On the Isovector Channels in Relativistic Point Coupling Models within the Hartree and Hartree-Fock Approximations

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Abstract

We investigate the consequences of Fierz transformations acting upon the contact interactions for nucleon fields occurring in relativistic point coupling models in Hartree approximation, which yield the same models but in Hartree-Fock approximation instead. Identical nuclear ground state observables are calculated in the two approximations, but the magnitudes of the coupling constants are different. We find for model studies of four-fermion interactions occurring in two existing relativistic point coupling phenomenologies that whereas in Hartree the isovector-scalar strength $\alpha_{TS}$, corresponding to $\delta$–meson exchange, is unnaturally small, indicating a possible new symmetry, in Hartree-Fock it is instead comparable to the isovector-vector strength $\alpha_{TV}$ corresponding to $\rho$–meson exchange, but the sum of the two isovector coupling constants appears to be preserved in both approaches. Furthermore, in Hartree-Fock approximation, both QCD-scaled isovector coupling constants are natural (dimensionless and of order 1) whereas in Hartree approximation only that of the isovector-vector channel is natural. This indicates that it is not necessary to search for a new symmetry and, moreover, that the role of the $\delta$–meson should be reexamined. This work presents the first comparisons of naturalized coupling constants coming from relativistic Hartree and relativistic Hartree-Fock solutions to the same Lagrangian.

Key words: Relativistic mean-field model, Hartree, Hartree-Fock, Fierz transformation

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1 Introduction

Relativistic mean field (RMF) models are quite successful in describing ground state properties of finite nuclei and nuclear matter properties. Such models describe the nucleus as a system of Dirac nucleons that interact in a relativistic covariant manner via mean meson fields [1,2,3,4] or via mean nucleon fields [5,6,7,8]. The meson fields are of finite range (FR) due to the meson exchange whereas the nucleon fields are of zero range (contact interactions or point couplings PC) together with derivative terms that simulate the finite range meson exchanges. A common element to the calculations referenced above is that they have all been performed in relativistic Hartree approximation.

The RMF-FR studies to date have generally considered three explicit meson fields. These are the isoscalar-scalar field due to exchange of the $\sigma$ meson, the isoscalar-vector field due to $\omega$ meson exchange, and the isovector-vector field due to $\rho$ meson exchange. The isovector-scalar field due to $\delta$ meson exchange has generally not been included because its contribution to the nuclear force from one-boson exchange is considered weak [10], given a relatively large mass of 983 MeV and a relatively small (but not well determined) coupling constant. It is, however, included in the RMF-PC studies of Refs. [5,6,8], where it is found that its contribution is very small. Furthermore, in a study [11] of the naturalness of the set of coupling coupling constants from Ref. [5] it was discovered that the isovector-scalar coupling constant is unnaturally small. This would presuppose a symmetry to preserve its small value.

Thus, in relativistic Hartree approximation the isovector-scalar channel may be neglected, and (perhaps) a symmetry may be identified to preserve the small value of its coupling constant. However, we have not found any such symmetry. Therefore, we instead examine our calculational approach, the relativistic Hartree approximation, and ask what happens to the magnitudes of the coupling constants in relativistic Hartree-Fock approximation where both the direct and exchange terms explicitly appear?

We believe that the investigation of exchange terms in an effective field theory for nucleons is meaningful even if one considers the RMF model as an approximation to the exact density functional in the spirit of the Hohenberg-Kohn theorem and Kohn-Sham theory [9], where exchange effects should be absorbed in the various coupling constants. This is because the Hartree-Fock theory can be viewed as a Kohn-Sham formalism with exact treatment of exchange (see Ref. [13], for example), which is then a different representation from the Hartree representation. It also has the correct one-particle limit and is a self-interaction free theory [12,13]. This is important for odd systems where the odd particle feels its own potential if exchange is ignored. The original idea to perform relativistic Hartree-Fock calculations by using contact interactions
is due to Ref. [14].

