Negative and positive refraction are not Lorentz covariant

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\textbf{Abstract}

The refraction of linearly polarized plane waves into a half-space occupied by a material moving at constant velocity was studied by directly implementing the Lorentz transformations of electric and magnetic fields. From the perspective of a co-moving observer, the moving material was a spatially local, pseudochiral omega material. Numerical studies revealed that whether or not negative refraction occurs in the moving material depends upon the speed of movement as well as the angle of incidence and the polarization state of the incident plane wave. Furthermore, the phenomena of negative phase velocity and counterposition in the moving material were similarly found not to be Lorentz covariant; both phenomena were also found to be sensitive to the angle of incidence and the polarization state of the incident plane wave.

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\section{Introduction}

The behaviour of plane waves at a planar interface between two disparate homogeneous mediums is a central topic in both fundamental and applied electrodynamics. In particular, the phenomenon of negative refraction \cite{1, 2, 3} has been the subject of intense research efforts for the past ten years, following experimental reports of this phenomenon in certain metamaterials \cite{4, 5}. Much of this effort has been motivated by the development of novel metamaterials, but negative refraction also arises in certain minerals \cite{6} and in biological structures \cite{7}. In addition, the prospects of negative refraction arising in relativistic scenarios — such as in uniformly-moving materials \cite{8, 9} or in strong gravitational fields \cite{10, 11, 12, 13} — is a matter of astrophysical and astronomical significance.

In the following we consider negative refraction induced by uniform motion. Earlier work relating to this topic relied on the Minkowski constitutive relations to describe the moving medium in a nonco–moving inertial reference frame \cite{14, 15, 16}, per the standard textbook approach \cite{17}. However, this approach is only appropriate to materials which are both spatially and temporally local \cite{18}. More recently, studies based on the uniform motion of realistic materials have been undertaken, using an approach in which the Lorentz transformations of the electric and magnetic fields are directly implemented \cite{8, 9}. These studies revealed

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that the phenomenons of negative phase velocity and counterposition\(^3\) — which are closely allied to negative refraction and similarly associated with certain metamaterials and relativistic scenarios — are not Lorentz covariant. Here we address the hitherto outstanding question: is negative (or positive) refraction Lorentz covariant? By means of a numerical analysis based on a uniformly moving pseudochiral omega material, we demonstrate that the answer to this question is ‘no’.

In the notation we adopt, 3–vectors are in boldface with the ` symbol denoting a unit vector. Double underlining signifies a 3×3 dyadic (i.e., a second rank Cartesian tensor) and the identity 3×3 dyadic is written as \[ \mathbb{I} = \hat{x} \hat{x} + \hat{y} \hat{y} + \hat{z} \hat{z}. \] The operators \( \text{Re} \) and \( \text{Im} \) deliver the real and imaginary parts of complex quantities; and \( i = \sqrt{-1}. \) The permittivity and permeability of free space are \( \epsilon_0 \) and \( \mu_0, \) respectively, with \( c_0 = 1/\sqrt{\epsilon_0 \mu_0} \) being the speed of light in free space.

## 2 Refraction into a moving pseudochiral omega material

### 2.1 Panewave analysis

Our attention is focussed on a spatially local, homogeneous material, characterized by the frequency–domain constitutive relations

\[
\begin{align*}
D' &= \epsilon' \cdot E' + \zeta' \cdot H' \\
B' &= \zeta' \cdot E' + \mu' \cdot H'
\end{align*}
\]

in the inertial reference frame \( \Sigma'. \) Herein, the 3×3 constitutive dyadics

\[
\begin{align*}
\epsilon' &= \epsilon_0 \begin{pmatrix}
\epsilon_x' & 0 & 0 \\
0 & \epsilon_y' & 0 \\
0 & 0 & \epsilon_z'
\end{pmatrix}, & \zeta' &= \frac{1}{c_0} \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}, \\
\mu' &= \mu_0 \begin{pmatrix}
\mu_{xx}' & 0 & 0 \\
0 & \mu_{yy}' & 0 \\
0 & 0 & \mu_{zz}'
\end{pmatrix}.
\end{align*}
\]

