Nuclear transition matrix elements for neutrinoless double-β decay within mechanisms involving light Majorana neutrino mass and right-handed current

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Employing the projected-Hartree-Fock-Bogoliubov (PHFB) model in conjunction with four different parametrizations of pairing plus multipolar effective two body interaction and three different parametrizations of Jastrow short range correlations, nuclear transition matrix elements for the neutrinoless double-β decay of $^{94, 96}$Zr, $^{100}$Mo, $^{110}$Pd, $^{128, 130}$Te and $^{150}$Nd isotopes are calculated within mechanisms involving light Majorana neutrino mass and right handed current. Statistically, model specific uncertainties in sets of twelve nuclear transition matrix elements are estimated by calculating the averages along with the standard deviations. For the considered nuclei, the most stringent extracted on-axis limits on the effective light Majorana neutrino mass $< m_\nu >$, the effective weak coupling of right-handed leptonic current with right-handed hadronic current $< \lambda >$, and the effective weak coupling of right-handed leptonic current with left-handed hadronic current $< \eta >$ from the observed limit on half-life $T^{0\nu}_{1/2}$ of $^{130}$Te isotope are $0.33$ eV, $4.57 \times 10^{-7}$ and $4.72 \times 10^{-9}$, respectively.

Keywords: Neutrinoless double beta decay; right-handed current; nuclear transition ma-

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1. Introduction

Observation of the lepton number $L$ violating neutrinoless double beta ($0^{\nu}\beta\beta$) decay is the most pragmatic approach to establish the Majorana nature of neutrinos. Arguably, the violation of lepton number $L$ conservation and Majorana nature of neutrinos are intimately related. In $0^{\nu}\beta\beta$ decay, the neutrino emitted from a nucleon is to be absorbed by another nucleon implying the existence of Majorana neutrino with finite mass. Alternatively, the occurrence of $0^{\nu}\beta\beta$ decay is also possible with the coexistence of right-handed $V+A$ and left-handed $V-A$ currents. In addition, the smallness of neutrino mass as explained by see-saw mechanism requires gauge groups with right-handed current. In several alternative mechanisms based on various gauge theoretical models beyond the standard model of electroweak unification, the conservation of lepton number $L$ is violated. Specifically, the exchange of light and heavy Majorana neutrinos involving left and right handed currents within the left-right symmetric model (LRSM) is one of such possibilities.

The rate of $0^{\nu}\beta\beta$ decay is a product of appropriate phase-space factors, nuclear transition matrix elements (NTMEs) and parameters of the underlying mechanisms. Recently, the phase-space factors have been calculated to good accuracy incorporating the screening correction. The extraction of accurate limits on the parameters of a particular mechanism depends on the reliability of NTMEs. The evaluation of reliable NTMEs is a challenging task. A suitable truncation of unmanageable Hilbert space into a manageable model space with appropriate single-particle energies (SPEs), and effective two-body interaction is required. In addition, alternative considerations of the finite size of nucleons (FNS), short range correlations (SRC) and the effective value of axial vector current coupling constant $g_A$ are also available.

The standard mass mechanism of $0^{\nu}\beta^-\beta^-$ decay has been extensively studied employing a large number of nuclear models, namely shell-model approach, QRPA, QRPA with isospin restoration, deformed QRPA, projected-Hartree-Fock-Bogoliubov (PHFB), energy density functional (EDF), covariant density functional theory (CDFT), and interacting boson model (IBM) with isospin restoration. The details about these theoretical studies have been excellently reviewed over the past years in Refs. and references there in. In spite of the fact that each model employs different model space, SPEs and two-body residual interactions, the calculated NTMEs $M^{(0\nu)}$ differ by a factor of 2–3.

Uncertainties in NTMEs for $0^{\nu}\beta^-\beta^-$ decay within mechanisms involving light Majorana neutrino mass, classical Majorons and sterile neutrinos have been estimated employing the PHFB approach in conjunctions with four different parametrizations of effective two-body interaction, form factors with two different parametrizations and three different parametrizations of the SRC. The uncer-
tainties in NTMEs for $0\nu\beta^-\beta^-$ decay involving heavy Majorana neutrino mass and new Majoron models have also been investigated. The main objective of the present work is to calculate sets of twelve NTMEs for the $0^+ \to 0^+$ transition of $0\nu\beta^-\beta^-$ decay involving light neutrino mass and right-handed current by employing sets of four different PHFB wave functions as well as three different parametrizations of SRC and estimate uncertainties therein.

