Current-driven dynamics of coupled domain walls in a synthetic antiferromagnet

Henri Saarikoski,1 Hiroshi Kohno,2 Christopher H. Marrows,3 and Gen Tatara1
1RIKEN Center for Emergent Matter Science (CEMS), 2-1 Hirosawa, Wako, Saitama 351-0198, Japan
2Department of Physics, Nagoya University, Nagoya 464-8602, Japan and
3School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, United Kingdom
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We develop the theory of magnetic domain wall motion in coupled double-layer systems where electrons can hop between the layers giving rise to an antiferromagnetic coupling. We demonstrate that the force from the interlayer coupling drives the walls and the effect of the extrinsic pinning is greatly reduced if the domain walls are initially separated. The threshold current density for metastable spin-aligned configurations is also much lower. We conclude that the interlayer coupling has a significant effect on domain wall mobility in double-layer systems.

I. INTRODUCTION

Domain walls in ferromagnetic materials involve magnetization reversal in a thin layer. The thickness of this layer is determined by the magnetic anisotropy energy and the exchange energy. Domain walls separate areas of different magnetization orientations and they are controllable using currents which create a spin torque that drives the wall. Therefore, devices using domain wall dynamics hold promise for future high speed, high density, and non-volatile data storage.1

However, domain wall motion is restricted by intrinsic and extrinsic pinning effects and current densities needed to move domain walls are typically high, of the order of $10^{12} \text{ A/m}^2$. The intrinsic pinning is due to the hard-axis magnetic anisotropy.2 The extrinsic pinning involves e.g. defects in individual layers. Theoretically, it was demonstrated that in the adiabatic limit, where the wall is driven solely by the spin-transfer torque, the wall has to overcome the energy barrier arising from the hard-axis anisotropy, and that the wall is intrinsically pinned.2 This energy barrier involves a threshold current below which a domain wall does not move or motion stalls soon after the current is turned on. In most cases, the threshold current of the intrinsic pinning is high.3 The intrinsic pinning effect was observed in a perpendicularly magnetized Co/Ni nanowire with reduced hard-axis anisotropy.4 The threshold current density was $2.5 \times 10^{11} \text{ A/m}^2$ and it was insensitive to the applied magnetic field which was consistent with theoretical predictions.5 Spin relaxation results in a torque orthogonal to the spin-transfer torque, and this torque, called the non-adiabatic torque, removes the intrinsic pinning effect and the threshold current is reduced.5,6,7 The threshold current is then determined by the extrinsic pinning potential and the non-adiabaticity parameter, $\beta$. In principle, the intrinsic pinning effect can be removed by fabricating a wire which has a cross-section of a perfect circle.2

However, most of the experiments have been carried out in the regime where extrinsic pinning effects dominate.8

For realizing fast domain wall motion and low threshold current density, several experimental attempts have been carried out. Lepadatu et al. controlled the value of non-adiabaticity parameter, $\beta$, by doping permalloy with vanadium.9 They showed that V-doping of 10% leads to an increase of $\beta$ by a factor of about two, but the threshold current did not improve since the spin polarization of the current was reduced by doping.10 Trilayer Pt/Co/MgO structures were studied by Miron et al.11,12 They were motivated by the idea that Rashba spin-orbit interaction would emerge in interfaces of layers of insulators and metals in the presence of heavy atoms with strong spin-orbit interaction. This interaction would realize very efficient wall motion since it acts as a large $\beta$ as predicted theoretically.13,14 The wall velocity in that trilayer system was 400 m/s, which is two orders of magnitude larger than in single layer systems, at current density of $3 \times 10^{12} \text{ A/m}^2$ (Ref. 12). However, it turned out that the mechanism for fast wall motion was not due to the Rashba interaction. In fact, systematic analysis on structures Pt/CoFe/MgO and Ta/CoFe/MgO indicated that the spin Hall effect in Pt and Ta layer injects spin current into the ferromagnetic layer and induces a substantial torque on the domain wall, resulting in fast motion.15,16 Due to high domain wall velocities artificial multi-layered structures are promising for designing devices with efficient domain wall motion.

