Non-Abelian gauge fields in circuit systems

Jiexiong Wu1,2,4, Zhu Wang1,2,4, Yuanchuan Biao1,2,4, Fucong Fei3, Shuai Zhang3, Zepeng Yin2, Yejian Hu2, Ziyin Song2, Tianyu Wu2, Fengqi Song3,5 and Rui Yu1,2

Circuits can provide a platform to study novel physics and have been used, for example, to explore various topological phases. Gauge fields—particularly, non-Abelian gauge fields—can play a pivotal role in the design and modulation of novel physical states, but their circuit implementation has so far been limited. Here we show that non-Abelian gauge fields can be synthesized in circuits created from building blocks that consist of capacitors, inductors and resistors. With these building blocks, we create circuit designs for the spin–orbit interaction and the topological Chern state, which are phenomena that represent non-Abelian gauge fields in momentum space. We also use the approach to design non-reciprocal circuits that can be used to implement the non-Abelian Aharonov–Bohm effect in real space.

Gauge fields are a key concept in modern physics, playing a role in high-energy physics, condensed-matter physics and electromagnetism. They can be categorized as Abelian and non-Abelian fields depending on whether their associated symmetry groups are commutative or not, with Abelian gauge fields generating distinct physics from non-Abelian ones. In recent years, non-Abelian gauge fields have been created in cold atoms1–5, photonic systems6–8, polaritonic systems9 and mechanical systems10. Various novel physical effects related to non-Abelian physics have also been explored, including the quantum anomalous Hall effect11, topological insulators12–14, non-Abelian monopoles15, the real-space non-Abelian Aharonov–Bohm effect16,17 and non-Abelian quantum simulation18.

A circuit that consists of lumped-parameter components is a system whose properties are determined by the parameters of the components and the topology of the electrical network. Since the wires are flexible and capable of making connections regardless of the spatial dimensions, the components can be connected with braided structures to achieve matrix-type tunnelling. As a result, circuits can be an ideal platform to study non-Abelian circuit systems. Physicists have previously been used to implement various topological phases, including topological insulators and semimetals19–28, high-order topological states29–31 and high-dimensional topological states32–35. In most of these studies, networks with capacitors and inductors are used to obtain a particular topological state protected by crystalline symmetry or time-reversal symmetry. However, the implementation of spin–orbit interaction (SOI) and the gauge field (including the non-Abelian gauge field) in such systems has so far received limited attention.

In this Article, we report a controllable approach to study the non-Abelian gauge field in circuit systems. We first provide circuit modules to implement non-Abelian tunnelling in the form of Pauli matrices. Then, using these building blocks, we create three representative physical systems: (1) the SOI, which is a time-reversal invariant; (2) the topological Chern state, which breaks time-reversal symmetry; our building blocks can be used to construct arbitrary forms of local SU(2) gauge fields; (3) we design non-reciprocal circuits and use them to achieve the real-space non-Abelian Aharonov–Bohm effect.

Building blocks of the non-Abelian gauge field

We start with the Yang–Mills Hamiltonian with non-Abelian gauge field $H_{YM} = \frac{1}{4g^2}[(p_x + A_x)^2 + (p_y + A_y)^2]$, where $m_i$ is the effective mass of the carrier and the gauge field component $A_{xi}$ denotes Hermitian matrices36. The vector potentials can take the form of $A_x = \sum_{\alpha=0}^2 \sigma_\alpha \sigma_0$ and $A_y = \sum_{\beta=0}^2 \beta \sigma_0$, where $\sigma_0$ is the identity matrix, $\sigma_{1,2,3}$ are the Pauli matrices acting on a twofold degenerate space and $\sigma_i$ and $\beta$ are coefficients. Therefore, the vector potentials are not commuted with each other. In the following, we will present a scheme to construct the twofold degenerate space and the vector potentials with the Pauli matrix type using basic electrical components. As shown in Fig. 1a, we consider three identical components (capacitors or inductors) connected head to tail to form a triangle in both cells $m$ and $n$. This configuration possesses $C_i$ rotational symmetry. Thus, the voltages at the nodes can be expanded as $v = v_0 \phi_1 + v_1 \phi_2 + v_2 \phi_3$, where $\phi_1 = (e^{i\sigma_3}, e^{i\sigma_1})$, $\phi_2 = (e^{i\sigma_1}, e^{i\sigma_3})$, $\phi_3 = (e^{i\sigma_3}, e^{i\sigma_1})$ and $\phi_0 = (e^{i\sigma_1}, e^{i\sigma_3})$. These are the basis functions of the irreducible representations of the $C_i$ group and $e = e^{i\pi/6}$ (Supplementary Table 1).

