Contour Following Accuracy Improvement—A Dynamic Fast Nonsingular Terminal Sliding Mode Control Approach

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ABSTRACT Servomechanisms and motion stages often encounter many mechanical transmission problems such as friction, backlash, and structural resonance, as well as other factors such as system nonlinearity, servo lags, and unknown disturbances. In contour following applications, these problems are the main causes of deterioration in contour following accuracy. As a result, the issue of dealing with the above problems so as to reduce tracking error and contour error is crucial. The Dynamic Fast Nonsingular Terminal Sliding Mode Control (DFNTSMC) scheme proposed in this paper combines the advantages of Fast Nonsingular Terminal Sliding Mode Control (FNTSMC) and Dynamic PID Sliding Mode Control (DSMC) while avoiding their drawbacks. The proposed DFNTSMC has attractive features such as improvement of contour following accuracy, chattering effect suppression, enhancement of robustness, and finite time convergence. The convergence of the proposed DFNTSMC is proved based on Barbalat’s lemma. Several contour following experiments are performed to assess the performance of the proposed DFNTSMC. Experimental results suggest that the proposed DFNTSMC outperforms the FNTSMC and DSMC also tested in the contour following experiment.

INDEX TERMS Contour error, contour following, sliding mode control, tracking error

I. INTRODUCTION

High precision motion stages and servomechanisms are commonly used in industrial applications [1]-[7]. When performing contour following on motion stages and servomechanisms, contour following accuracies are often hampered by issues such as friction [8], backlash [9], structure resonance, servo mismatch among different axes, servo lag and external disturbance. With high contour following accuracy gradually becoming a much sought-after goal in many manufacturing processes, the aforementioned problems need to be carefully dealt with. Contour following accuracy can typically be assessed using two error indices—tracking error and contour error. In general, tracking error is the difference between the current position and the current position command, while contour error is defined to be the shortest distance between the current position and the entire command path. The control methodologies commonly employed in improving contour following accuracy include iterative learning control [10], adaptive control [11], \( H_\infty \) control [12], and sliding mode control (SMC) [13]-[14], amongst others.

In particular, due to its ability in suppressing disturbance and robustness to modeling uncertainty, SMC has been adopted in many motion control and linear motor control applications [15]-[17]. Most conventional SMCs adopt the signum function to determine the switching force. However, when the system states approach the sliding surface, the signum function will result in the chattering phenomenon, thus deteriorating system performance [18]. Many approaches have been proposed to ease the chattering effects [19]-[24]. For example, Bartolini et al. proposed a second-order sliding mode control approach to produce a smoother control output [19], while Makrini et al. used a variable boundary layer to produce a smoother switching force [20]. In [21]-[22], fuzzy logic was employed to tune the switching force so as to eliminate chattering, while in [23], the continuous Sig function was adopted to generate a
smooth switching force when the system states approach the sliding surface.

Another significant drawback of the conventional SMC design based on the Lyapunov stability theory is that it only guarantees that the system states will converge to the sliding surface as time $t$ approaches infinity. In order to overcome this drawback, Man and Yu proposed the Terminal Sliding Mode Control (TSMC) such that the system states can converge to the sliding surface in finite time [25]. However, the TSMC approach has a singularity problem. In [26]-[28], the Nonsingular Terminal Sliding Mode Control (NTSMC) is proposed to avoid the singularity problem. However, both the TSMC approach and the NTSMC approach suffer from the chattering effect. In order to suppress the chattering effect and also ensure that the system states can converge more quickly, Zheng et al. proposed the Fast Nonsingular Terminal Sliding Mode Control (FNTSMC) [29] that utilizes a continuous Sig function [30] to generate the switching force. Although the FNTSMC has satisfactory performance in convergence rate and chattering effect suppression [29], [31]-[33], its performance in contour following accuracy is hampered by adverse effects due to friction. Recently, The Dynamic PID Sliding Mode Control (DSMC) [34] by Chu et al. use a double sliding surface design to achieve high motion/machining accuracy and ease the chattering effect. However, since a signum function is employed to generate the switching force, the DSMC still exhibits some chattering problems. In addition, when implementing DSMC into a motion control system, information such as jerk and the time derivative of external disturbance are essential. This will substantially increase the complexity of implementing the DSMC into a motion control system.