Here we develop theory of domain wall motion in coupled double-layer systems where electrons can hop between the layers giving rise to an antiferromagnetic coupling. Presence of antiferromagnetic coupling between the layers was demonstrated17 for ultrathin films in mid-1980’s and it is used in applications such as magnetic stabilization of magnetoresistive recording heads.18 The interlayer coupling induces an attractive force between the walls in the two different layers, and this force is expected to help depin the wall since the current drives both walls. We derive equations of motion for the system in the presence of force from the interlayer coupling and calculate domain wall dynamics from the resulting equations. It turns out that interlayer coupling indeed reduces the threshold current greatly if domain walls are initially separated at different pinning sites. The coupled layer systems are therefore promising for efficient domain wall motion not affected by localized random defects.
II. THEORETICAL MODEL

In this section we derive equation of motion for domain walls in a ferromagnetic double-layer system. We label the layers by \( i = 1, 2 \). The localized spin direction at position \( \mathbf{r} \) and time \( t \) in each ferromagnetic layer is denoted by a unit vector field, \( \mathbf{n}^{(i)}(\mathbf{r}, t) \). We define a coordinate system such that the wire lies in the \( x-z \) plane, extended along the \( z \)-direction, with the two layers stacked above each other in the \( y \)-direction (see Fig. 1). The magnetic easy axis is along the \( z \)-direction and \( y \)-direction is the magnetic hard axis. The spin Hamiltonian can then be written as

\[
H_S = \sum_{i=1,2} \int_{V_i} \frac{d^3r}{a^3} \times \left( \left( \frac{JS^2}{2} \nabla \mathbf{n}^{(i)} \right)^2 - \frac{K S^2}{2} \left( n_z^{(i)} \right)^2 + K_{\perp} S^2 \left( n_\perp^{(i)} \right)^2 \right),
\]

where \( J, K \) and \( K_{\perp} \) are the strength of the exchange interaction, the easy axis anisotropy energy and the hard axis anisotropy energy, respectively. In nanowires made from magnetically soft permalloy-like materials, these anisotropy constants arise from shape anisotropy. The magnitude of spin is \( S \), \( a \) is a lattice constant and \( V_i \) denotes volume of the ferromagnet \( i \). We consider the case in which material constants are the same for both ferromagnets.

Coupling between the two ferromagnets is mediated by electron hopping between the layers. The in-plane component of the coupling is here antiferromagnetic \( \Delta_{\parallel} \geq 0 \). We assume for generality that the out-of-plane component of the interlayer coupling \( \Delta_{\perp} \) is different from the in-plane component, since this coupling is affected by the demagnetization field. Therefore we consider here both antiferromagnetic \( \Delta_{\perp} > 0 \) and ferromagnetic \( \Delta_{\perp} < 0 \) out-of-plane couplings. We assume that the two ferromagnets are thin (compared with the domain wall thickness) and that the interlayer coupling acts uniformly on the whole spins. The interlayer coupling is thus represented by Hamiltonian

\[
H_1 = \sum_{i=1,2} \int_{V_i} \frac{d^3r_1}{a^3} \int_{V_2} \frac{d^3r_2}{a^3} \left[ \Delta_{\parallel} S^2 (n_z^{(1)}(r_1)n_z^{(2)}(r_2)) + \Delta_{\perp} S^2 n_{\perp}^{(1)}(r_1)n_{\perp}^{(2)}(r_2) \right].
\]

