Has Hawking radiation been measured?

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It is argued that Hawking radiation has indeed been measured and shown to possess a thermal spectrum, as predicted. This contention is based on three separate legs. The first is that the essential physics of the Hawking process for black holes can be modelled in other physical systems. The second is that the white hole horizons are the time inverse of black hole horizons, and thus the physics of both is the same. The third is that the quantum emission, which is the Hawking process, is completely determined by measurements of the classical parameters of a linear physical system. The experiment conducted in 2010 fulfils all of these requirements, and is thus a true measurement of Hawking radiation.

I. INTRODUCTION

In 1974 Hawking [1] predicted one of the most surprising phenomena in gravitational physics, and possibly in physics in general. That prediction was that black holes, objects whose spacetime structure was such that no radiation could propagate, even in principle, from inside the object to an outside observer, nevertheless produced radiation which gradually shrank the size of the black hole. Furthermore, that outgoing radiation had the spectrum of black body radiation, modified by an "albedo" factor. The temperature of the radiation for a non-rotating charged black hole was given by

$$T = \frac{\hbar c^3}{G k_B 8\pi M}$$

where $k_B$ is Boltzmann’s constant, $G$ is Newton’s gravitational constant, $\hbar$ is Planck’s constant, $c$ the velocity of light, and $M$ the mass of the black hole.

That the so called albedo was just that, and not a frequency dependent temperature, was demonstrated by showing that a black hole connected to a heat bath well outside the black hole would be in equilibrium if that outside temperature were given by that temperature. (This equilibrium is the so called Hartle Hawking “vacuum”).

That black holes could radiate was the first shock. Where did these emitted particles come from? They could not come from inside the black hole, since nothing can travel faster than light and even light cannot escape from inside. If they come from outside the horizon, exactly what creates them out there? The second shock was that this radiation was thermal. What causes this temperature? Does this mean that black holes are thermodynamic objects, like other hot objects? Do they have entropy, and what is the relation of the entropy of black holes to other forms of entropy? Is the second law of thermodynamics valid when this entropy is taken into account?

All of these questions have been some of the foremost topics in theoretical physics in the years since Hawking’s result, and are questions which still do not have universally accepted answers.

However, if one examines Hawking’s original calculation, there are some severe problems with his derivation. While mathematically unimpeachable, they are nonsense physically. The reason is intimately tied to the fact that nothing can escape from a black hole. Therefore, if one looks at that emitted radiation, and asks where it must have come from, since it is travelling away from the black hole now, it must have been closer to the black hole in the past. But it cannot have been inside the black hole. The equations for quantum field theory, used to predict the radiation, say that as time unwinds into the past, that radiation must have been closer and closer to the horizon, squeezed into a shorter and shorter distance, and thus a shorter and shorter wavelength. This is an exponential process, so that the wavelength decreases exponentially with the time into the past, with a time scale crudely set by the light crossing time of the black hole (i.e., the time taken for light to travel a distance equal to the circumference of the black hole). This process continues until finally one arrives at the time in the past when the black hole formed, presumably by the collapse of matter. That radiation then came, through the centre of the collapsing star, from the space outside the collapsing star.

Thus, if one follows Hawking’s calculation and one looks for the origins of that thermal radiation in the behaviour of the field in the distant past, the past aspects of the quantum field which creates the current radiation had wavelengths...
of order $GM e^{-c^2/4GM}$ and frequencies of order $c^3 e^{-c^2/4GM}$. Thus one second after a solar mass black hole forms, the radiation, produced by whatever the process is that produces Hawking radiation, originated from frequencies in the initial state of the uncollapsed system of order $e^{10^5}$, a number so absurdly large that any imaginable units would simply produce an insignificant change in that exponent. And the later the radiation one is considering is emitted, the larger and more absurd this factor becomes.

There is simply no way that the physical assumptions—namely that the quantum field which produces this radiation propagates linearly on the background unaltered spacetime—can be correct. Those frequencies which are needed to explain the radiation produced even one second after a solar mass black hole forms, correspond to energies which are $e^{10^5}$ times the energy of the whole universe. Such waves simply will not propagate with no effect on the background spacetime and will certainly not propagate as though the black hole were unaffected by its presence. That these fluctuations are “vacuum fluctuations” should make no difference to this observation.

