On the Use of Hydrogen Recombination Energy during Common Envelope Events

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Abstract

In this Letter we discuss what happens to hydrogen recombination energy that is released during regular common envelope (CE) events as opposed to self-regulated CE events. We show that the amount of recombination energy that can be transferred away by either convection or radiation from the regions where recombination takes place is negligible. Instead, recombination energy is destined to be used either to help CE expansion, as a work term, or to accelerate local fluid elements. The exceptions are donors that initially have very high entropy material, $S/(k_B N_A) > 37$ mol g$^{-1}$. The analysis and conclusions are independent of specific stellar models or evolutionary codes, and rely on fundamental properties of stellar matter such as the equation of state, Saha equation, and opacities, as well as on stellar structure equations and the mixing length theory of convection.

Key words: binaries: close

1. Introduction

A common envelope event (CEE) is a fate-defining episode in the life of a binary system. The phenomena of a common envelope (CE) occurs when the outer layers of one of the stars expands to engulf the companion (Webbink 1975, 1984; Paczynski 1976; Livio & Soker 1988). It is widely accepted that a CEE is the dominant mechanism by which an initially wide binary star can either become a very close binary star, or by which it merges (for a review on the current understanding of the CE and its importance for binary populations, see Ivanova et al. 2013). The outcome of a CEE depends on the energy budget during the interaction.

The foremost important topic in the consideration of the energy budget during CE events is which energy can and cannot be used to drive an envelope ejection. While the orbital energy release is taken as the unarguable primary source, this becomes less efficient as soon as the CE starts its expansion—the expanded envelope becomes tidally decoupled from a shrunken binary orbiting inside of it, and the completion of the CE ejection using purely orbital energy is hindered (Ivanova & Nandez 2016). Other energy sources that have been shown to help to complete a CE ejection are nuclear energy released during the interaction of the already shrunken binary (Podsiadlowski et al. 2010), and recombination energy (Nandez et al. 2015; Nandez & Ivanova 2016).

The use of the recombination energy seems to be a controversial subject. On the one hand, recombination energy conveniently kicks in when the envelope is both expanded and decoupled from the orbit, and if it takes place above the “recombination radius” (for a definition, see Ivanova & Nandez 2016), it would provide enough energy to complete the ejection. On the other hand, it has been argued that the energy released by hydrogen recombination is likely all transported to the surface and then lost as radiation (Soker & Harpaz 2003; Sabach et al. 2017; Grichener et al. 2018). The recombination energy is provided by both initially ionized hydrogen and helium, where helium provides about 60% of the energy that hydrogen provides. For the recombination energy from helium, which is produced deeper inside an envelope, there is less controversy over its use. Consequently, we will be not considering it in this Letter; if hydrogen recombination energy cannot be transported, then helium recombination energy is not transported.

For clarity, we define the efficient use of recombination energy as when this energy is predominantly used locally to help drive a CE ejection, either by acting as a work term, or being converted into kinetic energy by accelerating local fluid elements. Note that we separate the issue of the efficiency of recombination energy usage in regular CEEs from that in self-regulated CEEs, also known as slow spiral-ins (first considered in Meyer & Meyer-Hofmeister 1979). During a self-regulated CE, the envelope is allowed to readjust to match a moderate rate of orbital energy release to its surface luminosity. We stress that this implies that in the case of self-regulated CEEs the consideration of the efficiency of recombination energy is irrelevant—once the rate of orbital energy release is matched by the surface luminosity, the recombination profile freezes.

2. Fundamentals

The two fundamental stellar structure equations that determine the energy redistribution inside a star are as follows.

I. The energy transport equation:

$$\frac{L}{4\pi r^2} = F_{\text{rad}} + F_{\text{conv}},$$

(1)

Here, $L$ is the luminosity at the mass coordinate $m$, $r$ is the radial coordinate, and $F_{\text{rad}}$ and $F_{\text{conv}}$ are the radiative and the convective flux, respectively, at the same mass coordinate.

II. The energy equation. If there is no nuclear energy generation or neutrino losses, the net energy loss from a mass shell $dm$ is $dL$, and the energy equation is

$$\frac{dL}{dm} = -\frac{\partial e}{\partial t} + P \frac{\partial \rho}{\rho^2 \partial t}.$$ 

(2)

Here $t$ is time, $e$ is specific internal energy, $P$ is pressure, and $\rho$ is density. The two terms on the right-hand side of Equation (2) are often combined into a gravitational energy term. We write it in two components to emphasize that during expansion, the

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For Equations (1)-(4) we refer the reader to the appropriate texts; see, e.g., Kippenhahna et al. (2012).
second term acts as an “energy sink” due to the expansion of the shell, while the first term acts as a local energy source.

