Prediction of sand mold cutting force and identification of cutting force coefficients

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Abstract: Patternless casting technology is an advanced digital forming technology. Sand mold cutting is one of the most important techniques of patternless casting technology. In the process of sand mold cutting, the cutting force is the key factor affecting the quality of sand mold machining, and is an important prerequisite for the optimization of cutting parameters and the study of cutting stability. The accuracy of the cutting force prediction model depends largely on the identification of the cutting force coefficient. In the average cutting force model, the cutting force coefficient is considered as a constant. For identification of sand mold cutting force coefficient, a series of resin sand cutting experiments have been conducted in this paper, fixing spindle speed and depth of cutting, linearly changing the feed per tooth to measure the cutting forces, the tangential and radial cutting force coefficient are calculated by linearly fitting experimental data. The analysis results show that with the change of the cutting parameters the resin sand cutting force coefficient is basically the same.

1. Introduction
In sand cutting process, the cutting force is the key factor affecting the stability of machine tool and the dimensional accuracy of workpiece. To establish an accurate and reliable cutting force model by analyzing the cutting force is not only the basis of cutter stability prediction, but also the important premise of optimizing the cutting parameters, Identifying the cutting force coefficient of sand mould is the core of establishing the prediction model of sand mould cutting force.

The methods of cutting force modeling mainly include empirical method and mechanical modeling method. The empirical method cannot effectively reflect the characteristics of the numerical and directional fluctuation of cutting force with time cycle, so it is necessary to conduct a large number of cutting experiments and build a huge database to store the experimental results, once the cutting conditions change, the empirical formula is no longer applicable. Taylor, Koenigsberger[1-2] et al. measured the cutting force in the cutting process through a large number of cutting experiments, and established an empirical formula to describe the relationship between the cutting force, cutting conditions and tool geometric parameters. Mechanical modeling method first proposed by Sabberwal[3], which is a widely used in cutting. The cutting force is assumed to be proportional to the cutting cross-sectional area, and the proportional coefficient depends on the cutting conditions and material properties. Kline et al.[4-5] implied that the cutting force coefficients were constants. A model building procedure...
based on experimentally obtained average forces is presented and both instantaneous and average force system characteristics are described as a function of cut geometry and feed rate. Niu[6] and Ozkirimli[7] respectively established a multi-time delay dynamic model for tool cutting with variable tooth pitch and helical angle based on constant cutting force coefficient and considering cutter eccentric runout. Hayasaki[8] established the dynamic model of the cutting tool with variable helical angle, and optimized the tool parameters from the perspective of chatter suppression. Peng[9] established the dynamic model of rounded corners cutting and analyzed the cutting stability. Diez[10] built a two-degree-of-freedom dynamic model of thin-walled parts cutting system by using the exponential cutting force coefficient. Azeemeta[11] putforward the simplified method for determining the cutting force coefficients on aballend-mill, cutting force coefficients can be obtained from just one half-slot cutting, but the method requires that the tool axis be perpendicular to the workpiece surface. In terms of sand cutting, Xie et al. [12] studied the influence of cutting parameters on the surface quality of sand mold through cutting experiments. Song[13] et al. studied the cutting mechanism of sand mold. In the cutting process of sand mold, the study showed that under the cutting and extrusion action of the tool, the processed surface would appear brittle fracture. Under the high-speed collision of the tool, the "bonding bridge" would fracture the sand grain and make it break away from the surface of the sand mold.

Based on the above analysis, the average cutting force model has the characteristics of simple and effective, which can quickly calibrate the cutting force coefficient, and has been successfully applied in the metal cutting process. However, there are few relevant studies in non-metal cutting, especially in sand cutting. With the rapid development of patternless casting technology, the identification of cutting force coefficient for sand mould has a great practical value, which lays a theoretical foundation for the research of parameter optimization and chatter suppression.

