Vertical dissipation profiles and the photosphere location in thin and slim accretion disks

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ABSTRACT

As several authors in the past, we calculate optically thick but geometrically thin (and slim) accretion disk models and perform a ray-tracing of photons in the Kerr geometry to calculate the observed disk continuum spectra. Previously, it was common practice to ray-trace photons assuming that they are emitted from the Kerr geometry equatorial plane, $z = 0$. We show that the continuum spectra calculated with this assumption differ from those calculated under the assumption that photons are emitted from the actual surface of the disc, $z = H(r)$. This implies that a knowledge of the location of the thin disk effective photosphere is relevant for calculating the continuum emission. In this paper we investigate, in terms of a simple model, a possible influence of the (unknown, and therefore assumed ad hoc) vertical dissipation profiles on the vertical structure of the disk and thus on the location of the effective photosphere, and on the observed continuum spectra. For disks with moderate and high mass accretion rates ($\dot{m} < 0.01 \dot{m}_c$), we find that the photosphere location in the inner, radiation pressure dominated, disk region (where most of the radiation comes from) does not depend on the dissipation profile and therefore emerging disk spectra are insensitive to the choice of the dissipation function. For lower accretion rates, the photosphere location depends on the assumed vertical dissipation profile down to the disk inner edge, but the dependence is very weak and thus of minor importance. We conclude that the continuum spectra of optically thick accretion disks around black holes should be calculated with ray-tracing from the effective photosphere and that, fortunately, the choice of a particular vertical dissipation profile does not substantially influence the calculated emission.

Key words. accretion, accretion disks – black hole physics

1. Introduction

There is a consensus that the observed spectra of black hole binaries and active galactic nuclei should be explained by accretion of rotating matter onto black holes. However, the available theoretical models of accretion disks do not provide a sufficiently detailed and accurate description of all physical processes that are relevant. There is neither a quantitative description of the turbulent dissipation that generates entropy and transports angular momentum, nor a fully self-consistent method to deal with the radiative transfer in the accreted matter.

Only partial solutions exist. Magnetohydrodynamic simulations provide useful information on the “radial” (Hawley & Krolik 2002) and “vertical” (Turner 2004) turbulent energy dissipation, but cannot simultaneously deal with the radiative transfer. On the other hand, all existing methods of solving the radiative transfer adopt some simplifying assumptions. Usually, they divide the flow into separate rings of gas and assume that each ring is in hydrostatic equilibrium. They also assume an ad hoc energy dissipation law (e.g. Davis et al. 2005; Rozanska & Madej 2008; Idan et al. 2008). In most cases, radiative transfer with all the important absorption and scattering processes included is implemented on the disk surface, while the deepest parts of the disk are treated in the diffusion approximation. This simplification leads to a problem: the position of the disk photosphere is calculated only very roughly or is not calculated at all. In calculating the spectra, one usually assumes that all the emission takes place at the equatorial plane. However, in principle, ray-tracing routines (e.g. Bursa 2006) should account for the precise location of the emission, especially for moderate and high accretion rates.

The spectrum computed “vertically” for each ring (i.e. at each radius separately) has to be integrated over the disk surface. This calls for the need to know the global “radial” structure of the accretion disk, consistent with the rings’ “vertical” structures adopted in the radiative transfer calculations. Expanding along these lines, we are working on constructing a new vertical-plus-radial code that uses the sophisticated treatment of the radiative transfer from the works mentioned above, and at the same time fully incorporates our new radial, fully relativistic (in the Kerr geometry), transonic slim disk¹ code (described in Sadowski 2009).

This paper addresses two questions concerning the effective photosphere of an optically thick and geometrically thin or slim black hole (BH) accretion disk:

Is the knowledge of the exact photosphere location relevant in calculating the disk continuum spectra?

¹ Slim disk is a transsonic solution of an optically thick accretion disk described by vertically integrated equations with simplified (limited to vertical force balance at the surface) treatment of its vertical structure (Abramowicz et al. 1988).
Is the photosphere location sensitive to details of dissipative processes?

