Relation between Functional Complexity, Scalability and Energy Efficiency in WSNs

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Abstract—In order to understand the underlying mechanisms that lead to certain network properties (i.e. scalability, energy efficiency) we apply a complex systems science approach to analyze clustering in Wireless Sensor Networks (WSN). We represent different implementations of clustering in WSNs with a functional topology graph. Different characteristics of the functional topology provide insight into the relationships between system parts that result in certain properties of the whole system. Moreover, we employ a complexity metric - functional complexity (\( C_F \)) - to explain how local interactions give rise to the global behavior of the network. Our analysis shows that higher values of \( C_F \) indicate higher scalability and lower energy efficiency.

Keywords—Complex systems science, functional complexity, wireless sensor networks, clustering, scalability, energy efficiency

I. INTRODUCTION

As Wireless Sensor Networks (WSNs) represent large deployments of unattended sensors, which are disposable and expected to last until their energy drains, energy efficiency and scalability are critical factors involved in the design of communication protocols for these devices. The physical topology is dynamic because nodes exit due to the battery discharge resulting in a constant need for adding new nodes to the network. In order to adjust to the constant changes in the physical topology, clustering was proposed in the literature as a network reorganization technique.

Clustering partitions a network of nodes into a number of smaller groups (clusters). In addition to the energy efficiency, which explicitly affects the network lifetime, clustering algorithms have a great influence on scalability, load balancing, fault-tolerance, delay reduction, etc. [1]. Understanding the organizational and communication characteristics of different clustering algorithms allows us to comprehend which aspects of a specific implementation lead to certain characteristics, i.e. scalability and energy efficiency. The manner in which parts of the network share information and the extent to which information spreads throughout the network represent important aspects of a clustering algorithm, as they directly impact the system longevity and the scalability of the network. The complex systems science approach we adopt allows us to investigate the amount of information about the system subparts by examining the system as a whole and comparing this to the actual amount of information that exists within the system subparts. In other words, this allows us to quantify the amount of uncertainty of interaction that exists within smaller subparts of the system compared to the uncertainty of the whole system.

Different approaches to clustering are available in the literature. The authors of [2]–[14] introduced different approaches, which involve adaptive clustering, random competition-based clustering, Hierarchical Control Clustering (HCC), energy efficient hierarchical clustering, distributed clustering, Low Energy Adaptive Clustering Hierarchy (LEACH), and Hybrid Energy-Efficient Distributed (HEED) clustering. As highlighted in [1] and [15], the algorithms differ in properties like stability of the created clusters, objectives (e.g. scalability, fault-tolerance, connectivity, load balancing, redundancy elimination, rapid convergence, network lifetime), clustering criteria (e.g. identifier, position, cluster head frequency, residual energy), methodology (e.g. distributed, centralized, hybrid). We focus on the LEACH algorithm proposed in [13] and the HCC algorithm proposed in [12], due to the importance of these algorithms for WSNs (the LEACH algorithm is one of the most well-known clustering algorithms, and the HCC algorithm is the most popular multi-tier hierarchical clustering algorithm [1]).

Complex systems science focuses on the underlying local interactions between system parts which give rise to the global network behavior. In [16], we propose a framework which allows us to represent network functions with graphs called functional topologies. Therein, we also propose a metric to calculate the functional complexity of an implementation of a network function. Here, we employ our functional framework to model different implementations for clustering in WSNs. Our goal is to highlight the structural patterns present in the functional topology that result in certain properties (scalability and energy efficiency) of the implementation. This allows us to understand the underlying mechanisms that are a product of complex interactions between functional entities.

The main contributions of this paper are:

- We apply a complex systems science approach to analyze clustering in WSNs;
- Our study shows how the density of local connections in the functional topology affects the scalability of a clustering implementation;
- Our results highlight the structural patterns in the functional topology that lead to higher energy efficiency of the WSN;
- Our functional complexity allows us to analyze the trade-off between energy efficiency and scalability of a clustering implementation.

