Labor Supply within the Firm

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There is substantial variation in working time even within employer-employee matches, yet estimates of the Frisch elasticity of labor supply can be near zero. This paper proposes a tractable theory of earnings and working time to interpret these observations. Production complementarities attenuate the response of working time to idiosyncratic, or worker-specific, shocks, but firm-wide shocks are mediated by preference parameters. The model can be identified using firm-worker matched data, revealing a Frisch elasticity of around 0.5. A quasi-experimental approach that exploits only idiosyncratic variation would find an elasticity less than half this.

I. Introduction
Fluctuations in labor input occur along two margins. The extensive margin refers to the formation and termination of employment relationships,

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whereas the intensive margin describes the choice of working time conditional on being employed. Recent research has largely focused on the extensive margin.¹

The intensive margin is also active, though. In several European economies, fluctuations in working time per employee are as large as movements in employment (Llosa et al. 2014). In addition, plant-level data from the United States show that working time per person is as variable as employment (Cooper, Haltiwanger, and Willis 2015). This variability can seem puzzling in light of earlier findings showing individual working time to be highly inelastic (Hall 1999; Keane 2011).

This paper presents a tractable framework for studying working time and earnings of heterogeneous workers within a firm. The firm and its employees join in long-term relationships, bound together by the fact that extensive margin adjustments are costly. In this setting, an employee’s working time and earnings reflect her own preferences, the preferences of her colleagues, and the production technology. This framework informs a novel approach to the joint estimation of utility and production function parameters. In addition, our model can shed light on why working time may appear more inelastic in certain circumstances than implied by the utility parameters underlying structural labor supply elasticities.

A key ingredient of the model is that workers are (potentially) complements in production but have heterogeneous preferences over leisure. Under complementarities, variation in a worker’s own idiosyncratic labor supply incentives (i.e., leisure preferences) can yield small changes in working time given any utility parameter. Intuitively, the efficient response of working time is attenuated when one’s effort is not complemented by higher effort among coworkers. On the other hand, firm-level driving forces (i.e., firm productivity or product demand) act, in effect, to coordinate the responses of heterogeneous workers, eliciting the true intertemporal (Frisch) labor supply elasticity. Hence, the model can predict economically significant responses of working time to firm-level variation even if the reaction of individual working time to idiosyncratic events is more tepid.

In section II, we characterize the working time, earnings, and employment decisions in the model. An individual’s working time is set to maximize the surplus of the firm-worker match. The solution represents a balancing of two forces—production complementarities and preference heterogeneity.

¹ This literature includes search and matching models of unemployment (see Rogerson and Shimer 2011) and quantitative models of participation (Chang and Kim 2006; Erosa, Fuster, and Kambourov 2016).
The former acts to synchronize working time adjustments, whereas the latter drives them apart. If complementarities are strong enough, less of the dispersion in preferences passes through to working time.

Under complementarities, idiosyncratic variation is instead accommodated by the earnings bargain, which divides the match surplus. If a worker’s labor input is hardly adjusted despite an increase in her marginal value of time, she must be compensated for the added disutility. Thus, a telltale sign of strong complementarities is that employees’ working time adjustments to preference shocks are compressed relative to their earnings growth.

While we have highlighted the role of preferences, the model also allows for a worker-specific component of productivity. Interestingly, complementarities do not drive such a wedge between the elasticities of working time and earnings with respect to idiosyncratic productivity. Thus, the joint dynamics of working time and earnings depend crucially on the mix of idiosyncratic forces. We show how to use the covariance of working time and wages to distinguish between the two shocks, confirming that preference heterogeneity is especially critical to fitting our data.

The model is “closed” by the firm’s choice of employment demand. Employment adjustments will tend to crowd out working time, underscoring how an active intensive margin must be supported by frictions on the extensive margin. We consider realistic hiring and firing costs that imply the optimal policy takes an $S$ form, which can impart significant inertia to employment (Dixit 1997). The structure and size of adjustment frictions are based on direct evidence from the labor market we study in our empirical application.

To assess our theory, we turn in sections III and IV to a rich employee-employer matched dataset. The Veneto Worker History (VWH) database tracks the universe of workers and firms in the northern Italian region of Veneto from 1982 to 2001. The dataset includes employees’ earnings and days worked for each of their employers. Working days is an active margin: in a given year, more than 50% of workers adjust their days, and among these the typical change is between 10 and 19 days. Using simple regressions, we can apportion variation in days worked and earnings into firm-wide and idiosyncratic, or work-specific, components. These components have clear counterparts in the model and underlie our structural estimation.

In section V, we estimate the model’s parameters using the method of simulated moments (MSM). Our identification strategy relies on observing the distributions of earnings and working time inside firms. A key moment is the variance of working time adjustments within firms relative to the variance of earnings growth (again, inside firms). If this ratio is small, the model

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2 In Dixit (1997), the firm chooses capital and labor. In our case, the factors are distinct types of workers, each of which has a type-specific leisure preference and productivity.
infers that preference shocks, manifest in earnings, are being squeezed out of working time. Therefore, complementarities must be strong or, more exactly, the elasticity of substitution across jobs is low.

Whereas complementarities can be recovered from within-firm variation, the utility parameter behind the Frisch elasticity is more sharply revealed by firm-level fluctuations in working time. Intuitively, an increase in firm productivity elicits higher working time across the board, easing the diminishing returns that a lone worker faces. The remaining deterrent to adjusting working time is the rate of increase in marginal disutility, which can then be inferred from the firm-level response. We find a Frisch elasticity of working time of 0.483. This estimate is near the middle of the range cited by Chetty et al. (2011), who review several novel identification strategies that are relatively robust to idiosyncratic (preference) variation. Our relatively more structural approach is complementary to these efforts.

To close section V, we simulate policy interventions that draw out the implications of complementarities for labor supply. The policy can be interpreted as a temporary change in the labor income tax rate, although its effect is akin, more simply, to a one-off change in the disutility of labor. The policy is targeted to a fraction of a firm’s workforce in one scenario, whereas all workers face it in another. An individual’s working time falls twice as much when all her colleagues face the policy, attesting to the importance of complementarities. Conversely, if we used a model without complementarities to interpret results when the policy targets a fraction of workers, we would infer a Frisch elasticity of 0.2, or almost 60% smaller than our estimate.

Section VI evaluates the robustness of our results, with a focus on assessing our measure of working time. Recall that we observe paid days of work rather than total hours. Reassuringly, measures of days and hours worked in household surveys indicate that fluctuations in the former likely account for the vast majority of variation in the latter.

Finally, section VII concludes with a few remarks on how our framework could be applied, and extended, to consider other topics where complementarities play a role.

Related literature.—Our paper echoes research showing that working time can seem deceptively inelastic if one looks at the wrong driving forces. This point has been made in different contexts, including in Imai and Keane’s (2004) treatment of on-the-job learning and Rogerson and Wallenius’s (2009) model of nonlinear earnings-hours schedules (see also Keane and Rogerson 2012). In each case, variation in the relevant labor supply incentives (i.e., the present value of work in Imai and Keane 2004) elicits significant responses—the Frisch elasticity is nontrivial—even if working time generally varies (far) less than wages.

Second, our paper is reminiscent of a large literature on hours-adjustment frictions. Most recently, Chetty (2012) shows how to bound the Frisch elasticity if the friction can be thought of as a “small” deviation from standard
life cycle theory. (Our estimate is at the top end of Chetty’s range.) Other analyses are cast in more explicit models of hours constraints in which firms offer fixed hours-wage bundles (Altonji and Paxson 1988; Dickens and Lundberg 1993; Chetty et al. 2011). In our model, there are no constraints per se. Rather, coordination of labor supply emerges under complementarities.

Finally, there is a smaller literature that specifically considers the interaction of worker heterogeneity and complementarities. In Deardorff and Stafford (1976) and Yurdagul (2017), there is an outside sector (i.e., self-employment) to which workers move if hours under complementarities deviate too far from what they would choose on their own. Our model instead highlights the scope for renegotiating earnings (rather than necessarily separating) following idiosyncratic shocks. Indeed, this feature is crucial for interpreting certain moments of the data. In a recent empirical contribution, Labanca and Pozzoli (2022) find that a targeted income tax cut elevates working time only in firms with relatively diffuse hours (e.g., where complementarities appear weak). Our model also predicts that narrowly targeted policies may have little impact if there is a strong incentive to coordinate effort.

II. Theory
A. An Illustration

It may be helpful to first sketch a simplified version of the optimal working time problem that can still convey the essential message of the paper. We will relax a number of the restrictions later in this section.

In this labor market, firms and workers are heterogeneous. Firms differ with respect to productivity. Workers have heterogeneous preferences over leisure or, more broadly, different marginal values of time. Any such idiosyncratic variation across workers is a force for diffusion in their labor inputs.

At the firm level, however, suppose that production (potentially) requires the coordination of effort across workers. To formalize this notion, imagine that a firm’s output is produced by the execution of a fixed number \( N \) of jobs. For simplicity, we treat the firm’s workforce as given and assume that each worker performs one job (e.g., the workforce is also \( N \)). The firm’s output, \( \Gamma \), is assumed to be given by

\[
\Gamma = ZG(h) = Z \left( \sum_{j=1}^{N} b(j)^{\rho} \right)^{1/\rho},
\]

3 Whereas hours constraints are generally thought to apply at the firm level, Rogerson (2011) considers a macroeconomic model in which workers at all firms work a uniform level of hours.
4 Labanca and Pozzoli report that changes in hours are also more compressed in firms where the levels of hours are less diffuse. We will tend to emphasize changes in working time because complementarities cannot generally be identified by differences in levels alone, as we discuss in sec. IV.A.
where $Z$ is firm productivity, $b(j)$ is employee $j$’s working time, and $h = \{b(j)\}$ is an $N \times 1$ vector. The key structural parameter in equation (1) is $\rho$, which determines the elasticity of substitution across jobs. A value of $\rho = 1$ implies that jobs are perfect substitutes, whereas $\rho = -\infty$ implies perfect complements. Note that equation (1) exhibits constant returns with respect to $h$ but diminishing returns with respect to any individual $b(j)$.

Assume a worker’s marginal disutility of effort has the form $\xi(j)b(j)^{\varphi}$, where $\varphi > 0$ and $\xi(j)$ encompasses any shift in the marginal value of time of the worker who performs job $j$. For instance, $\xi$ would rise if a worker is needed at home to care for a family member. The utility parameter, $\varphi$, is a key object of interest in our paper. To convey the meaning of our findings for the broader literature, we will refer to $1/\varphi$ as the “implied Frisch elasticity” (even though workers are not wage-takers in our model, as we will see).