Magnetic anisotropy is very common in thin ferromagnetic films. Therefore we consider only Néel-type domain walls which have the domain wall solution

\[
\mathbf{n}^{(i)} = \begin{pmatrix} \sin \theta_i \cos \phi_i \\ \sin \theta_i \sin \phi_i \\ \cos \theta_i \end{pmatrix},
\]

where

\[
\cos \theta_i = (-)^i \tanh \frac{z - Z_i(t)}{\lambda}
\]

and \( \sin \theta_i = [\cosh \frac{z - Z_i(t)}{\lambda}]^{-1} \), where \( \theta \) is the angle between the moment and the \( z \)-axis and \( \phi_i(t) \) is the azimuthal angle around that axis and represents the out-of-plane angle of the spin in Fig. 1. The wall position is denoted by \( Z_i \) and \( \lambda \) is the thickness of the wall, given by \( \lambda = \sqrt{J/K} \). The topological charge of the domain wall, given by the sign in Eq. 4, differs for the two domain walls as a result of the antiferromagnetic in-plane coupling. This property is essential in the dynamics of the system of coupled walls. The geometry of the synthetic antiferromagnet under consideration is shown in Fig. 1.

Current applied in the direction of the wire gives rise to two important effects; the adiabatic spin-transfer torque effect, which induces a torque on the domain wall, and a non-adiabatic contribution which is described as a force on the wall. The adiabatic effect is given by the spin-transfer Hamiltonian,

\[
H_{ST} = -\sum_{i=1,2} \int_{V_i} \frac{d^3r}{a^3} \frac{e}{2} \mathbf{S} \mathbf{P} \mathbf{a} S \cdot \nabla \phi_i (\cos \theta_i - 1),
\]

where \( j \) is the electric current density, \( e < 0 \) is the electron charge, \( P \) is the spin polarization of the current. The non-adiabatic contribution as well as damping are inserted later in the equations of motion.

By collecting all the above contributions we obtain the Lagrangian of the coupled double-layer system under applied current

\[
L = \sum_{i=1,2} \int_{V_i} \frac{d^3r}{a^3} \hbar S \dot{\phi}_i (\cos \theta_i - 1) - H_S - H_1 - H_{ST},
\]

where the first term is the spin Berry’s phase term. We rewrite the Lagrangian in terms of the collective coordinates for the two walls, \( Z_i(t) \) and \( \phi_i(t) \) (see Fig. 1). The
where \( \nu_c = \frac{K_A \Delta S}{2} \) and \( \nu_c = \frac{p S^3}{2 M} \).

Now we include the effect of damping and non-adiabatic contribution of the current. The damping is included as \( \frac{\delta L}{\delta Z_i} = \alpha h N_i S \frac{\dot{Z}_i}{\lambda} \) and \( \frac{\delta L}{\delta \phi_i} = \alpha h N_i S \phi_i \).

The non-adiabatic torque, represented by a parameter \( \beta \), induces a force \( \frac{h N_i a^3}{2 e S \lambda^2} \beta P j \).

Since \( L = -h N_i S \frac{\dot{Z}_i}{\lambda} + h N_2 S \frac{\dot{Z}_2}{\lambda} + h - H \), the equations of motion obtained by differentiating with respect to \( Z_1 \) and \( Z_2 \) are

\[
\begin{align*}
-h N_1 S \frac{\dot{\phi}_1}{\lambda} &= F_1, \\
h N_2 S \frac{\dot{\phi}_1}{\lambda} &= F_2,
\end{align*}
\]

