A family of high voltage gain quasi-Δ-source impedance networks

Hamed Rezazadeh | Mohammad Monfared | Ali Nikbahar | Saeed Sharifi

Original Research Paper

Abstract

This paper proposes a class of impedance networks, called quasi-Δ-source, as an improvement to the successful Δ-source one. Compared to their origin, the three proposed networks offer higher voltage gains with a better magnetic circuit utilization and a smooth continuous input current. A lower magnetizing and total inductance required for the Δ-shaped coupled inductors of the proposed networks allows smaller and cheaper magnetic cores utilization. Also, a lower total power loss of the magnetically coupled elements is another interesting advantage of the proposed networks. Moreover, the total required capacitance of the proposed networks is the same and even slightly lower than that of the conventional Δ-source one. All these improve the performance of the proposed class of networks with the smaller size of components compared to their conventional competitor. The principles of operation are developed and the theoretical analysis is performed. Also, by using the circuit averaging technique, the small-signal models of the Δ-source and the proposed networks are derived as well as their control-to-output transfer function is achieved. Finally, the experiments on a 300 W rated power prototype of the proposed networks successfully confirm the theoretical achievements.

1 | INTRODUCTION

In recent years, many impedance networks are proposed as an attractive solution for high voltage boosting purposes using various techniques such as diode/capacitor assisted [1], switched boost networks [2–4], switched-capacitors (SCs) [5, 6], switched-inductors (SLs) [7, 8], and voltage lift [9]. The impedance source (or Z-source) networks are recently proposed for various applications such as uninterruptible power supplies (UPSs) [10, 11], adjustable speed drives [12], renewable energy systems [13–16], and electric vehicles [17, 18]. As the most recent advancement, the coupled inductors are employed within the Z-source networks, offering higher voltage gains with less number and smaller size of components. Consequently, the magnetically coupled impedance source networks, abbreviated MCIS networks, are proposed, in which the main constructions are the T/Γ-source [19], the Γ-Z-source [20], the flipped Γ-Z-source [21], the Y-source [22], and the sigma-Z-source [23]. The Y-source impedance network as an original topology can provide higher voltage gain compared to the conventional solutions with lower component count [21]. However, it suffers from a discontinuous input current and a bulky magnetic core due to a high peak magnetizing current. The aforementioned problems of the Y-source network are mitigated by the quasi-Y-source network [24], which can still keep the same voltage boosting capability as that of the original Y-source. Furthermore, some other well-developed networks are proposed in [25–27], proposing some methods of input current smoothing.

As another solution for reducing the magnetic element size and power losses, the successful Δ-source network [28] is derived from the reconfiguration of the coupled inductor of the original Y-source network. The Δ-source network offers the same voltage gain while reducing the leakage inductances, the magnetic element-related power losses, and the required core size. However, the current drawn from the input source of the Δ-source network is still discontinuous. Thus, to improve the performance of the converter for renewable and distributed power generation applications, an input filter is needed for smoothing the discontinuous current at the price of higher power loss. Also, the magnetic element is yet bulky and expensive for many applications due to a relatively high peak magnetizing current.

In order to further improve the Δ-source network, a class of three quasi-Δ-source networks is proposed in this work.
Compared to the Δ-source network, all these networks offer higher voltage gains with a continuous input current at the price of using one more inductor and one extra capacitor. Furthermore, the peak magnetizing current is considerably less than that of the conventional Δ-source network. This is mainly due to a negligible DC component of the magnetizing current and, at the same time, a lower magnetizing current ripple. In other words, the total inductance required for the coupled inductors of the proposed networks is lower than that of the Δ-source network. Thus, the aforementioned advantages lead to smaller magnetic element size and reduce its total power loss. In addition, the total required capacitance for the proposed networks is slightly lower than that of the Δ-source network, even with an additional capacitor. Besides, the circuit averaging technique is used to model the original Δ-source and the proposed networks for dynamic studies. The theoretical achievements of the proposed networks are confirmed through extensive tests on a 300 W prototype DC-DC converter. It is worth mentioning that for DC-DC conversion, there are other techniques with high boosting gain already proposed in [29–32], but these converters are only used for DC-DC applications, while the proposed networks can be employed for all conversions.

2 | PROPOSED NETWORKS

2.1 | Circuit analysis

The proposed quasi-Δ-source networks are presented in three types, as shown in Figure 1(b)–(d). These converters introduce an additional capacitor ($C_2$) and an inductor ($L_{in}$) to the conventional Δ-source converter of [28], shown in Figure 1(a). Consequently, the type I topology in Figure 1(b) is mainly inspired by the quasi-Z-source circuit already presented in [33]. Then, by reversing the dotted nodes of the windings $N_1$ and $N_2$ and changing the circuit position of capacitor $C_2$ and the diode $D_1$ of type I, type II of the proposed quasi-Δ-source, shown in Figure 1(c), is achieved. Finally, type III is obtained by connecting the cathode of $D_1$ to the capacitor $C_1$, as shown in Figure 1(d).

2.2 | Steady-state analysis

Like other impedance source converters, the proposed networks operate in two distinct states i.e. shoot-through (ST) and non-ST (NST).

Considering the volt-per-turn relation of an ideal three-winding transformer i.e. $V_1/N_1 = V_2/N_2 = V_3/N_3$, and writing KVL among Δ-shaped inductors, one can readily conclude

$$\begin{align*}
\text{Types I & Δ : } & N_1 = N_2 + N_3 \\
\text{Types II & III : } & N_2 = N_1 + N_3.
\end{align*}$$

2.2.1 | Proposed quasi-Δ-source (type I)

ST state

Figure 2(a) shows the equivalent circuit of the proposed quasi-Δ-source type I during the ST state. The switch $SW$ conducts
while the diode $D_1$ is reverse-biased, and the capacitors $C_1$ and $C_2$ deliver their stored energy into the inductors. The voltage equations of this state can be derived as

$$\begin{align*}
V_{N1} &= \frac{N_1}{N_3} V_C1 \\
V_{Lin} &= V_{in} + V_{C2}.
\end{align*}$$