2. The average cutting force model

2.1 Force analysis of cutting elements

In the mechanical modeling method, the instantaneous rigid force model can accurately predict the size and direction of cutting force at any moment in the machining process. The cutting edge of cutting cutter is divided into several small cutting edges, which are called “infinitesimal cutting unit”, As shown in Fig.1. According to Martellotti[14], the force exerted on the infinitesimal cutting unit on any cutting edge is equal to the product of the cutting force coefficient and the instantaneous cutting area. The cutting force is divided into shearing force caused by shearing in the shear zone, and blade force derived from flank surface friction of the cutting edge. The radial force $F_r$, tangential force $F_t$ and axial force $F_a$ are broken down into the $X$, $Y$, $Z$ three directions.

![Fig.1 Force analysis by a flat-end mill](image)

In the average cutting force model, cutting force coefficients are considered constant. The basic formula of the instantaneous rigid force model is:
In the formulae, $dF_t$, $dF_r$, $dF_a$ are respectively the tangential, radial and axial cutting force elements; $d(l)$, $d(a_z)$, $a_c$ are respectively the length of the cutting edge, the depth of axial cutting and the cutting thickness. $K_t$, $K_r$, $K_a$ are respectively the tangential, radial and axial cutting force coefficient; $K_{tc}$, $K_{rc}$, $K_{ac}$ are respectively the tangential, radial and axial blade force coefficient.

### 2.2 Identification of cutting force coefficients

As can be seen from the above force analysis, the cutter helical cutting edge only cuts in the effective contact area($\phi_e \leq \phi \leq \phi_a$). While the integral of the instantaneous cutting force is calculated when the spindle rotates for one cycle. The average cutting force between each tooth cycle can then be obtained by dividing the result by the pitch angle$^{15-16}$:

$$
\bar{F}_x = -\frac{1}{\phi_p} \int_{\phi_{st}}^{\phi_{ex}} F_x(\phi)d\phi
$$

$$
\bar{F}_y = -\frac{1}{\phi_p} \int_{\phi_{st}}^{\phi_{ex}} F_y(\phi)d\phi
$$

$$
\bar{F}_z = -\frac{1}{\phi_p} \int_{\phi_{st}}^{\phi_{ex}} F_z(\phi)d\phi
$$

The average cutting force derived from the integral of $X$, $Y$, $Z$ three directions is$^{17-19}$:

$$
F_x = -\left\{ \frac{Na_p a_j}{8\pi} [K_{tc} \cos 2\phi - K_{rc} (2\phi - \sin 2\phi)] + \frac{Na_p}{2\pi} [-K_{tc} (\sin \phi + K_{rc} \cos \phi)] \right\}_{\phi_e}^{\phi_a}
$$

$$
F_y = \left\{ \frac{Na_p a_j}{8\pi} [K_{tc} (2\phi - \sin 2\phi) + K_{rc} \cos 2\phi] - \frac{Na_p}{2\pi} [K_{tc} \cos \phi + K_{rc} \sin \phi] \right\}_{\phi_e}^{\phi_a}
$$

$$
F_z = -\left\{ \frac{Na_p}{8\pi} [-K_{tc} a_j \cos \phi + K_{ac} \phi] \right\}_{\phi_e}^{\phi_a}
$$

In Eq.3, where $\phi$ is the contact angle, $N$ is the number of teeth, and $a_p$ is the axial depth of cut. In down cutting, when the radius of end-mill $R$ is larger than the radial width of cut $a_c$, the entry_angle($\phi_{ae}$) and exit_angle($\phi_a$) of cutter can be represented as:

$$
\phi_{ae} = \pi - \arccos\left(1 - a_c / R\right)
$$

$$
\phi_a = \pi
$$

Slot cutting is used for convenience, $a_{ae} R$, $\phi_{ae} = 0$, $\phi_a = \pi$. In order to avoid the influence of cutter eccentricity during measurement, the total cutting force per revolution is measured, and the result then divided by the number of teeth. By substituting Eq.4 into Eq.3, the following equation results:
\[
\begin{align*}
F_x &= \frac{Na_p}{4} K_u a_f + \frac{Na_p}{\pi} K_{ue} \\
F_y &= \frac{Na_p}{4} K_u a_f + \frac{Na_p}{\pi} K_{ue} \\
F_z &= \frac{Na_p}{\pi} K_u a_f + \frac{Na_p}{2} K_{ue} 
\end{align*}
\]

