Localized-interaction-induced quantum reflection and filtering of bosonic matter in a one-dimensional lattice guide

I. Barbiero, B A Malomed and L Salasnich

Abstract

We study the dynamics of quantum bosonic waves in a one-dimensional tilted optical lattice. An effective spatially localized nonlinear two-body potential barrier is set at the center of the lattice. This version of the Bose–Hubbard model can be realized in atomic Bose–Einstein condensates, with the help of localized optical Feshbach resonance, controlled by a focused laser beam, and in quantum optics, using an arrayed waveguide with selectively doped guiding cores. Our numerical analysis demonstrates that the central barrier induces anomalous quantum reflection of incident wave packets, which acts solely on bosonic components with multiple onsite occupancies, while single-occupancy components pass the barrier, allowing one to distill them in the interaction zone. As a consequence, in this region one finds a hard-core-like state, in which the multiple occupancy is forbidden. Our results demonstrate that this regime can be attained dynamically, using relatively weak interactions, irrespective of their sign. Physical parameters necessary for the experimental implementation of the setting in ultracold atomic gases are estimated.

1. Introduction

Isolated quantum systems in out-of-equilibrium configurations have attracted a great deal of interest due to the possibility of observing new quantum effects [1]. An ideal platform to build such systems is offered by ultracold bosons in reduced dimensionality [2–4], where all parameters of the system can be controlled with a high level of accuracy and flexibility [5]. In this context, the band structure generated by optical lattices (OLs) [6] and the absence of dissipation have allowed the experimental observation of peculiar out-of-equilibrium effects [7–9] predicted several years ago [10]. Atomic motion induced by tilted OL potentials has been widely explored too, revealing remarkable quantum features [11, 12]. Furthermore, the study of the dynamics of bosonic waves in a continuous geometry opens the way to a novel applications in nonlinear optics [13, 14] and plasmas [15]. Scattering of bosonic solitary matter waves on narrow repulsive [16–21] and attractive [22, 23] potential barriers or wells has been extensively studied in a theoretical form too, suggesting experimental observations of the effect of the quantum reflection [24, 25]. In early work [26] and more recently [14, 27, 28], configurations where effective nonlinear potential barriers or wells are induced by spatially localized two-body interaction have been proposed as a possible mechanism to observe other various forms of the anomalous reflection and splitting [29].

In this work we combine the above-mentioned ingredients to study the scattering of wave packets, composed of non-interacting bosons in a tilted OL, on a localized interaction zone, by means of systematic simulations based on the time-dependent density-matrix-renormalization-group (t-DMRG) method. Exotic effects, such as selective quantum reflection, distillation and filtering, are revealed as a result of the scattering. In particular, we demonstrate that, even for a relatively small interaction strength, the nonlinear barrier acts as quantum filter, which almost completely reflects bosonic components with multiple onsite occupancies, while...
the components carrying the single occupancy (SO) are able to pass the barrier. In this way, a region where multiple occupancies (MO) are forbidden is found. We demonstrate that such a state can be distilled from the incident wave packet, using both repulsive and (rather unexpectedly) attractive localized interactions. Furthermore, our analysis reveals that the distillation effect, induced by the lattice’s band structure, features its most pronounced form, i.e., the total MO reflection, at relatively small interaction strengths, and it is not essentially affected by variation of the potential tilt which drives the incident wave packets.

2. The model

We study the evolution of an initially localized bosonic wave packet moving in a one-dimensional (1D) tilted OL, with onsite interaction acting in a finite region (‘barrier’) of size \( L_{\text{barr}} \) (measured in terms of the OL sites), where a part of incident waves may be trapped in the case of the attractive interaction, see figure 1. At \( t = 0 \), we place a Gaussian wave packet near the left edge of the whole lattice. Experimentally, the initial packet may be created by a very tight harmonic-oscillator trap, initially applied at the same spot, which is subsequently lifted. Potential tilt \( E \) can be produced and tuned by applying dc magnetic field along the vertical direction, with a gradient along the OL, its effect being to induce the accelerated motion of atoms towards the center, where a zone representing the nonlinear scatterer [26, 27, 29] is composed of a finite number of sites carrying onsite interaction strength \( (U = 0) \). Spatially non-uniform interactions in ultracold atomic gases were recently realized in experiments [30–32], with the help of the Feshbach resonance controlled by inhomogeneous external fields. These results motivate the consideration of various settings based on effective nonlinear potentials [26, 27, 29, 33–36], which includes the prediction of 1D quantum solitons in the Bose–Hubbard (BH) model with the strength of the onsite repulsive interaction \((U > 0)\) growing with the distance from the center, \( |i| \), at any rate faster than \( |i| \) [37]. On the other hand, it was shown in detail in the context of another physical setting in [29] that a confined interaction zone, extending over the width corresponding to a few OL sites, can be induced by means of the optical Feshbach resonance controlled by a laser beam shone onto the lattice in the perpendicular direction.

