Topological defects in triplet superconductors UPt$_3$, Sr$_2$RuO$_4$, etc.

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After a brief introduction on nodal superconductors, we review the topological defects in triplet superconductors such as UPt$_3$, Sr$_2$RuO$_4$, etc. This is in part motivated by the surprising discovery of Ana Celia Mota and her colleagues that in some triplet superconductors the flux motion is completely impeded (the ideal pinning). Among topological defects the most prominent is Abrikosov’s vortex with quantum flux $\phi_0 = \frac{hc}{2e}$. Abrikosov’s vortex is universal and ubiquitous and seen in both conventional and unconventional superconductors by the Bitter decoration technique, small angle neutron scattering (SANS), scanning tunneling microscopy (STM), micromagnetometer and more recently by Lorentz electron micrograph. In order to interpret the experiment by Mota et al a variety of textures are proposed. In particular, in analogy to superfluid $^3$He-A the $\ell$-soliton and $d$-soliton play the prominent role. We review these notions and point out possible detection of these domain walls and half-quantum vortices in some triplet superconductors.

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I. INTRODUCTION

We shall first survey the new world of nodal superconductors, which appeared on the scene in 1979. Indeed nodal superconductors are a child of the 21st century [1, 2, 3]. Although the presence of nodal superconductors in heavy-fermion superconductors like CeCu$_2$Si$_2$, UPt$_3$, UBe$_{13}$ and others was found in the late eighties [1], the systematic study of the gap function $\Delta(\mathbf{k})$ began only after the discovery of the high-$T_c$ cuprates La$_{2-x}$Ba$_x$CuO$_4$ by Bednorz and Müller [4] in 1986. The $d$-wave symmetry of high-$T_c$ cuprates YBCO, Bi-2212, etc. was established circa 1994 through the elegant Josephson interferometry [5, 6] and the powerful angle resolved photoemission spectrum (ARPES) [7] among others. In 1993 Volovik [8] derived the quasiparticle density of states of the vortex state in nodal superconductors within the semiclassical approximation. The surprising $\sqrt{H}$ dependence of the specific heat has been seen in YBCO [9], LSCO [10], and Sr$_2$RuO$_4$ [11, 12]. Later Volovik’s approach was extended in a variety of directions: a) the study of the thermal conductivity [13, 14]; b) for an arbitrary field direction [15]; and c) for different classes of $\Delta(\mathbf{k})$ [16]. These are summarized in Ref. [17]. Until now the powerful ARPES and Josephson interferometry have not been applied outside of high-$T_c$ cuprate superconductors.

Since 2001 Izawa et al have determined the gap function $\Delta(\mathbf{k})$ in Sr$_2$RuO$_4$ [18], CeCoIn$_5$ [19], $\kappa$-(ET)$_2$Cu(NCS)$_2$ [20], YNi$_2$B$_2$C [21], and PrOs$_4$Sb$_{12}$ [22, 23] via the angle-dependent magnetothermal conductivity. These $|\Delta(\mathbf{k})|^2$’s are shown in Figure 1. In addition, the gap function of UPt$_3$ was established around 1994-6 as $E_{2u}$ through the anisotropy in the thermal conductivity [24] and the constancy of the Knight shift in NMR [25]. Somewhat surprisingly all these superconductors are nodal and their quasiparticle density of states increases linearly in $|E|$ for $|E|/\Delta \ll 1$:

$$N(E) \sim |E|/\Delta$$

(1)

For example, this implies that the $p$-wave superconductivity in Sr$_2$RuO$_4$ as proposed in Ref. 26 is not consistent with
the specific heat data \[11\]. Also as discussed elsewhere \[27, 28\], the two gap model is of little help in this matter.

More recently the quasiparticle density of states in the vortex state in Sr\(_2\)RuO\(_4\) has been reported \[29\]. Indeed the observed quasiparticle density of states is very consistent with that predicted for an f-wave order parameter \[30\]. Also many of these superconductors are triplet: UPt\(_3\), Sr\(_2\)RuO\(_4\), (TMTSF)\(_2\)PF\(_6\), U\(_1-x\)Th\(_x\)Be\(_{13}\), URu\(_2\)Si\(_2\), PrOs\(_4\)Sb\(_{12}\), UNi\(_2\)Al\(_3\) and CePt\(_3\)Si, for example.

