Solar Oblateness from Archimedes to Dicke

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Summary. —
The non-spherical shape of the Sun has been invoked to explain the anomalous precession of Mercury. A brief history of some methods for measuring solar diameter is presented. Archimedes was the first to give upper and lower values for solar diameter in third century before Christ; we also show the method of total eclipses, used after Halley’s observative campaign of 1715 eclipse; the variant of partial eclipses useful to measure different chords of the solar disk; the method of Dicke which correlates oblateness with luminous excess in the equatorial zone.

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1. – Archimedes

Archimedes of Syracuse gave out an evaluation of the angle subtended by the Sun with the vertex on the observer’s eye. He knew that a perfect determination is not possible due to the observer bias and systematic errors, so he proposed to find out the
upper and lower angles allowed by his method: in this way we have a range within solar angular diameter lies. To do this we must have a big ruler (a strip of wood more likely) mounted on a basement in a place such that is possible to see the dawn (Syracuse in Sicily has an eastern free sea horizon). When the Sun rises up and it is still near the horizon, so that is possible to aim at it, we point the ruler toward Sun; suddenly we put on the ruler a rounded cylinder such that, if we look from one edge of the ruler pointed at the Sun, the Sun is hidden behind that. Afterwards we need to shift the cylinder on the ruler till we see a very little Sun limb from the edges of the cylinder. Now we block the cylinder in this position. Let’s find now, the lower evaluation angle. Archimedes argued: if the observer’s eye was point-like (fig.1), then the angle identified by the lines drawn from eye’s site such to be tangent to cylinder’s edges (the gray angle in fig.1), would be smaller than the angle made by the lines from the eye to the edges of the Sun (the black angle in fig.1). This is because we set the cylinder such that to see a portion of the Sun. Indeed the observer’s eye is not point-like but extended: we are overestimating the lower limit angle (see captions in fig. 2 and 3). This lead to the situation described in fig. 2 where we put, instead of the eye, a rounded surface with similar dimensions to those of the real eye. It is possible to find an appropriate rounded surface by taking two thin cylinders coloured black and white. Then, we need to put the black one close to the eye while the white one is placed more far in the line of sight. The right size of the surface is the one such that the black shape perfectly hides the white one and vice versa. In this way we’re sure to take a rounded surface not smaller than the eye. Now we need to find the upper limit angle: this is easier, we just need to shift the cylinder on the ruler until the Sun completely disappears and we block it. The angle identified by

Fig. 3. – Real eye geometry: the true solar angular diameter is obtained leading tangents from the eye border to the cylinder and up to the solar limb. Evaluating the solar angular diameter from the center of the eye yields an overestimate of it, as Archimedes pointed out in the Arenarius or Sand Reckoner.
Table I. – Last Contact timings of Total Eclipse of January 24, 1925

| Aperture (inches) | Magnification power | Timea |
|------------------|---------------------|-------|
| 2.75             | 33                  | h : 7 46m 55.6s |
| 3.4              | 80                  | h : 7 47m 02.4s |
| 5.0              | 55                  | h : 7 46m 47.3s |
| 6.0              | 60                  | h : 7 46m 59.9s |
| 20.0             | 175                 | h : 7 46m 59.5s |

a The first three of these times were noted by the correspondent observers with stop watches; the last two chronographically.

the line from the eye’s border tangent to the cylinder is surely bigger (or equal at least but not smaller) than the true one because now we can’t see the Sun no more (fig.3). Quantitatively, Archimedes measured the lower limit 1/200 times the squared angle and that the upper limit 1/164 times the squared angle, i.e. $27' \leq \Theta_\odot \leq 32'55''$. With such accuracy no oblateness was detectable; unless the apparent one due to atmospheric refraction, quantitatively studied by Tycho Brahe in sixteenth century.

2. – Measuring solar diameter with eclipses

The method of measuring the solar diameter with eclipses exploits the same principle of that one of the transits of Mercury: to recover the solar disk from more than three points. Those points correspond to the external (first and fourth) or internal (second and third, not present in a partial eclipse) contacts of the Moon or Mercury with the solar disk, as seen by different observers at different locations on the Earth (see figure 4).

