Parity asymmetry in the CMBR temperature power spectrum

Pavan K. Aluri\textsuperscript{1,*} and Pankaj Jain\textsuperscript{*}

\textit{Department of Physics, Indian Institute of Technology, Kanpur 208016, India}

Accepted 2011 October 10. Received 2011 September 15

ABSTRACT

We study the power asymmetry between even and odd multipoles in the multipolar expansion of cosmic microwave background temperature data from the \textit{Wilkinson Microwave Anisotropy Probe (WMAP)}, recently reported in the literature. We introduce an alternate statistic which probes this effect more sensitively. We find that the data are highly anomalous and consistently outside the 2\(\sigma\) significance level in the whole multipole range \(l = [2, 101]\). We examine the possibility that this asymmetry may be caused by the foreground cleaning procedure or by residual foregrounds. By direct simulations we rule out this possibility. We also examine several possible subdominant foregrounds, which might lead to such an asymmetry. However, in all cases we are unable to explain the signal seen in data. We next examine cleaned maps, using procedures other than that followed by the \textit{WMAP} science team. In particular, we analysed the maps cleaned by the Internal Power Spectrum Estimation (IPSE), needlets and the harmonic Internal Linear Combination (ILC) procedures. In all these cases, we also find a statistically significant signal of power asymmetry if the power spectrum is estimated from the masked sky. However, the significance level is found to be not as high as that in the case of the \textit{WMAP} best-fitting power spectrum. Finally, we test the contribution of low-\(l\) multipoles to the observed power asymmetry. We find that if we eliminate the first six multipoles, \(l = [2, 7]\), the significance falls below the 2\(\sigma\) confidence level. Hence, we find that the signal gets dominant contribution from low-\(l\) modes.

Key words: cosmic background radiation – cosmology: miscellaneous – cosmology: theory.

1 INTRODUCTION

The primary aim of the \textit{Wilkinson Microwave Anisotropy Probe (WMAP)} satellite has been to measure full-sky CMBR temperature anisotropies with great precision (Bennett et al. 2003a). The primary quantity of interest from these full-sky maps is the temperature power spectrum, which is used to constrain various cosmological parameters (Hinshaw et al. 2003; Spergel et al. 2003; Larson et al. 2011; Komatsu et al. 2011). It also gave TE cross power spectrum and E-mode power to a good precision (Kogut et al. 2003; Page et al. 2003; Komatsu et al. 2011; Larson et al. 2011). Since the release of \textit{WMAP} first-year data, many large-scale anomalies were reported in the data (Bennett et al. 2003b, 2011; Efstathiou 2003; de Oliveira-Costa et al. 2004; Eriksen et al. 2004a,b; Ralston & Jain 2004; Copi, Huterer & Starkman 2004; Schwarz et al. 2004; Land & Magueijo 2005b; de Oliveira-Costa & Tegmark 2006; Copi et al. 2007, 2010; Samal et al. 2008). Recently, Kim & Naselsky (2010) found that the cosmic microwave background (CMB) power in odd multipoles is anomalously more than that in the even multipoles (see also de Oliveira-Costa et al. 2004; Land & Magueijo 2005a; Gurzadyan et al. 2007; Ben-David, Kovetz & Itzhaki 2011; Gruppuso et al. 2011; Hansen et al. 2011). Some possible sources of such odd modulations to CMB, such as signals from an inflationary era and Solar system physics, were discussed in Groeneboom et al. (2010), Koivisto & Mota (2011) and Maris et al. (2011). This power asymmetry between even and odd multipoles is also called \textit{parity asymmetry}.

The CMBR temperature fluctuations on a sphere are usually expanded in terms of spherical harmonics, \(Y_{lm}\), as

\[ \Delta T(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi), \]

where \(a_{lm}\) are the multipolar expansion coefficients. The power spectrum of CMB is defined as

\[ C_l = \frac{1}{2l + 1} \sum_{m=-l}^{l} a_{lm}^* a_{lm}. \]

We denote \(\mathcal{C}_l = l(l+1)C_l/2\pi\). Kim & Naselsky (2010) defined the following two quantities:

\[ P^+ = \sum_{l=2}^{l_{\text{max}}} \frac{1 + (-1)^l}{2} \mathcal{C}_l, \]

\[ P^- = \sum_{l=2}^{l_{\text{max}}} \frac{1 - (-1)^l}{2} \mathcal{C}_l. \]
and

\[ P^+ = \sum_{l=2}^{l_{\text{max}}} \frac{1 - (-1)^l}{2} C_l. \]  

(4)

Here, \( P^+ \) and \( P^- \) represent the sum of power in the even and odd multipoles, respectively. The parity asymmetry statistic was defined as \( P^+/P^- \), which we refer to as \( P(l_{\text{max}}) \) for convenience. By comparing the data (Larson et al. 2011) with ‘pure’ realizations of the CMB signal generated based on the Λ cold dark matter (ΛCDM) model, Kim & Naselsky (2010) estimated the \( p \)-value to be 0.002. When this statistic was applied to full-sky \( C_l \) recovered from masked maps, using KQ85 mask from the WMAP seven-year data release, the minimum probability or \( p \)-value was found to be \( p = 0.003 \). The \( l_{\text{max}} \) corresponding to this \( p \)-value was found to be 22 from simulations, which is an a posteriori choice. By accounting for this posterior choice of \( l_{\text{max}} = 22 \), the probability was estimated to be reduced to 0.02.

