Calculation of Universal Amplitude Ratios 
in three-loop order

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Abstract

For the universality class of three-dimensional Ising systems the ratio of the high-
and low-temperature amplitudes for the correlation length and for the susceptibili-
ity are universal quantities. They can be calculated by renormalized perturbation
theory for scalar $\phi^4$ theory in fixed dimensions $D = 3$ in the symmetric phase and
in the phase of broken symmetry. In this article the amplitude ratios are calculated
in the three-loop approximation. Using the fixed point values of the coupling con-
stants we obtain $f_+/f_- = 2.013(28)$ and $C_+/C_- = 4.72(17)$.

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1 Introduction

Field-theoretic methods are a standard tool to obtain quantitative results about the
physics of second-order phase transitions. In particular they have been applied to the
determination of universal quantities like critical exponents or ratios of critical ampli-
tudes. These take the same values within large universality classes, characterized by the
dimensionality $D$ of space and the number $n$ of components of the order parameter.

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A quantity of theoretical and experimental interest is the ratio of correlation length amplitudes $f_+/f_-$. It is defined by the behaviour of the correlation length $\xi$ as a function of the temperature $T$ near the critical temperature $T_c$ through

$$\xi \sim \begin{cases} f_+ t^{-\nu}, & t > 0 \\ f_- (-t)^{-\nu}, & t < 0 \end{cases}; \quad t := \frac{T - T_c}{T_c}. \quad (1)$$

In this article we consider this amplitude ratio (as well as the susceptibility amplitude ratio) for the universality class of the three-dimensional Ising model. The corresponding field-theoretical model, which is assumed to be in the same class, is $\phi^4$ theory in $D = 3$ with a one-component ($n = 1$) real scalar field. The amplitude ratio can be calculated by means of renormalized perturbation theory in fixed dimensions $D = 3$. In a previous work [1] the calculation was done up to two loops. For the theoretical background and more details of the method we refer to that article and the references cited therein. In the present work we have extended the calculation to third order of perturbation theory and introduced a new scheme to reduce the relevant number of Feynman graphs.

The lagrangian density for the symmetric phase ($t > 0$) is given by

$$\mathcal{L}(\phi_0^+) = \frac{1}{2} (\partial \phi_0^+(x))^2 + \mathcal{V}(\phi_0^+)$$

$$\mathcal{V}(\phi_0^+) = \frac{1}{2} m_0^2 \phi_0^+(x) + \frac{1}{4!} g_0 \phi_0^4(x). \quad (2)$$

For the phase of broken symmetry ($t < 0$) we use a shift in the field variable and expand the potential around the minimum value $v_0 := \sqrt{3m_0^2/g_0}$ omitting constant terms.

$$\phi_0^-(x) := \phi(x) - v_0, \quad (3)$$

$$\mathcal{L}(\phi_0^-) = \frac{1}{2} (\partial \phi_0^-(x))^2 + \mathcal{V}(\phi_0^-)$$

$$\mathcal{V}(\phi_0^-) = \frac{1}{2} m_0^2 \phi_0^2-(x) + \frac{1}{3!} \sqrt{3g_0} m_0^3 \phi_0^3-(x) + \frac{1}{4!} g_0 \phi_0^4-(x). \quad (4)$$

Besides the $\phi^4$ self-interaction there is a cubic $\phi^3$ interaction. This term gives rise to a large number of additional Feynman graphs, many of them containing tadpole subgraphs. To simplify the calculation and to reduce the number of graphs, we have used a method to eliminate tadpole subgraphs by means of a modification of the mass and coupling parameters [2].

On the three-loop level no new divergencies arise. So we only have to handle subddivergencies, which are isolated by dimensional regularization.

