Meandering motions within the viscous sublayer
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Abstract
Impact of viscous sublayer scale roughness elements on large scale flows have not been fully understood and require high resolution 3D flow measurements to unravel. However, existing approaches fail to provide sufficient resolution for such measurements to fully resolve the sublayer. In this study, we use digital Fresnel reflection holography to capture 3D flows within the viscous sublayer at sub-viscous resolutions. The measurement highlights the presence of novel flow structures at the scale of the sublayer, with strong spanwise meandering motions, of 2-3 viscous wall units, indicating a highly unsteady and accelerating flow within. The probability distribution of accelerations shows a stretched exponential shapes characteristic of highly intermittent turbulence seen under isotropic flows. The presence of flow structures even at the scale of the sublayer, i.e., below $y^+\sim5$, points to the effectiveness of roughness elements in modulating the large scale flow.

Introduction
The viscous sublayer is the region of flow close to the wall where viscous forces dominate and presence of roughness at such scales are considered to be hydrodynamically smooth, having no impact on the flow [1]. However, some recent studies have highlighted the influence of sublayer scale roughness, both in decreasing skin friction [2] and reducing the size of the recirculation bubble [3]. To understand the underlying mechanism of their performance requires direct measurements of the flows within the sublayer, which typically span from millimeter to micrometer scales, at high resolution in 3D. Existing state-of-art measurement techniques including hot-film and laser doppler anemometry, though capable of performing high resolution measurement of turbulent flows, are limited in their accuracy when used for near wall flows [4,5]. Specifically, these techniques spatially average measurements over their sample volumes, defined by the size of probe, which are typically on the order of the viscous length scale (i.e., viscous wall unit $\delta_v$) or larger [6]. Likewise, the use of particle image based techniques including Tomographic PIV and Lagrangian particle tracking, routinely used to perform measurements within the buffer and logarithmic layers located further away from the wall [7,8], suffer from limited spatial resolutions to resolve the sublayer [9]. Even high resolution 3D flow measurements performed using Holographic PTV, with localized seeding is unable to obtain resolutions below 1 $\delta_v$ [10,11] limiting their ability to fully describe the small scale features. Thus, there exists need to perform high resolution 3D flow measurements within the sublayer, with aims to unravel the mechanism of large scale flow modifications introduced by sublayer scale roughness.

In this study, we employ Digital Fresnel Reflection Holography (DFRH), which records holograms by interfering backscattered light from tracers with the Fresnel reflection at the wall
acting as the reference to perform flow measurements within the viscous sublayer [12]. The experiments are performed in a turbulent channel at Reτ~400, the complete details for which are included in the supplementary materials. In addition, a comparison of flow statistics to a direct numerical simulation performed at identical conditions are also included, the details of which are also provided in the supplementary materials.

**Particle trajectories within the viscous sublayer**

The reconstructed particle trajectories (Fig. 1a) span a sample volume of 20×20×12δ, next to the wall, with particle motions even up to 0.1δ resolved by the measurement. Upon closer inspection, we can identify specific types of trajectories indicating the presence of flow structures at the scale of the viscous sublayer, which are highlighted in Figure 1. The instantaneous trajectories indicate intermittent high (Fig. 1b) and low (Fig. 1c) speed motions which resemble the structure of high and low speed streaks observed within the buffer layer [13,14]. The transition from high to low speeds are initiated by strong spanwise motions at scales larger than the measurement volume. Although we are not able to capture the full extent of these structures due to limited sample size, the streamwise length of the high speed structures can be estimated by the product of the mean convective velocity to the duration of transit, to be ~88δ, which falls close to the values reported in literature of a 100δ [13].

**Figure 1.** (a) Plot of particle trajectories captured by DFRH near the wall, with sample volume spanning 20×20×12 wall units. Individual trajectories sampled from below $y^+$~ 5 illustrating (b) high and (c) low speed motions similar to near wall high and low speed streaks in the buffer layer, (d) the wall normal motions and (e) meandering motions with spanwise motions in the scale of 2-3 wall units.

Despite the existence of strong spanwise variation of flows seen above, the scale of wall normal motions is much weaker within the sublayer, typically below ~1δ over the entire field of view (Fig. 1d). The angle subtended by the tracks are less than 3°, a value much smaller than standard wall normal inclination for hairpin type structures (~17-20°) within the buffer and
logarithmic layers [15–17]. Such weak wall normal motions also agree with the asymptotic analysis, with wall normal velocity fluctuations varying as $(y^+)^4$ while the corresponding streamwise and spanwise values vary as $(y^+)^2$ as $y^+ \to 0$, resulting in a of quasi-two dimensional flow near the wall [18].

