Combined effects of heat and mass transfer to magneto hydrodynamics oscillatory dusty fluid flow in a porous channel

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Abstract. The main aim of this article is to study the combined effects of heat and mass transfer to radiative Magneto Hydro Dynamics (MHD) oscillatory optically thin dusty fluid in a saturated porous medium channel. Based on certain assumptions, the momentum, energy, concentration equations are obtained. The governing equations are non-dimensionalised, simplified and solved analytically. The closed analytical form solutions for velocity, temperature, concentration profiles are obtained. Numerical computations are presented graphically to show the salient features of various physical parameters. The shear stress, the rate of heat transfer and the rate of mass transfer are also presented graphically.

1. Introduction

Dusty fluid flow plays an important role in many applications like waste water treatment, power plant piping, combustion and petroleum transport. Fluid flow under the influence of magnetic field and heat transfer appears in MHD accelerators, pumps and generators. This type of fluid is generally used in nuclear reactors, plasma studies, geothermal energy extraction and boundary layer control in the field of aerodynamics. The combined heat and mass transfer effects to MHD oscillatory dusty fluid flow through porous media has applications in the field of recovery of crude oil from pores of reservoir rocks, juice purification in sugar industry etc.

Saffman [1] proposed equations of motion for a binary mixture of fluid and dust particles. Makinde [2] made a detailed study of heat transfer to MHD oscillatory flow in a porous medium channel. Kulshretha and Puri [3] studied the wave structure in oscillatory Couette flow of a dusty gas. Gireesha et. al. [4] analyzed the heat transfer effects in an unsteady dusty fluid flow through a rectangular channel. Makinde [5] discussed the heat transfer transient of dusty fluid in a channel with variable physical properties and Navier slip condition. Prakash et. al. [6] investigated the MHD free convection flow of viscoelastic dusty gas through a semi infinite plate moving with velocity decreasing exponentially with time and radiative heat transfer. Govindarajan et. al. [7] examined the chemical reaction effects of MHD oscillatory flow in a porous medium channel. Vidhya et. al. [8] concentrated the chemical reaction effects of MHD oscillatory flow in an asymmetric channel. Om Prakash et. al. [9] studied the heat transfer effects to MHD oscillatory dusty fluid flow in a porous medium channel.
The main aim of this paper is to investigate the combined effects of heat and mass transfer to an electrically conducting, chemically reacting optically thin MHD oscillatory dusty fluid flow through a saturated porous medium channel. The temperature and concentration along the walls of the channel are not uniform. The governing equations of the fluid flow are solved mathematically using the relevant boundary conditions. The influence of various physical parameters on velocity, temperature and concentration profiles has been studied and numerical results obtained are presented graphically. The problem is formulated mathematically in Section 2. The solution of the problem is given in Section 3. The results are illustrated graphically and discussed in Section 4. The concluding remarks are given in Section 5.

2. Mathematical Formulation of the problem

Consider the optically thin, electrically conducting, heat generating, chemically reacting dusty fluid flow in a porous medium channel under the influence of an externally applied homogeneous magnetic field and radiative heat transfer as shown in Fig. 1.

Assume that the fluid conducts less electricity. Take a Cartesian coordinate system \((\bar{x}, \bar{y})\) where \(\bar{x}\) lies along the centre of the channel, \(\bar{y}\) is the distance measured in the normal direction.

The dust particles are solid, spherical, non-conducting equal in size and uniformly distributed in the flow region and their number density \(N_0\) is constant throughout the motion. The temperature between the particles is uniform throughout the motion. The interactions between the particles and chemical reaction effects are considered. The magnetic Reynolds number is considered to be very less so that induced uniform magnetic field is negligible and hence the Hall effects have been neglected. The dust particles are uniformly distributed and transported within the fluid such that the continuity equation is satisfied. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion, energy and concentration are as follows:

\[
\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k} u + \frac{N_0}{\rho} K_\rho (u_p - u) - \frac{\sigma B_0^2 u}{\rho} + g \beta (T - T_0) + g \beta^* (C - C_0) \tag{1}
\]

\[
\frac{\partial u_p}{\partial t} = K_\rho (u - u_p) \tag{2}
\]
\[
\frac{\partial T}{\partial t} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q}{\partial y} + \frac{Q}{\rho C_p} (T - T_0) \quad (3)
\]

\[
\frac{\partial C}{\partial t} = D_m \frac{\partial^2 C}{\partial y^2} - K'_c (C - C_0) \quad (4)
\]

together with the boundary conditions

\[ u(y, 0) = u_p(y, 0) = 0, T(y,0) = T_f, C(y, 0) = C_f \quad (5) \]

\[ u(a, t) = u_p(a, t) = 0, T(a, t) = T_f + (T_f - T_0) e^{i\omega t}, C(a, t) = C_f + (C_f - C_0) e^{i\omega t} \quad (6) \]

\[ u(0, t) = u_p(0,t) = 0, T(0, t) = T_0, C(0, t) = C_0 \quad (7) \]