In order to circumvent the aforementioned problems, this paper proposes the Dynamic Fast Nonsingular Terminal Sliding Mode Control (DFNTSMC) that combines the advantages of both FNTSMC and DSMC while avoiding their drawbacks. One of the attractive features of DSMC is that it uses two sliding surfaces, in which the proposed DFNTSMC also adopts a two-sliding-surface approach. However, the switching force of the conventional DSMC employs the signum function, which will result in chattering. In contrast, the sig function adopted in both the conventional FNTSMC and the proposed DFNTSMC can ease the chattering phenomenon. Moreover, unlike the conventional FNTSMC which only has one sliding surface, the proposed method employs two sliding surfaces. In summary, the proposed DFNTSMC has attractive features such as chattering effect suppression, good robustness, and high contour following accuracy, while the sliding surface can converge in finite time. In addition, when implementing the proposed DFNTSMC, information such as jerk and the time derivative of external disturbance are not indispensable. The control structure of the proposed DFNTSMC is simpler than that of the DSMC. Moreover, the convergence of the proposed DFNTSMC scheme has been theoretically proved based on Barbalat’s lemma. Several contour following experiments have been conducted to verify the effectiveness of the proposed DFNTSMC scheme.

The rest of the paper is organized as follows. Section II introduces the dynamic model of the experimental platform used in this paper. The proposed DFNTSMC scheme is introduced in Section III, while Section IV details the convergence proof of the proposed DFNTSMC scheme. The experimental setup and results are provided in Section V. Conclusions are given in Section VI.

II. DYNAMIC MODEL OF THE SERVOMECHANISM

Consider a class of nonlinear dynamical systems described by (1).

$$\dot{x}(t) = f(x(t)) + B \cdot u(t) - B \cdot (\tau, d_{n}, d_{non}) \quad (1)$$

where $x(t)$: $n \times 1$ vector of system state; $f$: $n \times 1$ vector field; $B$: $n \times 1$ input coefficient vector; $d_{non}$: nonlinearity; $d_{n}$: unknown disturbance and lump uncertainty; and $d_{non}$: unknown disturbance, lump uncertainty and nonlinearity that excludes viscous friction. Note that $\tau$ is assumed to be bounded. The dynamic models for most motion stages and servomechanisms used in industrial applications can be described by (1), including the experimental system employed in this paper as shown in Fig. 1.

Fig. 1. Experimental system used in this paper

III. THE PROPOSED DYNAMIC FAST NONSINGULAR TERMINAL SLIDING MODE CONTROL

External disturbance leads to the deterioration of contour following accuracy in motion stages and servomechanisms. As a result, designing a robust feedback controller that can effectively suppress this external disturbance is one of the major aims of this paper. Consequently, by adopting the advantages of both the FNTSMC [29] and the DSMC [34], this paper proposes the DFNTSMC scheme. The details concerning the derivation of the proposed DFNTSMC is elaborated in the following.
Consider a class of servomechanisms with dynamic behaviors governed by (1). The state error vector is defined as

\[ \ddot{x}(t) = x(t) - x_d(t) \]  

(2)

Design the first sliding surface \( S \) as

\[ S(\dot{x}(t)) = K \int_0^t \ddot{x}(r) dr \]  

(3)

where \( K \) is the gain constant vector and \( KB \) is nonsingular.

Next, let the sliding surface described by (3) be the new state variable and define the dynamic sliding surface \( \xi(t) \) as

\[ \xi(t) = G(X(t)) = \dot{S} + \lambda \text{sign}(S)^\gamma \]  

(4)

where \( X(t) \) is the new vector of state variables, and \( \gamma \) is used to avoid the singular point problem occurring in the controller, in which \( 1 < \gamma < 2 \). A larger \( \gamma \) leads to a faster convergence rate of the system state. However, the tolerance of the system to the error is lower. \( \lambda \) is a positive constant for which a larger \( \lambda \) leads to a faster response and better accuracy. In addition, the sign function is defined as follows [29]-[30]:

\[ \text{sign}(S)^\gamma = \begin{cases} S & \text{if } S > 0 \\ \text{sign}(S) \cdot S & \text{if } S < 0 \end{cases} \]  

(5)

In general, the sliding mode control scheme consists of equivalent force and switching force. The derivations of these two forces are given as follows. Differentiating the dynamic sliding surface \( \xi(t) \) with respect to \( t \) will yield

\[ \dot{\xi}(t) = \frac{dG(X(t))}{dt} = \dot{S} + \frac{d}{dt} \left( \lambda \text{sign}(S)^\gamma \right) \]  

\[ = \dot{S} + \lambda \left( \frac{\gamma - 1}{S} |S|^{\gamma - 2} S + \lambda |S|^{\gamma - 1} \dot{S} \right) \]  

\[ = \dot{S} + \lambda \gamma |S|^{\gamma - 1} \dot{S} = K(\dot{x} - \dot{x}_d) + \lambda \gamma |S|^{\gamma - 1} \dot{S} \]  

