Determination of the strain rate effect in electromagnetic forming processes

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Abstract. Effect of strain rate, quantity of plastic deformation and temperature of material on characteristics of material at electromagnetic forming is considered. It is shown that strength characteristics increase with growth of strain rate, and the plasticity increases with an increase in the value of plastic deformation and temperature. The variant of writing of the constitutive equation considering these changes is offered, and its use in calculations can significantly increase efficiency of calculations of operations of electromagnetic forming.

1. Introduction

When calculating technology of the electromagnetic forming (EMF) it is necessary to know not only static, but also dynamic parameters of workpiece material, their interrelation with changes of strain rate, quantity of plastic deformation and temperature.

In particular, at EMF loading of material has dynamic character, leads to big plastic deformations and is followed by significant heating of material. Strain rate of material at EMF changes from in the wide range, and reaches quantities about $10^3…10^4 \text{s}^{-1}$. Use of static dependences is inadmissible here as can lead to essential errors. In the mathematical aspect the processes, proceeding at EMF in the «equipment - inductor - workpiece» system, are described by the dynamic equations of electrodynamics and a thermo-elastic plasticity. At the same time it is essential that electromagnetic and thermomechanical groups of the equations are interconnected. In particular, pulse currents heat and change mechanical properties of materials of the inductor and workpiece, and deformation of workpiece leads to change of its geometry and electric parameters – inductance and resistance.

For such specific conditions of loading as EMF, deformation and strength properties of the applied materials are still insufficiently studied, therefore development and specification of mathematical models of workpiece forming processes is necessary.

Therefore, creation of the constitutive equations considering dependence of material characteristics on conditions of pulse loading is a relevant problem.

2. Problem definition

Many works are devoted to a problem of determination of dynamic yield strengths and durability. In particular, in experiments of F. Hauzer, J. Simmons and J. Dorn [1] confirmations on the effect of the strain rate on the type of loading diagrams have been received. In these experiments it is shown that increase in strain rate leads to significant increase in yield strength of material and its deformation hardening. Similar results have been received also for lead [2].

A number of experimental works on research of impact of plastic deformation and temperature of material at quantities of elastic modulus is published. In A.M. Zhukov’s experiments [3] cyclic stretching and unloading of tubular samples from various steel grades was investigated. During all phases of loading and unloading the sample elastic modulus was defined. In particular, at dynamic pulse loading of a sample from steel C1060 decrease in elastic modulus has been noted with a growth...
of quantity of plastic deformation. At dynamic plastic deformations a significant amount of heat, and in the conditions of high-speed loading its branch insignificant therefore material temperature increase significantly reduces its elastic modulus is distinguished. The experimental dependence of elastic modulus on material temperature for steel C1015 where it is shown that dynamic loading of characteristic of materials significantly change is given in work [4]. For practical use of these dynamic changes in calculations of processes of EMF it is necessary to have the mathematical description of interrelations between tension, deformations and their derivatives on time. But even for one-dimensional tension it is rather difficult to receive such mathematical description. Let's consider principal views of constitutive equations.

3. Theory
The most often used theory considering influence of strain rate on material parameters has been offered by L. Malvern in work [5]. The behavior of material was described by the equation:

\[ \sigma = f(\varepsilon) + a \cdot \ln(1 + b \cdot \dot{\varepsilon}_p), \]

(1)

where \( f(\varepsilon) \) is a function of quasistatic loading; \( a \) and \( b \) are coefficients, determined by processing of experimental data, \( \dot{\varepsilon}_p \) is the rate of plastic strain.

This equation can be presented in the form:

\[ \dot{\varepsilon}_p = \frac{1}{b} \left( e^{\frac{\sigma - f(\varepsilon)}{a}} - 1 \right) \]

Here the rate of plastic strain is provided as function of a difference between instantaneous tension and tension at quasistatic loading, which would turn out at the same deformation. This communication can be set in more general form:

\[ E \dot{\varepsilon}_p = F(\sigma - f(\varepsilon)), \]

where \( f \) is arbitrary function.

The same expression for determination of the rate of plastic strain was independently offered V.V. Sokolovsky in work [6]. In the Sokolovsky-Malvern's constitutive equation the full speed of deformation is presented in the form of the sum elastic and plastic components, and has an appearance:

\[ E \dot{\varepsilon} = \dot{\sigma} + F(\sigma - f(\varepsilon)) \cdot H(\sigma - f(\varepsilon)), \]

(2)

where \( H(\sigma - f(\varepsilon)) \) is Heaviside unit function.

P. Perzyna [7] has generalized the Sokolovsky-Malvern’s approach to the case of a complex stress state and has obtained the constitutive equation in the form

\[ \dot{\varepsilon}_{ij} = \frac{1}{2G} S_{ij} + \frac{1 - 2\mu}{E} \dot{\sigma} \delta_{ij} + \gamma(\Phi(F)) \frac{\partial f}{\partial \sigma_{ij}} \]

(3)

where \( \varepsilon_{ij} \) and \( \sigma_{ij} \) are respectively components of tensors of strains and tensions; \( S_{ij} \) are stress tensor deviator components; \( \sigma \) is hydrostatic pressure; \( G, \mu, E \) are constants of elasticity of material; \( \gamma \) is viscosity coefficient of the material; \( F \) is function of material plasticity (fluidity surface); \( f \) is static function of fluidity, depending on stress state \( \sigma_{ij} \) and a state of plastic deformation \( \varepsilon_{ij}^p \); \( \delta_{ij} \) are Kronecker's symbol; \( \Phi \) is non-linear function in the general case:

\[ \langle \Phi(F) \rangle = \begin{cases} 0 & F \leq 0 \\ \Phi(F) & F > 0 \end{cases} \]

Function \( F \) is defined by approximation of the experiments, investigating dynamic properties of materials, and in that specific case reflects influence of strain rate on material yield strength. For monoaxial stress state the equation (3) passes into Sokolovsky-Malverna's equation (2).