Here we consider the evolution of the atomic condensate governed by the following Hamiltonian of the BH type:

\[
H = -J \sum_{i=1}^{L} (b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i) + \frac{U}{2} \sum_{i=L_{\text{ref}}+1}^{L_{\text{ref}}+L_{\text{barr}}} n_i (n_i - 1) - E \sum_{i=1}^{L} n_i \tag{1}
\]

where \( b_i, (b_i^\dagger) \) is the bosonic annihilation (creation) operator for an atom at the \( i \)th site in the lattice of total length \( L \), and \( n_i \) is the atomic population at the site. The hopping of atoms between nearest lattice sites is controlled, as usual, by the respective probability \( J \), which sets scales for energy and time in the present system (i.e., \( J = 1 \) is set below), \( E \) is the potential tilt, and \( U \) the strength of the two-body interaction at those sites where it is present, i.e., \( L_{\text{ref}} + 1 \leq i \leq L_{\text{ref}} + L_{\text{barr}} \). Since we apply the interaction in a small part of the lattice, it is relevant to distinguish three different regions: the left region with \( L_{\text{ref}} \) sites, into which the incident atoms are reflected, the central region of the nonlinear barrier with \( L_{\text{barr}} \) sites, and the right region with \( L_{\text{tran}} \) sites, into which the atoms may be transmitted. Thus, the total number of sites in the lattice, which as a whole is embedded into a potential box, is \( L = L_{\text{ref}} + L_{\text{barr}} + L_{\text{tran}} \), as shown in figure 1.

**Figure 1.** The setting under the consideration. At \( t = 0 \), a bosonic wave packet is placed at the edge of the left part of the lattice, of size \( L_{\text{ref}} \) (in terms of the number of the OL sites), into which a part of the wave packet is reflected after the collision with the interaction zone \((U \neq 0)\). At \( t > 0 \), the evolution of the input is governed by Hamiltonian (1), with \( E \) driving the particles towards the central interaction zone (nonlinear barrier) of size \( L_{\text{barr}} \). Adjacent to it on the right-hand side, is a part composed of \( L_{\text{tran}} \) OL sites, into which transmitted particles will move.
To estimate a possibility of the experimental implementation of the proposed setting in ultracold gases, it is relevant to refer to recent experimental work [38], which used cesium atoms in the hyperfine ground state, \( |F = 3, m_F = 3\), for realizing regular and chaotic regimes of the superfluid flow in tilted OLs, created by laser beams with wavelength \( \lambda = 1.0645 \mu \text{m} \), with the corresponding depth equal to seven recoil energies \( (E_{\text{recoil}} = 1.325 \text{kHz}) \). This value of the depth translates into the atom’s hopping rate \( f = 52.3 \text{ Hz} \). Further, the scattering length \( a_s = 21.4 \) \( a_0 \) corresponds to the onsite interaction strength \( U = 102 \text{ Hz} \), which, by means of the Feshbach resonance, could be increased up to \( U = 533 \text{ Hz} \). Thus, the setting made it possible to easily realize values of the main control parameter, \( U/f \), ranging between 2 and 5. Values of this parameter which are essential to the results reported below are virtually the same, \( 2 < U/f < 6 \). The potential ramp was created in [38] using a combination of gravity and magnetic-field gradient, with values up to \( \nabla B = 31.1 \text{ G cm}^{-1} \). This method makes it possible to readily adjust values of \( E/f \) to values relevant to the present analysis, which are \( 0.1 < E/f < 0.5 \).