(6)

Substituting (1) into (6), and also ignoring the external disturbance \( d \), will result in

\[ \dot{\xi}(t) = K \cdot \left( f(x(t)) + B \cdot u(t) - \dot{x}_d \right) + \lambda \gamma |S|^{\gamma - 1} \dot{S} \]  

(7)

Let \( \dot{\xi}(t) = 0 \); one can obtain the equivalent force \( u_{eq} \) of the proposed DFNTSMC as described by (8).

\[ u_{eq}(t) = -(KB)^{-1} \left( \lambda \gamma |S|^{\gamma - 1} \dot{S} + K(f(x(t)) - \dot{x}_d) \right) \]  

(8)

By employing the technique developed in [29], the switching force of the proposed DFNTSMC scheme can be designed as follows:

\[ u_{sw}(t) = -\left( (KB)^{-1} \left( k_3 \dot{\xi}(t) + k_4 \text{sign}(\dot{\xi}(t))^\gamma \right) \right) \]  

(9)

where \( k_3, k_4 \) are positive gain constants and will be further discussed later on. By choosing the value of \( \rho \) such that \( 0 < \rho < 1 \), the system state can smoothly approach the dynamic sliding surface so as to suppress the chattering effect. Note that a larger \( \rho \) leads to a smoother switching force, whereas the system robustness becomes worse.

The total control \( u(t) \) can be expressed as:

\[ u(t) = u_{eq}(t) + u_{sw}(t) \]  

(10)

IV. CONVERGENCE PROOF OF THE PROPOSED DFNTSMC

The convergence of the proposed DFNTSMC scheme will be proved in the following.

Theorem 1: For the system described by (1) under the control laws (8)–(10), \( \xi(t) \) defined by (4) will converge to the region defined by

\[ |\xi| \leq \min(\phi_1, \phi_2) = \phi \]  

(11)

where \( \phi_1 = \frac{|d_2| KB}{k_3} \) and \( \phi_2 = \left( \frac{|d_2| KB}{k_4} \right)^{\frac{1}{\rho}} \).

Proof:

Define a Lyapunov function candidate \( V \) as described by (12).

\[ V = \frac{1}{2} \xi^2 \]  

(12)

Clearly, \( V \) is positive definite. Differentiating \( V \) along the system trajectory with respect to time \( t \) will yield

\[ \dot{V} = \xi \dot{\xi} = \xi \left( \frac{d}{dt} G(X(t)) \right) \]  

\[ = \xi \left( K \cdot \left( f(x(t)) + B \cdot u(t) - B \dot{x}_d \right) + \lambda \gamma |S|^{\gamma - 1} \dot{S} \right) \]  

(13)

Substituting (8), (9) and (10) into (13), after some mathematical manipulation will result in
\[ \dot{V} = \frac{d}{dt} G(X(t)) \]

\[ = \xi \left( K \cdot f(x(t)) + B \cdot u_{eq}(t) + B \cdot u_{sw}(t) - B_d \cdot \dot{x}_d + \gamma \right) \]

\[ = \xi \left( K \cdot \dot{x}(t) + K \cdot f(x(t)) - K \cdot \dot{x}_d + \gamma \right) \]

\[ = \xi \left( K \cdot \dot{x}(t) - KB \cdot (KB)^{-1} \left( \gamma \right)^{-1} \dot{x}_d + K \cdot f(x(t)) - K \cdot \dot{x}_d + \gamma \right) \]

\[ + KB \cdot u_{sw}(t) - K \cdot \dot{x}_d + K \cdot f(x(t)) - K \cdot \dot{x}_d + \gamma \left( \right)^{-1} \dot{x}_d \]

\[ = \xi \left( K \cdot \dot{x}(t) - KB \cdot (KB)^{-1} \left( \gamma \right)^{-1} \dot{x}_d - K \cdot \dot{x}_d + \gamma \right) \]

\[ = \xi \left( -k_3 \xi^2 + k_4 \sigma(\xi)^\rho \right) - d_e KB \xi \]

\[ = -k_3 \xi^2 - k_4 |\xi|^{\rho+1} - d_e KB \xi \]

\[ (14) \]

The following two strategies are employed in this paper to determine the suitable ranges of \( k_3 \) and \( k_4 \) such that \( \dot{V} \) described by Eq. (14) is negative definite [27].

