Self-Gravitational Corrections to the Cardy-Verlinde Formula of Achúcarro-Ortiz Black Hole

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Abstract

Recently, it was shown that the entropy of the black hole horizon in the Achúcarro-Ortiz spacetime can be described by the Cardy-Verlinde formula. In this paper, we compute the self-gravitational corrections to the Cardy-Verlinde formula of the two-dimensional Achúcarro-Ortiz black hole. These corrections stem from the effect of self-gravitation and they are derived in the context of Keski-Vakkuri, Kraus and Wilczek (KKW) analysis. The black hole under study is therefore treated as a dynamical background. The self-gravitational corrections to the entropy as given by the Cardy-Verlinde formula of Achúcarro-Ortiz black hole, are found to be positive. This result provides evidence in support of the claim that the holographic bound is not universal in the framework of two-dimensional gravity models.
Introduction

In 1992 Bañados, Teitelboim and Zanelli (BTZ) [1, 2] showed that (2 + 1)-dimensional gravity has a black hole solution. This black hole is described by two parameters, its mass $M$ and its angular momentum (spin) $J$. It is locally anti-de-Sitter space and thus it differs from Schwarzschild and Kerr solutions in that it is asymptotically anti-de-Sitter instead of flat spacetime. Additionally, it has no curvature singularity at the origin. AdS black holes, which are members of the two-parametric family of BTZ black holes, play a central role in AdS/CFT conjecture [3] and also in brane-world scenarios [4, 5]. Specifically AdS(2) black hole is most interesting in the context of string theory and black hole physics [6–8].

Concerning the quantum process called Hawking effect [9] much work has been done using a fixed background during the emission process. The idea of Keski-Vakkuri, Kraus and Wilczek (KKW) [10]-[13] is to view the black hole background as dynamical by treating the Hawking radiation as a tunnelling process. The energy conservation is the key to this description. The total (ADM) mass is kept fixed while the mass of the black hole under consideration decreases due to the emitted radiation. The effect of this modification gives rise to additional terms in the formulae concerning the known results for black holes [14]-[20]; a nonthermal partner to the thermal spectrum of the Hawking radiation shows up.

Holography is believed to be one of the fundamental principles of the true quantum theory of gravity [21,22]. An explicitly calculable example of holography is the much–studied anti-de Sitter (AdS)/Conformal Field Theory (CFT) correspondence. More recently, it has been proposed in a manner analogous with the AdS$_d$/CFT$_{d-1}$ correspondence, that quantum gravity in a de Sitter (dS) space is dual to a certain Euclidean CFT living on a spacelike boundary of the dS space [23] (see also earlier works [24]-[27]). Following this proposal, some investigations on the dS space have been carried out recently [26]-[49].

The Cardy-Verlinde formula recently proposed by Verlinde [50], relates the entropy of a certain CFT to its total energy and Casimir energy in arbitrary dimensions. In the spirit of AdS$_d$/CFT$_{d-1}$ and dS$_d$/CFT$_{d-1}$ correspondences, this formula has been shown to hold exactly for the cases of topological dS, Schwarzschild-dS, Reissner-Nordström-dS, Kerr-dS and Kerr-Newman-dS black holes.

Recently, much interest has been taken in computing the quantum corrections to the Bekenstein-Hawking entropy $S_{BH}$ [51–53]. In a recent work Carlip [54] has deduced the leading order quantum correction to the classical Cardy formula. The Cardy formula follows from a saddle-point approximation of the partition function for a two-dimensional CFT. This leads to the theory’s density of states which is related to the partition function by way of a Fourier transform. In [51], Medved has employed Carlip’s formulation to the case of a generic model of two-dimensional gravity with coupling to a dilaton field. In the
present paper we study the semi-classical gravitational corrections to the Cardy-Verlinde formula due to the effect of self-gravitation.

The remainder of this paper is organized as follows. In Section 1, we make a short review of the two-dimensional Achúcarro-Ortiz black hole. We present for the aforementioned black hole, expressions for its mass, angular momentum, angular velocity, temperature, area and entropy. In Section 2, we compute the self-gravitational corrections to the entropy of the two-dimensional Achúcarro-Ortiz black hole which is described by the Cardy-Verlinde formula. Finally, in Section 3 we briefly summarize our results and some concluding remarks are made.

