The effect of $B\pi$ continuum in the QCD sum rules for the $(0^+, 1^+)$ heavy meson doublet in HQET

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Abstract

We study the effect of $B\pi$ continuum in the QCD sum rule analysis of the heavy meson doublet $(0^+, 1^+)$ in the leading order of heavy quark effective theory. New sum rules are derived for the leading order binding energy $\bar{\Lambda}_{+, \frac{1}{2}}$ and pionic coupling constant $g'$. 

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1 Introduction

The QCD sum rules [1] for the masses of the excited heavy meson doublet ($B'_0, B'_1$) of spin parity $(0^+, 1^+)_{(+, \frac{1}{2})}$ have been studied in [2, 3, 4, 5], where the indices $(+, \frac{1}{2})$ denote the parity and spin of the light component $j_i$. Recently the $O(\alpha_s)$ correction to the $m(0^+)$ sum rule has been calculated in [6, 7]. In [1, 3] the masses of the $(0^+, 1^+)_{(+, \frac{1}{2})}$, together with those of the doublet $(1^+, 2^+)_{(+, \frac{1}{2})}$ were calculated up to the order of $O(1/m_Q)$ in the framework of heavy quark effective theory (HQET) [8]. Let $\bar{\Lambda}_{P, j_i} = m_{P, j_i} - m_b$, where $m_{P, j_i}$ is the mass of the the doublet in the leading order of $1/m_Q$ and $P, j_i$ are the parity and spin of the light component of the heavy mesons in the doublet. The results in [1] in the leading order of $\alpha_s$ are $\bar{\Lambda}_{+, \frac{1}{2}} = (1.15 \pm 0.10)$GeV if the usual interpolating current of the lowest dimension is used and $\bar{\Lambda}_{+, \frac{3}{2}} = (0.82 \pm 0.10)$GeV. Within errors the results in [1, 3, 4] are consistent with these results. This would imply that $0^+$ state lies $100 - 300$ MeV above $2^+$ state. The $O(1/m_Q)$ corrections calculated in [3] does not change much this mass difference. This result is inconsistent with the new experimental data [9] where $m(2^+)$ is about $100$ MeV larger than $m(0^+)$. It is unlikely that $O(\alpha_s)$ corrections can account for this discrepancy.

Recently Blok et al. made the following observation [10]. Due to the $S$-wave nature of the $B\pi$ intermediate state and the large coupling of the soft Goldstone particle, the contribution of the $B\pi$ continuum to the spectral density in the correlator of two $0^+$ currents is unusually large. It rises faster than the quark-gluon spectral density in the low
energy region and exceeds it in magnitude in that region. Thus, it may violate the naive quark-hadron duality if we integrate the spectra over a region below the lowest pole. It was proposed in [10] that this is the reason for the abnormal large value of the residue of the pole [2] obtained in the standard "lowest pole plus parton-like continuum model" for the QCD sum rules. Therefore, a better approach is to include the $B\pi$ continuum in the soft pion region in addition to the lowest $B'_0$ pole in the sum rule.

In this note we shall study in detail this effect in the sum rules for $\bar{\Lambda}^{1/2}$ and the residue of the $B'_0$ pole. Besides, the decay rates of $B'_0$ and $B'_1$ have been studied in [3, 11] with the light cone QCD sum rules (LCQSR) [12]. In view of the duality violation effect due to the intermediate states of $B\pi$ continuum in the sum rule we shall also re-investigate the LCQSR for pion coupling of the $(0^+, 1^+)$ doublet including the contribution of $B\pi$ states.

Section 2 is a short review of previous sum rules. New QCD sum rules with intermediate state contribution and the numerical analyses are presented in section 3. The last section is a short summary.

