Formation of a cavitation cluster in the vicinity of a quasi-empty rupture

E S Bol’shakova¹ and V K Kedrinskiy¹

¹Lavrentyev Institute of Hydrodynamics, Siberian Branch of Russian Academy of Sciences, Lavrentyevprosp.15, Novosibirsk, 630090, Russia

E-mail: kedr@hydro.nsc.ru

Abstract. The presentation deals with one of the experimental and numerical models of a quasi-empty rupture in the magma melt. This rupture is formed in the liquid layer of a distilled cavitating fluid under shock loading within the framework of the problem formulation with a small electromagnetic hydrodynamic shock tube. It is demonstrated that the rupture is shaped as a spherical segment, which retains its topology during the entire process of its evolution and collapsing. The dynamic behavior of the quasi-empty rupture is analyzed, and the growth of cavitating nuclei in the form of the boundary layer near the entire rupture interface is found. It is shown that rupture implosion is accompanied by the transformation of the bubble boundary layer to a cavitating cluster, which takes the form of a ring-shaped vortex floating upward to the free surface of the liquid layer. A p-k mathematical model is formulated, and calculations are performed to investigate the implosion of a quasi-empty spherical cavity in the cavitating liquid, generation of a shock wave by this cavity, and dynamics of the bubble density growth in the cavitating cluster by five orders of magnitude.

1. Introduction.

Explosive volcanic processes have been intensely investigated in both experiments and simulations [1, 2]. Geophysical data show that the magma melt is saturated by a high-pressure gas. The decrease in pressure induced by conduit opening violates the thermodynamic equilibrium state. Rarefaction waves are generated, diffusion processes behind the fronts of these waves arise, and intense development of bubble cavitation is observed, which can lead to magma melt rupture. The formation of such rupture can be considered as a hydrodynamic mechanism of cyclic eruptions. An experimental model of rupture formation in a liquid layer was proposed in [3]. Those experiments were performed in electromagnetic hydrodynamic shock tubes (EM HSTs) with the diapason of storage energies up to 0.8 – 1.2 kJ [3]. It was shown that the shock wave generated in the liquid layer by a membrane under the action of a pulsed magnetic field initiates the development of intense cavitation processes in the liquid layer after SW reflection from the free surface. In the presented problem (for small EM HST with storage energies up to 100 J) the pulsed motion of the membrane leads to the formation of a quasi-empty rupture already (in 10 µs) on the motionless membrane. The dynamics and structure of this rupture can be considered as a qualitative model of the above-mentioned process. Distilled water is a multiphase medium with very small gas bubbles, solid microparticles, and their combinations with sizes of about 1.5 µm and densities of the order of $10^6$ cm$^{-3}$ [4]. New experimental data on the dynamics of the state of the bubble boundary layer on the rupture interface and on its transformation to a cavitating cluster at the rupture implosion “point” are reported below. A two-phase mathematical model of the implosion of a quasi-empty spherical bubble under conditions of an intensely developing cavitating cluster is considered.

2. Statement of experiments

The test section of the small experimental setup (EM HST) includes a transparent cuvette 4 cm in diameter, which is filled with distilled water [4]. The bottom of the cuvette is a conducting Duralumin membrane, which is placed directly onto a flat helical coil connected via an electronic switch to a
high-voltage capacitor bank. When the switch is closed, a pulsed magnetic field is generated on the coil, which pushes the membrane due to the skin effect, thus, generating a powerful SW in the liquid (the SW amplitude is of the order of 10 MPa). The process is recorded by a high-speed video camera with a frequency of $10^5$ frames/s.

3. Experimental results: dynamics of the state of the quasi-empty rupture and bubble cluster

When the electronic switch is closed, an SW is generated in the liquid layer. This SW is reflected from the free surface of the layer in the form of a rarefaction wave and leads to the formation of an intense cavitation region (Fig. 1, a). In 0.5 ms, a rupture is formed on the membrane (Fig. 1, b). A bubble boundary layer on the surface, which covers the entire rupture interface, is clearly visible. The next two frames (Figs. 1, c and 1, d) demonstrate the dynamics of the collapsing rupture and SW radiation.

