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MHD mixed convection on an inclined stretching plate in Darcy porous medium with Soret effect and variable surface conditions

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Abstract: This work is concerned with a steady 2D laminar MHD mixed convective flow of an electrically conducting Newtonian fluid with low electrical conductivity along with heat and mass transfer on an isothermal stretching semi-infinite inclined plate embedded in a Darcy porous medium. Along with a strong uniform transverse external magnetic field, the Soret effect is considered. The temperature and concentration at the wall are varying with distance from the edge along the plate, but it is uniform at far away from the plate. The governing equations with necessary flow conditions are formulated under boundary layer approximations. Then a continuous group of symmetry transformations are employed to the governing equations and boundary conditions which determine a set of self-similar equations with necessary scaling laws. These equations are solved numerically and similar velocity, concentration, and temperature for various values of involved parameters are obtained and presented through graphs. The momentum boundary layer thickness becomes larger with increasing thermal and concentration buoyancy forces. The flow boundary layer thickness decreases with the angle of inclination of the stretching plate. The concentration increases considerably for larger values of the Soret number and it decreases with Lewis number. The skin friction coefficient increases for increasing angle of inclination of the plate, magnetic and porosity parameters, however it decreases for rise of thermal and solutal buoyancy parameters. In this double diffusive boundary layer flow, Nusselt and Sherwood numbers increase for rise of thermal and solutal buoyancy parameters, Prandtl number, but they behave opposite nature in case of angle of inclination of the plate, magnetic and porosity parameters. The Sherwood number increases for increasing Lewis number but it decreases for increasing Soret number.

Keywords: Soret effect, MHD mixed convection, inclined stretching porous plate, heat and mass transfer, Lie group of symmetry transformation.

1 Introduction

Similar to the natural convective flow, the mixed convective flow on stretching surface attracts focus of researchers for its vital applications in many realistic systems of serious interest such as thermal energy storage; geothermal energy utilization, recoverable systems, petroleum reservoirs, metallurgy, electro-chemistry, polymer processing, geophysics, etc. The flow of viscous Newtonian fluid in boundary layer for the motion of a flat sheet was initiated by Sakiadis [1]. Rather, there are many articles of boundary layer flow in a linearly as well as non-linearly flat stretching sheet [2–4] with various conditions. In a non-Newtonian Williamson fluid, Khan and Khan [5] pioneered the boundary layer flow.

Heat transfer is the energy transfer due to temperature difference in a medium or between two or more media. Heat can transfer in three ways, viz., conduction, convection, and radiation. Mass transfer is the movement of some other liquid mass in the fluid motion from one location to another. It occurs in many processes, such as absorption, evaporation, drying, etc. In a micropolar fluid, MHD boundary layer flow with heat and mass transfer was investigated by Chamkha et al. [6] and Khedr et al. [7]. Takhar et al. [8] analyzed unsteady MHD three dimensional boundary layer flow over a bidirectional linear stretching surface.

Darcy law [9] is a basic law for fluid flow through a porous medium. This law is valid when the porous mate-
Prasannakumara et al. [15] derived a porous medium with heat transfer over a non-linear stretching sheet with various physical conditions. Boundary layer nanofluid flow with heat transfer over a non-linear stretching sheet with porous medium was derived by Prasannakumara et al. [15]. Chamkha et al. [16, 17] obtained MHD boundary layer flow of a Newtonian fluid with convective heat transfer over an inclined porous medium. Chen et al. [18] discussed the MHD flow with natural convective heat and mass transfer from a permeable inclined surface for variable surface temperature and concentration.

When heat transfer and mass transfer simultaneously happen between fluxes, the flow causing potential is of more complicated type, as energy flux may be generated by composition gradient. The energy flux causes due to temperature gradient is known as the Dufour effect or diffusion-thermo effect. Whereas, temperature gradient can also produce mass fluxes, which is Soret effect or thermo-diffusion effect. Soret and Dufour effects may be ignored on the basis that they are of lesser order of magnitude compared to the effects illustrated by Fourier’s and Fick’s laws with some exceptions. Eckert and Drake [19] prescribed many cases when these effects can’t be neglected. Out of which, the Soret effect is an important one, which may be applicable to isotope separation and in a mixture of gases with very lesser molecular weight (H₂, He, etc.). The Dufour effect is considerable for medium molecular weight (N₂, air, etc.). There were some works regarding the Soret and Dufour effects on MHD boundary layer flow of Newtonian fluid [20–26]. The Soret-Dufour effects on the boundary layer flow for viscoelastic fluid was investigated by Hayat et al. [27] and Gbadeyan et al. [28]. Saritha et al. [29], pioneered boundary layer flow with heat and mass transfer of a power-law fluid over a porous flat plate taking Soret and Dufour effects. In a trapezoidal enclosures, Mudaf et al. [30] obtained unsteady double diffusive natural convection taking Soret and Dufour effect. In a convective nanofluid flow, Soret and Dufour effect was considered by Reddy et al. [31, 32].

