Entanglement entropy growth in stochastic conformal field theory and the KPZ class

DENIS BERNARD and PIERRE LE DOUSSAL

Laboratoire de Physique de l'Ecole Normale Supérieure, ENS, Université PSL, CNRS, Sorbonne Université, Université de Paris - 75005 Paris, France

received 22 May 2020; accepted in final form 30 June 2020
published online 5 August 2020

PACS 05.30.-d – Quantum statistical mechanics
PACS 05.10.-a – Computational methods in statistical physics and nonlinear dynamics
PACS 05.40.-a – Fluctuation phenomena, random processes, noise, and Brownian motion

Abstract – We introduce a model of effective conformal quantum field theory in dimension \(d = 1 + 1\) coupled to stochastic noise, where Kardar-Parisi-Zhang (KPZ) class fluctuations can be observed. The analysis of the quantum dynamics of the scaling operators reduces to the study of random trajectories in a random environment, modeled by Brownian vector fields. We use recent results on random walks in random environments to calculate the time-dependent entanglement entropy of a subsystem interval, starting from a factorized state. We find that the fluctuations of the entropy in the large deviation regime are governed by the universal Tracy-Widom distribution. This enlarges the KPZ class, previously observed in random circuit models, to a family of interacting many-body quantum systems.

Copyright © 2020 EPLA

The growth of entanglement entropy in some quantum chaotic systems modeled by random unitary circuits has been found [1,2] to exhibit features of the Kardar-Parisi-Zhang (KPZ) universality class. The KPZ class, which contains the continuum KPZ equation [3] modeling the classical stochastic growth of an interface, or equivalently the free energy of continuum directed paths in a random potential [4], is characterized in \(d = 1\) by super-diffusive dynamics \(x \sim t^{2/3}\), and by universal Tracy-Widom distributions [8] related to random matrix theory [9–14]. It contains a number of solvable discrete classical stochastic models, of, e.g., 1d particle transport, such as the asymmetric simple exclusion process (ASEP) [15], and directed polymers in random \(d = 1 + 1\) media [6]. Various KPZ class behaviors were also recently claimed in other quantum models, chaotic [16] or integrable [17–19], as well as in classical models, integrable [20–22] or not [23,24], with no obvious connections.

To which extent KPZ-like behaviors are universal for information spreading in noisy or chaotic many-body quantum systems is still unclear. In fact, apart from the random quantum circuits (in the limit of large on site dimension), most of the evidence is numerical, and a \textit{bona fide} derivation of KPZ behavior has been elusive. However, the model we are going to present yields extra supports for the robustness of such KPZ behaviors.

Recently, the class of quantum circuits has been extended to include solvable models [25,26] which show spectral form factor growths typical of chaotic systems but with operator spreading concentrated along the light cone. This points towards a possible connection between simple extended chaotic systems and, possibly noisy, conformal field theory (CFT).

In the present paper, we show that KPZ class behaviors emerge in yet another class of 1d stochastic quantum models. Specifically it manifests itself in the large deviations of certain time-dependent correlations, among which the entanglement entropy, and hence in particular in the entanglement entropy growth. Our results rely on using an exact representation of the quantum correlations in terms of classical diffusions in time-dependent random fields and the recently discovered connection [27–32] between such diffusion problems and the KPZ equation.

The models we consider are certain stochastic perturbations of CFT, which can be viewed as CFT in random geometry. They code for stochastic quantum dynamics as random quantum circuits do. They are continuous space analogs of models of spin chain submitted to stochastic baths, which were shown to provide quantum extensions.
of the simple symmetric exclusion process (SSEP) [33]. KPZ class behaviors were indeed claimed numerically in such stochastic spin chains [34]. Quantum extensions of the ASEPy are known to be obtainable [35] by promoting the noise to quantum noise [36]. These stochastic models code for fermions hopping along the chain with noisy amplitudes. Our stochastic CFT is thus defined by coupling an external noise to the energy-momentum tensor components, which generate left/right chiral moves within the CFT. As a consequence, operator spreading is predominantly concentrated close to the light cone, except for rare, but important, events.

Note that slightly different versions of stochastic CFTs were considered in [37] to model elastic scattering in a classically fluctuating environments1: a crossover from ballistic motion to diffusion and localization was found (see also [38] for a static version).

