Mathematical model of information process of protection of the social sector

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Mathematical model of information process of protection of the social sector

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Abstract. In work the mathematical model of information protection of society against distribution of extremist moods by means of impact on mass consciousness of information placed in media is investigated. Internal and external channels on which there is a dissemination of information are designated. The problem of optimization consisting in search of the optimum strategy allowing to use most effectively media for dissemination of anti-terrorist information with the minimum financial expenses is solved. The algorithm of a numerical method of the solution of a problem of optimization is constructed and also the analysis of results of a computing experiment is carried out.

1. Introduction
In modern conditions terrorism became a problem of world international level. The global nature of modern terrorism is caused by development of means of communication, increase, due to various reasons, in migration flows, dense contact of the different communities differing in world outlook, religious and other traditions. Social and religious tension is the fertile field for replenishment of ranks of the terrorist organizations from among the people who haven't found the place in modern society and people with the deformed mentality. At the same time leaders of the terrorist organizations often are not the fanatics, but the people who have got a good education, pursuing the far-reaching aims. Modern terrorism has perfectly learned to use modern technical means of communications (for example, the Internet) for distribution of the ideology and recruitment of followers, has adapted to use the opportunities given by the democratic institutes providing the rights and personal freedom to express the views. The international nature of the terrorism which isn't knowing national borders demands combination of efforts of all countries for fight against him.

The leadership of the Russian Federation pays considerable attention to fight against terrorism. One of the directions of this fight is elimination of the social factors promoting growth of extremist moods, especially among youth further capable to bring them into ranks of terrorists. Other direction is a preventing to distribution of the ideas of the terrorist organizations, recruitment of new members. In the Concept of counteraction to terrorism in the Russian Federation approved as the President of the Russian Federation on October 5, 2009 distribution of the ideas of terrorism and extremism through the information and telecommunication Internet and mass media is called one of the major external factors promoting emergence and distribution of terrorism in the Russian Federation.

Often mass media involuntarily promote distribution of the ideas of terrorism, coming across tricks of the terrorist organizations as it is already noted above, perfectly learned to use freedom to express
the opinion provided by institutes of democracy. For an opportunity to address wide audience terrorists manipulate public opinion, are covered with a mask of fighters against injustice, loosen moral and moral principles of society by substitution of concepts and ideals. Fight against similar terrorist activity demands the relevant activities, work with mass media in the direction of a point of their attention to need of increase in vigilance, professionalism and strengthening of attention to sources of providing information. The activity of terrorist promotion in mass media and the Internet needs to oppose effective counter-propaganda, namely, the purposeful work explaining to the population true tasks and the purposes of terrorism. Except professionals in the field of journalism and work with mass media it is necessary to attract widely the scientific capacity of the country for development of techniques of identification and blocking of distribution of sources of terrorist promotion. Development of methods of identification of information terrorist threats, their prevention and also assessment of a measure of their impact on the population is necessary.

In this work the model of distribution of counterterrorist promotion in society, including, on the Internet is investigated.

2. Theoretical part

2.1. Problem definition

We will consider some social group with a number $N^*$ which members are subject to information influence during the anti-terrorist promotion which is carried out through media on information channels which are important for this group [1-2].

Let $x(t)$ – the number of the persons who have apprehended information distributed by a source by the time $t$ of time, accepted the transferred ideas, norms, etc. We believe that at the initial moment $t = t_0$, $x(t_0) = x_0$.

We will assume that dissemination of information happens on two channels: external and internal.

The channel, external in relation to group, is characterized by size $u(t) \geq 0$ – number (intensity) of equivalent information acts of information transfer in unit of time.

The internal channel is defined by interpersonal communication of members of social group, at the same time for the characteristic of its intensity the parameter $\alpha \geq 0$ (a positive constant) is entered. As a result of such communication the members of the group who have already apprehended information, exerting impact on members yet not subject to information influence, make an own contribution to process of dissemination of information.