The firm and its workers choose an allocation of time $\{b(j)\}_{j=1}^{N}$. We suppose the parties bargain to the efficient outcome whereby each worker’s marginal value of time (outside the firm) is equated to her marginal product (inside the firm): $\xi(j)b(j)^{\varphi} = Z\partial G/\partial b(j)$. Optimal labor input therefore satisfies

$$b(j) = (\Omega(N)Z)^{1/\varphi} \xi(j)^{-1/(\varphi+1-\rho)}, \quad (2)$$

where $\Omega(N) \equiv (\sum_{i=1}^{N} \xi(i)^{-\rho/(\varphi+1-\rho)}(1-\rho)/\varphi)$.

Equation (2) imparts an important lesson about how to identify the (implied) Frisch elasticity, $1/\varphi$. Consider first the response of working time to a change in $\xi(j)$. In general, this elasticity depends on utility ($\varphi$) and production ($\rho$) parameters. The response of $b(j)$ unambiguously reflects $\varphi$ only in the special case of perfect substitutes, $\rho = 1$. If $j$ is complementary to other jobs, a change in $\xi(j)$ may instead have little impact on $b(j)$. Indeed, as $\rho$ declines toward $-\infty$, the response of $b(j)$ becomes increasingly attenuated for any $\varphi$. Intuitively, if complementarities are strong, diminishing returns to one’s own working time sets in rapidly. Even a slight increase in $b(j)$ following a reduction in $\xi(j)$ can be sufficient to drive $j$’s marginal product into line with the lower marginal cost of effort.

Whereas the influence of the Frisch elasticity is obscured in working time variation driven by $\xi(j)$, it clearly shapes the behavior of $b(j)$ in the presence of firm-level shocks, $Z$. When an employee adjusts her working time to a higher $Z$, her colleagues will match her higher effort, erasing the force of diminishing returns. The remaining deterrent to ramping up working time is the rate of increase in the disutility of effort. Thus, the elasticity of working time depends only on $1/\varphi$. This paper emphasizes that this distinction between idiosyncratic (e.g., $\xi$) and firm-wide ($Z$) variation can help shed light on labor supply dynamics.

$^5$ In the first-order condition, the marginal value of wealth is subsumed under $\xi(j)$. See sec. II.C.
In what follows, we expand on this setup along a number of dimensions. First, we demonstrate how to identify $\varphi$ and $\rho$ in the presence of both worker-specific preference and productivity differences. Second, we solve for an earnings bargain and show that while shifts in $\xi(j)$ are weakly passed through to changes in working time, they are more clearly manifest in the dispersion of earnings changes within the firm. Third, we endogenize employment, $N$, by introducing decreasing returns to scale at the firm level. We show how working time is shaped by both the returns to scale and frictions on employment adjustment.

B. The Environment

We now describe in detail workers’ preferences, firms’ production technology, and the structure of the labor market.

Preferences.—Utility is separable in consumption and leisure. In line with section II.A, the disutility from time worked $h$ is given by

$$\xi \nu(h) = \frac{h^{1+\varphi}}{1+\varphi},$$

(3)

where, to recall, $\varphi > 0$ and $\xi$ represents fluctuations in the worker’s marginal value of time.

In general, shifts in $\xi$ can impinge on consumption. To avoid this complication, we assume that each individual belongs to one of many large families (Merz 1995). By pooling members’ earnings, a family insure consumption against member-specific risk. The flow value of working will not depend directly on the degree of risk aversion but only on earnings and the cost of supplying labor, $\xi \nu(h)$ (see eq. [5] and app. C; apps. A–H are available online).

To preserve tractability, we make several simplifying assumptions concerning $\xi$. First, $\xi$ is independently and identically distributed (i.i.d.) across time and workers: at the start of each period, each worker selects anew a $\xi$ from a $K$-dimensional set, $\mathcal{X} \subseteq \mathbb{R}^K$. Second, we assume that types are drawn after hires have been made, but types are perfectly observed thereafter. Accordingly, firm and worker can contract (earnings and working time) on $\xi$. Finally, we assume that labor is divisible and so (ab)use a law of large numbers (Uhlig 1996) to eliminate any noise in the distribution of types within firm: a deterministic share $\lambda_\xi \in (0, 1)$ of a firm’s workforce draws type $\xi \in \mathcal{X}$, where $\sum_{\xi \in \mathcal{X}} \lambda_\xi = 1$ and $(1/K) \sum_{\xi \in \mathcal{X}} \xi$ is normalized to 1.

We revisit these assumptions later in the paper, but a few remarks now are worthwhile. First, the absence of persistence in $\xi$ will have no direct impact on working time or earnings. As we will see, optimal working time is a static condition, and the earnings bargain will take the same form when types are persistent. (We consider implications for the extensive margin in sec. VI.) Second, the assumption of perfect information is clearly stylized, but theories of hidden information are not necessarily consistent with our moments.
For instance, pooling equilibria under hidden information involve a compression of working time and earnings (Levin 2003), whereas the relative variability of earnings is a key feature of the data that emerges naturally within our framework. Finally, the assumption of divisible labor is made to aid analytical tractability.

Production.— Whereas equation (1) assumes that each worker performs a unique job, we now suppose that each type is assigned a unique set of jobs. The organization of production across a discrete number of types allows us to carry over the basic structure of equation (1) even when labor is divisible. Relative to equation (1), we also incorporate worker productivity heterogeneity.

A type is now a pair of preference \( \xi \) and productivity \( \theta \) levels. Analogously to preferences, we assume that productivity is i.i.d. and drawn from a \( L \)-dimensional set \( Y \subseteq \mathbb{R}^L \).\(^6\) A fixed share \( \lambda_\theta \) of the initial workforce will have productivity \( \theta \), so the fraction with pair \( (\xi, \theta) \) is \( \lambda_{\xi,\theta} = \lambda_\xi \lambda_\theta \). Note that in total there are \( M = K \times L \) pairs or types.

The efforts of heterogeneous types are combined to produce final output. Total labor input of a type \( (\xi, \theta) \) is \( n_{\xi,\theta} h_{\xi,\theta} \), where \( n_{\xi,\theta} \) is the measure of that type’s employment and \( h_{\xi,\theta} \) is the average supply of time among workers of that type. The type-specific labor inputs are aggregated via a constant elasticity of substitution production function,

\[
\Gamma = ZG(h, n) = Z \left( \sum_{\xi,\theta} \sum_{\gamma \in Y} (\theta n_{\xi,\theta} b_{\xi,\theta})^\alpha \right)^{1/\rho},
\]

where \( \rho \) again reflects the elasticity of substitution across jobs, \( n \equiv \{n_{\xi,\theta}\} \) and \( h \equiv \{b_{\xi,\theta}\} \) are \( M \times 1 \) vectors, and \( \alpha \in (0,1) \) is the returns to scale.\(^7\) The departure from constant returns \( \alpha < 1 \) ensures a well-defined notion of firm size, \( N = \sum_{\xi,\theta} n_{\xi,\theta} \). Note that since \( \alpha < 1 \), the limiting case of perfect substitutes refers to \( \rho = \alpha \); e.g., \( \rho \) must satisfy \( \rho < \alpha \). Finally, recall that \( Z \) is firm productivity, which will follow a first-order Markov process.

Equation (4) is a reduced-form structure, but we believe it gets the big picture right. Specifically, it captures the notion that the production of final output requires different jobs to be performed by different workers, each of

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\(^6\) Chang et al. (2020) instead consider correlated draws of preference and productivity shocks in order to match their moments on working time and wages. We can fit the comovement of these variables when \( \xi \) and \( \theta \) are i.i.d. (see sec. V.B).

\(^7\) We continue to interpret \( (1 - \rho)^{-1} \) as the elasticity of substitution across jobs, not across types per se. A simple example illustrates how eq. (4) can “inherit” \( \rho \) from a more primitive production function. Firm output \( \Gamma \) is an aggregate over jobs \( j \in [0,1] \), \( \Gamma = Z\kappa \int [\gamma(j) d\gamma]^{1/\rho} \), where \( \kappa = K^{(1-\rho)/\rho} \) is a normalizing constant and output of job \( j \), \( \gamma(j) \), is proportional to total person-hours. If types are allocated an equal share, \( 1/K \), of jobs, then \( \gamma(j) = Kn_{\xi,\theta} b_{\xi,\theta} \) for any \( j \) performed by type \( (\xi, \theta) \). Substituting for \( \gamma(j) \) in \( \Gamma \) yields eq. (4).
whom faces her own idiosyncratic circumstances (e.g., $\xi$). In reality, the fineness of this division of labor surely reflects more primitive forces, such as the costs of coordinating activities across jobs (Becker and Murphy 1992) and training workers on complex tasks (Costinot 2009). Nevertheless, these factors are likely to change slowly as new ideas and technologies are gradually developed, whereas we will be interested in year-to-year fluctuations in earnings and working time. Thus, a more explicit microfoundation of equation (4) would not necessarily alter our characterization of the basic trade-off between coordinating working time and accommodating heterogeneous preferences.

**Labor market frictions.**—Labor market frictions play a subtle but crucial role in the model. The costs of forming, and dissolving, matches imply a surplus to ongoing firm-worker relationships, creating scope for bargaining over earnings and working time. Following Roys (2016), we assume there is a matching friction such that neither firm nor worker can instantaneously replace the other. We assume that, in the firm's problem (see below), the matching friction is channeled entirely through the (broader) cost of hiring a worker, denoted by $\bar{c}$. For instance, if it takes longer to fill a job, the cost of recruiting is higher. Thus, conditional on $\bar{c}$, we do not need to elaborate further on matching. In addition, we assume there is a cost $\xi$ of firing a worker, which can be interpreted in our empirical application as mandated severance.

**C. Characterization**

This section characterizes the choices of working time, earnings, and employment.

1. **Firm and Worker Objectives**

**Workers.**—Consider the surplus from working as type $(\xi, \theta)$ at a firm with productivity $Z$ and workforce $n$. In the current period, the employee receives a return equal to earnings, $W_{\xi,\theta}(n, Z)$, less the disutility of effort, $\xi \nu(b_{\xi,\theta}(n, Z))$, and the value, $\mu$, of nonmarket time. Next period, productivity, $Z'$, is realized, and the worker draws a (potentially new) type $(x, y)$. If the match surplus is negative given $n$, separations occur at rate $s_{x,y}$. Putting these pieces together, the surplus from working, $W_{\xi,\theta}(n, Z)$, is

$$W_{\xi,\theta}(n, Z) = W_{\xi,\theta}(n, Z) - \xi \nu(b_{\xi,\theta}(n, Z)) - \mu$$

$$+ \beta \sum_{x,y} \lambda_{x,y} \mathbb{E}[(1 - s_{x,y}(n', Z')) W_{x,y}(n', Z') | Z].$$

(5)

A few remarks on equation (5) are warranted. First, in the absence of data on nonmarket incomes and activities, we simply treat $\mu$ as a fixed parameter. (For an approach to identifying heterogeneous nonmarket values using time use data, see Boerma and Karabarbounis 2021.) Second, the large-family
assumption means that the marginal value of wealth is invariant to worker- and firm-specific shocks and is, therefore, suppressed in equation (5).