In the next section we summarize the renormalization scheme. The results of the perturbative calculations are presented in section 3, and the numerical estimates are discussed in section 4.
2 Renormalized perturbation theory

The correlation length considered here is defined through the second moment of the connected two-point function.

\[ \xi^2 := \frac{1}{2D} \int d^D x x^2 G_c^{(2,0)}(x) = -\left. \frac{\partial}{\partial p^2} \frac{G_c^{(2,0)}(p)}{G_c^{(2,0)}(0)} \right|_{p=0}, \]  

(5)

\[ G_c^{(2,0)}(x) := \langle \phi_0(x) \phi_0(0) \rangle - \langle \phi_0(x) \rangle \langle \phi_0(0) \rangle. \]

(6)

The two-point function is related to the two-point vertex function \( \Gamma^{(2,0)} \) by

\[ -\Gamma_0^{(2,0)}(p) = \left( G_c^{(2,0)}(p) \right)^{-1}. \]  

(7)

The renormalized mass \( m_R \) is defined by

\[ m_R^2 := \left. \frac{\Gamma_0^{(2,0)}(p)}{\partial p^2 \Gamma_0^{(2,0)}(p)} \right|_{p=0} = \frac{1}{\xi^2}, \]  

(8)

and coincides with the inverse of the correlation length.

As \( -\Gamma_0^{(2,0)} = m_0^2 + p^2 - \Sigma \), where \( \Sigma(p) \) is the sum of all one-particle irreducible two-point graphs with amputated external legs, we can determine the correlation length \( \xi \) perturbatively.

The renormalization constants are defined through

\[ Z_3^{-1} := -\left. \frac{\partial \Gamma_0^{(2,0)}(p;m_0,g_0)}{\partial p^2} \right|_{p=0} \]

(9)

and

\[ Z_2^{-1} := -\Gamma_0^{(2,1)}(\{0;0\};m_0,g_0) = -\frac{\partial}{\partial m_0^0} \Gamma_0^{(2,0)}(0;m_0,g_0). \]  

(10)

In order to obtain the universal amplitude ratio \( f_+/f_- \) various quantities have to be expanded in powers of a dimensionless renormalized coupling \( u_R \). The following renormalization schemes are used in the two different phases.

Symmetric phase

The renormalized coupling constant in the symmetric phase \( g_R^{(4)} \) is, following [3, 4], defined by the value of the four-point vertex function at zero external momenta.

\[ g_R^{(4)} := -Z_3^2 \Gamma_0^{(4,0)}(\{0\};m_0,g_0). \]  

(11)

It is related to the dimensionless renormalized coupling \( u_R \) by

\[ g_R^{(4)} = m_R^{4-D} u_R. \]  

(12)
Additionally we define the coupling renormalization constant

\[
Z^{-1}_1 := -\frac{1}{g_0} \Gamma^{(4,0)}_0(\{0\}; m_0, g_0).
\] (13)

The complete set of renormalization conditions are

\[
\Gamma^{(2,0)}_R(0; m_R, u_R) = -m_R^2 \quad (14a)
\]

\[
\frac{\partial}{\partial p^2} \Gamma^{(2,0)}_R(p; m_R, u_R) \bigg|_{p^2=0} = -1 \quad (14b)
\]

\[
\Gamma^{(4,0)}_R(\{0\}; m_R, u_R) = -g_R^{(4)} \quad (14c)
\]

\[
\Gamma^{(2,1)}_R(\{0; 0\}; m_R, u_R) = -1. \quad (14d)
\]

**Phase of broken symmetry**

In the phase of broken symmetry the field has a non-vanishing expectation value

\[
v = v_0 + G^{(1,0)}_c, \quad \text{where} \quad v_0 = \sqrt{3m_0^2/g_0}. \]

The renormalized field expectation value is

\[
v_R := \frac{1}{\sqrt{Z_3}} v. \]

We define the renormalized coupling constant \( g_R \) through the one-point function according to

\[
g_R := \frac{3m_R^2}{v_R^2}. \] (17)

The dimensionless coupling \( u_R \) is again introduced by

\[
g_R = m_R^{4-D} u_R. \] (18)