We consider a quantum CFT in dimension $d = 1 + 1$ viewed as the low energy effective field theory of a gapless many-body system. The low energy states span a Hilbert space. It is equipped with its two-component energy-momentum tensor $T(x)$ and $\bar{T}(x)$, $x \in \mathbb{R}$ such that their sum $h_0(x) = v(T(x) + \bar{T}(x))$ is the energy density operator and the difference $p(x) = T(x) - \bar{T}(x)$ is the momentum density operator. In (unperturbed and noiseless) CFT, the dynamics is generated by a Hamiltonian

$$H_0 = \int dx h_0(x)$$

and the unitary evolution on the Hilbert space is described by the operator $U_0 = e^{-iH_0t}$. We now couple this system to space-time-dependent noise and define the flow between time $t$ and $t + dt$ of the perturbed unitary evolution as $U_{t+dt} := e^{-iH_0dt}$ with Hamiltonian increment

$$dH_t := H_0dt + \int dx (dW^+_t(x)T(x) + dW^-_t(x)\bar{T}(x)),$$

where $W^\pm_t(x)$ are two space-dependent stochastic processes. We choose their increments to be of the form

$$dW^\pm_t(x) := \xi^\pm_t(x)dt + \sqrt{D_0}dB^\pm_t,$$

where $B^\pm_t$ are two independent standard (spatially homogeneous) Brownian motions, $D_0$ the bare diffusion coefficient. The two random fields $\xi^\pm_t(x)$ are centered Gaussian space time white noise with covariance

$$\mathbb{E} [\xi^\pm_t(x)\xi^\mp_s(y)] = \rho_\delta \delta_\delta \delta_a \delta_a (x-y)\delta(t-s),$$

where $\delta$ is a short distance cutoff, and $\delta_\delta(x)$ is a mollifier of the delta function. Both $\rho_\delta$ and the Brownian with $D_0 > 0$ regularize the trajectories, as in turbulent transport [39–41]. Here we are interested in performing averages over $B^\pm$ and studying sample to sample fluctuations with respect to the realizations of the random fields $\xi^\pm_t(x)$.

We now show how to solve the equations of motion to make the link with trajectories in random environments. Since $T(x)$ and $\bar{T}(x)$ are generators of diffeomorphism on $\mathbb{R}$, the coupling (1) ensures that the dynamics of the chiral operators is linked to the trajectories associated to the random vector fields $W^\pm_t(x)$. In CFT it is sufficient to look at the primary operators with well defined scaling dimensions. These operators appear in two classes depending on their commutation relation with $T$ and $\bar{T}$. In pure CFT, they are connected along the light cone with velocities $\pm v$ (right/left movers). As shown in [42], the evolution of any chiral operator $\varphi(x)$ of dimension $\Delta$ is solved as

$$\varphi(x,t) := U^\pm_t \varphi(x)U_t = [X^+_t(X^+_t(x))]^\Delta \varphi(X^+_t(x)),$$

$$\tilde{\varphi}(x,t) := U^\pm_t \tilde{\varphi}(x)U_t = [\bar{X}^-_t(\bar{X}^-_t(x))]^\Delta \varphi(\bar{X}^-_t(x)),$$

where the prime denotes derivative with respect to $x$. Here $X^\pm_t$ are the processes specified by

$$X^\pm_{t+dt}(x \pm vdt \pm dW^\pm_t(x)) = X^\pm_t(x).$$

Equations (5) have a natural interpretation in terms of random trajectories. Consider the following problem of diffusion of a particle in a random field, whose position $x^+_u$ (resp. $x^-_u$) at time $u \in [0,t]$ obeys the Langevin equation

$$\frac{dx^\pm_u}{du} = \pm v + \xi^\pm_u(x^\pm_u) + \sqrt{D_0} \frac{dB^\pm_u}{du}.$$  

From a geometrical perspective, these two equations are those of the null geodesics in the random metric associated to these stochastic fields (see eq. (42) in [42]). It is clear that $X^\pm_t(x)$ is then the position at the initial time $t_0 = 0$ of the particle which will be at position $x^\pm$ at time $t$, and is associated to a trajectory $x^\pm_{u=t}$ such that $x^\pm_{u=t} = X^\pm_t(x)$ and $x^\pm_{u=t=0} = x$. See fig. 1.