are the forces. The equations of motion thus read

\[
\begin{align*}
\dot{Z}_1 - \alpha \lambda \dot{\phi}_1 &= \nu_c \sin 2 \phi_2 - \nu_c \\
&- \mu_1 u((Z_1 - Z_2)/2) (\Delta_+ \sin(\phi_1 - \phi_2) + \Delta_- \sin(\phi_1 + \phi_2)), \\
\dot{\phi}_1 - \alpha \lambda \dot{Z}_1 &= \frac{1}{\lambda} \nu_c + \frac{\mu_1}{2} \left[ \Delta_+ u'((Z_1 - Z_2)/2) \
&+ u'((Z_1 - Z_2)/2) (\Delta_+ \cos(\phi_1 - \phi_2) + \Delta_- \cos(\phi_1 + \phi_2)) \right], \\
\dot{Z}_2 - \alpha \lambda \dot{\phi}_2 &= \nu_c \sin 2 \phi_2 + \nu_c \\
&+ \mu_2 u((Z_1 - Z_2)/2) (\Delta_+ \sin(\phi_1 - \phi_2) - \Delta_- \sin(\phi_1 + \phi_2)), \\
\dot{\phi}_2 - \alpha \lambda \dot{Z}_2 &= \frac{1}{\lambda} \nu_c - \frac{\mu_2}{2} \left[ \Delta_+ u'((Z_1 - Z_2)/2) \
&+ u'((Z_1 - Z_2)/2) (\Delta_+ \cos(\phi_1 - \phi_2) + \Delta_- \cos(\phi_1 + \phi_2)) \right],
\end{align*}
\]

where \( \Delta_k \equiv \frac{1}{2} (\Delta_\parallel \pm \Delta_\perp) \) and \( \mu_i = N_i/N_1 \). The \( \mu_i \) parameters of the planes determine whether the system is a balanced synthetic antiferromagnet at \( \mu_1 = \mu_2 \) or an unbalanced synthetic ferrimagnet at \( \mu_1 \neq \mu_2 \).

III. EFFECT OF PINNING

Domain wall dynamics is affected by impurities, notches and other non-uniformities in the layers. We
model such non-uniformities using pinning forces on the domain walls. We are interested in calculating the terminal velocity of the domain walls under applied current when the domain walls are initially pinned in both layers. We consider therefore one pinning potential in each layer at distance $\ell$ to each other:  

$$F = -k_0^{(1)}(Z_1 - \ell)\theta(\xi - |Z_1 - \ell|) - k_0^{(2)}Z_2\theta(\xi - |Z_2|),$$  

(19)

where $k_0^{(i)} (i = 1, 2)$ are constants representing the strength of the potentials, $\xi$ is the width of the potential and $\theta(x)$ is a step function. We set the potential width $\xi$ in both layers. Using $k_i = \frac{\lambda}{N_x}k_0^{(i)}$, defining center of mass $Z_\pm$ and the difference $Z_-$ in the domain wall positions using $Z_\pm \equiv \frac{1}{2}(Z_1 \pm Z_2)$ as well as the average phase $\phi_\pm$ and the difference in the phase using $\phi_\pm \equiv \frac{1}{2}(\phi_1 \pm \phi_2)$, and denoting $\mu_\pm = (\mu_1 \pm \mu_2)/2$ we obtain the following equations for motion  

$$\dot{Z}_+ + \alpha \lambda \phi_+ = -\nu_e \cos 2\phi_+ \sin 2\phi_- + \nu_e + u(Z_-)(\mu_+ \Delta_+ \sin(2\phi_-) + \mu_- \Delta_- \sin(2\phi_+)), \quad (20)$$

$$\phi_- - \frac{\alpha}{\lambda} \dot{Z}_+ = \frac{k_1}{2}(Z_+ + Z_- - \ell)\theta(\xi - |Z_+ + Z_- - \ell|)$$

$$+ \frac{k_2}{2}(Z_+ - Z_-)\theta(\xi - |Z_+ - Z_-|) - \frac{\beta}{\alpha} \nu_e + \frac{\mu_-}{2} \times [\Delta_\parallel w'(Z_-) + u'(Z_-)(\Delta_+ \cos(2\phi_-) + \Delta_- \cos(2\phi_+))],$$

