Decays of $B$ mesons to light mesons are important for the study of CP violation in the standard model. In Ref. [1] it was suggested that since $m_b, E_M \gg \Lambda, m_M$ the amplitudes should factorize into simpler non-perturbative objects, and the proposed factorization theorem was checked at one-loop. This approach is often referred to as “QCD Factorization” (QCDF). Factorization has also been considered in the “perturbative QCD” (pQCD) approach [2]. These approaches rely on a perturbative expansion in $\alpha_s(E_M \Lambda)$ at leading order in $\Lambda/E_M$ and $\Lambda/m_b$. The results derived here apply even if $\alpha_s(E_M \Lambda)$ is not perturbative, and we prove that the physics sensitive to the $\Lambda/E_M$ scale is the same in $B \rightarrow M_1 M_2$ and $B \rightarrow M$ form factors. We argue that $c\bar{c}$ penguins could give long-distance effects at leading order. Decays to two transversely polarized vector mesons are discussed. Analyzing $B \rightarrow \pi\pi$ we find predictions for $B^0 \rightarrow \pi^0\pi^0$ and $|V_{ub}| f_{B^+\rightarrow\pi^+}(0)$ as a function of $\gamma$.

Using the soft-collinear effective theory we derive the factorization theorem for the decays $B \rightarrow M_1 M_2$ with $M_{1,2} = \pi, K, \rho, K^*$, at leading order in $\Lambda/E_M$ and $\Lambda/m_b$. The results derived here rely on model dependent assumptions about parameter values. 3) Does the power expansion converge? The soft-collinear effective theory (SCET) [6, 7] provides the necessary tools to address these issues. A first study of SCET factorization for $B \rightarrow \pi\pi$ has been made in [8]. In this paper we go beyond Refs. [1, 3, 8] in several ways. We first reduce the SCET operator basis to its minimal form and extend it to allow for all $B \rightarrow M_1 M_2$ decays (including two vectors). Our results show that all of the so-called “hard spectator” contributions are already present in the form factors, just with different hard Wilson coefficients. We also derive a form of the factorization theorem which does not rely on a perturbative expansion in $\alpha_s(E_M \Lambda)$, and show that the non-perturbative parameters are still the same as those in the $B \rightarrow M$ form factors. In our analysis long distance $c\bar{c}$ penguins [3, 10] are investigated, but are left unfactorized. For natural values of $m_b$ and $m_c$ we give an argument why these contributions can be leading order. This is contrary to expectations that they are power suppressed [1], but in agreement with expectations in [2, 3, 10]. The presence of these contributions could introduce large LO non-perturbative strong phases. Even in observables that are free from charming penguins our results differ phenomenologically from Ref. [1]. In particular while the power counting in Ref. [1] requires a hierarchy in parameters $\zeta_j^{BF} \ll \zeta_j^{B\pi}$, we show that SCET allows for other possibilities such as $\zeta_j^{B\pi} \sim \zeta_j^{B\pi}$. We demonstrate that the LO SCET results are in agreement with current $B \rightarrow \pi\pi$ data, and find current central values favor $\zeta_j^{B\pi} \gtrsim \zeta_j^{B\pi}$, albeit with fairly large uncertainties.

We set $M = P$ when discussing pseudoscalars, $M = V$ for vectors, and use an $M$ to denote either. The decays $B \rightarrow M_1 M_2$ are mediated in full QCD by the weak Hamiltonian, which for $\Delta S = 0$ reads

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{p = u,c} \lambda_{p}^{(d)} \left( C_1 O_1^p + C_2 O_2^p + \sum_{i=3}^{10,7,\gamma,8} C_i O_i \right),$$

where the CKM factor is $\lambda_p^{(f)} = V_{pd} V_{p*}^{d*}$ with $f = d$. The standard basis of $f = d$ operators are (with $O_1^p \leftrightarrow O_2^p$ relative to [11])

$$O_1^p = (\overline{\psi}_d)_{V-A}(\overline{d}p)_{V-A}, \quad O_2^p = (\overline{\psi}_b)_{V-A}(\overline{d}p)_{V-A},$$

$$O_{3,4} = \left\{ \overline{(d)}_{V-A}(\overline{q}q)_{V-A}, \overline{(d)}_{V-A}(\overline{q}q)_{V-A} \right\},$$

$$O_{5,6} = \left\{ \overline{(d)}_{V-A}(\overline{q}q)_{V+A}, \overline{(d)}_{V-A}(\overline{q}q)_{V+A} \right\},$$

$$O_{7,8} = \frac{3}{2} \left\{ \overline{(d)}_{V-A}(\overline{q}q)_{V+A}, \overline{(d)}_{V-A}(\overline{q}q)_{V+A} \right\},$$

$$O_{9,10} = \frac{3}{2} \left\{ \overline{(d)}_{V-A}(\overline{q}q)_{V-A}, \overline{(d)}_{V-A}(\overline{q}q)_{V-A} \right\},$$

$$O_{7,8} = \frac{m_b}{8\pi^2} \frac{\sigma_{\mu\nu}}{a_{\mu
u}} \left( e F_{\mu\nu} + g C_{\mu\nu} T^a \right) \left( 1 + \gamma_5 \right) b.$$

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**Keywords:** Factorization, Charming Penguins, Strong Phases, and Polarization
Here the sum over $q = u, d, s, c, b$ is implicit, $\alpha, \beta$ are color indices and $e_q$ are electric charges. The $\Delta S = 1$ $H_W$ is obtained by replacing $(f = d) \rightarrow (f = s)$ in Eqs. (12). The coefficients in Eq. (11) are known at NLL order [14]. In the NDR scheme taking $\alpha_s(m_Z) = 0.118$ and $m_b = 4.8 \text{ GeV}$ gives $C_{7-10}(m_b) = -0.317$, $C_{8g}(m_b) = -0.149$ and

$$C_{1-10}(m_b) = \{1.080, -1.177, 0.011, -0.033, 0.010, -0.040, 4.9 \times 10^{-4}, 4.6 \times 10^{-4}, -9.8 \times 10^{-3}, 1.9 \times 10^{-3}\}. \quad (3)$$

