See-Saw Realization of the Texture Zeros in the Neutrino Mass Matrix

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ABSTRACT

We study the see-saw realization of seven textures of the neutrino mass matrix, which were presented by Frampton, Glashow and Marfatia. Two of them (B₁ and B₂) are not realized in the see-saw mechanism without fine-tuning of parameters. We present some specific textures of the Dirac neutrino mass matrix and the right-handed Majorana neutrino one. In order to test these textures, we discuss the effect on the branching ratio of $\mu \rightarrow e\gamma$. We also study the $U(1)_X \times U(1)_{X'}$ flavor symmetry, in which $U(1)_X$ is anomalous and $U(1)_{X'}$ is non-anomalous, to reproduce texture zeros. We present examples of U(1) charges for two textures (A₁ and A₂).

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The Super-Kamiokande has almost confirmed the neutrino oscillation in the atmospheric neutrinos, which favors the $\nu_\mu \rightarrow \nu_\tau$ process \[1\]. For the solar neutrinos \[2, 3\], the recent data of the Super-Kamiokande and the Sudbury Neutrino Observatory also favor strongly the neutrino oscillation $\nu_e \rightarrow \nu_\mu, \tau$ with the large mixing angle(LMA) MSW solution \[4, 5\]. These results indicate the neutrino masses and mixings, especially, the bi-large flavor mixing. It is therefore important to investigate how the textures of lepton mass matrices can link up with the observables of the flavor mixing \[1, 4\].

Recently, Frampton, Glashow and Marfatia \[8\] found seven acceptable textures of the neutrino mass matrix with two independent vanishing entries in the basis with the diagonal charged lepton masses. The further study of these textures was presented by Xing \[9\]. Another insight has been also given focusing on texture zeros \[10\].

Since these textures are given for the light effective neutrino mass matrix, one needs to find the see-saw realization \[11\] of these textures from the standpoint of the model building. In this paper we have examined the see-saw realization of those seven textures. It is found that two of them are not given by the see-saw mechanism without fine tunings between parameters of the Dirac neutrino mass matrix and the right-handed Majorana neutrino one. Some specific textures of the Dirac neutrino mass matrix and the right-handed Majorana neutrino one are presented. These textures are discussed in terms of the branching ratio of $\mu \rightarrow e\gamma$ in order to test them experimentally. The $U(1)_X \times U(1)_{X'}$ flavor symmetry, in which $U(1)_X$ is anomalous and $U(1)_{X'}$ is non-anomolous, is discussed to reproduce texture zeros. In this symmetry, zeros in the mass matrix are derived from SYSY zeros, which are due to holomorphy of the superpotential.

There are seven acceptable textures with two independent zeros for the effective neutrino mass matrix as shown in Table 1. The cases $A_1$ and $A_2$ correspond to the hierarchical neutrino mass spectrum while $B_1 \sim B_4$ and $C$ correspond to the degenerate neutrino mass spectrum. Generally these textures are realized in the see-saw mechanism. However, as far as we exclude the possibility that these zeros are originated from accidental cancellations in the see-saw mechanism, the see-saw realization of these seven textures are not trivial. Then, these zeros should come from zeros of the Dirac neutrino mass matrix and the right-handed Majorana mass matrix. Therefore it is significant to study the see-saw realization of these seven textures.

In terms of the Dirac neutrino mass matrix $m_D$ and the right-handed Majorana neutrino mass matrix $M_R$, the effective neutrino mass matrix $M^e$ is given as

$$M^e = m_D M_R^{-1} m_D^T.$$ \[1\]

Therefore, each entry of the effective neutrino mass matrix $M^e$ is given in terms of $m_{ij}$, which is the component of $m_D$, and $M_{ij}$, which is the component of $M_R$. Zeros in $m_{ij}$ and $M_{ij}$ provide texture zeros in the effective neutrino mass matrix since we do not consider accidental cancellations between $m_{ij}$ and $M_{ij}$. The components of the effective neutrino mass matrix are written as

$$M^e_{ij} = \sum_{k, \ell=1}^3 m_{ik} M_{j\ell} (M_R^{-1})_{k\ell},$$ \[2\]

