Aharonov–Bohm effect, Center Monopoles and Center Vortices in $SU(2)$ Lattice Gluodynamics

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$SU(2)$ gluodynamics is investigated numerically and analytically in the (Indirect) Maximal Center gauge at finite temperature. The center vortices are shown to be condensed in the confinement phase and dilute in the deconfinement phase. A new physical object, center monopole, is constructed. We show that the center monopole condensate is the order parameter of deconfinement phase transition. The linking of the vortex worldsheets and quark trajectories is identified with the Aharonov–Bohm interaction in an effective Abelian Higgs theory. We conclude that the confinement in the Maximal Center gauge can be explained by a new mechanism called "the real superconductor mechanism".

1. Introduction

The investigation of the confinement in $SU(N)$ gauge theories is mainly based on a partial gauge fixing of the nonabelian gauge group up to its abelian subgroup. The popular abelian gauge, the so called "Maximal Center gauge", is proposed in Ref. [1]. In this gauge the nonabelian gauge group is fixed up to its center subgroup. In a center gauge the $SU(N)$ gauge theory is reduced to a $\mathbb{Z}_N$ gauge theory which contains vortex strings as topological defects. Lattice calculations [2] in the Maximal Center gauge show that the dynamics of these defects plays an important role in the color confinement. Below we study the central vortices and new topological defects, "center monopoles": and we discuss confinement mechanism in the Maximal Center gauge.

2. Center Vortices and Center Monopoles in the Maximal Center Projection

We study $SU(2)$ gluodynamics with the standard Wilson action. The Maximal Center gauge makes the link matrices $U$ as close to the center elements of $SU(2)$ group $(\pm I)$ as possible. This gauge is defined as follows [3]: first we fix the Maximal Abelian gauge by maximizing the field functional $\sum l \text{Tr}(U_l \sigma^3 U_l^* \sigma^3)$, over gauge transformations $U_{x,\bar{x}} = \Omega_x^\mu U_{x,\mu} \Omega_{x+\bar{x}}$, the summation is taken over links $l$ of the lattice, $\sigma^a$ are the Pauli matrices. Second we maximize the functional $\sum l \cos^2 \arg((U_l)^{11})$ over residual $U(1)$ gauge transformations, this fixing makes the link matrices close to the central elements $\pm I$.

The center vortices are defined as follows [1]. After fixing the Maximal Center gauge we define the $\mathbb{Z}_2$ plaquette variables $\sigma_P$:

$$\sigma_P \equiv (dn)_P = n_1 + n_2 - n_3 - n_4,$$

where links 1, $\ldots$, 4 form the boundary of the plaquette $P$ and $n_i = \text{sign(Tr}U_l\text{)}$. The worldsheets of the center vortex are defined on the dual lattice as a collection $^*\sigma$ of the plaquettes dual to the nonzero plaquettes $\sigma_P$, the worldsheets $^*\sigma$ are closed on the dual lattice ($^*\sigma = 0$).

The interaction of the center vortices with the Wilson loop is topological. To see this we represent the $SU(2)$ gauge field $U_l$ in the Maximal Center gauge as a product of the $\mathbb{Z}_2$ variable $\exp\{i\pi n_l\}$, $n_l = 0, 1$, and the $SU(2)/\mathbb{Z}_2$ variable $V_l$, $\text{Tr}V_l \geq 0$: $U_l = \exp\{i\pi n_l\} \cdot V_l$. Taking into account [1] we rewrite the Wilson loop for the contour $C$ as:

$$W_C = \text{Tr} \prod_{l \in C} U_l = \exp\{i\pi L(C, \sigma)\} \text{Tr} \prod_{l \in C} V_l,$$

where $L(C, \sigma)$ is the linking number of the quark trajectory $C$ with the string worldsheets $^*\sigma$ [1].

$$L(C, \sigma) = (\sigma, m[C]) = (\sigma, \Delta^{-1}dC),$$
The monopole trajectories are defined as follows:

\[ j = \frac{1}{2} \text{d} \left[ (dn) \mod 2 \right] = \frac{1}{2} \text{d} \left[ \sigma \mod 2 \right]. \tag{4} \]

The \( \mathbb{Z}_2 \) charge of the center monopole is conserved, the monopole trajectories are closed: \( \delta^* j = 0 \).

The important dynamical property of the center monopoles is the percolation probability \( C_{\text{mon}} \) which is defined as a probability for two different points of the lattice to be connected by the same center monopole trajectory \( C \). We observe, that \( C_{\text{mon}} \) vanishes in the deconfinement phase, and is non-zero in the confinement phase \( (C_{\text{mon}} \text{ is shown by boxes in Figure } 1(a)) \). We conclude that the confinement phase transition is accompanied by the condensation of the center monopoles. The monopoles are dual abelian degrees of freedom and their condensation means that the confinement phase corresponds to the dual superconductor phase for the Maximal Center gauge \[3\]. On the other hand, in the next Section we show that the confinement in the Maximal Center gauge might be explained by a different mechanism.

