PROBLEM-BASED LEARNING AND TEACHER TRAINING
IN MATHEMATICS

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ABSTRACT. Problem-based learning (PBL) is a constructivist learner-centred instructional approach based on the analysis, resolution and discussion of a given problem. It can be applied to any subject, indeed it is especially useful for the teaching of mathematics.

When compared to ‘traditional’ teaching, the PBL approach requires increased responsibility for the teachers (in addition to the presentation of mathematical knowledge, they need play the role of facilitators and engage students in gathering information and using their knowledge to solve given problems). It thus become crucial that the future teachers become aware of its effectiveness. One of the main obstacle to this awareness lies usually on the fact that future teachers did not find this methodology in their own training (neither pre- nor in-service). In this paper we will describe the attempt to introduce PBL in University courses so to have future maths teacher “experience mathematics” themselves.

1. PROBLEM-SOLVING, PBL, AND MATHEMATICS

A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on mind and character for a lifetime.’’ (Polya, 1945, p. v)

We like to start with this old quote from Polya, as these few line contain a meaningful view of mathematics and of mathematics education. Indeed, in Polya’s view the act of “solving problems” is the key to access true mathematical knowledge, and teaching mathematics effectively means having students become competent problem-solvers. This basic idea has been gaining more and more ground; simply put Polya is recognised as the founder of mathematical problem solving; his book “How to Solve It” planted the seed of the problem-solving ‘movement’ that flowered in the 1980s’ (Schoenfeld, 1992, p. 352). In Polya’s view teaching mathematics should not focus on the transmission of information, but rather on the development of the ability to use this information, or, in Polya’s words, on the transmission of know-how: “Our knowledge about any subject consists of information and know-how. Know-how is ability to use information; of course, there is no know-how without some independent thinking, originality, and creativity. Know-how in mathematics is the ability to do problems, to find proofs, to criticise arguments, to use mathematical language with some fluency, to recognise mathematical concepts in concrete situation” (Polya, 1962, vol. 2, p. 112). Polya’s works date back in time, a superficial look could target them as a bunch of old ideas, but this view is still valid and far from getting out of fashion. The need of the kind of competencies pointed out by Polya arise from today’s ‘real world’. When asked to speak at the latest Congress of the Unione Matematica Italiana (“Italian Mathematical Union”), employers stated that their industries need mathematicians that “can solve ill-structured problems, that are willing to look for flexible solutions, and are able to apply these solutions to parts of the problem that can arise in subsequent steps” (Barberis,
They suggested that mathematics instruction should “appoint students to solve not only classical problems, but also problems that are badly formulated, not well defined, and possibly with some contradictory data” (Barberis, 2007). Actually, research in education share Polya’s view, especially with the rise of the socio-constructivist movement. Constructivists aim at a neat evolution of the teaching practice, switching from a traditional model, based on teachers transmitting information to the pupils, to an innovative one, based on the construction of know-how. In many countries school reforms were promoted, inspired by these new pedagogical ideas. A clean example is the US “reform” stimulated by the National Council of Teachers of Mathematics’ Curriculum and Evaluation Standards for School Mathematics (1990). These standards promoted a view of mathematics that can be summarised by the following three points: mathematics instruction should lead students into

- “Seeking solutions, not just memorising procedures;
- Exploring patterns, not just memorising formulas;
- Formulating conjectures, not just doing exercises.” (National Research Council, 1989)

The standards suggested an “innovative” model of instruction that should give students “the opportunity to study mathematics as an exploratory, dynamic, evolving discipline, rather than as a rigid, absolute, closed body of laws to be memorised” (National Research Council, 1989). The US experience is indeed exemplary, and what so far appears as a clear picture of a neat evolution of the teaching practice does not reflect the truth. The “reform” gave rise to a big controversy well known as the “Math Wars”: a discussion full of hanger between supporters of “tradition” and supporters of “innovation”, involving not only the experts, but also the general public, was held on any kind of media (radio, local and national television, local, regional and national newspaper and magazines), but often the arguments given did not rely on solid grounds (Schoenfeld, 2004). The “war” has not yet come to a neat end, the whole story showing that the dispute about tradition and innovation is still far from being settled down and emphasizing a resistance to new methodologies. This resistance could also be caused by an unclear view of what “problem-solving” means: “‘problems’ and ‘problem-solving’ have had multiple and often contradictory meanings through the years—a fact that makes interpretation of the literature difficult” (Schoenfeld, 1992). In his review, Schoenfeld collects a list of often contradictory “goals for courses that were identified by respondents as ‘problem-solving’ courses:

- to train students to ‘think creatively’ and/or ‘develop their problem-solving ability’ (usually with focus on heuristic strategies);
- to prepare students for problem competitions such as the Putnam examinations or national or international Olympiads;
- to provide potential teachers with instruction in a narrow band of heuristic strategies;
- to learn standard techniques in particular domains, most frequently in mathematical modeling;
- to provide a new approach to remedial mathematics (basic skill) or to try to induce ‘critical thinking’ or ‘analytical reasoning’ skills.” (Schoenfeld, 1992, p. 337)

In other words, the term problem-solving can be misleading. On top of that we should recall that often the words “innovation”, “mathematical discovery”, and “problem-solving” were used as mere slogan system, rather than with full knowledge. It is remarkable that such an innovative researcher as Freudenthal has a very bitter comment: “‘problem-solving’ and ‘learn by discovery’ are now fashionable expression. I never liked them, neither at the beginning, when they were used as ‘slogan’, nor now, as I can see examples of. ‘Problem-solving’: that is solve the problem given by the teacher, or by the author of the textbook,
or by the researcher, using the tricks that they have in their mind [...] ‘learn by discovery’
that is find what has been hidden: like you do with Easter eggs” (Freudenthal, 1991).

