NUMERICAL STUDY OF THE LOWEST ENERGY CONFIGURATIONS FOR GLOBAL STRING-ANTISTRING PAIRS

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ABSTRACT

We investigate the lowest energy configurations for string - antistring pairs at fixed separations by numerically minimizing the energy. We show that for separations smaller than a critical value, a region of false vacuum develops in the middle due to large gradient energy density. Consequently, well defined string - antistring pairs do not exist for such separations. We present an example of vortex - antivortex production by vacuum bubbles where this effect seems to play a dynamical role in the annihilation of the pair. We also study the dependence of the energy of an string-antistring pair on their separation and find deviations from a simple logarithmic dependence for small separations.

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1. INTRODUCTION

Production and interaction of topological defects has been the subject of a large number of investigations. Existence of such defects in various condensed matter systems has been known for long time. Topological defects can also arise due to the spontaneous symmetry breaking in the early Universe. There are essentially two different sorts of mechanisms for the production of topological defects. They could be produced due to spatial fluctuations in the vacuum degrees of freedom of the Higgs field [1], or they could be produced directly due to energy fluctuations [2]. These two processes are unrelated and depending upon the situation, one may dominate over the other. (This is especially true for the global case. For the gauged case, the basic picture of [1] has been recently reanalyzed, see [3].) For example in the context of the early Universe, the number of global defects which exist in the broken phase as the universe cools through a phase transition will generally be dominated by the first kind of process as the number of thermally produced defects will be exponentially suppressed at low temperatures. On the other hand, if we were to consider say the heating of a sample of liquid crystal towards the transition temperature, then the production of defect-antidefect pairs may be dominated by the energy fluctuations.

When global defects are produced due to energy fluctuations then due to the fact that the topological quantum numbers are conserved in local fluctuations one needs to consider the production of defect-antidefect pairs due to local fluctuations in the energy. For 3+1 dimensions, one may consider monopole-antimonopole pairs, or small loops of strings whereas for 2+1 dimensions one may consider the production of vortex-antivortex pairs. Typically one will expect that a pair where defect and antidefect are far separated will be suppressed, first due to the local nature of the energy fluctuation and secondly, for global defects, the energy of a far separated pair is larger leading to additional suppression. One needs to, therefore, consider the production of a defect-antidefect pair with a given separation between the defect and the antidefect. Further, for a pair with a given defect-antidefect separation, various kinds of field configurations will exist and the one with lowest energy will dominate. As a defect-antidefect configuration is not a solution of static equations of motion, the above discussion amounts to finding the lowest energy configuration with the constraint that the distance between the defect and the antidefect be held fixed. We would like to mention here that similar kind of constrained minimization has been recently used for instanton-anti-instanton pairs in the context of determining multiparticle production cross-sections at high energies, see [4]. It may be interesting to
see if the approach we develop here can be used for those cases as well.

In this paper, we have carried out the numerical minimization of energy for global U(1) strings. We study a pair of parallel string and antistring, and find the lowest energy configuration for the pair by keeping the separation fixed. We find that such a configuration always does not exist. This is because when the separation between the string and the antistring is less than a certain critical value (which turns out to be about 8 - 9 times the inverse Higgs mass) then the gradient energy density in the intermediate region becomes very large such that it becomes possible to produce a pair of string-antistring in the intermediate region which annihilates, cancelling the initial winding numbers and thereby lowering the gradient energy. This suggests that the production of vortex-antivortex pairs separated by distances less than a certain critical value (or string loops with small diameters) will be suppressed in a thermal production process from what one might have expected. This is because the only way in which such a pair can have a distinct identification of a vortex and an antivortex is by having highly excited configuration. We also study the dependence of the energy of the string-antistring pair on their separation and find that, for small separations, the energy varies as proportional to \((\ln(R))^\alpha\), \(R\) being the separation between the string and the antistring, and \(\alpha\) numerically found to be \(\simeq 0.32\).

