Optimal Control for Aluminum Electrolysis Process Using Adaptive Dynamic Programming

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This work was supported in part by the Natural Science Foundation of Chongqing under Grant cstc2019jcyj-msxmX0080, in part by the Technical Innovation and Application Development of Chongqing under Grant cstc2019jscar-zdcxtxX0027, and in part by the Science and Technology Research Program of Chongqing Municipal Education Commission under Grant KJQN201801506 and Grant KJZD-K202001501.

ABSTRACT Optimal control of aluminum electrolysis production process (AEPP) has long been a challenging industrial issue due to its inherent difficulty in establishing an accurate dynamic model. In this paper, a novel robust optimal control algorithm based on adaptive dynamic programming (ADP) is proposed for the AEPP, where the system subjects to input constraints. First, to establish an accurate dynamic model for the AEPP system, recursive neural network (RNN) is employed to reconstruct the system dynamic using the input-output production data. To ensure input constraints are not to exceed the bound of the actuator, the optimal control problem of the AEPP is formulated under a new nonquadratic form performance index function. Then, considering the perturbation of the AEPP, the robust control problem is effectively converted to the constrained optimal control problem via system transformation. Furthermore, a single critic network framework is developed to obtain the approximate solution of the Hamilton-Jacobi-Bellman (HJB) equation. Finally, the proposed ADP controller is applied to the AEPP system to validate the effectiveness and performance.

INDEX TERMS Adaptive dynamic programming (ADP), optimal control, input constraints, aluminum electrolysis.

I. INTRODUCTION The production process of aluminum electrolytic industry is a strongly coupled and dynamic nonlinear process. AEPP mainly depends on the electrolytic reaction in the electrolytic cell. The quality of the electrolytic reaction determines the product quality of the aluminum. In actual production, AEPP cannot produce metal aluminum with high efficiency and high quality due to many problems exist in the AEPP. For example, the reaction in the electrolytic cell is affected by many factors, such as the temperature and the direct current (DC) voltage of the electrolytic cell. Hence, it is difficult to control the reaction process effectively. In the actual AEPP, engineer often based on equipment operating status and artificial experience to adjust the control parameters to achieve control requirements of the aluminum electrolysis. However, due to the limitations of this method and the limitation of manual operation, the product quality often fails to meet the requirements. Due to the complexity of the AEPP, how to establish an effective model and achieve efficient control of the AEPP has great academic value and engineering application value.

In the AEPP, a large amount of online and offline data are generated and stored such as online detection data, offline analysis data, operation statistics and so on. These production data contain rich system operation information [1]. Because it is difficult to establish an accurate aluminum electrolysis mechanism model, the use of data-driven control theory to solve the optimization and control of complex nonlinear aluminum electrolysis systems has become a hot spot in aluminum electrolysis optimization control. In [1], an intelligently optimized aluminum electrolytic manufacturing system was proposed for complex AEPP. In [2], a multi-target bacterial foraging algorithm was proposed, which can
maximize current efficiency and reduce resource consumption. In [3], a global double heuristic planning control strategy was proposed based on event-trigger, and which is applied to the optimization process of aluminum electrolytic production. Therefore, the data-driven can not only identify production processes with unknown aluminum electrolytic system models, but also can use online and offline data to achieve modeling and control between output variables and process variables [4]–[7].

Adaptive dynamic programming (ADP) known as a typical data-driven control method is proposed to approximately solve nonlinear optimal control problems [5]–[7]. Based on offline and online data, ADP uses non-linear function fitting methods to approximate the performance indicators of dynamic programming [8]–[12]. ADP is a powerful tool to solve HJB equation and overcome the severe difficulty of “curse of dimensionality”. In recent years, ADP is more and more widely used in industrial systems. However, it is difficult to establish an accurate mathematical model for the complex and uncertain non-linear production process. Hence, many data-driven ADP control methods have been presented, where offline or online input and output data are directly used to replace the model knowledge. In [13], data-driven method was used to establish a recursive neural network model for slag powder production process. A tracking controller was designed with control constraints and applied to the slag powder production process. In [14], a novel ADP method was proposed based on adaptive reinforcement learning for unknown nonlinear systems with input constraints. Simulation results verified the effectiveness of the proposed algorithm. In [15], a robust adaptive control algorithm was proposed based on reinforcement learning, which transformed the robust problem into an optimal control problem with constraints and guaranteed the stability of the nonlinear system. In [16], a data-driven robust approximate optimal tracking control scheme was proposed, where an unknown nonlinear system model was reconstructed and an approximate optimal tracking controller using the ADP method was designed. In [17], a data-driven adaptive dynamic programming method was proposed for a class of continuous-time nonlinear systems. By designing a multivariable tracking scheme, a simulation experiment of multivariable tracking control is realized. In [18], a novel data-drive neuro-optimal tracking control algorithm was proposed for unknown nonlinear systems. The proposed ADP controller was applied to a continuous stirred reactor system to verify its effectiveness and performance. In [19], an event-triggered approximate optimal control structure is proposed for a nonlinear continuous time system with control constraints. In [20] addressed the challenging industrial problem of natural gas desulfurization control, and proposed an improved unscented kalman filter assisted ADP method to solve the optimal control problem of the desulfurization system. Hence, data-driven adaptive control method, which can accurately identify the complex system and achieve optimal control, is widely used in actual industrial system [21].