II. CLUSTERING ALGORITHMS

Considering the limited energy resources and the size of WSNs, the best approach is to partition the network into in-
Fig. 1. The topology created according to the LEACH algorithm. Ordinary nodes transfer sensing information to the cluster-heads, which forward this information to the base station (The Figure is redrawn from [3]).

terconnected subgroups (clusters), whose local behavior gives rise to the global objective of the network (e.g. minimizing the energy consumption). Generally, clustering includes two phases:

- **Set-Up Phase** - this phase implies choosing cluster-heads. Cluster-heads are nodes that are responsible for the coordination of the clustering process.
- **Maintenance Phase** - this phase implies reorganization of the clusters in order to add new nodes to the cluster, deal with nodes that leave the cluster, or simply change the roles of nodes (cluster-head or ordinary node) in the cluster in order to achieve higher energy efficiency [17].

As the maintenance phase implies the reorganization of clusters in order to achieve higher energy efficiency and to adjust to changes in the topology, we focus on this phase. As mentioned previously, we consider two clustering algorithms (the LEACH and the HCC algorithms) proposed in [13] and [12].

The LEACH algorithm is based on rounds. Each round includes both phases (setup and maintenance phase). Each node in the network runs the algorithm, and decides randomly which role to play (cluster-head or ordinary node). When a node decides to be a cluster-head it broadcasts this information, whereas each node that decided to be an ordinary node listens to the broadcast messages and joins the closest cluster-head. After all ordinary nodes join one of the cluster-heads, the set-up phase finishes and the maintenance phase starts. In the maintenance phase nodes that belong to the same cluster transfer sensing information to the cluster-head (Figure 1), and as the LEACH algorithm involves rotating the role of cluster-heads between nodes in a cluster, the nodes that belong to the same cluster communicate to each other in order to choose the next cluster-head candidate.

The HCC algorithm is a multi-tier hierarchical clustering algorithm which has proven to be highly scalable. The set-up phase involves the Breath-First Search (BFS) tree discovery and cluster formation. The tree discovery is a distributed formation of a BFS tree rooted at the initiator node. The cluster formation is shown in Figure 2. The clustering formation process is distributed and it is executed on each node. Based on the tree discovery each node knows its parent node and its child nodes. Each node \( i \) discovers the size of its downstream subtree \( |V_i| \) and reports this to its parent node. The information about the subtree sizes allows us to create the clusters based on the defined cluster size \( k \) (number of nodes per cluster - this number is predefined). Each node compares its subtree size \( |V_i| \) to the defined cluster size \( k \) and if \( k \leq |V_i| < 2k \) the node initiates the cluster formation on its subtree. As the BFS tree denotes the routes to the base station, the sensing information of all sensors is transmitted using these routes. The maintenance phase involves just the maintaining up to date information on each node about its parent and child nodes.

### III. FUNCTIONAL FRAMEWORK

In order to analyze the underlying connectivity patterns of a clustering algorithm, we employ the functional framework introduced in [16]. The framework allows us to represent the implementation of a clustering algorithm with a graph called functional topology. We create functional topologies based on the functional connectivity between system parts, where each node represents a functional entity or any information source related to the implementation of the particular clustering algorithm, and each link indicates interactions between nodes. To quantify the variety of connection patterns between system entities and the roles that these entities have in the topology we employ a metric called functional complexity [16]. The multi-scale functional complexity is calculated with equation (1).

\[
C_F = \frac{1}{R-1} \sum_{r=1}^{R-1} \sum_{j=1+r}^{N} |I_r(A^j) - \frac{j}{N} I_r(A^N)|
\]  

(1)

The meaning of the terms appearing in equation (1) is shown in Table I. As WSNs rely on the communication established between one hop neighbors, in our analysis the maximum scale size will be equal to \( R = 2 \). Hence, we perform a single-scale
TABLE I.

| Symbol | Meaning |
|--------|---------|
| $N$    | total number of nodes in the functional topology |
| $j$    | subgraph size - number of nodes in the subgraph |
| $r$    | scale size |
| $R$    | maximum scale size, which is defined as the longest shortest path in the whole functional topology |
| $H(x_n)$ | entropy of node $n$ which indicates the uncertainty of interactions of node $n$ in the operation of a network function |
| $I_r(A^j_k)$ | $k^{th}$ subgraph with $j$ nodes |
| $I_r(N^r)$ | the total amount of information of the subgraph with $N$ nodes for scale $r$ |
| $⟨I_r(N^r)⟩$ | the average amount of information over all subgraphs with the size $j$ |