**Strategy #1**: Rewrite (14) as

\[ \dot{V} = -k_3 \xi^2 - k_4 |\xi|^{\rho+1} - d_e KB \xi \]

\[ = -\left( k_3 + \frac{d_e KB}{\xi} \right) \xi^2 - k_4 |\xi|^{\rho+1} \]

(15)

If \( k_3 \) is chosen such that \( k_3 > \frac{|d_e KB|}{\xi} \) (i.e. \( |\xi| > \frac{|d_e KB|}{k_3} \)), then one will have

\[ \left( k_3 + \frac{d_e KB}{\xi} \right) > 0 \]

(16)

Clearly, if inequality (16) is satisfied, then \( \dot{V} \) described by (15) is negative definite. Namely, if \( |\xi| > \phi_1 \), then \( \dot{V} \) is negative definite.

**Strategy #2**: Rewrite (14) as

\[ \dot{V} = -k_3 \xi^2 - k_4 |\xi|^{\rho+1} - d_e KB \xi \]

\[ = -k_3 \xi^2 - \left( k_4 + \frac{d_e KB}{\sigma(\xi)^\rho} \right) |\xi|^{\rho+1} \]

(17)

If \( k_4 \) is chosen such that \( k_4 > \frac{|d_e KB|}{\sigma(\xi)^\rho} \) (i.e. \( |\xi| > \left( \frac{|d_e KB|}{k_4} \right)^{\frac{1}{\rho}} \)), then one will have

\[ \left( k_4 + \frac{d_e KB}{\sigma(\xi)^\rho} \right) > 0 \]

(18)

Clearly, if inequality (18) holds, then \( \dot{V} \) described by (17) is negative definite. Namely, if \( |\xi| > \phi_2 \), then \( \dot{V} \) is negative definite.

With Strategy #1 and Strategy #2, one can conclude that as long as inequality \( |\xi| > \min(\phi_1, \phi_2) \) is satisfied, \( \dot{V} \) described by (14) is negative definite and the dynamic sliding surface \( \xi \) will converge to the region defined by

\[ |\xi| \leq \min(\phi_1, \phi_2) = \phi \]

(19)

However, Theorem 1 cannot be directly used to investigate the asymptotic property of \( \xi \) in the region defined by (11). In order to cope with this problem, Barbalat’s lemma is exploited [4], [35]-[37].

**Theorem 2**: For the system described by (1) under the control laws (8)–(10), as long as \( \xi(t) \) defined by (4) converges within the region defined by (11), then \( \lim_{t \to \infty} \xi(t) = 0 \).

Proof:

First, as \( \xi \) converges within the region defined by (11), the absolute value of \( \xi \) will be bounded by \( \phi \), indicating that \( \xi \in L_\phi \).

Then, with the fact that \( \xi \) is bounded and (4), one can conclude that both \( S \) and \( \dot{S} \) are bounded in the region defined by (11). In addition, recall that \( d \) is assumed to be bounded.

From (13) and (14), one can write

\[ \xi = -\left( k_3 \xi + k_4 \sigma(\xi)^\rho \right) - d_e KB \]

(19)

Since \( \xi, d, k_3, k_4, K, B \) are all bounded in the region defined by (11), one can conclude that \( \xi \) described by (19) is bounded in the region defined by (11) as well. That is, \( \xi \in L_\phi \).

In addition, rewrite (14) as

\[ \dot{V} = -\left( k_3 \xi^2 + k_4 |\xi|^{\rho+1} + d_e KB \xi \right) \]

(20)

Integrating both sides of (20) will give

\[ \int_0^\infty \dot{V} dt = \int_0^\infty k_3 \xi^2 dt + \int_0^\infty k_4 |\xi|^{\rho+1} dt + \int_0^\infty d_e KB \xi dt \]

(21)

Equation (21) can be further rewritten as

\[ V(0) - \max_{\xi(t)} V(t) \]

\[ = k_3 \int_0^\infty \xi^2 dt + k_4 \int_0^\infty |\xi|^{\rho+1} dt + KB \int_0^\infty d_e \dot{\xi} dt \]

(22)

Since \( \xi \) is bounded, from (12), it is clear that \( V \) is bounded in the region defined by (11). As a result, the three improper integrals on the right-hand side of (22) are finite. That is, \( \int_0^\infty \xi^2 dt < \infty \), \( \int_0^\infty |\xi|^{\rho+1} dt < \infty \) and \( \int_0^\infty d_e \dot{\xi} dt \) is finite.

