Convection is a fundamental physical process in the fluid cores of planets. It is the primary transport mechanism for heat and chemical species and the primary energy source for planetary magnetic fields. Key properties of convection—such as the characteristic flow velocity and length scale—are poorly quantified in planetary cores owing to the strong dependence of these properties on planetary rotation, buoyancy driving and magnetic fields, all of which are difficult to model using realistic conditions. In the absence of strong magnetic fields, the convective flows of the core are expected to be in a regime of rapidly rotating turbulence, which remains largely unexplored. Here we use a combination of non-magnetic numerical models designed to explore this regime to show that the convective length scale becomes independent of the viscosity when realistic parameter values are approached and is entirely determined by the flow velocity and the planetary rotation. The velocity decreases very rapidly at smaller scales, so this turbulent convective length scale is a lower limit for the energy-carrying length scales in the flow. Using this approach, we can model realistically the dynamics of small non-magnetic cores such as the Moon. Although modelling the conditions of larger planetary cores remains out of reach, the fact that the turbulent convective length scale is independent of the viscosity allows a reliable extrapolation to these objects. For the Earth’s core conditions, we find that the turbulent convective length scale in the absence of magnetic fields would be about 30 kilometres, which is orders of magnitude larger than the ten-metre viscous length scale. The need to resolve the numerically inaccessible viscous scale could therefore be relaxed in future more realistic geodynamo simulations, at least in weakly magnetized regions.

The very low fluid viscosity in planetary liquid cores implies that the convective flows are turbulent, but this turbulence differs both from three-dimensional (3D) turbulence owing to the anisotropy imposed by the rapid planetary rotation and from two-dimensional (2D) turbulence owing to the presence of Rossby waves. Conditions in planetary cores correspond to small Ekman numbers ($\text{Ek} = \nu/\Omega R^2$ with viscosity $\nu$, rotation rate $\Omega$ and core radius $R$), large Reynolds numbers ($\text{Re} = UR/\nu$ with flow speed $U$) and small Rossby numbers ($\text{Ro} = U/\Omega R = \text{Re} \times \text{Ek}$), with, for instance, $\text{Ek} \approx 10^{-15}$, $\text{Re} \approx 10^9$ and $\text{Ro} \approx 10^{-6}$ in the Earth’s core. Numerical models must employ a fluid viscosity that is orders of magnitude larger than realistic values to keep the range of time and length scales involved in the dynamics.

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**Fig. 1 | Flow in the 3D model.** Meridional and equatorial cross-sections of a snapshot of the axial vorticity in the 3D model for $\text{Ek} = 10^{-8}$, $\text{Ra} = 2 \times 10^{10}$ and $\text{Pr} = 10^{-2}$. Streamlines have been superimposed in the equatorial plane. In the colour scale, values of the axial vorticity are normalized by the planetary vorticity $2\Omega$. The kinetic energy of the velocity projected on a QG state $\langle (u_x) \rangle$, $\langle (u_z) \rangle$ in cylindrical polar coordinates (where the angle brackets denote an axial average) is within 0.2% of the total kinetic energy.
manageable, typically \( \text{Ek} \geq 10^{-7} \) and \( \text{Re} \leq 10^4 \). This however has the undesirable effect that convection properties are still controlled by the viscosity. In the viscous regime, convection takes the form of tall and narrow columns aligned with the rotation axis with an azimuthal length scale, \( L_\nu \), that depends on the viscosity as \( \text{Ek}^{1/3} \) (ref. 7), and so \( L_\nu \approx 10^m \) for Earth-like parameter values. When nonlinear effects become important in the rapidly rotating turbulent regime of large \( \text{Re} \) and low \( \text{Ro} \), the turbulent convective length scale \( L_\nu \) is expected to grow above the viscous length scale, up to a scale controlled by the flow velocity. The value of \( L_\nu \) is currently unknown for planetary cores.

The objective of this work is to provide an estimate of \( L_\nu \), under core conditions using an extensive numerical exploration of the low-viscosity regime. We use a combination of a state-of-the-art 3D model\(^{11}\) down to \( \text{Ek} = 10^{-8} \) supplemented by a simplified model of quasi-geostrophic (QG) rotating convection\(^{2,13}\) down to \( \text{Ek} = 10^{-11} \). The simplified QG model takes advantage of the Proudman–Taylor constraint\(^{14}\) by assuming that the axial vorticity is invariant along the rotation axis. The QG approximation is well supported by the results of the 3D model shown in Fig. 1. The numerical codes solve the governing equations of nonlinear Boussinesq convection driven by homogeneous internal heating in a full sphere geometry (see Methods). Magnetic fields are not included.

