Amplitude-squared squeezing of Schrödinger cat states via postselected von Neumann measurement

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Abstract We know that the original Schrödinger cat states have no amplitude-squared squeezing. In this paper, we investigate the amplitude-squared squeezing of Schrödinger cat states using postselected von Neumann measurement. The results show that after postselected von Neumann measurement, the amplitude-squared squeezing of Schrödinger cat states change dramatically and this can be considered a result of weak value amplification. The squeezing effect also investigated by studying the Wigner function of Schrödinger cat states after postselected measurement.

1 Introduction

The squeezing effect plays an essential role in the framework of quantum theory and its applications. The squeezed states of radiation fields have reduced uncertainty in specific field quadrature, i.e., quadrature fluctuations are below the level associated with the vacuum state or with coherent state [1]. Thus, the squeezed states of radiation fields which possesses the squeezing effect are considered truly quantum [2] and have no classical counterpart [3,4]. The study of squeezing, especially quadrature squeezing of radiation fields, has potential application in optical communication and information theory [5–16], gravitational wave detection [17], quantum teleportation [17–25], dense coding [26], resonance fluorescence [27], and quantum cryptography [28].

With the rapid development of techniques for making higher-order correlation measurements in quantum optics and laser physics, it is possible to define the higher-order squeezing effect of the field. By considering higher-order correlation functions of the amplitude, Hong and Mandel [29] defined a state to be squeezed to the $2^{N}$th order if the expectation value of the $2^{N}$th power of the difference between a field quadrature component and its average value is less than it would be in a coherent state. Hilley [30,31] defined another type of higher-order squeezing, named amplitude-squared squeezing (ASS) of the electromagnetic field. This type of squeezing arises in a natural way in second-harmonic generation and in a number of nonlinear optical processes. After that the ASS and more higher-order squeezing of radiation fields have been investigated in various physical systems [32–48], and these theoretical studies have suggested a possibility to extract information from an optical signal by higher-order correlation measurements. This has attracted our interest as a means of exploring states which possess higher-order squeezing. This purpose is related to the generation and optimization of various quantum states of light; since many of those states may possess important nonclassical properties like squeezing and sub-Poissonian photon statistics. In recent years, significant attention has been given to this purpose and Schrödinger cat states are a typical example. Schrödinger cat states which are defined as the quantum superposition of two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ have numerous theoretical and practical applications in research fields of quantum optics, quantum computation [49,50], and quantum information science [51–55]. Research has shown that the even Schrödinger cat state $(|\alpha\rangle + |-\alpha\rangle)$ exhibits normal squeezing but not sub-Poissonian statistics, while the odd Schrödinger cat state $(|\alpha\rangle - |-\alpha\rangle)$ exhibits sub-Poissonian statistics but has no normal squeezing effect [56]. It is a well-known fact that Schrödinger cat states have no ASS effect and seem unsuited to related higher-order correlation measurements. Thus, the question may arise: Is there any method to amplify or change the inherent properties of a quantum state such a Schrödinger cat state, so that it can repossess higher-order squeezing? This question can be addressed through the application of the weak value amplification technique.

The weak value amplification technique is related to the postselected weak measurement proposed by Aharonov, Albert, and Vaidman in 1988 [57]. One of the distinguished properties of weak measurement compared with traditional projective quantum measurement is that the induced weak value of the system observable can take large anonymous values [58]. The feature of the weak value is usually referred to as an amplification effect of postselected weak measurement and can be used to amplify tiny but useful information on physical systems [59–67]. For details of applications of weak measurement in signal amplification processes, we refer the reader to the recent overview of the field [68,69]. In weak measurement theory, the interaction strength between the system and the measurement is too weak, and it is enough only to consider the evolution of the unitary operator up to its first order.