The question thus arises— if the derivation relies on such absurd physical assumptions, can the result be trusted? If the physics of the emission process really does depend on the physics of the field at those frequencies, then surely one can regard the effect as at best highly speculative, and at worst almost certainly wrong.

When it was discovered, this process was seen to be something unique to black holes. Without a complete theory of quantum gravity, it would seem that one could not make any progress toward understanding this process. But in 1980, while teaching a course in fluid mechanics, I realised that there might be another way of approaching the problem. Many waves, including sound waves in a fluid, have a behaviour at low frequencies and long wavelengths which is almost identical to that of relativistic fields in a spacetime. Already in the 1920’s, Gordon[2] had realised that at low frequencies and long wavelengths, sound waves obey equations which obey a ”special relativity” set of transformations of space and time. If the background fluid were forced to flow, then that background fluid would alter the equations of motion of the sound waves precisely in the same way that a non-flat spacetime metric would alter the equations of motion for fields in the spacetimes corresponding to Einstein’s theory of gravity[2]. In particular, if one modelled the equations of motion of sound waves as an irrotational ($\nabla \times v = 0$ where $v_1$ is the first order perturbation of the flow away from the background flow $v$) perturbation of the fluid, then the velocity potential defined by $v_1 = \nabla \phi$ obeyed exactly the equations of motion of a scalar field in a metric

$$\frac{1}{\sqrt{|g|}} \partial_{\mu} \sqrt{|g|} g^{\mu\nu} \partial_\nu \phi = 0$$

where, in the case of the fluid, the metric coefficients were given by

$$\sqrt{|g|} g^{\mu\nu} = \rho \left( v^j \delta^{ij} - v^i v^j \right)$$

Here $g$ is the inverse of the determinant of the matrix $g^{\mu\nu}$, and $c$ is the velocity of sound in the fluid $\sqrt{\frac{\partial p}{\partial \rho}}$ (which may depend on position and time).

Since one can easily imagine the fluid somewhere flowing faster than the velocity of sound, sound waves from inside the surface on which the ”radial” velocity of the fluid is equal to the velocity of sound cannot escape that region, just as light cannot escape the black hole. The metric, whose components are the inverse of the matrix $g^{\mu\nu}$, can contain a horizon which is the exact analog of the horizon of a black hole.

By following Hawking’s derivation, line by line, for a fluid flow which accelerates to create such a “horizon”, one predicts that the quantum sound waves in such a fluid flow should also create quantum particles around the horizon, which should again have a temperature, in this case proportional to

$$T = \frac{\hbar}{k_B} \frac{1}{2\pi c} \frac{\partial(e^2 - v^2)}{\partial r}$$

evaluated on the surface where $c^2 = v^2$. 

Again, if one remains in the hydrodynamic approximation, the derivation suffers from the same difficulties as does that for the black hole radiation, namely that the radiation appears to depend on absurdly high frequencies and short wavelengths in the initial state of the system.

Unlike for gravity, however, for fluids we understand the short wavelength, high frequency physics, at least in principle. Fluids are made of molecules, and once the wavelength of the sound waves becomes comparable to the distance between the molecules, the hydrodynamic approximation fails. The equation of motion of the fluid particles are no longer continuum equations, but become finite difference type equations (assuming we can neglect special relativistic effects). While at wavelengths much longer than the inter-atomic spacing, continuum field theory type approaches are valid, at short wavelengths they no longer suffice. It was recognized by Jacobson[6] that one of the
key effects that this atomicity had was on the dispersion relation of the small fluctuations about some equilibrium flow of the fluid. In a fluid at rest, the relation between the frequency and wavelength was no longer the simple

$$\nu \lambda = c$$

(5)

where \(\nu\) is the frequency and \(\lambda\) the wavelength, but \(\nu\) has a much more complex relation to \(\lambda\).