In many laminar boundary layer flows, the symmetries are inherently present. Under symmetry transformations, the equations along with the boundary conditions and their solutions are expressed in the form of new variables. Using dimensional analysis, scaling invariances are found for many such flows and this process requires the knowledge about physical characters of the involved variables. The method, Lie symmetry analysis is a systematic one where there is no need to have prior assumptions and knowledge of physical characters of the considered equations and their invariant solutions. One of the key points of the aforesaid method is that it gives all possible symmetries of the system and it may determine whether a system of differential equations is solvable or not by known numerical or analytical techniques. Importantly, it provides physical perceptions about the system of equations. Many cases of fluid motion where invariant solutions are obtained using Lie symmetry can be found in Cantwell [33], Layek [34], Blumen and Kumei [35]. Remarkably, Layek and Sunita [36] and She et al. [37] used the above method to obtain scaling laws of mean velocity in turbulent boundary layer flows and pipe flows, respectively. Lie-group theoretic technique was applied in laminar boundary layer flows with heat and mass transfer by various researchers [38–44]. Sivasankaran et al. [45] studied recently heat and mass transfer in natural convection.

The group invariant solutions for MHD mixed convective boundary layer flow of Newtonian fluid on a stretching inclined porous plate in a porous medium with Soret effect and variable temperature and concentration at the wall are obtained. This type of treatment on the considered problem is original and novel. Special type magnetic field is required to determine similar equations of MHD double-diffusive system. The problem has wide application in engineering processes, such as, isotope separation, solar collectors, geothermal energy utilization flow in a desert cooler, etc. The obtained self-similar equations are solved numerically and all the variation in physical characteristics of the flow, heat and mass transfer are illustrated in this study.

2 Formulation of problem

Consider the mixed convective laminar boundary layer steady two-dimensional flow of an incompressible viscous electrically conducting Newtonian fluid with low electrical conductivity along an isothermal stretching semi-infinite inclined porous plate in a Darcy porous medium at an acute angle \( \alpha \) with the vertical. A specially generated strong variable magnetic field \( B(x) \) is applied along y-axis, which is perpendicular to the plate. Here, the induced magnetic field effect is neglected due to low conductivity of the liquid and it is very small in comparison to the externally applied magnetic field. Here, there is no interaction of charge particles and the velocity of the fluid is very
small in compare to the velocity of light. Hence, the Hall effect is very small and is neglected in this analysis. But the electro-magnetic force, the Lorentz force is taken into account which generally reduces the growth of velocity components and other flow quantities. Both wall temperature \( T_w \) and wall concentration \( C_w \) are taken as variables with the distance from the leading edge along the stretching plate. The uniform ambient temperature and concentration of the flow are \( T_\infty \) and \( C_\infty \), respectively. Also, the Soret effect which is important for liquid is considered. However, the Dufour effect which is important in gases and so its effect is not considered here. Under the usual boundary layer approximations, the governing equations for mass, momentum, energy, and concentration for the steady flow may be stated as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\frac{u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \cos \alpha + g\beta'(C - C_\infty) \cos \alpha - \frac{\sigma B^2(x)}{\rho} \frac{u - \nu}{k' u}, \tag{2}
\]

\[
\frac{u}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2}, \tag{3}
\]

\[
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} \tag{4}
\]

along with the boundary conditions

\[
u = U_w(x) = ax^m, \quad v = 0, \quad T = T_w(x) = T_\infty + bx^n,
\]

\[
C = C_w(x) = C_\infty + cx^p \quad \text{at} \quad y = 0 \tag{5}
\]

\[
u \to 0, T \to T_\infty, C \to C_\infty \quad \text{as} \quad y \to \infty, \tag{6}
\]

