Instantons and Magnetic Monopoles on $R^3 \times S^1$
with Arbitrary Simple Gauge Groups

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ABSTRACT

We investigate Yang-Mills theories with arbitrary gauge group on $R^3 \times S^1$, whose symmetry is spontaneously broken by the Wilson loop. We show that instantons are made of fundamental magnetic monopoles, each of which has a corresponding root in the extended Dynkin diagram. The number of constituent magnetic monopoles for a single instanton is the dual Coxeter number of the gauge group, which also accounts for the number of instanton zero modes. In addition, we show that there exists a novel type of the $S^1$ coordinate dependent magnetic monopole solutions in $G_2, F_4, E_8$. 

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In recent years there has been some interest in the connection between magnetic monopoles and instantons in theories with partially compactified space\cite{1,2,3}. Especially by employing the T-duality on D-branes, we have shown that a single instanton in the theory with SU$_n$ gauge group on space $R^3 \times S^1$ can be interpreted as a composite of $n$ distinct fundamental monopoles \cite{1}. On $R^3$, there are $n-1$ fundamental magnetic monopole solutions \cite{4}. On $R^3 \times S^1$, there exist an additional type of magnetic monopole solution associated with the lowest negative root, which plays the key role in making the net magnetic charge of the $n$ distinct monopoles to be zero.

In this note we generalize the above consideration to theories with arbitrary simple gauge group $G$ of rank $r$. Our consideration here is purely field theoretic. With nontrivial the Wilson loop $P \exp(i \int d x_4 A_4)$ along $S^1$, one can break the gauge symmetry maximally to abelian subgroups $U(1)^r$ with the rank $r$ of the gauge group. Similar to the SU($N$) case, we show that besides usual $r$ fundamental BPS monopoles \cite{4}, there exist an additional type of fundamental BPS monopoles associated with the lowest negative root. Especially for the $G_2, F_4, E_8$ groups, we find that these new monopoles will have $x_4$-dependence which cannot be gauged away. We then show that a single instanton solution is a composite of these $r + 1$ fundamental monopoles such that the net magnetic charge is zero. The set of constituent magnetic monopoles is unique and the number of the total fundamental magnetic monopoles turns out to be the dual Coxeter number, which is also consistent with the previously known zero mode counting \cite{5}.

Depending on how one views the compactified direction, different physics emerges. When we regard $S^1$ as the Euclidean time direction, physics is that of finite temperature Yang-Mills theories. Our relation between instantons and magnetic monopoles may shed some light on further understanding of chiral symmetry breaking and confinement in QCD. (For a current effort in lattice gauge theory community, see, for example, Ref. \cite{6}). When we view $S^1$ as the compactified space, our work may give further insights on the relation in some exact results in supersymmetric Yang-Mills theories in four and three dimensions \cite{6,7}. When we view $R^3 \times S^1$ as space and assume that there is additional time direction, our work has more direct connection with D-brane dynamics \cite{1}.

On $R^3 \times S^1$, the Euclidean path integral measure can be restricted to the gauge fields which are single-valued with respect to the coordinate $x_4$ of $S^1$. We choose the $x_4$ interval to be $[0, 2\pi]$ for convenience. Allowed gauge transformations $U(x)$ can be multivalued if the transformed gauge
field $UA_{\mu}U^\dagger - i\partial_{\mu}UU^\dagger$ remain single-valued. Such a large gauge transformation is possible only if the gauge group has a nontrivial center. Nontrivial Wilson loop arises as the fourth component of the gauge field takes nonzero expectation value. We can choose the direction of this to lie along the Cartan subalgebra $H_i (1 \leq i \leq r)$,

$$< A_4 >= h \cdot H. \quad (1)$$

For any finite action configuration, we can require the gauge field to approach the above asymptotic value at spatial infinity modulo gauge. One can always choose a set of simple roots, $\beta_i (1 \leq i \leq r)$, so that

$$h \cdot \beta_i \geq 0 \quad \text{for any } i. \quad (2)$$

This defines the so-called Weyl chamber in the root vector space.

