Screening Constant by Unit Nuclear Charge Photoionization of Rb$^{2+}$ Ions

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Abstract. Photoionization data of the trans-Fe element Rb$^{2+}$ are reported. Rydberg series $4s^24p^4(1D_2)nd$ and $4s^24p^4(3P_1)nd$ Rydberg series of Rb$^{2+}$ from the $^2P_{3/2}$ ground state and the $^2P_{1/2}$ metastable state of Rb$^{2+}$ converging respectively to the $4s^24p^4(1D_2)$ $4s^24p^4(3P_1)$ series limit in Rb$^{3+}$ are considered. Calculations are performed in the framework of the Screening constant by unit nuclear charge (SCUNC) method. Accurate data are tabulated up to $n = 40$. It is shown that the SCUNC analytical formulas reproduce with an excellent precision, recent ALS measurements of Macaluso et al., [J. Phys. B: At. Mol. Opt. Phys. 49 (2016), 235002; 50 (2017), 119501]. The energy deviations with respect to the ALS data are equal to 0.001 eV. New data are tabulated for $n = 21 – 40$.

Keywords. Photoionization; Rydberg series; Ground state; Metastable state; SCUNC

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1. Introduction

Photoionization studies of Rb ions (Sneden et al. [3]; Kilbane et al. [1]; Mueller et al. [2]) remain an active field of investigations due to their importance for modeling astrophysical objects such those in the asymptotic giant branch (AGB) region. It is widely believed that, a major source of discrepancy is the quality of the atomic data used in the modelling (Mishenina et al. [4]; Roederer et al. [5,6]; Frebel et al. [7]). Rb is one of the neutron-capture elements (Se, Cd, Ga, Ge, Rb, Kr, Br, Xe, Ba, Pb, etc.) (Pequignot and Baluteau [8]; Sharpee [9]; Sterling and Dinerstein [10]; Otsuka and Tajitsu [11]; García-Rojas et al. [12]; Sterling et al. [13]) produced by slow ($s$-process)
or rapid (r-process) neutron-capture nucleosynthesis in ionized nebulae. Photoionization study of Rb$^{2+}$ ions is especially crucial because it permits to provide benchmark data in connection with astrophysical applications. In addition such study permits to aid in the formulation of so-called “ionization correction factors” used in the modeling of planetary nebular emission lines of ions of Rb (Langanke and Wiescher [14]; Kwitter et al. [15]; Luridiana et al. [18,19]). Recently, Macaluso et al. [21] performed high-resolution photoionization cross section measurements of the for Rb$^{2+}$ over the photon energy range 37.31-44.08 eV using synchrotron radiation and the photo-ion, merged-beams technique at the Advanced Light Source at Lawrence Berkeley National Laboratory with a bandpass resolution of 13.5 ± 2.5 meV full width half maximum (FWHM). In tandem with the measurements, Breit-Pauli R-matrix calculations were performed in the intermediate coupling $jK$ to facilitate the identification of several highly excited Auger Rydberg resonance states of the Rb$^{2+}$ ions. Very recently, McLaughlin and Babb [17] used a fully relativistic approach within the Dirac-Coulomb R-matrix (DARC) approximation to calculate the cross sections for ground and metastable states. Although very good agreement are found between the DARC [17] and ALS measurements [19], calculations can be improved as maximum energy differences between theory and experiment at 0.008 eV are observed for the 4s$^2$4p$^4$(1D)$^2$nd and 4s$^2$4p$^4$(3P)$^1$nd Rydberg series. The motivation of this work is to use the Screening constant by unit nuclear charge (SCUNC) formalism (Sakho [20–22]; Ba et al. [23]; Badiane et al. [24]) to report precise high lying Photoionization data of the Rb$^{2+}$ ions reducing energy deviations with respect to the ALS data at a maximum of 0.001 eV.

The layout of this work is as follows. Section 2 presents a brief outline of the theoretical part of the work. Section 3 presents a discussion of the results obtained compared with the available literature data. Finally, in Section 4 we summarize and conclude the present study.

## 2. Theory

In the framework of the Screening Constant by Unit Nuclear Charge formalism, the total energy of the $(Nl, nl')^{2S+1}L^\pi$ excited states is expressed in the form (in Rydberg)

$$ E = -Z^2 \left( \frac{1}{N^2} + \frac{1}{n^2} \left[ 1 - \beta(Nl, nl',^{2S+1}L^\pi; Z) \right]^2 \right). $$

In this equation, the principal quantum numbers $N$ and $n$ are respectively for the inner and the outer electron of the helium-isoelectronic series. The $\beta$-parameters are screening constants by unit nuclear charge expanded in inverse powers of $Z$ and given by

$$ \beta(Nl, nl',^{2S+1}L^\pi; Z) = \sum_{k=1}^{q} f_k \left( \frac{1}{Z} \right)^k \tag{2.2} $$

where $f_k = f_k(Nl, nl',^{2S+1}L^\pi)$ are parameters to be evaluated empirically.