In addition to atomic Bose–Einstein condensates, the same BH system may be implemented as a quantum-optics model of an array of evanescently coupled parallel waveguides [39] (possibly, photonic nanowires [40]). In that case, the localized interaction zone can be created by means of selectively doping the respective guiding cores by a material which resonantly enhances the Kerr nonlinearity [41], while the potential ramp can be used by tapering individual cores. In the optics model, the evolution variable, \( t \), is replaced by the propagation distance, \( z \). Typically, the hopping rate corresponds to the inter-core coupling length \( f^{-1} \lesssim 1 \text{ cm} \), which makes it necessary to have the nonlinearity length as short as \( U^{-1} \sim 2 \text{ mm} \). This value is challenging, but the us of the resonantly enhanced nonlinearity may make it possible.

3. Numerical results

We report numerical results obtained by means of the \( t\)-DMRG technique [42] using 350 DMRG states in the time-evolution calculations and time step \( \Delta t = 0.01 \) (it was checked that taking smaller \( \Delta t \) does not affect the results). We simulated the system with \( L = 20 \) lattice sites and a variable number \( N \) of bosons, fixing the corresponding sizes in equation (1) as \( L_{\text{ref}} = L_{\text{tran}} = 8 \) and \( L_{\text{barr}} = 4 \). Although the total size used here, \( L = 20 \), is relatively small, it is comparable with that in experimentally realized systems [43]. It can be checked that the increase of \( L \) and, accordingly, of \( L_{\text{ref}}, L_{\text{barr}}, L_{\text{tran}} \) affects only characteristic time scales of the dynamical results reported below, but does not essentially alter outcomes of the scattering.

3.1. The flat repulsive barrier

Usually, reflection and transmission of wave packets is revealed by tracking expectation values \( \langle n_i \rangle \) of the density profile with respect to the evolving many-body quantum state, \( |\psi(t)\rangle \). Note that \( \langle n_i \rangle \) can be precisely measured in the experiment, by means of the recently developed \textit{in situ} imaging technique [44, 45]. The Gaussian shape of \( \langle n_i \rangle \) at \( t = 0 \) is localized on five lattice sites populated by \( N = 8, 10, 12 \) bosons, respectively, in the first, second and third row of figure 2. Once at \( t > 0 \) the bosons are free to move toward the central part of the system, the initial Gaussian density profile is deformed\(^7\), and its actual shape depends on \( N \), as is evident in the first column of figure 2.

Figures 2(a), (d) and (g) show typical quantum-reflection effects for \( E = 0.3 \) and \( U = 6.0 \). Indeed, it is seen that, as the bosons approach the interaction zone\(^8\) with \( U = 0 \), a large fraction of them bounce back, with only a small part being able to pass \( L_{\text{barr}} \). At the first glance, this behavior is similar to the one induced by the usual linear potential barrier, see, e.g., [25]. However, a crucial difference is that the nonlinear (interaction-induced) barrier in our setting acts only on two- and many-body states. Therefore, it is necessary to distinguish between the SO, and MO scattering behaviors. To this end, we define two operators acting on the many-body quantum state, \( |\psi(t)\rangle \). One operator counts SO sites, with occupancy \( \langle n_i \rangle \leq 1 \):\(^9\)

\[
\langle n_i \rangle \\frac{\langle \psi(t) \rangle}{\langle \psi(t) \rangle} = \alpha \frac{\langle \psi(t) \rangle}{\langle \psi(t) \rangle} \quad \text{with} \quad \alpha = \langle n_i \rangle \quad \text{if} \quad \langle n_i \rangle \leq 1, \quad \text{and} \quad \alpha = 0 \quad \text{if} \quad \langle n_i \rangle > 1.
\]

(2)

The operator counting MO sites is \( n_i^p = n_i(n_i - 1)/2 \), which acts according to\(^9\)

\[
\langle n_i \rangle \frac{\langle \psi(t) \rangle}{\langle \psi(t) \rangle} = \beta \frac{\langle \psi(t) \rangle}{\langle \psi(t) \rangle} \quad \text{with} \quad \beta = \langle (n_i(n_i - 1)/2) \rangle \quad \text{if} \quad \langle n_i \rangle > 1.
\]