In this paper consider the gauge symmetry is maximally broken to $U(1)^r$ so that the inequality in Eq. (2) holds strictly. We consider all generators of the gauge group in the adjoint representation. We normalize the generators so that

$$\text{tr} (H_i H_j) = c_2(G) \delta_{ij} \quad \text{and} \quad \text{tr} (E^\dagger_\alpha E_\beta) = c_2(G) \delta_{\alpha\beta}. \quad (3)$$

The normalization factor $c_2(G)$ is the quadratic Casimir for the adjoint representation because

$$c_2(G) = \text{tr}(T^a)^2 \quad \text{(no sum)}$$

$$= \frac{1}{d(G)} \sum_{a=1}^{d(G)} \text{tr} (T^a)^2 = \sum_{a=1}^{d(G)} (T^a)^2. \quad (4)$$

There is still an overall normalization to be fixed. With the choice that longest root vectors to have length one, $c_2(G)$ becomes the dual Coxeter number, which is an positive integer $\frac{d}{2}$.

The Yang-Mills action on $R^3 \times S^1$ has a lower bound $8\pi |m|/g^2$, where the topological charge is

$$m = \frac{1}{64\pi^2} \int d^4 x \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a$$

$$= \frac{1}{8\pi^2} \int_0^{2\pi} d\lambda \int_{S^2} dB_i^a \left[ B_i^a A_4^a - \frac{1}{2} \epsilon_{ijk} A_j^a A_k^a \right], \quad (5)$$

with $B_i^a = \frac{1}{2} \epsilon_{ijk} F_{ij}^a$. (Here we assume that the gauge fields do not have any singularities at interior region of space time.) When $m > 0$, the bound is saturated by the field configurations satisfying the self-dual equations,

$$B_i^a = D_i A_4^a - \partial_i A_i^a. \quad (6)$$
Magnetic monopole configurations with no $x_4$ dependence would have the topological charge contribution from the usual BPS term of Eq. (5), and instantons which goes to the pure gauge at infinity would have the the contribution from the last term of Eq. (5). In addition, the instanton topological charge would be integer since it measures the homotopy of this gauge from $S^1 \times S^2_\infty$ to $G$.

On $R^4$, one can find an instanton solution of minimum topological charge, $m = 1$, by embedding the $SU(2)$ solution along one of the longest root. The number of zero modes around a single instanton turns out to be $4c_2(G)$, [3]. The number of zero modes $4c_2(G)$ around a single instanton can be interpreted as a sum of five modes for the position and scale and the rest for the global gauge modes which changes the embedded solution. On $R^3 \times S^1$, one find a periodic instanton solution, which turns out to be a special limit where the gauge symmetry is partially restored [1, 9, 3]. We expect the number of zero modes to be identical to the $R^4$ case.

If we consider the self-dual configurations which are independent of $x_4$, Eq. (8) becomes the standard BPS equations for magnetic monopoles with $A_4$ playing the role of the Higgs field. For each root $\alpha$, there is a corresponding $SU(2)$ subgroup

\[
\begin{align*}
t^1(\alpha) &= \frac{1}{\sqrt{2\alpha^2}}(E_\alpha + E_{-\alpha}), \\
t^2(\alpha) &= -\frac{i}{\sqrt{2\alpha^2}}(E_\alpha - E_{-\alpha}), \\
t^3(\alpha) &= \alpha^* \cdot H,
\end{align*}
\]

where the dual of a root $\alpha$ is defined as

\[
\alpha^* = \alpha/\alpha^2.
\]

With our normalization, $\alpha^* = \alpha$ for any long root. We can embed the $SU(2)$ single monopole solution along $t(\alpha)$,

\[
A_\mu(x) = \left[ h - (h \cdot \alpha^*) \alpha \right] \cdot H \delta_{\mu 4} + t^a(\alpha) V^a_\mu(r; h \cdot \alpha),
\]

where

\[
\begin{align*}
V^a_4(r; u) &= \hat{r}^a \left[ \frac{1}{r} - u \coth ur \right], \\
V^a_i(r; u) &= \epsilon_{aij} \hat{r}_j \left[ \frac{1}{r} - \frac{u}{\sinh ur} \right].
\end{align*}
\]
The asymptotic behavior of this solution along the negative $x^3$ axis is given as $A_4 \to (h - \alpha^*/r) \cdot H$ and $B_i \to \delta_{i3} \alpha^*/r^2 \cdot H$, leading to the topological charge $m = h \cdot \alpha^*$. Especially each fundamental monopole corresponding to simple roots $\beta_i$ would have topological charge

$$m_i \equiv h \cdot \beta^*_i > 0. \quad (11)$$

It is well known that each fundamental monopole carries only four zero modes among any fluctuations independent of $x_4$ [4]. Any other BPS configurations would be composed with these monopoles. Of course on $R^4 \times S^1$, there can be additional zero modes in the Kaluza-Klein modes [4], more about which we will discuss later.

