Tkachenko waves, glitches and precession in neutron stars

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Abstract

Here I discuss possible relations between free precession of neutron stars, Tkachenko waves inside them and glitches. I note that the proposed precession period of the isolated neutron star RX J0720.4-3125 (Haberl et al. 2006) is consistent with the period of Tkachenko waves for the spin period 8.4 s. Based on a possible observation of a glitch in RX J0720.4-3125 (van Kerkwijk et al. 2007), I propose a simple model, in which long period precession is powered by Tkachenko waves generated by a glitch. The period of free precession, determined by a NS oblateness, should be equal to the standing Tkachenko wave period for effective energy transfer from the standing wave to the precession motion. A similar scenario can be applicable also in the case of the PSR B1828-11.

Keywords neutron stars; pulsars

1 Introduction

Isolated neutron stars (NSs) being non-spherical bodies are expected to demonstrate free precession (for a brief review see, for example, Link [2003]). However, examples of this phenomena are less than few, and even in rare cases when a precession-like behavior is observed different interpretations can be discussed (even not related to precession, see for example Ruderman and Gil [2006]).

The problem of long period free precession in NSs is a long standing one. A NS can precess if it is non-spherical and rotation axis does not coincide with a principal axis. Typically, biaxial objects are discussed, so deviation from spherical symmetry can be described by one parameter – oblateness (see Akgün et al. [2006] for a discussion of triaxial model). Expected values of NS oblateness (due to rotation or influence of strong magnetic fields) can naturally lead to precession periods about one year. The precession period is equal to $P_{\text{prec}} = P/\epsilon$. Here $P$ is the spin period of a NS, $\epsilon$ – its oblateness, and $P_{\text{prec}}$ – precession period. Measured precession periods require oblateness about $10^{-8}$.

However, discussing dynamics of NSs it is necessary to take into account the network of superfluid vortices inside them. The neutron superfluid liquid in the interior of a NS participates in rotation via formation of quantized vortex lines. The density of these lines per unit area is $n = 2\Omega/k$. Here $\Omega = 2\pi/P$ is spin frequency, and $k = h/2m_n$, where $m_n$ is a neutron mass (see, for example, Shapiro and Teukolsky [1983, Ch. 10]). The vortices exist in the core of a NS, where they can interact with superfluid (superconducting) protons and normal electrons, and in the crust, where they can pin to it.

Coupling of superfluid neutron vortices with electrons in a core results in damping of free precession (Alpar and Ogelman [1987]). But the time scale of this damping is long enough, according to these authors. For spin period about 1 second it is $\sim 400 - 10^4 P_{\text{prec}}$ (Alpar and Ogelman [1987]). Still, this time scale is much shorter than a NS age, so some excitation mechanism is necessary for precession. As it is discussed below, in the presented model excitation is due to a glitch.

A kind of pinning (“immobilization”) of vortices can also happen in the core due to interactions with magnetic flux tubes (see discussion, for example, in Link [2007]). In this case, the moment of inertia of “pinned” neutrons (which is about $I$) is about 10 times larger,
than the moment of the remaining parts of a NS, \( I_p \). So, \( P_{\text{prec}} \sim 0.1P \).

A different kind of problem appears if pinning in the crust is taken into account. For absolute pinning no long period precession is possible. Instead, the period of precession becomes equal to \( P(I/I_p) \), here \( I \) is NS moment of inertia, \( I_p \) is the moment of inertia of pinned superfluid in the crust. Typically it is expected that \( I_p/I \sim 10^{-2} \) (Shaham 1977), and the precession period is just \( \sim 100P \) if the absolute pinning is valid. However, Alpar and Ögelman (1987) showed that this is not the case due to finite temperature. Because of thermal effect always there is vortex creep which allow the pinned superfluid to follow precession.

The best example of a NS with precession-like behavior is PSR B1828-11. The proposed period is about 511 days with a harmonic at 256 days (Stairs et al. 2000). Most of discussions related to free precession deal with this source. In particular, the problem of non-existence of long period precession for strong pinning is typically confronted with observations of PSR B1828-11.