(3)

Thus we can separately take into account sites where the interaction, if present, is effective, i.e., \( \langle n_i^p \rangle \neq 0 \), and where it is not, i.e., \( \langle n_i \rangle = 0 \). In the second and third columns of figure 2, respectively, we show the evolution of the expectation values of operators \( n_i^p \) and \( n_i^p \). While it is evident from figures 2(b), (e), and (h) that the SO, represented by \( \langle n_i^p \rangle \), freely passes the nonlinear barrier, figures 2(c), (f), and (i) make it clear that the MO,

---

\(^7\) In a tilted lattice the motion of a wave packet with a permanent shape is impossible due to the constrain imposed by the energy conservation.

\(^8\) In all the cases shown, the \( U = 0 \) term is located only at sites with \( i = 9, 10, 11, 12 \).
represented by $\langle n^P_r \rangle$, bounces back from it. More precisely, we notice that, after an initial decrease due to the propagation in the non-interaction regime, $\langle n^P_r \rangle$ consistently grows at the right edge of $L_{ref}$ at intermediate times. The accumulation of the MO is followed by its nearly complete rebound. On the other hand, the SO features the behavior reverse to that of $\langle n^P_r \rangle$ in the $L_{ref}$ section of the lattice. In particular, dissociation (formation) of the MO is coupled to the increase (decrease) of $\langle n^r \rangle$.

The crucial point is the behavior of the bosons inside the central interaction zone, where, on the contrary to the MO, the SO can evidently reside. This fact is a drastic difference with respect to the usual settings, with a linear-potential barrier acting at the single-particle level. Thus, as already stated, the quantum transmission observed in figures 2(a), (d), and (g) is totally accounted for by the SO motion. In other words, the interaction zone acts as a quantum filter, which sends all the occupancies with $\langle n^r \rangle > 1$ back, and lets those with $\langle n^r \rangle < 1$ pass. In this way, the interaction zone, cleared of the MO, displays an effective hard-core on-site repulsion, with bosonic particles emulating fermions. A quantum gas where the pair- and multiple-occupations are forbidden due to the interaction is usually associated to the appearance of the Tonks–Girardeau (TG) regime.

The latter was originally predicted in a configuration preserving the Galilean invariance [46–48], but it has later been demonstrated both theoretically [49] and experimentally [50] that the presence of a lattice preserves the main features of the TG gas. Noticeably, in figure 2 the hard-core constraint is generated for all considered values of the boson number, $N$. The latter fact signals that the interaction strength, $U$, is responsible for the filtering effect. To check the efficiency of the filter, in figure 3 we plot densities which are, respectively, the observation values of $n^r$, $n^P_r$ and $n^r$, averaged over three different parts of the lattice, $L_{ref}$, $L_{barr}$, $L_{tran}$, for different values of the interaction strength, $U$:

$$
\rho = \frac{1}{L_{ref}} \sum_{k=1}^{L_{ref}} \langle n^r \rangle, \quad \rho_p = \frac{1}{L_{barr}} \sum_{k=1}^{L_{barr}} \langle n^P_r \rangle, \quad \rho_s = \frac{1}{L_{tran}} \sum_{k=1}^{L_{tran}} \langle n^r \rangle.
$$

(4)

It is clearly seen in figure 3 that, in the course of the evolution the value of $\rho$ is conspicuously different from zero in region $L_{barr}$ only for a relatively weak interaction strength, namely, $U = 2$. Once a stronger interaction acts in $L_{barr}$, the MO density practically vanishes. As a result, at intermediate values of time, a gas composed of the SO is stabilized in the distilled form inside the interaction zone. Obviously, the number of particles approaching the barrier does not depend on the interaction strength present in $L_{barr}$, as is evident in the first column of figure 3. In the same time, once the interaction capable to support the quantum filtering is applied in $L_{barr}$, the value of $\rho$ becomes independent of the interaction strength $U$. This is confirmed by the fact that in figure 3 we see that, for $U = 4$ and 6, $\rho_s$ is actually zero in region $L_{barr}$, hence $\rho_s$ has the same value for these two interaction strengths. As seen in the third column of figure 3, this aspect has its consequences also in the behavior of bosons in region $L_{tran}$. Indeed, the filtering process allows a larger number of bosons to enter $L_{tran}$, which means that, effectively,
the dynamically induced hard-core constraint increases the speed of the particles. In particular, our measurements yield \( r = \frac{1}{4} \), \( U = 4, 6 \), \( 2, 5 \) at \( t = 10 \). Interestingly this larger amount of particles allows the formation of higher SO and MO alike, see values of \( r_s \) and \( r_p \) in the third column of figure 3.

