Predicting Performances of Mutual Funds using Deep Learning and Ensemble Techniques

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Abstract
Predicting fund performance is beneficial to both investors and fund managers, and yet is a challenging task. In this paper, we have tested whether deep learning models can predict fund performance more accurately than traditional statistical techniques. Fund performance is typically evaluated by the Sharpe ratio, which represents the risk-adjusted performance to ensure meaningful comparability across funds. We calculated the annualised Sharpe ratios based on the monthly returns time series data for more than 600 open-end mutual funds investing in listed large-cap equities in the United States. We find that long short-term memory (LSTM) and gated recurrent units (GRUs) deep learning methods, both trained with modern Bayesian optimization, provide higher accuracy in forecasting funds’ Sharpe ratios than traditional statistical ones. An ensemble method, which combines forecasts from LSTM and GRUs, achieves the best performance of all models. There is evidence to say that deep learning and ensembling offer promising solutions in addressing the challenge of fund performance forecasting.

Keywords: deep learning, forecasting, fund performance, LTSM, GRU, ensemble.

1. Introduction

Investment funds are important intermediaries of the world’s financial market. Globally, there were more than 126,000 regulated mutual funds in 2020. In the United States alone, there were around 9,000 mutual funds with 24 trillion dollars of assets under management (AUM) in 2020, according to the Investment Company Fact Book by [IC] (2021). Choosing the right investment fund has always been a challenging task for both institutional and individual investors. Fund performance is driven by various factors, including the fund managers’ understanding of the macro environment, industry factors, stock performance as well as trading skills. While it is always written in most funds’ prospectus that past performance is not indicative of future performance, there has been ample empirical literature that documents fund performance persistence, for example, among fixed income funds as in [Kahn and Rudd (1995)], equity funds and real estate funds [Arnold et al. (2019)]. Some attribute performance persistence of mutual funds to managers’ skills [Berk and van Binsbergen (2015), Kacperczyk et al. (2014)] whereas others explain performance persistence by common factors affecting stock returns and persistent differences in fund expenses and transaction costs [Carhart (1997)] and capital flows [Lou (2012)].

While the asset pricing literature centers around factors explain fund performance and performance predictability, the topic of forecasting fund performance seems to be under-explored. It is therefore interesting to be able to answer the question of whether future fund performance can be forecast given past performance data, especially with the advanced techniques in deep learning for time-series
Being able to forecast fund performance with confidence will allow asset owners, typically individual investors, endowments, foundations, pension funds and sovereign funds, to make more informed decisions with regard to their future asset manager selection. It is well documented in the literature that investors recognize better skilled managers and reward outperformance with more fund flows (Berk and van Binsbergen, 2013). Better performance predictability will drive fund flows in the market to outperforming funds, at the same time creates pressure for under-performing funds to improve. This process enhances market efficiency.

Despite its significance, there has been lack of research in forecasting fund performances, especially research that involves forecasting the performance of multiple mutual funds as multiple simultaneous time series using one model. This research is intended to fill this gap. We aim to evaluate a range of techniques including modern deep learning algorithms, traditional statistical techniques, and the ensemble methods that combine model forecasts in different ways.

In this work, we investigate whether deep learning techniques are better at forecasting future fund performances based on past performance data than conventional statistical methods. Deep learning is comprised of sophisticated neural networks based methods designed to learn patterns from data. Deep learning has demonstrated its effectiveness in many prediction problems that involve natural language and vision datasets. For time series data, while there have been certain advances, there is still lack of convincing evidence that deep learning outperforms other simpler, statistics-based methods on different problem domains.

Past research used to conclude that neural networks were not well suited for time series forecasting (Hyndman, 2018). This may be attributable to the fact that numerous deep learning based algorithms require large amount of data to perform well. For example, the famous BERT model, a type of transformer used for natural language processing, was pre-trained on an extremely large text corpus including the English Wikipedia with over 2,500M words (Kenton and Toutanova, 2019). The nature of time series data does not permit such massive amount of data available for training deep learning models.

Contrarily, recent research in deep learning applied to the time series domain shows more promising results. In M4 forecasting competition, the hybrid approach that integrates exponential smoothing into advanced neural networks outperforms other statistical methods and wins the competition as in (Smyll, 2020). (Hewamalage et al., 2021) investigates the effectiveness of recurrent neural networks in forecasting a wide variety of datasets. The study concludes that RNNs are competitive alternatives to state-of-the-art univariate benchmark methods such as ARIMA and exponential smoothing (ETS).

What these studies have in common is that they investigate the recurrent neural network (RNN) algorithms that are well known approaches in sequence modeling. They also develop global deep learning models that are capable of learning from multiple time series to improve prediction accuracy for individual ones. This technique is known as cross-learning, which has been found as a promising alternative to traditional forecasting (Semenoglou et al., 2021).

Our study aims to address the question whether cross-learning with carefully tuned RNNs, specifically LSTM and GRUs, work more effectively than traditional univariate methods in the problem of forecasting performances for multiple mutual funds. The sample of mutual funds are relatively homogeneous in a sense that they adopt a similar investment strategy in the United States, which increases the chance that information in one time series would help predict the future values of other time series.

We compare the accuracy of ARIMA, Theta, and ETS statistical methods with that of deep learning algorithms in predicting fund performance. While ARIMA and ETS are popular univariate statistical forecasting models, Theta was the best-performing forecasting algorithm in the M3 competition according to (Makridakis and Hibon, 2000). In addition to individual deep learning and statistical algorithms, we propose a new method of ensembling with voting and investigate how this method as well as other methods for combining model forecasts lead to different performance gains.

In summary, the contributions of our research are:

- To the best of our knowledge, our research is the first that addresses the problem of forecasting performance of multiple mutual funds simultaneously using modern deep learning approaches.
- We present detailed methodology for training deep learning models, specifically, for LSTM and GRU algorithms, using modern Bayesian optimization.
- We propose a new method of ensembling with voting, in which the model predictions are combined based on a voting scheme.
- We experiment with other different ensembling methods for combining forecasts from all models and top performing ones including the use of local out-of-sample performance per time series per algorithm as well as global out-of-sample performance per algorithm in computing weighted averages of forecasts.
- We compare deep learning performances against a set of well-known statistical methods using rolling origin cross-validation with well-defined data splitting methods.

2. Study Background

This section presents the literature related to measuring and predicting performance of mutual funds and traditional univariate forecasting models and deep learning models for time series forecasting.
2.1. Predicting mutual fund performance

Fund performance is fundamentally assessed based on two aspects (1) expected return and (2) risk level, i.e., the variability of returns, which is measured by the standard deviation of return. Although the expected rate of return of a fund is a critical indicator, it alone does not provide an accurate picture of how the fund has performed relating to other funds as higher expected returns do come at a cost of having a higher level of risk. Different funds could exhibit different variability of returns due to difference in risk-taking strategies, diversification levels or management skills.

Since the 1960s, the Sharpe ratio introduced by Sharpe (1966) and modified in Sharpe (1994) has become the most popular performance statistics of mutual funds for considering the trade-off between fund’s risk and return. Sharpe ratio is also known as the reward-to-variability ratio as a fund’s excess return (the rate of return over and above the risk-free rate of return) is assessed against its level of risk, i.e., the variability of returns. Therefore, Sharpe ratio is unaffected by scale. The implied assumption of the Sharpe ratio is that all investors are able to invest funds at the risk-free interest rate, which represents the opportunity cost of investing in an investment fund or the cost of capital invested. Fund managers are under constant pressure to deliver high Sharpe ratio, otherwise risking losing capital allocated to their fund.