In this paper, we study this parity asymmetry in considerable detail. We first consider an alternate statistic to test for the parity asymmetry. This statistic is found to be more sensitive than that considered in Kim & Naselsky (2010). Next, we investigate whether this asymmetry might arise due to foregrounds, which are not symmetric under parity. Most of the foreground contamination, however, gets removed in the process of extracting the primordial power spectrum. Nevertheless, the residual foregrounds might be sufficiently large to cause parity asymmetry. Hence we use simulated foreground cleaned maps to test the significance of parity asymmetry in the WMAP CMBR data. We utilize both the ILC and IPSE cleaning procedures for this purpose. The simulated maps are generated using random realizations of the CMBR and the pre-launch Planck Sky Model (PSM) for foregrounds. We also allow for the possibility of some unknown foreground components. Another interesting anomaly found in the CMB data is the ecliptic dipolar modulation of the CMB power, discovered in Eriksen et al. (2004a). This signal is also parity asymmetric and hence one may suspect that there might be a relationship between this and the signal discovered in Kim & Naselsky (2010). We study the possibility that a dipole modulation of temperature anisotropy might lead to the observed parity asymmetry. Furthermore, we examine whether foreground cleaned maps obtained using alternate procedures such as the IPSE and needlet ILC also show the parity asymmetry observed in the WMAP best-fitting power spectrum. Finally, we determine the contribution of the modes at very low \( l \) to the observed asymmetry.

This paper is arranged as follows. In Section 2, a different statistic to understand this even–odd power asymmetry is presented. Then, in Section 3, we present our results obtained from mock cleaned data used to test the effect of foreground residuals. In Section 4, we explore the effect of unknown influences on the data which might be modulating the primordial signal to induce the observed power asymmetry. In Section 5, we present our analysis of the parity asymmetry in IPSE cleaned temperature data and the cleaned maps available using other procedures such as needlet ILC. In Section 6, our results from implementing different cuts at various low-\( l \) modes are shown. Finally, we conclude in Section 7.

2 AN ALTERNATE STATISTIC

In this section, we introduce a different statistic to quantify the parity asymmetry. As we shall see this statistic is a more sensitive probe than that given in equations (3) and (4). Instead of taking averages of \( I \) even or odd multipoles, we look at local \( I \) power asymmetry. It is defined as

\[ Q(l_{\text{odd}}) = \frac{2}{l_{\text{odd}} - 1} \sum_{l=3}^{l_{\text{max}}-1} \frac{C_{l_{\text{odd}}-1}}{C_l}. \]  

(5)

where the maximum, \( l_{\text{odd}} \), is any odd multipole \( l \geq 3 \) and the summation is over all odd multipoles up to \( l_{\text{odd}} \). Thus, \( Q(l_{\text{odd}}) \) is a measure of mean deviation of the ratio of power in an even multipole to its succeeding odd multipole from one, if it is present in the data. At low \( l \), since \( (l+1)C_l \sim \text{constant} \), statistically we expect our statistic to fluctuate about one, like \( P(l_{\text{max}}) \).

3 STATISTICAL SIGNIFICANCE OF PARITY ASYMMETRY USING PURE AND FOREGROUND CLEANED CMBR MAPS

We test the WMAP seven-year best-fitting CMB temperature power spectrum\(^2\) for anomalous parity asymmetry against both pure realizations of the CMBR and simulated cleaned maps. The pure CMB sky maps are generated as constrained realizations of the best-fitting theoretical CMB power spectrum from the ΛCDM model.\(^3\) The synfast facility of the freely available HEALPix\(^4\) software (Gorski et al. 2005) was used to produce full-sky pure CMB realizations at \( N_{\text{side}} = 512 \) of the HEALPix’s sky pixelization scheme. We then generate five raw maps corresponding to each frequency channel in which WMAP makes the observations. We do so by adding the pure CMB maps with synchrotron, thermal dust and free–free emission templates from the pre-launch PSM\(^5\) (Planck Blue Book 2005) available in each of the WMAP frequency bands. These maps were convolved with appropriate beam transfer functions of the \( K, Ka, Q, V \), and \( W \) bands of WMAP\(^6\) simulating the raw satellite data. We have also added Gaussian random noise in each pixel using the mean rms noise levels in each of the WMAP frequency channels provided in its seven-year data release (Jarosik et al. 2011). Strong foreground contamination to the observed cosmic CMB signal is assumed to be due to Galactic synchrotron, dust and free–free emissions. Synchrotron radiation is emitted from relativistic electrons in cosmic rays spiralling into the galactic magnetic field. When the dust grains in the interstellar medium get heated, they emit radiation due to vibrational mode transitions in infrared frequency which is the thermal dust emission. The free–free emission is due to the electron–ion interactions in the ionized medium between clusters of galaxies. The full-sky simulated raw maps thus generated were cleaned using the IPSE method as described below.

