Mathematical modeling of mutual operation of superconductive devices in autonomous power engineering

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Abstract. The paper presents results of mathematical modeling of mutual operation of electromechanical energy converters: high temperature superconductive (HTSC) synchronous generators and motors. It includes modes of operation of synchronous alternator and synchronous motor, mutual operation of alternators of comparable rating in an autonomous energy installation, mutual operation of synchronous HTSC motor and semi-conductive converter, influence of semi-conductive converter on the loss value in HTSC armature windings. In all cases specific features of HTSC electrical machines and their parameters and peculiarities of HTSC windings are accounted for with imposed restrictions on the current and magnetic flux variation velocity.

1. Introduction
Investigations of electro-technical devices, comprising elements with high-temperature superconductors (HTSC) display maximum economical effect in HTSC complexes and not in individual units. A complex may operate with air-liquefier, supplying all the installation with LN₂. Our team took part in the development of a number of autonomous electric power engineering complexes, intended for different purposes and including several HTSC devices. Their investigations revealed problems, associated with mutual operation of HTSC electrical machines or HTSC devices and semi-conductive converters. Presented below is the general view of autonomous energy installation (Figure 1,a) with synchronous alternator, comprising HTSC saddle-shape armature winding (Figure 1,b) and synchronous motor of a disc type with circular HTSC armature coils (Figure 1,c). These electrical machines and their mutual operation served the base for mathematical modeling, presented below.
2. Mutual operation of HTSC synchronous alternator and synchronous motor

Let us consider peculiarities of parallel operation of SC synchronous generator and SC synchronous motor of similar rating without a voltage regulator. Both machines are non-saturated and non-salient pole.

It should be noted that in contrast with conventional synchronous machines the value of synchronous reactance may be varied in a wide range from 0.1 to over 2.5 p.u. [1] without affecting the size and weight parameters. Therefore presented below is the way to obtain maximum rating and overload capacity in HTSC machines, operating in parallel.

Figure 2 presents the simplified diagrams of both alternator and motor ($x_G$ and $x_M$ – generator and motor synchronous reactances; $E_{0G}$ and $E_{0M}$ – vectors of EMF, induced by the rotor flux in armature windings, $\delta$ – load angle, $\varphi$ – the angle between the voltage and current vectors) [2].

Let us accept that superconductive (SC) synchronous generator and SC synchronous motor are connected by SC cable without intermediate resistance. Then according to vector diagrams (Figure 3)

\[ E_{0G} = U + j f x_G, \]  

(1)
\[ E_{M} = U + j \delta x_M. \]  

The following parameters may be obtained from the diagram (Figure 3) [3]:

\[ \frac{1}{2} E_{O_M} E_{O_M} \sin \delta = \frac{1}{2} UI (x_g + x_m) \cos \psi, \]  

\[ P_g = P_m = mUI \cos \psi, \]  

\[ P_{Qg} = P_{Qm} = \frac{m F_{Qg} F_{OM}}{x_g + x_m} \sin \delta. \]  

**Figure 3.** Vector diagram for parallel operation of synchronous alternator and synchronous motor

Maximum rating is achieved at \( \sin \delta = 1 \), that is, at \( \delta = \pi/2 \):

\[ P_{Q_{max}} = \frac{m F_{Qg} F_{OM}}{x_g + x_m}, \]  

The overload capacity is determined by the safety coefficient of the current in HTSC armature windings, which usually equals 1.5. Therefore the overload capacity is not to exceed this value.

Accounting for the possibility to vary the synchronous resistance on design stage, it is possible to choose the optimal value for both the alternator and motor with HTSC windings to obtain the maximum rating and overload capacity value with \( \delta_g + \delta_m = \pi/2 \) (6).

It should be noted that unlike parallel operation with the network of infinite power the rating of individual synchronous alternator or motor is determined not only by its excitation current value, but by the exaltation of other machines operating in parallel with it. The presented vector diagram permits to determine the values of the voltage, the armature current and \( \cos \varphi \). In case \( \delta = \pi/2 \) the expressions are very simple [3].

