Black Hole Quantum Mechanics in the Presence of Species

Gia Dvali,1,2,3,4,∗ Cesar Gomez,1,5,† and Dieter Lüst1,2,‡

1Arnold Sommerfeld Center for Theoretical Physics
Ludwig-Maximilians-Universität München, 80333 München, Germany
2Max-Planck-Institut für Physik,
Werner-Heisenberg-Institut, 80805 München, Germany
3CERN, Theory Department
1211 Geneva 23, Switzerland
4Center for Cosmology and Particle Physics
Department of Physics, New York University
5Washington Place, New York, NY 10003, USA
6Instituto de Física Teórica UAM-CSIC, C-XVI
Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain

Abstract

Recently within the context of a microscopic quantum theory, the Black Hole’s Quantum N-Portrait, it was shown that continuous global symmetries are compatible with quantum black hole physics. In the present paper we revise within the same framework the semi-classical black hole bound on the number of particle species $N_{\text{species}}$. We show that unlike the bound on global charge, the bound on species survives in the quantum picture and gives rise to a new fundamental length-scale, $L_{\text{species}} = \sqrt{N_{\text{species}}} L_P$, beyond which the resolution of species identities is impossible. This finding nullifies the so-called species problem. This scale sets the size of the lightest quantum black hole in the theory, Planckion. A crucial difference between the gravitational and non-gravitational species emerges. For gravitational species, the lightest black holes are exactly at the scale of perturbative unitarity violation, which is a strong indication for self-UV-completion of gravity. However, non-gravitational species create a gap between the perturbative unitarity scale and the lightest black holes, which must be filled by some unitarity-restoring physics. Thus, self-UV-completion of gravity implies that the number of non-gravitational species must not exceed the gravitational ones.

∗georgi.dvali@cern.ch
†cesar.gomez@uam.es
‡dieter.luest@lmu.de, luest@mppmu.mpg.de
I. INTRODUCTION

Semi-classically black holes are known to exhibit no-hair [1] and exact thermality of Hawking evaporation [2]. Despite the fact that these properties are derived in an idealized semi-classical limit in which black holes are infinitely heavy and infinitely long-leaved, one usually assumes that these are good (up to exponentially small deviations) approximations for real black holes of finite mass and lifetime, as long as they are much heavier than the Planck mass \((M_P)\).

Such extrapolation then leads to highly profound restrictions on the symmetry and the particle content of the theory. The two examples of such restrictions are: 1) The 'folk theorems” that state incompatibility of exact global symmetries with gravity; and 2) The restriction on the number of light particle species, \(N_{\text{species}}\) [3].

Both restrictions, when quantified lead to surprisingly similar constraints in the following sense. Let us denote by \(N_{\text{global}}\) the maximal possible dimensionality of a representation under a global symmetry group allowed in a given theory. Essentially \(N_{\text{global}}\) counts the maximal amount of the measurable global charge in a given theory. For continuous symmetries \(N_{\text{global}}\) is infinite, since a global charge (e.g., \(U(1)\) baryon number) can be increased without a limit by considering the states with arbitrary large occupation number of the charged particles (e.g., baryons).

Let us denote by \(N_{\text{species}}\) the number of particle species in the theory. For definiteness let us assume the species to be massless. Of course \(N_{\text{global}}\) and \(N_{\text{species}}\) are unrelated quantities. One counts the number of species, whereas the other counts a maximal charge, which can be arbitrarily large even if \(N_{\text{species}} = 1\). Despite this difference, in both cases, under the assumption of the validity of semi-classical properties to finite-mass black holes, one can design thought experiments that lead to the emergence of very similar length-scales below which the theory must be modified. For the species case the scale is [3]

\[
L_{\text{species}} = \sqrt{N_{\text{species}}} L_P .
\]

Whereas for global (or discrete gauge) symmetries the scale reads [4]

\[
L_{\text{global}} = \sqrt{N_{\text{global}}} L_P ,
\]

where \(L_P\) is the Planck length that in terms of the Planck mass and Newton’s constant can be expressed as \(L_P \equiv \frac{\hbar}{M_P} \equiv \sqrt{\hbar G_N}\). In the above semi-classical treatment, both scales appear to have universal and fundamental meaning, since these are the shortest scales beyond which black hole no-hair and thermality properties must be fully violated. Thus, both scales are treated as universal cutoff for applicability of Einstein gravity.

