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The 21-cm power spectrum after reionization

J. Stuart B. Wyithe\textsuperscript{1\,*} and Abraham Loeb\textsuperscript{2\,*}

\textsuperscript{1}School of Physics, University of Melbourne, Parkville, Victoria, Australia
\textsuperscript{2}Harvard–Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA

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ABSTRACT

We discuss the 21-cm power spectrum (PS) following the completion of reionization. In contrast to the reionization era, this PS is proportional to the PS of mass density fluctuations, with only a small modulation due to fluctuations in the ionization field on scales larger than the mean-free-path of ionizing photons. We derive the form of this modulation, and demonstrate that its effect on the 21-cm PS will be smaller than 1 per cent for physically plausible models of damped Ly\textalpha systems. In contrast to the 21-cm PS observed prior to reionization, in which H\textalpha regions dominate the ionization structure, the simplicity of the 21-cm PS after reionization will enhance its utility as a cosmological probe by removing the need to separate the PS into physical and astrophysical components. As a demonstration, we consider the Alcock–Paczynski test and show that the next generation of low-frequency arrays could measure the angular distortion of the PS at the per cent level for \( z \sim 3–5 \).

Key words: galaxies: high-redshift – intergalactic medium – diffuse radiation.

\section{1 INTRODUCTION}

Recently, there has been much interest in the feasibility of mapping the three-dimensional distribution of cosmic hydrogen through its resonant spin-flip transition at a rest-frame wavelength of 21 cm (Barkana & Loeb 2007; Furlanetto, Oh & Briggs 2006). Several experiments are currently being constructed (including MWA,\textsuperscript{1} LOFAR,\textsuperscript{2} PAPER,\textsuperscript{3} 21CMA,\textsuperscript{4} GMRT\textsuperscript{5}) and more ambitious designs are being planned (SKA\textsuperscript{6}).

One driver for mapping the 21-cm emission is the possibility of measuring cosmological parameters from the shape of the underlying power spectrum (PS; see Loeb & Wyithe 2008). During the epoch of reionization, the PS of 21-cm brightness fluctuations is shaped mainly by the topology of ionized regions, rather than by the PS of matter density fluctuations which is the quantity of cosmological interest (McQuinn et al. 2006; Iliev et al. 2007; Santos et al. 2008). As a result, the line-of-sight anisotropy of the 21-cm PS due to peculiar velocities must be used to separate measurements of the density PS from the unknown details of the astrophysics (Barkana & Loeb 2005; McQuinn et al. 2006). The situation is expected to be simpler both prior to the formation of the first galaxies (at redshifts \( z \gtrsim 20 \), Loeb & Zaldarriaga 2004; Lewis & Challinor 2007; Pritchard & Loeb 2008), and following reionization of the intergalactic medium (IGM; \( 1 \lesssim z \lesssim 6 \)) – when only dense pockets of self-shielded hydrogen, such as damped Ly\textalpha absorbers (DLA) and Lyman-limit systems (LLS) survive (Chang et al. 2008; Pritchard & Loeb 2008; Wyithe & Loeb 2008).

In this paper, we focus our discussion on the post-reionization epoch (Chang et al. 2008; Wyithe & Loeb 2008). The DLAs which contain most of the neutral hydrogen mass in the Universe at \( z \lesssim 6 \) are expected to be hosted by galactic mass dark matter haloes (Wolfe, Gawiser & Prochaska 2005). A survey of 21-cm intensity fluctuations after reionization would measure the modulation of the cumulative 21-cm emission from a large number of galaxies (Chang et al. 2008; Wyithe & Loeb 2008; Wyithe, Loeb & Geil 2008). Regarding the measurement of the 21-cm PS, this lack of identification of individual galaxies is an advantage, since by not imposing a minimum threshold for detection, such a survey collects all the available signal. This point is discussed in Pen et al. (2009), where the technique is also demonstrated via measurement of the cross-correlation of galaxies with unresolved 21-cm emission in the local Universe.

Studying the 21-cm PS after (rather than during) reionization offers two advantages. First, it is less contaminated by the Galactic synchrotron foreground, whose brightness temperature scales with redshift as \( (1 + z)^{2.6} \) (Furlanetto, Oh & Briggs 2006). Secondly, because the UV radiation field is nearly uniform after reionization, it should not imprint any large-scale features on the 21-cm PS that would mimic the cosmological signatures. In addition, on large spatial scales the 21-cm sources are expected to have a linear bias analogous to that inferred from galaxy redshift surveys.

\textsuperscript{*E-mail: swyithe@unimelb.edu.au; loeb@cfa.harvard.edu
\textsuperscript{1}http://www.haystack.mit.edu/ast/arrays/mwa/
\textsuperscript{2}http://www.lofar.org/
\textsuperscript{3}http://astro.berkeley.edu/dbacker/EoR/
\textsuperscript{4}http://web.phys.cmu.edu/past/
\textsuperscript{5}Pen et al. (2008)
\textsuperscript{6}http://www.skatelescope.org/
Most previous studies of post-reionization 21-cm fluctuations have assumed that the 21-cm emission traces perturbations in the matter density, and have considered peculiar motions only after a spherical average (Loeb & Wyithe 2008; Pritchard & Loeb 2008; Wyithe & Loeb 2008). The exception is a recent paper discussing measurements of the dark energy equation of state (Chang et al. 2008). Since the neutral gas resides within collapsed dark matter haloes, galaxy bias plays an important role in setting the 21-cm fluctuation amplitude. In this paper, we derive the 21-cm PS after reionization in the context of the formalism that has been developed to calculate it during reionization (Barkana & Loeb 2005; McQuinn et al. 2006; Mao et al. 2008). By framing the derivation this way, the relative merits of cosmological constraints from 21-cm surveys at redshifts before and after reionization can be more easily understood. In addition, this formalism provides a framework to describe the possible effect of fluctuations in the ionizing background. We then compute the Alcock–Paczynski (AP) effect (Alcock & Paczynski 1979) as an example for the cosmological utility of the post-reionization 21-cm PS. In our numerical examples, we adopt the standard set of cosmological parameters (Komatsu et al. 2008), with values of $\Omega_b = 0.24$, $\Omega_m = 0.04$ and $\Omega_\Phi = 0.76$ for the matter, baryon and dark energy fractional density, respectively, and $h = 0.73$ for the dimensionless Hubble constant.

