Quantum theory – essential from cosmos to consciousness

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Abstract. Quantum theory is the most successful physical theory. About one third of the gross national product in the developed countries results from its applications. But very often quantum theory is still declared as “crazy” or “not understandable”. However, quantum theory has a clear mathematical structure that expresses well-known experiences from every day life: A whole is often more than the sum of its parts, and not only the facts also the possibilities can act. If such structures become important then the consequences differ from the models of classical physics which rests on the fundamental differences between matter and motion, material and force, localization and extension, fullness and emptiness. From quantum theory one can learn that all these differences are useful in many cases but are not fundamental. There are equivalences between them, and these can be extended even to the equivalence between matter, energy and abstract quantum information. It is cosmological funded and is denominated as “Protyposis” to avoid the connotation of information and meaning. Protyposis enables a fundamentally new understanding of matter which can be seen as “formed”, “condensed” or “designed” abstract quantum information. One result of the Protyposis is a derivation of Einstein’s equations from the abstract quantum information. Another consequence is the ontological reality of the mind and its connection to a brain which can be explained without any dualistic model.

1. Introducing quantum theory
Quantum theory is the most successful physical theory. About one third of the gross national product in the developed countries results from its applications. The applications of this theory range from nuclear power to all tools for computing, laser, solar cells and so on. Up to now no border for its range of validity has been found. Quantum theory has a clear mathematical structure, so a student can it learn in a short time.

What are quanta? “Quantum” is a genus, like “fruit”. Precisely, there is never “fruit” in a bag, because seen in a detailed way there are apples, bananas, grapes etc. The same applies to quantum theory.

It seems to be useful to divide the quantum objects into three classes. Material quantum objects, like electrons, protons and so on and also macroscopic quantum systems have a rest mass, and therefore they can be at rest somewhere. Energetic objects like photons have no rest mass, so they have to move always with the velocity of light. Very interesting are structural quanta. Phonons, the quanta of vibrations of sound in solids, cannot appear outside of the solid body; nevertheless they are real in a certain sense. They interact with other quantum objects and are indispensable for an understanding of the structure of solid bodies. Also the quarks and gluons belong to the structural quanta. They also are indispensable for an understanding of the structure and properties of hadrons, even if they do not exist as free particles outside the hadrons. Also quantum bits represent essential structural quanta and also cannot exist in a localized form without a carrier.
Quantum theory has developed in an enormous manner in the last century. Today it comprehends much more than only good old quantum mechanics.

There are some essential steps in the evolution of this theory. The year 1900 is considered as the birth of quantum theory because Planck discovered the quantum of action. Einstein was the first one who discloses in 1905 the basic structure of quantum theory with his hypothesis of light quanta. In 1925 Heisenberg invented quantum mechanics. It was soon developed further by Schrödinger, Dirac and many other people. Already in 1930 Heisenberg and Pauli made some first approaches towards a quantum theory of force fields. The last essential step further can be seen in the fifties. In 1955 Weizsäcker started to think of a universal theory of abstract quantum information. Later on Finkelstein, Wheeler and others followed.

There exist some popular but insufficient ideas about the distinctions between quantum physics and classical physics. One misleading distinction is on the difference of microphysics and macrophysics. It is true that in microphysics only quantum theory is applicable; nevertheless there are also many macroscopic quantum phenomena. The difference between discontinuous and continuous effects should be another one. This neglects that there are also many operators in quantum physics that have continuous spectra. Popular but false is also the distinction between a fuzzy quantum theory and a sharp classical physics. It neglects that quantum theory does supply us with the most accurate description of nature we ever had. Classical physics lives with the illusion of exactness. Its mathematical basis stands on the assumption of an “arbitrarily smooth change” for any variable. This is a precondition for calculus, but it is by no means always fulfilled in nature. At very high precision in any case the quantum structure will become important because quantum physics is the physics of preciseness.

Often there is no need for the precision of quantum theory. At first sight, most of the processes in nature do appear smooth. But strictly speaking, all actions are quantized; they appear only in “numbers” or “steps”. Therefore one can say that all changes are strictly speaking quantum jumps. So the quantum jump is the smallest nonzero change in nature. Perhaps that makes this concept so attractive in politics and economics.