Recently, appeared another possible example of long period free precession in NSs. The existence of \( \sim 7 \) years precession period in one of a small group of isolated NSs (called XDINS – X-ray Dim Isolated NSs, or ICoNS – Isolated Cooling NSs, or the Magnificent Seven) – RX J0720.4-3125 – was suspected (Haberl et al. 2006). So, this object was added to the list, and the paradoxical situation of long precession in presence of superfluid vortices was reconsidered by Link (2006, 2007). This author proposed that either protons in NS interiors are type I superconductors, or neutrons in the outer core are normal (i.e., not superfluid). More recently Glampedakis et al. (2008) demonstrated that for long spin period and small precession angles NSs can have long precession periods (note, that for PSR B1828-11 the precession angle is proposed to be small, about few degrees (Stairs et al. 2000), but for RX J0720.4-3125 it can be larger, > 10 degrees (Haberl et al. 2006)). So, according to Glampedakis et al. (2008), the conclusion by Link (2006) and other authors that in the strong drag regime \( P_{\text{prec}} \sim 0.1P \) can be under doubt due to a short wavelength instability.

Clearly, the problem of free precession in NSs is far from being solved completely. In this brief note, based on coincidence between Tkachenko wave period and precession period in cases of PSR B1828-11 and RX J0720.4-3125, I discuss a mechanism to support precession in isolated NSs.

### 2 Tkachenko waves

A simple model for long period precession of isolated NSs proposed here is related to the so-called Tkachenko waves (Tkachenko 1966). These are displacement waves in the vortex line array that exist in rotating superfluid, or in other words a kind of sound waves propagating in the lattice of neutron vortices perpendicular to them. A good introduction to the Tkachenko waves physics can be found in the paper by Andereck and Glaberson (1982).

Already in early 70-s this phenomena was suggested to explain periodic modulations in NSs (Ruderman 1970, Dyson 1971). At that time motivation had been related to reported wobbling of the Crab pulsar, which was not confirmed by later observations. Then this approach was nearly forgotten, and only recently Noronha and Sedrakian (2008) returned to consideration of Tkachenko waves in NSs. In particular, they demonstrated that behavior of PSR B1828-11 can be explained by these waves.

According to Ruderman (1970) (see also an example given by Dyson 1971) the period of a standing Tkachenko wave in a NS can be estimated as:

\[
P_T = \frac{(2\pi/k)(1/V_T)}{1.77 R_6 P^{1/2} \text{ yrs}}.
\]

Here \( V_T \) – wave velocity, which in a simple case depends only on the spin period \( P \) and fundamental constants. \( R_6 \) – the core radius normalized to \( 10^6 \) cm (with such normalization the equation provides an estimate close to the upper limit for the period). The estimate is made for the mode with \( kR = 5 \). Spin period of a NS, \( P \), is given in seconds.

As one can see, this period is of order of those related to free precession.

### 3 Scenario for long period precession

The proposed precession periods for RX J0720.4-3125 and PSR B1828-11 are very similar to the Tkachenko wave periods for these stars. For RX J0720.4-3125 the period of precession is proposed to be equal to \( \sim 7 \) years (Haberl et al. 2006) or 4.3 years (van Kerkwijk and Kaplan 2007), spin period of this NS is equal to \( \sim 8.4 \) s (see, for example, Haberl 2007) for a review on XDINS). For PSR B1828-11 the precession period is equal to \( \sim 500 \) days, while the spin period is equal to 0.4 s, for this object the coincidence between precession and Tkachenko wave periods was already mentioned (see, for example, Gusev and Kitiashvili 2007).
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2004 [Noronha and Sedrakian 2008]. For $R$ about few – 10 km one obtains that the precession period is consistent with $P_T$ for both NSs (also the mode can be used as a parameter, however everywhere here I use $k R = 5$, as proposed by Ruderman 1970).

Moreover, in the case of RX J0720.4-3125 pulse profile modulations and spectral changes are observed. Period modulations related just to the Tkachenko waves alone hardly can be responsible for such evolution. Precession is necessary. The idea, proposed here is the following: energy stored in standing Tkachenko wave can power precession of a NS. The necessary condition for effective energy transfer can be the equality of $P_T$ and free precession period. The latter one depends on oblateness of a NS, which can be due to strong magnetic fields. The former one depends only on the spin frequency. Coincidence between the two characteristic time scales which depend on different quantities should not be a very frequent occasion. However, one more condition is necessary – it is necessary to generate the standing wave. The necessity of these two conditions can explain why precessing isolated NSs are so rare.

