (s', (\text{skip}, \emptyset)).$ Applying Fact 1 to this step.
step then gives \((s, cf_{init,Q^*}) \rightarrow^* (s', cf_{init,Q^*})\). Altogether, we now have the desired \(init_{Q^*} \rightarrow^* (s, cf_{init,Q^*}) \rightarrow^* (s', cf_{init,Q^*})\). □

**Proof (Corollary 3).** Let \(Q\) be a stateless program and let \((s, s') \in EF(Q^*)\) be one its effects. Now, pick some thread \(T' \in Q\). Then, Lemma 3 applied to \(Q, T'\) and \((s, s')\) gives some \(T\) with \((s, (T, emp)) \rightarrow (s', (\text{skip}, emp))\) as desired. □

### C.2 Proof of Lemma 1

Let \(P, Q\) be two programs such that \(Q\) is stateless and \(EF(T \parallel Q^*) \subseteq EF(Q^*)\) holds for all threads \(T \in P\). We show: \(EF(P) \subseteq EF(P \parallel Q^*) \subseteq EF(Q^*)\).

For the first inclusion, note that the effects of \(P\) are trivially included in the effects of \(P \parallel Q^*\) by the definition of effects; letting \(Q^*\) in \(P \parallel Q^*\) stutter resembles exactly \(P\). Hence, we focus on the second inclusion hereafter.

Towards the desired result, we show that the threads from \(P\) cannot distinguish whether they run in parallel with threads from \(P\) or with \(Q^*\). Formally, we show that, for every execution \(init_P \parallel Q^* \rightarrow^* (s, cf)\) and for every \(k \in dom(cf), T \in P \parallel Q^*\) with \(cf_{init,P \parallel Q^*}(k) = (T, emp)\), there is another execution with the same \(T\)-configuration but the remaining threads simulated by \(Q^*\): \(init_{T \parallel Q^*} \rightarrow^* (s, cf_{init,T \parallel Q^*}[0 \rightarrow cf(k)])\). Note that in the new execution we always let \(Q^*\) come back to its initial configuration to allow further iterations. We proceed by induction over the structure of \(P \parallel Q^*\) execution.

**IB:** The empty execution reaches only the initial program state. For \(P \parallel Q^*\) this is \((s_{\text{init}}, cf_{init,P \parallel Q^*})\). By definition, we have \(cf_{init,P \parallel Q^*}(k) = (T, emp)\) for every \(k, T\). Similarly, we have \(cf_{init,T \parallel Q^*}(k) = (T, emp)\) by definition. Hence, the empty execution of \(T \parallel Q^*\) reaches a state of the desired form.

**IH:** For every \(init_P \parallel Q^* \rightarrow^* (s, cf)\) and every \(k \in dom(cf), T \in P \parallel Q^*\) with \(cf_{init,P \parallel Q^*}(k) = (T, emp)\) there is \(init_{T \parallel Q^*} \rightarrow^* (s, cf_{init,T \parallel Q^*}[0 \rightarrow cf(k)])\).

**IS:** Consider now \(init_P \parallel Q^* \rightarrow^* (s, cf) \rightarrow_i (s', cf')\) for some \(i\). The semantics give \((s, cf(i)) \rightarrow (s', cf'(i))\). Let \(T_i \in P \parallel Q^*\) with \(cf_{init,P \parallel Q^*}(i) = (T_i, emp)\).

Then, the induction hypothesis for \(i\) and \(T_i\) gives:

\[
\begin{align*}
init_{T_i \parallel Q^*} & \rightarrow^* (s, cf_{init,T_i \parallel Q^*}[0 \rightarrow cf(i)]) \\
& \rightarrow_0 (s', cf_{init,T_i \parallel Q^*}[0 \rightarrow cf'(i)]).
\end{align*}
\]

The last step of thread 0 is valid here for the following reason. The state \((s, cf(i))\) is separated as for otherwise its step to \((s', cf'(i))\) would not be possible. Hence, Assumption 1 states that also \((s, cf(i))\) is separated. Since the owned heap of all threads \(j, j \neq i\), are empty, the concurrent result state of the execution is separated.