For a given Rydberg series originating from a $^{2S+1}L_J$ state, we obtain

$$ E_n = E_\infty - \frac{Z^2}{n^2} \left[ 1 - \beta(nl',s,\mu,\nu,^{2S+1}L^\pi; Z) \right]^2. \tag{2.3} $$
In this equation, \( \nu \) and \( \mu \) (\( \mu > \nu \)) denote the principal quantum numbers of the \( ^{2S+1}L_J \) \( nl \) Rydberg series used in the empirical determination of the \( f_k \)-screening constants, \( s \) represents the spin of the \( nl \)-electron (\( s = 1/2 \)), \( E_\infty \) is the energy value of the series limit, \( E_n \) denotes the resonance energy and \( Z \) stands for the atomic number. The \( \beta \)-parameters are screening constants by unit nuclear charge expanded in inverse powers of \( Z \) and given by

\[
\beta(Z, ^{2S+1}L_J, n, s, \mu, \nu) = \sum_{k=1}^{q} f_k \left( \frac{1}{Z} \right)^k
\]  

(2.4)

where \( f_k = f_k(^{2S+1}L_J, n, s, \mu, \nu) \) are screening constants to be evaluated empirically. In eq. (2.2), \( q \) stands for the number of terms in the expansion of the \( \beta \)-parameter. The resonance energy are the in the form

\[
E_n = E_\infty - \frac{Z^2}{n^2} \left\{ 1 - \frac{f_1(^{2S+1}L_J)}{Z(n-1)} - \frac{f_2(^{2S+1}L_J)}{Z} \pm \sum_{k=1}^{q} \sum_{k'=1}^{q'} f_{1k}^k F(n, \mu, \nu, s) \times \left( \frac{1}{Z} \right)^k \right\}^2.
\]  

(2.5)

In this equation, \( \pm \sum_{k=1}^{q} \sum_{k'=1}^{q'} f_{1k}^k F(n, \mu, \nu, s) \times \left( \frac{1}{Z} \right)^k \) is a corrective term introduce to stabilize the resonance energies with increasing the principal quantum number \( n \). In general, resonance energies are analyzed from the standard quantum-defect expansion formula

\[
E_n = E_\infty - \frac{R Z^2}{(n-\delta)^2}.
\]  

(2.6)

In this equation, \( R \) is the Rydberg constant, \( E_\infty \) denotes the converging limit, \( Z_{\text{core}} \) represents the electric charge of the core ion, and \( \delta \) means the quantum defect. In addition, theoretical and measured energy positions can be analyzed by calculating the \( Z^* \)-effective charge in the framework of the SCUNC-procedure

\[
E_n = E_\infty - \frac{Z^2}{n^2} R.
\]  

(2.7)

The relationship between \( Z^* \) and \( \delta \) is in the form

\[
Z^* = \frac{Z_{\text{core}}}{(1 - \frac{\delta}{n})}.
\]  

(2.8)

According to this equation, each Rydberg series must satisfy the following conditions

\[
\begin{align*}
Z^* &\geq Z_{\text{core}} \quad \text{if } \delta \geq 0 \\
Z^* &\leq Z_{\text{core}} \quad \text{if } \delta \leq 0 \\
\lim_{n \to \infty} Z^* &= Z_{\text{core}}
\end{align*}
\]  

(2.9)

Besides, comparing eq. (2.5) and eq. (2.7), the effective charge is in the form

\[
Z^* = Z \left\{ 1 - \frac{f_1(^{2S+1}L_J)}{Z(n-1)} - \frac{f_2(^{2S+1}L_J)}{Z} \pm \sum_{k=1}^{q} \sum_{k'=1}^{q'} f_{1k}^k F(n, \mu, \nu, s) \times \left( \frac{1}{Z} \right)^k \right\}.
\]  

(2.10)

Besides, the \( f_2 \)-parameter in eq. (2.2) can be theoretically determined from eq. (2.10) by neglecting the corrective term with the condition

\[
\lim_{n \to \infty} Z^* = Z \left( 1 - \frac{f_2(^{2S+1}L_J)}{Z} \right) = Z_{\text{core}}.
\]  