**2. What is the central structure for an understanding of quantum theory?**

To explain the central structure of quantum theory one has to look on the way in which composite systems are constructed.

For classical physics the composition of a many body system happens in an additive way. The state space of the composed system is the direct sum of the state spaces of the single particle systems. This results in a “Lego world view” of smallest building blocks – one or another kind of “atoms”. The world has to be decomposed into ultimately elementary objects – which never change – and the forces between such objects. This picture about the structure of reality was generally accepted for more than two millennia.

Composite systems in quantum physics are constructed in a fundamentally different way. The state space of a composed quantum system is the tensor product of the state spaces of the single particle systems.

To explain quantum theory we have to start with this structural difference. But “tensor product” is a very technical concept. Is it possible to relate it to something known in everyday life?

Here is to remind that “relations” create a product structure. One can say that the new states of a composed object are the relational structures between the states of its parts. Therefore quantum theory can be characterized as the physics of relations and appears as a clear mathematical implementation of a familiar life experience: A whole is often more than the sum of its parts.

Relational structures create networks, in the essence they are plurivalent. In such a net many different connections between two goals are possible. This explains a further characterization of quantum theory as “the physics of possibilities”. Quantum theory deals with possibilities only. As one consequence this explains that also possibilities can act – and not only the facts. If in a double slit experiment quantum particles have the possibility of going through both holes without controlling them then one will find behind the holes more than two bunches. If there is a controlling of the slits and thereby a change does happen from possibilities into facts then only two bunches will result. This is in the same manner as in every day life where controlling restricts possibilities and thereby influences human behavior.
3. The indissoluble relation between classical physics and quantum physics: The dynamic layering-process

Since the first days of quantum mechanics Bohr has insisted that classical physics is a precondition for speaking about quantum results. It is impossible to ignore that for mankind, there exist not only possibilities but also facts. Therefore for a good description of nature we need both parts of physics, classical physics and quantum physics. Its connection can be described as a “dynamic layering-process”. By going to the classical limit a quantum theoretical description changes into a classical one, by the process of quantization classical physics goes over to quantum physics.

About quantization there exist many concepts (a good overview can be found in [1]), so one may ask whether there is a hope to find a simple fundamental structure for it?

If we ignore, meanwhile, the canonical quantization then a general structure does appear. From a bit with its two states \{0,1\} to a qubit the quantization happens by constructing all the complex valued functions over two points, resulting in a two-dimensional complex space \(\mathbb{C}^2\). Path integral quantization can also be interpreted as constructing all functions over the classical pathways. At a first look second quantization seems to be different. But the construction of a state of the quantum field by states of quantum particles has the same structure as the construction of an analytical function from powers of the variable itself. The analytical functions are dense e.g. in the continuous or the measurable functions, even distributions can be explained as boundary values of analytical functions. So the analytical functions represent in some sense “all functions”, and a quantum field can be interpreted as “the functions over the quantum particles”.

Therefore quantization can be distinguished by the sentence: Quantization is the transition from the manifold of the facts to the possibilities over the facts – that are the functions over the facts. It seems possible to formulate: “Quantization is (actually) second quantization.” In this sense Einstein’s invention of photons was the first demonstration of quantization.

But how to interpret the canonical quantization from mechanics to quantum mechanics which is characterized by one or the other form of polarization, of a bisection of variables?

The explanation can be given as follows: Mechanics is a limit theory of quantum mechanics in such a way that all operators commute. In quantum mechanics the position operator acts multiplicatively on wave functions over position space and the momentum operator acts multiplicatively on wave functions over momentum space. A duplication from \(\Phi(x)\) or \(\Psi(p)\) to \(R[x,p]\) enables the commutation of position and momentum. Therefore classical mechanics acts over positions and momenta, polarization reverses this duplication.

The core of quantization can be explained by the sentence: Quantization is the transition from the facts to the relational network of possibilities which could develop from the facts – or in a more mathematical language – which span a linear space of functions over the facts.