For the low \( \text{Ek} \) values studied here, convection is always in a turbulent state, even near the nonlinear onset\(^{11,15}\), and \( \text{Re} \geq 10^4 \). The convection takes the form of vortical plumes that are radically elongated on scales much shorter than the outer radius (Fig. 2). At large radius, the steepening of the boundary slope inhibits vortical plume convection\(^5\). The dynamics there consists mainly of Rossby waves, which appear as elongated vortices with a prograde tilt\(^{16,17}\) (Fig. 2e). Their radial velocities are relatively small so conduction dominates the heat transport in the outer part of the equatorial plane\(^{15}\). Hereafter, we consider only the dynamics of the inner convective region, which grows wider with increasing Rayleigh number (Ra, which controls the buoyancy driving). The length scale of the convective flows decreases notably with increasing radius (Fig. 2f, g). We find that the convective length scale is controlled by \( \text{Ro} \), rather than by any viscous effect. The flows shown in Figs. 1, 2 are snapshots taken once the system has reached a statistically steady state, where the kinetic energy fluctuates around a constant mean value, and are entirely unlike the linear viscous mode at the onset of convection, which consists of drifting columns with narrow azimuthal length scale\(^{19} L_\nu \). The convective length scale increases with the buoyancy driving, as seen in the power spectra of the total and radial kinetic energies in Fig. 3. The peak of the radial kinetic energy moves to smaller azimuthal wavenumber \( m \) for increasing \( \text{Ra} \), as can be observed for the two different \( \text{Ra} \) shown at \( \text{Ek} = 10^{-10} \), and is located at a much smaller wavenumber \( (m = 133 \) and 106 for the smaller and larger \( \text{Ra} \), respectively) than the wavenumber of the marginal linear viscous mode at the onset of convection, which consists of drifting columns with narrow azimuthal length scale\(^{19} L_\nu \).
The scaling gives a convective length scale that depends on the flow velocity as $L_c \propto \left(\frac{\text{Ro}}{|\beta|}\right)^{1/2}$, where $\beta$ is a geometric factor related to the boundary slope (see Methods). This length scale is consistent with the $m^{-5}$ spectra of the kinetic energy. Assuming that the transport in the fluid bulk controls the heat transfer, the scaling uses a balance between the nonlinear advection of temperature and the transport of the mean temperature background to obtain $Re \propto Ra \times Ek/Pr$, or simply $Ro \propto Bu$, where $Bu = Ra \times Ek^2/Pr$ is the viscosity-free buoyancy parameter. The Prandtl number, $Pr$, is the ratio of viscosity to thermal diffusivity and is expected to be $0.01-0.1$ in liquid metal cores. The theoretical scaling law is tested in Fig. 4 against results obtained with the 3D and QG models and against published results obtained with a hybrid model that uses the QG approximation coupled to the 3D temperature. The characteristic convective length scale $L_c$ corresponds to the peak of the radial kinetic energy spectra. Points obtained at different Ek values collapse onto a single curve, especially for $Ek < 10^{-9}$, showing that the dependence of the results on the viscosity becomes negligible when core conditions are approached. Importantly, the good agreement obtained between the different numerical models supports the use of the QG approximation for modelling rapidly rotating convection. The data for the velocity and length scale, compensated by their respective theoretical scaling laws, align on a plateau at small $Ek$ values, indicating that the agreement between the simulations and the theoretical scaling improves progressively as $Ek$ decreases. The length scales show little dependence on $Pr$; for the velocity scaling law, the exponent is unaffected by $Pr$ but simulations with larger $Pr$ values tend to have a slightly smaller prefactor. To avoid the ‘shingling’ effect that occurs when using diffusion-free parameters, the scaling of the $Re$ is shown in Extended Data Fig. 1 and confirms the overlap of the data for $Ek \leq 10^{-9}$ and the good agreement with the exponent predicted by the theoretical scaling. The length scale $L_c$ corresponds to an azimuthal size in Fig. 4, and we further confirm in Extended Data Fig. 2 that the radial length scale obtained from radial correlations is in good agreement with this azimuthal scale. The radial dependence of the length scale observed in Fig. 2 is also in agreement with the theoretical dependence on $|\beta|^{1/2}$, as shown in Extended Data Fig. 3. Additional QG simulations performed with differential heating in the presence of an inner core (see Methods) show that the scaling law $L_c(Ro)$ of Fig. 4 is valid for other heating modes (Extended Data Fig. 4).