In this paper, a novel ADP control algorithm is proposed for the AEPP system with input constraints. First, recursive neural network is used to establish an accurate model for the AEPP. Then, a robust ADP algorithm with control constraints is proposed to obtain the optimal control law. The robust control problem is converted to the constrained optimal control problem. Furthermore, a single critic network framework is developed to obtain the approximate solution of the HJB equation. Experimental results show the effectiveness of the proposed algorithm. The major contributions of this paper include the following.

1) A novel robust optimal control algorithm is developed for the AEPP system, where only one critic NN is employed. Hence, the computation complexity is reduced.

2) This paper extends the work of [4] and [14] to develop an optimal controller for AEPP system with input constraints. Specifically, a new non-quadratic form performance index function is developed for the AEPP, which ensures the optimal control law not to exceed the bound of the actuator.

3) Typical industrial production process, the AEPP, is utilized to verify the effectiveness of the proposed method.

The rest of this paper is organized as follows: In Section II, the optimal control problem is formulated for nonlinear AEPP. In Section III, the dynamics of unknown nonlinear AEPP is reconstructed by RNN. Section IV develops the robust optimal control scheme in detail. In Section V constructs a single critic network to approximately solve the HJB equation. In Section VI, the proposed algorithm is applied to the AEPP, and experimental results are discussed. Finally, the conclusion is given in Section VII.

II. PROBLEM FORMULATION

A. AEPP DESCRIPTIONS

The AEPP is a complex reaction processes. First, alumina and cryolite are fused into the electrolytic cell. Then, physical and chemical reactions are occurred when high-voltage direct current is access to the electrolytic cell. Then, the produced liquid aluminum are extracted and clarified liquid aluminum are poured into aluminum ingots. As shown in Fig.1, raw materials of the AEPP system are including alumina, carbon anode, cryolite, and fluoride salt [22]. Among them, alumina is the key raw material. The reaction process is roughly as follows: Cryolite and fluoride together form an electrolyte melt. Cryolite is a good conductive melt, while the function of fluoride is to improve the molecular ratio of electrolyte and reduce the temperature of primary crystal. The aluminum electrolytic cell uses a carbon anode and a carbon cathode, and alumina is dissolved in the cryolite melt. Then, DC voltage input to the electrolytic cell. When the temperature of the cell reaches about 960°C, the electrolytic reaction will occur. Hence, liquid aluminum is obtained at the cathode and anode gas is generated at the anode. The reaction process is affected by the following factors: the temperature of the electrolytic cell, the voltage of the direct current, the concentration of alumina, the distance between the anode and the surface of...
In the AEPP, loss current is inevitable. We can reduce the loss current to improve the current efficiency.

The DC power consumption directly reflects the energy consumption of production one ton aluminum, which is also an important parameter to evaluate the technical level of AEPP. The relationship of DC power consumption, cell voltage and current efficiency is

$$p = 2980 \frac{U}{CE}$$

(2)

where \(p\) is DC power consumption of one ton aluminum, its unit is \(kW.h/t \rightarrow Al\); \(U\) is working voltage of electrolytic cell, its unit is \(V\); \(CE\) is current efficiency. It can be seen that the DC power consumption is inversely proportional to the current efficiency. The higher the current efficiency, the lower the DC power consumption. In addition, the slot voltage decreases and the DC power consumption will also decrease. Shortening the pole distance can reduce the tank voltage and thus reduce the DC power consumption, but excessively shortening the pole distance will reduce the current efficiency and increase the DC power consumption. Hence, the voltage of the electrolytic cell cannot be too low, we generally set at about 4-4.5 \(V\).

3) DYNAMIC DESCRIPTION OF AEPP

In real AEPP, the reaction process of the electrolytic cell is mainly related to the following parameters:

- The main controlled variables, such as working tank voltage \(V_1\), electrolytic cell working temperature \(C\), etc.
- The main control variables, such as the number of feeds \(u\), the molecular ratio, DC power consumption, aluminum output and current efficiency during the reaction.

In this paper, the number of feeds \(u\) is mainly considered as the control amount.

Hence, the AEPP control system can be described as

$$\dot{x} = f(x, \theta, u)$$

(3)

where \(u = [u]^T\), \(x = [x_1, x_2]^T\), \(\theta\) is a constant, \(x_1\) represents working tank voltage, \(x_2\) represents working temperature of electrolytic cell, \(u\) represents feeding number.