where \( x \)-direction is taken along the plate, \( y \)-direction is perpendicular to the plate, \( u \) and \( v \) are the velocity components along the \( x \)- and \( y \)-directions, respectively, \( g \) is the acceleration due to gravity, \( u \) is the kinematic viscosity, \( \beta' \) is the coefficient of expansion with concentration, \( \beta \) is the coefficient of thermal expansion, \( T \) is the temperature, \( C \) is the concentration, \( \rho \) is the constant fluid density, \( \sigma \) is the constant electrical conductivity of the fluid, \( \kappa \) is the thermal conductivity of the fluid, \( k' \) is the permeability of the porous medium, \( D_m \) is the coefficient of mass diffusivity, \( c_p \) is the specific heat at constant pressure, \( K_T \) is the thermal diffusion ratio, \( T_m \) is the mean fluid temperature, the parameters \( m, n, p \) are related to the stretching speed, temperature change and concentration change along the plate respectively. Moreover, \( a(>0) \), \( b \) and \( c(>0) \) are constants with \( b > 0 \) for heated stretching plate \( (T_w > T_\infty) \) and \( b < 0 \) for a cooled stretching plate \( (T_w < T_\infty) \). A physical sketch of the aforesaid problem along with the coordinate system is given in Figure 1.

Equations (1), (2), (3), (4) along with boundary conditions (5), (6) are transformed by introducing stream function \( \psi = \psi(x, y) \) for this 2D flow. The stream function satisfies the continuity equation by using \( u = \psi_y, v = -\psi_x \). This leads that the other equations (2)-(4) become

\[
\psi_y \psi_{xy} - \psi_x \psi_{yy} = \nu \psi_{yyy} + g\beta(T - T_\infty) \cos \alpha + g\beta'(C - C_\infty) \cos \alpha - \frac{\sigma B^2(x)}{\rho} \psi_y - \frac{\nu}{k'} \psi_y, \tag{7}
\]

\[
\psi_y T_x - \psi_x T_y = \frac{k}{\rho c_p} T_{yy}, \tag{8}
\]

\[
\psi_y C_x - \psi_x C_y = D_m C_{yy} + \frac{D_m K_T}{T_m} T_{yy}. \tag{9}
\]

The boundary conditions (5)-(6) are transformed into the following forms:

\[
\psi_y = U_w(x) = ax^m, \quad \psi_x = 0, \quad T = T_w(x) = T_\infty + bx^n,
\]

\[
C = C_w(x) = C_\infty + cx^p \quad \text{as} \quad y \to \infty. \tag{10}
\]

\[
\psi_y \to 0, T \to T_\infty, C \to C_\infty \quad \text{as} \quad y \to \infty. \tag{11}
\]

3 Symmetry analysis of the governing equations with boundary conditions

We now demonstrate symmetry analysis which determines invariant solutions via invariance principles of continuous
group of transformations operating on the independent and dependent variables of equations (7), (8), (9) along with the boundary conditions (10) and (11). (See the books Cantwell [17], Layek [18], Blueman and Kumei [19]). There are two independent variables in the equations, \( x, y \) and four dependent variables \( \psi(x, y), T(x, y), C(x, y), B(x) \). According to the group theory the infinitesimal generator has the form

\[
X = \xi_1 \frac{\partial}{\partial x} + \xi_2 \frac{\partial}{\partial y} + \eta_1 \frac{\partial}{\partial \psi} + \eta_2 \frac{\partial}{\partial T} + \eta_3 \frac{\partial}{\partial C} + \eta_4 \frac{\partial}{\partial B}.
\]  
(12)

The equations are of 3rd order, and so the above infinitesimal generator can extend three times and after extension the generator may be written in the following standard form