Both constraints would have far-reaching consequences. For example, \([2]\) immediately implies impossibility of having exact continuous global symmetries, since in such a case one would have \(N_{\text{global}} = \infty\) which would imply \(L_{\text{global}} = \infty\).

However, it is crucial to remember that in attributing a fundamental meaning to the scales \(L_{\text{global}}\) and \(L_{\text{species}}\), one is based on the very strong assumption that semi-classical properties can be extended to real (quantum) black holes of finite mass. Whether this is a valid assumption can only be answered in a microscopic fully-quantum description.

This question was addressed in [5] within the context of the recently-formulated quantum portrait of black holes [6]. It was shown there that the extension of semi-classical no-hair arguments to real black holes was incorrect, since black holes do carry hair even under
global charges to the order $N_{\text{global}}/N$, where $N$ is the occupation number of gravitons within the black hole, and $N_{\text{global}}$ is the amount of the global charge swallowed by the black hole. As a result, the basic assumptions leading to "folks theorems" about the non-existence of global charges are false. Within the black hole quantum portrait, the scale $L_{\text{global}}$ is fully reproducible, however it has no universal meaning. It simply marks a stage in black hole’s evolution history beyond which a particular black hole that initially swallowed $N_{\text{global}}$ units of a global charge depletes to the point where occupation numbers of gravitons and global charges equilibrates, i.e. $N_{\text{global}} \sim N$. Hence at this state, the global hair becomes fully important. At the same time a different black hole that was endowed by a zero global charge, will continue an almost-thermal depletion all the way till it reaches the size $\sim L_P$.

So far the discussion was constrained to the pure gravity case. However particle species, which are coupled to gravitons, exist in nature. Therefore in the present paper we shall extend the analysis of [5] applying it to the particle species hair. In particular, we will investigate the quantum meaning of the scale $L_{\text{species}}$. We shall discover that unlike $L_{\text{global}}$, the scale $L_{\text{species}}$ has an universal and a fundamental meaning. It is a scale beyond which an arbitrary black hole in a given theory, regardless of its initial composition, ceases to deplete at an approximately-thermal rate. Moreover, in full accordance with [7], $L_{\text{species}}$ is a lower cutoff beyond which the species identities cannot be resolved. As we shall see, the black hole quantum portrait reveals an important information about the underlying quantum physics behind this scale, which is impossible to read-off in the standard semi-classical treatment. Namely, the meaning of species scale is to set the size of lightest quantum black holes existing in the theory.

Our findings have a number of important implications.

First, in full accordance with [8] it nullifies the so-called species problem (for a review of species problem, see [9]). One incarnation of this problem is a seeming inconsistency between the Bekenstein and entanglement [10] entropies in the presence of many particle species. Our result confirms the claim of [8], that this problem is an artifact of not taking into the account the correct cutoff of the theory which is given by $L_{\text{species}}^{-1}$ rather than $M_P$.

Secondly, our findings have direct implication for the concept of self-UV-completion of gravity by black hole physics [11]. As explained in [11] for self-completion of gravity it is essential that the first quantum black holes exist right above the scale of perturbative unitarity violation. In this way, the black hole physics takes over and theory can classicalize exactly above the scale where perturbative unitarity fails.

As argued in [11], in pure Einstein gravity the two scales meet at $M_P$, and the existence of the lightest quantum black holes follows from the existence and the depletion of the heavier ones. In the language of the quantum $N$-portrait this claim becomes explicit in terms of occupation number of gravitons. The lightest quantum black holes are the ones with $N \sim 1$. Their mass is $M_P$, which coincides with perturbative scale of unitarity violation.

How the presence of species changes this balance? As already suggested in [11] the result must be sensitive to the nature of species. If species are gravitational (such as e.g., Kaluza-Klein gravitons) the property of having lightest black holes exactly at the unitarity violation scale does not change. On the other hand non-gravitational species create a gap between the two scales.