In Fig. 1b–d, we present the voltage waveforms of the basis functions on nodes 1 to 3 in cell $m$. With time-reversal symmetry, we choose $\phi_0$ and $\phi_3$, which are the complex conjugates of each other, to span the twofold degenerate pseudospin subspace. In this Article, cells $m$ and $n$ are referred to as pseudospin modules.

The designed connection modules that provide matrix-form vector potentials between the pseudospin modules are shown in Fig. 1e–h. Considering that the typical values of the parameters of capacitance, inductance and resistance are positive real numbers, we design a complete set of tunnelling modules that provide vector potentials in the form of $\pm \sigma_{0,1,2,3}$ and $\pm \sigma_{0,1,2,3}$. These modules enable the implementation of all forms of non-Abelian vector potentials for the two-band models. More information on these modules is given in Supplementary Section 1.

SOI in circuit

The vector potentials in $H_{YM}$ can take the form of $A_j = (\alpha' + \beta') \sigma_1$ and $A_j = (\alpha' - \beta') \sigma_2$. Substituting $A_{xy}$ into $H_{YM}$ and applying spin rotation $\sigma_1 \rightarrow -\sigma_2$ and $\sigma_2 \rightarrow \sigma_1$, we get the Rashba and Dresselhaus SOI Hamiltonian $H = \frac{\mu_e}{2m_e} - \alpha(p_x \sigma_2 - p_y \sigma_1) - \beta(p_x \sigma_2 + p_y \sigma_1) + \text{constant}$, where $\alpha = e^4 \mu_e / 16 \pi m_e$ and $\beta = e^4 \mu_e / 16 \pi m_e$. The Rashba SOI term $\alpha(p_x \sigma_2 - p_y \sigma_1)$ yields a real-space non-Abelian Aharonov–Bohm effect.

1Wuhan Institute of Quantum Technology, Wuhan, China. 2School of Physics and Technology, Wuhan University, Wuhan, China. 3National Laboratory of Solid State Microstructures, Collaborative Innovation Center of Advanced Microstructures and School of Physics, Nanjing University, Nanjing, China. 4These authors contributed equally: Jiexiong Wu, Zhu Wang, Yuanchuan Biao. 5E-mail: songfengqi@nju.edu.cn; yuru@whu.edu.cn
$p^2 = p_x^2 + p_y^2$ and $\sigma_z$ are the Pauli matrices acting on the pseudospin space. Here $a = a'/m_i$ and $\beta = \beta'/m_i$ are the Rashba and Dresselhaus SOI constants, respectively. For two-dimensional (2D) lattice systems, the Rashba and Dresselhaus SOI Hamiltonian can be written as

$$H_{\text{2D-SOI}}(k) = t_0 (\cos k_x + \cos k_y) - (\alpha + \beta) \sin k_x \sigma_2 + (\alpha - \beta) \sin k_y \sigma_1.$$

For one-dimensional (1D) lattice systems, we get

$$H_{\text{1D-SOI}}(k_x) = t_0 \cos k_x - (\alpha + \beta) \sin k_x \sigma_2.$$

Writing equations (1) and (2) in real space, one can obtain hopping terms proportional to $(\sigma_y \pm i \sigma_z)$ and $(\sigma_y \pm i \sigma_z)$ in the $x$ and $y$ directions, respectively.