2 21-CM POWER SPECTRUM AFTER REIONIZATION

The 21-cm PS after reionization is expected to be dominated by the neutral content of galaxies. In a scenario where fluctuations in the ionizing background can be ignored, the form of the PS could, therefore, be inferred directly from the galaxy PS (Loeb & Wyithe 2008). However, we also include here the possible influence of a fluctuating ionizing background on the 21-cm PS. Throughout our discussion, we assume the baryonic overdensity $\delta$ to equal the overdensity in dark matter on sufficiently large scales.

2.1 The 21-cm brightness temperature

The 21-cm brightness temperature fluctuation in a region of IGM is

$$\Delta T = 23.8 \left( \frac{1+z}{10} \right)^{1/2} \left[ 1 - \bar{x}_i (1 + \delta_i) (1 + \delta) (1 - \delta) \right] \mathrm{mK}, \quad (1)$$

where $\delta_i = \partial v_i / \partial r (Ha)$ and $\partial v_i / \partial r$ is the gradient of the peculiar velocity along the line of sight. The quantities $\bar{x}_i$ and $\bar{x}_o$ are the mean ionization fraction and the fluctuation in ionization fraction, respectively. We have assumed that the spin temperature of hydrogen is much higher than the cosmic microwave background (CMB) temperature [as expected for collisionally coupled gas in collapsed objects, and observed in some DLAs (Curran et al. 2007)]. Thus, we may neglect fluctuations in the kinetic and spin temperatures of the neutral hydrogen gas throughout the post-reionization epoch. To evaluate the brightness temperature fluctuation, we need to compute $\delta_i$. With this goal in mind, we first write the fluctuation in neutral hydrogen fraction as

$$\delta_{\text{HI}} = x_{\text{HI}}/\bar{x}_{\text{HI}} - 1, \quad (2)$$

where $\bar{x}_{\text{HI}}$ and $x_{\text{HI}}$ are the cosmic mean and local values for the neutral fraction (mass averaged).

2.2 Effect of the ionizing background

Next, we compute the effect of the ionizing background on the neutral fraction. Two regimes must be considered.

2.2.1 Systems which are optically thin to ionizing radiation

Low-density regions of the Universe contain optically thin, Lyα absorbers. The ionization fraction in this regime is controlled by the balance between the ionization rate owing to the UV background and the recombination rate at the local gas density. Given an ionization rate $\Gamma$, and a hydrogen density $n_\text{H}$ with a neutral fraction $x_{\text{HI}}$, the equilibrium in the optically thin regime is given by the condition

$$x_{\text{HI}} \sim \frac{n_{\text{H}} \alpha_f}{\Gamma}, \quad (3)$$

where $\alpha_f$ is the case-B recombination coefficient (Osterbrock & Ferland 2006). Assuming that the low-density regions of the hydrogen density field are unbiased with respect to the underlying dark matter density, we have

$$x_{\text{HI}} \propto \left( 1 + \frac{\Delta \Gamma}{\Gamma} \right)^{1}, \quad (4)$$

where $\Delta \Gamma$ is the perturbation in the intergalactic ionizing background. Note that the neutral fraction in optically thin regions is increased in overdense regions by recombinations, and decreased by the presence of an excess ionizing background.

2.2.2 Optically thick absorbers

Since optically thick systems are self-shielded, the effect of the UV background on their ionization state depends on the gas distribution within them. Below we discuss the effect of an ionizing background on the H i content of self-shielded systems. Since most of the hydrogen in such systems is known to be contained within DLAs, we focus our discussion on DLAs.

The nature of DLAs is not understood, although they are thought to be formed by dense, self-shielded gas in galaxies (Wolfe et al. 2005). The detailed modelling of DLAs is very uncertain, requiring numerical simulations that go beyond the scope of this paper. However, we can utilize results of simulations in the literature to estimate the range of possible strength for the influence of the UV background. The column density distribution of DLAs has a power-law form with a feature at $\sim 10^{20} \text{cm}^{-2}$ (e.g. Storrie-Lombardi & Wolfe 2000). This feature has been interpreted by a number of authors as being due to the effects of self-shielding of DLAs in a meta-galactic ionizing background (e.g. Corbelli, Salpeter & Bandiera 2001; Zheng & Miralda-Escudé 2002).

Zheng & Miralda-Escudé (2002) considered the column density distribution of damped Lyα systems modelled as spherical isothermal spheres. They solved for the self-shielded H i density profile numerically and found that the neutral fraction reaches $\sim 10^{-3}$ at the radius where the system becomes optically thick to meta-galactic ionizing radiation. Their results demonstrate that the transition from highly ionized to highly neutral gas occurs over a narrow region which is very small compared with the size of the absorbing system. Moreover, they find that the radius $r_s$ at which the gas becomes self-shielding is proportional to $\Gamma^{-1/3}$. For a spherical isothermal sphere, the hydrogen mass enclosed within radius $r$ scales as

$$M_{\text{HI}}(<r) \propto r, \quad (5)$$

and so the H i mass scales as

$$M_{\text{HI}} = M_{\text{HI}}(< r_s) \propto r_s \propto \Gamma^{-1/3}. \quad (6)$$
We, therefore, find that a given perturbation in the ionizing background produces a corresponding perturbation in the mass of neutral hydrogen within the self-shielded system of the form

$$\Delta M_{\text{HI}} \approx \frac{1}{3} \Delta \Gamma \Gamma.$$

(7)