In Sec. II we present the simplest possible Lagrangian containing scalar and vector fields of both isoscalar and isovector character, and we relate the coupling constants for this Lagrangian in relativistic Hartree approximation to the corresponding coupling constants in relativistic Hartree-Fock approximation. We apply our results in Sec. III to study the four-fermion contact interactions occurring in two existing realistic point coupling models determined (phenomenologically) in relativistic Hartree approximation. We then address the question of naturalness of the Hartree and Hartree-Fock coupling constants from these two models in Sec. IV. Our conclusions are given in Sec. V.

2 Four-Fermion Relativistic Point Coupling with Exchange

For comparing Hartree and Hartree-Fock representations we consider two-body contact interactions (four-fermion point couplings) in the mean field and no sea approximations, applied to the ground states of even-even nuclei:

\[
\mathcal{L} = -\frac{1}{2} \alpha_S (\bar{\psi} \psi)^2 - \frac{1}{2} \alpha_V (\bar{\psi} \gamma_{\mu} \psi) \mathbf{\bar{\psi}} \gamma_{\mu} \psi \\
- \frac{1}{2} \alpha_{TS} (\bar{\psi} \mathbf{\bar{r}} \psi) \cdot (\mathbf{\bar{r}} \bar{\psi}) - \frac{1}{2} \alpha_{TV} (\bar{\psi} \gamma_{\mu} \mathbf{\bar{r}} \psi) \cdot (\mathbf{\bar{r}} \bar{\psi}) 
\]

where \( \psi \) is the nucleon field and \( \mathbf{\bar{r}} \) is the isospin matrix. We wish to compare the same model ansatz in two different many-body approximations. Thus, our model space does not explicitly include pions because the pion field vanishes in the Hartree approximation, but contributes via its exchange terms in the Hartree-Fock approximation, which would then yield two different models. Implicitly the effects of the pion are nevertheless included because our coupling constants are determined by measured observables. Accordingly, we regard this work as a model study and do not construct a complete Hartree-Fock model.

Taking the normal ordered expectation value of \( \mathcal{L} \) in a Slater determinant \(|\Phi\rangle \) leads to the well-known direct and exchange terms that are to be solved in Hartree-Fock approximation, namely,

\[
\langle \Phi | \mathcal{L}_{HF} | \Phi \rangle = -\frac{1}{2} \alpha_S \rho_s S^2 - \frac{1}{2} \alpha_V \rho_v V^2 - \frac{1}{2} \alpha_{TS} \rho_{TS} S^2 - \frac{1}{2} \alpha_{TV} \rho_{TV} V^2 \\
+ \frac{1}{2} \alpha_{S_{ex}} S_{ex}^2 + \frac{1}{2} \alpha_{V_{ex}} V_{ex}^2 + \frac{1}{2} \alpha_{TS_{ex}} S_{ex}^2 + \frac{1}{2} \alpha_{TV_{ex}} V_{ex}^2 
\]

Here, we refer to the \( \{\alpha\} \) as coupling constants, \( \rho_S \) and \( \rho_V \) denote the isoscalar
scalar density and the time component of the isoscalar vector density, respectively, and $\rho_{TS}$ and $\rho_{TV}$ denote the corresponding isovector densities. The squares of the isoscalar scalar and vector exchange densities, $\rho_{S\text{ex}}$ and $\rho_{V\text{ex}}$, and those of the isovector scalar and vector exchange densities, $\rho_{TS\text{ex}}$ and $\rho_{TV\text{ex}}$, are given by [$a$ and $b$ are nucleon states]:

$$\rho_{S\text{ex}}^2 = \sum_{a,b} (\bar{\psi}_a \psi_b)(\bar{\psi}_b \psi_a)$$  \hspace{1cm} (3)$$

$$\rho_{V\text{ex}}^2 = \sum_{a,b} (\bar{\psi}_a \gamma_{\mu} \psi_b)(\bar{\psi}_b \gamma^\mu \psi_a)$$  \hspace{1cm} (4)$$

