Scheduling Opportunistic Links in Two-Tiered Reconfigurable Datacenters

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Abstract—Reconfigurable optical topologies are emerging as a promising technology to improve the efficiency of datacenter networks. This paper considers the problem of scheduling opportunistic links in such reconfigurable datacenters. We study the online setting and aim to minimize flow completion times. The problem is a two-tier generalization of classic switch scheduling problems. We present a stable-matching algorithm which is $2 \cdot (2/\varepsilon + 1)$-competitive against an optimal offline algorithm, in a resource augmentation model: the online algorithm runs $2 + \varepsilon$ times faster. Our algorithm and result are fairly general and allow for different link delays and also apply to hybrid topologies which combine fixed and reconfigurable links. Our analysis is based on LP relaxation and dual fitting.

I. INTRODUCTION

Given the popularity of data-centric applications and machine learning, the traffic in datacenters is growing explosively. Accordingly, over the last years, great efforts were made to render these networks more efficient, on various layers of the networking stack [1], including the physical network topology, e.g., [2]–[6]. As a next frontier toward more efficient datacenter networks reconfigurable optical topologies are emerging [7]–[13], and in particular, demand-aware topologies such as [9]–[12], [14]–[16] which can dynamically adapt towards the traffic patterns they serve. This is attractive as empirical studies show that datacenter workloads are indeed skewed and bursty, featuring much temporal and spatial structure [17]–[19], which may be exploited in adaptive infrastructures. For example, these technologies allow to flexibly transmit elephant flows via opportunistic links that provide shortcuts between the frequently communicating racks.

A key issue for the efficient operation of reconfigurable datacenter networks concerns the scheduling of the opportunistic links. As the number of these links is limited, they should be used for the most significant transmissions. This however is challenging as scheduling decisions need to be performed in an online manner, when the demand is not perfectly known ahead of time.

This paper studies this scheduling problem from a competitive analysis perspective: we aim to design an online scheduling algorithm which does not require any knowledge about future demands, but performs close to an optimal offline algorithm which knows the entire demand ahead of time. In particular, we consider a two-stage switch scheduling model as it arises in existing datacenter architectures, e.g., based on free-space optics [11]. In a nutshell (a formal model will follow shortly), we consider an architecture where traffic demands (modelled as packets) arise between Top-of-Rack (ToR) switches, while opportunistic links are between lasers and photodetectors, and where many laser-photodetector combinations can serve traffic between a pair of ToRs. The goal is to minimize the packet (i.e., flow) completion times over all packets in the system.

The problem is reminiscent of problems in classic switch scheduling [20], [21], as in each time step, an optical switch allows to “transmit a matching”. However, the two-stage version turns out to introduce several additional challenges (as also pointed out in [11]), which we tackle in this paper.

A. Our Contribution

This paper initiates the study of online switch scheduling algorithms for a multi-stage model which is motivated by emerging reconfigurable optical datacenter architectures. Our main contribution is an online stable-matching algorithm that is $2 \cdot (2/\varepsilon + 1)$-competitive against a powerful hindsight optimal algorithm, in a resource augmentation model in which the online algorithm runs $2 + \varepsilon$ times faster. Our analysis relies linear programming relaxation and dual fitting: we formulate primal and dual linear programs to which we will charge the costs of our online algorithm.

Our algorithm and result allow for different link delays and also apply to hybrid datacenter topologies as they are often considered in the literature: topologies which combine fixed and reconfigurable links.

We emphasize that resource augmentation is necessary for obtaining competitive algorithms: Dinitz et al. [22] prove that even in single tier networks, no randomized algorithm can be competitive against an adversary with matching transmission speed.

B. Overview of the Algorithm and Techniques: Analysis via Dual-Fitting

Our algorithm for the problem is based on a generalization of stable matching algorithm [23] for the two-tier networks. Informally, in our algorithm, each transmitter maintains a queue of packets that are not scheduled yet. The packets in the queue are sorted in the decreasing order of weights. At
each time step, our algorithm finds a stable matching between transmitters and receivers as follows: In our problem we are given a bipartite graph \( B = \{ (T \cup R), E \} \), between the set of transmitters \( T \) and set of receivers \( R \); the edge set \( E \) denotes the connections between transmitters and receivers. At time step \( \tau \), we assign the edge \( e = (t, r) \) connecting the transmitter \( t \) to the receiver \( r \) a weight \( w_e \), which is equal the highest weight packet in the queue of transmitter \( t \) at instant \( \tau \) which wants to use the edge \( e \). Taking the weights of edges as priorities, our algorithm simply computes a stable matching \( M \) in the graph \( B \), and schedules \( M \) at time step \( \tau \). (Note that since priorities in our algorithm are symmetric, one can compute a stable matching by a simple greedy algorithm.)

