Existence of Majorana fermions for M-branes wrapped in space and time

ANDREW CHAMBLIN

DAMTP, Silver Street
Cambridge, CB3 9EW, England
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Abstract

We show that it is possible to define Majorana (s)pinor fields on M-branes which have been identified under the action of the antipodal map on the adS factor of the throat geometry, or which have been wrapped on two-cycles of arbitrary genus. This is an important consistency check, since it means that one may still take the generators of supertranslations in superspace to transform as Majorana fermions under the adjoint action of $Spin(10, 1)$, even though the antipodally identified M2-brane is not space-orientable. We point out that similar conclusions hold for any p-branes which have the generic (adS) × (Sphere) throat geometry.
I. INTRODUCTION

Recently, there has been an enormous amount of interest in the information which may be contained in the near-horizon geometries of the p-branes which are a staple feature of the supergravity and string theory diet. In particular, it has recently been conjectured [2] that information about the dynamics of superconformal field theories (in the large \( N \) limit) may be obtained by studying the region near the horizon of certain D(p)-branes. Thus, the conjecture implies a correspondence between gauge theories in the large \( N \) limit and compactifications of supergravity theories. The correspondence is often called ‘holographic’ because the superconformal field theory (SCFT) lives on the causal boundary of adS. This boundary is the ‘horosphere’ at infinity - it is a timelike hypersurface with the topology \( S^1 \times S^p \), where the circle \( S^1 \) is the timelike factor.

Given this correspondence, one may search for new and interesting properties of SCFTs simply by investigating p-branes with unusual asymptotics. That is, one may consider p-branes where the throat geometry has been modified in some way. One obvious way to modify a given solution, is to identify the solution under the action of some freely acting discrete transformation group. Recently, Gibbons [1] has argued that such identifications may in fact be necessary in order to avoid fixed point singularities, in situations where one is ‘wrapping’ a p-brane on a toroidal cycle. In particular, he argues that one must compose any wrapping identifications with the antipodal map on the adS factor of the near-horizon geometry. Since the antipodal map is freely acting, the composition will be freely acting and the resulting identified brane will be free of fixed-point singularities.

Of course, whenever one identifies a manifold under the action of some freely acting involution, the resulting manifold may or may not be orientable. When the identified manifold is non-orientable, one has to be careful to check for the existence of fermions. That is, one needs to make sure that there exists a pin bundle with the right properties, so that any required fermionic fields can exist as sections of the bundle.

In this paper, we check that Majorana pin structure always exists for M-branes which are identified under the action of the antipodal map on the adS factor of the near-horizon geometry. Thus, wrapping M-branes in this way is not obstructed by the requirement of Majorana pin structure. We point out that similar considerations will hold for p-branes in any dimension, as long as the choice of representation for the parity inversion operator satisfies certain constraints. Finally, we conclude with some general remarks about the uniqueness of eleven dimensional SUGRA, and how M-theory may solve the old problem of the classification of fermions.

II. MAJORANA PINORS AND WRAPPING BRANES

We are working in eleven dimensions with the convention that the spacetime has signature \((-+\ldots++)\). \( D = 11 \), \( N = 1 \) supergravity is a theory which describes the interaction of gravity with a Majorana gravitino \( \Psi_A \) and a three-index gauge field \( A_{LMP} \). The theory has several continuous symmetries: Local \( N = 1 \) SUSY, \( D = 11 \) general covariance, Abelian gauge invariance for the three-form \( A_{LMP} \) and of course SO(10, 1) Lorentz invariance. It also has a discrete symmetry associated with the effect of spacetime reflections on the gauge field. This symmetry tells us [3] that the action and equations of motion in
eleven dimensions are invariant under an odd number of spatial (or temporal) reflections, together with the reversal of the sign of the gauge field:

\[ A_{LMP} \rightarrow -A_{LMP} \]

In fact, this discrete symmetry is essential whenever we consider non-orientable spacetime manifolds in M-theory. This is because we typically think of the four-form \( F_{LMNP} \) as being proportional to some volume form, or anti-symmetric tensor \( \epsilon_{LMNP} \). It follows that on a non-orientable manifold, \( F_{LMNP} \) will not have a definite sign - the sign will change when we propagate around a non-orientable loop. However, propagation around an orientation reversing loop also reflects everything through an odd number of spacetime dimensions, i.e., the equations of motion are still invariant even though the four-form is reduced to the status of a ‘pseudo-tensor’. This means that it still makes sense to talk about the eleven dimensional supergravity equations of motion on non-orientable spacetimes. For a further discussion of non-orientable configurations in M-theory, the reader should consult [5].