$$\nu = F \left( \frac{1}{\lambda} \right)$$

(6)

where \(F\) is some potentially complicated function of \(\frac{1}{\lambda}\) such that at large \(\lambda\) \(F\) becomes a linear function with slope \(c\). The phase velocity \(F\lambda\) and group velocity \(-\lambda^2 \frac{dF}{d\lambda}\) both will differ from \(c\) for small \(\lambda\). One can thus take a first step at understanding the dependence of the thermal radiation on the nature of the theory of the waves at short wavelengths by examining the behaviour of the prediction under changes in the dispersion relation of the waves at short wavelengths.

In a fluid with such a dispersion relation changes, we can again ask "What aspect of the state of the fluid in the past results in the thermal nature of the radiation emitted now?" The outgoing wave-packet projected back from the future is that as time unrolls into the past, the packet gets closer to the horizon, and its wavelength decreases just as in the back hole model. However, eventually its wavelength becomes small enough that the dispersion relation, and thus the group velocity of the wave changes. That wave-packet can no longer stay near the horizon (where the velocity of the fluid is now different from the changed velocity of the wave). As one goes back further into the past, that wave packet must have come from either inside the horizon (if the dispersion relation is such that the group velocity of the waves increases as the wavelength decreases) or outside (if the group velocity decreases as the wavelength decreases). This stops the exponential change in wavelength that the horizon brings about. The system has a natural cutoff to the decrease in the wavelength caused by the horizon. The radiation emitted now comes from aspects of the quantum field (the sound field) in the past which have much shorter, but not absurdly shorter than the wavelengths now. The wavelengths are not determined by an exponential of the time since the formation of the horizon, but rather by the dispersion relation. It is when the dispersion relation changes from the long wavelength, relativistic, regime, to the cutoff regime where the atomicity of matter becomes important.

A variety of numerical studies (eg, early ones are Unruh and Corley and Jacobson but see the references in Barcelo et al Living Reviews article) have shown that this system will still emit thermal quantum radiation. Changes in the short wavelength dispersion relation have no (or only a very small) effects on the temperature or thermal spectrum of the radiation emitted.

However, as always in physics, experiments are the final arbiter. Do other physical effects (viscosity in the fluid, turbulence, etc.) alter that thermal spectrum?

Since \(\hbar\) is so small, quantum effects, like Hawking’s prediction, will always be very small. Is it possible to measure such effects? For fluids, with typical laboratory velocities of meters per second, and changes on the scale of cm., the temperature of the radiation would be expected to be of the order of \(\frac{\hbar}{m} \frac{\Delta v}{\Delta x} \approx 10^{-10} K\). This is clearly extremely difficult to measure directly. However, as Hawking’s calculation already showed, the quantum emission follows directly from the classical behaviour of the system. Nowhere, except at the very end his calculation, did quantum mechanics play any role.
FIG. 2: The numerical result of calculating the thermal factor for a dispersive horizon. If the waves obey the thermal hypothesis, the three curves should be the same, which they are (except possibly at the longest wavelengths where the size of the calculation region is or the order of the wavelength, and the calculation is unreliable.)

For any linear system, the classical and the quantum behaviour, and classical equations and the quantum Heisenberg equations of motion are identical. It is only the interpretation of the symbols that occur in the calculation that differ. Instead of the field values, $\phi$ and conjugate momentum $\pi$ being interpreted as ordinary function, having some distinct real value at a point in spacetime, those symbols represent linear operators operating on a Hilbert space. Those operators obey non-trivial commutation relations, and it is those commutation relations that differentiate the classical and the quantum systems.

II. EXPERIMENT

In 2010, a group at the University of British Columbia (Silke Weinfurtner, Ted Tedford, Matt Penrice, Greg Lawrence, and I– a group of theoretical physicists and civil engineers) carried out an experiment to measure the spectrum of radiation produced by a horizon in an analog system following a suggestion of Schützhold and Unruh. The system of interest was water flowing in a flume (a long, narrow tank down which water flows). We had a 6m long tank, with a width of about 15cm, down which water, with a depth of about 20cm flowed. At the outflow end the water fell into a large storage tank. From this tank the water was pumped to the other end of the flume, where it entered the flume through a pipe and flowed through a progression of screens. These screen were to smooth out the flow of the water, so that the flow in the rest of the tank was as laminar as possible. (the screens convert gross turbulence into small scale turbulence which is rapidly damped out by the viscosity of the water.