Eq.5 can be derived in the following form:

\[
\begin{align*}
F_x &= \overline{F}_{xc} a_f + \overline{F}_{xe} \\
F_y &= \overline{F}_{yc} a_f + \overline{F}_{ye} \\
F_z &= \overline{F}_{zc} a_f + \overline{F}_{ze}
\end{align*}
\]

From Eqs. 5 and 6, the cutting force coefficients maybe expressed as:

\[
\begin{align*}
K_x &= \frac{4\overline{F}_{xc}}{Na_p} \\
K_y &= \frac{\pi \overline{F}_{ye}}{Na_p} \\
K_z &= \frac{4\overline{F}_{zc}}{Na_p} \\
K_{xe} &= \frac{\pi \overline{F}_{xe}}{Na_p} \\
K_{ze} &= \frac{2\overline{F}_{ze}}{Na_p}
\end{align*}
\]

The average cutting force is measured through repeated experiments and different feed rates are used in the sand cutting process. It can be seen from the above formula that the average cutting force in \( x \), \( y \) and \( z \) directions is a linear function of the feed per tooth.

3. Sand model cutting experiment

3.1 The experimental device and test system

As shown in Fig.2, “Digital precision forming machine without pattern casting” produced by China Academy of Machinery Science and Technology Co.Ltd. The machine is specialized in processing sand mold, which is selected for the experiments, and the cutters were a two-tooth diamond flat-end milling cutter(diameter 10mm). The sand mold workpiece is resin sand mixed by resin and adhesives, and the size range of the workpiece is \( 500 \times 500 \times 500 \)(mm).
Fig. 2 Digital precision forming machine and resin sand mold

Fig. 3 shows the connection diagram of the test system. Kistler9257B (the three-directional dynamometer) is fixed on the sand workpiece, Kistler5070A (multi-channel charge amplifier) was used to transform the dynamometer charge signal into a voltage signal, and Kistler5697 (the data acquisition card) is connected to the computer through the USB interface. DynoWare (Data acquisition software) is used to analyze the collected data results.

Fig. 3 The connection diagram of the experimental system

3.2 The experimental scheme
Patternless casting forming machine of model CAMTC-SMM2000S, which has the maximum spindle speed 18,000 rpm. Slot milling is used fixed spindle speed of 4500 rpm, axial depth of cut of 2mm, radial width of cut of 6.5mm, the feed per tooth was linearly changed, respectively 0.133mm/z, 0.200mm/z, 0.267mm/z, 0.333mm/z, and 0.400mm/z. The cutting parameters are shown in the Table 1:

| No. | Spindle speed \( n \) (r/min) | Axial width of cut \( a_y \) (mm) | Radial width of cut \( a_x \) (mm) | Feed rate \( v_f \) (mm/min) | Feed per Tooth \( a_f \) (mm) |
|-----|-------------------------------|---------------------------------|---------------------------------|----------------------------|-----------------|
| 1   | 4500                          | 3                               | 6.5                             | 1200                      | 0.133           |
| 2   | 4500                          | 3                               | 6.5                             | 1800                      | 0.200           |
| 3   | 4500                          | 3                               | 6.5                             | 2400                      | 0.267           |
| 4   | 4500                          | 3                               | 6.5                             | 3000                      | 0.333           |
| 5   | 4500                          | 3                               | 6.5                             | 3600                      | 0.400           |

