Simultaneous Robust State Estimation, Topology Error Processing, and Outage Detection for Unbalanced Distribution Systems

Zahra Soltani, Graduate Student Member, IEEE, Shanshan Ma, Member, IEEE, Mojdeh Khorsand, Member, IEEE, and Vijay Vittal, Life Fellow, IEEE

Abstract—This paper proposes an efficient algorithmic approach that overcomes the critical challenges in real-time unbalanced distribution system state estimation, topology error processing, and outage identification simultaneously. These challenges include (1) limited locations of measurement devices and unsynchronized measurement data as well as missing and bad data, (2) complicated mixed-phase switch actions and mutual impedances and shunt admittances, and (3) the nonlinear nature of unbalanced distribution system power flow with distributed energy resources (DERs). A single snapshot mixed-integer quadratic programming (MIQP) optimization framework is proposed to cope with these challenges, which simultaneously identifies real-time network topology, estimates system state, and detects the outages via analytical constraints. An AC optimal power flow (ACOPF) approach is proposed to accurately model unbalanced distribution systems. In the proposed MIQP formulation, nonlinearities due to the complicated mixed-phase switch operations and the ACOPF approach are linearized. The effectiveness of the proposed approach is verified on an actual distribution feeder in Arizona. The results illustrate that the proposed model is robust, accurate, and computationally efficient for implementation in a distribution management system (DMS).

Index Terms—Topology error processing, distribution system state estimation, outage detection, unbalanced distribution systems, AC optimal power flow, unsynchronized measurement data, bad data, missing data.

NOMENCLATURE

Sets and Indices
\( \mathcal{J} / n, m \) Set/Index for buses.
\( \mathcal{H} / \ell \) Set/Index of lines.
\( \mathcal{H}' / \mathcal{H}'' \) Set of lines with/without switch.
\( \psi / \phi \) Set/Index of phases.

Parameters and Constants
\( Z_{\ell}^{\phi,p} / Y_{\ell}^{\phi,p} \) Impedance/Shunt admittance of line \( \ell \) between phases \( \phi \) and \( p \).
\( R_{\ell}^{\phi,p} / X_{\ell}^{\phi,p} \) Resistance/Reactance of line \( \ell \) between phases \( \phi \) and \( p \).
\( N_{LTR} \) No load loss of transformer \( r \) at phase \( \phi \).
\( M \) A large positive number.

Variables
\( I_\ell^\phi \) Current of phase \( \phi \) in line \( \ell \).
\( I_{r,\ell}^{r,m,\phi} / I_{n,\ell}^{n,m,\phi} \) Real/Imaginary part of current in line \( \ell \) and phase \( \phi \).
\( V_n^\phi \) Voltage at bus \( n \) and phase \( \phi \).
\( V_{r,n}^{r,\phi} / V_{n,n}^{n,\phi} \) Real/Imaginary part of voltage at bus \( n \) and phase \( \phi \).
\( I_{r,n}^{r,\phi} / I_{n,n}^{n,\phi} \) Real/Imaginary part of current injection at bus \( n \) and phase \( \phi \).
\( P_{G,g,\phi} / Q_{G,g,\phi} \) Active/Reactive power of DER \( g \) at phase \( \phi \).
\( d_{p,l,\phi} / d_{g,l,\phi} \) Active power of load \( l \) at phase \( \phi \).
\( Q_{c,\phi} \) Reactive power of capacitor \( c \) at phase \( \phi \).
\( u_{\ell}^\phi \) Status of switch of line \( \ell \) at phase \( \phi \).
\( \Lambda_{\ell}^\phi \) Auxiliary binary variable.

I. INTRODUCTION

The variabilities and uncertainties of the high penetration of distributed energy resources (DERs) have complicated electric distribution system operation, resulting in changes of the network topology due to frequent reverse flow and protection action [1], [2]. To monitor the distribution system operation under such conditions, an accurate real-time distribution system state estimation (DSSE) constitutes a critical component of a distribution management system (DMS) to provide inputs for voltage and power flow control [1], [3], [4]. However, current formulations of the DSSE are based on a strong assumption that the DMS has precise knowledge of the distribution system topology [3], which necessitates that the topology error processor has to provide correct topology identification information in advance. This strong assumption may lead DSSE vulnerable to topology identification errors [5]. Most switches in distribution systems have no communication links to automatically report the...
switch status back to the DMS. At the same time, the complicated mixed-phase multiple switch actions frequently triggered to satisfy DERs and protection requirements may not be detected, may generate topology errors for the DSSE, and even result in line outages to isolate parts of the network automatically. Furthermore, the limited number of micro-Phasor Measurement Units (micro-PMUs) in the current distribution system and the unsynchronized measurement data from smart meters at the customer locations cannot provide sufficient real-time information for the DMS. Therefore, it is necessary to develop an efficient real-time distribution system topology processing, state estimation, and outage detection algorithm to correctly identify the network topology configuration, detect outage sections, and estimate system states with limited real-time measurements.

Many prior efforts have attempted to study distribution system topology processing. In [6], the authors presented a time-series signature verification method to detect the distribution network topology. The method explores possible topology configurations of a given distribution network to build a signal library and projects actual voltage phasor patterns onto the corresponding library. However, this topology verification method depends on the prior information of switch status and voltage measurement from micro-PMUs and the tuning of three parameters. If errors in the prior switch data or the variation of DER outputs and load become larger, it may generate errors in the current topology identification results. A statistical inference framework was proposed in [7] to verify the single-phase distribution grid topology by exploiting the voltage covariance from smart meter data. This approach fails to consider the unbalanced switch actions or multi-phase switch actions in the distribution system. A deep-learning-based topology identification approach was developed in [8] to identify multiple multi-phase switch actions with high accuracy. However, this deep-learning-based method requires massive amounts of data to train its identifier model. In summary, [5]–[8] identify the topology under normal conditions but cannot estimate the operational topology during outage conditions.

However, the research on outage detection plays the same important role as the operational network topology identification to monitor the real-time distribution system operation. The increased DER penetration can cause new protection issues such as desensitization and unintended islanding or tripping, which may result in single or multiple outages [9]. Researchers in [10]–[12] have investigated different approaches to detect single and multiple outages instantly. But most of these approaches require topology information before an outage occurs. For instance, the outage detection algorithm presented in [12] cannot identify and locate the outages in the distribution systems if the normal operational topology of the system is unknown. This calls for a general algorithmic approach to detect outage sections and the current operational network topology simultaneously. A mixed-integer linear program (MILP) formulation was developed in [13] to dynamically identify switch malfunctions and detect outages based on a balanced power flow. Authors in [14] also developed a MILP formulation to estimate the operational topology of a three-phase unbalanced power distribution system under both outage and normal operating conditions. Authors in [15] proposed a dynamic topology awareness approach to identify all possible topology changes without real-time DG measurements using the Kullback-Leibler (KL) divergence measure. Although these methods can identify all the possible topologies under different operation conditions, they cannot identify mixed-phase switch actions and estimate the unbalanced distribution system states at the same time.

Moreover, the approaches for the DSSE include optimization-based and data-driven methods. A weighted least square (WLS)-based DSSE model was proposed in [16] for three-phase four-conductor configured unsymmetrical medium voltage distribution systems. Although [16] developed an efficient state variable reduction approach to improve DSSE computational performance, the nonlinear nature of the WLS-based DSSE problem and unbalanced three-phase power flow makes it hard to obtain the global optimal solution or even to provide an acceptable solution for a large-scale distribution system. A linear DSSE algorithm was developed in [17] using the Taylor series of voltages in the interval form and solved by interval arithmetic techniques. However, this linear DSSE algorithm does not consider the unbalanced multi-phase power flow feature of distribution systems. Also, it is assumed in [17] that the node voltage magnitude comprises only the real part of voltage and eliminates the imaginary part of voltage, which is a strong assumption for unbalanced distribution systems. In [18], the nonconvex DSSE problem was reformulated as a rank-constrained semi-definite programming problem and solved by the rank reduction approach and the convex iteration approach. Reference [19] presented a data-driven learning-based optimization approach for the WLS-based DSSE problem. This approach uses historical load and energy generation data to train a shallow neural network to learn good initialization points for the Gauss-Newton algorithm. A deep learning approach was proposed in [20] based on Bayesian state estimation for an unobservable distribution system. However, data-driven methods in [19], [20] rely on historical data and may not adapt fast to sudden network changes.

As the observability of distribution systems has been increased by using micro-PMUs, Intelligent Electronic Devices (IEDs), voltage regulators, and smart inverter of DERs [21], some research efforts have integrated the topology error processing with the DSSE. Reference [22] proposed an algorithm based on changing branch statuses sequentially and performing branch current state estimation in each case to identify distribution system topology errors. Authors in [23] parallelly run the distribution system state estimation model with possible critical network configurations using measurement data. This approach uses a recursive Bayesian approach to build a probability model and identifies the distribution network configuration with the highest probability. However, the approaches of [22], and [23] may not be efficient as both of these methods rely on solving multiple state estimation problems; this inefficiency is worse for large systems, especially when multiple switch actions in different phases may happen simultaneously. In [24], a generalized state estimation approach was extended to identify distribution system topology changes by formulating the switch statuses as continuous state variables and adding additional soft operational constraints for each switch device. This formulation
does not consider the unbalanced multi-phase switch actions in the unbalanced distribution systems.

To deal with the deficiencies of the methods mentioned above, this paper proposes a single snap-shot mixed-integer quadratic programming (MIQP) formulation to identify distribution network topology, estimate system states, and detect outages simultaneously. A nonlinear AC optimal power flow (ACOPF) approach based on current-voltage (IV) considering the mutual impedances and admittances among different phases is proposed to model the unbalanced distribution systems accurately. The proposed ACOPF based on the IV formulation is novel and different from the widely-applied unbalanced DistFlow model, which assumes three-phase voltages are nearly balanced and ignores the power losses [25]. An iterative first-order approximation of the Taylor series expansion approach is used to linearize the nonlinear power balance injection constraints in the proposed IV-based ACOPF model. The complicated mixed-phase switch actions are formulated using the big-M method, and the outage detection algorithm is formulated by analytical constraints. A data integration approach is developed to integrate unsynchronized smart meters and synchronized micro-PMUs measurements with different sampling rates. Moreover, the lack of availability of reactive power measurement data, i.e., from smart meters, is solved by adding a power factor constraint in the formulation. The robustness of the proposed algorithm is evaluated against bad measurement data. The main contributions of this paper can be summarized as follows:

- Develop an accurate novel simultaneous topology error processing, state estimation, and outage detection algorithm via detailed unbalanced distribution system modeling in an IV-based ACOPF. The proposed model provides a more accurate DSSE for the DMS. Also, the proposed model is accurate for both normal and emergency, i.e., outage, conditions. The integrated tool can enhance real-time monitoring and control of distribution systems.

