The Mathematic Model Study of Optimum Align Method in Compass

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Abstract. In this paper, based on the study of the characteristics of horizontal well designed by optimum align method in compass software, we speculated that, the optimum align method is one kind of space circular arc method. Therefore, we built a horizontal well track design model based on spatial arc curve optimization alignment hypothesis. The orbit design problem of horizontal wells is transformed into the problem solving of multiple linear equations with trigonometric functions. If the kick off point, the target point and the dog leg degree of the first and second inclined sections are determined, the solution of the track parameters of the horizontal well is unique, and the track parameters of the horizontal well can be calculated accurately. The model result is the same as the optimum align method in compass program. Therefore, the algorithm essence of Compass Optimum Align design method is to solve the problem of multiple linear equations based on the optimal alignment of arc curves in space.

1. Introduction
In the Landmark Drilling Engineering design system, Compass is the kernel software for well-path design and analysis. It is also the most widely used well path design and anti-collision analysis software in modern drilling industry all around the world. In the well path design of horizontal well, Compass has ordered s-well, Build Turn, Dogleg Tool-face, and Optimum Align methods, etc. Among them, the Optimum Align method is the most widely used design method in the track design of horizontal Wells [1]. Drilling designers only need to input the predetermined dogleg degree of the first and second inclined section, and the software can calculate the length, inclination, azimuth and other parameters of different sections, so as to realize the automatic alignment of inclined section and horizontal section, which is not only convenient for operation, but also welcomed by the vast number of drilling designers. But the software algorithm is completely confidential, so the designer knows nothing about the programmed algorithm, which not only limits the design optimization, and it is also unfavorable to the directional technical staff's understanding of path design and field controlling. For these reasons, in this paper, we will introduce the Compass Optimum Align method study process and our algorithm design by us, and a practical model is verified. So, it is very useful for the well designers and drilling directional engineers.

2. mathematical model predicts of the optimal align method
The most crucial problem of path design for horizontal well is the track design method of inclined section. There are two kinds of track design methods in inclined well section, which are cylindrical spiral curve and space arc curve.
For the cylindrical helical curve method, a relatively constant tool face angle is used in the inclined section to complete the azimuth torsional and deflection operation, and the azimuth torsion rate is always constant regardless of the size of the inclination. This design method, once the oil reservoir is earlier than expected, the azimuthal angle may be very insufficient, so it would be very hard to reach the horizontal target accurately [2].

For the space arc curve method, only the curvature of inclined section (also known as the dogleg degree) is required to be constant, while the tool face angle of is not constant, so the space arc curve has the advantage of minimum curvature and shortest well segment. As long as a relatively constant curvature is maintained and tool surface angle is adjusted timely according to well inclination and azimuth trend, the directional operation can be completed very conveniently [3-5].

The optimization Align method divides the horizontal section into three parts: the first deflecting section, the stable slope section, and the second deflecting section, in which the and inclination and azimuth are changed. Users only need to preset the first and second dogleg degree, and the software can calculate the corresponding path parameters. The result can be output by it. The dogleg degree of the first section is the same to building rate. The second section’s dogleg degree is also constant, but the building rate and azimuth torsion rate are not constant. So the optimization Align method fits in well with the characteristics of the spatial arc curve method. According to that, we tentatively assume that, the Optimum Align algorithm is based on the space arc curve method.

3. Horizontal well path design model
Based on the assumption that the Optimum Align might be the method of spatial arc curve, the spatial model of horizontal well design in three-dimensional space is established. As we show in “Fig. 1”, the X-axis is the east-west coordinates of the horizontal plane, positive to the east and negative to the west. The Y-axis is the north-south coordinate of the horizontal plane, marked by +N (north) and -S (south), positive to the north and negative to the south. Z-axis is the vertical depth and downward is positive.

![Fig. 1: Horizontal Well Path Design Model](image)

3.1. The Model Parameters
O, Wellhead, the inclination angle is 0°, no azimuth.
Z, Kickoff point, the inclination angle is 0°, no azimuth.
C. The end of the first built section, also is the starting point of the stationary section.
D. The end of the stabilized section, also is the starting point of the second built section.
A. The end of the second built section, also is the starting point of horizontal segment.
B. End of horizontal segment.