Ruderman (1970) notes that Tkachenko waves can be generated by starquake glitches. Energy of precession is $E_{\text{prec}} = \frac{I \Omega_m \theta^2}{2}$, where $\theta$ is the precession amplitude [Jones 2004]. For PSR B1828-11 this energy is about $3 \times 10^{36}$ erg. This value is significantly smaller than typical glitch energy. In the case of RX J0720.4-3125 a glitch was proposed by van Kerkwijk et al. (2007). The glitch energy was estimated by them as $\sim 10^{37.5}$ erg ($\Delta \nu/\nu \sim 5 \times 10^{-8}$). Of course, it is necessary to say, that the energy of the glitch was estimated according to the fit with cubic model (van Kerkwijk et al. 2007), and so the value can be different if after the glitch the evolution is due to precession. Still, as an order of magnitude estimate the value from van Kerkwijk et al. (2007) can be used. Haberl et al. (2006) estimated the amplitude of precession $> 10^4$, but warn about uncertainties of their model. The period of precession is about 7 years. The energy of the precession motion then appears to be $\sim 3 \times 10^{35} (\theta/10^4)^2$ erg. With these values in hand the energy of the glitch is enough to drive precession even for large $\theta$ if efficiency of energy transfer is not very low.

van Kerkwijk et al. (2007) relate a “jump” in spectral properties of RX J0720.4-3125 to the glitch. In my opinion, this means that it is more probable that the glitch was due to a quake, not due to vortex lines unpinning (or accretion episode etc., see below).

Taking altogether, for RX J0720.4-3125 the following scenario is proposed: a glitch (most probably due to a starquake) generates Tkachenko waves; the period of a standing Tkachenko wave is equal to the free precession period for this NS; due to the standing wave precession starts after a glitch, or just there is an energy input into the pre-existing precession motion.

Tkachenko waves periodically change the spin frequency and moment of inertia of a NS. Waves move perpendicular to the vortex lines, which are parallel to the spin axis. The moment of inertia of a star can be non-symmetric respect to this axis, for example if oblateness is due to strong magnetic field. I speculate that periodic modulation of spin frequency and all components of moment of inertia in resonance with the precession period (determined by oblateness) would lead to energy transfer from Tkachenko waves to the precession motion.

4 Discussion

Absence of free precession in absolute majority of isolated NSs indicates that this phenomena needs some rare coincidence in properties of a NS. Here it is proposed that it is necessary to have:

- $P_T \approx P_{\text{prec}}$,
- a glitch to generate Tkachenko waves.

Instead of the proposal by Link (2007) – “A slowly-precessing neutron star cannot glitch” – I propose another: slow precession is powered by glitches via Tkachenko waves.

Observations of RX J0720.4-3125 are roughly consistent with this scenario. On the other hand, in the case of PSR B1828-11 no glitches have been observed. However, it is necessary to study for how long precession can survive after a glitch. If an old estimate by Alpar and Ögelman (1987), 400 – $10^4 P_{\text{prec}}$ is valid, then this time is long enough. If precession is periodically excited by glitches via Tkachenko waves even damping on a time scale of few precession cycles (Link 2006) would not contradict observations of RX J0720.4-3125.

The glitch in RX J0720.4-3125 reported by van Kerkwijk et al. (2007) by its consequences is similar to the one observed in an anomalous X-ray pulsar (AXP) CXOU J164710.2-455216 [Israel et al. 2007; Muno et al. 2007]. After the glitch the luminosity of the source was increased, and its spectrum changed. So, the jump in properties of the spectrum and luminosity of RX J0720.4-3125 proposed by van Kerkwijk et al. (2007) can be directly related to a glitch, which is weaker than in the case of CXOU J164710.2-455216 (still similarities in behavior of these sources can be considered as a kind of support to the hypothesis of a link between AXP's
and XDINS). But the evolution of the NS parameters after the “jump” requires precession.

Note, that the timing solution before MJD 52821, when a possible “glitch” happened according to \cite{vanKerkwijk2007}, can be relatively well described by the so-called cubic solution (the second derivative of $\nu$ is non-zero), see \cite{vanKerkwijk2007}. Spectral changes before this date are not very large \cite{Haberl2006, vanKerkwijk2007}. After MJD 52821 the timing solution is well described by a periodic function, see \cite{vanKerkwijk2007} (these authors studied several models with and without a glitch in their two papers), and spectral changes follow this law, too \cite{Haberl2006}. Based on that, I suggest that the timing residuals might be also explained by a model without (or with small) precession before the glitch, and strong precession after. However, this particular model has never been tested quantitatively against observational data.