Now, consider some arbitrary \(k \in dom(cf)\) and some arbitrary \(T_k \in P \parallel Q^*\) with \(cf_{init,P \parallel Q^*}(k) = (T_k, emp)\). In order to conclude the induction, we show that an execution of the desired form exists for \(k, T_k\). For the case \(k = i\) we are done because of the above execution. So consider \(k \neq i\).

By definition the above \(T_i \parallel Q^*\) execution gives \((s, s') \in EF(T_i \parallel Q^*). This yields \((s, s') \in EF(Q^*)\) due to the premise (in case of \(T_i \in P\) and Corollary 2) (in case...
of \( T_i \in Q^* \). Hence, \((s, cf_{\text{init}, Q^*}) \rightarrow^* (s', cf_{\text{init}, Q^*})\) follows from Corollary 1. Together with the induction hypothesis for \( k, T_k \), we get

\[
\text{init}_{T_k} \parallel Q^* \rightarrow^* (s, cf_{\text{init}, T_k} \parallel Q^*[0 \rightarrow cf(k)]) \\
\rightarrow^* (s', cf_{\text{init}, T_k} \parallel Q^*[0 \rightarrow cf(k)])
\]

which is a valid execution because the owned heaps of the intermediate \( Q^* \) states are always emp as shown by Lemma 3 (we skip a formal discussion as it would simply repeat the arguments of that lemma). This concludes the induction because we have \( cf(k) = cf'(k) \) due to \( k \neq i \) by the semantics.

Now, consider an effect \((s, s') \in EF(P \parallel Q^*)\). The effect stems from some execution of the form \( \text{init}_P \parallel Q^* \rightarrow^* (s_1, cf_1) \rightarrow (s'_1, cf'_1) \). By the above induction we know that there is another execution of the form \( \text{init}_{T \parallel Q} \rightarrow^* (s_2, cf_2) \rightarrow (s'_2, cf'_2) \) with \( T \in P \parallel Q^* \). This gives \((s, s') \in EF(T \parallel Q^*)\), as discussed in the induction step, (due to the premise and Corollary 2), and concludes the claim.

\[\square\]

C.3 Proof of Lemma 2

Let \( Q \) be a summary of \( P \). We show the equality in two parts:

(i) for every \( s \in R \) there is some \( T \in P \) such that \( s \in SH(T \parallel Q) \), and

(ii) for every \( T \in P \) and every \( s \in SH(T \parallel P) \) we have \( s \in R \).

Part (i). We show that every shared heap explored by the fixed point there is an execution that reaches this shared heap, too. We proceed by induction over the iterations of the fixed point.

**IB:** Let \((s, cf) \in X_0\). By definition, \( s = s_{\text{init}} \) and \( cf = (T, emp) \) for some \( T \in P \). The initial configuration of \( T \) in \( T \parallel Q^* \) is \( cf_{\text{init}, T \parallel Q^*}(0) = (T, emp) \). Hence, we have reachability in zero steps: \( \text{init}_{T \parallel Q^*} \rightarrow^0 (s, cf_{\text{init}, T \parallel Q^*}[0 \rightarrow cf]) \). Similarly, we get \( \text{init}_{Q^*} \rightarrow^0 (s, cf_{\text{init}, Q^*}) \).

**IH:** For every \((s, cf) \in X_i\) we have \( \text{init}_{Q^*} \rightarrow^* (s, cf_{\text{init}, Q^*}) \) and there is some \( T \in P \) with \( \text{init}_{T \parallel Q^*} \rightarrow^* (s, cf_{\text{init}, T \parallel Q^*}[0 \rightarrow cf]) \).

**IS:** Consider now some \((s', cf') \in X_{i+1}\). By definition we have \((s', cf') \in X_i\) or \((s', cf') \in \text{seq}(X_i)\) or \((s', cf') \in \text{interfere}(X_i)\). In the first case, we conclude by induction. In the second case, \((s', cf') \in \text{seq}(X_i)\), there is some \((s, cf)\) with \((s, cf) \rightarrow (s', cf')\). By Assumption 1 we know that \((s', cf')\) is separated. So by induction we get some \( T \in P \) with:

\[
\text{init}_{T \parallel Q^*} \rightarrow^* (s, cf_{\text{init}, T \parallel Q^*}[0 \rightarrow cf]) \rightarrow_0 (s', cf_{\text{init}, T \parallel Q^*}[0 \rightarrow cf'])
\]

where the last step is valid because separation holds as only the owned heap of \( cf' \) may be non-empty. This execution yields then the effect \((s, s') \in EF(T \parallel Q^*)\). Since \( Q \) is a summary of \( P \) we get \((s, s') \in EF(Q^*)\). Thus, Corollary 1 together with the induction hypothesis gives \( \text{init}_{Q^*} \rightarrow^* (s, cf_{\text{init}, Q^*}) \rightarrow^* (s', cf_{\text{init}, Q^*}) \).
This concludes this case.

