Patterned hydrodynamic slip flow control and thermal transport in a wavy microchannel through porous medium with combined Electromagnetohydrodynamic effect

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Abstract. A conceptual exploration is conducted to analyze the numerical experiment of the pattern hydrodynamic slip flow control and thermo-fluidic transport features coupled with the influence of the combined electromagnetohydrodynamic (EMHD) effect in a wavy microchannel through the porous medium. The constitutive equations for the velocity distribution and energy distribution are represented by a set of non-linear differential equations. These equations are solved numerically by employing a shooting technique based on the fourth-order Runge-Kutta method. The effect of Joule heating and temperature jump on the temperature distribution is also discussed in this study. The significant impacts of the electromagnetic force, the porosity of medium, Forchhemier inertia parameter, the impression of the wavy-walls amplitude, the effect of the wavenumber of the slip length on momentum, and thermal transport are discussed in this investigation. It is revealed that the velocity and the temperature both are augmented with the improvement of the amplitude of the wavy wall of the microchannel but the wave number of the slip length minimize the velocity as well as the temperature of the fluid.

Keywords: Electromagnetohydrodynamic flow, Porous medium, microchannel, Wavy wall, Numerical solution

1. Introduction

The promising utility of microfluidics plays a significant role in modern science and technology. For the wide applications in bio-medical areas, physiology, and medicine, microfluidics is taking the attention of researchers and engineers, not only that to design more advanced microfluidic device; microfluidic technology has being undertaken in so many fields such as biochemical analysis instruments, micro-heat exchanger, lab-on-chip devices, micro-pumps, micro-turbines etc. [1,2]. Moreover, in current years, various technological applications, including analysis and separation of different biological and chemical samples, material processing, and micro-electro-mechanical systems (MEMS) [3–5] are performing a significant role in the relevant newly emerging research.

Recently, researchers are investigated by various theoretical models of different flow problems to analyze the flow transportation phenomenon. So many researchers and engineers do many theoretical and experimental studies on microfluidics to develop very advanced devices like biomechanical devices, lab-on-chip devices, micropump, EMHD micropump, heat exchanger, etc. There are so many experimental and theoretical observations on EMHD flow in microfluidics are done by so many researchers [6-15]. The heat transfer performance is analyzed by Chakraborty et al. [16] under the influence of the applied magnetic field. Reza and Rana [17] reported an exact analytical solution of EMHD flow control of Casson fluid through the microchannel. Buren et al. [18] studied the corrugated channel flow through a micro parallel wall with EMHD effect by applying the Perturbation technique.
Very recently, EMHD flow of non-Newtonian second-grade fluid through a porous microchannel with a corrugated wall was investigated by Rashid et al. [19] by applying the perturbation technique. A comprehensive analysis of heat transfer of nanofluid through a microtube under the potential streaming effects is done by Zhao et al. [20]. Zhao et al. [21] represented the thermal transport of a nanofluid flow through a microchannel, which is influenced by pressure gradient effect and streaming potential. An analytical investigation on the thermal characteristics of interfacial slip flow through a porous microchannel is done by Reza et al. [22]. Electroosmotic flow transport through a microchannel with patterned hydrodynamic slip boundary condition is presented theoretically by Zhao and Yang [23].

The purpose of this numerical experiment is to illustrate the combined EMHD flow and heat transfer through a porous microchannel with wavy wall under a hydrodynamic patterned slip boundary condition by assuming that the slip length varies in space. We introduce the cosine wave variations in the wavy microchannel wall and the hydrodynamic pattern slip length. The velocity distribution and temperature distribution of the flow analyzed numerically under the very low Reynolds number $Re$. The impact of various non-dimensional parameters on velocity and temperature profiles are interpreted graphically.