We thus want to point out that when we refer to problem-solving we mean exactly Polya’s
view we started from, that is “learning to grapple with new and unfamiliar tasks when the
relevant solution methods (even if only partly mastered) are not known” (Schoenfeld, 1992,
p. 354), and we think that mathematics instruction should develop the students’ “ability to
solve problems in new contexts or to solve problems that differ from the ones one has
been trained to solve” (Schoenfeld, 2004, p. 262). Most of all, we would like to focus our
attention to the use of problems as a tool to stimulate learning, that is what is known as
Problem-based learning or PBL.

1.1. **Problem-based learning (PBL).** The socio-constructivist view of teaching and learn-
ing mathematics promotes new instructional strategies in which students are lead to learn
mathematics by personally and socially constructing mathematical knowledge. For a neat
definition of PBL we refer to (Savery, 2006) according to which PBL is an instructional
(and curricular) learner-centered approach that empowers learners to conduct research, in-
tegrate theory and practice, and apply knowledge and skills to develop a viable solution to
a defined problem”. Typically a PBL session follows these steps:

- pupils are given a problem;
- they discuss the problem and/or work on the problem on their own and/or in small
groups, collecting information useful to solve the problem;
- all the pupils gather together to compare findings and/or discuss conclusions; new
problems could arise from this discussion, in this case
- pupils go back to work on the new problems, and the cycle starts again.

Variation can occur, but there are a few key elements one should take care. We will briefly
report them here.

1.1.1. **Problems.** “Cognitive research and practical experience with PBL have made im-
portant strides in identifying the characteristics of a good problem” (Hmelo-Silver, 2004,
p. 244); “critical to the success of the approach is the selection of ill-structured problems
(often interdisciplinary)” (Savery, 2006, p. 12). To be more precise in order “to foster
flexible thinking, problems need to be complex, ill-structured, and open-ended; to support
intrinsic motivation, they must also be realistic and resonate with the students’ experiences.
A good problem affords feedback that allows students to evaluate the effectiveness of their
knowledge, reasoning, and learning strategies. The problems should also promote conjec-
ture and argumentation. Problem solutions should be complex enough to require many in-
terrelated pieces and should motivate the students’ need to know and learn” (Hmelo-Silver,
2004, p. 244). While we share the conviction that problems need to be “realistic”, and that
pupils will find no interest in problems dealing with a “cooking pot in the shape of a right
prism with base an equilateral triangle whose sizes are 1 meter long” (Dedó, 2001), in
our experience the connection of the problems with students’ experiences is not so crucial:
if the problems contains some “good maths”, the intellectual challenge is a good vehicle
for getting the students’ attention (e.g. see examples of problems in (Bonaiti et al, 2005),
(Cazzola, 2007), (Caronni et al, 2007), and (Bolondi, 2005)).

1.1.2. **Group-work.** Collaboration in essential. “As students generate hypotheses and de-
defend them to others in their group, they publicly articulate their current state of understand-
ing, enhancing knowledge construction and setting the stage for future learning” (Hmelo-Silver,
2004, p. 244). According to our experience, the group-work also motivate pupils, and in
the group they find support for dealing with difficult tasks they would not be able to tackle
by themselves.

Our source is in fact Freudenthal (1994).
1.1.3. *General discussion.* “A closing analysis of what has been learned from work with the problem and a discussion of what concepts and principles have been learned are essential” (Savery, 2006, p. 14). Indeed a general discussion in which all the groups gather together to compare findings and discuss solution is the step that really consolidate the learning. The effectiveness of the general discussion rely upon the fact that pupils know the details under discussion as they have been working themself on the problem.

1.1.4. *What do students learn?* A PBL approach improves students’ problem-solving abilities, e.g. see (Hmelo-Silver, 2004; Mergendoller et al, 2006), thus, in Polya’s view, give access to *true* mathematics. On the other hand “classroom instruction, which tend to focus almost exclusively on the knowledge base, deprives students of problem-solving knowledge” (Schoenfeld, 2004). Moreover, a PBL approach helps pupils to get abilities that go beyond the mathematical contents they have been assigned. By design, PBL helps students

- “construct an extensive and flexible knowledge base;
- develop effective problem-solving skills;
- develop self-directed, lifelong learning skills;
- become effective collaborators; and
- become intrinsically motivated to learn.” *Hmelo-Silver (2004)*

No need to stress the fact that these essential skills are what students will need in the “real world” after school.