The renormalization scheme in this phase is thus summarized by

\[
\Gamma^{(2,0)}_R(0; m_R, u_R) = -m_R^2 \quad (19a)
\]

\[
\frac{\partial}{\partial p^2} \Gamma^{(2,0)}_R(p; m_R, u_R) \bigg|_{p^2=0} = -1 \quad (19b)
\]

\[
\frac{3m_R^2}{v_R^2} = m_R^{4-D} u_R = g_R \quad (19c)
\]

\[
\Gamma^{(2,1)}_R(\{0; 0\}; m_R, u_R) = -1. \quad (19d)
3 Perturbation series

As the method was already developed in [1] the main problem in the present calculation was the number of diagrams in third order of perturbation theory. According to the renormalization conditions (14a-d, 19a-d) we had to calculate the two-point vertex function and its momentum derivative at zero momentum in both phases. In the symmetric phase the four-point function at zero momenta and in the phase of broken symmetry the vacuum expectation value of the field had to be calculated, too. From these series the masses and coupling constants in both phases are derived. In order to distinguish the parameters in the two phases we label them with an index + for the symmetric (high temperature) and − for the broken-symmetric (low temperature) phase. The natural expansion variables are the dimensionless renormalized couplings

\[ u_{R+} := \frac{g_{R+}^{(4)}}{m_{R+}^{4-D}} \quad \text{and} \quad u_{R-} := \frac{g_{R-}}{m_{R-}^{4-D}}. \]  

The bare dimensionless coupling is defined analogously with the bare parameters

\[ u_0 := \frac{g_0}{m_0^{4-D}}. \]  

The number of diagrams we encountered is already non-negligible: there are 204 one-particle irreducible diagrams contributing to the inverse propagator at the three-loop level, compared to 20 at two loops. After exploiting symmetries there are still 162 of them. Many of them contain tadpoles and their elimination by means of Dyson-Schwinger equations simplifies the book-keeping very much [2]. The reduced set of one-particle irreducible propagator-diagrams without tadpoles only contains 34 elements. Nevertheless we checked the calculation by means of the usual perturbation theory. The program QGRAF by P. Nogueira [6] was helpful in verifying the completeness of our list of diagrams.

The starting point of the calculation are the expansions of the renormalized masses and couplings in terms of the bare coupling. For this purpose a regularization scheme has to be used. The final results are independent of the choice of the regularization scheme. We decided to use dimensional regularization with \( D = 3 - \epsilon \).

In the symmetric phase the expansions are

\[ m_{R+}^2 = m_0^2 \left[ 1 - \frac{u_0}{8\pi} \left( 1 - \frac{\epsilon}{2} \left( \gamma + \ln \frac{m_0^2}{\pi} - 2 \right) + \mathcal{O}(\epsilon^2) \right) + \left( \frac{79}{162} - \frac{1}{3} B_{\text{div}}^{\text{div}} \right) \left( \frac{u_0}{8\pi} \right)^2 \right. \]
\[ \left. + \left( -\frac{131}{216} + \frac{71}{27} \ln \frac{4}{3} + \frac{32}{3} a + \frac{1}{6} B_1^{\text{div}} \right) \left( \frac{u_0}{8\pi} \right)^3 + \mathcal{O} \left( u_0^4 \right) \right], \]

\[ g_{R+}^{(4)} = g_0 \left[ 1 - \frac{3}{2} \frac{u_0}{8\pi} \left( 1 - \frac{\epsilon}{2} \left( \gamma + \ln \frac{m_0^2}{\pi} \right) + \mathcal{O}(\epsilon^2) \right) + \left( -\frac{2}{81} + 2(1 + \mathcal{O}(\epsilon)) \right) \left( \frac{u_0}{8\pi} \right)^2 \right. \]
\[ \left. + \left( \frac{199}{1296} - \frac{373}{54} \ln \frac{4}{3} - \frac{128}{3} a - C_{\text{Tet}} - \frac{1}{4} B_1^{\text{div}} \right) \left( \frac{u_0}{8\pi} \right)^3 + \mathcal{O} \left( u_0^4 \right) \right], \]
\[ u_{R^+} = u_0 \left[ 1 - \frac{u_0}{8\pi} \left( 1 - \frac{\epsilon}{2} \left( \gamma + \ln \frac{m_0^2}{\pi} + 2 \right) + \mathcal{O}(\epsilon^2) \right) + \left( \frac{329}{216} + \frac{1}{6} B_{\text{div}} \right) \left( \frac{u_0}{8\pi} \right)^2 \right. \\
+ \left. \left( -\frac{13}{9} - \frac{74}{9} \ln \frac{4}{3} - 48a - C^{\text{Tet}} - \frac{1}{3} B_{1}\text{div} \right) \left( \frac{u_0}{8\pi} \right)^3 + \mathcal{O} \left( u_0^4 \right) \right] \] 