where

$$M_R^{-1} = \frac{1}{D} \begin{pmatrix} M_{22}M_{33} - M_{23}^2 & M_{13}M_{23} - M_{12}M_{33} & M_{12}M_{23} - M_{13}M_{22} \\ M_{13}M_{23} - M_{12}M_{33} & M_{11}M_{33} - M_{23}^2 & M_{12}M_{13} - M_{11}M_{23} \\ M_{12}M_{23} - M_{13}M_{22} & M_{12}M_{13} - M_{11}M_{23} & M_{11}M_{22} - M_{12}^2 \end{pmatrix},$$ \[3\]
where $D$ denotes the determinant of $M_R$. In order to answer the question where zeros come from, we try to find textures with zeros for the Dirac neutrino mass matrix and the right-handed Majorana neutrino mass matrix by solving $M_{\nu i}^{\nu} = 0$ without cancellations among terms in eq.(2). This has been done by both analytic search and computer search.

At first, we present three remarks, in which the texture zeros are not obtained by the see-saw mechanism.

Remark 1: There is no see-saw realization in the cases of $B_1$ and $B_2$.

For the case $B_1$, putting constraints $M_{\nu 13}^{\nu} = M_{\nu 31}^{\nu} = M_{\nu 22}^{\nu} = 0$ with other non-zero components, we have searched textures with zeros of the Dirac and the right-handed Majorana neutrino mass matrices without fine tuning of the parameters. It is concluded that there is no solution to reproduce the case $B_1$. There is also no solution for $B_2$. Thus, $B_1$ and $B_2$ are unfavored for the model building with the see-saw mechanism. Therefore, the following studies focus on the five textures $A_1$, $A_2$, $B_3$, $B_4$ and $C$.

Remark 2: There is no see-saw realization for the five textures unless both Dirac neutrino mass matrix and right-handed Majorana neutrino one have zeros. In other words, the texture of $m_D$ or $M_R$ without zeros does not lead to desired five textures $A_1$, $A_2$, $B_3$, $B_4$ and $C$.

Remark 3: There is no see-saw realization for the five textures if the right-handed Majorana neutrino mass matrix is diagonal. This remark is important one since there are many discussions taking the diagonal basis for the right-handed Majorana neutrino mass matrix.

These remarks are useful guides to build models of the neutrino mass matrix.

If the Dirac neutrino mass matrix is specified, we can find possible textures of the right-handed Majorana neutrinos. In order to see the situation, we consider the Fritzsch texture in the Dirac neutrino mass matrix [12] for simplicity. In this case, there are three textures of the right-handed Majorana neutrinos only for the type $A_2$ as follows:

$$m_D = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}; \quad M_R = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}, \quad \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \quad \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad (4)$$

where $\times$ denotes non-zero entry. For other four textures $A_1$, $B_3$, $B_4$ and $C$, there is no solution. Thus, the type of $M^{\nu}$ and the texture of the right-handed Majorana neutrinos is selected once the texture of the Dirac neutrinos is specified.

Another simple example is the diagonal texture for the Dirac neutrino mass matrix. Then, the right-handed Majorana neutrino mass matrices are shown in Table 1. There are only two textures for $A_1$, $A_2$, $B_3$ and $B_4$, respectively, but nothing for $C$.

Now, let us fix the right-handed Majorana neutrino mass matrix. The first attempt is the simplest texture, which is parametrized by two parameters. This case corresponds to at least two degenerate right-handed Majorana neutrino masses. There are three types $a_0$, $b_0$, $c_0$ as follows:

$$M_R; \quad a_0 : \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & 0 \end{pmatrix}, \quad b_0 : \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad c_0 : \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}. \quad (5)$$

It is helpful to comment on the symmetry of these texture. The textures $b_0$ and $c_0$ are derived through the permutation of flavors of the right-handed neutrinos in the texture $a_0$. The per-
mutation of $2 \leftrightarrow 3$ and the cyclic permutation $3 \rightarrow 2 \rightarrow 1 \rightarrow 3$ in $a_0$ give $b_0$, while the $1 \leftrightarrow 2$ and the cyclic permutation $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$ in $a_0$ give $c_0$.

For $a_0$, $b_0$ and $c_0$ of $M_R$, there are ten textures of the Dirac neutrino mass matrix in $A_1$, $A_2$, $B_3$ and $B_4$, respectively while only two textures in $C$. In Tables 2 - 6, we present typical textures of the Dirac neutrino mass matrix for $a_0$ and $b_0$. As seen in the tables, textures of the Dirac neutrino mass matrix for $b_0$ are obtained by the permutation $2 \leftrightarrow 3$ in the right-handed sector (columns in the Dirac neutrino mass matrix). The cyclic permutation $3 \rightarrow 2 \rightarrow 1 \rightarrow 3$ also leads to another texture of the Dirac neutrino mass matrix, which is not presented in the tables.