However, how should we assign an incoming packet to a (transmitter, receiver) pair? In our algorithm, as soon as a packet arrives it is dispatched to a specific (transmitter, receiver) pair via which our algorithm commits to eventually transmitting the packet. This is the decision that complicates routing in two-tier networks. Our dispatch policy estimates the worst case impact of transmitting a packet via a specific (transmitter, receiver) pair, taking into account the set of queued packets in the system. In particular, we estimate how much latency of the system increases if a packet is transmitted via a (transmitter, receiver) pair. Finally, we chose the (transmitter, receiver) pair which has the least impact. We show that this greedy-dispatch policy coupled with stable matching is indeed competitive in the speed augmentation model [24], where we assume that the online algorithm can transmit the packets at twice the rate compared to the optimal offline algorithm. It is not hard to show that without speed augmentation, no online algorithm can be competitive [22].

Our algorithm and its analysis via dual fitting is inspired by scheduling for unrelated machines [25], and combines two disparate research directions: switch scheduling and scheduling for unrelated machines.

Our analysis via dual fitting works as follows. First we write a linear programming relaxation for the underlying optimization problem, and then we take the dual of the LP. This is done assuming that we know the entire input, which we can do because LP duality is only used in the analysis. The weak duality theorem states that any feasible solution to the dual is a lowerbound on the optimal primal solution (which in turn is a lowerbound on the optimal solution to our problem). The crux of the the dual-fitting analysis is to relate the cost of our algorithm to a feasible dual solution, thus allowing us to compare our cost to the optimal solution.

The dual of our LP for the problem has a rather interesting form. It consists of variables \( \alpha_p \) for each packet \( p \). For each time step \( \tau \), we also have variables \( \beta_{t, \tau} \) for each transmitter \( t \) and \( \beta_{r, \tau} \) for a receiver \( r \). We interpret \( \alpha_p \) as the latency seen by packet \( p \) and \( \beta_{t, \tau}, \beta_{r, \tau} \) variables as set of packets that are waiting to use the transmitter \( t \) and receiver \( r \) at time step \( \tau \). Clearly, sum over \( \alpha_p \) variables is equal to the total latency seen by packets. It is not hard to argue that the same also holds for \( \beta \) variables. However, the crucial part of the analysis is to show that indeed such an interpretation of the dual variables is a feasible solution. This is done by showing that our setting of dual variables satisfies all the dual constraints. Verifying the dual constraints crucially uses both our algorithmic decisions regarding dispatch policy and the stable matching algorithm. One could also interpret that our algorithmic decisions were in fact driven by the dual LP, in the sense of primal-dual algorithms [26].

C. Organization

The remainder of this paper is organized as follows. We introduce our formal model in Section II. Our algorithm is described in Section III and analyzed in Section IV. After reviewing related work in Section V, we conclude our contribution in Section VI.

II. Model

We consider a hybrid optical network which consists of a fixed and a reconfigurable topology. We model this network as a graph \( G = (V, E, d) \) where \( V \) is the set of vertices partitioned into the following four layers: sources \( S \), transmitters \( T \), receivers \( R \), destinations \( D \). Each transmitter \( t \in T \) is attached (has an edge) to a particular source \( \text{src}(t) \) and each receiver \( r \in R \) is attached to a particular destination \( \text{dest}(r) \) (a single source or destination may have multiple transmitters or receivers attached). The edges between transmitters and receivers form an optical reconfigurable network. For transmitter \( t \in T \), we denote the set of receivers adjacent to \( t \) in \( G \) by \( R(t) \); and for receiver \( r \in R \), \( T(r) \) is the set of transmitters adjacent to \( r \) in \( G \). The fixed part of the network is a set \( E_f \subseteq E \) of direct source-destination links.

At any time \( \tau \in \mathbb{N}_+ \), a transmitter \( t \) may have at most one active edge connecting it with a receiver from \( R(t) \), and each receiver \( r \) may have at most one active incoming edge from one of transmitters \( T(r) \). For any edge \( e \in E \), the delay of that edge is defined by \( d(e) \in \mathbb{N} \), that is, \( s \cdot d(e) \) is the time required to transmit a packet of size \( s \) through that edge. If \( e \) is a transmitter-receiver connection, then its delay is at least 1.

We study the design of a topology scheduler whose input is a sequence of packets \( \Pi \) arriving in an online fashion. A packet \( p \in \Pi \) of weight \( w_p \geq 0 \) which arrives at time \( a_p \) at source node \( \text{src}(p) \in S \), has to be routed to destination \( \text{dest}(p) \in D \). For packet \( p \in \Pi \), let \( E_p \) be the set of transmitter-receiver edges from the reconfigurable network that might be used to deliver \( p \), i.e., \( E_p = \{(t, r) \in T \times R : \text{src}(t) = \text{src}(p) \text{ and } \text{dest}(r) = \text{dest}(p)\} \). If there exists a link connecting \( \text{src}(p) \) with \( \text{dest}(p) \), the delay of that link is \( d_l(p) = d(\text{src}(p), \text{dest}(p)) \). By \( \Pi_r \) we denote the set of all packets that can be transmitted through the fixed network.

In this paper, we assume that packets are of uniform size. However, this assumption is without loss of generality in the speed augmentation model. By standard arguments [25], one can treat a packet \( p \) of size \( \ell_p \) as \( \ell_p \) unit-length packets each of weight \( w_p/\ell_p \). Hence, in the rest of the paper we assume that packets of uniform size.
The overall performance of the algorithm is measured with the standard notion of the competitive ratio, defined as the worst-case cost ratio, where $\text{OPT}$ is the optimal offline solution with limited transmission speed.