2. The calculated continuum spectra depend on the location of the effective photosphere

We start with a simple demonstration that the calculated thin disk continuum spectra depend on the location of the effective photosphere. For this purpose, we use the slim disk solutions that have been recently recalculated in the Kerr geometry by Sadowski (2009). We choose three particular models with the accretion rates

\[ 0.001 \dot{m}_C, \ 0.3 \dot{m}_C, \ 0.9 \dot{m}_C. \]  

(1)

Here the critical accretion rate \( \dot{m}_C \), defined as

\[ \dot{m}_C = \frac{64\pi G M}{c^2 k_{eff}} = 2.23 \times 10^{18} \frac{M}{M_\odot} \text{s}^{-1}, \]  

(2)

corresponds to the Eddington luminosity of an accretion disk for a non-rotating BH.

Disk shapes, i.e. the functions \( z = H(r) \) describing the vertical half-thickness, are presented in Fig. 1. For the lowest mass accretion rate \( 0.001 \dot{m}_C \), the \( H/r \) ratio for large radii is about 0.01 while for \( \dot{m} = 0.9 \dot{m}_C \) it reaches 0.13. Thus, for the accretion rates considered here, disks are always geometrically very thin,

\[ H(r) \ll r. \]  

(3)

For larger accretion rates, the slim disk thickness could be considerably higher. The models calculated by Sadowski (2009) provide the local flux of radiation \( F(r) \) in the frame of an observer comoving with the disk. In calculating the continuum spectrum, the standard assumptions have been adopted: (i) radiation from the effective photosphere is locally described by a modified black-body with hardening factor \( f = 1.7 \) (Shimura & Takahara 1995); (ii) the flux is limb-darkened by \( F_{\text{out}} \propto 2 + 3 \cos \theta_e \), where \( \theta_e \) is the emission angle relative to the effective photosphere normal vector. We ray-trace photons from the effective photosphere to an observer located 10 kpc away from the central black hole with mass \( M_{BH} = 9.4 M_\odot \) (the mass corresponds to the microquasar 4U 1543-47 whereas the order of magnitude of the distance can be considered typical for all Galactic BH binaries) and compute the observed continuum spectrum. We take into account all relativistic effects in the Kerr geometry (Bursa 2006).

For the three accretion rates (1), we calculate the disk spectrum twice: assuming that the effective photosphere coincides with the actual disk surface \( z = H(r) \), or that it coincides with the equatorial plane \( z = 0 \), as if the disk were infinitesimally thin. This means that in the first case we start ray-tracing from \( z = H(r) \), and in the second case from \( z = 0 \).

The resulting continuum spectra are plotted in Fig. 2. There are obvious differences between the spectra calculated with the two different assumptions about the location of the effective photosphere, i.e. with the photosphere either at \( z = H(r) \), or at \( z = 0 \). Even for the moderate mass accretion rate \( 0.3 \dot{m}_C \), the \( z = 0 \) spectral flux differs from the \( z = H(r) \) flux by about 5% at 20 keV, and for higher accretion rates the difference is much higher. Such differences should certainly be considered as highly significant.

Thus, our result demonstrates that knowing the precise location of the effective photosphere is necessary in calculating the BH slim disk continuum spectra. The effective photosphere lies somewhere between the \( z = H(r) \) and \( z = 0 \) surfaces.

In principle, the photosphere location could strongly depend on details of the vertical dissipation which are still not sufficiently well known and are therefore assumed ad hoc. This would be bad news. In the rest of this paper we argue that such a situation is unlikely.

Using a simple model for the vertical structure of a geometrically thin, optically thick accretion disk, we show that the photosphere location is not highly sensitive to dissipation. Note that for very thin stationary disks the same is true for the total flux \( F(r) \) emitted from a particular radial location – it does not depend on dissipation processes (Shakura & Sunyaev 1973).
These results strengthen one’s confidence in estimating the black hole spin by fitting the observed continuum emission of black hole sources to those calculated theoretically (see e.g. Shafee et al. 2006; Middleton et al. 2006; Gou et al. 2009). However, they also indicate that the fitting procedures should be improved to include ray-tracing from the actual location of the photosphere.