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However, if we want to connect the weak and strong measurement, investigate the measurement feedback of postselected weak measurement, and analyze experimental results obtained in nonideal measurements, the full-order evolution of the unitary operator is needed [70–72]. This kind of measurement is referred to as a postselected von Neumann measurement. The postselected von Neumann measurement can be used in state optimization and precision measurement problems [73–79]. Recently, one of the authors of this paper investigated the effects of postselected von Neumann measurement on the properties of single-mode radiation fields [78] and found that postselected von Neumann measurement changed the photon statistics and quadrature squeezing of radiation fields for different anomalous weak values and coupling strengths. However, to the best of our knowledge, the effects of postselected von Neumann measurement on higher-order squeezing phenomena of radiation fields have not been previously investigated.

In this work, motivated by our investigations [76,78,79], we study the effects of postselected von Neumann measurement on ASS of Schrödinger cat states. To achieve our goal, we take the spatial and polarization degrees of freedom of Schrödinger cat states as the measuring device (pointer) and system, respectively. Along with the standard process of weak measurement, we take pre- and post-selection on the system observable. By considering the full-order evolution of the unitary evolution operator of the total system, we determine the final state of the pointer after the postselected measurement. After checking the criteria for existence of ASS of the radiation field, we found that after using postselected measurement, the ASS of Schrödinger cat states changed more dramatically than the original one. We plot the related figures with allowed parameters and the analytical results indicated that the ASS effects on Schrödinger cat states are caused by signal amplification of the weak measurement technique. In order to provide more details of the squeezing phenomena of Schrödinger cat states after the postselected measurement, we evaluated the Wigner function for this state. When compared to the initial state, negative peaks as well as interference structures of Wigner functions in phase space changed significantly after postselected measurement. We also found that the shapes of the Wigner function of Schrödinger cat states are not only squeezed, but the negative regions increased with increasing coupling strength between the system and pointer. These results indicated that the nonclassicality of Schrödinger cat states is more pronounced with large weak values. We believe that since higher-order correlation measurements are necessary in some interdisciplinary fields such as quantum biology and quantum metrology, the current research motivated by the postselected weak measurement technique is of significant value and may provide some new, effective methods for implementations of related processes in those emerging research fields.

The rest of this paper is organized as follows. In Sect. 2, we outline the interpretation of ASS, introduce the postselected measurement model and present the normalized pointer final pointer state of our scheme. In Sect. 3, we study the ASS of Schrödinger cat states which occur after postselected von Neumann measurement. In Sect. 4, we investigate the Wigner function of Schrödinger cat states after postselected measurement to explain the squeezing phenomena. Finally, we conclude this work in Sect. 5.

2 Fundamental concepts

2.1 The definition of squeezing of squared field amplitude

As early as 1987, M. Hillery showed that squeezing of squared field amplitude (amplitude-squared squeezing) is a nonclassical effect and gave some specific examples [31]. Consider a single mode of electromagnetic field of frequency $\theta$ with creation and annihilation operator $a^\dagger$, $a$. The real and imaginary parts of the square of the field mode amplitude can be written as

$$Y_1 = \frac{(A^2 + A^\dagger A^2)}{2}, \quad Y_2 = i \frac{(A^2 - A^\dagger A^2)}{2},$$

where $A$ and $A^\dagger$ are slowly varying operators defined by $A = e^{i\theta \hat{a}}$, $A^\dagger = e^{-i\theta \hat{a}^\dagger}$ and obey the same commutation relations as $a$, $a^\dagger$. The operators $Y_1$ and $Y_2$ satisfy the commutation relationship [31]

$$[Y_1, Y_2] = i (2N + 1),$$

where $N$ is the number operator, $N = A^\dagger A$. Thus, the operators $Y_1$ and $Y_2$ obey the uncertainty relation

$$\Delta Y_1 \Delta Y_2 \geq \langle N + \frac{1}{2} \rangle.$$  \hspace{1cm} (3)

Here, $\Delta Y_{1,2} = \sqrt{\langle Y_{1,2}^2 \rangle - \langle Y_{1,2} \rangle^2}$ denotes the variance of $Y_{1,2}$ under arbitrary state $|\phi\rangle$. We say that the ASS exists in the variable $Y_i$ if it satisfies

$$(\Delta Y_i)^2 < \langle N + \frac{1}{2} \rangle \quad \text{for } i = 1 \text{ or } 2.$$  \hspace{1cm} (4)

In short, the system characterized by the wave function $|\phi\rangle$ may exhibit nonclassical features if it satisfies Eq. (4).