- Linearize the mixed-integer nonlinear joint DSSE and topology error processing model into a MIQP optimization problem with linear constraints and convex objective function. The proposed MIQP model can be solved by a commercial solver (e.g., Gurobi and CPLEX) and obtain the global optimal solution, which avoids numerical instability and initial condition sensitivity caused by the non-linearity.

- Use all available real-time data (e.g., smart meters and micro-PMUs) while addressing the concerns associated with using unsynchronized data. The MIQP formulation can avoid multiple runs of WLS-based DSSE procedure and the off-line data training and does not rely on historical information.

- The robustness of the proposed model is evaluated against missing data and bad data in the measurements of an actual distribution feeder in Arizona.

The proposed simultaneous topology error processing, state estimation, and outage detection algorithm is developed in Section II. Section III presents a sensor data integration approach. Simulation results are illustrated in Section IV. Concluding remarks are summarized in Section V.

II. SIMULTANEOUS TOPOLOGY ERROR PROCESSING AND STATE ESTIMATION FOR UNBALANCED DISTRIBUTION NETWORKS

In this section, a new ACOPF based on the IV formulation for three-phase unbalanced distribution systems is proposed and is utilized in the simultaneous topology error processing and state estimation model. Then, the nonlinear and proposed MIQP models of the simultaneous topology processing and state estimator that can be executed in the DMS are presented.

A. The Unbalanced Distribution System ACOPF Model

Consider an unbalanced distribution network with a graph representation of $\mathcal{G}=(\mathcal{J}, \mathcal{H})$, where $\mathcal{J}$ is a set of buses and $\mathcal{H}$ is a set of distribution lines. Every bus and line in $\mathcal{G}$ can have three phases, i.e., set $\psi = \{a, b, c\}$. $\mathcal{H}$ includes a set of lines without switches (denoted by $\mathcal{H}'$) and a set of lines with switches (denoted by $\mathcal{H}''$). In the proposed formulation, the inverse of $x$ is represented by $(x)^{-1}$. The voltage difference over a three-phase distribution line $\ell \in \mathcal{H}'$ in phase $\phi$ is formulated based on the self and mutual impedances and shunt admittances of the line as follows:

$$\begin{align*}
V_n^{\phi} - V_m^{\phi} &= Z_{\ell}^{\phi,\phi} I_{\ell}^{\phi} - \frac{1}{2} Z_{\ell}^{\phi,\phi} y_{\ell}^{\phi,\phi} V_n^{\phi} - \frac{1}{2} Z_{\ell}^{\phi,\phi} \sum_{k \in \psi, k \neq \phi} y_{\ell}^{k,\phi} V_k^{\phi} \\
&+ \sum_{p \in \psi, p \neq \phi} Z_{\ell}^{p,\phi} \left( I_{\ell}^{p} - \frac{1}{2} \sum_{k \in \psi} y_{\ell}^{k,p} V_k^{p} \right), \\
\forall \phi &\in \psi, \ell \in \mathcal{H}'
\end{align*}$$

where line $\ell$ connects bus $n$ to bus $m$. The current from bus $n$ to bus $m$ in the phase $\phi$ of a distribution line is defined based on (1a) as follows:

$$\begin{align*}
I_{\ell}^{\phi} &= (Z_{\ell}^{\phi,\phi})^{-1} \left[ V_n^{\phi} - V_m^{\phi} + \frac{1}{2} Z_{\ell}^{\phi,\phi} y_{\ell}^{\phi,\phi} V_n^{\phi} \\
&+ \frac{1}{2} Z_{\ell}^{\phi,\phi} \sum_{k \in \psi, k \neq \phi} y_{\ell}^{k,\phi} V_k^{\phi} \\
&- \sum_{p \in \psi, p \neq \phi} Z_{\ell}^{p,\phi} \left( I_{\ell}^{p} - \frac{1}{2} \sum_{k \in \psi} y_{\ell}^{k,p} V_k^{p} \right) \right], \forall \phi \in \psi, \ell \in \mathcal{H}'
\end{align*}$$

As illustrated in (1b), the current in the three-phase distribution line $\ell$ in the phase $\phi$ is obtained not only based on voltage, impedance, and admittance values of the phase $\phi$ but also on the current and voltage of the other phases as a result of the mutual impedances and admittances between the phase $\phi$ and the other two phases of the line. Based on (1b), $I_{\ell,n,m}^{\phi}$ has five terms, where the first two terms (i.e., $(Z_{\ell}^{\phi,\phi})^{-1} V_n^{\phi} - (Z_{\ell}^{\phi,\phi})^{-1} V_m^{\phi}$) represent the current flowing in the series self-impedance (i.e., $Z_{\ell}^{\phi,\phi}$) of the three-phase distribution line. The third term (i.e., $\frac{1}{2}(Z_{\ell}^{\phi,\phi})^{-1} Z_{\ell}^{\phi,\phi} y_{\ell}^{\phi,\phi} V_n^{\phi}$) shows the current of self-shunt admittance (i.e., $y_{\ell}^{\phi,\phi}$) of the line. The fourth term (i.e., $\frac{1}{2}(Z_{\ell}^{\phi,\phi})^{-1} Z_{\ell}^{\phi,\phi} \sum_{k \in \psi, k \neq \phi} y_{\ell}^{k,\phi} V_k^{\phi}$) shows the effect of the mutual shunt admittances between
phase $\phi$ and the other two phases on $I_n^\phi$. In the last term (i.e., $(Z_{\ell}^{\phi,\phi})^{-1} \sum_{\psi,\psi,\phi \neq \phi} Z_{\ell}^{\phi,\psi}(I_n^\phi - \frac{1}{2} \sum_{k \in \psi} y_{\ell}^{p,k} V_n^k)$), the current flowing in the other two phases (i.e., $I_n^\phi$) and also the self- and mutual shunt admittances of the other two phases ((i.e., $\frac{1}{2} \sum_{k \in \psi} y_{\ell}^{p,k} V_n^k$) appear in the formulation of $I_n^\phi$ due to the mutual impedances between the phase $\phi$ and the other two phases (i.e., $Z_{\ell}^{\phi,\psi}$). For instance, the current in phase $a$ of a line with sending end bus $n$ and receiving end bus $m$ is given by (1c) as:

$$I_n^a = \left( Z_{\ell}^{a,a} \right)^{-1} \left[ V_n^a - V_m^a + \frac{1}{2} Z_{\ell}^{a,a} V_n^a \right]$$

$$+ \frac{1}{2} Z_{\ell}^{a,a} (b_{\ell}^a V_n^a + y_{\ell}^{a,c} V_n^c)$$

$$- Z_{\ell}^{a,b} \left( I_n^b - \frac{1}{2} (y_{\ell}^{b,a} V_n^a + y_{\ell}^{b,b} V_n^b + y_{\ell}^{b,c} V_n^c) \right)$$

$$- Z_{\ell}^{a,c} \left( I_n^c - \frac{1}{2} (y_{\ell}^{c,a} V_n^a + y_{\ell}^{c,b} V_n^b + y_{\ell}^{c,c} V_n^c) \right) \right] \quad (1c)$$

The real and imaginary parts of the current in phase $\phi$ of line $\ell \in \mathcal{H}$ (i.e., $I_n^\phi$ in (1b)) are given in (1d)–(1e), respectively.

$$I_n^\phi = \left( R_{\ell}^{\phi,\phi} \right)^{-1} \left[ V_n^\phi - V_m^\phi + X_{\ell}^{\phi,\phi} I_n^{im,\phi} \right]$$

$$- \frac{1}{2} \left( R_{\ell}^{\phi,\phi} V_n^{im,\phi} + X_{\ell}^{\phi,\phi} V_n^{r,\phi} \right)$$

$$- \frac{1}{2} \sum_{p \in \psi, p \neq \phi} y_{\ell}^{p,k} (R_{\ell}^{p,\phi} V_n^{im,k} + X_{\ell}^{p,\phi} V_n^{r,k})$$

$$- \sum_{p \in \psi, p \neq \phi} R_{\ell}^{p,\phi} \left( I_n^{p,\phi} + \frac{1}{2} \sum_{k \in \psi} y_{\ell}^{p,k} V_n^{r,k} \right)$$

$$+ \sum_{p \in \psi, p \neq \phi} X_{\ell}^{p,\phi} \left( I_n^{im,p} + \frac{1}{2} \sum_{k \in \psi} y_{\ell}^{p,k} V_n^{r,k} \right) \right], \quad \forall \phi \in \psi, \ell \in \mathcal{H} \quad (1d)$$

$$I_n^{im,\phi} = \left( R_{\ell}^{\phi,\phi} \right)^{-1} \left[ V_n^{im,\phi} - V_m^{im,\phi} - X_{\ell}^{\phi,\phi} I_n^{r,\phi} \right]$$

$$+ \frac{1}{2} \left( R_{\ell}^{\phi,\phi} V_n^{r,\phi} - X_{\ell}^{\phi,\phi} V_n^{im,\phi} \right)$$

$$+ \frac{1}{2} \sum_{k \in \psi, k \neq \phi} y_{\ell}^{\phi,k} (R_{\ell}^{\phi,k} V_n^{r,k} - X_{\ell}^{\phi,k} V_n^{im,k})$$

$$- \sum_{p \in \psi, p \neq \phi} R_{\ell}^{p,\phi} \left( I_n^{im,p} + \frac{1}{2} \sum_{k \in \psi} y_{\ell}^{p,k} V_n^{r,k} \right)$$

$$- \sum_{p \in \psi, p \neq \phi} X_{\ell}^{p,\phi} \left( I_n^{r,p} + \frac{1}{2} \sum_{k \in \psi} y_{\ell}^{p,k} V_n^{r,k} \right) \right], \quad \forall \phi \in \psi, \ell \in \mathcal{H} \quad (1e)$$

The components of the injected current in phase $\phi$ of bus $n$ are shown in (1f)–(1g), where $\delta(n)$ is set of lines connected to bus $n$.

$$I_n^{r,\phi} = \sum_{\ell \in \delta(n)} I_n^{r,\phi}, \forall \phi \in \psi, n \in \mathcal{J} \quad (1f)$$

$$I_n^{im,\phi} = \sum_{\ell \in \delta(n)} I_n^{im,\phi}, \forall \phi \in \psi, n \in \mathcal{J} \quad (1g)$$

In distribution systems, DERs, e.g., inverter interfaced PV solar, are connected to the system via distribution transformers. For phase $\phi$ of bus $n \in \mathcal{J}$, the power balance formulations considering the DERs are given in (1h)–(1i).

$$\sum_{\psi \in \mathcal{J}} P_{g,\psi} - \sum_{\psi \in \mathcal{J}} d_{p,\psi,\ell} - \sum_{\psi \in \mathcal{J}} N \ell_{r,\psi} = V_n^{r,\phi} I_n^{r,\phi}$$

$$+ V_n^{im,\phi} I_n^{im,\phi}, \forall \phi \in \psi, n \in \mathcal{J} \quad (1h)$$

$$\sum_{\psi \in \mathcal{J}} Q_{g,\psi} + \sum_{\psi \in \mathcal{J}} Q_{c,\psi} - \sum_{\psi \in \mathcal{J}} d_{q,\psi,\ell} = V_n^{im,\phi} I_n^{r,\phi} - V_n^{r,\phi} I_n^{im,\phi}, \forall \phi \in \psi, n \in \mathcal{J} \quad (1i)$$

where $g(n), r(n), l(n),$ and $c(n)$ are set of DERs, loads, distribution transformers, and capacitors connected to bus $n$. The proposed accurate IV-based ACOPF can be used for different distribution system studies by considering different objective functions (e.g., minimizing operation cost, minimizing measurements error). In most of the prior distribution ACOPF models, such as the DistFlow model [25], line flow constraints are nonlinear, which are linearized by ignoring line losses in distribution systems. In the proposed ACOPF model (1), line losses are modeled, and the line flow constraints are inherently linear. Therefore, the proposed ACOPF model is more accurate for the topology detection and state estimation in distribution systems.