### Section III-B

A scheduler for packets in the reconfigurable network, which minimizes the total weighted (fractional) latency of its schedule, is the optimal online solution. The packets will use a direct connection or the reconfigurable links. In the latter case, the dispatcher assigns packet $p$ to some edge (i.e., a transmitter-receiver pair) that connects source and destination of $p$.

More precisely, the dispatcher attempts to minimize the weighted latency increase caused by $p$ (the latency of $p$, and the latencies of other packets). To this end, it needs to account for the different transmission times in the reconfigurable part of the network. The idea is to split packets into chunks, which can be transmitted in a single step (the size of a chunk depends on the delay of the assigned edge). The dispatcher is formally defined in Section III-B. The scheduler then greedily chooses the subset of chunks to be transmitted at each step. The set of edges associated with each chunk forms a stable matching. This process is presented in details in Section III-C.

#### A. Stable Matching and Blocking

A matching $M$ is stable with respect to symmetric weights $w$ if for any edge $e \notin M$, there exists edge $e' \in M$ adjacent to $e$ such that $w_{e'} \geq w_e$. We say that edge $e'$ blocks edge $e$. The scheduler at each time $\tau$, transmits a set of packets whose assigned edges form a stable matching. We will say that a chunk $c$ blocks another chunk $c'$ when $c$ is transmitted at time $\tau$ and $w_c \geq w_{c'}$, and edges assigned to $c$ and $c'$ share a transmitter or a receiver.

#### B. Dispatcher

At time $\tau$, the dispatcher handles packets that arrived since time $\tau - 1$. The packets are processed one by one. We will say that a packet $p'$ arrived before $p$ if $a_{p'} \leq \tau - 1$ or $a_{p'} = \tau$ and $p'$ was already handled by the dispatcher. Each packet assigned to the reconfigurable is split into chunks. For chunk $c$ we denote by $p(c)$ the packet whose part is chunk $c$. We say that a chunk is pending if it has not been transmitted through the reconfigurable network. For a set of chunks $C$ we denote the total weight of chunks in $C$ by $w(C)$.

For each packet $p$, let $B_p$ be the set of chunks of packets that arrived before $p$ in the input sequence and are pending at time when $p$ is processed. We define the impact of $p$ as the weighted latency of chunks from $B_p$ that are blocked by (chunks of) $p$.
plus the weighted latency of (chunks of) \( p \) incurred in rounds when \( p \) were blocked by a chunk from \( B_p \) or its own chunk. In particular, if packet \( p \) is transmitted through the fixed network, its impact is just the weighted latency of \( p \), that is \( w_p \cdot d(p) \).

Ideally, when packet \( p \) arrives, we would like to minimize the impact of this packet. This is, however, impossible to compute online: when the stable matching changes as more packets arrive, the impact of packet \( p \) might change as well, although the set \( B_p \) does not change (it is a property of the input sequence, not the algorithm). An example of such situation is depicted on Figure 2.

Instead, the algorithm minimizes the worst-case impact of \( p \). Namely, for each edge \( e \in E_p \), it computes, assuming that \( p \) is assigned to \( e \), how many chunks from \( B_p \) might block \( p \) in the future and how many chunks from \( B_p \) might be blocked by \( p \). Then, the algorithm minimizes the worst-case impact by assigning \( p \) to either the edge from the reconfigurable network to the direct fixed link between source and destination of \( p \) (only if such link exists).

Formally, when packet \( p \) is assigned to edge \( e \), it is split into \( d(e) \) chunks, each of size \( 1/d(e) \) and weight \( w_p/d(e) \). Let \( C_p(e) \) be the set of these chunks. Let \( A_p(e) \) be the set of chunks from \( B_p \) that are assigned to use edge adjacent to \( e \). We partition the set \( A_p(e) \) into two disjoint subsets: \( H_p(e) \) containing those chunks that may delay \( C_p(e) \) (i.e., at least as heavy as \( w_p/d(e) \)) and \( L_p(e) \) of chunks that might be delayed by \( C_p(e) \) (i.e., lighter than \( w_p/d(e) \)). Note that these definitions require that from two chunks of the same weight, the chunk of the earlier arriving packet is preferred. This will be preserved by the scheduler in the next section.

The worst-case impact of \( p \) assigned to \( e \) is then \( \Delta_p(e) = w_p \cdot \left( d(u) + \frac{d(e)+1}{2} + d(v) \right) + w_p \cdot |H_p(e)| + d(e) \cdot w(L_p(e)) \).

The first summand is the weighted latency of chunks of \( p \) (note that chunks \( C_p(e) \) delay each other). The remaining two summands account for the latency increase coming from \( p \) interacting with other chunks: i.e., they count the number of chunks from \( A_p(e) \) that might block \( p \), and the weighted latency of packets from \( A_p(e) \) that may be blocked by \( p \) (all \( |C_p(e)| = d(e) \) chunks of \( p \) might block \( L_p(e) \)).