1.1.5. *Teacher roles and teacher training.* With a PBL approach the teacher acts as a facilitator, guiding the the learning process and conducting the final debriefing at the conclusion of the learning experience. This new role requires inceased responsibility: the teacher needs to plan and set up the activities, but with this kind of approach not everything can be planned ahead. The teacher has to react to the pupils’ arguing and discovering, and this often requires building mathematical argumentations as the need arise. The teacher needs a strong mathematical background and the will to “get in the game”. Although research collects data proving the effectiveness of PBL, such an approach is not so well established in the teaching practice. There are of course criticism about PBL, a clear disadvantage to the use of PBL is that it can be really time consuming. Moreover “many lecturers have expressed concern about whether sufficient knowledge can be conveyed through a PBL format and whether students have sufficient prior experience to be able to benefit from the problem-solving situation. Another concern is that when students are initially confronted with this approach many are suspicious about its value, particularly if they have previously been used to teacher-centered approaches” (Taplin and Char, 2001). The resistance towards this kind of approach might be regarded as a symptom of the general resistance towards innovative constructivist approaches; in the following section we will discuss in teachers’ beliefs, that can also give an explanation. Indeed, the lack of training in this kind of activities deters teachers from choosing such a approach: “one barrier to using PBL in more diverse settings is the lack of a sufficient number of skilled facilitators in many settings” *Hmelo-Silver (2004)*. Models for teacher training in this kind of activity were developed. We can go back to LeBlanc (1982), as, although he actually focused on *problem-solving*, his model can be adapted to the training of PBL facilitators. Typically, the main lines of actions should be the following:

**solving problems:** teachers should be given some problems to solve, class interaction should focus also on how a problem might be solved rather than on the actual solution;

**having children solve problems:** teachers should learn to monitor their pupil actions and reactions;

**compiling a list of problems:** teachers should compile a list of problems for future use.
According to LeBlanc (1982), “teachers report that their own attitudes toward problem-solving and instruction have been changed by this training and that their students can and do solve problems and enjoy the process”. He concludes aiming researches would follow focusing on the analysis of the changes this kind of training induce in teachers attitudes.

1.2. Teachers’ beliefs. There is general acceptance of the idea that teachers’ beliefs are a crucial factor in the teaching practice: both educational beliefs and beliefs on the nature of the mathematical enterprise do shape teachers’ instructional practice and the nature of the learning environment they create. We particularly refer to Handal’s literature review (Handal, 2003), as he especially focuses on teachers’ beliefs when they face the choice of adopting an innovative teaching practice. What emerges is that teacher beliefs “act as a filter through which teachers make their decision rather than just relying on their pedagogical knowledge or curriculum guidelines” (Handal, 2003); in spite of having had “instruction about up-to-date methods of teaching mathematics, [teachers] often revert to teaching styles similar to those of their own teachers; they show little or no change in their conceptions of mathematics teaching despite their method courses” (Taplin and Chan, 2001). Indeed this reversion to “traditional teaching” is often due to strong beliefs that teachers acquired “symbiotically from their former mathematics school teachers after sitting and observing classroom lessons for literally thousands of hours throughout their past schooling”. Even worse, “once acquired, teachers’ beliefs are eventually reproduced in classroom instruction”, and “there is some evidence that, in some cases, teacher education programs are so busy concentrating on imparting pedagogical knowledge that little consideration is given to modifying these beliefs” (Handal, 2003). Indeed studies show that “a large population of teachers still believe that teaching and learning mathematics is more effective in the traditional model” (Handal, 2003, p. 50).

To change this trend, teacher training practice should switch from a traditional model to an innovative one, that is prospective teachers should be taught in a manner similar to the way they are supposed to teach.

2. THE EXPERIENCE

The experience is part of a “traditional” pre-service teacher training university course. When we say traditional, we mean that most of the teaching is carried on through frontal lectures addressed to a large audience (about 100 people). If we start from the assumption that “teachers teach the way they have been taught”, it is not surprising that once graduated these pre-service teachers hold the belief that frontal lecturing is an essential part of teaching. Even if the lectures they attended at university were all devoted to explaining how active learner-centered methodology are much more effective than traditional lectures. In other words, what we teach about teaching is completely different from the way we teach, and we should not be surprised if our students addressed us the objection “you are telling me beautiful things, but if I do not see you using such methodology, why should I believe it is worth the effort?”.

Our intention in carrying out this experience was to explore the extent to which the pre-service teacher developed beliefs in favor of the use of a PBL methodology and, at the same time, developed positive attitudes toward creative mathematical activities. Our experience is similar to the one carried out by Taplin and Chan (2001).

