Weak phase $\alpha$ from $B^0 \to a_1^+ (1260) \pi^\mp$

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Quasi two-body decays $B^0(t) \to a_1^+ (1260) \pi^\mp$ identified by four charged pions determine a phase $\alpha_{\text{eff}}$, which is equal to the weak phase $\alpha$ in the limit of vanishing penguin amplitudes. Applying SU(3) flavor symmetry to these decays and to $B \to a_1 K$ and $B \to K_1 \pi$, with $K_1$ an admixture of $K_1(1270)$ and $K_1(1400)$, we derive expressions providing bounds on $\alpha - \alpha_{\text{eff}}$. Higher precision in $\alpha$ may be achieved by an overall fit to a complete set of SU(3) related measurements. A method is sketched applying isospin symmetry to time-dependent invariant mass distributions in $B \to \pi^+ \pi^- \pi^0 \pi^0$.

I. INTRODUCTION

Hadronic $B$ decays from $\bar{b} \to \bar{u} u d \bar{d}$ transitions provide the most direct information about the weak phase $\alpha \equiv \arg(-V_{td} V_{tb}^*/V_{td} V_{ub}^*)$, governing the interference between $B^0 - \bar{B}^0$ mixing and $B$ decay amplitudes in these transitions. The current determination of $\alpha$ from time-dependent CP asymmetries in $B^0 \to \pi^+ \pi^-$, $B \to \rho^\mp \pi^\mp$ and $B \to \rho^\mp \rho^\pm$ involves a combined error at a level of $10^\circ$ \cite{1,2}.

Information on $\alpha$ can also be extracted from time-dependent decays $B^0(t) \to a_1^+ (1260) \pi^\mp$ \cite{3,4}. Recently the Babar \cite{3} and Belle \cite{4} collaborations have reported branching ratio measurements for these processes, where final states were identified through four charged pions,

$$B(B^0 \to a_1^+ (1260) \pi^\mp) = \left\{ \begin{array}{ll}
(40.2 \pm 3.9 \pm 3.9) \times 10^{-6} & \text{[5]}, \\
(48.6 \pm 4.1 \pm 3.9) \times 10^{-6} & \text{[6]}.
\end{array} \right. \hspace{1cm} (1)$$

These values, where charge-conjugation averaging is implied for initial and final states, are in agreement with a calculation based on naive factorization \cite{7}.

The difficulty in extracting $\alpha$, common to all the above modes, is the presence of subleading penguin amplitudes with a different weak phase than that of the dominant tree amplitudes. This difficulty can be overcome by using symmetries, either isospin \cite{8,9} or approximate SU(3) flavor \cite{10,11}. Applications of these symmetries to $B^0 \to a_1^+ \pi^\mp$ resemble applications to $B \to \rho \pi$, data, where isospin symmetry in a Dalitz plot analysis \cite{12,13} and flavor SU(3) for quasi two-body decays \cite{14,15} have already been used to determine $\alpha$.

An essential point in applying isospin to time-dependent decays into multibody final states is the existence of a final state which is common to several resonant channels having some overlap in phase space. This permits measuring relative phases between decay amplitudes for distinct resonant channels in $B^0$ and $\bar{B}^0$ decays. In $B \to \rho \pi$, decays to the final state $\pi^+ \pi^- \pi^0 \pi^0$ involve interference of $B^0(\bar{B}^0) \to \rho^+ \pi^-, B^0(\bar{B}^0) \to \rho^- \pi^+$ and $B^0(\bar{B}^0) \to \rho^0 \pi^0$ in the three corners of the Dalitz plot \cite{12,13}.