(21)

$$\dot{Z}_- + \alpha \lambda \phi_- = -\nu_e \sin 2\phi_+ \cos 2\phi_- + \nu_e + u(Z_-)(\mu_- \Delta_+ \sin(2\phi_-) + \mu_+ \Delta_- \sin(2\phi_+)), \quad (22)$$

$$\dot{\phi}_- - \frac{\alpha}{\lambda} \dot{Z}_- = \frac{k_1}{2}(Z_+ + Z_- - \ell)\theta(\xi - |Z_+ + Z_- - \ell|)$$

$$- \frac{k_2}{2}(Z_+ - Z_-)\theta(\xi - |Z_+ - Z_-|) + \frac{\mu_+}{2} \times [\Delta_\parallel w'(Z_-) + u'(Z_-)(\Delta_+ \cos(2\phi_-) + \Delta_- \cos(2\phi_+))],$$

(23)

IV. TERMINAL VELOCITY OF UNPINNED DOMAIN WALLS

In the absence of pinning potentials the terminal velocity of the domain wall can be analytically solved. We first assume that the domain wall separation remains small i.e. $|Z_-| \ll \lambda$ which gives approximately $u'(Z_-) = 0$ and $w'(Z_-) = 0$. We assume also that $\phi_\pm$ changes with time, resulting in vanishing of time averages of $\sin 2\phi_\pm$ and $\cos 2\phi_\pm$. After time-averaging we see that the terminal velocities are not affected by the interlayer coupling  

$$\langle \dot{Z}_+ \rangle = \frac{1}{1 + \alpha^2} \nu_e(1 + \alpha \beta)$$

$$\langle \dot{\phi}_- \rangle = \frac{1}{1 + \alpha^2} \nu_e(1 - \alpha \beta)$$

$$\langle \dot{Z}_- \rangle = 0$$

$$\langle \dot{\phi}_+ \rangle = 0.$$

(24)

We then assume that the separation of the domain wall grows with time, e.g., $|Z_-| \gg \lambda$. We can then approximate $u(Z_-) = u'(Z_-) = 0$ and $w'(Z_-) = 2 \operatorname{sgn}(Z_-)$. After time averaging the terminal velocities are then  

$$\langle \dot{Z}_+ \rangle = \frac{1}{1 + \alpha^2} \nu_e(1 + \alpha \lambda \mu_- \Delta_\parallel \operatorname{sgn}(Z_-))$$

$$\langle \dot{\phi}_- \rangle = \frac{1}{1 + \alpha^2} \nu_e(1 - \alpha \lambda \mu_- \Delta_\parallel \operatorname{sgn}(Z_-))$$

$$\langle \dot{Z}_- \rangle = -\frac{1}{1 + \alpha^2} \alpha \mu_+ \Delta_\parallel \operatorname{sgn}(Z_-)$$

$$\langle \dot{\phi}_+ \rangle = \frac{1}{1 + \alpha^2} \mu_+ \Delta_\parallel \operatorname{sgn}(Z_-).$$

(25)

We see that in this limit the velocity increases with interlayer coupling. This can be understood from the antiferromagnetic coupling of the walls which exerts a force on the walls. In practice unpinned walls which are first at finite distance from each other move fast until the separation vanishes. Then the walls start moving together at a lower velocity determined by Eq. (24). This typical behaviour is shown in numerical simulations in Fig. 3.

V. THRESHOLD CURRENT

Eqs. (20) - (23) are a group of first order differential equations. We integrate the solution from initial conditions using a numerical Runge-Kutta-Fehlberg 4th order method with a 5th order error estimator for the adaptive step size. We use dimensionless units in calculations by fixing $\nu_e = 1$ and setting the thickness of the wall $\lambda = 1$. The time is measured in terms of a dimensionless quantity, $\nu_e / \lambda = t K_\perp S / (2 \hbar)$. We consider separately the adiabatic and non-adiabatic regimes. In the former regime adiabatic torque on the wall dominates dynamics and in the latter case the non-adiabatic force gives the most important contribution.

