We study the possibility of measuring the muon Yukawa couplings in s-channel Higgs boson production at a muon collider with transversely polarized beams. We investigate sensitivity to the relative size of the CP-odd and CP-even muon Yukawa couplings. Provided the event rate observed justifies the operation of the $\mu^+\mu^-$ Higgs boson factory, we find that 40% polarization for both beams is sufficient to resolve the CP nature of a single resonance as well as disentangle it from two overlapping CP conserving resonances.
1 Introduction

In spite of the fact that the Standard Model (SM) of electroweak interactions has been tested with very high precision, its scalar sector still evades experimental confirmation. In particular, it is an open question if the proper theory should contain one or more physical Higgs bosons. Since the CP nature of Higgs particles is a model dependent feature its determination would not only provide information concerning the mechanism of CP violation but would also restrict possible extensions of the SM of electroweak interactions and therefore reveal the structure of fundamental interactions beyond the SM. A muon collider with transversely polarized beams is the only place where CP properties of a second generation fermion Yukawa coupling can be probed. This is the subject of our more complete analysis which is summarized in this talk. We follow the line of our previous works where we have tried to unveil the CP-nature of Higgs bosons in a model-independent way.

2 Production of Higgs Bosons

The attractive possibility of s-channel Higgs boson production at a muon collider has been discussed together with the possibility of the measurement of CP violation in the muon Yukawa couplings. The latter is based on the fact that in any muon collider design there is a natural beam polarization of the order of 20% that allows for the rare possibility of direct Higgs boson production with known polarization of the initial state particles.

The cross section for the Higgs boson resonance production, $\mu^+\mu^-\rightarrow R$, depends on the transverse $P_T^\pm$ and longitudinal $P_L^\pm$ beam polarizations and the $\bar{\mu}\mu \exp(i\delta\gamma_5)\mu$ muon Yukawa coupling in the following way:

$$\sigma_S(\zeta) = \sigma_S^0 \left[ 1 + P_L^+ P^-_L + P_T^+ P^-_T \cos(2\delta + \zeta) \right]$$

(1)

where $\zeta$ is the angle in the transverse plane between the beam polarizations and $\sigma_S^0$ is the unpolarized cross section. We stress that only the transverse polarization term is sensitive to $\delta$ of the muon Yukawa coupling. Since it is proportional to the product of the transverse polarizations, it is essential to have large $P_T^+$ and $P_T^-$, as obtained by applying stronger cuts while selecting muons from the decaying pions initially produced (which, however, causes a reduction of luminosity). To compensate, a more intensive proton source or the ability to repack muon bunches will be needed. Another speculative option, would be high, up to 50%, polarization obtained by a phase-rotation technique which would lead to less luminosity reduction.

While varying $\zeta$, one can observe a maximum at $\zeta = -2\delta$ and a minimum at $\zeta = \pi - 2\delta$. Thus, studying $\zeta$ dependence is essential for resolving the $\delta$ value. A muon collider offers the unique possibility of a setup which in a natural way provides a scan over different $\zeta$ values. We will not discuss this option here. Our results will correspond to a configuration with four fixed $\zeta$ values: 0, 90, 180 and 270 degrees. Even though this cannot be accomplished experimentally, due to the spin precession in the accelerator ring, it can be well approximated by a simple but realistic setup that yields the same results as the fixed $\zeta$ analysis at the expense of 50% luminosity increase.