II. TEXTURES IN TRIPLET SUPERCONDUCTORS

Here we consider possible textures in triplet superconductors. For simplicity we concentrate on two f-wave superconductors: UPt\(_3\) (3-dimensional) and Sr\(_2\)RuO\(_4\) (quasi 2-dimensional), which we understand well. Mota et al \[33, 34\] have discovered the ideal pinning in the B phase of UPt\(_3\) and below T=70 mK in Sr\(_2\)RuO\(_4\). Also these systems are characterized by \(\hat{\ell}\) and \(\hat{d}\) similarly to superfluid \(^3\)He-A \[35, 36\]. Here \(\hat{\ell}\) is the direction of the pair angular momentum. But unlike superfluid \(^3\)He-A \(\hat{\ell}\) is fixed to be parallel to one of the crystal axes: \(\hat{\ell} \parallel \pm \hat{c}\). \(\hat{\ell}\) can be called the chiral vector. On the other hand \(\hat{d}\) describes the spin configuration of the pair and is perpendicular to the pair spin \(\mathbf{S}_{\text{pair}}\). In the equilibrium configuration \(\hat{d} \parallel \pm \hat{\ell}\) in superfluid \(^3\)He-A, UPt\(_3\) and Sr\(_2\)RuO\(_4\) \[24\]. Also we believe that some triplet superconductors may not have \(\hat{\ell}\) and \(\hat{d}\) vectors, as observed in superfluid \(^3\)He-B. So it is very important to know if the superconductivity breaks the chiral symmetry, as observed in the experiments by Mota et al \[33, 34\] on the B phase in U\(_1-x\)Th\(_x\)Be\(_{13}\) (see Fig. 2). But it is possible that the superconductor in UBe\(_{13}\) does not have \(\hat{\ell}\) and \(\hat{d}\). Also we conjecture that the superconductivity in the B phase of PrOs\(_4\)Sb\(_{12}\) has \(\hat{\ell}\) and \(\hat{d}\) vectors \[24\] and that probably all other quasi 2-dimensional systems such as URu\(_2\)Si\(_2\), UNi\(_2\)Al\(_3\) and CePt\(_3\)Si do as well. Therefore we can think of a variety of domain walls as in superfluid \(^3\)He-A \[38, 39\]. Indeed the \(\hat{\ell}\)-soliton in Sr\(_2\)RuO\(_4\) has been considered by Sigrist and Agterberg \[40\]. The \(\hat{\ell}\)-soliton is created when in one side of the wall \(\hat{\ell} \parallel \hat{c}\), while in the other side \(\hat{\ell} \parallel -\hat{c}\). The chirality changes across the \(\hat{\ell}\)-soliton. However, unlike in superfluid \(^3\)He-A \(\hat{\ell}\) is practically fixed to be parallel to \(\pm \hat{\ell}\). For example, when \(\hat{\ell}\) is in the ab-plane there will be little superconducting order parameter left. Therefore we can estimate the \(\hat{\ell}\)-soliton energy per unit area as

\[
f_{\hat{\ell}} \simeq \frac{1}{4} N_0 \Delta^2(T)\xi(T) \tag{2}
\]

\[
\simeq \frac{1}{4} N_0 v_F \Delta(T) \tag{3}
\]

where \(\Delta(T)\) is the maximum value of the energy gap and \(v_F\) is the Fermi velocity.
On the other hand the \( \hat{d} \)-soliton may be much more easily created \cite{42}. The crucial element here is the spin-orbit energy which binds \( \hat{d} \) parallel to \( \hat{\ell} \). The relevant energy can deduced from the NMR data of UPt\(_3\) \cite{43} and Sr\(_2\)RuO\(_4\) \cite{44}. We estimate \( \Omega(T)/\Delta(T) \) for UPt\(_3\) and Sr\(_2\)RuO\(_4\): \( \Omega(T)/\Delta(T) \sim 0.5 \times 10^{-3} \) (B phase) and \( 0.2 \times 10^{-4} \) respectively. Here \( \Omega(T) \) is the characteristic frequency associated with the \( \hat{\ell} \) and \( \hat{d} \) coupling \cite{42}. Then the areal energy for the \( \hat{d} \)-soliton is given by

\[
 f_d \approx \frac{1}{4} N_0 v_F \Omega(T). \tag{4}
\]

