Salient Calculation at the Single Offshore Breakwater for a Wave Perpendicular to Coastline using Polynomial Approach

Syawaluddin Hutahaean

Ocean Engineering Program, Faculty of Civil and Environmental Engineering, Bandung Institute of Technology (ITB), Bandung 40132, Indonesia
syawaluddin1@ocean.itb.ac.id

Abstract — Coastal protection planning using offshore breakwater requires an estimation on the formed salient. There are research results using physical model as well as field observation on the relation between the length of breakwater and the position of breakwater with the formed salient. The relation, however, is qualitative in nature.

This research develops a calculation method for a formed salient in single offshore breakwater as a result of a wave that is perpendicular to the breakwater. The model is developed based on the characteristic of stable coastline, i.e. stable coastline that is parallel to the wave crestline forming it, whereas the salient equation is approached with polynomial.

The equation provides a good result, i.e. the measurement of salient that is very much in accordance with the result of previous research using physical model or field measurement.

Keywords — Offshore Breakwater, Coastal protection planning, Polynomial Approach, surf zone area.

I. INTRODUCTION

Many coastal protection using offshore breakwater or detached breakwater have been constructed. The construction is in the form of breakwater that is parallel to the coastline, within the surf zone area with a quite close distance with the coastline. At the coastline protected by offshore breakwater, sedimentation will occur where the sediment deposit is called salient (Fig.1). The efficiency of breakwater is measured from its salient condition. Although it is called offshore breakwater, the real location is quite close with the coastline in order to produce salient or tombolo, where the incoming wave is almost perpendicular or even perpendicular to the coastline. Therefore, this research formulated salient equation for the incoming wave perpendicular to breakwater and the coastline.

An important factor in offshore breakwater planning is the formation of salient, where the success of this coastal protection is in the formation of the salient. The dimension (small and large) of a salient is expressed by the distance of the top of salient with the original coastline, i.e. $Y_S$ (Fig.1.)

Fig.1. Offshore breakwater and salient

Quite a few researchers have conducted research on the measurement of salient $Y_S$, but it is only qualitative in nature. Those researchers are among others: Ahren and Cox (1990), Leo C. Van Rijn (2013), Inman and Proudy (1966), Nir (1982) and many more whose research results will be discussed in Chapter III. The result of the research is presented in the form of a comparison between the length of breakwater with the breakwater distance to original coastline $\left(\frac{l_s}{X}\right)$ with salient type, but it is only qualitative and is not expressed as a relation between $\frac{l_s}{X}$ with the salient height $Y_S$. It is stated that the bigger the value of $\frac{l_s}{X}$ the higher of salient height $Y_S$ will be where in a...
quite big value of \( \frac{L_s}{x} \) the tip of the salient will be so close to or reach breakwater, where the sediment is called tombolo. The salient calculation can be done using numerical model, using GENESIS software as in the US Army Corps of Engineer (1993).

\[
\sin \alpha \text{ tof } 0
\]

The peak ordinate of \( \alpha \), the tangent of its forming crestline (Fig.2).

\[
\frac{L_s}{x} \text{ parallel with the shadow zone, stable coastline }
\]

This research is developed based on the static equilibrium condition, where the tangent of the stable coastline is similar with the tangent of its forming crestline (Fig.2) with the goals of obtaining practical method in conducting salient measurement, i.e. the peak ordinate of salient \( y_s \). Salient equation is approached with polynomial of degree 10 so there are 11 polynomial coefficients that must be determined. The calculation of polynomial coefficient is done using the characteristic of stable coastline as has been mentioned.

**II. STUDY ON THE CHARACTERISTIC OF STABLE COASTLINE**

This section will show that stable coastline condition is parallel with crestline, using longshore sediment transport equation and an example of geometry stable coastline exists in the nature.

2.1. Review of longshore sediment transport equation formula.

Coastline changes are mainly caused by longshore sediment transport, where evolution coastline model, such as GENESIS uses longshore sediment transport equation as the basic equation. The longshore sediment transport equation is a function of the angle between crestline of the breaking wave against coastline, where if the crestline is parallel to the coastline, it will produce zero longshore sediment transport or no erosion and sedimentation or the coastline is in stable condition.

a. Kamphuis’ Longshore sediment transport formula, Kamphuis, J.W. (1991)

\[
Q_{ls} = (C_K H_b T) \sin(2 \alpha_b) \ldots \ldots \ldots (1)
\]