While there are other performance indicators introduced in academic literature, the Sharpe ratio has always been among the preferred choices of the industry practitioners when it comes to fund performance evaluation thanks to its simplicity. It is reported by most asset managers and included in large fund databases such as Morningstar or Lipper, cited reports from Elton and Gruber (2020). The Sharpe ratio of a fund is typically compared to the Sharpe ratio of the market. For a period, if the Sharpe ratio of a fund is higher than that of the market, the fund has outperformed the market and generated a positive alpha. Alternative to the market portfolio, the Sharpe ratio of a fund can also be compared to that of an index that represents the investment strategy of the fund - i.e., the benchmark index specified in the fund prospectus. Therefore, in this paper, we use the Sharpe ratio as an indicator of fund performance.

Given the scant literature on forecasting fund performance, the selection of a reasonable forecasting horizon remains an open question. Our considerations include a wide range of time windows in which performance persistence is found to hold as identified in the persistence literature stream. Persistence studies investigate whether funds’ relative performance is sustained over adjacent periods. Some find persistence to be short-lived (Bollen and Busse 2005), with performance persistence observed only within six months (Kacperczyk et al. 2014) or one year (Carhart 1997). Lou (2012) find that evidence for funds’ persistence reversed after six to twelve quarters. However, Irvine et al. (2022) provide evidence that outperformance persists for at least 17 to 24 months, depending on different asset pricing models. For longer time windows, Berk and van Binsbergen report large cross-sectional performance differences in managers’ skill persist for as long as ten years. Based on the diverse predictability and persistence time windows observed in the literature, we decide to use a forecasting horizon of 18 months with funds’ performance being forecast for each and every single month of the subsequent 18 months.

Most studies on forecasting of fund performance have used traditional time series forecasting models. Starting from the 1980s and 1990s, more advanced neural networks techniques were used to address financial forecasting questions. Specifically, the use of neural networks to forecast fund performance were introduced in the papers of Chiang et al. (1996) and Indro et al. (1999). Chiang et al. (1996) used neural networks to forecast mutual fund net asset value while Indro et al. (1999) employed the same approach to forecast mutual fund performance in terms of percentage returns.

A more recent paper is Wang and Huang (2010). While Wang and Huang (2010) also use Sharpe ratio to measure fund performance like ours, they compare the fast adaptive neural network classifier (FANNC) with the Back Propagation neural network (BPN) model and report that the FANNC is more efficient than the BPN in both classifying and predicting the performance of mutual funds. It should also be noted that Wang and Huang (2010) use manager’s momentum strategies and herding behavior as input variables for the prediction of funds’ Sharpe ratio whereas, in our approach, past Sharpe ratio is the input to forecast future one.

The study of Pan et al. (2018) is similar to the paper of Wang and Huang (2010) in which they use three inputs of fund size, annual management fee and fund custodian fee and the fund return as the output for data envelopment analysis, Back Propagation Neural Network and GABPN (new evolutionary method) for 17 open-end balance stock funds for the period August 31, 2015 to July 1, 2016. This study also uses five accuracy measures to evaluate forecast performance, analyzes the rate of return and builds the mutual fund net worth prediction model. Compared to Pan et al. (2018), our paper examined a substantially larger number of funds over a much longer time window using extensive forecasting methods such as deep learning, ensemble and traditional statistical techniques.

2.2. Traditional univariate forecasting models

Time series forecasting and especially stock price and fund performance forecasting have traditionally been a popular research topic in statistics and econometrics. Traditional methods range from simple seasonal decomposition (STL) and simple exponential smoothing (SEM) to more complex ones such as ETS (Hyndman et al. 2008) and ARIMA (Newbold 1983, Box et al. 2015). Non-linear
autoregressive time series using numerical forecasting procedure (n-step-ahead predictive) is also used in Cai (2005) and automatic ARIMA in Melard and Pauwels (2000) using Time Series Expert TSE-AX. Combining several traditional methods can be seen in Bergmeir and Benítez (2012), who used six traditional forecasting methods namely autoregression (AR), moving average (MA), combinations of those models (ARMA) and integrated ARMA as ARIMA, Seasonal and Trend decomposition using Loess (STL) as well as TAR (Threshold AR) based on cross-validation and evaluation using the series’ last part. They suggest that the use of a blocked form of cross-validation for time series evaluation should be the standard procedure.

The traditional uni-variate methods dominated other complex computational methods at many forecasting competitions, including M2, M3 and M4 as in Ahmed et al. (2010), Crone et al. (2011), Makridakis and Hibon (2000). Recent paper, Fiorucci and Louzada (2020) proposes the methodology for combining time series forecast models. They use weights from a cross-validation scheme which assigns higher weights to methods with more accurate in-sample predictions. The methodology is used to combine forecasts named generalized rolling origin evaluation from traditional forecasting methods as the Theta, Simple Exponential Smoothing (SES), and ARIMA models. Makridakis et al. (2020) reported 61 forecasting methods used in M4 competition with 100,000 time series. The paper of Li et al. (2020) used the ARIMA model with deep learning to forecast high-frequency financial time series.

The traditional uni-variate methods show that they work well when the data volume is not extensive (Bandaara et al. 2020; Xu et al. 2019 and Sezer et al. 2020) and a low number of parameters to be estimated.

2.3. Deep learning forecasting models

Recently, several papers include the traditional methods with deep learning (Li et al. 2020; Pannakkong et al. 2017, and Xu et al. 2019). Li et al. (2020) work on the Chinese CSI 300 index and compare method as Monte Carlo numerical simulation, ARIMA, support vector machine (SVM), long short-term memory (LSTM) and ARIMA-SVM models. Their results show that the enhanced ARIMA model based on LSTM not only improve the forecasting accuracy of the single ARIMA model in both fitting and forecasting, but also reduce the computational complexity of only a single deep learning model. Pannakkong et al. (2017) propose a hybrid forecasting model involving autoregressive integrated moving average (ARIMA), artificial neural networks (ANNs) and k-means clustering. Xu et al. (2019) use all linear model and deep belief network (DBN) model.

Several other papers, Jianwei et al. (2019), Cao and Wang (2019), and Sezer et al. (2020) used mixture models for financial commodity and stock forecasting. Jianwei et al. (2019) employed independent component analysis (ICA), gate recurrent unit neural network (GRUNN) named ICA-GRUNN among others to estimate gold prices. The paper shows that ICA-GRUNN produces prediction with high accuracy and outperforms the benchmark methods, namely ARIMA, radial basis function neural network (RBFNN), long short term memory neural network (LSTM), GRUNN and ICA-LSTM. Cao and Wang (2019) proposes convolutional neural network (CNN) and CNN-support vector machine (SVM) to forecast stock index. The paper concludes that neural networks method for financial prediction can deal with continuous and categorical variables and yield good prediction accuracy. Sezer et al. (2020) presents a review of financial time series forecasting methods. Machine Learning (ML) models (Artificial Neural Networks (ANNs), Evolutionary Computations (ECs), and Genetic Programming (GP), and Agent-based models), Deep Learning (DL) models (Convolutional Neural Networks (CNNs), Deep Belief Networks (DBNs), and Long-Short Term Memory (LSTM)) have appeared within the field, with results that significantly outperform their traditional ML counterparts for data like market indices, foreign exchange rates, and commodity prices forecasting.

Hewamalage et al. (2021) provides a big picture of the perspective direction for time series forecasting using recurrent neural network. The authors present an extensive study and provide a software framework of existing RNN architectures for forecasting as Stacked Architecture and Sequences to Sequence (S2S) for three RNN units including Elman RNN, LSTM and GRU. The main difference between S2S and Stacked with the dense layer and the moving window input format is that in the former, the error is calculated per each time step and the latter calculates the error only for the last time step.