### 3.2. The flat attractive barrier

The quantum reflection of the MO might seem a rather obvious consequence of the repulsive nature of the interaction. For this reason, it is interesting to consider the system with attractive interactions, i.e., \( U < 0 \), too. The analysis of static configurations for \( U < 0 \) and relatively large \( |U| \) has previously revealed collapsed states, see [27, 51] and references therein. This fact suggests that MO may not bounce back from the interaction zone, \( L_{\text{trap}} \), and get partly trapped in it. Nevertheless, figure 4, which displays the same characteristics of the dynamical scattering as in figure 3, but for \( U < 0 \), shows that this does not happen—in fact, the self-attraction zone does not accumulate the MO. Actually we observe that this system again stabilizes an effectively ‘distilled’ quasi-TG state in this zone, although with a higher density than in the case of \( U > 0 \).

The approximate symmetry between the cases of \( U > 0 \) and \( U < 0 \), revealed by the comparison of figures 3 and 4, agrees with findings of [52], where a similar symmetry was discovered in the transport of fermion atoms. In the present contexts, it is related to properties of the energy spectrum of lattice bosons, which demonstrates the symmetry with respect to \( U \leftrightarrow -U \). Moreover, the consideration of the attractive interaction helps one to understand how the present BH model gives rise to the quantum filtering, distillation, and rebound effects. First, it is obvious that, in either case, the system conserves the total energy (along with the total number of bosons). Further, the band structure produced by the OL imposes a limitation on possible values of the kinetic energy. In fact, the formation of MO in the interaction zone would induce energy variation that cannot be supported by the system in which any gain/loss in the potential energy must be converted into the opposite change of the kinetic energy. A precise many-body quantification of this effect is a very hard problem, due to the non-integrability of equation (1). Nevertheless, arguments regarding the two- [7–9] and three-body [53] bound states may be sufficient to explain many significant dynamical quantum effects in 1D lattice systems [7–9, 54]. In our case, the derivation of the two- and three-body energy spectrum is substantially complicated by the presence of the tilted potential. Nevertheless, the same energy arguments make it possible to explain the above-mentioned effect.

Actually, the rebound of the MO bosonic component from the self-attraction region, observed in figure 4, is alike to the commonly known effect of the partial reflection of an incident wave from a quantum-mechanical potential well [55], and also to the possibility of the rebound of a moving soliton from a potential well in the nonlinear model [56].

---

**Figure 3.** The three rows display, respectively, the evolution of average densities of the total number of bosons, SO (single occupancy), and MO (multiple occupancy), in the three sections of the lattice, which are defined as per equation (4). The three columns refer to sections \( L_{\text{ref}} \), \( L_{\text{bar}} \), and \( L_{\text{tran}} \), as indicated in the top line. The data were collected for \( N = 10 \) bosons and \( L = 20 \) sites, with \( N \) bosons placed, at \( t = 0 \), on 5 lattice sites. The other parameters are \( L_{\text{ref}} = L_{\text{bar}} = 8, L_{\text{tran}} = 4, E = 0.3 \), and different values of \( U \), as indicated in the figure.
3.3. A linearly shaped repulsive barrier