\( Q_{ls} \) longshore sediment transport rate, \( b \) = subscript denoting breaking condition; a complete information can be seen at Kamphuis, J.W. (1991). The concern of this equation is the element \( \sin(2 \alpha_b) \), where \( \alpha_b \) = angle of breaking waves to local shoreline. In this case, if \( \alpha_b = 0 \), the tangent of crestline is parallel or equal to the tangent of the coastline, then \( Q_{ls} = 0 \).

b. Longshore sediment transport of SPM (1984).

\[
Q_{ls} = \left( \frac{K_S}{16} \right) \sin(2 \alpha_b) \ldots \ldots \ldots (2)
\]

Similar to equation (1), the concern is the element \( \sin(2 \alpha_b) \) where \( \alpha_b = 0 \) then \( Q_{ls} = 0 \). Complete information on equation (2) can be seen at SPM (1984).

c. Longshore sediment transport formula of Hanson, H., and Kraus, N.C. (1989)

This longshore sediment transport equation from Hanson and Krauss is used at the widely used shoreline change model, i.e. GENESIS. The form of the equation is as follows.

\[
Q_{ls} = \left( H_b C_b \right) \sin(2 \alpha_b) \ldots \ldots \ldots (3)
\]

In the case of \( \frac{\partial H}{\partial x} = 0 \) or very small, then equation (3) becomes,

\[
Q_{ls} = \left( H_b C_b \right) \sin(2 \alpha_b) \ldots \ldots \ldots (4)
\]

In this equation (4) \( \alpha_b = 0 \), then \( Q_{ls} = 0 \). Complete information on equation (3), can be seen at Hanson, H., and Kraus, N.C. (1989). From the three longshore sediment transport equations, it can be stated that at the stable coastline, the tangent of the coastline is parallel or equal to the tangent of the crestline that forms the coastline. In an open area where the coast is formed by incoming wave, the tangent of the coastline is parallel with the crestline of the incoming wave, whereas at the shadow zone, stable coastline is parallel with the crestline diffracted wave.

2.2. Review on the form of stable coastline.

It has been known that in the nature there is geometrical form of the stable coastline in static equilibrium condition, and there are plenty of researches that have been done on...
the form of that stable coastline. There are several terminologies for the form of the stable coastline, Silverster, R. (1960) called it zeta bays, half-heart bay Silverster, R., Tsuchiya, Y.. ad Shibano, Y. (1980), crenulate shaped bays Silverster R, Hsu, J.R.C. (1993), Hsu, J.R.C., and Silverster R., Member et.al (1989). The form of the stable coastline, Silverster R, Hsu, J.R.C. (1993), Hsu, J.R.C., and Silverster R., Member et.al (1989) studying stable coastline between two headlands are as follows.

Stable coastline consists of two parts (Fig3.), i.e. coastline directly facing the incoming wave (BC line) and coastline facing the diffracted wave, AB line. At the segment of the coastline facing the incoming wave, the tangent of the stable coastline is equal to the tangent of the crestline of the incoming wave, whereas at the shadow zone facing the diffracted wave, the tangent of the coastline is parallel with the tangent of the crestline of the diffracted wave. From the review of the longshore sediment transport equation and the geometry of stable coastline, it can be concluded that stable coastline has a tangent that is parallel with the crestline forming it. For the coastline directly facing the incoming wave, the tangent stable coastline is equal to the tangent of the crestline of the incoming wave, whereas the coastline formed by the diffracted wave will have a tangent that is equal to the tangent of diffracted wave crestline. This condition will be used as boundary condition at the formulation of stable coastline equation.

III. SOME RESULTS OF PREVIOUS STUDIES
There are plenty of previous researches in the formulation of salient at the offshore backwater. This section will present some results of previous studies that will be used in the model development. The results of the research are in the form of qualitative relation \( \frac{L_s}{X} \) with the salient and do not mention about wave angle.