Our present analysis within the quantum portrait exactly confirms this picture. We see, that gravitational species, equally affect the depletion as well as self-sustainability properties of the graviton condensate. As a result, the lightest quantum black holes ($N \sim 1$) have the mass $L_{\text{species}}^{-1}$, which as we shall show exactly coincides with the scale of perturbative unitarity.
violation. Thus, in this case, the black hole classicalization regime smoothly merges with the perturbative unitarity regime without any gap.

In contrast, when species are non-gravitational the balance is violated. Non-gravitational species (similarly to gravitational ones) change the depletion law of the condensate, but fail to change the self-sustainability properties. As a result, we get a finite gap of order $\sim \Lambda_{\text{species}}^{3/4}$ between the scale of perturbative unitarity violation and the first quantum black hole.

Generalizing this result for arbitrary mix of gravitational and non-gravitational species, we reach the following powerful conclusion.

*The absence of a gap between the scale of perturbative unitarity violation and the lightest quantum black holes demands that the number of non-gravitational species should not exceed the number on gravitational ones.*

Taking the idea of self-completion seriously, the existence of about 100 non-gravitational species in the Standard model can be taken as indication of at least equal number of gravitational ones below the scale of $10^{17}$GeV.

The paper is organized as follows. In the next section we will generalize the Black Holes’s Quantum N-Portrait including besides the gravitons also other particle species into the black hole bound state. As a result we will see that the species scale is a fundamental, universal length in the quantum picture. We then give a physical meaning to this scale in terms of measurement of species identities. Finally, we discuss the crucial difference between gravitational and non-gravitational species, and the connection with self-UV-completion of gravity. Throughout the paper in the equations we shall ignore the order one numerical factors, which are unimportant for our analysis.

II. SPECIES HAIR

A. Quantum Meaning of the Species Scale

Before proceeding, let us set the stage by briefly formulating main aspects of the black hole quantum theory [6]. For a detailed discussion the reader is referred to the original papers. According to this picture, in pure gravity, i.e. in the absence of other species, the black holes represent a self-sustained bound-states (a Bose condensate) of weakly interacting long-wavelength gravitons. The remarkable thing about this bound-state is that the condition of self-sustainability can be satisfied for an arbitrary occupation number $N$ and fixes the characteristic wavelength of gravitons as $R = \sqrt{NL_P}$. All the other characteristics of the system, such as the total mass ($M$) and the coupling of individual gravitons ($\alpha$) are uniquely determined by $N$ as $\alpha = 1/N$ and total mass $M = \sqrt{N \frac{\hbar}{L_P}}$.\footnote{Notice, that these are the typical large-$N$ relations in ’t Hooft’s sense [12].} In this picture, we operate exclusively by quantum mechanical notions, such as quantum coupling, wave-length and occupation number. All the geometric and thermodynamical characteristics, such as horizon and temperature, are emergent as a result of semi-classical limit which corresponds
to double scaling limit

\[ N \to \infty, \quad L_P \to 0, \quad R \equiv \sqrt{N}L_P = \text{finite}, \quad \hbar = \text{finite}. \quad (3) \]

The notions of the geometric Schwarzschild radius \( R \) and of the temperature \( T = \hbar/R \) only emerge in this limit. Thus, the quantity \( R \), which quantum-mechanically is a characteristic wave-length of the condensate, acquires a geometric meaning only in the semi-classical limit.

Not surprisingly many of the intrinsically-quantum properties (such as black hair) become lost in this limit, unless one carefully monitors their scaling also. The above mentioned "folk theorems" about violation of global charges are precisely an artifact of not taking into the account the existence and scaling of the black hole hair [5].

Let us now assume that besides the graviton, the theory includes other massless species with total number of species being \( N_{\text{species}} \). Let us see how the existence of the extra species in the theory spectrum affects the depletion law. We start with a self-sustained condensate that is initially composed only out of \( N \) gravitons. In the absence of other species, the leading contribution to the depletion rate is coming from the process in which two graviton re-scatter in such a way that one of them gains the above-escape energy and leaks out of the condensate. The corresponding Feynman diagram is depicted in Fig.1

![Feynman diagram](image)