We now present a scheme to implement 1D and 2D SOI in the circuit using the building blocks given in Fig. 1e–h. We choose modules $m_{0i}$ and $m_{\pm i r_1}$ to realize the hopping terms and use the voltage followers (operational amplifier buffers) to control the hopping directions. As the resistors used in modules $m_{\pm i r_1}$ cause energy loss and yield non-Hermitian terms, we use a subcircuit denoted as $H$ to compensate for this energy loss and bring the system back to Hermitian. Based on these considerations, we construct the SOI circuits (Fig. 2a–c). Kirchhoff’s equations of the SOI circuit are given as

$$\langle h_1(k) \oplus H_{\text{circuit}}(k) \rangle \bar{v} = \omega^{-2} \tilde{(0 \oplus I_2)} \bar{v},$$

where $\oplus$ stands for the direct sum of the constant representation space and pseudospin space of $C_3$ group and $\bar{v} = (v_1, v_2, v_3)^T$ are the node voltages in the basis of the eigenfunctions of $C_3$ group. Also, $H_{\text{circuit}}(k)$ is the Hamiltonian in the pseudospin space. For the 1D SOI circuit, $H_{\text{circuit}}(k) = \sum_{i=0,2} f_i(k) \sigma_i$, where $f_0(k) = 2LC_i(1 - \cos k_i)/3 + LC_i/3$ and $f_2(k) = -2L \sin k_i/s_0 R_i$. For the 2D SOI circuit, $H_{\text{circuit}}(k) = \sum_{i=0} f_i(k) \sigma_i$, where $f_0(k) = 2LC_i(2 - \cos k_i - \cos k_i)/3 + LC_i/3, f_1(k) = -2L \sin k_i/s_0 R_i$, and $f_2(k) = -2L \sin k_i/s_0 R_i$. Here $R_{x,y}, C_{0,i}$, and $L$ are parameters of the components. For linear circuits, the non-Hermitian terms introduced by modules $m_{\pm i r_1}$ are linear. Therefore, in the $H$ module, we can use addition, subtraction, and multiplication functions of the operational amplifier to precisely compensate for these non-Hermitian terms and guarantee the Hermitian form of the Hamiltonian $H_{\text{circuit}}(k)$ and $H_{\text{circuit}}(k)$.

To verify our theoretical designs, we fabricate a 1D SOI printed circuit board (PCB) (Fig. 2d) and measure its eigenfrequency dispersions. As shown in Fig. 2e, the frequency bands indicate the SOI
type of splitting at the $k_x=0$ and $k_x=\pi$ points, which is consistent with the theoretical calculations. Details of the experiment are presented in Methods. The details of the 1D and 2D SOI circuits, proof of the stability of the circuits, derivation of equation (3) and spin information of the eigenstates are provided in Supplementary Section 2. Our scheme presented above can be easily generalized to lattice models in three dimensions or higher spatial dimensions to study various novel physics related to SOI.
The nodes of the connection modules are connected to the nodes of the neighbouring pseudospin modules. The $M$ modules in the pseudospin modules are used to generate topological mass terms and break time-reversal symmetry. We now discuss the Chern insulator's design scheme in a circuit that breaks the time-reversal symmetry. We start with a Chern insulator Hamiltonian as

$$H_{\text{Chern}}(k) = \sum_{i=1}^{3} d_i(k) \sigma_i,$$

(4)

where $d_i(k) = t_i \cos k_x, d_{i+1}(k) = t_i \cos k_y, d_{i+2}(k) = m_i + t_i \sin k_x \sin k_y$. Also, $t_{1,2,3}$ are the hopping parameters and $m_i$ is the mass term. The Hamiltonian has a non-zero Chern number if $0 < |m_i| < 2|t_i|$. The Chern model can be explored with a square lattice described by the following tight-binding Hamiltonian:

$$H_{\text{Chern}} = \sum_{m,n} (c_{m+1,n}^{+} \hat{U}_x c_{m,n} + c_{m,n+1}^{+} \hat{U}_y c_{m,n} + c_{m,n}^{+} \hat{M} c_{m,n} + \text{h.c.}),$$

(5)

where $\hat{U}_x = t_x/2\sigma_2 - it_y/2\sigma_3$ and $\hat{U}_y = t_x/2\sigma_1 - it_y/2\sigma_3$ are non-Abelian hopping operators. Also, $\hat{M} = m_0\sigma_3$ is the mass term and the two-component operator $c_{m,n}^{+}$ creates a particle at site $(m,n)$ in the pseudospin space.