According to Ogulu and Bestman [10], the radiative heat flux is given by

\[
\frac{\partial q}{\partial y} = 4\alpha^2 (T_0 - T) \quad (8)
\]

Introduce the dimensionless variables to non-dimensionalize the governing equations.

\[ \bar{x} = \frac{x}{a}; \quad \bar{y} = \frac{y}{a}; \quad \bar{u} = \frac{u}{U}; \quad \text{Re} = \frac{U a}{\nu}; \quad \theta = \frac{T - T_0}{T_f - T_0}; \quad \phi = \frac{C - C_0}{C_f - C_0}; \quad \bar{H}^2 = \frac{\sigma B_o^2 a^2}{\rho v}; \quad \bar{t} = \frac{t U}{a}; \quad \bar{p} = \frac{p a}{\rho v U}; \]

\[ Da = \frac{k}{a^2}; \quad \bar{s} = \frac{1}{Da}; \quad \text{Sc} = \frac{D_m}{a U}; \quad \text{Gr} = \frac{g \beta (T_f - T_0) a^2}{\nu U}; \quad \text{Pr} = \frac{v \rho C_p}{k}; \quad \text{Biot} = \frac{4\alpha^2 a^2}{K}; \]

\[ K_c = \frac{K_c a^2}{U}; \quad E = \frac{Q a^2}{K}; \quad M = \frac{\nu}{K_0 a^2}; \quad l = \frac{N_0 K_0 a^2}{\rho U} \quad (9) \]

3. Solution of the problem

The dimensionless governing equations together with the appropriate boundary conditions (after removing bars) can be written as

\[ \text{Re} \frac{\partial \bar{u}}{\partial t} = -\frac{\partial \bar{P}}{\partial \bar{x}} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - (\bar{s}^2 + \bar{H}^2 + l) \bar{u} + \lambda \bar{u}_p + Gr \bar{\theta} + Gc \bar{\phi} \quad (10) \]

\[ \text{Re} M \frac{\partial \bar{u}_p}{\partial t} = \bar{u} - \bar{u}_p \quad (11) \]

\[ \text{Re Pr} \frac{\partial \bar{\theta}}{\partial t} = \frac{\partial^2 \bar{\theta}}{\partial \bar{y}^2} + \left( N^2 + E \right) \bar{\theta} \quad (12) \]

\[ \frac{\partial \bar{\phi}}{\partial t} = \text{Sc} \frac{\partial^2 \bar{\phi}}{\partial \bar{y}^2} - K_c \bar{\phi} \quad (13) \]

together with the boundary conditions

\[ u(y, 0) = u_p(y, 0) = 0, \quad \theta(y, 0) = 1, \quad \phi(y, 0) = 1 \quad (14) \]

\[ u(1, t) = u_p(1, t) = 0, \quad \theta(1, t) = \text{e}^{i\omega t}, \quad \phi(1, t) = \text{e}^{i\omega t} \quad (15) \]

\[ u(0, t) = u_p(0, t) = 0, \quad \theta(0, t) = 0, \quad \phi(0, t) = 0 \quad (16) \]
For a purely oscillatory flow, we have the following conditions

\[- \frac{\partial P}{\partial x} = \lambda e^{i\omega t}, \quad u(y,t) = u_0(y) e^{i\omega t}, \quad u_p(y,t) = u_{p0}(y) e^{i\omega t}, \quad \theta(y,t) = \theta_0(y) e^{i\omega t}, \]

\[\phi(y,t) = \phi_0(y) e^{i\omega t}\]

where \(\lambda\) is constant oscillation amplitude for pressure gradient.