Table 1: Assuming the SM with $\delta = 0$, we give 1 and 3 $\sigma$ limits (in radians) on $\delta$ for the $b\bar{b}$ final state for various luminosity and polarization configurations. Beam energy spread 0.003% and $b\bar{b}$ tagging efficiency 54% have been assumed.

| $m_R$=110 GeV | $m_R$=130 GeV |
|----------------|----------------|
| $P[\%]$ | $L[pb^{-1}]$ | 1$\sigma$ | 3$\sigma$ | 1$\sigma$ | 3$\sigma$ |
| 20 | 150 | 0.94 | – | – | – |
| 39 | 75 | 0.30 | 1.14 | 0.41 | – |
| 48 | 75 | 0.20 | 0.64 | 0.27 | 0.93 |
| 45 | 150 | 0.15 | 0.50 | 0.21 | 0.69 |

In order to illustrate the ability to reject different Higgs boson CP scenarios we can assume that the measured data is mimicked by the SM Higgs boson. For given luminosity $L$ and total polarization...
$P$ for each of the beams, we can place 1 and $3 \sigma$ limits on the $\delta$ value for the observed resonance, assuming the $\delta = 0$ SM is input. The limits for Higgs boson masses of 110 and 130 GeV are presented in Table 2. For the expected yearly luminosity of $L = 150 \text{ pb}^{-1}$, even several years of running at the natural 20% polarization would be insufficient for useful limits. However, 1$\sigma$ limits for the $P = 39\%$ option (with reduced $L$) do give a rough indication of the CP nature of the resonance. 3$\sigma$ limits in 110-130 GeV mass range require either $> 40\%$ polarization or $< 50\%$ luminosity loss. (The requirements are less stringent for a 110 GeV Higgs boson.) We stress that there is no other way the measurement of the muon Yukawa $\delta$ can be done and that operation in the transverse polarization mode should not interfere with most of the other studies. For a heavier resonance, operation of a muon collider as an $s$-channel Higgs boson factory is justified only if the branching ratio $BR(R \to \mu^+\mu^-)$ is enhanced. Then, the analysis sketched above applies as well. Such enhancement arises in the Minimal Supersymmetric Standard Model (MSSM) at large $\tan \beta$. If the pseudoscalar mass is large ($m_A > 300$ GeV), the $H$ and $A$ masses will be similar. The increasing degeneracy with increasing $\tan \beta$ is illustrated for $m_A = 400$ GeV in Fig. 1, assuming squark masses of 1 TeV and no squark mixing and a beam energy spread of $R = 0.1\%$. Since the total widths of the $H$ and $A$ are substantial ($> 1$ GeV) for the $m_A$ and $\tan \beta$ values being considered, it is not guaranteed that we will be able to separate the peaks. The figure shows that we are able to observe two separate peaks (the $A$ peak being at lower mass than the $H$ peak) for moderate $\tan \beta \lesssim 6$. But, for higher $\tan \beta$ values the peaks begin to merge; for $\tan \beta > 8$, $|m_H - m_A| < 11$ GeV and one sees only a single merged peak. The picture changes if squark mixing is substantial; for instance, for $m_A = 300$ GeV, squark masses of 1 TeV and large squark mixing ($A_t = A_b = 3$ TeV), the $H$ and $A$ peaks actually cross at $\tan \beta \sim 5$. It would be crucial to distinguish such a case from a single CP-violating Higgs boson which may appear e.g. in the MSSM. Table 3 illustrates the very distinct event number pattern as a function of $\zeta$ that would yield the needed discrimination. The event rate for any single, CP conserving or CP violating Higgs boson, has a minimum and maximum as a function of $\zeta$. In contrast, overlapping CP-even and CP-odd resonances result in a pattern independent of $\zeta$.

Table 2: Event number pattern for different Higgs models as a function of $\zeta$, assuming $P_L^\pm = 0$ and $P_T^\pm = P$; see Eq. (3).

| ($\delta$) | $\zeta = 0$ | $\zeta = \pi/2$ | $\zeta = \pi$ | $\zeta = 3\pi/2$ |
|------------|-------------|-----------------|---------------|------------------|
| (0)        | $1+P^2$     | 1               | $1-P^2$       | 1                |
| ($\pi/4$)  | 1           | $1-P^2$         | 1             | 1                |
| ($\pi/2$)  | $1-P^2$     | 1               | 1             | 1                |
| (0) + ($\pi/2$) | 1     | 1       | 1             | 1                |