The detailed theoretical formalism of $0\nu\beta^-\beta^-$ decay within the mechanisms of LRSM, namely the exchange of light as well as heavy Majorana neutrino, admixture of $V^-A$ and $V^+A$ currents, and exchange of right handed heavy neutrino has been developed in Refs. 31–33. The theoretical formalism of the standard mass mechanisms has been extended by including the contribution of induced currents. Including the induced pseudoscalar terms in the nonrelativistic reduction of right-handed $V^+A$ current, the light neutrino exchange mechanism of $0\nu\beta^-\beta^-$ decay with left and right handed leptonic and hadronic currents has been investigated in detail. Presently, the NTMEs are calculated neglecting the induced pseudoscalar terms in the nonrelativistic reduction of right handed $V^+A$ current, which will not apparently change the final conclusions as seen in Ref. 6. However, this aspect will be dealt in future publication.

In Sec. 2, we present a brief theoretical formalism to study $0\nu\beta^-\beta^-$ decay involving light Majorana neutrino mass and right handed current. The calculated NTMEs required to study $0\nu\beta^-\beta^-$ decay of $^{94,96}\text{Zr}$, $^{100}\text{Mo}$, $^{110}\text{Pd}$, $^{128,130}\text{Te}$ and $^{150}\text{Nd}$ isotopes for the $0^+ \to 0^+$ transition and the uncertainties in NTMEs are presented in Sec. 3. Further, the extracted limits on the effective light Majorana neutrino mass $<m_\nu>$, the effective weak coupling of right-handed leptonic current with right-handed hadronic current $<\lambda>$, and the effective weak coupling of right-handed leptonic current with left-handed hadronic current $<\eta>$ from the largest available limits on half-lives of $0\nu\beta^-\beta^-$ decay $T_{1/2}^{(0\nu)}(0^+ \to 0^+)$ are presented in the same section. Conclusions are given in Sec. 4.

2. Theoretical Formalism

The general form of weak interaction Hamiltonian $H_W$ is given by

$$H_W = \frac{G}{\sqrt{2}} \left[ j_{L,R} J_{L,R}^{\mu\dagger} + \kappa j_{L,R} J_{R}^{\mu\dagger} + \eta j_{L,R} J_{L}^{\mu\dagger} + \lambda j_{L,R} J_{R}^{\mu\dagger} \right] + \text{h.c.}, \quad (1)$$

where $j_{L,R}$ and $J_{L,R}$ are left and right handed leptonic and hadronic currents, respectively. Further, $\kappa$, $\eta$ and $\lambda$ are the parameters for the admixture of $V - A$ and $V + A$ currents. The second term in the Eq. (1) is usually neglected as $\kappa$ enters into $\beta\beta$ decay amplitude always in the combination $1 \pm \kappa$ and it is expected that $|\kappa| \ll 1$.

Using the standard approximations of Ref. 31 with CP conservation, the rate
for the $0^+ \rightarrow 0^+$ transition of $0\nu\beta^+\beta^-$ decay is given by

$$T_{1/2}^{(0\nu)} = \frac{\langle m_\nu \rangle^2}{m_e} C_{mm} + \frac{\langle m_\nu \rangle}{m_e} \langle \lambda \rangle C_{m\lambda} + \frac{\langle m_\nu \rangle}{m_e} \langle \eta \rangle C_{m\eta} + \langle \lambda \rangle^2 C_{\lambda\lambda}$$

$$+ \langle \eta \rangle^2 C_{\eta\eta} + \langle \lambda \rangle \langle \eta \rangle C_{\lambda\eta},$$

(2)

where

$$\langle m_\nu \rangle = \sum_i U^2_{ei} m_i,$$

(3)

$$\langle \lambda \rangle = \lambda \left[ \sum_i \left( \frac{g_i^V}{g_i^A} \right) U_{ei} V_{ei} \right],$$

(4)

$$\langle \eta \rangle = \eta \left[ \sum_i U_{ei} V_{ei} \right],$$

(5)

and the nuclear structure factors $C_{xy}$ are written as

$$C_{mm} = G_{01} \left| M^{(0\nu)} \right|^2,$$

(6a)

$$C_{m\lambda} = M^{(0\nu)} (G_{03} M_{1^+} - G_{05} M_{2^-}),$$

(6b)

$$C_{m\eta} = M^{(0\nu)} (G_{03} M_{2^+} - G_{05} M_{1^-} - G_{05} M_P + G_{06} M_R),$$