An interesting implication of the existence of a critical separation between the string-antistring pair is for the case of string-antistring annihilation process. Generally in such studies the string and the antistring move with large velocities. Even if they start at rest, due to long range forces between global strings, they collide with large velocities. In most of such cases then, the above mentioned effect, namely, the intermediate region getting large energy density and leading to the annihilation, does not play any role. However, it may play the dominant role in the annihilation process in some special cases where the string and antistring are very close and the relative velocity is not too large as we will show in an example of the production of a vortex-antivortex pair by vacuum bubbles. The pair annihilation in that case proceeds by the evolution of the intermediate region into false vacuum and winding number disappears much faster than it could even if the vortices collided with speed of light.

We first discuss the constrained minimization of the energy of a string-antistring configuration in Sec. 2. The case of vortex-antivortex production by bubbles and subsequent annihilation is discussed in Sec. 3 and conclusions are presented in Sec. 4.
2. LOWEST ENERGY STRING-ANTISTRING CONFIGURATIONS

We consider a model with a single complex scalar field where strings arise due to the spontaneous breaking of a global U(1) symmetry

\[ L = \frac{1}{2} (\partial_\mu \Phi^*) (\partial^\mu \Phi) - \frac{\lambda^2}{4} (\Phi^* \Phi - \eta^2)^2 \] (1)

where \( \Phi = \Phi_1 + i \Phi_2 = \phi e^{i\theta} \). We will use the natural system of units with \( \hbar = c = 1 \). All distances will be measured in the units of \( (\eta \lambda)^{-1} \), energy density in the units of \( \lambda^2 \eta^4 \) and \( \phi \) in units of \( \eta \). We will use \( \lambda = \eta = 1 \) and therefore all these units are equal to 1.

The above model admits string solutions in the broken symmetry phase which, for a winding number one string parallel to the z axis, can be written as

\[ \Phi(r, \theta) = \phi(r) e^{i\theta} \] (2)

where \( r \) and \( \theta \) are respectively radial and azimuthal coordinates in the x-y plane. For the static case, \( \phi \) satisfies the following equation

\[ \phi'' + \frac{\phi'}{r} - \frac{\phi}{r^2} - \lambda^2 \phi (\phi^2 - \eta^2) = 0 \] (3)

The energy (per unit length, assuming z symmetry) associated with a given configuration is

\[ E = \int d^2 x \left[ \frac{1}{2} |\nabla \Phi|^2 + \frac{\lambda^2}{4} (\Phi^* \Phi - \eta^2)^2 \right] \] (4)

Our minimization procedure consists in starting with a given field configuration on a two dimensional lattice and then varying the field configuration at each lattice site. Since we are attempting to find the lowest energy configuration, we allow fields at the boundary to vary as well. We have tried out various minimization techniques and have found that over relaxation is very efficient for our case. This consists in first determining the most favorable fluctuation in \( \Phi \) at a given site by fluctuating \( \Phi \) there and considering the change in the energy density. The most suitable fluctuation corresponds to the minimum of the parabola which passes through these values of energy densities (corresponding to fluctuated values of \( \Phi \)). Then the actual change in \( \Phi \) is taken to be larger (by a certain factor) than this most suitable fluctuation. We have found that changing this factor in the range of 0.2 - 0.9 worked best for our case. Computations were carried out on Cray-2 and Cray X-MP computers at the Minnesota Supercomputer Institute.
We have tested our minimization code by finding the configuration of a single string. We take the form of $\Phi$ as given in Eq.(2) and prescribe some initial function for $\phi(r)$. We then minimize the energy (Eq.(4)) and determine $\phi(r)$ which gives the lowest energy configuration. We have found that even if the initial profiles for $\phi(r)$ prescribed are very different (for example we have tried out triangular form for $\phi(r)$), after about 200 iterations, $\phi(r)$ converges to the exact solution as obtained from Eq.(3). In Fig. 1 we have given the resulting $\phi(r)$. For this case we start with $\phi(r)$ given by $\phi(r) = \eta(1 - e^{-r/\delta})$ with $\delta$ chosen to be 0.4. We choose such a $\delta$ so that the initial profile is very different from the correct one. This initial profile is shown in Fig.1 by the dotted curve. The correct solution for $\phi(r)$ is obtained by numerically solving Eq.(3) using a Runge-Kutta algorithm of fourth order accuracy. The solution is shown in Fig. 1 by the solid curve. It is known [5] that for large $r$ the leading terms in a power series expansion of $\phi(r)$ are given by