In Chen et al. (2021), a multivariate model is proposed to explain Bitcoin price with other economic and technology variables, using both traditional statistical and deep learning models. They use RNN (Recurrent Neural Network) and RF (Random Forest) for the first stage to detect the importance of input variables. They propose a wide range of models for the second stage such as: ARIMA (Autoregressive Integrate Moving Average), SVR (Support Vector Regression), GA (Genetic Algorithm), LSTM (Long Short-Term Memory), ANFIS (Adaptive Network Fuzzy Inference System), for bitcoin forecast in different periods.

Our research fills the gap of the lack of modern deep learning research for predicting funds’ risk-adjusted performance measured by the Sharpe ratio. The results of our research show that deep learning and ensembling are promising approaches in addressing the challenge of predicting mutual funds’ performance.

3. Methodology

In this section, we present the methodology employed for this research, from cross-validation scheme, model selection to model training and other techniques to ensure the robustness of our results.
3.1. Cross-validation scheme

Cross-validation has been widely used in machine learning to estimate models’ prediction error as described in Ruppert (2004). This method estimates the mean out-of-sample error, also known as the average generalization error by Ruppert (2004).

For normal machine learning problems, where there is no time dimension, k-fold cross-validation is the common cross-validation technique applied in practice. This technique requires the dataset being randomly divided into a train set and a validation set, in k different settings. The random division makes each element of the dataset exist in the validation set exactly once and in training set at least once.

In time series forecasting, there is time dimension, and care must be taken to ensure that time-dependent characteristics are preserved during dataset setup for model evaluations. For example, we cannot simply and randomly assign an element into the validation set, since this may interrupt time order between observations and lead to data leakage, where future data may be used to predict past data.

The study of Bergmeir and Benítez (2012) showed that the use of cross-validation in time series forecasting led to more robust model selection. In this research, we follow the rolling origin scheme to divide our dataset into six cross-validation splits. Models are trained on each training set, and then evaluated on the respective validation set. The error measured in each validation set is the estimated generalization error for that particular fold. Averaging the generalization error estimated for all the folds will yield the estimated generalization error of the model. Model selection is then carried out by comparing average generalization errors across all the models trained.

Specifically, our cross-validation scheme is a slightly adjusted version of Generalized Rolling Origin Evaluation (GROE), as described in Fiorucci and Louzada (2020). Below are the properties of this GROE scheme in connection to our study:

- $p$ denote the number of origins, where a origin is as the index of the last observation of a train set. The origins are referred to as $n_1, n_2, ..., n_p$, which can be found recursively through the equation $n_{i+1} = n_i + m$. In this equation, $m$ represents the number of data points between two consecutive origins. In our study, the number of origins $p$ equals 6, which means original data are split in 6 different ways to produce 6 train/validation set pairs.
- $H$ represents the forecasting horizon which is 18 months in our study.
- $n$ is the length of each time series and $n_1$ is equal to $n - H$ if $n - H$ is greater than or equal a certain threshold.
- $m$ can be calculated as the biggest integer lower than or equal to $H/p$, which is problem dependent.

We recognize that if we strictly follow GROE, this will lead to validation sets with different sizes. For example, suppose the length of each time series $n$ in a dataset is 200, the forecast horizon $H$ is 18, the number of origins equals 6, then $m$ is calculated to be the biggest integer lower than or equal to $H/p$ which is 3. As $n_1$ equals to $n - H = 200 - 18 = 182$, the first train set consists of observations from the start until the 182th point, and the corresponding validation set consist of the remaining 18 (the forecast horizon) points in the dataset. The second origin $n_2 = 182 + 3 = 185$, and thus the second train set consists of 185 beginning observations, while there are only $200 - 185 = 15$ observations at the end for the second validation set.

We believe that setting the length of all the validation sets equal and equal to the forecast horizon will make the validation sets represent the actual forecast dataset when applying a forecasting model in practice better. Therefore, we adjust the method slightly by first determining the last train/validation sets, and then working backwards to determine the other train/validation sets. In our method, the last validation set is defined to consist of the last $H$ observations in the dataset, and the corresponding train set consists of all the observations in the dataset prior to the validation set. Then we shift the origin of the last train set, which is $n - H$, back in time $m$ data points to find the origin of the previous train sets, i.e. using $n_i = n_{i+1} - m$.

This modification ensures all validation sets have the same length, which is exactly equal to the forecast horizon. Our cross-validation scheme results in the following division of train and validation sets:

![Figure 1: Train and validation sets](image)

3.2. Model selection

Most competition-based research requires the use of a separate test set to evaluate the final model that is selected from the cross-validation process. Since our research is primarily concerned with model comparison across different algorithms, it does not require the use of such a test set. The model selection process as described in Archetti and Candelieri (2019) is the process that minimizes the loss function evaluated on the validation sets across algorithms, the essence of which are highlighted below.

A supervised learning algorithm $A$ maps feature data points $x_1, \ldots, x_n$ to their corresponding "targets" $y_1, \ldots, y_n$, where...
where $y$ is a continuous or categorical variable depending on whether the problem type is regression or classification. All the pairs $(x, y)$ are organized into a dataset $D = \{(x_i, y_i)\}_{i=1}^n$. The model selection problem is mathematically formulated as follows:

$$A^* \in \arg\min_{A \in \mathcal{A}} \frac{1}{k} \sum_{i=1}^{k} \mathcal{L}(A, D^{(i)}_{\text{train}}, D^{(i)}_{\text{valid}})$$

where $\mathcal{L}(A, D^{(i)}_{\text{train}}, D^{(i)}_{\text{valid}})$ is the loss achieved by algorithm $A$ when trained on $D^{(i)}_{\text{train}}$ and evaluated on $D^{(i)}_{\text{valid}}$. The set $\mathcal{A}$ contains all the relevant machine learning algorithms, $\mathcal{A} = \{A^{(1)}, \ldots, A^{(h)}\}$. $k$ represents the number of folds in a $k$-fold cross-validation scheme in which the dataset is split into $k$ equal-sized validation folds, $D^{(1)}_{\text{valid}}, \ldots, D^{(k)}_{\text{valid}}$, and associated training sets $D^{(1)}_{\text{train}}, \ldots, D^{(k)}_{\text{train}}$.

Ruppert (2004) distinguishes model selection from model assessment as two separate objectives of the model training process. Model selection is concerned with estimating the performance of different models to select the best one. When cross validation is used, the average prediction error that is estimated on the validation sets is compared across models for model selection. The other objective, model assessment, is to estimate the prediction error on new data, and this is performed only after we have chosen a final model. The test set is reserved for this type of model assessment.

Grounded on these literature, our research uses an appropriate cross-validation scheme to allow for the comparison of different algorithms and does not rely on a separate, final test set.

3.3. Recurrent neural networks

In this research, we employ two modern types of recurrent neural networks (RNNs) known as LSTM and GRU, and compare their effectiveness in forecasting fund performances against their statistical counterparts. In this section, we present the unit design and mathematics fundamentals of these techniques. In addition to LSTM and GRU, we also describe the vanilla RNNs which provide the foundation for more advanced RNN architectures.