Δ N: The north-south coordinate increment, m
Δ E: The east-west coordinate increment, m
Δ D: Vertical depth increment, m
D: Vertical depth, m
L: Well depth or length, m
ΔL: Depth increment, m
α: Inclination, (°)
φ: Azimuth, (°)
γ: Borehole Curvature (°) / 30 m
R: Radius of Curvature, m
a, b, c, d, z, o: Subscript of A, B, C, D, Z, O

3.2. The well path design steps

According to the above model, the point Z is an kickoff point, which belongs to known conditions. Points A and B are targets of horizontal well, so the coordinates and vertical depth are known conditions. The arc ZC segment’s length $L_{zc}$ is unknown, DA arc segment’s length $L_{da}$ is unknown, CD is the stationary section, its length $L_{cd}$ and azimuth $\phi_{cd}$ are unknown. Therefore, the path design becomes a process to solve the length and azimuth of arc ZC, DA, and line CD when the kickoff point, target, and the full Angle change rate (dog leg degree) of the first and second built are known. The number of unknown parameters is four.

1) **First step:** Based on the known vertical depth of A and B, calculate the horizontal inclination.

\[ \alpha = \frac{D_b-D_a}{\sqrt{(D_h-D_a)^2+(N_b-N_a)^2+(E_b-E_a)^2}} \]  

(1)

The azimuth is calculated according to the coordinate values of point A and B. The azimuth function is evaluated by the arctangent function.

\[ \phi = \arctan \left( \frac{E_b-E_a}{N_b-N_a} \right) \quad \text{when } E_b > E_a \]  

(2)

\[ \phi = 2\pi + \arctan \left( \frac{E_b-E_a}{N_b-N_a} \right) \quad \text{when } E_b \leq E_a \]  

(3)

When $N_b=N_a$ and $E_b>E_a$, $\phi=90^\circ$, otherwise $\phi=270^\circ$.

2) **Step 2:** Calculate the distance before the target and set the kickoff point. The curvature without the stationary segment can be estimated according to that. When estimating the curvature, it should be assumed that there is an ideal space arc curve from the kick point to the horizontal segment Point A (The value is only preliminarily estimated, so it is not accurate but has very important reference significance). The ideal space arc curve dogleg will be:

\[ \gamma_0 = \sqrt{\Delta\alpha^2 + \Delta\phi^2 (\sin \frac{\phi}{2})^2} \]  

(4)

Then, the rate of total angular change (dog leg degree) of the spatial curve is estimated as:

\[ \gamma = \frac{6\gamma_0}{\pi(E_b^2+N_b^2)^{0.5}} \]  

(5)

**Step 3:** Try to calculate the dogleg of curvature 1 and 2. $\gamma_1$ and $\gamma_2$ can be modified by cut-and-try method. Since the dogleg of ideal space arc do not take into account of the influence of the stationary segment, surplus the target distance algorithm is only an approximate solution, the well $\gamma_1$ and $\gamma_2$ can be preliminarily set to be slightly larger than the ideal $\gamma$.

On the basis of general experience, when it is necessary to lower the kickoff point, the $\gamma_1$ can be slightly greater than $\gamma_2$. Then, the dogleg of the first arc segment (ZC) and the second segment (DA) would be:

\[ \gamma_1 = \gamma + 2 \]  

(6)
\[ \gamma_2 = \gamma + 0.5 \] (7)

If you need to make a common double-increment path, the \( \gamma_1 \) should be lower than \( \gamma_2 \), if not, \( \gamma_1 \) can be a little larger than \( \gamma_2 \).

\[ \gamma_1 = \gamma + 0.5 \] (8)

1. \( \gamma_2 = \gamma + 1.5 \) (9)

\[ \gamma_{zc} = \gamma_1 = \arccos \left[ \cos \alpha_{oz} \cos \alpha_{cd} + \sin \alpha_{oz} \sin \alpha_{cd} \cos (\varphi_{cd} - \varphi_{oz}) \right] \] (10)

\[ \gamma_{da} = \gamma_2 = \arccos \left[ \cos \alpha_{cd} \cos \alpha_{da} + \sin \alpha_{cd} \sin \alpha_{da} \cos (\varphi - \varphi_{cd}) \right] \] (11)