In the real AEPP, the actuator is constrained by its own physical constraint. Hence, the control action \(u(t)\) should be limited in a specified range. The control action \(u(t)\) can be constrained as follows:

$$T_{min} \leq u(t) \leq T_{max}$$

(4)

where \(T_{min}\) and \(T_{max}\) are the minimum and maximum value of \(u(t)\), respectively.

The reaction in the electrolytic cell is a complex process. To make the reaction process smoothly, the temperature and the voltage of the electrolytic cell need to be controlled within the specified range. Therefore, the control target is to design an optimal control law \(u^*(t)\) to make system states track the desired temperature and voltage value.

However, it is difficult to obtain an optimal control law from system (3), due to the system function is unknown in the AEPP, loss current is inevitable. We can reduce the loss current to improve the current efficiency.

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However, it is difficult to obtain an optimal control law from system (3), due to the system function is unknown in
the real AEPP. Meanwhile, the corresponding desired control of the desired tracking state is also not easy to get from the unknown system. Therefore, we will present a new optimal tracking control method to obtain the controller for CSTR.

III. DATA-BASED MODELING FOR AEPP

In the real AEPP, the production system is a complicated nonlinear process. It is difficult to establish an accurate mathematical model for the AEPP. Hence, a large amount of production data in the AEPP are used to reconstruct system model by a RNN. Based on this, considering the asymmetric control constraints of AEPP, an optimal control strategy is designed to improve the production quality.

Based on input and output data, a RNN is used to identify the system dynamics. Hence, the system can be formulated as

\[ \dot{x}(t) = W_1x(t) + W_2^T f(x(t)) + W_3^T \mu(x(t))u(t) + \varepsilon(t) \]  

(5)

where system status \( x \in \mathbb{R}^n \), control law \( u \in \mathbb{R}^m \), \( W_1, W_2, W_3 \) are RNN ideal weight matrices. \( \varepsilon(t) \) is the bounded model reconstruction error. \( f(\cdot) \) is the activation function, \( \mu(\cdot) \) is a monotonically increasing function, and for any \( x, y \in \mathbb{R} \) exists \( k > 0 \), satisfies:

\[ 0 \leq f(x) - f(y) \leq k_1 (x - y), \]

\[ 0 \leq \mu(x) - \mu(y) \leq k_2 (x - y). \]

where \( \|f(x)\| \leq b_f(x), \|\mu(x)\| \leq b_\mu(x) \), in which \( b_f \) is a positive constant. In this paper, \( \mu(x) = \tanh(x), f(x) = \tanh(x). \) According to the formula (5), a data-driven model can be reconstructed as

\[ \hat{x}(t) = \hat{W}_1\hat{x}(t) + \hat{W}_2^T f(\hat{x}(t)) + \hat{W}_3^T \mu(\hat{x}(t))u(t) + v(t) \]

(6)

where \( \hat{x}(t) \) is the estimated value of system state vectors, \( \hat{W}_1, \hat{W}_2, \hat{W}_3 \) is the estimated value of desired weight \( W_1, W_2, W_3 \), respectively. \( v(t) \) satisfies:

\[ v(t) = \eta e_m(t) \]

(7)

where \( e_m(t) = x(t) - \hat{x}(t) \) is the model state error, \( \eta \) is adjustment parameter. Combining equations (5) and (6), the dynamic equation of model error can be derived as

\[ \dot{e}_m(t) = W_1 e_m(t) + \hat{W}_1 \hat{x}(t) \hat{x}(t) + W_2^T f + \hat{W}_2^T f(\hat{x}(t)) + \hat{W}_3^T \mu u(t) - \eta \hat{x}(t) + \varepsilon(t) + \hat{W}_3^T \mu(\hat{x}(t))u(t) \]

(8)

Considering the dynamic equation of model error, we have \( \hat{W}_1(t) = W_1(t) - \hat{W}_1(t), \hat{W}_2(t) = W_2(t) - \hat{W}_2(t), \hat{W}_3(t) = W_3(t) - \hat{W}_3(t), f = f(x(t)) - f(\hat{x}(t)), \mu = \mu(x(t)) - \mu(\hat{x}(t)). \)

The network weight matrix and adjustment parameters of the data-driven model (6) are updated according to the following learning law:

\[ \dot{\hat{W}}_1(t) = \Lambda_1 \hat{x}(t) e_m^T(t) \]

(9)

\[ \dot{\hat{W}}_2(t) = \Lambda_2 f(\hat{x}(t)) e_m^T(t) \]

(10)

\[ \dot{\hat{W}}_3(t) = \Lambda_3 \mu(\hat{x}(t))u(t) e_m^T(t) \]

(11)

where \( \Lambda_i, i = 1, 2, 3 \) is the corresponding positive definite matrix. The model error identification converges gradually when \( \lim_{t \to \infty} e_m(t) = 0 \), and \( \hat{W}_1(t), \hat{W}_2(t), \hat{W}_3(t) \) approaches the ideal matrix \( W_1, W_2, W_3 \), respectively.