A further characterization is: The quantization of a system is the transition from its nonlinear description in a low dimensional space having many or infinitely many degrees of freedom (e.g. classical mechanics or electromagnetic fields) to a linear description in an infinite dimensional space of many or infinitely many systems with few degrees of freedom (e.g. quantum bits respectively photons or other field quanta). This reminds of the exponential map and its conversion of products, which are nonlinear, into sums, which are linear.

It seems to be evident that quantum theory is the foundation for classical physics. For all of the objects that are handled so successfully by classical physics its existence can be understand only by quantum theory. It may be recalled that the existence of an atom is forbidden by classical electrodynamics. On the other hand classical physics is a precondition for the appearance of quantum properties. The quantum properties of a system become clear only if its entanglement with the environment is cut of. Such a cut can be modeled in classical physics only.

4. Quantum theory relativized distinctions

Quantum theory is based on every day experiences indeed, but in non-living nature they become essential only at a very high precision. At such a precision, however, effects may appear that are not so evident in every day experiences of the world around us.

Already in school the so-called wave-particle-duality is mentioned. In quantum theory depending on the circumstances one and the same quantum object can act as a wave or as a particle, as a localized or as an extended object. Quantum theory shows up that transformation between matter and motion or
between force and material is possible. Of course, Einstein’s famous formula $E = mc^2$ was found in special relativity, but in any experiment the pure quantum concept of antimatter is included. The distinction of force and material appears only as the difference between quanta of integer or half-integer spin. Dirac’s model shows that even the distinction between emptiness and plenitude is relativized by quantum theory. The central philosophical aspect of second quantization is that the distinction between object and attribute depends on the context. A quantum particle is an object in quantum mechanics and is an attribute of a quantum field.

5. The quantum theoretical equivalence of matter, energy and quantum information
In addition, quantum theory discloses a completely new perspective on the three entities matter, energy and abstract quantum information. Already since 1955 C. F. v. Weizsäcker has speculated on the possibility of founding physics on abstract quantum information. His “Ur-Theory” grows up from the intention [2] „Physik ist eine Erweiterung der Logik“ (Physics is an extension of logics). As the basis for a new reduction he has proposed quantized binary alternatives – Ur-Alternativen or Urs. Werner Heisenberg wrote on Weizsäcker’s concept [3] „…daß die Durchführung dieses Programmes ein Denken von so hoher Abstraktheit erfordert, wie sie bisher, wenigstens in der Physik, nie vorgekommen ist. « Ihm, Heisenberg, »wäre das sicher zu schwer«, aber Weizsäcker solle es mit seinen Mitarbeitern unbedingt versuchen. (… that the realization of this program “requires thinking at such a high degree of abstraction that up to now – at least in physics – has never happened.” For him, Heisenberg, “it would be too difficult”, but Weizsäcker and his coworkers should definitely do so.)

Why was for a long time no better resonance on this project? The first reason may be that it was far too abstract, and further that there were too little relations to experience. At this times the values that Weizsäcker has proposed have overburden the imagination of the physicists. That one proton is $10^{40}$ bit is till now hardly to communicate in physics. But a serious problem was that, at this time, the models were inconsistent with general relativity.

An essential step was to go beyond the Urs. Weizsäcker proposes [4] „Ein »absoluter« Begriff der Information hat keinen Sinn” (An »absolute« value of information is meaningless). But this is a contradiction to his claim [5]: “Materie ist Information” (Matter is information). Because matter has an absolute value for equivalence the information has to have one as well.

Therefore, there was a need to make the concept of »information« so abstract that neither emitter nor receiver and – of capital importance – also no meaning or knowledge is primarily included. This is a basic precondition for the equivalence of matter and information. To fulfill this it was necessary to show up the connection to modern theoretical und empirical structures of physics, especially to Bekensteins und Hawkins’s theory of the entropy of black holes and to cosmology. Physics is more than an »extension of logics« and in physics information is different from destination or meaning or knowledge. Meaning has always a subjective aspect as well, so meaning cannot be a basis for science and objectivity.