1 Achúcarro-Ortiz Black Hole

The black hole solutions of Bañados, Teitelboim and Zanelli [1, 2] in \((2 + 1)\) spacetime dimensions are derived from a three dimensional theory of gravity

\[
S = \int dx^3 \sqrt{-g} \left( R + 2\Lambda \right)
\]

with a negative cosmological constant \((\Lambda = 1/l^2 > 0)\).

The corresponding line element is

\[
ds^2 = -\left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right) dt^2 + \frac{dr^2}{-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}} + r^2 \left(d\theta - \frac{J}{2r^2} dt\right)^2
\]

It is obvious that there are many ways to reduce the three dimensional BTZ black hole solutions to the two dimensional charged and uncharged dilatonic black holes [55,56]. The Kaluza-Klein reduction of the \((2+1)\)-dimensional metric (2) yields a two-dimensional line element:

\[
ds^2 = -g(r) dt^2 + g(r)^{-1} dr^2
\]

where

\[
g(r) = \left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)
\]

with \(M\) the mass of the two-dimensional Achúcarro-Ortiz black hole, \(J\) the angular momentum (spin) of the afore-mentioned black hole and \(-\infty < t < +\infty\), \(0 \leq r < +\infty\), \(0 \leq \theta < 2\pi\).

The outer and inner horizons, i.e. \(r_+\) (henceforth simply black hole horizon) and \(r_-\) respectively, concerning the positive mass black hole spectrum with spin \((J \neq 0)\) of the line element (3) are given as

\[
r_{\pm}^2 = \frac{l^2}{2} \left( M \pm \sqrt{M^2 - \frac{J^2}{l^2}} \right)
\]
and therefore, in terms of the inner and outer horizons, the black hole mass and the angular momentum are given, respectively, by

\[ M = \frac{r_+^2}{l^2} + \frac{J^2}{4r_+^2} \quad (6) \]

and

\[ J = \frac{2r_+r_-}{l} \quad (7) \]

with the corresponding angular velocity to be

\[ \Omega = \frac{J}{2r^2} \quad (8) \]

The Hawking temperature \( T_H \) of the black hole horizon is [57]

\[
T_H = \frac{1}{2\pi r_+} \sqrt{\left(\frac{r_+^2}{l^2} + \frac{J^2}{4r_+^2}\right)^2 - \frac{J^2}{l^2}} = \frac{1}{2\pi r_+} \left( \frac{r_+^2}{l^2} - \frac{J^2}{4r_+^2} \right). \quad (9)
\]

The area \( A_H \) of the black hole horizon is

\[
A_H = \sqrt{2\pi l \left( M + \sqrt{M^2 - \frac{J^2}{l^2}} \right)^{1/2}} = 2\pi r_+ \quad (10) \]

and thus the entropy of the two-dimensional Achúcarro-Ortiz black hole, if we employ the well-known Bekenstein-Hawking area formula \( S_{BH} \) for the entropy [58–60], is given by

\[ S_{BH} = \frac{1}{4\hbar G} A_H \quad (12) \]

Using the BTZ units where \( 8\hbar G = 1 \), the entropy of the two-dimensional Achúcarro-Ortiz black hole takes the form

\[ S_{BH} = 4\pi r_+ \quad (13) \]

## 2 Self-gravitational corrections to Cardy-Verlinde formula

In this section we compute the self-gravitational corrections to the entropy of the two-dimensional Achúcarro-Ortiz black hole [13] described by the Cardy-Verlinde formula

\[
S_{CFT} = \frac{2\pi R}{\sqrt{ab}} \sqrt{E_C (2E - E_C)} \quad (14)
\]
The total energy $E$ may be written as the sum of two terms
\[ E(S, V) = E_E(S, V) + \frac{1}{2} E_C(S, V) \] (15)
where $E_E$ is the purely extensive part of the total energy $E$ and $E_C$ is the Casimir energy. The Casimir energy is derived by the violation of the Euler relation
\[ E_C = 2E - T_H S_{BH} - \Omega J \] (16)
which now will be modified due to the self-gravitation effect as
\[ E_C = 2E - T_{bh} S_{bh} - \Omega_{bh} J_{bh} . \] (17)

In the context of KKW analysis, it is easily seen that\(^1\)

\[
T_{bh}S_{bh} = T_H S_{BH} \left( 1 - \omega \frac{M}{2 \sqrt{M^2 - \frac{J^2}{l^2}} \left( M + \sqrt{M^2 - \frac{J^2}{l^2}} \right)} \right) \] (18)

\[
= T_H S_{BH} \left( 1 - \omega \frac{M l^4}{4 r_+^2 (r_+^2 - r_-^2)} \right) \] (19)

where $\omega$ is the emitted shell of energy radiated outwards the black hole horizon.