$$\phi(r) = \eta \left(1 - \frac{1}{2r^2}\right) \quad (5)$$

By fitting the large $r$ region of the solution (solid curve in Fig. 1) we find the exponent of $r$ in Eq.(5) to be 1.996 and the coefficient of $1/r^2$ to be 0.503. Starting with $\phi(r)$ as given by the dotted curve in Fig.1, after about 200 iterations we obtained the dashed curve in Fig.1 which is extremely close to the correct solution showing the efficiency of our energy minimization code.

We now continue to determine the lowest energy string-antistring configuration at a fixed separation. First, it is helpful to make a rough estimate of the gradient energy density contained in the region between the string and antistring. For this purpose we take the ansatz which we used in an earlier work for the case of global string loops [6]. (In [6], the core energy was neglected and only the gradient energy outside the core was considered. It does not matter, however, as the only thing we need from [6] is the extent of the region in which the gradient energy is concentrated at the midpoint of the loop.) It was assumed in [6] that all (or most) of the gradient energy is contained within the region bounded by two paraboloids. The gradient energy density is smallest near the center of the loop where the distance between either of the paraboloids from the center of the loop (called $Z_0$ in [6]) is largest. Taking $Z_0$ as a variational parameter, it was found in [6] that $Z_0 = \frac{\sqrt{3}}{4} D$ where $D$ is the diameter of the loop. As the loop was assumed to be azimuthally symmetric in [6], we can take its intersection by a plane normal to the loop and passing through it’s center. This gives us the Higgs phase distribution for
a parallel string-antistring pair separated by a distance $D$ such that all of the gradient energy is concentrated within two outermost parabolas, see Fig. 2. The distance between the two parabolas at the midpoint is $2Z_0$. The gradient energy density is smallest near the midpoint and if in that region it becomes larger than the false vacuum energy density, we will expect that well defined string-antistring configuration does not exist anymore. This will happen when

$$\frac{1}{2}\eta^2 \left(\frac{2\pi}{2Z_0}\right)^2 > \frac{\lambda^2}{4\eta^4}$$

(6)

Since $Z_0 = \sqrt{\frac{3}{4}} D$, this implies

$$D < \sqrt{\frac{2}{3}} \frac{4\pi}{\lambda \eta}$$

(7)

With our choice of parameters ($\eta = \lambda = 1$) this implies that when $D < 10.3$, a well defined string-antistring pair will not exist. We will see later that our numerical results confirm this estimate to a reasonable accuracy where we find the critical separation to be $\simeq 12.0$. [Actually the gradient energy density is higher near the strings which is where the false vacuum first develops, as we will see later. This may account for somewhat larger value of the critical separation we find.] This suggests that the ansatz used in [6] correctly represents the concentration of the gradient energy between the string-antistring pair (even though it does not correctly describe the field configuration near the cores of the individual strings).

We now consider the energy minimization for a string-antistring pair with their separation held fixed. All along we will take the string and antistring to be along $z$ axis and only present their profiles in the $x$-$y$ plane. Our results are thus also valid for vortex-antivortex pairs in 2+1 dimensions. We take the initial profiles of the string and the antistring to be the ones obtained by numerically solving the equations of motion (the solid curve in Fig.1). To give configuration of the pair we take the product ansatz [5]. If the string center is located at $\vec{r}_1$ and the antistring center at $\vec{r}_2$ then $\Phi$ for the pair is given by

$$\Phi_{\text{pair}}(\vec{r}) = \frac{1}{\eta} \Phi_{\text{string}}(\vec{r} - \vec{r}_1)\Phi_{\text{antistring}}(\vec{r} - \vec{r}_2)$$

(8)