\[
X^{[3]} = \xi_1 \frac{\partial}{\partial x} + \xi_2 \frac{\partial}{\partial y} + \eta_1 \frac{\partial}{\partial \psi} + \eta_2 \frac{\partial}{\partial T} + \eta_3 \frac{\partial}{\partial C} + \eta_4 \frac{\partial}{\partial B} \\
+ \xi_5 \frac{\partial}{\partial \psi_T} + \xi_6 \frac{\partial}{\partial \psi_T} + \xi_7 \frac{\partial}{\partial \psi_T} + \xi_8 \frac{\partial}{\partial \psi_T} + \xi_9 \frac{\partial}{\partial \psi_T} + \xi_{10} \frac{\partial}{\partial \psi_T} \\
+ \xi_{11} \frac{\partial}{\partial T} + \xi_{12} \frac{\partial}{\partial T} + \xi_{13} \frac{\partial}{\partial T} + \xi_{14} \frac{\partial}{\partial T} + \xi_{15} \frac{\partial}{\partial T} + \xi_{16} \frac{\partial}{\partial T} \\
+ \xi_{17} \frac{\partial}{\partial C} + \xi_{18} \frac{\partial}{\partial C} + \xi_{19} \frac{\partial}{\partial C} + \xi_{20} \frac{\partial}{\partial C} + \xi_{21} \frac{\partial}{\partial C} + \xi_{22} \frac{\partial}{\partial C}.
\]  
(13)

The coefficients are written as follows:

\[
\begin{align*}
\xi_1 &= D_x(\eta_1) - \psi_x D_y(\xi_1) - \psi_y D_x(\xi_2), \\
\xi_2 &= D_y(\eta_1) - \psi_x D_y(\eta_1) - \psi_y D_y(\eta_2), \\
\xi_5 &= D_x(\eta_7), \\
\xi_6 &= D_y(\eta_7), \\
\xi_7 &= D_x(\eta_8), \\
\xi_8 &= D_y(\eta_8), \\
\xi_9 &= D_x(\eta_9), \\
\xi_{10} &= D_y(\eta_9), \\
\xi_{11} &= D_x(\eta_{10}), \\
\xi_{12} &= D_y(\eta_{10}), \\
\xi_{13} &= D_x(\eta_{11}), \\
\xi_{14} &= D_y(\eta_{11}), \\
\xi_{15} &= D_x(\eta_{12}), \\
\xi_{16} &= D_y(\eta_{12}), \\
\xi_{17} &= D_x(\eta_{13}), \\
\xi_{18} &= D_y(\eta_{13}), \\
\xi_{19} &= D_x(\eta_{14}), \\
\xi_{20} &= D_y(\eta_{14}), \\
\xi_{21} &= D_x(\eta_{15}), \\
\xi_{22} &= D_y(\eta_{15}),
\end{align*}
\]  
(14)

The infinitesimals \( (\xi_1, \xi_2, \eta_1, \eta_2, \eta_3, \eta_4) \) are obtained by solving the defining equations \( X^{[3]}|_{F=0} = 0 \), with the help of Mathematica package \texttt{IntoToSymmetry.m} (Cantwell [18]) as

\[
\begin{align*}
\xi_1 &= b_1 + b_2 x, \\
\xi_2 &= a(x), \\
\eta_1 &= b_1 + b_2 \psi, \\
\eta_2 &= b_4 + b_2 T, \\
\eta_3 &= \frac{b_1}{b_4} - (C_\infty + \frac{b_3}{b_4} T_\infty) b_2 + b_2 C, \\
\eta_4 &= 0,
\end{align*}
\]  
(15)

where \( a \) is a polynomial function of \( x \) only.

Now we will find the similarity variables for self-similar solutions of the equations in (7), (8) and (9) along with boundary conditions in (10) and (11). The boundary value problem, the equations as well as boundary conditions remain invariant under the group of transformations. We now apply the invariant criteria \( X^{[3]}|_{F=0} = 0 \) to the boundary conditions. The invariance of the boundary condition \( y = 0 \) gives \( a(x) = 0 \), \( T = T_\infty(x) = T_\infty + bx^\rho \) gives \( b_1 = 0, n = 1 \), and \( T_\infty = \frac{b_3}{b_4} \). Also, \( C = C_\infty(x) = C_\infty + cx^p \) gives \( p = 1 \). So, the infinitesimals are transformed into

\[
\begin{align*}
\xi_1 &= b_2 x, \\
\xi_2 &= 0, \\
\eta_1 &= b_3 + b_2 \psi, \\
\eta_2 &= (T - T_\infty) b_2, \\
\eta_3 &= (C - C_\infty) b_2, \\
\eta_4 &= 0.
\end{align*}
\]  
(16)

Therefore, the equations (7), (8) and (9) along with boundary conditions in (10) and (11) admit two-parameter symmetry group of transformations and corresponding generators may be written as

\[
X_2 = \frac{dx}{\psi} + \frac{dy}{\psi_T} + \frac{dT}{\psi_T} + \frac{dC}{\psi_T} + \frac{dB}{\psi_T},
\]  
(17)

