Broadband spectroscopy of a dynamic impedance

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Abstract. Impedance spectrum of dynamic systems is time dependent. For example, fast impedance changes take place in high throughput microfluidic devices. Also, the impedance of cardiovascular system is dynamic. Measurements must be as short as possible to avoid significant impedance changes during the spectrum analysis and, at the same time, as long as possible for enlarging the excitation energy. The authors propose to use specific short chirp pulses for excitation. Thanks to unique properties of the chirp function, it is possible to meet the needs for spectrum bandwidth, measurement time, and signal-to-noise ratio so that the most accurate impedance spectrogram is obtained. The chirp wave excitation pulse can include thousands of cycles when the impedance changes slowly, but in the case of very high-speed changes it can be even shorter than a single cycle, preserving the same excitation bandwidth.

For example, 100 kHz bandwidth can be covered by the chirp pulse with duration from 10 \(\mu\)s to 1 s, only its excitation energy differs also 10\(^5\) times.

1. Introduction

Bioimpedance spectroscopy enables to characterize the biological matter, including label free identification of bioparticles in the microfluidic lab-on-a-chip (LoC) [1, 2], see figure 1. Broadband excitation is preferred when the speed of analysis is important [1-7]. Different signal waveforms are used [3, 4-7], the chirp wave is assumed to be the most suitable because of scalability [3-5].

The result of spectral analysis of dynamic system is not merely a spectrum but spectrogram – discrete time series of spectral snapshots. To avoid dynamic uncertainties, we have to perform short time measurement. On the other hand, we have to use long measurement time for maximizing the signal-to-noise ratio: (a) – through smoothing out the background noise, and (b) – via enlarging the excitation energy. The optimal solution is a balanced trade-off between process dynamics, measurement time, signal-to-noise ratio, and excitation bandwidth. A key factor for finding a successful compromise, is independent scalability in time (duration of excitation) and frequency (excitation bandwidth). The chirp signal provides such the double scalability [2-4]. A simple linear chirp has an instantaneous frequency \(\omega(t) = d\theta(t)/dt\), which changes linearly during the excitation interval \(T_{exc}\). Expressing \(\omega = 2\pi f\) and denoting \(f_1\) as an initial, and \(f_2\) as a final frequency and marking \(T_{ch}\) as a duration of the chirp pulse, we obtain the following expression for the linear chirp excitation:

\[
v_{ch}(t) = v_{sin}(t) = A\sin\left(2\pi \left(f_1 t + \left(f_2 - f_1\right)\frac{t^2}{2T_{ch}}\right)\right)
\]  

(1)
The excitation bandwidth $B_{\text{exc}} = f_2 - f_1$ remains the same [2] when the excitation time $T_{\text{exc}} = T_{\text{ch}}$ changes. Only the chirping rate $(f_2 - f_1)/T_{\text{ch}}$ and signal energy $E = (A^2)T_{\text{ch}}$ vary together with $T_{\text{ch}}$.

Figure 1. Generalized diagram of the LoC using the impedance spectroscopy method for sorting of bioparticles.

2. Proposed excitation waveforms
Traditionally is assumed that the chirp excitation contains a big number of cycles (rotations by $2\pi$ of the signal generating vector). The authors have analyzed shorter excitations and propose to use the chirp pulses having only some of cycles (e.g., 1 to 3) or even a part of a single cycle. They introduce a term “titlet” for denoting such very short chirplet type signals referring to short tweetings of tit (a bird *parus*, in Latin). The titlets allow to match requirements for spectrum bandwidth and excitation time for obtaining accurate impedance impedance spectrograms. As the lowest frequency $f_1$ is usually much smaller than the highest one, $f_2$, it is reasonable to introduce a simplification $f_1 = 0$ into (1):

$$v_{\text{sin}}(t) = \sin\left(2\pi \left(\frac{f_2}{2} \cdot t^2 / 2T_{\text{ch}}\right)\right)$$

(2)

Analysis of (2) shows ($f_1 = 0$) that the minimal excitation time is limited by the highest frequency $f_2$ in the excitation bandwidth $B_{\text{exc}}$ and by the number of cycles $p$ in the excitation signal:

$$\min T_{\text{exc}} = 2p / f_2$$

(3)

Moreover, the choice of bandwidth $B_{\text{exc}}$, duration $T_{\text{exc}} = T_{\text{ch}}$, and number of cycles $p$ of short linear chirps is limited with the discrete values (figure 2). For example, for $p = 1$ and $f_2 = 100\text{kHz}$, the minimal excitation time $T_{\text{exc}} = 20\mu\text{s}$. For $p = \frac{1}{2}$ and $p = \frac{1}{4}$, the excitation time reduces to 10 and 5 $\mu\text{s}$.