Along the flume is a smooth, aeroplane wing shaped obstacle on the bottom which forces the water to flow more rapidly over the top of the obstacle. The exit slope of the obstacle was adjusted to ensure that there was no stagnation in the flow as it left the obstacle. The flow was measured using Particle Image Velocimetry (in which neutrally buoyant particles are introduced into the water and their velocity measured by taking high resolution photographs of the particles illuminated by two closely spaced laser flashes) to ensure that flow separation did not occur. This showed that although the flow was laminar throughout, as the flow can down the trailing edge of the obstacle there was a decrease of the velocity with depth to about 50% of the velocity along the top of the stream.

In the experiment, we were interested in measuring the surface waves along the top of the flow. These surface waves are the analog of the field in the background metric determined both by the background flow of the water and the varying depth of the water (since the surface wave velocity depends on depth). In order to measure the depth we needed to accurately measure the surface of the water. We did this by dissolving the dye, rhodamin-C in the water and illuminating a narrow strip of the water with a green .5W laser whose beam was spread out to a length of about 2m along the surface and width of about 1mm. The rhodamine-C had a sufficient density in the water than the mean
FIG. 3: Diagram of the flume with the obstacle and the wave generator. The water falls over the weir at the end and is recirculated by the pump. The laser light is a narrow sheet 2 meters long and about 1mm wide along the centre of the flume from the top of the obstacle downstream.\[11\]

path of the light in the dyed water was only about 1mm. After absorbing the light, the dye fluoresced with a broad peak below the frequency of the green laser. Since the fluorescence was isotropic, this produced light in all directions, including almost perpendicular to the laser light beam. (The laser light itself tended to either specularly reflect at the surface or refract, neither of which produced light in the perpendicular direction). The fluorescence also destroyed the temporal and spatial coherence of the emitted light, and thus did not suffer from the “speckle” problem that visualization under laser light usually produces.

The bright surface fluorescent emission was then photographed with a digital camera (BW to obtain the maximum resolution that the number of pixels could produce, and with a 1980x1094 resolution) so that the full 2m illumination by the laser could be recorded.

Because of the softness of the lens (the focal resolution of the lens would smear out a point source of light to a size slightly larger than one pixel), one could determine the location of maximum brightness in the image to much better than 1 pixel by interpolation of the peak intensity from the pixels immediately adjacent to that brightest pixel. (This gave a resolution of about 1/5 of a pixel. Since the pixels themselves had a size of about 1mm when projected onto the water’s surface, this gave a single pixel resolution from the photographs of about .2mm in the vertical direction. Horizontal resolution was not as important since the wavelengths of the waves of interest were of the order of 10s of cm. After averaging by taking Fourier transforms of the surface waves, we could reliably detect waves on the surface with amplitudes down to about .01mm.

This surface resolution was important in that it allowed us to use very low amplitude waves in our experiments to ensure that we remained within the linear regime of wave propagation. As is well known, waves propagating up a shoaling beach have their amplitude amplified because of the decreasing velocity of the waves. (See for example the height and breaking of tsunami waves as they hit land, compared to their few cm height in the deep open ocean).

The experiment we carried out was not on the analog of black hole horizons, but rather on the analog of white hole horizons. White hole horizons are the time reverse of black hole horizons. Whereas a black hole horizon is a surface out of which no waves can come, a white hole horizon is one into which no waves can penetrate. Since physics is time symmetric, the physics of, and the quantum emission by, white hole horizons is the same as the time inverse of black hole horizons. However, while in at a black hole horizon, the quantum Hawking process creates low frequency, long wavelength outgoing modes, for a white hole horizon, the particles created are the time inverse. As argued above, the originating modes which create the Hawking radiation in the past were ultra high frequency, ultra short wavelength modes. In the inverse Hawking process it is these modes which are created. Fortunately in the analog systems, the dispersion relations ensure that these modes do not have the absurdly high frequencies, or absurdly short wavelengths that they do for the black holes.