Equations (1) and (2) show that the amount of energy that can be transferred away is limited by the fluxes. If the fluxes cannot transfer away the locally produced energy, then that energy has to be spent on the shell’s expansion. Hence, the intrinsic ability of the two fluxes to take the energy away determines the efficiency of recombination energy usage during a CE event. In addition to convective and radiative energy transport mechanisms, some energy can be transported by waves (see, e.g., Fuller 2017). Energy transported by waves will be not lost from the surface, but rather deposited somewhere closer to the surface, and hence likely contributes to the removal of the CE.

2.1. The Radiative Flux

The flux carried by radiation is

$$F_{\text{rad}} = \frac{4aG T^4 m}{3 \kappa P r^2} \nabla = \frac{4a}{3 \kappa P} g \nabla. \quad (3)$$

Here $a$ is the radiation density constant, $\kappa$ is the speed of light, and $G$ is the gravitational constant. $\nabla \equiv \frac{d \ln T}{d \ln P}$ is the actual gradient in the star; for radiative regions it cannot exceed $\nabla_{\text{ad}}$. $T$ is temperature, $\kappa$ is opacity, $g$ is gravitational acceleration. All of those quantities are local to the considered shell at the mass coordinate $m$.

2.2. The Convective Flux

The flux carried by convection within the mixing length theory (MLT) is

$$F_{\text{conv}} = \rho v_{\text{conv}} c_p T (\nabla - \nabla_e) \frac{1}{2H_p}. \quad (4)$$

Here $(\nabla - \nabla_e)$ is the excess of $\nabla$ above the variation of temperature in the convective element during its motion, $\nabla_e$. This difference is limited by the maximum value of the temperature gradient in the case of adiabatic convection $\nabla_{\text{ad}}$. This value is often 0.4, but in a zone of partial ionization it can become even less than 0.1. $c_p$ is the specific heat at constant pressure, $m_\text{conv}$ is the mixing length, and $H_p$ is the pressure height scale; in MLT they are usually connected via the mixing length parameter $a_{\text{str}} = \frac{m_\text{conv}}{H_p}$, which is often taken to be about 2. It is important that the MLT has been derived in the assumption that the convection is subsonic so that a convective eddy can always re-establish pressure equilibrium with its surroundings as it moves, $v_{\text{conv}}$, which is the velocity of a convective element, then can be considered as $v_{\text{conv}} = M_{\text{cv}} c_s$, where Mach number $M_{\text{cv}} < 1$ and $c_s$ is the local sonic velocity $c_s = \sqrt{1/\Gamma_1 P/\rho}$. $\Gamma_1 = \frac{\partial \ln P/\partial \ln P}{\partial \ln \rho}$ is the first adiabatic exponent. Currently, there is no stellar convection theory that would provide a valid result for a transonic or faster convection. We can now rewrite the convective flux as

$$F_{\text{conv}} = M_{\text{cv}} \rho c_s c_p T (\nabla - \nabla_e) \quad (5)$$

3. Ability of the Radiative Flux to Remove the Recombination Energy

Let us see if there are partially ionized regions where radiation might dominate over convection. First, in Figure 1 we show opacity in the $\rho-T$ plane for a typical stellar mixture with a mass fraction of hydrogen $X = 0.7$ and mass fraction of helium $Y = 0.28$. We also show there the contours of $H$ and $He$ partial ionization, as well as typical profiles of several giant stars. Typical values of opacities within the zone of partially ionized $H$ for stellar models are $\kappa > 10^3 \text{cm}^2 \text{g}^{-1}$. It is those high-opacity values that are expected to be the main cause of the weakness of the radiative flux to move energy, as compared to the convective flux.