The observations of the instants of totality (second and third contact) are not affected by atmospheric seeing, because of the sudden change of the overall luminosity. Conversely the determination of the instants of the external contacts during a partial eclipse (and all contacts during a transit), until the possibility of using CCD cameras or adaptive-optics techniques, was heavily affected by the seeing and the resolution of the telescope.

2.1. Historical data. – The effect due to the telescope resolution matching with the seeing conditions, is evident from the data retrieved in the 1925 total eclipse at the Chamberlin Observatory in Denver, Colorado [Table 1 from Howe (1925)]

From this dataset the eclipse has lasted more when observed with larger instruments, exception made for the second data.

2.2. New perspectives. – We propose to monitor the external contacts of a partial eclipse with CCD cameras whose frame rate $\Delta t \sim 0.01$ s is below the timescale of atmospheric seeing. Many solar photons can be gathered even with a semi-professional telescope of diameter $d \geq 0.2$ m with a bandpass filter. CCD frames can fix the instantaneous wavefront path. The presence of the lunar limb helps to evaluate the instantaneous point spread function for reconstructing the unperturbed wavefront according to current image-restoration techniques (Sanchez-Cuberes et al., 2000).

The identification of the lunar limb features and the solar limb near the contact event, in each image, and the absolute timing of each frame with WWV radio stations, will allow
Fig. 4. – In a partial eclipse we have only external contacts between solar and lunar limbs: (A) first and (B) last. Total and annular eclipses have also internal contacts which delimitate the phases of totality or annularity of the eclipse.

to know precisely the lunar feature and the time of the contact’s event.

If our partial eclipse method can be made to succeed in practice, it will open up more possibilities for measuring the solar diameter, especially because partial eclipses can be seen from large observatories relatively often.

2.3. Image-restoration techniques. – The combination between finite resolution of the telescopes, atmosphere’s turbulence and stray lights from other regions of the solar disk (both due to scattering in the Earth atmosphere and by optical surfaces) can be quantitatively studied during a partial eclipse. In fact the degradation effects made by the imaging system (atmosphere+telescope) are to be considered at the exact instant of the exposure, and when the Moon’s limb crosses the solar disk, it serves as a reference object to estimate the amplitudes of the instantaneous optical transfer function.

Once known that function, there are several methods developed to compensate for the atmospheric effects (Sanchez-Cuberes et al., 2000).

For example, with the 50 cm Swedish Vacuum Solar Telescope SVST at La Palma Löfdhal et al. (1997) used phase-diversity speckle restoration technique to study the evolution of bright points (0.2 arcsec of apparent dimension). The application of phase diversity restoration technique allows to reach the limits imposed by the diffraction to the instrument and can help adaptive optics to improve them (Criscuoli et al. 2001).

Recently Sanchez-Cuberes et al. (2000) have studied at high resolution (0.53 arcsec at the solar limb, and better in other regions) solar granulation features from CCD images of 13 ms of exposure taken with the SVST during the eclipse of May 10, 1994. Their idea was to match the lunar limb present in each frame to the numerical
simulation of the eclipse geometry, having included the lunar limb topography as given by the Watts’ profiles (Watts, 1963[7]).

Although past efforts to determine the solar diameter using observations of partial solar eclipses have failed due to atmospheric seeing, the possibility to restore video CCD images can succeed in the goal of determining with a great accuracy, for each observer whose position is known within 10 m of accuracy:

- the features of the lunar limb which firstly ‘hits’ the solar limb, and that one which is the last; with an accuracy of 0.2 arcseconds.
- the instant of the external contacts of the actual lunar limb with the solar limb, with an accuracy of 0.01 s.