Fig. 3 | Distribution of the kinetic energy at different length scales. Power spectra of the total kinetic energy (thin lines) and radial kinetic energy (thick lines) as a function of the azimuthal wavenumber $m$ at $s = 0.5$ for simulations with different Ek and Ra values for $Pr = 10^{-2}$ performed with the 3D and QG models. The kinetic energy is averaged in time and normalized by $\rho(LR)^2/2$. The length scale is inversely proportional to $m$. The dashed line represents a power law with exponent $-5$.

Fig. 4 | Scaling of the velocity and length scale. a, $Ro$ as a function of the buoyancy parameter $Bu$. b, Convective length scale $L_c$ as a function of $Ro$ using the 3D (green points), QG (blue) and hybrid (red) simulations. Marker colours correspond to $Ek$ (values given in the key) and shapes to $Pr$ (circles, $Pr = 10^{-2}$ and squares, $Pr = 10^{-1}$). In b, $L_c$ is radially averaged between $s \in [0.1, 0.6]$; the vertical bars give the standard deviation in this interval. The horizontal lines give the linear viscous length scale $L_v$, at $s = 0.5$ for given $Ek$ and with $Pr = 10^{-2}$. Insets, the data compensated by the theoretical scaling as a function of $Bu$ (a) and $Ro$ (b).
The smallest Ekman number computed with our simplified QG numerical model, \( Ek = 10^{-11} \), is approximately the value for the core of the Moon\(^{22} \). Non-magnetic convection in the lunar core is bracketed between the extinction of the dynamo, which occurs at \( Re \approx 10^7 \), corresponding to a critical magnetic \( Re \) of 10 (ref. 24) and the cessation of nonlinear convection, which occurs at \( Re \approx 10^9 \) (refs 11,15). Between these two events, our results predict that the turbulent length scale decreases as \( R^{1/2} \propto \alpha R^{1/2} \) from 0.1 \( R \approx 10 \) km to 0.001 \( R \approx 0.1 \) km, implying a large reduction in heat transport efficiency (see Methods).

Although non-magnetic, our study has interesting implications for the Earth’s core dynamics and geodynamo modelling. Characteristic flow speeds at the core–mantle boundary inferred from the geomagnetic secular variation have \( R_e \approx 10^{-6} \) (ref. 25), corresponding to \( L_c \approx 0.01 \) \( R \approx 30 \) km. This value is close to the magnetostrophic cross-over length scale\(^{26} \), a theoretical length scale below which magnetic forces become dynamically important; in the Earth’s core, this length scale is estimated to be 1–100 km. The geomagnetic field therefore probably affects core convection. In the presence of magnetic fields, the convective length scale is expected to increase\(^{27,28} \), so the 30-km scale will probably remain a lower limit for the energy-carrying length scales.

In the most recent geodynamo simulations\(^{28–30} \), the magnetic field is non-magnetic, our study has interesting implications for approaching a realistic force balance in geodynamo simulations.

### Online content

Any methods, additional references, Nature Research reporting summaries, source data, statements of data availability and associated accession codes are available at [https://doi.org/10.1038/s41586-019-1301-5](https://doi.org/10.1038/s41586-019-1301-5).

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METHODS

We model Boussinesq convection driven by homogeneous internal heating in a full-spherical geometry. This problem is relevant for planetary cores without a solid inner core, and is thus relevant for most of the Earth’s history. The model does not include magnetic fields. The sphere rotates at a rate $\Omega$ around the axis directed along the unit vector $\mathbf{e}_z$. The acceleration due to gravity $g$ is radial and increases linearly with the radius $r$ as $g = -\frac{GM}{r^2}$, where $M$ is the mass of the sphere.

The governing equations are written in a dimensionless form and are obtained by scaling length by $R$, scaling time by $R^2/\nu$, and scaling temperature by $\beta R^3/\alpha (6/\nu)^2$, where $\beta$ is the thermal diffusivity, $\nu$ the fluid kinematic viscosity, $\alpha$ the thermal expansion coefficient, and $C_p$ the heat capacity at constant pressure.