The firm.—The firm has an initial workforce $N_{-1}$. (A subscript $-1$ denotes a one-period lag, and a prime symbol denotes next-period values.) After productivity, $Z$, is realized, the firm may choose to hire. We assume a worker’s type $(\xi, \theta)$ is unknown at the time of hire. After hires (if any) are made, the firm’s workforce is denoted by $N$. Then all $N$ workers draw a type, and the firm and some of its workers may choose to separate. Let $n_{\xi,\theta}$ be the measure of type-$(\xi, \theta)$ workers retained. It follows that $N = \sum_{\xi,\theta} n_{\xi,\theta}$ is the measure of the workforce used in production and carried into the next period. Wages and time worked will be determined after separations (if any) are made.

It is helpful to first define the present value of a firm for a given allocation, $n = \{n_{\xi,\theta}\}$. Let $\pi$ stand for profit gross of firing and hiring costs,

$$
\pi(n, Z) = ZG(h(n, Z), n) - n^T W(n, Z),
$$

where $n^T$ is the transpose of $n$ and $W$ is the vector of earnings over types, $W = \{W_{\xi,\theta}\}$. The corresponding present value of the firm is then

$$
\Pi(n, Z) = \pi(n, Z) + \beta \int \Pi(N, Z^*)dF(Z^*|Z),
$$

where $\beta \in (0, 1)$ is the discount factor, $F$ is the distribution function of productivity $Z|Z$, and $\Pi$ is the continuation value.

Critically, $\Pi$ can be written as a function of just two state variables, $(N, Z)$, despite the heterogeneity across workers. This tractability is purchased by the assumption of i.i.d. types, which implies that we do not have to track individual types of workers over time.

The dynamic programming problem may now be written as follows. Consider, first, the problem for a given $N$. The firm’s problem at this stage is to decide separations and is characterized by the Bellman equation,

$$
\Pi^-(N, Z) = \max_n \left\{ \Pi(n, Z) - c \cdot \sum_{\xi,\theta} [\lambda_{\xi,\theta} N - n_{\xi,\theta}] \right\},
$$

subject to $\lambda_{\xi,\theta} N \geq n_{\xi,\theta}$ for each type $(\xi, \theta)$. Then step back one stage and consider the choice of hires, which brings the workforce up to a level, $N$. Since hires are anonymous, the value of the firm at this stage is

$$
\Pi(N_{-1}, Z) = \max_N \left\{ -\bar{c} \cdot [N - N_{-1}] + \Pi^-(N, Z) \right\},
$$

subject to $N \geq N_{-1}$.

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8 Note that under incomplete insurance, persistent shocks to $Z$ would shift the marginal value of wealth. Thus, the substitution effect (of a change in $Z$) would be partially offset by an income effect. In this case, the model would “need” a higher Frisch elasticity to match the variance of firm-wide working time.
Note that equations (7) and (8) imply that a firm may hire and separate workers in the same period. However, for realistic values of $c$ and $\bar{c}$, this will not happen: $Z$ must be quite low to warrant separations, in which case no hires are made. Thus, at firms that separate, $N = N_{-1}$, and at firms that hire, $n = \lambda N = \lambda N$.

2. Working Time

The firm and each of its workers jointly choose working time efficiently by equating the employee’s marginal disamenity to the marginal value of his time to the firm. The symmetry of workers within a type $(\xi, \theta)$ implies that their working time and earnings will be equal. Specifically, solving this first-order condition yields the following result.

**Proposition 1.** For any individual worker of type $(\xi, \theta)$, the efficient choice of working time is given by

$$h_{\xi,\theta} = (\alpha Z \Omega(n))^{1/(\varphi + 1 - \rho)} \cdot \left[\theta^\rho n_{\xi,\theta} / \xi\right]^{1/(\varphi + 1 - \rho)},$$

with $\Omega(n) = \left(\sum_{x, y} \sum_{\varphi + 1} [y^{\rho + 1} n_x \rho / x]^{\rho/(\varphi + 1 - \rho)}\right)^{(\alpha - \rho)/\rho}$.

Equation (9) indicates that the elasticity of $h_{\xi,\theta}$ with respect to $Z$ is $1/((\varphi + 1 - \alpha)$. As we saw in section II.A, this is decreasing in $\varphi$ or, equivalently, increasing in the Frisch elasticity. What is new is the role of the returns to scale, $\alpha$. The elasticity of working time is higher at higher $\alpha$, which signifies that diminishing returns sets in slowly. In what follows, we treat $\alpha$ as known and later parameterize it based on external evidence, as we lack much of the data (revenue, etc.) that would typically be used to estimate it.

Importantly, since the elasticity of $h_{\xi,\theta}$ with respect to $Z$ is independent of type, it applies to all types. In other words, the elasticity of mean working time with respect to $Z$ is also $1/((\varphi + 1 - \alpha)$. This is a simple but crucial point for estimation: although $h_{\xi,\theta}$ is unobserved, variation in average working time within the firm can be measured and informs the identification of $\varphi$.

Equation (9) also reveals the role of complementarities in shaping the reaction of working time to idiosyncratic events. To see this most clearly, consider a measure of workers of type $(\xi, \theta)$ that is small relative to the size of the firm (such that spillovers across types via $\Omega$ may be ignored for now). The corollary reports the effects of a perturbation to $\xi$ and $\theta$.

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9 As a result, the comparative static casts a sharper light on the roles of $\varphi$ and $\rho$. In general, the elasticities also depend on the size of the group with type $(\xi, \theta)$, e.g., the $n_{\xi,\theta}$ in $\Omega$. See app. C. In our estimated model, these scale effects are accounted for since the number of types is finite.
Corollary 1. (I) The elasticity of working time with respect to $\xi$, given by $-1/(\varphi + 1 - \rho) \leq 0$, tends to zero as $\rho \to -\infty$. (II) The elasticity of working time with respect to $\theta$, given by $\rho/(\varphi + 1 - \rho)$, is bounded above by $\alpha/(\varphi + 1 - \alpha) > 0$ and approaches $-1$ as $\rho \to -\infty$.

There are several aspects of corollary 1 that deserve attention. First, the responses of working time to changes in $\xi$ and $Z$ coincide only if $\rho = \alpha$, which implies that tasks are perfect substitutes. Otherwise, working time adjustments to $\xi$ are attenuated. Indeed, as we saw in section II.A, working time is increasingly invariant to $\xi$ as $\rho \to -\infty$ (e.g., in the limit where tasks are perfect complements). Crucially, this invariance emerges regardless of the value of the Frisch elasticity, $1/\varphi$.

By contrast, the elasticity of working time to $\theta$ does not vanish as $\rho \to -\infty$. The reason is that unlike a shift in $\xi$, a change in productivity, $\theta$, has a direct effect on the worker’s output. As a result, in the $\rho \to -\infty$ limit, $\ln h_{\xi,\theta}$ must move virtually one for one to offset shifts in $\ln \theta$ (and, thus, stabilize $\theta h_{\xi,\theta}$), or else the type’s marginal product would change precipitously. Still, even though working time is responsive in this limit, the change in $h_{\xi,\theta}$ becomes almost entirely detached from $\varphi$.

A final aspect of equation (9) concerns the relationship between the extensive and intensive margins. One can show that working time is declining in own-type employment but increasing in the employment of other types. These properties follow from concavity of the production function with respect to labor input $(n_{\xi,\theta}, h_{\xi,\theta})$ of any one type $(\xi, \theta)$ and supermodularity with respect to any two types. In addition, if there is a balanced expansion of employment of all types (e.g., $\Delta \ln n_{\xi,\theta}$ is identical $\forall (\xi, \theta)$), $\alpha < 1$ implies that $h_{\xi,\theta}$ falls $\forall (\xi, \theta)$.

Proceeding, the solution to working time (9) enables us to concentrate $h$ out of the firm’s problem. Substituting equation (9) into the revenue function (4) yields

$$ZG(h, n) = \hat{G}(n, Z) \equiv \alpha^{\alpha/(\varphi + 1 - \alpha)} Z^{\varphi + 1}/(\varphi + 1 - \alpha) \Omega(n) \left[\alpha(\alpha - \rho)/(\varphi + 1 - \alpha)\right].$$

(10)

Accordingly, period profit can be written as $\hat{\pi}(n, Z) \equiv \hat{G}(n, Z) - n^T W(n, Z)$. We now turn our attention to determining earnings $W(n, Z)$ and employment $n$.

3. Earnings

Following Cahuc, Marque, and Wasmer (2008), earnings are determined by splitting the marginal match surplus, awarding a share, $\eta \in (0, 1)$, to the worker. One can motivate marginal surplus splitting using the bargaining solution proposed by Stole and Zwiebel (1996) and developed rigorously in Brügemann, Gautier, and Menzio (2019).
The marginal surplus is the sum of the worker’s surplus, $W_{\xi,\theta}(n, Z)$, and the firm’s surplus, which has two parts. The first, denoted by $J_{\xi,\theta}(n, Z)$, is the marginal value of type-$(\xi, \theta)$ labor gross of adjustment costs. This term is obtained by differentiating the value function (6) (swapping out $\pi$ for $\hat{\pi}$) with respect to $n_{\xi,\theta}$,

$$J_{\xi,\theta}(n, Z) \equiv \frac{\partial}{\partial n_{\xi,\theta}} \hat{\pi}(n, Z) + \beta \int \Pi_N(N, Z')dF(Z'|Z),$$

where $N = \Sigma_{\xi,\theta} n_{\xi,\theta}$. In addition, the firm’s surplus accounts for the penalty $\xi$ that the firm incurs if an agreement is not reached, resulting in the worker’s separation. Accordingly, the surplus from retaining a worker is $J_{\xi,\theta}(n, Z) + \xi$, and earnings solve

$$W_{\xi,\theta}(n, Z) = \eta(W_{\xi,\theta}(n, Z) + J_{\xi,\theta}(n, Z) + \xi). \quad (11)$$

Our solution of equation (11), reported below, generalizes Cahuc, Marque, and Wasmer (2008) to an environment with endogenous separations and an intensive margin.

**Proposition 2.** The earnings bargain for a worker of type $(\xi, \theta)$ is given by

$$W_{\xi,\theta}(n, Z) = \eta \left( \frac{\partial G(n, Z)}{\partial n_{\xi,\theta}} + r\xi \right) + (1 - \eta)(\kappa \xi \nu(b_{\xi,\theta}(n)) + \mu), \quad (12)$$

where $r = 1 - \beta$, $b_{\xi,\theta}(n)$ solves equation (9), and $\kappa = (\varphi + 1 - \alpha)/(1 - \eta(1 - \alpha))(\varphi + 1) - \alpha > 1$ with $\eta \kappa < 1$ and $(1 - \eta)\kappa < 1$.