3.1 IPSE cleaning procedure

Here we briefly outline the cleaning procedure we employ to clean the simulated raw maps. The IPSE cleaning procedure (Tegmark, de Oliveira-Costa & Hamilton 2003; Saha, Jain & Souradeep 2006; Eriksen et al. 2007a; Saha et al. 2008; Samal et al. 2010) is a minimum variance optimization method better suited for multichannel CMB observations such as WMAP and Planck. It exploits the frequency dependence of astrophysical foregrounds received in various

\(^1\) While compiling references of this work, we learnt that a similar, but not identical, statistic was used by Land & Magueijo (2005a), which was referred to in Section 1.

\(^2\) Available at http://lambda.gsfc.nasa.gov/

\(^3\) Available at http://lambda.gsfc.nasa.gov/

\(^4\) Available at http://healpix.jpl.nasa.gov/

\(^5\) Available at http://www.planck.fr/heading79.html

\(^6\) Available at http://lambda.gsfc.nasa.gov/
The event–odd multipole power asymmetry in the WMAP's seven-year best-fitting temperature power spectrum in the multipole range \( l = [2, 101] \). The asymmetry is computed using both the \( P(l_{\text{max}}) \) statistic (lighter curve) and our parity asymmetry statistic, \( Q(l_{\text{odd}}) \) (darker curve).

Figure 1.

The even–odd multipole power asymmetry in the WMAP's seven-year best-fitting temperature power spectrum in the multipole range \( l = [2, 101] \). The asymmetry is computed using both the \( P(l_{\text{max}}) \) statistic (lighter curve) and our parity asymmetry statistic, \( Q(l_{\text{odd}}) \) (darker curve).

The method involves linearly combining various multi-channel maps in multipole space with appropriate weights as

\[
d_{\text{lm}}^{\text{clean}} = \sum_{i=1}^{n_c} \hat{a}_{i,lm} B_i^{-1}
\]

where \( d_{\text{lm}}^{\text{clean}} \) is the clean CMB signal extracted from the raw data, \( d_{\text{lm}} \), acquired from measurements in \( n_c \) frequency channels by linearly combining them with appropriate weights \( \hat{a}_{i,lm} \). The \( B_i \) factors are the symmetrized beam transfer functions in multipole space corresponding to an \( i \)th frequency channel. The weights are computed using the empirical covariance matrix

\[
\hat{C}_i = \frac{1}{2l+1} \sum_{m=-l}^{l} a_{lm}^* a_{lm}
\]

in the equation

\[
\hat{W}_i = \frac{e_0^T \hat{C}_i^{-1}}{e_0^T \hat{C}_i^{-1} e_0}
\]

where \( e_0 = (1, \ldots, 1)^T \) is a column vector with \( n_c \) unit elements and \( \hat{W}_i \) is also a column vector given by \((\hat{a}_{1,lm}, \ldots, \hat{a}_{n,lm})^T\). The clean power spectrum is then given by

\[
\hat{C}_i^{\text{clean}} = \frac{1}{e_0^T \hat{C}_i^{-1} e_0}
\]

Furthermore, taking into account the spatial variation of the foreground power across the sky, each map is divided into disjoint sky regions, and this procedure is applied iteratively in each of these sky partitions.

Using the IPSE cleaning procedure we generated an ensemble of 800 cleaned maps. A residual foreground bias correction was implemented on each of the cleaned maps by subtracting a bias map estimated from these simulations in pixel space (Bennett et al. 2003c). The power spectrum of each of these simulated maps was computed using the anafast facility of HEALPix and corrected for beam and pixel window effects. We used full-sky cleaned maps' power spectrum for the range \( l = [2, 10] \). The low-\( l \) power can be recovered reliably from full-sky cleaned maps using the IPSE method (Tegmark et al. 2003). For \( l \geq 11 \), we used full-sky \( C_l \) recovered from partial sky map we get after applying a galactic mask excluding the heavily contaminated regions in the sky. The KQ85 mask provided by the WMAP science team in their seven-year data release was applied to the simulated maps and we recovered the full-sky \( C_l \) using the MASTER of CMBR or the pseudo-\( C_l \) estimator (Hivon et al. 2002). At low \( l \), up to \( l = 32 \), the WMAP best-fitting power spectrum is estimated from a low-resolution ILC map by using the Blackwell–Rao likelihood estimator (Larson et al. 2011). For \( l > 32 \), Larson et al. (2011) estimated the multipole power using the same pseudo-\( C_l \) estimator that we use to estimate \( C_l \) at high \( l \). These power spectra form the basis of our analysis.