2 Previous sum rules

2.1 Previous mass sum rules

The interpolating current for the doublets $(0^+, 1^+)$ reads

$$J_{0^+, 1^+}^\dagger = \bar{h}_v q,$$

$$J_{1^+, 1^+}^\dagger = \bar{h}_v \gamma^5 \gamma^\alpha t q,$$

where $h_v(x)$ is the heavy quark field in HQET, $v_\mu$ is the heavy hadron velocity, $\gamma_i^\alpha = \gamma^\alpha - \hat{v}v^\alpha$, the indices $j, +, j_l$ in $J_{j, +, j_l}$ are the total angular momentum, the parity and the light component angular momentum respectively. Note there is a factor of $\frac{1}{\sqrt{2}}$ in the definitions of the interpolating currents in our previous work.

We consider the correlator

$$\Pi(\omega) = i \int d^4 x e^{ikx} \langle 0 | T \{ J_{0^+, 1^+}(x), J_{0^+, 1^+}^\dagger(0) \} | 0 \rangle$$

and define the overlapping amplitude $f_{+, 1^+}$ as

$$\langle 0 | J_{0^+, 1^+}(0) | B'_0 \rangle = f_{+, 1^+}.$$

Assuming the standard "pole plus parton-like continuum" model we get

$$\Pi(\omega) = \frac{1}{2} f_{+, 1^+}^2 e^{-\Lambda_{+, 1^+}/T} + \text{continuum}.$$  (5)

Here $\omega = v \cdot k$, which is a factor $\frac{1}{2}$ smaller than the $\omega$ used in [3]. We derive the sum rule for the $(0^+, 1^+)$ doublet [4]

$$\frac{1}{2} f_{+, 1^+}^2 e^{-\Lambda_{+, 1^+}/T} = \frac{3}{2\pi^2} \int_0^{\infty} \omega^2 e^{-\omega/T} d\omega + \frac{1}{2} \langle \bar{q}q \rangle - \frac{1}{32T^2} m_0^2 (\bar{q}q) - \frac{g^2 < (\bar{q} \gamma_\mu \frac{\omega}{T} q)^2 >}{2304 T^3},$$  (6)

where $m_0^2 (\bar{q}q) = \langle \bar{q} q \sigma_\mu G^{\mu\nu} q \rangle$. 

Using the following standard values for the condensates
\[
\langle \bar{q}q \rangle = -(0.225 \text{ GeV})^3, \\
\langle \alpha_s GG \rangle = 0.038 \text{ GeV}^4, \\
m_0^2 = 0.8 \text{ GeV}^2,
\]
and with \( s_0 = (1.5 \pm 0.1) \) GeV, \( T = 0.4 \sim 0.6 \) GeV we get from (3)
\[
\bar{\Lambda}_{+\frac{1}{2}} = (1.15 \pm 0.10) \text{ GeV},
\]
\[
f_{+\frac{1}{2}} = (0.570 \pm 0.08) \text{ GeV}^{3/2}.
\]

For comparison here we also write down the sum rules for the \((1^+,2^+)\) doublet.
\[
\frac{1}{2} f_{+3/2}^2 e^{-\bar{\Lambda}_{+\frac{3}{2}}/T} = \frac{1}{2\pi^2} \int_0^\infty \omega e^{-\omega/T} d\omega - \frac{1}{12} m_0^2 \langle \bar{q}q \rangle - \frac{1}{16} \langle \frac{\alpha_s}{\pi} G^2 \rangle T,
\]
where the following interpolating currents for the \((1^+,2^+)\) doublet are used
\[
J_{1+,\frac{3}{2}}^{1\alpha} = \sqrt{\frac{3}{2}} \bar{h}_v \gamma^5 (-i) \left( \mathcal{D}_t^\alpha - \frac{1}{3} \gamma_t^\alpha \mathcal{D}_t \right) q ,
\]
\[
J_{2+,\alpha_1 \alpha_2}^{1\alpha} = \frac{\bar{h}_v}{2} \left( \gamma_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} + \gamma_t^{\alpha_2} \mathcal{D}_t^{\alpha_1} - \frac{2}{3} g_t^{\alpha_1 \alpha_2} \mathcal{D}_t \right) q .
\]
These currents are also a factor \( \sqrt{2} \) larger than those in (3).