Due to the terms which depend on $Z_\pm$ and $\phi_\pm$ the time evolution of the system depends on the initial separation of the domain walls as well as the difference in their phases. A slight variation in the initial domain wall positions and phases is introduced in order to simulate experimental situations at finite temperature and in order to avoid special limiting solutions to the differential equations, for instance when terms on the right hand side vanish at $\phi_+ = \phi_- = 0$. This also smooths out the effect of discontinuous external pinning potentials. We use
tiny displacements to the initial domain wall positions and phases using random numbers from a uniform distribution giving a ±0.01 change in the wall position with respect to each other (in units of wall thickness λ). The domain wall velocity is evaluated after a sufficiently long time when the domain wall motion has stabilized.

The initial fluctuation of the wall position, δZ, corresponds to energy fluctuation of δE = 1/2 (δZ)^2 = NV_0 (δZ/ξ)^2, where the pinning potential depth per spin is V_0 ≡ ℏkBkξ^2 (we suppress here the suffix i = 1, 2 denoting the layer). In numerical calculations, the time is measured in terms of a dimensionless quantity, tv_c/λ = tK⊥S/(2ℏ), and thus a pinning strength k = 0.1 we use in the calculations would correspond to the pinning potential of V_0/K⊥ = kS^2ξ^2/(4ν_e) = 2.5 × 10^-2 if we choose S ≃ 1 and ξ ≃ λ. For permalloy wires, K⊥ ≃ 0.03 ~ 2.4 K (Ref. 20), and if we consider a wall with thickness of 100 nm in a wire of cross-sectional area of 400 nm × 5 nm, we have N = 1.3 × 10^7 (for a = 2.5 Å) as the number of spins in the wall. The fluctuation energy for δZ = 0.01 therefore is δE = 30K⊥ = 1 ~ 72 K. The initial fluctuations of ±0.01 in the calculations is therefore small in magnitude in comparison to those expected for permalloy wires at room temperature.

A. Extrinsic pinning

We insert pinning potentials in both layers as described in Section III. The initial conditions are chosen to fix the domain walls at the center of the pinning potentials. Details of the domain wall dynamics depend now on the relative strength of the parameters in the model. At finite values of the non-adiabatic torque β the extrinsic pinning potentials usually restrict domain wall motion and a large driving current is needed to depin the walls. This limits usefulness of magnetic domains in applications, and means to improve mobility has been in the focus of intense research efforts.

Figure 3 shows typical domain wall dynamics when the walls are pinned by the potentials and when the interlayer coupling is large enough to unpin the walls, respectively. If the force from the interlayer coupling (terms containing Δ∥ or Δ⊥ in Eqs. (20)–(23)) is not sufficiently large to unpin the walls the walls absorb the momentum leading to oscillations. The threshold current is the current at which depinning occurs. The depinning process is clearly aided by the force from the separation of the domain walls and once the walls clear the pinning potentials they start to travel together with the difference in phases eventually vanishing. In this limit the velocity decreases as discussed in Sec. IV. We calculate domain wall motion in the presence of pinning potentials from the velocity in this limit.

1. Weak non-adiabatic force (β < α)

We focus first on regime of weak force from the non-adiabatic torque β. We set k_{1,2} = 0.1, β = 0.005 and the damping term α = 0.01. We set the potential well width ξ = 1 which is comparable in size to the domain wall width. We find also that the potential well width does not significantly affect the results since the wall motion is coupled also inside very wide potential wells. The in-plane interlayer coupling is assumed to be antiferromagnetic Δ∥ > 0 and we first neglect the perpendicular component in the calculations setting Δ⊥ = 0. We investigate the effect of the perpendicular component later.

Figure 4 shows terminal domain wall velocity as a function of velocity of driving electrons ν_e and strength of the in-plane interlayer coupling Δ at different distances between the pinning potential sites. The velocity is calculated for asymmetric domain wall configurations (μ_1 = 1 and μ_2 = 1/2).

