Estimating electric current densities in solar active regions

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Overview

Background
Flares, currents, and vector magnetograms
Nonlinear force-free modelling

Method
A minimum current estimate for $J_z$

Test using simulated data
BCs from a NLFFF
Minimum current estimate
NLFFF reconstructions

Summary
Background: Flares, currents, and vector magnetograms

- Sunspot magnetic field energy power flares, CMEs
  - flares occur at sheared neutral lines
  - suggesting a connection with large scale currents
- Vector magnetograms provide maps of $\mathbf{B}$ at the photosphere
  - $\mathbf{B} = (B_x, B_y, B_z)$ in local coordinates ($z = 0$ is photosphere)
  - derived from spectro-polarimetric measurements (‘inversion’) (del Toro Iniesta 2003)
  - substantial uncertainties:
    - errors in polarisation measurements
    - inaccuracy in the model assumed for inversion
    - errors in ambiguity resolution (Leka et al. 2009)
    - inaccuracy due to rebinning of data (Leka & Barnes 2012)
Vertical current density at photosphere via Ampère’s law:

\[ \mu_0 J_z(x, y, 0) = v - u \]

with \( u = \left. \frac{\partial B_x}{\partial y} \right|_{z=0} \) and \( v = \left. \frac{\partial B_y}{\partial x} \right|_{z=0} \)

(1)

with field uncertainties \( \sigma_x, \sigma_y \) and centered differences:

\[ \frac{\sigma_u}{u} \sim \frac{L}{h} \frac{\sigma_x}{B_x} \quad \text{and} \quad \frac{\sigma_v}{v} \sim \frac{L}{h} \frac{\sigma_y}{B_y} \]

(2)

\( \bar{B}_i \) are characteristic values

\( h \) is grid spacing, \( L \) scale for field variation

the \( J_z \) estimates may be substantially uncertain

Field errors \( \sigma_x, \sigma_y \) are assigned based on fitting in inversion 
(e.g. SDO/HMI: Hoeksema et al. 2014)

these errors are likely to be underestimates

but they are useful indicators of the reliability of the \( J_z \) values
Nonlinear force-free fields (NLFFFs) are used as models:

\( \text{curl } \mathbf{B} = \alpha \mathbf{B} \quad \text{and} \quad \text{div } \mathbf{B} = 0 \) \hspace{1cm} (3)

where \( \alpha \) is the force-free parameter

BCs for Grad-Rubin (G-R) solution of Eqs. (3) in a half space:

\( B_z + \alpha = J_z / B_z \) over one polarity of \( B_z \)

the BCs are provided by vector magnetograms

In practice vector magnetogram BCs are changed

substantial change is needed to obtain a NLFFF solution

large (spurious?) \( \alpha \) values cause problems with methods

the changes introduced greatly exceed the uncertainties

e.g. ‘preprocessing’: changes of \( \lesssim 500 \) gauss \hspace{1cm} (Fuhrmann et al. 2011)

censoring of current densities in G-R methods: \( \langle \Delta B_h \rangle \approx 9\sigma_h \) \hspace{1cm} (Wheatland & Leka 2011)

changes due by errors and inconsistency of BCs with model

magnetic field unlikely to be force-free at the photosphere
Method: A minimum current estimate for $J_z$

- We want to estimate $J_z$ taking uncertainties into account
  - whilst avoiding very large values of $J_z$

- Consider the problem of minimizing $F$ wrt $u_{ij}$ and $v_{ij}$:

$$F = \sum_{ij} \frac{(u_{ij} - u_{ij}^{\text{est}})^2}{2\sigma_{u_{ij}}^2} + \frac{(v_{ij} - v_{ij}^{\text{est}})^2}{2\sigma_{v_{ij}}^2} + \lambda \sum_{ij} J_{z_{ij}}^2$$  \(4\)

  - where ‘est’ refers to observational estimates
    - e.g. obtained by differencing vector magnetogram values
  - the sums are over points in the magnetogram
  - $\lambda$ is a Lagrange multiplier

- Non-dimensional version ($B_s, L_s$ are characteristic values):

$$F = \sum_{ij} \frac{(u_{ij} - u_{ij}^{\text{est}})^2}{2\sigma_{u_{ij}}^2} + \frac{(v_{ij} - v_{ij}^{\text{est}})^2}{2\sigma_{v_{ij}}^2} + \Lambda \sum_{ij} (v_{ij} - u_{ij})^2$$  \(5\)