(2.11)
We get then $f_2 = Z - Z_{\text{core}}$, where $Z_{\text{core}}$ is directly obtain by the photoionization process from an atomic $X^{p+}$ system $X^{p+} + h\nu \rightarrow X^{(p+1)+} + e^-$. We find then $Z_{\text{core}} = p + 1$. Thus, for the Rb$^{2+}$ ions, $Z_{\text{core}} = 3$ and $f_2 = (37 - 3) = 34.0$. The remaining $f_1$-parameter is to be evaluated empirically using the ALS data of Macaluso et al. \cite{18, 19} for a given $(2S+1)L_f$ $\mu$ level with $\nu = 0$. The empirical procedure of the determination of the $f_1$-screening constant along with the corresponding uncertainty have been explained in details in our previous works (Sakho \cite{20–22}; Ba et al. \cite{23}; Badiane et al. \cite{24}). In the present work, all the energy resonances are calculated using the following simple expression (in Rydberg units)

- for the $4s^24p^4(3P)nd$ Rydberg series of Rb$^{2+}$ converging to the Rb$^{3+}$($3d^{10}4s^24p^3P_1$) threshold originating from the Rb$^{2+}$ ground $4s^24p^5P_{1/2}^{3/2}$ state and metastable $4s^24p^5P_{1/2}^{3/2}$ state, we get

$$E_n = E_\infty - \frac{Z^2}{n^2} \left(1 - \frac{f_1(3P_1)}{Z(n-1)} - \frac{f_2(3P_1)}{Z} \right) - \frac{f_1(3P_1) \times (n-\mu)}{Z^2(n-s-1)(n+\mu-s)} - \frac{f_1(3P_1) \times (n-\mu)^2}{Z^3(n-s-1)(n+\mu-s)} \right)^2 \quad (2.12)$$

- for the $4s^24p^4(1D_2)nd$ Rydberg series of Rb$^{2+}$ converging to the Rb$^{3+}$($3d^{10}4s^24p^31D_2$) threshold originating from the Rb$^{2+}$ ground $4s^24p^5P_{1/2}^{10}$ state and from the Rb$^{2+}$ metastable $4s^24p^5P_{1/2}^{10}$ state, we obtain

$$E_n = E_\infty - \frac{Z^2}{n^2} \left(1 - \frac{f_1(1D_2)}{Z(n-1)} - \frac{f_2(1D_2)}{Z} \right) - \frac{f_1(1D_2) \times (n-\mu)}{Z^2(n-s-2)(n+\mu+s)} - \frac{f_1(1D_2) \times (n-\mu)^2}{Z^3(n-s-2)(n+\mu+s)} \right)^2 \quad (2.13)$$

### 3. Results and Discussion

The present SCUNC calculations are listed in Tables 1-5. Comparisons are done with the Advanced Light Source (ALS) measurements of Macaluso et al. \cite{18, 19} and with the fully relativistic approach within the Dirac-Coulomb R-matrix (DARC) calculations of McLaughlin and Babb \cite{17}. Analysis of the values of the nuclear effective charge $Z^*$ listed in the first entry of each table indicate that $Z^*_{\text{max}} > Z_{\text{core}} = 3.0$. This means that the quantum defect is positive according to the SCUNC’s conditions analysis (2.9) in agreement with the sign of the theoretical and experimental quantum defects quoted in Tables 1-5. Besides, comparison of resonance energies indicate excellent agreements between theory and experiment. It should be mentioned that, the excellent SCUNC calculations with a maximum of energy deviation with respect to the ALS measurements at 0.001 eV. This allows one to expect the high lying data up to $n = 40$ to be useful benchmark data for astrophysical applications. In the work of McLaughlin and Babb \cite{17}, fully relativistic approach within the Dirac-Coulomb R-matrix calculations were performed in the intermediate coupling $jK$ Breit-Pauli approximation from the DARC 687 level calculations. In the present work, very precise photoionization data are obtained within the very simple formalism of the SUCNC method. The possibility to provide accurate photoionization data using the SCUNC constant is due to the validity of the formalism to treat correctly photoionization properties of multi-charged atomic systems as demonstrated in our previous works (Sakho \cite{20–22}; Ba et al. \cite{23}; Badiane et al. \cite{24}).
Table 1. Energy resonances ($E_n$, eV), quantum defect ($\delta$) and effective nuclear charge $Z^*$ of the $4s^24p^4(^3P_1)$ and Rydberg series of Rb$^{2+}$ converging to the Rb$^{3+}(3d^{10}4s^24p^4^3P_1)$ threshold originating from the Rb$^{2+}$ metastable $4s^24p^52p^\circ_{1/2}$ state. $f_1(^3P_1) = .797 \pm 0.071; \mu = 13$. The present SCUNC calculations are compared to the DARC calculations (McLaughlin and Babb [17]) and the ALS measurements (Macaluso et al. [18,19]). $|\Delta E|$ denotes the energy difference between the SCUNC calculations and the ALS measurements.