### 2.2 Previous sum rules for pionic couplings

Let us define the decay amplitudes of the doublet \((0^+,1^+)\) in full QCD (3)
\[
M(B'_0 \to B(k)\pi(q)) = I \sqrt{m_{B'_0} m_B} \frac{m_{B'_0}^2 - m_B^2}{2 m_{B'_0}^2} q ,
\]
\[
M(B'_1 \to B^*(k)\pi(q)) = I \{ \epsilon^* \cdot \eta g \sqrt{m_{B'_1} m_B} \frac{k \cdot q}{m_{B'_1}^2} + (k \cdot \eta)(q \cdot \epsilon^*) F \} ,
\]
where \( I = \sqrt{2} \), 1 for charged and neutral pion respectively. The structure \( F \) vanishes in the \( m_Q \to \infty \) limit.

In HQET these amplitudes have the simple form
\[
M(B'_0 \to B\pi) = I \ g',
\]
\[
M(B'_1 \to B^*\pi) = I \ \epsilon^* \cdot \eta g'
\]
where
\[
g' = -g(\bar{\Lambda}_{+\frac{1}{2}} - \bar{\Lambda}_{-\frac{1}{2}}).
\]

For deriving the sum rules for \( g' \) we consider the correlator
\[
\int d^4 x \ e^{-i k \cdot x} \langle \pi(q) | T\{ J_{0+,\frac{1}{2}}(0), J_{0-,\frac{1}{2}}(x) \} | 0 \rangle = I \ G_{B'_0 B}(\omega,\omega'),
\]

where $k' = k + q$, $\omega = v \cdot k$, $\omega' = v \cdot k'$, $q^2 = 0$ and $J_{0,\frac{1}{2}}^i = \bar{h}_v \gamma_5 q$.

Again after invoking the naive quark-hadron duality we have the double dispersion relation:

$$
\frac{f_{-\frac{1}{2}} f_{+\frac{1}{2}} g'}{4(\Lambda_{-\frac{1}{2}} - \omega')(\Lambda_{+\frac{1}{2}} - \omega')} + \frac{c}{\Lambda_{-\frac{1}{2}} - \omega'} + \frac{c'}{\Lambda_{+\frac{1}{2}} - \omega} + \text{continuum} .
$$

Expressing (18) with the pion wave functions [12] we obtain the LCQSR for $g'$ [11]:

$$
g' f_{-\frac{1}{2}} f_{+\frac{1}{2}} = 2F_\pi e^{\Lambda_{-\frac{1}{2}} + \Lambda_{+\frac{1}{2}}}{\varphi}(u(0)T^2 f_1(s_0) + \mu \varphi_P(u_0)T f_0(s_0) + g'_1(u_0)) ,
$$

where $F_\pi = 93$MeV, $\mu = -\frac{\sqrt{2} s_0}{F_\pi} = 1.32$GeV at the scale $\mu = 1$GeV, $\varphi_P(u)$ etc are the light cone pion wave functions defined by [12]

$$
<\pi(q)|\bar{d}(x)i\gamma_5 u(0)|0> = \sqrt{2} F_\pi \mu \int_0^1 du e^{i\mu q x} \varphi_P(u) .
$$

$\varphi'_{\pi}(u_0)$, $g'_1(u_0)$ are the first derivatives of $\varphi_{\pi}(u)$, $g_1(u)$ at $u = u_0$, $u_0 = \frac{T_1}{T_1 + T_2}$, $T = \frac{T_1 T_2}{T_1 + T_2}$, $T_1$, $T_2$ are the Borel parameters. We choose $T_1 = T_2$. Therefore $u_0 = \frac{1}{2}$. At this point $\varphi'_{\pi}(u_0)$ and $g'_1(u_0)$ vanish. The factor $f_{n}(x) = 1 - e^{-x} \sum_{k=0}^{n} \frac{x^k}{k!}$ is used to subtract the parton-like continuum contribution with the continuum threshold $s_0$.

