Mathematical model of a plain bearer lubricated with molten metal

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Abstract. The article describes the equation of the flow of incompressible lubricating fluid with micropolar properties for a “thin layer”, the continuity equation, the Darcy equation and the energy dissipation rate formula for determining function $F(\theta)$ caused by the molten surface of the bearing sleeve coated with molten metal of the bearing asymptotic solution according to the thermal parameter $K$. Using the exact solution for the zeroth and first approximations, speed and pressure fields in the lubricant and porosity layers and basic performance characteristics were determined.

1. Introduction
To increase power density of modern engines while increasing reliability and durability, it is necessary to improve the design of friction units, i.e. ensuring fluid hydrodynamic lubrication.

One of the ways to solve structural and operational problems is to use low-melting metal melt for coating the surface of a bearing sleeve [1–6].

An analysis of [7–12] shows that in order to ensure a fluid hydrodynamic regime, it is necessary to take into account the presence of a porous coating on the shaft surface.

This article aims to develop design models for a radial plain bearing with a porous coating of the shaft neck [13–14] and a low-melting metal coating used for the surface of bearing sleeves taking into account rheological properties of a micropolar lubricant and ensuring a fluid hydrodynamic friction.

2. Materials and methods
For a bearing sleeve with a porous coating of the shaft neck, having a pole in the center of the shaft (Fig. 1), the equation of the contour of shaft $C_0$, a shaft with a porous coating $C_1$, and a bearing sleeve coated with low-melting metal melt can be written as:

$$ r' = r_0 - H; \quad C_1: r' = r_0; \quad C_2: r' = r_0 (1 + H) + \lambda f(\theta), $$

where $H = \frac{1}{2} \varepsilon \sin^2 \theta + \ldots$, $\varepsilon = \frac{e}{r_0}$ - porous shaft radius; $r_1$ - radius of the bearer coated with fusible melt; $e$ - eccentricity; $\varepsilon$ - relative eccentricity; $H$ - porous layer thickness; $\lambda f(\theta)$ - limited function
Figure 1. Design scheme

The initial equation is a system of dimensionless equations of motion of a micropolar lubricant, Darcy’s law, continuity equations and the formula for the energy dissipation rate to determine function $F(\theta)$ determined by the melt of the bearing sleeve surface:

$$
\begin{align*}
\frac{\partial^2 u}{\partial r^2} + N_1^2 \frac{\partial \phi}{\partial r} = \frac{dP}{d\theta}, \quad \frac{\partial^2 u}{\partial r^2} = \frac{v}{N_1 \frac{\partial \phi}{\partial r}}, \quad \frac{\partial^2 v}{\partial \theta^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v}{\partial r} \right) = 0, \\
\frac{\partial^2 P}{\partial r^2} + 1 \frac{\partial P}{\partial r} + 1 \frac{\partial^2 P}{\partial \theta^2} = 0,
\end{align*}
$$

$$
\frac{d\phi(\theta)}{d\theta} = K \int_0^{\frac{r_1}{\delta}} \left( \frac{\partial v}{\partial r} \right)^2 dr,
$$

where $K = \frac{2\mu \Omega_0}{L \delta}, \quad \eta = \frac{e}{\delta}, \quad \eta_0 = \frac{\lambda r}{\delta}, \quad \Phi(\theta) = \eta_0 f(\theta)$.

For equations (2), the boundary conditions are as follows: $u = 0, v = 0, \nu = 0$ at

$$
\begin{align*}
r_1 = 1 + \eta \cos \theta + \Phi(\theta), \quad u(0) = 0, \quad \left. \frac{\partial P}{\partial r} \right|_{r = r_0} = \frac{\bar{M}}{H} \left( \frac{\bar{r}}{r} \right), \\
\nu(0) = 1, \quad p = P \quad \text{at} \quad r = \frac{n_0}{H}, \quad \left. \frac{\partial P}{\partial r} \right|_{r = \frac{n_0}{H}} = 0, \quad p(0) = p(2\pi) = \frac{p_e}{p'}
\end{align*}
$$

where $\bar{M} = -\frac{\kappa}{\bar{H} \delta^3} r_0^2$.