Last, consider \((s', cf') \in \text{interfere}(X_k)\). By definition there is some \(s, cf''\) such that \((s, cf') \in X_k\) and \((s, cf_{init,Q}) \rightarrow_k (s', cf'')\). Let \(T_k\) be some thread such that \(cf_{init,Q} = (T_k, emp)\). By induction, \(s \in SH(Q^*)\), thus \(cf''(k) = (\text{skip, emp})\) because \(Q\) is stateless. Then, together with the induction, we get:

\[
\begin{align*}
\text{init}_{Q^*} \rightarrow^* (s, cf_{init,Q^*}) &= (s, cf_{init,Q^*}[k \rightarrow (T_k^*, emp)]) \\
&\rightarrow (s, cf_{init,Q^*}[k \rightarrow (T_k^*; T_k^*, emp)]) \\
&\rightarrow (s', cf_{init,Q^*}[k \rightarrow (\text{skip}; T_k^*, emp)]) \\
&= (s', cf_{init,Q^*})
\end{align*}
\]

Hence, \((s, s') \in EF(Q^*)\). Now, Corollary 1 yields \(cf_{init,Q^*} \rightarrow^* (s', cf_{init,Q^*})\). So, \(init_{Q^*} \rightarrow^* (s', cf_{init,Q^*})\) holds by induction. Similar to the above, we invoke the induction again and get some \(T \in P\) and construct the following execution:

\[
\begin{align*}
\text{init}_{T \parallel Q^*} \rightarrow^* (s, cf_{init,T \parallel Q^*}) &= (s, cf_{init,T \parallel Q^*}[k \rightarrow (T_k^*, emp)]) \\
&\rightarrow (s, cf_{init,T \parallel Q^*}[k \rightarrow (T_k^*; T_k^*, emp)]) \\
&\rightarrow (s', cf_{init,T \parallel Q^*}[k \rightarrow (\text{skip}; T_k^*, emp)]) \\
&\rightarrow (s', cf_{init,T \parallel Q^*}[k \rightarrow (T_k^*, emp)]) \\
&= (s', cf_{init,T \parallel Q^*})
\end{align*}
\]

Note that the above execution is valid because \((s', cf')\) is separated by definition of \(\text{interfere}()\) together with \((s', cf') \in \text{interfere}(X_k)\), and by the fact that only the owned heap of \(cf'\) may be non-empty. This concludes the induction.

Now, consider some \(s \in \mathcal{R}\). By definition there is some \(cf\) such that \((s, cf) \in X_k\). Then, the above induction provides \(s \in SH(T \parallel Q^*)\) for some \(T \in P\).

\(\square\)

**Part (ii).** Let \(T \in P\) be some thread. We show that every reachable shared heap of \(T \parallel Q^*\) is explored by the fixed point. We proceed by induction over the structure of executions.

**IB:** The empty execution reaches only \(s_{init}\), and by definition \((s_{init}, cf_{init,T}) \in X_k\).

**IH:** For every execution \(init_{T \parallel Q^*} \rightarrow^* (s, cf)\) we have \((s, cf_{init,T}) \in X_k\).

**IS:** Consider now \(init_{T \parallel Q^*} \rightarrow^* (s, cf) \rightarrow (s', cf')\). This execution yields the effect \((s, s') \in EF(T \parallel Q^*)\). Since \(Q\) is a summary of \(P\), we can use the effect inclusion to get \((s, s') \in EF(Q^*)\). Then, Corollary 3 yields some \(T' \in Q\) with \((s, (T', emp)) \rightarrow (s', (\text{skip}, emp))\). By definition together with the semantics we get: \((s, cf_{init,Q}) \rightarrow (s', cf'')\) for some \(cf''\). And by induction we have \((s, cf_{init,T}) \in X_k\). Hence, \((s, cf_{init,T}) \in \text{interfere}(X_k) \subseteq X_k\) concludes the induction.