Correcting a numerical error in [5] we obtain from (20)

$$
g' = 3.6 \pm 0.7 .
$$

### 3 New sum rules with $B \pi$ intermediate states

#### 3.1 Mass sum rules

We have the dispersion relation for (3)

$$
\Pi(\omega) = \frac{1}{\pi} \int \frac{\rho(s)}{s - \omega - i\epsilon} ds ,
$$

where $\rho(s)$ is the spectral density in the limit $m_Q \to \infty$. At the quark level,

$$
\rho_q(s) = \frac{N_c}{2\pi} s^2 ,
$$

where $N_c = 3$ is the color number.

Due to the spontaneous chiral symmetry breaking, there exist $(N_f^2 - 1)$ massless Goldstone bosons, where $N_f$ is the light quark flavor number. The S-wave combination $B \pi$ has the same quantum numbers as the $B'_0$ meson. So the interpolating current (1) “sees” both $B \pi$ and $B'_0$. In other words, the contribution due to $B \pi$ intermediate states should be included explicitly when we write the spectral density at the phenomenological side. Otherwise the $B'_0$ pole contribution will be overestimated leading to an abnormal large residue.

$$
\rho(s) = \frac{\pi}{2} f_{+\frac{1}{2}}^2 \delta(s - \bar{\Lambda}_{+\frac{1}{2}}) + \rho_\pi(s) + \cdots ,
$$
where the first term is the $B'_0$ pole and the $\rho_\pi(s)$ is the $B\pi$ intermediate states contribution. The excited states and the continuum contribution is denoted by the ellipse.

We start from the full QCD Lagrangian to derive $\rho_\pi(s)$. It was shown in [10] that

$$\rho_\pi(p^2) = \frac{1}{8\pi}\left(\frac{f_{\pi}}{f}\right)^2(s - \bar{\Lambda}_{\pm\frac{3}{2}})\theta(s - \bar{\Lambda}_{-\frac{3}{2}})\theta(\bar{\Lambda}_{+\frac{3}{2}} - s)(N_f - \frac{1}{N_f}),$$

where we have used the relation $f_{\pm\frac{3}{2}} = \sqrt{m_B}f_B$ in the leading order of $1/m_Q$. Note $\rho_\pi(s)$ is a linear function of $s$ while the free parton level spectral density $\rho_\varphi(s)$ in (24) is quadratic in $s$. The latter is suppressed compared with the former in the lower energy region. So the spectral density is significantly disturbed by the presence of light Goldstone bosons. In (24) we have introduced the factor $\theta(\bar{\Lambda}_{+\frac{3}{2}} - s)$. The reason is that the soft pion theorem does not hold any more beyond the region $|q|_\pi < \bar{\Lambda}_{+\frac{3}{2}} - \bar{\Lambda}_{-\frac{3}{2}} \sim 350$ MeV. Moreover it was conjectured in [10] that $< 0|\bar{b}q|B\pi >$ drops when the total energy of $B\pi$ becomes larger than the mass of $B'_0$ so that the quark-hadron duality is restored after integrating the energy over a larger interval from zero to the continuum threshold.

Note $m_K = 498$ MeV and $m_\eta = 547$ MeV due to nonzero current quark mass. So in realistic case only $B\pi$ intermediate states contribute to $\rho_\pi(s)$ in (24) corresponding to $N_f = 2$. Now we arrive at the new sum rules after making Borel transformation:

$$\frac{1}{2}f_{+,1/2}^2e^{-\frac{1}{T}} + \frac{3}{16\pi^2}\left(\frac{f_{+,1/2}}{f}\right)^2\int_0^{\frac{1}{2}}(s - \bar{\Lambda}_{-\frac{3}{2}})e^{-s/T}ds$$

$$= \frac{3}{2\pi^2}\int_0^{s_0} s^2e^{-s/T}ds + \frac{1}{2}\langle \bar{q}q \rangle - \frac{1}{32T^2}m_B^2\langle \bar{q}q \rangle - \frac{g^2}{2304T^3},$$

where $s_0$ is the continuum threshold. Starting from $s_0$ we have modeled the phenomenological spectral density with the free parton-like one.
3.2 New sum rules for $g'$

Similarly we can write the double dispersion relation in the leading order of HQET for $\Pi_3.3$ as

$$\Pi(\omega, \omega') = \frac{1}{\pi^2} \int \frac{\rho(s, s')}{(s - \omega)(s' - \omega')} ds'ds + \cdots ,$$

where the ellipse denotes the subtraction terms.