| n  | $E_n$  | $|\Delta E|$ | $\delta$ | $\mu$ | $Z^*$ |
|----|--------|--------------|---------|------|------|
| 13 | 38.352 | 0.000        | 0.28    | 3.066|
| 14 | 38.458 | 0.001        | 0.28    | 3.061|
| 15 | 38.544 | 0.000        | 0.28    | 3.057|
| 16 | 38.614 | 0.000        | 0.28    | 3.053|
| 17 | 38.671 | 0.000        | 0.28    | 3.050|
| 18 | 38.719 | 0.000        | 0.28    | 3.047|
| 19 | 38.760 | 0.000        | 0.28    | 3.045|
| 20 | 38.794 | 0.000        | 0.28    | 3.042|
| 21 | 38.824 |            | 0.28    | 3.040|
| 22 | 38.850 |            | 0.28    | 3.038|
| 23 | 38.872 |            | 0.28    | 3.037|
| 24 | 38.891 |            | 0.28    | 3.035|
| 25 | 38.909 |            | 0.28    | 3.034|
| 26 | 38.924 |            | 0.28    | 3.032|
| 27 | 38.938 | 0.28        | 3.031   |
| 28 | 38.950 | 0.28        | 3.030   |
| 29 | 38.961 | 0.28        | 3.029   |
| 30 | 38.970 | 0.28        | 3.028   |
| 31 | 38.979 | 0.28        | 3.027   |
| 32 | 38.987 | 0.28        | 3.026   |
| 33 | 38.995 | 0.28        | 3.025   |
| 34 | 39.001 | 0.28        | 3.024   |
| 35 | 39.007 | 0.28        | 3.024   |
| 36 | 39.013 | 0.28        | 3.023   |
| 37 | 39.018 |            | 0.28    | 3.022|
| 38 | 39.023 |            | 0.28    | 3.022|
| 39 | 39.027 |            | 0.28    | 3.021|
| 40 | 39.031 |            | 0.28    | 3.021|
| ... | ...    |            | ...    |       |
| $\infty$ | 39.109 | 39.109 | 39.109 | ... | 3.000 |
Table 2. Energy resonances ($E_n$, eV), quantum defect ($\delta$) and effective nuclear charge $Z^*$ of the $4s^24p^4(^3P_1)$nd Rydberg series of Rb$^{2+}$ converging to the Rb$^{3+}$(3d$^{10}$4s$^24p^4^3P_1$) threshold originating from the Rb$^{2+}$ ground $4s^24p^5^3P_{3/2}$ state. $f_1(^3P_1) = -0.748 \pm 0.071$; $\mu = 13$. The present SCUNC calculations are compared to the DARC calculations (McLaughlin and Babb [17]) and the ALS measurements (Macaluso et al. [18][19]). $|\Delta E|$ denotes the energy difference between the SCUNC calculations and the ALS measurements.

| n  | SCUNC $E_n$ | DARC $E_n$ | ALS $E_n$ | $|\Delta E|$ | SCUNC $\delta$ | DARC $\delta$ | ALS $\delta$ | SCUNC $Z^*$ |
|----|------------|------------|-----------|-------------|-------------|-------------|-------------|-------------|
| 13 | 39.268     | 39.263     | 39.268    | 0.000       | 0.26        | 0.27        | 0.31        | 3.062       |
| 14 | 39.374     | 39.379     | 39.374    | 0.000       | 0.26        | 0.27        | 0.32        | 3.058       |
| 15 | 39.459     | 39.457     | 39.459    | 0.000       | 0.26        | 0.27        | 0.32        | 3.054       |
| 16 | 39.529     | 39.526     | 39.528    | 0.001       | 0.26        | 0.27        | 0.30        | 3.050       |
| 17 | 39.586     | 39.584     | 39.586    | 0.000       | 0.26        | 0.27        | 0.30        | 3.047       |
| 18 | 39.634     | 39.632     | 39.634    | 0.000       | 0.26        | 0.27        | 0.32        | 3.044       |
| 19 | 39.674     | 39.673     | 39.674    | 0.000       | 0.26        | 0.27        | 0.30        | 3.042       |
| 20 | 39.709     | 39.707     | 39.709    | 0.000       | 0.26        | 0.27        | 0.30        | 3.040       |
| 21 | 39.738     |           |           |             | 0.26        |             |             | 3.038       |
| 22 | 39.764     |           |           |             | 0.26        |             |             | 3.036       |
| 23 | 39.786     |           |           |             | 0.26        |             |             | 3.034       |
| 24 | 39.806     |           |           |             | 0.26        |             |             | 3.033       |
| 25 | 39.823     |           |           |             | 0.26        |             |             | 3.032       |
| 26 | 39.838     |           |           |             | 0.26        |             |             | 3.030       |
| 27 | 39.852     |           |           |             | 0.26        |             |             | 3.029       |
| 28 | 39.864     |           |           |             | 0.26        |             |             | 3.028       |
| 29 | 39.875     |           |           |             | 0.26        |             |             | 3.027       |
| 30 | 39.885     |           |           |             | 0.26        |             |             | 3.026       |
| 31 | 39.893     |           |           |             | 0.26        |             |             | 3.025       |
| 32 | 39.901     |           |           |             | 0.26        |             |             | 3.025       |
| 33 | 39.909     |           |           |             | 0.26        |             |             | 3.024       |
| 34 | 39.915     |           |           |             | 0.26        |             |             | 3.023       |
| 35 | 39.922     |           |           |             | 0.26        |             |             | 3.022       |
| 36 | 39.927     |           |           |             | 0.26        |             |             | 3.022       |
| 37 | 39.932     |           |           |             | 0.26        |             |             | 3.021       |
| 38 | 39.937     |           |           |             | 0.26        |             |             | 3.021       |
| 39 | 39.941     |           |           |             | 0.26        |             |             | 3.020       |
| 40 | 39.945     |           |           |             | 0.26        |             |             | 3.020       |
| $\infty$ | 40.023  | 40.023  | 40.023  | ...         | ...        | ...        | ...        | 3.000       |
Table 3. Energy resonances $(E_n, \text{eV})$, quantum defect ($\delta$) and effective nuclear charge $Z^*$ of the $4s^2 4p^4 (^1D_2)$nd Rydberg series of $\text{Rb}^{2+}$ converging to the $\text{Rb}^{3+} (3d^{10} 4s^2 4p^4 ^1D_2)$ threshold originating from the $\text{Rb}^{2+}$ metastable $4s^2 4p^5 ^2P^o_{1/2}$ state. $f_{1}^{(1D_2)} = -0.679 \pm 0.071$; $\mu = 8$. The present SCUNC calculations are compared to the DARC calculations (McLaughlin and Babb [17]) and the ALS measurements (Macaluso et al. [18, 19]). $|\Delta E|$ denotes the energy difference between the SCUNC calculations and the ALS measurements.

