A potential smoking gun for new physics in charm sector

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Abstract

We point out the decay mode $\Lambda_c^+ \to \Delta^+ K(t) \to \pi^+\pi^-$ is a potential smoking gun for new physics in charm sector. The Standard Model accommodated $CP$ asymmetry in this mode can be determined by branching fractions of the $\Lambda_c^+ \to \Delta^{++} K^-, \Lambda_c^+ \to \Delta^+ K^0_{S,L}$ and $\Lambda_c^+ \to \Delta^0 K^+$ modes without ad hoc assumptions. The only theoretical uncertainty is tiny isospin breaking. The ambiguities from the loop-induced quantities and $SU(3)_F$ breaking hadronic effects are avoided. Once the $CP$ asymmetry in the $\Lambda_c^+$ decays into $\Delta^+$ and neutral kaons is confirmed by experiments, we can check if it is beyond the Standard Model or not. Besides, it helps to verify the $CP$-violating effect resulted from interference between the Cabibbo-favored and doubly Cabibbo-suppressed amplitudes with the neutral kaon mixing. Future branching fractions measurements play a key role to reduce the theoretical uncertainties.

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I. INTRODUCTION

CP asymmetry in heavy quark weak decay provides a potential window to search for new physics. Charmed hadron system is the only platform to probe CP asymmetry in the up-quark sector. In 2019, the LHCb collaboration reported the discovery of CP asymmetry in the charm meson decays \cite{1},

\[ \Delta A_{CP} \equiv A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-) = (-1.54 \pm 0.29) \times 10^{-3}. \]

In theoretical aspect, the inevitable ambiguities in penguin topologies result in great difficulties in evaluating the CP asymmetries in the singly Cabibbo-suppressed D meson decays. The Quantum Chromodynamics (QCD) inspired approaches do not work well in charm scale. Harmful to the almost exact cancellation between the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $V_{cd}^*V_{ud}$ and $V_{cs}^*V_{us}$, the penguin topologies cannot be extracted from branching fractions. In literature, there are two controversial viewpoints for the observed CP asymmetry in charm: regarding it as a signal of new physics \cite{2–5}, or the non-perturbative QCD enhancements to penguin \cite{6–10}.

To avoid the challenge in predicting individual CP asymmetries, two sum rules for direct CP asymmetries in the first-order SU(3)$_F$ breaking corrections were proposed in Ref. \cite{11}. In these sum rules, the primary uncertainties induced by penguin topologies are eliminated. To uncover new physics effect, some coefficients of the sum rules, including strong phases of tree topologies, should be determined from the global fit to the data of branching fractions. However, there are too many parameters to fit limited data, the uncertainties are hard to reduce \cite{12}. This strategy is almost infeasible.

Because of the lack of knowledge of penguin topology, CP asymmetries generated from the interference between tree and penguin amplitudes might not be appropriate observables to test the SM. To circumvent the troubles caused by penguins, studying CP asymmetry induced by interference between the tree amplitudes is a significative attempt. In Ref. \cite{13}, the authors analyzed CP asymmetry in the $D^0 \to K^0_SK^0_S$ decay and conclude that $|A_{CP}^{dir}| \leq 1.1\%$ (95\% C.L.). If the future data exceed the up bound, it might be a signal of new physics. But a QCD enhancement of the penguin annihilation $PA$ cannot be excluded \cite{13}. Besides, the theoretical uncertainties cannot be well controlled since the tree-level decay amplitudes are arisen from the $SU(3)_F$ breaking effects. The similar problem also appears in the $D^0 \to K^0_SK^*0$ and $D^0 \to K^0_SK^*0$ modes. The extracted magnitudes of $E_P$ and $E_V$ amplitudes and relative strong phase $\arg(E_V/E_P)$ are suffered from
the uncontrollable $SU(3)_F$ breaking effects $^{14}$ $^{15}$. And a large penguin annihilation cannot be excluded either.

