Experimental Limits on the Width of the Reported \( \Theta(1540)^+ \)

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Using data on \( K^+ \) collisions on xenon and deuterium we derive values and limits on the width of the reported \( \Theta(1540)^+ \) exotic baryon resonance. The xenon experiment gives a width of \( 0.9 \pm 0.3 \) MeV. The other experiments give upper limits in the range 1 - 4 MeV.

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I. INTRODUCTION

The general features of the spectroscopy of mesons and baryons can be understood from the simple rules that a meson is made of a quark and an antiquark, while a baryon is made from three quarks. These rules are consistent with the principles of quantum chromodynamics, which show that physical particles are neutral under color-SU(3). However, QCD does not preclude the existence of other colorless configurations, including gluons or additional quarks and antiquarks. Recent results from a diverse collection of experiments [1–6] show evidence for a state \( \Theta(1540)^+ \), whose quantum numbers are those of the combination \( uudds \) and which thus cannot be composed simply of three quarks. Such states have been predicted [7] in a Skyrmion model.

The \( \Theta(1540)^+ \) has not been detected in data from a number of early experiments in which one might expect it to appear. However, definitive negative conclusions cannot be drawn in the absence of reliable predictions for production cross sections.

In situations where the \( \Theta(1540)^+ \) is or ought to be formed resonantly, as an intermediate state in a scattering experiment, it is possible to draw conclusions about its width from existing data. Such results can provide guidance to the structure of the resonance, or more important, to the likelihood that there truly is a such resonance.

The resonant cross section is determined entirely by \( \Gamma \), the width of the resonance and its branching ratios \( B_i \) and \( B_f \) into the initial and final channels according to the Breit-Wigner form:

\[
\sigma(m) = B_i B_f \sigma_0 \left[ \frac{\Gamma^2/4}{(m-m_0)^2 + \Gamma^2/4} \right] 
\]

With \( k \) as the CM momentum, \( s_1 \) and \( s_2 \) the incident spins and \( J \) the spin of the resonance, we have

\[
\sigma_0 = \frac{2J+1}{(2s_1+1)(2s_2+1)} \frac{4\pi}{k^2} = 68 \text{ mb} 
\]

where we have taken the values for \( K^+n \) collisions at a resonant mass \( m_0 \) of 1540 MeV and assumed that the resonance has \( J = 1/2 \). As we shall see, the mass resolution of the experiments is always broader than the natural width of the resonance so that the observable quantity is the integral of the resonant cross section:

\[
\int_{-\infty}^{\infty} dm \frac{B_i B_f \sigma_0}{\Gamma/2} \left[ \frac{\Gamma^2/4}{(m-m_0)^2 + \Gamma^2/4} \right] = B_i B_f \frac{\pi \Gamma}{2} \sigma_0 = (107 \text{ mb}) \times B_i B_f \Gamma \tag{3}
\]

II. \( K^+n \rightarrow K^0p \) IN XENON

In the DIANA experiment [2], in which a \( K^+ \) beam with momentum 750 MeV entered a xenon bubble chamber, the signal for the \( \Theta(1540)^+ \) was observed by measuring the \( pK_S \) invariant mass spectrum in the final state. If we treat the scattering as simply a two-body process, \( K^+n \rightarrow K^0p \), resonance occurs when the combination of the incident momentum of the \( K^+ \) and the Fermi momentum of the neutron give the invariant mass of the \( \Theta(1540)^+ \). Without the Fermi momentum, this would occur for a \( K^+ \) momentum of 440 MeV, to which the incident beam is reduced by ionization losses after penetrating a sufficient distance through the xenon. By observing the final-state invariant mass, reconstructed from the \( pK_S \), the effects of Fermi motion and incident beam degradation are removed, provided that we can ignore rescattering within the nucleus.

