A Usage of Probabilistic Methods in Decision Making for Wholesalers’ Problem

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Abstract. Decision making is an activity which is made in almost all economic and business activities. Specifically in trade, decision making on the supply of goods is made to maximize profit or minimize loss which becomes the problems which need to be studied. A fresh fruit trading system in supermarkets is simplified in a scenario which is aimed to determine the number of fresh fruit stocks each day. By random variable, an opportunity theoretical concept is used to approach the problem of how the amount of fresh fruit sold each day as the market demand in a trade system. Opportunity method and expectation concept were used to cope with the problems with the purpose of minimizing losses.

1. Introduction

Decision making is an activity which has to be made by every person in his life, either for individual needs or for the needs of groups. In general, it can be meant as a process to make one or more choices which are considered the best ones from some alternatives of the choice. Decision making is a systematic approach toward the truth of a problem and the gathering of facts and data. Full consideration of determination of alternatives and taking action with full consideration constitute the most accurate action [1]. The size of a certain problem in making a decision sometimes depends on the size of its risk which will be taken as the result of the final decision. Besides that, the complexity of the process of decision making often depends on the variety of alternatives which have to be taken and on the variety of rules as obstacles which have to be handled in obtaining the final choice to achieve what has been expected.

Systematic approach toward the truth of a certain problem by considering all involved elements to achieve what has been expected in making a decision needs to involve mathematical modelling. Mathematical modelling is a process to represent and explain physical and problematic systems in the real world in the form of mathematical statement in order to obtain simpler understanding and to make problem solving easier. Mathematical modelling, involving phenomena or random (non-deterministic) variable, needs to be approached by using the concept of opportunity theory. Non-deterministic means random; in other words, when a choice is made twice in the same system and situation, the result of the two choices may be similar or dissimilar. In the economic and business world such as in either big or small scale of trade, non-deterministic variable is often involved. In this research, mathematical modeling was carried out to analyze the problem of the number of stocks which had to be prepared by a supermarket which sold fresh fruit in a certain system and situation with the scenario and objective which would be explained in detail in the Chapter about the problem in the trade system and scenario.
Numerous models which had been approached probabilistically were carried out to accomplish the problem of optimization. One of the approaches of models was developed to determine optimal lot size in which each lot which was supplied by suppliers was full of imperfect goods with their probability density function was already known [2]. Meanwhile, $n$ component was combined to establish an integrated object by assuming each component to have opportunity density function and using the combination of opportunity distribution function [3]. Statistic quantity like expectation (mean) and variance played an important role in probabilistic analysis [4] so that the problems would be modelled in the probabilistic model.

2. Theoretical foundation

2.1. Decision theory

Statistics plays a very important role in making any decision. The phenomenon of uncertainty in shooting dice as being questioned by the Roman Emperor, Claudius, in his book, *How to Win in Shooting Dice*, has become the initial point of the development of the theory of opportunity. This theory as the branch of statistics then developed as the basis of the theory of statistical decision. Even though the development of statistical decision theory has significantly been found since the 1950s, but the basis of this science had been existed since the 18th century, pioneered by Thomas Bayes. Therefore, the statistical decision theory was often called Bayesian statistics. Bayesian statistics is a statistics which is focused on the process of decision making is different from classic statistics which is focused on parameter estimation. In general, each decision making will usually face four kinds of situation: certainty, uncertainty, risk, and conflict.

2.2. Probability method

The approach of decision making, using probability method, was done by the existence of uncertainty phenomenon which is often stated in a random variable. The characteristics of a random variable can be investigated by obtaining the distribution function of its opportunity. Generally, if $X$ is a random variable, the characteristics of $X$ random variable, for example the mean value of random variable, can be determined by finding out the distribution function of its opportunity. If random variable $X$ has opportunity distribution function $p(x)$, the mean value of random variable, which is called the expectation value of random variable $X$, is written as $E(X)$. This expectation value is defined by using the formula of $E(X) = \sum x p(x)$. Empirically, distribution function of the opportunity of random variable $X$ can be presented in a Table which is called the Table of random variable of opportunity distribution $X$. The characteristics of random variable $X$ can be obtained by using the table of opportunity variable. Decision making toward the number of fresh fruit stocks each day according to the scenario in the case above can be solved by using probability method.