**B. Nonlinear Simultaneous Topology Error Processing and State Estimation Model**

In this section, the proposed ACOPF model in (1) is extended for developing the simultaneous topology error processing and state estimation model. For an unbalanced distribution system with a set of lines $\mathcal{H}$, where $\mathcal{H} = \mathcal{H'} \cup \mathcal{H''}$, the line current constraints for each line $\ell \in \mathcal{H'}$ (i.e., a branch without switch) are defined in (1b) and (1d)–(1e). If a switch is installed on each phase of a three-phase distribution line, the formulation of the current in phase $\phi$ of the line presented in (1b) should be modified to model possible connectivity statuses of the switches (i.e., connected or disconnected). Fig. 1 shows the structure of a three-phase switchable distribution line with self and mutual impedances and admittances.

In order to model the status of a switch, a binary variable is introduced for each switch installed in each phase of the three-phase line $\ell \in \mathcal{H''}$ denoted as $u_{\ell,\phi}$. In this paper, $u_{\ell,\phi} = 0$ implies that the phase $\phi$ of the line $\ell$ is disconnected, and $u_{\ell,\phi} = 1$ denotes that phase $\phi$ of the line $\ell$ is connected. Therefore, the current in the phase $\phi$ of the line $\ell \in \mathcal{H''}$ from bus $n$ to bus $m$ with three
single-phase switches is defined using (2a).

\[
I^\phi_\ell = \left( Z^\phi_\ell \right)^{-1} u^\phi_\ell \left[ V_n^\phi - V^\phi_m \right]
+ \frac{1}{2} Z^\phi_\ell y^\phi_\ell V_n^\phi + \frac{1}{2} Z^\phi_\ell \sum_{k \in \Lambda, k \neq \phi} y^\phi_k u^\phi_k V_n^\phi
- \sum_{p \in \psi, p \neq \phi} Z^\phi_p u^p_\ell \left( I^p_\ell - \frac{1}{2} \sum_{k \in \Lambda} y^p_k \Lambda^k p V_n^k \right),
\]  
\forall \phi \in \psi, \ell \in \mathcal{H}'

(2a)

where \( \Lambda^k p \) is an auxiliary binary variable associated with the status of single-phase switches of the line as defined in (2b).

\[
\Lambda^k = \begin{cases} 
  u^k_\ell & \text{if } k \neq \phi \lor p \\
  1 & \text{if } k = \phi \lor p 
\end{cases}
\]

(2b)

where “or” is indicated by the symbol \( \lor \). In (2b), \( \Lambda^k p \) is equal to \( u^k_\ell \), when \( k \) is not equal to \( \phi \) (i.e., phase of \( I^\phi_\ell \)) or \( p \). As shown in (2a), \( I^\phi_\ell \) is not only determined based on the status of the switch installed in the phase \( \phi \) (i.e., \( u^\phi_\ell \)) but also on the status of switches installed in the two other phases because of mutual impedances and admittances. For instance, the current in phase \( a \) of the line \( \ell \in \mathcal{H}' \) with three single-phase switches is obtained using (2c).

\[
I^\phi_\ell = (Z^\phi_\ell)^{-1} u^\phi_\ell \left[ V_n^\phi - V^\phi_m + \frac{1}{2} Z^{a,a}_\ell u^a_\ell V_n^a + \frac{1}{2} Z^{b,b}_\ell u^b_\ell V_n^b - \frac{1}{2} \left( y^{b,a}_\ell V_n^a + y^{b,b}_\ell V_n^b + y^{c,c}_\ell V_n^c \right) \right] - \frac{1}{2} \sum_{k \in \Lambda} y^k_\ell \Lambda^k p V_n^k
- Z^{a,c}_\ell u^c_\ell \left( I^c_\ell - \frac{1}{2} \left( y^{c,a}_\ell V_n^a + y^{c,b}_\ell V_n^b + y^{c,c}_\ell V_n^c \right) \right)
\]

(2c)

As shown in (2c), the current in phase \( a \) of the line is not only dependent on voltage, impedance, admittance, and the status of the switch of phase \( a \) (i.e., \( u^a_\ell \)), but also voltage, current, and the status of the switches of phases \( b \) and \( c \) (i.e., \( u^b_\ell \) and \( u^c_\ell \)) as a result of the mutual impedances and admittances between the phase \( a \) and the other two phases of the line. In order to elucidate how \( \Lambda^k p \) in (2a) is determined based on (2b), we examine the last two terms in (2c). If we open (2a) for the given example in (2c) (i.e., phase \( a \) of the line \( \ell \)), the last term of (2a) with \( \Lambda^k p \) is presented in the last two terms of (2c) as follows:

\[
- \sum_{p \neq \phi, p \neq \phi} Z^\phi_p u^p_\ell \left( I^p_\ell - \frac{1}{2} \sum_{k \in \Lambda} y^p_k \Lambda^k p V_n^k \right)
- Z^{a,b}_\ell u^b_\ell \left( I^b_\ell - \frac{1}{2} \left( y^{b,a}_\ell V_n^a + y^{b,b}_\ell V_n^b + y^{b,c}_\ell V_n^c \right) \right)
- Z^{a,c}_\ell u^c_\ell \left( I^c_\ell - \frac{1}{2} \left( y^{c,a}_\ell V_n^a + y^{c,b}_\ell V_n^b + y^{c,c}_\ell V_n^c \right) \right) = A + B
\]

(2d)

For the purpose of explanation, the first and second terms of (2d) are named \( A \) and \( B \), respectively. First, we examine the part of \( A \) where \( \Lambda^k p \) is appeared (i.e., \( y^{b,a}_\ell V_n^a + y^{b,b}_\ell V_n^b + y^{b,c}_\ell V_n^c \)). For the term \( A \), the notations are \( p = b, \phi = a \), and \( k \in \{a, b, c\} \). Please note that although the term \( A \) is related to phase \( p = b \), however, due to complicated structure of distribution lines with mutual admittances (i.e., \( y^{b,a}_\ell \) and \( y^{b,c}_\ell \)), \( V_n^a \) and \( V_n^c \) are also appeared in the formulation. Let assume phase \( c \) of the distribution line \( \ell \) shown in Fig. 1 is disconnected. Therefore, the impact of mutual admittance of phase \( c \) (i.e., \( y^{b,c}_\ell V_n^c \)) should be neglected from the formulation of part \( A \). To this end, the binary variable associated with the status of phase \( c \) (i.e., \( u^c_\ell \)) is added to the term related to phase \( c \) (i.e., \( y^{b,c}_\ell V_n^c \)). In this regard, when the phase \( c \) of the distribution line \( \ell \) is disconnected, \( u^c_\ell \) will be equal to zero, and as a result \( y^{b,c}_\ell V_n^c \) will be discarded. In the case that the phase \( b \) of the distribution line \( \ell \) shown in Fig. 1 is disconnected, it is not needed to include the binary variable associated with the status of phase \( b \) (i.e., \( u^b_\ell \)) in the term related to the phase \( b \) (i.e., \( y^{b,b}_\ell V_n^b \)). The reason is that, \( u^b_\ell \) is already included at the beginning of \( A \) and is multiplied by \( y^{b,b}_\ell V_n^b \). Therefore, if the phase \( b \) of the distribution line \( \ell \) is disconnected, \( u^b_\ell \) will be equal to zero, and not only the term \( y^{b,b}_\ell V_n^b \) but also the whole term related to phase \( b \) (i.e., \( A \)) is eliminated from (2c). Also, if the phase \( a \) of the distribution line \( \ell \) is disconnected, there is no need to add \( u^a_\ell \) in the term related to phase \( a \) (i.e., \( y^{a,a}_\ell V_n^a \)). Because \( u^a_\ell \) is already modeled in the very beginning of (2c) and is multiplied by \( y^{a,a}_\ell V_n^a \). Therefore, if \( u^a_\ell = 0 \), all of the terms on the right side of (2c) will be eliminated. As a result, the current in the phase \( a \) of the line \( \ell \in \mathcal{H}' \) is obtained as zero (i.e., \( I^a_\ell = 0 \)). The auxiliary binary variable \( \Lambda^k p \) presented in (2b) is introduced to model this complexity. For the given example (i.e., term \( A \), where \( p = b, \phi = a \), and \( k \in \{a, b, c\} \)) if \( k = \phi \lor p \), the binary variables associated with their status (i.e., \( u^k_\ell \) and \( \Lambda^k p \)) will not be added and \( \Lambda^k p \) will be equal to 1. However, when \( k \neq \phi \lor p \) (i.e., \( k = c \)), the binary variables associated with its status (i.e., \( u^c_\ell \)) should be modeled. The same rule can be applied to part \( B \) of (2d). For the term \( B \), the notations are \( p = c, \phi = a \), and \( k \in \{a, b, c\} \). Based on (2b), when \( k = \phi \lor c \), \( \Lambda^k p \) will be equal to 1. If \( k \neq \phi \lor p \) (i.e., \( k = b \), \( u^b_\ell \) should be modeled.