2.1. Lectures. The course we used for the project was the 30-hours module Didattica della matematica (“Teaching mathematics”), at the University of Milano-Bicocca (Italy). The experience involved students in their third year, while in their previous career, they already attended two 30-hours modules on mathematical contents (Istituzioni di matematiche, i.e. “Elements of mathematics”), focusing on arithmetic (in their first year) and on Euclidean geometry (in their second year). For a description of the content of these courses

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2 The pre-service teacher training university degree course is a four years program.
**Instruction.** During this session you will be given a list of problems, problems you will have to solve during the first two hours working in group; the following time will be devoted to a general discussion with the other groups in order to confront your solutions and, of course, the methods you used to get your solutions. Be sure that every group has a member attending the general discussion.

At the end of the first two hours, hand in to me your written solution with the list of the group members.

The problems could be difficult: work together and seek confrontation in your group. In the final discussion any member of the group should be able to explain the group discoveries. And if the group has more than one solution it will be useful to write all of them down on the answer sheet and report all of them during the final discussion.

You will be given many problems: you are not required to solve them all, but it is important that when you work on a problem you have full concentration. So, make a choice within your group, tackle a single problem, and do not start working on a new one unless you have finished with the previous one.

Keep in mind that… these problems could be exam questions.

Good work!

**TABLE 1.** Instruction for the students

and of the whole pre-service teacher training university course we refer to Cazzola, 2004. Although devoted to pre-service teachers, these courses were attended by some in-service teachers too.

As we said, *Didattica della matematica* is a “standard” university course, nevertheless we attempt as much as possible to introduce dialogue in the lectures, but the dimension of the class makes things difficult. Anyway, the Faculty is open minded and experiments of innovative teacher methodologies are welcome. During their course of study, the pre-service teachers attend standard lectures and a series of pedagogical laboratories covering almost all curricular subjects. Moreover research on teaching is carried out by the centre matemática (Interuniversity research centre for the communication and informal learning of mathematics) hosted by the University.

As we were trying to adapt standard university lectures to a PBL methodology, we were, once more, inspired by Polya and his description of his experience as a teacher trainer “all the classes I have given to mathematics teachers were intended to be methods courses to some extent, the time was actually divided between subject matter and methods: perhaps nine tenths for subject matter and one tenth for methods, if possible, the class was conducted in dialogue form and some methodical remarks were injected incidentally, the derivation of a fact or the solution of a problem was almost regularly followed by a short discussion of its pedagogical implications; my main work, however, was to choose such problems as would illustrate strikingly some pattern of teaching” (Polya, 1962). However, university lectures are subject to many bonds: time constraint, large classes, syllabus to cover. Nevertheless these are exactly the obstacles that future teachers must deal with and we had the chance to show our students that such obstacles can be overridden. If we had given up, how could we expect our students to persist? As PBL is a methodology particularly effective for the learning of mathematics, it should be our first choice, as we need to give the pre-service teachers a solid mathematical content background.

2.2. **PBL sessions and problems.** During 2007, part of the curricular time was devoted to two PBL session in which students were presented with a series of problems. The two session lasted 4 hours, two of which were spent in small group-work on the problems, while the other two were devoted to the general discussion. We already mentioned the
(1) Given a set $A$ and a set $B$, how many functions from $A$ to $B$?
   (a) Let $A$ be the set consisting of the letters $a, b, c$ and $B$ be the set consisting of
       the numbers $1, 2$. List all the functions from $A$ to $B$.
   (b) It is possible to tell the number of functions of the previous question without
       actually writing down the list of all such functions?
   (c) Given a set $A$ with $n$ elements and a set $B$ with $m$ elements, how many
       functions from $A$ to $B$ is it possible to write down?

(2) Given a set $A$ of order $n$, how many subsets does $A$ have?
   (a) Given a set $A$ of order 5, how many subsets with 2 elements does $A$ have?
      And how many with 3 elements? And 4? And ...?
   (b) Given a set $A$ of order $n$, how many subsets with 2 elements does $A$ have?
      And how many with 3 elements? And how many with $n - 1$ elements?
   (c) During one of the lectures we discovered the “magic” of Tartaglia’s triangle

   \[
   \begin{array}{cccccc}
   & & & & & \\
   & & 1 & & & \\
   & 1 & & 1 & & \\
   & & 1 & & 2 & 1 & \\
   1 & & 3 & & 3 & & 1 \\
   & 4 & & 6 & & 4 & & 1 \\
   & & 5 & & 10 & & 10 & & 5 & & 1 \\
   & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 \\
   & & & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1 \\
   \end{array}
   \]

   Can you justify the fact that such a triangle contains exactly the numbers we
   are looking for?
   (d) What is the sum of each row of the Tartaglia’s triangle? Can you explain
      such a result?

(3) Given two sets $A$ and $B$ we say that $A$ has the same cardinality as $B$ if there exists
   a bijection from $A$ onto $B$.
   (a) The relation “to have the same cardinality as” is an equivalence relation?
   (b) A set is countable if it has the same cardinality as the set $\mathbb{N}$ of natural
      numbers. Is $\mathbb{Z}$, the set of integers, countable?
   (c) Are the subsets of $\mathbb{N}$ countable?
   (d) During standard lectures we proved that the set
       \[ I = \{ x \in \mathbb{R} \mid 0 < x < 1 \} \]
   is not countable (i.e. is uncountable). Do you think that the cartesian prod-
   uct $I \times I$ has the same cardinality as $I$? Does it have a larger or a smaller
   cardinality, whatever that means?