$CP$ asymmetry can also appear in the Cabibbo-favored (CF) and doubly Cabibbo-suppressed (DCS) charmed hadron decays into neutral kaons $^{16}$ $^{22}$. We studied time-dependent $CP$ asymmetry in the decay chains $D^+ \to \pi^+ K(t)(\to \pi^+\pi^-)$ and $D^+_s \to K^+ K(t)(\to \pi^+\pi^-)$ in Ref. $^{23}$, where $K(t)$ represents a time-evolved neutral kaon $K^0(t)$ or $\bar{K}^0(t)$ with $t$ being the time difference between the charm decays and the neutral kaon decays in the kaon rest frame. We reported a new $CP$-violating effect resulted from the interference between DCS and CF amplitudes with the mixing of final-state neutral kaons and estimated how large the total $CP$ asymmetries could be in the factorization-assisted topological-amplitude (FAT) approach $^{24}$. Unfortunately, the FAT approach seemingly cannot precisely describe the $SU(3)_F$ breaking effects and estimate the strong phases of tree amplitudes in the $D$ meson decays $^{25}$ $^{26}$. So the $CP$ asymmetry predictions are questioned.

Another window to search for new physics in charm is the neutral $D$ meson mixing system. Attributed to the charm scale being too heavy to apply the chiral perturbation theory and too light to apply the heavy quark expansion, and the strongly suppression by the GIM mechanism, the $D$ mixing system is sensitive to the high-order $SU(3)_F$ breaking effects $^{27}$. Theoretical evaluation of the $D$ mixing system in the SM is extremely difficult $^{28}$ $^{38}$.

To establish a "smoking gun" signal of new physics in charm, one need reliable SM predictions. With our inability to control the theoretical ambiguities for the charmed meson system, we turn to study the charmed baryons. Compared to the pseudoscalar and vector mesons, flavor symmetry in the baryon decuplet is more ideal, which would help to eliminate some theoretical uncertainties.

In this paper we study $CP$ asymmetry in the $\Lambda_c^+$ decays into $\Delta^+$ and neutral kaons. We find the hadronic effects can be quantified by the branching fractions of several $\Lambda_c \to \Delta K$ modes without $ad$ $hoc$ assumptions. The only theoretical uncertainty is the tiny isospin breaking. The ambiguities from the loop-induced quantities and $SU(3)_F$ breaking hadronic effects are avoided. Once the $CP$ asymmetry in the $\Lambda_c^+$ decays into $\Delta^+$ and neutral kaons is confirmed by experiments, we can check whether it is beyond the Standard Model or not. So it is a potential smoking gun for new physics in charm sector.
II. CP ASYMMETRY

We extend the analysis of CP asymmetries in the charmed meson decays into neutral kaons in Ref. \[23\] to the $\Lambda_c^+ \rightarrow \Delta^+ K(t)(\rightarrow \pi^+ \pi^-)$ mode. The mass eigenstates of neutral kaons $K^0_S$ and $K^0_L$ are linear combinations of the flavor eigenstates $K^0$ and $\bar{K}^0$,

$$|K^0_{S,L}\rangle = \frac{1 + \epsilon}{\sqrt{2(1 + |\epsilon|^2)}}|K^0\rangle \mp \frac{1 - \epsilon}{\sqrt{2(1 + |\epsilon|^2)}}|\bar{K}^0\rangle,$$

where $\epsilon$ is a complex parameter characterizing the indirect CP violation in the kaon mixing with $|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$ and $\phi_\epsilon = 43.52 \pm 0.05^\circ$ [39]. In experiments, a $K^0_S$ state is reconstructed via its decay into two charged pions at a time close to its lifetime. Hence, not only $K^0_S$, but also $K^0_L$ serve as the intermediate states in the $\Lambda_c^+ \rightarrow \Delta^+ K(t)(\rightarrow \pi^+ \pi^-)$ mode through the $K^0_S - K^0_L$ oscillation [21].