The above result allows to express time-dependent correlation functions of primary fields (chiral or anti-chiral) at points $x_i$ and time $t$, in terms of the initial correlations but at positions transported by the backward flow, i.e., at positions $X^\pm_i(x_i)$. Statistical properties of these quantum correlations reduce to those of the random trajectories $x^\pm_{u=t}$. We now use this property to calculate the Renyi entanglement entropy for an interval $[x_0, x]$ on the real axis. It is defined as

$$S_n = \frac{1}{\ln n} \log \text{Tr} \rho_{x_0,x}(t)^n$$

where $\rho_{x_0,x}(t)$ is the reduced density matrix at time $t$, obtained by tracing out the degrees of freedom in the domain complementary to the interval $[x_0, x]$. Within field theory, this Renyi entropy can be represented by a path integral on a $n$-sheet branched covering of the spacetime plane [43]. It leads to express $S_n$ in terms of quantum correlations as

$$\text{Tr} \rho_{x_0,x}(t)^n = e^{2A_n} \langle \Phi_n(x_0,t)\Phi_n(x,t)|\Phi_0 \rangle,$$

where $\Phi_n(x,t)$ and $\tilde{\Phi}_n(x,t)$ are the time evolved of the so-called conjugated twist operators, $\Phi_n(x)$ and $\tilde{\Phi}_n(x)$.

1But with more singular environments than the ones we consider in the present paper.

2A more local version of the model amounts to replace the Brownian noise in (17) by space-dependent noise, $B^\pm_t \rightarrow \xi^\pm_t(x)$, independent of $\xi^\pm_t(x)$. Interestingly, the results are unchanged as long as one restricts to observables involving at most one trajectory $X^\pm_t(x)$ at one point, such as $S_n$ (in the limit $x_0 \rightarrow -\infty$), or (24). They deviate however for correlations such as (26).
which implement the permutations of the sheets of the covering space at the branching points. Here $|\Psi_0\rangle$ is the initial state at $t_0 = 0$ of the full domain (here the real axis) and $a_0$ is an UV cutoff length. Decomposing the twist operators in chiral and anti-chiral components as $\Phi_n(x) = \phi_n(x)\bar{\phi}_n(x)$, and using the formula (3) for the primary fields $\phi_n$ and $\bar{\phi}_n$, one obtains

$$e^{(1-n)S_n} = J_t(x_0, x)^{\Delta_n} G_t(x_0, x),$$

where $J_t(x_0, x) = X_t^{-1}(x_0)X_t^+(x)X_t^{-1}(x)X_t^+(x)$ is the Jacobian associated to the stochastic flows (6), and $G_t(x_0, x)$ is equal to the following quantum correlation:

$$\langle \Psi_0 | \phi_n(X_t^{-1}(x_0))\bar{\phi}_n(X_t^+(x_0))\phi_n(X_t^{-1}(x))\bar{\phi}_n(X_t^+(x)) | \Psi_0 \rangle.$$  

The scaling dimension of the twist operators is $\Delta_n = \frac{\Delta}{2} (n-\frac{1}{2})$, with $c$ the CFT central charge.

We must now specify the initial state $|\Psi_0\rangle$. As in random circuit models and in quantum quench problems, we choose a gapped initial state with finite and small coherence length $\nu_0$. Within CFT we use the Calabrese-Cardy representation of such a state as [44,45]

$$|\Psi_0\rangle \propto e^{-\tau_0 H_0} |B\rangle,$$

where $|B\rangle$ is a (unnormalized) conformally invariant boundary state. This representation comes with rules for calculation of expectation values which involve analytic continuation of CFT correlation functions in a strip of width $2\nu_0$ conformally mapped to the upper half plane. Using these rules (see [42]) one obtains

$$G_t(x_0, x) = \left( \frac{\pi a_0}{2\nu_0} \right)^{4\Delta_n} \left[ \frac{(z_0^+ - z^+)^2}{(z_0^+ - z^-)^2} \right]^{\Delta_n} K(\eta),$$

where $z^\pm = \pm ie^{\pm \nu_0} X^\pm_t(x)$ and $z_0^\pm = \pm ie^{\pm \nu_0} X^\pm_0(x_0)$. The variable $\eta$ is the cross-ratio $\eta = \frac{(z_0^+ - z^-)(zt - z^+)}{(zt - z^-)(z_0^+ - z^+)}$ and $K(\eta)$ is called a conformal block. The above expression is in general difficult to evaluate but it simplifies in the limit of a large interval $x_0 \to -\infty$ with fixed endpoint $x$, because one can use the operator product expansion (OPE) in that limit. Indeed, we expect that $X_t(x_0) \to -\infty$ in that limit a.s. for any fixed $t$. Thus $z_0^\pm$ tend to zero along the imaginary axis, hence $\eta \ll 1$. One knows, from boundary OPE in CFT, the asymptotics of $K(\eta) \sim A^2 \eta^{-(2\Delta_n - \Delta_0)}$ as $\eta$ goes to zero, where $\Delta_0$ is the scaling dimension of the boundary operator produced by the OPE and $A_0$ is a universal amplitude. From refs. [44,45] one has $\Delta_0 = 0$. In this limit (11) becomes, to leading order,