(6c)

$$C_{\lambda\lambda} = G_{02} \left| M_{2^-} \right|^2 - \frac{2}{9} G_{03} (M_{1^+} M_{2^-}) + \frac{1}{9} G_{04} \left| M_{1^+} \right|^2,$$

(6d)

$$C_{\lambda\eta} = G_{02} \left| M_{2^+} \right|^2 - \frac{2}{9} G_{03} (M_{1^-} M_{2^+}) + \frac{1}{9} G_{04} \left| M_{1^-} \right|^2$$

$$- G_{07} (M_P M_R) + G_{08} \left| M_P \right|^2 + G_{09} \left| M_R \right|^2,$$

(6e)

$$C_{\lambda\eta} = -2 G_{02} (M_{2^+} M_{2^-}) + \frac{2}{9} G_{03} (M_{1^+} M_{1^-} + M_{2^-} M_{1^-})$$

$$- \frac{2}{9} G_{04} (M_{1^-} M_{1^+}).$$

(6f)

In addition, the combinations of NTMEs $M^{(0\nu)}$ and $M_{i\pm}$ ($i = 1, 2$) are defined as

$$M^{(0\nu)} = M_{GT} - M_F + M_T,$$

(7)

$$M_{1\pm} = M_{qGT} - 6 M_{qT} \pm 3 M_{qF},$$

(8)

$$M_{2\pm} = M_{\omega GT} \pm M_{\omega F} - \frac{1}{9} M_{1\mp}.$$  

(9)

Employing the generally agreed closure approximation in conjunction with the HFB wave functions, the NTMEs $M_\alpha$ ($\alpha = F, GT, T, \omega F, \omega GT, qF, qGT, qT, P$ and $R$) appearing in the expressions of nuclear structure factors $C_{xy}$ are calculated by
using the following expression\textsuperscript{20}

\[ M_\alpha = \left\langle 0^+_f \| O_\alpha (r, \sigma) \| 0^+_i \right\rangle \]

\[ = \left[ n'^i = 0 \right] n''_i = 0 \right]\frac{1}{\sqrt{2}} \int_0^\pi d\theta \sin \theta n_{(Z,N), (Z+2,N-2)}(\theta) \]

\[ \times \sum_{\alpha, \beta, \gamma, \delta} \langle \alpha \beta | O_\alpha (r, \sigma) | \gamma \delta \rangle \sum_{\varepsilon \eta} \frac{\left( f^{(1)}_{Z+2,N-2} \right)_{\varepsilon \beta}}{\left( 1 + F^{(1)}_{Z,N}(\theta) f^{(1)}_{Z+2,N-2} \right)_{\varepsilon \alpha}} \]

\[ \times \left[ \left( 1 + F^{(1)}_{Z,N}(\theta) f^{(1)}_{Z+2,N-2} \right)_{\varepsilon \eta} \right]. \] 

(10)

The calculation of \( n'^i, n_{(Z,N), (Z+2,N-2)}(\theta), f_{Z,N} \) and \( F_{Z,N}(\theta) \) require the intrinsic wave functions \( |\Phi_0\rangle \) of axially symmetric state with \( K = 0 \) expressed by the amplitudes \( (u_{im}, v_{im}) \) and expansion coefficients \( C_{ij,m} \), which are in turn obtained by minimizing the expectation value of the effective Hamiltonian given by\textsuperscript{20}

\[ H = H_{sp} + V(P) + V(QQ) + V(HH), \]

in a basis constructed by using a set of deformed states. In Eq. (11), the \( H_{sp}, V(P), V(QQ) \) and \( V(HH) \) denote the single particle Hamiltonian, the pairing, quadrupole-quadrupole and hexadecapole-hexadecapole parts of the effective two-body interaction, respectively. Further, the transition operators have the following general structure

\[ O_\alpha (r, \sigma, \tau) = S_\alpha (r, \sigma) \, \tau_\alpha^+ \, \tau^+ \frac{2R}{\pi} \int h_\alpha(qr)f_\alpha(q^2)dq. \]

(12)

The calculation of \( M^{(0\nu)} \) has already been discussed in Ref.\textsuperscript{22} Neglecting the induced pseudoscalar terms in the nonrelativistic reduction of right-handed \( V + A \) current\textsuperscript{22} the explicit structure of \( S_\alpha (r, \sigma), h_\alpha(qr) \) and \( f_\alpha(q^2) \) for the rest of the NTMEs \( M_\alpha \) is given in Table 1.