Let \( c = \arg\min_{e \in E_p} \Delta_p(e') \) be the edge that minimizes the worst-case impact of \( p \) among all edges of the reconfigurable network. If there exists a link \( e_t = (\text{src}(p), \text{dest}(p)) \) in \( E_t \), and if the weighted latency of sending \( p \) through \( e_t \) is smaller than the worst-case impact of \( p \) assigned to \( e \) (i.e., \( w_p \cdot d(e) \leq \Delta_p(e) \)), packet \( p \) is assigned to edge \( e_t \); otherwise, packet \( p \) is assigned to edge \( e \).

If packet \( p \) is not transmitted via a direct source-destination link, the edge \( e_p \), which will eventually transmit packet \( p \) is fixed. In the remaining part of the paper we will use \( H_p \) and \( L_p \) to denote the corresponding terms for the edge \( e_p \), that is, \( H_p(e_p) \) and \( L_p(e_p) \), respectively.

C. Scheduler

We describe how at time \( \tau \) packets released until time \( \tau \) are transmitted through the reconfigurable network. To this end, we construct the set \( M_\tau \) of chunks that will be transmitted in the interval \([\tau, \tau+1)\). The set of edges used by chunks from \( M_\tau \) forms a stable matching. We assume that each packet \( p \) is already assigned to an edge \( e_p \) and split into chunks such that a chunk \( c \) assigned to edge \( e \) can be transmitted in a single step, i.e., \( \text{size}(c) = 1/d(e) \). For a chunk \( c \), by \( p(c) \) we denote the packet whose part is \( c \). The weight of \( c \) is then \( w_c = w_p \cdot \text{size}(c) \).

The stable matching \( M_\tau \) is constructed greedily. Initially, the set \( M_\tau \) is empty. Then, for each pending chunk \( c \), in order of decreasing weights and increasing arrival times, if both endpoints of \( e_c \) are free (i.e., chunks from \( M_\tau \) do not use edges adjacent to \( e_c \)), chunk \( c \) becomes an element of \( M_\tau \) (it will be transmitted). Otherwise, if at least one of endpoints of \( e_c \) is already busy, then \( c \) is not transmitted. Observe that, due to our ordering with decreasing weights, \( c' \) blocks \( c \) as \( w_{c'} \geq w_c \).

When the algorithm processes all chunks, the matching \( M_\tau \) is transmitted.

IV. Competitive Analysis

In this section we prove that our algorithm ALG is \( 2 \cdot (2 + \varepsilon) / \varepsilon \)-competitive given a \( (2 + \varepsilon) \) speedup. In the analysis, instead of empowering the algorithm, we limit the capabilities of the optimum algorithm: For \( \varepsilon \geq 0 \) it transmission can take time at most \( 1/(2 + \varepsilon) \) in a single step. Although the schedule of the algorithm is non-migratory (a packet is assigned to and transmitted via exactly one path in \( G \)), the result holds against an optimal solution that is preemptive and migratory.

A. Linear Program Relaxation

For our analysis we will rely on the formulation of primal and dual linear programs (containing all feasible solutions transmitting packets with speed \( 1/(2 + \varepsilon) \)), to which we will be able to charge the costs of our online algorithm. This will eventually allow us to upper bound the competitive ratio (how far our algorithm is off from a best possible offline solution).

For packet \( p \in \Pi \), edge \( e = (t, r) \in E_p \) and time \( \tau \geq a_p \) we introduce a variable \( \pi_{p,e,\tau} \) interpreted as a fraction of packet \( p \) that is sent through the path \( \text{src}(p) \rightarrow t \rightarrow r \rightarrow \text{dest}(p) \) at time \( \tau \). Note that, this transmission takes time \( d(e) = d(\text{src}(t), t) + d(e) + d(r, \text{dest}(r)) \), which incurs weighted latency of \( w_p \cdot \pi_{p,e,\tau} \cdot (\tau + d(e) - a_p) \). To model fixed direct links between sources and destinations, for each packet \( p \) we introduce variable \( y_p \), which is interpreted as the amount sent through this direct connection. The weighted latency of this transmission is then \( w_p \cdot d(p) \cdot y_p \).

For time \( \tau \), let \( P(\tau) \) be the set of packets released earlier than \( \tau \). The linear program \( P \) shown in Figure 3 describes the set of feasible solutions (in particular, the optimal solution in a resource augmentation model).

Note that the number of variables and constraints is potentially infinite, but it is sufficient to consider only \( \tau \) smaller than \( \max_{p \in \Pi} a_p + \|\Pi\| \cdot \max_{e \in E} d(e) \). This is because if there is any pending packet, then any (reasonable) algorithm transmits at least one of them and transmitting packets one by one takes time at most \( \|\Pi\| \cdot \max_{e \in E} d(e) \).
Figure 2. The figure shows a graph and two inputs (sets of packets). In the graph, for each source, there is exactly one transmitter attached to it and for each destination, there is exactly one receiver (the transmitters and receivers are omitted on the picture). The label above each edge is the (only) packet that might be transmitted through it. Solid edges mark the stable matching (assuming the weight of an edge is the weight of the packet it can transmit) that would be transmitted if no more packets arrived. Upon arrival of a new packet \( p_4 \), the stable matching changes. As a result, packet \( p_2 \) is not blocked by \( p_3 \) and \( p_2 \) blocks \( p_1 \).