The dimensionless numbers are the Ekman number, $E_k = \nu/(2R^2)$, the Taylor number, $Ta = \alpha R^3/(6\nu C_p^2)$, and the Prandtl number, $Pr = \nu/\alpha$. This study focuses on $Pr$ values smaller than unity, which are relevant for the thermal convection of liquid metal cores.

The system of dimensionless equations is:

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u + \frac{2}{E_k} \left( \nabla^2_\perp + \nabla^2 \right) u = -\nabla p + \nabla^2 u + R \alpha \nabla r$$  \hspace{1cm} (1)

$$\nabla \cdot u = 0$$  \hspace{1cm} (2)

$$\frac{\partial \Theta}{\partial t} + u \cdot \nabla \Theta = \frac{2}{Pr} \mu_s = \frac{1}{Pr} \nabla^2 \Theta$$  \hspace{1cm} (3)

where $u$ is the velocity field, $p$ the pressure and $\Theta$ the temperature perturbation relative to the static temperature $\Theta_{\text{static}}(r)$. We use no-slip boundary conditions and a fixed temperature at the outer boundary.

**3D numerical model.** For the 3D simulations, we use the code XSHIELDS, which solves equations (1)–(3) using finite differences in the radial direction and spherical harmonic expansion. The input parameters and numerical resolutions used for the 3D simulations are given in the Supplementary Information. In the 3D simulations, $Pr$ is fixed at $10^{-4}$ and $E_k$ is varied between $10^{-4}$ and $10^{-8}$. The most computationally demanding simulations performed at $E_k = 10^{-4}$ were run with a numerical resolution of 2,016 radial grid points and truncation degree $L = 351$, and order $M = 319$ for the spherical harmonics. Hyperviscosity was used in all the 3D simulations, with viscosity depending on spherical harmonic degree, order and $Pr$.

**QG numerical model.** For simulations at smaller Ek values, we assume that the rotational constraint is such that the variations of the velocity along the axial direction are small compared with the variations along the orthogonal directions. We use a QG approximation for rapidly rotating spherical convection developed from the Busse annulus model and widely used in the context of planetary core convection. The dynamics are assumed to be dominated by the geostrophic balance, that is, the Coriolis force balances the pressure gradient at leading order. The leading-order velocity $u_s$ is invariant along $z$ and $u_s = (u_s, u_s, 0)$ in cylindrical polar coordinates. QG convection is driven by the cylindrical component of the velocity field. For a given spherical harmonic degree, order are small compared with the variations along the orthogonal directions.

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The velocity is described by a streamfunction $\psi$ that models the non-axisymmetric (that is, $\phi$-dependent) components with the addition of an axisymmetric azimuthal flow, $\psi_0$, where the overbar denotes an azimuthal average:

$$u_s = \frac{1}{H} \nabla \times (H \psi_0) + \tilde{u}_s \mathbf{e}_z$$  \hspace{1cm} (5)

This choice of the streamfunction accounts for mass conservation at the outer boundary. We assume that the axial velocity $u_s$ is linear in $z$ and has two contributions: the main contribution comes from mass conservation at the outer boundary and is proportional to $\beta = H^2/\nu$; the other contribution accounts for Ekman pumping, which is produced by the viscous boundary layer and scales as $E_k^{1/2}$. The Ekman pumping is parameterized by the formula obtained by asymptotic methods in the limit of small Ek for a linear Ekman layer.

The streamfunction $\psi$ describes only the non-axisymmetric motions, so $\psi_0$ is obtained by taking the azimuthal and axial averages of the $\phi$ component of the Navier–Stokes equation to give:

$$\frac{\partial \psi_0}{\partial t} + u_s \frac{\partial \psi_0}{\partial s} + \frac{\mu_s}{s} = \nabla^2 \psi_0 + \frac{1}{E_k^{1/2} H^{1/2}} \psi_0$$  \hspace{1cm} (6)

where the last term on the right-hand side corresponds to the Coriolis term simplified using mass conservation.

