Research on Three-Dimensional Interpolation Modeling Method of Micro- and Nanometer Controllable Surface

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Abstract. The fundamental cause of friction and wear is the effect of force, resulting in material heating, deformation, fatigue, adhesion and furrow effect, which is closely related to the micro-topography of the contact surface of friction pair. How to establish the real three-dimensional (3D) rough surface, using inter-polation surface reconstruction based on real data, especially in the three dimensional precision measurement data processing, etc, also the lack of research and practice, so in this paper, fractal interpolation and spline interpolation, Fourier interpolation surface modeling interpolation algorithm of three methods for modeling, a detailed analysis and expounds the advantages and disadvantages of the three ways of modeling with surface, and finally to the surface of the atomic force microscope to measure the actual extraction of a small number of sampling points are simulated.

Keywords: 3D rough surface, fractal interpolation, spline interpolation, Fourier interpolation.

1. Information
With the development of science and technology and the deepening of research, especially for functional surfaces with surface characteristics, the surface research scale has reached the level of micro and nano, so it is necessary to obtain the surface characteristics through instrument measurement. But now most of them adopt the resolution of the surface roughness measuring instrument far from atomic level, based on the morphology of instruments to collect data and calculate the parameters of difficult to objective and accurate characterization of the surface [1], especially when a certain measurement limitations, use instrument to measure the surface parameters obtained by the instrument resolution with the length, the influence of the parameters of the results there is a big error. Therefore, the measured data can be expanded, that is, interpolation reconstruction can be carried out on these data. At present, there is still a lack of research and practice in surface interpolation reconstruction based on real data, especially in the processing of 3D precision measurement data [2].

The existing interpolation methods include fractal interpolation, spline interpolation, Fourier interpolation and polynomial interpolation. For the reconstruction of real data, surface interpolation
can reflect the real condition of the surface more intuitively and accurately, which is more conducive to the subsequent study of surface functions and characteristics. In this paper, fractal interpolation, spline interpolation and Fourier interpolation surface modeling interpolation algorithms are analyzed in detail, and the advantages and disadvantages of the three modeling methods are described. Finally, a small number of sampling points are extracted from the actual surface measured by atomic force microscope for simulation.

2. Analysis of modeling algorithm

2.1. Fractal interpolation algorithm

Classical interpolation method is mainly aimed at Euclidean geometry description object, however sometimes for rough surface contour especially for micro/nano scale image Euclidean function is not a good description, using fractal interpolation function can close to these images, and the fractal dimension with the data of the fractal dimension can be within the scope of the appropriate scale to achieve consistently [3]. The essence of fractal interpolation algorithm is to construct an iterative function system so that its attractor A is the graph of interpolation function. As a new way to fit empirical data, fractal interpolation function, like elementary function, has its own geometric properties and can be expressed scientifically and calculated quickly. Then the basic interpolation surface function of 3D micro-nano surface simulated by fractal interpolation algorithm is described as follows:

Let the sampled data point be \((x_n, y_n, z_{n,m})\), \(n = 0,1,\ldots,N\); \(m = 0,1,\ldots,M\). Two-dimensional fractal surface interpolation function can be constructed \(f(x, y) = z_{n,m}\);

At the same time the X and Y direction is compressed and transformed into [3] \(\varphi_n(x) = a_n x + b_n\);
\(\varphi_m(y) = c_m y + d_m\).

And meet the conditions \(\varphi_n(x_0) = x_{n-1}, \varphi_n(x_N) = x_N; \varphi_m(y_0) = y_{m-1}, \varphi_m(y_M) = y_M\). From this calculate \(a_n, b_n, c_m, d_m\).

Let the Z-direction compression transformation be \(F_{n,m}(x, y, z) = e_{n,m} x + f_{x,y} y + g_{n,m} x y + a_{n,m} z + k_{n,m}\), where

\[
\begin{align*}
F_{n,m}(x_0, y_0, z_{0,0}) &= z_{n-1,m-1} \\
F_{n,m}(x_N, y_0, z_{N,0}) &= z_{n,m-1} \\
F_{n,m}(x_0, y_M, z_{0,M}) &= z_{n-1,m} \\
F_{n,m}(x_N, y_M, z_{N,M}) &= z_{n,m}
\end{align*}
\]

Where \(a_{n,m}\) is a free parameter to determine the fractal dimension of an interpolating surface, and \(0 \leq a_{n,m} < 1\).

Through the above fractal surface function formula, 3D micro-nano surface modeling can be carried out by MATLAB programming, and fractal interpolation surface has the following advantages and disadvantages.

Advantages: Fractal interpolation can overcome the traditional interpolation method cannot well reflect the local characteristics between the sampling points, using the fractal interpolation principle to conduct fractal simulation on the surface of micro and nano, can get more close to the actual surface 3D morphology than the traditional interpolation method; Fractal interpolation can be used to interpolate arbitrary data in the rectangular domain to obtain continuous fractal surfaces. The application of fractal interpolation algorithm to simulate micro-nano surface can greatly reduce the requirements of surface roughness measuring instru-ments and reduce the cost of experiments.