Thus, the second term in (17) can be written as
\[
T_{bh}S_{bh} = T_H S_{BH} - \omega \frac{M}{\left( M + \sqrt{M^2 - \frac{J^2}{l^2}} \right)} \] (20)

where
\[
T_H S_{BH} = 2 \left( \frac{r_+^2}{l^2} - \frac{J^2}{4 r_+^2} \right) . \] (21)

Additionally, one can easily check that in the context of KKW analysis the modified angular momentum (computed up to first order in $\omega$) is given as
\[
J_{bh} = J (1 - \epsilon_1 \omega) \] (22)

where $\epsilon_1$ is a parameter\(^2\)
\[
\epsilon_1 = \frac{1}{2} \sqrt{M^2 - \frac{J^2}{l^2}} \] (23)
\[
= \frac{l^2}{2 (r_+^2 - r_-^2)} . \] (24)

\(^1\)For the explicit computation of the modified thermodynamical quantities see [18]
\(^2\)This parameter is small for sufficiently large mass of the two-dimensional Achúcarro-Ortiz black hole (as it is expected for such a semiclassical analysis here employed since the radiating matter is viewed as point particles).
The modified angular velocity on the black hole horizon (computed also up to first order in $\omega$) is

$$\Omega_{bh} = \frac{\Omega_+}{(1 - \epsilon_1 \omega)}$$  \hspace{1cm} (25)$$

where $\Omega_+$ is the angular velocity evaluated on the black hole horizon

$$\Omega_+ = \frac{J}{2r_+^2}$$  \hspace{1cm} (26)$$

Therefore, the third term in (17) is given as

$$\Omega_{bh} J_{bh} = \Omega_+ J$$  \hspace{1cm} (27)$$

where

$$\Omega_+ J = \frac{J^2}{2r_+^2}$$  \hspace{1cm} (28)$$

At this point it is necessary to stress that we shall consider no self-gravitational corrections to the total energy $E$ of the system under study as well as to the radius which takes the form [61]

$$R = 2r_+ \left( \frac{1}{J} \right) \sqrt{ab}.$$  \hspace{1cm} (29)$$

The Casimir energy, substituting (20) and (27) in (17), is given as

$$E_C = \frac{J^2}{2r_+^2} + \omega \frac{M}{\left( M + \sqrt{M^2 - \frac{J^2}{l^2}} \right)}$$  \hspace{1cm} (30)$$

$$= \frac{J^2}{2r_+^2} + 2\epsilon_2 M \omega$$  \hspace{1cm} (31)$$

where $\epsilon_2$ is a parameter given by

$$\epsilon_2 = \frac{1}{2 \left( M + \sqrt{M^2 - \frac{J^2}{l^2}} \right)}$$  \hspace{1cm} (32)$$

$$= \frac{l^2}{4r_+^2}.$$  \hspace{1cm} (33)$$

Additionally, it is evident that the quantity $2E - E_C$ is given, by substituting again equations (20) and (27) in (17), as

$$2E - E_C = 2\frac{r_+^2}{l^2} - \omega \frac{M}{\left( M + \sqrt{M^2 - \frac{J^2}{l^2}} \right)}$$  \hspace{1cm} (34)$$

$$= 2\frac{r_+^2}{l^2} - 2\epsilon_2 M \omega.$$  \hspace{1cm} (35)$$

\footnote{This is also a small parameter for sufficiently large mass of the two-dimensional Achúcarro-Ortiz black hole.}
Apart from the Casimir energy, the purely extensive part of the total energy $E_E$ will also be modified due to the effect of self-gravitation. Thus, it takes the form

$$ E_E = \frac{r^2_+}{l^2} - \omega \left( \frac{M}{2 \left( M + \sqrt{M^2 - \frac{J^2}{l^2}} \right)} \right) $$