4. Experimental results and analysis
The graphs of dynamic cutting forces were obtained by dynamometer. The average cutting force in the \( X \) and \( Y \) directions is calculated based on the graphical data obtained from the above experimental measurements. The linear varying state between the average cutting force and the feed per tooth is shown
According to Eq.6, The linear regression function can be derived as in Eq.8, showing the linear relationship between the average cutting force and the feed per tooth.

\[
f_x = 37.1986a_f + 5.3909
\]
\[
f_y = 38.3808a_f + 5.1677
\]

Linear regression graph in Fig.4 showing the \(x\)- and \(y\)-directional cutting change in average cutting force with feed per tooth.

![Fig.4 Linear regression graph](image)

When these cutting force data are substituted into Eq.7, the cutting force coefficients can be calculated as in Eq.9:

\[
K_x = \left[4F_{xy}/(Na_f)\right] = 25.5872 (N/mm^2)
\]
\[
K_y = \left[4F_{yx}/(Na_f)\right] = 24.7991 (N/mm^2)
\]

To verify the correctness of cutting force coefficient, by changing the cutting depth, the cutting depth is changed to 2mm, and other parameters remain unchanged, the the relationship between the cutting force and the feed per tooth, as shown in Table 3:

| No. | Spindle speed \(n\) (r/min) | Axial width of cut \(a_o\) (mm) | Radial width of cut \(a_e\) (mm) | Feed rate \(V_f\) (mm/min) | Feed per Tooth \(a_f\) (mm) | Average force \(F_x\) (N) | Average force \(F_y\) (N) |
|-----|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 1   | 4500                        | 2                          | 6.5                         | 1200                        | 0.133                       | 6.07                        | 6.07                        |
| 2   | 4500                        | 2                          | 6.5                         | 1800                        | 0.200                       | 7.61                        | 8.47                        |
| 3   | 4500                        | 2                          | 6.5                         | 2400                        | 0.267                       | 9.36                        | 9.88                        |
| 4   | 4500                        | 2                          | 6.5                         | 3000                        | 0.333                       | 11.06                       | 12.26                       |
| 5   | 4500                        | 2                          | 6.5                         | 3600                        | 0.400                       | 12.46                       | 13.78                       |

A comparison of the cutting force data in Table 2 and the cutting force data in Table 3 reveals that when the spindle speed is 4500r/min and the axial depth of cut is 2mm and 3mm, the cutting force value in the Table 2:
measured in the $X$ and $Y$ direction rises proportionally to the increase in the axial depth of cut, with the ratios of the cutting force value to the feed per tooth being almost the same.

![Linear regression graph of cutting force in the $X$ direction and $Y$ direction when $a_p=2\,mm$ and $a_p=3\,mm$](image)

The cutting force coefficient when $a_p=2\,mm$ can be calculated as in Eq.10:

$$K_c = \frac{4F_{xc}}{(Na_p)} = 28.7983 \,(N/mm^2)$$

$$K_{rc} = \frac{4F_{xc}}{(Na_p)} = 25.4563 \,(N/mm^2)$$

From Fig.5, it can be seen from the identification results that the cutting force coefficients by different cutting depths are basically the same.

5. Conclusion

Based on the force analysis of the infinitesimal sand cutting unit, a series of slot milling experiments for resin sand have been performed. The experimental results show that the cutting force coefficient of sand model proved to be constant in the average cutting force model. When the axial depth of cut is the same, the linear functions of cutting force have the same slope. In other words, the gradient of change is basically the same. When the spindle speed is the same, the cutting force values measured in $X$ and $Y$ directions rise proportionally to the increase in the axial depth of cut. When the spindle and the axial depth of cut are the same, the cutting force increases linearly with the increase of feed per tooth. It can be seen that the cutting force coefficient mainly depends on the material of the workpiece and has little relation with the cutting parameters. According to above research, the cutting force prediction model was established, which laid a foundation for optimization of parameters and cutting chatter.

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