In (2a), if phase \( \phi \) of the line is disconnected, \( u^\phi_\ell \) will be equal to zero, which results in \( I^\phi_\ell \) being zero. Assume that phase \( \phi \) of the line is connected (i.e., \( u^\phi_\ell = 1 \)). In this regard, if any of the other two phases of the line are disconnected, the binary
variables corresponding to them (i.e., \( u_k^b \) or \( u_k^b \) in (2a)) will be equal to zero. The zero values for these binary variables in (2a) result in eliminating the effect of the voltages (i.e., \( V_{n}^r \) and currents (i.e., \( I_{\ell}^r \)) of the other two phases on the current in phase \( \phi \) of the line (i.e., \( I_{\ell}^\phi \)). For the given example in (2c), if phase \( b \) of the line \( \ell \) is not energized, \( u_k^r \) will be zero. Since every \( V_{n}^r \) and \( I_{\ell}^r \) is multiplied by \( u_k^r \) in (2c), the zero value of \( u_k^r \) results in vanishing of all terms including the phase \( b \) voltage and current. Constraints (2e)–(2f) include the real and imaginary parts of \( I_{\ell}^\phi \) in (2a).

\[
I_{\ell}^{\phi, \phi} = \left( R_{\ell}^{\phi, \phi} \right)^{-1} u_{k}^{\phi} \left[ V_{n}^{r, \phi} - V_{m}^{r, \phi} + X_{\ell}^{\phi, \phi} r_{\ell}^{im, \phi} \right] - \frac{1}{2} \gamma_{\ell}^{\phi, \phi} \left[ R_{\ell}^{\phi, \phi} V_{n}^{r, \phi} + X_{\ell}^{\phi, \phi} r_{\ell}^{im, \phi} \right] - \frac{1}{2} \sum_{p \in \psi, p \neq \phi} R_{\ell}^{p, p} u_{p}^{\phi} \left( I_{\ell}^{r, p} + \frac{1}{2} \sum_{k \in \psi} y_{k}^{\phi, k} u_{k}^{\phi} \right) + \frac{1}{2} \sum_{k \in \psi, k \neq \phi} y_{k}^{\phi, k} u_{k}^{\phi} \left( R_{\ell}^{\phi, \phi} V_{n}^{r, \phi} + X_{\ell}^{\phi, \phi} r_{\ell}^{im, \phi} \right) \right], \\
\quad \forall \phi \in \psi, \ell \in H'' \tag{2e}
\]

\[
I_{\ell}^{im, \phi} = \left( R_{\ell}^{\phi, \phi} \right)^{-1} u_{k}^{\phi} \left[ V_{n}^{im, \phi} - V_{m}^{im, \phi} \right] - X_{\ell}^{\phi, \phi} r_{\ell}^{im, \phi} + \frac{1}{2} \gamma_{\ell}^{\phi, \phi} \left[ R_{\ell}^{\phi, \phi} V_{n}^{r, \phi} + X_{\ell}^{\phi, \phi} r_{\ell}^{im, \phi} \right] + \frac{1}{2} \sum_{k \in \psi, k \neq \phi} y_{k}^{\phi, k} u_{k}^{\phi} \left( R_{\ell}^{\phi, \phi} V_{n}^{r, \phi} - X_{\ell}^{\phi, \phi} r_{\ell}^{im, \phi} \right) \right], \\
\quad \forall \phi \in \psi, \ell \in H'' \tag{2f}\]

More sensors such as micro-PMUs and smart meters are being installed in modern distribution systems to improve the observability and control of the distribution systems. Consider \( \mathcal{R} \) as a vector of measurements (e.g., micro-PMUs and smart meters) in an unbalanced distribution system, and \( \mathcal{R}(x) \) as a vector of their corresponding variables, which are functions of the system topology and states (i.e., \( x \)). The objective function of the proposed simultaneous topology processing and state estimation algorithm in an unbalanced distribution system is minimizing the weighted error between \( \mathcal{R} \) and \( \mathcal{R}(x) \) as expressed in (2g) using Euclidean norm (i.e., \( \| \cdot \|_2 \)).

\[
x^* = \arg \min \| \mathcal{W}(\mathcal{R}(x) - \mathcal{R}) \|_2^2 \tag{2g}
\]

where \( x^* \) is the vector of the obtained status of switches and voltage phasors. \( \mathcal{W} \) is a diagonal matrix including inverse of covariance of measurements noise. The objective function (2g) is subject to constraints (1d)–(1i), (2b), and (2e)-(2f). It should be noted that constraints (1d)–(1e) are considered for the lines without switches (i.e., \( \ell \in H' \)), and constraints (2e)–(2f) are modeled for lines with the switches (i.e., \( \ell \in H'' \)). Although the objective function in (2g) is convex, the constraints are nonlinear.

C. Proposed Simultaneous Topology Error Processing and State Estimation Model

The simultaneous topology processing and state estimation model in Section II-B is a mixed-integer nonlinear quadratic programming (MINQP) problem, which may not lead to the global optimal solution. In other words, the problem is a nonconvex nonlinear problem, which may obtain multiple solutions. The non-linearity in this problem is due to two sets of constraints (i.e., constraints (2e)–(2f) and (1h)–(1i)). In order to attain the global optimal solution, these two sets of constraints are linearized in the following sections using the big-M method and the Taylor series approach.

1) Linearizing Line Current Constraints With Switches: The current in the distribution line \( \ell \in H'' \) with three single-phase switches is given in (2e) and (2f), in which the fifth, sixth, and seventh terms of each of them are nonlinear. The non-linearity in the fifth term of (2e)–(2f) is due to the multiplication of the binary variable \( u_k^r \) with the continuous variables \( V_{n}^{im,k} \) and \( V_{n}^{r,k} \). Such non-linearity is linearized by applying the big-M method as shown in (3a)–(3d).

\[
-M \left( 1 - u_k^r \right) \leq \omega_k^r - V_{n}^{r,k} \leq M \left( 1 - u_k^r \right) \tag{3a}
\]

\[
-M u_k^r \leq \omega_k^r \leq M u_k^r \tag{3b}
\]

\[
-M \left( 1 - u_k^r \right) \leq \omega_k^r - V_{n}^{im,k} \leq M \left( 1 - u_k^r \right) \tag{3c}
\]

\[
-M u_k^r \leq \omega_k^r \leq M u_k^r \tag{3d}
\]

According to (3a)–(3d), if \( u_k^r = 1 \), the auxiliary variables \( \omega_k^r \) and \( \omega_k^r \) will be equal to \( V_{n}^{r,k} \) and \( V_{n}^{im,k} \), respectively. In the sixth and seventh terms of (2e)–(2f), if the auxiliary binary variable \( \omega_k^r \) is equal to 1 (i.e., \( k \) is equal to \( \phi \) or \( p \)), multiplication of binary variable \( u_k^r \) with continuous variables \( I_{\ell}^{r,p}, I_{\ell}^{im,p}, V_{n}^{r,k}, \) and \( V_{n}^{im,k} \) will appear in the constraints. To linearize these nonlinear terms, constraints (3e)-(3h) are added to the model.

\[
-M \left( 1 - u_k^r \right) \leq E_{\ell}^{r,p} - \left( I_{\ell}^{r,p} + \frac{1}{2} \sum_{k \in \psi, k \neq \phi/p} y_{k}^{\phi, k} V_{n}^{im,k} \right) \leq M \left( 1 - u_k^r \right) \tag{3e}
\]

\[
-M u_k^r \leq E_{\ell}^{r,p} \leq M u_k^r \tag{3f}
\]
where $E_p^p$ and $\hat{E}_p^p$ are auxiliary variables. In the linear constraints (3e)–(3h), if the switch installed in phase $p$ is disconnected (i.e., $u^p = 0$), $E_p^p$ and $\hat{E}_p^p$ will become zero. In this regard, the current in the line at phase $p$ and its corresponding mutual impedance (i.e., $R_{\ell}^{\phi,k}$ and $X_{\ell}^{\phi,k}$) will be removed from the formulations of current in phase $\phi$ (i.e., (2e)–(2f)). If the auxiliary binary variable $A^p_{\ell}$ is equal to $u^p_{\ell}$ (i.e., when $k$ is not equal to $\phi$ or $p$) in the sixth and seventh terms of (2e)–(2f), multiplication of two binary variables $u^p_{\ell}$ and $u^k_{\ell}$ with continuous variables $V_{n,r}^{r,k}$ and $V_{n}^{i,m,\phi}$ will be imposed on the model. First, multiplication of two binary variables $u^p_{\ell}$ and $u^k_{\ell}$ is linearized by substituting it with a new binary variable $\zeta_{\ell}^{p,k}$ and adding constraints (3i)–(3k).

\[
\zeta_{\ell}^{p,k} \leq u^p_{\ell}
\]

(3i)

\[
\zeta_{\ell}^{p,k} \leq u^k_{\ell}
\]

(3j)

\[
\zeta_{\ell}^{p,k} \geq u^p_{\ell} + u^k_{\ell} - 1
\]

(3k)

Based on (3i)–(3k), if both $u^p_{\ell}$ and $u^k_{\ell}$ are equal to 1, $\zeta_{\ell}^{p,k}$ will be obtained as 1. Assume $u^p_{\ell} = 0$ and $u^k_{\ell} = 0$, according to (3i)–(3j), $\zeta_{\ell}^{p,k}$ is also forced to be zero. Second, by replacing $u^p_{\ell} \times u^k_{\ell}$ with $\zeta_{\ell}^{p,k}$, multiplication of $\zeta_{\ell}^{p,k}$ with continuous variables $V_{n}^{i,m,\phi}$ and $V_{n,r}^{r,k}$ is linearized by introducing two auxiliary variables $F_{\ell}^{p,k}$ and $\hat{F}_{\ell}^{p,k}$ and the big-M approach as defined in (3l)–(3o).

\[
-M \left(1 - \zeta_{\ell}^{p,k}\right) \leq F_{\ell}^{p,k} - \frac{1}{2} y_{\ell}^{p,k} V_{n,r}^{r,k} \leq M \left(1 - \zeta_{\ell}^{p,k}\right)
\]

(3l)

\[
-M \zeta_{\ell}^{p,k} \leq F_{\ell}^{p,k} \leq M \zeta_{\ell}^{p,k}
\]

(3m)

\[
-M \left(1 - \zeta_{\ell}^{p,k}\right) \leq \hat{F}_{\ell}^{p,k} - \frac{1}{2} y_{\ell}^{p,k} V_{n}^{i,m,\phi} \leq M \left(1 - \zeta_{\ell}^{p,k}\right)
\]

(3n)

\[
-M \zeta_{\ell}^{p,k} \leq \hat{F}_{\ell}^{p,k} \leq M \zeta_{\ell}^{p,k}
\]

(3o)

In the constraints (3i)–(3o), if $\zeta_{\ell}^{p,k}$ is equal to 1, $F_{\ell}^{p,k} = \frac{1}{2} y_{\ell}^{p,k} V_{n,r}^{r,k}$ and $\hat{F}_{\ell}^{p,k} = \frac{1}{2} y_{\ell}^{p,k} V_{n}^{i,m,\phi}$. Considering the proposed linear constraints (3a)–(3o), the constraints (2e) and (2f) can be written as (3p) and (3q), respectively.

\[
I_{\ell}^{r,\phi} = \left( R_{\ell}^{\phi,k} \right)^{-1} u^\phi_{\ell} \left[ R_{\ell}^{\phi,k} V_{n}^{r,\phi} + X_{\ell}^{\phi,k} I_{\ell}^{i,m,\phi} \right] - \frac{1}{2} y_{\ell}^{\phi,k} \left( R_{\ell}^{\phi,k} V_{n}^{i,m,\phi} + X_{\ell}^{\phi,k} V_{n}^{r,\phi} \right)
\]