The relationship between dimensionless and dimensional quantities is given in the form

$$
\begin{align*}
r' = r_0 + \delta r, \quad \delta = r_t - r_0; \quad v' = \Omega_0 v, \quad u' = \Omega \delta u; \quad p' = p\frac{\delta}{r}, \quad p^* = \frac{(2\mu + \kappa) \Omega_0^2}{2\delta^2}, \quad \nu' = \nu; \\
\mu' = \mu; \quad \kappa' = \kappa; \quad \gamma' = \gamma; \quad N^2 = \frac{\kappa}{2\mu + \kappa}, \quad N_1 = \frac{2\mu f^2}{\delta^5 \kappa}, \quad f^2 = \frac{\gamma}{4\mu}.
\end{align*}
$$

In the porous layer: $P^* = \frac{p}{P} r', \quad r' = \bar{H} r^*, \quad k' = k^*$. Given the small gap and $\nu = 0$, in equation (2), let us average the second equation by the thickness of the lubricant layer:
\[
\frac{1}{h + \Phi} \int_{-\infty}^{h} \frac{\partial^2 \nu}{\partial r^2} dr = \frac{1}{N_i (h + \Phi)} \int_{-\infty}^{h} \nu dr + \frac{1}{N_i (h + \Phi)} \int_{-\infty}^{h} \frac{\partial u}{\partial r} dr.
\]  

(5)

Solution (5) with accuracy of \( O \left( \frac{\Phi}{N_i} \right) \), \( O \left( \frac{1}{N_i^2} \right) \), is as follows:

\[
\nu = \frac{1}{2N_i h} \left( r^2 - rh \right), \quad \frac{\partial \nu}{\partial r} = \frac{1}{2N_i h} \left( 2r - h \right), \quad A_i = \frac{1}{2N_i h}.
\]

(6)

Equation (2) taking into account (6) is as follows:

\[
\frac{\partial^2 u}{\partial r^2} + \frac{N_i^2}{2N_i h} \left( 2r - h \right) = \frac{dp}{d\theta}, \quad \nu = \frac{1}{2N_i h} \left( r^2 - rh \right), \quad \frac{\partial u}{\partial r} + \frac{\partial \nu}{\partial \theta} = 0,
\]

\[
\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r^2} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} = 0, \quad \frac{d\Phi(\theta)}{d\theta} = -K \int_{-\Phi(\theta)}^{\Phi(\theta)} \left( \frac{\partial \nu}{\partial \theta} \right)^2 d\theta.
\]

(7)

Function \( \Phi(\theta) \) will be determined as a set by small parameter \( K \):

\[
\Phi(\theta) = K\Phi_1(\theta) + K^2\Phi_2(\theta) + K^3\Phi_3(\theta) + ..., \quad H(\theta), \quad r = -\Phi(\theta)
\]

(8)

On contour \( r = \Phi(\theta) \) speed components \( u \) and \( v \) boundary conditions are as follows:

\[
v(1 + \eta \cos \theta + H(\theta)) = v(1 + \eta \cos \theta) + \left( \frac{\partial v}{\partial r} \right)_{r = 1, \eta \cos \theta} \cdot H(\theta) + \left( \frac{\partial^2 v}{\partial r^2} \right)_{r = 1, \eta \cos \theta} \cdot H^2(\theta) - ... = 0;
\]

\[
u(1 + \eta \cos \theta + H(\theta)) = \nu(1 + \eta \cos \theta) + \left( \frac{\partial \nu}{\partial r} \right)_{r = 1, \eta \cos \theta} \cdot H(\theta) - \left( \frac{\partial^2 \nu}{\partial r^2} \right)_{r = 1, \eta \cos \theta} \cdot H^2(\theta) - ... = 0.
\]

(9)

The asymptotic solution (7) taking into account (3) and (9) is as follows:

\[
v = v_0(r, \theta) + Kv_1(r, \theta) + K^2v_2(r, \theta) + ..., \quad u = u_0(r, \theta) + Ku_1(r, \theta) + K^2u_2(r, \theta) + ...;
\]

\[
\Phi(\theta) = -K\Phi_1(\theta) - K^2\Phi_2(\theta) - K^3\Phi_3(\theta) - ..., \quad p = p_0 + KP_1(\theta) + K^2P_2(\theta) + K^3P_3(\theta) ...
\]

(10)

Substituting (10) in (7) taking into account (3), we have:

- for zero approximation:
  \[
  \frac{\partial^2 u}{\partial r^2} + \frac{N_i^2}{2N_i h} \left( 2r - h \right) = \frac{dp}{d\theta}, \quad \frac{\partial v}{\partial r} + \frac{\partial u}{\partial \theta} = 0, \quad \frac{\partial^2 P}{\partial r^2} + \frac{1}{r^2} \frac{\partial P}{\partial r} + \frac{1}{r^2} \frac{\partial^2 P}{\partial \theta^2} = 0
  \]
  
  \[
  \text{and boundary conditions:}
  u_0 = 1, \nu_0 = 0, v_0 = 1, \quad r = 0
  u = 0, v_0 = 0, u_0 = 0, \quad r = 1 + \eta \cos \theta;
  \]