Therefore, using a large amount of offline data and after a long enough time for model identification, the nonlinear system can be expressed as

\[ \hat{x}(t) = W_1 x(t) + W_2^T f(x(t)) + W_3^T \mu(x(t))u(t) \]

(12)

IV. OPTIMAL CONTROL SCHEME BASED ON ADP

For the RNN model, a special index function is used to solve the asymmetric input bounded problem, and a critic network is employed to approximate the index function. Hence, an adaptive robust controller is developed to meet the control constraints [24].

A. HJB EQUATION FOR AEPP

According to equation(12), the aluminum electrolytic production system model with the perturbation term can be expressed as

\[ \dot{x}(t) = W_1 x(t) + W_2^T f(x(t)) + W_3^T \mu(x(t))u(t) + \Delta f(x(t)) \]

(13)

where \( \Omega_u = \{u | u \in \mathbb{R}^n, T_{\text{min}} \leq u(t) \leq T_{\text{max}} \} \) is the control inputs that satisfy constraints.

We assume that \( \hat{u}(t) \) is the expected control corresponding to expected \( \hat{x}(t) \), the control error is defined as

\[ u_e(t) = u(t) - \hat{u}(t) \]

(14)

where \( T_{\text{min}} - \hat{u}(t) \leq u_e(t) \leq T_{\text{max}} - \hat{u}(t). \)

For a constrained optimal control problem, the control goal is to find an optimal control law that satisfies the constraints to make the system progressively stable. Then, the performance index function can be formed as

\[ V_x(t) = \int_t^{\infty} \left[ d_M^2(x(s)) + r(x(s), u(s)) \right] ds (s \geq t) \]

(15)

where \( r(x, u) = x^T Q x + W(u) \), \( Q \) is positive definite matrix and \( W(u_e) \) is positive definite. In this paper, we choose a non-quadratic function for the AEPP system, which can be expressed as

\[ W(u_e) = 2k \int_a^b \psi^{-1}(\nu/k)^T R d\nu \]

\[ = 2k \sum_{i=1}^{m} \int_{a}^{b} \psi^{-1}(\nu_i/k) r_i d\nu_i \]

where \( a = (T_{\text{max}} + T_{\text{min}})/2 - \hat{u}(t), k = (T_{\text{max}} - T_{\text{min}})/2, R = \text{diag} \{r_1, r_2, \ldots, r_m\}, r_i > 0, i = 1, \ldots, m, \psi \in \mathbb{R}^m, \psi(\cdot) \) is the boundary limit and \( |\psi(\cdot)| \leq 1, \)
ψ⁻¹ (υ/k) = [ψ⁻¹ (υ₁/k) · · · ψ⁻¹ (υₘ/k)]ᵀ. It should be emphasized that W (υₙ) is positive definite since ψ⁻¹ (·) is a monotonic odd function and R is positive definite.

In this paper, we chose ψ (·) = tanh (·) to guarantee that the control inputs are bounded. Derivate time T to get Lyapunov equation

∇Vₜ (f (x) + g (x) u) + d₂M x (s) + r (x, u) = 0

(16)

The Hamilton operator equation that defines the control law u (x) and the value function V (x) :

H (x, Vₜ, u) = ∇Vₜ (f (x) + g (x) u) + d₂M x (s) + r (x, u).

(17)

Define the optimal value function as

Vₜ (x (t)) = \min_{u \in Ω} \int_{t}^{∞} \left[ d₂M (x (s)) + r (x (s), u (s)) \right] ds

(18)

The optimal value function can be obtained by solving the Hamilton function below

min_{u \in Ω} H (x, Vₜ, u) = 0

(19)

Suppose that the minimum value exists, the optimal control u (k) can be derived as

uₜ (t) = −k tanh \left( \frac{Vₜ g{T} (x)}{2k} \right) + a

(20)

Combining formula (19) and (20), we can write the equation of the nonlinear system as

Vₜ f (x) − 2k²Aᵀ (x) tanh A (x) + 2kaA (x) + d₂M (x)

0

+ xᵀ Qx + 2k \int_{a}^{T} \tanh⁻² (y − a) / kdγ = 0

(21)

where A (x) = (1/2k) g{T} (x) Vₜ.

Let A (x) = [A₁ (x), . . . , Aₘ (x)]ᵀ ∈ ℝᵐ and Aᵢ (x) ∈ ℝ, i = 1, . . . , m. According to [25], [26], we have

−k tanh (Aᵢ (x)) + a

2k \int_{a}^{T} \tanh⁻² (y − a) / kdγ

= 2k²Aᵀ (x) tanh A (x) + k² \sum_{i=1}^{m} \ln \left[ 1 − \tanh² (Aᵢ (x)) \right]

(22)

Through equation (22), equation (21) can be rewritten as

Vₜ f (x) + d₂M (x) + xᵀ Qx

+k² \sum_{i=1}^{m} \ln \left[ 1 − \tanh² (Aᵢ (x)) \right] + 2kaA (x) = 0.

