Applicability of the lattice Boltzmann method to determine the ohmic resistance in equivalent resistor connections

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Abstract. Knowing the ohmic resistance in the materials allow to know in advance its electrical behavior when a potential difference is applied, and therefore the prediction of the electrical performance can be achieved in a most certain manner. Although the Lattice Boltzmann method (LBM) has been applied to solve several physical phenomena in complex geometries, it has only been used to describe the fluid phase, but applicability studies of LBM on the solid-electric-conducting material have not been carried out yet. The purpose of this paper is to demonstrate the accuracy of calculating the equivalent resistor connections using LBM. Several series and parallel resistor connections are effected. All the computations are carried out with 3D models, and the domain materials are designed by the authors.

1. Introduction

During the last years, the structure of functional materials has been designed to obtain desirable responses when they are subjected to certain conditions in a system, e.g., high physical compression, flow complexity through the material, etc. Electrical and thermal transport parameters play an important role in transport processes through materials such as metal foam, porous materials, and/or semi-conductor materials.

Modeling of physical and mechanical process represents a useful tool to analyze the material properties, and it is a point of departure in the material design before the manufacturing process.

Several approaches applying the Lattice Boltzmann method (LBM) to model different transport phenomena have been carried out [1 - 3] to demonstrate the feasibility of the methodology. LBM has been applied to different physical and chemical systems. However, studies showing the applicability of LBM to determine the ohmic resistance in solid and solid/pore conducting material are scarce.

Considering the widely accepted idea of the equivalence between the electric an hydraulic system, several properties can be calculated from the hydraulic point of view, and then; its corresponding electric property being determined. Although there are some limitations when the electric-hydraulic analogy is used, it can facilitate the prediction of certain variables. The equivalences of the variables between the hydraulic and electric systems are shown in [4].
The purpose of this paper is to demonstrate the accuracy of calculating the equivalent resistor connections using LBM. Several series and parallel resistor connections are effected. All the computations are carried out with 3D models, and the domain materials are designed by the authors.

2. Methodology

The scheme applied to solve LBM to determine the ohmic resistance in solid conducting materials is D3Q19, which means a three-dimensional solution with nineteen linked velocities between the lattice elements. The NS equations, i.e., continuity and momentum equations, are recovered by solving the Lattice Boltzmann equation (LBE) and using the Bhatnagar–Gross–Krook (BGK) approximation as collision operator [5]. The discretized equation is expressed as follows:

$$\frac{\partial f_i(r,t)}{\partial t} + c_i \nabla f_i(r,t) = \frac{1}{\tau} \left[ f_i^{eq}(r,t) - f_i(r,t) \right]$$

(1)

Where $\tau$ corresponds to the relaxation parameter, $f$ is the denominated particle distribution function (PDF), $f_i^{eq}$ represents the equilibrium particle distribution, and $c$ is the bulk velocity of the fluid. The subscript $i$ is the number of linked velocities involved in the simulations, i.e., nineteen linked velocities, $r$ is the position vector and $t$ is the time. Once the LBE is solved, the velocity field over the domain is computed by using the following equation:

$$u(r,t) = \frac{\sum_i f_i c_i(r,t)}{\rho(r,t)}$$

(2)

Considering the equivalences presented in [4], the velocity values are equivalent to the current density in the electrical system. The first relationship used to analyze the applicability of the LBM to compute electrical variables is the widely known Ohm’s Law:

$$j = -\sigma \nabla \phi$$

(3)

where $j$ is the current density; in A/m$^2$, $\phi$ is the potential function; in V, and $\sigma$ is the material conductivity; in $\Omega \cdot m$.

Since the LBM is solving the equivalent hydraulic system, the ohmic resistance, considering the hydraulic variables presented in [4], is computed as follows:

$$R_{LBM} = \frac{\Delta p}{\phi_V}$$

(4)

where $\Delta p$ is the pressure drop applied to the ends of the material and $\phi_v$ corresponds to the volumetric flow rate. Using equation (4), several measurements are carried out to demonstrate the applicability of LBM for computing the ohmic resistance of solid and solid/pore conducting materials.

3. Results and discussion

Before to carry out different proofs to demonstrate the feasibility of using LBM to compute the ohmic resistance of conducting materials, there is a condition that should be matched related to the values of the velocities obtained during the modeling process. Given the characteristics of the methodology and according to [6], LBM leads to instabilities when high velocities, i.e., high current densities in electrical systems, are reached during the simulation process.