| $n$ | SCUNC | DARC | ALS | SCUNC | DARC | ALS | SCUNC |
|-----|-------|------|-----|-------|------|-----|-------|
| 8   | 38.446| 38.440| 38.446| 0.000 | 0.25 | 0.26 | 3.097 |
| 9   | 38.885| 38.884| 38.885| 0.000 | 0.25 | 0.25 | 3.087 |
| 10  | 39.196| 39.192| 39.196| 0.000 | 0.25 | 0.26 | 3.078 |
| 11  | 39.425| 39.424| 39.425| 0.000 | 0.25 | 0.26 | 3.070 |
| 12  | 39.598| 39.597| 39.598| 0.000 | 0.25 | 0.26 | 3.064 |
| 13  | 39.732| 39.731| 39.731| 0.001 | 0.25 | 0.26 | 3.058 |
| 14  | 39.838| 39.837| 39.837| 0.001 | 0.25 | 0.26 | 3.054 |
| 15  | 39.922| 39.922| 39.922| 0.000 | 0.25 | 0.26 | 3.050 |
| 16  | 39.992| 39.991| 39.991| 0.001 | 0.25 | 0.26 | 3.047 |
| 17  | 40.049| 40.048| 40.048| 0.001 | 0.25 | 0.26 | 3.044 |
| 18  | 40.097| 40.096| 40.096| 0.001 | 0.24 | 0.26 | 3.041 |
| 19  | 40.137|       |       |       | 0.24 | 0.26 | 3.039 |
| 20  | 40.171|       |       |       | 0.24 | 0.26 | 3.037 |
| 21  | 40.201|       |       |       | 0.24 | 0.26 | 3.035 |
| 22  | 40.226|       |       |       | 0.24 | 0.26 | 3.034 |
| 23  | 40.249|       |       |       | 0.24 | 0.26 | 3.032 |
| 24  | 40.268|       |       |       | 0.24 | 0.26 | 3.031 |
| 25  | 40.285|       |       |       | 0.24 | 0.26 | 3.029 |
| 26  | 40.300|       |       |       | 0.24 |       | 3.028 |
| 27  | 40.314|       |       |       | 0.24 |       | 3.027 |
| 28  | 40.326|       |       |       | 0.24 |       | 3.026 |
| 29  | 40.337|       |       |       | 0.24 |       | 3.025 |
| 30  | 40.347|       |       |       | 0.24 |       | 3.024 |
| 31  | 40.356|       |       |       | 0.24 |       | 3.024 |
| 32  | 40.364|       |       |       | 0.24 |       | 3.023 |
| 33  | 40.371|       |       |       | 0.24 |       | 3.022 |
| 34  | 40.378|       |       |       | 0.24 |       | 3.022 |
| 35  | 40.384|       |       |       | 0.24 |       | 3.021 |
| 36  | 40.389|       |       |       | 0.24 |       | 3.020 |
| 37  | 40.394|       |       |       | 0.24 |       | 3.020 |
| 38  | 40.399|       |       |       | 0.24 |       | 3.019 |
| 39  | 40.403|       |       |       | 0.24 |       | 3.019 |
| 40  | 40.408|       |       |       | 0.24 |       | 3.018 |
| ... | ...   | ...   | ...  | ...   | ...   | ... | ...   |
| $\infty$ | 40.485 | 40.485 | 40.485 | ... | ... | ... | 3.000 |
Table 4. Energy resonances ($E_n$, eV), quantum defect ($\delta$) and effective nuclear charge $Z^*$ of the $4s^24p^4(1D_2)$ Rydberg series of Rb$^{2+}$ converging to the Rb$^{3+}(3d^{10}4s^24p^4^3P_1)$ threshold originating from the Rb$^{2+}$ ground $4s^24p^5^5P^o_3/2$ state. $f_1(1D_2) = -0.748 \pm 0.071$; $\mu = 8$. The present SCUNC calculations are compared to the DARC calculations (McLaughlin and Babb [17]) and the ALS measurements (Macaluso et al. [18][19]). $|\Delta E|$ denotes the energy difference between the SCUNC calculations and the ALS measurements.