Consider now some \(s \in SH(T \parallel Q^*)\). By definition there is an execution of the form \(init_{T \parallel Q^*} \rightarrow^* (s, cf)\). The above induction now provides \((s, cf_{init,T}) \in X_k\). Hence, the desired \(s \in \mathcal{R}\) holds by definition.  

\(\square\)
C.4 Proof of Theorem 1
Let $P$ be a programs and $Q$ its summary. This provides the effect inclusion: $EF(T \parallel Q^*) \subseteq EF(Q^*)$. Then, by Lemma 1 we get $EF(P \parallel Q^*) \subseteq EF(Q^*)$. Because $EF(P) \subseteq EF(P \parallel Q^*)$ is always true, we have $EF(P) \subseteq EF(Q^*)$. By definition this yields $SH(P) \subseteq SH(Q^*)$ and concludes the first inclusion. For the remaining equality we proceed as follows. First, note that $SH(Q^*) \subseteq \bigcup_{T \in P} SH(T \parallel Q^*)$ is always true. Hence, due to Lemma 2 we have $SH(Q^*) \subseteq R$. Now, again by effect inclusion, we have $SH(T \parallel Q^*) \subseteq SH(Q^*)$. Consequently, we have $\bigcup_{T \in P} SH(T \parallel Q^*) \subseteq \bigcup_{T \in P} SH(Q^*) \subseteq SH(Q^*)$. Hence, Lemma 2 yields $R \subseteq SH(Q^*)$. Altogether, this gives the desired equality: $SH(Q^*) = R$.

C.5 Proof of Theorem 2
We show the individual implications.

(i) If $X_k$ satisfies the checks (CHK-MIMIC) and (CHK-STATELESS), then $Q$ is a summary of $P$.

(ii) If $Q$ is a summary of $P$, then $X_k$ satisfies (CHK-MIMIC) and (CHK-STATELESS).

Part (i). Let $X_k$ be the fixed point from Section 4 and let it satisfy the checks (CHK-MIMIC) and (CHK-STATELESS). First, we show an auxiliary statement, then, we show statelessness, and, last, we show the effect inclusion.

Auxiliary induction. Let $T \in P$ be some program. We show that the reachable states of the program $T \parallel Q^*$ are explored by the fixed point. We proceed by induction over executions. Note that we showed a similar property before. However, we cannot reuse it because it relies on the property we want to show, namely $Q$ being a summary of $P$.

IB: The empty execution reaches only $s_{init}$. By definition, $(s_{init}, cf_{init,T}) \in X_k$.

IH: For every execution $init_T \parallel Q^* \rightarrow^* (s, cf)$ we have $(s, cf(0)) \in X_k$ and moreover $cf(i) \in \{(\text{skip}, emp), (\text{skip}; T^*_i, emp), (T^*_i, emp), (T_i; T^*_i, emp)\}$ for every $i, T_i$ with $cf_{init,Q^*}(i) = (T^*_i, emp)$.

IS: Consider now $init_T \parallel Q^* \rightarrow^* (s, cf) \rightarrow_i (s', cf')$ for some $i$. Due to the semantics we have $cf(j) = cf'(j)$ for all $j \neq i$. Hence, the second proof obligation boils down to showing that $cf'(i)$ has the desired form. First, consider $i = 0$. This immediately satisfies the second proof obligation. Due to the semantics we have $(s, cf(0)) \rightarrow (s', cf'(0))$. Hence, $(s, cf(0)) \in \text{seq}(X_k) \subseteq X_k$ holds by induction. Consider now $i \neq 0$. We do a case distinction on $cf(i)$ according to the induction hypothesis.

Case $cf(i) = (\text{skip}, emp)$. This case cannot apply as it does not allow for the step to $(s', cf')$.