Finally, our measurements also capture the presence of strong meandering motions, with spanwise scales on the order of ~2-3δν as illustrated in Figure 1e. Such motions will not be resolved by state-of-art DNS simulations, which often limit grid resolutions in the streamwise and spanwise directions to be above ~5δν [19]. Although the presence of such meandering “sinuous” motions were reported by qualitative observations of Fage & Townend [20] very close to the wall ($r_R > 0.994$), no other study has since reported quantitative measurements of these motions within the sublayer. The presence of curvilinear tracks deep within the sublayer points to a highly unsteady flow undergoing strong accelerations, both in the streamwise and spanwise directions. Furthermore, they also indicate a strong coupling between streamwise and spanwise motions, with the instantaneous flow direction alternating between the two, but with no net spanwise flow. The existing measurements of near wall flow have found overwhelming evidence of streamwise wall normal flow coupling in the form of burst and sweep events [13], which do not appear to be the dominant structure within the viscous sublayer.

**Mean velocity statistics**

![Figure 2](image)

**Figure 2.** The mean (a) streamwise and (b) spanwise velocity profiles compared to the DNS prediction (solid line) at the same $Re_\tau$ of 400. The measured wall shear stress PDFs along the (c) streamwise and (d) spanwise directions are presented along with the corresponding DNS predictions (solid line).

The impact of these small scale structures within the sublayer, on the turbulence statistics are next investigated, by comparing the mean velocity profiles and the wall shear stress PDFs in the streamwise and spanwise directions to the corresponding estimates from DNS (which do not resolve them). The mean streamwise velocity (Fig. 2a) indicates a linear wall normal variation, matching both the DNS predictions as well as the hot-film [21–23] and laser doppler based measurements [24,25] reported in literature. On the other hand, the spanwise velocity (Fig. 2b) shows a zero mean profile, same as the DNS predictions, matching our observations of symmetric flows with the alternating direction of motion seen from the trajectories earlier.

The wall shear stress is calculated, for both experiment and DNS, as the instantaneous velocity gradient in viscous units ($u^+\delta_\tau^+$ or $w^+\delta_\tau^+$) for positions very close to the wall i.e., $y^+<1$ as the velocity varies linearly [26]. The stress fluctuation intensity for the two distributions, given by
\( \tau_{wall,x}' / \tau_{wall,x} \) and \( \tau_{wall,z}' / \tau_{wall,x} \) are 0.43 and 0.22, which are similar in magnitude to estimates from literature (e.g., 0.36 and 0.17 in [27]). In contrast, the skewness and flatness values of 1.38, 7.03 in the streamwise and -0.18 and 7.57 in the spanwise are much higher (0.95, 4.47 and 0.07 and 5.46 along x and z, respectively), possibly due to spatial averaging effect of micropillar based measurements in [27]. The streamwise PDF (Fig. 2c) shows a lower tail sharply falling to zero, due to absence of reverse flow events, which are extremely rare in a channel flow [28]. More importantly, the positive tail of the distribution shows an almost twice increase in probability compared to the DNS results, indicating the presence of extreme wall shear stress events not captured by the simulation. The spanwise wall shear stress PDF (Fig. 2d), on the other hand, shows a decrease in the tails by almost the same level (i.e. 2X). One reason for this difference in the streamwise and spanwise PDFs of similar scales can be attributed to the presence of the slow spanwise meandering motions observed in the experiment (Fig. 1e), which indicates a strong coupling between the streamwise and spanwise flows within the sublayer. The spanwise velocities are concentrated around zero, with both positive and negative values seen when flow alternates between the two directions. Such frequent flow changes at the smallest scales, which we quantify by the oscillation period of the average flow direction as a function of time to be ~10 viscous times (defined by \( \nu / u_{\tau}^2 \)), results in significant reorientation of flow between streamwise and spanwise directions. The curvilinear trajectories also lead to an increase of streamwise velocity by up to 2.5 times (see Supplementary Figure 2) resulting in the higher positive tail in streamwise wall shear stress PDF when compared to DNS predictions, which don’t resolve the motions at this scale. In addition, the distribution of the streamwise and spanwise velocity fluctuations (Supplementary Figure 3) indicating an increase of the streamwise and a decrease of the spanwise values below \( y^+ \sim 1 \), provides additional support to the observed changes in the wall shear stress distributions.