Now, a key thing to notice is that it really is not possible to consistently modify this structure in any way. In particular, the Majorana condition for the gravitino is precisely what one needs in order to match the number of bosonic and fermionic degrees of freedom. One cannot just flippantly introduce other representations for the fermions.

A pleasant feature of life in eleven dimensions is the fact that the real Clifford algebra may be written as

\[ \text{Cliff}(10, 1; \mathbb{R}) = \mathbb{R}(32) \]

\( \mathbb{R}(32) \) denotes the space of real \( 32 \times 32 \) matrices and \( \text{Cliff}(10, 1; \mathbb{R}) \) denotes the set of objects \( \gamma_{\mu} \) which satisfy the relation

\[ \gamma_{\mu} \gamma_{\nu} + \gamma_{\nu} \gamma_{\mu} = +2g_{\mu\nu} \]  

where \( g_{\mu\nu} \) is the metric on eleven dimensional Minkowski space with the signature prescribed above. In the usual way, these gamma matrices act on a 32 dimensional space of Majorana spinors, which are real with respect to the relevant charge conjugation operator \( C_{ij} = -C_{ji} \). Explicitly, such a spinor is just a 32 component column \( \psi_k, k = 1, 2, 3, ... 32 \).

It is essential for the construction of eleven dimensional supergravity that we are able to define, globally and consistently, these Majorana fermions in any eleven dimensional spacetime we wish to consider. Without such spinors, we can have no gravitino field with the right number of degrees of freedom and similarly we cannot define generators of supertranslations in superspace which will transform in the right way. If there is some topological anomaly or obstruction which prevents us from defining a globally well-defined spin bundle which has Majorana sections, then the entire structure will collapse.

Of course, up to now the spacetimes considered in most approaches to \( D = 11 \) SUGRA have had fairly trivial topological characteristics. As an example, consider the two objects which couple naturally to the three-form gauge field: The (electric) M2-brane and the (magnetic) M5-brane. The global causal structure of these M-branes is very familiar. The 2-brane interpolates between \( (adS)_4 \times S^7 \) (which is a supersymmetric compactification of the eleven dimensional theory) and flat Minkowski spacetime \( \mathbb{M}^{11} \). In a similar way, the 5-brane interpolates between \( (adS)_7 \times S^4 \) and \( \mathbb{M}^{11} \). In each case, as one falls down the throat
of the brane one moves into the region where the vacuum is a standard compactification of
the form \((adS)_{p+2} \times S^{11-p-2}\).

Given this picture, that p-branes are just solutions which describe vacuum interpolation,
one is led to several obvious and natural questions. For instance, is it possible to find p-
branes which interpolate between vacua which are exotic, or non-standard, compactifications
of the supergravity theory? What happens if we identify the known solutions, such as the
M2 and M5 branes, under the action of some discrete transformations? Do the resulting
‘orbifold’ branes still make sense?

One potential problem with identifying a given solution under the action of some freely
acting involution is the fact that the resulting orbifold brane may be non-orientable. In
particular, the existence of fermions on non-orientable spacetimes is a subtle problem. In
order to understand this problem, we need to first recall some elementary facts about the
‘pin’ groups.

Any proposal to quantize gravity via a path integral prescription, which includes a sum
over manifold topologies, will obviously force us to consider the effects of non-orientable
manifolds. A non-orientable manifold has the property that there exist closed loops in the
manifold, such that parallel propagation around a given loop results in a reversal of some
orientation. Thus, given a non-orientable manifold of signature \((p, q)\), it follows that the
tangent bundle of the manifold can at most be reduced to an \(O(p, q)\) bundle. When we
introduce fermions on the manifold, we ‘lift’ the tangent bundle to a bundle with fibers
given as the group which is the double-cover of the tangent bundle group. Thus, we need to
know what groups are the double-covers of \(O(p, q)\) in order to understand how to introduce
fermions on a non-oriented space.