Let us discuss about the ten textures of the Dirac neutrinos for cases $A_1$, $A_2$, $B_3$ and $B_4$. Two of them have three independent zeros, six of them have four zeros and remained two textures have five zeros. For $C$, there are only two textures with four zeros. We show these textures taking $a_0$ for the right-handed Majorana neutrino in the case of $A_1$:

$$m_D = \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & 0 & \times \end{pmatrix}, \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & \times & \times \end{pmatrix}, \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & 0 & \times \end{pmatrix}, \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix},$$ (6)

and other five textures are obtained by the symmetry such as:

$$m'_D = m_D \, P, \quad \text{with} \quad P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$ (7)

Since the case $c_0$ is also obtained from $a_0$ by the permutation $1 \leftrightarrow 2$ and the cyclic permutation $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$ in the right-handed sector, we have omitted results of $c_0$ in the tables.

The right-handed Majorana neutrino mass matrices in eq. (3) are modified by adding a new parameter. Since these have three independent zeros, the mass eigenvalues are not degenerate in general. The texture $a_0$ is modified as follow:

$$M_R; \begin{array}{l} a_1: \begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & 0 \end{pmatrix}, \quad a_2: \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}, \quad a_3: \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}, \quad a_4: \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}. \end{array}$$ (8)

For the texture $b_0$, we have

$$M_R; \begin{array}{l} b_1: \begin{pmatrix} \times & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad b_2: \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}, \quad b_3: \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}, \quad b_4: \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}, \end{array}$$ (9)

and for the texture $c_0$, we have

$$M_R; \begin{array}{l} c_1: \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix}, \quad c_2: \begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad c_3: \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad c_4: \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}. \end{array}$$ (10)
For textures of $M_R$ with one zero in the diagonal entries $(a_1, a_3, b_1, b_3)$, there are 6 textures of the Dirac neutrino mass matrix to reproduce $A_1$, $A_2$, $B_3$ and $B_4$. For textures of $M_R$ with two zeros in the diagonal entries $(a_2, a_4, b_2, b_4)$, there are 8 textures of the Dirac neutrino mass matrix for $A_1$, $A_2$, $B_3$ and $B_4$. It is remarked that there is no texture of the Dirac neutrino mass matrix to reproduce $C$. We show the typical textures of the Dirac neutrino mass matrix in Tables 2 - 6. Since the $b_i$ ($i = 1 - 4$) are also derived by the permutation $2 \leftrightarrow 3$ in $a_i$, the textures of the Dirac neutrino mass matrix correspond to the permutation $2 \leftrightarrow 3$ in their columns. The cyclic permutation $3 \rightarrow 2 \rightarrow 1 \rightarrow 3$ also leads to another texture of the Dirac neutrino mass matrix. We have also omitted the case of $c_1$ in tables.

What kind of experiments can test these textures? It is well known that the Yukawa coupling of the neutrino contributes to the lepton flavor violation (LFV). Many authors have studied the LFV in the minimal supersymmetric standard model (MSSM) with right-handed neutrinos assuming the relevant neutrino mass matrix [13, 16, 17, 18, 19, 20]. In the MSSM with soft breaking terms, there exist lepton flavor violating terms such as off-diagonal elements of slepton mass matrices and trilinear couplings (A-term). It is noticed that large neutrino Yukawa couplings and large lepton mixings generate the large LFV in the left-handed slepton masses. For example, the decay rate of $\mu \rightarrow e\gamma$ can be approximated as follows:

$$\Gamma(\mu \rightarrow e\gamma) \approx \frac{e^2}{16\pi} m_\mu^5 F \left| \frac{6 + 2a_1^2}{16\pi^2} (Y_\nu Y'_\nu)^{21} \ln \frac{M_X}{M_R} \right|^2,$$  

where the neutrino Yukawa coupling matrix $Y_\nu$ is given as $Y_\nu = m_D/v_2$ ($v_2$ is a VEV of Higgs) at the right-handed mass scale $M_R$, and $F$ is a function of masses and mixings for SUSY particles. In eq.(11), we assume the universal scalar mass ($m_0$) for all scalars and the universal A-term ($A_f = a_0m_0Y_f$) at the GUT scale $M_X$. Therefore the branching ratio $\mu \rightarrow e\gamma$ depends considerably on the texture of the Dirac neutrino [13, 19, 20].