$$\rho_{TS\text{ex}}^2 = \sum_{a,b} (\bar{\psi}_a \vec{\tau} \psi_b) \cdot (\bar{\psi}_b \vec{\tau} \psi_a)$$  \hspace{1cm} (5)$$

$$\rho_{TV\text{ex}}^2 = \sum_{a,b} (\bar{\psi}_a \gamma_{\mu} \vec{\tau} \psi_b) \cdot (\bar{\psi}_b \gamma^\mu \vec{\tau} \psi_a)$$  \hspace{1cm} (6)$$

We then apply Fierz transformations [15,16] in Dirac-iso space to the four exchange densities. This transformation expresses a product of nondiagonal matrix elements of Dirac $\Gamma$-matrices as an expansion into products of diagonal matrix elements, such as

$$(\bar{\psi}_a \Gamma_i \psi_b)(\bar{\psi}_b \Gamma_j \psi_a) = \sum_{k,l=1}^{16} c_{kl} (\bar{\psi}_a \Gamma_k \psi_a)(\bar{\psi}_b \Gamma_l \psi_b)$$  \hspace{1cm} (7)$$

where $\Gamma_i$ stands for one of the sixteen Dirac matrices $\{1, \gamma_\mu, \gamma_5, \gamma_5\gamma_\mu, \sigma_{\mu\nu}\}$ constituting a linearly independent basis in the space of complex $4 \times 4$ matrices, which may be coupled or uncoupled to isospin matrices $\vec{\tau}$. Applying Eq. (7) to Eqs. (3-6), one immediately sees that all terms containing $\gamma_5$ and $\gamma_5\gamma_\mu$ vanish because $\psi_a$ and $\psi_b$ are nucleon fields with good parity. We now make the (reasonable) approximation that the tensor ($\sigma_{\mu\nu}$) and iso-tensor ($\vec{\tau}\sigma_{\mu\nu}$) contributions are quite small and can be neglected [7]. Reordering the resulting terms leads to a Lagrangian that is formally identical to Eq. (2) without the four exchange terms, but with newly defined coupling constants instead:

$$\mathcal{L}_{\tilde{\text{HF}}} = -\frac{1}{2} \tilde{\alpha}_{SS} \rho_S^2 - \frac{1}{2} \tilde{\alpha}_{VV} \rho_V^2 - \frac{1}{2} \tilde{\alpha}_{TS} \rho_{TS}^2 - \frac{1}{2} \tilde{\alpha}_{TV} \rho_{TV}^2$$  \hspace{1cm} (8)$$

where the newly defined coupling constants are given by
\[
\begin{align*}
\tilde{\alpha}_S \equiv & \frac{7}{8} \alpha_S - \frac{1}{2} \alpha_V - \frac{3}{8} \alpha_{TS} - \frac{3}{2} \alpha_{TV} \\
\tilde{\alpha}_V \equiv & \frac{1}{8} \alpha_S + \frac{5}{4} \alpha_V - \frac{3}{8} \alpha_{TS} + \frac{3}{4} \alpha_{TV} \\
\tilde{\alpha}_{TS} \equiv & \frac{1}{8} \alpha_S - \frac{1}{2} \alpha_V + \frac{9}{8} \alpha_{TS} + \frac{1}{2} \alpha_{TV} \\
\tilde{\alpha}_{TV} \equiv & -\frac{1}{8} \alpha_S + \frac{1}{4} \alpha_V + \frac{1}{8} \alpha_{TS} + \frac{3}{4} \alpha_{TV} 
\end{align*}
\]