| $n$ | SCUNC | DARC | ALS | $|\Delta E|$ | SCUNC | DARC | ALS | $\delta$ | SCUNC | DARC |ALS | $Z^*$ |
|-----|--------|------|-----|----------|--------|------|-----|-----|--------|--------|------|-----|--------|
| 8   | 39.347 | 39.355 | 39.347 | 0.000   | 0.28   | 0.28 | 0.28 | 3.107 |
| 9   | 39.789 | 39.797 | 39.790 | 0.001   | 0.28   | 0.28 | 0.28 | 3.096 |
| 10  | 40.104 | 40.109 | 40.104 | 0.000   | 0.28   | 0.28 | 0.28 | 3.085 |
| 11  | 40.334 | 40.339 | 40.334 | 0.000   | 0.28   | 0.28 | 0.28 | 3.077 |
| 12  | 40.508 | 40.511 | 40.508 | 0.000   | 0.27   | 0.28 | 0.28 | 3.070 |
| 13  | 40.643 | 40.645 | 40.643 | 0.001   | 0.27   | 0.28 | 0.28 | 3.064 |
| 14  | 40.749 | 40.751 | 40.749 | 0.000   | 0.27   | 0.28 | 0.28 | 3.059 |
| 15  | 40.835 | 40.836 | 40.834 | 0.001   | 0.27   | 0.28 | 0.28 | 3.055 |
| 16  | 40.904 | 40.905 | 40.904 | 0.000   | 0.27   | 0.28 | 0.28 | 3.052 |
| 17  | 40.962 | 40.962 | 40.961 | 0.001   | 0.27   | 0.28 | 0.28 | 3.048 |
| 18  | 41.009 | 41.010 | 41.009 | 0.000   | 0.27   | 0.28 | 0.28 | 3.046 |
| 19  | 41.050 |        |        |         | 0.27   |      |      | 3.043 |
| 20  | 41.084 |        |        |         | 0.27   |      |      | 3.041 |
| 21  | 41.114 |        |        |         | 0.27   |      |      | 3.039 |
| 22  | 41.140 |        |        |         | 0.27   |      |      | 3.037 |
| 23  | 41.162 |        |        |         | 0.27   |      |      | 3.035 |
| 24  | 41.182 |        |        |         | 0.27   |      |      | 3.034 |
| 25  | 41.199 |        |        |         | 0.27   |      |      | 3.032 |
| 26  | 41.214 |        |        |         | 0.27   |      |      | 3.031 |
| 27  | 41.228 |        |        |         | 0.27   |      |      | 3.030 |
| 28  | 41.240 |        |        |         | 0.27   |      |      | 3.029 |
| 29  | 41.251 |        |        |         | 0.27   |      |      | 3.028 |
| 30  | 41.260 |        |        |         | 0.27   |      |      | 3.027 |
| 31  | 41.269 |        |        |         | 0.27   |      |      | 3.026 |
| 32  | 41.277 |        |        |         | 0.27   |      |      | 3.025 |
| 33  | 41.285 |        |        |         | 0.27   |      |      | 3.024 |
| 34  | 41.291 |        |        |         | 0.27   |      |      | 3.024 |
| 35  | 41.297 |        |        |         | 0.27   |      |      | 3.023 |
| 36  | 41.303 |        |        |         | 0.27   |      |      | 3.022 |
| 37  | 41.308 |        |        |         | 0.27   |      |      | 3.022 |
| 38  | 41.313 |        |        |         | 0.27   |      |      | 3.021 |
| 39  | 41.317 |        |        |         | 0.27   |      |      | 3.021 |
| 40  | 41.321 |        |        |         | 0.27   |      |      | 3.020 |
| ... | ...    |        |        | ...     | ...    |      |      | ...   |
| $\infty$ | 41.399 | 41.399 | 41.399 | ...    | ...    |      |      | 3.000 |
Table 5. Energy resonances ($E_n$, eV), quantum defect ($\delta$) and effective nuclear charge $Z^*$ of the $4s^24p^4({}^1D_2)$nd Rydberg series of Rb$^{2+}$ converging to the Rb$^{3+}$($3d^{10}4s^24p^4{}^3P_1$) threshold originating from the Rb$^{2+}$ ground $4s^24p^5{}^3P_{3/2}$ state. $f_1({}^1D_2) = -0.748 \pm 0.071$; $\mu = 8$. The present SCUNC calculations are compared to the DARC calculations (McLaughlin and Babb [17]) and the ALS measurements (Macaluso et al. [18],[19]). $|\Delta E|$ denotes the energy difference between the SCUNC calculations and the ALS measurements.