Case $cf(i) = (\text{skip}; T^*_i, emp)$. We get $s = s'$ and $cf'(i) = (T^*_i, emp)$ by the semantics. Hence, $cf(i)$ has the desired form. Moreover, $(s', cf'(0)) \in X_k$ holds by induction.
Consider now some effect \( \text{cf}(i) = (T^*, \text{emp}) \). Similarly to the previous case, we immediately conclude because we have \( s = s' \) and \( \text{cf}'(i) \in \{ (\text{skip}, \text{emp}), (T; T^*, \text{emp}) \} \).

Case \( \text{cf}(i) = (T; T^*, \text{emp}) \) with \( T \neq \text{skip} \). Due to the semantics we get \( (s, (T, \text{emp})) \to (s', \text{cf}'') \) for some \( \text{cf}'' \). Hence, \( (s, \text{cf}_{\text{init}, Q}(i)) \to (s', \text{cf}'') \) by definition. Moreover, the induction provides \( (s, \text{cf}(0)) \in X_k \). So we can invoke [CHK-STATELESS] and get \( \text{cf}'' = (\text{skip}, \text{emp}) \). As a consequence, we get \( \text{cf}'(i) = (\text{skip}; T^*, \text{emp}) \) of the desired form. Moreover, by definition of the fixed point, we have \( (s', \text{cf}(0)) \in \text{interfere}(X_k) \). Note here that \( (s', \text{cf}(0)) \) is separated because \( (s', \text{cf}') \) is separated by the semantics and we have \( \text{cf}(0) = \text{cf}'(0) \). This concludes the induction because of \( \text{interfere}(X_k) \subseteq X_k \).

Statelessness. Consider some thread \( T \in Q \), some reachable heap \( s \in \text{SH}(Q^*) \), and some transition \( (s, \text{cf}_{\text{init}, T}) \to (s', \text{cf}'') \). By definition we have \( s \in \text{SH}(T; Q^*) \) for some \( T' \in P \). So the auxiliary induction yields some \( (s, \text{cf}) \in X_k \). Moreover, we have \( \text{cf}_{\text{init}, Q}(i) = (T, \text{emp}) \) for some \( i \) by definition. Hence, [CHK-STATELESS] yields \( \text{cf}'' = (\text{skip}, \text{emp}) \) since \( \text{cf}_{\text{init}, T} = (T, \text{emp}) \). That is, \( Q \) is stateless by definition.

Effect inclusion. Let \( T \in P \) be some thread. Towards the effect inclusion we show that the reachable shared heaps of \( T \parallel Q^* \) are explored by \( Q^* \) alone. We proceed by induction over executions.

**IB:** The empty execution reaches only \( s_{\text{init}} \). By definition, \( \text{init}_{Q^*} = (s_{\text{init}}, \text{cf}_{\text{init}, Q^*}) \).

**IH:** For every execution \( \text{init}_{T \parallel Q^*} \to^* (s, \text{cf}) \) we have \( \text{init}_{Q^*} \to^* (s, \text{cf}_{\text{init}, Q^*}) \).

**IS:** Consider now \( \text{init}_{T \parallel Q^*} \to^* (s, \text{cf}) \to_i (s', \text{cf}') \) for some \( i \). Consider \( s \neq s' \) as we immediately conclude by induction otherwise. There are two cases.

Case \( i \neq 0 \). Due to the auxiliary induction together with the semantics, \( \text{cf}(i) \) must be of the form \( (T_i; T^*, \text{emp}) \) for some \( T_i \in Q \). So we have a valid step as follows: \( (s, (T_i, \text{emp})) \to (s', \text{cf}'') \) for some \( \text{cf}'' \). As shown above, \( Q \) is stateless; and so is \( T_i \in Q \). Hence, \( \text{cf}'' = (\text{skip}, \text{emp}) \) must hold because \( s \in \text{SH}(Q^*) \) by induction. Now, [Fact 1] yields \( (s, \text{cf}_{\text{init}, Q^*}) \to^* (s', \text{cf}_{\text{init}, Q^*}) \). This concludes the case by induction.