3. Problems based on trade system and scenario

A trade system and scenario in a supermarket which sells fresh fruit can be described as follows: The supermarket receives supply of fresh fruit in the unit of tons each day to be sold at a price which is, of course, higher than the purchasing price. The assumption on the fresh fruit will be: the fruit which has to be discarded by the time it is received will be thrown away with certain price of removal since it cannot be sold in the next day. Based on this trade system and scenario, it should be determined the number of fresh fruit stocks each day in the unit of tons in order to obtain maximal profit of the supermarket.

4. Discussions

A method of using opportunity concept by using the following steps was proposed to determine the solution of the problems in this trade system and scenario. Discussion on these steps would be presented in the following sub-chapters:

1. Translating trade system and scenario in mathematical model;
2. Determining opportunity distribution X empirically;
3. Applying decision mathematical model

4.1. Decision of Mathematical Model
The trade system and scenario above could be modeled mathematically as followed: It was assumed that X and Y were random variables which consecutively indicated the amount of fresh fruit in the unit of tons that could be sold (demand) each day in a supermarket and the number of its stocks (supply) in the unit of tons each day in the supermarket. Then, it was assumed that C was the price of fresh fruit per ton and C, was the sales price of fresh fruit per ton with C, > C. Meanwhile, the storage of the fresh fruit had high risk – the fresh fruit could not be sold or unsalable by the time it was received so that it had to be thrown away in the next day. In consequence, the supermarket would suffer financial loss in the amount of C per ton, plus the cost of its removal at C, rupiah per ton. The objective of this modeling was to obtain the model of the scenario of selling fresh fruit in the supermarket for determining the number of stocks in order to obtain maximal profit.

The purpose of maximizing profit could be formulated by minimizing loss. There were two types of loss in this problem: obsolescence losses (losses caused by the excessive stock which had to be thrown away in the next day and opportunity losses (losses caused by the lack of stock so that the profit could not be achieved properly). By using these two types of loss, the function of loss on conditional demand on stock, \( K_{X|Y}(x|y) \), could be modeled as follows:

\[
K_{X|Y}(x|y) = \begin{cases} 
(x - y)(c_t - c_0), & \text{if } x \geq y \\
(y - x)(c_0 + c_d), & \text{if } x < y 
\end{cases}, \text{ and } x, y > 0 \tag{1}
\]

As in the work expansion of Jerbi and Kharrat [5] for one dimension, the aim was to provide a formula of conditional expectation like equation (2) by adopting the concept of expectation and the function of conditional opportunity distribution with the result of loss expectation for each \( Y\in\mathbb{R} \) (possible number of stocks). This model could be presented in the formula as follows:

\[
E[K_{X|Y}(x|y)] = \sum k_{x|y} p_{x|y}(x|y), \text{ for all of possible } y \tag{2}
\]

with \( p_{x|y}(x|y) = p_X(x) \). After having been calculated \( E[K_{X|Y}(x|y)] \), for all of Y, the decision made was the value of Y with the minimum of \( \{ E[K_{X|Y}(x|y)] \} \).

4.2. Empirical opportunity distribution model
In this model, it was assumed that X and Y were discrete random variable which is i.i.d with the function of \( p(x) \) opportunity mass. If \( x_i \) indicated the amount of fresh fruit sale in the \( i \) day, \( i = 1,2, ..., n \), the distribution of random variable opportunity X could be empirically tabulated as it was found in Table 1, with \( x_i \) was statistic order to j; namely, the number of demands in the \( j \) order, \( j=1,2, ..., k \), where \( x_i \) was the minimum number of demands per day, \( x_s \) was the maximum number of demands per day, and \( f \) was the frequency, the number of days of sale in the \( j \) order so that \( \sum_{j=1}^{k} f_j = n \). Relative frequency (f) of sale was in the \( j \) variant if random variable opportunity was notated by \( p(x_i) \).
Table 1. Frequency Distribution and Random Variable Opportunity $X$

| $X_{ij}$ | $f_j$ | $f_r = p(x_{ij})$ |
|----------|-------|-----------------|
| $x_{(1)}$ | $f_1$ | $\frac{1}{n}f_1$ |
| $x_{(2)}$ | $f_2$ | $\frac{1}{n}f_2$ |
| ... | : | : |
| $x_{(k)}$ | $f_k$ | $\frac{1}{n}f_k$ |
| Total | $\sum_{i=1}^{k} f_i = n$ | 1 |

4.3. Application of decision mathematical model

The development of equation (1) on the requirement for random variable of realized stocks which was probably indicated by $y_{(1)}$, $y_{(2)}$, ..., $y_{(k)}$, could yield the value of random variable of conditional loss as seen in Table 2 below:

Table 2. Random Variable Modeling of Conditional Loss $K_{|y}$

| Number of Demands, $X$ | Number of Stocks, $Y_{(j)}$ | $y_{(1)}$ | $y_{(2)}$ | ... | $y_{(k)}$ |
|------------------------|-----------------------------|----------|----------|-----|----------|
| $x_{(1)}$              | ($C_b$+$C_d$)($y_{(2)}$- $x_{(1)}$) | ... | ($C_b$+$C_d$)($y_{(k)}$- $x_{(1)}$) |
| $x_{(2)}$              | ($C_b$-$C_s$)($y_{(2)}$- $y_{(1)}$) | 0 | ... | ($C_b$+$C_d$)($y_{(k)}$- $y_{(1)}$) |
| ...                    | :                           | :       | :       | :   |
| $x_{(k)}$              | ($C_b$-$C_s$)($y_{(k)}$- $y_{(k)}$) | ($C_b$-$C_s$)($y_{(k)}$- $y_{(k)}$) | ... | 0 |

The combination of Table 1 with Table 2 would yield Table 3, the table of opportunity distribution of conditional loss on the random variable of stock $Y$. This Table would be used to determine the loss expectation value based on the distribution of conditional demand opportunity on the number of stocks.

Table 3 Loss Opportunity Distribution of Conditional Random Variable

| $X$ | $K_{|y}(1)$ | $K_{|y}(2)$ | ... | $K_{|y}(k)$ | $f = p(x,y)$ |
|-----|-------------|-------------|-----|-------------|--------------|
| $x_{(1)}$ | 0 | ($C_b$+$C_d$)($y_{(2)}$- $x_{(1)}$) | ... | ($C_b$+$C_d$)($y_{(k)}$- $x_{(1)}$) | $\frac{1}{n}f_1$ |
| $x_{(2)}$ | ($C_b$-$C_s$)($y_{(2)}$- $y_{(1)}$) | 0 | ... | ($C_b$+$C_d$)($y_{(k)}$- $y_{(1)}$) | $\frac{1}{n}f_2$ |
| ... | : | : | : | : | : |
| $x_{(k)}$ | ($C_b$-$C_s$)($y_{(k)}$- $y_{(k)}$) | ($C_b$-$C_s$)($y_{(k)}$- $y_{(k)}$) | ... | 0 | $\frac{1}{n}$ |

The conditional loss expectation value of the number of stocks $Y$ could be calculated one by one as follows:

For stock $Y = y_{(1)}$

$$E[K_{|y}(1)|y_{(1)}] = 0 + \frac{1}{n}(C_b-C_s)(x_{(2)}-y_{(1)}) + \ldots + \frac{1}{n}(C_b-C_s)(x_{(k)}-y_{(1)})$$

For stock $Y = y_{(2)}$

$$E[K_{|y}(2)|y_{(2)}] = \frac{1}{n}(C_b-C_s)(y_{(2)}-x_{(1)}) + 0 + \ldots + \frac{1}{n}(C_b-C_s)(x_{(k)}-y_{(2)})$$

For stock $Y = y_{(k)}$
\[ E[K_X|Y(x)|y(2)] = \frac{1}{n}((C_r-C)(y_\omega-x_\omega)+\frac{1}{n}((C_r-C)(x_\omega-y_\omega)+...+0 \]

The decision made was based on the minimum value of loss expectation for the level of stock \(y_\omega, y_\omega\) which could be modeled with

\[ Y= \{ y_{(j)}| \min [E[K_X|Y(x)|y_{(j)}]] j=1, ..., k \} \]

5. Conclusion

This modeling used the function of opportunity distribution \(X\) as random variable of the demand for fresh fruit each day in order to determine conditional loss expectation toward the stock. Conditional loss random variable was modeled according to random variable of demand, \(X\) on condition that random variable of stock \(Y\) was in the trade system and scenario. The combination of conditional loss random variable and opportunity distribution of the number of demands \(X\) was used to determine the mean expectation value of conditional loss of the number of demands. The decision made was based on the minimum value of conditional loss expectation with the formula as follows:

\[ Y= \{ y_{(j)}| \min [E[K_X|Y(x)|y_{(j)}]] j=1, ..., k \} \]

The development of this model could be done by involving more than one variable of demand and stock, and by using different system and scenario for the next research materials.

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