Variable Switching Frequency Control-Based Six-Step Operation Method of a Traction Inverter for Driving an Interior Permanent Magnet Synchronous Motor for a Railroad Car

Jin-Ho Park¹, Woong Jo², Eun-Tak Jeon², Sang-Hun Kim³, Chang-Hee Lee², Jung-Hyo Lee³, Jun-Hee Lee⁴, Junsin Yi⁵, and Chung-Yuen Won⁶, Senior Member, IEEE

¹Interdisciplinary Program in Photovoltaic System Engineering, Sungkyunkwan University, Suwon, 16419 Korea
²Daewoony Co. Ltd., Anyang, 14051 Korea
³Department of Electrical Engineering, Kunsan National University, Kunsan, 54150 Korea
⁴Korean Railroad Research Institute, Uiwang, 16105 Korea
⁵Department of Information and Communication Engineering, Sungkyunkwan University, Suwon, 16419 Korea

Corresponding author: Chung-Yuen Won (e-mail: woncy@skku.edu).

ABSTRACT Most railroad cars employ a natural ventilation cooling method. For this reason, severe problems occur in the control devices for the railroad cars due to the heat generated by the high-power switching loss. Therefore, the traction inverter for the railroad car requires a control method that can reduce the switching loss. In this paper, we propose a control method for an interior permanent magnet synchronous motor in consideration of the driving conditions of the railroad cars. The proposed method combines a variable switching frequency-based synchronous pulse width modulation control method, a flux-weakening control method that considers the maximum voltage modulation index, and an optimized over-modulation method. Through the proposed method, the utilization rate of the DC-link voltage can be maximized while lowering the switching frequency of the traction inverter as much as possible, and the high-performance current control can be performed in the entire speed region of the railroad car. The validity and effectiveness of the proposed method are verified by the simulation results and the experimental results.

INDEX TERMS Railroad car, Traction, IPMSM, Six-step operation, Synchronous pulse width modulation, Variable switching frequency, Over-modulation.

I. INTRODUCTION

The strengthening of international environmental regulations has prompted many researches on interior permanent magnet synchronous motors (IPMSMs) as traction motors for eco-friendly vehicles [1]–[9]. In the case of railroad cars, induction motors (IMs) have been used as traction motors since 2000, and IMs have still been applied to new railroad cars. However, considering that most railroad cars can run for 20–30 years, some of the railroad cars should be replaced now, and the rest should be replaced in the future. For this reason, social demands for the design and control methods of large-capacity IPMSMs that can be used in the railroad industry have recently emerged. In addition, the efficiency of the IM used in the application field of this paper slightly exceeds 90%, whereas the efficiency of the IPMSM is 97% or higher. Furthermore, results of prior researches have shown that using the IPMSM instead of the IM can reduce the power consumption. Also, it is especially notable that a maximum of 20 traction motors is used in each railroad car. Therefore, although the manufacturing cost of the IPMSM slightly exceeds that of the IM, it is preferable to replace the IM with the IPMSM in consideration of the power consumption and service life of the railroad car.

A natural ventilation cooling method is generally applied to the traction inverter for the railroad car. Due to this reason, the heat generated during prolonged operation of power semiconductor devices and the high-power switching loss can lead to problems. Thus, low switching frequency operation of the power semiconductor device is required to solve the problems without increasing the volume/weight of heatsinks. For this reason, the six-step control method, by which the traction inverter is operated with a low switching frequency and the utilization rate of the DC-link voltage can be maximized, is highly suitable for the railroad car.

Hence, researches on the six-step control methods of the PMSM have been actively conducted. The researches include a study on a flux-weakening control algorithm for a quasi-six-step operation [10] and a study on performing the maximum-torque-per-ampere operation with a current controller in a constant torque region and switching to a voltage controller in a constant power region [11]. Additionally, in [12], a method of deactivating a current controller at a rated speed and applying the scaling gain to the voltage reference was proposed for the six-step operation. In [13], the enhancement of the output torque and the extension of the constant torque region were achieved by performing a method, which is a
a combination of a flux-weakening control algorithm and a dynamic over-modulation method. In [14], a voltage angle control-based six-step control method was proposed. In [15], the output current ripple was decreased by reducing low-frequency current oscillation during the six-step operation. However, in most of the researches mentioned above, experimental verifications were performed by fixing the switching frequency of the inverter at 5 kHz or higher.

When performing the over-modulation method for the six-step operation of the IPMSM, it is essential to modulate linearly the voltage reference corresponding to the output voltage in the over-modulation region. Thus, a control performance in the over-modulation region can be considered as the linearity of a voltage modulation index (MI). For this reason, various methods for ideally maintaining the linearity of the MI have been widely studied [16]–[22]. However, the computational processes of the methods proposed in the aforementioned studies lead to the computational burden of micro-controllers used in the railroad industry due to the complex calculation performed for the process in which the voltage reference is modulated. Therefore, it is necessary to simplify and optimize the over-modulation method.

Furthermore, under the condition that the inverter is operated with a low switching frequency as required in the application field of this paper, the number of carrier waves must be uniformly synchronized within one fundamental period of the inverter output voltage in order to solve problems caused by the unbalanced current [23]–[26]. This is because the number of carrier waves used within one fundamental period of the inverter output voltage is insufficient in a certain speed region when the inverter is operated with low carrier frequency (below 500 Hz). Thus, to prevent the problems mentioned above, if the fundamental frequency of the output voltage is higher than or equal to a preset frequency, the carrier frequency should be controlled so that the number of carrier waves is synchronized with a preset number during one fundamental period of the output voltage. Furthermore, in the variable speed drive system for the IPMSM, the phase of the voltage reference must be varied to control the phase of the output current in consideration of the characteristics of the IPMSM in the entire speed region. Therefore, it is essential to perform a control method for minimizing the synchronization error between the voltage reference and the carrier wave in consideration of the phase of the voltage reference, and the seamless switching between asynchronous PWM control mode and synchronous PWM control mode should be achieved.