If quantum information will become a basis for science it must be introduced as abstract quantum information, free of meaning. This is the Protyposis. It provides a base for a new understanding of the world that ranges from matter to consciousness. The Protyposis adds to $E=mc^2$, the equivalence of matter and energy, a further formula:

$$N = m c^2 t_{cosmos} 6\pi/\hbar.$$  

A mass $m$ or an energy $mc^2$ is equivalent to a number $N$ of qubits. The proportionality factor contains $t_{cosmos}$, the age of the universe. Today a proton is $10^{41}$ qubits.

The Protyposis opens new insights into the relations between quantum information and group theory. Two examples will be given. Weizsäcker [4, 5] and Drieschner [6] proposed that the three dimensional position space is a consequence of quantum theory – but their models disagreed with general relativity. This can be repaired by group theoretical considerations. Relativistic particles were represented by irreducible representations of the Poincaré group. Such representations can be constructed in a process of second quantization from quantum bits.
6. Cosmology and quantum bits
That the position space is a consequence of the symmetry for quantum bits was first proposed by Weizsäcker. An object is represented by all the decisions that are possible about it. Any decision that can be scientifically decided can be reduced to quantum bits. The states of a quantum bit are represented by its symmetry group. The symmetry group for a quantum bit is spanned by the groups $SU(2)$ and $U(1)$ and the complex conjugation. The essential part of the symmetry group for a quantum bit is the $SU(2)$, a three parameter compact group. Any number of quantum bits can be represented in the Hilbert space of measurable functions over the $SU(2)$, which is a $S^3$. This three dimensional space $S^3$ is identified with the three dimensional position space. This was already explained by Weizsäcker and Drieschner, but the interpretation of the qubits as “the smallest generally thinkable objects” (“die kleinsten überhaupt denkbaren Objekte”) [7] has set the qubits in a strong connection to spacelike atomistic associations. This has hindered the developing of the theory into a structure compatible to general relativity.

The connection to general relativity was established in [8, 9 and 10].

On a first sight it is not so obvious that a qubit is primarily a cosmic structure, but all localization properties arose only in connection with a material carrier and without carrier no localization. A qubit corresponds to a two-dimensional representation of $SU(2)$. Such a representation can behold as a subrepresentation of the regular representation. Then any state of a qubit is given by a function having only one knot surface on the $SU(2)$, the three-dimensional sphere $S^3$. Such a function divides the $S^3$ into two parts. Many qubits are represented by the tensor product of the two-dimensional representations of the $SU(2)$. Such a product representation can be reduced into irreducible representations. In this reduction scheme more localized functions appear. Therefore only many qubits can represent something that is localized on the $S^3$.

For an easier understanding we remind to the sinus. It divides the circle into two halves. With products of many sin-functions strongly localized functions can be constructed. The same is valid for the qubits.

Suppose there are $N$ qubits in the cosmos. The tensor product of the $N$ two-dimensional representations $D_{1/2}$ can be decomposed. The Clebsch-Gordan-series of the reduction into irreducible representations has the form

$$\left(2D_{1/2}\right)^{\otimes N} = \bigoplus_{j=0}^{[N/2]} \frac{N!(N+1-2j)}{(N+1-j)!j!} \left(\sqrt{N}^{-2j}\right) D_{\sqrt{N}^{-j}}.$$

The factors of multiplicities

$$f(j) = \frac{N!(N+1-2j)}{(N+1-j)!j!}$$

grow almost linearly from $f(N/2) = O(2^N N^{-3/2})$ for the largest wavelength $D_0$ respectively $D_{1/2}$ to its maximum at $D_{\sqrt{N}}$ with a value

$$f((1/2)(N - \sqrt{N})) = O(2N N^{3/2}) = (\sqrt{N}) O(2N N^{3/2}) = (\sqrt{N}) f(N/2).$$

Afterwards an exponential degree follows until the value $I$. For large $N$ the maximum is very sharp and the functions with a wavelength smaller than $\sqrt{N}$ almost never occur and can be ignored. By this group theoretical definition a smallest length on the $S^3$ is introduced, which will be identified with the Planck-length $\lambda_0$.