Alternatively, the distribution of gas in a DLA may be better represented by a self-gravitating disc. The effect of a meta-galactic background on the ionization structure and star formation has been investigated by a number of authors (Corbelli et al. 2001; Schaye 2004; Susa 2008). The hydrogen column depth at which the gas becomes self-shielding was shown to be proportional to $\Gamma^{1/3}$ as in the spherical isothermal case (Susa 2008). However, the exponential profile of the disc models implies that the ionized portion of the gas contains much less mass than in the case of a more extended power-law isothermal profile. Corbelli et al. (2001) compute the column depth of H I as a fraction of the column depth of hydrogen, at different intensities of ionizing background. For a disc with a column density of $10^{21} \text{ cm}^{-2}$, the fluctuation in H I mass owing to a fluctuation in the ionizing background is approximately

$$\frac{\Delta M_{\text{HI}}}{M_{\text{HI}}} \sim 2 \times 10^{-2} \Delta \Gamma \Gamma.$$

(8)

This effect is two orders of magnitude smaller than in the case of a spherical isothermal sphere.

To evaluate the fluctuation in neutral fraction associated with DLAs, we assume that the neutral gas is hosted within haloes of mass $M$ with associated galaxy bias $b$. We have

$$x_{\text{HI}} \propto \left(1 + b\delta\right) \left(1 - C \Delta \Gamma / \Gamma\right)$$

(9)

$$\sim 1 + b\delta - \frac{C \Delta \Gamma}{\Gamma},$$

(10)

where the last equality has reduced the expression to lowest order in $\delta$.

2.2.3 Combined neutral content of the IGM

The above estimates of mass fluctuation suggest that the neutral hydrogen content of the IGM is modified owing to fluctuations in the ionizing background ($\delta_J \equiv \Delta \Gamma / \Gamma$) by a factor of the form

$$s \approx (1 - C \delta_J),$$

(11)

where $C$ is a constant describing the magnitude of the effect. For optically thin regions $C = 1$, while for optically thick absorbers $C$ can take a range of values. In the case of an isothermal sphere, a fluctuation in the ionizing background leads to a fluctuation in the H I mass of a self-shielded system that is of comparable magnitude, indicating that the value of $C$ in equation (11) would be of order unity in this case. However, in haloes with virial temperatures larger than $10^4 \text{ K}$, hydrogen with an isothermal profile would be collisionally ionized, and hence have cooled into a much more concentrated profile. Since the host masses of DLAs are known to be greater than $10^{18} \text{ M}_\odot$ from clustering studies (Cooke et al. 2006), an isothermal profile does not provide a physically plausible model. On the other hand, modelling DLAs using an exponential disc, as is appropriate for gas-rich galaxies, implies a much smaller value of $C \sim 10^{-3} - 10^{-2}$. Thus, the effect of fluctuations in the ionizing background on the mass averaged H I density is expected to be small ($\lesssim 1$ per cent).

We can now add the effects of fluctuations in ionizing background on the neutral content of optically thin and optically thick absorbers within a region of large-scale overdensity $\delta$. We have

$$x_{\text{HI}} = F_{\text{thin}} x_{\text{HI}}(1 + \delta - \delta_J) + F_{\text{thick}} x_{\text{HI}}(1 + b\delta - \delta - C \delta_J)$$

$$\sim x_{\text{HI}} [1 + (F_{\text{thin}} + F_{\text{thick}} (b - 1)) \delta - (F_{\text{thin}} + C F_{\text{thick}})\delta_J],$$

(12)

where $F_{\text{thin}}$ and $F_{\text{thick}}$ are the fractions of H I in the optically thin and optically thick regimes, respectively. In the last equality, we assume the case where the optically thin component contains a negligible amount of the cosmic H I. Note that we ignore the optically thick component of the hydrogen in the remainder of this paper. However, we point out that the inclusion of this additional component changes only the values of the co-efficients of $\delta$ and $\delta_J$ in the above equations, but not the form of the expression.

2.2.4 Scale dependence of fluctuations in the ionizing background

Before deriving the fluctuation in neutral fraction, we next need to calculate the dependence of the parameter $\delta_J$ on scale. Ionizing radiation in the IGM has a finite mean-free-path which increases with time following the end of reionization. On scales larger than the mean-free-path ($\lambda_{\text{mfp}}$), with associated wave number $k < k_{\lambda_{\text{mfp}}} = 2\pi / \lambda_{\text{mfp}}$, all ionizing photons within a fluctuation were produced by galaxies that were also within that fluctuation. In this case, fluctuations in the ionizing background simply trace fluctuations in the density of galaxies so that the value of $\delta_J \propto b\delta$ is independent of scale. However, on scales smaller than the mean-free-path, ionizing photons were not produced by galaxies local to the fluctuation. As a result, $\delta_J / \delta$ is scale dependent. To derive this dependence we convolve the real space density field with a filter function to account for the effects of finite mean-free-path and the inverse square dependence of ionizing flux. The fluctuation in flux at a position $x$ becomes

$$\delta_J(x) \propto \int d^2 y G(x, x') \delta(x').$$

For simplicity we assume that all ionizing photons travel one mean-free-path, hence

$$G(x, x') = \frac{\Theta(|x - x'| - \lambda_{\text{mfp}})}{|x - x'|^2},$$

(13)

where $\Theta$ is the Heaviside step function. This may be rewritten

$$\delta_J(k) \propto g(k) \delta_k,$$

where $g(k)$ and $\delta_k$ are the Fourier transforms of the ionizing flux field and filter function respectively. We have

$$g(k) \propto \frac{\sin(k \lambda_{\text{mfp}})}{k},$$

(14)

and $k = |k|$. For small scales $k \lambda_{\text{mfp}} \gg 1$, we obtain

$$\delta_J(k) \propto \delta_k / k.$$  

For large scales we find $\delta_J(k) \propto \delta_k$ as expected from the argument described above.