The relevant scales in $B \rightarrow M_1 M_2$ are $m_b, m_c$, the jet scale $\sqrt{E_A}$ and $\Lambda$. Varying $\Lambda$ between $100 \sim 1000 \text{ MeV}$ the jet scale is numerically in the range $\sqrt{E_A} \simeq 0.5 \sim 1.6 \text{ GeV}$. Integrating out $\sim m_b$ fluctuations, the effective Hamiltonian in SCET$_0$ can be written as

$$H_W = \frac{2G_F}{\sqrt{2}} \sum_{n, \bar{n}} \left\{ \sum_i \int [d\omega_j] \frac{1}{4} c_i^{(j)}(\omega_j) Q_i^{(0)}(\omega_j) + \sum_i \int [d\omega_j] \frac{1}{4} b_i^{(j)}(\omega_j) Q_i^{(1)}(\omega_j) + Q_{\bar{c} \bar{c}} + \ldots \right\}, \quad (4)$$

where $c_i^{(j)}$ and $b_i^{(j)}$ are Wilson coefficients, the ellipses are higher order terms in $\Lambda/Q$, $Q = \{m_b, E\}$, and $Q_{\bar{c} \bar{c}}$ denotes operators appearing in long distance charm effects as in Fig. 1. Penguin contractions with light quark loops are included in matching onto $Q_i^{(0,1)}$ since their long distance contributions are power suppressed [1]. The long-distance contributions occur when one or both of the quark lines in the penguin loop become soft or collinear. In matching onto SCET these quark lines are left uncontracted and give rise to higher dimension operators which are power suppressed. An example which gives rise to a six quark operator is given in Fig. 2.

In penguin contractions with charm quarks the situation is different due to the threshold region. For the $\bar{c} \bar{c}$ system the offshellness depends on the value of $q^2 = m_b^2 x$, and long distance contributions from $x \rightarrow 0$ or $x \rightarrow 1$ are suppressed [2]. However, for $q^2 \sim 4m_b^2$ the charm quarks are moving non-relativistically. This region corresponds to momentum fractions $x \simeq 4m_b^2/m_c^2 \simeq 0.4$ in the middle of the distribution $\phi_M(x)$. These contributions have one $\alpha_s(2m_c)$, but can not be calculated perturbatively. Using NRQCD power counting they are “suppressed” by $O(\alpha)$ with $v \simeq 0.4 \sim 0.5$. Thus we conclude that these contributions may be leading order, and comparable in size to other penguin terms such as those from the small Wilson coefficients $C_3-6$. A rigorous account of these long distance $\bar{c} \bar{c}$ penguin contractions can only be obtained by deriving a factorization theorem for them, however we do not attempt to do so here, and therefore do not write down operators for $Q_{\bar{c} \bar{c}}$.

In Eq. (11) the $O(\lambda^0)$ operators are $[\sum_{q = u, d, s} Q^{(0)}_{1d}]$ obtained by swapping $\bar{d} \rightarrow \bar{s}$. In Eq. (5) the “quark” fields with subscripts $n$ and $\bar{n}$ are products of collinear quark fields and Wilson lines with large momenta $\omega_i$. For example

$$\bar{u}_{n, \omega} = \left[ \bar{e}^{(n)}_n W_n \delta(\omega - \bar{n} \cdot \mathbf{P}) \right], \quad (6)$$

where $\xi_n$ creates a collinear quark moving along the $n$ direction, or annihilates an antiquark. The $b_i$ field is the standard usoft HQET field with Lagrangian $\mathcal{L}_h = \bar{b}_i \mathbf{i} \nu \cdot \mathbf{D} b_i$. For a complete basis we also need operators with octet bilinears. We take these to be $Q_i^{(0)}$ with $T^A \otimes T^A$ color structure, for example

$$Q_i^{(0)} = \left[ \bar{u}_{n, \omega} \bar{d}_i P_L T^A b_i \right] \left[ \bar{d}_{\bar{n}, \omega} \bar{d}_i P_L T^A u_{\bar{n}, \bar{\omega}} \right]. \quad (7)$$

These $\bar{d} \bar{d}$ and $\bar{r} \bar{s}$ operators do not contribute to the decays $B \rightarrow M_1 M_2$ at leading order, but will in power corrections. Our basis of $Q_i^{(0)}$ operators can be directly related to the one derived in [8], except that we also included
\( Q_{n}^{(i)} \) which makes the basis sufficient to accommodate all electroweak penguin effects.

We also need the \( \mathcal{O}(\lambda) \) operators for the LO factorization. Defining

\[
ig B_{n,\omega}^{+\mu} = \frac{1}{(\omega)} \left[ W_{n}^{+}(i\bar{n} \cdot D_{c,n} \cdot iD_{\rho,n}^{\mu}) W_{n}(\omega - P^{\dagger}) \right] \tag{8}
\]

they are:

\[
Q_{1d}^{(i)} = \frac{-2}{mb} \left[ \tilde{u}_{n,\omega i} \, ig B_{n,\omega}^{+\mu} P_{L} b_{j} \right] \left[ \bar{d}_{n,\omega} q P_{L} u_{n,\omega} \right], \tag{9}
\]

\[
Q_{2d,3d}^{(i)} = \frac{-2}{mb} \left[ \tilde{u}_{n,\omega i} \, ig B_{n,\omega}^{+\mu} P_{L} b_{j} \right] \left[ \bar{u}_{n,\omega} q P_{L} u_{n,\omega} \right], \tag{10}
\]

\[
Q_{4d}^{(i)} = \frac{-2}{mb} \left[ \bar{q}_{n,\omega i} \, ig B_{n,\omega}^{+\mu} P_{L} b_{j} \right] \left[ \bar{d}_{n,\omega} \gamma_{\mu} P_{L} q_{n,\omega} \right], \tag{11}
\]

\[
Q_{5d,6d}^{(i)} = \frac{-2}{mb} \left[ \tilde{u}_{n,\omega i} \, ig B_{n,\omega}^{+\mu} P_{L} b_{j} \right] \left[ \bar{d}_{n,\omega} \gamma_{\mu} P_{L} u_{n,\omega} \right], \tag{12}
\]

\[
Q_{7d}^{(i)} = \frac{-2}{mb} \left[ \tilde{u}_{n,\omega i} \, ig B_{n,\omega}^{+\mu} P_{L} b_{j} \right] \left[ \bar{d}_{n,\omega} \gamma_{\mu} P_{L} u_{n,\omega} \right], \tag{13}
\]

\[
Q_{8d}^{(i)} = \frac{-2}{mb} \left[ \bar{q}_{n,\omega i} \, ig B_{n,\omega}^{+\mu} P_{L} b_{j} \right] \left[ \bar{d}_{n,\omega} \gamma_{\mu} P_{L} q_{n,\omega} \right]. \tag{14}
\]