(23)

B. PROBLEM TRANSFORMATION

In this section, by using Theorem 1 to prove that the robust control of the system (13) can be obtained by finding the optimal control solution for the value function (15) of the system (12) [27].

Assumption 1: The perturbation term Δf (x) satisfies the matching conditions such that Δf (x) = g (x) d (x). In addition, an unknown function d (x) ∈ ℝ is bounded by a known function dₘ (x), and \| d (x) \| ≤ dₘ (x). Meanwhile, d (0) = 0 and dₘ (0) = 0.

Assumption 2: f (x) + g (x) u is Lyapunov continuous on compact set Ω containing origin, namely, the system (13) is stable on compact set Ω. In addition, f (0) = 0.

Assumption 3: The control matrix g (x) is known to be bounded. For each x ∈ Ω, both exist constant gₘ and gₚ (0 < gₘ < gₚ) namely gₘ < \| g (x) \| < gₚ.

\hat{V}ₜ (x) = Vₜ f (x) + g (x) uₜ + Vₜ Δf (x)

(24)

Theorem 1: Consider (12) the nominal system described by the value function (15), suppose Assumption 1-3 holds. Then, the optimal control uₜ (t) design in (20) can guarantee the system (13) run steadily in the sense of uniform ultimate boundedness (UBB).

Proof: By formula (21), we have

Vₜ f (x) + g (x) uₜ

= −d₂M (x) − xᵀ Qx − 2k \sum_{i=1}^{m} uₜ \tanh⁻¹ ([γᵢ − a] / k) dγᵢ

(25)

where uₜ = [u₁ₜ, . . . , uₘₜ]ᵀ and uᵢₜ ∈ ℝ, i = 1, . . . , m.

Considering to Vₜ f and Vₜ g (x) = −2ktanh⁻¹ ([uₜ − a] / k), we have

Vₜ Δf (x) = −2ktanh⁻¹ ([uₜ − a] / k) d (x).

(26)

Combining these two formulas, we can get

\hat{V}ₜ (x) = −d₂M (x) − xᵀ Qx + β₁ (x)

−2ktanh⁻¹ ([uₜ − a] / k) d (x)

(27)

where \beta₁ (x) = −2k \sum_{i=1}^{m} \int_{a}^{T} \tanh⁻² ([γᵢ − a] / k) dγᵢ.

Denote τᵢ = tanh⁻¹ (1/(γᵢ − a) / k), i = 1, . . . , m. Hence, we get

\beta₁ (x) = −2k² \sum_{i=1}^{m} \int_{0}^{τᵢ} \tanh⁻² (τᵢ) dτᵢ

= 2k² \sum_{i=1}^{m} \tanh⁻¹ ([uᵢₜ − a] / k)

= −k² \sum_{i=0}^{m} \tanh⁻¹ ([uᵢₜ − a] / k)².

(28)
Due to
\[ \sum_{i=1}^{m} \left[ \tanh^{-1} \left[ (u_i^* - a) / k \right] \right]^2 = \tanh^{-T} \left[ (u^* - a) / k \right] \tanh^{-1} \left[ (u^* - a) / k \right] \quad (29) \]

According to (28) and (29), (27) can be rewritten as
\[ \dot{V}^* (x) = -d^2_{\tilde{M}} (x) - x^T Q x + d^T (x) d (x) \]

with \( \beta_2 (x) = 2 k^2 \sum_{i=1}^{m} \tanh^{-1} \left[ (u_i^* - a) / k \right] t_i \tanh^2 (t_i) \).

According to the median integral theorem, we can get
\[ \beta_2 (x) = 2 k^2 \sum_{i=1}^{m} \tanh^{-1} \left[ (u_i^* - a) / k \right] \theta_i \tanh^2 (\theta_i) \quad (31) \]

where the value of \( \theta_i \) is between 0 and \( \tanh^{-1} \left( u_i^* / k \right) \).

According to Assumption 3 and (29), we have
\[ \beta_2 (x) \leq 2 k^2 \tanh^{-T} \left[ (u^* - a) / k \right] \tanh^{-1} \left[ (u^* - a) / k \right] \]

Combining (30) and (32), we obtain
\[ \dot{V}^* (x) \leq -d^2_{\tilde{M}} (x) - x^T Q x + d^T (x) d (x) \]
\[ + \left[ \frac{1}{2} V^*_x (x)^{-T} g (x) g^T (x) V^*_x (x) \right] \]
\[ \leq -\lambda_{\min} (Q) \|x\|^2 + \frac{1}{2} g^2 M^2 \delta^2 M. \]

where \( \lambda_{\min} (Q) \) represents the minimum eigenvalue of the matrix \( Q \), which is positive definite, we have \( \lambda_{\min} (Q) > 0 \).