If the voltage changes and the frequency of rotation remains constant, the following vector diagram may be used for calculations (Figure 4).
The presented in Figure 4 modes of operation show, that the voltage increase results in variation of maximum rating and overload capacity with certain increase of the load angle from $\delta=\delta_1$ to $\delta=\pi/2$ and then decrease to $\delta_2$ and $\delta_3$.

3. Parallel operation of synchronous alternators of similar rating

Let us consider 2 similar synchronous alternators. It is well-known: to transfer the load from one generator to the other at mutual rating $P=\text{const}$, $U=\text{const}$ and $f=\text{const}$ it is necessary to accelerate the rotor of the alternator I and to decelerate the rotor of the alternator II (with $f=\text{const}$) [4, 5]. As the rating of the generator I increases due to load angle increase, the synchronizing rating will tend to return the rotor to the initial position. The velocity of the process depends on the steepness of the synchronizing power density curve. The synchronizing rating equals:

$$P_s = \frac{mF_U}{x_L} \cos \delta. \tag{7}$$

Vector diagram in Figure 8,a shows the initial mode of operation of both alternators. During the load transfer vectors $E_{oI}$ and $E_{oII}$ move around the arc of a circus (Figure 8,b). The load angle and the armature current of the alternator being loaded increase and respectively the same parameters of the unloaded alternator decrease. The velocity of armature current variation in both alternators must correspond to the current change velocity $dI/dt$, permitted for the superconductor applied. The important moment of the process lies in the fact that the current variation in HTSC armatures may be relatively slow due to these restrictions. The rating of the first generator increases, the rating of the second generator decreases according to the preliminary specified program.

Both alternators are to have equal values of $\cos \phi$, therefore the rate of EMF increase of the first generator and its decrease for the second generator are to keep the voltage value constant. It is necessary to vary the excitation currents as well (Figures 8,b,c). The rate of excitation current variation may be limited in case the rotor contains HTSC windings.

As a result, to transfer the rating from one alternator to the other it is necessary to accelerate the rotor of the alternator being loaded along with the increase of the excitation current and to slow the rotor of the generator being unloaded with subsequent decrease of its excitation current. The rate of these changes must keep the frequency and the voltage constant.
The synchronizing rating curve steepness depends on the parameters, obtained during the process of HTSC alternator designing: synchronous reactance $x_d$ and number of phases $m$. The curves are presented in Figure 9. The lower is synchronous reactance and the higher is the phase number, the faster is the stabilizing process.

It may happen that the synchronizing process will be relatively fast due to low synchronous reactance or increased phase number, but the process of the rating transfer will take increased time because of restrictions, imposed by the HTSC wire applied for windings manufacturing.

4. Mathematical model of HTSC motor operating with frequency converter and voltage inverter
The model was developed for the motor with permanent magnets on the rotor and HTSC armature windings fed from the frequency converter with voltage inverter. The developed functional model of the motor control is presented in Figure 10. The task for the shaft frequency of rotation is preliminary being received by the unit permitting to establish the desirable intensity of its growth and then by proportional integral frequency of rotation controller (PI-controller). The signal of rotor frequency is obtained by the PI-regulator as well. Depending on the disagreement the signal from the exit of PI-
The regulator is being sent to the unit of current calculation. The same unit receives the signal from the rotor position sensor. The unit forms the task for the phase A, B, and C currents with reference to the current rotor position.

The maximum motor torque value is obtained when the angle between the flux vector and overall current vector equals $\frac{\pi}{2}$. Zero angle value corresponds to the agreement of the above mentioned vectors. The data forms the base for the vectors of motor current calculations.

In the next unit the tasks for phase currents are being sent to the channel, forming the voltages of armature winding. Then the tasks for currents are being transformed into the pulses of the power switch of the inverter control in the units of pulse width modulation. Modeling of the motor modes of operation was based on application of simulation package Power System Simulink (MATLAB). The model is based on standard blocks of the package.