![Figure 1](image1.png)

**Figure 1.** Rupture dynamics. The loading energy $32 \text{ J}$, at $t = a) 0.1; b) 0.5; c) 1.04; d) 1.1 \text{ ms}$. 

The analysis of the rupture state dynamics shows that it acquires a shape close to a spherical segment. It is known that an implosion of an empty cavity in a homogeneous liquid near a solid wall is accompanied by the formation of a cumulative jet. In the present case, however, such a situation is not observed: the rupture implodes symmetrically, without changing its topology. It is of interest that the liquid flow on the membrane in the course of the rupture implosion forms a ring-shaped flow of a homogeneous liquid, which is closed under the rupture with a high velocity (Fig. 1, c). At the end of the rupture implosion, a dense core is formed, and the bubble boundary layer transforms to a bubble cluster surrounding the rupture. The state of the rupture surrounded by a dense bubble cluster at the instant of the implosion can be defined as a state with a high internal energy. This is confirmed by generation of a sufficiently powerful secondary SW after the rupture implosion (Fig. 1, d). Propagation of this SW and its interaction with the free surface leads to the formation of the secondary cavitation region (Fig. 1d). It was demonstrated earlier that the rupture retains its topology, i.e., a shape similar to a spherical segment. Based on this fact, we use the dimensional method, assuming that the basic parameters of the liquid state are the hydrostatic pressure $p_0$, and density $\rho$ of the liquid, the storage energy $E$, and the coefficient $\alpha$ characterizing the ratio of the maximum potential energy of the rupture to the loading energy $Q/E$. Let us find a combination of parameters that yields the time characteristics of the examined process:

$$T = \alpha^{2/3} E^{1/3} \rho^{1/2} p_0^{-5/6} \quad (1)$$

The formulas of the rupture dynamics have the form:

$$r \approx 13.4 + 6.4 \cdot t - 19 \cdot t^2, \quad h \approx 0.3 + 14 \cdot t - 12 \cdot t^2.$$
The value of $\alpha$ predicted by Eq. (1) is equal to the value for the maximum potential energy: $\alpha = 0.0032$ or $Q \approx 0.1$ J. It is of interest that the experimental results obtained for identical storage energy, but for different heights of the water layer in the cavity are close to each other. Thus, it is confirmed that the dynamics of cluster formation depends to a greater extent on the loading energy and is almost independent of the water layer height. Fig. 2 illustrates the cluster dynamics after the rupture implosion and generation of the secondary SW. It is seen that the cluster emerges to the water layer surface and possesses a sufficient amount of the initially stored energy to move to the liquid layer surface almost uniformly (1.5 cm in 2 ms). The character of the bubble cluster motion and the fact that a ring-shaped flow along the membrane is formed due to the rupture implosion allow us to assume that the bubble boundary layer transforms after the closure of the quasi-empty rupture to a ring-shaped cluster (vortex).

Figure 2. Dynamics of the bubble cluster. The loading energy is 32 J; the time instants are 1.3 (a), 1.7 (b), 2.1 (c), 2.7 (d), and 3.1 ms (e).

The results of the experimental analysis for the cavity width of 4 cm, water layer height of 1.5 cm, and loading energy of 18, 24.5, and 32 J are shown in Fig. 3. It is seen that the plot of motion is an almost straight line for the greatest energy examined in this study. As the energy decreases, the plot becomes curved, and the emergence velocity becomes smaller. The bubble cluster velocity is approximately equal to 7.5 m/s for the storage energy of 32 J.

Figure 3. Dynamics of bubble cluster emergence for different loading energies. The water layer height is 1.5 cm. The loading energy is 32 (blue), 24.5 (red), and 18 J (green).
4 Mathematical model of the imploding rupture in the intense cavitation region.

Formulation of the problem. A spherical cavity with a radius of 1 cm is placed into a space willed with a liquid, which is distilled water with known parameters of its heterogeneous state (radius of free gas microbubbles 1.5 µm, \( p_0 = 1 \text{ atm} \), and number density of these nuclei 10^6 cm^{-3} for the volume concentration of the gas phase \( k_0 = 10^{-6} \)). The pressure in the cavity (\( R \) radius) and microbubbles is equal to the hydrostatic pressure \( p_0 \approx 1 \text{ atm} \). The system is in the equilibrium state. At the time \( t = 0 \), the pressure inside the cavity instantaneously decreases to an extremely low value: \( p_0 = 10^{-8} \text{ atm} \). A cavitation process starts to develop near the cavity boundary under the action of the initial pressure in the bubbles.