3. A simple model of the vertical structure

Our simple model assumes that the accretion disk is geometrically thin (Eq. (3)) and that one may consider radial and vertical disc structure separately.

3.1. Radial equations

We do not solve radial equations, assuming instead that the rotation is strictly Keplerian (in the Kerr geometry),

$$\Omega = \Omega_K = \pm \sqrt{GM \over r^3} \pm a \sqrt{GM/c^2} = \sqrt{GM \over r^3} \pm \mathcal{B} \Omega_K$$

(4)

and that the flux $F(r)$ follows from the mass, energy and angular momentum conservation (i.e. does not depend on radial dissipation) and is given by the standard formula (Novikov & Thorne 1973; Page & Thorne 1974),

$$F(r) = \frac{3}{8} \frac{GM}{\pi r^5} \frac{Q}{BC^{1/2}}$$

(5)

Here $M$ is the black hole mass, $a$ is its specific angular momentum, and $\mathcal{A}(r, a), \mathcal{B}(r, a), C(r, a), D(r, a), E(r, a), Q(r, a)$ are relativistic correction factors, defined as explicit functions of their arguments in Page & Thorne (1974).

3.2. Vertical equations

We describe the vertical structure of the BH accretion disk in the optically thick regime by the following equations:

(i) The hydrostatic equilibrium:

$$\frac{dP}{dz} = -\rho \kappa \Omega_K^2 G z$$

(6)

where $P$ is the sum of the gas and radiation pressures:

$$P = P_{\text{gas}} + P_{\text{rad}} = k\rho T + \frac{4}{3}aT^4$$

(7)

and $\kappa \Omega_K^2 G z$ is the vertical component of the gravitational force of the central object calculated using the relativistic correction factor:

$$G = \frac{B^2 \mathcal{D} C}{\mathcal{A} R}$$

(8)

(ii) The energy generation equation:

$$\frac{dF}{dz} = \dot{Q}_\text{dis}$$

(9)

where $F$ is the flux of energy generated inside the disk at a rate given by $\dot{Q}_\text{dis}$ – the dissipation rate which will be discussed in Sect. 5.1.

(iii) The generated flux of energy is transported outward through diffusion of radiation and/or convection according to the following thermodynamical relation:

$$\nabla = \frac{\partial l_n T}{\partial l_n P}$$

(10)

where $\nabla$ is the thermodynamical gradient which can be either radiative or convective:

$$\nabla = \nabla_{\text{rad}} \quad \text{for} \quad \nabla_{\text{rad}} \leq \nabla_{\text{ad}}$$

$$\nabla_{\text{conv}} \quad \text{for} \quad \nabla_{\text{rad}} > \nabla_{\text{ad}}.$$  

(11)

The radiative gradient $\nabla_{\text{rad}}$ is calculated using diffusive approximation:

$$\nabla_{\text{rad}} = \frac{3\kappa_P PF}{16\sigma T^4 \kappa_K^2 G z}$$

(12)

where $\kappa_K$ is the mean Rosseland opacity (see Sect. 3.3) and $\sigma$ is the Stefan-Boltzmann radiation constant.

(iv) When the temperature gradient is steep enough to exceed the value of the adiabatic gradient $\nabla_{\text{ad}}$ we have to consider the convective energy flux. The convective gradient $\nabla_{\text{conv}}$ is calculated using the mixing length theory introduced by Paczyński (1969). We take the following mixing length:

$$H_{\text{ml}} = 1.0H_P$$

(13)

with pressure scale height $H_P$ defined as (Hameury et al. 1998):

$$H_P = \frac{P}{\rho \kappa_K^2 G z + \sqrt{P \delta \rho \Omega_K^2}}$$

(14)

The convective gradient is defined by the following formula:

$$\nabla_{\text{conv}} = \nabla_{\text{ad}} + (\nabla_{\text{rad}} - \nabla_{\text{ad}})y(y + V)$$