3.2 Statistical significance

We next compute the statistical significance using both the statistics, \( P(l_{\text{max}}) \) and \( Q(l_{\text{odd}}) \), given in equations (3)–(5). We used the best-fitting CMB temperature power spectrum from the WMAP seven-year data release as reference data. The values of both the statistics for the best-fitting power are shown in Fig. 1.

The random chance occurrence probability of getting a \( P(l_{\text{max}}) \) lower than that of the data for \( l_{\text{max}} \in [3, 101] \) is shown in Fig. 2. For the case of pure maps we used an ensemble of 10 000 simulated CMBR maps. We reproduce the results from Kim & Naselsky (2010) using the pure maps ensemble with the lowest probability.
Figure 2. Probability estimates of parity asymmetry seen in the data using the $P(l_{\text{max}})$ parity statistic, in the multipole range $l = [2, 101]$. The significances are computed using 10,000 pure maps and 800 cleaned maps, cleaned using the IPSE method. We find no significant difference between the two estimates. As can be seen, the most significant result occurs at $l = 22$ for both the cases and is beyond the 3σ CL.

at $l_{\text{max}} = 22$. From the 10,000 pure maps generated, the probability was found to be 0.0013. Also presented in the graph are significances computed using the cleaned maps ensemble. In this case, we used only 800 simulated maps due to constraints on computational time. As can be seen, the $p$-values from cleaned maps are slightly higher than the probability estimates from pure maps, but are relatively close. The foreground power has a strong even-parity preference, and Kim & Naselsky (2010) speculated that the observed asymmetry could be due to over-subtraction of foregrounds during foreground reduction. However, eventually they ruled out this possibility. Using cleaned maps, we confirm that this is indeed the case. Thus, any residual foreground contamination present in the cleaned maps may not induce a particular parity preference in the data. The contribution of noise is negligible to the power at low $l$. Since we studied this power asymmetry in a wider multipole range of up to $l = 101$, and incorporated noise in the simulated raw maps, we also conclude that noise cannot cause this asymmetry. From the ensemble of cleaned maps, the lowest probability for this parity asymmetry is again found to be at $l_{\text{max}} = 22$ with a chance probability of 0.13 per cent. This minimum value for significance is beyond the 3σ confidence level (CL) and quickly falls below the 2σ CL by around $l_{\text{max}} = 40$. Beyond $l_{\text{max}} = 40$, it largely stays below 2σ. Interestingly, the $P(l_{\text{max}})$ curve in Fig. 1 looks wavy, as if it was overlaid by some oscillations. It may be indicative of the presence of some underlying modulation (see e.g. Turner 1983; Martin & Ringeval 2004, 2006; Wang et al. 2005; Ichiki, Nagata & Yokoyama 2010).

With our estimator $Q(l_{\text{odd}})$, we find that in almost the entire multipole range $l = [2, 101]$, the significance of the parity asymmetry lies consistently outside the 2σ CL in both cases using the pure maps and the cleaned maps. These probability estimates are shown in Fig. 3. The only exceptions are the significances of the multipoles 63, 67 and 69 with the cleaned maps ensemble which are marginally inside the 95 per cent CL. Since $P(l_{\text{max}})$ involves the sum of all even or odd multipole powers up to a chosen $l_{\text{max}}$, which is equivalent to the mean power up to that $l_{\text{max}}$, it appears that their sum is hiding the true significance. From the plot we find that for $l = [18, 31]$, the significance of parity asymmetry using $Q(l)$ on the WMAP seven-year best-fitting temperature $C_l$ is outside 3σ, as estimated from pure maps with minimum at $l = 19$. In Kim & Naselsky (2010), $l_{\text{max}} = 22$ is specially singled out, for the $p$-value is lowest at that $l_{\text{max}}$. Here, we see that the $p$-values of $Q(l)$ in the range $l = [18, 31]$ remain close to their minimum. Hence, we do not attribute any special significance to a particular multipole where $Q(l)$ is minimum, but rather to the whole range $l = [18, 31]$. With the cleaned maps simulated set, we find that the $p$-value has slightly risen, but only slightly in comparison to that of pure maps. Hence, we argue that residuals in the cleaned data cannot cause the power asymmetry observed between even and odd multipoles. In the case of cleaned maps, the minimum probability for this parity asymmetry is found in the range $l = [22, 33]$ using our statistic.

In the above analysis, we used the IPSE cleaning procedure on the simulated maps. It is clearly better to use the same procedure for cleaning both the observed data and the simulated maps. We do this in Section 5.1, where we use the ILC cleaning procedure uniformly for the entire analysis. As we shall see, the results in that case are also consistent with those obtained in this section.