The signal in this experiment emerges only after making cuts that are believed to reduce the effect of rescattering. We make the assumption that, in the mass region near the resonance, it is the charge-exchange process on a single nucleon that is observed and that the cuts reduce the resonant and non-resonant charge-exchange processes by the same factor. The apparent resonant signal is contained within two 5-MeV bins. The background varies smoothly in this region at a value near 22 events per
bin. The resonant signal consists of about 26 events. We associate the background events with the $K^+d$ charge exchange cross section interpolated from off-resonance measurements, namely $4.1 \pm 0.3$ mb [8, 9]. In this way we determine the integral of the resonant cross section to be $(26/22) \times 5 \text{MeV} \times 4.1 \text{mb} = 24 \text{mb MeV}$. Using Eq. (3), and $B_i = B_f = 1/2$, appropriate for either $I = 0$ or $I = 1$, we deduce a width $\Gamma = 0.9 \pm 0.3$ MeV, where the quoted error is statistical only. There are systematic uncertainties, which we are unable to evaluate, associated with rescattering in the nucleus and with the cuts that isolate the signal. Of course, it is assumed that the excess events are indeed the result of a resonance $\Theta(1540)^+$ and not an artifact.

III. $K^+d$

If the decay products $K^+n$ or $K^0p$ are not measured precisely, the collision energy with a nuclear target is necessarily uncertain as a consequence of Fermi motion. In the case of a deuterium target, the Fermi motion can be treated quite completely. Let the momentum of the incident $K^+$ be $P_K$ and let the component of the neutron’s Fermi momentum in the beam direction be $p_z$. Then the CM energy squared is

$$s = m_K^2 + 2E_Km_N + m_N^2 - 2p_zP_K$$

(4)

where we have introduced $E_K^*$ as the beam energy that would make the resonance in the absence of Fermi motion. Because the resonance is narrow, we can write the Breit-Wigner form as

$$\sigma(E_K,p_z) = \sigma_0 B_i B_f \frac{\pi \Gamma}{2} m_0 \delta((E_K - E_K^*)m_N - p_zP_K)$$

(5)

The distribution $F(p_z)dp_z$ is related to the full Fermi momentum distribution $f(p)d^3p$ by

$$F(p_z) = 2\pi \int_{|p_z|}^{\infty} dp_\perp f(p)$$

(6)

In terms of $F(p_z)$, the resonant cross section, integrated over the distribution of Fermi momenta is

$$\sigma(E_K) = \sigma_0 B_i B_f \frac{\pi \Gamma}{2} m_0 F((E_K - E_K^*)m_N/P_K)$$

(7)

$$= (372 \text{mb}) \times B_i B_f \Gamma F((E_K - E_K^*)m_N/P_K)$$

The momentum-space wave function for the deuteron is easily computed from a spatial wave function in the Hulthén form [10]:

$$\phi(r) = N \frac{1}{r} (e^{-\alpha r} - e^{-\beta r})$$

(8)

Here $\alpha$ is related to the deuteron binding energy $\Delta E$ by $\alpha = \sqrt{m_N \Delta E} = 45.5$ MeV. A typical value for $\gamma = \beta/\alpha$ is 6.16. The momentum-space wave function is

$$\phi(p) = \int d^3r e^{-ip\cdot r} \frac{\sqrt{\alpha \beta}}{\pi} \frac{(\alpha + \beta)^{3/2}}{(\alpha^2 + p^2)(\beta^2 + p^2)} \frac{1}{(\gamma^2 - 1)^2} \frac{1}{1 + \frac{\gamma^2 + \delta^2}{\gamma^2 + \delta^2}}$$

(9)

The momentum distribution is $f(p) = |\phi(p)|^2$. Setting $F(p_z) = \alpha^{-1}G(\gamma, \delta)$, with $\delta = p_z/\alpha$, we find directly

$$G(\gamma, \delta) = (1 + \gamma)^3 \gamma \left\{ \frac{1}{(\gamma^2 - 1)^2} \frac{1}{1 + \delta^2} \frac{1}{\gamma^2 + \delta^2} \right\}$$

(10)

The function $G(\gamma, \delta)$ is shown for $\gamma = 6.16$ in Fig. 1.