(36)

$$ = \frac{r^2_+}{l^2} - \epsilon_2 M \omega $$

(37)

whilst it can also be written as [61]

$$ E_E = \frac{a}{4 \pi R} S^2_{bh} $$

(38)

$$ = \frac{4 \pi a}{R} r^2_{out} $$

(39)

$$ = \frac{4 \pi a}{R} r^2_+ (1 - 2 \epsilon_1 \omega) . $$

(40)

We substitute expressions (29), (31) and (35) which were computed to first order in $\omega$ (in the framework of KKW analysis) in the Cardy-Verlinde formula in order that self-gravitational corrections to be considered,

$$ S_{CFT} = 2 \pi \sqrt{ab} \left( \frac{1}{J} \sqrt{ab} \right) \sqrt{\left( \frac{J^2}{2r^2_+} + 2 \epsilon_2 M \omega \right) \left( \frac{2r^2_+}{l^2} - 2 \epsilon_2 M \omega \right) } $$

(41)

and consequently, the first-order self-gravitationally corrected Cardy-Verlinde formula of the two-dimensional Achúcarro-Ortiz black hole takes the form

$$ S_{CFT} = S_{BH} \sqrt{1 + \epsilon_3 \omega} . $$

(42)

where

$$ \epsilon_3 = \frac{4 M l^2}{J^2} \left( \frac{r^2_+}{l^2} - \frac{J^2}{4 r^2_+} \right) \epsilon_2 $$

(43)

$$ = 2 M \left( \frac{2 \pi l^2}{J} \right)^2 \frac{T_H}{S_{BH}} $$

(44)

It is easily seen that the parameter $\epsilon_3$ is positive and therefore the self-gravitational corrections are also positive.

It should be pointed out that in the context of KKW analysis the self-gravitational corrections to the entropy as described by the Cardy-Verlinde formula ($S_{CFT}$) of the two-dimensional Achúcarro-Ortiz black hole are different from the ones to the corresponding Bekenstein-Hawking entropy ($S_{BH}$). This is expected since in order to evaluate the corrections to the entropy as described by the Cardy-Verlinde formula ($S_{CFT}$), we have taken into account not only corrections to the Bekenstein-Hawking entropy but also to all quantities appearing in the Cardy-Verlinde formula, i.e. the Hawking temperature ($T_H$), the
angular momentum \((J)\) and the corresponding angular velocity \((\Omega_+)\). Furthermore, the entropy of the two-dimensional Achúcarro-Ortiz black hole \((S_{CFT})\) described in the context of KKW analysis by the semiclassically corrected Cardy-Verlinde formula violates the holographic bound \([21]\), i.e.

\[ S_{CFT} > S_{BH} > S_{bh} . \]  

## 3 Conclusions

In this work we have evaluated the semiclassical corrections to the entropy of two-dimensional Achúcarro-Ortiz black hole as described by the Cardy-Verlinde formula. These corrections are due to the self-gravitation effect. They are derived in the context of KKW analysis and we have kept up to linear terms in the energy of the emitted massless particle. The afore-mentioned gravitational background is treated as a dynamical one and the self-gravitational corrections to its entropy are found to be positive. This result is a direct violation to the holographic bound.

A couple of comments are in order concerning this violation. Firstly, it is known that the entropy of BTZ black hole does not violate the entropy bounds. Thus, it is expected that the Achúcarro-Ortiz black hole which is derived by a dimensional reduction of the BTZ black hole and shares the same thermodynamical formulas with BTZ black hole, will also respect the entropy bounds. On the contrary, in the context of KKW analysis it was proven\(^4\) that the modified entropy of Achúcarro-Ortiz black hole is different even to first order from the corresponding modified entropy of BTZ black hole. Secondly, Mignemi recently claimed \([62]\) that the existence of a holographic bound depends on the dynamics of the specific model of gravity in contrast to the Bekenstein bound which is inherent to the definition of black hole thermodynamics in any metric theory of gravity. Therefore, our result provides evidence in support of the claim that the holographic bound is not universal for 2D gravity models.

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\(^4\)Compare results in \([18]\) with the corresponding ones in \([19]\).
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