4 Unknown Foregrounds

The even–odd multipole power asymmetry we are studying is a point inversion (PI) symmetry violation. Therefore, we constructed some templates with explicit PI symmetry breakdown which may induce a power excess in odd multipoles and incorporate them in generating our simulated raw maps. In Dobler & Finkbeiner (2008a), an anomalous haze component was found in the WMAP
Figure 3. The p-values of $Q(l_{\text{odd}})$ for the WMAP seven-year best-fitting power spectrum. As can be seen, the data are consistently anomalous by being outside $2\sigma$ in the whole multipole range $l = [2, 101]$. For the case of cleaned maps, a value less than $1/800$ indicates that we obtained no simulated maps whose statistic was smaller than that of the observed data.}

data, which could have such asymmetry. Also, there are subdominant foregrounds in the microwave frequency region which are not well characterized yet (Kogut et al. 1996; de Oliveira-Costa et al. 2002; Dobler & Finkbeiner 2008b). However, instead of making any such identifications here, we pursue the analysis by including the new templates as some hitherto unknown components. A readily conceivable pattern with such a point symmetry violation is a hemispherical power asymmetry. This template is shown in Fig. 4 (top). We scale this template by a small factor before including it in the simulated raw data production pipeline so that its contribution remains subdominant and that the simulated raw maps conform visually to the accumulated raw data from observations. Our results were obtained using 350 simulations. Since we were expecting to find a lower significance of parity asymmetry in the presence of asymmetric foregrounds, 350 simulations are sufficient to probe it up to the $3\sigma$ CL.

We used this template in two ways: first, this template is scaled from the $K$ band to $W$ band of WMAP by a frequency-dependent power-law function and, second, as a constant asymmetric foreground component, constant in all frequency channels. In the former case, we chose the scaling factor to be 1/15th the monopole intensity of synchrotron from the PSM at 23 GHz (99 $\mu$K). It is further scaled to the $W$ band following a rigid frequency scaling (Bouchet & Gispert 1999) with a steep spectral index of $F(\nu) = F(\nu_0)(\nu/\nu_0)^{2.8}$, where $F(\nu_0)$ is the intensity distribution of a foreground component at a reference frequency $\nu_0$ extrapolated to another frequency $\nu$. We chose a large spectral index so that this effect dies off at higher frequencies (the $V$ or $W$ band) where CMB is supposed to be less contaminated by the foregrounds. These templates, generated at five frequency bands of WMAP, were added to raw maps and convolved with an appropriate beam function. In the latter case, where this asymmetric map is added as a fixed power in each pixel across all bands, it is scaled by 1/30th the synchrotron monopole intensity at 23 GHz. The scaling factor is chosen such that this excess power stays lower than the three dominant foregrounds in each channel. The results are presented in Fig. 5 using our statistic.

We find that in both cases the significance decreases but only slightly. These probability estimates are similar to either pure maps or cleaned maps with only the three dominant foreground components from the PSM. As a test for foreground components which may not be following polynomial scaling laws, we also used exponentially scaled prefactors for this template and found no increase in probability.

Then, we generated another template which is explicitly asymmetric in power between even and odd multipoles. We generated $a_{lm}$ with non-zero values for only $a_{l0}$ and zero otherwise, and the asymmetry dies (exponentially) with increasing $l$. The generated map is also shown in the middle panel of Fig. 4. Eriksen et al. (2004a) found a hemispherical power asymmetry between the north and south ecliptic hemispheres. Motivated by the similarity of the north–south power asymmetry and the PI symmetry violation, we explore whether there is any relation between these two phenomena. Therefore, we rotate this explicitly even–odd power asymmetric map such that the modulation axis is along the ecliptic poles. This template is scaled by 1/20th the monopole intensity of the synchrotron map from the PSM and added it as a constant ecliptic dipolar power excess. Here we used a slightly higher power to scale this template compared to the earlier constant hemispherical asymmetric template.

Again, we find no change in significance. It only decreases marginally. To find any significant effect with this template, we
Parity asymmetry in the CMBR

Figure 4. Various explicitly power asymmetric templates used in our analysis. These are generated by modifying some of the HEALPix routines. The middle and bottom panels are used in our studies to relate the north–south ecliptic power asymmetry found by Eriksen et al. (2004a) and the parity asymmetry that we are studying here.

had to add it at unrealistically high levels. With a low-intensity level, we do not find any increase in probability. The significance estimates with our statistic, $Q(l)$, for this dipolar modulation are also shown in Fig. 5. In Fig. 6, we presented the $p$-value estimates using the $P(l_{\text{max}})$ statistic.

4.1 Multiplicative modulation

So far we have explored the possibility of only ‘additive’ modulations to the data. Now, we consider ‘multiplicative’ modulations (Gordon et al. 2005; Eriksen et al. 2007b; Bunn & Bourdon 2008;
Figure 5. The $p$-value plot from the simulated ensemble set generated including our hitherto unknown foreground maps with explicit point-inversion asymmetry using the $Q(l_{\text{odd}})$ statistic. The significances given here are for (1) power-law scaled PI asymmetric map, (2) a constant power asymmetric map and (3) a foreground component which has explicit power asymmetry in its even and odd multipoles.

Figure 6. Same as Fig. 5, but for the $P(l_{\text{max}})$ statistic.