The time-dependent CP asymmetry in the $\Lambda_c^+ \rightarrow \Delta^+ K(t)(\rightarrow \pi^+ \pi^-)$ mode is defined as

$$A_{CP}(t) \equiv \frac{\Gamma_{\pi\pi}(t) - \Gamma_{\bar{\pi}\bar{\pi}}(t)}{\Gamma_{\pi\pi}(t) + \Gamma_{\bar{\pi}\bar{\pi}}(t)},$$

where

$$\Gamma_{\pi\pi}(t) \equiv \Gamma(\Lambda_c^+ \rightarrow \Delta^+ K(t)(\rightarrow \pi^+ \pi^-)),$$

$$\Gamma_{\bar{\pi}\bar{\pi}}(t) \equiv \Gamma(\Lambda_c^- \rightarrow \Delta^- K(t)(\rightarrow \pi^+ \pi^-)).$$

We write the ratio between $A(\Lambda_c^+ \rightarrow \Delta^+ K^0)$ and $A(\Lambda_c^+ \rightarrow \Delta^+ \bar{K}^0)$ as

$$A(\Lambda_c^+ \rightarrow \Delta^+ K^0)/A(\Lambda_c^+ \rightarrow \Delta^+ \bar{K}^0) = r e^{i(\phi + \delta)},$$

with the magnitude $r$, the relative strong phase $\delta$, and the weak phase $\phi = (-6.2 \pm 0.4) \times 10^{-4}$ in the SM [39].

Similarly to the D meson decays, the time-dependent CP asymmetry in the decay chain of $\Lambda_c^+ \rightarrow \Delta^+ K(t)(\rightarrow \pi^+ \pi^-)$ is derived to be

$$A_{CP}(t) \simeq \left(A_{CP\bar{K}}^{\Lambda_c^+}(t) + A_{CP\bar{K}}^{\Delta^+}(t) + A_{CP\bar{K}}^{\Delta^-}(t)\right)/D(t),$$

with

$$A_{CP\bar{K}}^{\Lambda_c^+}(t) = 2 e^{-\Gamma_{\bar{K}^0} t} Re(\epsilon) - 2 e^{-\Gamma_{K^0} t} \left(Re(\epsilon) \cos(\Delta m_K t) + Im(\epsilon) \sin(\Delta m_K t)\right),$$

where $\Delta m_K$ is the mass difference between charm and strange quarks.
\[ A_{\text{CP}}^\text{int}(t) = -4r \cos \phi \sin \delta \left[ e^{-\Gamma_{K}^0 t} \Im(e) - e^{-\Gamma_{K}^0 t} \right] \left( \Im(e) \cos(\Delta m_{K} t) - \Re(e) \sin(\Delta m_{K} t) \right), \]  
(9)

\[ A_{\text{CP}}^\text{dir}(t) = e^{-\Gamma_{K}^0 t} 2r \sin \delta \sin \phi, \]  
(10)

\[ D(t) = e^{-\Gamma_{K}^0 t} (1 - 2r \cos \delta \cos \phi), \]  
(11)

where the average of widths is \( \Gamma_{K} \equiv (\Gamma_{K}^0 + \Gamma_{K}^L)/2 \), and the differences of widths and masses are \( \Delta \Gamma_{K} \equiv \Gamma_{K}^0 - \Gamma_{K}^L \) and \( \Delta m_{K} \equiv m_{K}^0 - m_{K}^L \), respectively. The first term in Eq. (7), which is independent of hadronic parameters \( r \) and \( \delta \), is \( \text{CP} \) asymmetry in the neutral kaon mixing [21]. The second term is direct \( \text{CP} \) asymmetry induced by the interference between the tree level CF and DCS amplitudes. And the third term is the interference between the CF and DCS amplitudes with the neutral kaon mixing, a new \( \text{CP} \)-violating effect pointed out in Ref. [23]. Measurements of \( \text{CP} \) asymmetries depend on time intervals selected in experiments. The time-integrated \( \text{CP} \) asymmetry in the limit of \( t_1 \ll \tau_S \ll t_2 \ll \tau_L \) is

\[ A_{\text{CP}}(t_1 \ll \tau_S \ll t_2 \ll \tau_L) \simeq \left( A_{\text{CP}}^0 + A_{\text{CP}}^\text{dir} + A_{\text{CP}}^\text{int} \right) / D = \frac{-2\Re(e) + 2r \sin \phi \sin \delta - 4\Im(e) r \cos \phi \sin \delta}{1 - 2r \cos \phi \cos \delta}. \]  
(12)