Let us investigate $m_Dm_D^\dagger$ focusing on the process $\mu \rightarrow e\gamma$. Zeros in the Dirac neutrino mass matrix may suppress the $\mu \rightarrow e\gamma$ decay. There are many textures, which lead to $(m_Dm_D^\dagger)^{21} = 0$ as seen in eq.(2). Since the contribution of the neutrino Yukawa couplings on $\mu \rightarrow e\gamma$ is tiny in these textures, the branching ratio is safely predicted to be below the present experimental upper bound $1.2 \times 10^{-11}$ [21]. We call these textures as type II and other textures as type I, which can contribute considerably to the branching ratio. We present typical textures of the Dirac neutrino mass matrix and those numbers by classifying the type I and type II in Tables 2, 3, 4 and 5. The type I and II will be tested in the future experiments of the $\mu \rightarrow e\gamma$ decay.

Thus, we find a lot of sets which lead to five textures $A_1$, $A_2$, $B_3$, $B_4$ and $C$ through the see-saw mechanism without fine-tuning. The important problem is the origin of zeros. An interesting origin of zeros is the holomorphic zeros in the supersymmetry with the anomalous $U(1)$ flavor symmetry [13], which are called SUSY zeros. Although one can impose SUSY zeros in the neutrino mass matrix by assigning the relevant $U(1)$ charges, one cannot choose the diagonal basis in the charged lepton mass matrix. In other words, it is impossible to impose zeros of these textures keeping the diagonal charged lepton mass matrix in the case of the $U(1)$ flavor symmetry.

A simple way to avoid this difficulty is to have one anomalous $U(1)_X$ and another non-anomalous $U(1)_{X'}$, which are broken by two scalar fields $\phi_1$ and $\phi_2$ having the flavor charges $(-1, -1)$ and $(0, 1)$ [14]. Since the D-term potential guarantees $\langle \phi_1 \rangle = \langle \phi_2 \rangle$ in this model, the
effective interactions of the relevant sector are given as
\[
L_i \bar{\ell}_j H_1 \lambda^{x_i+y_j-x'_i-y'_j} \lambda^{x_i+y_j} + \frac{1}{M_R} L_i L_j H_2 \lambda^{x_i+x_j-x'_i-x'_j} \lambda^{x_i+x_j}, \quad (12)
\]
where
\[
\lambda \equiv \frac{\langle \phi_1 \rangle}{\Lambda} = \frac{\langle \phi_2 \rangle}{\Lambda}, \quad (13)
\]
and \((x_i, x'_i)\) are flavor charges of left-handed lepton doublets \(L_i\), and \((y_j, y'_j)\) are flavor charges of right-handed lepton singlets \(\ell_j\) for \(U(1)_X\) and \(U(1)_{X'}\), respectively. If the power of \(\lambda\) in eq.\((12)\) is negative, the interaction is forbidden. Then, the texture zero, which is SUSY zero, is reproduced in the mass matrix.

We find \(U(1)\) charges to realize the patterns of \(A_1\) and \(A_2\) as follows:

\[
A_1 : \quad (x_1, x_2, x_3) = (-3, 2, 4), \quad (x'_1, x'_2, x'_3) = (-11, 0, 4),
\]
\[
(y_1, y_2, y_3) = (3, 0, -4), \quad (y'_1, y'_2, y'_3) = (6, 2, -4), \quad (14)
\]
which give
\[
M^\nu \propto \lambda^8 \begin{pmatrix} 0 & 0 & \lambda \\ 0 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}, \quad M^e \propto \begin{pmatrix} \lambda^5 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (15)
\]
and

\[
A_2 : \quad (x_1, x_2, x_3) = (-3, 4, 2), \quad (x'_1, x'_2, x'_3) = (-11, 4, 0),
\]
\[
(y_1, y_2, y_3) = (3, -4, -2), \quad (y'_1, y'_2, y'_3) = (6, -6, 0), \quad (16)
\]
which give
\[
M^\nu \propto \lambda^8 \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \quad M^e \propto \begin{pmatrix} \lambda^5 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (17)
\]
where \(U(1)\) charges of Higgs scalars are taken to be zero, and \(\lambda \simeq 0.22\) is supposed. Since the magnitude of each entry has been estimated numerically by Xing [9], we can compare eqs.\((14)\) and \((16)\) with his numerical results. We have found that our results are completely consistent with them. Since the degenerate mass spectrum is unlikely realized in the \(U(1)\) flavor symmetry, we do not discuss the \(U(1)\) charges in the pattern of \(B_3\), \(B_4\) and \(C\).