2. Problem Statement and explanation of the mathematical model

Consider the electromagnetohydrodynamic flow transport characteristics of an electrolyte solution through a wavy microchannel with the porous medium by taking patterned hydrodynamic slip boundary conditions. The walls of the channel are considered as wavy surface geometry (Fig. 1), which can be modeled by considering wavy walls of the form: $H(x) = a - \lambda_a \cos^2 (\pi x / L)$, in which $L$ and $\lambda_a$ denote the characteristic length and the amplitude of the microchannel respectively and the constant height of the microchannel is denoted by $a$. It also assumes that the microchannel width ($W$) and length is much greater than the channel height ($L >> W >> 2H$). The electrolyte solution generates an EDL near the boundary wall, and the induced electric field in $x$-direction creates an electroosmotic flow. Also, the applied magnetic field is induced in the $y$-direction. The electroosmotic force is generated by the used electric field component $E_x$, due to the EDL near the microchannel wall, which is directed opposite to the fluid motion. The external applied magnetic field $B_y$ acts perpendicular to the fluid flow direction, and $E_z$ is imposed as an external transverse electric field in the $z$-direction. Assumed that the flow is steady and fully developed through the microchannel, which is filled with a porous material, in which the impenetrable walls are at $y = \pm H$ with constant permeability $K$.

![Fig. 1: Physical sketch of the problem](image-url)
2.1 Governing equations and flow analysis

In this study, the governing Brinkman momentum conservation equations of the flow can be expressed as:

\[-\nabla p + \mu \nabla^2 \overline{U} - \frac{\mu}{k} \overline{U} + \frac{C_f \rho}{\sqrt{\kappa}} |\overline{U}| \overline{U} + \overline{b} = 0\]

(1)

where \( p \) be the pressure, \( C_f \) is the inertial coefficient, \( K \) is the permeability of the porous medium, \( \mu \) is the viscosity of the fluid and \( \rho \) is the density of the fluid. The net body force \( \overline{b} \) due to the combined EMHD effect is given by:

\[\overline{b} = \rho_e \overline{E} + \overline{F}\]

(2)

where \( \rho_e \) be the net charge density. The body force is contributed by the combination the Lorentz force \( \overline{F} \) due to the imposed magnetic field and the applied electric field \( \overline{E} \). Here \( \overline{F} = \overline{J} \times \overline{B} \), where \( \overline{B} \) is the applied magnetic field, and \( \overline{J} \) is the current density, which may be written as:

\[\overline{J} = \sigma_e (\overline{E} + \overline{U} \times \overline{B})\]

(3)

where \( \sigma_e \) is called electric conductivity.

Assuming a one-dimensional velocity in the x-direction for the microchannel is given by \( \overline{V} = [u(y), 0, 0] \). In this case, the simplified form of the momentum conservation equation assuming a hydrodynamically fully developed flow through the porous medium is written as:

\[\frac{\partial p}{\partial x} + \mu \frac{u}{k} + \frac{C_f \rho}{\sqrt{\kappa}} u^2 - \mu \frac{d^2u}{dy^2} - \rho_e E_x + \sigma_e B_y^2 u - \sigma_e B_y E_z = 0\]

(4)

The appropriate boundary conditions are as follows:

\[\frac{du}{dy} = 0 \quad \text{at} \quad y = 0, \quad \quad u + \beta_s(x) \frac{du}{dy} = 0 \quad \text{at} \quad y = H, \]

(5)

where \( \beta_s(x) = \beta_0(1 + a e^{ik_0x}) \) is the slip length, which is hydrodynamically described. The variation of slip length is very small i.e. \( \alpha \ll 1 \), \( k_0 \) is the wave number, and \( \beta_0 \) is called the constant slip length. The net charge density \( \rho_e \) in the EDL depends on the EDL potential \( \psi \), and it is given by

\[\rho_e = -e k^2 \psi\]

(6)

The EDL potential \( \psi \) is well constructed by the linearized Poisson-Boltzmann equation

\[\frac{d^2\psi}{dy^2} = k^2 \psi\]

(7)

The boundary conditions for the EDL potential distribution are:

\[\psi = \zeta \quad \text{at} \quad \psi = -H, \quad \frac{d\psi}{dy} = 0 \quad \text{at} \quad y = 0,\]

(8)

where \( k = e z \left( \frac{2 \pi \sigma_0}{\kappa_B T} \right)^{\frac{1}{2}} \) is the Dybye-Hückle parameter and \( 1/k \) be the thickness of the EDL. Here \( \zeta \) be the constant zeta potential. By introducing the dimensionless variables, \( \psi^* = \psi/\zeta \), \( y^* = y/\alpha \), \( \omega = k\alpha \) and finally one can obtain the non-dimensional \( \psi^* \) as:

\[\psi^* = \cosh(\omega y^*)/\cosh(\omega h)\]