(15)

where $y$ is the solution of the equation:

$$\frac{9}{4} + \frac{\tau_{\text{ml}}^2}{3 + \tau_{\text{ml}}^2} + Vg^2 + V^2 - V = 0$$

(16)

with the typical optical depth for convection $\tau_{\text{ml}} = \kappa_{\text{eff}}H_{\text{ml}}$ and $V$ given by:

$$V = \frac{3 + \tau_{\text{ml}}^2}{3\tau_{\text{ml}}} \frac{\kappa_P^2 H_{\text{ml}}}{512\sigma^2 T^4 H_P} \left( \frac{\partial \ln \rho}{\partial \ln T} \right) \left( \nabla_{\text{rad}} - \nabla_{\text{ad}} \right).$$

(17)

The thermodynamical quantities $C_P, \nabla_{\text{ad}}$ and $(\partial \ln \rho / \partial \ln T)_P$ are calculated using standard formulae (e.g. Chandrasekhar 1967) assuming solar abundances ($X = 0.70, Y = 0.28$) and taking into account, when necessary, the effect of partial ionization of gas on the gas mean molecular weight.

(v) To close the set of equations describing the vertical structure of an accretion disk, we have to provide boundary conditions. At the equatorial plane ($z = 0$) we put $F = 0$ while at the disk photosphere we require $F = \sigma T^4$ with the flux $F$ given by Page & Thorne (1974).

3.3. Opacities

In this work, we consider optically thick accretion disks. Therefore, we use Rosseland mean opacities $\kappa_K$. The opacities as a function of density and temperature are taken from Alexander et al. (1983) and Seaton et al. (1994). For temperatures and densities out of both domains we interpolate opacities between these
two tables for intermediate values. Following other authors (e.g. Idan et al. 2008), we neglect expansion opacities, as the vertical velocity gradients in stationary thin disks are expected to be insignificant.

4. Numerical method

The set of ordinary differential equations defined in Sect. 3.2 is solved using the Runge-Kutta method of the 4th order. We start integrating from the equatorial plane (z = 0), where we assume some arbitrary central temperature $T_c$, density $\rho_c$ and set $F = 0$. We integrate the given set of equations until we reach the photosphere which is defined as a layer where the following equation is satisfied (corresponding to the optical depth $\tau = 2/3$):

$$\kappa R P = \frac{2}{3} \Omega_0^2 G z$$  \hspace{1cm} (18)

If the temperature $T$ and flux $F$ do not satisfy the outer boundary condition ($F = \sigma T^4$), we tune up the value of the assumed central density $\rho_c$ and integrate again, starting from the equatorial plane. Convergence is usually obtained after a few iteration steps. To obtain the photosphere profile $h(r)$, we look for the disk solutions at a number of radii in the range $r_{\text{ms}} < r \leq 200 M$ with $r_{\text{ms}}$ being the radius of the marginally stable orbit. At each radius, we additionally search for that value of the central temperature which determines the solution with the emitted flux equal to the Novikov & Thorne value.

5. The photosphere location for a few ad hoc assumed vertical dissipation profiles

We apply our simplified model of vertical structure (Sect. 3) to calculate the photosphere location for different dissipation prescriptions. We test four families of dissipation functions described in the following section.

5.1. Dissipation profiles

First we apply the standard $\alpha P$ prescription given by Shakura & Sunyaev (1973). The authors assumed that the source of viscosity in accretion disks is connected with turbulence in gas-dynamical flow, the nature of which was unknown at that time. They based their model on the following expression for the kinematic viscosity coefficient $v$:

$$v = v_{\text{turb}} h_{\text{turb}}$$  \hspace{1cm} (19)

(where $v_{\text{turb}}$ and $h_{\text{turb}}$ stand for typical turbulent motion velocity and length scale, respectively). Shakura & Sunyaev assumed that the velocity of turbulent elements cannot exceed the speed of sound $c_s$ as well as that their typical size cannot be larger that $\text{some size}$.