In the regions where convection is very efficient, $(\nabla - \nabla_e)$ could become as small as $\sim 0.01$. However, in the regions where convection competes with radiation, and the convection itself is not very effective, the ratio $\nabla / (\nabla - \nabla_e) \rightarrow 1$. As convective flux struggles to transfer energy away, it responds by boosting the local convective velocity (this is a consequence of the buoyancy force increasing when $(\nabla - \nabla_e)$ is no longer small), and $M_{\text{cv}} \rightarrow 1$. We introduce $f_{\text{ratio}}(\rho, T)$ such that

$$\frac{F_{\text{rad}}}{F_{\text{conv}}} \approx f_{\text{ratio}}(\rho, T) \left( \frac{g}{10^{-4}} \right)^{g}. \quad (6)$$

Figure 1. Envelope structures of three stellar models (AGB star with mass of $6 M_\odot$ and radius of 400 $R_\odot$, AGB star with mass of 2 $M_\odot$ and radius of 220 $R_\odot$, and RG star with mass of 1.6 $M_\odot$ and radius of 100 $R_\odot$) are plotted on a $T-\rho$ (temperature–density) plot. The solid dots indicate where the mass above this location on the $T-\rho$ curve is $M = 0.01, 0.05, 1.0, 2.0, 0.4, 0.8, 1.6, 3.2 M_\odot$ (all levels are applicable only for a $6 M_\odot$ AGB star). Colors correspond to opacities from Ferguson et al. (2005) generated for metal abundances from Grevesse & Sauval (1998), for $X = 0.7$ and $Z = 0.02$. The locations of hydrogen partial ionization zones ($H^+ / H = 0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 0.8, 0.9, 0.99$) and helium partial ionization zones ($He^+ / He = 0.01$ and $He'' / He = 0.99$) are found as in Kippenhahn et al. (2012). The dashed–dotted red lines (in the upper-left corner) indicate the inclinations that radiative and convective envelope solutions would have on the $T-\rho$ plane if radiative pressure is negligible and there is no partial ionization $(\nabla_{\text{ad}} = 0.4)$—above the recombination regions, all of the solutions are self-similar and are only shifted from each other. For details, see Kippenhahn et al. (2012). The shown stellar envelope profiles were obtained using MESA code (Paxton et al. 2011, 2013, 2015), revision 10108.
In Figure 2 we show $f_{\text{ratio}}(\rho, T)$ for a typical stellar mixture ($X = 0.7$, $Y = 0.28$). Please note that while we analyze the case of all giant stars, the location of the border between where each of the fluxes dominates is to the left of the most expanded stars. It therefore makes sense to scale to the case $g = 10^{-4} \, g_\odot$. The radiative flux is far less efficient than the convective flux where hydrogen ionization takes place at densities above $\rho_{\text{crit}} \sim 2 \times 10^{-10} \, g \, cm^{-3}$. Even changing $M_{\text{cv}}$ to 0.1 would affect results for $\rho_{\text{crit}}$ by a small amount. In the case of luminous massive stars, the effective local opacity in a three-dimensional star can be lower than that predicted by a one-dimensional model, due to the effect of the porosity (Owocki et al. 2004; Jiang et al. 2015). This effect may potentially play a role for $f_{\text{ratio}}$. We note that the effect of porosity needed to make radiation more effective than convection, for the region of interest, requires changing opacities by two orders of magnitude or more.

Radiation is capable of removing recombination energy only from a region where it is more efficient than convection. Sabach et al. (2017) have argued that a consideration of a random photon walk and a comparison of the time that it takes for a photon to escape can lead to most of the released hydrogen recombination energy being transported away. While the photons that are carried by radiation can indeed be taken away, their contribution to the overall energy transport from the zones of partial recombination is proportional to the ratio of the fluxes. Hence, the optical depth of the recombination zone, as well as the photon diffusion time and its comparison to a CE timescale, do not matter, so long as the convective flux dominates.

Radiation can take away the recombination energy only in cases when the envelope’s material, while expanding and cooling, passes through the recombination region while having density less than $\rho_{\text{crit}}$. To constrain which stars can do that, let us consider the envelope’s material evolution during a CEE.

The course of a fluid element on the $\rho$-$T$ plane can be split into three stages. First, there is an entropy increase due to either frictional orbital energy deposition or shocks. Second is the stage of adiabatic expansion while the plasma is still fully ionized. And finally, there is an expansion stage while recombining; this is accompanied by a decrease of plasma entropy. In all three stages, the track on the $\rho$-$T$ plan moves toward smaller densities and smaller temperatures.

We introduce the scaled entropy $s = S/(k_B N_A)$, where $k_B$ is the Boltzmann constant and $N_A$ is the Avogadro number. We show values of scaled entropy in Figure 3. One can see that the profiles of the stars in convective regions are aligned along the lines of constant entropy while wiggling in zones of partial ionization, as expected.