- for the first approximation:
  \[
  \frac{\partial^2 u}{\partial r^2} = \frac{1}{\mu} \frac{1}{d\theta} \frac{\partial \nu}{\partial r} + \frac{\partial u}{\partial \theta} = 0, \quad \frac{\partial^2 P}{\partial r^2} + \frac{1}{r^2} \frac{\partial P}{\partial r} + \frac{1}{r^2} \frac{\partial^2 P}{\partial \theta^2} = 0;
  \quad \frac{d\Phi_1(\theta)}{d\theta} = K \int_{0}^{1 + \eta \cos \theta} \left( \frac{\partial u_0}{\partial r} \right)^2 dr
  \]
  
  \[
  \text{and boundary conditions:}
  \nu_0 = 0, v_1 = \left( \frac{\partial u_0}{\partial r} \right)_{r = 0}, \quad \nu_1 = 0, v_1 = 0, \quad u_1 = 0, v_1 = 0, \quad u_1 = 0, v_1 = 0, \quad r = 1 + \eta \cos \theta;
  \]

(11)
\[ u_0(0) = \hat{M} \frac{\partial P}{\partial r} \bigg|_{r = \frac{r_0}{H}} \hat{r} = \frac{\partial P}{\partial r} \bigg|_{r = \frac{r_0}{H}} = 0 \]

\[ p_i(0) = p_i(2\pi) = 0; \quad K\phi_i(0) = K\bar{\phi}. \quad \Phi(0) = \Phi(2\pi) = \bar{\phi}. \quad (14) \]

For the zeroth approximation, we seek the exact solution in the form:

\[ U_0 = \frac{\partial \psi_0}{\partial \theta} + V_0(r, \theta); V_0 = -\frac{\partial \psi_0}{\partial \theta} + U_0(r, \theta); \psi_0(r, \theta, \xi) = \bar{\psi}_0(\xi), \bar{\psi} = \frac{r}{\bar{\theta}(0)}; \]

\[ V_0(r, \theta) = \bar{v}(\xi); U_0(r, \theta) = -\bar{u}_0(\xi) \cdot h'(\theta); \quad (15) \]

Substituting (15) in (11) and given (12), we have:

\[ \bar{\psi}_0(\xi) = \frac{\hat{C}_1}{2} (\xi^2 - \xi); \quad \bar{u}_0 = \frac{1}{\bar{C}_1} \frac{n^2 \xi^2}{2N_1} \left( \frac{\xi^2}{4} + \frac{\xi}{2} + 1 \right) \xi + 1; \quad \hat{C}_1 = 6. \quad (16) \]

From \( p_i(0) = p_i(2\pi) = \frac{P_s}{P} \) we have: \( \bar{C}_1 = \hat{C}_1 \quad (17) \)

Given (17) for pressure, we have \( p_i = \bar{C}_1 \eta \cos \theta + \frac{P_s}{P} \).

Given (18), pressure of the lubricant of the porous layer is determined by formula

\[ P(r^*, \theta) = R(r^*) \bar{C}_1 \eta \sin \theta + \frac{P_s}{P^*}. \quad (19) \]

Substituting (19) in (7) for \( R(r^*) \), we have:

\[ R(r^*) = \frac{r_0^2}{r^* - 2\bar{H}r + \bar{H}^2} + \frac{r_0^2}{r^* - 2\bar{H}r + \bar{H}^2} \]

Taking into account \( \hat{M} \frac{\partial P}{\partial r} \bigg|_{r = \frac{\bar{r}(\xi)}{\bar{r}}} = \int_0^1 \bar{u}(\xi) d\xi \) for \( \bar{C}_1 \) we have:

\[ \bar{C}_1 = \frac{6r_0^2}{12\bar{H}^2 \bar{M}} \frac{r_0^2 - 2\bar{H}r + \bar{H}^2}{\bar{r}^* - \bar{H}} \]

then \( p_0 \) we have:

\[ p_0 = \frac{6r_0^2}{12\bar{H}^2 \bar{M}} \frac{r_0^2 - 2\bar{H}r + \bar{H}^2}{\bar{r}^* - \bar{H}} \eta \sin \theta + \frac{P_s}{P^*}, \quad (22) \]