3. Results and Discussions

Employing the PHFB approach, four different sets of wave functions were generated with the consideration of four different parametrizations of the two body effective interaction\textsuperscript{23} The strength parameters of \( V(QQ) \), namely proton-proton, neutron-neutron and proton-neutron components are denoted by \( \chi_{2pp}, \chi_{2nn} \) and \( \chi_{2pn} \), respectively. Two different parametrizations, denoted by \( PQQ1 \) and \( PQQ2 \) were obtained by fitting the excitation energy \( E_{2^+} \) of the \( 2^+ \) state either by taking \( \chi_{2pp} = \chi_{2nn} \) and varying the strength of \( \chi_{2pn} \) or by taking \( \chi_{2pp} = \chi_{2nn} = \chi_{2pn}/2 \) and varying the three parameters together. Two additional parametrizations, namely \( PQQHH1 \) and \( PQQHH2 \) were obtained with the inclusion of the hexadecapolar \( HH \) part of the effective interaction.
TABLE 1. Explicit structure of $S_{\alpha}(\mathbf{r}, \mathbf{\sigma})$, $h_{\alpha}(q\mathbf{r})$ and $f_{\alpha}(q^2)$ of transition operator $O_{\alpha}(\mathbf{r}, \mathbf{\sigma}, \tau)$.

| NTME   | $S_{\alpha}(\mathbf{r}, \mathbf{\sigma})$ | $h_{\alpha}(q\mathbf{r})$ | $f_{\alpha}(q^2)$ |
|--------|------------------------------------------|--------------------------|------------------|
| $M_{\omega F}$ | 1                                        | $j_0(q\mathbf{r})$ | $g^2_{\alpha}(q^2)$ |
| $M_{\omega GT}$ | $\sigma_1 \cdot \sigma_2$               | $j_0(q\mathbf{r})$ | $g^2_{\alpha}(q^2)$ |
| $M_{\nu F}$   | 1                                        | $j_1(q\mathbf{r})q$ | $g^2_{\alpha}(q^2)$ |
| $M_{\nu GT}$  | $\sigma_1 \cdot \sigma_2$               | $j_1(q\mathbf{r})q$ | $g^2_{\alpha}(q^2)$ |
| $M_{T}$       | $3(\sigma_1 \cdot \mathbf{\tau}_{12})(\sigma_1 \cdot \mathbf{\tau}_{12}) - \sigma_1 \cdot \sigma_2$ | $j_1(q\mathbf{r})q$ | $g^2_{\alpha}(q^2)$ |
| $M_{P}$       | $\frac{R}{2q^2}(\sigma_1 - \sigma_2) \cdot \left(\frac{\mathbf{r} \times \mathbf{r}_+}{R}\right)$ | $q(q + \mathbf{A})$ | $g^2_{\alpha}(q^2)$ |
| $M_{R}$       | $\sigma_1 \cdot \sigma_2$               | $\frac{j_0(q\mathbf{r})q^2}{q(q + \mathbf{A})}$ | $\frac{1}{3m_N} \left(1 + \frac{g_M(q^2)}{g_V(q^2)}\right)g_A(q^2)$ |

Table 2. Change in the NTME $M_{\alpha}$ of $0\nu\beta^-\beta^-$ decay (in %) due to the exchange of light Majorana neutrino, and admixture of $V - A$ and $V + A$ currents, with the inclusion of FNS and SRC (SRC1, SRC2, and SRC3) for all four parametrizations of the effective two-body interaction.

| NTME   | FNS  | FNS+SRC |
|--------|------|---------|
|        |      | SRC1    | SRC2    | SRC3    |
| $M_{\omega F}$ | 13.1–17.7 | 11.6–17.3 | 0.1–1.1 | 3.1–3.8 |
| $M_{\omega F}$ | 25.8–37.7 | 2.9–5.9 | 3.5–5.3 | 4.2–6.6 |
| $M_{\omega GT}$ | 9.0–11.2 | 13.9–18.0 | 1.3–2.5 | 2.6–3.0 |
| $M_{\nu GT}$ | 18.9–24.2 | 5.3–7.6 | 3.1–4.0 | 4.3–5.8 |
| $M_{T}$ | 0.2–34.2 | 0.0–2.4 | 0.0–2.3 | 0.0–2.0 |
| $M_{P}$ | 10.8–42.5 | 2.9–17.8 | 3.8–14.1 | 4.5–17.9 |
| $M_{R}$ | 30.9–34.6 | 55.2–56.6 | 28.1–29.4 | 10.4–11.2 |

