The alternative to the incompressible fractional charge in quantum Hall effect: Comments on Laughlin and Schrieffer’s papers.

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Laughlin has found “exactly” the wave function which is ascribed to an excitation of fractional charge, such as e/3. We find that the exactness of the wave function is not destroyed by changing the charge to some other quantity, such as the magnetic field. Thus e/3 and H can be replaced by e and H/3. Therefore, the wave function need not belong to a quasiparticle of charge e/3.
1. Introduction

An effort is made to see if there is an alternative to the interpretation of the fractional charge in the quantum Hall effect. Laughlin has written the wave function for the quasiparticle of fractional charge. We examine to see if the wave function can be interpreted to describe integer charge and the blame of the fraction can be thrown on some other quantity. Laughlin has proposed a variational ground state which is a new state of matter, a quantum fluid, the elementary excitations of which are fractionally charged. The correctness of the wave functions is verified by direct numerical diagonalization of the many-body Hamiltonian. We wish to examine the alternative interpretation of the wave function without destroying the exactness of the calculation. In particular, we examine whether some other variable can take the blame of the fraction instead of the charge. If we bring a flux quantum near a solenoid, according to Laughlin, charge is affected whereas according to the correct answer, the field is affected. The charge density should automatically give the correct electric and magnetic vectors so that the magnetic field should be automatically correct. There are two variables, the charge and the field in the flux quantization. Therefore, Laughlin’s throwing the blame only on the charge is not correct and the automatic correction to the electric and magnetic vectors does not occur.

We find that several quantities arise as a factor of the charge so that the effect of the fraction need not be thrown on the charge. Since the charge multiplies the magnetic field, the factor arising in the field can also be read as a factor of the charge, then there is no trouble in defining a fractional charge. However, if charge is corrected without correcting the magnetic field, apparently there is no trouble, except that the choice of the variable is not unique.

In this comment, we explain the various choices available without disturbing the exactness and point out that field is better than the charge.

2. Theory

Let us consider a two-dimensional electron gas in the $x - y$ plane subjected to a magnetic field along $z$ direction. The eigen states of the single-body Hamiltonian can be
written as, $|m,n>$. The cyclotron frequency is, $\omega_c = eH_o/mc$. The magnetic length is $a_o = (h/m\omega_c)^{1/2}$ which upon substituting the cyclotron frequency becomes,

$$a_o = \left(\frac{\hbar c}{eH_o}\right)^{1/2}. \tag{1}$$

The energy levels of the type of a harmonic oscillator are produced when the applied field is along the $z$ direction,

$$H|m,n> = (n + \frac{1}{2})|m,n>. \tag{2}$$

The wave function of the lowest Landau level is written as a function of $z = x + iy$ with $|m>$ as an eigen state of angular momentum with eigen value $m$. The many-body Hamiltonian consists of, the kinetic energy with the vector potential included in the momentum, a potential due to the positively charged nuclei, $V(z_j)$, and the Coulomb repulsion between electrons. The wave functions composed of states in the lowest Landau level which describe the angular momentum $m$ about the center of mass are of the form,

$$\psi = (z_1 - z_2)^m(z_1 + z_2)^nexp[\left(-\frac{1}{4}\right)(|z_1|^2 + |z_2|^2)]. \tag{3}$$

Laughlin generalized this observation to $N$ particles by writing product of Jastrow functions,

$$\psi = \{\Pi_{j<k}f(z_j - z_k)\}exp\left(-\frac{1}{4}\Sigma_i|z_i|^2\right) \tag{4}$$

which minimizes the energy with respect to $f$. If $\psi$ is antisymmetric $f(z)$ must be an odd function, $f(z) = z^m$ with $m = odd$. To determine which $m$ minimizes the energy, we write,

$$|\psi_m|^2 = exp(-\beta\phi) \tag{5}$$

where $\beta = 1/m$ and $\phi$ is the classical potential energy which describes a system of $N$ identical particles of charge $Q=m$ with the neutralizing back ground charge density, $\sigma = (2\pi a_o^2)^{-1}$ per unit area. This is the classical one-component plasma (OCP). The solution of which is well known. For $\Gamma = 2\beta Q^2 = 2m > 140$, the OCP is a hexagonal crystal and fluid otherwise. Laughlin’s wave function describes a fluid of density,

$$\sigma_m = \frac{1}{m(2\pi a_o^2)} \tag{6}$$
which minimizes the energy. The charge density generated by (6) should be equal to that of the background charge so that there is overall charge neutrality.

