Confronting Finite Unified Theories with Low-Energy Phenomenology

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What happens as we approach the Planck scale?

How do we go from a fundamental theory to field theory as we know it?

How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?

How do particles get their very different masses?

What is the nature of the Higgs?
Search for understanding relations between parameters

**addition of symmetries.**

\[ N = 1 \text{ SUSY GUTs.} \]

Complementary approach: look for RGI relations among couplings at GUT scale \( \rightarrow \) Planck scale

\[ \Rightarrow \text{reduction of couplings} \]

\[ \Rightarrow \text{FINITENESS} \]

resulting theory: less free parameters \( \vdash \) more predictive

\( \text{scale invariant} \)
Dimensionless sector of all-loop finite $SU(5)$ model

**prediction for** $M_{top}$, **large** $\tan \beta$

Can be extended to Soft Supersymmetry Breaking (SSB) sector expressed only in terms of

- $g$ (gauge coupling) and
- $M$ (unified gaugino mass)

**too restrictive**

Constraint can be **relaxed**

- sum-rule for soft scalars
- better phenomenology

Confronting with low energy precision data

- Discriminate among different models
- $\implies$ **Prediction for Higgs mass and s-spectra**
Reduction of Couplings

A RGI relation among couplings $\Phi(g_1, \ldots, g_N) = 0$ satisfies

$$\mu \frac{d\Phi}{d\mu} = \sum_{i=1}^{N} \beta_i \frac{\partial \Phi}{\partial g_i} = 0.$$ 

$g_i =$ coupling, $\beta_i$ its $\beta$ function
Finding the $(N-1)$ independent $\Phi$'s is equivalent to solve the reduction equations (RE)

$$\beta_{g_i} \left( \frac{dg_i}{dg} \right) = \beta_i,$$

$i = 1, \cdots, N$

- completely reduced theory contains only one independent coupling and its $\beta$ function
- complete reduction: power series solution of RE
- uniqueness of the solution can be investigated at one-loop
The complete reduction might be too restrictive, one may use fewer $\Phi$’s as RGI constraints.

Reduction of couplings is essential for finiteness.

**finiteness:** absence of $\infty$ renormalizations

$$\Rightarrow \beta^N = 0$$

In SUSY no-renormalization theorems:

- $\Rightarrow$ only study one and two-loops

- guarantee that is gauge and reparameterization invariant at all loops
Finiteness

A chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group $G$ with gauge coupling constant $g$ has a superpotential

$$ W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k , $$

Requiring one-loop finiteness $\beta^{(1)}_g = 0 = \gamma^{(1)}_i$ gives the following conditions:

$$ \sum_i T(R_i) = 3 C_2(G) , \quad \frac{1}{2} C_{ipq} C^{ipq} = 2 \delta^i_j g^2 C_2(R_i) . $$

- restricts the particle content of the models
- relates the gauge and Yukawa sectors
One-loop finiteness $\implies$ two-loop finiteness

One-loop finiteness restricts the choice of irreps $R_i$, as well as the Yukawa couplings.

Cannot be applied to the susy Standard Model (SSM):
\[ C_2[U(1)] = 0 \]

The finiteness conditions allow only SSB terms.

It is possible to achieve all-loop finiteness $\beta^n = 0$:

1. One-loop finiteness conditions must be satisfied
2. The Yukawa couplings must be a formal power series in $g$, which is solution (isolated and non-degenerate) to the reduction equations
Supersymmetry is essential. It has to be broken, though...

\[-\mathcal{L}_{SB} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{2} (m^2)_i^{j} \phi^*^i \phi^j + \frac{1}{2} M \lambda \lambda + \text{H.c.}\]

The RGI method has been extended to the SSB of these theories.

- One- and two-loop finiteness conditions for SSB have been known for some time
  
  Jack, Jones, et al.