In the SM, \( A_{\text{CP}}^0 \) is well determined by parameter \( \epsilon \). \( A_{\text{CP}}^\text{int} \) and \( A_{\text{CP}}^\text{dir} \) are estimated to be \( \mathcal{O}(10^{-4}) \) and \( \mathcal{O}(10^{-5}) \), respectively. More details for \( \text{CP} \) asymmetries in the charmed hadron decays into neutral kaons can be found in Ref. [23].

From Eqs. (7)~(12), one can find there are only two hadronic inputs to the time-dependent and time-integrated \( \text{CP} \) asymmetries, \( r \) and \( \delta \). If \( r \) and \( \delta \) are well determined from branching fractions, the theoretical predictions for \( \text{CP} \) asymmetries of the \( \Lambda_c^+ \rightarrow \Delta^+ K(t) \rightarrow \pi^+ \pi^- \) mode could be very precise.

III. EXTRACTING HADRONIC PARAMETERS

In the \( \Lambda_c \rightarrow \Delta K \) modes, the initial state \( \Lambda_c^+ \) is an isospin singlet, the final states \( (\Delta^{++}, \Delta^+, \Delta^0, \Delta^-) \) form an isospin quartet, \( (K^+, K^0) \) and \( (\bar{K}^0, \bar{K}^-) \) form two isospin doublets. There are only two topologies contributing to the \( \Lambda_c \rightarrow \Delta K \) modes, the \( W \)-exchange diagram \( E' \) and the color-suppressed internal \( W \)-emission diagram \( C' \), which are displayed in Fig. [1]. Topological
FIG. 1: Topological diagrams contributing to the $\Lambda_c^+ \to \Delta^{++}K^-$ modes and $\Lambda_c^+ \to \Delta^+K^0(\Delta^0K^+)$ modes respectively.

decompositions of the $\Lambda_c \to \Delta K$ modes are

$$A(\Lambda_c^+ \to \Delta^{++}K^-) = -V_{cs}^* V_{ud} E_2'$$

$$A(\Lambda_c^+ \to \Delta^+K^0) = \frac{1}{\sqrt{3}} V_{cs}^* V_{ud} E_1'$$

$$A(\Lambda_c^+ \to \Delta^0K^+) = -\frac{1}{\sqrt{3}} V_{cd}^* V_{us} C_1'$$

$$A(\Lambda_c^+ \to \Delta^0K^+) = -\frac{1}{\sqrt{3}} V_{cd}^* V_{us} C_2'$$

Notice the only difference between $E_1'$ and $E_2'$ is that the $q\bar{q}$ generated from vacuum is $u\bar{u}$ or $d\bar{d}$. And the difference between $C_1'$ and $C_2'$ is an $u \leftrightarrow d$ exchange of the spectator quarks. The opposite sign in Eq. (13) and Eq. (14) is arisen from the quark components of kaons, $|K^-\rangle = -|s\bar{u}\rangle$ and $|\bar{K}^0\rangle = |s\bar{d}\rangle$. While the opposite sign in Eq. (15) and Eq. (16) is from the quark component of $\Lambda_c^+$ baryon, $|\Lambda_c^+\rangle = |(ud - du)c\rangle$. If isospin breaking is neglected in the topologies, i.e., $u(\bar{u}) = d(\bar{d})$, we get $E_1' = E_2' = E'$ and $C_1' = C_2' = C'$.