**FIG. 1:** A leading order process responsible for quantum depletion of graviton condensate.

The rate of this process goes as,

\[ \Gamma_{\text{leakage}} = \frac{1}{\sqrt{NL_P}} + L_P^{-1} \mathcal{O}(N^{-3/2}), \quad (4) \]

which gives the following depletion equation,

\[ \dot{N} = -\frac{1}{\sqrt{NL_P}} + L_P^{-1} \mathcal{O}(N^{-3/2}). \quad (5) \]

Notice that the quantity, which in the semiclassical limit acquires the meaning of Hawking temperature, is

\[ T \equiv \frac{\hbar}{\sqrt{NL_P}}. \quad (6) \]

In terms of this parameter the depletion equation can be written as

\[ \dot{N} = -\frac{T}{\hbar}. \quad (7) \]
Rewriting $N$ in terms of the black hole mass the depletion equation becomes a Stefan-Boltzmann law for a black hole,

$$\dot{M} = -\frac{T^2}{\hbar}.$$  \hfill (8)

Thus, the depletion imitates an approximately-thermal evaporation all the way until $N$ becomes order one, or equivalently, until the characteristic wave-length of the graviton condensate becomes of order $L_P$.

Let us now see how the presence of extra species changes this situation. In the presence of additional species, the contribution to the rate is enhanced by the processes, in which two gravitons now can annihilate also into the pair of two species. The corresponding diagram is shown in Fig. 2. As a result the depletion rate is now enhanced by factor of $N_{\text{species}}$.

![FIG. 2: A leading order process responsible for quantum depletion of graviton condensate into the species (represented by the red line).](image)

$$\Gamma_{\text{leakage}} = \frac{N_{\text{species}}}{\sqrt{N}L_P} + L_P^{-1} \mathcal{O}(N^{-3/2}),$$  \hfill (9)

which gives the following depletion equation,

$$\dot{N} = -\frac{N_{\text{species}}}{\sqrt{N}L_P} + L_P^{-1} \mathcal{O}(N^{-3/2}).$$  \hfill (10)

Notice, that the quantum precursor of temperature is still given by the quantity (6). Taking the time-derivative of this equation and using (10), we arrive at the following relation,

$$\frac{\dot{T}}{T} = \frac{N_{\text{species}}}{N} T.$$  \hfill (11)

This equation tells us that as soon as the number of gravitons in the condensate depletes to become comparable to $N_{\text{species}}$, the quantity $T$ cannot even approximately be interpreted as a temperature, since the rate of its change exceeds its own value. The corresponding wavelength of the graviton condensate is given by,

$$L_{\text{species}} = \sqrt{N_{\text{species}}} L_P.$$  \hfill (12)
Thus, we uncovered the quantum meaning of the species scale as the wave-length of the self-sustained condensate, for which \( N \sim N_{\text{species}} \). This scale is an universal characteristics of a given theory. Any black hole, irrespective of its initial composition, will lose even an approximate thermality ones its size will reach \( L_{\text{species}} \).

We thus see that the scale \( L_{\text{species}} \) maintains its universal meaning even in the quantum picture. It is also evident that this scale must have some fundamental meaning in the sense that the black holes must undergo a qualitative change in their nature beyond this scale. This is obvious from the fact that it makes no sense to talk about depleting self-sustained bound-state of gravitons beyond this point, because the depletion time is much shorter than the wave-length.

Moreover, let us show that we can never reach purely gravitational bound-states of sizes smaller than \( L_{\text{species}} \), even if we start out with a large black hole composed exclusively out of gravitons. In order to see this let us follow the evolution of a black hole which is initially composed only out of gravitons. The leading process of the depletion converts the black hole made out of \( N \) gravitons into the one with \( N - 2 \) gravitons and one new species. So the black hole gets endowed with species hair. The probability of getting rid of this hair is suppressed by a factor \( 1/(N - 2) \) relative to the probability of decreasing the number of gravitons. So after every step the occupation number of species in the black hole increases with the chance \( N_{\text{species}} \) to one. This process shall continue until the number of other species and gravitons in the black hole equilibrates. Moreover, the numbers of species and their anti-particles must be roughly the same. After this point the evolution of the condensate depends on the details of interaction between the species. But, in any case the departure from thermality for such a condensate is maximal. The study of its dynamics is a very interesting problem per se, but beyond the scope of the present paper. The lesson we would like to draw from this analysis is that it is highly unprobable that a purely graviton self-sustained bound-state can survive going through the scale \( L_{\text{species}} \). Thus the quantum portrait gives a simple microscopic rational for the universal nature of the species scale \( L_{\text{species}} \).