$$G_t(x_0, x) \simeq \frac{A^2}{\nu_0} \left( \frac{\pi a_0}{2\nu_0} \right)^{4\Delta_n} \frac{\cosh(\pi(\nu_0/4\nu_0) - x_0/z_0^+)}{\cosh(\pi(\nu_0/4\nu_0) - x/z_0^+)}^{2\Delta_n}.$$  

In the limit where the coherence length $\nu_0$ of the initial state $|\Psi_0\rangle$ is small, the non-vanishing of this correlation function conditions the two trajectories to start at nearby positions $X_t^\pm(x) \approx X_t^\pm(x)$. Estimating the probability of this event will thus be of importance below.

We are interested in various averages of the entropy $S_t$ over the Brownian, for a fixed random field configuration $\xi_t^x$ (i.e., fixed sample). Convenient averages have the form $-\frac{1}{q} \log \langle e^{-q S_t} \rangle$ as a function of the parameter $q$ varying from annealed average for $q = 1$ to the quenched average $\langle S_t \rangle_B$ for $q = 0$. Let us take the power $q/(n - 1)$ of (8), and average over the Brownian. Neglecting the terms containing $x_0$ in the limit $x_0 \to -\infty$ (see footnote 4) and setting for now the Jacobian factor to unity, we obtain

$$\langle e^{-q S_t} \rangle_B \simeq e^{-q s_0} \int dy^+dy^- P^+_{t_0}(x, y^+)P^+_{t_0}(x, y^-),$$

where $\delta_\eta = \frac{\Delta_n}{2} = \frac{\Delta}{2} (n-\frac{1}{2})$ and $s_0$ is the (non-universal) initial value of the entropy. We have introduced the following probability distribution function (PDF):

$$P^+_{t_0}(x, y) = \langle \delta(X^+_{t_0}(x) - y) \rangle_B.$$
Fokker-Planck equation as \( t_\circ \) is decreased,
\[
-\partial_t P_{t_\circ}(x,y) = \left[ \frac{D}{2} \partial_y^2 \pm \partial_y (v + \xi_{t_\circ}(y)) \right] P_{t_\circ}(x,y),
\]
(15)
in the time-reversed field \( \mp (v + \xi_{t_\circ}) \), with the condition \( P_{t_\circ}(x,y) = \delta(x-y) \), and where \( D \) is the coarse-grained diffusion coefficient (see [42] for details). Note that, as shown in [42], eqs. (13) and (15) can be extended to include the contributions of the Jacobian factor in eq. (8), however the latter are irrelevant at large scale and only renormalize a few amplitudes as detailed in [42].

