A Method for Fault Diagnosis Based on Multidimensional Scaling (MDS) in Analog Circuits

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Abstract. Considering different fault state of the circuit in the nonlinear analog circuit fault diagnosis, the output response of the circuit varies. A Method for Fault Diagnosis of Nonlinear Analog Circuits on the basis of Multidimensional Scaling (MDS) was put forward according to the variable output response. This method can transform the difference of output response under different fault states into a low-dimensional fitting composition and then distinguish the different fault types. Under the condition of single fault of the circuit, the simulation result proves that the method presented in this paper enjoys high accuracy in effectively classifying the different fault types, locating the faulty components and observing the difference among variable faults intuitively in the low-dimensional coordinate system.

1. Introduction
With the development of science and technology, there are a wide variety of circuits in the fields of industry, manufacturing, and military. In particular, nonlinear analog circuits still play an irreplaceable role in the development of science and technology. In order to ensure the normal operation and high reliability of analog and hybrid circuit systems, the research of nonlinear analog circuit test and fault diagnosis method becomes necessary. Therefore, fault diagnosis of analog circuits has become an important branch of circuit[1]. Analog circuit fault diagnosis is an indispensable part of the field of circuit testing. Faster and more accurate analog circuit fault diagnosis have been proposed with the increase of circuit integration and the improvement of processing speed. The difficulty of circuit fault diagnosis has been increased as the circuit manufacture process improve, the size becomes smaller, the number of nodes which can be measured is reduced. Therefore the technology of analog circuit fault diagnosis becomes the research focus among scholars[2], especially after entering the 21st century. The previous analog circuit fault diagnosis technology has hampered the development of circuit testing for its slow development.

Through the wider and deeper study on this field, Analog circuit fault diagnosis technology already has certain accumulation and a lot of research results. There are many theories updates from the network structure to the data processing technology. In order to classify these research results, we divided them into different types of fault diagnosis techniques and methods, such as the traditional fault diagnosis method including fault dictionary method[3], component parameter identification method[4], fault verification method[5], etc and the modern data processing techniques such as wavelet transform[6], support vector machine[7], neural network[8] and so on. These methods have greatly promoted the progress of fault diagnosis technology. Although the wavelet transform can adequately contain fault information, but in the practical application of wavelet transform, it is easy affected by the noise and the accuracy depends on the selection of mother wavelet. Support vector
machine (SVM) is based on VC theory of statistical learning theory and structural risk minimization principle, which can solve the small sample problem and nonlinear problem in classification[9]. Yet, there are drawbacks that a nonlinear SVM classifier in the practical application of SVM requires to solve complex quadratic programming problems and low speed. These drawbacks are particularly evident when dealing with large-scale data sets especially. Neural network is a method of abstracting and modeling human brain neuron network, which has been used in analog circuit fault diagnosis[10], but it is very difficult to obtain a large number of typical fault samples due to the limited number of measurable points of analog circuits. Moreover, there are some shortcomings in neural network such as slow convergence speed, easily falling into local optimum and being influenced by the complexity of network structure and the sample. Therefore the process parameters need to be adjusted constantly. These methods solve the problem of tolerance and non-linearity in analog circuit to a certain extent. However, there are some disadvantages such as poor universality, slow network convergence and insufficient theory of network design in the application of real-time fault diagnosis. The MDS in statistics is adopted in this paper, it is applied in the field of analog circuit fault diagnosis because of its strong ability of classification and its advantages of small sample size, strong generalization ability, fast diagnosis speed and high diagnostic accuracy, etc.

All thing above considered, a method for fault diagnosis of nonlinear analog circuits based on multidimensional Scaling (MDS) is proposed in this paper. Multidimensional scaling is a basic method of data compression and feature extraction, which constructs fitting points according to the distance matrix between the front and rear scale array as close as possible and similar to each other. The step response of the analog circuit is taken as an example in this paper, a new sample set formed by fault feature is extracted by MDS analysis of six sampling points on the output waveform, then clustering is carried out to realize the fault diagnosis of analog circuit. Few scholars has attempted this method. The fault diagnosis algorithm which is based on multidimensional scaling has its superiority compared with the previous fault diagnosis algorithms: the different information between the different types of faults in the analog circuit can be fully utilized to classify multiple fault types, fault detection and localization can also be achieved with a simple algorithm and high accuracy. The simulation results show that compared with other fault diagnosis methods, the multi-dimensional calibration method not only has a high fault diagnosis rate, but also can cluster each fault well.