More to this point, we perform an isospin analysis to the $\Lambda_c \to \Delta K$ modes. For the CF modes $\Lambda_c^+ \to \Delta^{++}K^-$ and $\Lambda_c^+ \to \Delta^+K^0$, the weak Hamiltonian changes isospin as $\Delta I = 1$ and $\Delta I_3 = 1$. Then we have an isospin amplitude $A_{1,1}$,

$$\langle \Lambda_c^+ | H_{CF} = \langle 0,0;1,1 | = \langle 1,1 |.$$
For the $\Delta K$ system in the isospin limit, we have

$$|\Delta^{++}K^-\rangle = \frac{1}{2}|2, 1\rangle + \frac{\sqrt{3}}{2}|1, 1\rangle,$$

$$|\Delta^{+}K^0\rangle = \frac{1}{2}|2, 1\rangle - \frac{1}{2}|1, 1\rangle. \quad (18)$$

The isospin conservation requires the initial and final states should have the same isospin. We can derive the isospin amplitudes of $\Lambda_c^+ \to \Delta^{++}K^-$ and $\Lambda_c^+ \to \Delta^{+}K^0$ modes as

$$A(\Lambda_c^+ \to \Delta^{++}K^-) = \frac{\sqrt{3}}{2}A_{1,1}, \quad (20)$$

$$A(\Lambda_c^+ \to \Delta^{+}K^0) = -\frac{1}{2}A_{1,1}. \quad (21)$$

Compared with the topological decomposition, we get $A_{1,1} = -2V_{cs}V_{ud}E'/\sqrt{3}$. For the DCS modes $\Lambda_c^+ \to \Delta^+K^0$ and $\Lambda_c^+ \to \Delta^0K^+$, the weak Hamiltonian changes isospin as $\Delta I = 1$ or $\Delta I = 0$, and $\Delta I_3 = 0$. There are two isospin amplitudes $A_{1,0}$ and $A_{0,0}$,

$$\langle \Lambda_c^+ |H_{DCS} = \langle 0, 0; 1, 0| + \langle 0, 0; 0, 0|$$

$$= \langle 1, 0| + \langle 0, 0|. \quad (22)$$

For the $\Delta K$ final states, we have

$$|\Delta^{+}K^0\rangle = \frac{1}{\sqrt{2}}|1, 0\rangle + \frac{1}{\sqrt{2}}|0, 0\rangle,$$

$$|\Delta^{0}K^+\rangle = \frac{1}{\sqrt{2}}|1, 0\rangle - \frac{1}{\sqrt{2}}|0, 0\rangle. \quad (23, 24)$$

The isospin amplitudes of $\Lambda_c^+ \to \Delta^+K^0$ and $\Lambda_c^+ \to \Delta^0K^+$ modes are derived to be

$$A(\Lambda_c^+ \to \Delta^{+}K^0) = \frac{1}{\sqrt{2}}A_{1,0} + \frac{1}{\sqrt{2}}A_{0,0}, \quad (25)$$

$$A(\Lambda_c^+ \to \Delta^{0}K^+) = \frac{1}{\sqrt{2}}A_{1,0} - \frac{1}{\sqrt{2}}A_{0,0}. \quad (26)$$

The isospin amplitude $A_{0,0}$ does not contribute to the $\Lambda_c^+ \to \Delta^+K^0$ and $\Lambda_c^+ \to \Delta^0K^+$ decays because the $u$ and $d$ quarks are antisymmetric in the weak Hamiltonian and symmetric in the baryon decuplet, $A_{0,0} = 0$. Thereby, $|A(\Lambda_c^+ \to \Delta^{+}K^0)| = |A(\Lambda_c^+ \to \Delta^{0}K^+)|$ is satisfied under the isospin symmetry.

The magnitudes of $E'$ and $C'$ can be extracted from branching fractions of the $\Lambda_c^+ \to \Delta^{++}K^-$ and $\Lambda_c^+ \to \Delta^0K^+$ modes respectively in the isospin symmetry. Then the ratio $r$ is determined according to Eq. (6). The magnitude of $C'$ can also be extracted from branching fractions of the
$\Lambda_c^+ \rightarrow \Delta^{++}K^-$ and $\Lambda_c^+ \rightarrow \Delta^+K^0_{S,L}$ modes via following relation,

$$|V_{cd}^*V_{us}|^2|\mathcal{C}|^2 = 3|\mathcal{A}(\Lambda_c^+ \rightarrow \Delta^+K^0_S)|^2$$

$$+ 3|\mathcal{A}(\Lambda_c^+ \rightarrow \Delta^+K^0_L)|^2 - |\mathcal{A}(\Lambda_c^+ \rightarrow \Delta^{++}K^-)|^2.$$