By comparing the theoretically calculated yrast spectra, the reduced $B(E2;0^+ \rightarrow 2^+)$ transition probabilities, deformation parameters $\beta_2$, static quadrupole moments $Q(2^+)$, gyromagnetic factors $g(2^+)$ and NTMEs $M_{2\nu}$ for the $0^+ \rightarrow 0^+$ transition with the available experimental data, the reliability of the wave functions had been ascertained in Ref.\textsuperscript{20} Moreover, the same wave functions had been employed for the study of $0\nu\beta^-\beta^-$ decay of $^{34,96}Zr, ^{98,100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te$ and $^{150}Nd$ isotopes within mechanisms involving exchange of light as well as heavy Majorona neutrinos, classical Majorons, sterile neutrinos\textsuperscript{21,22} and new Majorons\textsuperscript{23}.

In order to estimate average NTMEs $M_{\alpha}$ and uncertainties $\Delta M_{\alpha}$ statistically, sets of twelve NTMEs are calculated by using Eq.\textsuperscript{10} with the consideration of four different parametrizations of the two body effective interaction\textsuperscript{23} and three
Table 3. Deformation ratio $D_{\alpha}$ with the inclusion of FNS and SRC (SRC1, SRC2, and SRC3) for all four parametrizations of the effective two-body interaction.

| $D_{\alpha}$ | FNS+SRC | SRC1 | SRC2 | SRC3 |
|--------------|---------|------|------|------|
| $D_{\omega F}$ | 1.9–6.9 | 1.8–6.9 | 1.8–6.9 |
| $D_{q F}$ | 1.9–6.9 | 1.9–6.9 | 1.9–6.9 |
| $D_{\omega GT}$ | 1.9–7.2 | 1.9–7.2 | 1.9–7.2 |
| $D_{q GT}$ | 2.0–7.2 | 1.9–7.1 | 1.9–7.1 |
| $D_{q T}$ | -0.8–25.0 | -0.8–24.9 | -0.8–25.0 |
| $D_{P}$ | 1.8–13.6 | 1.8–12.1 | 1.8–11.9 |
| $D_{R}$ | 1.8–7.1 | 1.8–7.1 | 1.8–7.0 |

Different parametrizations of the SRC\cite{14} By considering a Jastrow form of short range correlations, three different parametrizations of SRC have been given by\cite{14}

$$f(r) = 1 - ce^{-ar^2}(1 - br^2), \tag{13}$$

where $a = 1.1 \, fm^{-2}, 1.59 \, fm^{-2}, 1.52 \, fm^{-2}$, $b = 0.68 \, fm^{-2}, 1.45 \, fm^{-2}, 1.88 \, fm^{-2}$ and $c = 1.0, 0.92, 0.46$ for Miller and Spencer parametrization, Argonne NN and CD-Bonn potentials, and are denoted by SRC1, SRC2 and SRC3, respectively. Specifically, sets of twelve NTMEs, namely $M_{\omega F,q F}, M_{\omega GT,q GT}, M_{q T}, M_{P}$, and $M_{R}$ are calculated within the approximations of point nucleons (P), nucleons having finite size (FNS) and also with SRC (FNS+SRC).