Step 4: Calculate the coordinate increments of different well segments. The vertical depth increment \( \Delta D \) of the first built section, north-south coordinate increment \( \Delta N \) and east-west coordinate increment \( \Delta E \) of the arc ZC segment are:

\[ \Delta D_{zc} = \frac{5400}{\pi \gamma_1} \tan \frac{\gamma_{zc}}{2} (\cos \alpha_{oz} + \cos \alpha_{cd}) \] (12)

\[ \Delta N_{zc} = \frac{5400}{\pi \gamma_1} \tan \frac{\gamma_{zc}}{2} (\sin \alpha_{oz} \cos \alpha_{oz} + \sin \alpha_{cd} \cos \alpha_{cd}) \] (13)

\[ \Delta E_{zc} = \frac{5400}{\pi \gamma_1} \tan \frac{\gamma_{zc}}{2} (\sin \alpha_{oz} \cos \varphi_{oz} + \sin \alpha_{cd} \sin \varphi_{cd}) \] (14)

In that formula:

\[ \alpha_{oz} = 0 \quad \alpha_{cd} = \frac{180}{\pi} \frac{L_{zc}}{R_1} \quad R_1 = \frac{5400}{\pi \gamma_1} \] (15)

Step 5: The increments of the vertical depth of the arc segment DA, the north-south coordinate and the east-west coordinate are:

\[ \Delta D_{da} = \frac{5400}{\pi \gamma_2} \tan \frac{\gamma_{zc}}{2} (\cos \alpha_{cd} + \cos \alpha) \] (16)

\[ \Delta N_{da} = \frac{5400}{\pi \gamma_2} \tan \frac{\gamma_{zc}}{2} (\sin \alpha_{cd} \cos \varphi_{cd} + \sin \alpha \cos \varphi) \] (17)

\[ \Delta E_{da} = \frac{5400}{\pi \gamma_2} \tan \frac{\gamma_{zc}}{2} (\sin \alpha_{cd} \cos \alpha_{cd} + \sin \alpha \sin \varphi) \] (18)

Step 6: The vertical depth increments, the east-west coordinate increment and the south-north coordinate increments from wellhead to point A are known values, they are:

\[ \Delta D_{zc} + \Delta L_{cd} \cos \alpha_{cd} + \Delta D_{da} = |D_a - D_z| \] (19)

\[ \Delta N_{zc} + \Delta L_{cd} \sin \alpha_{cd} \cos \varphi_{cd} + \Delta N_{da} = |N_a - N_z| \] (20)

\[ \Delta E_{zc} + \Delta L_{cd} \sin \alpha_{cd} \sin \varphi_{cd} + \Delta E_{da} = |E_a - E_z| \] (21)

Step 7: Combine the equations (10), (11), (19), (20) and (21). There are only four unknown parameters in the equations. With five equations, Lzc, Lcd, Lda and CD can be solved by equation solving.

4. EXAMPLE VERIFICATION

In order to verify the consistency of the above algorithm with the Optimum Align algorithm, it is assumed that the wellhead and target parameters and requirements of a horizontal well design as follows:

| Target | Coordinate | Layer | Depth (m) |
|--------|------------|-------|-----------|
|        | +N,-S      | +E,-W | Altitude  | TVD       |
| Well Head |            |       | 0         | 0         |
| A      | 200        | 300   | C6        | -580      | 580      |
| B      | 600        | 600   | C6        | -580      | 580      |

It is assumed that the depth of surface casing is 200 m, and the position of the kickoff point is 250 m.

1) First step: Based on the known vertical depth of A and B, calculate the horizontal inclination \( \alpha \).

\[ \alpha = \arctan \frac{D_{b} - D_{a}}{\sqrt{(D_{b} - D_{a})^2 + (N_{b} - N_{a})^2 + (E_{b} - E_{a})^2}} = \arctan \frac{580 - 580}{500} = 90^\circ \] (22)

To calculate the azimuth of the horizontal segment, the arctangent function is used to evaluate the azimuth:
\[ \varphi = \arctan \frac{E_B - E_A}{N_B - N_A} = \arctan \frac{600 - 200}{600 - 300} = 63.51^\circ \]  

Step 2: Set the kickoff point at 250 m, and calculate the target distance (the horizontal displacement from the wellhead to Point A) as follows:

\[ S_a = \sqrt{200^2 + 300^2} = 360.55 \]  