(27)

To extract the strong phase $\delta$, we define the $K^0_S - K^0_L$ asymmetry in the $\Lambda_c^+ \rightarrow \Delta^+K^0_{S,L}$ modes as

$$R(\Lambda_c^+, \Delta^+) \equiv \frac{\Gamma(\Lambda_c^+ \rightarrow \Delta^+K^0_S) - \Gamma(\Lambda_c^+ \rightarrow \Delta^+K^0_L)}{\Gamma(\Lambda_c^+ \rightarrow \Delta^+K^0_S) + \Gamma(\Lambda_c^+ \rightarrow \Delta^+K^0_L)},$$

(28)

which is derived to be [25]

$$R(\Lambda_c^+, \Delta^+) \simeq -2r \cos \delta.$$

(29)

The sub-leading terms are negligible since they are $O(10^{-4})$ compared to the leading term which is $O(10^{-2})$. If $R(\Lambda_c^+, \Delta^+)$ is measured by experiments, the strong phase $\delta$ is gotten by

$$\delta = \pm \arccos\left(\frac{|R(\Lambda_c^+, \Delta^+)/2r|}{2r}\right).$$

(30)

One can find the hadronic parameters of $CP$ asymmetries in the decay chain of $\Lambda_c^+ \rightarrow \Delta^+K(t)(\rightarrow \pi^+\pi^-)$, the magnitude ratio $r$ and the strong phase $\delta$, are quantified by branching fractions of the $\Lambda_c \rightarrow \Delta K$ modes under isospin symmetry without other model hypothesis. Since isospin symmetry is a very precise symmetry, the theoretical uncertainties of $r$ and $\delta$ can be well controlled. Once the $CP$ asymmetry in the $\Lambda_c^+ \rightarrow \Delta^+K(t)(\rightarrow \pi^+\pi^-)$ mode are confirmed by experiments, we can check if it is beyond the SM or not. Besides, precise measurements for $CP$ asymmetry in this mode can verify the new $CP$-violating effect resulted from interference between the CF and DCS amplitudes with the neutral kaon mixing. And the relative strong phase between two topologies in charmed baryon decays would help to understand the Quantum Chromodynamics.

Among the three terms contributing to total $CP$ asymmetry in the decay mode of $\Lambda_c^+ \rightarrow \Delta^+K(t)(\rightarrow \pi^+\pi^-)$, the direct $CP$ asymmetry is highly sensitive to new physics due to the small weak phase $\phi$ in the SM. Compared to the charm meson decay or mixing systems, the ambiguities from the loop-induced quantities and $SU(3)_F$ breaking effects are avoided in theoretical analysis, which would provide a clear signal of new physics. Thereby, the $\Lambda_c^+ \rightarrow \Delta^+K(t)(\rightarrow \pi^+\pi^-)$ mode is a potential smoking gun for new physics in charm sector.

IV. CONCLUSION

In summary, the SM accommodated $CP$ asymmetry in the $\Lambda_c^+$ decays into $\Delta^+$ and neutral kaons can be determined by the branching fractions of several $\Lambda_c \rightarrow \Delta K$ decays under the isospin

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symmetry without *ad hoc* assumptions. Thereby, it is a potential smoking gun to search for new physics in charm sector.