3.1. Ahrens and Cox (1990)
Ahrens and Cox (1990) used the beach response index classification scheme of Pope and Dean (1986) to develop a predictive relationship for beach response based on ratio of the breakwater segment length to breakwater distance from original shoreline. The relationship defining a beach response index \( I_s \) is:
\[
I_s = e^{(1.72 - \frac{L_s}{X})} \quad \text{ .......(5)}
\]
Table 1, shows the relationship between \( I_s \) and salient formation.

| \( I_s \) | \( \frac{L_s}{X} \) | Salient formation |
|---------|-----------------|------------------|
| 1       | 4.2             | Permanent tombolo |
| 2       | 2.5             | Periodic tombolo  |
| 3       | 1.52            | Well-developed salient |
| 4       | 0.81            | Subdued salient   |
| 5       | 0.27            | No sinuosity      |

3.2. Leo C. Van Rijn (2013)
The result of a research by Leo C. Van Rijn (2013), Table 2, related to this research is the relation between the value of \( \frac{L_s}{X} \) and the formation of salient, i.e.:
Table 2. The value of \( \frac{L_s}{X} \) and salient formation, Leo C. Van Rijn (2013)

\[
\frac{L_s}{X} \quad \text{Salient formation}
\]

\[
\begin{align*}
\frac{L_s}{X} & > 3 & \text{Permanent tombolo} \\
2 & < \frac{L_s}{X} & < 3 & \text{Permanent or periodic tombolo} \\
1 & < \frac{L_s}{X} & < 2 & \text{Well developed salient} \\
0.5 & < \frac{L_s}{X} & < 1 & \text{Weak to well developed salient} \\
0.2 & < \frac{L_s}{X} & < 0.5 & \text{Incipient to weak salient} \\
\frac{L_s}{X} & < 0.2 & \text{No effect}
\end{align*}
\]

3.3. Others
Inman and Frautschy (1966)
\[
\frac{L_s}{X} \leq 0.17 - 0.33 : \text{ no accretion}
\]
Nir (1982)
\[
\frac{L_s}{X} < 0.5 : \text{ no depositional condition}
\]
SPM (1984)
\[
\frac{L_s}{X} < 1 : \text{ tombolo formation prevented}
\]
$\frac{dy}{dx} > 2$ : tombolo formation certain

There is a conformity between the criteria from Ahren and Cox (1990) and the criteria of Leo C. Van Rijn (2013), whereas the criteria from Inman and Proudcy (1966), Nir (1982) and SPM (1984) also have a conformity with both.

IV. FORMULATION OF SALIENT EQUATION

4.1. Basic Equation and Boundary Conditions

Ordinate salient equation ($x$) is approached with polynomials of degree ten

$$y(x) = \sum_{i=0}^{10} c_i x^i \quad \ldots \ldots \ldots (6)$$

where the horizontal axis is coincided with the original coastline (Fig.4). The number of polynomial terms is an effort to obtain a unique solution, where the more the number of polynomial terms, the more will be the boundary conditions that are used so that they will increase the solution uniqueness.

There are two boundary conditions, i.e. the two ends of the salient, where in this section an assumption is done that coastline coordinate is fixed. Whereas at the interior points, the boundary condition of the stable coastline is done, i.e. coastline tangent or salient which is similar to the tangent of the wave forming it.

The wave forming salient is diffracted wave, whereas the direction of diffracted wave is defined as in Fig. 4. For a diffracted wave with direction $\overrightarrow{AD}$ towards a point $B$ at the original coastline with $x_B$ abscissa then the tangent of the crestline of the wave is

$$\beta_B = \tan \left( \frac{2x}{x_B} \right) \quad \ldots \ldots \ldots (7)$$

For a diffracted wave with direction $\overrightarrow{CD}$ towards a point $D$ at the original coastline with $x_D$ abscissa then the tangent of the crestline of the wave is

$$\beta_D = -\tan \left( \frac{x_D - x}{x} \right) \quad \ldots \ldots \ldots (8)$$

For $x_D = L_s - x_D$ produces $\beta_D = -\beta_B$. This characteristic produces a symmetrical characteristic at the salient with maximum point of salient at $x_{\text{max}} = \frac{L_s}{2}$ where $\beta = 0$.

Thus, the salient characteristic formed by diffracted is symmetrical. In this method there is an assumption that in the salient growth, the tangent of a point is still the same, from the beginning of the formation until the final condition.

To obtain the values of polynomial coefficients $c_0$, $c_1$, $c_2$, $c_3$, $c_4$, $c_5$, $c_6$, $c_7$, $c_8$, $c_9$ and $c_{10}$, boundary conditions are done at the points as presented in Table 3. The abscissa values of the boundary tangent condition points is the result of an experimentation to obtain a salient condition that in accordance with the Vanrijn and Ahren&Cox criteria.