2.2 Postselected von Neumann measurement and weak value

In Sect. 1, we mentioned the applications of postselected weak measurement. Here, we introduce the main idea of postselected von Neumann measurement with added related quantities used in our current work. In quantum measurement theory, the interaction
with value amplification, and the postselected weak measurement technique possesses numerous applications as mentioned in Sect. 1.

As we know, the weak measurement is characterized by pre- and post-selection of the system state and a weak value. We assume that initially the system and measuring device (pointer) are prepared to that initially the system and measuring device (pointer) are prepared to interaction and can take all allowed values in weak and strong measurement regimes. After this von Neumann type postselected state \(|\psi_f\rangle\otimes|\psi_1\rangle\) and obtain the information about a physical quantity \(\hat{\sigma}_x\) from the final pointer state by the following weak value

\[
\langle \hat{\sigma}_x \rangle_w = \frac{\langle \psi_f | \hat{\sigma}_x | \psi_f \rangle}{\langle \psi_f | \psi_f \rangle}.
\]

This is the definition of the weak value of the system observable. From Eq. (6), we know that when the preselected state \(|\psi_1\rangle\) and the postselected state \(|\psi_f\rangle\) are almost orthogonal, the absolute value of the weak value can be arbitrarily large. We call this feature weak value amplification, and the postselected weak measurement technique possesses numerous applications as mentioned in Sect. 1.

We can express the position operator \(\hat{X}\) and momentum operator \(\hat{P}\), in terms of the annihilation (creation) operators, \(\hat{a}\) (\(\hat{a}^\dagger\)) in Fock space representation as

\[
\hat{X} = \sigma (\hat{a}^\dagger + \hat{a}),
\]

\[
\hat{P} = \frac{i}{2\sigma} (\hat{a}^\dagger - \hat{a}),
\]

where \(\sigma = \sqrt{\hbar/2m\nu}\) is the size of the Gaussian ground state that depends on the mass of the pointer \(m\) and the frequency \(\nu\) with which the system oscillates, and \([\hat{a}, \hat{a}^\dagger] = \hat{I}\). Thus, we can write the unitary evolution operator \(e^{-i\hat{\sigma}_x \otimes \hat{P}}\) by using Eq. (8) as

\[
e^{-i\hat{\sigma}_x \otimes \hat{P}} = \frac{1}{2} (\hat{I} + \hat{\sigma}_x) \otimes D \left( \frac{\Gamma}{2} \right) + \frac{1}{2} (\hat{I} - \hat{\sigma}_x) \otimes D \left( -\frac{\Gamma}{2} \right),
\]

since the operator \(\hat{\sigma}_x\) satisfies \(\hat{\sigma}_x^2 = \hat{I}\). Here, the parameter \(\Gamma = g/\sigma\), and \(D(\mu)\) is a displacement operator with complex \(\mu\) defined by

\[
D(\mu) = e^{i\mu \hat{a}^\dagger - \mu^* \hat{a}}.
\]

Note that \(\Gamma\) characterizes the measurement strength, and we can say that the coupling between the system and pointer is weak (strong) if \(\Gamma < 1(\Gamma > 1)\). We assume throughout that the coupling constant \(\Gamma\) is an effective strength of the system and pointer interaction and can take all allowed values in weak and strong measurement regimes. After this von Neumann type postselected measurement, the total final system state is given as

\[
|\Theta\rangle = |\psi_f\rangle \otimes |\hat{I} \left( e^{-i\hat{\sigma}_x \otimes \hat{P}} |\psi_1\rangle \otimes |\phi\rangle \right)\rangle
= |\psi_f\rangle \otimes \langle \psi_f | e^{-i\hat{\sigma}_x \otimes \hat{P}} |\psi_1\rangle |\phi\rangle
= |\psi_f\rangle \otimes |\psi'\rangle.
\]