(3p)

\[
\forall \phi \in \psi, \ell \in H''
\]

\[
I_{\ell}^{i,m,\phi} = \left( R_{\ell}^{\phi,k} \right)^{-1} u^\phi_{\ell} \left[ R_{\ell}^{\phi,k} V_{n}^{r,\phi} - X_{\ell}^{\phi,k} I_{\ell}^{i,m,\phi} \right] + \frac{1}{2} y_{\ell}^{\phi,k} \left( R_{\ell}^{\phi,k} V_{n}^{i,m,\phi} - X_{\ell}^{\phi,k} V_{n}^{r,\phi} \right)
\]

(3q)

\[
\forall \phi \in \psi, \ell \in H''
\]

\[
I_{\ell}^{r,\phi} = \left( R_{\ell}^{\phi,k} \right)^{-1} u^\phi_{\ell} \left[ R_{\ell}^{\phi,k} V_{n}^{r,\phi} - X_{\ell}^{\phi,k} I_{\ell}^{i,m,\phi} \right] + \frac{1}{2} y_{\ell}^{\phi,k} \left( R_{\ell}^{\phi,k} V_{n}^{i,m,\phi} - X_{\ell}^{\phi,k} V_{n}^{r,\phi} \right)
\]

2) Linearization of Power Balance Constraints: The power balance equations (1h)–(1i) are nonlinear due to the products of current and voltage variables. In this paper, an iterative first-order approximation of the Taylor series expansion is used to develop the linear model of (1h)–(1i) as follows:

\[
\sum_{\gamma \in \omega_\phi} P_{\gamma,\psi,\phi} - \sum_{\phi \in \psi, \ell \in H''} S_{\phi,\psi,\phi} - \sum_{\gamma \in \omega_\phi} N L_{\gamma,\psi,\phi} \frac{T R_{\gamma,\psi,\phi} = V_{n,\gamma,\psi,\phi}^{r,\phi} + I_{\gamma,\psi,\phi}^{i,m,\phi} V_{n,\gamma,\psi,\phi}^{r,\phi} - V_{n,\gamma,\psi,\phi}^{r,\phi}}{V_{n,\gamma,\psi,\phi}^{r,\phi} + I_{\gamma,\psi,\phi}^{i,m,\phi} V_{n,\gamma,\psi,\phi}^{r,\phi} - V_{n,\gamma,\psi,\phi}^{r,\phi}}
\]

Authorized licensed use limited to the terms of the applicable license agreement with IEEE. Restrictions apply.
The objective function in (3z) should be changed to model any other types of norms and all constraints of the proposed model (i.e., (1d)-(1g), (3a)-(3y)) will be the same for modeling any norm type. The dashed box in Fig. 2 illustrates the proposed simultaneous topology error processing and state estimation model for an unbalanced distribution system in real-time. It should be noted that voltage measurements are obtained by solving an AC power flow via OpenDSS [26]. As shown in Fig. 2, measurements with added noise are fed to the proposed model. In the first iteration, the proposed algorithm uses flat start point. More accurate values of $V^r_{n,it-1}$, $V^im_{n,it-1}$, $I^r_{n,it-1}$, and $I^im_{n,it-1}$ can be obtained by solving the proposed simultaneous topology processing and state estimation algorithm iteratively, which results in improving the linearization approximations in the model.

D. Complementary Constraints for Real-Time Simultaneous State Estimation and Topology and Outage Detection

Outage detection enables the identification of the de-energized subnetwork parts of the distribution system network. It is the first and foremost step in the electric power distribution outage management system (OMS) to reduce the duration and sizes of power outages and accelerate the service restoration process [27]. The cause for this isolation could be faults, physical topology attacks, or DERs’ fluctuation triggering protection misoperation. However, the limited distribution system measurements challenge islanding and outage detection. This subsection develops a set of constraints for identifying the status of disconnected switches to combine with the topology error processing and state estimation constraints in Section II-C to simultaneously detect outage areas and identify topology in the energized regions.

The status of disconnected lines, which can lead to the outage and create the islands, must be identified as zero. However, in (3v)-(3w), when $I^r_{l,\phi} = 0$ and $I^im_{l,\phi} = 0$, the status of the switch in phase $\phi$ of the line $(u^\phi_{l})$ can be identified as 0 or 1. Hence, the proposed topology processing and state estimation model given in (3z) is improved by adding the following linear constraints in order to identify status of disconnected switches and outages in an unbalanced distribution system correctly.

\[
\begin{align*}
\epsilon - I^r_{l,\phi} &\leq \lambda (1 - \alpha^\phi_{l}) \\
\epsilon + I^r_{l,\phi} &\leq \lambda (1 - \beta^\phi_{l}) \\
\epsilon - I^im_{l,\phi} &\leq \lambda (1 - \gamma^\phi_{l}) \\
\epsilon + I^im_{l,\phi} &\leq \lambda (1 - \tau^\phi_{l}) \\
u^\phi_{l} &\leq \alpha^\phi_{l} + \beta^\phi_{l} + \gamma^\phi_{l} + \tau^\phi_{l}
\end{align*}
\]

where $\alpha^\phi_{l}, \beta^\phi_{l}, \gamma^\phi_{l}$, and $\tau^\phi_{l}$ are auxiliary binary variables, $\epsilon$ is a small positive number, and $\lambda$ is a large positive number. According to (4a)-(4e), if $I^r_{l,\phi}$ and $I^im_{l,\phi}$ are zero, $u^\phi_{l}$ will be forced to be zero. The formulation of the proposed simultaneous outage, topology, and state identification model is presented in (4f).

\[
x^* = \arg \min \| W(\Re(x) - \Re) \|_2^2
\]

s.t.  \ (1d) - (1g), (3a) - (3q), (3t) - (3y)

It should be noted that topology processor and state estimation are parts of the DMS, while outage detection is part of the OMS in the distribution systems. The proposed model can perform simultaneous topology processor, state estimation, and outage detection in a distribution network. The integrated tool can enhance real-time monitoring and control of distribution systems.

Authorized licensed use limited to the terms of the applicable license agreement with IEEE. Restrictions apply.
III. SENSORS DATA INTEGRATION APPROACH IN DISTRIBUTION SYSTEMS

Smart meters typically gauge electric energy consumptions. They can also provide average active power; however, they do not provide reactive power. To address the lack of reactive power measurements in distribution systems, pseudo reactive power measurements based on a random power factor are considered in the literature [7]. However, pseudo reactive power measurements introduce high errors in the model. To overcome the limitation of smart meters not providing reactive power, constraint (5a) is embedded in the proposed model.

\[
P_i \left( \frac{1}{PF_{i,\text{max}}} - 1 \right) \leq Q_i \leq P_i \left( \frac{1}{PF_{i,\text{min}}} - 1 \right), \quad \forall i \in S
\]

(5a)

where \( P_i \) is the variable associated with the average active power measured by the smart meter \( i \), \( Q_i \) is reactive power demand at each load, and \( S \) is the set of smart meters. \( PF_{i,\text{min}} \) and \( PF_{i,\text{max}} \) are the minimum and maximum power factor of each load and are considered to be 0.9 and 0.99, respectively. Another challenge in distribution system studies is that micro-PMUs provide synchronized instantaneous measurements of voltage and current phasors with a frequency of 120 Hz [28]; however, smart meters can provide an average of energy consumptions over a time period of 15 minutes, which are not synchronized instantaneous measurements. To address the issue of integrating measurements of these two devices with different sampling rates and approaches (i.e., average versus instantaneous), a statistical study is performed in this paper. In this regard, the residential load data of Pecan Street Inc. database are considered [29]. This dataset includes 1-second actual residential energy consumption data. Since the proposed model is conducted for the primary distribution system, the smart meter data are aggregated at the secondary side of each distribution transformer. Therefore, 1-second load data of different groups of customers (e.g., 7-8 houses) are collected at the secondary side of distribution transformers. Based on the conducted statistical study, the load variation between average active power during a 15 minutes time interval and instantaneous active power measurements follows a Gaussian distribution \( N = (0, \sigma^2) \) with zero mean and standard deviation \( \sigma \), where 99.73% of data are set in \( 3\sigma = 30\% \). Therefore, a 30% variation is considered in this work due to utilizing average power measured by smart meters versus instantaneous active power measurements due to the device sampling limitations.

IV. SIMULATION RESULTS

The proposed model is tested on an actual distribution feeder of a local electric utility in Arizona. The primary feeder of the test system as shown in Fig. 3 contains 2100 buses, 1790 distribution lines, and 371 distribution transformers. There are 23 switch cabinets with 157 single-phase switches. The total number of single-phase switches is 859, including 681 single-phase transformer switches and one three-phase switch at the substation. The number of aggregated loads with smart meters at the secondary of distribution transformers is 342, where 250 of these locations have aggregated rooftop PV meters measurements.

Measurement data of the distribution feeder of the local electric utility include one pseudo micro-PMU measurement, smart meter data and rooftop PV output power of load buses, substation measurements, and reactive power injection of capacitors. The micro-PMU is placed at the substation bus shown in Fig. 3. The micro-PMU measurements include three-phase voltage magnitude and angle of the bus with micro-PMU as well as three-phase current of the branch connected to the micro-PMU bus. The micro-PMU data do not include reactive power measurement. Accordingly, in per-phase representation, the micro-PMU data include 6 voltage measurements (magnitude and angle) and 6 current measurements (magnitude and angle). A per-phase representation of a bus in a three-phase distribution system is shown in Fig. 1. Reactive power measurements of four three-phase capacitors are available which result in 12 reactive power injection measurements and 12 active power zero-injection measurements. Substation sensor measurements in per-phase representation include three active power injections and three reactive power injections. Also, there are 1964 pairs of active and reactive zero-injection measurements in per-phase representation. It should be noted that the zero-injection measurements are modeled in the power balance constraints, not in the objective function. This paper assumes DMS knows the lines impedances and admittances and their connection based on the feeder information provided by the local electric utility. This is an acceptable assumption as the main focus of this work is on the primary distribution system. The proposed method identifies the status of switches in the distribution systems. It should be noted...
that the switches in the distribution feeder of the local electric utility have no communication links to automatically report the switch status back to the DMS. Hence, no switches status measurements are available. Since the distribution feeder of the local electric utility is an unbalanced three-phase system with various phase configurations (e.g., single-phase, three-phase), the number of buses is 2100, but there are 2325 nodes in per-phase representation. The total number of state variables in the proposed simultaneous topology processor, state estimation, and outage detection model is 2325 voltage phasors (i.e., 4650 voltage magnitude and angle) combined with 859 switch statuses (i.e., total 5509 state variables). The redundancy level is calculated by dividing the number of measurements by the number of state variables [30]. If the smart meters provide both active and reactive power measurements, there are 346 pairs of injection measurements (i.e., 692 measurements). Therefore, the total number of all aforementioned measurements is equal to 4662. According to [30], it can be seen that the distribution feeder of the local electric utility has a low measurement redundancy level. Moreover, as discussed in Section III, smart meters cannot provide reactive power information, therefore, the total number of all aforementioned measurements becomes 4316, which reduces the redundancy level more.