Thus the creation process for modes of the white hole analog horizons in our experiment will create modes with short wavelengths, which turn out to be about 20cm in our case. The corresponding black hole modes would have wavelengths of many 10s of meters which would be extremely difficult to measure in our 6m tank, of which only 2m was illuminated.

Thus, we had a wave generator, which consisted of a wire screen which was immersed more or less deeply into the water flow downstream of the obstacle. This screen would more or less impede the flow, producing a wave which would travel upstream toward the obstacle. If the water flow depth was appropriately adjusted (by means of the vertical weir– a adjustable vertical plate– at the end of the tank) then the upstream travelling waves would not travel over the barrier. They were blocked. The obstacle would both slow down the waves (whose wavelength was sufficiently
long that their velocity was $\sqrt{gh}$ where here $g$ is the acceleration of gravity and $h$ is the depth of the water. As the water shoals, the wave velocity decreased ($h$ get smaller) while the velocity of the water increased (due to the incompressibility of the water and the conservation of mass for the water flow, the velocity of the water times the depth is essentially constant). Thus, if the depth of the water is properly adjusted by the weir at the end of the tank, the velocity of the waves is less than the velocity of the water over the obstacle, and no waves could penetrate the region over the obstacle. The waves are blocked. At the “blocking point”, the point where the group velocity of the waves equalled the speed of the water, the waves cannot simply disappear. Instead, as in a black hole, they pile up there, with their wavelength steadily decreasing.

The dispersion relation of the surface waves

$$\omega = \sqrt{gk \tanh(kh)}$$

(7)

(where $k = \frac{2\pi}{\lambda}$ is the wave number and $\omega$ is the angular temporal frequency of the surface waves) means that when the wavelength became small enough, the group velocity of the waves will drop below the velocity of the water and the waves are swept away from the blocking point.

If we assume that the fluid flow is steady (time independent) then, for the small linear waves travelling over the surface of the fluid, the frequency of those waves in the lab frame will be constant. With the dispersion relation in the still fluid given by the above, the dispersion relation in flowing fluid will be

$$\omega = \sqrt{gk \tanh(kh)} - vk$$

(8)

While this equation assumes a constant velocity $v$, it should also be a reasonable approximation as long as $v$ does not change too fast. Thus at any point in the flow, there will in general be three possible values of $k$ for any small enough value of $\omega$ as long as $v$ is not too large. In figure 4 we plot the above dispersion relation for two values of $\nu$ corresponding to different locations in the flow. In the slowly flowing fluid, we have chosen a value of $\omega$ such that there are three possible values of $k$. The smallest value $k_p^+$ has a phase velocity $v_p = \frac{\omega}{k}$ and $v_0 = \frac{d\omega}{dk}$, the slope of the curve, which are both positive (taken in this case to refer to velocities to the left). This corresponds to the long wavelength ingoing wave. The other two solutions in the slow water regime $k_o^+$ with much shorter wavelengths, both have negative slopes (negative group velocities) which correspond to waves dragged away from the horizon. $k_o^+$ has positive phase velocity ($\omega$ and $k_o^+$ are both positive) while $k_o^−$ has negative phase velocity.

If we look at waves in the shallow, high-velocity region, there is only one solution to the dispersion relation with that same value of $\omega$. That wave has negative group and phase velocities, i.e., directed to the right toward the horizon. Thus, both this wave $k_o^−$, and the long wavelength possibility in the deeper water regime $k_o^+$ represent waves travelling toward the horizon, while both the short wavelength waves in the deeper water are travelling away.