**Lagrangian acceleration statistics**

![Figure 3](image)

**Figure 3.** (a) The spanwise curved particle trajectories colored by instantaneous normal acceleration \( a^+_n \) with extreme magnitudes, reaching 30 times the standard deviation \( \langle a'^2 \rangle^{1/2} \) at regions of strong curvatures. (b) The PDF of tangential (●) and normal (○) compared to a gaussian distribution (---) of similar standard deviation and measurements in isotropic turbulence (□) reported in Mordant et al. (2004)

Apart from the mean velocity statistics, DFRH measurements also enable measurements of
Lagrangian statistics including accelerations along the trajectories. In order to compare between tracks with and without such curvatures, we convert the cartesian components to the corresponding tangential and normal components along the trajectory. Figure 3a presents the spanwise meandering motions, shown earlier, colored by their instantaneous normal accelerations \( (a_n^+) \) as they move along the trajectories. The accelerations often increase as the particles move along, reaching peak values at the location of maximum curvature, where spanwise velocity flips directions. The values of instantaneous accelerations reach values greater than 30 times the standard deviation \( ((a'^2)^{1/2}) \), similar to observations of particle accelerations seen in an isotropic turbulence [29], with significant accelerations in the spanwise direction within the sublayer. Furthermore, the probability distributions of the acceleration show very good agreement to measurements performed under isotropic turbulence [30]. Specifically, they both exhibit a stretched exponential shape with tails extending much higher than a gaussian distribution of similar standard deviation, indicative of highly intermittent turbulence (HIT). The existence of similarity in the structure of acceleration within the viscous sublayer and isotropic turbulence is reported for the first time here and could ultimately serve as evidence of universality of turbulence at the smallest scales. Moreover, such a similarity may also support a possible connection between the energy cascade in an isotropic turbulence to the momentum cascade in wall bounded turbulent flows [31–33].

**Drag reduction by sublayer flow modulation**

![Figure 4](image_url)

**Figure 4.** A schematic of how an idealized flow structure can be influenced by roughness placed within the viscous sublayer, both at larger scales of \(~100\delta_v\) as well as those spanning \(~2\text{-}3\delta_v\) in the spanwise direction.

The presence of spanwise meandering motions at a scale of \(~100\delta_v\), corresponding to streamwise streaks, as well as smaller structures spanning \(~2\text{-}3\delta_v\) in the spanwise direction can help explain the impact of sublayer scale roughness on macroscopic flow. The interaction of a flow structures with roughness of an equivalent size will lead to an increase or decrease in the strength of the structure, based on the specific orientations and geometry used. For instance, the V-groove roughness of \(~200\delta_v\) width and \(~5\delta_v\) height in Sirovich & Karlsson [2] can intercept the \(~100\delta_v\) scale streak structures as illustrated in Figure 4, disrupting the transfer from larger energy
containing scales to the viscous dissipation scale. Furthermore, as described by the authors in [2], a random spanwise arrangement of roughness will introduce streamwise to spanwise flow reorganization, leading to a reduction in the streamwise velocity and through it reduce the skin friction. Similarly, if the roughness is placed in a regular grid they end up stabilizing such larger scale streamwise structures leading to an increase in drag. On the other hand, disrupting the smaller scale motions, that are on the order of 2-3δν in the spanwise direction, observed closer to the wall can be accomplished by roughness with separations of ~1δν. The existence of strong spanwise accelerating motions (see Supplementary Figure 4), as seen in our results earlier, indicates a significant contribution of spanwise motions to the rate of energy dissipation through the strain rate tensor (Sij). In addition, presence of spanwise motions introduces instabilities in the flow which when acted upon by an external stimulus (e.g., adverse pressure gradient) can trigger flow separation, forming a recirculation bubble [34]. By placing barriers at this scale to reduce the extent of spanwise motions (i.e., roughness arranged in a regular grid of ~1δν), and reorienting flow from spanwise to the streamwise direction helps increase the net streamwise flow velocity. Furthermore, this reduces the rate of energy dissipation due to spanwise motions, leading to drag reduction by decreasing the recirculation bubble size, as reported in Evans et al. [3].

Finally, the observations of such a highly unsteady flow with large accelerations in the streamwise and spanwise directions and no significant wall normal motions raises some important questions on the non-intrusiveness of several near wall flow measurement techniques, i.e., local seeding and micro fiber wall shear sensors. First, local seeding with wall normal jets to introduce tracer particles in Holographic or Lagrangian particle tracking [10,11,35] often compare the jet velocity to the centerline velocity to emphasize lack of flow influence. However, a more accurate comparison would be to the scale of wall normal velocity in the sublayer, which from our results are significantly weaker than even the spanwise flow, and are comparable to the jet velocity used. Thus, introducing such a strong wall jet would no doubt lead to modifications of the flow structures not only within the sublayer but also the buffer and logarithmic layers above. In the same vein, micro pillar based shear stress sensors, introduce by Brucker et al. [36] and extended for dynamic shear stress in Grosse et al. [37] needs to be reexamined in light of our observations. The authors specifically assume the sublayer flow to be steady and two dimensional varying predominantly in the wall normal direction which is incorrect based on our observations.

Acknowledgments

Funding for this research was provided by The Office of Naval Research. A large part of the computational work was performed using resources at the Minnesota Supercomputing Institute (MSI) at the University of Minnesota (http://www.msi.umn.edu) and the ACI at Penn State which the authors would also like to thank and acknowledge.