Hanson & Lewis (2009) which can break PI symmetry. The proposed modulation to the CMB is

$$\Delta T(\hat{n}) = \Theta(\hat{n})(1 + A \hat{\lambda} \cdot \hat{n}),$$

where $A$ is the modulation amplitude and $\hat{\lambda}$ is the preferred direction. We chose $\hat{\lambda}$ to lie along the axis of ecliptic poles. Thus, we generated a dipole modulation map of equation (10), as shown in the bottom panel of Fig. 4.

Even with an amplitude of $A = 0.3$, we find that it cannot induce a particular parity preference. The results from modulated pure maps are shown in Fig. 7. Also shown there are the pure map estimates for comparison. When the same modulation is applied to
Parity asymmetry in the CMBR

Figure 7. The $p$-values obtained after applying a dipolar multiplicative modulation to pure maps and raw maps before cleaning. The parity statistics of modulated pure maps are not different from the pure maps themselves. However, the modulated raw maps show a slight decrease in the significance of power asymmetry.

the raw maps, there is an enhancement in the parity asymmetry in the cleaned map (Fig. 7). This enhancement of power asymmetry in modulated raw maps suggests a measurement artefact rather than anything fundamental to the CMB radiation. Note that we use $A = 0.3$, which is a relatively large amplitude for modulation. Even with such a large amplitude, we do not find much change in the statistics of modulated pure maps compared to pure maps themselves.

5 IPSE AND OTHERS

Now we test the signal of parity asymmetry in maps cleaned by IPSE and several other procedures. In the case of IPSE, we perform a full-sky cleaning of the temperature raw data from the WMAP seven-year data release. The power spectrum is computed from the full-sky cleaned map up to $l = 10$ and used pseudo-$C_l$ estimator at higher $l$ after applying the WMAP KQ85yr7 mask. Later, we also estimate the power at low $l$ from the masked sky. This allows us to determine how the parity statistic is influenced by masking. For this power spectrum, the $Q(l)$ and $P(l_{\text{max}})$ values in the range $l = [2, 101]$ are shown in Fig. 8 and the probability estimates are shown in Fig. 9. These $p$-values are computed from 10 000 simulated pure maps.

We see that the IPSE cleaned data do not show significant power asymmetry. It was surprising to find this result given that the WMAP seven-year best-fitting power spectrum is found to be highly anomalous. Hence, we also applied the two statistics to other cleaned maps available to us. The maps considered here are: (1) a cleaned map from the WMAP five-year raw data obtained by using the procedure given in Tegmark et al. (2003); (2) a needlet ILC map$^9$ of Delabrouille et al. (2009) using the WMAP five-year data and (3) harmonic ILC$^{10}$ of Kim, Naselsky & Christensen (2008), which is also produced from the WMAP five-year data. We note that maps (1) and (2) are available at a resolution of the W band, just like the IPSE cleaned map. However, the harmonic ILC map is available at $1^\circ$ resolution. In all these cases, the power is obtained from the full-sky cleaned map up to $l = 10$ and pseudo-$C_l$ estimator for $l > 10$, as in the case of IPSE. The parity asymmetry statistic values of these maps are also shown in Fig. 8. As can be seen, all these maps give results close to each other, but do not agree with those obtained using the WMAP seven-year best-fitting temperature power spectrum. Hence, their significances are similar to the IPSE cleaned data, as shown in Fig. 9.

The power spectrum in all the cases analysed in this section is obtained from a full sky up to $l = 10$ and a masked sky for $l > 10$. This is in contrast to the WMAP best-fitting power spectrum which uses a masked sky over the entire multipole range. Hence, it is useful to determine how the results for the maps considered in this section change if we also use a pseudo-$C_l$ estimator for $l \leq 10$. In order to estimate parity asymmetry using a masked sky over the entire multipole range, we applied the WMAP KQ85yr7 mask to all these cleaned maps including our IPSE map. Then, we obtained the corresponding full-sky pseudo-$C_l$ values for these maps. The corresponding significance of the parity asymmetry, using both the statistics, is shown in Fig. 10. With our statistic, we find that all the different maps are relatively close to the WMAP best-fitting power

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$^9$ Available at http://www.apc.univ-paris7.fr/APC_CS/Recherche/Adamis/cmb_wmap-en.php

$^{10}$ Available at http://www.nbi.dk/~jkim/hilc/
The parity statistic values using $Q(l)$ (top panel) and $P(l)$ (bottom panel) applied to cleaned maps, using several different procedures. The maps used are: (1) the IPSE map; (2) Tegmark’s five-year cleaned map, obtained using the procedure described by Tegmark et al. 2003; (3) the needlet ILC five-year map and (4) the harmonic ILC five-year map. Also plotted are the power asymmetry statistic values for the WMAP seven-year best-fitting temperature power spectrum for comparison. All these maps show similar levels of parity asymmetry, but do not agree with the WMAP best-fitting data.

The $p$-values for parity asymmetry for the cleaned maps using several different procedures (see text).
that some of the large angle anomalies disappear in cut sky maps (Bielewicz et al. 2005; Bernui et al. 2007; Efstathiou, Ma & Hanson 2010; Pontzen & Peiris 2010; Copi et al. 2011). It is therefore encouraging that in the present case the signal is enhanced rather than diminished when we use masked sky.