Solving the Lie equations \( \frac{dx}{dy} = \xi(x, \psi) \) (where \( i = 1, 2 \) and \( \frac{dy}{dx} = \eta(x, \psi) \) (where \( j = 1, 2, 3, 4 \)) with the initial conditions at \( e \equiv (b_2, b_3) = (0, 0), \bar{\psi} \equiv (\bar{\psi}, \bar{T}, \bar{C}, \bar{B}) = (\psi, T, C, B) \), the group of transformation can be obtained as

\[
\begin{align*}
G_{b_1} : \bar{\psi} &= \psi, \\
T &= T_\infty + \frac{b_3}{b_4} (T - T_\infty), \\
C &= C_\infty + \frac{b_3}{b_4} (C - C_\infty), \\
B &= B, \\
G_{b_2} : \bar{\psi} &= \psi + b_3 T_\infty, \\
T &= T_\infty + b_3 T_\infty, \\
C &= C_\infty + b_3 T_\infty, \\
B &= B. (17)
\end{align*}
\]

Again, the boundary condition \( \psi = U_\infty(x) = ax^m \) under the scaling group \( G_{b_2} \) is invariant if and only if \( m = 1 \). Hence, we are left with a single dilatational group \( G_{b_2} \) and a single translational group \( G_{b_1} \). As two parameters Lie group is always solvable, therefore the Eqsns. (7), (8) and (9) with boundary conditions in (10) and (11) are surely solvable. So, the corresponding characteristic equations can be written as

\[
\frac{dx}{b_2 x} = \frac{dy}{b_3 + b_2 \psi} = \frac{dT}{(T - T_\infty) b_2} = \frac{dC}{(C - C_\infty) b_2} = \frac{dB}{0},
\]  
(18)

where \( b_0 = b_3/b_2 \). Solving the above characteristic equations, we get the invariants as \( y = \tau, \psi = -b_0 \chi x \), \( T = T_\infty + \chi G(\tau), C = C_\infty + \chi H(\tau), B = B_0 = \text{constant} \).

Substituting these, the equations (7), (8) and (9) are transformed into

\[
F^{2} - FF'' = \psi F'' + g\beta G \cos \alpha + g\beta H \cos \alpha - \frac{\sigma b_{0}^{2}}{\rho} F' - \frac{v}{k} F',
\]  
(19)
The boundary layer phenomena is analyzed from the physical quantity, namely, the skin friction coefficient ($C_f$), which is dimensionless form of wall shear stress and is expressed as $C_f = \frac{-\mu_x}{\frac{1}{2} \rho U^2}$, where $\tau_w = \mu \left( \frac{\partial \theta}{\partial y} \right)_{y=0}$ is the wall skin friction.

The heat transfer characteristic is obtained from the local Nusselt number ($Nu$), which physically indicates the rate of heat transfer and is defined as $Nu = \frac{q_w}{\kappa \left( \frac{\partial \theta}{\partial y} \right)_{y=0}}$ is the wall heat flux of the plate.

The mass transfer phenomena is analyzed from the local Sherwood number ($Sh$), which naturally gives the surface mass flux and is given by $Sh = \frac{j_w}{D_m \left( \frac{\partial \phi}{\partial y} \right)_{y=0}}$ is the mass transfer from the plate.

### 5 Numerical computation

The non-linear coupled boundary value problem (24), (25) and (26) with boundary conditions in (27) and (28) are solved using numerical shooting method [46–48] applicable to non-linear coupled equations. The BVP is converted into the following IVP:

\[
\begin{align*}
    f'(\eta) &= z(\eta), \\
    z'(\eta) &= p(\eta), \\
    p'(\eta) &= -f(\eta)p(\eta) + z^2(\eta) + Mz(\eta) + Kz(\eta) - \lambda \theta \cos \alpha - \lambda \phi \cos \alpha; \\
    \theta'(\eta) &= q(\eta), \\
    q'(\eta) &= Pr \left[ z(\eta)\theta(\eta) - f(\eta)q(\eta) \right]; \\
    \phi'(\eta) &= r(\eta), \\
    r'(\eta) &= Pr.Le \left[ z(\eta)\phi(\eta) - f(\eta)r(\eta) \right] - Sr.Le.Pr \left[ z(\eta)\theta(\eta) - f(\eta)q(\eta) \right] q,
\end{align*}
\]

with

\[
\begin{align*}
    f(0) &= 0, \quad z(0) = f'(0) = 1, \\
    p(0) &= z'(0) = f''(0) = \alpha_1 \text{(say)}, \\
    \theta(0) &= 1, \quad q(0) = \theta'(0) = \beta_1 \text{(say)}, \\
    \phi(0) &= 1, \quad r(0) = \phi(0) = y_1 \text{(say)}.
\end{align*}
\]