A conducting material with constant cross-sectional area, a square of 5 lu by side which allow us to get excellent agreement in hydraulics phenomena [7], and for three different lengths were submitted to
a range of potential difference at the ends of the material. As a result, the current-voltage curves of the material were obtained. The mentioned analysis and the I-V curve are presented in a previous study developed in [4].

According to the preliminary results, the I-V curve obtained, with no restriction on the potential difference applied, shows a similar behavior to the filament lamps [8], i.e., the current is increasing with the voltage but not linearly. However, due to the aim of this paper and taking into account that variations of temperature are not considered, a deeper analysis of the results is done to avoid such non-ohmic behaviors.

Analyzing the obtained results in detail, for a given cross-sectional area, there is a maximum value of potential difference which can give a numerical result, i.e., after this maximum value an instability is obtained. The maximum potential difference before no result is approximated 3.83 V, which for the given cross-sectional area, is the same independently of the material length. For each cross-sectional area, a maximum potential difference can be determined and the simulations can be effected.

3.1 Equivalent ohmic resistance for resistor connections

The validation of LBM as a tool to determine the ohmic resistance for conducting materials is obtained calculating the equivalent ohmic resistance of an arrangement of resistors. As largely known, if two resistors are connected in series, the equivalent resistance is calculated adding the resistance values. On the other hand, if they are connected in parallel, the reciprocal of the equivalent resistance is the result of adding the reciprocal values of each resistance value. The calculation of an equivalent resistor of a group of resistors is based on the voltage and current distribution in the circuit. Equivalent resistors for a series and parallel circuits can be determined by using the following expressions:

\[ R_{\text{series}} = \sum_{i=1}^{n} R_i \]  
\[ \frac{1}{R_{\text{parallel}}} = \sum_{i=1}^{n} \frac{1}{R_i} \]  

The ohmic resistance is computed using equation (4) from modeled domains with different dimensions, i.e., different length and different cross-sectional areas. All the resistance configurations are subjected to the same potential difference along the length direction, i.e., 0.03 V, the LBM is applied and the resistance is computed. Four different volume dimensions, representing resistors, were mimicked to calculate the individual ohmic resistance. The obtained results are presented in Table 1.

### Table 1. Computed ohmic resistance using LBM for four resistors with different dimensions (MxLxN)

| Name | Size (MxLxN) | Ohmic resistance x10^-3 (Ω)
|------|--------------|------------------------|
| R.   | 20x20x20     | 1.449                  |
| R.   | 20x15x20     | 1.087                  |
| R.   | 20x25x20     | 1.816                  |
| R.   | 25x15x20     | 0.736                  |

(M x N = cross-sectional area, L = length)

Various circuit configurations using the resistors presented in Table 1 were implemented. The ohmic resistance calculated by LBM, and the expected values using Eqs (5) and (6) are presented in Table 2. Last column shows the deviation error between the computed and expected values.
Table 2. Ohmic resistance configurations with their corresponding equivalent ohmic resistance computed using LBM and expected values from Eqs (5) and (6).

| Configuration | Ohmic resistance x10⁻³ (Ω) | Deviation error |
|---------------|-----------------------------|-----------------|
|               | LBM | Expected |                |
| (R, R)         | 2.929 | 2.828 | +3.57% |
| (R, R)         | 0.724 | 0.724 | --         |
| (R, R)         | 2.929 | 2.903 | +0.90% |
| (R, R)         | 4.389 | 4.390 | -0.02% |
| (R, R, R)      | 4.421 | 4.352 | +1.59% |

S=Series, P=Parallel.

Deviation errors between the computed equivalent resistance using LBM and Eqs (5) and (6) show a very good agreement, i.e. lower than 4.00%. Considering the obtained results, LBM shows better accuracy when the resistors are connected in parallel than in a series circuit.

4. Conclusions

LBM has proven to be a powerful tool for computing the ohmic resistance in solid and solid/pore materials [4]. Several simulations were carried out considering the conditions in which the LBM can show ohmic behavior of the materials. LBM results show a better agreement when the parallel circuits are computed in comparison with the serial circuits. However, for both cases the deviation errors fall in an acceptable range, i.e., lower than 4%. Although this computational work shows a very good agreement, there are some limitations that should be addressed in future investigations: the relaxation parameter in LBM can be related to the material resistivity, a more realistic electron flow can be achieved if an external force is added to the LBE solution.

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