Namely, \( \xi \in L_\phi \) and \( \xi \in L_\phi \). According to Barbalat’s lemma, since \( \dot{\xi} \in L_\phi \) and \( \xi \in L_\phi \), one can conclude that

\[ \lim_{t \to \infty} \xi(t) = 0 \].

#
Moreover, following the approach developed in [29], it can be proven that $\xi$ will converge in finite time.

**Theorem 3:** For the system described by (1) under the control laws (8)–(10), $\xi$ defined by (4) will converge to the region defined by (11) in finite time.

Proof:

Suppose that $\left(k_3 + \frac{d_1 KB}{\xi}\right) > 0$. There exist two positive constants $\alpha$ and $\beta$ such that $0<\alpha \leq \left(k_3 + \frac{d_1 KB}{\xi}\right)$ and $0<\beta \leq k_4$.

From (12) and (15), one will have

$$\dot{V} \leq -\alpha \xi^2 - \beta V^{(\rho+1)/2} \beta V^{(\rho+1)/2} \leq 0$$

or

$$\dot{V} + 2\alpha V + 2(\rho+1)/2 \beta V^{(\rho+1)/2} \leq 0 \tag{23}$$

As pointed out in [29] and [38]-[39], if a Lyapunov function candidate $V$ satisfies inequality (24)

$$V(t) + aV(t) + bV^\varepsilon(t) \leq 0 \tag{24}$$

where $a>0$, $b>0$, $0<\varepsilon<1$ and initial value $V(0)=V_0$, the convergence time satisfies the following inequality

$$T_s \leq \frac{1}{a(1-\varepsilon)} \ln \left(\frac{aV_0^{1-\varepsilon} + b}{b}\right) \tag{25}$$

From (23), (24) and (25), one can obtain the upper bound of the convergence time as described by (26).

$$T_s \leq \frac{1}{\alpha(1-\rho)} \ln \left(1 + \left(2V_0(1-\rho)\right)^{\alpha/\beta}\right) \tag{26}$$

That is, the dynamic sliding surface $\xi$ will converge to the region defined by (11) in finite time $T_s$.

**V. EXPERIMENTAL SETUP AND RESULTS**

The experimental platform used to assess the performance of the proposed control approach shown in Fig. 1 is an X-Y table system driven by two AC servomotors that have built-in 25000×4 pulses/rev encoders. The sampling period of the control system is set to 1 ms and the servo drives of both AC servomotors are set to torque mode. In general, the motions on the $x$-axis and $y$-axis of the X-Y table system can be seen as decoupled. Therefore, the dynamic equations that govern the motions on the $x$-axis and on the $y$-axis are very similar; only the case of the $x$-axis will be shown below as

$$\begin{bmatrix} \dot{v} \\ \dot{a} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -B_m/J_m \end{bmatrix} \begin{bmatrix} v \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J_m \end{bmatrix} \left(u - (d_{\text{friction}} - B_m v) - d_n\right) \tag{27}$$

where $J_m$: moment of inertia of the system; $B_m$: viscous coefficient; $u$: control input; $d_{\text{friction}}$: friction (the most common nonlinearity in an X-Y table system); $d_n$: unknown disturbance and lump uncertainty; $a$: acceleration; $v$: velocity; $p$: position; $d_d(d_{\text{friction}}, d_n) = d_{\text{friction}} + d_n - B_m v$: unknown disturbance, lump uncertainty and nonlinearity excluding viscous friction $B_m v$.

Comparing Eq. (27) with Eq. (1), one will have

$$x(t) = \begin{bmatrix} p_v \\ v \end{bmatrix}, \ f(x(t)) = \begin{bmatrix} v \\ B_m/J_m \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1/J_m \end{bmatrix}, \ x_d(t) = \begin{bmatrix} p_d \\ v_d \end{bmatrix}$$

In addition, the gain constant vector $K$ in (3) is set to $K=[k_2 \ k_1]$.