In the Chern circuit, the hopping terms are implemented with modules $m_{1,2,3}$ (Fig. 3a), and the mass term is realized by using the inverting operational amplifier (Fig. 3b). Kirchhoff’s equations of the Chern circuit are given as

$$\langle h_1(k) \oplus H_{\text{Chern}}(k) \rangle \tilde{v} = \omega^{-2} \left( 0 \oplus I_2 \right) \tilde{v},$$

(6)

where $H_{\text{Chern}}(k) = \sum_{i=1}^{3} g_i(k) \sigma_i$ with $g_1(k) = L(C_0 + 2C_1/3 + 2C_2 + 4C_3)$, $g_2(k) = -2LC_0 \cos k_y/\sqrt{3}$ and $g_3(k) = -2LC_2 \cos k_y/\sqrt{3}$. Also, $C_{1,2,3,0,1}$, $R_m$ and $L$ are parameters of the components. The mass term $-L/\sqrt{3}oR_m$ in $g_1(k)$ is contributed by the $M$ modules, which breaks the time-reversal symmetry and opens the topologically non-trivial gap. Hamiltonian $H_{\text{Chern}}$ (equation (4)) and $H_{\text{Chern}}$ (equation (6)) have the same mathematical structure except for the mass term, where the mass term in $H_{\text{Chern}}$ is related to eigenfrequency $\omega$. To identify the topological properties of the circuit system, we use Green’s function method to calculate the Chern number.
The obtained phase diagram for the Chern number as a function of $R_u$ is plotted in Fig. 3c.

We can examine the topological nature of the circuit by measuring the chiral edge states at the system’s boundary, which is related to the Chern number of the bulk system. We fabricate the Chern PCB containing 30 x 5 unit cells with periodic boundary conditions in the $x$ direction and open boundary conditions in the $y$ direction. The circuit structure in a unit cell is shown in Fig. 3d. The frequency dispersion is shown in Fig. 3e,f, where the black blocks are experimental results and the green dashed curves are theoretical results. The red and blue colours indicate the spin components in the $\pm y$ direction on the boundary state. It is clear that the frequency band structure has a bulk gap and exhibits chiral propagating edge modes that traverse the gap. The experimental data show that we can only detect the edge states located on the $y=1$ ($y=5$) boundary with the excitation source applied at the same boundary. In contrast, the edge states on the opposite boundary cannot be excited. Details about the experiment can be found in Methods. Due to the parasitic effects in electronic components, such as the internal resistance of inductors and connection wires, there is an inevitable broadening in the measured bands. This issue can be improved by choosing inductors with a lower internal resistance. The details of the Chern circuit, proof of its stability, derivation of equation (6) and calculation of Chern number can be found in Supplementary Section 3. In the Supplementary information, we also provide a scheme to realize a Chern circuit with a positive topological mass term using operational amplifiers with the integrator form.

**Non-reciprocal circuit and real-space non-Abelian Aharonov–Bohm effect**

Using the building blocks given in Fig. 1, we have implemented SOI and Chern insulators in the circuit system, which are novel phenomena.
generated by a non-Abelian gauge field in momentum space. However, an essential feature of the non-Abelian gauge field that the phase of the wave function is related to the order of the gauge fields that the wave passes through is not directly manifested in the above two phenomena. To explicitly characterize this property, we design non-reciprocal circuits and show that they tune the phase of the wave function is related to the order of the gauge fields that the wave passes through is not directly manifested in the above two phenomena. To explicitly characterize this property, we design non-reciprocal circuits and show that they tune the phase of the signals in a non-Abelian manner in real space.