In contrast, one can readily show that in $B \to a_1 \pi$ ($a_1 \to \rho \pi$) the final state $\pi^+ \pi^- \rho^+ \rho^-$, common to $B^0(\bar{B}^0) \to a_1^+ \pi^-, B^0(\bar{B}^0) \to a_1^+ \pi^+ + B^0(\bar{B}^0) \to a_1^0 \pi^0$, does not involve an overlap between the $a_1^+$ resonance bands and the $a_1^0$ resonance band. Each of the three resonant amplitudes does interfere with the dominantly longitudinal amplitudes \cite{14} for $B^0(\bar{B}^0) \to \rho^+ \rho^-$. (In Ref. \cite{4} cuts on $B^0 \to a_1 \pi$ were suggested to eliminate this interference.) Thus, in principle, a fit for $B^0(t) \to \pi^+ \pi^- \pi^0 \pi^0$ combining contributions from $B \to a_1 \pi$ and $B \to \rho \rho$ could permit measuring relative phases between the three $B^0 \to a_1 \pi$ amplitudes. The absence of a penguin amplitude in the $\Delta I = 3/2, I = 2$ linear combination of these amplitudes \cite{14} would then enable an extraction of $\alpha$ \cite{15}.

While in this respect the situation seems similar to $B \to \rho \pi$, one would face in this rather complex analysis the challenges of an additional $\pi^0$ in the final state and of an uncertainty in the $a_1$ resonance shape.

The purpose of this Brief Report is to propose an easier measurement of $\alpha$ in time-dependent decays $B^0(\bar{B}^0) \to a_1^+ (1260) \pi^\mp$ with four charged pions in the final state, which is the cleanest signal channel to reconstruct. We will study these decays in the quasi two-body approximation within a complete set of SU(3) related processes. While we will follow an analogous study of $B^0(t) \to \rho^+ \pi^-$ \cite{14}, a modification is required by the fact that $K_{1A}$, the SU(3) partner of $a_1(1260)$, is a mixture of two mass eigenstates, $K_1(1270)$ and $K_1(1400)$ \cite{17}.

In Section II we define decay amplitudes and time-dependent decay rates for $B^0(\bar{B}^0) \to a_1^+ \pi^\mp$ in terms of tree and penguin contributions, noting a measurable quantity $\alpha_{\text{eff}}$ which equals $\alpha$ in the limit of van-
ishing penguin contributions. Section III derives upper bounds on \( \alpha - \alpha_{\text{eff}} \) in terms of branching ratios for \( B \to a_1 K, B \to K_1(1270) \pi \) and \( B \to K_1(1400) \pi \). Section IV concludes, suggesting a determination of \( \alpha \) using an overall parameter fit to a complete set of SU(3) related observables.

II. AmpLitudes and time-Dependence

We borrow notations and conventions from Ref. [14]. \( B^0 \) decay amplitudes involve subscripts denoting the charge of the \( a_1 \), while \( B^0 \) amplitudes into charge conjugate states are denoted by \( \overline{B}^0 \) with the same subscripts,

\[
A_+ \equiv A(B^0 \to a_1^+ \pi^-), \quad A_- \equiv A(B^0 \to a_1^- \pi^+) \,
\]

\[
\overline{A}_+ \equiv A(\overline{B}^0 \to a_1^- \pi^+), \quad \overline{A}_- \equiv A(\overline{B}^0 \to a_1^+ \pi^-). \tag{2}
\]

The four decay amplitudes can be expressed in terms of a “tree” amplitude \( t \) and a smaller “penguin” amplitude \( p \). We adopt the c-convention, in which the top-quark has been integrated out in the \( b \to d \) penguin transition and unitarity of the CKM matrix has been used to move a \( V_{ub} V_{ud} \) term into the tree amplitude. We write

\[
A_\pm = e^{i\gamma} t_\pm + p_\pm, \quad \overline{A}_\pm = e^{-i\gamma} t_\pm + p_\pm, \tag{3}
\]

where dependence on the weak phase \( \gamma \) is displayed explicitly, while \( t_\pm \) and \( p_\pm \) contain strong phases.