TABLE 2. Problems for the first PBL session

similarity between our experience and the one carried out by Taplin and Chan (2001), the
main difference between the two experiences being the fact that they based the PBL session
on the discussion of pedagogical problems, while our idea was to transmit both pedagogical
and mathematical knowledge to our students at the same time. In other words, we
worked under the assumption that, by giving them mathematical problems in a PBL envi-
ronment, the pre-service teachers would have gained beliefs in favour of a PBL approach,
as they saw that with this setting they managed to grasp the mathematics in the given tasks,
or, in other words, they would have gotten the perception of the effectiveness of PBL as
they realized that in this way they did learn some good mathematics. Moreover, teach-
ers using a PBL approach need to experience mathematical discovery as they will have to
deal with students active learning, that often leads to conjecturing, proposing solution and,
sometimes, to making up unexpected questions: ‘the teacher should develop his students’
(1) A bag contains 4 red balls and 3 yellow balls.
   (a) We draw 2 balls out of the bag, one after the other, putting the drawn ball back in the bag. What is the probability that the two drawn balls are both red? That they are both yellow? That they are of different colours?
   (b) We draw 2 balls out of the bag, one after the other, without replacing the drawn ball back in the bag. What is the probability that the two drawn balls are both red? That they are both yellow? That they are of different colours?
   (c) We draw 2 balls together, at the same time, out of the bag. What is the probability that the two drawn balls are both red? That they are both yellow? That they are of different colours?

(2) (a) I was born in a non-leap year. Piero was born in a non-leap year too. What is the probability that Piero and I share our birthday?
   (b) In a group of 3 people, what is the probability that at least two of them share their birthday? And in a group of 4 people?
   (c) How many people should be present in this room in order to be sure that at least two of them share their birthday?
   (d) Are you willing to bet that at least two people in this room share their birthday?

(3) There are three closed doors. One of these hides a Ferrari, the other two just a box of chocolates.
   You can choose one of the doors and take away what’s hidden behind it. Aiming at getting a Ferrari, when you choose a door, a friend opens one of the two remaining doors, showing the box of chocolates hiding behind it.
   At this point you can make a new choice. You can either keep the door you have chosen first, or change it. What would you do? Why?

| TABLE 3. Problems for the second PBL session |
|---------------------------------------------|

know-how, theirs ability to reason; he should recognize and encourage creative thinking [. . . ], he should know what he is supposed to teach, he should show his students how to solve problems—but if he does not know, how can he show them?” [Polya 1962].

Lectures preceding the PBL sessions were devoted to illustrating the PBL approach. After the PBL sessions, students’ opinion were collected with a questionnaire. At the end of the course, students had to compile a list of problems as part of their final examination. The PBL sessions were introduced by the instructions given in Table 1. The students were given the problems and asked to work in groups of four. Instruction were ment as a guidance on how the session was supposed to work out, but also aimed at motivating the students. In this sense one should read the warning about difficult problems and the final remark “Keep in mind that… these problems could be exam questions”.

In each of the two PBL session students were given 3 problems and asked to make a choice. The decision of giving so many problems was dictated by the time constraints and the hope that in this way we would have covered a larger amount of mathematical contents. However it is a decision that has to be reviewed and we will discuss it later in this paper.

2.2.1. Problems. The problems (shown in Table 2 and 3) were chosen in order to cover part of the syllabus, which includes “Teaching probability” is a relevant part of it, combinatorics is propedeutic to probability, and, at the same times, gives a lot of examples that allow manipulation (and can give pre-service teachers useful ideas on activities that can be proposed to young children).

2.3. The questionnaire. The purpose of the questionnaire was to have the students review the experience, particularly testing the following aspects
   • check students’ ability to recognize strengths and weaknesses of the activity;
**Questionnaire**

On March 21 and March 28 we had two problem sessions in which we experimented a *problem-based learning* methodology.

1. What do you think you have learned in these sessions?  
2. What *did work* during the two lectures?  
   - Session of March 21  
   - Session of March 28  
   - Both session  
3. What *did not work*?  
   - Session of March 21  
   - Session of March 28  
   - Both session  
4. Are you likely to use a methodology of the type we experimented for the mathematics lessons in your future teaching profession?  
   - yes  
   - no  
   Why?  
   And for the other curricular subjects?  
   - yes  
   - no  
   Why?  
5. If you answered “yes” to the previous question, would you set up your lectures in a way different from the one we experimented?  
   - yes  
   - no  
   If yes, which changes?  
6. If you answered “no” to question 4 would you be willing to use such a practice only if you could modify it?  
   - yes  
   - no  
   If yes, how?  
7. If you answered “yes” to question 4, what part of the mathematical curricular time would you devote to a *problem-based learning* approach?  
   - From colleagues?  
   - From the school principal  
   - From the University?  
   - From others?  
8. What kind of support do you think you will need in order to use a *problem-based learning* approach?  
   - From colleagues?  
   - From the school principal  
   - From the University?  
   - From others?  
9. If you have any comments, write them here.  