Dynamics of the state of the cavitating liquid. By introducing a new function \( \zeta = p - p_k \) \( k^{1/6} \) and a variable \( \eta = r \alpha^{1/6} \) as a new independent variable, the system of the \( pk \)-model can be converted to the following form within the framework of some approximations under the condition of an incompressible liquid component

\[
\frac{d^2 \zeta}{dt^2} + \frac{\nu}{\eta} \left( \frac{d \zeta}{d\eta} \right) = 0, \quad \frac{\partial^2 k}{\partial t^2} = -\frac{3}{\rho_0 r^2} k^{1/3} \zeta + \frac{1}{6k} \left( \frac{\partial k}{\partial t} \right)^2,
\]

where \( \nu = 0, 1, 2 \) defines the type of the flow symmetry. In the spherical formulation, the first equation has the solution

\[
\zeta = \eta^{1/2} \left[ A f(\eta) + B K(\eta) \right],
\]

where \( A=0 \) because the solution is limited at infinity, and \( K(\eta) = (\pi/2\eta)^{-1/2}e^{-\eta} \) is a modified Bessel function.

Thus, we obtain

\[
\zeta = \eta^{1/2} \delta e^{\eta R} \left( \frac{\pi}{2\eta} \right)^{1/2} e^{-\eta R}. \]

Let us now return to the variables \( p-k \):

\[
p(t) - p_k = \left( p_0 \left( \frac{R}{R_0} \right)^{-3\gamma} - p_k \right) \frac{R}{\rho_0} \eta^{1/6} e^{-\eta R} \]

Substituting the resultant expression into the equation for \( k \), we obtain

\[
k = \frac{3k^{1/3} R}{\rho_0 r^2} \left( p_0 \left( \frac{R}{R_0} \right)^{-3\gamma} \right) e^{-\eta R} + \frac{1}{6k} \left( \frac{\partial k}{\partial t} \right)^2.
\]

Dynamics of the cavity in the cavitating liquid [5]. The dynamics of the spherical cavity can be described by using an analog of the Herring equation in a bubbly liquid

\[
R \ddot{R} + \frac{3}{2} \dot{R}^2 = H + \frac{\rho \dot{H}}{c_b \rho_0 \dot{t}}.
\]

In this equation, \( H \) is the enthalpy and \( c_b \) is the velocity of sound in the bubbly liquid, which is a function of the gas phase concentration in the cavitating liquid: \( c_b^2 = \gamma \rho_0 \left( \rho_0 k (1 - k) \right)^{1/2} \). The equation of state of the bubbly liquid with an incompressible component is \( pk / (1 - k) = a \).

The enthalpy \( H = \int \frac{d\rho}{\rho_b} \frac{\rho_0}{\rho_0 - \rho_b} \left( 1 + \frac{3}{\rho_0} p_k \right) \left( \frac{R}{R_0} \right)^{-3\gamma} \frac{\dot{R}}{R} \left( 1 + \frac{a \left( R_0 / R \right)^{-3\gamma}}{R_0} \right) \) - derivative determines the acoustic loss. Then the equation for a spherical cavity of radius \( R \) takes the form
Thus, the dynamics of the implosion of a quasi-empty cavity in a cavitating medium with a dynamically changing concentration of the bubbly liquid is described by the system of equations

\[
R \ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{p_g - p_\infty}{\rho_0} - \frac{3\gamma}{\rho_0} \rho_0 \left( \frac{R}{R_0} \right)^{-3\gamma} \frac{\dot{R}}{c_b} \left( 1 + \frac{a}{p_0} \left( \frac{R}{R_0} \right)^3 \right).
\]

The numerical solution of this system under the initial conditions \( k_0 = 10^{-6} \) and \( p_0 = 10^{-8} \) yields the following result (Figs. 4 and 5). Let us confine our consideration to the data on \( k \) at the cavity boundary \( r = R \).

\[
\begin{cases}
\dot{k} = \frac{3k_0^2}{\rho_0 \gamma^2} \left( \frac{p_\infty k^{-\gamma} - p_0 \left( \frac{R}{R_0} \right)^{-3\gamma}}{r \left( \frac{R}{R_0} \right)^{-3\gamma}} \right) e^{-a k^{1/6}(r-R)} + \frac{1}{6k} \left( \frac{\partial k}{\partial t} \right)^2.