Glitches naturally can produce thermal afterglows \cite{Hirano1997}. About $10^{38} - 10^{43}$ ergs can be released in a glitch \cite{vanKerkwijk2007} according to estimates of the increase in spin frequency by \cite{vanKerkwijk2007} this value is closer to $10^{38}$ erg). However, Hirano et al. showed that a thermal response of a NS to a glitch cannot produce a smooth temperature increase on the time scale of years. If surface temperature is increased just by few percent, as it is required by \cite{vanKerkwijk2007}, then the brightening lasts just for few days (this corresponds to weak energy release). If we require a temperature rise for a long time, then the effect is too strong \cite{Hirano1997}. So, I conclude that spectral changes on a long time scale should be attributed to precession of the NS.

Glitches of AXPs (and soft gamma-repeaters) can be different in nature with respect to radio pulsar glitches, as the former can be related to crust fracture due to superstrong magnetic field. Still, the origin of a glitch is not important for our discussion here. “Normal” glitches are quite common for long period pulsars, for example, PSR J1814-1744 with spin period about 4 seconds demonstrated a glitch \cite{Janssen2000}. So, RX J0720.4-3125 can glitch not only via the mechanism operating in magnetars, but also due to convenient mechanisms proposed for normal radio pulsars. For them one can estimate the recurrence time following \cite{Alpar1994}.

If the glitch of RX J0720.4-3125 is due to unpinning, then using standard formulae \cite{Alpar1994} one obtains that the recurrence time between two successive glitches is:

$$t_g = \frac{I_0}{\Omega^2}. \quad (2)$$

Parameter $\delta \Omega$ is the critical value of the difference between the rotation frequencies of normal matter and the superfluid at a boundary layer. $\delta \Omega$ itself can be estimated as \cite{Alpar1994}:

$$\delta \Omega = \Delta \Omega \frac{I}{I_p} \frac{\Omega}{\Omega_g}, \quad (3)$$

here $I_p$ is the effective moment of inertia of the region of a pinning layer.

Combining these two formulae one obtains the relation for the time between glitches:

$$t_g = \frac{2(A + B)\phi \Delta \Omega / \Omega}{I_0 \Omega}. \quad (5)$$

The estimate above was obtained assuming standard values \cite{Alpar1994} $A = 10^{32}$ erg, $\phi = 10^{-3}$, $B = 10^{48}$ erg, and $I \sim 10^{45}$ g cm$^2$.

Then, we can be just lucky to find a glitch in ~10 years of observations (but note, that it is not the only XDINS observed). Or, in XDINS quakes do not follow the formula for radio pulsars.

\cite{vanKerkwijk2007} proposed that an accretion episode can be responsible for spectral changes after a “glitch” in RX J0720.4-3125. I think that this is not a very probable reason for the origin of the glitch and corresponding changes. It is hardly possible to imagine that if we observe such an episode just after...
10 years of observations, other episodes were not frequent during the evolution of this source. With frequent episodes of accretion of light elements a NS should follow a slightly different cooling track (Kaminker et al. 2006). Such stars with accreted envelopes are hotter in their youth, but colder after they mature. This is more similar to the properties of 1E1207.4-5209 and Kes 79 (Gotthelf and Halpern 2007).

A remarkable difference between some NSs in supernova remnants (so-called CCOs) and XDINS in the solar vicinity can be due to the existence of accreted envelopes in the former. Note, that we do not observe descendants of 1E1207-like sources in our proximity. If they follow a standard cooling curve, like RX J0720.4-3125 and other XDINS, then they have to be observed. On the other hand, we do not see ancestors of XDINS in supernova remnants. This can be related to the fact that CCOs descendants are too cold at the age of XDINS to be easily detected by ROSAT, and vice versa ancestors of XDINS are not hot enough to be easily found in some supernova remnants.