In this paper, for the six-step operation of the IPMSM which is designed and manufactured for the railroad car, a synchronous PWM control method and an optimized single-mode over-modulation method are proposed in consideration of the synchronization error and the computational burden of micro-controllers. In addition, considering the maximum voltage modulation index, the flux-weakening control method is performed in the constant power region. As a result, through the combination of the proposed methods, a high-performance current control is achieved in the entire speed region of the railroad car, and a stable switching to the six-step operation is performed by maintaining the linearity of the MI in the over-modulation region. The validity and effectiveness of the proposed methods are demonstrated by simulations and experiments.

The remaining part of this paper is organized as follows. In Section II and III, the design of the synchronous PWM method and over-modulation method is explained. In Section IV, the proposed control system for driving the IPMSM is presented. In Section V and VI, simulation and experimental results are analyzed to demonstrate the performance of the proposed method. Finally, the conclusion of this study is summarized in Section VII.

II. SYNCHRONOUS PWM CONTROL STRATEGY

In this study, when performing the proposed synchronous PWM control, the control is conducted 18 times during one period of the voltage reference corresponding to nine periods of the carrier wave by applying a double-sampling method. Fig. 1 shows the adjustment process of the control period and carrier wave when performing the proposed synchronous PWM control method. In Fig. 1, the ideal control period (\(T_{\text{period ideal}}\)) means the sampling time corresponding to the interval of an electrical angle of 20° when assuming that the speed of the IPMSM is constant, and the “Part of the Voltage Theta” denotes a part of the phase angle of the voltage reference. In addition, (a) denotes the detected angle at the actual sampling point when assuming that synchronization error components exist, and (b) denotes the angle at the ideal sampling point when the interval of sampling points is ideally maintained at 20°. In this study, the error component between the detected angle at the actual sampling point and the ideal angle is defined as the phase error of ±\(\Delta\alpha\) corresponding to (d). In an actual system, the magnitude of the phase error becomes larger due to an additional error component generated when the current control is performed and the speed is varied, which leads to problems caused by unbalanced current and over-current. For this reason, the compensation for the phase error should be achieved at a next sampling point. As a result, when the proposed compensation algorithm is applied, a detected angle converges to (c) at the next sampling point.

The fundamental frequency of the inverter output voltage is calculated from the position information of the rotor to perform the proposed synchronous PWM control. Thus, as the speed of the IPMSM is varied, the control period (\(T_{\text{period}}\)) is updated to perform the synchronous PWM control. As a result, the control period to fix the number of carrier waves is expressed as:

\[
T_{\text{period}} = \frac{1}{Nf_1}
\]

where \(N\) denotes the number of sampling points, and \(f_1\) denotes the fundamental frequency of the inverter output voltage. In this system, \(N\) is set to 18 to perform 9-pulse PWM control during one period of the voltage reference.

Furthermore, to compensate the phase error, the phase of the voltage reference (\(\theta_h\)) is calculated from the d-q axis voltage references \((v_{d*}, v_{q*})\) of the stationary reference frame, and the variable transformed using \(N\) to perform the compensation algorithm is defined as \(a\), which is expressed as:
\[
\alpha = \text{atan} \left( \frac{v^*_{dq}}{v^*_{qs}} \right) \frac{N}{2\pi} = \frac{N\theta}{2\pi}
\]  
(2)

From (2), the rounding value of \( \alpha \) is defined as \( \alpha' \), and \( \alpha' \) can be considered as an ideal position information. Thus, the phase error (\( \Delta \alpha \)) can be expressed as:

\[
\Delta \alpha = \alpha' - \alpha
\]  
(3)

By multiplying (1) and (3), the correction value (\( \Delta T_{\text{comp}} \)) of the control period can be obtained as:

\[
\Delta T_{\text{comp}} = T_{\text{period}} \times \Delta \alpha
\]  
(4)

As a result, the compensated control period (\( T_{\text{sync}} \)), which is updated once every sampling period, is expressed as:

\[
T_{\text{sync}} = T_{\text{period}} + \Delta T_{\text{comp}} = \frac{T_{\text{period}}}{1 + \Delta \alpha}
\]  
(5)

The proposed synchronous PWM control is performed based on (5), by which the synchronization error can be minimized.

III. OPTIMIZATION OF THE OVER-MODULATION CONTROL METHOD

When performing the proposed synchronous PWM algorithm, the sampling period decreases as the speed increases. Due to this reason, margin of the computational period of a microcontroller for performing additional control algorithms becomes insufficient as the speed increases. Therefore, the optimization of an over-modulation method is required to stably conduct the six-step operation up to the maximum speed while retaining the high-performance current control.

The MI is generally defined as the ratio of the fundamental amplitude of the phase voltage, which is generated by the voltage reference vector \( V_r \), to \( 2V_{dc}/3 \) which is the maximum fundamental amplitude in the six-step operation. However, in this study, it is defined as the ratio of the magnitude of the voltage reference vector to \( 2V_{dc}/3 \) which is the maximum magnitude of the voltage reference vector that can be modulated, and it is expressed as:

\[
MI = \frac{|V_r|}{\frac{2}{3}V_{dc}}
\]  
(6)

where \( r \) is defined as \( r = |V_r|/V_{dc} \).

A. BASIC PRINCIPLE OF SVPWM

Fig. 2 shows the active voltage vectors (i.e., \( V_1, V_2, V_3, V_4, V_5, \) and \( V_6 \)) and the zero voltage vectors (\( V_0 \) and \( V_7 \)) according to the switching state of two-level voltage source inverters (VSIs). In addition, it is shown that the voltage reference vector (\( V_r \)) located in the boundary of the voltage hexagon is synthesized by the two adjacent active voltage vectors and the zero voltage vectors in Fig. 2. As a result, the voltage reference vector can be expressed as:

\[
\int_{T_1} V_v dt = \int_{T_1} V_n dt + \int_{T_1} V_{n+1} dt + \int_{T_1} V_{n+2} dt
\]  
(7)

where \( T_1 \) and \( T_2 \) mean the turn-on time of the active voltage vectors, and \( T_1 \) is the sampling period. Here, the turn-on time of the zero voltage vectors is defined as \( T_{07} \).