Consequently, \( V^* (x) < 0 \) as long as the state \( x (t) \) is not compact set
\[ \Omega_x = \left\{ x : \|x\| \leq \frac{g M \delta M}{2 \sqrt{2 \lambda_{\min} (Q)}} \right\}. \]

This shows that \( V^* (x) \) is a Lyapunov function for system (13) with the control \( u^* \), whenever \( x (t) \) lies outside the compact set \( x \). Therefore, the optimal control \( u^* (t) \) developed in (20) can ensure the trajectory of system (13) to be UUB.

V. CONTROLLER IMPLEMENTATION BASED ON NEURAL NETWORK

The key of the proposed ADP algorithm is to obtain the optimal value function and optimal control law. In this paper, we use a critical neural network to approximate the value function. According to the general properties of neural networks, we can represent the optimal value function as
\[ V^* (x) = W^*_c (x) + \epsilon (x) \quad (32) \]
where \( W^*_c \in R^{N_0} \) is the ideal neural network weight, \( \phi (x) = [\phi_1 (x), \ldots, \phi_{N_0} (x)]^T \in R^{N_0} \) is the activation function, \( N_0 \) is the number of neurons, \( \epsilon (x) \) is the function reconstruction error of neural network.

Derivative of formula (32), we have
\[ \dot{V}^*_c (x) = \nabla \phi^T (x) W^*_c + \nabla \epsilon (x) \quad (33) \]
where \( \nabla \phi (x) = \partial \phi (x) / \partial x \) and \( \nabla \phi (0) = 0 \).

Substituting formula (33) into formula (23), we get
\[ 0 = W^*_c \nabla \phi (x) + \nabla \epsilon (x) + d^2_{\tilde{M}} (x) + x^T Q x \]
\[ + k^2 \sum_{i=1}^{m} \ln \left[ 1 + \tanh^2 (\zeta_{i1} (x) + \sigma_i (x)) \right] \]
\[ + 2 k a (\zeta_{i1} (x) + \sigma_i (x)) \quad (34) \]

where \( \zeta_{i1} (x) = (1/2 k) g^T (x) \nabla \phi^T (x) W^*_c, \sigma_i (x) = (1/2 k) g^T (x) \nabla \epsilon (x), \zeta_{i1} (x) = [\zeta_{i1} (x), \ldots, \zeta_{im} (x)] \) with \( \zeta_{i1} (x) \in R, \sigma_i (x) = [\sigma_1 (x), \ldots, \sigma_m (x)]^T \) with \( \sigma_i (x) \in R, i = 1, \ldots, m \).

Using the median theorem, equation (34) can be rewritten as
\[ 0 = W^*_c \nabla \phi (f (x)) + d^2_{\tilde{M}} (x) + x^T Q x \]
\[ + k^2 \sum_{i=1}^{m} \ln \left[ 1 + \tanh^2 (\zeta_{i1} (x)) \right] + \epsilon_{HJB} \quad (35) \]
where \( \epsilon_{HJB} \) is the approximate error [25], [28], which can be formed as
\[ \epsilon_{HJB} = 2 k a (\zeta_{i1} (x) + \sigma_i (x)) + \nabla \epsilon (f (x)) \]
\[ + \sum_{i=1}^{m} 2 k^2 \tanh (\xi_{2i} (x)) \left( \tanh^2 (\xi_{2i} (x)) - 1 \right) \sigma_i (x) \]

where \( \xi_{1i} \in R \) chosen between \( 1 - \tanh^2 (A_i (x)) \) and \( 1 - \tanh^2 (\xi_{1i} (x)) \) , \( \xi_{2i} \in R \) chosen between \( A_i (x) \) and \( A_i (x) \).

As the number of neurons \( N_0 \) increases, \( \epsilon_{HJB} \) will approach 0. Namely, when \( \epsilon_{HJB} > 0 \) exists, there is a unique positive definite \( N_h \) and if \( N_0 > N_h \), there will be \( \| \epsilon_{HJB} \| \leq \epsilon_h \).

Hence, by using the median theorem, the optimal control law can be rewritten as follows:
\[ u^* (x) = -k \tanh (\xi_{1i} (x)) + \epsilon_u + a \quad (36) \]
where \( \epsilon_u = -\frac{1}{2} (1 - \tanh^2 (\xi)) g^T \nabla \epsilon \) and the value of \( \epsilon \in m \) limited to \( \xi_{1i} (x) \) and \( A (x) \).