If we define a new value of $a_o$, then $ma_o^2$ is replaced by $a_{new}^2$. Then the effect of $m$ is not on the charge but it is in the distance. That means that the effect of $m$ can occur on, (i)h, (ii)c,(iii) e or (iv) $H_o$. Thus there are at least four candidates to absorb the effect of $m$. Let us eliminate the Planck’s constant and the velocity of light. Then it is possible to affect either $e$ or $H_o$. It will surely be interesting if the effect of $m$ is thrown on the velocity of light. Then,

$$a_{new}^2 = \frac{\hbar(mc)}{eH_o}$$  \hspace{1cm} (7)

where for $m=3$, the quasiparticles will travel with the velocity of $3c$ which is faster than light. In the expression (1), the velocity of light $c$ is the value in vacuum so that the particles with velocity $3c$ will be travelling faster than light which will be noncausal or causality violating. Another interpretation is obtained by throwing the value of $m$ on the magnetic field,

$$a_{new}^2 = \frac{\hbar c}{e(H_o/m)}.$$  \hspace{1cm} (8)

Therefore, the number $m$ of Laughlin can be absorbed as a factor of magnetic field such as,

$$gH_o = H_o/m.$$  \hspace{1cm} (9)

It is perfectly allowed to devide the magnetic field by $m$ to define a new field. Out of the four options available, i.e., $h$, $c$, $e$ and $H_o$, Laughlin selected $e$ so that, the effective charge becomes,

$$e_{eff} = e/m$$ \hspace{1cm} (10)

where $h$, $c$ and $H_o$ are kept constant and $m$ is an odd integer. Therefore, Laughlin’s choice of variables is not unique. Let us ask, if some one had selected the velocity of light, out of the four variables, then what would have happened? The answer to this question is that we would have obtained the particles faster than light.

3. Exactness
The projection of $\psi_m$ for three and four particles onto the lowest energy eigen state of angular momentum $3m$ for $m=3$ and $m=5$ states is about 0.98 which shows the exactness of the wave functions. Therefore, the exactness of the calculation is not affected by changing the variable from $e$ to $H$, $h$ or $c$. The calculation is exact for any of the four variables. The total energy per particle can be written in terms of the distribution function of the one-component plasma (OCP),

$$U_{tot} = \pi \int_0^\infty \frac{e^2}{r}[g(r) - 1]rdr \simeq \frac{4}{3\pi} - 1) \frac{e^2}{R}$$ (11)

where integration domain is a disk of radius $R=(\pi \sigma_m)^{-1/2}$. At $\Gamma=2$, $g(r) = 1 - \exp[-(r/R)^2]$ which shows that for $m<9$, the $U_{tot}$ is deeper than for charge density waves. The excitations of $\psi_m$ are created by piercing the fluid at $z_o$ with an infinitely thin solenoid and passing through it a flux quantum $\Delta \varphi=hc/e$, adiabatically. This treatment does not treat the magnetic field correctly. Here again,

$$\text{field} \times \text{area} = hc/e$$ (12)

so that the variable is $e \times \text{field} \times \text{area}$ and hence the entire blame can not be thrown on $e$. It may appear that changing the $e$ will automatically take care of the electric vector $\mathbf{E}$ and the magnetic field $\mathbf{H}$ of the electromagnetic field but there is $\mathbf{H}$ in the flux quantization which is unchanged. A change in the charge density should change both $\mathbf{E}$ and $\mathbf{H}$. According to the adiabatic approximation, the flux quantum produces the quasiparticles. If this approximation is not valid, field will be produced not the quasiparticles. In fact the variables, $h$ and $c$ are also very good candidates. Increased value of the Planck’s constant will increase the cyclotron absorption energy and increased $c$ will give tycheons: noncausal faster than light particles. The effect of passing a flux quantum is to change the single-body wave function from,

$$(z - z_o)^m \exp(-\frac{1}{4}|z|^2) \text{ to } (z - z_o)^{m+1} \exp(-\frac{1}{4}|z|^2).$$ (13)