To transport energy by radiation from the recombination region, i.e., to start the recombination at densities comparable to $\rho_{\text{crit}}$, the scaled entropy of the material prior to the adiabatic expansion has to be more than $40 \, \text{mol} \, g^{-1}$. Only very expanded donors could have an entropy of their convective envelopes that large prior to a CEE.

During a CEE, stellar material is expected to get an entropy boost at the onset of its adiabatic expansion. What does that entropy boost mean? For an ionized gas, a boost by one unit in scaled entropy is equivalent to providing a specific heat $k_B N_A T$ to the same material. At the same time, its specific internal energy at the same temperature is $3/(2 \mu) \, k_B N_A T$. Note that an initially bound envelope implies that its internal energy is roughly half of the absolute value of its specific potential energy. Providing more than $\sim 2.5 k_B N_A T$ makes the envelope material immediately unbound. Therefore, if a stellar model prior to the onset of the CEE has its scaled entropy below about $37 \, \text{mol} \, g^{-1}$, radiation is not expected to remove the recombination energy.
Figure 4. Maximum possible convective flux for a stellar mixture with $X = 0.7$ and $Y = 0.28$ (see Equation (9)). Lines for hydrogen partial ionization zones are the same as in Figure 1.

4. Ability of the Convective Flux to Remove the Recombination Energy

Let us rewrite Equation (5) as

$$F_{\text{con}} = M_{\text{cv}} \frac{(\nabla - \nabla_e) F_{\text{max}}}{\nabla_{\text{ad}}} \rho_{\text{con}} c T_{\text{ad}}, \quad (8)$$

where $F_{\text{max}}$ is the “maximum possible convective flux”

$$F_{\text{max}} = \rho c_T T_{\text{ad}}. \quad (9)$$

Note that this maximum possible convective flux assumes that the convective velocity is the same as the local sonic velocity. In principle, an MLT-equipped stellar code could not produce a valid result for convective energy transport in this regime, but we still can use it as the upper limit. Conveniently, the quantity $F_{\text{max}}$ depends only on the equation of state (EOS; see Figure 4). Numerically, we find that we can approximate that, for most of the region of interest, the limiting flux $F_{\text{max}} \propto \rho^{0.8} r^{1.3}$.

Let us consider a fully ionized fluid element that is initially located above the hydrogen recombination region on the $\rho$–$T$ plane. Assume that it receives some energy (an entropy boost) from the shrinking binary orbit, and starts its expansion. Figure 4 shows that along an expansion track the value of $F_{\text{con}}$ for that fluid element decreases. During an adiabatic expansion of an ideal ionized gas, $T \propto \rho^{2/3}$, resulting in $F_{\text{max}} \propto \rho^{5/3}$. For a self-similar expansion, $\rho \propto r^{-\gamma}$. Then $L \propto M_{\text{cv}}((\nabla - \nabla_e)^{1/2} \propto M_{\text{cv}}((\nabla - \nabla_r) r^{-\gamma})$—as the CE expands, the local luminosity has to drop, unless both $M_{\text{cv}}$ and $(\nabla - \nabla_r)$ strongly increase, but their values are limited. The initial expansion of a CE, to some extent, becomes self-driving. This self-driving regime will break down once the thermal timescale of the expanding envelope becomes comparable to the dynamical timescale, and the adiabatic regime of the expansion stops, or when recombination starts to play a key role and an adiabatic expansion with $\gamma = 5/3$ is no longer a valid approximation.

Let us now consider the recombination and convection. Hydrogen recombination provides specific energy $E_{\text{rec,H}} = 1.3 \times 10^{13} f_h \text{ erg g}^{-1}$, where $X$ is the hydrogen mass fraction and $f_h = H^+ / H$ is the amount of hydrogen that is ionized. Now let us assume that recombination takes place on the same timescale as the fluid element moves through the ionization region, the recombination timescale $\tau_{\text{rec}}$. We can relate it to the local dynamical timescale $\tau_{\text{dyn}}$ as $\alpha_{\text{rec}} = \tau_{\text{rec}} / \tau_{\text{dyn}}$. For dynamical CEEs, $\alpha_{\text{rec}} \approx 1$, while for self-regulated CEEs, $\alpha_{\text{rec}} \lesssim 10$. The width of the hydrogen recombination zone can be written as $\delta_{\text{rec}} \approx \alpha_{\text{H}} r$. For giants, the width of the recombination zone is a substantial part of their total radius, with $\alpha_{\text{H}} \approx 0.1$–0.5, and $\alpha_{\text{H}}$ only increases as the CE expands. The local energy flux that is created due to recombination then can be found as

$$F_{\text{rec}} = \frac{E_{\text{rec,H}}}{\alpha_{\text{rec}} c_T} \rho dr = \frac{\alpha_{\text{H}}}{\alpha_{\text{rec}} c_T} \frac{E_{\text{rec,H}}}{\sqrt{2}}. \quad (10)$$