Time-dependent decay rates for initially \( B^0 \) decaying into \( a_1^0 \pi^\mp \) are given by [18]

\[
\Gamma(B^0(t) \to a_1^0 \pi^\mp) = e^{-\frac{1}{2}(C \pm \Delta C) \cos \Delta mt - (S \pm \Delta S) \sin \Delta mt}, \tag{4}
\]

where

\[
C \pm \Delta C = \frac{|A_\pm|^2 - |\overline{A}_\pm|^2}{|A_\pm|^2 + |\overline{A}_\pm|^2}, \tag{5}
\]

and

\[
S \pm \Delta S = \frac{2 \text{Re}(e^{-2i\gamma} \overline{A}_\pm A^*_\pm)}{|A_\pm|^2 + |\overline{A}_\pm|^2}. \tag{6}
\]

Here \( \Gamma \) and \( \Delta n \) are the average \( B^0 \) width and the neutral \( B \) mass difference, respectively. For initially \( \overline{B}^0 \) decays, the \( \cos \Delta mt \) and \( \sin \Delta mt \) terms in (4) have opposite signs. Thus, time-dependence in these decays determines the four quantities, \( S \pm \Delta S, C \pm \Delta C \).

We now define two phases which coincide with \( \alpha \) in the limit of vanishing penguin amplitudes [14], [18],

\[
\alpha_{\text{eff}}^\pm = \frac{1}{2} \text{arg} \left( e^{-2i\gamma} \overline{A}_\pm A^*_\pm \right). \tag{7}
\]

Whereas these two phases cannot be measured separately, their algebraic average \( \alpha_{\text{eff}} \) is measurable [14]:

\[
\alpha_{\text{eff}} \equiv \frac{1}{2} \left( \alpha_{\text{eff}}^+ + \alpha_{\text{eff}}^- \right) = \frac{1}{4} \left[ \arcsin \left( \frac{S + \Delta S}{\sqrt{1 - (C + \Delta C)^2}} \right) + \arcsin \left( \frac{S - \Delta S}{\sqrt{1 - (C - \Delta C)^2}} \right) \right]. \tag{8}
\]

The two shifts \( \alpha - \alpha_{\text{eff}}^\pm \) are expected to increase with the magnitudes of the corresponding penguin amplitudes, \( |p_\pm| \). The shifts may be expressed in terms of \( |p_\pm|, \gamma \) and corresponding CP-averaged rates and CP asymmetries in \( B^0 \to a_1^0 \pi^\pm \),

\[
\mathbf{\Gamma}(a_1^0 \pi^\mp) = \frac{1}{2} (|A_\pm|^2 + |\overline{A}_\pm|^2), \quad \mathbf{A}_{\text{CP}}^\pm = \frac{|A_\pm|^2 - |\overline{A}_\pm|^2}{|A_\pm|^2 + |\overline{A}_\pm|^2}. \tag{9}
\]

One finds [14], [19],

\[
\cos 2(\alpha - \alpha_{\text{eff}}^\pm) = 1 - 2|p_\pm|^2 \sin^2 \gamma / \mathbf{A}_{\text{CP}}^\pm. \tag{10}
\]

III. BOUNDS ON \( \alpha - \alpha_{\text{eff}} \) FROM FLAVOR SU(3)

The corrections \( \alpha - \alpha_{\text{eff}}^\pm \) caused by the penguin amplitudes \( p_\pm \) may be bounded by relating the decays \( B^0 \to a_1^0 \pi^\pm \) with corresponding \( \Delta S = 1 \) decays, \( B \to a_1 K \) and \( B \to K_1 a_1 \), where \( K_1 a_1 \) is a nearly equal admixture of the \( K_1(1270) \) and \( K_1(1400) \) resonances [17]. The bounds are effective because of a relative factor \( \lambda^2, \lambda = 0.23 \) between the ratios of penguin-to-tree amplitudes in \( \Delta S = 0 \) and \( \Delta S = 1 \) processes. The bounds become more restrictive for small values of \( |p_\pm|/|t_\pm| \). For instance, branching ratios for \( B \to a_1 K \) and \( B \to K_1 a_1 \) which are not much larger than \( B(a_1^0 \pi^\pm) \) would imply generically \( |p_\pm|/|t_\pm| \ll 1 \) (see discussion below), similar to what has been observed in \( B^0 \to \rho^0 \pi^0 \) [14].