The initial guess values $f''(0) = \alpha_1$, $\theta'(0) = \beta_1$, $\phi'(0) = y_1$ are needed to solve the above IVP. Here we have to choose the suitable finite value of $\eta \to \infty$, say, $\eta_{\infty} (= 10)$. By solving the above IVP using $4^{th}$ order Runge-Kutta method and comparing the computed values of $f''(\eta)$, $\theta(\eta)$, $\phi(\eta)$ at $\eta \to \eta_{\infty}$ with the prescribed boundary conditions $f'(10) = \theta(10) = \phi(10) = 0$, we can adjust the values of $f''(0)$, $\theta'(0)$, $\phi'(0)$ using Newton-Raphson method to get better accurate solutions of the
Validation of numerical scheme

To verify the accuracy of our present numerical scheme, we have compared wall shear stress $-f''(0)$ for some porosity parameter and wall temperature gradient $-\theta'(0)$ for some values of Prandtl number $Pr$ taking $\lambda = \lambda^* = M = K = 0$ with published papers Yih [10] and Hayat et al. [27] in Table 1 and Table 2, respectively and those data are in decent agreements.
Figure 7: Similar concentration profiles for various values of solutal buoyancy parameter $\lambda^*$ for fixed values of listed parameters.

Table 1: Comparison of wall shear stress $-f''(0)$ for some values of porosity parameter $K$ with $\lambda = \lambda^* = M = 0$.

| $K$ | Yih [10] (Numerical solution by Keller's box method) | Hayat et al. [17] (HAM solution) | Present study (Numerical solution by Shooting method) |
|-----|-------------------------------------------------|---------------------------------|-------------------------------------------------|
| 0.0 | 1.0000 | 1.000000 | 1.000000 |
| 0.5 | 1.2267 | 1.224767 | 1.226766 |
| 1.0 | 1.4542 | 1.454277 | 1.454296 |
| 1.5 | 1.5811 | 1.581467 | 1.581465 |
| 2.0 | 1.7321 | 1.732057 | 1.732055 |

Table 2: Comparison of wall temperature gradient $-\theta'(0)$ for some values of Prandtl number $Pr$ with $\lambda = \lambda^* = M = K = 0$.

| $Pr$ | Yih [10] (Numerical solution by Keller's box method) | Hayat et al. [17] (HAM solution) | Present study (Numerical solution by Shooting method) |
|-----|-------------------------------------------------|---------------------------------|-------------------------------------------------|
| 0.01 | 0.0097 | 0.01977 | 0.019778 |
| 0.72 | 0.80865 | 0.808635 | 0.808635 |
| 1.0 | 1.0000 | 1.000000 | 1.000000 |
| 3.0 | 1.9237 | 1.923743 | 1.923743 |
| 10.0 | 3.7207 | 3.720752 | 3.720752 |

Table 3: Values of $-f''(0)$ related to skin friction coefficient, $-\theta'(0)$ related to Nusselt number and $-\phi'(0)$ related to Sherwood number for some other parametric values.