Disadvantages: Because there are many methods to calculate the fractal dimension of the surface, the fractal dimension calculated by different methods is not the same, so the diversity of the micro-nano interpolation surface is generated. Moreover, when sampling data points are few, it is difficult to determine the fractal dimension of the surface, which can only be simulated by empirical values or multiple times, so the simulated surface obtained has certain blindness.
2.2. Spline interpolation algorithm

It is well known that spline functions have good properties and play an outstanding role in curve fitting [4]. Spline interpolation overcomes the non-smooth problem of piecewise interpolation function fitting and improves the smoothness of interpolation function [5]. The spline function can be applied by using the linear combination of spline function $N_m(t)$ to construct the approximate function $f'(t)$, and $f'(t)$ satisfies the mandatory requirements of function value and derivative value at the hourly node.

In the actual signal analysis and processing, the sufficient smoothness of the signal is often not satisfied, so the cubic spline interpolation can be used to approximate the original acquisition signal at multiple scales. Suppose known samples $\{f(x, y)\}$, through constructing spline interpolation coefficient of sample values $\{c_i\}$, hope $f'(t)$ interpolation function in the value of the node is a sample value $\{f(x, y)\}$. The interpolation function is

$$f'(t) = \sum c_i N_m(t + m/2 - l)$$

$$f'(x, y) = f(x, y)$$

Through the above algorithm formula, 3D micro-nano interpolation surface modeling can be carried out by MATLAB programming, and cubic spline interpolation surface has the following advantages and disadvantages. Because of the good recursive property of spline function itself, approximate interpolation functions with different shapes can be constructed according to different requirements to obtain different degrees of smooth micro-nano surface. However, if the sampling point is insufficient, although the extremely smooth surface can be obtained, the local characteristics of the surface cannot be reflected, resulting in the loss of surface information and misunderstanding of the subsequent surface characterization. In addition, the processing speed of cubic spline interpolation is slow, and errors will occur when the input data is not uniform or some points are too close to each other.

2.3. Fourier interpolation algorithm

In the 19th century, The French Fourier proposed that the periodic function $f(t)$ could be decomposed into the weighted sum of infinitely many positive and cosine functions with different frequencies, or the weighted sum of complex exponentials [6-9]. In Fourier analysis, the discrete Fourier transform (FT) has implicit periodicity and energy conservation in signal domain and frequency domain. Due to these characteristics of FT, Newland, You, Ehmann and Wu et al. conducted researches on rough surface modeling using FT based on surface discrete power spectrum in 1984, 1991, and 2000 respectively [10-12], demonstrating the feasibility of applying FT to surface modeling. When the actual signal is collected, the collected data cannot fully express the actual signal on the surface due to the large interval of collection due to the limitation of conditions. In this case, one-dimensional fast Fourier interpolation can be used to increase the sampling of the collected signal, and the obtained data is more in line with the data on the actual surface. The essence of one-dimensional fast Fourier interpolation is to transform the input data into the frequency domain through the FT, and then use more points of inverse FT to obtain the time domain, so as to augment the sampling of the data.

In this paper, an improved one-dimensional fast FT is adopted to carry out micro-nano 3D surface modeling. The basic algorithm is as follows: first for the acquisition of 3D signal of all row vector to

$$X(k) = \sum_{n=0}^{N-1} x(n) \left[ \exp\left(-j \frac{2\pi}{N} nk \right) \right]$$

formula as a template for the FT, then adopt the following formula as the sampling point to be designed to carry out inverse

$$F^{-1}(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \left[ \exp\left(-j \frac{2\pi}{N} nk \right) \right]^{-n}$$

we get the interpolating data of all the row vectors and the new 3D data, and similarly, we take the FT of all the column vectors of the new 3D data and the inverse FT of the design data sample points.

Through simulation can know, due to the characteristics of FT itself has, to improve the micro/nano surface of one-dimensional fast Fourier interpolation modeling of the surface, you can get with the
given texture and anisotropy, highly accord with certain probability distribution, and meet some given characteristics of micro/nano rough surface.

3. Simulation comparison
Using oxalic acid as electrolyte, porous anodic alumina film prepared by secondary anodic oxidation at room temperature was used as the surface roughness research object. The grayscale image of the micro-morphology of the film surface observed by atomic force microscope is shown in Figure 1, and Figure 2 is the AFM 3D image of the porous anodic alumina film.

The surface morphology data obtained from the scanning probe atomic force microscope were processed on the MATLAB platform to make the nanometer 3D base level image expression clearer. In this paper, only 250×250 sampling points in Figure 2 were extracted. Its image and height distribution histogram are shown in Figure 3 and 4.
In order to verify the feasibility of various algorithms, this paper uses interval extract the $10 \times 10$ points in the figure 3 as a sampling point, as shown in figure 5. A few collected $10 \times 10$ sampling points respectively for three kinds of interpolation algorithm simulation modeling are shown in the figure below.
**Figure 6.** Fractal interpolation 3D figure.

**Figure 7.** Fractal interpolation height histogram.

**Figure 8.** 3D diagram of cubic spline interpolation.

**Figure 9.** Height distribution histogram of cubic spline interpolation.
4. Conclusion
After the above summary and simulation based on real data, conclusions can be drawn:

(1) When the sampled surface does not have fractal characteristics, the surface height distribution obtained by using the improved one-dimensional Fourier interpolation algorithm is more consistent with that of the original micro-nano surface.

(2) When the surface is known to have certain fractal characteristics, that is, local features are similar, the improved one-dimensional Fourier interpolation combined with fractal interpolation can be used to get a surface that is closer to the real micro-nano surface.

(3) Although all of the three methods can be applied, it is still necessary to further explore which micro-nano surface processing technology each interpolation algorithm is suitable for.

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