Applying flavor SU(3) to \( B^0 \to a_1^0 \pi^\pm \) we will make two approximations, neglecting \( \Delta S = 1 \) annihilation amplitudes which are formally \( 1/m_t \)-suppressed [20], and neglecting nonfactorizable SU(3) breaking corrections when relating \( \Delta S = 0 \) and \( \Delta S = 1 \) amplitudes. Since we expect \( |p_\pm|/|t_\pm| \) to be small, these approximations have only a second order effect on the extracted value of \( \alpha \).

We start by discussing upper bounds on \( |\alpha - \alpha_{\text{eff}}| \) following from decay rates for \( B^+ \to a_1 K^0 \). Under the above-mentioned approximation one has

\[
A(B^+ \to a_1^+ K^0) = -\langle \lambda \rangle^{-1} f_K f_\pi p_-, \tag{11}
\]

\[
A(B^0 \to a_1^- K^+) = f_K f_\pi \langle \lambda \rangle^{-1} (\lambda^2 - e^{i\gamma}) \langle M \rangle, \tag{12}
\]

where \( \lambda = |V_{us}|/|V_{ud}| = |V_{cd}|/|V_{cs}| = 0.23, \) and \( f_\pi, f_K \) are decay constants [17]. We define two ratios of CP-averaged rates for these processes and for \( B^0 \to a_1^0 \pi^\pm \),
multiplied by $\mathcal{X}^2$,
\[
\mathcal{R}_+ \equiv \frac{\mathcal{X}^2 f_+ T(a_1^- K^0)}{f_K^2 T(a_1^- \pi^+)} , \quad \mathcal{R}_- \equiv \frac{\mathcal{X}^2 f_- T(a_1^+ K^0)}{f_K^2 T(a_1^- \pi^+)} .
\] (13)

Superscripts and subscripts denote the charges of the $B$ meson and the $a_1$ meson in the denominator. These definitions lead to bounds on $|p_-|/|t_-|$ as mentioned above,
\[
\frac{\sqrt{\mathcal{R}_-}}{1 + \sqrt{\mathcal{R}_-}} \leq \frac{|p_-|}{|t_-|} \leq \frac{\sqrt{\mathcal{R}_-}}{1 - \sqrt{\mathcal{R}_-}} ,
\] (14)
\[
\frac{\sqrt{\mathcal{R}_-} - \mathcal{X}^2}{1 + \sqrt{\mathcal{R}_-}} \leq \frac{|p_-|}{|t_-|} \leq \frac{\sqrt{\mathcal{R}_-} - \mathcal{X}^2}{1 - \sqrt{\mathcal{R}_-}} .
\] (15)

Eqs. (14)–(15) imply immediately
\[
\cos 2(\alpha - \alpha_{eff}^-) = \frac{1 - 2\mathcal{R}_- \sin^2 \gamma}{\sqrt{1 - (\mathcal{A}_{CP})^2}} ,
\] (16)
and therefore
\[
|\sin(\alpha - \alpha_{eff}^-)| \leq \sqrt{\mathcal{R}_-} \sin \gamma .
\] (17)

The CP-averaged rate for $12$ obeys $14$, $21$, $K^0(1400)$, and consequently,
\[
\cos 2(\alpha - \alpha_{eff}^-) \geq \frac{1 - 2\mathcal{R}_0^-}{\sqrt{1 - (\mathcal{A}_{CP})^2}} ,
\] (18)
and therefore
\[
|\sin(\alpha - \alpha_{eff}^-)| \leq \sqrt{\mathcal{R}_0^-} .
\] (19)

Similar considerations can be applied in order to obtain upper bounds on $|\sin(\alpha - \alpha_{eff}^-)|$ in terms of ratios of rates involving $K_{1A}$, the strange quark model $3P_1$ partner of $a_1$,
\[
\mathcal{R}_{+A} \equiv \frac{\mathcal{X}^2 f_+ T(K_{1A} \pi^+)}{f_K^2 T(a_1^- \pi^+)} , \quad \mathcal{R}_{-A} \equiv \frac{\mathcal{X}^2 f_- T(K_{1A} \pi^-)}{f_K^2 T(a_1^- \pi^-)} .
\] (20)