From the experience with the $SU_n$ case [4], we expect that there exist $x_4$-dependent self-dual solutions which can be identified as magnetic monopoles associated with the lowest negative root $\beta_0$. This root together with simple roots $\beta_i$ can be described by the extended Dynkin diagram. Note that $\beta_0$ is given uniquely by a linear combination of simple roots with positive integer coefficients, and it is one of the longest roots. In the $SU_n$ case, this BPS monopole solution for $\beta_0$ is equivalent to a $x_4$ independent solution under a large gauge transformation.

In the $SU_n$ case the presence of this additional magnetic monopole was crucial in showing that instantons are made of the $n$ distinct magnetic monopoles so that net magnetic charge is zero. Here we assume that it is possible to put these $n$ different self-dual solutions together nonlinear way. Especially in $SU_2$ case, the $\beta_0$ monopole has opposite magnetic charge even though the configuration is self-dual. The magnetic attractive force between $\beta_0$ and $\beta_1$ monopole cancels the Higgs repulsive force between them. In $SU_2$ case, we can construct the explicit field configuration for a single instanton as a composite of $\beta_0$ and $\beta_1$ monopoles by using the Nahm’s method [9].

For a general simple gauge group, there are several questions to be answered before one sees that instantons are composed of magnetic monopoles: (1) What is the $\beta_0$ magnetic monopole configuration? (2) What is the maximum range of $h$ in which all fundamental magnetic monopoles carry only four zero modes? (3) What is the constituent magnetic charge for a single instanton configuration? (4) How do the topological charge and the number of zero modes work out?

Let us first give the answer to the second question, which we will justify later. The range where
each of \( r + 1 \) fundamental monopoles have only four zero mode is given by the inequality
\[
\mathbf{h} \cdot \beta^*_0 > 0 \text{ and } \mathbf{h} \cdot (-\beta^*_0) < 1.
\] (12)

We call this as the fundamental cell. Since \( \beta_0 \) is a negative root, \( \mathbf{h} \cdot (-\beta_0) > 0 \). If the range of \( x_4 \) were \( R \) rather than \( 2\pi \), then the last inequality would be \( \mathbf{h} \cdot (-\beta^*_0) < 2\pi/R \).

To find the \( \beta_0 \) monopole solutions, we start by considering the group \( SU(3) \). In this case, the simple roots are \( \beta_1 = \mathbf{e}_1 - \mathbf{e}_2 \) and \( \beta_2 = \mathbf{e}_2 - \mathbf{e}_3 \) with \( \mathbf{e}_i \) such that \( \mathbf{e}_i \cdot \mathbf{e}_j = \delta_{ij}/2 \). The lowest root is \( \beta_0 = -\mathbf{e}_1 + \mathbf{e}_3 \). Its center \( Z_3 \) is generated by the gauge transformations
\[
C_j = e^{4\pi i \lambda_j \cdot \mathbf{H}},
\] (13)
where \( \lambda_i \) are fundamental weights satisfying
\[
2\lambda_i \cdot \beta^*_j = \delta_{ij}.
\] (14)

In \( SU_3 \), \( \lambda_1 = \frac{1}{3}(2\mathbf{e}_1 - \mathbf{e}_2 - \mathbf{e}_3) \) and \( \lambda_2 = \frac{1}{3}(\mathbf{e}_1 + \mathbf{e}_2 - 2\mathbf{e}_3) \). (One can easily see \( C_j E_{\beta_k} C_j^\dagger = E_{\beta_k} \) for all \( k \), implying that \( C_j \) belongs to the center.) On the fundamental representations, for example, \( C_1 |\lambda_1 > = e^{i\frac{4\pi}{3}} |\lambda_1 > \) and \( C_1 |\lambda_2 > = e^{i\frac{2\pi}{3}} |\lambda_2 > \). There are corresponding large gauge transformations,
\[
U_j = e^{2ix_4 \lambda_j \cdot \mathbf{H}},
\] (15)
such that \( U_j(2\pi) = C_j \). Under, for example, \( U_1^\dagger \), the asymptotic value \( \mathbf{h} \) transforms to
\[
\mathbf{h}_{\text{new}} = \mathbf{h} - 2\lambda_1.
\] (16)