| $\lambda$ | $\alpha$ | $M$ | $K$ | $Pr$ | $Le$ | $Sr$ | $-f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|-----------|----------|-----|-----|------|------|------|-----------|-------------|-----------|
| -0.5      | 0.5      | 1.0 | 0.8 | 0.71 | 3.0  | 0.6  | 1.63866   | 0.637968    | 0.742508  |
| 0         | 0.0      | 1.0 | 0.8 | 0.71 | 3.0  | 0.6  | 1.63866   | 0.637968    | 0.742508  |
| 0.5       | 0.5      | 1.0 | 0.8 | 0.71 | 3.0  | 0.6  | 1.63866   | 0.637968    | 0.742508  |
| 0.6       | 0.6      | 1.0 | 0.8 | 0.71 | 3.0  | 0.6  | 1.63866   | 0.637968    | 0.742508  |
| 0.5       | 0.5      | 1.0 | 0.8 | 0.71 | 3.0  | 0.6  | 1.63866   | 0.637968    | 0.742508  |
| 0.6       | 0.6      | 1.0 | 0.8 | 0.71 | 3.0  | 0.6  | 1.63866   | 0.637968    | 0.742508  |
| 0.5       | 0.5      | 1.0 | 0.8 | 0.71 | 3.0  | 0.6  | 1.63866   | 0.637968    | 0.742508  |
| 0.6       | 0.6      | 1.0 | 0.8 | 0.71 | 3.0  | 0.6  | 1.63866   | 0.637968    | 0.742508  |

$\Lambda^*$, $\lambda$, $\alpha$, $M$, $K$, $Pr$, $Le$, $Sr$.
7 Results and discussion

The computed numerical solutions are depicted graphically in the form of dimensionless velocity $f'\left(\eta\right)$, temperature $\theta\left(\eta\right)$, and concentration $\phi\left(\eta\right)$ for various values of parameters. The involved physical parameters are selected suitably, where $Pr = 0.71$ corresponds to air at $20^\circ C$, $Pr = 1.0$ corresponds to electrolyte solution, such as, salt-water and $Pr = 7.0$ corresponds to pure water.

At first, the most important aspect of investigation, i.e., the influences of thermal buoyancy parameter $\lambda$ and local solutal buoyancy parameter $\lambda^*$ on velocity, temperature and concentration profiles are presented in Figures 2–7. It is clear that $\lambda > 0$ corresponds to assisting flow (heated plate), $\lambda < 0$ corresponds to opposing flow (cooled plate) and $\lambda = 0$ corresponds to forced convection flow respectively. Also $\lambda^* > 0$ and $\lambda^* < 0$ corresponds to mass transfer from the plate to the fluid and reverse respectively. $\lambda^* = 0$ corresponds to the case where the mass transfer is absent. It can be noticed that for an increase of $\lambda$ or $\lambda^*$ (taking anyone fixed and others vary) the velocity profile $f'\left(\eta\right)$ along the plate increases but the temperature profile $\theta\left(\eta\right)$ and the concentration profile $\phi\left(\eta\right)$ (at a fixed $\eta$) along the plate decrease keeping other parameters unchanged. Consequently, the momentum boundary layer thickness increases but the thermal and concentration boundary layer thickness decrease for increasing $\lambda$ and $\lambda^*$.

Next, Figures 8–10 exhibit the self-similar velocity, temperature, and concentration profiles for different values of angle of inclination of the plate $\alpha$. It is worth noting that if the plate is rotated from vertical to horizontal with fixed origin without changing other parameters, the velocity profile along the plate decreases, but the temperature profile and the concentration profile increase. The
fluid has higher velocity when the surface is vertical than when inclined because of the fact that buoyancy force due to gravity and hence its effect decreases as the inclination of the plate becomes larger. So, the momentum boundary layer thickness reduces but the thermal and concentration boundary layer thicknesses increase with increasing angle of inclination of the plate $\alpha$.

The effects of magnetic parameter $M$ and porosity parameter $K$ on the velocity, temperature and concentration profiles are depicted in Figures 11–16. It is obvious that for an increase of $M$ or $K$ (taking anyone constant and others vary) the self-similar velocity along the plate decreases but the temperature profile and the concentration profile increase (at a fixed $\eta$) at constant other parameters. So, the fluid has higher velocity if the magnetic field is absent (i.e., $M = 0$) and the medium is non-porous (i.e., $K = 0$) but in this case heat and mass transfer decrease. The magnetic field and the porosity of the medium therefore have a tendency to resist the momentum transport and to increase the thermal and the concentration boundary layers.
Figures 17 and 18 illustrate the influence of the Prandtl number \( \Pr \) on the thermal and the concentration boundary layer thickness. The increase in \( \Pr \) reduces both the thermal and the concentration boundary layer thickness. Figure 19 reveals that the self-similar concentration profile decreases for increasing the Lewis number \( Le \) by taking other parameters constant. Thus, the increment of the Lewis number results to decrease the concentration boundary layer thickness. Whereas, Figure 20 displays the Soret effect on concentration profiles. It is observed that when \( Sr \) increases, the concentration profile also increases. So, the concentration boundary layer thickness increases for increasing \( Sr \). It is also important to mention here that the variations of velocity profile with Prandtl number, Lewis number, and Soret number are very small. Also, the variations in temperature profile for varying Lewis number and Soret number are not significant.