The problem of diffusion in a time-dependent random field was recently found to be related to the KPZ class. This was shown for discrete random walks models in a time-dependent random environment, through exact solutions [27,28] and in their weak disorder/continuum limit [29]. The continuum model was studied in [30] using physics arguments, and rigorously recently [31] (although the space-time white noise limit for \( \xi_{t_\circ} \) remains mathematically challenging). The main idea is as follows. For \( \xi_{t_\circ} \) white noise in time, the disorder average
\[
E[P_{t_\circ}(x,y)] = \frac{1}{\sqrt{2\pi Dt}} e^{-\frac{(x-v\theta_{t_\circ})^2}{2Dt}}
\]
is identical to the pure biased diffusion, i.e., \( \xi_{t_\circ} = 0 \), and gives the global shape of the PDF, which is centered around \( y = x \mp v(t-t_\circ) \). The KPZ physics arises away from this most probable direction, i.e., for \( y-x \sim (\mp v + \theta_{t_\circ}) t \) at large \( t \) (for \( t_\circ = 0 \) fixed) with \( \theta_{t_\circ} \neq 0 \). For \( y-x = o(t) \), i.e., \( \theta_{t_\circ} = \pm v \), the average profile at large time varies in space as \( e^{\mp (x-v\theta_{t_\circ}) t} \). Defining the fluctuation field around the average profile as \( Z_{t_\circ}(x,y) := P_{t_\circ}(x,y)e^{\mp (x-v\theta_{t_\circ}) t + \frac{v^2}{2D}(t-t_\circ)} \), one finds that \( Z_{t_\circ} \) satisfies the stochastic heat equation (SHE) as \( t_\circ \) is decreased,
\[
-\partial_t Z_{t_\circ} = \left[ \frac{D}{2\theta_{t_\circ}^2} \partial_y^2 - \frac{v}{\theta_{t_\circ}} \partial_y \right] Z_{t_\circ} + \ldots ,
\]
where the additional terms \( \ldots \partial_y (a(Z)) \) are connected to be irrelevant at large time in the region \( y-x = o(t) \) (since it contains higher gradients). This property is supported by recent results [31,32]. Here \( Z \) can be seen as the partition sum of a directed Polymer in the time-dependent random potential \( \frac{v}{\theta_{t_\circ}} \xi_{t_\circ}(y) \) and \( h_{t_\circ} = \log Z_{t_\circ} \) is the corresponding KPZ height field [42]. From known results on the KPZ equation, one expects, at large time \( t \), universal height fluctuations w.r.t. \( \xi_{t_\circ} \), scaling as \( \theta_{t_\circ} \sim t^{1/3} \), with space-time scaling \( x \sim t^{2/3} \). If the region \( y-x = o(t) \) dominates the integral we can thus write
\[
\langle e^{-qS_{t_\circ}} \rangle_B \sim e^{t \frac{v^2}{2D}(t-t_\circ)} \int dy^+ dy^- Z_{t_\circ}(x+y^+,y^-)Z_{t_\circ}(x-y^+,y^-) \cos \left( \frac{\pi (y^+-y^-)}{4v\theta_{t_\circ}} \right) \delta_{q\phi_{t_\circ}}.
\]
(17)

At large time the typical KPZ spatial scale is \( t^{2/3} \gg v t_\circ \), hence for fixed \( q > 0 \) one can approximate in both eq. (17) and (13) the factor \( 1/\cos \left( \frac{\pi (y^+-y^-)}{4v\theta_{t_\circ}} \right) \delta_{q\phi_{t_\circ}} \sim v t_\circ \delta(y^+ - y^-) \), which constrains the two trajectories \( x_{t_\circ}^\pm \) in (6) to be in an atypical configuration with near identical starting and ending points. Since \( \xi_{t_\circ} \) is uncorrelated fields, this allows to unfold the configuration by time-reversing one of the two trajectories, see fig. 2. The problem becomes equivalent in law to computing the probability density of return \( P_{t_\circ,t}(x,x) \) to the starting point \( x \), after time lapse \( 2t \), in a random vector field with constant bias \( v \). Hence,
\[
\langle e^{-q(y_{t_\circ}-y_s)} \rangle_B \sim v t_\circ P_{2t,0}(x,x) = v t_\circ Z_{2t,0}(x,x)e^{-\frac{v^2}{2D} t_\circ},
\]
(18)

similar to the point to point directed polymer partition function. From known results and universality in the KPZ class [46], we thus obtain that at large time
\[
\log \langle e^{-q(y_{t_\circ}-y_s)} \rangle_B \sim c_1 t + c_2 t^{1/3} A(\delta,\hat{x}),
\]
(19)

where \( c_1, c_2, c_3 \) are constants independent of \( q \) and \( n \), and \( A(\delta,\hat{x}) \) denotes the so-called Airy sheet process [47–49].

We recall that, for fixed \( \hat{y} \), it is equal to \( A(\hat{x},\hat{y}) \approx A_2(\hat{x}-\hat{y}) = \hat{y} \hat{v} \) where \( A_2(\hat{x}) \) is the Airy process [50] (describing, upon rescaling, the rightmost particle of the Dyson Brownian motion [51]). Its one point PDF for \( \chi = A(\hat{x},\hat{y}) \equiv_{intlaw} A_2(0) \) is given by the Gaussian unitary ensemble Tracy-Widom distribution (GUE-TW) [52]. Hence, we conclude that, at fixed \( q \), \( \log \langle e^{-qS_{t_\circ}} \rangle_B \) is distributed according to GUE-TW. The \( x \) dependence however of (19), i.e., with \( S_{t_\circ} \equiv S_n(x,t) \), is non-trivial and related to the Airy sheet.