The bargain in equation (12) is a weighted average of a (i) worker’s contribution to the firm and (ii) the value of his nonmarket time. The former (i) depends on the worker’s marginal product, $\partial G(n, Z)/\partial n_{\xi,\theta}$, and the annuitized firing cost, $r\xi$, which he “saves” the firm by continuing the match for another period.\(^{10}\) The latter (ii) also has two parts: outside of employment, a worker does not incur the disutility $\xi \nu(b_{\xi,\theta}(n))$, and he avails himself of $\mu$. Notably, solutions of certain collective bargaining games also admit a role for individual heterogeneity in the value of nonmarket time, which is a crucial element of equation (12) (Taschereau-Dumouchel 2020). This observation suggests that our theory may serve as a useful framework even when

\(^{10}\) The worker can use $\xi$ to negotiate a higher wage because the firm is subject to the severance cost as soon as he is hired. This is consistent with the labor contract that was most prevalent in Italy in our sample. See Mortensen and Pissarides (1999) for a discussion of bargaining under severance costs.
the structure of bargaining is more elaborate (e.g., there are collective and individual elements to it).\textsuperscript{11}

4. Comparing Earnings and Working Time Dynamics

Several of the model’s key implications for the joint dynamics of earnings and working time can be gleaned from equations (9) and (12). To this end, it is helpful to write out earnings more explicitly using equations (9) and (10),

\[
W_{t,\theta}(n, Z) = \kappa \alpha^{(\varphi + 1 - \alpha)} (Z \Omega(n))^{(\varphi + 1)/(\varphi + 1 - \alpha)} \left( \frac{\theta_n^{e+1}}{\xi} \right)^{\rho/(\alpha + 1 - \rho)} n_{\xi,\theta}^{-\varphi + 1)/(\alpha + 1 - \rho) + \omega, \tag{13}
\]

where \( \kappa \equiv \alpha(\eta \varphi + (1 - \eta))(\varphi + 1 - \alpha)/(\varphi + 1))/(1 - \eta(1 - \alpha))(\varphi + 1) - \alpha) \) is increasing in \( \eta \) and \( \omega \equiv \eta r + (1 - \eta)\mu \). Thus, \( \partial G/\partial n_{\xi,\theta} \) and \( \xi r(h_{\xi,\theta}) \) in equation (12) can be collected into a single term summarizing the variable portion of earnings. Note that a higher bargaining power \( \eta \) redistributes weight toward this term (since \( r \) in \( \omega \) is small), amplifying the effect of firm-level and idiosyncratic shocks.

Equation (13) clarifies the mapping between idiosyncratic events and earnings. Consider, for instance, an increase in the distaste for working, \( \xi \), and suppose jobs are complementary (\( \rho < 0 \)). Since the response of working time is suppressed, workers earn a premium for supplying costly effort, noting the term \( \xi^{-\alpha/(\varphi + 1 - \rho)} \) in equation (13). Indeed, earnings become increasingly responsive as \( \rho \to -\infty \). Thus, earnings should be more elastic than working time with respect to \( \xi \), at least if \( \omega \), the fixed portion of earnings, is not too large. The following corollary makes this intuition precise.\textsuperscript{12}

Corollary 2. The (absolute) elasticity of earnings with respect to \( \xi \), \( |\partial \ln W_{t,\theta}/\partial \ln \xi| \), increases relative to the (absolute) elasticity of working time, \( |\partial \ln h_{t,\theta}/\partial \ln \xi| \), as \( \rho \) falls away from zero and strictly exceeds the latter whenever \( \rho < -(1 - \omega/W_{t,\theta})^{-1} < -1 \).

Corollary 2 holds out the possibility of using data on earnings and working time to infer \( \rho \). If earnings are indeed relatively sensitive to idiosyncratic variation, we should see that the within-firm variance of earnings growth exceeds that of working time changes. Corollary 2 indicates that this excess variance of earnings growth is monotonically decreasing in \( \rho \).

\textsuperscript{11} To be clear, “collective” here refers to bargaining between a union and a firm. See sec. III.A for more on the structure of bargaining in Italy.

\textsuperscript{12} To simplify the presentation, we again suppose the mass of workers with \( (\xi, \theta) \) is “small.” Results similar to corollaries 2 and 3 obtain in the general case with “large” cohorts. See app. C.
The corollary refers only to a perturbation to $\xi$, however. Unlike $\xi$, which directly shifts $W_{t,\theta}$ via $\xi \nu(h_{t,\theta})$, the impact of productivity $\theta$ on earnings is channeled more through its effect on working time. In fact, insofar as $\rho$ shapes the elasticity of $h_{t,\theta}$ with respect to $\theta$, one can show that $\rho$ affects the response of earnings symmetrically. Thus, the change in earnings relative to the change in working time will not depend on $\rho$.\footnote{This statement is established in the proof of corollary 3. See app. A.}

Clearly, then, the mix of $\xi$ and $\theta$ will be critical for interpreting the volatility of earnings and working time. To identify the predominant source of variation (between $\xi$ and $\theta$), we can examine the comovement of the wage rate, $w_{t,\theta} = W_{t,\theta}/h_{t,\theta}$, and working time, $h_{t,\theta}$. A change in $\xi$ shifts earnings and working time in different directions, which implies that $w_{t,\theta}$ and $h_{t,\theta}$ move opposite one another.\footnote{To be more exact, $W_{t,\theta}$ and $h_{t,\theta}$ necessarily go in opposing directions if $\rho < 0$. Otherwise, they can move together, but the wage always moves opposite working time.} By contrast, changes in $\theta$ push earnings and working time in the same direction, which means the wage shifts in this direction only if $W_{t,\theta}$ moves more than $h_{t,\theta}$. This result will obtain as long as there is sufficient curvature in earnings with respect to $\theta$—for example, $\varphi$ is large enough (see eq. [13])—and/or the fixed portion of earnings, $\omega$, is small enough. Corollary 3 summarizes this discussion.

**Corollary 3.** (I) The responses of working time and the wage to changes in $\xi$ are, unambiguously, of the opposite sign. (II) A change in $\theta$ shifts working time and the wage in the same direction as long as $\varphi > (\omega/W_{t,\theta})^{-1} - 1)^{-1}$.

The comovement of working time and the wage rate places restrictions on the prevalence of the different shocks. The negative correlation of working time and wages in our data (see sec. IV) can be accommodated by variation in $\xi$ for any values of $\omega$ and $\varphi$. By contrast, changes in $\theta$ will drive wages and working time in opposing directions only if $\varphi$ is small, which means the Frisch elasticity is large. Indeed, since $\omega/W_{t,\theta} < 1/2$ at our estimated parameter values, the Frisch elasticity would have to exceed 1—a claim that our data on (firm-level) working time will not support. Thus, we infer that $\xi$ drives a substantial share of the movement in earnings and working time, and in this environment corollary 2 implies that the relative variability of earnings conveys critical identifying information about $\rho$.

5. Employment Demand

Thus far, we have taken employment as given. To complete our analysis, we now describe the solution to the dynamic employment demand problem (see app. A for details).
Consider the problem of a firm of initial size $N_{-1}$. Separation from a given type is optimal if that type’s marginal value of labor falls below $-\zeta$, where $\zeta$ is the cost of termination. Appendix A shows that this happens when firm productivity falls beneath a (type-specific) threshold value of $Z$. Naturally, the type of worker separated first is the type with the highest threshold, which is denoted by $Z_1(N_{-1})$.\(^{15}\)

If $Z$ falls further, the firm separates from a second type. As it does this, separations from the first type continue. This result reflects the supermodularity of the firm’s problem: as the firm reduces labor input of the second type, the marginal value of the first type falls further. The same idea applies when the firm separates from its third type and so forth.

The next piece of the optimal policy is the decision to hire. Given an initial size $N_{-1}$, the firm hires when $Z$ rises above a threshold $Z_0(N_{-1})$, at which point the marginal value of labor exceeds the marginal cost $\bar{\zeta}$. Since the firm hires before types are known, it simply chooses $N$, and each type’s size rises in proportion to its share in the population.

Figure 1 illustrates the labor demand policy with four types (e.g., there are two levels of preferences and productivities). Where $Z > Z_0(N_{-1})$, the firm hires, and each type’s employment is increased equally. Between $Z_0(N_{-1})$ and $Z_1(N_{-1})$, $N$ is held at $N_{-1}$. This space is a region of inaction in which shifts in $Z$ do not push the marginal value below $-\zeta$ or above $\bar{\zeta}$. As $Z$ falls below $Z_1(N_{-1})$, employment of one type is reduced. As $Z$ declines further, another type is separated jointly with all of the other types that were separated prior to it.

A final issue concerns the implications of employment demand for the intensive margin. Clearly, in the inaction region where $N = N_{-1}$, working time fully absorbs the effects of changes in $Z$. Outside of this region, changes in $Z$ have direct and indirect effects on working time. For given employment, the direct effect of, say, a higher $Z$ is stimulative. The indirect effect is channeled through employment adjustments, which dampen the reaction of working time (see sec. II.C.2). However, the direct effect dominates in the calibrated model because the frictions $\bar{\zeta}$ and $\zeta$ curtail the size of the adjustments to employment. Thus, an increase in $Z$ elevates both working time and employment (of all types).

### III. Data Source

Our data span the universe of private firms in Veneto, Italy, during the period 1982–2001. Located in the North East, Veneto is one of the largest and richest areas of Italy. Among the country’s 20 regions, Veneto ranked sixth in income per capita and fifth in population in our sample period.

\(^{15}\) In certain cases, one can infer this type’s identity, e.g., its $\xi$ and $\theta$. See app. C.
according to data from Istat. We make a case here that Veneto is a reasonable testing ground for our theory and then describe our data, the VWH files.

A. Institutional Context

As we take model to data, a potential concern for us is the role of national unions in Italy. Unions could, in principle, suppress contracting between firm and worker, which is a pillar of our theory. However, the de facto setting of wages and working time is broadly supportive of our modeling approach.

In the North East in particular, decision-making has been reasonably decentralized at the margin. The multitiered wage-setting process illustrates this point well. First, union-negotiated sector-wide contracts specify minimum wages, but in high-wage regions like Veneto, these rarely bind (Card, Devicienti, and Maida 2014). One tier down, workers’ representatives at the
firm negotiate “add-ons” to sector-wide contracts. These firm-level agreements are common among larger employers—half of firms with at least 20 workers have one—and the average premium (over industry minima) is about 25% (Guiso, Pistaferri, and Schivardi 2005; Card, Devicienti, and Maida 2014). Notably, firm-level negotiations in Veneto could be mediated by a self-organized committee of employees rather than union representatives. This observation reflects the relatively light touch of unions in the North East (Cattero 1989). Finally, management awards bonuses to individual workers (Erickson and Ichino 1995). Among the many employers with less than 20 workers, where firm-level contracts are less common, these individual premia are substantial—as high as 25%—and heterogeneous (Brusco 1982; Cattero 1989).