3. – Solar diameter measurements using total eclipses and transits

The method we present here is to be compared to the determination of the North-South diameter of the Sun (which is the polar one only when $P_0 = 0^\circ$, at apsides) from the analysis of total solar eclipses observed at the edges in order to recover secular variations in the solar diameter (Dunham and Dunham, 1973[19]; Fiala et al. 1994[2]) and to other determinations of the solar diameter based upon the observations of meridian transits of the Sun (see e.g. Winlock, 1853[3] and Ribes et al., 1988[16] accounting on the observations made by Picard in seventeenth century) or of the transits of Mercury across the photosphere (see Parkinson et al., 1980[5] and Maunder and Moore, 2000[6] for a complete historical review).

3.1. Total eclipses from centerline. – Totality occurs when the solar limb disappears behind the last valley of the eastern lunar limb and ends when the Sun reappears from another depression of the western lunar limb.

A source of error in the evaluation of solar diameter arises from the knowledge of Moon’s limb features. There are about 0.2 arcsec of uncertainty in Watts’ tables (1963[7], as determined from pairs of photoelectrically timed occultations (Van Flandern, 1970[17]; Morrison and Appleby, 1981[18]).

Therefore if one relies on Watts’ profile the best determinations with total solar eclipses can not reach an accuracy better than 0.2 arcsec. But the accuracy on the evaluation of the solar diameter can be considerably improved by measuring the times of dozens of Baily’s beads phenomena, involving a similar number of Watts’ points, thereby decreasing the error statistically.

3.2. Total eclipses from edges. – Even better is to make measurements relative to the same polar lunar valley bottoms at similar latitude librations, possible since all solar eclipses occur on the ecliptic with negligible latitude librations. It means to observe the total eclipse near the edges (Dunham and Dunham, 1973[19]).

Moreover, it is possible to exploit also situations of same longitudinal libration angle. The eclipses of 1925 and 1979 (after three complete Saros cycles, an ‘Exeligmos’ 54 years and 34 days) where also exactly at the same longitudinal libration angle: their comparison (Sofia et al., 1983[22]) removes the uncertainty on the measured variations of the solar diameter due to the Watts’ errors almost entirely.

With current lunar profile knowledge, then total and annular eclipses are better for determining the solar diameter, because they can produce polar Baily’s beads when observed at the edges of their totally (annularity) path.
An error of 10 m in the determination of the edges of the band of totality gives about 0.006 arcsec of uncertainty in the evaluation of solar diameter.

Regarding the timing of the beads events, the solar intensity goes to almost zero very quickly, then atmospheric seeing errors are more directly eliminated.

3.3. Transits. – The transits of Mercury of November 15, 1999, was a ‘grazing’ transit (Westfall, 1999[8]), not useful for an accurate measure of the solar diameter, because it did not allow to sample points of the solar disk enough spaced between them. The previous transit occurred in 1985 well before adaptive optics techniques and the large diffusion of CCD cameras. The transits of Mercury of May, 7 2003, and Venus (June 8, 2004 and 2012) have to be considered also for this purpose.

4. – Expected accuracy with partial eclipses evaluations

4.1. Positions of the observers. – An error of 10 m in the determination of the edges of the band of totality gives about 0.006 arcsec of uncertainty in the evaluation of solar diameter.

An accuracy of 10 m in geographical position of the observer can be achieved with about 10 minutes of averaging GPS.

4.2. Bandpass filter. – The observations have to be done with a filter with waveband of 6300 ± 800˚A, in order to have data always comparable between them, and in the same waveband of Solar Disk Sextant (SDS, see last paragraph).

4.3. Duration of the imaging of the external contacts. – The eclipse magnitude \( m \) is the fraction of the Sun’s diameter obscured by the Moon.

The relative velocity of the Moon’s limb over the Sun’s photosphere is about \( v = 0.5 \) arcsec per second, along the centerline of a total eclipse. For a partial eclipse the velocity of penetration of the dark figure of the Moon (perpendicularly to the solar radius) is \( v \sim 0.5 \cdot (1 - m) \) arcsec/s, then for having about 1 arcminute of Moon already in the solar disk it is necessary to continue to take images for \( \Delta t = 120/(1 - m) \) s after the first contact and before the last contact.