\section*{B. Smallest Pixels of Nature}

Finally, let us discuss another fundamental meaning of the species scale. As suggested in \cite{7} within the semi-classical treatment, the species scale represents a lower bound on the scale down to which the species identities can be resolved. Let us see how this argument goes through the quantum portrait.

The semi-classical argument of \cite{7} is structured as follows. Imagine that we would like to build a smallest pixel that could resolve the species identities in shortest possible time (that is, on the time-scale of its size). Let the size of the pixel be \( L_{\text{pixel}} \). The necessary condition that such a pixel must satisfy is to have localized sample of at least one member from each species (or an equivalent information). Thus, we have at least \( N_{\text{species}} \) particles localized within the pixel. The process of resolving the species identities is then a scattering process of an unknown particle at the pixel (see Fig. 3). This requirement gives the pixel a minimal mass,

\[
M_{\text{pixel}} = N_{\text{species}} \frac{\hbar}{L_{\text{pixel}}}.
\]  

(13)

Then any attempt of decreasing the size of the pixel below \( L_{\text{species}} \) would fail since it would result into the formation of a black hole of Schwarzschild radius exceeding \( L_{\text{species}} \). We thus arrive to the lower cutoff being \( L_{\text{species}} \).
Let us now reinterpret this reasoning in our quantum language. Quantum mechanically the pixel is a bound-state consisting of at least $N_{\text{species}}$ particles. In reality, one needs an extra interaction to localize the species wave-functions within $L_{\text{pixel}}$, which will increase the energy of the pixel beyond Eq. (13), however this only works in favor of our bound, since it increases the chances of black hole formation. Now, according to the quantum portrait [6], the energy of localized species necessarily results into the occupation number of gravitons, given by $N = N_{\text{species}}^2 \frac{L_P^2}{L_{\text{pixel}}^2}$, which is convenient to rewrite as

$$N = N_{\text{species}} \frac{L_{\text{species}}^2}{L_{\text{pixel}}^2}. \quad (14)$$

The latter form is very instructive, since it indicates that as soon as we try to decrease $L_{\text{pixel}}$ below $L_{\text{species}}$ the occupation number of gravitons will exceed the number of species, and moreover the gravitons will form a self-sustained bound-state, a black hole with the occupation number of other species being a small fraction of the number of gravitons, $N_{\text{species}} \gg N$. The wavelength of the resulting graviton bound-state (a size of a formed black hole) will be

$$R = \sqrt{NL_P} \gg L_{\text{species}}. \quad (15)$$

Of course, this black hole carries a detectable species hair, but the probability of extracting it is $1/N$ suppressed per species and thus takes much longer than the time $L_{\text{species}}$. Namely, to detect a hair of each individual species takes the time [5]

$$t_{\text{hair}} = N^{3/2}L_P \gg L_{\text{species}}. \quad (16)$$

Thus, we see why the scale $L_{\text{species}}$ is indeed a lower bound on the resolution capacity of the species identities.

The above reasoning shows that the scale $L_{\text{species}}$ is fundamental even in quantum-mechanical sense. In particular, all the virtual processes in which species counts differently must be cutoff at this scale. Thus, in a theory with species the cutoff scale is $L_{\text{species}}$ and not $L_P$. Necessity of such a cutoff immediately explains why there is no problem of in counting of the entanglement entropy in semi-classical limit [8].

III. NATURE OF SPECIES

So far we were not interested in the nature of species. We shall now become more specific and try to distinguish between the gravitational and non-gravitational species. As we shall
see, the behavior of smallest black holes is dramatically different in the two cases. In case of gravitational species the mass of the lightest black holes ("Planckions") is right at the scale at which the perturbative violation of unitarity takes place. As it was stressed in [11], this property is crucial for the self-completeness of theory, since the unitarity is restored by quantum black holes.