3. Real Superconductor Mechanism in the Maximal Center gauge

The partition function of the Maximal Center gauge is:

\[ Z_{g.f} = \int DU e^{-\beta \sum_p (1 - \frac{1}{2} \text{Tr} U_p) - S_{g.f}[U]}, \tag{5} \]

where the action \( S_{g.f} \) includes the Faddeev–Popov determinant and gauge fixing functionals.

The action \( S_{g.f}[U] \) is invariant under the transformations \( U \to -U \), therefore \( S_{g.f}[U] = S_{g.f}[V] \), where \( U = e^{i\pi n} V, \text{Tr} V > 0, n = 0, 1 \). Using this property the quantum average of the Wilson loop can be represented as follows:

\[ < W_C > = \frac{1}{Z_{g.f}} \int \text{Tr} V > 0 \text{ e}^{-S(V)} \text{Tr} \prod_{l \in C} V_l \sum_{\sigma(\mathbb{C}) \in \mathbb{Z}_2} e^{-\beta \sum_p \text{Tr} V_p - (1 - \cos(\pi \sigma_p)) + i\pi L(C, \sigma)} \tag{6} \]

To derive eq. (6) we used the definition of the vortex strings \[3\] and the representation for the Wilson loop \[3\].

The interaction proportional to linking number of the world sheet \( \ast \sigma \) and test particle world trajectory \( C \) is already known in the field theory \[3\]. This is the Aharonov–Bohm (AB) interaction of the test particle which scatters on the string carrying a magnetic flux. Below we show how to rewrite the considered theory in terms of the Abelian Higgs theory. The world sheets of the Abrikosov–Nielsen–Olesen (ANO) vortex strings \[3\] in this theory corresponds to variables \( \ast \sigma \) (center vortices).

The expectation value for the Wilson loop \[3\] can be represented as:

\[ < W_C > = \frac{1}{Z_{g.f}} \int \text{Tr} V > 0 \text{ e}^{-S(V)} \text{Tr} \prod_{l \in C} V_l \text{Tr} \prod_{l \in C} V_l \text{Tr} \prod_{l \in C} V_l \text{Tr} \prod_{l \in C} V_l \]
The fractal dimension $D_{\text{vort}}$ is defined as

$$D_{\text{vort}} = \lim_{\kappa \to \infty} \frac{\ln \frac{L}{\ell}}{\ln \kappa},$$

where the effective abelian action is

$$S(\theta, \varphi; V) = \beta \sum_{\rho} \text{Tr} V_\rho \cdot (1 - \cos (d\theta)_{\rho})$$

$$+ \kappa \sum_{l} (1 - \cos (d\varphi + 2\theta)).$$

Here $\theta$ is the compact abelian gauge field, $\varphi$ is the phase of the Higgs field $\Phi = |\Phi| e^{i\varphi}$ and the radial part of the Higgs field in the effective abelian theory is frozen (this corresponds to the London limit of the theory). The Higgs field carries the double charge. The couplings in the effective abelian theory fluctuate due to external integration over the field $V$.

To prove that (7) is equivalent to (6) we have to fix the unitary gauge, $d\varphi = 0$, and note that in the limit $\kappa \to \infty$ the integral over the variable $\theta$ is reduced to the sum over $\pi n$, $n = 0, 1$. The ANO vortex strings are defined as $\sigma = dn$.

The mechanism of confinement in the AHM representation (7) is as follows. The confinement phase of gluodynamics corresponds to the Coulomb phase of the effective AHM (7) since in the Coulomb phase the ANO vortices are condensed. The abelian Higgs field carries the electric charge $e = 2$, therefore the vortices carry the magnetic flux $2\pi e = \pi$. Since the ANO strings carry non-trivial flux they interact with the electrically charged particles (quarks) via AB effect: the interaction is proportional to the linking number of the vortex worldsheets with the world trajectory of the charged particle (3). According to numerical calculations (4), this topological interaction reproduces the $SU(2)$ string tension. We call the described mechanism of confinement as ”real superconductor mechanism” since in this picture the electrically charged particles are to be condensed in order to provide the suppression of the vortex strings in the deconfinement phase.

One may expect that the the AB interaction is strong in the confinement phase since the network of the ANO strings is percolating. The percolation probability for vortices $C_{\text{vort}}$ is defined similarly to that for center monopoles (see previous Section). We show the quantity $C_{\text{vort}}$ vs.

$\beta$ on $16^3 \times 4$ lattice in Figure 1(a) by circles. It is clearly seen that in the confinement phase the vortex strings are percolating with the maximal probability $C_{\text{vort}} = 1$. The quantity $1 - C_{\text{vort}}$ is an order parameter for the phase transition.

In Figure 1(b) we show the fractal dimension of the vortex string network, $D = 1 + 2A/L$ on $12^3 \times 8$ lattice Here $A$ is the number of plaquettes and $L$ is the number of links on the string. The fractal dimension $D$ is high at the confinement phase what is a characteristic feature of a percolating vortex network. In the deconfinement phase the value of this quantity is close to 2 since we have the dilute string ensemble.

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