\textbf{NST state}

During this state, the switch $SW$ is OFF, the diode $D_1$ conducts, the capacitors are charged from the input voltage source, and the inductors supply the load. According to Figure 2(b), the voltage equations are

$$\begin{align*}
V_{N1} &= -V_{C2} \\
V_{Lin} &= V_{in} + V_{C2} + \frac{N_2}{N_1} V_{N1} - V_{C1} \\
V_{out} &= V_{in} - V_{Lin} - V_{N1}.
\end{align*}$$

By applying the volt-second balance law on the input and magnetizing inductors, the capacitors voltage and the voltage gain are derived for type II, given in Equation (9).

$$\begin{align*}
V_{C1} &= \frac{1 - D_{ST}}{1 - (K_\Delta + 1)D_{ST}} V_{in} \\
V_{C2} &= \frac{K_\Delta D_{ST}}{1 - (K_\Delta + 1)D_{ST}} V_{in} \\
G &= \frac{V_{out}}{V_{in}} = \frac{1}{1 - (K_\Delta + 1)D_{ST}}.
\end{align*}$$

where $D_{ST}$ is the ST duty cycle and $K_\Delta = N_1/N_3$ for all $\Delta$-shaped networks.

\subsection{Proposed quasi-$\Delta$-source (type II)}

\textbf{ST state}

The equivalent circuit of the proposed network type II during ST state is shown in Figure 3(a). The voltage across the inductors during this state can be written as

$$\begin{align*}
V_{N1} &= \frac{N_1}{N_3} V_C1 \\
V_{Lin} &= V_{in} + V_{C2} + V_{N1}.
\end{align*}$$

\textbf{NST state}

Applying KVL to the equivalent circuit of the proposed network type II during NST state yields

$$\begin{align*}
V_{N1} &= -V_{C2} \\
V_{Lin} &= V_{in} + V_{C2} + \frac{N_2}{N_1} V_{N1} - V_{C1} \\
V_{out} &= V_{in} - V_{Lin}.
\end{align*}$$

By applying the volt-second balance law on the input and magnetizing inductors, the capacitors voltage and the voltage gain are derived for type II, given in Equation (9).

$$\begin{align*}
V_{C1} &= \frac{1 - D_{ST}}{1 - (K_\Delta + 1)D_{ST}} V_{in} \\
V_{C2} &= \frac{K_\Delta D_{ST}}{1 - (K_\Delta + 1)D_{ST}} V_{in} \\
G &= \frac{V_{out}}{V_{in}} = \frac{1}{1 - (K_\Delta + 1)D_{ST}}.
\end{align*}$$

\subsection{Proposed quasi-$\Delta$-source (type III)}

\textbf{ST state}

According to the ST state equivalent circuit of type III in Figure 3(c), the voltage across the winding $N_1$ and the input inductor can be written as

$$\begin{align*}
V_{N1} &= \frac{N_1}{N_3} V_C1 \\
V_{Lin} &= V_{in} + V_{C2} + V_{N1}.
\end{align*}$$
Figure 3 shows the equivalent circuit of the proposed network type III while SW is turned OFF by setting its gate pulses to zero. Accordingly, the voltage equations can be written as

\[
\begin{align*}
V_{N1} &= -\frac{N_1}{N_2} V_{C2} \\
V_{Lin} &= V_{in} - V_{C1} \\
V_{out} &= V_{C1} + V_{C2} + V_{N1}.
\end{align*}
\]

(11)

Finally, the capacitors voltage and the voltage gain can be obtained as Equation (13), by applying the volt-second balance law to the voltages of the inductors.

\[
\begin{align*}
V_{C1} &= \frac{1 - D_{ST}}{1 - (K_\Delta + 2)D_{ST}} V_{in} \\
V_{C2} &= \frac{(K_\Delta + 1)D_{ST}}{1 - (K_\Delta + 2)D_{ST}} V_{in} \\
G &= \frac{V_{out}}{V_{in}} = \frac{1}{1 - (K_\Delta + 2)D_{ST}}.
\end{align*}
\]

(13)

To better demonstrate the voltage boosting ability of the proposed networks along with the original one, their voltage gains are plotted versus \(D_{ST}\) in Figure 4 for various \(K_\Delta\) where the offering voltage gain of the conventional \(\Delta\)-source network can be obtained from Equation (14) [28].

\[
G = \frac{V_{out}}{V_{in}} = \frac{1}{1 - K_\Delta D_{ST}}.
\]

(14)

As a general conclusion, the proposed network type III offers the highest voltage gain among all topologies, while the voltage gain of the conventional \(\Delta\)-source network is the lowest. Consequently, if the proposed networks are implemented in the DC-AC application, the inverter can operate with a higher modulation index \(M\) due to the lower \(D_{ST}\) requirement in comparison to the conventional \(\Delta\)-source network. It means that the distortion in the output waveforms originated from low modulation index operation is considerably decreased. However, the impact of the impedance source network structure on the output waveforms quality is thoroughly investigated in [34, 35].

**3 | COMPONENT DESIGN**

The main circuit parameters to be designed are the input inductor \(L_{in}\), the magnetizing inductor \(L_{m}\), and the capacitors \(C_1\) and \(C_2\).
3.1 | Input inductance ($L_{in}$)

The input inductance of the proposed networks can be calculated based on its current ripple as

$$L_{in} = V_{in} \frac{\Delta i}{\Delta t_{in}}$$  \hspace{1cm} (15)

where $\Delta i_{in}$ is the ripple of the input current. Considering $\alpha\%$ as the maximum tolerable input current ripple, the input inductance of the proposed networks can be designed as

$$L_{in} = \frac{(G_i - 1)(1 + G_i K_D)}{G_i (1 + K_D)^2} \times L_{Base}$$

$$L_{in} = \frac{(G_{III} - 1)(1 + G_{III} K_D)}{G_{III} (2 + K_D)^2} \times L_{Base}$$

$$L_{in} = \frac{V_{in}^2 T_s}{\alpha \% P}$$  \hspace{1cm} (16)

where $T_s$ is the switching period and $P$ is the rated power.