The pole term is

$$\rho_p(s, s') = \frac{\pi^2}{4} g' f_{+1/2} f_{-1/2} \delta(s - \bar{\Lambda}_{-1/2}) \delta(s' - \bar{\Lambda}_{+1/2}) .$$

and the contribution of the $B\pi$ intermediate states in full QCD is

$$\rho_\pi(s, s') = \int \frac{dk}{(2\pi)^3} \int \frac{dl}{(2\pi)^3} \theta(k_0)\theta(l_0)\delta(k^2 - m_B^2)\delta(l^2) \frac{(2\pi)^3\delta(p - \bar{k} - \bar{l})\delta(p_0 - k_0 - l_0)\sum |\langle \pi(p) | j_B(0) | B(k)\pi(l) \rangle \langle B(k)\pi(l) | j_B(0) | 0 \rangle|}{(s - m_B^2)} .$$

Using the soft pion limit and $SU_3$ symmetry we find

$$\rho_\pi(s, s') = \frac{3}{32} \left( \frac{f_B f_{1/2} g'}{f_{1/2}^2} \right) m_B^2 m_b + m_q (1 - \frac{m_B^2}{s'}) \delta(s - m_B^2) .$$

Taking the heavy quark limit (34) is reduced to

$$\rho_\pi(s, s') = \frac{3}{32 f_{1/2}^2} g' f_{+1/2} f_{-1/2} (s' - \bar{\Lambda}_{-1/2}) \theta(s' - \bar{\Lambda}_{-1/2}) \theta(\bar{\Lambda}_{+1/2} - s') \delta(s - \bar{\Lambda}_{+1/2}) .$$

Finally we have a new sum rule for $g'$:

$$g' f_{-1/2} f_{+1/2} = 2 F_\pi e^{\bar{\Lambda}_{+1/2} / 2T} \{ e^{-\bar{\Lambda}_{+1/2} / 2T} + \frac{3}{8 \pi f_{1/2}^2} \int_{\bar{\Lambda}_{-1/2} - \frac{1}{2}}^{\bar{\Lambda}_{-1/2} + \frac{1}{2}} (s' - \bar{\Lambda}_{-1/2}) e^{-s' / 2T} ds' \}^{-1} \{ -\varphi_p(u_0) T^2 f_1(\frac{u_0}{8 \pi f_{1/2}^2}) + \mu_\pi \varphi_p(u_0) T f_0(\frac{u_0}{8 \pi f_{1/2}^2}) + g'_1(u_0) \} .$$

3.3 Numerical analysis

As input we need $\bar{\Lambda}_{-1/2} = 0.5$ GeV and $f_{-1/2} \approx 0.35$ GeV$^{3/2}$ at the order $\alpha_s = 0$ [13]. The numerical results for $\bar{\Lambda}_{+1/2}, f_{+1/2}$ and $\bar{\Lambda}_{+3/2}, f_{+3/2}$ in [2] were obtained by first applying the operator $\frac{d}{d\mu(1/T)}$ to (30) and (33) to extract $\bar{\Lambda}_{+1/2}$ and $\bar{\Lambda}_{+3/2}$, which were then used to obtain $f_{+1/2}$ and $f_{+3/2}$ respectively. Here we use a different procedure which appears to be better. This involves with simultaneously varying the parameters $\bar{\Lambda}_{+1/2}, f_{+1/2}, s_0$ etc to find the best fitting of the left hand side (L.H.S.) and right hand side (R.H.S.) of the sum rules. We work at the region $T > 0.4$ GeV for Eq. (30), where the power correction is under control. We allow the continuum threshold to vary from 1.06 GeV to 1.46 GeV.