\[
\begin{array}{ccc}
\text{packet} & \text{path} & \text{weight} & \text{impact} \\
p_1 & s_1 \rightarrow d_1 & 1 & w_{p_1} = 1 \\
p_2 & s_1 \rightarrow d_2 & 2 & w_{p_2} = 2 \\
p_3 & s_2 \rightarrow d_2 & 3 & w_{p_2} + w_{p_3} = 5 \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{packet} & \text{path} & \text{weight} & \text{impact} \\
p_1 & s_1 \rightarrow d_1 & 1 & w_{p_1} = 1 \\
p_2 & s_1 \rightarrow d_2 & 2 & w_{p_1} + w_{p_2} = 3 \\
p_3 & s_2 \rightarrow d_2 & 3 & w_{p_3} = 3 \\
p_4 & s_2 \rightarrow d_3 & 4 & w_{p_3} + w_{p_4} = 7 \\
\end{array}
\]

Figure 3. Linear program \( P \) describing all feasible solutions with reduced transmission speed in the reconfigurable network. We omit nonnegativity constraints of all variables.

\[
\begin{align*}
\text{min.} & \quad \sum_{p \in \Pi} \sum_{e \in E_p} \sum_{\tau \geq a_p} w_p \cdot x_{p,e,\tau} \cdot \left( \tau + \Delta(e) - a_p \right) + \sum_{p \in \Pi_T} w_p \cdot y_p \cdot d_{\ell}(p) \\
\text{s.t.} & \quad \sum_{e \in E_p} \sum_{\tau \geq a_p} x_{p,e,\tau} + y_p \geq 1 \quad \text{for all } p \in \Pi_\ell \\
& \quad \sum_{e \in E_p} x_{p,e,\tau} \geq 1 \quad \text{for all } p \in \Pi \setminus \Pi_\ell \\
& \quad \sum_{t \in T} \sum_{p \in \Pi_T} \sum_{\tau \geq a_p} d(t,r) \cdot x_{p,(t,r),\tau} \leq \frac{1}{2 + \varepsilon} \quad \text{for all } \tau, \ t \in T \\
& \quad \sum_{t \in T} \sum_{p \in \Pi_T} \sum_{\tau \geq a_p} d(t,r) \cdot x_{p,(t,r),\tau} \leq \frac{1}{2 + \varepsilon} \quad \text{for all } \tau, \ r \in R
\end{align*}
\]

Figure 4. Linear program \( D \) dual to \( P \). We omit nonnegativity constraints of all variables.

\[
\begin{align*}
\text{max.} & \quad \sum_{p \in \Pi} \alpha_p - \frac{1}{2 + \varepsilon} \left( \sum_{t \in T} \sum_{\tau} \beta_{t,\tau} + \sum_{r \in R} \sum_{\tau} \beta_{r,\tau} \right) \\
\text{s.t.} & \quad \alpha_p - d(e) \cdot \left( \beta_{t,\tau} + \beta_{r,\tau} \right) \leq w_p \cdot \left( \tau + \Delta(e) - a_p \right) \quad \text{for all } p \in \Pi, e = (t,r) \in E_p, \tau \geq a_p \\
& \quad \alpha_p \leq w_p \cdot d_{\ell}(p) \quad \text{for all } p \in \Pi_\ell
\end{align*}
\]

The first and the second sets of constraints force transmitting all packets either through the reconfigurable or the fixed network (if the latter is possible). The remaining two sets of constraints ensure that at any time, sets of edges corresponding to transmitted packets form a matching in the reconfigurable network and limit the transmission time to \( 1/(2 + \varepsilon) \) (recall that sending amount \( s \) through edge \( e \) takes time \( s \cdot d(e) \)).

However, we note that our algorithm will not give feasible solutions for the primal LP, as we are relying on resource augmentation. Hence, we next formulate the corresponding dual. The program \( D \) dual to \( P \) is shown in Figure 4.

**B. Solution to Dual Program**

From the weak duality, the value of a feasible solution to program \( D \) is a lower bound on the cost of the optimal solution to \( P \). We utilize this to prove that algorithm ALG is competitive. For the sake of analysis, we construct a feasible dual solution whose cost can be related to the cost of ALG.

In the solution to \( D \) used throughout the analysis, for packet \( p \in \Pi \) we set the value of \( \alpha_p \) to the worst-case impact of \( p \) estimated at the arrival of this packet: If packet \( p \) was transmitted through the direct source-destination link, we set \( \alpha_p = d_{\ell}(p) \). Otherwise, if packet \( p \) was sent through edge \( e_p \) in the reconfigurable network, the value of \( \alpha_p \) is set to \( \alpha_p = \Delta_{ep}(e_p) \).