Our basis in Eq. (9) is simpler than the one in \( 3 \) for several reasons. Terms with a \( B_{\perp}^{\perp} \) or \( D_{\perp}^{\perp} \) in the \( \bar{n} \)-bilinear can be reduced to Eq. (9) by Fierz transformations. This shows that hard-spectator and form factor contributions are related. Second, \( P_{\perp} Q_{i}^{(0)} = 0 \), so integration by parts allows a basis for \( Q_{i}^{(1)} \) with no \( n \)-covariant derivatives, so only field strengths \( B_{\perp}^{\perp} \) appear, plus \( [\tilde{u}_{n,\omega i} \gamma_{\mu} P_{L} b_{j}] P_{L}^{\mu} [\bar{d}_{n,\omega} \gamma_{\mu} P_{L} u_{n,\omega}] \) terms which give vanishing contributions. We suppress \( Q_{i}^{(1)} \)'s with octet bilinears that do not contribute at LO. The operators \( Q_{i}^{(0,1)} \) only contribute to \( SU(3)_{\bar{n}} \) singlet production and are not used below.

Next we determine the most general structure of the \( p^{2} \sim \mathcal{E}A \) contributions in \( \text{SCET}_{1} \). We decouple the usoft modes by making the field redefinitions \( Y_{n}^{\alpha} \rightarrow Y_{n}^{\alpha} \xi_{n}^{\dagger}, A_{n} \rightarrow A_{n} v_{n}^{\dagger} Y_{n}^{\alpha}, \) with \( v_{n} \) a Wilson line of \( n' A_{n} \), and \( n' = n \) or \( \bar{n} \). In \( Q_{i}^{(0,1)} \) all \( Y_{n}^{\alpha} \) cancel except for \( (Y_{n}^{\alpha} b) \) \( 3 \), and the operators factor into \( (n, v) \) and \( \bar{n} \) parts,

\[
Q_{i}^{(0,1)} = \tilde{Q}_{i}^{(0,1)} Q_{i}^{\bar{n}}. \tag{10}
\]

In Fig. 3 the \( M' \) meson only connects to the rest of the diagram at the scale \( p^{2} \sim Q^{2} \), through \( \tilde{Q}_{i}^{(0,1)} = \tilde{q}_{\bar{n},\omega 2} \Gamma_{\bar{n},\omega 3} \) for some flavors \( q, q' \) and Dirac structure \( \Gamma \). The shaded \( p^{2} \sim \mathcal{E}A \) region is required to generate the collinear \( M' \), similar to the \( B \rightarrow M \) form factors \( 12 \). At LO it is given by \( T \)-products of the remaining parts of the operators in Eq. (10). \( Q_{i}^{(0,1)} \) with one Lagrangian \( \mathcal{L}_{i}^{(1)} \) inserted on the spectator quark to swap from usoft to collinear:

\[
T_{1} = \int d^{4}y \, dy' \, T \left[ \tilde{Q}_{i}^{(1)}(y), a_{i} \mathcal{L}_{i}^{(1)}(y') \right] + i\mathcal{L}_{i}^{(1)}(y') + \int d^{4}y \, T \left[ \tilde{Q}_{i}^{(0)}(y), a_{i} \mathcal{L}_{i}^{(1)}(y) \right], \tag{11}
\]

\[
T_{2} = \int d^{4}y \, T \left[ \tilde{Q}_{i}^{(1)}(y), a_{i} \mathcal{L}_{i}^{(1)}(y) \right]. \tag{12}
\]

Here \( \mathcal{L}_{i}^{(1)} \) are \( q_{i} q_{i}^{\dagger} W_{i} \xi_{n} + h.c. \), and the form of our other \( \mathcal{L} \)'s can be found in \( 14 \).

Now we match \( \text{SCET}_{1} \) onto \( \text{SCET}_{2} \). A complete treatment of \( T_{1} \) is an open question due to endpoint singularities \( 12, 13, 16 \), but \( \left\langle V_{i}^{\dagger} T_{1} B \right\rangle = 0 \) and the nonzero matrix elements can be parameterized as

\[
\left\langle P_{i}^{\dagger} T_{1} B \right\rangle = m_{B} \zeta^{B \perp}, \quad \left\langle V_{i}^{\dagger} T_{1} B \right\rangle = m_{B} \zeta^{B \parallel}. \tag{13}
\]

For \( T_{2} \) the most general perturbative matching at \( \mu^{2} \sim \mathcal{E}A \) generates a set of operators with Wilson coefficients given by jet functions \( J \) and \( \bar{J} \) whose form is constrained by RPI, chirality, power counting and dimensional analysis. We suppress \( (\xi_{n})_{\perp}(\xi_{n})_{\parallel}(\xi_{n})_{\parallel} \) terms which make the basis sufficient to accommodate all electroweak penguin effects.

\[
\left\langle (\xi_{n})_{\perp}(\xi_{n})_{\parallel}(\xi_{n})_{\parallel} \right\rangle = \int d^{4}y \, \frac{V_{i}^{\dagger}(y)}{\int d^{2}y} \, \frac{V_{i}^{\dagger}(y)}{\int d^{2}y} \, \frac{V_{i}^{\dagger}(y)}{\int d^{2}y} = m_{B} \zeta^{B \parallel}. \tag{14}
\]

For \( T_{2} \) the most general perturbative matching at \( \mu^{2} \sim \mathcal{E}A \) generates a set of operators with Wilson coefficients given by jet functions \( J \) and \( \bar{J} \) whose form is constrained by RPI, chirality, power counting and dimensional analysis. We suppress \( (\xi_{n})_{\perp}(\xi_{n})_{\parallel}(\xi_{n})_{\parallel} \) terms which make the basis sufficient to accommodate all electroweak penguin effects.

\[
\left\langle (\xi_{n})_{\perp}(\xi_{n})_{\parallel}(\xi_{n})_{\parallel} \right\rangle = \int d^{4}y \, \frac{V_{i}^{\dagger}(y)}{\int d^{2}y} \, \frac{V_{i}^{\dagger}(y)}{\int d^{2}y} \, \frac{V_{i}^{\dagger}(y)}{\int d^{2}y} = m_{B} \zeta^{B \parallel}. \tag{15}
\]
Thus at LO only $A_{cc}$ could give transverse polarized vector mesons so

$$A(B \to V_1^{-}V_2^{+}) = \frac{2G_F}{\sqrt{2}} \langle V_1^{-}V_2^{+} | Q_{cc} | B \rangle. \quad (17)$$