Meanwhile, firms generally enjoyed discretion in negotiating working time, at least among full-time employees. Working time rules, including limits on overtime, were often eased in union agreements or loosely enforced, especially during the 1980s (Treu, Geroldi, and Maiello 1993; Lodovici 2000). Deviations from full-time, open-ended employment contracts were rare, though: part-time work as well as fixed-term contracts, which could be ended after 2 years at no cost, were uncommon. Consequently, firms could not use a temporary worker to replace the working time of an employee who draws a positive preference shock (a high $\xi$).

In this setting, where decision-making is generally diffused, we see unions and other national actors as parameterizing the bargaining process rather than deciding firm-level outcomes. For instance, the interplay between national unions and employers’ associations is likely to shape worker bargaining power ($\eta$), with the latter taken as an input into firm-level negotiations. We return to this point later when we take up a mid-1990s wage-setting accord that arguably enabled more flexible wage bargains, for example, a higher $\eta$ (see sec. VI.A).

B. The VWH Files

Our empirical analysis uses the VWH dataset that has been assembled by researchers at the University of Venice. The data are derived from Italian social security records, which track earnings and paid days of work for the purpose of calculating social insurance payments. Nearly every private sector employee in Veneto is covered by the data; public sector workers and the self-employed are excluded. The full sample contains 22.245 million worker-year observations over the years 1982–2001.

The VWH data have a number of features that recommend it for this analysis. The Veneto data stand out for reporting a measure of working time, namely, a worker’s annual paid days with each of his employers. Using paid days and earnings, we can also compute daily wages. Finally, the VWH specifies the calendar months for which a worker received any earnings from an employer, which enables us to track the worker’s tenure with a firm.
Table 1 provides a set of summary statistics for the full sample. Average daily earnings were around 120 euros. The average number of paid months per worker (per year) was 10, and within a paid month, days of work averaged between 23 and 24. There is a good reason why days paid per month is rather high: as Italy transitioned from 6- to 5-day weeks, its social security agency recorded a full week of work as 6 days in order to treat 5- and 6-day weeks equally for pension purposes.

Table 2 zeroes in on moments of the distribution of annual changes in paid work days. (These estimates pertain to the subsample of “stayers” used in our baseline analysis below.) While many workers do not adjust their days from one year to the next, 33% change the number of days worked by more than 10.16 Moreover, conditional on changing days, the typical size of the change is between 10 and 19, depending on whether some of the largest adjustments are included.

The VWH’s measure of paid days does not necessarily equate to days at work, although the link is reasonably tight. For instance, paid days does include leaves of absence paid by the firm, but if time off is taken each year (i.e., August vacation), we will still correctly measure changes in days at work. Other absences, such as disability, illness, and parental leave, are typically compensated by the state, and our data do not record state-remunerated time off as paid days (Filippi, Pacelli, and Villosio 2002). Likewise, spells of temporary layoff in which workers draw state benefits are not recorded as paid days.

More importantly, the VWH does not capture certain sources of variation in paid time worked. The most prominent omission is daily hours. In section VI, though, we use household survey data to show that variation

| Statistic                  | Mean  | SD   |
|---------------------------|-------|------|
| Paid days per month       | 23.65 | 5.25 |
| Job tenure (months)       | 53.10 | 53.71|
| Daily wage (2003 euros)   | 121.46| 426.76|
| Total days paid per year  | 243.88| 97.75|
| Months paid per year      | 9.96  | 3.38 |

Note.—A month is “paid” if an employee works at least one day for pay in the month. Moments are based on the full Veneto panel, 1982–2001. There are 22,245 million worker-year observations.

16 This inaction could reflect a cost of adjusting hours, which is not captured in the model. Such frictions would imply a nontrivial dynamic choice problem for $h_{t,t'}$. Our conjecture is that since an adjustment cost will eliminate small changes in $h_{t,t'}$, a higher Frisch elasticity may be needed to generate enough variance in working time conditional on adjusting.
in paid days is substantial relative to daily hours, and the conclusions from our analysis largely survive intact.\textsuperscript{17}

IV. Estimation Strategy

We estimate our model by MSM, which selects values for the parameters to minimize the distance between empirical and model-generated moments. Two broad considerations guide our choice of moments.

First, moments derived from first differences, rather than levels, are more likely to robustly identify the structural parameters. For instance, suppose firms operate a Leontief technology but idiosyncratic productivity (\( \theta \)) is permanent rather than (as assumed in the model) transitory. Since firms will equate efficiency units across jobs, working times will diffuse within a firm yet be unresponsive to \( \xi \); it is changes in working time that are compressed (indeed, equalized across jobs). Thus, first-differenced data correctly convey the degree of complementarities even if the nature of idiosyncratic productivity is misspecified.

Second, some parameters interact differently with firm-wide, as opposed to idiosyncratic, shocks. For example, recall the significance of \( Z \), but not \( \xi \), to the identification of \( \varphi \). Therefore, we want to distinguish firm-wide from within-firm components of changes in working time and earnings. In the next subsection, we illustrate how we do this.

A. Earnings and Working Time

We begin by describing moments pertaining to earnings and working time changes and then relate these to the structural parameters for which they are especially informative.

\textit{Empirical framework.}—Our empirical analysis centers around a simple regression designed to distinguish variation across workers within a firm

\begin{table}
\centering
\caption{Annual Changes in Days Worked (\( \Delta b \))}
\begin{tabular}{ll}
\hline
Statistic & Value \\
\hline
Share (\%) with \( \Delta b = 0 \) & 47.38 \\
Share (\%) with \( |\Delta b| > 10 \) & 33.15 \\
Average \( |\Delta b| \) if \( |\Delta b| \neq 0 \) & 19.06 \\
Average \( |\Delta b| \) if \( |\Delta b| \neq 0 \), excluding \( |\Delta b| > 50 \) & 9.75 \\
\hline
\end{tabular}
\end{table}

\textsc{Note.—}This table reports moments of the distribution of annual changes in days worked, denoted by \( \Delta b \). Statistics are derived from our sample of 2-year stayers, as defined in the main text (see also the note to table 3). There are 11.81 million worker-year observations.

\textsuperscript{17} As noted above, the data also do not capture the secular trend toward 5-day weeks. However, this development would seem to be orthogonal to the economic forces (i.e., intertemporal substitution) in the model and, thus, to the parameters that shape them. See app. B for more.
from firm-wide movements. Consider, first, working time. Let $\Delta \ln h_{ijt}$ denote the log change in days worked by employee $i$ in firm $j$ between years $t - 1$ and $t$. We estimate

$$\Delta \ln h_{ijt} = \chi_{it} C^h + \phi_{ijt}^h + \epsilon_{ijt}^h,$$  

(14)

where $\chi_{it}$ is a row vector of worker characteristics, $C^h$ is a conformable (column) vector of coefficients, and $\phi_{ijt}^h$ is a firm-year effect. Equation (14) is applied to a subsample of workers employed at the same firm for years $t - 1$ and $t$ (see below for more on sample selection). The elements of $\chi_{it}$ consist of a cubic in tenure (measured as of $t - 1$) and the change in broad occupation (between $t - 1$ and $t$). These controls help purge the data of observable heterogeneity in work schedules that is not modeled in our theory. The variation captured in $\phi_{ijt}^h$ and $\epsilon_{ijt}^h$ is what is used to estimate the structural model.

The parameter $\phi_{ijt}^h$ captures the mean log change in working time across employees in firm $j$ in year $t$. The variance of $\phi_{ijt}^h$ is thus our measure of the volatility of firm-wide working time. From the model’s perspective, it is natural to think of shocks to firm productivity as underlying $\phi_{ijt}^h$, although the latter can reflect other firm-level forcings (see below).

Meanwhile, the residual in equation (14), $\epsilon_{ijt}^h$, isolates shifts in working time across workers within a firm. Our structural model interprets the variation in $\epsilon_{ijt}^h$ as being driven by shocks to idiosyncratic preferences and productivity.

We also estimate a regression of the same form for earnings, which relates the log change in annual earnings, $\Delta \ln W_{ijt}$, to observables ($\chi_{it}$) and firm-year effects $\phi_{ijt}^W$,

$$\Delta \ln W_{ijt} = \chi_{it} C^W + \phi_{ijt}^W + \epsilon_{ijt}^W.$$  

(15)

The meanings of $\phi_{ijt}^W$ and $\epsilon_{ijt}^W$ are analogous to their counterparts in equation (14).

While we have laid out a simple mapping between the structural shocks and estimates in equations (14) and (15), the connection between the two is likely more subtle in practice. The reason is that the structural shocks behind the residuals, $\epsilon_{ijt}^h$ and $\epsilon_{ijt}^W$, do not generally average out among a finite sample of workers in a firm. The effect of a nonzero mean among idiosyncratic draws must be absorbed by the firm-year intercepts. Thus, a simple (quadratic-form) estimator of the variances of $\phi_{ijt}^h$ and $\phi_{ijt}^W$ reflects this finite-sample noise.19

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18 There are four broad occupations: blue-collar workers, who make up 65% of the sample; white-collar nonsupervisory workers (31%); apprentices (3%); and managers (1%).

19 Clearly, the first-best strategy is to explicitly incorporate this noise into the structural model. However, this approach deprives us of the tractability afforded by the law of large numbers (see sec. II.B).
Nevertheless, this estimator can be appropriate in our context. To see why, suppose a firm employs a handful of workers, each of whom takes independent draws of $\xi$. There may be times when workers draw the same $\xi$. From the perspective of our model, these common draws of $\xi$ do not exaggerate the variance of firm-level events; rather, they are a source of firm-level variation insofar as they elicit a common, or coordinated, response of workers’ labor supplies. Therefore, the diffusion in $\phi^b$ (as well as $\phi^w$) due to such events is properly treated as firm-level variation.

For completeness, appendix D estimates the variances of firm-year effects based on the finite-sample adjustment in Kline, Saggio, and Sølvsten (2020), which eliminates any spillovers of idiosyncratic variation into the $\phi$s. When the structural parameters are reestimated to fit these revised moments, we recover a Frisch elasticity (of just under 0.4) that is smaller than our baseline estimate (of 0.48), but not dramatically so.

Sample restrictions.—Equations (14) and (15) are estimated off a sample of workers who stay at the same firms in consecutive years. We define stayers in year $t$ as workers who were paid for at least one day in all months of the first quarter of year $t - 1$ and in all months of the last quarter of year $t$. Thus, these workers start and end the 2-year period with the same employer. After we remove firms in any year $t$ with only one employee—it would be awkward to analyze complementarities with these firms—we are left with 11.8 million worker-year observations. This is our sample of 2-year stayers.

While the construction of this sample allows for extended nonwork spells, workers’ absences from their employers are generally not recurrent. For instance, among workers who are not paid for a full month or more in year $t - 1$, most are paid for at least one day in every month of the next year. In this sense, these workers appear to have relatively strong attachments to their firms, which underlies our view that changes in their working time can be interpreted as intensive-margin adjustments.