Thus, we get the following dependence for the function $s$, which can now be written in terms of $\delta$,

$$s(\delta) = [1 - K(k) b \delta],$$

(16)

where we have explicitly written in the galaxy bias (noting that it is the galaxy density rather than the matter density which sources the ionization field), and defined the function

$$K(k) = K_0 \left(1 + \frac{k}{k_{\lambda_{\text{mfp}}}}\right)^{-1},$$

(17)
where $K_o \propto C$ is a new constant, which from the discussion following equation (11) is expected to be smaller than $\sim 0.01$. This fitting function interpolates smoothly between the limiting behaviour on small and large scales. We emphasize that this formulation is only valid on scales for which fluctuations in the density field are in the linear regime.

We can now express the fluctuation in the neutral hydrogen content, noting that this depends both on the density of galaxies and on the fluctuations in the ionizing background:

$$\delta_{HI} = [b(1 - K(k)) - 1]\delta.$$  

Similarly, we write the fluctuation in ionized fraction in terms of the local ionized hydrogen fraction ($x_i$) as

$$\delta_i \equiv x_i/\bar{x}_i - 1.$$  

(18)

Using the fact that $x_i + x_{HI} = 1$, we then find the ionization fluctuation in terms of the galaxy bias

$$\delta_i \approx [\bar{x}_{HI}/(\bar{x}_{HI} - 1)] [b(1 - K(k)) - 1]\delta.$$  

(19)

### 2.3 The 21-cm PS

We can now turn to calculating the 21-cm PS. To lowest order in Fourier space, the velocity fluctuation may be written as, $\delta_v(k) = -f \mu^2 \bar{\delta}$ where $\mu$ is the cosine of the angle between the $k$ and line-of-sight unit vectors (Kaiser 1987), and $f = d \log b/d \log (1 + z)$. To leading order in $\delta$, it then follows from equation (1) that the PS of brightness temperature fluctuations is given by,

$$P_{\delta T} = T_b^2 \{[\bar{x}_{HI} P_{SS} - 2 \bar{x}_{HI}(1 - \bar{x}_{HI}) P_{SH} + (1 - \bar{x}_{HI}) P_{SH}] + 2f \mu^2 [\bar{x}_{HI} P_{SS} - \bar{x}_{HI}(1 - \bar{x}_{HI}) P_{SH}] + f^2 \mu^4 (\bar{x}_{HI} P_{SS})\},$$  

(20)

where $T_b = 23.8(1 + z)/10^4$ mK, and $P_{SS}, P_{SH}$ and $P_{SS}$ are the PS of density fluctuations, the PS of ionization fluctuations and the cross-PS of ionization and density fluctuations, respectively. Our expression for $\delta_v$ allows the ionization PS and ionization-density cross-PS to be related to the density PS in a very simple way:

$$P_{\delta x} = -\frac{\bar{x}_{HI}}{1 - \bar{x}_{HI}} \{b[1 - K(k)] - 1\} P_{SS}$$  

(21)

and

$$P_{\delta x} = \frac{\bar{x}_{HI}}{(1 - \bar{x}_{HI})^2} \{b[1 - K(k)] - 1\}^2 P_{SS}.$$  

(22)

In addition, the ionization PS and ionization-density cross-PS are very simply related to each other

$$P_{\delta x} = -\frac{\bar{x}_{HI}}{(1 - \bar{x}_{HI})} \{[1 - K(k)] - 1\} P_{SS}.$$  

(23)

Upon substitution of equation (21) into equation (20), we obtain an expression for the angular dependence of the post-reionization 21-cm PS:

$$P_{\delta T} = T_b^2 \bar{x}_{HI}^2 \{b[1 - K(k)] + f \mu^2\}^2 P_{SS}.$$  

(24)

Thus, when combined equations (20) and (21) provide a simple relation between the density PS and the 21-cm PS involving ionization terms, allowing the full 21-cm PS to be utilized in deriving cosmological constraints. This compares favourably with the reionization epoch, during which either the $\mu^2$ term alone can be utilized (Barkana & Loeb 2005), or a functional form for the ionization PS must be assumed (Mao et al. 2008; Rhook, Geil & Wyithe 2009). On large scales, fluctuations in the ionization field will be correlated with density fluctuations. Thus, fluctuations in ionizing background would be degenerate with the galaxy bias $b$. However, on small scales, there is scale dependence in the function $K$ which breaks the degeneracy between ionization and galaxy bias. In addition, the angular dependence of the PS, which is due to gravitational infall and hence is independent of galaxy bias or ionization, breaks the degeneracy between galaxy bias and neutral fraction.

Equation (23) is valid on scales for which fluctuations are in the linear regime, and we can assume that galaxy bias is not scale dependent. In the case where $b = f = 1$ and $K_o = 0$ (i.e. unbiased sources at high redshift, with no modulation by the ionizing background), our expression for the PS reduces to the form of a uniformly ionized IGM, $P_{\delta T} = T_b^2 \bar{x}_{HI}^2 (1 + \mu^2)^2 P_{SS}$ (Barkana & Loeb 2005). We note that in the absence of a modulation of power due to the ionizing background, equation (23) could have been derived directly from considering the galaxy PS (once it is realized that effect of a line-of-sight velocity is the same for the optical depth in the 21-cm line and the galaxy number density).

Equation (23) is valid for a single halo mass. However, the DLAs reside within haloes having a range of masses. To account for this, we define an unknown $H_1$ mass weighted probability density function $p(b) db$ for the galaxy bias. By substituting $\delta_v$ into equation (1) and integrating over $p(b)$, it is easy to show that the PS for an arbitrary distribution of bias is

$$P_{\delta T} = T_b^2 \bar{x}_{HI}^2 \bar{x}_s \{b[1 - K(k)] + f \mu^2\}^2,$$  

(25)

where $b$ is the mean of the bias distribution $p(b)$. For completeness, we also calculate the spherically averaged PS:

$$P_{\delta T} = T_b^2 \bar{x}_{HI}^2 \bar{x}_s \{b[1 - K(k)] + f \mu^2\}^2 \times \left\{b^2[1 - K(k)]^2 + \frac{2}{3} f(b)[1 - K(k)] + \frac{1}{3} f^2\right\},$$  

(25)

which, in the case of $b = 1$, $f = 1$ and $K_o = 0$ reduces to the form for a uniformly ionized high-redshift IGM, $P_{\delta T} = 1.87 \times T_b^2 \bar{x}_{HI}^2 \bar{x}_s$ (Barkana & Loeb 2005). Note, although the relation between neutral hydrogen mass and host halo mass is unknown, the PS depends only on the mean of $p(b)$. For later use, we define $M(b)$ to be the halo mass corresponding to $b$.