The fact that a pseudo-$C_l$ estimator gives a higher significance of parity asymmetry in comparison to the power spectrum obtained from the full sky may lead one to suspect that the process of masking itself might generate some signal of parity asymmetry. In order to study this possibility, we determine the significance of parity asymmetry in the WMAP best-fitting power spectrum by using simulated masked random realizations of pure CMB. We use the KQ85yr7 mask for this purpose. The resulting significance levels for both the statistics are shown in Fig. 11. For comparison, we also show the results for the case when the power spectrum of the simulated maps is obtained from the full sky. We find that if we use the masked sky pseudo-$C_l$ estimator for the random samples, the significance level for parity asymmetry is slightly lower for both the statistics. Though there is a net rise in $p$-values due to masking, the relative change is marginal/low and the signal of anomalous parity asymmetry is still present.

5.1 Statistical significance using the ILC procedure for foreground removal

So far we have used the WMAP best-fitting power spectrum in our analysis. We computed the statistical significance by comparing the statistic for the best-fitting power with that obtained from the randomly generated pure CMB maps as well as simulated, foreground cleaned CMB maps. The simulated CMBR maps were cleaned using the IPSE procedure. We have also studied the parity asymmetry in other cleaned maps obtained using various procedures. In this section, we compute the statistical significance using the ILC procedure for foreground removal. The ILC procedure is used both for estimating the statistic for the WMAP data and for cleaning the simulated maps.

In the ILC procedure, foregrounds are removed by making a linear combination of maps at different frequencies in pixel space. Maps at different frequencies are smoothed to a common resolution of $1^\circ$ and added with suitable weights to minimize the foreground power in the combined map. The details of the ILC procedure are given in Bennett et al. (2003c). In Fig. 12 we compare the power extracted by our implementation of the ILC procedure with that obtained by WMAP. Both are in good agreement with each other.

In earlier papers, it has been shown that both the IPSE and ILC procedures are expected to have some bias at low $l$. The foreground cleaning procedure removes some extra power and hence the extracted signal is lower in comparison to the real signal. This effect is dominant at low multipoles $l = 2, 3$. Here we compute this bias for ILC using 600 simulations. The extracted bias is also shown in Fig. 12. As expected, we find a negative bias at low $l$ in power spectrum estimation (Hinshaw et al. 2007; Saha et al. 2008; Chiang, Naselsky & Coles 2009). However, we find that the bias is much smaller in comparison to that obtained using IPSE. The final power spectrum after removing this negative bias is also shown in Fig. 12.

In Fig. 13 we show both the statistics computed for the ILC cleaned map. The corresponding statistical significance using the two statistics is shown in Fig. 14. The results were presented for the WMAP seven-year ILC map, the ILC map obtained by us as well as the low-$l$ bias-corrected ILC power. We find that the statistical significance of all the three maps is comparable to one another. We note that the ILC map is reliable on angular scales greater than $10^\circ$ (Bennett et al. 2003c). It is available at a resolution of $1^\circ$ and so is the HILC map of Kim et al. (2008). Therefore, it will not be meaningful to assess the parity preference in those data at high $l$, even if it shows such an asymmetry, where its power spectrum deviates away from the theoretical CMB power spectrum.
Figure 11. The significance for the WMAP best-fitting power spectrum using masked sky random realizations of pure CMB. The results where the power spectrum of random realizations is estimated from full-sky maps are shown for comparison.

Figure 12. Plot of power spectrum of the ILC cleaned map from our implementation and from the WMAP seven-year ILC map. The curve below zero of the y-axis is the bias in the power spectrum from the ILC cleaning method. This is computed as an average over 600 simulated pure maps and clean maps generated at 1° resolution. Also shown is the best-fitting theoretical CMB temperature power spectrum for comparison. Our implementation and the WMAP cleaned ILC map agree with each other. Any difference could be due to our bias correction map estimated using the PSM. WMAP uses Maximum Entropy Method (MEM) foreground templates for generating the foreground bias map.
Figure 13. The two statistics applied to the ILC cleaned WMAP seven-year data. The statistics are shown for the ILC cleaned map as cleaned by us, bias-corrected ILC power and WMAP ILC seven-year map.

Figure 14. The $p$-values of the power asymmetry from the ILC cleaned data. Shown are the probability estimates for the two statistics from the ILC map cleaned by us, bias corrected ILC map from our implementation and the WMAP team cleaned seven-year ILC map. The probability of the parity asymmetry is lower compared to the WMAP seven-year best-fitting power spectrum. However, the probability rises slightly after correcting for the low-$l$ bias.