Still, one could consider a tighter definition of stayers, which requires even more consistent participation at the firm. To this end, we also report figures for an alternative sample, which we refer to as the 12/12 stayers. These workers are paid for at least one day in every month over years $t - 1$ and $t$.

Our restriction to stayers may raise concerns about selection bias. Note, however, that we will also select a sample of stayers from our model-generated data to form the relevant moments for MSM estimation. In this sense, we treat

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20 Indeed, one can confirm that if there is a common component to $\xi$, an increase in it is isomorphic in eq. (9) to a decrease in firm productivity, $Z$. In this sense, common components of the idiosyncratic draws are not problematic for the theory; they can be subsumed within our lone firm-level forcing variable.
the data and model symmetrically, which supports consistent estimation of the structural parameters (Smith 1993). 21

Regression estimates.—Table 3 summarizes estimates derived from equations (14) and (15). The first three rows pertain to within-firm (idiosyncratic) variation. Specifically, the first row reports \( \text{var}(e^w) \), the second shows \( \text{var}(e^h) \), and the third gives the ratio of the two. In the 2-year stayer sample, this ratio is 2.247—idiosyncratic earnings growth is more than twice as variable as idiosyncratic working time changes. The next three rows report the counterparts to these moments at the firm level, namely, \( \text{var}(\phi^W) \), \( \text{var}(\phi^h) \), and the ratio of the two. Note that given the mean of days worked in table 1, a value of \( \text{var}(\phi^h) = 0.078^2 \) represents 1.5–2 days per month. Variation in working time and earnings among the 12/12 stayers is less pronounced, which is unsurprising: the length of their nonworking spells in any one year is relatively abbreviated.

A more extended look at the moment, \( \text{var}(e^w)/\text{var}(e^h) \), is worthwhile. If this ratio is large, idiosyncratic shocks manifest in earnings are not passed through to working time, signifying that complementarities compress variation in working time. Accordingly, this moment will be critical to our strategy for identifying \( \rho \) (see below for further discussion).

| Moment Interpretation | Stayers |
|-----------------------|---------|
| \( \sqrt{\text{var}(e^w)} \) | 12/12 | 2-Year |
| Standard deviation of idiosyncratic component of \( \Delta \ln W \) | .162 | .210 |
| \( \sqrt{\text{var}(e^h)} \) | | |
| Standard deviation of idiosyncratic component of \( \Delta \ln h \) | .083 | .140 |
| \( \text{var}(e^w)/\text{var}(e^h) \) | Ratio of idiosyncratic variances | 3.798 | 2.247 |
| \( \sqrt{\text{var}(\phi^W)} \) | Standard deviation of firm component of \( \Delta \ln W \) | .114 | .132 |
| \( \sqrt{\text{var}(\phi^h)} \) | Standard deviation of firm component of \( \Delta \ln h \) | .057 | .078 |
| \( \text{var}(\phi^W)/\text{var}(\phi^h) \) | Ratio of firm-level variances | 3.989 | 2.885 |
| \( \text{cov}(\Delta \ln h, \Delta \ln w)/\text{var}(\Delta \ln w) \) | Projection of \( \Delta \ln h \) on \( \Delta \ln w \) | \(-.158 \) | \(-.169 \) |

NOTE.—W is annual earnings, \( h \) is paid days, and \( w \) is the daily wage (\( W/h \)). The 12/12 stayers are workers paid for at least 1 day in every month in 2 consecutive years. The 2-year stayers are paid for at least 1 day in each of the first 3 months in year \( t-1 \) and in each of the last 3 months in year \( t \).

21 Of course, stayers may differ from the average worker in unmodeled ways. One possibility is that some jobs are more critical to production, and firms are more likely to retain workers in these highly complementary jobs. By this logic, though, firms should compete hard to fill such jobs when they are vacant, which suggests that job movers will also work highly complementary jobs. A priori, then, it is unclear whether a worker’s status as a stayer or mover reveals the complementarity of her job.
Table 4 reports values of \( \text{var}(e_W) / \text{var}(e_h) \) for several subsamples. \(^{22}\) Seen through the lens of the model, working time appears to be compressed for broad classes of workers and sectors. These results underscore that our estimate on the full sample is reasonably robust, particularly for the 2-year stayers. The ratio \( \text{var}(e_W) / \text{var}(e_h) \) falls modestly if we drop public service–oriented sectors, such as health care, but rises modestly if we restrict attention to men. The ratio is also somewhat higher among larger firms.

Appendix G examines estimates of \( \text{var}(e_W) / \text{var}(e_h) \) in detailed (three-digit NAICS) industry data. Here, clear differences do emerge, but the patterns further underline the value of this ratio as a diagnostic for complementarities. For example, industry estimates of \( \text{var}(e_W) / \text{var}(e_h) \) are positively correlated with the incidence of “teamwork” in O*NET data. In addition, the ratio is generally higher in industries with larger male–female earnings differentials, a finding reminiscent of Goldin’s (2014) observation that the gender gap is pronounced where working time is less tailored to individual circumstances.

Targeting moments.—We now review several moments derived from equations (14) and (15) and sketch how they enable the identification of structural parameters. (Appendix E uses the sensitivity matrix of Andrews, Gentzkow, and Shapiro [2017] to guide a more extended discussion of identification.) The moments are based on the sample of 2-year stayers.

First, as a preliminary matter, note that there is no ex ante heterogeneity in the model and, thus, no counterpart to the covariates \( \chi \) in equations (14) and (15). In this setting, the unbiased estimate of the firm–year effect is the mean log change of working time (or earnings) within the firm. Therefore, this simple average is taken as the model analogue to \( \phi_h^b \) (or \( \phi_W^b \)). The deviation of a type’s outcome from the mean is the analogue to the residuals in equations (14) and (15).

Proceeding, the first moment is the variance of firm–year working time effects, or \( \text{var}(\phi^b) \). As a matter of accounting, this variance reflects the elasticity of average working time to (firm-level) shocks as well as the size of the

| Sample                                      | 12/12 Stayers | 2-Year Stayers |
|---------------------------------------------|---------------|----------------|
| Full sample                                 | 3.798         | 2.247          |
| Excluding women                             | 4.282         | 2.514          |
| Excluding small firms (<100 workers)        | 5.080         | 2.968          |
| Excluding health and education              | 3.592         | 2.078          |

Note.—This table shows the ratio of the variance of the idiosyncratic component of earnings growth to the variance of the idiosyncratic component of log working time changes for different subsamples. See app. G for an industry-level analysis of the ratio \( \text{var}(e_W) / \text{var}(e_h) \).

\(^{22}\) In each case, we use all observations to run eqqs. (14) and (15) but pool \( e_W^{jt} \) and \( e_h^{jt} \) across the relevant subsample.
shocks. In the structural model, the sensitivity of average working time to firm productivity, $Z$, hinges on the Frisch elasticity, $1/\varphi$. Thus, modulo the size of shocks, the moment $\text{var}(\phi^e)$ can be highly informative as to $\varphi$. We return shortly to discuss how to infer the variance of $Z$.

Next, we turn to earnings-related moments. According to equation (13), log changes in earnings reflect (i) fluctuations in the disutility of effort and marginal product and (ii) the pass-through of these changes to earnings. The pass-through rate in the latter (ii) is shaped, in part, by bargaining power $\eta$ and applies to any change in disutility and marginal product. Therefore, a higher $\eta$ amplifies fluctuations in both firm-level and idiosyncratic earnings. Furthermore, the volatility of earnings growth in general should rise relative to the variance of working time growth, since a higher $\eta$ has no direct impact on working time (see eq. [9]).

In addition, the model predicts that complementarities amplify the changes in disutility in the former (i) stemming from shifts in $\xi$. By mitigating the decline in working time following a rise in $\xi$, stronger complementarities (e.g., a “more negative” $\rho$) induce a larger increase in disutility, and the idiosyncratic element of earnings rises more to compensate a worker for supplying effort. This property underlies a monotone mapping between $\rho$ and the ratio of the variances of residuals, $\frac{\text{var}(\epsilon^w)}{\text{var}(\epsilon^h)}$, providing a clear way of identifying this parameter. This strategy also “frees up” $\eta$ to be used to target the relative volatility of firm-level earnings growth, that is, the ratio $\frac{\text{var}(\phi^w)}{\text{var}(\phi^h)}$.

To take stock, our strategy to identify parameters $\varphi$, $\eta$, and $\rho$ is centered around three moments. One reflects firm-level variation in working time, $\text{var}(\phi^h)$. The other two moments pertain to the relative variability of earnings at the firm and worker levels.

Given these moments, it remains to consider the volatility of idiosyncratic working time, or $\text{var}(\epsilon^h)$. Clearly, this moment bears on the variances of the two idiosyncratic shocks, denoted here by $\sigma_i^2$ and $\sigma_j^2$. To pin down both parameters, though, we need to supplement $\text{var}(\epsilon^h)$ with information beyond equations (14) and (15), a point we develop in the next section.

### B. Additional Moments

Next, we summarize three additional moments and discuss their information content for the model’s parameters. Table 5 lists all seven moments used in estimation.

To begin, we regress individual changes in log working time on log changes in daily earnings, with the latter given by $\Delta \ln w = \Delta \ln W - \Delta \ln h$. The estimated coefficient of $-0.169$ echoes studies such as Altonji (1986), who uncovered a coefficient of around $-0.3$. However, standard life cycle theory implies that one ought to be able to recover the Frisch elasticity from this regression. Earlier results were thought to reflect measurement error (see Borjas 1980), but even when instrumental variables were used to eliminate
the division bias, the regression returned coefficients that were small and often indistinguishable from zero (MaCurdy 1981; Altonji 1986). Our estimates using administrative data, which are far less subject to measurement error, reaffirm that this approach can fail to uncover a significantly positive Frisch elasticity.

From our model’s perspective, the regression suggests that the covariance of $\Delta \ln h$ and $\Delta \ln w$ heavily reflects idiosyncratic variation. Specifically, the ordinary least square coefficient points toward a crucial role for $\xi$. Intuitively, $\xi$ acts as a supply shock pushing working time and the wage in opposite directions, whereas $\theta$ is more akin to a demand shock, which induces working time and wages to move together (recall corollary 3). Thus, the projection

Table 5
Model Fit

| Moment                  | Model | Data |
|-------------------------|-------|------|
| $\text{var}(e^h)/\text{var}(e^e)$ | 2.247 | 2.247 |
| $\text{var}(\phi^\theta)/\text{var}(\phi^e)$ | 2.885 | 2.885 |
| $\sqrt{\text{var}(e^i)}$ | .140  | .140  |
| $\sqrt{\text{var}(\phi^i)}$ | .078  | .078  |
| $\text{cov}(\Delta \ln h, \Delta \ln w)/\text{var}(\Delta \ln w)$ | −.169 | −.169 |
| $\sqrt{\text{var}(\Delta \ln N)}$ | .175  | .175  |
| $\text{E}[N]$           | 17.130| 17.130|

| Parameter                | Symbol | Value         |
|--------------------------|--------|---------------|
| Elasticity of substitution across jobs | $1/(1 - \rho)$ | .338 (.0005) |
| Frisch elasticity of working time     | $1/\phi$ | .483 (.0006) |
| Worker bargaining power      | $\eta$  | .407 (.0005) |
| Flow value of nonemployment | $\mu$   | .210 (.0006) |
| Standard deviation of idiosyncratic preference | $\sigma_\xi$ | .294 (.0004) |
| Standard deviation of idiosyncratic productivity | $\sigma_\theta$ | .210 (.0007) |
| Standard deviation of shock to firm productivity | $\sigma_z$ | .203 (.0002) |

**Note.**—Data in panel A refer to 2-year stayers. Estimates are of our baseline model (see sec. V.B). Standard errors are in parentheses. Standard errors of $1/(1 - \rho)$ and $1/\phi$ are calculated by the delta method.