  - with $\Lambda = \lambda B_s^2/\mu_0 L_s^2$
The solution \((u_{ij}^{\text{min}}, v_{ij}^{\text{min}})\) corresponds to

\[
J_{z \ ij}^{\text{min}} = u_{ij}^{\text{min}} - v_{ij}^{\text{min}}
\]

\[
= \frac{J_{z \ ij}^{\text{est}}}{1 + 2\Lambda \sigma_{J \ ij}^2}
\]

where \(J_{z \ ij}^{\text{est}} = v_{ij}^{\text{est}} - u_{ij}^{\text{est}}\)

and \(\sigma_{J \ ij}^2 = \sigma_{u \ ij}^2 + \sigma_{v \ ij}^2\)

so \(J_z\) is reduced from the observational estimate by a factor

\[
f_{ij} = (1 + 2\Lambda \sigma_{J \ ij}^2)^{-1}
\]

more uncertain values are reduced more

we call Eq. (6) the ‘minimum current’ estimate for \(J_z\)

Using centered differences for \(u_{ij}^{\text{est}}, v_{ij}^{\text{est}}\) the error in \(J_z\) is

\[
\sigma_{J \ ij} = \frac{1}{\sqrt{2h}} \left( \sigma_{x \ ij+1}^2 + \sigma_{y \ ij-1}^2 + \sigma_{y \ i+1j}^2 + \sigma_{y \ i-1j}^2 \right)^{1/2}
\]
To demonstrate the method we need a test field

- a NLFFF is calculated from analytic $B_z$ and $\alpha$
  - a bipolar $B_z$ distribution with $\alpha \neq 0$ around the positive pole
  - calculated using the cfit code (Wheatland 2007)

- the boundary values of $\mathbf{B}$ for this NLFFF are used
BCs from NLFFF: L to R shows $B_z$, $B_h = (B_x^2 + B_y^2)^{1/2}$ and $J_z$

Gaussian noise is added to $B_x$ and $B_y$ at 10% of points
  - noise amplitude is $\sigma_x = \sigma_y = 0.25$
  - the $B_z$ values are unchanged

The minimum current method is then applied
  - $J_{zij}^{\text{est}}$ values (which are noisy) are obtained by differencing
  - minimum current estimates $J_{zij}^{\text{min}}$ are made
    - using $\sigma_{xij} = \sigma_{yij} = 0.25$ at points with noise
Test using simulated data: Minimum current estimate

- L to R shows $B_h$, $J_{z}^{\text{est}}$ and $J_{z}^{\text{min}}$
  - the Lagrange multiplier is $\Lambda = 5$
  - this choice produces small noise values in weak field regions

- The minimum current method recovers the values of $J_z$
  - except at locations affected by noise
- NLFFF reconstructions were made from the BCs
  - the cfit code fails using $B_z$ and $J_{z}^{\text{est}}$ as BCs
  - noise in the $\alpha$ estimates prevents G-R convergence
Test using simulated data: NLFFF reconstructions

- LH panel shows the original NLFFF (cropped)
  - the energy is $E/E_0 = 1.083$
- RH panel shows the P and N solutions using $J_{zij}^{\text{min}}$ values
  - P/N solution field lines are white/black
  - the G-R method converges
  - the solution energies are $E_P/E_0 = 1.064$ and $E_N/E_0 = 1.048$
    - reduced due to the smaller values of $J_z$
The ‘self-consistency’ method was also applied (Wheatland & Régnier 2007)

- this method produces a single NLFFF solution
  - involving a combination of P- and N-solution $\alpha$ maps
- so it uses information at both polarities
- and takes into account uncertainties in $\alpha$

**One self consistency cycle**
Convergence of self-consistency procedure

![Graph showing the convergence of a self-consistency procedure with iteration number on the x-axis and E/E₀ on the y-axis.](twin_monopole_jmin06_2)
- The LH panel below shows the resulting P and N solutions which are very similar (self-consistency is achieved)
  - the energies are $E_P/E_0 = 1.085$ and $E_N/E_0 = 1.086$
- The RH panel shows the boundary values of $J_z$
A new method is presented for estimating $J_z$ in active regions which alters the field gradients subject to uncertainties and minimizes $J_z^2$ across the magnetogram.

The resulting value for $J_z$ reduces the observational estimate by a factor $1 + \Lambda \sigma_J^2$, where $\sigma_J$ is the uncertainty in $J_z$.

So the method preserves $J_z$ values where they are well known.

Method is demonstrated on BCs from a NLFFF plus noise, a good approximation to the original $J_z$ distribution is obtained.

The method enables NLFFF reconstructions, the P and N solutions have reduced energies, the free energy is reduced by 25% and 40%.

A self-consistent NLFFF reconstruction is also shown, which recovers the free energy within 5%.

The method will be applied to vector magnetogram data.