The functional complexity compares the uncertainty of interactions for a smaller subset ($⟨I_r(N^r)⟩$) to the uncertainty which is expected from the calculation performed on the whole system ($I_r(A^N)$). $H(x_n)$ reaches its maximum if the probability of interaction with node $n$ is $p(x_n = 1) = 1/2$. As the distribution of links among nodes for a sparse graph is almost uniform, a sparse graph results in high values of $H(x_n)$. High values of $H(x_n)$ result in high values of $I_r(A^j_k)$. Therefore, the functional complexity is high for a sparse graph, with uniformly distributed links among nodes for subgraphs with the size $j < N$. The functional complexity is zero for a fully connected and for a disconnected graph. For more details about the functional framework and the complexity metric expressed by equation (1) the reader is referred to [16].

The graph shown in Figure 3 is an undirected graph created according to the Von Neumann neighborhood. It depicts an example of a physical topology of a Wireless Sensor Network (WSN). A wireless node $A$ can transmit information to another wireless node $B$ only if node $B$ is within the transmission radius $R_A$ of node $A$. In the case of WSNs, we consider the communication between two nodes established only if both nodes can transmit information to each other. In other words, we consider that two nodes can exchange information only if the distance between them is $d(A, B) \leq \min\{R_A, R_B\}$.

In [16], we presented an approach to map different frequency allocation algorithms into functional topologies. We apply the same approach to examine the functional topologies of the LEACH and HCC algorithms.

Our goal is to investigate the influence of the interactions among nodes after the clusters are established, on the objectives of clustering algorithms (specifically scalability and energy efficiency). We start with the LEACH algorithm. As discussed before, during the maintenance phase of the LEACH algorithm ordinary nodes that belong to the same cluster send their sensing information to the cluster-head, and they talk to each other in order to decide who is the next candidate for the cluster-head role. According to the approach in [16], we imagine a virtual decision maker entity that is moving from one node to another. At each ordinary node the decision maker entity communicates with the cluster-head and all other ordinary nodes that belong to the same cluster. At each cluster-head the decision maker entity communicates with each ordinary node that belongs to the same cluster and with the base station. This results in a functional topology with dense local connections (Figure 4).

To examine the functional topology of the HCC algorithm we use the same approach presented in the previous example. Again we focus on the maintenance phase of the algorithm. After the set-up phase the nodes establish connections to their neighboring nodes according to the BFS algorithm. Each node discovers its subtree, and exchanges information with its neighbors in the BFS tree. Again, we imagine a virtual decision maker entity that is moving from one node to another. At each
node the decision maker entity collects information from nodes that belong to its subtree and forwards this information to its parent. In other words, each node maintains its position in the BFS tree and therefore the functional topology of the HCC algorithm is equivalent to the BFS tree of the physical topology (Figure 5).

IV. Analysis

In order to transfer the information collected at a sensor node to a base station, WSNs establish paths in an ad-hoc manner. Two nodes can exchange information if and only if they are within the transmission radius of each other. However, after the set-up phase of the clustering algorithm finishes, nodes do not communicate to all other nodes that are within their coverage area. At this stage, the information exchange depends on the rules of the clustering algorithm.

The topology of WSNs is subject to constant changes due to the disposable nature of wireless sensors and the constant need to expand and densify the sensing area. Therefore, besides the energy efficiency, clustering algorithms for WSNs have to exhibit scalability. Scalability is the capability of the network to adapt to new nodes joining the network, existing nodes leaving the network, and other nodes migrating from one cluster to another [12].

The maintenance phase of the LEACH algorithm invokes the set-up phase, where nodes that belong to the same cluster talk to each other to decide which node is the next candidate for the cluster-head role, and each node transfers sensing information to the current cluster-head. Adding a new node to any cluster means establishing a connection to all nodes within the cluster because each node has to be aware of all other nodes that belong to the same cluster, in order to maintain the process of cluster-head elections.