3.2 | Magnetizing inductance ($L_{m}$)

Similar to the input inductance, the magnetizing inductance can be designed from the maximum allowed current ripple ($\Delta i_{m}$). Assuming $\beta\%$ as the magnetizing current ripple, one can write

$$L_{m} = \frac{K_D (G_i - 1)(1 + G_i K_D)}{G_i (1 + K_D)^2} \times L_{Base}$$

$$L_{m} = \frac{K_D (G_{III} - 1)(1 + G_{III} K_D)}{G_{III} (2 + K_D)^2} \times L_{Base}$$

$$L_{m} = \frac{V_{m}^2 T_s}{\beta \% P}$$  \hspace{1cm} (17)

3.3 | Capacitances

For proper selection of the capacitors, their maximum voltage ripple is a matter of concern along with the current flowing through them ($i_C$) and its time duration. Thus

$$C = \frac{i_C}{\Delta V_C} \Delta t$$  \hspace{1cm} (18)

where $\Delta V_C$ is the voltage ripple of the capacitors. Therefore, considering $\gamma\%$ as the maximum tolerable voltage ripple of the capacitors, the capacitances can be calculated as

$$C_1 = \frac{K_D (G_i - 1)}{G_i (1 + G_i K_D)} \times C_{Base}$$

$$C_2 = \frac{(G_{III} - 1)(1 + K_D)}{G_{III} (1 + G_{III} + G_{III} K_D)} \times C_{Base}$$

$$C_3 = \frac{1}{G_{III} (1 + K_D)} \times C_{Base}$$

$$C_{Base} = \frac{P T_s}{\gamma \% V_{in}^2}$$  \hspace{1cm} (19)

4 | NETWORKS COMPARISON

In this section, the average and peak magnetizing currents, the core size, the total required inductance and capacitance, the copper losses, and the input current ripple are investigated and compared with the conventional $\Delta$-source network of [28]. Also, for better comparison between proposed networks and some of the successful competitors, the parameters of the Y-source and quasi-Y-source, as origins of the MCIS networks, and the A-source, are compared with the proposed networks in Table 1.

4.1 | Magnetizing current

For a transformer with three windings, one can write

$$N_1 i_1 + N_2 i_2 + N_3 i_3 = N_1 i_m$$  \hspace{1cm} (20)

where $i_1$, $i_2$, and $i_3$ are the windings' current. According to the equivalent circuit of type I in Figure 5, it can be readily shown that the average of the capacitors' current over the ST state is

$$I_{C1} = \frac{-N_1 i_{mST}}{N_3}$$

$$I_{C2} = -I_{mST}$$  \hspace{1cm} (21)

where $I_{mST}$ and $I_{CST}$ are the averages of the input and magnetizing currents (referred to $N_1$) over the ST state, respectively.

For the NST state, the currents are,

$$I_{C1}^{NST} = I_{in}^{NST} - I_{m}^{NST}$$

$$I_{C2}^{NST} = I_{in}^{NST} - \frac{N_2}{N_1} I_{m}^{NST} - \frac{N_3}{N_1} I_{mST}$$  \hspace{1cm} (22)

where $I_{in}^{NST}$, $I_{C1}^{NST}$, and $I_{m}^{NST}$ are the average of the input, output, and magnetizing currents over the NST state, respectively.
TABLE 1 Comparison of various magnetically coupled impedance source networks with proposed one

| Converter        | \( K = \frac{1}{1 - K_D H} \) | \( V_{CI} / V_{in} \) | \( V_{C2} / V_{in} \) | \( V_{DI} / V_{in} \) | \( L_{m} / \omega \) | \( \Delta i_m / \Delta_i^{m \text{ref}} \) | \( \frac{I_{ST}^T}{I_{m}} / I_{m} \) |
|------------------|-------------------------------|-----------------------|-----------------------|-----------------------|-----------------|--------------------------|-----------------------|
| Proposed (Type I)| \( K_I = 1 + \frac{N_2}{N_1} \) | \((1 - D_{ST})G\)   | \((K_I - 1)D_{ST}G\) | \((K_I - 1)G\)   | 1               | \((K_I - 1)(1 - D_{ST})D_{ST}G\) | \(\frac{1}{D_{ST}}(1 - \frac{1}{G(1 - D_{ST})})\) |
| Proposed (Type II)| \( K_{II} = 1 + \frac{N_2}{N_3} \) | \((1 - D_{ST})G\)   | \((K_{II} - 1)D_{ST}G\) | \((K_{II} - 1)G\)   | 0               | \((K_{II} - 1)(1 - D_{ST})D_{ST}G\) | \(\frac{1}{D_{ST}}(1 - \frac{1}{G(1 - D_{ST})})\) |
| Proposed (Type III)| \( K_{III} = 2 + \frac{N_2}{N_3} \) | \((1 - D_{ST})G\)   | \((K_{III} - 1)D_{ST}G\) | \((K_{III} - 1)G\)   | \(\frac{1}{K_{II} - 1}\) | \((K_{III} - 1)(1 - D_{ST})D_{ST}G\) | \(\frac{1}{D_{ST}}(1 - \frac{1}{G(1 - D_{ST})})\) |
| \(\Delta\)-source \([28]\) | \(\Delta K = \frac{N_2}{N_3} \) | \((1 - D_{ST})G\)   | \((\Delta K - 1)G\)   | 1               | \(\Delta K(1 - D_{ST})D_{ST}G\) | \(\frac{1}{D_{ST}}(1 - \frac{1}{G(1 - D_{ST})})\) |
| \(Y\)-source \([22]\) | \(K_Y = \frac{N_1 + N_3}{N_2 - N_3} \) | \((1 - D_{ST})G\)   | \((\Delta K - 1)G\)   | \(1 + \frac{N_1}{N_2 - N_3}(1 - D_{ST})D_{ST}G\) | \(\frac{1}{D_{ST}}(1 - \frac{1}{G(1 - D_{ST})})\) |
| \(\Delta\)-source \([24]\) | \(K_{III} = \frac{N_1 + N_3}{N_2 - N_3} \) | \((1 - D_{ST})G\)   | \((K_{III} - 1)D_{ST}G\) | \((K_{III} - 1)G\)   | 0               | \(\frac{N_1}{N_2 - N_3}(1 - D_{ST})D_{ST}G\) | \(\frac{1}{D_{ST}}(1 - \frac{1}{G(1 - D_{ST})})\) |
| \(\Delta\)-source \([27]\) | \(K_A = 2 + \frac{N_2}{N_3} \) | \((1 - D_{ST})G\)   | \((K_A - 1)D_{ST}G\) | \((K_A - 1)G\)   | \(\frac{1}{K_{II} - 1}\) | \((K_A - 1)(1 - D_{ST})D_{ST}G\) | \(\frac{1}{D_{ST}}(1 - \frac{1}{G(1 - D_{ST})})\) |

* Magnetizing currents are referred to high turn number windings, \(\Delta_i^{m \text{ref}} = V_{in}T_fI_{m} \).