The groups which are double-covers of the group \(O(p, q)\) are called the \(pin\) groups. The
notation is meant to be humorous: Just as \(Spin(p, q)\) double-covers \(SO(p, q)\), so does \(Pin(p, q)\)
double-cover \(O(p, q)\). For an excellent review of the history of these things, the interested
reader should see [7].

In general, there are in fact eight distinct groups which double-cover \(O(p, q)\). These
different groups correspond to how one may choose to represent the discrete transformations
of parity inversion (P), time reversal (T), and the combination of the two (PT). More
precisely, since any of these discrete transformations squares to the identity in the tangent
bundle (i.e., \(P^2 = +1\) in the tangent bundle), it follows that there is a sign ambiguity in the
double-cover (i.e., \(P^2 = \pm 1\) in the pin bundle). It follows that there are \(2^3\) groups. Here,
we shall use the notation of Dabrowski [8] and write these double-covers as shown:

\[
h^{a, b, c} : \ Pin^{a,b,c}(p, q) \rightarrow O(p, q)
\]

with \(a, b, c \in \{+,-\}\). The signs of \(a, b,\) and \(c\) are defined to be the signs of \(P^2, T^2\) and \((PT)^2\)
respectively. This is all we will need to know about pin groups.

Now, if we are given a manifold which admits a globally well-defined pin-bundle, with
fibers \(Pin^{a,b,c}(p, q)\), then we shall say that the manifold admits a \(Pin^{a,b,c}(p, q)\)-structure. On a
given non-orientable manifold, Majorana fermions will be sections of a bundle corresponding
to some \(Pin^{a,b,c}(p, q)\)-structure. Thus, the existence of Majorana fermions is equivalent to
the existence of the relevant \(Pin^{a,b,c}(p, q)\)-structure. We therefore need to understand how
topology can obstruct the existence of a given pin structure.
III. OBSTRACTIONS TO MAJORANA PIN STRUCTURES ON WRAPPED M-BRANES

The obstructions to Cliffordian pin structures were worked out in [9]; this work was extended to include the obstructions to all pin structures in any dimension and any signature in [11]. In this short note, we will not go into the details of obstruction theory, or how the obstructions are derived. However, we do need to recall a small set of topological invariants in order to even write the obstructions down.

In order to do this, we first need some minimal notation. Let \( M \) denote the eleven-dimensional spacetime manifold, and \( g_L \) the Lorentzian metric (with signature as above) on \( M \). The obstructions which we will describe depend on these two basic objects. Calculating one of these obstructions amounts to calculating a number which is an element of the additive cyclic group \( \mathbb{Z}_2 = \{0, 1\} \). A given pin structure will exist if and only if the relevant obstruction vanishes.

An important invariant here is the second Stiefel-Whitney class, denoted \( w_2(M) \). This invariant, which is the obstruction to the existence of spin structure on \( M \), is an element of the second cohomology group \( H^2(M; \mathbb{Z}_2) \). That is to say, \( w_2 \) may be regarded as a form, which can be evaluated on two-dimensional cycles in \( M \) (the elements of \( H_2(M) \)). If \( w_2 \) is non-vanishing on a given two-cycle, it follows that there does not exist spin structure on \( M \). Explicitly, if \( w_2 \) did not vanish on some two-cycle, one would find that there was an anomaly in a given spinor field, as the spinor field was parallel propagated around on the two-cycle.

Next, we need the first Steifel-Whitney class, denoted \( w_1(M) \). This invariant is the obstruction to the orientability of \( M \), i.e., \( w_1 \) vanishes if and only if \( M \) is orientable. As the name suggests, this invariant is an element of the first cohomology group, \( H^1(M; \mathbb{Z}_2) \). \( w_1 = 1 \) on loops, or one-cycles, in \( M \) which are orientation reversing.