For time \( \tau \), transmitter \( t \in T \) and receiver \( r \in R \), let \( C_{t,\tau} \) and \( C_{r,\tau} \) be the set of all chunks assigned to use the edge adjacent to \( t \) and \( r \), respectively, that have not reached their destination until time \( \tau \). We set \( \beta_{t,\tau} = w(C_{t,\tau}) \) and \( \beta_{r,\tau} = w(C_{r,\tau}) \) to the total weight of chunks from corresponding sets.
C. ALG-to-Dual Ratio

The goal of this section is to relate the cost of ALG to the objective value of the dual solution. First, we show that the latency accumulated in $\beta$ variables is at most twice the cost of the algorithm. Second, we define a cost charging scheme from the weighted latency of ALG to variables $\alpha$. Third, by jointly considering these two relations, we get that the cost of the algorithm is at most $(2 + \varepsilon)/\varepsilon$ times the value of the dual solution.

**Lemma 1.** $\text{ALG} \geq \sum_{t \in T} \sum_{\tau} \beta_{t,\tau} = \sum_{r \in R} \sum_{\tau} \beta_{r,\tau}$.

**Proof.** The packets that use direct connections incur (positive) latency, but are not counted by the $\beta$ variables. Therefore, it is sufficient to prove that the sum of transmitters’ $\beta$ variables as well as the sum of receivers’ $\beta$ variables equals the weighted latency of packets (chunks) that are transmitted via the reconfigurable network.

Fix a chunk $c$ of packet $p$ that was transmitted via reconfigurable network. Let $A(c)$ denote the period when $c$ was active, that is, all the times $\tau$ from $a_p$ until the time $c$ reaches $\text{dest}(p)$. For any time $\tau$ in $A(c)$, chunk $c$ incurs cost $w_c$. Recall that chunk $c$ is assigned to exactly one edge $e(p(c))$ from $E_p(c)$ and thus to exactly one transmitter $t$ and receiver $r$ (the endpoints of edge $e$). Hence, for each $\tau \in A(c)$, it holds that $c \in C_{t,\tau}$ and $c \in C_{r,\tau}$, so for each time $\tau \in A(c)$, $w_c$ is counted towards $\beta_{t,\tau}$ and $\beta_{r,\tau}$. These are the only $\beta$ variables that count the latency of $c$ at transmission step $\tau$. The lemma follows by summing over all packets and their chunks in the input. \(\square\)

**ALG-to-$\alpha$’s charging scheme.** In this section we charge the cost of the algorithm (the weighted latencies) to packets. The goal is to show, that each packet is charged at most the value of the corresponding $\alpha$ variable. This will let us relate the cost of the algorithm to the dual solution.

Fix packet $p$. If $p$ was transmitted via the fixed network, then we simply charge its total latency to $p$ itself. Otherwise, $p$ was split into several chunks. The chunks of $p$ might delay other edges as edge $e_p$ can transmit just one of them in a single transmission. Therefore, we will focus on a single chunk of packet $p$ and charge its latency to other packets or packet $p$.

Let $c \in C_p(e_p)$ be a chunk of $p$. It incurs weighted latency of $w_c$ for each time $\tau \in A(c)$ before it reached $\text{dest}(p)$. When $c$ is being transmitted through any edge of the graph, we simply charge $w_c$ to $p$. It remains to charge latency of $c$ when it was waiting in the transmitter’s queue. For each such time $\tau$, there exists another chunk that blocked $c$. Let $B$ be the set of all chunks that blocked $c$.

For each chunk $c' \in B$, if $c'$ and $c$ are parts of the same packet $p$, the latency $w_c$ is charged, again, to $p$. Otherwise, we charge $w_c$ to $p$ or $p' = p(c)$ depending on which of these two packets arrived later. If $a_p < a_{p'}$, the latency of $w_c$ is charged to $p'$. Note that $c'$ is heavier than $c$ and hence $c \in L_{p'}$. If $a_p < a_{p'}$ we charge $w_c$ to packet $p$. If this is the case, it holds that $c' \in H_p$.

In the following lemma we bound the charges received by each packet.

**Lemma 2.** For packet $p$, let $c_p$ be the weighted latency charged to $p$. Then, it holds that $c_p \leq \alpha_p$.

**Proof.** Fix packet $p$. If $p$ is sent via the fixed network, the only charge it receives is $w_p \cdot d(p) = \alpha_p$. Otherwise, the charges received by $p$ are threefold:

- First, for each of its chunks, packet $p$ receives a charge of $w_c$ for every time $\tau$ when $c$ was not blocked (i.e., transmitted via any edge) or blocked by other chunk of $p$. The $i$-th (for $i \in \{1, 2, \ldots, d(p)\}$) delivered chunk of $p$ charges $w_c \cdot (d(src(p), t) + i + d(r, dest(p)))$. In total the latency charged in this case is equal to
  \[
  \sum_{i=1}^{d(p)} w_p \cdot (d(src(p), t) + i + d(r, dest(p)))
  \]
- Second, when $c$ blocked chunk $c'$ of a packet that arrived earlier than $p$, it receives charge $w_{c'}$. This charge can be received only from packets in set $L_p$. Note that $c'$ is delayed by all $d(e'_p)$ chunks of $p$.
- Third, when $c$ is blocked by some other chunk $c'$ of packet that arrived earlier than $p$, packet $p$ receives a charge of $w_c$. In this case, $c' \in H_p$.