To first order in the coupling constant \( u_0 \) we have to keep terms of order \( \epsilon \), because in second order there are divergent terms \( B_{\text{div}} \) which are proportional to \( \epsilon^{-1} \). By multiplication they give finite contributions in the third order.

The divergent terms \( B_{\text{div}} \) and \( B_{1}\text{div} \) cancel out in the final results and need not be displayed here. \( \gamma \) is Euler’s constant, and the other constants used in these equations are:

\[ a = \frac{\pi^2}{48} - \frac{1}{8} \ln^2 \left( \frac{4}{3} \right) - \frac{1}{3} \ln \left( \frac{4}{3} \right) - \frac{1}{4} \text{Li}_2 \left( \frac{4}{3} \right) = 0.0324645 \]  

with the dilogarithm

\[ \text{Li}_2(x) = -\int_0^x \frac{\ln(1-t)}{t} \, dt, \] 

and

\[ C^{\text{Tet}} = \pi^{-6} \int \text{d}^3 k_1 \text{d}^3 k_2 \text{d}^3 k_3 \Delta(k_1) \Delta(k_2) \Delta(k_3) \Delta(k_1-k_2) \Delta(k_2-k_3) \Delta(k_3-k_1) = 0.1739006, \]  

where \( \Delta(k) = (k^2 + 1)^{-1} \). This is the only integral which we could not solve analytically. The numerical value stems from [8, graph 12U4] and was confirmed by our own calculations.

In the broken-symmetry phase we get

\[ m_{R^-}^2 = m_0^2 \left[ 1 + \frac{3}{8\pi} \left( 1 - \frac{\epsilon}{2} \left( \gamma + \ln \frac{m_0^2}{\pi} - 10 \right) + \mathcal{O}(\epsilon^2) \right) + \left( \frac{3973}{5184} + \frac{1}{3} B_{\text{div}} \right) \left( \frac{u_0}{8\pi} \right)^2 \right. \\
+ \left. \left( -\frac{101245}{41472} + \frac{21535}{2592} \ln \frac{4}{3} - \frac{1723}{48} a - \frac{3345}{1024} C^{\text{Tet}} + \frac{1}{8} B_{1}\text{div} \right) \left( \frac{u_0}{8\pi} \right)^3 + \mathcal{O} \left( u_0^4 \right) \right], \]  

\[ g_{R^-} = g_0 \left[ 1 - \frac{7}{4\pi} \left( 1 - \frac{\epsilon}{2} \left( \gamma + \ln \frac{m_0^2}{\pi} - \frac{25}{50} \right) + \mathcal{O}(\epsilon^2) \right) + \frac{17099}{5184} \left( \frac{u_0}{8\pi} \right)^2 \right. \\
+ \left. \left( -\frac{4051}{576} + \frac{21319}{1296} \ln \frac{4}{3} - \frac{1045}{24} a - \frac{2849}{512} C^{\text{Tet}} + \frac{1}{12} B_{1}\text{div} \right) \left( \frac{u_0}{8\pi} \right)^3 + \mathcal{O} \left( u_0^4 \right) \right], \]  