3.3.1. Vanilla recurrent neural networks

RNNs are designed to address the problem of learning from and predicting sequences. They have been widely applied in the field of natural language processing. In the following figure and equations, we illustrate the internal workings of RNNs. The basic RNN cell has the structure as shown in Figure 2.

$$\begin{align*}
H_t &= \phi(H_{t-1} \cdot W_{h1} + X_t \cdot W_{h2} + b_h) \\
O_t &= H_t \cdot W_o + b_o
\end{align*}$$  

In equation 1, $H_t$ denotes the hidden state of the RNN cell, and $X_t$ denotes the input of the cell at the current time step (i.e. time step $t$). $W_{h1}$ and $W_{h2}$ are matrices of weights whereas $b_h$ denotes the bias vector for the hidden state. $\phi$ is the activation function of the hidden state. This equation describes the recurrent computation which gives rise to the term "recurrent", where the current hidden state $H_t$ is computed based on the previous hidden state $H_{t-1}$ and the current input $X_t$. This recurrence in computation leads to the ability of the hidden state $H_t$ to store information of the sequence up to time step $t$. In a similar way, the hidden state of the next time step $H_{t+1}$ is computed based on $H_t$ and the input $X_{t+1}$ at the next time step.

Equation 2 shows how the output is computed using the hidden state of the current time step. In equation 2, $O_t$ denotes the output of the cell at time step $t$, $W_o$ is the weight matrix, and $b_o$ represent the bias vector for the cell output.

Even though the recurrent computation makes it possible for RNNs to carry past information into the current time step, RNNs has limited capability in handling long-term dependencies in sequence data. This type of network suffers from the problems of vanishing and exploding gradients, making learning in long sequences difficult. Vanishing gradients happen when gradients during backpropagation become vanishingly small, and the weights cannot be updated adequately, whereas exploding gradients occur when large gradients accumulate during backpropagation which results in unstable models as model weights receive very large updates.

3.3.2. Long short-term memory

Long short-term memory (LSTM), introduced in Hochreiter and Schmidhuber (1997), extends the capacity of vanilla RNNs to be able to remember and effectively handle longer sequences. The computation of the hidden state in a LSTM is illustrated in Figure 3.
In Figure 3, there are three gates designed to regulate information flow through the memory cell. Specifically, the input gate controls how much information to read into the cell, the forget gate decides how much of the past information to forget and how much to retain, and the output gate reads output from the cell. This type of design enables the cell to decide when to remember and when to ignore inputs, which is essential in remembering useful information and forgetting less useful one.

Mathematically, these gates are computed as follows:

\[ I_t = \sigma (H_{t-1} \cdot W_{i1} + X_t \cdot W_{i2} + b_i) \]
\[ F_t = \sigma (H_{t-1} \cdot W_{f1} + X_t \cdot W_{f2} + b_f) \]
\[ O_t = \sigma (H_{t-1} \cdot W_{o1} + X_t \cdot W_{o2} + b_o) \]

In the above equations, \( I_t, F_t \) and \( O_t \) represent the input gate, forget gate and output gate at current time step \( t \). \( X_t \) is the input at time \( t \) and \( H_{t-1} \) is the hidden state at the previous time step. The \( W \)s represent the weight matrices and the \( b \)s are bias vectors. \( \sigma \) is the sigmoid activation function that makes the values of these gates in the range of \((0, 1)\).

The memory cell and the hidden state are computed as follows:

\[ \tilde{C}_t = \tanh (H_{t-1} \cdot W_{c1} + X_t \cdot W_{c2} + b_c) \]
\[ C_t = F_t \odot C_{t-1} + I_t \odot (\tilde{C}_t) \]
\[ H_t = O_t \odot \tanh (C_t) \]

In the above equations, \( \tilde{C}_t \) represents the candidate memory cell and \( C_t \) represent memory cell content. Similar to the equations of the gates, \( W \)s represent the weight matrices and the \( b \)s are bias vectors. The computation of candidate memory cell is similar to those of the gates, except that it uses the \( \tanh \) activation function instead of sigmoid.

The candidate memory cell captures both past information from \( H_{t-1} \) and information from the current input \( X_t \). The computation of memory cell content \( C_t \) is based on past memory cell state represented by \( C_{t-1} \) and the current memory cell candidate which serves as current input. The element-wise matrix multiplication denoted by \( \odot \) makes it possible for controlling how much of past information is forgotten (by multiplying \( F_t \) by \( C_{t-1} \)) and how much of current input is retained (by multiplying \( I_t \) by \( \tilde{C}_t \)). Both \( F_t \) and \( I_t \) use \( \sigma \) as the activation function, which produce values in the range \((0, 1)\) to control how much information is discarded/retained in element-wise matrix multiplication. For example, when values of \( F_t \) are close to zero, the result of element-wise matrix multiplication between \( F_t \) by \( C_{t-1} \) will make past information close to zero or in other words past information become forgotten.

The computation of the hidden state \( H_t \) depends on the memory cell \( C_t \) and how much of the memory cell is passed as output is controlled by the output gate \( O_t \).

Our study uses the above described design for LSTM, in contrast to Hewamalage et al. (2021) which uses LSTM with peephole connections. We also use the stacked architectures for both LSTM and GRU, where the term “stacked” means that different recurrent layers can be stacked on top of each other.

3.3.3. Gated recurrent units

Gated recurrent units (GRUs) (Cho et al., 2014) were introduced almost two decades after LSTM. GRUs possess a similar but simpler architecture than that of LSTM, which gives them the advantage of faster computation. GRU also use gates to regulate information flow, but it contains only the reset gate and the update gate.

The computation of the hidden state in GRU is described in Figure 4.

![Figure 4: Computation of hidden state in GRU](image-url)
\[ Z_t = \sigma (H_{t-1} \cdot W_{r1} + X_t \cdot W_{r2} + b_r) \]
\[ R_t = \sigma (H_{t-1} \cdot W_{r1} + X_t \cdot W_{r2} + b_r) \]

The above two equations show the mathematical mechanisms of the two gates in GRU: update gate \((Z_t)\) and reset gate \((R_t)\). The computation of both the update gate and reset gate at the current time step \(t\) is based on the hidden state at the previous time step \(H_{t-1}\) and the current input \(X_t\). The \(W\)s are the weight matrices and \(b\)s are the bias vectors. \(\sigma\) is the sigmoid activation function that makes the values of these gates in the range of \((0, 1)\).

The next equations show the computation of the candidate hidden state \((\tilde{H}_t)\) and the hidden state \((H_t)\).

\[ \tilde{H}_t = \tanh ( (R_t \odot H_{t-1}) \cdot W_{h1} + X_t \cdot W_{h2} + b_h) \]
\[ H_t = (1 - Z_t) \odot \tilde{H}_t + Z_t \odot H_{t-1} \]

The candidate hidden state at current time step \(\tilde{H}_t\) captures both past information from the previous hidden state \(H_{t-1}\) and information from the current input \(X_t\). The element-wise matrix multiplication of \(H_{t-1}\) and \(R_t\) makes it possible for the reset gate to control how much of past information in \(H_{t-1}\) is retained as the value of \(R_t\) is in the range \((0, 1)\). If, for example, \(R_t\) equals one, all of the information of \(H_{t-1}\) is retained, and if \(R_t\) equals zero, all previous hidden state information is discarded. The candidate hidden state, therefore, contains the current input information plus parts of previous hidden state information as controlled by the reset gate.

In the next equation, the current hidden state \(H_t\) is computed from the current candidate hidden state \(\tilde{H}_t\) and the hidden state from previous time step \(H_{t-1}\). The update gate is used to control how much information from these two components is passed to the current hidden state. When the update gate \(Z_t\) is close to one, the current hidden state takes most of the information from previous hidden state whereas when the update gate \(Z_t\) is close to zero, the current hidden state takes most of the information from the current candidate hidden state.