All the results presented above refer to a configuration where the bosons are subject to spatially uniform interactions in region $L_{\text{bar}}$. A essential issue is whether a barrier with spatially inhomogeneous interactions gives rise to similar filtering effects. In figure 5 we show the behavior of $\langle n_1 \rangle$, evaluated at different times in region $L_{\text{bar}}$ in the system with the interaction strength growing linearly with the distance. More precisely, we study a configuration where the bosons are subject to the site-dependent interaction $U(i)$, with minimum value $U_{\text{min}}$ at site $i = 9$, and maximum strength $U_{\text{max}}$ at $i = 12$, i.e. $U(i) = U_{\text{min}}(i - 8)$. As might be expected, it is observed in figure 5(a), corresponding to $U_{\text{min}} = 0.5$, that such a weak potential is not able to support any filtering. Noticeably, at site $i = 12$, where the strength is $U(i = 12) = 2$, we find that $\langle n_1 \rangle$ has the same value of $\rho_p$ following the definition of (4). Moreover, it is relevant to point out that the only site where MO is actually forbidden is the point where

![Figure 4](image_url)

**Figure 4.** The same as in figure 3, but for the system with the attraction ($U < 0$) acting in the interaction zone ($L_{\text{bar}}$).

![Figure 5](image_url)

**Figure 5.** The expectation value of $\langle n_1 \rangle$ in region $L_{\text{bar}}$, namely, at sites $i = 9, 10, 11, 12$, in the system with $N = 10$ bosons initially placed on 5 lattice sites, and $E = 0.3$. The local interaction strength, $U(i) = U_{\text{min}} \cdot (i - 8)$, grows linearly with the distance, with slope $U_{\text{min}} = 0.5, 1.0, 2.0$ in (a), (b) and (c), respectively.

3.3. A linearly shaped repulsive barrier

All the results presented above refer to a configuration where the bosons are subject to spatially uniform interactions in region $L_{\text{bar}}$. A essential issue is whether a barrier with spatially inhomogeneous interactions gives rise to similar filtering effects.

In figure 5 we show the behavior of $\langle n_1 \rangle$, evaluated at different times in region $L_{\text{bar}}$ in the system with the interaction strength growing linearly with the distance. More precisely, we study a configuration where the bosons are subject to the site-dependent interaction $U(i)$, with minimum value $U_{\text{min}}$ at site $i = 9$, and maximum strength $U_{\text{max}}$ at $i = 12$, i.e. $U(i) = U_{\text{min}}(i - 8)$. As might be expected, it is observed in figure 5(a), corresponding to $U_{\text{min}} = 0.5$, that such a weak potential is not able to support any filtering. Noticeably, at site $i = 12$, where the strength is $U(i = 12) = 2$, we find that $\langle n_1 \rangle$ has the same value of $\rho_p$ following the definition of (4). Moreover, it is relevant to point out that the only site where MO is actually forbidden is the point where

9 Note that we are here referring to the value of $\langle n_1 \rangle$ at the 12th site. It actually corresponds to having the barrier with interaction constant $U = 2$ on one lattice site. For this reason, we have $\langle n_1 \rangle = \rho_p$ following the definition of (4).
In the 1D isolated quantum system, to find a critical strength of the interaction able to give rise to the filtering precesses.

Actually we observe that the value of $\rho_p$ is more affected by $U$ in the case of the repulsive interaction.
4. Conclusion

We have introduced a version of the BH system composed of two sections which do not carry onsite two-body interactions, with an interaction zone sandwiched between them. The cases of spatially uniform repulsive and attractive interactions, as well as inhomogeneous interactions, were considered. This is a fully quantum counterpart of models with nonlinear potential barriers or wells, that were recently studied in optics and mean-field description of matter waves in atomic BEC. In those contexts, the spatially localized interactions may be induced, respectively, by means of selective doping, or by the Feshbach resonance controlled by an inhomogeneous external field. Using the quasi-exact numerically implemented t-DMRG method, we have considered the scattering problem, where the potential tilt sends a wave packet to collide with the effective nonlinear barrier (interaction zone). The result is that the nonlinear barrier, being transparent to the bosonic-wave component with the onsite SO, induces strong quantum reflection of the MO (multiple-occupancy) bosonic components. These properties make it possible to realize the quantum distillation of the SO component in the interaction zone, which is tantamount to inducing an effective on-site hard-core repulsion. The absence of the MO, which was experimentally demonstrated to be a characteristic feature of the TG state \(50, 57\), makes it possible to dynamically realize a similar state in the interaction zone of the present system. Furthermore, we have shown that, in contrast to the static configuration where the hard-core regime occurs for very strong repulsive interaction (while strong attraction may generate a highly excited state in the form of the super-TG gas \(58–61\)), our dynamical setting makes it possible to reach this hard-core-like regime, using relatively weak repulsion, or even weak attraction (which is an unexpected finding), in the interaction zone. Further investigations are currently in progress, to better characterize this peculiar regime. It has been demonstrated that the predicted results can be implemented using currently available experimental settings.