The full angle variation value of the space arc curve is:

\[ \gamma_a = \sqrt{\Delta \alpha^2 + \Delta \varphi^2} \left( \sin \frac{\alpha}{2} \right)^2 = \sqrt{90^2 + 63.51^2} \left( \sin \frac{90}{2} \right) = 104.65^\circ \]  

Then, the total angle change rate (dogleg degree) of the space curve is estimated as:

\[ \gamma = \frac{60 \gamma_a}{\pi (E_a^2 + N_a^2)^{0.5}} \approx 60 \times 104.65 \times \frac{3.142 \times (200^2 + 300^2)^{0.5}}{5.54^2/30} = 5.54^\circ/30m \]  

Step 3: Since the horizontal section is very shallow, it is necessary to depress the kickoff point, so \( \gamma_1 \) is slightly greater than \( \gamma_2 \), that is:

\[ \gamma_1 = (\gamma + 1.5) = 6.04^\circ/30m \]
\[ \gamma_2 = (\gamma + 0.5) = 5.54^\circ/30m \]

The total angular variation of the first and second built section are as follows:

\[ \gamma_1 = \gamma_1 + \gamma_2 = 11.58^\circ/30m \]
\[ \gamma_2 = \gamma_2 + \gamma_2 = 11.08^\circ/30m \]

Another equation can be established as:

\[ \cos \left( \frac{5.5L_{da}}{30} \right) = \arccos \left( \cos 0^\circ \cos \frac{6L_{zc}}{30} + \sin 0^\circ \sin \frac{6L_{zc}}{30} \cos (\varphi_{zc} - 90^\circ) \right) \]  

The dogleg of second built section is:

\[ \cos \left( \frac{5.5L_{da}}{30} \right) = \arccos \left( \cos \frac{6L_{zc}}{30} \cos 90^\circ + \sin \frac{6L_{zc}}{30} \sin 90^\circ \cos (53.13 - \varphi_{zd}) \right) \]

It can be simplified as:

\[ \cos \left( \frac{5.5L_{da}}{30} \right) = \arccos \left( \cos \frac{6L_{zc}}{30} \right) \]  

Step 4: Establish the coordinate increment equation of the first built segment. The vertical depth, north-south coordinate and East-west coordinate of the arc segment ZC are as follows:

\[ \Delta D_{zc} = \frac{5400}{\pi + 5.5} \tan \left( \frac{1}{2} \cos \frac{6L_{zc}}{30} \right) \times \left( 1 + \cos \frac{6L_{zc}}{30} \right) \]
\[ \Delta N_{zc} = \frac{5400}{\pi + 5.5} \tan \left( \frac{1}{2} \sin \frac{6L_{zc}}{30} \right) \times \left( \sin \frac{6L_{zc}}{30} \cos \varphi_{zd} \right) \]
\[ \Delta E_{zc} = \frac{5400}{\pi + 5.5} \tan \left( \frac{1}{2} \sin \frac{6L_{zc}}{30} \right) \times \left( \sin \frac{6L_{zc}}{30} \sin \varphi_{zd} \right) \]

Step 5: The increment of the vertical depth, the north-south coordinate and the east-west coordinate E is:

\[ \Delta D_{da} = \frac{5400}{\pi + 5.5} \tan \left( \frac{1}{2} \cos \frac{5.5L_{da}}{30} \right) \cos \frac{6L_{zc}}{30} \]
\[ \Delta N_{da} = \frac{5400}{\pi + 5.5} \tan \left( \frac{1}{2} \sin \frac{5.5L_{da}}{30} \right) \times \left( \sin \frac{6L_{zc}}{30} \cos \varphi_{zd} + \frac{3}{5} \right) \]
\[ \Delta E_{da} = \frac{5400}{\pi + 5.5} \tan \left( \frac{1}{2} \sin \frac{5.5L_{da}}{30} \right) \times \left( \sin \frac{6L_{zc}}{30} \sin \varphi_{zd} + \frac{4}{5} \right) \]