The maintenance phase of the HCC algorithm is simpler than that of the LEACH algorithm. Each node forwards the information of its children and transfers its own sensing information to its parent node. Therefore, adding a new node to the cluster simply means that the new node gets connected to one of the existing nodes which is going to be its parent node. The new node does not need to inform all nodes in the cluster about its arrival, which simplifies the process of adding nodes to the network.

The authors of [2]–[5] and [18] discuss different objectives of clustering algorithms. Among other things, the authors focus on scalability issues and network longevity (energy efficiency) of these algorithms. Hierarchical approaches proved themselves to be more scalable than their non-hierarchical counterpart. The authors of [4] showed that a trade-off exists between network scalability and energy consumption in clustering schemes. Our goal is to investigate the relationship between these objectives and the functional complexity, which would allow us to analyze them together. In [2]–[5] and [18], the authors emphasize that the LEACH algorithm is designed to extend the network longevity, whereas the HCC algorithm targets scalability as the main objective. In other words, the LEACH algorithm is less scalable and the HCC algorithm is less energy efficient.

As the authors of [1]–[5] agree that the communication between the cluster-head and the base station consumes most energy, we propose to calculate the energy efficiency of a clustering implementation as the ratio between the average number of intra-cluster connections and the number of links between the base station and each cluster-head in the functional topology. If the ratio increases the energy efficiency increases, due to the bigger number of intra-cluster connections as compared to the number of connections between the base station and the cluster-heads. As the scalability of the networks represents the adaptability to changes, we calculate it as the average number of messages sent when a new node joins the network. If the average number of messages increases the scalability of the network decreases.
Our analysis confirms the observation made by the authors of [4], which highlights the trade-off between scalability and energy efficiency. With our complex systems science approach we showed that these aspects of different clustering algorithms can be analyzed together by analyzing the functional complexity of the specific implementation. According to Table II increasing values of $C_F$ lead to the increase of scalability and decrease of energy efficiency.

V. CONCLUSION

The growing interest in WSNs due to the various applications in IoT results in a great variety of clustering algorithms that support these applications. Very often, one compares these algorithms based on certain properties (e.g. fault-tolerance,
delivery delay, energy efficiency, scalability). Comparing these algorithms based on a broad range of properties is very difficult. Additionally, a lack of understanding exists when we try to comprehend the mechanisms that lead to these properties. The aim of this paper is to apply a complex systems science approach to analyze these mechanisms. We use the functional framework that we proposed in [16] to model different implementations of clustering in WSNs. The framework allows us to represent the implementations of network functions with graphs, which are called functional topologies.

The next step after mapping the implementations of clustering into functional topologies is to calculate the functional complexity ($C_F$) of these topologies. The functional complexity captures the variety of structural patterns in the topology and quantifies the deviation in uncertainty of interactions for a smaller subset of the system from the uncertainty which is expected from the calculation performed on the whole system. In other words, $C_F$ quantifies how much information we cannot capture simply by studying the whole system, due to the complex relationships that exist between smaller subsets of the system.

In this paper we focus on two clustering algorithms, i.e. the LEACH and the HCC algorithms. Our goal is to investigate the mechanisms that provide high scalability in the case of the HCC algorithm and high energy efficiency for the LEACH algorithm. We also study the trade-off between these two network properties. After briefly introducing the algorithms, we map their implementations into functional topologies, and calculate the corresponding functional complexities.

We show that high functional complexity indicates greater scalability of the implementation. We also show that increasing values of the functional complexity with the number of clusters, indicates lower energy efficiency. We then highlight that a sparse functional topology results in higher complexity. Therefore, our functional topology explicitly explains both the higher scalability (sparse graph) and lower energy efficiency of the HCC algorithm as compared to the LEACH algorithm.