Table I shows the parameter values of the X-Y table system.

| TABLE I | PARAMETER VALUES OF THE X-Y TABLE SYSTEM |
|--------|----------------------------------------|
|        | x-axis | y-axis |
| Moment of inertia $J_m$ (N·m²/s²/rad) | 0.0000852102 | 0.00010783 |
| Viscous coefficient $B_m$ (N·m/s/rad) | 0.000308708 | 0.00032178 |

The experiment is conducted to compare the performance of three control schemes—FNTSMC, DSMC, and the proposed DFNTSMC. Throughout the experiment, the NURBS circle-shaped contour [40] shown in Fig. 2 and the NURBS hourglass-shaped contour [41] shown in Fig. 3 are used as the desired contours.

![Fig. 2. Desired NURBS circle-shaped contour](image-url)
Fig. 3. Desired NURBS hourglass-shaped contour

A. FNTSMC
The sliding surface of FNTSMC is designed as follows:

\[ S = (p - p_d) + \lambda \text{sig}(v - v_d) = e + \lambda \text{sig}(\dot{e}) \]  

(28)

The total control force of FNTSMC is:

\[ u = u_{eq} + u_{sw} \]

\[ u_{eq} = J_m a_d + B_m v - J_m \text{sign}(\dot{e}) e - J_m \left( k_1 S + k_2 \text{sig}(S)^3 \right) \]

(29)

B. DSMC
The 1st sliding surface of the DSMC is designed as follows:

\[ S = (v - v_d) + k_1 (p - p_d) + k_2 \int_0^t (p - p_d) d\tau \]

\[ = \dot{e} + k_4 e + k_2 \int_0^t e(\tau) d\tau \]  

(30)

The 2nd sliding surface of the DSMC is designed as follows:

\[ \xi = \dot{S} + k_3 S + k_4 \int_0^t S(\tau) d\tau \]  

(31)

The total control force of the DSMC is:

\[ u = \int_0^t \left( \dot{u}_{eq} + \dot{u}_{sw} \right) d\tau \]

\[ \dot{u}_{eq} = J_m \dot{a}_d - \left( k_1 + k_3 - \frac{B_m}{J_m} \right) (f - B_m v) + J_m \left( k_1 + k_3 \right) a_d \]

\[ -J_m \left( k_2 + k_3 k_1 + k_4 \right) \dot{e} + \left( k_3 k_2 + k_4 k_1 \right) e + k_4 k_2 \int_0^t e(\tau) d\tau \]

(32)

C. PROPOSED DFNTSMC
The 1st sliding surface of the proposed DFNTSMC is designed as follows:

\[ S = (v - v_d) + k_1 (p - p_d) + k_2 \int_0^t (p - p_d) d\tau \]

\[ = \dot{e} + k_4 e + k_2 \int_0^t e(\tau) d\tau \]  

(30)

The block diagram of the DSMC is illustrated in Fig. 5, while the parameter values of the DSMC used in Experiment #1 are listed in Table III.
The 2nd (i.e. dynamic) sliding surface of the proposed DFNTSMC is designed as follows:

$$
\xi = S + \lambda \text{sig}(S)^{\rho}
$$

(34)

Using Eq. (8) and Eq. (9), the total control force of the proposed DFNTSMC is:

$$
\begin{align*}
\begin{aligned}
    u &= u_{eq} + u_{SW} \\
    &= J_m a_d + B_m \dot{v} - J_m k_2 \frac{k}{k_1} \dot{e} - J_m \frac{\lambda \gamma}{k_1} |S|^{\rho-1} \dot{S} \\
    &\quad - J_m \frac{k_3 \xi + k_4 \text{sig}(\xi)^{\rho}}{k_1} \\
    &\quad - \frac{J_m k_2}{k_1} (\rho - 1) \int_0^t \frac{e(\tau) d\tau}{|S|^{\rho-1}} \\
    &= u_{eq} + u_{SW}
\end{aligned}
\end{align*}
$$

(35)

The block diagram of the proposed DFNTSMC is illustrated in Fig. 6, while the parameter values of the proposed DFNTSMC used in the experiment are listed in Table IV.