(9)

Then the simplified dimensionless form of the equation (4) is

\[\frac{d^2u^*}{dy^*} - \left( \frac{H\alpha^2 + \frac{1}{\alpha}}{\frac{1}{\alpha}} \right) u^* + \frac{F}{\alpha\sqrt{\alpha}} u^2 + \omega^2 \psi^* + HaS = ReP\]

(10)

The dimensionless variables and parameters are given by:
The following dimensionless parameters are introduced to make the energy equation for thermally fully developed flow under imposed constant wall heat flux dimensionless:

\[ Da = \frac{K^\ast}{\alpha}, \quad Ha = BpH \sqrt{\frac{\alpha}{\mu}}, \quad S = \frac{\alpha a}{\mu H}, \quad Re = \frac{\rho a u_{HS}}{\mu}, \quad F = \frac{\rho a cT_{HS}}{\mu}, \quad k = kL, \]

where \( u_{HS} \) is the steady Helmholtz-Smoluchowski velocity, \( Ha \) is the Hartman number, \( Da \) be the Darcy number, respectively. \( S \) is the transverse electric field, which indicates the strength of the electric field in \( x \)-direction, \( Re \) is the Reynolds number, and \( P \) is described as the constant dimensionless pressure gradient. The dimensionless boundary conditions for the equation (13) is given by

\[ u^\ast - \beta^\ast(x^\ast) \frac{du^\ast}{dy^\ast} = 0 \text{ at } y^\ast = -h, \quad u^\ast + \beta^\ast(x^\ast) \frac{du^\ast}{dy^\ast} = 0 \text{ at } y^\ast = h \quad (11) \]

where \( \beta^\ast(x^\ast) = \beta(1 + \alpha e^{i kx^\ast}) \) and \( h(x^\ast) = 1 - \lambda \cos^2(\pi x^\ast) \), \( \kappa \) is the non-dimensional wave number and \( \beta \) is called the dimensionless constant slip length.

### 2.2. Governing equations and thermal analysis

The energy equation for thermally fully developed hydrodynamic patterned slip flow through a porous microchannel with wavy wall is expressed as

\[ \rho C_p u \frac{dT}{dx} = k_{TH} \frac{d^2 T}{dy^2} + S_j \quad (12) \]

Here \( S_j \) is the heat generation due to Joule heating. Here \( S_j = \sigma_e (E_x^2 + B_y^2 u^2 - 2E_x B_y u) \) and \( k_{TH} \) is the thermal conductivity of the fluid, \( C_p \) is the specific heat of the liquid at constant pressure, \( T \) is the local temperature of the fluid. The appropriate boundary conditions may be expressed mathematically as

\[ \frac{dT(0)}{dy} = 0, \quad T(H) + \delta \frac{dT(H)}{dy} = T_w, \quad (13) \]

where \( \delta \) is the temperature jump. Introducing the non-dimensional temperature \( T^\ast = \frac{k_{TH}(T-T_w)}{q_w a} \) where the wall temperature is \( T_w \) and \( q_w \) is the constant wall heat flux. Further, for thermally fully developed flow under imposed constant wall heat flux, one may write \( \frac{dT}{dy} = \frac{dT_w}{dx} = \frac{dT_b}{dx} = \text{constant} \) and \( \frac{d^2 T}{dx^2} = 0 \) in which \( T_b \) is the bulk mean temperature. The overall energy balance of an elementary control volume of the fluid with the length of duct \( dx \) gives the following expression:

\[ \rho C_p H u_m \frac{dT_b}{dx} = q_w + \int_0^H S_j dy \quad (14) \]

So, the gradient of bulk mean temperature may be established as

\[ \frac{dT_b}{dx} = \frac{M}{\rho C_p} = \text{constant}. \quad (15) \]

where \( M = \frac{u_w}{u_m} \left( \frac{\sigma_e E_x^2}{u_m} + \frac{\sigma_B E_y^2}{u_m} - 2E_x \sigma_e B_y \right), G = \int_0^H u^2 dy, u_m = \int_0^H u dy. \)