The equation for the temperature perturbation $\Theta$ in the QG model is obtained by taking the axial average of the temperature equation and assuming that $\Theta$ is invariant along $z$ to obtain:

$$\frac{\partial \Theta}{\partial t} + u \cdot \nabla \Theta = \frac{4}{3 \Pr} \mu_s = \frac{1}{\Pr} \nabla^2 \Theta$$  \hspace{1cm} (7)

We use the gradient of the z-averaged static temperature profile, $(T_{\text{static}})_{z} = -4s/3\Pr$, rather than the gradient of the $z$-invariant static temperature profile, $(T_{\text{static}})_{z} = -3s/Pr$, to allow for a direct comparison of the Rayleigh numbers used in the different models. The assumption that $\Theta$ is invariant along $z$ is not rigorously justified and is used for numerical convenience; it permits us to treat the numerical problem in two dimensions, considerably reducing the computational load. The evolution equation for the streamfunction, the axisymmetric velocity and the temperature are solved on a 2D grid in the equatorial plane. The QG code uses a pseudo-spectral code with a Fourier decomposition in the azimuthal direction and a second-order finite-difference scheme in the radius, with irregular spacing.

**QG model with differential heating.** To test the dependence of our results on the heating mode and the presence of an inner core, we performed additional QG simulations using differential heating with fixed temperature boundary conditions and an inner core of radius $R_i = 0.35$. The temperature is scaled by $Pr\Delta T$. The equation for the temperature perturbation is solved in two dimensions with a static temperature gradient $(T_{\text{static}})_{z} = \gamma/(Pr \times \ln(R_i))$, where the constant $\gamma = 0.445$ is used to re-scale the $z$-invariant temperature profile so that it corresponds closely with the $z$-averaged static temperature profile.

**Number of output parameters.** The simulations are started from either a small temperature perturbation or the snapshot of a previous simulation performed at a different Rayleigh number in order to minimize the transient phase before saturation. All simulations are run to saturation, as shown in Extended Data Fig. 5, where we plot the time series of the kinetic energy density for one representative QG case at $E_k = 10^{-11}$ and one representative 3D case at $E_k = 10^{-2}$. For consistency, the kinetic energy density $K$ is defined in both cases as:

$$K = \frac{1}{2V} \int (u^2 + u^2)^1 dV$$  \hspace{1cm} (8)

where $V$ is the volume of the sphere, and the kinetic energy density of the axisymmetric velocity is:

$$K_{\psi_0} = \frac{1}{2V} \int (\psi_0^2)^1 dV$$  \hspace{1cm} (9)

A number of output parameters are given in the Supplementary Information. The characteristic velocity $U$ used to calculate $Ro$ and $Re$ is based on the root mean square of the radial velocity averaged in volume and time over at least ten convective turnover timescales.

The convective length scale is calculated as $L_c = \pi \sigma / m_\nu (\sigma)$, where $m_\nu$ is the wavenumber at the peak of the radial kinetic energy spectrum. The peak is determined by smoothing the time-averaged radial kinetic energy spectra with a polynomial of degree 14.

The radial length scale of the convective flow $L_c(\sigma)$ is calculated using the auto-correlation function $f$ of the radial component of the velocity field. For a given radius $s$, we calculate:

$$K_{\psi_0}(s) = \frac{1}{2V} \int (\psi_0)^1 dV$$  \hspace{1cm} (10)
\[ f(ds) = n(x, \phi, t) u (s + ds, \phi, t) \]  
(10)

where the overbar denotes an azimuthal average. Snapshots covering at least two dynamical timescales are used to compute the temporal average. \( L_s(s) \) is the full-width at half-maximum of \( f \).

**Inviscid scaling theory.** The theoretical scaling of the velocity and length scale assumes a triple inviscid balance in the axial vorticity equation between the vorticity advection, vortex stretching and vorticity generation by buoyancy:

\[ \frac{\text{Re}^2}{L_s} \approx \frac{|\beta| \text{Re}}{\text{Ek}} \approx \frac{\text{Ra} T}{L_s} \]  
(11)

where \( T \) denotes a typical temperature perturbation and we assume that the typical axial vorticity is \( \text{Re} / L_s \). The turbulent convective length scale then scales as \( L_s \propto (\text{Ro}/|\beta|)^{1/2} \). Assuming that, in rapidly rotating convection, the heat transfer is controlled by the transport in the bulk of the fluid rather than in the thermal boundary layers, we obtain a balance between the nonlinear advection of heat and the transport of the mean temperature background in the temperature equation:

\[ \frac{\text{Re} T}{L_s} \propto \frac{\text{Pr}}{L_s} \]  
(12)

Combining equations (11) and (12) leads to \( \text{Re} \propto \text{Ra} \times \text{Ek} / \text{Pr} \), where we neglect the geometric term \( s|/\beta| \). The efficiency of the heat transport can be measured by the ratio \( q / q_c \), where both the convective heat flux \( q = \text{Pr} T \text{Re} \) and the static heat flux \( q_c \propto 1 / \text{Pr} \) are dimensionless. The theoretical scalings of the velocity and temperature perturbation imply that \( q / q_c \propto L_s^2 \times \text{Pr} / \text{Ek} \).