An approximate representation of these states is chosen by Laughlin for the quasielectron and the quasihole. Laughlin writes $|\psi^{z_o}|^2$ as $\exp(-\beta \phi t)$ with $\beta=1/m$ and $\phi t=\phi-2\Sigma_\ell \ln|z_\ell|$.
\( z_o \) where \( \phi \) describes an OCP interacting with a phantom point charge at \( z_o \). The plasma screens this phantom by accumulating an equal and opposite charge near \( z_o \). The particle charge in the plasma is 1, rather than \( m \) so the accumulated charge is \( 1/m \), i.e., \( m_{e\text{eff}} = 1 \). The energy required to create a particle of Debye length \( a_o/\sqrt{2} \) is given by,

\[
\Delta_D = \frac{\pi}{4\sqrt{2}} \frac{e^2}{m^2 a_o}.
\]  

(14)

Note that the quantity which enters is again \( m^2 a_o \) and not just \( e/m \) so that the charge can be changed from \( e \) to \( e/m \) or the charge may be kept constant at \( e \) and \( a_o \) changed to \( m^2 a_o \). Therefore, two equivalent possibilities exist. One is to define the effective charge, \( e_{\text{eff}} = e/m \) and the other is to keep \( e \) unchanged and change \( a_o \) to \( a_{\text{eff}} = m^2 a_o \). The state described by \( \psi_m \) is incompressible because compressing it is equal to injecting particles and particles carry charge so the fractional charge will be destroyed. The incompressibility is not built in the hamiltonian and is extraneously imposed. It is easy to impose an external boundary condition that there is incompressibility but in the expression (14) such an incompressibility can not be enforced. Therefore, the incompressibility enforcement is not contained in the theory. The compressibility creates sound waves so the incompressibility causes the sound to be absent. This is equivalent to eliminating phonons, which has not been done, so the resistivity can touch zero value. In the BCS theory of superconductivity, the zero resistance is obtained not by introducing incompressibility but by eliminating phonons which makes the electrons attractive. We do not discuss the impurities or rotations because these are not contained in our hamiltonian. The Hall conductance is \( (1/m)e^2/h \) so that the effective charge is \( e_{\text{eff}} = (1/m)e \) and the factor \( e/h \) is caused by the units. The origin of this effective charge is not clear because of the other candidates and for \( m=3 \), the charge \( e/3 \) is not generated by the hamiltonian discussed by Laughlin but the antisymmetry surely requires that \( m \) is odd, such as 1, 3, 5, 7, ... etc. Therefore, odd aspect is a result of antisymmetry. Laughlin does not use spin so that there is spin-charge decoupling automatically built in the calculation. The spinless electrons may give unphysical results. Below eq. (13) it is argued that introducing a flux quantum \( \Delta \phi = hc/e \) results into accumulation of charge \( 1/m \). In fact, the charge need not
accumulate and only the field is modified as in (8). Therefore, the factor $1/m$ does not determine the charge of the excitations. In a different context Laughlin has pointed out that the bulk modulus of the Hartree-Fock ground state was zero. The bulk modulus is actually finite, as is usually the case for Jastrow-type trial wave functions for helium as admitted in an errata\textsuperscript{2}.

4. Missing charge.

If we agree to the choice of the charge and change it from $e$ to $e/3$, then what happened to the remaining charge of $2/3$? If we say that $e$ remains $e$ and $H$ changes to $H/3$, then we need not look for the missing charge. The wave function (4) does not conserve charge. Laughlin does not provide any prescription to find the missing charge.

5. Conclusions

The exact wave function, the excitations of which are fractionally charged does not determine the quasiparticle charge uniquely. The blame of the fraction need not be thrown on the quasiparticle charge. The introduction of the alternative choice of variable, instead of charge, does not destroy the exactness of the calculation. The magnetic field of the flux quantization condition has been left out.

It is clear that Laughlin’s wave function can not explain the experimental observations of the quantum Hall effect. It may be pointed out that the composite fermion model (CF) which requires that the flux quanta be attached to the electron is also not correct.\textsuperscript{3–5}

In the expression (13) of Arovas et al\textsuperscript{6} the quantity $eA_\phi$ can be replaced by $(e/m)A_\phi$. Now, multiply $A_\phi$ by $g$ so that the quantity of interest becomes $(e/m)gA_\phi$ with $g=1$. It is clear that $(e/m)$ and $(g/m)$ are not resolvable because $(e/m)gA_\phi$ is exactly equal to $e(g/m)A_\phi$. Therefore, whether the charge should be fractionalized or the unit flux should be corrected are not resolved. If charge is corrected, then fractional statistics will come otherwise the unit flux will be changed. This means that the fermion need not obey the “fractional statistics” and $\phi_\circ$ will be modified.

The correct theory of the quantum Hall effect is given in ref.7.
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