7. A cosmological model from the Protopysis
If it is acknowledged that the $S^3$ can be identified with position space, then with three physical plausible assumptions a cosmological model follows. [11]

- The energy of a quantum system is inversely to its extension.
The first law of thermodynamics is correct: 
\[ dU + pdV = 0. \]

There exists a universal and distinguished velocity: \( c \).

Because one qubit has a wavelength proportional to the radius \( R \) of the \( S^3 \)

\[ R = \lambda_0 \sqrt{N} \]

its energy must be proportional to

\[ 1/R = 1/\sqrt{N}. \]

Then the \( N \) qubits have an energy proportional to \( N/R = \sqrt{N} = R \). This means that the total energy \( U \) is proportional to \( R \).

The volume \( V \) of a \( S^3 \), the “surface” of a four dimensional sphere, is \( V = 2\pi^2 R^3 \). If one chooses the units for energy in such a way that \( U = 2\pi^2 R \), then the energy density behaves like \( \mu = 1/R^2 \).

From the first law of thermodynamics \( dU + pdV = 0 \) it follows

\[ 2\pi^2 (dR + p 3 R^2 dR) = 0 \]

or

\[ p = -1/(3 R^3) \]

and the state equation for qubits results as

\[ p = -\mu/3. \]

The energy-momentum-tensor is \( T^i_k = \text{diag}(\mu, \mu/3, \mu/3, \mu/3) \).

If there is – as supposed – a distinguished velocity \( c \), then it is natural that the cosmic radius \( R \) increases proportional to time \( t_{\text{cosmos}} \)

\[ R = c t_{\text{cosmos}}. \]

So for the Hubble-parameter \( H = (dR/dt)/R \) it follows

\[ H = 1 / t_{\text{cosmos}}. \]

The resulting cosmological model is a Friedman-Robertson-Walker-cosmos with the line element

\[ ds^2 = dt^2 - R^2(t) \left[ 1 - r^2 \right]^{-1} dr^2 + r^2 d\Omega^2 \]

with

\[ R(t) = c t. \]

The \( S^3 \)-cosmos expands with velocity of light.

From this metric the Einstein-tensor \( G^i_k \) can be computed, then

\[ G^i_k \sim T^i_k = \text{diag}(\mu, \mu/3, \mu/3, \mu/3). \]

If it is demanded that the proportionality between \( G^i_k \) and \( T^i_k \) should be conserved also for local variations of the energy then Einstein’s equations of general relativity appear as a consequence of the abstract quantum information.

The Protopysis can appear as matter and/or energy and/or vacuum and/or as a no-particle substance, that today is declared as dark energy.

\[ T^i_k (\text{Protopysis}) = T^i_k (\text{matter}) + T^i_k (\text{light}) + T^i_k (\text{vacuum}) + T^i_k (\text{no particles/dark energy}). \]

The energy density of the Protopysis is denoted by \( \mu \), the energy density of matter and light by \( \mu_{\text{(matter)}} \) and \( \mu_{\text{(light)}} \) and the energy density of the vacuum by \( \lambda \). The energy density of the vacuum behaves like an effective cosmological term. Because of \( \mu > 0, \mu_{\text{(matter)}} > 0, \mu_{\text{(light)}} > 0 \) it follows for the relation between matter and vacuum \( 2/\lambda \geq \mu_{\text{(matter)}} + \mu_{\text{(light)}} \geq \lambda \). It has the right order of magnitude. There is no restriction for the size of the no-particle form of the Protopysis.
A comparison with data shows that this model is not rejected by the observations. The cosmological model implicates $H_{\text{cosmos}} = 1$. The probability distribution for $H_0$ given by the SN Ia observations is tightly constrained to $0.96 \pm 0.04$, and is an approximating Gaussian curve [12]. And the supernova Data for higher z move towards the Protyposis-cosmology.