Suppose that the voltage reference vector is constant for the

![FIGURE 1. Adjustment process of the carrier wave and control period performed with/without synchronous compensation method](image-url)

VOLUME XX, 2017

This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see https://creativecommons.org/licenses/by/4.0/
sampling period, the voltage reference vector can be expressed as:

\[ \mathbf{V}_r = \mathbf{V}_a \frac{T_c}{T_1} + \mathbf{V}_m \frac{T_c}{T_2} \]  

(8)

Additionally, by using the phase angle of the voltage reference vector (\(\theta\)), the turn-on time (\(T_{0,7}-T_{2}\)) can be expressed as:

\[ T_i = T_c \frac{|V_r|}{2V_{dc}} \sin \left(\frac{\pi}{3} - \theta\right) = \sqrt{3}T_c r \sin \left(\frac{\pi}{3} - \theta\right) \]

\[ T_2 = T_c \frac{|V_r|}{2V_{dc}} \sin \left(\frac{\theta}{3}\right) = \sqrt{3}T_c r \sin \left(\frac{\theta}{3}\right) \]

\[ T_{0,7} = T_i - (T_i + T_2) \]

Furthermore, as depicted in Fig. 2, if the trajectory of the voltage reference vector is located inside the inscribed circle of the voltage hexagon, then it is defined as the linear modulation region, and the maximum value of the MI in this region is 0.866. Meanwhile, if the trajectory of the voltage reference vector lies on a region between the inscribed circle and circumscribed circle of the voltage hexagon, it is defined as the over-modulation region (0.866 < MI ≤ 1).

B. SIGNLE-MODE OVER-MODULATION CONTROL STRATEGY BASED ON SVPWM

From (8) and (9), it can be confirmed that voltage reference vectors located outside the voltage hexagon cannot be modulated. For this reason, when performing the over-modulation control, the voltage reference vectors, which exist in between the voltage hexagon and the circumscribed circle, should be adjusted to be located in the voltage hexagon. In addition, the linearity of the MI must be maintained by using the adjusted voltage vectors so that the fundamental amplitude of the inverter output coincides with the output of the controller. In [27], in order to perform the SVPWM-based single-mode over-modulation method, the modulation coefficient \(k\), which is a value between 0 and 1, is defined as:

\[ k = \frac{|V_r| - V_{dc}}{\sqrt{3}} = \frac{r - 1}{\sqrt{3}} \]

(10)

Fig. 3 shows the modulation process of the voltage reference vector \(\mathbf{V}_r\) based on a single-mode over-modulation method in Sector 1 of Fig. 2. As depicted in Fig. 3, a point on the trajectory corresponding to the circumscribed circle of the hexagon is denoted by \(H\), and a point on the trajectory of the voltage reference vector is denoted by \(I\). Then, there is \(\overline{IF}\) parallel to \(\overline{HB}\). In addition, a point of intersection of the two lines, which are \(\overline{IF}\) and the side of the voltage hexagon, is denoted by \(F\), and \(\overline{DF}\) parallel to \(\overline{CI}\) is determined. Here, considering the modulation coefficient \(k\) and the active voltage vector \(\mathbf{V}_a\) corresponding to the six vertexes of the hexagon, it is clear that \(|\mathbf{V}_r| = k|\mathbf{V}_a| + |\overline{DF}|\) because of \(|\mathbf{V}_r| = |\overline{CF}|\), which leads to \(|\overline{DP}| = |\mathbf{V}_r| - k|\mathbf{V}_a|\) and \(|\overline{P}| = e^{a\phi}\). As a result, the voltage reference vector is adjusted to be located in the voltage hexagon and the adjusted voltage vector \(\mathbf{V}_e\) can be expressed as:

\[ \mathbf{V}_e = k\mathbf{V}_a + (|\mathbf{V}_r| - k|\mathbf{V}_a|)e^{a\phi} \]

(11)

The SVPWM-based single-mode over-modulation method is performed based on (11), by which the fundamental amplitude of the voltage reference vector can be maintained [27].

C. PROPOSED SINGLE-MODE OVER-MODULATION CONTROL STRATEGY BASED ON CARRIER-BASED PWM

The SVPWM-based single-mode over-modulation method requires the complex computational processes such as the phase adjustment of the voltage vector, calculation of the turn-on time, and the sector discrimination. Moreover, it requires a high-performance processor, which increases the cost of the variable speed drive system for the IPMSM. To solve the problems, the single-mode over-modulation method based on the carrier-based PWM (CBPWM), which can reduce a complexity of the computational process and maintain the same performance, is proposed.

In Fig. 4, the basic principle of the CCBPWM method is explained. Suppose that the output voltage vector \(\mathbf{V}_a\) is located in Sector 1 of Fig. 2, the duty ratio and the switching signal

---

**FIGURE 2.** Basic Principle of SVPWM method

**FIGURE 3.** Single-mode over-modulation method based on SVPWM
can be depicted by comparing the three phase voltage references with the carrier wave during one period of the carrier wave as shown in Fig. 4. As a result, by using the duty ratio, the output voltage vector of the two-level VSI can be expressed as:

\[
\mathbf{V}_e = \frac{2}{3} \mathbf{V}_a - \mathbf{V}_b - \mathbf{V}_c = \frac{2}{3} \mathbf{V}_a - \mathbf{V}_b - \mathbf{V}_c. \tag{12}
\]

By rearranging (12), the output voltage vector can be rewritten as:

\[
\begin{bmatrix}
\frac{2}{3} \mathbf{V}_a \\
- \frac{1}{3} \mathbf{V}_b \\
- \frac{1}{3} \mathbf{V}_c
\end{bmatrix}
= \frac{1}{3} \mathbf{V}_d (D_a - D_b) + \frac{1}{3} \mathbf{V}_d (D_b - D_c) \tag{13}
\]

In addition, by using the turn-on time of the active voltage vectors and the zero voltage vector, the duty ratios for each phase voltage reference can be expressed as:

\[
D_a = \frac{T_i + T_c + T_s}{T_i},
D_b = \frac{T_s + T_c}{T_i},
D_c = \frac{T_s}{T_i}. \tag{14}
\]

By combining (13) and (14), the duty ratios can be rewritten as:

\[
\begin{aligned}
D_a &= \frac{1}{2} + \frac{V_a}{V_d} \\
D_b &= \frac{1}{2} + \frac{V_b}{V_d} \\
D_c &= \frac{1}{2} + \frac{V_c}{V_d}
\end{aligned} \tag{15}
\]

Eq. (15) can be simplified as:

\[
D_i = \frac{1}{2} + \frac{V_i}{V_d} \max \left( V_a, V_b, V_c \right) + \min \left( V_a, V_b, V_c \right), \quad i = a, b, c \tag{16}
\]

where \( V_i^* \) means the magnitude of the voltage reference vectors corresponding to three phase voltage references.