**Table 4. The questionnaire**

- monitor beliefs about the PBL approach, in particular get an idea if they had developed a willingness to implement a PBL teaching model;
- monitor beliefs about the role that universities can have in their future profession of teachers.

The questionnaire is shown in Table 4. Note that question 1, *what do you think you have learned in these sessions?*, was meant to be ambiguous, not specifying whether the “things learned” should be read as regarding mathematical or pedagogical contents. This was done in order let the student choose and let us better check their perception of the experience. Question about *what did work* and *what did not work* just wanted to stimulate in the students the reflection on the fact that the experience could be improved. Also, note that most of the question are sort of asked twice: clearly question 2 and question 4 are related, as question 3 and question 5 are.

Question 8 was meant to test the beliefs of the students and their awareness of possible difficulties in actually adopting a PBL approach, and test whether university is seen as a resource by prospective teachers.
express satisfaction with the experience 12
express changing in attitude towards ‘doing’ mathematics (“I used to do mathematics on my own”) 4
express negative emotions 1
learned to work in group, learned importance of group-work to create a comfortable atmosphere 80
recognize the two phases of a PBL (both group-work and general discussion with confrontation among groups) 34
new beliefs on the methodology (“we built up something that cannot be found in books”, “you can enjoy learning”) 17
learned to deal with time 11
you need good problems to use PBL 8
processes are more important than the mere results (“we got wrong result, but we learned some good maths anyway”) 7
learned some mathematical content (name mathematical contents without focusing on any in particular) 25
new beliefs on mathematics (“reasoning is a crucial part of mathematics”, “formulas learn by hearth worth nothing”, “you can try to guess first”) 22
learned a particular mathematical content 7

TABLE 5. Analysis of results: things learned

feeling of a lack of a conclusion, mostly due to the way the general discussion was conducted 37
logistics (e.g. small lectureroom) 31
negative comments due to expectations tipical of a ‘traditional’ didactical model 21
criticism about the groups (the chosen groups did not work out) 18
too many problems, not enough time (“all the problems were interesting, but we had not enough time to deal with them”) 16
too long, too tiring 4
no tutoring 3
everething worked smoothly 3
did not reach the PBL goals (“did not feel like I was part of the group discoveries”, “did not commit to the group”) 2
there was only a photocopy for each group 1

TABLE 6. Analysis of results: things that did not work

Some of the pre-service teachers were interwied to clarify part of their answer in the questionnaire
The questionnaire was submitted to the students shortly after the second PBL session, before making any comment on the experience with them; 94 questionnaire were collected and analyzed.

3. RESULTS

We will now examin the three different aspect we had in mind to monitor.
importance of group-work 32
this methodology makes mathematics interesting, engaging, pleasant, less boring; one understands he can make it; you give pupils a different view of mathematics in order to learn you need to co-operate and to confront with other people’s views this methodology is effective to produce long-life learning in mathematics you need to put theory into practice with this methodology you can monitor pupils’ progresses

| TABLE 7. Analysis of results: PBL strengths |
|--------------------------------------------|
| create groups more accurately 25          |
| better organize time 16                   |
| use a suitable classroom 14               |
| give solutions to all exercises (e.g. through the general discussion) 11 |
| more tutoring during group-work 5         |
| choose suitable problems 3                |
| use different materials 3                 |
| make smaller/bigger groups 3              |
| split the class in order to have less students 2 |
| better involve pupils in the general discussion (e.g. make everybody talk during the general discussion) 2 |
| have everybody work on the same problem 2 |
| have pupils write down carefully their solutions 1 |
| make a final test to evaluate the pupils 1 |
| let the pupils get to the solution, never tell them the solution 1 |
| read and explain the problems at the beginning of the session 1 |

| create groups more accurately 25          |
| better organize time 16                   |
| use a suitable classroom 14               |
| give solutions to all exercises (e.g. through the general discussion) 11 |
| more tutoring during group-work 5         |
| choose suitable problems 3                |
| use different materials 3                 |
| make smaller/bigger groups 3              |
| split the class in order to have less students 2 |
| better involve pupils in the general discussion (e.g. make everybody talk during the general discussion) 2 |
| have everybody work on the same problem 2 |
| have pupils write down carefully their solutions 1 |
| make a final test to evaluate the pupils 1 |
| let the pupils get to the solution, never tell them the solution 1 |
| read and explain the problems at the beginning of the session 1 |

| TABLE 8. Analysis of results: things would like to change/to put into action |
|--------------------------------------------|
| logistic (spaces, equipment) 51            |
| pedagogical support (both instruction, but also teaching material) 35 |
| economic support (e.g. provide stationery) 14 |
| competence (both pedagogical and mathematical, no clear distinction) 7 |
| support and collaboration in the working environment 7 |
| time 5                                     |
| appropriate number of students 2           |
| intellectual stimuli 2                     |

| TABLE 9. Analysis of results: general support needed |
|--------------------------------------------|
| cooperate (i.e. do activities together) 54 |
| share methodology/use same methodology in their lectures 19 |
| allow for extra time and spaces, if necessary 11 |
| respect for own choices 5                   |
| monitor pupils 1                           |

| TABLE 10. Analysis of results: support from future colleagues |
|--------------------------------------------|
| cooperate (i.e. do activities together) 54 |
| share methodology/use same methodology in their lectures 19 |
| allow for extra time and spaces, if necessary 11 |
| respect for own choices 5                   |
| monitor pupils 1                           |
agree on methodology and support choice 53
economic support (provide time, space, materials) 27
respect for freedom of choice and autonomy 12
organize/promote refresher courses 7
incentives 1
unimportant 1