| $n$ | SCUNC | DARC | ALS | SCUNC | DARC | ALS | SCUNC |
|-----|--------|------|-----|--------|------|-----|--------|
|     | $E_n$  | $\delta$ | $Z^*$ |
| 8   | 39.347 | 0.28  | 3.107 |
| 9   | 39.797 | 0.28  | 3.096 |
| 10  | 40.104 | 0.28  | 3.085 |
| 11  | 40.334 | 0.28  | 3.077 |
| 12  | 40.508 | 0.27  | 3.070 |
| 13  | 40.643 | 0.27  | 3.064 |
| 14  | 40.749 | 0.27  | 3.059 |
| 15  | 40.835 | 0.27  | 3.055 |
| 16  | 40.904 | 0.27  | 3.052 |
| 17  | 40.962 | 0.27  | 3.048 |
| 18  | 41.009 | 0.27  | 3.046 |
| 19  | 41.050 | 0.27  | 3.043 |
| 20  | 41.084 | 0.27  | 3.041 |
| 21  | 41.114 | 0.27  | 3.039 |
| 22  | 41.140 | 0.27  | 3.037 |
| 23  | 41.162 | 0.27  | 3.035 |
| 24  | 41.182 | 0.27  | 3.034 |
| 25  | 41.199 | 0.27  | 3.032 |
| 26  | 41.214 | 0.27  | 3.031 |
| 27  | 41.228 | 0.27  | 3.030 |
| 28  | 41.240 | 0.27  | 3.029 |
| 29  | 41.251 | 0.27  | 3.028 |
| 30  | 41.260 | 0.27  | 3.027 |
| 31  | 41.269 | 0.27  | 3.026 |
| 32  | 41.277 | 0.27  | 3.025 |
| 33  | 41.285 | 0.27  | 3.024 |
| 34  | 41.291 | 0.27  | 3.024 |
| 35  | 41.297 | 0.27  | 3.023 |
| 36  | 41.303 | 0.27  | 3.022 |
| 37  | 41.308 | 0.27  | 3.022 |
| 38  | 41.313 | 0.27  | 3.021 |
| 39  | 41.317 | 0.27  | 3.021 |
| 40  | 41.321 | 0.27  | 3.020 |
| ... | ...    | ...  | ... |
| $\infty$ | 41.399 | ... | 3.000 |
4. Conclusion

The screening constant by unit nuclear charge (SCUNC) is used to report accurate resonance energies belonging to the 4p → nd transitions from the $^2P_{3/2}^o$ ground state and the $^2P_{1/2}^o$ metastable state of Rb$^{2+}$ converging to the 4s$^2$4p$^4$ ($^1D_2$) and 4s$^2$4p$^4$ ($^3P_1$) series limit in Rb$^{3+}$. It is seen that the SCUNC formula established reproduces with an excellent precision less than 0.002 eV high-resolution measurements of Macaluso et al. (2017). New data $n = 21 – 40$ are tabulated as useful guidelines for the NIST data base and for future PI studies on Rb$^{2+}$ focussed on high excited levels.

Competing Interests

The authors declare that they have no competing interests.

Authors’ Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] D. Kilbane, F. Folkmann, J.-M. Bizau, C. Banahan, S. Scully, H. Kjeldsen, P. van Kampen, M. W. D. Mansfield, J. T. Costello and J. B. West, Phys. Rev. A 75 (2007), 032711, DOI: 10.1103/PhysRevA.75.032711

[2] A. Mueller, D. Macaluso, N. Sterling, A. Juarez, I. Dumitriu, R. Bilodeau, E. Red, D. Hardy and A. Aguilar, Bull. Am. Phys. Soc. 58 (2013), Q1.00141, URL: http://meetings.aps.org/Meeting/DAMOP13/Session/Q1.141

[3] C. Sneden, R. G. Gratton and D. A. Crocker, Astron. & Astrophys. 246 (1991), 354.