In the CBPWM-based control method, the voltage vector can be simply modulated using only the magnitude of the voltage reference vector and DC-link voltage without a complicated computational process as shown in (16). In order to apply this method to the over-modulation region, the modulation coefficient can be rewritten as (17) using \(|\mathbf{V}_e| = |\mathbf{V}_{\text{six}}| = 2V_{dc}/3\).

\[
k = \frac{|\mathbf{V}_e| - \sqrt{3} |\mathbf{V}_{\text{six}}|}{2V_{dc} / \sqrt{3}} = \frac{|\mathbf{V}_e| - 0.866 |\mathbf{V}_{\text{six}}|}{0.134 |\mathbf{V}_{\text{six}}|} \tag{17}
\]

where \( \mathbf{V}_{\text{six}} \) denotes the voltage vector for the six-step operation and is equal to the active voltage vector \( \mathbf{V}_x \).

The magnitude of the voltage reference vector \(|\mathbf{V}_e|\) can be obtained from (17) as:

\[
|\mathbf{V}_e| = k0.134 |\mathbf{V}_{\text{six}}| + 0.866 |\mathbf{V}_{\text{six}}| \tag{18}
\]

By substituting (18) into (11), the adjusted voltage vector can be expressed as:

\[
\mathbf{V}_e = k\mathbf{V}_{\text{six}} + \left( |\mathbf{V}_{\text{six}}| - k|\mathbf{V}_{\text{six}}| \right) e^{j\theta_n} = k\mathbf{V}_{\text{six}} + \left( k0.134 |\mathbf{V}_{\text{six}}| + 0.866 |\mathbf{V}_{\text{six}}| - k|\mathbf{V}_{\text{six}}| \right) e^{j\theta_n} = k\mathbf{V}_{\text{six}} + 0.866 \left( 1 - k \right)|\mathbf{V}_{\text{six}}| e^{j\theta_n} = k\mathbf{V}_{\text{six}} + \left( 1 - k \right) \mathbf{V}_{\text{sin}} \tag{19}
\]

where \( \mathbf{V}_{\text{sin}} \) denotes the voltage vector that moves along the trajectory corresponding to the inscribed circle in Fig. 2, and \( \mathbf{V}_{\text{sin}} \) can be expressed as:

\[
\mathbf{V}_{\text{sin}} = \frac{V_{dc} e^{j\theta_n}}{\sqrt{3}} = \frac{V_e}{\sqrt{3} |\mathbf{V}_e|} \tag{20}
\]

By the principle of the CBPWM method, the duty ratio of the adjusted voltage vector can be obtained from (19) as:

\[
D_i = kD_{\text{six}} + \left( 1 - k \right) D_{\text{six}} \tag{21}
\]

where \( D_{\text{sin}} \) and \( D_{\text{six}} \) denote the duty ratio of \( \mathbf{V}_{\text{sin}} \) and \( \mathbf{V}_{\text{six}} \), respectively.

By substituting (20) into (16), the duty ratio of \( \mathbf{V}_{\text{sin}} \) can be obtained as:

\[
D_{\text{sin},i} = \begin{cases} 
\frac{1}{2} + \frac{\sqrt{3} V_{\text{sin}}}{3 |\mathbf{V}_e|} V_i \max \left( \sqrt{V_{\text{sin}}, V_{\text{sin}}, V_{\text{sin}}, V_{\text{sin}}, V_{\text{sin}}, V_{\text{sin}} \right), & i = a, b, c \\
\frac{1}{2} + \frac{\sqrt{3} V_{\text{sin}}}{3 |\mathbf{V}_e|} V_i \min \left( \sqrt{V_{\text{sin}}, V_{\text{sin}}, V_{\text{sin}}, V_{\text{sin}}, V_{\text{sin}}, V_{\text{sin}} \right), & i = a, b, c
\end{cases} \tag{22}
\]
controller generates the d-q axis voltage references of the stationary reference frame.

When the proposed synchronous PWM control is performed, the generated voltage references are used to calculate the phase error, as in (2). As a result, the calculation result of (5) is used to generate the gate signals in PWM Generator. Additionally, the update of the control period means a variation of the carrier frequency and affects the bandwidth of proportional-integral (PI) controllers corresponding to the current controller and the flux-weakening controller. Therefore, in this system based on the variable switching frequency, the gain values of the controllers are conditionally updated according to the compensated control period.

When the MI reaches the maximum value in the linear modulation region, the flux-weakening controller performs a compensation for the d-axis current reference and the proposed over-modulation method is performed in consideration of the magnitude of the DC-link voltage and the maximum value of the MI for the six-step operation. In short, the compensation for the d-axis current reference is performed to maintain the magnitude of the adjusted voltage vector at \(2V_{dc}/3\), and the six-step control is achieved based on (26).

Fig. 6 shows the control period and the switching frequency in the entire speed region when the proposed control strategy is applied to the traction inverter for the IPMSM.