Substituting Eq. (17) in Eqs. (10) to (16), we obtain

\[\frac{d^2 u_0}{dy^2} - m_1^2 u_0 = -\lambda - Gr \theta_0 - Gc \phi_0\]

\[u_{p0} = \frac{u_0}{1 + i \omega \text{Re} M}\]

\[\frac{d^2 \theta_0}{dy^2} + m_2^2 \theta_0 = 0\]

\[\frac{d^2 \phi_0}{dy^2} - m_3^2 \phi_0 = 0\]

together with the boundary conditions

\[u_0 = u_{p0} = 0; \quad \theta_0 = \phi_0 = 1 \text{ on } y = 1\]

and \(u_0 = u_{p0} = 0; \quad \theta_0 = \phi_0 = 0 \text{ on } y = 0\)

where

\[m_1 = \sqrt{s^2 + H^2 + l + i \omega \text{Re} + \frac{l}{1 + i \omega \text{Re} M}}, \quad m_2 = \sqrt{N^2 + E - i \omega \text{Re} \text{ Pr}}, \quad m_3 = \sqrt{\frac{K_e + i \omega}{\text{Sc}}}\]

Equations (18), (20), (21) are solved using Eqs. (22) and (23).

We obtain the temperature, concentration and velocity of the dusty fluid as

\[\theta(y,t) = \frac{\sin (m_1 y)}{\sin (m_1)} e^{i\omega t}\]

\[\phi(y,t) = \frac{\sinh (m_3 y)}{\sinh (m_3)} e^{i\omega t}\]
For dusty fluid, the skin friction or shear stress at the walls is given by

\[ u(y,t) = \left( \frac{\lambda}{m_2^2} - \frac{\lambda}{m_2^2} \left( \frac{\sinh (m_2 y)}{\sinh m_2} \right) - \frac{\lambda}{m_2^2} \left( \frac{\sinh (m_2 (1-y))}{\sinh m_2} \right) \right) e^{i\omega t} + \frac{Gr}{m_1^2 + m_2^2} \left[ \frac{\sin (m_1 y)}{\sin m_1} - \frac{\sinh (m_2 y)}{\sinh m_2} \right] e^{i\omega t} \]

The dust particles are having velocity

\[ u_0(y,t) = \frac{1}{1 + i\omega \text{Re} M} \left( \frac{\lambda}{m_2^2} + \frac{\lambda}{m_2^2} \left( \frac{\sinh (m_2 y)}{\sinh m_2} \right) - \frac{\lambda}{m_2^2} \left( \frac{\sinh (m_2 (1-y))}{\sinh m_2} \right) \right) e^{i\omega t} + \frac{Gr}{m_1^2 + m_2^2} \left[ \frac{\sin (m_1 y)}{\sin m_1} - \frac{\sinh (m_2 y)}{\sinh m_2} \right] e^{i\omega t} \]

Skin Friction

For dusty fluid, the skin friction or shear stress at the walls is given by

\[ \tau = \left[ \frac{\partial u}{\partial y} \right] \text{at} \ y = 0, \ y = 1 \]

On simplification, we get

\[ \tau = \left[ -\frac{\lambda}{m_2} \left( \cosh (m_2 y) \right) + \frac{\lambda}{m_2} \left( \cosh (m_2 (1-y)) \right) + \frac{Gr}{m_1^2 + m_2^2} \left[ \frac{m_1 \cos (m_1 y)}{\sin m_1} - \frac{m_2 \cosh (m_2 y)}{\sinh m_2} \right] \right] e^{i\omega t} \]

For dusty particles, the skin friction is given as

\[ \tau = \frac{1}{1 + i\omega \text{Re} M} \left[ -\frac{\lambda}{m_2} \left( \cosh (m_2 y) \right) + \frac{\lambda}{m_2} \left( \cosh (m_2 (1-y)) \right) + \frac{Gr}{m_1^2 + m_2^2} \left[ \frac{m_1 \cos (m_1 y)}{\sin m_1} - \frac{m_2 \cosh (m_2 y)}{\sinh m_2} \right] \right] e^{i\omega t} \]

Nusselt Number

The rate of heat transfer across the channel is given by

\[ Nu = \left[ -\frac{\partial \theta}{\partial y} \right] \text{at} \ y = 0, \ y = 1 \]
The rate of mass transfer across the channel is given by

$$ \frac{m_i \cos (m_i, y)}{\sin (m_i)} e^{i\omega t} $$

at $y = 0, y = 1$

**Sherwood Number**

The rate of mass transfer across the channel is given by

$$ Sh \left[ \frac{\partial \phi}{\partial y} \right] \text{ at } y = 0, y = 1 $$

$$ Sh = \left( \frac{m_i \cosh (m_i, y)}{\sinh (m_i)} \right) e^{i\omega t} \text{ at } y = 0, y = 1 $$