23 One distinction between our analysis and earlier studies is that we observe daily earnings rather than the hourly wage. We return to this point in sec. VI.B.
of $\Delta \ln h$ on $\Delta \ln w$ is informative as to the size of preference relative to the size of productivity shocks. This moment, together with the variance of $\epsilon^h$, helps identify both $\sigma_\epsilon$ and $\sigma_\epsilon$.

The last two moments refer to employment. The first is the standard deviation of employment growth across firms. Note that the latter is employment weighted, so it is representative of the volatility faced by a typical worker. The second moment is mean firm size, $E[N]$. These two moments are conceptually linked to two parameters in particular. The dispersion of employment growth reflects the size of firm-level shocks and, thus, anchors the choice of $\sigma_Z$, the standard deviation of innovations to firm productivity (see below). The size of firms is strongly influenced by workers’ outside option, $\mu$. Intuitively, if $\mu$ is small, the rents from a match are large, and so more hires are made.

Finally, we have also examined the model’s fit with respect to several nontargeted moments. These include the persistence of average working time and earnings—the model matches these moments rather well—as well as the latter’s correlation with employment, which the model somewhat overstates. See appendix D for a fuller discussion.

V. Model Estimation

Seven parameters are estimated. They are $\rho$, which governs the elasticity of substitution across jobs; the utility parameter, $\varphi$; worker bargaining power, $\eta$; the worker’s outside option, $\mu$; and the standard deviations of the shocks, namely, $\sigma_{\epsilon^h}$, $\sigma_\epsilon$, and $\sigma_Z$. We choose the values of other parameters on the basis of outside evidence. In this section, we first report how we set the latter parameters and then discuss the estimation results.

A. Preliminaries

We start with the firm productivity process, which is assumed to follow a geometric first-order autoregression,

$$\ln Z = \xi \ln Z_{-1} + \epsilon_Z, \quad \text{with } \epsilon_Z \sim N(0, \sigma_Z^2).$$

To pin down $\xi$ and $\sigma_Z$, one could draw from research that studies total factor productivity (in our data, we cannot). At the same time, these parameters will likely have important implications for some of our moments. Our strategy is to split the difference: we treat $\sigma_Z$ as a free parameter but fix $\xi = 0.8$ (Foster, Haltiwanger, and Syverson 2008). We opt to estimate the former

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24 After the completion of this project, we learned of Pozzi and Schivardi (2016), who examine data for three manufacturing sectors in Italy. Their estimates (tables 2 and 4) yield an autocorrelation of revenue total factor productivity (TFP) of 0.763, or just slightly less than our choice of $\xi$. They do estimate a more persistent process for physical TFP, which would imply a higher Frisch elasticity in our model (see sec. VI.A).
because it has a more direct impact on the volatility of working time and earnings, which follow static decision rules.

We pin down four more parameters on the basis of external information. First, the returns to scale are set to $\alpha = 0.67$, which will be consistent, given our estimate of $\eta$, with Italy’s labor share of around three-quarters (ILO and OECD 2015). Second, we choose a discount factor of $\beta = 0.941$, which is consistent with the average annual real interest rate in Italy over our sample. Third, the hiring cost, $\bar{c}$, is equivalent to 2.5 months of average earnings, according to a survey of plants in Veneto’s neighboring region of Lombardy (Del Boca and Rota 1998). Finally, the severance cost, $c$, represents a little over 7 months of earnings. The latter is a synthesis of multiple separation costs in Italy (see app. H).

Idiosyncratic preferences, $\xi$, and productivities, $\theta$, are independent discrete random variables. We assume that each is drawn from a uniformly weighted three-point distribution. Specifically, $\ln \xi \in \mathcal{X} = \{-X, 0, X\}$ and $\ln \theta \in \mathcal{Y} = \{-Y, 0, Y\}$, where $X$ and $Y$ are implied immediately by $\sigma^2_x$ and $\sigma^2_\theta$, respectively. In total, then, we have nine pairs of $\varsigma = (\xi, \theta)$, with each cohort equally represented in the population—for example, $\lambda_{\xi,\theta} = 1/9$ for each $(\xi, \theta)$. Appendix D argues that our conclusions are likely to hold when $\mathcal{X}$ and $\mathcal{Y}$ are higher dimensional and demonstrates this claim when $\xi$ and $\theta$ are drawn from four-point distributions. We also considered alternative shapes for the distributions of $\xi$ and $\theta$, but for reasons discussed in section VI.C, our moments appear to favor the assumption of uniformity.

Given these choices and initial guesses for the parameters, we simulate earnings, employment, and working time outcomes within firms. We generate 220 years of data and compute the moments on the basis of the last 20. Structural parameters are then updated to minimize the equal-weighted quadratic loss between the model-implied and empirical moments. 25

B. Main Results

Table 5 summarizes our results. The top panel confirms that the model, which is just identified, perfectly reproduces the targeted moments. The bottom panel reports estimates of the structural parameters. We discuss each of the parameter estimates in turn.

Elasticity of substitution.—Our estimate of $\rho = -1.962$ implies an elasticity of substitution across jobs of $(1 - \rho)^{-1} = 0.338$. To interpret this result, consider the response of working time to a one log point increase in $\xi$, holding fixed the employment of each type. Given an estimate of $\varphi$ (see below) and using equation (9), working time declines by approximately

25 When we constructed the moments within the model, we experimented with several different sample sizes. We settled on 10 independent panels of 20,000 firms because increases beyond these numbers had almost undetectable impacts on our estimates.
$(\varphi + 1 - \rho)^{-1} \approx 0.2$ log points. We further draw out the implications of $\rho < 0$ for making inferences about labor supply behavior in the next subsection.

Frisch elasticity.—We find a Frisch elasticity of $1/\varphi = 0.483$. This value is two to three times larger than earlier estimates in the life cycle literature (see Keane 2011) but within the range of results in more recent papers (see Chetty et al. 2011) whose research designs are less confounded by idiosyncratic variation. For example, Pistaferri (2003) shows that one can robustly identify the Frisch elasticity in a life cycle context using the response of hours to expected earnings growth, finding $1/\varphi = 0.7$. Pistaferri’s strategy can be interpreted within our framework by noting that if $\xi$ is relatively transitory, the expected path of earnings will more clearly reflect firm-level driving forces ($Z$). There are also a few estimates of the Frisch elasticity based on large-scale policy reforms, although no consensus has emerged. Sigurdsson (2021) finds a Frisch elasticity of just under 0.4 based on a tax holiday in Iceland (see also Bianchi, Gudmundsson, and Zoega 2001), but Martínez, Saez, and Siegenthaler (2021) do not find any labor input response to temporary regional tax rate changes in Switzerland.

Worker bargaining power.—Our finding of $\eta = 0.407$ implies a flexibility of earnings that is within the range of estimates in related research. On the one hand, it is somewhat below the $\eta = 0.52$ in Roys (2016), who estimates a model featuring a similar bargaining problem on French micro data. On the other hand, the elasticity of daily earnings with respect to average product implied by our $\eta$ is at the top end of estimates in Card, Devicienti, and Maida (2014), who also use the VWH (linked to other company account data). Card et al.’s results indicate an elasticity between 0.06 and 0.20, whereas our model-implied analogue is 0.22.26

Flow outside option.—The outside option $\mu$ represents 70% of mean earnings. To interpret this, it is helpful to see $\mu$ as the sum of two parts (although this was unnecessary for estimation): (i) income (i.e., transfers) per period of unemployment and (ii) the average discounted surplus of future employment. Appendix C shows how one can back out the former (i) using the estimated model, finding it to be almost 50% of average earnings. This figure is somewhat higher than unemployment insurance replacement rates in Italy—the latter are likely closer to 40% (see app. H)—but we take this to be an encouraging result for an out-of-sample test.

Shocks.—Our estimate of $\sigma_Z$ implies a standard deviation of productivity growth ($\Delta \ln Z$) equal to 0.214. This is similar to estimates based on plant-level TFP in other advanced European economies (Asker, Collard-Wexler, 

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26 Card et al. translate their results into estimates of bargaining power, which are substantially smaller than our value of $\eta$. However, since their static bargaining framework is unlike our setup, the more apples-to-apples comparison concerns the elasticity of wages with respect to average product.
and De Loecker 2014). At the same time, the implied standard deviation of \( \ln Z \) of 0.338 is somewhat higher than in Foster, Haltiwanger, and Syverson (2008), whose estimates are for the United States and are centered around 0.23. Finally, the dispersion of \( \ln Z \) is slightly less than total idiosyncratic variation as measured by \((\sigma^2_x + \sigma^2_v)^{1/2} \approx 0.361\).

C. The Importance of Complementarities for Labor Supply

In this section, the estimated model is used to implement two counterfactual simulations. Labor supply incentives are altered for only a fraction of the workforce in one counterfactual but for all workers in the other. The difference in outcomes (among participants in both counterfactuals) illustrates the impact of complementarities on the dynamics of labor supply.

Each counterfactual features an unanticipated and temporary change in \( y \). While such an experiment may seem a little abstract, we present the counterfactual in this way to maximize simplicity and transparency. In appendix F, we show that a temporary change in \( y \) can in fact stand in for a temporary change in a labor income tax rate (e.g., a tax holiday or a one-time tax surcharge).\(^{27}\) Intuitively, a larger wedge between the worker’s marginal product and marginal value of time has the same labor supply implications as a larger \( y \). Nevertheless, to make our point, it suffices to directly adjust \( y \).

We first perturb \( \xi \) for just the median cohort in the firm, \((\xi, \theta) = (1, 1)\). To illustrate, we elevate \( \xi \) by 25 log points, but the implied working time elasticities are robust to other choices. The affected employees, who make up one-ninth of the workforce, cut their time worked by 5.1%. If we viewed this result through the limiting case of \( \rho = \alpha = 1 \), we would infer a Frisch elasticity of \( \Delta \ln h / \Delta \ln \xi = 0.204 \) (corollary 1). In other words, by neglecting complementarities, we would mistakenly infer a Frisch elasticity that is 60% less than our estimate in section V.B. Notably, this result is only modestly affected if the cohort size is smaller than one-ninth of the workforce (see app. F).