- It is also possible to have all-loop RGI relations in the finite and non-finite cases
  
  Kazakov; Jack, Jones, Pickering
SSB terms depend only on $g$ and the unified gaugino mass $M$

universality conditions

$$h = -MC, \quad m^2 \propto M^2, \quad b \propto M_\mu$$

Very appealing! But too restrictive; it leads to phenomenological problems:

- The lightest susy particle (LSP) is charged.  
  Yoshioka; Kobayashi et al
- It is incompatible with radiative electroweak breaking.  
  Brignole, Ibáñez, Muñoz

Possible to relax the universality condition to a sum-rule for the soft scalar masses

$$\Rightarrow \text{better phenomenology.}$$

Kobayashi, Kubo, Mondragón, Zoupanos
Soft scalar sum-rule for the finite case

Finiteness implies

\[ C^{ijk} = g \sum_{n=0} \rho_{(n)}^{ijk} g^{2n}, \]

The one- and two-loop finiteness for \( h \) gives

\[ h^{ijk} = -MC^{ijk} + \cdots = -M \rho_{(0)}^{ijk} g + O(g^5). \]

Assume that lowest order coefficients \( \rho^{ijk}_{(0)} \) and \( (m^2)^i_j \) satisfy diagonality relations

\[ \rho^{ipq}_{(0)} \rho^{jpq}_{(0)} \propto \delta^j_i, \quad (m^2)^i_j = m^2_j \delta^i_j \quad \text{for all } p \text{ and } q. \]

We find the the following soft scalar-mass sum rule

\[ \left( m_i^2 + m_j^2 + m_k^2 \right) / MM^\dagger = 1 + \frac{g^2}{16\pi^2} \Delta^{(1)} + O(g^4) \]

for \( i, j, k \) with \( \rho^{ijk}_{(0)} \neq 0 \), where \( \Delta^{(1)} \) is the two-loop correction,

\[ \Delta^{(1)} = -2 \sum_i [(m_i^2 / MM^\dagger) - (1/3)] T(R_i), \]

which vanishes for the universal choice.
All-loop sum rule

One can generalize the sum rule for finite and non-finite cases to all-loops!!

Possible thanks to renormalization properties of $N = 1$ susy gauge theories.

The sum-rule in the NSVZ scheme is

$$m_i^2 + m_j^2 + m_k^2 = |M|^2 \left\{ \frac{1}{1 - g^2 C(G)/(8\pi^2)} \frac{d \ln C^{ijk}}{d \ln g} + \frac{1}{2} \frac{d^2 \ln C^{ijk}}{d(\ln g)^2} \right\} + \sum_l m_l^2 T(R_l) \frac{d \ln C^{ijk}}{C(G) - 8\pi^2/g^2} d \ln g.$$

Interesting: Finite sum rule satisfied also in certain certain class of orbifold models in which the massive states are organized into $N = 4$ supermultiples, if $d \ln C^{ijk}/d \ln g = 1$. 

Kazakov et al; Jack, Jones et al; Yamada; Hisano, Shifman

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Several aspects of Finite Models have been studied

- *SU(5)* Finite Models studied extensively
  
  Rabi et al; Kazakov et al; López-Mercader, Quirós et al; M.M, Kapetanakis, Zoupanos; etc

- One of the above coincides with a non-standard Calabi-Yau *SU(5) × E8*
  
  Greene et al; Kapetanakis, M.M., Zoupanos

- Finite theory from compactified string model also exists (albeit not good phenomenology)
  
  Ibáñez

- Criteria for getting finite theories from branes exist
  
  Hanany, Strassler, Uranga

- Realistic models involving all generations exist
  
  Babu, Eckbahrt, Gogoladze

- Some models with *SU(N)^k* finite \iff 3 generations, good phenomenology with *SU(3)^3*
  
  Ma, M.M, Zoupanos

- Relation between commutative field theories and finiteness studied
  
  Jack and Jones

- Proof of conformal invariance in finite theories
  
  Kazakov
**SU(5) Finite Models**

We study two models with $SU(5)$ gauge group. The matter content is

$$3\ 5 + 3\ 10 + 4\ \{5 + \bar{5}\} + 24$$

The models are finite to all-loops in the dimensionful and dimensionless sector. In addition:

- The soft scalar masses obey a sum rule
- At the $M_{GUT}$ scale the gauge symmetry is broken and we are left with the MSSM
- At the same time finiteness is broken
- The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs $\{5 + \bar{5}\}$ which couple to the third generation