When one send a long wavelength wave at the horizon from the right, it will eventually be stopped by the flow, and be converted into the two outgoing short wavelength waves. No outgoing wave can enter into the fast flow region, because there is no solution there with group velocity away from the horizon. (there do exist waves with imaginary wave-number there of course, which would correspond to exponentially damped solutions and which would in general be needed to satisfy the boundary conditions at the horizon.)

What we measured in our experiment was precisely that conversion of the ingoing waves into outgoing waves. In particular the amplitudes of those various outgoing waves was the crucial output of this experiment.

III. NORM

Before continuing with the description of the experiment, I must return to quantum mechanics. The above experiment sounds completely classical. But the Hawking effect is surely a quantum effect—$\hbar$ occurs in the formula for the temperature. How could this classical experiment have anything to do with quantum mechanics?

For any linear system (i.e., a system with a quadratic Hamiltonian), as those surface waves are, there is a conserved norm for complex solutions of the wave equations. If $\phi_i$ and $\pi_i$ are the field variable and conjugate momentum, then

$$(\bar{q}, q) = \frac{i}{2} \sum_i (\bar{\pi}_i q_i - \bar{q}_i \pi_i)$$

(9)

defines an inner product between the solutions $\bar{q}_i$, $\bar{\pi}_i$ and $q_i$, $\pi_i$. which is conserved in time even if the Hamiltonian is explicitly time dependent.

This inner product can be used to define a norm for complex solutions

$$< q_i, q_i > = \frac{i}{2} \sum_i (\pi_i^* q_i - q_i^* \pi_i)$$

(10)
FIG. 4: The dispersion relations for the surface waves in regions where the flow is faster and slower than the speed of the long wavelength surface waves. For a given frequency, designated by the horizontal line above zero, \( k_i^+ \) is the positive norm long wavelength wave whose group velocity is incoming toward the white hole horizon in the slow flow region, while \( k_i^- \) is the negative norm mode whose group velocity is incoming toward the horizon in the fast flow region. \( k_o^- \) and \( k_o^+ \) are waves whose group velocities carries them away from the horizon by the flow, and represent the waves which are created by the horizon from the incoming waves \( k_i \). The wave \( k_o^- \) is a negative norm wave, and \( k_o^+ \) a positive norm wave, and their intensity ratio is the ratio of Bogoliubov coefficients, and should have a thermal character if the Hawking analysis is correct. In our experiment, the mode corresponding to \( k_i^+ \) is generated, and the amplitudes of the resultant waves \( k_o^\pm \) are measured.

since the Hamiltonian is real, and thus if \( q_i, \pi_i \) is a solution, so is its complex conjugate.

This norm is not positive definite, and the norm of real solutions is zero. However one can choose a set of solutions \( \{ \hat{q}_i^x \} \) which have positive norm, are orthogonal to each other and to the associated set of complex conjugate solution, and such that the whole set of solutions are a complete set of solutions. We will call this set the positive and negative norm mode solutions. If the \( q_i \) are normalized \( \langle q^x, q^y \rangle = \delta^{xy} \), and we define annihilation and creation operators \( a^x, a^{x\dagger} \) such that

\[
[a^x, a^{y\dagger}] = \delta^{xy} \tag{11}
\]

then the quantum operators

\[
Q_i = \sum_x a^x q_i^x + a^{x\dagger} q_i^{x*} \tag{12}
\]

\[
\Pi_i = i \sum_x (-a^x q_i^x + a^{x\dagger} q_i^{x*}) \tag{13}
\]

obey the standard equal time commutation relations

\[
[Q_i(t), \Pi_j(t)] = i\delta_{ij} \tag{14}
\]

Ie, these operators obey the Heisenberg equations of motion for the Hamiltonian (because the solutions \( q_i^x \) all do) and obey the commutation relations for configuration and conjugate momentum.

If we know what the classical solutions of the equations of motion are we also know what the full solutions to the quantum system are.