At the salient equation and its boundary condition there is no wave height impact or diffraction coefficient since crestline tangent is not determined by wave height or diffraction coefficient. Whereas salient tangent is determined by crestline tangent.
4.2. Relationof Breakwater distance $X$ and Breakwater length $L_s\left(\frac{L_s}{X}\right)$ with Salient height $Y_s$

As has been stated that there is a relation between salient height $Y_s$with the value $\frac{L_s}{X}$ or in other words there is an impact of $X$ and $L_s$ on the formation of salient. The relation can be explained as follows

Fig.5 shows a diffracted wave towards a point P at the original coastline at different breakwater $X$ position. The farther away the position of breakwater, the smaller the coastline tangent, and the smaller the tangent crestline, the smaller also the coastline tangent and the smaller the coastline tangent the smaller the size of salient $Y_s$.

![Fig.5: The comparison of tangent crestline of diffracted wave at different breakwater position.](image)

Fig.6 shows that for different breakwater length, crestline diffracted wave towards the same point P have different tangent against original coastline, where the shorter the length of breakwater, the smaller the tangent of the crestline.

V. THE RESULT OF THE EQUATION

Fig 7. presents the result of a model for breakwater length of $L_s = 60 \text{ m}$ with the position of $X = 30 \text{ m}$ from the original coastline where $\frac{L_s}{X} = 2.0$. Salient that is formed has a measurement of $Y_s = 12.37 \text{ m}$. Both Ahren and Van Rijn stated that this salient is a well-developed style.

![Fig.7. Salient at $L_s = 60 \text{ m}$, $X = 50 \text{ m}$, $\frac{L_s}{X} = 1.2$, and $X = 30 \text{ m}$, $\frac{L_s}{X} = 2.0$](image)

Table 4 presents the result of calculation for a number of $\frac{L_s}{X}$ values with fixed $L_s$. It shows that the result of the model is in accordance with the result of Ahren as well as Van Rijn’s research, where perfect tombolo is formed at $\frac{L_s}{X} = 3$.

On the other hand, Table 5 presents the result of the calculation with changing breakwater length $L_s$, whereas the breakwater distance $X$ is fixed at $30 \text{ m}$, which also provides a result that is in accordance with the result of the research by Ahren and Van Rijn. Thus, it can be concluded that the model that is developed provides a very good result.

The comparison between the result of the calculation in Table 4 with the result of the calculation in Table 5 shows that at the same value of $\frac{L_s}{X}$ different $Y_s$ value was obtained, i.e. $Y_s$ is bigger at bigger $L_s$, as an example for $\frac{L_s}{X} = 1.5$, in Table 4, $(L_s = 60 \text{ m}, X = 40 \text{ m})$, $Y_s = 9.28 \text{ m}$, meanwhile in Table 5, $(L_s = 45 \text{ m}, X = 30 \text{ m})$, $Y_s = 6.95 \text{ m}$.

| $\frac{L_s}{X}$ | $X$ (m) | $L_s$ (m) | $Y_s$ (m) |
|-----------------|--------|-----------|-----------|
| 0.25            | 240    | 60        | 1.55      |

Table 4. The result of the calculation of salient $Y_s$ for $L_s$ constant of 60 m.
Table 5. The result of the calculation of salient $Y_s$, for $X$ constant at 30 m.

| $L_s$ (m) | $X$ (m) | $L_s$ (m) | $Y_s$ (m) |
|-----------|---------|-----------|-----------|
| 0.25      | 30      | 7.5       | 0.19      |
| 0.5       | 30      | 15        | 0.77      |
| 0.75      | 30      | 22.5      | 1.74      |
| 1         | 30      | 30        | 3.09      |
| 1.25      | 30      | 37.5      | 4.76      |
| 1.5       | 30      | 45        | 6.95      |
| 1.75      | 30      | 52.5      | 9.46      |
| 2         | 30      | 60        | 12.37     |
| 2.25      | 30      | 67.5      | 15.69     |
| 2.5       | 30      | 75        | 19.04     |
| 2.75      | 30      | 82.5      | 22.95     |
| 3         | 30      | 90        | 27.78     |
| 3.25      | 30      | 97.5      | 30        |

VI. CONCLUSION

There is a conformity between the result of the model with the result of the previous research which is the result of field observation as well as the result of physical model in the laboratory. Thus it can be concluded that the method that was developed is capable of modeling the formation of salient and tombolo well.

However, the obstacles in this method is the determination of interior abscissa of the boundary condition points was done like an experiment, where with different interior abscissa of the boundary condition points will result in different result. Therefore, further development needed is formulating equation for determining the boundary condition interior points or looking for additional equilibrium equation so that the result of the model is no longer dependent on the location of the interior boundary condition points.

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