The following dimensionless parameters are introduced to make dimensionless the equation (12),

\[ \gamma = \left( \frac{M u_{HS}}{q_w} + 2HaSBr \right), Br = \frac{u_{HS}}{aq_w}, S_j = \frac{\sigma_e E_x^2 a}{q_w}. \]

where \( \gamma \) is denoted by the ratio of the heat generation owing to the electric and magnetic fields interaction with heat conduction, \( Br \) (Brinkman number) describes the ratio of heat which is produced by viscous dissipation and molecular conduction, \( S_j \) is non-dimensional joule heating parameter due to the conduction of heat. Then by using (15) the dimensionless form of the equation (13) is obtained as

\[ \frac{d^2 r^\ast}{dy^2} = \gamma u^\ast - Br Ha^2 u^2 - S_j \quad (16) \]

The corresponding non-dimensional boundary conditions are expressed as

\[ \frac{dT^\ast(0)}{dy^\ast} = 0, \quad T^\ast(h) + \delta \frac{dT^\ast(h)}{dy^\ast} = 0, \quad (17) \]

where \( \delta = \delta_j/\alpha \) represents the measure of temperature jump.

### 3. Numerical Methods

The momentum equation (10) and the thermal energy equation (16) have been solved numerically with the corresponding boundary conditions (11) and (17), respectively. These nonlinear ODEs are solved...
numerically using the shooting technique based on the fourth-order Runge-Kutta method in MATLAB environment.

![Figure 2](image1.png)  ![Figure 2](image2.png)

**Figure 2:** Velocity distribution profiles for several value of (a) Hartmann number when Da=0.02 (b) Darcy number when Ha=1.0 and other Parameter values $\omega = 5, P = 0.5, \beta = 0.01, \kappa = 1, x^* = 0.2096, \lambda = 0.05, \alpha = 0.01, S = 10, F = 1$

4. Results and Discussions

This study deals the numerical solutions of the velocity and temperature profiles of a patterned hydrodynamic slip flow under the influence of combined EMHD effects through a porous microchannel with wavy walls. The variation of dimensionless velocity and temperature distributions with different parameter values of the flow is discussed in this section. The interactive influences of different non-dimensional parameters on velocity and temperature are examined. The impacts of electromagnetic effect, the effect of the porous medium, the effect of Forchhemier inertia parameter, the impression of the amplitude of the wavy wall, the effect of the wave number of the slip length, and the influence temperature jump parameter are discussed on the flow velocity and temperature distribution. Further, the Joule heating effect is also examined on the temperature distribution.

![Figure 3](image3.png)  ![Figure 3](image4.png)

**Figure 3:** Velocity distribution profiles for several value of (a) inertia parameter for Parameter values $\omega = 5, P = 0.5, \beta = 0.01, \kappa = 1, x^* = 0.0628, Da = 0.02, \alpha = 0.01, S = 10, Ha = 0.5$, (b) dimensionless amplitude of wavy walls for Parameter values $\omega = 5, P = 0.5, \beta = 0.01, \kappa = 1, x^* = 0.0628, Da = 0.2, \alpha = 0.01, S = 10, Ha = 0.5$
Figure 4: Velocity distribution profiles for several value of (a) wave number (b) slip parameter for Parameter values $\omega = 5, P = 0.5, \lambda = 0.05, F = 1, x^* = 0.0628, Da = 0.2, \alpha = 0.01, S = 10, Ha = 0.5$