In Fig. 1, we show examples of the dimensionless 21-cm PS after reionization [$\Delta = k^3 P_{\delta T}/(2\pi^2)$]. We assume $M(b) = 10^{13} M_\odot$, use the Sheth–Tormen fitting function for the bias (Sheth, Mo & Tormen 2001), and consider three redshifts, $z = 2.5, 3.5$ and $5.5$, at which the mean-free-path for ionizing photons is taken to be $\lambda_{\text{mfp}} = 600, 300$ and $100$ comoving Mpc, respectively (Bolton & Haehnelt 2007; Faucher-Giguère et al. 2008). There is at least a factor of 2 uncertainty in the value of the mean-free-path. However, our results are insensitive to the exact value because the mean-free-path is larger than the scales accessible to 21-cm PS observations (owing to the requirements of foreground removal) at all redshifts except those closest to reionization.

Two models are shown to illustrate the effect of the ionizing background on 21-cm fluctuations. In the first, we assume a fiducial model for which the ionizing background has no effect on the fluctuations in neutral fraction ($K_o = 0$). This is shown by the dotted curves in Fig. 1. We also show examples of a model in which $K_o = 1$, a value which is unphysically large but is chosen so as to

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8 The quantity $f$ is close to unity at high redshifts, taking values of 0.974, 0.988 and 0.997 at $z = 2.5, 3.5$ and $5.5$. 

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clearly illustrate the qualitative effect of fluctuations in the ionizing background on the shape of the 21-cm PS. In this model, the fluctuations in neutral fraction due to the ionizing background are as strong as (and hence cancel) the effect of the galaxy bias on large scales. Fig. 1 illustrates that the fractional modulation of the 21-cm PS is of the order of $K_0$, while as discussed in Section 2.2.3, physically plausible values of $K_0$ are of the order of $10^{-2}$. Since the fractional modification of the PS is of the order of 2 $K_0$, and our results indicate a fractional change in amplitude which is smaller than a factor of 2 at $k \sim 0.1$ over the redshift range considered, we therefore find that fluctuations in the ionizing background should modify the 21-cm PS after reionization by $\lesssim 1$ per cent on the scales and redshifts of interest.

### 2.4 Discussion

The effect of the ionizing background on the 21-cm PS can be understood qualitatively as follows. First, we have argued that the ionizing background follows the galaxy density field on large scales. As a result, on large scales, regions of overdensity (underdensity) will have a higher (lower) than average ionizing background. In our linear formulation, the effect of the ionizing background is degenerate with galaxy bias on large scales, because in the case of $K_0 > 0$, it lowers (raises) the contribution to the 21-cm intensity from galaxies with a particular halo mass. This modification of the 21-cm intensity of galaxies reduces the amplitude of the PS, at all scales $k \ll k_{\text{adp}}$. We emphasize that because the fluctuations in the ionizing background are strongly correlated with density on large scales, they suppress rather than add additional power to the 21-cm PS. The suppression becomes greater for smaller values of $k/k_{\text{adp}}$. Thus, at fixed scale, the effect of a fluctuating background is larger at higher redshift where the mean-free-path is smaller.

On small scales $k \gg k_{\text{adp}}$, only some fraction of the ionizing background is generated locally within the fluctuation. The remainder of the ionizing background is generated within a region which averages over many fluctuations, and so has a mean value that is near that of the background. As a result, the suppression of power described above, owing to a correlation between the ionizing background and overdensity, is not as strong on small scales. Our derivation yields the variation of the suppression with scale ($\propto k^{-1}$). It should be noted that our formulation ignores the possible Poisson contribution to fluctuations in the ionizing background due to quasars. Poisson fluctuations introduce additional power into the 21-cm PS beyond the component associated with the underlying density field of galaxies. The effect of Poisson fluctuations could become important at low redshift ($z \lesssim 3$), where quasars contribute significantly to the ionizing background. We do not consider the possible Poisson contribution in the remainder of this paper which focuses on redshifts above the peak of quasar activity ($z \gtrsim 2.5$).

We emphasize that our derivation of the 21-cm PS after reionization is only valid on scales where the density fluctuations are in the linear regime. It should, therefore, be noted that there are potential complications that arise due to non-linearities in the mass and velocity fields on small scales. Derived analysis of the galaxy PS is a result of $N$-body simulations has shown (Seo & Eisenstein 2005) that the PS can be treated as linear on scales greater than 15 comoving Mpc (i.e. $k_{\text{max}} \lesssim 0.4$ $\text{Mpc}^{-1}$) at $z = 3.5$, increasing towards higher redshifts. However, weak oscillatory features in the PS, such as the baryonic acoustic oscillations, are suppressed on even larger scales because matter moves across distances on the order of $\sim 5$–$10$ Mpc over a Hubble time.\(^9\) As groups of galaxies form, the linear-theory prediction for the location of each galaxy becomes uncertain, and as a result noise is added to the correlation among galaxies and hence to the measurement of the mass PS. The noise associated with the movement of galaxies smears out the acoustic peak in the correlation function of galaxies in real space (Eisenstein, Seo & White 2007; Seo et al. 2008). The associated reduction of power in the baryonic acoustic oscillations is found to be in excess of 70 per cent on scales smaller than $k_{\text{max}} \sim 0.4$ $\text{Mpc}^{-1}$ at $z = 3.5$, corresponding to a length-scale of $\sim \pi/(2k_{\text{max}}) = 3.9$ comoving Mpc (Seo et al. 2008).