This is a bianisotropic, Lorentz–reciprocal [19] material, known as a pseudochiral omega material [20]. Constitutive relations of this form have been used to describe certain negatively refracting metamaterials, assembled from layers of split–ring resonators [21]. Several different designs of metamaterials are based on this general configuration [22, 23, 24]. As the pseudochiral omega material is presumed to be dissipative, the constitutive parameters \( \epsilon_{x,y,z}', \zeta' \) and \( \mu_{x,y,z}' \) are complex–valued functions of the angular frequency \( \omega'. \)

Suppose that the pseudochiral omega material fills the half-space \( z > 0, \) while the half–space \( z < 0 \) is vacuous. The inertial reference frame \( \Sigma' \) translates at constant velocity \( v = v \hat{v} \) with respect to the inertial reference frame \( \Sigma, \) in the plane of the interface \( z = 0. \) In keeping with an earlier study [9], we take \( \hat{v} = \hat{x}. \) The Lorentz transformations [17]

\[
\begin{align*}
E &= (E' \cdot \hat{v}) \hat{v} + \gamma \left( (\mathbb{I} - \hat{v}\hat{v}) \cdot E' - v \times B' \right) \\
B &= (B' \cdot \hat{v}) \hat{v} + \gamma \left( (\mathbb{I} - \hat{v}\hat{v}) \cdot B' + v \times E' \right) \\
H &= (H' \cdot \hat{v}) \hat{v} + \gamma \left( (\mathbb{I} - \hat{v}\hat{v}) \cdot H' + v \times D' \right) \\
D &= (D' \cdot \hat{v}) \hat{v} + \gamma \left( (\mathbb{I} - \hat{v}\hat{v}) \cdot D' - \frac{v \times H'}{c_0^2} \right)
\end{align*}
\]

with the real–valued scalars

\[
\gamma = \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta = \frac{v}{c_0},
\]

\(^3\)See Sec. 2.2 for definitions of negative refraction, negative phase velocity and counterposition.
relate the electromagnetic field phasors in the frame $\Sigma$ to those in the frame $\Sigma'$.

Now suppose that the vacuous half-space $z < 0$ contains a line source, which is stationary with respect to the frame $\Sigma$. The source extends infinitely in directions parallel to the $y$ axis, and it is located at a great distance from the interface $z = 0$. Let us consider one plane wave incident on the interface $z = 0$, as a representative of the angular spectrum of plane waves launched by the source. With respect to the frame $\Sigma$, this plane wave is described by the electric and magnetic field phasors

$$
E_i = e_i \exp[i (k_i \cdot r - \omega t)], \quad H_i = h_i \exp[i (k_i \cdot r - \omega t)], \quad z \leq 0.
$$

(5)

Herein, the wavevector

$$
k_i = \kappa \hat{x} + k_0 \cos \theta \hat{z},
$$

(6)

with the real-valued scalar

$$
k = k_0 \sin \theta \in (-k_0, k_0),
$$

(7)

the free-space wavenumber $k_0 = \omega \sqrt{\varepsilon_0 \mu_0}$ and $\omega$ being the angular frequency with respect to $\Sigma$.

With respect to the frame $\Sigma'$, the incident plane wave is represented by

$$
E_i' = e_i' \exp[i (k_i' \cdot r' - \omega t')], \quad H_i' = h_i' \exp[i (k_i' \cdot r' - \omega t')], \quad z \leq 0,
$$

(8)

wherein the phasor amplitudes $\{e_i', h_i'\}$ are related to $\{e_i, h_i\}$ via the Lorentz transformations (3), while

$$
k_i = \gamma \left(k_i' \cdot \hat{v} + \frac{\omega'}{c_0} \hat{v} + (I - \hat{v} \hat{v}) \cdot k_i'\right),
$$

$$
r = \left[I + (\gamma - 1) \hat{v} \hat{v}\right] \cdot r' + \gamma v t',
$$

$$
\omega = \gamma (\omega' + k_i' \cdot \hat{v}),
$$

$$
t = \gamma \left(t' + \frac{v \cdot r'}{c_0^2}\right).
$$

(9)