To determine \( \Phi_i(\theta) \) taking into account equation (16) and \( K\Phi_i(\theta) = K\bar{\phi} \) we use the following equation:

\[ \Phi_i(\theta) = \frac{2}{\sqrt{1 - \eta^2}} \arctan \left[ \frac{1 + \eta^2 \theta}{\sqrt{1 - \eta^2}} \left( \frac{\hat{C}_1^2 + 2 - 4\eta^2 + 4\eta^2}{12} + \frac{\hat{C}_1^2 (1 - 3\eta^2)}{6(1 - \eta^2)} + \frac{\hat{C}_1^2 (1 - 3\eta^2)}{720N_1^2} \right) \right] \]

\[ + \frac{\ln \frac{\theta}{2}}{(1 - \eta^2) \ln \frac{\theta}{2} + 1 + \eta^2} \left[ \frac{\hat{C}_1^2 (28 - 31\eta^2 + 9\eta^2)}{4(1 - \eta^2)} + \frac{\hat{C}_1^2 (1 - 3\eta^2)}{6(1 - \eta^2)} + \frac{\hat{C}_1^2 (4 - 5\eta^2 + 3\eta^2)}{24(1 - \eta^2)(1 - \eta)} \right] \left( \frac{1}{(1 - \eta) \ln \frac{\theta}{2} + 1 + \eta^2} \right)^2 \bar{\phi}. \quad (23) \]

For the first approximation, we found an exact self-similar solution in the same way as for the zeroth approximation (taking into account the melt); as a result we have:
\[ \psi_i(\xi) = \frac{c_i}{2} (\xi^2 - \xi), \quad \hat{u}_i(\xi) = \frac{c_i}{2} \xi^2 + M \xi + M, \quad (24) \]

Based on \( p_i(0) = p_i(2\pi) = 0 \) we have: \( \hat{c}_2 = -M \hat{c}_1, \)

\[ (25) \]

where \( M = \sup_{0 \leq \theta \leq 2\pi} \left( \frac{\partial u_{0}}{\partial r} \right) \Phi_i(\theta) \)

In view of (25), similarly to the zeroth approximation for dimensionless pressure, we have:

\[ p_i = \frac{6M_0 \left( 2\varphi_0^2 - 2\tilde{H}_0 + \tilde{H}^2 \right)}{12\tilde{H}^2 \sin(\theta) + 2\left( 2\varphi_0^2 - 2\tilde{H}_0 + \tilde{H}^2 \right)} \quad (26) \]

Given (11), (13), (22) and (26) for the friction force and the component of the supporting force vector, we have:

\[ R_y = \frac{\mu \Omega_{01}^2 \delta}{\delta^2} \int_0^{2\pi} \left( p_0 - \frac{P}{r^2} + K p_i \right) \sin \theta d\theta = \frac{M_0 \left( 2\varphi_0^2 - 2\tilde{H}_0 + \tilde{H}^2 \right)}{12\tilde{H}^2 \sin(\theta) + 2\left( 2\varphi_0^2 - 2\tilde{H}_0 + \tilde{H}^2 \right)}; \]

\[ R_x = \frac{\mu \Omega_{01}^3 \delta^3}{\delta^2} \int_0^{2\pi} \left( p_0 - \frac{P}{r^2} + K p_i \right) \cos \theta d\theta = 0. \]

\[ L_{r_p} = \left[ \frac{\hat{c}_2}{6} \xi^2 + K \frac{\hat{c}_2}{6} \xi^2 \right] d\theta = \frac{M_0 \left( 2\varphi_0^2 - 2\tilde{H}_0 + \tilde{H}^2 \right)}{6\tilde{H}^2 \sin(\theta) + 2\left( 2\varphi_0^2 - 2\tilde{H}_0 + \tilde{H}^2 \right)} \left( 1 + K M + \frac{N^2 \pi}{4N_1} \right). \]

\[ (27) \]

3. Results and Discussion

Based on the results of numerical analysis, the following diagrams were built:

**Figure 2.** The dependence of the friction force on parameters \( K \) determined by melt \( N_1 \) characterizing the size of the molecules of a micropolar lubricant and bond parameter \( N_2^2 \).

A significant contribution of structural parameters \( K \) determined by melt \( N_1 \) characterizing the size of the molecules of a micropolar lubricant and bond parameter \( N_2^2 \) was shown. With an increase in the design parameter \( K \) (at \( K = 0 \) and \( K \neq 0 \)), the friction coefficient decreases by 21 %, and the bearing capacity increases by 12 %.

The dependence of the coefficient of friction on structural parameter \( K \) is close to linear in the range of 0.0009–0.0035.
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