In Region (1), the switching frequency is fixed at 400 Hz to perform the asynchronous PWM control. In this region, although the switching frequency is very low, the IPMSM can be driven without problems because the carrier frequency is considerably higher than the fundamental frequency of the voltage reference. As the speed increases, there is a point in time that nine periods of the carrier wave are included within one period of the voltage reference. From this point at which Region (2) starts, as the speed increases, the switching
frequency increases because the synchronous PWM control is performed by fixing nine periods of the carrier wave and varying the carrier frequency, namely, the control period. In addition, in Region ②, a compensation algorithm, which uses the phase information of the voltage reference, is performed from a point at which the synchronization error between the carrier waves and the voltage reference is minimum. This point corresponds to the starting point of Region ③. As the speed continuously increases, the MI reaches the maximum value in the linear modulation region. From this point, the over-modulation method and the flux-weakening control are performed. This point corresponds to the starting point of Region ④. As a result, the over-modulation method, the flux-weakening control, the compensation algorithm, and the synchronous PWM control are performed in Region ④ corresponding to the constant power region. In addition, the control period continuously decreases due to the synchronous PWM control, and the actual switching frequency is equal to the frequency of the rotor due to the six-step operation.

Fig. 7 shows the ideal trajectory of voltage reference vectors in the stationary reference frame when the proposed control algorithms are performed. As shown in Fig. 7, Region ① is an asynchronous PWM region where the switching frequency is fixed at 400 Hz, and Region ② is a synchronous PWM region where the control is performed 18 times during one fundamental period of the inverter output voltage. In Region ③, the compensation algorithm for the phase error is performed. As a result, it is shown that the points on the trajectory are uniformly located on the vector plane at intervals of 20°. Finally, Region ④ is the over-modulation region, in which the points on the trajectory converge to the six vertexes of the hexagon based on (26) for the six-step operation.
FIGURE 9. Simulation results performed with/without application of the compensation method under the condition of the inertia load (full waveform, enlarged waveform in transient state, and enlarged waveform in steady state): (a) without compensation of the phase error and (b) with compensation of the phase error
In this section, the control performance is analyzed by the simulation results. Specifically, PSIM 9.1 provided by PowerSim is used as a simulation tool. The detailed specifications and parameters of the IPMSM designed and manufactured for the railroad car are listed in Table 1.

Fig. 8 shows the simulation results performed with and without the application of the compensation algorithm for the phase error in a region where the speed increases (about 1,420 rpm). The simulation is performed under the conditions that a load machine is driven as the speed control mode and the IPMSM is controlled in the torque control mode. The IPMSM is driven by the torque control at a continuous rated torque of 636 N·m and the load machine is accelerated by the speed control up to 2,200 rpm. As shown in Fig. 8(a), when the compensation algorithm is not applied, the phase error exists between an ideal angle and a detected angle. However, when the method proposed in Section II is applied to reduce the phase error, it is shown that the ripple of the d-q axis current of the rotor reference frame is improved due to the reduction of the phase error, as shown in Fig. 8(b).

Fig. 9 shows the simulation results performed with and without the application of the compensation algorithm for the phase error in the entire speed region under the conditions which are the DC-link voltage of 1,500 V and a continuous rated torque of 636 N·m, and the control performance is analyzed by applying the inertia load in consideration of the actual railroad car system. Generally, in the application field of this paper, the railroad cars are operated with an average acceleration of 3.0 km/h/s based on a wheel diameter of 0.82 m and gear ratio of 7.07. However, in this simulation, a low inertia load is applied to obtain fast dynamic responses. As shown in Fig. 9(a), when the compensation algorithm using the phase information of the voltage reference is not performed, the synchronous error, which corresponds to the phase error calculated from (3), is continuously generated as the inverter output frequency increases. Moreover, as shown in the comparison results of the transient and steady states when switching to the six-step operation, it is not possible to stably perform the current control due to the accumulated phase errors that lead to the unbalanced current and over-current. Thus, the proposed synchronous PWM method must be considered to conduct the proposed over-modulation method with a low carrier frequency. Fig. 9(b) shows the simulation results when all of the proposed control methods are combined. As shown in the simulation results, the phase error converges to zero as the carrier frequency is varied to maintain the preset number of pulses after switching to the synchronous PWM control mode at about 900 rpm. After that, the CBPWM-based single-mode over-modulation method and the flux-weakening control are simultaneously performed at about 3,000 rpm, and then a stable
switching to the six-step operation is achieved and the MI is maintained at 1.

Fig. 10 shows the trajectory of the voltage vectors in the stationary reference frame corresponding to the results of Fig. 9. As shown in Fig. 10(a), the sampling points on the trajectory are not located on the vector plane at intervals of 20° and the linearity of the MI is not guaranteed in the over-modulation region due to the accumulated phase errors. As a result, it is expressed as the asymmetry of the trajectory as shown in Fig. 10(a).

Fig. 10(b) shows the trajectory when the proposed methods are combined. As shown in Fig. 10(b), the sampling points are uniformly synchronized within one period of the voltage reference, and the linearity is maintained by a simple computational approach based on (26) in the over-modulation region. Moreover, the switching to the six-step operation simply is performed with the proposed over-modulation method and the flux-weakening algorithm. Additionally, the results of simulation analysis show that Fig. 10(b) is almost similar to Fig. 7, which shows the ideal trajectory of the voltage reference vectors. As a result, it is demonstrated that an excellent performance of current controller can be retained by performing the proposed methods in the variable speed drive system based on a low switching frequency.

VI. EVALUATION BY EXPERIMENTATION

The experimental verifications of the proposed methods are performed using the test equipment at the Korea Railroad Research Institute (KRRI). Fig. 11 shows the test environment, including the IPMSM, the load machine, and the traction inverter. In order to analyze the control performance, the IPMSM is coupled to the load machine and controlled in the torque control mode, and the load machine is used as an inertia load or a speed load. The specifications and parameters of the IPMSM are listed in Table I.

A. EXPERIMENT RESULTS

Fig. 12 shows the experimental results of an accelerated test performed with the proposed control methods, and the data of control variables are periodically stored in a memory device every 40 ms using Ethernet communication. The test is performed under the conditions that are the DC-link voltage of 1,500 V and the continuous rated torque command. As shown in Fig. 12, after the input voltage of 1,500 V is applied to the inverter, the DC-link capacitor is charged and the torque command is applied at 233 s. Then, the IPMSM is driven based on the d-q axis current reference of the rotor reference frame generated from a lookup table.