This is smaller than \( f_d \) for UPt\(_3\) and Sr\(_2\)RuO\(_4\) by factors of \( 0.5 \times 10^{-3} \) and \( 0.2 \times 10^{-4} \), respectively. Also \( \xi_d = v_F/\Omega(T) \) gives the spatial extension of the domain well. This ranges from \( 10 \mu m \sim 1 \) mm.

In the presence of \( \hat{\ell} \)-solitons vortices may enter into the superconducting state as observed in the vortex sheet of superfluid \( ^3\text{He}-\text{A} \) \cite{45, 46}. Otherwise the motion of the vortex across the \( \hat{\ell} \)-soliton is completely impeded as discussed in \cite{40}. When an Abrikosov vortex encounters a \( \hat{d} \)-soliton, the vortex appears to split into two half-quantum vortices (HQV) in the vicinity of \( T \approx T_c \). At lower temperatures it appears that Abrikosov’s vortex should tunnel through the \( \hat{d} \)-soliton. This type of HQV was first predicted in the context of superfluid \( ^3\text{He}-\text{A} \) \cite{47, 48, 49}. However, these HQV’s have not yet been observed in superfluid \( ^3\text{He}-\text{A} \) \cite{50}. Therefore half-quantum vortices may be first observed in triplet superconductors.

### III. HALF-QUANTUM VORTICES

Here we shall consider a pair of half-quantum vortices (HQV) attached to a \( \hat{d} \)-soliton \cite{42}. The texture free energy is given by

\[
 f_{pair} = \frac{1}{2} \chi_N C^2 \int dy \, dz (\nabla \phi)^2 + \sum_{i,j} |\partial_i \hat{d}_j|^2 + \xi_d^{-2} \sin^2(\Psi) \tag{5}
\]

where \( C \) is the spin wave velocity, \( \xi_d = C(T)/\Omega(T) \) and

\[
 K = \frac{\rho_s}{\rho_{s,\text{spin}}} = \frac{1 + \frac{1}{2} F_1 1 + \frac{1}{2} F_1^0 (1 - \rho_0^0)}{1 + \frac{1}{2} F_1^0 1 + \frac{1}{2} F_1 (1 - \rho_0^0)} \tag{6}
\]

where \( \rho_s \) and \( \rho_{s,\text{spin}} \) are the superfluid density and the spin superfluid density respectively. Here \( F_1 \) and \( F_1^0 \) are the Landau parameters and \( \rho_0^0(T) \) is the superfluid density when \( F_1 = F_1^0 = 0 \). The superfluid density for UPt\(_3\) and Sr\(_2\)RuO\(_4\) are shown in Fig. 3. In this analysis we assumed that \( |\Delta(k)| \sim |\cos \theta \sin^2 \theta| \) and \( |\cos \chi| \) in UPt\(_3\) \cite{51} and Sr\(_2\)RuO\(_4\) respectively. We note that \( \rho_0^0 \) in Sr\(_2\)RuO\(_4\) is the same as in d-wave superconductivity \cite{52}. In particular, for \( T \) in the vicinity of \( T_c \) we obtain

\[
 \rho_s^0 \approx -\frac{4}{3} \ln \left( \frac{T}{T_c} \right) \text{ for Sr}_2\text{RuO}_4 \tag{7}
\]

\[
 \rho_s^0 \approx -\frac{143}{105} \ln \left( \frac{T}{T_c} \right) \text{ for UPt}_3 \tag{8}
\]