Taking into account the expression for the $t_{\phi}$ component of the viscous stress tensor in a Newtonian accretion disk:

$$t_{\phi} = \frac{3}{2} \rho v \Omega$$  \hspace{1cm} (20)

and the standard form of the vertical equilibrium:

$$h = \left( \frac{P}{\rho} \right)^{1/2} \left( \frac{\rho}{GM} \right)^{1/2} \approx \frac{c_s}{\Omega}$$  \hspace{1cm} (21)

one can limit the $t_{\phi}$ in the following way:

$$- t_{\phi} \leq \rho \Omega c_s h \approx \rho c_s^2 \approx P.$$  \hspace{1cm} (22)

Therefore, Shakura & Sunyaev (1973) introduced a dimensionless viscosity parameter $\alpha$ satisfying the condition $\alpha \leq 1$:

$$t_{\phi} = - \alpha P.$$  \hspace{1cm} (23)

This expression for viscosity is called the $\alpha$ prescription and has been widely and successfully used for many years in accretion disk theory. In this case, the flux generation rate $dF/dz$ (Eq. (9)), given in the local rest frame by:

$$\frac{dF}{dz} = \sigma (r_{\text{K}})^2 \Omega_0 = - \frac{3}{2} \frac{DB}{C} \Omega_0 K$$  \hspace{1cm} (24)

is expressed as:

$$Q_{\text{dis}} = \frac{dF}{dz} = \frac{3}{2} \frac{DB}{C} \alpha P \Omega_0 K.$$  \hspace{1cm} (25)

In this work, we apply the regular $\alpha P$ prescription for a few arbitrary values of the $\alpha$ parameter: $\alpha = 0.001, 0.01, 0.1, 0.2, 0.5, 1.0$.

In the last decade, several authors (e.g. Sincell & Krolik 1998; Hubeny et al. 2001; Davis et al. 2005; Hui et al. 2005) have applied a constant dissipation rate per unit mass. We follow this investigation by using a few models which have a constant dissipation rate for the flux generation rate:

$$Q_{\text{dis}} = \frac{dF}{dz} = \frac{F_0}{m_0} \rho.$$  \hspace{1cm} (26)

Due to the fact that this expression requires not only the total emitted flux $F_0$ but also the total surface density $m_0$, one has to divide the latter based on other disk solutions. In our work, we use models calculated using the regular $\alpha P$ prescription for different values of $\alpha$.

Recent numerical MHD simulations of stratified accretion disks in the shearing box approximation (Davis et al. 2009) show that dissipation rate can be well approximated by the following formula:

$$Q_{\text{dis}} = \frac{dF}{dz} = \frac{1}{2} \frac{F_0}{\sqrt{m_0}} \frac{1}{\sqrt{m}}.$$  \hspace{1cm} (27)

We apply it to another class of models ($m^{-1/2}$ profile). As in the constant dissipation rate case, we impose surface density profiles obtained using $\alpha P$ models.

Several authors have investigated accretion disks with heat generation concentrated near the equatorial plane or close to the disk surface (e.g. Paczyński & Abramowicz 1982; Paczyński 1980). Therefore, we implement one more class of dissipation prescriptions with an arbitrarily positioned dissipation maximum. We use the following expression:

$$Q_{\text{dis}} = \frac{dF}{dz} = \frac{dF}{dz} \exp^{-\frac{r - r_p}{d P}} = \frac{3}{2} \frac{DB}{C} \alpha P \Omega_0 K \exp^{-\frac{r - r_p}{d P}}$$  \hspace{1cm} (28)

which is the flux generation rate for $\alpha P$ models re-scaled with a Gaussian function centered at $r_0$ with dispersion parameter $d$ given by:

$$z_0/r = d_{\text{max}} \quad d/r = d_{\text{disp}}$$  \hspace{1cm} (29)

where $d_{\text{max}}$ and $d_{\text{disp}}$ are constant for a given disk model.