Let us discuss now the PDF of \( S_n \). Taking the logarithm in (8) and (12) we have
\[
S_n = s_n + 2\delta_n \log \cosh \left( \frac{\pi (X^+_n(x) - X^-_n(x))}{4v t_\circ} \right)
\]
(20)

\[
\approx s_n + 2\delta_n \frac{\pi(X^+_n(x) - X^-_n(x))}{4v t_\circ}.
\]
(21)

For any given \( x \), \( X^+_n(x) - X^-_n(x) \) has the same statistics as a diffusion in a biased random field for time duration.
2t. Indeed, since $\xi^+$ and $\xi^-$ are independent vector fields, we can reflect one of them around the space slice at time $t$ to define a Gaussian white noise vector field on a doubled time interval. As a consequence, $[X^s_i(x) - X^s_j(x)] \equiv [\tilde{x}_2t - \tilde{x}_0 ]$, where $\tilde{x}_s$ are the corresponding trajectories (see [42] for detail).

Let us now focus on the typical behavior of $S_n$. We can use arguments and exact results from the diffusion problem. The typical value of $\tilde{x}_2t$ is $2vt$ with variance $2tD$, hence the typical value of $S_n$ is $S^{typ}_n = \frac{\pi \delta_0}{\tau_0} + \frac{\theta}{\tau} t$ with a variance $\mathbb{E}[S_n^2] = D_t t$ with $D_s = \frac{(4\pi \tau)^{-1/2}}{2s}$ (note that the effect of the Jacobian is to renormalize $D_s$ [42]). As a function of the end point position, $x$, when varied over regions $x = O(t^{1/2})$, $S_n(x,t)$ exhibits subleading sample to sample fluctuations $O(t^{1/2})$ described by the Edwards-Wilkinson equation (i.e., the KPZ equation without the non-linear term) [42].

For atypical fluctuations, the large deviations $S_n$ can be obtained from the large deviations of $\tilde{x}_2t - \tilde{x}_0$. These have been studied in the context of diffusion in random environments [27,30-32] and one obtains at large time\(^5\) (see [42]) for $\theta \geq -v$

$$\log \text{Prob}(S_n > \frac{\pi \delta_0}{\tau_0} + (\theta + \frac{\theta}{\tau} t) ) \simeq \frac{\theta}{\tau}
$$

Here $t^* = \kappa^2 / D^3$ and $\theta^* = D^2 / \kappa$ are characteristic scales of the diffusion in a Brownian field. Note that at small $\theta$ one has $J(\theta) = \frac{\theta}{\tau} t^* + O(\theta^4)$ and $G(\theta) \simeq \frac{1}{2\tau_0} \theta^4 / (1 + O(\theta^2))$ from the universal KPZ regime [42].

One now asks how the result (22) for the probability matches the result (19) for the exponential moments. This will allow to specify the domain of validity of (19) as a function of $q$ for a small but finite $\tau_0$. It shows that there is a phase transition at a critical value $q_c$, which corresponds to a change in the geometry of the contributing atypical trajectories. At large time we can evaluate the integral on $\theta$ representing the exponential moments using a saddle point method, which gives the estimate

$$\log \mathbb{E}[e^{-q(S_n - s_0)}] \simeq - \min_\theta \left[ \frac{\pi \delta_0}{2t\tau_0} + \frac{\theta}{\tau} t + \frac{2t}{\tau} \left( \frac{\theta}{\tau} t^* \frac{\theta}{\tau} t^{1/3} G(\theta) \right) \frac{\theta}{\tau} t^{1/3} \right].$$

There is a transition as a function of $q$ in this minimisation problem. There exists a $q_c > 0$ such that for $q > q_c$ the minimum is frozen at $\theta = -\frac{\theta}{\tau} t$ independent of $q$, in which case formula (19) holds, with $c_1 = (2/t^*)^2 (1 - v)$ and $c_2 = (2/t^*)^{1/3} \mathbb{E}[\varphi(\theta)]$. For $q < q_c$, the maximum is at $\theta = \theta_c(q) > -v$. Let us estimate $q_c$ in the case where $\beta_c = \frac{2}{\kappa^2 D^3}$.

\(^5\)Note that the probability for the equality for $S_n$ is expected to follow the same large deviation law.

As detailed in [42] the analysis of the trajectories allows to exhibit KPZ-type large deviations.