For this new asymptotic value, we note that \( \mathbf{h}_{\text{new}} \cdot \beta^*_0 = 1 + \mathbf{h} \cdot \beta^*_0 > 0 \) and \( \mathbf{h}_{\text{new}} \cdot \beta_2 = \mathbf{h} \cdot \beta_2 \) in the fundamental cell. On the other hand \( \mathbf{h}_{\text{new}} \cdot \beta_1 < 0 \). Thus with respect to \( \mathbf{h}_{\text{new}} \), \( \beta_0 \) and \( \beta_2 \) are simple roots. One can then construct the \( x_4 \)-independent \( \beta_0 \) monopole solution. This solution has the topological charge
\[
m_0 = 1 + \mathbf{h} \cdot \beta^*_0.
\] (17)

Gauge transforming back to the original \( \mathbf{h} \) by \( U_1 \), we get the time-dependent \( \beta_0 \) magnetic monopole solution,
\[
A_\mu(x; \beta_0) = [\mathbf{h} - (\mathbf{h}_{\text{new}} \cdot \beta^*_0)\beta_0] \cdot \mathbf{H} \delta_{\mu 4} + U_1 t^a(\beta_0) U_1^\dagger V_\mu(r; m_0),
\] (18)
where we used the fact $\beta^*_0 = \beta_0$. Its topological charge is gauge invariant and so is still $m_0$.

This turns out to be a general feature for all groups with nontrivial center. Table I shows the center of gauge groups. Clearly there is a relation between the symmetry of the extended Dynkin diagram and the center of the group. Except for $G_2, F_4, E_8$, there is always at least one simple long root, say $\beta_1$, which symmetric to $\beta_0$. The corresponding global gauge transformation $U_1$ in Eq. (15) with the corresponding fundamental weight $\lambda_1$ can be used to construct the $\beta_0$ solution and given again by Eq. (18). (Since we know all simple roots in terms of orthogonal vectors, we can check this explicitly with some labor.)

\[
\begin{array}{|c|c|}
\hline
\text{Group} & Z(G) \\
\hline
SU_n, n \geq 1 & Z_n \\
SO_{2n+1}, n \geq 3 & Z_2 \\
SO_{2n}, n \text{ even}, n \geq 4 & Z_2 \oplus Z_2 \\
SO_{2n}, n \text{ odd}, n \geq 5 & Z_4 \\
Sp_{2n}, n \geq 2 & Z_2 \\
E_6 & Z_3 \\
E_7 & Z_2 \\
E_8, F_4, G_2 & \{1\} \\
\hline
\end{array}
\]

\textbf{Table I:} The center of the universal covering group $G$

For $G_2, F_1, E_8$, their center is trivial and the above method fails to give the $\beta_0$ monopole solution. The monopole solution for $\beta_0$ for these groups should be genuinely $x_4$-dependent, which cannot be gauged away. By using the fact that $[2\lambda_1 \cdot H, E\beta_0] = -E\beta_0$ in the $SU_3$ case, we rewrite the $x_4$-dependent solution (18) as

\[
A_\mu(x; \beta_0) = (h - (h_{\text{new}} \cdot \beta_0^*) \beta_0) \cdot H_{5\mu} \\
+ e^{-ix_4 t^2(\beta_0)} t^a(\beta_0) e^{ix_4 t^2(\beta_0)} V_\mu(r; h \cdot \beta_0).
\]

Clearly the above solution is single-valued in $x_4$ since $t^a(\beta_0)$ transforms with $t = 1$ under itself. For all other gauge groups with nontrivial center, the above argument works also. Now we notice that the above solution is perfectly acceptable even in $G_2, F_4, E_8$ cases as it is single-valued. Indeed along the negative $x_3$ axis, the above spherically symmetric solution has exactly right asymptotics for the $\beta_0$ monopoles. Its topological charge in $G_2, F_4, E_8$ are still $m_0$ since the topological charge for this solution lies just in $SU(2)$ sector and its value is invariant under this $SU(2)$. 

