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Majority-vote model with heterogeneous agents on square lattice

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We study a nonequilibrium model with up-down symmetry and a noise parameter \( q \) known as majority-vote model of M.J. Oliveira 1992 with heterogeneous agents on square lattice. By Monte Carlo simulations and finite-size scaling relations the critical exponents \( \beta/\nu \), \( \gamma/\nu \), and \( 1/\nu \) and points \( q_c \) and \( U^* \) are obtained. After extensive simulations, we obtain \( \beta/\nu = 0.35(1) \), \( \gamma/\nu = 1.23(8) \), and \( 1/\nu = 1.05(5) \). The calculated values of the critical noise parameter and Binder cumulant are \( q_c = 0.1589(4) \) and \( U^* = 0.604(7) \). Within the error bars, the exponents obey the relation \( 2\beta/\nu + \gamma/\nu = 2 \) and the results presented here demonstrate that the majority-vote model heterogeneous agents belongs to a different universality class than the nonequilibrium majority-vote models with homogeneous agents on square lattice.

Keywords: Monte Carlo; Majority vote; Nonequilibrium; noise.

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1. Introduction

A community of people where each person has a characteristic (for example, an opinion on a particular subject and this opinion can be expressed in a binary form, in favor (+1) or against (−1) a particular issue in question, and this opinion can be influenced by the vicinity of this individual) can be modeled using some simple models as the equilibrium Ising model \[ \text{[12]} \] that has become an excellent tool to study models of social application \[ \text{[3]} \]. Many works these nature are well described in a thorough review \[ \text{[4]} \], a more recent summary by Stauffer \[ \text{[5]} \] and the following papers in these special issues on sociophysics in this journal. The majority-vote model (MVM) of Oliveira \[ \text{[6]} \] is a nonequilibrium model of social interaction: individuals of a certain population make their decisions based on the opinion of the majority of their neighbors. This model has been studied for several years by various researchers in order to model social and economic systems \[ \text{[7,8,9,10,11]} \] in regular structures \[ \text{[12,13,14,15]} \] and various other complex networks \[ \text{[16,17,18,19,20,21,22,23]} \].

There are also applications to real elections in which similar models of opinion dynamics have been explored in the literature, such as Araújo \textit{et al.} \[ \text{[24]} \].
In the present work, we study the critical properties of MVM with random noise on a square lattice $SL$. Here, we start with each individual or agent having their characteristic noise $q_i$ randomly selected within a range from 0 to $q$. Thus each agent does not have an opinion in the presence of a constant noise $q$ as in the traditional MVM \cite{6}, but instead each agent has intrinsic resistance, $q_i$, to the opinion of their neighborhood on $SL$. The effective dimension using the exponents ratio $\beta/\nu$ and $\gamma/\nu$ is also determined for MVM with random noise. Finally, the critical exponents calculated for this model are compared with the results obtained by Oliveira \cite{6}.

2. Model and simulation

In the MVM on $SL$, the system dynamics traditional is as follows. Initially, we assign a spin variable $\sigma$ with values $\pm 1$ at each node of the lattice. At each step we try to spin flip a node. The flip is accepted with probability

$$w_i = \frac{1}{2} \left[ 1 - (1 - 2q) \sigma_i \cdot S \left( \sum_j \sigma_j \right) \right],$$

where $S(x)$ is the sign $\pm 1$ of $x$ if $x \neq 0$, $S(x) = 0$ if $x = 0$. To calculate $w_i$ our sum runs over the $k = 4$ nearest neighbors of spin $i$ on square lattice. Eq. (1) means that with probability $(1 - q)$ the spin will adopt the same state as the majority of its neighbors. The control parameter $0 \leq q \leq 1$ plays a role similar to the temperature in equilibrium systems: the smaller $q$, the greater the probability of parallel aligning with the local majority.