A previous study conducted to evaluate GRU and LSTM algorithms on the sequence modeling tasks of polyphonic music modeling and speech signal modeling found GRU to be comparable to LSTM (Chung et al., 2014). However, it is unclear which type of these algorithms performs better in forecasting fund performance. Therefore, our research aims to evaluate and compare these two deep learning methods in the context of fund performance forecasting. Our study implements these methods using the Keras and Tensorflow libraries in Python where Bayesian optimization is used for hyper-parameter optimization, which will be described in a later section.

### 3.3.4. Ensemble methods

Ensemble methods are ways to combine forecasts from different models with the aim to improve models’ performance. There are different methods to combine forecasts. The simplest method would be averaging the model predictions for different algorithms. Other methods can make use of weighted averages. The study of Fiorucci and Louzada (2020) combines time series forecast models with the weights proportional to the quality of each model’s in-sample predictions in a cross-validation process. Their findings reveal that forecast results improve compared to methods that use equal weights for combination. However, all of the methods used for combination in their study are statistical methods, and in-sample prediction qualities are used to determine weights.

Our approach in ensembling forecast results is different from the above study in that in addition to combining forecasts of top performing models, we also combine forecasts for models belong to different approaches, i.e. all deep learning models combined with all statistical models. According to Petropoulos et al. (2022), combined forecasts work more effectively if the methods that generate forecasts are diverse, which leads to less correlated errors. Another difference is that we use out-of-sample model prediction qualities to determine weights, which we believe are more objectively representative of models’ performance than using in-sample prediction qualities. The reason is that deep learning models may easily overfit the training data and produce high quality in-sample predictions while the corresponding performances can significantly drop on out-of-sample data.

In this study, we propose a comparison of four ensembling methods:

- The first method uses equal weights in averaging model forecasts. This is the simple form for combining forecasts, known as simple average.
- In the second method, we introduce a new method of ensembling with a voting scheme. As time series predictions in our study may consist of both negative, and positive values, we use zero to determine the boundary to apply the voting scheme. If there are \(n\) models to combine forecasts, we first compute the number of models that produce negative predictions and non-negative predictions. If for a particular data point, the majority of the models produce non-negative predictions, the voting scheme would select those models for ensembling and ignore the rest. Voting works similarly when the majority of the models produce negative predictions. In this study, we restrict ensembling with voting to the case where the number of models \(n\) is odd to ensure there is a clear majority voting result for each data point.
- The third method calculates weighted average of model forecasts where the weights are inversely proportional to the final out-of-sample MASE (our main error
metric used for evaluation of forecasting models) result of each algorithm. The weights represent the overall (or global) average out-of-sample model quality across all time series for each algorithm.

- The forth method calculates weighted average of model forecasts where the weights are inversely proportional to the average out-of-sample MASE result of each algorithm for the particular time series in question. The weights represent the specific (or local) average out-of-sample model quality for each time series in each algorithm.

### 3.4. Traditional statistical models for time series forecasting

To compare against deep learning algorithms, we select three well-known traditional statistical methods for time series forecasting, which are ARIMA, ETS, and Theta. The Theta method of forecasting, introduced by Fiorucci et al. (2016), is a special case of simple exponential smoothing with drift. Further detail about the three models can be seen in Hyndman and Athanasopoulos (2018).

In our experiments, we use an optimized version of the Theta model proposed by Fiorucci et al. (2016). We use the well-known forecast R package introduced by Hyndman and Khandakar (2008) for training ETS and ARIMA models, and the forrTheta R package for training the optimized Theta models. These models are trained using the parallel processing capabilities provided in the furrr R package.

### 4. Experiment setup

#### 4.1. Data description

Our dataset started with the monthly returns of more than 1200 mutual funds investing in listed large-cap equities in the United States available on Morningstar Direct. We required at least 20 years of data up to October 2021 and obtained 634 funds. These funds are relatively homogeneous as they share the same investment strategy and are all domiciled in the United States. From the monthly returns series, we computed the annualised Sharpe ratios for each fund on a rolling basis. Table 1 summarises the statistics of these time series.

Looking at the details, the table illustrates five fund examples with the annualised Return (percent) and the annualised Sharpe ratio, in terms of their mean, standard deviation, minimum and maximum values. The five funds are respectively the minimum average return, 25th percentile, 50th percentile, 75th percentile and the maximum average return over the period. The risk-free rate of return is the market yield on the 3-month U.S. Treasury Securities. The last two rows represent the overall average return and respectively the Sharpe ratio of all the funds used in the sample.

|          | Mean  | Std   | Min   | Max   |
|----------|-------|-------|-------|-------|
| us45890e7395 | Return | 4.351 | 24.969 | -129.960 | 62.540 |
|          | Sharpe | 0.213 | 0.542 | -1.444 | 2.132 |
| us5529813791 | Return | 7.922 | 23.741 | -99.820 | 70.080 |
|          | Sharpe | 0.295 | 0.555 | -1.448 | 1.780 |
| us82301q7759 | Return | 8.149 | 22.494 | -100.080 | 69.620 |
|          | Sharpe | 0.301 | 0.571 | -1.444 | 2.507 |
| us6499175063 | Return | 8.801 | 29.045 | -114.100 | 95.900 |
|          | Sharpe | 0.304 | 0.584 | -1.288 | 3.129 |
| us1253254072 | Return | 15.065 | 35.222 | -136.560 | 125.460 |
|          | Sharpe | 0.296 | 0.535 | -1.155 | 4.768 |
| Riskfree  |       | 1.464 | 1.713 | 0.011 | 6.356 |
| Overall   | Return | 8.226 | 23.272 | -176.220 | 126.320 |
| Overall   | Sharpe | 0.290 | 0.556 | -2.446 | 4.708 |

Table 1: Time series descriptive statistics

Within the primary “large-cap equities” strategy that categorizes the funds, there are three major sub-strategies, namely growth, value and blend. Those adopting a “growth” sub-strategy focuses on growth stocks listed in the United States, i.e., stocks with strong earnings growth potential, whereas “value” sub-strategy means the investment targets value stocks, those evaluated to be undervalued. “Blend” represents a mix of growth and value sub-strategies. The study sample of 634 funds includes all the 3 sub-categories mentioned above.

#### 4.2. Data preprocessing

##### 4.2.1. Data splitting

The time series data are split into 6 train and validation sets using the cross-validation scheme described in section 3.1 of Methodology. These cross-validation scheme allows for robust and reliable model selection based on their average out-of-sample performances.

##### 4.2.2. Removing trend and seasonality

We transform each time series in the train set of each cross-validation split into its stationary form by two commonly used techniques known as log transformation and differencing. Log transformation stabilizes the variance of the time series (Hyndman and Athanasopoulos, 2013). Since negative Sharpe ratios exist, an offset is added into all time series, to ensure each one of them are all positive, before applying log transformation.

While log transformation is effective in handling time series variances, differencing can help remove changes in the level of a time series, and is therefore effective in eliminating (or reducing) trend and seasonality (Hyndman and Athanasopoulos, 2013). Subsequent to log transformations, we examine each time series to see which one needs differencing and the number of differences required to transform them into stationary form. The result shows that the majority of the time series have been stationary after log transformation and 68 of them need to be applied first differencing. These 68 time series are applied first differencing in each cross-validation split and their last train observation is saved for later inverse transformation back to their original scales.
4.2.3. Data postprocessing

Since each time series has been transformed prior to entering modeling stage, the models’ outputs are not in the original scale and this does not allow for direct comparison of the models’ outputs with the validation sets to obtain error metrics. Therefore, models’ predictions are transformed back to the original scales following the inverse order that we apply in the preprocessing steps. In detail, predictions for those time series that are previously differenced receive inverse differencing; after this step, all time series are inverse log transformed, and then the offset that was previously added is subtracted from each time series.