In the asynchronous speed region under about 900 rpm, it can be seen that the switching frequency is 400 Hz through the computation time of 75 μs and the margin of computational period of 1,173 μs. The control mode switches to the synchronous PWM control mode at 242 s when the speed reaches about 900 rpm. The ripple of the margin of...
computational period indicate that the proposed synchronous PWM control is performed. In addition, the flux-weakening control is performed at 251 \( s \) when the speed reaches about 3,000 rpm, and the MI is maintained at 1 in the six-step operation. The maximum computation time is approximately 84 µs under the condition that all of the control algorithms, including the compensation algorithm and the optimized over-modulation algorithm, are performed. As a result, it is possible to stably drive up to a maximum speed of 4,810 rpm in consideration of the computational burden of a microcontroller, and the high performance current control is retained in the entire speed region.

Fig. 13 shows the DC-link voltage, line-to-line voltage, phase currents and the enlarged waveform under the condition that the six-step control is carried out without the application of the compensation algorithm for the phase error. As shown in the enlarged waveform, the accumulated phase errors lead to the unstable control performance such as unbalanced current in the transient state when switching to the six-step operation.

Fig. 14 and Fig. 15 show the line-to-line voltage, three-phase currents and the enlarged waveform under the condition that the proposed methods are conducted. The enlarged waveforms of region ① and ② in Fig. 14 show the stable switching to the six-step operation and the transient state of the line-to-line voltage and phase currents. Fig. 15 shows the enlarged waveform for each speed region. In Fig. 15, the region ① and ② correspond to the linear modulation region and the over-modulation region, respectively, and the region ③ shows the six-step operation through the line-to-line voltage waveform. As shown in Fig. 15, the IPMSM is stably controlled in the entire speed region using the 9-pulse synchronous PWM method and CBPWM-based single-mode over-modulation method under the condition of the accelerated test.

**B. ANALYSIS RESULTS**

The output phase currents of the inverter are applied to the stator of the IPMSM and affect the core loss of the motor due to harmonic components resulting from the PWM control. For this reason, the researches on harmonic loss have been conducted [28]–[29], and the results show that the total harmonic distortion (THD) of the output phase current affects the motor loss.

Fig. 16 shows the fast Fourier transform (FFT) results of phase currents when the proposed method, by which the six-
step operation is performed, and the conventional method by which the MI is maintained at 0.866 in the flux-weakening region are performed with the fixed carrier frequency of 5 kHz. In order to analyze the FFT results, the experiments are carried out by controlling the load machine with the constant speed command of 4,810 rpm and driving the IPMSM under the continuous rated torque condition. The analyzed results of Fig. 16 are listed in Table II. Table II shows the comparison results of the THD and amplitudes of the harmonics.

Fig. 17 shows the performance analysis results of the proposed and the conventional method in the entire speed region. The experiments are performed by controlling the load machine with the same acceleration from standstill to 4,810 rpm and driving the IPMSM under the continuous rated torque condition. As shown in Fig. 17, the enhancement of the output torque is achieved by the six-step operation. As a result, the output torque of the IPMSM for the railroad car is increased by about 146% at the maximum speed. In addition, the carrier frequencies are 5 kHz and 2,164.5 Hz, respectively, and the switching losses are 770 W and 42.6 W, respectively. In this system, the switching loss is decreased by 94.5%, and the switching device is an IGBT (MBN450FS33F-HITACHI). As a result, the validity and effectiveness of the proposed method are verified by the experimental results and comparisons.

VII. CONCLUSION
In this paper, a variable switching frequency-based six-step control method is proposed for driving the IPMSM used in the railroad car. The proposed control method combines a variable switching frequency-based synchronous PWM control method, the flux-weakening algorithm, and the CBPWM-based over-modulation method. In the proposed synchronous PWM control method, a scheme to calculate and compensate the synchronization error between the voltage reference and the carrier waves is applied to the inverter output frequency-based synchronous PWM control method by using the phase of the voltage reference that changes in the variable speed drive system. As a result, the carrier waves are synchronized with the
rotating frequency of the IPMSM by varying the sampling time, namely, the control period. Additionally, the flux-weakening algorithm and the optimized over-modulation method are performed in consideration of the maximum utilization rate of the DC-link voltage and the six-step operation. As a result, the high-performance current control and a stable switching to the six-step operation are achieved in the over-modulation region. In addition, it is verified that the proposed methods can be applied to the micro-controller used in the railroad industry by identifying sufficient margin of the computational period with a design that considers the process for performing the proposed control algorithms. Therefore, the results of this study are expected to contribute to the technology development and commercialization of the traction inverter for the IPMSM through the application of the proposed method. The validity and effectiveness of the proposed method are verified by the simulation and experimental results.

REFERENCES

[1] G. Pellegrino, A. Vagati, B. Boazzo and P. Guglielmi, “Comparison of Induction and PM Synchronous Motor Drives for EV Application Including Design Examples,” IEEE Trans. Ind. Appl., vol. 48, no. 6, pp. 2322–2332, Nov/Dec. 2012.

[2] G. Pellegrino, A. Vagati, P. Guglielmi and B. Boazzo, “Performance Comparison Between Surface-Mounted and Interior PM Motor Drives for Electric Vehicle Application,” IEEE Trans. Ind. Electron., vol. 59, no. 2, pp. 803–811, Feb. 2012.

[3] C. Calleja, A. Lopez-de-Heredia, H. Gatzanaga, L. Aldasoro and T. Nieva, “Validation of a Modified Direct-Self Control Strategy for PMSM in Railway-Traction Applications,” IEEE Trans. Ind. Electron., vol. 63, no. 8, pp. 5143–5155, Aug. 2016.

[4] S. Bolognani, S. Calligaro and R. Petrella, “Adaptive Flux-Weakening Controller for Interior Permanent Magnet Synchronous Motor Drives,” IEEE J. Emerg. Sel. Topics Power Electron., vol. 2, no. 2, pp. 236–248, Jun. 2014.