In Fig. 2 we plot for $m_A = 300\text{GeV}$ $\Delta \chi^2$ obtained for the four polarization-luminosity options (I)-(IV): (I) $P = 0.2$, $L = 3.0 \text{ fb}^{-1}$, (II) $P = 0.39$, $L = 1.5 \text{ fb}^{-1}$, (III) $P = 0.48$, $L = 1.5 \text{ fb}^{-1}$, (IV) $P = 0.45$, $L = 3.0 \text{ fb}^{-1}$. We emphasize that (for $R = 0.1\%$) options (I) and (II) do not require over-design of the proton source. The $\Delta \chi^2$ plots show that good discrimination is obtained even for option (I) once $\tan \beta > 10$. Option (II) would be needed for good discrimination if $\tan \beta \sim 5$.

We have found that for simple MSSM test cases with $m_A = 300-400\text{ GeV}$ and $\tan \beta > 8$ (for which we cannot see separate resonance peaks) even natural 20% polarization will allow us to distinguish two overlapping resonances from any single one at more than the $3\sigma$ level. Higher polarization will allow for a precise measurement of the relative contribution from the CP-even and the CP-odd component.

\*To test a model A against B we introduce $\Delta \chi^2 = \Sigma_i \frac{(N_i^A-N_i^B)^2}{N_i^B}$, where $N_i^{A/B}$ denotes the number of events in the $i$th bin calculated within the model $A/B$; see Grzadkowski, Gunion and Pliszka for details. The background generated by $\gamma^*$ and $Z$ exchange is taken into account.
Figure 1: We plot $b\bar{b}$ (solid) and $t\bar{t}$ (dashed) event rates for total integrated luminosity of $L = 7 \text{ pb}^{-1}$ coming from $\mu^+\mu^- \rightarrow H + A$ as a function of $\sqrt{s}$, assuming $m_A = 400 \text{ GeV}$. Each window is for the specific $\tan\beta$ value noted. These event rates are to be multiplied by a factor of 1000 for the expected yearly integrated luminosity of 7 fb$^{-1}$ for the beam energy spread $R = 0.1\%$. We employ squark masses of 1 TeV and no squark mixing. Supersymmetric decay channels are assumed to be closed.
Figure 2: In the upper left window, we plot the $b\bar{b}$ event rates in the MSSM for the $H$ and $A$ (the $A$ rate is the larger of the two) as a function of $\tan \beta$ for $m_A = 300$ GeV, assuming squark masses of 1 TeV, no squark mixing and integrated luminosity of $L = 2 \text{ fb}^{-1}$. Also shown is the (relatively small) background rate. In the remaining windows we plot $\Delta \chi^2$, after including precession and increasing $L$ to $L = 3 \text{ fb}^{-1}$, as a function of $\tan \beta$ for three different cases in which we forcibly lower $m_H$ to 300 GeV (for exact degeneracy). (1) We adjust the $H$ and $A$ event rates so that each is exactly equal to the average of the $A$ and $H$ rates as predicted by the MSSM, and compute $\Delta \chi^2$ for $H + A$ vs. a single Higgs resonance (of any type) with the same total event rate, employing only the $b\bar{b}$ channel. (2) We use the actual $H$ event rate and compute $\Delta \chi^2$ for $H + A$ vs. a single CP-even resonance yielding the same $b\bar{b}$ event rate. (3) As in (2), but vs. a single CP-odd resonance. In cases (1)-(3), we give results as a function of $\tan \beta$ for the four polarization–luminosity situations (I)-(IV) (as labelled on the curves) described in the text.
3 Summary and Conclusions

We have presented results of a realistic study of measuring the CP properties of the muon Yukawa couplings in Higgs boson production at a muon collider with transversely polarized beams. We have found that transverse polarization is essential for determining the CP nature of the muon Yukawa couplings. In particular, a collider with $P \approx 40\%$ and at least 50\% of the original luminosity retained will ensure that the CP nature of the produced scalar resonance will be revealed.

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