Figure 2. Nomogram of relationships between the excitation time $T_{\text{exc}}$ (the same value as duration $T_{\text{ch}}$ of the chirp pulse), upper frequency $f_2$ (practically the same as excitation bandwidth $B_{\text{exc}}$, because the lower frequency $f_1 < f_2$ can be neglected, see (1) and (2)), and the number $p$ of cycles (full-turn rotations by $2\pi$ of the generating vector) in the linear chirp pulse excitation signal.

Remark: excitation signals can contain only a fraction of the full cycle, e.g., $p = 0.5$ corresponds to a half-turn $\pi$, and $p = 0.25$ to a quarter of turn $\pi/2$. 
3. Waveforms and spectral densities of the short-chirp titlet excitations

The chirp wave excitation pulse with linear frequency sweep can cover a wide frequency span $B_{exc}$ from $f_1 \geq 0$ to $f_2$, and include lots of cycles if the speed of changes is low. More than 90% of signal energy $E = (A^2/2) T_{exc}$ [2] falls into $B_{exc}$ and, therefore, the power spectral density (PSD) of a voltage signal is near to $E/B_{exc}$, $V^2/Hz$. The spectral density of RMS value, RMSSD, is equal to $(PSD)^{1/2}$, $V/Hz^{1/2}$.

Figure 4 describes a chirp with moderate duration (200 µs), which contains $p = 10$ cycles. The shape of its normalised RMSSD (figure 5) is only slightly distorted in comparison with spectral density of long chirps ($p = 1000$ to 100 000) having almost ideally flat spectrum [3, 6] within $B_{exc} \approx f_2 = 100$ kHz.

In figure 6 we can find a single cycle titlet ($p = 1$, the final value of rotation phase $\theta_{fin} = 2\pi$) with duration 20 µs having the same bandwidth 100 kHz. The flatness of spectrum (figure 7) is somewhat distorted in comparison with the previous case ($p = 10$) and its actual energy is 10 times lower.

Figure 8 describes a 1/2-cycle titlet ($p = 1/2$, $\theta_{fin} = \pi$) with 10 µs duration, which has almost perfectly flat spectrum within the excitation bandwidth $B_{exc} \approx f_2 = 100$ kHz, only its energy is 2 times lower than in the previous case ($p = 1$). The spectral density drops off steeply (-40 dB/decade) outside the excitation bandwidth.
Finally, figure 10 describes a 1/4-cycle titlet ($p = \frac{1}{4}$, $\theta_{\text{fin}} = \pi/2$) with 5 µs duration. Flatness of its spectrum (figure 11) is very near to ideal within the excitation bandwidth $B_{\text{exc}} = f_2 = 100$ kHz, only the spectral density drops slowly (~20 dB/decade) outside the bandwidth and the signal energy is lower.

![Figure 10](image1.png)  
**Figure 10.** A quarter-cycle ($\theta_{\text{fin}}=\pi/2$) titlet with duration 5 µs and bandwith $f_2 = 100$ kHz.  

![Figure 11](image2.png)  
**Figure 11.** Normalized RMS spectral density of the quarter-cycle titlet in figure 10.

### 4. Conclusions

The short chirp pulse excitation (known also as a chirplet or titlet) is a perspective candidate for implementation in wideband spectroscopy of dynamic impedances. The spectral density of excitation is flat beginning from very low frequencies (fractions of one Hz or even DC) up to tens of MHz range. More than 90% of generated energy falls into the useful excitation frequency band $B_{\text{exc}}$, which remains invariant when the duration of excitation $T_{\text{exc}}$ changes in tens and hundreds of times.

In other words, we can choose and change the excitation bandwidth and the excitation time independently in the best way by using the scalability in time and frequency domain and taking into account the speed of impedance changing, noise level, and the excitation energy maximally allowed. As a result, we can maximize the amount of information derived from the data of spectral measurements.

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