Because the ideal neural network weight is unknown, formula (36) cannot be calculated in the actual control process.
Therefore, we choose a critic network to approximate the value function. Then the value function can be represented as
\[ \hat{V}(x) = \hat{W}_c^T \varphi(x) \] (37)
where \( \hat{W}_c \) is the estimated value of \( W_c \). The error value of the weight can be defined as \( \hat{W}_c - W_c \), and the estimated value of the optimal control using equation (37) can be represented as follows
\[ \hat{u}(x) = k \tanh\left( -\frac{1}{2k} g^T(x) \nabla \varphi^T W_c \right) + a \] (38)

According to formulas (17), (36), and (38), the approximate Hamiltonian can be represented as
\[ H \left( x, W_c^T \right) = \hat{W}_c^T \nabla \varphi f(x) + d^2 \alpha(x) + x^T Q x + k^2 \sum_{i=1}^m \ln \left[ 1 + \tanh^2(\xi_{2i}(x)) \right] + 2k a \left( \xi_2(x) + \sigma(x) \right) \Delta = e \] (39)
where \( \xi_2(x) = -(1/2k) g^T(x) \nabla \varphi^T W_c \), \( \xi_{2i}(x) = \left[ \xi_{21}(x), \ldots, \xi_{2m}(x) \right] \) with \( \xi_{2i}(x) \in \mathbb{R}, i = 1, \ldots, m \).

From formula (35) and formula (39), we can get
\[ e = -\hat{W}_c^T \nabla \varphi f(x) + \sum_{i=1}^m k^2 \left[ \Upsilon(\xi_{2i}) - \Upsilon(\xi_{1i}) \right] - \epsilon_{HJB} \] (40)
where \( \Upsilon(\xi_{ai}) = \ln \left[ 1 - \tanh\left( \Upsilon(\xi_{ai}) \right) \right] \), \( \alpha = 1, 2 \), so for \( \forall \xi_{ai}(x) \in \mathbb{R}, \xi_{ai}(x) \) can be expressed as
\[ \Upsilon(\xi_{ai}) = -2 \ln \left[ 1 + \exp(-2\xi_{ai}(x) \text{ sgn} \left( \xi_{ai}(x) \right)) \right] - 2\xi_{ai}(x) \text{ sgn} \left( \xi_{ai}(x) \right) + \ln 4 \] (41)
where \( \text{ sgn} \left( \xi_{ai}(x) \right) \in \mathbb{R} \) is symbolic function. It should be noted that
\[ \sum_{i=1}^m \Upsilon(\xi_{ai}) = -2 \sum_{i=1}^m \ln \left[ 1 + \exp(-2\xi_{ai}(x) \text{ sgn} \left( \xi_{ai}(x) \right)) \right] - 2\xi_{ai}^T(x) \text{ sgn} \left( \xi_{ai}(x) \right) + m \ln 4 \] (42)

Hence, combining (40) and (42), we have
\[ e = 2k^2 \left[ \xi_1^T(x) \text{ sgn} \left( \xi_1(x) \right) - \xi_2^T(x) \text{ sgn} \left( \xi_2(x) \right) \right] - \hat{W}_c^T \nabla \varphi f(x) + k^2 \Psi_\xi - \epsilon_{HJB} \]
\[ = k \left[ W_c^T \nabla \varphi f(x) \text{ sgn} \left( \xi_1(x) \right) - \hat{W}_c^T \nabla \varphi^T \left( \alpha(x) \right) \right] - \hat{W}_c^T \nabla \varphi f(x) + k^2 \Psi_\xi - \epsilon_{HJB} \]
\[ = -\hat{W}_c^T \left[ \nabla \varphi f(x) - k \nabla \varphi^T \left( \alpha(x) \right) \right] + \nu(x) \] (43)

where \( \Psi_\xi = 2 \sum_{i=1}^m \ln \left[ 1 + \exp(-2\xi_{ai}(x) \text{ sgn} \left( \xi_{ai}(x) \right)) \right] \)
\[ \nu(x) = kW_c^T \nabla \varphi^T \left( \alpha(x) \right) \left[ \text{ sgn} \left( \xi_1(x) \right) - \text{ sgn} \left( \xi_2(x) \right) \right] + k^2 \Delta_\xi - \epsilon_{HJB} \]

In order to obtain the minimum value \( e \), we need to choose \( \hat{W}_c \) to minimize the squared residual error \( E = (1/2)e^2 \).

Use the gradient descent method to get the control law of the evaluation function in the form [14]–[16], [26], [29]:
\[ \hat{W}_c = -\frac{\partial E}{\left( 1 + \theta^T \theta \right)^2} \partial \hat{W}_c = -\frac{\partial \theta}{\left( 1 + \theta^T \theta \right)^2} e \] (44)

Among them, \( \theta = \nabla \nabla \left( f(x) + g(x) \hat{u} \right), \theta > 0 \) is a tuning design value, and \( \left( 1 + \theta^T \theta \right)^2 \) is used for normalization.

From the expressions (36) and (38), we can find that the value function and the critic network have the same weight. Hence, if the value function can be approximated by the critic neural network given in (36), the control strategy is obtained by (38).