Next consider $B \to V_1V_2$, $B \to V_1P$ and $B \to PP$ decay. Now it is the $J$ term in Eq. (13) that contributes along with possible long distance charm penguins. Due to the form of our operators the $J$ term is identical to the analysis of the $B \to M$ form factors. The LO factorization formula for $A = \langle M_1M_2 | H_W | B \rangle$ which determines $B^0, B^- \to M_1M_2$ with $M_{1,2}$ pseudoscalars or longitudinal vectors is

$$A(B \to M_1M_2) = \lambda_{c\bar{c}}^{(f)} A_{cc} M_1M_2 + \frac{G_F m_B^2}{\sqrt{2}} \left\{ f_{M_2} \zeta^{BM_1} \right. \times \int_0^1 du T_{2\zeta}(u) \phi^{M_2}(u) + f_{M_1} \zeta^{BM_2} \int_0^1 du T_{1\zeta}(u) \phi^{M_1}(u) \bigg\} \times \phi^{M_1}(x) \phi^{M_2}(u) + T_{1j}(u, z) \phi^{M_1}(x) \phi^{M_1}(u) \bigg\} \phi^+_B(k_+), \quad (18)$$

where $A_{cc}$ denote possible long distance charm penguin amplitudes which contribute in channels where $c_i^{(d,s)}$ are present. For each decay mode the set of hard coefficients $T_{1\zeta}$ and $T_{2\zeta}$ can be obtained from Table I.

A new result from our analysis is that the jet function $J$ in Eq. (13) is the same as that appearing in the factorization formula for $B \to M$ form factors (19). We quote here two of these formulas, one for the standard $B \to P \bar{\nu} \nu$ form factor $f_+(E)$, and one for the form factor $A_{ii}$ for $B \to V_i^{-}V_i^{+}$ decays,

$$A_{ii}(E) = \frac{1}{m_V} \left[ \frac{m_B E A_2(E)}{m_B + m_V} \right] A_1(E), \quad (19)$$

where

$$E = \frac{m_B^2 + m_V^2 - q^2}{2m_B}. \quad (20)$$

At LO in SCET and QCDF (12, 13, 16, 17, 18, 20)

$$f_+(E) = T^{(+)}(E) \zeta^{BP}(E) + N_0 \int_0^1 dz \int_0^1 dx \int_0^\infty dk_+ \times C_{j}^{(+)}(z, E) J_{j}(x, k_+), \quad (21)$$

where $N_0 = f_B f_B m_B/(4E^2)$, and the functions $T^{(+)}(E), C_{j}^{(+)}(z)$ are combinations of SCET Wilson coefficients and can be found in (18). In that paper the jet functions $J_{j}^{(l)}(z, x, k_+)$ in Eq. (13) are denoted by $J_{j}^{(l)}(z, x, k_+)$ in Ref. (18). Finally it is clear that possible meson fluctuations (21) can not spoil factorization in $Q_{ij}^{(0,1)}$ which have color singlet $\pi$-bilinear, and so their role will be identical to that in the form factors.

At this point we compare our result in Eq. (19) with the result in QCDF (11). From Eq. (25) of (11) the LO factorization theorem is

$$\langle M_1M_2 | O_i | B \rangle = \left\{ f_{B \to M_1}(0) f_{M_2} \int dt T_{M_2,i}(t) \phi_{M_2}(u) + (1 \leftrightarrow 2) \right\} + f_{M_1} f_{M_2} f_B \int dt dx \int_0^\infty dk_+ \times T_{ii}^{(l)}(x, u, k_+) \phi_{M_1}(x) \phi_{M_2}(u) \phi_B(k_+), \quad (23)$$

where the parameters are the QCDF form factors $F_{B \to M}(0), \phi_{M_1}$, and $\phi_B$ (other parameters appear when
power suppressed terms from annihilation or chirally enhanced corrections are included). In the QCD power counting the second term is suppressed relative to the first by a factor of $\alpha_s$. The result in Eq. (22) is quite similar to the SCET formula derived in Eq. (13). However, there are several important differences, which we comment on. The two things that are most important for phenomenology are that QCDF does not allow for a leading order $A_{\pi\pi}^{P\pi}$ contribution, and that the SCET analysis suggests that the contribution from $\zeta$ and $\zeta_J$ are comparable in size, rather than $\zeta_{BP}^{\pi \pi} \ll \zeta_{BP}$ as in QCDF. As discussed later, current data on $B \to \pi\pi$ seems to support $\zeta_{BP}^{\pi \pi} \sim \zeta_{BP}$, albeit with large uncertainties. This difference has significant phenomenological ramifications, as it implies that even in absence of leading order charming penguin effects the perturbative strong phases predicted in [1] would receive $O(100\%)$ corrections. Besides these points there are several technical differences between the two formulas. Using $F^{B \to M}(0)$ in Eq. (22) rather than $\zeta_{BP}^{M}$ does not completely separate out all contributions from the hard scale. Also, in Eq. (22) $T^1$ and $T^{11}$ include perturbative contributions from both the $\mu^2 \simeq Q^2$ and $\mu^2 \simeq E\Lambda$ scales [24]. In the result in Eq. (13) these scales are separated in $T_{ij}$ and $J$ respectively. If $\zeta_{BP}$ is independent of the $\mu^2 \simeq E\Lambda$ scale as argued in Ref. [10], then the scales are also completely separated in the $T_{ij} \zeta_{BP}$ term, otherwise $\zeta_{BP}$ still encodes physics at both the jet scale $E\Lambda$ and the scale $\Lambda^2$.