We hold a string (or antistring) fixed by fixing the field configuration at few nearby points on the lattice. Only fixing $\Phi$ at the center of the string does not work since it only fixes the magnitude of $\Phi$ without fixing it’s winding number and it becomes energetically favorable for the pair to let the winding slip out of the fixed centers and annihilate it in
the middle. We consider the center of the string (antistring) to lie at the midpoint of an elementary plaquette and then hold Φ fixed at the four corners of this plaquette thereby fixing the winding number. [We have also tried fixing Φ in somewhat larger region by considering the center of the string at a lattice site and holding Φ fixed at the corners of all four plaquettes which have one vertex common with the center of the string. The results are essentially the same.] We use $400 \times 400$ lattice with the physical size of $80 \times 80$. The choice of the lattice size was governed by the fact that if a string is too close to the boundary then it becomes energetically favorable for it to have its gradient energy concentrated towards the boundary. To avoid this “boundary” effect we considered lattice size such that the separation between the string-antistring is smaller than the distance of either of them from the lattice boundary.

Fig. 3a shows the energy density plot for a string-antistring pair with separation equal to 14 units. In order to show string-antistring configuration clearly we will always plot only the central portion of the lattice with physical size of $20 \times 20$. Minimization is carried out until the energy is almost stationary. The energy density plot for the final configuration is shown in Fig. 3b. The strings get little squeezed towards the middle region and energy density gets little peaked near the centers of the strings (due to holding the strings fixed). The profile of $\phi$ does not change much (except little squeeze towards middle) and we do not show it here. We therefore see that for this separation (as well as for larger separations) it is possible to find the lowest energy configuration for a well defined string-antistring pair. We then continue the minimization but now by letting the strings move. String-antistring successively approach each other and annihilate as shown by Fig.3c and Fig.3d.

We now consider a pair with smaller separation. Fig. 4a shows the energy density plot for a pair with separation equal to 12 units. Fig. 4b is the plot of $\eta - \phi$ for this pair. We carry out the minimization but in this case we do not achieve any stationary value of the energy and the energy keeps decreasing. We show the plots at various intermediate stages. As the minimization proceeds, the energy density becomes highly peaked near the centers of the strings as shown in Fig. 4c. (If the peak becomes higher than 1.2 then we truncate it for plotting convenience.) Again this happens because we have held $\Phi$ near the string center fixed while $\Phi$ in the neighboring region is distorting to keep energy lowest. Fig. 4d shows the plot of $\eta - \phi$ at the same stage (as in Fig. 4c) showing clearly that the whole profile of string and the antistring has squeezed towards the intermediate region and two peaks have developed in the intermediate region. These peaks are actually a pair of string-antistring which get created in the middle region. This happens as the
Higgs phase gradient energy concentrates more and more in the middle region and given that the gradient energy is highest near the strings, it becomes favorable for $\Phi$ to develop zeros in those regions. The distortion near the string centers is so large that the winding number near the original string centers disappears (due to finite lattice spacing) and is carried by the new pair. Figs.4e-4g show the successive stages where the intermediate peaks (the new string-antistring pair) come towards each other and annihilate. The two remaining peaks in Fig. 4g are due to holding the string centers fixed. When we let $\Phi$ vary everywhere then these peaks quickly decay away as shown in Fig. 4h. Fig. 4i is the Higgs phase plot at the initial stage corresponding to Fig. 4a and shows the windings of the string and antistring. Fig. 4j corresponds to the stage as in Fig. 4g showing clearly that all the windings have disappeared. Exactly near the initial locations of the centers of string-antistring the winding numbers are still held fixed. However, due to finite lattice spacing, the continuity can be broken if it is energetically favorable which is what happens here. The peaks in the energy density (e.g. Figs. 4d and 4g) are precisely due to this rapid change in the Higgs phase near the string centers (these peaks disappear in Fig. 4h when $\Phi$ is not held fixed anymore).