The instants \( t_1 \) and \( t_4 \) of the external contacts can be determined with an accuracy better than the frame rate \( \Delta t \leq 10^{-2} \)s. In fact \( t_1 \) and \( t_4 \) can be deduced by interpolating the motion of the rigid Moon’s profile, which becomes better defined as the eclipse progresses.

In this way each observer (2 at least are needed) can fix two points on the Moon’s limbs and two instants for the contacts.

4.4. Expected accuracy in the solar diameter measurements. – The accuracy of the determination of the lunar features producing the external contacts for a given observer is therefore limited by the Watts’ profiles errors (0.2 arcsec).

Two observers enough distant (500 to 1000 Km in latitude for a East-West path of the eclipse) allow to have 4 points and 4 times for recovering the apparent dimensions of the solar disk at the moment of the eclipse within few hundredth of arcsecond of accuracy.

The accuracy becomes worse as the points sample a smaller part of the solar circle. The following table shows how the error on the determination of the solar diameter changes from having three points within 60 degrees to 240 degrees.
That accuracy can allow the detection of the oblateness of the Sun. Therefore more than three observers can allow improvement of the detection of the shape of the Sun by minimizing the residuals of the best fitting ellipse.

5. – Perspectives on eclipse methods

We have proposed an accurate measurement of the solar diameter during partial solar eclipses. This method is the natural extension of the method of measuring the solar diameter during total eclipses. It exploits modern techniques of image processing and fast CCD video records to overcome the problems arising from atmospheric turbulence. With this method professional and semi-professional observatories can be involved in such measurements, much more often than in total eclipses.

Moreover this method can be used for obtaining data useful for the absolute calibration of measurements by instruments that are balloon-borne (Sofia et al., 1994; 1996) or satellite-borne (Damé et al., 1999) with a precision of $\Delta D \leq 40 \times 10^{-3}$ arcsec.

It is also to note that from the first to the fourth contact of eclipses there are about two hours. The apparent solar diameter changes with a maximum hourly rate up to $25 \times 10^{-3}$ arcsec/hr due to the orbital motion of the Earth; this effect is strongly reduced around the apsides on July 4th and January 4th, $\leq 2 \times 10^{-3}$ arcsec/hr and this is a favourable case for eclipses in December-January or June-July.

In the future Watts’ tables can be substituted by the upcoming (2004) data of the Selene Japanese spacecraft, and the systematic errors arising from them will be avoided.

6. – Secular variations of solar diameter

It was the 3rd of May 1715 when solar eclipse was observed in England from both edges of the paths of totality. Following Dunham and Dunham method it is possible to extract solar radius information by determining the edges of the path of the totality. Unluckily there are elements of uncertainty on the effective positions of the observers on the edges and this causes a remarkable error on the radius determination using 1715’s eclipse data. Another eclipse on January 24, 1925 was very accurately observed by more than 100 employees of the Affiliated Electric Companies of NY City and many other advanced amateurs in response to the campaign led by E.W. Brown and a detailed study was made after the observation. Sofia, Fiala et al. used Brown’s data and found a correction of $(0.21 \pm 0.08)$ arcsec for the standard solar radius value of 959.63 arc sec at a distance of 1UA, for 1925 eclipse. Analyses of the eclipse in Australia in 1976 and of the eclipse in North America in 1979, were made by Sofia, Fiala et al. but no appreciable changes in the solar radius were found between those two eclipses. However, the solar radius determined for 1715 was found to be $(0.34 \pm 0.2)$ arcsec larger than 1979 value. On the other hand, Sofia, Fiala, Dunham and Dunham found that between the 1925 and the 1979 eclipses, the solar radius decreased by 0.5 arcsec but the solar size between 1925 and 1715 did not significantly changed. Therefore they concluded that the solar radius changes are not secular. Eddy and Boornazian in the same year reported results over observations made between 1836 and 1953 at the Royal Greenwich Observatory. They found a secular decrease trend in the horizontal solar diameter amounting to more than 2 arc sec/century while the solar vertical diameter seemed to change with about half of this rate. With the same data Sofia et al. had found out that any secular changes in the solar diameter in the past century, could not have exceeded 0.25 arc sec. The disagreement between the results of different groups depends on the different data.
selection criteria and on different solar and lunar ephemerides adopted, as it is shown in [2] for the analyses of the annular eclipse of May 30, 1984. Another measurement of the solar radius, independent on lunar ephemerides, was made by Shapiro [24] who analyzed data from 23 transits of Mercury between 1736 and 1973. His conclusion was that any secular solar radius decrease was below 0.15 arc sec/century. This method has been criticized for the black drop effect which affects the exact determination of the instants of internal contacts, first pointed out by Captain Cook during Venus’ transit of 1769.