This result already shows that in the Hartree-Fock approach, due to the exchange effect, all original terms contribute to all channels of the effective interaction. And the formal structure of Eq. (8) is identical to the Hartree approximation for the same model with redefined coupling constants. However, this Lagrangian, when considering all terms arising from the Fierz transformations, is a self-interaction free theory. The inverse solution of Eq. (9) is

\[
\begin{align*}
\alpha_S = & \frac{34}{21} \tilde{\alpha}_S + \frac{4}{21} \tilde{\alpha}_V + \frac{6}{21} \tilde{\alpha}_{TS} + \frac{60}{21} \tilde{\alpha}_{TV} \\
\alpha_V = & \frac{1}{21} \tilde{\alpha}_S + \frac{31}{21} \tilde{\alpha}_V + \frac{15}{21} \tilde{\alpha}_{TS} - \frac{39}{21} \tilde{\alpha}_{TV} \\
\alpha_{TS} = & \frac{2}{21} \tilde{\alpha}_S + \frac{20}{21} \tilde{\alpha}_V + \frac{30}{21} \tilde{\alpha}_{TS} - \frac{36}{21} \tilde{\alpha}_{TV} \\
\alpha_{TV} = & \frac{5}{21} \tilde{\alpha}_S - \frac{13}{21} \tilde{\alpha}_V - \frac{9}{21} \tilde{\alpha}_{TS} + \frac{57}{21} \tilde{\alpha}_{TV} 
\end{align*}
\]

Given the above results, if one determines the coupling constants of a relativistic point coupling Lagrangian in Hartree approximation, then the set of coupling constants \{\tilde{\alpha}\} in Eqs. (8) and (9) has been determined. Use of these coupling constants in Eq. (10) then yields the original Hartree-Fock coupling constants \{\alpha\}. The two sets of coupling constants yield identical predictions of the nuclear ground state observables, but their magnitudes and physical interpretation are different because the former set implicitly accounts for exchange processes whereas the latter set explicitly accounts for exchange processes.

3 Exchange Effects in Relativistic Point Coupling Models Determined from Measured Observables in Hartree Approximation

We examine coupling constants occurring in two realistic relativistic point coupling models that have been determined in Hartree approximation. The four terms of Eq. (1) are included, but higher order terms (six- and eight-fermion point couplings) and derivative terms are included as well. Here we focus our attention on the four four-fermion point couplings alone.
Table 1
Four-Fermion Relativistic Hartree $\{\tilde{\alpha}\}$ and Hartree-Fock $\{\alpha\}$ Coupling Constants in Two Realistic Lagrangians (PC-LA and PC-F4) [$10^{-4}$ MeV$^{-2}$].

| Force   | $\tilde{\alpha}_S$ | $\tilde{\alpha}_V$ | $\tilde{\alpha}_{TS}$ | $\tilde{\alpha}_{TV}$ | $\alpha_S$  | $\alpha_V$  | $\alpha_{TS}$ | $\alpha_{TV}$ |
|---------|---------------------|---------------------|------------------------|------------------------|-------------|-------------|-------------|-------------|
| PC-LA   | -4.508              | 3.427               | 7.421 $\times 10^{-3}$ | 3.257 $\times 10^{-1}$ | -5.712      | 4.244       | 2.286       | -2.314      |
| PC-F4   | -3.834              | 2.594               | -5.924 $\times 10^{-2}$ | 3.937 $\times 10^{-1}$ | -4.608      | 2.872       | 1.345       | -1.425      |

The two models are: PC-LA containing 9 coupling constants that appears in Ref. [5] published in 1992, and PC-F4 containing 11 coupling constants that appears in Ref. [8] published in 2002. As explained above, the four four-fermion coupling constants appearing in these tables are taken as the set $\{\tilde{\alpha}\}$ in Eqs. (8) and (9). The inverse solution Eq. (10) then yields the Hartree-Fock coupling constants $\{\alpha\}$. We show both the Hartree $\{\tilde{\alpha}\}$ and Hartree-Fock $\{\alpha\}$ coupling constants in Table 1.