More importantly for experiments that aim to measure the PS shape is the non-linear correction to the PS due to virialized groups of DLAs (e.g. Tinker, Weinberg & Zheng 2006; Tinker 2007). The group velocity dispersion, $\sigma_{\text{vir}}$, introduces the so-called ‘finger-of-god’ in redshift space which is unrelated to the peculiar velocity according to linear theory. We can estimate the wave number at which this effect becomes important relative to the Hubble flow as $k_{\text{vir}} \sim \pi H(z)/(2\sigma_{\text{vir}})$. At $z \sim 3.5$, we find $k_{\text{vir}} \sim 10$ $\text{Mpc}^{-1}$. In the future, an improvement to our analysis could be made by including the analytic model for the redshift space galaxy two-point correlation function described by Tinker (2007). This model is constructed within the framework of the halo occupation distribution, and quantifies galaxy bias on linear and non-linear scales. In addition, the\(^9\) This characteristic scale of displacement follows from the fact that $\sigma_{\text{g}}$, the normalization of the PS on $8$ $h^{-1}$ Mpc, is of the order of unity at the present time.
model describes redshift space distortions and clustering on both linear and non-linear scales. Finally, other non-linear effects may arise that are not present in galaxy redshift surveys, owing to internal rotation curve of the neutral hydrogen. However, the corresponding wave numbers are very large, \( k \gg 10 \, \text{Mpc}^{-1} \).

### 2.5 Sensitivity to the 21-cm PS after reionization

Before proceeding to discuss the cosmological potential of the post-reionization 21-cm PS, we compute the sensitivity with which it could be detected.

To compute the sensitivity \( \Delta P_{\Delta T}(k, \mu) \) of a radio-interferometer to the 21-cm PS, we follow the procedure outlined by McQuinn et al. (2006) and Bowman, Morales & Hewitt (2007) (see also Wyithe et al. 2008). The important issues are discussed below, but the reader is referred to these papers for further details. The uncertainty comprises of components due to the thermal noise, and due to sample variance within the finite volume of the observations. We also include a Poisson component due to the finite sampling of each mode (Wyithe 2008), since the post-reionization 21-cm PS is generated by discrete clumps rather than a smooth IGM. However, we find that Poisson noise dominates only when \( M_{\odot} > 10^{11} \, \text{M}_{\odot} \) (see Wyithe 2008). We assume that foregrounds can be removed over 8-MHz bins, within a bandpass of 32 MHz (McQuinn et al. 2006) [foreground removal, therefore, imposes a maximum on the wave-number accessible of \( k \sim 0.07 \left[ (1+z)/4.5 \right]^{-1} \, \text{Mpc}^{-1} \), and consider a hypothetical follow-up telescope to the MWA which would have 10 times the total MWA collecting area (see Wu et al. 2007). This telescope is assumed to have an antenna density distributed as \( r(r) \propto r^{-2} \) within a diameter of 2 km and a flat density core of radius 80 m (see McQuinn et al. 2006). The antennae design is taken to be optimized at the redshift of observation (so that the physical collecting area of the array equals its effective collecting area), with each of 5000 phased arrays (tiles) consisting of 16 cross-dipoles. An important ingredient is the angular dependence of the number of modes accessible to the array (McQuinn et al. 2006). We assume three fields are observed for 1000 h each. For the experiment described, the signal-to-noise ratio of the PS will be largest over the decade of scales around \( k \sim 0.1 \, \text{Mpc}^{-1} \) (McQuinn et al. 2006; Wyithe et al. 2008). The sensitivity curves (within bins of \( \Delta k = k/10 \) are plotted as the solid grey lines in Fig. 1. We find that both the 21-cm PS, as well as the effect of a non-zero value of \( K_{\alpha} \) would be easily measured using the MWA 5000.

We note that the unprecedented power of cosmic-variance limited 21-cm surveys for constraining cosmological parameters is made possible by the wide fields of view for the low-frequency telescopes under construction, combined with the very large volumes available at high redshift (Loeb & Wyithe 2008; Mao et al. 2008). For example, in the units of the Sloan Digital Sky Survey volume \( (V_{\text{sd}} = 5.8 \times 10^6 \, \text{Mpc}^3) \), the telescope described would observe \( V/V_{\text{sd}} \sim 2 \), \( V/V_{\text{sd}} \sim 7.6 \) and \( V/V_{\text{sd}} \sim 9.8 \) for \( z = 2.5 \), 3.5 and 5.5, respectively. These large volumes are obtained because of the very wide field of view available to an array with a design like MWA. The array described would have a primary beam of \( A_{\theta} \sim 2000 \, \text{deg}^2 \).

### 3 APPLICATION OF THE ALCOCK–PACZYNSKI TEST

Like traditional galaxy redshift surveys, the observed 21-cm PS is sensitive to only one underlying PS \( (P_{\Delta T}) \), in addition to the four parameters, \( \tilde{x}_{\delta z}, (b), k_{\text{amp}} \) and \( K_{\alpha} \). These parameters are related to properties of DLAs and LLSs, and can be measured through independent means via quasar absorption line studies (particularly true for \( \tilde{x}_{\delta z}, (b) \) and \( k_{\text{amp}} \)). This situation should be contrasted with the reionization era where the observed 21-cm PS is also sensitive to long range, non-gravitational fluctuations through \( P_{\delta} \) and \( P_{\nu} \). Provided that the non-linear evolution of the PS can be properly accounted for, this dominant dependence on the density PS provides a considerable advantage for the purposes of cosmological constraints.