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REFERENCES

[1] A. A. Abbasi and M. Younis, “A survey on clustering algorithms for wireless sensor networks,” Computer Communications, vol. 30, no. 14-15, pp. 2826–2841, 2007.
[2] M. M. Afzar and M. H. Tayarani-N, “Clustering in sensor networks: A literature survey,” Journal of Network and Computer Applications, vol. 46, pp. 198–226, 2014.
[3] M. Aslam, N. Javaid, A. Rahim, U. Nazir, A. Bibi, and Z. A. Khan, “Survey of extended LEACH-based clustering routing protocols for wireless sensor networks,” Proceedings of the 14th IEEE International Conference on High Performance Computing and Communications, HPCC-2012 - 9th IEEE International Conference on Embedded Software and Systems, ICESS-2012, pp. 1232–1238, 2012.
[4] C. Jiang, D. Yuan, and Y. Zhao, “Towards Clustering Algorithms in Wireless Sensor Networks-A Survey,” 2009 IEEE Wireless Communications and Networking Conference (WCNC2009), pp. 1–6, 2009.
[5] X. Liu, “A survey on clustering routing protocols in wireless sensor networks,” Sensors, vol. 12, no. 8, pp. 11113–11153, 2012.
[6] R. Nagpal and D. Coore, “An algorithm for group formation in an amorphous computer,” in Proc. 10th International Conference on Parallel and Distributed Computing Systems (PDCSâ ˘A ´Z98). Citeseer, 1998.
[7] A. Ephremides, J. E. Wieselthier, and D. J. Baker, “Design Concept for Reliable Mobile Radio Networks With Frequency Hopping Signaling.” Proceedings of the IEEE, vol. 75, no. 1, pp. 56–73, 1987.
[8] K. K. Xu and M. Gerla, “A heterogeneous routing protocol based on a new stable clustering scheme,” MILCOM 2002 Proceedings, vol. 2, pp. 838–843, 2002. [Online]. Available: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1179583
[9] D. J. Baker and A. Ephremides, “The Architectural Organization of a Mobile Radio Network via a Distributed Algorithm,” IEEE Transactions on Communications., vol. 29, no. 11, pp. 1694–1701, 1981.
[10] C. R. Lin and M. Gerla, “Adaptive clustering for mobile wireless networks,” IEEE Journal on Selected Areas in Communications, vol. 15, no. 7, pp. 1265–1275, 1997.
[11] S. Bandyopadhyay and E. Coyle, “An energy efficient hierarchical clustering algorithm for wireless sensor networks,” INFOCOM 2003. Twenty-Second Annual Joint Conference of the IEEE Computer and Communications., vol. 00, no. C, pp. 1713–1723, 2003. [Online]. Available: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1209194
[12] S. Banerjee and S. Khuller, “A Clustering Scheme for Hierarchical Control in Wireless Networks,” Proceedings of IEEE INFOCOM 2001, pp. 1028–1037, 2001.
[13] W. B. Heinzelman, A. P. Chandrakasan, and H. Balakrishnan, “An application-specific protocol architecture for wireless microsensor networks.” IEEE Transactions on Wireless Communications, vol. 1, no. 4, pp. 660–670, 2002.
[14] O. Younis and S. Fahmy, “Distributed clustering in ad-hoc sensor networks: A hybrid, energy-efficient approach,” in INFOCOM 2004. Twenty-third Annual Joint Conference of the IEEE Computer and Communications Societies, vol. 1. IEEE, 2004.
[15] O. Younis, M. Krunz, and S. Ramanathan, “Node clustering in wireless sensor networks: Recent developments and deployment challenges,” IEEE Network, vol. 20, no. 3, pp. 20–25, 2006.
[16] M. Dzaferagic, N. Kaminski, I. Macaluso, and N. Marchetti, “A Functional Complexity Framework for the Analysis of Telecommunication Networks,” The paper is submitted to the IEEE Systems Journal (currently under review), pp. 1–9, 2016. [Online]. Available: http://arxiv.org/abs/1607.02020
[17] S. Basagni, “Distributed clustering for ad hoc networks,” Proceedings Fourth International Symposium on Parallel Architectures, Algorithms, and Networks (I-SPAN’99), pp. 1–15, 1999.
[18] N. A. Pantazis, S. A. Nikolidakis, and D. D. Vergados, “Energy-efficient routing protocols in wireless sensor networks: A survey,” IEEE Communications Surveys & Tutorials, vol. 15, no. 2, pp. 551–591, 2013.