We consider four circuits consisting of cells 1, 2, 3 and two connection modules for each (Fig. 4a). In circuit 31, cells 1 and 2 are connected by modules $m_{03}$ and $m_{03}$. In the spin subspace, the connection between cells 1 and 2 can be written in the form of $m_{03} = i(\omega C_{11} \sigma_0 + R_{11}^{-1} \sigma_1) = i\alpha e^{i\beta \sigma_3}$, where $\alpha = \sqrt{\beta^2 + 3}, \beta = \sqrt{\omega^2 C_{11}^2 + 3 \omega^2 C_{33}^2}$ and $\theta_3 = \arctan(-3 C_{33}/C_{11})$. Also, $C_{11}$, $R_{11}$, $C_{33}$ and $C_{13}$ are the parameters of components in the connection modules. Exchanging the order of $m_{03}$ and $m_{03}$ in circuit 31 gives circuit 13. Since $m_{03}$ and $m_{03}$ in circuit 31 and circuit 13 will yield different outputs for the same input signals. To rigorously prove this result, we consider inputting currents at cell 1 and detecting the voltages at cell 3. The transfer impedance equation for the input and output is

$$\tilde{v}_{out}^{31} = (z_3 \oplus Z_3^{13}) \tilde{v}_{in}^{31} = (z_3 \oplus z_2 e^{i(\theta_3 - \theta_3)\sigma_3}) \tilde{v}_{in}^{31}$$  (7)

for circuit 31 and

$$\tilde{v}_{out}^{13} = (z_3 \oplus Z_3^{13}) \tilde{v}_{in}^{13} = (z_3 \oplus z_2 e^{i(\theta_3 - \theta_3)\sigma_3}) \tilde{v}_{in}^{13}$$  (8)

Fig. 5 | Real-space non-Abelian Aharonov–Bohm effect. a, Schematic of the circuit for the real-space non-Abelian Aharonov–Bohm effect. Modules A and B are circuits 13 and 31 or 12 and 21, respectively, as designed in Fig. 4. The $\Sigma$ module is an operational amplifier voltage adder. b, Details of the $\Sigma$ module, where the input-output relation is given as $v_o = v_i$. c, d, Theoretical (c) and experimental (d) results of the contrast functions $\rho$ with the A and B modules are composed of circuits 13 and 31. e, f, The same data as c (e) and d (f), except for circuits 12 and 21.
In circuit 21, the connection modules are $m_{\sigma_1} + m_{\sigma_2}$ between cells 1 and 2 and $m_{\sigma_1} + m_{\sigma_2}$ between cells 2 and 3. The order of the connection modules is exchanged in circuit 12 (Fig. 4a). The transfer impedance equations are obtained as

$$Z^{21}_{\text{in}} = (z_3 + Z^2_{\text{in}}) \delta_{\text{in}} = (z_3 + z_{\text{in}}e^{i\varphi_{\text{in}}}e^{-i\rho_{\text{in}}(\beta Z) e^{i\varphi_{\text{in}}}}) e^{i\varphi_{\text{in}}} \delta_{\text{in}}$$

(9)

for circuit 21 and

$$Z^{12}_{\text{in}} = (z_3 + Z^2_{\text{in}}) \delta_{\text{in}} = (z_3 + z_{\text{in}}e^{i\varphi_{\text{in}}}e^{-i\rho_{\text{in}}(\beta Z) e^{i\varphi_{\text{in}}}}) e^{i\varphi_{\text{in}}} \delta_{\text{in}}$$

(10)

for circuit 12, where $\varphi$ and $\rho$ are functions of $\theta_1$ and $\theta_2$. The processes for the same initial state modulating by circuits 21 and 12 and then are collected by the voltage adder (Fig. 5b). Modules A and B are circuits 13 and 21, respectively. The current source provides the input currents with non-Abelian Aharonov–Bohm effect. In Fig. 5a, we design the circuit system to implement the non-Abelian Aharonov–Bohm effect in real space. The current source provides the input currents with controllable phase and amplitude. The $\Sigma$ module is an operational amplifier voltage adder (Fig. 5b). Modules A and B are circuits 13 and 31 or circuits 12 and 21, respectively. The signals $i_{\text{out}}$ flowing through modules A and B undergo phase modulation (equations (7)–(10)) and then are collected by the $\Sigma$ modules to produce the final output voltages $v_{\text{out}}$. In the pseudospin space, the entire process described above can be expressed with the following equation:

$$v_{\text{out}} = (z_3 + z_{\text{in}}) \delta_{\text{in}} = (z_3 \delta_{\text{in}} + z_{\text{in}}e^{i\varphi_{\text{in}}}e^{-i\rho_{\text{in}}(\beta Z) e^{i\varphi_{\text{in}}}}) e^{i\varphi_{\text{in}}} \delta_{\text{in}}$$

(11)

We define the intensity contrast function $\rho = |v_{\text{out}}|/|v_{\text{in}}|$ to examine the interference of the output signals, where $v_{\text{in}}$ ($v_{\text{out}}$) is the spin-up (spin-down) component of the output voltages. For modules A and B taking circuits 13 and 31, we get

$$v_{\text{in}} = (\cos \theta_{1} + \cos \theta_{2} + \cos \theta_{1} - \phi)$$

(12)

and

$$v_{\text{out}} = (\sin \theta_{1} + \sin \theta_{2} + \sin \theta_{1} - \phi) e^{-i\rho_{\text{in}}}$$

(13)

where the spin orientation in the input currents is characterized by angles $\theta$ and $\phi$ on the Bloch sphere as $i_{\text{in}} = (i_{\text{in}} \cos \eta/2, i_{\text{in}} \sin \eta/2)$. The intensity contrast $\rho$ with respect to $\eta$ and $\phi$ is shown in Fig. 5c–f, where the theoretical and experimental results are in good agreement. The expressions of $v_{\text{in}}$, in the case of chosen circuits 12 and 21 and more details about the non-Abelian Aharonov–Bohm circuit are presented in Supplementary Section 4.

Conclusions

We have reported a scheme to implement non-Abelian gauge fields in circuit systems, which we use to create the SOI, topological state and real-space non-Abelian Aharonov–Bohm effect. In the alternative Chern circuit scheme, which involves only inductors and capacitors, at least six nodes in one unit cell are required to create the complex hoppings for a two-band Chern insulator. Our scheme successfully reduces the degrees of freedom in the unit cell at the price of introducing an operational amplifier, which increases the complexity of the circuit structure to a degree. However, this allows us to use the computational functions of the operational amplifier in combination with the designed modules to generate rich physical states in the circuit. In the SOI and Chern circuits, we showed that the operational amplifiers can be used to transform a system from non-Hermitian to Hermitian and design topological time-reversal symmetry-breaking terms. Our scheme could also be used to study non-Hermitian physics as well as a wide variety of non-Abelian physics, including the non-Abelian Aharonov–Casher effect, the Hofstadter’s moth and the motion of particles in non-Abelian gauge fields.

Methods

**PCB preparation of 1D SOI circuit.** The 1D SOI circuit was implemented on a FR4 PCB (Fig. 2a). The parameters of the devices are chosen as follows. Inductor $L$ is 1.8 $\mu$H with $\pm 5\%$ tolerance and 53 $\Omega$ series resistance. Resistor $R_{\text{m}}$ modules $m_{\sigma_1} = 202 \Omega$ with $\pm 1\%$ tolerance. Capacitor $C_{\text{m}}$ in modules $m_{\sigma_2} = 2.7$ nF with $\pm 5\%$ tolerance. Capacitor $C_{\text{m}} = 1$ nF with $\pm 5\%$ tolerance. Resistor $R_{\text{m}}$ in module $\mathcal{H}$ is realized by connecting two $R_{\text{m}}$ in parallel. Resistor $R_{\text{m}}$ is 806 $\Omega$ with $\pm 1\%$ tolerance. The operational amplifiers are AD8057. The 1D SOI PCB with 20 unit cells is shown in Supplementary Fig. 3d. The voltages at each node, including the amplitude and phase, are probed by the Rohde & Schwarz vector network analyser ZN26 5 kHz–6 GHz.