\[ u_{R^-} = u_0 \left[ 1 - \frac{31}{16\pi} \left( 1 - \frac{\epsilon}{2} \left( \gamma + \ln \frac{m_0^2}{\pi} - \frac{44}{31} \right) + \mathcal{O}(\epsilon^2) \right) + \left( \frac{40957}{13824} - \frac{1}{3} B_{\text{div}} \right) \left( \frac{u_0}{8\pi} \right)^2 \right. \\
+ \left. \left( -\frac{284719}{73728} + \frac{21247}{1728} \ln \frac{4}{3} - \frac{819}{32} a - \frac{8051}{2048} C^{\text{Tet}} + \frac{31}{24} B_{1}\text{div} \right) \left( \frac{u_0}{8\pi} \right)^3 + \mathcal{O} \left( u_0^4 \right) \right]. \]  

For the calculation of the universal amplitude ratio of correlation lengths we need the functions

\[ F_{\pm}(u_{R^\pm}) := \left. \frac{\partial m_{R^\pm}^2}{\partial m_{0^\pm}^2} \right|_{g_0}. \]
When the bare masses and couplings are expressed in terms of the renormalized ones the perturbation series are

$$ F_+(u_{R+}) = 1 - \frac{1}{2} \frac{u_{R+}}{8\pi} - \frac{1}{6} \left( \frac{u_{R+}}{8\pi} \right)^2 + \left( \frac{13}{27} - \frac{71}{54} \ln \frac{4}{3} - \frac{16}{3} a \right) \left( \frac{u_{R+}}{8\pi} \right)^3 + O \left( \frac{u^4_{R+}}{} \right), \quad (32) $$

$$ F_-(u_{R-}) = 1 + \frac{3}{16} \frac{u_{R-}}{8\pi} - \frac{233}{768} \left( \frac{u_{R-}}{8\pi} \right)^2 + \left( -\frac{297256}{663552} - \frac{21535}{5184} \ln \frac{4}{3} + \frac{1723}{96} a + \frac{3345}{2048} C_{Tet} \right) \left( \frac{u_{R-}}{8\pi} \right)^3 + O \left( \frac{u^4_{R-}}{} \right). \quad (33) $$

At this point we still have two coupling constants, $u_{R+}$ and $u_{R-}$. Following [1] we introduce a new coupling $\bar{u}_R$, which is defined in both phases, such that the corresponding $\beta$-functions in both phases and consequently the numerical values of the fixed point couplings are equal. Previous experience suggests to choose $\bar{u}_R$ such that it coincides with the usual coupling in the low-temperature phase: $\bar{u}_R \equiv u_{R-}$. Therefore we generally denote it by $u_{R-}$ in the following. The other coupling constant expressed as a series in $u_{R-}$ is

$$ u_{R+}(u_{R-}) = u_{R-} \left[ 1 + \frac{1}{4} \frac{u_{R-}}{8\pi} + \frac{1633}{2592} \left( \frac{u_{R-}}{8\pi} \right)^2 + \left( \frac{1011239}{165888} - \frac{30271}{1296} \ln \frac{4}{3} + \frac{7}{8} a + \frac{2337}{512} C_{Tet} \right) \left( \frac{u_{R-}}{8\pi} \right)^3 + O \left( \frac{u^4_{R-}}{} \right) \right]. \quad (34) $$

This relation allows to express all renormalized perturbation series in terms of a single coupling constant $u_{R-}$. In particular for the ratio $F_-/F_+$ we obtain

$$ \Phi_-(u_{R-}) := \frac{F_-(u_{R-})}{F_+(u_{R+})} = 1 + \frac{11}{16} \frac{u_{R-}}{8\pi} + \frac{85}{256} \left( \frac{u_{R-}}{8\pi} \right)^2 + \left( \frac{-109217}{663552} - \frac{14719}{5184} \ln \frac{4}{3} + \frac{745}{32} a + \frac{3345}{2048} C_{Tet} \right) \left( \frac{u_{R-}}{8\pi} \right)^3 + O \left( \frac{u^4_{R-}}{} \right). \quad (35) $$