![Fig. 6. Block diagram of the proposed DFNTSMC](image)

**TABLE IV**

| Equivalent control parameters | x-axis | y-axis |
|------------------------------|--------|--------|
| $k_1$                        | 5      | 20     |
| $k_2$                        | 1000   | 1500   |
| $\lambda$                    | 250    | 200    |
| $\gamma$                     | 1.801  | 1.801  |
| $k_3$                        | 150    | 100    |
| $k_4$                        | 0.015  | 10     |
| $\rho$                       | 0.95   | 0.95   |

D. NURBS Circle-shaped Contour Following Experiment

Under each control scheme, the X-Y table is controlled to move along the desired NURBS circle-shaped contour five consecutive times (i.e. iterations). For each control scheme, the experimental data are recorded and compared. According to the experimental results shown in Figs. 7~15 and Tables V~VIII, the proposed DFNTSMC clearly has the best performance in the NURBS circle-shaped contour following experiment among the three tested control schemes.
Fig. 11. Tracking error of DSMC; left: x-axis; right: y-axis

Fig. 12. Contour error of DSMC

Fig. 13. Control force of the proposed DFNTSMC; left: x-axis; right: y-axis

Fig. 14. Tracking error of the proposed DFNTSMC; left: x-axis; right: y-axis

Fig. 15. Contour error of the proposed DFNTSMC

TABLE V
RMS VALUES OF TRACKING ERROR (µm) FOR EACH ITERATION OF NURBS CIRCLE-SHAPED CONTOUR FOLLOWING FNTSMC, DSMC, DFNTSMC

| Iteration No. | x-axis | y-axis | x-axis | y-axis | x-axis | y-axis |
|---------------|--------|--------|--------|--------|--------|--------|
| 1st           | 11.61  | 12.53  | 1.718  | 2.739  | 1.326  | 2.000  |
| 2nd           | 11.58  | 12.47  | 1.761  | 2.503  | 1.386  | 1.890  |
| 3rd           | 11.68  | 12.47  | 1.775  | 2.474  | 1.419  | 1.893  |
| 4th           | 11.70  | 12.38  | 1.787  | 2.479  | 1.321  | 1.877  |
| 5th           | 11.54  | 12.33  | 1.766  | 2.525  | 1.393  | 1.906  |

TABLE VI
RMS VALUES OF CONTOUR ERROR (µm) FOR EACH ITERATION OF NURBS CIRCLE-SHAPED CONTOUR FOLLOWING FNTSMC, DSMC, DFNTSMC

| Iteration No. | FNTSMC | DSMC | DFNTSMC |
|---------------|--------|------|---------|
| 1st           | 16.04  | 3.025| 2.329   |
| 2nd           | 15.89  | 3.055| 2.331   |
| 3rd           | 16.02  | 3.039| 2.365   |
| 4th           | 16.02  | 3.051| 2.414   |
| 5th           | 15.91  | 3.074| 2.348   |

TABLE VII
MAX VALUES OF TRACKING ERROR (µm) FOR EACH ITERATION OF NURBS CIRCLE-SHAPED CONTOUR FOLLOWING FNTSMC, DSMC, DFNTSMC

| Iteration No. | FNTSMC | DSMC | DFNTSMC |
|---------------|--------|------|---------|
| 1st           | 15.63  | 18.95| 11.274  |
| 2nd           | 15.67  | 19.35| 11.174  |
| 3rd           | 16.60  | 17.41| 11.526  |
| 4th           | 16.20  | 17.30| 11.570  |
| 5th           | 16.02  | 17.41| 11.747  |

TABLE VIII
MAX VALUES OF CONTOUR ERROR (µm) FOR EACH ITERATION OF NURBS CIRCLE-SHAPED CONTOUR FOLLOWING FNTSMC, DSMC, DFNTSMC

| Iteration No. | FNTSMC | DSMC | DFNTSMC |
|---------------|--------|------|---------|
| 1st           | 21.21  | 16.55| 12.309  |
| 2nd           | 21.07  | 16.81| 11.931  |
| 3rd           | 21.57  | 16.83| 11.577  |
| 4th           | 20.81  | 16.62| 12.415  |
| 5th           | 20.85  | 16.84| 12.610  |
E. NURBS Hourglass-shaped Contour Following Experiment

Under each control scheme, the X-Y table is controlled to move along the desired NURBS hourglass-shaped contour five consecutive times (i.e. iterations). For each control scheme, the experimental data are recorded and compared. According to the experimental results shown in Figs. 16–24 and Tables IX–XII, the proposed DFNTSMC clearly has the best performance among the three tested control schemes in the NURBS hourglass-shaped contour following experiment.