** \(I_{ST}^T\): Shoot-through current.

\[ D_{ST} I_{ST}^T = \frac{N_1}{N_1}(1 - D_{ST})(I_{in}^{NST} - I_{ST}^{NST}) \] \(\text{(23)}\)

\[ (1 - D_{ST})I_{m}^{NST} = d_{ST} I_{in}^{ST} + \frac{N_2}{N_1}(1 - D_{ST})I_{ST}^{NST} + \frac{N_3}{N_1}(1 - D_{ST})I_{ST}^{NST} \] \(\text{(24)}\)

From Equations (23) and (24), the average magnetizing current \(I_{m}^o\) is derived as

\[ I_{m}^o = D_{ST} I_{ST}^T + (1 - D_{ST})I_{ST}^{NST} = I_{in} \] \(\text{(25)}\)

where \(I_{in}\) is the average input current.

Following a similar approach, the average magnetizing current (referred to \(N_1\)) of the types II and III networks are calculated as

\[ I_{m}^o = D_{ST} I_{ST}^T + (1 - D_{ST})I_{ST}^{NST} = 0 \] \(\text{(26)}\)

According to Equation (25), the average magnetizing current of the proposed network type I is the same as that of the conventional \(\Delta\)-source. Interestingly, the average DC component of \(I_{in}\) of type II network, which is already expected to be negligible, is zero from Equation (26). Besides, the average magnetizing current of type III is less than the average input current, because \(K_A\) is decided to be more than unity to attain the required voltage gain.

As given in Equation (27), for calculating the magnetizing current ripple \((\Delta_i)\), the voltage across the winding \(N_1\) and the dwell time of its corresponding state of operation are required.

\[ \Delta_i = \frac{\Delta t \times V_{in}N_1}{L_{in}}. \] \(\text{(27)}\)

Accordingly, for the ST state of all types of the proposed networks and the conventional \(\Delta\)-source, one can write

\[ \Delta_i = K_{\Delta}(1 - D_{ST})G V_{in}D_{ST}T_i \] \(\text{(28)}\)

Assuming the same \(K_{\Delta}\), for achieving the same voltage gain, all proposed networks require lower duty cycles compared to the conventional \(\Delta\)-source network. Thus, according to Equation (28), the magnetizing current ripple of all proposed networks is lower than the conventional \(\Delta\)-source, assuming the same \(V_{in}\), \(L_{m}\), and \(T_i\) that directly leads to lower core losses \([36]\).
4.2 Core size

According to [37], the maximum core stored energy mainly determines the core size. This energy relates to the product of the square of the peak magnetizing current and the magnetizing inductance as

$$W_{\text{max}} \propto L_m I_{\text{m, max}}^2, \quad (29)$$

Assuming the same and constant peak magnetizing current for all the circuits under study, Equation (29) can be simplified to

$$W_{\text{max}} \propto L_m \quad (30)$$

where

$$\begin{aligned}
L_{\Delta} & = \frac{(G - 1)(1 + G K_\Delta - G)}{G K_\Delta} \frac{V_{\text{in}} T_i}{2(I_{\text{m, max}} - P/V_{\text{in}})} \\
L_I & = \frac{K_\Delta (G - 1)(1 + G K_\Delta)}{G(1 + K_\Delta)^2} \frac{V_{\text{in}} T_i}{2I_{\text{m, max}} - P/V_{\text{in}}} \\
L_{II} & = \frac{K_\Delta (G - 1)(1 + G K_\Delta)}{G(1 + K_\Delta)^2} \frac{V_{\text{in}} T_i}{2I_{\text{m, max}}} \\
L_{III} & = \frac{K_\Delta^2 (G - 1)(1 + G K_\Delta + G)}{G(2 + K_\Delta)^2} \frac{V_{\text{in}} T_i}{2(K_\Delta I_{\text{m, max}} - P/V_{\text{in}})}. \\
\end{aligned} \quad (31)$$

For the core size comparison among the proposed and the conventional \(\Delta\)-source networks, the magnetizing inductances of Equation (31) are plotted in Figure 6(a). This figure is plotted assuming the same peak magnetizing current for all networks, as \(I_{\text{m, max}} = 8\) A, which can be derived from experimental conditions for the conventional \(\Delta\)-source network. As seen from this figure, the required magnetizing inductance of all the proposed networks is lower than that of the conventional \(\Delta\)-source one. In addition, among the proposed networks, type II offers the lowest magnetizing inductance, while type I has the highest required value for \(L_m\). Thus, the proposed networks interestingly need much smaller core sizes compared to their conventional competitor.

4.3 Total inductance

As an important practical requirement, the required total inductance of the \(\Delta\)-shaped inductors is calculated for all networks as follows

$$\begin{aligned}
L_{\text{total}}^\Delta & = L_{\text{m}} \left( 1 + \left( \frac{K_\Delta - 1}{K_\Delta} \right)^2 + \left( \frac{1}{K_\Delta} \right)^2 \right) \\
L_{\text{total}}^I & = L_{\text{m}} \left( 1 + \left( \frac{K_\Delta + 1}{K_\Delta} \right)^2 + \left( \frac{1}{K_\Delta} \right)^2 \right). \quad (32)
\end{aligned}$$

Figure 6(b) depicts the required total inductances given in Equation (32) versus voltage gain, which are calculated by substituting \(L_{\text{m}}\) from Equation (31) into Equation (32). As it is obvious from this figure, the required total coupled inductances of all proposed networks are less than that of the conventional \(\Delta\)-source, which significantly reduces the copper loss of the coupled inductor windings. Also, as done in [28], Table 2 shows the results of the sum of \(N_j i_j^2 r_{\text{rms}}\) as the copper loss indicator and the sum of \(N_j i_{j, \text{rms}}^2\) as the indicator of the windings’ volume or weight. As seen from this table, the windings’ copper loss of types I–III of the proposed networks is lower than the conventional \(\Delta\)-source network by 19.3%, 21.7%, and 31.5%, respectively. Also, for types I–III, the windings’ volume or weight is lower than the original competitor by 9.3%, 18.2%, and 25%, respectively.