4.2.4. Forecasting multiple outputs

Unlike univariate statistical methods, deep learning algorithms can potentially take advantage of cross-learning, in which patterns from multiple time series are learnt to improve forecast accuracy for individual time series. In this work, we aim to compare forecasting methods for predicting 634 mutual funds, which involve 634 time series of a homogeneous group of funds. Each time series requires multi-step-ahead forecasts for a forecasting horizon of 18 months. Therefore, we approach the problem as a multiple-output and multi-step-ahead forecasting problem, where inputs are fed into the neural networks, and after training, vectors of outputs produced directly by the model for 18 months ahead. This strategy has been demonstrated effective by the works of Taieb et al. (2012) and Wen et al. (2017) and also adopted by Hewamalage et al. (2021).

Supervised deep learning maps inputs to outputs, where in our time series context, inputs are fed into the model and outputs produced by the model following a sliding window approach. This sliding window scheme is used in the work of Hewamalage et al. (2021). This approach considers each data sample as consisting of an input window of past observations of all time series that is mapped to an output window of future observations which immediately follow the input window in terms of time order relationship. In our experiments, each train set is preprocessed to form multiple sliding input and output windows with these characteristics, where each sliding window of the next consecutive time step has the same format as the previous one but is shifted by one time step. This scheme allows for the use of past (lagged) time series in predicting future time series in each data observation.

As patterns are learnt from inputs to predict outputs, we believe it is necessary to set the length of input sequences to be at least equal to the length of output sequences. This would suggest the deep learning algorithms have sufficient information from the past to predict the future. The length of each input window should not be too large, since the use of too long input windows would significantly reduce the number of training samples and probably affect model performance. In our study, we select the length of input windows to be 20 months, given the length of output windows determined by the forecast horizon chosen which is 18 months. These choices balance the need for covering sufficient information in the input windows and the need to retain as many training samples as possible.

4.3. Training deep learning models

The performance of deep learning models depends on a number of factors, including, among others, the right choice of the loss function, the optimization procedure within the training loop, and the outer optimization procedure that selects the best combination (or configuration) of hyper-parameters. Our training procedure emphasizes extensive hyper-parameter tuning based on careful consideration for hyper-parameter configurations. This section describes in detail these training and optimization procedures.

4.3.1. Loss function

We use \( \text{rmse} \) as the loss function for training deep learning models. This loss function will be optimized when training mini-batches of data samples. This type of loss function has the advantage of restricting large, outlier errors, since large errors may result in really large squared errors. This loss function is also the metric that we optimize in the Bayesian optimization loops.

4.3.2. Hyper-parameter search setup

Hyper-parameters refer to the parameters that models cannot learn during the training process, but may have effect on models’ performance. For RNNs, the number of hidden layers, units in each hidden layer, and learning rate, etc., are important hyper-parameters. The process of identifying which hyper-parameter configuration results in optimal generalization performance is known as hyper-parameter optimization. We employ Bayesian optimization for this optimization procedure, which will be detailed in the next section.

In our experiments, the following hyper-parameters are optimized:

- Learning rate
- Number of hidden layers
- Units in a hidden layer
- Dropout rate for inputs
- Dropout rate for hidden layers
- Mini-batch size
- Whether to use batch-normalization or not
- If using batch-normalization, placing it either before or after dropout layers
- Optimizer functions
• Activation functions
• Weight decay
• Number of epochs

Unlike [Hewamalage et al. (2021)], we consider dropout rate as an important hyper-parameter for optimization. Dropout is an effective technique for addressing the over-fitting problem in deep neural networks [Srivastava et al., 2014]. This technique may be even more appropriate for time series forecasting, where datasets are usually relatively small compared to those in other fields such as NLP or Computer Vision.

We also provide an option to employ batch normalization [Ioffe and Szegedy, 2015] to further regularize and improve training speed, which allow for the possible training of deeper networks. Originally, batch normalization was understood to work by reducing internal covariate shift. Later research demonstrated this concept was a misunderstanding. The work of Santurkar et al., 2018 clarified that the effectiveness of batch normalization lied on its ability to make the optimization landscape significantly smoother.

In the training process that uses batch normalization, this normalization is performed for each mini-batch. By using batch normalization, we can use higher learning rates, and be more relaxed about network initialization. As a regularizer, in some cases, we may not need to use dropout if batch normalization is already in use. In our work, it is, however, uncertain that batch normalization can replace the need for dropout, especially when our training data are limited. Therefore, we choose to optimize both of these hyper-parameters.

There is also additional consideration for the order of batch normalization and dropout in actual implementation. The combination of batch normalization and dropout works well in some cases, but decreases the model performance in some others, and thus their order should be carefully considered [Li et al., 2019].

To work around this issue, and to possibly take advantage of both dropout and batch normalization, in our experiments, we add another hyper-parameter that controls whether batch normalization layers are added to the model before or after dropout layers. The range of dropout rates in our experiments includes zero, which means the case of no dropout is covered.

Another regularization method tuned is weight decay. Weight decay, also called L2 regularization, is probably the most common technique for regularizing parametric machine learning models [Zhang et al., 2021]. This technique works by adding a penalty term (or regularization term) into the model’s loss function, so that the learning process will minimize the prediction loss plus the penalty term. The updated loss function with weight decay is then given by:

$$L = Error + \psi \sum_{i=1}^{P} w_i^2$$

L2 regularization

Applied to our context, Error denotes the root mean square error from the network outputs, $w_i$ represents the trainable parameters of the network with $p$ the number of all such parameters.

Another important hyper-parameter is the number of hidden layers, which controls the network depth. The use of regularization methods described above can help reduce models’ overfitting, which as a result may allow for deeper networks to be trained. Therefore, we set the range of number of hidden layers from one to five, with five hidden layers represent a quite deep network given the limited amount of training data.

In addition to the above, we choose to optimize other hyper-parameters including learning rate, mini-batch size, units which represent the dimensionality of the output space in a hidden layer, number of epochs, optimizers and activation functions. For each numeric hyper-parameter, we choose to optimize them within a range of values for the search, without making the computation cost too high. For example, the learning rate ranges between 0.0005 and 0.2, and the dropout rate ranges between 0.0 and 1.0. To control the computation cost within reasonable limits, we limit the range of the number of epochs to between 1 and 30.

4.3.3. Hyper-parameter optimization

Prior studies have investigated different methods for executing this optimization procedure. Among these, grid search, random search and Bayesian optimization provide three alternatives for optimizing hyper-parameters. Bayesian optimization has demonstrated to be more effective and will be chosen in our experiments.

4.3.4. Grid search

The grid search method finds the optimal hyper-parameter configuration based on a predetermined grid of values. This requires careful consideration in the choice of grid values, as there might be no limit to the number of possible hyper-parameter configurations. For example, a learning rate that ranges between 0.001 and 0.1 can take countless possible values, and when combined with several other hyper-parameters, the search space would be so vast, such that covering all possible configurations in the search is impossible. To limit the choice of hyper-parameters in a grid, we can rely on expert knowledge and experience, but this cannot guarantee success in many cases, especially when there are a large number of hyper-parameters for tuning in deep learning models.

4.3.5. Random search

Random search narrows down the number of possible hyper-parameters to search by selecting a random subset of all possible configurations. By narrowing down the search space, this algorithm reduces training time and has proved to be effective in many cases [Putatunda and Rama, 2019]. The research of Bergstra and Bengio, 2012 and [Putatunda and Rama, 2019] show that random search
is more effective than grid search as a hyper-parameter optimization method.