Here, \(|\psi'\rangle\) the final state (not normalized) of the measuring device, and its explicit expression can be written as

\[
|\psi'\rangle = \langle \psi_f | e^{-i\hat{\sigma}_x \otimes \hat{P}} |\psi_1\rangle |\phi\rangle
= \frac{1}{2} \left[ (\langle \psi_f | (\hat{I} + \hat{\sigma}_x) |\psi_1\rangle D \left( \frac{\Gamma}{2} \right) + \langle \psi_f | (\hat{I} - \hat{\sigma}_x) |\psi_1\rangle D \left( -\frac{\Gamma}{2} \right) |\phi\rangle
= \frac{1}{2} \left[ (1 + \langle \sigma_x \rangle_w) D \left( \frac{\Gamma}{2} \right) + (1 - \langle \sigma_x \rangle_w) D \left( -\frac{\Gamma}{2} \right) \right] |\phi\rangle.
\]

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This is the general expression of the final pointer state which initially prepared in $|\phi\rangle$ in our measurement scheme. In the present work, we assume that the pre- and post-selected states are

$$|\psi_i\rangle = \cos \frac{\theta}{2} |H\rangle + e^{i\phi} \sin \frac{\theta}{2} |V\rangle,$$

and

$$|\psi_f\rangle = |H\rangle,$$

and then, the weak value of the observable $\hat{\sigma}_x$ which defined in Eq. (6) is expressed as

$$\langle \sigma_x \rangle_w = e^{i\phi} \tan \frac{\theta}{2}.$$

Here, $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$. As mentioned earlier, the weak value can take an anomalous value and it is accompanied by low successful postselection probability $P_s = |\langle \psi_f | \psi_i \rangle|^2 = \cos^2 \frac{\theta}{2}$. The pre- and postselected states which we introduced in this paper can be implemented in optical experiments by using polarizing beam splitter and half/quarter wave plates [80,81].

3 The ASS of Schrödinger cat states

In this section, we study ASS of Schrödinger cat states after postselected von Neumann measurement. The Schrödinger cat state is a typical quantum state which is composed of the superposition of two coherent correlated states moving in opposite directions. This state has many applications in quantum information processing. In this study, we take the spatial and polarization degrees of freedom of Schrödinger cat states as the measuring device and system, respectively, and investigate the effects of postselected measurement on the polarization and spatial components of the Schrödinger cat state beams. The mathematical expression of normalized Schrödinger cat states is [82]

$$|\Phi\rangle = K \left( |\alpha\rangle + e^{i\delta} |\alpha\rangle - |\alpha\rangle \right),$$

where

$$K = \left[ 2 + 2e^{-2|\alpha|^2} \cos \omega \right]^{-\frac{1}{2}}$$

is the normalization coefficient, and $\alpha = |\alpha|e^{i\delta}$ is an arbitrary complex number with modulus $|\alpha|$ and argument $\delta$. Here, we would like to mention that $\omega \in [0, 2\pi]$, when $\omega = 0 (\omega = \pi)$ it is called the even (odd) Schrödinger cat state, and when $\omega = 2\pi$, it is also called the Yurke–Stoler state.

By using the fundamental concepts introduced in Sect. 2, we can get the final normalized state of the pointer after changing $|\phi\rangle$ in Eq. (12) to the Schrödinger cat states $|\Phi\rangle$, and this can be expressed as

$$|\Psi\rangle = \frac{\kappa}{2} \left[ (1 + \langle \sigma_x \rangle_w) D \left( \frac{\Gamma}{2} \right) + (1 - \langle \sigma_x \rangle_w) D \left( -\frac{\Gamma}{2} \right) \right] |\Phi\rangle,$$

with normalization constant

$$\kappa = \frac{1}{2} \left[ 1 + \langle \sigma_x \rangle_w^2 \right] + K^2 \left[ 1 - \langle \sigma_x \rangle_w^2 \right] \cos(2\Gamma \text{Im}[\alpha]) e^{-\frac{\Gamma^2}{2}}$$

$$+ K^2 \left[ \text{Re}(1 - \langle \sigma_x \rangle_w^2 - 2\Gamma \text{Im}[\langle \sigma_x \rangle_w]) \right]$$

$$\left( e^{i\omega} e^{-\frac{1}{2} |2\alpha + \Gamma|^2} + e^{-i\omega} e^{-\frac{1}{2} |2\alpha - \Gamma|^2} \right)^{\frac{1}{2}}.$$