Combining all three cases, we obtain that the latency charged to $p$ is at most

\[
c_p \leq w_p \cdot \left( d(src(p), t) + \frac{d(e_p) + 1}{2} + d(r, dest(p)) \right) + w_p \cdot |H_p| + d(e) \cdot w(L_p) = \alpha_p
\]

**Lemma 3.** For any $\varepsilon > 0$ it holds that $\text{ALG} \leq (2 + \varepsilon)/\varepsilon \cdot D$.

**Proof.** Fix $\varepsilon > 0$. If we sum the guarantees from Lemma 2 over all packets $p \in \Pi$, we obtain $\text{ALG} \leq \sum_p \alpha_p$. This combined with Lemma 1 immediately yields the lemma, as

\[
D \geq \sum_{p \in \Pi} \frac{\alpha_p}{2 + \varepsilon} \cdot \left( \sum_{t \in T} \sum_{\tau} \beta_{t,\tau} + \sum_{r \in R} \sum_{\tau} \beta_{r,\tau} \right)
\geq \text{ALG} - \frac{2}{2 + \varepsilon} \cdot \text{ALG} = \frac{\varepsilon}{2 + \varepsilon} \cdot \text{ALG}.
\]

**D. Dual-to-Opt Ratio**

By weak duality, the value of any feasible solution to $D$ is a lower bound on the cost of $\text{Opt}$. Our assignment of $\alpha$ and $\beta$ variables does not necessarily constitute a feasible solution to $D$, that is, some constraints might be violated. However, these constraints are "almost feasible", i.e., the solution obtained by halving each variable is feasible. Therefore, the value of our
dual solution is at most twice the optimum, which together with the results of the previous section, will lead to the bound on the competitive ratio of ALG.

The following lemma shows that constraints in $\mathcal{D}$ corresponding to packet $p$ and edge $e$ are violated by a factor of 2 if instead of $\alpha_p$ we take the value of precomputed impact of $p$ assigned to edge $e$.

**Lemma 4.** For packet $p$, edge $e = (t, r) \in E_p$ and time $\tau \geq \alpha_p$ it holds that

$$\Delta_p(e) - d(e) \cdot (\beta_{t, r} + \beta_{r, \tau}) \leq 2 \cdot w_p \cdot \left( t + \hat{d}(e) - \alpha_p \right).$$

**Proof.** Fix packet $p$, edge $e = (t, r) \in E_p$ and time $\tau \geq \alpha_p$. We start with proving that

$$L := w_p \cdot |H_p(e)| + d(e) \cdot w(L_p(e)) \leq d(e) \cdot (\beta_{t, r} + \beta_{r, \tau}) + 2 \cdot w_p \cdot (\tau - \alpha_p).$$

To this end observe that the contribution of a single chunk $c \in A_p(e)$ (i.e., chunk that uses edge adjacent to $e$) towards $L$ is at most

$$\min \left( d(e) \cdot w_c, w_p \right).$$

Two nontrivial relations follow directly from definitions of sets $H_p(e)$ and $L_p(e)$. If $c \in H_p(e)$, then $w_p \leq w_c \cdot d(e)$ and when $c \in L_p(e)$, then $d(e) \cdot w_c \leq w_p$.

Let $P$ be the set of all chunks in $A_p(e)$ that have not reached their destination by time $\tau$. By (2), the contribution of chunks from $P$ towards $L$ is at most $d(e) \cdot w(P) \leq d(e) \cdot (\beta_{t, \tau} + \beta_{r, \tau})$.

It remains to bound the contribution of packets in $Q = A_p(e) \setminus P$. Again, by (2), chunk $c \in Q$ contributes at most $w_p$ towards $L$. The proof of (1) is concluded by observing that the set $Q$ contains at most $2 \cdot (\tau - \alpha_p)$ chunks as endpoints of edge $e$ transmit at most one chunk in each transmission step.

The lemma follows by combining inequality (1), the fact that $d(e) \geq 1$, and the definition of $\Delta_p(e)$:

$$\Delta_p(e) = w_p \cdot \left( d(t) + \frac{d(e) + 1}{2} + d(r) \right) + w_p \cdot |H_p(e)| + d(e) \cdot w(L_p(e)) \leq w_p \cdot \hat{d}(e) + d(e) \cdot (\beta_{t, \tau} + \beta_{r, \tau}) + 2 \cdot w_p \cdot (\tau - \alpha_p) < 2 \cdot w_p \cdot \left( \tau + \hat{d}(e) - \alpha_p \right) + d(e) \cdot (\beta_{t, \tau} + \beta_{r, \tau}).$$

In the next lemma we combine weak duality to relate the value of our dual solution to the value of optimum.

**Lemma 5.** The value of our solution to dual program is at most twice the value of optimal solution, i.e., $\mathcal{D} \leq 2 \cdot \text{OPT}$. 

**Proof.** We prove that the solution to $\mathcal{D}$ obtained by halving each variable $\alpha$ and $\beta$ is feasible. To this end we show that the solution defined in Section IV-B violates constraints by a factor at most 2.