respectively. Here we assumed that \( \mathbf{H} \parallel \hat{a} \) and that the domain wall extends in the \( y-z \) plane. Here we consider 2 typical cases as shown in Fig. 4a) and b). In Fig. 4a) the \( \hat{d} \)-soliton runs parallel to the \( c \) axis while in Fig. 4b) it runs parallel to the \( b \)-axis. For the first configuration (i.e. a) we parametrize \( \hat{d} = \cos \psi \hat{\zeta} + \sin \psi \hat{\gamma} \) with

\[
 \psi(y, z) = \frac{1}{2} \left( \arctan \left( \frac{z + (R/2)}{y} \right) - \arctan \left( \frac{z - (R/2)}{y} \right) \right) \tag{9}
\]

with 2 HQV located at \( (y, z) = (0, R/2) \) and \( (0, -R/2) \) while in the second configuration (b)

\[
 \psi(y, z) = \frac{1}{2} \left( \arctan \left( \frac{y + (R/2)}{z} \right) + \arctan \left( \frac{y - (R/2)}{z} \right) \right) \tag{10}
\]

Also \( \Phi \) in Eq.(7) is the phase of the order parameter \( \Delta(k) \). Then the total free energy reduces to

\[
 f_{pair} = \frac{1}{2} \chi_N C^2 (\pi K \ln(\lambda/R) + \pi \ln(R/\xi) + \pi \frac{R}{2 \xi_d} \times \ln(4 \xi_d/R)) \tag{11}
\]
FIG. 3: The superfluid density for the B phase of UPt$_3$ (solid line) and Sr$_2$RuO$_4$ are shown.

FIG. 4: The spatial orientation of the $\hat{d}$-vector.

where $\lambda$ and $\xi$ are the magnetic penetration depth and the coherence length respectively.

By minimizing $f_{\text{pair}}$ with respect to $R$, we obtain

$$R_0^2 = 2\xi_d^2(K - 1)/[\ln(\frac{4\xi f}{\sqrt{e}R_0})] > 0$$  \hspace{1cm} (12)

where $R_0$ is the optimal distance of a pair of HQVs. First in order to have a pair of HQVs we need $K > 1$, which is guaranteed when $F_1 > F_1^c$ and $T \simeq T_c$. Also it is necessary to have

$$R_0 < \frac{4}{\sqrt{e}}\xi_d$$  \hspace{1cm} (13)

In particular when $K - 1 \ll 1$, we obtain

$$\frac{R_0}{\xi_d} \simeq \frac{4}{\sqrt{e}} - \frac{\sqrt{e}}{2}(K - 1)$$  \hspace{1cm} (14)

where $e = 2.71828\ldots$. Also the separation between a pair of HQV should be of the order of $\xi_d \sim 10\mu m \sim 1$ mm.
IV. CONCLUDING REMARKS

We have described an abundance of triplet superconductors. Many of their order parameters possess the \( \hat{\ell} \) and \( \hat{d} \) vectors: \( \Delta(\mathbf{k}) \) in the B phase of UPt3, Sr2RuO4, both the A and B phase of PrOs4Sb12 and perhaps many other systems. In these systems the \( \hat{d} \)-soliton is the most common domain wall. The presence of the \( \hat{d} \)-soliton can impede the flux motion in a variety of ways. The most intriguing is the splitting of an Abrikosov vortex into a pair of half-quantum vortices as discussed in [42]. We expect that some of the techniques used to observe Abrikosov’s vortex can be used in the present circumstances. These techniques include the Bitter decoration technique [54], small angle neutron scattering (SANS) [55], scanning tunneling microscopy (STM) [55], micromagnetometer [56] and more recently Lorentz electron micrograph [57]. We expect the exploration of these topological defects in triplet superconductors will enhance our understanding of these exotic superconductors. Also they will provide ideal laboratories to check rich field-theoretical concepts at moderately low temperatures from 1 ~ 100 mK. Therefore the future of topological defects in nodal superconductors is still wide open.

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