In Table 2, the relative changes in NTMEs $M_{\alpha}$ (in %) due to the different approximations are presented. Due to FNS, the maximum change in $M_{\omega F,\omega GT}$, $M_{q F,q GT,q T,P}$ and $M_{R}$ is about 18%, 40%, and 35%, respectively. With the inclusion of SRC, the NTMEs $M_{\alpha}$ change by about 18%, 2.5% and 4% due to SRC1, SRC2 and SRC3, respectively. The observed changes in $M_{q F,q GT}$ with the inclusion of SRC1, SRC2 and SRC3 are of the same order and the maximum change is about 7%. Due to the inclusion of SRC, the change in $M_{q T}$ is about 2% and $M_{P}$ can change between 2%-18%. The maximum change in $M_{R}$ due to SRC1, SRC2 and SRC3 is about 57%, 29% and 11%, respectively. To quantify the effect of deformation on $M_{\alpha}$, the quantity $D_{\alpha} = M_{\alpha}(\zeta_{qq} = 0)/M_{\alpha}(\zeta_{qq} = 1)$ has been defined as the ratio of $M_{\alpha}$ at zero deformation ($\zeta_{qq} = 0$) and full deformation ($\zeta_{qq} = 1$)\cite{34}. In the Table 3, we tabulate the values of $D_{\alpha}$ for $\alpha = \omega F, \omega GT, q F, q GT, q T, P$ and $R$. In the mass range $A = 90–150$, the NTMEs $M_{\alpha}$ are suppressed by factor of about 2–7 ($D_{q T}$ and $D_{P}$ are suppressed by a factor of about 25 and 14, respectively) due to deformation effects and hence, a proper consideration of deformation of participating nuclei is quite crucial in the nuclear structure aspects of $0\nu\beta – \beta$- decay.

In Table 4, the averages and standard deviations of seven NTMEs, namely $M_{\omega F,q F}, M_{\omega GT,q GT}, M_{q T}, M_{P}$ and $M_{R}$ are presented. It is observed that the maximum uncertainty in $M_{\omega F,q F}$, $M_{\omega GT,q GT}$ and $M_{P}$ is about 15% but for $^{150}$Nd, in which the standard deviation of $M_{P}$ is about 40%. In $^{94}$Zr, $^{100}$Mo, and $^{110}$Pd
Presently, we reevaluate them for leptonic current with right-handed hadronic current factors $C$ coupling of right-handed leptonic current with left-handed hadronic current $\langle \lambda \rangle$ factors of the effective mass of light neutrino $T$ are extracted from the largest observed limits on half-lives of isotopes, the NTMEs $M_{\eta\eta}$ are quite uncertain due to change of sign in the case of $PQQHH1$, $PQQHH2$, and $PQQ2$ parametrizations. The maximum uncertainty in $M_R$ is about 30%. In Ref. [22] NTMEs $M_{(0\nu)}$ have already been calculated. Presently, we reevaluate them for $g_A = 1.2701$ and sets of twelve nuclear structure factors $C_{m\lambda}$, $C_{m\eta}$, $C_{\lambda\lambda}$, $C_{\lambda\eta}$, $C_{\eta\eta}$ and $C_{\lambda\eta}$ are computed for $^{96}Zr$, $^{100}Mo$, $^{110}Pd$, $^{128,130}Te$ and $^{150}Nd$ isotopes using the phase space factors calculated by Štefánik et al.[23] The averages of these six nuclear structure factors are reported in Table 5. To exhibit the relative role of NTMEs due to different mechanisms, we define $M_{eff}$ and $M_{eff}^{(0\eta)}$ as

$$C_{\lambda\lambda} = G_{01} \left| M_{eff}^{(0\lambda)} \right|^2,$$

$$C_{\eta\eta} = G_{01} \left| M_{eff}^{(0\eta)} \right|^2,$$

and present the NTMEs $M_{eff}^{(0\lambda)}$ and $M_{eff}^{(0\eta)}$ along with $M_{(0\nu)}$ reevaluated for $g_A = 1.2701$ in Table 6. It is observed that NTMEs $M_{eff}^{(0\lambda)}$ are about twice of $M_{(0\nu)}$ and NTMEs $M_{eff}^{(0\eta)}$ are larger by two orders in magnitude than the latter.