To illustrate for cosmological constraints from the anisotropy of the 21-cm PS after reionization, we calculate the AP effect. Our approach is to specify the general result of Barkana (2006) for the distortion of the true PS \( \left[ P_{\Delta T}(k, \mu) \right] \) resulting from an incorrect choice of cosmology. This is parametrized in terms of the dilution parameters \( \alpha \) and \( \alpha_{\perp} \), which describe the distortions between the transverse and line-of-sight scales, and in the overall scale, respectively. These are defined such that \( (1 + \alpha) = \frac{\alpha}{\alpha + \alpha_{\perp}} \), \( \alpha_{\perp} \) is the ratio between the assumed and true values of the angular diameter distance, \( D_A \). In the AP test, the correct cosmology is inferred by finding cosmological parameters for which \( \alpha = \alpha_{\perp} = 0 \).

To calculate the AP effect, we apply equation (8) in Barkana (2006):

\[
P_{\Delta T}(k, \mu) = \left( 1 + \alpha - 3\alpha_{\perp} \right) P_{\delta}^\perp \left( \mu \right) + \left( \alpha \mu^2 - \alpha_{\perp} \right) \frac{\partial P_{\Delta T}^\perp}{\partial \ln k} + \alpha \left( 1 - \mu^2 \right) \frac{\partial P_{\Delta T}^\perp}{\partial \ln \mu} \tag{26}
\]

To the PS in equation (24). Here, the PS \( P_{\Delta T} \) and \( P_{\Delta T}^\perp \) are evaluated at the observed \( k \). This procedure results in a modified PS that is related to the true density PS \( \left( P_{\delta}^\perp \right) \) via

\[
P_{\Delta T}(k, \mu) = T_{\delta}^\perp x_{\delta z}^\perp P_{\delta}^\perp \left( \mu \right) \left( b \left( 1 - K(k) \right) + \mu^2 \right) \frac{\partial P_{\Delta T}^\perp}{\partial \ln k} \tag{27}
\]

Barkana (2006) expanded the general form of this equation to find the linear combinations of the true PS \( P_{\delta}^\perp, P_{\delta}^\perp, P_{\delta} \) and \( P_{\delta}^\perp \) which form the coefficients of terms containing \( \mu^0, \mu^2, \mu^6, \mu^8 \). His result shows that in general, the parameter \( \alpha \) (which corresponds to anisotropy) mixes \( P_{\delta}^\perp, P_{\delta}^\perp, P_{\delta} \). As a consequence, the \( \mu^6 \) term, which arises through the AP effect, must be isolated in order to probe the anisotropy parameter \( \alpha \). On the other hand, equation (27) is sensitive only to \( P_{\delta}^\perp \). As a result, the coefficients of all powers of \( \mu \) in the post-reionization PS can be utilized in the AP effect to find the cosmological parameters that yield \( \alpha = \alpha_{\perp} = 0 \).

### 3.1 AP constraints on the PS

We next use equation (27) to calculate the permissible region of parameter space \( \mathcal{P} = \left( \alpha, \alpha_{\perp}, \tilde{x}_{\delta z}, M_{\odot}, K_{\alpha} \right) \) around a true solution with PS \( P_{\delta}^\perp \) and \( p_{\nu} = \left( 0, 0, 0.02, 10^{11} \, \text{M}_{\odot} \right) \). The fiducial value of \( M_{\odot} \) is chosen to lie in the middle of the mass range for DLAs measured from the cross-correlation analysis of Cooke et al. (2006).
We have assumed that the mean-free-path of ionizing photons is known a priori, and do not fit it as a free parameter. As part of this procedure, we assume an estimate for the sensitivity of the future low-frequency interferometer described in Section 2.5 to the 21-cm PS \( \Delta P_{ST}(k, \mu) \), and construct likelihoods

\[
\ln L(p) = -\frac{1}{2} \sum_{k, \mu} \left[ \frac{P_{ST}(k, \mu, p) - P_{ST}(k, \mu, p_o)}{\Delta P_{ST}(k, \mu)} \right]^2,
\]

where the sum is over bins of \( k \) and \( \mu \). To estimate the constraints achievable via the AP effect, we modify the linear PS to include the non-linear erasure of the acoustic peaks based on the work of Eisenstein et al. (2007). Specifically, we use

\[
P_{\delta\delta}(k) = \left[ P_{\delta\delta}(k) - P_{\delta\delta}^{\text{smth}}(k) \right] \exp \left( -k^2 \Sigma_{nl}^2 / 2 \right) + P_{\delta\delta}^{\text{smth}}(k),
\]

where \( P_{\delta\delta}^{\text{smth}} \) is the ‘no wiggle’ form from Eisenstein & Hu (1999), and \( \Sigma_{nl} = 3.9[(1+z)/4.5]^{-1} \) Mpc in the high-redshift limit. In addition, to account for the possibility of non-linearity in the smooth PS at small scales, we restrict our fitting to wave numbers \( k_{\text{max}} < 0.4 \) Mpc\(^{-1} \) (see Section 2.4).

The results are shown in Fig. 2 for three redshifts, \( z = 2.5, 3.5 \) and 5.5. Likelihood contours are shown at each redshift for the parameter sets \((\alpha, \alpha_\perp), (\bar{x}_{HI}, M_{b})\) and \((K_o, M_{b})\). For each set, the likelihood is marginalized over the remaining three parameters assuming flat prior probabilities. The exception is \( \bar{x}_{HI} \approx 0.02 \pm 0.002 \), which we have assumed to be known a priori (with Gaussian errors) from future observations of quasar absorption spectra. The assumed uncertainty in \( \bar{x}_{HI} \) corresponds to a factor of \( \sim 2 \) improvement over existing measurements (Wolfe et al. 2005). Fig. 2 shows that the angular dependence of the PS breaks degeneracy between different parameters, allowing \( \bar{x}_{HI}, M_{b}, \alpha \) and \( \alpha_\perp \) to each be measured from the fitting. In addition, the departure from the underlying shape of PS allows the value of \( K_o \) to also be constrained.