We find that the threshold current for domain wall motion decreases rapidly with increasing antiferromagnetic interlayer coupling at finite distance between the pinning potentials. The threshold current is lowered at large distances between the pinning potential sites. Assuming that impurities can be modeled using pinning potentials with a random distribution and no correlations between
Therefore even a small displacement for one of the walls induces a force on the coupled wall and decreases the threshold current density. As a consequence large initial domain wall separation assists the depinning process slightly and smooths out the sharp boundary between the regimes. In this cross-over regime some initial domain wall configurations lead to depinning of the walls. Finite temperature in experiments may therefore assist the depinning process. In the case of strong pinning potentials the threshold current has only a weak dependence on the pinning potential strength. We find that even in this regime the interlayer coupling decreases the threshold current.

2. Strong non-adiabatic force ($\beta \gg \alpha$)

Next we consider the regime where $\beta$ is large and the non-adiabatic torque predominates. The domain wall motion is then driven by the force exerted by this torque. Threshold current density is low and the terminal domain wall velocity above the threshold current is proportional to $\beta/\alpha$ (Ref. 3). In this regime mobility is high and the threshold current depends on the interlayer coupling and the distance between the pinning potential sites. Figure 5 shows terminal domain wall velocity when the pinning potentials are located at different positions with respect to each other ($\ell = 0, 1, 2$). We find that the non-adiabatic force from the interlayer coupling drives the walls and the effect of the extrinsic pinning is greatly reduced at finite $\ell$. In this regime the threshold current is strongly reduced by even a weak interlayer coupling.

At high driving currents ($\nu_e > 0.2$) and weak pinning strength the domain wall mobility is reduced due to a mechanism which is analogous to the Walker breakdown in magnetic fields. The combination of strong non-adiabatic driving and the interlayer coupling increases domain wall mobility significantly at finite $\ell$. We see a factor of 5 improvement in the threshold current at the coupling strength $\Delta_\parallel = 0.1$. Otherwise, the behaviour is similar to the case of weak non-adiabatic driving force and consistent with the analytical calculation in Sec. 3.

So far we have neglected the out-of-plane component of the interlayer coupling $\Delta_\perp$. The out-of-plane component is affected by the demagnetization field and therefore it can differ from the in-plane coupling in experiments. However, calculations at fixed in-plane coupling strength $\Delta_\parallel = 0.5$ with ferromagnetic and antiferromagnetic out-of-plane coupling strength ($\Delta_\perp = +0.5$ and $\Delta_\perp = -0.5$, respectively) show little effect on the threshold current (Fig. 5). Antiferromagnetic out-of-plane coupling gives the highest domain wall velocity close to the regime of Walker breakdown. We find also that the potential well width $\xi$ does not significantly affect the results since the wall motion is coupled also inside very wide potential wells.
FIG. 5. Averaged terminal domain wall velocity (given by the color bar) in double-layer systems in the regime of strong non-adiabatic driving ($\beta/\alpha = 6$) and different distances between the pinning potential sites $\ell$. In this regime unpinned domain wall velocity is proportional to $\beta/\alpha$ until the point of Walker breakdown at $\nu_e \approx 0.2$. In-plane interlayer coupling is antiferromagnetic ($\Delta = \Delta_\parallel > 0$) and improves domain wall mobility at finite $\ell$. The layers have unequal thickness $\mu_1 = 1, \mu_2 = 1/2$ and $\alpha = 0.01$.

FIG. 6. Averaged terminal domain wall velocity in double-layer systems in the non-adiabatic driving regime with fixed in-plane coupling strength $\Delta_\parallel = +0.5$ and ferromagnetic ($\Delta_\perp = -0.5$), vanishing ($\Delta_\perp = 0$), and antiferromagnetic ($\Delta_\perp = +0.5$) out-of-plane component, respectively. Layer thickness $\mu_1 = 1$ and $\mu_2 = 0.5$. Distance between the pinning potential sites $\ell = 2$.