In Fig. 3 we present dissipation profiles for a few illustrative models. The upper panel presents the flux generation rate...
versus the vertical coordinate. The $aP$ model (solid line) has a bell-shaped dissipation profile with most of the energy generated at $z < 0.5H$. The model with a constant dissipation rate per unit mass (thin dashed line) exhibits a very similar behavior. The dissipation is no longer concentrated around the equatorial plane for the $m^{-1/2}$ model (thick dashed line) – the maximal flux generation is located at $z = 0.4H$. However, the dissipation profile is still extended through the whole disk thickness. The arbitrarily positioned models (dot-dashed lines) show the opposite behavior: the energy generation is concentrated around a given arbitrary location. In the case of the models presented in Fig. 3, the dissipation is confined to altitudes close to the equatorial plane ($z = 0$) and $z = 0.5H$.

Another point of view on the dissipation profiles is presented in the bottom panel of Fig. 3. The profiles are plotted against the fractional mass surface density $m/m_0$. The right edge of the figure ($m = m_0$) corresponds to the equatorial plane. Two families of models, with dissipation profiles constant and proportional to $m^{-1/2}$ are represented by straight lines (they are power-law functions of $m$). As in the upper plot, one can notice a similarity between the $aP$ and constant dissipation rate models. The maximum of the flux generation rate for the arbitrarily positioned model centered at $z = 0.5H$ corresponds to the mass depth $m = 0.1m_0$. Obviously, the model centered at the equatorial plane has its maximal dissipation rate at $m = m_0$.

![Fig. 3. Dissipation profiles for a few representative models at $r = 20M$.](image)

The upper panel presents dissipation functions versus the vertical coordinate $z$ while the bottom panel the former versus surface density $m(z) = \int_0^H \rho(z') dz'$. In both panels, left and right edges correspond to the disk surface and equatorial plane, respectively. The models represent four families: $aP$ (A3, solid line), constant dissipation rate per unit mass (B2, thin dashed line), $m^{-1/2}$ profile (C2, thick dashed line) and arbitrarily positioned dissipation function (D1 and D5, dash-dotted lines).

### Table 1. Model assumptions for $m = 0.1m_\odot$.

| Model | Model family | $\alpha$ | $d_{\text{max}}$ | $d_{\text{isp}}$ | $a^*$ |
|-------|--------------|---------|-----------------|------------------|-----|
| A1    | $aP$         | 0.001   | --              | --               | 0.0 |
| A2    | $aP$         | 0.01    | --              | --               | 0.0 |
| A3    | $aP$         | 0.1     | --              | --               | 0.0 |
| A4    | $aP$         | 0.2     | --              | --               | 0.0 |
| A5    | $aP$         | 0.5     | --              | --               | 0.0 |
| A6    | $aP$         | 1.0     | --              | --               | 0.0 |
| AS1   | $aP$         | 0.01    | --              | --               | 0.0 |
| AS2   | $aP$         | 0.1     | --              | --               | 0.0 |
| AS3   | $aP$         | 0.5     | --              | --               | 0.0 |
| B1    | constant diss. rate | 0.01 | --              | --               | 0.0 |
| B2    | constant diss. rate | 0.1 | --              | --               | 0.0 |
| B3    | constant diss. rate | 0.5 | --              | --               | 0.0 |
| C1    | $m^{-1/2}$   | 0.01    | --              | --               | 0.0 |
| C2    | $m^{-1/2}$   | 0.1    | --              | --               | 0.0 |
| C3    | $m^{-1/2}$   | 0.5    | --              | --               | 0.0 |
| D1    | arbitrarily positioned | 0.1 | 0.0 0.001 | --               | 0.0 |
| D2    | arbitrarily positioned | 0.1 0.02 | 0.001 | --               | 0.0 |
| D3    | arbitrarily positioned | 0.1 | 0.0 0.001 | --               | 0.0 |
| D4    | arbitrarily positioned | 0.1 | 0.0 0.001 | --               | 0.0 |
| D5    | arbitrarily positioned | 0.1 | 0.0 0.0005 | --               | 0.0 |
| D6    | arbitrarily positioned | 0.1 | 0.0 0.005 | --               | 0.0 |

$^a$ Details of model assumptions are given in Sect. 5.1. 