[5] J. Wu, J. Wang, C. Gan, Q. Sun and W. Kong, “Efficiency Optimization of PMSM Drives Using Field-Circuit Coupled FEM for EV/HEV Application,” IEEE Access, vol. 6, pp. 15192–15201, Mar. 2018.

[6] A. V. Sant, V. Khadikar, W. Xiao and H. H. Zeineddin, “Four-Axis Vector-Controlled Dual-Rotor PMSM for Plug in Electric Vehicles,” IEEE Trans. Ind. Electron., vol. 62, no. 5, pp. 3202–3212, May. 2015.

[7] J. Lara, J. Xu and A. Chandra, “A Novel Algorithm Based on Polynomial Approximations for an Efficient Error Compensation of Magnetic Analog Encoders in PMSM for EVs,” IEEE Trans. Ind. Electron., vol. 63, no. 6, pp. 3377–3388, Jun. 2016.

[8] D. Fodorean, L. Idoumghar, M. Brevilliers, P. Minicuicescu and C. Irimia, “Hybrid Differential Evolution Algorithm Employed for the Optimum Design of a High-Speed PMSM Used for EV Propulsion,” IEEE Trans. Ind. Electron., vol. 66, no. 12, pp. 9824–9833, Dec. 2017.

[9] J. Lara and A. Chandra, “Performance Investigation of Two Novel HSFSI Demodulation Algorithms for Encoderless FOC of PMSMs Intended for EV Propulsion,” IEEE Trans. Ind. Electron., vol. 65, no. 2, pp. 1074–1083, Feb. 2018.

[10] T. S. Kwon, G. Y. Choi, M. S. Kwak and S. K. Sul, “Novel Flux-Weakening Control of an IPMSM for Quasi-Six-Step Operation,” IEEE Trans. Ind. Appl., vol. 54, no. 6, pp. 1722–1731, Nov/Dec. 2015.

[11] M.-S. Huang, K.-C. Chen, C.-H. Chen, Z.-F. Li and S.-W. Hung, “Torque control in constant power region for IPMSM under six-step voltage operation,” IET Electric Power Appl., vol. 13, no. 2, pp. 181–189, Feb. 2019.

[12] S. H. Kim and J. K. Seok, “Maximum Voltage Utilization of IPMSMs Using Modulating Voltage Scalability for Automotive Applications,” IEEE Trans. Power Electron., vol. 28, no. 12, pp. 5639–5646, Dec. 2013.

[13] Y. C. Kwon, S. M. Kim and S. K. Sul, “Six-Step Operation of PMSM With Instantaneous Current Control,” IEEE Trans. Ind. Appl., vol. 50, no. 4, pp. 2614–2625, July/Aug. 2014.

[14] J. Liu, W. Zhang, F. Xiao, C. Lian and S. Gao, “Six-Step Mode Control of IPMSM for Railway Vehicle Traction Eliminating the DC Offset in Input Current,” IEEE Trans. Power Electron., vol. 34, no. 9, pp. 8981–8993, Sep. 2019.

[15] Z. Ke, J. Zhang and R. Raich, “Low-Frequency Current Oscillation Reduction for Six-Step Operation of Three-Phase Inverters,” IEEE Trans. Power Electron., vol. 32, no. 4, pp. 2948–2956, Apr. 2017.

[16] D. C. Lee and G.-M. Lee, “A Novel Overmodulation Technique for Space-Vector PWM Inverters,” IEEE Trans. Power Electron., vol. 13, no. 6, pp. 1144–1151, Nov. 1998.

[17] J. Holtz, W. Lotakat and A. M. Khambadkone, “On Continuous Control of PWM Inverters in the Overmodulation Range Including the Six-Step Mode,” IEEE Trans. Power Electron., vol. 8, no. 4, pp. 546–553, Oct. 1993.

[18] C. Bharatraj, S. Jeevananthan and J. L. Munda, “A Timing Correction Algorithm-Based Extended SVM for Three-Level Neutral-Point-Clamped MLIs in Over Modulation Zone,” IEEE J. Emerg. Sel. Topics Power Electron., vol. 6, no. 1, pp. 233–245, Mar. 2018.

[19] H. Fang, X. Feng, W. Song, X. Ge and R. Ding, “Relationship between two-level space-vector pulse-width modulation and carrier-based pulse-width modulation in the over-modulation region,” IET Power Electron., vol. 7, no. 1, pp. 189–199, Jan. 2014.

[20] F. Jung and F.-Y. He, “A Simplified Carrier-Based Pulse-Width Modulation Strategy for Two-level Voltage Source Inverters in the Over-modulation Region,” Journal of Power Electronics (in Korea), vol. 17, no. 6, pp. 1480–1489, Nov. 2017.

[21] S. Bolognani and M. Zigliotto, “Novel Digital Continuous Control of SVM Inverters in the Overmodulation Range,” IEEE Trans. Ind. Appl., vol. 33, no. 2, pp. 525–530, Mar/Apr. 1997.

[22] H. K. Lee, S. M. Hong, J. W. Choi, K. H. Nam and J. H. Kim, “Sector-Based Analytic Overmodulation Method,” IEEE Trans. Ind. Electron., vol. 66, no. 10, pp. 7624–7632, Oct. 2019.

[23] A. R. Beig, “Synchronized SVPWM Algorithm for the Overmodulation Region of a Low Switching Frequency Medium-Voltage Three-Level VSI,” IEEE Trans. Ind. Electron., vol. 59, no. 12, pp. 4548–4554, Dec. 2012.

[24] H. Yang, Y. Zhang, G. Yuan, P. D. Walker and N. Zhang, “Hybrid Synchronized PWM Schemes for Closed-Loop Current Control of High-Power Motor Drives,” IEEE Trans. Ind. Electron., vol. 66, no. 9, pp. 6920–6929, Sep. 2017.

[25] S. K. Sahoo and T. Bhattacharya, “Rotor Flux-Oriented Control of Induction Motor With Synchronized Sinusoidal PWM for Traction Application,” IEEE Trans. Power Electron., vol. 31, no. 6, pp. 4429–4439, Jun. 2016.