Finally, one can conclude that the required \(\Delta\)-shaped elements of the proposed networks are comparably smaller, cheaper, and with lower power losses than that of the conventional one.
TABLE 2  Comparison of copper loss and windings volume (weight) indicators

| Converter          | $i_{1,\text{rms}} (N_1)$ | $i_{2,\text{rms}} (N_2)$ | $i_{3,\text{rms}} (N_3)$ | $\sum N_j i_{j,\text{rms}}^2$ | $\sum N_j i_{j,\text{rms}}$ |
|--------------------|---------------------------|---------------------------|---------------------------|-------------------------------|------------------------------|
| Conventional Δ-source | 3.26 (120)               | 5.21 (90)                 | 7.65 (30)                 | 5473                          | 1090                         |
| Proposed (type I)  | 3.10 (120)               | 4.63 (90)                 | 6.67 (30)                 | 4417                          | 989                          |
| Proposed (type II) | 4.84 (90)                | 1.89 (120)                | 7.64 (30)                 | 4288                          | 892                          |
| Proposed (type III)| 4.14 (90)                | 1.74 (120)                | 7.84 (30)                 | 3750                          | 817                          |

4.4  | Total capacitance

Figure 6(c) shows the normalized total capacitance required for the conventional Δ-source and the proposed networks assuming the same voltage gains for all. As can be seen from Figure 6(c), the total required capacitance for the type III of the proposed networks is the lowest for $G > 3.8$. However, as a general conclusion from this figure, the proposed networks require the same and even slightly lower total capacitance than the conventional Δ-source network.

4.5  | Input current ripple

The continuous input current is one of the advantages of the proposed networks compared to the conventional Δ-source one, which makes them popular for renewable energy applications. Besides, the input current ripple is an important characteristic of these networks, which should be investigated. The current ripple of the inductor is related to its voltage during charging or discharging and the dwell time as Equation (33)

$$\Delta i_{\text{in}} = \frac{\Delta t \times V_{\text{Lin}}}{L_{\text{in}}}.$$  (33)

By substituting the input inductors’ voltage of the proposed networks from the steady-state analysis, the input current ripple of these networks can be obtained as

\[
\begin{align*}
\Delta i_{\text{I}}^{\text{in}} &= (1 - D_{\text{ST}}) D_{\text{ST}} G \times \Delta i_{\text{Base}}^{\text{in}} \\
\Delta i_{\text{II}}^{\text{in}} &= (1 + K_{\Delta}) (1 - D_{\text{ST}}) D_{\text{ST}} G \times \Delta i_{\text{Base}}^{\text{in}} \\
\Delta i_{\text{III}}^{\text{in}} &= (1 + K_{\Delta}) (1 - D_{\text{ST}}) D_{\text{ST}} G \times \Delta i_{\text{Base}}^{\text{in}} \\
\Delta i_{\text{Base}}^{\text{in}} &= \frac{V_{\text{Lin}} T_s}{L_{\text{in}}}.
\end{align*}
\]  (34)

The normalized input current ripple of the proposed networks versus voltage gain is plotted for various $K_{\Delta}$ in Figure 7. The input current ripple of the proposed network type I is considerably lower than the other two proposed networks. Also, the proposed network type III offers a lower input current ripple than that of the proposed network type II. Moreover, unlike types II and III, the input current ripple of type I is reduced by an increase of $K_{\Delta}$.

5  | SMALL-SIGNAL MODEL

Since there are various nonlinear elements in the power electronic converters, the small-signal modeling is used to approximate their behavior with linear equations to design a reliable closed-loop controller. For the sake of simplicity, the small-signal model of Δ-shaped networks is derived for the DC-DC application, shown in Figure 8.

5.1  | Δ-Source network

The circuit averaging technique involves averaging currents and voltages of the nonlinear switching devices over a switching period. For the Δ-source network, the average of the diodes’
voltage and the switch current can be written as

\[
\begin{align*}
V_{D1} &= \frac{(K_d - 1) D_{ST}}{1 - K_d D_{ST}} V_{in} = \alpha_1 V_{in} \\
I_{SW} &= K_d D_{ST} I_{in} = \alpha_2 I_{in} \\
V_{D2} &= \frac{D_{ST}}{1 - K_d D_{ST}} V_{in} = \alpha_3 V_{in}.
\end{align*}
\]

By considering the values with a \( \sim \) as small perturbations around the steady-state operation point, one can write

\[
V_{D1} + \tilde{V}_{D1} = \frac{(K_d - 1) (D_{ST} + \tilde{D}_{ST})}{1 - K_d (D_{ST} + \tilde{D}_{ST})} (V_{in} + \tilde{V}_{in}).
\]

The large-signal diode voltage expression in Equation (36) can be simplified to

\[
V_{D1} + \tilde{V}_{D1} = \frac{(K_d - 1) D_{ST}}{1 - K_d D_{ST}} V_{in} + \frac{(K_d - 1) D_{ST}}{1 - K_d D_{ST}} \tilde{V}_{in}
+ \frac{(K_d - 1) V_{in}}{(1 - K_d D_{ST})^2} \tilde{D}_{ST} = \alpha_1 V_{in} + \alpha_1 \tilde{V}_{in} + \beta_1 \tilde{D}_{ST}.
\]

Similarly, the large-signal expressions of the switch current and output diode voltage are

\[
\begin{align*}
I_{SW} + \tilde{I}_{SW} &= K_d D_{ST} I_{in} + K_d D_{ST} \tilde{I}_{in} \\
+ K_d I_{in} \tilde{D}_{ST} &= \alpha_2 I_{in} + \alpha_2 \tilde{I}_{in} + \beta_2 \tilde{D}_{ST} \\
V_{D2} + \tilde{V}_{D2} &= \frac{D_{ST}}{1 - K_d D_{ST}} V_{in} + \frac{D_{ST}}{1 - K_d D_{ST}} \tilde{V}_{in}
+ \frac{V_{in}}{(1 - K_d D_{ST})^2} \tilde{D}_{ST} = \alpha_3 V_{in} + \alpha_3 \tilde{V}_{in} + \beta_3 \tilde{D}_{ST}.
\end{align*}
\]

Using the circuit averaging technique, according to Equations (37)–(39), the switch and diodes are replaced by the controlled current and voltage sources, respectively. Figure 9(a) shows the large-signal model of the \( \Delta \)-source network and the derived dc and ac small-signal models can be seen in Figure 9(b) and (c), respectively.