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Meandering motions within the viscous sublayer: Supplementary materials
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Supplementary Figure 1. (a) Schematic and (b) image of the experimental setup for digital Fresnel reflection holography consisting of a laser, beam splitter, imaging lens and a camera.

All of the turbulent flow experiments are performed in the recirculating flow facility located in the Flow Field imaging lab. The facility (Figure 1) consists of a 1200 mm long, 50×50 mm square test section made out of stainless steel, with a settling chamber located upstream to limit flow disturbances. Optical access ports on all four sides made from acrylic are located near the end of the test section, with a stainless steel insert which allows one of the wall sections to be replaceable. I use this insert to replace the standard acrylic walls used with the AR-coated acrylic for the DFRH measurements. A 0.5 hp centrifugal pump (Dayton) with a variable frequency drive allows the flow speed within the facility to be varied continuously.

The experimental system for DFRH used in previous experiments is set up on an optical table near the flow facility as shown in Figure 1b. The 100mW green diode laser (OptoEngine Inc.) is steered by two turning mirrors to reach the beam splitter placed in front of the AR coated acrylic window. The channel is uniformly seeded with 13 μm silver coated hollow glass spheres at a volume fraction Φ ~0.001. A NAC HX-5 high speed camera equipped with a 10x long working distance objective lens (Mitutoyo Inc.) is used to record the backscattered holograms from the particles within the test section at a resolution of 1 μm/pixel over a 1×1 mm field of view. The sampling rate and exposure time for the camera are fixed at 3000 fps and 5 μs, respectively, to ensure particle displacements are on the order of 5-10 pixels per frame for accurate tracking. In addition, I also capture burst data at 50 Hz, capturing 10 frames at 3000 fps, for the calculation of mean statistics.
over a sufficiently long duration for proper statistical convergence. In total I record a total of 9 repetitions of the burst data as well as 3 repetitions of the time resolved measurements.

Numerical processing:

We implement a coherence based enhancement (based on Molei & Sheng [1]) which uses cross correlations to identify holograms with matching backgrounds to calculate a time average background image, to eliminate the interference pattern and signature of stationary particles from the hologram. After enhancement, the holograms are reconstructed using the same RIHVR algorithm, which solves an ill posed inverse problem using $l_1$ regularization, to obtain the 3D intensity fields [2]. To extract useful quantitative information from the reconstructed 3D intensity fields requires some additional post processing steps. A 3D Gaussian smoothing is applied to improve the accuracy of manual threshold segmentation and position extraction by an intensity weighted centroid. The extracted positions are tracked in time using TrackPy [3,4] to obtain Lagrangian trajectories of particles that represent the local fluid velocity. As the tracking algorithm is based on nearest neighbor particle matching, whenever there is a loss of particle in a frame due to drop in signal strength of a particle over time, the code ends up linking two different tracks together. Even if this occurs over a small fraction of the data, we need to eliminate such spurious tracks to limit their influence on the calculated statistics. A neural network based filter is used to validate tracks based on manually selected features, including the mean and standard deviations of streamwise, spanwise velocity, track length and the change in angular displacement, over the entire track. Since over a 10 frame time scale the motion of particles are predominantly along straight lines, the angular displacements should thus be close to each other for a valid track. On the other hand, spurious tracks that are linked randomly, across the spanwise direction will include large changes in the angular displacement which are unphysical. A manually labeled test data set of over 1000 tracks, with half each of valid and invalid tracks, sampled randomly from the entire recorded data is used to train a five layer dense neural network that calculates the probability that a track is valid with an accuracy and recall of $\sim 93\%$ and $\sim 91\%$, respectively. The validated tracks are smoothed using a second order piecewise polynomial filter (Savitzky Golay) to eliminate any spurious high frequency fluctuations in the position data, and differentiated in time to obtain the velocity and acceleration at each time step. A separate piecewise polynomial filter for both the velocity and acceleration are applied to suppress noise from the numerical differentiation. The estimated velocities are then ensemble averaged with wall normal bins of 10 $\mu$m to calculate the mean velocity profile within the viscous sublayer. Finally, the wall position ($y=0$) is estimated by extrapolating the linear mean velocity profile to zero and the friction velocity and the viscous wall unit calculated using the slope at the wall.
Supplementary Figure 2. Spanwise meandering trajectories colored by normalized streamwise velocity illustrating acceleration in the streamwise direction.

Supplementary Figure 3. Wall normal profile of (a) streamwise and (b) spanwise velocity fluctuations with insets highlighting the data below $y^+ \sim 1$.

Supplementary Figure 4. The PDF of streamwise (●) and spanwise (○) acceleration compared to a gaussian distribution (---) of similar standard deviation and Lagrangian acceleration measurements in isotropic turbulence (□) reported in Mordant et al. [5]
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