In contrast, in case of non-gravitational species there is a mass gap between the scale of perturbative unitarity-violation and the first available quantum black holes. UV-completion of theory demands that this gap be filled up by some non-black hole physics.

Let us now highlight this fundamental difference in more details.

**A. Gravitational Species**

We shall define *gravitational species* as the ones that just like graviton are sourced by the energy momentum of the existing particles in the theory. Essentially, this set includes all the elementary spin-2 species that may exist in the theory. Of course, by general covariance all such states except one (the "true" graviton) must be massive, and their number will obey a certain distribution. We shall parameterize this distribution by denoting $N_{\text{species}}(m)$ the number of all species at and below a mass level $m$. A familiar example of such a distribution is provided by Kaluza-Klein theories, although we shall keep our discussion independent of geometric considerations.

The crucial property of gravitational species is that they not only affect the depletion law, but also the self-sustainability condition of the condensate. In order to make this clear, we shall work in a particular example of extreme democratic sourcing, in which all the species universally source each other with the same strength as the massless graviton, $\alpha \equiv L_p^2/\lambda^2$, where $\lambda$ is a characteristic wave-length in the process. Of course, in concrete cases the strength and the type of graviton couplings can (and will) obey certain selection rules, as it happens for example in Kaluza-Klein theories, however for our purposes this idealized approximation is good enough as it maximally sharpens the point.

In such a situation for the gravitons in a self-sustained bound-state of wave-length $\lambda$ the effective gravitational coupling becomes

$$\alpha_{\text{eff}} = N_{\text{species}}(\lambda) \alpha.$$  \hfill (17)

This fixes the self-sustainability relation between the wave-length and the occupation number $N$ as,

$$\lambda = \sqrt{NN_{\text{species}}} L_p = \sqrt{NL_{\text{species}}} ,$$  \hfill (18)

where the $L_{\text{species}} = L_p\sqrt{N_{\text{species}}(\lambda)}$ has to be understood as "running" species scale due to the wave-length dependence of the number of species involved.

The depletion rate in such a case is

$$\Gamma = \lambda^{-1} (L_p^2/\lambda^2)^2 N^2 N_{\text{species}}^2 ,$$  \hfill (19)

where the $N_{\text{species}}^2$ factor comes from the fact that, due to democratic inter-sourcing, each constituent pair can annihilate into $N_{\text{species}}^2$ pairs, as it is depicted on Fig. 4.

Taking into the account (18), the above rate can be rewritten in a very simple form,

$$\Gamma = \frac{1}{\sqrt{NL_{\text{species}}}} .$$  \hfill (20)
Comparing with (4) we see that depletion law is the same as for a single species, but with $L_P$ being replaced by $L_{\text{species}}$.

This is an extremely important fact, which tells us that for gravitational species the depletion law remains approximately thermal all the way till $N \sim 1$. Notice, that the state $N \sim 1$, is a lowest mass member of the black hole tower.

The mass and the characteristic wavelength of such a lightest black hole is,

$$M_{\text{min}} = L_{\text{species}}^{-1}, \quad \lambda_{\text{min}} = L_{\text{species}}.$$

Notice that this coincides with the scale of perturbative unitarity violation in the theory. Indeed, consider a tree-level $2 \to 2$ scattering process depicted in Fig. 5, in which a two initial particles annihilate into any of the two species.

At center of mass energy $E$, the rate of the process goes as

$$\Gamma_{2\to2} = E \frac{E^4}{M_P^4} N_{\text{species}}^2 = E (EL_{\text{species}})^4,$$

where the $N_{\text{species}}^2$ factor appears due to summation over both internal (green) and external (red) lines. Obviously, this process violates perturbative unitarity at the energy

$$M_{\text{unitary}} = L_{\text{species}}^{-1}$$
Comparing with (21) we see that the quantum black holes enter into the game right above the unitarity violating scale $L_{\text{species}}$. The picture is essentially the same as for single species, with the only difference that everything happens at a lower energy scale, with $L_{\text{species}}$ which takes up the role of $L_P$. Thus, the necessary condition for self-completion \[11\] is satisfied. The quantum black holes are readily available in the spectrum right above the unitarity scale!