The jet function $J$ depends on physics at the intermediate scale, so its perturbative expansion in $\alpha_s(\sqrt{E\Lambda})$ is not as convergent as for the $T_{ij}$ and $T_{ic}$ which are expanded in $\alpha_s(Q)$. In fact perturbation theory may fail for $J$ all together. This can be tested both by experiment [22] and by additional perturbative calculations. Using SCET we can still obtain an expression for $A(B \to M_1 M_2)$ without expanding $J$ perturbatively,

$$A = \frac{G_F m_B^2}{\sqrt{2}} \left\{ f_{M_1} \int_0^1 du \int_0^1 dz \, T_{11}(u, z) \phi_{BM_2}^{M_1}(z) \phi_{M_1}(u) \right\} + \left\{ 1 \leftrightarrow 2 \right\} + \lambda f \, A_{\phi_{M_1}^{M_1} M_2},$$

where power counting implies $\zeta_{BP} \sim (\Lambda/Q)^{3/2}$. Here the non-perturbative parameters $\zeta_{BM_1}, \zeta_{BM_2}(z)$, and $\phi_{M_1}(u)$, still all occur in the $B \to M$ semileptonic and rare form factors. For a model independent analysis they need to be determined from data. Note that it was possible for us to derive Eq. (24) because in Eq. (13) we separated the scales $Q^2$ and $E\Lambda$ into $T$'s and $J$'s respectively. The corresponding results for the form factors in Eq. (21) are

$$f_+ = T^{(+)}(E) \zeta_{BP}(E) + N_0 \int_0^1 dz \, C_{J_{BP}}^{(+)}(z) \zeta_{BM_2}(z, E),$$

$$A_\parallel = T^{(A_\parallel)}(E) \zeta_{BP}(E) + N_\parallel \int_0^1 dz \, C_{J_{BP}}^{(A_\parallel)}(z) \zeta_{BM_2}(z, E).$$

The two form factors in Eq. (21) can be obtained from data on $B \to (P, V)_{ij} \nu$, giving important information on the $\zeta_{BP}^{BM}$ appearing in Eq. (13). Note that in Eqs. (15) and (24) the $\zeta$'s are evaluated at $E = m_B/2$. Eq. (13) and (24) are the main results of our paper.

Using Eq. (24) still requires matching the full theory $O_i$'s onto the $Q_i^{(0)}$ to determine the Wilson coefficients $c_i^{(f)}$ and $b_i^{(f)}$. For the coefficients of $Q_i^{(1)}$ we find $[f = d, s]$

$$c_1^{(f)} = \lambda f \left[ C_1 + \frac{C_2}{N_c} \right] - \lambda f \left[ \frac{3}{2} C_10 + \frac{C_9}{N_c} \right] + \Delta c_1^{(f)},$$

$$c_2^{(f)} = \lambda f \left[ C_2 + \frac{C_1}{N_c} \right] - \lambda f \left[ \frac{3}{2} C_9 + \frac{C_{10}}{N_c} \right] + \Delta c_2^{(f)},$$

$$c_3^{(f)} = -\lambda f \left[ \frac{3}{2} C_2 + \frac{C_8}{N_c} \right] + \Delta c_3^{(f)},$$

$$c_4^{(f)} = -\lambda f \left[ C_4 + \frac{C_3}{N_c} - \frac{C_{10}}{2 N_c} - \frac{C_9}{2 N_c} \right] + \Delta c_4^{(f)}.$$
of whether a quantity is contaminated depends on the relative size of \( \zeta_{BM} \) and \( \zeta_{BM}^j \). If \( \zeta_{BM} \gg \zeta_{BM}^j \) as in QCDF then any \( f = d \) decay in Table I that is independent of \( c_1^{(d)} \) could receive large corrections, making quantities such as \( Br(B^0 \to \pi^0 \pi^0) \) contaminated. Here the most problematic are large power corrections proportional to \( C_1 \Lambda/E \) which is \( \sim C_2 \) and \( \gg C_{2\gamma} \). These can arise for example from T-products involving the \( Q_{2\gamma}^{(0)} \) operators.

The situation is much better in the case \( \zeta_{BM} \sim \zeta_{BM}^j \) since any decay depending on \( c_1^{(d)}, b_1^{(d)}, \) or \( b_2^{(d)} \) will not be contaminated and can be expected to have power corrections of normal size, \( \sim 20\% \). Our analysis of \( B \to \pi \pi \) below favors this situation, in which case \( Br(B^0 \to \pi^0 \pi^0) \) is not contaminated.

At leading order in \( \Lambda/E \) there are only two sources of strong phases: the one-loop \( \Delta c_1, \Delta b_1 \) which can become complex, and the unfactorized \( A_{c\bar{c}} \) charming penguin. Additional final state phases come from power corrections \( \sim \Lambda/E \). It is known from \( B^0 \to D^0 \pi^0 \) decays that \( \Lambda/E \) corrections produce \( \sim 30\% \) non-perturbative strong phases in agreement with dimensional analysis \([22]\). These large phases have nothing to do with a \( \Lambda/m_c \) expansion so we expect strong phases of similar size from power corrections in \( B \to M_1 M_2 \). For contaminated decays, such as \( B \to K K, \) non-perturbative strong phases \( \sim C_1 \) could be order unity.

The factorization theorems in Eqs. \([18,24]\) can be used to make quantitative predictions for nonleptonic \( B \to M M' \) decays. There are many applications; a few of the more important categories are: i) Decay modes which are independent of charming penguin contributions are determined by \( \zeta \) and \( \zeta_j \) which can be extracted from semileptonic form factors. ii) SCET implies SU(3) relations beyond those following from \( H_W \) in Eq. \([4]\) with full QCD. It also simplifies the structure of SU(3) breaking corrections. iii) For \( B \to V V' \) SCET allows us to analyze penguin effects. iv) Using isospin SCET makes predictions for matrix elements whose quantum numbers differ from the reduced set of \( A_{c\bar{c}}^{M_1 M_2} \) amplitudes. In the remainder of the paper we discuss examples in each of these categories. In particular we show that Eq. \([24]\) gives a reasonable fit to the current \( B \to \pi \pi \) data.

The parameters \( \zeta_{BM} \) and \( \zeta_{BM}^j \) in Eq. \([24]\) for nonleptonic decays are common to those appearing in \( B \to M \) form factors Eq. \([25]\). Decays that do not depend on \( A_{c\bar{c}} \) include all combinations in Table I that are independent of \( c_1 \) and \( b_4 \), such that \( B^+ \to \pi^0 \pi^- \) and \( B^+ \to \rho^+ \rho^- \) once isospin is used. For example,

\[
\sqrt{2} A(B^- \to \pi^- \pi^0) = \frac{G_F m_B^2}{\sqrt{2} \pi} f_\pi \left( \int_0^1 du dz (u_1^{(d)} + b_2^{(d)} - b_3^{(d)}) (u, z) \zeta_{BM}^\pi (z) \phi_\pi^\pi (u) + \zeta_{BM}^\pi \int_0^1 du \left( (c_2^{(d)} - c_3^{(d)}) (u) \phi_\pi^\pi (u) \right) \right),
\]

At tree level the \( b_1^{(j)} \)'s are independent of \( \zeta \) and this relation gives a clean constraint on \( \zeta_{BM}^\pi \) and \( \zeta_{BM}^\pi = \int dz \zeta_{BM}^\pi (z) \).