Step 6: The vertical depth increment, the east-west coordinate increment, and the south-north coordinate increment from the wellhead to Point A are known values, they are:

\[ \Delta D_{cd} \]
\[ \Delta N_{cd} \]
\[ \Delta E_{cd} \]

\[ \varphi = \arctan \frac{E_B - E_A}{N_B - N_A} = \arctan \frac{600 - 200}{600 - 300} = 63.51^\circ \]  

\[ S_a = \sqrt{200^2 + 300^2} = 360.55 \]  

\[ \gamma_a = \sqrt{\Delta \alpha^2 + \Delta \varphi^2} \left( \sin \frac{\alpha}{2} \right)^2 = \sqrt{90^2 + 63.51^2} \left( \sin \frac{90}{2} \right) = 104.65^\circ \]  

\[ \gamma = \frac{60 \gamma_a}{\pi (E_a^2 + N_a^2)^{0.5}} \approx 60 \times 104.65 \times \frac{3.142 \times (200^2 + 300^2)^{0.5}}{5.54^2/30} = 5.54^\circ/30m \]  

\[ \gamma_1 = (\gamma + 1.5) = 6.04^\circ/30m \]
\[ \gamma_2 = (\gamma + 0.5) = 5.54^\circ/30m \]

\[ \gamma_1 = \gamma_1 + \gamma_2 = 11.58^\circ/30m \]
\[ \gamma_2 = \gamma_2 + \gamma_2 = 11.08^\circ/30m \]

\[ \cos \left( \frac{5.5L_{da}}{30} \right) = \arccos \left( \cos 0^\circ \cos \frac{6L_{zc}}{30} + \sin 0^\circ \sin \frac{6L_{zc}}{30} \cos (\varphi_{zc} - 90^\circ) \right) \]

\[ \cos \left( \frac{5.5L_{da}}{30} \right) = \arccos \left( \cos \frac{6L_{zc}}{30} \cos 90^\circ + \sin \frac{6L_{zc}}{30} \sin 90^\circ \cos (53.13 - \varphi_{zd}) \right) \]

It can be simplified as:

\[ \cos \left( \frac{5.5L_{da}}{30} \right) = \arccos \left( \cos \frac{6L_{zc}}{30} \right) \]  

\[ \Delta D_{zd} \]
\[ \Delta N_{zd} \]
\[ \Delta E_{zd} \]
Step 7: Combine equations of (31), (33), (40), (41), (42). There are only four unknown parameters in the equations set, they can be solved by Mathcad for Lzc, Lcd, Lda, φcd. Due to the cyclical characteristics of trigonometric functions, constraints must be added to the solution of the equation, that is:

\[ 0^\circ \leq \phi_{cd} \leq 360^\circ \]

According to the above method, if γ1=6.04°/30m, γ2=5.54°/30m, then:

\[ \Delta L_{zc}=240.62, \quad L_{cd}=30.12, \quad L_{da}=273.46, \quad \alpha_{cd}=48.44^\circ, \quad \phi_{cd}=68.65^\circ. \]

So, what about the Compass Align method? We use the same target, dogleg degree and kickoff point, the soft result is:

| Table 2 | The design parameters using compass Align method |
|---------|--------------------------------------------------|
| Table head | Depth | Inc | Az | TVD | Vsec | DLS | Build | Turn |
|           | m     | Deg | deg | m   | m    | deg/30m | deg/30m | deg/30m |
| Well head | 0     | 0   | 0   | 0   | 0    | 0       | 0       | 0       |
| Kick off  | 250   | 0   | 0   | 250 | 0    | 0       | 0       | 0       |
| First built | 490.6 | 48.4 | 68.7 | 463 | 87.8 | 6       | 6       | 0       |
| holding   | 520.7 | 48.4 | 68.7 | 482.9 | 108.4 | 0     | 0       | 0       |
| Second built(A) | 794.2 | 90 | 36.9 | 580 | 353.6 | 5.5     | 4.6     | -3.5    |
| Horizontal(B) | 1294.2 | 90 | 36.9 | 580 | 848.5 | 0       | 0       | 0       |

According to the above table:

- Length of the first built section: \( L_{zc}=490.62-250=240.62m \)
- Length of the stationary section: \( L_{cd}=520.74-490.62=30.12m \)
- Length of the second built section: \( L_{da}=490.62-250=240.62m \)
- Angle of the stationary section: \( \alpha_{cd}=48.44^\circ \)
- Azimuth of the stationary section: \( \phi_{cd}=68.65^\circ \)

The results of the design model are exactly the same as those of the Compass software.