7. – The method of Dicke for measuring solar oblateness

Around 1961, R. H. Dicke and others [25] tried to point out the possible effects due to existence of a scalar field in the framework of Einstein’s General Relativity. The presence of such a scalar field would have important cosmological effects. The gravitational deflection of light and the relativistic advancement of planetary perihelia are two effects that could have been influenced by a scalar field: with respect to classical General Relativity both effects were expected to be about 10% less in the case the scalar field would be present. For this reason Dicke showed that the advancement of the line of apsides of Mercury was not to be considered as a good test for General Relativity, which was believed before, because of the entanglement of its causes (scalar field and classical General Relativity) [26]. A small solar oblateness ($\Delta R/R \sim 5 \cdot 10^{-5}$) caused by internal rotation in the Sun would cause the 10% effect of perihelion advancement without invoking any relativistic effect. It was clear that until such oblateness could be excluded or confirmed from observational data, the interpretation of the advancement of Mercury’s line of apsides would was ambiguous. The Einstein relativistic motion of the longitude of the perihelion is

$$\dot{\pi} = \frac{1}{T a e^2 (1-e^2)^{3/2}}$$

where $a$ is the planetary semimajor axis, $e$ is the eccentricity and $T$ is the period; on the other hand we have the rotation of the perihelion due to an oblate Sun which is

$$\dot{\pi} = \frac{\Delta}{T a e^2 (1-e^2)^{3/2}}$$

where $\Delta$ is the ratio between (solar equatorial radius - polar radius) and (mean radius). The scalar-tensor theory of gravitation could have been brought in agreement with observational data, if the Sun possessed a small oblateness and a mass quadrupole moment.

In 1966, Dicke and Goldenberg [27] measured the difference in flux between the equator and polar limb of the Sun. The idea was simple: using a chopper with apertures made to show only a small section of the solar limb (see fig. 5), they measured the flux at the poles and at the equator of the Sun. We must consider two iipothesis:

- If the temperature at the pole is equal to the one at the equator, finding a flux difference can only mean that there is an oblateness such that the radius at the pole $R_p$ and the radius at the equator $R_e$, differs from a quantity $\Delta R$. Then the flux difference $\Delta F$ should be constant if we change the exposed limb by changing the chopper’s aperture.

- On the contrary, if there is no oblateness but we still have a flux difference $\Delta F$, means that there is a temperature gradient. Then $\Delta F$ should be proportional to the amount of exposed limb.

Dicke and Goldenberg found that $\Delta F$ remained about constant so that the Sun should have a small oblateness. The theory of the measurement is well explained in [28]. II.
Fig. 5. – Measurement of the flux $F$ within the chopper mask at the solar poles. By changing the fraction $f_l$ of exposed limb it is possible to detect if there is any $\Delta F$ between polar and equatorial diameters, and if it changes with $f_l$. Only if $\Delta F \neq 0$ and it is constant with $f_l$ it is a consequence of the solar oblateness, otherwise it would be a consequence of temperature gradients.

The instrument used and the measurement procedure is also explained in [28], IV, V, VI and VII. Mainly both the instrument and the measuring procedure were designed to eliminate systematic errors. Dicke found for $\Delta R$ to be $\Delta R = 43.3 \pm 3.3 \times 10^{-3}$ arcsec.

The oblateness of $\Delta R / R = (4.51 \pm 0.34) \times 10^{-5}$ implies a quadrupole moment of $J = (2.47 \pm 0.23) \times 10^{-5}$ [29].