The incident plane wave gives rise to two refracted plane waves in the half-space $z > 0$, and one reflected plane wave in the half-space $z < 0$. In the frame $\Sigma'$, the refracted plane waves are represented by the electric and magnetic phasors

$$
E_i' = e_i' \exp[i (k_i' \cdot r' - \omega t')], \quad H_i' = h_i' \exp[i (k_i' \cdot r' - \omega t')], \quad z \geq 0, \quad (j = 1, 2),
$$

(10)

wherein the wavevectors

$$
k_{tj} = (k_i'^t \cdot \hat{x}) \hat{x} + k_{zj} \hat{z}, \quad (j = 1, 2)
$$

(11)

comply with Snell’s law [17]. The wavevector components $k_{zj}'$ as well as the relationships between the phasor amplitudes $e_{tj}'$ and $h_{tj}'$, are deduced by combining the constitutive relations (1) with the source-free Maxwell curl postulates in $\Sigma'$ [17]. We find [29]

$$
k_{z1}' = \omega' \sqrt{\varepsilon_0 \mu_0} \mu_x \left(\epsilon_x' \left(\frac{(k_i' \cdot \hat{x})^2}{\omega'^2 \varepsilon_0 \mu_0} - \frac{(k_i' \cdot \hat{x})^2}{\omega'^2 \varepsilon_0 \mu_0}ight)\right)
$$

(12)

$$
k_{z2}' = \omega' \sqrt{\varepsilon_0 \mu_0} \epsilon_z' \left(\epsilon_{zj}' \mu_y' - \xi'^2\right) - \frac{(k_i' \cdot \hat{x})^2}{\omega'^2 \varepsilon_0 \mu_0}.
$$
Notice that since \( k''_{zj} \) are generally complex-valued, the refracted plane waves are nonuniform.

The reflected plane wave is represented by the electric and magnetic phasors

\[
\begin{align*}
E'_r &= e'_r \exp[i(\mathbf{k}'_r \cdot \mathbf{r}' - \omega t')] \\
H'_r &= h'_r \exp[i(\mathbf{k}'_r \cdot \mathbf{r}' - \omega t')] 
\end{align*}
\]

in the frame \( \Sigma' \), with the wavevector of the reflected plane wave being

\[
k'_r = (\mathbf{k}'_r \cdot \hat{x}) \hat{x} - k''_{zr} \hat{z}.
\]

The source-free Maxwell curl postulates in \( \Sigma' \) yield an expression for \( k''_{zr} \) and relationships between the phasor amplitudes \( e'_r \) and \( h'_r \).

By invoking the standard boundary conditions across the plane \( z = 0 \), i.e., [17]

\[
\begin{align*}
(e'_r + e''_r) \cdot \hat{x} = e'_{ij} \cdot \hat{x} & \quad (e'_r + e''_r) \cdot \hat{y} = e'_{ij} \cdot \hat{y} \\
(h'_r + h''_r) \cdot \hat{x} = h'_{ij} \cdot \hat{x} & \quad (h'_r + h''_r) \cdot \hat{y} = h'_{ij} \cdot \hat{y}
\end{align*}
\]

the phasor amplitudes \( \{e'_r, h'_r\} \) and \( \{e''_r, h''_r\} \) can be found. The reflected and the refracted plane waves in the frame \( \Sigma \) may then be deduced by applying the Lorentz transformations (3) and (9).

We represent the refracted plane wave in the frame \( \Sigma \) by the electric and magnetic field phasors

\[
\begin{align*}
E_t &= e_{it} \exp[i(\mathbf{k}_{ij} \cdot \mathbf{r} - \omega t)] \\
H_t &= h_{it} \exp[i(\mathbf{k}_{ij} \cdot \mathbf{r} - \omega t)] 
\end{align*}
\]

wherein the wavevectors

\[
k_{ij} = \kappa \hat{x} + k_{zj} \hat{z}, \quad (j = 1, 2),
\]

while the corresponding reflected plane wave is represented by

\[
\begin{align*}
E_r &= e_r \exp[i(\mathbf{k}_r \cdot \mathbf{r} - \omega t)] \\
H_r &= h_r \exp[i(\mathbf{k}_r \cdot \mathbf{r} - \omega t)]
\end{align*}
\]

with the wavevector

\[
k_r = \kappa \hat{x} - k_0 \cos \theta \hat{z}.
\]

### 2.2 Negative refraction, negative phase velocity and counterposition

We highlight three phenomena which the uniformly moving pseudochiral omega medium can support: namely, negative refraction, negative phase velocity and counterposition. All three phenomena are closely associated with certain metamaterials [29]. We consider these phenomena referred to the frame \( \Sigma \).