8. Relativistic particles from quantum bits

For a precise definition of a particle one has to work in Minkowski space. A relativistic particle is then represented by an irreducible representation of the Poincaré group. Such a representation can be constructed from quantum information by Parabose creation and destruction operators for qubits and anti-qubits (Urs and Anti-Urs) with states running from 1 to 4.

Let be $r,s,t \in \{1,2,3,4\}$, $|\Omega\rangle$ the vacuum for qubits and $p$ the order of Parabose statistics. The commutation relations for Parabose are

$$\left[\hat{a}_r,\hat{a}_s^+\right] = -2\delta_{rs}\hat{a}_t^+ - 2\delta_{rt}\hat{a}_s^+, \quad \left[\hat{a}_r^+,\hat{a}_s\right] = -2\delta_{rs}\hat{a}_t - 2\delta_{rt}\hat{a}_s, \quad \left[\hat{a}_r^+,\hat{a}_s^+\right] = -2\delta_{rs}\hat{a}_t^+$$

$$\left[\hat{a}_r^+,\hat{a}_s^+\right] = 0, \quad \left[\hat{a}_r^+,\hat{a}_s\right] = 0, \quad \left[\hat{a}_r,\hat{a}_s\right] = 0$$

$$\hat{a}_r^+ \hat{a}_s^+ |\Omega\rangle = \delta_{rs} p |\Omega\rangle.$$

The vacuum of Minkowski space $|0\rangle$ is an eigenstate of the Poincaré group with mass, energy, momentum and spin equal to zero. It can be constructed over the vacuum of the qubits $|\Omega\rangle$: [14]
Another example [15, 16] is a massive fermion at rest with mass \( m = p_0 \neq 0 \), momentum \( P_1 = P_2 = P_3 = 0 \), spinz = \(-\frac{1}{2}\), Parabose-order \( p > 1 \):

\[
|0\rangle = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (-1)^{n_1+n_2} \left( \frac{\hat{a}_1^+ \hat{a}_1^* + \hat{a}_2^+ \hat{a}_2^*}{2} \right)^{n_1} \left( \frac{\hat{a}_3^+ \hat{a}_3^* + \hat{a}_4^+ \hat{a}_4^*}{2} \right)^{n_2} |\Omega\rangle.
\]

Matter can be seen indeed as a special form of abstract quantum information. For everyday life an appropriate differentiation may be:

- Matter is inactive, it offers resistance against change.
- Energy can move matter.
- Information can trigger Energy.

9. Conclusions for life and consciousness

The Protoposis enables clear and accessible perceptions for quantum phenomena. Suitable imaginations for quantum behaviour cannot be found among the facts of the objects and the forces, but rather in circumstances, where quantum physics acts on the macroscopic world, where possibilities exist as reality, e.g. in quantum controlling processes of/on something unstable, like our body.

Life is characterized by controlling and timing via quantum information. Only unstable systems can be controlled. Living systems are unstable because they are far from the thermodynamical equilibrium. In the process of self-regulation – which in the progression of biological evolution extend even to consciousness – at organisms quantum effects can become operative macroscopically. Consciousness is quantum information with a living brain as its carrier, quantum information that experiences and knows himself. This is intended not as an analogy but as a physical characterization. Quantum information is of equal reality like e.g. the quarks. Of course, a human personality as we know from daily life needs a living brain as its carrier, but the consciousness of a person is much more than only “a function” of his brain. That consciousness is widely based on quantum physics becomes clear also in many psychic phenomena. We can perceive many possibilities at the same time, we can feel ambivalence, and we can have contradictory thoughts about the same object at the same time, and so on. Controlling and timing of the body happen by mental information. The unconscious is physically seen a form of quantum information just like the consciousness. For the unconscious does not exist – like for any unknown quantum system – no time, no clear and certain „NO“. The ambivalence, a characteristic of any quantum system so to say, is much more expressed in the unconscious than in our conscious thoughts.

At last it should be mentioned that all the experimental tryouts about the information processing at living brains, like fNMR or PET, are based on quantum physics.

Quantum theory offers by the Protoposis an understandable new view of the world, mankind and consciousness. More can be found in the Literature. [17, 18]

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