2. Multidimensional scaling fault diagnosis

Multidimensional scaling is an effective method to analyze the data in statistics, which mirrors the data from the original high-dimensional space $\mathbb{R}^p$ to a low-dimensional space $\mathbb{R}^d$ through a certain matrix operation so as to realize extraction of data characteristics. After dimension reduction, the original key information of the data can be preserved, making the data easier to process and identify. In fault diagnosis of analog circuits, considering that different fault points have different data characteristics, there is a certain distance between them in multidimensional space. Therefore, it is possible to judge and classify different types of fault points according to the distance.

The specific steps of Multidimensional scaling to achieve fault diagnosis are as follows:

1. Calculate $b_{ij}$ according to the distance array data.

Assume that the original data sample set $Y_{\text{exp}}$ consists of the fault characteristics of the analog circuit (where $n$ is the number of samples, $p$ is the number of fault features). The matrix representation is $Y_{\text{exp}}=(Y_1, Y_2, \ldots, Y_n)^T$, the coordinates of $Y_i$ are denoted by $Y_i=(Y_{i1}, Y_{i2}, \ldots, Y_{ip})$, $i= 1, 2, \ldots, n$.

Definition 2.1 If a matrix $D=(d_{ij})_{n \times n}$ satisfies the condition: (1) $D=D^T$; (2) $d_{ij} \geq 0$, $d_{ii}=0$, $i$, $j=1, 2, \ldots, n$. then the distance array $D$ is the generalized distance matrix, $d_{ij}$ is the Euclidean distance between the i-th point and the j-th point.
Here, \( D = \begin{pmatrix} 0 & d_{12} & \cdots & d_{1n} \\ d_{21} & 0 & \cdots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & \cdots & 0 \end{pmatrix} \)

\[
d_{ij}^2 = (Y_i - Y_j)^T (Y_i - Y_j) = (Y_{i1} - Y_{j1})^2 + (Y_{i2} - Y_{j2})^2 + \cdots + (Y_{ip} - Y_{jp})^2
\]

(1)

The matrix representation of the \( n \) points in the \( k \)-dimensional space is \( X_{n \times k} = (X_1, X_2, \ldots , X_n)^T \), and the coordinates of \( X_i \) are denoted by \( X_i = (X_{i1}, X_{i2}, \ldots , X_{ik}) \). We call \( X_{n \times k} \) is a fitting composition of distance array \( D \) in multidimensional scaling technique, the distance matrix \( \tilde{D} \) in \( k \)-dimensional space is the fitting distance array of \( D \), \( \tilde{D} \) and \( D \) are required to be as close as possible.

Where \( \tilde{D} = \begin{pmatrix} 0 & \tilde{d}_{12} & \cdots & \tilde{d}_{1n} \\ \tilde{d}_{21} & 0 & \cdots & \tilde{d}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{d}_{n1} & \tilde{d}_{n2} & \cdots & 0 \end{pmatrix} \)

\[
\tilde{d}_{ij}^2 = (X_i - X_j)^T (X_i - X_j) = (X_{i1} - X_{j1})^2 + (X_{i2} - X_{j2})^2 + \cdots + (X_{ik} - X_{jk})^2
\]

(2)

Then \( b_{ij} = \frac{1}{2} (-d_{ij}^2 + \sum_{j=1}^n d_{ij}^2 + \sum_{i=1}^n d_{ij}^2 - \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n d_{ij}^2) \)

(3)

2. The centric inner product matrix \( B \) of \( X \) is constructed according to \( b_{ij} \).

\[
B = (b_{ij})_{n \times n} = \begin{pmatrix} (X_1 - \overline{X})^T \\ \vdots \\ (X_n - \overline{X})^T \end{pmatrix} (X_1 - \overline{X}, \ldots , X_n - \overline{X}) , \text{where, } \overline{X} = \frac{1}{n} \sum_{i=1}^n X_i
\]

(4)

Then \( B \) is called the centric inner product matrix of \( X \).

3. Calculate the eigenvalues and eigenvectors of the matrix \( B \) and select the main eigenvalues and the corresponding eigenvectors.