Exactly the same behavior, as we observed above for separation equal to 12, was observed for separations smaller than 12 as well. For separation equal to 13 the situation was not very clear as the energy seems to become stationary after a large number of variational steps. However, the profile of $\phi$ develops edges near the centers of the strings (somewhat similar to the one in Fig. 4d, but the second peak being extremely close to the original string). Separation equal to 13 thus seems like the border line case. Clear distinct behavior is observed between separation $\geq 14$ case (where the string and antistring remain almost unchanged, except little squeeze towards the middle region) and separation $\leq 12$ case where it is not possible to hold string and antistring separately. Note that this is in good agreement with the estimate of the cutoff separation in Eq.(7). What this means is that the lowest energy configurations for separations less than 12 do not resemble in any way to string-antistring configurations. In a thermal production one will generally expect that pairs with smaller and smaller separations will be more and more abundant (especially for vortex-antivortex pairs). This does not seem to be the case though as pairs with distances less than 12 (for general parameters this cutoff distance will be of order of what is given in Eq.(7)) will exist only if they are highly excited. One may thus expect the number density of string-antistring pairs to not keep increasing as a function of separation and (at best) level off at a cutoff separation. For string loops these results suggest that, again due to the concentration of the gradient energy in the inside of the
loop, there will be a critical diameter such that well defined, lowest energy, configurations for loops with smaller (fixed) diameters will not exist.

We now study the dependence of the energy of the string-antistring pair on their separation. We found the lowest energies for separations $R$ equal to 14, 16, 20, 24, 28 and 32. For larger values of $R$ (actually even for $R = 32$), the strings are closer to the boundary and the values of energy are affected by the boundary cutoff (as we mentioned earlier). For large $R$, $E$ is supposed to vary in the following manner [5],

$$ E = A \ln \left( \frac{R}{\lambda} \right) $$

with $A$ of the order of $\pi$ and $\lambda$ of the order of inverse Higgs mass. (Energies of the cores can be absorbed by redefining $\lambda$).

Fig. 5 shows the variation of the energy $E$ obtained by our minimization code, as a function of $\ln(R)$. Small squares show the values of $E$ at the above mentioned values of $R$. There is a clear deviation from a straight line. We have fitted the two different segments of straight lines (shown by the two dashed line segments) to these points. For small $R$, best fit to first three points gives $A \simeq 6.20$ and $\lambda \simeq 0.73$, while for large $R$, best fit to last three points gives $A \simeq 4.35$ and $\lambda \simeq 0.17$. Theoretically expected value of $\lambda$ is $\simeq 0.71$ (corrections in the definition of $\lambda$ due to core energy are small). Although the value of $A$ obtained by fitting points at large $R$ is closer to theoretically expected value, the value of $\lambda$ is not. The increase in the slope of the energy curve for smaller values of $R$ may be related to the existence of the critical separation. However, we again emphasize that values of $E$ at large $R$ are affected by boundary cutoffs which may affect the slope. We also attempted to fit a curve where $A$ in Eq. (9) was replaced by $AR^\beta$. However, we could not find a good fit for any choice of (positive or negative) $\beta$, $A$ and $\lambda$. The curve which describes a good fit to all six points is given by

$$ E = A (\ln(R/\lambda))^{\alpha} $$

with $A \simeq 19.3, \lambda \simeq 6.0$ and $\alpha \simeq 0.32$. This is shown by the solid curve in Fig. 5. Larger values of $\alpha$ do not fit all the points so well. However, we would like to mention that the value of $\alpha$ here may be affected by boundary effects as we mentioned above.

3. VORTEX-ANTIVORTEX PRODUCTION BY BUBBLES

As we had mentioned earlier, the above results also have implications for string-antistring annihilation processes. However in most such cases the intermediate region
does not have time to evolve due to large velocities of the strings. One has to then find special cases where the string-antistring get created very close to each other and with small relative velocities. We now present an example where this is what seems to happen and the annihilation of string-antistring pair appears to be completely dominated by the sort of behavior we discussed above. We study a first order phase transition case in 2+1 dimensions where the vortices are produced by the collision of vacuum bubbles (see [7], for details). The Lagrangian density is given by

\[ L = \frac{1}{2} \partial_\mu \Phi^* \partial^\mu \Phi - \frac{\lambda}{4} \phi^2 (\phi - \phi_0)^2 + \frac{\lambda}{2} \epsilon \phi_0 \phi^3 \] (11)

where \( \phi \) is the magnitude of \( \Phi \) (\( \Phi = \phi e^{i\theta} \)). There is a metastable vacuum at \( \phi = 0 \) and the true vacuum is at \( \phi = \eta' \) where \( \eta' \) is the vacuum expectation value of \( \Phi \) which spontaneously breaks the \( U(1) \) global symmetry leading to the existence of global strings. Following results are for parameter choices \( \epsilon = 0.1, \phi_0 = 4.0 \) and \( \lambda = 4.0 \). We will measure spatial and temporal coordinates in terms of the inverse Higgs mass which for these values of parameters is equal to 0.12 (we continue to use the natural system of units).