#### 5.2. The photosphere location

We test a number of models representing all four classes of dissipation profiles for a moderate mass accretion rate $m = 0.1m_\odot$. Details of model parameters are given in Table 1. The location of the disk photospheres is presented in Fig. 4. The first four panels present results for four classes of dissipation profiles in the case of a non rotating BH. A common behaviour is clearly visible: all photosphere locations coincide at radii shorter than $30M$. Outside this radius, the location of the photosphere depends on the assumed dissipation profile and varies between $H/r \approx 0.03$ for the $aP$ model with the highest value of $\alpha = 1.0$ and $H/r \approx 0.06$ for the lowest value $\alpha = 0.001$. It is of major importance to understand why all photosphere profiles coincide in the inner region, which corresponds to the radiation pressure and electron scattering dominated regime of an accretion disk. The location of the photosphere is defined by Eq. (18). Wherever the radiation pressure and electron scattering dominate in a disk, the left hand side of that formula depends only on the temperature, which is connected at the photosphere to the outgoing flux as $F = \sigma T^4$. Therefore, the formula for the energy flux (Eq. (5)) uniquely determines the location of the photosphere in the inner region of a disk independently of the assumed dissipation function. Under these assumptions, we get from Eq. (18):

$$H = \frac{2F_{\text{keq}}}{\Omega_k^2 G c}$$  \hspace{1cm} (30)

which describes perfectly the photosphere profiles presented in Fig. 4 for $r < 30M$.
general in accretion disks that exhibit radiation pressure dominated regimes. This statement is of paramount importance due to the fact that most of the emission from an accretion disk takes place (in the non-rotating case) between 6 and 20 $M$. As we have proven, for such radii the photosphere location should not depend on an assumed dissipation profile and, therefore, the spectrum is also likely to be independent of dissipation profile. However, one cannot make sure of this without performing full ray-tracing, which would correctly account for flux emitted from dissipation dependent regions. The results of this procedure will be discussed in the following section.

As Shakura & Sunyaev (1973) have shown, for the lowest mass accretion rates the gas dominated region of an accretion disk may extend all the way to the inner edge of a disk. For such a case, the behavior described in the previous paragraph is not expected: the photosphere profile should depend on the dissipation function at all radii. To account for this fact, we compare photosphere profiles of accretion disks with low mass accretion rates ($\dot{m} = 0.1\dot{m}_C$) for the non-rotating case. The BH mass was $9.4 M_\odot$. Model parameters are given in Table 1.

![Fig. 4. Photosphere profiles for different model families for $\dot{m} = 0.1\dot{m}_C$. The uppermost panel presents profiles for the $\alpha P$ models, the second one for models with constant dissipation rate per unit mass, the third one for the $m^{-1/2}$ dissipation profile. The fourth is for arbitrarily positioned dissipation while the bottommost panel is for $\alpha P$ models in the case of a rotating BH ($a^* = 0.9$). The BH mass was 9.4 $M_\odot$. Model parameters are given in Table 1.](image)

The emerging spectrum should be insubstantial. This is studied in detail in the following section.

5.3. Continuum spectra

To assess the importance of different vertical dissipation profiles on emerging continuum spectra we calculate the latter based on photosphere profiles obtained using our model. For comparison, we also examine spectra calculated assuming emission from the equatorial plane ($z_{em} = 0$) as well as from the photosphere surface ($z_{em} = H$) for non-rotating ($a^* = 0$) and highly-spinning ($a^* = 0.9$) BHs. The bottom panel presents the ratio of the spectral flux (defined as above) calculated from the photosphere to the flux calculated from the equatorial plane for the models with extreme photosphere profiles (see Sect. 5.2). The inclination angle $i = 60^\circ$.