On a Lorentzian manifold, the first Steifel-Whitney class decomposes into two 'sub'-classes, which may be regarded as the obstructions to space and time orientability separately. In particular, there is the 'spacelike' Steifel-Whitney class, denoted \( w_1^S(M; g_L) \), which is the obstruction to the orientability of the spacelike sub-bundle of the tangent bundle, and likewise there is a timelike class, denoted \( w_1^T(M; g_L) \), which is the obstruction to time orientability. Obviously,

\[
w_1(M; g_L) = w_1^S(M; g_L) + w_1^T(M; g_L)
\]

i.e., if you go around a loop and simultaneously reverse both the space and time orientations, then the overall orientation of the spacetime manifold is not reversed. Throughout this paper we will assume that spacetime is at least time orientable; it follows that \( M \) is non-orientable if and only if it is not space orientable. This is all of the topological information which we shall need.

Now, we need to decide which pin structure corresponds to the Majorana fermions described above. Since all of the orbifold branes which we will consider here will be time orientable but not space orientable, this means that we need to make a choice about how we are going to represent the parity inversion operator. Our choice, which is the simplest ansatz that will give a unitary representation of \( O(10, 1) \), is the Cliffordian representation:

\[
P = \gamma_1\gamma_2\gamma_3\gamma_4\gamma_5\gamma_6\gamma_7\gamma_8\gamma_9\gamma_{10}
\] (3.1)
This is the Cliffordian choice in the sense that this is how you would represent, in the Clifford algebra itself, inversion through all of the spacelike coordinates simultaneously. (Note: We could just as easily take $\mathcal{P} = \gamma_0$, as discussed in [13], [7]. It should be obvious - from what we say below - that this would not affect the obstruction theory). On the surface this may seem innocuous, but there are some real subtleties here. First, the choice (3.1) for $\mathcal{P}$ forces us to make the corresponding Cliffordian choice for time reversal:

$$\mathcal{T} = \gamma_0$$

(3.2)

This is fine; however, we have to decide whether we want to represent time reversal using a unitary operator or an anti-unitary operator. Explicitly, we have to ask ourselves: Do we want to just multiply by $\gamma_0$ and reverse the sign of $t$ when we apply $\mathcal{T}$ to a pinor field (this would give us a unitary operator), or do we also take the charge conjugate of the field (this would give us an anti-unitary operator)? Wigner [12] argued that we should use an anti-unitary operator to represent time reversal, since then time reversal would map positive energy states to positive energy states, in the quantum mechanical Hilbert space. Thus, in the Wignerian approach one no longer works with strictly unitary representations of $O(10,1)$; instead, one works with what Wigner called corepresentations, which are basically just like unitary representations only some of the operators are allowed to be anti-unitary. Here, we are not worrying about these subleties because we are not trying to do quantum mechanics - we are just looking for some choices of $\mathcal{P}$ and $\mathcal{T}$ which will move us around in the fibre of the pin bundle in the appropriate way. In any event, taking $\mathcal{T}$ to be anti-unitary will not affect the obstruction theory. All that matters for our purposes is that $\mathcal{P}^2 = -1$.

Given all of this, we can now work out the pin structure we are working with for these Majorana fermions. Given the signature of spacetime, one calculates

$$\mathcal{P}^2 = -1, \mathcal{T}^2 = -1, (\mathcal{PT})^2 = -1$$

which means that these Majorana pinors require the existence of a Pin$^{−−−−}(10,1)$-structure. Actually, there is one final subtlety here, namely, we could reverse the signs of all of the gamma matrices simultaneously. This would change the sign of $(\Gamma_{11})^2$, and so we would be working with a Pin$^{−−++}(10,1)$-structure. This choice would not affect the obstruction theory, or anything else, and need not concern us here.