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[1] R. Aaij *et al.* [LHCb], Phys. Rev. Lett. 122, no.21, 211803 (2019).
[2] M. Chala, A. Lenz, A. V. Rusov and J. Scholtz, JHEP 1907, 161 (2019).
[3] A. Dery and Y. Nir, JHEP 1912, 104 (2019).
[4] L. Calibbi, T. Li, Y. Li and B. Zhu, JHEP 10, 070 (2020).
[5] A. J. Buras, P. Colangelo, F. De Fazio and F. Loparco, JHEP 10, 021 (2021).
[6] H. N. Li, C. D. Lu and F. S. Yu, [arXiv:1903.10638](https://arxiv.org/abs/1903.10638) [hep-ph].
[7] H. Y. Cheng and C. W. Chiang, Phys. Rev. D 100, no. 9, 093002 (2019).
[8] Y. Grossman and S. Schacht, JHEP 07, 020 (2019).
[9] S. Schacht and A. Soni, Phys. Lett. B 825, 136855 (2022).
[10] D. Wang, JHEP 03, 155 (2022).
[11] S. Müller, U. Nierste and S. Schacht, Phys. Rev. Lett. 115, no. 25, 251802 (2015).
[12] S. Müller, U. Nierste and S. Schacht, Phys. Rev. D 92, no.1, 014004 (2015).
[13] U. Nierste and S. Schacht, Phys. Rev. D 92, no. 5, 054036 (2015).
[14] U. Nierste and S. Schacht, Phys. Rev. Lett. 119, no.25, 251801 (2017).
[15] H. Y. Cheng and C. W. Chiang, Phys. Rev. D 104, no.7, 073003 (2021).
[16] I. I. Y. Bigi and H. Yamamoto, Phys. Lett. B 349, 363-366 (1995).
[17] Z. Z. Xing, Phys. Lett. B 353, 313-318 (1995) [erratum: Phys. Lett. B 363, 266 (1995)].
[18] H. J. Lipkin and Z. z. Xing, Phys. Lett. B 450, 405-411 (1999).
[19] G. D’Ambrosio and D. N. Gao, Phys. Lett. B 513, 123-129 (2001).
[20] S. Bianco, F. L. Fabbri, D. Benson and I. Bigi, Riv. Nuovo Cim. 26, no.7-8, 1-200 (2003).
[21] Y. Grossman and Y. Nir, JHEP 04, 002 (2012).
[22] B. R. Ko *et al.* [Belle], Phys. Rev. Lett. 109, 021601 (2012) [erratum: Phys. Rev. Lett. 109, 119903 (2012)].
[23] F. S. Yu, D. Wang and H. n. Li, Phys. Rev. Lett. 119, no.18, 181802 (2017).
[24] H. n. Li, C. D. Lu and F. S. Yu, Phys. Rev. D 86, 036012 (2012).
[25] D. Wang, F. S. Yu, P. F. Guo and H. Y. Jiang, Phys. Rev. D 95, no.7, 073007 (2017).
[26] M. Ablikim et al. [BESIII], [arXiv:2202.13601 [hep-ex]].
[27] R. L. Kingsley, S. B. Treiman, F. Wilczek and A. Zee, Phys. Rev. D 11, 1919 (1975).
[28] A. Datta and D. Kumbhakar, Z. Phys. C 27, 515 (1985).
[29] H. Georgi, Phys. Lett. B 297, 353-357 (1992).
[30] T. Ohl, G. Ricciardi and E. H. Simmons, Nucl. Phys. B 403, 605-632 (1993).
[31] I. I. Y. Bigi and N. G. Uraltsev, Nucl. Phys. B 592, 92-106 (2001).
[32] A. F. Falk, Y. Grossman, Z. Ligeti and A. A. Petrov, Phys. Rev. D 65, 054034 (2002).
[33] E. Golowich and A. A. Petrov, Phys. Lett. B 625, 53-62 (2005).
[34] H. Y. Cheng and C. W. Chiang, Phys. Rev. D 81, 114020 (2010).
[35] T. Jubb, M. Kirk, A. Lenz and G. Tettlmatzi-Xolocotzi, Nucl. Phys. B 915, 431-453 (2017).
[36] H. Y. Jiang, F. S. Yu, Q. Qin, H. n. Li and C. D. Lü, Chin. Phys. C 42, no.6, 063101 (2018).
[37] H. N. Li, H. Umeeda, F. Xu and F. S. Yu, Phys. Lett. B 810, 135802 (2020).
[38] A. Lenz, M. L. Piscopo and C. Vlahos, Phys. Rev. D 102, no.9, 093002 (2020).
[39] P. A. Zyla et al. [Particle Data Group], PTEP 2020, no.8, 083C01 (2020).