TABLE 11. Analysis of results: support from the school principal

training (both pre-service, and in-service, no clear distinction) 49
produce teaching materials and activities for the classroom 16
do research 15
cooporation and involvement in activities (not specified) 1
financial support (to promote research and activities conducted by the teachers) 1
focus on their present experience as university students 11

TABLE 12. Analysis of results: support from the university

parents 48
pupils 5
museums and research centers 3
other school workers 2

TABLE 13. Analysis of results: who else can be of help?

3.1. Ability to recognize strengths and weaknesses of the experience. With the questionnaire we aimed to test the students’ attitude towards the experience. In particular, what the students recognized as the activity strengths are highlighted in Table 5, that contains categorized answers to questions 1 (What do you think you have learned in these sessions?) and 2 (What did work during the two lectures?). Looking at the answer given to question 1, we see that the students include pedagogical contents too, thus enforcing our hypotheses that dealing with mathematical contents would have succeeded in having them gain awareness of the methodology. A comparison between Table 5 and Table 7, the latter containing categorized answers to the open part of question 4 (Why would you use a PBL approach?), show that the students put a big emphasis on “group-work” rather than on the whole PBL approach (even if during their course of studies they had many pedagogical content courses explaining distinctions). In the interview some students admitted that when they answered the questionnaire they had not yet “studied the textbooks”. Althought we do not want to open a discussion on “affects”, we think it is significat to point out that some students felt the need to express their personal satisfaction with the experience, when answering question 1 (as shown in Table 5). Switching to what were seen as the weaknesses of the approach, Table 6 collects answers to question 3 (What did not work?), while Table 8 contains answers to question 5 and question 6 (What changes would you apply to the approach?); some students answered to question 6 even if they were not supposed to do so. It could be useful to compare the two tables. In our opinion, most of the criticism listed in Table 6 are truly due to a misconconduction of the experience. We take the blame for this, but wish to defer comments later in this paper (the real surprise came from the 3 people answering “everything worked smoothly”). By the category “negative comments due to
expectations typical of a ‘traditional’ didactical model” we mean answer of the type “we were given no textbooks to study ahead”, “we were not able to solve the problem because we hadn’t studied”, in other words clearly focusing on a lack of information as opposed to Polya’s know-how. While we accept most of the criticism contained in Table 6, we had the chance to interview the student who answered “there was only a photocopy for each group”, and explain her that this was exactly the trick that helped to get the group members work together.

3.2. Willingness to use a PBL approach. In order to test the willingness to use a PBL approach, we do not think that the yes/no answer to question 4 (Are you likely to use a methodology of the type we experimented for the mathematics lessons in your future teaching profession?) is somehow significant: the questionnaire was nominative and students still had to face us for taking their final exams (just for the record, everybody answered “yes”). From this point of view it is much more interesting to look at the answers they gave to question 5 (would you set up your lectures in a way different from the one we experimented?) and 6 (would you be willing to use such a practice only if you could modify it?), collected in Table 8 because you can get the feeling that the students were really “getting in the game”. The pre-service teachers do think as they were really to plan a PBL session. Also we feel that the answer given to question 7 (what part of the mathematical curricular time would you devote to a problem-based learning approach?) is really indicative of a good attitude towards PBL, although we need to point out that the analysis of the answers was more difficult than expected. The question asked for a “ratio” of the partition of curricular time between traditional lecturing and PBL activities, but it turned out, once again, that the language of fractions is not such a well mastered language as it should. Some students actually found it difficult to answer because they had no idea of how many curricular hours they are supposed to have. We had to filter and categorize these answers: approximately 39 students stated that they would use PBL activities for more than 50% of the curricular time devoted to mathematics. We can add to these a further 24 that think of a weekly appointment devoted to a PBL activity. In other words a wide majority of the 94 pre-service teachers stated they would like to use PBL activities in a significant part of the time devoted to mathematics. Some of the more articulated answers indicated that such activities should be used to introduce any new topic (“let the problem introduce the need for the theory”). In a completely different direction a small, not marginal number of students (8) stated that PBL activities should be used at the end of each topic.