Case \( i = 0 \). By the semantics we have \( (s, \text{cf}(0)) \to (s', \text{cf}'(0)) \). The auxiliary induction gives \( (s, \text{cf}(0)) \in X_k \). Hence, [CHK-MIMIC] gives the step \( (s, \text{cf}_{\text{init}, Q}) \to_k (s', \text{cf}'') \) for some \( k, \text{cf}'' \). By statelessness of \( Q \) and \( s \in \text{SH}(Q^*) \) by induction we must have \( \text{cf}''(k) = (\text{skip}, \text{emp}) \). That is, there is some \( T_k \in Q \) with \( (s, (T_k, \text{emp})) \to (s', (\text{skip}, \text{emp})) \). Now, we invoke [Fact 1] again which yields \( (s, \text{cf}_{\text{init}, Q^*}) \to^* (s', \text{cf}_{\text{init}, Q^*}) \). This concludes the case by induction.

Consider now some effect \( (s, s') \in \text{EF}(T \parallel Q^*) \). This effect stems from an execution of the form \( \text{init}_{T \parallel Q^*} \to^* (s, \text{cf}) \to (s', \text{cf}') \). By the above induction we have \( \text{init}_{Q^*} \to^* (s, \text{cf}_{\text{init}, Q^*}) \). Moreover, with an analogous reasoning as for the induction step we get some thread \( T_k \in Q \) with \( (s, (T', \text{emp})) \to (s', (\text{skip}, \text{emp})) \) and
\[ cf_{\text{init},Q^*}(k) = (T_k^*, \text{emp}) \] Hence, the following is a valid execution:

\[
\begin{align*}
\text{init}_{Q^*} \rightarrow^* (s, cf_{\text{init},Q^*}) &= (s, cf_{\text{init},Q^*}[k \rightarrow (T_k^*, \text{emp})]) \\
&\rightarrow (s, cf_{\text{init},Q^*}[k \rightarrow (T_k^*; T_k^*, \text{emp})]) \\
&\rightarrow (s', cf_{\text{init},Q^*}[k \rightarrow (\text{skip}; T_k^*, \text{emp})]).
\end{align*}
\]

That is, \((s, s') \in \text{EF}(Q^*)\). This establishes the effect inclusion and together with statelessness from above shows that \(Q^*\) is a summary of \(P\).

**Part (ii).** Let \(Q^*\) be a summary of \(P\). First we show that \((\text{CHK-STATELESS})\) holds, afterwards we tackle \((\text{CHK-MIMIC})\).

\((\text{CHK-STATELESS})\) Consider some \(s \in \mathcal{R}\) and some transition \((s, cf_{\text{init},Q^*}) \rightarrow (s', cf')\). By definition, we have \(cf_{\text{init},Q^*} = (T, \text{emp})\) and thus \((s, (T, \text{emp})) \rightarrow (s', cf')\). Moreover, by the induction of the Proof of \(\text{Lemma 2} \), we have \(s \in SH(Q^*)\). Hence, we immediately conclude \(cf' = (\text{skip}, \text{emp})\) because \(Q^*\) is stateless.

\((\text{CHK-MIMIC})\) Consider some \((s, cf) \in X_k\) and some transition \((s, cf) \rightarrow (s', cf')\). By the induction of the Proof of \(\text{Lemma 2} \) Part (i), there is some thread \(T \in P\) with \(cf_{\text{init},T\parallel Q^*} \rightarrow^* (s, cf_{\text{init},T\parallel Q^*}[0 \rightarrow cf])\). So by the semantics, we have

\[
\begin{align*}
&cf_{\text{init},T\parallel Q^*} \rightarrow^* (s, cf_{\text{init},T\parallel Q^*}[0 \rightarrow cf]) \rightarrow (s', cf_{\text{init},T\parallel Q^*}[0 \rightarrow cf'])
\end{align*}
\]

because \((s', cf')\) is separated by \((\text{Assumption 1})\) and thus also the resulting state of the execution, \((s', cf_{\text{init},T\parallel Q^*}[0 \rightarrow cf'])\), is separated. That is, \((s, s') \in \text{EF}(T\parallel Q^*)\) holds by definition. Hence, we get \((s, s') \in \text{EF}(Q^*)\) because of the effect inclusion (which is provided by the fact that \(Q^*\) is a summary of \(P\)). Now, \((\text{Corollary 3})\) yields some \(T' \in Q\) with \((s, (T', \text{emp})) \rightarrow (s', (\text{skip}, \text{emp}))\). Hence, we have \((s, cf_{\text{init},Q}) \rightarrow (s', cf'')\) for some \(cf\). This concludes the \((\text{CHK-MIMIC})\). \(\Box\)