Wave propagation in elastic bi-materials with a doubly periodic array of interface cracks

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Abstract. In the present study, the scattering of plane elastic waves by a doubly periodic array of planar interface cracks is investigated. The problem is solved using the boundary integral equation method. The influences of the shape of cracks and lattice in the periodic array on wave transmission through an interface are studied. It is revealed that introduction of the periodic array of interface cracks allows to increase wave energy transmission compared to the interface without cracks. It is shown that the phenomenon is related to cut-off frequencies location and decrease in the amplitudes of the crack-opening-displacements.

1. Introduction

Manipulation of ultrasonic waves is an important issue for modern communications, sound insulation, vibration reduction, sensing technologies [1, 2]. Elastic composites with periodically arranged components called acoustic/elastic metamaterials or phononic crystals can be used for wave energy manipulation [3, 4, 5]. Though voids and inclusions are usually employed in elastic metamaterials to adjust wave propagation through it, cuts, cracks or slits artificially introduced into an elastic structure can also be applied for this purpose [6, 7].

Reflection and blocking of elastic waves by periodic or random distributions of crack-like defects have been intensively investigated in recent years [7, 8, 9, 10, 11]. In the present work, introduction of a doubly periodic array of planar cuts or cracks to change transmission/reflection properties of an interface between two dissimilar elastic half-spaces is investigated.

2. Mathematical model

A doubly periodic array of cracks is located at the interface $x_3 = 0$ between two dissimilar isotropic elastic half-spaces $V_1$ ($x_3 < 0$) and $V_2$ ($x_3 > 0$) as shown in Fig. 1. The material properties of the two elastic half-spaces are determined by the mass densities $\rho_j$, longitudinal and transverse wave velocities $v_{Lj}$ and $v_{Tj}$ ($j = 1, 2$). All interface cracks or cuts $\Omega^{(j_1j_2)}$ have the same arbitrary shape and size. Here, the superscripts $j_1$ and $j_2$ denote the number of the cell, where the crack $\Omega^{(j_1j_2)}$ is situated. The cracked media can be divided into unit-cells, each crack lies in one cell of the oblique lattice given by the parameters $s_1, s_2, s_3$ as shown in Fig. 1b.
Propagation of plane waves normally incident from lower half-space $V_1$ is considered. The total displacement field $u$ is a sum of the incident field $u^{in}$ and the wave-field $u^{out}(x)$ scattered by each crack $\Omega^{(j_1j_2)}$ in the doubly periodic array. According to the periodicity of cracks, the total scattered wave-field $u^{sc}(x)$ satisfies the equality

$$u^{sc}(x_1, x_2, x_3) = u^{sc}(x_1 - s_1j_1 - s_2j_2, x_2 - s_3j_2, x_3), \quad j_1, j_2 \in \mathbb{Z}. \quad (1)$$

The cracked interface can be virtually divided into unit-cells $G^{(j_1j_2)}$, where the reference unit-cell $G^{(00)}$ is centered at the origin of the coordinate system. The displacement vector satisfies the governing equations of wave motion. The solution is obtained using the boundary integral equation method [12, 8]. The wave scattering problem is reduced to a boundary integral equation with an unknown crack-opening-displacement (COD) of the reference crack $\Omega^{(00)}$.

The periodicity of the cracks location leads to the occurrence of cut-off frequencies, which are determined by the properties of the lattice and wave velocities $v_{kj}$ ($k = T, L$) in the elastic half-spaces $V_j$ as follows [7]:

$$\omega^{(j_1j_2)}_{kj} = \frac{2\pi v_{kj}}{s_1s_2} \sqrt{(s_2j_1)^2 + (s_1j_2 - s_3j_1)^2}, \quad j_1, j_2 = 0, 1, 2, \ldots \quad (2)$$

For studying the wave propagation in a layered media with cracks, an analysis of elastic wave energy transmission is provided. Let us introduce the energy flux transferred by the incoming plane wave through the interface in the absence of cracks $E_0$ and the energy flow transmitted through the interface $E^+$ and reflected by the interface $E^-$ (see [7] for more details). The energy reflection and transmission coefficients are defined as: $\kappa^\pm = E^\pm/E^0$. Another important characteristic of the scattering called the energy scattering coefficient $\Xi$ and used further in analysis is defined as the ratio of the energy flux scattered by the cracks to the energy $E_0$.