\(^6\)In (23) to obtain $\theta_c$ we used $|\theta + v| = \theta + v$. There is another saddle point for the choice $|\theta + v| = -(\theta + v)$, $\theta_c = \frac{1}{\theta} v$ for $q < -q_c$. However its contribution is subdominant.
In conclusion, we have introduced a stochastic version of $d = 1 + 1$ CFT modeling random unitary dynamics in interacting many-body systems. We have analysed the statistical behaviors (the typical and atypical behaviors) of various quantum correlations, including the entanglement entropy of a subsystem. Geometrically, the rare events we analysed correspond to null geodesics converging to nearby points, as caustics do. Within stochastic CFT, these behaviors are universal in the sense that they only rely on the conformal symmetry acting on the physical Hilbert space of the model. We have been able to decipher them by mapping their analysis to that of random trajectories in random fields. For systems initially prepared in short-range correlated states, we found that the large deviations of the fluctuations of the entanglement entropy are controlled by the KPZ class universality. The mechanism for the emergence of KPZ behavior appears to be different from the one unveiled in the studies of random quantum circuits. Understanding the extent by which such KPZ-like behaviors are universal for information or operator spreading in noisy or chaotic many-body quantum systems remains an important question.

***

We especially thank G. Barraquand for very helpful discussions. We are also grateful to B. Doyon and A. Nahum, for enlightening discussions. PLD acknowledges support from ANR under the grant ANR-17-CE30-0027-01 RaMaTraF.