The values of local skin friction coefficient, Nusselt number, Sherwood number are directly proportional to \( f''(0) \), \( \theta'(0) \), and \( \phi'(0) \). The values of \( -f''(0), -\theta'(0) \), and \( -\phi'(0) \) are
–ϕ′(0) for various values of other parameters are given in Table 3. It is found that the value of –f″(0) increases for increasing α, M, K, Pr, Le, however it decreases for rise of λ, Λ′, Sr. We have noticed that the magnitude of –θ′(0) and –ϕ′(0) increase for rise of λ, Λ′, Pr, but they behave opposite nature in case of a, M, K. So the magnitude of –θ′(0) and –ϕ′(0), the similar effect occurs in case of parameters λ, Λ′, Pr, a, M, K. But for the parameters Le, Sr the opposite behavior occurs for the magnitude of –θ′(0) and –ϕ′(0). The values of –ϕ′(0) increases for increasing Le, but it decreases for increasing Sr.

8 Concluding remarks

The double-diffusive boundary layer flow of an electrically conducting fluid along a stretching inclined plate embedded in a Darcy porous medium is investigated. The governing equations are formulated and appropriate boundary conditions are prescribed. By adopting Lie group of continuous transformations, we obtain scaling laws and similarity equations. The self-similar equations are then solved numerically by using shooting method applicable to nonlinear ODEs. The results are plotted graphically. We can summarize the following important points:

1. The momentum boundary layer thickness increases with increasing thermal buoyancy parameter and solutal buoyancy parameter but it decreases with increasing angle of inclination of the plate, magnetic parameter, and porosity parameter.

2. Thermal boundary layer thickness decreases with increasing thermal buoyancy parameter, solutal buoyancy parameter and Prandtl number but it increases with increasing angle of inclination of the plate, magnetic parameter and porosity parameter.

3. The concentration boundary layer thickness decreases with increasing thermal buoyancy parameter, solutal buoyancy parameter and Prandtl number and Lewis number, but it increases with increasing angle of inclination of the plate, magnetic parameter and porosity parameter.

4. Though there are not many variations in temperature and velocity profiles due to Soret effect, but it causes an increment in concentration inside the boundary layer considerably.

5. The skin friction coefficient increases for increasing angle of inclination of the plate, magnetic and porosity parameters, however it decreases for rise of thermal and solutal buoyancy parameters.

6. Nusselt and Sherwood numbers increase for rise of thermal and solutal buoyancy parameters, Prandtl number, but they behave oppositely in case of increments of angle of inclination of the plate, magnetic and porosity parameters. Lewis number has tendency to increase the values of Sherwood number, but Soret number has opposite tendency.

Nomenclature

a, b, c, m, n, p constants
K porosity parameter
C concentration
Le Lewis number
C1 material fluid parameter
M magnetic parameter
Cw concentration at the wall
Nu Nusselt number
C∞ free stream concentration
Pr Prandtl number
cp specific heat at constant pressure
Re Reynolds number
Cf skin friction coefficient
Sh Sherwood number
Dm coefficient of mass diffusivity
Sr Soret number
f dimensionless stream function
T temperature
f′ dimensionless velocity
Tm mean fluid pressure
g acceleration due to gravity
Tw temperature at the wall
Gr local Grashof number
T∞ free stream temperature
kT thermal diffusion ratio
u, v velocity in x, y directions
k′ permeability of the porous medium
x, y Cartesian coordinates

Greek symbols
τxy shear stress tensor
Λ thermal buoyancy parameter
β coefficient of thermal expansion
Λ′ solutal buoyancy parameter
β′ coefficient of concentration expansion
ϕ stream function
μ coefficient of viscosity
η similarity variable
x thermal conductivity of the fluid
ν kinematic viscosity
\( \rho \) density of the fluid
\( \theta \) dimensionless temperature
\( \phi \) dimensionless concentration

**Superscript**
\( ' \) differentiation with respect to \( \eta \)

**Subscript**
\( w \) condition at the surface
\( \infty \) condition at infinity

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