We note that by including the uncertainties in the AP effect (through \( \alpha \) and \( \alpha_\perp \)), our derived uncertainties for non-cosmological parameters \((M_{b}, \bar{x}_{HI} \) and \( K_o)\) include the uncertainty in the underlying matter PS. An exception is that the uncertainty in the PS amplitude (which is proportional to the normalization of the primordial PS, \( \sigma_8 \)) is degenerate with \( \bar{x}_{HI} \). However, since the error in \( \bar{x}_{HI} \) is comparable to or larger than the fractional error in \( \sigma_8 \), this does not add additional uncertainty relative to the constraints shown in Fig. 2. An important additional uncertainty may also be introduced through the uncertainty in the primordial PS index or through a running spectral index, which we have not considered in this study.

At \( z \lesssim 3.5 \), the neutral fraction constraints are improved over the assumed prior information from the IGM. The mass of the DLA hosts is constrained to high precision (tens of per cent, Wyithe...
21-cm emission after reionization

Figure 3. Central and right-hand panels: constraints on the pre-reionization PS distortions achievable via the AP test at two redshifts. The results correspond to an optimistic high-redshift case, with \( \bar{x}_{HI} = 1 \) and \( P_{xx} = P_{\delta\delta} = 0 \). The panels show the parameter set \((\alpha, \alpha_\perp)\). In each case, likelihood contours are shown at 60 and 7 per cent of the maximum likelihood with each set marginalized over \( \bar{x}_{HI} \) (which we have assumed to be unconstrained a priori). The projections of these contours on to individual parameter axes correspond to the 68.3 and 90 per cent confidence ranges, respectively. For comparison, (left-hand panel) the \( K_\alpha = 0 \) case at \( z = 3.5 \) is repeated from Fig. 2.

2008), with more accurate estimates at lower redshifts. In addition, the value of \( K_\alpha \) is constrained to be smaller than a few hundredths (with minimal redshift dependence), although this in not at a level comparable to the physically expected value of \( \sim 10^{-2} \). Thus, on scales larger than the mean-free-path, the perturbation due to the ionizing background \((b) K_\perp \delta \) is constrained at a level that is smaller than \((\delta^2)^{1/2} \). On scales smaller than the mean-free-path, this perturbation is suppressed by \((k/k_{\text{mfp}})^{-4} \). The cosmological constraints follow from the precision with which the distortion of the 21-cm PS (as described by \( \alpha \) and \( \alpha_\perp \)) can be measured. We find that distortions owing to the assumption of an incorrect cosmology could each be constrained at a per cent level.

3.2 Comparison with high-redshift constraints
Before concluding, we show for comparison (Fig. 3) the constraints on the parameter set \((\alpha, \alpha_\perp)\) for optimistic high-redshift cases \((z = 12.5 \text{ and } 15)\), where we assume \( \bar{x}_{HI} = 1 \) and fluctuations dominated by the density field \((P_{\delta\delta} = P_{xx} = 0)\). These cases have fitted parameters \( p = (\alpha, \alpha_\perp, \bar{x}_{HI}) \), and correspond to the IGM prior to the formation of galaxies and ionized bubbles. The smaller \( \bar{x}_{HI} \) at \( z < 6 \) relative to the pre-reionization epoch is offset by the galaxy bias factor enhancement as well as the growth of structure, and by lower foreground contamination. With respect to constraining cosmological parameters, we find that the post-reionization 21-cm PS would be competitive with the most optimistic expectations for the 21-cm PS at high redshift.

4 CONCLUSIONS
In this paper, we have derived the 21-cm PS following the completion of reionization. Our approach is to derive the PS within the formalism that has been developed to describe the 21-cm PS during reionization (Barkana & Loeb 2005; McQuinn et al. 2006; Mao et al. 2008). By framing the derivation this way, we are able to directly compare the relative merits of cosmological constraints from 21-cm surveys at redshifts prior to and post-reionization. We have derived expressions for the components of the post-reionization 21-cm PS that are due to the density field, the ionization field and their cross-correlation. As is the case prior to reionization, we find that these components contribute to the observed PS in proportions that depend on the angle relative to the line of sight along which the power is measured. However, in difference from the situation prior to reionization, we have shown that all components of the 21-cm PS are directly proportional to the PS of the underlying matter fluctuations, with a small but predictable modulation on scales below the mean-free-path of ionizing photons. We have derived the form of this modulation, and have shown that its effect on the observed PS will be at less than the 1 per cent level for physically plausible astrophysical models of DLA systems. The simplicity of the 21-cm PS after reionization stands in contrast to the astrophysical uncertainty during the reionization epoch, where \( \text{HI} \) regions dominate the 21-cm signal. This simplicity will enhance the utility of the 21-cm PS after reionization as a cosmological probe (Loeb & Wyithe 2008) by removing the need to separate the PS into physical and astrophysical components (Barkana & Loeb 2005).

To illustrate the utility of the 21-cm PS after reionization as a cosmological probe, we have examined the AP (Alcock & Paczynski 1979) test. Our calculations show that the next generation of low-frequency arrays could measure the angular distortion of the PS to around \( \sim 1 \) per cent at \( z \sim 3.5 \). It has previously been shown that the scale of baryonic acoustic oscillations, which constitutes a standard ruler (Blake & Glazebrook 2003; Eisenstein et al. 2005; Seo & Eisenstein 2005; Padmanabhan et al. 2007), can be also be used to independently probe \( H \) and \( D_A \) in 21-cm PS, and hence to measure the equation of state of the dark energy (Chang et al. 2008; Wyithe et al. 2008). With the caveat that non-linear evolution of the 21-cm PS must be quantitatively understood, our simple analysis indicates that the precision achievable via the AP (Alcock & Paczynski 1979) test using the 21-cm PS after reionization could be better than those available via a 21-cm fluctuation measurement of the acoustic scale of baryonic oscillations for a given observing strategy (Shoji, Jeong & Komatsu 2009). More generally, our analysis shows that the 21-cm PS after reionization shares the same favourable features as a galaxy redshift survey. The advantage of using 21-cm fluctuations lies in the fact that individual sources need not be resolved. This would allow PS measurements using 21-cm fluctuations to be extended to higher redshifts.

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