Acknowledgments

We appreciate a valuable discussion with M D Lukin. This work was supported by MIUR (FIRB 2012, Grant No. RBFR12NLNA-002; PRIN 2013, Grant No. 2010LLKJBX). L B thanks CNR-INO BEC Center in Trento for CPU time.

References

[1] Polkovnikov A, Sengupta K, Silva A and Vengalattore M 2011 Rev. Mod. Phys. 83 863
[2] Salasnich L, Parola A and Reatto I 2002 Phys. Rev. A 65 045316
[3] Mateo Muñoz A and Delgado V 2008 Phys. Rev. A 77 033617
[4] Cazalilla M A, Citro R, Giamarchi T, Orignac E and Rigol E 2011 Rev. Mod. Phys. 83 1405 and references therein
[5] Bloch I, Dalibard J and Zwerger W 2008 Rev. Mod. Phys. 80 885
[6] Jaksch D, Bruder C, Cirac I J, Gardiner C W and Zoller P 1998 Phys. Rev. Lett. 81 3108
[7] Winkler K, Thalhammer G, Lang F, Grimm R, Hecker-Denschlag J, Daley A J, Kantian A, Büchler A P and Zoller P 2006 Nature 441 853
[8] Strohmaier N, Greif D, Jordens R, Tarruell L, Moritz H, Eslinger T, Sensarma R, Pekker D, Altman E and Demler E 2010 Phys. Rev. Lett. 104 080401
[9] Mark M J, Haller E, Lauber K, Dansel J G, Janisch A, Büchler H P, Daley A J and Nägerl H C 2012 Phys. Rev. Lett. 108 215302
[10] Hubbard J 1963 Proc. R. Soc. A 276 238
[11] Simon J, Bakr W S, Ma R, Tai M E, Preiss M P and Greiner M 2011 Nature 472 307
[12] Meinert F, Mark M J, Kirilov E, Lauber K, Weinmann P, Gröbner M, Daley A J and Nägerl H C 2014 Science 344 1259–62
[13] Kivshar Y S and Agrawal G 2003 Optical Solitons: From Fibers to Photonic Crystals (San Diego, CA: Academic)
[14] Kartashov Y V, Malomed B A and Torner L 2011 Rev. Mod. Phys. 83 247