FIG. 1: The function $G(\gamma, \delta)$, which gives the density of nucleons in the deuteron as a function of momentum in a given direction $p_z = \alpha \delta$, where $\alpha = 45.5$ MeV.

Exactly at resonance we have $G(\gamma = 6.16, \delta = 0) = 0.438$ and thus $F(0) = 0.0096 \text{MeV}^{-1}$. For a $K^+$ beam that is nearly tuned to the resonant momentum of 440 MeV we then have for the cross section averaged over Fermi momenta,

$$\sigma(E_K) = (3.6 \text{mb/MeV}) B_i B_f \Gamma$$

(11)

Charge exchange in $K^+d$ collisions has been measured by Slater et al. [8] at incident momenta of 376 MeV and 530 MeV and by Damerell et al. [9] at a momentum of 434 MeV. From Fig. 1 a direct calculation shows that the former two measurements are sufficiently far from the resonance to be unaffected, while the last is so close that we can take $p_z = 0$. The measurements are listed in Table I.

Comparison of the cross sections shows no sign of an enhancement near 440 MeV. We take a conservative limit of 1 mb on the resonant cross section. Using $B_i = B_f = 1/2$, we derive a limit of 1.1 MeV.

A similar analysis can be made using the total cross section data of Bowen [11]. The measurements are shown
TABLE I: Charge-exchange cross section in $K^+d$ reactions measured by Refs. [8, 9].

| $P_K$ (MeV) | $\sigma_{CEX}$ (mb) |
|-------------|---------------------|
| 376 [8]     | 3.1 ± 0.4           |
| 434 [9]     | 4.0 ± 0.15          |
| 530 [8]     | 6.5 ± 0.6           |

in Table II. Here linear interpolation shows an excess of $0.60 \pm 0.30$ mb at the resonant momentum, though this might well be simply a sign of a gradual deviation from linearity. We take $1.5$ mb as a conservative upper limit on the resonant cross section. Then using $B_i = 1/2$ and $B_f = 1$, we find an upper limit of $\Gamma = 0.8$ MeV.

| $P_K$ (MeV) | $\sigma_{tot}$ (mb) |
|-------------|---------------------|
| 366         | 21.41 ± 0.30        |
| 440         | 23.46 ± 0.24        |
| 506         | 24.16 ± 0.23        |

TABLE II: Total $K^+d$ cross section measured by Ref. [11].

An even more conservative limit is obtained by taking the entire $I = 0$ cross section at $P_K = 440$ MeV to be resonant. This cross section is reported by Bowen [11] to be 9.4 mb, while Carroll et al. [12] find 13 mb. This latter gives an upper limit for $\Gamma$ of 3.6 MeV.

Our results are consistent with those obtained by Arndt, Strakovsky, and Workman [13], who, using their partial wave analysis of $K^+N$ data, exclude a $\Theta(1540)^+$ with a width of more than a few MeV. Our results are also consistent with, but more stringent than those obtained by Nussinov [14] and by Haidenbauer and Krein [15].

IV. COMMENTS

A width of 1 MeV is quite uncommon for a hadronic decay. For comparison we consider the $\Lambda(1520)$, which decays by $d$ wave to $\Lambda N$ with a partial width of 7.2 MeV. If the $\Theta(1540)^+$ decays via $p$ wave, it might be expected to be somewhat broader than the $\Lambda(1520)$. Instead it is evidently much narrower.

It is not possible to make quantitative statements of the same sort using the photoproduction data reported by CLAS [3, 4] or SAPHIR [5]. However, qualitatively, the very small apparent width suggests that nonresonant production cross sections should be quite small, while the data of these experiments seem to show quite visible effects.

The value for the width inferred from the DIANA and the limits derived from the charge-exchange and total-cross-section measurements in deuterium are not inconsistent. However, they point to such a narrow width that, if the $\Theta(1540)^+$ truly exists, it is exotic dynamically as well as in its quantum numbers.

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