Next, \( \xi \) is raised uniformly for all workers in the firm. Average time worked falls 9.7%, or almost twice as much as when just one cohort is affected. Even if we (wrongly) assumed \( \rho = \alpha = 1 \), we would recover a Frisch elasticity of 0.39, which is much closer to our estimate. Interestingly, the elasticity of working time in this counterfactual is in line with Sigurdsson’s (2021) analysis of Iceland’s (nationwide) tax holiday (again, see app. F).

More broadly, these results can help reconcile a nontrivial Frisch elasticity with a relatively muted response to purely idiosyncratic variation in labor supply incentives. A seminal example of the latter is a series of randomized

\(^{27}\) To be more exact, introducing a labor income tax \( t \) is equivalent to scaling \( \xi \) and \( \mu \) by \((1 - t)\). However, the change in \( \mu \) has little quantitative effect because it operates on working time only indirectly (by modestly altering the choice of employment).
control trials known as the negative income tax (NIT) experiments, which contributed to an earlier consensus on the inelasticity of (male) labor supply. The structure of the NITs is too intricate for us to fully capture, but it represented, in short, a temporary shift in tax rates and transfers that were “personal to the worker” (Hall 1999). Our counterfactuals illustrate how, under complementarities, these outcomes understate the scope for intertemporal substitution. Policy changes that are broadly applied, rather than narrowly targeted, are more likely to elicit working time responses indicative of the underlying preference parameters.

VI. Robustness

This section probes the robustness of our estimates. We examine the roles of (a) preset parameters and sample periods, (b) measurement error in working time, and (c) the assumed distributions of idiosyncratic types.

A. Preset Parameters and Sample Period

We reestimated the model given a higher firing cost, \( c \); a lower persistence of productivity, \( \xi \); and higher returns to scale, \( \alpha \). In another exercise, we fit a more recent subsample of the data. The results are reported in tables B1 and B2 (tables B1, B2, D1–D5, E1, G1–G4 are available online). Taken together, they point to a Frisch elasticity \( (1/\varphi) \) between 0.321 and 0.591 and an elasticity of substitution \( (1/(1 - \rho)) \) between 0.280 and 0.454. The midpoints of these ranges are very close to our baseline estimates. The last row of each table presents the counterfactuals: with one exception, working time continues to respond almost twice as much when all workers face the higher \( y \).

Larger frictions and less persistent productivity have similar effects. A firing cost of one year’s earnings compresses changes in employment, which has two implications. First, since employment crowds out working time (see sec. II.C.2), smaller changes in the former are matched by larger movements in the latter. In addition, the larger firm-level shocks needed to reproduce the observed variance of \( \Delta \ln N \) further exaggerate fluctuations in working time as well as earnings. Therefore, a lower Frisch elasticity, \( 1/\varphi \), and bargaining power, \( \eta \), are needed to match the data. Like a higher \( c \), less persistent productivity induces smaller changes in employment: if hiring and firing are costly to reverse, firms attenuate responses to relatively transitory shocks. For \( \xi = 0.6 \), which is at the bottom of the range cited in Syverson (2011), the model “needs” a higher \( \sigma_Z \) and lower values of \( 1/\varphi \) and \( \eta \).

Many parameters move in the opposite direction when \( \alpha \) is raised. We set \( \alpha = 0.835 \), which is halfway between 1 (constant returns) and our baseline of \( \alpha = 0.67 \).28 A higher \( \alpha \) makes labor demand more elastic, and smaller

\[^{28}\text{Our baseline of } \alpha = 0.67 \text{ implicitly treats capital as if it were fixed. Any degree of capital adjustment will imply a (reduced-form) elasticity of output with respect}\]
shifts in $Z$ are needed to match the variance of $\Delta \ln N$. It follows that $1/\varphi$ must rise to generate realistic volatility in working time.

We also reestimated the model over 1994–2001. The results suggest some changes in the contours of wage and working time setting during this period. First, we observe more variance in wage and, thus, earnings growth, perhaps reflecting a 1993 accord among policy makers, employers, and unions that is thought to have enabled more flexible wage bargains (Lodovici 2000). This change alone implies a higher bargaining power, $\eta$, but has modest implications for the key preference and production parameters, $\varphi$ and $\rho$, that shape the counterfactuals. Table B2 also shows, though, a coincident decline in working time variability, which is not directly linked to the accord. These moments imply a lower Frisch elasticity, which is needed to reduce $\text{var}(\phi^h)$, and a higher elasticity of substitution, which ensures that $\text{var}(e^w)/\text{var}(e^b)$ does not rise too much (when $\text{var}(e^h)$ falls). Consistent with these changes, there was a separate push by workers around this time to temper firms’ overtime use while enhancing individuals’ scheduling flexibility (D’Aloia, Olini, and Pelusi 2006). Finally, a higher elasticity of substitution contributes to a smaller gap between firm-level and idiosyncratic labor supply responses, but even so, the former is 50% larger.

B. Measurement Error in Working Time

The VWH lacks data on daily hours. As a result, it likely understates the variance of working time and, thus, may overstate the relative variance of earnings growth (since earnings in the VWH do reflect all remunerated time). This subsection examines the implications of mismeasuring these moments for our baseline results.

Our analysis draws on Italy’s Labor Force Survey (LFS), which has a uniquely helpful feature: it asks about weekly hours and days worked in the survey reference week. We can then match self-reported job stayers across adjacent years and compute the role of days in weekly hours fluctuations. Appendix B shows that the days margin accounts for virtually all of weekly hours growth if we include (reference) weeks with no paid days, which reflect, in part, weeks of layoff. In such cases, though, we cannot confirm that the respondent returns to the same job. If we drop weeks with no paid days, the importance of the days margin is reduced by almost half. For the sake of sensitivity analysis, we simply take days to make up roughly 75% of hours fluctuations, or the midpoint between these two results.

to labor input that exceeds 0.67. Therefore, we consider a higher, rather than lower, $\alpha$.

29 To this end, the agreement encouraged firm-level contracts to increase the share of wages tied to firm-level performance. In addition, the accord formally abolished a wage indexation scheme, but the latter had been substantially weakened long before (Manacorda 2004).
Appendix B examines the implications of missing one-quarter of hours variation. Suppose the latter is distributed across \( \text{var}(\varepsilon^h) \) and \( \text{var}(\phi^h) \) in proportion to each moment’s share in the total variance of working time in the VWH. The resultant rise in the idiosyncratic variance implies weaker complementarities, which yields a higher elasticity of working time to idiosyncratic events. However, the higher implied firm-level variance points to a bigger Frisch elasticity, which amplifies the response to firm-level shocks. It turns out that these two changes virtually offset one another: working time still responds twice as much to firm-level as to idiosyncratic events, consistent with results in section V.

Relatedly, the lack of hours data means that we cannot separate out the components of daily earnings. The overall elasticity of days to daily earnings can be decomposed into two parts that reflect the comovement of days with (i) daily hours and (ii) hourly earnings. However, the former (i) has no counterpart in the model. In appendix B, we use the LFS to infer the component due to the latter (ii), which has a more natural connection to the comovement of working time and wages in the model. Whereas the overall elasticity is \(-0.169\) (see table 5), we peg the contribution of the latter (ii) to be \(-0.130\). This result suggests that the overall elasticity largely reflects the comovement of working time and wages.

C. Distributions of Idiosyncratic Types

We now assess the importance of two of our assumptions about idiosyncratic types, namely, \( \xi \) and \( \theta \) are (1) purely transitory and (2) uniformly distributed.

First, the bottom line of our results is likely to be robust to the introduction of persistent types. Persistence has a direct impact on neither optimal working time, which is an intratemporal condition, nor earnings, which takes the same form as in equation (12) (see app. C) when types are persistent. On the extensive margin, persistence would diminish the motive for labor hoarding, leading to more turnover after draws of \( \xi \) and \( \theta \). Excess turnover would have to be offset in the model by a lower \( \sigma_Z \), which means that a higher Frisch elasticity would be needed to match the volatility of average working time.\(^{30}\)

Second, appendix D shows that our moments favor uniform types insofar as modest deviations from uniformity weaken the model’s fit. The reason is that under nonuniform distributions, changes in type often yield changes in own-type employment, \( n_{\xi,\theta} \). Since diminishing returns means that \( h_{\xi,\theta} \) and \( w_{\xi,\theta} \) are each declining in \( n_{\xi,\theta} \), changes in own-type employment push working time and the daily wage in the same direction. The result is a strongly positive, and highly counterfactual, correlation between \( h_{\xi,\theta} \) and \( w_{\xi,\theta} \).

\(^{30}\) The effect of persistence on the hiring margin is somewhat harder to predict because it shapes the composition of the pool of potential hires. See app. D for a discussion.
VII. Conclusion

This paper has argued that production complementarities compress working time adjustments within a firm, squeezing out the influence of idiosyncratic, or worker-specific, shocks. As a result, working time elasticities derived from idiosyncratic variation are attenuated relative to the Frisch elasticity. By contrast, firm-level variation acts to coordinate employees’ working time decisions and thereby elicits a response more consistent with utility parameters. Indeed, our estimates imply that an identification strategy based on idiosyncratic variation would recover a Frisch elasticity that is biased down by almost 60%. More generally, our results suggest that more aggregate-level variation, such as broad-based policy changes, is likely to better inform estimation of preference parameters.

Our framework can shed light on other economic questions where complementarities play a role. For instance, it can inform the study of housing wealth effects (on labor supply) when housing prices change unevenly across workers (see Guerrieri, Hartley, and Hurst 2013). In addition, it can aid in assessing public policies that target the intensive margin, such as paid family leave (for a review, see Olivetti and Petrongolo 2017). The cost to the firm of adjusting to a worker’s absence depends on the elasticity of substitution across employees. Thus, our framework could inform the cost-benefit analysis of such policies.

At the same time, certain extensions to our model would be worthwhile. A challenging but valuable task is to incorporate imperfect information over types. Suppose, for instance, that firms have only a noisy signal of the preference shifter, $\xi$. We conjecture that working time and earnings will respond less to workers’ reports of higher $\xi$s, leading to potentially larger extensive margin adjustments. This approach would help bridge the divide between our paper, in which there is substantial scope for renegotiating over $\xi$, and related models with competitive labor markets and, thus, no space for bargaining (see Yurdagul 2017).

Another profitable extension addresses the choice of complementarities over the long run. Goldin and Katz (2016) have argued that changes in information technology and market structure have supported the adoption of new modes of production with generally weaker complementarities. By integrating the choice of production structure into the firm’s problem, our framework could engage long-run trends in working time and earnings as well as the short-run dynamics on which this paper focused.

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