### 5.2 Proposed quasi-\( \Delta \)-source networks

Following a similar approach, the large-signal switch current and diodes’ voltage for the proposed networks can generally be expressed as

\[
\begin{align*}
V_{D1} + \tilde{V}_{D1} &= \alpha_1 V_{in} + \alpha_1 \tilde{V}_{in} + \beta_1 \tilde{D}_{ST} \\
I_{SW} + \tilde{I}_{SW} &= \alpha_2 I_{in} + \alpha_2 \tilde{I}_{in} + \beta_2 \tilde{D}_{ST} \\
V_{D2} + \tilde{V}_{D2} &= \alpha_3 V_{in} + \alpha_3 \tilde{V}_{in} + \beta_3 \tilde{D}_{ST}
\end{align*}
\]

where the coefficients \( \alpha_i \) and \( \beta_i \) (\( i = 1, 2, 3 \)) are calculated and summarized in Table 3. Figure 10(a)–(c) shows the ac small-signal model of types I–III of the proposed networks, respectively. The impedances of the inductors, capacitors, and load by considering their equivalent series resistances (ESRs) can be lumped and expressed as

\[
\begin{align*}
Z_{Lin} &= s L_{in} + R_{Lin}, \quad Z_{C1} = \frac{1}{s C_1} + R_{C1}, \\
Z_{C2} &= \frac{1}{s C_2} + R_{C2}, \quad Z_{e} = \left( \frac{1}{s C_o} + R_{Co} \right) | \frac{R_o}{Z_m} = s L_{out}, \\
Z_{K1} &= s L_{K1} + R_{1}, \quad Z_{K2} = s L_{K2} + R_{2}, \quad Z_{K3} = s L_{K3} + R_{3}
\end{align*}
\]

where \( R_{Lin}, R_{C1}, R_{C2}, \) and \( R_{Co} \) are the ESRs of the input inductor, capacitors \( C_1, C_2, \) and \( C_o, \) respectively. Also, \( L_{K1}, L_{K2}, \) and \( L_{K3} \) are the leakage inductances and \( R_{1}, R_{2}, \) and \( R_{3} \) are the ESRs.
of the windings $N_1$, $N_2$, and $N_3$ of the coupled inductor, respectively.

Using the small-signal models of the proposed networks shown in Figure 10, any required transfer functions of the proposed networks for design and control purposes can be obtained. For instance, the control-to-output transfer function of the proposed networks can be derived by ignoring perturbation $\tilde{v}_{in}$ ($\tilde{v}_{in} = 0$) in the small-signal models of Figure 10. For the proposed network type I, one can write the following KVL and KCLs in S-domain from Figure 10(a).

\[
\begin{align*}
KVL_1 : Z_{L_{in}} \tilde{i}_{in} - Z_{C_2} \tilde{i}_{C_2} - \beta_3 \tilde{d}_{ST} + \tilde{v}_{out} & = 0 \\
KCL_1 : \tilde{i}_{C_1} = (1 - \alpha_2) \tilde{i}_{in} - \beta_2 \tilde{d}_{ST} - \frac{\tilde{v}_{out}}{Z_o} \\
KCL_2 : \tilde{i}_1 + \tilde{i}_2 = \tilde{i}_{in} + \tilde{v}_{C_2} \\
KCL_3 : \tilde{i}_1 + \tilde{i}_3 = \tilde{v}_{C_2} + \alpha_2 \tilde{i}_{in} + \beta_2 \tilde{d}_{ST} + \frac{\tilde{v}_{out}}{Z_o}.
\end{align*}
\]

Also, by applying KVL to the loop of the $\Delta$-shaped coupled inductor of Figure 10(a) and considering $N_1 = N_2 + N_3$, one can conclude that

\[
Z_{K_1} \tilde{i}_1 - Z_{K_2} \tilde{i}_2 - Z_{K_3} \tilde{i}_3 = 0. \tag{43}
\]

From Equation (20), the magnetizing current (referred to $N_1$) can be calculated as

\[
\tilde{i}_m = \tilde{i}_1 + \frac{N_2}{N_1} \tilde{i}_2 + \frac{N_3}{N_1} \tilde{i}_3. \tag{44}
\]

Applying KVL to the loop consisting of the input inductor, diodes’ branch, coupled inductor winding $N_2$, and capacitor $C_1$ yields

\[
Z_{L_{in}} \tilde{i}_{in} - \beta_1 \tilde{d}_{ST} + \frac{N_2}{N_1} Z_{m} \tilde{i}_m + Z_{K_2} \tilde{i}_2 - Z_{C_1} \tilde{i}_{C_1} = 0. \tag{45}
\]

By applying KVL to the loop including input inductor, diode branch, coupled inductor winding $N_1$, and output load in Figure 10(a), one can obtain

\[
Z_{L_{in}} \tilde{i}_{in} - \beta_1 \tilde{d}_{ST} + Z_{m} \tilde{i}_m + Z_{K_1} \tilde{i}_1 - \beta_3 \tilde{d}_{ST} + \tilde{v}_{out} = 0. \tag{46}
\]

Finally, by solving Equations (42)–(45) for $\tilde{i}_1$, $\tilde{i}_2$, $\tilde{i}_3$, $\tilde{i}_m$, and $\tilde{i}_{in}$ and substituting them in Equation (46), the control-to-output transfer function of the proposed network type I can be calculated. This transfer function is numerically calculated in Equation (47) with the parameters of experimental tests (refer to Table 4).

The control-to-output transfer function of the proposed network types II and III are derived by a similar procedure and the results are also summarized in Equation (47).

The Bode plots of the transfer functions calculated in Equation (47) are shown in Figure 11. As can be seen, the proposed networks are high-order complex systems with almost the same magnitude and phase in a wide range of frequencies. Therefore, one may conclude that they have very similar transient behaviors. Besides, the controller and the circuit parameters must be carefully designed according to the special requirements of the application that the proposed networks are employed for.