The phase transition in this case proceeds by the nucleation of critical vacuum bubbles whose profile is given by the solutions of the Euclidean equations of motion. The actual details of this are not relevant here and we refer the reader to Ref. [7]. It was shown in [7] that when three critical bubbles collide then depending on the values of Higgs phases inside the bubbles, a vortex may form at the collision point. We had also found in [7] that vortex can form even in the collision of two critical bubbles and a subcritical bubble; a subcritical bubble being a small bubble which collapses (and then bounces back before collapsing again). We had found in [7] that the vortex found in this manner invariably escapes out of the bubbles because of the large momentum of the walls of critical bubbles compared to the momentum of the wall of the subcritical bubble. [Here we may mention again that use of classical equations of motion for subcritical bubbles is really justified only for the case of thermal production. One may thus consider the case of thermal production and take our critical and subcritical bubbles as just representing a class of expanding and collapsing bubbles respectively.] We had also found in [7] that if the Higgs phase distribution is asymmetric in the three colliding critical bubbles then in order to minimize the gradient energy, the vortex develops large velocity towards the direction of larger phase gradient.

We use these results now and consider the collision of two critical bubbles and a subcritical bubble such that most of the phase gradient energy is concentrated towards
the critical bubbles (from the collision point) which should then counter the effect of large momentum of critical bubble walls. The idea being that this way one may be able to stop the vortex from escaping out of the bubble. What we find however is that although the vortex itself does not escape out of the bubble, an antivortex gets created at the bubble wall which moves in and annihilates this vortex. The whole thing being consistent with the fact that the final field configuration in space is the one given only by the two critical bubbles.

Fig. 6a is the plot of $\eta' - \phi$ and shows the initial profiles of bubbles. Left one is the subcritical bubble while the two on the right are the critical bubbles. In the plots of $\eta' - \phi$, the x axis will be from top to bottom and the y axis from left to right. Fig. 6b shows the initial distribution of the Higgs phase $\theta$ for these bubbles. Starting with the bubble with smallest value of y (which is the subcritical bubble) and going counter clockwise, the values of $\theta$ are respectively $180^0$, $100^0$, and $260^0$. $\theta$ for the subcritical bubble will flip to $\theta = 0$ after the bubble collapses and bounces back, see [7]. Thus the variation of $\theta$ is maximum between the top two (critical) bubbles. Fig. 6c shows the situation at $t = 20.2$ when all three bubbles have coalesced and a vortex is formed in the collision region. Figs. 6d - 6g show closeup of the region of the vortex. Fig. 6e shows an antivortex separating from the bubble wall at $t = 24.37$ (which will become clear when we show plots of Higgs phase). We see clearly that the region between the vortex and the bubble wall evolves to the false vacuum. Vortex-antivortex annihilate each other by $t = 24.76$ as shown in Fig. 6f and finally decay away, see Fig. 6g.

Let us now follow the annihilation process by following the plots of the Higgs phase. Fig. 7a-7c are the plots of the region containing the vortex. Fig. 7a shows only one vortex whereas Fig. 7b shows the presence of a well defined pair of vortex-antivortex near $X \simeq 56.0$ (corresponding to the plot in Fig. 6e). Vortex being at $y \simeq 49.0$ and the antivortex at $y \simeq 43.0$. [The lengths of vectors in these figures are large for large $\phi$ and vectors are not plotted where $\phi$ is extremely small. Fig. 7b therefore shows that vortex and the antivortex are clearly separated by region where $\phi$ is different from zero.] Fig. 7c shows the case when the vortex and antivortex have disappeared. If this annihilation had proceeded by the vortex and antivortex moving towards each other, it will imply a relative velocity of about 14 times the speed of light, clearly an absurd number. What instead happens here is that the region in between the vortex and antivortex evolves to $\phi = 0$ and essentially dissolves the vortex and antivortex. This is the same sort of behavior we had observed in the variational study discussed in Sec.2.
4. CONCLUSIONS