Here $\nu_{\text{esc}} = \sqrt{2Gm/r}$ is the local escape speed. We can form the ratio between the maximum possible local convective flux and the energy flux due to recombination:

$$\frac{F_{\text{con}}}{F_{\text{rec}}} = \frac{\sqrt{2} c_T c_T T_{\text{ad}} \alpha_{\text{rec}}}{\nu_{\text{esc}} E_{\text{rec,H}} \alpha_{\text{H}}}. \quad (11)$$

The maximum possible convective flux can remove the released recombination energy only if that ratio is more than one; otherwise, the shell will expand. The first term, $\sqrt{2} c_T / \nu_{\text{esc}}$, is about one for any star that is stable at the onset of a CE. The quantity $c_T T_{\text{ad}} / E_{\text{rec,H}}$ depends only on the EOS and is plotted in Figure 5, only for the part of the $\rho$–$T$ plane where the recombination energy can be released. As can be seen, the convective flux is trapped in the low ionization regions. For example, if $f_h < 0.2$, in order to take away the energy from the recombination zone, either the timescale for the envelope expansion has to be up to 100 times larger than the local dynamical timescale, or the width of the recombination zone $\alpha_{\text{H}}$ has to be less than 0.01. Note that it is only on short, dynamical
timescales that the recombination energy cannot be removed by fluxes. This is why stars evolving from the main sequence to the red giant phase do not lose their envelopes when they go through hydrogen and helium recombinations.

Note that in giants convection is strongly subsonic prior to the CE. An increase of the convective velocity from subsonic to sonic speeds implies that part of the released recombination energy goes into the kinetic energy of convective eddies. For typical subsonic convection prior to the CE, the energy that becomes kinetic energy is about \( \epsilon_s^2 / 2 = \Gamma_s / (2 \mu_s) k_\text{B} n_s T_s \); i.e., if a fluid element has to be sped up to sonic velocity, its specific kinetic energy becomes comparable to its specific internal energy (note that a slightly supersonic fluid element is becoming locally unbound). Therefore, it will store most of the released recombination energy as kinetic energy. Stellar codes usually ignore the kinetic energy contained in the convective eddies, as they do not carry much energy in the case of subsonic convection, nor is the acceleration of the convective eddies a process that converts thermal energy into kinetic energy on a dynamical timescale. However, for CEEs this effect has to be taken into account, at least when the ability of the convective flux to carry energy is considered.

5. Conclusions

We have considered whether radiative or convective flux is capable of removing the energy that is released during hydrogen recombination during regular CEEs. We limit ourselves to regular (dynamic) CEEs, as self-regulated CEEs similar to, for example, those considered in Grichener et al. (2018), cannot in principle provide insight into the efficiency of recombination energy usage, due to their intrinsic feature of freezing the envelope recombination profile in order to match the constant energy source.

For regular CEEs, we have shown that the convective flux is incapable of removing the released recombination energy, even in the limit of transonic convection. As a result, the released recombination energy is fated to be utilized as a work term. Also, for convection to become transonic or faster, it has to convert most of the released recombination energy into kinetic energy. In both cases, through the work term or via kinetic energy boost, the recombination energy is used locally to help expand and eject the CE. We also anticipate that if the stellar codes that are used to model CEE show that the convection has to be transonic or faster, the results of the calculations might be invalid, as in this case where the MLT is used in a regime beyond its validity.

On the other hand, the radiative flux can potentially only transfer the recombination energy from donors where, prior to CE, their scaled entropy is \( s \gtrsim 37 \text{ mol g}^{-1} \). In donors with a lower value of entropy in their envelopes, envelope material does not pass through the region where radiative transfer is effective enough to remove the recombination energy; for them, convection always dominates. For those donors, if their envelope receives an entropy boost large enough to allow the matter to start recombination in the radiation-dominated regime, the envelope material will become unbound before the recombination starts. An evaluation of the recombination energy losses by radiative diffusion, whether by photon diffusion time or by the optical depth of the recombination zone, should not be done when the radiative transfer itself is not dominating the overall energy transfer.

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