\section*{B. Non-Gravitational Species}

It is obvious that situation is very different for non-gravitational species. Such species affect the depletion law, but at the same time do not make binding force stronger. As a result, the black hole goes out of thermal regime while still being a multi-particle state, much heavier than the perturbative-unitarity violation scale. Indeed, since non-gravitational species only affect the depletion law and not the self-sustainability condition of the graviton condensate. \footnote{This should not to be confused with the perturbative loop contribution to the Newton’s constant. This contribution exists for both gravitational and non-gravitational species and it is simply summed up in fixing the effective Newton’s constant once and for all.} The mass of the smallest black hole in such a case is $M_{\text{min}} = L_{\text{species}} M_P^2 = \sqrt{N_{\text{species}}} M_P$. Notice that this exceeds the perturbative unitarity violating scale, which for non-gravitational species is at least as low as,

$$M_{\text{unitary}} = M_P / N_{\text{species}}^{1/4}.$$  \hfill (24)

This unitarity bound comes from a $2 \rightarrow 2$ scattering process between the species via graviton exchange, depicted in Fig.6. The rate of this process goes similar to eq.(22), except there is one less power of $N_{\text{species}}$ due to the fact that there is only one intermediate graviton exchange.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig6.png}
\caption{A process giving a perturbative unitarity bound in which two initial particles are scattered into two final species represented by red lines.}
\end{figure}

So the question is what fills the unitarity gap between this scale and the smallest black holes? One way or another, we need some non-black hole physics for curing the unitarity problem in the interval of energies

$$M_P / N_{\text{species}}^{1/4} < E < \sqrt{N_{\text{species}}} M_P.$$  \hfill (25)
This situation is schematically depicted in Fig 7. The red lines represented regions of perturbative unitarity, whereas the green ones the regions in which unitarity is restored by quantum black holes. The dashed region in the left is a non-unitarity window in case of non-gravitational species.

FIG. 7: A schematic confrontation between the cases of gravitational (left) versus non-gravitational (right) species. The red and green lines represent the pertubative unitarity and black hole regions respectively. The dashed black region on the right represents a non-unitarity window in case of non-gravitational species.

Our analysis can be easily generalized to a mixed situation when species consist of both gravitational and non-gravitational ones. The particular details of course change, but the following conclusion persists. Whenever number of non-gravitational species is much larger than the gravitational ones, there is gap between the unitarity scale and the masses of the smallest black holes.

This suggest the following surprisingly powerful conclusion:

Self-UV-completion of gravity demands that the number of non-gravitational species be less or equal than the gravitational ones (normalized per strength of Einstein’s graviton).

C. Discussions and Outlook

In this paper we have reconsidered the semi-classical black hole bound on the number of particle species \[ \frac{1}{2} \] within the microscopic quantum portrait and have shown that the bound persists in full quantum picture.

Introduction of \( N_{\text{species}} \) in the theory creates a new fundamental length scale, \( L_{\text{species}} \), which is an ultimate lowest cutoff on the black hole size. Within quantum portrait this scale acquires a very clear physical meaning, as the characteristic black hole size (or to be more precise, the characteristic wave-length of the black hole constituents) beyond which the depletion process not even approximately can mimic the Hawking thermal spectrum. Essentially, no black holes of the smaller size exist in a given theory.

Furthermore, we have shown that physics of the cutoff is highly sensitive to the nature of species. For gravitational species, the scale \( L_{\text{species}}^{-1} \) simultaneously marks the upper bound on the scale of perturbative unitarity, as well as the lowest bound on the black hole
mass. This is a strong indication in favor of self-UV-completion of gravity by classicalization through quantum black holes. Indeed, the quantum black holes that are crucial for unitarizing scattering amplitudes are readily available right at the scale of perturbative unitarity violation.

Situation is dramatically different for non-gravitational species (such as e.g., extra neutrino species). In this case there is a mass gap between the mass of the lightest black holes and the scale of perturbative unitarity. This gap, unless filled by some yet unknown physics, indicates an intrinsic inconsistency of the theory.

We are thus, lead to the conclusion that in any consistent theory of gravity the number of gravitational species must be the dominant one. All the known examples of deformation of Einstein gravity, such as Kaluza-Klein or perturbative string theory, satisfy this property.

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