![Fig. 5. Photosphere profiles for $\alpha P$ models for three different mass accretion rates ($\dot{m} = 0.10, 0.05$ and $0.01\dot{m}_C$) and two values of the $\alpha$ parameter (0.1 and 1.0). The regions where the photosphere profiles for a given accretion rate coincide are radiation pressure dominated. The BH was non-rotating with $M = 9.4 M_\odot$.](image)

![Fig. 6. Emerging continuum spectra calculated for different models. The upper panel presents the number of photons per energy unit crossing a unit surface every second assuming $d = 10$ kpc. Spectra calculated assuming emission from the equatorial plane ($z_{em} = 0$) as well as from the photosphere surface ($z_{em} = H$) are presented for non-rotating ($a^* = 0$) and highly-spinning ($a^* = 0.9$) BH. The bottom panel presents the ratio of the spectral flux (defined as above) calculated from the photosphere to the flux calculated from the equatorial plane for the models with extreme photosphere profiles (see Sect. 5.2). The inclination angle $i = 60^\circ$.](image)
extracting more energetic photons. Spectra for all the photosphere profiles almost coincide and differ significantly from those calculated assuming emission from the equatorial plane. This is clearly visible in the lower panel where we plot the ratio of spectral flux emitted from the photosphere to that emitted from the equatorial plane for the two most extreme photosphere profiles (compare Fig. 4) for each value of BH angular momentum. The spectral profiles are almost identical at energies between 2–20 keV. One may conclude that for accretion disks with sufficiently high accretion rate to form the inner radiation pressure supported region, the spectra depend only insignificantly on the vertical dissipation profile in the most interesting range of energies (2.5 ÷ 20 keV; Gou et al. 2009). As was discussed in Sect. 5.2 for the lowest mass accretion rates (<0.05 $\dot{m}_C$), the inner, radiation pressure dominated disk region never appears and the photosphere location depends on the dissipation profile down to the disk inner edge. As it has been pointed out, such a regime of accretion rates implies small values of the H/R ratio. To assess the impact of the dissipation profiles on the continuum spectra, we plotted ratios of corresponding spectral profiles for low and very low mass accretion rates (Fig. 7). The upper panel presents the ratios as observed by an observer perpendicular to the disk plane. The ratios approach 1 with decreasing mass accretion rate. The convergence is very slow due to the fact that the disk thickness in the disk outer region (gas pressure and free-free scattering dominated) depends weakly on the mass accretion rate ($H \propto m^{3/20}$, Shakura & Sunyaev 1973). The difference in spectral flux between models with $m = 0.01 \dot{m}_C$ and 0.0001 $\dot{m}_C$ is no greater than 0.1% at 20 keV. For an inclined observer (the bottom panel), the departures from the equatorial plane model are much more significant (up to 5% at 20 keV). However, the difference between the spectral fluxes for the models with the two lowest mass accretion rates (corresponding to $H/R = 0.007$ and 0.015, respectively) is still of the order of 1% or even smaller. We conclude that in the lowest accretion rate regime, changes of the photosphere location caused either by different mass accretion rates or different dissipation profiles are insignificant. However, one has to keep in mind that even for mass accretion rates as low as 0.0001 $\dot{m}_C$, taking into account the non-zero photosphere location is important for inclined observers as the spectral flux at high energies approaches the equatorial plane limit very slowly.

6. Discussion

The results presented here are very encouraging in the context of fitting the calculated optically thick continuum spectra of slim accretion BH disk to the observed continuum emission. Our results imply that the major uncertainties of the accretion disk models, in particular the vertical dissipation profiles, have a rather small influence on the calculated continuum spectra of steady disks with low accretion rates. This is good news for those who use spectral fitting to estimate the spin of black holes.

One should have in mind, however, that there are several effects that have not been taken into account in our simple model, and more theoretical work is needed before fully believable methods of calculating slim disk continuum spectra will be at hand. One of the most obvious (and also quite challenging) theoretical developments needed is a self-consistent and simultaneous solution of the vertical and radial structures of slim, optically thick black hole accretion disks. We are currently working on this problem.

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Fig. 7. Ratios of spectral profiles (photosphere to equatorial) for models with low mass accretion rates for the perpendicular (i = 0°, upper panel) and i = 60° (lower panel) observers.