It should be noted that in the traditional state estimation method solved by iterative derivative-based approaches such as the Gauss-Newton approach, the number of measurements should be greater than the number of states [31]. This implies that the traditional state estimation method can be used for networks with high measurement redundancy. However, most distribution systems have low measurement redundancy [15], [20], [32], as shown for the studied test system. Therefore, the low measurement redundancy cannot satisfy the conditions of the traditional state estimation [32]. Moreover, the traditional state estimation method may not converge and obtain the global optimal solution. It will be shown in the next sections that while the number of measurements is limited in the distribution feeder of the local electric utility with low measurement redundancy, the proposed integrated model can accurately estimate system states, detect topology and outages, and enhance observability and monitoring of the distribution system in real-time.

The Gaussian distribution function is utilized to model the measurement noise of the pseudo micro-PMU with a \( \text{TotalVectorError} \leq 0.05\% \) [6]. The measurement data of smart meters, rooftop PV sensors, capacitors, and substation sensor device include noise, which is considered as a Gaussian distribution with zero mean and 10%, 1%, and 0% error, respectively [33]. The proposed model is programmed in JAVA and solved by Gurobi 9.0.0 solver on a computer with an Intel Core i7 CPU @ 3.10 GHz and 16 GB of RAM. Topology processing in utility distribution feeders is more challenging compared to the IEEE test case systems due to switch cabinets with various phase configurations, as demonstrated in Fig. 4, and other detailed models of the electric devices. To describe the test system in detail, switch cabinets with various phase configurations and no-load loss of transformers are modeled in the proposed simultaneous topology processing and state estimation tool.

Table I shows the accuracy of the proposed simultaneous topology processing and state estimation model in identifying the status of switches in the test system in a normal state of the system without an outage. The metric \( M_{su} \) (i.e., \( M_{su} = 100 \times (1 - \left[ \sum_{E \in \Omega w} \sum_{\phi} \left| u^{\phi}_{E} - u^{\phi}_{E} / |\Omega_{su}| \right| \right] ) \)) is used in Table I to assess the accuracy of the proposed model in the detection of the system topology. \( u^{\phi}_{E} \) is the actual status of switch \( E \), \( \Omega_{su} \) is set of switches, and \( su \) is index of the set. The metric \( M_{su} \) is reported in Table I for two different sets: (1) set \( \Omega_{1} \) including the status of all switches in the system, (2) set \( \Omega_{2} \) including the status of all switches in the system excluding those connected to presently no-load region (i.e., not connected to any device in the system) and normally open switches (NOSs).

The locations of 12 errors reported in Table I are demonstrated in Fig. 5 using five orange circles. Yellow rhombuses in Fig. 5 show termination of the laterals presently connected to no-load region. It can be observed that most of the errors (i.e., 9 errors in circles 3-5) are inside switch cabinets that are located in parts of the system, which are connected to presently no-load regions. In other words, they are not connected to any device (e.g., load, distribution transformer, capacitor, rooftop PV unit) in the network. These locations are related to locations where the local electric utility has extended the feeder to serve future customers, but no development has taken place in those regions yet. The remaining three errors are shown in circles 1-2, which are in parts of the system that a NOS connects different phases together. While the proposed algorithm identifies the status of NOSs correctly as disconnected, the status of switches right before the NOSs are identified wrongly as disconnected due to the current being zero. It can be observed in Table I that the proposed model is able to identify the real-time topology of the distribution feeder with 100% accuracy for all of the regions with actual load and/or DERs even with few numbers of measurement.

**Table I**

| Measurement noise | Yes |
|-------------------|-----|
| Total number of single-phase switches | 859 |
| Wrong statuses (connected to presently no-load region and NOS) | 12 |
| Undetectable statuses | 0 |
| \( M_1 \) accuracy (%) | 100 |
| \( M_2 \) accuracy (%) | 98.60 |
| Captured feeder load (%) | 100 |
| Simulation time (s) | 20 |

Fig. 4. (Left) switch cabinet with 9 single-phase switches and three fuses, (Right) switch cabinet with 6 single-phase switches and 6 fuses.
devices (e.g., one micro-PMU) and consideration of measurement noise; moreover, the accuracy of the proposed model is high even while considering the status of switches connected to presently no-load regions and NOSs. With high interconnection structure accuracy both 100% or 98.6%, 100% of the local electric utility feeder load is captured as shown in Table I. The state estimation results including voltage magnitude error and voltage angle error for each node of the local electric utility network are depicted in Fig. 6. In Fig. 6, each point represents the voltage error of phase \(a\), \(b\), or \(c\) of buses. According to Fig. 6, the proposed simultaneous topology processing and state estimation model estimates states of the test system very precisely during a normal condition (maximum voltage magnitude error \(< 0.17\%\) and maximum voltage angle error \(< 0.0024\) radian). The average simulation time is 20 seconds, which implies that the proposed tool is fast for the real-time applications. To further illustrate the performance of the proposed simultaneous topology processing and state estimation model, four other sets of cases are investigated in the following sections. These four case studies include studying outages in the test system, modeling bad data, practical consideration of micro-PMU and smart meter devices, and evaluating the robustness of the proposed tool against the missing data.

### A. Simultaneous Identification of Outage, Topology, and States of Distribution Systems

Outages in distribution systems can occur due to switch malfunctions or faults in the system. As a result of the outages, the network is divided into different energized regions and de-energized parts. The proposed model is able to detect not only the outage areas but also the topology of the energized regions. Four cases are analyzed for simultaneously detecting outage areas and topology of the energized sections in the test system. In cases A1 and A2, one switch inside two different switch cabinets in the test system feeder is disconnected leading to two different outage areas in the system with 46 and 37 islanded nodes, respectively. In case A3, multiple simultaneous outages are simulated by considering cases A1 and A2 together. In case 4, a large outage as a result of disconnecting a switch inside another switch cabinet in the test system is simulated with 381 islanded nodes as illustrated in Fig. 7 (the red color shows the parts of the system with outage). Table II shows the accuracy of the proposed topology processing model for four simulated

![Fig. 5. Location of 12 topology processing errors.](image)

![Fig. 6. State estimation error for a normal state of the local electric utility system: (Left) voltage magnitude (Right) voltage angle for each node.](image)

![Fig. 7. Islanded region of the local electric utility system in case A4.](image)
outage cases. As shown in Table II, the proposed model can detect topology and outages of the test system with high accuracy in cases A1-A4. It should be noted that the locations of 12 errors in the status of single-phase switches in Table II for the four cases are the same as the locations shown in Fig. 5. Moreover, the undetectable switch status in cases A1 and A3-A4 in Table II is the switch located right before the disconnected switch in each case. While this switch is connected in each case, no current is flowing in it due to the disconnection of the downstream feeder. The zero current makes the status of such switches undetectable. However, the status of this switch does not impact load and feeder connectivity and the ability to control the feeder. With the high accuracy of the proposed approach in detecting the outage and topology of the system, 100% of the local electric utility feeder load is captured in cases A1-A4 for the energized areas. Due to the fact that the proposed tool identifies the system states simultaneously with the topology and outage areas of the system, the error of the obtained voltage magnitude and angle states are illustrated in Fig. 8 for case A4 with the largest outage. It can be seen that the system states are estimated with high accuracy even while having a large outage in the network. The range of simulation time for cases A1-A4 is from 15 seconds to 21 seconds.

B. Bad Data Modeling

In this section, two bad data analysis studies are conducted to assess the robustness of the proposed simultaneous topology processing and state estimation model considering: 1) fixed large error in the measurements; 2) probabilistic error with bad data detection.

1) Bad Data Analysis With Fixed Large Error: In order to evaluate the robustness of the proposed tool in the case of bad data, three cases (Table III) are studied, where the aggregated smart meter data at the secondary of the distribution transformers are randomly vitiated for the bad data modeling. In this regard, 60% error in the aggregated smart meter measurements connected to the largest loads in the network for the bad data modeling, the measurement noise of meters in the system (e.g., micro-PMU noise, 10% error in smart meter data) are modeled. The information of the magnitude of errors (i.e., 60%) of bad data measurements and the measurement noise of meters are not communicated with the proposed simultaneous topology processor and state estimation algorithm. For instance, the active power error in all aggregated loads of the system in case B3 is shown in Fig. 9. As shown in Fig. 9, 40 aggregated smart meters data have an error equal to 60% for modeling bad data. The rest of the aggregated smart meters data shown in Fig. 9 are subject to 10% error due to the smart meters’ precision error modeled using the Gaussian distribution. Fig. 9 is only given to show the severity of the bad data measurements error and meters precision error for the case study B3. The input to the proposed model is only the measurements values of the smart meters, while the values of the associated error of the measurements (shown in Fig. 9) are unknown to the model.

Table III shows the results of the proposed topology processing in the case of bad data in the system. Both $M_1$ and $M_2$ metrics for evaluation of the topology processing accuracy in cases B1-B3 are the same as results of Table I without considering bad data in the measurements. Therefore, it can be inferred that the proposed tool is significantly robust in performing topology processing in the case of considering both measurement noise and bad data. At the same time, the proposed approach with the high topology detection accuracy captures 100% of the local electric utility feeder load in cases B1-B3, as shown in Table III. The errors of estimated values of system states for case B3 with the highest number of aggregated smart meter measurements with the bad data (i.e., 40) are depicted in Fig. 10. By comparing Fig. 6 with Fig. 10, it can be observed that state estimation errors for most of the nodes in the test system are comparable, and only the error for those nodes with the bad data is slightly higher.

![Image](image_url)
(e.g., maximum voltage magnitude error in case B3 < 0.9%). This comparison implies that the proposed tool is also robust in terms of estimating system states considering bad data in the system measurements. The range of simulation time for cases B1-B3 is from 17 seconds to 22 seconds.