In this note I neglect (as most, if not all, other authors who studied Tkachenko waves in NSs) the influence of interaction between neutron vortex lines and magnetic flux tubes. This interaction can significantly affect the velocity of waves, and so their period, and to damp them. The velocity of a Tkachenko wave can be estimated as:

\[ V_T = \frac{1}{2} \left( \frac{b \Omega}{2 \pi m_{\text{pair}}} \right)^{1/2} = \left( \frac{\Omega k}{8 \pi} \right)^{1/2} \sim \frac{b}{P}. \] (6)

Here \( m_{\text{pair}} = 2 m_n \), and \( b \) is the distance between vortex lines. So, \( P_T \sim (R/b)P \) in the simple case when there are no interactions with flux tubes or other complications. When vortices are able to “communicate” with the help of numerous magnetic flux tubes, the velocity of the wave can be larger, so the period of Tkachenko-like wave would be shorter. This question should be explored.

To conclude, in this short note I proposed that long period precession in RX J0720.4-3125 can be related to Tkachenko waves, generated in a recent glitch. In the proposed model before a glitch precession could be negligible. The critical condition for the free precession excitation is the equality between Tkachenko wave period and the period of free precession.

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References

Akgün, T., Link, B., Wasserman, I.: Mon. Not. R. Astron. Soc. 365, 653 (2006)
Alpar, A.M., Ögelman, H.: Astron. Astrophys. 185, 196 (1987)
Alpar, A.M., Baykal, A.: Mon. Not. R. Astron. Soc. 269, 849 (1994)
Andereck, C.D., Glaberson, W.I.: Low Temp. Phys. 48, 257 (1982)
Dyson, F.: Fermi Lectures 1970. Accademia Nazionale dei Lincei, Roma (1971)
Glampedakis, K., Andersson, N., Jones, D.I.: Phys. Rev. Lett. 100(8), 081101 (2008)
Gottelf, E.V., Halpern, J.P.: Astrophys. J. Lett. 664, L35 (2007)
Gusev, A., Kitiashvili, I.: J. Dyn. Control Syst. 10, 120 (2004)
Haberl, F.: Astrophys. Space Sci. 308, 181 (2007)
Haberl, F., Turolla, R., de Vries, C.P., Zane, S., Vink, J., Méndez, M., Verbunt, F.: Astron. Astrophys. 451, L17 (2006)
Hirano, S., Shibazaki, N., Umeda, H., Nomoto, K.: Astrophys. J. 491, 286 (1997)
Israel, G.L., Campana, S., Dall’Osso, S., Muno, M.P., Cummings, J., Perna, R., Stella, L.: Astrophys. J. 664, 448 (2007)
Janssen, G.H., Stappers, B.W.: Astron. Astrophys. 457, 611 (2006)
Jones, P.B.: Phys. Rev. Lett. 92(14), 149001 (2004)
Kaminker, A.D., Gusakov, M.E., Yakovlev, D.G., Gnedin, O.Y.: Mon. Not. R. Astron. Soc. 365, 1300 (2006)
Link, B.: In: Bailes, M., Nice, D.J., Thorsett, S.E. (eds.) Radio Pulsars. Astronomical Society of the Pacific Conference series, vol. 302, p. 241. Astron. Soc. Pac., San Francisco (2003)
Link, B.: Astron. Astrophys. 458, 881 (2006)
Link, B.: Astrophys. Space Sci. 308, 435 (2007)
Muno, M.P., Gaensler, B.M., Clark, J.S., de Grijs, R., Poley, D., Stevens, I.R., Portegies Zwart, S.F.: Mon. Not. R. Astron. Soc. 378, L44 (2007)
Noronha, J., Sedrakian, A.: Phys. Rev. D 77(2), 023008 (2008)
Ruderman, M.: Nature 225, 619 (1970)
Ruderman, M., Gil, J.: Astron. Astrophys. 460, L31 (2006)
Shaham, J.: Astrophys. J. 214, 251 (1977)
Shapiro, S.L., Teukolsky, S.A.: Black holesm white dwarfs, and neutron stars: the physics of compact objects. Wiley-Interscience, New York (1983)
Stairs, I.H., Lyne, A.G., Sheamar, S.L.: Nature 406, 484 (2000)
Tkachenko, V.K.: Sov. J. Exp. Theor. Phys. 23, 1049 (1966)
vKerkwijk, M.H., Kaplan, D.L.: Astrophys. Space Sci. 308, 191 (2007)
vKerkwijk, M.H., Kaplan, D.L., Pavlov, G.G., Mori, K.: Astrophys. J. Lett. 659, L149 (2007)