Assuming that the eigenvalue of \( B \) is \( \lambda_i \) and the corresponding eigenvector is \( e_i \), the largest \( r \) eigenvalues are \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r \) and the corresponding eigenvector is \( e_1, e_2, \ldots , e_r \). Let \( \Gamma = (e_1, e_2, \ldots , e_r) \) be a matrix of eigenvectors. There are two ways to determine the value of \( r \): (1) Determine \( r = 1, 2 \) or \( 3 \) in advance. (2) Determine \( r \) by calculating the proportion \( \delta \) of the previous \( r \) eigenvalues greater than zero to the absolute value of the total eigenvalue.

\[
\delta = \frac{\lambda_1 + \lambda_2 + \cdots + \lambda_r}{|\lambda_1| + |\lambda_2| + \cdots + |\lambda_r|} \geq \delta_0
\]

(5)

Where \( \delta_0 \) is the proportion of the variance contributed in advance.

4. Calculate \( X_{n \times k} \) according to the following formula (8) to obtain \( r \)-dimensional fitting composition. Let \( \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots , \lambda_r) \), then
After that required dimensionality reduction data set is calculated, the clustering of the fault points of different types of analog circuits in low-dimensional space can be performed better. From the above description, MDS algorithm is based on the characteristics of the Euclidean distance matrix obtained. The characteristic is described as follows: \( B \) is the inner product matrix of the distance matrix \( D \) only when \( D \) is the Euclidean distance square matrix. It is assumed that the input distance matrix \( D \) is European, and the corresponding inner product matrix is obtained by the doubly centering transformation of \( D \), and then the reconstruction of the fault node coordinates can be obtained by decomposition. However, \( D \) is not the Euclidean distance matrix due to the presence of measurement errors in the actual sample collection. So the MDS algorithm is designed to solve the following problem:

\[
\min_{D \in \text{EDM}} \| A D A - A \hat{D} A \|_2^2
\]

(8)

where EDM is the set of distance measurement matrix; \( \hat{D} \) is the Euclidean distance matrix corresponding to \( D \); \( A \) is the doubly centering matrix, given by \( A = I - (e^T_n e_n)/n \), here, \( e_n \) is all ones \( n \)-dimensional column vector, \( I \) is \( n \)-dimensional identity matrix; \( \| \cdot \|_2 \) is Frobenius norm.

This is an indirect solution, that is, the distance matrix \( D \) in the non-similarity space is transformed into \( B \) in the similarity space by the doubly centering transformation, and the reconstruction of the node coordinates is obtained by dimension reduction. Thus, the cost function of the MDS algorithm can be expressed as:

\[
J = \| B - X X^T \|_2^2
\]

(9)

The MDS algorithm is solved according to the least squares criterion in a similar space by constructing a non-similarity matrix and performing a doubly centering transformation. If the measurement noise is Gaussian white noise, the process of constructing non-similarity matrix and the doubly centering transformation make the error of the elements in the similarity matrix not subject to the Gaussian distribution. At this point the LS-based estimate is no longer optimal, and the doubly centering transformation can cause error diffusion. Which will lead to the robustness of the MDS algorithm and the clustering accuracy of the sample points are affected when the sample error is large.

The solution to the above problem is to find the Euclidean distance matrix \( \hat{D} \) corresponding to the matrix \( D \) in the non-similar space without doubly centering transformation. Since the L1 norm has better anti-impulse noise than the L2 norm, so the LAD algorithm is often more robust than the LS algorithm[11]. Therefore the cost function can be changed to the form of L1 norm, using LAD criteria:

\[
J = \| D - \hat{D} \|_1
\]

(10)

The solution method under the LAD criterion is the minimization of the equation (11). To facilitate solving, which was converted to semi-definite programming problem. Eq. (11) is used to find the Euclidean distance matrix \( \hat{D} \) closest to the square matrix \( D \), which can be expressed as:

\[
\min_{\hat{D} \in \text{EDM}} \| D - \hat{D} \|_1
\]

(11)