3. Results and discussion

At first, the influences of the shape of cracks, the type of lattice and the frequency $\omega$ on the wave propagation through an interface with a doubly periodic array of cracks are analysed. Fig. 2 demonstrates the energy transmission coefficient $\kappa^+(\omega)$ and the energy scattering coefficient $\Xi(\omega)$ for a doubly periodic array of square, rectangular and circular cracks. In the numerical calculations, all the crack distributions are assumed to have the same crack density $C = S(G^{(j_1j_2)})/S(G^{(j_1j_2)} \cap \{x_3 = 0\}) = 0.5625$, and they are situated at the interface between aluminium and brass ($\rho_1 = 2700 \text{ kg/m}^3$, $\rho_2 = 8500 \text{ kg/m}^3$, $v_{1L} = 6198 \text{ m/s}$, $v_{1L} = 4240 \text{ m/s}$, $v_{2T} = 3122 \text{ m/s}$ and $v_{2T} = 2140 \text{ m/s}$). Two different lattices are considered here: square ($s_1 = s_2$, $s_3 = 0$ in Figs. 2a,c) and oblique ($s_1 = s_2$, $s_3 = 0.25s_2$ in Figs. 2b,d). In both cases, one can see a monotonic decay of $\kappa^+(\omega)$ and increase of $\Xi(\omega)$ at lower frequencies up to the first cut-off frequency $\omega_{T2}^{(01)}$, which is independent on the shape and the crack density $C$. At the frequencies

![Figure 1. An interface with a doubly periodic array of rectangular cracks (a) and unit-cell of the periodic array (b).](image_url)
Figure 2. The energy transmission coefficient $\kappa^+$ and the energy scattering coefficient $\Xi$.

Figure 3. Components of the COD $\Delta u_k$ for square, rectangular and circular cracks at $k_{T1}s_1 = 4.2$ (full reflection) and $k_{T1}s_1 = 4.32$ (full transmission).

slightly higher than the first cut-off frequency $\omega_{T2}^{(01)}$, vice versa, a sharp increase of $\kappa^+(\omega)$ and a sharp decrease of $\Xi(\omega)$ are observed after the first cut-off frequency $\omega_{T2}^{(01)}$. A similar behaviour in the coefficients $\kappa^+(\omega)$ and $\Xi(\omega)$ is also observed at higher frequencies, where most local extrema
of the energy transmission coefficients appear near cut-off frequencies.

A horizontal dashed line in Fig. 2 shows the value of the transmission coefficient $k^+ = k^+(0)$ in the absence of cracks. The values of the transmission coefficients for square and oblique lattices have a maximum at $k_{T1}s_1 = 4.32$, which is slightly above the cut-off frequency $\omega_{T2}^{(0)}$. In a certain frequency range in the vicinity of $k_{T1}s_1 = 4.32$, the transmission coefficient becomes larger than that in the absence of the cracks. Let us consider the COD at frequencies $k_{T1}s_1 = 4.2$ and $k_{T1}s_1 = 4.32$ near the cut-off frequency $\omega_{T2}^{(0)}$, where nearly full reflection and full transmission are respectively observed. Since the coefficient $\Xi$ has its local minimum at the frequency, where almost full transmission is possible, it can be concluded that transmission is accompanied by a lower scattering at the cracks (smaller amplitudes of the COD). To reveal and analyse this effect more detailed, the surface plots of three components $\Delta u_k$ of the COD for square, rectangular and circular arrays of cracks are given in Fig. 3 for the square lattice ($s_1 = s_2, s_3 = 0$). Indeed, the wave-forms for the two components $\Delta u_1$ and $\Delta u_2$ of the COD corresponding to in-plane wave motion are very similar in both cases in contrast to $\Delta u_3$. In the case of full transmission, the out-of-plane component $\Delta u_3$ of the COD has approximately the same values as the in-plane components $\Delta u_1$ and $\Delta u_2$, but several times smaller compared with $\Delta u_3$ in the case of full reflection at $k_{T1}s_1 = 4.2$.

4. Conclusions

Though the introduction of the periodic array of cracks at the interface leads to extra wave reflections, narrow frequency ranges, where the wave transmission is increased due to the periodic array of cracks, are revealed. It is shown that this effect is related with the cut-off frequencies of the waveguide composed of two semi-infinite rod-like structures with a cross-section in the form of the unit-cell of the considered doubly periodic array of cracks. It should be noted, that the COD at frequencies slightly below the cut-off frequency are much larger compared to that at the frequencies in the narrow frequency range, where a nearly full transmission is possible.

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