VI. EXPERIMENTS AND DISCUSSION

In this section, actual production data of 300KA low-energy aluminum electrolytic cell of Chongqing Tiantai Aluminum Co., Ltd. are used to show the effectiveness and performance of our proposed algorithm.

In the experiment, we collected various historical data of the aluminum electrolytic cell production process from January 2017 to February 2018. Due to errors or human factors, the data collected directly from the production process inevitably exit errors and noise. After error elimination, 1680 sets of data are used to experiment. From the above analysis, it can be known that the AEPP is a multi-variable and strongly coupled nonlinear processes. In real AEPP, the temperature of the electrolytic cell and the DC power consumption are important indicators to measure the quality of aluminum products. In the experiment, temperature and average DC voltage of the aluminum electrolytic cell are used as the state variables, feeding number \( u \) is the control variable. Experimental simulation setup is shown in Fig.2.

FIGURE 2. Experimental setup of aluminum electrolytic system.

In order to maintain the quality of the product and maintain the stability of the AEPP, each control variable must reach a certain range. At the same time, according to the constraints of the actuator and the experience of field engineers, each control variable has an allowable range of change.

In order to obtain an accurate model, four-layer RNN with structures 2-2-2-2 is used to identify the input and output data.
The neural network initial value is $x_0 = [-0.5, 0.5]^T$. The identification parameters are: $\eta=20$, $\Lambda_1=[1, 0.5; 0.5, 1]$, $\Lambda_2=[1, 0.2; 0.2, 1]$, $\Lambda_3=[1, 0.1; 0.1, 1]$.

The identification error is shown in Figs. 3 to 5. From Fig.3 and Fig.4, it can be seen that the proposed method can identify the state of the system well. Fig.5 shows the model identification error. In the initial stage, the model error is large due to the inappropriate initial value. After a period of time, the model error converges to zero. From Fig.5, we can say that the proposed identifier network can effectively approximate the unknown nonlinear AEPP system.
The stability parameters in the obtained identification model (12) are: $W_1 = \begin{bmatrix} -0.9475 & -2.1183 \\ 0.4692 & -0.4939 \end{bmatrix}$, $W_2 = \begin{bmatrix} -0.4945 & -1.4223 \\ 0.7137 & -0.1438 \end{bmatrix}$, $W_3 = \begin{bmatrix} 0.4145 & 0.5758 \\ 0.1958 & 0.4049 \end{bmatrix}$.

According to the experience in the AEP, when the temperature of the electrolytic cell is controlled at 945°C and the average voltage is controlled at about 3645 millivolts, the reaction reaches a dynamic equilibrium and the quality of the produced aluminum can meet the requirements. In real production, by adjusting the feeding number can make the temperature and average voltage of the electrolytic cell within the required range. That is, the system state can reach the desired value by controlling the feeding number.

Keeping the weight of the RNN unchanged, we choose the activation function of the evaluation network as:

$$\varphi(x) = [x_1^2, x_2^2, x_1x_2, x_1^4 - x_2^4, x_1x_2^2, x_1^2x_2^2, x_1x_2^3, x_1^3]$$

The number of neurons here is obtained by computer simulation. The initial evaluation network weight can be set to 0, the initial state is set to $x_0 = [0.4178, -0.3776]^T$, and the parameters are set to $k = 1$, $\gamma=0.95$.

The tracking results of average voltage and temperature of electrolytic cell are shown in Figs. 6 and 7, respectively. The red dashed lines in the figure indicate the desired control state values. It is obvious that the tank average voltage and temperature can quickly converge to the desired stable state. Weight convergence of the critic NNs is shown in Fig.8. It can be seen that the weight of the critic network can converge to the stable state.

To ensure the stability of the system, the control input in the actual production is limited to a certain range. In order to verify the input bounded robust control strategy proposed in this paper, the experiments compare the case without input constraints and with input constraints. Under the controller with no input constraint, when the control input exceeds the constraint value, the control input is set to the constraint value. The experimental results are shown in Fig.9 and Fig.10.

From Fig.9, it is obvious that the control input has exceeded the constraint value of 450. In contrast, in Fig.10, we can see that the control input can still achieve better control results when the control input does not exceed the constraint value. Hence, experimental results show that the proposed algorithm can achieve the constrained optimal control very well.

VII. CONCLUSION

In this paper, a novel optimal control method based on ADP was developed for AEP system with control constraints. To overcome the challenge of establishing an accurate mathematical model for the AEP system, a data-driven model based on the production data was established using RNN. Then, an optimal robust controller based on ADP was designed to ensure the control input is constrained in order not to exceed the bound of the actuator. Furthermore, the robust control problem is effectively transformed to the constrained optimal control problem via system transformation. In particular, the proposed optimal control algorithm is implemented via a single critic network framework. Experimental results demonstrated the effectiveness of the proposed algorithm, where temperature and average voltage of the aluminum electrolytic cell can quickly converge to the desired stable state.

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