2) Bad Data Analysis With Probabilistic Error and Bad Data Detection: In Section IV. B. 1, the bad data analysis is conducted in the presence of the bad data in the measurements without detecting and replacing them. In this section, a bad data analysis is conducted to evaluate the robustness of the proposed simultaneous topology processing and state estimation model in the presence of the bad data, while a bad data detection method is utilized to identify and replace the measurements with the bad data. To this end, a pre-state estimation bad data detection method based on a composite hypothesis testing is utilized [20]. Most of the bad data detection methods detect bad data by residual errors after solving the state estimation. The bad data detection based on the hypothesis testing detects and identifies bad data prior to the state estimation and is suitable for distribution systems with low measurement redundancy [20]. The null hypothesis $H_0$ is considered for the measurement $r$ without the bad data with Gaussian distribution with mean $\mu_0$ and standard deviation $\sigma_0$. The alternative hypothesis $H_1$ is considered for measurement $r$ with the bad data with different distribution than the measurement under $H_0$. The Wald test is used to decide the rejection of $H_0$ by assessing whether the normalized distance of a measurement from $\mu_0$ is beyond a threshold. The size $\alpha$ Wald test using the Gaussian tail probability function, i.e., $Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} exp(-u^2/2)du$, is presented in (6a) [20].

$$\frac{r - \mu_0}{\sigma_0} \begin{cases} \mathcal{H}_1, & z_{\alpha/2} := Q^{-1}_{\alpha/2} \\ \mathcal{H}_0, & \end{cases}$$  \hspace{1cm} (6a)$$

In the Wald test (6a), the size $\alpha$ is mostly considered to be 0.05, which implies that the probability of the false alarm is less than 5%. Three cases, i.e., cases C1-C3, in which measurements are beyond their standard deviation but not detectable through simple comparisons are considered. For the bad data modeling, the aggregated smart meter data at the secondary of 35 distribution transformers (out of 342 load buses) of the local electric utility system are corrupted probabilistically using a Gaussian distribution with zero mean and standard deviation $\sigma_1$, which is larger than $\sigma_0$ under null hypothesis $H_0$. In case C1, the standard deviation of the measurements with the bad data is considered as $2\sigma_0 \leq \sigma_1 \leq 3\sigma_0$. In case C2 and case C3, the standard deviation of the measurements with bad data is considered as $\sigma_1 = 3\sigma_0$ and $\sigma_1 = 4\sigma_0$, respectively. It should be noted that the standard deviation of the measurements with bad data, i.e., $\sigma_1$, is unknown to the bad data detection and the proposed simultaneous topology processing and state estimation model in all cases C1-C3. Table IV shows the accuracy of the proposed topology processing in cases C1-C3 (i) with the presence of bad data without detecting, filtering, or replacing them, (ii) with the presence of bad data utilizing the bad data detection and replacement algorithm. The total number of bad data detected out of 35 aggregated smart meter measurements with bad data using the Wald test in cases C1-C3 are shown in Table IV. Each identified bad data measurement by the Wald test with bad data using the Wald test in cases C1-C3 are shown in Table IV.

![Fig. 10. State estimation error for case B3 (Left) voltage magnitude (Right) voltage angle for each node of the local electric utility network.](Image)

![Fig. 11. State estimation error for case C3 in the presence of bad data without bad data detection and replacement (Left) voltage magnitude (Right) voltage angle for each node of the local electric utility network.](Image)

![Fig. 12. State estimation error for case C3 with bad data detection and replacement (Left) voltage magnitude (Right) voltage angle for each node of the local electric utility network.](Image)

### TABLE IV

| Case               | C1   | C2   | C3   |
|--------------------|------|------|------|
| Number of aggregated smart meter measurements modeled as bad data | 35   | 35   | 35   |
| Measurement noise  | Yes  | Yes  | Yes  |
| Number of bad data detected by Wald test | 16   | 23   | 23   |
| $M_1$ accuracy (%) without bad data detection | 100  | 100  | 100  |
| $M_2$ accuracy (%) without bad data detection | 98.60| 98.60| 98.60|
| Captured feeder load (%) without bad data detection | 100  | 100  | 100  |
| $M_1$ accuracy (%) | 100  | 100  | 100  |
| $M_2$ accuracy (%) | 98.60| 98.60| 98.60|
| Captured feeder load (%) | 100  | 100  | 100  |
with the bad data detection and replacement algorithm are equal to the results of Table I without any measurements with the bad data. Therefore, it can be inferred that the proposed topology processing model is robust with the presence of bad data in the measurements.

Also, the state estimation results of case C3 without and with bad data detection and replacement algorithm are shown in Fig. 11 and Fig. 12, respectively. By comparing Fig. 11 and Fig. 12, it can be observed that although the proposed simultaneous state estimation and topology processing method is robust in the presence of the bad data in the measurements in case C3 (shown in Fig. 11), the accuracy of the proposed state estimation model is further enhanced by utilizing the bad data detection and replacement algorithm (shown in Fig. 12). The state estimation results of cases C1 and C2 are the same as case C3, all of which are accurate and robust in the presence of the bad data in the measurements and without bad data detection (maximum voltage magnitude and absolute angle errors in case C1 are 0.524% and 0.0052 rad, and in case C2 are 0.799% and 0.0103 rad, respectively). Similar to case C3, the accuracy of the model in cases C1 and C2 is increased further by using the bad data detection and replacement algorithm (maximum voltage magnitude and absolute angle errors in case C1 are 0.506% and 0.004 rad, and in case C2 are 0.373% and 0.003 rad, respectively).

C. Practical Challenges of Utilizing Smart Meter and Micro-PMU in Distribution Systems

In the results of Tables I–III, it is assumed that the smart meters can provide active and reactive power at the same time that the micro-PMU measurements are received at the feeder head. In this section, the practical challenges of exploiting the smart meters and the micro-PMUs measurements in the distribution system are examined: (1) smart meters usually measure the electric energy consumption over a 15 minutes interval without the reactive power of load; (2) integration of instantaneous synchronized micro-PMUs measurements with unsynchronized average smart meters measurements. In order to address these challenges, constraint (5a) is added to the proposed model for estimating the reactive power of the smart meters. Moreover, in addition to the 10% measurement error of the smart meter devices, 30% variation using Gaussian distribution is modeled for smart meters measurements due to integrating them with the micro-PMU measurements with different sampling rates and approaches. The modeled active load deviations of the smart meters of the test system are shown in Fig. 13. It should be noted that all measurement noise of the devices in the system (e.g., micro-PMU, PV sensors) are also modeled. In this case, the total number of undetectable and wrongly identified (presently connected to no-load region and NOSSs) status of switches are zero and 12, respectively. Accordingly, \( M_1 \) is obtained as 100% and \( M_2 \) is equal to 98.60%. The 12 wrong statues, in this case, are the same as those presented in Table I without considering the practical challenges of smart meters and micro-PMUs. These same results convey that the proposed primary topology processing model is highly precise even when considering the measurements noise of all sensors and integration of synchronized instantaneous measurements of micro-PMU with average active power measurements of smart meter devices. Also, the identified topology correctly captures 100% of the system load. The average simulation time for this case is 28 seconds. The error of state estimation using the proposed model for each node of the system is demonstrated in Fig. 14. Comparing Fig. 6 (i.e., results of the case without considering practical challenges of smart meters and micro-PMUs) with Fig. 14 illustrates that the accuracy of the proposed simultaneous topology processing and state estimation tool even while considering measurement devices with different sampling rates and approaches is significantly high.

D. Missing Data Analysis

Another practical challenge of the real-time topology processing and state estimation tool is missing AMI data due to communication issues, the meters malfunction, or bad data. There are around 6-8 smart meters aggregated at the secondary of the distribution transformers in the test system. In order to analyze the robustness of the proposed real-time tool, a case is considered, where some of the smart meter reading data at 246 buses (out of 342 load buses) are missing randomly. It is assumed that the maximum missed smart meter reading data is 50% of the aggregated smart meters in each primary node. In this case study, all measurements noise of sensor devices (e.g., micro-PMU, smart meter) in the test system are modeled. Also, all other practical challenges presented in Section IV-C, i.e., 30% load variations due to the integration of smart meters and micro-PMU data and inability of the smart meters for providing the reactive power data of the loads, are considered.
In this paper, an approach is proposed for dealing with the missing data to enhance the accuracy of the proposed model. In this regard, missing smart meter measurement of each load is estimated based on the measurement of the previous time interval. The simulation is conducted on July 15, 2019, at 5:00 pm. The results of the proposed topology processing considering estimating missed smart meter data approach are $M_1 = 100\%$ and $M_2 = 98.60\%$. The total number of undetectable status of switches is zero. The total number of wrong statuses is 12, which their locations are shown in Fig. 5. The average simulation time for this case is 29 seconds. The error of estimated states of the electric utility system is depicted in Fig. 15. Comparing Fig. 6, Fig. 14, and Fig. 15 confirm that the proposed approach is robust to missing data while considering all measurement noise of the sensors and the practical challenges of the smart meters and the micro-PMU.

### E. Choice of Objective Function

The proposed simultaneous outage, topology, and state identification model is a MIQP problem with an Euclidean norm squared objective function. As mentioned in Section II-C, such an objective function is inherently convex, which can be solved using a branch and bound method in which the global optimal solution can be achieved by setting the optimality gap equal to zero. There are other types of norms used for solving state estimation and topology processor problems, the main among which is the absolute-value norm. The proposed simultaneous outage, topology, and state identification model using the absolute-value norm objective function can be written as follows:

$$x^* = \arg \min |W(\Re(x) - \Re)| \quad (7a)$$

s.t. \((1d)-(1g), (3a)-(3q), (3t) - (3y), (4a) - (4e)\)

The absolute-value norm objective function of \((7a)\) is a nonlinear nonconvex norm, which makes the model a mixed-integer nonlinear programming (MINLP) optimization problem. Such problems may not converge and may get stuck at a local optimal solution. In order to overcome this issue, a linearization approximation of the model is required. The linear approximation of the objective function in \((7a)\) is presented in \((7b)\) with added constraints \((7c)\) and \((7d)\).

$$x^* = \arg \min (W(\gamma + \beta)) \quad (7b)$$

$$R(x) - R = \gamma - \beta \quad (7c)$$

$$\gamma \geq 0 \text{ and } \beta \geq 0 \quad (7d)$$

where $\gamma$ and $\beta$ are nonnegative variable vectors. Hence, the proposed simultaneous outage, topology, and state identification model can be formulated as a MILP problem with a linearized objective function \((7b)\) and constraints \((1d)-(1g), (3a)-(3q), (3t) - (3y), (4a) - (4e),\) and \((7c)-(7d)\). The MILP model with linearized absolute-value norm objective function is tested on a normal condition of the local electric utility system, and its results are compared with those of the proposed MIQP model with Euclidean norm squared objective function shown in Table I and Fig. 6. It should be noted that the optimality gap is set to 0 in the Gurobi solver for solving the MILP problem as well. The results of topology processing using the MILP model are $M_1 = 100\%$ and $M_2 = 98.60\%$, which are the same as the results of MIQP in Table I. Fig. 16 shows the results of the state estimation using the MILP model. By comparing Fig. 6 and Fig. 16, it can be seen that the accuracy of the state estimation model using the MILP model is less than the proposed MIQP model. The reason is that the linear reformulation of the absolute-value norm objective function is subject to error, which leads to an inaccurate solution of the state estimation. However, the objective function of the proposed MIQP is inherently convex and no reformulation is needed. In the future work, the proposed model with Euclidean norm squared objective function will be compared with other types of norms, which only requires changing the objective function of the proposed model while all of the constraints of the proposed model will be intact.