The topological obstruction to the existence of this pin structure was worked out in [11]. One refers to the relevant theorem, and finds that there exists Pin$^{−−−−}(10,1)$-structure on a time orientable spacetime $(M, g_L)$ if and only if the below obstruction vanishes:

$$w_2(M) + w_1^S(M; g_L) \sim w_1^S(M; g_L) = \mathcal{O}(M)$$

(3.3)

where $\sim$ denotes the cup-product (see, e.g., [10]). Thus, we see that as long as $w_2(M) = 0$ and $w_1^S(M; g_L) = 0$, $\mathcal{O}(M) = 0$ and so Majorana structure will exist. Of course, if $w_2(M) = 0$ then $M$ is a spin manifold anyway and, as we have already pointed out, many of the orbifold branes will not be space orientable and hence $w_1^S(M; g_L) \neq 0$ in general. This obstruction is therefore non-trivial, and has to be checked for each orbifold brane.

Here, we are primarily concerned with M-branes which have been identified under the action of the antipodal map on the adS factor of the near-horizon geometry. Such antipodally identified branes are said to be ‘wrapped in space and time’ [1], because the antipodal map
on adS involves an identification of the timelike coordinate. Explicitly, if we write the metric on \((adS)_{p+2}\) in static coordinates:

\[
ds^2 = -\cosh^2 \chi dt^2 + d\chi^2 + \sinh \chi d\Omega_p^2
\]  

(3.4)

where \(d\Omega_p^2\) is the round metric on the sphere \(S^p\) and \(0 \leq t \leq 2\pi\), then the antipodal map (denoted \(J\)) may be written

\[
J : (t, \chi, n) \mapsto (t + \pi, \chi, -n)
\]

Recently, Gibbons [1] has introduced this involution and argued that it may be an essential ingredient in any scenario where one is wrapping a p-brane around a toroidal cycle. Explicitly, he shows that it is not possible to find any finite freely generated abelian group (acting as spatial translations on the coordinates ‘tangent’ to the brane) which acts freely. Thus, any naive attempt to wrap a p-brane on a torus would result in fixed-point singularities. However, the antipodal map \(J\) always acts freely, and so one possible way to obtain a non-singular wrapped brane is to compose the action of the lattice of spatial translations with the involution \(J\).

Of course, in situations involving several p-branes one often requires the universal covering space of adS, denoted \(\text{CadS}\), because several distinct adS patches may be required. \(\text{CadS}\) is obtained by ‘unwrapping’ the time coordinate for adS; that is to say, on \(\text{CadS}\) the variable \(t\) in (3.4) is allowed to run over the whole real line. One may then extend the action of \(J\) on \((adS)_{p+2}\) to a \(\mathbb{Z}\)-graded action on \((\text{CadS})_{p+2}\) as shown [1]:

\[
J^n : (t, \chi, n) \mapsto (t + n\pi, \chi, (-1)^n n)
\]

With all of this in mind, let us now turn to the question of the existence of Majorana spinors on the M-branes. As we remarked above, it would appear [1] that whenever we wrap an M2 or M5 brane on a toroidal cycle, we will have to simultaneously identify the adS factor of the throat geometry of the brane if we want to avoid singularities. Thus, before we do anything we should check for the existence of Majorana fermions on the orbifold M-branes. Let’s begin with the M5 brane. As was pointed out in [1], the action of \(J\) on an odd-dimensional \((adS)_{p+2}\) is not only orientation preserving, but \(J\) actually lies in the identity component of the conformal group for \((p+1)\)-dimensional Minkowski space. It follows that \(w_1^S(M;gL) = 0\) and \(w_2(M) = 0\) for the antipodally identified M5-brane, i.e., these branes in fact admit spin structure so all of this concern about pin structure does not really apply here.

The real issue is whether or not one can put Majorana structure on the antipodally identified M2-brane. Indeed, if we antipodally identify the \((adS)_4\) factor of the M2-brane, we obtain a non-space orientable manifold. Indeed, if we antipodally identify the \((adS)_4\) factor of the M2-brane, we obtain a non-space orientable manifold. \footnote{Of course, if we require the existence of singletons then we can only identify the covering space under the action of \(J^4\), which is orientation preserving [1]. Thus, the existence of singletons will imply the existence of a spin structure. Nevertheless, the discussion presented here is relevant because it will apply in any scenario where the M2-brane worldvolume is non-orientable, regardless of whether or not there exist singletons or doubletons.}

In fact, the \(S^2\)-factor in the adS geometry is
converted, under the action of $J$, into a two-cycle with the topology of $\mathbb{RP}^2$ (the ‘cross-cap’). $w_2 = 1$ on this two-cycle, and so this orbifold M2-brane does not admit a spin structure. On the other hand, one also calculates that $w_1^S \sim w_1^S = 1$ on this two-cycle. It follows that the total obstruction $\mathcal{O}(M)$ actually vanishes, i.e., the antipodally identified M2-brane does admit Majorana pinors.