The difference between the two models is the way the Higgses couple to the $24$

Kapetanakis, Mondragón, Zoupanos; Kazakov et al.
The superpotential which describes the two models takes the form

\[ W = \sum_{i=1}^{3} \left[ \frac{1}{2} g_i^u \ 10_i 10_i H_i + g_i^d \ 10_i 5_i \ H_i \right] + g_{23}^u \ 10_2 10_3 H_4 \]

\[ + g_{23}^d \ 10_2 5_3 \ H_4 + g_{32}^d \ 10_3 5_2 \ H_4 + \sum_{a=1}^{4} g_a^f H_a 24 \ H_a + \frac{g^\lambda}{3} (24)^3 \]

find isolated and non-degenerate solution to the finiteness conditions
The finiteness relations give at the $M_{GUT}$ scale

| Model A | Model B |
|---------|---------|
| $g_t^2 = \frac{8}{5} g^2$ | $g_t^2 = \frac{4}{5} g^2$ |
| $g_{b,\tau}^2 = \frac{6}{5} g^2$ | $g_{b,\tau}^2 = \frac{3}{5} g^2$ |
| $m_{Hu}^2 + 2m_{10}^2 = M^2$ | $m_{Hu}^2 + 2m_{10}^2 = M^2$ |
| $m_{Hd}^2 + m_5^2 + m_{10}^2 = M^2$ | $m_{Hd}^2 - 2m_{10}^2 = -\frac{M^2}{3}$ |

- 3 free parameters: $M$, $m_5^2$ and $m_{10}^2$
- 2 free parameters: $M$, $m_5^2$
Phenomenology

The gauge symmetry is broken below $M_{GUT}$, and what remains are boundary conditions of the form $C_i = \kappa_i g$, $h = -MC$ and the sum rule at $M_{GUT}$, below that is the MSSM.

- We assume a unique susy breaking scale
- The LSP is neutral
- The solutions should be compatible with radiative electroweak breaking
- No fast proton decay

We also

- Allow 5% variation of the Yukawa couplings at GUT scale due to threshold corrections
- Include radiative corrections to bottom and tau, plus resummation (very important!)
- Estimate theoretical uncertainties
We look for the solutions that satisfy the following constraints:

- Right masses for top and bottom
- The anomalous magnetic moment of the muon $g - 2$
- The decay $b \rightarrow s \gamma$
- The branching ratio $B_s \rightarrow \mu^+ \mu^-$
- Cold dark matter density $\Omega_{CDM} h^2$

The lightest MSSM Higgs boson mass
The SUSY spectrum

FeynHiggs, Suspect, FUT
FUTA: $M_{top} \sim 183$ GeV
FUTB: $M_{top} \sim 172$ GeV

Theoretical uncertainties $\sim 4\%$
Δb and Δtau included, resummation done

FUTB μ < 0 favoured
uncertainties ∼ 8 %
Higgs

FUTB: \( M_{\text{Higgs}} = 122 \sim 126 \text{ GeV} \)
Uncertainties \( \pm 3 \text{ GeV} \) (FeynHiggs)

\[ \Omega_{\text{CDM}} h^2 < 0,3 \]

\[ 0,094 < \Omega_{\text{CDM}} h^2 < 0,129 \]
Results

When confronted with low-energy precision data

**only FUTB $\mu < 0$ survives**

No solution for g-2, very constrained from dark matter

- $M_{top} \sim 172 \text{ GeV} \quad 4\%$
- $m_{bot}(M_Z) \sim 2.8 \text{ GeV} \quad 8\%$
- $M_{Higgs} \sim 122 - 126 \text{ GeV} \quad 3\text{ GeV}$
- $\tan \beta \sim 44 - 46$

Extension to 3 fams on its way with flavour symmetry;
with $R \Rightarrow$ neutrino masses

in this case dark matter candidate is not LSP, results may change
Conclusions

- Finiteness: powerful, interesting and intriguing principle \(\Rightarrow\) reduces greatly the number of free parameters
- Completely finite theories i.e. including the SSB terms, that satisfy the sum rule.
- Confronting the SU(5) models with low-energy precision data does distinguish among models:
  - FUTB \(\mu < 0\) survives (remarkably)
  - large tan \(\beta\)
  - s-spectrum starts above \(200 \sim 300\) GeV
  - a prediction for the Higgs \(M_h \sim 122 - 126\) GeV
  - no solution for \(g - 2\), constrained from dark matter
- Extension to three fams with \(R\) on its way
- Detailed study of finite SU(3)\(^3\) \(\iff\) 3 generations in progress