Using the average nuclear structure factors $C_{m\lambda}$, $C_{\lambda\lambda}$, $C_{\eta\eta}$, on-axis limits on the effective mass of light neutrino $\langle m_\nu \rangle$, the effective weak coupling of right-handed leptonic current with right-handed hadronic current $\langle \lambda \rangle$, and the effective weak coupling of right-handed leptonic current with left-handed hadronic current $\langle \eta \rangle$ are extracted from the largest observed limits on half-lives $T_{1/2}^{(0\nu)}$ of $0\nu\beta^-\beta^-$ decay (Table 7). The extracted limits on $\langle m_\nu \rangle$, $\langle \lambda \rangle$, and $\langle \eta \rangle$ for $^{130}Te$ ($^{100}Mo$) nuclei are 0.33 eV (0.38 eV), $4.57 \times 10^{-7}$ ($4.39 \times 10^{-7}$) and $4.72 \times 10^{-9}$ ($5.23 \times 10^{-9}$), respectively. In the last two columns of the same Table 7, the predicted half-lives $T_{1/2}^{(0\nu)}$ of $0\nu\beta^-\beta^-$ decay of $^{96}Zr$, $^{100}Mo$, $^{110}Pd$, $^{130}Te$ and $^{150}Nd$ isotopes are given.
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Table 5. Average nuclear structure factors $C_{\alpha\beta}$, $C_{\alpha\lambda\lambda}$, $C_{\alpha\eta\eta}$, $C_{\lambda\lambda}$, $C_{\lambda\eta}$ and $C_{\lambda\eta}$ for the $0\nu\beta^-\beta^-$ decay of $^{96}$Zr, $^{100}$Mo, $^{110}$Pd, $^{130}$Te and $^{150}$Nd isotopes.

| Nuclei | $C_{\alpha\beta}$ | $C_{\alpha\lambda\lambda}$ | $C_{\alpha\eta\eta}$ | $C_{\lambda\lambda}$ | $C_{\lambda\eta}$ |
|--------|------------------|--------------------------|-------------------|-------------------|------------------|
| $^{96}$Zr | 4.37$\times$10$^{-13}$ | 1.62$\times$10$^{-12}$ | 5.02$\times$10$^{-11}$ | 1.51$\times$10$^{-12}$ | -1.54$\times$10$^{-12}$ |
| $^{100}$Mo | -2.26$\times$10$^{-13}$ | -8.45$\times$10$^{-13}$ | 1.80$\times$10$^{-10}$ | 4.76$\times$10$^{-12}$ | 3.52$\times$10$^{-10}$ |
| $^{110}$Pd | 5.02$\times$10$^{-11}$ | 1.80$\times$10$^{-10}$ | 9.14$\times$10$^{-11}$ | 8.21$\times$10$^{-13}$ | 1.45$\times$10$^{-8}$ |
| $^{130}$Te | 1.51$\times$10$^{-12}$ | 4.76$\times$10$^{-12}$ | 8.21$\times$10$^{-13}$ | 1.21$\times$10$^{-12}$ | 1.19$\times$10$^{-8}$ |
| $^{150}$Nd | 5.02$\times$10$^{-11}$ | 1.80$\times$10$^{-10}$ | 9.14$\times$10$^{-11}$ | 8.21$\times$10$^{-13}$ | 1.45$\times$10$^{-8}$ |

Table 6. Effective NTMEs $M_{\mu\nu}^{(0\alpha)}$ and $M_{\mu\nu}^{(0\lambda)}$ along with $M_{\mu\nu}^{(0\nu)}$ for the $0\nu\beta^-\beta^-$ decay of $^{96}$Zr, $^{100}$Mo, $^{110}$Pd, $^{130}$Te and $^{150}$Nd isotopes.

| Nuclei | $M_{\mu\nu}^{(0\alpha)}$ | $M_{\mu\nu}^{(0\lambda)}$ | $M_{\mu\nu}^{(0\nu)}$ |
|--------|--------------------------|--------------------------|--------------------------|
| $^{96}$Zr | 2.85 | 5.30 | 463.07 |
| $^{100}$Mo | 6.25 | 10.71 | 920.40 |
| $^{110}$Pd | 7.15 | 8.08 | 1072.80 |
| $^{130}$Te | 4.05 | 5.71 | 567.26 |
| $^{150}$Nd | 2.84 | 5.26 | 338.85 |

Table 7. Experimental limits on half-lives $T_{1/2}^{(0\nu)}$ for the $0\nu\beta^-\beta^-$ decay of $^{96}$Zr, $^{100}$Mo, $^{110}$Pd, $^{130}$Te and $^{150}$Nd isotopes along with the extracted off-axis limits on the effective mass of light neutrino $\langle m_\nu \rangle$, $\langle \lambda \rangle$ and $\langle \eta \rangle$. Predicted half-lives $T_{1/2}^{(0\nu)}$ of $0\nu\beta^-\beta^-$ decay for two sets of parameters (i) $\langle m_\nu \rangle = 50$ meV (Case I) and (ii) $\langle m_\nu \rangle = 50$ meV, $\langle \lambda \rangle = 10^{-7}$ and $\langle \eta \rangle = 10^{-9}$ (Case II).