IV. RESULTS

The Bogoliubov coefficients are the relation between the ingoing modes and the outgoing modes. In particular, if and ingoing positive norm mode \( \phi_i \) is converted into a linear combination of outgoing positive and negative norm
FIG. 5: The plot of the intensities of the radiation as a function of wave-number for the waves incident on the barrier. The peak near \( k = 0 \) is the incoming wave, while the two peaks, one at positive and one at negative \( k \) represent the positive and negative norm waves produced by the interaction of the incoming wave with the horizon. Note that the wavelengths of the incoming waves are much longer than the illuminated region of the top of the water, the waves are travelling over an uneven bottom, and there is no wavelength matching of the analyzed region, making the peaks broad, even though only one frequency was incoming. The logarithm of the ratio of the intensities is plotted in the other graph. If the thermal hypothesis is correct, then that plot should be a straight line, which, to experimental accuracy, it is. Unfortunately the temperature corresponding to the slope of the graph \( T = \frac{h\nu_0}{k_B} \) where \( \nu_0 \) is the inverse slope, corresponds to a temperature of about \( 10^{-12}K \), slightly cooler than the water we used in the experiment.

modes \( \phi_+^o, \phi_-^o \) such that

\[
\phi_i \rightarrow \alpha \phi_+^o + \beta \phi_-^o
\]

where all three have unit (positive or negative) norm, then because of the conservation of norm, \( |\alpha|^2 - |\beta|^2 = 1 \) and the ratio

\[
\frac{|\beta|^2}{|\alpha|^2} = e^{-\frac{\bar{\hbar} \omega}{k_B T}}
\]

defines the temperature of the emitted quantum radiation. If \( \ln(|\beta|^2) \) is linear in \( \omega \) the temperature is a constant, and in Hawking’s calculation, related to the properties of the horizon.

Thus, this experiment give strong support to the hypothesis that horizons, whether black hole, sonic, or other will produce a quantum noise with a thermal spectrum, whose temperature is determined by the behaviour of the horizon.

V. FUTURE

What remains? Clearly it would also be good to be able to directly see the thermal quantum noise created by a horizon. As in the above experiment, this is extremely difficult. The temperature scales directly with the velocity
of the waves, and inversely with the scale over which the horizon is created. This suggests that horizons which are created by light (high velocity and thus high temperature) are preferable, and also horizons which are created over very short length scales are preferred. One suggestion is that one look at non-linear effects in optical media to create horizons. Schuetzhold and I[12] made described one possibility in which the effective velocity of electromagnetic waves in a wave guide could be altered and used to create a (moving) horizon. This was extended by Leonhardt[13] who suggested using the non-linear Kerr effect (the change in the refractive index of a transparent medium by an intense pulse of radiation in the medium). One would use an intense pulse at one frequency to change the refractive index for a different frequency such that at that other frequency, the velocity of the pulse was higher than the velocity of the light within that pulse. This would create a black hole/white hole pair of horizons in the rest frame of the pulse. This is probably the experiment which is closest to fruition, but still faces immense obstacles. The because of the weak non-linearities of any known medium, the intensity of the pulse which changes the refractive index need to be so high that it begins to damage the material if it is to create a sufficiently large change in the index of refraction within the pulse. This damage can created radiation noise in a broad range of frequencies[14]. However, the hope is that in the next 10 years or less, the first direct detection of radiation created by an analog horizon will have been seen[15].

Furthermore in the analog experiment which we carried out, the behaviour of the waves was in some sense too predictable. The modes were all in a regime in which the physics is reasonably well understood (if we ignore turbulence and viscos effects). It would be great if one could carry out an experiment where at least some of the modes were in a regime in which the physics was poorly understood. For example, in liquid He, sound waves, or rather modes of vibration, whose wavelengths are comparable to the inter-atomic spacing in the liquid are very poorly understood. If one could run an experiment in which the black hole horizon radiation originated with modes which lay within that region of atomic wave-length “sound waves”, it would strengthen the evidence that the thermal radiation from horizons really was as ubiquitous as it seems to be. Unfortunately such experiments still seem a long way off.

It is of course true that even the direct observation of thermal quantum radiation from an analog horizon does not prove that black holes will radiate. Something could make the gravitational system behave differently from any analog system. It is however very hard to imagine what that something could be. The derivation of the thermal radiation from analog horizons follows so closely the derivation of thermal radiation from black hole horizons that it is very hard to imagine how the one could occur but not the other.