This function finally yields the desired amplitude ratio via

$$ \frac{f_+}{f_-} = \left[ 2\Phi_-(u_{R-}^*) \right]^\nu, \quad (36) $$

where $u_{R-}^*$ is the fixed point value of the coupling and $\nu$ is the correlation length exponent. Both $u_{R-}^*$ and $\nu$ can be obtained in perturbation theory too, but more precise values are available in the literature and we shall make use of them.
4 Numerical results

What is needed for the amplitude ratio of correlation lengths is the value of $\Phi_-(u_{R-})$ at $u_{R-} = u_{R-}^*$. For the fixed point $u_{R-}^*$ we take an estimate $u_{R-}^* = 14.73(14)$ from low-temperature series [9] and another estimate $u_{R-}^* = 15.1(1.3)$ used in [10]. For comparison we note that the zero of the 3-loop $\beta$-function is located at $u_{R-}^* = 14.2$, using a (2,1)-Padé-Borel approximation. The entries in table 1 result from the possible Padé approximants

| Fixed point $u_{R-}^*$ | $\Phi_-(u_{R-}^*)$ [3,0]-Padé | [2,1]-Padé | [1,2]-Padé | [0,3]-Padé |
|------------------------|---------------------------------|------------|------------|------------|
| 14.73 [9]              | 1.5288                          | 1.5301     | 1.5002     | 1.5149     |
| 15.1 [10]              | 1.5456                          | 1.5471     | 1.5133     | 1.5301     |

Table 1: $\Phi_-$ as a function of the low-temperature fixed point $u_{R-}^*$

to $\Phi_-(u_{R-})$ evaluated at $u_{R-}^*$. The values are quite close together and the mean value is 1.526(26), where the error represents the maximal deviation. The numerical convergence of the series at the fixed point is rather good,

$$\Phi_-(u_{R-}^*) = 1 + 0.410 + 0.118 + 0.012,$$

although it is expected to be asymptotic only. An application of the usual Padé-Borel summation method does not improve the result. On the contrary the uncertainty is more than doubled.

Using the high-temperature coupling ($u_{R+}^* \approx 24$) as an expansion parameter is much worse. For $\Phi_+(u_{R+})$ we get a mean value of 1.39(34) with Padé, and 1.48(24) with Padé-Borel summation, respectively, so the maximal deviation is more than ten times higher as above. This is related to the poor numerical convergence:

$$\Phi_+(u_{R+}^*) = 1 + 0.657 + 0.146 - 0.396.$$  

Therefore we consider the estimate from $\Phi_-$ as more reliable.

For the critical exponent $\nu$ we used Monte Carlo results ($\nu = 0.624(2)$) [11] and values of renormalized perturbation theory ($\nu = 0.6300$) [12]. After exponentiation with these values for $\nu$ we get the universal amplitude ratio for the correlation length $\xi$ according to (36),

$$f_+ = 2.013(28).$$

Using the high-temperature coupling instead we would get a value of 1.98(20) employing Padé-Borel approximations. The error is ten times larger than in (36). As discussed above the low-temperature coupling appears to be the better expansion parameter.
With a similar method we have calculated the universal amplitude ratio of the susceptibility. As before we get best results with the low-temperature coupling, namely

$$\frac{C_+}{C_-} = 4.72(17).$$  \hspace{1cm} (40)

The same ratio has been calculated by means of three-dimensional perturbation theory in \[13\] with the result $C_+/C_- = 4.77(30)$.

5 Conclusion

Our third order calculation of the amplitude ratio of correlation lengths (39) is a confirmation and improvement of the two-loop result of 2.03(4) \[1\]. Theoretical estimates in the $\epsilon$-expansion (1.91 \[14\]), high- and low-temperature expansions (1.96(1) \[17\], 1.94(3) \[9\]) are lower. Experimental values (2.05(22), 2.22(5) \[16\], 1.9(2), 2.0(4) \[17\]) and Monte Carlo results (2.06(1) \[18\]) are above or close to our results.

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