**Data availability**

Source data for Figs. 3, 4 are provided with this paper. The data generated during this study are included in the Supplementary Information file. Any additional data that support the findings of this study are available from the corresponding author on reasonable request.

**Code availability**

The 3D numerical code XSHELLS is freely available at https://bitbucket.org/nschaeff/xshells and is distributed under the open source CeCILL License (http://www.cecill.info/licences/Licence_CeCILL_V2.1-en.html). The QG numerical code is available from the corresponding author on request.

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Extended Data Fig. 1 | Scaling of the Reynolds number. Re as a function of $Ra \times Ek/Pr$ in simulations performed with the 3D model (green data points) for $Ek \in [10^{-8}, 10^{-6}]$, the QG model (blue data points) for $Ek \in [10^{-11}, 10^{-8}]$, and the hybrid model (red data points) for $Ek \in [10^{-8}, 10^{-7}]$. Marker colours correspond to Ekman numbers (values given in the key) and marker shapes correspond to Prandtl numbers (circles, $Pr = 10^{-2}$ and squares, $Pr = 10^{-1}$). The dashed line represents $Re = 0.6 Ra \times Ek/Pr$. Inset, the same data compensated by theoretical scaling as a function of $Ra \times Ek/Pr$. 
Extended Data Fig. 2 | Comparison of the radial length scale with the azimuthal length scale. Radial scale of the convective flows $\mathcal{L}_r(s)$ as a function of the azimuthal length scale $\mathcal{L}(s)$ obtained with the QG model at different radii $s$. Marker colours correspond to Ekman numbers (with Pr = $10^{-2}$) and marker shapes correspond to the given radii. The radial scale is calculated from auto-correlation functions of the radial velocity, and the convective length scale corresponds to an azimuthal scale calculated from the peak of the power spectra of the radial kinetic energy at radius $s$. The dashed line represents $\mathcal{L}_r(s) = \mathcal{L}(s)$. 
Extended Data Fig. 3 | Variation of the convective length scale with radius. Convective length scale $L(s)$ as a function of $\text{Ro}(s)/|\beta|$ obtained with the QG model at different radii $s$. Marker colours correspond to Ekman numbers, solid-colour markers correspond to $\text{Pr} = 0.01$, dotted markers to $\text{Pr} = 0.1$, and marker shapes correspond to the given radii.

The convective length scale corresponds to an azimuthal scale calculated from the peak of the power spectra of the radial kinetic energy at radius $s$. The dashed line represents $L(s) = 6(\text{Ro}(s)/|\beta|)^{1/2}$. Inset, the length scale compensated by theoretical scaling as a function of $\text{Ro}(s)/|\beta|$. 
Extended Data Fig. 4 | Effect of the heating mode on the convective length scale. Convective length scale $L$ as a function of $Ro$ obtained with the QG model for internal heating (IH, same points as in Fig. 4) and differential heating (DH) with an inner core of radius $R_i = 0.35$. $Ek \in [10^{-11}, 10^{-7}]$ and $Pr \in \{10^{-2}, 10^{-1}, 1\}$ are given in the key.

The convective scale is averaged over radii between $s = 0.1$ and 0.6 and the vertical error bars give the standard deviation in this interval. The dashed line represents $L = 11Ro^{1/2}$. Inset, the same data compensated by theoretical scaling as a function of $Ro$. 
Extended Data Fig. 5 | Time series of the kinetic energy density for two representative simulations. a, b. Time series of the kinetic energy density $K$ and the kinetic energy density of the axisymmetric flow $K_{\text{axi}}$ for $E_k = 10^{-11}$, $Pr = 0.01$ and $Ra = 3.75 \times 10^{13}$ using the QG model (a) and $E_k = 10^{-8}$, $Pr = 0.01$ and $Ra = 2 \times 10^{10}$ using the 3D model (b). Time is given in units of a viscous timescale.