To investigate the ASS of Schrödinger cat states, we have to follow the condition mentioned in the previous section about the existence of the ASS of a single mode electromagnetic field with frequency $\theta$. After some simple algebra, the condition for existence of ASS of a single mode radiation field, Eq. (4), can be rewritten as

$$R = (\Delta K_1)^2 - \langle a^2 \rangle + \frac{1}{2}.$$

$$= \frac{1}{2} \text{Re}[\langle a^4 \rangle] + \frac{1}{2} \langle a^2 \rangle - \left( \text{Re}[\langle a^2 \rangle] \right)^2 < 0.$$

Here, $\langle \cdot \rangle$ denotes the expectation values of corresponding quantities under the state $|\Psi\rangle$. It can be seen that the negativity of the variable $R$ reveals the ASS phenomenon of the state. In this paper, we only examine the $Y_1 = \frac{1}{2} \langle a^2 \rangle$ component of the field. To achieve our goal, first of all we have to calculate the above related quantities and their explicit expressions are listed below.

1. The expectation value $\langle a^2 \rangle$ under the state $|\Psi\rangle$ is given by

$$\langle a^2 \rangle = \frac{\kappa^2 K^2}{4} \left[ |1 + \langle \sigma_x \rangle_w^2|^2 H_1(\Gamma) + |1 - \langle \sigma_x \rangle_w|^2 H_1(-\Gamma) \right]$$
of Schrödinger cat states have the same trend with increasing

However, as indicated in Fig. 1, the

To further confirm our claims, we plotted the variation curves of

2. The expectation value \( \langle a^2 a^2 \rangle \) under the state \(|\Psi\rangle\) is given by

\[
\langle a^2 a^2 \rangle = \frac{k^2 \lambda^2}{4} \left[ \left(1 + \langle \sigma_x \rangle \right)^2 \left(1 - \langle \sigma_x \rangle \right) + 2 \Re \left[ \left(1 - \langle \sigma_x \rangle \right) \left(1 + \langle \sigma_x \rangle \right) \left(K_2(\Gamma) + K_2(-\Gamma)\right) \right] \right],
\]

with

\[
K_1(\Gamma) = \left| \alpha + \frac{\Gamma}{2} \right|^4 + \left| \alpha - \frac{\Gamma}{2} \right|^4 + 2 \Re \left[ e^{i\omega} \left( \frac{\alpha^4 + \frac{\Gamma}{2} \left( \alpha - \frac{\Gamma}{2} \right)^2 \left( \alpha + \frac{\Gamma}{2} \right)^2 \right) \right],
\]

and

\[
K_2(\Gamma) = e^{i\omega} \left| \alpha + \frac{\Gamma}{2} \right|^4 e^{-2i\alpha^2 + 2} + e^{\frac{2i\lambda\omega}{\lambda}} e^{-2i\alpha^2} \left( \frac{\alpha^4 + \frac{\Gamma}{2} \left( \alpha - \frac{\Gamma}{2} \right)^2 \left( \alpha + \frac{\Gamma}{2} \right)^2 \right) \right] \right],
\]

3. The expectation value \( \langle a^4 \rangle \) under state \(|\Psi\rangle\) is given by

\[
\langle a^4 \rangle = \langle \Psi | a^4 | \Psi \rangle = \frac{k^2 \lambda^2}{4} \left[ \left(1 + \langle \sigma_x \rangle \right)^2 G_1(\Gamma) + \left(1 - \langle \sigma_x \rangle \right) G_1(-\Gamma) \right] + (1 - \langle \sigma_x \rangle) \left(1 + \langle \sigma_x \rangle \right)^4 G_1(\Gamma) + (1 + \langle \sigma_x \rangle) \left(1 - \langle \sigma_x \rangle \right)^4 G_2(-\Gamma) \right],
\]