**PCB preparation of Chern circuit.** The Chern circuit was implemented on a FR4 PCB (Fig. 3a). Inductor $L$ is 1.8 $\mu$H with $\pm 5\%$ tolerance and 53 $\Omega$ series resistance. Capacitor $C_{\text{m}}$ in module $m_{\sigma_1} = 2.7$ nF with $\pm 5\%$ tolerance. Capacitor $C_{\text{m}} = 1$ nF with $\pm 5\%$ tolerance. Resistor $R_{\text{m}}$ in module $\mathcal{H}$ is realized by connecting two $R_{\text{m}}$ in parallel. Resistor $R_{\text{m}}$ is 806 $\Omega$ with $\pm 1\%$ tolerance. The operational amplifiers are AD8057. The Chern PCB with 15 $\times 5$ unit cells is shown in Supplementary Fig. 3c. The voltages at each node, including the amplitude and phase, are probed by Rohde & Schwarz vector network analyser ZN26 5 kHz–6 GHz.

**PCB preparation and measurement of real-space non-Abelian Aharonov–Bohm effect.** The circuit for the real-space non-Abelian Aharonov–Bohm effect is implemented on a breadboard (Figs. 4a and 4b). Inductor $L$ is 1.8 $\mu$H with $\pm 5\%$ tolerance and 53 $\Omega$ series resistance. Capacitor $C_{\text{m}}$ in module $m_{\sigma_1} = 2.7$ nF with $\pm 5\%$ tolerance. Capacitor $C_{\text{m}} = 1$ nF with $\pm 5\%$ tolerance. Resistor $R_{\text{m}}$ and $R_{\text{m}}$ are the same, where their resistance is 1 $\Omega$ with $\pm 1\%$ tolerance. Due to the limited size of the breadboard, where two 1 $\Omega$ resistors are needed in parallel, we replaced them with a resistor of 50 $\Omega$ with $\pm 1\%$ tolerance. Where three 1 $\Omega$ resistors are needed in parallel, we replaced them with a 30 $\Omega$ resistor with $\pm 1\%$ tolerance. The operational amplifier is AD8047. The current frequency is 15 kHz, which is generated by a current source (Keithley 6221). The voltage is measured by Rigel oscilloscope MSO5354. To compare with the theoretical results, we deduct the d.c. components of the experimental data, which are $\pm 30$ mV for circuit 31, $\pm 24$ mV for circuit 13, $\pm 12$ mV for circuit 21 and $\pm 17$ mV for circuit 12. The d.c. errors arise for two reasons. One is that the vertical accuracy of the oscilloscope is on the order of 10 mV. Another reason is that in our experiment, we use a single current source to supply current at the three input ports in three separate steps. Because the circuit is linear, the total output voltage is the sum of the output voltages obtained from the three measurements described above. However, the cost of this measurement method is that a large cumulative error is obtained. If three current sources simultaneously supply the input currents and a more accurate oscilloscope is used, the d.c. components of the measurement will be reduced. Nevertheless, since the important information in the experimental
data is the amplitude and phase of the a.c. components, the presence of the d.c. components does not affect our conclusions.

Data availability
The data that support the findings of this study are available from the corresponding authors upon reasonable request.

Code availability
The codes that support the findings of this study are available from the corresponding authors upon reasonable request.

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Author contributions
R.Y. supervised the project. J.W., Z.W. and R.Y. drew the circuit diagram of the SOI PCB. Z.W. and R.Y. measured the experimental data of the SOI circuit. Z.W., Y.H. and R.Y. drew the circuit diagram of the Chern PCB. Z.W., T.W., Y.H. and Z.S. measured the experimental data of the Chern circuit. Z.W., Y.B., S.Z. and F.F. measured the experimental data of the non-Abelian Aharonov–Bohm effect. J.W. wrote part of the data acquisition program for the experimental instruments. F.S. and R.Y. analysed the data.

Competing interests
The authors declare no competing interests.

Additional information
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Correspondence and requests for materials should be addressed to Fengqi Song or Rui Yu.
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