First, in the scenario under consideration here, negative refraction occurs when the real part of the wavevector \( k_{zj} \) casts a negative projection onto the positive axis \( z \); i.e., when \( \text{Re} \, k_{zj} < 0 \) [2]. Second, the phase velocity of the refracted wave is classified as negative when the real part of the wavevector \( k_{zj} \) casts a negative projection onto the corresponding cycle-averaged Poynting vector \( \mathbf{P}_j \) [1]. For our purposes here, it suffices to consider \( \mathbf{P}_j \) in the limit \( |\mathbf{r}| \to 0 \), as provided by

\[
\mathbf{P}_j \big|_{|\mathbf{r}| \to 0} = \frac{1}{2} \left[ (\text{Re} \, e_{ij}) \times (\text{Re} \, h_{ij}) + (\text{Im} \, e_{ij}) \times (\text{Im} \, h_{ij}) \right], \quad (j = 1, 2).
\]

The signs of the square roots in eqs. (12) are selected in order to ensure that \( \mathbf{P}_j \) casts a positive projection onto the positive \( z \) axis. Thereby, energy flow directed into the \( z > 0 \) half-space is assured. Third, counterposition occurs when the real part of the wavevector \( k_{zj} \) and the corresponding cycle-averaged Poynting vector \( \mathbf{P}_j \)
are oriented on opposite sides to the normal to the interface \( z = 0 \) \[25, 26, 27\]. Because here \( \mathbf{P}_j \cdot \hat{z} > 0 \), the conditions for counterposition are

\[
(\text{Re} \mathbf{k}_{ij} \cdot \hat{z}) (\mathbf{P}_j \cdot \hat{x}) < 0 \quad \text{for} \ \kappa > 0
\]
\[
(\text{Re} \mathbf{k}_{ij} \cdot \hat{z}) (\mathbf{P}_j \cdot \hat{x}) > 0 \quad \text{for} \ \kappa < 0
\]

\[
(j = 1, 2).
\]  

(21)

For uniform plane wave propagation in an isotropic dielectric material, negative refraction and negative phase velocity are effectively synonymous \[1, 2\]. However, this is not the case for more complex materials \[28\]. In particular, for the pseudochiral omega material considered here, it has been established that negative refraction, negative phase velocity and counterposition are independent phenomenons \[29\].

### 2.3 Numerical studies

We now explore the phenomenons of negative refraction, negative phase velocity and counterposition for a specific numerical example, chosen to allow direct comparison with an earlier study \[29\]. Let the pseudochiral omega medium occupying the half-space \( z > 0 \) be characterized by the constitutive parameters: \( \epsilon'_e = 0.1 + 0.03i; \ \epsilon'_p = 0.14 + 0.02i; \ \epsilon'_s = 0.13 + 0.07i; \ \mu'_x = -0.29 + 0.09i; \ \mu'_y = -0.18 + 0.03i; \ \mu'_z = -0.17 + 0.6i; \) and \( \zeta' = 0.11 + 0.05i \).

In the following we consider two polarization states for the incident plane wave. As described in the Appendix, the incident \( s \)-polarization state, as characterized by

\[
\mathbf{e}_i = a_s \mathbf{s} \equiv a_s \hat{y},
\]

gives rise to the refracted plane wave with wavevector \( \mathbf{k}_{r1} \), whereas the incident \( p \)-polarization state, as characterized by

\[
\mathbf{e}_i = a_p \mathbf{p}_+ \equiv a_p (-\cos \theta \hat{x} + \sin \theta \hat{z}),
\]

gives rise to the refracted plane wave with wavevector \( \mathbf{k}_{r2} \). The corresponding reflectances and transmittances are presented in the Appendix.