Here, in order to make the model more realistic in a social context we associate to each agent its characteristic noise $q_i$. Thus the agent has not only opinion, but also an individual resistance to the opinion of this neighborhood. Therefore, the new rate of reversal of the spin variable is

$$w_i = \frac{1}{2} \left[ 1 - (1 - 2q_i) \sigma_i \cdot S \left( \sum_j \sigma_j \right) \right],$$

where the noise parameter $q_i$, associated with the site $i$, satisfies the probability distribution

$$P(0 < q_i < q) = 1/q$$

and takes real values randomly in the interval $[0, q]$.

To study the critical behavior of the model we define the variable $m \equiv \sum_{i=1}^{N} \sigma_i/N \ (N = L \times L)$. In particular, we are interested in the magnetization $M$, susceptibility $\chi$ and the reduced fourth-order cumulant $U$

$$M_L(q) \equiv \langle |m| \rangle,$$

$$\chi_L(q) \equiv N \left( \langle m^2 \rangle - \langle m \rangle^2 \right),$$

$$U_L(q) \equiv N \left( \langle m^4 \rangle - 3 \langle m^2 \rangle^2 \right).$$
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\[ U_L(q) = 1 - \frac{\langle m^4 \rangle}{3 \langle m^2 \rangle^2}, \]  

(4c)

where \( \langle \cdots \rangle \) stands for a thermodynamic average. The results are averaged over the \( N_{\text{run}} \) independent simulations.

These quantities are functions of the noise parameter \( q \) and obey the finite-size scaling relations

\[ M_L(q) = L^{-\beta/\nu} f_m(x), \]  

(5a)

\[ \chi_L(q) = L^{\gamma/\nu} f_\chi(x), \]  

(5b)

\[ \frac{dU_L(q)}{dq} = L^{1/\nu} f_U(x), \]  

(5c)

where \( \nu, \beta, \) and \( \gamma \) are the usual critical exponents, \( f_{m,\chi,U}(x) \) are the finite size scaling functions with

\[ x = (q - q_c)L^{1/\nu} \]  

(5d)

being the scaling variable. Therefore, from the size dependence of \( M \) and \( \chi \) we obtained the exponents \( \beta/\nu \) and \( \gamma/\nu \), respectively. The maximum value of susceptibility also scales as \( L^{\gamma/\nu} \). Moreover, the value of \( q^* \) for which \( \chi \) has a maximum is expected to scale with the lattice size \( L \) as

\[ q^* = q_c + bL^{-1/\nu} \text{ with } b \approx 1. \]  

(6)

Therefore, the relations (5c) and (5d) may be used to get the exponent \( 1/\nu \). We also have applied the calculated exponents to the hyperscaling hypothesis

\[ 2\beta/\nu + \gamma/\nu = D_{\text{eff}} \]  

(7)

in order to get the effective dimensionality, \( D_{\text{eff}} \), and to improve the \( \beta/\nu \) and \( \gamma/\nu \) exponents ratio for \( D_{\text{eff}} = 2 \) on \( SL \).

We performed Monte Carlo simulation on \( SL \) with various lattice sizes \( L \) (100, 200, 300, 400, 500 and 1000). We took \( 2 \times 10^5 \) Monte Carlo steps (MCS) to make the system reach the steady state, and then the time averages are estimated over the next \( 2 \times 10^5 \) MCS. One MCS is accomplished after all the \( N \) spins are investigated whether they flip or not.

The results are averaged over \( N_{\text{run}} \) (100 \( \leq N_{\text{run}} \leq 500 \)) independent simulation runs for each lattice size and for given set of parameters \( (q, L) \).

3. Results and Discussion

In Figs. 1, 2, and 3 we show the dependence of the magnetization \( M \), susceptibility \( \chi \), and Binder cumulant \( U \) on the noise parameter \( q \), obtained from simulations on \( SL \) with \( L \) ranging from \( L = 100 \) to 1000 lattice size \( (N = 10,000 \text{ to } 1,000,000 \text{ sites}) \). The shape of \( M(q) \), \( \chi(q) \), and \( U(q) \) curve, for a given value of \( L \), suggests the
presence of a second-order phase transition in the system. The phase transition occurs at the critical value $q_c$ of the noise parameter $q$. This parameter $q_c$ is estimated as the point where the $U_L(q)$ curves for different lattice sizes $L$ intercept each other. Then, we obtain $q_c = 0.1589(4)$ and $U^* = 0.604(7)$ for $SL$.