## 6 EXPERIMENTAL RESULTS

The performance of the proposed networks is confirmed through extensive tests on a 300 W DC-DC converter test rig, shown in Figure 12, with the parameters in Table 4. According to this table and Equation (1), for the conventional $\Delta$-
TABLE 4  Experimental Parameters

| Parameter                  | Value                |
|----------------------------|----------------------|
| Output power               | 300 W                |
| Output voltage             | 200 V                |
| Input voltage              | 50 V                 |
| Switching frequency        | 25 kHz               |
| $D_{ST}$                   | 0.1875 ($\Delta$ & type II) 0.15 (types I & III) |
| Magnetic core              | 0077615A7            |
| Winding turns              | 120:90:30            |
| $L_{K1}, R_1$ (120 turns)  | 5 µH, 0.24 Ω         |
| $L_{K2}, R_2$ (90 turns)   | 1.67 µH, 0.19 Ω      |
| $L_{K3}, R_3$ (30 turns)   | 2.47 µH, 0.07 Ω      |
| Magnetizing inductor, $I_m$| 1.2 mH ($\Delta$ & type I) 675 µH (types II & III) |
| Capacitors $C_1C_2Co$      | 48 µF (1 mΩ)16 µF (5 mΩ)16 µF (5 mΩ) |
| Input inductor, $I_{in}$   | 1.5 mH (60 mΩ)       |
| Switch $SW$                | IPW60R037CSFD        |
| Diode $D_1$                | RHRG75120            |
| Diode $D_2$                | APT30D60B            |

From Table 1, for the desired voltage gain of $G = 4$, the ST duty cycle is derived as $D_{ST} = 0.1875$ for the

conventional $\Delta$-source and type II of the proposed networks, while it is calculated as $D_{ST} = 0.15$ for both types I and III. Figures 13–16 are the experimental waveforms of the conventional $\Delta$-source and the types I–III, respectively. The input/output voltages and currents are presented in Figures 13–16(a). As it is observed from these figures, all networks have an output voltage of approximately 180 V instead of 200 V. This is because the voltage gains are designed according to the steady-state analysis under ideal lossless components, while the practical voltage gain is seriously affected by the parasitic resistances in passive
components and the non-ideal performance of the semiconductors, which is investigated in [38]. Moreover, unlike the conventional Δ-source network, a continuous current waveform is drawn from the source by the proposed networks. Besides, as already expected from the comparison section and Figure 7, the input current ripple of type I of the proposed networks is considerably less than that of the other ones. The continuous input current of the proposed networks and their high voltage gains are more suitable for the renewable and distributed power generation. Also, Figures 13–16 (b) show the measured capacitors and output voltages of all networks under study. A low voltage ripple across the capacitors is recognized and measured as almost less than 3%, which was already expected from the capacitors’ design equations of Section 3. The gate signal of the switch SW and the blocking voltages of the switching devices are presented in Figures 13–16 (c). As can be seen, the peak voltage stresses across the switching devices are almost the same as those already derived from the theoretical equations given in Table 1. As already expected, SW turns ON in ST state while the diodes D1 and D2 block. On the contrary, these diodes conduct during the NST state, and SW blocks the output voltage. Finally, it is worth mentioning that in all circuits, the diode D1 experiences voltage ringings once it starts blocking due to the presence of the parasitic switch capacitance and the inductances of the windings. This problem and the relevant solution are already well investigated in [28].

Figures 13–16(d) depict the windings and the magnetizing currents. As already expected from the theoretical analysis, the peak magnetizing currents of the proposed networks are lower than that of their origin.

In order to investigate the transient behavior of the proposed impedance networks in response to the input voltage
step-change, the open-loop transient responses of the input and the output voltages and currents are measured and reported in Figure 17. As can be seen, the input voltage is increased from 50 to 60 V (20%). It is evident from the comparison of the time divisions of Figure 17 that the proposed network type II can successfully recover both voltage and current waveforms in response to the step-change in about 5 ms, while this time for the two other networks is about 10 ms.

Figure 18 shows the measured efficiencies of all converters at different output powers. The output voltage is fixed to 200 V by adjusting the ST duty cycle for each circuit, while the input voltage is 50 V. At the rated power of 300 W, the efficiencies are 90.61%, 90.38%, 90.76%, and 90.90% for the conventional $\Delta$-source, types I–III, respectively. As seen from this figure, the efficiencies of the proposed networks (types II and III) are almost the same and slightly higher than the conventional $\Delta$-source network. It reveals that while the added inductor and capacitor of the proposed networks contribute to the losses compared to the conventional circuit, the total losses are lower, especially because of the lower losses in the coupled inductors of the proposed networks that are shown in Figure 19. This figure depicts the distribution of the analytically calculated losses based on the method in [39] among different components of the $\Delta$-shaped networks at the nominal power (300 W).

7 | CONCLUSION

This paper proposes a class of improved networks inspired by the conventional successful $\Delta$-source network. Three proposed
networks, called type I–III quasi-Δ-source networks, provide higher voltage gains with remarkably smaller Δ-shaped magnetic element core size and required total inductance. A lower power loss of the coupled inductors and, at the same time, maintained total required capacitance are the other benefits of the proposed class of impedance networks over their origin. These features of the proposed networks allow utilizing them for many applications such as electric and hybrid electric vehicles (EVs and HEVs) and adjustable speed drives. Moreover, their continuous input current makes them a good choice for renewable energy source applications. The steady-state analysis, the small-signal modeling, and the comparison among some of the other successful MCIS networks are also presented. Finally, the superiority of the proposed class of quasi-Δ-source networks is confirmed through experimental tests on a laboratory prototype.