with

\[
G_1(\Gamma) = 2\alpha^4 + 3\Gamma^2 \alpha^2 + \frac{\Gamma^4}{8} + 2 \cos \omega e^{-2i\alpha^2} \left( \frac{\alpha^4 - 2\Gamma^2 \alpha^2 + \frac{3}{2} \Gamma^2 \alpha^2 - 2 \Gamma^3 \alpha + \frac{4}{16} \right),
\]

and

\[
G_2(\Gamma) = e^{2i\lambda\omega} \left( \frac{\alpha - \frac{\Gamma}{2} \right)^2 e^{-2i\alpha^2} + e^{i\omega} \left( \frac{\alpha + \frac{\Gamma}{2} \right)^2 e^{-2i\alpha^2} \right],
\]

To check the effects of postselected von Neumann measurement on ASS of Schrödinger cat states, we plot \( R \) as a function of \(|\alpha|\) for different \(\omega\) and the analytical results are shown in Fig. 1. We know that there is no ASS of initial Schrödinger cat states. However, as indicated in Fig. 1, the \( R \) can be below zero after postselected von Neumann measurement and can change dramatically for large values of \(|\alpha|\) with large weak values. Figure 1 also shows that \( R \) can take negative values periodically, and \( R \) of three kinds of Schrödinger cat states have the same trend with increasing \(|\alpha|\).

As mentioned in Sect. 2, in our scheme the coupling strength \( \Gamma \) can take any values in weak and strong measurement regimes. To further confirm our claims, we plotted the variation curves of \( R \) as a function of \(\Gamma\) for different weak value for the even (odd) Schrödinger cat state and Yurke–Stoler state, respectively. It is clearly shown in Fig. 2 that there is no ASS when the interaction strength \(\Gamma = 0\) (no interaction), for all three Schrödinger cat states. This is proof of the fact that there is really no ASS for Schrödinger cat states initially. From Fig. 2a, we observe that the ASS of the even Schrödinger cat state behaves more and more strongly as the interaction strength \(\Gamma\) increases. Figure 2b shows the \( R \) of the Yurke–Stoler state. We observed that in most of the regions, \( R \) takes on negative values, especially for large real values, and the curves all trend to below zero as \(\Gamma\) increases. This means that the ASS of the Yurke–Stoler state behaves more stably for strong measurement with large weak values. On the contrary, we saw in Fig. 2c that although the variable \( R \) takes negative values when \(\Gamma\) takes relatively small values, the ASS of odd Schrödinger cat state totally disappears when \(\Gamma\) increases, regardless of the change of the weak value. In addition, it is evident from Fig. 2 that increasing of the weak value has a positive effect on the ASS phenomenon of Schrödinger cat states, which we believe also results from the signal amplification effect of the weak value.
In our previous work [78], we discussed the effects of postselection von Neumann measurements on the ordinary squeezing properties of Schrödinger cat states. The results showed that the quadrature squeezing (ordinary squeezing) effect of the Schrödinger cat states increases with increasing interaction strength for anomalous weak values compared to the initial pointer state. Therefore, combined with the above results, we believe that the dramatic changes brought by the postselected von Neumann measurements on the ordinary and second-order squeezing of the Schrödinger cat state should not be underestimated.

In order to better explain the effects of the postselected von Neumann measurement on the nonclassicality of Schrödinger cat states including ordinary and ASS effects, in the next section we will discuss the Wigner function of Schrödinger cat states with state $|\psi\rangle$. 

4 Wigner function of Schrödinger cat states after postselected measurement

The Wigner function is the earliest quasi-probability distribution function in phase space, and it is also a very interesting structure in quantum mechanics. The state when the Wigner function takes negative values in phase space is a nonclassical state, and it is a direct general, this Wigner function is real, and it is bounded in phase space. We plot in Fig. 3, the Wigner functions with different parameters of interest for the Schrödinger cat states as mentioned above.