(30)

4. **Graphical Results and Discussions**

The velocity $u$, temperature $\theta$ and concentration $C$ profiles are depicted graphically against $y$ for different physical parameters: Grashof number for heat transfer, Grashof number for mass transfer, radiation parameter $N$, Hartmann number $M$, Reynolds number $Re$, Schmidt number $Sc$ and chemical reaction parameter $K_c$ for a MHD oscillatory dusty fluid flow in a porous channel. The graphs are plotted using MATLAB.

![Graphical Results](image)

**Figure 2.** Temperature profiles with increasing $N$ and

$E=1; \ w=1; \ Re =1; \ Pr=1; \ =0:0.0001:1; \ t=1$

It is evident that the dusty fluid temperature raises with increase in the value of radiation parameter. This increase in temperature may contribute to internal radiative heat generation. Also the maximum value of the temperature lies in the centre region of the channel.
It is clear that the increase in radiation parameter decreases the rate of heat transfer of the dusty fluid upto the centre of the channel and then the rate of heat transfer gradually increases.

Figure 4. Concentration profile with increasing $K_c$ and $S_c = 1; w = 1; y = 0:0.001:1; \ t = 1$

From the above diagram, we understand that the concentration of the dusty fluid decreases with the increase in chemical reaction parameter.

Figure 5. Concentration profile with increasing $S_c$ and $K_c = 1; w = 1; y = 0:0.001:1; \ t = 1$

The graph depicts that the rise of Schmidt number increases the concentration of the dusty fluid.
Figure 6. Sherwood number with increasing $Sc$ and $Kc=1$; $w=1$; $y=0:0.001:1$; $t=1$

The graph gives the pictorial representation of increase in Schmidt number with the rate of mass transfer. It is well said that initially the rate of mass transfer of the dusty fluid decreases up to the central region and then it gradually increases up to the other end of the channel.

Figure 7. Velocity of dusty fluid with increasing $Gr$ and $N=0.1$; $E=1$; $Kc=1$; $Sc=1$; $w=1$; $Re =1$; $Pr=1$; $s =1$; $H=1$; $M=1$; $partcon=1$; $Gr= 1$; $Gc =1$; $y=0:0.0001:1$; $t=1$; $\lambda=1$;

Figure 8. Velocity of dusty fluid with increasing $Gc$

It is evident from Figures 7 and 8, that the increase in the values of $Gr$ and $Gc$ increases the velocity of the dusty fluid.
Figure 9. Velocity of dust particles with increasing particle concentration parameter $l$

Figure 10. Effect of skin friction with increasing particle concentration parameter $l$

It is obvious from Figure 10 that the increasing particle concentration parameter shows a reverse effect on shear stress.

5. Conclusion

The main findings are summarized below:

- Increase in radiation parameter $N$ increases temperature profile.
- The rate of heat transfer of dusty fluid decreases up to the centre of the channel and then it rises, if we increase radiation parameter $N$.
- Rise in the chemical reaction parameter $K_c$ decreases concentration profile, but concentration of the dusty fluid increases with rise of Schmidt parameter $S_c$.
- Velocity of dusty fluid increases with both $Gr$ and $Gc$.
- Velocity of dust particles falls with increase in particle concentration parameter.
- Shear stress falls with increase in particle concentration parameter.

If we neglect chemical reaction term in this paper, then the results in this paper coincides with the results obtained by Om Prakash and Makinde [9].
6. Nomenclature

| Symbol | Description |
|--------|-------------|
| $T_f$  | fluid initial temperature |
| $T_0$  | left wall temperature |
| $T_w$  | right wall temperature |
| $u$    | velocity of fluid particles in the x-direction |
| $u_p$  | velocity of dusty particles in the x-direction |
| $U$    | flow mean velocity |
| $K_c$  | chemical reaction parameter |
| $K_0$  | $= 6\pi \rho D$ | Stoke’s constant |
| $D$    | Average radius of dust particles |
| $Q$    | heat absorption/generation at the channel |
| $l$    | particle concentration parameter |

7. Greek Symbols

| Symbol | Description |
|--------|-------------|
| $\alpha$ | heat source/sink parameter |
|         | (or) mean radiation absorption coefficient |
| $\tau$  | skin friction |

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