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Having constructed explicit configurations for all $r + 1$ fundamental monopoles, we are now in the position to ask whether they have only four zero modes. The zero mode analysis around spherically symmetric magnetic monopoles solutions can be done by introducing spinors $\psi = \delta A_4 + i\sigma_i \delta A_i$, which satisfy the Dirac equation

$$(-i\sigma_j D_j + D_4)\psi = 0.$$  \hspace{1cm} (20)

This is equivalent to the linear fluctuation equations of the self-dual equation and the background gauge condition $D_\mu \delta A_\mu = 0$. For the time-independent solution (18), the analysis is quite similar to Ref. [4], except the fluctuations can now depend on $x_4$, though single-valued, so that $\psi \sim e^{ilx_4}$ with an integer $l$. The generators of the Lie algebra can be split into irreducible representations of $t(\beta_i)$. The two relevant parameters along a root $\alpha$ [4] are the isospin $t_3 = \beta_i^* \cdot \alpha$ and the hypercharge

$$y = \frac{\mathbf{h} \cdot \alpha}{\mathbf{h} \cdot \beta_i} - t_3 + \frac{l}{\mathbf{h} \cdot \beta_i}.$$  \hspace{1cm} (21)

The number of normalizable zero mode depends on the value of $y$ and the isospin $t$ of the $\alpha$ multiplet with respect to $t(\beta_i)$. As given in Eq.(C.5) of Ref. [4], the $|y|$ ranges for the nonzero number of zero modes for each $t$ are

$$t = \frac{1}{2}; \text{ one for } |y| < \frac{1}{2};$$

$$t = 1; \text{ two for } |y| < 1,$$

$$t = \frac{3}{2}; \text{ four for } |y| < \frac{1}{2}, \text{ three for } \frac{1}{2} < |y| < \frac{3}{2}.$$  \hspace{1cm} (22)

Otherwise with $t \leq 3/2$ there is no zero mode. (At the boundary of $|y|$ range, the matter is subtle but does not concern in this note.) For example in $SU_3$, when $\mathbf{h}$ lies in the fundamental cell (12), one can see easily that each of time-independent $\beta_i$ monopoles has only four zero modes. Outside this cell the monopoles have more zero modes due to the Kaluza-Klein modes, suggesting that it is not fundamental. Indeed, it is a composite of instantons and fundamental monopoles as shown in Ref. [1]. Depending on the gauge group, the fundamental cell may not be the smallest cell for which all of $x_4$-independent $\beta_i$ monopoles have only four zero modes.

For the $x_4$-dependent $\beta_0$ monopole solutions, let us proceed first with the (18) solution. For these solutions we can gauge away the $x_4$-dependence by a large gauge transformation, and then analysis is identical as before. The fluctuation should be single-valued in $x_4$ and one use the parameter $y$ of Eq. (21) with $\mathbf{h}_{\text{new}}$ of Eq. (16). The number of these fundamental monopoles still have four zero modes in the fundamental cell.
Instead we could use the solution (19), which is also only option for the $G_2, F_4, E_8$ cases. To do fluctuation analysis around this solution, we notice that the fluctuation should be still single-valued. Again the fluctuation equation (20) can be split into the irreducible representations with respect to $t(\beta_0)$. Only $t = 1$ fluctuations are along $\beta_0$ itself, since $\beta_0$ is one of the longest. Any other nontrivial fluctuations have $t = \frac{1}{2}$. From the fluctuation equation (20), we can gauge away $e^{-ix_4t^3(\beta_0)}$ from the background field, as change of variables, which makes the $t = 1$ fluctuations still single-valued but the $t = 1/2$ fluctuations double-valued. The $y$-parameter is then

$$y = \frac{h \cdot \alpha}{m_0} - \beta_0^* \cdot \alpha + \frac{\beta^* \cdot \alpha + l}{m_0}.$$  

Thus the $l$ parameter in Eq. (21) would be integer for $t = 1$ fluctuations and half-integer for $t = 1/2$ fluctuations. Of course this analysis in theories with group of nontrivial center is consistent with the analysis done before by using a large gauge transformation. For all gauge groups including $G_2, F_4, E_8$, one can check after some labor that a single $\beta_0$ monopole described by Eq. (19) has indeed only four zero modes if $h$ lies in the fundamental cell. Also one can show that the fundamental cell is the maximal region after some labor.