4.3.6. Bayesian optimization

Bayesian optimization (BO) is a modern hyper-parameter optimization technique that is effective for searching through a large search space that may involve a large number of hyper-parameters. It has demonstrated superior performance to the random search and grid search approach in a variety of settings (Wu et al., 2019; Putatunda and Rama 2019).

The essence of BO is the construction of a probabilistic surrogate model that models the true objective function, and the use of this surrogate model together with the acquisition function to guide the search process. Its procedure first defines an objective function to optimize, which may be the loss function, or some other function as deemed more appropriate for model selection. In training models where evaluation of the objective function is costly, the surrogate which is cheaper to evaluate is used as approximation of the objective function (Bergstra et al., 2011).

The whole process of Bayesian optimization is sequential by nature, since the determination of next promising points to search is dependent on what is already known about the previously searched points. In addition to the surrogate model, another important component of BO is the acquisition function, which is designed to guide the search toward potential low values of the objective function (Archetti and Candelieri, 2019). The acquisition function allows for the balance between exploitation and exploration, where exploitation means that searching is performed near the region of the current best points and exploration refers to the searching in the regions that have not been explored.

The initial probabilistic surrogate model is constructed by fitting a probability model over a sample of points selected by random search or some other sampling method. In the next step, a new promising point to search are identified using the acquisition function. The objective function is then evaluated and then the probabilistic surrogate model updated with the new information. The next step uses the acquisition function to suggest a further promising point. This process is repeated in a sequential manner until some termination condition is satisfied.

Gaussian Process (GP) is a popular choice for the surrogate model. The GP can be understood as a collection of random variables which satisfies the condition that if any finite number of these random variables are combined, the result will be a joint Gaussian distribution (Archetti and Candelieri, 2019). Another surrogate model is Tree-structured Parzen Estimator (TPE). Unlike GP which models \( P(y|x) \) directly, the TPE approach models \( P(x|y) \) and \( P(y) \) (Bergstra et al., 2011), where \( x \) represents hyper-parameters, and \( y \) represents the associated evaluation score of the objective function. The hyper-parameter search space can be defined by a generative process, of which the TPE replaces the prior distributions of the hyper-parameter configuration with specific non-parametric densities; the substitutions become a learning algorithm that then create various densities over the search space (Bergstra et al., 2011).

The TPE algorithm is implemented in the well-known hyperopt Python package in Bergstra et al., (2013). In our experiments, we utilize this library for hyper-parameter optimization using the TPE algorithm of the Bayesian optimization framework, where the number of iterations is set to be 1,000 for each deep learning method. All deep learning models are trained using a server with the following characteristics: 8-core CPU, 16 GB RAM and Linux Ubuntu 20.04.4.

4.3.7. Consideration about shuffling training data

By default, the Keras library shuffles the training data before each epoch. In our study, data are time series data where the observations (or sliding windows) may exhibit dependencies over time. Shuffling training data in this case may disturb the time dependencies among the training observations, but it will not affect the time dependencies among the time steps within the same training observations. It is therefore not obvious whether shuffling training data or not will lead to better results. In our study, we run experiments for LSTM and GRUs in two conditions, either training data is shuffled or not shuffled before each epoch. We believe the result is beneficial for future similar research.

4.4. Forecast Accuracy Measures

In this section, we present the metrics used to compute forecast accuracy that enables performance comparison among the models studied.

Let \( X_t \) denote the observation at time \( t \) and \( F_t \) denote the forecast of \( Y_t \). Then the forecast error \( e_t = X_t - F_t \). Let have \( k \) forecasts and that observation of data at each forecast period.

We use the same notation as in Hyndman and Koehler (2006) mean \( (x) \) to denote the sample mean of \( \{x\} \) over the sample. Analogously, we use median \( (x) \) for the sample median.

The most commonly used scale-dependent measures are based on the absolute errors or squared errors. Let \( X_t \) denote the observation at time \( t \) and \( F_t \) denote the forecast of \( Y_t \). Then the forecast error \( e_t = X_t - F_t \). Let define Mean Square Error \( MSE = \text{mean} (e_t^2) \).

Root Mean Square Error:

\[
RMSE = \sqrt{\text{mean} (e_t^2)}
\]

Mean Absolute Error:

\[
MAE = \text{mean} (|e_t|)
\]

Often, the RMSE is preferred to the MSE as it is on the same scale as the data. Historically, the RMSE and MSE
have been popular, largely because of their theoretical relevance in statistical modeling. However, they are more sensitive to outliers than MAE or MDAE (Median Absolute Error).

Comparing to absolute error, percentage errors have the advantage of being scale-independent, and so are frequently used to compare forecast performance across different datasets. Let defined percentage error as \( p_t = 100e_t/X_t \), The Symmetric Median Absolute Percentage Error:

\[
SMDAPE = \text{median} \left( 200 \frac{|X_t - F_t|}{(X_t + F_t)} \right)
\]

The problems arising from small values of \( X_t \) may be less severe for SMDAPE. However, usually when \( X_t \) is close to zero, \( F_t \) is also likely to be close to zero. Thus, the measure still involves division by a number close to zero.

**Scaled errors** is defined as:

\[
q_t = \frac{1}{N} \sum_{i=1}^{N} \frac{e_t}{X_{t-1}}
\]

which is independent of the scale of the data. A scaled error is less than one if it arises from a better forecast than the average one-step naïve forecast computed in-sample. Conversely, it is greater than one if the forecast is worse than the average one-step naïve forecast computed in-sample (see Hyndman and Koehler 2006).

The famous scaled error is the Mean Absolute Scaled Error:

\[
MASE = \text{mean} (|q_t|)
\]

When MASE < 1, the proposed method gives, on average, smaller errors than the one-step errors from the naïve method. If multi-step forecasts are being computed, it is possible to scale by the in-sample MASE computed from multi-step naïve forecasts.

The recent paper of Kim and Kim (2016) investigated and provided practical advantages of the new accuracy measure MAPE, the mean arctangent absolute percentage error:

\[
\text{MAAPE} = \frac{1}{N} \sum_{t=1}^{N} \text{AAPE}_t = \frac{1}{N} \sum_{t=1}^{N} \arctan \left( \frac{|X_t - F_t|}{X_t} \right)
\]

Although MAAPE is finite when response variable (i.e. \( X_t \)) equals zero, it has a nice trigonometric representation. However, because MAAPE’s value is expressed in radians, this makes MAAPE less intuitive. Note that MAPE does not have a symmetric version, since division by zero is no longer a concern. The MAPE is also scale-free because its values are expressed in radians.

5. Findings and analysis

For each algorithm, we first calculate the average accuracy metrics for each time series across the cross validation splits. The results for all time series of an algorithm are then averaged to yield the final metrics’ values representing the performance of that particular algorithm. For the latter step, we also compute the median and standard deviation in addition to the mean.