This is reassuring, because it means we can wrap M-branes in space and time without worrying about whether we might be selecting out the fermions which are essential for the construction of the eleven-dimensional theory. We can perform a similar analysis for membranes wrapped on a surface of arbitrary genus \cite{[14]}; there, one checks that cycles of arbitrary genera admit the Majorana pin structure, i.e., $\mathcal{O}(M)$ vanishes on any spacelike two-cycle, regardless of the genus and orientation.

In general, the arguments presented here will go through for antipodally identified p-branes in any dimension, as long as one is careful to choose a representation for which $\mathcal{P}^2 = -1$. This sign ensures that the obstruction has the form (3.3), and hence that the right sort of cancellation will occur for the non-orientable branes.

This does not mean, however, that Majorana pinors are always allowed. One can certainly imagine scenarios where one performs an exotic orbifold projection on the transverse directions of a p-brane, or wraps a p-brane around a cycle with an exotic topology, so that the resulting spacetimes will not admit Majorana structure.

Furthermore, it is worth pointing out that more stringent topological conditions arise when one considers Type II string compactifications. In particular, it has recently been shown \cite{[15]} that in order for a D-brane to consistently wrap a given cycle, the normal bundle of the cycle must admit a Spin$^c$ structure. The obstruction theory for Spin$^c$ structures has been discussed recently in \cite{[16]}, where it has been shown (among other things), that the normal bundle for a SUSY cycle is generically Spin$^c$.

IV. CONCLUSIONS

We have shown that it is possible to put Majorana pinors on M-branes which are wrapped in space and in time. If one chooses a natural, Cliffordian choice for the representation of parity inversion then it would seem that Majorana fermions select a unique pin structure. In fact, given any choice of representation of $\mathcal{P}$, the Majorana condition selects a unique pin group. This is because, once we have made a choice for the representations of $P$ and $T$, we are not allowed to introduce any complex numbers (this would violate the Majorana condition) and we are not allowed to do any parity doubling (then the fermions would have the wrong number of degrees of freedom for SUSY). But these are the only two mechanisms which we can use to generate other representations for $P$ and $T$! In other words, there is always only one choice of $P$ and $T$ consistent with the Majorana/SUSY conditions in eleven dimensions. Does this uniqueness perhaps imply that M-theory can solve the old problem of ‘the classification of fermions’?

The problem of how one classifies fermions is simply this: Does it make sense, or is it meaningful, to classify fermions according to their behaviour under the action of the full inhomogeneous Lorentz group? What would be the experimental consequences (if any) of such a classification scheme? Ever since Wigner \cite{[12]} introduced the different corepresentations of $O(3, 1)$ for the Dirac equation, people have wondered about these things (see \cite{[3]} for
a modern viewpoint).

Suppose that we do classify fermions according to their behaviour under the action of $P$ and $T$. Then there are in principle eight distinct particle ‘types’, where the type is determined by the pin group which acts on the fermion at a point of spacetime. It is not hard to see that most of the observed elementary particles can only come in one type. For example, suppose that there existed two types of electron, a ‘plus’ type and a ‘minus’ type. The Pauli exclusion principle would allow you to place a plus electron and a minus electron in the same state. Obviously, this would seriously mess up most of known chemistry unless the electromagnetic interaction coupled only to one type, and the other type was decoupled from known matter! Thus, it would seem that nature has selected a particular pin structure for the description of elementary particles. From a four-dimensional point of view, it is unclear why or how nature makes such a selection.

From the point of view of M-theory, however, the choice is obvious - Majorana selects a unique pin bundle. Four-dimensional multiplets, the descendants of the unique eleven-dimensional structure, then inherit this choice. This elegant solution of the classification problem is just another example of the power of M-theory.

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