First, for packet $p \in \Pi$, the constraint corresponding to primal variable $y_p$ (i.e., routing through the fixed network) is not violated as $\alpha_p$ minimizes the impact of routing $p$ through any path, in particular, the direct source-destination link and hence $\alpha_p \leq w_p \cdot d_t(p)$.

Second, fix the dual constraint corresponding to primal variable $x_{p,e, \tau}$ for packet $p \in \Pi$, edge $e = (t, r) \in E_p$ and time $\tau \geq \alpha_p$.

Applying the definition of $\alpha_p$ and Lemma 4 to the left-hand side of the dual constraint we obtain:

$$\alpha_p - d(e) \cdot (\beta_{t, \tau} + \beta_{r, \tau}) \leq \Delta_p(e) - d(e) \cdot (\beta_{t, \tau} + \beta_{r, \tau}) \leq 2 \cdot w_p \cdot \left( \tau + \hat{d}(e) - \alpha_p \right).$$

Therefore, the dual solution created by halving each variable is feasible. The lemma follows from weak duality.

By combining Lemma 3 and Lemma 5, we derived the following theorem:

**Theorem 1.** For any $\varepsilon > 0$, ALG is $2 \cdot (2/\varepsilon + 1)$-competitive with speedup $(2 + \varepsilon)$. That is, for any input $I$, if ALG works $2 + \varepsilon$ times faster than the optimal offline algorithm OPT, the cost of ALG is bounded by $\text{ALG} \leq 2 \cdot (2/\varepsilon + 1) \cdot \text{OPT}$.

V. RELATED WORK

Reconfigurable optical topologies [7], [8], [11], [13], [27]–[32] have recently received much attention in the literature as an alternative to traditional static datacenter topologies [2], [3], [6], [33]–[36]. It has been demonstrated that already demand-oblivious reconfigurable topologies can deliver unprecedented bandwidth efficiency [7], [8], [13]. By additionally exploiting the typical skewed and bursty structure of traffic workloads [17]–[19], [37]–[40], demand-aware reconfigurable topologies can be further optimized, e.g., toward elephant flows. To this end, existing demand-aware networks are based on traffic matrix predictions [41]–[43] or even support per-flow or “per-packet” reconfigurations [9]–[11], [14], [30], [44], [45]. Our focus on this paper is on the latter, and we consider fine-grained scheduling algorithms.

While most existing systems are mainly evaluated empirically, some also come with formal performance guarantees. However, to the best of our knowledge, besides some notable exceptions which however focus on single-tier models [15], [22], [46]–[48], hardly anything is known about the achievable competitive ratio by online packet scheduling algorithms. In particular, our paper is motivated by the multi-tier ProjecToR architecture [11], to which our analysis also applies.

Our model and result generalizes existing work on competitive switch scheduling [20], [21]: In classic switch scheduling, packets arriving at a switch need to be moved from the input buffer to the output buffer, and in each time step, the input buffers and all their output buffers must form a bipartite matching. A striking result of Chuang, Goel, McKeown, and Prabhakar showed that a switch using input/output queueing with a speed-up of 2 can simulate a switch that uses pure output queueing [21]. Our model generalizes this problem to a multi-tier problem, and we use a novel primal-dual charging scheme.
Venkatakrishnan et al. [15] initiated the study of an offline scheduling variant of the circuit switch scheduling problem, motivated by reconfigurable datacenters. They consider a setting in which demand matrix entries are small, and analyze a greedy algorithm achieving an (almost) tight approximation guarantee. In particular, their model allows to account for reconfiguration delays, which are not captured by traditional crossbar switch scheduling algorithms, e.g., relying on centralized Birkhoff-von-Neumann decomposition schedulers [49].

Schwartz et al. [48] recently presented online greedy algorithms for this problem, achieving a provable competitive ratio over time. This line of research however is technically fairly different from ours: the authors consider a maximization problem, aiming to maximize the total data transmission for a certain time window, whereas in our model, we aim to minimize completion times (i.e., *all* data needs to be transmitted). Furthermore, while we consider a multi-tier network (inspired by architectures such as [11]), these works assume a complete bipartite graph. Last but not least, we also support a simple form of hybrid architectures in our model. Besides these differences in the model, our model differs significantly from [15], [48] in terms of the used techniques. While prior work relies, among other, on randomized rounding, we study an online primal-dual approach.

In general, online primal-dual approaches have received much attention recently, after the seminal work by Buchbinder and Naor [26]. Unlike much prior work in this area, we however do not use the online primal-dual approach for the design of an algorithm, but only for its analysis. In this regard, our approach is related to scheduling literature by Anand et al. [25], and the interesting work by Dinitz et al. [22] on reconfigurable networks. We generalize the analysis of [25] to a more general graph where we can have conflicts at receivers of bicriteria scheduling algorithms. Furthermore, it would be interesting to explore randomized scheduling algorithms.

**VI. Conclusion**

We presented a competitive scheduling algorithm for reconfigurable datacenter networks which generalizes classic switch scheduling problems and whose analysis relies on a dual-fitting approach. We understand our work as a first step and believe that it opens several interesting avenues for future research. In particular, it would be interesting to study the optimality of bicriteria scheduling algorithms. Furthermore, it would be interesting to explore randomized scheduling algorithms.

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