Numerically we have

$$\bar{\Lambda}_{+1/2} = 0.85 \pm 0.15 \text{ GeV} ,$$

$$f_{+1/2} = 0.36 \pm 0.10 \text{ GeV}^{3/2} .$$

With these parameters the left hand side and right hand side agree within five percent in the region $0.5 < T < 0.8$ GeV as can be seen from FIG. 1. Typically at $T = 0.4$ GeV the
sum of the $B'_0$ pole and $B\pi$ intermediate states constitutes about 60% of the whole sum rule. The continuum starting from $s_0$ is about 40%.

It is important to notice that the $B\pi$ intermediate states contribute about 15% to the left hand side of (30). If we use this fitting method in the numerical analysis of the old sum rules we reproduce the results in [2]

\begin{align*}
\bar{\Lambda}_{+,\frac{1}{2}} &= (1.2 \pm 0.2)\text{GeV}, \\
f_{+,\frac{1}{2}} &= (0.75 \pm 0.15)\text{GeV}^{\frac{3}{2}}, \\
s_0 &= (1.8 \pm 0.2)\text{GeV}.
\end{align*}

Note in [2] no error is given for $f_{+,\frac{1}{2}}$. The value and error in (40) is the result of our reanalysis. We see that both $f_{+,\frac{1}{2}}$ and $\bar{\Lambda}_{+,\frac{1}{2}}$ are significantly reduced after taking into account $B\pi$ intermediate states.

We can also apply the fitting method to the analysis of the sum rules for the $(1^+, 2^+)$ doublet. The fitted curve of the L.H.S. and the parton level curve of the R.H.S. of Eq. (11) are shown in FIG. 2 in the region $T > 0.4\text{GeV}$ with the following most suitable parameters $\bar{\Lambda}_{+,\frac{3}{2}}, f_{+,\frac{3}{2}}, s_0$.

\begin{align*}
\bar{\Lambda}_{+,\frac{3}{2}} &= (0.95 \pm 0.10)\text{GeV}, \\
f_{+,\frac{3}{2}} &= (0.263 \pm 0.06)\text{GeV}^{\frac{3}{2}}, \\
s_0 &= (1.3 \pm 0.2)\text{GeV}.
\end{align*}

With these parameters the fitting is excellent, typically with an accuracy within one percent in the large interval $0.5 < T < 1.4\text{GeV}$ as can be seen from FIG. 2.

The central value of $\bar{\Lambda}_{+,\frac{3}{2}}$ is about 100 MeV lower than that of $\bar{\Lambda}_{+,\frac{1}{2}}$, in good agreement with the experimental data [3].

Now we are ready to extract $g'$. Using the same pion wave functions as in [11] we have

\begin{equation}
g' f_{-,\frac{1}{2}} f_{+,\frac{1}{2}} = (0.36 \pm 0.05) \text{ GeV}^3,
\end{equation}

where the error refers to the variations with $T$ and $s_0$. And the central value corresponds to $T = 0.9\text{GeV}$ and $s_0 = 1.26\text{GeV}$. The variation of the left hand side of (36) with $T$ and $s_0$ is presented in FIG. 2. Finally we get

\begin{equation}
g' = 2.8 \pm 0.5.
\end{equation}

The decay width formulas in the leading order of $1/m_Q$ are

\begin{align*}
\Gamma(B'_0 \to B\pi) &= \frac{3}{8\pi} g'^2 |\vec{q}|_{\pi}, \\
\Gamma(B'_1 \to B^*\pi) &= \frac{3}{8\pi} g'^2 |\vec{q}|_{\pi}.
\end{align*}

In order to include the large $1/m_Q$ correction in the kinematical factors, we use the decay width formulas with finite $m_Q$ instead of (47), (48).