The orientation angle of the real part of the wavevector \( \mathbf{k}_{ij}, (j = 1, 2) \), as defined by \( \tan^{-1}(\kappa / \text{Re} k_{zj}) \), is plotted in Fig. 1 versus the relative speed \( \beta \in (-1, 1) \) for the angle of incidence \( \theta \in \{0^\circ, 5^\circ, 25^\circ\} \). For normal incidence, we see that \( \text{Re} \mathbf{k}_{ij} \) is also normal to the \( z = 0 \) interface, for both incident \( s \)- and \( p \)-polarization states. However, for \( \theta = 5^\circ \) and \( \theta = 25^\circ \), the orientation of \( \text{Re} \mathbf{k}_{ij} \) relative to the interface normal is highly sensitive to \( \beta \), for both incident \( s \)- and \( p \)-polarization states. For example, for \( \theta = 5^\circ \) with \( \mathbf{e}_i = \mathbf{s} \), negative refraction occurs for \(-1 < \beta < -0.03\) and for \(0.21 < \beta < 1\), and the refractive is positive otherwise. Similarly, for \( \theta = 25^\circ \) with \( \mathbf{e}_i = \mathbf{s} \), and for \( \theta \in \{5^\circ, 25^\circ\} \) with \( \mathbf{e}_i = \mathbf{p}_+ \), whether the refraction is negative or positive depends upon the relative speed \( \beta \). The real part of \( \mathbf{k}_{ij} \) is oriented normally to the \( z = 0 \) interface in the limits \( \beta \to \pm 1 \), for both \( j = 1 \) and \( 2 \).

In Fig. 2, the quantity \( \mathbf{P}_j \big|_{|r| = 0} \cdot \text{Re} \mathbf{k}_{ij}, (j = 1, 2) \), which determines whether the phase velocity is positive or negative, is plotted against relative speed \( \beta \in (-1, 1) \) for the angle of incidence \( \theta \in \{0^\circ, 5^\circ, 25^\circ\} \). The numerical results echo those of Fig. 1 insofar as the sign of the phase velocity is highly sensitive to \( \beta \) for both states of incident polarization, for \( \theta \in \{5^\circ, 25^\circ\} \). Unlike the case of negative/positive refraction, the sign of the phase velocity is highly sensitive to \( \beta \) even at normal incidence, for both the \( s \)- and the \( p \)-polarization states of the incident plane wave. In the limits \( \beta \to \pm 1 \), \( \mathbf{P}_j \big|_{|r| = 0} \) becomes oriented parallel to the interface \( z = 0 \) for both the \( s \)- and the \( p \)-polarization states of the incident plane wave and the phase velocity of the refracted plane wave therefore becomes orthogonal.

Lastly, we turn to the quantity \( (\mathbf{P}_j \cdot \hat{x}) \text{Re} (\mathbf{k}_{ij} \cdot \hat{z}) \), which determines whether or not the cycle–averaged Poynting vector and the real part of the wavevector for the refracted plane wave are counterposed, per the conditions (21). This quantity is plotted against relative speed \( \beta \in (-1, 1) \) for the angle of incidence \( \theta \in \{0^\circ, 5^\circ, 25^\circ\} \) in Fig. 3. In a manner similar to that represented by Figs. 1 and 2, we see that the sign of \( (\mathbf{P}_j \cdot \hat{x}) \text{Re} (\mathbf{k}_{ij} \cdot \hat{z}) \) is highly sensitive to \( \beta \) for both \( j = 1 \) and \( 2 \). Furthermore, for all angles of incidence considered with \( \mathbf{e}_i = \mathbf{s} \) and with \( \mathbf{e}_i = \mathbf{p}_+ \), counterposition arises in the limit \( \beta \to 1 \) whereas it does not arise in the limit \( \beta \to -1 \).
3 Concluding remarks

The main conclusions to be drawn from this study are that, for the uniformly moving pseudochiral omega material under consideration,

(i) negative refraction is not Lorentz covariant;
(ii) negative phase velocity is not Lorentz covariant; and
(iii) counterposition is not Lorentz covariant.