### 3. Simulation experiment

In order to verify the effectiveness of the above diagnosis method, the first diagnostic circuit in this paper is a CTSV filter (Continuous-Time State-Variable Filter) selected from ITC ’97 international standard circuit (show in Figure 1).
The nominal value of each component has been marked in the circuit, the input node is \( V_{in} \), the output node is \( V_{out} \). In the simulation experiment, the normal tolerance of resistor and capacitor are set to \( \pm 5\% \) of the nominal value. Since the probability of single fault occurring in analog circuit is about 80\%–90\% of the total number of faults, therefore we only consider the case of the single failure of the passive components in the circuit occurs. It is assumed that the component is faulty when it deviates from of its nominal value. Taking into account the changes in each component values have different effects on the output circuit, the sensitivity of the circuit to be measured is analyzed. For the characteristics of the circuit in this paper, select the higher sensitivity components \( R_1 \), \( R_5 \), \( R_6 \), \( C_2 \) as a fault component. Where the fault state deviates from the nominal value of +50\% is denoted by \( R_1^+ \), \( R_5^+ \), \( R_6^+ \), \( C_2^+ \), and the fault state from the nominal value of -50\% is denoted by \( R_1^- \), \( R_5^- \), \( R_6^- \), \( C_2^- \). Therefore, we only discuss \( R_1^+ \), \( R_5^+ \), \( R_6^+ \), \( C_2^+ \), \( R_1^- \), \( R_5^- \), \( R_6^- \), \( C_2^- \) and normal state, the type of fault shown in Table 1.

### Table 1. Circuit fault types.

| Fault type | normal | \( R_1^+ \) | \( R_1^- \) | \( R_5^+ \) | \( R_5^- \) | \( R_6^+ \) | \( R_6^- \) | \( C_2^+ \) | \( C_2^- \) |
|------------|--------|------------|------------|------------|------------|------------|------------|------------|------------|
| Nominal value | \ldots | 10k\( \Omega \) | 10k\( \Omega \) | 10k\( \Omega \) | 3k\( \Omega \) | 3k\( \Omega \) | 20nF | 20nF | 3k\( \Omega \) |
| Fault value | \ldots | 15k\( \Omega \) | 5k\( \Omega \) | 15k\( \Omega \) | 5k\( \Omega \) | 4.5k\( \Omega \) | 1.5k\( \Omega \) | 30nF | 10nF |

According to Table 1, the normal state and the fault state of the circuit are simulated respectively by the circuit simulation software Multisim10. 0, assuming that only the circuit single fault occurs at a certain time and other components random changes within the tolerance range. In the normal state all components vary within a tolerance range. A step signal of 5V is applied to the CTSV filter circuit, and then the step response curve is acquired from the output node \( V_{out} \). Six data points are collected evenly in the first 2ms of the step response curve, and the fault feature vector \( Y=(Y_{i1}, Y_{i2}, Y_{i3}, Y_{i4}, Y_{i5}, Y_{i6}) \) with six components is constructed. The Monte-Carlo analysis was performed 30 times for each state, and 270(30×9) vector samples were obtained. Figure 2 shows a partial step response curve for nine states (five samples for each state).
Figure 2. The partial step response curve of CTSV filter circuit in five states.

In the MATLAB2014a simulation environment, the collected fault samples are simulated by the method proposed in this paper. The feature projection of the sampled data of output node $V_{out}$ is obtained as shown in Figure 3(a). It can be seen from the simulation results that the proposed method in this paper can identify and cluster each fault type successfully, and obtain the ideal result.

It can be seen from figure 3 that the clustering of the fault R6+ and C2+ is too close, which can not be distinguished in the clustering results of the PCA method compared with the MDS method.
However, the MDS method is more clear about the clustering of each fault type. Compared with the MDS method, the K-means method can not completely correct diagnose the fault R6+ and the correct rate is inferior to the MDS method. The Isomap method exist the wrong diagnosis between the fault R7 and the normal state, and the correct rate is inferior to the MDS method.

4. Conclusion
In this paper, a new fault diagnosis method based on MDS algorithm for analog circuits is presented. This method makes full use of the similarity between different fault points and displays the clustering results of different fault type intuitively in low dimensional space. The step response of analog circuits is taken as an example in this paper. The feature extraction and data compression of each fault state sample are done by using multidimensional scaling technique. The transformation from high-dimensional input to low-dimensional feature is completed and a new test sample set is formed, which reduces the complexity of the sample point clustering. Compared with other methods, this method has the advantages of simple algorithm and requires less measurable nodes with fast fault detection, location and high accuracy. In the follow-up work, in order to apply it to more complex analog circuit fault diagnosis, the algorithm will be improved and more comprehensive judgments will be considered.

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