In this case the speed of change of number of the members of the group who have accepted the imparted norms is the sum of speeds of external and internal influences. This dependence is expressed by the equation [1]:

$$\dot{x}(t) = (u(t) + \alpha x(t))(N^* - x(t)), \quad t \in [0, T].$$

(1)

We believe that the resource of the operating influence is limited on the set period $[0, T]$:

$$u(t) \leq u_{\text{max}}.$$ 

As criterion of optimality we consider maximizing number of the members of the group who have apprehended information at minimization of costs of these purposes on the fixed period. In this case criterion function will take a form:

$$J(u) = \int_0^T (x(t) - ku(t))^2 \, dt \rightarrow \text{max},$$

(2)

where $k$ – coefficient of cost of external influence (a payment for placement of information in media, financial and time expenditure).

As a result we come to a problem of optimum control:
\[ J(u) = \int_0^T (x(t) - ku(t))^2 \, dt \rightarrow \text{max}, \quad (3) \]

at dynamic restriction:
\[ \dot{x}(t) = (u(t) + \alpha(t))(N^* - x(t)), \quad t \in [0,T], \quad (4) \]

restriction for management:
\[ 0 \leq u(t) \leq u_{\text{max}}, \quad t \in [0,T], \quad (5) \]

entry condition:
\[ x(t_0) = x_0. \quad (6) \]

2.2. Method of solution

We will apply the principle of a maximum of Pontryagin to the decision. Pontryagin's function tasks (3)-(6) have an appearance:
\[ H(t,x,u,p) = -(x(t) - ku(t))^2 + p(t)((u(t) + \alpha(t))(N^* - x(t))) \quad (7) \]

According to the principle of a maximum [3]-[4], the interfaced function \( p(t) \) meets conditions:
\[ \dot{p}(t) = -\frac{\partial H}{\partial x(t)} = \left( \alpha N^* - 2x(t)\alpha - u(t) \right)p(t) - 2k\alpha + 2x(t), \quad t \in [0,T], \quad (8) \]
\[ p(T) = 0 \quad (9) \]

We will enter switching function:
\[ \psi(t) = \frac{2x(t)k - p(t)(N^* - x(t))}{2k^2} - \frac{u_{\text{max}}}{2}, \quad t \in [0,T]. \quad (10) \]

Then optimum control \( \overline{u}(t) \) meets a condition:
\[ \overline{u}(t) = \begin{cases} u_{\text{max}} \psi(t) \leq 0, \\ 0, \psi(t) > 0, \end{cases} \quad t \in [0,T]. \quad (11) \]

The regional problem of the principle of a maximum has an appearance:
\[ \begin{dcases} \dot{x}(t) = (\overline{u}(t) + \alpha(t))(N^* - x(t)), \\ \dot{p}(t) = -\frac{\partial H}{\partial x(t)} = \left( \alpha N^* - 2x(t)\alpha - \overline{u}(t) \right)p(t) - 2k\alpha + 2x(t), \end{dcases} \quad x(t_0) = x_0, \quad p(T) = 0, \quad (12) \]

где
\[ \overline{u}(t) = \begin{cases} u_{\text{max}} \psi(t) \leq 0, \\ 0, \psi(t) > 0, \end{cases} \quad t \in [0,T]. \quad (13) \]

Thus, the solution of the task of optimum control (3)-(6) can be found as the solution of system of differential equations (12)-(13).
In some cases the system of a look (12)-(13) can be solved analytically. However, in the considered task the analytical decision is rather labor-consuming, besides even the most insignificant modification of model (for example, input of additional restrictions for a phase variable $x(t)$) does the decision only by an analytical method difficult. At the same time numerical methods of the decision (a method of a projection of a gradient, the interfaced directions, Newton, etc.) show high efficiency at the solution of the considered class of tasks by methods of nonlinear programming [3-19].

We approximate the considered task (3)-(6) problem of nonlinear programming and we will construct an algorithm of her numerical decision with use of a method of a projection of a gradient.

2.3. Discrete approximation
We will break a piece $[0, T]$, $q$-l point on $q$ partial intervals of length $\Delta t = T / q$, we will put $t^i = i\Delta t$, $t^0 = 0$, $t^q = T$, $x(t^i) = x^i$, $u(t^i) = u^i$, $i = 0, q - 1$.