Flavor SU(3) symmetry is a powerful tool for studying nonleptonic B decays. In one particular application, Ref. \([23]\) proposed using flavor SU(3) symmetry to determine \( \gamma \) from \( B^+ \to K^+ \pi^- \pi^0 \). Corrections to this approach are available from SU(3) breaking effects and are typically \( \sim 30\% \). The factorization relation Eq. \([24]\) implies enhanced SU(3) relations beyond those in QCD. For example, in QCD all \( B \to PP \) decays to pseudoscalar octet mesons are parameterized in the SU(3) limit by 5 complex amplitudes. Using the SCET factorization formula Eq. \([18]\) this number is reduced to one complex amplitude \( A_{c\bar{c}} \), one real number \( \zeta \) and one real function \( \zeta_j (z) \). In the language of Ref. \([22]\) the operators in Eq. \([4]\) do not generate the \( E, A \), and \( PA \) amplitudes, so these are power suppressed.

In certain cases the SU(3) breaking can be also computed. Such an example is the determination of two SU(3) breaking parameters \( R_{1,2} \) appearing in a SU(3) relation used to extract \( \gamma \) \([23]\)

\[
A(B^- \to K^0 \pi^-) + \sqrt{2} A(B^- \to K^- \pi^0) = \sqrt{2} |V_{u}\rangle \langle V_{ud}| (R_1 - \delta_{EW} e^{-\gamma} R_2) A(B^- \to \pi^- \pi^0).
\]

Here \( \delta_{EW} \) parameters the largest electroweak penguin effects and is calculable. The parameters \( R_{1,2} \) can be expressed in terms of \( \zeta_{BM}^\pi, \zeta_{BM}^{BK}, \zeta_{BM}^{BK} (z), \zeta_{BM}^{BK} (z) \) and calculable Wilson coefficients and do not involve \( A_{c\bar{c}} \) or \( A_{c\bar{c}}^{K^*} \).

Polarization measurements in decays to two vector mesons have received much attention recently. These decays were studied in Ref. \([17]\) and it was argued that factorization implies \( R_T \sim 1/m_b^2 \) and \( R_L / R_T = 1 + O(1/m_b) \), where \( R_{T,L} \) denote the longitudinal, transverse, perpendicular and parallel polarization fractions \( R_T = R_L + R_T, R_0 + R_T = 1 \). Using SCET we find that \( R_T \) is power suppressed in agreement with \([17]\), unless the charming penguin amplitude \( A_{c\bar{c}} \) spoils this result. We can not resolve the validity of the \( R_L / R_T \) relation working only at LO in \( 1/m_b \). Experimentally, one finds \([24,27]\)

\[
\begin{align*}
R_0(B^+ \to \rho^0 \rho^-) & = 0.975 \pm 0.045, \\
R_0(B^0 \to \rho^+ \rho^-) & = 0.98 \pm 0.02 \pm 0.03, \\
R_0(B^0 \to \phi K^*) & = 0.49 \pm 0.06.
\end{align*}
\]

It has been argued that the large transverse polarization observed in the \( \phi K^* \) mode might provide a second hint at new physics in \( b \to s s s \) channels beyond sin(2\( \beta \)) from \( B \to \phi K_S \). Unfortunately this conclusion could be spoiled by a contribution from \( A_{c\bar{c}} \) at leading order. \( A_{c\bar{c}} \) does not contribute to \( B^+ \to \rho^+ \rho^- \), but can affect \( B^0 \to \phi K^* \) and \( B^0 \to \rho^+ \rho^- \). Until charming penguins
are better understood the polarization measurements do not provide a clean signal of physics beyond the standard model.

We finally examine in some detail the predictions of this paper for $B \to \pi \pi$ decays, and show that they reproduce the existing data. The present world averages are

$$S_{\pi\pi} = -0.74 \pm 0.16, \quad C_{\pi\pi} = -0.46 \pm 0.13,$$

$$BR(B^+ \to \pi^0 \pi^+) = (5.2 \pm 0.8) \times 10^{-6},$$

$$BR(B^0 \to \pi^+ \pi^-) = (4.6 \pm 0.4) \times 10^{-6},$$

$$BR(B^0 \to \pi^0 \pi^0) = (1.9 \pm 0.5) \times 10^{-6},$$

where the branching fractions are CP averages. The amplitudes are naturally divided into two pieces with different CKM factors, as $A = \lambda^{(d)}_b T + \lambda^{(d)}_t P$, where $T$ and $P$ are usually called “tree” and “penguin” amplitudes. The decay amplitudes for $B \to \pi \pi$ can be written in a model-independent way as

$$A(\bar{B}^0 \to \pi^+ \pi^-) = \lambda^{(d)}_b T(1 + r_c e^{i\delta_c} e^{i\gamma}),$$

$$A(\bar{B}^0 \to \pi^0 \pi^0) = \lambda^{(d)}_t T(1 + r_n e^{i\delta_n} e^{i\gamma}),$$

$$\sqrt{2}A(B^- \to \pi^0 \pi^-) = \lambda^{(d)}_n T,$$

where $(r_c, \delta_c)$ and $(r_n, \delta_n)$ parameterize the ratio of tree to penguin contributions to $B^0 \to \pi^+ \pi^−$ and $B^0 \to \pi^0 \pi^0$, respectively. We have neglected small electroweak penguin contributions. Isospin gives the relations

$$T = T_c + T_n, \quad T_c r_c e^{i\delta_c} + T_n r_n e^{i\delta_n} = 0,$$

leaving only 5 independent strong interaction parameters in Eq. (31).

In the first step of the analysis, we assume that $\beta, \gamma$ are known, use this to disentangle the tree and penguin amplitudes, and thus extract the five parameters in Eq. (31). In a second step, these parameters are compared with the leading order predictions from SCET, and used to extract the nonperturbative parameters appearing in the factorization formula Eq. (24), working at tree level in matching at the hard scale. The resulting SCET parameters are then used to predict values for $|V_{ub}| f_\pi(0)$ and $Br(B^0 \to \pi^0 \pi^0)$ as functions of $\gamma$.