REFERENCES

[1] Nahum A., Ruhman J., Vijay S. and Haah J., Phys. Rev. X, 7 (2017) 031016.
[2] Zhou T. and Nahum A., Phys. Rev. B, 99 (2019) 174205 (arXiv:1804.09737).
[3] Kardar M., Parisi G. and Zhang Y. C., Phys. Rev. Lett., 56 (1986) 889.
[4] Kardar M., Nucl. Phys. B, 290 (1987) 582.
[5] Huse D. A., Henley C. L. and Fisher D. S., Phys. Rev. Lett., 55 (1985) 2924.
[6] Johansson K., Commun. Math. Phys., 209 (2000) 437 (arXiv:math/9903134); Transversal fluctuations for increasing subquantum on the plane, arXiv:math/9910146.
[7] Halpin-Healy T. and Takeuchi K. A., J. Stat. Phys., 160 (2015) 794 (arXiv:1505.01910).
[8] Tracy C. A. and Widom H., Commun. Math. Phys., 159 (1994) 151 (arXiv:hep-th/9211141).
[9] Piraforer M. and SpoHN H., Phys. Rev. Lett., 84 (2000) 4882.
[10] Baik J. and Rains E. M., J. Stat. Phys., 100 (2000) 523.
[11] Kriecherbauer T. and Krug J., J. Phys. A: Math. Theor., 43 (2010) 403001.
[12] Calabrese P. and Le Doussal P., Phys. Rev. Lett., 106 (2011) 250603.
[13] Corwin I., Macdonald processes, quantum integrable systems and the Kardar-Parisi-Zhang universality class, in Proceedings of the ICM, arXiv:1403.6877.
[14] Quastel J. and Spohn H., J. Stat. Phys., 160 (2015) 965 (arXiv:1503.06185).
[15] Derrida B., Phys. Rep., 301 (1998) 65.
[16] Nahum A., Vijay S. and Haah J., Phys. Rev. X, 8 (2018) 021014 (arXiv:1705.08975).
[17] Litobitina M., Znidaric M. and Prosen T., Phys. Rev. Lett., 122 (2019) 210602 (arXiv:1903.01329).
[18] De Nardis J., Medenjak M., Karrasch C. and Ilievski E., Anomalous spin diffusion in one-dimensional antiferromagnets, arXiv:1903.07959.
[19] Gopalkrishnan S., Vasseur R. and Ware B., Proc. Natl. Acad. Sci. U.S.A., 116 (2019) 16250 (arXiv:1904.01039).
[20] Krainzik K. and Prosen T., Kardar-Parisi-Zhang physics in integrable rotationally symmetric dynamics on discrete space-time lattice, arXiv:1909.03799.
[21] Das A., Kulkarni M., Spohn H. and Dhar A., Kardar-Parisi-Zhang scaling for the Faddeev-Takhtajan classical integrable spin chain, arXiv:1906.02760.
[22] Das A., Damle K., Dhar A., Huse D. A., Kulkarni M., Mendol C. B. and Spohn H., Nonlinear fluctuating hydrodynamics for the classical XXZ spin chain, arXiv:1901.00024.
[23] Spohn H., The 1 + 1 dimensional Kardar-Parisi-Zhang equation: More surprises, arXiv:1909.09403.
[24] Roy D. and Pandit R., The one-dimensional Kardar-Parisi-Zhang and Kuramoto-Sivashinsky universality class: Limit distributions, arXiv:1908.06007 (2019).
[25] Bertini B., Kos P. and Prosen T., Phys. Rev. X, 9 (2019) 021033 (arXiv:1812.05090).
[26] Bertini B., Kos P. and Prosen T., Phys. Rev. Lett., 121 (2018) 264101 (arXiv:1805.00031).
[27] Barraquand G. and Corwin I., Probab. Theory Relat. Fields, 167 (2017) 1057 (arXiv:1503.04117).
[28] Thiery T. and Le Doussal P., J. Phys. A, 50 (2016) 4 (arXiv:1605.07538).
[29] Corwin I. and Gu Y., Kardar-Parisi-Zhang equation and large deviations for random walks in weak random environments, arXiv:1606.07332.
[30] Le Doussal P. and Thiery T., Phys. Rev. E, 96 (2017) 010102 (arXiv:1705.05159).
[31] Barraquand G. and Rychnovsky M., Large deviations for sticky brownian motions, arXiv:1905.10280.
[32] Barraquand G. and Le Doussal P., J. Phys. A: Math. Theor., 53 (2020) 215002 (arXiv:1912.11085).
[33] Bernard D. and Jin T., Phys. Rev. Lett., 123 (2019) 080601 (arXiv:1904.01406).
[34] Knap M., Phys. Rev. B, 98 (2018) 184416 (arXiv:1806.04666).
[35] Jin T., Krajenbrink A. and Bernard D., Phys. Rev. Lett., 125 (2020) 040603 (arXiv:2001.04278).
[36] PartHASARATHY K. R., An Introduction to Quantum Stochastic Calculus, Vol. 85 (Birkhäuser) 2012.
[37] Bernard D. and Doyon B., Phys. Rev. Lett., 119 (2017) 110201 (arXiv:1612.05956).
[38] Langmann E. and Moosavi P., Phys. Rev. Lett., 122 (2019) 020201 (arXiv:1807.10239).
[39] Gawedzki K. and Horvai P., J. Stat. Phys., 116 (2004) 1247.
[40] Bernard D., Gawedzki K. and Kupiainen A., J. Stat. Phys., 90 (1998) 519 (arXiv:cond-mat/9706035).
[41] Le Jan Y. and Raimond O., Ann. Probab., 30 (2002) 826; 32 (2004) 1247.
[42] See the supplemental material in Bernard D. and Le Doussal P., arXiv:1912.08458.
[43] Cardy J. L., Castro-Alvaredo O. A. and Doyon B., J. Stat. Phys., 130 (2008) 129 (arXiv:0706.3384).
[44] Calabrese P. and Cardy J., Phys. Rev. Lett., 96 (2006) 136801 (arXiv:cond-mat/0601225).
[45] Calabrese P. and Cardy J., J. Stat. Mech., 2016 (2016) 064003 (arXiv:1603.02889).
[46] Matetski K., Quastel J. and Remenik D., The KPZ fixed point, arXiv:1701.00018.
[47] Corwin I., Quastel J. and Remenik D., J. Stat. Phys., 160 (2012) 815 (arXiv:1103.3422).
[48] Dauvergne D., Ortmann J. and Virag B., The directed landscape, arXiv:1812.00099.
[49] Borodin A., Gorin V. and Wheeler M., Shift-invariance for vertex models and polymers, arXiv:1912.02957.
[50] Prahofer M. and Spohn H., J. Stat. Phys., 108 (2002) 1071.
[51] Prolhac S. and Spohn H., J. Stat. Mech., 2011 (2011) P03020 (arXiv:1101.4622).
[52] Tracy C. A. and Widom H., Commun. Math. Phys., 159 (1994) 151.
[53] Calabrese P. and Cardy J., J. Stat. Mech., 2007 (2007) P06008 (arXiv:0704.1880).
[54] Calabrese P. and Cardy J., J. Stat. Mech., 2007 (2007) P10004 (arXiv:0708.3750).