6 PARITY SIGNIFICANCE WITH LOW-$l$ CUTS

We see from Fig. 1 that the values of both the statistics, $Q(l)$ and $P(l_{\text{max}})$, are much lower at small ‘$l$’. This is also reflected in Figs 2 and 3, where the $p$-values are found to be relatively small at low $l$. Also, as mentioned earlier, many studies found that the low-$l$ values are associated with various anomalies. Hence, it is reasonable to assess the parity asymmetry neglecting some of the low-$l$
Figure 15. The parity preference estimators $Q(l)$ and $P(l_{\text{max}})$ with various low-$l$ cuts as applied to the WMAP seven-year best-fitting power spectrum. We see that the estimators steadily rise close to one with increasing low-$l$ cut.

multipoles. In order to get a better insight into this parity asymmetry issue and avoid any ‘anomalous low-$l$’ concerns, we discard some low-$l$ values in this analysis and compute both the statistics for the WMAP data and assess its significance. Thus, our statistic now becomes

$$Q(l_{\text{odd}}) = \frac{2}{l_{\text{odd}} - l_{\text{cut}} + 1} \sum_{l = l_{\text{cut}}}^{l_{\text{odd}}} \frac{C_l - 1}{C_l},$$

where $l_{\text{cut}}$ is any odd $l > 3$ and the summation is again over all odd multipoles $\leq l_{\text{odd}}$. We implement a similar $l$-cut for $P(l_{\text{max}})$ at low $l$ in computing $P^+$ and $P^-$. The result of applying the two statistics to the WMAP best-fitting power with various low-$l$ cuts is shown in Fig. 15. As can be seen, both the asymmetry statistics rise closer to one with increasing multipole cuts. With different low-$l$ cuts, the $p$-values at various $l$ in the range $l = [l_{\text{cut}}, 101]$ are computed for both the statistics and the results are given in Figs. 16 and 17. We show that the significance of parity asymmetry immediately starts decreasing and falls below $2\sigma$ just by ignoring the first six multipoles ($l = 2, \ldots, 7$). This happens with both the statistics. It shows that the dominant contribution to the parity asymmetry arises from a few low-$l$ multipoles only. This raises the interesting possibility that this might be related to the other low-$l$ anomalies seen in the WMAP data. These might arise from a common physical origin.

7 CONCLUSIONS

We have analysed the signature of parity asymmetry, recently found in the WMAP best-fitting temperature power spectrum, in considerable detail in the multipole range $l = [2, 101]$. For this purpose, we used the statistic, $P(l_{\text{max}})$, introduced earlier as well as a new measure, $Q(l_{\text{odd}})$, which appears to be more sensitive. We confirm the signal of parity asymmetry at the significance level of $3\sigma$. By comparing an ensemble of simulated foreground cleaned maps with the WMAP best-fitting power spectrum, we deduce that the observed parity asymmetry cannot be attributed to foreground cleaning or residual foregrounds. Here we use the foreground templates from the PSM. The PSM may not correctly model some subdominant or unknown foregrounds. Hence, we created some templates which explicitly violate parity and included them in our analysis as both additive and multiplicative modulation to the CMB. We again find that these cannot explain the observed power asymmetry. The level of asymmetry present in data can be obtained by introducing an unrealistically large value of these foreground components. Hence, we find that we are unable to attribute the observed signal to foreground cleaning or residual foregrounds.

We then tested the presence of this signal of parity asymmetry in several other foreground cleaned maps such as the IPSE, cleaned map of Tegmark et al. 2003, needlet ILC and harmonic ILC maps. We find that these maps also show a signal of parity asymmetry provided we use the pseudo-$C_l$ estimator after applying a mask in order to eliminate the heavily foreground contaminated regions. The significance level for these maps, however, is found to be not as high as that in the case of the WMAP best-fitting power spectrum. The ILC map also shows parity asymmetry with results closer to those obtained with the WMAP best-fitting power spectrum.

Finally, we tested the WMAP data for parity asymmetry by eliminating some of the low-$l$ modes. The low-$l$ multipoles are known to show some anomalous results such as low quadrupole power (Bennett et al. 2003b), alignment of various multipoles (de Oliveira-Costa et al. 2004) etc. Hence, it is possible that the parity asymmetry
Figure 16. The $p$-values of our parity preference estimator, $Q(l)$, as applied to the WMAP best-fitting power with various low-$l$ cuts, using an ensemble of 10,000 pure CMB realizations. The significance falls below 95 per cent CL just by ignoring the first six multipoles.

Figure 17. Same as Fig. 16 but for the $P(l)$ statistic. Note that the significance we found earlier disappears by ignoring the first six multipoles.

might also get a large contribution from these multipoles. We found that the parity asymmetry disappears by just ignoring the first six multipoles ($l = 2, \ldots, 7$). Hence, we conclude that the low-$l$ multipoles give dominant contribution to the signal of parity asymmetry. It is, therefore, possible that all the low-$l$ anomalies, including the parity asymmetry, might have a common origin.

**ACKNOWLEDGMENTS**

We acknowledge the use of the WMAP data available from NASA’s LAMBDA website (http://lambda.gsfc.nasa.gov/). We also used the publicly available HEALPix software (Gorski et al. 2005) for map handling and for extracting relevant information from these maps.
We thank John P. Ralston for useful discussions. We also thank Pavel Naselsky for a useful communication.

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