| Nuclei | $T_{1/2}^{(0\nu)}$ (Exp) | $\langle m_\nu \rangle$ | $\langle \lambda \rangle$ | $\langle \eta \rangle$ | $T_{1/2}^{(0\nu)}$ (I) | $T_{1/2}^{(0\nu)}$ (II) |
|--------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $^{96}$Zr | 9.2$\times$10$^{21}$ | 8.09 | 8.53$\times$10$^{-6}$ | 1.00$\times$10$^{-7}$ | 2.39$\times$10$^{26}$ | 3.00$\times$10$^{25}$ |
| $^{100}$Mo | 1.1$\times$10$^{24}$ | 0.38 | 4.39$\times$10$^{-7}$ | 5.23$\times$10$^{-9}$ | 6.45$\times$10$^{25}$ | 9.34$\times$10$^{24}$ |
| $^{110}$Pd | 6.0$\times$10$^{23}$ | 827 | 1.43$\times$10$^{-3}$ | 1.10$\times$10$^{-5}$ | 1.63$\times$10$^{26}$ | 2.84$\times$10$^{25}$ |
| $^{130}$Te | 4.0$\times$10$^{24}$ | 0.33 | 4.57$\times$10$^{-7}$ | 4.72$\times$10$^{-9}$ | 1.72$\times$10$^{26}$ | 2.95$\times$10$^{25}$ |
| $^{150}$Nd | 2.0$\times$10$^{22}$ | 3.17 | 3.35$\times$10$^{-6}$ | 5.36$\times$10$^{-8}$ | 7.88$\times$10$^{25}$ | 1.25$\times$10$^{25}$ |

for two sets of parameters (i) $\langle m_\nu \rangle = 50$ meV and (ii) $\langle m_\nu \rangle = 50$ meV, $\langle \lambda \rangle = 10^{-7}$ and $\langle \eta \rangle = 10^{-9}$. It is noticed that the predicted half-lives $T_{1/2}^{(0\nu)}$ are smaller for the latter parametrization than those of pure mass mechanism. By defining $\left[ T_{1/2}^{(0\nu)} \right]^{-1} = C^{(0\nu)}$, it is seen that in total $C^{(0\nu)}$, the the contribution of mass mechanism is about 13%–17%, the $\lambda$-term contributes 23%–57% and the $\eta$-term contributes 24%–41%. Further, the contributions of $m_\lambda$ and $m_\eta$-term are about 7%–8% and 13%–25%, respectively, while the $\lambda\eta$-term contribute less than 1%.
4. Conclusions

Using HFB wave functions generated with four different parametrization of pairing plus multipolar type of effective two body interaction, and three different parametrizations of Jastrow SRC, sets of twelve NTMEs, namely $M_{\omega F,qF}$, $M_{\omega GT,qGT}$, $M_{qT}$, $M_P$, and $M_R$ are calculated to study the $0\nu\beta\beta$ decay of $^{94,96}\text{Zr}$, $^{100}\text{Mo}$, $^{110}\text{Pd}$, $^{128,130}\text{Te}$ and $^{150}\text{Nd}$ isotopes within mechanisms involving the light Majorana neutrino, and right handed $V+A$ current. The effect due to FNS is maximum (about 40%) for $M_{qF,qGT,qT,P}$. Due to SRC1, SRC2 and SRC3, the maximum change in $M_R$ is about 57%, 29% and 11%, respectively. Effects due to deformation reduce the NTMEs by a factor of 2–7.

The maximum uncertainty in $M_{\omega F,qF}$, $M_{\omega GT,qGT}$ and $M_P$ is about 15% albeit the standard deviation of $M_P$ for $^{150}\text{Nd}$ is about 40%. In the case of $M_R$, the maximum uncertainty is about 30%. The NTMEs $M_qT$ are quite uncertain. Using the average nuclear structure factors $C_{mm}$, $C_{\lambda\lambda}$, and $C_{\eta\eta}$, the most stringent on-axis extracted limits on $\langle m_\nu \rangle$, $\langle \lambda \rangle$, and $\langle \eta \rangle$ from the largest observed limits on half-lives $T_{1/2}^{0\nu}$ of $^{130}\text{Te}$ isotope are 0.33 eV, $4.57 \times 10^{-7}$ and $4.72 \times 10^{-9}$, respectively.

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