While conclusions (ii) and (iii) are consistent with findings previously reported for a uniformly moving isotropic dielectric material [9, 14], conclusion (i) reveals a previously unknown facet of refraction. This has far reaching consequences for researchers exploring the refraction of electromagnetic waves in astrophysical applications, as well as those investigating fundamental aspects of electromagnetic theory. In particular, the absence of Lorentz covariance of negative refraction further vindicates the 3+1 approach to electromagnetic wave propagation in strong gravitational fields, which has recently been usefully exploited to elucidate gravitationally induced negative phase velocity in vacuum [10, 11, 12, 13].

Appendix

Here we present the reflectances and transmittances for the reflection–transmission scenario described in Sec. 2. In terms of linearly polarized states, let the amplitude of the electric phasor of the incident plane wave be expressed as

\[ \mathbf{e}_i = a_s \mathbf{s} + a_p \mathbf{p}_+, \]  

where the unit vectors \( \mathbf{s} = \hat{y} \) and \( \mathbf{p}_+ = -\hat{x} \cos \theta + \hat{z} \sin \theta \). The corresponding vector amplitude for the reflected plane wave may be written as

\[ \mathbf{e}_r = r_s \mathbf{s} + r_p \mathbf{p}_-, \]  

where the unit vector \( \mathbf{p}_- = \hat{x} \cos \theta + \hat{z} \sin \theta \), while for the refracted plane wave we have

\[ \mathbf{e}_t = t_1 \mathbf{s} + t_2 \mathbf{p}_-, \]  

where the vector \( \mathbf{p} \) lies in the plane of incidence and satisfies \( \mathbf{p} \cdot \mathbf{p}^* = 1 \). The relative amplitudes \( r_{s,p}/a_{s,p} \) and \( t_{1,2}/a_{s,p} \) may be deduced by following the strategy described in Sec. 2.1; i.e., by transforming to the frame \( \Sigma' \), then invoking the boundary conditions (15), and finally transforming back to the frame \( \Sigma \). By so doing, we find that

\[ \begin{align*}
a_s &= 0 \implies r_s = t_1 = 0 \\
a_p &= 0 \implies r_p = t_2 = 0
\end{align*} \]  

Thus, there are no cross–polarization terms.

For the particular example considered in Sec. 2.3, the reflectances \( |r_{s,p}/a_{s,p}|^2 \) and \( |r_p/a_p|^2 \), and the transmittances \( |t_1/a_s|^2 \) and \( |t_2/a_p|^2 \), are plotted against relative speed \( \beta \in (-1, 1) \) in Fig. 4 for the angle of incidence \( \theta \in \{0^\circ, 5^\circ, 25^\circ\} \). We note that, for all angles of incidence considered, the reflectance \( |r_p/a_p|^2 \) tends to unity in the limits \( \beta \to \pm 1 \) whereas this is not the case for the reflectance \( |r_s/a_s|^2 \). In a similar vein, for all angles of incidence considered, the transmittance \( |t_1/a_s|^2 \) tends to zero in the limits \( \beta \to \pm 1 \) whereas the transmittance \( |t_2/a_p|^2 \) does not.

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Figure 1: The orientation angle of $\mathbf{R}_{\mathbf{k}}^j$ (in degree) plotted against relative speed $\beta \in (-1, 1)$ for the angles of incidence $\theta = 0^\circ$ (broken curve, red), $\theta = 5^\circ$ (solid curve, green), and $\theta = 25^\circ$ (broken dashed curve, blue). Plots are shown for the $s$–polarization state ($j = 1$) and the $p$–polarization state ($j = 2$) of the incident plane wave.

Figure 2: As Fig. 1 except that the quantity plotted against $\beta$ is $P_j \cdot \mathbf{R}_{\mathbf{k}}^j$ (normalized), ($j = 1, 2$).
Figure 3: As Fig. 1 except that the quantity plotted against $\beta$ is $(P_j \cdot \hat{x}) \text{ Re } (k_{tj} \cdot \hat{z})$ (normalized), $(j = 1, 2)$.

Figure 4: As Fig. 1 except that the quantities plotted against $\beta$ are the reflectances $|r_s/a_s|^2$ and $|r_p/a_p|^2$, and the transmittances $|t_1/a_s|^2$ and $|t_2/a_p|^2$. 