In order to present the see-saw realization of the \(U(1)\) symmetry, we show two examples including the \(U(1)\) charges of right-handed neutrinos. For \(A_1\), \(U(1)\) charges of right-handed neutrinos \(N_i\) and \(N'_i\) are taken as
\[
(N_1, N_2, N_3) = (3, -3, 0), \quad (N'_1, N'_2, N'_3) = (6, -6, -5), \quad (18)
\]
which with eq.\((14)\) give
\[
M_R \propto \begin{pmatrix} 0 & 1 & \lambda^5 \\ 1 & 0 & 0 \\ \lambda^5 & 0 & \lambda^{10} \end{pmatrix}, \quad m_D \propto \lambda^4 \begin{pmatrix} \lambda & 0 & 0 \\ 0 & 0 & \lambda^5 \\ 0 & 1 & \lambda^5 \end{pmatrix}. \quad (19)
\]
For $A_2$, we take

\[
(N_1, N_2, N_3) = (3, -3, 0), \quad (N_1', N_2', N_3') = (4, -4, -1),
\]

which with eq.(16) give

\[
\begin{align*}
M_R &\propto \begin{pmatrix} 0 & 1 & \lambda^3 \\ 1 & 0 & 0 \\ \lambda^3 & 0 & \lambda^2 \end{pmatrix}, \\
m_D &\propto \lambda^2 \begin{pmatrix} \lambda^5 & 0 & 0 \\ 0 & 1 & \lambda^3 \\ \lambda^4 & 0 & \lambda^3 \end{pmatrix}.
\end{align*}
\]

(21)

Thus, the $U(1)_X \times U(1)_{X'}$ flavor symmetry gives the neutrino mass matrix which is consistent with the experimental data.

The summary is given as follows. We have studied the see-saw realization of seven textures of the neutrino mass matrix, which were presented by Frampton, Glashow and Marfatia. Two of them ($B_1$ and $B_2$) are not realized in the see-saw mechanism without fine-tuning of parameters. Some specific textures of the Dirac neutrino mass matrix and the right-handed Majorana neutrino one have been presented. In order to test these textures experimentally, the branching ratio of $\mu \rightarrow e\gamma$ has been discussed.

The $U(1)_X \times U(1)_{X'}$ flavor symmetry, in which one $U(1)_X$ is anomalous and $U(1)_{X'}$ is non-anomalous, has been studied to reproduce the texture zeros. An example of U(1) charges has been found for $A_1$ and $A_2$, respectively, which give hierarchical neutrino masses. Other approaches are also necessary in the model building of two-zeros textures [22].

It is expected that the texture of the neutrino mass matrix is more constrained by the further coming experiments. Therefore, the systematic study becomes also available in the framework of the see-saw mechanism in the near future.

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Table 1: Seven patterns of the neutrino mass matrix $M^\nu$ with two independent vanishing entries. Possible right-handed Majorana mass matrices are presented in the case of the diagonal Dirac neutrino mass matrix.

| Pattern | Texture of $M^\nu$ | Dirac $m_D$ | Majorana $M_R$ |
|---------|-------------------|-------------|----------------|
| A_1     | $\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$ | $\begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$ | $\begin{pmatrix} \times & \times & \times \\ \times & \times & 0 \\ \times & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & \times & \times \\ \times & \times & 0 \\ \times & 0 & 0 \end{pmatrix}$ |
| A_2     | $\begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}$ | $\begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$ | $\begin{pmatrix} \times & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$ |
| B_1     | $\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$ | $\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$ | $\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$, $\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$ |
| B_2     | $\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$ | $\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$ | $\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$, $\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$ |
| B_3     | $\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$ | $\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$ | $\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$, $\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$ |
| B_4     | $\begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & \times & 0 \end{pmatrix}$ | $\begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & \times & 0 \end{pmatrix}$ | $\begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & \times & 0 \end{pmatrix}$, $\begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & \times & 0 \end{pmatrix}$ |
| C       | $\begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$ | $\begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$ | $\begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$ | nothing |
Table 2: In the case of $A_1$, typical Dirac neutrino textures of types I and II are presented for the right-handed Majorana neutrino textures $a_i$ and $b_i$. Textures $c_i$ are omitted. The total $\sharp$ denotes the numbers of possible Dirac neutrino textures, $\ddagger$ denotes the numbers of Dirac neutrino textures of types I and II, respectively.