We calculate target functionality with use of a formula of the left rectangles:

$$ I(u) = \sum_{i=0}^{q-1} \left( x^i - ku^i \right)^2 \Delta t. $$ (14)

For approximation of derivatives we use Euler's formula of the 1st order of accuracy:

$$ \dot{x}(t) \approx \frac{x^{i+1} - x^i}{\Delta t}, \quad x^0 = x_0. $$ (15)

The task approximating an initial problem of optimum control with an accuracy $O(\Delta t)$, has an appearance:

$$ I(u) = \sum_{i=0}^{q-1} \left( x^i - ku^i \right)^2 \Delta t \to \max, $$ (16)

at restrictions:

$$ x^{i+1} = x^i + \left( u^i + \alpha \dot{x} \right) \left( N^* - x^i \right) \Delta t, \quad x^0 = x_0, $$ (17)

$$ 0 \leq u^i \leq u_{\max}, \quad i = 0, q - 1. $$ (18)

We build Lagrange's function:

$$ L(x, c, u, p) = -\sum_{i=0}^{q-1} \left( x^i - ku^i \right)^2 \Delta t + \sum_{i=0}^{q-1} p^{i+1} \left( x^{i+1} - x^i - \left( u^i + \alpha \dot{x} \right) (N^* - x^i) \Delta t \right). $$ (19)

From stationarity conditions: $\frac{\partial L}{\partial x^i}(x, c, u, p) = 0$, $i = 0, q$, we receive recurrence relations for the interfaced variables $p^i$, $i = 0, q - 1$:

$$ p^i = p^{i+1} - \left( (N^* - 2x^i \alpha - u^i) \right) p^i + 2k \alpha + 2x^i \Delta t, \quad i = 0, q - 1, \quad p_0^q = 0. $$ (20)

We calculate derivative functions of Lagrange in the direction $u$:

$$ \frac{\partial L}{\partial u^i} = \frac{2x^i k \Delta t - p^i \left( N^* - x^i \right) \Delta t}{2k^2} - \frac{u_{\max}}{2} \Delta t, \quad i = 0, q - 1. $$ (21)
Taking into account the received formulas the algorithm of the numerical decision can be developed [3].

2.4. Gradient projection method algorithm

1. We set admissible approach of a vector of management \( u^{(0)} \) on a zero step:

\[
0 \leq (u^i)^{(0)} \leq u_{\text{max}}, \quad i = 0, q - 1. \tag{22}
\]

2. We count the initial trajectory corresponding to him: \((x^i)^{(0)}, \quad i = 0, q\), on a formula:

\[
(x^{(i+1)})^{(0)} = (x^{(i)})^{(0)} + \left( (u^{(i)})^{(0)} + \alpha (x^{(i)})^{(0)} \right) \left( N^{*} - (x^{(i)})^{(0)} \right) \Delta t, \quad i = 0, q - 1, (x^{(0)})^{(0)} = 0. \tag{23}
\]

3. We calculate initial value of functionality:

\[
I^{(0)} = -\sum_{i=0}^{q-1} \left( (x^{(i)})^{(0)} - k (u^{(i)})^{(0)} \right)^2 \Delta t. \tag{24}
\]

We define the interfaced variables \( p^i \):

\[
p^i = p^{i+1} - \left( \alpha N^{*} - 2 (x^{(i)})^{(0)} \alpha - (u^{(i)})^{(0)} \right) p^{i+1} + 2k \alpha - 2 (x^{(i)})^{(0)} \Delta t, \quad i = 0, q - 1;
\]

\[
p^q = 0. \tag{25}
\]

4. We set initial value of size of a step of gradient descent \( \alpha^{(k)} \) also we will organize a cycle on steps.

5. We calculate management \((u)^{(k+1)}\), corresponding \((k+1)\) iterations:

\[
(u)^{(k+1)} = (u)^{(k)} - \alpha^{(k)} \frac{\partial L}{\partial u^i} (u)^{(k)} , \quad i = 0, q - 1. \tag{26}
\]

6. For new value \((u)^{(k+1)}\) vector of management we check an admissibility condition:

\[
0 \leq (u^i)^{(k+1)} \leq u_{\text{max}} , \quad i = 0, q - 1. \tag{27}
\]

7. If the condition for some hub \( i \) isn’t satisfied, then we carry out a projection of management to an admissible set.