The SU(3) decompositions of the amplitudes in the numerators are similar to $11$ and $12$,
\[
A(B^+ \to K_{1A} \pi^+) = \frac{-\mathcal{X}}{f_{a_1} f_{K_1}} f_K ,
\] (21)
\[
A(B^0 \to K_{1A} \pi^-) = \frac{f_{K_1}}{f_{a_1}} (-\mathcal{X}) f_K + e^{i\chi_{K_{1A}}} .
\] (22)

This implies bounds on $|p_+|/|t_+|$ similar to $12$ and $16$ with $\mathcal{R}_{+0}$ replaced by $\mathcal{R}_{+A}$. Instead of $16$ and $19$ one now has
\[
|\sin(\alpha - \alpha_{eff}^-)| \leq \sqrt{\mathcal{R}_{+A}} \sin \gamma ,
\] (23)
\[
|\sin(\alpha - \alpha_{eff}^+)| \leq \sqrt{\mathcal{R}_{+A}} .
\] (24)

We now discuss upper bounds on $\mathcal{R}_{+A}$ and $\mathcal{R}_{-A}$ in terms of physical processes involving the mass eigenstates $K_1(1270)$ and $K_1(1400)$. The state $K_{1A}$ is an almost equal admixture of these states,
\[
K_{1A} = \cos \theta K_1(1400) + \sin \theta K_1(1270) ,
\] (25)
with $33^\circ < \theta < 57^\circ 22$, while the orthogonal state, the strange SU(3) $^1P_1$ partner of $b_1(1236)$, is
\[
K_{1B} = -\sin \theta K_1(1400) - \cos \theta K_1(1270) ,
\] (26)
This implies
\[
A(B^+ \to K_{1A} \pi^+) = \cos \theta A(K_1^0(1400) \pi^+) + \sin \theta A(K_1^0(1270) \pi^+) ,
\] (27)
where the two pure penguin amplitudes, identified by their final states, involve an arbitrary relative strong phase. Thus, one has an upper bound on $\mathcal{R}(K_{1A} \pi^+)$,
\[
\mathcal{R}(K_{1A} \pi^+) \leq \sqrt{\frac{\mathcal{R}(K_1^0(1400) \pi^+)}{\sin \theta \sqrt{\mathcal{R}(K_1^0(1270) \pi^+)}}} ,
\] (28)
which determines an upper bound on $\mathcal{R}_{+A}$ defined in $20$.

A similar expression holds for an upper bound on $\mathcal{R}_{-A}$, in terms of the mixing angle $\theta$ and CP-averaged decay rates for $B^0 \to K_{1A}^+(1400) \pi^-$ and $B^0 \to K_{1A}^+(1270) \pi^-$. This bound can be shown to hold in spite of the fact that these processes involve both penguin and tree amplitudes.

Finally, one combines the two separate upper bounds on $|\alpha - \alpha_{eff}^-|$ and $|\alpha - \alpha_{eff}^+|$ to obtain a bound on $|\alpha - \alpha_{eff}|$,
\[
|\alpha - \alpha_{eff}| \leq \frac{1}{2}(|\alpha - \alpha_{eff}^-| + |\alpha - \alpha_{eff}^+|) .
\] (29)

IV. CONCLUSION

We have studied the extraction of $\alpha$ from time-dependent decays $B(t) \to a_1^\pm(1260)\pi^+$ in the quasi two-body approximation. The four observables, $S \pm \Delta S$ and $C \pm \Delta C$, determine the angle $\alpha_{eff}$ in Eq. $8$ up to a fourfold discrete ambiguity. A twofold ambiguity in $\alpha_{eff}$ may be resolved either by other constraints on $\alpha$, or by assuming that the two added angles on the right-hand side of Eq. $8$ differ by much less than $180^\circ$. This follows from $|\arg(|t_-|/|t_+|)| < 90^\circ$, valid to leading order in $1/m_b$ $21$, and an assumption of small $|p_+|/|t_+|$, testable through relations such as Eqs. $14$ and $15$.