R \ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{p_0 \left( \frac{R}{R_0} \right)^{-3\gamma} - p_\infty}{\rho_0} - \frac{3\gamma}{\rho_0} \rho_0 \left( \frac{R}{R_0} \right)^{-3\gamma} \frac{\dot{R}}{c_b} \left( \rho_0 \frac{k(1-k)}{\gamma p_\infty} \right)^{1/2} \left( 1 + \frac{a}{p_0} \left( \frac{R}{R_0} \right)^3 \right).
\end{cases}
\]

The numerical solution of this system under the initial conditions \( k_0 = 10^{-6} \) and \( p_0 = 10^{-8} \) yields the following result (Figs. 4 and 5). Let us confine our consideration to the data on \( k \) at the cavity boundary \( r = R \).

![Figure 4](image)

**Figure 4.** a) Dynamics of the spherical cavity; b) dynamics of the gas phase concentration.

It is seen that the concentration of the gas phase surrounding the imploding cavity increases under the action of the initial internal pressure in microbubbles. However, at the cavity implosion instant, the pressure in the cavity increases and the concentration rapidly decreases. Fig. 5.a illustrates the dynamics of the acoustic losses in the cavity. It is seen that the cavity implosion is accompanied by pulsed release of the SW energy in a narrow region near the cavity implosion point. The integral acoustic loss of energy (Fig. 5.b) is described by the formula

\[
I = \int_{R_0}^R \frac{3\gamma}{\rho_0} \rho_0 \left( \frac{R}{R_0} \right)^{-3\gamma} \frac{\dot{R}}{c_b} \left( \frac{\rho_0 k(1-k)}{\gamma p_\infty} \right)^{1/2} \left( 1 + \frac{a}{p_0} \left( \frac{R}{R_0} \right)^3 \right) 2R^2 dR
\]
Figure 5. a) Dynamics of acoustic losses; the signal width is $\Delta t=1.2*10^{-8}$ s; b) integral acoustic loss of energy.

In dimensionless variables, the initial potential energy of the quasi-empty cavity is $2/3$. If the initial internal pressure in the cavity is extremely low, the entire energy transforms to pulsed radiation (Fig. 5, b).

5. Conclusions

The analysis of the experimental data shows that shock wave loading of the liquid layer leads to the rupture development on the bottom of the cuvette. The rupture shape is close to a spherical segment, and there is a bubble boundary layer on the rupture interface. After the implosion, this boundary layer transforms to a bubble cluster. The rupture implosion generates the secondary shock wave, which is confirmed by the formation of the secondary cavitation region and which forms the cluster structure. After shock wave generation, the bubble cluster emerges to the liquid layer surface. The character of the cluster dynamics indicates the formation of a ring-shaped vortex moving upward to the liquid surface. It is shown that the cluster motion is almost independent of the liquid layer height and is mainly determined by the loading energy. A $p$-$\kappa$ mathematical mode is formulated, and calculations are performed to investigate the implosion of a full quasi-empty spherical cavity in the cavitating liquid, generation of a shock wave by this cavity, and dynamics of the bubble density growth in the cavitating bubbly cluster by five orders of magnitude. It is of interest that the integral acoustic loss of energy is equal to the initial maximal potential energy of cavity.

Acknowledgement

This work was supported by the Russian Foundation for Basic Research (Grant No. 15-05-03336a) and partly by Siberian Branch of the Russian Academy of Sciences (Project No. III.22.3.1).

References

[1] Woods W.: The dynamics of explosive volcanic eruptions// Reviews of geophysics, V. 33, No. 4, p. 495–530, (1995)
[2] Gonnermann, H.M., Manga M. The fluid mechanics inside a volcano Annu. Rev. Fluid Mech.2007,39 P. 321–56
[3] Bolshakova E. S., Kedrinskiy V. K., Journal of Physics: Conference Series – 2016. – V. 754. № 4, P. 042003
[4] BesovA. S., KedrinskiyV. K., PalchikovE. I. Letters in JTPh. – 1984. –V. 10, № 4. P. 67–70
[5] Kedrinskiy V. K, Bolshakova E. S. The Journal of the Acoustical Society of America , – 2015. – V. 138. №3, Pt. 2, P. 1829.