Finally question 8 (What kind of support do you think you will need in order to use a problem-based learning approach?) tries to test beliefs, showing that these pre-service teachers acknowledge the difficulties they are likely to meet. It is well known that “the context of school instruction obliges practising elementary and secondary teachers to teach traditional mathematics even when they may hold alternative views about mathematics and about mathematics teaching and learning; parents and professional colleagues, for example, expect teachers to teach in a traditional way” (Handal 2003), not to forget the role of parents and of the public opinion in the episode of the “Math wars”. Students’ answers are collected in Tables 9 to 13. The authenticity of students’ efforts in filling in the questionnaire emerges with this question: they are accurate, aware and careful about the details; they identify most of the point outlined by the researchers. Note that “parents” was not among the given choices, and was well recognised as a strong counterpart.

3.3. Role of the University. Currently in Italy there is a debate on teacher training and evaluating. It is well recognized the role of the University as the institution research and training is demanded, but somehow the world of school hardly tolerate actions by the University as an interference with its own life. At the time of writing there is a plan for a reform and the future of the pre-service teacher training university course we are working in is uncertain. Things such a High School of Education are planned, but no sign
of start yet. Also the clean distinction between academic and school domains can be seen in the fact that Universities do not get involved in the editing of school textbooks. This is probably the case in many other European countries, as pointed out by Wittmann (2007), and probably due to the academy as well (significantly enough the production of school teaching material is taken in very little consideration for the career of university professor). It is a fact that the mathematical community seldom even get to see maths school texts, and namely mathematicians look at school textbooks only when they happen to have their own children in school age. We questioned our students about what in their opinion should be the role of the University in their future profession as school teachers, and their answers are collected in Table 12. They recognize the role of the University not only for research, but, above all, for training too, although it is not clear whether they mean pre-service or in-service training (often the two aspects are mixed). There are explicit requests of the activation of a sort of helpdesk for teachers. The pre-service teachers who undergo an university course get acquainted with the University and do not feel it as an external body. A small, but significant, portion of the students think that the University can fill the lack of teaching materials. Perhaps this answer is influenced by the fact that these students know the center matematica and its proposals (among which the before mentioned textbooks) (Bonaiti et al, 2005), (Caronni et al, 2007), and Bolondi, 2005).

3.4. Does and don't's. After we asked our students to review the experience, we want to make our personal analysis. As we already stated, this was one of the first attempts to introduce PBL activities in our standard mathematics University courses. The constraints were very strong. The activity was planned so to give our students a small chance, but this was done at the expenses of some aspects that probably were equally important (and some expert will disagree with our implementation). The PBL session have involved a far too large class: we let free choice to the students whether to attend or not the PBL sessions, and we ended up with more than 100 students showing up. Whoever organized PBL activities knows too well that this was a big problem. The classroom was large enough to host all those people, but obviously the small group-work caused a disturbing constant noise, and the stress on logistic in Tables 6, 8, and 9 is not a surprise. Moreover, the high number of students compared to the number of tutors has meant that there was almost no tutoring at all and the groups were let to work completely on their own. The message of the importance of tutoring did not get to the students. If we look at Table 5 only 3 people recognised the importance in such activities of the teacher actions. Also the number of problems given for each session was too large. The choice of giving so many problems was in part due to the wish to cover a larger amount of mathematical content, but also to the fact that such problems had not been tested yet, and we could not foresee how long students would have taken to solve them, so we wanted to provide extra material for early finishers. This had the side effect of distracting students, as clearly emerges from Table 6. The final discussion suffered from this too, as students highlighted the fact that the discussion on problems they hadn’t been working on was not easy to follow. For future implementations we will defenetely give the students a single problem to work on.

Even if the experience had clear faults, we still think that it was useful for the students: at least they learned that some things should not be done. We can add that this experience actually acted on our own beliefs: we learned that this kind of activities nicely fit in the university routine, and, even if we will have to discuss with our faculty in order to get the necessary resources to build “proper” PBL environments, it is worth the effort.

4. Final remark

With this kind of activities we hope to encourage our students to actually use the current theories of learning and teaching, although we know that there will be obstacles (lack of materials, of time and of collaboration among peers; size of room; non-supporting administration and parents). University education cannot do much about most of these factors, as
we are facing a broader cultural issue. From a certain point of view something is changing; in response to the concern that a teacher might not be motivated to introduce innovative activities in class “when that teacher knows that his or her students will be tested on basic skills in a district proficiency exam” (Handal, 2003) we can observe that the international PISA-OECD tests tend to assess the kind of skills that Polya put in the category of know-how.

Anyway the more we work with pre-service teachers, the more we understand their request for support, as emerges from their answers to the questionnaire. Probably the University can play a significant role in breaking that self-perpetuating cycle that maintains traditional beliefs and practices. Maybe just making small changes in its own teaching style. Some of the students involved in the experience asked us to participate in the building of a collection of the mathematics teaching proposals they produced during the course, in order to have them ready to use in their future working experience, and this collection is now available through a website. Moreover, we are currently supervising a small portion of the student (6) that are currently proposing PBL activities to primary school children in order to produce a research for their final dissertation.

By analyzing the answers to the questionnaires and collecting sensations received from conversations with our students, we are lead to think that this experience has constituted a small contribution in strengthening our students’ beliefs in favour of a PBL teaching model.

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