Fig. 16. Control force of FNTSMC; left: x-axis; right: y-axis

Fig. 17. Tracking error of FNTSMC; left: x-axis; right: y-axis

Fig. 18. Contour error of FNTSMC

Fig. 19. Control force of DSMC; left: x-axis; right: y-axis

Fig. 20. Tracking error of DSMC; left: x-axis; right: y-axis

Fig. 21. Contour error of DSMC

Fig. 22. Control force of the proposed DFNTSMC; left: x-axis; right: y-axis
The Dynamic Fast Nonsingular Terminal Sliding Mode Control (DFNTSMC) scheme proposed in this paper has several attractive features such as chattering effect suppression, enhancement of robustness, contour following accuracy improvement, and finite time convergence. Moreover, when applied to the X-Y table system employed in this paper, the proposed DFNTSMC does not need the jerk information, and the time derivative of the external disturbance is not essential. Therefore, the structure of the proposed DFNTSMC is simpler than that of DSMC. Experimental results indicate that the proposed DFNTSMC outperforms both FNTSMC and DSMC in performance indices such as tracking error and contour error. As a result, we can conclude that the proposed DFNTSMC scheme is the best among the three tested control schemes.

VII. CONCLUSION

Table: Table IX

| Iteration | FNTSMC | DSMC | DFNTSMC |
|-----------|--------|------|---------|
| x-axis    | y-axis | x-axis | y-axis   |
| 1st       | 12.88  | 11.81 | 4.716   | 3.985   | 2.809   | 3.463   |
| 2nd       | 12.74  | 12.01 | 4.564   | 3.960   | 2.690   | 3.593   |
| 3rd       | 12.76  | 12.03 | 4.612   | 3.924   | 2.702   | 3.483   |
| 4th       | 12.68  | 12.00 | 4.602   | 3.951   | 2.695   | 3.427   |
| 5th       | 12.61  | 12.00 | 4.605   | 3.919   | 2.735   | 3.485   |

Table: Table X

| Iteration | FNTSMC | DSMC | DFNTSMC |
|-----------|--------|------|---------|
| x-axis    | y-axis | x-axis | y-axis   |
| 1st       | 13.39  | 5.946 | 4.305   |
| 2nd       | 13.44  | 6.034 | 4.485   |
| 3rd       | 13.46  | 6.045 | 4.382   |
| 4th       | 13.43  | 6.041 | 4.375   |
| 5th       | 13.36  | 6.030 | 4.441   |

Table: Table XI

| Iteration | FNTSMC | DSMC | DFNTSMC |
|-----------|--------|------|---------|
| x-axis    | y-axis | x-axis | y-axis   |
| 1st       | 28.94  | 22.83 | 18.43   |
| 2nd       | 30.06  | 22.80 | 18.12   |
| 3rd       | 30.01  | 22.84 | 18.45   |
| 4th       | 29.63  | 22.66 | 18.99   |
| 5th       | 30.82  | 23.50 | 18.35   |

REFERENCES

[1] E. C. Park, H. Lim, and C. H. Choi, “Position control of X-Y table at velocity reversal using presliding friction characteristics,” IEEE Trans. Control Syst. Technol., vol. 11, no. 1, pp. 24-31, Jan. 2003.

[2] Y. Zhang and Q. Xu, “Adaptive sliding mode control with parameter estimation and Kalman filter for precision motion control of a piezo-driven microgripper,” IEEE Trans. Control Syst. Technol., vol. 25, no. 2, pp. 728-735, Mar. 2017.

[3] Xinxin Shi, YangQuan Chen, Jiacai Huang, “Application of fractional-order active disturbance rejection controller on linear motion system,” Control Eng. Pract., vol. 81, pp. 207–214, 2018.

[4] Shubo Wang, Haiheng Yu, Jinpeng Yu, “Robust adaptive tracking control for servo mechanisms with continuous friction compensation,” Control Eng. Pract., vol. 87, pp. 76–82, 2019.

[5] Antonella Ferrara, Gian Paolo Incroponi, Bianca Sangiovanni, “Tracking control via switched integral sliding mode with application to robot manipulators,” Control Eng. Pract., vol. 90, pp. 257–266, 2019.

[6] Jianfei Pan; Pengfei Fu; Shuangxia Niu; Can Wang; Xiaodong Zhang, “High-precision coordinated position control of integrated permanent magnet synchronous linear motor stations,” IEEE Access, vol. 8, pp. 126253 - 126265, July 2020.

[7] Changlin Zhu; Quanzhang Tu; Chengming Jiang; Ming Pan; Hao Huang, “A cross coupling control strategy for dal-motor speed synchronous system based on second order global fast terminal sliding mode control,” IEEE Access, vol. 8, pp. 217967 - 217976, Dec. 2020.