|    | Majorana $M_R$ | total $\sharp$ | Typical $m_D$ of type I | $\ddagger$ | Typical $m_D$ of type II | $\ddagger$ |
|----|----------------|----------------|--------------------------|-----------|--------------------------|-----------|
| $a_0$ | $\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & 0 \end{pmatrix}$ | 10 | $\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$ | 6 | $\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & \times & \times \end{pmatrix}$ | 4 |
| $a_1$ | $\begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & 0 \end{pmatrix}$ | 6 | $\begin{pmatrix} \times & 0 & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix}$ | 4 | $\begin{pmatrix} \times & 0 & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix}$ | 2 |
| $a_2$ | $\begin{pmatrix} 0 & \times & \times \\ \times & \times & 0 \\ \times & 0 & 0 \end{pmatrix}$ | 8 | $\begin{pmatrix} 0 & \times & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix}$ | 4 | $\begin{pmatrix} 0 & \times & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix}$ | 4 |
| $a_3$ | $\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}$ | 6 | $\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$ | 4 | $\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & \times & \times \end{pmatrix}$ | 2 |
| $a_4$ | $\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$ | 8 | $\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$ | 4 | $\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & \times & \times \end{pmatrix}$ | 4 |
| $b_0$ | $\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$ | 10 | $\begin{pmatrix} 0 & \times & 0 \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$ | 6 | $\begin{pmatrix} 0 & \times & 0 \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$ | 4 |
| $b_1$ | $\begin{pmatrix} \times & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$ | 6 | $\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}$ | 4 | $\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}$ | 2 |
| $b_2$ | $\begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & \times \end{pmatrix}$ | 8 | $\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}$ | 4 | $\begin{pmatrix} 0 & \times & 0 \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$ | 4 |
| $b_3$ | $\begin{pmatrix} 0 & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$ | 6 | $\begin{pmatrix} 0 & \times & 0 \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$ | 4 | $\begin{pmatrix} 0 & \times & 0 \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$ | 2 |
| $b_4$ | $\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$ | 8 | $\begin{pmatrix} 0 & \times & 0 \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$ | 4 | $\begin{pmatrix} 0 & \times & 0 \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$ | 4 |
Table 3: In the case of $A_2$, typical Dirac neutrino textures of types I and II are presented.