In order to further verify the consistency of the design model with the Compass Optimization Align method, we preset in Optimum Align \( \gamma_1=7^\circ/30m \) and \( \gamma_2=5.5^\circ/30m \), the design path obtained by the software is as follows:

| Table 3 | The design parameters using compass Align method |
|---------|--------------------------------------------------|
| Table head | Depth | Inc | Az | TVD | Vsec | DLS | Build | Turn |
|           | m     | Deg | deg | m   | m    | deg/30m | deg/30m | deg/30m |
| Well head | 0     | 0   | 0   | 0   | 0    | 0       | 0       | 0       |
| Kick off  | 250   | 0   | 0   | 250 | 0    | 0       | 0       | 0       |
| First built | 424.0 | 40.6 | 72.7 | 409.8 | 52.4 | 7.0     | 7.0     | 0       |
| holding   | 474.4 | 40.6 | 72.7 | 448.1 | 81.4 | 0       | 0       | 0       |
| Second built(A) | 791.6 | 90 | 36.9 | 580 | 353.6 | 5.5     | 4.7     | -3.4    |
| Horizontal(B) | 1291.6 | 90 | 36.9 | 580 | 848.5 | 0       | 0       | 0       |

According to the above table, the Compass’s result is:

- Length of the first built section: \( L_{zc}=424.03-250 = 174.03m \)
- Length of the stationary section: \( L_{cd} = 474.43 - 424.03 = 50.4m \)
- Length of the second built section: \( L_{da} = 791.59 - 474.43 = 317.16m \)
- Angle of the stationary section: \( \alpha_{cd}=40.61^\circ \)
- Azimuth of the stationary section: \( \phi_{cd}=72.69^\circ \)

If use the algorithm of this paper, the design result is: \( \Delta L_{zc}=174.03m, \quad L_{cd}=50.40m, \quad L_{da}=317.16m, \quad \alpha_{cd}=40.61^\circ, \quad \phi_{cd}=72.69^\circ \), that two methods’ results are identical.
5. conclusions

5.1. The Characteristics of Optimum Align Method in Compass
For the horizontal well path formed by the method of optimum align, the dogleg degree in the first built segment is in line with the build rate, and the build rate in the second built segment is constant, so the azimuth turn rate is not constant. Moreover, the closer to point A, the lower azimuth turn rate was needed. Because the dogleg degree of the second built section is finite, it can be realized that in the smaller inclination section, it has larger azimuth turn rate, and in bigger inclination section has a smaller azimuth turn rate, that is very beneficial to the field track control and much easier to touch the horizontal target.

5.2. The Core Method of Optimum Align Method in Compass
The core algorithm of the optimal alignment is how to optimize alignment of arc curves in space. The first built section is a two-dimensional curve and the second segment is a spatial arc curve, in which the inclination increases and the azimuth turns. The beginning of the stationary segment is tangent to the end of the first built segment, and the end is tangent to the beginning of the second built segment.

5.3. The Technical Achievements in This Paper
In this paper, according to the study of horizontal well path design, we made the problem of horizontal well path design into a solution procedure of a system of equations with multiple variables. You just need to set the kickoff point, targets, the two dogleg degrees of that two built sections, it will have uniqueness of wellbore path parameters, and the calculation process is very simple.

Acknowledgement
Research institute of Yanchang petroleum(group)Co.ltd.

Reasons for recommendation:
In this paper, the authors had carried out a very meaningful research on the design of horizontal well path. According to the study of the algorithm of the optimal alignment method, it made us understand that, the methods to optimize alignment of arc curves in space are the cores of optimum align method in compass software. So, the core algorithm of the optimal alignment is how to optimize alignment of arc curves in space. The first built section is a two-dimensional curve and the second segment is a spatial arc curve, in which the inclination increases and the azimuth turns. The beginning of the stationary segment is tangent to the end of the first built segment, and the end is tangent to the beginning of the second built segment. Therefore, The trajectory design of horizontal wells is transformed into the problem solving of multiple linear equations with trigonometric functions. If the kickoff point, the target point and the dog leg degree of the first and second inclined sections are determined, the solution of the trajectory parameters of the horizontal well is unique, and the track parameters of the horizontal well can be calculated accurately.

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