We have used flavor SU(3) to obtain upper bounds $17$, $19$, $23$ and $24$ on $|\alpha - \alpha_{eff}^\pm|$. This requires measuring CP-averaged rates for either $B^+ \to a_1^+ K^0$ or $B^0 \to a_1^- K^+$ and CP-averaged rates for either $B^+ \to K_{1A}^0(1270) \pi^+$ and $B^+ \to K_{1A}^0(1400) \pi^+$ or $B^0 \to K_{1A}^+(1270) \pi^-$ and $B^0 \to K_{1A}^+(1400) \pi^-$. The resulting
upper bound, Eq. (29), assumes an unknown relative sign between \( \alpha - \alpha_{\text{eff}} \) and \( \alpha - \alpha_{\text{eff}}' \). In \( B^0 \to \rho^\pm \pi^\mp \) these two shifts are expected to have opposite signs because \( |p_{\pm}|/t_{\pm} \) are small and \( \arg(p_{\pm}/t_{\pm}) \) lie in opposite hemispheres [22], as shown in a global SU(3) fit to \( B \to VP \) decays [22] and in QCD factorization including \( 1/m_b \)-suppressed terms [22]. This reduces the bound on \( |\alpha - \alpha_{\text{eff}}| \) in \( B^0 \to \rho^\pm \pi^\mp \) by a factor of two [22]. It is unclear whether a similar argument holds in \( B^0 \to \pi^0 \pi^- \). Instead of using SU(3) to obtain upper bounds on \( |\alpha - \alpha_{\text{eff}}| \) one may perform a fit to all the observables in \( B^0(t) \to \rho^\pm \pi^\mp \) and in SU(3) related modes. This study is expected to reduce errors and to resolve ambiguities in \( \alpha \), as has been shown in the case of \( B^0 \to \rho^\pm \pi^\mp \) by performing a \( \chi^2 \) fit [11]. Since the states \( K_{1A} \) and \( K_{1B} \) in [26] and [28] mix, a complete set of processes includes also the decay \( B^0 \to b_1^+(1235)\pi^- \) described by amplitudes \( t_{b_1^+} \) and \( p_{b_1^+} \) in analogy with [3]. Information from \( B^0 \to b_1^+(1235)\pi^- \) is not needed since the corresponding \( \Delta S = 1 \) decays \( B \to b_1 K \) are unrelated to \( B \to a_1 K \).

Amplitude decompositions are given in Eqs. (9), (11), (12), (21), and (22) and by corresponding expressions for \( A(B^0 \to b_1^+ \pi^-), A(B \to K_{1B} \pi) \), with the replacements \( t_{+} \to t_{b_1^+}, p_{+} \to p_{b_1^+}, K_{1A} \to K_{1B} \). The total number of observables is seventeen, including \( S, C, C' \) and the two CP-averaged rates in \( B^0(t) \to a_1^+ \pi^- \), the CP-averaged rates and asymmetries in \( B^0 \to b_1^+ \pi^-, a_1^- K^+, K_1^-(1400)\pi^-, K_1^+(1270)\pi^- \), and the rates for \( B^+ \to a_1^+ K^0, K_1^+(1400)\pi^+, K_1^0(1270)\pi^+ \). The seventeen observables are described in terms of twelve parameters, the magnitudes and relative phases of \( t_{b_1^+}, p_{b_1^+}, t_{+}, p_{+} \) and the weak phase \( \alpha \). A simplification, \( t_{b_1^+}^+ \approx 0 \), occurs by assuming factorization of tree amplitudes, which holds at leading order in \( 1/m_b \) [21, 25], and by using the \( G \)-parity of \( b_1 \) [20]. This implies a vanishing asymmetry in \( B^0 \to b_1^+ \pi^- \) and a small rate for this process unless \( p_{b_1^+}^+ \) is enhanced by nonperturbative effects.

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