| Majorana $M_R$ | total $\ell$ | Typical $m_D$ of type I | Typical $m_D$ of type II | |
|---------------|--------------|--------------------------|--------------------------|---|
| $a_0$ | \[
\begin{pmatrix}
0 & 0 & \times \\
0 & \times & 0 \\
\times & 0 & 0
\end{pmatrix}
\] | 10 | \[
\begin{pmatrix}
0 & 0 & \times \\
\times & \times & \times \\
0 & \times & \times
\end{pmatrix}
\] | 6 | \[
\begin{pmatrix}
0 & 0 & \times \\
\times & 0 & 0 \\
0 & \times & \times
\end{pmatrix}
\] | 4 |
| $a_1$ | \[
\begin{pmatrix}
\times & 0 & \times \\
0 & \times & 0 \\
\times & 0 & 0
\end{pmatrix}
\] | 6 | \[
\begin{pmatrix}
\times & 0 & 0 \\
\times & \times & \times \\
\times & \times & 0
\end{pmatrix}
\] | 3 | \[
\begin{pmatrix}
\times & 0 & 0 \\
0 & \times & 0 \\
\times & \times & 0
\end{pmatrix}
\] | 3 |
| $a_2$ | \[
\begin{pmatrix}
0 & \times & \times \\
\times & \times & 0 \\
\times & 0 & 0
\end{pmatrix}
\] | 8 | \[
\begin{pmatrix}
\times & 0 & 0 \\
\times & \times & \times \\
\times & \times & 0
\end{pmatrix}
\] | 4 | \[
\begin{pmatrix}
\times & 0 & 0 \\
0 & \times & \times \\
\times & \times & 0
\end{pmatrix}
\] | 4 |
| $a_3$ | \[
\begin{pmatrix}
0 & 0 & \times \\
0 & \times & 0 \\
\times & 0 & 0
\end{pmatrix}
\] | 6 | \[
\begin{pmatrix}
0 & 0 & \times \\
\times & \times & \times \\
0 & \times & \times
\end{pmatrix}
\] | 3 | \[
\begin{pmatrix}
0 & 0 & \times \\
\times & 0 & \times \\
0 & \times & \times
\end{pmatrix}
\] | 3 |
| $a_4$ | \[
\begin{pmatrix}
0 & \times & \times \\
0 & \times & \times \\
\times & 0 & 0
\end{pmatrix}
\] | 8 | \[
\begin{pmatrix}
0 & 0 & \times \\
\times & \times & \times \\
0 & \times & \times
\end{pmatrix}
\] | 4 | \[
\begin{pmatrix}
0 & 0 & \times \\
\times & 0 & \times \\
0 & \times & \times
\end{pmatrix}
\] | 4 |
| $b_0$ | \[
\begin{pmatrix}
0 & \times & 0 \\
\times & 0 & 0 \\
0 & \times & 0
\end{pmatrix}
\] | 10 | \[
\begin{pmatrix}
0 & \times & 0 \\
\times & \times & \times \\
0 & \times & \times
\end{pmatrix}
\] | 6 | \[
\begin{pmatrix}
0 & \times & 0 \\
\times & 0 & \times \\
0 & \times & \times
\end{pmatrix}
\] | 4 |
| $b_1$ | \[
\begin{pmatrix}
\times & \times & 0 \\
\times & 0 & 0 \\
0 & \times & 0
\end{pmatrix}
\] | 6 | \[
\begin{pmatrix}
\times & 0 & 0 \\
\times & \times & \times \\
0 & \times & \times
\end{pmatrix}
\] | 3 | \[
\begin{pmatrix}
\times & 0 & 0 \\
0 & \times & \times \\
0 & \times & \times
\end{pmatrix}
\] | 3 |
| $b_2$ | \[
\begin{pmatrix}
0 & \times & \times \\
\times & 0 & 0 \\
\times & 0 & 0
\end{pmatrix}
\] | 8 | \[
\begin{pmatrix}
\times & 0 & 0 \\
\times & \times & \times \\
\times & 0 & \times
\end{pmatrix}
\] | 4 | \[
\begin{pmatrix}
\times & 0 & 0 \\
\times & 0 & \times \\
\times & 0 & \times
\end{pmatrix}
\] | 4 |
| $b_3$ | \[
\begin{pmatrix}
0 & \times & 0 \\
\times & \times & 0 \\
0 & \times & 0
\end{pmatrix}
\] | 6 | \[
\begin{pmatrix}
0 & \times & 0 \\
\times & \times & \times \\
0 & \times & \times
\end{pmatrix}
\] | 3 | \[
\begin{pmatrix}
0 & \times & 0 \\
\times & 0 & \times \\
0 & \times & \times
\end{pmatrix}
\] | 3 |
| $b_4$ | \[
\begin{pmatrix}
0 & \times & 0 \\
\times & 0 & \times \\
0 & \times & \times
\end{pmatrix}
\] | 8 | \[
\begin{pmatrix}
0 & \times & 0 \\
\times & \times & \times \\
0 & \times & \times
\end{pmatrix}
\] | 4 | \[
\begin{pmatrix}
0 & \times & 0 \\
\times & 0 & \times \\
0 & \times & \times
\end{pmatrix}
\] | 4 |
Table 4: In the case of B\textsubscript{3}, typical Dirac neutrino textures of types I and II are presented.