8. We count the trajectory corresponding to this management \((x)^{(k+1)}\):

\[
(x^{(i+1)})^{(k+1)} = (x^{(i)})^{(k+1)} + \left( (u^{(i)})^{(k+1)} + \alpha (x^{(i)})^{(k+1)} \right) \left( N^{*} - (x^{(i)})^{(k+1)} \right) \Delta t ,
\]

\[
(x^{(0)})^{(k+1)} = 0. \tag{28}
\]

9. We calculate the next approach of criterion function:

\[
I^{(k+1)} = -\sum_{i=0}^{q-1} \left( (x^{(i)})^{(k+1)} - k (u^{(i)})^{(k+1)} \right)^2 \Delta t. \tag{29}
\]
10. We carry out check of a condition of monotony in a method of gradient descent. If the
condition is satisfied: \( I^{(k)} < I^{(k+1)} \), that we reduce a step of gradient descent also we pass to a step 5,
otherwise we believe: \( \alpha^{(k+1)} = \alpha^{(k)} \) also we pass to a step 10.

11. Iterative process continues until the set accuracy of calculations is reached \( \varepsilon \).

12. If \( |I^{(k+1)} - I^{(k)}| < \varepsilon \), we pass that to a step 12, otherwise we believe \( (u)^{(k)} = (u)^{(k+1)} \),
\( (x)^{(k)} = (x)^{(k+1)} \) also we repeat all listed algorithm stages, since a step of the 5th algorithm where \( \varepsilon \) –
the required accuracy of calculation of functionality.

13. \( I^{(k+1)}, (x)^{(k+1)}, (u)^{(k+1)} \) – solution of a task.

3. Computing experiment
The considered algorithm is realized in the environment of Microsoft Visual Studio 2013 in the form
of Windows – the application. The numerical solution of a task at the choice of parameters is
constructed: \( N^* = 100 \), \( u_{\text{max}} = 0.1 \), \( \alpha = 0.001 \), \( b = 0.8 \), \( k = 1 \), \( T = 10 \), \( q = 1000 \), \( \varepsilon = 10^{-5} \). The optimal
solution is presented in figures 1-2.

Figure 1. Schedule \( \bar{u}(t) \).

Figure 2. Schedule \( \bar{x}(t) \).
At the choice of values of parameters: \( N^* = 100 \), \( u_{\max} = 0.1 \), \( \alpha = 0.001 \), \( b = 0.8 \), \( k = 100 \), \( T = 10 \), \( q = 1000 \), \( \varepsilon = 10^{-5} \) the optimal solution presented in figures 3-4 is received.

The solution of a task at values is provided on figures 5-6: \( N^* = 100 \) , \( u_{\max} = 0.1 \), \( \alpha = 0.001 \), \( b = 0.8 \), \( k = 500 \), \( T = 10 \), \( q = 1000 \), \( \varepsilon = 10^{-5} \).
Figure 6. Schedule $x(t)$.

From the analysis of schedules it is possible to draw a conclusion that at increase in a payment for placement of information in media in some timepoint the management which is responsible for distribution on the external channel is turned off and distribution continues only due to personal communications in group. Shutdown of the external channel happens the earlier, than cost is higher. Established facts will quite be coordinated with common sense that emphasizes adequacy of model of the considered situation.

4. Conclusion

Thus, at insignificant costs of placement of information in media strategy with the maximum intensity of external influence is optimum. At increase in cost of placement external influence initially as much as possible, and then is turned off. At the same time the activity of an internal channel of communication between members of social group remains. The results of the solution of a task received in number show high efficiency of the applied computing methods which keep the relevance in various modifications of model (for example, at input of the additional parameters expressing a oblivion of the transferred information, heterogeneity of social group, etc.) In this regard specification of model and also the analysis of productivity of numerical methods at the solution of the considered task is of special interest. The practical value of a research consists in development of the software which can be used at the organization of information protection of society against influence of ideology of extremism and also promotion of other socially important norms.

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