STRUCTURAL BEHAVIOUR ON SLOPE NUMBER OF JAHANGIR GRAPHS

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Abstract. The slope number is defined to be the minimum number of slopes required to draw the graph. In the present paper, investigation on slope number of Jahangir graph is focused and studied elaborately, since the defined graph has an excellent application on transmitting confidential information between nuclear sites. In this paper, slope number of Jahangir graph J_{2,m} is determined and studied elaborately. This is an optimization problem and is NP-hard to determine for any arbitrary graph.

1. Introduction

The slope number problem was first introduced by Wade and Chu in 1994. They proved results for complete graph of k_n and executed an algorithm [1]. Much results on characterization of slopes for regular polygons was investigated by Jamison [2]. Our study is motivated by the various results on slope number which minimizes the number of slopes. Motivation comes from the fact that the investigation on the slope parameter for several graphs were dicussed by Ambrus et.al [3]. Any cubic graph can be drawn in the plane using four basic slopes \( \left\{ 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{-\pi}{4} \right\} \) [4]. The slope number is atleast \( \left\lceil \frac{d}{2} \right\rceil \) only when the graph has a vertex of degree d. With its rich history many have turned their attention towards the study of slope number problem. This problem is often helpful to conceptualize required number of slopes in graph drawing. In order to obtain the slope number the vertices are placed in a regular polygon. Motivated by this fact, we would like to concentrate on Jahangir graphs because of its nice structural outlook. In the present paper, slope number of jahangir graphs is investigated and studied in a descriptive manner. Interesting results are obtained which exhibits the results on characterization of the slopes of defined graph.

2. Preliminaries
Let $G=(V,E)$ be a graph with no isolated vertex. A sub-graph $H$ of $G$ is a graph consists of $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. A graph $G$ is said to be a connected graph, if every pair of vertices of $G$ has a path from one vertex to another. Consider the graph which are finite, connected and undirected without loops and multiple edges. In a straight line drawing, the vertices and edges are represented as points and straight line segments respectively.

3. Slope Number

The minimum number of distinct edge slopes required to draw the graph $G$ is called the slope number. It is written as $sl[G]$. Figure 1 is the illustration of the slope number.

![Figure 1. slope number of $G$ is 4](image)

The following lemmas and theorems contributes the fundamental concept of slope number.

3.1 Theorem: $sl[k_n] = \begin{cases} 
0, & \text{if } n = 1 \\
1, & \text{if } n = 2 \\
n & \text{if } n \geq 3 
\end{cases} \quad [1]

4. Jahangir Graph

Jahangir graphs $J_{n,m}$ for $n \geq 2$, $m \geq 2$, is a graph on $nm + 1$ vertices consisting of a cycle $C_{nm}$ with one additional vertex which is adjacent to $m$ vertices of $C_{nm}$ at distance $n$ to each other on $C_{nm}$. Figure 2 shows Jahangir graph $J_{2,8}$. [5]
The main results of Jahangir graphs are as follows. Here, we consider the maximum degree of G to obtain the result.

4.1 Theorem

Let \( J_{2, m} \) be a jahangir graph. Then \( sl[J_{2, m}] = \begin{cases} \frac{m}{2}, & \text{if } m \text{ is even} \\ m, & \text{if } m \text{ is odd} \end{cases} \)

Proof:

Let G be a jahangir graph. To prove slope number of \( J_{2, m} = m/2 \)

Case (i) If \( m \) is even, 
Since \( m \) is even, \( nm \) is even and hence \( C_{nm} \) is an even cycle. Choose the central vertex which is adjacent to \( m \) vertices of \( C_{nm} \). Since the maximum degree of the central vertex is \( m+1 \), the opposite edges of maximum degree shares the same slope. i.e each slope of drawing appears exactly twice among the edges that are incident to maximum degree.

Therefore, the slope number of \( J_{2, m} \) is \( m/2 \).

Case (ii) If \( m \) is odd
Since \( m \) is odd, \( nm \) is even and hence \( C_{nm} \) is an even cycle. Choose the central vertex which is adjacent to \( m \) vertices of \( C_{nm} \). The maximum degree of the central vertex is \( m+1 \) but the edges of maximum degree does not share the same slope. Since the maximum degree of G have different slopes. Here all the edges of \( m \) shares distinct slopes. i.e each slope of drawing appears exactly once among the edges that are incident to maximum degree.

Therefore, the slope number of \( J_{2, m} \) is \( m \).

Hence the proof.

4.2 Theorem
Let G be a jahangir graph. Then \( \text{sl}[J_{n,m}] = \frac{m+n}{4} \) for \( m = n \), if \( m \) and \( n \) are even.

Proof:
Let G be a jahangir graph.
Assume that if \( m \neq n \).

Case (i) \( m \) is even, \( n \) is odd such that \( m < n \)

The edges incident with the maximum degree shares same slope. Also the edges of cycle \( C_{nm} \) will be parallel to atleast one pair of opposite edges of central vertex. Hence \( m < n \)

Therefore, \( \text{sl}[J_{n,m}] = \frac{m+n}{4} \) for \( m = n \)

Case (ii) \( n \) is even, \( m \) is odd such that \( m > n \)

The edges incident with the maximum degree shares different slope. But the edges of cycle \( C_{nm} \) be parallel to the edges of incident with central vertex. Also the edges of cycle \( C_{nm} \) will be parallel to atleast one pair of opposite edges of central vertex. Hence \( m > n \)

Therefore, \( \text{sl}[J_{n,m}] = \frac{m+n}{4} \) for \( m = n \)

4.3 Theorem

Let G be a jahangir graph. Then \( \text{sl}[J_{n,m}] = m + n \), if and only if \( m = n \).

References

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