| Majorana $\mathbf{M}_R$ | total $\#$ | Typical $\mathbf{m}_D$ of type I | $\#$ | Typical $\mathbf{m}_D$ of type II | $\#$ |
|---------------------------|-----------|----------------------------------|-----|----------------------------------|-----|
| $a_0$                     | 10        | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | 6   | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | 4   |
| $a_1$                     | 6         | $\begin{pmatrix} x & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | 4   | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | 2   |
| $a_2$                     | 8         | $\begin{pmatrix} 0 & x & x \\ x & x & 0 \end{pmatrix}$ | 4   | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | 4   |
| $a_3$                     | 6         | $\begin{pmatrix} 0 & 0 & x \\ 0 & 0 & 0 \end{pmatrix}$ | 4   | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | 2   |
| $a_4$                     | 8         | $\begin{pmatrix} 0 & 0 & x \\ x & x & 0 \end{pmatrix}$ | 4   | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | 4   |
| $b_0$                     | 10        | $\begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \end{pmatrix}$ | 6   | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | 4   |
| $b_1$                     | 6         | $\begin{pmatrix} x & x & 0 \\ x & 0 & 0 \end{pmatrix}$ | 4   | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | 2   |
| $b_2$                     | 8         | $\begin{pmatrix} x & x & 0 \\ x & 0 & 0 \end{pmatrix}$ | 4   | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | 4   |
| $b_3$                     | 6         | $\begin{pmatrix} 0 & x & 0 \\ x & 0 & 0 \end{pmatrix}$ | 4   | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | 2   |
| $b_4$                     | 8         | $\begin{pmatrix} 0 & x & 0 \\ x & 0 & 0 \end{pmatrix}$ | 4   | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | 4   |
Table 5: In the case of $B_4$, typical Dirac neutrino textures of types I and II are presented.

| Majorana $M_R$ | total $\#$ | Typical $\boldsymbol{m_D}$ of type I | # | Typical $\boldsymbol{m_D}$ of type II | # |
|----------------|------------|---------------------------------------|---|---------------------------------------|---|
| $a_0$          |            | $\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & 0 \end{pmatrix}$ | 10 | $\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ \times & 0 & 0 \end{pmatrix}$ | 10 |
| $a_1$          |            | $\begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & 0 \end{pmatrix}$ | 6  | $\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ \times & 0 & 0 \end{pmatrix}$ | 5  |
| $a_2$          |            | $\begin{pmatrix} 0 & \times & \times \\ \times & \times & 0 \\ \times & 0 & 0 \end{pmatrix}$ | 8  | $\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ \times & 0 & 0 \end{pmatrix}$ | 5  |
| $a_3$          |            | $\begin{pmatrix} 0 & 0 & \times \\ \times & 0 & \times \\ 0 & \times & 0 \end{pmatrix}$ | 6  | $\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & 0 & 0 \end{pmatrix}$ | 5  |
| $a_4$          |            | $\begin{pmatrix} 0 & \times & \times \\ \times & \times & 0 \\ \times & 0 & 0 \end{pmatrix}$ | 8  | $\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & 0 & 0 \end{pmatrix}$ | 5  |
| $b_0$          |            | $\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & \times & 0 \end{pmatrix}$ | 10 | $\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & 0 & 0 \end{pmatrix}$ | 10 |
| $b_1$          |            | $\begin{pmatrix} \times & \times & 0 \\ \times & 0 & 0 \\ 0 & \times & 0 \end{pmatrix}$ | 6  | $\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & 0 & 0 \end{pmatrix}$ | 5  |
| $b_2$          |            | $\begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$ | 8  | $\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & 0 & 0 \end{pmatrix}$ | 5  |
| $b_3$          |            | $\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & \times & 0 \end{pmatrix}$ | 6  | $\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & 0 & 0 \end{pmatrix}$ | 5  |
| $b_4$          |            | $\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$ | 8  | $\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & 0 & 0 \end{pmatrix}$ | 5  |
In the case of C, typical Dirac neutrino textures of type I are presented. In this case, there is no type II and there are 2 textures of the Dirac neutrino mass matrix only for $a_0$ and $b_0$.

| Majorana $M_R$ | total $\sharp$ | Typical $m_D$ of type I $\sharp$ | Typical $m_D$ of type II $\sharp$ |
|----------------|----------------|---------------------------------|---------------------------------|
| $a_0$          | 0 0 $\times$  | 2 $\times \times \times$        | 2 nothing                       |
|                | 0 $\times$ 0  |                                 |                                 |
|                | $\times$ 0 0  |                                 |                                 |

| $b_0$          | 0 $\times$ 0  | 2 $\times \times \times$        | 2 nothing                       |
|                | $\times$ 0 0  |                                 |                                 |
|                | 0 0 $\times$  |                                 |                                 |