FIG. 7. a) A metastable initial state with parallel spin orientations at the center of the walls ($\phi_1 = \phi_2 = 0$). b) Averaged terminal domain wall velocity for the initial metastable states. The in-plane interlayer coupling is antiferromagnetic $\Delta_\parallel > 0$ and the out-of-plane component $\Delta_\perp = 0$. Figure shows the regime of weak non-adiabatic force ($\beta/\alpha = 0.5$) and the regime of strong non-adiabatic driving ($\beta/\alpha = 6$). The threshold current decreases with increasing interlayer coupling in both cases. The layers have unequal thickness $\mu_1 = 1, \mu_2 = 1/2$, the pinning potential strength $k_1 = k_2 = 0.1$ and the pinning sites are at the same position $\ell = 0$.

3. **Evolution of metastable states**

Next we study metastable states corresponding to parallel spin alignment at the center of the walls ($\phi_1 = \phi_2 = 0$) (see Fig. 7a). Energy associated with this initial spin alignment increases with interlayer coupling and helps depin the domain walls. Simulations show then that the system evolves until the final states have antiparallel spin alignment. The threshold current decreases strongly with the increasing interlayer coupling even at $\ell = 0$ in the regimes of both weak and strong non-adiabatic driving (Fig. 7b). We find an order of magnitude difference in the threshold current density when the interlayer coupling strength is large.

**B. Intrinsic pinning**

In the case of adiabatic driving ($\beta \to 0$) the domain wall dynamics is driven purely by the spin transfer torque. The intrinsic pinning effects dominate domain wall dynamics over extrinsic effects. Intrinsic pinning is caused by the hard-axis magnetic anisotropy. We investigate here whether the interlayer coupling modifies the in-
Intrinsic pinning within our model at $\beta = 0$ assuming that there are no extrinsic pinning potentials $k_1 = k_2 = 0$. Figure 8 shows the domain wall terminal velocity as a function of velocity of driving electrons $\nu_e$ at $\Delta = 0$, 0.25, and 0.5 at equal layer thickness $\mu_1 = \mu_2 = 1$ shown at left. The threshold $\nu_e$ at different layer thicknesses $\mu_2$ fixing $\mu_1 = 1$ shown at right.

FIG. 8. Terminal domain wall velocity under intrinsic pinning in the absence of extrinsic pinning ($k_1 = k_2 = 0$). Interlayer coupling is assumed to be antiferromagnetic and isotropic $\Delta_{||} = \Delta_{\perp} = \Delta$. The domain wall velocity as a function of the velocity of driving electrons $\nu_e$ at $\Delta = 0, 0.25,$ and $0.5$ at equal layer thickness $\mu_1 = \mu_2 = 1$ shown at left. The threshold $\nu_e$ at different layer thicknesses $\mu_2$ fixing $\mu_1 = 1$ shown at right.

VI. DISCUSSIONS

In our theory the force from the interlayer coupling facilitates significantly domain wall motion in the regime where extrinsic pinning effects dominate. Even a low interlayer coupling improves mobility and in the limit of high interlayer coupling our theory predicts a much lower threshold current for the domain walls pinned at random impurity sites. A domain wall separation which is of the order of the wall width is large enough to significantly improve domain wall mobility. This effect is further expected to be enhanced at elevated temperatures due to thermal fluctuations in the domain wall positions. In contrast to the extrinsic pinning regime, the interlayer coupling enhances effectively the hard-axis isotropy giving rise to no mobility improvement in the regime where intrinsic pinning effects dominate.

We have shown theoretically that interlayer coupling improves domain wall mobility in correlated bilayer systems. The interlayer coupling greatly reduces the effective pinning potential depth when the pinning potentials are uncorrelated in the two layers. Bilayer systems are thus promising candidates for realization of efficient domain wall control with low current densities.

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