[4] T. V. Mishenina, V. V. Kovtyukh, C. Soubiran, C. Travaglio and M. Busso, Astron. & Astrophys. 396 (2002), 189, DOI: 10.1051/0004-6361:20021399

[5] I. U. Roederer, C. Sneden, I. B. Thompson, G. W. Preston and S. A. Shectman, Astrophys. J. 711 (2010), 573, DOI: 10.1088/0004-637X/711/2/573

[6] I. U. Roederer, A. F. Marino and C. Sneden. Astrophys. J. 742 (2011), 37, DOI: 10.1088/0004-637X/742/1/37

[7] A. Frebel, J. D. Simon and E. N. Kirby, Astrophys. J. 786 (2014), 74, DOI: 10.1088/0004-637X/786/1/74

[8] D. Pequignot and J. P. Baluteau, Astron. & Astrophys. 283 (2) (1994), 593–625.

[9] B. Sharpee, Y. Zhang, R. Williams, E. Pellegrini, K. Cavagnolo, J. A. Baldwin, M. Phillips and X.-W. Liu, Astrophys. J. 659 (2007) (preprint astro-ph/0612101), DOI: 10.1086/515165

[10] N. C. Sterling and H. L. Dinerstein, Astrophys. J. Suppl. 174 (2008), DOI: 10.1086/520845

[11] M. Otsuka and A. Tajitsu, Astrophys. J. 778 (2013) (preprint 1310.1151), DOI: 10.1088/0004-637X/778/2/146

[12] J. García-Rojas, S. Madonna, V. Luridiana, N. C. Sterling, C. Morisset, G. Delgado-Inglada and L. Toribio San Cipriano, Mon. Not. Roy. Astro. Soc. 452 (2015) (preprint 1506.07079), DOI: 10.1093/mnras/stv1415
[13] N. C. Sterling, H. L. Dinerstein, K. F. Kaplan and M. A. Bautista, Astrophys. J. 819 (2016), (preprint 1602.03188), DOI: 10.3847/2041-8205/819/1/L9

[14] K. Langanke and M. Wiescher, Rep. Prog. Phys. 64 (2001) 1657, DOI: 10.1088/0034-4885/64/12/202

[15] K. B. Kwitter, R. H. Méndez, M. Peña, L. Stanghellini, R. L. M. Corradi, O. De Marco, X. Fang, R. B. C. Henry, A. I. Karakas, X.-W. Liu, J. A. López, A. Manchado and Q. A. Parker, Rev. Mex. Astron. Astro. 50 (2014), 203 (preprint 1403.2246), https://arxiv.org/abs/1403.2246v1

[16] V. Luridiana, C. Morisset and R. A. Shaw, Astron. & Astrophys. 573 (2015), A42, DOI: 10.1051/0004-6361/201323152

[17] B. M. McLaughlin and J. F. Babb, J. Phys. B: At. Mol. Opt. Phys. 52 (2019), 125201, DOI: 10.1088/1361-6455/ab1e99

[18] D. A. Macaluso, K. Bogolub, A. Johnson, A. Aguilar, A. L. D. Kilcoyne, R. C. Bilodeau, M. Bautista, A. B. Kerlin and N. C. Sterling, J. Phys. B: At. Mol. Opt. Phys. 49 (2016), 235002, DOI: 10.1088/0953-4075/49/23/235002

[19] D. A. Macaluso, K. Bogolub, A. Johnson, A. Aguilar, A. L. D. Kilcoyne, R. C. Bilodeau, M. Bautista, A. B. Kerlin and N. C. Sterling, J. Phys. B: At. Mol. Opt. Phys. 50 (2017), 119501, DOI: 10.1088/1361-6455/aa6d1b

[20] I. Sakho, At. Data. Nuc. Data Tables 117-118 (2017), 425, DOI: 10.1016/j.adt.2016.12.001

[21] I. Sakho, J. Electron Spectro & Related Phenomena 222 (2018), 40, DOI: 10.1016/j.elspec.2017.10.001.

[22] I. Sakho, The Screening Constant by Unit Nuclear Charge Method, Description & Application to the Photoionization of Atomic Systems, ISTE Science Publishing Ltd., London, and John Wiley & Sons, Inc. USA (2018), ISBN: 978-1-119-47694-8.

[23] M. D. Ba, A. Diallo, J. K. Badiane, M. T. Gning, M. Sow and I. Sakho, Rad. Phys. Chem. 153 (2018), 111, DOI: 10.1016/j.radphyschem.2018.09.010

[24] J. K. Badiane, A. Diallo, M. D. Ba, M. T. Gning, M. Sow and I. Sakho, Rad. Phys. Chem. 158 (2019), 17, DOI: 10.1016/j.radphyschem.2019.01.008