The non-dimensional velocity distribution profiles for different pertinent parameters are displayed in Figures 2-4. The non-dimensional velocity profiles for different values of the Hartmann number is shown in figure 2(a). In the presence of the transverse electric field (i.e. $S = 10$), it gives an increasing trend on the velocity profile with an increase of $Ha$, which is shown in the figure 2(a). Also, the velocity distribution profiles for several value of Darcy number can be depicted from figure 2(b). In the presence of a transverse electric field, when the Darcy number rises then the fluid velocity is also increasing which is represented in figure 2(b). So, it is very interesting to know that the flow velocity is always enhanced in high porous medium. The dimensionless velocity distribution for different non-dimensional inertia parameters is illustrated in figure 3(a). From this figure, it can be observed that the velocity is increased when the non-dimensional inertia parameter value is increased. Also, it can be noted that when there is no effect of inertia, i.e. $F = 0$, then the velocity of the fluid becomes slower than the other cases. The increment of the dimensionless amplitude of the wavy walls decreases the fluid velocity, which is displayed in figure 3(b). It is also found from the figure that the velocity is maximum at $\lambda = 0$. There is no appreciable change in the velocity near to the boundary with the variation of wavenumber of the pattern slip but the velocity has an increasing trend in the middle layers of the flow, which is represented in figure 4(a). The increment of slip parameter increases the velocity profile, and the flow becomes slower for no-slip boundary condition i.e. $\beta = 0$. (Fig-4(b))

Figure 5: Temperature distribution profiles for several value of (a) Hartmann number (b) Darcy number for Parameter values $\omega = 5, P = 0.5, \lambda = 0.05, F = 1, x^* = 0.2096, Br = 0.04, St = 0.6, \alpha = 0.01, S = 10, \delta = 0.01.$
Figure 6: Temperature distribution profiles for several value of (a) inertia parameter (b) wave number for Parameter values \( \omega = 5, P = 0.5, \lambda = 0.05, Ha = 0.5, x^* = 0.0628, Br = 0.04, S_J = 0.6, \alpha = 0.01, S = 10, \delta = 0.01. \)

The temperature distribution of the flow is described in figure-4-8 for different other parameters. The temperature profiles (Figure-5(a)) are increased with the variation of the applied magnetic field which is embodied by Hartmann number. From figure 5(a), it can be observed that the temperature rises due to enhancement of flow aiding component of the imposed applied electromagnetic field. Figure-5(b) exhibits the influences of the dimensionless temperature distributions with different Darcy numbers. The temperature profile has an increasing trend when the Darcy number is increasing, which is shown in figure 5(b). So, it was found that the temperature becomes high in a high porous medium. The influence of the inertia parameter is analyzed in figure 6(a). From this figure it can be noted that the temperature profile is reduced with the improvement of inertia parameter. The temperature becomes greater when \( \kappa = 0 \) i.e. when the wave number of pattern slip has vanished, which is shown in figure 6(b). The increment of the dimensionless amplitude of the wavy walls decreases the temperature of the fluid which is displayed in figure 7(a). It also be found from the figure that the temperature is maximum at \( \lambda = 0 \). Temperature profiles for different values of the temperature jump parameter is depicted in figure 7(b). The increment of Brinkman number enhanced the temperature profiles, which is displayed in figure 8(a). The joule heating effects is playing a very important role in thermal transport characteristics. Figure 8(b) illustrates the influence of the Joule heating effect on the temperature profile. It may be observed that the temperature profile is enhanced with the increment of dimensionless joule heating parameter.

Figure 7: Temperature distribution profiles for several value of (a) dimensionless amplitude of wavy walls (b) temperature jump parameter for Parameter values \( \omega = 5, P = 0.5, Ha = 0.5, x^* = 0.0628, Br = 0.04, S_J = 0.6, \alpha = 0.01, S = 10. \)
Figure 8: Temperature distribution profiles for several value of (a) inertia parameter (b) wave number for Parameter values $\omega = 5, P = 0.5, \lambda = 0.05, Ha = 0.5, Da = 0.2, x^* = 0.0628, \alpha = 0.01, S = 10, \delta = 0.01$.

5. Conclusion:

In this present study, the velocity and temperature distribution is investigated numerically for the patterned hydrodynamic slip flow through a porous microchannel with wavy walls under the combined EMHD effects. The following observations can be drawn from this numerical study:

- The flow is not a fully developed flow at low porous medium, but it is fully developed large value of Darcy number.
- The velocity and temperature both have an enhancement with the improvement of applied magnetic field in the presence of transverse electric field.
- The velocity and temperature both are proportional to the Darcy number. That means both are increasing in high porous medium.
- The velocity and temperature both are augmented for increase of the given dimensionless amplitude of the wavy walls.
- The improvement of slip length enhanced the velocity of the fluid but the dimensionless wave number reduces the velocity of the flow.

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