We emphasize again that the phenomena we discuss here, namely the region in between the string-antistring playing a crucial role in the annihilation process, will generally be obscured in the studies where the string and antistring move with large velocities. In those cases, the annihilation will proceed by the string and antistring approaching each other. Under very special situations, such as the one discussed above for the case of bubbles, it may happen that the antistring just gets created close to the string (with little relative velocity) and one may be able to observe such effects. Our results for the lowest energy configurations of string-antistring pairs (vortex-antivortex pairs in 2+1 dimensions) show the existence of a cutoff separation and suggest that if defect-antidefect pairs were thermally produced then their number density should not keep increasing and at best may flatten out for separations smaller than a critical value. For string loops our results imply that loops with diameters less than a critical diameter may be suppressed compared to naive expectations. We have also studied the variation of the energy of a string-antistring pair on the separation $R$ and find that, for small $R$, it deviates from a simple logarithmic dependence. These results should have consequences for defect production in phase transitions (such as the one studied in [8]) and may be testable in condensed matter experiments. All these qualitative features should clearly exist for other global defects as well, such as global monopoles etc.

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FIGURE CAPTIONS

Figure 1 : Solid curve shows the string profile obtained by solving equations of motion. Dotted curve is the initial profile used for minimization of energy and dashed curve gives the string profile after the minimization is completed.

Figure 2 : Two small circles denote cross-sections of the string and antistring. Solid curves bound the region inside which most of the gradient energy of the Higgs phase is concentrated in the middle.

Figure 3 : (a) Energy density plot for initial string-antistring pair with separation equal to 14.0. (b) String-antistring after energy minimization is completed with separation fixed. (c) String-antistring pair after further minimization with separation allowed to change. (d) Annihilation of string-antistring.

Figure 4 : (a) and (b) give the plots of energy density and $\eta - \phi$, respectively for initial string-antistring pair with separation equal to 12.0. (c) and (d) are similar plots at an intermediate stage of the energy minimization showing the formation of a new string-antistring pair (which carries the winding numbers of the initial pair) in the middle region. (e) and (f) are plots at a later stage of the minimization showing the situation when this new pair is about to annihilate. (g) Field configuration after the annihilation is completed. The two remaining peaks are due to holding the original configuration fixed in those regions. (h) Field quickly decays away when minimization is continued while letting $\Phi$ vary everywhere. (i) Plot of the initial distribution of Higgs phase showing the windings of the string and the antistring. Higgs phase is equal to the azimuthal angle of a vector. (j) Final plot of the Higgs phase showing that the winding numbers have disappeared.

Figure 5 : Plot of energy $E$ vs. $\ln(R)$. Small squares show the values of $E$ obtained by the minimization code for various values of $R$. Two segments of dashed lines show best fit for points at large $R$, and small $R$ respectively. Solid curve denotes the fit $E = A(ln(R/\lambda)\alpha)$.

Figure 6 : (a) Plot of $\eta' - \phi$ showing initial configuration of bubbles. (b) Initial distribution of Higgs phase. (c) Profile of the vortex after bubbles have coalesced at $t = 20.20$. (d) Closeup of the region of the vortex at $t = 23.58$. (e) Vortex at $t = 24.37$. Small peak separating from the bubble wall is an antivortex. (f) Annihilation of the vortex and antivortex at $t = 24.76$. (g) Decayed configuration at $t = 27.5$.

Figure 7 : (a) Higgs phase plot. Vortex is located near $Y \simeq 49.0$, $X \simeq 56.0$. (b) Antivortex has formed near $Y \simeq 43.0$, $X = \simeq 56.0$. (c) Vortex and antivortex have disappeared and there is no winding present.