\begin{equation}
\Gamma(B'_0 \to B\pi) = \frac{3}{32\pi} g'^2 \frac{m_B (m_{B'_0}^2 - m_B^2)^2}{m_{B'_0}^3} |\vec{q}|_{\pi},
\end{equation}

\begin{equation}
\Gamma(B'_1 \to B^*\pi) = \frac{3}{32\pi} g'^2 \frac{m_B (m_{B'_1}^2 - m_B^2)^2}{m_{B'_1}^3} |\vec{q}|_{\pi}.
\end{equation}
\[
\Gamma(B'_1 \rightarrow B^*\pi) = \frac{1}{32\pi} \frac{g^2}{m^2_B} \frac{m^2_{B^*} - m^2_{B'}}{m^4_{B'_1}} [2 + \frac{(m^2_{B'_1} + m^2_{B^*})^2}{4m^2_{B'_1}m^2_{B^*}}] |\bar{q}|_\pi \ .
\]

(50)

Numerically we have

\[
\Gamma(B'_0 \rightarrow B\pi) \approx 250\text{MeV} ,
\]

(51)

\[
\Gamma(B'_1 \rightarrow B^*\pi) \approx 250\text{MeV} ,
\]

(52)

with \(m_{B'_1} = m_{B^*} + 350\text{MeV}, m_{B'_0} = m_B + 350\text{MeV} \).

4 Summary

In summary, we have reanalyzed the QCD sum rules for both the \((0^+, 1^+)\) mass and its pionic decay amplitude. The contribution of the \(B\pi\) intermediate states in the soft pion region are investigated in detail. The spectral density is disturbed significantly by the presence of Goldstone bosons. After subtracting the contribution of these intermediate states, we have obtained new results for the binding energy \(\bar{\Lambda}_{+,\frac{1}{2}}\) and decay widths for the \((0^+, 1^+)\) doublet in the leading order of HQET. Numerically \(\bar{\Lambda}_{+,\frac{1}{2}} = (0.85 \pm 0.15)\text{GeV}\), which is about 100 MeV smaller than \(\bar{\Lambda}_{+,\frac{1}{2}} = (0.95 \pm 0.10)\text{GeV}\) extracted with the same fitting method, in good agreement with the most recent experimental data [9]. The \((0^+, 1^+)\) decay width is around 250 MeV, which remains to be larger than the experimental result \((76 \pm 28(\text{stat}) \pm 15(\text{syst}))\text{ MeV}\) in [9]. The origin of this discrepancy is not clear at present. We want to point out that the same contamination from the Goldstone bosons exists for the sum rules for the \((1^+, 2^+)\) doublet [4, 5]. But in this case the \(B\pi\) intermediate states disturb the spectral density only slightly for they are in the D-wave state.

The errors for our numerical results given in Sec. 3.3 include only those from the variation of the Borel parameter \(T\) and the continuum threshold \(s_0\) in the window. They don’t include those from the uncertainty of the condensates and intrinsic errors of the QCD sum rule approach. In our analysis we have neglected the contribution of \(B\pi\) intermediate states to the spectral density for \(s > \bar{\Lambda}_{+,\frac{1}{2}}\) since the soft pion theorem does not hold any more and there is not a reliable way to estimate the matrix element \(<0|\bar{b}q|B(k)\pi(q)>\). This is another source of uncertainty. After we submitted the original version of this paper, we learned that CLEO collaboration have measured the \(D'_0\) mass and width to be \(2461\text{ MeV}\) and \(200 \sim 400\text{ MeV}\) respectively [14].

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Figure Captions

FIG. 1. The variation of the right and left hand side of Eq. (30) with Borel parameter $T$ is plotted as solid and dotted curves respectively with the fitting parameters in (37)-(38).

FIG. 2. The variation of the right and left hand side of Eq. (10) with $T$ using the central values of the fitting parameters in (42)-(44).

FIG. 3. The sum rules for $g(f_{-}^{1/2} f_{+}^{1/2})$ with $s_{0} = 1.36, 1.26, 1.16$ GeV respectively.
FIG. 1