### V. Conclusion

In this paper, an efficient MIQP-based optimization model is proposed to simultaneously identify real-time network topology, estimate system state, and detect outages of unbalanced distribution systems. An ACOPF approach based on current-voltage (IV) is developed, which models unbalanced distribution systems precisely. Moreover, the challenges of integrating micro-PMUs and smart meters data with different sampling rates and approaches (i.e., synchronized instantaneous versus unsynchronized average) is studied using a statistical study. The proposed model is further developed to overcome the limitation of the lack of reactive power measurements from smart meters. The results illustrate that the proposed model can identify different...
topologies and outages of an actual electric utility system in Arizona with a high accuracy greater than 98% considering the challenges and measurement noise of the sensors in the system. Also, the proposed model identifies the states of the system simultaneously with topologies and outages in real-time accurately and fast (average simulation time is in the range of 15 seconds to 29 seconds). The robustness of the proposed approach is evaluated by modeling missing data and bad data in the sensors measurements. The results show that the proposed algorithm is significantly robust against bad measurement data and missing data in terms of identifying the topology and states of the system. Also, 100% of the local utility feeder load is captured using the proposed tool in all case studies. In future work, we will extend our current approach to consider uncertainty of network connections and line parameters.

References

[1] B. Hua, Y. Li, A. Liu, S. Wang, J. Xu, and S. Shi, “Topology analysis of distribution network based on multi-source measurement data,” in Proc. Asia Energy Elect. Eng. Symp., 2020, pp. 531–537.

[2] O. Ardakanian et al., “On identification of distribution grids,” IEEE Trans. Control Netw. Syst., vol. 6, no. 3, pp. 950–960, Sep. 2019.

[3] S. Grotas, Y. Yakoby, I. Gera, and T. Routtenberg, “Power systems topology and state estimation by graph blind source separation,” IEEE Trans. Signal Process., vol. 67, no. 8, pp. 2036–2051, Apr. 2019.

[4] L. Zhao, W. Song, L. Tong, Y. Wu, and J. Yang, “Topology identification in smart grid with limited measurements via convex optimization,” in Proc. IEEE Innov. Smart Grid Technol. - Asia, 2014, pp. 803–808.

[5] B. Hayes, A. Escalera, and M. Prodanovic, “Event-triggered topology identification for state estimation in active distribution networks,” in Proc. IEEE PES Innov. Smart Grid Technol. Conf. Europe, 2016, pp. 1–6.

[6] G. Cavraro and R. Arghandeh, “Power distribution network topology detection with time-series signature verification method,” in Proc. IEEE Trans. Power Syst., vol. 33, no. 4, pp. 3500–3509, Jul. 2018.

[7] G. Cavraro, V. Kekatos, and S. Veeramachaneni, “Voltage analytics for power distribution network topology verification,” IEEE Trans. Smart Grid, vol. 10, no. 1, pp. 1058–1067, Jan. 2019.

[8] N. Duan and E. M. Stewart, “Deep-learning-based power distribution network switch action identification leveraging dynamic features of distributed energy resources,” IET Gener. Transmiss. Distrib., vol. 13, no. 14, pp. 3139–3147, 2019. [Online]. Available:https://digital-library.theiet.org/content/journals/10.1049/iet-gtd.2018.6195

[9] O. Ardakanian, Y. Yuan, R. Dobbe, A. von Meier, S. Low, and C. Tomlin, “Event detection and localization in distribution grids with phasor measurement units,” in Proc. IEEE Power Energy Soc. Gen. Meeting, 2017, pp. 1–5.

[10] Y. Jiang, C.-C. Liu, M. Diedesch, E. Lee, and A. K. Srivastava, “Outage management of distribution systems incorporating information from smart meters,” IEEE Trans. Power Syst., vol. 31, no. 5, pp. 4144–4154, Sep. 2016.

[11] R. A. Sevlian, Y. Zhao, R. Rajagopal, A. Goldsmith, and H. V. Poor, “Outage detection using load and line flow measurements in power distribution systems,” IEEE Trans. Power Syst., vol. 33, no. 2, pp. 2053–2069, Mar. 2018.

[12] B. Alnajjab, A. N. Samudrala, C. Chen, R. S. Blum, S. Kar, and E. M. Stewart, “Outage detection for distribution networks using limited number of power flow measurements,” J. Modern Power Syst. Clean Energy, vol. 8, no. 2, pp. 315–324, 2020.

[13] M. Farajollahi, A. Shahsavari, and H. Mohsenian-Rad, “Topology identification in distribution systems using line current sensors: An MILP approach,” IEEE Trans. Smart Grid, vol. 11, no. 2, pp. 1159–1170, Mar. 2020.

[14] A. Gandhurru, S. Poudel, and A. Dubey, “Joint estimation of operational topology and outages for unbalanced power distribution systems,” IEEE Trans. Power Syst., vol. 35, no. 1, pp. 608–617, Jan. 2020.

[15] X. Li, B. Han, G. Li, L. Luo, K. Wang, and X. Jiang, “Dynamic topology awareness in active distribution networks under DG uncertainties using GMM-PSEs and KL divergence,” IEEE Trans. Sustain. Energy, vol. 12, no. 4, pp. 2086–2096, Oct. 2021.

[16] Y. Liu, J. Li, and L. Wu, “State estimation of three-phase four-conductor distribution systems with real-time data from selective smart meters,” IEEE Trans. Power Syst., vol. 34, no. 4, pp. 2632–2643, Jul. 2019.

[17] Y. Zhang and J. Wang, “Towards highly efficient state estimation with nonlinear measurements in distribution systems,” IEEE Trans. Power Syst., vol. 35, no. 3, pp. 2471–2474, May 2020.

[18] Y. Yao, X. Liu, D. Zhao, and Z. Li, “Distribution system state estimation: A semidefinite programming approach,” IEEE Trans. Smart Grid, vol. 10, no. 4, pp. 4369–4378, Jul. 2019.

[19] A. S. Zaman, X. Fu, and N. D. Sidiropoulos, “Data-driven learning-based optimization state estimation in distribution systems,” IEEE Trans. Power Syst., vol. 34, no. 6, pp. 4796–4805, Nov. 2019.

[20] K. R. Mestav, J. Luengo-Rozas, and L. Tong, “Bayesian state estimation for unobservable distribution systems via deep learning,” IEEE Trans. Power Syst., vol. 34, no. 6, pp. 4910–4920, Nov. 2019.

[21] A. Primadianto and C. Liu, “A review on distribution system state estimation,” IEEE Trans. Power Syst., vol. 32, no. 5, pp. 3875–3883, Sep. 2017.

[22] M. E. Baran, J. Jung, and T. E. McDermott, “Topology error identification using branch current state estimation for distribution systems,” in Proc. Transmiss. Distrib. Conf. Exposition: Asia Pacific, 2009, pp. 1–4.

[23] R. Singh, E. Manitssas, B. C. Pal, and G. Strbac, “A recursive bayesian approach for identification of network configuration changes in distribution system state estimation,” IEEE Trans. Power Syst., vol. 25, no. 3, pp. 1329–1336, Aug. 2010.

[24] G. N. Korres and N. M. Manousakis, “A state estimation algorithm for monitoring topology changes in distribution systems,” in Proc. IEEE Power Energy Soc. Gen. Meeting, 2012, pp. 1–8.

[25] D. B. Arnold, M. Sankur, R. Dobbe, K. Brady, D. S. Callaway, and A. V. Meier, “Optimal dispatch of reactive power for voltage regulation and balancing in unbalanced distribution systems,” in Proc. IEEE Power Energy Soc. Gen. Meeting, 2016, pp. 1–5.

[26] EPRI, “Simulation tool: Opensim.” [Online]. Available: https://smartgrid.epri.com/SimulationTool.aspx

[27] A. Arif, Z. Wang, J. Wang, and C. Chen, “Power distribution system outage management with co-optimization of repairs, reconfiguration, and DG dispatch,” IEEE Trans. Smart Grid, vol. 9, no. 5, pp. 4109–4118, Sep. 2018.

[28] A. Von Meier, E. Stewart, A. McEachern, M. Andersen, and L. Mehrmehrnan, “Precision micro-synchrophasors for distribution systems: A summary of applications,” IEEE Trans. Smart Grid, vol. 8, no. 6, pp. 2926–2936, Nov. 2017.

[29] “Pecan street inc. dataset,” 2019. [Online]. Available: https://dataset pecanstreet.org

[30] Y. Lin and A. Abur, “Robust state estimation against measurement and network parameter errors,” IEEE Trans. Power Syst., vol. 33, no. 5, pp. 4751–4759, Sep. 2018.

[31] A. Abur and E. A. Gomez, Power System State Estimation: Theory and Implementation. Boca Raton, FL, USA: CRC press, 2004.

[32] R. Dobbe, W. van Westering, S. Liu, D. Arnold, D. Callaway, and C. Tomlin, “Linear single- and three-phase voltage forecasting and bayesian state estimation with limited sensing,” IEEE Trans. Power Syst., vol. 35, no. 3, pp. 1674–1683, May 2020.

[33] J. Liu, J. Tang, F. Ponci, A. Monti, C. Muscas, and P. A. Pegoraro, “Trade-offs in PMU deployment for state estimation in active distribution grids,” IEEE Trans. Smart Grid, vol. 3, no. 2, pp. 915–924, Jun. 2012.

Zahra Soltani (Graduate Student Member, IEEE) received the B.Sc. degree in electrical engineering from the Arak University, Arak, Iran, in 2014, and the M.Sc. degree in electrical engineering from the Amirkabir University of Technology, Tehran, Iran, in 2018. She is currently working toward the Ph.D. degree in electrical engineering with Arizona State University, Tempe, AZ, USA. Her research interests include optimization, distributed energy resources modeling and scheduling, and power system operation including topology processing and state estimation.
Shanshan Ma (Member, IEEE) received the B.S. degree in information and electrical engineering from Zhejiang University City College, Hangzhou, China, in 2012, the M.S. degree from the Department of Electrical Engineering and Computer Science, South Dakota State University, Brookings, SD, USA, in 2015, and the Ph.D. degree with the Department of Electrical and Computer Engineering, Iowa State University, Ames, IA, USA, in 2020. She is a Postdoctoral Research Scholar with the School of Electrical, Computer and Energy Engineering, Arizona State University, Tempe, AZ, USA. Her research interests include optimization and control in power distribution systems.

Mojdeh Khorsand (Member, IEEE) received the B.Sc. degree in electrical engineering from the University of Mazandaran, Babolsar, Iran, in 2007, the M.Sc. degree in electrical engineering from the Iran University of Science and Technology, Tehran, Iran, in 2010, and the Ph.D. degree in electrical engineering from Arizona State University, Tempe, AZ, USA, in 2017. She is currently an Assistant Professor with the School of Electrical, Computer, and Energy Engineering, Arizona State University. Her research interests include power system operations, advanced distribution management systems, electric energy markets, transient stability, and protection systems.

Vijay Vittal (Life Fellow, IEEE) received the B.E. degree in electrical engineering from the B.M.S. College of Engineering, Bangalore, India, in 1977, the M.Tech. degree from IIT Kanpur, India, in 1979, and the Ph.D. degree from Iowa State University, Ames, IA, USA, in 1982. From 2005 to 2020, he was the Director with the Power System Engineering Research Center. He is currently a Regents’ Professor and the Ira A. Fulton Chair Professor with the Department of Electrical, Computer, and Energy Engineering, Arizona State University, Tempe, AZ, USA. Dr. Vittal is a member of the National Academy of Engineering.