[26] J. Y. Park, S. H. Jung and J. I. Ha, “Variable Time Step Control for Six-Step Operation in Surface-Mounted Permanent Magnet Machine Drives,” IEEE Trans. Power Electron., vol. 33, no. 2, pp. 1501–1513, Feb. 2018.

[27] Z. Liwei, W. Xuhui, L. Jun and T. Q. Zheng, “Math Demonstration and Practical Application of Fundamental Voltage Amplitude Linear Output Based SVPWM Overmodulation Control,” in Proc. IEEE Conf. Ind. Appl., Oct. 2006.

[28] W. Ni, D. Xu, G. Wang, L. Ding, G. Zhang and L. Qu, “Maximum Efficiency Control of PMSM Drives Considering System Losses Using Gradient Descent Algorithm Based on DC Power Measurement,” IEEE Trans. Ind. Electron., vol. 62, no. 4, pp. 2135–2143, Apr. 2015.

[29] A. Balamurali, G. Feng, C. Lai, J. Tyong and N. C. Kar, “Maximum Efficiency Per Annum Control of Permanent-Magnet Synchronous Machines,” IEEE Trans. Energy Convers., vol. 33, no. 4, pp. 2240–2249, Dec. 2018.
JIN-HO PARK was born in Korea in 1986. He received the B.S. degree in Control and Measurement Engineering from Korea National University of Transportation, Chungju, Korea, in 2009, and the M.S. degree in Mechatronics Engineering from Sungkyunkwan University, Suwon, Korea, in 2011. Since 2015, he has been a Member of the Staff of DAWONSYS, CO., LTD., where he is currently a Deputy General Manager and team leader in the Electrical Equipment R&D Team. His research interest includes electric vehicles and railway propulsion systems.

WOONG JO was born in Korea in 1989. He received the B.S. degree in Intelligent Robot Engineering from Bucheon University, Bucheon, South Korea, in 2016, and the M.S. degree in Electrical Engineering from Hanyang University, Seoul, South Korea, in 2019. Since 2019, he has been a Member of the Staff of DAWONSYS, CO., LTD., where he is currently an Assistant Manager in the Electrical Equipment R&D Team of Innovation R&D Center. His research interest includes motor drive, such as electric vehicles and railway propulsion systems.

EUN-TAK JEON was born in Korea in 1991. He received the B.S. degree in Electrical and Information Engineering from Seoul National University of Science and Technology, Seoul, Korea, in 2017. Since 2017, he has been a Member of the Staff of DAWONSYS, CO., LTD., where he is currently an Assistant Manager in the Electrical Equipment R&D Team of Innovation R&D Center. He is currently working toward the M.S. degree in Electronics and Electrical Engineering at Dankook University, Yongin, Korea. His research interest is control of synchronous machine drives and power conversion systems.

SANG-HUN KIM (S’20) was born in Korea in 1994. He received the B.S. and M.S. degrees in electrical and computer engineering from Ajou University, Suwon, South Korea, in 2019 and 2021, respectively. Since 2021, he has been a Member of the Staff of DAWONSYS, CO., LTD., where he is currently an Assistant Manager in the Electrical Equipment R&D Team of Innovation R&D Center. His research interest is control of synchronous machine drives and power conversion systems.

CHANG-HEE LEE was born in Korea in 1975. He received the B.S. and M.S. degrees in Electrical Engineering from Chungbuk University, Cheongju, Korea, in 2001 and 2003, respectively. He received the Ph. D degree in Electrical Engineering from Hanyang University, Seoul, Korea, in 2020. Since 2011, he has been a Member of the Staff of DAWONSYS, CO., LTD., where he is currently a Director of Innovation R&D Center. His current research interests include power converters control and electric railway system.

JUNG-HYOUNG LEE was born in Seoul, Korea, in 1978. He received the B.S. degree in Electrical Engineering from Konkuk University, Seoul, Korea, in 2006, and the M.S. and Ph.D. degrees in Electrical Engineering from Sungkyunkwan University, Suwon, Korea, in 2008 and 2013, respectively. From 2013 to 2016, he was a senior researcher of automotive component R&D Team in LG Innotek. From 2016, he has been an associate professor of Electrical Engineering department in Kunsan National University. His research interests include converters and inverters for motor drive application.

JUNE-HEE LEE (S’17–M’18) received his B.S. and Ph.D. degrees in Electrical and Computer Engineering from Ajou University, Suwon, South Korea, in 2013 and 2018. Since 2018, he has been with the Korea Railroad Research Institute, Uijeongbu, South Korea. His research interests include grid-connected systems, high power electric machine drive, power conversion systems.

JUN-SIN YI was born in Seoul, Korea, in 1962. He received the B.S. degree in Electronic and Electrical Engineering from Sungkyunkwan University, Korea in 1989. He received the M.S. and Ph. D. degree in Electronic and Electrical Engineering from The State University of New York, University at Buffalo, U.S.A, in 1991 and 1994, respectively. He is currently working as a professor at Sungkyunkwan University, Suwon. His main research interest is solar cells, Thin Film Transistor and their applications.

CHUNG-YUEN WONG (S’85–M’88–SM’05) was born in Korea in 1955. He received the B.S. degree in electrical engineering from Sungkyunkwan University, Suwon, Korea, in 1978, and the M.S. and Ph.D. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1980 and 1987, respectively. From 1990 to 1991, he was with the Department of Electrical Engineering, University of Tennessee, Knoxville, TN, USA, as a Visiting Professor. Since 1988, he has been a Member of the Faculty of Sungkyunkwan University, where he is currently a Professor in the College of Information and Communication Engineering. Also, from 2008 to 2013, he was the Director of Samsung Energy Power Research Center. He was the President of the Korean Institute of Power Electronics in 2010. Since 2016, he has been the Director of the DC Distribution Research Center. His current research interests include the power electronic of electric machines, electric/hybrid vehicle drives, and power converters for DC distribution system.