**Figure 7.** Structural scheme of synchronous motor.

The model used for calculation is not presented as it is relatively huge. Figures 11-12 present the calculation results for the motor start by screw characteristics $n = n_s$ (graphs for $W_m$ - velocity, $M_m$ - torque, $I_{abc}$ - phase current, $U_{ab}$ - motor linear voltage).

There is witnessed a certain armature current value jump. In our case it does not exceed the safety coefficient with reference to the superconductor critical current. But it should be accounted for to exclude transition of the superconductor to the normal state.
Figure 8. The calculation results for the motor start by screw characteristics.

Figure 9. The calculation results for the motor start by screw characteristics.

5. Modelling of influence of semi-conductive converter on the level of loss in HTSC armature winding

It is well-known that discreteness of operation of semi-conductive converter elements results in appearance of high harmonics in current and voltage curves. In the model the converter is accomplished as a three-phase bridge circuit with f smoothing filter and current inverter. Results of modeling of operation of HTSC windings with a simi-conductive converter for the no-load of the converter and rated load are presented in Figures 13-14 (a – phase current curves, b – their spectral content). Spectral content, corresponding to the no-load mode, contains except $k = 6n \pm 1$ harmonics of the order $k = 2$, $k = 3$, $k = 4$, $k = 6$. As the load increases, the even harmonics start to disappear and the 3-d harmonic decreases with reference to the 1-st one.

Numerical and relative values of current harmonics from mains supply for the no-load mode are presented in Table 1.
Table 1. Numerical and relative values of current harmonics for the no-load mode

| $K$ | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 11  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $I_k$, A | 12,1 | 1,2 | 0,6 | 0,25 | 7,09 | 0,25 | 2,4 | 1,6 |
| $I_k$, p.u. | 1,0 | 0,1 | 0,05 | 0,02 | 0,58 | 0,02 | 0,2 | 0,135 |

Numerical and relative values of current harmonics from mains supply for the rated mode are presented in Table 2.

Table 2. Numerical and relative values of current harmonics for the rated mode

| $K$ | 1   | 3   | 5   | 7   | 11  |
|-----|-----|-----|-----|-----|-----|
| $I_k$, A | 82,0 | 1,48 | 14,8 | 6,71 | 1,48 |
| $I_k$, p.u. | 1,0 | 0,018 | 0,18 | 0,08 | 0,018 |

Figure 10. Operation mode simulation results of the converter at no-load duty.
Ia, Ib

Figure 11. Results of modeling the rated operating mode of the converter.

Therefore the results of modeling show that the high harmonic content will not result in substantial loss in HTSC armature windings under load but it can cause increased losses in the no-load mode. The comparison of the loss values is to be compared for each particular case of mutual operation of HTSC synchronous motor and a semi-conductive converter. Therefore the results of modeling show that the high harmonic content will not result in substantial loss in HTSC armature windings under load but it can cause increased losses in the no-load mode. The comparison of this loss values is to be compared for each particular case of mutual operation of HTSC synchronous motor and a semi-conductive converter.

6. Conclusions
The results of modeling of mutual operation of HTSC synchronous alternators and synchronous motors in autonomous power installations show that they can fulfill the main requirements demanded the electrical equipment modes of operation, but there exist a lot of limitations, imposed by the type of superconductor applied, permissible current variation and impossibility to exceed the critical current or magnetic flux density to exclude transition of the superconductor to normal state.
References

[1] Chubraeva L I 1991 *Generators of nonconventional design (research calculation methods)* (Leningrad: Science. Leningrad Branch) p 246
[2] Vajnov A I 1968 *Electrical machines* (Leningrad: Energy) p 768
[3] Zimin V I 1960 *Ship electrical machines of alternating current* (Moscow: Military publishing house of the USSR Ministry of Defense) p 463
[4] Mezin E K 1985 *Ship electrical machines. Textbook* (Leningrad: Shipbuilding) p 320
[5] Kitaev E and Greftsev N 1960 *Course of general electrical engineering*. (Leningrad: Publication of the fifth revised and supplemented state union shipbuilding publishing house)

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