However our first experimental demonstration that a horizon produces a thermal spectrum together with the very elementary arguments that, for linear systems, the classical behaviour determines the quantum behaviour, is at least a first step to solidifying the truth of Hawking’s observation that horizons are associated with thermal radiation, despite the problematic nature of his original derivation.

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[1] Hawking, S.W., “Black hole explosions”, Nature 248, 30 (1974) Hawking, S.W., “Particle creation by black holes”, Commun. Math. Phys., 43, 199 (1975).
[2] Gordon, W., “Zur Lichtfortpflanzung nach der Relativitstheorie”, Ann. Phys. (Leipzig), 72, 421–456, (1923)
[3] Unruh, W.G., “Experimental Black-Hole Evaporation”, Phys. Rev. Lett., 46, 1351 (1981).
[4] The equation with a varying velocity of sound, c was derived in Visser, M. “Acoustic black holes: horizons, ergospheres and Hawking radiation” Class. Quantum Grav. 15 1767 (1998)
[5] Unruh, W.G., “Sonic analogue of black holes and the effects of high frequencies on black hole evaporation”, Phys. Rev. D51 2827 (1994). [http://arXiv.org/abs/gr-qc/9409008]
[6] Jacobson, T.A., “Black-hole evaporation and ultrashort distances”, Phys. Rev. D 44, 1731 (1991)
[7] Corley, S., and Jacobson, T.A., “Hawking Spectrum and High Frequency Dispersion”, Phys. Rev. D 54, 1568 (1996) [http://arXiv.org/abs/hep-th/9601073]
[8] Carlos Barcelo, Stefano Liberati and Matt Visser, “Analog Gravity” , Living Rev. Relativity 14, 3 (2011) [http://www.livingreviews.org/lrr-2011-3]
[9] Schutzhold, R. and Unruh, W.G., “Gravity wave analogues of black holes”, Phys. Rev. D66, 044019 (2002)
[10] For a more extensive discussion, including its application to black hole evaporation, see Unruh, W.G. “Quantum Noise in Amplifiers and Hawking/Dumb-Hole Radiation as Amplifier Noise” to appear in Analog Spacetimes. The first 30 years ed V.M.S. Cardoso, L.C.B. Crispino, S. Liberati, E.S. Olleira, M. Visser, Livra da Fisica (Sao Paulo) (2014) and arXiv:1107.2669
[11] Weinfurtner, S., Tedford, E.W., Penrice, M.C.J., Unruh, W.G. and Lawrence, G.A., “Measurement of stimulated Hawking emission in an analogue system”, Phys. Rev. Lett. 106 021302, (2011)

[12] Schützhold, R. and Unruh, W.G., “Hawking Radiation in an Electromagnetic Waveguide”, Phys. Rev. Lett., 95, 031301, (2005)

[13] Philbin, T.G., Kuklewicz, C., Robertson, S., Hill, S., Knig, F. and Leonhardt, U., “Fibre-optical analogue of the event horizon”, Science 319, 1367?1370, (2008).

[14] F. Belgiorno, S. L. Cacciatori, M. Clerici, V. Gorini, G. Ortenzi, L. Rizzi, E. Rubino, V. G. Sala, and D. Faccio, Hawking Radiation from Ultrashort Laser Pulse Filaments, Phys. Rev. Lett. 105, 203901 (2010). but also see Schützhold, R. Unruh,W. “Comment on Hawking Radiation from Ultrashort Laser Pulse Filaments” Phys. Rev. Lett. 107 149401 (2011). The experiments are close but have not quite yet achieved the observation of Hawking radiation.

[15] See also papers in Analogue Gravity Phenomenology Analogue Spacetimes and Horizons, From Theory to Experiment Faccio, D.; Belgiorno, F.; Cacciatori, S.; Gorini, V.; Liberati, S.; Moschella, U. (Eds.) Lecture Note in Physics vol 870 (Springer Verlag, Berlin) (2013)