![Fig. 1. Magnetization $M$ as a function of the noise parameter $q$, for $L = 100, 200, 300, 400, 500,$ and 1000 lattice size.](image)

In Fig. 1 we plot the dependence of the magnetization $M^* = M(q_c)$ vs. the lattice size $L$. The slope of curve corresponds to the exponent ratio $\beta/\nu$ according to Eq. (3a). The obtained exponent is $\beta/\nu = 0.35(1)$ for our $SL$.

The exponent ratio $\gamma/\nu$ at $q_c$ and $q_{\chi_{\text{max}}}(L)$ is obtained from the slope of the straight line with $\gamma/\nu = 1.23(8)$ and 1.01(9), respectively as presented in Fig. 5 for $SL$.

To obtain the critical exponent $1/\nu$, we used the scaling relation (6). The calculated value of the exponent $1/\nu$ are $1/\nu = 1.05(5)$ for $SL$ (see Fig. 6). We plot $ML^{\beta/\nu}$ versus $(q - q_c)L^{1/\nu}$ in Fig. 7 using the critical exponents $1/\nu = 1.23(8)$ and
\[ \beta/\nu = 0.35(1) \] for lattice size \( L = 300, 400, 500, \) and 1000 for \( SL \). The good collapse of the curves for five different lattice sizes corroborates the estimate for \( q_c \) and the critical exponents \( \beta/\nu \) and \( 1/\nu \).

In Fig. 8 we plot \( \chi_{L}^{1-\gamma/\nu} \) versus \( (q - q_c) L^{1/\nu} \) using the critical exponents \( \gamma/\nu = 1.01(9) \) and \( 1/\nu = 1.05(5) \) for lattice size \( L = 300, 400, 500, \) and 1000 for \( SL \). Again, the good collapse of the curves for five different lattice size corroborates the estimation for \( q_c \) and the critical exponents \( \gamma/\nu \) and \( 1/\nu \).

4. Conclusion

Finally, we remark that our MC results obtained on \( SL \) for MVM with random noise show that critical exponent ratios \( \beta/\nu = 0.35(1) \) and \( \gamma/\nu = 1.01(9) \) are different from the results of MVM for regular lattice \( \beta/\nu = 0.125(5) \) and \( \gamma/\nu = 1.73(5) \) \cite{6} and equilibrium 2D Ising model \cite{2}. On the other hand, we show also that the critical exponent \( 1/\nu = 1.05(5) \) and Binder cumulant \( U^* = 0.604(7) \) are similar to the MVM for regular lattice \cite{4}. We also showed that the effective dimension \( D_{eff} \) (within error bars) is close to 2. The agreement in \( D_{eff} \) and \( 1/\nu \) but not in the two exponent ratios \( \beta/\nu \) and \( \gamma/\nu \) remains to be explained.

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Fig. 3. Same as Fig. 1 but now for the Binder cumulant $U$.

Fig. 4. Log-log plot of magnetization $M^* = M(q_c)$ vs. the linear lattice size $L$ for $SL$.

Fig. 5. Log-log plot of susceptibility at $q_c$ and $q_{\chi_{\text{max}}}(L)$ versus $L$ for $SL$. 
Fig. 6. Log-log plot of $\ln |q_c(L) - q_c|$ versus the lattice size $L$ for $SL$.

Fig. 7. Data collapse of the magnetisation $M$ for the lattice size $L = 300, 400, 500, \text{ and } 1000$ for $SL$. The exponents used here were $\beta/\nu = 0.35(1)$ and $1/\nu = 1.05(5)$.

Fig. 8. Data collapse of the susceptibility for the lattice size $L = 300, 400, 500, \text{ and } 1000$ for $SL$. The exponents used here were $\gamma/\nu = 1.01(9)$ and $1/\nu = 1.05(5)$. 