Figure 7. Evolution of \(\langle n_i^p \rangle\) in the system with \(N = 10, E = 0.3\), and \(U = 6\) in the central part of the lattice. (a) and (b) refer to a configuration where the initial Gaussian profile is localized over 3 or 7 sites, respectively.
[15] Stasiewicz K, Shukla P K, Gustafsson G, Buchert S, Lavraud B, Thibe B and Klos Z 2003 Phys. Rev. Lett. 90 085002
[16] Helm J L, Billam T P and Gardiner S A 2012 Phys. Rev. A 85 053626
[17] Martin A D and Ruostekoski J 2012 New J. Phys. 14 043040
[18] Cuevas J, Kevrekidis P G, Malomed B A, Dyke P and Hulet R 2013 New J. Phys. 15 063006
[19] Polo J and Ahufinger V 2013 Phys. Rev. A 88 053628
[20] Helm J L, Rooney S J, Weiss C and Gardiner S A 2014 Phys. Rev. A 89 033610
[21] Helm J L, Cornish S L and Gardiner S A 2015 Phys. Rev. Lett. 114 134101
[22] Lee C and Brand J 2006 Eur. Phys. Lett. 73 321
[23] Ernst T and Brand J 2010 Phys. Rev. A 81 033614
[24] Marchant A L, Billam T P, Wiles T P, Yu M M H, Gardiner S A and Cornish S L 2013 Nat. Commun. 4 1865
[25] Marchant A L, Billam T P, Yu M M H, Rakonjac A, Helm J L, Polo J, Weiss C, Gardiner S A and Cornish S L 2016 Phys. Rev. A 93 021604
[26] Malomed B A and Ya Azbel M 1993 Phys. Rev. B 47 10402
[27] Dör R and Malomed B A 2011 Phys. Rev. A 83 033828
[28] Maor O, Dör R and Malomed B A 2013 Opt. Lett. 38 5454
[29] Sakaguchi H and Malomed B A 2016 New J. Phys. 18 025020
[30] Bauer D M, Lettner M, Vo C, Rempe G and Dürr S 2009 Nature Phys. 5 339
[31] Yamazaki R, Taie S, Sugawa S and Takahashi Y 2010 Phys. Rev. Lett. 105 050405
[32] Clark L W, Ha L-C, Xu C-Y and Chin C 2015 Phys. Rev. Lett. 115 155501
[33] Matveevanuyoo T, Malomed B A and Dong G 2008 Phys. Rev. A 78 053601
[34] Borovkova O V, Kartashov Y V, Malomed B A and Torner L 2011 Opt. Lett. 36 3088
[35] Borovkova O V, Kartashov Y V, Torner L and Malomed B A 2011 Phys. Rev. E 84 035602(R)
[36] Gligoric G, Maluckov A, Hadzievski I and Malomed B A 2013 Phys. Rev. E 88 032905
[37] Barbiero L, Malomed B A and Salasnich L 2014 Phys. Rev. A 90 063611
[38] Heinert F, Mark M J, Kirilov E, Lauber K, Weinnmann P, Grobner M and Nager H C 2014 Phys. Rev. Lett. 112 193003
[39] Krimer D O and Khomeriki R 2011 Phys. Rev. A 84 041807
[40] Ye F, Mihalache D, Hu B and Panoiu N C 2010 Phys. Rev. Lett. 104 106802
[41] Hukriede J, Runde D and Kip D 2003 J. Phys. D: Appl. Phys. 36 R1
[42] White S R and Feiguin A E 2004 Phys. Rev. Lett. 93 076401
White S R and Feiguin A E 2005 Phys. Rev. B 72 020404(R)
[43] Fukuhara T, Sahauss P, Endress M, Hild S, Cheneau M, Bloch I and Gross C 2013 Nature 502 76
[44] Baks W S, Gillen J L, Peng A, Foellinger S and Greiner M 2009 Nature 462 74
[45] Sherson J F, Weitenberg C, Endres M, Cheneau M, Bloch I and Kuhr S 2010 Nature 467 68
[46] Tomk L 1936 Phys. Rev. 50 953
[47] Bijl A 1936 Physica 4 329
[48] Girardeau M 1960 J. Math. Phys. 1 516
[49] Cazalilla M A 2003 Phys. Rev. A 67 053606
Cazalilla M A 2004 Phys. Rev. A 70 041604(R)
[50] Paredes B, Widera A, Murg V, Mandel O, Folling S, Cirac I J, Shlyapnikov G W, Hansch T W and Bloch I 2004 Nature 429 277
[51] Barbiero L and Salasnich L 2014 Phys. Rev. A 89 063605
[52] Schneider U et al 2012 Nat. Phys. 8 213
[53] Johnson R, Tiesinga E, Porto J V and Williams C J 2009 New J. Phys. 11 093022
[54] Barbiero L, Menotti C, Recati A and Santos L 2015 Phys. Rev. B 92 180406(R)
[55] Landau L D and Lifshitz E M 1974 Quantum Mechanics (Moscow: Nauka)
[56] Ernst T and Brand J 2010 Phys. Rev. A 81 033614
[57] Kinoshita H, Wenger T T and Weiss D S 2004 Science 305 1125
[58] Chen S, Guan L, Yin X, Hao Y J and Guan X M 2010 Phys. Rev. A 81 031609
[59] Muth D and Fleischhauer M 2010 Phys. Rev. Lett. 105 150403
[60] Girardeau M D and Astrakharchik G E 2012 Phys. Rev. Lett. 109 235305
[61] Panfil M, De Nardis J and Caux J S 2013 Phys. Rev. Lett. 110 125302