Comparing the relativistic Hartree coupling constants of the models PC-LA and PC-F1 with the relativistic Hartree-Fock coupling constants one observes the following: (a) whereas the exponents in the Hartree coupling constants range from -7 to -4, those of the Hartree-Fock are all -4; (b) the Hartree-Fock isovector-scalar coupling constant is much larger than its Hartree counterpart, and has changed sign in PC-F4; and (c) the Hartree-Fock isovector-vector coupling constant has changed sign and its absolute magnitude has also increased in comparison to its Hartree counterpart.

The Hartree-Fock isovector coupling constants have a larger role than those of Hartree. In fact, the four Hartree-Fock coupling constants are of the same order of magnitude and, furthermore, the magnitudes of the two isovector coupling constants are roughly equal. But none of this is true in the Hartree case. The Hartree-Fock coupling constants from both models have the same signs (not true for the Hartree case). In addition, it appears from the two models that the sum of the isovector coupling constants is better determined by the ground state observables than are the individual values, as was also learned in Ref. [8].

Finally, we can ask what are the errors (uncorrelated and correlated) in the determination of the sets of coupling constants in the two calculations? We find that (a) the uncorrelated and correlated errors in the isovector coupling constants are significantly diminished in Hartree-Fock approximation (some by roughly two orders of magnitude), and (b) in this approximation, all four of the coupling constants are equally well determined, unlike in the Hartree approximation. This result is a consequence of the fact that in Hartree-Fock approximation the four four-fermion coupling constants contribute in each of the four channels, due to the explicit treatment of exchange processes.
Therefore, the magnitudes of the four four-fermion Hartree-Fock coupling constants, in the two point-coupling models studied, are well determined and they are comparable. This immediately brings to mind the question of the naturalness of these coupling constants to which we now turn our attention.

4 The Quest for Naturalness

The naturalness of the coupling constants relates to the question as to whether QCD scaling and chiral symmetry apply to finite nuclei. In 1990, Weinberg [17] showed that Lagrangians with (broken) chiral symmetry predict the suppression of N-body forces. He accomplished this by constructing the most general possible chiral Lagrangian involving pions and low-energy nucleons as an infinite series of allowed derivative and contact interaction terms and then using QCD energy (mass) scales and dimensional power counting to categorize the terms of the series. This led to a systematic suppression of the N-body forces.

We use the scaling procedure of Manohar and Georgi [18] but without pion fields. Explicit pionic degrees of freedom are absent in RMF Hartree theory, but can be present in RMF Hartree-Fock theory where Eq. (11) then also contains the pion field and pion mass as in Eq. (1) of Ref. [11]. The scaled generic Lagrangian term of the (physical) series is, without pions,

\[ \mathcal{L} \sim -c_{l_{n}} \left[ \frac{\bar{\psi} \psi}{f_{\pi}^{2} \Lambda} \right]^{l} \left[ \frac{\partial^{\mu}}{\Lambda} \right]^{n} f_{\pi}^{2} \Lambda^{2} \]  

where \( \psi \) is a nucleon field, \( f_{\pi} \) is the pion decay constant, 92.5 MeV, \( \Lambda = 770 \text{ MeV} \) is the QCD large-mass scale taken as the \( \rho \) meson mass, and \( (\partial^{\mu}) \) signifies a derivative. Dirac matrices and isospin operators (we use \( \vec{t} \) here rather than \( \vec{\tau} \)) have been ignored. Chiral symmetry demands [19]

\[ \Delta = l + n - 2 \geq 0 \]  

such that the series contains only positive powers of \( 1/\Lambda \). If the theory is natural [18], the Lagrangian should lead to dimensionless coefficients \( c_{l_{n}} \) of order unity. Our more stringent definition [8] is that a set of QCD-scaled coupling constants is natural if their absolute values are distributed about the value 1 and the ratio of the absolute maximum value to the absolute minimum value is less than 10. Thus, all information on scales ultimately resides in the \( c_{l_{n}} \). If they are natural, QCD scaling works.