Assuming values for the CKM angles $\beta$ and $\gamma$ we can use the 5 pieces of experimental data given in Eq. (31) to determine the 5 parameters in Eq. (32). Using $(\beta, \gamma) = (23^\circ, 64^\circ)$ and the data for the CP asymmetries we find for the penguin parameters $r_c$ and $\delta_c$.

$$r_c = 0.75 \pm 0.35, \quad \delta_c = -44^\circ \pm 12^\circ.$$ 

This is in good agreement with the recent determinations of these parameters in Refs. [27]. Using the branching ratio data as input, we can determine the tree parameters

![Diagram showing constraints on the triangle of tree amplitudes](image)

**FIG. 4:** Constraints on the triangle of tree amplitudes $T/T_c - T_n/T_c = 1$ from current world averaged data on $B \to \pi \pi$. The shaded regions show the two 1-σ regions for $\gamma = 64^\circ$ including the error correlation between $|t|$ and $|t_n|$. The central values for $\gamma = 54^\circ$ and $\gamma = 74^\circ$ are also shown.

as well. We find

$$|T| = N_\pi (0.29 \pm 0.02) \left( \frac{3.9 \times 10^{-3}}{|V_{ub}|} \right),$$

$$|t| = 2.07 \pm 0.42, \quad |t_n| = \begin{cases} 1.15 \pm 0.33 \quad (I) \\ 1.42 \pm 0.35 \quad (II) \end{cases},$$

where $N_\pi = \frac{G_F m_B^2 f_\pi}{\sqrt{2}}$ and we defined

$$t = \frac{T}{T_c}, \quad t_n = \frac{T_n}{T_c},$$

Some of the errors in Eqs. (31) and (32) have sizeable correlations. The results for the tree triangle are shown graphically in Fig. 4. The two $\gamma = 64^\circ$ solutions correspond to those in Eq. (35) and the ellipses denote 1σ contours. Also shown in this figure is the isospin tree triangle, which for the reduced tree level amplitudes reads $1 + t_n = t$. There are two strong phases in this triangle which are also shown in the figure, namely $\theta$ between $T$ and $T_c$ and $\theta_n$ between $T_n$ and $T_c$.

As a second step the extracted amplitudes are compared with the predictions of this paper at leading order in $\Lambda/m_b$ and tree level in the SCET Wilson coefficient $c_i^{(d)}$ and $b_i^{(d)}$. At this order our result has four independent parameters. The tree amplitudes $T, T_c$ are given by the factorization relation Eq. (15) and depend on the
FIG. 5: Model independent results for $\zeta_{B\pi}$, $\zeta_{B}\pi$, and the $B \to \pi$ form factor $f_+(q^2 = 0)$ as a function of $\gamma$. The shaded bands show the 1-$\sigma$ errors propagated from the $B \to \pi\pi$ data.

The amplitude $T = N_\pi \frac{1}{3} (C_1 + C_2) \left[ 4 \zeta_{B\pi} + (4 + \langle \bar{u}^{-1}\rangle_\pi) \zeta_{B}\pi \right]$, $T_c = N_\pi \left[ \left( C_1 + C_2 \frac{2}{3} + C_4 + C_3 \frac{3}{3} \right) \zeta_{B\pi} \right] + \left( C_4 + (1 + \langle \bar{u}^{-1}\rangle_\pi) \frac{C_2}{3} + C_3 \right) \zeta_{B}\pi$, where $\langle \bar{u}^{-1}\rangle_\pi = \int_0^1 \phi_\pi(u)/(1-u)$, and $\zeta_{B\pi} = \int dz \zeta_{B\pi}(z)$. The penguin amplitude also gets a contribution from the complex $A_{cc\pi}$ amplitude, so

$$P = -\left[ \frac{\lambda_{d}^{(d)}}{\lambda_{d}^{(s)}} \right] T_c r_c e^{i \delta_c} = N_\pi \left[ \left( C_4 + C_3 \frac{3}{3} \right) \zeta_{B\pi} \right] + \left( C_4 + (1 + \langle \bar{u}^{-1}\rangle_\pi) \frac{C_2}{3} + C_3 \right) \zeta_{B}\pi + \frac{1}{N_\pi} A_{cc\pi}. \quad (38)$$

The amplitude $T_n$ is given by the isospin relation Eq. (38) as $T_n = T - T_c$. At tree level in SCET Wilson coefficients the $B \to \pi$ form factor at $q^2 = 0$ is

$$f_+(0) = \zeta_{B\pi} + \zeta_{B}\pi. \quad (39)$$

Neglecting the $O(\alpha_s(m_h))$ corrections introduces an error of about 10% for the $T$ amplitudes, which is smaller than the expected size of the power corrections $O(\Lambda/E)$. Eq. (37) implies that the tree amplitudes $T, T_c$ are calculable in terms of the $\zeta, \zeta_{B}$ parameters, and their relative strong phase are small $\theta, \theta_n \sim O(\alpha_s(m_h), \Lambda/E)$. On the other hand, the penguin amplitude $P$ can have an $O(1)$ strong phase due to the charming penguin amplitude $A_{cc\pi}$. The pattern of results in Fig. 4 supports these predictions for the tree amplitudes $T, T_c$, for the upper hand solution. In particular, within the experimental uncertainty the phases $\theta$ and $\theta_n$ are still consistent with being small and compatible with order $O(\Lambda/E)$ effects.