The variants of writing the constitutive equations considered above are written in the isothermal formulation. Besides, in them elastic modules \( E, G \) are accepted by constants, though at the big quantity of plastic strain and high temperature of material they significantly decrease [3, 4].
We use a variant of writing the defining equation in the form of P. Perzyna's equation (3), which implicitly takes into account the change in the yield strength and the coefficient of deformation hardening with an increase in the rate of plastic deformation, and the elastic modulus of the material with an increase in the amount of plastic deformation, as well as the change in these characteristics with an increase in the temperature of the material.

We apply a variant of writing the constitutive equation in the form of P. Perzyna's equation (3), which implicitly takes into account the change of yield strength and strain-hardening coefficient with an increase in the quantity of plastic strain rate and also the material elastic modulus with increase in the quantity of plastic deformation, as well as the change in these characteristics with an increase in the temperature of the material [8]:

\[
\dot{\varepsilon}_{ij} = \frac{1}{2G} \dot{S}_{ij} + \frac{1 - 2\mu}{E} \sigma_{ij} + \eta \Phi \left( \sqrt{J_2} \left( \frac{\partial e}{\partial \varepsilon_i} - 1 \right) \right) \frac{S_{ij}}{2\sqrt{J_2}}.
\]

where \( \eta \) is viscosity coefficient; \( \Phi \) is experimentally defined function; \( f_2 \) is the second invariant of deviator tensions; \( \partial e = \partial e(W_p) \) is hardening parameter (the diagram of static loading \( f(e) \) is most often used); \( W_p \) is work of plastic deformation.

Viscosity coefficient \( \eta \) and \( \Phi(\sqrt{J_2}/\partial e - 1) \), which enter the equation (4), are defined by processing of experimental data.

For accounting of change of strain-hardening coefficient, functions, which approximate test results, are added to the constitutional equation as P. Perzyna's equation (4). The most commonly used linear function has the form:

\[
\Phi \left( \sqrt{J_2}/\partial e - 1 \right) = \sqrt{J_2} - \frac{\partial e}{\partial \varepsilon_i} N(\varepsilon_i).
\]

Let’s introduce the notation

\[
N(\varepsilon_i) = \frac{\partial e}{\eta}.
\]

then the plastic component of deformation speed will take a form:

\[
\dot{\varepsilon}_{ij}^p = \sqrt{J_2} - \frac{\partial e}{N(\varepsilon_i)}.
\]

Expressing value \( \sqrt{J_2} \) from the equation (6), we obtained:

\[
\sqrt{J_2} = \partial e + N(\varepsilon_i) \dot{\varepsilon}_{ij}^p.
\]

Accuracy of approximation can be increased, even when maintaining linear nature of function \( \Phi \), by introduction of the additional member to the equation (7):

\[
\sqrt{J_2} = \partial e + M(\varepsilon_i) + N(\varepsilon_i) \cdot \dot{\varepsilon}_{ij}^p.
\]

The equation (8) is the approximating dependence for charts of dynamic loading.

Accuracy of approximation can be increased, even when maintaining linear nature of function \( \Phi \) approximation, the analytical equation is necessary for function definition \( E(\varepsilon_p, T) \). It is offered to use dependence [9]:

\[
E(\varepsilon_p, T) = (E_0 - E_\varepsilon) \cdot e^{-q \varepsilon_p} + E_\varepsilon - K \cdot T,
\]

where \( E_0 \) is elastic modulus at deformation, \( \varepsilon_p = 0; E_\varepsilon, q, K \) are the material constants, defined from an experiment; \( T \) is temperature change.
4. Discussion of results
Let's consider variant of receiving the constitutional equation (4) for the case of a one-dimensional stress state of axial pressure for material steel C255 [10]. In this case the equation (4) and a condition (8) register so:

\[
\dot{\varepsilon} = \frac{1}{N(\varepsilon)} \dot{\sigma} + \frac{\sigma - f(\varepsilon) - M(\varepsilon)}{N(\varepsilon)} \left(1 - \frac{1}{E(\varepsilon, \Delta T)} \frac{\partial E(\varepsilon^p, \Delta T)}{\partial \varepsilon^p} \sigma\right)
\]

\[
\sigma = f(\varepsilon) + M(\varepsilon) + N(\varepsilon) \cdot \dot{\varepsilon}^p,
\]

where \(\sigma\) and \(\varepsilon\) are the monoaxial tension and compressive strain respectively.

When processing experimental data [9] for material steel C255 the least-squares method has received the approximating dependences, which are included into structure of the defining equations (10):

\[
f(\varepsilon) = 241 + 2196 \cdot \varepsilon,
\]

\[
M(\varepsilon) = 266 - 103 \cdot \varepsilon,
\]

\[
N(\varepsilon) = 0.193 + 0.17 \cdot \varepsilon.
\]

Results of approximation are shown in the Figure. Continuous lines are experimental data, dotted lines are built on dependence (11) taking into account (12).

![Figure 1](image-url)

**Figure 1.** Dynamic curves of loading and their approximations for material steel C255. Continuous lines are experimental data, dotted lines are built on equation (11) taking into account (12).

It is shown that use for this material of linear approximation of function \(\Phi\) is sufficiently proved. The difference from experimental data does not exceed 4% that allows to recommend this approach for calculations of technological processes of EMF.

5. Conclusions
The constitutional equation is proposed, that takes into account the change of strain rate, the quantities of plastic deformation and temperature under dynamic loading. Its use in design calculations for operations of electromagnetic forming can significantly increase efficiency of the applied calculations and technologies.

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