Table 2 presents these accuracy metrics measured for two methods of deep learning (LSTM, GRUs) and three statistical methods (ARIMA, ETS and Theta).

| Algorithm | LSTM | GRU | Arima | ETS | Theta |
|-----------|------|-----|-------|-----|-------|
| mae       | 1.362| 1.421| 1.539 | 1.721| 1.762 |
| median    | 1.324| 1.368| 1.500 | 1.700| 1.699 |
| sd        | 0.176| 0.187| 0.172 | 0.159| 0.225 |
| rmse      | 0.402| 0.413| 0.454 | 0.503| 0.513 |
| median    | 0.393| 0.402| 0.445 | 0.498| 0.498 |
| sd        | 0.049| 0.049| 0.044 | 0.043| 0.053 |
| mae       | 0.327| 0.341| 0.370 | 0.414| 0.424 |
| median    | 0.322| 0.332| 0.363 | 0.411| 0.411 |
| sd        | 0.039| 0.041| 0.037 | 0.034| 0.049 |
| smdape    | 99.946| 97.301| 111.320| 133.133| 132.561|
| median    | 95.628| 90.920| 107.999| 130.480| 129.424|
| sd        | 16.357| 19.257| 19.848| 23.878| 20.804|
| mae       | 74.759| 76.074| 77.763| 81.963| 82.685|
| median    | 75.284| 75.789| 76.613| 82.901| 83.061|
| sd        | 4.603| 4.038| 4.402| 3.217| 3.873|

Table 2: Accuracy measures of 5 forecasting models

Overall, LSTM has the best performance with the smallest mean, median and standard error for almost all accuracy metrics except for the smdape metric which yielded a higher mean and median for the model LSTM compared to GRU model. The results clearly show that LSTM is significantly superior to the other four forecasting methods. ARIMA is the best performing traditional statistical approach but still has much lower accuracy than the GRU model. The metrics’ mean results mainly show the same information as the median and the standard error. Therefore in the latter table, we present only the mean value of five different accuracy metrics.

It can be concluded that the methods of deep learning provide significantly higher accuracy results than the statistical traditional ones and this confirms our research hypothesis. This table presents the results where deep learning models are trained without shuffling training data before each epoch, which provides better accuracy than the method that shuffles training data as shown in Table 2 in the Appendix. The results show that LSTM model with non-shuffled training data provides lower errors than the one with shuffled training data for 4 metrics (mae, rmse, mae, smdape). Whereas GRU model with shuffled training data provide a lower error than the one with non-shuffled training data. Especially, GRU with shuffled training data shows the best accuracy measure of smdape and mape.
(even better than the LSTM with non-shuffled training data).

These results show that researcher should carefully choose the accuracy metrics for evaluating models. In our paper, we use mase as the main metric which leads to the choice of the LSTM model with non-shuffled training data as the best model among all deep learning and statistical models.

| Algorithm       | mase   | rmse   | mae    | smdape | maape |
|-----------------|--------|--------|--------|--------|-------|
| simple average  | 1.462  | 0.434  | 0.351  | 110.026| 76.041|
| global weights  | 1.457  | 0.433  | 0.350  | 109.485| 75.977|
| local weights   | 1.456  | 0.432  | 0.350  | 109.411| 75.955|
| voting          | 1.451  | 0.427  | 0.348  | 101.910| 76.521|

| Algorithm       | mase   | rmse   | mae    | smdape | maape |
|-----------------|--------|--------|--------|--------|-------|
| simple average  | 1.350  | 0.394  | 0.324  | 95.755 | 75.356|
| global weights  | 1.350  | 0.394  | 0.324  | 95.780 | 75.358|
| local weights   | 1.509  | 0.463  | 0.363  | 133.792| 75.109|

| Algorithm       | mase   | rmse   | mae    | smdape | maape |
|-----------------|--------|--------|--------|--------|-------|
| simple average  | 1.636  | 0.482  | 0.393  | 128.122| 79.113|
| global weights  | 1.633  | 0.481  | 0.392  | 127.658| 79.011|
| local weights   | 1.620  | 0.488  | 0.389  | 144.893| 77.631|
| voting          | 1.711  | 0.500  | 0.411  | 131.276| 81.883|

| Algorithm       | mase   | rmse   | mae    | smdape | maape |
|-----------------|--------|--------|--------|--------|-------|
| simple average  | 1.376  | 0.405  | 0.331  | 98.426 | 75.157|
| global weights  | 1.375  | 0.405  | 0.330  | 98.290 | 75.160|
| local weights   | 1.373  | 0.404  | 0.330  | 98.122 | 75.149|
| voting          | 1.394  | 0.408  | 0.335  | 98.132 | 75.626|

Table 3: Accuracy measures of Ensemble models

Table 3 provides the results of the ensemble methods applied to combine forecasts in different groups of models: all models (all algorithms in the table), top two models (LSTM and GRUs), top three models (LSTM, GRUs and Arima) and statistical models (Stats in the table). For each choice of ensemble models, we use a different method for combining forecasts as mentioned in the methodology section which includes simple average, weighted average with global weights (global weights in the table), weighted average with local weights (local weights in the table) and ensemble with voting (voting in the table).

The best results come from the ensemble models of GRUs and LSTM only. Then the second best from the three best methods LSTM, GRUs and Arima. The ensembles of all algorithms provide better accuracy for all error metrics than ensembling using the only three statistical methods. Furthermore, ensembling provides lower error metrics than the individual ones for deep learning methods, for simple average and global weights methods. For example, means of mase for LSTM and GRUs are respectively 1.362 and 1.421, whereas the highest mean of ensemble methods of simple average and global weights is only 1.35. Same results are also obtained for other error metrics as rmse, mae and smdape. However the local weights ensemble of GRUs and LSTM seem to be worst than the individual ones (true with mase, rmse, mae and smdape). The maape metric again provides different results where the local weights method provides even better result than simple average and global weights methods, which in the middle of the individual mean of LSTM and GRU values.

The ensemble of the three best methods LSTM, GRUs and Arima for four different methods of weighting provide significantly better accuracy than the individual GRUs and the Arima (mean of mase is 1.394 lower than the one of GRUs and Arima 1.421 and 1.539 respectively). Unexpectedly, the ensemble of all algorithm provide much better accuracy measures than the ensemble of statistical models, but still not better than each of individual deep learning models.

At the beginning, we expect that the ensemble of all models should provide the best accuracy measure as, theoretically, this involves the most diverse set of models that can result in less correlated errors. However, the ensembles of GRUs and LSTM using simple average and global weights provide the best models with the lowest mean of all error metrics except for maape.

Another interesting result is although our proposed method of ensembling with voting does not achieving the best overall performance, it achieves the best performance among all ensemble methods when combining forecasts from all models. This indicates that this method is worth further investigation in future research.

6. Conclusion

The most interesting purpose of this study is to address the challenge of forecasting performances of multiple mutual funds using modern deep learning approaches with a comparison against popular traditional statistical approaches. In addition, we use different ensemble methods to combine the forecasting predictions of both traditional and modern approaches. The results clearly show that the ranking order of model quality for the studied methods are Ensemble, LSTM, GRU, ARIMA, ETS, and Theta respectively.

We find an interesting technical issue in implementing deep learning for time series forecasting which shows that the training method that uses non-shuffled training data provides much higher accuracy than the method that uses shuffled training data, which shuffles the training data before each epoch.

We conclude that deep learning and ensemble methods offer promising solutions to the problem of forecasting performances of multiple mutual funds measured by Sharpe ratios.

7. Appendix

8. Acknowledgement

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| Algorithm | LSTM | GRU |
|----------|------|-----|
| mase     | 1.400 | 1.390 |
| mean     | 1.340 | 1.351 |
| median   | 0.192 | 0.166 |
| rmse     | 0.421 | 0.415 |
| mean     | 0.411 | 0.410 |
| median   | 0.048 | 0.044 |
| mae      | 0.336 | 0.334 |
| mean     | 0.325 | 0.329 |
| median   | 0.041 | 0.036 |
| smdape   | 103.506 | 95.454 |
| mean     | 99.399 | 90.512 |
| median   | 16.103 | 16.004 |
| maape    | 74.562 | 73.750 |
| mean     | 74.343 | 73.568 |
| median   | 4.560 | 3.741 |

Table 4: Accuracy measures with shuffling data

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