Applying Eq. (11) to the dimensioned relativistic Hartree and Hartree-Fock coupling constants of Table 1, we obtain the corresponding sets of QCD-
Table 2
Relativistic Hartree \{\tilde{\alpha}\} and Relativistic Hartree-Fock \{\alpha\} Naturalized Coupling Constants \{c_n\} for the Four-Fermion Point Couplings in Two Realistic Lagrangians (PC-LA and PC-F4).

| Coup. Const. | \(c_n(\text{PC-LA})\) | \(c_n(\text{PC-F4})\) |
|--------------|----------------------|----------------------|
| \(\tilde{\alpha}_S\) | -1.928               | -1.641               |
| \(\tilde{\alpha}_V\) | 1.466                | 1.109                |
| \(\tilde{\alpha}_{TS}\) | 0.013                | -0.101               |
| \(\tilde{\alpha}_{TV}\) | 0.557                | 0.674                |
| # natural | 3                    | 3                    |
| | \(|\max|/|\min|\) | 152. | 16.2 |
| \(\alpha_S\) | -2.443               | -1.971               |
| \(\alpha_V\) | 1.815                | 1.229                |
| \(\alpha_{TS}\) | 3.912                | 2.301                |
| \(\alpha_{TV}\) | -3.958               | -2.438               |
| # natural | 4                    | 4                    |
| | \(|\max|/|\min|\) | 2.18 | 1.98 |

scaled coupling constants listed in Table 2. This table shows that, whereas the Hartree coupling constants are not natural, the Hartree-Fock coupling constants are natural. The culprit is the isovector-scalar channel (corresponding to \(\delta\) meson exchange) in the Hartree approximation. Additional studies with toy Lagrangians consisting only of four-fermion interactions, not presented herein, show that naturalness is recovered in the Hartree-Fock representation from Hartree solutions where either of the isovector coupling constants is unusually small. The culprit can apparently be either of the isovector coupling constants provided their sum remains approximately constant. The exchange process, however, dominates in both isovector channels to the extent that naturalness is recovered in the Hartree-Fock approximation in either case.

The explicit inclusion of pion terms in the Hartree-Fock Lagrangian should not affect these results. This is because the generic chiral Lagrangian of Eqs. (11) and (12) has an important property: refining the model by adding new terms, such as pions, will change all of the naturalized coupling constants, but naturalness will still apply, that is, naturalness is largely independent of the details, such as adding pions, provided the physics is introduced via the measured observables in the framework of these equations.
5 Conclusions

We have extracted relativistic Hartree-Fock coupling constants from the coupling constants for relativistic Hartree calculations by use of Fierz relations and contact interactions. Identical observables are calculated with these two approximations, but the coupling constants and their physical interpretation are different because, whereas in the Hartree approximation they implicitly account for the exchange processes, in the Hartree-Fock approximation they explicitly account for the exchange processes.

We have learned three things. First, viewing Hartree and Hartree-Fock as two different representations we have shown that the smallness of the isovector-scalar coupling constant $\alpha_{TS}$ may be an artifact of the Hartree representation rather than the signature of a new symmetry and, therefore, the search for the symmetry is not needed. Second, while several relativistic Hartree studies have concluded that the observables determine a single isovector coupling constant (that of the $\rho$-meson), which is natural, we have learned that it is the sum of two isovector coupling constants that is preserved in relativistic Hartree and that individually these are both natural and their sum is also preserved in relativistic Hartree-Fock. Given that the $\delta$-meson is now playing a role in recent asymmetric nuclear matter studies [20] it is relevant that its coupling constant is natural. Finally, the Hartree-Fock approximation may constitute a physically more realistic framework for power counting and QCD scaling than the Hartree approximation.

Our conclusions are subject to the assumptions that: (a) we are able to neglect tensor contributions, (b) our results for four-fermion contact interactions will not be strongly affected by Fierz transformations on the remaining higher-order and derivative terms, and (c) explicit inclusion of pion interactions will change all of the coupling constants while retaining their naturalness.

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