We plot in Fig. 1 ASS of Schrödinger cat states. Here, we take $\theta = \frac{\pi}{2}$, $\delta = 0$, $\psi = \frac{2\pi}{3}$, $\Gamma = 2$. 

![Graph](image)

Each row represents the even (odd) Schrödinger cat state and the Yurke–Stoler state, respectively. The three plots in each row correspond
ASS of Schrödinger cat states. Here, we take $|\alpha| = 1$ and other parameters are the same as those used in Fig. 1 a $\omega = 0$; b $\omega = \frac{\pi}{2}$; c $\omega = \pi$ from left to right represents the cases for $\Gamma_1 = 0$, 0.5, 2, and we also took the large weak value ($\varphi = \frac{7}{9}\pi$) corresponding to the most pronounced ASS phenomenon in Fig. 2. The Wigner functions in Fig. 3 all exhibit redundancy and highly nonclassical characteristics in phase space. By comparing the three plots in each row, we can see that the shapes of the Wigner functions are not only squeezed as the measured intensity $\Gamma$ increases, but we can clearly see the quantum interference structures formed between the peaks. Moreover, by comparing each histogram we can see that there are some differences in the peaks of the Wigner functions for different states at the same interaction strength $\Gamma$. In the strong measurement region ($\Gamma = 2$), the Wigner functions of the three types of Schrödinger cat states exhibited the most pronounced coherence properties.

In addition to this, one can observe the color of the shadow for the peaks on the lower plane and can judge the sizes of the corresponding negative values, with the darker color corresponding to a more negative value of $W(z)$. The negative regions of the Wigner functions of the Schrödinger cat states increased in strong measurement regions, and these results obtained by the Wigner function corroborate our observations in Sect. 3. In short, we can achieve a better ASS effect as the nonclassicality of the state increases.

5 Conclusion and remarks

We know that the ASS phenomenon does not exist in the initial Schrödinger cat states. In this paper, we investigated the effects of postselected von Neumann measurement on the ASS of Schrödinger cat states. For our purpose, we first give a mathematical expression for the final state of the measured pointer, and then find the exact expression for the variable $R$ associated with ASS by calculations and show the analytical curves for different system parameters. However, through the analysis, we observed a dramatic
change in the ASS of the Schrödinger cat states after considering the measurement which is characterized by postselection and weak value. We found that as the interaction strength $\Gamma$ increases, the ASS phenomenon of three types of Schrödinger cat states exhibit interesting behavior. Among them, the ASS of the even Schrödinger cat state behaves more positively after the measurement, while the odd Schrödinger cat state behaves exactly the opposite way. Moreover, the larger the weak value is, the more pronounced the ASS is under the same conditions. This indicates that the signal amplification effect of the weak values play an important role in this process.

To explain the squeezing effects of Schrödinger cat states more clearly, we reconstructed the Wigner functions for each state after postselected measurement and analyzed the Wigner functions for different related parameters. It was observed that not only are the peaks squeezed, but they also exhibit coherent structure in specific coupling strength regions. Among them, the Wigner function of the even Schrödinger cat state exhibits the most obvious coherence properties after postselected measurement.

In the present work, the results clearly showed that the postselected von Neumann measurement has a positive effect on the nonclassicality of the Schrödinger cat states, especially on its ASS. Therefore, we hope that the results of the present study can provide new methods to study the quantum information processes related to ASS of radiation fields.

In this study, we only investigate the effects of postselected measurements on the second-order squeezing of Schrödinger cat states, and it belongs to the state optimization process. State optimization is a effective method to increase the implementation efficiency of related processes. On the other hand, as previous works indicated, the higher-order squeezing, named ASS of the electromagnetic field, is a natural a way to generate the higher-oder squeezed states [31], and it can be used for reducing the noise in the output of certain nonlinear optical devices [30,31]. Thus, in order to expand the practical applications of postselected von Neumann measurements, it would be interesting to study the effects of postselection von Neumann measurement on the properties of a wide variety of quantum states including entanglement, noise reduction and state control. Work along these lines is currently in progress, and we anticipate that our results will be presented in the near future.

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