Thus there exist $r + 1$ distinct self-dual magnetic monopoles, one for each root in the extended Dynkin diagram. For $h$ in the fundamental cell, each of these monopole solutions have only four zero modes. In addition, the topological charge $m_i, 0 \leq i \leq r$ are positive in the fundamental cell. Their magnetic charge is $4\pi \beta_i^*$. Since the instanton carries zero magnetic charge, one can ask whether there is a linear combination of these magnetic charges such that the total magnetic charge is zero. It turns out that there is a unique minimum set of positive integers $(n_0, n_1, ..., n_r)$ as shown in Table II, such that

$$\sum_{i=0}^{r} n_i \beta_i^* = 0.$$  

(24)

Especially the number of $\beta_0$ monopoles is $n_0 = 1$. We call these positive integers to be Dynkin numbers, whose sum turns out to be the dual Coxeter number

$$c_2(G) = \sum_{i=0}^{r} n_i.$$  

(25)

Let us now imagine nonlinearly superposed field configurations for fundamental monopoles whose numbers are prescribed by Dynkin numbers. Then their net magnetic charge is equal to zero and their total topological charge is

$$\sum_{i=0}^{r} n_i m_i = 1 + h \cdot \sum_{i=0}^{r} n_i \beta_i^* = 1.$$  

(26)
The number of zero modes is $4c_r(G)$. They satisfy the self-dual equations. This is exactly what one expects for a single instanton solution.

| Group | $r$ | $d(G)$  | $c_2(G)$ | $(n_0, n_1, ..., n_r)$ |
|-------|-----|---------|----------|-----------------------|
| $SU_{n+1}$ | $n$ | $n^2 + n$ | $n + 1$ | $(1, 1, ..., 1)$ |
| $SO_{2n+1}$ | $n$ | $n(2n + 1)$ | $2n - 2$ | $(1, 1, 1, 2, 2, ..., 2)$ |
| $SO_{2n}$ | $n$ | $n(2n - 1)$ | $2n - 2$ | $(1, 1, 1, 2, 2, ..., 2)$ |
| $Sp_{2n}$ | $n$ | $n(2n + 1)$ | $n + 1$ | $(1, 1, ..., 1)$ |
| $G_2$ | 2 | 14 | 4 | $(1, 1, 2)$ |
| $F_4$ | 4 | 52 | 9 | $(1, 1, 2, 2, 3)$ |
| $E_6$ | 6 | 78 | 12 | $(1, 1, 1, 2, 2, 2, 3)$ |
| $E_7$ | 7 | 133 | 18 | $(1, 1, 1, 2, 2, 2, 3, 3, 4)$ |
| $E_8$ | 8 | 248 | 30 | $(1, 2, 2, 3, 3, 4, 4, 5, 6)$ |

Table II: The rank $r$, dimension $d(G)$, the dual Coxeter number $c_2(G)$, and Dynkin numbers, for all simple compact Lie groups.

To conclude, we have shown that there exist $r + 1$ distinct self-dual magnetic monopoles on $R^3 \times S^1$ and that instantons on $R^3 \times S^1$ are made of these magnetic monopoles. Quite reasonable is our assumption that all self-dual solutions with the same boundary condition can be put together. It would be interesting to see whether this relation between instantons and magnetic monopoles can be explored further in the various area mentioned at the beginning: the finite temperature QCD [1], the generalization of Witten-Seiberg results of $N = 2$ or $N = 1$ supersymmetric theories on $R^3 \times S^1$ [6, 8], and D-brane and M-theory [11]. Especially recent works by Vafa et.al. [8] on $N = 1$ supersymmetric theories on $R^3 \times S^1$ contains the dual Coxeter numbers and Dynkin numbers by using $F$ and $M$ theories, which we believe can be interpreted as the effect due to the presence of constituent magnetic monopoles for instantons.

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