Using the numbers in Eq. (55) for $|T|$ and $|t|$ and the SCET results in Eqs. (74), we can extract the nonperturbative parameters $\zeta, \zeta_{B}$. Taking LL order for the coefficients ($C_1 = 1.107, C_2 = -0.248, C_3 = 0.011, C_4 = -0.025$ at $\mu = 4.8$ GeV) and $\langle \bar{u}^{-1}\rangle_\pi = 3$ GeV, we find

$$\zeta_{B\pi} |_{\gamma = 64^\circ} = (0.05 \pm 0.05) \left( \frac{3.9 \times 10^{-3}}{|V_{ub}|} \right), \quad (40)$$

$$\zeta_{B}\pi |_{\gamma = 64^\circ} = (0.11 \pm 0.03) \left( \frac{3.9 \times 10^{-3}}{|V_{ub}|} \right),$$

where the quoted errors are propagated from the experimental errors from $|T|$ and $|t|$ in Eq. (35). Using the results for $r_c$ and $\delta_c$ in Eq. (34) and $|V_{cb}| = 0.041$ the penguin amplitude is

$$P \bigg|_{\gamma = 64^\circ} = (0.043 \pm 0.013) e^{i(136^\circ \pm 12^\circ)}. \quad (41)$$

The $\zeta_{B\pi}$ and $\zeta_{B}\pi$ terms in Eq. (38) contribute 0.002 to $P/N_\pi$, which is only a small part of the experimental result. The perturbative corrections from the $\Delta c_i^{(f)}$'s or particularly the $\Delta b_i^{(f)}$'s can add terms whose rough size is estimated to be $\sim \zeta_{B\pi} C_1 \alpha_s(m_h)/\pi \sim 0.007$. After removing these contributions, the sizeable remainder would be attributed to $A_{cc\pi}$. Since $A_{cc\pi}$ can have a large nonperturbative strong phase, the large phase in Eq. (11) supports the conclusion that this term contributes a substantial amount to $P/N_\pi$.

The extraction of the above parameters allows us to make two model independent predictions with only $\gamma$ and $|V_{ub}|$ as input. First a prediction for the semileptonic $B \to \pi$ form factor $f_+(0)$ is possible. Combining Eq. (40) with Eq. (39) we find

$$f_+(0) \bigg|_{\gamma = 64^\circ} = (0.17 \pm 0.02) \left( \frac{3.9 \times 10^{-3}}{|V_{ub}|} \right). \quad (42)$$

In Fig. 5 we show results for $\zeta_{B\pi}$, $\zeta_{B}\pi$, and $f_+(0)$ for other values of $\gamma$, thus generalizing the results in Eqs. (10) and (12). Note that including the correlation in the errors for $\zeta_{B\pi}$ and $\zeta_{B}\pi$ has led to a smaller uncertainty for $f_+(0)$. Theory uncertainty is not shown in Eq. (12) or Fig. 5 and the most important source are power corrections which we estimate to be $\pm 0.03$ on $f_+(0)$. One loop $\alpha_s(m_h)$ corrections are also not yet included. Varying $\mu = 2.4 - 9.6$ GeV in the LL coefficients changes $f_+(0)$ by only a small amount $\mp 0.01$.

It is interesting to note that the central values from our fit to the data give $\zeta_{B}\pi \gtrsim \zeta_{B\pi}$ which differs from the hierarchy used in QCD. Furthermore our central value for $f_+(0)$ is substantially smaller than the central values obtained from both QCD sum rules [29] ($f_+(0) = 0.26$), from form factor model based fits to the semileptonic data [30] ($f_+(0) = 0.21$), or those used in the QCDF analysis $f_+(0) = 0.28$ or 0.25.
Our analysis can also be used to make a prediction for $Br(B^0 \to \pi^0 \pi^0)$. At tree level in SCET $|t_n| = |t| - 1$ which gives

$$\frac{\Gamma(B^0 \to \pi^0 \pi^0)}{\Gamma(B^0 \to \pi^0 \pi^-)} = \left(\frac{|t|-1}{|t|}\right)^2 + \frac{r_e^2}{|t|^2} \frac{2r_e}{|t|} \left(1 - \frac{1}{|t|}\right) \cos(\delta_e) \cos(\gamma).$$

(43)

Thus we predict

$$Br(B^0 \to \pi^0 \pi^0) = \begin{cases} (1.0 \pm 0.7) \times 10^{-6}, & \gamma = 54^\circ \\ (1.3 \pm 0.6) \times 10^{-6}, & \gamma = 64^\circ \\ (1.8 \pm 0.7) \times 10^{-6}, & \gamma = 74^\circ \end{cases}$$

(44)

These results are all in reasonable agreement with the current world average. The uncertainty quoted in Eq. (44) is only from the inputs in Eq. (43), and will be directly reduced when the first four measurements in Eq. (31) improve. Since the $\zeta B^\pi$ term in Eq. (40) is $\sim \zeta B^\pi$ our results for $Br(B^0 \to \pi^0 \pi^0)$ are not contaminated and we expect that theoretical uncertainty from power corrections plus $\alpha_s(m_h)$ corrections will add a $\sim 20-30\%$ uncertainty to the results in Eq. (43). Note that one can turn the analysis in Eq. (43) around and use the data on $B \to \pi\pi$ in Eq. (31) to give a new method for determining the value of $\gamma$, where the theoretical input from factorization is that the tree triangle is flat.

Our values in Eq. (43) are somewhat larger than the central values predicted in QCDF ($\sim 0.3 \times 10^{-6}$ [4] or pQCD ($\sim 0.2 \times 10^{-6}$ [31]). For $\gamma = 54^\circ$ the first term in Eq. (43) dominates our result, while the $r_e^2$ penguin term has a large cancellation with the interference term $\cos(\gamma)$. For larger $\gamma$'s this cancellation becomes less effective and $Br(B^0 \to \pi^0 \pi^0)$ increases. In QCDF $\zeta B^\pi$ dominates over a small $\zeta J$, but has a small coefficient $\propto C_9/C_1/3$, so the first term in Eq. (43) is small. In pQCD the $M_{a,e}$ terms which are multiplied by $C_1$ are also small for $B \to \pi^0 \pi^0$.

In this paper we have used SCET to derive a factorization theorem for $B \to M_1 M_2$ decays and explored the theoretical and phenomenological implications. Several issues for $B \to M_1 M_2$ still remain to be resolved. A factorization formula for the charming penguin contribution should be worked out, and polarization effects should investigated beyond leading order. It needs to be shown that the $n-\bar{n}$ factorization is not spoilt by Glauber degrees of freedom. The one loop $\Delta \eta$'s need to be computed, as well as a resummation of Sudakov logarithms which are given by the evolution equations for the SCET operators. Charming penguin effects need to be better understood in an effective theory approach, and a full factorization theorem for the $A_{1/2}$ amplitude should be worked out. Finally, power corrections (including so called chirally enhanced terms, annihilation contributions, and $C_1/E$ terms) should be studied using SCET.

This work was supported in part by the DOE under DE-FG03-92ER40701, DOE-ER-40682-143, DEAC02-6CH03000, and the cooperative research agreement DF-FC02-94ER40818. I.S. was also supported by a DOE OJI award.

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