At the end of his analysis, Dicke found that new independent measurements of the solar oblateness were needed, to make comparison between data taken with different faculae activity on the Sun. In 1975, new observations were made by Hill and Stebbins [30]. They considered a complication raised in Hill’s work [31] of a time varying excess of equatorial brightness due to sunspots and faculae. It is clear that to measure the difference between the polar radius and the equatorial radius, we must first be sure on which point to take as equatorial solar edge. The point is to give out a consistent definition of the solar limb. This can be done by using a proper limb darkening function. Hill et al. demonstrated that the excess brightness can be easily monitored by using a proper analytic definition of the solar edge, using the FFTD [32]. It was pointed out that the main problem in this kind of measurements is identifying some point on the limb darkening curve as the solar edge. It is clear that if more points on the darkening curve can be taken as solar edge, many different definition of solar radii can be gave and many different measures of solar oblateness done. The differences between these values will contain information about the shapes of the limb profiles. The value obtained from Hill for the intrinsic visual oblateness is $(18.4 \pm 12.5) \times 10^{-3}$ arcsec which is obviously
in conflict with the value of Dicke-Goldenberg. In this confused situation another group decided to construct an instrument to measure long term changes: the Solar Diameter Monitor (SDM) at The High Altitude Observatory. Their purpose was to determine which kind of solar diameter variation was taking place, if any, within a reasonable period of time (3-5 years). The SDM began operation in Aug. 1981. An accurate discussion on the measured duration of solar meridian transit during six years between 1981 and 1987 is made in [34] where Brown and Christensen-Dalsgaard adopted adjustments to the modified IAU value for the astronomical unit (value of $1.4959787066 \cdot 10^5$ Mm, US Naval Observatory, 1997) to take into account for the mean displacements between the telescope’s noontime location and the Earth’s centre. They also corrected for the displacements of the Sun’s centre relative to the barycentre of the Earth-Sun system. They found the solar radius to be $R_\odot = (695.508 \pm 0.026)$ Mm which is about 0.5 Mm smaller than the Allen (1973) value of 695.99 Mm. Moreover, Brown and Christensen-Dalsgaard found no significant variations in the solar diameter during their observational period: their annual averages for the years 1981-1987 all agree within $\pm 0.037$ Mm. Toulmonde [35] discussed about 71000 measurements regarding almost 300 years of data: he did not find evidence of any secular variation in his data.

8. – Solar Disk Sextant measurements

Further attempts to measure the solar oblateness have been made with the Solar Disk Sextant (SDS) which is an instrument made to monitor the size and shape of the Sun. The principle of the instrument is well described in Sofia, Maier and Twigg work [36]. Basically a prism with an opening angle very stable along the years is posed in front of the objective of a telescope, and it produces two images of the Sun at focal plane. The distance between the center of those images is depending on the focal length of the telescope, while the gap between the two limbs depends on the angular diameter of the Sun. The same idea is exploited in using two pinholes instead of one and has been proposed for simpler prototypes of SDS [37], whose images are unaffected by optical distortions. Considered that the solar radius changes until now reported are to be of the order of 1 arc sec per century, the SDS instrumental accuracy was asked to keep calibrated on 0.01 arc sec/year and a stability of 0.003 arc sec/year was reached. The really good feature of the SDS consists in the fact that the instrument accuracy requirements are for relative rather than absolute values of the radius which led to a solar edge point detection accurate to 1/10 pixel on the instrument. With statistical methods one can have a further reduction of a 10 factor. The SDS early version was developed to be carried into space during Space Shuttle flights, but unlikely the Challenger accident took place. This led to the needs to change strategy avoiding important delays. SDS was mounted on a system for ground based observations but it was soon clear that no valuable scientific data could be obtained from ground because the atmosphere's influence. So the SDS was mounted on a stratospheric balloon and it measured solar oblateness [11]. A complete analysis of his 4 flights data (1992, 1994, 1995 and 1996) is still in progress.

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