REFERENCES

1. Gajanayake, C.J., et al.: Extended-boost Z-source inverters. IEEE Trans. Power Electron. 25(10), 2642–2652 (2010)
2. Sharifi, S., Monfared, M.: Series and tapped switched-coupled-inductors impedance networks. IEEE Trans. Ind. Electron. 65(12), 9498–9508 (2018)
3. Asl, E.S., et al.: High voltage gain half-bridge quasi-switch boost inverter with reduced voltage stress on capacitors. IET Power Electron. 10(9), 1095–1108 (2017)
4. Hasan Babay Nozadian, M., et al.: Switched Z-source networks: A review. IET Power Electron. 12(7), 1616–1633 (2019)
5. Li, D., et al.: Generalized multicell switched-inductor and switched-capacitor Z-source inverters. IEEE Trans. Power Electron. 28(2), 837–848 (2013)
6. Nguyen, M.K., et al.: Switched-capacitor quasi-switched boost inverters. IEEE Trans. Ind. Electron. 66(8), 5970–5978 (2019)
7. Sharifi, S., Monfared, M.: Modified series and tapped switched-coupled-inductors quasi-Δ-source networks. IEEE Trans. Ind. Electron. 66(8), 5970–5978 (2019)
8. Sharifi, S., et al.: Generalized three-winding switched-coupled-inductor impedance networks with highly flexible gain. IEEE Trans. Ind. Electron. (2020)
9. Li, L., Tang, Y.: A high set-up quasi-Δ-source inverter based on voltage-lifting unit. In: 2014 IEEE Energy Conversion Congress and Exposition (ECCE), Pittsburgh, PA, 1880–1886 (2014)
10. Zhou, Z.Y., et al.: Single-phase uninterruptible power supply based on Z-source inverter. IEEE Trans. Ind. Electron. 55(8), 2997–3004 (2008)
11. Rymarski, Z., Bernacki, K.: Influence of Z-source output impedance on dynamic properties of single-phase voltage source inverters for uninterruptible power supply. IET Power Electron. 7(8), 1978–1988 (2014)
12. Peng, F.Z., et al.: Z-source inverter for motor drives. IEEE Trans. Power Electron. 20(4), 857–863 (2005)
13. Bansal, R.C., et al.: Steady-state and small-signal models of a three-phase quasi-Δ-source AC–DC converter for wind applications. IET Renew. Power Gener. 10(7), 1033–1040 (2016)
14. Li, Y., et al.: Modeling and control of quasi-Z-Source inverter for distributed generation applications. IEEE Trans. Ind. Electron. 60(4), 1532–1541 (2013)
15. Tang, Y.P., et al.: Grid-tied photovoltaic system with series Z-source inverter. IET Renew. Power Gener. 7(3), 275–283 (2013)
16. Liu, Y., et al.: Modelling and controller design of quasi-Z-source inverter with battery-based photovoltaic power system. IET Power Electron. 7(7), 1665–1674 (2014)
17. Shen, M., et al.: Comparison of traditional Inverters and Z-source inverter for fuel cell vehicles. IEEE Trans. Power Electron. 22(4), 1453–1463 (2007)
18. Peng, F.Z., et al.: Application of Z-source inverter for traction drive of fuel cell—battery hybrid electric vehicles. IEEE Trans. Power Electron. 22(3), 1054–1061 (2007)
19. Qian, W., et al.: Trans-Z-source inverters. IEEE Trans. Power Electron. 26(12), 3453–3463 (2011)
20. Loh, P.C., et al.: F-Z-source inverters. IEEE Trans. Power Electron. 28(11), 4880–4884 (2013)
21. Loh, P.C., Bhaaljerg, F.: Magnetically coupled impedance-source inverters. IEEE Trans. Ind. Appl. 49(5), 2177–2187 (2013)
22. Swiakato, Y.P., et al.: Y-source impedance network. IEEE Trans. Power Electron. 29(7), 3250–3254 (2014)
23. Soon, J.J., Low, K.S.: Sigma-Z-source inverters. IET Power Electron. 8(5), 715–723 (2015)
24. Swiakato, Y.P., et al.: Quasi-Y-source boost DC–DC converter. IEEE Trans. Power Electron. 30(12), 6514–6519 (2015)
25. Swiakato, Y.P., et al.: New magnetically coupled impedance (Z-) source networks. IEEE Trans. Power Electron. 31(11), 7419–7435 (2016)
26. Adamowicz, M., et al.: New type LCCT-Z-source inverters. In: 2011 14th European Conference on Power Electronic Applications, Birmingham, 1–10 (2011)
27. Ayachit, A., et al.: Steady-state and small-signal analysis of a source converter. IEEE Trans. Power Electron. 33(8), 7118–7131 (2018)
28. Hakemi, A., et al.: Δ-Source impedance network. IEEE Trans. Ind. Electron. 64(10), 7842–7851 (2017)
29. Wu, T.E., et al.: Boost converter with coupled inductors and buck–boost type of active clamp. IEEE Trans. Ind. Electron. 55(1), 154–162 (2008)
30. Wai, R.J., Duan, R.Y., High step-up converter with coupled-inductor. IEEE Trans. Power Electron. 20(5), 1025–1035 (2005)
31. Li, W., He, X.: Review of nonisolated high-step-up DC/DC converters in photovoltaic grid-connected applications. IEEE Trans. Ind. Electron. 58(4), 1259–1260 (2011)
32. Rezazadeh, H., Monfared, M.: Quadratic Δ-source impedance network. In: 2020 11th Power Electronics, Drive Systems, and Technologies Conference (PEDSTC), Tehran, 1–5 (2020)
33. Anderson, J., Peng, F.Z.: Four quasi-Z-source inverters. In: 2008 IEEE Power Electronics Specialists Conference, Rhodes, 2743–2749 (2008)
34. Rymarski, Z., Bernacki, K.: Drawbacks of impedance networks. Int. J. Circuit Theory Appl. 46(6), 612–628 (2018)
35. Rymarski, Z., Bernacki, K., Doya, L.: Decreasing the single-phase inverter output voltage distortions caused by impedance networks. IEEE Trans. Ind. Appl. 55(6), 7586–7594 (2019)
36. Melyman, C.W.: Transformer and Inductor Design Handbook. CRC Press, Boca Raton, FL (2011)
37. Sharifi, S., et al.: Highly efficient single-phase direct AC to AC converter with reduced semiconductor count. IEEE Trans. Ind. Electron. (2020)
38. Kong, X., et al.: Effects of parasitic resistances on magnetically coupled impedance-source networks. IEEE Trans. Power Electron. 35(9), 9171–9183 (2020)
39. Nguyen, M.K., et al.: Improved trans-Z-source inverter with continuous input current and boost inversion capability. IEEE Trans. Power Electron. 28(10), 4500–4510 (2013)